Couplings and Scales in Strongly Coupled Heterotic String Theory

Tom Banks

Department of Physics and Astronomy
Rutgers University, Piscataway, NJ 08855-0849

Michael Dine

Santa Cruz Institute for Particle Physics
University of California, Santa Cruz, CA 95064

If nature is described by string theory, and if the compactification radius is large (as suggested by the unification of couplings), then the theory is in a regime best described by the low energy limit of $M$-theory. We discuss some phenomenological aspects of this view. The scale at which conventional quantum field theory breaks down is of order the unification scale and consequently (approximate) discrete symmetries are essential to prevent proton decay. There are one or more light axions, one of which solves the strong CP problem. Modular cosmology is still problematic but much more complex than in perturbative string vacua. We also consider a range of more theoretical issues, focusing particularly on the question of stabilizing the moduli. We give a simple, weak coupling derivation of Witten’s expression for the dependence of the coupling constants on the eleven dimensional radius. We discuss the criteria for the validity of the long wavelength analysis and find that the “real world” seems to sit just where this analysis is breaking down. On the other hand, residual constraints from $N = 2$ supersymmetry make it difficult to see how the moduli can be stabilized while at the same time yielding a large hierarchy.

4/96
1. Introduction and Summary

The only vacuum independent quantitative predictions of weakly coupled heterotic string phenomenology are a relation between the Planck mass, the four dimensional gauge coupling and the string tension, and a relation between the unification scale and the scale of compactification. This last prediction is rather troubling. For if we suppose that the successful supersymmetric unification of couplings is not an accident, then one predicts that the scale of compactification is a factor of 20 or so below the string scale. This, in turn, implies that the dimensionless coupling of string theory is of order $10^7$, so that a weak coupling description surely does not make sense.\footnote{These statements are valid for more or less isotropic Calabi-Yau manifolds. In \cite{1} we argued that highly anisotropic manifolds could resolve this problem. There have also been attempts to extract conventional four dimensional unified gauge theories from string theory.}

On the other hand, it has been argued for many years that string theory cannot be weakly coupled if it describes nature. In the weak coupling region, the dilaton potential almost certainly cannot be stabilized\cite{2}. So perhaps we should simply accept the facts as they appear, and suppose that the compactification scale, $R$, is large, in heterotic string tension units, and the theory is strongly coupled. One might worry that, by duality, such a strong coupling region would be mapped into a weakly coupled region of some other string theory (or of $M$ theory) and that this region would suffer from some version of the dilaton runaway problem. However, in ref. \cite{3}, it was pointed out that all of the known dualities map the region of large radius, strong coupling, and fixed four dimensional coupling to other strongly coupled theories (or at least theories in which the couplings are not arbitrarily small).

Witten has recently taken this viewpoint to its logical conclusion\cite{4}. At strong coupling, the heterotic theory is described, at low energies, by 11-dimensional supergravity. More generally, the strong coupling limit of the theory has been called $M$-theory\cite{5}. Witten has argued that M-theory might well provide a better description of nature than weakly coupled strings. The $M$-theory description is valid, as we will see shortly, when a certain parameter, which we will call $\epsilon$, is small. This parameter seems to be of order one in the real world, so the $M$ theory description is likely to be at least qualitatively much better.
than the weak coupling string description.

Compactification of the $E_8 \times E_8$ heterotic theory on a Calabi-Yau space, $X$, is dual to $M$ theory compactified on $X \times S^1/Z_2$. Using formulas presented in [4], one finds the following connections between the 11 dimensional Planck mass, $M_{11}$ (defined in terms of the coefficient of the Einstein lagrangian in 11 dimensional supergravity, as $M_{11} = \kappa_{11}^{-2/9}$), the 11-dimensional radius, $R_{11}$, and the compactification radius, $R = V^{1/6}$, where $V$ is the volume of the Calabi-Yau space on the boundary with unbroken $E_6$ gauge group:

$$R_{11}^2 = \frac{\alpha_{GUT}^3 V}{512 \pi^4 G_N^2},$$  \hspace{1cm} (1.1)

where $G_N$ is the four dimensional Newton’s constant;

$$M_{11} = R^{-1} \left(2(4\pi)^{-2/3} \alpha_{GUT}\right)^{-1/6}. \hspace{1cm} (1.2)$$

Substituting reasonable phenomenological values, one finds that the eleven dimensional Planck length is roughly half the compactification radius, while the eleven dimensional radius is about ten times the compactification scale! So one might hope that eleven (or five)-dimensional supergravity provides at least a crude approximation to the real world. Moreover, if this viewpoint is correct, dramatic new physics occurs long before one encounters the four dimensional Planck scale. The universe first looks five dimensional, then eleven dimensional. The four dimensional Planck scale, $M_4$, is simply a parameter of low energy physics; there is no interesting new dynamics at this scale! Quantum Gravitational (more properly Quantum M theoretical) effects, become important at the unification scale. This has possible implications for many questions, including issues of early universe cosmology. These are among the issues we will explore in this paper.

The qualitative physics of this $M$ theory regime is quite different from that of weakly coupled heterotic strings, which are no longer the lowest energy excitations. The fact that the compactification scale is large in string tension units is a consequence of the fact that heterotic strings are membranes stretched between the two walls of the eleven dimensional world. The fundamental energy scale in this regime is the eleven dimensional Planck mass $M_{11}$. The membrane tension is one in these units but the heterotic membrane is large because the eleventh dimension is an order of magnitude larger than $l_{11} = M_{11}^{-1}$. The
heterotic string tension, $T_h$ is $M_{11}^3 R_{11}$. The compactification radius is of order one in $l_{11}$ units, and this is what determines the unification scale.

While the $M$-theory description should be qualitatively much better than the weak coupling string description, the universe is probably not in a regime where one can simply compute in the classical, low energy eleven dimensional supergravity theory. In the classical supergravity theory, the expansion parameter is

$$\epsilon = \kappa^{2/3} R_{11} / R^4. \quad (1.3)$$

This number is of order one. So we might expect unknown quantum $M$-theory corrections to be of order one. (This should be compared with the situation in the weakly coupled string theory description, where the “small parameter” is of order $10^7$.) As we will discuss, this is just as well, since in the weak coupling limit one could not understand the stabilization of the moduli.

Even before exploring any detailed dynamics, the view that the universe is approximately five dimensional has interesting consequences. Consider, for example, the strong CP problem. It is well known that in four dimensional string models, there is always a “model-independent” axion, the partner of the usual dilaton, with the potential to solve the strong CP problem. The superpartners of the Kahler, $(1,1), \text{moduli of X}$ provide other axion candidates. In weakly coupled string theory, world-sheet instanton effects break the Peccei-Quinn (PQ) symmetries of these Kahler axions, by amounts of order $e^{-R^2}$. Usually, since $R$ is assumed to be a number of order one in $T_h$ units, these breaking effects are also taken to be of order one. However, if $R$ is large, this factor can be extremely small. Indeed, in the five dimensional picture, these axions lie in vector multiplets and the associated PQ symmetries are “would-be five dimensional gauge symmetries” that are broken only by boundary effects and by membrane instantons. The latter are highly suppressed because of the large size of the membranes (actually, because it occurs in a holomorphic superpotential this effect can be calculated by extrapolating weak coupling formulae, as we will see below). We will argue that in a class of M theory vacua the dominant boundary effect is Quantum Chromodynamics, and a linear combination of the Kahler axions is a QCD axion. A second phenomenological issue arises from the fact that the unification scale is
so close to $l_{11}$. The most sensitive probe of such large scales is proton decay. Exact or approximate symmetries will be essential in understanding why the proton is so stable.

A careful examination of the four dimensional low energy effective theory gives rise to other interesting observations. The Kahler axion multiplet naturally gives rise to a no scale model with broken SUSY and vanishing cosmological constant as the leading term in a systematic computation of the effective potential. A natural explanation of squark mass universality can also be obtained in this model. The no scale structure and squark degeneracy are only valid in leading order in $(R_{11}M_{11})^{-1} \sim 0.1$. It is unclear how far we can rely on these results as explanations of phenomena in the real world.

One of the most fundamental issues in string theory is the question of how the moduli are stabilized. In the weak coupling limit of string theory, moduli are either exact or unstable. If the theory describes nature, one must hope that the moduli are stabilized at a point in moduli space where semiclassical reasoning is not valid. This raises the worry that one will not be able to predict anything from string theory. There will be no small parameter to explain a small scale of supersymmetry breaking, for example, and the smallness of the gauge couplings and their apparent unification must be accidents. In ref. [1], a solution to this problem was suggested, exploiting the holomorphy of the superpotential and gauge coupling functions, and certain discrete symmetries. It was assumed that the compactification radii are of order one in string units, and that string perturbation theory breaks down even for small values of the dimensionless string coupling. More precisely, the model-independent dilaton $S$ was supposed large, while the other moduli were of order one. Stringy non-perturbative effects in the gauge couplings and the superpotential were shown to behave as powers of $e^{-S}$. As a result, one can understand why supersymmetry breaking is small and the gauge couplings are unified, and one predicts only tiny corrections to the lowest order superpotential for matter fields. Because one is in a region where the dilaton superpotential is monotonically decreasing, stabilization of the moduli must arise through large corrections to the Kahler potential.

The view that the compactification scale is large and that the string coupling is very strong requires a reassessment of this picture. In the limit that the low energy, 11-dimensional supergravity description is valid, the theory suffers from instabilities simi-
lar to those at weak coupling, as we will see in some detail. On the other hand, as we have
said, taking the 11 dimensional parameters from the “observed” four dimensional ones,
nature would seem to reside in precisely the regime where the long wavelength description
breaks down. So it would seem reasonable to hope that quantum $M$-theory effects are
responsible for the stabilization of the moduli. One of the goals of the present work is to
explore this possibility, and to ask what weak-coupling predictions, if any, survive into the
strong coupling regime.

In order to do this, it is necessary to understand as well as possible the structure of the
low energy theory in the small $\epsilon$ regime. One of the main results of ref. [4] is a computa-
tion of the gauge couplings from an eleven dimensional perspective. Studying the classical field
equations, Witten finds that the Calabi-Yau volume on the $E_8$ side decreases linearly with
$R_{11}$, so that, for fixed $E_6$ coupling, the $E_8$ coupling blows up at a finite value of $R_{11}$. In
section 2.2, we point out that, exploiting the holomorphy of the gauge coupling function,
these functions can be computed by weak coupling methods. The imaginary parts of the
chiral fields $\vec{T}$ and $S$ (the usual moduli whose real parts describe the internal radii and
the four dimensional gauge coupling, respectively) are the axions we have spoken of above.
They can be normalized so that the theory is invariant under $2\pi$ shifts of these fields. As
a result, the gauge coupling functions are necessarily of the form

$$f^a = \frac{1}{32\pi^2} (m^a S + \vec{m}^a \cdot \vec{T}) + O(e^{-(rS + \vec{s} \cdot \vec{T})})$$  (1.4)

where $m$, $\vec{m}$, $r$ and $\vec{s}$ represent sets of integers. Perturbative heterotic string physics is
valid when $S \gg 1; S \gg T$ and $S/T^3 \gg 1$. M theory is well approximated by classical
supergravity (SUGRA) when $S$ and $T$ are both large such that $S/T^3$ is small. However,
we must avoid regions where $S$ and $|T|$ are comparable and certain linear combinations of
them are small. In these regions physics on one of the boundaries of the eleven dimensional
world is strongly coupled. By holomorphy we can just as well calculate the linear terms in
the gauge kinetic functions at weak coupling. Such weak coupling calculations have been
performed in the past for a variety of theories [4] such as orbifold models, and the difference
of the $E_6$ and $E_8$ couplings has been calculated for Calabi-Yau spaces. In section 2.2, we
point out that the couplings themselves can be computed directly for Calabi-Yau spaces,
by dimensionally reducing the Green-Schwarz counterterms introduced in ten-dimensions to cancel anomalies. This gives the couplings of the axions in the $T$ multiplets to $F\tilde{F}$ and is easily supersymmetrized to give the coupling of the full multiplet. We find, as in ref. [4], that the sign of these couplings is such that, for fixed $S$, the $E_8$ coupling blows up at a finite value of the radius.

This fact is already quite striking. It was probably ignored in the past because it corresponds to a point of strong string coupling. As stressed in [4], the blowing up of the coupling may have something to do with the stabilization of the moduli. As we will discuss in some detail, in the weak coupling regime, one can determine the potential for the moduli completely. Gluino condensation gives rise to a superpotential which behaves as a power of $e^{-S+\vec{\alpha}\cdot \vec{T}}$. In the semiclassical SUGRA regime, we will fully determine the Kahler potential for the moduli and matter fields. Gluino condensation then leads to a potential which grows with radius for fixed coupling of the standard model gauge fields, i.e. at weak coupling the dynamics tends to shrink the eleventh dimension. This is rather surprising, since one might have expected that for widely separated walls, the eleven-dimensional dynamics would become free. In the regime where M theoretical dynamics reduces to supergravity, there is no way to prevent the shrinkage. Thus, quantum M theory is crucial to the stabilization of the radius of the eleventh dimension. Similar remarks can be made about the size of $X$. Phenomenology indicates that it is quite close to $l_{11}$. Consequently the stabilization of this modulus probably also requires the intervention of quantum M theory.

Given that the parameter $\epsilon$ is of order 1, it is not unreasonable to expect that quantum $M$-theory dynamics stabilize the moduli at their observed values. Note that unlike the situation analyzed in [4], there is no mystery here about the weakness of the standard model gauge couplings. The latter play no role in the stabilization of the moduli. Nor do we need to invoke a premature breakdown of perturbation theory. Weak coupling arises from geometrical factors of order one, primarily a factor of 2 (which gets raised to the sixth

---

2 The weak coupling calculation which we will perform here – and thus, in some sense, Witten’s eleven dimensional calculation, has actually been performed some time ago by L. Ibanez and P. Nilles, [7]
power) between the linear size of the Calabi Yau manifold on the $E_6$ boundary and $l_{11}$. We will find, however, that residual constraints from $N = 2$ supersymmetry – the no-scale structure we alluded to earlier – raise puzzles about how some of the moduli are stabilized.

As in ref. [1] we can attempt to use holomorphy of the superpotential and gauge coupling functions, together with exact discrete shift symmetries for the moduli, to argue that certain semiclassical predictions received only exponentially small corrections even at strong coupling. Now, however, the situation is more complicated. We have noted above that in the M theory regime certain linear combinations of the moduli can be small, and exponentials of these are no longer suppressed. By studying physics deep in the semiclassical regime, where the Calabi Yau volumes are everywhere large, we will argue below that these unsuppressed exponentials do not infect the predictions of gauge coupling unification and ratios of Yukawa couplings. A crucial ingredient of this argument is the use of holomorphy to extrapolate semiclassical results into the regime of phenomenological relevance.

The rest of this paper is organized as follows. In the next section we review the effective theory in five dimensions which results from compactification of $M$ theory on a Calabi-Yau space. We pay particular attention to certain approximate symmetries which will survive in four dimensions as Peccei-Quinn symmetries. We then reduce the theory to four dimensions. We give a weak coupling string calculation of the dependence of the coupling on the 11-dimensional radius. We discuss the form of the resulting Kahler potentials, including restrictions inherited from the approximate five-dimensional supersymmetry. We point out that there are several approximate Peccei-Quinn symmetries which hold to an extremely high degree of accuracy. The axion associated with one of these symmetries solves the strong CP problem, but will violate the conventional cosmological bounds. In section 3, we discuss the problem of stabilizing the moduli, exhibiting the intriguing yet rather problematic no-scale structure. We offer some speculations about how stabilization might occur and about the possible origin of a hierarchy. Finally, in section 4, we discuss some phenomenological and cosmological implications of these observations.
2. Some Effective Field Theories

2.1. Effective Field Theory in Five Dimensions

To begin, let us be more precise about the numerical values of various parameters. We do this not because of any illusion that the tree level calculation of these parameters is immune to corrections, but in order to orient ourselves. The tree level fit to the fine structure constant and the unification scale gives

\[ R = 2l_{11} = (3 \times 10^{16} GeV)^{-1}. \]  

(2.1)

\[ R_{11}M_{11} = 8. \]  

(2.2)

Here, \( L \) is the sixth root of the volume of \( X \), \( R_{11} \) is the length of the eleventh dimension (\( \pi \rho \) in Witten’s notation), \( M_{11} = l_{11}^{-1} \), and \( l_{11} \) is the ninth root of \( \kappa^{2} \), the coefficient of the eleven dimensional Einstein action. The fit of M theory to the real world suggests that six of the dimensions are very small, one is one order of magnitude larger and the rest are at least as large as our horizon volume.

We can also write a formula for the heterotic string tension in terms of eleven dimensional quantities. For large \( R_{11} \) we have an approximate five dimensional SUSY, and this is a BPS formula which receives no corrections. Boundary effects and other breaking of SUSY down to four dimensional \( N = 1 \), will give corrections to this formula of order \( (R_{11}M_{11})^{-1} \), which we will neglect. The string tension formula can be obtained by the following reasoning. In ten dimensions, one has expressions for the gauge and gravitational couplings in terms of the tension [8]:

\[ \kappa_{10}^{2} = \frac{1}{4} \lambda^{2}(2\alpha')^{4} \]  

(2.3)

\[ g_{10}^{2} = \lambda^{2}(2\alpha')^{3}, \]  

(2.4)

where \( \lambda \) is the dimensionless string coupling. Comparing with the eleven dimensional expressions for these quantities yields

\[ 2\alpha' = \frac{(\kappa_{11})^{2/3}}{\pi R_{11}(4\pi)^{2/3}} \]  

(2.5)
Alternatively, we can use Polchinski’s formula for the Dirichlet two brane tension in Type IIA string theory, and the fact that the heterotic string is just a two brane stretched between the walls of the world. We also need the Kaluza Klein relation between the ten and eleven dimensional gravitational constants. This calculation gives the same result as above, if one is careful about factors of 2 coming from the relation between compactifications of M theory on a circle and an orbifold.

Given the relatively large size of $R_{11}$, it is appropriate to consider an effective five dimensional action for physics at length scales larger than $l_{11}$ but smaller than $R_{11}$. We will then reduce this to a four dimensional effective action for scales longer than $R_{11}$. In the bulk, the five dimensional theory has full five dimensional SUSY, and its lagrangian has been worked out by Antoniadis et. al., following [10]. The volume of X is in a hypermultiplet along with some of the purely internal and the dual of the purely external components of the three form gauge potential. The complex structure moduli also pair up into hypermultiplets, with internal components of the three form. The quaternionic metric on this space of hypermultiplets is not determined by general considerations. For large volume it can be computed by Kaluza Klein technology. However, equation (2.1) tells us that the volume is not large. We expect M theory to give corrections to this metric of order $l_{11}R^{-1}$.

On the other hand, the volume preserving Kahler moduli, are in vector multiplets, along with the integrals of the three form over nontrivial $(1,1)$ cycles. The bosonic part

$$\approx \frac{1}{136} M_{11}^{-2}. \quad (2.6)$$

\footnote{We thank J. Polchinski for guidance through the conventions of this paper.}
of the lagrangian for these multiplets is given by

\[ L_{\text{vec}} = G_{ab} [F_{\mu\nu}^a F^{\mu\nu b} + \partial_\mu X^a \partial_\mu X^b] + C_{abc} e^{\mu\nu\lambda\kappa\sigma} A^a_\mu F_{\nu\lambda}^b F_{\kappa\sigma}^c \] 

(2.7)

Here \( G_{ab} = -\partial_a \partial_b \ln N \), where \( N = C_{abc} X^a X^b X^c \), with \( C_{abc} \) the intersection numbers of the corresponding \((1,1)\) forms. The fields \( X^a \) are constrained to satisfy \( N(X) = 1 \).

This lagrangian is invariant under local gauge transformations of the \( h_{1,1} \) \( U(1) \) gauge fields which vanish at the boundaries of the fifth dimension. Now consider the transformation \( \delta A_5^a = \partial_5 c^a \) with \( c^a \) a linear function which vanishes only on the \( E_8 \) boundary. More microscopically, we view this transformation as originating from transformations of the eleven dimensional three form gauge field by \( \delta A_{i\bar{j}11} = \partial_{11} d^a_{i\bar{j}} \). Here we choose the gauge function so that \( \partial_{11} d^a_{i\bar{j}} = b^a_{i\bar{j}} \), with \( b^a \) one of the harmonic \((1,1)\) forms on the Calabi-Yau fiber at \( x^{11} \), and so that \( d^a_{i\bar{j}} \) vanishes on the boundary. \( c^a \) is the integral of \( d^a \) over the \( a \)th \((1,1)\) cycle on the manifold (and we have renamed the eleventh dimension the fifth). This transformation is not a symmetry of the system. However, it is broken only by nonperturbative physics which involves the \( E_6 \) boundary. Loosely speaking, nonperturbative effects on the \( E_6 \) boundary arise from membranes stretched between the two boundaries, and Euclidean 5-branes wrapped around the Calabi-Yau manifold on this boundary. These approximate symmetries become Peccei-Quinn symmetries of the effective four dimensional theory. We will estimate the dominant symmetry breaking effects below.

2.2. Four Dimensional Effective Field Theory

We now want to reduce our resolving power and obtain a description of the world on length scales longer than \( R_{11} \). This will be an \( N = 1 \) locally supersymmetric four

---

4 Micha Berkooz has pointed out to us that the nontrivial background fields calculated by Witten, break \( d = 5 \) SUSY. Thus, there may be corrections to this lagrangian. However, when \( R_{11} \) is much larger than \( l_{11} \) the unknown dynamics of short distance M theory should not be affected by this soft breaking of \( d = 5 \) SUSY. The corrections should be calculable in low energy supergravity. That is, integrating out the unknown massive degrees of freedom of quantum M theory, should give us a Lagrangian which is \( d = 5 \) supersymmetric to leading order in \( R_{11}^{-1} \). The fields which Witten calculates to have \( N = 2 \) SUSY breaking VEVs are all in hypermultiplets and they do not effect the vector multiplets to leading order in the long distance expansion.
dimensional field theory. We first address the question of the gauge couplings in this theory. Witten has given us an eleven dimensional calculation of the blowup of the $E_8$ coupling when $R_{11}$ reaches a critical value. It is a remarkable example of the power of holomorphy\textsuperscript{12} that this calculation can be exactly reproduced by extrapolation of results for the weakly coupled heterotic string.

Witten determines the dependence of the $E_6$ and $E_8$ gauge couplings on the volume of the Calabi-Yau space and the radius of the eleventh dimension. However, if the four dimensional effective coupling is small, while the Calabi-Yau radius is large, it should be possible to obtain this dependence from a weak coupling computation. The point is that there is a regime of large radius ("$T$") and small coupling (large "$S$"), such that the dimensionless string coupling is small, and these couplings can be computed in perturbation theory. The gauge coupling functions are holomorphic functions of $S$ and $T$. They must also be invariant under discrete shifts of $S$ and $T$. With the normalizations we will use, these shifts are,

$$S \to S + 2\pi i \quad T \to T + 2\pi i.$$  \tag{2.8}

As a result, up to terms which are exponentially small for large $S$, the gauge couplings functions $f_a$, must be given by

$$f_a = m_a S + n_a T,$$  \tag{2.9}

where $m_a$ and $n_a$ are integers. The $m_a$'s are determined by the central terms, $k_a$, in the Kac-Moody algebras. The $n_a$'s can be obtained from a one loop computation.

These couplings have been evaluated in the literature for many special cases\textsuperscript{13}. For large radius Calabi-Yau compactifications, a formula has been presented for the difference of the $E_6$ and $E_8$ couplings\textsuperscript{13}. However, for large radius, the separate couplings are well defined and it is actually a simple matter to determine them. The point is that for large radius, these couplings can be obtained by reduction of the ten-dimensional effective action. In particular, in terms of component fields, these couplings imply couplings of certain "axion-like" fields to $F \tilde{F}$. These axions correspond to particular excitations of the antisymmetric tensor field, $B_{MN}$, with indices in the internal space. Such couplings are necessarily linear in $B$ and involve products of $F_{\mu\nu}$, i.e. from a ten-dimensional perspective
they are precisely the terms which appear in the Green-Schwarz counterterms. So it is only necessary to reduce the Green-Schwarz counterterms to four dimensions.

Before examining the Green-Schwarz counterterms themselves, a few preliminaries are necessary. First, we must determine the excitations of the $B$ field corresponding to the various axions, and how they fit into chiral multiplets. The necessary expressions appear in ref. [14]. The axions are in one to one correspondence with harmonic $(1,1)$ forms, $b_{i,ar{i}}^{(a)}$. These are conventionally normalized so that

$$
\left(\int_{\Sigma_a} b^{(b)} \right) = \delta_a^b
$$

(2.10)

where $\Sigma_a$ are a basis of nontrivial closed two-dimensional sub-manifolds. In terms of these, and adopting units with $2\alpha' = 1$, the action takes the form

$$
I = -\frac{i}{2\pi} \int d^2z \left( (r_a + i\theta_a) b_{\bar{i}i}^{(a)} \bar{\partial}X^\bar{i} \partial X^i + ((r_a - i\theta_a) b_{\bar{i}i}^{(a)} \bar{\partial}X^\bar{i} \partial X^i \right).
$$

(2.11)

By virtue of the normalization of the $b^{(a)}$s, the coefficients of the $\theta_a$’s are quantized, and $\theta_a$ has period $2\pi$. As we will now show, $\theta_a$ is the imaginary part of the chiral field whose real part is $r_a$. Note that $2\pi r_a$ is what one would call the radius-squared of the internal space.

In order to determine the structure of the four dimensional chiral fields, it is necessary to adopt some conventions. We take the ten-dimensional fields to satisfy $\Gamma_{11} = 1$, where $\Gamma_{11} = \Gamma_1 \ldots \Gamma_{10}$. In making the reduction to four dimensions, we introduce three complex coordinates, $x_i$ and $\bar{x}_i$ (this was implicit in the discussion above), and a corresponding set of $\gamma$ matrices. In particular, if we define

$$
X^1 = x^1 + ix^2 \quad X^\bar{1} = x^1 - ix^2
$$

(2.12)

eq_\text{etc.}, and if we define corresponding six dimensional $\gamma$ matrices, $d^i$ and $d^{\bar{i}}$, then we can define “states” by

$$
|0\rangle \quad |\bar{i}\rangle = d^{\bar{i}}|0\rangle \quad |k\rangle = d^{\bar{i}}d^{\bar{j}}|0\rangle \quad |\bar{0}\rangle = d^{\bar{i}}d^{\bar{j}}d^{\bar{k}}|0\rangle.
$$

(2.13)

Calling $d^{\bar{7}} = -id^1 \ldots d^6$, $|0\rangle$ has $d^{\bar{7}}=1$, and the chiralities of the other states follow immediately. In particular, the states $|i\rangle$ have chirality one both internally and in four dimensions. So vertex operators of the form

$$
V_{27} = b_{i\bar{i}}^{(a)} \bar{\lambda}^\bar{i} DX^i
$$

(2.14)
are vertex operators for 27’s with positive chirality. Note, however, that when trying to identify these operators with fields, it must be remembered that the vertex operators are like creation operators, i.e. they are like complex conjugates of fields. Similarly, we can read off the operators for the moduli, from eqn. (2.11). In particular, the chirality plus field is the one which multiplies $DX^i$, but complex conjugated as described above, i.e. $r_n + i\theta_n$

Now we can turn to the Green-Schwarz term. This term has been evaluated in various places. We choose to take the result from ref. [15]:

$$\frac{1}{24 \times 12} \frac{1}{(2\pi)^3} \int B[TrF^4 - \frac{1}{300} (TrF^2)^2 - \frac{1}{10} TrF^2 trR^2 + 3TrR^4 + \frac{3}{4} tr(R^2)^2] \quad (2.15)$$

We can dimensionally reduce this immediately. Break up $F$ into parts with indices in four dimensions and indices in the internal six dimensions. Replace $B$ by $2\pi\theta_a b^{(a)}$. Recall that $\text{Tr}(F^4) = \frac{1}{100} (\text{Tr}F^2)^2$, and $\text{Tr}F^2 = 30\text{tr}F^2$. One then obtains, for the $E_8$ coupling to the axion,

$$\theta_a \frac{1}{32\pi^2} \int d^4xF \tilde{F} \int \frac{b^{(a)} \wedge F \wedge F}{8\pi^2} \quad (2.16)$$

For the $E_6$ coupling, one obtains the same result but with the opposite sign.

In order to finally determine the sign of the coupling of the modulus to the gauge fields, one notes that that the sign of the coupling of the imaginary part to $F\tilde{F}$ is opposite to that of the coupling of the chiral field to $W^2_a$ [13]. So we see that the $E_8$ fields couple to $S - T \int \frac{b \wedge F \wedge F}{8\pi^2}$ while the $E_6$ fields couple to the same combination but with the opposite sign for the $T$ term.

Finally, we can compare this with Witten’s result. Using the formula for $\alpha'$, eqn. (2.3), and Witten’s expression for the difference of the $E_8$ and $E_6$ couplings,

$$\delta \alpha^{-1} = \frac{2}{(4\pi)^{4/3} \kappa^{2/3}} 2\pi^2 R \int \frac{1}{8\pi^2} \omega \wedge (F \wedge F - \frac{1}{2} R \wedge R) \quad (2.17)$$

we have, in units with $2\alpha' = 1$

$$= \frac{1}{8\pi^2} \int \frac{\omega \wedge F \wedge F}{8\pi^2} \quad (2.18)$$
where we have taken the spin connection to equal the gauge connection. To obtain the corresponding term in the action involving $F \tilde{F}$, one needs to multiply this expression by $\frac{1}{16\pi}$. Again, the properly normalized fluctuation of $B(\omega)$ contains a factor of $2\pi$, so the difference of the two couplings is the same as expected from eqn. 2.12. However, it is also clear that we cannot identify the volume on the $E_6$ side with the weak coupling $S$; it would appear to be something like $S - cT$.

In order to be more precise about the comparison between the weak coupling results and Witten’s, we must pay more attention to the proper definition of four dimensional chiral superfields in terms of higher dimensional geometry. In the weakly coupled region, there is only one Calabi Yau volume, while in the M theory regime, we must specify precisely what average over the fifth dimension we are using. Thus, as we have seen above, it is wrong to identify the real part of the chiral superfield $S$ of the weakly coupled heterotic string with the volume of the Calabi Yau manifold on the $E_6$ boundary, which Witten uses to parameterize his results. The weak coupling calculation shows that the $E_6$ coupling is a linear combination of $S$ and the $T^\alpha$, and it is this linear combination which is identified with the volume on the $E_6$ boundary. We have found it useful to pass through 5 dimensions in our search for a good parameterization of the space of chiral superfields in the four dimensional effective field theory. In particular, we want to keep track of the $h_{(1,1)}$ approximate $U(1)$ symmetries which are unbroken by strong $E_8$ dynamics. These act on chiral superfields which are defined in terms of functions with boundary conditions on the $E_8$ boundary. That is, shifts of the imaginary part of these superfields are would be five dimensional gauge transformations with gauge function defined to vanish on the $E_8$ boundary. We will call the $h_{1,1}$ axion chiral multiplets $Y^a$. It is convenient then to parameterize the volume and complex structure moduli by their values on the $E_8$ boundary as well. As noted in a previous section, these belong to five dimensional hypermultiplets. However, only one chiral field from each hypermultiplet survives the breaking of SUSY that accompanies the reduction to four dimensions. We will denote the superfield whose real part is proportional to the volume of $X$ on the $E_8$ boundary by $S$. The normalization is fixed so that shifts of the imaginary part of $S$ by $2\pi$ are exact symmetries of the theory. The complex structure moduli on the $E_8$ boundary are denoted by $C_\alpha$. Note that although
all of these fields are defined in terms of boundary conditions, they are what we will later describe as bulk moduli. The boundary conditions determine the behavior of the classical vacuum configuration throughout the fifth dimension. The action for making a small spacetime dependent deformation of these boundary conditions will be proportional to $R_{11}$.

With these definitions we can write our weak coupling results for the gauge kinetic functions in terms of the fields $S$ and $Y^a$ in the M theory regime. We have

$$S = S - T^a \int \frac{b_a \wedge F \wedge F}{8\pi^2}.$$

To summarize, the basic phenomenon observed by Witten, i.e. that gauge couplings can blow up in the region of moduli space where the Calabi Yau volume is larger than the string scale, is evident in extant weak coupling calculations. It has probably been ignored in the past because it only occurs when the heterotic string is strongly coupled, but analyticity and discrete symmetries allow us to reliably compute in this region. The perturbative computation reproduces the fact that the term in the $E_8$ coupling function linear in the moduli vanishes (and thus the gauge coupling becomes strong) at a point in the M theory regime. In addition it enables us to identify the weakly coupled moduli fields as particular linear combinations of the fields $S$ and $Y^a$ which have simple properties in the M theory region. Once the $E_8$ coupling becomes strong however, we can no longer neglect possible exponential terms in the gauge coupling function. In the low energy $E_8$ gauge theory, an accidental $U(1)$ symmetry prevents the occurrence of such terms, but in M theory we do not expect to have such a symmetry. Thus, although we know that the coupling becomes strong, we do not know that it becomes infinitely strong. In the strong coupling region we do not have a reliable calculation either of the $E_8$ coupling itself, or of the superpotential for the moduli which is generated by the strongly coupled dynamics.

One may worry that similar incalculable effects will infect the computation of the coupling functions on the $E_6$ boundary. This could completely ruin predictions of coupling unification. We know of no symmetry argument which rules this out, but we believe that the following physical argument is plausible. Let us examine the region where the Calabi Yau volume is much larger than $l_{11}^5$. In this regime, the $E_8$ coupling becomes strong only at very large $R_{11}$. Thus, there is a regime in which $R_{11}$ is large and the $E_8$ coupling is
still small enough that nonperturbative dynamics is well approximated by a very dilute gas of small instantons. In addition, since the instanton density is exponential in the coupling, the average instanton spacing can be taken much larger than $R_{11}$. In this limit, the dominant effect on the lagrangian of the $E_6$ boundary will come from the local influence of single instantons. $E_8$ instantons are 5 branes in eleven dimensional space. Their effect on the $E_6$ boundary must fall like $R_{11}^{-3}$ as $R_{11}$ is increased. Thus they cannot give rise to effects on the $E_6$ coupling functions which grow exponentially with $R_{11}$. Indeed, Green’s functions made up of fields which live purely on the $E_6$ boundary cannot soak up the $E_8$ instanton zero modes, and get no contribution from these nonperturbative configurations. We have made this argument for very large $V$ and $R_{11}$, but holomorphy tells us that if the growing exponentials are not present in this regime, they are not present at all. Coupling unification is a prediction in the M theory region of moduli space.

One advantage of our weak coupling calculation of vacuum polarization functions is that we can easily extend it to the case where $E_8$ is broken by Wilson lines. In fact, it is not difficult to see that the result is unchanged in the presence of Wilson lines. At large radius, on the torus, one must compute an expectation value of the form

$$\langle V_B V_A V_A V_A \rangle$$  \hspace{1cm} (2.19)

where $V_B$ is a vertex operator for the antisymmetric tensor, and $V_A$ is a vertex operator for a gauge field. One can take, say, $V_B$ in the $-1$ superconformal ghost number picture, and the $V_A$’s in the zero ghost picture. As in the flat space calculation, the term with an $\epsilon$ tensor arises from the sector with $(P, P)$ boundary conditions for the right movers. In the large $R$ limit, there is an (approximate) zero mode for each of the $\psi_I$’s. This is just the correct number of zero modes to be soaked up by the five vertex operators in eqn. (2.19). The momentum factors in the four gauge boson vertex operators then give $F^4$. Because the fundamental group of the non simply connected Calabi Yau manifold acts freely on its covering space, at large radius one just has an ordinary momentum integral to do, up to terms which are down by powers of $1/R$. Such terms have the wrong $R$ dependence to correct the modulus-dependence of the gauge couplings.

The rest of the calculation is as in ten dimensions. The right moving boson and
fermionic contributions cancel. Level matching then implies that only states with $L_0 = 0$ contribute on the left. This is identical to the situation without the Wilson line.

2.3. Kahler Potentials

The dynamics of SUSY breaking in the M theory regime is, as usual in string theory, intimately connected with the stabilization of the moduli. In the M theory regime, the moduli break up into several distinct classes. All moduli originate as dimensionless deformations of a supersymmetric classical ground state of a theory with fundamental mass scale $M_11$. However, fields that originate in the bulk of eleven dimensional spacetime, have kinetic terms in the effective four dimensional theory which are proportional to $R_{11}$. In particular this is the case for the four dimensional metric and this is part of Witten’s proposal for the origin of the large ratio between the four dimensional Planck mass $M_4$ and the eleven dimensional Planck scale $M_{11}$. Thus, we should imagine that in the conformal frame fixed by 11 dimensional SUGRA the Kahler potential for all of the bulk moduli has a coefficient of order $\frac{M_{11}^2}{8\pi} \equiv m_4^2$. When we rescale these fields, $B_i$, to give them their proper dimension, their lagrangian will be a function of $\frac{B_i}{m_4}$. Note that it is the reduced Planck mass $m_4$ that we choose in this formula rather than the Planck mass itself. Historically, it has been natural to associate the mass associated with Newton’s constant as the fundamental mass scale of quantum gravity. However, in the M theory regime at least, it is a low energy artifact. $m_4$ is the parameter which appears in all formulae in the M theory regime.

In writing a supersymmetric four dimensional lagrangian, it is convenient to choose a conformal frame in which the Einstein term does not depend on the chiral superfields. This is the frame in which textbook expressions for the supergravity potential are written. In this frame, the coefficient of the Einstein term and of the Kahler potential for dimensionless bulk moduli fields is $M_{11}^2$. We will refer to this as the canonical frame. Note that this is different from the Einstein frame, where the coefficient of the four dimensional Einstein lagrangian is $m_4^2$. This is a consequence of the fact that $M_{11}$ is the fundamental scale,

---

5 This is the term which we use to describe chiral superfields which originate as modes of bulk fields in five dimensions.
while $m_4$ is a function of the moduli.

Among the bulk moduli will be those that descend from components of vector multiplets in 5 dimensions. $h_{1,1}$ of these can be associated with Kahler deformations of the Calabi Yau manifold. The way in which these emerge from the 5 dimensional lagrangian has been described in [11]. Remember that the five dimensional theory contained $h_{1,1}+1$ vector multiplets, whose scalar components live on a manifold with coordinates $X^a$ satisfying the constraint $N(X) = 1$. The dimensionally reduced theory is conveniently described in terms of the complex fields $Y^a = R_{11}X^a + iA_5^a$ which are the scalar components of chiral superfields. These fields are unconstrained and have (in canonical conformal frame) the Kahler potential $-\ln N(ReY^a)$. In this approximation, the theory is invariant under continuous shifts of the imaginary parts of the $Y^a$. In the quantum theory, we expect this to be broken to a discrete shift symmetry. However we have argued above that the symmetry breaking is entirely due to stretched membranes and to fivebranes embedded in the $E_6$ boundary.

We can estimate the size of the stretched membrane contribution in two ways. First a naive eleven dimensional calculation suggests a PQ symmetry breaking term of order $e^{-cM_{11}^3R^2R_{11}}$. This is the same form as the PQ breaking term which arises from a single worldsheet instanton in the weak coupling theory. We can in fact, reproduce this result by analytically continuing the world sheet instanton contribution of weakly coupled string theory. This has the form $e^{-cR^2}$ in units with $2\alpha' = 1$. Inserting the formula for the string tension in terms of eleven dimensional quantities we get $e^{-c2^{2/3}\pi^{5/3}M_{11}^3R^2R_{11}}$. The latter derivation allows us to compute the precise coefficient, $c$, in the exponent for specific Calabi Yau manifolds. It also leads us to another example in which symmetry and holomorphy arguments are enhanced by an appeal to physical intuition. Symmetry and holomorphy would allow us to add a term to the space time superpotential which breaks the axion shift symmetries and vanishes only when the $E_8$ coupling is weak. This could give the axions a large mass when the $E_8$ coupling is strong. The physical picture of stretched membranes assures us that this does not occur. Strong $E_8$ coupling might modify the contribution of a membrane instanton on its boundary. This should appear as a multiplicative factor of order one in the instanton amplitude, and will not change our estimate of its order of
To estimate the value of the axion mass, we plug in the values of $R$ and $R_{11}$ from our fit. These are determined in terms of $M_{11}$ so we must also use the expression, eqn. (2.3), for $\alpha'\theta$

The axion mass vanishes in the limit of supersymmetry breaking; it is thus expected to be of order the SUSY breaking scale to the fourth power. Assuming that this scale is of order $10^{11}$ GeV, yields an axion potential of order

$$V_a = e^{-544c}10^{44} \text{GeV} \sim 10^{(44-236c)}\text{GeV}.$$  

(2.20)

This should be compared with the Quantum Chromodynamic contribution to the axion potential which is $\sim 10^{-4}$ in GeV units. For $c > 0.1$, the QCD contribution dominates, and the model will solve the strong CP problem. In orbifold examples, $c \sim (2\pi)^2$ and the stretched membrane contribution is completely negligible. It appears then to be a general feature of the M theory region of moduli space that there are $h_{1,1}$ axion fields which get their mass mainly from nonperturbative effects on the $E_6$ boundary. The strongest such effect, if the gauge group is broken to the standard model, is QCD, and one of the axions will solve the strong CP problem. Others will get their mass only from weak instantons, and from stretched membranes. These very light axions will have Compton wavelengths of astrophysical magnitudes. However, their coherent couplings to matter may be suppressed relative to gravity by as much as low energy CP violation. In this case we believe that they may be compatible with observation. If not, M theory will only describe the real world if $h_{1,1} = 1$. In any event, in the M theory region of moduli space, axions solve the strong CP problem. The relevant invisible axion violates the cosmological axion bound. We will comment on this in the cosmology section below.

The low energy spectrum also includes fields which originate as modes on the boundary of the five dimensional world. Apart from the gauge fields, there are the moduli of the $E_8$ gauge bundle on the $E_6$ boundary, and quark, lepton and Higgs superfields, as well as possible exotic matter. We denote the generic chiral multiplet originating on the boundary as an edge field, $E_I$. The Kahler potential for these fields is of order $M_{11}^2$, so that when

---

6 However, see the comments about boundary axions below.
they are made dimensionful, their lagrangian will depend on $\frac{E}{M_{11}}$. In general, it will depend on the bulk moduli as well, and will be a correction to the Kahler potential of these fields. When $R_{11}$ and $R$ are large, but $\epsilon$ is small, it is a simple matter to determine the Kahler potential for these edge states. It is, in fact, precisely the same as on the weakly coupled string side. To see this, one simply has to consider the lagrangian for the edge states, which for the bosonic fields takes the form:

$$\mathcal{L}_e = -\frac{1}{8\pi (4\pi\kappa^2)^{2/3}} \int d^{10}x \sqrt{g} \text{tr} F^2.$$  \hspace{1cm} (2.21)

Reducing this lagrangian is similar to reducing the usual ten-dimensional supergravity lagrangian on a Calabi-Yau space. The factors of $R_{11}$ work out correctly. In particular, if one first reduces to ten-dimensions, it is necessary to rescale the ten-dimensional metric by $g_{MN} \to R_{11}^{-1/4} g_{MN}$. This gives $R_{11}^{-3/4}$ in front of the gauge term, which is the conventional form of the ten-dimensional action.

It is curious that the Kahler potentials for all of the fields have the same form at both extremely weak and extremely strong string couplings. It is not clear that this is enforced by any symmetry. Moreover, we have seen that the identification of the fields $S$ and $\vec{T}$ is different in the two regimes. Nevertheless, perhaps it holds some deeper meaning.

Finally, let us note that the boundary moduli may provide us with another candidate for the invisible axion. Indeed, in [14], it was shown that many $(2,0)$ moduli might receive masses of order $e^{-\frac{T_h}{R^2}}$. This is a superpotential calculation, and may be analytically extrapolated into the M theory regime. Since it refers to fields which live on the $E_6$ boundary, it will not be affected by strong coupling dynamics on “the other side of the world”. If these $(2,0)$ moduli affect the $E_6$ gauge couplings at one loop in heterotic perturbation theory, as is almost certainly the case, then they will provide another contribution to the QCD axion. The true axion will be a linear combination of these, and the $h_{1,1}$ moduli discussed above. However, because the boundary moduli have decay constants of order $M_{11}$ rather than $m_4$, the dominant component will be a boundary modulus. This will ameliorate the cosmological axion problem.
3. Mechanisms for Stabilizing the Moduli

In the limit that the classical eleven dimensional description is good, we expect to find
the usual problem of runaway in the various moduli. If we simply consider compactification
with gauge group $E_6 \times E_8$, we can compute the potential due to gluino condensation of
the “far side.” We do not need to think carefully about the interactions between the two
walls to do this, since we have already determined the four dimensional Kahler potential,
and the superpotential due to gluino condensation follows, as in ref. [17] from symmetry
considerations. One obtains, then, a potential identical to that at weak coupling. It tends
to zero as

$$V \approx |Y|^{-3}e^{-S/b_o}$$

This potential favors large Calabi-Yau volume on the $E_8$ boundary and large $R_{11}$. This is a
region where the supergravity analysis should be completely valid, so we have encountered
the eleven dimensional version of the stability problem. Perhaps, however, the fact that,
for fixed $E_6$ gauge coupling, the potential forces $R_{11}$ to zero is a hopeful sign. This follows
from the fact that for fixed $E_6$ coupling, the $E_8$ coupling (and thus the strength of the
gaugino condensate) decreases with $R_{11}$.

We turn, then, to a discussion of what sorts of physics might stabilize the moduli. We
begin by discussing the dynamics of the strongly coupled gauge theory on the $E_8$ boundary.
The proximity of the phenomenologically determined value of $R_{11}$ to the strong coupling
point motivates us to search for a mechanism involving the strong gauge dynamics which
freezes some of the fields.

As we argued in the previous section, the fields associated with five dimensional vector
multiplets do not participate in the strong dynamics. The superpotential generated by $E_8$
and other quantum effects in M theory will be a function of $S$ and perhaps of the complex
structure moduli, but will not depend on the fields $Y^a$. Label the fields on which it does
depend $Z^A$. Then the potential will have the form

$$V = M_{11}^4 e^K (K^{AB} F_A \bar{F}_B + [G^{ab} G_a G_b - 3]|W|^2)$$

Here $G \equiv -\ln N$ and $G_a, G_{ab}, etc.$ refer to derivatives with respect to $ReY^a$ ($G^{ab}$ is the
inverse metric). This expression is the first term in an asymptotic expansion of the potential
for large $Y^a$. The equations $F_A = 0$ have a solution at $S = \infty$, the weak coupling region referred to above. Generically, we may expect them to have a solution for finite values of $S$ as well. When $S$ is small, the theory is strongly coupled and the Calabi Yau volumes everywhere small (at finite $R_{11}$), and we can calculate neither the superpotential nor the Kahler potential. It is reasonable to postulate the existence of a discrete set of solutions to these $k$ equations for $k$ complex unknowns. Furthermore, generically, $W$ will not vanish at these points. In regions where $S$ is relatively large it may be a good approximation to neglect higher order terms in the superpotential, while retaining the complicated Kahler potential. The leading term in the superpotential has the form $e^{-\frac{S}{b_0}}$ where $b_0$ is the first coefficient in the renormalization group beta function. The corrections are powers of $e^{-S}$ multiplied by the leading term or by 1.

We now note the remarkable property\cite{18} of the Kahler potentials for the axion multiplets, which has been widely exploited in no scale models: the term in square brackets in (3.2) vanishes identically for any $W$ and any value of $Y^a$. As a consequence, the submanifold with $F_A = 0$ of the full moduli space is, in the current approximation, a stationary manifold of the potential, with broken supersymmetry and vanishing cosmological constant. Moreover, the scale of SUSY breaking is as yet undetermined, since it depends on the values of the $Y^a$.

The $Y^a$ will be determined by terms higher order in the $Y$ expansion of the Kahler potential. At order $\frac{1}{|Y|}$ we also encounter terms in the Kahler potential that involve the boundary fields. These include quarks, $Q^i$, and moduli of the gauge bundle that breaks $E_8$ to $E_6$. To this order, the Kahler potential will have the form

$$G = -\ln N(ReY) + h(Y, E) + h_{ij}(Y)Q^{i*}Q^j$$ (3.3)

where $h$ and $h_{ij}$ are homogeneous of degree minus one in $Y^a$. They can also depend on the gauge bundle moduli, $E_I$. We will assume that there is a solution of the equations

$$\frac{\partial h(Y, E)}{\partial E_I} = 0,$$ (3.4)

which fixes the value of the gauge bundle moduli. With this assumption, there is only one term of order $Y^{-1}$ and quadratic in quarks which appears to depend on a matrix other
than $h_{ij}$. It is proportional to

$$L_a L^{ab} h_{ij,a} (Q^i)^* Q^j$$  \hspace{1cm} (3.5)$$

where $L \equiv \ln N$. $N$ is a homogeneous polynomial, so $L_a = -L_{ab} Y^b$ and the dangerous term is proportional to $Y^a h_{ij,a}$. To leading order in $Y^{-1}$, this is proportional to $h_{ij}$ itself. Thus the squark mass matrix is proportional to the matrix in the quark kinetic term and we have universality. Corrections to this will be of relative order $\frac{1}{|Y|} \sim 10^{-1}$ (here we use the phenomenological fit to the value of $|Y| \sim R_{11}$ since we are not yet able to calculate it theoretically).

The value of the $Y^a$ will be determined by minimizing the potential with $Q^i = 0$ and $E_I$ determined by equation (3.4). This procedure will have the usual philosophical problem discussed in [2]. Minimization is achieved only by balancing terms of different orders in $Y$, even though $Y$ is large. There are several differences from the analogous problem in weakly coupled string theory. There one is forced to contemplate cancellations between different exponentials of a large number. Here we have a Laurent series in $Y^a$, and $|Y|$ must be of order 10 in order to explain the ratio between the unification scale and the Planck scale. A second contrast with the weakly coupled problem is that we seem to have solved at least one of the stability problems of the weakly coupled theory. $S$ is presumed to be fixed in the strong coupling region by the equation $F_S = 0$. Note that this is completely compatible with the fact that the gauge theory on the $E_8$ boundary is weakly coupled at the unification scale.

Unfortunately, this argument leaves us with a puzzle about the scale of SUSY breaking. In the true strong coupling regime, the superpotential generated by nonperturbative dynamics on the $E_8$ boundary is of order $M_{11}^3$. The gravitino mass is then fixed to be of order $|W|/M_{11}|Y|^{-1} M_{11} \sim 10^{15}$ GeV. In order to get the right scale of SUSY breaking, we must assume that the superpotential generated by the strong $E_8$ dynamics is of order $10^{-12}$ in eleven dimensional units. This suggests that the coupling is not terribly strong and very probably that the gauge group is smaller than $E_8$. For a gauge group $G$ with $k$ instanton zero modes in the adjoint representation, the implied $G$ fine structure constant at the unification scale is $\frac{0.45}{k}$.

Another problem, which loses none of its severity through familiarity, is that we do not
have an explanation of the value of the cosmological constant. The no scale cancellation actually works through order $|Y|^{-1}$ but fails at higher order. The fact that the vacuum energy density will be smaller by a factor of 100 than in a typical hidden sector model with the same value of the gravitino mass is perhaps suggestive, but hardly represents a solution of the cosmological constant problem.

To conclude the discussion of this scenario, we briefly note the properties of the moduli. The bulk moduli coming from $S$ and the complex structure of $X$ will have masses of order the gravitino mass. Their kinetic terms are of the same order as the Einstein term in the action, so their couplings to ordinary matter will be suppressed by powers of $m_4$. In Einstein frame (the frame in which the coefficient in front of the Einstein lagrangian is $m_4^2$) they will have potential energies of order $m_4^2 m_c^2$. The boundary moduli $E_I$ have potentials of order $|Y|^{-1} m_4^2 M_{11}^2$ in the canonical frame lagrangian. However, in this frame their kinetic terms also carry an inverse power of $|Y|$. In Einstein frame this means that they have potentials of the form $m_4^2 M_{11}^2 V \left( \frac{E}{M_{11}} \right)$. Thus their masses are of order $m_4^2$ and their couplings to matter are inversely proportional to $M_{11}$.

The bulk moduli associated with the real parts of the axion multiplets have a potential which is suppressed by two powers of $|Y|$ relative to the other bulk moduli. Thus, their mass is of order $10^{-1} m_4^2$ or 100 GeV. Their couplings to matter are nonrenormalizable and scale with $m_4$. The QCD axion has a mass of order $10^{-10}$ eV and decay constant of order $m_4$. Its coherent couplings to matter are further suppressed by the same factors that suppress any low energy CP violation. If $h_{11} > 1$ there will be more of these multiplets. Now however the axions will be extremely light as noted above.

If there are also boundary contributions to the QCD axion then the true axion decay constant will be $M_{11}$. We will also have a definite prediction of a very light axion which would contribute to long range spin dependent forces and very weak (compared to gravity) long range coherent forces.

4. Low Energy Constraints on Planck Scale Physics

The replacement of the Planck scale $M_4$ by $M_{11}$ as the threshold for as yet incalculable
quantum gravitational effects sharpens the constraints on physics at ordinary scales from possible higher dimension operators.

The most important such effect is the lowering of the scale of dimension five baryon number violating operators by two or three orders of magnitude. Discrete symmetries which eliminate or suppress dimension five operators become absolutely imperative. The constraint from gravitational physics is now of the same order as that conventionally quoted for grand unified models. In a similar manner, we find a new estimate for gravitational contributions to neutrino masses.

We also find a stronger constraint on models which invoke pseudogoldstone bosons of accidental continuous symmetries. Previously, one argued that a renormalizable theory at scale $f$ might spontaneously break an accidental continuous symmetry, producing a Goldstone boson with decay constant $f$. If gravitational physics breaks all global symmetries (this is certainly the case in string theory) we expect a Goldstone boson mass to be generated. It will be of order $\frac{f^{\frac{d}{2}(d-4)+1}}{M_G^{\frac{1}{2}(d-4)}}$ where $M_G$ is the scale of gravitational effects and $d$ is the dimension of the leading operator which breaks the symmetry $[19]^{7}$

For example, in an attempt to build a QCD axion model based on accidental symmetries we must require that the gravitationally induced mass be smaller than that coming from QCD. In equations, we must have $\Lambda_{QCD}^2 M_G^{\frac{1}{2}(d-4)} > f^{\frac{1}{2}(d-4)+3}$. For an axion decay constant of order $10^{11}$ GeV, this requires $d > 16$. Similar restrictions apply to majoron models.

5. Cosmology

Here we will make only the briefest remarks about cosmology in the M theory region of moduli space. The first thing to note is that the large vacuum energy densities typical of many inflation models are uncomfortably close to the eleven dimensional Planck scale. This raises the disturbing (or perhaps exciting) possibility that the inflationary era can only be studied with the unknown machinery of quantum M theory.

7 Usually the gauge symmetries of the renormalizable model allow operators of dimension 5 or 6, but discrete gauge symmetries can be invoked to push $d$ to larger values. In the M theory region of moduli space, $M_G \sim 3 \times 10^{16}$ GeV.
Indeed, in the scenario we have presented in this paper for nonperturbative physics and SUSY breaking, the natural scales of energy density in the low energy four dimensional theory are all much lower than $M_{11}$. The vacuum energy density is of course moduli dependent, so we can always imagine that inflation takes place in a region of moduli space where the energy density is close to $M_{11}^4$. We will then have to deal with the “cosmic overshoot” problem described by Brustein and Steinhardt [20]. The initial energy density of the system is much larger than the barriers that separate the inflationary region of moduli space from the extreme weak coupling region where string theory contradicts observation. In [21] it was suggested that this problem might be less severe than it had first appeared. In a region of steeply falling potential, the moduli lose energy exponentially in the distance covered by the trajectory on moduli space. It requires a detailed knowledge of the lagrangian on moduli space to determine whether the system really crosses the barrier into the weak coupling region.

We also note that the natural candidates for inflatons in the M theory regime are the bulk moduli. They have self couplings which scale with powers of $m_4$ so that the natural size of the forces restoring these moduli to their equilibrium values is of the same order as gravitational friction.

Assuming that we can construct a satisfactory inflationary model, we will certainly have to face a cosmological moduli problem. Many of the bulk moduli have masses of the same order of magnitude as squarks in strongly coupled heterotic string theory. Despite the replacement of $M_4$ by $M_{11}$ as the fundamental gravitational scale, $M_4$ (or perhaps $m_4$) is the parameter which determines the couplings of the moduli to ordinary matter. We will have to borrow one of the existing mechanisms for solving this problem [22] [21] or come up with a new one. Note that we also have a QCD axion with Planck scale decay constant. Most mechanisms (with the notable exception of [23]) for solving the cosmological moduli problem will not help with the axion. However, the very existence of the moduli will change the nature of the axion problem. The very early universe will be cold and matter dominated, so the usual analysis of axion history above the QCD phase transition may not be relevant.

It should be clear furthermore that the cosmology of strongly coupled heterotic string
theory is considerably more complicated than models that have been considered in the literature. In addition to more or less conventional bulk moduli and QCD axion fields, the model also has boundary moduli. These have mass of order the gravitino mass, but couplings to matter suppressed only by powers of $M_{11}^{-1}$. Their reheat temperature is about 1 MeV. We also have scalar partners for the axions, which will be a form of late decaying dark matter, and probably have to have very small density at nucleosynthesis if they are not to ruin classical cosmology. The distributions of energy among the various scalar fields may lead to a rich and complicated cosmological scenario. We will also have to sort out the question of whether the QCD axion is dominantly a boundary modulus in generic regions of moduli space in order to embark on a detailed study of the cosmology of M theory.

6. Conclusion

Strongly coupled heterotic string theory retains most of the attractive features of the weakly coupled region but provides a better fit to the parameters of the real world. There is no longer a discrepancy between string theory and supersymmetric coupling unification. In the strongly coupled region there is always a QCD axion and the strong CP problem is resolved. The axion decay constant violates cosmological bounds, but we view this as a challenge rather than a definitive failure of the theory. Indeed, the most serious phenomenological problem of string theory, in any region of moduli space is the cosmological moduli problem. Several solutions to this have been proposed, but Linde’s seems to be the only one which could resolve the axion problem. The axion is of course also an attractive dark matter candidate. If $h_{1,1} > 1$, the theory predicts a number of essentially stable axionlike particles with Compton wavelengths of astrophysical magnitude. If boundary moduli contribute to the QCD axion then we will certainly have at least one of these particles. Observations measuring the number of such light axions would be of the utmost interest. They would amount to measurements of topology of the six compactified dimension. Alternatively, if no such particles are found, and if there are boundary contributions to the QCD axion, the entire M theory region of moduli space would be ruled out.

We have also proposed a scenario for SUSY breaking in the strong coupling region. The fundamental reason for the discrepancy between the Planck scale and the unification
scale is the existence of a fifth “large” dimension an order of magnitude larger than the unification scale. As a consequence, certain fields of the theory exhibit an approximate 5 dimensional supersymmetry which is broken by terms of order inverse powers of the radius of the fifth dimension. There are $h_{1,1}$ chiral superfields in the low energy four dimensional theory which descend from vector multiplets in five dimensions. The axions are the imaginary parts of these fields. Approximate $N=2$ SUSY produces an approximate no scale scenario for SUSY breaking, in which the F terms of the axion multiplets are the order parameters. The R symmetry breaking which triggers SUSY breaking comes from nonperturbative physics on the strongly coupled boundary. We argue that in this scenario squark degeneracy naturally arises to leading order in the inverse radius of the fifth dimension. The scenario also leads to $h_{1,1}$ scalar axion superpartners, with masses of order 100 GeV and Planck scale couplings to matter. These are a form of late decaying dark matter and are constrained by classical cosmology.

The scenario is unacceptable as it stands. If we make the natural assumption that strongly coupled physics does not introduce any small parameters into the superpotential, then we predict the SUSY breaking scale to be much too large. Otherwise, we must resort to the sort of Kahler stabilization of some of the moduli that we advocated in [1] for the regime of weakly coupled string theory. Apart from this, we must also invoke higher order terms in the expansion in the inverse radius of the fifth dimension to explain the stabilization of the radius. This is precisely the sort of procedure that was criticized in [2]. Here however the expansion parameter is only of order 0.1. It is plausible then that the expansion breaks down for the values of the moduli at which the minimum is achieved. It is also reasonable to use the expansion as evidence for the existence of a SUSY breaking minimum (though not of course to understand why the cosmological constant is zero). However, we do not see how to save the prediction of squark mass universality which follows from the no scale structure at large $R_{11}$.

It is fairly clear from this discussion that we do not yet understand the mechanism of SUSY breaking in the M theory regime. We suspect that this may be closely connected with another phenomenological issue that has not yet been explored, the quark mass matrix. Most successful theories of the quark mass matrix are based on horizontal symmetries.
In string theory, an attractive origin for horizontal symmetries has been suggested by a number of authors[24]. They originate as $U(1)$ gauge symmetries which have Fayet-Iliopoulos D-terms. We feel certain that the dynamics of cancellation of the D-term will influence the breaking of supersymmetry and the stabilization of the moduli. Perhaps it will help to resolve some of the puzzles we have uncovered.

In the long term, if the M theory region of moduli space has anything to do with the real world, the most striking feature of its phenomenology will be the low scale at which interesting gravitational phenomena become accessible. At energies of order $10^{15}$ GeV, “experiments” will reveal an extra bosonic dimension of spacetime, and discover that some of the degrees of freedom live on “the other wall of the world”. At energies one or two orders of magnitude higher we will encounter true quantum mechanical manifestations of gravity and find out what M is. We have already indicated that the low scale of gravitational phenomena forces us to envisage discrete symmetries which forbid the leading gravitational corrections to the standard model. It is to be hoped that further study will reveal interesting signatures of M theory that can be probed at low energies, or in the early universe.

Acknowledgements

We thank M.Berkooz, R.Leigh, Y.Nir, A. Rajaraman, N. Seiberg, S. Shenker, Y. Shirman, L. Susskind, S. Thomas, P. Townsend and E. Witten for conversations. The work of M.D. was supported in part by the U.S. Department of Energy. The work of T. Banks was supported in part by the Department of Energy under grant #DE – FG0296ER40559.
References

[1] T. Banks and M. Dine, “Coping with Strongly Coupled String Theory,” hep-th/9406132, Phys. Rev. D50 (1994) 7454.
[2] M. Dine and N. Seiberg, Phys. Lett. 162B, 299 (1985), and in Unified String Theories, M. Green and D. Gross, Eds. (World Scientific, 1986).
[3] M. Dine and Y. Shirman, “Truly Strong Coupling and Large Radius in String Theory,” SCIPP-96-07, hep-th/9601175
[4] E. Witten, “Strong Coupling Expansion of Calabi-Yau Compactification, hep-th/9602070.
[5] M.J. Duff, P. Howe, T. Inami and K.S. Stelle, Phys. Lett. 191B (1987) 70; C.M.Hull, P.K.Townsend, “Unity of Superstring Dualities”, Nucl. Phys. B438, (1995),409, hep-th/9410167; E. Witten, “String Theory Dynamics in Various Dimensions,” Nucl. Phys. B443 (1995) 85, hep-th/9503124; J. Schwarz, “The Power of M Theory,” Phys. Lett. B367 (1996) 97, hep-th/9510086; P. Horava and E. Witten, “Heterotic and Type I String Dynamics From Eleven Dimensions”, Nucl. Phys. B460, (1996),506, hep-th/9510209; “ Eleven-Dimensional Supergravity on a Manifold with Boundary,” hep-th/9603142.
[6] V. Kaplunovsky and J. Louis, “On Gauge Couplings in String Theory,” hep-th/9502077, Nucl. Phys. B451 (1995) 53.
[7] L. Ibanez and P. Nilles, Phys. Lett. 180B (1986) 354.
[8] D.J. Gross, J.A. Harvey, E. Martinec and R. Rohm, Nucl. Phys. B267 (1986) 75
[9] J.Polchinski, “Dirichlet Branes and Ramond Ramond Charges”, Phys. Rev. Lett. 75, (1995), 4724, hep-th/9510017.
[10] I. Antoniadis, S. Ferrara, T.R. Taylor, “N=2 Heterotic Superstring and Its Dual Theory in Five Dimensions,” Nucl.Phys. B460 (1996) 489, hep-th/9511108
[11] M.Gunyadin, G.Sierra, P.K.Townsend, Nucl. Phys. B242 (1984) 244; Nucl. Phys. B253 (1985) 573.
[12] N. Seiberg, “The Power of Holomorphy,” hep-th/9506077.
[13] L. Dixon, V. Kaplunovsky and J. Louis, Nucl. Phys. B355 (1991) 649; J.P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, Nucl. Phys. B372 (1992) 145; I. Antoniadis, K. Narain and T. Taylor, Phys. Lett. B267 (1991) 37; V. Kaplunovsky and J. Louis, “On Gauge Couplings In String Theory,” Nucl. Phys. B444 (1995) 191, hep-th/9509204.
[14] M. Dine, N. Seiberg, X.-G. Wen and E. Witten, Nucl. Phys. B289 (1987) 319, Nucl. Phys. B278 , (1986),769.
[15] W. Lerche, B.E.W. Nilsson, and A.N. Schellekens, Nucl. Phys. B289 (1987) 609.
[16] J. Wess and J. Bagger, Supersymmetry and Supergravity, Princeton University Press, Princeton (1983).

30
[17] J.P. Derendinger, L.E. Ibanez and H.P. Nilles, Phys. Lett. 155B (1985) 65; M. Dine, R. Rohm, N. Seiberg and E. Witten, Phys. Lett. 156B (1985) 55.
[18] E.Cremmer, S. Ferrara, C.Kounnas, D.V.Nanopoulos, Phys. Lett. 133B, (1983), 61; J. Ellis, A.B. Lahanas, D.V. Nanopoulos and K. Tamvakis, Phys. Lett. 134B (1984) 429; J. Ellis, C. Kounnas and D.V. Nanopoulos, Nucl. Phys. B241 (1984) 406; Phys. Lett. 143B (1984) 410; J. Ellis, K. Enqvist and D.V. Nanopoulos, Phys. Lett. 147B (1984) 99.
[19] M. Kamionkowski and J. March-Russell, Phys. Lett. 282B (1992) 137; R. Holman et al., Phys. Lett. 282B (1992) 132; S.M. Barr and D. Seckel, Phys. Rev. D46 (1992) 539.
[20] R. Brustein and P. Steinhardt, Phys. Lett. B302 (1993) 196.
[21] T. Banks, M. Berkooz, and P. Steinhardt, Phys. Rev. D52 (1995) 705.
[22] L. Randall and S. Thomas, “Solving the Cosmological Moduli Problem with Weak Scale Inflation,” Nucl. Phys. B449 (1995) 229, hep-ph/9407248; M. Dine, L. Randall and S. Thomas, “Supersymmetry Breaking in the Early Universe,” Phys. Rev. Lett. 75 (1995) 398, hep-th 9507453; D.H.Lyth, E.D.Stewart, Phys. Rev. D53, (1996),1784, Phys. Rev. Lett. 75, (1995),201, hep-ph/9510204/9502417.
[23] A. Linde, “Relaxing the Cosmological Moduli Problem,” Phys. Rev. D53 (1996) 4129.
[24] J.L. Lopez and D.V. Nanopoulos, Nucl. Phys. B338 (1990) 421; L. Ibanez, Phys. Lett. B303 (1993) 55; L. Ibanez and G.G. Ross, Phys. Lett. B332 (1994) 100; A.E. Farragi, Phys. Lett. B274 (1992) 47, Phys. Rev. D47 (1993) 5021; A.E. Faraggi and E. Halyo, Phys. Lett. B307 (1993) 305; Nucl. Phys. B416 (1994) 63; P. Bineutry and P. Ramond, Phys. Lett. B350 (1995) 49.