Implications of $b \rightarrow s\gamma$ for CP Violation in Charged Scalar Exchange

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Abstract

In models of three or more scalar doublets, new CP violating phases appear in charged scalar exchange. These phases affect CP asymmetries in neutral $B$ decays, even if Natural Flavor Conservation holds. The recent upper bound on the decay $b \rightarrow s\gamma$ constrains the effect to be at most of order a few percent. Modifications of constraints on the CKM parameters open an interesting new region in the $\sin 2\alpha - \sin 2\beta$ plane even in the absence of new phases.

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A detailed investigation of $B$-meson decays is a promising way to discover or severely constrain New Physics beyond the Standard Model. In particular, $B$ physics is sensitive to extensions of the scalar sector: in most models scalars couple more strongly to heavier quarks and thus may affect bottom (and top) decays while having a negligible effect on lighter quark decays.

Models of three or more scalar doublets allow for new CP violating phases in charged scalar exchange [1]. It is conventional wisdom that if Natural Flavor Conservation (NFC) [2] is imposed on the Yukawa couplings, then these phases do not affect CP asymmetries in neutral $B$ decays [3, 4]. We show that this is not the case: the new phases have an effect on $B - \bar{B}$ mixing and consequently on CP asymmetries. All existing bounds from CP violating processes — $\epsilon$, $\epsilon'$ and $D_n$ (the electric dipole moment of the neutron) — do not exclude strong effects in CP asymmetries in $B$ decays. However, the CP violating couplings contribute also to the (CP conserving) radiative decay $B \to X_s \gamma$. We find that the recent upper bound on this decay [5] does constrain the shift in CP asymmetries to be small, at most 0.02.

We also investigate the modifications in the predictions for the CP asymmetries that result from the different constraints on CKM parameters. These arise because there are contributions from scalar mediated diagrams to $B - \bar{B}$ mixing and to $\epsilon$. We find that a combined measurement of the CP asymmetries in $B$ decays into $\psi K_S$ and $\pi\pi$ may probe new contributions to $B - \bar{B}$ mixing even if there are no new phases involved.

To explain how the new effects arise and to study how they are constrained by experimental data, it is simplest to work in the framework of three doublet models, and to assume that one of the two charged scalars is much heavier than the other. However, our results hold for any multi-scalar model (with at least three doublets) where NFC is implemented by requiring that only one doublet couples to each quark sector and that it is a different one in each sector (model II [6]). The couplings of the two physical charged scalars $H_{i}^{\pm}$ ($i = 1, 2$) to quarks are given by
(see e.g. ref. [7])

$$L_H = \frac{G_{F}^{1/2}}{2^{1/4}} \sum_{i=1,2} [H^+_i U(Y_i M_q V(1 - \gamma_5) + X_i V M_d(1 + \gamma_5))D + \text{h.c.}].$$  \hspace{1cm} (1)

Here $V$ is the CKM matrix, while $X$ and $Y$ are complex numbers that depend on mixing parameters in the charged scalar sector. The contributions to $B - \bar{B}$ mixing from box diagrams with intermediate $W$-bosons and $H_1$-scalars (we assume that the heavier scalar $H_2$ contributes negligibly) are of the form [8]

$$M_{12}^B = \frac{G_{F}^{2}}{64\pi^2} (V^*_t V_b)^2 [I_{WW} + I_{HH} + 2I_{WH}].$$  \hspace{1cm} (2)

The Standard Model contribution is $I_{WW}$ while box diagrams with two $H_1$-propagators or one $H_1$- and one $W$-propagator give $I_{HH}$ and $I_{WH}$, respectively. The potentially large and interesting contributions are contained in $I_{HH}$:

$$I_{WW} = m_t^2 I_0(x_t) V_{LL},$$

$$I_{HH} \approx m_t^2 y_t [ |Y|^4 I_1(y_t) V_{LL} + (XY^*)^2 y_b I_2(y_b) S_{LL}],$$

where $x_q \equiv m_q^2/m_W^2$, $y_q \equiv m_q^2/m_H^2$, and $m_H$ is the mass of the lightest charged scalar. The $I_i$ functions are given by

$$I_0(x) = 1 + \frac{9}{1 - x} - \frac{6}{(1 - x)^2} - \frac{6x^2 \ln x}{(1 - x)^3},$$

$$I_1(y) = \frac{1 + y}{(1 - y)^2} + \frac{2y \ln y}{(1 - y)^3},$$

$$I_2(y) = \frac{8}{(1 - y)^2} + \frac{4(1 + y) \ln y}{(1 - y)^3}. \hspace{1cm} (4)$$

We calculate the various matrix elements in the vacuum insertion approximation [9]:

$$V_{LL} \equiv \langle \bar{B} | (\bar{d} \gamma^\mu (1 - \gamma_5) b)^2 | B \rangle = \frac{4}{3} f_B^2 m_B,$$

$$S_{LL} \equiv \langle \bar{B} | (\bar{d} (1 - \gamma_5) b)^2 | B \rangle \approx -\frac{5}{6} f_B^2 m_B. \hspace{1cm} (5)$$

In eq. (3) we gave only the terms most relevant to our argument. However, in our calculations we use the full expressions.
Within the Standard Model, CP asymmetries in $B$ and $B_s$ decays depend on the angles $\alpha$, $\beta$ and $\gamma$ of the unitarity triangle only [10]. In the multi-scalar framework, they are affected by the new phase in charged scalar exchange through the dependence of $B-\bar{B}$ mixing on $\text{Im}(X^*Y)$. (The effects on decay amplitudes are negligible.) The argument that there is no such effect results from approximating $y_b = 0$ in eq. (3). This is not always justified. What $I_{HH}$ (and $I_{WH}$) really depend on is the Yukawa coupling to the bottom quark, $m_b/\langle \phi_d \rangle$. It is a rather attractive option in multi scalar models to have $m_b/m_t \sim \langle \phi_d \rangle/\langle \phi_u \rangle$. In such a case, the Yukawa coupling of the bottom quark is as large as that of the top quark, and the term proportional to $y_b$ is as important as the one that is not. It is in this region of parameter space that CP violation from charged scalar exchange may have large effects on CP asymmetries in $B$ decays. Note that in the neutral $K$ system, the corresponding terms are proportional to the Yukawa coupling of the strange quark and therefore the CP violating effect is negligible.

From eq. (3) we can learn in what region of the $(X,Y)$ parameter space the new effects may play a significant role. If $|X| \lesssim |Y|$, then the term proportional to $I_1$ dominates $I_{HH}$. This terms contributes with the same phase as $I_{WW}$ and thus the Standard Model predictions for the CP asymmetries are not modified. A necessary condition for large effects is then

$$\frac{|X|}{|Y|} \gtrsim \frac{m_t}{m_b}. \quad (6)$$

However, if (6) is fulfilled with $|XY| \lesssim 1$, then $I_{WW}$ dominates over $I_{HH}$ and the Standard Model predictions are still unchanged. Thus we also need

$$|XY| \gg 1. \quad (7)$$

To present the way in which the Standard Model predictions are modified, we define a phase

$$\theta_H = \arg(I_{WW} + I_{HH} + 2I_{HW}). \quad (8)$$

Then, if the Standard Model predicts that a certain asymmetry equals $\sin(\theta_{SM})$, 

in a multi scalar model the prediction is modified to $\sin(\theta_{SM} + \theta_H)$, e.g.

$$a_{CP}(B \to \psi K_S) = -\sin(-2\beta + \theta_H),$$
$$a_{CP}(B \to \pi^+\pi^-) = \sin(2\alpha + \theta_H).$$

(The overall minus sign in $a_{CP}(B \to \psi K_S)$ arises because the final $\psi K_S$ state is CP-odd.) If the phase $\theta_H \ll \theta_{SM}$, the shift $\Delta a_{CP}$ from the Standard Model prediction is

$$\Delta a_{CP} \lesssim \sin(\theta_H).$$

A few points are in order:

a. The CKM phase is factored out in (2), so that $I_{WW}$ is real. Then, when $I_{HH}$ and $I_{HW}$ are real, the Standard Model result is reproduced, as it should.

b. As the modification is in the phase of the mixing amplitude and not in the decay amplitude, the shift of the measured angle is universal, namely independent of the decay mode.

c. Eqs. (6) and (7) imply that the effect is never very large in $B_s$ decays: When $|X|$ is large, an additional term in $I_{HH}$,

$$I_{HH}^q = m_t^2 y_6 y_q |X|^4 I_1(y_t) V_{LL},$$

(q = d or s for $B_d$ or $B_s$, respectively) becomes important in $B_s$ decays, and it contributes with the same phase as the Standard Model diagram. For very large $|X|$, the corrections in $B_s$ decays are small. Consequently, the angles “$\beta$”, “$\alpha$” and $\gamma$ deduced naively from $B \to \psi K_S$, $B \to \pi\pi$ and $B_s \to \rho K_S$, respectively, will sum up to approximately $\pi$, even though the first two do not correspond to angles of the unitary triangle anymore. This is an example of a general result that holds when the phase of $B_s - \bar{B}_s$ mixing is the same as in the Standard Model [11].

To see if indeed large effects in $B$ decays are possible, we now study the experimental constraints on $X$ and $Y$. For the charged scalar mass, we use $m_H \gtrsim m_Z/2$
Both $|X|$ and $|Y|$ are constrained by the requirement of perturbativity [13]:

$$ |X| \lesssim 120, \quad |Y| \lesssim 6 \quad \Rightarrow \quad \text{Im}(XY^*) \leq |XY| \lesssim 720. \quad (12) $$

The value of $|Y|$ is constrained by $B - \bar{B}$ mixing [13, 14],

$$ |Y| \lesssim \begin{cases} 2 & m_H \sim m_Z/2, \\ 3 & m_H \sim 2m_Z. \end{cases} \quad (13) $$

The value of $|X|$ is constrained by $B \to X\tau\nu$ [15],

$$ |X| \lesssim \frac{m_H}{0.54 \text{ GeV}}, \quad (14) $$

but only if it is one and the same doublet scalar that couples to the charged leptons and to down quarks. We thus consider below also values of $|X|$ that do not fulfill (14). A direct bound on $\text{Im}(XY^*)$ comes from CP violating processes. The strongest among these comes from $D_n$ [9, 16],

$$ \text{Im}(XY^*) \lesssim \begin{cases} 20 & m_H \sim m_Z/2, \\ 100 & m_H \sim 2m_Z. \end{cases} \quad (15) $$

(This bound arises from quark electric dipole moment operators. Bounds from the three gluon operator may be stronger, but suffer from larger hadronic uncertainties.)

The strongest constraint on $\text{Im}(XY^*)$, however, comes – somewhat surprisingly* – from a CP conserving process, the decay $b \to s\gamma$ [5]:

$$ \text{BR}(B \to Xs\gamma) \leq 5.4 \times 10^{-4}. \quad (16) $$

Within multi-scalar models with NFC, this branching ratio is given by [9]

$$ \text{BR}(B \to Xs\gamma) = C|\eta_2 + G_W(x_t) + (|Y|^2/3)G_W(y_t) + (XY^*)G_H(y_t)|^2, \quad (17) $$

* This situation was actually foreseen in ref. [9].
where
\[ C \equiv \frac{3\alpha \eta_1^2 \, \text{BR}(B \to X_c \ell \nu)}{2\pi F_{ps}(m_{\ell}^2/m_b^2)} \approx 3 \times 10^{-4}. \] (18)

\( F_{ps} \sim 0.5 \) is a phase space factor, \( \eta_1 \sim 0.66 \) and \( \eta_2 \sim 0.57 \) are QCD corrections factors [17]. The expressions for the \( G \)-functions can be found in ref. [9]. In the two Higgs-doublet model [17,18], \( XY^* = 1 \) and the bound (16) gives a lower bound on \( m_H \) almost independently of \( Y \) [19]. In the three Higgs-doublet model, \( m_H \sim m_Z/2 \) is still allowed.

The upper bound on \( \text{Im}(XY^*) \) corresponds to a case where the real part of the new diagrams cancels the Standard Model contributions and the upper bound (16) is saturated by the imaginary part of these diagrams:
\[ \text{Im}(XY^*) \lesssim \sqrt{\frac{5.4 \times 10^{-4}}{C}} \frac{1}{G_H(y_t)}. \] (19)

The results are presented in Fig. 1. For \( m_t \sim 140 \text{ GeV} \) we get
\[ \text{Im}(XY^*) \lesssim \begin{cases} 2 & \text{if } m_H \sim m_Z/2, \\ 4 & \text{if } m_H \sim 2m_Z. \end{cases} \] (20)

For heavier (lighter) top mass, the bounds are stronger (weaker).

The upper bound on \( \text{Im}(XY^*) \) implies that charged scalar exchange can make only a negligible contribution to \( \epsilon \) and \( \epsilon' \) and cannot be the only source of CP violation. (For recent attempts to allow this possibility see ref. [20]. The combination of the upper bound on \( D_n \) and the lower bound on \( m_H \) made it very unlikely [9,16,21,\ldots].) On the other hand, the contribution to \( D_n \) can still be close to the experimental upper bound.

The bound (20) by itself does not imply that the effects of charged scalar exchange on CP asymmetries are small. However, the upper bound on \( b \to s \gamma \) also implies that the conditions (6) and (7) cannot be simultaneously fulfilled. Take, for example, the case that \( |Y| \ll 1 \). Then the term proportional to \( |Y|^2 \) in (17) is
negligible. An upper bound on $|XY^*|$ is derived when $\arg(XY^*) \sim \pi$: $|XY^*| \lesssim 2$, in contradiction to (7). A survey of the $(X,Y)$ values consistent with (16) leads to the results shown in Fig. 2. We find

$$\theta_H \leq 1.2^\circ \implies \Delta a_{CP} \lesssim 0.02.$$ \hspace{1cm} (21)

The effect is smaller for heavier charged scalar or for $Y$-values different from those presented in Fig. 2. An effect of the magnitude (21) is still larger than the hadronic
uncertainties in $B \to \psi K_S$. However, it is too small to be unambiguously observed in $B$-factories, where the accuracy in $a_{CP}(B \to \psi K_S)$ is expected to be of $O(0.05)$ [10].

Modifications of the Standard Model predictions for CP asymmetries in $B$ decays may also arise from the different constraints on CKM parameters. This holds even for two scalar doublet (type I and type II) models where indeed there are no new phases. The most significant effect is that the lower bounds on $|V_{tb}V_{td}^*|$ from $B - \bar{B}$ mixing and from $\epsilon$ are relaxed, because charged scalar exchange may contribute significantly [14]. This opens up a region in the plane of $\sin 2\alpha - \sin 2\beta$ forbidden in the Standard Model, as shown in Fig. 3. We used here the same input parameters as in ref. [22].

We find an interesting result, which goes beyond the specific extension of the Standard Model investigated here: If experiment finds a relatively low value of $\sin 2\beta$ (below 0.5) and a negative value of $\sin 2\alpha$, it may be an indication that there are significant contributions from new physics to $B - \bar{B}$ mixing, even if these contributions carry no new phases! We should also emphasize that a nice feature of the Standard Model – that at low $\sin 2\beta$ values there is a strong correlation between $\sin 2\beta$ and $\sin 2\alpha$ and, in particular, $|\sin 2\alpha|$ cannot be small – is maintained even in the presence of the new effects discussed here.

\* We improved upon the analysis of ref. [22] by working in the three dimensional parameter space of $(\rho, \eta, |V_{cb}|)$ rather than integrating over the allowed range for $|V_{cb}|$. We find that the Standard Model lower bound on $\sin 2\beta$ is even stronger than the one given in [22]: $\sin 2\beta \geq 0.23.$
In terms of the CP-violating measure $J$ [23], the multi scalar-doublet model allows lower values:

$$J \geq \begin{cases} 
1.4 \times 10^{-5} & \text{Standard Model,} \\
5.5 \times 10^{-6} & \text{Multi-Scalar,}
\end{cases} \quad (22)$$

while the upper bound, $J \leq 6.3 \times 10^{-5}$, remains unchanged.

To summarize, in models of three or more scalar doublets and natural flavor conservation, new phases in charged scalars exchange may affect CP asymmetries in $B$ decays. The recent upper bound on the decay $B \rightarrow X_s \gamma$ constrains both CP violating and CP conserving charged scalar couplings, implying that the deviations from the Standard Model predictions cannot exceed a few percent.

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FIGURE CAPTIONS

1) The upper bound on $\text{Im}(XY^*)$ as a function of the lightest charged scalar mass $m_H$. The three curves correspond to $m_t = 90$ (solid), 140 (dashed) and 180 (dotted) GeV.

2) The upper bound on $\theta_H$, the shift in the phase measured in CP asymmetries in $B$ decays (see (8)), as a function of $\text{arg}(XY^*)$ for $m_H \sim m_Z/2$ and $Y = 0.1$. The three curves correspond to $m_t = 90$ (solid), 140 (dashed) and 180 (dotted) GeV.

3) The allowed region in the $\sin 2\alpha - \sin 2\beta$ plane in the Standard Model (solid) and in multi-scalar models (dot-dashed).