Hydrodynamization in systems with detailed transverse profiles

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Abstract
The observation of fluid-like behavior in nucleus-nucleus (AA), proton-nucleus (pA) and high-multiplicity proton-proton (pp) collisions motivates systematic studies of how different measurements approach their fluid-dynamic limit. We have developed numerical methods to solve the ultra-relativistic Boltzmann equation for systems of arbitrary size and transverse geometry. Here, we apply these techniques for the first time to the study of azimuthal flow coefficients $v_n$ including non-linear mode-mode coupling and to an initial condition with realistic event-by-event fluctuations. We show how both linear and non-linear response coefficients extracted from $v_n$ develop as a function of opacity from free streaming to perfect fluidity. We note in particular that away from the fluid-dynamic limit, the signal strength of linear and non-linear response coefficients does not reduce uniformly, but that their hierarchy and relative size shows characteristic differences.

Introduction. Hydrodynamization denotes the transition to hydrodynamics of systems that carry fluid- and non-fluid-dynamic degrees of freedom and that therefore do not need to behave fluid dynamically at all times and under all conditions. The observation of strong signs of collectivity in ultra-relativistic nucleus-nucleus (AA), proton-nucleus (pA) and proton-proton (pp) collisions \cite{1, 2, 3} has motivated in recent years many studies of hydrodynamization in strongly- and weakly-coupled models of quark-gluon plasma \cite{4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31}. Their ultimate aim is to provide a rigorous underpinning of the fluid-dynamic interpretation of collective flow in AA, pA and pp collisions, and to delineate the limitations of any such interpretation.

Most studies of hydrodynamization profit from simplified set-ups that do not reflect all phenomenological complications but that exhibit general features in great clarity. In particular, most studies of hydrodynamization to date assume exact Bjorken boost invariance, employ conformally symmetric collective dynamics and focus on dimensionally reduced 1 + 1D systems \cite{4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31} (for studies extending this framework, see \cite{13, 18, 20, 21, 22, 23, 24, 25, 26, 32}). Within this setting, one has reached in recent years a thorough understanding of the off-equilibrium evolution of simple observables in various models. For instance, the asymmetry $p_T/p_L$ between longitudinal and transverse pressure and the higher longitudinal momentum moments of the stress-energy tensor are known to approach rapidly their universal attractor solution in kinetic theory \cite{33, 26, 16}. The mathematical structures behind this behaviour continue to be studied in the context of resurgence \cite{9, 10, 27, 28}.

The lessons learnt from these 1 + 1D systems are expected to carry over to the phenomenological reality in 3 + 1D. For instance, the early-time dynamics of $p_T/p_L$ in boost-invariant 3 + 1D systems is known to be governed locally in the transverse plane by an effective 1 + 1D evolution, and the 1 + 1D universal attractor for $p_T/p_L$ is therefore of relevance for the 3 + 1D dynamics. However, very few observables of phenomenological relevance can be studied in 1 + 1D systems, and some important questions have therefore received little attention so far in the debate of hydrodynamization. One of them is whether all bulk observables hydrodynamize under conditions comparable to those under which $p_T/p_L$ hydrodynamizes, or whether some classes of observables require systems of longer lifetime, larger spatial extent and/or higher density to approach the values they attain under conditions of almost perfect fluidity. Of particular interest in this context are the conditions for hydrodynamization of the azimuthal momentum anisotropies $v_n$ of soft multi-particle production, as these are amongst the most abundant and most precisely measured signatures of collective behavior in AA, pA and pp collisions. Here, we analyze their hydrodynamization in a boost invariant...
conformally symmetric 3 + 1D kinetic transport theory, whose 1 + 1D variants have been used repeatedly in studies of hydrodynamization.

Up until this point, only the linear response coefficients have been studied in full kinetic theory because of the technical challenges related to solving Boltzmann equations for a distribution functions in complex geometries [37, 35, 26], though some results exist for perturbative solutions around free-streaming [38, 37]. We have developed numerical techniques to solve such systems and present here the first non-linear response coefficients, and we present the first solution to the Boltzmann equation for an initial condition with realistic event-by-event fluctuations.

**Kinetic Theory.** We consider massless, boost-invariant kinetic theory in the isotropization-time approximation, and we restrict the discussion to the first momentum moments \( F(\vec{x}_\perp, \Omega, \tau) = \int \frac{d^2p}{(2\pi)^2} p f \) of the distribution function \( f \).

Here, \( p \) is the modulus of the three-momentum, the velocity is \( v_\mu = p_\mu / p \) with \( p_\mu p^\mu = 0 \) and \( v^0 = 1 \), and \( \Omega \) denotes the angular phase space of \( v_\mu \). \( F \) defines the energy momentum tensor \( T^\mu_\nu = \int d\Omega v^\mu v^\nu F \), as well as arbitrary higher \( v_\mu \)-moments that lie beyond hydrodynamics. It satisfies the equations of motion [26]

\[
\partial_\tau F + \vec{v}_\perp \cdot \partial_{\vec{x}_\perp} F - \frac{v_\gamma}{\tau} (1 - v_\gamma^2) \partial_{\eta} F + \frac{4v_\gamma^2}{\tau} F = -C[F] = -\gamma \epsilon^{1/4}(x) [v_\gamma d\gamma^4](F - F_{\text{iso}}),
\]

where \( \epsilon \) is the local energy density. Fluid-like and particle-like excitations are known to coexist in this kinetic transport and their properties can be calculated analytically. In particular, the coupling \( \gamma \) is related to the specific shear viscosity \( \frac{\eta}{s} = \frac{1}{3\gamma \tau_R} \), and \( F \) relaxes locally on a time scale \( \tau_R = \frac{\gamma}{\gamma \tau} \) to the isotropic distribution \( F_{\text{iso}}(\tau, \vec{x}_\perp; \Omega) = \left( \frac{sT}{\mu_0 v_\gamma} \right)^4 \) whose functional form is fixed by symmetries and by the Landau matching condition, \( \partial_\tau T_\mu^\nu = -\epsilon \delta_\mu^\nu \).

As the dynamics [1] is scaleless, dimensionful characteristics of the collision system can enter only via the initial conditions, and they can affect results only in dimensionless combinations. For a system of transverse r.m.s. size \( R \) and energy density \( \epsilon_0 \) at initial time \( \tau_0 \), it follows that the opacity \( \hat{\gamma} = \gamma R^{3/4}(\epsilon_0 \tau_0)^{1/4} \) is the unique model parameter. Eq. [1] interpolates between free-streaming in the limit of vanishing opacity \( \hat{\gamma} \to 0 \) and ideal fluid dynamics in the limit \( \hat{\gamma} \to \infty \).

We initialize [1] with two different classes of initial conditions. We first study linear and non-linear response coefficients based on the simple Gaussian ansatz

\[
F(\tau, \vec{x}_\perp; \phi, v_\gamma) = 2\epsilon_0 \delta(v_\gamma) \exp \left[ -\frac{\vec{x}_\perp^2}{2R^2} \right] \left[ 1 + \sum_n \delta_n \left( \frac{f}{R} \right)^n \cos(n \theta - n \phi_\mu) \exp \left[ -\frac{\vec{\theta}_\perp^2}{2R^2} \right] \right].
\]

The exponential \( -\frac{\vec{x}_\perp^2}{2R^2} \) multiplying the cos-term ensures that the distribution stays positive everywhere for sufficiently small \( \delta_n \)'s. The initial spatial azimuthal asymmetries are proportional to the real factors \( R_n \), and they are oriented along the azimuthal directions \( \phi_\mu \). Alternatively, we initialize [1] also with the “realistic” initial conditions arising from the T9KENTo model by replacing the radial profile with that arising from the initial state model.

For both classes of initial conditions, we quantify azimuthal anisotropies in terms of the complex-valued spatial eccentricities for \( n > 1 \),

\[
e_n \equiv -\int d\theta d\varphi r^\mu r^\nu \exp \left[ in\theta \right] F(\tau, \vec{x}_\perp; \Omega) \int d\theta d\varphi r^\mu r^\nu F(\tau, \vec{x}_\perp; \Omega) \equiv |e_n| \epsilon \exp \left[ in\varphi \right].
\]

Evolving with eq. [1] the initial conditions [2], we obtain the evolution of the energy-momentum tensor \( T^\mu_\nu \) and the transverse energy flow \( dE_\perp \) at late times

\[
\frac{dE_\perp}{dt} \equiv \int dp_\perp^2 \frac{dN}{dp_\perp^2 dp_\parallel} \left[ 1 + 2 \sum_{n=1}^\infty v_n \cos(n \phi - n \phi_\mu) \right].
\]

This determines the energy flow coefficients \( V_n = v_\parallel \epsilon \exp \left[ in\varphi \right], \) where \( \phi_\parallel \) is the azimuthal orientation of the energy flow. In contrast to flow coefficients extracted from particle distributions \( dN \), our study focuses on energy-flow coefficients which are not affected by hadronization since hadronization conserves energy and momentum.

The viscous fluid-dynamic limit of eq. [1] is restricted to the evolution of seven fluid-dynamic fields which may be identified with those seven components of \( T^\mu_\nu(\tau, \vec{x}_\perp) = \int d\Omega v^\mu v^\nu F \) that do not vanish under boost-invariance. We are interested in the apparently simple kinetic theory [1] for \( F(\tau, \vec{x}_\perp; \phi, v_\gamma) \) away from the fluid dynamics limit since it
Ellicptic flow: \( v_2 / 2 \), Quartic flow: \( v_4 / 2 \)

Our first main result is to observe that the non-linearities are more important for large opacity, as the lines in Fig. 1 obtained for a momentum angle \( \phi \). According to eq. (5), the response coefficients are of particular interest, since they help to disentangle this ambiguity.

As a first example, we consider initial conditions in which a single mode \( \delta_2 \) is excited (\( \delta_n = 0 \) for \( n \neq 2 \)). In the course of the evolution, the non-linear mode-mode coupling of this initial second harmonic with itself excites 4th, 6th, 8th, ... harmonics, but also the 0th harmonic. In turn, these higher harmonics affect the non-linear response coefficients numerically by seeding the initial conditions in a way.

In close analogy, we determine other linear and non-linear response coefficients numerically by seeding the initial conditions.
conditions with suitable choices of eccentricities. To determine the linear response coefficients $w_{n,n}(\hat{\gamma})$, $n \leq 5$, shown in Fig. 2, we run simulations seeded with a single $n$-th harmonic for different values of $\epsilon_n$, and we extrapolate to limit $n \rightarrow 0$ ($\epsilon_n^2$), see Fig. 2. For the non-linear response coefficients $w_{m,n,m_2}$ (where $n = m_1 + m_2$ or $|m_1 - m_2|$), displayed in Fig. 3, we pick initial data with non-vanishing $\epsilon_{m_1}$, $\epsilon_{m_2}$, and all other eccentricities vanishing. Extrapolating from simulations for different initial values of $\epsilon_{m_1}$, $\epsilon_{m_2}$, we determine $w_{m,n,m_2} = \lim_{n \rightarrow 0, \epsilon_{m_1} \rightarrow 0, \epsilon_{m_2} \rightarrow 0}$.

We ask next how the linear and non-linear response coefficients in Figs. 2 and 3 hydrodynamize, i.e., how they approach their fluid-dynamic limit with increasing opacity $\hat{\gamma}$. To this end, we relate the opacity that characterizes kinetic transport to quantities accessible in viscous fluid dynamics. The definition $\hat{\gamma} = \gamma R^{3/2} (\epsilon_0 \tau_0)^{1/4}$ assumes that the early-time evolution is given by free-streaming which is not the case for viscous fluid dynamics. We therefore have to work with an equivalent definition that can be expressed in terms of quantities measured at a time at which the flow builds up and fluid dynamics may be operational. To this end, we write

$$\hat{\gamma} = \gamma R \left( \frac{\epsilon_R}{f_{0-R}(\hat{\gamma})} \right)^{1/4},$$

(6)

where, for the Gaussian background in the initial condition $f_2$, $\epsilon_0$ and $\epsilon_R$ denote central ($r = 0$) energy densities at times $\tau_0$ and $R$, respectively. The function $f_{0-R}(\hat{\gamma}) = \frac{\epsilon_R}{\epsilon_0}$ is defined as the ratio of the energy per unit time at time $\tau = R$ to the energy that the system would have if it were free-streaming $f_0$. We calculate $f_{0-R}(\hat{\gamma})$ from kinetic theory for $\hat{\gamma} \leq 10$, and we match for larger $\hat{\gamma}$ to the known asymptotic large-$\hat{\gamma}$ behavior $f_{0-R} \sim \hat{\gamma}^{-4/9}$.

With $f_{0-R}(\hat{\gamma})$ known, we relate viscous fluid-dynamic calculations to $\hat{\gamma}$ by specifying $\epsilon_R$ and $\eta(s)$ from fluid dynamics and solving eq. (6) for $\hat{\gamma}$. In particular, we use the kinetic relation between the interaction strength $\gamma$ and the shear viscosity $\nu \eta$ to determine $\epsilon_R$ and $\hat{\gamma}$, and to extract from the transverse energy flow at late times the energy-flow coefficients $\nu_e$. In general, these results depend on $\tau_0$. That the $\tau_0 \rightarrow 0$-limit of $\nu_e(\hat{\gamma})$ exists is a direct consequence of the fact that viscous fluid dynamics, like kinetic theory, has a universal attractor solution at arbitrarily early times $\tau_0$. While the attractor of kinetic theory keeps $\epsilon R$ fixed leading to the scaling of $\hat{\gamma} = \gamma R^{3/2} (\epsilon_0 \tau_0)^{1/4}$, the attractor of the viscous (Israel-Stewart) hydrodynamics considered here keeps $\epsilon R \hat{\gamma} (\nu^2)$ constant. Therefore taking the $\tau_0 \rightarrow 0$ limit while keeping $\gamma$ as defined in eq. (6) fixed corresponds to scaling initial energy densities by $\epsilon R \tau_{0} \to \hat{\gamma} (\nu^2)$.

This non-standard procedure differs from the common phenomenological practise, it allows for a particularly clean comparison between kinetic theory and fluid dynamics by eliminating the unphysical model parameter $\tau_0$. The difference between the kinetic theory and fluid-dynamic results obtained this way do not inform us on the validity or the breakdown of the current phenomenological practise. Instead it emphasizes the importance of the early-time attractor.

Figure 2: Left panel: the linear response coefficients $w_{n,n} = \lim_{n \rightarrow 0} \frac{\epsilon_n \to R}{\epsilon_n \to R}$ calculated for the kinetic theory $f_{2,2}$ as a function of opacity $\hat{\gamma}$ (thick lines). Arrows at the right indicate values in the ideal-fluid limit corresponding to $\hat{\gamma} \to \infty$. Right panel: Same as in left panel but in semi-logarithmic presentation and overlaid with results from viscous fluid dynamics (thin dashed lines).
which differs from kinetic theory and the fluid dynamics) for the physical observables measured in experiments and it informs us about the extent to which the entire signal $v_n$ is or is not build up by the degrees of freedom encoded in viscous fluid dynamics. For linear response coefficients, this comparison is shown in the right panel of Fig. 2.

Technically, we evolve the viscous fluid-dynamic equations as described in Ref. [39, 40] by splitting all fluid dynamic fields into an azimuthally symmetric background and an azimuthally anisotropic perturbation and solving for them to first order in initial eccentricites. In the same way, we set up a control calculation for the much simpler ideal fluid dynamics to second order in eccentricities (arrows in Fig. 3). Within the range $\hat{\gamma} \to \infty$, several cubic response coefficients in the limit $\hat{\gamma} \to \infty$ are or is not build up by the degrees of freedom encoded in viscous fluid dynamics. For linear response coefficients, this comparison is shown in the right panel of Fig. 2. Results for $w_{2,2}$ and $w_{3,3}$ differ somewhat from those reported in [26] since the initial conditions are different.

As expected from general reasoning, the viscous fluid-dynamic results for $\frac{v_n(\hat{\gamma})}{\hat{\gamma}}$ in the limit $\tau_0 \to 0$ asymptote for $\hat{\gamma} \to \infty$ to the ideal fluid-dynamic results in the same $\tau_0 \to 0$ limit, see Fig. 2. Remarkably, the hierarchy between the elliptic and triangular linear response coefficient gets inverted as a function of $\hat{\gamma}$: kinetic theory at low $\hat{\gamma}$ shows $w_{2,2} > w_{3,3}$ while ideal fluid dynamics shows $w_{2,2} < w_{3,3}$. Viscous fluid dynamics accounts for this inversion qualitatively: for very small specific shear viscosity $\frac{\gamma}{\hat{\gamma}}$, i.e., very large opacity $\hat{\gamma}$, it is consistent with ideal fluid dynamics, but the hierarchy changes as a function of opacity, see right panel of Fig. 2. Also the results from kinetic theory hint at such an inversion, as the slope of $w_{3,3}(\hat{\gamma} = 10)$ is larger than the slope of $w_{2,2}(\hat{\gamma} = 10)$.

As seen from Fig. 2, viscous fluid dynamics reproduces the main qualitative trends of kinetic theory (hierarchy of response coefficients) at $\hat{\gamma} \sim O(10)$, but significant quantitative differences persist. On general grounds, we expect that kinetic theory matches quantitatively to viscous fluid dynamics at sufficiently large $\hat{\gamma}$ when the fluid dynamic gradient expansion becomes quantitatively reliable. All data shown here are consistent with this expectation. It would clearly be interesting to extend the numerical calculations in kinetic theory to larger $\hat{\gamma}$ and to determine the $\hat{\gamma}$-scale at which a seamless matching to viscous fluid dynamics is found. However, with increasing $\hat{\gamma}$, the numerical evaluation becomes more expensive, and within the scope of the present letter, we were not able to push to higher $\hat{\gamma}$.

We have extended this analysis to a set of quadratic and cubic response coefficients, see Fig. 3. To make some statements about their hydrodynamization we determine the quadratic response coefficients in the limit $\hat{\gamma} \to \infty$ by solving ideal fluid dynamics to second order in eccentricites (arrows in Fig. 3). Within the range $\hat{\gamma} < 10$, several quadratic response coefficients are seen to cross, and at $\hat{\gamma} = 10$, the hierarchy of the numerically large response coefficients ($w_{5,23} > w_{4,22} > w_{6,24}$) found in kinetic theory is consistent with that of ideal fluid dynamics. In the range $\hat{\gamma} > 10$, the numerically smaller response coefficients $w_{3,25}$ and $w_{2,53}$ need to cross. These observations give further support to the conclusions reached from Fig. 2.

In a remarkable note [38], it was observed already that in the dilute limit of kinetic theory far from equilibrium, linear and quadratic response coefficients grow linearly in the average number of rescatterings $\bar{N}_{\text{resc}}$ while cubic ones have a quadratic dependence. In Ref. [38], this scaling was established for elastic two-to-two collision kernels. The
Figure 4: Upper panels: Energy density in the transverse plane initialized with the TRENTo model at $\tau_0/R = 0.05$ and evolved for $\hat{\gamma} = 2$ with the kinetic theory up to times $\tau/R = 0.05, 0.5, 2.0$ and $4.0$, respectively. Lower panel: The value of the elliptic and triangular flow coefficients evaluated for the same TRENTo event and for different opacities $\hat{\gamma}$. Results for the full event including harmonics $n \leq 7$ are compared to simplified events in which only specific harmonics are kept. (For $\hat{\gamma} = 1$, the circles for $n \leq 3$ and full event overlap in the left plot.)

line of arguments of Ref. [38] does not apply to the collision kernel [1]. However, a perturbative expansion of [1] in $\hat{\gamma}$ can be viewed as an expansion in the average number of scattering centers [37], and it is therefore natural to test whether our results show this same scaling, too. For linear and quadratic coefficients, we know already from the perturbative analysis in [37] that they do. For cubic response coefficients, however, we observe small violations of the scaling. In the neighborhood of $\hat{\gamma} = 0$, the cubic coefficients in the right panel of Fig. 3 show a small linear component, though the quadratic one can be dominant.

**Evolving initial conditions with realistic event-by-event fluctuations in kinetic theory.** We now apply our newly developed numerical machinery to the first exploratory study of a realistic initial condition that would be one single event in an event sample of an event-by-event analysis. The initial condition is a typical TRENTo event [41] in the $5 - 10\%$ centrality class smoothened such that only initial $\epsilon_n$'s for $n \leq 7$ are kept. We have checked that the finesse of our discretization allows for the stable propagation of such events. A typical time evolution is shown in the upper panel of Fig. 4 with $\hat{\gamma} = 2$. It illustrates that the Boltzmann equation can be solved non-perturbatively for distribution functions representing realistic initial conditions.

The radial profile of the TRENTo event studied here differs from [2] and this can affect the value of linear and non-linear response coefficients. To quantify the difference, we compare the $w_{2,2}^{(TRENTo)}$ extracted for these two profiles and find the following numbers $w_{2,2}^{(TRENTo)} = \frac{1}{2} \epsilon_2^{Trento,n=2} = 0.156, 0.239, 0.288$ and $0.319$, compared to $w_{2,2}(\hat{\gamma}) = 0.166, 0.266, 0.327$ and $0.372$ taken from Fig. 2 for $\hat{\gamma} = 1, 2, 3, 4$. Technically, $w_{2,2}^{(TRENTo)}$ is not a linear response coefficient, since it was extracted at finite eccentricity, but Fig. 1 informs us that the numerical contribution arising from finite...
eccentricity is negligible for small opacity. We checked this for the T$_{R}$ENTo profile as well (data not shown). We observe that the dependence on the radial profile in the linear response coefficients ranges from 5% to 15% in this $\gamma$-range. The analogous study of $\nu_{3,3}$ shows a 2% to 10% difference in the same $\gamma$-range. Therefore, the open circles in the lower panel of Fig. 4 are accounted for within 2% - 15% accuracy by the linear response coefficients calculated from the simplified profile (2). The remaining difference between open circles and full results in Fig. 4 result from mode-mode couplings of different harmonics. We see that while the linear response covers the bulk of the results, non-linearities have to be included to go reliably beyond 20%-30% accuracy. The non-linearities generated by the lowest harmonics $n \leq 3$ account for half of all the non-linearities.

This paper is motivated by the wealth of studies of hydrodynamization and thermalization in simplified settings. We have developed the necessary machinery for overcoming many of these simplifications and to facilitate studies of hydrodynamization in complex realistic geometries, and to thus push the study of hydrodynamization from in vitro to in vivo. The ability to solve the Boltzmann equation for ultra-relativistic systems with realistic initial geometries and including all non-linear mode-mode couplings provides insight into how the characteristic features of fluid dynamics emerge gradually with increasing interaction strength. Away from the fluid dynamic limit, signals of collectivity are not simply reduced uniformly in size, but their relative strength varies characteristically with opacity, the hierarchy of the dominant linear response coefficients is inverted and so is the hierarchy of several non-linear ones. This may provide novel possibilities for characterizing to what extent systems of different size do or do not hydrodynamize. In event samples, and to study Boltzmann equations with other phenomenologically relevant complications.

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