Analysis of the $pp \to pp$, $\pi d \to \pi d$, and $\pi d \to pp$ scattering data

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Abstract

A combined analysis of the main reactions of the two-baryon system ($pp \to pp$, $\pi d \to \pi d$, and $\pi d \to pp$) over the $\sqrt{s}$ interval from pion threshold to 2.4 GeV has been completed. The overall phase in $\pi d \to pp$ has now been determined. The combined analysis has resulted in an improved fit to the $\pi d$ elastic and $\pi d \to pp$ databases.

INTRODUCTION

Below 2 GeV, in the proton laboratory kinetic energy $T_p$ for the $NN$ system, the dominant channels contributing to $NN$ inelasticity are $\pi d$ and $N\Delta$. In Fig. 1 we display the total cross sections for $pp$ and $\pi d$ scattering broken into their components. At these energies, it is useful to employ a multi-channel formalism in analyzing all existing data simultaneously. In the present work, we have used the K-matrix formalism in order to unify the analysis of coupled reactions, $pp \to pp$, $\pi d \to \pi d$, and $\pi d \to pp$, which we have, in the past, considered separately. The range of $\sqrt{s}$ was chosen to include all of our previous results for the pion-induced reactions ($T_\pi = 0 - 500$ MeV, or $T_p = 288 - 1290$ MeV).

The present analysis differs from those carried out previously in a number of important respects. We did not restrict our study to partial-waves containing interesting structures. For $pp$ elastic scattering, all waves with $J \leq 7$ were used. Partial waves with $J \leq 5$ were retained for both $\pi d$ elastic scattering and $\pi d \to pp$. In addition, the K-matrix parameters were determined solely from our fits to the available databases for each separate reaction. No results of outside analyses or any model approaches were used as constraints. As a result, the amplitudes found in our K-matrix fits are as “unbiased” as those coming from the separate analyses.

FORMALISM

We have constructed a K-matrix formalism coupling the $pp$, $\pi d$ and $N\Delta$-channels, in order to analyze the reaction $\pi d \to pp$ along with elastic $pp$ and $\pi d$ scattering. The $N\Delta$-channel was added to account for all channels other than $pp$ and $\pi d$. This choice ensured that unitarity would not be violated in our global fit. As the elastic $pp$ partial-wave analysis is far superior to the $\pi d$ elastic and $\pi d \to pp$ analyses, we have carried out fits in which the $pp$ partial-waves were held fixed. As described in Ref.(7), the $pp$ amplitudes were used to fix some elements of the K-matrix, while the others were determined from a fit to the combined $\pi d$ elastic and $\pi d \to pp$ databases.

States of a given total angular momentum and parity were parameterized by a (4x4) K-matrix which coupled to an appropriate $N\Delta$-channel. Spin-mixed (2x2) $pp$-states couple to unmixed $\pi d$-states, and unmixed $pp$-states couple to spin-mixed (2x2) $\pi d$-states, so the $\pi d - pp$ system is always represented by a (3x3) matrix. The associated T-matrix elements are then expanded as polynomials in the pion energy times appropriate phase-space factors.

PARTIAL-WAVE AMPLITUDES

We have fitted the amplitudes for $pp \to pp$ and the existing databases for $\pi d \to pp$, and $\pi d \to \pi d$, using the K-matrix formalism outlined in the previous Section. The $\pi d$ elastic and $\pi d \to \pi d$ databases, used in this analysis, are described in Refs.(3) and(4), and are available from the authors. The overall $\chi^2$ for our combined analysis is actually superior
to that found in our separate analyses (Table 1). This is due to the improved parameterization scheme. We should emphasize that the amplitudes for \( pp \) elastic scattering are the same as those given in Ref.(2). (No difference was found in C500 using the \( pp \) solution WI96, which was limited to \( T_p = 1600 \text{ MeV} \), vs our recent solution SM97, which was extended to \( T_p = 2500 \text{ MeV} \).)

The results for our combined/separate analyses of \( \pi d \) elastic scattering are also qualitatively similar, up to the limit of our single-energy analyses. In Fig. 2a we compare the main partial-waves from our separate analysis (solution SM94)\(^{(3)}\) and our combined analysis (solution C500). Significant differences begin to appear above a pion laboratory kinetic energy of 300 MeV or 2.3 GeV in \( \sqrt{s} \). The upper limit to our single-energy analyses is due to a sharp cutoff in the number of data. This is apparent in Fig. 2 of Ref.(3). Much additional data above 300 MeV will be required before a stable solution to 500 MeV can be expected.

A comparison of results for \( \pi d \to pp \) reveals the most pronounced differences. One reason for this is the overall phase which was left undetermined in Ref.(4). There, we arbitrarily chose the \( ^3P_1S \) wave to be real. In the present analysis, the overall phase has been determined. Given that the overall phase is found to be about 60° in the combined analysis, we have chosen to compare the partial-wave amplitudes from the separate and combined analyses in terms of their moduli (Fig. 2b). As was the case for \( \pi d \) elastic scattering, differences are most significant above approximately 2.3 GeV in \( \sqrt{s} \). A similar lack of data exists above this energy.

In general, we see a good agreement for the dominant amplitudes found in the separate and combined analyses. Single-energy analyses were done in order to search for structures which may be missing from the energy-dependent fit. Those corresponding to the combined fit are displayed in Fig. 2. Many of the partial-wave amplitudes from C500 show rising imaginary parts near 500 MeV (Fig. 2), a feature absent in the analysis of \( \pi d \) elastic data alone.

**SUMMARY AND CONCLUSIONS**

We have obtained new partial-wave amplitudes for \( \pi d \) elastic scattering and the reaction \( \pi d \to pp \), using a K-matrix method which utilized information from our elastic \( pp \) scattering analysis. In addition to producing amplitudes more tightly constrained by unitarity, we have resolved the overall phase ambiguity existing in our separate \( \pi d \to pp \) analysis.

The combined analysis has resulted in a improved fit to the \( \pi d \) elastic and \( \pi d \to pp \) databases. The most noticeable differences in the partial-waves appear at higher energies where the existing data are sparse. It is difficult to find cases where the fit has been dramatically improved. One exception is the \( \pi d \) total cross sections above 300 MeV. Here the combined analysis is much more successful in reproducing the energy dependence (Fig. 3a).

The excitation function of \( iT_{11} \) for \( \pi d \to pp \) (Fig. 3b) is also suggestive, though the uncertainties are large. Much additional data for \( \pi d \) elastic and \( \pi d \to pp \) above 300 MeV will be

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### Table 1. Comparison of the combined analysis (C500) and our previous (separate) analyses: WI96 for \( pp \to pp \)\(^{(4)}\), SM94 for \( \pi d \to \pi d \)\(^{(3)}\), and SP96 for \( \pi d \to pp \)\(^{(4)}\). The relevant energy ranges are: \( T_\pi = 0 - 500 \text{ MeV} \), \( T_p = 288 - 1290 \text{ MeV} \), and \( \sqrt{s} = 2015 - 2440 \text{ MeV} \).

| Reaction    | Separate \( \chi^2/\text{Data} \) | Combined \( \chi^2/\text{Data} \) |
|-------------|-----------------------------------|----------------------------------|
| \( pp \to pp \) | 17380/10496                      | 17380/10496                     |
| \( \pi d \to \pi d \) | 2745/1362                          | 2418/1362                       |
| \( \pi d \to pp \) | 7716/4787                          | 7570/4787                       |
required to extend combined analysis to the corresponding range of \( pp \) elastic scattering, say to \( \sqrt{s} = 3 \text{ GeV} \). The present analysis has also resulted in a unified description of the resonancelike behavior previously noted in our separate analyses of \( pp \) and \( \pi d \) elastic scattering, and the reaction \( \pi d \to pp \). This behavior has been variously described as “resonant” (due to the creation of dibaryon resonances) and “pseudo-resonant” (due to the \( N\Delta \) intermediate state). We expect that our combined analysis will further constrain models based on these two mechanisms.

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C500/SM94

\[ \pi d \rightarrow \pi d \]

\[ \sigma \text{ (mb)} \]

\[ T \text{ (MeV)} \]

(a)

C500/SP96

\[ \pi d \rightarrow pp \]

\[ \theta = 30 \pm 2.5^\circ \]

\[ iT_{11} \]

(b)
