NEW MODEL-INDEPENDENT METHOD TO TEST THE CURVATURE OF THE UNIVERSE

H. YU 1 AND F. Y. WANG 1,2

1 School of Astronomy and Space Science, Nanjing University, Nanjing 210093, China; fayinwang@nju.edu.cn
2 Key Laboratory of Modern Astronomy and Astrophysics (Nanjing University), Ministry of Education, Nanjing 210093, China

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ABSTRACT

We propose a new model-independent method to test the cosmic curvature by comparing the proper distance and transverse comoving distance. Using the measurements of the Hubble parameter \( H(z) \) and the angular diameter distance \( d_A \), the cosmic curvature parameter \( \Omega_K \) is constrained to be \(-0.09 \pm 0.19\), which is consistent with a flat universe. We also use a Monte Carlo simulation to test the validity and efficiency, and find that our method can give a reliable and efficient constraint on cosmic curvature. Compared with other model-independent methods testing the cosmic curvature, our method can avoid some drawbacks and give a better constraint.

Key words: cosmological parameters – cosmology: observations

1. INTRODUCTION

The cosmic curvature is a fundamental parameter for cosmology. Whether the space of our universe is open, flat, or closed is important for us to understand the evolution of our universe and the dark energy equation of state (Ichikawa et al. 2006; Ichikawa & Takahashi 2006; Clarkson et al. 2007; Zhao et al. 2007; Weinberg et al. 2013). Due to the strong degeneracy between the curvature and the dark energy equation of state, it is difficult to study a non-flat \( \Lambda \)CDM model (Clarkson et al. 2007). Besides, a significant detection of a non-zero curvature will affect the fundamental theory of cosmology because most observations support a flat \( \Lambda \)CDM model, including the latest Planck result, which gives \(|\Omega_K| < 0.005\) (Planck Collaboration et al. 2015). However, most of these constraints are not in a direct geometric way. Therefore, determining the cosmic curvature with model-independent methods is very important.

In order to constrain the cosmic curvature in a direct geometric way, some definitions of cosmological distance should be introduced. Several distance definitions, such as the proper distance \( d_P \), luminosity distance \( d_L \), angular diameter distance \( d_A \), and transverse comoving distance \( d_M \) are defined to investigate cosmology (Hogg 1999; Coles & Lucchin 2002; Ichikawa et al. 2006; Weinberg 2008; Weinberg et al. 2013). Under the assumption of Friedmann–Lemaître–Robertson–Walker metric, the proper distance can be expressed as

\[
d_P(r) = a_0 \int_0^r \frac{dr'}{\sqrt{1 - Kr'^2}} = a_0f(r),
\]

with \( f(r) = \sin^{-1}r, r, \) or \( \sinh^{-1}r \) for curvature \( K = +1, 0, \) or \(-1\), \( a_0 \) is the present scale factor, and \( r \) is the comoving coordinate of the source. Under the definition of the Hubble parameter \( H(z) = \ddot{a}/a \), it can also be expressed as

\[
d_P(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{E(z')},
\]

where \( z \) is the redshift, \( H_0 \) is the Hubble constant, \( c \) is the speed of light, and \( E(z) = H(z)/H_0 \). Similarly, the transverse comoving distance can be expressed as

\[
d_M(z) = a_0 r(z) = \frac{c}{H_0\sqrt{-\Omega_K}} \sin \left[ \sqrt{-\Omega_K} \int_0^z \frac{dz'}{E(z')} \right],
\]

where \( \Omega_K \) is the energy density of cosmic curvature \((-i \sin(\pi x) = \sinh(x)\) if \( \Omega_K > 0\)). With the definition of \( d_M \), \( d_L \) and \( d_A \) can be derived through \( d_L = d_M(1 + z) \) and \( d_A = d_M/(1 + z) \), respectively.

Numerous works have been done to determine the curvature parameter \( \Omega_K \) using different methods, some of which are model-independent. Bernstein (2006) proposed a model-independent method using the weak lensing and baryon acoustic oscillation (BAO) data to constrain \( \Omega_K \) based on the distance sum rule, which was used to test the FLRW metric in Räsänen et al. (2015; hereafter, called the DSR method). The basic principle of the DSR method is that the relation between \( d(z_1) \) and \( d(z_1) + d(z_2, z_2) \) depends on the cosmic geometry (see Figure 1 of Bernstein 2006). The value of \( d(z_1, z_2) \) can be calculated from gravitational lensing. However, the large uncertainty in the gravitational lens system restricts its efficiency on constraining the curvature (Räsänen et al. 2015). Another important model-independent method was proposed in Clarkson et al. (2007), by comparing the Hubble parameter \( H(z) \) and the derivative function of transverse comoving distance \( d_M \) gained from \( d_A \), which has been used in many works (Clarkson et al. 2008; Li et al. 2014; Yahya et al. 2014; Cai et al. 2016; hereafter, the C07 method).

The basis of this method is that one can determine the curvature by combining measurements of the Hubble parameter \( H(z) \) and the transverse comoving distance \( d_M(z) \)

\[
\Omega_K = \frac{[H(z)d_M'(z)]^2 - c^2}{[H_0d_M(z)]^2},
\]

where \( ' \) means the derivative with respect to redshift \( z \). However, in this method, one needs to determine the derivative function of transverse comoving distance \( d_M \) from a fitting function, which will introduce a large uncertainty.

Therefore, in order to avoid the drawbacks of the two methods, we propose a new direct geometric method to test the cosmic curvature. This method is based on the comparison between proper distance \( d_P \) obtained from the Hubble parameter measurement, and transverse comoving distance \( d_M \) obtained from the angular diameter distance \( d_A \) measurement. The structure of this paper is organized as follows. In Section 2, we introduce our new model-independent method to test the cosmic curvature using \( d_P \) and \( d_M \). In Section 3, we
give our constraint on $\Omega_K$ using the Hubble parameter and angular diameter distance measurements. In Section 4, we test the validity and efficiency of our method with the Monte Carlo simulation. In Section 5, we discuss its advantages compared to other methods. Finally, a summary will be given in Section 6.

2. METHOD TO TEST $\Omega_K$

Comparing the definitions of proper distance $d_P$ and transverse comoving distance $d_M$, one can find that the difference between them is only caused by the curvature of the universe. This gives the basis to test the cosmic curvature using the comparison of $d_P$ and $d_M$. From Equations (2) and (3), $\Omega_K$ can be derived from

$$H_0 d_M \sqrt{-\Omega_K} = \sin \left( \frac{H_0 d_P}{c} \sqrt{-\Omega_K} \right).$$

Equation (5) gives the direct relation among $\Omega_K$, $d_P$, and $d_M$. Once the $d_P$ and $d_M$ are determined, $\Omega_K$ can be calculated through this equation. If the value of redshift and $\Omega_K$ are not large, $H_0 d_P \sqrt{-\Omega_K}/c$ is less than one. From the Taylor expansion, Equation (5) can be approximated as

$$\Omega_K = \frac{6c^2 d_M - d_P}{H_0^2 \frac{d_P}{d_M}},$$

from which $\Omega_K$ can be determined directly.

Figure 1 shows the key principle of our method in the $\Omega_K < 0$ case. In this figure, the arc OS is the proper distance between source S and observer O, while the transverse comoving distance between them is $d_M = a_0 \sin \left( \frac{d_P}{c} \right)$. In this case, it is obvious that the $d_M$ of an object has an upper limit $a_0$ and it is less than $d_P$. In contrast, in an open universe ($\Omega_K > 0$), we have $d_M > d_P$. $d_M = d_P$ only happens in a flat universe. Therefore, the cosmic curvature can be derived by comparing $d_M$ and $d_P$. With the assumption of $\sqrt{|\Omega_K|} I \ll 1$, we can obtain

$$\delta = \frac{d_P - d_M}{d_P} = -\frac{\Omega_K I^2}{6},$$

with $I = \int_0^z \frac{d\zeta}{E(\zeta)}$. The $\delta$ means the relative difference between $d_P$ and $d_M$. The value of $|\delta|$ gives the requirement on the accuracy of measurement, which means the $\Omega_K$ cannot be constrained if the observed uncertainty is much larger than $|\delta|$. Besides, if the total relative error of the measurement sample of $d_M$ and $d_P$ is $\sigma$, one can expect that the tightest constraint on $\Omega_K$ will have an error of about $\sigma \Omega_K \sim \sigma I/2$. In other words, Equation (7) gives the constraining limit of this method.

To obtain the transverse distance $d_M$, we choose the angular diameter distance $d_A$ measurement based on BAO in several previous works (Blake et al. 2012; Xu et al. 2013; Samushia et al. 2014; Delubac et al. 2015). These data and their references are listed in Table 1. The detailed information about these data can be found in their references. The $d_M$ can be easily derived with the direct relation between them $d_M = d_A(1 + z)$. The next important issue is how to measure $d_P$. From Equation (2), the proper distance $d_P$ only depends on the $H(z)$ function. Therefore, in order to derive the proper distance $d_P$, one can construct the $H(z)$ function from Hubble parameter measurements. Then $d_P$ can be derived from Equation (2). There are tens of Hubble parameter measurements derived from differential ages of galaxies and the radial BAO in the previous literature, which are listed in Table 2. In order to make our method model-independent, the Gaussian Process (GP) method is used to reconstruct the $H(z)$ function. The GP method is a powerful tool to reconstruct a function from data directly without any assumption of the function form and is used widely in astronomy (Holsclaw et al. 2010; Shafieloo & Clarkson 2010; Bilicki & Seikel 2012; Seikel et al. 2012a; Shafieloo et al. 2012). Therefore, with the GP method, we don’t need any prior cosmological model. There is a good python package for the GP method called Gapp developed by Seikel et al. (2012a), which was used in many works (Bilicki & Seikel 2012; Seikel et al. 2012b; Cai et al. 2016). It can reconstruct the function as long as observed data was input. More detailed information about the GP method and Gapp can be found in Seikel et al. (2012a).

![Figure 1. Illustration of the proper distance $d_P$ and transverse comoving distance $d_M$ in a closed universe. It is obvious that $d_M < d_P$.](image)

Table 1

| $z$  | $d_A(z)$ (Mpc) | References        |
|------|----------------|-------------------|
| 0.44 | 1205 ± 114     |                   |
| 0.6  | 1380 ± 95      | Blake et al. (2012) |
| 0.73 | 1534 ± 107     |                  |
| 0.35 | 1050 ± 38      | Xu et al. (2013)  |
| 0.57 | 1380 ± 23      | Samushia et al. (2014) |
| 2.34 | 1662 ± 96      | Delubac et al. (2015) |
The data of the Hubble parameter $H(z)$ and their references are given in Table 2. These Hubble parameters at different redshifts are derived using differential ages of galaxies and the radial BAO method. Using these observed data and the GP method, the $H(z)$ function can be reconstructed, which is shown as the blue curve in Figure 2. Hereafter, this $H(z)$ function is called GP-$H(z)$. For comparison, we also fit the observed data based on the $\Lambda$CDM model, which is shown as the green curve in Figure 2. Hereafter, this $H(z)$ function is called Fit-$H(z)$. We can find that the Fit-$H(z)$ is well covered by the GP-$H(z)$ function and its 1σ confidence region. With the reconstructed GP-$H(z)$ function, one can use Equation (2) to derive the proper distance at a certain redshift. The derived $d_P(z)$ functions from GP-$H(z)$ and Fit-$H(z)$ are shown in Figure 3. Hereafter, they are called GP-$d_P(z)$ and Fit-$d_P(z)$ respectively. Comparing with Figure 2, one can find that the derivation between GP-$d_P(z)$ and Fit-$d_P(z)$ becomes smaller than that between GP-$H(z)$ and Fit-$H(z)$.

Using Equation (5), one can derive the $\Omega_k$ from the $d_P$ and $d_A$ at the same redshift. Figure 4 shows the result of derived $\Omega_k$ at different redshifts. The average $\Omega_k$ and its error bar are derived from Equations (8) and (9). The result is listed in Table 3. The average $\Omega_k$ constrained by these six data is $\Omega_k = -0.09 \pm 0.19$. There is no significant deviation from a flat universe. From Figure 2, it is obvious that the high-redshift measurement gives a tighter constraint on $\Omega_k$ than the low-redshift measurement. The reason is that the $I$ term in $H(z)$ function is called Fit-$H(z)$. We can find that the Fit-$H(z)$ is well covered by the GP-$H(z)$ function and its 1σ confidence region. With the reconstructed GP-$H(z)$ function, one can use Equation (2) to derive the proper distance at a certain redshift. The derived $d_P(z)$ functions from GP-$H(z)$ and Fit-$H(z)$ are shown in Figure 3. Hereafter, they are called GP-$d_P(z)$ and Fit-$d_P(z)$ respectively. Comparing with Figure 2, one can find that the derivation between GP-$d_P(z)$ and Fit-$d_P(z)$ becomes smaller than that between GP-$H(z)$ and Fit-$H(z)$.

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Equation (7) is larger at high redshift, which will decrease the error of $\Omega_k$ through $\sigma_{\Omega_k} \sim 6\sigma f^2$.

4. SIMULATION

In order to test the validity and efficiency of our method, we perform a Monte Carlo simulation. The route of the simulation is as follows: (1) creating mock $H(z) - z$ and $d_M - z$ data sets based on a prior cosmological model; (2) reconstructing the GP-$H(z)$ function from GP method; (3) using the GP-$H(z)$ function to derive the GP-$d_P(z)$ function through Equation (2); (4) using Equations (5) and (8) to constrain $\Omega_k$ and its average value; and (5) simulating $10^4$ times for each prior cosmological model and give the distribution of determined average $\Omega_k$.

For simulation, we choose the $\Lambda$CDM model as the prior cosmological model. The model parameters are chosen as $H_0 = 70 \text{ km s}^{-1}\text{ Mpc}^{-1}$, $\Omega_M = 0.3$, and $\Omega_\Lambda = 1 - \Omega_M - \Omega_k$, where $\Omega_k = -0.1$, 0 and 0.1 for different cases. In each simulation, there are 20 mock $H(z) - z$ and $d_M - z$ data sets respectively. The redshifts of these mock data are chosen equally in $\log{(1+z)}$ space in the redshift range $0.1 \leq z \leq 5.0$. The relative uncertainty of these mock data is 1%, which will be realized in future observations (Weinberg et al. 2013).

Figure 5 gives an example of the simulations in the $\Omega_k = 0$ case. The three panels show the mock Hubble parameter data with the GP-$H(z)$ function, mock $d_M$ data with the GP-$d_P(z)$ function and the final determined $\Omega_k$. From this figure, we can see that the GP method can reconstruct the $H(z)$ function well. In this case, the final derived average $\Omega_k$ is $\Omega_k = 0.0001 \pm 0.0092$. Figure 6 shows the posterior distributions of $\Omega_k$ for three $\Omega_k$ cases. From this figure, one can find that our method can give a reliable and tight constraint on the prior $\Omega_k$. The uncertainty is $\sigma_{\Omega_k} \approx 0.011$. This result means that if there are 20 $d_A$ and $H(z)$ measurements with 1% uncertainty, our method can give a constraint on $\Omega_k$ at the 1% level. In the future, there will be more accurate measurements of $d_A$ and $H(z)$, and a tighter constraint on $\Omega_k$ can be expected.

5. COMPARED WITH OTHER METHODS

In this section, we compare our method with other model-independent methods. Just as introduced in the first section, there are two model-independent methods to constrain the curvature of the universe proposed in previous literature (Bernstein 2006; Clarkson et al. 2007), which have been used in many works (Clarkson et al. 2008; Li et al. 2014; Yahya et al. 2014; Räisänen et al. 2015; Cai et al. 2016). The first one is the DSR method based on the distance sum rule proposed by Bernstein (2006) and the other is the C07 method based on the Equation (4) (Clarkson et al. 2007).

The basic principle of the DSR method is the distance sum rule, which means that if there are two sources $S_1$ and $S_2$ at redshift $z_1$ and $z_2$ (assuming $z_1 < z_2$), it has $d(z_2) = d(z_1) + d(z_1, z_2)$ if our universe is flat. Otherwise, $d(z_2) > d(z_1) + d(z_1, z_2)$ or $d(z_2) < d(z_1) + d(z_1, z_2)$ for $\Omega_k > 0$ or $\Omega_k < 0$ respectively. The distance between $S_1$ and $S_2$ can be determined by a gravitational lens (Bernstein 2006; Räisänen et al. 2015). Compared to Figure 1 of Bernstein (2006) with our Figure 1, it can be found that the difference between $d_P$ and $d_M$ is larger at high redshift, which will decrease the uncertainty significantly, especially when the function form is unknown and the data is not enough. In order to check the efficiency of the C07 method, we also use the Monte Carlo simulation to test it, and the simulation is same as introduced in Section 4. Figure 7 gives an example of the simulations in the $\Omega_k = 0$ case, which is similar to Figure 5. Instead of $H(z)$ and $d_P(z)$ functions, we show the $d_M(z)$ function in the top panel of Figure 7. Because the $H(z)$ and $d_P(z)$ functions are similar to those in Figure 5. From Figure 7, it is obvious that the $d_M(z)$ function derived from the GP method has a large derivation with the theoretical one, and the determined $\Omega_k$ are not as good as those in the bottom panel of Figure 5. Figure 8 shows the posterior distributions of $\delta_{\Omega_k}$ determined with the C07 method for three $\Omega_k$ cases. From this figure, one can find that the C07 method gives a large uncertainty on the determined $\Omega_k$.

6. SUMMARY

We have proposed a new model-independent method to test the cosmic curvature in this paper. The main principle of our method is to compare the proper distance $d_P$ and transverse comoving distance $d_M$ at the same redshift (Figure 1 gives an illustration). Using Equation (5), one can derive the $\Omega_k$ if $d_P$ and $d_M$ are obtained. With the measurements of the Hubble...
parameter, we use the GP method to reconstruct the $H(z)$ function and use Equation (2) to derive the $d_P(z)$ function. Using the measurements of angular diameter distance, the transverse comoving distance can be calculated easily through $d_M = d_A(1 + z)$. We used the $H(z)$ and $d_A$ measurements collected from previous literature. The reconstructed $H(z)$ function and the derived $d_P(z)$ function are shown in Figures 2 and 3. In order to compare with the $\Lambda$CDM model, the best-fitted $H(z)$ and $d_P(z)$ functions are also shown in these figures. The comparison shows that the GP method can give a reliable reconstructed function from observed data. Figure 4 shows the derived $\Omega_k$ at several different redshifts, which are also listed in Table 3. Using Equations (8) and (9), the average $\Omega_k$ can be obtained, which is $\Omega_k = -0.09 \pm 0.19$. This result shows that $\Omega_k$ has no significant derivation from zero.

To check the validity and efficiency of our method, we use the Monte Carlo simulation to test it. For $\Lambda$CDM model with three different $\Omega_k$, $-0.1$, $0$, and $0.1$, we simulate $10^4$ times for each case. Figure 5 gives an example of the simulations for the $\Omega_k = 0$ case and Figure 6 gives the posterior distributions of $\Omega_k$ determined with our method for the three $\Omega_k$ cases. These two figures show that our method can give a reliable and efficient constraint on $\Omega_k$. We also compared our method with the DSR and C07 method. We find that DSR method needs a higher accuracy of measurement than our method. More

| $z$   | 0.35   | 0.44   | 0.57   | 0.60   | 0.73   | 2.34   | Average |
|-------|--------|--------|--------|--------|--------|--------|---------|
| $\Omega_k$ | 1.24 ± 2.78 | 0.63 ± 3.74 | 0.29 ± 0.75 | −0.28 ± 1.59 | 0.13 ± 1.18 | −0.13 ± 0.20 | −0.09 ± 0.19 |
importantly, the systematic uncertainty of the gravitational lens system parameter $f$ significantly restricts its efficiency on constraining the curvature. For the C07 method, we also test it with simulations. The result is shown in Figures 7 and 8. From Figure 7, we find that the first derivative function of $d_{L}(z)$ derived from the GP method has a large derivation with the theoretical one. So the determined $\Omega_k^*$ is not reliable. Figure 8 shows the posterior distributions of $\Omega_k$ determined by the C07 method for the three $\Omega_k$ cases. Meanwhile, the C07 method will give a large uncertainty on the determined $\Omega_k$ with observed data at same accuracy level.

Future observations will improve the constraint on the cosmic curvature. The Extended Baryon Oscillation Spectroscopic Survey (eBOSS) will compile 250,000 new, spectroscopically confirmed luminous red galaxies, which yield measurements of $d_A$ with 1.2% precision and measurements of $H(z)$ with 2.1% precision (Dawson et al. 2016). HETDEX will perform a survey of 800,000 Ly$\alpha$ emission-line galaxies at $1.8 < z < 3.7$ (Hill et al. 2006). The precision on $d_A$ and $H(z)$ is of the order of 2% using BAO. The BAO analysis from the Wide Field Infrared Survey Telescope (WFIRST) will yield about 1.0% measurements of the angular diameter distance $d_A$ and Hubble parameter $H(z)$ by the 17 million galaxies redshift survey in the redshift range of $1.3 < z < 2.7$ (Green et al. 2012). Meanwhile, the Euclid satellite with a survey area of approximately 14,000 deg$^2$ and a redshift range of $0.7 < z < 2.0$ will measure the Hubble parameter $H(z)$ with 1.5% precision (Laureijs et al. 2011). Weinberg et al. (2013) had predicted the accuracy of future measurements of the Hubble parameter and angular diameter distance through the full sky BAO survey and gave a encouraging forecast that the relative error on $H(z)$ and $d_A$ would be less than 1% at redshift $z > 0.5$. Therefore, with our method, the curvature parameter $\Omega_k$ can be constrained at a very high accuracy level in a model-independent way.

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