Frequency Domain Fatigue Assessment of Vehicle Component under Random Load Spectrum

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Abstract. This research is focused on the application of frequency domain based fatigue life predict methods on vehicle component. The basic theory of these approaches is based on the frequency-based signals, the probability density function (PDF) of signals and Miner cumulative damage criterion. A typical suspension virtual prototype model is established to derive dynamic loading arisen from random road exciting. Several kinds of fatigue life predicting approaches in frequency domain are applied and compared. The influence factors for these methods, such as PSD average methods, frequency ranges and frequency intervals are also discussed. Appropriate results can be obtained at last.

1. Introduction

Over the past 30 years there has been a heightened interest in the prediction of fatigue life for vehicle components due to global competition and higher customer demands for safety, durability and reliability of the products [1]. The prediction of fatigue life can be carried out based on fatigue tests or on virtual prototype simulation, in the domain of time or frequency.

In order to assess the fatigue damage of structures two different approaches can be adopted. If the stress time history can be captured very easy from experimental measurements or numerical simulation, time domain fatigue damage assessment approach will be applied usually. In this approach, all input loading and output stress or strain response are time-based signals [1]. The approach is based on cycle counting schemes and damage accumulation models, such as rainflow count and Palmgren-Miner linear damage rule [3]. Usually many stress/strain time histories are needed to formulate reliable statistical considerations about the distribution of dynamic loading, the fatigue damage and service life, this makes this approach, even if easy from a theoretical point of view, clearly costly and time consuming.

In some situations, however, response stress and input loading are preferably expressed as frequency-based signals, usually in the form of power spectral density (PSD). In this case, a frequency domain analysis based on the so-called spectral approach is much popular. This is a very important approach in solution to random vibration fatigue problem. In this approach, the irregular stress time

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history is modelled as a random process with the assumption of stationary and Gaussian. Compared to the previous one, the approach can reduce the computational time consuming significantly and can be applied to solve fatigue damage problem of components subjected to complex random loading. The approach typical applied in offshore oil platform and space aircraft fatigue damage assessment. Because as hugely complex structures they usually subjected from many kinds of random loading, such as wind and wave loading.

Several methods can be applied to assess fatigue damage in frequency domain. In 1964 Bendat proposed the first significant step towards a method of determining fatigue life from PSDs. Bendat assumed that the probability density function (PDF) of peaks for a narrow band signal tended towards a Rayleigh distribution. The Bendat’s narrow band solution is extremely conservative because it overrated the probability density of peaks for wider band time histories. On the ground of Bendat’s method, several amended methods were proposed, including Wirsching model, Hancock model, Kam-dovor model, Steinberg model and Tunn model. In 1985 Dirlik proposed an empirical closed form solution to the broad band problem following extensive computer simulations using the Monte Carlo technique, preferable accuracy is acquired. The Dirlik’s empirical formula was verified theoretically by Bishop on the base of smooth gauss random hypothesis. Dirlik’s empirical formula is computational less complexity but shows little reduce on accuracy over Bishop’s method, which is used widely for fatigue analysis in frequency domain.

Vehicles components suffered a lot from the irritation of external loading mainly originated from irregular road profile. Usually, as the irregularity of road profile is modelled in the frequency domain also as a random process, the stress response of vehicle components generally seems stationary and Gaussian distribution in practice. As a long time domain signal must be recorded in order to accurately describe the random response. It is very difficult to assessment fatigue damage of vehicle components in time-domain, but more convenient to analysis based on the signal in the frequency domain. Therefore, it is fit to assess fatigue damage of vehicle components in frequency domain.

In the paper, in order to application the frequency domain based fatigue damage predict methods on vehicle components, a typical suspension control arm of vehicle is taken for example; different kinds of frequency domain based methods are applied. The results of different approaches are compared and the influence factors for theses approaches are also discussed.

2. Fatigue loading testing based on numerical simulation

The stress loading responses on structures in time domain or frequency are very important for the assessment of fatigue damage of components, which can be acquired from experimental measurements or numerical simulations. The former method has been widely used to obtain accurate loading time histories of vehicle components for fatigue damage assessment. But this method depends much on proving ground testing or on road testing and is extremely time and money consuming. Usually, it has difficult to implement in many cases, especially in the early stages of vehicle design. With the increasing application of virtual prototype technology in vehicle dynamic simulation, numerical simulation method has been very popular in loading testing. In order to acquire loading responses on the structure, the virtual prototype model of a type of McPherson suspension is established. The irritation of irregular road profile also is considered in the model.

2.1. Dynamic model of suspension system

Rigid flexible coupled multi-body dynamic method is adopted to establish the virtual prototype model of a so-called McPherson suspension in the paper. The structure of McPherson suspension is composed of 5 components including control arm, knuckle, upper strut, tire, spring and damper, as shown in Figure 1. The control arm connects to the ground with two revolute joints at point 3 and 4, and can rotate relative to the ground. The knuckle connects to control arm with a spherical joint at point 2 and connects to tire with a revolute joint. The upper strut connects to the ground with a universal joint. Damper and spring are two force elements, damper force acts between control arm and
ground, spring force acts between knuckle and upper strut. And only the translational movement 
between knuckle and upper strut is allowed with the acting of spring force.

In order to consider deformation of control arm in numerical simulation, finite element method 
(FEM) is adopted to establish the model of control arm. Take time saving and model simplification 
into account, the modified Craig-bampton Component Mode Synthesis method (CMS) has been used. 
The CMS method is one kind of model reduction methods, and can be applied to reduce the degrees of 
freedom of FEM model to a very large extent. The FEM model of control arm is shown in Figure 1 
(on the right); it has 4383 nodes and 13489 tetra4 elements. The reduced model of control arm can be 
integrated to multi-body model using a so-called modal neutral file (mnf). FEM model and reduced 
model is established in Nastran and rigid flexible coupled multi-body model is established in 
ADAMS.

![Figure 1. McPherson suspension model and FEM control arm](image)

2.2. Model of road profile

The irregularity of road profiles has mainly influence the loading variations acting on vehicle 
components, especially for suspension structures. Road profile can be described commonly as 
longitudinal profile or PSD. The longitudinal profile contains the most information, and a PSD 
function succinctly shows roughness as a distribution of mean-square variations over wave numbers. 
The PSD will be used to generate road roughness profiles if the actual road roughness profiles are not 
available.

A popular method for generating a random profile that is approximately stationary from a PSD 
function is by summing a large number of sinusoids. The amplitudes are determined by the PSD 
function and a frequency interval, and the phase angles are random.

That is, 

$$z(\xi) = \sum_{i=1}^{n} \left[ \sqrt[2]{\Delta v_i G_{zz}(v_i)} \cos(2\pi v_i \xi + \phi_i) \right]$$  

(1)

Where \( \Delta v_i = (v_{i+1} - v_{i-1}) / 2 \)

\( z(\xi) \) is longitudinal profile of roads, \( \xi \) is longitudinal distance, \( n \) is the number of sinusoidal 
components used to generate the profile, \( G_{zz}(v_i) \) is road PSD, \( v_i \) is the spatial frequency associated 
with the \( i^{th} \) component of the summation, \( \Delta v_i \) is the bandwidth of the \( i^{th} \) component, and \( \phi_i \) is a 
random phase angle.

The number of components \( n \) in equation (1) should be large enough so that the exact values of 
the spatial frequency, \( v_i \), within a frequency window \( \Delta v_i \) are not significant. The spatial frequency,
v_1, usually covers a range from 9.84 \times 10^{-3} \text{ to } 3.28 \text{ cycle m}^{-1}, or wavelengths from 0.21 \text{ to } 91.46 \text{ m cycle}^{-1}, with a \Delta v_1 interval of 9.84 \times 10^{-3} \text{ cycle m}^{-1} for low frequencies and 0.33 \text{ cycle m}^{-1} for higher frequencies. The profile obtained with this method is approximately stationary (due to the uniform distribution of random phase) and Gaussian (due to the central limit theorem). For computational efficiency, the sum of sinusoids can be replaced with the FFT. A typical single road roughness profile generated from the PSD is shown in Figure 2.

![Figure 2. Time domain and frequency domain road model](image)

2.3. Dynamic simulation and loading calculation
Dynamic simulation is an effective approach to acquire loading spectra in early stage of design. Based on rigid-flex coupled dynamic model, forces and moments of each direction at joint points can be obtained. It can be seen from analysis that forces in Z direction of point 1 and point 2 have dominant influence on stress time history and fatigue damage of control arm, and are selected in subsequent fatigue life calculation. The flowchart of loading calculates and extract from dynamic simulation is shown in Figure 3.

Based on the rigid flexible coupled model, stress time history of each nodes of control arm also can be calculated by the so-called modal stress recovery (MSR) method. As a kind of modal transient analysis method, MSR method can be applied to calculate the dynamic stress at each time step in simulation. Stress distributing contour of control arm at a particular time step of maximal stress occur is shown in Figure 4, from which the damageable area can be seen clearly.

![Figure 3. Flowchart of loading calculates and extract from dynamic simulation](image)

![Figure 4. Stress contour of control arm derived from modal stress recovery approach](image)
3. Fatigue life calculate and influencing factors analysis

3.1. Fatigue life analysis in time domain
To predict fatigue life in time domain, two essential components are needed: loading cycle counting methods and fatigue damage rules. We can apply the classical time domain analysis based on cycle counting schemes and damage accumulation model.

Vehicle components usually subjected to random applied loads, therefore, the load time history is generally complex and cannot be applied directly in a fatigue calculation. Consequently, cycle counting methods are employed to decompose this complex load history. Different cycle counting strategies are available; however, they can lead to different counts. Rainflow counting method was proposed by Matsuishi and Endo\cite{1} in 1968 to count the number of cycles of each load range in a load history. The method was demonstrated accurately in identifying hysteresis loops in a variable amplitude histogram.

Output from a Rainflow cycle counting usually expressed as a range mean histogram which is shown in Figure 5. The load range of each cycle is given along the x axis, the load mean is shown on the y axis and the z axis gives the number of cycles contained in the time history for each range and mean.

The Palmgren-Miner’s hypothesis is one of the most widely used damage accumulation models. It assumes a linear damage accumulation. Although other nonlinear models have been proposed, they are more complicated than the Miner’s rule and cannot provide consistently better results.

In the paper, the fatigue life analysis in time domain is on the ground of Rainflow cycle counting and Miner’s damage accumulation rule.

![Figure 5 Range-mean histogram derived from time history by Rainflow cycle counting](image)

3.2. Fatigue life analysis in frequency domain
Frequency domain fatigue life analysis is based on total life approach and accepts multi-location input files from finite element results. The results of fatigue damage can be either a combination of transfer function from a frequency response analysis with corresponding loading input PSDs or the output response PSDs from a transient response analysis.

Because at present an exact theoretical framework is not available for the case of broadband Gaussian processes, usually, empirical approaches are used to calculate the damage caused by broadband random signals. Dirlik’s empirical approach has been proved to be the most accuracy method for fatigue damage assessment at present\cite{4,5}.

The Dirlik’s formulation is given in equation (2).

$$N(S) = E[P|T_p(S)]$$ (2)
Where, $N(S)$ is the number of stress cycles of range $S$ N/mm$^2$ expected in time $T$ sec. $E[P]$ is the expected number of peaks, can be obtained by $E[P] = \frac{m^2}{m_4}$. $p(S)$ is PDF of peaks stress range, which can be expressed as follows:

$$p(S) = \frac{D_1 e^{-\frac{s^2}{2}} + D_2 Z e^{\frac{-s^2}{2R^2}} + D_3 Z e^{-\frac{s^2}{2}}}{2\sqrt{m_0}}$$  \hspace{1cm} (3)

$$D_1 = \frac{2(x_m - \gamma^2)}{1 + \gamma^2}, \quad D_2 = \frac{1 - \gamma - D_1 + D_2^2}{1 - R}, \quad D_3 = 1 - D_1 - D_2, \quad Z = \frac{S}{2\sqrt{m_0}}$$

$$Q = \frac{1.25(y - D_1 - D_2 R)}{D_1}, \quad R = \frac{1}{1 - y - D_1 + D_2^2}, \quad \gamma = \frac{m_2}{\sqrt{m_0 m_4}}, \quad x_m = \frac{m_1}{m_0} \sqrt{\frac{m_2}{m_4}}$$

Where $m_n$ is the $n^{th}$ moment of area of the PSD, which is obtained as:

$$m_n = \int f^n \cdot G(f) df$$ \hspace{1cm} (4)

Where $G(f)$ is the value of the single side PSD at frequency $f$ Hz.

The damage is calculated using Palmgren-Miner rule:

$$D = \sum_i \frac{n_i}{N(S_i)} = \frac{S_i}{K} \int S^m P(S) dS$$ \hspace{1cm} (5)

Where: $n_i$ is the actual counted number of cycles. $N(S_i)$ is the number of cycles associated to the load level. $S_i$ is the total number of cycles in required time.

3.3. Comparison among frequency domain approaches

As mentioned in the preceding, several approaches can be used to calculate fatigue life in frequency domain. A summary of the results derived from different approaches is given in Table 1. The first column of the table implies the model is irritated from different road profile or also means different load cases, and the second column of the table involves different nodes of control arm. (If fatigue life in time domain means $A$, and fatigue life in frequency domain means $A'$, the results in Table 1 means $\gamma = A'/A$)

From the Table, it can be seen that Narrow band approach is very conservative compared to other methods, and Dirlik approach is remarkably credible than other approach. The average discrepancy between Dirlik approach and the fatigue life in time domain is only 16.9%. Usually, vehicle components subjected not only to narrow band random vibration, so Dirlik method is recommended as the most suitable method for frequency domain fatigue damage assessment.

Table 1. Comparison between different frequency domain fatigue life analyses ($\gamma$)
3.4. Influencing factor of fatigue damage results in frequency domain

3.4.1. The influence of PSD averaging method

Compared to time domain approach, PSDs of loading will be applied to predict fatigue damage in frequency domain. Thereby the accuracy of frequency domain based fatigue life analysis depends on credible PSDs.

PSDs can be derived from fourier analysis of stress time history directly. When dealing with PSDs, two ways can be applied to carry out spectral estimates for each of many FFT buffers. Linear average is executed in the first method. This method is very simple, but could introduce some disadvantages. Because it does linear average on the component values over the number of calculate buffers. So the very fast high amplitude events that occur in a single buffer will be reduced. Peak average is applied in the second method. In this method the largest FFT for each component is retained and disadvantages can be eliminated.

The results of PSDs at Z direction of point 1 are shown in Figure 6; they are derived from these two different ways. Significant difference can be seen between the two methods, the maximal, means and RMS values of Peak average method are much higher than Linear average method.

The fatigue damage results based on the two PSDs average method are shown in Table 2. Obviously, the peak average method can obtain more reasonable results especially for Dirlik, Tunna, and Kam-dovor approaches, but linear average method is more suitable for Narrow Band, Hancock and Steinberg approaches.

| Load case | Node | Dirlik | Narrow Band | Tunna | Wirsching | Hancock | Kam-dovor | steinberg |
|-----------|------|--------|-------------|-------|-----------|---------|-----------|-----------|
| 1781      | 1.783| 0.057  | 1.461       | 1.337 | 0.082     | 1.783   | 0.068     |
| 1746      | 1.558| 0.090  | 2.302       | 1.145 | 0.129     | 1.527   | 0.107     |
| **Rank_6**| 3602 | 1.667  | 0.145       | 3.685 | 1.194     | 0.208   | 1.592     | 0.171     |
| 260       | 0.417| 0.036  | 0.920       | 0.299 | 0.052     | 0.398   | 0.043     |
| 870       | 0.300| 0.075  | 1.737       | 0.181 | 0.107     | 0.241   | 0.094     |
| 1781      | 0.712| 0.022  | 0.663       | 3.396 | 0.031     | 4.679   | 0.026     |
| 1746      | 2.035| 0.029  | 0.942       | 2.816 | 0.043     | 3.913   | 0.035     |
| **Rank_7**| 3602 | 1.746  | 0.058       | 1.781 | 3.233     | 0.085   | 4.453     | 0.069     |
| 260       | 0.762| 0.009  | 0.287       | 0.518 | 0.013     | 0.717   | 0.011     |
| 870       | 0.715| 0.025  | 0.816       | 0.471 | 0.037     | 0.655   | 0.030     |
| **Average**| 1.169| 0.055  | 1.459       | 1.459 | 0.079     | 1.996   | 0.065     |

![Figure 6. PSDs of Z direction force at point 1 derived from two average methods](image1)

![Figure 7. The influence of frequency range on fatigue life results](image2)
Table 2. Results comparison between different PSDs average methods (γ)

| Average method | Load case | Dirlik | Narrow Band | Tunna | Wirsching | Hancock | Kam_dovor | Steinberg |
|---------------|-----------|--------|-------------|-------|-----------|---------|-----------|-----------|
| **Peak average** | Rank_6    | 1.778  | 0.057       | 1.460 | 1.335     | 0.081   | 1.778     | 0.068     |
|                | Rank_7    | 0.712  | 0.022       | 0.663 | 3.398     | 0.031   | 4.674     | 0.026     |
|                | Average   | 1.245  | 0.0395      | 1.0615| 2.3665    | 0.056   | 3.226     | 0.047     |
| **Linear average** | Rank_6   | 10.966 | 1.313       | 36.307| 7.557     | 1.903   | 10.227    | 1.534     |
|                | Rank_7   | 100.082| 4.875       | 157.722| 68.681   | 7.184   | 95.493    | 5.807     |
|                | Average  | 55.524 | 3.094       | 97.0145| 38.119   | 4.5435  | 52.86     | 3.6705    |

3.4.2. The influence of frequency ranges and intervals
The frequency ranges and intervals in frequency response analysis not only influence the size of result files and calculate time consumed, but influence the fatigue damage results of structures. If frequency ranges are too broad or frequency intervals are too dense, the results of frequency response analysis will become too larger and could not be imported to calculate fatigue damage in frequency domain. With the increase of range and the decrease of interval of frequency, the size of result and model files augmented quickly, as show in Table 3.

Table 3. Size of result file and model file of differ frequency ranges and intervals

| Frequency range intervals=0.5Hz | Size of Op2 result file (Mpa) | Size of db model file (Mpa) | Frequency intervals (Hz) | Size of Op2 result file Range=50Hz (Mpa) | Size of db model file (Mpa) |
|--------------------------------|-------------------------------|-----------------------------|-------------------------|------------------------------------------|-----------------------------|
| 10                             | 75.3                          | 64.3                        | 1.5                     | 125                                      | 103                         |
| 20                             | 146                           | 123                         | 1.2                     | 153                                      | 126                         |
| 30                             | 217                           | 181                         | 1.0                     | 181                                      | 149                         |
| 40                             | 288                           | 238                         | 0.8                     | 254                                      | 206                         |
| 50                             | 359                           | 295                         | 0.5                     | 359                                      | 295                         |
| 75                             | 536                           | 440                         | 0.4                     | 448                                      | 365                         |
| 100                            | 714                           | 584                         | 0.25                    | 808                                      | 654                         |

The influence of frequency range on fatigue damage results at node 1781 is shown in Figure 7, when the frequency ranges of analysis is lower than 30Hz, it has great influence on fatigue life results. With the increase of frequency range, fatigue life results become to converge. It also can be explained by Figure 6, the main frequency of loadings act on the structures mainly located between 0 to 30Hz. The influence of frequency interval is shown in Table 4. It also has great influence on fatigue life results from frequency domain approach.

Table 4. Results comparison between different Frequency intervals (γ)

| Frequency interval (Hz) | Dirlik | Narrow Band | Tunna | Wirsching | Hancock | Kam_dovor | Steinberg |
|-------------------------|--------|-------------|-------|-----------|---------|-----------|-----------|
| 0.25                    | 0.712  | 0.022       | 0.663 | 3.398     | 0.031   | 4.674     | 0.026     |
| 0.4                     | 1.410  | 0.028       | 0.855 | 3.900     | 0.040   | 5.377     | 0.033     |
| 0.5                     | 1.534  | 0.029       | 0.956 | 3.986     | 0.043   | 5.563     | 0.035     |
| 0.8                     | 1.735  | 0.031       | 0.649 | 4.144     | 0.044   | 5.334     | 0.037     |
| 1.0                     | 0.799  | 0.023       | 0.583 | 3.470     | 0.032   | 4.631     | 0.027     |
| 1.2                     | 1.254  | 0.026       | 0.881 | 3.751     | 0.039   | 5.253     | 0.031     |
| 1.5                     | 250.921| 24.088      | 808.683| 167.759   | 35.703  | 235.149   | 28.390    |
| 1.8                     | 400.432| 65.762      | 2430.756| 334.171   | 90.180  | 654.721   | 66.427    |
4. Conclusions

Take a suspension control arm of vehicle for example; different kinds of frequency domain based methods are applied on fatigue life assessment of typical vehicle components. The results of different approaches are compared and the influence factors for theses approaches are also discussed. The results are as follows:

1. The fatigue lives from diverse methods looks like very discrete, which indicate that the current frequency domain based methods are still have large deviations with traditional time domain method.

2. Dirlik method can obtain the most consistent results with time domain based method. And the treatment of random road exciting as narrow-band random broadband signal will give a conservative result, such as Narrow Band model, Hancock model and Steinberg model. So it is recommended to use Dirlik method in the prediction of fatigue damage for vehicle components in frequency domain.

3. PSD average methods, frequency ranges and intervals have great influence on frequency domain based methods. It should be careful to select proper parameters for the assessment of fatigue damage for vehicle components. Appropriate validation with experiment data or simulative time history data is recommended before the application of suitable frequency domain based methods.

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