On the possibility of $f_0$ observation in low energy $pp$ collisions*

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Received: date / Revised version: date

Abstract. Within the meson-exchange model we calculate $f_0$-meson production cross section in $\pi N$ and $NN$ reactions and investigate the possibility for $f_0$ observation via the $K\bar{K}$ decay mode in $pp$ collisions. Our studies indicate that an extraction of the $f_0$ signal is unlikely due to the large background from other reaction channels.

PACS. 13.75.Cs Nucleon-nucleon interactions – 13.75.Gx Pion-baryon interactions

1 Introduction

The status of the scalar $f_0$-meson is still an open problem in particle physics. In the 1996 Review of Particle Physics [1] the $f_0$ decay modes were announced as $78.1 \pm 2.4\%$ for the $\pi\pi$ and $21.9 \pm 2.4\%$ for the $K\bar{K}$-channel. In the 1998 Review [2], however, the $f_0 \rightarrow \pi\pi$ mode is established as dominant, while the $f_0 \rightarrow K\bar{K}$ mode is stated as seen.

A recent theoretical status of the problem has been presented by Oller and Oset [3] and Krehl, Rapp and Speth [4]. Here we do not attempt to add a further summary on the problem, but discuss the possibility for a direct observation of the $f_0$-meson, though keeping in mind the simplicity of the BW approximation as e.g. pointed out by Janssen et al. [18].

2 The reaction $\pi N \rightarrow f_0 N$

In order to test the validity of the approach [16, 17] we start with the $\pi N \rightarrow f_0 N \rightarrow K\bar{K}$ reaction. The relevant one-pion exchange diagram is shown in Fig. 1; the corresponding differential cross section can be calculated as

$$\frac{d\sigma}{dM} = \int_{m_{\pi \pi}}^{t_{\text{max}}^2} dt \frac{1}{2\pi^3s} \frac{|k|}{|q|^2} |M_{f0}|^2,$$

where $M$ is the invariant $K\bar{K}$-mass, $s$ is the total energy of the pion-nucleon system squared, $q$ is the pion three-momentum in the $\pi N$ center-of-mass frame, while $k$ is the kaon three-momentum in the $f_0$ rest frame and $|k| = \sqrt{M^2 - 4m_f^2}/2$. In (1) $t$ stands for the transferred four-momentum squared $t = (p'_{2} - p_2)^2$, where $p_2, p'_2$ are the four-momenta of the nucleons in the initial and final states.

The matrix element in Eq. (1) is given by

$$M_{if} = g_{\pi NN} \bar{u}(p'_2) i \frac{\gamma_5 u(p_2)}{2} \frac{A_{\pi\pi\rightarrow K\bar{K}}(M)}{t - m_f^2} \times F_{\pi NN}(t) F_{f0\pi\pi}(t).$$

In principle, the $\pi\pi \rightarrow K\bar{K}$ amplitude can be taken as a $K$-matrix solution from the coupled channel analysis of the experimental data on $\pi p$ and $\bar{p}p$ reactions (cf. Anisovich et al. [14]). Another way is to adopt the Breit-Wigner approach and to define the $\pi\pi \rightarrow K\bar{K}$ amplitude as

$$A_{\pi\pi\rightarrow K\bar{K}}(M) = \frac{g_{f_0\pi\pi} f_{f_0KK}}{M^2 - m_{f0}^2 + i m_{f0}^\prime \Gamma_{\text{tot}}(M)},$$

* Supported by Forschungszentrum Jülich, BMBF and DFG
where $g_{f_0\pi\pi}$ and $g_{f_0KK}$ denote the coupling constants, while $m_{f_0}$ and $\Gamma_{tot}$ are the mass and width of the $f_0$-meson, respectively. Finally, the squared matrix element – averaged over the initial and summed over the final states – is given as

$$|M_{\pi f_0}|^2 = \frac{g_{\pi NN}^2 g_{f_0\pi\pi}^2 g_{f_0KK}^2}{(M^2 - m_{f_0}^2)^2 + m_{f_0}^4 \Gamma_{tot}^2(M)} \times \frac{-t}{(t - m_{f_0}^2)^2} F_{\pi NN}(t) F_{f_0\pi\pi}(t),$$

where $F_{\pi NN}$ is the form factor at the $\pi NN$ vertex taken in the monopole form

$$F(t) = \frac{A^2 - m_{f_0}^2}{A^2 - t}$$

with a cut-off parameter $A = 1.05$ GeV [20]. The $\pi NN$ coupling constant is $g_{\pi NN}^2/\Lambda^4 = 14.4$ [19]. $F_{f_0\pi\pi}$ is the form factor at the $f_0\pi\pi$ vertex taken as in [6] with $\Lambda = 1.05$ GeV again. Note that (4) is valid only for low pion energies, since at high energies one needs to Reggeize the reaction amplitude similar to [14,21]. Since the $\pi NN \to f_0NN \to K\bar{K}N$ cross section depends upon the product $g_{f_0\pi\pi}^2 g_{f_0KK}^2$ of the squared couplings and not their values itself, one can fit only the product of the coupling constants by experimental data.

The dominant $f_0$-meson decay channels are the pion and kaon modes [6]. Neglecting other possible modes with extremely small decay branching ratios $Br$, one has to saturate the unitarity condition:

$$Br(f_0\to\pi\pi) + Br(f_0\to K\bar{K}) = 1. \quad (6)$$

The branching ratios $Br(f_0\to\pi\pi)$ and $Br(f_0\to K\bar{K})$ are given by integrals of the Breit-Wigner distribution over the invariant mass of the final particles [22]:

$$Br(f_0\to\pi\pi) = \int_{2m_{\pi}} 2dM \pi \frac{M m_{f_0} \Gamma_{f_0\pi\pi}(M)}{(M^2 - m_{f_0}^2)^2 + m_{f_0}^4 \Gamma_{tot}^2(M)} \quad (7)$$

$$Br(f_0\to K\bar{K}) = \int_{2m_K} 2dM \pi \frac{M m_{f_0} \Gamma_{f_0KK}(M)}{(M^2 - m_{f_0}^2)^2 + m_{f_0}^4 \Gamma_{tot}^2(M)},$$

where the total width of the $f_0$-meson is defined as

$$\Gamma_{tot}(M) = \begin{cases} \Gamma_{f_0\pi\pi}(M), & \text{if } M \leq 2m_K, \\ \Gamma_{f_0\pi\pi}(M) + \Gamma_{f_0KK}(M), & \text{if } M \geq 2m_K, \end{cases} \quad (8)$$

and the partial decay widths $f_0\to\pi\pi$ and $f_0\to K\bar{K}$ are related to the relevant coupling constants as

$$\Gamma_{f_0\pi\pi}(M) = \frac{g_{f_0\pi\pi}^2 \sqrt{M^2 - 4m_{\pi}^2}}{16\pi}, \quad \Gamma_{f_0KK}(M) = \frac{g_{f_0KK}^2 \sqrt{M^2 - 4m_{K}^2}}{16\pi}. \quad (9)$$

Substituting (4) and (8) into (6) one finds that formula (6) provides a unique relation between the coupling constants $g_{f_0\pi\pi}$ and $g_{f_0KK}$. Fig. 2 shows the result of our numerical solution of (6). The upper part of Fig. 2 displays $g_{f_0KK}$ as a function of $g_{f_0\pi\pi}$ in a wide range. The maximum value of $g_{f_0\pi\pi}$ is found to be $\approx 3.33$ and it approaches an asymptotic value of 1.93, whereas $g_{f_0KK}$ always increases with $g_{f_0\pi\pi}$ for values below 3.33. The lower part of Fig. 2 shows the latter range for $g_{f_0\pi\pi}$ and $g_{f_0KK}$ on a larger scale.

In order to fix the $f_0\pi\pi$ and $f_0KK$ coupling constants individually one needs the explicit knowledge of one of the branchings. For instance, taking $Br(f_0\to\pi\pi) = 78.1\%$ [11] we obtain $g_{f_0\pi\pi}$=3.05 GeV and $g_{f_0KK}$=4.3 GeV; according to (6) and (8) the total $f_0$ width then amounts to 233 MeV. Fig. 2 shows this solution as set $B$.

The $\pi^-p\to f_0n\to K^+K^-n$ total cross section calculated with set $B$ is shown by the solid line in Fig. 3 and overestimate the experimental data collected in Refs. [23,24]. Fig. 3 also shows
the data [23] for the $\pi^- p \to K^+ K^- n$ cross section, which is substantially above the $\pi^- p \to f_{0n}$ data since it includes other production mechanisms as e.g. proposed in Ref. [24].

In principle, to fit the $\pi^- p \to f_{0n} \to K^+ K^- n$ data with the coupling constants from set $B$ one might adjust the cut-off $\Lambda$ in (5) at the $f_{0\pi\pi}$ vertex as a free parameter. Fig. 3 shows the calculations with $\Lambda=0.3$ GeV and $\Lambda=0.1$ GeV, which are roughly in line with the absolute magnitude for the $\pi^- p \to f_{0n}$ cross section but contradict its energy dependence.

We conclude that it is not possible to describe the experimental data on the $\pi^- p \to f_{0n} \to K^+ K^- n$ reaction adopting the $f_{0\pi\pi}$ and $f_{0KK}$ coupling constants from set $B$. Moreover, as was shown above, set $B$ yields a large total width for the $f_{0\pi\pi}$-meson (233 MeV) that is out of the range $\Gamma_{f_0}=40$-100 MeV quoted in the 1996-estimation from the Particle Data Group [3].

We now fit the data [8,9] on the $\pi^- p \to f_{0n} \to K^+ K^- n$ cross section by taking the product of the $g_{f_{0\pi\pi}}$ and $g_{f_{0KK}}$ coupling constants as a free parameter. The result is shown in Fig. 4. Our solution for the product of the coupling constants is shown in Fig. 2 as set $A (g_{f_{0\pi\pi}}=1.49$ GeV, $g_{f_{0KK}}=0.82$ GeV) that leads to the following $f_{0\pi\pi}$-meson properties:

$$Br(f_{0 \to \pi\pi}) = 98\%$$
$$Br(f_{0 \to KK}) = 2\%$$
$$\Gamma_{tot} = 44.3\text{ MeV},$$

which are in nice agreement with the numbers from the recent Review of Particle Physics [3].

Fig. 5 displays the $KK$ invariant mass spectrum from $\pi^- p$ collisions at a beam momentum of 3.2 GeV/c in comparison to the experimental data from [8]. The solid line in Fig. 5 indicates our calculation with the parameters from set $A$ which reasonably describes the experimental spectrum.

3 The reaction $NN \to f_{0}NN$

The relevant diagrams for the $NN \to f_{0}NN \to K \bar{K} NN$ reaction are shown in Fig. 3: the corresponding differential cross section is given as

$$\frac{d\sigma}{dM} = \int dE_1 d\eta_1 d\cos \theta_1 d\varphi_1 \frac{1}{2^{11}\pi^{10}} \frac{|k|^2}{|p_1|^2} |M_{1f}|^2,$$

with the matrix element taken as the sum of the direct and exchange terms in Fig. 2.

$$M_{1f} = g^2_{\pi NN} \bar{u}(p')_1 i \gamma_5 u(p_1) \frac{A_{\pi \pi \to KK}(M)}{(q_1^2 - m_{\pi}^2)(q_2^2 - m_{\pi}^2)} \times \bar{u}(p'_2) i \gamma_5 u(p_2) F_{\pi NN}(q_1^2) F_{\pi NN}(q_2^2)$$
$$- g^2_{\pi NN} \bar{u}(p'_1) i \gamma_5 u(p_2) \frac{A_{\pi \pi \to KK}(M)}{(q_1^2 - m_{\pi}^2)(q_2^2 - m_{\pi}^2)} \times \bar{u}(p'_2) i \gamma_5 u(p_1) F_{\pi NN}(q_1^2) F_{\pi NN}(q_2^2),$$
where \( k \) is the kaon three-momentum in the \( f_0 \)-rest frame, \( p_1 \) and \( p_2 \) are the four-momenta of the initial nucleons, while \( p'_1 \) and \( p'_2 \) are the four-momenta of the final nucleons. Moreover, \( p_1 \) is the three-momentum of the initial nucleon in their center-of-mass frame (cms), \( E'_1 \) is the energy of the final nucleon in the cms, \( q_0 \) and \( q \) are the energy and three-momentum of the kaon pair in the cms, respectively. In (12) \( q \) is the polar angle of the vector \( q \) in the cms defined as \( \theta_q = q \cdot p_1 / p_1 \), while \( \varphi_q \) is the azimuthal angle of \( q \) in the cms.

The square of the matrix element (12) – averaged over the initial and summed over the final states – is given by

\[
|M_{ij}|^2 = \frac{g_{\pi NN}^4 g_{f_0 \pi}^2 g_{f_0 KK}^2}{(M^2 - m_{f_0}^2)^2 + m_{f_0}^2 F_{tot}^2(M)} \times F_{\pi NN}(q^2_1) F_{\pi NN}(q^2_2) \frac{q^2_1 q^2_2}{(q^2_1 - m_{\pi^\pm}^2)^2(q^2_2 - m_{\pi^\pm}^2)^2} + \frac{g_{\pi NN}^2 g_{f_0 \pi}^2 g_{f_0 KK}^2}{(M^2 - m_{f_0}^2)^2 + m_{f_0}^2 F_{tot}^2(M)} \times F_{\pi NN}(q^2_1) F_{\pi NN}(q^2_2) \frac{q^2_1 q^2_2}{(q^2_1 - m_{\pi^\pm}^2)^2(q^2_2 - m_{\pi^\pm}^2)^2} + \text{interference term}
\]

Actually one has to introduce a form factor at the \( f_0 \pi \pi \) vertex since both pions are off their mass-shell. Following the assumption from Refs. [25,26] we use the form

\[
F\pi\pi(q^2_1, q^2_2) = F_{\pi NN}(q^2_1) F_{\pi NN}(q^2_2),
\]

where the \( \pi NN \) form factor was taken as in [5] with a cut-off parameter \( A=1.05 \) GeV. The form factor (14) is normalized to unity at \( q^2_1 = m_{\pi^\pm}^2 \) and \( q^2_2 = m_{\pi^\pm}^2 \), which is consistent with the kinematical conditions for the determination of the \( f_0 \pi \pi \) coupling constant.

The dotted line in Fig. 7 shows the \( pp\rightarrow f_0pp\rightarrow K^0\bar{K}^0 pp \) cross section calculated with the coupling constants from set \( A \) and with the form factor (14) in comparison to the experimental data [23] for the \( pp\rightarrow K^0\bar{K}^0 pp \) reaction. The dashed line shows the calculations within the pion and kaon exchange model from Ref. [24] for \( KK \) production. To estimate the maximal \( f_0 \) production cross section we neglect the form factor at the \( f_0 \pi \pi \) vertex and show the result in terms of the solid line in Fig. 7. Actually, the contribution from \( f_0 \) production to the total \( pp\rightarrow K^0\bar{K}^0 pp \) cross section is almost negligible at high energies. However, a possible way for \( f_0 \) observation is due to the low energy part of the \( K\bar{K} \) invariant mass spectrum.

We thus calculate the \( K^+K^- \) invariant mass spectrum from the \( pp\rightarrow K^+K^- pp \) reaction at a beam energy of 2.85 GeV, which corresponds to the kinematical conditions for the DISTO experiment at SATURNE [5]. Since at this energy the \( \phi \)-meson production becomes possible we include its contribution to the \( K^+K^- \) spectrum. The \( pp\rightarrow \phi pp \) total cross section was taken from Ref. [27] and the \( K^+K^- \) invariant mass was distributed.
according to the Breit-Wigner resonance prescription with a full \( \phi \)-meson width \( \Gamma_\phi = 4.43 \) MeV and the branching ratio \( Br(\phi \rightarrow K^+K^-) = 49.1\% \) [3].

The dotted line in Fig. 9 shows the \( K^+K^- \) invariant mass spectrum for the \( pp \rightarrow \phi pp \) reaction while the dash-dotted line indicates the spectrum from the \( pp \rightarrow K^+K^- pp \) reaction, which was calculated as in Ref. [24] on the basis of pion and kaon exchange diagrams. The solid line in Fig. 8 shows the total \( K^+K^- \) spectrum.

To test the possibility for a direct \( f_0 \) observation via the \( K^+K^- \) spectrum from \( pp \) collisions one should compare the total \( K^+K^- \) production cross section from meson-exchange diagrams and \( \phi \)-decay (denoted as background) with the explicit contribution from the \( pp \rightarrow f_0 pp \rightarrow K^+K^- pp \) reaction. The solid line in Fig. 9 shows the background while the dashed line indicates the \( K^+K^- \) spectrum calculated with the coupling constants from set \( A \) and without form factor at the \( f_0\pi\pi \) vertex. If the \( f_0\pi\pi \) and \( f_0K\bar{K} \) coupling constants are determined by the set \( A \), then it is quite obvious that the \( f_0 \)-meson cannot be directly detected in \( pp \) collisions by using the \( K^+K^- \)-mode. Note, that when introducing a form factor [14] at the \( f_0\pi\pi \) vertex the contribution from \( pp \rightarrow f_0 pp \rightarrow K^+K^- pp \) becomes even smaller.

To test the sensitivity of the model upon the \( f_0 \) parameters we also perform the calculation with set \( B \) and show the result in terms of the dotted line in Fig. 9. Indeed, in that case the \( f_0 \) contribution is very strong at low \( K^+K^- \) invariant mass. Thus experimental data from DISTO might be crucial for the examination of the \( f_0 \) properties.

We also test the possibility for \( f_0 \) detection by use of the \( K^+K^- \) spectrum from \( pp \) collisions at energies very close to the \( pp \rightarrow K^+K^- pp \) reaction threshold, i.e. for the kinematical conditions available at the COSY accelerator. Fig. 10 shows our calculations for the excess energies \( \sqrt{s} = 2m_N - 2m_K \) of 5 and 50 MeV. The dashed lines in Fig. 10 show the \( K^+K^- \) production calculated again in accordance with [24]. The solid lines correspond to our results obtained with the \( f_0\pi\pi \) and \( f_0K\bar{K} \) coupling constants from set \( A \), while the dotted lines are calculations with set \( B \). Note that the results shown in Fig. 10 are obtained without a form factor at the \( f_0\pi\pi \) vertex, thus they should be considered as upper limits for the \( f_0 \)-meson contribution. Furthermore, at an excess energy of 5 MeV strong final state interactions between the two protons should enhance the yield substantially. However, these final state interactions are of similar strength in both reaction channels and may be disregarded in their ratio, which is the relevant quantity here.

We conclude that at an excess energy of 5 MeV, i.e. very close to the \( pp \rightarrow K^+K^- pp \) reaction threshold, the contribution from \( f_0 \)-meson production is almost negligible and cannot be separated from the background processes. At \( \sqrt{s} = 2m_N - 2m_K = 50 \) MeV the contribution from the \( pp \rightarrow f_0 pp \rightarrow K^+K^- pp \) reaction, as a maximal estimation, is a few times less than the contribution from \( K^+K^- \)-pair production due to pion and kaon exchange diagrams.
4 Conclusions

We have investigated the production of $f_0$-mesons in $pp$ interactions and the possibility for its observation via the $f_0 \rightarrow K\bar{K}$ mode. Our calculations have been based upon the one-pion exchange model and a Breit-Wigner prescription for the $f_0$ resonance which allows for a quantitative estimate. The coupling constants at the $f_0\pi\pi$ and $f_0K\bar{K}$ vertices have been constraint by experimental data on the $\pi N \rightarrow f_0 N$ reaction; this approach gives a full $f_0$-meson width of 44.3 MeV and the branching ratios $Br(f_0 \rightarrow \pi\pi) = 98\%$, $Br(f_0 \rightarrow K\bar{K}) = 2\%$. Our estimation is in line with the $f_0$ properties from the recent 1998 Review of Particle Physics [2], but substantially contradicts the numbers from the 1996 Review [1].

It is found that the $K^+K^-$ invariant mass distribution from the $pp \rightarrow K^+K^-pp$ reaction at a beam energy of 2.85 GeV, which corresponds to the experimental condition for the DISTO experiment at SATURNE, might be sensitive to the $f_0$-meson properties. With the $f_0$ properties given by the 1996 Review of Particle Physics [1] the $f_0$ signal should be seen as an enhancement in the low energy part of the $K^+K^-$ mass spectrum. However, following our estimation, we do not expect such an enhancement and predict a $K^+K^-$ invariant mass spectrum as shown in Fig. 9 (set A).

The possibility for the $f_0$-meson observation in $pp \rightarrow K\bar{K}pp$ reactions at near-threshold energies available at the COSY accelerator was studied, too. Our calculations indicate that no $f_0$ signal might be extracted from the $K^+K^-$ invariant mass spectrum due to the large background contribution from other reaction channels [24] as arising from pion and kaon exchanges.

We appreciate stimulating discussions with C. Hanhart, J. Haidenbauer, W. Kühn, J. Ritman and M.A. Smondyrev.

References

1. Particle Data Group, Phys. Rev. D 54, 1 (1996)
2. Particle Data Group, Eur. Phys. J. C 3, 1 (1998)
3. J.A. Oller, E. Oset, hep-ph/9809337
4. O. Krehl, R. Rapp, J. Speth, Phys. Lett. B 390, 23 (1997)
5. F. Balestra et al., Phys. Rev. Lett. (1998), in press
6. J. Smirsky, T. Lister, et al., COSY Proposal No. 46.1 (1997)
7. S.V. Bashinskii, R.L. Jaffe, Nucl. Phys. A 625, 167 (1997)
8. O.I. Dahl, et al., Phys. Rev. 163, 1377 (1967)
9. W. Beusch, et al., Phys. Lett. B 25, 357 (1967)
10. A.J. Pawlicki, et al., Phys. Rev. D 12, 631 (1975)
11. N.M. Cason, et al., Phys. Rev. Lett. 41, 271 (1978)
12. D. Cohen, et al., Phys. Rev. D 22, 2595 (1980)
13. A. Etkin, et al., Phys. Rev. D 25, 1786 (1982)
14. V.V. Anisovich, et al., Phys. Lett. B 355, 363 (1995)
15. S.V. Bashinskii, B. Kerbikov, Phys. Atom. Nucl. 59, 1979 (1996)
16. D. Morgan, M.R. Pennington, Phys. Rev. D 48, 1185 (1993)
17. D. Morgan, M.R. Pennington, Phys. Rev. D 48, 5422 (1993)
18. G. Janssen, B.C. Pearce, K. Holinde, J. Speth, Phys. Rev. D 52, 2690 (1995)
19. R. Machleid, K. Holinde, Ch. Elster, Phys. Rep. 149, 1 (1987)
20. A. Sibirtsev, K. Tsushima, A.W. Thomas, Phys. Lett. B 421, 59 (1998)
21. N.N. Achasov, G.N. Shestakov, Phys. At. Nucl. 60, 1522 (1997)
22. N.N. Achasov, V.V. Gubin, Phys. Lett. B 363, 106 (1995)
23. Landolt-Börnstein, New Series, ed. H. Schopper, I/12 (1988).
24. A.A. Sibirtsev, W. Cassing, C.M. Ko, Z. Phys. A 358, 101 (1997)
25. W.S. Chung, G.Q. Li, C.M. Ko, Nucl. Phys. A 625, 371 (1997)
26. K. Nakayama, A. Szczurek, C. Hanhart, J. Haidenbauer, J. Speth, Phys. Rev. C 57, 1580 (1998)
27. A.A. Sibirtsev, Nucl. Phys. A 604, 455 (1996)