Cosmological backreaction of heavy string states

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Abstract

We propose a mechanism to have a smooth transition from a pre-Big Bang phase to a standard cosmological phase. Such transition is driven by gravitational production of heavy massive string states that backreact on the geometry to stop the growth of the curvature. Close to the string scale, particle creation can become effective because the string phase space compensate the exponential suppression of the particle production. Numerical solutions for the evolution of the Universe with this source are presented.
1 Introduction

The standard cosmological model, even in its inflationary extension, is plagued by an “initial condition problem” [1], since it must emerge from an initial singularity. One of the most successful proposal to overcome this problem was the introduction of the so-called pre-Big Bang scenario [2] (for a review, see [3]), developed in the context of string theory, in which the Universe is supposed to emerge from a string vacuum (so the initial geometry is flat), perturbed by quantum fluctuations of the dilaton (other proposals were put forward in different contexts, such as brane cosmology and Loop Quantum Gravity, see for example [4–10]). This is achieved by implementing purely stringy symmetries on the string cosmology equations. Those symmetries suggest the existence of a phase of growing curvature. However, it is not clear, a priori how to connect smoothly the pre-big bang and the post-big bang phases, since, at the classical level, the two phases are still disconnected by a singularity. Many ideas were proposed to overcome this problem, all involving further assumptions on the dynamics of the Universe, namely higher order correction (either loop or $\alpha'$ contribution) and negative energy density contribution to obtain the desired “repulsive gravity” (an incomplete list would include [11–16]).

In this paper we will propose a mechanism to obtain a regular evolution of the curvature that rely on production of heavy massive string states by gravitational backreaction. It is well known [17] that an evolving Universe will produce particles because of the squeezing of the ground state. It has been speculated in the literature that this effect, though being negligible, could actually have some physical consequences [18–20]. The idea is that the exponential suppression of the energy density of particles created during the evolution of the Universe is compensated by the exponential growing of the multiplicity of states above the string scale (the Hagedorn spectrum), thus giving a sensible contribution to the total energy density. Of course, dealing with gravitationally induced particle production brings one to deal with a sort of semiclassical realization of quantum gravity, which is not under control, and in fact the authors themselves of the mentioned papers admit some of the ideas are rather speculative. Here we want to go one step forward with speculations, and assume that the gravitationally produced energy backreacts locally on the geometry of the Universe. We will see that even a rough estimate such as the one we are going to show here is enough to stop the growing of the Ricci curvature and induce a bell-shaped behaviour. Let us stress that all the analysis is performed under the assumption that the universe stays in a low-energy (low curvature) regime in which supergravity is a valid approximation, and the only ingredient taken from stringy physics is the behaviour of the high energy spectrum that influences the phase space of low curvature universe. In fact, we are going to treat string massive modes as scalars, assuming however that their multiplicity is controlled by the Hagedorn spectrum. The validity of this assumption is verified by checking that the
curvature found with the modified cosmological equations do not exceed the string scale (which has been set to unity in the numerical evaluations).

We are aware that some of the passages we will show, though physically well-motivated, can be challenged on a formal basis. Nevertheless, since our goal was only to check if a backreaction mechanism could provide for a graceful exit without invoking any further assumptions and higher order modifications of the string cosmology equations, we just skipped over formal complexities to have an estimate of the order of magnitude of the effect. We plan to come back on all the mathematical issues (among other things we will discuss below) in a forthcoming paper.

The paper is organized as follows: Section 2 is devoted to some formal developments that will lead us to write the energy density (and pressure) produced by gravitationally excited string states in term of a time dependent Bogoliubov coefficient. In section ?? we will use this expression to find, under certain assumption, a sound expression for the energy density, to be put into the string cosmology equations which are solved numerically in section 4. Finally, in section 5 we will comment on our results and the (rather strong) approximations assumed in obtaining them, and on how to further develop the present study in several directions.

2 QFT approach to gravitational backreaction

Let us then consider a massive scalar field living in a curved space. The action is

\[ S = \frac{1}{2} \int d^4x \sqrt{|g|} \left( g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - m^2 \phi^2 \right). \]

(1)

Let us assume that the geometry of the Universe is a conformally flat FRW; so, in the conformal time gauge, the metric is

\[ ds^2 = a^2(\eta) \left( d\eta^2 - dx^2 \right), \]

(2)

and the action can be written as

\[ S = \frac{1}{2} \int d\eta d^3x \ a^2(\eta) \left( \phi'^2 - (\nabla \phi)^2 - m^2 a^2(\eta) \phi^2 \right). \]

(3)

where \((\nabla \phi)^2\) stands for \(\sum_i(\partial_i \phi)^2\). Now we introduce the canonical field \(\varphi(\eta, x) = a(\eta) \phi(\eta, x)\), so that the action can be rewritten as

\[ S = \frac{1}{2} \int d\eta d^3x \left[ \eta^\mu\nu \partial_\mu \varphi \partial_\nu \varphi - \left( m^2 a^2(\eta) - \frac{a''(\eta)}{a(\eta)} \right) \varphi^2 \right]. \]

(4)

This action can be interpreted as describing a scalar field living on a flat space-time, with a potential which controls its gravitational interactions. From the action (4) we can evaluate
the (canonical) energy momentum tensor.

\[ T_{\mu \nu} = \frac{\partial L}{\partial (\partial_\nu \varphi)} \partial_\mu \varphi - L \delta_{\mu \nu} \]

\[ = \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} \left[ (\partial_\mu \varphi)^2 - \left( m^2 a^2 - \frac{a''}{a} \right) \varphi^2 \right] \delta_{\mu \nu}. \]  

(5)

The diagonal elements of this tensor are the energy density and the pressure. The field conjugate to \( \varphi \) is

\[ \pi(\eta, x) = \frac{\partial L}{\partial \varphi'}, \]  

(6)

so the Hamiltonian density, to be identified with the energy density, and the pressure along the \( i \)th direction are, according to (5)

\[ \rho_m(\eta, x) = \frac{1}{2} \left[ \pi^2 + (\nabla \varphi)^2 + V(\eta) \varphi^2 \right], \]

\[ p_{m, i}(\eta, x) = \frac{1}{2} \left[ \pi^2 + (\nabla \varphi)^2 - V(\eta) \varphi^2 \right], \]  

(7)

where, of course, \( V(\eta) = m^2 a^2(\eta) - \frac{a''(\eta)}{a(\eta)} \).

Now we expand the fields in Fourier modes and, following conventional quantization procedure, we promote (\( \eta \)-dependent) Fourier coefficients to operators:

\[ \varphi(\eta, x) = \frac{1}{(2\pi)^{3/2}} \int d^3k \varphi_k(\eta) e^{ik \cdot x}, \]

\[ \pi(\eta, x) = \frac{1}{(2\pi)^{3/2}} \int d^3k \pi_k(\eta) e^{ik \cdot x}, \]  

(8)

where \( \pi_k(\eta) = \varphi'_k(\eta) \) and \( \varphi_k(\eta) \) satisfies the (operatorial) evolution equation:

\[ \varphi''_k(\eta) + \omega^2(\eta) \varphi_k(\eta) = 0, \]  

(9)

with \( \omega_k(\eta) = \sqrt{k^2 + V(\eta)} \). Moreover, since the field \( \varphi \) is real, and thus Hermitean, the modes must satisfy the relation

\[ \varphi_k^\dagger(\eta) = \varphi_{-k}(\eta), \]

\[ \pi_k^\dagger(\eta) = \pi_{-k}(\eta). \]  

(10)

We can then write down the energy density and pressure in terms of these modes. By defining the vector

\[ \zeta_k(\eta) = \begin{pmatrix} \varphi_k(\eta) \\ \pi_k(\eta) \end{pmatrix}, \]  

(11)

the densities (7) can be rewritten as

\[ \rho_m(\eta, x) = \frac{1}{2} \int \frac{d^3k \; d^3p}{(2\pi)^3} \zeta_k^\dagger(\eta) P_{k,p}(\eta) \zeta_p(\eta) e^{i(k-p) \cdot x}, \]

\[ p_{m, i}(\eta, x) = \frac{1}{2} \int \frac{d^3k \; d^3p}{(2\pi)^3} \zeta_k^\dagger(\eta) Q_{k,p}^{(i)}(\eta) \zeta_p(\eta) e^{i(k-p) \cdot x}, \]  

(12)
with
\[
P_{k,p}(\eta) = \begin{pmatrix} k \cdot p + V(\eta) & 0 \\ 0 & 1 \end{pmatrix},
\]
\[
Q^{(i)}_{k,p}(\eta) = \begin{pmatrix} k \cdot p - V(\eta) & 0 \\ 0 & 1 \end{pmatrix}.
\]
This expression is useful, because we know [21, 22] that the evolution for \(\zeta_k(\eta)\) is given by:
\[
\zeta_k(\eta) = U_k(\eta, \eta_0)\zeta_k(\eta_0),
\]
where the propagator can be written as
\[
U_k(\eta, \eta_0) = \begin{pmatrix} A_k(\eta, \eta_0) & B_k(\eta, \eta_0) \\ C_k(\eta, \eta_0) & D_k(\eta, \eta_0) \end{pmatrix},
\]
and the four coefficients of the matrix are
\[
A_k(\eta, \eta_0) = i [g_k(\eta_0)f_k^*(\eta) - g_k^*(\eta_0)f_k(\eta)],
\]
\[
B_k(\eta, \eta_0) = i [f_k(\eta)f_k^*(\eta_0) - f_k^*(\eta)f_k(\eta_0)],
\]
\[
C_k(\eta, \eta_0) = i [g_k(\eta_0)g_k^*(\eta) - g_k^*(\eta_0)g_k(\eta)],
\]
\[
D_k(\eta, \eta_0) = i [g_k(\eta)f_k^*(\eta_0) - g_k^*(\eta)f_k(\eta_0)].
\]
Here \(f_k(\eta)\) is solution of the canonical evolution equation (9) for modes of the field \(\varphi\) and \(g_k(\eta)\) is solution of the mode equation (which we have not reported) for the conjugate momentum \(\pi\) (so, of course, the relation \(g_k(\eta) = f_k(\eta)\) holds), while \(\eta_0\) is a suitable time on which we will define initial condition. We will assume that \(\eta_0\) is far enough in the past so that it is possible to consider the space to be Minkowski. Using eq. (14) we can thus write:
\[
\rho_m(\eta, \mathbf{x}) = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \frac{d^3p}{(2\pi)^3} \zeta_k^\dagger(\eta_0) S_{k,p}(\eta, \eta_0) \zeta_p(\eta_0) e^{i(k-p) \cdot \mathbf{x}},
\]
\[
p_{m,i}(\eta, \mathbf{x}) = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \frac{d^3p}{(2\pi)^3} \zeta_k^\dagger(\eta_0) T_{k,p}^{(i)}(\eta, \eta_0) \zeta_p(\eta_0) e^{i(k-p) \cdot \mathbf{x}},
\]
where,
\[
S_{k,p}(\eta, \eta_0) = U_k^\dagger(\eta, \eta_0) P_{k,p} U_k(\eta, \eta_0),
\]
\[
T_{k,p}^{(i)}(\eta, \eta_0) = U_k^\dagger(\eta, \eta_0) Q_{k,p}^{(i)} U_k(\eta, \eta_0).
\]
Now we want to express this in terms of creation-annihilation operators. Notice that on an actual Minkowski space, there are no ambiguities in building a Fock space; so we define:
\[
\zeta_k(\eta_0) = \left( \frac{1}{\sqrt{2\omega_k}} \right) a_k + \left( \frac{1}{i\sqrt{2\omega_k}} \right) a_k^\dagger,
\]
where \(\omega_k = \sqrt{p^2 + \frac{m^2}{c^4}}\).
with \( a_k \) and \( a_k^\dagger \) satisfying the usual canonical commutation rules and \( \omega_k \equiv \omega_k(\eta_0) \). Inserting this expression in (17) we get

\[
\rho_m = \frac{1}{2} \int \frac{d^3k \ d^3p}{(2\pi)^3} \left[ S^{(1)}_{k,p} a_k^\dagger a_p + S^{(2)}_{k,p} a_k^\dagger a_{-p}^\dagger + S^{(3)}_{k,p} a_{-k} a_p + S^{(4)}_{k,p} a_{-k} a_{-p}^\dagger \right] e^{i(k-p)\cdot x},
\]

\[
p_{m,i} = \frac{1}{2} \int \frac{d^3k \ d^3p}{(2\pi)^3} \left[ T^{(i,1)}_{k,p} a_k^\dagger a_p + T^{(i,2)}_{k,p} a_k^\dagger a_{-p}^\dagger + T^{(i,3)}_{k,p} a_{-k} a_p + T^{(i,4)}_{k,p} a_{-k} a_{-p}^\dagger \right] e^{i(k-p)\cdot x},
\]

where the \( S^{(i)} \) and \( T^{(i,j)} \) are calculated from the bilinear product of the matrices \( S_{k,p} \) and \( T_{k,p}^{(i)} \) respectively, with the vectors defined in (19). In taking the VEV of (20), only the term with the correct order survives. Then the \( \delta \) function allows us to integrate over one of the two (equal) momenta. Moreover, in order to handle with this expression in a cosmological context (as we are going to do), we assume that particles are created homogeneously throughout the space, so that the integral in (20) depends only on the square momentum \( k^2 \) of the excitation. We thus get

\[
\langle 0 | \rho_m(\eta) | 0 \rangle = 2\pi \int_0^{+\infty} dk \ k^2 S^{(4)}_{k,k}(\eta, \eta_0),
\]

\[
\langle 0 | p_{m,i}(\eta) | 0 \rangle = 2\pi \int_0^{+\infty} dk \ k^2 T^{(i,4)}_{k,k}(\eta, \eta_0).
\]

The coefficients \( S^{(4)} \) and \( T^{(i,4)} \) are easily calculated. After a little algebra we get:

\[
S^{(4)}_{k,k}(\eta, \eta_0) = \frac{1}{2\omega_k} \left[ \left( k^2 + V(\eta) \right) |A_k(\eta, \eta_0)|^2 + |C_k(\eta, \eta_0)|^2 \right] + \frac{\omega_k}{2} \left[ \left( k^2 + V(\eta) \right) |B_k(\eta, \eta_0)|^2 + |D_k(\eta, \eta_0)|^2 \right],
\]

\[
T^{(i,4)}_{k,k}(\eta, \eta_0) = \frac{1}{2\omega_k} \left[ \left( k^2 - V(\eta) \right) |A_k(\eta, \eta_0)|^2 + |C_k(\eta, \eta_0)|^2 \right] + \frac{\omega_k}{2} \left[ \left( k^2 - V(\eta) \right) |B_k(\eta, \eta_0)|^2 + |D_k(\eta, \eta_0)|^2 \right].
\]

This expression can be further specified by noting that, if the space-time is flat in \( \eta = \eta_0 \), solutions for the evolution equation (9) can be written as:

\[
f_k(\eta \to \eta_0) = \frac{1}{\sqrt{2\omega_k}} e^{-i\omega_k(\eta-\eta_0)},
\]

\[
g_k(\eta \to \eta_0) = -i \sqrt{\frac{\omega_k}{2}} e^{-i\omega_k(\eta-\eta_0)},
\]

with \( \omega_k = \sqrt{k^2 + m^2} \) (\( m \) is the mass of the excitation). Furthermore, we get a better insight on the expressions of energy density and pressure by writing the mode functions \( f_k \) and \( g_k \).
in terms of time-dependent Bogoliubov operators \([19, 23]\):

\[
\begin{align*}
  f_k(\eta) &= \frac{\alpha_k(\eta)}{\sqrt{2\omega_k(\eta)}} \exp\left(-i \int_{\eta_0}^{\eta} dx \omega_k(x)\right) + \frac{\beta_k(\eta)}{\sqrt{2\omega_k(\eta)}} \exp\left(i \int_{\eta_0}^{\eta} dx \omega_k(x)\right), \\
  g_k(\eta) &= -i \sqrt{\frac{\omega_k(\eta)}{2}} \alpha_k(\eta) \exp\left(-i \int_{\eta_0}^{\eta} dx \omega_k(x)\right) + i \sqrt{\frac{\omega_k(\eta)}{2}} \beta_k(\eta) \exp\left(i \int_{\eta_0}^{\eta} dx \omega_k(x)\right),
\end{align*}
\]

(24)

with \(\alpha_k\) and \(\beta_k\) satisfying the condition \(|\alpha_k|^2 - |\beta_k|^2 = 1\), which follows from the Wronskian condition on \(f_k\) and \(g_k\). Substituting this in (22), and plugging everything into the expressions for the energy density and pressure (21) we get:

\[
\begin{align*}
  \rho_m(\eta) &= 2\pi \int_0^{+\infty} dk k^2 \left(1 + 2|\beta_k(\eta)|^2\right) \omega_k(\eta), \\
  p_m(\eta) &= 2\pi \int_0^{+\infty} dk \frac{k^4}{\omega_k(\eta)} (1 + 2|\beta_k(\eta)|^2).
\end{align*}
\]

(25) (26)

In the next section, we will use this expressions to obtain the energy density as a function of the geometry of the on-shell universe.

### 3 Energy density of the gravitationally excited string states

In the previous section, we obtained a formal expression for the energy density and pressure of massive scalar modes during the evolution of the universe. At this stage we can already notice that, since (26) is suppressed by a factor \(m\) with respect to (25), the pressure will be negligible with respect to the energy density for large mass (non-relativistic) modes. Therefore, from now on we will assume that the pressure is exactly zero, leaving a more detailed treatments for future works.

Asymptotically, the Bogoliubov coefficient \(\beta_k(\eta)\) can be interpreted as the number density of particles produced via gravitational interaction from an initial vacuum state. Let us stress that, by setting \(\beta_k(\eta) = 0\), which is true for \(\eta \to \eta_0\), expression (25) reduces just to the unrenormalized zero point energy of the field. Thus we are led to consider a properly renormalized form for (25) as the energy density of massive excitations of the field under consideration, generated by gravitational interaction. We choose to regularize the energy density by subtracting the time-dependent zero-point energy of the oscillations, so to get

\[
\rho_m(\eta) = 4\pi \int_0^{+\infty} dk |\beta_k(\eta)|^2 \omega_k(\eta).
\]

(27)

Such expression for the energy density has the key feature of vanishing at \(\eta = \eta_0\) (as it is expected since we want a vacuum solution to start with), and in general whenever the space-time is flat. In addition, it has the expected form of a sum over energies of each
mode\(^1\), following the interpretation of \(\beta\) as the number density of gravitationally produced particles, with the spectrum modified by gravitational interaction.

We now need to evaluate \(\beta_k(\eta)\), which is in general a very difficult task. If we assume that the evolution of the Universe is “slow” enough, so that we are in an adiabatic regime [17], we can use for \(\beta_k(\eta)\) the approximate expression [20]

\[
\beta_k(\eta) = \int d\eta \frac{\omega_k^2(\eta)}{2\omega_k(\eta)} \exp \left( -2i \int_{\eta_0}^{\eta} d\eta' \omega_k(\eta') \right). \tag{28}
\]

The integral can be evaluated, to the same level of approximation, with the steepest descent method [19, 20] giving:

\[
|\beta_k(\eta)|^2 = \exp \left[ -4 \frac{\omega_k^2(\eta)}{V(\eta)\sqrt{\frac{6H_{\text{eff}}^2(\eta) - R_{\text{eff}}(\eta)}{6m^2}}} \right], \tag{29}
\]

where \(H_{\text{eff}}\) and \(R_{\text{eff}}\) are respectively the Hubble parameter and the scalar curvature, calculated starting from an effective scale factor

\[
a_{\text{eff}}(\eta) = \sqrt{a^2(\eta) - \frac{a''(\eta)}{m^2 a(\eta)}}. \tag{30}
\]

With this expression for \(\beta_k\), the integral in (27) converges and can be evaluated exactly. After some technical manipulations we get:

\[
\rho_m(\eta) = -\frac{\pi^2}{8} V^2(\eta) \sqrt{\frac{6H_{\text{eff}}^2(\eta) - R_{\text{eff}}(\eta)}{6m^2}} \left( \frac{i}{H_{\text{eff}}^2(\eta) - \frac{R_{\text{eff}}(\eta)}{6}} \right) \exp \left( -\frac{2m}{\sqrt{H_{\text{eff}}^2(\eta) - \frac{R_{\text{eff}}(\eta)}{6}}} \right),
\]

where \(H_{\text{eff}}^{(1)}\) is the Hankel function of the first kind. For heavy massive states we can approximate the Hankel function with its large argument expansion [24], thus getting:

\[
\rho_m(\eta) = \frac{\pi^{3/2}}{8} V^2(\eta) \left( \frac{6H_{\text{eff}}^2(\eta) - R_{\text{eff}}(\eta)}{6m^2} \right)^{3/4} \exp \left( -\frac{4m}{\sqrt{H_{\text{eff}}^2(\eta) - \frac{R_{\text{eff}}(\eta)}{6}}} \right). \tag{32}
\]

As expected, the contribution for each massive mode is exponentially suppressed at low (sub-Planckian) energies. Incidentally, being at sub-Planckian energies justifies our quantum field theory approach to production of string modes (which, of course, have masses above that limit), as we stressed in the introduction. Now, as we already mentioned above, the multiplicity of string modes is exponentially growing [19] \(N \propto \exp(m/T_H)\) (\(T_H\) being the Hagedorn temperature), and this can possibly compensate the exponential suppression

\(^1\)Note that \(\omega_k(\eta)\) is not exactly the energy of the single mode, since it contains also the “pump field” term.
of each produced mode. In fact, the total energy density generated by all the stringy massive modes is:

$$\rho_{\text{tot}}(\eta) = \sum_{m \geq m_s} \rho_m(\eta) \exp\left(\frac{m}{T_H}\right).$$

(33)

Again, this expression can be estimated by noting that $a_{\text{eff}}(\eta) \simeq a(\eta)$ for heavy enough states, and approximating the series as an integral, substituting $m \simeq T_H x$, with $x > 0$. We get:

$$\rho_{\text{tot}}(\eta) \simeq \frac{\pi^3}{8} T_H^{5/2} a^4 \left(\frac{H^2 - \frac{R}{6}}{x^{5/2}}\right)^{3/4} \int_0^{+\infty} dx \, x^{5/2} \exp \left[-x \left(\frac{T_H}{\sqrt{H^2 - \frac{R}{6}}} - 1\right)\right].$$

(34)

Since the energy density is a scalar, turning to cosmic time is straightforward. We finally get:

$$\rho_{\text{tot}}(t) \simeq T_H^4 a^4(t) \left(\frac{H^2(t) - \frac{R(t)}{6}}{T_H^2}\right)^{5/2}.$$  

(35)

In the next section, we will use this expression as a source in string cosmology equations.

4 Cosmology with the backreacting energy density

In the last section we have found an analytic expression for the energy density massive string states excited by the gravitational interaction with an evolving universe. Now we assume that this energy density backreacts on the geometry of the Universe to favor a smooth transition between a phase of superinflationary accelerated expansion with growing curvature to a (standard) phase of decelerated expansion and decreasing curvature. To see this, we plug the expression (35) into the pre-big bang equations. We assume that the extra dimension are compactified down to the Plank scale, and there is a mechanism for the stabilization of the modula. The string cosmology equations then read [3]

$$\dot{\phi}^2 - 6H \dot{\phi} + 6H^2 = \frac{2}{T_H^2} e^\phi \rho_{\text{tot}},$$

(36)

$$2\ddot{\phi} + 6H \dot{\phi} - \dot{\phi}^2 - 6\dot{H} - 12H^2 = 0.$$  

(37)

(we have identified the string mass with $T_H$) where $\rho_{\text{tot}}$ is given by (35) and we have assumed that matter is minimally coupled to the metric of the string frame, without any direct dilaton coupling, see, e.g. [25] (we will comment more on this later). Of course, these equations cannot be solved analytically, so we must turn to numeric. Plots for the interesting physical quantities are shown in Fig. 1. Numerical integration has been carried out by fixing a
dummy initial timescale so that the maximum value of the curvature occurs at $t = 0$, and by setting suitable (small, in units of string mass) initial condition for the Hubble parameter and the dilaton energy density. As we can see from panel (b), the curvature reaches a maximum and decreases, evolving towards a phase of De Sitter evolution (i.e. with our notations, a constant curvature regime). It is possible to check, in addition, that all the approximation requirements are met. On the other hand, the dilaton energy density, depicted in panel (a) with the squared Hubble parameter is not under control and blows up, after reaching a first local maximum at the same time age of the curvature (see also plot (d)). This is not surprising, since we have not considered the coupling of the generated massive states with the dilaton and the (possible) corresponding dilaton stabilization. Motivated by the successful transition to a phase of decelerated expansion we found in our approach, we seek to extend the present analysis to the dilatonic sector in a forthcoming paper, so to
have a full-fledged graceful exit solution.

5 Conclusions and Outlook

Particle creation generated by gravitational backreaction seems to provide a cogent mechanism to stop superinflationary accelerated expansion and to drive the Universe to a (standard) phase of decelerated expansion. Calculations we have presented in this paper, though being speculative in some of their aspects, show that, as the curvature grows, there is the possibility that gravitational energy excites heavy string states, thus producing some sort of “friction” effect. This can actually slow down and eventually stop the growth of the curvature and prevent the occurrence of the singularity. This can happen before the curvature reaches the string scale, so before the universe enters in a regime in which quantum field theory and supergravity are no longer valid.

However, the analysis we have performed needs to be further developed in many directions: First of all, some of the mathematical derivations need to be posed on a more steady ground (for example, our derivation rely on the assumption that the metric is conformally flat, so that formal results valid in a flat space can be straightforwardly extended to our curved model). In addition, as we already noted, the particle creation mechanism has to include a direct coupling to the dilaton. In fact, even though the geometry is regular everywhere in the toy model we have presented, the dilaton still blows up, indicating a breakdown of the validity of our approximations. Nevertheless, since the dilaton itself is a string mode, it is expected to couple with massive states, so high energetic dilaton modes will somehow excite heavy modes as well. This will probably provide a mechanism to stabilize the dilaton after the transition.

Moreover, to actually enter into a phase of standard evolution, one should consider some sort of “reheating process”. The idea is that string states generated via gravitational backreaction decays into radiation and matter, to start the standard phase. Another issue worth to be pursued is to have a complete dynamical system analysis. In fact, our calculations confirm the known result that late-time evolution of the Universe falls toward a de Sitter phase. It is not clear, however, if the de Sitter curvature scale is controlled by the string scale, or can be influenced by backreaction. In the second case, maybe one can trigger the mechanism (in addition to other contributions coming from a correct approach to modula and extra-dimension stabilization) to lower the de Sitter scale towards some realistic value. In this way we could possibly use the same tools that cure the pathological behaviour of the early Universe also to address problems of our late time Universe.
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