The different baryonic Tully-Fisher relations at low masses.

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ABSTRACT

We compare the Baryonic Tully-Fisher relation (BTFR) of simulations and observations of galaxies ranging from dwarfs to spirals, using various measures of rotational velocity \( V_{\text{rot}} \). We explore the BTFR when measuring \( V_{\text{rot}} \) at the flat part of the rotation curve, \( V_{\text{flat}} \), at the extent of HI gas, \( V_{\text{last}} \), and using 20\% (\( W_{20} \)) and 50\% (\( W_{50} \)) of the width of HI line profiles. We also compare with the maximum circular velocity of the parent halo, \( V_{\text{DM}} \), within dark matter only simulations. The different BTFRs increasingly diverge as galaxy mass decreases. Using \( V_{\text{last}} \) one obtains a power law over four orders of magnitude in baryonic mass, with slope similar to the observed BTFR. Measuring \( V_{\text{flat}} \) gives similar results as \( V_{\text{last}} \) when galaxies with rising rotation curves are excluded. However, higher rotation velocities would be found for low mass galaxies if the cold gas extended far enough for \( V_{\text{rot}} \) to reach a maximum. \( W_{20} \) gives a similar slope as \( V_{\text{last}} \) but with slightly lower values of \( V_{\text{rot}} \) for low mass galaxies, although this may depend on the extent of the gas in your galaxy sample. \( W_{50} \) bends away from these other relations toward low velocities at low masses. By contrast, \( V_{\text{DM}} \) bends toward high velocities for low mass galaxies, as cold gas does not extend out to the radius at which halos reach \( V_{\text{DM}} \). Our study highlights the need for careful comparisons between observations and models: one needs to be consistent about the particular method of measuring \( V_{\text{rot}} \), and precise about the radius at which velocities are measured.

Key words: galaxies: evolution - formation - haloes cosmology: theory - dark matter

1 INTRODUCTION

Galaxies follow a tight relation between optical luminosity and the width of the 21 cm line of neutral hydrogen HI (Tully & Fisher 1977). The 21 cm line-width is a measure of rotation velocity \( V_{\text{rot}} \) which reflects total mass within a given radius, while luminosity is a reflection of stellar mass. Other measures of \( V_{\text{rot}} \) have subsequently been used in deriving the Tully-Fisher relation, using information from the full rotation curves of galaxies (e.g. Verheijen 2001; McGaugh 2005; Kuzio de Naray et al. 2006; Noordermeer & Verheijen 2007; Yegorova & Salucci 2007; Oh et al. 2011), and luminosity is often converted to stellar masses, making the relation between \( V_{\text{rot}} \) and stellar mass more explicit.

For low mass galaxies, which become increasingly gas rich (Geha et al. 2006; Bradford et al. 2015), a tighter relation is found when \( V_{\text{rot}} \) is plotted against the total observable baryonic mass \( M_b \), i.e. stellar mass plus cold gas mass (Freeman 1999; McGaugh et al. 2000). As argued in McGaugh (2012), the gas dominance in low mass galaxies minimises the importance of the error in stellar mass, allowing relatively accurate measurements of \( M_b \) when deriving the baryonic Tully-Fisher relation (BTFR).

However, low mass galaxies are more problematic when it comes to determining \( V_{\text{rot}} \). For high mass galaxies, rotation curves generally rise sharply (Roberts & Rots 1973; Rubin et al. 1978), and some then drop before flattening at the outer parts (e.g. Sofue & Rubin 2001; Noordermeer & Verheijen 2007), at a velocity which can then be defined as \( V_{\text{flat}} \), a common measure of \( V_{\text{rot}} \) used in BTFRs. By contrast, rotation curves of low mass galaxies rise more gently (e.g. Lelli et al. 2013) and although they do start to flatten beyond about 2 disc scale-lengths, they are often still rising at the last measured point (e.g. Catinella et al. 2006; Swaters et al. 2009). To extend the \( V_{\text{flat}} \) BTFR to low mass galaxies, Stark et al. 2009 select galaxies with rotation curves that are approximately flat in their outer regions, according to some flatness criteria.

The low mass end of the BTFR is also problematic when measuring \( V_{\text{rot}} \) using HI line widths. The discrepancy between measuring the HI line-width at 20 per cent (\( W_{20} \)) and 50 per cent (\( W_{50} \)) of its peak value is typically 25 km s\(^{-1}\) (Koribalski et al. 2004; Brad-

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Table 1. Properties of the MaGICC galaxies ordered by halo mass. $R_{\text{HI}}$ and $V_{\text{last}}$ are measured at $N_{\text{HI}}=1M_{\odot}/pc^2$ and $N_{\text{HI}}=10^{15}cm^{-2}$

| Name     | $M_{\text{halo}}$ ($M_{\odot}$) | $M_{*}$ ($M_{\odot}$) | $M_{\text{HI}}$ ($M_{\odot}$) | $h_f$ (kpc) | $R_{\text{HI}}^\text{e}$ (kpc) | $R_{\text{HI}}^\text{f}$ (kpc) | $V_{\text{last}}^\text{e}$ (km/s) | $V_{\text{last}}^\text{f}$ (km/s) | $R_{\text{flat}}$ (kpc) | $V_{\text{flat}}$ (km/s) | $W_{20}$ (km/s) | $W_{50}$ (km/s) | $R_{\text{DM}}^\text{max}$ (kpc) | $V_{\text{DM}}^\text{max}$ (km/s) |
|----------|---------------------------------|------------------------|-------------------------------|-------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| g15784_MW | 1.49x10^{12}                   | 5.67x10^{10}           | 4.04x10^{10}                  | 3.23        | 31.9                          | 46.3                          | 225.7                         | 212.0                         | 26.8             | 218.8          | 500.5           | 404.8           | 54.90           | 183.0           |
| g21647_MW | 8.24x10^{11}                   | 2.51x10^{10}           | 8.51x10^{10}                  | 1.30        | 14.5                          | 33.9                          | 164.4                         | 150.7                         | 17.1             | 157.5          | 390.0           | 350.0           | 38.15           | 161.6           |
| g1536_MW   | 7.10x10^{11}                   | 2.36x10^{10}           | 1.20x10^{10}                  | 3.46        | 33.2                          | 42.0                          | 165.8                         | 157.6                         | 8.48             | 170.8          | 386.2           | 349.9           | 52.86           | 142.9           |
| g5664_MW   | 5.39x10^{11}                   | 2.74x10^{10}           | 7.25x10^{10}                  | 2.34        | 17.4                          | 42.4                          | 162.4                         | 137.6                         | 12.6             | 169.0          | 399.8           | 368.5           | 39.15           | 124.5           |
| g7124_MW   | 4.47x10^{11}                   | 6.30x10^{9}            | 6.99x10^{9}                   | 2.79        | 15.3                          | 30.7                          | 126.0                         | 120.7                         | 12.3             | 118.2          | 218.6           | 154.8           | 38.15           | 135.1           |
| g15807_Mrr | 2.82x10^{11}                   | 1.46x10^{10}           | 7.66x10^{9}                   | 1.94        | 17.0                          | 25.9                          | 134.7                         | 123.3                         | 10.4             | 137.0          | 289.4           | 179.2           | 32.99           | 101.0           |
| g15784_Mrr | 1.70x10^{11}                   | 4.26x10^{9}            | 5.11x10^{9}                   | 2.27        | 12.8                          | 19.6                          | 110.4                         | 104.1                         | 12.2             | 104.8          | 210.4           | 183.6           | 27.45           | 91.51           |
| g22437_Mrr | 1.10x10^{11}                   | 7.44x10^{8}            | 1.87x10^{8}                   | 1.88        | 7.87                          | 12.4                          | 75.95                         | 77.14                         | 6.44             | 69.4           | 141.0           | 104.9           | 27.32           | 66.69           |
| g21647_Mrr | 9.65x10^{10}                   | 1.98x10^{8}            | 9.55x10^{7}                   | 1.75        | 7.74                          | 21.2                          | 60.51                         | 61.89                         | 7.70             | 56.1           | 111.5           | 80.0            | 19.08           | 80.81           |
| g1536_Mrr  | 8.04x10^{10}                   | 4.46x10^{8}            | 1.25x10^{8}                   | 1.70        | 7.73                          | 14.9                          | 64.97                         | 68.63                         | 9.14             | 61.66         | 94.15           | 65.37           | 26.43           | 71.49           |
| g5664_Mrr  | 5.87x10^{10}                   | 2.36x10^{8}            | 7.50x10^{7}                   | 1.66        | 6.82                          | 13.6                          | 54.85                         | 61.14                         | 8.92             | 55.27          | 105.3           | 58.36           | 19.58           | 62.31           |
| g7124_Mrr  | 5.23x10^{10}                   | 1.32x10^{8}            | 7.13x10^{7}                   | 1.16        | 7.19                          | 11.1                          | 51.53                         | 54.19                         | 7.37             | 48.05          | 81.40           | 52.73           | 19.08           | 67.56           |
| g15807_drr | 3.04x10^{10}                   | 1.60x10^{7}            | 1.31x10^{7}                   | 1.26        | 2.96                          | 7.77                          | 29.93                         | 41.86                         | 8.00             | 41.22          | 53.70           | 37.36           | 16.50           | 50.49           |
| g15784_drr | 1.77x10^{10}                   | 8.98x10^{6}            | 2.12x10^{7}                   | 0.73        | 1.48                          | 2.06                          | 23.26                         | 27.41                         | 3.92             | 34.09          | 39.38           | 16.88           | 13.73           | 45.75           |
| g22437_drr | 1.19x10^{10}                   | 5.05x10^{6}            | 1.61x10^{7}                   | 0.17        | 1.23                          | 1.59                          | 21.92                         | 23.52                         | 1.08             | 20.09          | 27.75           | 12.79           | 13.66           | 36.17           |
| g1536_drr  | 9.69x10^{9}                    | 7.20x10^{5}            | 1.79x10^{7}                   | 0.17        | 1.04                          | 1.58                          | 21.72                         | 25.33                         | 1.58             | 24.10          | 41.96           | 16.81           | 13.22           | 35.74           |

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There are 16 galaxies, separated into three sub-sets, labelled as Milky-Way (MW), irregular (Irr) and dwarf irregular (dIrr) types, with stellar masses ranging $5 \times 10^6$-$5 \times 10^{10}$ $M_\odot$. Resolution varies depending on the "type": MW types have $m_{\text{star}}=4.0 \times 10^6 M_\odot$, $m_{\text{gas}}=5.7 \times 10^5 M_\odot$, $m_{\text{dm}}=1.1 \times 10^6 M_\odot$ and a gravitational softening length of $c=312$ pc (for all particle types); Irr's have $m_{\text{star}}=4.3 \times 10^6 M_\odot$, $m_{\text{gas}}=7.1 \times 10^5 M_\odot$, $m_{\text{dm}}=1.4 \times 10^6 M_\odot$ and $c=156$ pc; and dIrr's have $m_{\text{star}}=5.7 \times 10^5 M_\odot$, $m_{\text{gas}}=1.1 \times 10^6 M_\odot$, $m_{\text{dm}}=1.7 \times 10^6 M_\odot$ and $c=78$ pc.

The halos are identified using AHF (Knollmann & Knebe 2009) with halo masses defined within a sphere containing $\Delta_{\text{vir}} \simeq 250$ times the cosmic matter density at $z=0$. The bulk of the analysis is done using pynbody (Pontzen et al. 2013).

For the simulations, we use baryon mass $M_\text{bary}=1/8 M_\odot$ with the factor of 4/3 used to account for forms of gas other than HI, in a manner that mimics observational assumptions of the comparison data (e.g. McGaugh 2012). $M_\text{HI}$ remains an approximation since an accurate model of HI mass would require full radiative transfer. In particular self shielding from the UV background is not included. This uncertainty in the fraction of gas classified as HI will not affect our rotation curve shapes, but may affect the extent of the HI discs and hence the radius at which we measure $V_{\text{rot}}$. Disk scale lengths are derived from exponential fits to the surface brightness profiles in the I band: each stellar particle from the simulation represents a single stellar population (SSP) for which an absolute magnitude in a particular bandpass is calculated, interpolating between a grid of SSP luminosities from Girardi et al. (2010) and Marigo et al. (2008) for various stellar ages and metallicities. We note that scale-lengths in Swaters et al. 2009 and Lelli et al. 2014 use the R band, which won't significantly effect the comparison.

Basic galaxy parameters are shown in Table 1 including halo mass ($M_{\text{halo}}$), stellar mass ($M_\ast$), HI gas mass ($M_{\text{HI}}$), disc scale length $h$, the extent of the HI disc $R_{\text{HI}}$, defined as the radius at which the HI density of the galaxy falls to 1 $M_\odot$ pc$^{-2}$, which is adopted in several observational studies (e.g. Broeils & Rhee 1997, Swaters et al. 2002, Noordermeer et al. 2005, Lelli et al. 2014), although we note that the Begum et al. (2008) data set which comprises a significant fraction of the observed low mass galaxies, uses a threshold of $10^{19}$ cm$^{-2}$. We examine both thresholds in Fig 1 which compares the sizes of the simulated galaxies with observations, in HI gas and star light. Observational data come from Broeils & Rhee (1997), Swaters et al. (2002), Noordermeer et al. (2005), Lelli et al. (2014) and Begum et al. (2008).

Fig 1 plots $D_{\text{HI}}$ against $M_{\text{HI}}$ and $D_6$. Green and red circles in the top and bottom panels show the simulations when defining the diameter ($D_{\text{HI}}$) as where the HI surface density drops below 1 $M_\odot$ pc$^{-2}$ and $10^{19}$ cm$^{-2}$ respectively. Fits to the observed $D_{\text{HI}}$-$M_{\text{HI}}$ relation are shown as red lines. We will use this relation to define a radius $R_{\text{HI}}=D_{\text{HI}}/2$ where we measure the circular velocity $V_{\text{last}}$, which results in a measure of $V_{\text{rot}}$ for each simulation at a radius that is similar to that of observed galaxies of the same $M_{\text{HI}}$. $D_6$ is defined as 6.4 times the disc scale length, and is used as an indication of the extent of star light in galaxies. This is considered better than using an isophotal diameter at e.g. 25 B-mag arcsec$^{-2}$, as only a small fraction of the disk is enclosed within the isophotal diameter in low surface brightness galaxies (Swaters et al. 2002).

The simulations fall reasonably within the range of observed galaxies, giving some confidence in the mass distributions of the baryons (see also Santos-Santos et al. 2016). Larger, uniform samples of simulations and observations are required to make careful statistical comparisons including determination of scatter. Regardless, the focus of this paper is on how different measures of $V_{\text{rot}}$ give different BTFRs, with which we will proceed.

3 ANALYSIS AND RESULTS

3.1 Rotation curves

Figure 2 shows the circular velocity rotation curves of the simulated galaxies, computed using the gravitational potential of the simulation along the mid-plane of the aligned disc. This is more accurate than just assuming a spherically symmetric potential and averaging mass inside spherical shells (i.e. the classical $V^2=GM/r$).

The contribution from each mass component is plotted as a different linestyle: cold gas thick solid; stars dashed; DM dot-dashed; total, thin solid with circles marking integer multiples of $h$. Rotation curves are plotted out to $R_{\text{HI}}$, or to $R_{\text{flat}}$, whichever is larger. Vertical dotted (dashed) lines show $R_{\text{HI}}$ in cases where $R_{\text{flat}}<R_{\text{HI}}$, using density threshold of 1 $M_\odot$ pc$^{-2}$ ($10^{19}$ cm$^{-2}$).

Whether the shapes of the rotation curves match, in detail, the distribution of observed galaxy rotation curve shapes requires a larger sample of simulations and is beyond the scope of this paper.

Figure 1. Comparisons of the sizes of the simulated galaxies with observations, in HI gas and star light. Observational data sets are shown in the legends. Top Panel: Green circles show $M_{\text{HI}}$ of the simulations as a function of the diameter ($D_{\text{HI}}$) where the HI surface density drops below 1 $M_\odot$ pc$^{-2}$. Middle panel: $D_6$, as a function of $D_{\text{HI}}$ where $D_6$ is 6.4 times the disc scale length. Bottom panel: Red circles show $D_{\text{HI}}$ where the HI surface density drops below $10^{19}$ cm$^{-2}$ versus $M_{\text{HI}}$.

\begin{align}
\log M_{\text{HI}} &= 1.89 \log D_{\text{HI}} + 6.63 \\
\log M_{\text{HI}} &= 1.77 \log D_{\text{HI}} + 6.29
\end{align}
We note that Oman et al. (2015) found that the EAGLE simulations (Schaye et al. 2015) were not able to reproduce the variation in rotation curve shapes that is observed, in particular they could not simulate galaxies with slowly rising rotation curves. They showed that the MaGICC simulations, as used here, did form galaxies with such slowly rising rotation curves, as can be seen in Fig 2. Yet some observed low mass galaxies have more steeply rising rotation curves (e.g. Swaters et al. 2009). A larger suite of simulations will allow us to determine whether or not we can recover the full variation in rotation curve shapes that are seen in observations, which we will explore in a forthcoming study.

3.2 HI linewidths

The observations of $V_{\text{rot}}$ relevant to this study are based on HI radio data, so we constructed mock HI data cubes. We derive HI line widths for the simulated galaxies, using inclinations $45^\circ$, $60^\circ$ and edge-on. In Fig 3 we show the global HI line profiles for inclinations of $60^\circ$, indicating 20% and 50% of the maximum flux by horizontal dashed and dot-dashed lines respectively. The quoted values of $W_{50}$ and $W_{20}$ in Table 1 are calculated by taking the average of the inclination corrected line-widths from the different inclinations $45^\circ$, $60^\circ$ and edge-on. The results we present do not change significantly if any single inclination is chosen.

Fig 4 shows the ratio $W_{50}/W_{20}$ as a function of $W_{20}/2$ for the simulations (red circles) and HIPASS data (Meyer et al. 2004 black dots). The relation $W_{20}=W_{50}+25$ is also shown (cyan line), the typical difference between the observed $W_{20}$ and $W_{50}$ (Koribalski et al. 2004; Bradford et al. 2015). The offset between $W_{50}$ and $W_{20}$ becomes significant for lower velocities/masses, with the relation $W_{50}-W_{20}-25$ implying that at $W_{20}/2=[50,40,30]$, $W_{50}/2$ is [25,31,42] per cent lower. Our simulations suggest that this simple relation holds down to very low velocities/masses.
3.3 Measurements of rotation velocity

We note some nomenclature used in this paper:

- $V_{\text{flat}}$: the circular velocity at the flat part of the rotation curve.
- $R_{\text{flat}}$: the radius at which $V_{\text{flat}}$ is reached.
- $V_{\text{last}}$: the circular velocity at the last point of the HI disc.
- $R_{\text{HI}}$: the extent of the HI disc: defines where $V_{\text{last}}$ is measured.
- $D_{\text{HI}}$: diameter of the HI disc: $2 \times R_{\text{HI}}$.
- $V_{\text{DM max}}$: the $V_{\text{max}}$ from the DM only simulations.
- $R_{\text{DM max}}$: the radius at which $V_{\text{DM max}}$ is reached.
- $h$: disc scale length.

The criteria for selecting flat rotation curves in Stark et al. (2009) requires that $V_{\text{rot}}$ measured at 3 disc scale lengths ($3h$) is 85 per cent of $V_{\text{last}}$, or in cases where $R_{\text{HI}}$ is greater than $4h$, then $V_{\text{rot}}$ measured at $4h$ is required to be 90 per cent of $V_{\text{last}}$. This definition has a degree of arbitrariness, as it depends on the relative distance between $R_{\text{HI}}$ and integer multiples of $h$. In this study, we define rotation curves to be flat in the regions where $V_{\text{rot}}$ changes by less than 5 per cent between integer multiples of $h$. We define $V_{\text{flat}}$ at the flattest of these flat regions, i.e. where the change in $V_{\text{rot}}$ is the smallest, within the HI disc (in regions where radius < $R_{\text{HI}}$). $V_{\text{flat}}$ is measured at the centre of the flattest region within the HI disc.

Our flatness criteria are better defined than those used in Stark et al. (2009), and are also stricter for galaxies with $R_{\text{HI}} < 5h$, (69 per cent of galaxies in the Swaters et al. 2009 data set from which Stark et al. 2009 was largely drawn), is at least as strict for galaxies with $5h < R_{\text{HI}} < 6h$ (a further 9 per cent of Swaters et al. 2009 galaxies), but may not be as strict for galaxies with $R_{\text{HI}} > 6h$ (12 per cent of Swaters et al. 2009 sample).

Some of our low mass simulated galaxies do not have any flat regions at radii lower than $R_{\text{HI}}$. These are deemed to be galaxies with slowly rising rotation curves. In such cases, marked with † in Table 1, we continue beyond $R_{\text{HI}}$ until we find the first flat region of the rotation curve, i.e. a region where $V_{\text{rot}}$ increases by less than 5 per cent as radius increases by $h$. $V_{\text{flat}}$ is then measured at the central point of this flat region. Galaxies with rising rotation curves will be identified in our BTFR plot, and are analogous to those
Figure 4. The ratio $W_{50}/W_{20}$ plotted as a function of $W_{20}/2$ for the simulations (red circles) overplotted on HiPass data (black dots), with the relation $W_{20}=W_{50}+25$. also shown as a cyan line.

galaxies that are excluded in studies of observed $V_{\text{flat}}$ BTFRs (e.g. Stark et al. [2009] McGaugh [2012] McGaugh & Schombert [2015]).

3.4 The Baryonic Tully-Fisher relation

Figure 5 shows the different BTFRs for the different $V_{\text{flat}}$ measurements of our simulated suite of galaxies. These are: $V_{\text{flat}}$ (orange circles), $V_{\text{last}}$ (green squares), $V_{\text{DHI}}$ (blue stars), $W_{20}/2$ (pink triangles), $W_{50}/2$ (red diamonds) and $V_{\text{DM}}$ (black crosses).

Overplotted are observational BTFRs: the solid line shows the fit found in Lelli et al. (2016) using $V_{\text{flat}}$ from observations of gas-rich galaxies $\log(M_\odot) = 3.95 \log(V_{\text{flat}}) + 1.86$, a relation very similar to that found in McGaugh & Schombert (2015); the dashed line shows the relation found by Bradford et al. (2015) using $W_{20}$ measurements of galaxies selected from the Sloan Digital Sky Survey, $\log(W_{20}/2) = 0.277 \log(M_\odot) - 0.672$; the dotted line shows the $W_{50}$ relation determined by using $W_{50} = W_{20} - 25$. We found a similar $W_{50}$ BTFR using local volume data as compiled by Karachentsev et al. (2013) but do not overplot here as it adds to clutter without adding information. We also show a theoretical (dot-dashed) line from Di Cintio & Lelli (2016), which uses a semi-empirical model within a Cold Dark Matter cosmology, measured in the flat region of the rotation curve.

For simulated galaxies with $M_\odot \gtrsim 2 \times 10^9 M_\odot$ the $V_{\text{flat}}, V_{\text{last}}$ and $W_{20}$ BTFRs are similar. This corresponds to galaxies with $V_{\text{rot}} \lesssim 45 \text{ km} s^{-1}$ living in halos with $M_{\text{halo}} \lesssim 5 \times 10^{11} M_\odot$. $W_{20}$ and $W_{50}$ are integrated measurements that are intrinsically more sensitive to the rising part of the rotation curve at small radii, which contains most of the HI flux, explaining their difference with $V_{\text{last}}$. The difference $W_{50} - W_{20} - 25$ holds over a large range of masses, meaning that the $W_{50}$ BTFR does not have a large offset from the other measures of $V_{\text{rot}}$ for massive galaxies: at $W_{20}/2 = [100, 150, 200]$, $W_{50}/2$ is about [12.9, 6] per cent lower than the other measures of $V_{\text{rot}}$.

The $V_{\text{DHI}}$ BTFR differs from $V_{\text{last}}$, indicative of the sensitivity of low mass galaxies to the radius at which $V_{\text{rot}}$ is measured. This $V_{\text{DHI}}$ BTFR also shows increased scatter at the the HI mass end, but this is a region where scatter in the observed BTFRs also increases (see e.g. McGaugh 2012; Lelli et al. 2014).

For galaxies with $M_\odot \lesssim 10^9 M_\odot$, the different forms of the BTFR increasingly diverge. The $V_{\text{last}}$ using threshold of $10^{10} \text{ cm}^{-2}$ BTFR of the simulations follow closely the $V_{\text{flat}}$ value of McGaugh & Schombert (2015), which is also close to the $V_{\text{flat}}$ BTFR from Stark et al. (2009).

The $V_{\text{flat}}$ BTFR from the simulations also follows the observed $V_{\text{flat}}$ BTFR if the galaxies with rising rotation curves are neglected, as they are in the observational derivations. Such galaxies are marked with red outlines in Fig 5. Our model suggests that low mass galaxies with rising rotation curves would fall off the $V_{\text{flat}}$ BTFR, having excessively high values of $V_{\text{rot}}$ if their gas discs would extend far enough for $V_{\text{flat}}$ to be measured.

We also show results from the corresponding dark matter only simulations, which shows how the measured $V_{\text{rot}}$ of disc galaxies becomes a smaller fraction of $V_{\text{max}}$. This is due to two related effects: primarily, $R_{\text{HI}}$ becomes a smaller fraction of $R_{\text{DM}}$ for low mass galaxies, which are confined to the inner regions of DM halos (compare $R_{\text{HI}}$ to $R_{\text{DM}}$ in Table 1, and see also Fig 7 of Brook & Shankar 2015); secondly, baryons are able to cool to the central regions of halos, so galaxies with high baryonic masses, i.e. high mass galaxies, have more steeply rising rotation curves in general, meaning that $V_{\text{rot}}$ becomes close to $V_{\text{max}}$ at relatively small radii.

The shape of dark matter halos can be affected by baryons by either adiabatic contraction (Blumenthal et al. 1986), or expansion (see Pontzen & Governato 2014 for a review). The competing processes mean that the final profile can depend on the amount of gas inflowing to the central region causing contraction, and the timescale and mass of gas outflows causing expansion. Simulations suggest that this results in dark matter density profile shapes being dependent on galaxy mass (Di Cintio et al. 2014b) Chan et al. 2015).
The rotation curve shapes in the galaxies of this study have been affected by these processes, but as stated above, the detailed study of these shapes is beyond the scope of this paper.

In fact, correspondence with observations requires the existence of low mass galaxies that have rising rotation curves, as was found in our simulations.

The rising rotation curves of many low mass galaxies means that one needs to take particular care when comparing models and theory. Measuring the $V_{\text{flat}}$ BTFR for low mass galaxies requires a selection, with galaxies with rising rotation curves discarded. It is not clear whether this introduces a bias. For example, do low mass galaxies with extended HI discs and flat rotation curves fit on the average $M_\ast$-$M_{\text{halo}}$ relation?

Measuring the $V_{\text{last}}$ BTFR, on the other hand, is somewhat arbitrary, with the position of measurement determined by the radial extent of the cold gas. The large difference between $R_{\text{HI}}$ and $R_{\text{DHI}}$ highlights that $V_{\text{rot}}$ may increase significantly in some cases, if HI discs extended further and allowed measurements at larger radii.

In fact, even when our low mass simulated galaxies meet the criteria for $V_{\text{flat}}$, i.e. they rise by less than five per cent when radius increases by $h$, they are still measured at a radius that can be a small fraction of $R_{\text{max}}$. In these cases, even small gradients in the rotation curve can result in significant difference between the measured $V_{\text{flat}}$ and the $V_{\text{rot}}$ that would be reached if the cold gas extended far enough to measure rotation velocities at their peak value.

To proceed in confronting models with observations, it is suggested to test the BTFR as measured in different ways, at a variety of radii. This also facilitates the evaluation of the variation of rotation curve shapes, which provide further important constraints on galaxy formation models. We will undertake such a procedure in a forthcoming study (Katz, H.

| $V_{\text{rot}}$ | $a$ | $b$ |
|-----------------|-----|----|
| $V_{\text{DM max}}$ | 5.19 | -5.2 |
| $W_{20}/2$ | 3.13 | 3.78 |
| $W_{50}/2$ | 2.70 | 4.83 |
| $V_{\text{last}}[N_{\text{HI}}]=10^{19} \text{cm}^{-2}$ | 3.51 | 2.85 |
| $V_{\text{flat}}[N_{\text{HI}}]=10^{20} \text{cm}^{-2}$ | 4.02 | 1.85 |
| $V_{\text{DHI}}$ | 3.56 | 2.80 |
| $V_{\text{flat}}$ | 3.72 | 2.43 |

Our hydrodynamical simulations of galaxy formation can explain the various forms of the BTFR within a $\Lambda$CDM context. It is instructive to compare our study with two recent $\Lambda$CDM cosmological models that have used the BTFR as a constraint. [Trujillo-Gomez et al. (2011)] and [Dutton (2012)].

In [Trujillo-Gomez et al. (2011)], a model population was made by matching $M_\ast$ to $M_{\text{halo}}$ following abundance matching, and giving each model galaxy an empirically motivated gas fraction and scale-length. A BTFR was derived by measuring circular velocities at 10kpc. For galaxies at the high mass end of our study, correspondence with [Trujillo-Gomez et al. (2011)] is good. This is expected because $R_{\text{HI}}$ extends far enough to reflect the value of $V_{\text{rot}}$ at 10kpc. For low mass galaxies, the [Trujillo-Gomez et al. (2011)] BTFR bends toward values of relatively high velocity, in a manner that is very similar to our $V_{\text{max}}$ BTFR. This is also expected, because $R_{\text{max}}$ is $\sim$10kpc for such galaxies, as seen in our Table 1, and such galaxies have very low baryon fractions meaning that DM only simulations are not greatly different from their model galaxies. [Dutton (2012)] use a semi-analytic model population of galaxies, tuned to match the empirical $M_\ast$-$M_{\text{halo}}$ and specific angular momentum-$M_{\text{halo}}$ relations. They define $V_{\text{flat}}$ as the circular velocity measured at the radius that contains 80 per cent of the cold gas. This is analogous to our $V_{\text{last}}$, so it is not surprising that both studies match the $V_{\text{flat}}$ BTFRs from [Stark et al. (2009), Dutton (2012)] does not test whether all their model galaxies’ rotation curves are actually flat; the rotation curves in some low mass galaxies may still be rising at the point they measure ‘$V_{\text{flat}}$’. In fact, correspondence with observations requires the existence of low mass galaxies that have rising rotation curves, as was found in our simulations.

The different Baryonic Tully Fisher relations
et al. in prep), comparing a ΛCDM model that assumes an NFW profile (Navarro et al. [1996]) with a model where DM halo shapes are modified by baryons (Di Cintio et al. [2014a]).

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