Fine structure of the exciton electroabsorption in semiconductor superlattices

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Abstract

Wannier-Mott excitons in a semiconductor layered superlattice (SL) of period much smaller than the 2D exciton Bohr radius in the presence of a longitudinal external dc electric field directed parallel to the SL axis are investigated analytically. The exciton states and the optical absorption coefficient are derived in the tight-binding and adiabatic approximations. Strong and weak electric fields providing spatially localized and extended electron and hole states, respectively, are studied. The dependencies of the exciton states and the exciton absorption spectrum on the SL parameters and the electric field strength are presented in an explicit form. We focus on the fine structure of the ground quasi-2D exciton level formed by the series of closely spaced energy levels adjacent for higher frequencies. These levels are related to the adiabatically slow relative exciton longitudinal motion governed by the potential formed by the in-plane exciton state. It is shown that the external electric fields compress the fine structure energy levels, decrease the intensities of the corresponding optical peaks and increase the exciton binding energy. A possible experimental study of the fine structure of the exciton electroabsorption is discussed.
I. INTRODUCTION

In recent paper of Suris [1] the effect of the centre of mass motion of the Wannier-Mott exciton on its binding energy in a layered semiconductor superlattices (SL) has been studied analytically. The SL period was assumed to be much smaller than the 2D excitonic Bohr radius. As a result the electron and hole motions decompose into two parts: the fast longitudinal motion parallel to the SL axis and the adiabatically slow transverse in-plane motion governed by the SL potential and the 2D quasi-Coulomb exciton field, respectively. The fine structure of each 2D exciton energy level occurring for adjacent but higher energies was found to occur. This energy group consists of closely spaced satellite energy levels which in turn relate to the adiabatically slow longitudinal relative exciton motion in the triangular quantum well. This well is formed by the quasi-uniform electric field caused by the corresponding in-plane exciton state. A similar structure was studied originally in the pioneering work of Kohn and Luttinger [2] when considering the donor states in the Ge and Si bulk crystals with extremely anisotropic isoenergetic surfaces. The effect of the longitudinal uniform external dc electric fields on the exciton states and the exciton optical absorption spectrum in the semiconductor SL associated with the transitions to these satellite states are in this context certainly of relevance.

In the present paper this effect is investigated for the case that the period of the SL is much smaller than the 2D exciton Bohr radius. In the effective mass approximation the overlapping exciton wave function is expanded over the SL Wannier functions that in turn allows us using the tight-binding and the adiabatic approximations to calculate analytically the exciton states in the SL subject to the longitudinal external electric field. The dependencies of the exciton states and the exciton absorption coefficient on the SL parameters (period and minibands widths) and on the external electric field strength are presented explicitly. The regime of a strong electric field providing the Wannier-Stark spatial localization of the carriers and the more interesting regime of weak electric field which destroy the exciton fine structure, generate the extended longitudinal electron and hole states and increase the 2D exciton binding energies are considered. In conclusion we discuss the applicability conditions of the obtained results and estimate the expected experimental
The paper is organized as follows. In Section 2 in the tight-binding and adiabatic approximations the equation describing the exciton in the SL in the presence of the longitudinal external dc electric fields is derived. The exciton energies and wave functions are presented in an explicit form in Section 3. In Section 4 we calculate the exciton absorption coefficient and trace its dependencies on the SL parameters and on the external electric field strengths as well as discuss the applicability of the obtained results. We also estimate the expected experimental values. Section 5 contains the conclusions.

II. GENERAL APPROACH

We consider a Wannier-Mott exciton in a semiconductor SL subject to an uniform external electric field \( \vec{F} \) directed parallel to the SL \( z \)-axis. The semiconductor energy bands are taken to be parabolic, nondegenerate, spherically symmetric and separated by a wide energy gap \( E_g \). In the effective mass approximation the envelope wave function \( \Psi \) of the exciton consisting of the interacting electron (\( e \)) and hole (\( h \)) with the effective mass \( m_j \), charges \( e_j (e_e = -e_h = -e) \) and positions \( \vec{r}_j (\vec{\rho}_j, z_j) \) \( j = e, h \) obeys the equation

\[
\left\{ \sum_{j=e,h} \left[ -\frac{\hbar^2}{2m_j} \nabla_j^2 + V_j(z_j) + e_j F z_j \right] + U(\vec{r}_e - \vec{r}_h) \right\} \Psi(\vec{r}_e, \vec{r}_h) = E \Psi(\vec{r}_e, \vec{r}_h). \tag{1}
\]

In eq. (1)

\[
U(\vec{\rho}_e - \vec{\rho}_h, z_e - z_h) = -\frac{e^2}{4\pi\varepsilon_0\varepsilon\sqrt{(\vec{\rho}_e - \vec{\rho}_h)^2 + (z_e - z_h)^2}} \tag{2}
\]

is the Coulomb potential of the electron-hole attraction in the semiconductors with the dielectric constant \( \varepsilon \), \( E \) is the total exciton energy, \( V_j(z_j) = V_j(z_j + nd) \), \( n = 0,1,\ldots,N \) are the periodic model potentials of the SL formed by a large number \( N \gg 1 \) of quantum wells of width \( d \), separated by \( \delta \)-function type potential barriers (see Ref. [3]). The chosen model correlates well with the nearest neighbor tight-binding approximation for the interwell tunneling and enables us to perform calculations in an explicit form. Since we follow the original approach to the problem presented in details in Ref. [1], only a brief outline of the
calculations will be provided.

First we assume that the energies of the size-quantization \( b_j \) considerably exceed the exciton Rydberg constant \( R_y \), determined by the exciton Bohr radius \( a_0 \), the miniband widths \( \Delta_j \) and the distance \( eFd \) between the Wannier-Stark (W-S) energy levels, i.e.

\[
\Delta_j, \quad R_y, \quad eFd \ll b_j, \tag{3}
\]

where

\[
b_j = \frac{\hbar^2 \pi^2}{2m_j d^2}; \quad R_y = \frac{\hbar^2}{2\mu a_0^2}; \quad a_0 = \frac{4\pi \varepsilon_0 \hbar^2}{\mu e^2}; \quad \mu^{-1} = m_e^{-1} + m_h^{-1}, \quad j = e, h.
\]

The imposed conditions (3) decompose the particle motion into two components: the fast longitudinal parallel to the SL \( z \)-axis and the slow transverse in-plane \( \vec{\rho} \)-motions governed, respectively, by the SL potentials \( V_j(z_j) \) and the electric fields \( F \), and the exciton attraction (2). This allows us to employ the approximation of isolated, namely, ground minibands and expand the exciton function \( \Psi \) over the orthonormalized basis set of the SL Wannier functions \( w(z_j - n_j d) \)

\[
\Psi(\vec{r}_e, \vec{r}_h) = \sum_{n_e, n_h} w(z_e - dn_e)w(z_h - dn_h)\Phi(z_e, \vec{r}_e; z_h, \vec{r}_h), \tag{4}
\]

where \( z_{e,h} \) are replaced by \( n_{e,h} = z_{e,h}d \) in the envelope \( \Phi \)-function due to its weak dependence on these variables. Clearly, the total exciton momentum with the transverse \( \vec{P} \) and longitudinal \( Q \) components is kept constant. The wave function \( \Phi \) can be written in the form

\[
\Phi(n_e, \vec{r}_e; n_h, \vec{r}_h) = \exp \left\{ i \left[ \vec{P} \vec{R}_\perp + \frac{1}{2} Qd(n_e + n_h) - \gamma dn \right] \right\} \chi(n, \vec{\rho}), \tag{5}
\]

where

\[
n = n_e - n_h, \quad \gamma(Q)d = \arctan \left[ \frac{\Delta_e - \Delta_h}{\Delta_e + \Delta_h} \tan \left( \frac{1}{2} Qd \right) \right];
\]

\[
\vec{R}_\perp = \frac{m_e \vec{r}_e + m_h \vec{r}_h}{M}, \quad \text{and} \quad \vec{\rho} = \vec{r}_e - \vec{r}_h, \quad M = m_e + m_h,
\]
are the transverse centre of mass ($\vec{R}_\perp$) and relative ($\vec{\rho}$) coordinates, respectively. Substitution of functions (4) and (5) into eq. (1) and subsequent projection yields the equation for the function $\chi(n, \vec{\rho})$

\[-\frac{eFd}{2} \left\{ \left( n + \frac{\mathcal{E}}{eFd} - \beta \right) 2\chi(n, \vec{\rho}) + \beta [\chi(n + 1, \vec{\rho}) + \chi(n - 1, \vec{\rho})] \right\} +
\left[ -\frac{\hbar^2}{2\mu} \vec{\nabla}_\rho^2 + U(n, \vec{\rho}) - W \right] \chi(n, \vec{\rho}) = 0,
\]

where

$$\beta = \frac{\Delta_{eh}(Q)}{2eFd}, \quad W = E - \tilde{E}_g - \frac{P^2}{2M} - T(Q) - \mathcal{E}, \quad \tilde{E}_g = E_g + b_e + b_h,$$

$$U(n, \vec{\rho}) = \langle w(z_e - d_{ne})w(z_h - d_{nh}) \mid U(\vec{\rho}_e - \vec{\rho}_h, z_e - z_h) \mid w(z_e - d_{ne})w(z_h - d_{nh}) \rangle.$$  

The energy

$$T(Q) = \frac{1}{2} [\Delta_e + \Delta_h - \Delta_{eh}(Q)]; \quad \Delta_{eh}(Q) = [(\Delta_e + \Delta_h)^2 - 2\Delta_e \Delta_h (1 - \cos Qd)]^{1/2}$$

is the energy of the centre of mass longitudinal motion. Here it is reasonable to introduce the parameters

$$m_{\parallel}(Q) = \frac{2\hbar^2}{\Delta_{eh}(Q)d^2}, \quad \text{and} \quad a_F(Q) = \left( \frac{\hbar^2}{2m_{\parallel}(Q)eF} \right)^{1/2},$$

which are the reduced longitudinal effective mass ($m_{\parallel}(Q)$), determining the relative z-motion of the electron-hole pair and the effective length ($a_F(Q)$) of the corresponding state. On solving eq. (6) the exciton wave function $\Psi$ (11) and the exciton energy $E$ can be calculated in principle.

III. EXCITON STATES

Wannier-Stark (W-S) regime $a_F(Q) \leq d$ ($\beta \leq 1$)
Below we consider sufficiently strong electric fields $F$ providing $z$-localization of the carriers within several periods and quantization of its longitudinal states (W-S levels). The solution to eq. (6) can be written in the adiabatic form

$$\chi(n, \vec{\rho}) = R_{p(k)}(n, \vec{\rho})\psi_\nu(n),$$

where $R_{p(k)}(n, \vec{\rho})$ is the 2D exciton wave function of the discrete ($p$) (continuous ($k$)) states determined by the potential $U(n, \vec{\rho})$ and corresponding to the energies

$$W_p = -\frac{Ry}{(p+\delta p+\frac{1}{2})^2}, \; p = 0, 1, 2, \ldots, \; \delta p \simeq \frac{2}{3} \frac{d}{a_0} \ll 1, \; \text{and} \; W_k = \frac{\hbar^2 k^2}{2\mu}. \quad (8)$$

Quantum defects $\delta p$ have been calculated earlier in Ref. [5] in the framework of the chosen model potentials $V_j(z_j)$ [3]. The solution $\psi_\nu$ in the function $\chi(n, \vec{\rho})$ (7) to the difference equation (6) for a vanishing of the term in the curly brackets, is well known (see for example Ref. [6] and references therein)

$$\psi_\nu(n) = J_{-(n+\nu)}(\beta), \; \varepsilon_\nu = eFd\nu + \frac{1}{2} \Delta_{eh}, \; \nu = 0, \pm 1, \pm 2, \ldots, \quad (9)$$

where $J_m(x)$ and $\varepsilon_\nu$ are the Bessel functions and the W-S energy levels, respectively. Eqs. (8), (9), (7) and (5) lead to the total exciton energies

$$E_{\nu,p(k)}(P, Q) = \tilde{E}_g + \frac{P^2}{2M} + T(Q) + \varepsilon_\nu + W_{p(k)} \quad (10)$$

and the orthonormalized wave functions $\Psi$ (see eq. (4))

$$\Psi_{\nu,p(k)}^{(P, Q)}(z_e, z_h; \vec{\rho}, \vec{R}_\perp) = \frac{e^{iP\vec{R}_\perp}}{\sqrt{SN}} \sum_n e^{i\left(\frac{2}{3}Q-\gamma\right)dn} J_{-(n+\nu)}(\beta)R_{p(k)}(n, \vec{\rho})G^{(Q)}_n(z_e, z_h), \quad (11)$$

where

$$G^{(Q)}_n(z_e, z_h) = \sum_{n_h} w[z_e - d(n+n_h)]w[z_h - dn_h]e^{iQn_h},$$

$S$ is the area of the SL layer. The wave functions $\Psi_{\nu,p(k)}^{(P, Q)}$ satisfy
\[ \langle \Psi_{\nu_p(k)}(\vec{P},Q) | \Psi_{\nu_p'(k')}(\vec{P}',Q') \rangle = \delta_{\vec{P}\vec{P}'}\delta_{QQ'}\delta_{\nu\nu'}\delta_{p(k)p'(k')}. \]

Eqs. (10) and (11) at \( \vec{P} = Q = 0 \) coincide with those obtained for the exciton energies and wave functions, respectively, in Ref. [3].

**Continuous regime** \( a_F(Q) \gg d (\beta \gg 1) \)

Weak electric fields \( F \) generate extended longitudinal exciton states, for which the continuous limit implying in eq. (6)

\[ nd = z, \; \chi(n+1,\vec{\rho}) + \chi(n-1,\vec{\rho}) - 2\chi(n,\vec{\rho}) = d^2 \frac{\partial^2 \chi(z,\vec{\rho})}{\partial z^2} \]

becomes applicable. The function \( \chi(z,\vec{\rho}) \) obeys the equation

\[
\left[ -\frac{\hbar^2}{2m_\parallel(Q)} \frac{\partial^2}{\partial z^2} - eFz - \frac{\hbar^2}{2\mu} \vec{\nabla}^2_{\vec{\rho}} + U(z,\vec{\rho}) - W \right] \chi(z,\vec{\rho}) = 0;
\]

\[ W = E - \tilde{E}_g - \frac{\rho^2}{2M} - T(Q). \quad (12) \]

Since \( m_\parallel(Q) \gg \mu \) (see condition (3)) the slow \( z \)- and fast \( \vec{\rho} \)-motions are separated adiabatically which in turn enables us to take \( \chi(z,\vec{\rho}) = R_p(\vec{\rho})\psi_p(z), \; p = 0, 1, \ldots \), where the functions \( R_p(\vec{\rho}) \) describe the exciton states governed by the 2D Coulomb potential \( U(0,\vec{\rho}) \sim \rho^{-1} \) \( (2) \) with the exact Rydberg energies \( W_p \) \( (8) \) at \( \delta p = 0 \). Below to be definite we consider the ground 2D state with \( R_0(\vec{\rho}) = (2\pi)^{-\frac{1}{2}} a_0^{-1} \exp(-2\rho/a_0) \) and \( W_0 = -4Ry \). For the wave function \( \psi_0(z) \) we obtain from eq. (12)

\[ -\frac{\hbar^2}{2m_\parallel(Q)} \psi_0''(z) + \left[ \tilde{U}_0(z) - eFz - \varepsilon_0 \right] \psi_0(z) = 0; \; \varepsilon_0 = W - W_0, \quad (13) \]

where

\[ \tilde{U}_0(z) = \langle R_0(\vec{\rho}) | U(\rho, z) - U(\rho, 0) | R_0(\vec{\rho}) \rangle = 8Ry \left( |u| + \frac{1}{2} u^2 \ln |u| - \frac{1}{3} |u|^3 \right); \; |u| = \frac{4|z|}{a_0} \ll 1. \quad (14) \]
Equation (13) at $F = 0$ for the region of small $z$-coordinate has been considered by Kohn and Luttinger [2] and Suris [1] for the cases of a bulk crystal and SL, respectively. The symmetric triangular potential well

$$\bar{U}_0(z) = eF_0|z|, \quad F_0 = \frac{32Ry}{ea_0}$$

formed by a quasi-uniform exciton electric field $F_0$ and the potential of the uniform external electric field $F \leq F_0$ form the asymmetric triangular potential well. This enable us to present the exact solution to eq. (13) in terms of the Airy functions $Ai(x)$ [7]

$$\psi_{0s}(z) = \begin{cases} 
Ai\left(\frac{z-a_+}{a_+}\right); & z \geq 0; \\
Ai\left(\frac{z-a_-}{a_-}\right); & z \leq 0,
\end{cases}$$

where

$$z_{+,-} = \frac{\mathcal{E}_{0s}}{e(F_0 \mp F)}; \quad a_{+,-} = a_{F_0}(Q) \left(1 \mp \frac{F}{F_0}\right)^{\frac{1}{3}}; \quad a_{F_0}(Q) = \left(\frac{\hbar^2}{2m_{\|}(Q)eF_0}\right)^{\frac{1}{3}};$$

$$C^2 = \left(\frac{2\pi^2}{2s+1}\right)^{\frac{1}{3}} \frac{1}{a_{F_0}(Q)} \left\{ \left(1 + \frac{F}{F_0}\right)^{\frac{1}{3}} + \left(1 - \frac{F}{F_0}\right)^{\frac{1}{3}} \right\}^{-1}.$$}

The satellite energies

$$\mathcal{E}_{0s} = (12\pi)^{\frac{2s}{3}} Ry \left(\frac{\mu}{m_{\|}(Q)}\right)^{\frac{1}{3}} \left[\left(1 - \frac{F^2}{F_0^2}\right)(2s+1)\right]^{\frac{2}{3}}; \quad s = 0, 1, 2, \ldots$$

adjacent to the energy level $W_0$ determine the total exciton energies

$$E_{0s}(\vec{P}, Q) = \tilde{E}_g + \frac{\vec{P}^2}{2M} + T(Q) + \mathcal{E}_{0s} + W_0.$$ (17)

Eqs. (15) and (5) lead to the orthonormalized total exciton wave functions (4)

$$\tilde{\Psi}_{0s}^{(\vec{P}, Q)}(z_e, z_h; \vec{P}, \vec{R}_\perp) = \frac{e^{i(\vec{P}\vec{R}_\perp + QZ - \frac{\Delta_e - \Delta_h}{2\Delta_e + \Delta_h}Qz)}}{\sqrt{SNd}} R_0(\vec{P})\psi_{0s}(z),$$

with
\[ \langle \Psi_{0s}^{(\vec{P},Q)} | \Psi_{0\nu'}^{(\vec{P}',Q')} \rangle = \delta_{\vec{P} \vec{P}'} \delta_{Q Q'} \delta_{ss'}. \]

**IV. SPECTRUM OF THE EXCITON ABSORPTION: RESULTS AND DISCUSSION**

The relation between the transition rate \( \Pi \) and the exciton optical absorption coefficient \( \alpha \) is as follows [4]

\[ \alpha = \frac{n_0 \hbar \omega \Pi}{cU SN d}; \quad \Pi = \frac{1}{t} \sum_{e,h} | \frac{1}{i \hbar} \int_0^t \Psi_0(\vec{r}_e, \vec{r}_h) \mathcal{P}_{eh}(\tau) \Psi^*(\vec{r}_e, \vec{r}_h) e^{i \vec{F} \cdot \vec{r}_e} d\vec{r}_e d\vec{r}_h |^2, \quad (19) \]

where \( n_0 \) is the refractive index, \( c \) is the speed of light, \( \tilde{U} = \varepsilon_0 n_0^2 F_0^2 \) is the optical energy density caused by an oscillating electric field of frequency \( \omega \) and magnitude \( F_0 \) polarized parallel to the SL axis. \( \sum_{e,h} \) is a sum over all band states involved in optical transitions and \( \Psi_0(\vec{r}_e, \vec{r}_h) = \delta(\vec{r}_e - \vec{r}_h) \) is the wave function of the initial electron-hole state [8]. \( \Psi(\vec{r}_e, \vec{r}_h) \) and \( E \) are the exciton wave function and energy, respectively,

\[ \mathcal{P}_{eh}(t) = \frac{i 2 \hbar e F_0 p_{ehz}}{m_0 E_g} \cos \omega t; \quad (E_g \approx \hbar \omega) \]

is the operator of allowed electric dipole transitions [9], determined by the free electron mass \( m_0 \) and momentum \( z \)-component matrix element \( p_{ehz} \) calculated between the amplitudes of the Bloch functions of the electron and hole bands.

**Wannier-Stark regime** \( a_F(Q) \leq d \ (\beta \leq 1) \)

Substituting the exciton wave function \( \Psi \) with \( R_{p(k)}(n, \vec{p}) \) from Ref. [4] and energy \( E \) into eq. (19) we obtain

\[ \alpha = \sum_{\nu} \left[ \frac{\pi e^2 \hbar}{2 \varepsilon_0 n_0 V m_0 c} \sum_{p=0}^{\infty} f_{2p}^{(2n)} \delta \left( \hbar \omega - \tilde{E}_g - \varepsilon_{\nu} + \frac{R_y}{(p + \delta p + \frac{1}{2})^2} \right) \right. \\
\left. + \alpha^{(0)} J^2_{\nu}(\beta) \frac{2 \mu}{\pi \hbar^2 \cosh \xi_{\nu}(\omega)} \Theta(h \omega - \tilde{E}_g - \varepsilon_{\nu}) \right] \]

\[ , \quad (20) \]
where

\[
f_{\nu p}^{(2D)} = J_{-\nu}(\beta) \frac{4V\hbar\omega |p_{ehz}|^2}{\pi a_0^2 (p + \delta p + \frac{1}{2})^3 m_0 dE_g^2}; \quad \alpha^{(0)} = \frac{2\pi \hbar^2 \omega^2 |p_{ehz}|^2}{\varepsilon_0 n_0 m_0^2 c dE_g^2};
\]

\[
\xi_{\nu}(\omega) = \pi \left( \frac{\hbar \omega - \tilde{E}_g - \varepsilon_{\nu}}{Ry} \right)^{-\frac{1}{2}},
\]

(21)

\[f_{\nu p}^{(2D)}\] is the oscillator strength of the transition to the \(\nu p\) state and \(\Theta(x)\) is the Heavyside step function.

**Continuous regime** \(a_F(Q) \gg d (\beta \gg 1)\)

At this stage we take in eq. (19) eqs. (18) and (17) for the wave function \(\Psi\) and energy \(E\), respectively. The coefficient of absorption becomes

\[
\alpha = \frac{\pi e^2 \hbar}{2 \varepsilon_0 n_0 V m_0 \epsilon_c} \sum_s f_{0s} \delta \left( \hbar \omega - \tilde{E}_g + E_{0s}^{(b)} \right)
\]

(22)

where

\[
f_{0s} = \frac{16V\hbar\omega |p_{ehz}|^2}{\pi m_0 E_g^2 a_0^2 a_{F_0}(0)} \left( \frac{2}{\pi} \right)^{\frac{3}{4}} \Lambda_s(F)
\]

(23)

with

\[
\Lambda_s(F) = \frac{\sin^2 \left[ \frac{\pi}{3} (2s + 1) \left( 1 + \frac{F}{F_0} \right) + \frac{\pi}{4} \right]}{(2s + 1)^{\frac{2}{3}}} \frac{2 \left( 1 - \frac{F}{F_0} \right)^{\frac{3}{4}}}{\left( 1 - \frac{F}{F_0} \right)^{\frac{3}{2}} + \left( 1 + \frac{F}{F_0} \right)^{\frac{3}{2}}}
\]

and

\[
E_{0s}^{(b)} = 4Ry \left[ 1 - \left( \frac{\mu}{m_{||}(0)} \right)^{\frac{1}{2}} \left( \frac{3\pi}{2} \right)^{\frac{1}{4}} \Omega_s(F) \right]
\]

(24)

with

\[
\Omega_s(F) = \left[ (2s + 1) \left( 1 - \frac{F^2}{F_0^2} \right) \right]^{\frac{3}{4}}
\]
are the oscillator strengths of the transitions to the 0s states \((f_{0s})\) and corresponding binding energies \((E_{(0s)}^{(b)})\), respectively. The oscillator strengths \(f_{0s}\) scaled with the 2D oscillator strength \(f_{00}^{(2D)}\) calculated from eqs. (21) for \(\beta \ll 1\) \((J_0(\beta) = 1)\) and the blue shifts \(\hbar \omega - (\bar{E}_g - 4Ry)\) of the ground \(s = 0\) and excited \(s = 1, 2, 3\) satellite exciton peaks scaled with the 2D exciton binding energy \(4Ry\) can be expressed via the functions \(\Lambda_s(F)\) and \(\Omega_s(F)\), respectively, by

\[
\frac{f_{0s}}{f_{00}^{(2D)}} = \frac{d}{a_0} \left( \frac{16m^\parallel(0)}{\pi^2 \mu} \right)^\frac{1}{3} \Lambda_s(F) \tag{25}
\]

and

\[
\frac{\hbar \omega - (\bar{E}_g - 4Ry)}{4Ry} = \left( \frac{9\pi^2 \mu}{4m^\parallel(0)} \right)^\frac{1}{3} \Omega_s(F). \tag{26}
\]

Since the exciton electroabsorption in the W-S regime has been discussed in details by Zhilich [3] we comment only briefly on eq. (20) below. The absorption spectrum consists of a periodic sequence of equidistant blocks each of them is formed by the Rydberg series of discrete \(p\)-peaks and continuous branches \(\xi_\nu(\omega)\) adjacent to the W-S threshold \(\hbar \omega_\nu = \bar{E}_g + \varepsilon_\nu\ (T(0) = 0)\) from low and high frequencies, respectively. The distance between the neighboring blocks \(\hbar \Delta \omega = eFd\) considerably exceeds the energy width \(Ry\) of the series. Within the Rydberg series the ground peak \(p = 0\) dominates others, which decrease in intensity as \(\sim (p + \frac{1}{2})^{-3}\) with increasing indices \(p\). In the presence of sufficiently strong electric fields \((\beta < 1)\) the exciton series corresponding to the threshold \(\hbar \omega_0 = \bar{E}_g + \frac{1}{2}\Delta_{eh}\) becomes more pronounced, while intensities of others \(\sim \beta^{2|\nu|}\) decrease with increasing the index of the subband \(\nu\).

For weak electric fields \(F\), providing the continuous regime, the absorption spectrum consists of Rydberg series formed by the \(p\)-peaks, each of them has the fine structure represented by the satellite \(s\)-lines located above the energy \(\hbar \omega_p = \bar{E}_g - \frac{4Ry}{(2p+1)^2}\). In the absence of the electric field \(F\) the oscillator strengths of the satellite peaks \(f_{0s}\) related to the ground exciton state \(p = 0\) are presented in Fig.1. The intensities of the 0s satellite lines oscillate with increasing the quantum number \(s\). The longitudinal motion with the finite effective mass \(m^\parallel(0)\) reduces the binding energy \(E_{0s}^{(b)}\), blue shifts the exciton peaks and causes the os-
cillator strengths $f_{0s}$ (23) to be small compared to the corresponding strictly 2D values $f_{00}^{(2D)}$ (see eq. (27)). For the ground exciton state $p = s = 0$ the correction to the binding energy $E_{b0}^{(b)} - 4Ry$ scaled with the 2D binding energy $4Ry$ and the ratio of the peak intensities are

\[
\frac{E_{b0}^{(b)} - 4Ry}{4Ry} = -\left(\frac{9\pi^2 \mu}{4m\| (0)}\right)^{\frac{1}{3}} \text{ and } \frac{f_{00}}{f_{00}^{(2D)}} = \frac{d}{a_0} \left(\frac{16m\| (0)}{\pi^2 \mu}\right)^{\frac{1}{3}} \sin^2 \frac{7\pi}{12},
\]

respectively. The oscillator strengths and the blue shifts of the ground and some excited satellite exciton peaks versus the electric fields $F$ are depicted in figures 2 and 3, respectively. With increasing electric field $F$ the $0s$ maxima oscillate, reduce in magnitude and red-shift towards the energy $h\omega_0$.

At $F \simeq F_0$ the fine structure of the optical spectrum turns into a quasi-continuous subband that in turn allows us to replace in eqs. (22)- (24)
FIG. 2. The oscillator strengths of the ground \((f_{00})\) and excited \((f_{01}, f_{02}, f_{03})\) satellite exciton peaks scaled with the 2D oscillator strength \(f_{00}^{(2D)}\) calculated from eqs. (23) and (21) at \(\beta \ll 1\), respectively, (see eq. (25)) as a function of the relative electric fields \(F/F_0\).

\[
\sum_s \text{ by } \int \frac{ds}{dE_{0s}^{(b)}} dE_{0s}^{(b)} \quad \text{with} \quad \frac{ds}{dE_{0s}^{(b)}} = \frac{3}{16Ry} \left(2s + 1\right) \frac{m_i(0)}{9\pi^2 \mu} \frac{1}{2} \left(1 - \frac{F}{F_0}\right)^{-\frac{3}{2}}. \text{ This gives, as expected, the quasi-2D exciton absorption in an electrically unbiased SL in the vicinity of the bottom of the miniband branching from the ground \((p = 0)\) exciton peak } \alpha \sim Ry(\Delta_e + \Delta_h)^{-\frac{1}{2}}[\hbar \omega - (\tilde{E}_g - 4Ry)]^{-\frac{1}{2}} \text{ (see for example Ref. [10]). The limiting case } F = F_0 \text{ is excluded by the conditions (27).}
\]

The continuous regime implies the adiabatic separation of the fast \(\vec{\rho}\)-motion in the heteroplane governed by the 2D Coulomb potential and the slow motion parallel to the SL \(z\)-axis within the asymmetric triangular quantum well. This requires the extent of the wave function \(\psi(z)\) \((13)\) \(a_{+,-}(F)\) to be much less than the 2D exciton Bohr radius \(\frac{1}{2}a_0\). Taking into account the basic condition \((3)\) we obtain
FIG. 3. The dependencies of the blue shifts $\hbar\omega - (\tilde{E}_g - 4\text{Ry})$ of the ground $s = 0$ and excited $s = 1, 2, 3$ satellite exciton peaks scaled with the 2D exciton binding energy $4\text{Ry}$ (see eq. (26)) on the relative electric fields $F/F_0$. The blue shifts are calculated from eqs. (22) and (24) and counted from the position of the ground ($p = 0$) 2D peak $\hbar\omega_0 = \tilde{E}_g - 4\text{Ry}$.

$$d \ll a_0 \left( \frac{\mu}{32m_{\parallel}(0)} \right)^{\frac{1}{3}} \left( 1 - \frac{F}{F_0} \right)^{-\frac{1}{3}} \ll \frac{1}{2} a_0. \quad (27)$$

In addition, the effect of the longitudinal motion of the exciton with the finite reduced mass $m_{\parallel}(0) \sim \Delta_{e,h}^{-1}d^{-2}$ should be a small perturbation to the 2D exciton binding energy $4\text{Ry}$ in eq. (24) for $E_{0s}^{(b)}$ that in turn requires

$$\left( \frac{\mu}{m_{\parallel}(0)} \right)^{\frac{1}{3}} \left( \frac{3\pi}{2} \right)^{\frac{1}{2}} \left( 1 - \frac{F^2}{F_0^2} \right)^{\frac{1}{4}} \ll 1. \quad (28)$$

The applicability of the obtained results needs individual discussions. Eqs. (27) and (28) impose strong restrictions on the SL parameters $\Delta_{e,h}, d, \mu$ and the electric fields $F$ that in turn prevents us from detailed numerical estimates of the expected experimental values for
the concrete SLs. In particular eq. (28) implies on the one hand a small reduced effective mass \( \mu \), narrow miniband \( \Delta_{eh} \) and short SL period \( d \), while on the other hand \( \Delta_{eh} \approx \mu^{-1}d^{-2} \). However, as pointed out in Ref. [1] this contradiction can be lifted by the tunneling factor governed by the optimal relationship between the barrier and well widths. This factor can be considered as independent of the lattice parameters. As for the electric fields \( F \) that its are on the one hand promote the fulfillment of eq. (28) and continual regime and on the other hand prevent use of the adiabatic approximation both reflected in eq. (27).

It follows from eq. (22) that the red shift \( \omega(F) - \omega(0) \) of the ground \((s = 0)\) satellite peak corresponding to the ground Rydberg series \((p = 0)\) for weak electric fields \( F << F_0 \) reads

\[
\omega(F) - \omega(0) = -\frac{4Ry}{\hbar} \left( \frac{2\pi^2 \mu}{3m_\parallel(0)} \right)^\frac{1}{3} \left( \frac{F}{F_0} \right)^2
\]

and allows us to calculate the longitudinal effective reduced mass \( m_\parallel(0) \). Subsequently the width of the total electron-hole miniband \( \Delta_{eh}(0) = \Delta_e + \Delta_h \) affecting the interband electronic, optical and transport properties of the semiconductor SLs can be derived.

Focussing on possible experiments estimates of the expected values can be made for the parameters of the GaAs/Al\(_x\)Ga\(_{1-x}\)As \((x = 0.3)\) SL of period \( d = 3 \text{ nm} \) with the parameters \( \mu = 0.060m_0 \), \( \epsilon = 13.2 \), \( Ry = 4.7 \text{ meV} \), \( a_0 = 11.5 \text{ nm} \) [11]. The exciton electric field determining the longitudinal exciton potential \( \tilde{U}_0(z) \) (14) is \( F_0 = 130 \text{ kV/cm} \). As pointed out above in view of the conditions (27) and (28) at \( F = 0 \) this SL with the realistic miniband width does not seem to be a candidate for an experimental study. However, in the case of the miniband width \( \Delta_{eh} \approx 30 \text{ meV} \) and \( F = 0.88F_0 \) the binding energy \( E_{00}^{(b)} \) (24) becomes \( E_{00}^{(b)}/4Ry \approx 0.58 \). At the same time this electric field considerably reduces the corresponding peak intensities to give for the oscillator strengths (23) \( f_{00}(F) = 0.062f_{00}(0) \). Clearly, the electric fields with strengths \( F \approx F_0 \) narrow down the distances between the satellite peaks for different quantum numbers \( s \) and compose the quasi-continuous absorption band. Note that the terms \( \sim u^2, |u|^3 \) in eq. (14) reduce the effect of the potential \( \tilde{U}_0(z) \) on the 2D exciton binding energy \( 4Ry \) in eq. (24) for \( E_{0s}^{(b)} \), contribute to the condition (28) and thus facilitate the search for suitable lattices. We believe that the continuing technological
progress in fabricating narrow miniband short period SLs renders our analytical results relevant and can contribute to the understanding of the nature of the SL exciton spectra as well as further promote its application to optoelectronics.

V. CONCLUSIONS

We have derived analytically the exciton wave functions and energies and the exciton absorption coefficient in semiconductor superlattices in the presence of external dc electric fields directed parallel to the SL axis. The SL period was taken to be much smaller than the exciton Bohr radius. Strong and weak electric fields providing the spatially localized and extended exciton states, respectively, were considered in the adiabatic approximation. In weak electric fields regime we focus on the fine structure of each of the 2D exciton energy levels grouped into the Rydberg series. This structure is related to the bound electron-hole states in the quasi-uniform longitudinal exciton electric field that in turn reduces the exciton binding energy. It was shown that the external electric field compensates the exciton electric field, considerably modifies the exciton states and corresponding optical exciton fine structure and recovers the strictly 2D exciton binding energy and the 2D exciton absorption in an unbiased SL.

VI. ACKNOWLEDGMENTS

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