Efficient and Scalable Recommendation via Item-Item Graph Partitioning

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Abstract—Collaborative filtering (CF) is a widely searched problem in recommender systems. Linear autoencoder is a kind of well-established method for CF, which estimates item-item relations through encoding user-item interactions. Despite the excellent performance of linear autoencoders, the rapidly increasing computational and storage costs caused by the growing number of items limit their scalabilities in large-scale real-world scenarios. Recently, graph-based approaches have achieved success on CF with high scalability, and have been shown to have commonalities with linear autoencoders in user-item interaction modeling. Motivated by this, we propose an efficient and scalable recommendation via item-item graph partitioning (ERGP), aiming to address the limitations of linear autoencoders. In particular, a recursive graph partitioning strategy is proposed to ensure that the item set is divided into several partitions of finite size. Linear autoencoders encode user-item interactions within partitions while preserving global information across the entire item set. This allows ERGP to have guaranteed efficiency and high scalability when the number of items increases. Experiments conducted on 3 public datasets and 3 open benchmarking datasets demonstrate the effectiveness of ERGP, which outperforms state-of-the-art models with lower training time and storage costs.

Index Terms—Collaborative Filtering, Recommender System, Graph Partitioning

I. INTRODUCTION

The rapid development of the Internet has given rise to recommender systems which focus on alleviating the information overload problem by providing personalized recommendations. The core task of recommendation is to capture user preferences through the interactions between the users and the recommended objects, i.e., the items. This task, known as collaborative filtering (CF) [1], [2], has been extensively studied in recent years by academia and industry.

A straightforward way to implement CF is to estimate the relationship for each pair of users or items. The neighborhood-based method [3] and linear autoencoders [4], [5] attempt to estimate this kind of relationship by constructing a similarity matrix within the set of users or items, where item similarity matrices show better performance on CF tasks than user similarity matrices [6]. While neighborhood-based approaches build the similarity matrix by nonparametric statistical measures [7], linear autoencoders solve an optimization problem to learn the weight matrix by encoding the user-item interaction matrix, which usually demonstrates better recommendation performance. Although linear autoencoders are powerful in modeling direct relationships between items, they suffer from efficiency and scalability problems in practical recommendation scenarios when the scale of the item set increases. There are studies that try to improve training and recommendation efficiency by incorporating sparsity regularization [4] or refining the optimization process [8], but the limitations in high computation and storage complexity in the model training still exist.

Since each entry in the user-item interaction matrix in CF can be naturally considered as an edge between nodes in a bipartite graph, graph-based methods, including graph convolutional networks (GCN) [9], [10] and graph spectral methods [11], [12] are proposed to explore high-order relationships between users and items. Compared to the one-hop relationships estimated in the neighborhood-based methods and linear autoencoders, graph-based methods seek to model multi-hop neighborhood information to learn node representations for users and items. Graph-based methods have achieved success in CF in recent years, demonstrating competitive performance with high scalability compared to linear autoencoders. A recent study [13] shows that both linear autoencoders and neighborhood-based methods can be incorporated in a unified framework of graph convolution. This provides the feasibility of improving the efficiency of linear autoencoders with a graph-based approach. In linear autoencoders, the efficiency of constructing item-item relationships depends mainly on the size of the item set. An intuitive idea is then to control the size of the relationship estimation problem by selecting a subset from all items. A similar idea can be found in the recently proposed UltraGCN [14], which retains a certain number of neighbors on the item-item graph based on edge weights and improves training efficiency. After conducting a theoretical analysis on the graph filtering strategy of UltraGCN, we find that it is a special case of graph partitioning. This strategy has been shown to be effective in improving training efficiency and reducing noise on item-item graphs, but there are still limitations, such as a fixed number of neighbors assigned to all items. This gives us motivation to refine the graph partitioning strategy applied in CF tasks.

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Inspired by the previous works, we propose a linear autoencoder model with guaranteed efficiency and high scalability, named Efficient and scalable Recommendation via item-item Graph Partitioning (ERGP). In particular, we propose a recursive strategy for partitioning on an item-item graph and learn linear autoencoders within each partition. The size of the partition can be specified and is independent of the total number of items, allowing our model to achieve a guaranteed training efficiency and high scalability. To cope with the information loss caused by the partitioning, we assume that the item-item weight matrix is a combination of the learned sparse block diagonal linear autoencoders and a low-rank matrix containing global item-item relationships. The low-rank matrix can be derived by spectral analysis and can act on the optimization of the linear autoencoder, aiming to learn fine-grained local item relations in each partition. Experimental studies on 3 public datasets and 3 open benchmarking datasets demonstrate the effectiveness of our proposed ERGP, which can achieve significant performance gains at lower storage and time costs.

The contribution of this work is summarized as follows:

- We theoretically illustrate the commonalities of graph partitioning and graph filtering strategies in existing CF models, and propose a recursive graph partitioning strategy to divide the item set into subsets with guaranteed size constraints.
- We propose a novel linear autoencoder model that learns item-item relations over the partitioned item set, resulting in a significant improvement in computational and storage efficiency of the model training.
- We introduce global information into linear autoencoders to alleviate the information loss produced by graph partitioning, and propose two data augmentation strategies that act as regularization in the optimization process.
- Experiments conducted on 3 public datasets and 3 open benchmarking datasets demonstrate that our proposed model outperforms state-of-the-art CF models while reducing time and space complexity.

II. RELATED WORKS

Collaborative Filtering (CF) is a widely searched problem in recommender systems. In CF, user-item interactions can be represented as several equivalent forms [14]: discrete user-item pairs, a sparse user-item interaction matrix, or a user-item bipartite graph. Consequently, there are several paradigms to achieve recommendations, including history-based models and embedding-based models [4], [5].

A. History-based CF

History-based CF models consider the historical interacted items as the user features, and recommend items by inputting user features to a precomputed or learned item-item matrix. The item-item matrix can be obtained by calculating the item similarity, which falls into the neighborhood-based method [3], or by optimizing the encoding problem of the user-item interaction matrix, which falls into the linear autoencoders [4], [5] or deep autoencoders [15]. Ning et al. first propose sparse linear method (SLIM) to introduce linear autoencoders to CF task, which demonstrates great strength in modeling item-item relations. Subsequent studies attempt to improve the performance and efficiency of SLIM, including refining the optimization method [8] and applying the denoising module [16]. However, the storage cost and computational efficiency of such methods are highly dependent on the size of the item set, which limits their performance in practical scenarios with increasing item numbers.

B. Embedding-based CF

Unlike history-based methods, embedding-based CF methods map the users and items to embedding vectors, which are used to generate recommended items by sorting the dot product of a user embedding and all item embeddings. Here the embeddings are trained with the user-item interactions. A classical type of model is matrix factorization [17], [18] which considers interactions as discrete user-item pairs. This way of processing interaction data is later fed into deep neural networks [1] to achieve better recommendation performance. In recent years, Graph Convolution Network (GCN) have received much attention. Due to the natural graphical properties of user-item interaction, GCN have been applied to CF [9], [10], [14], [19] to learn user and embeddings through multi-layer graph convolution. In the early proposed NGCF [9], heavy designs are used in the standard GCN, including non-linear activation of feature transformation, which has been shown to be unnecessary in CF by LightGCN [10]. Due to the introduction of linear convolutional layers, the process of representation learning can be approximated, thus reducing the training cost imposed by multi-layer convolution [14]. In addition, a recent work [13] points out the commonality among the linear graph convolution, matrix factorization, and history-based methods, and proposes a non-parametric history-based model GF-CF, which yields competitive performance with small computational cost.

C. Partitioning-based CF

In CF, users are likely to interact with items based on their own preferences, which can lead to the structure of communities [20]. Therefore, existing studies attempt to apply graph partitioning methods and clustering methods to identify the localized correlations within users and items. Depending on the type of methods, the learned partition information can be used to generate user and item embeddings [20], [21] or can be used as a structural prior for the structure of deep neural networks [22]. Results in such studies reveal the contribution of partition information in enhancing the recommendation performance.

In models that directly model item-item relationships, like neighborhood-based methods and linear autoencoders, applying partitioning brings a more significant impact by modifying the scale of the model learning. Establishing the relationship between items only within the same partition reduces the computational costs and leads to the block diagonalization of
the item-item relationship matrix on the entire item set \[22\]. However, such method results in the loss of global information outside the partition and shows a trade-off between the performance and scalability \[23\]. Other linear autoencoder models, such as \[24\], \[25\], attempt to approximate the block diagonal properties by adding regularization terms to the learning of SLIM. They successfully improve the performance of SLIM but fail to achieve efficiency and scalability gains because the relationship matrix is still learned on the entire item set.

Different from the existing studies, we first propose a recursive partitioning strategy that ensures the size constraint of model learning, and first introduce the global patterns into the localized item relationship modeling. These allow the proposed ERGP to fully utilize the partition information, and achieve recommendation performance gains with high scalability and efficiency.

III. PRELIMINARIES

This section presents the formulation of CF problem and the definitions used in two types of CF methods: graph-based methods and linear autoencoders.

A. Problem Formulation

Suppose a user set \( \mathcal{U} \), an item set \( \mathcal{I} \), and the observed interaction set \( \mathcal{R} = \{ (u, i) | u \in \mathcal{U}, i \in \mathcal{I} \} \) between users and items. For each user \( u \), Collaborative Filtering (CF) aims to recommend top-\( k \) items from \( \mathcal{I} \) this user has not interacted with.

B. Graph-based Methods

The Graph Convolutional Networks (GCN) have recently illustrated excellent performance in CF tasks. Most GCN-based CF methods perform graph convolution on the user-item interaction matrix \( \mathcal{R} \in \{ 0, 1 \}^{\mathcal{U} \times \mathcal{I}} \) with implicit feedback, which is defined as

\[
R_{ui} = \begin{cases} 1, & \text{if interaction } (u, i) \text{ is observed,} \\ 0, & \text{otherwise.} \end{cases}
\]  

As a common practice, standard normalization is applied to ensure the stability of graph convolution on multiple layers. The normalized interaction matrix is defined \[26\] in as follows:

\[
\tilde{R} = D^{-\frac{1}{2}} R D^{-\frac{1}{2}},
\]

where \( D = \text{diag}(1^T R) \) is the row sum of the interaction matrix \( R \). Similarly, the column sum of \( R \) is defined as \( D_R = \text{diag}(R 1^T) \). On the other hand, the item-item adjacency matrix is also explored to identify the relationships between items. Depending on whether the normalization strategy is used in \( R \), there are two styles for constructing item-item adjacency matrix:

Definition 1 (Unnormalized Item-Item Graph \[14\]): Suppose \( \mathcal{R} \) a bipartite graph between user nodes and item nodes, then the unnormalized item-item adjacency matrix is defined as

\[
Q = R^T R.
\]

Definition 2 (Normalized Item-Item Graph \[13\]): Suppose \( \mathcal{R} \) an incidence matrix of a hypergraph \( \mathcal{R} = \{ \mathcal{I}, \mathcal{U} \} \), where a hyperedge \( u \in \mathcal{U} \) is incident with a vertex \( i \in \mathcal{I} \) when \( i \in u \). Then the normalized item-item adjacency matrix of \( \mathcal{R} \) is defined as

\[
\tilde{Q} = \tilde{R}^T \tilde{R}.
\]

It is important to note that although \( \tilde{R} \) can be easily converted from \( R \), \( \tilde{Q} \) cannot be obtained by direct conversion from \( Q \).

C. Linear Autoencoders

Linear autoencoder is a class of neighborhood-based CF methods. The basic linear autoencoder model aims to solve a ridge regression problem to learn the item-item weight matrix \( C \) \[5\]:

\[
\arg \min_C \frac{1}{2} \| R - RC \|_F^2 + \frac{\theta_2}{2} \| C \|_F^2 + \frac{\theta_1}{2} \| C \|_1.
\]

Here, \( C \in \mathbb{R}^{\mathcal{I} \times \mathcal{I}} \) denotes the learned item-item weight matrix. Suppose \( r_u \) the \( u \)-th row of \( R \), recommendations for user \( u \) is generated by sorting the vector \( r_u C \). In practical applications, \( C \in \mathbb{R}^{\mathcal{I} \times \mathcal{I}} \) can be very large due to the numerous candidate items. One item may only show significant correlations with a limited number of other items, making the weight matrix sparse. Therefore, SLIM \[4\] introduces the \( l_1 \)-norm to control the sparsity of the weight matrix:

\[
\arg \min_C \frac{1}{2} \| R - RC \|_F^2 + \frac{\theta_2}{2} \| C \|_F^2 + \theta_1 \| C \|_1 \text{ s.t. } \text{diag}(C) = 0, C \geq 0,
\]

where the diagonal-zero constraint is added to prevent trivial solution. \[5\] has the closed-form solution, while \[6\] requires the iterative optimization. However, both approaches suffer from high computational costs in time (\( O(|\mathcal{I}|^2 \cdot 376) \) for matrix inversion) and storage (\( O(|\mathcal{I}|^2) \) for the storage of dense matrix) during the optimization process \[5\], which leads to the scalability problem when the number of items increases.

IV. METHODOLOGY

In this section, we propose our framework ERGP as illustrated in Fig.1. In general, it uses a recursive graph partitioning strategy and a partition-aware linear autoencoder model to exploit item-item correlations with high efficiency and high scalability. Then we introduce ERGP in detail.

A. Item-Item Graph Partitioning

Compared to the highly sparse user-item interaction matrix \( R \), the item-item adjacency matrix \( Q \) is much denser. This adds storage and time expense to the model training process. To retain the high training efficiency, UltraGCN \[14\] performs filtering in \( Q \) to select the most informative item-item pairs. Top-\( k \) item-item pairs in the row \( i \) of \( Q \) are selected according to

\[
\arg \max_{j \in \mathcal{I}} \omega_{ij},
\]

where the weight \( \omega_{ij} \) is defined as

\[
\omega_{ij} = \frac{Q_{ij}}{\sqrt{p_i}} \cdot \sqrt{\frac{p_i}{p_i - Q_{ii}}},
\]

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where \( p_i = \sum_k Q_{ik} \) is the sum of \( i \)-th row in \( Q \). This filtering strategy has been shown to be effective in improving recommendation efficiency and performance, revealing the feasibility of sample selection from the adjacency matrix of items. However, limitations still exist and can result in loss of information. First, the filtering strategy is carried out locally in each row, which ignores the influence of other neighboring nodes in the item-item graph. Second, reserving a fixed number of neighbors for all items does not match the real scenario, where the number of neighbors varies in relation to the item popularity.

Next, we show that the item filtering strategy in UltraGCN is a special case of graph partitioning. Finding a cut-set in the graph, which is known as the graph partitioning problem, has been widely searched in the graph theory. An effective approach to the graph partitioning problem is modularity maximization \([27]\). The modularity of an undirected weighted graph \( G = \{V, E\} \) is defined as

\[
M(G) = \frac{1}{2W} \sum_{i,j} (A_{ij} - \frac{d_i d_j}{2W}) \mathbf{1}_{C_i = C_j}, \tag{9}
\]

where \( W \) is the sum of all edge weights in \( G \), \( A_{ij} \) is the edge weight between node \( i \) and node \( j \), \( d_i \) and \( d_j \) are degrees of node \( i \) and node \( j \), respectively. \( \mathbf{1}_{C_i = C_j} \) is the indicator function which equals 1 when node \( i \) and node \( j \) belong to the same partition and equals 0 otherwise.

Consider there is a subgraph \( G_i \) that contains node \( i \) and all its neighbors in the item-item adjacency graph. Edges exist only between node \( i \) and its neighbor nodes, where the edge weights equal to \( \omega_{ij} \) defined in \([8]\). In \( G_i \), the total edge weights \( W \) equals to the degree of node \( i \), and the degree of the node other than node \( i \) is equal to the edge weight between it and node \( i \). Then \( \ref{9} \) can be simplified to

\[
M(G_i) = \frac{1}{2d_i} \sum_{i,j} \omega_{ij} - \frac{1}{2d_i} \mathbf{1}_{C_i = C_j} = \frac{1}{4d_i} \sum_{i,j} \omega_{ij} \mathbf{1}_{C_i = C_j}, \tag{10}
\]

Then, the graph filtering strategy \( \ref{7} \) is equivalent to iterative modularity maximization graph partitioning in the above sub-graph. Each iteration the edge with the smallest weight will be removed until node \( i \) has \( k \) neighbors left.

The limitations of the neighbor filtering strategy in UltraGCN have been discussed above. In linear autoencoder models, \( Q \) is also involved during the model training \([8]\), leading to the same efficiency and scalability problem. UltraGCN gives us the inspiration that the size of the problem can be reduced by partitioning the item-item graph. In the partitioned graph, multiple connected components can be generated and used to train linear autoencoders independently. The storage and computational complexity of the optimization process will depend on the size of the item sets in each partition, which can be guaranteed by controlling the size limit of partitions.

Therefore, we propose a recursive partitioning strategy to ensure that the partition size does not exceed a set upper limit. As shown in Fig. \( 1 \) in each iteration, a graph is divided into two partitions by the adopted spectral method proposed in \([28]\), which utilizes the eigenvector of the normalized graph adjacency matrix with the second largest eigenvalue. Suppose \( V \in \mathbb{R}^{|V| \times n} \) the eigenvectors of the normalized item-item adjacency matrix \( \tilde{Q} \) with the top-\( n \) largest eigenvalues, the partition of node \( i \) can be derived through

\[
i \in \begin{cases} \text{Partition 1,} & \text{sign}(V_{i,2}) = +1, \\ \text{Partition 2,} & \text{sign}(V_{i,2}) = -1, \end{cases} \tag{11}
\]

where \( V_{i,2} \) is the \( i \)-th element of the eigenvector with the second largest eigenvalue of \( \tilde{Q} \). This algorithm is shown to be a good approximation of modularity maximization graph partitioning \([28]\). If the number of items in a partition exceeds the set limit, graph partitioning strategy \( \ref{11} \) will be performed recursively in this partition, until the number of items in all partitions is less than the set limit. After recursion, several partitions can be obtained, whose sizes are guaranteed and independent of the total number of items.
B. Partition-aware Linear Autoencoder

By partitioning item-item graph to several unconnected components, the linear autoencoder can be learned in each partition by optimizing the problem like \( \Theta \). However, the learned block diagonal item-item weight matrix has all zero value of \( C_{ij} \) when item \( i \) and item \( j \) belongs to different partitions. It means the recommendation results are highly depending on the partitions. If a user has never clicked on an item in a partition, all items in that partition will not be recommended. To alleviate the impact of partitioning on recommendation performance, optimization of the linear autoencoder should consider the item relationships across partitions. We assume the item-item weight block is not only block diagonal, but a combination of a low-rank matrix \( \hat{W} \) and a block diagonal sparse matrix \( S \). In [12], the author formulates CF as a low-rank matrix factorization problem in the scope of graph signal processing. Suppose the constrained matrix factorization problem is

\[
\arg\min_{U,V} \| \hat{R} - UV^T \|_F^2, \quad s.t. V^TV = I. \tag{12}
\]

Here, the solution \( V \) contains the top-\( n \) eigenvectors of \( \hat{Q} \) which has been derived in Section IV-A. And the low-rank approximation of \( R \) is derived by

\[
D_U^{\frac{1}{2}}UV^TD_V^{\frac{1}{2}} = D_U^{\frac{1}{2}}U(V^TV)V^TD_V^{\frac{1}{2}} = RD_I^{\frac{1}{2}}VV^TD_I^{\frac{1}{2}}. \tag{13}
\]

Therefore, we set

\[
W = D_I^{-\frac{1}{2}}VV^TD_I^{\frac{1}{2}}, \tag{14}
\]

and assume the item-item weight matrix \( C \) is the combination of \( W \) and a block diagonal sparse matrix \( S \). Unlike the linear autoencoder problem formulated in [3], the \( l_1 \) regularization term, the diagonal zero constraint, and the non-negative constraint act only on \( S \) instead of \( C \). Due to the introduction of \( W \), we modify the form of \( l_1 \) regularization term and add a new regularization term to balance the effect of \( S \) and \( W \). Then, we define the final problem of ERGP as

\[
\arg\min_S \frac{1}{2} \| R - R(\lambda W + S) \|_F^2 + \frac{\theta_2}{2} \| \lambda W + S \|_F^2
+ \theta_1 \| S \|_1 + \frac{\eta}{2} \| 1^T - 1^T(\lambda W + S) \|_F^2,
\]

\[
s.t. \text{diag}(S) = 0, \quad S \geq 0, \quad S_{i,j}[g(i) \neq g(j)] = 0, \tag{15}
\]

where \( g(i) \) is the partition item \( i \) is assigned to, \( \lambda \) is used to adjust the weight of \( W \). In Section IV-C, we will show that the two new regularization terms \( \| (\lambda W + S)D_I \|_F^2 \) and \( \| 1^T - 1^T(\lambda W + S) \|_F^2 \) are equivalent to the data augmentation to the original training dataset. The top-\( k \) recommendation result for user \( u \) is generated as

\[
\arg\max_r \sum_{i=1}^{k} r_u(\lambda W + S), \tag{16}
\]

where \( r_u \) is the \( u \)-th row of \( R \).

C. Data Augmentation

1) Weighted \( l_2 \) regularization: Let \( C = \lambda W + S \), the \( l_2 \) regularization term in (6) can be written as

\[
\| \lambda W + S \|_F^2. \tag{17}
\]

Suppose there is a new user who only interacted with item \( i \), the linear autoencoder is optimized as

\[
\arg\min_S \| I_I^T - I_I^T(\lambda W + S) \|_F^2
= \arg\min_S (1 - (\lambda W_{ii} + S_{ii}))^2 + \sum_{j \neq i} (\lambda W_{ji} + S_{ji})^2
= \arg\min_S \sum_j (\lambda W_{ji} + S_{ji})^2 + c
= \arg\min_S \| \lambda W + S \|_F^2 + c,
\]

where \( c \) is a constant, and \( S_{ii} \) is 0 due to the zero diagonal constraint. Then, the regularization term (17) can be considered as the augmentation of the data samples, where the user only interacted with one item. As the occurrence number of items varies in the training set, we assign weights to each column of \( S \) based on the degree matrix \( D_I \), which is shown in (15).

2) Sum-one Regularization: Several linear autoencoder models [29] add sum-one constraint \( 1^T C = 1^T \) to learn a normalized weight matrix. Here we relax it by adding a regularization term to the original problem in (6)

\[
\| 1^T - 1^T(\lambda W + S) \|_F^2. \tag{19}
\]

This regularization term can be considered as a data augmentation, which is equivalent to add new users who interacted with all items.

Instead of concatenating the augmented user samples into \( R \), we can directly modify \( Q \) to achieve the equivalent effect in the optimization of linear autoencoders. The augmented item-item adjacency matrix is defined as

\[
\hat{Q} = R^T R + \theta_2 D_I + \eta. \tag{20}
\]

D. Optimization

With the constraint \( S_{ij,g(i) \neq g(j)} = 0 \), the problem (15) can be divided to several sub-problem in each partition. Here we use the skim to denote the parameters in one partition. For example, we denote item-item weight matrix in a partition as \( S' \), and \( S \) is derived by the concatenation of \( S' \) in all partitions:

\[
S = \text{diag}(S'_1, ..., S'_k). \tag{21}
\]

Then we focus on the optimization in one partition. This constraint optimization problem can be solved by Alternating Direction of Multiplier Method (ADMM) [8], [30] by converting the problem (15) to

\[
\arg\min_{Z', S'} \frac{1}{2} \| R' - R'(\lambda W' + Z') \|_F^2 + \frac{\theta_2}{2} \| (\lambda W' + Z')D_I \|_F^2
+ \theta_1 \| S' \|_1 + \frac{\eta}{2} \| 1^T - 1^T(\lambda W' + Z') \|_F^2,
\]

\[
s.t. \text{diag}(Z') = 0, \quad Z' \geq 0, \quad Z' = S'. \tag{22}
\]
1) **Update Z′:** The update of $Z′$ at $k + 1$-th iteration can be written as

$$
Z′(k+1) = \arg\min_{Z′} \frac{1}{2} \| R′ - R(\lambda W′ + Z′) \|_F^2 + \frac{\theta_2}{2} \| (\lambda W′ + Z′) D1 \|_F^2 + \mu^T \text{diag}(Z′) + \frac{\eta}{2} \| T^1 - T(\lambda W′ + Z′) \|_F^2 + \frac{\rho}{2} \| Z′ - S′(k) + \Phi′(k) \|_F^2,
$$

(23)

where $\mu$ is the vector of the augmented Lagrange multiplier for the constraint $\text{diag}(Z′) = 0$, $\Phi$ is the dual variable, and $\rho$ is the augmented Lagrange multiplier for the constraint $Z′ = S′$ with the dual variable $\Phi$. By introducing the data augmentation operations discussed in Section IV-C, we replace $Q$ with $\tilde{Q}$ and rewrite the solution of (23) as

$$
Z′(k+1) = (\tilde{Q}' + \rho I)^{-1}(\tilde{Q}' (I - \lambda W) + \rho (S′(k) - \Phi′(k)) - \text{diagMat}(\mu)),
$$

(24)

where $\text{diagMat}(\cdot)$ denotes the conversion from a vector to the diagonal matrix. Here the multiplier $\mu$ can be removed by the equality constraint $\text{diag}(Z′(k+1)) = 0$:

$$
\mu = \text{diag}((\tilde{Q}' + \rho I)^{-1}(\tilde{Q}' (I - \lambda W) + \rho (S′(k) - \Phi′(k)) \odot \text{diag}(\tilde{Q}' + \rho I)^{-1}),
$$

(25)

where $\odot$ denotes the element-wise division. Replace $\mu$ in (24) with (25), the update of $Z′$ at $k + 1$-th iteration is derived.

2) **Update S′:** The update of $S′$ at $k + 1$-th iteration can be written as

$$
S′(k+1) = \arg\min_{S′} g(S′)
= \arg\min_{S′} \theta_1 I^T S′ I + \frac{\rho}{2} \| Z′(k+1) - S + \Phi′(k) \|_F^2,
$$

(26)

where the domain of $g(S′)$ is $S′ \geq 0$. The solution of (26) can be written as

$$
S′(k+1) = (Z′(k+1) + \Phi′(k) - \frac{\theta_1}{\rho} +),
$$

(27)

where $(\cdot)_+$ denotes the non-negative mapping $f(x) = \max(x, 0)$.

3) **Update $\Phi$′:** The update of dual variable $\Phi′$ at $k + 1$-th iteration can be written as

$$
\Phi′(k+1) = \Phi′(k) + Z′(k+1) - S′(k+1).
$$

(28)

When optimization is finished, $S′$ is returned as the item-item weight matrix for this partition. In the actual optimization process, we filter out small values in the sparse matrix $S′$ after the optimization to reduce the noise. Generally, setting the filtering threshold between $1e-3$ and $5e-3$ is tested to be effective in reducing the number of parameters and has little effect on the recommendation performance. The proposed method is summarized in Algorithm 1.

![Algorithm 1 Efficient and Scalable Recommendation via Item-Item Graph Partitioning (ERGP)](https://openbenchmark.github.io/BARS/)

**Require:** item set $I$, interaction matrix $R$, hyperparameters $\lambda, \theta_1, \theta_2, \eta$, and the size limit of partition $\text{size\_limit}$

**Ensure:** top-k recommended items for user $u$

**procedure** PART($I, R, \text{size\_limit}$)

$\hat{Q} \leftarrow [4]$  
$V \leftarrow \text{EigenDecomposition}(\hat{Q})$  
Derive $I_1, I_2$ from $I$ through [11]

5:  
$I_P \leftarrow \text{EmptySet}()$

for $\bar{I}$ in $\{I_1, I_2\}$ do

if $|\bar{I}| \geq \text{size\_limits}$ then

Derive $R$ from $R$ according to $\bar{I}$

$I_P \leftarrow \text{Merge}(I_P, \text{PART}(\bar{I}, R, \text{size\_limit}))$

else

$I_P \leftarrow \text{Merge}(I_P, \bar{I})$

end if

end for

return $I_P$

15:  
end procedure

$Q \leftarrow [4]$  
$V \leftarrow \text{EigenDecomposition}(\hat{Q})$  
$W \leftarrow [14]$  
$I_P \leftarrow \text{PART}(I, R, \text{size\_limit})$

20:  
$S_P \leftarrow \text{EmptyList}()$

for $\bar{I}$ in $I_P$ do

Derive $R$ from $R$ according to $\bar{I}$

$\hat{Q} \leftarrow [20]$  
for $k = 1; k \leq \text{max\_iter}; k ++$ do

$Z′ \leftarrow [24], [25]$  
$S′ \leftarrow [27]$  
$\Phi′ \leftarrow [28]$  
end for

$S_P \leftarrow \text{Append}(S_P, S′)$

end for

$S \leftarrow [21]$  
Recommend items for user $u$ by [16]

---

V. Experiments

**A. Experimental Setup**

1) **Datasets and Evaluation Metrics:** We conduct experiments on three public datasets: Pinterest, Amazon-cds, and Douban. The data sets are divided into training sets, validation sets, and test sets, with a ratio of 0.7: 0.15: 0.15. Two common evaluation metrics are used: NDCG@K and Recall@K, denoted as N@K and R@K, respectively. K is set to 10 and 20. To conduct unbiased evaluation, we repeat each experiment for 7 times with different random data split, and report the average result.

Furthermore, we test our proposed ERGP model on the benchmarking BARS[1][51], including 3 datasets: Gowalla, Yelp2018, and Amazon-books. The original data splits are used.

[1]https://openbenchmark.github.io/BARS/
to ensure fair comparison on 2 evaluation metrics: NDCG@20 and Recall@20. The statistics of all 6 datasets are summarized in Table I.

2) Baselines: We compare our proposed model with several types of CF methods:

- Embedding-based methods: MF-BPR [13], LightGCN [10], UltraGCN [14], and SimGCL [19].
- History-based methods: GF-CF [13], BISM [25].

We do not compare the traditional linear autoencoder models such as SLIM [4], EASE [5] and ADMMSLIM [8] due to the high storage cost in datasets with a large number of items, which leads to out-of-memory problems. Instead, we add BISM [25] to the comparison. BISM adds constraints to the optimization of linear autoencoder and learns an item-item weight matrix that exhibit properties close to a block diagonal matrix. Moreover, BISM presents an element-wise iterative optimization approach, achieving better performance than SLIM and EASE with relatively low storage cost.

3) Parameter Settings: For all embedding-based methods, the embedding size for users and items is set to 64, the learning rate is set to 1e-3, and the training batch size is set to 2048. The number of training epochs is set to 1000, and the best performance model in the validation set will be used for testing.

For history-based method, each method contains an item-item weight matrix stored as a sparse matrix, except for GF-CF [13] which has no trainable parameters. The number of eigenvectors extracted in ERGP is set to 256 to maintain consistency with GF-CF. For ERGP, we conduct parameter sensitivity analysis on $\lambda$ and $\theta_1$ in Section V-D, and tune $\theta_2$ in [0.1, 0.2, 0.5, 1, 2, 5] and $\eta$ in [1e-2, 1e-1, 1]. In linear autoencoders, the sparsity of the learned matrix cannot be set explicitly, but can be changed by adjusting the hyperparameter for $l_1$ regularization. Therefore, we list the number of model parameters in the performance comparison to make a comparison with embedding-based methods. All sparse matrices are stored in compressed sparse row (CSR) format, which contains approximately twice the parameter numbers as the number of non-zero values (NNZ) in the sparse matrix.

B. Performance Comparison

Table I reports the performance comparison on 3 public datasets with statistical analysis. In each metric, the models that achieve the best performance are marked in bold, and the second-best models are underlined. The highlights are listed as follows.

- The proposed ERGP model achieves the overall best performance on 3 datasets. The results of significant testing indicates that ERGP achieves significant improvement on all metrics of Amazon-cds and Pinterest datasets, and NDCG metrics of Douban dataset ($p$-value < 0.01). This demonstrates the effectiveness of ERGP in modeling item-item relationships by the assumption of a low-rank matrix and a block diagonal sparse matrix. While the low-rank matrix preserves global relationships across the item partitions, the linear autoencoder inside each partition can learn fine-grained relationships between the nearest neighbors of items. These two critical components together contribute to the outstanding performance of ERGP.
- ERGP is more effective in reducing the model size and training cost of linear autoencoders. The traditional linear autoencoder model BISM [25] also achieves favorable performance compared to embedding-based methods, especially on Amazon-cds and Douban datasets. However, learning item relationships on the full item-item adjacency matrix leads to high storage costs during model training and a huge number of model parameters. ERGP successfully addresses this limitation through the recursive graph partitioning strategy, which reduces the size of linear autoencoder optimization problem and yields better performances with a lower number of parameters than embedding-based methods.

Moreover, we test our proposed ERGP model on 3 benchmarking datasets and compare to the state-of-the-art CF methods summarized by the open benchmarking project BARS [31]. In addition to BPR, LightGCN, GF-CF, UltraGCN, we add several graph-based methods NGCF [9] and DGCF [32], two-tower method SimpleX [33], and matrix factorization method MF-CCL [33]. The number of parameters of the ERGP model is set smaller than all embedding-based methods to ensure a fair comparison. Table II reports the comparison results. It can be seen that our proposed ERGP model achieves first place in four metrics and second place in two metrics, which further demonstrates the power of ERGP in CF task.

C. Ablation Analysis

To validate the contribution of each component in the proposed ERGP model, we conduct experiments on 3 variants of ERGP listed as follows:

- ERGP($\theta_2 = 0$): it removes the term (17) from the optimization problem of $S$.
- ERGP($\eta = 0$): it removes the term (19) from the optimization problem of $S$.
- ERGP($\lambda = 0$): it removes the low-rank matrix $W$ and makes $C = S$, which only contains item relationships in each partition.

Here we do not set $\theta_1$ to 0 as a variant, because it is necessary to guarantee the sparsity of the learned item-item weight.
### Table II
**Performance Comparison on 3 Public Datasets**

| Dataset   | Amazon-cds | Douban | Pinterest |
|-----------|------------|--------|-----------|
| Metrics   | R@10 R@20 N@10 N@20 #Param | R@10 R@20 N@10 N@20 #Param | R@10 R@20 N@10 N@20 #Param |
| BPR       | 0.0741 0.1111 0.0467 0.0569 5.04 M | 0.0901 0.1347 0.0861 0.0966 2.26 M | 0.0959 0.1423 0.0637 0.0846 4.17 M |
| LightGCN  | 0.0905 0.1346 0.0380 0.0701 5.04 M | 0.1067 0.1571 0.1106 0.1206 2.26 M | 0.0972 0.1578 0.0730 0.0956 4.17 M |
| UltraGCN  | 0.1046 0.1487 0.0686 0.0806 5.04 M | 0.1379 0.1925 0.1462 0.1556 2.26 M | 0.0986 0.1604 0.0750 0.0976 4.17 M |
| SimGCL    | 0.1016 0.1468 0.0657 0.0781 5.04 M | 0.1211 0.1699 0.1257 0.1346 2.26 M | 0.0981 0.1590 0.0739 0.0965 4.17 M |
| GF-CF     | 0.0931 0.1350 0.0609 0.0725 - | 0.1229 0.1719 0.1276 0.1365 - | 0.1007 0.1620 0.0758 0.0987 - |
| BISM      | 0.1121 0.1541 0.0759 0.0874 111 M | 0.1597 0.2158 0.1837 0.1889 158 M | 0.0995 0.1594 0.0754 0.0978 24.0 M |
| ERGP      | 0.1149 0.1576 0.0780 0.0896 3.87 M | 0.1595 0.2095 0.1931 0.1950 2.14 M | 0.1030 0.1650 0.0779 0.1010 1.66 M |

### Table III
**Performance Comparison on 3 Benchmarking Datasets**

| Dataset | Gowalla | Yelp2018 | Amazon-books |
|---------|---------|----------|---------------|
| Metrics | R@20 N@20 | R@20 N@20 | R@20 N@20 |
| BPR     | 0.1627 0.1378 | 0.0576 0.0468 | 0.0338 0.0264 |
| MF-CCL  | 0.1837 0.1493 | 0.0698 0.0572 | 0.0559 0.0447 |
| NGCF    | 0.1570 0.1327 | 0.0579 0.0477 | 0.0344 0.0263 |
| LightGCN| 0.1820 0.1547 | 0.0653 0.0532 | 0.0411 0.0318 |
| DGCF    | 0.1842 0.1561 | 0.0654 0.0534 | 0.0422 0.0324 |
| GF-CF   | 0.1849 0.1518 | 0.0697 0.0571 | 0.0710 0.0584 |
| SimpleX | 0.1872 0.1557 | 0.0701 0.0575 | 0.0583 0.0468 |
| UltraGCN| 0.1862 0.1580 | 0.0683 0.0567 | 0.0681 0.0556 |
| ERGP    | 0.1884 0.1566 | 0.0703 0.0584 | 0.0717 0.0580 |

Fig. 2. Performance of ERGP with different $\lambda$ on (a) Amazon-cds dataset and (b) Pinterest dataset.

matrix in ERGP. Table IV reports the results of the ablation analysis. It can be seen that ERGP consistently outperforms all variants on the test datasets. On the other hand, comparisons between variants show that different components of ERGP contribute differently on each dataset. ERGP($\lambda = 0$) shows a large performance degradation in the Pinterest and Yelp2018 datasets, while ERGP($\eta = 0$) degrades most significantly on Douban dataset. On Amazon-cds dataset, all three variants show a significant performance degradation compared to the full version of ERGP. This verifies the effectiveness of the individual components in ERGP.

### D. Parameter Sensitivity

1) Impact of $\lambda$: We test the impact of $\lambda$ by setting it to different values between 0.1 and 1.0 with a step size of 0.1. Fig. 2 shows the performance on Amazon-cds and Pinterest datasets. As the $\lambda$ increases from 0.1 to 1.0, the performance of ERGP rises and then falls. ERGP maintains great performance when setting the $\lambda$ between 0.2-0.5 on both datasets, while the optimal performance is achieved with a different $\lambda$ value on each dataset. This phenomenon confirms the role of low-rank matrix in introducing global information in the optimization of linear autoencoder in each partition, while the effectiveness varies depending on the characteristics of the dataset.

2) Impact of $\theta_1$: We test the impact of $\theta_1$ by setting it to different values between 0.2 and 2.0 with a step size of 0.2. Fig. 3 shows the performance on Yelp2018 dataset. As $\theta_1$ decreases gradually from 2.0, the sparsity constraint of the matrix becomes weaker and the number of parameters increases. At the same time, the performance of ERGP first rises and then stabilizes, or even declines. This suggests that more parameter numbers in linear autoencoders may introduce noise and lead to performance degradation. On the other hand, although the performance of ERGP decreases as the $\theta_1$ increases, it still achieves better performance when the number of parameters is much smaller than that of the newest graph-based approach. This demonstrates the flexibility of ERGP in balancing efficiency and performance in large-scale recommender systems.
### Table IV

| Dataset     | Amazon-cds | Douban | Pinterest | Yelp2018 |
|-------------|------------|--------|-----------|----------|
| Metrics     | R@20       | N@20   | R@20      | N@20     |
| ERGP ($\theta_2 = 0$) | 0.1539 0.0870 | 0.2084 0.1943 | 0.1624 0.0996 | 0.0692 0.0574 |
| ERGP ($\gamma = 0$) | 0.1540 0.0873 | 0.2046 0.1909 | 0.1643 0.1006 | 0.0702 0.0582 |
| ERGP ($\lambda = 0$) | 0.1542 0.0887 | 0.2085 0.1945 | 0.1612 0.0991 | 0.0662 0.0556 |
| ERGP        | 0.1576 0.0896 | 0.2095 0.1950 | 0.1650 0.1010 | 0.0703 0.0584 |

Fig. 3. Performance of ERGP with different $\theta_1$ on Yelp2018 dataset: (a) Recall@20 and (b) NDCG@20.

#### Efficiency Analysis

In this subsection, we conduct theoretical and experimental analysis to illustrate the major highlight of ERGP: the efficiency and scalability improvement in both time and storage brought by the graph partitioning strategy.

1) **Computational Complexity and Storage Cost:** The major contributions to the computational complexity of ERGP are the generation of low-rank matrix $W$ and the optimization of partitioned linear autoencoder $S$. $W$ is generated by the top-$k$ eigenvectors on $\hat{Q}$, which can be obtained by calculating the top-$k$ singular values of $\hat{R}$, which is known as the truncated singular value decomposition (SVD) performed on the sparse matrix $\hat{R}$. Several algorithms are designed to solve this problem, like Lanczos method [34] and LOBPCG [35]. This kind of method optimally yields the computational complexity of $O(T(k^3 + k|T|))$ [13], [36], where $T$ denotes the iteration numbers and $|T|$ is the interaction numbers in the training set, which equals to the non-zero element numbers in $R$. Because only matrix $\hat{R}$ is needed in the truncated SVD instead of $Q$, the storage cost is $O(|T|)$.

The optimization of linear autoencoders requires a computational complexity of at least $O(|T|^2)$ contributed by the matrix inverse [5], and a storage cost of $O(|T|^2)$. In ERGP, the optimization of the linear autoencoder is performed within each partition and the size of the partition can be set explicitly, which is independent of $|T|$. Suppose the size limit of a partition is $p < |T|$, then the upper bound of the total computational complexity is $O(|T|^2/p^{3.376})$, and the storage cost is $O(|T|^2/p)$. These are rough upper bounds which assumes all partitions have the same size $p$. The actual produced partitions usually have uneven sizes, which leads to lower computing and storage costs. The efficiency and scalability improvement of ERGP are also revealed in Fig. 1. Instead of storing and computing on the full matrix $S$, ERGP only needs to perform on several block diagonal matrices, which effectively alleviates the scalability problem in linear autoencoders.

On the other hand, ERGP requires computing the first two eigenvectors of the item adjacency matrix at each time of partitioning. The eigenvector used for the first partitioning can be obtained when calculating $W$, while the subsequent partitioning processes have the computational complexity $O(T(k^3 + k|T'|))$, where $k$ is set to 2 and $T'$ is a subset of $T$, and the storage cost is $O(|T'|)$. Both of them contribute little to the overall computational and storage cost and can be neglected.

2) **Experimental Analysis:** To verify the efficiency improvement of the recursive partitioning strategy, we run ERGP model by setting the size limit of the partition to different values. Fig. 1 shows the results on Gowalla dataset. When the size of the partition is reduced to 8000 (about 20% of the items in Gowalla dataset), the model performance changes slightly, while the training time is significantly reduced. Continuing to decrease the size limit, the model performance starts to decrease with the increase in the number of partitions. This demonstrates that when the size limit of the partition is set appropriately, ERGP can significantly improve the training efficiency of the linear autoencoders, while maintaining excellent recommendation performance. Furthermore, we compare the training time of ERGP and several baseline methods, which is reported in Table IV. It can be seen that ERGP achieves a training speed close to that of the non-parametric
method GF-CF and more than 10 times speedup improvement compared to the graph-based methods. Also, ERGP achieves a speedup of more than 10 times compared to BISM which learns the linear autoencoder on the entire item set with high storage cost. These observations show that the graph partitioning strategy proposed in ERGP brings a significant improvement in efficiency, allowing it to be superior in both speed and performance, which are the main concerns in large-scale recommender systems.

VI. Conclusion

In this paper, we propose a novel efficient and scalable recommendation model named ERGP, which applies graph partitioning to item-item graphs to learn item-item relationships with a guaranteed computational efficiency and high scalability. The proposed recursive partitioning strategy reduces the size of the linear autoencoder optimization problem and successfully addresses the scalability problem of linear autoencoders in large-scale recommendation scenarios. ERGP also introduces global information to the linear autoencoder optimization inside each partition to cope with information loss caused by graph partitioning, bringing significant improvements in recommendation performance. Experimental results validate the effectiveness of each component in ERGP, and demonstrate the superiority of ERGP in terms of efficiency and performance compared to state-of-the-art models in CF tasks.

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