Aspects of spacetime-symmetry violations

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Abstract
The violation of spacetime symmetries provides a promising candidate signal for underlying physics, possibly arising at the Planck scale. This talk gives an overview over various aspects in the field, including some mechanisms for Lorentz breakdown, the SME test framework, and phenomenological signatures for such effects.

1 Introduction
Although phenomenologically successful, the Standard Model of particle physics leaves unanswered a variety of theoretical questions. At present, significant theoretical work is therefore directed toward the search for an underlying theory that includes a quantum description of gravity. However, observational tests of such ideas face a major obstacle of practical nature: most quantum-gravity effects in virtually all leading candidate models are expected to be extremely small due to Planck-scale suppression.

During the last decade, minuscule violations of Lorentz and CPT invariance have been identified as promising Planck-scale signals [1]. The basic idea is that these symmetries hold exactly in established physics, are amenable to ultrahigh-precision tests, and may be broken in a number of approaches to quantum gravity. As examples, we mention strings [2], spacetime foam [3, 4], nontrivial spacetime topology [5], loop quantum gravity [6], noncommutative geometry [7], and cosmologically varying scalars [8].

The low-energy effects associated with Lorentz and CPT violation are described by the Standard-Model Extension (SME) [9]. The SME is an effective field theory at the level of the usual Standard Model and general relativity. Its flat-spacetime limit has provided the basis for modern experimental [10] and theoretical investigations of Lorentz and CPT violation involving mesons [11–14], baryons [15–17], electrons [18–20], photons [21], muons [22], and the Higgs sector [23]. It is interesting to note that neutrino-oscillation experiments offer discovery potential [9, 24, 25].
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The present talk discusses some topics in this field of research. In Sec. 2, we briefly review various mechanisms for Lorentz violation that have been proposed in the literature. We specifically focus on Lorentz breaking through cosmologically varying scalars: this effect highlights the interplay of translation and rotation/boost invariance. It is also phenomenologically interesting because many cosmological models contain novel scalar fields with time dependencies driven by the expansion of the universe. An explicit example of such a model motivated by $N = 1$ supergravity is given in Sec. 3. This talk is summarized in Sec. 4.

2 Some mechanisms for Lorentz violation

Lorentz breaking can occur in a variety of candidate underlying models. This section gives a brief overview of a subset of theoretical ideas along these lines. We focus on the mechanisms for Lorentz violation mentioned in the introduction. Those (and most other) models are based on a completely Lorentz-invariant Lagrangian; symmetry breakdown occurs because the ground-state solution of the respective equations of motion does not exhibit Lorentz invariance. This leads to various immediate consequences. For example, spacetime remains Lorentzian, so that different inertial coordinate systems are still linked by the usual Lorentz transformations. Moreover, conventional spinors and tensors still represent physical quantities. However, the vacuum contains a structure that acts like a background field selecting a preferred direction. Then, the outcome of an experiment can depend on the orientation and velocity of the laboratory implying the violation of particle Lorentz symmetry.

Spontaneous Lorentz and CPT violation in string theory. From a theoretical perspective, spontaneous symmetry breaking (SSB) is a particularly attractive mechanism for Lorentz violation. SSB is experimentally well established in condensed-matter systems, and in the electroweak model it is responsible for mass generation. The basic idea is that a symmetric zero-field state is not the lowest energy configuration. Non-zero vacuum expectation values (VEV) are, in fact, more favorable energetically. Within the field theory of the open bosonic string, it has been demonstrated [2] that SSB can trigger VEVs of vector and tensor fields, which would then select preferred spacetime directions. There is also theoretical evidence indicating the presence of spontaneous Lorentz violation in relativistic point-particle field theories with nonpolynomial interactions [26].

Spacetime foam. The basic idea behind this mechanism is that Planck-scale fluctuations could result in a sea of microscopic virtual black holes and other topologically nontrivial spacetime configurations in the vacuum. Besides violations of conventional unitary quantum mechanics, this could lead to Lorentz-breaking dispersion relations for particles propagating in such backgrounds. The emergence of Lorentz violation is intuitively reasonable because the thermal black-hole sea has a rest frame, which selects a preferred (timelike) direction. In a subset of these approaches, the dispersion-relation modifications are interpreted as resulting from recoil effects on quantum matter in such black-hole or D-particle backgrounds [3]
Nontrivial spacetime topology. This approach studies the physics resulting from the compactification of one of the three spatial dimensions [5]. On observational grounds, the compactification radius must be very large. Note also that the local structure of flat Minkowski space is maintained. The finite size of the compactified dimension leads to periodic boundary conditions, which implies a discrete momentum spectrum in this direction and a Casimir-type vacuum. It is then intuitively reasonable that this vacuum possesses a preferred direction along the compactified dimension.

Loop quantum gravity. Another idea how Lorentz violation can arise has been investigated in loop quantum gravity. To analyze the the classical limit of the theory, one considers coherent states peaked around the classical solution for the metric. However, one can only take into consideration coherent states that do not oscillate at transplanckian scales where Einstein’s theory of gravitation is known to be invalid. This procedure introduces an absolute distance into such classical limits, which is incompatible with special relativity. As a sample consequence, the Maxwell equations are modified leading to a Lorentz-breaking plane-wave dispersion relation [6].

Noncommutative field theory. A popular approach to underlying physics is noncommutative field theory. The key idea is that the Minkowski coordinates $x^\mu$ are no longer ordinary real numbers. They are promoted to operators on a Hilbert space satisfying commutation relations of the form $[x^\mu, x^\nu] = i\theta^{\mu\nu}$. Here, $\theta^{\mu\nu}$ is a spacetime-constant real-valued tensorial parameter. The presence of the nondynamical $\theta^{\mu\nu}$ in this framework typically leads, for example, to vacuum anisotropies and is therefore associated with Lorentz violation [7].

Cosmologically varying scalars. A varying scalar, regardless of the mechanism causing the spacetime dependence, typically implies the violation of translational invariance. Since translations and Lorentz transformations are closely intertwined in the Poincaré group, it is unsurprising that the translation-symmetry breakdown can also affect Lorentz invariance.

Consider, for example, the angular-momentum tensor $J^{\mu\nu}$, which generates rotations and Lorentz boosts:

$$J^{\mu\nu} = \int d^3x \left( \theta^{\mu\nu} x^\alpha - \theta^{\alpha\nu} x^\mu \right). \tag{1}$$

Note that this definition contains the energy–momentum tensor $\theta^{\mu\nu}$, which is no longer conserved when translational symmetry is violated. Typically, $J^{\mu\nu}$ will now exhibit a nontrivial dependence on time, so that the conventional time-independent Lorentz-transformation generators can cease to exist. As a result, Lorentz and CPT invariance are no longer guaranteed.

More intuitively, the violation of Lorentz symmetry in the presence of a varying scalar can be seen as follows. The 4-gradient of the scalar has to be nonzero in some region of spacetime. This gradient then selects a preferred direction in such a spacetime region. Consider, for instance, a particle that has interactions with the scalar. Then, its propagation features may be different in the directions perpendicular and parallel to the gradient, and physically inequivalent directions signal the violation of rotation invariance. Since rotations are contained in the
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Lorentz group, Lorentz symmetry must be broken.

Lorentz violation induced by spacetime-dependent scalars can also be established at the level of the Lagrangian. As an example, consider a system with varying coupling $\xi(x)$ and two scalar fields $\phi$ and $\Phi$, such that the Lagrangian $\mathcal{L}$ includes a kinetic-type term $\xi(x) \partial^\mu \phi \partial_\mu \Phi$. A partial integration of the action of this system (e.g., with respect to the first partial derivative in the above term) leaves unaffected the equations of motion. The resulting equivalent Lagrangian $\mathcal{L}'$ then contains a term

$$\mathcal{L}' \supset - K^\mu \phi \partial_\mu \Phi,$$

where $K^\mu \equiv \partial^\mu \xi$ is an external nondynamical 4-vector, which clearly breaks Lorentz invariance. Note that for spacetime dependencies of $\xi$ on cosmological scales, such as the claimed variation of the fine-structure parameter [27], $K^\mu$ is constant to an excellent approximation locally—say on solar-system scales.

Example: a supergravity cosmology

In this section, we illustrate the above results within a specific supergravity model that generates the variation of two scalars $A$ and $B$ in a cosmological context. It results in a fine-structure parameter $\alpha$ and an electromagnetic $\theta$ angle that depend on spacetime. Our analysis is performed within the framework of $\mathcal{N} = 4$ supergravity in four spacetime dimensions. Although this model is unrealistic in detail, one can gain qualitative insights into candidate fundamental physics because it is a limit of $\mathcal{N} = 1$ supergravity in eleven dimensions, which is contained in M-theory.

When only one graviphoton $F^{\mu\nu}$ is excited, the bosonic part of the pure $\mathcal{N} = 4$ supergravity Lagrangian is determined by [28]

$$\kappa \mathcal{L}_{sg} = -\frac{1}{2} \sqrt{g} R + \sqrt{g} (\partial_\mu A \partial_\nu A + \partial_\mu B \partial_\nu B) / 4B^2 - \frac{1}{2} \kappa \sqrt{g} M F_{\mu \nu} F^{\mu \nu} - \frac{1}{4} \kappa \sqrt{g} N F_{\mu \nu} \tilde{F}^{\mu \nu}. $$

(3)

Here, the $M$ and $N$ are functions of the scalars $A$ and $B$ given by

$$M = \frac{B(A^2 + B^2 + 1)}{(1 + A^2 + B^2)^2 - 4A^2}, \quad N = \frac{A(A^2 + B^2 - 1)}{(1 + A^2 + B^2)^2 - 4A^2}. $$

(4)

The dual field-strength tensor is denoted by $\tilde{F}^{\mu \nu} = \varepsilon^{\mu \nu \rho \sigma} F_{\rho \sigma} / 2$, and $g = - \det(g_{\mu \nu})$. In what follows, we rescale $F^{\mu \nu} \rightarrow F^{\mu \nu} / \sqrt{\kappa}$, so that the gravitational coupling $\kappa$ disappears in the equations of motion.

The next step is to gauge the internal SO(4) symmetry of the full $\mathcal{N} = 4$ supergravity Lagrangian. This supports the interpretation of $F^{\mu \nu}$ as the electromagnetic field-strength tensor. The resulting potential for the scalars $A$ and $B$ is known to be unbounded from below [29]. However, we take a phenomenological approach and assume that in a realistic situation stability must be ensured by additional fields and interactions. At leading order, we can then model the potential for the scalars with mass-type terms:

$$\delta \mathcal{L} = -\frac{1}{2} \sqrt{g} (m_A^2 A^2 + m_B^2 B^2). $$

(5)
We add these terms to our Lagrangian $\mathcal{L}_{\text{sg}}$ in Eq. \ref{eq:ls}. The complete $N = 4$ supergravity Lagrangian also includes fermionic matter \cite{28}. In the present cosmological model, we can represent the fermions by the energy–momentum tensor $T_{\mu \nu}$ of dust describing galaxies and other matter:

$$T_{\mu \nu} = \rho u_{\mu} u_{\nu}. \quad \tag{6}$$

Here, $\rho$ is the energy density of the matter and $u^{\mu}$ is a unit timelike vector orthogonal to the spatial hypersurfaces, as usual.

We are now in a position to determine cosmological solutions of our supergravity model. We make the usual assumption of an isotropic homogeneous flat ($k = 0$) Friedmann–Robertson–Walker universe with the conventional line element

$$ds^2 = dt^2 - a^2(t) \left( dx^2 + dy^2 + dz^2 \right). \quad \tag{7}$$

Here, $a(t)$ is the scale factor and $t$ denotes the comoving time. The assumption of isotropy prohibits our electromagnetic field from acquiring nonzero expectation values on large scales, so that we can set $F_{\mu \nu} = 0$. Then, our cosmological model is governed by the equations of motion for the scalars $A$ and $B$ and the Einstein equations. We remark that the fermionic matter is uncoupled from the scalars at tree level, so that $T_{\mu \nu}$ is approximately conserved by itself. We then find $\rho(t) = c_n / a^3(t)$, where $c_n$ is an integration constant.

In special cases, this cosmological model admits a variety of analytical solutions \cite{8}. In general, however, numerical integration is necessary. A physically interesting scenario is shown in Figs. \ref{fig:1} and \ref{fig:2}. The input data for this solution are \cite{8}

$$m_A = 2.7688 \times 10^{-42} \text{ GeV},$$
$$m_B = 3.9765 \times 10^{-41} \text{ GeV},$$
$$c_n = 2.2790 \times 10^{-84} \text{ GeV}^2,$$
$$a(t_n) = 1,$$
$$A(t_n) = 1.0220426,$$
$$\dot{A}(t_n) = -8.06401 \times 10^{-46} \text{ GeV},$$
$$B(t_n) = 0.016598,$$
$$\dot{B}(t_n) = -2.89477 \times 10^{-45} \text{ GeV}, \quad \tag{8}$$

where the dot denotes differentiation with respect to the comoving time, and the subscript $n$ indicates the present value of the quantity.

For purposes of this talk, the details of this particular solution are less interesting. It is important to note, however, that the scalars $A$ and $B$ have acquired a nontrivial dependence on the comoving time $t$: they vary on cosmological scales. Thus, we have established the first requirement for observable Lorentz violation. Note that this feature is common to many other, more realistic cosmological models.

Next, consider excitations of $F_{\mu \nu}$ in the background cosmological solution $A_b$ and $B_b$, which is depicted in Fig. \ref{fig:2}. Experiments are often confined to small
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Figure 1. Scale factor $a(t)$ versus fractional comoving time $t/t_n$. The priors given in Eq. (8) are chosen such that the expansion history of this model matches closely the one observed for our universe.

Figure 2. Time dependence of the scalars $A$ and $B$. Although at late times the scalars approach constant values, they do exhibit a nontrivial dependence on the comoving time. This model is therefore a candidate for exhibiting Lorentz violation.

spacetime regions, so it is appropriate to work in a local inertial frame. In such a frame, the effective Lagrangian $L_{\text{cosm}}$ for localized $F_{\mu\nu}$ fields follows from Eq.
Here, $M_b$ and $N_b$ are determined by the time-dependent cosmological solutions $A_b$ and $B_b$. Comparison with the conventional electrodynamics Lagrangian

$$L_{em} = -\frac{1}{4} e^2 F_{\mu\nu} F^{\mu\nu} - \frac{1}{16\pi^2} \theta F_{\mu\nu} \tilde{F}^{\mu\nu}.$$  

(10)

shows that $e^2 \equiv 1/M_b$ and $\theta \equiv 4\pi^2 N_b$. Since $M_b$ and $N_b$ depend on the varying-scalar background $A_b$ and $B_b$, the electromagnetic couplings $e$ and $\theta$ are no longer constant in general. In light of the Webb data set [27], a time-dependent fine-structure parameter $\alpha$ is intriguing by itself. However, here we are interested in the fact that our cosmologically varying scalar is coupled to a conventional Standard-Model particle—the photon. Thus, the second requirement for observable Lorentz violation is satisfied.

To establish the breakdown of Lorentz symmetry in our effective electrodynamics more clearly, we can look at the modified Maxwell equations resulting from Lagrangian (9):

$$\frac{1}{e^2 \partial^\mu F_{\mu\nu}} - \frac{2}{e^4} (\partial^\mu e) F_{\mu\nu} + \frac{1}{4\pi^2} (\partial^\mu \theta) \tilde{F}_{\mu\nu} = 0.$$  

(11)

In our cosmological supergravity model, the gradients of $e$ and $\theta$ appearing in Eq. (11) are nonzero, approximately constant in local inertial frames, and act like a nondynamical external background. This vectorial background selects a preferred direction in the local inertial frame breaking Lorentz invariance.

We remark that the term containing the gradient of $\theta$ can be identified with a Chern–Simons-type contribution to our modified electrodynamics. Such a term, which is included in the minimal SME, has received substantial attention recently [30]. For instance, it typically leads to vacuum Čerenkov radiation [31]. We also point out that a Lorentz-violating Chern–Simons-type term for gravity can be constructed [32]. This term can be generated in a model similar to ours, which also contains a cosmologically varying scalar [8].

4 Summary

This talk has discussed various aspects of spacetime-symmetry violations. The idea is that various approaches to quantum gravity can lead to Lorentz-violating ground states, which are characterized by backgrounds that select one or more preferred directions. We have briefly discussed a few explicit examples leading to such vacua. One of these examples involves scalars with a nontrivial spacetime dependence on cosmological scales. We have argued that the involved breakdown of translational invariance is typically associated Lorentz violation. This specific mechanism might be of particular interest in light of recent cosmological models involving scalar fields and recent claims of a variation of the fine-structure parameter.
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