Correction

Correction to: Coupled Self-Organized Hydrodynamics and Stokes Models for Suspensions of Active Particles

Pierre Degond, Sara Merino-Aceituno, Fabien Vergnet and Hui Yu

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This note provides a list of errata and their correction for Reference [1].

- There is a typo in Eq. (5.12): the term in \((\tilde{\lambda} - 1)\) should come with a negative sign. This expression becomes:

  \[
  i\rho_0 (-\alpha + V_0 \cdot k) + \gamma |k|^2 \rho_0 - \frac{(\tilde{\lambda} - 1)\tilde{b} \rho_0^2 k_0^2}{2|k|^2} \Omega
  \]

  \[
  \bar{\Omega} = \left[ \frac{1}{2} \rho_0 \left( 2\tilde{\lambda}k_0\bar{p} + \tilde{b}(\tilde{\lambda} + 1)(\rho_0\bar{k} + \tilde{b}\rho_0) \right) - \frac{i}{\kappa} \bar{\rho} \right] k^\perp.
  \]  

- In the statement of Th. 5.1 (Linear Stability Analysis), we have that Eq. (5.5) is not true. As a consequence equation (5.6) is slightly modified into

  \[
  D(\alpha, k) = \left\{ \frac{\tilde{b} \rho_0}{2|k|^2} \left[ \left( -4\tilde{\lambda} \frac{k_0^2}{|k|^2} + \tilde{\lambda} + 1 \right) (-\alpha + U_0 \cdot k) \right. \right.
  \]

  \[
  -c_1 k_0 \left( -2\tilde{\lambda} \frac{k_0^2}{|k|^2} + \tilde{\lambda} + 1 \right) \left. \right] + \frac{i}{\kappa} c_1 \left( |k|^2 - k_0^2 \right)
  \]

  \[
  -(-\alpha + U_0 \cdot k) \left[ i(-\alpha + V_0 \cdot k) - \frac{(\tilde{\lambda} - 1)\tilde{b} \rho_0 k_0^2}{2|k|^2} + \gamma |k|^2 \right],
  \]  

when \( \bar{k} \neq 0 \). And Eq. (5.7) is valid when \( \bar{k} \neq 0 \). When \( \bar{k} = 0 \) we have a different scenario where \( \bar{\rho} = 0 \) and \( \bar{\Omega} \neq 0 \) arbitrary and

  \[
  \alpha = V_0 \cdot k + i \left( \frac{(\tilde{\lambda} - 1)\tilde{b} \rho_0 k_0^2}{2|k|^2} - \gamma |k|^2 \right),
  \]  

which gives only stable modes since \( \tilde{\lambda} \in [-1, 1] \) and so \( \text{Im}(\alpha) \leq 0 \). (Finally, notice that, the parameter \( \eta \) in the original article does not play a role any more in this corrected version.)

The original article can be found online at https://doi.org/10.1007/s00021-019-0406-9.
To obtain these results we modify the proof of Th. 5.1 Case B, which we rewrite fully next (notice that the number of the equations that do not start by 0 refer to the number of the equations as they appear in the original article).

**Proof. Case (B)** Suppose that \( k^\perp \neq 0 \). Doing the inner product of Eq. (0.1) with \( k \) and using that \( k^\perp \cdot k = |k|^2 - k^2_0 \), the dispersion relation is given by (using Eq. (5.11)):

\[
\begin{align*}
\left\{ \tilde{b} \frac{\rho_0}{2|k|^2} \left[ \rho_0 \bar{k} \left( -4\bar{\lambda} \frac{k^2_0}{|k|^2} + \bar{\lambda} + 1 \right) + \bar{\rho} k_0 \left( -2\frac{\bar{\lambda} k^2_0}{|k|^2} + \bar{\lambda} + 1 \right) \right] - \frac{i}{\kappa} \bar{\rho} \right\} (|k|^2 - k^2_0) \\
= \left[ i\rho_0(-\alpha + V_0 \cdot k) - (\bar{\lambda} - 1) \frac{\bar{b} \rho_0}{2} \frac{k^2}{|k|^2} + \gamma |k|^2 \rho_0 \right] \bar{k},
\end{align*}
\]

and from Eq. (5.10b) we have the relation

\[
(-\alpha + U_0 \cdot k)\bar{\rho} + \rho_0 c_0 \bar{k} = 0.
\]

Next we distinguish between the cases \( \bar{k} \neq 0 \) and \( \bar{k} = 0 \). Suppose that \( \bar{k} \neq 0 \), then from Eq. (0.5) we have that \(-\alpha + U_0 \cdot k \neq 0 \) and multiplying Eq. (0.4) by \(-\alpha + U_0 \cdot k \neq 0 \) and using Eq. (0.5), we get the dispersion relation in Eq. (0.2), after simplifying \( \bar{k} \).

If, on the contrary, \( \bar{k} = 0 \), then one can check that \( k^\perp \cdot \bar{\Omega} = 0 \) (thanks to Eq. (5.10a)) and, therefore, they are normal. Since by assumption \( k^\perp \neq 0 \) from Eq. (0.1) we conclude that the coefficient in front of \( k^\perp \) on the right hand side must be zero. Rewriting this term using that \( \bar{k} = 0 \) and Eq. (5.11) we have that

\[
\bar{\rho} I = 0,
\]

with

\[
I := \frac{\bar{b} \rho_0}{2|k|^2} k_0 \left( - \frac{2\lambda k_0^2}{|k|^2} + \bar{\lambda} + 1 \right) - \frac{i}{\kappa}.
\]

From here we deduce that \( \bar{\rho} = 0 \) because otherwise we should have that \( I = 0 \) but the imaginary part of \( I \) is non-zero, so \( \bar{\rho} = 0 \). In particular this implies that Eq. (0.5) is fulfilled and that \( \bar{\Omega} \neq 0 \) (otherwise we would have null perturbation). Since \( \bar{\Omega} \neq 0 \), from Eq. (0.1) again (remembering that \( k^\perp \perp \bar{\Omega} \)) we must have that the coefficient in front of \( \bar{\Omega} \) is equal to zero. This gives the dispersion relation (0.3).

Now, we go back to the case \( \bar{k} \neq 0 \). To simplify the analysis we will restrict ourselves to the case where \( k^\perp = k \), i.e. \( k_0 = k \cdot \bar{\Omega}_0 = 0 \) and \( \bar{k} \neq 0 \). This implies, in particular, that \( U_0 \cdot k = V_0 \cdot k = v_0 \cdot k \). With these considerations one can simplify the dispersion relation (0.2) into

\[
\tilde{D}(\alpha, k) = 0,
\]

where \( \tilde{D}(\alpha, k) \) is given in Eq. (5.7).

- There are two typos at the end of page 21: in the third line of Case (A) part b) should read \( \text{Im}(\alpha) \) instead of \( \text{Im}(\omega) \); and in the fifth line should read “\( \Omega \) is arbitrary with \( \Omega \cdot \Omega_0 = 0 \)” rather than “\( \bar{\Omega} \) is arbitrary with \( \bar{\Omega}, \bar{\Omega}_0 \neq 0 \)”.

**Compliance with ethical standards**

**Conflict of interest** The authors declare they have no conflict of interest.

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**Reference**

[1] Degond, P., Merino-Aceituno, S., Vergnet, F., Yu, H.: Coupled self-organized hydrodynamics and stokes models for suspensions of active particles. J. Math. Fluid Mech. 21, 6 (2019). https://doi.org/10.1007/s00021-019-0406-9

Pierre Degond and Sara Merino-Aceituno  
Department of Mathematics  
Imperial College London  
London SW7 2AZ  
UK  
e-mail: pdegond@imperial.ac.uk

Sara Merino-Aceituno  
e-mail: s.merino-aceituno@sussex.ac.uk; sara.merino@univie.ac.at

Sara Merino-Aceituno  
Faculty of Mathematics  
University of Vienna  
Oskar-Morgenstern-Platz 1  
1090 Vienna  
Austria

and

Department of Mathematics  
University of Sussex  
Falmer Brighton BN1 9RH  
UK

Fabien Vergnet  
Laboratoire de mathématiques d’Orsay (LMO), Université Paris-Sud, CNRS  
Universit Paris-Saclay  
15 rue Georges Clémenceau  
91405 Orsay Cedex  
France  
e-mail: fabien.vergnet@math.u-psud.fr

Hui Yu  
Institut für Geometrie und Praktische Mathematik  
RWTH Aachen University  
52062 Aachen  
Germany  
e-mail: huiyu@tsinghua.edu.cn

and

Mathematical Sciences Center  
Tsinghua University  
Haidian District Beijing 100084  
China