Electrical performances of pyroelectric bimetallic strip heat engines describing a Stirling cycle

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Abstract. This paper deals with the analytical modeling of pyroelectric bimetallic strip heat engines. These devices are designed to exploit the snap-through of a thermo-mechanically bistable membrane to transform a part of the heat flowing through the membrane into mechanical energy and to convert it into electric energy by means of a piezoelectric layer deposited on the surface of the bistable membrane. In this paper, we describe the properties of these heat engines in the case when they complete a Stirling cycle, and we evaluate the performances (available energy, Carnot efficiency…) of these harvesters at the macro- and micro-scale.

Introduction

The development of bimetallic strip heat engines has been pursued as an alternative to the Seebeck thermoelectric generators based on the properties of semiconductors like bismuth tellurides, or to the conventional heat engines using fluids as a heat transfer medium. Proofs of concept have been reported in [1-5]. These devices exploit the sudden displacement of a bimetallic membrane switching from a critical equilibrium position to a stable one, to harvest a part of the thermal energy flowing through the bimetal and to convert it into kinetic energy. The existence of a hysteretic behavior, consequence of their bistability, enables to cycle beams between a hot source and a cold one (Fig. 1a,b), causing then an astable behavior that can be modeled like a switched thermal capacitor [6]. They can be coupled with a piezoelectric transducer deposited on the surface of the bistable membrane in order to transform the strain energy of the membrane into electric energy. Up to now, the modeling of these harvesters has been limited to the study of the properties of the thermal snap-through and of the efficiency of the thermo-mechanical transduction performed by the bimetallic strip [7]. In this paper we address the issue of modeling the performances of the thermo-mechano-electrical transduction realized by a piezoelectric bimetallic strip heat engine. Given that thermo-mechanically bistable beams can be fabricated at the macroscale and at the microscale (Fig. 1a-b), we evaluate the efficiency of these devices whatever their sizes. Our model is based on the architecture represented in Fig. 1d, where we assume that the membrane behaves like a plate strip.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{(a) Principle of the heat engine. (b) Bistable NC4-Invar bimetallic membrane at the macroscale. (c) Bistable Al-SiON beams at the microscale. (d) Simplified representation of the piezoelectric harvester studied in this paper.}
\end{figure}
1. Modeling of the Stirling heat engine

The modeling of the thermo-mechanical instability of simply-supported initially-buckled composite beams was presented in [8], where the use of the Galerkin method enabled to demonstrate that the equilibrium was described by a third-order polynomial of the beam’s deflection amplitude. In this paper, we extend this method to the case of partially-clamped beams, according to the technique presented in [7-9], in order to predict the behavior of the heat engine at the microscale. The boundary conditions are modeled by a finite torsional stiffness $\gamma$ varying from 0 (simply-supported beams) to $\infty$ (clamped-clamped beams). As explained in [7], the choice of a rigidity verifying $\gamma.L=36.7$ is enough to describe a clamped-clamped beam and authorizes first-order buckling modes. Whatever the value of the clamp stiffness, the beam’s shape at the equilibrium can be approximated by (1a) where $k$ depends on the $\gamma$ according to (1b)

$$w(x) = w_c \left( \cos(k \cdot x) - \cos \left( k \frac{L}{2} \right) \right)$$  

(1a)

$$k \cos \left( k \frac{L}{2} \right) + \gamma \sin \left( k \frac{L}{2} \right) = 0$$  

(1b)

The equilibrium is then described by (2a), where $n_e$ is the beam’s axial stiffness, $i_e$ the beam’s bending stiffness, $n_u$ the beam thermal expansion, $m_u$ the asymmetry of the thermal expansion. The effect of the residual stress is modeled by two parameters, $F_o$ being the total residual stress and $M_o$ the asymmetry of the residual stress. Thus the thermo-mechanical bistability appears only if the effective residual stress $P_o$ exceeds the Euler buckling load (2b).

$$n_e \frac{(k^2L - k \cdot \sin(k \cdot L))^2}{8L} \cdot w_e^2 + \frac{k^2L - k \cdot \sin(k \cdot L)}{2} (i_e k^2 - P_o) w_e + 2k \sin \left( k \frac{L}{2} \right) (M_o - m_u) T = 0$$  

(2a)

$$P_o = n_u - F_o \geq i_e k^2$$  

(2b)

Figure 2. (a) Hysteretic behavior of a Al-Invar beam in the diagram $(w_c,T)$. (b) Evolution of the mechanical energy of the bistable Al-Invar beam. (c) Evolution of the entropy of the bistable Al-Invar beam.

The beam’s thermal hysteresis and the beam’s deflection amplitude $w_c$ of the unsnapped state at $T_o$ can be found by deriving (2a). The evolutions of the deflection, total strain energy and entropy of a bistable Al-Invar bimetallic beam ($L=4\text{cm}$, $v=0.3\text{cm}$, $t_1=t_2=200\mu\text{m}$) covered by a PZT layer ($t_p=100\mu\text{m}$) are represented in Fig.2. In Fig.2b we observe that the variations of the energy during the beam’s snap are weak because it corresponds to the sum of the energy variations of every layers, which see positive or negative variations of their strain energy due to internal stresses either tensile or compressive.

$$\Delta T = 2 \sqrt{\frac{(k^2L - k \cdot \sin(k \cdot L)) \cdot \frac{1}{k \cdot \sin \left( k \frac{L}{2} \right)}}{27 \cdot n_e \cdot m_u}} \left( \frac{P_o - i_e k^2}{2} \right)^3$$  

(3a)

$$w_c = \frac{4 \cdot L \cdot m_u \cdot \sin \left( k \frac{L}{2} \right) \cdot \Delta T}{k \cdot (k \cdot L - \sin(k \cdot L))^2 \cdot n_e}$$  

(3b)

We now describe the electrical behaviour of a piezoelectric layer of thickness $t_p$ deposited on the bistable bimorph beam, either on the passive layer (low thermal expansion material) or the active layer of the bistable membrane (high thermal expansion material). The constitutive equation of this layer is (4).

$$d\vec{E}_3 = p_{33} \cdot \vec{P} + \varepsilon_{31} \cdot d\vec{S}_1 + \varepsilon_{31}^p \cdot d\vec{E}_3$$  

(4)

In weak coupling, the effect of the stress induced by the polarization of the piezoelectric material is neglected: the equilibrium equation (2a) is not modified and the occurrence of the thermal snap-through does not depend on the electric field inside the piezoelectric layer. By integrating (4), we find the electrical equation of the piezoelectric
bimetallic strip heat engine, linking the electric charge $Q_e$ carried by the piezoelectric layer to its output voltage $U_p$, the temperature of the system and the deflection of the beam (5). Knowing that $w_1$ depends on the temperature (see (2a)), we can interpret the effect of the thermomechanical instability like a tertiary nonlinear pyroelectric effect.

$$dQ_e = C_p \left(-dU_p + \frac{\varepsilon_3}{\varepsilon_3^3} \frac{t_p}{e_3} \frac{3}{e_3} . dT + \frac{t_p}{e_3} \frac{e_31}{e_33} \frac{k^2 . L - k . \sin(k . L)}{2} \cdot w_1 + 2 \cdot (x_p - z_0) \cdot k . \sin \left( k \frac{L}{2} \right) . dW_1 \right) \quad (5)$$

In this paper, we only study the performances of the Stirling thermodynamic cycle due to its high theoretical efficiency and to the possibility to easily implement it with a SECE power management circuit [10,11]. The Stirling cycle generically describes a cycle made of two isochoric transformations (heating and cooling of the engine) and two isothermal transformations (phases of compression and expansion). In the case of a pyroelectric heat engine, the extensive parameter is the electric charge: the isochoric transformations are replaced by open-circuited transformations and the isothermal transformations by electrical charges and discharges of the piezoelectric capacitor. We consider three different working scenarios, depending on how and when the harvester is polarized.

Given that the piezoelectric layer can be deposited on the active layer or on the passive layer of the bistable beam, six different Stirling cycles are presented in Fig.3 in the case of the previous Al-Invar-PZT beam. In our study, we only consider static equilibrium positions and we neglect the voltage peaks observed when the beam snaps and snaps back.

![Figure 3. (a) Diagram (Q,V) of the Stirling cycles Ia and Ip. (b) Diagram (Q,V) of the Stirling cycles IIa and IIp. (a) Diagram (Q,V) of the Stirling cycles IIIa and IIIp.](image)

In the first scenario (Fig.3a), the “isochoric” heating of the bistable mover happens between state 4 and 2, meaning that the bistable beam is open-circuited before its unsnaps and until before it snaps. Similarly the “isochoric” cooling of the device happens between 2 and 4. The isothermal charge and discharge of the piezoelectric capacitance are performed in 2 and 4. $i_a$ represents the case of a piezoelectric layer deposited on the active layer of the beam (Al) and $i_p$ the case of a layer deposited on the passive layer of the beam (Invar). For the cycles $i_a$ and $i_p$ (Fig.3b), the “isochoric” heating and cooling are realized from 1 to 3 and from 3 to 1. The isothermal transformations are realized in 1 and 3. The last Stirling cycle ($IIIa$ and $IIIp$) is a combination of the two other cycles: the piezoelectric capacitance is open-circuited from 1 to 2, then it is quickly discharged in 2. The composite beam snaps in open circuit from 2 to 3 and is discharged in 3. We show that the available electric energy obeys to (6a) for the cycles $i_a$ and $i_p$, to (6b) for $i_a$ and $i_p$ and to (6c) for $IIIa$ and $IIIp$.

$$W_e = C_p \left(\frac{p^2}{\varepsilon_3} t_p \Delta T \right)^2 + 4 \cdot \frac{e_31}{e_33} \cdot \left( x_p - z_0 \right) \cdot k \cdot \sin \left( k \frac{L}{2} \right) . w_c^2 \quad (6a)$$

$$W_e = C_p \left(\frac{p^2}{\varepsilon_3} t_p \Delta T + 8 \cdot \frac{e_31}{e_33} \cdot \left( x_p - z_0 \right) \cdot k \cdot \sin \left( k \frac{L}{2} \right) . w_c^2 \right)^2 \quad (6b)$$

$$W_e = C_p \left(\frac{p^2}{\varepsilon_3} t_p \Delta T \right)^2 + \frac{3}{4} \cdot \frac{e_31}{e_33} \cdot \left( k^2 . L - k . \sin(k . L) \right) . w_c^2 + 2 \cdot \frac{e_31}{e_33} \cdot \left( x_p - z_0 \right) \cdot k \cdot \sin \left( k \frac{L}{2} \right) . w_c^2 \quad (6c)$$

In the next parts we compare the performances of these Stirling engines at the macroscale and microscale, and we study the influence of the thermo-mechanical instability and the impact of the piezo-/pyroelectric properties of the piezoelectric layer on the efficiency of the energy harvester.
2. Performances at the macroscale

In order to find the most efficient Stirling cycle, we plot in Fig.4a the available electric energy as a function of the thermal hysteresis of an Al-Invar-PZT beam whose thermal hysteresis is centered on the medium value 70°C. This figure confirms the observations made on Fig.2 where the area enclosed by the working cycle is higher for \( Ip \). The energy produced by \( Ip \) progressively decreases because for highly unstable beams, the stress is tensile when the beam is snapped and unsnapped, thus reducing the electric charge variation. By comparing the amount of energy produced by the harvester with the energy produced by a classical pyroelectric engine based on the pyroelectric effect only, we can see the potential benefit of using a bistable beam to create a pseudo-tertiary pyroelectric effect to boost the performance of the generator, and to harvest heat from thermal gradients instead of heat from temporal variation of the temperature. For a 3K hysteresis, the available energy of \( Ip \) is 75.6µJ, corresponding to an energy surface density of 18.9µJ.cm\(^{-2}\). According to [3], the cycling frequency of a harvester with a membrane working with \( T_{th}=67°C \) and \( T_{h}=70°C \) is of around 0.25Hz, meaning that a piezoelectric harvester with a 3K-hysteresis beam could deliver up to 18.9µW, enough to supply a Wireless Sensor Node [12].

Given that the thermo-mechano-electrical coupling factor is weak, the dependence of the heat capacitance on the mechanical stress and on the piezoelectric effect is weak, and the latent heat is negligible. In that conditions, the heat supplied by the hot source can be approximated by

\[
Q_h \approx C_v \Delta T
\]

We thus define the thermo-electrical efficiency of the device by (8a), and the Carnot efficiency of the heat engine by (8b). Given that our model does not take into account the thermal transfers inside the harvester, the efficiency (8a) is normalized by the ratio of the thermal hysteresis to the snap temperature instead of the true hot source temperature and cold source temperature. The formula (8b) thus gives the maximal Carnot efficiency but corresponds to the working conditions that generate the lowest electrical power.

\[
\eta_{abs} = \frac{W_e}{C_v \Delta T}
\]

\[
\eta_{rel} = \frac{W_e}{C_v \Delta T^2} \left( \frac{m_r}{M_r} + \frac{\Delta T}{2} + 298K \right)
\]

Fig 4b. plots the evolution of these two efficiencies as functions of the thermal hysteresis for the cycle Ip. It shows that after reaching a minimal value around 2K, the efficiency increases as the thermal hysteresis widens. However, the rapid fall of the Carnot efficiency for highly hysteretic devices means that the transformation becomes less reversible. This performances degradation is mainly linked to the efficiency of the thermo-mechanical transduction according to [7]. Typical Carnot efficiencies of around 0.4% are obtained with a 3K hysteresis. In order to quantify the impact of the piezoelectric properties on the efficiency of the heat engine, the available energy and the Carnot efficiency of various piezoelectric materials are represented as a function of the electro-thermal coupling factor defined in [11]. The fact that PZT gives better results than PVDF shows that the performances of the heat engine are more linked to the piezoelectric coupling than to the pyroelectric effect. Knowing that the electro-thermal coupling factor represents the Carnot efficiency of a classical pyroelectric generator, we can see that the pyroelectric bimetallic strip heat engine can be more efficient than a classical pyroelectric generator when PZT is used as an electro-mechanical transducer.

Figure 4. (a) Evolution of the available energy of a complete cycle as a function of the thermal hysteresis for an Al-Invar-PZT beam (L=4cm v=1cm) (b) Evolution of the efficiencies of the system. (c) Available energy vs electro-thermal coupling factor [11] of various piezoelectric materials for a 3K hysteresis. (c) Carnot efficiency vs electro-thermal coupling factor.

3. Efficiency of the harvester at the microscale

Since miniaturized beams are fabricated by successive depositions of materials layers, the beams edges are can be considered as clamped to the substrate and the beams cannot continue being modeled as simply-supported beams:
the clamp stiffness is therefore taken equal to $\gamma=36.7/L$ [7]. In these simulations, we model the performances of the stacks PZT-Al-SiO$_2$ and Al-SiO$_2$ PZT ($L=400\mu m$, $v=100\mu m$, $t_{Al}=2\mu m$, $t_{SiO2}=2\mu m$, $t_{PZT}=1\mu m$), corresponding to scaling factor of 100 compared to the macro-harvester. Fig.5a first represents the evolution of the energy generated by the Stirling cycle $H_p$ as a function of the instability of the beam represented by its thermal hysteresis. In order to quantify the impact of the torsional stiffness, the performances of a simply-supported beam with a single capacitor are compared with those of a clamped-clamped beam with a single capacitor. The significant decline of the generated energy (factor 15 for a hysteresis of 3K) is due to the coexistence of tensile stresses and compressive stresses inside the piezoelectric layer that the charges created by the tertiary non-linear pyroelectric effect. The energy is generated by the pyroelectric effect. By using three series piezoelectric capacitors, we can compressive stresses inside the piezoelectric layer that the charges created by the tertiary non-linear pyroelectric effect.

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![Figure 5](image_url)  
(a) Electrical energy per cycle produced by an Al-SiO$_2$-PZT beam at the microscale as a function of the thermal hysteresis. (b) Carnot efficiency of the beam at the microscale. (c) Comparison of various piezoelectric beams ($L=400\mu m$, $v=100\mu m$, $t=5\mu m$) at the microscale for a hysteresis of 3K.

4. Conclusion

In this paper, we presented the results of the modeling of pyroelectric bimetallic strip heat engine. By using the Galerkin method, we found simple laws describing the non-linear behavior of thermo-mechanically bistable beams and its impact on the electric behavior of the generator and we defined it like a tertiary non-linear pyroelectric effect. We then studied the performances of the energy harvester when it describes a Stirling cycle and we show that energy densities of around 20µJ.cm$^{-2}$ can be obtained at the macroscale with a PZT-based heat engine (hysteresis of 3K), corresponding to a Carnot-relative efficiency of 0.4%, superior to the efficiency of a classical pyroelectric heat engine. This shows that the use of non-linear energy harvesting techniques can also be used to increase the performances of pyroelectric heat engines.

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