Fuzzy Logic in Selection of Maritime Search and Rescue Units

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Abstract: The article discusses methods of ships assessment when determining their suitability for search and rescue action (SAR) at sea. Selection of the most preferable ships is one of the action planning elements. Due to various construction and equipment the civilian ships can only perform rescue task to a certain degree. According to the Multi-Criteria Decision Analysis (MCDA), many parameters and data have to be compared in order to create a ranking of vessels ordered according to the coordinator’s preferences. When data are missing, incomplete or uncertain, a similar effect can be obtained using fuzzy logic. The author discussed the nature of the criteria, evaluation methods and presented a simulation of a ship study using fuzzy logic. The author developed fuzzy rules and presented the principle of operation of the controller. The article deals with the main principles of a decision support system (DSS) for the selection of ships in SAR operations.

Keywords: search and rescue; fuzzy logic; selection criteria; decision support systems; SRU

1. Introduction

Sea transport and shipping are vulnerable to many risks, such as fire, collision, grounding, flooding, cargo shifting, man overboard, etc. In spite of introducing new safety rules and standards, accidents at sea are still common. According to the data published by the European Maritime Safety Agency [1], in 2014–2019, as many as 19,414 casualties and incidents occurred on ships flying a flag of one of the EU Member States within the EU Member States’ territorial sea or internal waters (as defined in UNCLOS), resulting in 496 deaths and 6210 injuries.

In distress, when the safety of the ship or its crew and/or passengers are in danger, abandonment of the ship is inevitable. Survival in the water or on-board a lifeboat or life raft is challenging, with hypothermia and drowning being the most common causes of death.

A quick and efficient search and rescue (SAR) operation can prevent a disaster or at least increase the chances of survival. SAR operations are organised and carried out by a Rescue Coordination Centre (RCC) or another vessel in the vicinity. Crucial decisions are made by the SAR coordinator, and their assessment of the situation largely determines the success or failure of the operation.

One of the decisions made at the stage of planning a SAR operation is the selection of vessels fit for the task [2]. Specialised search and rescue as well as civilian vessels can participate in an SAR operation. Specialised vessels are multifunctional, well equipped, and manned with a qualified crew. Varied technical and operational parameters of civilian vessels (e.g., cargo, passenger, fishing, leisure, research ships) either increase or reduce their potential for carrying out search and rescue tasks, being defined by the intended use of the ship, its structure, manoeuvrability, equipment, and experience of the crew.

Suitability of vessels for an SAR operation is assessed based on a variety of criteria. The most important ones are speed, seaworthiness, range of operation, availability of detection equipment, equipment for the recovery of survivors, and ability to provide assistance, among other [3,4]. The SAR coordinator is assigned with the task of selecting the most suitable vessels—a task equally difficult and stressful, performed under pressure of time.
and responsibility for one’s decision. Their decision largely determines the success or failure of the operation, which can mean life or death for many. The overriding objective is to maximise effectiveness while minimising the risk.

An SAR coordinator based at an RCC can deploy available specialised vessels or assign other civil vessels in the vicinity of the distress. The first choice is the specialised vessels at the RCC. However, in certain circumstances, when the time to distress is long (e.g., when the distress position is far away from the shore), assistance of civilian vessels in the vicinity of the site is essential. In Mass Rescue Operations (MRO), carried out for passenger vessels, the number of specialised rescue vessels available can be too small and assistance of civilian vessels indispensable.

In selecting suitable vessels, the SAR coordinator uses their best knowledge and experience to compare vessels’ potential. In one of the previous papers, the author discusses a mathematical model supporting the selection of suitable vessels. The model, based on the Multi-Criteria Decision Analysis (MCDA), is used to assess vessels under analysis and suggest those which best meet the SAR coordinator’s requirements. Various criteria, even ones that are contradictory, expressed in different measurement units, and carrying different weights, can be compared. The result comes as a ranking of alternatives, sorted by the decision-maker’s preferences. The method can be part of a Decision Support System (DSS) for Search and Rescue at Sea.

In this paper, the author discusses modification of the method with the use of fuzzy logic to build the criteria evaluation methodology. In the standard MCDA, assessments (levels) have fixed boundaries. Assignment of certain values to the input data is possible provided that the latter is reliable, available and sufficiently detailed. Application of the fuzzy logic method can help improve the evaluation result where certain data are missing or unreliable.

Application of fuzzy logic and the MCDA can boost effectiveness of vessel selection for an SAR operation. The proposed method is aimed to streamline planning of SAR operations, with a mathematical solution supporting the SAR coordinator in the decision-making process.

2. Literature Overview (State of the Art)

Organisation and coordination of SAR operations is equally complex in all sea areas all over the world. To improve the safety of rescue operations, international standard procedures and guidelines have been published in the International Aeronautical and Maritime Search and Rescue (IAMSAR) Manual. The International Maritime Organisation (IMO) promotes the development of enhancements, modifications and activities aimed at improving the effectiveness of SAR operations at sea.

The majority of research papers in this field refer to the planning of SAR operation, e.g., modelling of the drift of an object at sea. There is also an increasing amount of research focusing on other aspects such as optimising the deployment of resources, using new technologies and equipment and enhancing the importance of decision support systems in solving maritime rescue problems. Examples of research work on improving the effectiveness of planning and conducting search and rescue operations at sea are given below.

For the purpose of supporting the planning and monitoring of rescue operations at sea, a Bayesian network application method is proposed by [5]. For this purpose, the principles for determining the reliability of the raft are presented. In this way, it is possible to assess the safety of the conducted action. This enables more successful action planning. Another example of using mathematical assumptions in relation to maritime rescue is the optimisation algorithm presented in [6]. The discussed algorithm simulates human behaviour during search and rescue operations. The method is applicable to the operation of constraints during problem solving.

In the previous articles of the author and co-authors [4,7] the way of using AIS (Automated Identification System) sensors in rescue operations was presented. The AIS system is used to detect ships and collect data on their parameters. A method of decoding
data from the system has been proposed. In addition, experiments were conducted to select vessels for specific search and rescue operations. Multi-criteria analysis (MDCA) was used for this purpose.

Another approach for supporting rescue planning is presented in [8]. The described method allows for optimal allocation of resources in a specific area. The method is based primarily on search theory and gradient search methods. Thanks to this, it is possible to improve the efficiency of planning and reduce the costs of action. The topic of resource allocation is also addressed in [9]. The authors presented a model that takes into account the uncertainty of demand and allows for a balanced distribution of resources. The method solves the problem for various objective function term weights. The proposed model primarily improves the performance of the SAR system. Another study on resource allocation confirms that the multi-criteria analysis is applicable for search and rescue planning [10]. The model developed allows vessels with different characteristics to be considered. The simulation model is based on historical data and is designed to solve organisational problems.

Research on the use of fuzzy logic in decision-making systems is presented in [11]. The study describes a way to distribute personnel for crisis management purposes. A fuzzy expert system and a decision tree were used for this purpose. The model is based on different criteria and, as a result, supports the process of organizing the action. Fuzzy logic principles have also been applied in advanced technology for search and rescue planning. Publication [12] discusses an aspect of using Unmanned Aerial Vehicles for search. The flight path of the drone is determined by calculating the location of the waypoints to be tracked. Another example demonstrating how to combine the principles of multi-criteria analysis and fuzzy logic is [13]. The described case is not related to the field of maritime rescue, but the proposed methodology shows how to support the decision making process when selecting a solution based on several criteria assessment.

For better detection and identification of vessels, data fusion principles from the Automated Identification System and Synthetic Aperture Radar have been developed and described in [14,15]. Ships can be detected by satellites because they cause higher backscatter in images compared to the background. When combined with data from an identification system, it is possible to use it for safety or security purposes, but also to support the engagement of vessels in search and rescue operations.

3. Materials and Methods
3.1. Standard MCDA

The Multi-Criteria Decision Analysis (MCDA) is a tool facilitating the decision-making process. Using mathematical models, the MCDA sorts the available solutions taking into consideration the decision-maker’s preferences.

The author proposes to use the method in the process of vessel selection for SAR operations. Vessels are selected based on an analysis of their characteristics and parameters. The decision-maker (typically, the SAR coordinator) specifies preferences and defines them as vessel evaluation criteria. Tested by means of the MCDA mathematical models, the available alternatives (here: vessels) are sorted, in a descending order, by the decision-maker’s preferences. One of the great merits of the MCDA is the ability to use a set of diversified criteria, expressed in various units of measurement (e.g., m/s or h), each assigned with its own weight and evaluation coefficient. The result of the analysis is typically shown as a ranking of alternatives.

The MCDA uses numerous mathematical models, such as AHP, ANP, UTA, Promethee II, Electre III, Oreste, etc. The procedure of analysis for Promethee II (Preference Ranking Organisation Method for Enrichment Evaluations) is discussed below.

Pairs of available solutions (here: vessels) are compared for their compliance with the evaluation criteria. The criteria are minimised or maximised and assigned weights and preference functions. The preference function determines the difference for a criterion between the evaluations obtained by two possible decisions into a preference degree
ranging from 0 to 1. The method uses six standard preference functions and is implemented in four phases [16].

Phase 1 consists of the calculation of the degree of preference for each pair of alternatives and the value for each criterion. Where \( g_i(a) \) is the \( i \)th criterion value for the \( a \) alternative, the difference between the \( i \)th criterion value for two alternatives, \( a \) and \( b \), can be determined:

\[
d_i(a, b) = g_i(a) - g_i(b)
\]  
(1)

The value of the degree of preference of the \( i \)th criterion for two decisions, \( a \) and \( b \), is denoted as \( H_i(a, b) \). The preference function, used for the calculation of the degree of preference, is defined as follows:

\[
H_i(a, b) = F(d_i(a, b)) \quad \text{with} \quad \forall x \in [-\infty, \infty], \quad 0 \leq F(x) \leq 1
\]  
(2)

In phase 2, the degrees of preference of all the criteria are summed up and a global preference index is determined for each pair of alternatives. Where \( C \) denotes the criterion and \( ij \) its weight, the global preference index for each pair of alternatives, \( a \) and \( b \), is calculated as follows:

\[
\pi(a, b) = \sum_{i \in C} w_i \times H_i(a, b)
\]  
(3)

Phase 3 consists of the calculation of outranking flows. The positive flow \( \varphi^+(a) \) and the negative flow \( \varphi^-(a) \) are determined for all the alternatives. Where \( A \) is a set of alternatives and \( n \)—the number of possible alternatives, the following formulas apply:

\[
\varphi^+(a) = (n - 1)^{-1} = \sum_{x \in A} \pi(a, x)
\]  
(4)

\[
\varphi^-(a) = (n - 1)^{-1} = \sum_{x \in A} \pi(x, a)
\]  
(5)

In phase 4, the final evaluation (net flow) is calculated for each alternative with the use of the previously calculated outranking flows. The final evaluation determines the positions of alternatives in the ultimate ranking. The net flow is calculated with the following formula:

\[
\varphi(a) = \varphi^+(a) - \varphi^-(a)
\]  
(6)

The higher an alternative’s position is in the ranking, the better its compliance with the decision-maker’s preferences. The ranking can support the SAR coordinator in the selection of vessels. They may decide to choose one or more top-ranking vessels. Further examination of the pre-selected vessels can be done via, e.g., radio communications. For the final decision on whether or not to engage a civilian vessel, the SAR coordinator may need other complementary data, which are not taken into consideration in the mathematical selection (e.g., vessel’s own risk, readiness of the crew/ship to get engaged in the operation, etc.).

Prior to performing the calculations referred to above, it is necessary to:

1. Determine a set of alternatives (here: gather information about vessels in the vicinity of the distress position),
2. Determine a set of assessment criteria (a coherent family of criteria ensuring an exhaustive, coherent and non-excessive evaluation),
3. Model the decision-maker’s preferences (determine the criteria evaluation methodology, including, inter alia, define the weight of each criterion and choose the preference function).

3.2. Preference Function

Six preference functions are used in the Promethee II method. One of them must be selected for each criterion: type 1—a normal preference function; type 2—a U-shaped preference function; type 3—a V-shaped preference function; type 4—a level preference
function; type 5—a linear preference function; type 6—a Gaussian preference function. Moreover, with the preference functions, evaluation thresholds are in use: the indifference (balance or incomparability) threshold \( q \), the preference threshold \( p \), and the Gaussian threshold \( s \).

The level preference function is discussed below (Figure 1). It assigns levels to particular features of an alternative. The balance \( q \) and preference \( p \) thresholds must be determined and the mean value between the thresholds must be calculated. If the difference between the values of assessments \( g_i(a) - g_i(b) \) falls between the two thresholds, the preference of one alternative over the other is weak and the function has a value of 0.5. If the difference \( g_i(a) - g_i(b) \) is below the balance threshold \( q \), there is no preference (the function has a value of 0). With the difference \( g_i(a) - g_i(b) \) above the preference threshold \( p \), the preference of one alternative over the other is strong (the function has a value of 1).

\[
H_i(a, b) = \begin{cases} 
0 & g_i(a) - g_i(b) \leq 0 \\
\frac{1}{2} & q_i \leq g_i(a) - g_i(b) \leq p_i \\
1 & g_i(a) - g_i(b) > p_i 
\end{cases}
\]

**Figure 1.** Level preference function—MCDA Promethee II.

The level preference function is well suited for qualitative criteria, when the decision-maker wishes to modulate the degree of preference by the deviation between the evaluation levels.

### 3.3. Vessel Selection Criteria

When selected for an SAR operation, ships should be assessed for their search or rescue capabilities. Depending on the profile of the operation, vessels can have different potential. The profile of an SAR operation is essential in the building of the set of criteria and modelling the decision-maker’s preferences, as it determines the relation between the type of distress and the type of assistance required. Some examples of SAR operation profiles are: search and rescue of a single survivor in the water, search for a missing ship, etc. Since each profile has its own key features and parameters, there is no universal vessel equally suitable for all profiles. Therefore, the selection of vessels suitable for a particular SAR operation profile is necessary and aimed to boost effectiveness of the operation.

In search operations (for survivors in the water, lifesaving craft, missing ships) the following features of the ship are of utmost importance:

- Precise location capabilities (detection equipment)
- Effective observation capabilities (high observation altitude or detection equipment)
- Relatively high speed of the vessel
- Relatively large range of operation
- Manoeuvrability (ability to carry out search patterns)
- Seaworthiness
- Relatively short time to scene (depending on the current position of the vessel)
- Experience and training of the crew

In rescue operations (recovery of survivors from the water, picking up persons from survival crafts or vessels in distress) the following features of the ship are of utmost importance:
• Equipment for the recovery of survivors from the water
• Equipment for providing medical assistance
• Ability to accommodate a certain number of survivors
• Communications equipment
• Ability to participate in the operation

The criteria referred to above are qualitative or quantitative. An example evaluation of the criteria is shown in Table 1. Levels of compliance with a certain criterion (defined in the form of a description or range of values) correspond to certain grades. The grading scale can be defined arbitrarily and should be adjusted to the nature of the criterion.

Table 1. Methodology of criterion evaluation.

| Criterion C | Grade            |
|-------------|------------------|
| full compliance with the expectation/ or the highest range value | Level 1 |
| average compliance with the expectation/ or the medium range value | Level 2 |
| low compliance with the expectation/ or the lowest range value | Level 3 |

When comparing ships, their parameters need to be collected and evaluation matrix needs to be created. The SAR coordinator faces the challenge of gathering data within a short time span. The priority is to select the most suitable vessels as soon as possible and commence the search and/or rescue without further delay.

Part of the data can be downloaded from vessel traffic monitoring systems, such as, e.g., the commonly used Automatic Identification System (AIS). The AIS provides the vessel’s position, type, navigational status (e.g., underway, anchored, fishing, disabled, etc.), current speed and course, and technical parameters (length overall, beam, draught). Data downloaded from the AIS can be approximate, inaccurate, or incomplete. Moreover, the AIS does not provide information about the vessel’s equipment, which is crucial for the examination of search and rescue capabilities.

Therefore, evaluation of compliance with the criteria may be challenging. For example, the SAR coordinator may wish to evaluate the vessel’s compliance with the qualitative criterion of time to distress on a three-point scale—high, medium, and low. The exact time to distress is unknown, while the estimated time to distress is within 40 to 60 min. The boundary between Level 1 and Level 2 is at the value of 50 min. Another example—the coordinator wants to evaluate compliance with the qualitative criterion of vessel’s equipment for the recovery of survivors from the water on a three-point descriptive scale (full, medium, low compliance). Their knowledge in this respect is limited (e.g., the type of lifeboats and launching method are known, but there is no information about other lifesaving appliances available on-board the vessel). The examples above show that the coordinator may find it difficult to unequivocally assign a compliance level.

Effectiveness of the evaluation may be compromised due to missing information. What is worse, insufficient information may lead to a wrong decision, undue delay, or even failure of the operation. In consideration of the above, the author proposes that the criteria evaluation be supported with the use of fuzzy logic.

In conclusion, in order to select ships, it is proposed to build ship rankings based on evaluation criteria. For this, it is necessary to collect data on available ships, to conduct modelling of the decision maker’s preferences and to carry out calculations. The computation can be done using MCDA, fuzzy logic or a combination of both methods.

3.4. Fuzzy Logic for Criteria Evaluation

Imprecise information is very common. We naturally use imprecise (non-numerical, unreliable) notions to explain the occurring phenomena—hence the term ‘fuzzy informa-
tion’. In the maritime industry, examples of fuzzy information are expressions such as ‘high speed’, ‘medium range’ or ‘good seaworthiness’. Although they do not refer to any particular values, they can effectively convey an important message. We also tend to describe the nature of mechanisms or processes by means of rules, such as, e.g., ‘If the search speed is high, the search effectiveness is high’. Interestingly, imprecise terms can be described using the language of mathematics. Understood by computers, the language of mathematics can be applied in decision-making support systems. Zadeh [17] proposes fuzzy sets as a way of presenting a problem. Mamandi [18] has developed a way of employing such sets in decision-making and process control. In fuzzy logic, there is an array of intermediate values between false (0) and true (1), determining the elements’ degree of membership in a set. Finding how close an element is to 0 or 1 can convey a sufficiently significant piece of information. Determining the degree to which a vessel complies with a certain criterion can be applied at the stage of planning an SAR operation (selecting the most suitable vessels for a certain type of distress). Selection of vessels based on their features and parameters can improve the effectiveness of SAR operations and accelerate the decision-making process.

The procedure discussed below (8, 9) is used to produce the mathematical description. A fuzzy set \( A \) in a certain (not empty) space \( X \), denoted as \( A \subseteq X \), is a set of pairs [19]:

\[
A = \{(x, \mu_A(x)); x \in X\} \quad (7)
\]

where

\[
\mu_A : x \rightarrow [0, 1] \quad (8)
\]

is the membership function of fuzzy set \( A \). Each element is assigned a degree of membership in set \( A \).

A well-defined fuzzy set requires a properly constructed membership function. The function serves as a tool encrypting the expert’s knowledge (their perception of the phenomenon). Some of the typical membership functions are (Figure 2):

- \( s \)-type, modelling (many, a lot of, fast, e.g., high speed);
- \( z \)-type, modelling (few, little, slowly, e.g., small range);
- trapezoidal, modelling (about, on average, e.g., average height).

![Typical membership functions.](image)

Fuzzy sets are typically defined as trapezoidal or triangular curves. Each value has a slope where it rises, falls or peaks (1).

The system is controlled using fuzzy reasoning rules and fuzzy controllers. Fuzzy reasoning rules are built on knowledge and experience of a relevant process or phenomenon. In an SAR operation, experts are individuals with work experience at sea, knowledge of ships’ construction and operation, and advanced skills of organising and carrying out operations at sea.

The reasoning rules are typically expressed by experts in imprecise utterances using vague terms (linguistic variables), such as, e.g., ‘When the vessel slightly drifts off course, make a small correction of the rudder angle’. Fuzzy reasoning values are represented by If-Then sentences:

- IF \( x \) is \( A \) THEN \( y \) is \( B \)
- IF \( x_1 \) is \( A_1 \) AND \( x_2 \) is \( A_2 \) AND,..., THEN \( y \) is \( B \)
- IF \( x_1 \) is \( A_1 \) OR \( x_2 \) is \( A_2 \) OR,..., THEN \( y \) is \( B \)
Where A and B represent linguistic values defined as fuzzy sets, X and Y represent universes, x represents the input variable and y—the output variable. Another example of a fuzzy reasoning rule is presented below:

- IF speed is excellent OR altitude is medium THEN suitability is high.

A fuzzy system is composed of many rules and each of them can generate a result. Therefore, aggregation (accumulation) of all the available results is necessary in order to generate a single response of the fuzzy system. The process consists of accumulating all the output rules in one fuzzy set (9):

$$R = \bigcup_{i=1}^{m} R_{i}$$  \hspace{1cm} (9)

The final stage is referred to as defuzzification. The fuzzy value obtained by each rule in the process of fuzzification must be converted into a real value. This is an operation transforming the output signals from qualitative to quantitative. The most commonly used defuzzification operators are:

- Centre of area/gravity
- Centre of sums/largest area
- First of maxima
- Middle of maxima
- Max criterion

4. Case Study

In order to demonstrate the application of fuzzy logic, a simulation of the planning of an SAR operation has been conducted. The simulation covers the evaluation of a vessel’s suitability, based on selected criteria.

Examination of ships’ parameters in single-criterion and multi-criteria analyses will provide data to create a ranking of vessels, sorted by their suitability in a descending order (from the one which best meets the SAR coordinator’s preferences to the least useful one).

The ultimate goal is to provide the SAR coordinator with information on the vessel’s potential and support him/her in the decision-making process.

Two cases were considered during the simulation. In the first criteria evaluation, a combination of MCDA and fuzzy logic results were used. In the second criteria evaluation, the test result without the use of fuzzy logic was presented. The outcome was compared and discussed.

4.1. Case Characteristic

The simulated SAR operation is the recovery of one person from the water. A man overboard incident may occur on-board any type of vessel, e.g., a passenger or merchant vessel, yacht, fishing vessel, etc. The incident has been reported over the radio to RCC.

Operations of the search and rescue of a single person in the water require sharp lookout, as the object is small and difficult to be sighted. Vessels with a high superstructure (a high observation altitude) are considered well suited for this type of task. Another important parameter is the vessel’s ability to recover the survivor from the water (including its manoeuvrability and equipment). In addition, the following are also important: time required for response, speed, facilities for providing medical assistance (rooms, medical equipment, qualified staff), etc.

The coordinator therefore has to identify vessels in the vicinity, assess their utility according to his/her preferences and select the most favourable vessels. Assessment of sharp (precise) parameters is possible using MCDA. Fuzzy logic is used to evaluate non-sharp parameters.
4.2. Procedure Description

The method consists of the following stages: collection of information about vessels available in the vicinity of distress, conducting the modelling of the decision-maker’s preferences and carrying out a computational experiment.

Stage 1: Data gathering

A group of civilian ships which could possibly be employed in the SAR action has been identified in the vicinity of the distress by means of the Automatic Identification System (AIS). The data for the simulation are inserted in Table 2. The values for length, beam, draught and speed are numerical. Information on vessel type and navigational status is presented in descriptive form. In the original, all data are coded in accordance with NMEA (National Marine Electronics Association) 0183/2000 standard.

| Vessel | Length over all (LOA) | Beam (B) | Draught (T) | Navigational Status | Type          | Speed |
|--------|-----------------------|----------|-------------|---------------------|---------------|-------|
| 1      | 84 m                  | 12 m     | 4.7 m       | underway using engine | general cargo vessel | 15 kn |
| 2      | 250 m                 | 44 m     | 7.7 m       | underway using engine | passenger vessel  | 19 kn |
| 3      | 144 m                 | 23 m     | 6.4 m       | underway using engine | general cargo vessel | 14 kn |
| 4      | 120 m                 | 11 m     | 5.2 m       | at anchor           | general cargo vessel | 18 kn |
| 5      | 67 m                  | 10 m     | 3.2 m       | underway using engine | military        | 12 kn |

Stage 2: Modelling the preferences

Preferences represent the expert’s (decision-maker’s, SAR coordinator’s) opinion on the vessel’s parameters which are the most useful in a SAR operation of a certain profile.

The modelling of preferences consists primarily in determining the set of criteria and the weights of the individual criteria. Furthermore, the type of preference function, the direction of preference and the thresholds were defined. This step was carried out in the form of an expert study. A group of three experts defined the action profile and the criteria for assessing vessels: speed, draught, vessel type, navigation status, observation height, medical equipment.

The AIS system allows the first four parameters to be determined almost immediately. The other two require extended investigation. The expert group proposes to use a new criterion “quality”. This criterion will be established by fuzzing out the criteria of observation height and medical supplies. Details are given in Table 3. An explanation regarding the criteria choice is placed below.

The operational speed of the ship has a significant impact on the effect of the action. In the case of the hypothermia risk, quick action and rapid implementation can increase the chances of saving the survivor’s life. Criterion is maximised. The criterion is the most important (weighting 0.275).

The draught of a ship can be important for two reasons. The first is the presence of shallows or other underwater hazards (e.g., wrecks, rocks). Which may increase the risk of the vessel. The second is the ability to carry out search patterns. When searching for a single person in the water, manoeuvrability and a small circulation radius will matter. Search tracks will be laid out at close distances. Vessels with large draught may experience problems. The criterion is minimised. The criterion is the least important (weighting 0.125).
Table 3. Criteria evaluation methodology.

| No | Criterion          | Mode of Assessment | Preference Direction | Weight, Preference Function | Methodology of Assessment                  |
|----|--------------------|--------------------|----------------------|----------------------------|--------------------------------------------|
| C1 | Speed              | quantitative       | max                  | 27.5%, linear              | Numerical value                           |
| C2 | Draught            | quantitative       | min                  | 12.5%, usual               | Numerical value                           |
| C3 | Navigational status| qualitative        | min                  | 22.5%, usual               | Level 1 Underway using engine, Underway sailing |
|    |                    |                    |                      |                            | Level 2 At anchor, Moored, Engaged in fishing |
|    |                    |                    |                      |                            | Level 3 Not under command, Restricted manouevrability, Constrained by her draught, Aground |
| C4 | Type               | qualitative        | min                  | 17.5%, usual               | Level 1 Cargo, Military, etc.              |
|    |                    |                    |                      |                            | Level 2 Tanker, Passenger, etc.            |
|    |                    |                    |                      |                            | Level 3 Fishing, Towing, etc.              |
| C5 | Quality            | quantitative       | max                  | 25%, linear                | Numerical value based on results of defuzzification |

Navigation status affects response time. Ships underway are ready for immediate action. Ships busy with work may require an extended response time. Ships in a particular state may not be fit to undertake any task and are usually the least preferred by the coordinator. The criterion shall be evaluated qualitatively. The criterion is minimised. The weighting of the criterion is significant (0.225).

The criterion of the type of vessel provides information about how the vessel is operated. Cargo vessels are the most suitable for the job, as they usually meet the SOLAS and STCW requirements concerning the navigational, communications, and rescue equipment, as well as training of the crew. Ships with a specially trained crew (e.g., military) are equally preferred. Passenger ships and some ships carrying dangerous goods (e.g., tankers) have similar characteristics to cargo ships but also have a higher risk of being used (e.g., presence of large numbers of people). They are less preferred than regular cargo ships. The least useful are fishing vessels and pleasure yachts. Most of them do not meet the STCW standards in terms of equipment and qualifications of the crew. Communications problems can be expected (they use shorter range communications equipment) as well as language problems (in some parts of the world). Compared to merchant vessels, the assessment of small vessels in terms of suitability for participation in an SAR operation is poorer. It must be noted here, however, that exceptions may apply. The criterion shall be evaluated qualitatively. The criterion is minimised. The weighting of the criterion is significant but less than the Navigational status (0.175).

The AIS does not provide data on superstructure height or vessel equipment. Deductions in this regard can be drawn on the basis of general knowledge of ships, analysis of known parameters (e.g., length over all, beam, type) or direct communication with the ship. A defuzzification method was used to determine the values of the new criterion ‘quality’. The method is discussed in the next section.

Defuzzification Method

Experts have developed reasoning rules for obtaining a new criterion value. This was done based on the ability to detect objects and to provide medical assistance. The developed controller is aimed to classify the vessel into one of three qualitative categories. The input
linguistic variables are the observation altitude and medical equipment, while the output variable has been denoted as quality (Figure 3). The output quality is represented by three sets (Low, Average, High Quality) within a range of 0 to 100%.

Based on experience, the expert has developed the following decision rules:

1. IF the observation altitude is low AND the medical equipment is basic THEN the quality is low.
2. IF the observation altitude is low AND the medical equipment is advanced THEN the quality is average.
3. IF the observation altitude is medium AND the medical equipment is basic THEN the quality is average.
4. IF the observation altitude is medium AND the medical equipment is advanced THEN the quality is high.
5. IF the observation altitude is high AND the medical equipment is basic THEN the quality is average.
6. IF the observation altitude is high AND the medical equipment is advanced THEN the quality is high.

Based on the above, the criterion ‘quality’ can be identified. The criterion is minimised. The criterion is very important but less significant than speed (weighting 0.25).

Operation of the controller for the Vessel 1 is shown in Figure 4. The input value of the observation altitude criterion is \( o.a. = 12 \) metres, the medical equipment is medium (on-board hospital, oxygen, AED, medicines, staff) and evaluated at \( m.e. = 0.5 \). On this basis, suitability of the vessel can be evaluated.

After rule aggregation, defuzzification was performed. Application of the implication operator allowed the following calculations:

\[
\mu_{\text{quality low}}(o.a. = 12, m.e. = 0.5) = \min(\mu_{\text{low}}(12), \mu_{\text{basic}}(0.5)) = \min(0.87, 0.5) = 0.5
\]
\[
\mu_{\text{quality average}}(o.a. = 12, m.e. = 0.5) = \min(\mu_{\text{medium}}(12), \mu_{\text{basic}}(0.5)) = \min(0.13, 0.5) = 0.13
\]
\[
\mu_{\text{quality high}}(o.a. = 12, m.e. = 0.5) = \min(\mu_{\text{medium}}(12), \mu_{\text{advanced}}(0.5)) = \min(0.13, 0.5) = 0.13
\]

The results are as follows: the quality is low = 0.5, the quality is average = 0.5 (max(0.13, 0.5)) and the quality is high = 0.13. The centroid method was used to calculate the final value of fuzzification. The Centre of Gravity \( \omega^* \) is equal 42.5%. This value is used as a parameter for the quality criterion.

Observation altitude is the point above the water surface from which the lookout scans the area. The higher the eyes of the lookout are, the greater the distance from the horizon and the better the range of search. Surface vessels have a fixed observation altitude and the lookout is posted on the navigation bridge or in another spot. Observation altitude is an important coefficient determining the sweep width. An appropriately adjusted sweep width has a significant impact on the success of search. It should be noted here that the observation altitude on surface vessels may vary within a range of 2 to 55 metres. Small units, such as yachts, fishing boats or small merchant vessels, have a small superstructure, and the lookout’s eyes can be at a height of a few metres above the water surface (ca. 10 m). Medium-sized merchant vessels, passenger ships, and specialised vessels have higher superstructures with an observation altitude of 20 to 30 m. Ultra-large vessels, such as the biggest cruise ships, tankers, container ships and car carriers, can have an observation altitude of more than 40 or even 50 m. The vessel may be classified into Low, Medium and/or High sets (Figure 3).

The vessel’s medical equipment determines its ability to provide medical assistance. Civilian vessels may vary greatly in this respect. The equipment of small units is usually very basic and may include means of thermal protection and a first aid kit. Larger units may have a room where survivors can be provided with shelter. Medical equipment includes, inter alia, defibrillators, oxygen bottles and respirators, spineboards, vacuum mattresses, infusion pumps, intubation and thorax drainage equipment, labour kits, etc. Several levels of readiness to provide medical assistance can be distinguished, e.g.,

- The vessel has an advanced medical module and a doctor on board
- The vessel has equipment and rooms for providing medical assistance (e.g., an on-board hospital)
- The vessel has standard medical equipment and is able to provide some medical assistance (medicines, oxygen, AED)
- The vessel has thermal protection means and/or a first aid kit
- The vessel is not able to provide any medical assistance

The author proposes two sets where medical equipment is evaluated numerically on a scale from 0 to 1. The vessel can be classified in the Basic and/or Advanced set (Figure 3).
On this basis, the input values for observation altitude (o.a.) and degree of medical equipment (m.e.) of the vessels were determined:

\[ V_1 = (12, 0.5), V_2 = (45, 0.65), V_3 = (30, 0.5), V_4 = (24, 0.5), V_5 = (16, 0.8) \]

Based on experience, the expert has developed the following decision rules:

1. IF the observation altitude is low AND the medical equipment is basic THEN the quality is low.
2. IF the observation altitude is low AND the medical equipment is advanced THEN the quality is average.
3. IF the observation altitude is medium AND the medical equipment is basic THEN the quality is average.
4. IF the observation altitude is medium AND the medical equipment is advanced THEN the quality is high.
5. IF the observation altitude is high AND the medical equipment is basic THEN the quality is average.
6. IF the observation altitude is high AND the medical equipment is advanced THEN the quality is high.

Based on the above, the criterion ‘quality’ can be identified. The criterion is minimised. The criterion is very important but less significant than speed (weighting 0.25).

Operation of the controller for the Vessel 1 is shown in Figure 4. The input value of the observation altitude criterion is o.a. = 12 metres, the medical equipment is medium (on-board hospital, oxygen, AED, medicines, staff) and evaluated at m.e. = 0.5. On this basis, suitability of the vessel can be evaluated.

![Figure 4. Operation of the controller.](image)

After rule aggregation, defuzzification was performed. Application of the implication operator allowed the following calculations:

- \( \mu_{\text{quality-low}} (\text{o.a.} = 12, \text{m.e.} = 0.5) = \min (\mu_{\text{low}}(12), \mu_{\text{basic}}(0.5)) = \min(0.87, 0.5) = 0.5 \)
- \( \mu_{\text{quality-average}} (\text{o.a.} = 12, \text{m.e.} = 0.5) = \min (\mu_{\text{low}}(12), \mu_{\text{advanced}}(0.5)) = \min(0.87, 0.5) = 0.5 \)
- \( \mu_{\text{quality-average}} (\text{o.a.} = 12, \text{m.e.} = 0.5) = \min (\mu_{\text{medium}}(12), \mu_{\text{basic}}(0.5)) = \min(0.13, 0.5) = 0.13 \)
- \( \mu_{\text{quality-high}} (\text{o.a.} = 12, \text{m.e.} = 0.5) = \min (\mu_{\text{medium}}(12), \mu_{\text{advanced}}(0.5)) = \min(0.13, 0.5) = 0.13 \)

The results are as follows: the quality is low = 0.5, the quality is average = 0.5 (max(0.13, 0.5)) and the quality is high = 0.13. The centroid method was used to calculate the final result...
value of fuzzification. The Centre of Gravity ($\omega^*$) is equal 42.5%. This value is used as a parameter for the quality criterion.

$$\omega^* = \frac{\sum_{i=1}^{n} x_i \mu(x_i)}{\sum_{i=1}^{n} \mu(x_i)} = 42.5$$

The same procedure was applied analogously to vessels 2–5. The final results are presented in the Table 4. The lowest score was achieved by Vessel 1 (42.5) and the highest score was achieved by Vessel 5 (79.05). The sharpened values of the quality criterion are added to the set of criteria. All ship assessments as well as preferences are shown in Figure 5.

Table 4. Results for defuzzification.

| Vessel   | $\omega^*$ | Vessel 2 | Vessel 3 | Vessel 4 | Vessel 5 |
|----------|------------|----------|----------|----------|----------|
| Vessel 1 | 42.5       | 76.5     | 51.96    | 61.95    | 79.05    |

Figure 5. Dialog window of the Visual Promethee software.

Stage 3: The computational test and the results

Multi-criteria decision analysis can be done by different methods. The expert group chose the Promethee II method for the simulation. This method has many advantages. It quantitatively measures the distances between alternative solutions. It allows for one-person selection (lower number of decision makers). It has detailed preferences, but uses an uncomplicated preference model. It allows to conduct selection on a small group of alternatives and allows results to be obtained in graphical form. Does not require much computational effort.

Calculations were performed using Visual Promethee software. First, vessel (alternatives) data and coordinator preferences were entered into the software dialog window (Figure 5). The columns represent the criteria. The first row contains the units in which the (quantitative) criteria are expressed. Qualitative criteria are expressed on a scale. The following rows contain the preferences of the decision-maker (direction of preferences, preference function and thresholds, if applicable). The sign n/a indicates ”not applicable” and has been inserted where the threshold value is not specified. The next section presents the statistics of the results. The values of the individual vessel scores are entered in the
lower section. The best values for the criterion are marked in green, the worst in red. Black is used for intermediate values.

Then the calculations were carried out. The results were presented in the form of a net flow table and network (Figure 6). The table shows the outcomes in the form of a ranking of the alternatives. Vessels are ranked in order of decreasing potential search and rescue performance (net flow value).

In the case of a large number of alternatives (e.g., several dozen ships in the vicinity of the distress), this can affect the quickness of the selection, but also the effectiveness of the decision.

Figure 6. Results of the analysis (table and network). Left: Case 1, Right: Case 2.

In Figure 6 (Case 1), we can see that Vessel 5 (Phi = 0.1252) comes first and Vessel 4 (Phi = 0.01191) comes last. The numerical result also makes it possible to assess whether the vessels can be treated equally. Vessel 3 and Vessel 1 achieved a similar score. The table also includes data on entering and leaving flow (partial rankings). These values influence the horizontal arrangement of alternatives in the network (lower part of the Figure 6). Alternatives closer to the right edge of the network have higher negatives values. It means they have been assessed lower in the single-criterion assessment than those located to the left.

Vessel 5 is a military ship with very good medical equipment. Although it is not fast and the observation height is medium, in the overall assessment it meets the coordinator’s requirements to the greatest degree. If the coordinator needs one vessel for action then this will potentially be the most useful. The coordinator may also consider the cooperation of two units (e.g., Vessel 5 and Vessel 2). The second vessel is a passenger ship with a high superstructure and high speed. Instead of Vessel 2 the coordinator may choose to pair Vessel 3 if he/she feels that the risk of Vessel 2 is too high (this is due to the poorer rating of the vessel type criterion). Vessel 2 is located to the right of Vessel 3 on the network.

Both forms of presentation of results are a great help to the coordinator. He/she can decide to choose the first vessel or select the group of vessels with the highest potential (upper left corner of the network).

Such a complete assessment (including all necessary criteria) was only possible by supplementing the data with the quality criterion (which is based on observation altitude and medical equipment). To compare the results of the analysis without taking these parameters into account, Case 2 was developed.
In this case (Figure 6) we can notice that Vessel 5 and Vessel 2 which were at the top of the previous ranking are now much lower. The evaluation is based on fewer criteria and weaker ships are now are higher ranked. Vessel 1 and Vessel 3 are cargo ships with medium speed and medium superstructure height.

In the case of a large number of alternatives (e.g., several dozen ships in the vicinity of the distress), this can affect the quickness of the selection, but also the effectiveness of the decision.

5. Discussion and Conclusions

The results obtained clearly indicate that there is a possibility to support the vessel selection process using fuzzy techniques. Fuzzy logic allows solving problems of missing or uncertain data, which are common in maritime rescue.

In the case studied, the experts defined the criteria (ship parameters) necessary to assess the ships. They had full knowledge of some parameters and partial knowledge of other parameters. The known data included ship dimensions (length, beam draught), speed, navigational status and ship type. This information was provided and decoded from the AIS. However, information on medical equipment and observation altitude was missing.

The principles of multi-criteria analysis (MCDA) were used to conduct the experiments. In the first experiment, the potential capabilities of five vessels were investigated based on known parameters. In the second experiment, in addition to the known values, the results obtained during the fuzzy test for the last two criteria were used. Significant differences in the outcomes were shown.

In the first experiment, potentially weaker ships obtained better results because their assessment was based on a limited number of criteria. In the second case, more information was taken into account and therefore the results better represent the preferences of the coordinator.

The development of fuzzy rules requires expert knowledge. The role of experts can be played by action coordinators and researchers.

In the case studied, a fuzzy rule base was created for two linguistic variables. The altitude of the observations was defined in sets of low, medium and high. The degree of medical equipment was defined in the basic and advanced sets. An output value in the form of quality was developed. The quality value represents missing data. The popular centre of gravity method was used to sharpen the final value. The obtained result became a new additional criterion for testing with the MCDA method.

The experiment shows that the method is uncomplicated, requires little computational effort and gives good results.

Knowledge of the precise parameters of vessels in the vicinity of the incident may be limited. Data may be unknown or incomplete. Making an analysis based on a small number of criteria does not give a complete picture. The selection of vessels using MCDA is very effective. However, it requires specific data. Combining this method with the advantages of fuzzy logic can improve the effectiveness of SAR planning.

It is recommended to develop a wide range of event scenarios and to define appropriate assessment criteria for different action profiles. Next, it is recommended to build a database of fuzzy rules. All information can be used to create a decision support system for the selection of vessels for the rescue task based on MCDA principles.

To summarise the study carried out, a scheme for the operation of the ship selection module in the DSS decision support system was developed (Figure 7). The system consists of the following components:

1. Expert knowledge base. Data collection consists of modelling the preferences of the decision maker. The expert research concerns the determination of the correct set of criteria, the weighting of the criteria and the preference function (or other indicators depending on the method applied).
2. Vessel database. The vessel parameters shall be determined during the initial stages of a search and rescue operation. Data can be retrieved from various monitoring systems, e.g., AIS.

3. Parameter matrix. The matrix represents the vessel data for each evaluation criterion defined by the expert.

4. Choice of calculation method. Computational methods require specific data. MCDA methods require a full parameter matrix. Fuzzy logic copes with missing, uncertain or inexact data.

5. Carry out testing (computational experiment). If the parameter matrix is complete, it is possible to obtain results using both MCDA and fuzzy logic. In case of a lack of certain data, the parameter matrix may be supplemented with a new criterion, for which the data will be obtained by fuzzy deduction. The outcome of the latter can also serve as a final result for the selection of ships.

6. Results. The results will be presented in the form of a ranking of ships ordered from those which to the greatest degree possess the desired characteristics and parameters.

7. Selection of ships. The coordinator shall decide which vessels are most suited to his/her preferences. In other words: which ones best meet the expectations for a given action profile.

Figure 7. Decision support system blocks for selection of maritime search and rescue units.

The method discussed above easily eliminates vessels which do not sufficiently meet the SAR coordinator’s requirements and selects a group of the most suitable (optimal) vessels. Thus, effectiveness of the SAR operation can be improved and the risk involved minimised. Application of fuzzy logic and the MCDA in decision-making support systems reduces is a modern digital aid to the coordinator and can increase the chance of saving lives.

Quick planning of successful SAR operations is the ultimate goal of the SAR coordinator. The mathematical models and decision-making support systems which have been applied in maritime rescue so far mostly relate only to certain aspects of the SAR operation planning (e.g., calculation of the object’s drift caused by current and wind). Less focus has been given to the selection of vessels suitable for certain profiles of the SAR operation.

Situations where civilian ships are involved are not very common. However, when an accident occurs far from shore or the scale of the accident exceeds the capacity of the SAR system, civilian ships are indispensable.

The selection of vessels for an SAR operation must take into consideration a variety of criteria expressed as a coherent set of preferences. Depending on the profile of the SAR operation, the SAR coordinator’s preferences and requirements differ. When searching
for and rescuing a single person in the water, parameters such as vessel’s speed, detection abilities, range of operation, recovery and medical equipment must be considered.

Evaluation of vessels on the basis of one criterion is usually insufficient. Application of a multi-criteria analysis enables us to obtain numerical results and sort vessels by key preferences. By employing fuzzy logic, we can overcome the challenge posed by unreliable and incomplete data.

Support for the planning and coordination of both search and rescue operations is advised by international organisations, such as the IMO. The development of decision-making support systems has become a trend in many industries. Modern maritime search and rescue needs innovative solutions. Fuzzy logic can be applied in the evaluation of vessel’s suitability for an SAR operation and support of the selection of vessels at the stage of planning an SAR operation.

Further development of decision support systems in maritime rescue can be based on both MCDA and fuzzy logic. This will require deeper analysis of criteria, action profiles, event scenarios, fuzzy reasoning and testing with different methods. It is proposed to deepen research into the selection of vessels and methods for assessing their suitability for search and rescue operations. Fuzzy logic has great potential for application. Studies have shown the importance of this method in improving the effectiveness of SAR operations. Thus, the ability to respond in an emergency will be increased. More people can be saved, the consequences of accidents can be reduced. Selecting the right vessel for the task will also help to reduce the risk of being involved.

Despite the introduction of modern technology and safety procedures, there are still a large number of accidents at sea. The rescue service, despite its advanced equipment, is not able to act on its own in all accidents. Engaging civilian ships is then essential. It is the coordinator’s responsibility to choose the most suitable ship. This task can sometimes be difficult and stressful. Decision support systems are therefore a good solution and it is necessary to invest in their development.

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