Yilmaz Theory of SNe 1a Redshift

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Abstract

A redshift-luminosity distance relation in excellent agreement with observations is calculated here for SNe 1a using the Yilmaz gravitational theory. In contrast to the current conventional explanation based on general relativity, the Yilmaz theory does not require a cosmological constant term that implies the existence of “dark energy”. The Yilmaz theory requires only one parameter; a mean mass-energy density of the cosmos. The required value is essentially the same as the critical density for a Friedmann-Robertson-Walker cosmological metric. The Yilmaz theory therefore still requires the existence of non-baryonic dark matter.

1 Introduction

The redshifts exhibited by distant SNe 1a can be encompassed by the addition of a cosmological constant to the field equations of general relativity. These can be written as

$$G^j_i = -(8\pi G/c^4)T^j_i - \delta^j_i \Lambda/c^2$$

(1)

where $G^j_i$ is the Einstein tensor (not to be confused with Newton’s gravitational constant in the right member) and $T^j_i$ is the matter energy-momentum tensor. $\Lambda$ is Einstein’s cosmological constant. $\Lambda c^2/(8\pi G)$ can be interpreted as a constant energy density of the cosmological vacuum. It is a constant “dark energy” density that uniformly pervades cosmological spacetime. With the adoption of a Friedmann-Robertson-Walker (FRW) metric and an appropriate value for $\Lambda$, the solutions of Eq. 1 provide a good description of an expanding and accelerating universe. Fitting the solution to the luminosity distance vs. redshift data for SNe 1a yields a numerical value for $\Lambda$ that implies that “dark energy” must constitute about 73% of the mass-energy density of the universe (Suzuki et al. 2012). About 23% of the remaining mass-energy must consist of “non-baryonic dark matter”, which leaves about 4% ordinary baryonic matter.

If “dark energy” represents the ground state oscillations of all of the quantum fields within the cosmos, we might expect its value to be roughly 120 orders of magnitude larger (Carroll 2004, Sec. 4.5) than the $\sim 10^{-8}$ erg cm$^{-3}$ that is needed to explain the cosmological redshift observations. In addition to this rather glaring discrepancy, the energy density of matter would decrease in an expanding universe, which would allow only one coincidental moment of time in which matter and vacuum energy densities might be of comparable magnitude. Thereafter the expansion of the universe must accelerate. In view of the discomfort caused by the size discrepancy and the coincidence problem, there is reason to consider a different approach to understanding the cosmological redshifts.

2 Yilmaz Theory

Though well established in the research literature, (Yilmaz 1971, 1982, 1992, Mizobuchi 1985) the Yilmaz gravity theory is neither well known nor widely utilized, but it has passed all of
the previously known observational tests and remains as a viable gravity theory. The Yilmaz theory is a metric theory of gravity in a preferred set of coordinates. It differs from Einstein’s general relativity primarily by the role of the metric coefficients, $g_{ij}$.

In the Einstein theory, the metric coefficients are generalizations of the gravitational potential of Newton’s theory. In the Yilmaz theory, $g_{ij}$ is merely the generalization of $\eta_{ij}$, the Lorentzian metric of special relativity. The gravitational potential, $\phi$ is also extended to become a tensor field, $\phi^\alpha_\beta(x^k)$ such that the curved spacetime metric is a function of the $\phi^\alpha_\beta$; i.e.

$$g_{ij} = g_{ij}(\phi^\beta_\alpha(x^k)).$$

In this way, gravity is regarded as a field in its own right, and not something manifest solely by spacetime curvature. In general, $\phi^\alpha_\beta \neq 0$ in matter free space and also $(\partial_\nu \phi^\beta_\alpha)^2 \neq 0$ in general. Thus there will be localized field energy in matter free space. It is of second degree in the derivatives of gravitational potentials and automatically of the appropriate sign and magnitude (see Appendix). It also contributes to spacetime curvature in a way that was forbidden by fiat in Einstein’s theory.

In the Yilmaz theory, the presence of free fall is indicated by a locally Minkowskian metric. Removal of constraining forces that prevent free fall is represented by choice of reference position for potentials. Thus, unlike general relativity, free-fall is not achieved by a coordinate transformation, but rather by the choice of reference position for potential, such that $\phi \rightarrow \phi + C \rightarrow 0$ and $g_{ij} \rightarrow 1$ resulting in a locally Minkowskian metric.

Particles of mass-energy are the sources of the potentials. The metric coefficients are functions of the potentials; i.e. $g_{ij} = g_{ij}(\phi^\mu_\nu(x^k))$ and the potentials obey generalized coordinates d’Alembertian equations. For a parcel of proper mass density $\rho$ and speed $u_\mu$, $T^\nu_\mu = \rho u_\mu u^\nu$ and:

$$\Box^2 \phi^\nu_\mu = (4\pi G/c^4)T^\nu_\mu$$

where $\Box^2 = (\sqrt{-g})^{-1}\partial_i(\sqrt{-g} g^{ij} \partial_j)$ is the d’Alembertian operator. It is apparent that where spacetime curvature is negligible, these equations reduce to those of a special relativistic field theory.

One of the central tenets of the Yilmaz theory is that the speed of light in space free of matter must be isotropic. This condition can be enforced in part by adhering to the use of “harmonic coordinates” for which

$$\partial_i(\sqrt{-g} g^{ij}) = 0$$

It ensures that the phase speed of light will be the same in every direction.\textsuperscript{2}

In his first presentation of the theory in terms of tensor potentials, in 1971, Yilmaz stated the relation between the fields $\phi^\nu_\mu$ and the metric as a functional differential equation

$$dg_{\mu\nu} = 2(g_{\mu\nu} d\phi - g_{\mu\alpha} d\phi^\alpha_\nu - g_{\nu\alpha} d\phi^\alpha_\mu)$$

where $\phi$ is the trace of $\phi^\nu_\mu$. It was later shown (Yilmaz 1975, 1992) that this can be integrated to yield a metric form which is exact for many cases of physical interest:

$$g_{\mu\nu} = (\hat{\eta}e^{2(\phi I - 2\hat{\phi})})_{\mu\nu}$$

\textsuperscript{2}A plane wave of the form $\psi = e^{i(\omega t - k \cdot r)}$ propagating in the space of Eq. 7 should satisfy a generalized d’Alembertian equation, $\Box^2 \psi = (1/\sqrt{-g})\partial_i(\sqrt{-g} g^{ij} \partial_j \psi) = 0$. This will generate nonzero terms of the form $\psi k_\lambda \partial_i(\sqrt{-g} g^{ij})$ and make the phase speed of light depend on its direction of travel unless $\partial_i(\sqrt{-g} g^{ij}) = 0$. This harmonic gauge condition is assumed to hold for the metric of Eq. 7 and must also hold if $\psi$ were to represent a gravitational wave.
where \( \hat{\eta} \) is the metric of the Minkowskian background, \( \phi = \phi^k_k \) is the trace of \( \hat{\phi} = \phi^\mu_\mu \) and \( \hat{I} \) is the identity matrix. The exponential function is defined in terms of its ordered Taylor expansion and \( \hat{\eta} \) is the Minkowskian metric. (In rectangular coordinates, \( \hat{\eta} \) is diagonal with elements \((1, -1, -1, -1)\).)

Although the equations of the Yilmaz theory are decidedly nonlinear, they are very easy to use in weak field situations. One of their striking features is that gravitating masses move in potentials that obey a superposition principle and they interact via a field stress-energy tensor, \( t^j_i \). Thus multiple body interactions are easily encompassed in the Yilmaz theory (Yilmaz 1992, 1994). In contrast there are no easy multiple body solutions in general relativity.

For a metric in coordinates \((t, x, y, z)\) one would normally need at least the potentials \( \phi^0_0, \phi^1_1, \phi^2_2, \text{ and } \phi^3_3 \), however, for a cosmological description, we will assume that the metric has the isotropic form:

\[
ds^2 = e^{\nu} c^2 dt^2 - e^{\lambda}(dx^2 + dy^2 + dz^2)
\]  

(7)

for which \( g_{00} = e^{\nu} \) and \( g_{ii} = -e^{\lambda} \).

In this case, we find from Eq. 6 that \( \phi^2_2 = \phi^3_3 = \phi^1_1 \) is required and

\[
\nu(t, x, y, z) = 6\phi^1 - 2\phi^0
\]  

(8)

and

\[
\lambda(t, x, y, z) = 2\phi^1 + 2\phi^0
\]  

(9)

The harmonic coordinate condition, Eq. 4, provides two relations

\[
\partial_0 e^{(3\lambda - \nu)/2} = \partial_0 e^{4\phi^0_0} = 0
\]  

(10)

\[
\partial_i e^{(\lambda + \nu)/2} = \partial_i e^{4\phi^1_1} = 0
\]  

(11)

It is apparent from Eqs. 10 & 11 that we must have \( \phi^0_0 = \phi^0_0(x, y, z), \) independent of time, and \( \phi^1_1 = \phi^1_1(t), \) must be independent of \( x, y, z \). Eq. 3 then gives the equations to be solved for these potentials as:

\[
\nabla^2 \phi^0_0 = -(4\pi G/c^4) \rho u^0 u^0 e^{(2\phi^0_0 + 2\phi^1_1)}
\]  

(12)

and

\[
e^{-\nu} \phi^1_1 = (4\pi G/c^4) T^1_1
\]  

(13)

As Eq. 12 stands, it is inconsistent because \( \phi^0_0 \) is supposed to have no time dependence, yet \( \phi^1_1 \) has time dependence in the right member. This inconsistency can be removed by choosing our observation point to be located at \( r = 0, t = 0 \), where the present value of mass-energy density would be \( \rho_0 \). and requiring that

\[
\rho = \rho_0 e^{-2\phi^1_1}
\]  

(14)

Although \( \rho_0 = \rho_0(r) \) would remove the inconsistency, it would describe an inhomogeneous universe. To avoid this, we require \( \rho_0 = constant. \) This removes the inconsistency and leaves Eq. 12 as

\[
\nabla^2 \phi^0_0 = -(4\pi G/c^4) \rho_0 u^0 u^0 e^{2\phi^0_0}
\]  

(15)
3 Cosmological Red Shifts

In this section we will obtain a solution of Eqs. 13 & 15 and a relation between redshift and luminosity distance for SNe 1a for a model universe consisting of an expanding, isotropic, spherically symmetric, pressureless cosmic dust comprised of galaxy sized dust particles. We will use the metric form of Eq. 7 as applied by an observer located at \( r = 0 \) at the present time, \( t = 0 \). There is no “universal” time for the universe in this approach nor is there a scale factor for the entire universe. There is only the local time of an observer at the origin of coordinates. For these present conditions, \( \lambda = \nu = 0 \) at the observer’s location and the observer’s local spacetime is Minkowskian. Photons emitted at earlier (negative) times and at large distances, \( r \), will be detected as redshifted by \( 1 + z = e^{-\nu/2} \). Photons emitted at earlier times into a particular solid angle will be spread over a larger aperture as the universe expands while they are in transit. As a result, they will appear to have come from a more distant source. In conventional FRW cosmology, the measured photon flux is diminished by two factors of \( (1 + z) \). The individual photons redshift by a factor \( (1 + z) \), and the photons hit the detector less frequently, due to time dilation. In the present approach, one of these factors is taken into account in \( \nu \neq 0 \). The apparent luminosity distance will be enlarged and given by \( d_L = (1 + z)r \).

For this model universe, it is assumed that motions of the dust particles are always very small compared to the speed of light. Thus only \( T_0^0 \) is nonzero. \( T_0^0 = \rho u_0 u^0 = \rho c^2 \), where \( \rho \) is the average mass density of “dust particles” in the universe. Taking \( T_1^1 = 0 \), Eq. 13 shows that \( \phi_0^0 \) will vary linearly with time. Taking \( \phi_1^1 = C_1 t \), with \( C_1 \) a positive integration constant, satisfies Eq. 13. Eq. 14 then provides for a cosmos with a matter density that decreases with time; i.e., an expanding universe.

Defining

\[
R_0 = \sqrt{c^2/(4\pi G \rho_0)}, \quad x = r/R_0, \quad T = ct/R_0
\]

and converting \( \nabla \) to spherical coordinates, Eq. 15 becomes

\[
d^2 \phi_0^0 / dx^2 + (2/x) d\phi_0^0 / dx = - e^{2\phi_0^0}
\]

A low order solution can be obtained by expanding the exponential function of the right member. Assuming that \( \phi_0^0 = \Sigma a_n x^{n+2} \), we find,

\[
\phi_0^0 = -x^2/6 + x^4/60 - x^6/687.3 + x^8/3565 \ldots
\]

This fits well to \( x \sim 1 \), but numerical solutions are needed for larger values of \( x \). The numerical solutions of Eqs. 13 and 17 allow us to determine the metric functions

\[
\lambda = 2C_1 T + 2\phi_0^0
\]

\[
\nu = 6C_1 T - 2\phi_0^0
\]

\( C_1 \) is proportional to the Hubble constant as will be seen by considering the gravitational red shift that would be expected for a photon emitted at some previous time and detected now at our location \( x = 0, \; T = 0 \). A null geodesic photon path taken from \( r \) to zero and time \( T \) in the past to the present will have \( ds^2 = 0 \). Thus, \( e^{\nu/2} c dt = - e^{\lambda/2} dr \), where the negative sign is taken because \( r \) decreases as \( t \) increases from the time of emission to our detecting it at the present time. Substituting the solutions for \( \lambda \) and \( \nu \) into this last relation and rearranging, we obtain

\[
\int_T^0 e^{2C_1 T} dT = (1 - e^{2C_1 T})/2C_1 = - \int_x^0 e^{2\phi_0^0} dx
\]
For large values of \( x \), the integral on the right must be evaluated numerically, but it is instructive to first consider the expansions to lowest orders, for which we obtain

\[
T + C_1 T^2 = -x + x^3/9
\]  

(22)

To lowest order, we have \( T = -x \), or \( t = -r/c \), as expected. The redshift of a photon, to lowest order, would be

\[
z = e^{-\nu/2} - 1 \approx 3C_1 x = 3C_1 \sqrt{4\pi G \rho_0} \frac{r}{c}
\]  

(23)

It is now apparent that the Hubble constant, \( H_0 \), is given by

\[
H_0 = 3C_1 \sqrt{4\pi G \rho_0}
\]  

(24)

By numerically solving Eq. 17 for \( \phi_0^0 \) and integrating numerically, the integral on the right side of Eq. 21 is found to have the limiting value of -2.1405 for very large \( x \). For the corresponding time, \( T \to -\infty \), we find from Eq. 21 that \( \frac{1}{2C_1} = 2.1405 \), or \( C_1 = 0.2336 \).

This is a self-consistent choice for \( C_1 \) that leaves only one free parameter, \( \rho_0 \), to be chosen to fit the redshift-luminosity data. With the value of \( C_1 \) now determined, the appropriate time, \( T \), for any \( x \), can be found from

\[
T = \ln\left[1 - 2C_1 \int_0^x e^{2\phi_0^0} dx\right]/2C_1
\]  

(25)

Once \( T \) is known, the values of \( \phi_1^1 \), \( \nu \), \( \lambda \), \( z \) and \( d_L = r(1 + z) \) can be computed. Numerical solution data for \( C_1 = 0.2336 \) and \( \rho_0 = 1.06 \times 10^{-29} \text{ g cm}^{-3} \) are given in Table 1. This choice for \( \rho_0 \) was based on a Hubble constant obtained by a least squares fit to 166 data points for \( z \leq 0.1 \) that yielded \( H_0 = 64.5 \pm 0.7 \text{ km s}^{-1} \text{ Mpc}^{-1} \). (Data from The Supernova Cosmology Project, (Amanullah et al. 2010, Suzuki et al. 2012)) Eq. 24 was then used to calculate \( \rho_0 = 1.06 \times 10^{-29} \text{ g cm}^{-3} \). This provides a very good fit to the supernova redshift data over the whole range of observed redshifts as shown in Figure 1. The value of \( \rho_0 \) is essentially the same as the critical density that would be obtained for a FRW metric.

4 Discussion

Three other attempts have been made to apply the Yilmaz theory to cosmology. Yilmaz (1958) developed a metric for a static universe. Increasing evidence of the inadequacy of this approach led him to the extensions in his 1971 theory. Mizobuchi (1985) applied the 1971 theory to a cosmological model consisting of a perfect fluid. This was not a central point of an otherwise very informative article, but it appears to have been based on the erroneous inclusion of a factor of \( \sqrt{-g} \) in \( T^\nu_{\mu} \), where \( (-g) \) is the determinant of the metric. \( T^\nu_{\mu} \) should have been taken to be just the diagonal matter tensor of a perfect fluid, \( T^\nu_{\mu} \to (\rho c^2, -\rho, -P, -P) \). The approach taken here was motivated by that of Mizobuchi (1985) but correcting the error leads to significantly different results.

A third attempt to apply the Yilmaz theory to cosmology was provided by Ibison (2006). Ibison assumed the correctness of the flat-space FRW metric

\[
ds^2 = dt^2 - a(t)^2(dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\Phi^2)
\]  

(26)

and found a coordinate transformation to the form of Eq. 7. This transformation was shown to satisfy the harmonic coordinate condition, but only at the expense of leaving \( \lambda \) and \( \nu \) dependent.
Table 1: Parameters of redshift and distance calculations

| $x$  | $a \phi_0^0(x)$ | $\int_0^x e^{2\phi_0} dx$ | $T$  | $z$  | $b d_L (Mpc)$ |
|------|----------------|-----------------------------|------|------|---------------|
| 0.000| 0.000          | 0.000                       | 0.000| 0.000| 0.000         |
| 0.010| -0.002         | 0.100                       | -0.102| 0.073| 350           |
| 0.200| -0.007         | 0.199                       | -0.209| 0.150| 750           |
| 0.300| -0.015         | 0.297                       | -0.320| 0.233| 1206          |
| 0.400| -0.026         | 0.393                       | -0.434| 0.321| 1722          |
| 0.500| -0.041         | 0.487                       | -0.552| 0.414| 2304          |
| 0.600| -0.058         | 0.577                       | -0.673| 0.512| 2958          |
| 0.700| -0.078         | 0.665                       | -0.796| 0.616| 3687          |
| 0.800| -0.100         | 0.748                       | -0.921| 0.725| 4498          |
| 0.900| -0.125         | 0.828                       | -1.047| 0.838| 5394          |
| 1.000| -0.152         | 0.904                       | -1.175| 0.957| 6380          |
| 1.100| -0.180         | 0.976                       | -1.303| 1.080| 7459          |
| 1.200| -0.211         | 1.043                       | -1.431| 1.208| 8637          |
| 1.300| -0.242         | 1.107                       | -1.559| 1.339| 9915          |
| 1.400| -0.275         | 1.167                       | -1.686| 1.475| 11296         |
| 1.500| -0.309         | 1.222                       | -1.812| 1.614| 12783         |
| 1.600| -0.344         | 1.275                       | -1.937| 1.756| 14377         |
| 1.700| -0.379         | 1.323                       | -2.061| 1.901| 16079         |
| 1.800| -0.415         | 1.368                       | -2.182| 2.049| 17890         |
| 1.900| -0.451         | 1.410                       | -2.302| 2.198| 19810         |
| 2.000| -0.487         | 1.450                       | -2.420| 2.350| 21840         |
| 3.000| -0.845         | 1.718                       | -3.473| 3.896| 47880         |
| 4.000| -1.164         | 1.853                       | -4.299| 5.353| 82840         |
| 5.000| -1.434         | 1.928                       | -4.945| 6.625| 124280        |
| 6.000| -1.662         | 1.973                       | -5.459| 7.704| 170250        |
| 7.000| -1.856         | 2.003                       | -5.878| 8.618| 219480        |
| 8.000| -2.023         | 2.024                       | -6.228| 9.399| 271200        |
| 9.000| -2.168         | 2.039                       | -6.525| 10.075| 324900      |
| 10.000| -2.296       | 2.050                       | -6.783| 10.670| 380400     |

\(^a\) All values calculated for $C_1 = 0.2336$.  \(^b\) These values calculated for $\rho_0 = 1.06 \times 10^{-29} \text{ g cm}^{-3}$ are shown by the solid line on Fig. 1.
Figure 1: Luminosity distance vs redshift for SNe 1a. Data from The Supernova Cosmology Project, (Amanullah et al 2010, Suzuki et al. 2012). The fitted curve is determined by only one free parameter, $\rho_0 = 1.06 \times 10^{-29} \, g \, cm^{-3}$ that was obtained from the Hubble Constant fitted for $z \leq 0.1$ (see text).
only on time with no position dependence. Ibison’s transformation, $dt = a(\zeta)^3 d\zeta$, produces a result that is equivalent to setting $\phi_0^\beta = 0$ and is incapable of fitting the SNe 1a redshift data. Fig. 1 shows that the redshift data is nicely encompassed by the Yilmaz theory and the metric of Eq. 7.

The Cosmological Principle asserts that the universe is spatially homogenous and isotropic, but it does not demand strict adherence to the FRW metric. The FRW metric mathematically ensures a translational invariance that would leave the universe with the same appearance to all observers at the same “cosmic time”, but that is not the only way to obtain consistency with the principle. Form invariance of Eq. 7, the requirement that $\phi^\beta_\alpha = 0$ hold at the observer’s location and the requirement that $\rho_0 = constant$ satisfy the requirements. In this case, however, a “cosmic time” would have no meaning.

Yilmaz (1971) showed that his theory would change Einstein’s field equations (Eq. 1) to the form

$$G^i_j = -(8\pi G/c^4)(\rho u_i u^j + t^i_j)$$

where $t^i_j$ is a stress-energy tensor of the gravitational field. This addition of $t^i_j$ is about the smallest correction that one might imagine for Einstein’s general relativity. Since $t^i_j$ adds only second order corrections, leaving the first order theory basically intact, it passes all of the weak-field tests that have been taken as confirmation of Einstein’s theory. Without endorsing a particular form such as the Yilmaz $t^i_j$, Lo (1995) has shown that the inclusion of a gravitational field stress-energy term is necessary in order for Einstein’s field equations to correctly encompass his gravitational radiation formula.

$t^i_j$ is generally of second order in the derivatives of the potentials $\phi^\beta_\alpha$ and is small. For example, with $x = r/R_0$ in the present calculations (see Appendix),

$$t^0_0 = (\rho_0 c^2/2)(e^{-\nu}3C_1^2 - e^{-\lambda}(\partial_x \phi^0_0)^2)$$

has a value of about $8 \times 10^{-10}$ erg cm$^{-3}$ at the observer’s location at the origin of coordinates. Of course, it has larger values at distant locations and earlier times when the matter density of the universe was larger.

The replacement of a cosmological constant with $t^i_j$ provides a term that is of the correct order of magnitude and sign without regard to any other properties of the quantum vacuum. In this way, the Yilmaz theory eliminates the need for “dark energy”, however, the density $\rho_0 = 1.06 \times 10^{-29} g$ cm$^{-3}$, obtained here is at least an order of magnitude larger than the known baryonic mass density of the cosmos. Thus a need for a considerable amount of dark matter remains in the Yilmaz theory.

Finally, it should be noted that the solution of the Yilmaz equations found here fails to conserve energy. The conservation law of the Yilmaz theory is the Freud identity (Yilmaz 1982, 1992). In rectangular coordinates, it would be written as

$$\partial_i(\sqrt{-g} T^i_k) = 0$$

With $T^i_k = T^0_0 = \rho_0 c^2 e^{-2\phi^0_1}$ and $\sqrt{-g} = e^{6\phi^0_1 + 2\phi^0_0}$, this would require $\partial_0 \phi^1_0 = 0$ and energy-momentum would be conserved only in a static universe.
Appendix

5 A. Central body metric

Since the Yilmaz theory is neither well-known nor widely used, it seems appropriate to discuss some of its features here. While a few of the basic concepts of the theory have been presented here, the theory is well developed and informative discussions of various aspects of the theory have also been provided by Alley (1995), Menzel (1976), Mizobuchi (1985) and Yilmaz (1975). Perhaps the most well-known idea of the theory is the exponential metric for the space beyond a static, spherically symmetric mass, $M$. In this case, $T_0^0 = 0$. $\phi_0 = \phi = GM/c^2 r$ is the solution of Eq. 3 and the metric is

$$ds^2 = e^{-2\phi} c^2 dt^2 - e^{2\phi}(dx^2 + dy^2 + dz^2)$$

(30)

That there are no black holes in this metric may be the most widely known aspect of the Yilmaz theory. Instead of an event horizon at $r = 2GM/c^2$, there is a photon orbit at this location. In another respect of interest to astrophysics, the innermost marginally stable orbit for a particle in orbit around $M$ occurs at $r = 5.24GM/c^2$ rather than the $6GM/c^2$ of the Schwarzschild metric.

This would accommodate most models of accretion disks for compact objects. In addition, it should be noted that it might be possible for objects to become so compact that $\phi \gg 1$. In this circumstance, photons emitted at the surface would be extremely red shifted as observed distantly and very little, if any, luminosity would be observed for the object. A correct treatment of realistic compact objects of astrophysical interest will require consideration of the trapped radiation fields, especially in cases involving an active collapse process.

Although it is unlikely that massive point particles actually exist, $g_{00} = e^{-2\phi}$ would be zero only for $r = 0$. This should be regarded as just a classical physics point particle singularity rather than an event horizon. The Kretschmann invariant is zero rather than divergent at $r = 0$. There is no curvature singularity there.

6 Gravitational field stress-energy tensor

The Yilmaz gravitational energy expression, $t_i^j$, is essentially Einstein’s gravitational stress-energy pseudotensor expressed in terms that eliminate pseudotensors and leave a true tensor quantity (Yilmaz 1992, Yilmaz & Alley 1999). First define terms:

$$g^{ij} = \sqrt{-K}g^{ij} \quad g_{ij} = g_{ij}/\sqrt{-K}$$

(31)

and

$$W^i_j = (1/8\sqrt{-K})g^{jk}(\bar{\partial}_k g_{ab}\bar{\partial}_i g^{ab} - 2\bar{\partial}_k \sqrt{-K}\bar{\partial}_i (1/\sqrt{-K}) - 2\bar{\partial}_a g_{kb}\bar{\partial}_i g^{ab})$$

(32)

Here the overbar represents a covariant derivative with respect to local Minkowskian coordinates which share the same origin and orientation as those of the general metric. $\sqrt{-K} = \sqrt{-g}/\sqrt{-\eta}$, where $\sqrt{-g}$ is the determinant of the metric and $\sqrt{-\eta}$ is the determinant of the metric of the Minkowskian background. In rectangular coordinates $(x,y,z,t)$, $\sqrt{-\eta} = 1$ and all Christoffel symbols vanish, leaving a normal partial derivative.

This elaborate derivative procedure is necessary to eliminate pseudotensors that might otherwise arise. The pseudotensor problem has been discussed in detail (Yilmaz 1992). This procedure eliminates them, but it is merely a mathematical artifice. No bimetric theory is intended...
here and there is no further need of a Minkowskian metric. Particle motions under the influence of only gravitational forces would follow geodesics of the general metric, not the Minkowskian metric. Pseudotensor problems can be avoided in two ways. The first simply consists of the use of rectangular coordinates, in which they never appear. The second is to take derivatives as covariant derivatives in local Minkowskian coordinates. Preliminary considerations aside, \( t^j_i \) is given as a true tensor quantity as

\[
  t^j_i = W^j_i - \frac{1}{2} W^k_k
\]  

(33)

Yilmaz often used expressions that incorporated a harmonic coordinate condition. The expressions above for \( W^j_i \) and \( t^j_i \) were based on Pauli’s decomposition of the Einstein tensor with no harmonic coordinate conditions included.

Evaluating Eq. 33 for the metric of Eq. 7 and complete spherical symmetry for which \( r \) is the only spatial variable yields

\[
  t^0_0 = -t^1_1 = (c^4/8\pi G)(e^{-\nu}3\dot\lambda^2/4 + e^{-\lambda}(\lambda'\nu'/2 + \lambda^2/4))
\]  

(34)

and

\[
  t^2_0 = t^3_0 = (c^4/8\pi G)(-e^{-\nu}3\dot\lambda^2/4 + e^{-\lambda}(\lambda'\nu'/2 + \lambda^2/4).
\]  

(35)

Here dots represent partial derivatives with respect to time, and primes represent partial derivatives with respect to the radial coordinate, \( r \). Then using the expressions of Eqs. 34 & 35 for the right member of Eq. 27, the field equations for \( G^0_{00}, G^1_{11}, \) and \( G^2_{20} = G^3_{30} \) become

\[
  e^{-\lambda}[(1/\nu^2)\partial_r(r^2\partial_r\lambda) + \lambda'(\lambda' + \nu')/2] = -(8\pi G/c^4)T^0_0
\]  

(36)

\[
  e^{-\nu}(\ddot\lambda + \dot\lambda(3\dot\lambda - \dot\nu)/2) + e^{-\lambda}(\lambda' + \nu')/r = -(8\pi G/c^4)T^1_1
\]  

(37)

\[
  -e^{-\nu}(\ddot\lambda + \dot\lambda(3\dot\lambda - \dot\nu)/2) + (1/2)e^{-\lambda}[\nu'' + \lambda'' + (\lambda' + \nu')/r + (\lambda' + \nu')^2/2] = -(8\pi G/c^4)T^2_2
\]  

(38)

The generalized d’Alembertian Eqs. 3 have time dependence that is not evident in Eq. 36. If one wished to use harmonic coordinates and the relations, \( \lambda = 2\phi_1 + 2\phi_0 \) and \( \nu = 6\phi_1 - 2\phi_0 \), the Eqs. 3 would yield

\[
  \Box^2\lambda = (8\pi G/c^4)(T^0_0 + T^1_1)
\]  

(39)

and would include second time derivatives \( e^{-\nu}\dot\lambda \) that are not in Eq. 36. As noted by Lo (1995), the more general time dependence of the d’Alembertian equations is necessary in order to encompass gravitational waves. The metric form of Eq 6 and the d’Alembertian equations have been shown to describe them well (Mizobuchi 1985). Further, in cases with no time dependence, but where \( T^1_1, T^2_2 \) or \( T^3_3 \) would be nonzero, one finds that the use of harmonic coordinates would lead to \( \lambda' + \nu' = 0 \) and no solution for Eqs. 37 & 38. While Eq. 27 leads to Eqs. 36 & 37 and these reduce to Eqs. 13 & 17 under the same assumed conditions, Eq. 27 must be regarded as having only limited applicability.

7 Quantum possibilities

The question of how quantum mechanics and general relativity might be reconciled has recently been sharpened by considering what happens to a freely falling particle of matter approaching an event horizon. The possibility that it might meet a radiative “firewall” has recently become a very active research topic (e.g., Abramowicz, Kluzniak & Lasota 2013, Anastopoulos
& Savvidou, 2014, Hawking 2014). This is a problem of such importance that we should consider all aspects; however, the necessity of event horizons seems not to have been questioned in astrophysics. They have been accepted without proof. Although there are many astronomical objects that are known to be compact and massive enough to be black holes, if event horizons exist, none have been shown to possess this quintessential feature of a black hole.

Einstein developed general relativity with the aim of explaining gravitational phenomena as manifestations of spacetime curvature alone. In his field equations he included all forms of energy as sources of gravitation and curvature but expressly rejected a separate gravitational field as a source of energy. Instead of having separate gravitational potentials, the metric coefficients of general relativity take the dual roles of potentials and descriptors of spacetime geometry. One of the problems that this presents for quantum theory is that the covariant derivatives of the metric tensor are identically zero. Potentials such as $\phi_0$ and $\phi_1$ that exist separately from the metric may provide a path to a quantum theory of gravity (Yilmaz 1995, 1997, 1980). This needs further exploration.

Although there have been indications of small things amiss with general relativity, such as the failure to have a complete correspondence limit with special relativity (Yilmaz 1975, Alley 1995), they have not generally led to serious consideration of rival theories. Even serious difficulties such as the failure to encompass the quadrupole gravitational radiation formula (Wald 1984; Yu 1992, Lo 1995) have been ignored. To the contrary, astrophysicists have stretched the applications of the theory to the point of accepting the existence of black holes, singularities and dark energy. In view of the ease with which the Yilmaz theory removes these, they may not be necessary at all. While “black holes” have become a part of the mystique of astrophysics that may persist as descriptive terminology even if event horizons are abandoned, dark energy has much the same appeal as adding more epicycles. It may soon be forgotten.

8 Compact objects

Removing event horizons from the astrophysical menagerie does, however, leave a need for a new understanding of the nature of the gravitationally collapsed and compact objects that are presently thought by many to be black holes. The luminosity differences between a very large redshift, $z$, and the $z = \infty$ of a black hole might be small and subtle. Other differences, such as the presence of magnetic fields, might betray the lack of an event horizon. Robertson & Leiter (2002) presented evidence for the existence of intrinsic magnetic moments in stellar mass black hole candidates. They later devised a magnetic, eternally collapsing object (MECO) model that could account for the observations and extended its application to active galactic nuclei, including Sgr A* (Robertson & Leiter 2003, 2004, 2006, 2010). Additional observational evidence for magnetic moments in AGN has also been found (Schild, Leiter & Robertson 2006, 2008). The MECO model needs minor revision to incorporate the Yilmaz exponential metric.

Since the objects presently considered to be black holes are too massive and compact to be supported by neutron degeneracy pressure, they most likely would collapse to a size that can be supported by internal radiation pressure (Mitra 2006). They might well become quark-gluon plasmas. At the same time, their surface emissions must occur with such extreme redshifts that their distantly observed luminosity would be quite low. In this regard, the ECO (e.g., Mitra 2000-2006) or MECO (e.g., Robertson & Leiter 2002-2006) models, which only need large gravitational redshifts and/or intrinsic magnetic fields to function may possibly be encompassed within the Yilmaz theory. This remains to be worked out for spacetimes dominated by electromagnetic radiation fields.
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