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Marked point process for modelling seismic activity (case study in Sumatra and Java)

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Abstract. Earthquake is a natural phenomenon that is random, irregular in space and time. Until now the forecast of earthquake occurrence at a location is still difficult to be estimated so that the development of earthquake forecast methodology is still carried out both from seismology aspect and stochastic aspect. To explain the random nature phenomena, both in space and time, a point process approach can be used. There are two types of point processes: temporal point process and spatial point process. The temporal point process relates to events observed over time as a sequence of time, whereas the spatial point process describes the location of objects in two or three dimensional spaces. The points on the point process can be labelled with additional information called marks. A marked point process can be considered as a pair \((x, m)\) where \(x\) is the point of location and \(m\) is the mark attached to the point of that location. This study aims to model marked point process indexed by time on earthquake data in Sumatra Island and Java Island. This model can be used to analyse seismic activity through its intensity function by considering the history process up to time before \(t\). Based on data obtained from U.S. Geological Survey from 1973 to 2017 with magnitude threshold 5, we obtained maximum likelihood estimate for parameters of the intensity function. The estimation of model parameters shows that the seismic activity in Sumatra Island is greater than Java Island.

1. Introduction

Indonesia is in a position that is vulnerable to the threat of natural disasters. This is due to the geographical position of Indonesia which is at the meeting of three large plates of earth namely the Eurasian plate, the Indo-Australian plate and the Pasific plate. The encounter and movement of three large earth's plates creates an active volcano ranks and earthquake potentials in most of the Indonesian archipelago.

Sumatra Island and Java Island are two areas of Indonesia that are often affected earthquake. Among the various regions in Sumatra, the areas with high earthquake potential are Aceh and West Sumatra. Based on data from United State Geological Survey (USGS) in 1960 to June 2017 in Aceh Province area, there are at least 2400 earthquakes with magnitude size above 4 on Richter scale. As
for the area of West Sumatra at the same time interval, there are at least 900 earthquakes with magnitude measurements above 4 on the Richter scale. Since 1990 until June 2017, there have been recorded 473 earthquakes that occurred in Java area with magnitude grater than or equal to 4 on Richter scale. Examples of major earthquakes that occur on the Java Island include the Yogyakarta earthquake which occurred on May 2006 with magnitude 5.9 on the Richter scale. A month later, an earthquake accompanied by a tsunami occurred in Pangandaran, Ciamis Regency, West Java. Then in September 2009, a major earthquake also occurred in Tasikmalaya Regency, West Java.

Point process is a major subject in statistical seismology. Point process is a part of stochastic process that can be used to describe events in a particular pattern. In this process, earthquakes are viewed as a random collection of points in a space, where each point represents the time or location of an event [2]. The occurrences of earthquakes in certain locations can be viewed as points, whereas the size associated with the occurrence of an earthquake is the magnitude or depth [10]. In this process, the occurrence of the event is memory less and mutually independent from occurrences of other events.

In seismological research, some models that can be used to describe the aftershock pattern are Omori's law and Omori-Utsu's law. These mathematical models are not always accurate in modeling earthquakes for long periods of time. The relation between mainshock and aftershocks can be illustrated by other mathematical models which are the development of Omori-Utsu's Law [5,8,9]. Further research related to stochastic modeling to analyze seismic activity in a region with point process approach has also been done by some experts [7,12]. In this article we discuss the marked point process which is indexed by time. This process is presented by an intensity function which considering the occurrence of mainshock and its aftershocks. Then we will estimate this function through maximum likelihood method with considering the history process.

2. Marked Point Process

The point process is defined as a random collection of points located on a particular region. The points of this process can be expressed as events, time of events, location of events, or time and location of events [6]. The construction of a point process can be done by several approaches, one of them is through a counting process. The state space and the parameter space of this counting process are non negative integer sets and subsets of $\mathbb{R}^d$ with $d \geq 1$. A counting process $X(t)$ is defined as the number of events occurring during time $t$. The counting process $X(t)$ satisfies the following properties:

1) $X(t) \geq 0$
2) $X(t)$ is integer
3) If $s<t$, then $X(s) \leq X(t)$
4) For $s<t$, then $X(t) - X(s)$ denotes the number of events occurring in the interval $(s, t]$.

The counting process has independent increments property. It means that the number of occurrences between time $s$ and $t$, $X(t) - X(s)$, independent of the number of events occurred up to time $s$. The counting process has stationary increments property if the distribution of the number of occurrences at an interval depends only on the length of the interval. There are two types of point processes, i.e. the temporal point process and spatial point process. The temporal point process relates to events observed at a sequence of time, while the spatial point process relates to the location of the object in a two or three-dimensional space [11].
The points of the point process are often labeled with additional information called marks. A marked point process can be considered as a pair \((X, m)\) where \(x\) is the point of location and \(m\) is the mark or quantity attached to the point.

**Definition 1.** A marked point process in a space \(S\) with the mark in a space \(M\) is a point process \(Y\) at \(S \times M\) such that \(N_Y(K \times M) < \infty, \) for all compact sets \(K \subset S.\)

This definition describes that the projected process (from unmarked points) is finite local [2].

**Theorem 2.** Let \(Y\) be the marked point process on \(S\) with marks in \(M\). Let \(X\) be projected at \(S\) (from unmarked points), then

1) \(X\) is a Poisson process in \(S\) with intensity \(\mu,\) and given \(X,\) the mark attached to point in \(X\) independently and identically distributed with joint distribution \(Q\) on \(M;\)

2) \(Y\) is a Poisson process in \(S \times M\) with intensity measure \(\mu \otimes Q.\)

3. **Conditional Intensity Function**

An important concept of this temporal marked point process is the conditional intensity function on the history of a time-dependent process. This concept describes the probability of an event in the future which depends on previously information up to the present time. In seismology this function can describe seismic activity in a region for a certain period of time. The ground intensity function is the rate of events depend on time, and not only influenced by the present time but also influenced by the previous events, or history process [3]. This function describes the instant Poisson rate as a function that depends only on time. Let \(N_\delta(t)\) be the number of occurrences at the time interval \([t, t + \delta)\), the ground intensity function can be written as

\[
\lambda(t, \theta_1, \ldots, \theta_m | H_t) = \lim_{\delta \to 0} \frac{1}{\delta} P(N_\delta(t) > 0 | H_t)
\]

where \(N_\delta(t)\) is the number of occurrences at the time interval \([t, t + \delta).\)

Furthermore we discuss the intensity function on the marked point process indexed by time with the magnitude as its marked. According to Kagan [4], the rate of aftershocks is defined as

\[
N(t) = \frac{C_1}{t}
\]

where \(C_1\) is parameter and \(t\) is time measured from the beginning of a mainshock. Nowadays a more complex equation is used to estimate the rate of aftershocks:

\[
N(t) = \frac{C_1}{(C_2 + t)^p}
\]

where \(C_1, C_2,\) and \(p\) are parameters. Equation (2) is a modification from Omori’s Law (1) and called modified Omori’s Law or Omori-Utsu’s Law.

In the epidemic model, the number of individuals living at time \(t\) is controlled by the rate of immigration, birth rate and death rate. In the case of earthquake, immigration refers to the occurrence of independent events, whereas birth is associated with the sequence of events triggered by the previous event. The process of birth and death depends on age for each individual \(x\) living at time \(t.\)
For the next interval \((t, t + dt)\), there are probability of one birth, \(g(x)dt\), and probability of one death, \(h(x)dt\). Birth and death are independent of each other. If the birth process considers immigration at rate \(\mu\) per unit time and there is no death, \(h(x) = 0\) then the process has the following conditional intensity:

\[
\dot{\lambda}(t) = \mu + \sum_{i} g(t_i-t)
\]  
(3)

where \(t_i\) is the \(i^{th}\) occurrence time and \(N(t)\) is the number of occurrences, \(\{t_i\}\), in \((0, t)\).

Equation (3) can be extended to the multivariate point process, \(\{t_i^{m}\}\), and the conditional intensity function for the discrete magnitude value of \(j\) and \(m\) is

\[
\dot{\lambda}_j(t) = \mu_j + \sum_{m} \sum_{i} g_{jm}(t-t_i)
\]  
(4)

If we assume that \(g_{jm}(t) = c(m)g_j(t)\) and the point process \(N(t) = \sum_{m} N_{m}(t)\) then the conditional intensity is given by \(\dot{\lambda}(t) = \sum_{j} \dot{\lambda}_j(t)\) and

\[
\dot{\lambda}(t) = \mu + \sum_{i} c(m_i) g(t_i-t)
\]  
(5)

where \(t_i\) is the occurrence time of \(N(t)\), \(m_i\) is the magnitude corresponding to \(t_i\), and \(g(t) = \sum_{j} g_j(t)\).

Based on the Omori-Utsu law (2) and the epidemic type model (5), we require the following assumptions:

1) The ground seismic rate in a region is constant \(\mu\), which means that the ground seismic occurs according to the stationary Poisson process with constant rate \(\mu\).

2) All earthquake events, including aftershocks, may cause subsequent aftershocks, and any subsequent aftershocks generate the second subsequent independently. The number of aftershocks of a major earthquake with magnitude \(M\) follows Poisson distribution with average \(c(M)\):

\[
c(M) = \kappa e^{\alpha (M - M_0)}
\]  
(6)

where \(M_0\) is the magnitude threshold while \(\kappa\) and \(\alpha\) are parameters.

3) Aftershocks of a mainshock with magnitude \(M\) occurring at certain time intervals appropriate with Poisson process with rate \(c(M)g(t-t_0)\) with \(g(t)\) is a normalized Omori function at a certain time so that \(\int g(t)dt = 1\) and is expressed as

\[
g(t) = \frac{p-1}{c} \left(1 + \frac{t}{c}\right)^{-p}
\]  
(7)

where \(c\) and \(p\) are parameters, while \(t_0\) is the occurrence time of the mainshock.

4) Magnitude distribution is independent with rate of occurrence. By using the explicit form of the Gutenberg-Richter relationship, the probability density function of the magnitude is

\[
f(M) = \beta e^{\beta(M-M_0)}
\]

where \(\beta\) corresponds with \(b\), \(\beta = 2.30b\).

By substituting \(c(m_i)\) with equation (6) and \(g(t-t_i)\) with equation (7) in equation (5), the conditional intensity function of marked point process is
4. Seismicity Modeling for Sumatra Island and Java Island

In this section we estimate the conditional intensity function for earthquake data in Sumatra and Java. The earthquake data is a secondary data sourced from the United States Geological Survey (USGS). The earthquake data contains times of $i^{th}$ earthquake occurrence, $t_i$, and magnitude of $i^{th}$ earthquake occurrence, $m_i$. We use earthquake data occurred from January 1973 to April 2017 with a magnitude of $\geq 5$ and a depth of $\leq 70$ km.

4.1 Sumatra Earthquake Data

In the Sumatra Island for the period time January 1973 to April 2017 with magnitude of $\geq 5$ and depth of $\leq 70$ km there are 2,114 earthquakes. Based on the maximum likelihood estimate method, the conditional intensity function (8) for Sumatra earthquake data can be written as

$$
\lambda(t) = \mu + \kappa \sum_{i=0}^{n} e^{4(M_i - M_0)} (1 + \frac{t - t_i}{c})^{-p}
$$

(8)

![Figure 1. Plot of Magnitude and Time of Sumatra Earthquake Data](image-url)
Figure 2. Plot of Logarithmic of Conditional Intensity Function for Sumatra Earthquake Data

Plot of magnitude and time of Sumatra earthquake data is presented in Figure 1 while the plot of logarithmic of conditional intensity function is presented in Figure 2. Based on Figure 1 it can be seen that on March 11, 2012, September 12, 2007, March 28, 2005, March 11, 2012 and December 26, 2004 there was an earthquake with large magnitude occurring in off the west coast of northern Sumatra, southern Sumatra, northern Sumatra, and off the west coast of northern Sumatra. Based on Figure 2 it appears that these areas also have high intensity functions.

4.2 Java Earthquake Data
In Java Island for the time period from January 1973 to April 2017 with magnitude ≥ 5 and depth ≤ 70 km there are 488 earthquakes. Based on maximum likelihood estimate method, the intensity function of marked point process for Java earthquake data can be written as:

\[
\lambda(t) = 0.0152 + 10.8592 \sum_{t_i < t} e^{0.0875(M_i - M_0)} \left(1 + \frac{t - t_i}{0.0043}\right)^{-1.0484}
\]

Plot of magnitude and time of Java earthquake data is presented in Figure 3 whereas plot of the logarithmic of conditional intensity function is presented in Figure 4. Based on Figure 3 it can be seen that on September 7, 1974, April 3, 2011, October 25, 2000, September 2, 2009 and July 17, 2006, there was an earthquake with large magnitude occurring in Southern Java and the Sunda Strait. Based on Figure 4, it is seen that in those areas the conditional intensity function for Java earthquake data is also high.
4.3 Seismic Activity

In this section seismic activity on Sumatra and Java islands is compared by considering all parameters of their intensity function. Table 1 shows that parameter value $\mu$ of Sumatra earthquake data is greater than Java which means that the base seismic rate of Sumatra Island greater than Java Island.

The parameter value $\kappa$ of Java earthquake data is greater than Sumatra which means that the productivity of aftershocks in Java is higher than Sumatra, while the parameter value $\alpha$ of Sumatra earthquake data is greater than Java which means that the efficiency of earthquake with certain magnitude in producing aftershocks in Sumatra is higher than Java. Furthermore, the parameter value $c$ of Sumatra earthquake data is greater than Java which means that in time scale, the decay rate of aftershocks in Sumatra is higher than Java, and parameter $p$ of Java earthquake data is greater than Sumatra which means that the decay rate of aftershocks in Java is higher than Sumatra.
Table 1. Parameter Estimate of Conditional Intensity Function for Earthquake Data in Sumatra and Java

| Parameter | Sumatra | Java   |
|-----------|---------|--------|
| $\mu$     | 0.0291  | 0.0152 |
| $\kappa$  | 0.2240  | 10.8592|
| $\alpha$  | 2.4417  | 0.0875 |
| $c$       | 0.0199  | 0.0043 |
| $p$       | 0.9930  | 1.0484 |

5. Conclusion
Intensity function on a marked point process can modeling seismic activity in a region. By using maximum likelihood estimate method we can apply this model to determine earthquake characteristic in Sumatra and Java islands. Base seismic rate in Sumatra is greater than the one in Java. Although decay rate of aftershocks in time in Sumatra is higher than the one in Java, but impact of earthquake occurrences in Sumatra is greater than the one in Java. This process have not yet considered the spatial component for its intensity function, so the improvement of intensity function (5) can be done to model seismic activity in a region by considering magnitude, time occurrence, and location.

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