THEORETICAL STUDY ON \( pp \to pn\pi^+ \) REACTION AT MEDIUM ENERGIES

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The \( pp \to pn\pi^+ \) reaction is a channel with the largest total cross section for pp collision in COSY/CSR energy region. In this work, we investigate individual contributions from various \( N^* \) and \( \Delta^* \) resonances with mass up to about 2 GeV for the \( pp \to pn\pi^+ \) reaction. We extend a resonance model, which can reproduce the observed total cross section quite well, to give theoretical predictions of various differential cross sections for the present reaction at \( T_p = 2.88 \) GeV. It could serve as a reference for identifying new physics in the future experiments at HIRFL-CSRR.

1. Introduction

The study of excited nucleon states is very important for understanding the internal structure of nucleon and the strong interaction in the nonperturbative QCD domain [1]. In the early years, our investigation on the \( N^* \) and \( \Delta^* \) baryon spectroscopy was mainly based on \( \pi N \) experiments, which made observations unsatisfactory [2-3]. An outstanding problem is that, in many of its forms, the quark model
predicts a large amount of “missing” \( N^* \) and \( \Delta^* \) states around 2 GeV/c\(^2\), which have not to date been observed\(^\text{345}\). Therefore, it is of necessity to search for these “missing” \( N^* \) and \( \Delta^* \) states from other production processes. Moreover, even for those well-established resonance states, properties like mass, width and branching ratios still suffer large experimental uncertainties\(^6\), which also need further studies in more other production processes. Here we propose to look for “missing” \( N^* \) and \( \Delta^* \) resonances in \( pp \rightarrow pn\pi^+ \) reaction. At COSY/Jülich, Experiments for studying \( N^* \) and \( \Delta^* \) resonances through \( pp \) collisions are being carried out, but there is a lack of a good 4\( \pi \) detector for complete measurement of diverse differential cross sections. At present, a heavy ion cooler-storage ring HIRFL-CSR—an accelerator system of the same beam energy region with maximum incoming-proton kinetic energy up to 2.88 GeV\(^\text{7}\) has already been completed at Lanzhou. With its scheduled 4\( \pi \) hadron detector\(^\text{7}\), it will have a special advantage for studying excited nucleon states through \( pp \) collisions.

Recently, BES collaboration has produced quite a few novel findings on \( N^* \) resonances by using various \( N^* \) production processes from \( J/\psi \) or \( \psi' \) decays\(^8\). Measurements of the \( J/\psi \rightarrow p\pi^-\bar{n} + \text{c.c.} \) decay by BES collaboration showed two new, clear \( N^* \) peaks in the \( \pi\pi \) invariant mass spectrum around 1360 MeV/c\(^2\) and 2065 MeV/c\(^2\), respectively\(^9\). Of them the former one was identified as the first direct observation of the \( N^* \)(1440) peak in the \( \pi N \) invariant mass spectrum, which was confirmed by the CELSIUS-WASA Collaboration in their \( n\pi\pi \) invariant mass spectrum of \( pp \rightarrow np\pi^+ \) reaction\(^\text{13}\). For the latter one, it is very likely to be a long-sought missing \( N^* \) peak around 2 GeV/c\(^2\). However, similar searches for it in \( \psi' \) decays are inconclusive\(^11\). Therefore, it is of necessity to look for the new \( N^* \) resonance in other reaction processes, such as the \( pp \rightarrow pn\pi^+ \) reaction.

Furthermore, in Ref.\(^\text{14}\), the authors found that the \( \Delta^{++} \)(1620) resonance gives an overwhelmingly large contribution in the \( pp \rightarrow nK^+\Sigma^+ \) reaction by t-channel \( \rho^+ \) exchange. If so, it is also expected to make a significant contribution in the \( pp \rightarrow pn\pi^+ \) reaction, as can be checked in the present work. In Ref.\(^\text{15}\), we have studied the \( pp \rightarrow pn\pi^+ \) reaction for beam energies below 1.3 GeV. Here we extend the study of this reaction to higher energies and investigate individual contributions from various \( N^* \) and \( \Delta^* \) resonances with mass up to 2 GeV/c\(^2\) for this reaction. We extend a resonance model, which can describe the experimental data of the total cross section for beam energies ranging from 0.8 GeV to 3.0 GeV quite well, to give theoretical prediction of various differential cross sections for the \( pp \rightarrow pn\pi^+ \) reaction at \( T_p = 2.88 \) GeV. It can be used for the subsequent comparison with the experimental results at COSY and HIRFL-CSR. Meanwhile, it could serve as a reference for the construction of the scheduled 4\( \pi \) hadron detector at HIRFL-CSR.

2. Formalism and ingredients

We study the \( pp \rightarrow pn\pi^+ \) reaction within an effective Lagrangian approach. In our model, all the mesons, baryons and resonances are treated as fundamental fields.
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All the basic Feynman diagrams involved in our calculation for this reaction are depicted in Fig. 1. In view of overall system invariant mass about 3 GeV for $T_p = 2.88$ GeV, we have checked contributions from all the well-established $N^*$ and $\Delta^*$ resonances (overall status 3 or 4 stars) below 2 GeV/$c^2$, but only present the results of the relatively significant ones in next section. Meanwhile, we investigate the contribution from $N^*(2065)$ resonance for the present reaction. Explicitly, we list in Table 1 all the $N^*$, $\Delta^*$ resonances and the meson exchanges considered in our present calculation.

For $N^*(2065)$, according to results in Ref. 9, its spin-parity is limited to be $1/2^+$ and $3/2^+$, and it is more likely that both are needed. In the quark model there are predictions for the existence of $N^*$ resonances with spin-parity $1/2^+$ and $3/2^+$ between 2.0 and 2.1 GeV/$c^2$ [14,15]. Since the spin-parity of the new resonance(s)
was not well determined, we assume that this peak consists of exactly those resonances with \( J^p = 1/2^+, 3/2^+ \) predicted in Ref. 5, which are \( N^*(1975)(J^p = 1/2^+) \), \( N^*(2030)(J^p = 3/2^+) \) and \( N^*(2065)(J^p = 1/2^+) \). Among them \( N^*(2065) \) has much stronger coupling to \( \pi N \) than the other two, which is in accord with BES results. As in Ref. 16, here we also treat the observed \( N^*(2065) \) peak as an effective \( N^*(2065)(J^p = 1/2^+) \) resonance which represents all contributions of the three resonances \( N^*(1975)(J^p = 1/2^+) \), \( N^*(2030)(J^p = 3/2^+) \) and \( N^*(2065)(J^p = 1/2^+) \). In so doing, the coupling constant \( g^{2\pi N^*N(2065)}_{\pi N} / 4\pi \) is scaled by a factor of 1.122. See Ref. 16 for details of this effective treatment. Of course, we have used the BES observed values for the mass and width of \( N^*(2065) \) to determine its relevant coupling constant, see Table I. In view of scanty information for its decay branching fractions, here we regard \( N\pi \) as the dominant decay mode of \( N^*(2065) \), whereas the \( N^*(2065) \) peak in invariant mass \( M_{\pi\pi} \) spectrum is so strong and highly significant. So, in our calculations we have taken an artificial branching ratio up to 1 for \( N\pi \) decay mode.

The effective Lagrangian densities involved for describing the meson-\( NN \) vertices are:

\[
\mathcal{L}_{\pi NN} = -\frac{f_{\pi NN}}{m_\pi} \overline{u}_N \gamma_5 \gamma_\mu \vec{\tau} \cdot \partial^\mu \psi_\pi u_N, \tag{1}
\]

\[
\mathcal{L}_{\eta NN} = -ig_{\eta NN} \overline{u}_N \gamma_5 \psi_\eta u_N, \tag{2}
\]

\[
\mathcal{L}_{\sigma NN} = g_{\sigma NN} \overline{u}_N \psi_\sigma u_N, \tag{3}
\]

\[
\mathcal{L}_{\rho NN} = -g_{\rho NN} \overline{u}_N (\gamma_\mu + \frac{\kappa}{2m_N} \sigma_{\mu\nu} \partial^\nu) \vec{\tau} \cdot \psi_\rho u_N. \tag{4}
\]

At each vertex a relevant off-shell form factor is used. In our computation, we take the same form factors as used in the well-known Bonn potential model 17:

\[
F_{MN}(k_M^2) = \left( \frac{\Lambda^2}{\Lambda^2 - k^2_M} \right)^n \tag{5}
\]

with \( n=1 \) for \( \pi, \eta \) and \( \sigma \) mesons and \( n=2 \) for \( \rho \) meson. \( k_M, m_M \) and \( \Lambda_M \) are the 4-momenta, mass and cut-off parameter for the exchanged meson \( (M) \), respectively. The coupling constants and the cut-off parameters are taken as the following ones 14,15,17,18,19,20:

\[
g_{\pi NN}^2/4\pi = 14.4, \quad g_{\eta NN}^2/4\pi = 0.4, \quad \Lambda_\pi = \Lambda_\eta = 1.3 \text{ GeV}, \quad g_{\sigma NN}^2/4\pi = 5.69, \quad \Lambda_\sigma = 2.0 \text{ GeV}, \quad g_{\rho NN}^2/4\pi = 0.9, \quad \Lambda_\rho = 1.85 \text{ GeV}, \quad \kappa = 6.1.
\]

Note that the constant \( g_{\pi NN} \) is related to \( f_{\pi NN} \) of Eq. (1) as \( g_{\pi NN} = (f_{\pi NN}/m_\pi)2m_N \) 20.

To calculate the amplitudes of diagrams in Fig. 1 within the resonance model, we also need to know interaction vertices involving \( N^* \) and \( \Delta^* \) resonances. In Ref. 21 a Lorentz covariant orbital-spin scheme for \( N^*NM \) couplings has been described in detail, which can be easily extended to describe all the couplings that appear.
in the Feynman diagrams depicted in Fig. 1. By using that scheme, we can easily
obtain the effective couplings:

\[
\mathcal{L}_{\pi N \Delta(1232)} = g_{\Delta(1232)N} \frac{g^{\mu} \gamma^{\nu}}{q^2} \psi^\mu u_{\Delta(1232)}(1232)^\mu + h.c.,
\]

\[
\mathcal{L}_{\pi NN^*(1440)} = g_{N^*(1440)NN} \frac{g^{\mu} \gamma^{\nu}}{q^2} \psi^\mu u_{N^*(1440)} + h.c.,
\]

\[
\mathcal{L}_{\sigma NN^*(1440)} = g_{N^*(1440)NN} \frac{g^{\mu} \gamma^{\nu}}{q^2} \psi^\mu u_{N^*(1440)} + h.c.,
\]

\[
\mathcal{L}_{\pi NN^*(1520)} = g_{N^*(1520)NN} \frac{g^{\mu} \gamma^{\nu}}{q^2} \psi^\mu u_{N^*(1520)} + h.c.,
\]

\[
\mathcal{L}_{\rho NN^*(1520)} = g_{N^*(1520)NN} \psi^\mu u_{N^*(1520)} + h.c.,
\]

\[
\mathcal{L}_{\pi NN^*(1535)} = g_{N^*(1535)NN} \psi^\mu u_{N^*(1535)} + h.c.,
\]

\[
\mathcal{L}_{\rho NN^*(1535)} = g_{N^*(1535)NN} \psi^\mu u_{N^*(1535)} + h.c.,
\]

\[
\mathcal{L}_{\pi NN^*(1600)} = g_{N^*(1600)NN} \frac{g^{\mu} \gamma^{\nu}}{q^2} \psi^\mu u_{N^*(1600)} + h.c.,
\]

\[
\mathcal{L}_{\pi NN^*(1620)} = g_{N^*(1620)NN} \psi^\mu u_{N^*(1620)} + h.c.,
\]

\[
\mathcal{L}_{\rho NN^*(1620)} = g_{N^*(1620)NN} \frac{g^{\mu} \gamma^{\nu}}{q^2} \psi^\mu u_{N^*(1620)} + h.c.,
\]

\[
\mathcal{L}_{\pi NN^*(1650)} = g_{N^*(1650)NN} \psi^\mu u_{N^*(1650)} + h.c.,
\]

\[
\mathcal{L}_{\rho NN^*(1650)} = g_{N^*(1650)NN} \psi^\mu u_{N^*(1650)} + h.c.,
\]

\[
\mathcal{L}_{\pi NN^*(1700)} = g_{N^*(1700)NN} \frac{g^{\mu} \gamma^{\nu}}{q^2} \psi^\mu u_{N^*(1700)} + h.c.,
\]

\[
\mathcal{L}_{\rho NN^*(1700)} = g_{N^*(1700)NN} \psi^\mu u_{N^*(1700)} + h.c.,
\]

\[
\mathcal{L}_{\pi NN^*(1700)} = g_{N^*(1700)NN} \frac{g^{\mu} \gamma^{\nu}}{q^2} \psi^\mu u_{N^*(1700)} + h.c.,
\]

\[
\mathcal{L}_{\rho NN^*(1700)} = g_{N^*(1700)NN} \psi^\mu u_{N^*(1700)} + h.c.,
\]

\[
\mathcal{L}_{\pi NN^*(1710)} = g_{N^*(1710)NN} \frac{g^{\mu} \gamma^{\nu}}{q^2} \psi^\mu u_{N^*(1710)} + h.c.,
\]

\[
\mathcal{L}_{\rho NN^*(1710)} = g_{N^*(1710)NN} \psi^\mu u_{N^*(1710)} + h.c.,
\]

\[
\mathcal{L}_{\pi NN^*(1720)} = g_{N^*(1720)NN} \frac{g^{\mu} \gamma^{\nu}}{q^2} \psi^\mu u_{N^*(1720)} + h.c.,
\]

\[
\mathcal{L}_{\rho NN^*(1720)} = g_{N^*(1720)NN} \psi^\mu u_{N^*(1720)} + h.c.,
\]

\[
\mathcal{L}_{\pi NN^*(1905)} = g_{N^*(1905)NN} \frac{g^{\mu} \gamma^{\nu}}{q^2} \psi^\mu u_{N^*(1905)} + h.c.,
\]

\[
\mathcal{L}_{\rho NN^*(1905)} = g_{N^*(1905)NN} \psi^\mu u_{N^*(1905)} + h.c.,
\]

\[
\mathcal{L}_{\pi NN^*(1910)} = g_{N^*(1910)NN} \frac{g^{\mu} \gamma^{\nu}}{q^2} \psi^\mu u_{N^*(1910)} + h.c.,
\]

\[
\mathcal{L}_{\rho NN^*(1910)} = g_{N^*(1910)NN} \psi^\mu u_{N^*(1910)} + h.c.,
\]

\[
\mathcal{L}_{\pi NN^*(1920)} = g_{N^*(1920)NN} \frac{g^{\mu} \gamma^{\nu}}{q^2} \psi^\mu u_{N^*(1920)} + h.c.,
\]

\[
\mathcal{L}_{\rho NN^*(1920)} = g_{N^*(1920)NN} \psi^\mu u_{N^*(1920)} + h.c.,
\]

\[
\mathcal{L}_{\pi NN^*(1930)} = g_{N^*(1930)NN} \frac{g^{\mu} \gamma^{\nu}}{q^2} \psi^\mu u_{N^*(1930)} + h.c.,
\]

\[
\mathcal{L}_{\rho NN^*(1930)} = g_{N^*(1930)NN} \psi^\mu u_{N^*(1930)} + h.c.,
\]

\[
\mathcal{L}_{\pi NN^*(2065)} = g_{N^*(2065)NN} \psi^\mu u_{N^*(2065)} + h.c.,
\]
For the relevant vertices involving $N^*$ and $\Delta^*$ resonances, the off-shell form factors are adopted as follows:

$$F_M(k_M^2) = \left( \frac{\Lambda^*_M^2 - m_{M^*}^2}{\Lambda^*_M^2 - k_M^2} \right)^n$$

(37)

where $n=1$ for all the resonances except for $n=2$ for $\Delta(1232)$. All the coupling constants and cut-off parameters used in the present paper are listed in Table I.

In addition, we also introduce form factors for the off-shell baryon resonances as in Refs. 22, 23, 24

$$F_R(q) = \frac{\Lambda^4}{\Lambda^4 + (q^2 - m_R^2)^2},$$

(38)

with $\Lambda = 0.8$ GeV.

The propagators can be written as

$$G(q) = \frac{\not{q} + m_R}{q^2 - m_R^2 + im_R \Gamma_R}$$

(39)

for the spin-\(\frac{1}{2}\) resonances,

$$G_{\mu\nu}(q) = \frac{-P_{\mu\nu}(q)}{q^2 - m_R^2 + im_R \Gamma_R}$$

(40)

with

$$P_{\mu\nu}(q) = -(\not{q} + m_R)[g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3m_R}(\gamma_\mu q_\nu - \gamma_\nu q_\mu) - \frac{2}{3m_R^2} q_\mu q_\nu],$$

(41)

for the spin-\(\frac{3}{2}\) resonances, and

$$G_{\mu\nu\alpha\beta}(q) = \frac{-P_{\mu\nu\alpha\beta}(q)}{q^2 - m_R^2 + im_R \Gamma_R}$$

(42)

with

$$P_{\mu\nu\alpha\beta}(q) = - (\not{q} + m_R)[\frac{1}{2}(\tilde{g}_{\mu\alpha}\tilde{g}_{\nu\beta} + \tilde{g}_{\mu\beta}\tilde{g}_{\nu\alpha}) - \frac{1}{5} \tilde{g}_{\mu\nu} \tilde{g}_{\alpha\beta}$$

$$+ \frac{1}{10}(\tilde{\gamma}_\mu \tilde{\gamma}_\alpha \tilde{g}_{\nu\beta} + \tilde{\gamma}_\nu \tilde{\gamma}_\beta \tilde{g}_{\mu\alpha} + \tilde{\gamma}_\mu \tilde{\gamma}_\beta \tilde{g}_{\nu\alpha} + \tilde{\gamma}_\nu \tilde{\gamma}_\alpha \tilde{g}_{\mu\beta})],$$

(43)

for the spin-\(\frac{5}{2}\) resonances.

After the effective Lagrangians, coupling constants and propagators fixed, the amplitudes for various diagrams can be written down straightforwardly by following the Feynman rules. And the total amplitude is just their simple sum. Here we give...
Table 1. Relevant parameters of $N^*$ and $\Delta^*$ included in our calculations. The widths and branching ratios are taken from PDG\textsuperscript{6} and the cut-off parameters are from Refs.\textsuperscript{14,15,18,25}. Here the $g^2/4\pi$ for $N^*(2065)N\pi$ vertex has already been scaled by a factor of 1.122.

| Resonance   | Width/GeV | Decay mode | Branching ratio | $g^2/4\pi$ | Cut-off/GeV |
|-------------|-----------|------------|-----------------|------------|-------------|
| $\Delta(1232)$ | 0.118     | $N\pi$    | 1.0             | 19.54     | 0.6         |
| $N^*(1440)$  | 0.3       | $N\pi$    | 0.65            | 0.51      | 1.3         |
|             |           | $N\sigma$ | 0.075           | 3.20      | 1.1         |
| $N^*(1520)$  | 0.115     | $N\pi$    | 0.6             | 1.73      | 0.8         |
|             |           | $N\rho$   | 0.09            | 1.32      | 0.8         |
| $N^*(1535)$  | 0.15      | $N\pi$    | 0.45            | 0.037     | 1.3         |
|             |           | $N\eta$   | 0.525           | 0.34      | 1.3         |
|             |           | $N\rho$   | 0.02            | 0.097     | 1.3         |
| $\Delta^*(1600)$ | 0.35   | $N\pi$    | 0.175           | 1.09      | 0.8         |
| $\Delta^*(1620)$ | 0.145  | $N\pi$    | 0.25            | 0.06      | 1.3         |
|             |           | $N\rho$   | 0.14            | 0.37      | 1.3         |
| $N^*(1650)$  | 0.165     | $N\pi$    | 0.775           | 0.06      | 0.8         |
|             |           | $N\eta$   | 0.065           | 0.026     | 0.8         |
|             |           | $N\rho$   | 0.02            | 0.015     | 0.8         |
| $N^*(1675)$  | 0.15      | $N\pi$    | 0.4             | 2.16      | 0.8         |
| $N^*(1680)$  | 0.13      | $N\pi$    | 0.675           | 5.53      | 0.8         |
| $N^*(1700)$  | 0.1       | $N\pi$    | 0.1             | 0.075     | 0.8         |
|             |           | $N\rho$   | 0.07            | 0.043     | 0.8         |
| $\Delta^*(1700)$ | 0.3     | $N\pi$    | 0.15            | 1.02      | 0.8         |
|             |           | $N\rho$   | 0.125           | 0.69      | 0.8         |
| $N^*(1710)$  | 0.1       | $N\pi$    | 0.15            | 0.012     | 0.8         |
|             |           | $N\eta$   | 0.062           | 0.042     | 0.8         |
|             |           | $N\sigma$ | 0.25            | 0.085     | 1.1         |
| $N^*(1720)$  | 0.2       | $N\pi$    | 0.15            | 0.12      | 0.8         |
|             |           | $N\eta$   | 0.04            | 0.28      | 0.8         |
| $\Delta^*(1905)$ | 0.33   | $N\pi$    | 0.12            | 1.74      | 0.8         |
| $\Delta^*(1910)$ | 0.25  | $N\pi$    | 0.225           | 0.076     | 0.8         |
|             |           | $N\rho$   | 0.37            | 0.29      | 0.8         |
| $\Delta^*(1920)$ | 0.2     | $N\pi$    | 0.125           | 0.18      | 0.8         |
| $\Delta^*(1930)$ | 0.36  | $N\pi$    | 0.1             | 1.06      | 0.8         |
| $N^*(2065)$  | 0.165     | $N\pi$    | 0.057           | 0.37      | 1.3         |

explicitly the individual amplitude corresponding to $N^*(1440)(\pi^0$ exchange), as an example,

$$
\mathcal{M}(N^*(1440), \pi^0) = \sqrt{2} \frac{f_{\pi NN}}{m_\pi} g_{N^*N\pi} \bar{u}_n(p_n, s_n)\gamma_5 \slashed{p}_\pi G^N(1440)(q)\gamma_5 \bar{k}_\pi u_1(p_1, s_1) \frac{i}{k_{\pi}^2 - m_\pi^2} \bar{u}_3(p_3, s_3)\gamma_5 \slashed{k}_\pi u_2(p_2, s_2) + (\text{exchange term with } p_1 \leftrightarrow p_2),
$$

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where $u_{n}(p_{n},s_{n})$, $u_{3}(p_{3},s_{3})$, $u_{1}(p_{1},s_{1})$, $u_{2}(p_{2},s_{2})$ denote the spin wave functions of the outgoing neutron, proton in the final state and two initial protons, respectively. $p_{\pi}$ and $k_{\pi}$ are the 4-momenta of the outgoing and the exchanged pion mesons. $p_{1}$ and $p_{2}$ represent the 4-momenta of the two initial protons. The coupling constant appearing herein can be determined from the experimentally observed partial decay width of $N^{*}(1440)$ resonance as follows,

$$\Gamma_{N^{*}(1440)\rightarrow N\pi} = \frac{3g_{N^{*}(1440)N\pi}^{2}}{4\pi} \frac{m_{\pi}^{2}(E_{N} - m_{N})}{m_{N^{*}(1440)}} + 2(p_{cm}^{N})^{2},$$

(47)

with

$$p_{cm}^{N} = \sqrt{\frac{(m_{N^{*}(1440)}^{2} - m_{N}^{2})^{2} - (m_{N} + m_{\pi})^{2}(m_{N}^{2} - m_{\pi}^{2})}{4m_{N^{*}(1440)}^{2}}},$$

(48)

$$E_{N} = \sqrt{(p_{cm}^{N})^{2} + m_{N}^{2}}.$$

(49)

All the other coupling constants can be obtained similarly. See Refs. 14, 15 for details.

Then the calculation of the cross section $\sigma(pp \rightarrow pm\pi^{+})$ is straightforward,

$$d\sigma(pp \rightarrow pm\pi^{+}) = \frac{1}{4F} \sum_{s_{i},s_{f}} |M|^{2} \frac{m_{p}d^{2}p_{3}d^{2}p_{\pi}m_{n}d^{2}p_{n}}{2E_{3}E_{\pi}E_{n}} \delta^{4}(p_{1} + p_{2} - p_{3} - p_{\pi} - p_{n})$$

(50)

with the flux factor

$$F = (2\pi)^{5} \sqrt{(p_{1} \cdot p_{2})^{2} - m_{p}^{4}}.$$

(51)

The factors 1/4 and $\sum_{s_{i},s_{f}}$ emerge for the simple reason that the polarization of initial and final particles is not considered.

3. Numerical results and discussion

With the formalism and ingredients discussed in the former section, we computed the total cross section versus the kinetic energy of the proton beam ($T_{P}$) for the $pp \rightarrow pm\pi^{+}$ reaction by using a Monte Carlo multi-particle phase space integration program. The results for $T_{P}$ ranging from 0.8 to 3.0 GeV are shown in Fig. 2 along with experimental data 26 for comparison.

As one can see from Fig. 2, the experimental data of total cross section are reproduced reasonably well by our theoretical calculations over the entire energy range. Note that we have considered the interference terms between the direct amplitudes (diagram a,c in Fig. 1) and the corresponding exchange amplitudes (diagram b,d in Fig. 1) in our calculations. However, the interference terms between different resonance-excitation processes and between various meson-exchange diagrams are ignored. We also show contributions of various components which are large and not
Fig. 2. Total cross section and contributions from various $N^*$ (left) and $\Delta^*$ (right) resonances as a function of $T_P$ for the $pp \rightarrow pn\pi^+$ reaction with the solid line as the simple incoherent sum of all contributions, compared with data. Left: the dashed, dotted, dot-dashed, dot-dot-dashed, short-dashed and short-dotted lines represent individual contributions from $N^*(1440)$, $N^*(1520)$, $N^*(1650)$, $N^*(1675)$, $N^*(1680)$, and $N^*(2065)$, respectively. Right: the dashed, dotted, dot-dashed, dot-dot-dashed, and short-dashed lines represent individual contributions from $\Delta(1232)$, $\Delta^*(1600)$, $\Delta^*(1620)$, $\Delta^*(1700)$, and $\Delta^*(1905)$, respectively.

negligible there, $N^*$ contributions in Fig. 2 (left) and $\Delta^*$ contributions in Fig. 2 (right), respectively. Individual contributions from $N^*(1440)$, $N^*(1520)$, $N^*(1650)$, $N^*(1675)$, $N^*(1680)$, and $N^*(2065)$ are presented in Fig. 2 (left) by dashed, dotted, dot-dashed, dot-dot-dashed, short-dashed and short-dotted lines, respectively. And contributions from $\Delta(1232)$, $\Delta^*(1600)$, $\Delta^*(1620)$, $\Delta^*(1700)$, and $\Delta^*(1905)$ are shown in Fig. 2 (right) by dashed, dotted, dot-dashed, dot-dot-dashed, and short-dashed lines, respectively. One can find that contributions from $\Delta(1232)$ and $N^*(1440)$ are still dominant in present energy region and the contribution from $N^*(1680)$ becomes significant for kinetic energy above 2.0 GeV. We also give our predictions of invariant mass spectra and Dalitz plot in Fig. 3 and the momentum and angular distributions of the final charged particles in Fig. 4 for $pp \rightarrow pn\pi^+$ reaction at $T_P = 2.88$ GeV.

The $pp \rightarrow pn\pi^+$ reaction is a channel with the largest total cross section for pp collision in the present energy region. Since the kinetic energy $T_P$ of the proton beam at HIRFL-CSR can reach 2.88 GeV with luminosity above $10^{32}$ cm$^{-2}$s$^{-1}$, the event rate will be so large that it is easy to collect enough events and to get high statistics data for this channel at HIRFL-CSR. The theoretical prediction of various observables given in this work would provide us more knowledge on the relevant physics when compared to the future experimental results. Meanwhile, a good measurement of invariant mass spectra and Dalitz plot will play the key role for identifying new resonances and determining their parameters. In this regard, the scheduled 4$\pi$ hadron detector at HIRFL-CSR will be particularly competent.

As mentioned above, the spin-parity of the $N^*(2065)$ peak was not well deter-
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Fig. 3. Theoretical prediction of invariant mass spectra and Dalitz plot for $pp \rightarrow pn\pi^+$ reaction at $T_p = 2.88$ GeV. The dashed curves stand for pure phase space distribution while the solid curves include the interaction amplitudes.

mined by the BES collaboration, therefore we have used an effective treatment for the $N^*(2065)$ resonance(s) as the authors did in Ref. [16]. This effective description would indeed be a very good approximation for the total cross section, but generally speaking, it might reproduce the differential cross sections not so well. However, due to the dominance of $N^*(2065)$ (its magnitude is much stronger than the other two) among the three resonances predicted in Ref. [5] hence even for the description of various differential cross sections, it would be acceptable. Of course, this issue still waits for an exact answer from future experimental results at HIRFL-CSR.

To sum up, in this paper we investigate individual contributions from diverse $N^*$ and $\Delta^*$ resonances up to 2 GeV$/c^2$ for the $pp \rightarrow pn\pi^+$ reaction. We extend a resonance model, which can describe the observed total cross section for beam
energies ranging from 0.8 GeV to 3.0 GeV quite well, to give theoretical prediction of various differential cross sections for this reaction at $T_p = 2.88$ GeV. It can be used for identifying new physics in the future experiments at HIRFL-CSR. It could also serve as a reference for the construction of the scheduled $4\pi$ hadron detector at HIRFL-CSR, which is quite possible to offer more physical information and to help us understanding the relevant physics better.

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