Incentive Mechanism Design for Emergency Frequency Control in Multi-Infeed Hybrid AC-DC System

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Abstract—In multi-infeed hybrid AC-DC (MIDC) systems, the emergency frequency control (EFC) with LCC-HVDC systems participating is of vital importance for system frequency security. Nevertheless, when regional power systems are operated by different decision-makers, the LCC-HVDC systems and their connected AC systems might be unwilling to participate in the EFC due to the costs and losses. In this paper, to incentivize the LCC-HVDC systems and their connected adjacent AC systems to participate in the droop-based EFC, a novel control-parameter-based incentive mechanism is proposed, which can flexibly deal with various possible power imbalances. Then, a non-cooperative-based incentive game model is formulated to quantitatively analyze the incentive mechanism in the MIDC system. An algorithm for seeking the Nash equilibrium is designed, and the uniqueness of Nash equilibrium is proven. Moreover, the individual rationality, incentive compatibility and social optimality of the proposed mechanism are analyzed and proven. The effectiveness of the proposed incentive mechanism is verified through a case study.

Index Terms—Incentive mechanism, emergency frequency control, multi-infeed hybrid AC-DC system, LCC-HVDC system.

NOMENCLATURE

A. Abbreviations

EFC Emergency frequency control.
MIDC Multi-infeed hybrid AC-DC system.
LCC Line-commutated-converter-based.
AM AC main system.
AD Adjacent AC system.
OAM Optimization problem of AM system.
OAD Optimization problem of AD system.

B. Indices and Sets

i Index of LCC-HVDC/AD systems and generators in AM system.
h Index of generators in AD i system.
j Index of emergency faults.
t Index of iterations in SE-FP algorithm.
D Set of LCC-HVDC/AD systems.
G am Set of generators in AM system.
G ad Set of generators in AD i system.
F Set of emergency faults.
Ω R, Ωγ Feasible set of reward R/virtual price γ.
Ω D i Feasible set of droop coefficient k D i.

C. Variables and Functions

k D i Droop coefficient of LCC-HVDC i.
R Reward provided by AM system.
γ Virtual price provided by AM system.
P D,i Power control order of LCC-HVDC i.
ΔP E i Actual occurring power imbalance.
ΔP G i Power regulation of LCC-HVDC i.
ΔP E,i/h Power regulation of generator i/h.
ω am Expected frequency deviation of AM system.
ω c,i Frequency deviation of AD i system.
ω ad,i Margin response coefficient of AM system.
a,eγ,e k Errors in SE-FP algorithm.
λ Lagrangian multiplier.
S j Variables under emergency fault j.
P E,i Variables corresponding to ΔP E,i.
S E,i Variables corresponding to ΔP E,i.
S ad,i Modified variables and functions in Section III.B.
F am() Disutility function of AM system.
F ad,i() Disutility function of AD i system.
Y() Response function of AM system.

D. Parameters

n D,m Number of LCC-HVDC/SE-LCC systems.
n G Number of generators in G am.
n ad,i Number of generators in G ad,i.
\begin{align*}
& n_F \quad \text{Number of faults in } F. \\
& P_{ij}^D \quad \text{Nominal power of LCC-HVDC } i. \\
& P_{ij}^G \quad \text{Nominal power of generator } il/h. \\
& T \quad \text{Execution cycle of incentive mechanism.} \\
& \Delta P_{Ej} \quad \text{Power imbalance of emergency fault } j. \\
& p_j \quad \text{Fault ratio of emergency fault } j. \\
& \Delta P_{E}^\text{exp} \quad \text{Expected power imbalance.} \\
& \Delta P_{E}^\text{np} \quad \text{Nearest-to-expected power imbalance.} \\
& \alpha_{ij}^\text{e/h} \quad \text{Cost coefficient of generator } il/h. \\
& k_{ij}^\text{e/h} \quad \text{Drop coefficient of generator } il/h. \\
& \varepsilon_{\gamma_1}, \varepsilon_{\text{min}} \quad \text{Error tolerances.} \\
& (\cdot)^* \quad \text{The value of } (\cdot) \text{ at Nash equilibrium.} \\
& (\cdot) \quad \text{The value of } (\cdot) \text{ at social optimum.} \\
& (\cdot)^0 \quad \text{Initial value of } (\cdot) \text{ in SE-FP algorithm.} \\
& (\cdot), (\cdot) \quad \text{Lower/upper bound of } (\cdot). 
\end{align*}

I. INTRODUCTION

A. Motivation

CONVENTIONAL AC power systems have been gradually transformed into complex hybrid AC-DC power systems due to the development of HVDC technologies [1], [2]. In China, the line-commutated-converter-based HVDC (LCC-HVDC) systems [3] are widely applied as the tie-lines between regional grids, which forms asynchronous multi-infeed hybrid AC-DC (MIDC) systems [4]. In an MIDC system, one AC main system is connected with multiple adjacent AC systems through multiple LCC-HVDC systems (refer to Fig. 1 for detailed topology). The complex dynamics of the MIDC system bring challenges to the secure and stable operation.

Considering the frequency security of the MIDC system, the traditional frequency regulation strategies might not meet the frequency security requirements because: 1) The feeding of multiple LCC-HVDC systems might lead to the frequency regulation reserve shortage and inertia shortage of the AC main system. 2) Emergency faults (e.g., HVDC block faults or AC-DC cascading faults) with considerable power imbalances are prone to occur in the MIDC system [5], [6]. Therefore, the MIDC system requires emergency frequency control (EFC) strategy. Compared with the conventional EFC strategies, i.e., the load shedding and generator tripping operations [7], [8], the EFC strategies with LCC-HVDC systems participating are more economic and effective for the MIDC system, which utilize the active power fast adjustability [9] and the short-time overload capability [10] of LCC-HVDC systems.

Considering the EFC strategies in MIDC systems, the existing works focus on how to design EFC strategies to meet various control objectives in engineering practice [11], [12], under the assumption that the entire power system is operated by one decision-maker and all the LCC-HVDC systems and adjacent AC systems are obliged to participate in the EFC of the AC main system. For instance, in our previous work [13], a decentralized droop-based EFC strategy is proposed. Nevertheless, considering the future development and marketization of the power grid [14], [15], the AC main system, LCC-HVDC systems and adjacent AC systems might belong to different decision-makers. In addition, participating in the EFC brings generation costs, reserve costs and frequency deviation losses to the adjacent AC systems, and also brings equipment losses caused by fast power regulation to the LCC-HVDC systems. Therefore, the adjacent AC systems and LCC-HVDC systems might be unwilling to participate in the EFC due to their self-interests, which need incentives from the AC main system. In this paper, to reasonably incentivize the adjacent AC systems and LCC-HVDC systems to participate in the droop-based EFC, a novel control-parameter-based incentive mechanism is proposed. Note that the design idea of the proposed incentive mechanism can be extended to other EFC strategies, and the incentive mechanism can also provide reference for the power grid with single decision-maker.

B. Literature Review

Considering that the EFC strategy in MIDC systems is a special frequency control strategy, we first review the various design ideas of incentive mechanism for frequency control in the existing literature, and the main findings are as follows. In [16], a control performance standard (CPS)-based incentive mechanism is proposed to incentivize power producers to participate in the automatic generation control (AGC). The definition of the CPS index requires synchronous interconnection between the regional grid providing power support and the regional grid with power imbalance [17]. However, considering the EFC in MIDC system, the adjacent AC systems which provide power support are asynchronously connected with the AC main system through LCC-HVDC systems, thus, a CPS-like index cannot be defined in the scenario of MIDC system, and the mechanism in [16] is not applicable. In [18], an incentive mechanism for distributed frequency control implementation is proposed to enforce the system stability, which is based on the Vickrey-Clarke-Groves (VCG) mechanism [19]. Nevertheless, the VCG-based mechanism is not self-budget balancing and extra bonuses are required to balance the budget.

Subsequently, we introduce the relevant works about the frequency regulation reserve market mechanism, since which can be regarded as a reserve-capacity-based incentive mechanism for frequency control [20]. There are two significant steps in the frequency regulation reserve market mechanism, which are reserve capacity determination and reserve market clearing, respectively [21]. The methods for reserve capacity determination mainly include the reliability-based method [22], the conditional-value-at-risk (CVaR) based approach [23] and the method based on cost-benefit analysis [24], [25], etc. And the reserve market clearing methods mainly contain the bilateral-contract-based method [26], the game-based approach [27] and the bidding-based approach [28], etc. The above various methods make the frequency regulation reserve market mechanism widely applied in the engineering practice, but this kind of reserve-capacity-based incentive mechanism must be cleared according to a specific reserve capacity demand. Considering the unpredictable emergency faults with random occurrences and considerable power imbalances, the aforementioned reserve-capacity-based approach usually conservatively selects the severest fault as the reserve capacity demand for market clearing, which causes economic losses and waste of reserve.
resources. Therefore, the less conservative incentive mechanisms which can flexibly deal with various power imbalances are urgently required.

Moreover, considering the incentive mechanisms for the demand side with frequency regulation potential, in [29], a user-oriented double-incentive mechanism is proposed to improve the reserve service capability of electric vehicles (EVs), in which the semi-managed response mode effectively simplifies the response process. In [30], [31], the co-optimization method for demand response aggregators to participate in the energy and reserve market is studied. From the perspective of individual frequency regulation resource, the above works focus on the optimal decision-making and operation methods for the demand side resources when involved in the frequency regulation market, so as to incentivize them to participate in the frequency regulation. Nevertheless, in the incentive mechanism for EFC, which is to be studied in this paper, in addition to considering the individual optimal decision-making of the frequency regulation resources participating in EFC, several system-level constraints (e.g., the frequency security constraints) should also be considered to guarantee the effectiveness of the incentive mechanism in case of emergency faults. However, these system-level constraints are barely considered in the above research.

C. Contribution

According to the literature review, to design a incentive mechanism which is applicable to the droop-based EFC of asynchronous MIDC systems, the key challenge is how to enable the incentive mechanism to deal with various unpredictable active power imbalances caused by emergency faults, with frequency security constraints considered. In this paper, a control-parameter-based idea is utilized to design the incentive mechanism for droop-based EFC, and a non-cooperative game model (i.e., the incentive game) is formulated, with frequency security constraints and frequency regulation constraints considered. Then, an algorithm for seeking the Nash equilibrium of the incentive game is designed for practical engineering applications, and the uniqueness of the equilibrium is proven.

D. Organization

The rest of this paper is organized as follows. Section II introduces the control-parameter-based incentive mechanism for droop-based EFC strategy. Section III formulates and analyzes the incentive game model. Section IV discuss the properties of the proposed incentive mechanism. In Section V, an MIDC system case is tested and the effectiveness of the proposed incentive mechanism is verified. Section VI concludes this paper.

II. INCENTIVE MECHANISM FOR DROOP-BASED EMERGENCY FREQUENCY CONTROL

In this section, we first introduce some preliminaries on the droop-based EFC strategy in the MIDC system. Then, the control-parameter-based incentive mechanism for droop-based EFC strategy is proposed.

Generally, the topology of an asynchronous-interconnected MIDC system is shown in Fig. 1, where the AC main system (AM system for short) is connected with \( n_D \) LCC-HVDC systems.
Specifically, there are \( m \) LCC-HVDC systems transmitting power from sending-end (SE) systems to the AM system while \( (n_D - m) \) LCC-HVDC systems transmitting power from the AM system to receiving-end (RE) systems, which are called SE-LCC systems and RE-LCC systems respectively. The SE \( 1 \sim m \) and RE \( (m+1) \sim n_D \) systems are collectively called the adjacent AC systems (AD systems for short). For the topology of the MIDC system, we make the following assumptions:

**Assumption 1:** 1) One AD system is only connected with one LCC-HVDC system. In engineering practice, there exist multiple LCC-HVDC systems connected to the same AD system, and we can make them equivalent to one LCC-HVDC system. 2) The AD systems operate asynchronously, i.e., there is no AC tie-line between any two AD systems. In fact, if there exist AC tie-lines between two AD systems, we can regard these two systems as one AD system.

The set of LCC-HVDC systems is denoted by \( \mathcal{D} \), which can also represents the set of AD systems due to Assumption 1. The set of generators in the AM system is denoted by \( \mathcal{G}^\text{am} \), and the set of generators in AD \( i \) system is denoted by \( \mathcal{G}_i^\text{ad} \), where \( i \in \mathcal{D} \). We denote the numbers of generators in the above sets with \( n^\text{am}_i \) and \( n^\text{ad}_{Gi} \), respectively. Based on the described MIDC system, we design the incentive mechanism for droop-based EFC strategy.

### A. Preliminaries on Droop-Based EFC Strategy

The authors have proposed a decentralized droop-based EFC strategy to address the emergency frequency problems in the MIDC system [13], where the active power-frequency (P-f) droop control for the LCC-HVDC system is designed, and a coordinated droop mechanism is introduced to ensure that the LCC-HVDC droop control only works in emergency situations as supplementary support for conventional generators’ primary frequency control. Moreover, considering the designed EFC strategy, a Lyapunov-function-based stability analysis illustrates the asymptotic stability of the closed-loop equilibrium. Benefiting from the decentralized design logic, the droop-based EFC strategy guarantees a fast response in case of emergency faults, and can easily be applied to practical engineering projects. For more details about the droop-based EFC strategy, we refer to [13]. The P-f droop control equation of LCC-HVDC \( i \) is shown as follows:

\[
P^D_{\text{ord},i} = P^D_i - k^D_i \omega^am, \quad i \in \mathcal{D}
\]

where \( P^D_i \) is the nominal active power of LCC-HVDC \( i \), which is positive for SE-LCC or negative for RE-LCC, \( P^D_{\text{ord},i} \) is the active power control order of LCC-HVDC \( i \), \( k^D_i > 0 \) is the droop coefficient of LCC-HVDC \( i \), and \( \omega^am \) is the frequency deviation from the nominal frequency of the AM system.

In the scenario where the AM system and AD systems are operated by different decision-makers, supposing that the AM system has requirements for the steady-state frequency after emergency faults, i.e., \( \omega^am \) is a given constant, and ignoring the upper and lower bounds of \( P^D_{\text{ord},i} \), the power support of LCC-HVDC \( i \) under the designed EFC strategy is \( \Delta P^D_i = -k^D_i \omega^am \), which is proportional to \( k^D_i \). Thus, the contribution of LCC-HVDC \( i \) to the frequency security of the AM system can be characterized by its droop coefficient \( k^D_i \). Then, we design the control-parameter-based incentive mechanism in the following subsection, and further explain that the design idea can be extended to various EFC strategies.

### B. Control-Parameter-Based Incentive Mechanism Design

In the design of incentive mechanism, we first make the following assumption:

**Assumption 2:** 1) Considering the consistency between AD \( i \) system and its connected LCC-HVDC \( i \) in decision-making and utility, we regard them as one decision-maker, and represent this decision-maker as AD \( i \) in the mechanism design. 2) The LCC-HVDC systems and generators can provide enough power support to cover the power imbalances caused by emergency faults, and so the load shedding operations are ignored in the mechanism design. 3) The impact of the voltage and reactive power is ignored in the mechanism design, which is reasonable for the transmission networks [32], [33].

In this paper, we focus on how to incentivize the AD systems to participate in the EFC and implement the designed LCC-HVDC droop control. Different from the reserve-capacity-based incentive mechanism in the existing literature, a control-parameter-based incentive mechanism is proposed in this section. The main idea of the incentive mechanism is that the AD system with more contribution to the system frequency security obtains more amount of reward. Since the contribution can be characterized by the droop coefficient, in the proposed incentive mechanism, the AM system provides a total reward \( R \) to the AD systems, and the allocated reward to each AD system is proportional to the droop coefficient of its connected LCC-HVDC system. The proposed incentive mechanism for droop-based EFC is shown in Fig. 2, which follows these steps below.

1. **Step 1. Emergency Fault Set Selection:** The AM system first selects the set of emergency faults \( \mathcal{F} \) by enumeration approach. The set \( \mathcal{F} \) is supposed to contain all the concerned emergency faults, e.g., the HVDC block faults and the generator tripping faults, and the number of faults in \( \mathcal{F} \) is denoted by \( n_F \). The power imbalances corresponding to the faults in \( \mathcal{F} \) are \( \{ \Delta P_{E_j} \}, j \in \mathcal{F} \), where \( \Delta P_{E_j} > 0 \) for power shortage or \( \Delta P_{E_j} < 0 \) for power redundancy. Then, the fault ratios \( \{ p_j, j \in \mathcal{F} \} \) (defined as the ratio of the occurrence times of fault \( j \) to the total occurrence times of all faults in \( \mathcal{F} \)) can be estimated via the historical data. We have \( \sum_{j \in \mathcal{F}} p_j = 1 \). The execution cycle \( T \) of the designed mechanism is the average time period of one emergency fault occurrence.

2. **Step 2. Incentive Game Formulation and Solution:** For each \( \Delta P_{E_j} \) in \( \mathcal{F} \), the corresponding reward \( R_j \) for AD systems and the corresponding droop coefficients \( \{ k^D_{ij} \}, i \in \mathcal{D} \) for LCC-HVDC systems can be determined by the Nash equilibrium of a game among the AM system and AD systems, which is called the incentive game. The incentive game is organized by a supporting platform. The formulation and solution of the incentive game are introduced in detail in Section III, where the frequency regulation constraints and frequency security constraints are considered, and an algorithm for seeking the Nash equilibrium is proposed. Then, by respectively connecting the scatter points...
Several points worth mentioning about the designed incentive mechanism for EFC:

1. Considering the advantages of the designed mechanism, since the droop coefficients of LCC-HVDC systems can be adjusted immediately according to the practical power imbalance, the proposed incentive mechanism can flexibly deal with various possible power imbalances. Moreover, in the proposed mechanism, the short-time overload capability of LCC-HVDC systems is utilized to provide power support for EFC; in normal conditions, LCC-HVDC systems have no need to reserve capacity, which ensures the operation economy. Note that the typical short-time overload capability of LCC-HVDC systems is approximately 1.1-1.2 p.u. [37, 38, 39]; thus, due to the large transmission power of LCC-HVDC systems, this short-term overload capability is considerable in the MIDC system.

2. Besides the droop-based EFC, after some modifications, the control-parameter-based design idea for incentive mechanism can be extended to other types of EFC strategies. The modifications mainly include: (i) An index characterizing the controller’s contribution to system frequency security should be defined according to the control parameters of the EFC strategy. (ii) In the following incentive game model, the expression of the power regulation of the LCC-HVDC system needs to be modified according to the EFC strategy.

3. Although the reward allocation rule of the proposed mechanism is based on the droop coefficient, we also consider the fault ratios \{p_j\} in Fig. 3 are \(R^n\) and \(\{k^{Dn}\}\), respectively. At the start of the incentive mechanism execution, the AM system pre-pays the reward \(R^n\) to the AD systems. Then, the AD systems implement the designed droop-based EFC strategy in the LCC-HVDC systems in advance and set their droop coefficients with \(\{k^{Dn}\}\), and allocate the reward \(R^n\) in proportion to the droop coefficients.

**Remark 1:** Several points worth mentioning about the designed incentive mechanism for EFC:

1. Considering the advantages of the designed mechanism, since the droop coefficients of LCC-HVDC systems can be adjusted immediately according to the practical power imbalance, the proposed incentive mechanism can flexibly deal with various possible power imbalances. Moreover, in the proposed mechanism, the short-time overload capability of LCC-HVDC systems is utilized to provide power support for EFC; in normal conditions, LCC-HVDC systems have no need to reserve capacity, which ensures the operation economy. Note that the typical short-time overload capability of LCC-HVDC systems is approximately 1.1-1.2 p.u. [37, 38, 39]; thus, due to the large transmission power of LCC-HVDC systems, this short-term overload capability is considerable in the MIDC system.

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3. Although the reward allocation rule of the proposed mechanism is based on the droop coefficient, we also consider

((\Delta P_{Ej}, R_j), j \in \mathcal{F}) and (\Delta P_{Ej}, k^{Dj}), j \in \mathcal{F} in turn, the \Delta P_{E-R} curve and the \Delta P_{E-\{k^D\}} curves can be derived, as shown in Fig. 3. Based on the above curves, the following pre-payment and real-time adjustment are carried out.

**Step 3. Pre-Payment for Control Implementation:** Base on the fault ratios \{p_j\}, we define the expected power imbalance as \(\Delta P_{E}^{ex} = \sum_{j \in \mathcal{F}} (p_j \Delta P_{Ej})\), and the nearest-to-expected value of power imbalance \(\Delta P_{E}^{n} \in \mathcal{F}\) is defined as the solution of the following optimization problem, i.e.

\[
\Delta P_{E}^{n} = \arg \min_{\Delta P_{Ej} \in \mathcal{F}} |\Delta P_{Ej} - \Delta P_{E}^{ex}|
\]

The reward and droop coefficients corresponding to \(\Delta P_{E}^{n}\) in Fig. 3 are \(R^n\) and \(\{k^{Dn}\}\), respectively. At the start of the incentive mechanism execution, the AM system pre-pays the reward \(R^n\) to the AD systems. Then, the AD systems implement the designed droop-based EFC strategy in the LCC-HVDC systems in advance and set their droop coefficients with \(\{k^{Dn}\}\), and allocate the reward \(R^n\) in proportion to the droop coefficients.

**Step 4. Real-Time Adjustment of Droop Coefficients:** Supposing that an emergency fault with power imbalance \(\Delta P_{E}^{r}\) occurs during the incentive mechanism execution cycle. If \(\Delta P_{E}^{r} \leq \Delta P_{E}^{n}\) (e.g., \(\Delta P_{E}^{r}\) in Fig. 3(b)), the EFC strategy with the pre-set droop coefficients can satisfy the frequency security constraints and there is no need to adjust the droop coefficients. If \(\Delta P_{E}^{r} > \Delta P_{E}^{n}\) (e.g., \(\Delta P_{E}^{r}\) in Fig. 3(b)), the pre-set droop coefficients cannot meet the frequency security constraints and need real-time adjustments according to the practical \(\Delta P_{E}^{r}\).
the power regulation margin constraint of the LCC-HVDC system in the following incentive game modeling. Therefore, an LCC-HVDC system with small power regulation margin will not provide a large droop coefficient.

III. INCENTIVE GAME FORMULATION AND ANALYSIS

In this section, the incentive game model is formulated to quantitatively analyze the proposed incentive mechanism. Then, an algorithm for seeking the Nash equilibrium is proposed, which can be easily applied to engineering practice. Moreover, the uniqueness of the Nash equilibrium is analyzed.

A. Incentive Game Model

For each power imbalance $\Delta P_{Ej}$ in $\mathcal{F}$, the incentive game model is formulated to derive the corresponding $R_j$ and $\{k_{ij}\}$. In the following model, we omit the subscript $j$ for brevity. In the incentive game, the AM system and the AD systems solve their respective optimization problems to minimize the disutilities.

For the AM system, the optimization objective consists of two parts: 1) minimizing the reward $R$ to the AD systems, 2) minimizing the total power regulation costs of the generators in the AM system. We define the cost function of generator $i \in \mathcal{G}_{am}$ in a classic form $\frac{1}{2}\alpha_i(\Delta P_i^G)^2$ [40], where $\Delta P_i^G$ is the power regulation, and $\alpha_i$ is the cost coefficient for generator $i$. The optimization problem of the AM system (OAM problem) is shown in (3).

\[ \min_R F_{am}^R(R) = R + \sum_{i \in \mathcal{G}_{am}} \frac{1}{2}\alpha_i(\Delta P_i^G)^2 \]  \hspace{1cm} (3a)
\[ \text{s.t. } \Delta P_i^G = -k_i^G\omega_{am}, \quad i \in \mathcal{G}_{am} \]  \hspace{1cm} (3b)
\[ R = Y(\omega_{am} - \omega_c^{am}) \]  \hspace{1cm} (3c)
\[ P_i^G \leq P_i^T + \Delta P_i^G \leq T_i^G, \quad i \in \mathcal{G}_{am} \]  \hspace{1cm} (3d)
\[ \omega_{am} \leq \omega_{am}^{\alpha} \leq \omega_{am} \]  \hspace{1cm} (3e)
\[ R \leq R \leq R \]  \hspace{1cm} (3f)

where $R$ is the decision variable, $F_{am}^R(R)$ is the disutility function of the AM system, $k_i^G$ is the droop coefficient of generator $i$, $\omega_{am}^{\alpha}$ is the calculated value of the frequency deviation according to the given $\{k_i^G\}$, which shows whether the given $\{k_i^G\}$ by the AD systems can satisfy the frequency security constraints, $\omega_{am}$ is the expected steady-state frequency deviation, $Y(\cdot)$ is a response function, $P_i^G$ is the nominal power of generator $i$, $\Delta P_i^G$ is the frequency regulation equation of the AM system, (3c) is the droop control equation of generator $i$. (3d) describes the response of the AM system to $\omega_{am}-\omega_{am}^{\alpha}$, and in the following subsection, we make an analytical assumption on the response function $Y(\cdot)$, (3e) is the power regulation constraint of $\Delta P_i^G$, (3f) is the frequency security constraint of the AM system, (3g) shows the value range of $R$.

For AD $i$, the optimization objective also consists of two parts: 1) maximizing the shared reward in proportion to $k_i^D$, 2) minimizing the total costs of the generators in AD $i$ system. The shared reward of AD $i$ can be represented as $\left(\frac{k_i^D}{\sum_{i \in \mathcal{D}} k_i^D}\right)R$, and the generators in AD $i$ also use the classic cost function as aforementioned. The optimization problem of AD $i$ system (OAD $i$ problem) is as follows.

\[ \min_{k_i^D} F_{ad}^i(k_i^D) = -\left(\frac{k_i^D}{\sum_{i \in \mathcal{D}} k_i^D}\right)R + \sum_{h \in \mathcal{G}_{ad}} \frac{1}{2}\alpha_h(\Delta P_h^G)^2 \]  \hspace{1cm} (4a)
\[ \text{s.t. } \Delta P_i^D = -k_i^D\omega_{ad}^m = -\left(\sum_{h \in \mathcal{G}_{ad}} k_h^G\right)\omega_{ad}^m \]  \hspace{1cm} (4b)
\[ \Delta P_h^G = -k_h^G\omega_{ad}^m, \quad h \in \mathcal{G}_{ad} \]  \hspace{1cm} (4c)
\[ P_i^D \leq P_i^D + \Delta P_i^D \leq T_i^D \]  \hspace{1cm} (4d)
\[ P_h^G \leq P_h^G + \Delta P_h^G \leq T_h^G, \quad h \in \mathcal{G}_{ad} \]  \hspace{1cm} (4e)
\[ \omega_{ad} \leq \omega_{ad}^m \leq \omega_{ad} \]  \hspace{1cm} (4f)

where $k_i^D$ is the decision variable, $F_{ad}^i(k_i^D)$ is the disutility function of AD $i$ system, $\alpha_h$, $k_h^G$, $\Delta P_h^G$ and $P_h^G$ are respectively the cost coefficient, primary droop coefficient, power regulation, and nominal power of generator $h \in \mathcal{G}_{ad}$, $\omega_{ad}^m$ is the frequency deviation of the AD $i$ system, $\Delta P_i^D$ is the power regulation of LCC-HVDC $i$. (4b) includes the P-f droop control equation of LCC-HVDC $i$ and the frequency regulation equation of AD $i$ system, (4c) is the droop control equation of generator $h$, (4d) and (4e) are the power regulation constraints of $\Delta P_i^D$ and $\Delta P_h^G$ respectively, and (4f) is the frequency security constraint of AD $i$ system.

In the designed mechanism, the self-interested AM system and AD systems have different optimization objectives and each system wants to minimize its own disutility. Thus, the process of AM system and AD systems solving the above optimization problems naturally presents a non-cooperative game, i.e., the incentive game. The three key elements of the incentive game are summarized as follows [41], [42]:

- **Players:** The AM system and the AD systems.
- **Strategies:** The reward $R \in \Omega_R$ for the AM system, and the droop coefficient $k_i^D \in \Omega_i^D$ for AD $i$ system. The $\Omega_R$ and $\Omega_i^D$ are the feasible sets of the strategies, which can be determined by the constraints (3e)–(3g) and (4d)–(4f), respectively.
- **Payoffs:** The disutilities $F_{am}^R(R)$ for the AM system, and $F_{ad}^i(k_i^D)$ for AD $i$ system.

We denote the vector $k^D = \{k_i^D, i \in \mathcal{D}\}$, then, the definition of the Nash equilibrium of the incentive game is given:

**Definition 1 (Nash Equilibrium):** A strategy profile $x^* = (R^*, k^{D*})$ is a Nash equilibrium of the incentive game if:

\[ F_{ad}^i(R^*, k_i^{D*}) \leq F_{ad}^i(R^*, k_i^{D, j}), \quad \forall k_i^D \in \Omega_i^D, \quad i \in \mathcal{D} \]
\[ F_{am}^R(R^*, k^{D*}) \leq F_{am}^R(R, k^{D, j}), \quad \forall R \in \Omega_R \]  \hspace{1cm} (5)
where \((R^*, k^D, d^D)\) represents the strategy profile where the AD i system adopts an arbitrary \(k^D_i\) in \(\Omega^D\) while the other players adopt the strategies at the Nash equilibrium, \((R, k^D)\) represents the strategy profile where the AM system adopts an arbitrary \(R\) in \(\Omega^R\) while the AD systems adopt the strategies at the Nash equilibrium. (5) shows that when at the Nash equilibrium, no player can further reduce its disutility by changing its own strategy. Thus, when at the Nash equilibrium \((R^*, k^D)\), due to the self-interests of the players, the AM system will not provide more reward for the EFC with droop coefficients \(k^D\), and the AD systems will not set larger droop coefficients for the given reward \(R^*\), which is the physical significance of the Nash equilibrium. Subsequently, an algorithm for seeking the Nash equilibrium of the incentive game is designed, which can be easily applied to engineering practice.

B. Algorithm for Seeking the Nash Equilibrium

For the incentive game, there are two common approaches to seek the Nash equilibrium: 1) the centralized approach; 2) the iterative approach. In the centralized approach, the Karush-Kuhn-Tucker (KKT) conditions of the optimization problems of all the players are jointly solved to derive the Nash equilibrium; hence, the centralized approach requires the global parameter information, which is difficult to obtain due to individual privacy preserving. Therefore, it is difficult to implement the centralized approach in the practical engineering. In this subsection, a simple but practical iterative approach is utilized to design the algorithm for seeking the Nash equilibrium, i.e., the fixed point method [43], in which the players of the game iteratively solve their respective optimization problems to find the equilibrium.

When one player solves its optimization problem, the decision variables of the other players are regarded as given constants. Note that there exist couplings among the decision variables \(<D^D_i>\) in the first items of (4a), i.e., one AD system needs the information of all the other AD systems’ decision variables to solve its OAD problem. In this situation, seeking the Nash equilibrium directly by the fixed point method might result in problems such as initial value dependence and decision order dependence [44]. Therefore, a virtual price variable \(\gamma\) is introduced to eliminate the couplings in the OAD problems, which is defined as:

\[
\gamma = \frac{R}{\sum_{i\in D} k^D_i} \tag{6}
\]

When \(k^D\) is given by the AD systems, the reward \(R\) can be determined independently by \(\gamma\); thus, in the algorithm design, we regard \(\gamma\) as the decision variable of the AM system in place of \(R\). According to (6), the first item in (4a) can be represented as \(-\gamma k^D_i\), which is independent of the decision variables of other AD systems; hence, the definition of \(\gamma\) eliminates the aforementioned couplings and benefits the seeking for the Nash equilibrium.

Then, based on the defined \(\gamma\), a linear response function \(\hat{Y}(\cdot)\) is adopted to describe the constraint (3d) in the OAD problem, which is shown as follows:

\[
\gamma = \hat{Y}(\omega^am - \omega^e) = a(\omega^am - \omega^e) + \gamma^{t-1} \tag{7}
\]

where \(a\) is the marginal response coefficient with \(0 < a < 1\), \(\gamma^{t-1}\) is the virtual price value from the last fixed-point iteration. In engineering practice, since the AM system can determine the expected steady-state frequency after emergency faults according to the technical guide for power system security and stability, we make the assumption that the expected steady-state frequency deviation \(\omega^am\) is a constant given by the AM system, and the impact of \(\omega^am\) on the Nash equilibrium is analyzed in the case study. (7) describes the decision-making willingness of the AM system, i.e., when the calculated frequency deviation \(\omega^am\) is different from \(\omega^am\), the AM system will adjust the virtual price \(\gamma\) according to (7).

When \(\omega^am\) is assumed to be a constant, the constraints (3c) and (3e)–(3f) do not contain the decision variable, and the second item of (3a) also becomes a constant; hence, the above constraints are only the requirements for the AM system to determine \(\omega^am\), and are independent of the solution of the OAD problem. Therefore, by substituting (6)–(7) into (3), and removing constraints (3e), (3e)–(3f) and the second item of (3a), the modified OAM (MOAM) problem in the compact form is shown in (8):

\[
\begin{align*}
\min_{\gamma} & \hat{F}^{am}(\gamma) = \left(\sum_{i\in D} k^D_i\right) \gamma \tag{8a} \\
\text{s.t.} & \Delta P_E = -\left(\sum_{i\in D} k^D_i + \sum_{i\in G} k^G_i\right) \omega^am - \omega^e + \gamma^{t-1} \tag{8b} \\
& \gamma \in \Omega^\gamma \tag{8d}
\end{align*}
\]

where \(\Omega^\gamma\) is the feasible set of \(\gamma\) which can be derived by (3g) and \(0 < a < \bar{d}\). Substitute (6) and (4b)–(4c) into (4a), and the modified OAD (MOAD) i problem in the compact form is shown in (9):

\[
\begin{align*}
\min_{k^D_i} & \hat{F}^{ad}(k^D_i) \\
& = -\gamma k^D_i + \frac{(\omega^am)^2 \sum_{h\in G^{ad}} h^\gamma (k^G_h)^2 (k^D_i)^2}{\sum_{h\in G^{ad}} (k^G_h)^2} \tag{9a} \\
\text{s.t.} & k^D_i \in \Omega^D_i \tag{9b}
\end{align*}
\]

where the feasible set \(\Omega^D_i = \{k^D_i | k^D_i \leq k^D \leq k^D_i\}\) is defined in Section III-A. In the algorithm for seeking the equilibrium with the fixed point method (SE-FP algorithm), the modified Nash equilibrium \(\hat{x}^* = (\gamma^*, k^Dm)\) is first derived by the fixed point approach. Then, the Nash equilibrium \(x^* = (R^*, k^D)\) can be obtained by (6). The details of the algorithm for seeking the Nash equilibrium is shown in Algorithm 1.

Considering the implementation of the formulated incentive game with the SE-FP algorithm, Fig. 4 shows the framework of the incentive game, where the AM system and the AD systems realize information transmission through a platform. In Fig. 4, only a small amount of information (i.e., the decision variables \(\gamma\) and \(k^D_i\), and given constant \(\omega^am\)) needs to be transmitted, thus, the incentive game with the SE-FP algorithm can be easily implemented in practical projects, and the private information (e.g.,
This subsection, the uniqueness of the Nash equilibrium of the incentive game is analyzed, which ensures that there will not appear multiple equilibriums when applying the proposed incentive mechanism. We make the following assumptions.

**Assumption 3:** For the incentive game, there exists a Nash equilibrium \( x^* = (R^*, k^{D*}) \), \( R^* \in \Omega^R \), \( k^{D*} \in \Omega^D \).

**Assumption 4:** The droop coefficients of the LCC-HVDC systems satisfy \( \sum_{i \in D} k_i^D < \sum_{i \in D} k_i^G < \sum_{i \in D} \bar{k}_i^D \).

Then, we have the following proposition for the uniqueness of the equilibrium.

**Proposition 1:** If Assumptions 3–4 hold, the Nash equilibrium \( x^* = (R^*, k^{D*}) \) of the incentive game is unique.

**Proof:** The Nash equilibrium \( x^* \) is unique if and only if the modified Nash equilibrium \( \bar{x}^* = (\bar{\gamma}^*, k^{D*}) \) is unique, thus, we just need to analyze the uniqueness of \( \bar{x}^* \).

Before the uniqueness analysis of the equilibrium, we first analyze the monotonicity of the optimal decision \( k^{D, opt}_i \) with respect to the virtual price \( \gamma \). According to the optimality condition (KKT condition) [45], the optimal decision of the MOAD problem is:

\[
k^{D, opt}_i = \left[ \frac{(\sum_{h \in G^ad_i} K_h^G)^2 \gamma}{2(\omega_{am})^2 \sum_{h \in G^ad_i} \alpha_h (k_h^G)^2} \right] \Omega^D_i
\]

(11)

where \( \lfloor \cdot \rfloor \Omega^D_i \) represents the projection on \( \Omega^D_i \). According to (11), for each \( i \in D \), \( k^{D, opt}_i \) is monotonically increasing with respect to \( \gamma \) when \( k^{D, opt}_i \) is an interior point of \( \Omega^D_i \), and \( k^{D, opt}_i \) tends to a saturated value \( \bar{k}_i^D \) when \( \gamma \) continuously increases. Then, if Assumption 4 holds, there exists at least one AD \( i \) system with \( k^{D}_i < k^{D, opt}_i < \bar{k}_i^D \), thus, the sum \( \sum_{i \in D} k^{D, opt}_i \) is monotonically increasing with respect to the virtual price \( \gamma \).

Based on the monotonicity, we further analyze the uniqueness of the modified Nash equilibrium. According to the modified incentive game with the SE-FP algorithm, one of the conditions at the equilibrium is:

\[
|\hat{\gamma}^* - \gamma^{t-1}| < \varepsilon_{\gamma}
\]

(12)

where \( \varepsilon_{\gamma} \approx 0 \) is a very small number. Then, combining (8c) and (12), we have \( \omega_{am}^\bar{\gamma} = \omega_{am}^\gamma \), which yields the following so-called equilibrium condition:

\[
\sum_{i \in D} k_i^{D*} = \frac{\Delta P^E}{\omega_{am}} - \sum_{i \in \bar{D}} k_i^G
\]

(13)

If Assumption 3 holds, there exists a \( k^{D*} \) satisfying (13). From the monotonicity, there exists at most one virtual price \( \gamma^* \) which is corresponding to the \( k^{D*} \). Therefore, the modified Nash equilibrium \( \bar{x}^* = (\gamma^*, k^{D*}) \) is unique, which completes the proof.

**Remark 2:** In engineering practice, if the power imbalance is too large, all the droop coefficients might reach their saturated values (upper bounds), and the Assumptions 3–4 might not hold in the incentive game. In fact, at this time, the LCC-HVDC systems and generators cannot provide enough power support to cover the emergency power imbalance. To deal with this scenario, the incentive game with the SE-FP algorithm can give the saturated reward \( R \) corresponding to the saturated droop coefficients (as shown in Section V-C), and the uncovered part of the power imbalance needs to be undertaken coordinately by the load shedding operations.
IV. PROPERTIES OF THE INCENTIVE MECHANISM

In this section, several fine properties of the designed incentive mechanism for EFC are analyzed. In this mechanism, the pre-payments and the pre-settings of the droop coefficients are all based on the Nash equilibrium of the incentive game with the nearest-to-expected power imbalance, thus, we show the properties of the incentive mechanism by analyzing the properties of the incentive game model.

A. Individual Rationality and Incentive Compatibility

The individual rationality means that AD systems can obtain more utilities by participating in the designed incentive mechanism than not, thus, they are willing to participate in the mechanism [46]. Considering the disutility function \( F^{ad}(k_i^D) \) of the AD \( i \) system in the incentive game model, when AD \( i \) does not participate in the incentive game (i.e., \( k_i^D = 0 \)), we have \( F^{ad}(0) = 0 \) according to (9a). Then, we have the following proposition about the individual rationality of the incentive game.

**Proposition 2:** The incentive game is individual rational for AD systems, i.e., given \( \gamma > 0 \), the disutility with the optimal decision satisfies \( F^{ad}(k_i^{D, opt}) < 0 \).

**Proof:** Obtain the derivative of \( F^{ad}(k_i^D) \) with respect to \( k_i^D \), which is:

\[
\frac{d F^{ad}(k_i^D)}{d k_i^D} = -\gamma + \frac{2(\omega \gamma)^2 \sum_{h \in G_{ad}} \frac{1}{2} \alpha_h (k_h^G)^2}{(\sum_{h \in G_{ad}} k_h^G)^2} k_i^D
\]  

(14)

Since \( 0 \in \Omega_i^D \), according to (11), we have:

\[
\frac{d F^{ad}(k_i^D)}{d k_i^D} < 0, \forall k_i^D \in (0, k_i^{D, opt})
\]  

(15)

Thus, \( F^{ad}(k_i^D) \) is monotonically decreasing in the interval \((0, k_i^{D, opt})\), which yields:

\[
F^{ad}(k_i^{D, opt}) < F^{ad}(0) = 0
\]  

(16)

which completes the proof.

The individual rationality analysis shows that the designed incentive mechanism is reasonable and can provide positive incentives for the AD systems to participate in the game.

Moreover, the incentive compatibility [47] in the proposed incentive mechanism for EFC means that each AD system is willing to give the true droop coefficient instead of a false one, i.e., giving the true droop coefficient will obtain more utility than not. In fact, the incentive compatibility can be satisfied since the incentive mechanism is based on the Nash equilibrium of the incentive game. According to Definition 1, when at the Nash equilibrium, there is no player being able to obtain more utility by changing its own strategy, i.e.,

\[
F^{ad}(R^a, k^{D^*}) \leq F^{ad}(R^a, k^{D,f}, k^{D,f}_i), \forall k_i^{D,f} \in \Omega_i^D
\]  

(17)

where \( k_i^{D,f} \) is a false droop coefficient. Therefore, the AD systems are willing to give the true droop coefficients by optimal decision-making, which guarantees the incentive compatibility.

B. Social Optimality

To analyze the social efficiency of the proposed incentive mechanism, we first define the social welfare problem [48]. When an emergency fault occurs and the droop-based EFC strategy works, the optimization objective of the social planner is minimizing the total power regulation costs of the generators in all the AD systems. Since the social welfare problem is considered from the perspective of the whole power system, we ignore the reward \( R \) in this problem. The social welfare problem can be represented as (18).

\[
\min_{\{k_i^D\}} \sum_{i \in D} \sum_{h \in G_{ad}} \frac{1}{2} \alpha_h (\Delta P_h^G)^2
\]  

s.t. \( \Delta P_E = - \left( \sum_{i \in D} k_i^D + \sum_{i \in \Omega_i^D} k_i^G \right) \omega \gamma
\]  

(18a)

\[
\Delta P_i^D = - k_i^D \omega \gamma = - \left( \sum_{h \in G_{ad}} k_h^G \right) \omega \gamma, i \in D
\]  

(18c)

\[
k_i^D \in \Omega_i^D, i \in D
\]  

(18e)

**Definition 2 (Social Optimum):** The droop coefficients \( \bar{k}_i^D = \{k_i^D, i \in D\} \) is the social optimum if \( \bar{k}_i^D \) is the optimal solution of (18).

Then, we have the following proposition about the social efficiency of the incentive game.

**Proposition 3:** If Assumption 3–4 holds, for the incentive game and the social welfare problem, the following statements hold:

1) the \( k_i^{D^*} \) at the Nash equilibrium of the incentive game is identical to the social optimum \( \bar{k}_i^D \).

2) the virtual price \( \gamma^* \) at the Nash equilibrium is equal to the negative Lagrangian multiplier of constraint (18b) at the social optimum.

3) if for \( \forall i \in D, k_i^D \) is an interior point of \( \Omega_i^D \), the social optimum \( \bar{k}_i^D \) (i.e., the \( k_i^{D^*} \)) can be derived analytically.

**Proof:** We first simplify the social welfare problem. Substitute (18c)–(18d) into (18a), the objective of the social welfare problem is represented as:

\[
\sum_{i \in D} \frac{(\omega \gamma)^2 \sum_{h \in G_{ad}} \frac{1}{2} \alpha_h (k_h^G)^2}{(\sum_{h \in G_{ad}} k_h^G)^2} (k_i^D)^2
\]  

(19)

Let:

\[
\omega \gamma = \sum_{i \in D} k_i^G
\]  

(20a)

\[
W = - \Delta P_E - \sum_{i \in \Omega_i^D} k_i^G
\]  

(20b)
We have \( u_i > 0, W > 0 \). Then, the social welfare problem is simplified as (21):

\[
\begin{align*}
\min_{\{k_i^D\}} & \quad \sum_{i \in D} u_i(k_i^D)^2 \\
\text{s.t.} & \quad \sum_{i \in D} k_i^D = W, \lambda \\
& \quad k_i^D \in \Omega^D, i \in D
\end{align*}
\]

(21a)

(21b)

(21c)

where \( \lambda \) is the Lagrangian multiplier of constraint (21b).

Considering the conditions satisfied at the modified Nash equilibrium \( \hat{x}^* = (\gamma^*, k^{D*}) \), \((\gamma^*, k^{D*})\) is the optimal solution of the MOAD \( i \) problem (9), which satisfies the following KKT condition [49] (with the variable substitutions (20)):

\[
0 \in 2u_i k_i^{D*} - \gamma^* + N_{\Omega^D}(k_i^{D*}), i \in D
\]

(22)

Moreover, \((\gamma^*, k^{D*})\) satisfies the following condition according to (13):

\[
\sum_{i \in D} k_i^{D*} = W
\]

(23)

Note that if we identify the Lagrangian multiplier \( \lambda \) of constraint (21b) with \(-\gamma\), the combination of (22) and (23) is also the KKT condition of the social welfare problem (21), which means that \( k^{D*} \) is identical to \( \hat{k}^D \) and \( \gamma^* \) is equal to \(-\lambda\), where \( \lambda \) is the value corresponding to \( k^{D*} \).

If \( k_i^D = k_i^{D*} \) is an interior point of \( \Omega^D \), \( \forall i \in D \), we have \( \{0\} = N_{\Omega^D}(k_i^{D*}) \). Then, by (22) and (23), the unique \( \hat{k}^D \) and the modified Nash equilibrium \( \hat{x}^* \) can be derived analytically, as shown in (24).

\[
\begin{align*}
\hat{k}_i^D &= k_i^{D*} = \frac{W}{u_i \sum_{i \in D} (1/u_i)}, i \in D \\
\hat{\lambda} &= -\gamma^* = -\frac{2W}{\sum_{i \in D} (1/u_i)}
\end{align*}
\]

(24a)

(24b)

Remark 3: Based on the above analysis, the Nash equilibrium under the incentive mechanism satisfies the social optimality, which shows a fine property. According to Proposition 3, when all the parameters of the entire system are available, seeking the Nash equilibrium of the incentive game is equivalent to solving a convex social welfare problem, which also verifies the uniqueness of the Nash equilibrium. Moreover, in the scenario where the entire power system is operated by a single decision-maker, since the social optimality is satisfied, the incentive mechanism proposed in this paper can also provide reference on how to set the droop coefficients and how to reward the AD systems participating in the EFC strategy.

V. CASE STUDY

In this section, the effectiveness of the incentive game is illustrated, and the properties of the proposed mechanism are verified by a case study. Moreover, the impact of the expected frequency deviation \( \omega_{ann} \) is discussed.

### TABLE I

| No. | \( P^G_{i} \text{(MW)} \) | \( P^G_{i} \text{ (MW)} \) | \( \alpha_i \text{ (p.u./MW)} \) | \( k^G_i \text{ (MW/Hz)} \) |
|-----|-----------------|-----------------|-----------------|-----------------|
| G1  | 320             | 500             | 0.1             | 500             |
| G2  | 350             | 500             | 0.1             | 500             |
| G3  | 370             | 500             | 0.1             | 500             |
| G4  | 380             | 500             | 0.1             | 500             |
| G5  | 400             | 500             | 0.1             | 500             |
| G6  | 425             | 600             | 0.1             | 600             |
| G7  | 450             | 600             | 0.1             | 600             |
| G8  | 470             | 600             | 0.1             | 600             |

#### A. System Description

The topology of the MIDC test system is shown in Fig. 1. In the test system, the AM system contains eight generators, and is connected with four LCC-HVDC systems, in which LCC1, LCC2, LCC3 are SE-LCC systems and LCC4 is an RE-LCC system. There are four AD systems corresponding to the four LCC-HVDC systems, and each AD system contains three equivalent generators. Considering the incentive game model, the related parameters of the generators in the AM system are shown in Table I, and we set the generation cost per unit active power of G1 as the base-value of costs and disutilities. The related parameters of the LCC-HVDC systems and the generators in corresponding AD systems are shown in Table II. The above parameters of the test system are partly from the 4-HVDC modified IEEE New England system in [13], [50].

We set the upper and lower bounds of \( \omega_{ad}^{d} \) with \( \omega_{ad}^{d} = 0.2 \) Hz and \( \omega_{ad}^{d} = -0.2 \) Hz. We set the upper and lower bounds of the marginal response coefficient with \( \pi = 20 \) p.u./MW and \( q = 10 \) p.u./MW. The expected frequency deviation is set with \( \omega_{ann} = -0.2 \) Hz for emergency faults with power shortages, and the impact of \( \omega_{ann} \) is analyzed in Section V-D.

In this case study, we select all the single generator tripping faults in the AM system to compose the emergency fault set \( \mathcal{F} \), i.e., \( \mathcal{F} = \{F_1, \cdots, F_8\} \), where the power imbalance \( \Delta P_{Fi} \) of Fi is corresponding to the nominal power \( P^G_{i} \) of generator Fi. When solving the incentive game with \( \Delta P_{Fi} \), the corresponding droop coefficient \( k^G_i \) of the tripping generator is set to be zero. Then, based on the above settings, we present the following results.

#### B. Effectiveness of the Incentive Game

First, for each emergency fault in \( \mathcal{F} \), we derive the Nash equilibrium \( x^* = (R^*, k^{D*}) \) by the proposed SE-FP algorithm. The Nash equilibriums corresponding to emergency faults are shown in Table III. It can be seen from Table III that for each fault in the selected fault set, the SE-FP algorithm can always seek an equilibrium of the incentive game, which illustrates the effectiveness of the incentive game model and the SE-FP algorithm. Then, according to the Nash equilibriums, the \( \Delta P_{Ed}^{eq} \) curves and the \( \Delta P_{Ed}^{eq} \{k_i^{D*}\} \) curves for the test system can be derived, as shown in Fig. 5. Further, according to the proposed incentive mechanism, given the fault ratios \( \{p_i, i \in \mathcal{F}\} \), the pre-payment and pre-settings of droop coefficients can be carried out. Supposing that the fault ratios in the fault set of...
TABLE II
RELATED PARAMETERS OF LCC-HVDC SYSTEMS AND GENERATORS IN AD SYSTEMS

| No. | $P_i^a$(MW) | $P_i^d$, $P_i^p$(MW) | $P_i^C$(MW) | $P_i^C$ (MW) | $\alpha_i$(p.u./MW$^2$) | $k_i^C$(MW/Hz) |
|-----|-------------|----------------------|-------------|--------------|-----------------|----------------|
| LCC1/AD1 | 645 | 750, 550 | (610, 540, 650) | (700, 650, 750) | (500, 450, 550) | (0.9, 1.0, 1.1) | (130, 100, 150) |
| LCC2/AD2 | 630 | 750, 550 | (600, 550, 620) | (700, 650, 750) | (500, 450, 550) | (1.0, 1.0, 0.8) | (155, 120, 140) |
| LCC3/AD3 | 660 | 750, 550 | (580, 530, 630) | (700, 650, 750) | (500, 450, 550) | (0.9, 1.0, 0.8) | (150, 115, 150) |
| LCC4/AD4 | 500 | 600, 400 | (590, 560, 640) | (700, 650, 750) | (500, 450, 550) | (1.1, 1.0, 0.8) | (140, 110, 145) |

TABLE III
NASH EQUILIBRIUMS CORRESPONDING TO EMERGENCY FAULTS

| No. | $\Delta P_E$(MW) | $\gamma^*$ (p.u.-Hz/MW) | $k_i^D$ (MW/Hz) | $k_i^D$ (MW/Hz) | $k_i^D$ (MW/Hz) | $R^*$ (p.u.) |
|-----|-----------------|----------------------|----------------|----------------|----------------|-------------|
| F1  | 320             | 2.25                 | 162.88         | 179.38         | 188.55         | 174.18      | 1589.17     |
| F2  | 350             | 2.75                 | 198.69         | 218.82         | 230.00         | 212.48      | 2364.77     |
| F3  | 370             | 3.12                 | 225.26         | 248.08         | 260.76         | 240.89      | 3039.51     |
| F4  | 380             | 3.28                 | 236.81         | 260.81         | 274.13         | 253.24      | 3359.24     |
| F5  | 400             | 3.61                 | 261.07         | 287.52         | 302.21         | 279.18      | 4082.72     |
| F6  | 425             | 4.08                 | 294.57         | 324.42         | 340.99         | 315.01      | 5197.74     |
| F7  | 450             | 4.43                 | 319.99         | 352.41         | 370.41         | 342.18      | 6133.28     |
| F8  | 470             | 4.81                 | 347.71         | 382.94         | 402.51         | 371.83      | 7242.13     |

Fig. 5. Curves for test system. (a) $\Delta P_E$-$R$ curve. (b) $\Delta P_E$-$\{k^D_i\}$ curve.

Second, to investigate the convergence of the SE-FP algorithm, we set five different initial values of the virtual price $\gamma^0$ between 0 p.u.-Hz/MW to 10 p.u.-Hz/MW to iteratively solve the Nash equilibrium with power imbalance $\Delta P_{E2}=350$ MW. The iteration processes of $\gamma$ and droop coefficient of LCC1 $k_i^D$ are shown in Fig. 6. It can be seen that with different initial value settings, the $\gamma$ and $k_i^D$ both converge to the same equilibrium, which verifies the convergence of the SE-FP algorithm. Based on the above results, the incentive mechanism with the SE-FP algorithm has practical significance for two reasons: 1) The convergence of the SE-FP algorithm guarantees that the Nash equilibrium of the incentive game can be effectively solved. 2) The SE-FP algorithm only needs a small amount of information transmission (i.e., the decision variables $\gamma$ and $k_i^D$, and constant $\omega_{am}$) to derive the Nash equilibrium, which can be easily implemented in practice.
Moreover, considering the speed of stabilizing the AM system frequency with the incentive mechanism, since the implementation of the droop-based EFC in LCC-HVDC systems increases the bandwidth of the closed-loop system, the post-fault system frequency can be stabilized more quickly [51]. For the time-domain simulation results about the system frequency dynamics, please refer to our previous work [13].

C. Properties Verification

We first verify the uniqueness of the Nash equilibrium by showing the monotonicity of the optimal droop coefficient with respect to the virtual price $\gamma$. We set $\gamma$ to vary from 3 p.u.-Hz/MW to 7 p.u.-Hz/MW, and the optimal droop coefficients of LCC-HVDC systems can be derived by solving the MOAD problems (9), as shown in Fig. 7. The results in Fig. 7 not only show the aforementioned monotonicity and uniqueness, but also illustrate that the optimal droop coefficients from MOAD problems tend to saturated values (upper bounds) as $\gamma$ continuously increases. Therefore, when all the droop coefficients reach their saturated values due to a considerable power imbalance, the incentive game can give the saturated reward $R$ corresponding to the saturated $\gamma$ in Fig. 7, as mentioned in Remark 2.

Then, for each emergency fault in $F$, we show the disutilities of the AD systems at the Nash equilibrium in Fig. 8 to verify the individual rationality of the proposed incentive mechanism. As shown in Fig. 8, the disutilities of AD systems with various power imbalances are all less than zero, which means that by participating in the incentive mechanism, the AD systems can obtain more utilities than not. Thus, the individual rationality is verified, and the AD systems are willing to participate in the designed mechanism, which facilitates the application of the mechanism in the MIDC system with multiple decision-makers.

In addition, we verify the social optimality of the incentive mechanism by comparing the droop coefficients at the Nash equilibriums and social optimums, and comparing the virtual prices at Nash equilibriums and the Lagrangian multipliers at the social optimums, as shown in Fig. 9. The social optimums are derived by solving the social welfare problems (18) with various power imbalances. In Fig. 9(a), for each power imbalance, the LCC1 droop coefficient at Nash equilibrium is identical to that at the social optimum. And in Fig. 9(b), for each power imbalance, the virtual price at Nash equilibrium is equal to the negative Lagrangian multiplier of constraint (18b). Thus, the social optimality of the proposed mechanism holds, and in the scenario where the entire MIDC system is operated by one single decision-maker, the designed incentive mechanism can also provide reference on how to set the droop coefficients and how to reward the AD systems.

D. Impact of the Expected Frequency Deviation

In Section III-B, to effectively seek the Nash equilibrium, we assume that the expected frequency deviation $\omega^{\text{am}}$ is a constant determined by the AM system. In this subsection, we further discuss the impact of $\omega^{\text{am}}$ on the Nash equilibrium of the incentive game, in order to provide reference for the AM system to determine the $\omega^{\text{am}}$.

We set $\omega^{\text{am}}$ to vary from $-0.25$ Hz to $-0.12$ Hz, and respectively solve the Nash equilibrium of the incentive game with power imbalance $\Delta P_{E2} = 350$ MW. The related variables at Nash equilibrium are shown in Fig. 10. As the $\omega^{\text{am}}$ increases, the
reward $R^*$ in Fig. 10(a) and the droop coefficients in Fig. 10(b) also increase, while the virtual price $\gamma^*$ presents a tendency of increasing first and then decreasing. The change tendency of the virtual price can be explained by the analytical expression (24b), where $\gamma^*$ is a quadratic function with respect to $\omega^{nm}$. Referring to the above tendencies, the AM system can further determine the expected frequency deviation according to its own requirements.

VI. Conclusion

In this paper, a control-parameter-based incentive mechanism is proposed to incentivize the LCC-HVDC systems and their connected adjacent AC systems to participate in the droop-based EFC of the MIDC system. The pre-payments in this mechanism promote that the LCC-HVDC systems implement the controllers and set the control parameters in advance. And benefitting from the immediate adjustability of the droop coefficients of LCC-HVDC systems, the proposed incentive mechanism is able to flexibly deal with various possible power imbalances. Then, to quantitatively analyze the proposed mechanism in the MIDC system, a non-cooperative-based incentive game model is formulated, and a fixed-point-based algorithm for seeking the Nash equilibrium is designed, which can be easily applied in engineering practice. The uniqueness of the Nash equilibrium is rigorously proven. And several fine properties of the designed incentive mechanism, i.e., the individual rationality, incentive compatibility and social optimality, are analyzed and proven. In the case study for a MIDC test system, the effectiveness of the proposed incentive mechanism is illustrated, and the properties of the mechanism are verified.

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