Cross-Coupling Correction for LaCoste & Romberg Airborne Gravimeter

SUN Zhongmiao  XIA Zheren  LI Yingchun

Abstract  The cross-coupling corrections for the LaCoste & Romberg airborne gravimeter are computed as a linear combination of 5 so-called cross-coupling monitors. The weight factors (coefficients) determined from marine gravity data by the factory are obviously not optimal for airborne application. These coefficients are recalibrated by minimizing the difference between airborne data and upward continued surface data (external calibration) and by minimizing the errors at line crossings (internal calibration) respectively. An integrating method to recalibrate the above-mentioned coefficients and the beam scale factor simultaneously is also presented. Experimental results show that the systemic errors in the airborne gravity anomalies can be greatly reduced by using any of the recalibrated coefficients. For example, the systemic error is reduced from 4.8 mGal to 1.8 mGal in Datong test.

Keywords  airborne gravimetry; cross-coupling correction; airborne gravimeter; systemic error

CLC number  P223.4

Introduction

The key component of the first Chinese airborne gravimetry system is a LaCoste & Romberg air/sea gravimeter, which is used to measure the total acceleration (called specific force) which includes the gravity field signal. The specific force data in the output of the gravimeter were smoothed by a “5-minute” filter and had a 5-minute delay[1]. The accuracy and resolution of these smoothed data are enough to meet the requirements of marine gravimetry, as the velocity of the ship is low and the wave motion is relatively regular. However, they are usually not suitable for airborne gravimetry due to the rapid velocity and turbulence. Thus, the specific force has to be computed by other observations of the gravimeter. One of them is the cross-coupling correction (CC-correction).

Cross-coupling errors are often classified into two categories: inherent and imperfection[1]. Inherent cross-coupling is a fundamental property of the beam type gravimeter and cannot be eliminated, except by reducing the beam motion to zero. However, it can be reduced somewhat by restricting the beam motion through increased damping. Nevertheless, this method is limited in application as the beam velocity is also reduced and so is the resolution of the gravity measurement. Imperfection cross-coupling arises because the suspension is not stiff enough. An ideal system is one that has very low compliance in the vertical direction and is perfectly rigid in all other directions. This ideal situation can never be realized in practice since all mechanical systems exhibit some

Received on June 20, 2007.
SUN Zhongmiao, Xi’an Research Institute of Surveying and Mapping, 1 Middle Yanta Road, Xi’an 710054, China.
E-mail:sun_szm@sina.com
elasticity. This is especially true for instruments as delicate as gravity meters.

The cross-coupling corrections for the LaCoste & Romberg airborne gravimeter can be computed as a linear combination of five so-called cross-coupling monitors\(^\text{[1]}\), which are parts of the gravimeter outputs. The weight factors (coefficients) provided by the factory were determined from marine data series by a cross correlation technique\(^\text{[2]}\). However, such a determination will depend on the conditions under which the data are collected. Although the various situations of the wave-motions and the changes of the sea-surface were considered as much as possible, the determination may not be optimal for airborne application. If the CC-corrections from the outputs of the gravimeter were directly adopted, it might introduce some systemic errors in the results. In this study, the coefficients of the cross-coupling monitors are recalibrated by three methods respectively.

1 Data processing models

The fundamental equation for the airborne scalar gravimetry is\(^\text{[3,4]}\):

\[
\Delta g = g_s + (f_z - f_z^B) - \dot{v}_u + \delta a_z + \delta a_y + \delta a_x - \gamma_0 \tag{1}
\]

where \(\Delta g\) is the gravity anomaly of the sampling point at flight altitude; \(g_s\) is the apron gravity value; \(f_z\) is the specific force in the gravimeter outputs; \(f_z^B\) is the base reading; \(\dot{v}_u\) is the vertical acceleration; \(\delta a_z\) is Eötvös correction; \(\delta a_y\) is the correction for misalignment of the platform; \(\delta a_x\) is free-air correction; and \(\gamma_0\) is normal gravity.

Details on the mathematical models and the filtering techniques for the above-mentioned corrections can be found in References \([5,6]\). Whereas the model of the specific force \(f_z\) is:

\[
f_z = G(S + KB' + CC) \tag{2}
\]

where coefficient \(G\) converts from units of count unit to units of mGal; \(S\) is the spring tension; \(K\) is the beam scale factor (\(K\)-factor); \(B'\) is beam velocity; CC is cross coupling.

The linear relation between the CC-correction and its five monitors for the L & R airborne gravimeter can be approximated as:\(^\text{[1]}\):

\[
CC = a_i \left(\{z^i\}^B + a_2 \{y^i\}^B + a_3 \{x^i\}^B + a_4 \{y^i\}^B + a_5 \{z^i\}^B\right) \tag{3}
\]

where \(a_i (i = 1, \ldots, 5)\) represents the coefficients for CC-corrections; the \(x, y\) and \(z\) axes are transverse to the beam, along the beam, and vertical, respectively; \(\triangleright\) denotes the average operator; "\(^B\)" and "\(^i\)" represent the first and the second differentiator respectively.

Each of the five monitors has been assigned names in the outputs of the gravimeter, namely:

\[
\left\{\begin{align*}
\{z^0\}^B &= VE \propto B^2, \\
\{y^0\}^B &= VCC \propto f_x \cdot B', \\
\{x^0\}^B &= AX \propto f_y \cdot B', \\
\{y^0\}^B &= AX^2 \propto f_z^B \cdot B'
\end{align*}\right.
\]

where \(f_x\) and \(f_y\) are the observations of the cross and long accelerometer; VE, VCC, AX and AX2 represent the names of the five monitors, respectively.

Therefore, Eq.(3) can be expressed as:

\[
CC = a_1 \cdot VE + a_2 \cdot VCC + a_3 \cdot AX + a_4 \cdot AL + a_5 \cdot AX2 \tag{4}
\]

or

\[
CC = b_1 \cdot B'^2 + b_2 \cdot f_x \cdot B + b_3 \cdot f_y \cdot B' + b_4 \cdot f_z^B + b_5 \cdot f_z^B \cdot B' \tag{5}
\]

The difference between Eqs.(4) and (5) is that the different raw observations are used. Thus, the coefficients related to Eqs.(4) and (5) are also different. Being the equivalency of the coefficients \(a_i\) and \(b_i\), here we only discuss the calibration methods of the coefficients \(a_i\).

Substituting Eq.(2) and Eq.(4) into Eq.(1), we get:

\[
\Delta g = g_s - f_z^B - \dot{v}_u + \delta a_z + \delta a_y + \delta a_x - \gamma_0 + G \cdot S + GB' \cdot K + G \cdot [VE \ VCC \ AX \ AL \ AX2]^T \cdot [a_1 \ a_2 \ a_3 \ a_4 \ a_5]^T \tag{6}
\]

2 Calibration of CC-coefficients

2.1 External calibration

If there were dense and precise surface gravity data in the surveying area, the upward-continued reference of each sample can be obtained by Poisson integral. Assuming that the external reference for sample point \(P\) is \(\Delta g_P\), we get from Eq.(6):

\[
(VE \ VCC \ AX \ AL \ AX2)^T \cdot [a_1 \ a_2 \ a_3 \ a_4 \ a_5]^T = \Delta g_P \tag{7}
\]
where
\[
\Delta g_i = (\Delta g_r - g_b + f^2_0 + \bar{v} - \delta a_x - \delta a_H - \delta a_F + \gamma_0 - G \cdot S - G B' \cdot K) / G
\]
(8)

The coefficients \(a_i\) can be determined by means of the least squares estimation based upon lots of sample points.

### 2.2 Internal calibration

In general, there are no or only sparse surface gravity data in the surveying area. Therefore, the external calibration of the CC-coefficients is no longer suitable for the case. If the flight lines are designed in cross format and the crossover points are enough, then the CC-coefficients can be determined by minimizing the errors at line crossings. The models are as follows.

Assuming that flight line \(i\) and \(j\) intersect at point \(P\), we obtain from equation Eq.(6):
\[
(\Delta g_r)_i = \delta g_i + G \cdot (VE \cdot VCC \cdot AX \cdot AL \cdot AX2) \cdot [a_1 \ a_2 \ a_3 \ a_4 \ a_5]^T
\]
(9)
\[
(\Delta g_r)_j = \delta g_j + G \cdot (VE \cdot VCC \cdot AX \cdot AL \cdot AX2) \cdot [a_1 \ a_2 \ a_3 \ a_4 \ a_5]^T
\]
(10)
where
\[
\delta g = g_b - f^2_0 - \bar{v} - \delta a_x + \delta a_H + \delta a_F - \gamma_0 + G S + G B' \cdot K
\]
(11)

Therefore, the crossover error \(V_{ij}\) at point \(P\) is:
\[
V_{ij}^2 = G ([VE]) - (VE) \cdot (VCC) - (VCC) \cdot (AX) - (AX) \cdot (AL) - (AL) \cdot (AX) - (AX) \cdot (AX2) \cdot (AX2) \cdot [a_1 \ a_2 \ a_3 \ a_4 \ a_5]^T + (\delta g_i - \delta g_j)
\]
(12)

### 2.3 Integrating calibration with \(K\)-factor

In Sections 2.1 and 2.2, if the \(K\)-factor also regards \(a\) as an unknown parameter, we can simultaneously estimate the CC-coefficients and the \(K\)-factor after some modifications of Eq.(7) and Eq.(12).

### 3 Test results and analysis

#### 3.1 Test description

Test data came from the Datong airborne gravimetry that was accomplished in 2002. There was dense and precise ground gravity with higher resolution, the accuracy of the upward-continued reference was better than 2.0 mGal\(^6\).

The test area belongs to a medium mountain, the range is about 1°40'×2°00'. The terrain undulation is steeper in the eastern side and the highest altitude is about 2 800 m. The western part is highland and the average height is about 1 000 m. The maximal difference of the height in the whole area is about 2 100 m. The change of the gravity anomalies in the flight altitude is ~50-80 mGal. Two kinds of flight lines were designed to test the achievable resolution of the airborne gravimetry. One is high line, which was flown at an average height of 3 400 m and covered the whole area (total 30 lines). Another is low line, which was flown at an average height of 2 800 m and covered only the northwest area (total 20 lines). The line spacing was 5' for high lines and 2.5' for low lines. The survey was performed using a medium-size, pressurized cabin aircraft. The mean ground speed of the aircraft during the operation was around 360 km/h. There were five GPS ground stations operated at a sampling rate of 1 s and two GPS antennae mounted on the cabin roof during the survey. All flights were done at 7 am-12 pm local time. It was just the windy periods, so the flight conditions were quite turbulent and bad.

#### 3.2 Results of external calibration

To explain details, we used three data sets, namely all lines, high lines and low lines to determine the CC-coefficients. The airborne gravity anomalies can be obtained using each group of CC-coefficients according to Eq.(6) and then filtered by a multistage Butterworth filter with a half power point at 200 s\(^6\). Comparing the filtered results to the upward-continued references, we can evaluate the validity of the new CC-coefficients. It should be noted that the CC-corrections for the high-lines that were obtained with the new CC-coefficients recalibrated by low-lines could be regarded as an external check and vice versa.

For convenience, the raw CC-corrections in the outputs of the gravimeter were marked as CC_R, the CC-corrections computed by new coefficients recalibrated by all-lines, high-lines and low-lines were marked as CC_A, CC_H, and CC_L, respectively.
The CC-corrections calibrated by internal calibration were marked as CC_C.

The statistics of the differences between airborne gravity anomaly calculated by each CC-corrections and the upward-continued references are given in Table 1. Fig.1 depicts the airborne gravity anomaly for line WE05_D327 and line NS46_D410 that are compared with their corresponding references.

| Calibration method | CC-correction | All lines | High lines | Low lines |
|--------------------|---------------|-----------|------------|-----------|
|                    | Mean          | Standard deviation | Mean          | Standard deviation | Mean          | Standard deviation |
| External           | CC_R          | -4.78     | 6.38       | -4.75       | 6.40       | -4.84       | 6.34       |
|                    | CC_A          | -1.85     | 6.25       | -1.81       | 6.34       | -1.92       | 6.07       |
|                    | CC_H          | -1.91     | 6.39       | -1.89       | 6.45       | -1.95       | 6.28       |
|                    | CC_L          | -1.52     | 6.36       | -1.43       | 6.59       | -1.69       | 5.90       |
| Internal           | CC_C          | -2.25     | 6.34       | -2.37       | 6.33       | -2.02       | 6.34       |

It can be seen from Table 1 and Fig.1 that the mean difference is reduced from 4.8 mGal to 1.8 mGal using any of the recalibrated coefficients, namely the systemic errors in the airborne gravity anomalies have been greatly reduced. However, since it aims to reduce or remove the systemic errors by a calibration process, the standard deviations of the differences are almost the same as those resulting from raw CC-corrections. Moreover, the standard deviation of the 148 crossovers is reduced from 6.63 mGal to 6.34 mGal when we use CC_A instead of CC_R.

### 3.3 Results of internal calibration

As an example, we use the 148 crossovers for all high lines to recalibrate the coefficients $a_i$. The statistics of the differences between airborne gravity anomaly calculated by new CC-corrections (CC_C) and the upward-continued references are listed in row 7 of Table 1. Obviously, similar conclusions can be drawn as described in Section 3.2, namely the systemic errors in the airborne gravity anomalies have been greatly reduced.

To show the accuracy of the internal calibration in detail, the CC-corrections CC_H and CC_C obtained from external and internal calibration, respectively, are depicted in Fig.2. For comparison, the raw CC-corrections CC_R are also shown in Fig.2. We can see that the results of external and internal calibration coincide; the standard deviation of the differences is less than 0.5 mGal.

The standard deviation of the crossovers for high lines is reduced from 6.63 mGal to 6.21 mGal when we use CC_C instead of CC_R, which is consistent with the least-squares principle. However, in this case,
the standard deviation of the crossovers for low lines is dropped from 6.23 mGal to 6.18 mGal. The validity of the new CC-coefficients is proved again.

Since the internal calibration requires only the measurement to be well designed, with a large and well-distributed number of line crossings, and it does not require any external information, it will be useful and valuable. Although the systemic errors are less reduced than those of external calibration, this can be acceptable for the required accuracy of 6 mGal.

### 3.4 Results of integrating calibration with K-factor

We can evaluate the accuracy of the integrating calibration using the same methods as those of the individual calibration. The compared results of the integrating calibration are listed in Table 2. It could be seen from Table 1 and Table 2 that the standard deviation is somewhat less than that of individual calibration, while the reduction of the systemic error is almost identical.

| Calibration method          | CC-correction | All lines | High lines | Low lines |
|-----------------------------|---------------|-----------|------------|-----------|
|                             | Mean | Standard deviation | Mean | Standard deviation | Mean | Standard deviation |
| Integrating calibration     |       |               |       |               |       |               |
| (external)                  | CC_R | -4.78       | 6.38   | -4.75       | 6.40   | -4.84       | 6.34 |
|                             | CC_A | -1.93       | 6.08   | -1.98       | 6.07   | -1.84       | 6.11 |
|                             | CC_H | -1.51       | 6.16   | -1.61       | 6.07   | -1.33       | 6.31 |
|                             | CC_L | -2.39       | 6.08   | -2.38       | 6.21   | -2.41       | 5.84 |
| Integrating calibration     |       |               |       |               |       |               |
| (internal)                  | CC_C | -2.12       | 6.04   | -2.23       | 5.95   | -1.91       | 6.19 |

# 4 Conclusions

Cross-coupling is a fundamental property of the beam type gravimeter. The cross-coupling corrections for the LaCoste & Romberg airborne gravimeter can be expressed as a linear combination of five monitors, whose coefficients were provided by the manufacturer. However, these coefficients are not optimal for airborne application (especially for large change of the CC-correction). In this paper, these coefficients are recalibrated by minimizing the difference between airborne data and upward-continued surface data and by minimizing the errors at line crossings respectively. Some initial conclusions can be drawn based on the analyses of the numerical results.

1) The systemic error of the airborne gravity could be reduced greatly by use of the new coefficients, which are determined by external or internal calibration. The systemic error is reduced from 4.8 mGal to 1.8 mGal in the Datong survey.

2) The resulting CC-corrections from external and internal calibration are coincident; the standard deviation of their differences is less than 0.5 mGal.

3) The accuracy of the integrating calibration is somewhat better than that of the individual calibration.

4) The internal calibration method will be useful and valuable, although the possible systemic error is less reduced than that of external calibration.

# References

[1] Valliant H D(1991) The LaCoste and Romberg air/sea gravity meter: an overview [M]. 2nd ed. London: CRC Press

[2] LaCoste L J B(1993) Cross correlation method for evaluating and correcting shipboard gravity data [J]. Geophysics, 38: 701-709

[3] Schwarz K P, Li Y C(1996) An introduction to airborne gravimetry and its boundary value problems [C]. IAG International Summer School, Como, Italy

[4] Olesen A V(2002) Improved airborne scalar gravimetry for regional gravity field mapping and geoid determination [D]. Copenhagen: University of Copenhagen

[5] Sun Zhongmiao, Xia Zheren, Shi Pan, et al.(2004) Filtering and processing for the airborne gravimetry data [J]. Progress in Geophysics, 19(1):119-124 (in Chinese)

[6] Sun Zhongmiao(2004) Theory, methods and applications of airborne gravimetry[D]. Zhengzhou: Information Engineering University (in Chinese)