Nanomechanical-resonator-assisted induced transparency in a Cooper-pair box system

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Abstract. We propose a scheme to demonstrate the electromagnetically induced transparency (EIT) in a system consisting of a superconducting Cooper-pair box (CPB) coupled to a nanomechanical resonator (NR). In this scheme, the NR plays an important role to contribute additional auxiliary energy levels to the CPB so that the EIT phenomenon could be realized in such a system. We call it, in this paper, resonator-assisted induced transparency (RAIT). The RAIT technique provides a detection scheme in a real experiment to measure physical properties, such as the vibration frequency and the decay rate, of the coupled NR.

Contents

1. Introduction 2
2. Model and calculations 3
3. Experimental parameters and results 8
4. Conclusion 13
Acknowledgments 13
References 13

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1. Introduction

To be able to observe quantum phenomena in mesoscopic (macroscopic) physical systems which contain many millions of atoms is of great importance in quantum mechanics and also quantum information science. Currently, there are experimental and theoretical groups devoted to observing quantum effects in truly solid-state mechanical oscillators or cantilever systems [1]–[21]. In the regime when the individual mechanical quanta are of the order of or greater than the thermal energy, quantum effects become important and the motion of the mechanical resonator or cantilever is close to or on the verge of the quantum limit. Coupling such a mechanical system to an electrical motional transducer might enable us to observe and control the motional quantum state of the mechanical system.

A superconducting Cooper-pair box (CPB) [23]–[28] consists of a superconducting island (box) weakly linked to a superconducting reservoir by a Josephson tunnel junction. The coherent controls and manipulations of the effective two-level quantum system of a CPB (a charge qubit) have been demonstrated [23, 25, 26]. Due to its controllability, a CPB has been proposed to be one of the ideal candidates to act as a transducer or a mediator to couple to a nanomechanical resonator (NR). With appropriate quantum state controls of a CPB, the CPB has been proposed to be used to produce the desired NR Fock state and perform a quantum non-demolition (QND) measurement of the NR Fock state [29], to drive an NR into a superposition of spatially separated states and probe their decoherence rate [30], to cool the NR to its ground state and generate squeezed state of the NR [31], to probe the quantum mechanical feature of tiny motions of an NR [32], to integrate a superconducting transmission line resonator with an NR [33], and to demonstrate progressive quantum decoherence [34].

Recently, a high-frequency mechanical resonator beam that operates in the gigahertz (GHz) range has been reported [35]. For an NR operating at the fundamental frequency of GHz and at a temperature of 10–100 mK, some interesting phenomena close to or on the verge of the quantum limit may be observed. In this paper, we propose a scheme to demonstrate the electromagnetically induced transparency (EIT) in a system consisting of a superconducting CPB qubit coupled to an NR. The conventional EIT effect occurs in an ensemble of three-level \( \Lambda \)-type atomic systems with two lower states coupled, respectively, to an excited state with two laser fields (control and probe fields) [36, 37]. A typical experiment is conducted by scanning the probe laser frequency and measuring its transmitted intensity. The transparency of the medium takes place when the absorption on both transitions is suppressed due to destructive interference between excitation pathways to the common upper level. In addition to the absorption eliminated via quantum interference, the EIT effect has also been shown to be an active mechanism to slow down or stop a light pulse completely in various systems, such as an ultracold gas of sodium atoms [38], rare-earth-ion-doped crystals [39], semiconductor quantum wells [40], quantum dot exciton systems [41], and systems with four-level or multi-level cells [42]. Recently, it has been proposed to use a superconductive analogy to EIT in a persistent-current flux qubit biased in a \( \Lambda \) configuration to probe small qubit errors due to decoherence or imperfect state preparation [43, 44]. Here, we show that the EIT phenomenon could be realized in an effective two-level superconducting CPB charge qubit coupled to an NR. The capacitive CPB–NR coupling plays an important role to contribute additional auxiliary energy levels for EIT to occur. As a result, the resonator-assisted induced transparency (RAIT) technique provides a detection scheme in a real experiment to measure physical properties, such as the vibration frequency and
Figure 1. Schematic diagram of an NR coupled to a CPB qubit. The microwave currents (with frequencies ωs and ωc) and a direct current (Ib) are applied to flow along the MW line beside the CPB to control the magnetic flux through the CPB loop.

the decay rate of the coupled NR, or the decay and decoherence rates of the CPB qubit, if one set of values of either the CPB or NR properties is known by other means.

2. Model and calculations

In our model, we assume that an NR couples capacitively to a CPB qubit (see figure 1). The tunnel junction of the CPB shown in figure 1 is split into two to form a SQUID loop [23, 25], which allows us to control its effective Josephson energy with a small external magnetic field or magnetic flux. Two microwave currents are applied in a microwave (MW) line [45] beside the CPB to induce oscillating magnetic fields in the Josephson junction SQUID loop of the CPB qubit. We call one of them the control current with frequency ωc and amplitude Ec. The other is called the signal (probe) current with frequency ωs and amplitude Es. In addition, a direct current Ib is also applied to the MW line to control the magnetic flux through the SQUID loop and thus the effective Josephson coupling of the CPB qubit. The Hamiltonian of the total system can be written as [29, 33]:

\[
H = H_{\text{CPB}} + H_{\text{NR}} + H_{\text{int}},
\]

\[
H_{\text{CPB}} = \hbar \omega_q S_z - \hbar E_j \cos \left[ \frac{\pi \phi_0(t)}{\phi_0} \right] S_x,
\]

\[
H_{\text{NR}} = \hbar \omega_r a^+ a,
\]

\[
H_{\text{int}} = 2\hbar \lambda (a^+ + a) S_z.
\]

Here, \(H_{\text{CPB}}\) and \(H_{\text{NR}}\) are, respectively, the Hamiltonians of the CPB qubit and the NR. \(H_{\text{int}}\) is the interaction between them [29]. The operators \(a\) and \(a^+\) denote the creation and annihilation
operators for the NR with frequency \( \omega_i \) and mass \( m \). The two-level system of the CPB qubit can be characterized by the pseudospin-1/2 operators \( S_i \) and \( S_\pm = S_+ + S_- \). \( \omega_0 = 4E_C(2n_g - 1) \) is the electrostatic energy and \( E_j \) is the maximum Josephson energy. Here, \( E_C = e^2/(2C_\Sigma) \) is the charging energy with \( C_\Sigma \) being the total CPB capacitance and \( n_g = (C_bV_b^2 + C_gV_g^2)/(2e) \) is the dimensionless gate charge, where \( C_g \) and \( V_g \) are, respectively, the gate capacitance and gate voltage of the CPB qubit, and \( C_b \) and \( V_b \) are, respectively, the capacitance and voltage between the NR and the CPB island. The capacitive interaction strength can be written as \( \lambda = 4E_C^2C_gV_g/(2de\sqrt{2}\hbar m\omega_0) \), where \( d \) is the distance between the NR and the CPB [29]. The coupling between the MW line and the CPB qubit in the second term of equation (2) results from the total externally applied magnetic flux \( \phi_s(t) = \phi_q(t) + \phi_0 \) through the CPB qubit loop of an effective area \( S \) with \( \phi_0 = \hbar/(2e) \) being the flux quantum [33]. Here,

\[
\phi_q(t) = \mu_0 S I(t)/(2\pi r),
\]

with \( r \) being the distance between the MW line and the qubit and \( \mu_0 \) being the vacuum permeability [33]. \( \phi_q(t) \) and \( \phi_0 \) are produced, respectively, by the microwave current

\[
I(t) = E_c \cos(\omega_c t) + E_s \cos(\omega_0 t + \delta) \tag{6}
\]

and the direct current \( I_\delta \) in the MW line. For simplicity, we assume the phase factor \( \delta' = 0 \) as it is not difficult to show that the results of this paper do not depend on the value of \( \delta' \).

Choosing the direct current \( I_\delta \) and the microwave current \( I(t) \) such that \( \phi_0 \gg \phi_q(t) \) and \( \pi \phi_0/\phi_0 = \pi/2 \), we have

\[
\hbar E_j \cos \left( \frac{\pi \phi_q(t)}{\phi_0} \right) \approx -\hbar E_j \frac{\pi \phi_q(t)}{\phi_0}. \tag{7}
\]

We can work in a frame rotating at the frequency \( \omega_c \) of the control current and the total Hamiltonian in this rotating frame becomes

\[
H_c = \hbar \Delta S_z + \hbar \omega_0 a^+a + 2\hbar \lambda(a^+a)S_z + \hbar \Omega(S_+ + S_-) + \mu E_s(S_z e^{-i\delta t} + S_- e^{i\delta t}). \tag{8}
\]

In analogy to the case of a two-level atom driven by bichromatic electromagnetic waves, here

\[
\mu = \mu_0 ShE_j/(4r\phi_0) \tag{9}
\]

is the effective ‘electric dipole moment’ of the qubit,

\[
\Omega = \mu E \hbar \tag{10}
\]

is the effective ‘Rabi frequency’ through the control current,

\[
\Delta = \omega_q - \omega_c \tag{11}
\]

is the detuning between the CPB qubit resonance frequency and control current frequency, and

\[
\delta = \omega_s - \omega_c \tag{12}
\]

is the detuning between the signal (probe) current frequency and the control current frequency.

Furthermore, we may take into account the decoherence and relaxation of the CPB qubit and NR by including their coupling to external environments into the Hamiltonian [46]–[49]. We assume that the environments to which the CPB and NR couple, respectively, could be described as independent ensembles of harmonic oscillators with respective spectral densities characterizing their properties. We also assume that the NR interacts bilinearly with the environment through their position operators, and the CPB interacts through \( S_z \) operator and
$S_z$ operator with the environment position operators. The $S_x$ coupling to the environment models the relaxation (and thus also decoherence) process of the CPB qubit, whereas the $S_z$ coupling to the environment models the pure dephasing process of the CPB qubit \[46\]–\[49\]. Generally speaking, the value of $\omega_q$ of the CPB qubit is considerably greater than the value of $\omega_r$ of the NR. Therefore, it may be plausible to apply the rotating-wave approximation to the CPB–environment coupling term, but not for the NR–environment coupling term in the system–environment interaction Hamiltonian. By following the standard procedure \[46\]–\[49\], it is then straightforward to derive the Born–Markovian master equation of the reduced density matrix of the CPB–NR system, $\rho(t)$, through tracing out the environmental degrees of freedom as

$$\frac{d\rho}{dt} = -\frac{i}{\hbar} [H_c, \rho] + A t \{[S_-, [S_+, \rho]] + h.c.) + B \{[S_-, \{S_+, \rho]\} + h.c.\} + E [S_z, [S_z, \rho]] + D [Q, [Q, \rho]] + G [Q, \{P, \rho\}] + \frac{i}{\hbar} L [Q, \{P, \rho]\}, \quad (13)$$

where $Q = -2\lambda(a^+ + a)$ and $P$ are the position and momentum operators of the NR, respectively. The coefficients $A, B, E, D, G$ and $L$ are related to the characteristics of the coupling, and to the structure and properties of the environments. Their explicit forms can be written as

$$A = -\frac{1}{2\hbar} \left\{ \frac{\gamma_1}{2} \left( 1 + 2N(\omega_q) \right) \right\}, \quad (14)$$

$$B = \frac{1}{2\hbar} \left\{ \frac{\gamma_2}{2} \right\}, \quad (15)$$

$$E = -\frac{1}{\hbar} \left\{ \frac{\gamma_2}{2} \left( 1 + 2N(0) \right) \right\}, \quad (16)$$

$$D = -\frac{1}{4\hbar} \gamma_3 \left( 1 + 2N(\omega_r) \right), \quad (17)$$

$$G = -\frac{1}{2\hbar \bar{\omega}_r} \Delta_3, \quad (18)$$

$$L = -\frac{1}{4m\omega_r^2} \gamma_3, \quad (19)$$

where

$$\gamma_1 = 2\pi J_x (\omega_q), \quad (20)$$

$$\gamma_2 = 2\pi J_z (0), \quad (21)$$

$$\gamma_3 = 2\pi J_r (\omega_r), \quad (22)$$

$$\Delta_3 = \mathcal{P} \int_0^\infty d\omega \frac{J_R (\omega)}{\omega - \omega_r} (1 + 2N(\omega)). \quad (23)$$

Here, $J_x, J_z$ and $J_r$ are the spectral densities of the respective environments coupled through $S_x$ and $S_z$ to the CPB, and through $Q$ to the NR, respectively. $N(\omega) = 1/[\exp(\hbar\omega/k_B T) - 1]$ is the Boltzmann–Einstein distribution of the thermal equilibrium environments. $\mathcal{P}$ denotes
the principal value of the argument. Note that \( \omega_c \) and \( \omega_q \) in \( H_c \) in equation (13) should be regarded as the real physical frequencies in which the renormalization and Lamb shifts due to the interactions with the environments have been included. With the master equation (13), we can obtain the equation of motion for the mean (or expectation value) of any physical operation \( O \) of the CPB–NR system by calculating \( \langle \dot{O}(t) \rangle = Tr[\dot{\rho}(t)] \). For convenience, in the following we denote variable \( O(t) \) as its expectation value \( \langle O(t) \rangle \), and will clarify its definition when there is a potential confusion. We thus have

\[
\frac{dS_-}{dt} = \left[ -\frac{1}{T_2} - i(\Delta + Q) \right] S_- + 2i\Omega S_z + 2\frac{i}{\hbar} \mu S_x e^{-i\delta t},
\]

(24)

\[
\frac{dS_z}{dt} = -\frac{1}{T_1} \left( S_z + \frac{1}{2} \right) - i\Omega (S_+ - S_-) - i\frac{\mu}{\hbar} (S_x e^{-i\delta t} - S_- e^{i\delta t}),
\]

(25)

\[
\frac{d^2 Q}{dt^2} + \gamma \frac{dQ}{dt} + \omega_r^2 Q = -8\omega_r^3 \lambda_0 S_z,
\]

(26)

where

\[ \lambda_0 = \frac{\lambda^2}{\omega_r^2}. \]

(27)

Please note that the variables \( S_- \), \( S_z \) and \( Q \) in equations (24)–(26) stand for the expectation values of their corresponding operators, respectively, except that the product of the variables \( Q \) and \( S_- \) in equation (24) should be regarded as \( \langle QS_- \rangle \). The decoherence time \( T_2 \) and excited-state relaxation time \( T_1 \) of the CPB qubit, and the decay rate \( \gamma \) of the NR are derived microscopically as

\[
T_2 = \left[ \frac{1}{\hbar} \left\{ \frac{\gamma_1}{2} \left( 1 + 2N(\omega_q) \right) \right\} + \frac{4}{\hbar} \left\{ \frac{\gamma_2}{2} \left( 1 + 2N(0) \right) \right\} \right]^{-1},
\]

(28)

\[
T_1 = \left[ \frac{2}{\hbar} \left\{ \frac{\gamma_1}{2} \left( 1 + 2N(\omega_q) \right) \right\} \right]^{-1},
\]

(29)

\[
\gamma = \frac{1}{2m \omega_r^2} \gamma_3.
\]

(30)

Note that if the pure dephasing coupling were dropped, i.e. \( \gamma_2 = 0 \), then \( T_2 = 2T_1 \). This is often the case for a two-level atomic system. After defining \( p = \mu S_- \), \( k = 2S_z \), we have

\[
\frac{dp}{dt} = \left[ -\frac{1}{T_2} - i(\Delta + Q) \right] p + i\frac{\mu^2 k \mathcal{E}}{\hbar},
\]

(31)

\[
\frac{dk}{dt} = -\frac{1}{T_1} \left( k + 1 \right) - 4 \Im(p \mathcal{E}^*) \frac{1}{\hbar},
\]

(32)

\[
\frac{d^2 Q}{dt^2} + \gamma \frac{dQ}{dt} + \omega_r^2 Q = -4\lambda_0 \omega_r^3 k,
\]

(33)

where \( \mathcal{E} = \mathcal{E}_c + \mathcal{E}_x e^{-i\delta t} \) and \( \mathcal{E}_c = \hbar \Omega / \mu \). Note that the \( Qp \) term in equation (31) should be regarded as the expectation value of \( \langle Qp \rangle \). The above equations cannot be solved as they are not closed.
In order to solve these equations, we first take the semiclassical approach by factorizing the NR and CPB qubit degrees of freedom, i.e. \( \langle Q p \rangle = \langle Q \rangle \langle p \rangle \). This ignores any entanglement between these systems. We will study the EIT behavior in the context of a weak signal (probe) current amplitude \( E_s \) in the presence of a strong control current amplitude \( E_c \). To obtain an analytical solution, we make the ansatz
\[
p = p_0 + p_1 e^{-\imath \lambda t} + p_{-1} e^{\imath \lambda t},
\]
\[
k = k_0 + k_1 e^{-\imath \lambda t} + k_{-1} e^{\imath \lambda t},
\]
\[
Q = Q_0 + Q_1 e^{-\imath \lambda t} + Q_{-1} e^{\imath \lambda t}.
\]
By substituting equations (34)–(36) into equations (31)–(33) and working to the lowest order in \( E_s \), we finally obtain in the steady state the following solutions:
\[
k_1 = \frac{2(T_1 / T_2) \Omega_0^2 k_0 \theta}{(T_1 / T_2) \delta_0 - 1 - 2(T_1 / T_2) \beta} \frac{E_s}{E_c}.
\]
\[
p_1 = \frac{[(4 \lambda_0 k_1 \omega_0^3) / (-\delta^2 - \imath \gamma \delta + \omega_0^2)] \cdot [(\mu^2 k_0 E_c) / (\Delta - 4 \lambda_0 \omega_0 k_0 - (\imath / T_2))] + \imath \mu^2 (k_0 E_s + k_1 E_c)}{\imath \hbar (\Delta - 4 \lambda_0 \omega_0 k_0 - \delta) + (\hbar / T_2)},
\]
where
\[
\theta = \frac{1}{\imath (\Delta_c - 4 \lambda_0 \omega_0 k_0 - \delta_0) + 1} + \frac{\imath}{\Delta_c - 4 \lambda_0 \omega_0 k_0 + \imath},
\]
\[
\beta = \frac{4 \lambda_0 \omega_0 \eta [(\Omega_0^2 k_0)(\Delta_c - 4 \lambda_0 \omega_0 k_0 - \imath) + \Omega_0^2 k_0]}{\imath (\Delta_c - 4 \lambda_0 \omega_0 k_0 - \delta_0) + 1} + \frac{4 \lambda_0 \omega_0 \eta [(\Omega_0^2 k_0)(\Delta_c - 4 \lambda_0 \omega_0 k_0 + \imath)] + \Omega_0^2 k_0}{-\imath (\Delta_c - 4 \lambda_0 \omega_0 k_0 + \delta_0) + 1},
\]
\[
\eta = \frac{\omega_0^2}{\omega_0^2 - 1 \gamma \delta_0 - \delta^2}.
\]
and dimensionless variables \( \omega_0 = \omega / T_2 \), \( \gamma_0 = \gamma / T_2 \), \( \delta_0 = \delta / T_2 \), \( \Omega_c = \Omega / T_2 \) and \( \Delta_c = \Delta / T_2 \) are introduced for later numerical convenience. The zero-order population inversion of the CPB is determined by the following equation:
\[
(k_0 + 1)[(\Delta_c - 4 \lambda_0 \omega_0 k_0)^2 + 1] + 4 \frac{T_1}{T_2} \Omega_0^2 k_0 = 0. \tag{42}
\]
The cubic equation (42) has either a single or three real roots. The latter case just corresponds to the intrinsic bistable states, which we will not discuss here.

To observe the EIT phenomena, we calculate the absorption power of the signal (probe) current as a function of its frequency. We note that the second term containing \( S_1 \) in equation (2) describes the Cooper-pair tunneling energy controlled by the external flux which is induced by the applied MW line current. By using the approximation of equation (7), the absorption power of the signal current can be expressed as
\[
P_{\text{abs}} = \pi \frac{\hbar E_j}{\phi_0} S_1 \frac{d\phi_s}{d\tau}, \tag{43}
\]
where $\phi_s = \mu_0 SI_s/(2\pi r)$ is the magnetic flux produced by the signal current $I_s = E_s \cos(\omega_s t)$. There are several terms in $S$, as indicated in equation (34) which is written in the frame rotating at the control current frequency $\omega_c$. It is obvious that only the terms $p_1 e^{-i\omega_c t}$ and $p_1^* e^{i\omega_c t}$ in the laboratory frame have nonzero contributions to the time-averaged absorption power which can be measured in experiments. In this way, after the time average, we have

$$P_{abs} = -2\mu E_s \omega_c \text{Im}(p_1),$$

(44)

where Im denotes taking the imaginary part, and $\mu = \mu_0 S\hbar E_j/(4r\phi_0)$.

3. Experimental parameters and results

We discuss how the EIT phenomena could be designed and realized in the CPB–NR system with realistically reasonable parameters. The typical values of the charging energy $E_c$ and the Josephson coupling energy $E_j$ of a CPB charge qubit are often designed such that $E_c \gg E_j$, so $E_c = 40$ GHz and $E_j = 2$ GHz are chosen. The values $\lambda = 50$ MHz and $\omega_r = 1$ GHz are used as in [4, 29, 30, 32]. The decay (relaxation) rate and decoherence rate of the CPB system depend on temperatures and the qubit operational points which could be controlled by the external gate voltage and magnetic flux. It has been reported that the relaxation rate $1/T_1 = 4$ MHz [31, 50] and the decoherence time $T_2$ can reach the order of a microsecond at the degeneracy point [51]. In our system, the CPB qubit may not be tuned at the degeneracy point, so it could be sensitive to the inevitable charge noise that is present in the circuits. Hence, we choose the decoherence rate conservatively to be $1/T_2 = 20$ MHz. The dominant mechanism for the damping of the resonator mode may come from the coupling to the phonon modes of the support, which could lead to the decay rate $\gamma = 0.01$ MHz [31]. For numerical convenience, we use dimensionless quantities for these quantities as follows: $\gamma_0 = \gamma T_2 = 5 \times 10^{-4}$, $\omega_0 = \omega_r T_2 = 50$ and $\lambda_0 = \lambda^2/\omega_r^2 = 2.5 \times 10^{-3}$. For $S = 1 \mu m^2$, $r = 1 \mu m$ and $E_c = 200 \mu A$, we have $\mu/\hbar = \mu_0 SE_j/(4r \phi_0) \approx 300$ GHz A$^{-1}$ and $\Omega_c = \Omega T_2 = (\mu/\hbar) E_j T_2 = 3$.

Figure 2 plots the absorption power of the signal current $P_{abs}$ of equation (44) as a function of the detuning $\Delta_s = (\omega_s - \omega_0) T_2$). The absorption power is scaled in units of $P_0$, which is the maximum value of $P_{abs}$ when $\Omega_c = 0$. In the absence of the control current ($\Omega_c = 0$), the solid curve shows a standard resonance absorption profile of the signal current in the CPB system, with the center of the curve shifted from the resonance $\omega_s = \omega_0$ a bit. This is due to the coupling $\lambda_0$ between the CPB and the NR [29, 32]. Furthermore, the resonant frequency shift increases with the increase in the coupling constant $\lambda_0$. When the control current is turned on ($\Omega_c = 3$), a narrow non-absorption hole appears at $\Delta_s = 0$ as shown in the dashed curve in figure 2. This indicates that the signal current has a narrow peak of induced transparency (no absorption). Without the coupling $\lambda_0$ between the CPB and NR, such a phenomenon will not appear. This can be seen from figure 3 in which a standard absorption resonance profile for $\lambda_0 = 0$ (solid line) appears. The central resonance peak also shifts a little bit from $\Delta_s = 0$ (i.e. $\omega_s = \omega_0$) due to the presence of the control current ($\Omega_c = 3$). When $\lambda_0 \neq 0$ (dashed and dot–dashed curves), the coupling prevents absorption of the signal (probe) current in a narrow portion of the resonance profile. Therefore, we call it RAIT. As we have chosen $\Delta_s = \omega_0$, the minima of the non-absorption holes (valleys) appear at $\Delta_s = 0$ in figures 2 and 3. Otherwise, they will move away from the point $\Delta_s = 0$. Recently, Radeonychev et al [52] have shown that resonant transparency could occur in a two-level quantum system induced via mechanical (acoustical) harmonic vibration of a solid medium along the propagation of multi-frequency

[New Journal of Physics 10 (2008) 095016 (http://www.njp.org/)]
laser radiation. They concluded that the atomic acoustical vibration plays a key role in the so-called acoustically induced transparency (AIT).

Alternatively, we may also understand how the RAIT could occur in the CPB and NR system as follows. We first consider the NR and CPB system without the application of the control and signal MW currents. The Hamiltonian, excluding the coupling to the external environments, from equation (8) is then

\[ H_c = \hbar \omega_q S_z + \hbar \omega_r a^+ a + 2\hbar \lambda (a^+ + a) S_z. \]  

(45)
\[ \Delta = \omega_r - \omega_c = \omega_q, \]

where \( \omega_r \) is the resonant frequency of the NR and \( \omega_q \) is the resonant frequency of the CPB qubit. A weak signal (probe) current with a tunable frequency \( \omega_s \) is applied, and its absorption power profile near the resonant frequency of the CPB qubit is measured.

The capacitive coupling between the CPB and the NR indicates that one charge state of the CPB will attract the NR and shift its equilibrium position near the CPB, whereas the other charge state will repel the CPB. Therefore, a displaced oscillator basis will allow us to use the ordinary harmonic oscillator formalism within each displaced potential well. The Hamiltonian of equation (45) can then be diagonalized \[53, 54\] in the eigenbasis of

\[ |\pm, N_{\pm}\rangle = |\pm\rangle_z \otimes e^{\frac{\lambda}{\omega_r}(a^+ - a)}|N\rangle, \] \( (46) \)

with the eigenenergies

\[ E_{\pm} = \pm \hbar \omega_q + \hbar \omega_r (N - \lambda_0), \] \( (47) \)

where the CPB qubit states \( |\pm\rangle_z \) are eigenstates of \( S_z \) with the excited state \( |+\rangle_z = |e\rangle \) and the ground state \( |-\rangle_z = |g\rangle \), the oscillator states \( |N_{\pm}\rangle \) are position-displaced Fock states, and \( \lambda_0 \) is defined in equation (27). Note that \( |+, N_+\rangle \) and \( |-, N_-\rangle \) form, respectively, an orthonormal basis with \( \langle M_+|N_+\rangle = \delta_{MN} \) and \( \langle M_-|N_-\rangle = \delta_{MN} \), but the states \( |N_+\rangle \) and \( |N_-\rangle \) are not mutually orthogonal and their inner products are given by \[53, 54\]

\[ \langle M_-|N_+\rangle = e^{-2\lambda_0} \left( \frac{2\lambda_0}{\omega_r} \right)^{N-M} \sqrt{\frac{M!}{N!}} L_M^{N-M}(4\lambda_0), \] \( (48) \)

where \( L^j_i(y) \) is the associated Laguerre polynomial. Thus, the coupling to NR could provide the CPB qubit with additional auxiliary energy levels to realize the EIT phenomena. For the parameters used in the simulations, the quantum three levels that may realize the EIT phenomena could be chosen as \( |-, N_-\rangle, |-, (N + 1)_-\rangle \) and \( |+, N_+\rangle \), illustrated in figure 4. In figure 4, the state \( |-, N_-\rangle \) has the same parity as state \( |-, (N + 1)_-\rangle \), and thus the effective electric–dipole transition is forbidden. On the other hand, the state \( |+, N_+\rangle \) is of opposite parity and thus has a nonzero effective electric–dipole coupling to both \( |-, N_-\rangle \) and \( |-, (N + 1)_-\rangle \) states. These conditions satisfy the specific restrictions on the configuration of the three levels (states) in atoms to realize EIT. That is, two of the three possible transitions between the states must be dipole-allowed, i.e. the transitions can be induced by an oscillating electric field. The third transition should be dipole-forbidden. In our setup, the strong control field (current)
is tuned on resonance between the $|+, N_+\rangle$ and $|-, (N+1)\rangle$ transition. The weak probe or signal current is tuned near resonance between the two states, $|+, N_+\rangle$ and $|-, N_\rangle$. Then the signal–current absorption power profile of the transition is measured. As in a conventional three-level \Lambda-type atomic system [36, 37], the strong control field current with the detuning equal to the vibration frequency of the NR drives the coupled CPB–NR system. As a result, the dressed CPB–NR coupled system becomes transparent for a weak signal current with a frequency matching the resonant frequency of CPB qubit. We note that the selection rule here is somewhat different from that of the superconducting flux-qubit circuit, which shows an EIT phenomenon in a \Lambda configuration [43, 44]. There, dipole-like coupling is allowed between all pairs of levels due to the symmetry breaking of the potential of the flux qubit [55].

Note that the decay rates of levels $|+, N_+\rangle$ are much larger than those of levels $|-, N_\rangle$ for arbitrary values of $N$ although they have roughly the same decoherence rates. This is because $|-, \rangle = |g\rangle$ is assumed to be the lowest electronic energy state of the CPB qubit and the phonon number decay rate $\gamma$ is taken to be much smaller than $T_1^{-1}$ and $T_2^{-1}$ ($\gamma T_2 = \gamma_0 = 5 \times 10^{-4}$ and $T_2/T_1 = 0.2$). Due to these decay and decoherence rates, the condition [37] for which all the important features of the RAIT remains considerably observable requires $\Omega^2 \gg (T_1 T_2)^{-1}$ or equivalently $\Omega^2 \gg (T_2/T_1)$. For the parameters chosen in figures 2 and 3 for our CPB–NR system, the condition is well satisfied. If, however, the value of $\omega_k$ decreases and the value of the ratio $(T_2/T_1)$ increases, then the EIT absorption hole (dip) will become shallow. In a two-level atomic system driven by a strong, resonant field, when the Rabi frequency of the driving field is greater than the atomic decay rate, the resonance fluorescence spectrum exhibits a three-peak structure, called a Mollow three-peak spectrum [56, 57]. If we set $\lambda_0 = 0$ (no interaction between the CPB qubit and the NR) and $\Delta_c = 0$ (i.e. the control current resonantly interacting with the CPB qubit) with $\Omega_c$ much larger than the decay rate of the CPB qubit, a Mollow three-peak spectrum but with different relative peak widths compared with a two-level atomic system could be expected in our CPB qubit system. The difference in the peak widths lies in the fact that usually $T_2/T_1 = 2$ in an atomic system (see equations (28) and (29) if the pure dephasing rate $\gamma_2 = 0$), whereas a typical value of $T_2/T_1 = 0.2$ is chosen for the CPB qubit system. If we apply a weak probe field (signal current) and calculate the signal-current absorption power profile, we find that the signal-current absorption power profile takes on negative values (see figure 5), representing stimulated emission rather than absorption [58, 59]. This amplification of the signal current may be understood to happen primarily at the expense of the strong driving (control) current, which experiences an increased attenuation rate. A similar amplification of the probe-field profile in a strongly driven two-level atomic system at optical frequencies was predicted in [58] and experimentally observed and verified in [59].

The essential point of RAIT in our system is that the absorption power of the signal current goes abruptly to zero (almost) for $\omega_k = \omega_c + \omega_\nu$. This gives us a method to measure the vibration frequency of the NR with high precision. The procedure is as follows. Fixing the frequency of the control current which is close to the resonant frequency of the CPB, and changing the frequency of the signal current, when the signal current becomes transparent, obviously we have $\omega_k = \omega_c + \omega_\nu$, i.e. the difference between the frequency of the control current and the signal current is the frequency of the NR, $\omega_\nu$. One can also see from figure 3 that the width of the absorption hole (valley) increases with the increasing values of $\lambda_0$. This can be understood as follows. With the increase in the coupling strength $\lambda_0$, the absorption peak shifts to the right, and the minima of the non-absorption hole (valley) appears, however, at $\Delta_s = 0$ (as we have chosen $\Delta_c = \omega_0$ mentioned above). As a result, the width of the absorption hole (valley) increases.
Figure 5. Signal current absorption profile for $\lambda_0 = 0$ (no interaction between the CPB and NR), $\Delta_c = 0$ (driven exactly on resonance) and $\Omega_c = 3$ (by a strong control field). The negative values of the absorption profile represent stimulated emission, i.e. amplification of the signal current. Other parameters used are $\gamma_0 = 5 \times 10^{-4}$ and $(T_2/T_1) = 0.2$.

Figure 6. Scaled absorption power of the signal current as a function of the NR decay rate $\gamma_0$ for $\Delta_s = 0$. Other parameters used are $\Omega_c = 3$, $\lambda_0 = 2.5 \times 10^{-3}$ and $\Delta_c = \omega_0 = 50$.

In figure 6, we draw the absorption power as a function of the NR decay rate $\gamma_0$ for $\Delta_s = 0$. Again, the absorption power is scaled in units of $P_0$, which is the maximum value of $P$ for the parameters chosen in figure 6. It shows that the absorption power of the signal current increases with the increase in the decay rate of the NR. In this way, we can determine the decay rate of the NR and investigate its relation with the environmental temperatures according to the absorption power of the signal current.

New Journal of Physics 10 (2008) 095016 (http://www.njp.org/)
4. Conclusion

In conclusion, we have demonstrated the EIT phenomena in a system consisting of a CPB qubit coupled to an NR. Though the CPB is an effective two-level system, the NR contributes additional auxiliary energy levels so that the EIT phenomena can be realized in such a system. Without the NR, the EIT phenomena will disappear. So we call our scheme RAIT. Our proposal, which is within the reach of current experimental technology, provides a detection scheme to measure physical properties, such as the vibration frequency and the decay rate of the coupled NR, or the decay and decoherence rates of the CPB qubit, if one set of values of either the CPB or NR properties is known by other means. We have specifically demonstrated here a case to measure the vibration frequency and the decay rate of the NR using the RAIT technique. Such a high-contrast and high-accuracy NR frequency measurement could potentially provide an alternative to other NR measurement schemes, such as magnetomotive detection.

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References

[1] Bocko M F and Onofrio R 1996 Rev. Mod. Phys. 68 755
[2] Roukes M 2001 Phys. World 14 (2) 25
   Craighead H G 2000 Science 290 1532
[3] Blencowe M 2004 Phys. Rep. 395 159
[4] Schwab K C and Roukes L C 2005 Phys. Today 58 36
[5] Knobel R G and Cleland A N 2003 Nature 424 291
[6] LaHaye M D, Buu O, Camarota B and Schwab K C 2004 Science 304 74
[7] Naik A, Buu O, LaHaye M D, Armour A D, Clerk A A, Blencowe M P and Schwab K C 2006 Nature 443 193
[8] Marshall W, Simon C, Penrose R and Bouwmeester D 2003 Phys. Rev. Lett. 91 130401
[9] Mancini S, Giovannetti V, Vitali D and Tombesi P 2002 Phys. Rev. Lett. 88 120401
[10] Santamore D H, Goan H-S, Milburn G J and Roukes M L 2004 Phys. Rev. A 70 052105
[11] Hensinger W K, Utami D W, Goan H-S, Schwab K, Monroe C and Milburn G J 2005 Phys. Rev. A 72 041405
[12] Bose S and Agarwal G S 2006 New J. Phys. 8 34
[13] Peano V and Thorwart M 2006 New J. Phys. 8 21
[14] Pirandola S, Vitali D, Tombesi P and Lloyd S 2006 Phys. Rev. Lett. 97 150403
[15] Xue F, Liu Y-X, Sun C P and Nori F 2007 Phys. Rev. B 76 064305
[16] Audenaert K, Eisert J, Plenio M B and Werner R F 2002 Phys. Rev. A 66 042327
[17] Plenio M B, Hartley J and Eisert J 2004 New J. Phys. 6 36

New Journal of Physics 10 (2008) 095016 (http://www.njp.org/)
