Irreversibility analysis in Marangoni forced convection flow of second grade fluid

Sohail A Khan, T Hayat, Ahmed Alsaedi and Q M Zaigham Zai

1 Department of Mathematics, Quaid-I-Azam University, 45320, Islamabad 44000, Pakistan
2 Nonlinear Analysis and Applied Mathematics (NAAM) Research Group, Faculty of Science, King Abdulaziz University, P O Box 80207, Jeddah 21589, Saudi Arabia
3 Department of Mathematics, COMSAT University Islamabad, Park Road, Tarlai Kalan, Islamabad, Pakistan

E-mail: sohailahmadkhan93@gmail.com

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Abstract
Marangoni forced convective MHD flow of second grade liquid is scrutinized. Heat source/sink, Joule heating and dissipation are addressed in energy equation. Physical aspects of entropy optimization with binary chemical reaction are addressed. Energy and entropy expressions are computed. Marangoni convection influenced on the surface pressure difference is calculated through temperature gradient, magnetic field and concentration gradient. Nonlinear PDE’s are reduced to ordinary one through suitable variables. Nonlinear system is computed for convergent solution by employing of OHAM. Characteristics of different influential parameters on entropy generation, concentration, temperature, Bejan number and velocity are graphically deliberated. Velocity enhances via Marangoni ratio parameter. Velocity and temperature have reverse effects for higher approximation of magnetic variable. For higher second grade fluid parameter the velocity is augmented. An increment occurs in temperature against higher values of Brinkman number and fluid parameter. Concentration decrease versus higher Marangoni ratio parameter. Entropy optimization upsurges for rising values of fluid parameters. Some relevant applications of Marangoni convection effect include atomic reactor, semiconductor processing, thin-film stretching, silicon wafers, soap films, material sciences, nanotechnology and applied physics etc. Entropy supports to progress the importance of numerous engineering and electronic devices development.

1. Introduction

Improvement of Marangoni forced convection is usually the dissipative boundary layer between two phase fluid flow like gas-liquid and liquid-liquid boundaries. Mass transportation along an interface between two liquids due to surface tension gradient is called as Gibbs-Marangoni effect (Marangoni effect). On the other hand if there is thermal dependence case, then the phenomenon called Bénard-Marangoni convection (thermocapillary convection). Marangoni convection depends upon the difference of surface pressure computed by gradient of temperature, magnetic effect and concentration gradients. Some important applications of Marangoni convection effect like atomic reactor, semiconductor processing, thin-film stretching, silicon wafers, soap films, material sciences, nanotechnology and applied physics etc. Marangoni convection is extensively used in the coloring on the ground, for instance, fine art mechanism. The most important manufacturing applications of the Marangoni convection concept are melting and welding processes. Basic concept of mass and heat transportation phenomenon in Marangoni boundary layer flow are comprehensively discussed. Impact of Brownian movement and thermophoresis effect in viscous liquid subject to Marangoni forced convection is highlighted by Sheikholeslami and Chamkha [1]. Rassol et al [2] scrutinized the Marangoni convection in MHD flow of second grade nanoliquid. Behavior of Marangoni forced convection in MHD Casson liquid flow is studied by Mahanthesh et al [3]. Hayat et al [4] scrutinized Marangoni convection impact in water based CNTs by heated impermeable stretchable surface. Characteristics of space dependent heat source/sink in MHD
radiative carbon nanotubes flow subject to an infinite disk with Marangoni convection impact is investigated by Mahanthesh et al [3]. For instance the MHD Marangoni forced convective flow of Casson nanoliquid with dissipation is illustrated by Shafiq et al [6]. Chen [7] reported Marangoni convection behavior in time-dependent power-law liquid flow by a stretchable surface. Impact of surface tension in magnetohydrodynamic unsteady viscous liquid flow with Marangoni convection effect is exemplified by Rudraiah et al [8]. Thermocapillarity effect in time dependent fluid flow over a stretchable sheet is studied by Dandapat et al [9]. Lin et al [10] reported the Marangoni effect in pseudoplastic nanofluid flow. Zhang et al [11] reported Marangoni convection flow with external pressure using Padé approximant.

Second law of thermodynamics states that irreversibility can be created in any processes and never destroyed in a system. Entropy rate is used to augments the system performance. An important aspect of thermodynamic second law is that entropy rate of any process in system is constantly positive or zero. Entropy production is produced because of heat fluxes, friction between solid surfaces, Joule heating, diffusion, mass fluxes, dissipation and Joule-Thomson effect etc. For higher irreversibility in a thermodynamical system there are additional probabilities of entropy generation and consequently significant of thermodynamical system diminishes. Thermodynamics second law affords efficient method and entropy optimization to reduce resistance in the system. It supports to progress the importance of numerous engineering and electronic devices development. Primarily Bejan [12] adapted the irreversibility analysis technique. Irreversibility analysis in MHD dissipative flow of non-Newtonian nanoliquid with Brownian diffusion thermophoresis impact over a stretchable sheet is explored by Hayat et al [13]. Impact of entropy optimization in viscous dissipative flow subject to radial magnetic field in infinite concentric cylinders is highlighted by Yusuf and Oni [14]. Li et al [15] explored entropy rate in force convective flow of nanoliquid with helical twisted tape. Impact of entropy rate in Prandtl-Eyring nanofluid by a stretchable surface is addressed by Khan et al [16]. Characteristics of heat transportation in MHD flow of Casson liquid with irreversibility exploration by a vertical cylinder is deliberated by Reddy et al [17]. Irreversibility exploration in Darcy-Forchheimer nanofluid flow by a curved stretchable surface is demonstrated by Hayat et al [18]. Irreversibility in radiative flow of second grade nanomaterial by a stretchable surface with dissipation is explored by Sithole et al [19]. Alsaadi et al [20] addressed the entropy rate in MHD non-Newtonian nanoliquid flow with homogenous and heterogeneous chemical reactions. Sobamowo and Akinshilo [21] scrutinized entropy rate and thermal flux in fourth grade liquid with variable viscosity. Some attempts about irreversibility analysis are mentioned in [22–38].

The abovementioned study witness that no effort has been made to deliberate the influence of entropy optimization in Marangoni forced convection in MHD flow of second grade fluid. However in recent times there are numerous researchers and scientists that scrutinize the Marangoni forced convective flow of second grade fluid with Joule heating and dissipation effects. Here our prime objective is to analyze the irreversibility exploration in Marangoni forced convection in MHD flow of second grade fluid. The solutal and thermal capillaries behaviors are vital factors in Marangoni convection of liquid and nanoliquid. Joule heating, viscous dissipation and heat generation/absorption are considered in energy equation. Physical aspects of irreversibility with binary chemical reaction are addressed. Optimal homotopy analysis technique is implemented for convergent series solutions [39–44]. Characteristics of several involved variables on velocity, Bejan number, concentration, entropy rate and temperature are deliberated.

2. Statement of problem

Marangoni forced convection magnetohydrodynamic flow of second grade liquid is addressed. Flow is generated due to concentration and temperature gradients. Marangoni influence is exploited to apprehend the flow of liquid in forward direction. Joule heating, heat generation/absorption and dissipation in energy equation are discussed. First order chemical reaction is present. Salient effects of entropy rate and binary chemical reaction are examined. Let x–direction is along surface and y–axis being normal to sheet. Magnetic field (B_0) is exerted normal to the sheet. Temperature and concentration both as taken as the functions of x. The flow geometry is highlighted in figure 1.

Governing equations satisfy [1–4]:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \alpha_1 \left( \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} + \frac{\partial^3 u}{\partial x \partial y^3} - \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^3 u}{\partial y^3} \right) - \frac{\sigma B_0^2}{\rho} \frac{\partial u}{\partial y},
\]
\[
\begin{align*}
\frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} &= \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{(\rho \chi)} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\alpha_1}{(\rho \chi)} \left( \frac{\partial^2 u}{\partial y \partial x} + \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} \right) \\
&+ \frac{\sigma_1 B_0^2}{(\rho \chi)} u^2 \tau + \frac{Q_0}{(\rho \chi)} (T - T_\infty)
\end{align*}
\]

(3)

\[
\begin{align*}
\frac{\partial C}{\partial x} + \nu \frac{\partial C}{\partial y} &= D_B \frac{\partial^2 C}{\partial y^2} - k_C (C - C_\infty)
\end{align*}
\]

(4)

with

\[
\begin{align*}
\left. \mu \frac{\partial u}{\partial y} \right|_{y=0} &= \left. \frac{\partial u}{\partial y} \right|_{y=0} = \sigma_0 \left( \gamma_T \frac{\partial T}{\partial x} \right)_{y=0} + \gamma_C \frac{\partial C}{\partial x} \right|_{y=0}, \\
v(x, 0) &= 0,
\end{align*}
\]

(5)

In above expression \(u\) and \(v\) represent the velocity components, \(\mu\) the dynamic viscosity, \(\rho\) the density, \(\alpha\) the second grade liquid parameter, \(\sigma_1\) the electrical conductivity, \(\sigma\) the surface tension, \(\alpha\) the thermal conductivity, \(T\) the temperature, \(c_p\) the specific heat, \(T_\infty\) the ambient temperature, \(L\) the reference length, \(Q_0\) the heat generation/absorption coefficient, \(T_0\) the wall temperature, \(D_B\) the mass diffusivity, \(C\) the concentration, \(C_\infty\) the ambient concentration, \(k_C\) the reaction rate and \(C_0\) the wall concentration.

Let surface tension \((\sigma)\) as a linear function of temperature and concentration [1, 2]:

\[
\sigma = \sigma_0 - \gamma_T (T - T_\infty) - \gamma_C (C - C_\infty),
\]

(6)

with

\[
\gamma_T = -\frac{\partial \sigma}{\partial T} \bigg|_{T=T_\infty}, \quad \gamma_C = -\frac{\partial \sigma}{\partial C} \bigg|_{C=C_\infty}.
\]

(7)

where \(\gamma_T, \sigma_0\) and \(\gamma_C\) show the positive constants.

Considering

\[
\begin{align*}
\psi &= \nu x f (\eta), \\
u &= \frac{\partial \phi}{\partial x}, \\
v &= -\frac{\partial \phi}{\partial x}, \\
\eta &= \frac{x}{L}, \\
T &= T_\infty + T_0 X^2 \theta (\eta), \\
C &= C_\infty + C_0 X^2 \phi (\eta),
\end{align*}
\]

(8)

we get

\[
\begin{align*}
f'''' + f'''' - f''^2 + \beta (2 f' f'' - f'') - M f'' = 0,
\end{align*}
\]

(9)

\[
\begin{align*}
\theta'' + Pr \theta'' - 2 Pr \theta' + Br \theta'' + \beta \theta (2 f' f'' - f'') + M f'' + Pr Q \theta = 0,
\end{align*}
\]

(10)

\[
\begin{align*}
\phi'' + Sc f \phi'' - 2 Sc f' \phi - \gamma S c \phi = 0,
\end{align*}
\]

(11)

\[
\begin{align*}
f (0) = 0, \\
f'' (0) = -2 (1 + M_a), \\
\theta (0) = 1, \\
\phi (0) = 1, \\
f' (\infty) = 0, \\
\theta (\infty) = 0, \\
\phi (\infty) = 0.
\end{align*}
\]

(12)
Here \( M \left( \frac{L^2n^2b^2}{\rho} \right) \) shows the modified Hartman number, \( \beta \left( \frac{\alpha_1}{\mu^2} \right) \) the second grade fluid parameter, \( M_a \left( \frac{\gamma}{\alpha_2 \mu} \right) \) Marangoni ratio parameter, \( Pr \left( \frac{\nu}{\alpha} \right) \) the Prandtl number, \( Q \left( \frac{Qd^2}{\gamma\rho} \right) \) the heat generation/absorption parameter, \( Ec \left( \frac{\nu c^2}{\alpha} \right) \) the Eckert number, \( Sc \left( \frac{\nu}{D} \right) \) the Schmidt number, \( Br \left( \frac{Pr}{Ec} \right) \) the Brinkman number and \( \lambda \left( \frac{kLr}{\rho} \right) \) the chemical reaction parameter.

3. Entropy modeling

Entropy production is produced because of heat fluxes, friction between solid surfaces, Joule heating, diffusion, mass fluxes, dissipation and Joule-Thomson effect etc. Here entropy is produced due to heat and mass irreversibilities, liquid friction irreversibility and Joule heating irreversibility [13–15] i.e.

\[
S_G = \frac{k}{\theta^2} \left( \frac{\partial \theta}{\partial y} \right)^2 + \mu \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\alpha_1}{\theta} \left( \frac{\partial u}{\partial y} \right)^2 \left( \frac{\partial V}{\partial x} \right) \left( \frac{\partial V}{\partial y} \right) + \frac{\sigma B_i^2}{\theta^2} u^2 \right). 
\]

or

\[
N_o = \theta^2 + \frac{Br}{A} f n^2 + \frac{Pr}{A} f^2 f^2 - f f f f + M B r f^2 + L \theta^2 \phi' + L \phi'^2. \]

4. Bejan number

It is the ratio of heat and mass irreversibility to total irreversibility.

\[
\text{Bejan number} = \frac{\text{Entropy rate due to heat and mass transfer}}{\text{Total entropy}}, 
\]

or

\[
Be = \frac{\theta^2 + \frac{Br}{A} f n^2 + \frac{Pr}{A} f^2 f^2 - f f f f + M B r f^2 + L \theta^2 \phi' + L \phi'^2}{\theta^2 + \frac{Br}{A} f n^2 + \frac{Pr}{A} f^2 f^2 - f f f f + M B r f^2 + L \theta^2 \phi' + L \phi'^2}. 
\]

Here \( N_o \left( \frac{Sc \nu^2}{k} \right) \) shows the entropy rate, \( L \left( \frac{k \nu c^2}{k} \right) \) the diffusion parameter and \( A \left( \frac{\nu c^2}{k} \right) \) the dimensionless parameter.

5. Solutions

The initial guesses and linear operators for OHAM solutions can be expressed [39–44]:

\[
\begin{align*}
   f_0(\eta) &= 2(1 + r)(1 - e^{-\eta}), \\
   \theta_0(\eta) &= e^{-\eta}, \\
   \phi_0(\eta) &= e^{-\eta}, \\
   L_f &= \frac{\partial}{\partial \eta^2} - \frac{\partial}{\partial \eta}, \\
   L_\theta &= \frac{\partial}{\partial \eta^2} - 1, \\
   L_\phi &= \frac{\partial}{\partial \eta^2} - 1, \\
\end{align*}
\]

with

\[
\begin{align*}
   L_f &= [z_0 + z_1 e^\eta + z_2 e^{-\eta}], \\
   L_\theta &= [z_4 e^\eta + z_5 e^{-\eta}], \\
   L_\phi &= [z_6 e^\eta + z_7 e^{-\eta}], \\
\end{align*}
\]

in which \( c_i (i = 1, 2, 3, ..., 7) \) denote the arbitrary constants.

Considering \( h_f, h_\theta \) and \( h_\phi \) as the auxiliary variables and \( q \) the embedding parameter \( q \in [0, 1] \) one has the following zero order deformation problems:

\[
(1 - p) L_f[F(\eta; p) - f_0(\eta)] = ph_f \Theta_f L_f[F(\eta; p)], 
\]

(20)
The average square residual errors as given by Liao
\[ (1 - p) L_n[\theta(\eta; p) - \theta_0(\eta)] = ph\theta R_n L \theta(\eta; p) \],
\[ (1 - p) L_n[\phi(\eta; p) - \phi_0(\eta)] = ph\phi R_n L \phi(\eta; p) \],
\[ F''(0; p) = -2(1 + \tau), \quad F(0; p) = s, \quad F'(\infty; p) = 0, \quad \theta(0; p) = 1, \quad \phi(0; p) = 1, \quad \phi(\infty; p) = 0. \]
\[ \theta(\infty; p) = 0, \quad \phi(0; p) = 1, \quad \phi(\infty; p) = 0. \]

Definitions of operators are
\[ L_f = \frac{\partial^2 F(\eta; p)}{\partial \eta^2} + F(\eta; p) \frac{\partial^2 F(\eta; p)}{\partial \eta^2} - \left( \frac{\partial F(\eta; p)}{\partial \eta} \right)^2 - M \left( \frac{\partial F(\eta; p)}{\partial \eta^2} \right)^2 \]
\[ \beta \left[ 2 \frac{\partial F(\eta; p)}{\partial \eta} \frac{\partial^2 F(\eta; p)}{\partial \eta^2} - F(\eta; p) \frac{\partial^2 F(\eta; p)}{\partial \eta^2} \right] + Pr \theta(\eta; p), \]
\[ \beta \left[ \frac{\partial^2 \phi(\eta; p)}{\partial \eta^2} + Sc \left( \frac{\partial \phi(\eta; p)}{\partial \eta} \right) - 2Sc \left( \frac{\partial \phi(\eta; p)}{\partial \eta} \right) \right] - \gamma Sc \phi(\eta; p). \]

The problem for mth order are
\[ L_1 f_m - \chi_m f_{m-1} = h_f R^f_m, \]
\[ L_2 \theta_m - \chi_m \theta_{m-1} = h_\theta R^\theta_m, \]
\[ L_2 \phi_m - \chi_m \phi_{m-1} = h_\phi R^\phi_m, \]
\[ \frac{\partial^2 f_m}{\partial \eta^2} \bigg|_{\eta=0} = 0, \]
\[ \frac{\partial f_m}{\partial \eta} \bigg|_{\eta=0} = 0, \]
\[ \frac{\partial f_m}{\partial \eta} \bigg|_{\eta=0} = 0, \]
\[ R^f_m = f''_{m-1} + \sum_{k=0}^{m-1} f_{m-1-k} f''_k + \beta \left( \sum_{k=0}^{m-1} f''_{m-1-k} f''_k - \sum_{k=0}^{m-1} f_{m-1-k} f''_k \right), \]
\[ R^\theta_m = \theta''_{m-1} + \sum_{k=0}^{m-1} \theta_{m-1-k} \theta''_k - 2Pr \sum_{k=0}^{m-1} \theta_{m-1-k} \theta''_k + MBr \sum_{k=0}^{m-1} f''_{m-1-k} f''_k + PrQ \theta_{m-1}, \]
\[ R^\phi_m = \phi''_{m-1} + \sum_{k=0}^{m-1} \phi_{m-1-k} \phi''_k - 2Sc \sum_{k=0}^{m-1} \phi_{m-1-k} \phi''_k = \gamma Sc \phi_{m-1}, \]
\[ x_m = \begin{cases} 0, & m \leq 1 \\ 1, & m > 1 \end{cases} \]

6. Convergence analysis

The average square residual errors as given by Liao [39–44] satisfy:
\[ \varepsilon^f_m = \frac{1}{k+1} \sum_{j=0}^{k} N_j \left( \sum_{j=0}^{m} f(\eta) \right)^2, \]
Total squared residual error satisfies \([39-44]\):

\[
\varepsilon_m^f = \varepsilon_m^f + \varepsilon_m^\theta + \varepsilon_m^\phi,
\]

in which \(\varepsilon_m^t\) denotes total squared residual error.

Total squared residual error is indicated in figure 1. Individual averaged squared residual errors versus the convergence control parameters are given in table 1.

### Table 1. Computational iterations for individual averaged squared residual errors.

| \(m\) | \(\varepsilon_m^f\) | \(\varepsilon_m^\theta\) | \(\varepsilon_m^\phi\) |
|------|----------------|----------------|----------------|
| 2    | 0.00129108     | 0.00021776     | 0.00016193     |
| 3    | 1.24757 \times 10^{-6} | 2.6565 \times 10^{-7} | 4.97071 \times 10^{-6} |
| 6    | 3.15714 \times 10^{-9} | 2.45307 \times 10^{-4} | 5.73078 \times 10^{-2} |
| 10   | 1.20177 \times 10^{-11} | 2.81922 \times 10^{-10} | 9.66539 \times 10^{-9} |
| 14   | 5.94258 \times 10^{-14} | 4.10366 \times 10^{-10} | 1.97396 \times 10^{-9} |
| 18   | 3.57711 \times 10^{-16} | 7.00303 \times 10^{-11} | 4.54003 \times 10^{-9} |
| 22   | 2.51698 \times 10^{-18} | 1.32938 \times 10^{-13} | 1.13256 \times 10^{-9} |
| 26   | 2.00573 \times 10^{-20} | 2.72049 \times 10^{-12} | 2.9978 \times 10^{-10} |

Figure 2. Total residual error.
7. Discussion

Here we implemented optimal homotopic analysis technique to get the convergence series solution for nonlinear system. Prominent behavior of influential parameters for velocity, Bejan number, concentration, entropy rate and temperature are analyzed.

7.1. Velocity

Influence of various pertinent parameter like (\(\beta\)), (\(M\)) and (\(Ma\)) on velocity \(f'(\eta)\) are shown in figures 3–5, Figure 3 depicts the effect of (\(\beta\)) on \(f'(\eta)\). Here \(f'(\eta)\) enhances versus \(\beta\). It is because of higher estimation of \(\beta\) the viscosity of liquid reduces and thus velocity \(f'(\eta)\) upsurges. Figure 4 reveals the characteristics of \(M\) on velocity \(f'(\eta)\). For higher magnetic parameter \(M\) the more Lorentz force opposed fluid motion and thus velocity decays. Figure 5 displayed the effect of Marangoni ratio variable \(Ma\) on velocity. One can find that velocity boosts against Marangoni ratio parameter.

7.2. Temperature

Figures 6–10 are developed for influences of \(Ma\), \(\beta\), \(M\), \(Q\) and (Br) on temperature \(\theta(\eta)\). Figure 6 evaluates characteristic of Marangoni ratio parameter on \(\theta(\eta)\). Temperature is increased versus higher Marangoni ratio parameter. Characteristics of \(\beta\) on \(\theta(\eta)\) are portrayed in figure 7. Here temperature upsurges against larger \(\beta\). Impact of \(M\) on \(\theta(\eta)\) is shown in figure 8. Clearly higher magnetic parameter yields more Lorentz forces which improve resistance to liquid flow and therefore temperature boosts. Figure 9 elucidated impact of \(Q\) on \(\theta(\eta)\). Clearly temperature augmented against \(Q\). Effect of \(Br\) on \(\theta(\eta)\) is displayed in figure 10. Clearly temperature is increased against \(Br\). It is because of higher Brinkman number corresponds to slow the heat produced by dissipation effect and hence temperature upsurges.

7.3. Concentration

Influence of Marangoni ratio parameter on \(\phi(\eta)\) is illustrated in figure 11. For higher \(Ma\) the concentration decays. It is due to the surface tension created by concentration and temperature gradients. Variation of \(Sc\) on

| Pr | Sit hole et al. [19] | Olanrewaju et al. [45] | Recent results |
|----|---------------------|------------------------|----------------|
| 0.5 | 0.21441547          | 0.214368               | 0.214365       |
| 0.7 | 0.24976956          | 0.250142               | 0.250141       |
| 1.0 | 0.28782508          | 0.289161               | 0.289157       |
| 2.0 | 0.35519994          | 0.356176               | 0.356169       |
concentration is displayed in figure 12. Here mass diffusivity decays for higher Sc and consequently reduction occurs in concentration. Figure 13 examines the behavior of (γ) on φ(η). Clearly for higher (γ) the liquid behaves thicker and thus concentration decays.
7.4. Entropy optimization and Bejan number

Influence of Brinkman number on $N_G$ and $Be$ is interpreted in figures 14, 15. In fact $Br$ is the thermal generated source within the liquid flow region. The heat created together with the thermal transmission from the wall.
improves the disorderness of the thermal system and therefore entropy generation rises. $Be$ decays versus higher Brinkman number. We clearly noticed that the viscous effect is dominant over thermal irreversibility. Influence of $(\beta)$ on $Be$ and $N_G$ is highlighted in figures 16, 17. Clearly for higher approximation of $(\beta)$ $N_G$ is augmented.
Bejan number diminished against larger $\beta$. Variation of $N_G$ and $Be$ versus magnetic parameter ($M$) are revealed in figures 18, 19. Clearly higher magnetic parameter ($M$) yield more Lorentz forces which improves the resistance to liquid flow It augments the disorderedness in thermal system and consequently $N_G$ augmented.
One can clearly find that for larger \((M)\) the Bejan number reduced. In fact Joule heating irreversibility dominants over thermal and solutal irreversibility. Figures 20, 21 are developed to examine the effect of \((L)\) on \(Be\) and \(N_G\). As expected both \(N_G\) and \(Be\) upsurge for larger diffusion parameter \((L)\).
8. Conclusions

Marangoni forced convection magnetohydrodynamic flow of second grade liquid is addressed. Flow is generated due to concentration and temperature gradients. Entropy generation is developed through second law of thermodynamics. A physical feature of irreversibility exploration is examined. Marangoni influence is
exploited to examine the flow of liquid in forward direction. Joule heating, heat generation/absorption and dissipation in energy equation are discussed. First order chemical reaction is present. The key result are given below.

- For higher approximation of ($\beta$) the velocity ($f'(\eta)$) upsurges.
- Velocity has opposite effect against ($M$) and ($M_b$).
- For higher ($M_b$) the temperature is enhanced.
- Similar impact of temperature is observed via ($\beta$) and ($Br$).
- $\theta(\eta)$ augments for larger ($M$) and ($Q$).
- Concentration decays for higher ($M_b$).
- $\phi(\eta)$ is a decreasing function of ($Sc$) and ($\gamma$).
- $N_b$ and $Be$ are increased for higher ($L$).
- $N_b$ and $Be$ have opposite effects for larger ($M$).
- For higher second grade fluid parameter ($\beta$), both $N_b$ and $Be$ have reverse effects.

The current effort is basic and modeling about such problem can be done for third grade fluid model subject to diffusion-thermo and thermal-diffusion effects, dissipation and Joule heating effects with rheological characteristics and stretchable phenomenon with porous medium through modified Darcy’s laws.

ORCID iDs

Sohail A Khan © https://orcid.org/0000-0001-8240-6044

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