PAPER

Darcy-Forchheimer flow of MHD nanofluid thin film flow with Joule dissipation and Navier’s partial slip

Muhammad Jawad¹, Zahir Shah¹, Saeed Islam¹, Ebenezer Bonyah² and Aurang Zeb Khan³

¹ Department of Mathematics, Abdul Wali Khan University, Mardan, KP, Pakistan
² Department of mathematics Education, University of Education Winneba-(Kumasi Campus), Kumasi 00233, Ghana
³ Department of Physics, Abdul Wali Khan University, 23200 Mardan, KP, Pakistan

E-mail: ebbonya@gmail.com

Keywords: nanofluids, liquid films, unsteady stretching surface, magnetic field, HAM

Abstract

In this paper investigation is carried out on two dimensional liquid film with heat generation/absorption and variable heat transmission of nanofluid MHD flow on an unsteady stretching sheet. Flow of nanofluid phenomenon is model from the basic governing time-dependent equations. By the use of suitable similarity transformation, these basic equations are transformed to differential equations system. The nanofluid is supposed to slip along the boundary of the sheet. To find the solution of the transformed modeled equations Homotopy Analysis technique is used. A numerical survey is presented for the convergence of the implemented technique. Effects of variations of different influential parameters like Nu number and Cfₓ for fluid flow of liquid film with mass and heat transfer is observed. The effect of unsteadiness parameter S over thin film is explored analytically for different values. It is investigated that for large values of M that the nanofluid films velocity distribution decreases, where increase in the value of K₁, a reduction in the porous medium permeability. Thickness of thermal boundary layer decreases with increasing values of S, while increase of radiation parameter, the Nusselt number also increases. Furthermore, the embedded parameters used for comprehension of the physical presentation, like inertial parameter F₁, magnetic parameter M, permeability parameter K₁, Eckert number Ec, Prandtl number Pr, and parameters ε₁, ε₂ and γ has been presented by graphs and discussed in detail.

Nomenclature

Pr Prandtl number
Cfₓ Skin friction coefficient
Ec Eckert number
Qₓ Heat flux
ϕ Nanoparticle volume friction
S Unsteadiness parameter
K Permeability parameter
F₁ Inertia parameter
K₁ Permeability parameter
Tₛ Slit Temperature
Tᵣ Position Temperature
uₓ, v Velocity components (m/s⁻¹)
Cₚ Specific heat (J/kgK)

© 2018 The Author(s). Published by IOP Publishing Ltd
1. Introduction

Enrichment of heat transferred by the accumulation of nano size atoms to a base liquid is a fast-growing field of attention and is used in numerous industries from floor heating and micro channel cooling to heat revival systems, which has thrived in the recent decades. Nano fluid have single-phase heat transfer coefficients and higher thermal conductivity than the bottom liquids. In a porous media, the convection flow has been extensively investigated in present years.

The stretching surface is formed by boundary layer flows frequently occur in many engineering use such as in fake fibers, metal spinning, permanent casting, glass blow and in the sketch of plastic films etc The nano-scaled particles have suggested specific consideration as significance of barriers in pressure drop or making the mixture homogenous for all particle dimensions. Since these particles are almost in the identical size of the base fluid molecules, they highly identify stable suspensions during a long period of time.

Khan [1] has described nano fluid flow with mass and heat transfer for Buongiorno’s model. Mahdy et al [2] have studied an unsteady contracting cylinder, through which nano fluid flows in occurrence of heat transfer by applying Buongiorno’s model. Malvandi et al [3] have discussed nano fluid flow through a vertical annular pipe.
In [4–6] scientists have investigated nanofluid flow over a stretching sheet. Ellahi et al [7] have deliberated non-Newtonian nanofluid flow with MHD effect through a pipe. Nadeem et al [8] have discussed nanofluid flow through a cone. Fakour et al [9] have described nanofluid thin film flow by transfer of heat from stretching sheet. Abolbashari et al [10] have described Casson nanofluid flow for analytical modelling of entropy generation. Nadeem et al [11] have investigated the flow of nano particles present on stretching sheet of MHD Maxwell fluid. Rokni et al [12] have studied nanofluid MHD flow in occurrence of heat transfer involving two plates. Shehzad et al [13] have described the nanofluid MHD flow with convective boundary condition of Jeffrey fluid model. Mahmood et al [14] have discussed nanofluid flow for cooling purposes. Fakour et al [15] have designated nanofluid flow in a channel having porous walls by incomes of heat conduction. Hatami et al [16] have described laminar flow of nanofluid with heat transfer passes through rotating and contracting disks. Nadeem et al [17] have discussed Non-Newtonian and non-orthogonal nanofluid flow at stagnation point in occurrence of heat transfer. Sheikholeslami et al [18] have described a semi-porous channel nanofluid flow with MHD effect. Akbar et al [19] have discussed nanofluid flow in the existence of buoyancy and viscosity effects with MHD from a straight up stretching sheet. Fakour et al [20] have investigated nanofluid through an erect channel. Akbar et al [21] have described nano particles flow through stretching sheet with water based stagnation point. Kumar et al [22] have described nanofluid flow in a thermal field. Recently Shah et al [23–26] have investigated rotating nanofluid flow between parallel plates. The most current investigational and theoretical research of Sheikholeslami on nanofluids using dissimilar phenomena, with modern application, possessions and properties with usages of diverse approaches can be studied in [27–31] in which he shown importance of nanofluid in nanotechnology. Ellahi et al [32] have investigated nanoliquid on MHD Poiseuille flow with variable thermal conductivity.

The flow exploration of thin film flow has got significant dedication due to its enormous usages and application in range of technology, industries and engineering in the current few years. Flow problems on thin film realized in many fields, and expanded from flow in lungs to lubrication problems, which may be unique of the major subfield of thin film problems. The study of applied applications of thin film flow can be an exciting interaction among fluid mechanics, structural mechanics, and theology. In observation of the above mentioned uses, it become a significant topic for researchers to the development of the study of fluid film on stretching surface. Fluid film flow was paramount investigated for viscous flow and further it is extended to Non-Newtonian fluids. Wua et al [33] have studied MHD thin film instability of fluid. Hameed et al [34] have described thin film non-Newtonian fluid flow with MHD effect on vertically moving belt. Shankar et al [35] have investigated thin fluid films in the presence of electric field. Lampe et al [36] have described the influence dynamics of drops on thin film of viscoelastic worm. Sandeep et al [37] have described heat transfer of thin film flow with graphene nanoparticles. Aziz et al [38] have studied flow of thin film with transfer of heat on stretching unsteady sheet. Chen [39] has studied non-Newtonian liquid film with viscous dissipation effect and transfer of heat through an unsteady stretching sheet. Magahra [40] has discussed thin Casson fluid flow with variable heat flux and heat transfer on stretching sheet. Wang [41] has described flow on stretching sheet of fluid film. Andersson et al [42] have examined power law fluid film over an unsteady stretching sheet. Anderssona et al [43] have discussed flow thin film flow with transfer of heat over an unsteady stretching sheet. Seth et al [44] have studied Casson thin fluid flow in non-Darcy porous medium in the presence of Navier’s partial slip and Joule dissipation. Mohsan et al [45] have studied ferrofluid flow with heat transfer which effects the particle shape. Khan et al [46] have discussed Fourth grade fluid propagating with mass transport through a curved channel in the presence of magnetic effects. Bhatti et al [47] have described effect of heat transfer on MHD flow through a Darcy-Brinkman-Forchheimer Porous medium. Ellahi et al [48, 49] have discussed Slip on heat transfer boundary layer flow over a moving plate based on specific entropy generation in the presence of MHD. Fetecau et al [49] have investigated magnetic effects and Combine porous of Newtonian fluids over an infinite plate.

In field of engineering and science Mathematical problems are multi part in its nature and the exact solution of such type of problem is too difficult. Numerical and analytical methods are used to calculate an approximate solution. The Homotopy Analysis Method is one of the widespread and important technique for the solution of such type of problems. Liao in (1992) [50] has observed that this technique is fast convergent to the approximate solution and it is best fit for the solution of nonlinear problems. A solution obtained by this method is a series solution of a particular variable function [51–54].

2. Problem formulation

In this research, two dimensional flow of nanofluid through an unsteady elastic thin sheet is considered. The effect of heat transfer and constant magnetic field of strength B₀ is considered in the flow. Stretching sheet moves with a velocity \( \hat{U}_0 (x, t) \), given as
\[ \tilde{U}_x(x, t) = \frac{\gamma x}{1 - \zeta t}, \]

Where \( \zeta \) and \( \gamma \) represents stretching parameters and \( y \)-axis is vertical to it. Temperature of the wall of the liquids is taken, and is defined as [44]

\[ T_s = T_0 - T_r \left( \frac{dx^m}{k\sqrt{c/\nu}} \right) (1 - \zeta t)^{-n}, \]

Here \( \nu \) denotes kinematic viscidness of liquid, \( \zeta \) stretching parameter, \( c \) and \( k \) are arbitrary constant of thermal conductivity, \( r \) and \( m \) space indices and \( T_r, T_s \) represents the slit and position temperatures correspondingly.

The external magnetic field is defined as

\[ \tilde{B}(t) = \tilde{B}_0(1 - \zeta t)^{-0.5}, \]

Where \( \tilde{B}_0(t) \) denotes the applied magnetic field.

Furthermore, the surface heat flux is supposed to be changing with respect to time and distance, specified by the equation as [44]

\[ q(x, t) = -kT_y = -T_r \frac{dx^\nu}{(1 - \zeta t)^{m+\frac{1}{2}}}. \]

For two dimensional nanofluid flow equation (5) is reduced to the form [37–44]

\[ u_x + u_y = 0, \]

In the light of our assumptions equation (9) reduces to [37–44]

\[ u_t + uu_x + vv_y = \frac{\mu_{nf}}{\rho_{nf}} u_{yy} = \frac{\sigma B_0^2 u(t)}{\rho} - \frac{\nu_{nf}}{k} u(t) - Fu^2(t), \]

Here \( u \) and \( v \) are the velocity vectors constituents along \( x \) and \( y \)-axis correspondingly, and \( \rho_{nf} \) represents liquid density. Thermal energy equations is reduced as [37–44]

\[ T_t + uu_x + vv_y = \frac{k}{\rho_p} u_{yy} - \frac{\sigma B_0^2 u(t)}{\rho_p} + \frac{\nu_{nf}}{k} u(t) + \frac{\mu_{nf}}{\rho_p} u^2. \]

Where the velocity components in \( x \) and \( y \) directions are shown by \( u \) and \( v \), respectively. \( \alpha_{nf}, \rho_{nf} \) and \( \mu_{nf} \) are the thermal diffusivity, density and viscosity of nanofluid and is defined as:

\[ \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s, \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}. \]

In equation (7), the term \( Q \) represents the heat generation or absorption, which given by

\[ Q = \left( \frac{kU}{\chi\nu} \right) A^*(T - T_0). \]

Where the temperature dependent heat generation/absorption coefficient is denoted by \( A^* \).
The mathematical relation explaining the volume fraction \( \phi \), heat capacitance \( k_{nf} \) and thermal conductivity \( (\rho C_p)_{nf} \) are given as

\[
(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s \quad k_{nf} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + 2\phi(k_f - k_s)}.
\]  

(10)

The suitable boundary conditions for the flow configuration are taken as [44]

\[
u = U(x, t) + N_t u_p, \quad \nu = 0, -kT_p = q(x, t) \text{ at } y = 0, \\
u_p = T_p = 0, \quad \nu = h(t), \quad \text{at } y = h(t).
\]

(11)

Familiarising the succeeding similarity transformations [37–44]

\[
u = \frac{cx}{1 - \alpha t} f'(\eta), \quad \nu = \left(\frac{-\alpha \nu^2}{1 - \alpha t}\right) f(\eta), \quad \eta = y(1 - \alpha t)^\frac{1}{2}\left(\frac{\nu}{c}\right)^\frac{1}{2}
\]

\[
T = T_0 - T_{nf}\left(\frac{dx}{k}\left(\frac{\nu}{c}\right)^\frac{1}{2}\right)(1 - \alpha t)^{-\frac{1}{2}\nu}, \quad \text{and } \theta(\eta) = \frac{T - T_0}{T_i - T_0}.
\]

(12)

Implementing the similarity transformation to equations (6)–(8) realizes the continuity equality, and the left over equations are converted to system of non-linear differential equations as

\[
\varepsilon_1 f''' + ff'' - (M + k_f) f' - (1 + F_1x) f'^2 - s\left(f' + \frac{\eta}{2} f''\right) = 0,
\]

(13)

\[
\varepsilon_2 \theta'' + ME_c f'^2 + \left(\frac{A_s}{pr}\right) e^s + \sigma f'^2 - \sigma\left(m\theta + \frac{\eta\theta'}{2}\right) - \gamma \theta f' + f\theta' = 0.
\]

(14)

After implementing the similarity transformations the boundary conditions of the problem takes the form

\[
f = 0, \quad f' = 1 + \lambda f'' , \quad \theta' = -1, \quad \text{at } \eta = 0,
\]

\[
f'' = 0, \quad \theta'' = 0, \quad f = \frac{\gamma \gamma}{2}, \quad \text{at } \eta = \gamma.
\]

(15)

(16)

Here \( \varepsilon_1 \) and \( \varepsilon_2 \) are the two constant explained in the following form

\[
\varepsilon_1 = \frac{1}{(1 - \phi)^2(1 - \phi + \phi\rho_i/\rho_f)}, \quad \varepsilon_2 = \frac{k_{nf}/k_f}{(1 - \phi + \phi(\rho C_p)_s/(\rho C_p)_f)}.
\]

(17)

The physical constraints after generalization are obtained as, \( s = \frac{\alpha}{\varepsilon} \) is the non-dimensional measure of unsteadiness, \( F_1 = C_0 x/\sqrt{K} \) is local inertia parameter, \( K_1 = C_0 Re_x/\mu U \) is permeability parameter, \( M = \sigma\beta_0^2(1 - \alpha t)/(\rho c) \) represents the magnetic parameter, \( Pr = \frac{\mu_0^2}{K} \) is Prandtl number and \( Ec = U^2/((\rho c)(T_p - T_0)) \) represent Eckert number, while \( \gamma \) represents the value of similarity variable \( \eta \) and at the free surface and defined as

\[
\gamma = (c/\nu^2)(1 - \alpha t)^\frac{1}{2}\nu(t).
\]

(18)

3. Physical quantities of interest

The physical quantities of interest such as mass flux, heat flux and Skin friction have abundant uses in engineering fields. For micropolar nanofluid flow problem Skin friction is defined as

\[
C_{fx} = \frac{\tau_w}{\rho_f(U_w)^2},
\]

(19)

Where \( \tau_w \) is given as

\[
\tau_w = \left(\frac{\partial\nu}{\partial y}\right).
\]

(20)

The Nusselt number is denoted as \( Nu = \frac{q_x}{k_s(T_p - T_0)/y} \) the heat flux and \( Q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \). The dimensionless form of \( C_{fx} \) and \( Nu \) are attained as
Where \( \text{Re}_x \) is the local Reynolds number and defined as \( \text{Re}_x = \frac{a_x}{v} \).

### 4. Solution by HAM

By using the dependable boundary conditions (15), (16) the solutions of equations (13), (14), are obtained by Homotopy analysis method. The preliminary deductions are selected as follows

\[
\hat{f}_0(\eta) = \frac{2(\gamma + s\lambda)\eta + (s - 2)\eta^2}{2(\gamma + 2\lambda)}, \quad \hat{\theta}_0(\eta) = 1.
\]  

(22)

The linear operatives represented by \( L_f, L \)

\[
L_f(\hat{f}) = \hat{f}''' + \hat{f}'' \hat{\theta}' = \hat{f}''', \quad L(\hat{\theta}) = \hat{\theta}''.
\]

Which have the subsequent applicability

\[
L_f(e_1 + e_2\eta + e_3\eta^2) = 0, \quad L(\hat{\theta}_2 + e_3\eta) = 0.
\]

(23)

(24)

Where \( e_i (i = 1 - > 7) \) denotes the coefficients involve in the general solution.

The corresponding non-linear operators \( N_f, N_\theta \) are carefully selected in the form

\[
N_f[\hat{f}(\eta; \zeta)] = e_1 \hat{f}''' + \hat{f}'' + (M + k_0)\hat{f}' - (1 + F_lx)\hat{f}'' - s\left(\hat{f}' + \frac{\eta}{2}\hat{f}''\right),
\]

\[
N_\theta[\hat{f}(\eta; \zeta), \hat{\theta}(\eta; \zeta), \zeta] = e_2\hat{\theta}'' + ME_n\hat{f}'' - \frac{1}{pr}\hat{\theta}'' + E_c\hat{f}'' - \frac{m\hat{\theta}'' + \eta\hat{\theta}'}{2} - \gamma\hat{\theta}'' + \hat{f}'\hat{\theta}'.
\]

(25)

(26)

The 0th-order scheme is

\[
(1 - \zeta)L_f[\hat{f}(\eta; \zeta)] - \hat{f}_0(\eta) = p h_fN_f[\hat{f}(\eta; \zeta)],
\]

\[
(1 - \zeta)L(\hat{\theta}(\eta; \zeta)) - \hat{\theta}_0(\eta) = p h_\thetaN_\theta[\hat{f}(\eta; \zeta), \hat{\theta}(\eta; \zeta)].
\]

(27)

(28)

The correspondent boundary constrains are

\[
\hat{f}(\eta; 0) = \hat{f}(\eta; \gamma) = \hat{f}(\eta; 1), \quad \hat{\theta}(\eta; 0) = \hat{\theta}(\eta; \gamma) = \hat{\theta}(\eta; 1).
\]

(29)

The dependable boundary constrains are

\[
\hat{f}(\eta_0) = 0, \quad \hat{\theta}(\eta_0) = 0, \quad s(1 + F_lx)\sum_{j=0}^{n-1} (\hat{f}'_{n-1}, \hat{f}'_j) - s\left(\hat{f}_j + \frac{\eta}{2}\hat{f}_{n-1}\right).
\]

(32)

Expand the velocity field \( \hat{f}(\eta; \zeta) \) and temperature field \( \hat{\theta}(\eta; \zeta) \) in Taylor’s series about \( \zeta = 0 \)

\[
\hat{f}(\eta; \zeta) = \hat{f}_0(\eta) + \sum_{n=1}^{\infty} \hat{f}_n(\eta)\zeta^n,
\]

\[
\hat{\theta}(\eta; \zeta) = \hat{\theta}_0(\eta) + \sum_{n=1}^{\infty} \hat{\theta}_n(\eta)\zeta^n.
\]

(31)

The dependable boundary constrains are

\[
\hat{f}_0(\eta_0) = \hat{f}'(0) - \chi_{f}^n(0) = \hat{\theta}_0(\eta_0) = -1, \quad \text{at} \ \eta = 0,
\]

\[
\hat{f}_n(\gamma) = \hat{\theta}_n(\gamma) = 0, \text{at} \ \eta = \gamma.
\]

(33)

Here

\[
\varphi_{f}(\eta) = e_1 \hat{f}''(\eta) - (M + k_0)\hat{f}'(\eta) - s(1 + F_lx)\sum_{j=0}^{n-1} (\hat{f}'_{n-1}, \hat{f}'_j) - s\left(\hat{f}_j + \frac{\eta}{2}\hat{f}_{n-1}\right).
\]  

(34)
Where

\[ \chi_n = \begin{cases} 
0, & \text{if } \zeta \leq 1 \\
1, & \text{if } \zeta > 1. 
\end{cases} \] (36)

5. Convergence of HAM solution

The convergence of the equations (27)–(28), wholly be particular by the secondary restrictions \( h_{\gamma}, h_{\theta} \). It is a choice in a way to control and converge the series answer. The probability sector of \( h \) are design \( h \)-curves of \( \frac{dh}{dy} \), \( \frac{d\theta}{dy} \) for 25th order approximated HAM solution. The effective regions of \( h \) are \(-1.5 < h_{\gamma} < 0.0\) and \(-1.5 < h_{\theta} < 0.0\). The convergence of the HAM technique by \( h \)-curves is used for velocity and temperature fields have been represented in figure 2. Table 1 displays the numerical values of HAM solutions at dissimilar approximation using dissimilar values of parameters. It is observable from the table that HAM method is a quickly convergent techniques.

6. Results and discussion

In this segment we deliberate the physical assets of dissimilar embedding parameters of the modelled problems and there influence on velocity and temperature profile which are showed in figures (3–15). Graphical view of problem is presented in figure 1.

6.1. Velocity profile \( \frac{dh}{dy} \)

The present work concentrates on the explanation of the nanoliquid film flow through modelled parameters. Mostly, numerical computations are accomplished for temperature and velocity field distribution contained by the boundary layer film collected with wall velocity and wall temperature gradient, to obtain vision in flow regime of the physics involved of flow parameters for numerous values, which illustrate the structure of flow. Figures 3 to 7 represents the performance of liquid velocity \( \frac{dh}{dy} \) in the influence of \( S \), inertial parameter \( F_1 \), magnetic field parameter \( M \), and permeability parameters \( K_1 \) and \( \varepsilon_1 \) explicitly. We noticed in figure 3 that an increase in the value of \( S \) leads to decrease the thickness of film, instantaneously, the inside velocity of thin film increases the surface velocity on raising \( S \). As a result an increase in \( S \) increases the stretching velocity \( U(x, t) \). While slip velocity increases due to raise in slip parameter, and is clearly results in decrease of liquid velocity. We examined in figure 4 that inside thin film liquid, velocity is increased as \( M \) increases as well as to reduce radically the film thickness. The reason at the back of such influence of \( M \) is due to the introduction of weak body force
which is represented as Lorentz force, thin film is an electrically conducting fluid due to the presence of $M$. This force acts to both the fields in a vertical direction. As viscous force and body force is the ratio of hydro magnetic as suggested by $M$, higher values of magnetic field shows a greater body force which has the capability to slow down the liquid flow. Figure 5 represent the effect of enlarging of $K_1$, which causes in the reduction the fluid flow.
velocity. Since we observed from the appearance of an increase in the value of $K_1$, a reduction in the porous medium permeability. Therefore, small gap is obtainable for fluid to flow and therefore, we examine that velocity of thin film is reduced. Figure 6 illustrates that on rising $F_1$, inner thin film fluid velocity is decreased, while there is no influence of $F_1$ on thickness of film. There is hardly an influence of $F_1$ on free surface velocity which is obvious from figure 6. In state of porous gap with larger pores sizes, and porous medium expended by fluid-solid interaction, which increases the viscous interference. Hence, an increase in $F_1$ causes a better flow resistance, so velocity of fluid is reduces. Figure 7 identifies that $\varepsilon_1$ has no influence on fluid film thickness, the reduction of $\varepsilon_1$ reduces the fluid velocity.

6.2. Temperature profile $\theta(\eta)$

Figures 8 to 15 are planned to observe the behaviour of $\theta$ against the parameters $M$, $K_1$, $Pr$, $Ec$, $F_1$, $S$, and $\gamma$ respectively. It is examined from figures 8 to 9 that for larger values of $M$ and $K_1$, temperature of thin film is increased all over in the region of film. While the attainment of permeability parameter and $M$ has led to decrement in the fluid velocity in region of thin film. Thus, additional work is done to slow the fluid against these three physical objects dissipates in the form of energy and therefore enlarged $\theta$ is examined in the thermal boundary layer. It is obvious from figure 10 that larger Prandtl number $Pr$ compels to decrease the thin film region the fluid temperature. Increasing Pr, decreases the thickness of thermal boundary layer. Since Pr is a
Figure 7. Effect of $\epsilon_1$ on $\frac{df}{dh}$ for dissimilar values, when $K_i = 0.4$, $M = 0.5$, $F_1 = 0.6$, $s = 1.2$.

Figure 8. Effect of $M$ on $\theta(\eta)$ for dissimilar values, when $K_i = 0.5$, $\epsilon_2 = 1$, $F_1 = 0.6$, $s = 1.2$, $Pr = 1$, $Ec = 0.4$, $\gamma = 0.5$.

Figure 9. Effect of $K_1$ on $\theta(\eta)$ for dissimilar values, when $M = 0.4$, $\epsilon_2 = 1$, $F_1 = 0.6$, $s = 1.2$, $Pr = 1$, $Ec = 0.4$, $\gamma = 0.2$. 
measure of relative significance of thermal diffusivities and momentum, and hence larger the Prandtl number lesser the thermal diffusion. Therefore the results of physical meaning of Prandtl number are in great conformity. Figure 11 identifies that the θ attainment is enlarged on increasing the Ec, which actually supports the physics. Because the ratio of kinetic energy to enthalpy is called Eckert number, heat stored in the liquid is dissipated by increasing Ec, by which the θ is increased. Figure 12 show that an increase F1 means a greater resistance to the flow, therefore, the fluid velocity is getting decreased. Figure 13 shows that fluid temperature is dependent on S. One can examine that when unsteadiness in the stretching increases, which as a result decreases the fluid temperature of thin film as well as free surface temperature. Increasing the values of S a thickened thermal boundary layer is produced. It is exciting to observe at this time that for larger values of S, results in the fluid temperature to fall radically, while the thermal boundary layer get thicker with raise in it. Figure 14 demonstrates that the fluid velocity is decreased on decreasing ε2. Figure 15 shows the effect of γ, which is the similarity variable at free surface given by equation (18), and depends on α. So, in this way that an enlargement in α results in the increment in the value of γ. The values of α is increases due to increase in S and decreases on the increasing values of Eckert number. Therefore, the increasing values of S increases the thickness of the thermal boundary layer. The residual error concentration for velocity and heat profiles are shown in figures (16) and (17) respectively.
Figure 12. Effect of $F_1$ on $\theta(\eta)$ for dissimilar values, when $M = 0.4$, $\varepsilon_1 = 1$, $S = 1.2$, $Ec = 0.4$, $K_2 = 0.5$, $Pr = 1$, $\gamma = 0.5$.

Figure 13. Effect of $S$ on $\theta(\eta)$ for dissimilar values, when $M = 0.4$, $\varepsilon_1 = 1$, $F_1 = 0.6$, $Ec = 0.4$, $K_2 = 0.5$, $Pr = 1$, $\gamma = 0.6$.

Figure 14. Effect of $\varepsilon_2$ on $\theta(\eta)$ for dissimilar values, when $M = 0.4$, $S = 1.2$, $F_1 = 0.6$, $Ec = 0.4$, $K_2 = 0.5$, $Pr = 1$, $\gamma = 0.5$. 
Figure 15. Effect of $\gamma$ on $\theta(\eta)$ for dissimilar values, when $M = 0.4$, $S = 1$, $F_1 = 0.6$, $Ec = 0.1$, $K_4 = 0.5$, $Pr = 1$, $\epsilon_2 = 1$.

Figure 16. Impact of $h$ curve of the residuals for the velocity.

Table 2. The Skin friction coefficient for different values of $M$, $k$, $\varepsilon_1$ and $S$.

| $M$ | $k$ | $\varepsilon_1$ | $S$  | $C_{f_0}$ |
|-----|-----|-------------------|------|-----------|
| 0.1 | 0.5 | 0.1               | 1.5  | -1.922542 |
| 0.5 |      |                   |      | -1.960871 |
| 1.0 |      |                   |      | -2.00879  |
| 1.5 | 0.1 |                   |      | -2.22145  |
|      | 0.5 |                   |      | -0.859375 |
| 1.0 |      |                   |      | -1.18416  |
| 1.5 | 0.1 |                   |      | -1.47810  |
|      | 0.5 |                   |      | -1.74479  |
| 1.0 |      |                   |      | -1.11228  |
| 1.5 | 0.1 |                   |      | -0.669792 |
|      | 0.5 |                   |      | -1.37964  |
| 1.0 |      |                   |      | -1.70833  |
6.3. Table discussion

Tables 2–4 describe the impact of $C_{fx}$ and $Nu$ under the effect of different parameters. The properties of $M$, $Ec$, $\varepsilon_2$, and $s$ on skin friction for the different values are shown in Table 2. It is examined that the increasing rates of magnetic $M$ and unsteady parameter $S$ blows the Skin-friction coefficient. The higher values of stretching parameter $\xi$ and thickness constraint $\beta$ reduces the $C_{fx}$ coefficient. The effects of $M$, $s$, $Ec$ and Pr on the $Nu$ for the different values are shown in Table 3. It is observed that increasing rates of unsteady parameter $S$ and magnetic parameter $M$ reduces the $Nu$, while the unsteady parameter $s$, and $\beta$ increase the $Nu$. The effects of $M$, $Rd$, $Pr$ and
s on the Nu is shown in table 4. It is noticed that Nu increases due to an increase in thermoporetic parameter, while increasing unsteady parameter and Pr causes a decreases Nu.

7. Conclusion

Concerning to frequent uses in the engineering field and the study of industrial devices, we have presented a mathematical model for defining the fluid film flow of non-Newtonian nano-fluids through malleable viscidness and thermal conductivity in occurrence of uniform magnetic field. The model of the flow is invented for nano-liquids over a stretching sheet. We are strongly trusted in this modification efforts to be worthy enough for the investigators to classify the unlimited thermal conductive, non-Newtonian fluid with unlimited thermal conductive nanoparticles. The key determined comments of this work are:

- It is investigated that for large values of $M$ that the nanofluids films velocity distribution decreases.
- Thickness of thermal boundary layer decreases with increasing values of $S$, while increase of radiation parameter, the Nusselt number also increases.
- It is observed that increase in the value of $K_1$, a reduction in the porous medium permeability.
- The increasing values of Pr, increases the surface temperature, where inverse influence is create for $S$, that the surface temperature reduces due to large values of $S$.
- An increase temperature profile is observed for greater values of Eckert number $Ec$ and vice versa.
- The mass flux decreases due to greater values of $S$, while a decrease in $S$ causes an increase in the mass flux.
- HAM method convergence with the variations in physical parameters is examined numerically.

Competing interests

All the authors declare that they have no challenging interests.

ORCID iDs

Zahir Shah https://orcid.org/0000-0002-5539-4225
Saeed Islam https://orcid.org/0000-0001-5263-4871
Ebenezer Bonyah https://orcid.org/0000-0003-0808-4504

References

[1] Khan W A 2013 Buongiorno model for nano fluid Blasius flow with surface heat and mass fluxes J. Thermophys. Heat Transfer 27 134–41
[2] Mahdy A and Chamkha A 2015 Heat transfer and fluid flow of a non-Newtonian nano fluid over an unsteady contracting cylinder employing Buongiorno’s model Int. J. Numer. Methods Heat Fluid Flow 25 703–23
[3] Malvandi A, Moshiri S A, Soltani E G and Ganji D D 2014 Modified Buongiorno’s model for fully developed mixed convection flow of nanofluids in a vertical annular pipe Comut. Fluids 89 124–32
[4] Hayat T, Ashraf M B, Shehzad S A and Abouelmagd E I Int. J. Numer. Methods Heat Fluid Flow 2015 Three dimensional flow of Erying Powell nanoluid over an exponentially stretching sheet Flow 25 593–616
[5] Nadeem S, Haq R U, Akbar N S, Lee C and Khan Z H 2013 Numerical study of boundary layer flow and heat transfer of oldroyd-B nanoluid towards a stretching sheet Plosone 8 1–6
[6] Rosmila A B, Kandasamy R and Muhamin I 2012 Lie symmetry groups transformation for MHD natural convection flow of nanoluid over linearly porous stretching sheet in presence of thermal stratification Appl. Math. Mech. Engl. Ed. 33 593–604
[7] Ellahi R 2013 The effects of MHD an temperature dependent viscosity on the flow of non-Newtonian nanoluid in a pipe analytical solutions Appl. Math. Modelling 37 1451–67
[8] Nadeem S and Saleem S 2014 Series solution of unsteady Erying Powell nanoluid flow on a rotating cone Indian J. Pure Appl. Phys. 52 725–37
[9] Abolbashari M H, Freidoonimehr N and Rashidi M M 2015 Analytical modeling of entropy generation for Cassonnano-fluid flow induced by a stretching surface Adv. Powder Technol. 6 542–52
[10] Nadeem S, UHaq R and Khan Z H 2014 Numerical study of MHD boundary layer flow of a Maxwell fluid past a stretching sheet in the presence of nanoparticles J. Taiwan Inst. Chem. Eng. 45 121–6
[11] Rohini H B, Alsaed D M and Valipour P 2016 Electro hydrodynamic nanoluid flow and heat transfer between two plates J. Mol. Liq. 216 583–9
[12] Shehzad S A, Hayat A and Alsaedi A 2014 MHD flow of Jeffrey nanoluid with convective boundary conditions Braz. Soc. Mech. Sci. Eng 3 873–83
[13] Mahmoodi M and Kandelousi S H 2016 Kerosene—alumina nanofluid flow and heat transfer for cooling application J. Cent. South Univ 23 983–90
[14] Fakour M, Ganji D D and Abbasi M 2014 Scrutiny of underdeveloped nanofluid MHD flow and heat conduction in a channel with porous walls Int. J. Case Studies Therm. Eng. 4 202–14
[15] Fakour M, Rahbari A and Khodabandeh E 2018 Nanofluid thin film flow and heat transfer over an unsteady stretching elastic sheet by LMS J. Mech. Sci. Tech. 32 177–83
[16] Hatami M, Sheikholeslami M and Ganji D D 2014 Laminar flow and heat transfer of nanofluid between contracting and rotating disks by least square method Power Technol 253 769–79
[17] Nadeem S, Mehmood R and Akbar N S 2013 Nonorthogonal stagnation point flow of a nano non-Newtonian fluid towards a stretching surface with heat transfer Int. J. Heat Mass Transf. 57 679–89
[18] Sheikholeslami M, Hatami M and Ganji D D 2013 Analytical investigation of MHD nanofluid flow in a semi-porous channel Powder Technol. 246 327–36
[19] Akbar N S, Tripathi D, Khan Z H and Beg O A 2016 A numerical study of magneto hydrodynamic transport of nanofluids over a vertical stretching sheet with exponential temperature-dependent viscosity and buoyancy effects Chem. Phys. Lett. 661 20–30
[20] Fakour M, Vahabzadeh A and Ganji D D 2014 Scrutiny of mixed convection flow of a nanofluid in a vertical channel Int. J. Case Studies Therm. Eng. 4 15–23
[21] Akbar N S, Mustafa M T and Khan Z H 2016 Stagnation point flow study with water based nanoparticles aggregation over a stretching sheet: numerical solution J. Comput. Theor. Nanosci. 13 8615–9
[22] Kumar S, Prasad S K and Banerjee I 2010 Analysis of flow and thermal field in nanofluid using a single phase thermal dispersion model Appl. Math. Model. 34 573–92
[23] Shah Z, Islam S, Ayaz H and Khan S 2018 Radiative heat and mass transfer analysis of micropolar nanofluid flow of Casson fluid between two rotating parallel plates with effects of Hall current ASME Journal of Heat Transfer accepted (https://doi.org/10.1115/1.4004145)
[24] Shah Z, Islam S, Gul T, Bonyah E and Khan M A 2018 The electrical MHD and hall current impact on micropolar nanofluid flow between rotating parallel plates Results Phys 9 1201–14
[25] Shah Z, Islam S, Gul T, Bonyah E and Khan M A 2018 Three dimensional third grade nanofluid flow in a rotating system between parallel plates with Brownian motion and thermophoresis effects Results Phys 10 36–45
[26] Shah Z, Gul T, Khan A M, Ali I and Islam S 2017 Effects of hall current on steady three dimensional non-Newtonian nanofluid in a rotating frame with brownian motion and thermophoresis effects J. Eng. Technol. 6 280–96
[27] Sheikholeslami M 2017 Influence of magnetic field on nanofluid free convection in an open porous cavity by means of Lattice Boltzmann method J. Mol. Liq. 234–364
[28] Sheikholeslami M 2017 Numerical simulation of magnetic nanofluid natural convection in porous media Physics Letters A 381 494–503
[29] Sheikholeslami M 2014 Numerical study of heat transfer enhancement in a pipe filled with porous media by axisymmetric TLB model based on GP. Eur. Phys. J. Plus 129 248
[30] Sheikholeslami M and Rokni H B 2017 Simulation of nanofluid heat transfer in presence of magnetic field: a review Int. J. Heat Mass Transfer 115 1203–33
[31] Sheikholeslami M and Ganji D D 2016 Nanofluid convective heat transfer using semi analytical and numerical approaches a review J. Taiwan Inst. Chem. Eng. 65 63–77
[32] Ellahi R, Zeeshan A, Shehzad N and Sultan Z A 2018 Structural impact of kerosene-Al2O3 nanoliquid on MHD poiseuille flow and heat transfer with water based nanoparticles aggregation over a stretching sheet Int. J. Heat Mass Transfer 115 1203–33
[33] Ellahi R, Zeeshan A, Shehzad N and Sultan Z A 2018 Structural impact of kerosene-Al2O3 nanoliquid on MHD poiseuille flow and heat transfer with water based nanoparticles aggregation over a stretching sheet Int. J. Heat Mass Transfer 115 1203–33
[34] Ellahi R, Zeeshan A, Shehzad N and Sultan Z A 2018 Structural impact of kerosene-Al2O3 nanoliquid on MHD poiseuille flow and heat transfer with water based nanoparticles aggregation over a stretching sheet Int. J. Heat Mass Transfer 115 1203–33
[35] Ellahi R, Zeeshan A, Shehzad N and Sultan Z A 2018 Structural impact of kerosene-Al2O3 nanoliquid on MHD poiseuille flow and heat transfer with water based nanoparticles aggregation over a stretching sheet Int. J. Heat Mass Transfer 115 1203–33
[36] Ellahi R, Zeeshan A, Shehzad N and Sultan Z A 2018 Structural impact of kerosene-Al2O3 nanoliquid on MHD poiseuille flow and heat transfer with water based nanoparticles aggregation over a stretching sheet Int. J. Heat Mass Transfer 115 1203–33
[37] Ellahi R, Zeeshan A, Shehzad N and Sultan Z A 2018 Structural impact of kerosene-Al2O3 nanoliquid on MHD poiseuille flow and heat transfer with water based nanoparticles aggregation over a stretching sheet Int. J. Heat Mass Transfer 115 1203–33
[38] Ellahi R, Zeeshan A, Shehzad N and Sultan Z A 2018 Structural impact of kerosene-Al2O3 nanoliquid on MHD poiseuille flow and heat transfer with water based nanoparticles aggregation over a stretching sheet Int. J. Heat Mass Transfer 115 1203–33
[39] Chen C H 2006 Effect of viscous dissipation on heat transfer in a non-Newtonian liquid flow over an unsteady stretching sheet J. Non-Newtonian Fluid Mech 135 128–35
[40] Megaha A M 2015 Effect of slip velocity on Casson thin film flow and heat transfer due to unsteady stretching sheet in presence of variable heat flux and viscous dissipation Appl. Math. Mech. -Engl. Ed 36 1273–84
[41] Wang C Y 1990 Liquid film on an unsteady stretching surface Q. Appl. Math. 84 601–10
[42] Andersson H L, Aarsseth J B, Braud N and Dandapat B S 1996 Flow of a power-law fluid film on an unsteady stretching surface J. Non-Newtonian Fluid Mech. 62 1–8
[43] Andersson H L, Aarsseth J B and Dandapat B S 2000 Heat transfer in a liquid film on an unsteady stretching sheet Int. J. Heat Mass Transfer 43 69–74
[44] Seth G S, Tripathi R and Mishra M K 2017 Hydromagnetic thin film flow of Casson fluid in non-Darcy porous medium with Joule dissipation and Navier’s partial slip Appl. Math. Mech. -Engl. Ed. 38 1618–20
[45] Mohsan H, Ahmad Z, Aqib M and Rahmat E 2017 Particle shape effects on ferrofluids flow and heat transfer under influence of low oscillating magnetic field J. Magn. Magn. Mater. 443 36–44
[46] Khan A A, Masood F, Ellahi R and Bhatti M M 2018 Mass transport on chemi-normal fourth-grade fluid propagating peristaltically through a curved channel with magnetic effects J. Mol. Liq. 258 186–95
[47] Bhatti M M, Zeeshan A, Ellahi R and Shit G C 2018 Mathematical modeling of heat and mass transfer effects on MHD peristaltic propulsion of two-phase flow through a Darcy-Brinkman-Forchheimer Porous medium Advanced Powder Technology 29 1189–97
[48] Ellahi R, Sultan Z A, Abdul B and Majeed A 2018 Effects of MHD and Slip on heat transfer boundary layer flow over a moving plate based on specific entropy generation J. Taibah Univ. Sci. 12 476–82
[49] Majeed A, Zeeshan A, Sultan Z A and Ellahi R 2018 Heat transfer analysis in ferromagnetic viscoelastic fluid flow over a stretching sheet with suction Near. Comp. Appl. 30 1947–55
[50] Fetecau C, Ellahi R, Khan M and Nehad A S 2018 Combine porous and magnetic effects on some fundamental motions of Newtonian fluids over an infinite plate J. Porous Media 21 589–605
[51] Liao S J 1999 An explicit, totally analytic approximate solution for blasius viscous flow problems Int. J. Non-Linear Mech. 34 759–78
[52] Hammed H, Haneef M M, Shah Z, Islam S, Khan W and Muhammad S 2018 The combined magneto hydrodynamic and electric field effect on an unsteady Maxwell nanofluid flow over a stretching surface under the influence of variable heat and thermal radiation Appl. Sci. 8 160
[53] Shah Z, Bonyah E, Islam S, Khan W and Ishaq M 2018 Radiative MHD thin film flow of Williamson fluid over an unsteady permeable stretching Heliyon 4 e00825
[54] Zuhra S, Saeed Khan N, Shah Z, Islam S and Bonyah E 2018 Simulation of bioconvection in the suspension of second grade nanofluid containing nanoparticles and gyrotactic microorganisms AIP Adv. 8 105210