INTERSTELLAR FILAMENTS AND THE STATISTICS OF GALACTIC H I

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ABSTRACT

This paper presents a statistical explanation of filament formation in the Galactic atomic hydrogen. We claim that even in the absence of dynamical factors, the Gaussian field corresponding to the measured values of the spectrum of random density should develop filamentary structure, the existence of which has long been claimed. Therefore, this paper shows that the filamentary structure of HI is consistent with the shallow three-dimensional spectrum of random density obtained by Lazarian.

Subject headings: ISM: general — ISM: structure — radio lines: ISM

1. INTRODUCTION

The usually accepted picture of the interstellar medium (ISM) consists of several interacting phases (see Shull 1987), with cold (~100 K), warm (~10^4 K), and hot (~10^6 K) phases having different properties (McKee & Ostriker 1977; McCray & Snow 1979). In the present paper, we deal with cold neutral medium, specifically with neutral hydrogen (H I) in the disk of our Galaxy (see Burton 1992), and propose a new explanation of filament formation in this phase.

Filaments are entities widely spread in the ISM. In molecular gas, filaments are often associated with shocked gas and outflows (see Wiseman & Ho 1996; Bally 1996). The aim of the present paper is to discuss an alternative mechanism that can be responsible for the formation of large-scale filaments in H I.

Our research was motivated by recent advances in the theory of the interstellar turbulence, being in its embryonic state (see An advantage of this approach is that these descriptors do not depend on the particular nature of forces leading to H I overdensity. Consequently, we expect that the filaments observed in 21 cm will be seen at 100 μm also. However, for the sake of simplicity, further on we will speak about H I filaments observed at 21 cm.

In what follows, we remind our reader of the statistical technique used in L95 for studying the density structure of random field. The spectrum was found to be shallow, corresponding to k^-z (0 < z < 1), which is different from the Kolmogorov one. At the same time, we believe that many filaments within Galactic H I originate via the statistical process we describe.

It is worth noting that our predicted filaments are real physical entities and not an artifact of data handling. Therefore, we expect a tight correlation in gas and dust emission if dust and gas are coupled together (Verschuur 1994). In other words, we expect that the filaments observed in 21 cm will be seen at 100 μm also. However, for the sake of simplicity, further on we will speak about H I filaments observed at 21 cm.

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2. STATISTICS OF H I

The structure of H I can be characterized best by statistical descriptors. Descriptors such as the density spectrum and the correlation function of density have been used successfully for decades in the studies of both hydrodynamic turbulence (see Monin & Yaglom 1971) and large-scale structure (see Peebles 1980). An advantage of this approach is that these descriptors do not depend on the particular nature of the random density field, since the theory of the interstellar turbulence, being in its embryonic state (see Scalo 1985), would not give us reliable clues about...
density arising from a supernova explosion, or statistical, e.g., arising from statistical properties of the random density field. In the latter case, forces that cause density enhancements are not correlated with the filament length.

The fact that filaments are ubiquitous in the ISM was stressed by V91a and V91b. His papers provoked a stream of attempts to explain filament formation by dynamical causes. A number of magnetohydrodynamic effects leading to filament formation were described in Lazarian (1993b) and Elmegreen (1994), but these attempts failed to provide a universal explanation for the phenomenon. Exotic schemes involving the Bennett pinch (Carlquist 1988; Verschuur 1995) have been considered also.

While dynamical formation of filaments is undoubtedly important in particular circumstances (see Bally 1996), we will dwell on the statistical origin of filaments. This implies that the filaments are not caused by forces acting coherently on extended portions of gas but are merely a transient phenomenon arising from overlapping peaks of hydrogen density, which is the result of uncorrelated dynamical impacts.

We assume that the random density field in H I is Gaussian and stationary. The first assumption (Gaussian) is a natural one to start with because whatever the stochastic sources of density fluctuations are, acting independently in different parts of the Galactic disk, they are likely to provide a Gaussian distribution of density for any given wavenumber k (see Gardiner 1983). As for the condition of stationarity, we consider that the collapse of clouds is compensated by their expansion after the onset of star formation².

The properties of a Gaussian field are determined by its spectrum (see Peebles 1980), and in what follows we will use the shallow spectrum obtained in L95 to predict the statistical properties of structures that can be formed in H I.

4. GAUSSIAN FIELDS AND FORMATION OF FILAMENTs

Rather complex computations are required to obtain the properties of structures arising within Gaussian density fields. We are fortunate that such computations have been done recently by researchers interested in the formation of the large-scale structure of the universe. Therefore, whenever possible, we refer to their studies.

Following customary procedures, we separate the density ρ(x) into the mean ̄ρ and the fluctuation δ(x). The fluctuations cover a whole range of spatial scales and amplitudes. A Gaussian random field of density fluctuations can be represented as a superposition of Fourier modes with random amplitudes,

\[ δ(x) = (2π)^{-3/2} \int d^3k a_k P^{1/2}(k) W(kR_0)e^{ikx}, \]  

(1)

where independent random quantities \( a_k \) are normalized and orthogonal, \( \langle a_k a^*_k \rangle = δ_{kk} \). The properties of the field are defined by the power spectrum \( P(k) = E(k)/4πk^2 \), which is related to the two-point correlation function

\[ \langle δ(x)δ(x') \rangle \equiv ξ(r = |x - x'|, R_0) = (2π)^{-3} \int d^3k P(k)W^2(kR_0)e^{ik(x-x')} . \]  

(2)

¹ This signal is essentially a one-dimensional Fourier transform of the intensity collected within the diagram of an individual telescope forming an interferometric pair.

² This corresponds to the notion of rapid reprocessing of different ISM phases.
We consider isotropic random fields where $P(k)$ and therefore $\zeta(r)$ do not depend on the direction of their arguments. Looking for smoothed properties of the field, we allowed for an appropriate window function $W^2(kR_f)$ with filtering scale $R_f$, e.g., Gaussian $W(kR_f) = \exp \left[-(kR_f)^2/2\right]$. The dispersion of density fluctuations is given by

$$\sigma_0^2(R_f) = \langle \delta(x)\delta(x) \rangle = \frac{1}{2\pi} \int k^2 dk P(k)W^2(kR_f) . \quad (3)$$

Our goal is to show that, although density fluctuations are statistically isotropic, each realization of the random density field naturally contains anisotropic, predominantly filamentary structures on density levels $\geq 1\sigma_0$. One numerical example of the random field with $P(k) \propto k^{-1}$, shown in Figure 1, illustrates this statement. To provide a quantitativ treatment, we follow the way of reasoning introduced in Bond, Kofman, & Pogosyan (1996b).

The most visible objects in the random field are the high-amplitude maxima of the field, which we call peaks. In the first instance, the peaks are characterized by their scale $R_{pk}$ and dimensionless height $v$, so that at the peak’s location one finds a maximum with density value, $\delta(x_{pk}) = v\sigma_0(R_{pk})$ after smoothing the random field with the filter scale $R_f = R_{pk}$. In terms of $H I$ density, such peaks probably correspond to the isolated high-contrast entities that give rise to the concept of ”cloud.” The enhancement of density may make such an entity gravitationally bound and collapsing (see Moschovias 1991).

The statistics of rare peaks is well studied in the literature (see Peebles 1980; Bardeen et al. 1986). In particular, the density profile near a peak of height $v_{pk}$ follows, on average,
the correlation function
\[ \langle \delta(r) \mid v_p \rangle = \frac{v}{\sigma_0(R_{pk})} \xi(r, R_{pk}) \approx \nu \sigma_0(R_{pk}) \left( 1 - \frac{r^2}{2r_c^2} \right), \tag{4} \]
where \( r = x - x_{pk} \). Here we have introduced the notation of a conditional mean density field \( \langle \delta(x) \mid v \rangle \), given the constraint that its value at peak position is fixed at \( v_{pk} \) after smoothing with a filter \( R_{pk} \). The last approximation in equation (4) is obtained by expanding the correlation function near the peak up to quadratic terms in distance \( r \). It provides a definition of the density correlation length \( r_c \).

The above formulae describe the angle-average shape of the high-density enhancement. To go beyond the spherical approximation, we consider statistical properties of the tensor field:
\[ e_i(x) = (2\pi)^{-3/2} \int d^3 k a_k P^{1/2}(k) \left( \frac{k_i k_j}{k^2} \right) W(k R_c) e^{ikx}, \tag{5} \]
(note the \( k^{-2} \) factor, so that \( e_i \) has the same dimensionality as density). Clearly, the density is given by the trace \( \sum e_{ii} \), and so we can express the eigenvalues \( \lambda_1 \leq \lambda_2 \leq \lambda_3 \) of \( e_{ii} \) via two new parameters \( \epsilon_1 \) and \( \epsilon_2 \):
\[ \begin{align*}
\lambda_1 &= \frac{1}{2} \delta(1 - 3\epsilon_1 + \epsilon_p), \\
\lambda_2 &= \frac{6}{5} \delta(1 - 2\epsilon_2), \\
\lambda_3 &= \frac{3}{2} \delta(1 + 3\epsilon_1 + \epsilon_p).
\end{align*} \tag{6, 7, 8} \]
By definition, \( \epsilon \geq 0 \) and \( -\epsilon \leq \epsilon_p \leq \epsilon \).

The role of \( \epsilon_1 \) and \( \epsilon_2 \) becomes clear if we calculate the mean density profile around the peak with given values of \( \epsilon_1 \) and \( \epsilon_2 \). In the coordinate system with axes parallel to the principal axes of \( e_{ii}(x_{pk}, R_{pk}) \), the second-order approximation gives (BKP96)
\[ \langle \delta(r) \mid v, \epsilon_1, \epsilon_2, \epsilon_p \rangle = \nu \sigma_0(R_{pk}) \left( 1 - \frac{r_1^2}{2r_c^2} - \frac{r_2^2}{2r_c^2} - \frac{r_3^2}{2r_c^2} \right), \tag{9} \]
with correlation length now different in different directions
\[ r_{c1} = \frac{r_c}{\sqrt{1 - 3\epsilon_1 + \epsilon_p}}, \tag{10} \]
\[ r_{c2} = \frac{r_c}{\sqrt{1 - 2\epsilon_2}}, \tag{11} \]
\[ r_{c3} = \frac{r_c}{\sqrt{1 + 3\epsilon_1 + \epsilon_p}}. \tag{12} \]

One can see that isosurfaces of constant overdensity near the peak, with fixed values of \( \epsilon_1 \) and \( \epsilon_2 \), are given by triaxial ellipsoids with ellipticity \( \epsilon \) and prolaticity \( \epsilon_p \).

The parameters \( \epsilon_1 \) and \( \epsilon_2 \) are discussed in Bond (1987a, 1987b) and Bond & Myers (1996a, 1996b, 1996c), and their mean expected values for a peak of height \( v \) are shown to be
\[ \langle \epsilon_1 \mid v \rangle \approx 0.54v^{-1}, \tag{13} \]
\[ \langle \epsilon_2 \mid v \rangle = 0. \tag{14} \]
This means that, although peaks tend to be more and more spherical as their height increases, the peaks of moderate height are expected to be significantly anisotropic. For example, for a \( 2\sigma_0 \) peak, the density correlation length in the longest first direction \( r_{c1} \) is typically 2.3 times larger than angle-averaged \( r_c \), and 3 times longer than in the third direction \( r_{c3} \), where the density falls the fastest.

An analysis of the shape of the density field in the vicinity of peaks already gives us an indication of the development of anisotropic entities of enhanced density elongated along one axis. Now, to understand the remarkable scale of filaments shown on the lower density threshold in Figure 1, we should consider the behavior of the random field in between high maxima. Although the regions of high-density enhancement are not connected with each other, Figure 1 shows that as one lowers the density threshold, the bridges between high peaks form a joint filamentary structure, for an overdensity threshold approaching the \( 1\sigma_0 \) level. In fact, the percolation theory (see Shandarin 1983) confirms that random Gaussian fields percolate on a \( 1\sigma_0 \) threshold.

Equations (9)-(14) provide a hint for a very extended correlation length in one direction if moderate overdensities with \( v \sim 1.5 \) are considered. The imaginary values of coherence scale for \( \epsilon > \frac{1}{2} \) are interpreted as the density increase from the central point, which shows that the point with such high \( \epsilon \) is not typically a maximum but a saddle point between higher peaks. Of course, the second-order expansion breaks as \( r_{c1} \) goes to infinity, and one has to use more elaborate methods to describe lower density enhancements.

Extending the analysis of the mean density behavior to approximate a single peak, it was proposed in BKP96 that the conditional mean density field (under the constraints that the properties of several peaks are fixed) could be used as a tool for studying how density enhancements arise in between the field maxima.

The peak-patch theory developed in Bond & Myers (1995, 1996a, 1996b, 1996c) shows that it is likely statistically that neighboring peak patches will be oriented preferentially along the same 1-axis. This means that the enhancement of overdensity is likely to bridge the filaments.

The physical reason for this is discussed in Bond (1989). In terms of superposition of random density waves, the latter result can be understood as follows: the existence of the elongated peak patch means a higher density of waves, with their crests along the longer peak-patch axis. This entails a slower decoherence along this axis than along the others. When the peak separation increases, the overdensity bridges between the peaks gradually weaken. This effect is exemplified in Figure 2, where we show the mean expected density profile under the constraint that there are two peaks with largely aligned ellipticity at varying distances. One can see that the bridging enhancement of density, which is strong when two peaks are close together and tightly aligned, weakens when the separation and/or the angle between the longest axes increase(s).

To define the overall system of filaments, of which a bridge between a pair of peaks is a single segment, one requires the calculation of the conditional probability of density, provided that the properties of density peaks are defined in a number of points. This is a problem from the domain of numerical computation, and further on we provide the results of such calculations.

\footnote{While deriving eqs. (9)-(14), we have not explicitly imposed the condition that second derivatives of the density field are positive at \( r_{ck} \). This is satisfied automatically for sufficiently high \( v \) and low \( \epsilon \), with the result that the point is actually a peak.}
The formation of filaments is vivid for the conditional probability of density for 10 peaks (see Fig. 3). It is evident from Figure 3 that such Gaussian peaks provide a distinguishable filamentary pattern. Note that these filaments are not determined by a “filament-forming” force.

It is also evident from Figure 3 that the filaments are rather irregular. Similar irregularity was interpreted in V91b as evidence of existing instability. In our example, these structures appear because of the statistical properties of the Gaussian field only.

The natural question that arises is whether other topological structures are possible. Indeed, our experiments with the conditional probability of density for three peaks show that the probability of having enhanced density is higher over all the places between the three peaks, forming an overdense sheet. However, the density contrast of such a sheet is lower than the density contrast for the filaments (see Fig. 4).

In other words, some sort of “mountain analogy” is applicable to the random density of H I. Individual peaks of the highest density are rare and disconnected from each other at high-density contrast. However, at lower density contrast, the peaks are connected with each other by ridges of enhanced density. The membranes of enhanced density corresponding to high mountain plateaus in our analogy are less probable than ridges.

It is easy to see how mapping from Galactic coordinate space to the velocity space distorts the web structure. We limit ourselves to some general remarks here.

Let us consider a new coordinate system where velocity component \( v_z \) serves as the line-of-sight \( z \)-coordinate. The gaseous velocity \( v \) can be presented as the sum of a smooth part, determined by the Galactic rotation curve \( \bar{v}(x, y, z) \) and the random turbulent component \( u \),

\[
v = \bar{v} + u ;
\]

\( \bar{v} \) is assumed to be invertible, in the sense that from measured \( x, y, v_z \), one can find the “mean” H I \( z \)-position by solving the equation \( v_z = \bar{v}(x, y, z) \). In other words, we assume that the restricted map given by \( \bar{v} \) is single valued. Without loss of generality, the random field \( u(x, y, z) \) can be considered to have zero mean and zero average divergence.

Previously, we have dealt with the random H I density field in real space \( \rho(x, y, z) \). The density \( \rho(x, y, v_z) \) in the velocity space is a random field as well; its properties are governed by both \( \rho(x, y, z) \) and turbulent velocity component \( u \) via the obvious transformation \( \rho(x, y, v_z) = \rho(x, y, z)(\bar{v}(x, y, z) + u(x, y, z)) \). The denominator is just the Jacobian of the map from \( (x, y, v_z) \) to \( (x, y, z) \). If average densities are defined to satisfy \( \bar{\rho} = \rho/\bar{v}_{z} \), the density fluctuation fields \( \delta, \delta_z \) satisfy

\[
\delta_z = \frac{\delta - u_z/\bar{v}_{z}}{1 + u_z/\bar{v}_{z}} .
\]

The typical value of \( \bar{v}_{z}^{-1} \) in the region is a natural scale with which to compare random \( u_z \). Simplification is achieved if

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4. The velocity-space mapping that we discuss here is not dissimilar to the redshift-space mapping that arises in the large-scale structure studies (Kaiser 1987).

5. When \( v \) is not single valued, several regions at different distances from the observer overlap. The web structures from these regions can be distinguished if the ratio of the distances is large.
Fig. 3.—The Gaussian field (left) and its reconstruction with 20 of the largest peaks (right). The threshold is shown at 2σ (top), 1.5σ (middle), and 1σ (bottom).
\[ \delta_z = \delta - u_{z,z}, \] 

and \( u_{z,z} \) are strongly correlated to be in phase. Such correlation is expected in \( \text{H} \, \text{I} \), but it is difficult to estimate its degree.

On the other hand, the full map (eq. [15]) may be multivalued for some regions of \((x, y, z)\) if the (derivative of) turbulent component is large enough. The most dense structures formed are caustics in the velocity space where \( \bar{v}_{z,z} + u_{z,z} = 0 \). For a proper analysis, one should use the theory of singularities in differentiable mappings (see Arnold, Gussien-Zade, & Varchenko 1988). These structures and their formation will be discussed elsewhere.

In general, we argue that outside the parameter space where the map (eq. [15]) is multivalued, the same statistical mechanisms we described in real space lead to structure creation in the density field in \( xyz \) space.\(^6\) Dealing with the velocity space, however, one should expect some statistical

\(^6\) In fact, observations in \( xyz \) space were done by V91a and V91b, who discovered the filamentary structure of \( \text{H} \, \text{I} \).
anisotropy between \( x \), \( y \) and the line-of-sight \( z \) directions, the presence of non-Gaussian features in \( \delta_n \) due to nonlinearity of the transformation (eq. [16]), and the absence of a one-to-one correspondence between filaments in \( xyv \) and Galactic space.\(^7\)

All in all, we predict the existence of filaments that can be visualized using contemporary data cubes, but we warn that the observed pattern is distorted from the original one because of the problems intrinsic to the velocity-space mapping (see Burton 1992).

5. DISCUSSION

Our results testify that the increase of resolution and sensitivity of telescopes is bound to reveal more fine structure in the distribution of \( H \ I \). Indeed, at first only the entities of the highest contrast were found, and those were identified as clouds (see Verschuur 1993). Further research, however, provided evidence of the existence of filaments and shells (Heiles 1996; V91a; V91b).

The visibility of filaments and other structures in \( H \ I \) depends on the intensity of fluctuations observed, i.e., on \( \sigma_0 \). Naturally, in our simple model, which ignores self-gravity, the equilibrium distribution would correspond to uniform density. Therefore, the more energy is injected locally by star winds or supernova explosions, the higher is the level of fluctuations and the sharper is the expected contrast of filaments.

Since the density enhancements correlate with the regions of energy injection, the issue of distinguishing the dynamical and statistical origin of filaments has to be clarified. The individual enhancements of density have a dynamical origin. However, the coalescence of these enhancements is statistical by nature. In other words, no particular long-ranged forces are needed to create filaments in the random density field.

Another factor that influences the properties of \( H \ I \) filaments is the spectrum of the density field. Figure 5 shows the structure of the filaments corresponding to different spectral indices \( \alpha \) of the random density field. It is evident from this figure that \( \alpha \approx 0 \), which, according to L95, is close to the spectral index of the \( H \ I \) density field, provides a clearly distinguished filamentary pattern.

In this study, we adopted a simple model of Galactic \( H \ I \) to elucidate the basic features of statistical formation of large-scale filaments in \( H \ I \); we did not intend to provide a detailed description of the \( H \ I \) web structure. For instance, we disregarded the dynamical evolution of \( H \ I \) peaks under self-gravity. Computations in BKP96 show that the contrast of filaments goes up if self-gravity is accounted for. This corresponds to intuitive expectation, since self-gravity tends to contract overdense regions. At the same time, we do not expect self-gravity to be a dominant factor, since only an insignificant part of the Galactic \( H \ I \) undergoes gravitational collapse at a given moment (see Elmegreen 1992), while the \( H \ I \) filaments seems ubiquitous. In fact, the observations in V91a and V91b indicate that large-scale \( H \ I \) filaments are not self-gravitating.

Our simplified model assumes that the density field is Gaussian. This assumption is not valid for a density field with variance above the mean density because of the physical restriction that density cannot be negative. An analysis of generic properties of non-Gaussian fields is not conceivable at the present time; however, we believe that the main conclusion of the existence of the rich geometrical structures of pure statistical origin holds for a wide range of random fields. A non-Gaussian density field that is produced from a Gaussian one by substituting zero wherever the field becomes negative should produce more distinct and wider separated filaments compared with a Gaussian field. We expect that future work on filament statistics will include a systematic study of non-Gaussian density fields.

Within our model, filaments are not aligned. In other words, the large-scale web of filaments spreads isotropically in all directions. However, the gas in the Galactic disk is distributed anisotropically. First of all, gaseous density changes with distance from the Galactic plane. Then the Galactic magnetic field and differential rotation introduce large-scale anisotropies. A challenging problem of including these factors in a model of filament formation is beyond the scope of the present paper. Therefore, we discuss these anisotropies only qualitatively.

If the power spectrum of random density does not change with distance from the Galactic disk, the topology of the percolation pattern does not change. However, the decrease of density may make the filaments more difficult to detect. Therefore, a tendency of the observed filament pattern to lie within the Galactic plane is expected. There the differential rotation tends to stretch statistically formed structures, providing a better correlation in the azimuthal direction.

The Galactic magnetic field, which is also disregarded in our model, can be more important. Unfortunately, the data on the field structure are far from compelling, and this limits the extent to which quantitative modeling can reproduce the \( H \ I \) filamentary structure. Studies of the large-scale field by starlight polarimetry (see Heiles 1996; Zweibel 1996), synchrotron intensity (see Lazarian 1992), and synchrotron polarization (see Spietsra & Brouw 1976) provide rather conflicting results and leave us groping for the ratio of the regular to random magnetic field. A customary adopted assumption that the regular and random fields are of equal strength is insufficiently supported by observational data.

However, there are ambiguities even within our simplified picture. Yes, it is likely that the regular magnetic field can influence the statistics of elongated density peaks. However, the question remains unclear as to how it does so. While an ad hoc assumption corresponds to the hydrogen flowing along magnetic field lines and forming elongated entities, recent numerical computations in Gammie & Ostriker (1996) have shown that cold \( H \ I \) tend to be concentrated in narrow sheets that are perpendicular to the direction of the magnetic field.

Observational evidence of the web anisotropy is inconclusive so far. Although attempts to find the anisotropy of \( H \ I \) density spectrum by Green (1994) were unsuccessful, his data cannot say much about anisotropy in the distribution of filaments. Indeed, interferometric measurements by Green (1994) are dominated by small-scale fluctuations, while filaments are features of the large-scale pattern. We also suspect that mapping from the line of observations to the velocity space of \( H \ I \) mitigates the anisotropy imposed by the regular magnetic field. Indeed, such a mapping inev-
Fig. 5.—The patterns for different spectra of random density obtained at threshold 1σ. The whole box of simulation is shown. The top left figure corresponds to the spectral index $\beta = x - 2 = 0$ and shows cloudlike structures. Filamentary structures are revealed by the top right figure, corresponding to the spectral index $\beta = -1$. Sheets and filaments are more in the bottom right figure, corresponding to the spectral index $\beta = -2$. The smoothing is the same as in Fig. 1, and the spectra are normalized so that the dispersion $\sigma$ is the same for all three cases. Everywhere in the figure, the random realization of the amplitudes $a_k$ is chosen to be the same in order to visualize how the structures are affected by the redistribution of power in the spectrum.

Notably entails averaging over the variations of the magnetic field direction along the line of sight.

To understand the last point, let us recall that one attempts to recover the information using a Galactic rotation curve (see Kerr & Lynden-Bell 1986). The precision of this curve is limited, apart from other factors, by the gas turbulent motion. Therefore, patches of gas at different distances from the observer are mapped onto one point in the velocity space. If these patches have a different direction of magnetic field, the anisotropies of gas density distribution are partially averaged out.

Complications related to the space-velocity mapping also preclude us from predicting the exact structure of the H I web. It seems, however, that the expected correlations between density enhancements and the gradient of random velocity $u_{zz}$ are likely to make filaments more distinct. Additional enhancements of density are expected from singularities in differential mapping, which we discussed in § 4. We expect that these effects will be more important compared with, for example, self-gravity. Future research in the field is likely to reveal a rich variety of structures that form in H I statistically.9 Surely, the effects of possible deviations from the Gaussian assumptions, as well as the dynamical formation of filaments, can be important in particular regions of H I. Therefore, our aim in this paper was not to explain the detailed structure of filaments observed in H I, but to attract the attention of the researchers in the field to the possibility of a purely statistical origin of filaments.

We expect that further research in the field will account for singularities in velocity-space mapping, non-Gaussian

Note that the filaments discussed above are not related to any intermittent turbulence but appear in a purely Gaussian random field.
features of random density field, the Galactic magnetic field, etc., and will reveal a rich variety of observed structures.

6. CONCLUSIONS

Our application of the theory of percolation of the Gaussian density fields, developed in Bardeen et al. (1986) and BKP96, to Galactic H I resulted in predicting a purely statistical mechanism of filament formation. For the shallow spectrum of H I density fluctuations obtained in L95, we found a pronounced large-scale filamentary pattern that should arise in the Galactic atomic hydrogen because of this mechanism. This may provide an explanation for the intriguing results obtained by V91a and V91b.

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Note added in proof.—Our further analysis proved that the velocity fluctuations substantially distort the density spectrum observed. For instance, the bottom right figure in Figure 5 corresponds to the Kolmogorov turbulence in xyz space with density spectral index $\beta = \alpha - 2 \approx -3$. Therefore, results in L95 do not exclude Kolmogorov turbulence in H I.