Dynamical Evolution of Planets in Disks

Planets in resonant Orbits

W. Kley (wilhelm.kley@uni-tuebingen.de)
Astronomie und Astrophysik, Abt. Computational Physics, Universität Tübingen, D-72076
Tübingen, Germany

Abstract. We study the evolution of a system consisting of two protoplanets still embedded in a protoplanetary disk. Results of two different numerical approaches are presented. In the first kind of model the motion of the disk material is followed by fully viscous hydrodynamical simulations, and the planetary motion is determined by N-body calculations including exactly the gravitational potential from the disk material. In the second kind we only solve the N-body part and add additional analytically given forces which model the effect of the torques of the disk. This type of modeling is of course orders of magnitudes faster than the full hydro-model. Another advantage of this two-fold approach is the possibility of adjusting the otherwise unknown parameters of the simplified model.

The results give very good agreement between the methods. Using two different initial setups for the planets and disk, we obtain in the first case a resonant trapping into the 3:1 resonance, and in the second case a trapping into the 2:1 resonance. Resonant capture leads to a rise in the eccentricity and to an alignment of the of the spatial orientation of orbits. The characteristics of the numerical results agree very favorably with those of 3 observed planetary systems (GJ 876, HD 82943, and 55 Cnc) known to be in mean motion resonances.

1. Introduction

Since the first discovery in 1995, during the last years the number of detected extrasolar planets orbiting solar type stars has risen up to about 100 (see eg. http://www.obspm.fr/encycl/encycl.html by J. Schneider for an always up to date list). It has been found that among those there are 11 systems with 2 or more planets. With further observations to come, this number may still increase, as for some systems trends in the radial velocity curve have been found. Among these multiple planet extrasolar systems there are now 3 confirmed cases, GJ 876 (Marcy et al., 2001), HD 82943 (the Coralie Planet Search Programme, ESO Press Release 07/01), 55 Cnc (Marcy et al., 2002) where the planets orbit their central star in a low order mean motion resonance, where the orbital periods have nearly exactly the ratios 2:1 or 3:1. The parameter of these planetary systems are displayed below in Table I. This implies that about 1/4 of planetary systems, or even more, may be in resonance, a fraction which is even higher if secular resonances, as for example present in υ And (Chiang and Murray, 2002), are also taken into account.

The formation of such resonant planetary systems can be understood by considering the joint evolution of proto-planets together with in the protoplanetary disk from which they formed. By local linear analysis it was shown
that the gravitational interaction of a protoplanet with the disk may lead to torques resulting in the migration of a planet (Goldreich and Tremaine, 1980; Lin and Papaloizou, 1986; Ward, 1997; Tanaka et al., 2002). Additionally, for planetary masses of around one Jupiter mass, gap formation as result of angular momentum transfer between the (viscous) disk and the planet was considered (Lin and Papaloizou, 1993). Fully non-linear hydrodynamical calculations (Kley, 1999; Bryden et al., 1999; Lubow et al., 1999; Nelson et al., 2000) for Jupiter sized planets confirmed the expectations and showed clearly that this interaction leads to: 

i) the formation of spiral shocks waves in the disk, whose tightness depends on the sound-speed in the disk, 

ii) an annular gap, whose width is determined by the equilibrium between gap opening tidal torques and gap closing viscous and pressure forces. 

iii) an inward migration on a timescale of $10^5$ yrs for typical disk parameter in particular disk masses corresponding to that of the minimum mass solar nebula, 

iv) a possible mass growth after gap formation up to about $10 M_{\text{Jup}}$ when finally gravitational torques overwhelm, and finally v) a prograde rotation of the planet.

Recently, these single planet calculations were extended to calculations with multiple planets. Those have shown already (Kley, 2000; Bryden et al., 2000; Snellgrove et al., 2001; Nelson and Papaloizou, 2002) that during the early evolution of protoplanetary systems, when the planets are still embedded in the disk, different migration speeds may lead an approach of the planets and eventually to resonant capture. Pure N-body calculations including additional damping terms to model disk-planet interaction effects have been calculated for example by (Snellgrove et al., 2001; Lee and Peale, 2002; Murray et al., 2002).

Here we present new numerical calculations treating the evolution of a pair of two embedded planets in disks. We consider both, fully hydrodynamic and simplified N-body calculations to model the evolution. In the first approach, the motion of the disk is followed by solving the full time dependent Navier-Stokes equations simultaneously with the motion of the planets. Here the motion of the planet is determined by the gravitational potential of the other planet, the star, and that of the disk. In the latter approach we take a simplified assumption and perform 3-body (star and 2 planets) calculations augmented by additional (damping) forces which take the gravity of the disk approximately into account. These two approaches allow a direct comparison of the methods, and will enable us to determine in detail the damping constants required for the simpler (and much faster) second model.

2. The Observations

The basic orbital parameter of the 3 systems in mean motion resonance are stated in Table I. Two of them, GJ 876 and HD 82943 are in a nearly exact
Table I. The orbital parameter of the 3 systems known to be in a mean motion resonance. $P$ denotes the orbital period, $M \sin i$ the mass of the planets, $a$ the semi-major axis, $e$ the eccentricity, $\omega$ the longitude of periastron, and $M_*$ the mass of the central star. It should be noted, that the orbital elements for the planets may vary on secular time scales. Thus in principle one should always state the epoch corresponding to these osculating elements. The values quoted for GJ 876 are based such on a dynamical fit to the data (Lee and Peale, 2002; Laughlin and Chambers, 2001), where the planetary mass refers to the real masses assuming $\sin i = 0.78$.

| Nr | Per | $M \sin i$ | $a$ | $e$ | $\omega$ | $M_*$ |
|----|-----|------------|-----|-----|---------|--------|
|    | [d] | $M_{Jup}$  | [AU]| [deg]|         | $M_\odot$ |
| GJ 876 (2:1) |     |            |     |       |         |        |
| c  | 30.569 | 0.766     | 0.13 | 0.24 | 159     | 0.32   |
| b  | 60.128 | 2.403     | 0.21 | 0.04 | 163     |        |
| HD 82943 (2:1) |   |           |     |       |         |        |
| b  | 221.6 | 0.88      | 0.73 | 0.54 | 138     | 1.05   |
| c  | 444.6 | 1.63      | 1.16 | 0.41 | 96      |        |
| 55 Cnc (3:1) |     |            |     |       |         |        |
| b  | 14.65 | 0.84      | 0.11 | 0.02 | 99      | 0.95   |
| c  | 44.26 | 0.21      | 0.24 | 0.34 | 61      |        |
| d  | 5360 | 4.05      | 5.9  | 0.16 | 201     |        |

2:1 resonance. In both cases we note that the outer planet is the more massive one by a factor of about two (HD 82943), and more than three (GJ 876). The eccentricity of the inner (less massive) planet is larger than that of the outer one. For the system GJ 876 the alignment of the orbits is such that the two periastra are pointing in nearly the same direction. The values quoted in Table I are based on the dynamical orbit calculations to match the combined Keck+Lick data (Lee and Peale, 2002; Laughlin and Chambers, 2001). For the system HD 82943 these data have not been clearly identified, due to the much longer orbital periods, but do not seem to very different from each other. The last system, 55 Cnc, is actually a triple system. Here the inner two planets orbit the star very closely and are in a 3:1 resonance, while the additional, more massive planet orbits at a distance of several AU (Marcy et al., 2002). For other longer period systems where only trends in the radial velocity curve have been observed further observations may reveal even more systems in a resonant configuration.
3. The Models

It is our goal to determine the evolution of protoplanets still embedded in their disks. To this purpose we employ two different methods which supplement each other. Firstly, a fully time-dependent hydrodynamical model for the joint evolution of the planets and disk is presented. Because the evolutionary time scale may cover several thousands of orbits these computations require sometimes millions of time-steps, which translates into an effective computational times of up to several weeks.

Because often the main interest focuses only on the orbital evolution of the planet and not so much on the hydrodynamics of the disk, we perform additional simplified 3-body computations. Here the gravitational forces are augmented by additional damping terms designed in such a way as to incorporate in a simplified way the gravitational influence of the disk. Through a direct comparison with the hydrodynamical model it is then possible to infer directly the necessary damping forces.

3.1. Full Hydrodynamics

The first set of coupled hydrodynamical-N-body models are calculated in the same manner as described in detail in (Kley, 1998), (Kley, 1999) for single planets and in (Kley, 2000) for multiple planets, and the reader is referred to those papers for more details on the computational aspect of the simulations the computations. Other similar models, following explicitly the motion of single and multiple planets in disks, have been presented by (Nelson et al., 2000; Bryden et al., 2000; Snellgrove et al., 2001). During the evolution material is taken out from the centers of the Roche-lobes of the two planets, which is monitored and assumed to have been accreted onto the two planets. We present two runs: one (model B) where the mass is added to the planet, and another one (model A) where this mass is not added to the dynamical mass of the planets, i.e they always keep their initial mass. They are allowed to migrate (change their semi-major axis) through the disk according to the gravitational torques exerted on them. This assumption of constant planet mass throughout the computation is well justified, as the migration rate depends, at least for type II drift, only weakly on the mass of the planet (Nelson et al., 2000). The initial hydrodynamic structure of the disk, which extends radially from \( r_{\text{min}} \) to \( r_{\text{max}} \), is axisymmetric with respect to the location of the star, and the surface density scales as \( \Sigma(r) = \Sigma_0 r^{-1/2} \). The material orbits initially on purely Keplerian orbits \( v_r = 0, v_\phi = GM_\star/r^{1/2} \). Deviations from Kepler rotation due to pressure gradients or self-gravity are typically less than 1% thus this initial velocity setup is well justified. The fixed temperature law follows from the constant vertical height \( H/r = 0.05 \) and is given by \( T(r) \propto r^{-1} \).
Table II. Planetary and disk parameter of the models. The mass of the Planet is given in Jupiter masses ($M_{\text{Jup}} = 10^{-3} M_\odot$), the viscosity in dimensionless units, the disk mass located between $r_{\text{min}}$ and $r_{\text{max}}$ in solar masses, and the minimum and maximum radii in AU.

| Model | Mass1 | Mass2 | Viscosity | $M_{\text{disk}}$ | $r_{\text{min}}$ | $r_{\text{max}}$ |
|-------|-------|-------|-----------|-------------------|------------------|------------------|
| A     | 3     | 5     | $\alpha = 10^{-2}$ | 0.01              | 1                | 30               |
| B     | 1 (Var) | 1 (Var) | $\nu = \text{const.}$ | 0.01              | 1                | 20               |

The kinematic viscosity $\nu$ is given by an $\alpha$-description $\nu = \alpha c_s H$, with the sound speed $c_s$. We present two models:

i) Model A, having a constant $\alpha = 0.01$, which may be on the high side for protoplanetary disks but allows for a sufficiently rapid evolution of the system to identify clearly the governing physical effects. The two embedded planets have a mass of 3 and 5 $M_{\text{Jup}}$ and are placed initially at 4 and 10 AU, respectively.

ii) Model B with a constant $\nu$, equivalent to an $\alpha = 0.004$ at 1 AU. Here the two embedded planets each have an initial masses of 1 $M_{\text{Jup}}$, and are placed initially at 1 and 2 AU, respectively. This model is in fact a continuation of the one presented in (Kley, 2000).

All the relevant model parameters are given in Table II.

3.2. DAMPED N-BODY

As pointed out, the full hydrodynamical evolution is computationally very time consuming because ten-thousands of orbits have to be calculated. Hence, we perform also pure N-body calculations of a planetary system, consisting only of a star and two planets. The influence of the surrounding disk is felt here only through additional (damping) forces. For those we assume, that they act on the semi-major axis $a$ and the eccentricity $e$ of the two planets through an explicitly specified relations $\dot{a}(t)$ and $\dot{e}(t)$, which may depend on time. Here, as customary in solar system dynamics, it is assumed that the motion of the two planets can be described at all times by approximate Kepler ellipses where the time-dependent parameter $a(t)$ and $e(t)$ represent the values of the osculating orbital elements at the epoch $t$.

This change of the actual semi-major axis $a$ and the eccentricity $e$ caused by the gravitational action of the disk can be translated into additional forces changing directly the position $\vec{x}$ and velocity $\vec{u}$ of the planets. In our implementation we follow exactly (Lee and Peale, 2002), who give the detailed explicit expressions for these damping terms in their appendix. As a first test
of the method we recalculated their model for GJ 876 and obtained identical results.

Using the basic idea of two planets orbiting inside of a disk cavity (see Fig. 1 below), we only damp $a$ and $e$ of the outer planet. Here we choose a general given functional dependence of the form $g(t) = g_0 \cdot \exp\left[-\left(t/t_0\right)^p\right]$, with $g \in (a, e)$, where $a_0$ and $e_0$ are just the initial values. The values of the exponent $p$ and the timescale $t_0$ are adjusted to match the results of the full hydrodynamic calculations. Comparative results of the two methods are displayed in Section 4.3.

4. Results

The basic evolutionary sequence of two planets evolving simultaneously with the disk has been calculated and described by (Kley, 2000) and (Bryden et al., 2000). Before concentrating on details of the resonant evolution we first summarize briefly the main results.

4.1. Full Hydrodynamic Evolution: Overview

At the start of the simulations both planets are placed into the axisymmetric disk, where the density is initialized such that in addition to the radial density profile partially opened gaps are superimposed. Upon starting the evolution the two main effects are:

a) As a consequence of the accretion of gas onto the two planets the radial regime in between them will be depleted in mass and finally cleared. This phase typically takes only a few hundred orbital periods. At the same time the region interior of the inner planet will lose material due to accretion onto the central star. Thus, after an initial transient phase we typically expect the configuration of two planets orbiting inside of an inner cavity of the disk, see Fig. 1, and also (Kley, 2000).

b) After initialization the planets quickly (within a few orbital periods) create non-axisymmetric disturbances, the spiral features, in the disk. In contrast to the single planet case these are no longer stationary in time, because there is no preferred rotating frame. The gravitational torques exerted on the two planets by those density perturbations induce a migration process for the planets.

Now, the different radial location of the planets within the cavity has a distinct influence on their subsequent evolution. As a consequence of the clearing process the inner planet is no longer surrounded by any disk material and thus cannot grow any further in mass. In addition it cannot migrate anymore,
Figure 1. Overview of the density distribution of model A after 1250 orbital periods of the inner planet. Higher density regions are brighter and lower ones are darker. The star lies at the center of the white inner region inside of $r_{\text{min}} = 1$. The location of the two planets is indicated by the white dots, and the Roche-lobe sizes are also drawn. Clearly seen are the irregular spiral wakes generated by the planets. Only outside of the 2nd planet the regular inter-twined two spiral arms are visible.

because there is no torquing material in its vicinity. The outer planet on the other hand still has all the material of the outer disk available, which exerts negative (Linblad) torques on the planet. Hence, in the initial phase of the computations we observe an inwardly migrating outer planet and a stalled inner planet with a constant semi-major axis, see the first 5000 yrs in the top panel of Fig. 2.

This decrease in separation causes an increase of the gravitational interaction between the two planets. When the ratio of the orbital periods of the planets has reached the fraction of two integers, i.e. they are in a mean motion resonance, this may lead to a resonant capture of the inner planet by the outer one. Whether this happens or not depends on the physical conditions in the
Figure 2. The semi-major axis (AU), eccentricity and position of the periastron of the orbit versus time for Model A. In this example, the planets have fixed masses of $3 \, M_{\text{Jup}}$ and $5 \, M_{\text{Jup}}$, and are placed initially at 4 and 10 AU, respectively. In the beginning, after the inner gap has cleared, only the outer planet migrates inward, and the eccentricities of both planets remain relatively small, less than $\approx 0.02$. After about 6000 years the outer planet has reached a radius with a period exactly three times that of the inner planet. The periodic gravitational forcing leads to a capture of the inner planet into a 3:1 resonance by the outer one. This is indicated by the dark reference line (labeled 3:1), which marks the location of the 3:1 resonance with respect to the inner planet. Upon resonant capture the eccentricities grow, and the orbits librate with a fixed relative orientation of $\Delta \omega = 110^\circ$. 
Figure 3. Left: The difference in the direction of the periastron, $\Delta \omega = \omega_1 - \omega_2$, of the two planets vs. time. Right: Ratio of eccentricities $e_1/e_2$ versus periastron difference. The indices 1,2 refer to the inner and outer planet, respectively. The color and symbol coding is identical for the left and right panel. During the evolution into resonance the dots are 'captured' to eventually circle around the equilibrium value $\Delta \omega = 110^\circ$ and $e_1/e_2 = 0.9$.

disk (eq. viscosity) and the orbital parameter of the planets. If the migration speed is too large for example, there may not be enough time to excite the resonance, and the outer planet just continues its migration process, see eg. (Haghighipour, 1999). Also, if the initial eccentricities are too small, then there may be no capture as well, see also (Lee and Peale, 2002).

4.2. 3:1 RESONANCE: MODEL A

In model A this capture happens at $t \approx 6000$ when the outer planet captures the inner one in a 3:1 resonance (see dark line in top panel of Fig. 2). From that point on, the outer planet, which is still driven inward by the outer disk material, will also be forcing the inner planet to migrate inwards. The typical time evolution of the orbital elements, semi-major axis ($a$), eccentricity ($e$) and direction of the periastron ($\omega$), of such a case are displayed in Fig. 2 for model A.

We summarize the following important features of the evolution after resonant capture:

a) The inner planet begins to migrate inward as well, forced in by the outer planet. Thus both planets migrate inward simultaneously, always retaining their resonant configuration. As a consequence, the migration speed of the outer planet slows down, and their radial separation declines.

b) The eccentricities of both planets grow initially very fast and then settle to oscillating quasi-equilibrium values which change slowly on a secular time scale. This slow increase of the eccentricities on the longer timescale
is caused by the growing gravitational forces between the planets, due to the decreasing radial distance of the two planets on their inward migration process.

c) The ellipses (periastrae) of the planets librate at a constant angular speed. Caused by the resonance, the speed of libration for both planets is identical, which can be inferred from the parallel lines in the bottom panel of Fig. 2. The orientation of the orbits is phase locked with a constant separation of the periastrae by a fixed phase-shift $\Delta \omega$.

More detail of the capture into resonance and the subsequent libration of the orbits is illustrated in Fig. 3 for model A. It is seen (left panel) that the difference of the periastrae settles to the fixed average value of $\Delta \omega = 110^\circ$, a libration amplitude of about $15^\circ$, and libration period of about 3000 yrs. The right panel shows the evolution in the $e_1/e_2$ vs $\Delta \omega$ using the same color and symbol coding coding. During the initial process of capturing the points (open squares) approach the final region from the top right region of the diagram. At later times the points circle around the equilibrium point. We note that additional models, not displayed here, with different planet masses and viscosities all show approximately the same shift in $|\Delta \omega|$, if capture occurs into 3:1 resonance. The capture in such an asymmetric 3:1 resonance has been studied for the 55 Cnc case by (Lee and Peale, 2003), and the stability of these resonant configurations has recently been discussed by (Beauge et al., 2002).

4.3. 2:1 RESONANCE: MODEL B

The second model setup is taken directly from (Kley, 2000). Here we continued exactly that model for a little longer time, to infer some more characteristics of the intrinsic dynamics of that planetary system. The evolution of the orbital elements $a$ and $e$ is displayed in Fig. 4. Here the planets are placed on initially tighter orbits with a semi-major axis ratio of only 2. The initial orbital evolution is similar to the first model, i.e. an inwardly migrating outer and a stalled inner planet. However, caused by the reduced initial radial distance higher resonances are not available, and the resonant capture occurs into the 2:1 resonance, see reference line in left panel. The eccentricities of both planets rise again upon capture but this time the mass of the inner planet $m_1 = 1M_{Jup}$ is, due to the explained starvation, much smaller than that of the outer one $m_2 = 3.1M_{Jup}$. This leads to to much larger rise in eccentricity, yielding a ration $e_1/e_2 \approx 4$. In Fig. 5 the alignment of the orbits is indicated. This time, as seems typical for this type of 2:1 resonances (Snellgrove et al., 2001; Lee and Peale, 2002), the separation in the periastrae is centered around zero, $\Delta \omega = 0$, with a libration amplitude of up to about $20^\circ$. 
Figure 4. The evolution of semi-major axis (left) and eccentricity (right) for model B. The planets had an initial radial location of 1 and 2 AU, and masses of $1 \, M_{Jup}$ each, which were allowed to increase during the computation. The results of the full hydrodynamic evolution are shown by the dashed lines. A reference line, with respect to the inner planet, indicating the location of the 2:1 resonance is shown. Before $t = 4500$ only very few data points are plotted, thereafter they are spaced much more densely, which explains the different looking curves. The solid curves are obtained using the simplified damped 3-body evolution as described in the text.

Figure 5. Left: The difference in the direction of the periastrae of the two planets vs. time. Right: Ratio of eccentricities versus periastron difference. The data points are spaced equally in time with a distance of approximately $\delta t = 3/4$ years. Shown is only the very last section of the evolution of model B, from 4500 to 5100 yrs, which covers nearly two and a half libration periods. In this case of a 2:1 resonance, the capture leads to a complete alignment of the orbits with $\Delta \omega = 0$. 
For comparison and test, we modeled the evolution of the model B also using the 3-body method, which is briefly outlined above and compared this to the full hydrodynamical evolution. As outlined in Sect. 3.2, only the outer planet is damped in its semi-major axis $a$ and eccentricity $e$. It turned out that for a good agreement the damping time scale $t_0$ is identical for $a$ and $e$. Here in model B, we used $t_0 = 75,000$ yrs and $p = 0.75$ to obtain the displayed results, and to reach the agreement with the full hydrodynamical evolution. This contrasts the results of (Lee and Peale, 2002) where a shorter timescale was used for the eccentricity damping. The difference may be caused for example by not modeled damping processes or possible disk dissipation. In further models we plan to relate the parameter $t_0$ and $p$ to physical quantities of disk such as viscosity, temperature.

These additional results are also displayed in Fig. 4 with the solid lines. For the semi-major axis (left panel) the agreement is very good indeed. The obtained eccentricities are in good agreement as well. The only difference is the lack in eccentricity oscillations in the simplified damped 3-body model, which may again be caused by a change in eccentricity damping in the full model which is not modeled properly in the simplified 3-body version.

5. Summary and Conclusion

We have performed full hydrodynamical calculations simulating the joint evolution of a pair of protoplanets together with their surrounding protoplanetary disk, from which they originally formed. The focus lies on massive planets in the range of a few Jupiter masses. For the disk evolution we solve the Navier-Stokes equations, and the motion of the planets is followed using a 4th order Runge-Kutta method, considering their mutual interaction, the stellar and the disk’s gravitational field. These results were compared to simplified (damped) N-body computations, where the gravitational influence of the disk is modeled through analytic damping terms applied to the semi-major axis and eccentricity.

We find that both methods yield comparable results, if the damping constants in the simplified models are adjusted properly. These constants should be obtained from the full hydrodynamical evolution.

Two types of resonant situations are investigated. In the first case the initial radial separation of the two planets was sufficiently far that they were captured into a 3:1 resonance. In this case, the capture leads to an orbit alignment of $\Delta \omega = 110^\circ$. In the second case (model B) the capture is in 2:1 with $\Delta \omega = 0^\circ$. These difference in $\Delta \omega$ for varying types of resonances seems to be a robust and a generic feature, which is supported by the observations of all three resonating planets. Additionally, we find that the inner planet preferably has lower mass caused by the inner disk gap, and finally the lower
mass planet should have a larger eccentricity. These findings are indeed seen in the observed 2:1 resonant planetary systems GJ 876 and HD 82943.

References

Beauge, C., S. Ferraz-Mello, and T. Michtchenko: 2002, ‘Extrasolar Planets in Mean-Motion Resonance: Apses Alignment and asymmetric stationary Solutions’. Astrophys. J., submitted pp. astro-ph/0210577.

Bryden, G., X. Chen, D. N. C. Lin, R. P. Nelson, and J. C. B. Papaloizou: 1999, ‘Tidally Induced Gap Formation in Protostellar Disks: Gap Clearing and Suppression of Protoplanetary Growth’. Astrophys. J. 514, 344–367.

Bryden, G., M. Różycka, D. N. C. Lin, and P. Bodenheimer: 2000, ‘On the Interaction between Protoplanets and Protostellar Disks’. Astrophys. J. 540, 1091–1101.

Chiang, E. I. and N. Murray: 2002, ‘Eccentricity Excitation and Apsidal Resonance Capture in the Planetary System υ Andromedae’. Astrophys. J. 576, 473–477.

Goldreich, P. and S. Tremaine: 1980, ‘Disk-satellite interactions’. Astrophys. J. 241, 425–441.

Haghighipour, N.: 1999, ‘Dynamical friction and resonance trapping in planetary systems’. Mon. Not. R. Astron. Soc. 304, 185–194.

Kley, W.: 1998, ‘On the treatment of the Coriolis force in computational astrophysics’. Astron. & Astrophys. 338, L37–L41.

Kley, W.: 1999, ‘Mass flow and accretion through gaps in accretion discs’. Mon. Not. R. Astron. Soc. 303, 696–710.

Kley, W.: 2000, ‘On the migration of a system of protoplanets’. Mon. Not. R. Astron. Soc. 313, L47–L51.

Laughlin, G. and J. E. Chambers: 2001, ‘Short-Term Dynamical Interactions among Extrasolar Planets’. Astrophys. J. Letter 551, L109–L113.

Lee, M. H. and S. J. Peale: 2002, ‘Dynamics and Origin of the 2:1 Orbital Resonances of the GI 876 Planets’. Astrophys. J. 567, 596–609.

Lee, M. H. and S. J. Peale: 2003, ‘Extrasolar Planets and Mean-Motion Resonance’. In: Scientific Frontiers in Research on Extrasolar Planets. pp. astro-ph/0209176.

Lin, D. N. C. and J. Papaloizou: 1986, ‘On the tidal interaction between protoplanets and the protoplanetary disk. III - Orbital migration of protoplanets’. Astrophys. J. 309, 846–857.

Lin, D. N. C. and J. C. B. Papaloizou: 1993, ‘On the tidal interaction between protostellar disks and companions’. In: Protostars and Planets III. pp. 749–835.

Lubow, S. H., M. Seibert, and P. Artyomowicz: 1999, ‘Disk Accretion onto High-Mass Planets’. Astrophys. J. 526, 1001–1012.

Marcy, G. W., R. P. Butler, D. Fischer, S. S. Vogt, J. J. Lissauer, and E. J. Rivera: 2001, ‘A Pair of Resonant Planets Orbiting GI 876’. Astrophys. J. 556, 296–301.

Marcy, G. W., R. P. Butler, D. A. Fischer, G. Laughlin, S. S. Vogt, G. W. Henry, and D. Pourbaix: 2002, ‘A Planet at 5 AU around 55 Cancri’. Astrophys. J. 581, 1375–1388.

Murray, N., M. Paskowitz, and M. Holman: 2002, ‘Eccentricity Evolution of Migrating Planets’. Astrophys. J. 565, 608–620.

Nelson, R. P. and J. C. B. Papaloizou: 2002, ‘Possible commensurabilities among pairs of extrasolar planets’. Mon. Not. R. Astron. Soc. 333, L26–L30.

Nelson, R. P., J. C. B. Papaloizou, F. . . Masset, and W. Kley: 2000, ‘The migration and growth of protoplanets in protostellar discs’. Mon. Not. R. Astron. Soc. 318, 18–36.

Snellgrove, M. D., J. C. B. Papaloizou, and R. P. Nelson: 2001, ‘On disc driven inward migration of resonantly coupled planets with application to the system around GI876’. Astron. & Astrophys. 374, 1092–1099.
Tanaka, H., T. Takeuchi, and W. R. Ward: 2002, ‘Three-Dimensional Interaction between a Planet and an Isothermal Gaseous Disk. I. Corotation and Lindblad Torques and Planet Migration’. *Astrophys. J.* **565**, 1257–1274.

Ward, W. R.: 1997, ‘Protoplanet Migration by Nebula Tides’. *Icarus* **126**, 261–281.

*Address for Offprints:* W. Kley, Astronomie und Astrophysik, Abt. Computational Physics, Universität Tübingen, D-72076 Tübingen, Germany, wilhelm.kley@uni-tuebingen.de