Examining a Stretching and Shrinking Wedge to Determine the Predictive Role of Nanoparticles in Shaping Nanofluid Heat Transfer in Nano-Darcy Mixed Convective Settings

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Abstract-- The aim of this study was to determine how nanoparticles tend to affect the heat transfer and velocity parameters during nanofluid flow processes. The study gained insights from the context of Non-Darcy mixed convection systems. Also, a wedge was considered for investigation, with specific features under examination involving surface stretching and shrinkage. Also, the coordinate transformation technique was used to obtain ordinary differential equations, which were also obtained after the conversion of governing partial differential equations. Similar, the fourth order Runge Kutta technique was used to solve the transformed equations, complemented by the shooting technique. Upon achieving the temperature and velocity field results, they were presented both in tabular and graphical forms. The motivation was to provide room for studying the movement of water, gas, and oil through the gas field or oil reservoir, as well as water purification and groundwater migration. From the results, it was established that an increase in the rate of nanoparticles concentration heat transfer causes a decrease in the friction factor. On the other hand, an increase in nanoparticles concentration caused an increase in Nusselt number or the rate of heat transfer. Additional findings demonstrate that an increase in the suction parameter causes an increase in the rate of heat transfer, as well as the friction factor. Lastly, this study established that as the wedge angle increases, the Nusselt number or rate of heat transfer increases, with a similar trend observed regarding the correlation between the wedge angle and the friction factor.

1. Introduction
In sciences and engineering, some of the conventional fluids that are experienced include oil and water [1]. Given the fluids’ thermal conductivities, they tend to be limited in relation to heat transfer performance enhancement [2, 3]. Indeed, nanofluids constitute nanoparticles suspensions occurring in base fluids [4]. In the recent past, there has been growing interest in the use of nanofluids to improve base fluid thermal conductivity [5, 6]. The perfect thermal performance that has been associated with the fluids arises from high surface to volume ratio and other distinguished thermal features [7]. In the previous studies, two models that have been proposed include dispersion models and homogeneous models [8]. This study relied on a non homogeneous model to understand the transport phenomena. In many engineering applications, mixed convection heat transfer has been observed. Some of these applications include the cooling of buildings, cooling of electric equipment, and heat exchangers [9, 10]. In the nanofluid community, most of the previous scholarly studies have examined some of the beneficial effects of nanoparticles dispersion into base fluids in relation to heat transfer enhancement [4-6], especially in mixed convection contexts. As such, there is a considerable previous development and research effort geared towards enhancing the rate of heat transfer in materials. However, the recent past situation has seen the demand for more robust and efficient heat transfer fluids grow dramatically [8, 9], especially due to the need for higher thermal conductivity – compared to the traditional fluids [10]. Therefore, this study examine how nanoparticles are likely to affect wedge surface nanofluid behaviors, with surface stretching and shrinking also considered as parameters that could play moderating roles.
2. Methodology

Given a wedge onto which nanofluid flew or interacted, this study considered a steady and laminar mixed convective flow. Also, the study considered the effects of viscosity and density variation and how they could shape Boussinesq approximation and the momentum equation. Given the wedge, the y axis reflected the normal to the surface while the x-axis was taken along it (the wedge). The wedge’s surface injection or suction was also included. Given a boundary layer’s approximation, the following governing equations were utilized:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
\rho_{nf} \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu_{nf} \frac{\partial^2 u}{\partial y^2} + U \frac{du}{dx} - \mu_{nf} (u - U)
\]

\[
F(u^2 - U^2) + \beta \theta_{nf} (T - T_\infty) \sin \left( \frac{\alpha}{2} \right) = 0
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\alpha_{nf}}{(\rho c_p)_{nf}} \frac{\partial^2 T}{\partial y^2} + \mu_{nf} \frac{\partial u}{\partial y}^2 = 0
\]

\[
u = 0, \frac{\partial u}{\partial y} = T = T_w \text{ at } y = 0
\]

\[
u = U(x), T = T_\infty \text{ at } y \rightarrow \infty
\]

The further analysis of the parameters in the equations above led to transformations in the form:

\[
\eta(x, y) = y \frac{(1+m)}{2} \sqrt{\frac{2}{\nu x}} \cdot \psi(x, y) = \frac{2x}{1+m} \int f(x, \eta), U(x) = Ax^m
\]

In this case,

\[
\frac{\partial \psi}{\partial y} = -\frac{\partial \psi}{\partial x} \theta = \frac{T - T_\infty}{T_w - T_\infty}
\]

Substituting into the previous equations led to:

\[
\mu_{nf} f''' - \rho_{nf} f'' \frac{2}{1+m} (1-f'')^- + \beta_{nf} \theta \sin \left( \frac{\alpha}{2} \right) - \mu_{nf} (f' - 1) = 0
\]

\[
\alpha_{nf} \theta'' - \frac{2}{1+m} \theta f' + f \theta' + \mu_{nf} \theta^2 = 0
\]

Also, the transformed boundary conditions yielded:

\[
f(0) = s, f'(0) = h, \theta(0) = 1, \eta = 0
\]

\[
f'(\infty) = 1, \theta(\infty) = 0 \eta \rightarrow \infty
\]

3. Results and Discussion

As mentioned earlier, the motivation of this study was to determine how nanoparticles could affect the performance of nanofluids, especially those interacting with a stretching and shrinking wedge. From the results, one of the specific areas that were investigated involved the relationship between velocity distribution and nanoparticles concentration in the selected boundary layer. Indeed, the study established that when there is an increase in the concentration of the nanoparticles, whether in the case of shrinking or stretching wedge, there was a decrease in the velocity. Another area that was investigated involved the correlation between temperature distribution and nanoparticle concentration, given the selected or target boundary layer. From the analysis, this study established that whether in the
case of shrinking or stretching wedge, the temperature increased with an increase in nanoparticle concentration.

Another parameter that was investigated involved the possible role of the wedge angle on the velocity distribution of the nanofluid. Indeed, findings demonstrated that in both shrinking and stretching, there was a decrease in velocity with an increase in the wedge angle. Apart from the correlation between the wedge angle and material velocity, the study investigated how the parameter (wedge angle) could affect the distribution of temperature in the target material. Indeed, it was found that the temperature decreases with an increase in the wedge angle. These findings were obtained both for the shrinking and stretching case.

With crucial insights gained from the correlations above, additional investigation strived to understand how the concentration of the nanoparticle interacted with the friction factor of the nanofluid. From the results, it was evident that an increase in nanoparticle concentration in the nanofluid caused a decrease in the material’s friction factor. However, the increase in nanoparticle concentration caused an increase in the Nusselt number or rate of heat transfer, reflecting a direct correlation (unlike the previous case where an inverse correlation was established relative to the relationship or interaction between nanoparticle concentration and the friction factor).

Also, an increase in the suction parameter caused an increase in the rate of heat transfer and the friction factor, hence a direct relationship. However, comparing the wedge angle and the friction factor yielded different, inverse outcomes. Specifically, an increase in the wedge angle caused a decrease in the material friction factor. However, there was a direct relationship between the wedge angle and the Nusselt number or rate of heat transfer in such a way that an increase in the former parameter caused an increase in the latter parameter.

4. Conclusion

In summary, this study analyzed a boundary layer. The objective of the investigation was to determine how various parameters operating on the part of nanoparticles could affect fluid flow behaviors over stretching and shrinking wedge. The study’s setting entailed non-Darcy medium. For practical applications, the study’s findings are projected to gain use in examining material movements such as situations involving water, gas, and oil through the gas field and oil reservoirs. Also, the study’s findings are projected to gain practical application in the context of groundwater migration, as well as water purification. Specific results demonstrated that as the concentration of nanoparticles increases, there is a decrease in the friction factor. On the other hand, the study established that as the concentration of nanoparticles increases, there is a decrease in the Nusselt number of the rate of heat transfer. However, there was a direct relationship among interactions of three parameters. Specifically, the study’s findings demonstrated that with an increase in the suction parameter, there is an increase in the rate of heat transfer, as well as the friction factor. However, the role of the wedge angle on material behavior posed mixed results. For example, the investigation found that as the wedge angle increases, the Nusselt number or rate of heat transfer tends to increase. On the other hand, an increase in the wedge angle was observed to cause a decrease in the friction factor. From a practical point of view, it could be seen that in thermal systems and devices, their efficiencies depend on the rates of heat transfer. Also, the rates of heat transfer are seen to be shaped by the working fluids’ thermal conductivity. Hence, the contribution of the study to sciences and engineering fields is that it paved the way for the consideration of some of the moderating factors that are worth considering while seeking to develop robust and more efficient heat transfer fluids whose thermal conductivities might be significantly higher than traditional fluids.

5. References

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