Fast Radio Bursts from Reconnection in a Magnetar Magnetosphere

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Abstract

The nearly 100% linear polarization has been reported for a few fast radio bursts (FRBs). This finding places severe limits on the emission mechanism. I argue that the completely polarized radiation could be generated in the course of relativistic magnetic reconnection in the outer magnetosphere of the magnetar. At the onset of the magnetar flare, a large-scale magnetic perturbation forms a magnetic pulse, which propagates from the flare site outward. The pulse strongly compresses magnetospheric plasma and pushes it away. The high-frequency MHD waves are generated when the magnetic pulse reaches the current sheet separating, just beyond the light cylinder, the oppositely directed magnetic fields. Coalescence of magnetic islands in the reconnecting current sheet produces magnetosonic waves, which propagate away on the top of the magnetic pulse and escape in the far zone of the wind as radio waves polarized along the rotational axis of the magnetar. I estimate parameters of the outgoing radiation and show that they are compatible with the observed properties of FRBs.

Unified Astronomy Thesaurus concepts: Non-thermal radiation sources (1119); Magnetohydrodynamics (1964); Plasma astrophysics (1261)

1. Introduction

Extraordinary properties of fast radio bursts (FRBs) impose a high requirement on the emission process. Two mechanisms are mostly discussed in the literature, the curvature emission of “bunches” in a neutron star magnetosphere (Cordes & Wasserman 2016; Kumar et al. 2017; Ghisellini & Locatelli 2018; Katz 2018; Yang & Zhang 2018; Lu & Kumar 2019; Wang et al. 2019) and the synchrotron maser emission (Lyubarsky 2014; Beloborodov 2017, 2019; Ghisellini 2017; Waxman 2017; Gruzinov & Waxman 2019; Metzger et al. 2019; Margalit et al. 2020).

The available models of the curvature emission just assume that “bunches” with the required properties exist in the magnetosphere; the emission of these bunches is estimated by applying formulas of vacuum electrodynamics. Even leaving aside the question of how the bunches are formed and whether they are able to survive in spite of an immense repulsive potential, one could not ignore the fact that their radiation properties are strongly affected by the plasma (Gil et al. 2004). The required large charge of the bunches implies a very high plasma density, such that the plasma frequency, $\omega_p$, is well above the emission frequency, $\omega$. Then, if the bunches move together with the plasma, they are completely shielded and do not radiate at all. If they move with respect to the plasma such that their velocity is out of the range of the plasma thermal velocities, their emission rate is only a fraction $(\omega/\omega_p)^2$ of the vacuum emission rate, so that at the inferred parameters the curvature emission is suppressed by many orders of magnitude. The models ignoring plasma effects could not be considered as physically viable.

The synchrotron maser could provide, at reasonable conditions, the observed radiation power. However, this mechanism is hardly compatible with the 100% linear polarization observed in a few FRBs (Gajjar et al. 2018; Michilli et al. 2018; Oslowski et al. 2019). The synchrotron emission is totally polarized only in the plane of the rotating particles; the out-of-plane emission has an electric field component parallel to the magnetic field so that the polarization of the total emission never reaches 100%. In the simulations of Gallant et al. (1992), a close to 100% polarization of the synchrotron maser emission from a relativistic shock has been observed; however, this result seems to be an artifact of 2D simulations. Namely, their simulation domain was resolved in the direction of the normal to the shock front and in the direction of the magnetic field (parallel to the shock plane) so that all parameters were homogeneous in the direction perpendicular to the magnetic field in the shock plane. This means that radiation was generated in fact by currents homogeneous in this direction. Such currents naturally produce 100% linearly polarized emission. In effect, the synchrotron maser emission is produced by charged bunches rotating in the magnetic field; therefore, their out-of-plane emission should have a component polarized along the field. Only 3D simulations could provide a realistic picture of the polarized synchrotron maser emission.

The totally linearly polarized radiation implies that one of the two polarization modes is totally suppressed in the source or on the way out. Such a condition is achieved if MHD waves with an appropriate frequency are generated in a magnetosphere, where the magnetic energy density exceeds the plasma energy, including the rest energy. Any MHD perturbation excites Alfvén and fast magnetosonic (FMS) waves. However, the group velocity of Alfvén waves is directed along the magnetic field so that only FMS waves could propagate outward, across the magnetic field lines. The condition that the energy in the magnetosphere is dominated by the magnetic energy is crucially important because in this case the FMS waves are smoothly converted into vacuum electromagnetic waves when propagating toward the decreasing plasma density. In FMS waves, the electric field is directed perpendicular to the background magnetic field; therefore, they give rise to the radio emission polarized perpendicularly to the background magnetic field.

High-frequency MHD waves could be generated in the course of magnetic reconnection in a current sheet separating oppositely directed magnetic fields. In the reconnection process, the current sheet breaks into a system of linear currents, which are called magnetic islands because of their
island-like appearance in 2D simulations. Inasmuch as the parallel currents are attracted one to another, the islands continuously merge (see, e.g., the review by Kagan et al. 2015). The merging of two islands perturbs the surrounding magnetic field producing an FMS pulse propagating radially from the merging site. This mechanism was recently proposed as a source of radio emission from the Crab and Crab-like pulsars (Uzdensky & Spitkovsky 2014; Lyubarsky 2019; Philippov et al. 2019). In this paper, a model for FRBs is developed, in which the radio emission is produced by the magnetic reconnection at the onset of the magnetar flare. A rotating magnetar continuously emits relativistic, magnetized wind, similar to the pulsar wind. The oblique rotator introduces in the wind zone an oscillating structure. In the equatorial zone of the wind, the field changes sign every half of a period; therefore, the regions of opposite polarity are separated by a corrugated current sheet. When a magnetar flare shakes the magnetosphere, a strong magnetic perturbation produces a magnetic pulse that propagates from the flare site outward. Just beyond the light cylinder, the pulse catches up with the current sheet, and the strong compression triggers violent reconnection, which gives rise to the radio emission via merging of magnetic islands.

The paper is organized as follows. In Section 2, I estimate parameters of the magnetic pulse and the characteristic frequency and the total energy of the MHD waves generated when the pulse triggers reconnection in the equatorial current sheet. In Section 3, parameters of the magnetar wind are presented and propagation of the magnetic pulse in the wind zone is described. In Section 4, I consider escape of the MHD waves from the magnetic pulse and their conversion to the radio emission. Conclusions are presented in Section 4.

2. Generation of FMS Waves

At the onset of the magnetar flare, the twisted magnetic configuration becomes unstable, which yields a sudden rearrangement of the magnetic field (see, e.g., the recent review by Kaspi & Beloborodov 2017 and references therein). The rapid motion of overtwisted magnetic field lines produces a large-scale MHD perturbation that propagates outward, opens the magnetosphere, and propagates farther out into the magnetar wind (Parfrey et al. 2013; Carrasco et al. 2019). The formation and propagation of such a perturbation, a magnetic pulse, have already been discussed by Lyubarsky (2014), Beloborodov (2017, 2019), Metzger et al. (2019), and Margalit et al. (2020) in the context of FRB production by a magnetar flare. In this paper, I show that the FRB may be produced in the course of magnetic reconnection triggered, just outside the light cylinder, by the magnetic pulse. Namely, GHz magnetosonic waves are generated in the course of reconnection, and these waves give rise to powerful radio emission.\(^1\)

The outward-propagating perturbation is in fact an FMS pulse. The amplitude of the pulse may be conveniently presented as

\[ B_{\text{pulse}} = \sqrt{\frac{L_{\text{pulse}}}{c}} = 3.8 \times 10^{8} \frac{L_{\text{isot}}^{1/4}}{R_{L}} \text{ G}, \]

where \( L_{\text{pulse}} \) is the isotropic luminosity associated with the pulse, \( R_{L} = 4.8 \times 10^{12} \text{ cm} \) is the light cylinder radius, and \( P \) is the magnetar period. Here and hereafter I employ the shorthand notation \( q = q/10^{5} \) in cgs units, e.g., \( L_{\text{isot}} = L/(10^{47} \text{ erg s}^{-1}) \). The total energy in the pulse is

\[ E = L_{\text{pulse}} \tau = 10^{44} L_{\text{pulse}}^{4/3} \tau^{-3} \text{ erg}, \]

where \( \tau = l/c, \) with \( l \) the width of the pulse. The duration of the observed FRB will be shown to be about \( \tau \).

Just beyond the light cylinder, the pulse enters the magnetar wind. The pulse propagates through the wind practically with the speed of light. The nonlinear theory of the pulse propagation is presented in Appendix B; here I give only a short outline. The plasma is squeezed in the pulse and pushed forward; the plasma velocity with respect to the wind is estimated as the velocity of the zero electric field frame, \( v' = E'/B' = B_{\text{pulse}}/(B_{\text{wind}} + B_{\text{pulse}}) \), where \( B_{\text{wind}} \) is the magnetic field of the wind; the prime refers to quantities in the wind frame. The corresponding Lorentz factor is

\[ \Gamma = \frac{B_{\text{wind}}'}{2B_{\text{wind}}} = \frac{1}{2} \frac{B_{\text{pulse}}}{B_{\text{wind}}} = 100 \frac{L_{\text{isot}}^{1/4}}{P^{1/3}}. \]

Here the magnetic field of the wind is expressed via the magnetic momentum of the magnetar, \( \mu \):

\[ B_{\text{wind}} = B_{L} \frac{R_{L}}{R}; \quad B_{L} = \frac{\mu}{R_{L}^{3}} \]

\[ = 9 \times 10^{3} \frac{\mu^{1/3}}{P^{3}} \text{ G}. \]

Note that \( \Gamma \) is the Lorentz factor of the plasma within the pulse. The pulse itself, i.e., the waveform, moves with so large a Lorentz factor that one can safely assume that the pulse moves with the speed of light; see Appendix B.

In the magnetar wind, a current sheet separates two magnetic hemispheres, with the shape of the sheet having been likened to a ballerina’s skirt. When the magnetic pulse arrives at the sheet, the sharp acceleration and compression cause the violent reconnection. Moreover, the current sheet is destroyed by the Kruskal–Schwarzschild instability (Lyubarsky 2010; Gill et al. 2018), which is the magnetic counterpart of the Rayleigh–Taylor instability, so that the field line tubes with the oppositely directed fields fall into the magnetic pulse forming multiple small current sheets scattered over the body of the pulse. Within each of the small current sheets, the reconnection process occurs via formation and merging of magnetic islands, which gives rise to an FMS noise. This noise is converted, as will be shown below, into the radio emission. Since the sources of the noise are distributed in the body of the magnetic pulse but not concentrated at the front part, the duration of the observed radiation burst is of the order of the duration of the pulse, \( \tau \sim l/c \). Let us estimate the total energy of the generated waves and their characteristic frequencies.

### The emitted energy

The total reconnecting magnetic flux may be estimated as \( B_{L} R_{L}^{2} \). Within the pulse, the stripe with the oppositely directed field is compressed \( B_{\text{pulse}}/B_{\text{wind}} \) times.
Then, the total energy of annihilated fields is roughly

$$\varepsilon \sim (B_{\text{pulse}}/B_{\text{wind}}) B_{\text{p}}^2 R_{\text{e}}^2.$$  (5)

According to the simulations by Philippov et al. (2019), the fraction $f \sim 0.01$ of the reconnecting magnetic energy is emitted in the form of FMS waves. Then, the energy of the radio burst may be estimated as

$$\varepsilon_{\text{FRB}} = f \varepsilon = 3.8 \times 10^{39} \frac{f - \frac{2}{3} \mu_3^3 L_{\text{pulse}}^{1/2}}{P} \text{erg}.$$  (6)

The characteristic frequencies.—The characteristic frequency of the emitted waves is determined by the collision time of two merging islands. The size of the islands is 10–100 times larger than the width of the current sheet, $a'$ (Philippov et al. 2019); therefore, one can write

$$\omega' = \frac{c}{\xi a'},$$  (7)

where $\xi \sim 10$–100. Here and hereafter, quantities measured in the plasma comoving frame are marked by a prime. The width of the sheet may be estimated as it has been done for the pulsar current sheet (Lyubarskii 1996; Uzdensky & Spitkovsky 2014). Within the sheet, the pressure of the external magnetic field is balanced by the pressure of “hot” pairs,

$$N' m_e c^2 \gamma'_T = \frac{B_{\text{p}}^2}{8\pi}.'$$  (8)

Here $N'$ is the pair density within the sheet, $\gamma'_T$ is the characteristic Lorentz factor of the pairs, and $m_e$ is the electron mass. The reconnection occurs via the collisionless tearing instability; therefore, the width of the sheet, $a'$, is of the order of a few particle Larmor radii,

$$a' = \frac{\zeta m_e c^2 \gamma'_T}{e B_{\text{p}}},$$  (9)

where $\zeta$ is a few. The synchrotron cooling time is very short at the inferred parameters; therefore, within the sheet, the reconnection energy release is balanced by the synchrotron cooling,

$$\varepsilon B_{\text{p}}^2/4\pi = N' \sigma_T B_{\text{p}}^2/4\pi \omega' a'$$  (10)

Here $\varepsilon \sim 0.1$ is the reconnection rate. Eliminating from the last three equations $\gamma_T$ and $N$ in favor of $a$, one gets

$$a' = \left( \frac{3 \varepsilon \zeta}{r_e} \right)^{1/2} \left( \frac{c}{\omega'_B} \right)^{3/2},$$  (11)

where $r_e$ is the classical electron radius and $\omega'_B = e B_{\text{p}}/m_e c$ is the cyclotron frequency. Now the emitted frequency in the observer’s frame may be estimated as

$$\nu = 2\Gamma \frac{\omega'}{2\pi} = \frac{1}{\pi \zeta} \left( \frac{r_e}{3 \varepsilon \zeta \Gamma} \right)^{1/2} \omega_B^{3/2} = 3 \frac{\mu_3^3 L_{\text{pulse}}^{1/8}}{\xi_3 \Gamma_3 \epsilon_{\text{p}}^{1/2} P_{\text{p}}^{1/2}} \text{GHz}.$$  (12)

The emitted FMS waves propagate on the top of the magnetic pulse and escape from it far away from the magnetar. On the way out, different processes could affect their propagation. Let us consider the fate of the FMS waves and find at what conditions they eventually escape as radio waves.

3. The Magnetar Wind and Propagation of the Magnetic Pulse

In order to study propagation and escape of the FMS waves, let us consider the properties of the pulsar wind and of the magnetic pulse, within which the waves are generated. The particle flux in the wind from a strongly twisted magnetar magnetosphere could reach (Beloborodov 2019)

$$N \sim \frac{c \mu}{e R_3 R_{\pm}},$$  (13)

where $\mu \sim 10^3$ is the pair multiplicity and $R_\pm \sim 5 \times 10^6 \mu_3^{1/3}$ cm is the distance from the star where the magnetic field falls to $10^{13}$ G such that the pair production stops. With the above particle flux, the magnetization parameter of the magnetar wind, which is defined as the ratio of the Poynting flux to the rest-mass energy flux in the wind, is

$$\eta = \frac{B_3^2 R_3^2}{N_3 m_e c^3} = 2.5 \times 10^4 \frac{\mu_3^{4/3}}{M_3 P^3}. $$  (14)

The magnetization parameter is in fact the maximal Lorentz factor achievable by the wind if the magnetic energy is converted to the kinetic energy.

Beyond the light cylinder, the strongly magnetized wind is accelerated linearly with the distance, $\gamma \sim R/R_\pm$, until it reaches the FMS point, $\gamma \sim \eta^{1/3}$. Beyond the FMS point, the wind accelerates very slowly, $\propto (\ln R)^{1/3}$ (Beskin et al. 1998), if there is no dissipation. In this case, one can take the Lorentz factor of the wind in the far zone, $R \gg \eta^{1/3} R_\pm$, to be

$$\gamma_{\text{wind}} = 3 \eta^{1/3} = 90 \frac{\mu_3^{4/9}}{M_3 P^{1/3}}.$$  (15)

The magnetic dissipation could lead to a gradual acceleration of the wind in the equatorial belt, where the magnetic field changes sign every half of a period (Lyubarsky & Kirk 2001; Kirk & Skjæraasen 2003); then, the Lorentz factor of the wind exceeds $\gamma_{\text{wind}}$ and may even reach $\eta$.

The wind is terminated at a strong shock formed when the dynamic pressure of the wind is balanced by the plasma pressure within the nebula surrounding the magnetar. The radius of the wind termination shock is estimated as

$$R_3 = \left( \frac{4\pi e^2}{P c^3 \mu} \right)^{1/2} = 1.2 \times 10^{15} \frac{\mu_3^{1/2}}{P_{15}^{1/2}} \text{cm}.$$  (16)

Here I normalized the pressure within the nebula, $P$, to $10^{-4}$ dyn cm$^{-2}$; such a pressure is found in the nebula surrounding the repeater FRB 121102 (Beloborodov 2017).

The magnetic pulse moves through the magnetar wind as a propagating wave (see Appendix B) so that the plasma enters the pulse through the front part and eventually leaves it through the rear part. Within the pulse, the plasma moves with respect to the wind with the Lorentz factor (3). In the lab frame, the
Lorentz factor of the plasma within the pulse is
\[ \gamma_{\text{pulse}} = 2\gamma_{\text{wind}} \Gamma = \gamma_{\text{wind}} \sqrt{\frac{B_{\text{pulse}}}{B_{\text{wind}}}} \]
\[ = 1.8 \times 10^4 \frac{L_{\text{pulse}}^{1/4}}{\mu_{33}^{1/18} M_{3}^{1/3} \gamma_{\text{wind}}} \]  
(17)

Inasmuch as the plasma acquires a very high Lorentz factor, it is dragged within the pulse to a large distance and is substituted by the wind plasma very slowly.

The pulse picks up the magnetospheric plasma at the region where the magnetic flux in the pulse,
\[ \Phi_0 = B_{\text{pulse}} l R = l \frac{L_{\text{pulse}}}{c} \]  
(18)

becomes equal to the magnetospheric flux, \( B R^2 \), where \( B = \mu/R^3 \). This occurs at the distance
\[ R_0 = \frac{\mu}{\sqrt{\tau L_{\text{pulse}} c}} = 2 \times 10^7 \frac{\mu_{33}^{3/13}}{\tau_{-3} \gamma_{\text{wind}}^{1/27}} \text{ cm}, \]  
(19)
i.e., well within the magnetosphere. Let us estimate the amount of plasma available in this region.

The magnetar magnetosphere is filled by an electron–positron plasma generated in cascades induced by slow untwisting of magnetospheric field lines (Thompson et al. 2002; Beloborodov & Thompson 2007; Thompson 2008; Beloborodov 2013). The pair injection rate is determined by currents in the magnetosphere. In a strongly twisted magnetosphere, the magnetic field lines with the apex radii \( \sim R \) carry the current \( j \sim cB/R \) because the “toroidal” component of the magnetic field could not exceed, by stability considerations, the poloidal one. The theory of pair production in magnetars predicts the plasma density
\[ N \sim M j / e c \sim M B / e R, \]  
(20)

where \( M \sim 100–1000 \) is the pair multiplicity (Beloborodov 2013). Then, the total number of particles in the region \( R \sim R_0 \) is estimated as
\[ N' \sim NR^3 = \frac{M \mu}{e R_0}. \]  
(21)
The pulse picks up these particles and drags them out of the magnetosphere. One can estimate the ratio of the Poynting to the kinetic energy fluxes in the pulse in the far zone as
\[ \sigma_{\text{pulse}} = \frac{\xi}{m_e c N' \gamma_{\text{pulse}}} = 4.5 \times 10^{10} \frac{L_{\text{pulse}}^{1/4} \mu_{33}^{1/18} \gamma_{\text{wind}}}{M_{3}^{3/33} \gamma_{\text{wind}}}. \]  
(22)

This estimate was obtained assuming that the plasma density in the magnetosphere remained the same as in a quiet state. However, one can expect that already at the onset of the pulse some amount of plasma is produced. Then, \( \sigma_{\text{pulse}} \) decreases but still remains extremely high.

The new plasma enters the pulse through the front part, together with the new magnetic flux. Since the magnetar wind runs away with a relativistic velocity, the newly accumulated flux is estimated as
\[ \Phi = \int B_{\text{wind}} (c - v_{\text{wind}}) dt = \int \frac{B_{\text{wind}}}{2 \gamma_{\text{wind}}} dR. \]  
(23)

In the striped part of the wind (in the equatorial belt), only the average magnetic flux is accumulated, the alternating part of the field being annihilated either in the wind before the pulse arrives (and then the wind is accelerated) or when compressed within the pulse. Therefore, one has to substitute \( B_{\text{wind}} \) in this estimate by \( \xi B_{\text{wind}} \), where \( B_{\text{wind}} \) is given by Equation (4) and the latitude-dependent coefficient \( \xi < 1 \) is the ratio of the mean to the total field. Now the accumulated flux may be presented as
\[ \Phi = \frac{\xi B_{\text{wind}} R_{0}}{2 \gamma_{\text{wind}}^{2}}. \]  
(24)
The new flux and plasma form a layer in the front part of the pulse. The width of the layer, \( \Delta \), is presented as
\[ \Delta = \frac{\Phi}{\Phi_0} = 0.2 \xi M_{3}^{2/3} \mu_{33}^{1/9} \left( \frac{\gamma_{\text{wind}}}{\gamma_{\text{wind}}^{1/27}} \right)^2 R_0. \]  
(25)

One can expect that at the distances of interest, at least a part of the alternating field is annihilated in the wind so that \( \gamma_{\text{wind}} > \gamma_{\text{wind}}^{1/27} \). Taking this into account, and also that \( \xi < 1 \) (at the equator, \( \xi = 0 \)), one can expect that when the pulse arrives at the wind termination shock, the layer with the new plasma and flux fills only a small part of the pulse, the mean body of the pulse being still filled with the plasma picked up in the magnetosphere.

The pulse may be considered as a propagating wave until the shock is formed owing to the nonlinear steepening of the pulse. The nonlinear steepening occurs because the wave velocity depends on the local density and magnetic field, which vary across the pulse. In a highly magnetized plasma, the nonlinearity is weak, and therefore the nonlinear steepening scale in the comoving plasma frame is large (Lyubarsky 2003; see also Appendix B),
\[ s_{\text{steep}} \sim \sigma_{\text{pulse}} l'. \]  
(26)

Transforming to the lab frame and making use of Equation (17), one finds that the shock formation distance is
\[ R_{\text{steep}} = 4 \sigma_{\text{pulse}} \gamma_{\text{pulse}}^{2} l' = 4 \times 10^{16} \sigma_{\text{pulse}} \times \frac{L_{\text{pulse}}^{1/4} \mu_{33}^{1/18} \gamma_{\text{wind}}}{M_{3}^{3/33} \gamma_{\text{wind}}^{1/27}} \text{ cm}. \]  
(27)

Taking into account that \( \sigma_{\text{pulse}} \gg 1 \) (see above), one concludes that the shock could hardly be formed while the pulse propagates within the magnetar wind.

The above considerations assume implicitly that the mean field in the wind is roughly in the same direction as the initial field of the pulse. Taking into account that the mean field has opposite signs in two hemispheres, one can imagine a situation when the accumulated flux is opposite to the flux in the pulse. In an ideal case, this does not affect the obtained results because one can easily imagine a pulse within which the magnetic field changes sign such that a current sheet separates domains of the opposite polarity. The electromagnetic stress is a quadratic function of the fields; therefore, a solution of ideal MHD equations is not affected if one reverses fields in any bundle of the magnetic field lines and inserts an appropriate current sheet provided that the plasma in the sheet is light enough (so that the overall inertia is not affected). In reality, the
current sheet is unstable, and one can expect that narrow magnetic tubes of the opposite polarity fall into the main body of the pulse and dissipate there, as was discussed in Section 2. In this case, one can expect that the overall magnetization of the pulse decreases and the pulse is accelerated because the heated plasma expands, transforming heat to kinetic energy. The detailed analysis of this process is beyond the scope of this paper. As was discussed above, the pulse accumulates a relatively small new flux before it arrives at the wind termination shock or at the previously ejected baryonic cloud. Therefore, one can expect that the above results remain valid independently of the relative magnetic polarity in the pulse and in the wind.

4. Escape of the Waves

FMS waves are in fact sound waves in which longitudinal plasma oscillations are maintained by the magnetic pressure. In the relativistic case, their phase velocity is found as (e.g., Appl & Camenzind 1988)

$$\frac{\omega}{k} = c \sqrt{\frac{\sigma}{1 + \sigma}},$$

(28)

where $\sigma = B^2 / 4\pi \rho c^2$. The electric field in the wave, $E = \frac{1}{c} \mathbf{v} \times \mathbf{B}$, is directed perpendicularly to the propagation direction; therefore, this is a transverse electromagnetic wave. At $\sigma \to \infty$, the phase velocity of the wave goes to the speed of light. The conductivity current in the wave is found by substituting a harmonic wave with the above dispersion law into the Maxwell equations,

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}; \quad \nabla \times \mathbf{B} = \frac{4\pi}{c} j + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t},$$

(29)

and taking into account that in this wave $k \cdot \mathbf{E} = 0$. Then, one finds that the ratio of the conductivity to the displacement current is

$$\frac{j}{i\omega \mathbf{E}} = \frac{1}{4\pi(1 + \sigma)}.$$  

(30)

One sees that at $\sigma \to \infty$ the conductivity current vanishes so that when the plasma density goes to zero, the FMS wave becomes a vacuum electromagnetic wave.

This could be easily understood as follows. Consider a vacuum electromagnetic wave superimposed on a homogeneous magnetic field such that the electric field of the wave is perpendicular to the background field. A charged particle in such a system experiences drift motion in the crossed electric and magnetic fields, the direction of motion being independent of charge. Therefore, if a small amount of plasma is added to the system, no electric current appears in the system, and the wave propagates as in vacuum.

The FMS waves propagate outward on the top of the magnetic pulse, which runs through the magnetar wind. The pulse propagates practically with the speed of light. Recall that the velocity of the pulse is the velocity of the wave shape; the plasma moves a bit slower, even though highly relativistically (see Equation (17)). The wave shape propagates with the FMS velocity with respect to the plasma so that the Lorentz factor of the wave shape is extremely large, $\sim \gamma_{\text{pulse}} \sqrt{\sigma_{\text{pulse}}}$ (see Appendix B). Therefore, any deviations of the pulse velocity from the speed of light are negligible. The pulse is decelerated only when it collides with a stationary or slowly moving plasma. This happens either at the wind termination shock, when the pulse enters the magnetar nebula (Lyubarsky 2014), or, in the case of repeaters, when the pulse collides with the previously ejected baryonic material (Beloborodov 2017, 2019; Metzger et al. 2019; Margalit et al. 2020). When the pulse is decelerated, the FMS waves escape as radio waves. The observed duration of the FRB is determined by the width of the pulse, and therefore it is of the order of $\tau$. Different processes could affect these waves on the way out; let us consider them.

First of all, let us consider the cyclotron absorption. Making use of Equation (1), one finds that the cyclotron resonance, $\omega' = \omega_{\text{B}} \equiv eB_{\text{pulse}} / m_e c$, is reached at the radius

$$R_{\text{cycl}} = 5 \times 10^5 \frac{L_{\text{pulse},47}^{1/2}}{\nu_9} \text{ cm},$$

(31)

which is comparable to the radius of the termination shock (16), and therefore in some cases the cyclotron absorption should be taken into account. The resonant cross section is

$$\sigma_{\text{res}} = 4\pi^2 \frac{e^2}{m_e c} \delta(\omega' - \omega_{\text{B}}).$$

(32)

Now the cyclotron absorption depth is found as

$$\tau_{\text{cycl}} = \int \sigma_{\text{res}} N' dR' = \frac{4\pi^2 e N'}{\sigma_{\text{pulse}} \gamma_{\text{pulse}} c},$$

(33)

where $N'$ is the lepton density in the comoving frame. Making use of the estimate $dB' / dR' \sim B_{\text{pulse}} / R$, one finds

$$\tau_{\text{cycl}} \sim \frac{\pi \omega_{\text{B}} R_{\text{cycl}}}{\sigma_{\text{pulse}} \gamma_{\text{pulse}} c} = \frac{\pi \sigma_{\text{pulse}} R_{\text{cycl}}}{2\sigma_{\text{pulse}} \gamma_{\text{pulse}}^2 c} = 1.6 \times 10^7 \left( \frac{L_{\text{pulse},47}^{1/3}}{\nu_9} \right) \left( \frac{\gamma_{\text{pulse}}}{\gamma_{\text{cycl}}^2} \right)^2,$$

(34)

where I used Equations (1) and (17) and the general relation for the ratio of the Poynting to the kinetic energy flux $\sigma = \omega_{\text{B}}^2 / \omega_{\text{B}}^2$.

One sees that the synchrotron absorption is weak if $\sigma_{\text{pulse}} > 10^7$. According to the estimate (22), the initial magnetization of the pulse satisfies this constraint. However, when the wind matter enters the pulse, the magnetization becomes significantly lower, and therefore the cyclotron absorption may be significant in the layer where the wind plasma is accumulated. As was argued above, only a fraction of the pulse is filled by the wind matter at the cyclotron resonance point (31); therefore, only a fraction of the radiation is absorbed, the rest escaping freely. Another option is that the waves escape the pulse before the cyclotron resonance radius is reached. The last occurs when the radius of the magnetar wind termination shock (16) is smaller than the cyclotron resonance radius. The pulse is sharply decelerated just beyond the termination shock (Lyubarsky 2014), and the waves escape into the nebula, where the magnetic field is significantly smaller than within the pulse so that $\omega > \omega_{\text{B}}$. In repeating FRBs, the termination shock is pushed outward by the previous bursts, and therefore it may be well beyond the cyclotron resonance radius. In this case, however, the magnetic pulse is sharply decelerated when it enters the mildly relativistic baryonic material ejected by the previous flare (Beloborodov 2019; Metzger et al. 2019; Margalit et al. 2020). This occurs at
relatively small distances from the magnetar, \( \sim 10^{12} - 10^{14} \) cm; therefore, the waves leave the pulse well before the cyclotron resonance radius is reached.

Now let us consider the **nonlinear interactions of MHD waves**, which could produce a wave cascade that transfers the wave energy out of the observed spectral range. The nonlinear interactions of force-free MHD waves were studied by Thompson & Blaes (1998) and Lyubarsky (2019). The strongest are the three-wave interactions:

\[
\omega = \omega_1 + \omega_2; \quad k = k_1 + k_2. \tag{35}
\]

In the force-free MHD, the dispersion relation for the FMS waves is \( \omega = ck \), and for the Alfvén waves \( \omega = ck \cos \theta \), where \( \theta \) is the angle between the background magnetic field and the direction of the wave. Substituting these dispersion laws into the conservation laws, one sees that only interactions involving both types of waves are possible:

\[
fms \approx \text{fms} + \text{Alfvén} \quad \text{and} \quad fms \approx \text{Alfvén} + \text{Alfvén}. \tag{36}
\]

The rate of the interaction may be roughly estimated, in the zero electric field frame, as

\[
q' \sim \left( \frac{\delta B'}{B'} \right)^2 \omega', \tag{37}
\]

where \( \delta B \) is the amplitude of the waves and \( B \) is the background field.

The induced decay of FMS waves (36) is possible only in the presence of Alfvén waves. However, they do not propagate across the magnetic field lines and therefore remain close to the reconnection cites. Far from the reconnection cites, they grow exponentially from small fluctuations with the rate (37), which requires many e-folding times. Therefore, the condition that the FMS waves do not decay significantly may be written as

\[
\tau_{NL} \equiv \int_{R_L}^R q'dr' < 10. \tag{38}
\]

As soon as the pulse propagates in the accelerating wind, the Lorentz factor of the plasma in the pulse also increases linearly with the distance. The ratio \( \delta B'/B' \) is independent of the distance and the Lorentz factor of the flow; however, the proper wave frequency decreases as \( \gamma^{-1} \sim r^{-1} \) and the proper time goes as \( dr' \propto dr/r \). Therefore, as long as the flow is accelerated, \( \tau_{NL} \) is determined by the initial region \( R \sim R_L \), where the wind is still mildly relativistic and the pulse Lorentz factor is given by Equation (3). Taking into account that the ratio \( (\delta B'/B')^2 \) is just the ratio of the FMS energy (6) to the total energy of the magnetic pulse (2), one gets

\[
\tau_{NL} \sim \frac{\epsilon_{\text{FRB}}}{E} \frac{\omega R_L}{2 \Gamma^2 c} = 1.9 \frac{f_2 \mu_{33}^{10/9} v_{9}}{L_{\text{pulse,47}} \Gamma_{-3} P_{-3}}, \tag{39}
\]

which means that the FMS signal is not affected significantly by the three-wave decay process in the vicinity of the magnetar.

In the far zone, where the wind stops accelerating or accelerates slowly, the nonlinear optical depth begins to increase:

\[
\tau_{NL} = \frac{\epsilon_{\text{FRB}}}{E} \frac{\omega R_L}{2 \Gamma^2 c} = 50 \frac{f_2 \mu_{33}^{10/9} v_{9} \mathcal{M}_{50}^{1/3}}{L_{\text{pulse,47}} \Gamma_{-3} P_{-3}} (\frac{\gamma_{\text{wind}}}{\gamma_{\text{wind}}})^2 R_{15}. \tag{40}
\]

One sees that at large enough distances from the magnetar, the nonlinear wave–wave interaction could have removed the wave energy from the observed spectral range. However, the MHD approximation is already violated at these distances. Namely, the MHD approximation assumes that the proper wave frequency is less than both the plasma frequency, \( \omega_p = (4\pi e^2 N^2/m_e)^{1/2} \), and the cyclotron frequency, \( \omega_B = eB/m_e \). Making use of the relation \( \sigma = (\omega_B/\omega_p)^2 \), the ratio of the proper wave frequency to the plasma frequency in the pulse is estimated, with the aid of Equation (1), as

\[
\frac{\omega'}{\omega_p} = \frac{\omega' \sigma^{1/2}}{\omega_B} = 0.2 \sigma^{1/2} \frac{R_{15} v_{9}}{L_{\text{pulse,47}}}. \tag{41}
\]

One sees that the MHD approximation is violated (in fact, the Alfvén waves do not exist at \( \omega' > \omega_p \)) before the MHD nonlinear interactions become significant if \( \sigma \) exceeds a few dozens, which is not a severe constraint.

Another important effect is the **nonlinear steepening**, which could lead to formation of shock waves and their subsequent decay. An important point is that for FMS waves this effect works even when the wave frequency exceeds the plasma frequency, i.e., when the MHD approximation is generally violated. The dispersion relation for electromagnetic waves polarized perpendicularly to the background magnetic field looks like (e.g., Melrose 1997)

\[
\omega'^2 = k'^2 c^2 - \frac{\omega^2_{\gamma} \omega'^2}{\omega_B^2 - \omega^2_p}. \tag{42}
\]

This relation is reduced to the dispersion of the FMS waves (28) at \( \omega' \ll \omega_B \); therefore, in the magnetically dominated plasma, \( \omega_B \gg \omega_p \), this wave behaves as an FMS wave even at \( \omega_p < \omega' < \omega_B \), when the MHD approximation is generally inapplicable. Therefore, the nonlinear steepening proceeds until the cyclotron resonance radius (31). The shock formation scale for a strong pulse of a width \( l \) is given in the comoving plasma frame by Equation (26). For waves with amplitude smaller than the background field this equation is modified to

\[
s_{\text{step}}' \sim \frac{B'}{\delta B'} \frac{c}{\omega'}. \tag{43}
\]

Transforming to the lab frame and making use of Equations (2), (6), and (17), one finds that the decay of the FMS waves due to the nonlinear steepening could occur at the distances

\[
R_{\text{step}} = 4 \sigma \gamma_{\text{pulse}}^2 \frac{E}{\epsilon_{\text{FRB}} \Gamma} \frac{c}{\omega} = 3 \times 10^{14} \frac{\sigma}{L_{\text{pulse,47}} \Gamma_{-3} P_{-3}} \text{ cm}. \tag{44}
\]

The waves survive if this distance exceeds the cyclotron resonance distance (31), which implies a not very restrictive condition \( \sigma > 10 \).
Now let us consider the \emph{induced scattering}. Within the cyclotron resonance radius, there is no scattering at all because the particles experience only drift oscillations in the crossed electric field of the waves and the magnetic field of the background. Inasmuch as the drift motion is independent of the particle charge, there is no oscillating electric current that produces scattered waves. Beyond the cyclotron resonance, $R > R_{\text{cyl}}$, the induced scattering may be estimated from the Kompaneets equation

$$\frac{\partial n_\nu}{\partial t} = \frac{\sigma_T N^\prime h}{m_e c^2} \frac{1}{\nu^2} \frac{\partial}{\partial \nu^2} (\nu^4 n_\nu^2).$$  \hfill (45)$$

Here $\sigma_T$ is the Thomson cross section, $h$ is the Planck constant, and $n_\nu$ is the photon occupation number. One sees that the induced scattering rate may be presented, in the comoving plasma frame, as

$$\dot{q}' \sim \sigma_T N^\prime c \frac{h \nu^2}{m_e c^2} \frac{1}{\nu^2} \frac{\omega_p^4}{\omega^3 m_e c^2 N^\prime},$$  \hfill (46)$$

where $U_{\text{rad}}' \sim h \nu^2 n_\nu / c^3$ is the radiation energy density. Making use of the relations $\omega_p^2 = \sigma \omega^2$ and $\frac{\omega_{\text{rad}}'}{m_e c^2 N^\prime} = \frac{U_{\text{rad}}'}{B_{\text{pulse}}^2 / 4\pi} = 1 \frac{\varepsilon_{\text{FRB}}}{E}$, one gets the estimate for the induced scattering optical depth

$$\frac{\omega_p^4}{\omega^3} \frac{\varepsilon_{\text{FRB}}}{\varepsilon} \frac{\sigma_{\text{pulse}} R}{c \gamma_{\text{pulse}}} \frac{\sigma_{\text{pulse}} R}{\varepsilon} \frac{\varepsilon_{\text{FRB}}}{\sigma_{\text{pulse}} R_{16}^3} \frac{f_{-2} M_{\text{p}}^{10/9} \Lambda_f^{1/3} p_{\text{pulse}}}{\tau_{-3} \gamma_{\text{wind}} P} \left( \frac{\gamma_{\text{wind}}}{\tau_{\text{wind}}} \right)^2.$$  \hfill (47)$$

One sees that the induced scattering may be neglected at a moderately large magnetization, $\sigma_{\text{pulse}} \gtrsim 100$. One concludes that at quite general conditions, FMS waves could escape as radio waves.

### 5. Polarization of the Outgoing Radiation

Let us now consider polarization of the outgoing radiation. In this paper, the source of radiation is assumed to be the magnetic reconnection producing FMS waves. In the FMS wave, the electric field is directed perpendicularly to the background magnetic field. When an FMS wave propagates toward decreasing plasma density, it is converted into the electromagnetic wave polarized perpendicularly to the local magnetic field. The waves are generated in the reconnection process triggered by a large-scale magnetic pulse produced by a magnetar flare and propagating outward in the magnetar magnetosphere and the wind. The FMS waves propagate on the top of the pulse and escape as radio waves far away from the magnetar, when the pulse is decelerated colliding with the surrounding plasma. The magnetic pulse is formed well within the magnetosphere (see Equation (19)), and the magnetic flux from this region is transferred by the pulse outward. Therefore, the direction of the magnetic field within the pulse is determined by the rotational phase of the magnetar. When the pulse travels in the magnetar wind, it picks up the azimuthal magnetic field, which is accumulated at the front of the pulse (see the discussion at the end of Section 3). The radiation passes through this layer on the way out. If the polarization vector is adiabatically adjusted to the local magnetic field, the outgoing radiation becomes eventually completely polarized perpendicularly to the field in the wind, i.e., along the rotational axis of the magnetar. In the opposite case, the waves are split into two normal modes corresponding to the local magnetic field so that the radiation is depolarized.

In the comoving plasma frame, the condition for adiabatic adjustment of the polarization vector is written as (e.g., Ginzburg 1970)

$$\frac{\omega}{c} \Delta n \delta > 1,$$  \hfill (49)$$

where $\delta$ is the characteristic width of the polarization vector rotation zone and $\Delta n$ is the difference between the refraction indexes of the two normal modes in the system. In the electron–positron plasma, the modes are linearly polarized. One of them is polarized perpendicularly to the background magnetic field and is described by the dispersion law (42). The second mode is polarized along the magnetic field, and therefore it does not “feel” the magnetic field; it is described by the usual dispersion law, $\omega^2 = k^2 c^2 + \omega_p^2$. Taking into account that $\omega \lesssim \omega \omega_p$ in the wave escape zone, one could take $\Delta n \sim (\omega_p / \omega)^2$.

The minimal width of the transition zone where the background magnetic field turns may be estimated from the condition that there are enough charge carriers to provide the required curl of the magnetic field,

$$\frac{B'}{\delta} < 4\pi e N' .$$  \hfill (50)$$

Substituting this inequality into the left-hand side of the inequality (49), one gets

$$\frac{\omega}{c} \Delta n \delta > \frac{\omega_p^2}{c a^4} \frac{B'}{4\pi e N' \omega} = \frac{\omega_p^3}{\omega} .$$  \hfill (51)$$

One sees that if the radiation escapes the magnetic pulse before the cyclotron resonance radius (31) is reached, the 100% polarized emission could be formed. This happens if the magnetar wind termination shock lies within the cyclotron resonance radius or if the pulse collides with the mildly relativistic baryonic material ejected by the previous flare. The last is relevant only for repeating FRBs. An essential point is that in this case the polarization is directed along the rotation axis of the magnetar, and therefore the position angle does not change from one burst to another. If the waves escape the magnetic pulse beyond the cyclotron resonance radius, the radiation is depolarized, the final degree of polarization being dependent on the angle between the magnetic field in the pulse and in the wind.

### 6. Conclusions

In this paper, I outlined a model, which attributes the radio emission of FRBs to the magnetic reconnection during the magnetar flare. The work was motivated by the discovery of
100% polarization in a few FRBs (Gajjar et al. 2018; Michilli et al. 2018; Osłowski et al. 2019), which imposes severe restrictions on the emission process because one of the two polarization modes should be suppressed completely. The reconnection is accompanied by generation of an FMS noise, which could be converted into the electromagnetic radiation if the magnetic energy in the system exceeds the plasma energy. The FMS waves polarized perpendicularly to the background magnetic field, and they are converted into 100% polarized electromagnetic waves provided that depolarization processes do not come into play.

One can expect the violent reconnection at the onset of the magnetar flare when a strong, large-scale magnetic pulse from the flare reaches the current sheet separating, just beyond the light cylinder, the oppositely directed magnetic fields. It was shown in the paper that the energy and characteristic frequency of the generated FMS waves are compatible with the observed parameters of FRBs. These waves propagate outward on the top of the magnetic pulse. I considered different processes, which could hinder propagation of the waves on the way out, such as cyclotron absorption, nonlinear wave interactions, and induced scattering, and showed that the wave could escape as electromagnetic waves. The waves are polarized perpendicularly to the background magnetic field, and unless the depolarization occurs, the outgoing radiation acquires 100% polarization degree.

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**Appendix A**

**FMS Waves in the Inner Magnetosphere**

At the onset of the magnetar flare, the violent reconnection occurs in the inner part of the magnetosphere; therefore, one can expect that FMS waves are generated there. The plasma scales in the inner magnetosphere are microscopically small; therefore, it is not evident that the waves in the GHz frequency band could be generated. In this appendix, I show that even if we assume that the FMS waves with the required frequencies and power are produced deep inside the magnetosphere, they would decay via the nonlinear interactions (36) so that no radiation would escape.

The nonlinear interaction rate is given by Equation (37). Denoting the luminosity of the FMS waves as $L_{FRB}$, one gets the estimate for the wave amplitudes,

$$\delta B = \frac{L_{FRB}}{c R}, \quad (A1)$$

so that

$$\frac{\delta B}{B} = \frac{L_{FRB} R^2}{c \mu} \quad (A2)$$

The MHD cascade develops at the condition (38), which in our case is written just as

$$\left( \frac{\delta B}{B} \right)^2 \frac{c}{R} < 10. \quad (A3)$$

Substituting the above estimate, one finds that FMS waves could not propagate beyond the distance

$$R = 1.1 \times 10^7 \left( \frac{\mu^2}{L_{FRB} \Delta \nu^2} \right)^{1/5} \text{cm}, \quad (A4)$$

which is well within the magnetosphere. Note that the magnetospheric magnetic field exceeds the perturbation field (1) at this distance, so that the use of the dipole formula for the background field in the estimate (A2) is justified.

**Appendix B**

**Propagation of a Magnetic Pulse through the Magnetar Wind**

The magnetic perturbation from a magnetar flare propagates outward as a large-scale FMS pulse. The propagation of a nonlinear wave in a highly magnetized wind is described as follows (Lyubarsky 2003). In the far zone of the wind, the magnetic field may be considered as purely azimuthal and the pulse as spherical. It follows from the continuity equation,

$$\frac{\partial}{\partial t} N + \frac{1}{R^2} \frac{\partial}{\partial R} R^2 N v = 0, \quad (B1)$$

and the frozen-in condition,

$$\frac{\partial B}{\partial t} + \frac{1}{R} \frac{\partial}{\partial R} R v B = 0, \quad (B2)$$

that the magnetic field may be presented as

$$B = b R N, \quad (B3)$$

where $b$ is a constant. The dynamic equation may be presented in the form of the energy equation

$$\frac{\partial}{\partial t} T_{00} + \frac{1}{R^2} \frac{\partial}{\partial R} R^2 T_{01} = 0, \quad (B4)$$

where the components of the energy—momentum tensor are

$$T_{01} = w \gamma^2 v + \frac{B^2}{4 \pi} \gamma^2 v, \quad (B5)$$

$$T_{00} = w \gamma^2 - p + \frac{1 + v^2}{8 \pi} B^2 \gamma^2. \quad (B6)$$

Here $w$ and $p$ are the plasma enthalpy and the pressure, respectively.

When the plasma is cold, $w = m N c^2$, $p = 0$, where the prime refers to the plasma comoving frame, one can define the new variable,

$$\tilde{N} = NR^2, \quad (B7)$$

and write Equations (B1)–(B6) as a set of the 1D hydrodynamics equations

$$\frac{\partial \tilde{N}}{\partial t} + \frac{\partial \tilde{N}}{\partial R} = 0, \quad (B8)$$

$$\frac{\partial T_{00}}{\partial t} + \frac{\partial T_{01}}{\partial R} = 0, \quad (B9)$$

$$T_{01} = \mathcal{W} \gamma^2 v; \quad T_{00} = \mathcal{W} \gamma^2 - \mathcal{P}, \quad (B10)$$
for the medium with the effective equations of state and energy
\[ P = \frac{R^2 B^2}{8\pi} = \frac{b^2 N^2}{8\pi}; \quad \mathcal{E} = m\tilde{n} c^2 + \frac{b^2 \tilde{N}^2}{8\pi}; \]  

\[ \mathcal{W} = \mathcal{E} + P = m\tilde{n} c^2 + \frac{b^2 \tilde{N}^2}{4\pi}. \]  

Now the solution may be found in the form of a simple wave (e.g., Landau & Lifshitz 1987).

The nonlinear simple wave is found from the condition that all dependent variables are functions of one of them, e.g., \( \tilde{N} \). This means that Equations (B8) and (B9) should be equivalent, which implies
\[ \frac{dT_{01}}{dT_{00}} = \frac{d(\tilde{N}v)}{d(\tilde{N})}. \]  

The last equation reduces to
\[ \frac{dv}{d\tilde{N}} = \frac{s_{\text{FMS}}}{\gamma^2 \tilde{N}(1 + vs_{\text{FMS}}/c^2)}, \]  

where
\[ s_{\text{FMS}} = \left( \frac{dP}{dc} \right)^{1/2} c = \left( \frac{b^2 \tilde{N}^2}{4\pi m\tilde{n}^2 c^2 + b^2 N^2} \right)^{1/2} c, \]  

is the FMS velocity. Making use of Equations (B3) and (B7), one presents \( s_{\text{FMS}} \) in the standard form:
\[ s_{\text{FMS}} = \left( \frac{B^2}{4\pi m\tilde{n}^2 c^2 + B^2} \right)^{1/2} c = \left( \frac{\sigma}{1 + \sigma} \right)^{1/2} c, \]  

where \( \sigma \) is the magnetization parameter. For a small wave amplitude, \( \delta n \ll \tilde{n} \), one obtains the linear FMS wave propagating with the velocity \( s_{\text{FMS}} = \text{const} \). For a high-amplitude pulse, \( s_{\text{FMS}} \) depends on the local density; therefore, the wave becomes nonlinear. However, in the strongly magnetized case, \( \sigma \gg 1 \), \( s_{\text{FMS}} \) is close to the speed of light; therefore, even high-amplitude waves are nearly linear. The physical reason for this is that the displacement current significantly exceeds the conductivity current (see Equation (30)).

Substituting \( s_{\text{FMS}} = c \) and \( v = (1 - 1/(2\gamma^2))c \) into Equation (B14), one gets
\[ \frac{d\gamma}{d\tilde{N}} = \frac{\gamma}{2\tilde{N}}, \]  

which yields
\[ \gamma = \left( \frac{\tilde{N}}{N_{\text{wind}}} \right)^{1/2} \gamma_{\text{wind}} = \left( \frac{B}{B_{\text{wind}}} \right)^{1/2} \gamma_{\text{wind}}. \]  

Here the index “wind” refers to quantities in the magnetar wind outside of the pulse. Note that \( v \) and \( \gamma \) are the velocity and the Lorentz factor of the plasma in the pulse, i.e., of the zero electric field frame (see Equations (3) and (17)). The pulse itself, i.e., the waveform, moves along the characteristics of Equations (B8) and (B9),
\[ \frac{dx}{dt} = \frac{d(\tilde{N}v)}{d(\tilde{N})} = \frac{v + s_{\text{FMS}}}{1 + vs_{\text{FMS}}/c^2} = \left( 1 - \frac{1}{8\gamma^2} \right) c. \]  

The last equality in this expression is obtained for \( \sigma \gg 1 \), \( \gamma \gg 1 \). One sees that the waveform moves with the Lorentz factor \( 2\gamma \sqrt{\sigma} \), so that at each point the waveform moves with respect to the plasma with the local FMS velocity. Inasmuch as the velocity of the waveform varies along the pulse, the pulse shape also varies, which could lead to formation of a shock front. However, the corresponding distance, \( \sim 8\gamma^2 \sigma \), is so large that for the parameters of the pulse discussed in Section 3, the nonlinear steepening could be neglected for any reasonable distances from the magnetar. Therefore, one can safely assume that the pulse moves with the speed of light.

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