AN EXACT AND EXPLICIT FORMULA FOR PRICING LOOKBACK OPTIONS WITH REGIME SWITCHING

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Abstract. This paper investigates the pricing of European-style lookback options when the price dynamics of the underlying risky asset are assumed to follow a Markov-modulated Geometric Brownian motion; that is, the appreciation rate and the volatility of the underlying risky asset depend on states of the economy described by a continuous-time Markov chain process. We derive an exact, explicit and closed-form solution for European-style lookback options in a two-state regime switching model.

1. Introduction. Option pricing is an important field of research in financial economics from both a theoretical and practical point of view. The pioneering work of [1] laid the foundations of the field and stimulated important research in option pricing theory and its mathematical models. The Black-Scholes formula has been documented in many studies that it does not explain the empirical behavior of the implied volatility smile and smirk. Consequently, many extensions to the Black-Scholes model have been introduced. Such models include the stochastic volatility models, jump-diffusion models and models driven by Lévy process.

Recently, there has been considerable interest in applications of regime switching models driven by a Markov chain to various financial problems. For an overview of Markov chains, see [4]. Many papers those investigated option pricing problems under a regime switching model include [6], [3], [2] and [14]. There is no closed-form solution to exotic options under a regime switching model. The numerical methods to solve a system of pricing partial differential equations is complex and computational time could be substantial. In our pricing model, we assume that the knowledge of the current state of the chain is already known before a unique option price can be obtained. If the chain is observable, then of course we know the current state of the chain. On the other hand, if the chain is unobservable, the current state of the chain needs to be filtered from the observations. Alternatively,
one can condition on the current state of the chain and then average over the possible states of the chain to get a market price, see [7].

In this paper, we investigate the pricing of European-style lookback options when the price dynamics of the underlying risky asset are governed by a Markov-modulated Geometric Brownian motion. The Markov-modulated model can provide a more realistic way to describe and explain the market environment. It provides one possible way to model the situation where the market parameters depend on a market mode which switches among a finite number of states and reflects the state of the underlying economy, the macro-economic condition, the general mood of the investors in the market, business cycles and other economic factors (See [12]). We derive an analytical solution for lookback options by means of the homotopy analysis method (HAM). HAM was initially suggested by [10] and has been successfully used to solve a number of heat transfer problems, see [9]. [13] proposed to adopt HAM to obtain an analytic pricing formula for American options in the Black-Scholes model. [8] used HAM to derive an analytic formula for lookback options under Heston’s stochastic volatility model. In this paper, we consider the regime-switching model and it gives rise to a couple of partial differential equations as pricing equations. We applied HAM to solve these couple of partial differential equations and obtain a closed-form solution in infinite series.

This paper is organized as follows: Section 2 describes the dynamics of the asset price under the Markov-modulated Geometric Brownian motion. Section 3 formulates a floating-strike lookback option. Section 4 derives an exact, closed-form solution for the floating-strike lookback option. Section 5 briefly discusses a fixed strike lookback option. The final section draws a conclusion.

2. Asset Price Dynamics. Consider a complete probability space \((\Omega, \mathcal{F}, \mathcal{P})\), where \(\mathcal{P}\) is a real-world probability measure. Let \(\mathcal{T}\) denote the time index set \([0, T]\) of the model. Write \(\{W_t\}_{t \in \mathcal{T}}\) for a standard Brownian motion on \((\Omega, \mathcal{F}, \mathcal{P})\). Suppose the states of an economy are modelled by a finite state continuous-time Markov chain \(\{X_t\}_{t \in \mathcal{T}}\) on \((\Omega, \mathcal{F}, \mathcal{P})\). Without loss of generality, we can identify the state space of \(\{X_t\}_{t \in \mathcal{T}}\) with a finite set of unit vectors \(X := \{e_1, e_2, \ldots, e_N\}\), where \(e_i = (0, \ldots, 1, \ldots, 0) \in \mathbb{R}^N\). We suppose that \(\{X_t\}_{t \in \mathcal{T}}\) and \(\{W_t\}_{t \in \mathcal{T}}\) are independent.

Let \(A\) be the generator \([a_{ij}]_{i,j=1,2,\ldots,N}\) of the Markov chain process. From [4], we have the following semi-martingale representation theorem for \(\{X_t\}_{t \in \mathcal{T}}\):

\[
X_t = X_0 + \int_0^t AX_s ds + M_t ,
\]

where \(\{M_t\}_{t \in \mathcal{T}}\) is an \(\mathbb{R}^N\)-valued martingale increment process with respect to the filtration generated by \(\{X_t\}_{t \in \mathcal{T}}\).

We consider a financial model with two primary traded assets, namely a money market account \(B\) and a risky asset or stock \(S\).

The instantaneous market interest rate \(\{r(t, X_t)\}_{t \in \mathcal{T}}\) of the bank account is given by:

\[
r_t := r(t, X_t) = \langle r, X_t \rangle ,
\]

where \(r := (r_1, r_2, \ldots, r_N)\) with \(r_i > 0\) for each \(i = 1, 2, \ldots, N\) and \(\langle \cdot, \cdot \rangle\) denotes the inner product in \(\mathbb{R}^N\).
In this case, the dynamics of the price process $\{B_t\}_{t \in \mathcal{T}}$ for the bank account are described by:
\[
    dB_t = r_t B_t dt , \quad B_0 = 1 .
\]
(2.3)
Suppose the stock appreciation rate $\{\mu_t\}_{t \in \mathcal{T}}$ and the volatility $\{\sigma_t\}_{t \in \mathcal{T}}$ of $S$ depend on $\{X_t\}_{t \in \mathcal{T}}$ and are described by:
\[
    \mu_t := \mu(t, X_t) = \langle \mu, X_t \rangle > , \quad \sigma_t := \sigma(t, X_t) = \langle \sigma, X_t \rangle > ,
\]
(2.4)
where $\mu := (\mu_1, \mu_2, \ldots, \mu_N)$, $\sigma := (\sigma_1, \sigma_2, \ldots, \sigma_N)$ with $\sigma_i > 0$ for each $i = 1, 2, \ldots, N$ and $\langle \cdot, \cdot \rangle >$ denotes the inner product in $\mathbb{R}^N$.

We assume that the price dynamics of the underlying risky asset $S$ are governed by the Markov-modulated Geometric Brownian motion:
\[
    dS_t = \mu_t S_t dt + \sigma_t S_t dW_t , \quad S_0 = s_0.
\]
(2.5)

3. **Lookback options.** In this section, we now turn to the pricing of lookback options in a regime switching model. In particular, we consider a floating strike lookback option under a regime switching model. Assume we are already working under a risk neutral measure $Q$. The price dynamics of the underlying risky share under $Q$ are governed by
\[
    dS_t = r_t S_t dt + \sigma_t S_t d\tilde{W}_t .
\]
(3.1)
The payoff of this option is the difference between the maximum asset price over the time between initiation and expiration and the asset price at expiration. The maximum of the asset price up to time $t$ is denoted by
\[
    Y(t) = \max_{0 \leq u \leq t} S_u.
\]
(3.2)
Then the payoff of the lookback option at expiration time $T$ is $V(T) = Y(T) - S_T$.

Given $S_t = s$, $Y(t) = y$ and $X(t) = x$, a price of the lookback option $V$ is:
\[
    V(t, s, y, x) = E^Q[e^{-\int_t^T r_u du}(Y(T) - S_T)|S_t = s, Y(t) = y, X(t) = x] .
\]
(3.3)
Applying the Feynman-Kac formula to the above equation, then $V(t, s, y, x)$ satisfies the system of partial differential equations (PDEs)
\[
    \frac{\partial V}{\partial t} + r_t s \frac{\partial V}{\partial s} + \frac{1}{2} \sigma_t^2 s^2 \frac{\partial^2 V}{\partial s^2} - r_t V + \langle V, Ax \rangle = 0 ,
\]
(3.4)
in the region $\{(t, s, y); 0 \leq t < T, 0 \leq s \leq y\}$ and satisfies the boundary conditions
\[
    V(T, s, y, x) = f(s, y) = y - s , \quad 0 \leq s \leq y , \quad \frac{\partial V}{\partial y}(t, y, y, x) = 0 , \quad 0 \leq t < T , \quad y > 0
\]
(3.5)
Consequently, if $X(t) := e_i$ ($i = 1, 2, \ldots, N$),
\[
    \mu_t = \mu_i , \quad V(t, s, x) = V(t, s, e_i) := V_i ,
\]
(3.6)
and $V_i$ ($i = 1, 2, \ldots, N$) satisfy the following system of PDEs:
\[
    -r_i V_i + \frac{\partial V_i}{\partial t} + r_i s \frac{\partial V_i}{\partial s} + \frac{1}{2} \sigma_i^2 s^2 \frac{\partial^2 V_i}{\partial s^2} + \langle V_i, A e_i \rangle = 0 ,
\]
(3.7)
with the boundary conditions:
\[
    V(T, s, y, e_i) = f(s, y) = y - s , \quad 0 \leq s \leq y , \quad i = 1, 2, \ldots, N
\]
(3.8)
4. A closed-form formula. In this section, we restrict ourselves to a special case with the number of regimes $N$ being 2 in order to simplify our discussion. By means of the homotopy analysis method, we derive a closed-form solution for a floating strike lookback option under a regime switching model. The payoff of the floating strike lookback option has a linear homogeneous property:

$$f(s, y) = sg \left( \ln \left( \frac{y}{s} \right) \right), \quad \text{where} \quad g(z) = e^z - 1, \quad z = \ln \left( \frac{y}{s} \right).$$

This linear homogeneous property along with the transformation $z = \ln \left( \frac{Y}{s} \right)$ and $U_i = \frac{Y_i}{s}$, $i = 1, 2$ transform the system of equations (3.6)-(3.8) into

$$\begin{align*}
\mathcal{L}_i \partial_t U_i(t, z) &= a_{1i}(U_2(t, z) - U_i(t, z)) \\
U_i(T, z) &= g(z) \\
\partial_{z} \left( \frac{\partial U_i}{\partial z} \right) \mid_{z=0} &= 0
\end{align*} \quad \text{for } i = 1, 2 \quad \text{and} \quad \partial_{z} \left( \frac{\partial U_i}{\partial z} \right) \mid_{z=0} = 0 \quad \text{for } i = 1, 2 \quad (4.1)$$

where

$$\mathcal{L}_i = \frac{\partial}{\partial t} + \frac{1}{2} \sigma_i^2 \frac{\partial^2}{\partial z^2} - (r_i + \frac{\sigma_i^2}{2}) \frac{\partial}{\partial z}, \quad i = 1, 2$$

$0 \leq t \leq T$ and $z > 0$. Following the same line as [8], the homotopy analysis method is adopted to solve $U_i(t, z), i = 1, 2$ from equation (4.1).

Now we introduce an embedding parameter $p \in [0, 1]$ and construct unknown functions $ar{U}_i(t, z, p), i = 1, 2$ that satisfy the following differential systems:

$$\begin{align*}
(1 - p) \mathcal{L}_1 [\bar{U}_1(t, z, p) - \bar{U}_1^0(t, z)] &= -p \left\{ \mathcal{A}_1[\bar{U}_1(t, z, p), \bar{U}_2(t, z, p)] \right\} \\
\bar{U}_1(t, z, p) &= g(z) \\
\partial_{z} \left( \frac{\partial \bar{U}_1}{\partial z} \right) \mid_{z=0} &= (1 - p) \partial_{z} \left( \frac{\partial \bar{U}_1^0}{\partial z} \right) \mid_{z=0}
\end{align*} \quad \text{for } i = 1, 2 \quad (4.3)$$

$$\begin{align*}
(1 - p) \mathcal{L}_2 [\bar{U}_2(t, z, p) - \bar{U}_2^0(t, z)] &= -p \left\{ \mathcal{A}_2[\bar{U}_1(t, z, p), \bar{U}_2(t, z, p)] \right\} \\
\bar{U}_2(t, z, p) &= g(z) \\
\partial_{z} \left( \frac{\partial \bar{U}_2}{\partial z} \right) \mid_{z=0} &= (1 - p) \partial_{z} \left( \frac{\partial \bar{U}_2^0}{\partial z} \right) \mid_{z=0}
\end{align*} \quad \text{for } i = 1, 2 \quad (4.4)$$

Here $\mathcal{A}_i, i = 1, 2$ are functionals defined as

$$\mathcal{A}_1[\bar{U}_1(t, z, p), \bar{U}_2(t, z, p)] = \mathcal{L}_1(\bar{U}_1) - a_{11}(\bar{U}_1 - \bar{U}_2),$$

$$\mathcal{A}_2[\bar{U}_1(t, z, p), \bar{U}_2(t, z, p)] = \mathcal{L}_2(\bar{U}_2) - a_{22}(\bar{U}_2 - \bar{U}_1).$$

With $p = 1$, we have

$$\begin{align*}
\mathcal{L}_1(\bar{U}_1) &= a_{11}(\bar{U}_1 - \bar{U}_2) \\
\bar{U}_1(t, z, 1) &= g(z) \\
\partial_{z} \left( \frac{\partial \bar{U}_1}{\partial z} \right) \mid_{z=0} &= 0
\end{align*} \quad \text{and} \quad \begin{align*}
\mathcal{L}_2(\bar{U}_2) &= a_{22}(\bar{U}_2 - \bar{U}_1) \\
\bar{U}_2(t, z, 1) &= g(z) \\
\partial_{z} \left( \frac{\partial \bar{U}_2}{\partial z} \right) \mid_{z=0} &= 0
\end{align*} \quad (4.5)$$

Comparing with (4.1), it is obvious that $\bar{U}_i(t, z, 1), i = 1, 2$ are equal to our searched solutions $U_i(t, z), i = 1, 2$.

Now we set $p = 0$, the equations (4.3) and (4.4) become

$$\begin{align*}
\mathcal{L}_1 \partial_t \bar{U}_1(t, z, 0) &= \mathcal{L}_1 \left[ \frac{\partial \bar{U}_1}{\partial t}(t, z) \right] \\
\bar{U}_1(t, z, 0) &= g(z) \\
\partial_{z} \left( \frac{\partial \bar{U}_1}{\partial z} \right) \mid_{z=0} &= \frac{\partial \bar{U}_1^0}{\partial z} \mid_{z=0}
\end{align*} \quad \text{and} \quad \begin{align*}
\mathcal{L}_2 \partial_t \bar{U}_2(t, z, 0) &= \mathcal{L}_2 \left[ \frac{\partial \bar{U}_2}{\partial t}(t, z) \right] \\
\bar{U}_2(t, z, 0) &= g(z) \\
\partial_{z} \left( \frac{\partial \bar{U}_2}{\partial z} \right) \mid_{z=0} &= \frac{\partial \bar{U}_2^0}{\partial z} \mid_{z=0}
\end{align*} \quad (4.6)$$
\( \tilde{U}_i(t, z, 0), i = 1, 2 \) will be equal to \( \tilde{U}_i^0(t, z) \) when \( \tilde{U}_i^0(T, z) = g(z), i = 1, 2 \). \( \tilde{U}_i^0(t, z) \) is known as the initial guess of \( U_i(t, z) \). Following the same line as [8], \( \tilde{U}_i^0(t, z) \) is chosen as the solution of the following PDEs:

\[
\begin{align*}
\mathcal{L}_1[\tilde{U}_1^0(t, z)] &= 0 \\
\tilde{U}_1^0(0, z) &= g(z) \\
\frac{\partial \tilde{U}_1^0(t, z)}{\partial z}
\bigg|_{z=0} &= 0
\end{align*}
\begin{align*}
\mathcal{L}_2[\tilde{U}_2^0(t, z)] &= 0 \\
\tilde{U}_2^0(0, z) &= g(z) \\
\frac{\partial \tilde{U}_2^0(t, z)}{\partial z}
\bigg|_{z=0} &= 0
\end{align*}
\tag{4.7}
\]

Note that \( s\tilde{U}_i^0(t, z) \) is the price of the floating strike lookback put option under the Black-Scholes-Merton model. Its explicit, closed-form formula is given in [5]:

\[
\tilde{U}_i^0(t, z) = e^s e^{-r_i(T-t)} N(-d_i^*) - N(-d_i^*) + \frac{\sigma_i}{2r_i} \left[ N(d_i^*) - e^{-r_i(T-t)} \phi \right],
\tag{4.8}
\]

where

\[
d_i^* = -z + \frac{(r_i \pm \sigma_i^2)(T-t)}{\sqrt{\sigma_i^2(T-t)}}, \quad d_i^* = d_i^* - \frac{2r_i}{\sqrt{\sigma_i}} \sqrt{T-t}
\]

and \( N(.) \) is the cumulative distribution function for standard normal distribution.

To find the values of \( \tilde{U}_i(t, z, 1), i = 1, 2 \), we can expand the functions \( \tilde{U}_i(t, z, p) \) as a Taylor’s series expansion of \( p \)

\[
\begin{align*}
\tilde{U}_1(t, z, p) &= \sum_{m=0}^{\infty} \frac{\tilde{U}_1^m(t, z)}{m!} p^m \\
\tilde{U}_2(t, z, p) &= \sum_{m=0}^{\infty} \frac{\tilde{U}_2^m(t, z)}{m!} p^m
\end{align*}
\tag{4.9}
\]

where

\[
\begin{align*}
\tilde{U}_1^m(t, z) &= \frac{\partial^m}{\partial p^m} \tilde{U}_1(t, z, p) \bigg|_{p=0} \\
\tilde{U}_2^m(t, z) &= \frac{\partial^m}{\partial p^m} \tilde{U}_2(t, z, p) \bigg|_{p=0}
\end{align*}
\tag{4.10}
\]

To find \( \tilde{U}_1^m(t, z) \) and \( \tilde{U}_2^m(t, z) \) in equation (4.9), we put (4.9) into (4.3) and (4.4) respectively and obtain the following recursive relations:

\[
\begin{align*}
\mathcal{L}_1[\tilde{U}_1^m] &= a_{11} (\tilde{U}_1^{m-1} - \tilde{U}_2^{m-1}) \\
\tilde{U}_1^m(T, z) &= 0 \\
\frac{\partial \tilde{U}_1^m(t, z)}{\partial z}
\bigg|_{z=0} &= 0
\end{align*}
\begin{align*}
\mathcal{L}_2[\tilde{U}_2^m] &= a_{22} (\tilde{U}_2^{m-1} - \tilde{U}_1^{m-1}) \\
\tilde{U}_2^m(T, z) &= 0 \\
\frac{\partial \tilde{U}_2^m(t, z)}{\partial z}
\bigg|_{z=0} &= 0
\end{align*}
\tag{4.11}
\]

\( m = 1, 2, \ldots \). We introduce the following transformations:

\[
\tau = T - t, \quad \alpha_i = \frac{2r_i}{\sigma_i} \quad \text{and} \quad \tilde{U}_i^m(t, z) = e^{-\frac{1}{2} \sigma_i^2 (\alpha_i + 1)^2 \tau + \frac{1}{2} (\alpha_i + 1) \sigma_i^2 \tau} \tilde{U}_i^m(t, z).
\]

We can rewrite equation (4.11) in the form of standard non-homogeneous diffusion equations

\[
\begin{align*}
\frac{\partial \tilde{U}_1^m}{\partial \tau} - \frac{1}{2} \sigma_1^2 \frac{\partial^2 \tilde{U}_1^m}{\partial \alpha_1^2} &= a_{11} e^{-\frac{1}{2} \sigma_1^2 (\alpha_1 + 1)^2 \tau + \frac{1}{2} (\alpha_1 + 1) \sigma_1^2 \tau} \left( \tilde{U}_1^{m-1} - \tilde{U}_2^{m-1} \right) \\
\tilde{U}_1^m(0, 0) &= 0 \\
\frac{\partial \tilde{U}_1^m}{\partial \alpha_1} (\tau, 0) + \frac{1}{2} (\alpha_1 + 1) \tilde{U}_1^m (\tau, 0) &= 0
\end{align*}
\tag{4.12}
\]

\[
\begin{align*}
\frac{\partial \tilde{U}_2^m}{\partial \tau} - \frac{1}{2} \sigma_2^2 \frac{\partial^2 \tilde{U}_2^m}{\partial \alpha_2^2} &= a_{22} e^{-\frac{1}{2} \sigma_2^2 (\alpha_2 + 1)^2 \tau + \frac{1}{2} (\alpha_2 + 1) \sigma_2^2 \tau} \left( \tilde{U}_2^{m-1} - \tilde{U}_1^{m-1} \right) \\
\tilde{U}_2^m(0, 0) &= 0 \\
\frac{\partial \tilde{U}_2^m}{\partial \alpha_2} (\tau, 0) + \frac{1}{2} (\alpha_2 + 1) \tilde{U}_2^m (\tau, 0) &= 0
\end{align*}
\tag{4.13}
\]
The system of PDEs (4.12) and (4.13) has a well-known closed-form solution respectively:

\[
\hat{U}^m_1(\tau, z) = a_{11} \int_0^\tau \int_0^\infty \exp \left\{ -\frac{1}{8} \sigma_1 (\alpha_1 + 1)^2 u + \frac{1}{2} (\alpha_1 + 1) \xi \right\}
\times \left( \hat{U}^{m-1}_1(T - u, \xi) - \hat{U}^{m-1}_2(T - u, \xi) \right) G_1(\tau - u, z, \xi) d\xi du,
\]

\[
(4.14)
\]

\[
\hat{U}^m_2(\tau, z) = a_{22} \int_0^\tau \int_0^\infty \exp \left\{ -\frac{1}{8} \sigma_2 (\alpha_2 + 1)^2 u + \frac{1}{2} (\alpha_2 + 1) \xi \right\}
\times \left( \hat{U}^{m-1}_2(T - u, \xi) - \hat{U}^{m-1}_1(T - u, \xi) \right) G_2(\tau - u, z, \xi) d\xi du,
\]

\[
(4.15)
\]

where

\[
G_1(t, z, \xi) = \frac{1}{\sqrt{2\pi \sigma_1 t}} \left\{ \exp \left[ \frac{-(x - \xi)^2}{2\sigma_1 t} \right] + \exp \left[ \frac{-(x + \xi)^2}{2\sigma_1 t} \right] \right\}
+ 2\kappa_1 \int_0^\infty \exp \left[ \frac{-(z + \xi + \eta_1)^2}{2\sigma_1 t} + \kappa_1 \eta_1 \right] d\eta_1 \right\},
\]

\[
(4.16)
\]

\[
G_2(t, z, \xi) = \frac{1}{\sqrt{2\pi \sigma_2 t}} \left\{ \exp \left[ \frac{-(x - \xi)^2}{2\sigma_2 t} \right] + \exp \left[ \frac{-(x + \xi)^2}{2\sigma_2 t} \right] \right\}
+ 2\kappa_2 \int_0^\infty \exp \left[ \frac{-(z + \xi + \eta_2)^2}{2\sigma_2 t} + \kappa_2 \eta_2 \right] d\eta_2 \right\},
\]

\[
(4.17)
\]

and

\[
\kappa_i = \frac{1}{2} (\alpha_i + 1), \quad \eta_i = e^{-\frac{1}{2} \sigma_i (\alpha_i + 1)^2 \tau + \frac{1}{8} (\alpha_i + 1) z}, \quad i = 1, 2
\]

5. Fixed strike lookback options. The payoff of a fixed strike lookback option does not have a linear homogeneous property. Consequently, the dimension of the corresponding PDEs cannot be reduced. However, a model independent put-call parity for lookback options, proposed by [11] may be used to price a fixed strike lookback option. Denote a fixed strike lookback call by \( C_{fix}(t, s, y, K, x) \), then the put-call parity is given by

\[
C_{fix}(t, s, y, K, x) = V(t, s, y, x) + Ke^{-r(T-t)}. \quad (5.1)
\]

6. Conclusion. We consider the pricing of the floating strike lookback option in a two-state regime switching model. The closed-form analytical pricing formulas for the floating strike lookback option is derived by the means of the homotopy analysis method.

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