Finite Temperature Effective Potential for the Abelian Higgs Model to the Order $e^4, \lambda^2$

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Abstract

A complete calculation of the finite temperature effective potential for the abelian Higgs model to the order $e^4, \lambda^2$ is presented and the result is expressed in terms of physical parameters defined at zero temperature. The absence of a linear term is verified explicitly to the given order and proven to survive to all orders. The first order phase transition has weakened in comparison with lower order calculation, which shows up in a considerable decrease of the surface tension.
1 Introduction

The electroweak phase transition \cite{1,2,3} has recently attracted much interest due to the hope to explain the observed baryon asymmetry of the universe within the minimal standard model \cite{4,5}. Much work has also been devoted to the phase transition in the abelian Higgs model, the simplest gauge theory with spontaneous symmetry breaking \cite{6,7}, because it is believed to exhibit the main features of the electroweak phase transition. An important prerequisite to the understanding of the mechanism of the phase transition is the knowledge of the effective potential at finite temperature \cite{6,8}. It has already been calculated for the abelian Higgs model in \cite{9,10} to the order $e^3$, $\frac{\lambda^3}{2}$ and in \cite{11} to the order $e^4$, $\lambda$. In the last publication the assumption $\lambda \sim e^3$ is made and the scalar masses $m_\phi$ and $m_\chi$ are counted as order $\frac{\lambda^{1/2}}{2}$, which can only be justified close to the critical temperature. Other approaches to the effective potential are found in \cite{12,13}.

In this paper, assuming formally $\lambda \sim e^2$, a complete calculation of the effective potential in the abelian Higgs model at finite temperature to the order $e^4$, $\lambda^2$ is presented. Coupling constants appearing in the denominator through infrared divergences \cite{14,15} are taken into consideration and the full dependence on the Higgs field $\hat{\phi}$, the zero temperature vacuum expectation value $v$ and $T$ is kept. Therefore this calculation supplies the potential given in \cite{11} with $\lambda$-corrections, which change the potential at $T_c$ and the surface tension significantly, in spite of their numerical smallness.

The approach is based on the Dyson-Schwinger equation for the derivative $\partial V/\partial \phi$, i.e. the tadpole diagrams are summed \cite{2,16}. In section 1 the principal method of the calculation, which makes also use of the gap equations for effective masses \cite{9}, is explained.

The explicit formula for $V(\phi, T)$ is given in section 2. It reproduces the results of \cite{3,10} and \cite{11} when taking the appropriate limits.

Section 3 addresses the problem of the linear term \cite{9,12,17,18,19}. The expression for $V$ to the order $e^4$, $\lambda^2$ shows that a nontrivial cancellation leads to $\lim_{\phi \to 0} \frac{\partial V}{\partial \phi} = 0$, i.e. the absence of a linear term. It is proven that this feature survives to all orders of perturbation theory.

To get rid of the arbitrary scale $\bar{\mu}$, which is introduced when regularizing the theory in the dimensional scheme, a transition to physical parameters is performed in section 4. However this finite renormalization at zero temperature proves to be not very important numerically.

The numerical analysis in the last section concentrates on the surface tension \cite{20,21}, because this easily accessible parameter gives a first characteristic of the strength of the first order phase transition, which is important for the generation of the baryon asymmetry in the standard model. The surface tension is found to be generally smaller than the lower order calculation \cite{9} suggests. This decrease is dramatic for large $\lambda$, but in contrast to the $e^4$, $\lambda$-potential \cite{11}, the potential discussed here does not show a complete change to a second order phase transition in the considered domain of $\lambda$. 

2
2 Calculation of the Effective Potential using Dyson-Schwinger Equations

2.1 Explanation of the Method in $\lambda \phi^4$-Theory

After describing the calculation in some detail in simple $\lambda \phi^4$-theory the extension to the abelian Higgs model is shown to be straightforward.

The euclidean Lagrangian has the form

$$\mathcal{L} = -\frac{1}{2}(\partial \phi)^2 + \nu \phi^2 - \frac{\lambda}{4} \phi^4,$$

where $\nu = \lambda v^2$ is counted as order $\lambda$. Using the familiar zero temperature technique of Dyson-Schwinger equations [22] it is easy to obtain the relation

$$-\frac{\partial}{\partial \phi}(V - V_{\text{tree}}) = A + B = +.$$

Here the 3-vertex in the first term arises from the shift $\phi \to \phi + \bar{\phi}$. The two different sorts of blobs symbolize the full propagator and the full 3-vertex.

The next step is to investigate the first term

$$A = -\lambda \phi \int \frac{dk}{k^2 + m_{\text{tree}}^2 + \Pi(k)},$$

where the tree level mass square $m_{\text{tree}}^2 = \lambda \phi^2 - \nu$ is assumed to be of order $\lambda$. For a calculation to order $\lambda^2$ it is sufficient to know $\Pi(k_0 \neq 0, \tilde{k})$ and $\Pi(0, \vec{y}m)$ to order $\lambda$ and $\lambda^{3/2}$ respectively. Here the need for different treatment of the contribution with zero Matsubara frequency can be understood by performing the substitution $\bar{k} \to \bar{y}m$ in the integral. (See [3, 4, 5] for the correct way of counting the order of infrared contributions here and below.) Therefore in the Dyson-Schwinger equations for $\Pi(k)$ the vertex correction can be neglected [3]:

$$-\Pi(k) = -(\Pi_a(k) + \Pi_b(k)) = +.$$


With the definitions
\[ m_3^2 = m_{\text{tree}}^2 + \Pi_a(0)|_{\lambda^{3/2}} + \Pi_b(0)|_{\lambda}, \quad \Pi_{02} = \Pi_b(0)|_{\lambda}, \quad \Pi_{03} = \Pi_b(0)|_{\lambda^{3/2}} - \Pi_b(0)|_{\lambda} \]
\[ \Pi_1(k) = \Pi(k) - \Pi(0), \]
where the powers of \( \lambda \) symbolize the accuracy to which a certain term has to be calculated, the following relation holds:
\[ m_3^2 + \Pi(k) = m_3^2 + \Pi_{03} + \Pi_1(k) + O(\lambda^2). \] (6)

Of course \( \Pi_{02} \) vanishes in \( \lambda \phi^4 \)-theory, but keeping this term formally makes the extension to the abelian gauge theory with power counting rule \( \lambda \sim e^2 \) more simple. After substituting (6) into equation (3) the integrand is expanded neglecting contributions of order higher than \( \lambda^2 \), which results in the formula
\[ A = -\lambda \dot{\phi} \sum \int dk \left( \frac{1}{k^2 + m_3^2} - \frac{\Pi_{03} + \Pi_1(k)}{(k^2 + m^2)^2} \right) = \]
\[ = -\lambda \dot{\phi} \sum \int dk \left( \frac{1}{k^2 + m_3^2} + \frac{\Pi_{02}}{(k^2 + m^2)^2} - \frac{\Pi_{02} + \Pi_{03} + \Pi_1(k)}{(k^2 + m^2)^2} \right). \] (7)

Here \( m^2 = m_3^2|_{\lambda} \) is the leading order mass term including the temperature correction. Observing that in term \( B \) of equation (3) the vertex need not be corrected to obtain the full \( \lambda^2 \)-result it becomes obvious that the third term in the right hand side of (7) together with \( B \) is equal to the derivative of the two-loop diagram
\[ \frac{\partial}{\partial \phi} \{ V_s \} = \frac{\partial}{\partial \phi} \left\{ \right\}. \] (8)

with leading order mass corrections in the propagators. So the final expression for the potential is
\[ V = V_{\text{tree}} + \int d\phi' \lambda \phi' \sum \int dk \left( \frac{1}{k^2 + m_3^2} + \frac{\Pi_{02}}{(k^2 + m^2)^2} \right) + V_s. \] (9)

A similar way of combining the different contributions to \( V \) has been considered in [23].
2.2 Extension to the Abelian Higgs Model

The abelian Higgs model in Landau gauge includes, from the topological point of view, exactly the same graphs, because it does also contain 3-and 4-vertices. Consider the euclidian Lagrangian

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - |D_\mu \Phi|^2 + \nu |\Phi|^2 - \lambda |\Phi|^4, \quad \Phi = \frac{1}{\sqrt{2}} (\hat{\varphi} + \varphi + i \chi). \]  

(10)

Applying the formal power counting rule \( \lambda \sim e^2 \) the main difference from the purely scalar model lies in the low order of the vector-scalar-scalar vertex. This manifests itself through the appearance of nonzero terms \( \Pi_{02}^{\tau}, \Pi_{02}^{L} \) with \( T, L \) referring to contributions to the transverse and longitudinal parts of the vector propagator \[9, 24\] from the diagram in fig. 1.

Fig. 1

But this feature has already been accounted for in the last subsection by keeping \( \Pi_{02} \). Therefore the contributions to the potential can be classified analogously to the purely scalar case described above. This can also be shown starting directly from the Dyson-Schwinger equations of the abelian Higgs model. Of course at each step it is necessary to convince oneself that no contributions of order \( \lambda^2, e^4 \) are lost. The final result has exactly the same structure as the potential of the last subsection, but it involves more terms due to the particle content of the theory.

\[ V = V_{\text{tree}} + V_1 + \cdots + V_8 + V_{3,\Pi} + V_{4,\Pi} \]  

(11)

Here

\[ V_1 = \int \varphi' \lambda \varphi' \mathcal{I}(m_{\chi,3}) \quad , \quad V_2 = \int \varphi' 3\lambda \varphi' \mathcal{I}(m_{\varphi,3}) \quad , \]  

\[ V_3 = \int \varphi' (2 - 2\epsilon) e^2 \varphi' \mathcal{I}(m_{T,3}) \quad , \quad V_4 = \int \varphi' e^2 \varphi' \mathcal{I}(m_{L,3}) \]  

(12)

with the standard temperature integral \[3\]

\[ \mathcal{I}(m) = \sqrt{2} \int \frac{dk}{k^2 + m^2} \]  

(13)

to be evaluated in \( n - 1 = 3 - 2\epsilon \) dimensions. The masses are defined in analogy with equation \( [3] \) by the appropriate zero momentum parts of the Dyson-Schwinger equations, which have to be iterated once to obtain the result to order \( e^3, \lambda^{3/2} \) \[3\]. Corrections from the momentum dependent part of the Dyson-Schwinger equations are needed for the gauge boson only:

\[ V_{3,\Pi} = \int \varphi' (2 - 2\epsilon) e^2 \varphi' \sqrt{2} \int \frac{d\Pi_{02}^{\tau}}{(k^2 + m_{\tau}^2)^2}, \quad V_{4,\Pi} = \int \varphi' e^2 \varphi' \sqrt{2} \int \frac{d\Pi_{02}^{\tau}}{(k^2 + m_{\tau}^2)^2}. \]  

(14)
Finally $V_5$ through $V_8$ represent the diagrams of figure 2.

Furthermore the contributions generated by the counterterm Lagrangian have to be added.

### 3 Explicit Result to Order $e^4, \lambda^2$

The calculations needed for the temperature dependent masses to order $e^3, \lambda^{3/2}$ can essentially be taken from [9]. Differences arise because the loop with two propagators does only contribute in lowest order due to definition (5). Also it is necessary to keep the $\epsilon$-dependence in leading order. The integrals in $V_5$ through $V_8$ have already been done in [11, 25] and won’t be given here explicitly. After adding all the terms up, which proves to be rather laborious, the final result, using the $\overline{\text{MS}}$-scheme, reads

$$
V(\varphi) = \frac{\varphi^2}{\beta^2} \left[ -\frac{\beta^2 \nu}{2} + \frac{\lambda}{6} e^2 + \frac{e^4}{8} \left( -\frac{11}{9} \ln \mu^2 \beta^2 + \frac{2}{3} c + 3c_2 - \frac{13}{9} \right) 
+ \frac{e^4}{16\pi^2} \left( \ln \frac{2m_L + m_\varphi}{2m + m_\varphi} + \frac{1}{2} \ln \frac{m + m_\varphi + m_\chi}{m + m_\varphi} + 3 \ln \frac{\beta(m + m_\varphi + m_\chi)}{3} \right) + \frac{\beta^2 \nu \lambda}{8\pi^2} \left( \ln \mu^2 \beta^2 - c + \frac{3}{2} \right) 
+ \frac{\lambda^2}{16\pi^2} \left( \frac{1}{2} \ln \mu^2 \beta^2 + \frac{1}{2} c - c_2 + \frac{1}{3} - 4 \ln \frac{\beta(m + m_\varphi + m_\chi)}{3} - 3 \ln \frac{2m + m_\varphi}{m + m_\varphi} \right) 
+ \frac{\lambda^2}{16\pi^2} \left( -\frac{5}{3} \ln \mu^2 \beta^2 + \frac{2}{3} c + c_2 + 2 + \ln \frac{\beta(m_\varphi + 2m_\chi)}{3} + 3 \ln \beta m_\varphi + 2 \ln \frac{m + m_\varphi + m_\chi}{m_\varphi + m_\chi} \right) \right] 
+ \varphi^4 \left[ \frac{\lambda}{4} + \frac{1}{64\pi^2} (10\lambda^2 + 3e^4) (c - \frac{3}{2} - \ln \mu^2 \beta^2) + \frac{e^4}{32\pi^2} \right] 
+ \frac{m_\varphi^4}{64\pi^2 \beta^2 \varphi^2} \ln \frac{m_\varphi (2m + m_\varphi)}{(m + m_\varphi)^2} - \frac{M_0^2 e^2}{16\pi^2 \beta^2} \left( 2 \ln \frac{\beta(m + m_\varphi + m_\chi)}{3} + \ln \frac{2m + m_\varphi}{m + m_\varphi} \right) 
+ \frac{1}{32\pi^2 \beta^2} \left[ (e^2 + \lambda) m_\varphi m_\chi + e^2 m_L (m_\varphi + m_\chi) + 2\lambda m (m_\chi - m_\varphi) + e^2 m (m_\chi + 2m_\varphi) \right] 
- \frac{1}{12\pi \beta} (m_\varphi^3 + m_\chi^3 + 2m^3 + m_L^3).\right]
$$
Here the lowest order masses are

\[ m_\varphi^2 = 3\lambda \varphi^2 + M_0^2 , \quad m_\chi^2 = \lambda \varphi^2 + M_0^2 , \quad M_0^2 = -\nu + \frac{4\lambda + 3e^2}{12\beta^2} , \quad (16) \]

\[ m^2 = e^2 \varphi^2 , \quad m_L^2 = e^2 \varphi^2 + \frac{e^2}{3\beta^2} \]

and the constants \( c \) and \( c_2 \) arise from the temperature integral \( I(m) \) (see [3]) and the scalar two loop integral in (8) calculated in [25]

\[ c = \frac{3}{2} + 2 \ln 4\pi - 2\gamma \approx 5.4076 , \quad c_2 \approx 3.3025 . \quad (17) \]

Dropping terms of order \( e^4 \) (with power counting rule \( \lambda \sim e^2 \)) the effective potential from [9] is recovered. When changing the power counting to \( \lambda \sim e^3 \) and \( m_\varphi^2, m_\chi^2 \sim \lambda \) the result of Arnold and Espinosa [11] is obtained after dropping terms of order higher than \( e^4 \). It should be noted that the term in (15) containing an explicit \( 1/\varphi^2 \)-factor does not show singular behaviour near \( \varphi = 0 \) because of the logarithm, which decreases fast enough.

4 Absence of a linear Term

At first sight the contribution to the potential (15) proportional to \( m(m_\chi + 2m_\varphi) \) seems to produce a linear behaviour for small \( \varphi \). But in this limit the logarithmic terms have to be expanded at the point \( \varphi = 0 \), which results in linear terms exactly cancelling the one mentioned above. In fact this feature, already discussed by several authors [9, 12, 17, 18] starting from lower order calculations, can be shown to survive to all orders of small couplings perturbation theory :

\[ \lim_{\varphi \rightarrow 0} \frac{\partial V}{\partial \varphi} = 0 \] to all orders in \( e \) and \( \lambda \).

This can be proven in the following way (compare the argument in [17]) : Global U(1)-symmetry implies

\[ \frac{1}{\varphi} \frac{\partial V}{\partial \varphi} = m_\chi^2(q^2 = 0) . \quad (18) \]

Obviously it suffices to show, that the self energy \( \Pi_\chi(q^2 = 0) \) is finite for \( \varphi \to 0 \). Above the barrier temperature singularities can only arise from the transverse gauge boson propagator due to the temperature masses of \( \chi \) and \( \varphi \). Therefore diagrams of the kind shown in fig. 3 have to be investigated. Here the wavy lines symbolize full vector propagators and the blobs are vertices without internal vector lines, meaning the sum of all diagrams built from scalar propagators with the correct number of external vector lines.
If $\varphi = 0$, a gauge covariance argument, completely analogous to the zero temperature case, shows for the vertices with external vector lines only, that

$$\Gamma_{\alpha\beta \ldots \mu \nu}^{2n} \sim |\vec{k}_1| \ldots |\vec{k}_2^n|$$

for small $|\vec{k}_i|$ and $k_i^0 = 0$, $\alpha \beta \ldots \mu \nu \in \{1, 2, 3\}$. \hspace{1cm} (19)

If $\varphi \neq 0$, diagrams not covered by the previous gauge covariance argument because of explicit $\varphi$-factors at the vertices have to be added to $\Gamma_{\alpha\beta \ldots \mu \nu}^{2n}$. They however vanish not slower than $\varphi^2$ in the limit $\varphi \to 0$, which is clear from the fact that the unbroken theory has no vertices with an odd number of scalar lines. Therefore in the case $|\vec{k}| \sim \varphi$ ($\varphi$ being the natural infrared cutoff introduced by the transverse vector mass $e\varphi$) the sum of both contributions, i.e. the complete vertex $\Gamma_{\alpha\beta \ldots \mu \nu}^{2n}$, can be counted as $\varphi^2$ when searching for small-$\varphi$ singularities.

Consider the most dangerous lowest power of $\varphi$ stemming from the maximal infrared divergence, which is obtained by setting $k^0 = 0$ for all transverse vector propagators. It can be calculated by scaling the loop momenta according to $\vec{k} \to \vec{y}\varphi$. Counting the vector vertices as $\varphi^2$ the following formula for the minimal overall power of $\varphi$ is obtained (compare the argumentation in appendix A of [15]):

$$n_{\varphi} = 3L - 2I + 2(V - 2) + 2$$

(20)

Here $L$, $I$ and $V$ denote the number of vector loops, vector propagators and full vertices, symbolized by blobs in fig. 3, respectively. The last term $+2$ follows from a closer look at the contribution of the full vertices with external $\chi$-lines: Fig. 3 shows examples of the two different structures to be investigated. If there are two such "$\chi$-vertices", each will contribute a factor $\varphi$, resulting in the correction $+2$. If there is only one with two external $\chi$-lines, it may have no explicit $\varphi$-factor. In the latter case however the contribution of the vector vertices $2(V - 2)$ has to be replaced by $2(V - 1)$, which again corresponds to a correction $+2$. Therefore equation (20) is valid in the general case. Now using the well-known formula $V + L - I = 1$ it follows immediately that

$$n_{\varphi} = L \geq 0,$$

(21)

or equivalently: There is no divergence for $\varphi \to 0$. If not all of the vector propagators are infrared divergent, the vertices connected by "$\text{heavy}$" lines may be formally fused. Now repetition of the above argument leads again to the desired result thus completing the proof.
5 Transition to physical Parameters

To get rid of the arbitrary scale \( \bar{\mu} \) the potential is rewritten in terms of physical parameters defined at zero temperature. Such parameters are the Higgs and vector masses and the vacuum expectation value of the Higgs field. To stay closer to the previous notation they can be expressed through new coupling constants \( \bar{\lambda} \) and \( \bar{e} \), defined by

\[
m^2_{\phi,\text{phys}} = 2\bar{\lambda} v^2, \quad m^2_{\text{phys}} = \bar{e}^2 v^2, \quad \frac{\partial V}{\partial \phi_{\text{phys}}} \Bigg|_{\phi_{\text{phys}} = v} = 0.
\]  

(22)

Effectively a finite renormalization of the form

\[
\phi^2 = \phi^2_{\text{phys}} (1 + c), \quad \lambda = \bar{\lambda} + \delta \lambda, \\
e^2 = \bar{e}^2 + \delta \bar{e}^2, \quad \nu = \bar{\lambda} v^2 + \delta \nu_{\text{phys}}
\]  

(23)

has to be performed. The physical \( \phi \)-propagator is

\[
\frac{1}{(1 + c)(q^2 - m^2_{\phi,\text{phys}}) - \Pi_{\phi}(q^2)},
\]  

(24)

therefore the on-shell definitions of the new parameters follow from

\[
c = \frac{\partial}{\partial q^2} \text{Re} \Pi_{\phi}(q^2) \bigg|_{q^2 = m^2_{\phi,\text{phys}}}, \\
m^2_{\phi} + \text{Re} \Pi_{\phi}(m^2_{\phi,\text{phys}}) = m^2_{\phi,\text{phys}}, \quad m^2 + \text{Re} \Pi(m^2_{\text{phys}}) = m^2_{\text{phys}}
\]  

(25)

together with the last equality in (22). The zero temperature effective potential to the order \( \lambda^2, e^4 \) needed here is given by

\[
V = -\frac{\nu}{2} \phi^2 + \frac{\lambda}{4} \phi^4 - \frac{m^4}{64\pi^2} \left( \frac{3}{2} + \ln \frac{\bar{\mu}^2}{m^2_{\phi}} \right), \\
\quad -\frac{m^4}{64\pi^2} \left( \frac{3}{2} + \ln \frac{\bar{\mu}^2}{m^2_{\phi}} \right) - \frac{3m^4}{64\pi^2} \left( \frac{5}{6} + \ln \frac{\bar{\mu}^2}{m^2} \right).
\]  

(26)

Contributions to the self energy corrections \( \Pi_{\phi}(q^2) \) and \( \Pi(q^2) \) come from the usual zero temperature one loop diagrams. In view of the principal features of the potential considered in this paper the numerical effect of the performed finite renormalization is not very important (see fig. 5 in the last section). Therefore the complete formula for \( V \), which is easy to obtain, is not given here explicitly.
6 Numerical Results and Discussion

In the previous sections a complete calculation of the finite temperature effective potential to the order $e^4, \lambda^2$ has been performed, including the transition to physical parameters defined at zero temperature. The gauge coupling is chosen to be $e = 0.3$ and the influence of $\lambda$, which corresponds to the Higgs mass, is investigated. At first sight the calculated potential seems to ensure a first order phase transition in a wide range of the parameter $\lambda$, but reliability of perturbation theory has to be questioned. This becomes obvious from fig. 4, where different approximations of the effective potential at their respective critical temperatures are shown. (Here and below the dimensionful quantities are given in units of $v$ and its powers.) Some insight can be gained from a comparison with the results to order $e^3, \lambda^{3/2}$ obtained in [9, 10]. A reasonable physical quantity to be calculated from both potentials is the surface tension [20, 21], which can be seen as a measure of the strength of the phase transition:

$$\sigma = \int_{\varphi^+}^\varphi d\varphi \sqrt{2V(\varphi, T_c)},$$  \hspace{1cm} (27)

where $\varphi_+$ is the position of the second degenerate minimum of $V$ and the potential is normalized to ensure $V(\varphi = 0) = 0$. The numerical results are shown in fig. 5. It includes besides the surface tension from potentials to the order $e^3, \lambda^{3/2}$ [9] and to the order $e^4, \lambda^2$ also the results calculated from a potential to order $e^4, \lambda$ [11], where according to the power counting rule $\lambda \sim m^2, \varphi \sim m^2$ all terms of order higher than $e^4$ have been neglected. Obviously the shift introduced by the transition to zero temperature physical parameters is not important for the present discussion. The fact that perturbation theory is not reliable for large $\lambda$, already stressed in [9], can be clearly read off from fig. 5 ($\sigma$ changes by an order of magnitude when adding the last term in the perturbation series). Somewhat surprisingly the perturbation series cannot be trusted too much for small $\lambda$ either. Here $\sigma$ decreases by a factor of $\sim 2$ at least. Two different higher order terms are mainly responsible this for this change. In the region of extremely small $\lambda$ it is essentially the $e^4\varphi^4$ term (see equation (15)), which cannot be viewed as a small correction to the tree level term $\lambda\varphi^4/4$. This enhancement of the $\varphi^4$-term is a temperature effect and therefore cannot be removed by zero temperature renormalization. For $m_\varphi^2/m^2 \sim 0.4$ the logarithmic mass dependence of the $e^4\varphi^2$ term seems to be more important. The great influence of this numerically small contribution can be understood by recalling that at the critical temperature the leading order $\varphi^2$-terms essentially cancel and that a $\varphi$-dependence in a coefficient of $\varphi^2/\beta^2$ cannot be absorbed in a correction of $T_c$.

It is interesting to compare the above discussion with another method to investigate the reliability of the perturbation series: Applying the $\xi$-conditions introduced in [1] in the more restrictive form of [13] (i.e. using the higher order expression
for the masses) the region of reliability $m_{\phi}^2/m^2 \lesssim 0.1$ is obtained for $\xi=2$. For the largest permissible Higgs mass fig. 5 suggests $\Delta \sigma/\sigma = 0.57$ which signals the breakdown of the perturbation series. This error does not decrease for smaller Higgs masses, in contrast to the standard model calculation in lower order of $[15]$, due to the shift of $\lambda$ through an $e^4$-term discussed above. However, this enhancement of the $\phi^4$-contribution, which is invisible in the $\xi$-conditions, does not threaten the first order of the phase transition. Therefore perturbation theory seems to ensure a first order phase transition for small $\lambda$ at least, in spite of the still unknown exact value of the surface tension.

The potential to the order $e^4, \lambda$ from $[11]$ does not give rise to a first order phase transition for $\lambda \gtrsim 0.01$. The value of $\sigma$ calculated using this potential differs significantly from the result presented here for $\lambda \gtrsim 0.007$. This is partially due to contributions of the form $e^4 \varphi^2 \ln(m + m_{\phi})$ and the like, already mentioned above. Counting $m_{\phi}$ as higher order correction results in the $\ln m$-contributions found in the potential from $[11]$, which is obviously a significant change for small $\phi$.

Fig. 6 shows the vacuum expectation value in the asymmetric phase at $T_c$. This parameter does not reflect the dramatic change of the surface tension by higher order corrections.

It would be interesting to extend this approach to the standard model, which seems to be straightforward, and to try to estimate the influence of expected corrections beyond $e^4, \lambda^2$.

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Fig. 4 Different approximations of the effective potential plotted at their respective critical temperatures with $\lambda=0.01$ (the $e^4, \lambda$-potential is a result of $[11]$)

Fig. 5 Dependence of the surface tensions calculated from the different potentials on the zero temperature mass relation $m_{\phi}^2/m^2 = 2\lambda/e^2$ with $e = 0.3$

Fig. 6 Dependence of the position of the second minimum of different potentials on the zero temperature mass relation $m_{\phi}^2/m^2 = 2\lambda/e^2$ with $e = 0.3$
References

[1] D. A. Kirzhnits and A. D. Linde, Phys. Lett. B42 (1972) 421
[2] S. Weinberg, Phys. Rev. D9 (1974) 3357
[3] L. Dolan and R. Jackiw, Phys. Rev. D9 (1974) 3320
[4] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B155 (1985) 36
[5] G. R. Farrar and M. E. Shaposhnikov, preprint CERN-TH.6732/93 and RU-93-11 (1993)
[6] D. A. Kirzhnits and A. D. Linde, Ann. Phys. 101 (1976) 195
[7] A. D. Linde, Rep. Prog. Phys. 42 (1979) 389
[8] M. E. Carrington, Phys. Rev. D45 (1992) 2933
[9] W. Buchmüller, T. Helbig and D. Walliser, preprint DESY-92-151
[10] P. Arnold, Phys. Rev. D46 (1992) 2628
[11] P. Arnold and O. Espinosa, Phys. Rev. D47 (1993) 3546
[12] G. Amelino-Camelia, preprint BUHEP-93-12 (1993)
[13] A. Jakovac and A. Patkos, preprint BI-TP-93-18 (1993)
[14] T. Helbig, Ph.D. Thesis, Hamburg University (1993)
[15] W. Buchmüller, Z. Fodor, T. Helbig, D. Walliser, preprint DESY 93-021 (1993)
[16] D. A. Kirzhnits and A. D. Linde, JETP 40 (1974) 628
[17] M. Dine, R.G. Leigh, P. Huet, A. Linde and D. Linde, Phys. Rev. D46 (1992) 550
[18] J.R. Espinosa, M. Quiros and F. Zwirner, preprint CERN-TH-6577/92 (1992)
[19] C.G. Boyd, D.E. Brahm and S.D.H. Hsu, preprint CALT-68-1795 (1992)
[20] S. Coleman, Phys. Rev. D15 (1977) 2929
[21] A. D. Linde, Nucl. Phys. B216 (1983) 421
[22] R.J. Rivers, Path integral methods in quantum field theory (Cambridge Univ. Press, 1987)

[23] C.G. Boyd, D.E. Brahm and S.D.H. Hsu, preprint CALT-68-1858, HUTP-93-A011 and EFI-93-22 (1993)

[24] J.I. Kapusta, Finite temperature field theory (Cambridge Univ. Press, 1989)

[25] R.R. Parwani, Phys. Rev. D45 (1992) 4695