Generalized Rarita-Schwinger Equations

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Abstract The Rarita-Schwinger equations are generalised for the delta baryon having spin 3/2 and isospin 3/2. The coupling of the nucleon and the delta fields is studied. A possible generalisation of the Walecka model is proposed.

1 Introduction

In a previous paper [1] the problem of spin 3/2 has been discussed. More specifically the generalisation of the Rarita-Schwinger equations has been presented for particles having isospin as internal degree of freedom.
The description of particles with spin 3/2 can be given by the help of the Bargmann-Wigner [2] procedure introducing the multispinor fields

$$\Psi_{\alpha\beta\gamma}(x),$$

(1)

which must be completely symmetric in all of their indices and the Dirac-equation must be satisfied for all indices:

$$\left(\gamma_{\mu} \partial^{\mu} + M\right)_{\alpha\alpha'}\Psi_{\alpha'\beta\gamma}(x) = 0,$$

(2)

$$\left(\gamma_{\mu} \partial^{\mu} + M\right)_{\beta\beta'}\Psi_{\alpha\beta'\gamma}(x) = 0,$$

(3)

$$\left(\gamma_{\mu} \partial^{\mu} + M\right)_{\gamma\gamma'}\Psi_{\alpha\beta\gamma'}(x) = 0.$$  

(4)

These requirements lead to the Rarita-Schwinger equations given [3] by

$$\left(\gamma_{\kappa} \partial^{\kappa} + M\right)\psi^{\mu}(x) = 0.$$  

(5)

$$\gamma_{\mu}\psi^{\mu} = 0.$$  

(6)

The completely symmetric function $\Psi_{\alpha\beta\gamma}(x)$ can be built from $\psi^{\mu}(x)$. As it is seen all of the vector components of $\psi^{\mu}(x)$ satisfy Dirac-type equations, however they are not independent, they must fulfill the constraint given by Eq. (6). This constraint coming from the symmetry condition projects out the spin 1/2 components and the remaining components describe particles with spin 3/2.

Here we use the notation $x_4 = it$ and summation is implied for every two identical indices. The Dirac-matrices $\gamma_{\mu}$ satisfy the anticommutator relations:

$$\gamma_{\mu}\gamma_{\nu} + \gamma_{\nu}\gamma_{\mu} = 2\delta_{\mu\nu} \quad (\mu, \nu = 1, 2, 3, 4)$$  

(7)

2 Rarita-Schwinger Equations for Particles with Isospin

2.1 Symmetry Conditions

Using the Bargmann-Wigner procedure [2] we have generalised the Rarita-Schwinger equations assuming that all the indices $\alpha$, $\beta$, $\gamma$ of the field $\Psi_{\alpha\beta\gamma}(x)$ contain
an isospin index too, labeling the eigenstates of the third component of isospin $1/2$. The field $\Psi_{\alpha\beta\gamma}(x)$ must be completely symmetric in its indices. In the 8-dimensional space there exist 36 linearly independent matrices which are symmetric in $\alpha$ and $\beta$ and can be given as follows:

$$(TC)_{\alpha\beta}, \quad (i\gamma_5 TC)_{\alpha\beta}, \quad (\gamma_\mu i\gamma_5 TC)_{\alpha\beta},$$

$$(\gamma_\mu \tau_m TC)_{\alpha\beta}, \quad \left(\frac{1}{2}\Sigma_{\mu\nu} \tau_m TC\right)_{\alpha\beta}. \quad (8)$$

Here the matrices $\Sigma_{\mu\nu}$, $C$ and $T$ are defined by

$$\Sigma_{\mu\nu} = (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)/(2i), \quad (9)$$

$$C = \gamma_2 \gamma_4, \quad (10)$$

$$T = \tau_1 \tau_3, \quad (11)$$

with the Pauli-matrices $\tau_m$ $(m = 1, 2, 3)$.

Using these matrices the multispinor field is defined in the following form:

$$\Psi_{\alpha\beta\gamma}(x) = (TC)_{\alpha\beta}\psi_\gamma^0 + (i\gamma_5 TC)_{\alpha\beta}\psi_\gamma^5 + (\gamma_\mu i\gamma_5 TC)_{\alpha\beta}\psi_\gamma^{\mu5}$$

$$+ (\gamma_\mu \tau_m TC)_{\alpha\beta}\psi_\gamma^{\mu m} + \left(\frac{1}{2}\Sigma_{\mu\nu} \tau_m TC\right)_{\alpha\beta}\psi_\gamma^{\mu\nu m}. \quad (12)$$

The task is to determine the functions $\psi^0(x)$, $\psi^5(x)$, $\psi^{\mu5}(x)$ and $\psi^{\mu\nu m}(x)$. Up till now the field $\Psi_{\alpha\beta\gamma}(x)$ is symmetric only in the indices $\alpha$ and $\beta$. To guarantee the complete symmetry one must eliminate the components antisymmetric in respect of the second and third indices. This requirement is fulfilled if the contraction of $\Psi_{\alpha\beta\gamma}(x)$ with all of the matrices antisymmetric in the indices $\beta$ and $\gamma$ are vanishing.

In 8-dimensional space there exist 28 antisymmetric, linearly independent matrices, which can be defined as follows:

$$(C^{-1}T^{-1}\tau_n)_{\beta\gamma}, \quad (C^{-1}T^{-1}i\gamma_5 \tau_n)_{\beta\gamma}, \quad (C^{-1}T^{-1}\gamma_\lambda i\gamma_5 \tau_n)_{\beta\gamma},$$

$$(C^{-1}T^{-1}\gamma_\lambda)_{\beta\gamma}, \quad (C^{-1}T^{-1}\frac{1}{2}\Sigma_{\lambda\rho})_{\beta\gamma}. \quad (13)$$

Thus the requirement of the complete symmetry leads to the following 28 conditions:

$$\gamma_\mu \psi^{\mu m} = \tau^m(\psi^0 - i\gamma_5 \psi^5), \quad (14)$$

$$\frac{1}{2}\Sigma_{\mu\nu} \psi^{\mu\nu m} = \tau^m(\psi^0 + i\gamma_5 \psi^5), \quad (15)$$

$$i\gamma_\mu \psi^{\mu m} = -\psi^{\mu m} + \tau^m(\tau_m \psi^{\mu m} + \gamma^\nu i\gamma_5 \psi^5), \quad (16)$$

$$i\gamma_5 \psi^{5} = -\tau_m \psi^{\mu m} + \gamma^\mu(\psi^0 - i\gamma_5 \psi^5), \quad (17)$$

$$\tau_m \psi^{\mu m} = i\tau_m(\gamma^\nu \psi^{\nu m} - \gamma^\mu \psi^{\mu m}) + \Sigma^{\mu\nu}(\psi^0 + 2i\gamma_5 \psi^5). \quad (18)$$
2.2 Decomposition of the Dirac Equation

As it was stated in the Introduction the field $\Psi_{\alpha\beta\gamma}(x)$ must satisfy the Dirac equation with respect of its all three indices.

By applying the Dirac-operator $(\gamma_\kappa \partial^\kappa + M)$ on the first index $\alpha$ of $\Psi_{\alpha\beta\gamma}(x)$ the following equation is obtained:

\[
(\gamma_\kappa \partial^\kappa + M)_{\alpha'\alpha}(TC)_{\alpha'\beta}\psi^0_\gamma(x) + \\
(\gamma_\kappa \partial^\kappa + M)_{\alpha'\alpha}(i\gamma_5 TC)_{\alpha'\beta}\psi^5_\gamma(x) + \\
(\gamma_\kappa \partial^\kappa + M)_{\alpha'\alpha}(\gamma_\mu i\gamma_5 TC)_{\alpha'\beta}\psi^{\mu 5}_\gamma(x) + \\
(\gamma_\kappa \partial^\kappa + M)_{\alpha'\alpha}(\gamma_\mu \tau_m TC)_{\alpha'\beta}\psi^{\mu m}_\gamma(x) + \\
(\gamma_\kappa \partial^\kappa + M)_{\alpha'\alpha}\left(\frac{1}{2}\Sigma_{\mu\nu} \tau_\mu \tau_\nu TC\right)_{\alpha'\beta}\psi^{\mu \nu m}_\gamma(x) = 0. \tag{19}
\]

This equation is fulfilled if its 36 linearly independent components vanish. To separate these components we contract this equation with the following 36 matrices

\[
(C^{-1}T^{-1})_{\beta\alpha}, \quad (C^{-1}T^{-1}i\gamma_5)_{\beta\alpha}, \quad (C^{-1}T^{-1}\gamma_\lambda i\gamma_5)_{\beta\alpha}, \\
(C^{-1}T^{-1}\gamma_\lambda \tau_n)_{\beta\alpha}, \quad (C^{-1}T^{-1}\frac{1}{2}\Sigma_{\lambda\rho} \tau_n)_{\beta\alpha}, \tag{20}
\]

which are linearly independent and symmetric in their indices $\beta$ and $\alpha$.

The result is the following 36 equations:

\[
M\psi^0 = 0, \tag{21} \\
M\psi^5 = -\partial_\kappa \psi^{\kappa 5}, \tag{22} \\
M\psi^{\lambda 5} = -\partial^\lambda \psi^5, \tag{23} \\
M\psi^{\mu \lambda n} = i\partial_\kappa \psi^{\kappa \lambda n}, \tag{24} \\
M\psi^{\rho \lambda n} = i(\partial^\lambda \psi^{\rho \kappa n} - \partial^\rho \psi^{\lambda \kappa n}). \tag{25}
\]
As it is seen the resulting equations form a coupled set of differential equations for the determination of the functions \( \psi^0(x), \psi^5(x), \psi^{\lambda 5}(x), \psi^{\lambda n}(x) \) and \( \psi^{\rho \lambda n}(x) \).

The functions \( \psi^{\rho \lambda n} \) are not independent quantities since they can be derived from \( \psi^{\rho m} \) by the help of the equation (25). The equation (21) shows that the function \( \psi^0 \) must be zero:

\[
\psi^0 = 0. \quad (26)
\]

The rest of these equations together with the conditions (14)–(18) can be reduced to the following set of equations:

\[
(\gamma_\kappa \partial^\kappa + M) \psi^{\mu m}(x) = 0, \quad (27)
\]
\[
(\gamma_\kappa \partial^\kappa + M) \psi^{\mu 5}(x) = 0, \quad (28)
\]
\[
(\gamma_\kappa \partial^\kappa + M) \psi^{5}(x) = 0, \quad (29)
\]
\[
\gamma_\mu \psi^{\mu m} = -\tau^m i \gamma_5 \psi^5, \quad (30)
\]
\[
\tau_m \psi^{\mu m} = i \gamma_5 \psi^{\mu 5} - \gamma^\mu i \gamma_5 \psi^5. \quad (31)
\]

This can be simplified by assuming trivial solutions for \( \psi^5 \) and \( \psi^{\lambda 5} \):

\[
\psi^5(x) = 0, \quad \psi^{\lambda 5}(x) = 0. \quad (32)
\]

In this way the number of components is decreased, however, the number of independent components does not. Thus the conditions given above and the remaining differential equations are reduced to the following set of equations:

\[
(\gamma_\kappa \partial^\kappa + M) \psi^{\mu m}(x) = 0, \quad (33)
\]
\[
\gamma_\mu \psi^{\mu m} = 0, \quad (34)
\]
\[
\tau_m \psi^{\mu m} = 0. \quad (35)
\]

These are the generalised Rarita-Schwinger equations.

The field \( \psi^{\mu m}(x) \) is transformed as a direct product of a Lorentz-vector \( (\mu) \) and a Dirac-spinor \( (\gamma) \) multiplied with a direct product of an isovector \( (m) \) and an
isospinor \( \gamma \). As a consequence of these properties it contains components of spin \( \frac{3}{2} \) and \( \frac{1}{2} \) and isospin \( \frac{3}{2} \) and \( \frac{1}{2} \). The constraint (34) projects out from \( \psi^{\mu m} \) the spin \( \frac{1}{2} \) components, while the constraint (35) projects out the isospin \( \frac{1}{2} \) components.

\section{3 Solutions of the Generalised Rarita-Schwinger Equations}

In this Section we study the plane wave type solutions of the generalised Rarita-Schwinger equations, defined by

\[ \psi^{\mu m}(x) = u^{\mu m}(\vec{k}, E)e^{ikx} \]  

(36)

The amplitude \( u^{\mu m}(\vec{k}, E) \) must satisfy the following algebraic equation:

\[ (i\vec{k} \vec{\gamma} - E\gamma_4 + M)u^{\mu m}(\vec{k}, E) = 0 \]  

(37)

If \( E > 0 \) then in the rest frame, where \( \vec{k} = 0 \) and \( E = M \) this equation is reduced to the following condition:

\[ \gamma_4 u^{\mu m}(0) = u^{\mu m}(0) \]  

(38)

where the shorthand notation \( u^{\mu m}(\vec{k} = 0, E = M) = u^{\mu m}(0) \) has been introduced. This condition means that the small components vanish. The constraints

\[ \gamma_\mu u^{\mu m}(0) = 0 \quad \text{and} \quad \tau_m u^{\mu m}(0) = 0 \]  

(39)

lead to the following results

\[ u^{4m}(0) = 0, \quad \sigma_a u^{am}(0) = 0, \quad \tau_m u^{am}(0) = 0, \]  

(40)

where \( a = 1, 2, 3 \) and \( m = 1, 2, 3 \).

Apparently we have \( 9 \times 4 = 36 \) constraints for the \( 12 \times 4 = 48 \) components of \( u^{\mu m}(0) \). However these constraints are not independent: \( \sigma_a \tau_m u^{am}(0) = \tau_m \sigma_a u^{am}(0) \).
consequently the number of the constraints is only 32 and so the number of
the independent components of \( u^{\mu m} \) in the rest frame is 16. This means that the Bargmann–
Wigner procedure provides the description of a particle with spin 3/2 and isospin 3/2, as it will be discussed below.

First of all we note that it is possible to introduce total isospin eigenstates
instead of \( u^{\mu m}(\vec{k}, E) \) by the help of Clebsch-Gordan coefficients. If the total isospin
is 1/2 then the eigenstate vanishes identically because of the constraint (35). The
other linearly independent eigenstate characterised by the total isospin 3/2 can be
given as
\[
\begin{align*}
  u^{\mu t}(\vec{k}, E) &= \sum (1-m_\gamma/2)u^{\mu m}(\vec{k}, E),
\end{align*}
\]
where \( m_\gamma \) is the isospinor index suppressed up till now and the eigenvalue of the
third component of the total isospin is denoted by \( t \).

Similarly to the total isospin we may introduce the total spin eigenstates
\( u^{\sigma \mu t} \), where the eigenvalue of the third component of the total spin is denoted by \( \sigma \).
For fixed values of \( \vec{k}, E \) and that of the indices \( a \) and \( t \) 4 independent solutions,
characterised by \( \sigma = +3/2, +1/2, -1/2, -3/2 \) exist which can be given as follows:
\[
\begin{align*}
  u^{\sigma \mu t}_{+3/2}(\vec{k}, E) &= \varepsilon_{+1}^{a}u^{t}_{+1/2}, \quad (42) \\
  u^{\sigma \mu t}_{+1/2}(\vec{k}, E) &= \sqrt{\frac{1}{3}}\varepsilon_{+1}^{a}u^{t}_{+1/2} - \sqrt{\frac{2}{3}}E\varepsilon_{-1}^{a}u^{t}_{+1/2}, \quad (44) \\
  u^{\sigma \mu t}_{-1/2}(\vec{k}, E) &= -i\sqrt{\frac{2}{3}}E\varepsilon_{+1}^{a}u^{t}_{-1/2}, \quad (45) \\
  u^{\sigma \mu t}_{-3/2}(\vec{k}, E) &= \varepsilon_{+1}^{a}u^{t}_{+1/2}, \quad (48) \\
  u^{\sigma \mu t}_{-1/2}(\vec{k}, E) &= +i\sqrt{\frac{2}{3}}E\varepsilon_{+1}^{a}u^{t}_{-1/2}, \quad (47) \\
  u^{\sigma \mu t}_{-3/2}(\vec{k}, E) &= 0. \quad (49)
\end{align*}
\]

Here the amplitudes \( u^{\sigma \mu t}_{\Sigma} \) and \( \varepsilon_{S}^{a} \) are defined as orthonormalised helicity eigen-
states obtained from the following equations
\[
\begin{align*}
  \frac{1}{2} \sum_{\Sigma} \bar{u}^{\sigma \mu t}_{\Sigma} u^{\sigma \mu t}_{\Sigma} &= \sum_{\Sigma} u^{\sigma \mu t}_{\Sigma}, \quad (\Sigma = +1/2, -1/2). \quad (50)
\end{align*}
\]
\[ \frac{\vec{S}_k}{|k|} \varepsilon^a_S = S \varepsilon^a_S, \quad (S = -1, 0, +1). \]  

(51)

The operators \( \Sigma_i \) and \( S_i \) are defined as follows

\[ \Sigma_i = -i/2 \epsilon_{ijk} \gamma^j \gamma^k, \quad (52) \]

\[ (S_i)_{jk} = -i \epsilon_{ijk}, \quad (53) \]

where the completely antisymmetric Levi-Civita symbol is denoted by \( \epsilon \).

4 Non-Strange Baryons

Up till now no elementary particle with spin 3/2 has been observed. This means that no real application exists for the Rarita-Schwinger equations. On the other hand among the baryons the delta resonances, observed by Fermi [5], have spin 3/2. They are, however composite particles, built up from quarks. Nevertheless they are generally considered in nuclear physics as structureless objects with spin 3/2 and isospin 3/2. In the quark model of Gell-Mann [4] the delta, as a baryon, is a bound state of three quarks described by the symmetric wave function

\[ [\phi_\alpha(1)\phi_\beta(2)\phi_\gamma(3)] \]

(54)

where the single particle functions \( \phi_\alpha(i) \) depend on the space, spin and isospin variables of a single quark. It has to be noted that the antisymmetry of the three-quark state is guaranteed by the antisymmetric color factor which is omitted here. The quarks are confined to the volume of the delta and in the ground state they are in a state symmetric with respect of the interchange of space coordinates, consequently the ground state must be symmetric in the spin-isospin variables. Among the circumstances of low energy phenomena, when the quark degrees of freedom are frozen yet, we can forget about the space-time coordinates of the individual quarks and can introduce the space-time coordinates \( x \) of the delta as a whole. Then the delta can be described by the wave function \( \Psi_{\alpha\beta\gamma}(x) \) which is symmetric with respect of the interchange of the spin-isospin indices \( \alpha, \beta \) and \( \gamma \). The nucleon in its ground
state having spin $1/2$ and isospin $1/2$ can be described similarly by a function $\Psi_\alpha$ satisfying the Dirac equation.

The relativistic wave functions of the non-interacting, non-strange baryons, namely those of the nucleon and the delta satisfy the following Dirac-type equations:

\[
\begin{align*}
(\gamma_\kappa \partial^\kappa + M_N)\psi(x) &= 0, \quad (55) \\
(\gamma_\kappa \partial^\kappa + M_\Delta)\psi^{\mu m}(x) &= 0, \quad (56) \\
\gamma_\mu \psi^{\mu m} &= 0, \quad (57) \\
\tau_m \psi^{\mu m} &= 0. \quad (58)
\end{align*}
\]

5 Quantum hadrodynamics

In order to have a complete quantum field theory of the non-strange hadrons appropriate mesonic fields and interactions among the baryonic and mesonic fields must be introduced in the spirit of the Walecka model [7]. This model, the so-called Quantum hadrodynamics turned out to be the unique framework for reformulating of our whole knowledge on nuclear physics [8]. In the framework of the Quantum hadrodynamics a great number of interesting properties of the hadronic matter has been studied rather successfully [1]. Among others the role of the delta resonances has been studied very extensively [6], [10]. It seems to be worthwhile to reexamine these problems in the context of the generalised Rarita-Schwinger equations. We assume that the interactions among non-strange baryons are mediated by non-strange pseudoscalar and vector mesons described by the isovector $\pi_m(x)$ and $\rho_{\mu m}(x)$, and the isoscalar $\eta(x)$ and $\omega_{\mu}(x)$ fields. The Lagrangian density of the system can be written as follows:

\[
L = \bar{\psi}(\gamma_\kappa \partial^\kappa + M_N - ig_\pi \gamma_5 \tau^m \pi_n - ig_\rho \gamma^\kappa \tau^m \rho_n - ig_\eta \gamma^5 \eta - ig_\omega \gamma^\kappa \omega_n)\psi \\
+ \bar{\psi}_{\mu m}(\gamma_\kappa \partial^\kappa + M_\Delta - ig_\pi \gamma_5 \tau^m \pi_n - ig_\rho \gamma^\kappa \tau^m \rho_n - ig_\eta \gamma^5 \eta - ig_\omega \gamma^\kappa \omega_n)\psi_{\mu m} \\
- i\bar{\psi}(g'_{\rho} \rho_{\mu m} + g'_\pi \gamma^5 \partial_\mu \pi_m)\psi_{\mu m} - i\bar{\psi}_{\mu m}(g'_{\rho} \rho_{\mu m} + g'_\pi \gamma^5 \partial_\mu \pi_m)\psi_{\mu m} \\
+ L_\pi + L_\rho + L_\eta + L_\omega, \quad (59)
\]
where $L_\pi$, $L_\rho$, $L_\eta$, and $L_\omega$ are the Lagrangians of the free $\pi$, $\rho$, $\eta$, and $\omega$ fields, resp., and the constraints

$$\gamma_\mu \psi^{\mu m} = 0, \quad \tau_m \psi^{\mu m} = 0$$

must be taken into account. Here it is assumed, that direct couplings between the nucleon and the delta are produced by the $\rho_{\mu m}(x)$ fields and by the derivative of the pion fields $\partial_\mu \pi_m(x)$.

As it was discovered by K. Novobátky [11] the quantisation in the presence of constraints can be performed in a consistent way if the independent degrees of freedom are separated before the quantisation procedure and only the independent, so called dynamical variables are quantised. Using this method the loss of the explicit covariance is the price we have to pay as a definite frame of reference must be chosen. If we express the field $\psi^{\mu m}(x)$ in terms of the $3/2$ isospin and $3/2$ helicity eigenstates then the constraints are satisfied automatically, consequently in the quantisation procedure only the components of the delta field are involved.

6 Elimination of the Delta Degrees of Freedom

If the system is at high temperature, as it is the case in high energy heavy ion collision, then the delta contribution to the baryon density may be substantial. At low temperature, however, as in the ground state of nuclear matter, the contribution of the delta to the baryon density is much more reduced, therefore at low temperature we may perform a formal elimination of the delta field.

Starting from the coupled field equations, given by

$$\begin{align*}
(\gamma_\kappa \partial^\kappa + M_\Delta - ig_\pi \gamma^5 \tau^n \pi_n - ig_\rho \gamma^\kappa \tau^n \rho_{\kappa n} - ig_\eta \gamma^5 \eta - ig_\omega \gamma^\kappa \omega_{\kappa}) & \psi_{\mu m} \\
- i(g'_{\mu} \rho_{\mu m} + g'_{\pi} \gamma^5 \partial_\mu \pi_m) & = 0,
\end{align*}$$

$$\begin{align*}
(\gamma_\kappa \partial^\kappa + M_N - ig_\pi \gamma^5 \tau^n \pi_n - ig_\rho \gamma^\kappa \tau^n \rho_{\kappa n} - ig_\eta \gamma^5 \eta - ig_\omega \gamma^\kappa \omega_{\kappa}) & \psi_{\mu m} \\
- i(g'_{\mu} \rho_{\mu m} + g'_{\pi} \gamma^5 \partial_\mu \pi_m) & = 0,
\end{align*}$$

we can solve Eq. (62) for $\psi_{\mu m}$:

$$\psi_{\mu m} = (\gamma_\kappa \partial^\kappa + M_N - ig_\pi \gamma^5 \tau^n \pi_n - ig_\rho \gamma^\kappa \tau^n \rho_{\kappa n} - ig_\eta \gamma^5 \eta - ig_\omega \gamma^\kappa \omega_{\kappa})^{-1}$$
and substituting this into Eq. (63), we obtain:

\[ (\gamma_\kappa \partial^\kappa + M_N - ig_\pi \gamma^5 \tau^n \pi_n - ig_\rho \gamma^\rho \gamma^\kappa \tau^n \rho_{kn} - ig_\eta \gamma^5 \eta - ig_\omega \gamma^\kappa \omega_{\kappa} + \Phi)\psi = 0, \]  

(65)

where the term \( \Phi \) is defined as

\[ \Phi = (g'_\rho \rho_{\mu m} + g'_\pi \gamma^5 \partial_\mu \pi_m) \]

(66)

Using a complete set of 8x8 matrices the term \( \Phi \) can be expanded. This decomposition produces contributions to the interaction terms of Eq. (65). The contribution to \( M_N \) can be identified with the sigma field introduced by Walecka [7]. As it is seen the elimination of the degrees of freedom associated with the delta gives rise to an effective scalar-isoscalar interaction. The procedure followed here seems to be a natural way for the introduction of the ‘nonexisting’ sigma field, furthermore it supports the traditional interpretation of the sigma particle as a correlated meson pair.

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