Four-body eikonal approach to three-body halo nuclei scattering

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Halo nuclei projectiles can undergo into a fragmentation process when scattered by a nucleus target. The corresponding core from exotic nuclei is usually observed while the others fragments of the reaction are not. We use the recently proposed theory by [1] with a four-body (4B) description of the inclusive breakup reaction where the projectile is 20C described like a two-neutron halo nucleus. The momentum of both neutrons are integrated out giving a generic description of the core angular distribution. In this preliminary study we perform an analysis of the inclusive inelastic breakup cross-section using the eikonal approximation for the distorted wave function of the projectile. A study of the inclusive inelastic cross-sections of 20C from the collision with different targets are presented.

I. INTRODUCTION

The neutron halo of unstable nuclei close to the drip line has been observed indirectly by some experimental groups [2,3] using different reaction processes. Some works like those investigate Borromean two-neutron halo nuclei as, for example 22C, and non-Borromean type such as 23C, which are interesting by the fact that the many-body effects can be disentangled from the few-body dynamics. The large spatial distribution of the weakly bound halo of these exotic nuclei should bring some distinctive features to the reaction cross-sections, and the observed large reaction cross-sections were associated to a possible extended structure of two-neutron halo in the carbon isotopes.

To contribute to the investigation of the two-neutron halo nuclei collision problem, we address the theoretical analysis of inelastic breakup reactions when weakly bound nuclei are employed as projectiles. The four-body theory proposed to describe this kind of reactions, namely the formalism developed to calculate inclusive breakup cross-sections and used here was described in details in Ref. [1]. This new theory can be applied to treat reactions with stable/unstable projectiles composed by three-fragments (3B), such as two-neutron halo nuclei. It is an extension of the inclusive three-body breakup model for incomplete fusion reactions developed for two fragment projectiles by Ichimura, Autern and Vincent (IAV) [4], Udagawa and Tamura (UT) [5] and Hussein and McVoy (HM) [6].

The relevance of the four-body theory developed in [1] comes from the fact that it allows to obtain the fragment (neutron) yield in the reaction. In this framework, the inclusive breakup cross-section is a sum of the four-body elastic plus inclusive inelastic breakup cross-sections involving the absorption cross-sections of the neutrons.

Particularly interesting is the contribution of the two-fragment correlation to the inclusive inelastic breakup cross-section through a three-body absorptive interaction, which appears naturally in the four-body formalization. We present preliminary results by showing an eikonal analysis with the São Paulo potential used to describe the core-target interaction in 20C reactions with a target. The internal wave function of the incoming projectile is obtained from a three-body renormalized model [7]. We compute the inclusive inelastic breakup cross-section in which the halo neutrons, labelled as x1 and x2, are not observed. The cross-sections are estimated for the collisions of a 20C projectile with 12C and 208Pb targets (A), schematically represented as A(a, b)X, where a = x1 + x2 + b and b is the core.

II. THEORETICAL FORMALISM

We provide a briefly description of the formalism used, for a complete discussion see [1].

The inclusive breakup cross-section is a sum of two distinct terms, the elastic breakup and the inelastic breakup cross-sections

$$d^2\sigma_{b} = d^2\sigma_{b}^{EB} + d^2\sigma_{b}^{INEB}$$

our interest is second quantity, the inelastic part of the reaction spectra. This can be computed by

$$d^2\sigma_{b}^{INEB} = \frac{2}{\hbar \omega_{a}} \rho_{b}(E_{b}) \langle \hat{\rho}_{x_{1}, x_{2}} | W_{x_{1}} + W_{x_{2}} + W_{3B} | \hat{\rho}_{x_{1}, x_{2}} \rangle,$$

where $W_{x_{1}}$, $W_{x_{2}}$ and $W_{3B}$, are the imaginary part of the optical interaction potential. The position of the weakly bound neutrons of the incident nucleus provide the source function

$$\hat{\rho}_{X}(r_{x_{1}}, r_{x_{2}}) = \left( \chi_{b}^{(-)}(r_{b}) \right)^{\dagger} \Psi_{0}^{4B(+)}(r_{b}, r_{x_{1}}, r_{x_{2}})$$

containing the four body (two neutrons + halo + target) description. The full four-body scattering state is $\Psi_{0}^{4B(+)}(r_{b}, r_{x_{1}}, r_{x_{2}})$, which should be ideally obtained by
solving the Faddeev-Yakubovsky equations in the continuum with optical potentials for the fragments and the target, in addition to the inter-fragment potentials. The boundary condition for the projectile-target four-body scattering contains the initial three-body wave function of the halo projectile, which in particular can be obtained by the three-body renormalized model [7].

The inelastic part of the breakup cross-section of the weakly bound projectile is given by

\[
\frac{d^2 \sigma^{INEB}_{b}}{dE_b d\Omega_b} = \rho_b(E_b) \frac{k_a}{E_a} \left[ \frac{E_{x_1}}{k_{x_1}} \sigma_{R}^{x_1} + \frac{E_{x_2}}{k_{x_2}} \sigma_{R}^{x_2} + \frac{E_{CM}(x_1, x_2)}{(k_{x_1} + k_{x_2})} \sigma_{R}^{3B} \right],
\]

where \( a \) and \( x \) correspond to projectile and fragment labels, respectively. Finally, the single fragment inclusive cross-sections are obtained by

\[
\sigma_{R}^{x} = \frac{k_{x}}{E_{x}} \langle \hat{\rho}_{x_1, x_2} | W_{x} | \hat{\rho}_{x_1, x_2} \rangle,
\]

and the double fragment inclusive cross-section is

\[
\sigma_{R}^{3B} = \frac{(k_{x_1} + k_{x_2})}{E_{CM}(x_1, x_2)} \langle \hat{\rho}_{x_1, x_2} | W_{3B} | \hat{\rho}_{x_1, x_2} \rangle
\]

which represents the two-fragment irreducible inelastic processes.

### III. CROSS SECTIONS

The source function computed with the Hussein and McVoy (HM) model [3] is written as

\[
\langle r_{x_1}, r_{x_2} | \hat{\rho}_{HM} \rangle = \hat{S}_b(r_{x_1}, r_{x_2}) \chi_{x_1}^{(+)}(r_{x_1}) \chi_{x_2}^{(+)}(r_{x_2})
\]

where the four-body scattering state is approximated by the product of the distorted waves of the three fragments and the projectile bound state wave function. The \( \hat{S} \)-matrix can be given by (for simplicity, we drop the \( x \) index for fragments)

\[
\hat{S}(r_{x_1}, r_{x_2}) = \int dr_{b} \hat{\phi}_{3B}(r_{x_1}, r_{x_2}, r_{b}) e^{i q \cdot r_{b}} \langle \hat{\psi}^{(-)}_{p_1}(r_{b}) | \hat{\psi}^{(+)}_{p_2}(r_{b}) \rangle,
\]

where \( \hat{\phi}_{3B} \) is the incoming three-body wave function computed with the renormalized three-body model, projected in four-body coordinates now. The core distorted waves are computed with the straight-line trajectory in the eikonal approximation,

\[
\hat{S}^{Eik}_{p_1, p_2} = \psi^{(-)}_{p_1}(r_{b}) \psi^{(+)}_{p_2}(r_{b})
\]

\[
= \exp \left[ -\frac{i \mu}{\hbar p_{b}} \int_{-\infty}^{-z_b} dz V \left( \sqrt{b_{b}^2 + z^2} \right) -\frac{i \mu}{\hbar p_{b}} \int_{z_b}^{\infty} dz V \left( \sqrt{b_{b}^2 + z^2} \right) \right].
\]

The scattering dynamics for this case is shown in Fig. 1, where cylindrical coordinates are employed with \( r_b = \sqrt{x_b^2 + y_b^2} \). The São Paulo (SP) optical potential is taken to represent the core-target interaction \( V \) in the Eq. [8]. Although the comparison to different optical potentials is out of the scope of this preliminary work, we stress that the heavy-ion São Paulo potential has been successfully applied to several cases, e.g. elastic, inelastic and transfer reactions [8] [9]. The SP potential takes into account nucleon-nucleon interactions folded into nucleon densities products providing at the end a complex and energy depend interaction for a chosen nucleus. One of our goals is the study of the \( T \)-matrix that can be obtained in the limit

\[
\hat{T}^{Eik} = \lim_{z_b \to \infty} \hat{S}^{Eik} - 1.
\]

For this, we let scattering to be in the \( x - z \) plane and write

\[
q_b \cdot r_b = q_b \cdot b_b + q_b \cdot z_b \hat{k}
\]

\[
= b_b \ k_b \ sin \theta_b \ cos \varphi_b + (k_b - k_b' \ cos \theta_b) z_b.
\]

The single fragment inclusive cross-section in the four-body theory can then be obtained by

\[
\frac{E_{x_1}}{k_{x_1}} \sigma_{R}^{x_1} = \int dr_{x_1} \int dr_{x_2} | \hat{S}_b(r_{x_1}, r_{x_2}) |^2 | \chi_{x_1}^{(+)}(r_{x_1}) |^2 \times W^{nT}(r_{x_1}) | \chi_{x_2}^{(+)}(r_{x_2}) |^2 .
\]

where \( \kappa = 1, 2 \) refers to the neutrons.

The weakly bound nucleons of the projectile interact with the target nucleus via optical potential represented in this model by \( W^{nT} \). Since the São Paulo potential was developed for nucleus-target interactions (meanly heavy nucleus), we represent, as a first approach, the neutrons-target interaction as a Woods-Saxon-like function. This, provides a good approximation for the interaction shape of potentials usually obtained by more sophisticated optical potentials. We take the Woods-Saxon potential as

\[
W^{nT}(r) = \frac{W_{01}}{1 + \exp ((r - R)/a_I)},
\]

and the following parameters for interaction \( n - ^{12}\text{C} \):

\( W_{0R} = 49.9395 \text{ MeV} \), \( W_{0z} = 1.8256 \text{ MeV} \), \( a_R = a_I = \)
0.676 fm and $R_I = R_R = 2.5798$ fm; from Ref. [10]. For the two-fragment irreducible inclusive cross-section, we have that

$$\sigma^{3B}_R = \int dr_{x_1} \int dr_{x_2} \left| \hat{S}(r_{x_1}, r_{x_2}) \right|^2 \left| \chi_{x_2}^{(+)}(r_{x_2}) \right|^2 \times W^{3B}(r_{x_1}, r_{x_2}) \left| \chi_{x_1}^{(+)}(r_{x_1}) \right|^2,$$

(12)

where Woods-Saxon potential is also used for $W^{3B}$ interaction.

### IV. EIKONAL ANALYSIS

The transition matrix obtained within the eikonal approximation [3] is presented in Figure 2. We show the $\hat{T}$-matrix computed for a $^{20}$C projectile on the target of $^{12}$C as a function of the impact parameter $b_b$. The most of the absorption occurs around the target nucleus radius, vanishing for larger values of impact parameter $b_b$. The preliminary results the structure and results obtained have been helping us to understand the role of the projectile in the scattering dynamics. Our next step is the addition the two fragments distorted waves. This will provide a complete description of all ranges of action of the nuclear potential, one may expect it to be sensible to the target surface. Although we present preliminary results the structure and results obtained have been helping us to understand the role of the projectile halo in the scattering dynamics. For bigger values of the impact parameter the contribution of this configuration vanishes but a reaction closer to the elastic limit.

The inelastic effect can be noted by the Fig. 5. For a fixed incoming energy $E_b = 50$ MeV/nucleon, one choose values for outcoming energy such that blue line represents $E'_b = 0.8E_b$, and black-dotted line $E'_b = 0.5E_b$. It is possible to see the cross-section oscillation increases as one has a reaction closer to the elastic limit.

### V. CONCLUSIONS

The structure of the reaction cross-sections for the absorption of one of the interacting fragments removes the ambiguity about the difference between the 4B and 3B cases, which we find to be damped by the absorption effect of the other fragment. We observed, as shown in the eikonal analysis, the core scattering process is more sensible to the target surface. Although we present preliminary results the structure and results obtained have been helping us to understand the role of the projectile halo in the scattering dynamics. Our next step is the addition the two fragments distorted waves. This will provide a complete description of all ranges of action of the nuclear potential, one may expect it to be very important specially for targets with large number of protons. We are working to add the Coulomb scattering like some works for on neutron-halo [11]. This interaction plays a role when the three body wave function comes in, small effects can become present specially for large val-
FIG. 4. Inclusive four-body fragment cross-section, Eq. (11) as function of scattering angle for fixed core for incoming energy at 50 MeV/nucleon (and outcoming is $E'_b = 0.8 E_b$) for targets of $^{12}$C (top) and $^{208}$Pb (bottom).

FIG. 5. Angular distribution from inclusive four-body fragment cross-section, Eq. (11) for fixed core incoming energy at 50 MeV/nucleon for different outcoming core energies showing the inelastic effect.

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