Distributed User Scheduling for MIMO-Y Channel

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Abstract

In this paper, distributed user scheduling schemes are proposed for the multi-user MIMO-Y channel, where three $N_T$-antenna users ($N_T = 2, 3$) are selected from three clusters to exchange information via a three-antenna amplify-and-forward (AF) relay. The proposed schemes effectively harvest multi-user diversity (MuD) without the need of global channel state information (CSI) or centralized computations. In particular, a novel reference signal space (RSS) is proposed to enable the distributed scheduling for both cluster-wise (CS) and group-wise (GS) patterns. The minimum user-antenna (Min-UA) transmission with $N_T = 2$ is first considered. Next, we consider an equal number of relay and user antenna (ER-UA) transmission with $N_T = 3$, with the aim of reducing CSI overhead as compared to Min-UA. For ER-UA transmission, the achievable MuD orders of the proposed distributed scheduling schemes are analytically derived, which proves the superiority and optimality of the proposed RSS-based distributed scheduling. These results reveal some fundamental behaviors of MuD and the performance-complexity tradeoff of user scheduling schemes in the MIMO-Y channel.

Index Terms

Multi-user scheduling, MIMO-Y channel, Analog network coding, Multi-way relay, Multi-user diversity.

I. INTRODUCTION

The multi-way relay channel (MWRC) [1] has been considered as a fundamental building block for future cooperative communications. Currently, two basic representatives of the MWRC have

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been focused. The first one is the two-way relay channel (TWRC), which has been extensively studied with various wireless network coding (WNC) techniques [2], [3]. The second one is the MIMO-Y channel [4], which is a novel extension of the TWRC with multiple independent unicast transmissions among three users. As compared to the TWRC, the MIMO-Y channel requires more sophisticated signal processing with WNC and spatial-resource management. Specifically, the basic MIMO-Y channel has been proposed with a new concept of signal space alignment (SSA) [4], which is a novel application of the principle of interference alignment [5]. SSA aligns the bi-directional information from two users at the relay to maximize the utility of the relay antenna, and it also enables the WNC [2], [3] for efficient transmission with the half-duplex relay. Because of its fundamental role and novel transmission schemes, the MIMO-Y channel is now attracting increasing attentions.

The MIMO-Y channel can find many interesting applications in various three-party communication scenarios. For example, in ad-hoc networks, three geographically isolated nodes can exchange messages with the help of the relay; in cellular networks, a group of three users can share information via the relay with flexible cooperative or device-to-device communication protocols; in satellite communication, the satellite often serves as a relay to enable information exchanges among three earth stations. Inspired by the wide range of potential applications, many efforts have been devoted to understanding the fundamental limit of MIMO-Y channel. For example, the achievable degrees-of-freedom and capacity of the MIMO-Y channel have been studied with various antenna configurations at the users or relay [4], [6]–[8]. Beyond the concerns of the fundamental limit, some practical schemes have been also proposed to enhance the transmission reliability of MIMO-Y channel in the wireless fading environment [9]–[11]. Of particular interests are the diversity-achieving beamforming schemes, which employ more than two antennas at the user to perform selective or iterative beamforming optimization [9], [10]. Although these schemes show significant performance improvements as compared to the proof-of-concept scheme in [4], solely relying on the user’s multiple antennas for a scalable diversity gain is not always practical. The limited size and power supply of the user’s equipment are practical constraints. Therefore, other diversity-achieving schemes are also demanded to complement these beamforming techniques.

Multi-user diversity (MuD), which is known as an important source to combat wireless fading [12], [13], can be potentially exploited for the MIMO-Y channel. It has been noted that although the number of antenna is limited for each user’s equipment, a system potentially has multiple
users requiring data transmission. Therefore, by carefully scheduling the users’ transmissions, significant performance gain can be obtained. The multi-user scheduling has been studied for the traditional broadcasting [14] and interference channels [15], [16]. Regarding the general MWRC, the comprehensive solution of user scheduling is still open. Some initial researches have been done for the TWRC with a variety of system configurations [17]–[22], and they offer valuable insights to inspire new applications. However, the designs of efficient user scheduling schemes for the MIMO-Y channel have different challenges than TWRC. In general, the MIMO-Y channel calls for new user scheduling methods for its unique system and traffic configurations, i.e., each user has multiple antennas to support two independent unicast information flows [4]. In particular, the unique SSA-oriented MIMO-Y transmission requires more sophisticated transmit/receive beamforming designs [4], which are often coupled with the multi-user scheduling metrics [23]. Such coupling may significantly increases the system overheads for CSI and the computation complexity of the scheduling center. Taking the scheduling methods [14], [17]–[23] for example, they are all conducted in a centralized fashion with global CSI and require relatively complicated computations at the scheduling center. In fact, even in the cellular network with high user density, asking the base station to learn the global CSI is costly [13]. For the MIMO-Y channel, which often fits into the low-complexity and structure-less networks, the assumption of a powerful dedicated scheduling center is not always feasible, especially when one node or user just serves as the immediate relay. Therefore, novel cost-effective scheduling methods are needed for the MIMO-Y channel.

In this paper, we consider a basic multi-user MIMO-Y channel, where one three-antenna relay helps information exchange among three selected $N_T$-antenna ($N_T = 2, 3$) users from three clusters. Such basic configuration is sufficient to capture the essential of the MIMO-Y transmission; it also simplifies MuD analysis for clear insight. In particular, we propose low-complexity distributed user scheduling schemes for the MIMO-Y channel with two scheduling patterns, namely, cluster-wise scheduling (CS) and group-wise scheduling (GS). For the CS, a cluster representative is selected from each cluster, and the three selected representatives conduct information exchange via the relay. Such scheduling may find applications in the wireless ad-hoc or sensor networks. For example, when some globally critical events are observed by many on-site nodes at three isolated places, one node is selected from each cluster to perform information exchange. For the GS, three users (each from a different cluster) are associated within a predefined group before transmission, and one group is scheduled to exchange information via
the relay. Such scheduling may be useful in the cellular networks or device-to-device networks where a group of three users wish to share information within their social network.

Moreover, depending on the number of required antennas equipped at the user, two possible MIMO-Y transmission schemes are considered. Specifically, the transmission scheme with the minimum number of user antenna (Min-UA) $N_T = 2$ is first considered with a variable-gain AF relay. It is noted that after user scheduling the Min-UA transmission adopts a joint beamforming to achieve SSA at the relay, where the three selected users and the relay needs to known the three-party CSI. Aiming at reducing CSI overhead, the user antenna is slightly increased as $N_T = 3$, and the transmission with an equal number of relay and user antenna (ER-UA) is proposed with a fixed-gain AF relay. The ER-UA transmission allows distributed beamforming at the user with local CSI, which reduces the CSI overhead. In contrast to the centralized scheduling schemes [14], [17]–[23], the proposed schemes can distribute the computations of scheduling metrics to the users with local CSI. Therefore, the scheduling center enjoys very low implementation complexity without global CSI.

The objective of this paper is to study low-complexity distributed CS and GS for MIMO-Y channel with both Min-UA and ER-UA transmissions. Specifically, the key contributions are summarized as follows.

1) A novel reference signal space (RSS) is proposed to guide the distributed scheduling with both Min-UA and ER-UA transmissions. The RSS is a predefined signal space which is known to all the nodes in the network. Under the guidance of RSS, each user can calculate its individual scheduling metric with local CSI, which enables several distributed scheduling schemes with global benefits.

2) RSS-based distributed CS and GS are proposed for Min-UA transmission. Noting that the optimal CS and GS are not decomposable for distributed implementations with Min-UA transmission [23], two sub-optimal angle-based scheduling strategies are proposed with RSS, which enable distributed implementations of CS and GS. Specifically, each user can calculate its angle coordinate within the RSS by using only local CSI, and the angle-based coordinate is used to infer the relative positions of the pair-wise aligned signal vectors or directions within the SSA-resultant signal space at the relay. It is interesting to note that the selected users can generate a near-orthogonal SSA-resultant signal space at the relay, which results in improved system performance.

3) RSS-based distributed CS and GS are proposed for ER-UA transmission. Aiming at uti-
lizing only local CSI, RSS is used to guide both distributed beamforming and scheduling with ER-UA transmission. In particular, each user can calculate its beamforming matrix as well as the individual scheduling metric with local CSI and RSS. It is noted that the locally calculated individual scheduling metric is equivalent to the link gain of MIMO-Y channel. Therefore, such individual metric has a straightforward connection to the optimal (centralized) scheduling metric that is a function of all the link gains. By using the local and individual scheduling metric, effective distributed scheduling schemes are shown to achieve near-optimal performances.

4) The performances of the proposed schemes are analyzed. Specifically, RSS-based distributed CS and GS are carefully analyzed for ER-UA transmission, because of their near-optimal performances and tractability. It is interesting to note that the distributed scheduling achieves the same MuD order as the centralized scheduling under ER-UA transmission. This observation is theoretically proved by studying the network’s outage probabilities and the achievable diversity-multiplexing tradeoffs (DMTs) \[24\] with both centralized and distributed scheduling schemes. The explicit MuD orders are obtained as \(d_{CS}^* = \min(M_1, M_2, M_3)\) for both distributed and centralized CS, and \(d_{GS}^* = M\) for both distributed and centralized GS, where \(M_k\) is the number of candidates in the \(k\)-th cluster \(k \in \{1, 2, 3\}\), and \(M\) is the number of candidate groups. These results not only prove the optimality of the proposed distributed scheduling in terms of MuD order, but also shed light into the MuD behaviors in MIMO-Y channel, which have not been well studied so far.

**Organization:** Section II introduces the system model and the general MIMO-Y transmission. Section III describes the distributed CS and GS with Min-UA transmission. Section III details the ER-UA transmission and the corresponding distributed CS and GS, and Section IV analyzes the outage probabilities and the achievable DMTs. Numerical results and complexity analysis are summarized in Section VI, and Section VII concludes this paper.

**Notations:** The integer set \(\{1, 2, \ldots, K\}\) is abbreviated as \([1, K]\), \([A]_{m,n}\), \((A)^*\), \((A)^T\), \((A)^H\), \((A)^{-1}\), \(\text{Tr}(A)\), \(\text{vec}(A)\), \(\text{Range}(A)\) and \(\lambda_{\text{min}}(A)\) are the \((m,n)\)-th entry, conjugate, transpose, conjugate transpose, inverse, trace, vectorization, range and minimum eigenvalue of a matrix \(A\). \(I_{m\times m}\) and \(0_{m\times m}\) represent the \(m \times m\) identity matrix and all-zero matrix. \(\text{Span}(a, b)\) denotes the subspace spanned by vectors \(a\) and \(b\), \(\|a\|\) and \(\langle a \rangle = a/\|a\|\) are the Euclidean-norm and the
normalization operation of vector $\mathbf{a}$. $\angle(\mathbf{a}, \mathbf{b}) = \cos^{-1}\left(\frac{|\mathbf{a}^H \mathbf{b}|}{|\mathbf{a}||\mathbf{b}|}\right)$ is the acute angle between vector $\mathbf{a}$ and $\mathbf{b}$. $\mathbb{C}$ represents the set of complex numbers. $\mathcal{CN}(\mathbf{m}, \Sigma)$ denotes a complex Gaussian random vector with mean $\mathbf{m}$ and covariance matrix $\Sigma$. $E\{\cdot\}$ is the expectation operator. $C^n_k$ is the number of $k$-combinations from a given set of $n$ elements. $\dot{=} = \exp$ is the exponential equality, e.g., $f(x) \dot{=} x^a$ represents $a = \lim_{x \to \infty} \frac{\log(f(x))}{\log(x)}$.

II. SYSTEM MODEL AND MIMO-Y TRANSMISSION

As shown in Fig. 1, a MIMO-Y network comprises a three-antenna relay $R$ and three clusters of $N_T$-antenna ($N_T = 2, 3$) users $\{S_{jk}, j_k \in [1, M_k], k \in [1, 3]\}$, $j_k$ and $M_k$ are the intra-cluster user index and the number of candidates within the $k$-th cluster. It is assumed that there is no direct link between any two users in different clusters, and the half-duplex AF relay helps information exchange among clusters. Time-division duplex (TDD) mode is assumed, therefore channel reciprocity holds. The channels of $S_{jk} \rightarrow R$ and $R \rightarrow S_{jk}$ are denoted as $\mathbf{H}_{jk} \in \mathbb{C}^{3 \times N_T}$ and $\mathbf{H}_{jk}^T \in \mathbb{C}^{N_T \times 3}$, respectively, whose entries are independent and identically distributed (i.i.d.) $\mathcal{CN}(0, 1)$. User scheduling is the focus of this paper, and only one user is selected from each cluster. The three selected users then exchange information through the basic MIMO-Y channel [4]. Before presenting the specific transmission and scheduling schemes, the outline of the general MIMO-Y transmission is reviewed in this section. For the ease of exposition, the intra-cluster index of each user is temporarily neglected, and the selected user in the $k$-th cluster is denoted as $S_k$. Accordingly, the relevant channels of $S_k$ are denoted as $\mathbf{H}_k$ and $\mathbf{H}_k^T$, respectively. In addition,
the information symbols from \( S_k \) to another two users \( \{S_l\}_{l \in \mathcal{L}_k} \) are collected in \( \{d_{l,k}\}_{l \in \mathcal{L}_k} \), \( \mathcal{L}_k = [1, 3] \setminus \{k\} \).

Analog network coding (ANC) \(^2\) is employed for efficient AF relaying, which consists of the multiple access (MAC) and broadcasting (BC) transmission phases. During the MAC phase, user \( S_k \) employs the transmit beamforming matrix \( V_k = [v_{1,k} v_{l_2,k}] \in \mathbb{C}^{N_T \times 2} \) for data transmission, and the transmitted symbol vector is \( s_k = V_k d_k \in \mathbb{C}^{N_T \times 1} \), where \( d_k = [d_{1,k} d_{l_2,k}]^T \in \mathbb{C}^{2 \times 1} \) and \( \mathbb{E}\{d_k d_k^H\} = I_{2 \times 2} \), \( k \in [1, 3] \), \( l_1 \), \( l_2 \in \mathcal{L}_k \), \( l_1 \neq l_2 \). A transmit power constraint is imposed as \( \text{Tr} (V_k V_k^H) \leq P_T \), where \( P_T \) is the average transmit power of each user. Then, the relay \( R \) receives

\[
y_R = \sum_{k=1}^{3} H_k s_k + n_R, \tag{1}
\]

where \( n_R \in \mathbb{C}^{3 \times 1} \) is the additive white Gaussian noise (AWGN) vector distributed as \( \mathcal{CN} (0, \sigma^2_R I_{3 \times 3}) \).

During the BC phase, the transmitted signal of the AF relay is given as \( (s_R)^* = (Wy_R)^* = (G_R \tilde{W} y_R)^* \in \mathbb{C}^{3 \times 1} \), where \( \tilde{W} \in \mathbb{C}^{3 \times 3} \) is the relay processing matrix and \( G_R \) is the power controlling coefficient to be specified later. Then, the received signal at \( S_k \) is expressed as

\[
y_k = H_k^H s_R + n_k, \tag{2}
\]

where \( n_k \in \mathbb{C}^{N_T \times 1} \) is the AWGN vector distributed as \( \mathcal{CN} (0, \sigma^2_S I_{3 \times 3}) \). According to the ANC protocol, \( S_k \) needs to perform self-interference cancellation before extracting the useful information sent by \( \{S_l\}_{l \in \mathcal{L}_k} \) with the receive beamforming matrix \( U_k \in \mathbb{C}^{N_T \times 2} \), which is described as

\[
\hat{y}_k = U_k^H (y_k - H_k^H \tilde{W} H_k s_k) = \hat{y}_{k,S} + \hat{y}_{k,N}, \tag{3}
\]

where \( \hat{y}_{k,S} = U_k^H H_k^H \tilde{W} \sum_{l \in \mathcal{L}_k} H_l V_l d_l \in \mathbb{C}^{N_T \times 1} \) and \( \hat{y}_{k,N} = U_k^H (H_k^H \tilde{W} n_R + n_k) \in \mathbb{C}^{N_T \times 1} \) are the signal component and the noise component of the decision variable \( \hat{y}_k \). With a proper design of transmit/receive beamforming matrices \( \{U_k, V_k\}_{k=1}^3 \) at users and \( \tilde{W} \) at the relay, \( S_k \) can have interference-free reception and recover the useful information as \( (\hat{d}_{k,l_1} \hat{d}_{k,l_2}) = f (\hat{y}_k), l_1, l_2 \in \mathcal{L}_k, l_1 \neq l_2 \), where \( f \) represents the decoding process. In the next sections, we will show the detailed designs of \( \{U_k, V_k\}_{k=1}^3 \) and \( \tilde{W} \) for Min-UA and ER-UA transmissions as well as their corresponding CS and GS.

\(^1\)Similar to the treatment in \(^2\), the signal model at the user is simplified with the conjugate operation, where the channel matrix \( H_k^H \) can be in the form of hermitian transpose in \(^3\).
III. DISTRIBUTED USER SCHEDULING WITH MIN-UA TRANSMISSION

In this section, the distributed user scheduling schemes are studied with the Min-UA transmission, where each user is equipped with $N_T = 2$ antennas. It is noted that in this scenario the instantaneous three-party CSI is required by the three selected users and the relay for joint beamforming \[4\]. Because of this coupling, the calculation of the optimal scheduling metric, i.e., the post-processing signal-to-noise-ratio (SNR), cannot be easily decomposed, and the design of an effective distributed user scheduling is very challenging. To this end, a novel RSS is introduced to guide the distributed CS and GS. To be more specific, the RSS is a predefined 3-dimension signal space $\Omega = \text{Span} (e_1, e_{II}, e_{III})$, whose normalized orthogonal basis $E = [e_1 e_{II} e_{III}] \in \mathbb{C}^{3 \times 3}$ is assumed to be known by all the users in this network. In fact, $E$ can be designed off-line or broadcasted by the relay. Before presenting the RSS-based distributed user scheduling schemes, the Min-UA transmission is briefly reviewed in the following subsection.

A. Min-UA Transmission

Without loss of generality, let us assume three users $\{S_1, S_2, S_3\}$ are randomly selected to perform Min-UA MIMO-Y transmission. In particular, SSA is required for the bi-directional information exchange between the pair $S_l$ and $S_k$, $k \in [1, 3]$, $l \in L_k$, and the transmit beamforming matrix of each user is jointly designed with another two users using the three-party CSI. More specifically, the pair-wise transmit beamforming vectors of $S_l$ and $S_k$ can be jointly designed by solving the null-space problem \[4\] as

$$[H_l - H_k] \tilde{v}_{m=\pi(l,k)} = 0,$$

where $\tilde{v}_m = \text{vec} (\tilde{v}_{k,l}, \tilde{v}_{l,k}) \in \mathbb{C}^{4 \times 1}$ contains the pair-wise transmit beamforming vectors with a total power normalization as $||\tilde{v}_m||^2 = 1$. For simplicity, a total power constraint $P_T$ is imposed on this pair-wise transmit beamforming vectors, and the effective transmit beamforming vector for $d_{l,k}$ is expressed as

$$v_{l,k} = \sqrt{P_T} \tilde{v}_{l,k}.$$

It is easy to check that each unicast stream of $S_k$ has an average power of $E \{||v_{l,k}||^2\} = \frac{1}{2} P_T$, therefore, the average transmit power of each user is $P_T$. According to (4) and (5), it is noted that

\footnote{For notational clarity, a permutation function over the source index pair $(l, k)$ is introduced as $m = \pi(l, k) = \pi(k, l)$, $m, k \in [1, 3]$, $l \in L_k$. Specifically, $\pi$ is defined as $\pi(1, 2) = I$, $\pi(1, 3) = II$, $\pi(2, 3) = III$.}
the three-party CSI is necessary for the beamforming matrix $V_k = [v_{l_1,k} \ v_{l_2,k}]$ of $S_k$, $l_1, l_2 \in \mathcal{L}_k$, $l_1 \neq l_2$. Employing the pair-wise transmit beamforming vectors $v_{k,l}$ and $v_{l,k}$, the bi-directional signal between $S_l$ and $S_k$ is then aligned along a direction $f_{m=\pi(k,l)} = H_l v_{k,l} = H_k v_{l,k} \in \mathbb{C}^{3 \times 1}$ within the signal space of $R$, and the received signal at $R$ is given by (cf. (1))

$$y_R = \sqrt{P_T F d_+} + n_R,$$

where $F = [f_{I} f_{II} f_{III}] \in \mathbb{C}^{3 \times 3}$ is the signal space seen by $R$ as shown in Fig. 2, and $d_+ = [d_{+,1} \ d_{+,2} \ d_{+,3}]^T \in \mathbb{C}^{3 \times 1}$ is the superimposed signal with element $d_{+,m} = d_{l,k} + d_{k,l}$. According to [9], the zero forcing (ZF)-based relay processing matrix $\tilde{W} = F^{-H} F^{-1} \in \mathbb{C}^{3 \times 3}$ is employed at $R$, and the power controlling factor of the variable-gain AF relay is given by (7)

$$G_R = \sqrt{\frac{P_R}{P_T} \text{Tr} (\tilde{W}) + \sigma_R^2 \text{Tr} (\tilde{W} \tilde{W}^H)},$$

where $P_R$ is the average transmit power of $R$. At each user $S_k$, the received signal is given by (2), then the remaining signal after subtracting self-interference is given by (8)

$$\tilde{y}_k = \sqrt{P_T H_k^H W F_k d_{+,k}} + H_k^H W n_R + n_k,$$

where $W = G_R \tilde{W}$, $F_k = [f_{m_1} f_{m_2} f_{m'}] \in \mathbb{C}^{3 \times 3}$, and $d_{+,k} = [d_{k,l_1} d_{k,l_2} d_{+,m'}]^T \in \mathbb{C}^{3 \times 1}$, $m_1 = \pi(l_1, k)$, $m_2 = \pi(l_2, k)$, $l_1, l_2 \in \mathcal{L}_k$ and $m' \in \{1, 3\} \setminus \{m_1, m_2\}$. Then the receive beamforming $U_k = [v_{l_1,k} \ v_{l_2,k}] \in \mathbb{C}^{2 \times 2}$ is applied to obtain $\hat{y}_k = U_k^H \tilde{y}_k$, and each desired signal stream within $\hat{y}_k$ is given by (9)

$$\hat{y}_{k,l} = \sqrt{P_T G_R d_{k,l}} + G_R f_{m}^T F n_R + v_{l,k}^H n_k,$$
where the elements of $\mathbf{1}_m \in \mathbb{C}^{3 \times 1}$ are all zeros except the $m$-th element that equals to one. After briefing the Min-UA transmission, the corresponding user scheduling schemes are discussed in the next subsection.

**B. Problem Formulation and Centralized Scheduling**

In this subsection, the optimal CS and GS are formulated with Min-UA Transmission, which serves as a preliminary and reference for distributed scheduling. Some necessary notations and performance metrics are first introduced. For ease of description on CS, an ordered set $J = \{j_1, j_2, j_3\}$ is used as a collection of the user indexes, $j_k \in [1, M_k]$, $k \in \{1, 2, 3\}$, and all the possible realizations of $J$ are enumerated in another ordered set $J^Q = \{J_q\}_{q=1}^Q$, $Q = M_1 M_2 M_3$.

Moreover, when $M_1 = M_2 = M_3 = M$, a subset $J' \subset J$ is used to facilitate the description for GS, which is defined as $J' = \{J'_p\}_{p=1}^M$ and $J'_p = \{j'_1, j'_2, j'_3\}$ subject to $Ic(j'_1) = Ic(j'_2) = Ic(j'_3) = p$.

Regarding the system performance metrics, the overall outage probability of the network with arbitrary selected users $\{S_{j_1}, S_{j_2}, S_{j_3}\}$ is defined as

$$P_{\text{out}}(\rho) = \mathbb{P}(\rho_{\text{min}} \leq \rho_{th}) \quad (10)$$

where $\rho_{\text{min}}$ is the minimum post-processing SNR (min-SNR) of the network with selected users in $J$. Then, $\rho_{\text{min}}$ is defined as

$$\rho_{\text{min}} = \min_{k \in \{1, 3\}, l \in \mathcal{L}_k} (\rho_{j_k,j_l}) \quad (11)$$

where $\rho_{j_k,j_l}$ represents the end-to-end post-processing SNR of link $S_{j_l} \rightarrow R \rightarrow S_{j_k}$, $l \in \mathcal{L}_k$, and $\rho_{j_k,j_l}$ is calculated with reference to (9) as

$$\rho_{j_k,j_l} = \frac{\text{SNR}_R \text{SNR}_T}{N_{j_k,j_l}} \quad (12)$$

where $N_{j_k,j_l} = \text{SNR}_R \left[ \hat{\mathbf{W}}^{(j)} \right]_{m,m} + \| \mathbf{v}_{j_l,j_k} \|^2 \text{SNR}_T \text{Tr} \left( \hat{\mathbf{W}}^{(j)} \right) + \| \mathbf{v}_{j_l,j_k} \|^2 \text{SNR}_T \text{Tr} \left( \hat{\mathbf{W}}^{(j)} \left( \hat{\mathbf{W}}^{(j)} \right)^H \right)$, SNR$_T = P_T/\sigma_R^2$ and SNR$_R = P_R/\sigma_S^2$ are the average transmit SNR of the user and the relay. Based on the aforementioned notations, the centralized CS is given by

$$J_C = \{j_1^*, j_2^*, j_3^*\} = \arg \max_{J \in \mathcal{J}} \left( \rho_{\text{min}}^{(J)} \right) \quad (13)$$

$Ic(j_k)$ represents the intra-cluster index of a user $S_{j_k}$. It is noted that the index used for GS could have been subsumed under $J$ or simplified with less symbols, however, we keep this seemingly redundant form to unify the descriptions on user scheduling criteria and the subsequential analysis.
Fig. 3. The geometrical interpretations on the RSS-based distributed user scheduling with Min-UA transmission. In this example, we consider the cluster-wise scheduling and assume that there are two users \( \{ S_{1_k}, S_{2_k} \} \) in the \( k \)-th cluster, \( k \in [1, 3] \). The angles between each user’s characterization direction \( r_{jk} \) and the basis of RSS \( \{ e_I, e_{II}, e_{III} \} \) can be calculated by the user in a distributed manner. Employing the proposed distributed user scheduling, the pattern of users’ characteristic directions can be directly optimized so that the SSA-resultant signal space \( F \) can be indirectly shaped to be more orthogonal.

and the centralized GS is given by

\[
J'_C = \left\{ j'_1^*, j'_2^*, j'_3^* \right\} = \arg \max_{J' \in J'} \left( \rho^{(J')}_{\text{min}} \right). \tag{14}
\]

Similar to the scheduling in [23], both centralized CS and GS involve global CSI and high computational complexity at the scheduling center.

C. RSS-based Distributed scheduling with Min-UA

In this subsection, RSS-based distributed CS and GS are proposed with Min-UA transmission. Before presenting the specific scheduling schemes, the geometrical interpretations on the SSA-resultant signal space and RSS are reviewed to motivate our design. As shown in Fig. 3, the SSA-resultant signal space \( F \) is composed of three equivalent channel vectors \( \{ f_m \}_{m=1}^{\text{III}} \). Note that the ZF-based transceiver \( \tilde{W} \) is employed at \( R \), which involves the orthogonal decomposition and projection on \( F \) and may cause significant power loss especially when \( F \) is ill-conditioned. Intuitively, if one can choose the user group to shape \( F \) such that the equivalent channel vectors
in $F$ are almost orthogonal, it is likely that the ZF-resultant power loss can be mitigated and the system performance can be improved.

The interpretations on the orthogonality among channels may have impact on the design of user scheduling. A straightforward observation is on the orthogonality among $\{f_m\}^m_{m=1}$. However, this observation is not instructive for an efficient distributed scheduling with just local CSI. It is noted that $f_m$ lies in the joint space of $H_{jk}$ and $H_{ji}$, i.e., $\text{Span}(f_m) = \text{Range}(H_{jk}) \cap \text{Range}(H_{ji})$, $m = \pi(l, k)$, and the measurement for orthogonality between $f_m$ and $f_{m'}$, $m' \neq m$, requires the three-party CSI. Therefore, new methods should be developed to enable distributed scheduling. Considering only local CSI, i.e., the channel $H_{jk}$ of user $S_{jk}$, we first introduce the characteristic direction of the channel as $r_{jk} \in \text{Null}(H_{jk})$, $\|r_{jk}\| = 1$. As shown in Fig. 3, if we can make $\{r_{jk}\}^3_{k=1}$ orthogonal, then the channels $\{H_{jk}\}^3_{k=1}$ are orthogonal, and eventually the joint spaces of theses channels, i.e., $\{f_m\}^m_{m=1}$, are orthogonal. It is interesting to note that the shaping of $\{f_m\}^m_{m=1}$ can be achieved in a distributed manner with RSS and local CSI. To make this intuitive observation more concrete, the RSS-based distributed CS and GS are detailed in the following subsections.

1) RSS-based distributed CS: Recall that the RSS is a predefined 3-dimension signal space $\Omega_R = \text{Span}(e_I, e_{II}, e_{III})$, which is known by all the users in the network. Then, each user can calculate the angular-coordinate of its characteristic direction within RSS with only local CSI as $\phi_{jk} = [\phi_{jk,I} \phi_{jk,II} \phi_{jk,III}]$, where $\phi_{jk,m} = \angle(r_{jk}, e_m)$. The proposed RSS-based distributed CS aims to find the best user from each cluster based on the angular-coordinate. Specifically, the RSS-based distributed CS is a three-round sequential scheduling scheme. Let $\mathcal{K}(m) \subseteq [1, 3]$ be the set of candidate cluster-indexes for the $m$-th round, $m \in \{I, II, III\}$. Then, the procedure of distributed CS with Min-UA transmission is summarized as follows:

1) Initialization: Let $\mathcal{K}(1) = [1, 3]$.
2) User selection: During the $m$-th round, the user whose characteristic direction is mostly aligned with $e_m$ is selected from the $\{k \in \mathcal{K}(m)\}$ cluster(s) as

$$j^+_\mu(m) = \arg \min_{\{k \in [1, M_k], k \in \mathcal{K}(m)\}} (\phi_{jk,m}) ;$$

where $\mu(m)$ represents the cluster-index of the selected user during the $m$-th round.
3) Update: After the $m$-th round selection, the users in the $\mu(m)$ cluster are informed to keep silent afterwards, i.e., setting $\mathcal{K}(m+1) = \mathcal{K}(m) \setminus \mu(m)$ and $m = m + 1$.
4) End control: If $m \leq III$, go to step 2; else, end.
After the selections, the indexes of the selected users are collected in $J_D = \{j_{\mu(1)}^+, j_{\mu(2)}^+, j_{\mu(3)}^+\}$. As shown in Fig. 3, the proposed RSS-based scheduling is able to generate the favorable patterns of characteristic directions, i.e., $\{r_{jk}\}_{k=1}^3$ are respectively aligned with distinct directions of $\{e_m\}_{m=1}^{\text{III}}$. Then the channels as well as the SSA-resultant signal space are shaped under the guidance of RSS.

Next, the detailed implementations of the 2) and 3) steps of the proposed distributed CS are further elaborated. It is assumed that all the users are synchronized to a common clock, such as the Global Positioning System (GPS) signal, and a timer that lasts proportionally to $\phi_{jk,m}$ is installed in $S_{jk}$. More specifically, during the $m$-th round, $R$ broadcasts a beacon signal with the index of the successful candidate of the $(m-1)$-th round, i.e., $j_{\mu(m-1)}^+$, and $j_{\mu(0)}^+ = \text{null}$. Upon receiving the beacon, each user first checks if $\mu(m-1)$ equals to its own cluster-index. If so, the user keeps silent for the rest of the scheduling period; if not, the user tries to compete during this round. The competing user $S_{jk}$ obtains the value of $m$ from its own counter, and uses its $m$-th angle $\phi_{jk,m}$ to trigger the timer for the response to the beacon. The first time-out user is selected in each round.

Remark 1: The distributed CS enjoys very low implementation complexity without explicit feedback from the users. Although a relatively large number of candidates are necessary to achieve a distinct performance gain, the proposed scheme still enjoys a good performance-complexity tradeoff. It will be shown later that the system overhead is fixed for the proposed distributed CS, which is independent to the number of candidate users. In contrast, the complexity and CSI overheads of the centralized CS increase very fast as the number of users increases.

2) RSS-based distributed GS: Similar to the aforementioned CS, the proposed distributed GS relies on the angular-coordinate calculated by each user with RSS and local CSI. It is noted that the GS always need a certain metric to evaluate the group performance, which requires the centralized decision. Still, the proposed scheme aims to distribute the computations to the users, so that the scheduling center $R$ can be designed as simple as possible. In particular, the RSS-based distributed GS employs a progressive feedback protocol, which consists of two phases. In the first phase, $S_{jk}'$ uses local CSI to check which direction of RSS is mostly aligned with its characteristic direction $r_{jk}'$, and feeds back the index of the most aligned RSS direction (using only two bits) to $R$ as

$$
m_{jk}' = \arg \min_{m \in \{1, II, III\}} (\phi_{jk,m}). \tag{16}$$
Then R collects the indexes of each group $J'$ in a set $M_{J'} = \{m_{j_1}', m_{j_2}', m_{j_3}'\}$, and check if the elements in $M_{J'}$ have distinctive values. This checking serves as a coarse judgment on the orthogonality of the SSA-resultant signal space $F$, and only when the elements in $M_{J'}$ are of distinctive values the related users continue to compete the channel. To offer some intuitions, Fig. 3 offers two user combinations, namely $\{S_{11}, S_{12}, S_{13}\}$ and $\{S_{21}, S_{22}, S_{23}\}$. The first combination can pass the coarse selection; but the second can not, since both $r_{21}$ and $r_{22}$ are more aligned with $e_1$. Let us collect the surviving groups in a set $J'' \subseteq J'$, in the second phase, R informs each surviving user to feed back the individual scheduling metric, which is the largest angle of its angular-coordinate within RSS, i.e.,

$$\phi_{j_k', \text{min}} = \min_{m \in \{\text{I, II, III}\}} (\phi_{j_k', m}) , \quad j_k' \in J' \in J''. \quad (17)$$

Then R synthesizes $\{\phi_{j_k', \text{min}}\}_{k=1}^{3}$ to generate the GS scheduling metric $\phi_{\text{sum}}^{(J')} = \sum_{j_k' \in J'} \phi_{j_k', \text{min}}$ of one surviving group $J' \in J''$, and the preferred user group is selected as

$$J_D' = \{j_1', j_2', j_3'\} = \arg \min_{J' \in J''} \left( \phi_{\text{sum}}^{(J')} \right). \quad (18)$$

Finally, it is note that if no surviving users exist after the first phase, random selection can be used to pick one group out of $J'$.

**Remark 2:** The progressive feedback is an opportunistic feedback scheme, which enables the distributed GS with low system overheads. Noting the symmetry of the channel’s statistic properties, it is easy to check that the characteristic directions have equal chances to align with every direction in $E$ of RSS, then the average surviving ratio of a candidate group can be calculated as $3!/3^3 = 2/9$ after the first round of feedback on the aligned direction within RSS (16). Therefore, only $2/9$ of the user groups need feed back the scheduling metrics (17) on average. In this sense, the proposed GS is suitable for the networks where the candidates are abundant and the low-complexity scheduling is demanded.

**Remark 3:** In fact, the proposed distributed CS/GS can only shape the SSA-result signal space $F$, which is not a straightforward optimization towards the post-processing SNR. Since $F$ is coupled with the three-party channels within a user group, any further descriptions on $F$ may require the three-party CSI. To this end, it seems that shaping $F$ is perhaps the best thing one can do with local CSI and RSS. It is noted that the statistical behaviors of $E$ is unknown with user scheduling in general [23], and also because of the reasons explained by Remark 4 below, we do not perform theoretical analysis on CS and GS with Min-UA transmission in this paper.
Remark 4: Similar to the distributed CS, the distributed GS with Min-UA transmission can only harvest partial MuD gain when the number of candidate groups is large; and the performance gap between the centralized and distributed scheduling schemes are distinct, which will be shown later in Section VI. These observations motivate us to look into the ER-UA case, where near-optimal distributed scheduling is possible as shown in the following sections.

IV. DISTRIBUTED SCHEDULING WITH ER-UA TRANSMISSION

In this section, the distributed user scheduling schemes are proposed for ER-UA transmission, where both relay and users are equally equipped with $N_T = 3$ antennas. As compared with the Min-UA transmission, one extra antenna is added at the user, which offers enough dimensions to achieve an active signal alignment with predefined direction. Specifically, by venturing an extra antenna to each user, the application of RSS can be extended to guide both distributed beamforming and user scheduling. Moreover, it is interesting to note that, unlike the scheduling with Min-UA transmission, the distributed user scheduling schemes achieve comparable performances as their centralized counterparts with ER-UA transmission. Again, the transmission scheme is first introduced before presenting the user scheduling schemes.

A. ER-UA MIMO-Y Transmission

Again, let us assume three users $\{S_1, S_2, S_3\}$ are randomly selected to exchange information. It is noted that the ER-UA MIMO-Y transmission relies on the RSS, which enables each user to design its transmit beamforming vectors with only local CSI and achieve SSA at the relay. Specifically, aiming at the pair-wise signal alignment in RSS $\Omega_R$ at $R$, the reference direction $e_m$ is allocated to guide the pair-wise transmit beamforming at $S_k$ and $S_l$, $m = \pi(l,k)$. Note that the RSS-guided transmit beamforming vectors $v_{l,k}$ and $v_{k,l}$ can be solved separately with local CSI as

$$v_{l,k} = \sqrt{P_T / 2} \langle H_k^{-1} e_m \rangle, \quad v_{k,l} = \sqrt{P_T / 2} \langle H_l^{-1} e_m \rangle,$$

(19)

and the power constraint is imposed as $\|v_{l,k}\|^2 = \|v_{k,l}\|^2 = P_T / 2$ per unicast stream. During the MAC phase, it is observed that SSA is achieved under the RSS, i.e., $\text{Span}(H_k v_{l,k}) = \text{Span}(H_l v_{k,l}) = \text{Span}(e_m)$ as shown in Fig. 4. the received signal at $R$ (cf. (1)) is given by

$$y_R = \sqrt{P_T / 2} \tilde{E} \tilde{d}_+ + n_R,$$

(20)
where $\tilde{d}_{+} := [\tilde{d}_{+,1} \tilde{d}_{+,2} \tilde{d}_{+,3}]^T$ is the vector of the superimposed signals and $E = [e_1 e_\Pi e_\III]'$ is the RSS as well as the equivalent MIMO channel seen by $R$. The $m$-th element of $\tilde{d}_{+}$ is $\tilde{d}_{+,m} = \tilde{d}_{l,k} + \tilde{d}_{k,l},$ where $\tilde{d}_{l,k} = \alpha_{m,k} d_{l,k}, m = \pi (l, k)$, and $\alpha_{m,k} = ||H_k^{-1}e_m||^{-1}$ is the equivalent channel coefficient for $d_{l,k}$ in the MAC phase. Aiming at a simpler implementation, the fixed-gain AF relay is used here. The relay processing matrix is then simplified as $\tilde{W} = I_{3 \times 3}$. During the BC phase, the relay transmits $s_R = G_R y_R$ with the long-term power controlling coefficient

$$G_R = \sqrt{P_R/E\left\{||y_R||^2\right\}} = \sqrt{2P_R/\left(P_T \alpha_{\text{sum}}^2 + 6\sigma_R^2\right)},$$

(21)

where $\alpha_{\text{sum}}^2 = \sum_{k=1}^{3} \sum_{m=1}^{3} \sum_{l \in \mathcal{L}_k} \alpha_{\pi(l(k),k)}^2$ and $\overline{\alpha}_{m,k}^2 = E\left\{||H_k^{-1}e_m||^{-2}\right\}$ is the channel-averaged equivalent channel gain for $d_{l,k}$. Note that $\alpha_{m,k}^2$ is only related to the long-term channel statistics, $G_R$ is thus treated as a constant and assumed to be known by all the nodes in this network. Then the self-interference-free signal $\tilde{y}_k = y_k - G_R H_k^H \tilde{W} k \tilde{y} L_k S_k$ at $S_k$ is expanded as

$$\tilde{y}_k = \sqrt{P_T/2G_R H_k^H E_k \tilde{d}_{+,k} + G_R H_k^H n_R + n_k},$$

(22)

where $E_k = [e_{m_1} e_{m_2} e_{m'}] \in \mathbb{C}^{3 \times 3}$, and $\tilde{d}_{+,k} = [\tilde{d}_{l_1,k} \tilde{d}_{l_2,k} \tilde{d}_{+,k'}]^T \in \mathbb{C}^{3 \times 1}$, $m_1 = \pi (l_1, k)$, $m_2 = \pi (l_2, k)$, $l_1, l_2 \in \mathcal{L}_k$ and $m' \notin \{I, \Pi, \III\} \setminus \{m_1, m_2\}$. Based on (22), the receive beamforming $U_k = G_R^{-1} [H_k^{-1}e_{m_1} H_k^{-1}e_{m_2}] \in \mathbb{C}^{3 \times 2}$ is used to obtain $\hat{y}_k = U_k^H y_k$, where the aligned interference regarding $\tilde{d}_{+,m'}$ is discarded, and each desired signal stream within $\hat{y}_k$ is given by

$$\hat{y}_{k,l} = \sqrt{P_T/2\alpha_{m,l} d_{k,l}} + n_{k,l}, l \in \mathcal{L}_k,$$

(23)

where $n_{k,l} = e_{m_1}^H n_R + \frac{1}{G_R} e_{m_1}^H (H_k^{-1})^H n_k$ is the noise item. Then it is easy to extract the useful information from $\hat{y}_{k,l}$.

Remark 5: It is noted that the Min-UA transmission scheme requires joint transmit/receive beamforming design with the three-party CSI at users and relay, which involves high CSI overheads and relatively complicated signal processing. In contrast to Min-UA, ER-UA transmission employs an extra user antenna to enable the simple RSS-based distributed transmit/receive beamforming design with local CSI. Therefore, the system overhead for CSI exchanging is significantly reduced. Another advantage of ER-UA transmission is that it enables the near-optimal and low-complexity RSS-based distributed user scheduling, which will be shown in the next subsection.
B. Problem Formulation and Centralized Scheduling

Similar to Section III-B, the user index set $J = \{j_1, j_2, j_3\}$ is reintroduced and the relevant overall outage probability is defined as $P_{\text{out}}^{(J)}(\rho_{th}) = \Pr\left(\rho_{\text{min}}^{(J)} \leq \rho_{th}\right)$ and the overall post-processing SNR is defined as $\rho_{\text{min}}^{(J)} = \min_{k \in [1,3], l \in L_k} (\rho_{j_k,j_l}^{(J)})$, where $\rho_{j_k,j_l}$ is the end-to-end post-processing SNR of link $S_{j_l} \rightarrow R \rightarrow S_{j_k}$, $l \in L_k$. From (23), $\rho_{j_k,j_l}$ is calculated as

$$\rho_{j_k,j_l} = \frac{1}{2} \frac{\rho_{j_k,j_l,1}^{(J)} \rho_{j_k,j_l,2}^{(J)}}{\rho_{j_k,j_l,1}^{(J)} + \rho_{j_k,j_l,2}^{(J)} + c},$$

where $\rho_{j_k,j_l,1} = \text{SNR}_T \alpha_{m,j_l}^2$ and $\rho_{j_k,j_l,2} = \text{SNR}_R \alpha_{m,j_k}^2$ are treated as the equivalent SNRs of the first hop and second hop, respectively, and $c = 3 (\text{SNR}_T + 1)$ is calculated with statistics of $\alpha_{\text{sum}}^2$. After introducing necessary notations, the centralized CS and GS are first considered as benchmarks, which are respectively given by

$$J_C = \{j_1^*, j_2^*, j_3^*\} = \arg \max_{J \in \mathcal{J}} \left(\rho_{\text{min}}^{(J)}\right),$$

and

$$J'_C = \{j_1^{**}, j_2^{**}, j_3^{**}\} = \arg \max_{J' \in J'} \left(\rho_{\text{min}}^{(J')}\right).$$

Again, it is noted that the CSI overheads and the computational complexities involved in the centralized CS and GS are high. To this end, a simplified scheduling is required.

\footnote{We may reuse some of the notations appeared in the previous sections to convey similar concepts, if not causing confusion.}
C. RSS-based Distributed User Scheduling with ER-UA

In this subsection, RSS-based distributed CS and GS are proposed with ER-UA transmission. From the previous subsection, it is observed that the end-to-end post-processing SNR of link (24) is an increasing function of the equivalent channel gains (ECGs) regarding the two hops. Intuitively, the considered distributed scheduling exploits the symmetry of the reciprocal bi-directional channel. Let us take the link $S_k \leftrightarrow R \leftrightarrow S_l$ for example, when the first-hop of $S_k \xrightarrow{1st} R \xrightarrow{2nd} S_l$ is optimized for the $k$-th cluster, the second-hop of $S_l \xrightarrow{1st} R \xrightarrow{2nd} S_k$ is anonymously optimized for the $l$-th cluster. In this way, the scheduling that takes the first-hop ECG as an optimization objective can benefit the overall transmission, as shown in Fig. 5. Moreover, the first-hop ECG can be calculated with local CSI and RSS; therefore, efficient distributed implementations of CS and GS are possible with ER-UA, which are detailed in the following subsections.

1) RSS-based Distributed CS: Employing RSS, each user can not only design its transmit beamforming as (19) to ensure SSA at the relay, but also calculate the scheduling metric, i.e., the minimum-ECG, to enable distributed CS. It is noted the RSS-based distributed CS is
independently conducted in each cluster, which aims to find the user with the maximal minimum-ECG from each cluster, as shown in Fig. 5. In particular, the minimum-ECG of $S_{jk}$ is defined as $\alpha_{jk}^2 = \min\left(\alpha_{m1,jk}^2, \alpha_{m2,jk}^2\right)$, $m_1 = \pi(l_1, k)$, $m_2 = \pi(l_2, k)$, which can be calculated with the local CSI $H_{jk}$ and RSS. Then, the efficient distributed scheduling is given by $J_D = \{j_1^+, j_2^+, j_3^+\}$, where the preferable user of the $k$-th cluster is selected according to the following criterion

$$j_k^+ = \arg \max_{j_k \in [1, M_k]} \left(\alpha_{jk}^2\right).$$ \hfill (27)

The distributed implementation of the proposed scheme is simple. Similar to distributed CS with Min-UA transmission, we assume all users are synchronized to a common clock. To start the scheduling, $R$ broadcasts a beacon and $S_{j_k}$ calculates $\alpha_{jk}^2$ with local CSI $H_{jk}$; then a timer that lasts inverse-proportionally to $\alpha_{jk}^2$ is used by $S_{j_k}$, and the first time-out user must be $S_{j_k}^+$. 

2) RSS-based Distributed GS: Similar to the Min-UA scenario, for the distributed GS with ER-UA transmission, each user exploits local CSI to calculate the individual scheduling metric and feeds it back to $R$ for final decision. The distributed GS is first given by

$$J_D' = \{j_1', j_2', j_3'\} = \arg \max_{j' \in J'} \left(\gamma_{\min}(J')\right),$$ \hfill (28)

where $\gamma_{\min}(J') = \frac{\text{SNR}_T(\alpha_{[3]}^{(J')}\gamma_{\min})^2 \text{SNR}_R(\alpha_{[2]}^{(J')}\gamma_{\min})^2}{\text{SNR}_R(\alpha_{[2]}^{(J')}\gamma_{\min})^2 + c}$ is the GS metric synthesized by $R$ and is defined as the equivalent SNR of the user group $J'$, and $\alpha_{[n]}^{(J')}$ is the $n$-th largest element of $\{\alpha_{j_1'}, \alpha_{j_2'}, \alpha_{j_3'}\}$. It is noted that user $S_{j_k}$ can calculate $\alpha_{jk}^2$ with local CSI, and the value is fed back to the relay $R$. With a total of $3M$ feedback of individual metrics, $R$ forms the set $\left\{\gamma_{\min}(J'_p)^M\right\}_{p \in [1, M]}$ and makes a centralized decision to choose the preferred user group.

Remark 6: The individual scheduling metric and the synthesized GS metric are critical. In the proposed distributed GS, $\alpha_{jk}^2$ characterizes the quality of the weaker link between $S_k$ and $R$, and $\gamma_{\min}(J')$ is actually constructed as a lower bound of $\rho_{\min}(J')$ to be shown later. Therefore, the distributed GS aims to improve the lower bound of the overall system performance.

Remark 7: Unlike the scheduling schemes with the Min-UA transmission, the proposed distributed CS and GS achieve comparable performances as their centralized counterparts with the ER-UA; therefore, the proposed distributed scheduling schemes enjoy very good performance-complexity tradeoffs with the ER-UA. In the next section, these observations are theoretically analyzed.
V. Performance Analysis with ER-UA Transmission

In this section, outage performances of the proposed distributed CS, GS and their centralized counterparts are quantified with ER-UA transmission. In order to facilitate the analysis, let us denote the min-SNRs (cf. (10)) of centralized CS (25) and GS (26) as $\rho_{\min,CS}^{(JC)}$ and $\rho_{\min,GS}^{(JC)}$; similarly, we denote the min-SNRs of distributed CS (27) and GS (28) as $\rho_{\min,CS}^{(JD)}$ and $\rho_{\min,GS}^{(JD)}$.

It is noted that the analysis with the distributed scheduling schemes are difficult, because the optimization objectives of these schemes are not exactly the min-SNRs of the network. To this end, tractable lower bound (LB) and upper bound (UB) are first established for 1) $\rho_{\min,CS}^{(JC)}$ and $\rho_{\min,CS}^{(JD)}$ with CS, then for 2) $\rho_{\min,GS}^{(JC)}$ and $\rho_{\min,GS}^{(JD)}$ with GS, respectively. Next, the bounds of outage probabilities are developed with tractable theoretical results, and their high SNR approximations are analyzed to extract the DMT and MuD orders. Without loss of generality, $\text{SNR}_R = \text{SNR}_T = \text{SNR}$ and $M_1 \leq M_2 \leq M_3$ are assumed to ease the derivations.

A. Bounding the outage probabilities

To begin with, the following proposition is introduced to bound the min-SNRs with CS.

**Proposition 1.** Using ER-UA transmission, the min-SNRs of centralized and distributed CS are bounded as

$$\rho_{\min,CS} \overset{(a1)}{\leq} \rho_{\min,CS}^{(JD)} \overset{(a2)}{\leq} \rho_{\min,CS}^{(JC)} \overset{(a3)}{\leq} \rho_{\min,CS},$$

where the LB and UB are $\rho_{\min,CS} = g \left( \tilde{\lambda}_{J_1}^{[3]}, \tilde{\lambda}_{J_2}^{[2]} \right)$ and $\rho_{\min,CS} = g \left( \alpha_2 I_{j_1^*}, \alpha_2 I_{j_2^*} \right)$, respectively, and $g \left( x, y \right) = \frac{x y \text{SNR}^2}{2 y + 6 (1 + \text{SNR}^{-1})}$, $x, y > 0$. For $\rho_{\min,CS}$, $\tilde{\lambda}_{J_1}^{[n]}$ is the $n$-th largest element of $\{ \tilde{\lambda}_{j_1^*}, \tilde{\lambda}_{j_2^*}, \tilde{\lambda}_{j_3^*} \}$, where $J = \{ j_1^*, j_2^*, j_3^* \}$, $j_k^* = \arg \max_{j_k \in [1, M_k]} \left( \tilde{\lambda}_{j_k} \right)$ and $\tilde{\lambda}_{j_k} = \lambda_{\min} \left( H_{j_k} H_{j_k}^H \right)$, $j_k \in [1, M_k]$. For $\rho_{\min,CS}$, $j_k^* = \arg \max_{j_k \in [1, M_k]} \left( \alpha_{I_{j_k^*}} \right)$, $k = 1, 2$.

**Proof:** See Appendix I.

Similarly, the following proposition is used to bound the min-SNRs with GS.

**Proposition 2.** Using ER-UA transmission, the min-SNRs of centralized GS and distributed GS are bounded as

$$\rho_{\min,GS} \overset{(b1)}{\leq} \rho_{\min,GS}^{(JD)} \overset{(b2)}{\leq} \rho_{\min,GS}^{(JC)} \overset{(b3)}{\leq} \rho_{\min,GS},$$

where the LB and UB are $\rho_{\min,GS} = \max_{J' \in J'} \left\{ g \left( \tilde{\lambda}_{J'}^{[3]}, \tilde{\lambda}_{J'}^{[2]} \right) \right\}$ and $\rho_{\min,GS} = g \left( \alpha_2 I_{j_1^*}, \alpha_2 I_{j_2^*} \right)$,
respectively. For $\rho_{\text{min,GS}}$, $\tilde{\lambda}(j')$ is the $n$-th largest element of $\{\tilde{\lambda}_{j_1}, \tilde{\lambda}_{j_2}, \tilde{\lambda}_{j_3}\}$, where $J' = \{j'_1, j'_2, j'_3\} \subseteq J$, $\tilde{\lambda}_{j_k} = \min_{j_k \in \mathbb{J}_k} \left( H_{j_k}^H H_{j_k} \right)$. For $\bar{P}_{\text{min,GS}}$, $\hat{j}_k = \arg \max_{j_k \in [1, M]} \left( \sigma_{2, j_k}^2 \right)$, $k = 1, 2$.

Proof: See Appendix II.

From Proposition 1 and Proposition 2, the common LB and UB for $P_{\text{out,CS}}^{(JC)}$ and $P_{\text{out,CS}}^{(JD)}$ with CS are defined as

$$P_{\text{out,CS}}^{\text{LB}} \leq P_{\text{out,CS}}^{(JC)} \leq P_{\text{out,CS}}^{(JD)} \leq P_{\text{out,CS}}^{\text{UB}},$$

where $P_{\text{out,CS}}^{\text{LB}}(\rho_{th}) = \Pr \left( \bar{P}_{\text{min,CS}} \leq \rho_{th} \right)$, $P_{\text{out,CS}}^{\text{UB}}(\rho_{th}) = \Pr \left( \rho_{\text{min,CS}} \leq \rho_{th} \right)$. And similarly, the common LB and UB for $P_{\text{out,GS}}^{(JC)}$ and $P_{\text{out,GS}}^{(JD)}$ with GS are given by

$$P_{\text{out,GS}}^{\text{LB}} \leq P_{\text{out,GS}}^{(JC)} \leq P_{\text{out,GS}}^{(JD)} \leq P_{\text{out,GS}}^{\text{UB}},$$

where $P_{\text{out,GS}}^{\text{LB}}(\rho_{th}) = \Pr \left( \bar{P}_{\text{min,GS}} \leq \rho_{th} \right)$, $P_{\text{out,GS}}^{\text{UB}}(\rho_{th}) = \Pr \left( \rho_{\text{min,GS}} \leq \rho_{th} \right)$. Then, these bounds are further quantified. Particularly, an ordered set $\mathcal{M}_\pi = \{\mathcal{M}_i\}_{i=1}^{6}$ is introduced to collect all the permutations of the three elements in $\mathcal{M} = \{M_1, M_2, M_3\}$, where $\mathcal{M}_i$ is also an ordered set and the $n$-th element of $\mathcal{M}_i$ is denoted as $M_i, n \in [1, 3]$. The detailed derivations are collected in Appendix III and the key results of $P_{\text{out,CS}}^{(JC)}(\rho_{th})$, $P_{\text{out,GS}}^{(JC)}(\rho_{th})$ and $P_{\text{out,CS}}^{\text{UB}}(\rho_{th})$ are given by

$$P_{\text{out,CS(GS)}}^{\text{LB}}(\rho_{th}) = 2M_2 \sum_{q=0}^{M_2-1} \left[ \left( \frac{K_1(2\sqrt{p(q+1)b})}{q+1} \right)^{2} \right],$$

and

$$P_{\text{out,CS(GS)}}^{\text{UB}}(\rho_{th}) \approx \sum_{i=1}^{6} \left( M_{i,2} M_{i,3} \left( 1 - e^{-3\mu} \right) + M_{i,2} M_{i,3} \left( 1 - e^{-3\mu} \right) \right) \left[ 1 - \left( 1 - e^{-3\mu} \right) \right],$$

respectively, where $K_1(x)$ is the modified Bessel function of the second kind [26], $a(\rho_{th}) = \frac{2\rho_{th}}{\text{SNR}}$, $b(\rho_{th}) = \frac{6\rho_{th}(\text{SNR}+1)}{\text{SNR}^2}$ and $\mu = \frac{1}{2} (a + \sqrt{a^2 + 4b})$ is the positive root of the quadratic equation $y^2 = ay + b$, $M = M_1 + M_2 + M_3$. It is noted that the integral $\int_{\mu}^{\infty} e^{-3(\mu^3-y)dy}$ can be efficiently evaluated with software like MATLAB or Mathematica. Finally, it is noted that $P_{\text{out,CS}}^{\text{LB}}(\rho_{th})$ can be obtained from (33) by setting $M_1 = M_2 = M$, therefore, the equation of (33) is reused for conciseness.
B. High SNR Analysis

In this subsection, the high SNR analysis on the bounds of outage probabilities are given, including the asymptotic approximations and the DMT. Only the key results are provided here while all the derivations and proofs are collected in Appendix IV and Appendix V.

1) High SNR Approximations of $P_{out,CS}^{LB}$ and $P_{out,CS}^{UB}$:

a) $P_{out,CS}^{LB}$: The high SNR approximation of $P_{out,CS}^{LB} (\rho_{th})$ is given as follows

$$
\begin{align*}
& G_{CS,1}^{LB} \left( \frac{2\rho_{th}}{SNR} \right) \frac{d_{CS}^{LB}}{SNR} \ln \left( \frac{SNR}{2\rho_{th}} \right), \quad M_2 = M_3, \\
& G_{CS,2}^{LB} \left( \frac{2\rho_{th}}{SNR} \right) \frac{d_{CS}^{LB}}{SNR}, \quad M_2 \neq M_3,
\end{align*}
$$

where the diversity UB is $d_{CS}^{UB} = M_3$, and the power gains are $G_{CS,1}^{LB} = M_3^3M_3^3$, $G_{CS,2}^{LB} = \varphi (M_3, M_2, 1)$. Here $M_{[n]}$ is the $n$-th largest element of $\{M_1, M_2, M_3\}$, and $\varphi (N_1, N_2, \tau)$ is given by

$$
\varphi (N_1, N_2, \tau) = N_2 \sum_{q=0}^{N_2-1} \sum_{p=1}^{N_1} \frac{(-1)^{q+p} C_{q}^{N_2-1} C_{p}^{N_1} (e_{p,q} (\tau) e_{p,q} (\tau))}{(q + 1)},
$$

where $e_{p,q} (\tau) = [c_{p,d'}, \tau), c_{p,d'\cdot 1}, \tau), \ldots, \tau)]^T \in \mathbb{C}^{(d'+1) \times 1}$ with element $e_{p,n} (\tau) = \frac{(-\tau p)^n}{n!}$, $n \in [0, d']$, $d' = \min (N_1, N_2)$, and $e_{p,q} (\tau) = [0, e_{p,q,1} (\tau), \ldots, e_{p,q,d'} (\tau)]^T \in \mathbb{C}^{(d'+1) \times 1}$ with element

$$
e_{p,q,n} (\tau) = \left( b_{1,n-1} \ln \left( 2\tau \sqrt{3p (q + 1)} \right) + b_{2,n-1} \right) \\
\times (12\tau^2 p (q + 1))^n, \quad n \in [1, d'],
$$

where the coefficients in $e_{p,q,n} (\tau)$ are $b_{1,t} = \frac{\sqrt{2}t^{t-1}}{t(t+1)^{t-1}}$, $b_{2,t} = b_{1,t} \left[ -\ln (2) - \frac{t}{2} (\psi (t + 1) + \psi (t + 2)) \right]$, $t = n - 1$, and $\psi (x)$ is the digamma function [26].

b) $P_{out,CS}^{UB}$: Then the high SNR approximation for $P_{out,CS}^{UB} (\rho_{th})$ is given as

$$
\begin{align*}
& G_{CS,1}^{UB} \left( \frac{2\rho_{th}}{SNR} \right) \frac{d_{CS}^{UB}}{SNR} \ln \left( \frac{SNR}{2\rho_{th}} \right), \quad M_1 = M_2 = M_3, \\
& G_{CS,2}^{UB} \left( \frac{2\rho_{th}}{SNR} \right) \frac{d_{CS}^{UB}}{SNR}, \quad M_1 \geq M_2 = M_3, \\
& G_{CS,3}^{UB} \left( \frac{2\rho_{th}}{SNR} \right) \frac{d_{CS}^{UB}}{SNR}, \quad M_1 = M_2 \geq M_3, \\
& G_{CS,4}^{UB} \left( \frac{2\rho_{th}}{SNR} \right) \frac{d_{CS}^{UB}}{SNR}, \quad M_1 \neq M_2 \neq M_3,
\end{align*}
$$

where the diversity LB is $d_{CS}^{LB} = M_3$, and the power gains are $G_{CS,1}^{UB} = 6M_3^3M_3^3$, $G_{CS,2}^{UB} = 2M_3^3M_3^3$, $G_{CS,3}^{UB} = \varphi (M_2, M_3, 3) + \varphi (M_3, M_2, 3)$, and $G_{CS,4}^{UB} = \sum_{i=1}^{2} \varphi (M_1, M_3, 3) + \varphi (M_3, M_1, 3)$.

2) High SNR Approximations of $P_{out,GS}^{LB}$ and $P_{out,GS}^{UB}$:
a) $P_{\text{out,GS}}^{LB}$: As shown in (33), $P_{\text{out,GS}}^{LB}(\rho_{\text{th}}) = P_{\text{out,CS}}^{LB}(\rho_{\text{th}})$, when $M_{i\in[1,3]} = M$, then the high SNR approximation regarding $P_{\text{out,CS}}^{LB}(\rho_{\text{th}})$ in (36) can be modified to obtain

$$P_{\text{out,GS}}^{LB}(\rho_{\text{th}}) \approx G_{GS}^{LB} \left( \frac{2\rho_{\text{th}}}{\text{SNR}} \right)^{d_{GS}^{LB}} \ln \left( \frac{\text{SNR}}{2\rho_{\text{th}}} \right),$$

(38)

where $d_{GS}^{UB} = M$ and $G_{GS}^{LB} = M3^{M}$.

b) $P_{\text{out,GS}}^{LB}$: The derivations of $P_{\text{out,GS}}^{UB}(\rho_{\text{th}})$ are given in Appendix IV, and the result is given by

$$P_{\text{out,GS}}^{UB}(\rho_{\text{th}}) \approx G_{GS}^{UB} \left( \frac{2\rho_{\text{th}}}{\text{SNR}} \ln \left( \frac{\text{SNR}}{2\rho_{\text{th}}} \right) \right)^{d_{GS}^{UB}},$$

(39)

where $d_{GS}^{UB} = M$ and $G_{GS}^{UB} = (6 \times 3^{3})^{M}$.

3) DMT Analysis: In this subsection, the results of high SNR approximations are further generalized and unified within the DMT framework [24]. The DMT analysis gives a comprehensive description of the tradeoff between transmission reliability and spectral efficiency with adaptive data-rate. For the whole system with adaptive data-rate, a target sum-rate is defined as $R_{t}(\text{SNR}) = r \log_{2}(1 + \text{SNR})$ and $r$ is the multiplexing gain. Then, the outage probability of the system is redefined with $R_{th}$ as

$$P_{\text{out}}(\text{SNR}) = \Pr \left( R (\text{SNR}) \leq R_{t} (\text{SNR}) \right),$$

(40)

where $R (\text{SNR}) = \frac{1}{2} \sum_{k\in[1,3]} \sum_{l \in \mathcal{L}_{k}} \log_{2} (1 + \rho_{k,l}(\text{SNR}))$ is the instantaneous sum-rate of the system, and $\rho_{k,l}$ is the end-to-end post-processing SNR of link $S_{l} \rightarrow R \rightarrow S_{k}$. Finally, the DMT is defined as

$$d(r) = -\lim_{\text{SNR} \to \infty} \frac{\log (P_{\text{out}}(\text{SNR}))}{\log (\text{SNR})}.$$

(41)

After these definitions, a proposition is given to summarize the DMT results.

**Proposition 3.** Using ER-UA transmission, both centralized CS and distributed CS achieve the same DMT as

$$d_{CS}(r) = \min (M_{1}, M_{2}, M_{3}) (1 - r/3)^{+},$$

(42)

and both centralized GS and distributed GS achieve the same DMT as

$$d_{GS}(r) = M (1 - r/3)^{+}.$$

(43)

**Proof:** See Appendix V.
Remark 8: The maximum MuD orders are obtained as $d_{CS}^* = \min(M_1, M_2, M_3)$ for both distributed and centralized CS, and $d_{GS}^* = M$ for both distributed and centralized GS. If the random scheduling is employed, the maximum MuD order would be just 1. Therefore, the proposed schemes obtain scalable MuD orders. Moreover, by showing that both the outage-optimal centralized scheduling and the proposed distributed scheduling achieve the same DMT, the optimality of the proposed distributed scheduling schemes are established with ER-UA transmission. Finally, given a total number of candidates, it is shown that the symmetric user configuration is most efficient from the DMT’s perspective, which equally distributes the candidates in three clusters.

VI. NUMERICAL RESULTS

In this section, numerical results are presented to show the effectiveness of the proposed schemes and validate the theoretical derivations. The i.i.d. Rayleigh fading channels are assumed. The overall outage probability is used as an effective metric to evaluate the transmission reliability of the network. Specifically, the SNR threshold is set as $\rho_{th} = 1$ for each unicast stream, which corresponds to a target rate of $0.5 \times 6 \times \log_2 (1 + \rho_{th}) = 3$ bit per channel use of the MIMO-Y channel. The symmetric SNR is assumed for the relay system as $P_T/\sigma_R^2 = P_R/\sigma_S^2 = \text{SNR}$. The triplet $(M_1, M_2, M_3)$ is used to represent the number of users in all the three clusters, and it is simplified as $(M)$ for the GS.

A. Performance and Complexity Comparisons

In this test case, both the distributed and the centralized scheduling schemes are compared for the Min-UA and the ER-UA MIMO-Y transmissions. The symmetric user configuration, i.e., $(M, M, M)$, is assumed. The random selection is used as a reference, which achieves a MuD order of 1 regardless the user configuration. Therefore, the random selection is also indicated by user configuration (1,1,1) for CS or simplified as (1) for GS.

1) Min-UA Transmission: Focusing on the Min-UA transmission, Fig. 6 and Fig. 7 present the overall outage performances of the CS and the GS, respectively. It is observed that the proposed distributed scheduling schemes are inferior to the centralized schemes. Only when $M$ is relatively large, the distributed scheduling shows distinctive performance improvement as compared to the random selection.
Fig. 6. Overall outage probability of the centralized and distributed CS with Min-UA transmission, where $N_R = 3$, $N_T = 2$ and $\rho_{th} = 1$.

Fig. 7. Overall outage probability of the centralized and distributed GS with Min-UA transmission, where $N_R = 3$, $N_T = 2$ and $\rho_{th} = 1$.

2) **ER-UA Transmission:** Fig. 8 and Fig. 9 present the overall outage performances of CS and GS with ER-UA transmission. As shown in these figures, the proposed distributed user scheduling schemes and their centralized counterparts achieve comparable performances. Moreover, a scalable MuD order is observed for both the centralized scheduling and the proposed distributed scheduling. These observations prove the optimality of the distributed scheduling in terms of MuD order.
Fig. 8. Overall outage probability of the centralized and distributed CS with ER-UA transmission, where $N_R = N_T = 3$ and $\rho_{th} = 1$.

Fig. 9. Overall outage probability of the centralized and distributed GS with ER-UA transmission, where $N_R = N_T = 3$ and $\rho_{th} = 1$.

3) Complexity and CSI Overhead: To further appreciate the proposed distributed scheduling, the computational complexities and the CSI overheads of all the considered scheduling schemes are compared. In particular, the complexity is measured in terms of the number of floating point operations (flops), where the basic calculations follow the results from [16], [27], [28]. As shown in Table I, the centralized scheduling schemes require global CSI of all the candidates, which involves high CSI feedback overheads. The computational complexity at the scheduling center is also relatively high, as shown in Table II. In contrast to the centralized scheduling,
the proposed distributed methods allow each user to calculate its own scheduling metric. For GS, such metric is explicitly fed back; for CS, only the orders of these metrics are relevant and no explicitly feedback is necessary. Therefore, the distributed schemes significantly reduces the computational complexity at the relay. It is also noted that Min-UA and ER-UA transmission schemes show different performance-complexity tradeoffs. For the Min-UA transmission, the low implementation complexity is achieved at the cost of insufficiently utilized MuD gain; only when the number of users is large, the distributed scheduling shows distinctive performance improvement. On the other hand, the ER-UA transmission allows the near-optimal distributed scheduling; therefore, it is sufficient to apply the distributed scheduling to effectively harvest the full MuD order.

B. Validating Theoretical Derivations for ER-UA Transmission

Fig. 10 and Fig. 11 validate the theoretical derivations of outage probability bounds and the corresponding high SNR approximations for CS and GS respectively. It is shown that the derived
Fig. 10. Overall outage probabilities of the centralized/distributed CS and their bounds with high SNR approximations, where the user configuration is \((2, 3, 4)\) and SNR threshold is \(\rho_{th} = 1\).

Fig. 11. Overall outage probabilities of the centralized/distributed GS and their bounds with high SNR approximations, where the user group configuration is \(M = 3\) and the SNR threshold is \(\rho_{th} = 1\).

bounds in Proposition 1, 2, and the relations in (31) and (32) are correct. It is also noted that the developed bounds are loose due to several approximations, e.g., using the minimum eigenvalue to obtain the upper bounds and relaxing the interval of integration for more tractable results, and etc. Fortunately, these bounds are still useful and correct, because they enable the tractable and explicit MuD order in Proposition 3. As verified in the figures, both asymptotic results regarding the UB and LB show the same diversity order, accurately bounding the MuD orders of both the centralized and distributed scheduling schemes with CS and GS, respectively.
VII. CONCLUSION

The distributed cluster-wise scheduling (CS) and group-wise scheduling (GS) have been studied for the MIMO-Y channel with two transmission schemes which have different requirements on the minimum number of user antennas. The RSS has been employed to guide the distributed CS and GS with the Min-UA transmission; and these low-complexity distributed scheduling schemes obtain notable MuD gains when the candidates are abundant. With a simpler yet effective implementation, the RSS-based ER-UA MIMO-Y transmission has been proposed, and the corresponding distributed CS and GS are theoretically proved to achieve the comparable performances as their centralized counterparts. By comparing a variety of scheduling schemes with Min-UA and ER-UA transmissions, the performance-complexity tradeoffs of user scheduling has been revealed for the MIMO-Y channel. Moreover, DMT analysis with ER-UA transmission shows that the achievable MuD gain is limited by the minimum number of users in the three clusters, which sheds light into the fundamental behavior of MuD in the MIMO-Y channel. Extending the distributed scheduling to the more general MWRCs is a promising future work, while the analysis for the explicit MuD behaviors with the Min-UA transmission is still open.

APPENDIX

APPENDIX I PROOF OF PROPOSITION 1

Proof: For inequality (a1), recall the definition of the equivalent channel coefficient as 

\[ \alpha_{m,jk} = \left\| H^{-1}_{jk} e_m \right\|^{-1} \] where \( \| e_m \| = 1 \), then \( \lambda_{\min} \left( H_{jk} H_{jk}^H \right) \leq \alpha_{m,jk}^2, \forall m \in \{ \pi (l, k) : l \in \mathcal{L}_k \} \) holds \[29\]. Therefore, \( \lambda_{\min} \left( H_{jk} H_{jk}^H \right) \leq \min_{m} \{ \alpha_{m,jk}^2 \} := \alpha_{jk}^2 \). By using the monotonicity of \( g(x,y) \), it is shown that

\[
g \left( \tilde{\lambda}^{(J_\alpha)}_{[3]}, \tilde{\lambda}^{(J_\alpha)}_{[2]} \right) = \min_{k \in [1,3]} \min_{l \in \mathcal{L}_k} \left( g \left( \tilde{\lambda}_{jk}, \tilde{\lambda}_{jk}^c \right) \right) \leq \min_{k \in [1,3]} \min_{l \in \mathcal{L}_k} \left( g \left( \alpha_{jk}^2, \alpha_{jk}^2 \right) \right) \leq \min_{k \in [1,3]} \min_{l \in \mathcal{L}_k} \left( g \left( \alpha_{jk}^2, \alpha_{jk}^2 \right) \right) = \rho_{\min,CS}^{(J_D)} \]

where the last inequality is based on the fact \( \alpha_{jk}^2 \leq \alpha_{jk}^2 \), i.e., \( j_k^1 \) is defined to be optimal for \( \alpha_{jk}^2 \) (cf. (27)). The inequality (a2) is straightforward by noting that \( J_C \) is optimal for \( \rho_{\min}^{(J_C)} \). For inequality (a3), it is observed that

\[
\rho_{\min}^{(J_C)} \leq \rho_{j_1^1,j_1^2} = g \left( \alpha_{j_1^1,j_1^2}, \alpha_{j_1^1,j_2} \right) \leq g \left( \alpha_{j_1^1,j_1^2}, \alpha_{j_1^2,j_2} \right). \]
where \( \{j_1^1, j_2^1\} \subset J_C \) and the last inequality is based on the monotonicity of \( g(x, y) \) and the fact \( \alpha_{2,j_k}^2 \leq \alpha_{1,j_k}^2 \), i.e., \( j_k^1 \) is defined to be optimal for \( \alpha_{2,j_k}^2 \), then the proof is finished.

### Appendix II Proof of Proposition 2

**Proof:** Starting from the inequality (b1), it is noted that

\[
\rho_{j_k,j_k} = g(\alpha_{m,j_k}^2, \alpha_{m,j_k}^2) \\
\quad \geq \frac{\alpha_{j_k}^2 \alpha_{j_k}^2 \text{SNR}}{2 \max \{\alpha_{j_k}^2, \alpha_{j_k}^2\} + 6 (1 + \text{SNR}^{-1})} \\
\quad \geq g \left( \left( \frac{\alpha_{[3]}^2}{\alpha_{[2]}^2} \right)^2, \left( \frac{\alpha_{[2]}^2}{\alpha_{[3]}^2} \right)^2 \right) = \gamma_{\min}^{(J')} \\
\quad \geq g \left( \frac{\tilde{\lambda}_{(j')}^2}{\alpha_{[3]}^2}, \frac{\tilde{\lambda}_{(j')}^2}{\alpha_{[2]}^2} \right) : = \zeta_{\min}^{(J')},
\]

where \( \left( \frac{\alpha_{[n]}^2}{\alpha_{[m]}^2} \right)^2 \) is the \( n \)-th largest element of \( \left\{ \alpha_{j_1^1,j_1^1}^2, \alpha_{j_2^1,j_2^1}^2, \alpha_{j_3^1,j_3^1}^2 \right\} \), \( J' = \{j_1^1, j_2^1, j_3^1\} \) and \( \alpha_{j_k}^2 = \min_m \left\{ \alpha_{m,j_k}^2 \right\} \). In (46), (c1) is obtained according to the definition of \( \alpha_{j_k}^2 \) and the monotonicity of \( g(x, y) \); (c2) is constructed by comparing all elements in \( \left\{ g \left( \frac{\alpha_{j_k}^2}{\alpha_{j_k}^2} \right), k \in [1,3], l \in L_k \right\} \); (c3) is based on the fact that \( \alpha_{j_k}^2 \geq \tilde{\lambda}_{(j')}^2 \) and the monotonicity of \( g(x, y) \), where \( \tilde{\lambda}_{(j')}^2 \) is the \( n \)-th largest element of \( \left\{ \tilde{\lambda}_{j_1^1}, \tilde{\lambda}_{j_2^1}, \tilde{\lambda}_{j_3^1} \right\} \). Based on (46), it is easy to show that

\[
\rho_{\min}^{(J')} \geq \gamma_{\min}^{(J')} \geq \zeta_{\min}^{(J')}, \quad \forall J' \in \mathcal{J}',
\]

then we proceed to obtain the following inequalities

\[
\rho_{\min}^{(J')} \geq \gamma_{\min}^{(J')} \geq \zeta_{\min}^{(J')}, \quad \forall J' \in \mathcal{J}',
\]

where \( J_c \) is defined as \( J_c = \arg \max_{J' \in \mathcal{J}'} \left\{ \zeta_{\min}^{(J')} \right\} \). Here, (c4) and (c6) are based on the basic relations in (47), and (c5) is based on the optimality of \( J_c^{(J')} \) for \( \left\{ \gamma_{\min}^{(J')} \right\} \). It is noted that the inequalities (c1)-(c6) have been used to prove the inequality (b1) within (30). Then, the inequality (b2) is based on the fact that \( J_{c}^{(J')} \) is optimal for \( \rho_{\min}^{(J')} \), where \( J' \in \mathcal{J}' \). For the inequality (b3), note that \( \rho_{\min}^{(J_c)} \leq \rho_{\min}^{(J_c)} \) due to the fact \( \mathcal{J}' \subset \mathcal{J} \), then by modifying the upper bound in Proposition 1, the proof is finished.

### Appendix III Derivations for \( P_{out,GS}^{LB}, P_{out,CS}^{UB} \) and \( P_{out,GS}^{UB} \)

As the preliminary, the relevant distributions of the key random variables (RVs) are given. First, according to Lemma 1 in Appendix VI, it is known that \( \alpha_{m,j_k}^2 \) is exponentially distributed with
the probability density function (PDF) as \( f_{\alpha m, j}^2 (x) = e^{-x} \), therefore, the order statistic \([30]\) is applied to these i.i.d. RVs, and we obtain the PDF of \( \alpha^2 m, j^2 \) as \( f_{\alpha^2 m, j}^2 (x) = M_k (1 - e^{-x})^{M_k - 1} e^{-x} \).

According to \([31]\), \( \lambda_{j_k} = \lambda_{\min} (H_j H_j^H) \) is also exponentially distributed as \( f_{\alpha m, j}^2 (x) = e^{-3x} \), therefore, the PDF of \( \lambda_{j_k} \) is given by \( f_{\lambda_{j_k}} (x) = M_k (1 - e^{-3x})^{M_k - 1} e^{-3x} \). Then, we continue the derivations for \( P_{LB, out, CS(GS)} \), \( P_{UB, out, CS} \) and \( P_{UB, out, GS} \).

\[ P_{LB, out, CS(GS)} : \] Let us first define \( X = \alpha^2 j_i^1 \) and \( Y = \alpha^2 j_i^2 \), then we can express the LB of outage probability as \( P_{LB, out, CS} (\rho_{th}) = \int_0^\infty f_Y (y) \left( \int_0^{\alpha^2 y} f_X (x) \, dx \right) \, dy \), where \( a (\rho_{th}) = \frac{2\rho_{th}}{\text{SNR}} \) and \( b (\rho_{th}) = \frac{6 \rho_{th} (\text{SNR} + 1)}{\text{SNR}^2} \). After some straightforward calculations, \( P_{LB, out, CS} (\rho_{th}) \) is given in (33).

\[ P_{UB, out, CS} : \] With a slight abuse of notations, let us define \( X = \lambda_{j_i^1}, Y = \lambda_{j_i^2} \) and \( Z = \lambda_{j_i^3} \), and introduce the ordered set \( \Theta = \{ \Theta_i \}_{i=1}^6 \) to collect all the permutations among \( \{ X, Y, Z \} \), where \( \Theta_i = \{ \theta_{i,n} \}_{n=1}^3 \), e.g., \( \Theta_1 = \{ \theta_{1,1}, \theta_{1,2}, \theta_{1,3} \} = \{ X, Y, Z \} \), \( \Theta_2 = \{ \theta_{2,1}, \theta_{2,2}, \theta_{2,3} \} = \{ X, Z, Y \} \) and so on. We also associate \( \Theta_i \) with an event as \( O_i \), which orders the elements of \( \Theta_i \), e.g., \( \Theta_1 = \{ X, Y, Z \} \leftrightarrow O_1 = \{ X \geq Y \geq Z \} \). Then we divide the whole integral region into six subregions to facilitate the calculation, where each subregion is associated with \( O_i, \, i \in [1, 6] \).

Based on such division, \( P_{UB, out, CS} (\rho_{th}) \) is re-expressed as

\[ P_{UB, out} (\rho_{th}) = \sum_{i=1}^6 \Pr \{ E_i (\rho_{th}) \}, \tag{49} \]

where \( E_i (\rho_{th}) = \{ g (\theta_{i,[3]} \theta_{i,[2]} \leq \rho_{th}, O_i \} \), \( \theta_{i,[n]} \) is the \( n \)-th largest element of \( \Theta_i \). Without loss of generality, let us focus on the probability of \( E_1 (\rho_{th}) \), given by \( \Pr \{ E_1 (\rho_{th}) \} = \sum_{i=1}^3 \int \int D_{1,t} f (x, y, z) \, dx \, dy \, dz = \sum_{i=1}^3 I_{1,t} \), where \( f (x, y, z) = f_X (x) f_Y (y) f_Z (z) \) due to independence, and the integral region of \( I_{1,t} \) is \( D_{1,t} = \{(x, y, z) | x \in X_{1,t}, y \in Y_{1,t}, z \in Z_{1,t} \} \), i.e., \( \int_{X_{1,t}} \int_{Y_{1,t}} \int_{Z_{1,t}} \). Then \( I_{1,1} = \int_0^\mu \int_0^\mu \int_0^\mu f (x, y, z) \, dx \, dy \, dz \) and \( I_{1,2} = \int_0^\mu \int_0^\mu f (x, y, z) \, dx \, dy \, dz \) are obtained after some calculations as

\[ I_{1,1} = \frac{M_2 M_3 (1 - e^{-3\mu})^{M_3}}{(M_1 + M_2) M_3}, \tag{50} \]

\[ I_{1,2} = \frac{M_2 (1 - e^{-3\mu})^{M_1 + M_2} \left[ 1 - (1 - e^{-3\mu})^{M_3} \right]}{M_1 + M_2}, \tag{51} \]

where \( \mu = \frac{1}{2} (a + \sqrt{a^2 + 4b}) \) is the positive root of the quadratic equation \( y^2 = ay + b \), \( a (\rho_{th}) = \frac{2\rho_{th}}{\text{SNR}} \), \( b (\rho_{th}) = \frac{6 \rho_{th} (\text{SNR} + 1)}{\text{SNR}^2} \) and \( M_\Sigma = M_1 + M_2 + M_3 \). It is noted that \( I_{1,3} = \int_0^\mu \int_0^\mu \int_0^\mu f (x, y, z) \, dx \, dy \, dz \) is not easy to calculate with tractable results, thus the following
approximations is used as $I_{1,3} < I_{1,3}' = \int_0^\infty \int_0^\infty \int_0^{a+b+y} f(x, y, z) \, dx \, dy \, dz$, where the integral region regarding variable $y$ has been enlarged. After some calculations, the result of $I_{1,3}'$ is given by

$$I_{1,3}' = \left[ 3M_2 \sum_{p=0}^{M_2-1} \sum_{q=0}^{M_1} (-1)^q C_p^{M_2-1} C_q^{M_1} e^{-3pa} \int_{\mu}^{\infty} e^{-3((p+1)y-qb+\frac{y}{2})} \, dy \right] \left[ 1 - (1 - e^{-3\mu})^{M_3} \right],$$

(52)

then $\Pr \{ E_1 (\rho_{th}) \} \approx I_{1,1} + I_{1,2} + I_{1,3}'$ is obtained. It is also noted that the permutation of $\{X, Y, Z\}$ and the user configuration $\{M_1, M_3, M_3\}$ have a fixed matching pattern in $E_i (\rho_{th})$; therefore, we can modify the results of $\Pr \{ E_1 (\rho_{th}) \}$ in (50), (51) and (52) to get $\Pr \{ E_i (\rho_{th}) \}$, $i \in [2, 6]$, and finally arrive at the result in (34).

$P^{UB}_{out,GS}$: Applying the order statistics of i.i.d. RVs, $P^{UB}_{out,GS} (\rho_{th})$ is expanded as

$$P^{UB}_{out,GS} (\rho_{th}) = \Pr \{ \rho_{min,GS} \leq \rho_{th} \} = \left[ \Pr \{ \zeta_{min}^{(j)} \leq \rho_{th} \} \right]^M,$$

(53)

where $\zeta_{min}^{(j)}$ is defined in (46). It is also noted that the approximation of $\Pr \{ \zeta_{min}^{(j)} \leq \rho_{th} \}$ can be easily obtained by substituting $M_1 = M_2 = M_3 = 1$ into (50), (51), (52), and after some calculations, we have the result in (35).

**APPENDIX IV DERIVATIONS FOR THE HIGH SNR APPROXIMATIONS**

The high SNR approximations aim to facilitate the extraction of the MuD gain. Although the results in (34) and (35) are easy to be evaluated with software, they involve integral of exponential functions and are not easy to be further analyzed. To this end, some approximations are applied before the high SNR analysis. Specifically, $I_{1,3}'$ in (52) is further approximated as $I_{1,3}'' = \int_0^\infty \int_0^\infty \int_0^{a+b+y} f(x, y, z) \, dx \, dy \, dz$, where the integral intervals of $x$ and $y$ have been enlarged from $x, y \in [\mu, \infty)$ to $x, y \in [0, \infty)$. The rational of this approximation is that when SNR is high, the condition $\mu \to 0$ holds, and such approximation will not influence the asymptotic behaviors that are relevant to the MuD gain. Then the result of $I_{1,3}''$ is given by

$$I_{1,3}'' = 2M_2 \sum_{q=0}^{M_2-1} \left[ \frac{(-1)^q C_q^{M_2-1}}{2(q+1)} \right] + 3 \sum_{p=1}^{M_1} C_q^{M_2-1} C_p^{M_1} (-1)^q e^{-3pa} \left[ \frac{pb}{(q+1)} \right] K_1 \left( \frac{6\sqrt{p(q+1)b}}{b} \right) \left[ 1 - (1 - e^{-3\mu})^{M_3} \right],$$

(54)

By using $I_{1,3}'$, $\Pr \{ E_1 (\rho_{th}) \}$ is further approximated as $\Pr \{ E_1 (\rho_{th}) \} < \tilde{P} \{ E_1 (\rho_{th}) \} = I_{1,1} + I_{1,2} + I_{1,3}''$. Following the same lines of the derivations of $P^{UB}_{out,CS}$, we have the approximation $P^{UB}_{out,CS} < \tilde{P}^{UB}_{out,CS} = \sum_{i=1}^6 \tilde{P} \{ E_i (\rho_{th}) \}$. It is noted that $\tilde{P}^{UB}_{out,CS}$ can be obtained by replacing the second item of summation in (34) with a slight modification of $I_{1,3}'$, i.e., changing $M_k$ into $M_{i,k}$, $k \in [1, 3]$. Due to limited space, the explicit expression of $\tilde{P}^{UB}_{out,CS}$ is omitted. On the other
hand, by substituting \( M_1 = M_2 = M_3 = 1 \) in \( P_{\text{out,CS}}^{UB} \) and following the derivation of \( P_{\text{out,GS}}^{UB} \), we have \( P_{\text{out,GS}}^{UB} < P_{\text{out,GS}}^{UB} \), and \( P_{\text{out,GS}}^{UB} \) is obtained by replacing the third item in (35) with \( 6e^{-3\mu} \left( 1 - e^{-3a}6\sqrt{bK_1(6\sqrt{b})} \right) \). Then, \( P_{\text{out,C(G)}}^{LB} \) and \( P_{\text{out,CS(G)}}^{UB} \) are actually used to continue the high SNR analysis.

For the high SNR analysis, we first approximate \( b = \frac{6\rho_{th}\text{SNR}+1}{\text{SNR}^2} \approx \frac{6\rho_{th}}{\text{SNR}} = 3a \) and \( \mu \approx \frac{1}{2} \left( a + \sqrt{a^2 + 12a} \right) \), consequently, \( P_{\text{out,C(G)}}^{LB} \) and \( P_{\text{out,CS(G)}}^{UB} \) can be defined as functions of \( a \). Then the power series of the exponential function \( e^x \) and the modified Bessel function \( K_1(x) \) are used to express \( P_{\text{out,C(G)}}^{LB} \) and \( P_{\text{out,CS(G)}}^{UB} \) with the polynomial forms regarding \( a \). By finding the first nonzero derivative orders of these polynomials and discarding the higher order infinitesimal terms when \( a \to 0 \), the key results are obtained and shown in (36)-(39).

**APPENDIX V PROOF OF PROPOSITION 3**

**Proof:** Let us focus on the CS scenario first, the LB of \( P_{\text{out,CS}}(\text{SNR}) \) is given by

\[
P_{\text{out,CS}}(\text{SNR}) \geq \Pr \left( R_{\text{CS}}^{UB} \leq R_{th}(\text{SNR}) \right) = \Pr \left( \rho_{\min,CS} \leq (1 + \text{SNR})^\frac{1}{3} - 1 \right) = P_{\text{out,CS}}^{LB}(\text{SNR}),
\]

(55)

where \( R_{\text{CS}}^{UB} = 3 \log_2 \left( 1 + \rho_{\min,CS} \right) \) is the UB of \( R \). Then, according to the high SNR analysis in (36) and Lemma 2 in Appendix VI, it is easy to check that

\[
P_{\text{out,CS}}^{LB}(\text{SNR}) \approx \text{SNR}^{-d_{CS}^L(1-\frac{1}{3})}.
\]

(56)

One the other hand, the UB of \( P_{\text{out,CS}}(\text{SNR}) \) is given by

\[
P_{\text{out,CS}}(\text{SNR}) \leq \Pr \left( R_{\text{CS}}^{LB} \leq R_{th}(\text{SNR}) \right) = \Pr \left( \rho_{\min,CS} \leq (1 + \text{SNR})^\frac{1}{3} - 1 \right) = P_{\text{out,CS}}^{UB}(\text{SNR}),
\]

(57)

where \( R_{\text{CS}}^{LB} = 3 \log_2 (1 + \rho_{\min,CS}) \) is the lower bound of \( R \). Based on (37) and Lemma 2, it is shown that

\[
P_{\text{out,CS}}^{UB}(\text{SNR}) \approx \text{SNR}^{-d_{CS}^U(1-\frac{1}{3})}.
\]

(58)

Noting \( d_{CS}^L = d_{CS}^U = \min \{ M_1, M_2, M_3 \} \), the DMT in (42) for CS scenario is obtained by combining (55) to (58). The DMT analysis for the GS scenario follows the similar procedures as the CS scenario, and we omit this part for limited space. The proof is then finished.
Lemma 1. Given $e_m$, the reference direction from the orthogonal basis $E$ of the RSS, and the channel matrix $H_{jk}$ with i.i.d. $CN(0,1)$ entries, the equivalent channel gain $\alpha_{m,jk}^2 = \| H_{jk}^{-1} e_m \|^2$ of link $S_{jk} \rightarrow R$ is exponentially distributed as $f_{\alpha_{m,jk}^2}(x) = e^{-x}$.

Proof: Firstly, it is noted that $e_m = E_1 m$, where $E_1 m \in \mathbb{C}^{3 \times 1}$ has all zero elements except $[E_1 m]_m = 1$, then $\alpha_{m,jk}^2$ is re-expressed as

$$\alpha_{m,jk}^2 = \| H_{jk}^{-1} E_1 m \|^2 = \left[ \left( \tilde{H}_{jk} \tilde{H}_{jk}^H \right)^{-1} \right]^{-1}_{m,m}, \quad (59)$$

Note that $H_{jk} H_{jk}^H$ and $\tilde{H}_{jk} \tilde{H}_{jk}^H = EH_{jk} H_{jk}^E$ have the same statistic properties, because of the independence between $E$ and $H_{jk}$, and the unitarily invariant of the central Wishart matrix $[33]$. According to [34], we know $\left[ (H_{jk} H_{jk}^H)^{-1} \right]^{-1}_{m,m}$ is exponentially distributed with a PDF $f(x) = e^{-x}$, so is $\alpha_{m,jk}^2$ as defined in (59), and the proof is finished.

Lemma 2. $(\frac{c}{SNR})^d \ln \left( \frac{SNR}{c} \right) \approx SNR^{-d}$

Proof: Using the l’Hospital’s rule, it is easy to check that $(\frac{c}{SNR})^d = o \left( \left( \frac{c}{SNR} \right)^d \ln \left( \frac{SNR}{c} \right) \right)$ and $(\frac{c}{SNR})^d \ln \left( \frac{SNR}{c} \right) = o \left( \left( \frac{c}{SNR} \right)^{d-\epsilon} \right)$ when SNR $\rightarrow \infty$ with $\forall \epsilon > 0$, and the inequality

$$c_1 \lim_{SNR \rightarrow \infty} \left( \frac{c}{SNR} \right)^d < \lim_{SNR \rightarrow \infty} \left( \frac{c}{SNR} \right)^d \ln \left( \frac{SNR}{c} \right) < c_2 \lim_{SNR \rightarrow \infty} \left( \frac{c}{SNR} \right)^{d-\epsilon}, \quad (60)$$

holds for some $c_1$ and $c_2$. Let $\epsilon \rightarrow 0$ in (60), then $(\frac{c}{SNR})^{d-\epsilon} \rightarrow (\frac{c}{SNR})^d$ and $(\frac{c}{SNR})^d \ln \left( \frac{SNR}{c} \right) \approx SNR^{-d}$, and the proof is finished.

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