Sum-Rate Capacity Scaling Law in Massive MIMO With Antenna Selection

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Abstract—Antenna selection can handle the cost and complexity issues in massive multiple-input multiple-output (MIMO) channels. In this article, the sum-rate capacity of a multiuser massive MIMO uplink channel is analyzed. A mathematically tractable sum-rate capacity upper bound is derived for the considered system. Furthermore, for a sufficiently large base station (BS) antenna number, a deterministic equivalent (DE) of the sum-rate capacity bound is derived. Based on this DE, the sum-rate capacity is shown to grow double logarithmically with the number of BS antennas. Numerical experiments confirm the validity of the analytical results.

Index Terms—Antenna selection, capacity scaling law, massive multiple-input multiple-output (MIMO), sum-rate capacity.

I. INTRODUCTION

To meet the immense demand for mobile data traffic, massive multiple-input multiple-output (MIMO) has emerged as a critical enabling technology in wireless networks due to its good spectral efficiency performance [1]. By deploying a large-scale antenna array at the base station (BS), massive MIMO offers unprecedented degrees of freedom (DoFs) to the wireless channel. However, to reap the maximal transmission rate, each BS antenna element needs to be accompanied by a dedicated radio frequency (RF) chain, which significantly increases the hardware cost, power consumption, and signal processing complexity of the system [2]. Numerous methods have been recently proposed as cost-effective alternatives to alleviate these issues; see [3]–[8] and the references therein. Among them is antenna selection (AS) [3]–[5], a technique to activate an antenna subset for communications. As is proved by theoretical analyses and testing data, AS can reduce the number of RF chains while preserving the DoFs offered by the large-scale antenna array [3].

In AS-aided MIMO systems, an intriguing question is "how the channel capacity scales as the BS antenna number grows large?", namely the capacity scaling law (CSL) or the capacity scaling rate. The CSL plays a critical role in defining the capacity limits of AS in MIMO channels, which, thus, provides important system design insights. Because of its importance and extensive applications, the CSL has been one of the research focuses in AS in recent years [9]–[14].

A. Related Works

In a single-user (SU) MIMO downlink channel where a single transmit antenna with the most substantial channel gain is selected, the capacity was shown to scale with the BS antenna number at a double logarithmic rate [9]. This work was extended to another downlink case with a single-antenna receiver served by a multi-antenna BS, where multiple BS antennas with the most potent channel coefficients are selected for communications [10]. Essentially, these two works characterized the CSL achieved by AS in large-scale multiple-input single-input (MISO) systems. As a further advance, the CSL was unveiled for massive SU-MIMO uplink systems using the sub-array switching (SAS) architecture, where multiple receive antennas are selected via an exhaustive search (ES) [11]. The SAS architecture means that each RF chain is connected to one disjoint sub-array of the BS. This is in contrast to the full-array switching (FAS) architecture, where each RF chain is connected to all the BS antennas via a switching network. Additionally, the CSLs achieved by the greedy search-based AS method [12] and the channel gains-based AS method [13] were also studied for SU-MIMO channels. It is noteworthy that the works mentioned above did not consider the CSL in multiuser systems. To fill this gap, researchers discussed the CSL in a multiuser MIMO (MU-MIMO) multicast channel where multiple transmit antennas are selected via the ES-based method [14].

B. Motivations and Main Contributions

Although the above works have laid a solid foundation for understanding the CSL in massive MIMO systems, one can improve them in three aspects. Firstly, multiuser unicast transmission is one of the most typical applications of massive MIMO, while the CSL therein has not yet been fully understood when multiple antennas are selected. Secondly, some existing CSLs were derived for either specific switching architectures [11] or specific selection algorithms [12], [13], which makes the analytical results lack sufficient generality. Last but not least, most existing works assumed the channels suffer from Rayleigh fading [9]–[14]. Although Rayleigh is the most popular fading model, a more accurate fading model for a realistic radio environment is the Nakagami- $m$ fading that encompasses Rayleigh as a particular case [15]. Yet, the CSL under the Nakagami fading model has received little attention.

To overcome these limitations, we analyze the CSL in a FAS architecture-based multiuser massive MIMO channel with multiple BS antennas selected by the ES-based method. Considering the Nakagami- $m$ fading model, we derive an analytically tractable bound to characterize the sum-rate ca-
capacity. We obtain a deterministic equivalent (DE) of the sum-rate capacity bound by setting the BS antenna number to infinity. On this basis, we show that the sum-rate capacity of the considered system grows double logarithmically with the number of BS antennas. It is worth mentioning that the derived CSL is applicable for the FAS architecture and the ES-based AS method [11], both being capacity-optimal for MIMO channels. As a result, the derived CSL serves as an upper bound for the CSLs achieved by other switching architectures and AS methods.

C. Notation

Scalars, vectors, and matrices are denoted by non-bold, bold lower-case, and bold upper-case letters, respectively; \( \mathbb{C} \) stands for the complex plane. The Hermitian of matrix \( A \) is indicated with \( A^H \) and \( \mathbf{I}_N \) is the \( N \times N \) identity matrix; \( \det(\cdot) \), \( \mathbb{E}\{\cdot\} \), and \( \mathbb{V}\{\cdot\} \) denote the determinant, expectation, and variance operators, respectively; \( \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp\left(-t^2\right) dt \) denotes the Gauss error function. In addition, the real Gaussian distribution having mean \( \eta \) and variance \( \sigma^2 \) is represented by \( \mathcal{N}(\eta, \sigma^2) \). Finally, \( x \sim \mathcal{CN}(0, \mathbf{X}) \) denotes a circularly symmetric complex Gaussian vector having zero mean and covariance matrix \( \mathbf{X} \).

II. System Model

A. Massive MIMO Channel

The considered massive MIMO uplink communication system consists of \( K \) users and one BS, as depicted in Fig. 1. We assume that each user \( k \in \mathcal{K} = \{1, \ldots, K\} \) is equipped with a single transmit antenna to convey signals while the BS has \( N \gg K \) antennas for receiving. The received vector at the BS is given by

\[
y = \sqrt{p_u} \mathbf{Gx} + \mathbf{n} = \sqrt{p_u} \mathbf{HD}^{1/2} \mathbf{x} + \mathbf{n},
\]

where \( \mathbf{x} \in \mathbb{C}^{K \times 1} \) is the vector of symbols transmitted by all users and the input covariance matrix is given by \( \mathbf{R}_x = \mathbb{E}\{\mathbf{xx}^H\} = \mathbf{I}_K \); \( p_u \) is the average transmit power of each user terminal; \( \mathbf{H} = [h_{n,k}] \in \mathbb{C}^{N \times K} \) and \( h_{n,k} \) models the fast fading from user \( k \) to the \( n \)th antenna of the BS; \( \mathbf{D} = \text{diag}\{\beta_1, \ldots, \beta_K\} \in \mathbb{C}^{K \times K} \) and \( \beta_k \) models both geographic attenuation and shadowing from user \( k \) to the BS; \( \mathbf{G} = \mathbf{HD}^{1/2} = [g_1, \ldots, g_K] \in \mathbb{C}^{N \times K} \) is the channel matrix from the users to the BS with \( g_k \in \mathbb{C}^{N \times 1} \) representing the channel vector from user \( k \) to the BS; \( \mathbf{n} \sim \mathcal{CN}(0, \sigma_n^2 \mathbf{I}_N) \) is the additive white Gaussian noise (AWGN) with \( \sigma_n^2 \) being the noise power. The diagonal elements of \( \mathbf{D} \), i.e., \( \{\beta_k\}_{k=1}^K \), are assumed to be constant across the antenna array, whereas the elements of \( \mathbf{H} \) are assumed to be independent but not identically distributed (i.i.d.) each with Nakagami-\( m \) distributed magnitude and uniformly distributed phase on \( [0, 2\pi) \). Particularly, the probability density function (PDF) of \( |h_{n,k}|^2 \) is given by

\[
f_{n,k}(x) = \frac{1}{\Gamma(m_k)} m_k^{m_k} \exp\left(-m_k x\right), \quad x > 0, \quad (2)
\]

where \( m_k \geq 0.5 \) [15] and \( \Gamma(z) = \int_0^\infty t^{z-1} \exp\left(-t\right) dt \) is the complete gamma function.

To evaluate the theoretical system performance upper bound, we assume that the channel state information (CSI) is perfectly known at the BS. As is widely known, exploiting a minimum mean square error with successive interference cancellation (MMSE-SIC) decoder can achieve the sum-rate capacity of an uplink multiuser channel [16]. By denoting \( \rho_u = \frac{p_u}{\sigma_n^2} \), the sum-rate capacity of the considered system is given by [16]

\[
R = \log_2 \det \left( \mathbf{I}_N + \rho_u \sum_{k=1}^{K} \mathbf{g}_k \mathbf{g}_k^H \right) \quad (3)
\]

\[
= \log_2 \det \left( \mathbf{I}_N + \rho_u \mathbf{GG}^H \right). \quad (4)
\]

It is worth noting that the capacity expression presented in (4) has many meanings.

- Generally, equation (4) calculates the sum-rate capacity of the considered uplink MU-MIMO channel [16].
- When \( \beta_1 = \ldots = \beta_K \) and \( m_1 = \ldots = m_K \), equation (4) calculates the channel capacity of a SU-MIMO channel subject to equal transmit power allocation.
- When \( \rho_u \to \infty \), equation (4) calculates the asymptotic sum-rate capacity of a dirty paper coding-based downlink MU-MIMO channel [17].

The above arguments imply that the CSL derived in the following pages also applies to the SU-MIMO and downlink MU-MIMO channels.

B. Antenna Selection

Since the BS has complete channel information of \( \mathbf{G} \), it can exploit receive antenna selection (RAS) to reduce the hardware implementation complexity. To guarantee spatial multiplexing gain for the \( K \) users, we assume that \( L \geq K \) BS antennas are selected, and an independent RF chain feeds each antenna. Moreover, we assume that the BS adopts a FAS-based architecture, where each RF chain is connected to all the BS antennas via a switching network [11], as shown in Fig. 1. Let \( \mathbf{G} \in \mathbb{C}^{L \times K} \) denote the resulting sub-matrix of \( \mathbf{G} \) after the RAS. Without loss of generality, we resort to the sum-rate capacity maximization as the criterion of antenna selection. As a result, the optimal channel sub-matrix satisfies

\[
\mathbf{G}_{\text{opt}} = \arg \max_{\mathbf{G} \in \mathcal{S}} \log_2 \det \left( \mathbf{I}_L + \rho_u \mathbf{G} \mathbf{G}^H \right), \quad (5)
\]

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where $\mathcal{S}$ denotes the full set of the candidate channel sub-matrices with $|\mathcal{S}| = \binom{N}{K}$. For simplicity, we define $\mathcal{R}_o(K) \triangleq \log_2 \det (L_1 + \rho_o \mathbf{G}_o \mathbf{G}_o^H)$. It is worth mentioning that the problem defined in (5) is NP-hard and $\mathbf{G}_o$ can be found via the ES-based AS method having exponential searching complexity $\mathcal{O}(N^K)$.

### III. Sum-Rate Capacity Upper Bound

While the previous section has established the fundamental model for RAS-aided massive MIMO uplink channels, in the following sections, we discuss more properties of the sum-rate capacity to unveil more system insights. It is worth noting that the calculation of $\mathcal{R}_o(K)$ requires an ES, which makes the subsequent analyses of $\mathcal{R}_o(K)$ intractable. As a compromise, our efforts will focus on an upper bound of $\mathcal{R}_o(K)$. This bound is obtained by relaxing the system model described in (1) to the aggregation of $K$ independent single-input multiple-output sub-channels and selecting the best $L$ out of $N$ antennas for maximal-ratio combining (MRC) in each sub-channel. Afterward, $\mathcal{R}_o(K)$ will be upper bounded by the summation of the capacity of the $K$ sub-channels [18]. Particularly, let $\{\hat{h}_{n,k}\}_{n=1}^N$ denote the ordered set of $\{h_{n,k}\}_{n=1}^N$, i.e., $|\hat{h}_{1,k}| \geq |\hat{h}_{2,k}| \geq \cdots \geq |\hat{h}_{N,k}|$. Then, the sum-rate capacity upper bound can be expressed as

$$\mathcal{R}_u \triangleq \sum_{k=1}^K \log_2 \left( 1 + \rho_k \beta_k \sum_{l=1}^L |\hat{h}_{l,k}|^2 \right) \geq \mathcal{R}_o(K),$$

(6)

where the equality in $\ast$ holds for $K = 1$.

Having defined the upper bound of $\mathcal{R}_o(K)$, we now move on to characterizing its statistical properties by analyzing its mean for the sake of gleaning further insights. As stated before, the PDF of $|h_{n,k}|^2$ is given by

$$f_{\nu_n}(x) = \frac{1}{2} \frac{1}{\Gamma(m_k)} y^{m_k-1} \exp(-y) \mathbf{1}(y > 0),$$

for the joint PDF of the ordered variables $\{x_{n,k} = |\hat{h}_{n,k}|^2\}_{n=1}^N$ is given by $f_{\nu_n}(x_{1,k}, \cdots, x_{N,k}) = N! \prod_{n=1}^N f_{\nu_n}(x_{n,k})$. Accordingly, the characteristic function of $\mathcal{R}_k \triangleq \log_2 \left( 1 + \rho_k \beta_k \sum_{l=1}^L x_{l,k} \right)$ can be expressed as

$$\Phi_k(\omega) = \int_0^\infty \cdots \int_0^\infty \exp(j\omega \mathcal{R}_k) f_{\nu_n}(x_{1,k}, \cdots, x_{N,k}) \, dx_{1,k} \cdots dx_{N,k},$$

(7)

With the characteristic function at hand, the mean of $\mathcal{R}_u$ can be numerically calculated, which yields

$$\mathbb{E}\{\mathcal{R}_u\} = \sum_{k=1}^K \frac{1}{j} \frac{1}{d\omega} \Phi_k(\omega) \bigg|_{\omega = 0}.$$  

(8)

Nevertheless, we notice that although (8) can be used to calculate $\mathbb{E}\{\mathcal{R}_u\}$, the $K$ $N$-fold integrations therein involve a huge computation burden, especially in massive MIMO settings where $N$ is a large value. Furthermore, due to the mathematical intractability of (8), it is challenging to use this calculation expression to gather any system insights. To handle this difficulty, we intend to derive a more concise expression to approximate $\mathbb{E}\{\mathcal{R}_u\}$, based on which the sum-rate capacity scaling law is explored.

Before further derivations, let us introduce some key preliminary results that will be useful in constructing the approximation of $\mathbb{E}\{\mathcal{R}_u\}$.

#### Lemma 1. Let $x_1, x_2, \cdots, x_N$ be independent and identically distributed, each with the cumulative distribution function (CDF) $F(x)$ that satisfies: 1) $\lim_{x \to +\infty} [F(x) - F(-x)] = 1$, and 2) $\forall \alpha > 0$, $\sup\{x : F(x) \leq \alpha\} = \inf\{x : F(x) \geq \alpha\}$. Here, $\sup A$ and $\inf A$ return the supremum and infimum of set $A$, respectively. Let $(x_1) \geq (x_2) \geq \cdots \geq (x_N)$ denote the order of the sample $\{x_n\}_{n=1}^N$. Then as $N \to +\infty$ and for fixed $L (L < N)$, $Y_N^L = \sum_{l=1}^L x_{l,1}$ converges in distribution to $Y \sim \mathcal{N} (\mu_y, \sigma_y^2)$, where

$$\mu_y = \frac{N}{L} \int_v^\infty x \, dF(x),$$

(9)

$$\sigma_y^2 = \frac{L}{N} \int_v^\infty x^2 \, dF(x) - \frac{\mu_y^2}{L^2},$$

(10)

Proof: Please refer to [9, pp. 473–475].

We now intend to leverage the asymptotic properties stated in Lemma 1 to investigate the statistical features of $\mathcal{R}_u$ in the limit of large number of BS antennas. Particularly, based on Lemma 1, $\sum_{l=1}^L x_{l,k}$ converges in distribution to a Gaussian random variable $X_k \sim \mathcal{N}(\mu_k, \sigma_k^2)$ as $N$ approaches infinity, where

$$\mu_k = \frac{N}{L} \int_{v_k}^\infty f_{x_{n,k}}(x) \, dx = \frac{N}{L} \int_{v_k}^\infty f_{x_{n,k}}(x) \, dx,$$

(11)

and where $\mathcal{R}_u \triangleq \sum_{K=1}^K \log_2 (1 + \rho_k \beta_k X_k)$, we can obtain

$$\lim_{N \to \infty} \mathbb{E}\{\mathcal{R}_u\} = \mathbb{E}\{\mathcal{R}_a\}.$$  

(17)

Based on Lemma 1, $\mathbb{E}\{\mathcal{R}_a\} = \mathbb{E}\{\mathcal{R}_a\}$ holds as $N$ grows infinitely large. However, as will be shown in Section VI, $\mathbb{E}\{\mathcal{R}_a\}$ can provide an accurate approximation of $\mathbb{E}\{\mathcal{R}_u\}$ even for finite system dimensions, such as $N = 128$. As a result, the sum-rate capacity upper bound can be approximated as $\mathbb{E}\{\mathcal{R}_u\} \approx \mathbb{E}\{\mathcal{R}_a\}$ [20], [21], which suggests that

$$\mathbb{E}\{\mathcal{R}_a\} \approx \sum_{K=1}^K \int_{v_k}^\infty \frac{1}{\sqrt{2\pi \sigma_k^2}} \exp \left( -\frac{(x-\mu_k)^2}{2\sigma_k^2} \right).$$

(18)

It is worth mentioning that the random variable $x_{l,k} = \sum_{l=1}^L |\hat{h}_{l,k}|^2$ takes positive values for sure. Yet, its approx-
imation, $X_k \sim N(\mu_k, \sigma_k^2)$, can take negative values for a small probability. This is due to the fact that $X_k$ is only an approximation of $\sum_{i=1}^L x_{i,k}$ when $N < \infty$. As will be detailed in Appendix, $\lim_{N \to \infty} \mu_k = \infty$ and $\lim_{N \to \infty} \sigma_k^2 = 0$, which indicates that

$$\lim_{N \to \infty} \Pr(X_k < 0) = \lim_{N \to \infty} \left[1 + \text{erf} \left(-\frac{\mu_k}{\sigma_k^2}\right)\right] / 2 \quad (19)$$

$$= \frac{1}{2} \left[1 + \text{erf} \left(-\infty\right)\right] / 2 = 0. \quad (20)$$

The above arguments imply that the deviation of the approximated distribution $N(\mu_k, \sigma_k^2)$ from the exact distribution of $\sum_{i=1}^L x_{i,k}$ vanishes as $N$ grows infinitely large. Considering this fact, we can also approximate the $R_u$ as

$$\mathbb{E}\{R_u\} \approx \sum_{k=1}^K \int_{-\infty}^{\infty} \log_2 \left(1 + \rho_u \beta_k (x) \right) f_{X_k}(x) \mathrm{d}x. \quad (21)$$

We notice that both (18) and (21) only involve $K$ single integrations, which, thus, are much more computationally tractable than the calculation formula presented in (8).

IV. Sum-Rate Capacity Scaling Law

The fact of $\lim_{N \to \infty} \mathbb{E}\{R_u\} = \mathbb{E}\{R_u\}$ suggests that $\mathbb{E}\{R_u\}$ presents the same scaling law as $\mathbb{E}\{R_u\}$ with the increment of $N$. accordingly, it makes sense to rely on $\mathbb{E}\{R_u\}$ to explore the scaling law of $\mathbb{E}\{R_u\}$. By setting $N$ as infinity, the following theorem can be found.

**Theorem 1.** For fixed $L$ ($L < N$),

$$\lim_{N \to \infty} \frac{\sum_{k=1}^K \log_2 \left(1 + \rho_u \beta_k \frac{L}{m_k} \log N\right)}{\mathbb{E}\{R_u\}} = 1$$

and $\lim_{N \to \infty} \mathbb{V}\{R_u\} = 0$.

**Proof:** The proof is detailed in Appendix. $\blacksquare$

In statistics, the ratio of the standard deviation to the mean is termed as the coefficient of variation (CV), which measures the dispersion of a probability distribution. As derived in Theorem 1, the squared CV (SCV) of the capacity upper bound, i.e., $rac{\mathbb{V}\{R_u\}}{\mathbb{E}\{R_u\}^2}$, converges to zero as $N$ grows, which suggests that the dispersion of $R_u$ collapses to zero in the large-system limit. Based on Theorem 1, we can write $R_u$ as

$$R_u = X \sum_{k=1}^K \log_2 \left(1 + \rho_u \beta_k \frac{L}{m_k} \log N\right) \quad (22)$$

with the random variable $X$ satisfying $\lim_{N \to \infty} \mathbb{E}\{X\} = 1$ and $\lim_{N \to \infty} \mathbb{V}\{X\} = 0$. Therefore, it can be easily shown that as $N \to \infty$,

$$R_u = \sum_{k=1}^K \log_2 \left(1 + \rho_u \beta_k \frac{L}{m_k} \log N\right) \xrightarrow{a.s.} 0, \quad (23)$$

where $a.s.$ denotes almost sure convergence. This means that $\sum_{k=1}^K \log_2 \left(1 + \rho_u \beta_k \frac{L}{m_k} \log N\right)$ serves as a DE for $R_u$. $\blacksquare$

**Corollary 1.** The sum-rate capacity, $R_u^{(K)}$, scales with $N$ at a double logarithmic rate.

**Proof:** The results in (23) indicate that $R_u = O(\log(\log N))$ as $N \to \infty$. Let $R_1$ denote the capacity bound for $K = 1$. Thus, we have $R_1 = O(\log(\log N))$ and $R_u \geq R_u^{(K)} \geq R_u^{(1)} = R_1$. Using the Sandwich Theorem, we get $R_u^{(K)} = O(\log(\log N))$ and the proof is completed. $\blacksquare$

**Remark 1.** The results in (23) indicate that $R_u$ converges to $\sum_{k=1}^K \log_2 \left(1 + \rho_u \beta_k \frac{L}{m_k} \log N\right)$ with probability 1 in the large system limit, which can be treated as a consequence of the channel hardening [23].

**Remark 2.** The results in (23) also indicate that a larger value of the sum-rate capacity bound can be attained in a richer scattering environment with a smaller Nakagami parameter $m$ [15]. The reason is that more gains are possible due to antenna selection for smaller values of $m$. More specifically, due to the larger channel variations involved with richer scattering, the system sum-rate capacity can be more improved by properly choosing a candidate subset of antennas. Thus, a smaller value of $m$ yields a larger selection diversity gain. When $m$ tends to infinity, the channel becomes the AWGN channel, and no selection diversity gain can be exploited because there is no channel variation.

**Remark 3.** Note that $R_u$ is achieved by the FAS-based architecture and the ES-based selection method, both being capacity-optimal. Hence, it can be concluded that the sum-rate achieved by other switching architectures and selection methods will grow no faster than double logarithmically with the BS antenna number.

V. Special Cases

We now intend to discuss some special cases of the derived results to unveil more system insights. For convenience of discussion, let $R_u^{(K,L)}$ and $R_u^{(K,L)}$ denote the exact capacity and capacity upper bound achieved by $L$ active antennas and $K$ users, respectively.

1) **Special Case I:** We first consider the case when $K = 1$. Under this circumstance, the uplink MU-MIMO channel degenerates into a single-user hybrid antenna selection and MRC (H-S/MRC) channel [10], [21]. As stated before, $R_u^{(1,L)} = R_u^{(1,L)}$. Using Theorem 1, we can get $\mathbb{E}\{R_u^{(1,L)}\} = \log_2 \left(1 + \rho_u \beta_1 \frac{L}{m_1} \log N\right) \xrightarrow{a.s.} 0$ as $N$ grows infinitely large. Hence, one concludes that the average channel capacity of a single-user H-S/MRC channel scales with $N$ at a double logarithmic rate.

2) **Special Case II:** We then consider another special case when $\beta_1 = \ldots = \beta_K = \beta$ and $m_1 = \ldots = m_K = m$. In this case, the MU-MIMO channel degenerates into a SU-MIMO channel where the transmitter has $K$ antennas and adopts the uniform power allocation scheme. Based on Theorem 1, it is manifest that the capacity upper bound satisfies $\mathbb{E}\{R_u^{(K,L)}\} \geq K \log_2 \left(1 + \rho_u \beta \frac{L}{m} \log N\right) \xrightarrow{a.s.} 0$ as $N$ approaches infinity.

3) **Special Case III:** Finally, we consider the case of $m_1 = \ldots = m_K = 1$, namely the Rayleigh fading channel. In this case, the constant $v_k$ in (16) satisfies $\int_{-\infty}^{\infty} f_n(x)(-x) \mathrm{d}x = \int_{-\infty}^{\infty} \exp(-x) \mathrm{d}x = \exp(-v_1) = \frac{1}{\beta}$, and thus $v_k = \log \frac{\beta}{2}$. Upon plugging $v_k = \log \frac{\beta}{2}$ and $f_n(x) = \exp(-x)$ into (15) to (16), we can get $\mu_k = L \left(1 + \log \frac{\beta}{2}\right)$ and $\sigma_k^2 = L \left(2 - \frac{\beta}{2}\right)$. Let $L$ be fixed and it follows that $\lim_{N \to \infty} \frac{\sigma_k^2 \mu_k}{\rho_u \beta} = 0$. Consequently, we can get

$$\mathbb{E}\{R_u^{(K,L)}\} = \sum_{k=1}^K \log_2 \left(1 + \rho_u \beta_k \frac{L}{m_k} \log N\right) \xrightarrow{a.s.} 0,$$

which yields a double logarithmic CSL. Moreover, the following corollaries are concluded.
Corollary 2. When \( K = 1 \), the capacity bound degenerates into the exact channel capacity of a H-S/MRC channel over Rayleigh fading. In this case, the asymptotic capacity is given by

\[
\log_2 \left( 1 + p_u \beta_1 L \left( 1 + \log \frac{N}{L} \right) \right),
\]

which is consistent with the results shown in [10, eq. (4)].

Corollary 3. When \( K = 1 \) and \( L = N \), the capacity bound degenerates into the exact channel capacity of a MISO channel over Rayleigh fading. The resulting asymptotic capacity is given by

\[
\log_2 \left( 1 + p_u \beta_1 N \right),
\]

which serves as a special case of [23, eq. (14)].

VI. NUMERICAL RESULTS

In this section, numerical results are provided to verify the correctness of the above theoretical analyses. The users are assumed to be distributed uniformly over a hexagonal cell with a radius of 1000 m. The free-space path loss model is

\[
-10 \log_{10} \beta_k = 92.5 + 20 \log_{10} [f_0 \ \text{Hz}] + 20 \log_{10} [d_k \ \text{km}],
\]

where \( f_0 = 4 \ \text{GHz} \) is the carrier frequency and \( d_k \) is the distance between the BS and user \( k \). Besides, we set \( \sigma^2 = -100 \ \text{dBm}. \) All the simulation results are obtained via \( 5 \times 10^5 \) independent Monte-Carlo trials.

To verify the correctness of (18), Fig. 2 plots the exact sum-rate capacity, the simulated capacity bound, and the asymptotically approximated capacity bound (calculated by (18)) in terms of the transmit power for selected numbers of \( L \). It can be observed that the asymptotically approximated results match perfectly with the simulations, which, thus, verifies our previous derivations. Moreover, the capacity bound becomes tighter for a larger value of \( L \). As Fig. 2 shows, the sum-rate capacity upper bound is tight and presents a similar changing trend to the exact capacity. Accordingly, one can conclude that this bound is a good alternative for the exact capacity in system sum-rate performance evaluation. Actually, the Gaussian-distribution approximation stated in Lemma 1 is based on the assumption of \( N \to \infty \). Yet, it can be seen from Fig. 2 that this approximation works well even for a limited value of \( N \), i.e., \( N = 128 \), which indicates the robustness of Lemma 1 as well as the satisfying precision of (18).

Fig. 3 shows the mean of the capacity bound versus \( N \) for a fixed value of \( L \). For reference, the curves representing the exact sum-rate capacity and \( \sum_{k=1}^{K} \log_2 \left( 1 + p_u \beta_k \frac{L}{m_k} \log N \right) \) (labeled as ‘\( \mathcal{O} \left( \log (\log N) \right) \)’) are also plotted. As shown, both the capacity and its upper bound grow like \( \mathcal{O} \left( \log (\log N) \right) \), which supports our conclusions in Corollary 1. Then, we assume \( m_k = m, \ \forall k \in K \), and plot the SCV of \( R_u \), i.e., \( \frac{\sigma_{R_u}^2}{\mathbb{E}[R_u]^2} \), versus \( N \) for selected values of \( m \) in Fig. 4. As this graph shows, the SCV tends towards zero as \( N \) increases, which supports the conclusion drawn in Theorem 1. Moreover, as stated before, a smaller Nakagami-\( m \) fading parameter corresponds to larger channel variations, yielding a larger value of the SCV and a slower channel hardening speed. This is also validated by Fig. 4.

VII. CONCLUSION

The sum-rate capacity achieved by receive antenna selection in massive MIMO uplink systems has been discussed. For mathematical tractability, a low-complexity sum-rate capacity upper bound has been proposed. A deterministic equivalent of this bound has been found to characterize its asymptotic behavior in the limit of large number of BS antennas. On this basis, the sum-rate capacity is shown to scale with the BS antenna number at a double logarithmic rate.

The results of this study can be used to investigate antenna selection-aided massive MIMO systems in various respects,
such as the energy efficiency, spectral efficiency, and resource efficiency, which will be addressed in the extended version of this work.

APPENDIX

It follows from (16) that \( \lim_{N \to \infty} \int_{-\infty}^{\infty} f_{n,k}(x) \, dx = \lim_{N \to \infty} \frac{1}{N} = 0 \) and thus \( \lim_{N \to \infty} v_k = k \). Using [24, eq. (8.11.2)], we have

\[
\lim_{N \to \infty} \frac{f_{n,k}(x)}{L/N} = \lim_{N \to \infty} \frac{\Gamma(m_k \nu_k k^{-1})}{\Gamma(m_k \exp(m_k \nu_k))} = 1, \tag{25}
\]

which yields

\[
\lim_{N \to \infty} \frac{\log \left( \Gamma(m_k \nu_k k^{-1}) \right)}{\log L - \log N} = \lim_{N \to \infty} \frac{m_k \nu_k}{k} = 1. \tag{26}
\]

Similarly, based on (15), we can get

\[
\lim_{N \to \infty} \frac{\mu_k}{L \nu_k} = \lim_{N \to \infty} \frac{N \nu_k \Gamma(m_k \exp(m_k \nu_k))}{\nu_k L} = 1, \tag{27}
\]

where the equality in \( \circ \) holds for (25). By continuously following the above steps, we have \( \lim_{N \to \infty} \frac{\sigma_k^2}{L \nu_k} = 0 \). As stated before, \( \lim_{N \to \infty} \frac{\sigma_k^2}{L \nu_k} = 0 \). Hence, \( \frac{W_k}{\nu_k L} \) converges to 1 with probability one as \( N \to \infty \), which together with (27) and the fact of \( \lim_{N \to \infty} \frac{\nu_k L}{m_k \log N} = 1 \), suggests that \( \lim_{N \to \infty} \frac{\log \left( \Gamma(m_k \exp(m_k \nu_k)) \right)}{\log L - \log N} = 1 \). Note that

\[
\lim_{N \to \infty} \frac{\log \left( \Gamma(m_k \exp(m_k \nu_k)) \right)}{\log L - \log N} = 1. \tag{28}
\]

which completes the proof of the first part of Theorem 1. Following a similar approach as that in obtaining (28), we find that \( P_k \) satisfies

\[
\lim_{N \to \infty} P_k = \lim_{N \to \infty} \frac{\log \left( \Gamma(m_k \exp(m_k \nu_k)) \right)}{\log L - \log N} = 1. \tag{29}
\]

Therefore, the second part of Theorem 1 is also proved.

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