Quenched scalar-meson correlator with Domain Wall Fermions

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We study the $\bar{q}q$ singlet and non-singlet scalar-meson masses using domain wall fermions and the quenched approximation. The singlet mass is found to be smaller than the non-singlet mass and indicates that the lowest singlet meson state could be lighter than 1 GeV. The two-point functions for very small quark masses are compared with expectations from the small-volume chiral perturbation theory and the presence of fermionic zero modes.

1. Introduction
The nature of the lightest scalar mesons is not well understood. The observed $\sigma$ resonance at 600 MeV with $m_\pi < 480$ MeV and $\Gamma_\pi \approx 320$ MeV [1] can be either a manifestation of the $\pi\pi$ scattering or a distinct $0^{++}$ state. Recent lattice simulations explore whether the lightest singlet $0^{++}$ state is a $\bar{q}q$ state, a mixture of $\bar{q}q$ and glueball or a $q^2\bar{q}^2$ state [2,3,4,5]. A dynamical simulation of $\bar{q}q$ in Ref. [2] indicates $m^{\text{lat}}_\sigma \sim m_\rho^{\text{lat}}$, while Ref. [3] finds $m^{\text{lat}}_\sigma$ as low as $m^{\text{lat}}_\pi$ indicating possible restoration of chiral symmetry at finite volume. In quenched studies, $\bar{q}q$-glueball mixing renders the lowest scalar mass both below 1 GeV [6] and above 1 GeV [5].

The lightest observed non-singlet scalar states are $a_0(980)$ and $a_0(1450)$. Previous lattice simulations indicate that the lowest scalar ($\bar{q}q$)$_{I=1}$ state is heavier than 1 GeV [6,3,4]. This opens the question on the nature of the $a_0(980)$, proposed to be a $q^2\bar{q}^2$ state in [4]. Dynamical simulation [6] observes an indication for the $a_0 \to \eta'\pi$ decay.

Those simulations were using Wilson [3,6,4] and Kogut-Susskind [6] fermions. We re-examine the light scalar spectrum using Domain Wall fermions (DWF), which allow us to study lighter quark masses. Good chiral properties of DWF are especially welcome in the study of the $\sigma$ meson, which is intimately related to chiral symmetry breaking.

A light $\sigma$ could play an important role as an intermediate state in $K\to\sigma\to(\pi\pi)_{I=0}$, for example.

2. Lattice simulation
We use the quenched approximation together with DWF ($L_s = 16$) and RG-improved gauge action (DBW2). This renders the chiral symmetry breaking parameter $m_{res}$ smaller than 1 MeV. Our lattice parameters are shown in Table 1.

| $a^{-1}$ | $V$ | conf. | $m_{\pi}a$ | $m_0$ [MeV] |
|---------|-----|-------|------------|-------------|
| 2 GeV   | 16$^3$2 | 300   | [0.18, 0.26] | 800±40      |
| 1.3 GeV | 16$^3$2 | 100   | [0.30, 0.9]  | 740±35      |
| 1.3 GeV | 8$^3$24 | 400   | [0.33, 0.9]  | 700±25      |

Table 1: Simulation parameters, ranges of $m_{\pi}a$ and results for $m_0$.

The disconnected quark diagram is simulated with the Kuramashi technique [5]. This technique is applied also for the connected diagram, so that a meson has a point source on every lattice site.

3. Connected diagram and quenching effects
The connected two-point function exhibits conventional exponential decay for $m_q > m_\pi$ and gives us the $a_0$ mass consistent with [4].

The connected two-point function has a striking quenched effect for $m_q < m_\pi$ - it is negative at larger times! Fig. 1 shows that the negativity is more pronounced for smaller quark masses and smaller volumes. We also find that the correlation function is negative only for non-zero topological charge $Q$. This is consistent with the effects of fermionic zero modes, which are not suppressed in the quenched approximation and whose contributions are related by [3]

$$\int d^4x (a_0(x)a_0^\dagger(0))_0 = \int d^4x (\pi(x)\pi^\dagger(0))_0 = -\frac{|Q|}{V m_\pi^2}.$$
The negativity can be understood also within quenched chiral perturbation theory (Q\chi\!\!PT). The small-volume version of Q\chi\!\!PT \cite{3} respects the relation \cite{6} and is discussed in the Section 5. It is applicable when pion wavelength \(\lambda_\pi > L\).

We extract \(m_{a0}\) using the large-volume version of Q\chi\!\!PT where \(\lambda_\pi < L\) \cite{6}. The dominant negative contribution to \(\langle a_0(t)a_0^\dagger(0)\rangle\) in this approach arises from the \(\pi\eta'\) intermediate state \cite{6} (Fig. 2). This diagram dominates at larger \(t\), where the \(e^{-m_{a0}^t}\) term is sub-dominant. The \(\bar{q}q - \pi\eta'\) coupling is \(m_\pi^2/m_q\). The finite volume is taken into account by replacing the integral over momenta with a finite sum. The value of the \(m_0\) insertion is determined from the disconnected pseudoscalar diagram and is given in Table 1. The value of \(a^{-1} = 1.3\) GeV. Lines are Q\chi\!\!PT predictions based on the diagram in Fig. 2.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Connected scalar two-point function at \(a^{-1} = 1.3\) GeV. Lines are Q\chi\!\!PT predictions based on the diagram in Fig. 2.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{fig2.png}
\caption{This diagram gives the dominant quenched effect to connected scalar correlator.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{fig3.png}
\caption{Singlet and non-singlet scalar masses for our largest lattice \(16^3 \times 32\) at \(a^{-1} = 1.3\) GeV.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{fig4.png}
\caption{Disconnected scalar two-point function at \(16^3 \times 32\) and \(a^{-1} = 2\) GeV together with the fits to the formula given in Section 4.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{fig5.png}
\caption{The disconnected two-point function on the quark and the meson level.}
\end{figure}

\begin{table}
\centering
\begin{tabular}{|c|c|}
\hline
\(m^2\) & \(m(a^{-1} = 1.3\) GeV) \(m(a^{-1} = 2\) GeV) \(m(a^{-1} = 3\) GeV) \(m(a^{-1} = 4\) GeV) \\
\hline
\(m_{a0}\) & \(0.025\) & \(0.020\) & \(0.015\) & \(0.010\) \\
\hline
\(m_a\) & \(0.040\) & \(0.032\) & \(0.024\) & \(0.016\) \\
\hline
\end{tabular}
\caption{Singlet and non-singlet scalar masses for our largest lattice \(16^3 \times 32\) at \(a^{-1} = 1.3\) GeV.}
\end{table}
the possibility of \(a_0(980)\) being a \(\bar{q}q\) state.

4. Disconnected diagram and \(\sigma\) meson

Our data for the disconnected two-point function is given in Fig. 4. We analyze it in terms of the meson diagrams in Fig. 5: \(\langle \sigma^0(t)\sigma^0(0) \rangle - \langle \sigma^0(t) \rangle \langle \sigma^0(0) \rangle = [\sqrt{8}f_{\sigma}m_{\sigma}/m_\pi]^2/\left(p^2 + m_\sigma^2\right)^2.\) The values of \(m_{a_0}\) and \(f_{\sigma}\) are taken from the connected diagram above. We have checked also that \(m_{a_0}\), determined from the shape of the disconnected two-point function, is consistent with \(m_{a_0}\) obtained from the connected two-point function. A one-parameter fit to \(m_\pi^2\) gives \(m_\pi^2a_\sigma^2 = -0.11 \pm 0.04\) at \(m_\pi a = 0.3003 \pm 0.0019\) for our largest lattice, consistent with results from the other two lattices. The sum of the diagrams with an arbitrary number of \(m_\pi^4\) insertions in the dynamical theory gives the singlet scalar mass \(m_{a_0}^2 = m_{a_0}^2 + m_\pi^2\). The sign of the disconnected diagram in our data has an important physical consequence as it implies that \(m_\pi^2 < 0\) and \(m_\sigma < m_{a_0}\). The resulting \(m_\sigma\) is shown in Fig. 5.

5. \(Q\chi PT\) at small volume

Fig. 6 shows the comparison between predictions of small-volume \(Q\chi PT\) and our data for two-point functions at fixed topology and very small \(m_q\). The dominance of the fermionic zero modes as \(V \rightarrow 0\) and \(m_q \rightarrow 0\) (Eq. 1) is reflected in our data as well as the effective theory. Note that \(Q\chi PT\) gives a negative prediction for the pion correlator at this \(V\) for larger \(t\), although a pion correlator with a point source and sink should be positive on every configuration by definition. This probably indicates that the expansion parameter \(1/(L_f\pi)\) in \(Q\chi PT\) is not small enough at \(8^3 \times 24\) with \(a^{-1} = 1.3\) GeV and the comparison has to be performed at larger volume.

6. Conclusion

We examined the lightest \(\bar{q}q\) singlet and non-singlet scalar meson masses using Domain Wall fermions in the quenched approximation. Our conclusions on the scalar spectra are based on the lattice with the largest volume and are displayed in Fig. 4. The singlet is lighter than non-singlet - this is an inevitable consequence of the sign of the disconnected two-point function. Our results \(m_{a_0} = 1.04 \pm 0.07\) GeV and \(m_\sigma = 0.9 \pm 0.1\) GeV at the smallest quark mass indicate that the lightest singlet \(\bar{q}q\) state could be lighter than 1 GeV and cannot rule out that the \(a_0(980)\) is a \(\bar{q}q\) state.

REFERENCES

1. E.M. Aitala et al., E791 Coll., Phys. Rev. Lett. 86 (2001) 770.
2. S. Muroya et al., SCALAR Coll., these proceedings, hep-lat/0112012.
3. C. McNeile and C. Michael, these proceedings, Phys. Rev. D 63 (2001) 114503.
4. M. Alford and R.L. Jaffe, Nucl. Phys. B 578 (2000) 367.
5. W. Lee and D. Weingarten, Phys. Rev. D 61 (2000) 014015.
6. W. Bardeen et al., Phys. Rev. D 65 (2002) 014509.
7. C. Bernard et al., Phys. Rev. D 64 (2001) 054506.
8. Y. Kuramashi et al., Phys. Rev. Lett. 72 (1994) 3448.
9. T. Blum et al., RBC Coll., hep-lat/0007038.
10. P. Damgaard et al., Nucl. Phys. B 629 (2002) 445.