Superconformal $6D$ $(2,0)$ theories in superspace

C. Grojean$^a$ and J. Mourad$^{b,c}$

$^a$ Service de Physique Théorique, CEA-Saclay
F-91191 Gif/Yvette Cedex, France

$^b$ Laboratoire de Physique Théorique et Modélisation,
Université de Cergy-Pontoise
Site Saint-Martin, F-95032 Cergy-Pontoise, France

$^c$ Laboratoire de Physique Théorique et Hautes Energies
Université de Paris-Sud, Bât. 211, F-91405 Orsay Cedex, France

Abstract

A geometrical construction of superconformal transformations in six dimensional $(2,0)$ superspace is proposed. Superconformal Killing vectors are determined. It is shown that the transformation of the tensor multiplet involves a zero curvature non-trivial cochain.

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$^1$Laboratoire associé au CNRS-URA-D0063.
1 Introduction

Many reasons motivate the study of supersymmetric six dimensional chiral theories with sixteen supercharges \[1\]. The more recent one is that the worldvolume of the five-brane \[2, 3, 4, 5, 6\] of \(M\)-theory \[7\] is described by such a theory. Not much is known about \(M\)-theory besides that it contains membranes and five-branes and that by compactification it reduces to superstring theory. A recent conjecture by Maldacena \[8\] states that \(M\)-theory on \(AdS_7 \times S^4\) with radii \(R_{sph} = R_{AdS}/2 = l_p(\pi N)^{1/3}\) (\(l_p\) is the eleventh dimensional Planck length) is dual to the superconformal worldvolume theory describing \(N\) coincident five-branes. Some consequences of this conjecture were examined in \[9\]. The study of six dimensional \((2,0)\) theories may thus provide important clues concerning the still mysterious \(M\)-theory. Another conjecture on \(M\)-theory is that of Matrix theory \[10\]; here too, \((2,0)\) six dimensional theories appear from Matrix theory compactified on \(T^4\) \[11\].

The aim of this paper is the geometrical study of six dimensional \((2,0)\) theories in superspace. The \((2,0)\) multiplet contains five scalars, one Weyl-symplectic Majorana spinor and an anti-self dual three form \[12\]. In section 2, we recall the superfield description of this multiplet \[13\] with one superfield in the vector representation of the \(R\)-symmetry group \(SO(5)\). This superfield is subject to a constraint which, as shown in section 2, reproduces the correct multiplet and the equations of motions. In section 3, we show that there exists an alternative formulation with the aid of a closed super three-form which is subject to some constraints. These constraints are somewhat similar to those of the 10D super Yang-Mills constraints \[14\] and to the six dimensional \((1,0)\) constraints for the tensorial multiplet \[15\]. For \((1,0)\) six dimensional theories it is possible to find nontrivial sigma models living on a quaternionic target space \[16\]. In section 4, we show that a generalization to sigma models of the free theory leads only to trivial conformally flat target spaces. This illustrates the rigidity of \((2,0)\) theories. In section 5, we define superconformal transformations as supercoordinate transformations leaving the super-flat metric invariant up to a scale. A similar construction for \(N=1 4D\) theories is considered in \[17\] and references therein. We calculate the resulting super-Killing vectors and show that their Lie algebra is that of \(OSp(6, 2|2)\). An algebraic construction of the superconformal \((2,0)\) algebra was given in ref \[18\], it relies on the triality property of the six-dimensional conformal group \(SO(6, 2)\) and results in the orthosymplectic group \(OSp(6, 2|2)\). Our geometrical construction provides a realisation of the generators of \(OSp(6, 2|2)\) in superspace and facilitates the study of superconformal invariance in a manifestly supersymmetric context. In section 6, we determine the transformation of the scalar superfield under superconformal transformations. We find that this transformation involves a zero curvature 1-cochain on the superconformal Lie algebra. We determine explicitly this cochain and show that it is non-trivial. We illustrate the usefulness of the formalism of section 3 by showing that the transformation
of the super-three form is purely geometrical and simpler than that of the scalar superfield. We collect our conclusions in section 7. Our notations and some technical results which are used in the text can be found in the Appendix.

2 Free supermultiplet

The on-shell \((2,0)\) supermultiplet comprises five scalars \(\phi^i\) transforming as a vector of \(SO(5)\), one symplectic Majorana-Weyl fermion and a two-form with self dual field strength. The goal of this section is to provide a manifestly supersymmetric description of this multiplet \([13]\). One may try to consider a scalar superfield transforming in the \(5\) of \(SO(5)\) whose \(\theta = 0\) component is the scalar field. Such a superfield would have the general form\(^2\)

\[
\Phi^i(x, \theta) = \phi^i(x) + \bar{\theta} \psi^i + \ldots,
\]

where \(\ldots\) stand for terms with two and more \(\theta\). We see that at the one \(\theta\) level we have five independent fermions. Since the on-shell supermultiplet has only one fermion we have to constrain the superfield in such a way that only one out of the five \(\psi^i\) be independent. An \(SO(5)\) covariant way of doing this is to impose that

\[
\psi^i = \Gamma^i \psi \text{ with } \psi = \frac{1}{5} \Gamma^i \psi^i.
\]

Note that \(\psi^i\) may be written as the \(\theta = 0\) component of \(D\hat{\alpha} \Phi^i = (\partial_{\hat{\alpha}} - (\Gamma^i \theta) \hat{\alpha} \partial_{\mu}) \Phi^i\), so that a manifestly supersymmetric and \(SO(5)\) covariant constraint on the superfield is

\[
D\Phi^i = \frac{1}{5} \Gamma^i \Gamma_j D\Phi^j,
\]

or equivalently

\[
D\Phi^i = \frac{1}{4} \Gamma^i \gamma_j D\Phi^j.
\]

When indices are made explicit this constraint reads

\[
D_{\alpha a} \Phi^i = \frac{1}{4} (\gamma^i)_{a b} D_{\alpha b} \Phi^j.
\]

Our notations can be found in the appendix. The rest of this section is devoted to the analysis of the constraint \((2.5)\). We will show that it reproduces the \((2,0)\) supermultiplet and the equations of motion.

\(^2\)In ref \([13]\) the scalar superfield is in the antisymmetric representation of \(Sp(2)\), \(\Phi_{[ab]}\). It is related to ours by \(\Phi^i = (\gamma^i)_{ab} \Phi^{[ab]}\).
Let $\Psi_{\alpha a} = (\gamma_i)_{\alpha}^b D_{ab} \Phi^i / 5$ then the supersymmetric transformation of $\phi^i$ is given by the $\theta = 0$ component of $D\Phi^i = \Gamma^i\Psi$. In order to get the quadratic terms in $\theta$ we take the $D_{\beta b}$ of the constraint (2.5). The following decomposition of the product of two derivative is useful

$$D_{\alpha a} D_{\beta b} = - (\gamma^\mu)_{[\alpha\beta]} \Omega_{[ab]} \partial_\mu + \Omega_{[ab]} D_{\alpha\beta} + (\gamma^i)_{[ab]} D_{i(\alpha\beta)} + (\gamma^{ij})_{(ab)} D_{[ij]\alpha\beta}, \quad (2.6)$$

where $(\alpha\beta)$ or $[\alpha\beta]$ means that the quantity is symmetric or antisymmetric. The quantities appearing in (2.6) are defined, for $N = \emptyset, i$ and $ij$, by

$$D_{N\alpha\beta} = \frac{1}{8} [D_{\alpha a}, D_{\beta b}] (\gamma_{N}^{ba}) \quad (2.7)$$

Taking the supersymmetric derivative of (2.5) and using the identities

$$\gamma^{ij} \gamma^l = 2 \eta_l{}^{i}{}_{j}, \quad \gamma^{ij} \epsilon_{kml} = 4 \eta_l^{[i} \eta_j^{j]}, \quad \gamma^{ij} \eta_{j}^{[i} \epsilon_{kml]} = 4 \eta_l^{[i} \eta_j^{j]}, \quad (2.8)$$

we get the following equations

$$D_{(\alpha\beta)} \Phi^i = 0, \quad D_{k(\alpha\beta)} \Phi^j = \frac{1}{5} \eta_k{}^{j} D_{i(\alpha\beta)} \Phi^i; \quad (2.10)$$

$$D_{kl[\alpha\beta]} \Phi^j = \frac{1}{2} (\eta_k{}^{j} (\gamma^\mu)_{\alpha\beta} \partial_\mu \Phi_l - \eta_l{}^{j} (\gamma^\mu)_{\alpha\beta} \partial_\mu \Phi_k). \quad (2.11)$$

Taking the $\theta = 0$ part of these equations shows that the only new field that appears at this level is the $\theta = 0$ component of

$$H_{\alpha\beta} \equiv D_{i(\alpha\beta)} \Phi^i. \quad (2.12)$$

Regarded as a matrix, this superfield $H$ can be decomposed in the basis made by the antisymmetrised products of the Dirac matrices. Taking into account the symmetry and chirality properties ($\alpha$ and $\beta$ are both of the opposed chirality compared to the $\theta$'s), only products of three Dirac matrices appear in this decomposition. So $H$ is equivalent to an anti self-dual three-form given by

$$h_{\mu_1\mu_2\mu_3} = (\gamma_{\mu_1\mu_2\mu_3})^{\alpha\beta} H_{\alpha\beta}. \quad (2.13)$$

$h_{\mu_1\mu_2\mu_3}$ is anti self-dual because

$$\epsilon_{\mu_1\mu_2\mu_3} \epsilon_{\mu_4\mu_5\mu_6} \gamma_{\mu_4\mu_5\mu_6} = -6 \gamma_{\mu_1\mu_2\mu_3}, \quad (2.14)$$

when acting on chiral spinors. In order to get the transformation of the spinor $\psi$, we take the supersymmetric derivative of the equation $\Psi = \gamma_i D\Phi^i / 5$ to get

$$D_{\beta b} \Psi_{\alpha a} = - (\gamma^\mu)_{[\beta a]} (\gamma_i)_{[ba]} \partial_\mu \Phi^i + \frac{1}{5} \Omega_{[ba]} H_{\beta a}. \quad (2.15)$$
In order to look for possible new fields at the level of the product of three \( \theta \)'s we have to calculate the supersymmetric derivative of \( H_{\alpha \beta} \). The consistency of (2.15) gives

\[
2 (\Gamma^\mu)_{\gamma \beta \delta} \partial_\mu \Psi_{\alpha \alpha} - (\Gamma^\mu)_{\beta \lambda \sigma}(\gamma_i)_c^d \partial_\mu \Psi_{\gamma d} - (\Gamma^\mu)_{\gamma \alpha \delta}(\gamma_i)_b^d \partial_\mu \Psi_{\beta d} - \frac{1}{3} \Omega_{ab} D_{\gamma e} H_{\beta \alpha} - \frac{1}{5} \Omega_{ac} D_{\beta b} H_{\gamma \alpha} = 0. \tag{2.16}
\]

Taking the symmetric part in \( ac \) and multiplying by \((\Omega^{-1})^{ba}\) we get

\[
D_{\gamma a} H_{\alpha \beta} = 5(\gamma^\mu)_{\gamma \beta} \partial_\mu \Psi_{\alpha \alpha} + 5(\gamma^\mu)_{\gamma \alpha} \partial_\mu \Psi_{\beta \alpha} \tag{2.17}
\]

which gives the supersymmetric transformation of the three-form and shows that no new degrees of freedom appear at the three \( \theta \)'s as well as at higher levels. Multiplying (2.17) by \( C^{\alpha \beta} \) gives the equation of motion of the fermion \( \gamma^\mu \partial_\mu \Psi = 0 \) which when used in (2.15) gives the bosonic equations \( \partial_\mu \partial_\mu \Phi^i = 0 \) and \( \gamma^\nu \gamma^\alpha \partial_\nu H_{\alpha \beta} = 0 \). The simplest way to obtain the latter is to notice that

\[
H_{\alpha \beta} = -\frac{5}{4} D_{\beta b} \Psi^b \tag{2.18}
\]

and use the Dirac equation. The equation of motion of \( H_{\alpha \beta} \) reads for the three-form \( h \) defined in (2.13) as \( dh = d^* h = 0 \) which gives the Bianchi identity and the equations of motion of the three-form. This equation assures that the three-form \( h \) is the field strength of a two-form which can be identified with the two-form of the tensoriel supermultiplet. It remains to prove that equations (2.16) have no other content than equations (2.17). This is easily proved when the relation

\[
(\gamma^i)_{ab}(\gamma_i)_c^d = 2\delta_a^d \Omega_{cb} - 2\delta_b^d \Omega_{ca} - \delta_c^d \Omega_{ab} \tag{2.19}
\]

is used.

3 Super two-form

In this section we formulate the (2,0) free theory with a super two-form \( B = \frac{i}{2} B_{MN} E^M E^N \) with field strength \( H = dB \). Here \( M = (\mu, \dot{\alpha}) \), \( E^\mu = dx^\mu - \theta \Gamma^\mu d\theta \) and \( E^{\dot{\alpha}} = d\theta^{\dot{\alpha}} \) are the basis of 1 super-forms invariant under global supersymmetries. In a way similar to Yang-Mills theories [14], we impose the constraints

\[
H_{\dot{\alpha} \dot{\beta} \dot{\gamma}} = 0, \tag{3.1}
\]

\[
H_{\mu \dot{\alpha} \dot{\beta}} = (\Gamma_\mu \Gamma_i)_{\dot{\alpha} \dot{\beta}} \bar{\Phi}^i, \tag{3.2}
\]

and solve the Bianchi identity \( dh = 0 \). Here \( \bar{\Phi}^i \) is a scalar superfield which belongs to the vectorial representation of \( SO(5) \). We shall prove that we can identify \( \bar{\Phi}^i \) with the scalar superfield \( \Phi^i \) of the previous section.
Terms with four spinorial indices in the Bianchi identity give $H_{\mu(\hat{\alpha}\hat{\beta})(\Gamma^\nu)_{\hat{\delta}\hat{\lambda}}} = 0$ which is satisfied thanks to the relation

$$\langle \Gamma_{\mu\hat{i}} \rangle_{\hat{\alpha}\hat{\beta}} \langle \Gamma^\nu \rangle_{\hat{\delta}\hat{\lambda}} = 0, \quad (3.3)$$

which is proved in the Appendix.

Terms with three spinorial indices in the Bianchi identity give

$$D_{\hat{\alpha}} H_{\hat{\beta}\hat{\gamma}\mu} + 2H_{\nu\mu(\hat{\alpha}(\Gamma^\nu)_{\hat{\beta}\hat{\gamma}})} = 0. \quad (3.4)$$

Taking into account the Dirac matrices property

$$\langle \Gamma_{\mu\nu} \rangle_{\hat{\alpha}\hat{\beta}} \langle \Gamma^\iota \rangle_{\hat{\delta}\hat{\lambda}} + \langle \Gamma^\iota \rangle_{\hat{\alpha}\hat{\beta}} \langle \Gamma_{\nu\iota} \rangle_{\hat{\delta}\hat{\lambda}} = 0, \quad (3.5)$$

which is proved in the Appendix, the solution to (3.4) can be given with the aid of a spinorial superfield $\tilde{\Psi}_{\hat{\alpha}}$ as

$$2H_{\nu\mu\hat{\beta}} = \langle \Gamma_{\nu\mu} \rangle_{\hat{\beta}\hat{\delta}} \tilde{\Psi}_{\hat{\delta}}, \quad (3.6)$$

$$D_{\hat{\gamma}} H_{\hat{\alpha}\hat{\beta}\mu} = \langle \Gamma_{\mu\iota} \rangle_{\hat{\alpha}\hat{\beta}} \langle \Gamma^\iota \rangle_{\hat{\gamma}\hat{\delta}} \tilde{\Psi}_{\hat{\delta}}. \quad (3.7)$$

Comparing (3.7) with (3.2) we see that $\tilde{\Phi}^i$ and $\tilde{\Psi}_{\hat{\alpha}}$ must be related by

$$\langle \Gamma^\iota \rangle_{\hat{\alpha}\hat{\beta}} \tilde{\Psi}_{\hat{\beta}} = D_{\hat{\alpha}} \tilde{\Phi}^i, \quad (3.8)$$

which is equivalent to the constraint (2.5) obeyed by the scalar superfield $\Phi^i$ allowing us to identify the two.

The terms with two spinorial indices in the Bianchi identity lead to

$$\partial_{\mu} H_{\nu(\hat{\alpha}\hat{\beta})} + D_{(\hat{\alpha}} H_{\hat{\beta})\mu\nu} - H_{\mu\nu\rho(\Gamma^\rho)_{\hat{\delta}\hat{\beta}}} = 0, \quad (3.9)$$

which is satisfied provided we make the identification

$$H_{\mu\nu\rho} = h_{\mu\nu\rho} \quad (3.10)$$

and use equation (2.15). The term with one spinorial index is identically zero due to (2.17) and finally the term with no spinorial indices in the Bianchi identity is zero due to the equation of motion of $h_{\mu\nu\rho}$ and the identification (3.10).

In brief, the constraints (3.4) and (3.2) for the closed super three-form $H$ are equivalent to the constraints (2.5): from the superfield $\Phi^i$ we can construct a closed super three form verifying (3.1) and (3.2), and, conversely, from the constraints (3.1) and (3.2) on a closed super three-form, we get a scalar superfield verifying (2.3).
4 Sigma model

In this section, we search for sigma-model generalizations of the constraint (2.5). For (1,0) theories, this was done in [16]. The five dimensional target space is assumed to be described by a moving basis $e^I_i(\Phi)$, where $I = 1, \ldots, 5$ is a flat index of $SO(5)$ and $i$ is a curved space index. We generalise the constraint (2.5) to have the form

$$e^I_i(\Phi)D_{aoa}\Phi^i = \frac{1}{4}(\gamma^I_J)_a^b e^J_j(\Phi)D_{ab}\Phi^j. \quad (4.1)$$

From (4.1), we can deduce, as in section 2, the transformations of the fields under supersymmetry, their equations of motions and, in addition, the constraints on the geometry of the target space.

The transformation rule for $\phi^i$ can be deduced from the $\theta = 0$ component of the equation

$$D_{aa}\Phi^i = e^I_i(\gamma^I_J)_a^b \Psi_{ab}. \quad (4.2)$$

In order to get the constraints on the moving basis $e^I_i(\Phi)$, we take the spinorial derivative of (4.1) to obtain

$$e^I_i(\Phi)D_{ac}\Phi^i - \frac{1}{4}(\gamma^I_J)_a^b e^J_j(\Phi)D_{ac}\Phi^j = f^I_{\gammaaca}, \quad (4.3)$$

where we used the definition

$$f^I_{\gammaaca} = \left(\frac{1}{4}\gamma^I_J - \eta^I_J\right)_a^b e^J_j(\Phi)D_{ac}\Phi^j. \quad (4.4)$$

It is convenient, for the analysis of (4.3), to use the following decomposition on the $SO(5)$ gamma matrices

$$f^I_{\gammaaca} = f^I_{\gamma\Omega ca} + f^I_{\gamma\gamma \Omega} + f^I_{JK\gamma\gamma J\Omega} \quad (4.5)$$

Then equation (4.3), using the decomposition (2.6), gives the following relations which replace eqs (2.10) and (2.11)

\[
\begin{align*}
    e^I_J D_{(\gamma\alpha)}\Phi^j &= f^I_{(\gamma\alpha)}, \\
    e^I_J D_{K(\gamma\alpha)}\Phi^j &= \frac{1}{5}\eta_{IK}\epsilon^J_j D_{J(\gamma\alpha)}\Phi^j + \frac{16}{15} f_{JK(\gamma\alpha)} - \frac{4}{15} f_{KI(\gamma\alpha)}, \\
    e^I_J D_{MN[\gamma\alpha]}\Phi^j &= \frac{1}{2} \left[ \eta_{MIJ}\epsilon^N_j (\gamma^\mu)\gamma_\alpha \partial_\mu \Phi^j - \eta_{NJM}(\gamma^\mu)\gamma_\alpha \partial_\mu \Phi^j \right] \\
    &- \frac{1}{5} \epsilon^{JK}_{IMN} f_{JK[\gamma\alpha]} + \frac{2}{5} (\eta_{MI}\epsilon_{N[\gamma\alpha]} - \eta_{NJ}\epsilon_{M[\gamma\alpha]}) + \frac{4}{5} f_{IMN[\gamma\alpha]}, \\
\end{align*}
\]
as well as the following constraints on \( f^I \gamma_a c_a \):

\[
f^I I_a = 0, \quad f(1K)[\gamma_a] = 0, \quad f(I)J[I][\gamma_a] = 0, \quad f^I J[I][\gamma_a] = -2f^I J[I][\gamma_a],
\]

\[
f^I J[I][\gamma_a] = 2f^I J[I][\gamma_a], \quad 6f[IJM][\gamma_a] = -\epsilon_{IMN}^J f[IJM][\gamma_a].
\]

(4.9)

The structure of \( f^I \gamma_a c_a \) given in (4.4) implies a number of constraints which are identically satisfied. These are given by

\[
f^I I_a = 2f^I J[I][\gamma_a], \quad f^I I_a = 0, \quad 6f[IJM][\gamma_a] = -\epsilon_{IMN}^J f[IJM][\gamma_a].
\]

(4.10)

(4.11)

So only the second constraint in (4.9) and (4.11) are not identically satisfied. It turns out that these two imply that the moving basis must be such

\[
de^I \wedge e^I + de^I \wedge e^I - \frac{2}{3} \eta^I J \eta K L de^K \wedge e^L = 0.
\]

(4.12)

It is possible to solve equations (4.13) to get \( e^I = \Sigma d^I \) for some functions \( \Sigma \) and \( \Upsilon^I \) which means that the target space is conformally flat. By a change of coordinates in the target space \( \Phi^I \rightarrow \Upsilon^I \) the theory is transformed to the free theory of section 2. So no non-trivial sigma-models are allowed by (2,0) supersymmetry.

5 Superconformal transformations

In this section we give a geometrical construction of the superconformal transformations and determine explicitly the realisation of the generators in the (2,0) superspace.

The flat supersymmetric metric in superspace is given by

\[
g = \eta_{\mu\nu} E^\mu \otimes E^\nu.
\]

(5.1)

Notice that the other term appearing \( a priori \) in the (2,2) superspace, \( C_{\alpha \beta} E^\alpha \otimes E^\beta \), is forbidden by chirality in (2,0) theories. A supercoordinate transformation is generated by an even vector field \( \xi = \xi^\mu E_\mu + \xi^\beta E_\beta \), where \( \xi^\mu \) is even and \( \xi^\beta \) is odd. Under this transformation, the supercoordinates \( Z^M \) transform as

\[
\delta x^\mu = \xi^\mu + \theta^\alpha (\Gamma^\mu)_{\dot{a}\dot{b}} \xi^{\dot{b}}, \quad \delta \theta^{\dot{a}} = \xi^{\dot{a}},
\]

(5.2)

and the metric varies as

\[
\delta g = L_\xi g,
\]

(5.3)
where $L_\xi$ is the Lie derivative with respect to the vector $\xi$. A superconformal transformation is defined by the requirement that the transformed metric be proportional to the initial one, that is

$$\delta g = \alpha(Z)g,$$

(5.4)

where $\alpha$ is a priori an arbitrary superfield. The vector field $\xi$ is said to be a superconformal Killing vector. The use of the relation $[L_\xi, L_{\xi'}] = L_{[\xi, \xi']}$, where $[,]$ is the Lie bracket, shows that if $\xi$ and $\xi'$ are two superconformal Killing vector fields with scales $\alpha$ and $\alpha'$ then $[\xi, \xi']$ is also a superconformal Killing vector with scale $\xi(\alpha') - \xi'(\alpha)$, so that the set of all $\xi'$'s forms a Lie algebra.

In order to determine explicitly the superconformal Killing vectors we first calculate the variation of the basis super one-forms:

$$\delta E^\mu = (d\iota_\xi + \iota_\xi d)E^\mu = (\partial_\nu \xi^\mu)E^\nu + (D_\hat{\alpha} \xi^\mu + 2(\Gamma^\mu)_{\hat{\alpha}\hat{\beta}} \xi^{\hat{\beta}})E^{\hat{\alpha}},$$

(5.5)

so the condition (5.4) is verified provided

$$D_\hat{\alpha} \xi^\mu + 2(\Gamma^\mu)_{\hat{\alpha}\hat{\beta}} \xi^{\hat{\beta}} = 0,$$

(5.6)

$$\partial_\mu \xi_\nu + \partial_\nu \xi_\mu = \alpha \eta_{\mu\nu}.$$  

(5.7)

We shall show that the equation (5.7) is a consequence of (5.6), so that the solutions of the latter determine all superconformal Killing vectors. Note that, by equation (5.6), $\xi^{\hat{\beta}}$ is determined in terms of $\xi^\mu$ by

$$\xi^{\hat{\beta}} = -\frac{1}{12} (\Gamma^\mu)^{\hat{\beta}\hat{\alpha}} D_\hat{\alpha} \xi^\mu.$$  

(5.8)

This allows to write equation (5.6) in terms of $\xi^\mu$ as

$$D_\hat{\alpha} \xi^\mu = \frac{1}{6} (\Gamma^{\mu\nu})^{\hat{\beta}} D_\hat{\beta} \xi_\nu,$$

(5.9)

which is very similar to the constraint of the scalar superfield (2.3), the vectorial structure in $SO(5)$ being replaced by the same one in $SO(1,5)$. In order to analyse equation (5.9) it is convenient to decompose the product of two spinorial derivatives as

$$D_\hat{\alpha}D_\hat{\beta} = -(\Gamma^{\nu})_{(\hat{\alpha}\hat{\beta})} \partial_\nu + (\Gamma^{\nu ij})_{(\hat{\alpha}\hat{\beta})} D_{\nu[ij]}$$

$$+ (\Gamma^{\mu_1\mu_2\mu_3})_{(\hat{\alpha}\hat{\beta})} D_{\mu_1\mu_2\mu_3} + (\Gamma^{\mu_1\mu_2\mu_3})_{(\hat{\alpha}\hat{\beta})} D_{\mu_1\mu_2\mu_3},$$

(5.10)

then taking the spinorial derivative of (5.9) yields

$$\partial_\mu \xi_\nu + \partial_\nu \xi_\mu = \frac{1}{3} \partial_\rho \xi^\rho \eta_{\mu\nu},$$

(5.11)

$$D_{\nu[ij]} \xi_\mu = \frac{1}{6} D_{\rho[ij]} \xi^\rho \eta_{\nu\mu},$$

(5.12)

$$D_{\mu_1\mu_2\mu_3} \xi_{\mu_4} = -\frac{1}{4} \left( \eta_{\mu_4[\mu_1} \partial_{\mu_2} \xi_{\mu_3]} - \frac{1}{6} \xi_{\mu_1\mu_2\mu_3} \eta_{\mu_4[\nu_1} \partial_{\nu_2} \xi_{\nu_3]} \right),$$

(5.13)

$$D_{\mu_1\mu_2\mu_3\nu} \xi_{\mu_4} = 0.$$  

(5.14)
The first equation is equivalent to (5.1) which shows that (5.7) is contained in (5.6). Let $\zeta_\mu(x)$, $\zeta^\dot{\alpha}(x)$ and $\zeta^{ij}(x)$ be the $\theta = 0$ components of $\xi_\mu$, $\xi^\dot{\alpha}$ and $D_\rho[x][\xi^\rho]$, then equations (5.11, 5.12, 5.13) and (5.14) show that the superfield $\xi_\mu$ is determined in terms of the $\zeta$'s. These are solutions to the following decoupled equations which are consequences of (5.11)

$$
\partial_\mu \zeta_\nu + \partial_\nu \zeta_\mu = \frac{1}{3} \partial_\rho \zeta^\rho \eta_{\mu\nu},
$$

(5.15)

$$
(\Gamma_\mu)_{\dot{\alpha}\dot{\beta}} \partial_\nu \xi^\dot{\beta} + (\Gamma_\nu)_{\dot{\alpha}\dot{\beta}} \partial_\mu \zeta^\dot{\beta} = \frac{1}{3} \eta_{\mu\nu} (\Gamma_\rho)_{\dot{\alpha}\dot{\beta}} \partial^\rho \zeta^\dot{\beta};
$$

(5.16)

$$
\partial_\mu \xi^{ij} = 0.
$$

(5.17)

The solutions to (5.15) are the well-known conformal Killing vectors

$$
\zeta_\mu = a_\mu + a_{(\mu\nu)} x^\nu + \lambda x_\mu + (x^2 \eta_{\mu\nu} - 2 x_\mu x_\nu) k^\nu,
$$

(5.18)

where $a_\mu$, $a_{\mu\nu}$, $\lambda$ and $k_\mu$ are parameters of infinitesimal translations, Lorentz transformations, dilatations and special conformal transformations. Similarly, the solutions to (5.16) are determined in terms of two constant spinors $\epsilon$ and $\eta$ (respectively simplectic-Majorana-Weyl and anti-simplectic-Majorana-Weyl spinors) as

$$
\zeta^\dot{\beta} = \epsilon^\dot{\beta} + x_\mu (\Gamma_\mu)^{\dot{\beta}}_{\dot{\alpha}} \eta^{\dot{\alpha}},
$$

(5.19)

$\epsilon$ is the parameter of a supersymmetry transformation and $\eta$ that of a special supersymmetry transformation. Finally the solution of (5.17) is given by

$$
\xi^{ij} = \frac{1}{4} \epsilon^{[ij]},
$$

(5.20)

where $\epsilon^{[ij]}$ are constants which represent infinitesimal $SO(5)$ rotations. The complete $\theta$ expansion of the superfield $\xi_\mu$ follows from the solutions (5.18, 5.19, 5.20) and from the equations (5.11, 5.12, 5.13) after some tedious algebra as

$$
\xi_\mu = \zeta_\mu - 2 \overline{\theta} \Gamma_\mu \zeta + \theta \left( \Gamma^{ij} \zeta_{ij} + \frac{1}{4} \Gamma^{\mu\nu\rho\sigma} \partial_\mu \zeta_{\nu\rho} \right) \theta
$$

$$
+ \frac{1}{2} \theta \Gamma_{\mu\nu\rho} \partial_{\mu} \zeta_{\nu\rho} - \frac{1}{64} \theta \Gamma_{\mu\nu\rho\sigma} \partial_\mu \zeta_{\nu\rho} \partial_{\sigma} \partial^\rho \zeta_{\nu\sigma}.
$$

(5.21)

In order to have the complete expression of the superconformal Killing vector field $\xi$ we have to calculate $\xi^{\dot{\alpha}}$ from equation (5.8). the result, after some arrangements is

$$
\xi^{\dot{\alpha}} = \left[ \zeta - \Gamma_{ij} \theta \zeta^{ij} + \frac{1}{12} \theta \partial_\mu \zeta^\mu - \frac{1}{4} \Gamma^{\mu\nu} \theta \partial_\mu \zeta_{\nu}
$$

$$
- \frac{1}{6} \theta \overline{\theta} \Gamma^{\mu\nu} \partial_\mu \zeta - \frac{1}{2} \Gamma^{\mu\nu} \theta \partial_\mu \zeta_{\nu} - \frac{1}{24} \Gamma_{\mu\nu\rho\sigma} \partial_\mu \zeta_{\nu\rho} \partial_\sigma \partial^\rho \zeta_{\nu\sigma}
$$

$$
+ \frac{1}{32} \Gamma^{\mu\nu\rho} \partial_{\mu} \zeta_{\nu\rho} \theta \partial_{\sigma} \partial^\sigma \zeta_{\nu\sigma}ight]^{\dot{\alpha}}.
$$

(5.22)
It is also possible to determine the scale $\alpha$,

$$\alpha = \frac{1}{3} \left[ \partial_\mu \zeta^\mu - 2 \partial \Gamma^\mu \partial_\mu \zeta \right]. \quad (5.23)$$

The Lie algebra of superconformal transformations can be deduced from the calculation of the Lie bracket of two super-Killing vectors $\xi_1$ and $\xi_2$ which we denote by $\xi_3$. Let the parameters determining the $\xi_a$ ($a = 1, 2, 3$) be given by $a_\mu^a$, $a_\mu^{a\nu}$, $\lambda_a$, $k_\mu^a$, $\epsilon_a$, $\eta_a$ and $\epsilon_{ij}^a$ then the parameters of $\xi_3$ are given by

$$a_3^\mu = a_1^\mu a_2^\nu - a_2^\mu a_1^\nu + \lambda_2 a_1^\mu - \lambda_1 a_2^\mu - 2 \bar{\epsilon}_1 \Gamma^\mu \epsilon_2,$$
$$a_3^{\mu\nu} = 2 \left( a_1^\mu k_2^\nu - a_1^\nu k_2^\mu - a_2^\mu k_1^\nu + a_2^\nu k_1^\mu + \bar{\epsilon}_2 \Gamma^{\mu\nu} \eta_1 - \bar{\epsilon}_1 \Gamma^{\mu\nu} \eta_2 \right) + a_1^\mu a_2^\rho - a_2^\nu a_1^\rho,$$
$$\lambda_3 = -2 \left( a_1^\mu k_2^\nu - a_2^\mu k_1^\nu + \bar{\epsilon}_1 \eta_2 - \bar{\epsilon}_2 \eta_1 \right),$$
$$k_3^\mu = a_2^\mu k_1^\nu - a_1^\mu k_2^\nu + \lambda_1 k_2^\mu - \lambda_2 k_1^\mu - 2 \bar{\eta}_1 \Gamma^{\mu\nu} \eta_2,$$
$$\epsilon_3 = a_1 \Gamma^{\mu\nu} \eta_2 - a_2 \Gamma^{\mu\nu} \eta_1 + \frac{1}{4} \left( \epsilon_{ij}^{k} \Gamma_{ij} \epsilon_2 - \epsilon_{ij}^{k} \Gamma_{ij} \epsilon_1 \right) - \frac{1}{2} \left( \lambda_1 \epsilon_2 - \lambda_2 \epsilon_1 \right)$$
$$- \frac{1}{4} \left( a_1 \mu \Gamma^{\mu\nu} \eta_2 - a_2 \mu \Gamma^{\mu\nu} \eta_1 \right),$$
$$\eta_3 = k_1 \mu \Gamma^{\mu\nu} \epsilon_2 - k_2 \mu \Gamma^{\mu\nu} \epsilon_1 + \frac{1}{4} \left( \epsilon_{ij}^{k} \Gamma_{ij} \eta_2 - \epsilon_{ij}^{k} \Gamma_{ij} \eta_1 \right) + \frac{1}{2} \left( \lambda_1 \eta_2 - \lambda_2 \eta_1 \right) - \frac{1}{4} \left( a_1 \mu \Gamma^{\mu\nu} \eta_2 - a_2 \mu \Gamma^{\mu\nu} \eta_1 \right),$$
$$\epsilon_{ij}^3 = \epsilon_{ik}^3 \epsilon_{jk}^3 - \epsilon_{ik}^3 \epsilon_{jk}^3 - 4 \left( \epsilon_1 \Gamma^{\mu\nu} \eta_2 - \epsilon_2 \Gamma^{\mu\nu} \eta_1 \right). \quad (5.24)$$

These relations encode the Lie algebra of superconformal transformations. It can be readily verified that this Lie algebra is that of $OSp(6,2|2)$ given in reference [18].

6 Superconformal invariance

The vector fields $\xi$, in general, do not commute with $D_\bar{\alpha}$; using the relation (5.4) on the Killing vector, the precise commutation relations is given by

$$[D_{\bar{\alpha}}, \xi] = (D_{\bar{\alpha}} \xi)_{\bar{\beta}} D_{\bar{\beta}}. \quad (6.1)$$

This equation shows that under a superconformal transformation the vector fields $D_{\bar{\alpha}}$ are not invariant. However they transform in such a way as not to mix with the $\partial_\mu$. This could have been also a starting point for the definition of superconformal transformations. For future use it is important to explicit the right hand side of (6.1) as

$$D_{\bar{\alpha}} \xi_{\bar{\beta}} = -\frac{1}{4} \left( \Gamma^{\mu\nu} \right)_{\bar{\beta}\bar{\delta}} \partial_\mu \xi_\nu + \frac{1}{12} C_{\bar{\beta}\bar{\gamma}} \partial_\sigma \xi^\sigma + \frac{1}{12} \left( \Gamma^{ij} \right)_{\bar{\beta}\bar{\sigma}} D_{\sigma[ij]} \xi^\sigma, \quad (6.2)$$
where $D_{\sigma[ij]}\xi^\sigma$ is given, from (5.21), by

$$D_{\sigma[ij]}\xi^\sigma = -12\zeta_{ij} + 2\bar{\theta}\Gamma_{ij}^\sigma \partial_\sigma \zeta - \frac{3}{4} \bar{\theta}\Gamma_{\nu}^i \Gamma_{ij}^\nu \partial_\sigma \partial^\sigma \zeta^\nu. \quad (6.3)$$

From equation (6.1) we can easily deduce that the constraint (2.5) is not invariant under the transformations $\delta\Phi^i = \xi(\Phi^i)$. This motivates the introduction of a connection $\Lambda^i_j(\xi)$ so that the transformation of $\Phi^i$ becomes

$$\delta_\xi\Phi^i = \xi(\Phi^i) + \Lambda^i_j(\xi)\Phi^j. \quad (6.4)$$

We shall explicitly construct $\Lambda(\xi)$ later. Here, we examine some of its mathematical properties. In fact $\Lambda$ is not strictly a connection because in general we do not have $\Lambda(f\xi) = f\Lambda(\xi)$, it is only a cochain on the Lie algebra of superconformal transformations realised with the superconformal Killing vectors. It is valued in the tensor product of the algebra of superfields and the algebra of $5 \times 5$ matrices. This cochain is however not arbitrary. The requirement $[\delta_\xi, \delta_\xi'] = \delta[\xi, \xi']$ gives the following consistency condition on $\Lambda(\xi)$

$$\xi(\Lambda(\xi')) - \xi'(\Lambda(\xi)) - \Lambda([\xi, \xi']) + [\Lambda(\xi), \Lambda(\xi')] = 0, \quad (6.5)$$

where we have used a matrix notation for $\Lambda$. In order to make the algebraic meaning of (6.5) clearer, recall that given a $n$-cochain on a Lie algebra, $g$, $\alpha(g_1, \ldots, g_n)$ which is valued in a $g$-module $\mathcal{M}$ then the Chevalley exterior derivative $s\alpha$ is a $n+1$-cochain given by

$$s\alpha(g_1, \ldots, g_{n+1}) = \sum_{1 \leq i \leq n+1} (-1)^{i-1} g_i \alpha(g_1 \ldots \hat{g}_i \ldots g_{n+1})$$

$$+ \sum_{1 \leq i < j \leq n+1} (-1)^{i+j} \alpha([g_i, g_j]g_1 \ldots \hat{g}_i \ldots \hat{g}_j \ldots g_{n+1}). \quad (6.6)$$

where the notation $\hat{g}_i$ means that the element $g_i$ has been omitted. The Chevalley exterior derivative verifies $s^2 = 0$ and the graded Leibniz rule. This allows to write condition (6.5) as

$$s\Lambda + \Lambda^2 = 0, \quad (6.7)$$

where the exterior product of two cochains is defined in a way similar to the exterior product of two forms. The algebraic interpretation of condition (6.5) is thus that $\Lambda$ has vanishing curvature. A particular solution to (6.7) is given by a "pure gauge"

$$\Lambda = \beta^{-1}s\beta, \quad (6.8)$$

where $\beta$ is a 0-cochain. Equation (6.8) reads explicitly

$$\Lambda(\xi) = \beta^{-1}\xi(\beta). \quad (6.9)$$
Note that if $\Lambda$ is a pure gauge then if we define $\Phi'$ by $\beta \Phi$ we get $\delta \Phi' = \xi(\Phi')$, so that a rescaling of the superfield allows the elimination of $\Lambda$. We shall show below that the correct $\Lambda$ is not a pure gauge but has vanishing curvature.

In order to explicitly determine $\Lambda(\xi)$, we demand that if $\Phi$ verifies the constraint (2.5) then so does $\delta \Phi = \xi(\Phi) + \Lambda(\xi) \Phi$. This is true provided

\[
\begin{align*}
D^\alpha \Lambda^i_j - \frac{1}{4} (\Gamma^i_k)_\alpha ^\beta D^\beta_j \Lambda^k_j = 0, \\
(\gamma^i_j)_a^b D_{ab} \xi^{bc} - D_{aa} \xi^{\beta b}(\gamma^i_j)_b^c - ((\gamma^i_j)_a^c \Lambda^k_j - \Lambda^i_k(\gamma^k_j)_a^c) \delta^\alpha_\beta = 0.
\end{align*}
\]

(6.10)

By replacing $D^\alpha \xi_\beta$ in equation (6.11) by its expression (6.2) we determine $\Lambda_{ij}$ up to an arbitrary function $\chi$ as

\[
\Lambda_{ij} = -\frac{1}{3} D^\nu [ij] \xi^\nu + \eta_{ij} \chi.
\]

(6.12)

This expression shows that $\Lambda$ is actually valued in the Lie algebra of $u(1) \oplus \mathfrak{so}(5)$ rather than $\mathfrak{gl}_5$. The function $\chi$ is calculated with the aid of equation (6.10) after using the explicit expression of $D^\nu [ij] \xi^\nu$ given in (6.3) as well as its spinorial derivative which is given by

\[
D^\alpha D^\beta [ij] \xi^\gamma = 2(\Gamma^\nu \Gamma^i_j)_{\alpha \beta} \partial_\nu \xi^\gamma.
\]

(6.13)

The resulting expression for $\chi$ turns out to be very simple

\[
\chi = \frac{1}{3} \partial_\mu \xi^\mu + \lambda' = \alpha + \lambda',
\]

(6.14)

where $\lambda'$ is an arbitrary constant which has to be set to zero in order to have $\Lambda(0) = 0$. Finally, we are in position to deduce the complete expression for the 1-cochain $\Lambda_{ij}$ as

\[
\Lambda_{ij}(\xi) = 4\xi_{ij} - \frac{2}{3} \theta \Gamma^i_j \Gamma^\sigma \partial_\sigma \xi + \frac{1}{4} \theta \Gamma^\nu \Gamma^i_j \theta \partial_\nu \partial^\sigma \xi^\nu + \frac{1}{3} \eta_{ij} \partial_\mu \xi^\mu.
\]

(6.15)

The above expression for $\Lambda$ shows clearly that it is a non-trivial zero-curvature 1-cochain because it cannot be written as a “pure gauge”-cochain of the form (6.9). It remains to check that $\Lambda$ indeed verifies the consistency condition (6.7).

Note first that the $u(1)$ part of (6.7) reads

\[
s\chi = 0,
\]

(6.16)

which is true because $\chi$ is given by equation (6.14) and $\alpha$ is closed : $\alpha([\xi, \xi']) = \xi(\alpha(\xi')) - \xi'(\alpha(\xi))$. The antisymmetric part of $\Lambda$ has also a vanishing curvature. This can be verified by a direct calculation using (5.24) but a more simple way is to examine the transformation property of the super three-form.
We shall show that the simple geometrical transformation
\[ \delta H = L_\xi H \] (6.17)
leaves the constraints (3.1) and (3.2) invariant. Note first that the transformation of the moving basis \( \hat{E}^\mu \) does not involve the spinorial basis \( \hat{E}^\alpha \), so the constraint (3.1) is left invariant by (6.17). The transformation of the \( \mu\hat{\alpha}\hat{\beta} \) components of \( H \) under (6.17) reads
\[ \delta H_{\mu\hat{\alpha}\hat{\beta}} = \xi(H_{\mu\hat{\alpha}\hat{\beta}}) + H_{\nu\hat{\alpha}\hat{\beta}} \partial_\mu \xi^\nu + H_{\mu\hat{\alpha}\delta} D_\beta \xi^\delta + H_{\mu\beta\delta} D_\alpha \xi^\delta. \] (6.18)
The use of relation (6.2) allows to write
\[ \delta H_{\mu\hat{\alpha}\hat{\beta}} = (\Gamma_\mu \Gamma_i)_{\hat{\alpha}\hat{\beta}} \delta \Phi^i, \] (6.19)
with \( \delta \Phi^i \) given by
\[ \delta \Phi^i = \xi(\Phi^i) + \frac{1}{3} \left( \partial_\mu \xi^\mu \Phi^i - D_\sigma \xi^\sigma \Phi^i \right). \] (6.20)
Relation (6.19) shows that the constraint (3.2) is left invariant by the transformation (6.17). In addition we can identify the transformation of \( \Phi^i \) which agrees, as it should, with the previously calculated one. This geometrical construction of \( \Lambda \) assures the vanishing curvature property due to the Lie structure of the transformations of the three-form \( H \).

7 Conclusion

We have given two equivalent formulations of the tensorial multiplet of 6D (2,0) theories. We have illustrated the rigidity of (2,0) supersymmetries by showing the triviality of the sigma-model. We note here that this triviality can be understood from the super three-form point of view; indeed, the generalization of the constraints (3.2) to a curved target space manifold, \( H_{\mu\hat{\alpha}\hat{\beta}} = (\Gamma_{\mu I})_{\hat{\alpha}\hat{\beta}} \varepsilon^I_i \varepsilon^j \Phi^j \) does not lead to an interacting sigma-model. Note that the five scalars \( \phi^i \) describe the transverse fluctuations of the five-brane. The constraint (2.5) coincides with the equation obtained in [3] using the superembedding formalism applied for a five-brane in a super-flat background, in the physical gauge and in the linearised approximation. This suggests the existence of a formulation of the full non-linear equations of motion of the five-brane which is analogous to the one presented in section 3 and which is based on constraints on a three-form in a non super-flat background. We hope to come back to this issue in more details elsewhere.

We have also realised the superconformal transformations as derivations in superspace. This gives a geometric construction of the superconformal Lie algebra.
compared to the algebraic one presented in [18]. Moreover, this allows to realise, in a manifestly supersymmetric way, the transformation of the supermultiplet. This gives an alternative proof to the one relying on the component formalism presented in [18] that the linearised equations of motion of the five-brane are superconformally invariant.

We have found that the transformation of the three-form involves only coordinate reparametrisations whereas, for the scalar superfield, the introduction of a cochain is in addition needed. This suggests that the formulation in terms of super three-forms may be helpful in the construction of superconformal interacting theories and shed some light on the conjecture [8] relating them to $M$-theory. In this respect, recently it has been shown [19] that the bosonic action for $M$-theory five-branes in their near horizon background have a non-linearly realised conformal invariance. Our results may be useful in extending this invariance to a superconformal one.

### A Conventions

In this Appendix we collect our conventions and notations. The 6D $(2,0)$ supersymmetry algebra is conveniently described as a chiral truncation of the reduction of the 11D algebra. The 11D superalgebra reads

$$\{Q_{\hat{\alpha}}, Q_{\hat{\beta}}\} = 2(\Gamma^{\hat{\mu}} C)_{\hat{\alpha}\hat{\beta}} P_{\hat{\mu}}, \quad (A.1)$$

where $\hat{\alpha} = 1, \ldots, 32$, $\hat{\mu} = 0, \ldots, 10$ and $C_{\hat{\alpha}\hat{\beta}}$ is an antisymmetric matrix verifying

$$C^{-1} \Gamma^{\hat{\mu}} C = -\Gamma^{\hat{\mu}T}. \quad (A.2)$$

The reality condition on 11D fermions reads

$$\Psi = C\bar{\Psi}^T, \quad (A.3)$$

or equivalently

$$\bar{\Psi}^{\hat{\alpha}} = C^{\hat{\alpha}\hat{\beta}} \Psi_{\hat{\beta}} \equiv \Psi^{\hat{\alpha}}, \quad (A.4)$$

where $C^{\hat{\alpha}\hat{\beta}}$ is the inverse of $C_{\hat{\alpha}\hat{\beta}}$. We shall use $C$ to raise and lower indices and the notation $(\Gamma^{\mu})_{\hat{\alpha}\hat{\beta}}$ for $(\Gamma^{\mu} C)_{\hat{\alpha}\hat{\beta}}$. Under reduction to six dimensions the spinorial index $\hat{\alpha}$ decomposes as $aa$ where $\alpha$ is an $SO(5,1)$ spinorial index and $a$ an $SO(5)$ spinorial index. A representation of the Gamma matrices is conveniently given by

$$\Gamma^{\mu} = \gamma^{\mu} \otimes 1, \quad \Gamma^{5+i} = \tilde{\gamma} \otimes \gamma^i, \quad (A.5)$$
where \( \mu \) and \( i \) are respectively six and five dimensional vector indices, and \( \tilde{\gamma} \) is the chirality matrix in six dimensions:

\[
\tilde{\gamma} = \gamma^0 \ldots \gamma^5 .
\]  
(A.6)

We shall also denote \( \Gamma^{5+i} \) by \( \Gamma^i \). In this representation the charge conjugaison matrix \( C \) may be written as

\[
C = C \otimes \Omega ,
\]  
(A.7)

where \( C \) is symmetric and verifies

\[
C^{-1} \gamma^\mu C = -\gamma^\mu T ,
\]  
(A.8)

whereas \( \Omega \) is antisymmetric and verifies

\[
\Omega^{-1} \gamma^i \Omega = \gamma^i T .
\]  
(A.9)

Spinorial indices of \( SO(5, 1) \) and \( SO(5) \) can be raised and lowered with repctively \( C \) and \( \Omega \). The reduction of the algebra (A.1) to six dimensions leads to the \((2,2)\) algebra

\[
\{ Q^+, Q^+ \} = 2\Pi^+ \gamma^\mu P_\mu \Pi^+ ,
\]  
(A.10)

\[
\{ Q^-, Q^+ \} = 2\Pi^- \gamma^i c_i \Pi^+ ,
\]  
(A.11)

\[
\{ Q^-, Q^- \} = 2\Pi^- \gamma^\mu P_\mu \Pi^- ,
\]  
(A.12)

where the five-dimensional momentum appears as a central charge, and \( \Pi^\pm \) are the projectors on fermions of a given six-dimensional chirality. The \((2,2)\) algebra is invariant under the transformation \( Q^- \rightarrow -Q^-, c \rightarrow -c \). Modding out with respect to this symmetry leads to the desired \((2,0)\) algebra. This is obtained by setting \( Q^- = c^i = 0 \) in the above formulae. As is evident from its construction this algebra has a \( Spin(5) = Sp(2) \) R-symmetry. The 11D Majorana fermion becomes a \( Spin(5) \) Majorana-Weyl six dimensional fermion :

\[
\Psi = C\bar{\Psi}^T ,
\]  
(A.13)

which reads in components

\[
\Psi_a = \Omega_{ab} C\bar{\Psi}^b T .
\]  
(A.14)

The antisymmetrised product of \( n \) gamma matrices is denoted by \( \Gamma^{\mu_1 \ldots \mu_n} \). We have

\[
(\Gamma^{\mu_1 \ldots \mu_n} C)^T = -(\text{1})^{n(n+1)/2} \Gamma^{\mu_1 \ldots \mu_n} C ,
\]  
(A.15)
from which we get
\[
(\gamma^{\mu_1 \cdots \mu_n} C)^T = (-1)^{n(n+1)/2} \gamma^{\mu_1 \cdots \mu_n} C, \tag{A.16}
\]
\[
(\gamma^{i_1 \cdots i_n} \Omega)^T = -(-1)^{n(n-1)/2} \gamma^{i_1 \cdots i_n} \Omega. \tag{A.17}
\]

A useful relation is the Fierz rearrangement formula which reads for four Weyl-Majorana fermions \cite{20}
\[
(\bar{\epsilon}_1 \Pi^+ \epsilon_2) (\bar{\epsilon}_3 \Pi^+ \epsilon_4) = \]
\[- \sum_{n_1=0,2, n_2=0,1,2} \frac{2}{c_{n_1} \bar{c}_{n_2}} (\bar{\epsilon}_1 \Gamma^{\mu_1 \cdots \mu_n} \Gamma^{i_1 \cdots i_n} \Pi^+ \epsilon_4) (\bar{\epsilon}_3 \Gamma_{\mu_1 \cdots \mu_n} \Gamma_{i_1 \cdots i_n} \Pi^+ \epsilon_2), \tag{A.18}
\]
with coefficients \( c_n \) and \( \bar{c}_n \) given by
\[
c_n = 8 (-1)^{n(n-1)/2} \quad \text{and} \quad \bar{c}_n = 4 (-1)^{n(n-1)/2}. \tag{A.19}
\]

Using the Fierz rearrangement for commuting spinors \( E \) and \( F \), we get the useful relations
\[
(\bar{E} \Gamma_{\mu} E) (\bar{E} \Gamma^\mu E) = 0, \tag{A.20}
\]
and
\[
(\bar{F} \Gamma_{\mu} E) (\bar{E} \Gamma^\nu E) + (\bar{F} \Gamma^i E) (\bar{E} \Gamma_{\mu i} E) = 0. \tag{A.21}
\]
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