ELECTROWEAK MEASUREMENTS AND TOP QUARK MASS LIMITS

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ABSTRACT

The agreement of electroweak measurements with theory places limits on the masses of the top quark and the W boson. It is shown how these limits arise and what constraints various measurements (particularly a top quark mass determination) would provide on the theory. The degree to which present and future measurements can constrain the Higgs boson mass is examined.

1. Introduction

The unified description of weak and electromagnetic interactions has led to a series of predictions which are in accord with all present data, including recent measurements from $e^+e^-$ collisions at the Z mass, $p\bar{p}$ collisions at high energies, deep inelastic scattering of neutrinos, and parity violation in atoms. This agreement is so good that it constrains higher-order effects, particularly that of the top quark mass. In the present paper we examine such constraints in the light of recent data, and extend the analysis of a recent review [1] to parameters $S$ and $T$ which describe effects of new physics [2].

2. Electroweak theory and radiative corrections

The electroweak theory replaces the Fermi coupling constant, $G_F = 1.16637 \pm 0.00002 \times 10^{-5}$ GeV$^{-2}$, by combinations of dimensionless couplings and masses:

$$ G_F \sqrt{2} = \frac{g^2}{8M_W^2}, \quad G_F \sqrt{2} = \frac{g^2 + g'^2}{8M_Z^2}, $$

(1)

where $(g, g')$ are the SU(2) and U(1) couplings. The electric charge $e$ is related to $g$ and $g'$ by $e = g \sin \theta = g' \cos \theta$, so that

$$ M_W^2 = \frac{\pi \alpha}{\sqrt{2}G_F \sin^2 \theta}, \quad M_Z^2 = \frac{\pi \alpha}{\sqrt{2}G_F \sin^2 \theta \cos^2 \theta}. $$

(2)

The value of $M_Z = 91.187 \pm 0.007$ GeV [3] then can be used to predict the value of $\theta$ in the lowest-order theory, leading to a value of $M_W$.

The electromagnetic charge when probed at the scale of $M_W$ or $M_Z$ is slightly stronger than that at long distances as a result of vacuum polarization effects.

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The fine-structure constant, instead of being about 1/137, is about 1/128. This modification is crucial in obtaining a value of $M_W$ from the above procedure which is close to the experimental average \[4, 5\].

The major effect of a top quark mass is a modification of the relation between $G_F$ and $M_Z$:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2 + g'^2}{8M_W^2}, \quad \rho \simeq 1 + \frac{3G_Fm_t^2}{8\pi^2\sqrt{2}}.$$  \hfill (3)

The quadratic dependence on $m_t$ comes from the top quark’s contribution to $W$ and $Z$ self-energy diagrams. No such quadratic dependence appears in the photon vacuum polarization because of electromagnetic gauge invariance. The relation for $M_Z$ in terms of $\theta$ now becomes

$$M_Z^2 = \frac{\pi\alpha}{\sqrt{2}G_F\rho \sin^2\theta \cos^2\theta}. \hfill (4)$$

Higgs boson contributions to $W$ and $Z$ self-energies lead to an additional term

$$\Delta \rho = -\frac{3}{8\pi \cos^2\theta} \ln \frac{M_H}{M_W} \hfill (5)$$

in $\rho$. Now, $\theta$, $M_W$, and other electroweak observables depend on both $m_t$ and $M_H$. This dependence, along with present bounds on $M_W$ \[4, 5\], leads for $M_H < 1$ TeV to a crude upper limit of $m_t < 200$ GeV. The lower bound on $m_t$ (95% c.l.) is 91 GeV \[6\]. A measurement of $m_t$ to ±5 GeV and $M_W$ to ±50 MeV will begin to distinguish among predictions for various Higgs masses.

Additional terms logarithmic in $m_t$ lead to modifications of the relations written previously:

$$\frac{G_F}{\sqrt{2}} = (1 + \Delta Z_W)\frac{g^2}{8M_W^2}, \quad \frac{G_F\rho}{\sqrt{2}} = (1 + \Delta Z_Z)\frac{g^2 + g'^2}{8M_Z^2} \hfill (6)$$

where $\Delta Z_W$ and $\Delta Z_Z$ represent the effects of variation with momentum transfer between $q^2 = 0$ (where $G_F$ is measured) and the $W$ and $Z$ poles (where coupling constants and masses are defined). They may be expressed in terms of quantities of order 1:

$$\Delta Z_W = \frac{\alpha S_W}{4 \sin^2\theta}; \quad \Delta Z_Z = \frac{\alpha S_Z}{4 \sin^2\theta \cos^2\theta} \hfill (7)$$

Similarly, $\rho = 1 + \alpha T$ can be expressed in terms of a parameter $T$ of order 1.

If one expands around nominal values of $m_t$ and $M_H$, one finds \[7\]

$$T \simeq \frac{3}{16\pi \sin^2\theta} \left[ \frac{m_t^2 - (140 \text{ GeV})^2}{M_W^2} \right] - \frac{3}{8\pi \cos^2\theta} \ln \frac{M_H}{100 \text{ GeV}} \hfill (8)$$

$$S_W = \frac{1}{6\pi} \left[ \ln \frac{M_H}{100 \text{ GeV}} - 2 \ln \frac{m_t}{140 \text{ GeV}} \right], \quad S_Z = \frac{1}{6\pi} \left[ \ln \frac{M_H}{100 \text{ GeV}} + 4 \ln \frac{m_t}{140 \text{ GeV}} \right]. \hfill (9)$$

3. Electroweak observables and fits

The precise value of the $Z^0$ mass entails a value of $\sin^2\theta \equiv x_0 = 0.2323 \pm 0.0002 \pm 0.0005$ for $m_t = 140$ GeV, $M_H = 100$ GeV. We expand a set of electroweak observables
A function of $m$ and $Z$ width of the $S$ for the forward-backward asymmetry for low Higgs mass occurs. This tilt has been traced to slightly different input values in contrast to other fits \cite{9, 10} in which a slight (but not significant) tilt in favor of $M$ about this value; details are to be found in Ref. \cite{1}. For example, we have \cite{8}

$$\sin^2 \theta - x_0 = \frac{\alpha}{1 - 2x_0} \left[ \frac{1}{4} S_Z - x_0(1 - x_0)T \right] = (3.65 \times 10^{-3}) S_Z - (2.61 \times 10^{-3}) T , \quad (10)$$

and corresponding other expressions for $Z$ partial widths, neutral-current to charged-current ratios in deep inelastic neutrino scattering, and weak charges as measured in atomic parity violation. The data are summarized in Table I.

Based on the data in Table I, we obtain $\chi^2$ for specific values of $M_H$ as a function of $m_t$. The results are shown in Fig. 1. The minimum $\chi^2$ values for $M_H = (100, 300, 1000)$ GeV are $(4.34, 4.33, 4.33)$, corresponding to $m_t = (144 \pm 16, 160 \pm 15, 177 \pm 14)$ GeV. The lack of preference for any particular Higgs boson mass stands in contrast to other fits \cite{3, 10} in which a slight (but not significant) tilt in favor of low Higgs mass occurs. This tilt has been traced to slightly different input values for the forward-backward asymmetry for $b$ quark production and for the leptonic width of the $Z$.

A fit based on the degrees of freedom $S$ and $T$ of Ref. \cite{2} was also performed. (Here we have assumed $S = S_W = S_Z$, as occurs when one has extra degenerate

| Quantity | Reference | Experimental Value | Nominal Theory$^a$ | Expt. $\div$ theory |
|----------|-----------|--------------------|-------------------|---------------------|
| $Q_W$ (Cs) | b) | $-71.04 \pm 1.81$ | $-73.20 \pm 0.970 \pm 0.025$ |
| $M_W$ (GeV) | c) | $80.14 \pm 0.27$ | $80.21^{b)} \pm 0.999 \pm 0.003$ |
| $N_\nu$ [from $\Gamma(Z \rightarrow \nu\bar{\nu})$] | d) | $3.04 \pm 0.04$ | $3 \pm 1.013 \pm 0.013$ |
| $\Gamma(Z \rightarrow l^+ l^-)$ (MeV) | e) | $83.52 \pm 0.33$ | $83.6 \pm 0.999 \pm 0.004$ |
| $\Gamma(Z \rightarrow \text{all})$ (MeV) | d) | $2492 \pm 7$ | $2488 \pm 6 \pm 1.001 \pm 0.004$ |
| $\bar{x}$ (asymms., $\tau$ pol.) | d) | $0.2324 \pm 0.0011$ | $0.2322 \pm 0.1001 \pm 0.005$ |
| $\bar{x}(q\bar{q}$ asymm.) | d) | $0.2323 \pm 0.0032$ | $0.2322 \pm 0.1000 \pm 0.014$ |
| $\bar{x}(\bar{e}D)$ | b) | $0.224 \pm 0.020$ | $0.2322 \pm 0.965 \pm 0.086$ |
| $\bar{x}(\bar{e}C)$ | b) | $0.20 \pm 0.05$ | $0.2322 \pm 0.86 \pm 0.22$ |
| $\bar{x} \left[ \sigma(\nu \nu^{-}) \right]$ | f) | $0.232 \pm 0.009$ | $0.2322 \pm 1.000 \pm 0.04$ |
| $M_W$ (MeV) from $R_\nu$ | g) | $80.32 \pm 0.32$ | $80.21 \pm 0.1001 \pm 0.004$ |
| $R_\nu$ | b) | $0.387 \pm 0.009$ | $0.376 \pm 1.02 \pm 0.02$ |
| $\bar{x}$ ($A_{LR}$ at SLC) | h) | $0.2378 \pm 0.0056$ | $0.2322 \pm 1.024 \pm 0.024$ |

$^a$For $m_t = 140$ GeV, $M_H = 100$ GeV.

$^b$As in Ref. \cite{4}.

$^c$Raised from value in Ref. \cite{3} as a result of new $M_Z$ measurement \cite{4}.

$^d$Ref. \cite{8}.

$^e$F. Merritt, Seminar, Univ. of Chicago, April, 1992.

$^f$New CHARM II value: Ref. \cite{3} and G. Rädel, this conference.

$^g$Based on CCFR value of $1 - (M_W/M_Z)^2 = 0.2242 \pm 0.0057$ \cite{3}.

$^h$C. Baltay, this conference. Value not included in fit.
The elongated nature of the ellipses illustrates the absence of any preference for a specific Higgs mass. One can change the Higgs mass without much penalty as long as the top quark mass changes in a compensating way. For $M_H < 1$ TeV (an approximate upper bound resulting from unitarity), we see from Fig. 2 that one can still only conclude $m_t < 200$ GeV, but with 90% confidence.

4. Conclusions

We have shown that the top quark mass is limited by today’s electroweak data to be less than about 200 GeV. Stronger limits are to be mistrusted. A plot in $S$ and $T$ shows no particular preference for any sign of $S$ but implies $S < 1$ at the 90% confidence level. The discovery of the top quark and the measurement of its mass remain the highest priority for obtaining further information about the electroweak theory.

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6. References

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Figure 2: Fit to parameters $S$ and $T$ based on data of Table I. Plotted point corresponds to minimum $\chi^2$; inner and outer ellipses correspond to 68% and 90% c.l. limits. Standard model curves correspond, from left to right, to $M_H = 100, 300, \text{ and } 1000 \text{ GeV}$. Ticks on these curves, from bottom to top, correspond to $m_t = 100, 140, 180, 220, \text{ and } 260 \text{ GeV}.$

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