Deterministic remote preparation of arbitrary two qubit state via GHZ-like

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Abstract. We proposed a novelty and scheme for quantum teleportation of deterministic of arbitrary two qubit state via GHZ-like as quantum channel. Compared with previous scheme, the advantage of our scheme is that Alice’s unitary operation simpler and the total probability of successful and fidelity reaches to 1.

1. Introduction

Information theory has experienced significant development. The main purpose of classical information theory is to understand the source of information remotely as well as modern information theory. Modern information theory has been developed in a quantum system called quantum information theory. One of the successes of quantum information theory is quantum teleportation. Quantum teleportation was first proposed by [1], where Alice (sender) wants to send a quantum state to the receiver (Bob). Classic steps can be taken by Alice, which measures the quantum state and makes a copy, then the copy is sent to Bob and Bob reconstructs it to restore the quantum state. But in quantum, one cannot make precise measurements of the state of a particle. Then it’s impossible to do by Alice. Then, it is known that there is an entanglement state (EPR) [2] of two separate particles, if one is affected, the other is affected. So, by utilizing of entanglement principle and classical communication, Alice can transfer an unknown quantum states to Bob, without having to send particles physically.

Bouwmeester [3] has successfully observed three qubit state entanglement, GHZ entanglement experimentally. Pati [4] proposed a different scheme with teleportation [1]. The difference is that Alice knows exactly what state she wants to send (e.g qubit in equatorial of Bloch Sphere), which in this scheme requires less classical communication than teleportation. this scheme is called as Remote State Preparation (RSP). Lo [5] learned about the amount of classical communication needed to process quantum information. Bennet [6] shows that classical RSP protocol for the general state to be sent is asymptotic, i.e one bit per qubit - half teleportation - and even less if teleporting the entangled state. The main goal of RSP is Alice wants to help Bob in his laboratory to prepare the state from remote distance. In the teleportation protocols said that both of parties (sender and receiver) is oblivious of the state, but in the remote state preparation we assume that Alice have known the state which wants to send to Bob, but the next development is not necessarily oblivious to Bob [7].

Devetak studied RSP by approaching RSP in the low entanglement region with the rate distortion method [8] and then RSP has been successfully realized in experimental using nuclear magnetic resonance by Peng [9] and show that RSP of a special ensemble was successfully completed by a
maximally entangled channel with one classical bit communication. Entanglement has a very important role in quantum information theory. Liu has successfully proposed the RSP scheme for two entangled state \[10\] and remote preparation of arbitrary two and three qubit states by using two bipartite partially entangled states as the quantum channel, it is show that the probability of success increases 4 times for 2 qubits and 8 times for 3 qubits, and this is the same as the maximal state of the resource entanglement \[11\]. Recently, Zhan proposed new scheme for Deterministic remote preparation of arbitrary two and three qubit states \[12\].

In this paper, we reconsider the scheme for remotely preparation of arbitrary 2 qubits in ref. \[12\]. we propose a scheme for RSP of arbitrary 2 qubit with GHZ-like as its quantum channel. first we use the maximally entangled channel and our scheme is used to send an arbitrary 2 qubits with complex coefficients to Bob. In the scheme we have proposed, we use the 6 qubits state as a quantum channel, which consists of the interaction of 2 GHZ-like states. where, the first 4 qubits are Alice’s and the last 2 qubits are Bob’s. Alice measured 2 times on her qubit, the first measurement was made on first Alice’s 2 qubits from the channel, then the next measurement was done on last Alice’s 2 qubits of channel. And then in the first measurement Alice can check the coefficient deviation before sent to Bob, and then Alice can perform unitary operation to correct the coefficient. After done Alice perform second measurement and sent the result to Bob. Bob perform unitary operation to his qubit base on the Alice’s measurement result.

2. Deterministic Remote preparation of arbitrary two qubit state via GHZ-like

we assume that the state which Alice want sent to Bob is:

\[|\phi> = \lambda_0|00> + \lambda_1 e^{i\delta_1}|01> + \lambda_2 e^{i\delta_2}|10> + \lambda_3 e^{i\delta_3}|11>\]  

(1)

And use two GHZ-like as quantum channel

\[|\psi_1> = \frac{1}{2} \begin{vmatrix} |001> + |010> + |100> + |111> \end{vmatrix} \]

\[|\psi_1> = \frac{1}{2} \begin{vmatrix} |001> + |010> + |100> + |111> \end{vmatrix} \]

(2)

where qubit 1,2,4 and 5 is Alice’s while 3 and 6 is Bob’s. Because Alice have known about the state which want sent to Bob precisely, so Alice can construct any measurement basic which one she wants, the measurement basic can be construct by mutually orthogonal basic vectors (MOBVs), that is:

\[
\begin{pmatrix} |00> \\ |01> \\ |10> \\ |11> \end{pmatrix} = F^\dagger \begin{pmatrix} |\phi_0> \\ |\phi_1> \\ |\phi_2> \\ |\phi_3> \end{pmatrix}
\]

Where

\[
F^\dagger = \begin{pmatrix}
\lambda_0 & \lambda_3 & \lambda_2 & \lambda_3 \\
-\lambda_4 e^{i\delta_1} & -\lambda_3 e^{i\delta_1} & -\lambda_2 e^{i\delta_1} & -\lambda_1 e^{i\delta_1} \\
\lambda_1 e^{i\delta_2} & \lambda_2 e^{i\delta_2} & \lambda_3 e^{i\delta_2} & \lambda_4 e^{i\delta_2} \\
-\lambda_2 e^{i\delta_3} & -\lambda_3 e^{i\delta_3} & -\lambda_4 e^{i\delta_3} & -\lambda_5 e^{i\delta_3}
\end{pmatrix}
\]

(4)

then obtained the basis of Alice measurement for qubit 1 and 4 as follows:

\[|00>_{14} = (\lambda_0|\phi_0> + \lambda_1|\phi_1> + \lambda_2|\phi_2> + \lambda_3|\phi_3>\]

\[|01>_{14} = (\lambda_1 e^{i\delta_1}|\phi_0> - \lambda_0 e^{i\delta_1}|\phi_1> - \lambda_3 e^{i\delta_1}|\phi_2> + \lambda_2 e^{i\delta_1}|\phi_3>\]

\[|10>_{14} = (\lambda_2 e^{i\delta_2}|\phi_0> + \lambda_3 e^{i\delta_2}|\phi_1> - \lambda_0 e^{i\delta_2}|\phi_2> - \lambda_1 e^{i\delta_2}|\phi_3>\]

\[|11>_{14} = (\lambda_3 e^{i\delta_3}|\phi_0> - \lambda_2 e^{i\delta_3}|\phi_1> + \lambda_0 e^{i\delta_3}|\phi_2> + \lambda_1 e^{i\delta_3}|\phi_3>)\]
By eq.(2),(3),(4), and (5), we get the joint system can be written as

\[
|\psi > = |\psi_1 > \otimes |\psi_2 >
\]

\[
= \frac{1}{16} (|00 >_14 |l_0 > + |01 >_14 |l_1 > + |10 >_14 |l_2 > + |11 >_14 |l_3 >)
\]

(6)

Where

\[
|l_0 > = (|0010 > + |0110 > + |1001 > + |1101 >)_{2356}
\]

\[
|l_1 > = (|0100 > + |0111 > + |1000 > + |1011 >)_{2356}
\]

\[
|l_2 > = (|0001 > + |0010 > + |1101 > + |1110 >)_{2356}
\]

\[
|l_3 > = (|0000 > + |0011 > + |1100 > + |1111 >)_{2356}
\]

(7)

If the equation 5 is substituted into the 6 equation then obtained

\[
|\psi > = |\phi_o >_14 |p_o > + |\phi_1 >_14 |p_1 > + |\phi_2 >_14 |p_2 > + |\phi_3 >_14 |p_3 >
\]

(8)

Where

\[
|p_0 > = \frac{1}{4} (\lambda_0 |l_0 > + \lambda_1 e^{i\delta_1} |l_1 > + \lambda_2 e^{i\delta_2} |l_2 > + \lambda_3 e^{i\delta_3} |l_3 >)_{2356}
\]

\[
|p_1 > = \frac{1}{4} (\lambda_1 |l_0 > - \lambda_0 e^{i\delta_1} |l_1 > + \lambda_3 e^{i\delta_2} |l_2 > - \lambda_2 e^{i\delta_3} |l_3 >)_{2356}
\]

\[
|p_2 > = \frac{1}{4} (\lambda_2 |l_0 > - \lambda_3 e^{i\delta_1} |l_1 > - \lambda_0 e^{i\delta_2} |l_2 > + \lambda_1 e^{i\delta_3} |l_3 >)_{2356}
\]

\[
|p_3 > = \frac{1}{4} (\lambda_3 |l_0 > + \lambda_2 e^{i\delta_1} |l_1 > - \lambda_1 e^{i\delta_2} |l_2 > - \lambda_0 e^{i\delta_3} |l_3 >)_{2356}
\]

(9)

then Alice perform the first measurements to eq. (8) by projecting to base $|\phi_l >_{14}$ it will get the measurement result $|p_l >_{2356}$, where $l = 0,1,2,3$. and then we will discuss about Alice’s first measurement result one by one

2.1. If Alice’s First Measurement Result is $|p_o >$

The first, because Alice have known about the information of the state will be sent to Bob precisely, so Alice can make sure about the coefficient of first measurement result before perform second measurement. In the eq. (9), it is known that no coefficient deviation for state $|p_o >$. And based on the eq. (8) and (9) so the state $|p_o >$ can be written as:

\[
|p_o > = |00 >_{25} |q^{(o)}_0 > + |01 >_{25} |q^{(o)}_1 > + |10 >_{25} |q^{(o)}_2 > + |11 >_{25} |q^{(o)}_3 >
\]

(10)

Where

\[
|q^{(o)}_0 > = \frac{1}{4} (\lambda_0 |00 > + \lambda_1 e^{i\delta_1} |10 > + \lambda_2 e^{i\delta_2} |01 > + \lambda_3 e^{i\delta_3} |11 >)_{36}
\]

\[
|q^{(o)}_1 > = \frac{1}{4} (\lambda_0 |10 > + \lambda_1 e^{i\delta_1} |11 > + \lambda_2 e^{i\delta_2} |00 > + \lambda_3 e^{i\delta_3} |01 >)_{36}
\]

\[
|q^{(o)}_2 > = \frac{1}{4} (\lambda_0 |01 > + \lambda_1 e^{i\delta_1} |00 > + \lambda_2 e^{i\delta_2} |11 > + \lambda_3 e^{i\delta_3} |10 >)_{36}
\]

\[
|q^{(o)}_3 > = \frac{1}{4} (\lambda_0 |00 > + \lambda_1 e^{i\delta_1} |01 > + \lambda_2 e^{i\delta_2} |10 > + \lambda_3 e^{i\delta_3} |11 >)_{36}
\]

(11)

After that, Alice can perform the second measurement of qubit 2 and 5 in the eq. (10) by projecting to the basic measurement $\{0,1\}$, and then get measurement result in the state of $|q^{(o)}_l >$. If Alice's
measurement to basic $|00>_{25}$ so the qubit 3 and 6 will be collapsed to $|q^{(o)}_0>$. If Alice observe the result of measurement basic $|01>_{25}, |10>_{25}$ dan $|11>_{25}$, the qubit 3 and 6 will be collapsed to the state $|q^{(o)}_1>, |q^{(o)}_2>, $ and $|q^{(o)}_3>$. The successful probability and the fidelity of each result is 1/16. Then, Bob can perform the suitable unitary operation to his qubit, so he gets the same state with the original state which Alice sent.

This protocol is appropriate with a quantum teleportation protocol, but any little problem, because when Alice perform measurement of eq. (10) by projecting to basic {0, 1}, that is should the state $|q^{(o)}_o>$. which received by Bob is the same of the original state, so Bob just do nothing, in other words, Bob only perform $I \otimes I$ as unitary operation to his qubit (3 and 6).

Because Alice have known about the state which want sent to Bob, so Alice can evaluate the state of $|p^{(o)}_o>$ in the eq (10). So this problem can be solved by an qubit flipped operation performed by Alice on qubit 2 and 5 in the eq. (10). Alice defined the unitary flipped operator as follows:

$$\sigma_{x25} = \sigma_x \otimes I \otimes \sigma_x \otimes I$$

And then Alice perform that operator to eq. (10), so get:

$$|p^{(o)}_o> = \sigma_{x25} |p^{(o)}_o> = \sigma_{x25} |p^{(o)}_o> = |00>_{25} |q^{(o)}_o> + |01>_{25} |q^{(o)}_1> + |10>_{25} |q^{(o)}_2> + |11>_{25} |q^{(o)}_3>$$

Where

$$|q^{(o)}_0> = \frac{1}{4}(\lambda_0 |00> + \lambda_1 e^{i\delta_1} |01> + \lambda_2 e^{i\delta_2} |10> + \lambda_3 e^{i\delta_3} |11>)_{36}$$

$$|q^{(o)}_1> = \frac{1}{4}(\lambda_0 |01> + \lambda_1 e^{i\delta_1} |00> + \lambda_2 e^{i\delta_2} |11> + \lambda_3 e^{i\delta_3} |10>)_{36}$$

$$|q^{(o)}_2> = \frac{1}{4}(\lambda_0 |10> + \lambda_1 e^{i\delta_1} |11> + \lambda_2 e^{i\delta_2} |00> + \lambda_3 e^{i\delta_3} |01>)_{36}$$

$$|q^{(o)}_3> = \frac{1}{4}(\lambda_0 |11> + \lambda_1 e^{i\delta_1} |10> + \lambda_2 e^{i\delta_2} |11> + \lambda_3 e^{i\delta_3} |00>)_{36}$$

As before, Alice can perform the second measurement of qubit 2 and 5 in the eq. (13) by projecting to the basic measurement {0,1}, and then get measurement result in the state of $|q^{(o)}_1>$. Then, Bob can perform the suitable unitary operation to his qubit, so he gets the same state with the original state which Alice sent, for the state $|q^{(o)}_0>, |q^{(o)}_1>, |q^{(o)}_2>, $ and $|q^{(o)}_3>$. Bob’s suitable unitary is $I \otimes I$, $I \otimes \sigma_x$, $\sigma_x \otimes I$ and $\sigma_x \otimes \sigma_x$.

2.2. If Alice’s First Measurement Result is $|p_1>, |p_2>, $ and $|p_3>$

As previous Section, Alice apply operator $\sigma_{x25}$ on the state $|p_1>$ and get,

$$|p^{(o)}_1> = \sigma_{x25} |p_1> = \frac{1}{4}(\lambda_1 (|1111> + |1100> + |0011> + |0000>)_{2356}$$

$$- \frac{1}{4}(\lambda_0 e^{i\delta_1} (|1110> + |1101> + |0010> + |0001>)_{2356}$$

$$+ \frac{1}{4}(\lambda_3 e^{i\delta_2} (|1011> + |1000> + |0111> + |0100>)_{2356}$$

$$- \frac{1}{4}(\lambda_2 e^{i\delta_2} (|1010> + |1001> + |0110> + |0101>)_{2356}$$

(15)
Next, Alice can be checked the coefficient of $|p'_{1} >$, because Alice have known about the original state precisely. In other words, Alice checked the coefficient deviation of the state $|p_{1} >, |p_{2} >$, and $|p_{3} >$ before she performs the second measurement. Because in that state, the coefficient has deviation, so Alice can make unitary operation to make the coefficient become like an original state coefficient.

She can introduce the unitary operator to make coefficient become like an original state coefficient:

$U_{1} = (\begin{pmatrix} e^{i\delta_{1}} & 0 & 0 & 0 \\ -e^{-i\delta_{1}} & 0 & 0 & 0 \\ 0 & e^{i(\delta_{3}-\delta_{2})} & 0 & 0 \\ 0 & 0 & 0 & -e^{-i(\delta_{3}-\delta_{2})} \end{pmatrix})$ (16)

she can perform that unitary operator in eq. (20) to $|p'_{1} >$ in eq. (15) and get

$|p_{1} > = U_{1}|p'_{1} >$

$= |00 >_{25} |q_{0}^{(1)} > + |01 >_{25} |q_{1}^{(1)} > + |10 >_{25} |q_{2}^{(1)} > + |11 >_{25} |q_{3}^{(1)} >$ (17)

Where

$|q_{0}^{(1)} > = \frac{1}{4} (\lambda_{0} |01 > + \lambda_{1} e^{i\delta_{1}} |00 > + \lambda_{2} e^{i\delta_{2}} |11 > + \lambda_{3} e^{i\delta_{3}} |10 >)_{36}$

$|q_{1}^{(2)} > = \frac{1}{4} (\lambda_{0} |00 > + \lambda_{1} e^{i\delta_{1}} |01 > + \lambda_{2} e^{i\delta_{2}} |10 > + \lambda_{3} e^{i\delta_{3}} |11 >)_{36}$

$|q_{2}^{(3)} > = \frac{1}{4} (\lambda_{0} |11 > + \lambda_{1} e^{i\delta_{1}} |10 > + \lambda_{2} e^{i\delta_{2}} |01 > + \lambda_{3} e^{i\delta_{3}} |00 >)_{36}$

$|q_{3}^{(4)} > = \frac{1}{4} (\lambda_{0} |10 > + \lambda_{1} e^{i\delta_{1}} |11 > + \lambda_{2} e^{i\delta_{2}} |00 > + \lambda_{3} e^{i\delta_{3}} |01 >)_{36}$ (18)

Now, Alice can perform the second measurement of qubit 2 and 5 in the eq. (17) by projecting to the basic measurement $\{0, 1\}$, and then get measurement result in the state of $|q_{l}^{(1)} >$. Then, Bob can perform the suitable unitary operation to his qubit, so he gets the same state with the original state which Alice sent and Bob’s suitable unitary is $I \otimes \sigma_{z}, I \otimes I, \sigma_{x} \otimes \sigma_{z}$ and $\sigma_{x} \otimes I$.

Similar procedure, Alice can apply the same method as above. If the result of first measurement is collapsed to $|p_{2} >$ and $|p_{3} >$ so the unitary operator to remove the coefficient deviation is:

$U_{2} = (\begin{pmatrix} e^{i\delta_{2}} & 0 & 0 & 0 \\ 0 & -e^{i(\delta_{3}-\delta_{1})} & 0 & 0 \\ 0 & 0 & -e^{-i\delta_{2}} & 0 \\ 0 & 0 & 0 & e^{i(\delta_{3}-\delta_{1})} \end{pmatrix})$ (19)

And

$U_{3} = (\begin{pmatrix} e^{i\delta_{3}} & 0 & 0 & 0 \\ 0 & e^{i(\delta_{2}-\delta_{1})} & 0 & 0 \\ 0 & 0 & -e^{-i(\delta_{3}-\delta_{2})} & 0 \\ 0 & 0 & 0 & e^{-i\delta_{3}} \end{pmatrix})$ (20)

then the suitable of Bob’s unitary operation can be describes in the table as follows:
Table 1. The Correlation of the first Alice’s measurement results (M1) for qubits 1 and 4, the second Alice’s measurement for qubits 2 and 5, and Bob’s unitary operation ($U_b$) for qubits 3 and 6

| M1  | M2  | $U_b$  | M1  | M2  | $U_b$  |
|-----|-----|--------|-----|-----|--------|
| $\phi_0$ | $|00\rangle_{25}$ | $I \otimes I$ | $\phi_2$ | $|00\rangle_{25}$ | $\sigma_x \otimes I$ |
|      | $|01\rangle_{25}$ | $I \otimes \sigma_x$ |      | $|01\rangle_{25}$ | $\sigma_x \otimes \sigma_x$ |
|      | $|10\rangle_{25}$ | $\sigma_x \otimes I$ |      | $|10\rangle_{25}$ | $I \otimes I$ |
|      | $|11\rangle_{25}$ | $\sigma_x \otimes \sigma_x$ |      | $|11\rangle_{25}$ | $I \otimes \sigma_x$ |
| $\phi_1$ | $|00\rangle_{25}$ | $I \otimes \sigma_x$ | $\phi_3$ | $|00\rangle_{25}$ | $\sigma_x \otimes \sigma_x$ |
|      | $|01\rangle_{25}$ | $I \otimes I$ |      | $|01\rangle_{25}$ | $\sigma_x \otimes I$ |
|      | $|10\rangle_{25}$ | $\sigma_x \otimes \sigma_x$ |      | $|10\rangle_{25}$ | $I \otimes \sigma_x$ |
|      | $|11\rangle_{25}$ | $\sigma_x \otimes I$ |      | $|11\rangle_{25}$ | $I \otimes I$ |

3. Conclusion
We have proposed the novelty scheme of deterministic remote preparation of arbitrary two qubit via GHZ-like as quantum channel. Different from the scheme [10] is in our scheme, before Alice perform second measurement to qubit 2 and 5, she can make unitary flipped qubit operation, defined by operator $\sigma_{x \otimes 25}$. it makes Alice’s unitary operation is simpler than use Hadamard. And we get the total probability of successful and fidelity reach to 1.

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