The Truncated Moment Problem
for Unital Commutative \( \mathbb{R} \)-Algebras

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Abstract

Let \( A \) be a unital commutative \( \mathbb{R} \)-algebra, \( K \) a closed subset of the character space of \( A \), and \( B \) a linear subspace of \( A \). For a linear functional \( L : B \to \mathbb{R} \), we investigate conditions under which \( L \) admits an integral representation with respect to a positive Radon measure supported in \( K \). When \( A \) is equipped with a submultiplicative seminorm, we employ techniques from the theory of positive extensions of linear functionals to prove a criterion for the existence of such an integral representation for \( L \). When no topology is prescribed on \( A \), we identify suitable assumptions on \( A \), \( K \), \( B \) and \( L \) which allow us to construct a seminormed structure on \( A \), so as to exploit our previous result to get an integral representation for \( L \).

Our main theorems allow us to extend some well-known results on the Classical Truncated Moment Problem, the Truncated Moment Problem for point processes, and the Subnormal Completion Problem for 2-variable weighted shifts. We also analyze the relation between the Full and the Truncated Moment Problem in our general setting: we obtain a suitable generalization of Stochel’s Theorem, which readily applies to Full Moment Problems for localized algebras.

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Keywords: truncated moment problem, full moment problem, measure, integral representation, linear functional.