Thermal buoyancy on magneto hydrodynamic flow over a vertical saturated porous surface with viscous dissipation

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Abstract: In the present article, we studied the magnetohydrodynamic flow induced heat transfer from vertical surface embedded in a saturated porous medium in the presence of viscous dissipation. Appropriate similarity transformations are used to transmute the non-linear governing partial differential equations to non-linear ODE. To solve these ordinary differential equations (ODE) we used the well-known integral method of Von Karman type. A comparison has been done and originates to be in suitable agreement with the previous published results. The tabulated and graphical results are given to consider the physical nature of the problem. From this results we found that the magnetic field parameter depreciate the velocity profiles and improves the heat transfer rate of the flow.

1. Introduction:

The analysis of heat transfer due to magnetic field effect in porous media is of important significant due to its Erect number and energy-related engineering applications such as groundwater pollution, magnetohydrodynamic accelerators, power generation, power plants, missile re-entry, etc. The objective of the present article is to contract the necessary observe of the phenomenon of natural convection heat transfer near a vertical surface embedded in a fluid-saturated porous medium. Magnetic field effect is due to the variation of velocity and temperature across the boundary layer. To investigate the present problem use the Von-Karman type integral method has been adopted from the Nakayama and Hossain [6]. On explanation of the afore-mentioned proofs only, Merkin [1] made a systematic analysis of heat transfer connected results. Bejan and Khair [2] studied the heat and mass transfer along a wall entrenched in a saturated porous medium with continuous concentration and temperature. Y¨ucel [3] analyzed the heat transfer results in a vertical surface set in porous media. Lai et al[4] studied the results of heat and mass transfer along slender bodies embedded in porous media by natural convection. Lai and Kulacki [5] analyzed the general study on the Bejan and Khair article. Nakayama and Hossain [6] used integral treatment to studied the results of heat and mass transfer in a porous media by natural convection. Singh and Queeny [7] applied the free convection on heat and mass transfer along a vertical surface in porous medium. Singh and Chandarki [8] invested results on heat and mass transfer from
cylinder in a porous medium using integral method. The researchers [9, 10] have made a analysis MHD effect on stretching sheet in a porous media. Haq et al. [12] studied MHD effect on Cassonnanofluid over a shrinking sheet for observed the heat transfer. Srinivasacharya and Kumar [13] describe the prerequisites of the radiative heat transfer. Sarojamma and Vendabai [14] studies the heat absorption of boundary layer of a Cassonnanofluid stretching cylinder in the presence of transverse magnitude. Mebarek-Oudina [15] observed the heat transfer of liquid metal between Vertical Coaxial Cylinders by magneto hydrodynamic natural convection. NasreenBano and Singh[16] made a detailed analysis on coupled heat and mass transfer in vertical thin needle using integral treatment by natural convection. Sayyed al et[17] used integral method for observe the heat transfer in vertical surface by natural convection along radiation embedded in a saturated porous media. Raju et al [18] studied the heat transfer of MHD unsteady flow in cone packed with alloy nanoparticles by natural convection. Raju and Sandeep [18] made an investigation on Unsteady Casson-nanofluid flow in a rotating surface by used numerical method.

2. Mathematical formulation:

Let us study the boundary layer flow over a vertical superficial embedded fluid-flow in saturated porous medium. The temperature in the surface as \( T_0 \). The porous medium has been maintained temperature as \( T_\infty \) and the surface is far-off from this temperature. The uniform magnetic field controls the flow and induced the viscous dissipation to transfer the heat at the boundary layer. The boundary layer approximations and Darcy’s law have been used, with these suppositions the governing partial differential equations are

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

\[
u \left[ \beta_T (T - T_\infty) \right] = \frac{\sigma \beta_0^2}{\rho} u
\]  

\[
\rho C_p \left[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = k \frac{\partial^2 T}{\partial y^2} + \mu \left( \frac{\partial u}{\partial y} \right)^2
\]

Where \( \beta_T \) is thermal coefficients, \( \nu = \frac{\mu}{\rho} \) is kinematic velocity, Darcian velocities of \( u, v \) are in \( x, y \) directions of the fluid.

The boundary conditions for temperature and velocity at the surface are

\[
y = 0 : \quad v = 0, \quad T = T_w
\]  

\[
y \rightarrow \infty : \quad u = 0, \quad T \rightarrow T_\infty
\]

Where \( k \) is the thermal conductivity, \( \alpha = \frac{k}{\rho C_p} \) is porous medium thermal diffusivity.
The following are the simultaneous equations are using to transform the governing non-linear PDE in to the ODE

\[
\eta = \frac{y(R_a)^{1/2}}{\lambda}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \\
u = \frac{a(R_a)^{1/2}}{2} (f - \eta f'), \quad \alpha = \frac{R_a \alpha f'}{x}
\]  

(5)

Where \( R_a = \frac{k \beta g (T_w - T_\infty)}{\nu \alpha} \) is local Rayleigh number.

Using Eq. (5), the Eq.(2) and Eq.(3) reduce to

\[
\left(1 + M^2 \right) f''(\eta) = -\theta'(\eta)
\]

(6)

\[
\theta'(\eta) + E_c (f'(\eta))^2 = \frac{f(\eta)\theta'(\eta)}{2}
\]

(7)

Through boundary conditions

\[
\eta = 0: \quad \theta(\eta) = 1, \quad f = 0
\]

(8i)

\[
\eta = \infty: \quad \theta(\eta) = 0, \quad f' = 0
\]

(8ii)

Where \( M^2 = \frac{\sigma \beta^2}{\rho} \) is the magnetic field, \( E_c = \frac{g^2 \beta^2 \mu}{\nu^2 k} (T_w - T_\infty) \) is the Eckert number.

2.1. Integral method:

Apply the integration on the energy transformation Eq.7 with respect to \( \eta \) with the limits \( \eta \to 0 \) to \( \eta \to \infty \)

\[
\int_0^\infty \theta'(\eta) d\eta = \frac{1}{2} \int_0^\infty \theta'(\eta) f(\eta) d\eta - E_c \int_0^\infty (f')^2 d\eta
\]

(9)

\[
\left[ -\theta'(0) \right] = -\frac{1}{2} \int_0^\infty f'(\eta) \theta(\eta) d\eta - E_c \int_0^\infty (f')^2 d\eta
\]

(10)

Express the temperature profile in the form of exponential and it satisfies the boundary conditions (8i), (8ii).

\[
\theta = e^{-\frac{\eta}{\xi}}
\]

(11)
Where $\delta_t$ is the thickness of the thermal boundary layer

Solving Eq.10, using Eq.6 and Eq.11 we obtain

$$\frac{1}{\delta_t^2} = \frac{(1 + M^2)}{2\left[2(1 + M^2)^2 + E_c\right]}$$

(12)

The local Nusselt number is

$$-\theta'(0) = \frac{Nu}{(R_a)^{\frac{1}{3}}} = \frac{1}{\delta_t} = 0.5 \left[\frac{(1 + M^2)}{2(1 + M^2)^2 + E_c}\right]^{\frac{1}{2}}$$

(13)

Comparing the acquired accuracy temperature profile from this problem against with the solution obtained by Sayeed et al [17]

$$\frac{Nu}{(R_a)^{\frac{1}{3}}} = 0.444 \left[\frac{(1 + M^2)}{2(1 + M^2)^2 + E_c}\right]^{\frac{1}{2}}$$

(14)

Eq.13 is approximate heat transfer equation and Eq.14 is exact formula of heat transfer.

3. Results and discussions

Table1: Variation of heat transfer for various values of $M$ and $E_c$ with several of $\eta$

| $\eta$ | $M$ | $E_c$ | $\frac{Nu}{(R_a)^{\frac{1}{3}}}$ |
|-------|-----|------|-------------------------------|
| 0     |     | 0.1  | 0.2485                        |
| 2     |     | 0.5  | 0.2425                        |
| 6     |     | 1    | 0.2357                        |
| 10    |     | 2    | 0.2236                        |
| 12    |     | 3    | 0.2132                        |
| 14    | 1   | 4    | 0.2041                        |
| 18    |     | 10   | 0.1667                        |
| 22    |     | 100  | 0.0680                        |
| 0     |     | 0.1  | 0.1118                        |
| 2     |     | 0.5  | 0.1117                        |
| 6     |     | 1    | 0.1115                        |
| 10    |     | 2    | 0.1112                        |
| 12    |     | 3    | 0.1110                        |
| 14    |     | 4    | 0.1107                        |
| 18    |     | 10   | 0.1091                        |
| 22    |     | 100  | 0.0913                        |
From Table 1 we observe that heat transfer varies while variation of magnetic field and Erect number. If magnetic field rises along with Erect number then heat transfer reduces. In Table 2 comparison of the temperature in present article and the temperature various in article Syeed[17], if the Erect increases then

| η | E_1 | θ(η) | E_2 | θ(η) | E_3 | θ(η) | E_4 | θ(η) | E_5 | θ(η) |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 0.6245 | 0.6241 | 0.6533 | 0.6529 | 0.6757 | 0.6755 | 0.7098 | 0.7096 |
| 6 | 0.2444 | 0.2431 | 0.2785 | 0.2783 | 0.3086 | 0.3083 | 0.3577 | 0.3574 |
| 10 | 0.0951 | 0.0947 | 0.1190 | 0.1186 | 0.1408 | 0.1407 | 0.1805 | 0.1800 |
| 12 | 0.0595 | 0.0591 | 0.0775 | 0.0774 | 0.0952 | 0.0950 | 0.1279 | 0.1277 |
| 14 | 0.0370 | 0.0369 | 0.0516 | 0.0515 | 0.0643 | 0.0642 | 0.0907 | 0.0906 |
| 18 | 0.0145 | 0.0144 | 0.0215 | 0.0215 | 0.0295 | 0.0293 | 0.0456 | 0.0456 |

**Table 2:** Comparison of temperature Variation (θ(η)) with temperature variation in Syeed[17] for various values of $E_c$. 

From Table 1 we observe that heat transfer varies while variation of magnetic field and Erect number. If magnetic field rises along with Erect number then heat transfer reduces. In Table 2 comparison of the temperature in present article and the temperature various in article Syeed[17], if the Erect increases then
temperature also increases. The present article gives the good agreement results compare with the publish articles related with this article.

Figure. 1 Temperature profile for various values of $E_c$ taking $M = 1$

Figure. 2 Temperature profile for various values of $M$ taking $E_C = 3$

Figure. 3 Temperature profile for various values of $\rho$ taking $M = 0.0125$ and $E_C = 0.1$
Figure. 4 Velocity profile for various values of $M$ taking $Ec = 3$

Figure. 5 Velocity profile for various values of $E_c$ taking $M = 1$

Figure. 6 Velocity profile for various values of $\rho$ taking $M = 0.0125$ and $Ec = 0.1$

From Table.1 comparison of heat transfer of the present paper to the article Syeed et al. [17] taking absent of buoyancy and radiation effect and took the magnetic field effect. Magnetic field is raising the
heat transfer decrease while increase the Erect number. This result is best agreement. Fig. 1 gives the variation between the temperature and the Eckert number. The temperature has been raised while raise the Erect number. Fig. 2 presents the temperature and magnetic field, in this graph we observed magnetic field effect raise then temperature also increase. Fig. 3 shows the temperature reduced while density increases. Fig. 4 represents the velocity of the fluid increases while magnetic field increases certain stage and velocity reduces when \( M \) increases. Usually magnetic field value increases, it produces opposite force to flow direction. This force is says Lorentz force. So in this case we have seen velocity field decrement in being there of magnetic field. Fig. 5 and Fig. 6 are plotted to observe the velocity nature under the Eckert number and density. In this case velocity reduced when \( E_i \) increases and \( \rho \) increases.

4. Concluding remarks:

MHD flow of natural convection on heat transfer investigated numerically and through the graphs. The inclusion of Eckert number and magnetic field has expressively influenced the solutions. The important points of this study are shortened as below:

- The variation in velocity field with an increase in Eckert number when magnetic field effects are absent.
- Local Nusselt number has inverse correlation with the Eckert number parameter and magnetic field parameter.
- Temperature increases when magnetic field and Eckert number parameter, but it is opposite trends follows in density.

References:

[1] Merkin 1979, Free convection boundary layers on axi-symmetric and two-dimensional bodies of arbitrary shape in a saturated porous medium, *Int. J. Heat Mass Transfer* 22, 1461-1462.

[2] Bejan, K., R. Khair, Heat and mass transfer by natural convection in a porous medium, Int.J.Heat Mass Transfer 28, 909-918.

[3] Yuce1918, Heat and mass transfer about vertical surfaces in saturated porous media, AIChE Symp. Ser. 269, 85 344.

[4] F. C. Lai, C.Y. Choi, F. A. Kulacki, Coupled heat and mass transfer by natural convection from slender bodies of revolution in porous media, *Int. Commun. Heat Mass Transfer* 17(15) 609-620.

[5] F. C. Lai, F. A. Kulacki 1991, Coupled heat and mass transfer by natural convection from vertical surfaces in porous medium, Int. J. Heat Mass Transfer 34(4-5) 1189-1194.

[6] A. Nakayama, M. A. Hossain1995, An integral treatment for combined heat and mass transfer by natural convection in a porous medium, Int. J. Heat Mass Transfer 38(4) 791-765.

[7] P. Singh1977, Queeny, Free convection heat and mass transfer along a vertical surface in a porous medium, Acta Mech. 123(1-4) 69-73.

[8] Singh 2009, I. M. Chandarki, Integral treatment of coupled heat and mass transfer by natural convection from a cylinder in porous media, Int. Commun. Heat Mass Transfer 36(3) 269-273.

[9] P. Vyas, N. Shrivastava 2010, Radiative MHD flow over a non-isothermal stretching sheet in a porous medium, Appl. Math. Sci. 4(50) 2475-2484.

[10] Vyas, A. Ranjan, Dissipative MHD boundary layer flow in a porous medium over a sheet stretching non-linearly in the presence of radiation, Appl. Math. Sci. 4(63) 3133-3142.

[11] Haq, S. Nadeem, Z.H. Khan, T.G. Okedayo 2014 Convective heat transfer and MHD effects on Cassonnanofluid flow over a shrinking sheet Cent. Eur. J. Phys, 12 (12) pp. 862-871.
[12] Srinivasacharya, P. V. Kumar 2015, Radiation effect on natural convection over an inclined wavy surface embedded in a non-Darcy porous medium saturated with a nanofluid, Journal of Porous Media 18 (8) 777-789.

[13] Sarojamma, K. Vendabai 2015, Boundary layer flow of a Casson nanofluid past a vertical exponentially stretching cylinder in the presence of a transverse magnetic field with internal heat generation/absorption Int. J. Mech. Aerosp. Ind. Mechatr. Eng, 9 (1)

[14] Mebarek-Oudina, Magneto hydrodynamic Natural Convection of Liquid Metal between Vertical Coaxial Cylinders, J. of Applied Fluid Mechanics 9(4) (2016) 1655-1665.204 Diffusion Foundations Vol.

[15] NasreenBano, B. B. Singh, An integral treatment for coupled heat and mass transfer by natural convection from a radiating vertical thin needle in a porous medium, Int. Commun. Heat Mass Transfer 84(2017) 41-48.

[16] Sayyed, B. B. Singh and NarseenBano 2017, An Integral Treatment for Dissipative Boundary Layer Flow along a Radiating Vertical Surface by Natural Convection in a Porous Medium., doi:10.4028/www.scientific.net/DF.11.191.

[17] Raju, Mohammad MainulHoque, NisatNowroz Anika 2017, S. U. Mamatha, and Pooja Sharma. Natural convective heat transfer analysis of MHD unsteady Carreaunanofluid over a cone packed with alloy nanoparticles. Powder Technology 317 408-416.

[18] Raju, & N. Sandeep(2017), Unsteady Casson nanofluid flow over a rotating cone in a rotating frame filled with ferrous nanoparticles: A Numerical study. Journal of Magnetism and