Effect of supersymmetric phases on lepton dipole moments and rare lepton decays

A. Bartl\textsuperscript{1}, W. Majerotto\textsuperscript{2}, W. Porod\textsuperscript{3}, D. Wyler\textsuperscript{3}

\textsuperscript{1} Institut für Theoretische Physik, Universität Wien, A-1090 Vienna, Austria
\textsuperscript{2} Institut für Hochenergiephysik der Österreichischen Akademie der Wissenschaften A-1050 Vienna, Austria
\textsuperscript{3} Institut für Theoretische Physik, Universität Zürich, CH-8057 Zürich, Switzerland

October 31, 2018

Abstract

We study the effect of SUSY phases on rare decays of leptons and on their magnetic and electric dipole moments. We consider the most general mass matrices for sleptons within the MSSM including left–right mixing, flavour mixing and complex phases. We show that the phases also affect CP even observables. Moreover, we demonstrate that contrary to common belief the phase of $\mu$ can be large even for slepton masses as small as 200 GeV provided the lepton flavour violating parameters are complex.

1 Introduction

The observed neutrino oscillations \cite{1, 2, 3} are a clear indication for non-vanishing neutrino masses and violation of individual lepton numbers. Therefore, one expects flavour violating effects also for charged leptons. Furthermore, in analogy to quarks, lepton flavour violation may also be related to CP violation.

Lepton flavour violation (LFV) in the charged lepton sector is severely constrained by the stringent experimental bounds on the branching ratios $BR(\mu \to e\gamma) < 1.2 \cdot 10^{-11}$, $\textsuperscript{4}$, $BR(\tau \to e\gamma) < 2.7 \cdot 10^{-6}$, $BR(\tau \to \mu\gamma) < 1.1 \cdot 10^{-6}$ and rare processes such as $\mu - e$ conversion $\textsuperscript{5}$. The limits on leptonic CP violation, such as the bound of $10^{-27}$ cem on the electric dipole moment (EDM) of the electron are also quite strong. However, within a standard model (SM) framework, they are somewhat less significant because the leptonic dipole moments, being a three–loop effect, are generically small $\textsuperscript{6}$. 

\textsuperscript{4}
In supersymmetric (SUSY) extensions of the standard model, LFV and CP violation can also originate in the slepton sector and the corresponding effects can be generically large. Consequently, rare processes and CP violation impose significant bounds on the flavour violating terms in the slepton mass matrices. The various phenomenological implications of LFV with real mass matrices for sleptons and sneutrinos were extensively studied (see e.g. [7, 8, 9, 10]). The main result is that despite the stringent experimental bounds on flavour violating lepton decays, large lepton flavour violating signals are predicted in production and decays of supersymmetric particles, in particular in final states containing $e^\pm \tau^\mp$ pairs. On the other hand, studies with complex parameters were largely limited to specific SUSY models, for example the mSUGRA model [11, 12, 13, 14, 15], or only some parameters were taken to be complex [16, 17].

In the present paper we study flavour changes and CP violation in the lepton sector in the general situation, where all parameters can be complex, in particular the LFV entries of the slepton mass matrices. This important generalization is quite natural and is motivated by the close analogy between quarks and leptons and their supersymmetric partners. In the CKM matrix the phase is quite large; the smallness of certain CP-violating observables (in the K-system) is not a result of small phases but of the structure of the theory.

Because CP-violating effects such as electric dipole moments can be quite large in SUSY, the present experiments impose rather stringent bounds on phases and it is often suggested that they are small altogether [13]. Since this view is in a sense contradictory to the large phase of the standard model, it is desirable to carry out a general study of flavour and CP violation with complex parameters in order to see whether the restrictions can be softened. Furthermore, large leptonic CP violation together with leptogenesis [19] may also be the key to the baryon asymmetry of the universe. One goal of our work therefore is to determine whether large phases are indeed possible and not in contradiction with experiment. We will demonstrate that in the presence of complex flavour violating parameters, the usual bounds on the phase of the $\mu$ parameter are no more valid even for slepton masses as small as 200 GeV.

In general supersymmetric models with soft breaking terms there is a large number of (complex) parameters. Consequently, each observable can have contributions from several parameters and no clear statements on their allowed ranges may be possible. As a second goal of our study, we want to show that, nevertheless, important results can be obtained because the present limits on rare processes in the lepton sector are so strong. Furthermore, several experiments with substantially increased sensitivity are planned for the near future and will lead to even more decisive information.

As there are many parameters involved, we use in this study the first of the so called Snowmass points [20] as a starting point and add flavour violating parameters as well as possible phases. The processes we will study are the rare leptonic decays $\mu \to e\gamma$, $\tau \to e\gamma$ and $\tau \to \mu \gamma$ and the electric (EDM) and magnetic dipole moments (MDM) of $e$, $\mu$ and $\tau$. We will use the present experimental bounds, $d_e < 1.5 \cdot 10^{-27}$ ecm [21], $d_\mu < 1.5 \cdot 10^{-18}$ ecm [22], $d_\tau < 1.5 \cdot 10^{-16}$ ecm [5]. For the magnetic moments we assume that the supersymmetric contribution is limited by the experimental errors of $\pm 10^{-12}$ and $\pm 0.058$ for $a_e$ and $a_\tau$ respectively. For the muon, there are new measurements of $a_\mu$ [23], but there are still several uncertainties in the theoretical value of $a_\mu^{SM}$ [24, 25]. We will take the very conservative range $a_\mu^{exp} - a_\mu^{SM} = 43 \cdot 10^{-10}$, which corresponds to the largest deviation in the calculations. Future measurements of $d_e$ [26] and $d_\mu$ [27] may substantially improve the sensitivity to $10^{-29}$ and $10^{-24}$, respectively. Also new
experiments for the search of the rare decay $\mu \rightarrow e\gamma$ at the level of $10^{-14}$ [28] are underway.

The paper is organized as follows: In the next section we define the parameters and fix the notation. First we consider in section 3 a situation without flavour violation but with complex parameters. In section 4 the general situation with lepton flavour violation and complex parameters is studied. Some conclusions are drawn in section 5.

# 2 The basic parameters

We assume a general supersymmetric $SU(2) \times U(1) \times SU(3)$ model with the standard soft breaking mass parameters and trilinear scalar couplings [29]. In the electroweak gaugino sector and in the slepton sector the soft breaking part of the Lagrangian reads as:

$$\mathcal{L} = M_{L,ij} \tilde{l}_L \tilde{l}_j + M_{E,ij} \tilde{e}_L \tilde{e}_j + A_{ij} H_1 \tilde{l}_L \tilde{l}_j + A^{\dagger}_{ij} H_1^* \tilde{l}_L \tilde{l}_j$$

$$+ M_1 \tilde{b} \tilde{b} + M_1^* \tilde{b} \tilde{b} + M_2 \tilde{w}^a \tilde{w}^a + M_2^* \tilde{w}^a \tilde{w}^a$$

(1)

$M_1$ and $M_2$ are the $U(1)$ and $SU(2)$ gaugino mass parameters, respectively. $M_{L}^2$ and $M_{E}^2$ are the soft SUSY breaking mass matrices for left and right sleptons, respectively, and the $A_{ij}$ are the trilinear soft SUSY breaking couplings of the sleptons and Higgs boson. $M_1$, $M_2$, $M_{L,ij} = (M_{L,ji})^*$, $M_{E,ij} = (M_{E,ij})^*$ and $A_{ij}$ are complex; note that $A_{ij} \neq A_{ji}^*$ for $i \neq j$. The most general charged slepton mass matrix including left-right mixing as well as flavor mixing is usually written in the form

$$M^2_i = \begin{pmatrix} M_{LL}^2 & M_{LR}^2 \\ M_{LR}^2 & M_{RR}^2 \end{pmatrix},$$

(2)

where the entries are $3 \times 3$ matrices. In terms of the parameters introduced in (1), they are given by

$$M_{LL,ij}^2 = M_{L,ij}^2 + \frac{v_d^2 y_{E}^i y_{E}^j}{2} + \frac{(g^2 - g^2) (v_d^2 - v_u^2) \delta_{ij}}{8},$$

(3)

$$M_{LR,ij}^2 = \frac{v_d A_{ij}^* - \mu v_u y_{E}^i}{\sqrt{2}},$$

(4)

$$M_{RR,ij}^2 = M_{E,ij}^2 + \frac{v_d^2 y_{E}^i y_{E}^j}{2} - \frac{g^2 (v_d^2 - v_u^2) \delta_{ij}}{8}.$$  

(5)

The indices $i, j, k = 1, 2, 3$ characterize the flavors $e, \mu, \tau$. $\mu$ and the $y_{E}^i$ are the usual $\mu$ parameter and the lepton Yukawa couplings such that $m_l = v_d y_{E}^l$. $v_u$ and $v_d$ are the vacuum expectation values of the neutral Higgs fields (with $\tan \beta = v_u / v_d$). In what follows we will work in a basis where $M_2$ is real and where the lepton Yukawa coupling is real and flavour diagonal. Both assumptions can be done without loss of generality, because (i) only phase differences matter and (ii) there are no right-handed neutrinos in the low energy spectrum. The mass eigenstates $\tilde{l}_n$ of (2) are given by $\tilde{l}_n = R_{nm} \tilde{l}'_m$ with $\tilde{l}'_m = (\tilde{e}_L, \tilde{\mu}_L, \tilde{\tau}_L, \tilde{e}_R, \tilde{\mu}_R, \tilde{\tau}_R)$.

Similarly, one finds for the sneutrinos

$$M_{\tilde{\nu},ij}^2 = M_{L,ij}^2 + \frac{(g^2 + g^2) (v_d^2 - v_u^2) \delta_{ij}}{8}$$

(6)
and the corresponding mass eigenstates $\tilde{\nu}_i = R_{ij}^L \tilde{\nu}_j$ and $\tilde{\nu}'_i = (\tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau)$. We have not explicitly written neutrino Yukawa couplings or (Majorana) masses and corresponding soft terms, although neutrino flavour violations were partly motivating this study. If one assumes the sea-saw mechanism for the (light) left-handed neutrino masses, the right-handed neutrinos and sneutrinos are very heavy and they practically decouple. The 'left-handed' sneutrino mass terms will contain contributions from the neutrino Yukawa couplings, but these will be very tiny playing a role only in the special case where all soft terms are exactly degenerate; we therefore neglect them in this work.

The relevant interactions for this study are

$$\mathcal{L} = \bar{l}_i (c_{ikm}^L P_L + c_{ikm}^R P_R) \chi_k^0 + \bar{l}_i (d_{ikr}^L P_L + d_{ikr}^R P_R) \tilde{\chi}_i \tilde{\nu}_r + h.c.$$  (7)

and the couplings $c_{ikm}^L$, $c_{ikm}^R$, $d_{ikm}^L$, $d_{ikm}^R$ are given in the Appendix. \footnote{Similar interactions in the neutrino sector would give rise to neutrino decays such as $\nu \rightarrow \nu' \gamma$ if kinematically allowed. Such decays are of interest in cosmology. However, the estimated lifetimes in our framework are so large that no interesting effect is expected.}

The couplings in (7) give, at the 1-loop level, contributions to the anomalous magnetic moments of the leptons $a_i$, the electric dipole moments $d_l$ and to rare lepton decays such as $l_j \rightarrow l_i \gamma$ if individual lepton number is not conserved. All these observables are induced by the same amplitude

$$T = i e e^{\mu\nu} \frac{g^\nu}{2m_{l_j}} \bar{l}_i \sigma_{\mu\nu} (a_{ij}^L P_L + a_{ij}^R P_R) l_j$$  (8)

arising from the diagrams shown in Fig. 1. Here we take $i \leq j$.

The coefficients $a_{ij}^L$ and $a_{ij}^R$ are given by

$$16\pi^2 a_{ij}^L = \sum_{k=1}^{4} \sum_{r=1}^{6} \left( c_{ikr}^L c_{jkr}^L \frac{m_{ij}^2}{m_{\chi_k^0}} + c_{ikr}^R c_{jkr}^R \frac{m_{ij}^2}{m_{\chi_k^+}} \right) F_1 \left( \frac{m_{i}^2}{m_{\chi_k^0}} \right) + c_{ikr}^L c_{jkr}^R \frac{m_{ij} m_{i}^2}{m_{\chi_k^0}^2} F_3 \left( \frac{m_{i}^2}{m_{\chi_k^0}^2} \right)$$

$$+ \sum_{k=1}^{2} \sum_{r=1}^{3} \left( d_{ikr}^L d_{jkr}^L \frac{m_{ij}^2}{m_{\chi_k^0}} + d_{ikr}^R d_{jkr}^R \frac{m_{ij}^2}{m_{\chi_k^+}} \right) F_2 \left( \frac{m_{i}^2}{m_{\chi_k^0}^2} \right) + d_{ikr}^L d_{jkr}^R \frac{m_{ij} m_{i}^2}{m_{\chi_k^0}^2} F_4 \left( \frac{m_{i}^2}{m_{\chi_k^0}^2} \right)$$

$$a_{ij}^R = a_{ij}^L (L \leftrightarrow R)$$  (9)

resulting in the following formulas for $\Delta a_i$, $d_i$ and $l_j \rightarrow l_i \gamma$:

$$\Delta a_i = \frac{1}{2} \text{Re} \left( a_{ii}^L + a_{ii}^R \right)$$

$$\frac{1}{e} d_i = \frac{1}{2} \text{Im} \left( -a_{ii}^L + a_{ii}^R \right)$$

$$\Gamma(l_j \rightarrow l_i \gamma) = \frac{\alpha m_{l_j}}{16} \left( |a_{ij}^L|^2 + |a_{ij}^R|^2 \right)$$  (12)

where in the last equation we have neglected terms of order $O(m_{l_j}/m_{l_j})$. The functions $F_i$ and the explicit form of the couplings are listed in the Appendix. We consider for consistency only the 1-loop contributions to the EDMs and the MDMs neglecting the partial 2-loop results for the EDMs presented in 30.
Figure 1: Generic diagrams contributing to $\Delta a_l, d_l, l_j \to l_i \gamma$.

3 The flavour conserving case

We begin our investigation with the point SPS#1a $^{20}$ which is defined by $M_{L,11}^2 = M_{L,22}^2 = 202.3^2 \text{ GeV}^2$, $M_{L,33}^2 = 201.5^2 \text{ GeV}^2$, $M_{E,11}^2 = M_{E,22}^2 = 138.7^2 \text{ GeV}^2$, $M_{E,33}^2 = 136.3^2 \text{ GeV}^2$, $A_{11} = -7.567 \cdot 10^{-3} \text{ GeV}$, $A_{22} = -1.565 \text{ GeV}$, $A_{33} = -26.326 \text{ GeV}$, $M_1 = 107.9 \text{ GeV}$, $M_2 = 208.4 \text{ GeV}$, $\mu = 365 \text{ GeV}$, tan$\beta = 10$. Note that the $A$ parameters are already multiplied by the lepton Yukawa couplings. This gives (for real parameters) the following SUSY contributions to the observables:

$d_e = d_\mu = d_\tau = 0$, $\Delta a_e = 6.8 \cdot 10^{-14}$, $\Delta a_\mu = 2.9 \cdot 10^{-9}$, $\Delta a_\tau = 8.4 \cdot 10^{-7}$.

The flavour conserving contributions to the leptonic MDM and EDM were extensively discussed in $^{31, 32, 18, 33, 34}$. In the flavour conserving case the phase $\varphi_\mu$ of the parameter $\mu$ is severely constrained $^{18, 33, 34, 35, 36}$ by the EDMs of the electron and the neutron. There are some regions in parameter space, where $\varphi_\mu$ can be about $\pi/10$ for slepton masses as light as $\mathcal{O}(200) \text{ GeV}$ if there are cancellations between the chargino and neutralino contributions $^{35}$. In the case of the electron EDM such a cancellation is due to an interplay of the phases $\varphi_{A_{11}}$ and $\varphi_{U_{1}}$, where $\varphi_{U_{1}}$ is the phase of the $M_1$ parameter. In Fig. 2a we show the range of the $\varphi_\mu$--$\varphi_{U_{1}}$ plane allowed by the electron EDM; $\varphi_{A_{11}}$ is varied in the full range. If one fixes this phase, then the two bands collapse to lines. Similar results have been found in $^{36}$. We see how the inclusion of the phase $\varphi_{A_{11}}$ enlarges the allowed region, but not too much. In Fig. 2b, the SUSY contribution $\Delta a_\mu$ to the anomalous magnetic moment of the muon is shown. The two bands correspond to the cases where $\varphi_\mu$ is centered near 0 and near $\pi$. We see that while the EDM leaves a twofold ambiguity for the phase $\varphi_\mu$, the $CP$-conserving anomalous magnetic moment discriminates between the two values - indeed the lower band is already excluded and $\varphi_\mu$ must be near $\pi$. This has been observed before $^{32}$. This analysis shows that phases are also important for $CP$-conserving observables and that a combined analysis of all effects is necessary. In case the theoretical treatment of $a_\mu$ is improved, it might be even possible to exclude the situation without flavour violation.

Despite the new freedom, $\mu$ is still basically real. Unless the $A$ parameters are substantially larger, this conclusion remains. However, bigger values of $|A_{11}|$ are in contradiction with stability arguments for the potential. As we will see, flavour violating complex parameters change this picture. In addition to SPS#1a we have considered also other Snowmass points and found similar results.
Figure 2: a) Allowed regions in the $\phi_\mu$–$\phi_{U_1}$ plane by the electron EDM; b) SUSY contribution $\Delta a_\mu$ to the anomalous magnetic moment of the muon.

4 Including flavour violation

We begin the analysis with a study of pairwise flavour mixings between two generations to get an idea which parameters have the largest effect on which observables. The new parameters that need to be introduced (say $M^2_{L,12}$, etc.) are chosen such that the bounds on all observables are saturated.

4.1 $\tilde{e}$–$\tilde{\mu}$ mixing

Starting from our reference point, we add the following flavour violating terms: $M^2_{L,12} = 0.1$ GeV$^2$, $M^2_{E,12} = 0.1$ GeV$^2$, $A_{12}^1 = 10^{-3}$ GeV, $A_{21}^2 = 10^{-3}$ GeV. With these parameters we get $\text{BR}(\mu \to e \gamma) = 1.1 \cdot 10^{-12}$. As mentioned, these parameters saturate the limits on the branching ratio (we have disregarded possible subtle cancellations).

Due to the relative smallness of the off-diagonal parameters the effect of the phases is small, as can be seen in Fig. 3. The effect of the phases of $A_{12}^1$ and $A_{21}^2$ is nearly the same as that of $\phi_{M^2_{E,12}}$ and is not shown. The magnetic and the electric dipole moments are practically independent of the phases of the flavour violating parameters and are therefore not explicitly shown.

4.2 $\tilde{e}$–$\tilde{\tau}$ mixing

Now the flavour violating terms $M^2_{L,13} = 1500$ GeV$^2$, $M^2_{E,13} = 2000$ GeV$^2$, $A_{31} = A_{13} = 20$ GeV are introduced. This yields $\text{BR}(\tau \to e \gamma) = 1.05 \cdot 10^{-6}$. The effect of the phases is shown in Fig. 4. Each individual contribution from the various phases $\phi_{M^2_{E,13}}$, $\phi_{M^2_{L,13}}$, $\phi_{A_{13}}$ and $\phi_{A_{31}}$ is similar in size to that of $\phi_\mu$. If only one of these phases would generate the electron EDM, it would have to be very near zero or $\pi$ because the effect is of order $10^{-23}$ ecm as seen in the
Figure 3: $10^{12} \text{BR}(\mu \to e \gamma)$ as a function of $\varphi_{M_{E,12}^2}$ (full line) and $\varphi_{M_{L,12}^2}$ (dashed line). The phase $\varphi_\mu$ is set equal to zero.

Figure 4: a) $10^{23} \cdot d_e$ and b) SUSY contributions to $a_e$ as a function of $\varphi_{M_{E,13}^2}$ (full line), $\varphi_{M_{L,13}^2}$ (dashed line), $\varphi_{A_{13}}$ (dashed dotted line) and $\varphi_{A_{31}}$ (long-short dashed line) for $\varphi_\mu = 0$. 
figure. But if there are several contributions, the phases can be arbitrarily large, since various contributions can cancel each other. Such a cancellation is not obvious, because the bounds on \( \Delta a_e \) and BR(\( \tau \to e \gamma \)) must be satisfied and the parameters are already constrained. The dependence of these quantities on the phases looks similar to that of BR(\( \mu \to e \gamma \)) in Fig. 3 and are not shown here.

In Fig. 5 we show the contour plot for \( d_e \) as a function of \( \varphi_\mu \) and \( \varphi_{M^2_{L,13}} \). One can see that a very small \( d_e \) consistent with the experimental upper bound can be obtained for all (!) values of \( \varphi_\mu \) provided that also the phase \( \varphi_{M^2_{L,13}} \) of the flavour violating parameter \( M^2_{L,13} \) is large. There are roughly two allowed regions. In one region, the two phases are equal and opposite and there is a cancellation between the lepton flavour conserving and the lepton flavour violating contributions. In this case, the phase of \( \mu \) can be large indeed. In the other region, \( \varphi_\mu \) is around \( \pi \) and there is only a weak dependence on \( \varphi_{M^2_{L,13}} \). In this situation, the contribution from \( M^2_{L,13} \) is not important for the dipole moment. Note that here only the phases shown in the plot are varied, while the others are set equal to 0.

As can be seen from Fig. 5b, the decay rate for \( \tau \to e \gamma \) varies within an order of magnitude over the plot. However, if further experiments would establish a considerably lower limit or measure the branching ratio with 50% or better, the phases could be severely constrained. This underlines clearly the strength of a combined analysis, once the basic supersymmetric parameters are known. We also note that the ‘landscape’ in the plots is quite dramatic and that the allowed regions are narrow.

In Fig. 6 the situation for the phases of the flavour violating parameters \( M^2_{E,13} \) and \( A_{13} \) are shown. These terms have a strong interplay with the phase of \( \mu \) as seen by the ‘dramatic’ landscape. In both cases, the phase is essentially limited to zero and \( \pi \) (recall, phases not shown in the plot are set equal to zero). On the other hand, the phase of \( \mu \) is not restricted.
4.3 $\bar{\mu}$-$\bar{\tau}$ mixing

In this subsection we consider mixing between the second and the third generation by adding the following flavour violating terms to our reference point: $M^2_{E,23} = 1500 \text{ GeV}^2$, $M^2_{E,23} = 2000 \text{ GeV}^2$, $A_{32} = A_{23} = 20 \text{ GeV}$, yielding $\text{BR}(\tau \rightarrow \mu \gamma) = 1.0 \cdot 10^{-6}$. The effect of the phases is shown in Fig. 7. We see the branching ratio $\text{BR}(\tau \rightarrow \mu \gamma)$ can vary by a factor two when changing the phases from 0 to $\pi$. If one takes $\varphi_{\mu} = \pi/2$ then one finds $d_{\mu} = (-2.7 \pm 0.1) \times 10^{-22}\text{eem}$ varying only slightly with the flavour violating phases. Note that this value is clearly in the reach of future experiments [26]. In the case of $\text{BR}(\tau \rightarrow \mu \gamma)$ we find a range of $2.5 \cdot 10^{-7}$–$8.5 \cdot 10^{-7}$ and the functional dependence on the phases are shifted by $\pi/2$ for $\varphi_{M_{E,23}}$ and $-\pi/2$ in the remaining cases.

4.4 The three generation case

Finally, we allow for the most general mixing in the slepton sector. We take our reference point and add all possible phases and generation mixing terms. The moduli of the off-diagonal terms are between zero and the following upper bounds: $|M^2_{E,12}|, |M^2_{L,12}| \leq 10 \text{ GeV}^2$, $|M^2_{E,13}|, |M^2_{E,23}| \leq 1000 \text{ GeV}^2$, $|A_{12}|, |A_{21}| \leq 0.05 \text{ GeV}$, $|A_{13}|, |A_{31}|, |A_{23}|, |A_{32}| \leq 20 \text{ GeV}$. All phases are varied in the range between 0 and $2\pi$.

Fig. 8 is a scatter plot of the allowed values of $\varphi_{\mu}$ and $\varphi_{U_1}$ obeying all constraints from the EDMs and the rare lepton decays. Comparing Fig. 8 with Fig. 2 one sees that maximal values for both $\varphi_{\mu}$ and $\varphi_{U_1}$ are possible. This is again due to cancellations between the lepton flavour conserving and the lepton flavour violating contributions. Note that such cancellations are possible even for slepton masses as small as 200 GeV. In Fig. 8 we show the SUSY contribution $\Delta a_{\mu}$ to the anomalous magnetic moment of the muon as a function of $\varphi_{U_1}$, varying all parameters and phases in the range given above and fulfilling the constraints from the EDMs and the rare lepton decays.
Figure 7: a) $d_\mu$ and b) BR($\tau \rightarrow \mu \gamma$) as a function of $\varphi_{M^2_{E,23}}$ (full line), $\varphi_{M^2_{L,23}}$ (dashed line), $\varphi_{A_{23}}$ (dashed dotted line) and $\varphi_{A_{32}}$ (long-short dashed line) for $\varphi_\mu = 0$.

As pointed out in [16], LFV leads to the violation of the naive scaling relations like $d_e/d_\mu \simeq m_e/m_\mu$. Similarly one also expects deviations from the relation $\Delta a_e/\Delta a_\mu \propto (m_e/m_\mu)^2$, in particular as a consequence of the phases of the parameters. Fig. 9 shows our results for $\Delta a_e$ versus $\Delta a_\mu$ and $d_e$ versus $d_\mu$. One sees that the naive relation $\Delta a_e/\Delta a_\mu \propto (m_e/m_\mu)^2$ is largely maintained after imposing the experimental constraints arising from EDMs and rare decays even if one allows for the most general flavour structure. However, there are parameter points where the simple $\Delta a_\mu - \Delta a_e$ scaling is violated, as has also been noted by the authors of Refs. [16, 12]. Of interest is the 'hole' in Fig. 9a which excludes vanishing corrections.

The situation is completely different in the case of the electric dipole moments where the correlation between $d_e$ and $d_\mu$ is completely destroyed once all possible flavour violating parameters are taken into account.\footnote{Other possibilities of violations of the scaling relations have been presented in [37].} The reason for the difference between EDMs and the MDMs is that in the case of the $d_e$ cancellations of at least of two orders of magnitude are required to satisfy the experimental bounds implying that $d_e$ is no longer proportional to $m_e$. We have checked that in the case, where a larger modulus of $d_e$ is allowed, the proportionality to $m_e$ is restored except for the region around 0. In addition we have checked that the ratio $d_\mu/d_\tau$ is still proportional to $m_\mu/m_\tau$.

Fig. 10 shows the allowed regions for the complex parameters $M^2_{E,13}$, $M^2_{L,13}$, $A_{13}$ and $A_{31}$ respectively, for $\varphi_\mu = \pi/2$. All other phases have been varied in the range $(0, 2\pi)$. Again, the allowed regions are large. Note, that the $|M^2_{E,13}|$, $|M^2_{L,13}|$ can go up to 5% of $|M^2_{E,33}|$ and $|M^2_{L,33}|$, respectively. $|A_{13}|$ and $|A_{31}|$ can have the same order of magnitudes as $|A_{33}|$. In case of $\varphi_\mu = 0$ roughly the same areas would be allowed. The major differences compared to $\varphi_\mu = \pi/2$ are:

(i) The moduli of the $A$ parameters are smaller by about 25%. (ii) There are less points with large moduli.
Figure 8: a) Allowed regions in the $\varphi_{\mu} - \varphi_{U_1}$; b) SUSY contribution $\Delta a_{\mu}$ to the anomalous magnetic moment of the muon. Here we have taken the most general form for the slepton mixings.

Figure 9: Correlation between a) $\Delta a_e$ and $\Delta a_{\mu}$ and b) $d_e$ and $d_{\mu}$. 
Figure 10: Real and imaginary parts of $M_{E,13}^2$, $M_{L,13}^2$, $A_{13}$, and $A_{31}$ allowed by the experimental constraints. We have taken $\varphi_\mu = \pi/2$ and the remaining phases have been varied as described in the text.
5 Conclusions

In this paper we have studied the most general mass matrices for sleptons within the MSSM, including left–right mixing, flavour mixing and complex phases. In particular, we have analysed the implications on the phases coming from the experimental restrictions on anomalous magnetic and electric dipole moments of the charged leptons and on the rare decays $\mu \to e\gamma$, $\tau \to e\gamma$ and $\tau \to \mu\gamma$. For the basic SUSY parameters such as the masses of the super-particles we have used the values discussed in the Snowmass report [20].

Since there are many free parameters in a general scenario, we have first considered several special situations:

(i) If flavour violation in the slepton sector is negligible (no off-diagonal matrix elements) we recover the known results for the phases: In general, all possible phases, especially the phase $\varphi_\mu$ of the $\mu$ parameter must be small (or $\pi$) in order to be consistent with the electric dipole moments. Only in the case that the phases of $M_1$ and $A_{11}$ are correlated to the phase of $\mu$, the phases of $M_1$ and $A_{11}$ can be maximal due to cancellations of different contributions to $d_e$.

(ii) In case there is flavour mixing only in the selectron–smuon sector, the moduli of the flavour violating parameters in the mass matrices have to be small to satisfy the experimental bound on $\mu \to e\gamma$. Therefore, the effects of their phases is small even if they are maximal.

(iii) If there is only mixing between selectrons and staus, each individual contribution to $d_e$ due to the phases $\varphi_\mu$, $\varphi_{U_{13}}$, $\varphi_{M_{E_{13}}}$, $\varphi_{M_{L_{13}}}$, $\varphi_{A_{13}}$, and $\varphi_{A_{31}}$ is of similar size. If only one of these phases is non-vanishing, it must be small if the slepton masses are $O(100)$ GeV. However, if two or more phases are present, all of them including $\varphi_\mu$ could be large because the various contributions to $d_e$ may cancel each other.

(iv) And finally, if only smuons and staus mix, the present experiments do not limit the phases of the mixing parameters. However, the planned new measurement of the muon EDM being sensitive to $d_\mu \simeq 10^{-24}$ ecm [26] could give restrictions on phases if only one is present, otherwise combinations of various phases will be constrained.

In the general case with arbitrary three–generation mixing, cancellations between various LFC and LFV contributions to $d_e$ are easily possible. The numerical analysis has lead to two main results:

(i) Despite the large number of unknown parameters, significant restrictions on the allowed ranges are obtained. A good example is given in Fig. 8. From Fig. 8 we see that the allowed range for the new contributions to $g – 2$ are limited by the two wiggly bands. So if the theoretical analysis of $g – 2$ in the standard model would yield that $\Delta a$ is in the range $(1 – 2) \cdot 10^{-9}$, the phase $\phi_{U(1)}$ would have to be near 90 or 270 degrees. Similarly Fig. 10 a and c limit the values of $M_{E_{13}}$ and $M_{L_{13}}$ to about $2 \cdot 10^3 GeV^2$, about 2–3 % of the values of the diagonal elements; in contrast, $A_{13}$ and $A_{31}$ are much less constrained.

Fig. 9 a) shows an unexpected hole in the allowed ranges of the muon and electron $g – 2$, and Fig. 9 b) the surprising independence of the respective electric dipole moments. This is a clear indication [10] that the presence of lepton flavour violating phases leads to large deviations of the scaling relations such as $d_e/d_\mu \simeq m_e/m_\mu$, but to much smaller modifications for the scalings of $\Delta a_e/\Delta a_\mu$ etc. In the case of the electron, the mass is so small that other contribution are also important and may swamp out the mass dependence almost completely. Therefore, it is

If the range in Fig. 9 is increased, a weak mass dependence is visible.
possible that the EDM’s of $\mu$ and $\tau$ are larger than expected from "naive" scaling.

(ii) The lepton flavour violating parameters as well as $\mu$, $M_1$ and $A_{ii}$ ($i=1,2,3$) can have large phases, despite the stringent limits on CP violation. In particular, the phase of the parameter $\mu$ can be maximal even for O(100) GeV slepton masses, in contrast to naive expectations. Therefore, the phases of the supersymmetric parameters can be as large as those in the standard model and need not be artificially small. While we have used one of the Snowmass points for our presentation of detailed numerical results, we have checked that the qualitative features of our results do not depend on this specific choice.

Acknowledgments

We thank W. Bernreuther for useful discussions. This work was supported by the European Community’s Human Potential Programme under contract HPRN-CT-2000-00149, by the ‘Fonds zur Förderung der wissenschaftlichen Forschung’ of Austria, projects No. P13139-PHY and No. P16592-N02 and by the Swiss ‘Nationalfonds’. W.P. has been supported by the Erwin Schrödinger fellowship No. J2272 of the ‘Fonds zur Förderung der wissenschaftlichen Forschung’ of Austria.

A Slepton couplings

Here we collect the formulas for the various slepton couplings in the most general form. The couplings $l_i-\tilde{\chi}^\pm_j-\tilde{\nu}_k$ are given by

$$d_{ijk}^L = -gV_{j1} \sum_{r=1}^{3} (R_{kr}^\nu)^* R_{L,ir}^l$$

$$d_{ijk}^R = U_{j2} \sum_{r,s=1}^{3} (R_{kr}^\nu)^* Y_{rs}^E R_{R,is}^l$$

where $U$ and $V$ are the chargino mixing matrices, $R_{\nu}^l$ is the sneutrino mixing matrix, $R_{L}^l$ and $R_{R}^l$ are the left and right mixing matrix of the left and right charged leptons, respectively. Note that in the absence of right handed neutrinos the latter two can be chosen to be the unit matrix without loss of generality. The couplings $l_i-\tilde{\chi}_j^0-\tilde{l}_k$ are given by

$$c_{ijk}^L = f_j^R \sum_{r=1}^{3} (R_{k,3+r}^l)^* R_{L,ir}^l - N_{j3}^* \sum_{r,s=1}^{3} (R_{k,r}^l)^* Y_{rs}^E R_{R,is}^l$$

$$c_{ijk}^R = f_j^L \sum_{r=1}^{3} (R_{k,r}^l)^* R_{L,ir}^l - N_{j3}^* \sum_{r,s=1}^{3} (R_{k,3+r}^l)^* (Y_{sr}^E)^* R_{L,is}^l$$

$$f_j^R = -\sqrt{2}g' N_{j1}$$

$$f_j^L = \frac{1}{\sqrt{2}} (g' N_{j1} - g N_{j2})$$

where $N$ is the neutralino mixing matrix and $R_{\tilde{l}}^l$ is the mixing matrix of the charged sleptons.
B Loop functions

For completeness we display here the loop functions used:

\[ F_1(x) = \frac{-2 + 3x - 6x^2 + 3x^3 + 6x \log x}{6(1 - x)^4} \]  (20)

\[ F_2(x) = \frac{1 - 6x + 3x^2 + 2x^3 - 6x^2 \log x}{6(1 - x)^4} \]  (21)

\[ F_3(x) = \frac{-1 - x^2 + 2x \log x}{(1 - x)^3} \]  (22)

\[ F_4(x) = \frac{1 - 4x + 3x^2 - 2x^2 \log x}{(1 - x)^3} \]  (23)

References

[1] Y. Fukuda et al. [Super-Kamiokande Collaboration], Phys. Rev. Lett. 81 (1998) 1562; S. Fukuda et al. [Super-Kamiokande Collaboration], Phys. Rev. Lett. 86 (2001) 5651.

[2] Q. R. Ahmad et al. [SNO Collaboration], Phys. Rev. Lett. 87 (2001) 071301.

[3] K. Eguchi et al. [KamLAND Collaboration], Phys. Rev. Lett. 90 (2003) 021802.

[4] M. L. Brooks et al. [MEGA Collaboration], Phys. Rev. Lett. 83 (1999) 1521.

[5] K. Hagiwara et al. [Particle Data Group Collaboration], Phys. Rev. D 66 (2002) 010001.

[6] E.P. Shabalin, Sov.J.Nucl.Phys.28:75,1978, Yad.Fiz.28:151-155,1978; A. Czarnecki and B. Krause, Acta Phys.Polon. B28 (1997) 829-834; Phys. Rev. Lett. 78 (1997) 4339.

[7] N. Arkani-Hamed et al., Phys. Rev. Lett. 77, 1937 (1996); N. Arkani-Hamed et al., Nucl. Phys. B 505, 3 (1997).

[8] H. Baer et al., Phys. Rev. D 63, 095008 (2001); J. Hisano et al., Phys. Rev. D 60, 055008 (1999); D. Nomura, Phys. Rev. D 64, 075001 (2001); M. Guchait, J. Kalinowski and P. Roy, Eur. Phys. J. C 21, 163 (2001).

[9] N. V. Krasnikov, Phys. Lett. B 388, 783 (1996).

[10] W. Porod and W. Majerotto, Phys. Rev. D 66 (2002) 015003.

[11] R. Barbieri, L. J. Hall and A. Strumia, Nucl. Phys. B 445 (1995) 219.

[12] A. Romanino and A. Strumia, Nucl. Phys. B 622 (2002) 73.

[13] I. Masina and C. A. Savoy, Nucl. Phys. B 633 (2002) 139.

[14] G. C. Branco, D. Delepine and S. Khalil, hep-ph/0304164

[15] I. Masina, hep-ph/0304299
[16] J. L. Feng, K. T. Matchev and Y. Shadmi, Nucl. Phys. B 613 (2001) 366.
[17] T. F. Feng, T. Huang, X. Q. Li, X. M. Zhang and S. M. Zhao, hep-ph/0305290.
[18] G. Eyal and Y. Nir, Nucl. Phys. B 528 (1998) 21; S. Abel, S. Khalil and O. Lebedev, Nucl. Phys. B 606 (2001) 151.
[19] M. Fukugita and T. Yanagida, Phys. Lett. B 174 (1986) 45; W. Buchmüller and M. Plümacher, Int. J. Mod. Phys. A 15 (2000) 5047.
[20] B. C. Allanach et al., Eur. Phys. J. C 25 (2002) 113.
[21] E. D. Commins, S. B. Ross, D. DeMille and B. C. Regan, Phys. Rev. A 50 (1994) 2960. B. C. Regan, E. D. Commins, C.J. Schmidt and D. DeMille, Phys. Rev. Lett. 88 (2002 071805.
[22] J. Bailey et al. [CERN-Mainz-Daresbury Collaboration], Nucl. Phys. B 150 (1979) 1.
[23] E. P. Sichtermann [the Muon g-2 Collaboration], hep-ex/0301003.
[24] F. Jegerlehner, J.Phys.G29 (2003) 101.
[25] M. Davier, S. Eidelman, A. Höcker and Z. Zhang, hep-ph/0208177.
[26] See e.g. L.R. Hunter, Talk at the workshop Tests of Fundamental Symmetries in Atoms and Molecules, Harvard, 2001. Available online http://itamp.harvard.edu/fundamentalworkshop.html.
[27] Y. K. Semertzidis et al.; to appear in the proceedings of KEK International Workshop on High Intensity Muon Sources (HIMUS 99), Tsukuba, Japan, 1-4 Dec 1999, hep-ph/0012087.
[28] T. Mori et al., R-99-05, http://meg.web.psi.ch.
[29] H. P. Nilles, Phys. Rept. 110 (1984) 1; H. E. Haber and G. L. Kane, Phys. Rept. 117 (1985) 75; J. F. Gunion and H. E. Haber, Nucl. Phys. B 272 (1986) 1 [Erratum-ibid. B 402 (1993) 567].
[30] T. Kadoyoshi and N. Oshimo, Phys. Rev. D 55 (1997) 1481; D. Chang, W. Y. Keung and A. Pilaftsis, Phys. Rev. Lett. 82, 900 (1999) [Erratum-ibid. 83, 3972 (1999)].
[31] M. Carena, G.F. Giudice and C.E.M. Wagner, Phys. Lett. B 390 (1997) 234; E. Gabrielli and U. Sarid, Phys. Rev. Lett. 79 (1997) 4752.
[32] J.L. Feng and K.T. Matchev, Phys. Rev. Lett. 86 (2001) 3480; L. Everett, G.L. Kane, S. Rigolin and L. Wang, Phys. Rev. Lett. 86 (2001) 3484; T. Ibrahim, U. Chattopadhyay and P. Nath, Phys. Rev. D 64 (2001) 016010; J. Ellis, D.V. Nanopoulos and K.A. Olive, Phys. Lett. B 508 (2001) 65; S. Komine, T. Moroi and M. Yamaguchi, Phys. Lett. B 507 (2001) 224; A. Bartl et al., Phys. Rev. D 64 (2001) 076009; Z. Chacko and G.D. Kribs, Phys. Rev. D 64 (2001) 75015; D.G. Cerdeno, E. Gabrielli, S. Khalil, C. Munoz and E. Torrente-Lujan, Phys. Rev. D 64 (2001) 093012; U. Chattopadhyay and P. Nath, hep-ph/0208012. S.P. Martin, J.D. Wells, hep-ph/0209309.
[33] T. Ibrahim and P. Nath, Phys. Rev. D 58 (1998) 111301; T. Falk and K. Olive, Phys. Lett. B 439 (1998) 71.

[34] U. Chattopadhyay, T. Ibrahim and P. Roy, Phys. Rev. D 64 (2001) 013004; V. Barger et al., Phys. Rev. D 64, 056007 (2001); T. Ibrahim and P. Nath, Phys. Rev. D 64 (2001) 093002.

[35] A. Bartl, T. Gajdosik, W. Porod, P. Stockinger and H. Stremnitzer, Phys. Rev. D 60 (1999) 073003.

[36] M. Brhlik, G. J. Good and G. L. Kane, Phys. Rev. D 59 (1999) 115004. M. Brhlik, L. L. Everett, G. L. Kane and J. Lykken, Phys. Rev. Lett. 83 (1999) 2124; Phys. Rev. D 62 (2000) 035005.

[37] K. S. Babu, B. Dutta and R. N. Mohapatra, Phys. Rev. Lett. 85, 5064 (2000); A. Pilaftsis, Nucl. Phys. B 644, 263 (2002).