Separation Axioms in $N_{nc}$ Topological Spaces via $N_{nc}$ e-open Sets

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Abstract. The main idea of this research is to define a new neutrosophic crisp points in neutrosophic crisp topological space namely $(N_{nc}P_N)$, the concept of $N_{nc}$ limit point was defined using $(N_{nc}P_N)$, with some of its properties, the separation axioms $(N_{nc}e\tau_i$-space $(i = 0, 1, 2)$ were constructed in neutrosophic crisp topological space using $(N_{nc}P_N)$ and examine the relationship between them in details.

Keywords and phrases: Keywords: $N_{nc}$ topological spaces, $N_{nc}$ limit point, separation axioms.

1. Introduction

Smarandache’s neutrosophic system have wide range of real time applications for the fields of Computer Science, Information Systems, Applied Mathematics, Artificial Intelligence, Mechanics, decision making, Medicine, Electrical & Electronic, and Management Science etc [1, 2, 3, 4, 20, 21]. Topology is a classical subject, as a generalization topological spaces many types of topological spaces introduced over the year. Smarandache [16] defined the Neutrosophic set on three component Neutrosophic sets (T-Truth, F-Falsehood, I-Indeterminacy). Neutrosophic topological spaces (nts’s) introduced by Salama and Alblowi [11]. Lellies Thivagar et al. [9] was given the geometric existence of $N$ topology, which is a non-empty set equipped with $N$ arbitrary topologies. Lellis Thivagar et al. [10] introduced the notion of $N_{nc}$-open (closed) sets and $N_{nc}$ topological spaces. Al-Hamido [5] explore the possibility of expanding the concept of neutrosophic crisp topological spaces into $N$-neutrosophic crisp topological spaces and investigate some of their basic properties. Several generalized forms of strongly open and strongly closed functions in topological spaces have been introduced and investigated over the course of years. Certainly, it is hard to say whether one form is more or less important than another. Functions and of course strongly open and strongly closed functions stand among the most important and most researched points in the whole of mathematical science. Various interesting problems arise when one considers openness and closeness. Its importance is significant in various areas of mathematics and related sciences. In 2008, Erdal Ekici [7] introduced a new class of generalized open sets called e-open sets and studied several fundamental and
interesting properties of \(e\)-open sets and introduced a new class of continuous functions called \(e\)-continuous functions into the field of topology. In 2020, Vadivel and co-authors [18, 19] the concept of \(N\)-neutrosophic \(\delta\)-open, \(N\)-neutrosophic \(\delta\)-semiopen, \(N\)-neutrosophic \(\delta\)-preopen and \(N\)-neutrosophic \(e\)-open sets are introduced.

Salama et al. [12, 14] put some basic concepts of the neutrosophic crisp set and their operations, and because of their wide applications and their grate flexibility to solve the problem, we used these concepts to define new types of neutrosophic points, that we called neutrosophic crisp points (briefly, \(ncpt\)’s). Finally, we used these points (\(ncpt\)’s) to define the concept of neutrosophic crisp \(e\) limit point, with some of its properties and construct the separation axioms (\(N_{nc}\epsilon_T\) space, \(i = 0, 1, 2\)) in neutrosophic crisp topological and examine the relationship between them in details.

2. Preliminaries

Salama and Smarandache [15] presented the idea of a neutrosophic crisp set in a set \(U\) and defined the inclusion between two neutrosophic crisp sets, the intersection (union) of two neutrosophic crisp sets, the complement of a neutrosophic crisp set, neutrosophic crisp empty (resp., whole) set again and discover a few properties.

**Definition 2.1** Let \(U\) be a non-empty set. Then \(H\) is called a neutrosophic crisp set (in short, \(ncs\)) in \(U\) if \(H\) has the form \(H = (H_1, H_2, H_3)\), where \(H_1, H_2, H_3\) are subsets of \(U\).

The neutrosophic crisp empty (resp., whole) set, denoted by \(\phi_n\) (resp., \(U_n\)) is an \(ncs\) in \(U\) defined by \(\phi_n = (\phi, \phi, U)\) (resp. \(U_n = (U, U, \phi)\)). We will denote the set of all \(ncs\)'s in \(U\) as \(ncS(U)\). In particular, Salama and Smarandache [13] classified a neutrosophic crisp set as the followings.

A neutrosophic crisp set \(H = (H_1, H_2, H_3)\) in \(U\) is called a neutrosophic crisp set of Type 1 (resp. 2 & 3) (in short, \(ncs\)-Type 1 (resp. 2 & 3) ), if it satisfies \(H_1 \cap H_3 = H_2 \cap H_3 = H_3 \cap H_1 = \phi\) (resp. \(H_1 \cap H_2 = H_2 \cap H_3 = H_3 \cap H_1 = \phi\) and \(H_1 \cup H_2 \cup H_3 = U\) & \(H_1 \cap H_2 \cap H_3 = \phi\) and \(H_1 \cup H_2 \cup H_3 = U\)). \(ncS_1(U)\) (\(ncS_2(U)\) and \(ncS_3(U)\)) means set of all \(ncs\) Type 1 (resp. 2 and 3).

**Definition 2.2** Let \(H = (H_1, H_2, H_3), M = (M_1, M_2, M_3) \in ncS(U)\). Then \(H\) is said to be contained in (resp. equal to) \(M\), denoted by \(H \subseteq M\) (resp. \(H = M\)), if \(H_1 \subseteq M_1, H_2 \subseteq M_2\) and \(H_3 \subseteq M_3\) (resp. \(H \subseteq M\) and \(M \subseteq H\)); \(H^c = (H_3, H_2, H_1)\); \(H \cap M = (H_1 \cap M_1, H_2 \cap M_2, H_3 \cap M_3)\); \(H \cup M = (H_1 \cup M_1, H_2 \cup M_2, H_3 \cup M_3)\). Let \((U_j)_{j \in J} \subseteq ncS(U)\), where \(U_j = (H_{j_1}, H_{j_2}, H_{j_3})\). Then \(\bigcap_{j \in J} U_j\) (simply \(\bigcap U_j\)) = \((\bigcap H_{j_1}, \bigcap H_{j_2}, \bigcup H_{j_3})\); \(\bigcup_{j \in J} U_j\) (simply \(\bigcup U_j\)) = \((\bigcup H_{j_1}, \bigcup H_{j_2}, \bigcap H_{j_3})\).

The following are the quick consequence of Definition 2.2.

**Proposition 2.1** [8] Let \(L, M, O \in ncS(U)\). Then

(i) \(\phi_n \subseteq L \subseteq U_n\),

(ii) if \(L \subseteq M\) and \(M \subseteq O\), then \(L \subseteq O\),

(iii) \(L \cap M \subseteq L\) and \(L \cap M \subseteq M\),

(iv) \(L \subseteq L \cup M\) and \(M \subseteq L \cup M\),

(v) \(L \subseteq M\) iff \(L \cap M = L\),

(vi) \(L \subseteq M\) iff \(L \cup M = M\).

Likewise the following are the quick consequence of Definition 2.2.

**Proposition 2.2** [8] Let \(L, M, O \in ncS(U)\). Then
Remark 2.1
(a) \((DeMorgan’s laws)\) : 
\[L \cup (M \cap O) = (L \cup M) \cap O,\]
\[L \cap (M \cup O) = (L \cap M) \cup O,\]
(b) \((Absorption laws)\) : 
\[L \cup (L \cap M) = L,\]
\[L \cap (L \cup M) = L.\]
(c) In general, \(L \cup L^c = U,\) \(L \cap L^c = \phi.\)

Proposition 2.3 \[8\] Let \(L \in ncS(U)\) and let \((L_j)_{j \in J} \subseteq ncS(U).\) Then
(i) \((\bigcap L_j)^c = \bigcup L_j^c, (\bigcup L_j)^c = \bigcap L_j^c,\)
(ii) \(L \cap (\bigcup L_j) = \bigcup (L \cap L_j), L \cup (\bigcap L_j) = \bigcap (L \cup L_j).\)

Definition 2.3 \[6\] Let \(U\) be a non-empty set and the \(nc\) sets \(H \& M\) in the form \(H = \langle H_1, H_2, H_3 \rangle, M = \langle M_1, M_2, M_3 \rangle\) then the additional new ways for the intersection, union and inclusion between \(H \& M\) are
\(H \cap M = (H_1 \cap M_1, H_2 \cap M_2, H_3 \cap M_3)\)
\(H \cup M = (H_1 \cup M_1, H_2 \cup M_2, H_3 \cup M_3)\)
\(H \subseteq M \iff H_1 \subseteq M_1, H_2 \subseteq M_2 \text{ and } H_3 \subseteq M_3.\)

Definition 2.4 \[6\] For all \(u, v, w\) belonging to a non-empty set \(U.\) Then the \(nc\) points related to \(u, v, w\) are defined as follows:
(i) \(u_{P_1} = \langle \{ u \}, \phi, \phi \rangle\), is called a \(ncpt\) \((ncpt_{P_1})\) in \(U.\)
(ii) \(v_{P_2} = \langle \phi, \{ v \}, \phi \rangle\), is called a \(ncpt\) \((ncpt_{P_2})\) in \(U.\)
(iii) \(w_{P_3} = \langle \phi, \phi, \{ w \} \rangle\), is called a \(ncpt\) \((ncpt_{P_3})\) in \(U.\)

The set of all \(nc\) points \((ncpt_{P_1}, ncpt_{P_2}, ncpt_{P_3})\) is denoted by \(ncPt.\)

Definition 2.5 \[6\] Let \(U\) be a non-empty set and \(u, v, w \in U.\) Then the \(ncpt:\)
(i) \(u_{P_1}\) is belonging to the \(nc\) set \(L = \langle L_1, L_2, L_3 \rangle\), denoted by \(u_{P_1} \in L,\) if \(u \in L_1,\) wherein \(u_{P_1}\) does not belong to the \(nc\) set \(L\) denoted by \(u_{P_1} \notin L,\) if \(u \notin L_1.\)
(ii) \(v_{P_2}\) is belonging to the \(nc\) set \(L = \langle L_1, L_2, L_3 \rangle,\) denoted by \(v_{P_2} \in L,\) if \(v \in L_2.\) In contrast \(v_{P_2}\) does not belong to the \(nc\) set \(L,\) denoted by \(v_{P_2} \notin L,\) if \(v \notin L_2.\)
(iii) \(w_{P_3}\) is belonging to the \(nc\) set \(L = \langle L_1, L_2, L_3 \rangle,\) denoted by \(w_{P_3} \in L,\) if \(w \in L_3.\) In contrast \(w_{P_3}\) does not belong to the \(nc\) set \(L,\) denoted by \(w_{P_3} \notin L,\) if \(w \notin L_3.\)

Remark 2.1 \[6\] If \(L = \langle L_1, L_2, L_3 \rangle\) is a \(nc\) set in a non-empty set \(U\) then:
\(L \setminus u_{P_1} = \langle L_1 \setminus \{ u \}, L_2, L_3 \rangle, L \setminus u_{P_1}\) means that the component \(L\) does not contain \(u_{P_1}.\)
\(L \setminus v_{P_2} = \langle L_1, L_2 \setminus \{ v \}, L_3 \rangle, L \setminus v_{P_2}\) means that the component \(L\) does not contain \(v_{P_2}.\)
\(L \setminus w_{P_3} = \langle L_1, L_2, L_3 \setminus \{ u \} \rangle, L \setminus w_{P_3}\) means that the component \(L\) does not contain \(w_{P_3}.\)

Remark 2.2 \[6\] If \(L = \langle L_1, L_2, L_3 \rangle\) is a \(nc\) set in a non-empty set \(U\) then:
\(L = (\langle u_{P_1} : u_{P_1} \in L \rangle) \cup (\langle v_{P_2} : v_{P_2} \in L \rangle) \cup (\langle w_{P_3} : w_{P_3} \in L \rangle) = (\langle \{ u \}, \phi, \phi : u \in U \rangle) \cup (\{ v \}, \phi, \phi : v \in U \rangle) \cup (\{ w \}, \phi, \phi : w \in U \rangle)\)
\(L = \langle \{ u_{P_1} : u_{P_1} \in L \rangle \cup \langle v_{P_2} : v_{P_2} \in L \rangle \cup \langle w_{P_3} : w_{P_3} \in L \rangle = \langle \{ u \}, \phi, \phi : u \in U \rangle \cup \langle \{ v \}, \phi, \phi : v \in U \rangle \cup \langle \{ w \}, \phi, \phi : w \in U \rangle).\)
Definition 2.6 [13] A neutrosophic crisp topology (briefly, ncT) on a non-empty set $U$ is a family $\tau$ of nc subsets of $U$ satisfying the following axioms

(i) $\phi_n, U_n \in \tau$.
(ii) $H_1 \cap H_2 \in \tau \ \forall \ H_1 \ & H_2 \in \tau$.
(iii) $\bigcup_a H_a \in \tau$, for any $\{H_a : a \in J\} \subseteq \tau$.

Then $(U, \tau)$ is a neutrosophic crisp topological space (briefly, ncTs) in $U$. The $\tau$ elements are called neutrosophic crisp open sets (briefly, ncos) in $U$. A ncs $C$ is closed set (briefly, ncs) iff its complement $C^c$ is ncos.

Definition 2.7 [5] Let $U$ be a non-empty set. Then $nc\tau_1, nc\tau_2, \ldots, nc\tau_N$ are $N$-arbitrary crisp topologies defined on $U$ and the collection $N_{nc}\tau = \{S \subseteq U : S = (\bigcup_{j=1}^N H_j) \cup (\bigcap_{j=1}^N L_j), H_j, L_j \in nc\tau_j\}$ is called $N$ neutrosophic crisp (briefly, $nc$) topology on $U$ if the axioms are satisfied:

(i) $\phi_n, U_n \in N_{nc}\tau$.
(ii) $\bigcap_j U_j \in N_{nc}\tau \ \forall \ \{U_j\}_{j=1}^n \subseteq N_{nc}\tau$.
(iii) $\bigcup_j U_j \in N_{nc}\tau \ \forall \ \{U_j\}_{j=1}^n \subseteq N_{nc}\tau$.

Then $(U, N_{nc}\tau)$ is called a $N_{nc}$-topological space (briefly, $N_{nc}$ts) on $U$. The $N_{nc}\tau$ elements are called $N_{nc}$-open sets ($N_{nc}$os) on $U$ and its complement is called $N_{nc}$-closed sets ($N_{nc}$cs) on $U$. The elements of $U$ are known as $N_{nc}$-sets ($N_{nc}s$) on $U$.

Definition 2.8 [5] Let $(U, N_{nc}\tau)$ be any $N_{nc}$ts. Let $H$ be an $N_{nc}s$ in $(U, N_{nc}\tau)$. Then $H$ is said to be a $N_{nc}$-regular open [17] set (briefly, $N_{nc}$ros) if $H = N_{nc}\text{int}(N_{nc}\text{cl}(H))$. The complement of an $N_{nc}$ros is called an $N_{nc}$-regular closed set (briefly, $N_{nc}$rcs) in $U$.

The family of all $N_{nc}$ros (resp. $N_{nc}$rcs) of $U$ is denoted by $N_{nc}$ROS$(U)$ (resp. $N_{nc}$RCS$(U)$).

Definition 2.9 [18] A set $H$ is said to be a

(i) $N_{nc}\delta$ interior of $H$ (briefly, $N_{nc}\delta\text{int}(H)$) is defined by $N_{nc}\delta\text{int}(H) = \bigcup\{S : S \subseteq H \ & S$ is a $N_{nc}$ros$\}$.
(ii) $N_{nc}\delta$ closure of $H$ (briefly, $N_{nc}\delta\text{cl}(H)$) is defined by $N_{nc}\delta\text{cl}(H) = \bigcup\{u \in U : N_{nc}\text{int}(N_{nc}\text{cl}(L)) \cap H \neq \phi, u \in L \ & L$ is a $N_{nc}$os$\}$.

Definition 2.10 A set $H$ is said to be a

(i) $N_{nc}\delta$-open (briefly, $N_{nc}\delta o$) set [18] if $H = N_{nc}\delta\text{int}(H)$.
(ii) $N_{nc}$e-open (briefly, $N_{nc}$e$o$) set [19] if $H \subseteq N_{nc}\text{cl}(N_{nc}\delta\text{int}(H)) \cup N_{nc}\text{int}(N_{nc}\delta\text{cl}(H))$.

The complement of an $N_{nc}$dos (resp. $N_{nc}$eos) is called an $N_{nc}\delta$ (resp. $N_{nc}$e) closed set (briefly, $N_{nc}$dos (resp. $N_{nc}$eos)) in $U$.

The family of all $N_{nc}$eos (resp. $N_{nc}$ecs) of $U$ containing a point $u \in U$ is denoted by $N_{nc}$eOS$(U, u)$ (resp. $N_{nc}$eCS$(U, u)$). The family of all $N_{nc}$dos (resp. $N_{nc}$ecs, $N_{nc}$eos and $N_{nc}$ecs) of $U$ is denoted by $N_{nc}$dos$(U)$ (resp. $N_{nc}$dCS$(U)$, $N_{nc}$eos$(U)$ and $N_{nc}$eCS$(U)$).

Definition 2.11 [6] Let $(U, N_{nc}\tau)$ be $N_{nc}$ts, $P \in N_{nc}$pt in $U$, a $N_{nc}$ set $L = \{L_1, L_2, L_3\} \in N_{nc}$eOS$(U)$ is called $N$ neutrosophic crisp $e$ neighbourhood (briefly, $N_{nc}$enh$d$) of $P$ in $(U, N_{nc}\tau)$ if $P \in L$. 

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3. $N_{nc}e$ limit point

**Definition 3.1** Let $(U, N_{nc})$ be $N_{nc}$ts, $P \in N_{nc}pt$ in $U$, a $N_{nc}$ set $L = \langle L_1, L_2, L_3 \rangle \in N_{nc}eOS(U)$ is called $N_{nc}eohd$ of $P$ in $(U, N_{nc})$, if there is $N_{nc}eo$ set $H = \langle H_1, H_2, H_3 \rangle$ containing $P$ such that $H \subseteq L$.

Every $N_{nc}eohd$ of any point $P \in N_{nc}pt$ in $U$ is $N_{nc}eohd$ of $P$, but in general the inverse is not true.

**Example 3.1** Let $U = \{u, v, w\}$, $N_{nc} = \{\phi_N, U_N, H, L, C\}$, $N_{nc} = \{\phi_N, U_N\}$. $H = \{\{u\}, \phi, \phi\}$, $L = \{\{v\}, \phi, \phi\}$, $C = \{\{v, w\}, \phi, \phi\}$, then we have $2_{nc} = \{\phi_N, U_N, H, L, C\}$. If we take $U = \{\{u, v\}, \{w\}, \phi\}$. Then $C = \{\{u, v\}, \phi, \phi\}$ is an $N_{nc}eo$ set containing $p = w_{P_1} = \{\{u\}, \phi, \phi\}$ and $C \subseteq U$. That is $U$ is a $N_{nc}eohd$ of $p$ in $(U, N_{nc})$, while it is not a $N_{nc}eohd$ of $p$.

**Definition 3.2** Let $(U, N_{nc})$ be $N_{nc}$ts and $L = \langle L_1, L_2, L_3 \rangle$ be $N_{nc}$ set of $U$. A $N_{nc}pt P$ in $U$ is called a $N_{nc}e$ limit point (briefly, $N_{nc}ept$) of $L = \langle L_1, L_2, L_3 \rangle$ if every $N_{nc}eo$ set containing $P$ must contains at least one $N_{nc}pt$ of $L$ different from $P$. It is easy to say that the $N_{nc}pt P$ is not $N_{nc}ept$ of $L$ if there is a $N_{nc}eo$ set $O$ of $P$ and $P \cap (O \backslash P) = \phi_n$.

**Definition 3.3** The set of all $N_{nc}ept$’s of a $N_{nc}$ set $L$ is called $N_{nc}e$ derived set of $L$, denoted by $N_{nc}eD(L)$.

**Example 3.2** In Example 3.1, if we take $D = \{\{u, v\}, \phi, \phi\}$. Then $p = w_{P_1} = \{\{u\}, \phi, \phi\}$ is the only $N_{nc}ept D$, i.e. $N_{nc}eD(D) = \{w_{P_1}\}$.

**Remark 3.1** (i) Let $L$ be any $N_{nc}$ set of $U$, if $P = \{\{u\}, \phi, \phi\} \in N_{nc}eOS(U)$ in any $N_{nc}ts (U, N_{nc})$, then $P \in N_{nc}eD(L)$.

(ii) Let $L$ be any $N_{nc}$ set of $U$, the following facts is true:

$N_{nc}eD(L) \not\subseteq L$, $L \not\subseteq N_{nc}eD(L)$, and sometimes $N_{nc}eD(L) \cap L = \phi_n$ or $N_{nc}eD(L) \cap L \neq \phi_n$.

(iii) In any $N_{nc}ts (U, N_{nc})$, we have $N_{nc}eD(\phi) = \phi_n$.

**Theorem 3.1** Let $(U, N_{nc})$ be $N_{nc}$ts and $L = \langle L_1, L_2, L_3 \rangle$ be a $N_{nc}$ set of $U$, then $L$ is $N_{nc}e$ set if $N_{nc}eD(L) \subseteq L$.

**Proof.** Let $L$ be $N_{nc}e$ set, then $(U \backslash L)$ is $N_{nc}eo$ set this implies that for each $N_{nc}pt P \in N_{nc}Pt$ in $(U \backslash L)$, $P \notin L$, there is a $N_{nc}eo$ set $O$ of $P$ and $O \subseteq (U \backslash L)$. Since $L \cap (U \backslash L) = \phi_n$, then $P$ is not $N_{nc}ept$ of $L$, thus $O \cap L = \phi_n$, which implies that $P \notin N_{nc}eD(L)$. Hence $N_{nc}eD(L) \subseteq L$.

Conversely, assume that $P \notin N_{nc}eD(L)$, implies that $P$ is not $N_{nc}ept$ of $L$, hence, there is a $N_{nc}eo$ set $O$ of $P$ and $O \cap L = \phi_n$ which means that $O \subseteq (U \backslash L)$ and since $(U \backslash L)$ is a $N_{nc}eo$ set. Hence $L$ is $N_{nc}e$ set.

**Theorem 3.2** Let $(U, N_{nc})$ be $N_{nc}$ts, $L$, $O$ be a $N_{nc}$ sets of $U$, then the following properties hold:

(i) $N_{nc}eD(\phi_n) = \phi_n$

(ii) If $L \subseteq O$, then $N_{nc}eD(L) \subseteq N_{nc}eD(O)$

(iii) $N_{nc}eD(L \cap O) \subseteq N_{nc}eD(L) \cap N_{nc}eD(O)$

(iv) $N_{nc}eD(L \cup O) = N_{nc}eD(L) \cup N_{nc}eD(O)$.

**Proof.** (i) The proof is, directly.

(ii) Assume that $N_{nc}eD(L)$ be a $N_{nc}$ set containing a $N_{nc}pt P \in N_{nc}Pt$ then by Definition 3.2, for each $N_{nc}eo$ set $V$ of $P$, we have $L \cap V \backslash P \neq \phi_n$, but $L \subseteq O$, hence $O \cap V \backslash P \neq \phi_n$, this means that $P \in N_{nc}eD(O)$. Hence, $N_{nc}eD(L) \subseteq N_{nc}eD(O)$. 


(iii) Since
\[ L \cap O \subseteq L, \text{ then by (ii) } N_{nc}eD(L \cap O) \subseteq N_{nc}eD(L) \]
(1)
\[ L \cap O \subseteq O, \text{ implies } N_{nc}eD(L \cap O) \subseteq N_{nc}eD(O) \]
(2)
from (1) \& (2) \( N_{nc}eD(L \cap O) \subseteq N_{nc}eD(L) \cap N_{nc}eD(O) \).

(iv) Let \( P \in N_{nc}Pt \) such that \( P \notin N_{nc}eD(O) \), then either \( P \notin N_{nc}eD(L) \) and \( P \notin N_{nc}eD(O) \), then there is a \( N_{nc}o \) set \( K \) of \( P \) and \( L \cap K \not\subseteq \phi_n \) and \( O \cap K \not\subseteq \phi_n \), this implies that \( (L \cup O) \cap K \not\subseteq \phi_n \), i.e \( P \notin N_{nc}eD(L \cup O) \), hence
\[ N_{nc}eD(L \cup O) \subseteq N_{nc}eD(L) \cup N_{nc}eD(O). \]
(3)

Conversely, since \( L \subseteq L \cup O, O \subseteq L \cup O, \) then by property (ii) \( N_{nc}eD(L) \subseteq N_{nc}eD(L \cup O) \) and \( N_{nc}eD(O) \subseteq N_{nc}eD(L \cup O) \), thus
\[ N_{nc}eD(L \cup O) \supseteq N_{nc}eD(L) \cup N_{nc}eD(O) \]
(4)
from (3) and (4) we have \( N_{nc}eD(L \cup O) = N_{nc}eD(L) \cup N_{nc}eD(O) \).

**Remark 3.2** In general, the inverse of property (ii) \& (iii) in Theorem 3.2 is not true. The following examples act as an evidence to this claim.

**Example 3.3** Let \( U = \{u, v, w\}, \) \( \text{nc}\tau_1 = \{\phi_N, U_N, H\}, \) \( \text{nc}\tau_2 = \{\phi_N, U_N\}, \) \( H = \{\phi, \{u\}, \phi\}, \) then we have \( 2_{nc}\tau = \{\phi_N, U_N, H\}. \) If we take \( L = \{\phi, \{u\}, \phi\}, C = \{\phi, \{v\}, \phi\}, \) \( 2_{nc}eD(L) = \{\phi, \{v\}, \phi\}, 2_{nc}eD(C) = \{\phi, \{v\}, \phi\}, \) and \( 2_{nc}eD(H) \subseteq 2_{nc}eD(C) \), but \( L \not\subseteq C \).

**Example 3.4** In Example 3.3, \( 2_{nc}eD(H \cap C) = \phi. \) Therefore, \( 2_{nc}eD(H \cap C) \neq 2_{nc}eD(H) \cap 2_{nc}eD(C) \).

**Theorem 3.3** For any \( N_{nc} \) set \( L \) over the universe \( U, \) then \( N_{nc}ecl(L) = L \cup N_{nc}eD(L) \).

**Proof.** Let us first prove that \( L \cup N_{nc}eD(L) \) is a \( N_{nc}ec \) set that is \( U_n \cap (U_n \setminus N_{nc}eD(L)) \) is a \( N_{nc}eO \) set. Now for a \( N_{nc}Pt \) \( P \in (U_n \setminus L) \cap (U_n \setminus N_{nc}eD(L)), \) then \( P \notin (U_n \setminus L) \) and \( P \notin (U_n \setminus N_{nc}eD(L)), \) thus \( P \notin L \) and \( P \notin N_{nc}eD(L). \) So by Definition 3.3, there is a \( N_{nc} \) set \( R \) of \( P \) such that \( R \cap L = \phi_n, \) hence \( R \subseteq U_n \setminus L \). Now for each \( P_1 \in R, \) then \( P_1 \notin N_{nc}eD(L), \) then \( R \cap N_{nc}eD(L) = \phi_n, \) this implies that \( R \subseteq U_n \setminus N_{nc}eD(L), \) i.e \( R \subseteq (U_n \setminus L) \cap (U_n \setminus N_{nc}eD(L)). \) Thus \( (U_n \setminus L) \cap (U_n \setminus N_{nc}eD(L)) \) is a \( N_{nc}ecl \) of all its elements and hence \( (U_n \setminus L) \cap (U_n \setminus N_{nc}eD(L)) \) is a \( N_{nc}eO \) set and thus \( L \cup N_{nc}eD(L) \) is a \( N_{nc}ec \) set containing \( L, \) therefore \( N_{nc}ecl(L) \subseteq L \cup N_{nc}eD(L). \) Since \( N_{nc}ecl(L) \) is a \( N_{nc}ec \) set (see Definition 3.3 ) and \( N_{nc}ecl(L) \) contains all its \( N_{nc}ecl \). Thus \( N_{nc}eD(L) \subseteq N_{nc}ecl(L) \) and \( L \subseteq N_{nc}ecl(L), \) hence \( N_{nc}ecl(L) = L \cup N_{nc}eD(L) \).

4. Separation axioms in a \( N_{nc} \) topological space

**Definition 4.1** A \( N_{nc}eCl (U, N_{nc}\tau) \) is called:

(i) \( P_1-N_{nc}e\tau_0 \)-space if \( \forall u_{P_1} \neq v_{P_1} \in U \exists a \ N_{nc}eO \) set \( O \) in \( U \) containing one of them but not the other.

(ii) \( P_2-N_{nc}e\tau_0 \)-space if \( \forall u_{P_2} \neq v_{P_2} \in U \exists a \ N_{nc}eO \) set \( O \) in \( U \) containing one of them but not the other.

(iii) \( P_3-N_{nc}e\tau_0 \)-space if \( \forall u_{P_3} \neq v_{P_3} \in U \exists a \ N_{nc}eO \) set \( O \) in \( U \) containing one of them but not the other.

(iv) \( P_4-N_{nc}e\tau_1 \)-space if \( \forall u_{P_4} \neq v_{P_4} \in U \exists a \ N_{nc}eO \) sets \( O_1, O_2 \) in \( U \) such that \( u_{P_1} \in O_1, \) \( v_{P_1} \not\subseteq O_1 \) and \( u_{P_1} \not\subseteq O_2, v_{P_1} \not\subseteq O_2. \)
(v) $P_2$-$N_{nc}e\tau_1$-space if $\forall u_{P_2} \neq v_{P_2} \in U \exists$ a $N_{nc}e\sigma$ sets $O_1, O_2$ in $U$ such that $u_{P_2} \in O_1$, $v_{P_2} \notin O_1$ and $u_{P_2} \notin O_2$, $v_{P_2} \in O_2$.

(vi) $P_3$-$N_{nc}e\tau_1$-space if $\forall u_{P_3} \neq v_{P_3} \in U \exists$ a $N_{nc}e\sigma$ sets $O_1, O_2$ in $U$ such that $u_{P_3} \in O_1$, $v_{P_3} \notin O_1$ and $u_{P_3} \notin O_2$, $v_{P_3} \in O_2$.

(vii) $P_1$-$N_{nc}e\tau_2$-space if $\forall u_{P_1} \neq v_{P_1} \in U \exists$ a $N_{nc}e\sigma$ sets $O_1, O_2$ in $U$ such that $u_{P_1} \in O_1$, $v_{P_1} \notin O_1$ and $u_{P_1} \notin O_2$, $v_{P_1} \in O_2$ with $O_1 \cap O_2 = \phi$.

(viii) $P_2$-$N_{nc}e\tau_2$-space if $\forall u_{P_2} \neq v_{P_2} \in U \exists$ a $N_{nc}e\sigma$ sets $O_1, O_2$ in $U$ such that $u_{P_2} \in O_1$, $v_{P_2} \notin O_1$ and $u_{P_2} \notin O_2$, $v_{P_2} \in O_2$ with $O_1 \cap O_2 = \phi$.

(ix) $P_3$-$N_{nc}e\tau_2$-space if $\forall u_{P_3} \neq v_{P_3} \in U \exists$ a $N_{nc}e\sigma$ sets $O_1, O_2$ in $U$ such that $u_{P_3} \in O_1$, $v_{P_3} \notin O_1$ and $u_{P_3} \notin O_2$, $v_{P_3} \in O_2$ with $O_1 \cap O_2 = \phi$.

Definition 4.2 A $N_{nc}e\sigma\tau$ is called:

(i) $N_{nc}e\sigma_0$-space if $(U, N_{nc}e\sigma\tau) = P_1$-$N_{nc}e\sigma_0$-space, $P_2$-$N_{nc}e\sigma_0$-space and $P_3$-$N_{nc}e\sigma_0$-space.

(ii) $N_{nc}e\sigma\tau_1$-space if $(U, N_{nc}e\sigma\tau) = P_1$-$N_{nc}e\sigma_1$-space, $P_2$-$N_{nc}e\sigma_1$-space and $P_3$-$N_{nc}e\sigma_1$-space.

(iii) $N_{nc}e\sigma\tau_2$-space if $(U, N_{nc}e\sigma\tau) = P_1$-$N_{nc}e\sigma_2$-space, $P_2$-$N_{nc}e\sigma_2$-space and $P_3$-$N_{nc}e\sigma_2$-space.

Remark 4.1 For a $N_{nc}e\sigma\tau\tau$ $(U, N_{nc}e\sigma\tau)$

(i) Every $N_{nc}e\sigma_0$-space is $P_1$-$N_{nc}e\sigma_0$-space.

(ii) Every $N_{nc}e\sigma_0$-space is $P_2$-$N_{nc}e\sigma_0$-space.

(iii) Every $N_{nc}e\sigma_0$-space is $P_3$-$N_{nc}e\sigma_0$-space.

(iv) Every $N_{nc}e\sigma_1$-space is $P_1$-$N_{nc}e\sigma_1$-space.

(v) Every $N_{nc}e\sigma_1$-space is $P_2$-$N_{nc}e\sigma_1$-space.

(vi) Every $N_{nc}e\sigma_1$-space is $P_3$-$N_{nc}e\sigma_1$-space.

(vii) Every $N_{nc}e\sigma_2$-space is $P_1$-$N_{nc}e\sigma_2$-space.

(viii) Every $N_{nc}e\sigma_2$-space is $P_2$-$N_{nc}e\sigma_2$-space.

(ix) Every $N_{nc}e\sigma_2$-space is $P_3$-$N_{nc}e\sigma_2$-space.

(x) Every $N_{nc}e\sigma_2$-space is $N_{nc}e\sigma_0$-space.

(xi) Every $N_{nc}e\sigma_2$-space is $N_{nc}e\sigma_1$-space.

But not conversely.

Proof. The proof is directly from Definition 4.2.

The inverse of Remark 4.1 is not true, the following example explain this state.

Example 4.1 Let $U = \{u, v\}, N_{nc}=1 = \{\phi_N, U_N, H\}, N_{nc}=2 = \{\phi_N, U_N\}, H = \{\{u\}, \phi, \phi\}$, then we have $2_{nc\tau} = \{\phi_N, U_N, H\}, \sigma_1 = \{\phi_N, U_N, L\}, \sigma_2 = \{\phi_N, U_N\}, L = \{\phi, \{v\}, \phi\}$, then we have $2_{nc\sigma} = \{\phi_N, U_N, C\}, \mu_1 = \{\phi_N, U_N, C\}, \mu_2 = \{\phi_N, U_N\}, C = \{\phi, \phi, \{u\}\}$, then we have $2_{ncH} = \{\phi_N, U_N, C\}$.

(i) $(U, 2_{nc\tau})$ is $P_1$-$2_{nc\sigma}$ and it is not $2_{nc\tau_1}$-space.

(ii) $(U, 2_{nc\sigma})$ is $P_2$-$2_{nc\tau_1}$-space but it is not $2_{nc\tau_0}$-space.

(iii) $(U, 2_{nc\mu})$ is $P_3$-$2_{nc\tau_2}$-space but it is not $2_{nc\tau_0}$-space.

Example 4.2 Let $U = \{u, v\}, N_{nc}=1 = \{\phi_N, U_N, H, L\}, N_{nc}=2 = \{\phi_N, U_N\}, H = \{\{u\}, \{v\}, \phi\}$, then we have $2_{nc\tau} = \{\phi_N, U_N, H, L\}, \sigma_1 = \{\phi_N, U_N, C, D\}, \sigma_2 = \{\phi_N, U_N\}, C = \{\phi, \phi, \{u\}\}, D = \{\phi, \phi, \{v\}\}$, then we have $2_{nc\sigma} = \{\phi_N, U_N, C, D\}$.

(i) $(U, 2_{nc\tau})$ is $P_1$-$2_{nc\tau_1}$ (resp. $P_1$-$2_{nc\tau_2}$)-space but it is not $2_{nc\tau_1}$ (resp. $2_{nc\tau_2}$)-space.

(ii) $(U, 2_{nc\sigma})$ is $P_2$-$2_{nc\tau_1}$ (resp. $P_2$-$2_{nc\tau_2}$)-space but it is not $2_{nc\tau_1}$ (resp. $2_{nc\tau_2}$)-space.
(iii) \( (U, 2_{nc}) \) is \( P_3\)-\( 2_{nc} e \tau_1 \) (resp. \( P_3\)-\( 2_{nc} e \tau_2 \))-space but it is not \( 2_{nc} e \tau_1 \) (resp. \( 2_{nc} e \tau_2 \))-space.

**Example 4.3** Let \( U = \{u, v\}, \ 2_{nc} \tau_1 = \{\phi_N, U_N, H, L, C\}, \ 2_{nc} \tau_2 = \{\phi_N, U_N\}, \ H = \{\{u\}, \phi, \phi\}, \ L = \langle \phi, \{v\}, \phi \rangle, \ C = \langle \phi, \phi, \{u\} \rangle \), then we have \( 2_{nc} \tau = \{\phi_N, U_N, H, L, C\} \). Then \( (U, 2_{nc} \tau) \) is \( 2_{nc} e \tau_0 \)-space but not \( 2_{nc} e \tau_1 \)-space.

**Example 4.4** Let \( U = \{u, v\}, \ 2_{nc} \tau_1 = \{\phi_N, U_N, H, L\}, \ 2_{nc} \tau_2 = \{\phi_N, U_N\}, \ H = \{\{u\}, \phi, \phi\}, \ L = \{\{u, v\}, \phi, \phi\} \), then we have \( 2_{nc} \tau = \{\phi_N, U_N, H, L\} \). Then \( (U, 2_{nc} \tau) \) is \( 2_{nc} e \tau_1 \)-space but not \( 2_{nc} e \tau_2 \)-space.

5. Conclusion

In this paper we have defined a new \( N_{nc} \) points in \( N_{nc} \) limit point, with some of its properties. Further, we constructed the separation axioms \( (N_{nc} e \tau_i\text{-space } i = 0, 1, 2) \) in \( N_{nc} \) topological and examine the relationship between them in details.

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