Holographic dark energy linearly interacting with dark matter

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We investigate a spatially flat Friedmann-Robertson-Walker (FRW) cosmological model with cold dark matter coupled to a modified holographic Ricci dark energy through a general interaction term linear in the energy densities of dark matter and dark energy, the total energy density and its derivative. Using the statistical method of χ2-function for the Hubble data, we obtain $H_0 = 73.6$km/sMpc, $\omega_x = -0.842$ for the asymptotic equation of state and $z_{acc} = 0.89$. The estimated values of $\Omega_{de}$ which fulfill the current observational bounds corresponds to a dark energy density varying in the range $0.25R < \rho_x < 0.27R$.

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I. INTRODUCTION

Many different observational sources such as the Supernovae Ia, the large scale structure from the Sloan Digital Sky survey and the cosmic microwave background anisotropies have corroborated that our universe is currently undergoing an accelerated phase. The cause of this behavior has been attributed to a mysterious component called dark energy and several candidates have been proposed to fulfill this role. For example, a positive cosmological constant $\Lambda$, explains very well the accelerated behavior but it has a deep mismatch with the theoretical value predicted by the quantum field theory. According to this principle, the entropy of a system does not scale with its volume but with its surface area and so in cosmological principle to the cosmology. Among all the interesting holographic dark energy models proposed so far, here we focus our attention on a modified version of the well known Ricci scalar cutoff. Besides, there could be a hidden non-gravitational coupling between the dark matter and dark energy without violating current observational constraints and thus it is interesting to develop ways of testing an interaction in the dark sector. Interaction within the dark sector has been studied mainly as a mechanism to solve the coincidence problem. We will consider an exchange of energy or interaction between dark matter and dark energy which is a linear combination of the dark energy density $\rho_x$, total energy density $\rho$, dark matter energy density $\rho_c$, and the first derivate of the total energy density $\rho'$.

II. THE INTERACTING MODEL

In a FRW background, the Einstein equation for a model of cold dark matter of energy density $\rho_c$ and modified holographic Ricci dark energy having energy density $\rho_x = \left(2\dot{H} + 3\alpha H^2\right)/\Delta$, reads

$$3H^2 = \rho = \rho_c + \rho_x,$$

where $\alpha, \beta$ are constants and $\Delta = \alpha - \beta$.

In terms of the variable $\eta = 3\ln(a/a_0)$, the compatibility between the global conservation equation

$$\rho' = d\rho/d\eta = -\rho_c - (1 + \omega_x)\rho_x,$$

and the equation deduced from the expression of the modified holographic Ricci dark energy

$$\rho' = -\alpha \rho_c - \beta \rho_x,$$

namely, $(\rho_c + \gamma_x \rho_x) = (\alpha \rho_c + \beta \rho_x)$, gives a relation between the equation of state of the dark energy component $\omega_x = \gamma_x - 1$ and the ratio $r = \rho_c/\rho_x$.

$$\omega_x = (\alpha - 1)r + \beta - 1.$$

Solving the system of equations (1) and (3) we get $\rho_c$ and $\rho_x$ in terms of $\rho$ and $\rho'$ as

$$\rho_c = -(\beta \rho + \rho')/\Delta, \quad \rho_x = (\alpha \rho + \rho')/\Delta.$$
The interaction between both dark components is introduced through the term $Q$ by splitting the Eq. (3) into $\rho_c + \alpha \rho_c = -Q$ and $\rho_x + \beta \rho_x = Q$. Then, differentiating $\rho_c$ or $\rho_x$ in (3) and using the expression of $Q$ we obtain a second order differential equation for the total energy density $\rho_{13}$

$$\rho'' + (\alpha + \beta)\rho' + \alpha \beta \rho = Q \Delta. \quad (6)$$

For a given interaction $Q$, solving Eq. (6) gives us the total energy density $\rho$ and the energy densities $\rho_c$ and $\rho_x$ after using Eq. (5).

The general linear interaction $Q_{13}$, linear in $\rho_c$, $\rho_x$, $\rho$, and $\rho'$, can be written as

$$Q = c_1 \frac{(\gamma_s - \alpha)(\gamma_s - \beta)}{\Delta} \rho + c_2 (\gamma_s - \alpha) \rho_c$$

$$- c_3 (\gamma_s - \beta) \rho_x - c_4 \frac{(\gamma_s - \alpha)(\gamma_s - \beta)}{\gamma_s \Delta} \rho'$$

where $\gamma_s$ is constant and the coefficients $c_i$ fulfill the condition $c_1 + c_2 + c_3 + c_4 = 1_{13}$.

Now, using Eqs. (5) we rewrite the interaction (8) as a linear combination of $\rho$ and $\rho'$,

$$Q = \frac{u\rho + \gamma_s^{-1}(u - (\gamma_s - \alpha)(\gamma_s - \beta))\rho'}{\Delta} \quad (8)$$

where $u = c_1 (\gamma_s - \alpha)(\gamma_s - \beta) - c_2 \beta (\gamma_s - \alpha) - c_3 \alpha (\gamma_s - \beta)$. Replacing the interaction (8) into the source equation (5), we obtain

$$\rho'' + (\gamma_s + \gamma^+)\rho' + \gamma_s \gamma^+ \rho = 0. \quad (9)$$

where the roots of the characteristic polynomial associated with the second order linear differential equation (9) are $\gamma_s$ and $\gamma^+ = (\beta \alpha - u) / \gamma_s$. In what follows, we adopt $\gamma^+ = 1$ for mimicking the dust-like behavior of the universe at early times. In that case, the general solution of (9) is $\rho = b_1 a^{-3\gamma_s} + b_2 a^{-3}$ from which we obtain

$$\rho_c = \frac{(\gamma_s - \beta) b_1 a^{-3\gamma_s} + (1 - \beta) b_2 a^{-3}}{\Delta} \quad (10a)$$

$$\rho_x = \frac{(\alpha - \gamma_s) b_1 a^{-3\gamma_s} + (\alpha - 1) b_2 a^{-3}}{\Delta} \quad (10b)$$

Interestingly, Eqs. (10) tell us that the interaction (5) seems to be a good candidate for alleviating the cosmic coincidence problem because the ratio $\Omega_c / \Omega_x$ becomes bounded for all times.

$Q = (1 - \alpha) \rho_c$

Let us consider the particular case in which the interaction $Q$ is proportional to the energy density of the dark matter $\rho_c$ in such a way that $\rho_c + \rho_c = \rho_x + (1 + \omega_x) \rho_x = 0$. That is, each fluid separately, satisfies an equation of conservation. Here the constants $u$ and $\gamma_s$ defined above, correspond to $u = \beta (\alpha - 1)$ and $\gamma_s = \beta$ with which the expressions (10) for the energy densities of the dark matter and dark energy are written as functions of the redshift as

$$\rho_c = \frac{(1 - \beta) b_2 (1 + z)^3}{\Delta}, \quad (11a)$$

$$\rho_x = \frac{(\alpha - \beta) b_1 (1 + z)^3 + (\alpha - 1) b_2 (1 + z)^3}{\Delta}. \quad (11b)$$

The ratio between both components $r = \rho_c / \rho_x$ turns out to be

$$r = \frac{b_2 (1 - \beta)(1 + z)^3(1 - \beta)}{b_1 \Delta + b_2 (\alpha - 1)(1 + z)^3(1 - \beta)} \quad (12)$$

and shows that in the early universe, both components behave as dust. In the final stages the ratio tends to zero and therefore it does not solve the problem of the coincidence. This example of interaction, between non-relativistic dark matter and the modified holographic Ricci dark energy, is important because allows to show that the holographic forms of the dark energy are always interacting with the non-holographic component. This behavior can be observed in the equation (11b) and is due to the functional dependence of the holographic equation of state with the ratio $r$ of the energy densities,

$$\omega_x = \frac{(\beta - 1) b_1 \Delta}{b_1 \Delta + b_2 (\alpha - 1)(1 + z)^3(1 - \beta)}. \quad (13)$$

each time $\alpha$ is different from 1.

III. OBSERVATIONAL CONSTRAINTS

The transition redshift $z_{acc}$ that satisfies the equation $\dot{H} + H^2 = 0$ and the actual Hubble factor $H_0$ allow us to express the coefficients $b_i$ in equations (10) as $b_1 = 3H_0^2 - b_2$ and $b_2 = 3H_0^2 (2 - 3\gamma_s) / [2 - 3\gamma_s + (1 + z_{acc})^3(1 - \gamma_s)]$, so that the Hubble function, reads

$$H(z) = \frac{H_0 (1 + z_{acc})^{3/2}}{\sqrt{2 - 3\gamma_s + (1 + z_{acc})^3(1 - \gamma_s)}} \times \sqrt{\frac{(1 + z)^{3\gamma_s}}{(1 + z_{acc})^{3\gamma_s}} + (2 - 3\gamma_s) \frac{(1 + z)^3}{(1 + z_{acc})^3}} \quad (14)$$
We apply the $\chi^2$–statistical method to the Hubble data for constraining the cosmological parameters of the Hubble function. The three-dimensional confidence regions $1\sigma$ and $2\sigma$ are shown in the left panel of Fig.1 where the sphere indicates the best fit values $z_{acc} = 0.89$, $H_0 = 73.6 $km/s/Mpc and $\gamma_s = 0.158$ with a minimum value of the $\chi^2$ function per degree of freedom $\chi^2_{dof} = 0.846$.

Interesting in the sense that we now know the values $z_{acc}$ and $H_0$ of the current Hubble parameter predicted by our model, nevertheless this information does not allow to determine the values of $\alpha$ and $\beta$ best fitted to the observational data. We must bear in mind that a feasible model of the dark sector has dark components with positive definite energy densities, accelerated expansion and non phantom dark energy. These requirements are fulfilled when $b_1$ and $b_2$ are positive constants, which correspond to $\alpha \geq 1$ and $0 \leq \beta < 2/3$. To determine the most acceptable ranges of the parameters $\alpha$ and $\beta$ we note that this constants are involved in the expressions of the partial energy densities $\rho_c$ and $\rho_x$ and so, we apply the $\chi^2$–statistical method as above but now using the expressions

$$H^2(z) = H_0^2 (\omega_s - \omega_x)(1 + z)^3 + \omega_x (1 + z)^3 \gamma_s,$$

$$\omega_0 = \alpha \Omega_{c0} + \beta \Omega_{x0} - 1, \quad \Omega_{i0} = \frac{\rho_{i0}}{3H_0^2}, \quad \Omega_{c0} + \Omega_{x0} = 1.$$ (15)

The results of this procedure, included in Table I, show that the holographic case $\alpha = 4/3$ and $\beta = 1$ has a very poor statistical adjustment $\chi^2_{dof} = 22.23$, whereas inversely, the models with $\alpha = 4/3$ and $\beta < 0.1$, that is $0.25 \rho_c < \rho_x < 0.27 \rho_c$, behave reasonably well leading to $\chi^2_{dof} = 0.761 < 1$. The constants $b_1$ and $b_2$ have two sets of expressions, as they are written in terms of $z_{acc}$ and $H_0$, used in (14), or in terms of the current parameters of density $\Omega_{c0}$ and $\Omega_{x0}$, as used in (15). These sets allow to express the constant $\omega_0$ as

$$\omega_0 = \frac{\omega_x}{1 - (1 + 3\omega_x)(1 + z_{acc})^3 \omega_x},$$ (16)

and verify that the first line of the Table I gives the correct values of $\alpha$ and $\beta$. In the next subsections, we will use these values $\alpha = 1.15$, $\beta = 0.01$ and $\Omega_{c0} = 0.3$ in the figures and expressions for the partial densities and their ratio.
A. The crisis of the age

The age of the universe in units of $H_0^{-1}$ can be obtained as a function of the redshift $z$ with the expression

$$ t(z) = \int_z^\infty \frac{dv}{(1 + v)H(v)} $$

(17)

We depict this age-redshift relation in the right panel of Fig.1. The parametric curve of cosmological time $t(z)$ is drawn from (17) in units of $H_0^{-1}$ for the best values $z_{acc} = 0.89$, $H_0 = 73.6$ km/s/Mpc and $\gamma_s = 0.158$. Because the cosmological constraints with the Hubble data only cover redshifts over the range $0 \leq z < 2$, the comparison with cosmic milestones will be trustworthy in this range only, and for that reason we consider only two old stellar sources such as the 4 Gyr old galaxy LBDS 53W069 at redshift $z = 1.43^{+0.02}_{-0.02}$ and the 3.5 Gyr old galaxy LBDS 53W091 at redshift $z = 1.55^{+0.02}_{-0.02}$. We find that at low redshift $z < 2$, the Ricci-like holographic dark energy model seems to be free from the cosmic-age problem, namely, the universe cannot be younger than its constituents.

B. The magnitude-redshift relation

It is well known that observations of type Ia supernova (SNe Ia) have predicted and confirmed that our universe is passing through an accelerated phase of expansion. Since then, the observational data coming from these standard candles have been taken seriously. It is commonly believed that measuring both, their redshifts and apparent peak flux, gives a direct measurement of their luminosity distances and thus SNe Ia data provides the strongest constraint on the cosmological parameters. The theoretical distance modulus is defined as

$$ \mu(z) = 5 \log_{10} D_L + \mu_0 $$

(18)

where $\mu_0 = 43.028$, and $D_L$ is the Hubble-free luminosity distance, which for a spatially flat universe can be recast as

$$ D_L(z) = (1 + z)H_0 \int_0^z \frac{dz'}{H(z')} $$

(19)

Using the best fit values of $\omega_s$ and $z_{acc}$ in Eqs. (18)–(19) we get the theoretical distance modulus $\mu(z)$ that we draw in the left panel of Fig.2 together with the observational data $\mu_{obs}(z)$ and their error bars. The theoretical distance modulus (18) will strongly depend on the model used so taking into account a particular cosmology and comparing its $\mu(z)$ with $\mu_{obs}(z)$ one can judge the plausibility of the cosmological model. As we see from Fig.2 our model shares an excellent agreement with the observational data in the zones corresponding to small redshift $[z \leq 0.1]$ and large redshift $[0.1 \leq z \leq 1.5]$.

C. The deceleration parameter, equations of states and density parameters

There are magnitudes that do not depend explicitly on the pair of constants $(\alpha, \beta)$ which selects one particular form for the energy density of the dark energy $\rho_x = (2H^2 + 3\alpha H^2)/(\alpha - \beta)$, but on the linear combination $\omega_0$ defined in (15). These are: the total energy density $\rho$, the deceleration parameter $q$ and the global equation of state $\omega$, whose explicit expressions can be written in terms of the transition redshift $z_{acc}$ and the asymptotic equation of state $\omega_s$ by (14) and the functions

$$ q(z) = -1 + \frac{3}{2} \frac{(\omega_s - \omega_0) + \omega_0(1 + \omega_s)(1 + z)^{3\omega_s} - (\omega_s - \omega_0) + \omega_0(1 + z)^{3\omega_s}}{(\omega_s - \omega_0) + \omega_0(1 + z)^{3\omega_s}}, $$

(20)

$$ \omega(z) = \frac{2q(z) - 1}{3} $$

(21)

with $\omega_0$ given by (10). In the right panel of Fig.2 the deceleration parameters for all our like-holographic models with $\omega_0 = -0.65$ and $\omega_s = -0.84$, $q(z)$ (solid line), are compared with the deceleration parameter of the $\Lambda$CDM model $q_{\Lambda CDM}(z)$ (dashed line) that holds $\Omega_c = 0.3$. There we can see that the deceleration parameter of our models vanishes near $z_{acc} = 0.84$, so these universes enter in the accelerated phase more earlier than the $\Lambda$CDM model with actual density parameters $\Omega_c = 0.3$ and $\Omega_{\phi} = 0.7$. The effective equation of state $\omega(z)$, plotted in the right panel of Fig.3 and looking there, we conclude that our models have $-1 < \omega(z) < 0$ in the interval $z \geq 0$. More precisely, $\omega(z)$ begins like non-relativistic matter, decreases rapidly around $z = 2$ and ends with the asymptotic value $\omega_s = -0.84$.

Instead, the density parameters $\Omega_c = \rho_c/3H^2$ and $\Omega_\phi = \rho_\phi/3H^2$, their ratio $r = \Omega_\phi/\Omega_c$, the equation of state for the dark energy $\omega_\phi$ of Eq. (2) and also the interaction used $Q$ of Eq. (5), are described explicitly in terms of $\alpha$ and $\beta$ by the expressions

$$ \Omega_c(z) = \frac{(1 - \beta)(\omega_s - \omega_0) + \omega_0(1 + \omega_s)(1 + z)^{3\omega_s}}{\Delta(\omega_s - \omega_0) + \omega_0(1 + z)^{3\omega_s}} $$

(22)

$$ \Omega_\phi(z) = \frac{(\alpha - 1)(\omega_s - \omega_0) + \omega_0(\alpha - 1 - \omega_s)(1 + z)^{3\omega_s}}{\Delta(\omega_s - \omega_0) + \omega_0(1 + z)^{3\omega_s}} $$

(23)

$$ r = \frac{\omega_0(\beta - \omega_s - 1) + (1 + z)^{3\omega_s}}{\omega_0(\beta - \omega_s + 1) + (1 + \beta)(\omega_s - \omega_0)} $$

(24)

$$ \omega_\phi(z) = \frac{\omega_0(1 + \alpha)(1 + z)^{3\omega_s} + (\alpha - 1)(\omega_s - \omega_0)}{\omega_0(\beta - \omega_s - 1)(1 + z)^{3\omega_s} + (1 + \beta)(\omega_s - \omega_0)} $$

(25)

$$ Q = \frac{(\alpha \beta - 1 - \omega_s)\rho + (\alpha + \beta - 2 - \omega_s)\rho'}{\alpha - \beta} $$

(26)
The density parameters $\Omega_c$ and $\Omega_x$ and their ratio $r(z)$ are plotted in the left panel of Fig. 3 for the best values $\alpha = 1.15$, $\beta = 0.01$, $\omega_0 = -0.65$ and $\omega_s = -0.84$, where we can see that the general linear interaction $Q$ helps to alleviate the coincidence problem. The later is drawn in the right panel of Fig. 3 together with the dark energy equation of state $\omega_x(z)$ which has $-1 < \omega_x(z) < 0$ for $z \geq 0$. The linear interaction $Q$ of Eq. (26) corresponds to the choice $u = \alpha\beta - 1 - \omega_s$ in the Eq. (8) and its curve is always negative satisfying the second law of thermodynamics that requires the energy flow goes from dark energy to dark matter.\(^{17}\)

IV. CONCLUSIONS

We have examined a modified holographic Ricci dark energy coupled with cold dark matter and found that this scenario describes satisfactorily the behavior of the energy densities of both dark components alleviating the problem of the cosmic coincidence. We have shown that the compatibility between the modified and the global conservation equations restricts the equation of state of the dark energy component relating it to the ratio of energy densities. This constrain makes the holographic density always interacts with the non-holographic component except in the unlikely event that $\alpha = 1$, which is forbidden for positive energy densities. From the observational point of view we have obtained the best fit values of the cosmological parameters $z_{acc} = 0.89$, $H_0 = 73.6$ km/s/Mpc and $\gamma_s = 0.158$ with a $\chi^2_{dof} = 0.761 < 1$.\(^{12}\)
per degree of freedom. The $H_0$ value is in agreement with the reported in the literature and the critical redshift $z_{acc} = 0.89$ is consistent with BAO and CMB data. We have found that in the redshift interval where is trustworthy compared with old stellar sources the model is free from the cosmic-age problem.

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