Employment of a Monte Carlo method in the dynamic characterization of pressure transducers

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Abstract. Dynamic calibration is a multidisciplinary theme which is attracting interest from the metrological community. Methods and devices are being tested, such as Shock Tube and Drop-weigh systems. There is a great demand for research into this issue, in particular regarding uncertainty analysis in dynamic calibration. This paper deals with the use of a Monte Carlo method in the dynamic characterization of pressure transducers. We analyse the process to characterize the transducer Kistler 701A. The results showed that the method can be efficient for evaluating the uncertainties involving the dynamic characterization of piezoelectric transducers that cover frequency ranges from 1 to 60 kHz and a pressure range from 1 to 1000 kPa.

1. Introduction

We use the words “dynamic calibration” to define the process that characterizes a dynamic pressure transducer, determining some properties, such as natural frequency, damping, peak time, stabilization time and sensitivity [1].

In this calibration method we consider a theoretical signal generated by a device, such as a shock tube, as the standard. Based on the characteristics of this device, a known and repetitive signal is generated and this signal is compared with that captured by the transducer under calibration [1].

Due to the demand for dynamic measurement pressure in areas such as the aeronautics and automotive industries [2], research centres and large corporations have an interest in improving its measurement processes. Some examples are the National Physical Laboratory, NPL, in England; the Physikalisch-Technische Bundesanstalt (PTB), in Germany, Rolls-Royce, Volkswagen and Porsche [3].

To find a standardized method that serves for several pressure and frequency ranges is a challenge for the dynamic calibration of pressure transducers. Another challenge is to determinate the uncertainties of this process [4].

Therefore, this article deals with the use of a Monte Carlo Method to determine the uncertainties in the dynamic characterization of pressure transducers. The Monte Carlo Method is used to estimate the mean values of the experimental parameters and associated uncertainties. The properties described in this paper were peak time, natural frequency and damping ratio, all identified in time domain.

From the analysis of the results we see that a Monte Carlo method can be efficient for evaluating the uncertainties involving the dynamic characterization of the piezoelectric pressure transducers that cover frequency ranges from 1 to 60 kHz and pressure ranges from 1 to 1000 kPa [1]. The sensor employed in the test is a Kistler model 701A.

The article is the result of studies carried out by the Institute of Aeronautics and Space, IAE, Brazil.
2. Facility and Mathematical Modelling

2.1. Facility

The quality of the signal generated for calibration is a determining factor for the success of the dynamic calibration. Thus, a good laboratory structure can make all the difference. The facility used in the study is the Henry T. Nagamatsu Laboratory of the Aerodynamics and Hypersonic Division, of the Institute of Advanced Studies, IEAv, of Department of Aerospace Science and Technology, DCTA, Brazil [1].

A shock tube (figure 1) is a device that contains two compartments which are separated by a diaphragm. The first compartment is called "driver" where a gas (generally air) is stored with \( P_1 \) pressure. The second compartment is the "driven" where the \( P_2 \) and \( P_2' \) sensors are located. This compartment contains a \( P_2 \) pressure.

When the diaphragm is broken by pressure difference between \( P_1 \) and \( P_2 \), it generates a shock wave that is used to calculate a step signal that will be employed in the characterization of the transducers [5].

Figure 1. Shock tube and sensors.

2.2 Mathematical Modelling

The start of the dynamic characterization process of the sensor occurs immediately after the rupture of the diaphragm separating the driver and driven tubes. Thus, firstly we calculate the velocity of the front shock wave which passes by sensors \( P_2 \) and \( P_2' \) (see figure 1). Using a theoretical model (equation (1)) and based on the value of the velocity of the wave front \( (L/t) \) where \( L \) is the distance between the two sensors, \( P_2 \) and \( P_2' \) and \( t \) is the time of the passage of the front wave between them, we estimate the value of the pressure step signal, \( \Delta P \) [3].

\[
\Delta P = P_2 - P_1 = \frac{7}{6} P_1 \left[ \frac{L}{\gamma_1 R_1 T} \right]^2 - 1 \tag{1}
\]

In equation (1) \( \gamma_1 \) is the ratio of specific heat, equal to 1.4 for the air considered as a perfect gas; \( R_1 \) is the perfect gas constant, equal to 287 J/(kg.K) for the air; and \( T_1 \) is the temperature in the driven section of the tube, expressed in kelvin [5].

We can represent the dynamic behaviour of the pressure sensor membrane by a mass-spring system with damping, a second order system. The Laplace Transform of the output function for this system can be the result of the multiplication between the Laplace Transform of the input function and the Laplace Transform of the differential equation which represents the mass-spring system behaviour.

Assuming that the input signal is a unit step and by applying the Laplace Inverse Transform to the output function, one obtains the temporal response of the sensor expressed by equation (2), estimating the damping ratio \( \zeta \) and the natural frequency \( \omega_n \) values. The gain \( K \) of the system can be determined in the steady state regime [6].

\[
x(t) = K \left[ 1 - e^{-\omega_0 t} \left( \cos \zeta \omega_n \sqrt{1 - \zeta^2} t + \frac{\zeta}{\sqrt{1 - \zeta^2}} \sin \omega_n \sqrt{1 - \zeta^2} t \right) \right] \tag{2}
\]

The damping ratio, \( \zeta \), can be obtained experimentally employing equation (3) [6]. Where \( x(t) \) is a two consecutives peaks above the mean value of the steady signal, \( x(t_{\infty}) \)
In figure 2, we can see the points used in Equation (3) to calculate the damping ratio value.

\[ \zeta = \frac{1}{1 + \left( \frac{2.728}{\log_{10}\left( \frac{x(t_{\max 1}) - x(t_{\infty})}{x(t_{\infty}) - x(t_{\max 2})} \right)} \right)^{2}} \]  

(3)

Figure 2. Two consecutives peaks and mean value of the steady signal.

The natural frequency, \( \omega_n \), can be determined by peak time, \( t_p \), or by the time for the signal to achieve overshoot, \( x(t_{\max 1}) \), and by damping, according to equation (4) [6].

\[ \omega_n = \frac{\pi}{t_p \sqrt{1 - \zeta^2}} \]  

(4)

3. Results and discussions

The Monte Carlo method is based on the Probability Distribution Functions, PDF, of the input quantities. These distributions are propagated \( M \) times, independently, according to each mathematical model of the measurement and the mean and standard deviations of the results are estimated.

Its main advantage is that it can be used to calculate uncertainties with asymmetric distribution. As such, the uncertainty is determined by the lower and upper limits of the PDF resulting from the measurement [7, 8].

Three tests were carried out on a shock tube using the same configuration to obtain the parameters to perform a Monte Carlo simulation. Mean and standard deviation values are seen in Table 1.

| Parameters | Mean values | Standard deviation | Units |
|------------|-------------|--------------------|-------|
| \( T \)    | 469         | 5                  | \( \mu s \) |
| \( L \)    | 0.400       | 0.014              | m     |
| \( T_1 \)  | 20.40       | 0.5                | °C    |
| \( P_1 \)  | 95.90       | 0.12               | kPa   |
| \( t_p \)  | 109 \( \times 10^{-7} \) | 12 \( \times 10^{-7} \) | s     |
| \( x(t_{\infty}) \) | 2.85 | 0.05 | V |
| \( x(t_{\max 1}) \) | 4.09 | 0.16 | V |
| \( x(t_{\max 2}) \) | 3.68 | 0.09 | V |
Using the values from Table 1, assuming a t-Student distribution, we performed $10^6$ trials for each measurement model (equations (1), (3) and (4)) to estimate the mean values and uncertainties of the parameters $\Delta P$, $\zeta$, $\omega_n$, for a 95% level of confidence, as shown in Table 2 [8].

**Table 2 - Input quantities for propagation distributions to estimate $\Delta P$, $\zeta$ and $\omega_n$.**

| Output parameters | Input parameters | Mean values | Standard deviation | Uncertainties lower limits | Uncertainties upper limits | Output units |
|-------------------|------------------|-------------|--------------------|---------------------------|---------------------------|-------------|
| $\Delta P$        | $t$, $L$, $T_1$, $P_1$ | $5.8 \times 10^2$ | $0.5 \times 10^2$ | $4.8 \times 10^2$ | $6.8 \times 10^2$ | kPa         |
| $\zeta$           | $x (t_0)$, $x (t_{\max 1})$, $x (t_{\max 2})$ | $0.58 \times 10^{-1}$ | $0.21 \times 10^{-1}$ | 0 | $1.1 \times 10^{-1}$ | -           |
| $\omega_n$        | $t_p$            | $290 \times 10^4$ | $3 \times 10^4$ | $25 \times 10^4$ | $36 \times 10^4$ | rad/s       |

The histograms of figures 3 (a), (b) and (c) respectively represent the resulting PDF for outputs $\Delta P$, $\zeta$ and $\omega_n$ considering the input values of Table 1 applied to Equations (1), (3) and (4).

In these histograms, the red line represents a normal distribution, according to mean values and standard deviation of the quantities. The blue bars represent the results of a Monte Carlo simulation considering the input parameters and model adopted.

![Figure 3 (a). Histogram of $\Delta P$](image1)

![Figure 3 (b). Histogram of $\zeta$](image2)

![Figure 3 (c). Histogram of $\omega_n$](image3)
In figures 3 (a), (b) and (c) we can see an acceptable agreement between the red line, which represents a normal distribution, and the blue area, which represents the experimental data. It can be seen that uncertainties values for damping and natural frequency are asymmetric, as was also observed in Table 2.

Comparing the uncertainties limits shown in columns five and six of the Table 2, it can be seen that uncertainties values of the $\Delta P$ quantities are symmetric as shown in figure 3 (a). Thus, when using a coverage factor $k = 2$ [9], we have a coverage interval for (the) $\Delta P$ value which is similar to (the) uncertainties limits obtained according to (a) Monte Carlo Method.

Concerning values of the uncertainties limits for $\zeta$ and $\omega_n$, when using a coverage factor $k = 2$ [9] we have a coverage interval different from that obtained according to (the) Monte Carlo Method, in other words, a Monte Carlo Method supplies asymmetric values in relation to (the) mean values, as seen in figures 3 (b) and 3 (c).

4. Conclusion

In this paper we discussed the use of a Monte Carlo method for estimating uncertainties in dynamic calibration of a pressure transducer. Firstly we considered the applications of the pressure sensors. Then we briefly the facility of the Institute of Advanced Studies, IEAv, in Brazil, and the mathematical modeling used in this study.

The Monte Carlo Method was used to determine the mean values and the uncertainties of the peak time, natural frequency and damping ratio parameters. This Monte Carlo method proved to be capable of evaluating the uncertainties involving the dynamic characterization of the piezoelectric pressure transducers that cover frequency ranges from 1 to 60 kHz and pressure range from 1 to 1000 kPa.

5. References

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