Doping and energy dependent microwave conductivity of kinetic energy driven superconductors with extended impurities

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Within the framework of the kinetic energy driven superconducting mechanism, the effect of the extended impurity scatterers on the quasiparticle transport of cuprate superconductors in the superconducting state is studied based on the nodal approximation of the quasiparticle excitations and scattering processes. It is shown that there is a cusplike shape of the energy dependent microwave conductivity spectrum. At low temperatures, the microwave conductivity increases linearly with increasing temperatures, and reaches a maximum at intermediate temperature, then decreases with increasing temperatures at high temperatures. In contrast with the dome shape of the doping dependent superconducting gap parameter, the minimum microwave conductivity occurs around the optimal doping, and then increases in both underdoped and overdoped regimes.

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I. INTRODUCTION

After over 20 years extensive studies, it has become clear that superconductivity in doped cuprates results when charge carriers pair up into Cooper pairs as in the conventional superconductors, then these charge carrier Cooper pairs condensation reveals the superconducting (SC) ground-state. However, as a natural consequence of the unconventional SC mechanism that is responsible for the high SC transition temperatures, the charge carrier Cooper pairs in cuprate superconductors have a dominated d-wave symmetry. In particular, this d-wave SC state remains one of the cornerstones of our understanding of the physics in cuprate superconductors. However, in spite of the unconventional SC mechanism, the angle-resolved photoemission spectroscopy (ARPES) experimental results have unambiguously established the Bogoliubov-quasiparticle nature of the sharp SC quasiparticle peak in cuprate superconductors, then the SC coherence of the low energy quasiparticle peak is well described by the simple Bardeen-Cooper-Schrieffer (BCS) formalism with the d-wave SC gap function \( \Delta(k) = \Delta(\cos k_x - \cos k_y)/2 \). In this d-wave case, the characteristic feature is the existence of four nodal points \( \pm \pi/2, \pm \pi/2 \) (in units of inverse lattice constant) in the Brillouin zone, where the SC gap function vanishes, then the most physical properties of cuprate superconductors in the SC state are controlled by the quasiparticle excitations around the nodes. In particular, the key signature of the nodal quasiparticle transport appears in the microwave conductivity \( \sigma(\omega, T) \), which is essentially electromagnetic absorption by the quasiparticles excited out of the condensation (either thermally excited quasiparticles or excitations created by the impurity scattering).

Understanding the role of impurities in cuprate superconductors has taken many years of great effort. This follows from that the physical properties of cuprate superconductors in the SC state are extreme sensitivity to the impurity effect than the conventional superconductors due to the finite angular-momentum charge carrier Cooper pairing. Experimentally, By virtue of systematic studies using the microwave conductivity measurements, some essential features of the evolution of the quasiparticle transport of cuprate superconductors with energy and temperature in the SC state have been established: (1) at low temperatures, the experimental results show the existence of the very long-live excitation deep in the SC state, as evidenced by the sharp cusplike energy dependent microwave conductivity spectrum, where the width of the sharp peak is nearly temperature independent, and main behaviors of the microwave conductivity are governed by thermally excited quasiparticles being scattered by impurities or other defects; (2) at low energies, the temperature dependent microwave conductivity increases linearly with increasing temperatures at low temperatures, and reaches a maximum (a large broad peak) around intermediate temperature, then decreases with increasing temperatures at high temperatures. In particular, this broad peak shifts to higher temperatures as the energy is increased. Theoretically, an agreement has emerged that the BCS formalism with the d-wave SC gap function is useful in the phenomenological description of the quasiparticle transport of cuprate superconductors in the SC state, although the SC pairing mechanism is beyond the conventional electron-phonon mechanism. In this case, the microwave conductivity of cuprate superconductors has been phenomenologically discussed in the zero temperature and energy by including the contributions of the vertex corrections. Recently, these discussions have been generalized to study the temperature and energy dependence of the quasiparticle transport of cuprate superconductors in the SC state. To the best of our knowledge, the microwave conductivity of cuprate superconductors has not been treated starting...
from a microscopic SC theory, and no explicit calculations of the doping dependence of the microwave conductivity has been made so far.

In this paper, we start from the kinetic energy driven SC mechanism and study the effect of the extended impurity scatterers on the microwave conductivity of cuprate superconductors. We evaluate explicitly the microwave conductivity of cuprate superconductors within the nodal approximation of the quasiparticle excitations and scattering processes, and qualitatively reproduced some main features of the microwave conductivity measurements on cuprate superconductors in the SC state. It is shown that there is a cusplike shape of the energy dependent microwave conductivity spectrum. At low temperatures, the microwave conductivity increases linearly with increasing temperatures, and reaches a maximum at intermediate temperature, then decreases with increasing temperatures at high temperatures. In contrast with the dome shape of the doping dependent SC gap parameter, the minimum microwave conductivity occurs around the optimal doping, and then increases in both underdoped and overdoped regimes.

This paper is organized as follows. We present the basic formalism in Sec. II, and then discuss the energy, temperature, and doping dependence of the quasiparticle transport of cuprate superconductors in the SC state in Sec. III, where we show that the quasiparticle transport of cuprate superconductors in the SC state can be qualitatively understood within the framework of the kinetic energy driven SC mechanism by considering the effect of the extended impurity scatterers. Finally, we give a summary in Sec. IV.

II. FORMALISM

In cuprate superconductors, the single common feature is the presence of the two-dimensional $\text{CuO}_2$ plane, and it is believed that the unconventional physical properties of cuprate superconductors is closely related to the doped $\text{CuO}_2$ planes. It has been argued that the $t$-$J$ model on a square lattice captures the essential physics of the doped $\text{CuO}_2$ plane: \[ H = -t \sum_{i,j} \hat{C}^\dagger_{i\sigma} \hat{C}_{j+i\sigma} + \mu \sum_{i\sigma} \hat{C}^\dagger_{i\sigma} \hat{C}_{i\sigma} + J \sum_{i\eta} \hat{S}_i \cdot \hat{S}_{i+\eta}, \] (1)

where $\hat{n} = \pm \hat{x}, \pm \hat{y}$, $\hat{S}_i = \hat{C}^\dagger_{i\uparrow} \tau \hat{C}_{i\downarrow}/2$ is spin operator with $\tau = (\tau_1, \tau_2, \tau_3)$ as Pauli matrices, the constrained electron operator $\hat{C}_{i\sigma} = C_{i\sigma} (1 - n_{i-\sigma})$ with $n_{i\sigma} = C^\dagger_{i\sigma} C_{i\sigma}$, and $\mu$ is the chemical potential. In the constrained electron operator, the operators $C^\dagger_{i\sigma}$ and $C_{i\sigma}$ are to be thought of as operating within the full Hilbert space, while the constrained electron operator $\hat{C}^\dagger_{i\sigma} (\hat{C}_{i\sigma})$ does not create (destroy) any doubly occupied sites, and therefore represents physical creation (annihilation) operator acting in the restricted Hilbert space without double electron occupancy. The strong electron correlation in the $t$-$J$ model manifests itself by the restriction of the motions of the electrons in the restricted Hilbert space without double electron occupancy, which can be treated properly in analytical calculations within the charge-spin separation (CSS) fermion-spin theory, where the constrained electron operators are decoupled as $\hat{C}_{i\sigma} = h^\dagger_{i\sigma} S_{i\sigma}^-$ and $\hat{C}_{i\sigma} = h^\dagger_{i\sigma} S_{i\sigma}^+$, with the spinful fermion operator $h_{i\sigma} = e^{-i\hat{\phi}_{i\sigma}} h_i$ describes the charge degree of freedom together with some effects of spin configuration rearrangements due to the presence of the doped hole itself (dressed holon), while the spin operator $S_i$ describes the spin degree of freedom (spin), then the motions of electrons are restricted in the restricted Hilbert space without double electron occupancy. Moreover, these dressed holon and spin are gauge invariant, and in this sense, they are real and can be interpreted as the physical excitations. In this CSS fermion-spin representation, the low-energy behavior of the $t$-$J$ model (1) can be expressed as,

\[ H = t \sum_{\eta} (h^\dagger_{i+\eta\uparrow} h_{i\sigma} S_{i+\eta\downarrow} S_{i\sigma}^+ + h^\dagger_{i+\eta\downarrow} h_{i\uparrow} S_{i\sigma}^+ S_{i+\eta\downarrow}) - \mu \sum_{\sigma\eta} h^\dagger_{i\sigma} h_{i\eta} + J_{\text{eff}} \sum_{\eta} \hat{S}_i \cdot \hat{S}_{i+\eta}, \] (2)

with $J_{\text{eff}} = (1 - \delta)^2 J$, and $\delta = \langle h^\dagger_{i\sigma} h_{i\eta} \rangle = \langle h^\dagger_{i\sigma} h_i \rangle$ is the hole doping concentration. As an important consequence, the kinetic energy term in the $t$-$J$ model has been transferred as the dressed holon-spin interaction, which reflects that even the kinetic energy term in the $t$-$J$ Hamiltonian has strong Coulombic contribution due to the restriction of the motions of electrons in the restricted Hilbert space without double electron occupancy.

Recently, we have developed a kinetic energy driven SC mechanism based on the CSS fermion-spin theory, where the dressed holon-spin interaction from the kinetic energy term in the $t$-$J$ model (2) induces the dressed holon pairing state with the d-wave symmetry by exchanging spin excitations, then the electron Cooper pairs originating from the dressed holon pairing state are due to the charge-spin recombination, and their condensation reveals the d-wave SC ground-state. Moreover, this d-wave SC state is controlled by both SC gap function and quasiparticle coherence, then the maximal SC transition temperature occurs around the optimal doping, and decreases in both underdoped and overdoped regimes. In particular, we have shown that this SC state is the conventional BCS like with the d-wave symmetry, so that the basic BCS formalism with the d-wave SC gap function is still valid in quantitatively reproducing all main low energy features of the ARPES experimental measurements on cuprate superconductors, although the pairing mechanism is driven by the kinetic energy by exchanging spin excitations, and other exotic magnetic scatterings is beyond the BCS formalism. Following our previous discussions, the full dressed holon Green’s function in the SC state can be obtained.
in the Nambu representation as,

\[
\tilde{g}(\mathbf{k}, \omega) = Z_{hF} \frac{1}{\omega^2 - E_{hF}^2} \left( \omega + \xi_\mathbf{k} \right) \Delta_{hF}(\mathbf{k}) \Delta_{hZ}(\mathbf{k}) - \xi_\mathbf{k}, \quad \xi_\mathbf{k} = Zt\chi_\mathbf{k} - \mu,
\]

the spin correlation function \( \chi = \langle S^+_i S^-_{i+\eta} \rangle \),

\[
\begin{align*}
1 = & \frac{(Zt)^2}{N^3} \sum_{\mathbf{k}, \mathbf{p}, \mathbf{p'} } \gamma^{(d)}(\mathbf{k}, \mathbf{p}, \mathbf{p'}, \mathbf{p'}) Z_{hF}^2 \frac{B_p B_{p'}}{E_{hF} \omega_{p} \omega_{p'}} \left( \frac{F_1^{(1)}(\mathbf{k}, \mathbf{p}, \mathbf{p'})}{(\omega_{p'} - \omega_{p})^2 - E_{hF}^2} + \frac{F_1^{(2)}(\mathbf{k}, \mathbf{p}, \mathbf{p'})}{(\omega_{p'} + \omega_{p})^2 - E_{hF}^2} \right), \\
1 = & 1 + \frac{(Zt)^2}{N^2} \sum_{\mathbf{p}, \mathbf{p'}} \gamma_{\mathbf{p} \mathbf{k}} \xi_{\mathbf{p} \mathbf{k}} Z_{hF} B_{\mathbf{p}} B_{\mathbf{p'}} E_{hF} \omega_{\mathbf{p}} \omega_{\mathbf{p'}} \left( \frac{F_2^{(1)}(\mathbf{p}, \mathbf{p'})}{(\omega_{\mathbf{p'}} - \omega_{\mathbf{p}} - E_{hp - \mathbf{p'}} + k_0)^2} + \frac{F_2^{(2)}(\mathbf{p}, \mathbf{p'})}{(\omega_{\mathbf{p'}} + \omega_{\mathbf{p}} + E_{hp - \mathbf{p'}} + k_0)^2} \right),
\end{align*}
\]

\[\gamma_\mathbf{k} = \frac{1}{Z} \sum_\mathbf{q} e^{ik_\mathbf{q}}, Z \text{ is the number of the nearest neighbor sites, the renormalized dressed holon d-wave pair gap function } \Delta_{hF}(\mathbf{k}) = Z_{hF} \Delta_{hF}(\mathbf{k}), \text{ where the effective dressed holon d-wave pair gap function } \Delta_{hF}(\mathbf{k}) = \Delta_{hF}(\mathbf{k}) \text{ with } \gamma_\mathbf{k}^{(d)} = \frac{\cos k_x - \cos k_y}{2}, \text{ and the dressed holon quasiparticle spectrum } E_{hF}(\mathbf{k}) = \sqrt{E_{hF}^2 + |\Delta_{hF}(\mathbf{k})|^2}, \]

\[\text{while the dressed holon quasiparticle coherent weight } Z_{hF} \text{ and effective dressed holon gap parameters } \Delta_{hF} \text{ are determined by the following two equations.}^{20,25,27,28,29,30,31}\]

\[\begin{align*}
1 = & \frac{(Zt)^2}{N^3} \sum_{\mathbf{k}, \mathbf{p}, \mathbf{p'} } \gamma^{(d)}(\mathbf{k}, \mathbf{p}, \mathbf{p'}, \mathbf{p'}) Z_{hF}^2 \frac{B_p B_{p'}}{E_{hF} \omega_{p} \omega_{p'}} \left( \frac{F_1^{(1)}(\mathbf{k}, \mathbf{p}, \mathbf{p'})}{(\omega_{p'} - \omega_{p})^2 - E_{hF}^2} + \frac{F_1^{(2)}(\mathbf{k}, \mathbf{p}, \mathbf{p'})}{(\omega_{p'} + \omega_{p})^2 - E_{hF}^2} \right), \\
1 = & 1 + \frac{(Zt)^2}{N^2} \sum_{\mathbf{p}, \mathbf{p'}} \gamma_{\mathbf{p} \mathbf{k}} \xi_{\mathbf{p} \mathbf{k}} Z_{hF} B_{\mathbf{p}} B_{\mathbf{p'}} E_{hF} \omega_{\mathbf{p}} \omega_{\mathbf{p'}} \left( \frac{F_2^{(1)}(\mathbf{p}, \mathbf{p'})}{(\omega_{\mathbf{p'}} - \omega_{\mathbf{p}} - E_{hp - \mathbf{p'}} + k_0)^2} + \frac{F_2^{(2)}(\mathbf{p}, \mathbf{p'})}{(\omega_{\mathbf{p'}} + \omega_{\mathbf{p}} + E_{hp - \mathbf{p'}} + k_0)^2} \right),
\end{align*}
\]

where \( A_1 = \alpha C^2 + (1 - \alpha)/(4Z), \) \( A_2 = \alpha C + (1 - \alpha)/(2Z) \), and the spin correlation functions \( C = (1/Z^2) \sum_{\eta, \bar{\eta}} \langle S^+_i S^-_{i+\eta} \rangle \) and \( C^2 = (1/Z^2) \sum_{\eta, \bar{\eta}} \langle S^+_i S^-_{i+\eta} \rangle \). In order to satisfy the sum rule of the correlation function \( \langle S^+_i S^-_{i+\eta} \rangle = 1/2 \) in the case without the antiferromagnetic long-range-order, an important decoupling parameter \( \alpha \) has been introduced in the above MF calculation.\^{20,25,27} \text{ which can be regarded as the vertex correction. These two equations in Eqs. (4a) and (4b) must be solved simultaneously with other self-consistent equations, then all order parameters, decoupling parameter } \alpha, \text{ and chemical potential } \mu \text{ are determined by the self-consistent calculation.}

In the CSS fermion-spin theory\^{24,25}, the electron Green’s function is a convolution of the spin Green’s function and dressed holon Green’s function. Following our previous discussion\^{20,25,27}, we can obtain the electron diagonal and off-diagonal Green’s functions in the SC state as,

\[
G(\mathbf{k}, \omega) = \frac{1}{N} \sum_\mathbf{p} Z_{hF} B_\mathbf{p} \left\{ \coth \frac{1}{2} \beta \omega_\mathbf{p} \left( \frac{U_{hp + \mathbf{k}}^2}{\omega + E_{hp + \mathbf{k}} - \omega_\mathbf{p}} + \frac{U_{hp + \mathbf{k}}^2}{\omega + E_{hp + \mathbf{k}} + \omega_\mathbf{p}} \right) + \frac{V_{hp + \mathbf{k}}^2}{\omega - E_{hp + \mathbf{k}} - \omega_\mathbf{p}} + \frac{V_{hp + \mathbf{k}}^2}{\omega - E_{hp + \mathbf{k}} + \omega_\mathbf{p}} \right\} + \text{tanh} \left\{ \frac{1}{2} \beta E_{hp + \mathbf{k}} \left( \frac{U_{hp + \mathbf{k}}^2}{\omega + E_{hp + \mathbf{k}} + \omega_\mathbf{p}} \right) - \frac{U_{hp + \mathbf{k}}^2}{\omega + E_{hp + \mathbf{k}} - \omega_\mathbf{p}} + \frac{V_{hp + \mathbf{k}}^2}{\omega - E_{hp + \mathbf{k}} - \omega_\mathbf{p}} - \frac{V_{hp + \mathbf{k}}^2}{\omega - E_{hp + \mathbf{k}} + \omega_\mathbf{p}} \right\}.
\]
function \( \Sigma \)

Following their discussions comes from the \( \pi, \pi \) level malism with the d-wave SC gap function in terms of and (6b) can be approximately reduced as the BCS for-

pendence of the microwave conductivity of cuprate su-

fer the spins center around the \( \pi, \pi \) point in the MF level20,23,27, then the main contributions for the spins comes from the \( \pi, \pi \) point. In this case, the electron diagonal and off-diagonal Green’s functions in Eqs. (6a) and (6b) can be approximately reduced as the BCS formalism with the d-wave SC gap function in terms of electron quasiparticle coherence factors \( U_{nk} = (1 + \xi_k / E_{nk}) / 2 \) and \( V_{nk}^2 = (1 - \xi_k / E_{nk}) / 2 \). These convolutions of the spin Green’s function and dressed holon diagonal and off-diagonal Green’s functions reflect the charge-spin recombination. Since the spins center around the \( \pi, \pi \) point in the MF level, the electron quasiparticle coherent weight \( Z \) where the electron quasiparticle co-

herence factors \( U_{nk} \approx V_{nk}^2 \approx (1 + \xi_k / E_{nk}) / 2 \) and \( V_{nk} \approx \xi_k / E_{nk} \), with \( \xi_k = Z_t \gamma_k + \mu \), and \( \gamma_k = \pi, \pi \) have been transferred into the electron quasiparticle coherence factors \( U_{nk} \) and \( V_{nk} \) and electron quasiparticle spectrum \( E_k \approx E_{nk} \) have been transferred into the electron quasiparticle coherence factors \( U_{nk} \) and \( V_{nk} \) and electron quasiparticle spectrum \( E_k \). Respectively, by the convolutions of the spin Green’s function and dressed holon Green’s functions due to the charge-spin recombination. This means that within the kinetic energy driven SC mechanism, the dressed holon pairs condense with the d-wave symmetry in a wide range of the hole doping concentration, then the electron Cooper pairs originating from the dressed holon pairing state are due to the charge-spin recombination, and their condensation automatically gives the electron quasiparticle character. For the convenience in the following discussions, these electron Green’s functions in Eq. (7) in the SC state can be expressed in the Nambu representation as,

\[
\tilde{G}(k, \omega) = Z_F \left( \frac{\omega + \bar{\Delta}(k)}{\omega^2 - E_k^2 - \bar{\Delta}(k)} \right),
\]

With the help of this BCS formalism under kinetic energy driven SC mechanism, now we can discuss the effect of the extended impurity scatterers on the quasiparticle transport in cuprate superconductors. In the presence of impurities, the unperturbed electron Green’s function in Eq. (8) is dressed via the impurity scattering,

\[
\tilde{G}_I(k, \omega)^{-1} = \tilde{G}(k, \omega)^{-1} - \tilde{\Sigma}(k, \omega),
\]

where \( \Sigma(k, \omega) = \sum_\alpha \Sigma_\alpha(k, \omega) \tau_\alpha \). It has been shown that all but the scalar component of the self-energy function can be neglected or absorbed into \( \Delta_Z(k) \). In this case, the dressed electron Green’s function in Eq. (9) can be explicitly rewritten as,

\[
\tilde{G}_I(k, \omega) = Z_F \left( \frac{\omega - \Sigma_0(k, \omega) \tau_0 + \Delta_Z(k) \tau_1 + [\bar{\xi}_k + \Sigma_3(k, \omega)] \tau_3}{\omega^2 - \Sigma_0(k, \omega)^2 - \bar{\xi}_k^2 - \Delta_Z^2(k)} \right).
\]

Based on the phenomenological d-wave BCS-type electron Green’s function20,22, the energy and temperature dependence of the microwave conductivity of cuprate superconductors has been fitted23, where the electron self-energy functions \( \Sigma_0(k, \omega) \) and \( \Sigma_3(k, \omega) \) have been treated within the framework of the T-matrix approximation. Following their discussions22,27, the electron self-energy function \( \tilde{\Sigma}(k, \omega) \) can be obtained approximately as,

\[
\tilde{\Sigma}(k, \omega) = \rho_i \tilde{T}_{kk}(\omega),
\]

where \( \rho_i \) is the impurity concentration, and \( \tilde{T}_{kk}(\omega) \) is the diagonal element of the T-matrix,

\[
\tilde{T}_{kk}(\omega) = V_{kk} \tau_3 + \sum_{k'} V_{kk'} \tau_3 \tilde{G}_I(k', \omega) \tilde{T}_{kk'}(\omega),
\]

where \( V_{kk'} \) is the impurity scattering potential. As mentioned in Sec. I, there is no gap to the quasiparticle excitations at the four nodes for the d-wave SC state of cuprate superconductors, therefore the quasiparticles are generated only around these four nodes. It has been
shown\(^{12}\) that this characteristic feature is very useful when considering the impurity scattering, since the initial and final momenta of a scattering event must always be approximately equal to the \(k\)-space location of one of the four nodes in the zero temperature and zero energy, while the impurity scattering potential \(V_{kk'}\) varies slowly over the area of a node. In this case, a general scattering potential \(V_{kk'}\) need only be evaluated in three possible cases: the intranode impurity scattering \(V_{kk'} = V_1 (k \text{ and } k' \text{ at the same node}),\) the adjacent-node impurity scattering \(V_{kk'} = V_2 (k \text{ and } k' \text{ at the adjacent nodes}),\) and the opposite-node impurity scattering \(V_{kk'} = V_3 (k \text{ and } k' \text{ at the opposite nodes}),\) then the impurity scattering potential \(V_{kk'}\) in the \(T\)-matrix can be effectively reduced as\(^{12}\),

\[
V_{kk'} \rightarrow V = \begin{pmatrix}
V_1 & V_2 & V_3 & V_2 \\
V_2 & V_1 & V_2 & V_3 \\
V_3 & V_1 & V_2 & V_2 \\
V_2 & V_3 & V_1 & V_2
\end{pmatrix}.
\] (13)

At the zero temperature and zero energy, these nodes reduce to points. In this case, this nodal approximation for the impurity potential can reproduce any impurity potential. However, at finite temperatures and energies, there is a limitation on the forward scattering character of the impurity potential because this nodal approximation assumes the Brillouin zone quadrant around a particular node\(^{12}\). It has been shown\(^{12}\) that although the strict forward scattering limit can therefore not be reached at finite temperatures and energies, this nodal approximation is still appropriate to treat the intermediate range scatterers. Therefore in the following discussions, we employ the simplified impurity scattering potential in Eq. (13) to study the impurity scattering effect on the quasiparticle transport of cuprate superconductors. Substituting Eq. (13) into Eq. (12), the \(T\)-matrix can be obtained as a \(4 \times 4\)-matrix around the nodal points,

\[
\hat{T}_{jj'}(\omega) = V_{jj'} \tau_3 + \hat{I}_G(\omega) \tau_3 \sum_{jj''} V_{jj''} \hat{T}_{jj''}(\omega),
\] (14)

where \(\hat{I}_G(\omega)\) is the integral of the electron Green’s function, and can be obtained as,

\[
\hat{I}_G(\omega) = \frac{1}{N} \sum_k \hat{G}_I(k, \omega) \approx \hat{G}_{10}(\omega) \tau_0 + \hat{G}_{13}(\omega) \tau_3,
\] (15)

with \(\hat{G}_{10}(\omega)\) and \(\hat{G}_{13}(\omega)\) are given by,

\[
\hat{G}_{10}(\omega) = \frac{1}{N} \sum_k Z_F \frac{\omega - \Sigma_0(\omega)}{\omega^2 - E_k^2},
\] (16a)

\[
\hat{G}_{13}(\omega) = \frac{1}{N} \sum_k Z_F \frac{\delta_k + \Sigma_3(\omega)}{\omega^2 - E_k^2},
\] (16b)

and the self-energy functions \(\Sigma_0(\omega)\) and \(\Sigma_0(\omega)\) are evaluated as,

\[
\Sigma_0(\omega) = \frac{\rho_i}{4} \left( \frac{2 \hat{G}_{10}(\omega)V_{13}^2}{[1 - \hat{G}_{13}(\omega)V_{13}]^2 - [\hat{G}_{10}(\omega)V_{13}]^2} + \frac{\hat{G}_{10}(\omega)(V_{123})^2}{[1 - \hat{G}_{13}(\omega)V_{123}]^2 - [\hat{G}_{10}(\omega)V_{123}]^2} \right)
\] (17a)

\[
\Sigma_3(\omega) = \frac{\rho_i}{4} \left( \frac{2 V_{13} [1 - \hat{G}_{13}(\omega)V_{13}]}{[1 - \hat{G}_{13}(\omega)V_{13}]^2 - [\hat{G}_{10}(\omega)V_{13}]^2} + \frac{V_{123} [1 - \hat{G}_{13}(\omega)V_{123}]}{[1 - \hat{G}_{13}(\omega)V_{123}]^2 - [\hat{G}_{10}(\omega)V_{123}]^2} \right),
\] (17b)

where \(V_{13} = V_1 - V_3, V_{123} = V_1 - 2V_2 + V_3,\) and \(V_{123} = V_1 + 2V_2 + V_3.\)

In the framework of the linear response theory, the microwave conductivity of cuprate superconductors can be calculated by means of the Kubo formula as\(^{23}\),

\[
\sigma(\omega, T) = \frac{\text{Im} \Pi(\omega, T)}{\omega},
\] (18)

with \(\Pi(\omega, T)\) is the electron current-current correlation function in the SC state, and can be expressed as,

\[
\Pi(\tau - \tau') = - < T_\tau \mathbf{J}(\tau) \cdot \mathbf{J}(\tau') > .
\] (19)

In the CSS fermion-spin representation\(^{24,25}\), the electron polarization operator can be evaluated as,

\[
P = \sum_i \mathbf{R}_i \tilde{n}_i = \sum_{i, \sigma} \mathbf{R}_i \hat{C}^\dagger_{i, \sigma} \hat{C}_{i, \sigma} = \frac{1}{2} \sum_{i, \sigma} \mathbf{R}_i \hat{h}_{i, \sigma} \hat{h}_{i, \sigma}^\dagger,
\] (20)

then within the \(t-J\) model (2), the current density of electrons is obtained by the time derivation of this polarization operator using the Heisenberg’s equation of motion as,
\[ J = ie[H, P] = \frac{1}{2} e \tau \sum_{\sigma} \eta(h_{i+\bar{q}1}^\dagger h_{i+\bar{q}1} S_{i+\bar{q}}^+ S_{i+\bar{q}0}^+ + h_{i+\bar{q}1}^\dagger h_{i+\bar{q}1} S_{i+\bar{q}0}^- S_{i+\bar{q}}^-) = -\frac{1}{2} e \tau \sum_{\nu\sigma} \eta \hat{G}_\nu^\dagger \hat{C}_{i+\bar{q}\sigma} \approx \frac{e v_f}{\sqrt{2}} \sum_{k\sigma} k \hat{C}^\dagger_{k\sigma} \hat{C}_{k\sigma}, \quad (21) \]

with \( v_f = \sqrt{2t} \) is the electron velocity at the nodal points. According to this current density (21), the current-current correlation function in Eq. (19) can be obtained as,

\[
\Pi(i\omega_n, T) = \frac{e^2 v_f^2}{2} \frac{1}{N} \sum_\beta \frac{1}{i\omega_n} \text{Tr}[\hat{G}(k, i\omega_n') \times \hat{G}(k, i\omega_n)] 
\times \hat{G}(k, i\omega_n'), (22)\]

\[
J(\omega, \omega') = \frac{I_0^{(0)} + L_A[I_0^{(0)} I_3^{(3)} + I_0^{(3)} I_3^{(0)}]}{[1 - (L_A I_0^{(0)} + L_B I_3^{(0)})][1 - (L_A I_3^{(3)} + L_B I_0^{(3)})] - [L_A I_0^{(0)} + L_B I_3^{(3)}][L_A I_3^{(0)} + L_B I_0^{(3)}]}, \quad (24)\]

with the functions,

\[
L_A(\omega, \omega') = [T_{11}^{(0)}(\omega)T_{11}^{(0)}(\omega + \omega') + T_{11}^{(3)}(\omega)T_{11}^{(3)}(\omega + \omega')] - T_{13}^{(0)}(\omega)T_{13}^{(0)}(\omega + \omega') - T_{13}^{(3)}(\omega)T_{13}^{(3)}(\omega + \omega'), \quad (25a)\]

\[
L_B(\omega, \omega') = [T_{11}^{(0)}(\omega)T_{11}^{(0)}(\omega + \omega') + T_{11}^{(3)}(\omega)T_{11}^{(3)}(\omega + \omega')] - T_{13}^{(0)}(\omega)T_{13}^{(0)}(\omega + \omega') - T_{13}^{(3)}(\omega)T_{13}^{(3)}(\omega + \omega'), \quad (25b)\]

while the functions \( I_0^{(0)}(\omega, \omega') \) and \( I_3^{(3)}(\omega, \omega') \) are given by,

\[
I_0^{(0)}(\omega, \omega') \tau_0 + I_0^{(3)}(\omega, \omega') \tau_3 = \frac{1}{N} \sum_k \hat{G}_I(k, \omega) \hat{G}_I(k, \omega + \omega'), \quad (26a)\]

\[
I_3^{(3)}(\omega, \omega') \tau_0 + I_3^{(0)}(\omega, \omega') \tau_3 = \frac{1}{N} \sum_k \hat{G}_I(k, \omega) \hat{G}_I(k, \omega + \omega'). \quad (26b)\]

Substituting Eq. (23) into Eq. (18), the microwave conductivity of cuprate superconductors is obtained explicitly as,

\[
\sigma(\omega, T) = e^2 v_f^2 \int_{-\infty}^{\infty} d\omega' \frac{n_F(\omega') - n_F(\omega + \omega')}{2\pi} [\text{Re} J(\omega' - i\delta, \omega' + \omega + i\delta) - \text{Re} J(\omega' + i\delta, \omega + \omega + i\delta)]. \quad (27)\]

We emphasize that based on the nodal approximation of the quasiparticle excitations and scattering processes, this microwave conductivity of cuprate superconductors in Eq. (27) is obtained within the kinetic energy driven SC mechanism, although its expression is similar to that obtained within the phenomenological BCS formalism with the d-wave SC gap function.

### III. MICROWAVE CONDUCTIVITY OF CUPRATE SUPERCONDUCTORS

In cuprate superconductors, although the values of \( J \) and \( t \) is believed to vary somewhat from compound to compound, however, as a qualitative discussion, the commonly used parameters in this paper are chosen as \( t/J = 2.5 \), with an reasonably estimative value of \( J \sim 1000K \). We are now ready to discuss the doping, energy, and temperature dependence of the quasiparticle transport of cuprate superconductors in the SC state with extended impurities. We have performed a calculation for the microwave conductivity \( \sigma(\omega, T) \) in Eq. (27), and the results of \( \sigma(\omega, T) \) as a function of energy with temperature \( T = 0.002J = 2K \) (solid line), \( T = 0.004J = 4K \) (dashed line), \( T = 0.008J = 8K \) (dash-dotted line), and \( T = 0.01J = 10K \) (dotted line) under the slightly strong impurity scattering potential with \( V_1 = 58J, V_2 = 49.32J, \) and \( V_3 = 40.6J \) at the impurity concentration \( \rho = 0.000014 \) for the doping concentration \( \delta = 0.15 \) are plotted in Fig. 1 in comparison with the corresponding experimental results of cuprate superconductors in the SC state (inset). Obviously, the energy evolution of the microwave conductivity of cuprate superconductors is qualitatively reproduced. In particular, a low temperature cusplike shape of the microwave conductivity is obtained for cuprate superconductors in...
The microwave conductivity $\sigma(\omega, T)$ as a function of energy with $T = 0.002J = 2K$ (solid line), $T = 0.004J = 4K$ (dashed line), $T = 0.008J = 8K$ (dash-dotted line), and $T = 0.01J = 10K$ (dotted line) at $\rho = 0.000014$ for $t/J = 2.5$, $V_1 = 58J$, $V_2 = 49.32J$, and $V_3 = 40.6J$ in $\delta = 0.15$. Inset: the corresponding experimental result of cuprate superconductors in the SC state taken from Ref. [14].

For a better understanding of the physical properties of the microwave conductivity $\sigma(\omega, T)$ in cuprate superconductors, we have studied the doping evolution of the microwave conductivity, and the result of $\sigma(\omega, T)$ as a function of doping with temperature $T = 0.002J = 2K$ and energy $\omega = 0.000087J \approx 1.81GHz$ under the slightly strong impurity scattering potential with $V_1 = 58J$, $V_2 = 49.32J$, and $V_3 = 40.6J$ at the impurity concentration $\rho = 0.000014$ is plotted in Fig. 2 (solid line). For comparison, the corresponding result of the SC gap parameter of cuprate superconductors is also shown in the same figure (dashed line). Our result shows that in contrast to the dome shape of the doping dependent SC gap parameter, the microwave conductivity $\sigma(\omega, T)$ decreases with increasing doping in the underdoped regime, and reaches a minimum in the optimal doping, then increases in the overdoped regime. This doping dependent behavior of the low energy microwave conductivity $\sigma(\omega, T)$ at low temperatures is also qualitatively consistent with the universal microwave conductivity limit $\sigma \propto 1/\Delta$ at low energy as temperature $T \to 0$, if this SC gap parameter $\Delta$ in the phenomenological BCS formalism has the similar dome shape doping dependence.

In the above discussions, we mainly study the effect of the extended impurity scatterers on the quasiparticle transport at low temperatures ($T \ll T_c$) and low energies ($\omega \ll \Delta$). Now we discuss the temperature dependence of the quasiparticle transport of cuprate superconductors, where $T$ may approach to $T_c$ from low temperature side. In this case, it has been shown that the inelastic scattering process should be considered at higher temperatures, such as the quasiparticle-quasiparticle scattering. This is followed from the fact that the inelastic quasiparticle-quasiparticle scattering process is suppressed at low temperatures due to the large SC gap parameter in the quasiparticle excitation spectrum. However, the contribution from this inelastic quasiparticle-quasiparticle scattering process is increased rapidly when $T$ approaches to $T_c$, from low temperature side, since there is a small SC gap parameter near $T_c$. In particular, it has been pointed out that the contribution from the quasiparticle-quasiparticle scattering process to the transport lifetime is exponentially suppressed at low temperatures, and therefore the effect of this inelastic quasiparticle-quasiparticle scattering can be considered by adding the inverse transport lifetime into the imaginary part of the self-energy function $\Sigma_0(\omega)$ in Eq. (17a), then the total self-energy function $\Sigma_0^{\text{tot}}(\omega)$ can be expressed as

$$
\Sigma_0^{\text{tot}}(\omega) = \Sigma_0(\omega) - i[2\tau_{\text{inel}}(T)]^{-1},
$$

with $\tau_{\text{inel}}(T)$ has been chosen as $[2\tau_{\text{inel}}(T)]^{-1} = 91.35(T - 0.005)^4 J$. Using this total self-energy function $\Sigma_0^{\text{tot}}(\omega)$ to replace $\Sigma_0(\omega)$ in Eq. (27), we have
performed a calculation for the microwave conductivity \( \sigma(\omega, T) \), and the results of \( \sigma(\omega, T) \) as a function of temperature \( T \) with energy \( \omega = 0.0000547J \approx 1.14 \text{GHz} \) (solid line), \( \omega = 0.0001094J \approx 2.28 \text{GHz} \) (long dashed line), \( \omega = 0.0006564J \approx 13.4 \text{GHz} \) (dash-dotted line), \( \omega = 0.001094J \approx 22.8 \text{GHz} \) (dotted line), and \( \omega \approx 75.3 \text{GHz} \) (short dashed line) at \( \rho = 0.000014 \) for \( t/J = 2.5, V_1 = 58J, V_2 = 49.32J, \) and \( V_3 = 40.6J \) at the impurity concentration \( \delta = 0.15 \) are plotted in Fig. 3 in comparison with the corresponding experimental results of cuprate superconductors in the SC state taken from Ref. 14.

FIG. 3: The microwave conductivity as a function of temperature with energy \( \omega = 1.14 \text{GHz} \) (solid line), \( \omega = 2.28 \text{GHz} \) (long dashed line), \( \omega = 13.4 \text{GHz} \) (dash-dotted line), \( \omega = 22.8 \text{GHz} \) (dotted line), and \( \omega \approx 75.3 \text{GHz} \) (short dashed line) under the impurity scattering potential with energy \( \omega \approx 0.000014 \) for the doping concentration \( \delta = 0.15 \). Inset: the corresponding experimental result of cuprate superconductors in the SC state taken from Ref. 14.

Within the framework of the kinetic energy driven d-wave cuprate superconductivity\(^{20}\), our present results of the energy and temperature dependence of the microwave conductivity by considering the effect of the extended impurity scatterers are qualitatively similar to the earlier attempts to fit the experimental data by using a phenomenological d-wave BCS formalism\(^{8,9,15-18,19}\). Establishing this agreement is important to confirming the nature of the SC phase of cuprate superconductors as the d-wave BCS-like SC state within the kinetic energy driven SC mechanism. It has been shown\(^{7,11,12,13,14}\) that there are some subtle differences for different families of cuprate superconductors, and these subtle differences may be induced by the other effects except the impurity scattering. However, we in this paper are primarily interested in exploring the general notion of the effect of the extended impurity scatterers on the kinetic energy driven cuprate superconductors in the SC state. The qualitative agreement between the present theoretical results and experimental data also show that the presence of impurities has a crucial effect on the microwave conductivity of cuprate superconductors.

**IV. SUMMARY**

In conclusion we have shown very clearly in this paper that if the effect of the extended impurity scatterers is taken into account in the framework of the kinetic energy driven d-wave superconductivity, the microwave conductivity of the \( t-J \) model calculated based on the nodal approximation of the quasiparticle excitations and scattering processes per se can correctly reproduce some main features found in the microwave conductivity measurements on cuprate superconductor in the SC state\(^{7,11,12,13,14}\), including the energy and temperature dependence of the microwave conductivity spectrum. The theory also predicts a V-shaped doping dependent microwave conductivity, which is in contrast with the dome shape of the doping dependent SC gap parameter, and therefore should be verified by further experiments.

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