The Hubble Tension Persists Beyond Slow-roll Inflation

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For a standard ΛCDM universe with a power-law primordial power spectrum, the discrepancy between early- and late-universe measurements of the Hubble constant continued to grow, and recently reached 5.3σ. During inflation, events beyond slow-roll often lead to features in the primordial power spectrum, hence breaking the power-law assumption in the derivation of the Hubble tension. We investigate, in a very model-independent way, whether such inflationary “glitches” can ease the Hubble tension. The recently released Planck temperature and polarization data and the 2019 SH0ES+H0LiCOW joint constraint on the Hubble constant are combined to drive a blind Daubechies wavelet signal search in the primordial power spectrum, up to a resolution Δ ln k ~ 0.1. We find no significant detection of any features beyond power-law. With 64 more degrees of freedom injected in the primordial power spectrum, the Hubble tension persists at a 4.9σ level.

INTRODUCTION

The Λ cold dark matter (ΛCDM) model has been taken as the standard cosmological paradigm since the discovery of late-universe acceleration [33, 35]. It is a remarkable success in terms of explaining the temperature and polarization anisotropies of the cosmic microwave background (CMB) that have been accurately measured by the Planck satellite [3, 4], the baryon acoustic oscillation features in the galaxy redshift survey data [1, 17, 15], the weak gravitational lensing of galaxies [24], the Type Ia supernovae luminosity distances [40], and many others.

Recently, the local distance-ladder measurement of Hubble constant (SH0ES) [35, 37–39], followed by independent support from time delay of strong-lensing quasars images (H0LiCow) [16], starts to challenge the “concordance” ΛCDM picture. Assuming a minimal six-parameter ΛCDM model, SH0ES and H0LiCOW results together provide a 5.3σ difference of H_0 with the CMB measurement. This inconsistency, often referred to as “Hubble tension”, may indicate new physics beyond ΛCDM. Simple one-parameter extensions of ΛCDM, however, were found insufficient to resolve the Hubble tension [20, 28]. More sophisticated models are hence proposed to take the challenge. The list includes but is not limited to modified gravity [25, 26, 42], early dark energy [8, 23, 34], interacting dark components [10, 14, 15, 47, 38], and extra relativistic species [9, 12, 13, 19]. It has also been claimed that the Hubble tension may just be a relativistic non-linear effect in the standard ΛCDM paradigm [11].

Adhikari and Huterer proposed that non-Gaussian CMB covariance from a strong coupling between long-wavelength modes and short-wavelength modes can resolve the Hubble tension [2]. We repeated their calculation and found the same results. However, we noticed that in this model the posterior amplitude of matter fluctuations (σ_8) is significantly higher than ΛCDM value, which is already at the upper edge of the bounds from late-universe observations of galaxy clustering and weak gravitational lensing [3]. Moreover, it is yet to be shown that the prediction of polarization and the large tri-spectrum in this model is consistent with Planck data.

Nevertheless, the idea that Hubble tension may be due to some anomalies in primordial conditions is worth further investigation.

In the concordance picture, the initial seeds of cosmological fluctuations are assumed to originate from vacuum quantum fluctuations during early-universe inflation. For simplest single-field slow-roll inflation models, the predicted primordial metric fluctuations are almost perfectly Gaussian, and has a slightly tilted power-law primordial scalar power spectrum \( P(k) = A_s \left( \frac{k}{k_{\text{pivot}}} \right)^{n_s - 1} \), where \( k \) is the comoving wave number and \( k_{\text{pivot}} = 0.05 \text{Mpc}^{-1} \) is the pivot scale. The standard analysis of CMB and large-scale structure data is usually established on this featureless power-law primordial power spectrum. The global deviation from power-law shape, is bounded by Planck data within a sub-percent level: \( \frac{d\ln P}{d\ln k} \approx -0.0041 \pm 0.0067 \), which is fully consistent with the single-field slow-roll prediction \( \frac{d\ln P}{d\ln k} \lesssim 10^{-3} \). The Planck collaboration also studied a broad class of inflation models as well as many phenomenological parametrizations, but found no evidence beyond the single-field slow-roll scenario [6, 7]. Neither does a blind node expansion with cubic-spline interpolation favor any smooth non-power-law features with a resolution \( \Delta \ln k \sim 1 \). These results are supported by many other independent works [16, 27, 49]. In summary, the CMB data do not favor any global periodic oscillations or any broad smooth features with resolution \( \Delta \ln k \sim 1 \).

The apparently missing ingredient - sharper local features with \( \Delta \ln k \ll 1 \) are as well motivated from the theoretical perspectives. Note that \( \ln k \) roughly corresponds to physical time or number of expansion e-folds during inflation. Many slow-roll-breaking processes during inflation, such as crossing a step in the inflaton potential,
has strong impact only for $\sim \text{a few} \times 0.1$ efoldings. These models can then produce sharp ($\Delta \ln k \sim \text{a few} \times 0.1$) features that are typically local in time ($\ln k$) domain and band-limited in frequency (Fourier conjugate of $\ln k$) domain. One way to study these sharp features is the top-down approach, that is, to parameterize and constrain the predicted features, in a model-by-model manner. For a few templates from popular models, the Planck collaboration, again, found null results [6]. See also Refs. [21, 22, 43, 45] for earlier works. The other way, which is missing for the latest Hubble-tension related data, and will be done in this work, is the bottom-up approach that model-independently covers a much broader class of models.

We apply a wavelet analysis, a statistical tool specifically designed to study local and band-limited signals, to search for sharp features in the primordial power spectrum. Similar analysis has been done for earlier CMB data from COBE and WMAP satellites [29–32, 41], before Planck data drove the Hubble tension. The purpose of our re-examination in the latest Planck data is to investigate whether the Hubble tension is driven by a primordial sharp feature that manifests itself in high-$\ell$ multipoles that are only accurately measured by Planck.

**METHOD**

The Daubechies wavelet basis takes the form:

$$
\Psi_{n,m}(t) = 2^{n/2}\Psi_{0,0}(2^n t - m), \quad n, m \in \mathbb{Z}
$$

where $\Psi_{0,0}$ is the mother function of Daubechies wavelet. The basis functions are complete, compactly supported and orthogonal with respect to both the scale $n$ and the position $m$ indices. They are moving kernels with hierarchical resolutions, with each resolution level a factor of 2 finer than the previous one. As shown in Fig. 1, the Daubechies mother functions are not unique. The smoothness of the Daubechies mother function increases with its order. In this work, we use the 4th order Daubechies basis, and check the robustness of our result with 2nd order Daubechies basis.

To blindly search features in the primordial scalar power spectrum $P(k)$, we decompose its deviation from power-law shape into Daubechies wavelets

$$
\ln \frac{P(k)}{P_{\text{ref}}(k)} = \sum_{n=0}^{3} \sum_{m=-2^{n+1}}^{2^{n+1}} A_{n,m} \Psi_{n,m} \left( \ln \frac{k}{k_{\text{pivot}}} \right),
$$

where the reference power-law is $P_{\text{ref}}(k) = A_s \left( \frac{k}{k_{\text{pivot}}} \right)^{n_s-1}$. The lower and upper bounds of the scale index $n$ are chosen such that the resolution in $\ln k$ is limited to $0.1 \lesssim \Delta \ln k \lesssim 1$, to match features from slow-roll-breaking processes during inflation. The lower and upper bounds of the position index $m$ are chosen such that CMB scales measured by Planck are well covered.

We use the publicly available software CosmoMC [24] to run Markov Chain Monte Carlo (MCMC) simulations and to estimate the marginalized bounds of cosmological parameters, which include the standard six built-in parameters ($\Omega_b h^2, \Omega_c h^2, \theta, \tau_{re}, A_s, n_s$) and the sixty-four $A_{n,m}$ coefficients defined in Eq. (2). Here $\Omega_b h^2$ and $\Omega_c h^2$ are baryon and CDM densities, respectively; $\theta$ is the angular extension of sound horizon on the last scattering surface; $\tau_{re}$ is the reionization optical depth. The Hubble constant $H_0$ can be derived from these parameters. Flat priors are applied to all the parameters including the $A_{n,m}$ coefficients.

To explicitly extract Hubble-tension-driven wavelet signals, we use jointly the SH0ES + H0LiCow constraint $H_0 = 73.82 \pm 1.10 \text{ km s}^{-1} \text{Mpc}^{-1}$ [40] and the Planck final release of TT,TE,EE + lensing likelihood [4].

**RESULTS AND CONCLUSIONS**

Examining the posterior of all the $A_{n,m}$ parameters, we find no significant (> 3$\sigma$) detection of any of the wavelet components. In Fig. 2 we directly visualize the non-deviation from power-law by showing the reconstructed $P(k)$ trajectories. Compared to the 12-knot cubic spline reconstruction in section 6.3 of Ref. [6], our wavelet analysis, by construction, picks out more local and sharper features.

The high-frequency wiggling in $P(k)$ is driven, or at least partially driven by the statistical fluctuations in CMB power spectrum. In Fig 3 we show how the wavelet
At higher ell’s, the trajectories converge due to a much smaller cosmic variance. These features can also be seen in the left and middle parts of Fig 2. The large scattering in the right part of Fig 2 corresponds to the unconstrained power on small scales (high-k) beyond Planck resolution.

For the Hubble constant, we obtain a Planck + SH0ES + H0LiCow joint constraint: $H_0 = 69.4 \pm 0.7 \text{km s}^{-1}\text{Mpc}^{-1}$. Because the posterior is very close to Gaussian, we can approximately remove the SH0ES + H0LiCow contribution and obtain a Planck-only constraint, as shown in Fig. 4. For comparison, we also plot the Planck constraint for the standard ΛCDM power-law case as well as the 12-knot-spline case, which we obtain by repeating the calculations in Ref. [6]. We find that allowing more features in the primordial power spectrum, either local and band-limited as in the wavelet case, or just low-pass filtered as in the 12-knot-spline case, in general pushes the mean $H_0$ towards an even smaller value, which balances out the increased uncertainty and keeps the Hubble tension at roughly the same level. More specifically, the tension between Planck and SH0ES + H0LiCow is 4.9σ for the wavelet analysis, and 5.3σ for the 12-knot-spline.

Finally, we repeated the calculation with 2nd order Daubechies basis, and found no significant variations in the results. We thus arrive at the conclusion - the Hubble tension is unlikely caused by any events beyond slow-roll during early-universe inflation.

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