An Accelerated Procrustean Markov Process Model with Coherent Constraint for Non-Rigid Structure from Motion

Ying Zhang, Xia Chen, Zhan-Li Sun*, Member, IEEE, Kin-Man Lam, Senior Member, IEEE, and Zhigang Zeng, Senior Member, IEEE

Abstract—Non-Rigid Structure from Motion (NRSfM) is the task of reconstructing the 3D point set of a non-rigid object from an ensemble of images with 2D correspondences, which has been a long-lasting challenging research topic. Compared to the state-of-the-art methods for NRSfM, the Procrustean Markov Process (PMP) model has obtained a relatively good performance. However, the estimation error and the convergence time of the PMP model will increase simultaneously when noise is present. To address this problem, in this paper, a coherent constraint is constructed to suppress the noise in the initialization step of the PMP algorithm. Moreover, an Accelerated Expectation Maximization (AEM) algorithm is devised to optimize the PMP estimation model. Experimental results on several widely used sequences demonstrate that our proposed algorithm achieves state-of-the-art performance, as well as its effectiveness and feasibility.

Index Terms—Non-rigid structure from motion, accelerated expectation maximization algorithm, coherent constraint.

1 INTRODUCTION

Reconstructing the 3D object shapes from a set of 2D images has become a valuable approach to enhance the tasks in computer vision, such as virtual reality [1], object recognition [2], biometrics [3], human-computer interaction [4], [5], etc. Non-Rigid Structure from Motion (NRSfM) provides a useful approach to simultaneously estimate the 3D time-varying deformed object and the relative camera motion from the corresponding 2D observation points in a sequence of images. Although many effective algorithms have been proposed for NRSfM in the past few decades, it is still a very complex and ill-posed problem due to the lack of prior information about the 3D structure.

In order to solve the uncertainty in NRSfM, many different a priori information, assumptions and constraints have been utilized in reconstructing the 3D shapes. Inspired by the factorization technique for Structure from Motion (SfM) [6], a low-rank constraint was proposed in [7] to model the unknown time-varying deformable 3D shapes, represented as a linear combination of a small number of 3D shape bases. In the matrix factorization method, the 2D observed matrix was factored into a 3D pose matrix and a 3D shape basis matrix. Subsequently, many works have been proposed based on the low-rank shape model. In [8], a closed-form solution was reported, which considers both the low-rank constraint and the rotation constraint. An approximate rank-3 solution was derived in [9] by utilizing a Gaussian prior and a probabilistic principal component analysis shape model. In [10], an approximate rank-3 solution was proposed to solve NRSfM by considering very small semi-definite programming and a nuclear-norm minimization problem. Furthermore, a multilinear factorization algorithm was presented in [11], which incorporates the shape basis assumption and a time-independent latent smoothness characteristic of the unknown 3D non-rigid shapes.

A dual approach to the shape basis representation was proposed in [12] to reduce the number of unknown parameters. The dual approach assumes that the 3D point trajectories are constrained to lie in a linear trajectory space. The linear space is compactly spanned by 3K predefined independent basis trajectories, obtained via the Discrete Cosine Transform (DCT). Nevertheless, the rank-3K constraint has limited capability to model high-frequency deformation, represented by trajectories.

In [13], [14], a better reconstruction of high-frequency deformation was achieved without relaxing the rank-3K constraint, by modeling a smoothly deforming 3D shape as a single point moving along a smooth time-trajectory within a linear shape space. The predefined DCT was applied to represent the coefficients of shape basis.

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Ying Zhang and Xia Chen are with School of Electrical Engineering and Automation, Anhui University, China, and Hefei ZC Optoelectronic Technologies Ltd., Anhui Province Key Laboratory of Non-Destructive Evaluation, China. (Joint first authors, the authors contributed equally to this work.) Zhan-Li Sun is with School of Electrical Engineering and Automation, Anhui University, China.

Kin-Man Lam is with School of Electronic and Information Engineering, The Hong Kong Polytechnic University, Hong Kong, China.

Zhigang Zeng is with the School of Automation, Huazhong University of Science and Technology, Wuhan 430074, China, and also with the Key Laboratory of Image Processing and Intelligent Control of Education Ministry of China, Huazhong University of Science and Technology, Wuhan 430074, China.

*Corresponding author. (e-mail: zhlsun2006@126.com)
Based on the trajectory representation, a scalable monocular surface reconstruction method was proposed in [15] to solve the NRSfM problem, for both sparse and dense data. The optimized solution was obtained through singular value thresholding, proximal gradient and alternating direction method of multipliers. In [16], the dense NRSfM problem with complex non-rigid deformations was solved based on the Grassmann manifold. The complex non-rigid deformation was assumed lying on a union of local linear subspaces, both spatially and temporally. In addition, a scalable, efficient, and accurate solution was proposed in [17] to solve the NRSfM problem, by combining the existing point-trajectory low-rank models with a probabilistic framework for matrix normal distributions. For the trajectory-based methods, how to determine the optimal number of shape bases is a difficult problem. In [18], a Procrustean Normal Distribution (PND) was proposed to represent the distribution of shape deformations by strictly separating the motion and deformation components. The 3D structure can be accurately reconstructed via an EM algorithm, without requiring any additional constraints or prior knowledge. Although [18] and the improved version (PND2) [19] have achieved a relatively good reconstruction performance on most commonly used datasets, they do not work well for shapes with some large drastic deformations and noise, due to the lack of smoothing constraints.

In [20], a Procrustean Markov Process (PMP) model was proposed to enforce the smoothness constraint between two adjacent frames. The sequence of 3D shapes is considered as a simple stationary Markov process based on Procrustes alignment. Nevertheless, the convergence of the EM algorithm is relatively slow. Moreover, the PMP model is sensitive to noise. In this paper, an accelerated PMP model with a coherent constraint is proposed to improve the robustness and the convergence speed of the EM algorithm. Experimental results on several commonly used sequences verify the effectiveness and feasibility of the proposed algorithm.

The key contributions of the proposed approach are two aspects, as follows:

- In order to suppress the noise, a coherent constraint, corresponding to a displacement function, is proposed to preserve the global structure of each shape by constraining adjacent points to move coherently.
- An Accelerated Expectation Maximization (AEM) algorithm is proposed to achieve faster convergence, when optimizing the PMP estimation model.

The remainder of the paper is organized as follows. A detailed description of the proposed method is presented in Section 2. Experimental results are given in Section 3. Finally, conclusions are made in Section 4.

2 METHODOLOGY

The proposed algorithm is composed of three main components: formulation of the PMP model [20], initialization of the PMP model with a coherent constraint, and the optimization of the PMP model using the proposed accelerated EM algorithm.

2.1 Formulation of the PMP model

For the $i$th ($i = 1, \ldots, n_s$) frame in an image sequence, the observed 3D structure $D_i$ can be represented as a collection of 3D coordinates $(x, y, z)$ of $n_p$ feature points, i.e.,

$$D_i = \begin{bmatrix} x_{i,1} & x_{i,2} & \cdots & x_{i,n_p} \\ y_{i,1} & y_{i,2} & \cdots & y_{i,n_p} \\ z_{i,1} & z_{i,2} & \cdots & z_{i,n_p} \end{bmatrix}. \hspace{1cm} (1)$$

Under the orthographic projection model, the $z$ coordinates are unknown for $D_i$. Define a $3 \times n_p$ binary weight matrix $W_i$, whose elements in the first two rows and the third row are all ones and zeros, respectively. Then, (1) can be represented as,

$$D_i = W_i \odot (X_i - t_i1^T) + m_i, \hspace{1cm} (2)$$

where the $3 \times n_p$, hidden variable $X_i$ denotes the true 3D shape of the $i$th frame, $t_i \in \mathbb{R}^{3 \times 1}$ is the translation, $1 \in \mathbb{R}^{n_p \times 1}$ is a vector with elements of one, and $m_i \in \mathbb{R}^{3 \times n_p}$ is a zero-mean Gaussian noise with standard deviation $\sigma$. The operator $\odot$ denotes the Hadamard product.

As in [20], given the scale $s_i$ and the rotation matrix $R_i \in \mathbb{R}^{3 \times 3}$, $X_i$ can be aligned to $Y_i \in \mathbb{R}^{3 \times n_p}$, as follows:

$$Y_i = s_i R_i X_i. \hspace{1cm} (3)$$

For $Y_i$, the first-order linear Markov process can be presented as follows:

$$\text{vec}(Y_i) = \alpha \text{vec}(Y_{i-1} - \overline{Y}) + \text{vec}(\overline{Y}) + n_i, \hspace{1cm} (4)$$

where $\text{vec}(Y_i)$ is a vectorization form of $Y_i$. $Y_{i-1}$ and $\overline{Y} \in \mathbb{R}^{3 \times n_p}$ are the $(i-1)$th aligned 3D shape and the mean shape of $Y_i(i = 1, \ldots, n_s)$, respectively [20]. The smoothness parameter $\alpha$ is the transition probability of $Y_i$ moving through the successive time periods. The noise term $n_i \in \mathbb{R}^{3n_p \times 1}$ is a Gaussian random vector with independent and identical distribution. In (4), the smoothness assumption can effectively reduce the effect of large deformation.

The aligned 3D shapes $Y_i$ obeys the procrustean normal distribution [18], i.e.,

$$p(Y_i) \sim \mathcal{N}_P(\overline{Y}, \Sigma_R), \hspace{1cm} (5)$$

where the symbol $\mathcal{N}_P(\cdot, \cdot)$ denotes the procrustean normal distribution, and $\Sigma_R \in \mathbb{R}^{3n_p \times 3n_p}$ denotes the covariance matrix of $Y_i(i = 1, \ldots, n_s)$. As done in [18], in order to solve the singular problem, $\Sigma_R$ is decomposed as,

$$\Sigma_R = Q \Sigma Q^T, \hspace{1cm} (6)$$

where $Q \in \mathbb{R}^{3n_p \times (3n_p-7)}$ and $\Sigma \in \mathbb{R}^{(3n_p-7) \times (3n_p-7)}$ are an orthogonal matrix and a non-singular positive definite symmetric matrix, respectively.

Combining (4) and (5), the distribution of $n_i$ is given as follows:

$$n_i \sim \mathcal{N}(0, QHQ^T), \hspace{1cm} (7)$$
where the symbol $\mathcal{N}(\cdot, \cdot)$ denotes the normal distribution, and $H \in \mathbb{R}^{(3n_p - 7) \times (3n_p - 7)}$ is an unknown positive definite symmetric matrix [20]. Furthermore, the mean $\mu_{Y_i|Y_{i-1}}$ and the variance $\Sigma_{Y_i|Y_{i-1}}$ of the conditional probability $p(Y_i|Y_{i-1})$ can be computed as follows:

$$\mu_{Y_i|Y_{i-1}} = \alpha \text{vec}(Y_{i-1}) + \text{vec}(\bar{Y}), \quad (8)$$

$$\Sigma_{Y_i|Y_{i-1}} = \text{QHQ}^T, \quad (9)$$

Considering (5), (8) and (9), the probability $p\{Y_i\}$ ($i = \{1, 2, ..., n_s\}$) can be given as,

$$p(Y_1, Y_2, \ldots, Y_{n_s}|\Theta) = p(Y_1|\Theta) \prod_{j=2}^{n_s} p(Y_j|Y_{j-1}, \Theta), \quad (10)$$

where $\Theta = \{\bar{Y}, R_i, s_i, \alpha, H, \Sigma, Q, \sigma\}$. Referring to [20], the unknown parameters $\Theta$ can be estimated by maximizing the following log-likelihood function,

$$\log(p\{\{D_i, X_i\}\}|\Theta)) = \log(p(X_1, X_2, ..., X_{n_s}|\Theta)) + \sum_{i=1}^{n_s} \log(p(D_i|X_i, \Theta)). \quad (11)$$

### 2.2 Initialization of PMP model with a coherent constraint

Considering the coherent constraint, a good initial value for $\Theta$ can be obtained for PMP via the following optimization model,

$$\min_{\Psi} \sum_{i=1}^{n_s} \left\| s_i R_i X_i + t_i 1^T + C_i G_i - \bar{X} \right\|_F^2 + \text{tr}(C_i G_i C_i^T)$$

s.t. $R_i^T R_i = I_3$, $\left\| \bar{X} \right\|_F = 1, \quad (12)$

where $C_i \in \mathbb{R}^{3 \times n_p}$ is a coefficient matrix, $\bar{X}$ is the mean matrix of $X_i (i = 1, \ldots, n_s)$, $\Psi$ is a collection of unknown parameters $\Psi = \{s_i, R_i, t_i, \bar{X}, X_i, C_i\}$, and $I_3$ is a $3 \times 3$ identity matrix. The matrix $G_i \in \mathbb{R}^{n_p \times n_p}$ is a kernel matrix, whose element $g_{mn}$ is computed as follows:

$$g_{mn} = \text{G}(X_{i,m}, X_{i,n}) = \exp(-\frac{\|X_{i,m} - X_{i,n}\|_F^2}{2\beta}), \quad (13)$$

where $X_{i,m}$ and $X_{i,n}$ are the $m^{th}$ and the $n^{th}$ point in $X_i$, respectively; and $\beta$ defines the width of the Gaussian kernel function [25]. The transformation $C_i G_i$ is assumed to be a displacement function. The regularization term $\text{tr}(C_i G_i C_i^T)$ is a global structure constraint, following the motion coherence theory [23], [24], which can constrain the smoothness of $C_i G_i$ [25]. The parameter $\lambda$ makes a trade-off between Procrustes alignment and regularization.

First, $\bar{X}$ and $t_i$ can be obtained by combining (12) and the constraint term $\bar{X} = 0,$

$$\bar{X} = \frac{1}{n_s} \sum_{i=1}^{n_s} (s_i R_i X_i + t_i 1^T + C_i G_i), \quad (14)$$

$$t_i = -\frac{1}{n_p} (s_i R_i X_i + C_i G_i)1. \quad (15)$$

Substitute (15) into (12) and let $B = I_{n_p} - \frac{1}{n_p} 1 1^T$, where $B \in \mathbb{R}^{n_p \times n_p}$, we can get

$$\min_{R_i} \left\| s_i R_i X_i - \bar{X} - C_i G_i B \right\|_F^2. \quad (16)$$

For (16), considering the orthogonal Procrustes problem, we have

$$R_i = V_i U_i^T, \quad (17)$$

where $U_i A_i V_i^T = \text{svd} (X_i B (\bar{X} - C_i G_i B)^T)$, and $\text{svd}(\cdot)$ represents the singular value decomposition [26].

As each shape variation is assumed to be orthogonal to the mean shape [18], we have

$$\text{vec}(s_i R_i X_i + C_i G_i - \bar{X})^T \text{vec} X = 0. \quad (18)$$

Considering $\left\| \bar{X} \right\|_F = 1$, we have

$$\text{vec}(s_i R_i X_i + C_i G_i)^T \text{vec}(\bar{X}) = 1. \quad (19)$$

According to (19), $s_i$ can be computed as follows

$$s_i = \frac{1 - \text{vec}(C_i G_i)^T \text{vec}(\bar{X})}{\text{vec}(R_i X_i B_i)^T \text{vec}(\bar{X})}. \quad (20)$$

As in [20], the true 3D shape, $X_i$, can be decomposed as follows:

$$X_i = D_i + L(z_i), \quad (21)$$

where $L(z_i)$ transforms $z_i$ into a $3 \times n_p$ matrix, in which the elements of the first two rows are zeros and the elements of the third row are $z_i$. Furthermore, $\text{vec}(L(z_i)) = Wz_i$, where $W$ is a truncated version of $(I - \text{diag}(\text{vec}(W)))$ removing all-zero columns.

Considering (15) and (21), (12) can be rewritten as follows:

$$\sum_{i=1}^{n_s} \left\| (s_i R_i (D_i + L(z_i)) + C_i G_i) B - \bar{X} \right\|_F^2 + \lambda \text{tr}(C_i G_i C_i^T). \quad (22)$$

Then, we compute the one-order partial derivative of (22) with respect to $C_i$ and $z_i$, respectively. As a result, $C_i$ and $z_i$ can be derived by setting these two partial derivatives to zeros, as follows:

$$z_i = \left[ W^T (B \otimes I_3) \right]^T \text{vec} \left( \frac{1}{s_i} R_i^T (\bar{X} + C_i G_i - D_i) \right), \quad (23)$$

$$C_i = \left[ \frac{1}{s_i} R_i^T (\bar{X} - X_i B) \right] \left[ GB + \frac{1}{s_i} I_{3n_p} \right]^{-1}, \quad (24)$$

where the operators $\otimes$ and $\dagger$ denote the Kronecker product and the pseudo inverse, respectively.

### 2.3 The PMP model optimization using an accelerated EM algorithm

For the model (11), an accelerated EM algorithm is proposed to derive the solutions. Let $\mu_{i|n_x}$ and $C_{i|n_x}$ be the mean and covariance of $p(X_i|D_i, ..., D_{n_x})$, respectively. The cross-covariance of $\text{vec}(X_i)$ and $\text{vec}(X_{i+1})$
is denoted as $C_{i,i+1|n_s}$. The variables $\mu_{i|n_s}$, $C_{i|n_s}$ and $C_{i,i+1|n_s}$ can be computed by the Kalman smoothing in the E-step [20].

In the M-step, all the unknown parameters $\Theta$ in (11) are updated by maximizing the expectation of (11), i.e.

$$J(\Theta|\Theta^t) = E \left[ \log(p(\{D_t, X_t\}|\Theta)) | \Theta^t \right].$$

(25)

Then, each element of $\Theta^t$ at the $(t+1)^{th}$ iteration can be obtained as follows:

$$\Theta^{t+1} = \arg \max_{\Theta \in \Theta} J(\Theta|\Theta^t).$$

(26)

The E-step and M-step of the original EM algorithm are repeated to produce a series of estimates $(\Theta^{t+1}, \Theta^t, \Theta^{t-1})$.

Denote $\phi$ as a vectorized variable of $\Theta$. Referring to [27], for the accelerated EM algorithm, $\phi$ can be updated as follows:

$$\phi_{new}^t = \phi^t + \left[ [\phi^t - \phi^t_0]^{-1} + [\phi^t_0 - \phi^t]^{-1} \right]^{-1} \cdot$$

(27)

where the operation $[\cdot]^{-1}$ is defined as follows:

$$[\cdot]^{-1} = \frac{\cdot}{\|\cdot\|^2}.$$  

(28)

In (27), a problem is addressed here. In (11), $H$ and $\Sigma$ are both required to be positive definite symmetric matrices. In order to satisfy this condition, the upper or lower triangular part of $H^t$ and $\Sigma^t$ are first extracted and vectorized. After updated by (27), they are transformed into the corresponding positive definite symmetric matrices.

As a result, we can obtain a set of new variables $\Theta_{new}^t$ according to (26) and (27). Denote $b_{new}^t$ as,

$$b_{new}^t = \left[ \text{vec}(Y_{new}^t); \text{vec}(R_{new}^t); \text{vec}(s_{new}^t); \alpha_{m_{new}}^t; \right.$$  

$$\text{vec}(S_{new}^t); \text{vec}(Q_{new}^t); \sigma_{m_{new}}^t] \cdot$$

(29)

where $b_{new}^t \in R^{c \times 1}$, and $c = (3n_p - 7)(6n_p - 7) + 3n_p + 10n_s + 2$.

The iterations are repeated until,

$$e_\Theta = \|b_{new}^t - b_{old}^t\| < \rho, \quad \text{or} \quad t > \tau,$$

(30)

where $\tau$ is the maximum number of iterations. The pseudocode of the PMP-CAEM algorithm is given in Algorithm 1.

### Algorithm 1: The pseudocode of the PMP-CAEM algorithm.

1. Initialize $\Theta^0 = \{Y^0, R^0, s^0, \alpha^0, H^0, \Sigma^0, Q^0, \sigma^0\}$, $b^0$.
2. Set $\rho = 1e - 05$, $\tau = 1e + 03$.
3. $\Theta^1 \leftarrow \Theta^0$, $b^1 \leftarrow b^0$.
4. $\Theta_{old} \leftarrow \Theta^t$, $b_{old} \leftarrow b^t$.
5. $t \leftarrow 0$.
6. repeat
7. Compute $\Theta_{new}^t$ by (26) and (27),
8. Compute $e_\Theta$ by (30),
9. $\Theta_{old} \leftarrow \Theta_{old}^t$, $b_{old} \leftarrow b_{new}^t$,
10. $\Theta^t \leftarrow \Theta^t$.
11. Update $t \leftarrow t + 1$.
12. until $e_\Theta < \rho$ or $t > \tau$.

### Table 1: The numbers of frames ($n_s$) and the numbers of point tracks ($n_p$) for eleven motion capture sequences.

| Number | Sequence  | $n_s$ | $n_p$ |
|--------|-----------|-------|-------|
| 1      | walking   | 260   | 33    |
| 2      | jaws      | 240   | 91    |
| 3      | dance     | 264   | 75    |
| 4      | face1     | 74    | 37    |
| 5      | face2     | 352   | 40    |
| 6      | pickup    | 350   | 41    |
| 7      | stretch   | 370   | 41    |
| 8      | yoga      | 307   | 41    |
| 9      | drink     | 1102  | 41    |
| 10     | FRGC      | 400   | 62    |
| 11     | capoeira  | 250   | 41    |

In order to evaluate the reconstruction performance, the normalized error $\varepsilon$ of the 3D coordinates between the estimated 3D shape ($X_t$) and the ground-true 3D shape ($X_0$) is used as the performance index, i.e.,

$$\varepsilon = \frac{1}{n_s} \sum_{i=1}^{n_s} \frac{\|X_i - \tilde{X}_i\|_F}{\|X_0\|_F}.$$  

(31)

Smaller $\varepsilon$ means that the estimation is more accurate.

### 3 Experiments

#### 3.1 Experiment Datasets and Set-up

The performance of the proposed method is evaluated on eleven widely used motion sequences: walking, jaws, dance, face1, face2, pickup, stretch, yoga, drink, Face Recognition Grand Challenge (FRGC), and capoeira. These sequences are publicly available from [9], [12], [18], [21]. Note that the FRGC is a 3D facial-landmark dataset from [18], by adding random rotation and scaling to the original FRGC 2.0 database without temporal dependence [22]. For these sequences, the corresponding number of frames ($n_s$) and the number of points tracked ($n_p$) are listed in Table 1. Figure 1 shows one frame of these eleven image sequences. All simulations were conducted using MATLAB, running on an ordinary personal computer.

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$$\varepsilon = \frac{1}{n_s} \sum_{i=1}^{n_s} \frac{\|X_i - \tilde{X}_i\|_F}{\|X_0\|_F}.$$  

(31)

Smaller $\varepsilon$ means that the estimation is more accurate.

#### 3.2 Comparison to Recently Reported Results

In order to evaluate the effectiveness of the proposed method, denoted as PMP-CAEM, we compare it with several state-of-the-art NRSfM algorithms, including the well-known block matrix method (denoted as BMM) [10], the column-space-fitting method (denoted as CSF) [13], the CSF2 method [14], the procrastanean normal
distribution method (denoted as PND2) [19] and the procrustean Markov process method (denoted as PMP) [20].

Among these methods, the low-rank parameter $K$ has a significant influence on the final estimation performance for CSF, CSF2 and BMM. For a fair comparison, the parameter $K$ is successively set at $\{1, 2, \cdots, 13\}$ for these three methods. The parameter value corresponding to the smallest estimation error is selected as the approximate optimum parameter value of $K$.

**TABLE 3:** The computation runtimes (seconds) of eleven sequences without noise for the six methods.

| Sequence | CSF  | CSF2 | BMM   | PND2 | PMP  | PMP-CAEM |
|---------|------|------|-------|------|------|-----------|
| walking | 9.7  | 18.2 | 1381.8| 281.3| 237.4| 146.5     |
| jaws    | 0.1  | 18.5 | 192.4 | 629.9| 353.2| 477.2     |
| dance   | 35.5 | 34.2 | 1083.0| 531.7| 510.2| 213.7     |
| face1   | 4.5  | 4.0  | 8.8   | 34.3 | 29.5 | 13.6      |
| face2   | 6.4  | 37.3 | 45.9  | 126.0| 117.0| 73.5      |
| pickup  | 10.2 | 107.2| 134.9 | 211.2| 159.6| 102.2     |
| stretch | 12.1 | 81.0 | 1099.4| 217.4| 168.5| 139.0     |
| yoga    | 21.2 | 47.3 | 843.2 | 236.8| 123.6| 49.6      |
| drink   | 47.2 | 563.6| 1439.9| 577.6| 390.4| 138.0     |
| FRGC    | 31.1 | 32.2 | 96.1  | 318.4| 258.0| 238.1     |
| capoeira| 7.4  | 12.6 | 111.9 | 149.7| 117.3| 68.7      |

Table 2 shows the 3D reconstruction errors $\varepsilon$ on eleven sequences without noise for the six methods. In order to easily compare the performances of different algorithms, the best result and the second-best result in Table 2 are highlighted in red and blue, respectively. The reconstruction errors of PND2, PMP and PMP-CAEM are generally lower than that of other methods for most sequences. Moreover, the reconstruction errors of PMP-CAEM are close to that of PMP. Table 3 shows the computation times (seconds) of the different methods on the eleven sequences without noise. We can see that the computation runtimes of PND2, PMP and PMP-CAEM are obviously longer than that of other methods. However, the computation times of PMP-CAEM are significantly lower than that of PMP. This shows that the computation runtimes required by PMP can be greatly reduced by the use of the accelerated expectation maximization algorithm.

In order to investigate the robustness to noise, we conducted the experiments with the addition of the Gaussian noise on the original sequences. The standard deviation or level of the Gaussian noise is set as $\sigma_{\text{noise}} = a_{\text{noise}} \max_{i,j,k} \{|d_{ijk}|\}$, where the noise rate $a_{\text{noise}}$ is set at $\{0.2, 0.22, 0.24, 0.26, 0.28\}$, respectively, and $d_{ijk}$ represent the observed ground truth and the points with noise, respectively.
TABLE 2: The 3D reconstruction error $\varepsilon$ of eleven sequences without noise for six different, and the corresponding mean and standard deviation ($\mu \pm \sigma$) for each method on all the sequences.

| Sequence  | CSF   | CSF2  | BMM   | PND2  | PMP   | PMP-CAEM |
|-----------|-------|-------|-------|-------|-------|----------|
| walking   | 0.1050 | 0.0695 | 0.0805 | 0.0407 | 0.0424 | 0.0428   |
| jaws      | 0.0048 | 0.0259 | 0.1448 | 0.0272 | 0.0096 | 0.0096   |
| dance     | 0.1808 | 0.1397 | 0.1360 | 0.1247 | 0.1278 | 0.1280   |
| face1     | 0.0433 | 0.0343 | 0.0362 | 0.0215 | 0.0198 | 0.0204   |
| face2     | 0.0677 | 0.0206 | 0.0197 | 0.0150 | 0.0166 | 0.0162   |
| pickup    | 0.0170 | 0.0213 | 0.0302 | 0.0133 | 0.0127 | 0.0126   |
| stretch   | 0.0246 | 0.0219 | 0.0235 | 0.0150 | 0.0124 | 0.0123   |
| yoga      | 0.0211 | 0.0207 | 0.0223 | 0.0128 | 0.0128 | 0.0129   |
| drink     | 0.0659 | 0.0071 | 0.0149 | 0.0031 | 0.0018 | 0.0019   |
| FRGC      | 0.1909 | 0.1909 | 0.1147 | 0.0731 | 0.0727 | 0.0726   |
| capoeira  | 0.2258 | 0.3309 | 0.2544 | 0.3116 | 0.3132 | 0.3126   |
| $\mu \pm \sigma$ | 0.0803 ± 0.0103 | 0.0806 ± 0.0065 | 0.0798 ± 0.0038 | 0.0598 ± 0.0082 | 0.0584 ± 0.0085 | 0.0596 ± 0.0080 |

TABLE 4: The 3D reconstruction errors $\varepsilon$ of the six methods on eleven sequences when $a_{noise}$ is set at 0.26.

| Sequence  | CSF   | CSF2  | BMM   | PND2  | PMP   | PMP-CAEM |
|-----------|-------|-------|-------|-------|-------|----------|
| walking   | 0.5112 | 0.5137 | 0.4759 | 1.0359 | 0.4503 | 0.4130   |
| jaws      | 0.7233 | 1.6459 | 0.5616 | 0.7870 | 0.4527 | 0.4381   |
| dance     | 0.5359 | 1.9010 | 0.5149 | 1.0398 | 0.4570 | 0.4571   |
| face1     | 0.5380 | 0.4944 | 0.3627 | 0.7121 | 0.3402 | 0.3364   |
| face2     | 0.6021 | 0.6467 | 0.6493 | 0.9700 | 0.5471 | 0.5473   |
| pickup    | 0.4390 | 0.4143 | 0.4662 | 0.6883 | 0.3502 | 0.3523   |
| stretch   | 0.4696 | 0.4572 | 0.4561 | 0.7111 | 0.4365 | 0.4076   |
| yoga      | 0.4571 | 0.4393 | 0.4709 | 0.6681 | 0.2967 | 0.2940   |
| drink     | 0.3685 | 0.3548 | 0.3917 | 0.6339 | 0.3221 | 0.3218   |
| FRGC      | 0.5915 | 0.5917 | 0.4451 | 0.8351 | 0.3720 | 0.3717   |
| capoeira  | 0.5094 | 0.4957 | 0.4930 | 0.7329 | 0.4619 | 0.4604   |
| $\mu \pm \sigma$ | 0.5223 ± 0.0090 | 0.7233 ± 0.2792 | 0.4625 ± 1.0064 | 0.8013 ± 0.0221 | 0.4081 ± 0.0088 | 0.4030 ± 0.0055 |

TABLE 5: The number of iterations for the EM algorithm in PMP and PMP-CAEM when $a_{noise}$ is set at 0.26.

| Sequence  | PMP    | PMP-CAEM |
|-----------|--------|----------|
| walking   | 1000   | 273      |
| jaws      | 457    | 164      |
| dance     | 698    | 438      |
| face1     | 93     | 77       |
| face2     | 267    | 238      |
| pickup    | 816    | 678      |
| stretch   | 693    | 272      |
| yoga      | 469    | 358      |
| drink     | 1000   | 1000     |
| FRGC      | 243    | 183      |
| capoeira  | 507    | 432      |

is the $(j,k)$th elements of $D_i$, where $i = 1, \cdots, n_s$ and $j = 1, 2, 3; k = 1, \cdots, n_p$. Figure 2 shows one frame of the sequences jaws and stretch with and without noise. The symbols ‘-’ and ‘~’ represent the points of the ground truth and points with noise, respectively. We can see that the positions of the points are randomly changed when noise is added to the original data.

As an example, Table 4 shows the 3D reconstruction errors $\varepsilon$ of the six methods on the eleven sequences for the six methods when $a_{noise}$ is set at 0.26. We can see from Tables 2 and 4 that the reconstruction errors of the various algorithms are significantly increased when noises are added. The reconstruction errors of PMP and PMP-CAEM are obviously lower than that of other methods for most of the sequences. For CSF, CSF2, and BMM, a 3D shape is assumed to be composed by a linear combination of $K$ shape bases. Such a model cannot achieve a satisfactory result because the deformation and translation caused by the noise are random and irregular for the different points.

For PMP and PMP-CAEM, the smoothing constraint can suppress the partial deformation and deviation caused by noise. Different from PMP and PMP-CAEM, the smoothing constraint is not considered in the PND2 model. Therefore, the noise has a more serious effect on its final estimation results. From Table 4, it can be seen that the performance of PND2 is not yet as good as PMP and PMP-CAEM. Moreover, the reconstruction errors of PMP-CAEM are lower than that of PMP.

Table 5 shows the number of iterations for the EM algorithm used in PMP and the accelerated EM used in PMP-CAEM, when $a_{noise}$ is set at 0.26. The number of iterations of PMP-CAEM is obviously lower than that of PMP. Therefore, the accelerated expectation maximization algorithm can significantly decrease the convergence time of PMP.

Figure 3 shows the 3D reconstruction errors $\varepsilon$ of the six methods on the eleven sequences, when $a_{noise}$ is set at different values. Table 6 tabulates the correspond-
Fig. 3: The 3D reconstruction errors $\varepsilon$ of the six methods on the eleven sequences, when $a_{\text{noise}}$ is set at different values.

TABLE 6: The mean and standard deviation ($\mu \pm \sigma$) of 3D reconstruction errors $\varepsilon$ of the six methods on the eleven sequences, when $a_{\text{noise}}$ is set at different values.

| Sequence   | CSF    | CSF2   | BMM    | PND2   | PMP    | PMP-CAEM |
|------------|--------|--------|--------|--------|--------|----------|
| walking    | 0.4794 ± 0.0518 | 0.4751 ± 0.0586 | 0.4656 ± 0.0346 | 0.9535 ± 0.1349 | 0.4051 ± 0.0443 | 0.3992 ± 0.0401 |
| jaws       | 0.6522 ± 0.0992 | 1.0253 ± 0.5666 | 0.5407 ± 0.0340 | 0.7419 ± 0.0628 | 0.4179 ± 0.0511 | 0.4083 ± 0.0490 |
| dance      | 0.4680 ± 0.0653 | 1.0492 ± 0.7814 | 0.4935 ± 0.0318 | 0.8921 ± 0.1186 | 0.4335 ± 0.0434 | 0.4303 ± 0.0403 |
| face1      | 0.5039 ± 0.0530 | 0.4996 ± 0.0495 | 0.3330 ± 0.0469 | 0.6422 ± 0.0942 | 0.3144 ± 0.0409 | 0.3119 ± 0.0389 |
| face2      | 0.5727 ± 0.0451 | 0.8674 ± 0.6579 | 0.6095 ± 0.0522 | 0.9054 ± 0.1046 | 0.5332 ± 0.0239 | 0.5278 ± 0.0103 |
| pickup     | 0.4190 ± 0.0325 | 0.3899 ± 0.0351 | 0.4419 ± 0.0390 | 0.6355 ± 0.0833 | 0.3236 ± 0.0462 | 0.3259 ± 0.0482 |
| stretch    | 0.4436 ± 0.0420 | 0.4286 ± 0.0464 | 0.4294 ± 0.0323 | 0.6534 ± 0.0878 | 0.3939 ± 0.0733 | 0.3861 ± 0.0707 |
| yoga       | 0.4324 ± 0.0390 | 0.4103 ± 0.0437 | 0.4466 ± 0.0268 | 0.6148 ± 0.0825 | 0.2769 ± 0.0379 | 0.2749 ± 0.0364 |
| drink      | 0.3470 ± 0.0339 | 0.3332 ± 0.0349 | 0.3791 ± 0.0352 | 0.5842 ± 0.0782 | 0.2959 ± 0.0430 | 0.2965 ± 0.0429 |
| FRGC       | 0.5558 ± 0.0407 | 1.4037 ± 0.7356 | 0.4408 ± 0.0446 | 0.7724 ± 0.0994 | 0.3361 ± 0.0523 | 0.3399 ± 0.0511 |
| capoeira   | 0.4955 ± 0.0301 | 0.4891 ± 0.0274 | 0.4702 ± 0.0388 | 0.6884 ± 0.0707 | 0.4438 ± 0.0263 | 0.4445 ± 0.0258 |

4 Conclusions

In this paper, an accelerated PMP model, with a coherent constraint, is proposed for non-rigid structure from motion. The experimental results demonstrated that the proposed method can simultaneously decrease the estimation error and the convergence time of EM algorithm for PMP.

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