Suppression of Lepton Flavour Violation from Quantum Corrections above $M_{GUT}$

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Abstract

We study the predictions for sfermion masses and Lepton Flavour Violation (LFV) for the WMAP preferred parameter space in $b - \tau$ Yukawa-unified models with massive neutrinos. A soft term structure as predicted by an Abelian flavour symmetry combined with $SU(5)$ RGEs for scales above $M_{GUT}$, results to an efficient suppression of the off-diagonal terms in the scalar soft matrices, particularly for $m_0 < 100$ GeV. Using the WMAP bounds, this implies $35 \leq \tan\beta \leq 45$, $350$ GeV $\leq m_{1/2} \leq 1$ TeV, with the higher $\tan\beta$ values being favored. Within this framework, $SU(5)$ unification becomes compatible with the current experimental bounds, in contrast to the conventional case where the soft terms are postulated at the GUT scale.

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1 Introduction

The pattern of fermion masses and mixings is one of the most compelling mysteries in particle physics. The large hierarchies in the fermion mass matrices and the origin of mixing terms remain unclear. Neutrino oscillations render this problem even more peculiar since, data from atmospheric [1] and solar [2] neutrinos confirms the existence of neutrino oscillations with near-maximal \(\nu_\mu - \nu_\tau\) mixing and large \(\nu_e \rightarrow \nu_\mu\) mixing [3].

In recent years, several attempts to explain the observed fermion structure have been put forward in the literature. Among them, flavour symmetries are particularly appealing [4]; in these models, only third generation entries are non-zero as long as the family symmetry remains unbroken, whereas the remaining entries are generated through non-renormalizable terms after symmetry breaking by fields acquiring non-zero vacuum expectation values (vevs).

The flavour problem is particularly challenging in supersymmetric (SUSY) theories, where soft breaking terms involve off-diagonal entries and complex phases that may lead to unacceptably large Flavour Changing Neutral Currents (FCNC) and CP-violating vertices. Two popular solutions usually adopted in the literature to solve this problem are either to consider universal soft terms at the high scale [5] or to invoke some kind of alignment among the Yukawa textures and the soft terms [6]. Whichever option is taken, however, we should keep in mind that RGE evolution from the high scale down to low energies also generates additional off-diagonal contributions, since it is not possible to simultaneously diagonalise neutrino, charged lepton and slepton mass matrices [7]. These contributions also imply violation of the corresponding charged-lepton numbers [8, 9, 10, 11, 12, 13], generating process forbidden in the SM, such as \(\mu \rightarrow e\gamma\), \(\mu - e\) conversions, \(\tau \rightarrow \mu\gamma\) and \(\tau \rightarrow e\gamma\) decays.

As it turns out, the stringent bounds from LFV are hard to satisfy; in fact, it was shown that models with flavour symmetries based on \(SU(5)\) with hierarchical Yukawa textures and the lepton mixing arising mainly from the charged-lepton sector, tend to predict too large rates [14, 15]. The data from WMAP [16] and the resulting bounds on Cold Dark Matter (CDM) further constrain theoretical models and make the potential consequences of Grand Unified Theories (GUT) for Dark Matter worth exploring [17, 18, 19, 20]. Imposing Yukawa unification, as expected in GUTs, the solutions become even more predictive, with additional constraints on the model parameters [21, 22].

The purpose of this work is to study the predictions for LFV in models where \(SU(5)\) unification is combined with flavour symmetries, taking into account the RGE evolution above the GUT scale, and focusing on the WMAP preferred area presented in [18]. We show that, by postulating the mass matrices at a high scale \(M_X\) and evolving them down to \(M_{GUT}\), the pathological situation encountered in the conventional models [14, 15] may be remedied. For regions of the parameter space with a low \(m_0\), the pattern we end up with exhibits a sizeable suppression in the off-diagonal terms as compared to the textures at \(M_X\), yielding acceptable LFV predictions. We show that this is true, even in the case of maximal mixing in the charged lepton sector, which is the most dangerous one as far as LFV is concerned.

The paper is organized as follows: In Section 2 we summarise the origin of flavour violation in a generic \(SU(5)\) framework. In section 3 we discuss fermion and sfermion mass matrices in \(SU(5)\) unification with an Abelian flavour symmetry. Section 4 describes
the running procedure and the results. The conclusions are presented in Section 5.

2 Lepton Flavour Violation in SUSY-SU(5) with see-saw neutrinos

In SUSY theories, charged lepton flavour violation may be generated at the loop-level, even in models with universal soft terms at a high scale. This is also true for the MSSM, when extended with a see-saw mechanism to generate small neutrino masses [8, 11]. In this case, LFV terms are generated radiatively, since it is not possible to simultaneously diagonalise neutrino, charged lepton and slepton mass matrices [7].

In order to explain the observed lepton hierarchies by suitable Yukawa textures, we introduce flavour symmetries, which may also predict the structure of the soft mass terms. LFV processes like \( l_i \rightarrow l_j \gamma \) impose severe constraints on the allowed patterns [14, 15]. In this work we focus on the implications of massive neutrinos in SU(5) Yukawa unification with an additional \( U(1)_F \) family symmetry, including RGE effects not only below, but also above \( M_{\text{GUT}} \).

We start by considering the following SUSY \( SU(5) \) superpotential:

\[
W_X = T^T \mathcal{Y}_u^\delta T_1 H + T^T \tilde{Y}_d \tilde{T}_1 \tilde{H} + \tilde{F}_1^T \gamma_\nu^\delta S_1 H + S^T \bar{M}_R S_1,
\]

where \( \mathcal{Y}_\alpha (\alpha = u, d, \nu) \) are the Yukawa matrices for the up-type quarks, down-quarks/charged-leptons and Dirac neutrinos, respectively. \( M_R \) is the heavy Majorana mass matrix. The symbol \( \delta \) stands for diagonal, the original fields rotated as

\[
T = U_{10} T_1, \quad \tilde{F} = U_{\nu L} \tilde{F}_1, \quad S = U_{\nu R} S_1,
\]

with the rotating matrices defined as

\[
\mathcal{Y}_u = U_{10} \mathcal{Y}_u^0 U^T_{10}, \quad \mathcal{Y}_d = U_{5L}^\dagger \mathcal{Y}_d^\delta U_{5R}, \quad \mathcal{Y}_\nu = U_{\nu L}^\dagger \mathcal{Y}_\nu^\delta U_{\nu R};
\]

and

\[
\tilde{\mathcal{Y}}_d = V_{CKM}^* \mathcal{Y}_d^0 V_{E}^T.
\]

Here, \( V_{CKM} = U_{10}^T U_{5L} \) and \( V_E = U_{\nu L} U_{5R} \) denote the mixings in the quark and lepton sectors, while \( \bar{M}_R = U_{\nu R}^T M_R U_{\nu R} \).

The off-diagonal contributions to slepton mass matrices, when the superfields are rotated so that charged leptons become diagonal, can be understood through three rotations at different energy scales:

- \( M_X \). The rotations in the superpotential fields lead to the following transformation of the soft terms:

\[
m_{10}^2 = U_{10}^T m_{10}^2 U_{10} \quad \tilde{m}_5^2 = U_{\nu L}^T m_5^2 U_{\nu L}
\]

- \( M_{\text{GUT}} \). Assuming that \( SU(5) \) is broken down to the MSSM gauge group, the superpotential becomes

\[
W_{\text{MSSM}} = Q^T \mathcal{Y}_u^\delta U H_2 + Q^T (V_{CKM}^* \mathcal{Y}_d^\delta) D H_2 + L^T (V_{E}^* \mathcal{Y}_\nu^\delta) E H_2 + L^T \mathcal{Y}_\nu^\delta S H_2 + S^T \bar{M}_R S
\]

3
where we have absorbed the matrices $V^*_E$ and $V^*_CKM$ in the definitions of the superfields $E$ and $D$ respectively. The scalar soft masses then become:

$$m^2_E = V^*_CKM$$

$$m^2 = m^2_{\nu},$$

$$m^2_Q = m^2_{U} = \bar{m}^2_{\nu},$$

$$m^2_D = V^*_E m^2_{\nu} V_E,$$

(7)

We write $m^2_D$ and $m^2_E$ in the following way:

$$m^2_D = V^*_E m^2_{\nu} V_E$$

$$= U^\dagger_{5R} U_{\nu L}^\dagger m^2_{\nu} U_{\nu L} U^\dagger_{5R}$$

$$\simeq U^\dagger_{5R} m^2_{\nu} U_{5R}$$

$$m^2_E = V^*_CKM m^2_{\nu} V_{CKM}$$

$$= U^\dagger_{5L} U^{\dagger}_{10} m^2_{\nu} U_{10} U^{\dagger}_{10} U_{5L}$$

$$\simeq U^\dagger_{5L} m^2_{\nu} U_{5L}$$

(8)

where the last step holds if radiative corrections to the rotation matrices are neglected.

• $M_N$. $M_N$ is the scale at which the heavy right handed neutrinos decouple. Below this scale, the particle content is just the one of the MSSM complemented with the neutrino mass operator resulting from the see-saw mechanism. Consequently, the superpotential can be written in a basis where the charged-lepton mass matrix becomes diagonal and the left slepton mass matrix becomes

$$\bar{m}^2_L = V^*_E m^2_L V_E \simeq U^\dagger_{5R} m^2_{\nu} U_{5R}$$

(9)

RGE effects play a significant role in the calculation of flavour-violating processes. Even in case of universal soft terms at $M_X$, RGE runs between $M_X$ and $M_{GUT}$ (arising mainly through superpotential terms of the form $\bar{E}UH$ where $H$ is a colour-triplet Higgs field) give rise to one-loop diagrams that also renormalise the right-handed slepton masses (which in the CMSSM would remain to a large extent diagonal). In the leading-logarithmic approximation these corrections are given by [23]

$$ (m^2_L)_{ij} \simeq -\frac{3}{8\pi^2} \sum_v V^{3i}_{v3} V^{3j}_{v3} V_{CKM}^* m^2_0 + a^2_0 \log \frac{M_X}{M_{GUT}} $$

(10)

for $i \neq j$, and, as we mentioned, are suppressed due to the smallness of $V_{CKM}$; this holds in the minimal supersymmetric $SU(5)$, since in extensions of the theory this mixing may be further amplified [24]. On the contrary, runs from $M_{GUT} \rightarrow M_N$ are crucial. In the leading-logarithmic approximation, the non-universal renormalization of the soft supersymmetry-breaking scalar masses is given by

$$ (m^2_L)_{ij} \simeq -\frac{1}{8\pi^2} \left( \sum_v V^{2i}_{v3} V^{2j}_{v3} \log \frac{M_X}{M_{\nu3}} + \sum_v V^{*2i}_{v3} V^{*2j}_{v3} \log \frac{M_X}{M_{v2}} \right) (3m^2_0 + a^2_0) $$

(11)

implying that the corresponding corrections to left-handed slepton masses are proportional to $V_E$ (the Dirac neutrino mixing matrix in the basis where the $d$-quark and charged-lepton masses are diagonal). In this approach, non-universality in the soft supersymmetry-breaking left-slepton masses is much larger than the one in the right-slepton masses.
3 SU(5) textures

Having defined the general framework, the next step consists of summarising SU(5) Yukawa textures that match the fermion data and may also predict the pattern of soft terms to be expected. The mass matrices are constructed by looking at the field content of SU(5) representations, namely: three families of \((Q, u^c, e^c)\), three families of \((L, d^c)\), \(\bar{E}\) representations, and heavy right-handed neutrinos in singlet representations. This model has therefore the following properties: (i) the up-quark mass matrix is symmetric, and (ii) the charged-lepton mass matrix is the transpose of the down-quark mass matrix, which relates the mixing of the left-handed leptons to that of the right-handed down-type quarks. Since the CKM mixing in the quark sector is due to a mismatch between the mixing of the left-handed up- and down-type quarks, it is independent of mixing in the lepton sector, easily reconciling the large atmospheric neutrino mixing angle with the observed small \(V_{CKM}\) mixing. Following the \(U(1)_F\) charge assignment in \([25]\), the Yukawa matrices have the form

\[
\mathcal{Y}_u \propto \begin{pmatrix} \varepsilon^6 & \varepsilon^5 & \varepsilon^3 \\ \varepsilon^5 & \varepsilon^4 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{pmatrix}, \quad \mathcal{Y}_e^T \propto \mathcal{Y}_d \propto \begin{pmatrix} \varepsilon^4 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon & 1 & 1 \end{pmatrix}, \quad \mathcal{Y}_\nu \propto \begin{pmatrix} \varepsilon^1 & \varepsilon^2 & \varepsilon^3 \\ \varepsilon^2 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^3 & \varepsilon^3 & 1 \end{pmatrix}
\]

(12)

where \(n_i\) stand for the heavy Majorana neutrino charges. As discussed in the Introduction, we will assume that the entire lepton mixing is arising from the charged-lepton sector, which is potentially the most dangerous case as far as LFV is concerned.

The rotation matrices that diagonalise \(\mathcal{Y}_u\) and \(\mathcal{Y}_d^T \propto \mathcal{Y}_d\) are

\[
U_{10} = \begin{pmatrix} -1 + \frac{\varepsilon^2}{2} & \varepsilon & 0 \\ -\varepsilon & -1 + \frac{\varepsilon^2}{2} & \varepsilon^2 \\ \varepsilon^2 & \varepsilon & 1 \end{pmatrix}, \quad U_{10} = \begin{pmatrix} -1 + \frac{\varepsilon^2}{2} & \varepsilon & 0 \\ -\varepsilon & -1 + \frac{\varepsilon^2}{2} & \varepsilon^2 \\ \varepsilon^2 & \varepsilon & 1 \end{pmatrix}
\]

(13)

\[
U_{5L} = \begin{pmatrix} -1 + \frac{\varepsilon^2}{2} & \varepsilon & 0 \\ -\varepsilon & -1 + \frac{\varepsilon^2}{2} & \varepsilon^2 \\ \varepsilon^2 & \varepsilon & 1 \end{pmatrix}, \quad U_{5R} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} - \frac{\varepsilon}{2\sqrt{2}} & \frac{1}{\sqrt{2}} - \frac{\varepsilon}{2\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} + \frac{\varepsilon}{2\sqrt{2}} & \frac{1}{\sqrt{2}} + \frac{\varepsilon}{2\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}
\]

(14)

While there is no unique choice of the right handed neutrino charges \(n_1, n_2, n_3\) (several choices may lead to correct low energy neutrino data) representative choices can be made, and among the simplest patterns is the one provided by the assignment \(\{n_1, n_2, n_3\} = \{1, 1, 1\}\). In this case,

\[
V_E = U_{\nu L}^\dagger U_{5R} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} + \frac{\varepsilon}{2\sqrt{2}} & \frac{1}{2} + \frac{\varepsilon}{2\sqrt{2}} \\ -\frac{1}{\sqrt{2}} + \frac{\varepsilon}{2} & \frac{1}{2} + \frac{\varepsilon}{2} & -\frac{1}{2} + \frac{\varepsilon}{2\sqrt{2}} \\ \frac{1}{2} + \frac{\varepsilon}{2} & -\frac{1}{2} + \frac{\varepsilon}{2} & \frac{1}{2} - \frac{\varepsilon}{2\sqrt{2}} \end{pmatrix}
\]

(15)

where \(U_{\nu L}\) is the left rotation matrix for the Dirac neutrino sector.

In addition, flavour symmetries generally imply non-universal soft terms \([13]\), since the structure of the soft terms is linked to the family charges. For the Yukawa textures in
Eq. (12) the soft mass matrices $m_{10}^2$ and $m_5^2$ become

$$m_{10}^2 \propto \begin{pmatrix} 1 & \varepsilon & \varepsilon^3 \\ \varepsilon & 1 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{pmatrix} m_0^2, \quad m_5^2 \propto \begin{pmatrix} 1 & \varepsilon & \varepsilon \\ \varepsilon & 1 & 1 \\ \varepsilon & 1 & 1 \end{pmatrix} m_0^2,$$

(16)

The diagonalizations performed on the superfields also influence the soft mass terms, thus we must rotate the textures accordingly:

$$\bar{m}_{10}^2 = \begin{pmatrix} 1 & \varepsilon & \varepsilon^3 \\ \varepsilon & 1 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{pmatrix} m_0^2, \quad \bar{m}_5^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & \varepsilon \\ 0 & \varepsilon & 1 \end{pmatrix} m_0^2$$

(17)

In what follows, we will analyze the predictions of the above textures for on LFV processes of the type $l_i \rightarrow l_j + \gamma$, considering that all textures initially arise at a scale $M_X > M_{\text{GUT}}$.

4 RGE runs and results

Let us briefly discuss the running procedure. We use a top-down approach, the scale $M_X$ being the starting point. At this scale, both the Yukawa textures and the soft mass matrices are determined by the family symmetry charges. We evolve the 3rd generation parameters down to $M_{\text{GUT}}$ using the $SU(5)$ RGEs. In this way, both the $V_{\text{CKM}}$ and the $V_{E}$ mixing matrices are predicted. Then, we further evolve the corresponding RGEs (including the right-handed neutrino mass scale $M_N$ and the SUSY threshold corrections) down to the electro-weak scale. At $M_Z$ we impose the experimental constraints on the gauge couplings, as well as acceptable fermion masses and mixings. From this point, we employ a bottom-up approach to evolve the RGEs, using the experimental constraints, up to the GUT scale (properly re-obtaining the SUSY scale and introducing the right-handed neutrino modes). At the GUT scale we end up with $V_{\text{CKM}}$ and $V_{E}$ that are to be compared with the ones computed in the top-down method. Such a comparison allows us to extract information about the off-diagonal soft terms at the high scale $M_X$.

As in Ref. [18] the values we use are $M_X = 2 \cdot 10^{17}$ GeV, $M_N = 3 \cdot 10^{14}$ GeV. The coupling $\lambda_{\nu_3}$ is determined such that $m_{\nu_3} \sim 0.05$ eV; $m_b(M_Z) = 2.92$ GeV and $\alpha_s = 0.1172$. The evaluation of the LFV observables is done by performing a full diagonalization of the slepton mass matrices (for instance, see [8]), inserting the full rotation matrices in the lepton-slepton-gaugino vertices and summing over all the mass eigenstates of the exchanged particles. Soft terms are computed in the basis where the charged leptons are diagonal. Our results agree with other updated estimates of the branching ratios, such as those given in Ref. [26].

4.1 Runs above $M_{\text{GUT}}$

The introduction of a non-trivial flavour structure for the slepton soft terms at $M_{\text{GUT}}$, as predicted by the family symmetry that also generates Yukawa couplings, typically
Figure 1: Prediction for the charged-lepton flavour violating branching ratios showing the difference of taking either $M_X$ or $M_{GUT}$ as the starting point of the runs.

results to a large violation of the bounds on $l_j \rightarrow l_i \gamma$ \cite{14,15}. This picture may be remedied by taking into account RGE effects from a scale $M_X > M_{GUT}$. In this case, the cosmological requirement of having a neutral particle as the LSP imposes low values on $m_0$, such that $m_{\tilde{\tau}} > m_\chi$ \cite{17,18,19} (diagonal terms in the soft mass matrices have a large RGE growth, while non-diagonal elements remain almost unaffected by the runs). Thus, even assuming non-diagonal soft terms with matrix elements of the same order of magnitude at $M_X$, the corresponding matrix at $M_{GUT}$ exhibits dominant diagonal elements. To some extent, the RGE effect is similar to the action of closing an umbrella: the general non-universal soft terms at $M_X$ resemble an open umbrella that approaches a diagonal matrix at the GUT scale.

In Fig. 1 we show the differences between the following: i) SU(5) RGE evolution of the soft terms from a high scale $M_X$ down to $M_{GUT}$ and then to the MSSM with see-saw neutrinos (solid lines), and ii) Soft SUSY breaking terms given at $M_{GUT}$ and then the MSSM with see-saw neutrinos (dash-lines). In case ii) we stop the lines at the value of $m_0$ below which $m_{\tilde{\tau}}$ becomes the LSP. In contrast, $m_0$ can even vanish at $M_X$ in case i). The textures and soft terms we use are similar to Ref. \cite{15}. However, unlike these authors, we decouple the right-handed neutrinos below $M_{GUT}$. As a result, the predicted BR’s do not vanish in the limit $m_0 = 0$. We also observe the presence of one peak for each decay; the origin of such peaks can be traced back to the cancellations coming from the RR sector, in agreement with \cite{26}.

The advantageous feature of runs above the GUT scale relies on the increase of the mass of the lightest stau, such that the condition $m_\chi < m_{\tilde{\tau}}$ is achieved even at low values of $m_0$. These values are sufficiently low to predict rates for charged lepton violation within the current experimental bounds and a relic density on the WMAP range \cite{17,18}.

We can provide an explicit example of the growth of the diagonal terms of the slepton
mass matrix in models with interesting predictions for both LFV and $\Omega h^2$. Let us consider the $0 < m_0 < 100$ GeV region. In the area of the parameter space where the WMAP bounds are satisfied due to $\tau - \chi$ Co-annihilations, we find that $m_{1/2}$ is essentially a linear function of $m_0$, $m_{1/2} \sim a_1^1 + a_2^1 m_0$, where $i$ runs over the multiplets. Taking into account that the radiative corrections to the off-diagonal entries of the soft mass matrices are subdominant as compared with those of the diagonal ones, these diagonal elements can be expressed as follows:

$$m_{S_i}^2 \simeq C_i^2 (m_0) m_0^2,$$  \hspace{1cm} (18)

where we have defined

$$C_i^2 (m_0) \equiv \frac{144}{20 \pi \alpha_5} \left( (\frac{a_i^1}{m_0})^2 + 2 a_i^1 a_i^2 \right) \ln \left( \frac{M_X}{M_{GUT}} \right)$$  \hspace{1cm} (19)

and $S_i$ stands for the supermultiplets 10 and $\bar{5}$. As stated, Eq.(18) implies a large enhancement only for the diagonal entries of the soft matrices, further suppressing the off-diagonal elements. It turns out indeed that for values of $m_0 \simeq 60 - 80$ GeV at $M_X$ such an enhancement at the GUT scale is as large as $\simeq 100$. As a consequence, the soft mass matrices $\bar{m}_{10}^2$ and $\bar{m}_5^2$ at GUT scale read as

$$\bar{m}_{10}^2 = \begin{pmatrix} 1 & \varepsilon^3 \\ \varepsilon^3 & \varepsilon^5 \end{pmatrix} C^2 (m_0) m_0^2,$$

$$\bar{m}_5^2 = \begin{pmatrix} 0 & 0 \\ 0 & \varepsilon^3 \end{pmatrix} C^2 (m_0) m_0^2$$  \hspace{1cm} (20)

clearly exhibiting the suppression on the off-diagonal terms (as compared with textures (17)).

Before going through the main results, it will be instructive to analyse the dependence of the LFV rates on the universal soft parameter $A_0$. In Fig. 2 we used again the textures and soft terms of Ref.[15] with the first set of right-handed neutrino charges.

![Figure 2: Variation of the charged-lepton flavour violating branching ratio for $\mu \rightarrow e\gamma$ with $A_0$.](image-url)
discussed above. We observe that, for fixed $m_0$ and $M_{1/2}$, the LFV rates yield unacceptable predictions beyond some $A_0$, hence further constraining the allowed parameter space (note that the allowed range for $m_0$ increases with $\tan \beta$). Recall that the upper limit on $A_0$ arises from the appearance of tachyonic soft masses.

4.2 Runs below $M_{\text{GUT}}$ and LFV rates

An immediate question is how sensitive the results are upon variations of $m_0$. As shown in Fig. 3 the applied constraints imply that solutions only exist between $\tan \beta \simeq 35$ and $\tan \beta \simeq 45$. Naturally, smaller values of $m_0$ lead to an enhancement of the allowed parameter space (while eq. 19 indicates how a higher $M_X$ enhances the allowed values of $m_0$). Furthermore, small values of $m_0$ become favoured once LFV processes rates are taking under consideration and the “umbrella effect” emerges. In particular, it can be seen that no reliable parameter combination survives above $m_0 \simeq 150$ GeV.

Figure 3: We plot the allowed parameter space resulting from applying the constraints, as explained in the text, for $m_0 = 0, 50, 100, 150$ (upper plot). The BR’s in terms of $\tan \beta$ computed all along the allowed parameter space is also plotted (lower three plots).
In the following, we shall show how the textures defined in the previous section yield acceptable charged-LFV rates once the umbrella effect is considered. As observed from Figs. 4 the predictions for the branching ratios for $\ell_i \rightarrow \ell_j + \gamma$ decays lie within the current experimental bounds for properly chosen parameters.

We should stress that the depicted ranges for both $m_0$ and $M_{1/2}$ are the cosmologically preferred parameter space, as found in [18]. We can see that the case $\tan \beta = 35$ is ruled out, as there is no overlapping region for the three decays. This is because of the RR sector-induced cancellations mentioned above, which arises for much larger values of $m_0$ for $\tau \rightarrow e\gamma$ than for the other two processes. The case $\tan \beta = 40$ does possess a common area for $20 < m_0 < 50$ GeV. However, such an area lies outside the cosmologically preferred region, as shown on the left panel of Fig. 5. Thus, we conclude that the case $\tan \beta = 40$ is only marginally allowed. Finally, for $\tan \beta = 45$
the whole range for $m_0 < 100$ GeV is allowed, when suitable values for $M_{1/2}, A_0$ are chosen. Moreover, as shown on the right panel of Fig. 5 there exists an overlapping region when considering the cosmologically favoured parameter space (values of any parameters involved in these plots ($m_0, M_{1/2}, A_0$) beyond the ranges shown lead to tachyonic soft masses (see also comment before Fig. 2)). Thus, GUT runs efficiently suppress the off-diagonal entries, yielding charged-LFV rates that render the $SU(5)$ model compatible with current experimental bounds.

Figure 5: Allowed parameter space when both LFV and cosmological constraints are taken into account for $\tan \beta = 40$ and 45.

5 Conclusions

In this work, we have shown that $SU(5)$ runs above the GUT scale, naturally suppress the off-diagonal entries of the soft matrices, through what has been called the umbrella effect, leading to a nearly flavour-independent contribution for $m_0 < 100$ GeV. Such low values of $m_0$ are discarded if $SU(5)$ runs are not taken into account, since in this case the lightest stau becomes the LSP. Within this framework, $SU(5)$ runs lead to acceptable LFV rates for the cosmologically preferred parameter space found in [18], favoring values of $\tan \beta$ around 45. Significant deviations from this value, in either way, are harder to reconcile with cosmological data.

In order to illustrate the above, we have studied LFV predictions in the case that conventional $SU(5)$ is enhanced by an Abelian family symmetry. This is in fact one of the potentially most dangerous scenarios as far as charged-lepton flavour violation is concerned, particularly in the simple realisations where lepton mixing (at the GUT multiplet basis) is dominated by the charged lepton sector. Our results indicate that even in this case, the umbrella effect leads to suppressions to LFV rates, leading to a very significant enhancement of the available parameter space, as compared to the conventional schemes, with runs below $M_{GUT}$.

Let us say a few words on future perspectives. Our next step is a more elaborate analysis of the umbrella effect, also including flavour violating processes from squark mixing. It would also be desirable to make a comparative analysis with other GUT
theories, including left-right symmetric models, which can potentially lead to further enhancements of the allowed parameter space. A final point to address would be model-dependent features that could depend on the details of the heavy Majorana neutrino sector. In this respect, flavour violating decays may shed some light to the mass patterns of right handed neutrinos, which are not easily constrained by the fermion data alone.

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Appendix

In this appendix we summarise the RGEs that are most relevant for the purposes of the work addressed in this paper. For runs above the GUT scale the equations involving the Yukawa couplings and the soft mass terms corresponding to the 10 and \( \bar{5} \) representations of \( SU(5) \), for the 3\(^{rd} \) generation, take the form \[23\]

\[
16\pi^2 \frac{d\lambda_N}{dt} = \left[-\frac{48}{5} g_5^2 + 7\lambda_N^2 + 3\lambda_t^2 + 4\lambda_b^2\right] \lambda_N ,
\]

\[
16\pi^2 \frac{d\lambda_d}{dt} = \left[-\frac{84}{5} g_5^2 + 10\lambda_d^2 + 3\lambda_t^2 + \lambda_N^2\right] \lambda_d ,
\]

\[
16\pi^2 \frac{d\lambda_b}{dt} = \left[-\frac{96}{5} g_5^2 + 9\lambda_t^2 + 4\lambda_d^2 + \lambda_N^2\right] \lambda_t ,
\]

\[
16\pi^2 \frac{dm_{10}^2}{dt} = -\frac{144}{5} g_5^2 M_5^2 + (12\lambda_t^2 + 4\lambda_d^2) m_{10}^2 + 4 \left[(m_5^2 + m_{10}^2) \lambda_d^2 + A_d^2\right] + 6 \left(\lambda_t^2 m_h^2 + A_t^2\right) ,
\]

\[
16\pi^2 \frac{dm_{\bar{5}}^2}{dt} = -\frac{96}{5} g_5^2 M_5^2 + 2 \left(4\lambda_t^2 + \lambda_N^2\right) m_{\bar{5}}^2 + 8 \left[(m_{10}^2 + m_{\bar{5}}^2) \lambda_d^2 + A_d^2\right] + 2 \left(\lambda_N^2 m_h^2 + \lambda_N^2 m_{\bar{5}}^2 + A_N^2\right)
\]

For runs from \( M_{GUT} \) to \( M_N \), the equations for the Yukawa matrices are \[27\]:

\[
16\pi^2 \frac{d\lambda_N}{dt} = -\left[\left(\frac{3}{5} g_1^2 + 3g_2^2\right) I_3 - (4\lambda_N^2 + 3\lambda_t^2 + \lambda_b^2)\right] \lambda_N ,
\]

\[
16\pi^2 \frac{d\lambda_t}{dt} = -\left[\left(\frac{9}{5} g_1^2 + 3g_2^2\right) I_3 - (4\lambda_t^2 + 3\lambda_d^2 + \lambda_N^2)\right] \lambda_t .
\]
\[ 16\pi^2 \frac{d\lambda_t}{dt} = - \left[ \left( \frac{13}{5} g_1^2 + 3 g_2^2 + \frac{16}{3} g_3^2 \right) I_3 - (6\lambda_t^2 + \lambda_\nu^2) \right] \lambda_t \] (28)

Since the neutrino has no coupling to the bottom quark, the Yukawa matrix corresponding to the latter remains unchanged with respect to the MSSM case.

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