Research on Modeling and Control of Tilting Three-rotor UAV

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Abstract. The tilting trirotal UAV is a new type of aircraft that combines the characteristics of fixed-wing UAVS and multi-rotor UAVS. To study and solve the control problems of tilting rotor UAVs, This paper focuses on the mechanical analysis of the rotor mode, and establishes the mathematical model. Simulation of rotor flight mode in MATLAB/SIMULINK environment, provides strategies for the design of tilting rotor UAV control systems.

1. Introduction
The concept of tilting rotorcraft was proposed around the 1940s. As a new type of aircraft in the context of war, its advantages of high-speed flight and vertical take-off & landing are unmatched by traditional fixed-wing aircraft and helicopters. The earliest tilt-rotor aircraft was developed by Transcendental Aircraft Company to surpass 1-G. During the flight test, the rotor was converted from vertical to 70°. It was found that the aircraft was unstable due to airflow interference [1]. From the mid-1980s to the 1990s, Bell and Boeing Helicopter developed the Osprey V-22, which has the advantages of high speed, long-distance, heavy load and low fuel consumption [2]. Throughout the 1990s, UAVs have become more powerful and are considered semi-automatic power multipliers. Bell's tilt-rotor UAV "Eagle Eye" was used by the Coast Guard for search and reconnaissance. In 2002, the Korea Air-Conditioning Research Institute developed the Smart Tilt-rotor UAV, which has a VTOL flight envelope and high cruising speed [3]. In 2010, the tilting three-rotor Panther UAV developed by Israel Aerospace Industries (IAI) was unveiled. In 2015, South Korea released a new tilt-rotor UAV TR-60 and plans to mass-produce it in 2024 [4]. At present, the UAVs that can complete the vertical take-off and landing are mainly divided into tilting wing type, tilting rotor type, tailstock type and hybrid type, the tilting three-rotor UAV adopts a single power system to avoid Redundancy reduces the take-off weight, making the conversion flexible and more practical. Flight mechanics modeling and controller design are the key to realize the function of tilting rotor UAV. This paper combines the UAV platform built in the laboratory, as shown in Figure 1, analyzes the mechanical characteristics of its rotor mode ,establishes the model and designs a controller in three-rotor mode.
2. Analysis of mechanical properties
Because of the different flight modes, the tilt-rotor UAVs have different force analysis in different modes. In the multi-rotor mode, the balance is mainly achieved by the tension and gravity of the rotor motor. In fixed-wing mode, the power of the aircraft is mainly derived from the pressure difference between the upper and lower surfaces of the wing. In the transition mode, the aircraft is subjected to both the rotor tension and the aerodynamic force of the wing. The forward speed increases with the rotation of the rotor, and the aerodynamic force also changes. Accurate mechanical analysis is the basis of dynamic modeling and control design. The following focuses on the mechanical properties of the three-rotor mode. For the convenience of discussion, the actuators are defined as follows. The motor installed in the right tilting mechanism is M1, the steering gear is S1, the motor and steering gear mounted on the left side of the tilting mechanism are M2 and S2, and the tailstock motor is M3. The steering gear of the right aileron is S3, the steering gear of the left aileron is S4, and the steering gears on the right and left sides of the control type of the V-shaped tail are S5 and S6, as is shown in Fig. 2.

2.1. Mechanical analysis under three-rotor mode
According to the knowledge of rotor analysis and leaf theory, the external forces and moments of the rotor in the three-rotor flight mode have lift $F$, anti-torsion torque $Q$, roll moment $L$ and resistance $D$ [5]. The specific formulas are as follows:

$$F = \frac{1}{2} \rho AC_T R^2 \omega^2$$  \hspace{1cm} (1)

$$Q = \frac{1}{2} \rho AC_Q R^2 \omega^2$$  \hspace{1cm} (2)
\[ L = \frac{1}{2} \rho A C_L R^2 \omega^2 \]  

\[ D = \frac{1}{2} \rho A C_D R^2 \omega^2 \]  

Where \( \rho, A, R, \omega \) are air density, paddle area, blade radius, and propeller angular velocity, respectively, \( C_L, C_Q, C_I, C_D \) are the lift coefficient, torque coefficient, roll moment coefficient, and drag coefficient of the rotor, respectively. Their expressions are as follows:

\[ \frac{C_L}{2\sigma a} = \left( \frac{1}{6} + \frac{1}{4} \mu^2 \right) \theta_s - \frac{1}{4} \lambda \]  

\[ \frac{C_Q}{2\sigma a} = \frac{1}{8h} (1 + \mu^2) \overline{C_d} + \lambda \left( \frac{1}{6} \theta_s - \frac{1}{4} \lambda \right) \]  

\[ \frac{C_I}{2\sigma a} = \mu \left( \frac{1}{6} \theta_s - \frac{1}{8} \lambda \right) \]  

\[ \frac{C_D}{2\sigma a} = \frac{1}{4h} \mu \overline{C_d} + \frac{1}{4} \lambda \mu \theta_s \]  

Where \( a \) is a constant, \( \overline{C_d} \) is the average of the backward force coefficients of the primitives, and \( \sigma \) is the rotor solidity.

In the three-rotor flight mode, the lift and torque generated by the rotor are ignored, and the resulting lift and torque can be expressed as [6]:

\[ F = k_1 \omega^2 \]  

\[ Q = k_2 \omega^2 \]  

Let the unit space vector of the rotor generating the pulling force in the body coordinate system be \( e = [0 \ 0 \ -1]^T \). The angular velocities of the three rotor motors are \( \omega_1, \omega_2 \) and \( \omega_3, \alpha_1 \) represents the angle of rotation of the tilting motor, Then the total tension \( F_r \) of the three rotors in the body coordinate system is:

\[ F_r = F_1 + F_2 + F_3 = \begin{bmatrix} -k_1 \omega_1^2 \sin \alpha_1 - k_1 \omega_2^2 \sin \alpha_2 \\ 0 \\ -k_1 \omega_1^2 \cos \alpha_1 - k_1 \omega_2^2 \cos \alpha_2 - k_1 \omega_3^2 \end{bmatrix} \]  

The torque \( Q_r \) of the three rotors in the body coordinate system is:

\[ Q_r = r_1 \times F_1 + r_2 \times F_2 + r_3 \times F_3 + Q_1 + Q_2 + Q_3 \]

\[ = k_1 \omega_1^2 \begin{bmatrix} -r_{y1} \cos \alpha_1 \\ -r_{y1} \sin \alpha_1 \\ r_{x1} \sin \alpha_1 \end{bmatrix} + k_1 \omega_2^2 \begin{bmatrix} -r_{y2} \cos \alpha_2 \\ -r_{y2} \sin \alpha_2 \\ r_{x2} \cos \alpha_2 \end{bmatrix} + \\ k_1 \omega_3^2 \begin{bmatrix} -r_{y3} \\ r_{x3} \cos \alpha_3 \\ 0 \end{bmatrix} - k_2 \omega_1^2 \begin{bmatrix} -\sin \alpha_1 \\ 0 \\ -\cos \alpha_1 \end{bmatrix} - k_2 \omega_2^2 \begin{bmatrix} -\sin \alpha_2 \\ 0 \\ -\cos \alpha_2 \end{bmatrix} - k_2 \omega_3^2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \]  

\[ (12) \]
2.2. Gyro Torque Analysis in Three-Rotor Mode

When the rotor on the front side of the wing rotates at a high speed around the axis and tilts under the action of the steering gear, the motor generates a gyro moment of the impedance rotating shaft. The gyro moment is also generated due to the rotation of the UAV body at an angular velocity \( \omega = [p \ q \ r]^T \). In order to better cancel the torque, the two motors rotate in opposite directions. According to the gyro torque calculation formula:

\[
Q_{gM1} = J_{M1} \omega_1 \times (\omega + \omega_{s1}) = J_{M1} \omega_1 \begin{bmatrix} (q + \omega_{s1}) \cos \alpha_1 \\
p \cos \alpha_1 + r \sin \alpha_1 \\
-(q + \omega_{s1}) \sin \alpha_1 \end{bmatrix} \\
Q_{gM2} = J_{M2} \omega_2 \times (\omega + \omega_{s2}) = J_{M2} \omega_2 \begin{bmatrix} -(q + \omega_{s2}) \cos \alpha_2 \\
p \cos \alpha_2 + r \sin \alpha_2 \\
(q + \omega_{s2}) \sin \alpha_2 \end{bmatrix} \tag{13, 14}
\]

Where \( J_{M1} \) represents the moment of inertia of the rotating part, \( \omega \) represents the angular velocity of the UAV rotating around the body coordinate system, \( \omega_1, \omega_2, \omega_3 \) are the angular velocities of the three motors, \( \omega_{s1} \) represents the tilting angular velocity of the tilting motor.

Then the total gyro moment of the three rotors is:

\[
Q_{gM} = J_{M1} \omega_1 \begin{bmatrix} (q + \omega_{s1}) \cos \alpha_1 \\
p \cos \alpha_1 + r \sin \alpha_1 \\
-(q + \omega_{s1}) \sin \alpha_1 \end{bmatrix} + J_{M2} \omega_2 \begin{bmatrix} -(q + \omega_{s2}) \cos \alpha_2 \\
p \cos \alpha_2 + r \sin \alpha_2 \\
(q + \omega_{s2}) \sin \alpha_2 \end{bmatrix} + J_{M3} \omega_3 \begin{bmatrix} q \\
p \\
0 \end{bmatrix} \tag{15}
\]

In the three-rotor mode, although the aircraft has a speed with the surrounding air, the flight speed in this mode is low and the aerodynamic force is much smaller than the pulling force of the motor, so the aerodynamics are ignored when we consider this mode of flight.

3. Dynamics modeling

According to the flight principle and mechanical characteristics of the UAV, the Newton-Eulerian method is used to derive the mathematical model of the three-turn rotary rotor UAV. Establishing a suitable coordinate system description vector can obtain a simple mathematical model. The coordinate system of the tilting rotor body described in this paper mainly includes: ground coordinate g system, body coordinate b system, airflow coordinate a system, rotor coordinate r system, as is shown in the figure. 3. The attitude of the UAV is represented by three Euler angles: roll \( \phi \), yaw \( \psi \), pitch \( \theta \), The control system controls the UAV through the attitude angle. The attitude angle is based on the ground coordinate system, and the accelerometer gyroscope takes the body coordinate system as the reference object. The transition matrix of the ground coordinate system to the body coordinate system:

\[
R^b_g(\phi, \theta, \psi) = \begin{bmatrix}
c \theta c \psi & c \theta s \psi & -s \theta \\
s \theta c \psi s \theta - s \psi c \theta & s \theta s \psi s \theta + c \psi c \theta & c \theta s \theta \\
s \theta c \psi c \theta + s \psi s \theta & s \theta s \psi c \theta - c \psi s \theta & c \theta c \theta
\end{bmatrix} \tag{16}
\]

Where \( c \) stands for cos and \( s \) stands for sin.
Make the following assumptions: (1) The UAV is a rigid body and the mass is constant; (2) Assume that the ground is the inertial reference system, the ground coordinates are the inertial coordinates; (3) It is assumed that the gravity acceleration does not change with the altitude of the flight; (4) The ObXbZb plane of the body coordinate system is the aircraft symmetry plane and the geometry and mass distribution of the UAV symmetry, \( J_{\phi \theta \psi} = J_{\theta \phi \psi} = J_{\theta \psi \phi} = J_{\psi \theta \phi} = 0 \).

According to the relationship between the Euler angle of the UAV and the angular velocity of rotation, the conversion relationship between the body coordinate system and the ground coordinate system, the angular motion equation of the UAV motion can be obtained:

\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} =
\begin{bmatrix}
1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi \sec \theta & \cos \phi \sec \theta
\end{bmatrix}
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
\] (17)

The line motion equation of the UAV is:

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix} =
\begin{bmatrix}
c \phi \psi & s \phi \theta \psi - c \theta \psi & -c \phi \psi - c \theta \phi \psi + c \theta \phi \psi \\
c \phi \theta \psi & s \phi \phi \psi + c \theta \psi & -s \phi \psi + c \theta \phi \psi + c \theta \phi \psi \\
-s \phi & c \theta \\
c \phi \theta & s \phi \phi & c \theta
\end{bmatrix}
\begin{bmatrix}
u \\
v \\
w
\end{bmatrix}
\] (18)

Where \( c \) stands for \( \cos \) and \( s \) stands for \( \sin \). \( \omega = [p \ q \ r]^T \) represents the aircraft angular velocity vector. \( V = [u \ v \ w]^T \) represents the velocity vector of the aircraft in the body coordinate system.

4. Controller design
The parameters such as the Euler angle of the drone are obtained by the sensing device such as the IMU and are filtered. In order to reduce the complexity of the controller model, it is common to linearize the hover state for a six-degree-of-freedom nonlinear model [7]. For the analysis of the tilting three-rotor UAV flight control system, the control of the aircraft is divided into an inner speed ring and an outer position ring. The attitude controller is divided into an external angle controller and an internal angular velocity controller. The established model is divided into PID controller implementation, control quantity allocation and drone model part. The initial attitude angles of the three channels are all 0.5 rad simulation results as shown in the figure4:
Conclusion
In this paper, the mechanical characteristics of the three-rotor flight mode are designed. The mathematical model of the aircraft is established. The dual-loop PID controller of the rotor flight mode is designed and the control requirements are met.

References
[1] Foster M. Evolution of Tiltrotor Aircraft [J]. Aiaa Journal, 2013.
[2] Bolkcom C. V-22 Osprey Tilt-Rotor Aircraft [J]. Congressional Research Service Reports, 2005.
[3] Yoo C S, Kang Y S, Park B J. Hardware-In-the-Loop simulation test for actuator control system of Smart UAV [C]. / International Conference on Control Automation & Systems. IEEE, 2010.
[4] Kang Y S, Park B J, Cho A, et al. Envelop expansion flight test of flight control systems for TR-60 tilt-rotor UAV [J]. 2013: 1866-1871.
[5] Prouty R W. Helicopter performance, stability, and control [M]. Krieger Pub. Co. 1995.
[6] Bayrakceken M K, Arisoy A. An educational setup for nonlinear control systems: Enhancing the motivation and learning in a targeted curriculum by experimental practices [Focus on Education] [J]. Control Systems IEEE, 2013, 33 (2): 64-81.
[7] ARDUPILOT. http://ardupilot.org/, [Online; accessed August, 2018]