Time Centrality in Dynamic Complex Networks

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Abstract

There is an ever-increasing interest in investigating dynamics in time-varying graphs (TVGs). Nevertheless, so far, the notion of centrality in TVG scenarios usually refers to metrics that assess the relative importance of nodes along the temporal evolution of the dynamic complex network. For some TVG scenarios, however, more important than identifying the central nodes under a given node centrality definition is identifying the key time instants for taking certain actions. In this paper, we thus introduce and investigate the notion of time centrality in TVGs. Analogously to node centrality, time centrality evaluates the relative importance of time instants in dynamic complex networks. In this context, we present two time centrality metrics related to diffusion processes. We evaluate the two defined metrics using a real-world dataset representing an in-person contact dynamic network. We validate the concept of time centrality showing that diffusion starting at the best classified time instants (i.e. the most central ones), according to our metrics, can perform a faster and more efficient diffusion process.
1 Introduction

We witness an ever-increasing interest in investigating the dynamics of complex networks (i.e., changes in nodes or edges over time) representing them as time-varying graphs (TVGs) \[4, 7, 10\]. In this context, a number of recent efforts investigate new centrality definitions to capture the relative node importance in TVGs \[2, 8, 11, 13, 14, 16, 18\]. For some TVG scenarios, however, more important than identifying the central nodes under a given definition, is identifying the key time instants for taking certain actions. Deciding when, and not only where from, to start a diffusion process can be of upmost importance for a more effective outcome.

This paper introduces and investigates the notion of time centrality in dynamic complex networks. Analogously to node centrality, time centrality assesses the relative importance of a given time instant within a TVG, which can be defined in different ways. In this context, we present two metrics, namely cover time and time-constrained coverage, to assess time centrality in TVGs from different perspectives. In doing so, we also show the relevance and relative importance (i.e., centrality) that some time instants may reach in comparison with other (less important) time instants. Both metrics regard different perspectives related to diffusion processes in complex dynamic networks.

We evaluate both time centrality metrics we define relying on a real-world dataset of an in-person contact network in a hospital environment (details in Section 5). Our results show that start spreading information at the top ranked (i.e., the most central) time instants, according to our metrics, can make diffusion processes in TVGs faster and more efficient. For example, compared with a random choice, a diffusion process based on time centrality metrics in the adopted dataset can reach 10% of the studied TVG about 3 times faster in the median case. Likewise, a diffusion process starting at the most central time instants can cover more nodes in the TVG with a limited number of allowed time steps for spreading when compared with random chosen time instants.

The paper is organized as follows. Section 2 discusses related work on temporal centrality in complex networks. In Section 3, we present the model we use to represent TVGs. Section 4 introduces the notion of time centrality and formalize two time centrality metrics related to diffusion processes. In Section 5, we present results on analyzing time centrality in a real-world dataset representing a in-person contact network. Finally, we summarize results and discuss the outlook of future work based on time centrality in Section 6.

2 Related Work

A large number of works on dynamic complex networks concerns both data structures to model TVGs and different definitions of node centrality in that context. The first works that attempted to represent TVGs were influenced by traditional graph modeling. For instance, a number of works model TVGs through a sequence of static subgraphs that represents the network dynamics.
in a discrete way as time passes \[3, 5, 7, 15, 17\].

Recent works tend to represent TVGs in different manners. For example, Casteigts et al. \[4\] use a presence function defined over continuous time intervals. The model proposed by Kostakos \[10\] is based on the idea that a class of links represent instantaneous interactions between distinct nodes while another class represents a waiting state of a given node. Kim and Anderson \[8\] suggest the use of edges connecting nodes at different time instants (temporal and mixed dynamic edges). For any of these TVG models, recent proposals investigate adaptations of the node centrality notion from traditional graphs to the time-varying context, leading to different notions of temporal node centrality targeted at particular applications \[2, 8, 11, 13, 14, 16, 18\].

As far as we know, this paper is the first to propose the notion of time centrality in dynamic complex networks. In contrast to previous work, this paper intends to assess the most important time instants, thus exploring the notion of time centrality.

3 Modeling Time-Varying Graphs

We model TVGs as a particular case of a MultiAspect Graph (MAG) \[19\] in which the vertices and time instants are the key features (i.e., aspects) to be represented by the model. A MAG is a structure capable of representing multilayer and time-varying networks while also having the property of being isomorphic to a directed graph. The MAG structural form resembles the multilayer structure recently presented by \[9\], since in both cases the proposed structure has a construction similar to an even uniform hypergraph associated with an adjacency concept similar to the one of simple directed graphs.

Formally, a MAG can be defined as an object \(H = (A, E)\), where \(E\) is a set of edges and \(A\) is a finite list of sets, each of which is called an aspect. In our case, for modeling a TVG, we have two aspects, namely vertices and time instants, i.e. \(|A| = 2\). For the sake of simplicity, this 2-aspect MAG can be regarded as representing a TVG as an object \(H = (V, E, T)\), where \(V\) is the set of nodes, \(T\) is the set of time instants, and \(E \subseteq V \times T \times V \times T\) is the set of edges. As a matter of notation, we denote \(V(H)\) as the set of all nodes in \(H\), \(E(H)\) the set of all edges in \(H\), and \(T(H)\) the set of all time instants in \(H\).

An edge \(e \in E(H)\) is defined as an ordered quadruple \(e = (u, t_a, v, t_b)\), where \(u, v \in V(H)\) are the origin and destination nodes, respectively, while \(t_a, t_b \in T(H)\) are the origin and destination time instants, respectively. Therefore, \(e = (u, t_a, v, t_b)\) should be understood as a directed edge from node \(u\) at time \(t_a\) to node \(v\) at time \(t_b\). If one needs to represent an undirected edge in the TVG, both \((u, t_a, v, t_b)\) and \((v, t_b, u, t_a)\) should be in \(E(H)\).

We also define a temporal node as an ordered pair \((u, t_a)\), where \(u \in V(H)\) and \(t_a \in T(H)\). The set \(VT(H)\) of all temporal nodes in a TVG \(H\) is given by the cartesian product of the set of nodes and the set of time instants, i.e. \(VT(H) = V(H) \times T(H)\). As a matter of notation, a temporal node is represented by the ordered pair that defines it, e.g. \((u, t_a)\).
The usage of the object $H = (V, E, T)$ to represent a TVG is formally introduced in [20]. Therein, the representation of the TVG based on temporal nodes is proven to be isomorphic to a directed static graph. This is an important theoretical result since this allows the use of the isomorphic directed graph as a tool to analyze both the properties of a TVG and the behavior of dynamic processes over a TVG, as done is this work. This model is also shown to unify the representation of several previous (classes of) models for TVGs of the recent literature, which in general are unable to represent each other [20].

Importantly, the adopted TVG model is a particular case of a MAG [19]. Therefore, adding new aspects, such as different network layers, is straightforward. This opens perspectives of extending the analysis of the time centrality concept we introduce in Section 4 to more complex dynamic networked systems, such as time-varying multilayer networks [1,9,19]. We intend to further explore such perspectives in our future work.

4 Time Centrality

We here introduce the notion of time centrality. As with node centrality, different definitions of time centrality can reflect different notions of importance for time instants targeting distinct applications.

We present two time centrality metrics that can assess diffusion in TVGs from different perspectives. To define such metrics, we consider a generic TVG $H = (V, E, T)$ (see Section 3). Let $N = |T(H)|$ be the size of set $T(H)$. Each time instant $t_i \in T(H)$, where $0 \leq i < N$, integrates the set of sequential time instants described by $T(H) = \{t_0, t_1, \ldots, t_{N-1}\}$. For the sake of clarity, the illustrative figures in this section show sequences of times instants as disconnected snapshots, although in the adopted model temporal edges connect nodes in different time instants, easing the temporal analysis.

4.1 Cover Time (CT)

The Cover Time (CT) metric from a time centrality perspective assesses the minimum number of time instants (steps) a diffusion takes to reach a given fraction of the nodes. More formally, the cover time $CT(t_i, \tau)$ returns the average amount of time instants (steps) for a diffusion starting at time $t_i$ to reach a given percentage $\tau$ of nodes. If the fraction $\tau$ of nodes is not reached for a diffusion process starting on any node at $t_i$, then

$$CT(t_i, \tau) = \begin{cases} \frac{1}{|V(H)|} \sum_{u \in V(H)} d_t(t_i, u, \tau), & \text{if } \tau \text{ reached,} \\ \infty, & \text{otherwise.} \end{cases}$$

(1)

where $d_t(t_i, u, \tau)$ is the number of time instants (steps) for a diffusion starting on node $u$ at time instant $t_i$ to reach a fraction $\tau$ of nodes. If the fraction $\tau$ of nodes is not reached for a diffusion process starting on any node at $t_i$, then
$CT(t_i, \tau)$ is considered $\infty$. The results presented in Section 5 consider only the finite results of cover time.

We consider the diffusion process to follow a Breadth-First Search (BFS), i.e. a generic node $(u, t_i)$ starts diffusion by sending information to all of its adjacent nodes. Then, the adjacent temporal nodes relay information for their own adjacent nodes in the next time instant, and so on. Information is distributed until the percentage $\tau$ of nodes is reached and the time needed for diffusion is stored. This evaluation is repeated for every node $(u, t_i) \in VT(H)$. The average diffusion time is the cover time centrality of the time instant $t_i$.

As an example, the top part of Figure 1 shows an illustrative diffusion process starting at $t_a$ and finishing at $t_a+10$, whereas Figure 1 presents another diffusion process starting at $t_b$ and finishing at $t_b+9$. Intuitively, the time instant $t_b$ is more important (central) in regard of its cover time, since the diffusion starting at it is faster on average.

![Illustrative cover time on TVGs.](image)

Figure 1: Illustrative cover time on TVGs.

### 4.2 Time-Constrained Coverage (TCC)

The *Time-Constrained Coverage (TCC)* metric from a time centrality perspective evaluates the TVG coverage achieved by a diffusion process after a limited
number of time steps. Here we assume the same diffusion process defined for the
cover time (Section 4.1). More precisely, for a given time instant $t_i$, $TCC(t_i, \phi)$
returns the average percentage of the TVG nodes reached by the diffusion pro-
cess in $\phi$ of time steps. More formally,

$$TCC(t_i, \phi) = \frac{1}{|V(H)|^2} \sum_{u \in V(H)} d_c(t_i, u, \phi),$$

(2)

where $d_c(t_i, u, \phi)$ is the number of nodes reached from node $u$ after $\phi$ time steps
when the diffusion starts at time $t_i$. For example, Figure 2 presents two distinct
illustrative diffusion processes occurring in $\phi$ time steps. At the leftmost part,
the process starting at $t_a$ reaches less nodes than the process starting at $t_b$ at the
rightmost part after $\phi$ time steps. The latter diffusion process actually reaches
all nodes in the TVG at the $t_b + \phi$ time instant. Therefore, the time instant $t_b$ is
more important (central) in regard of its time-constrained coverage, since the
diffusion starting at it is more effective in reaching a larger number of nodes on
average after $\phi$ time steps.

Figure 2: Illustrative time-constrained coverage on TVGs.

5 Time Centrality Results

We evaluate the time centrality metrics we propose using a TVG dataset col-
lected in the context of the MOSAR project [12]. MOSAR (Mastering hOSpital
Antimicrobial Resistance and its spread) is a scientific collaboration project
that comprises several medical, biochemistry, and computing research institu-
tions. The MOSAR project focuses on antimicrobial-resistant bacteria (AMRB)
transmission dynamics in high-risk environments, such as intensive care units
and surgical centres. The adopted dataset consists of the records of in-person
contacts (from physicians, nurses, staff members, and patients) in a certain med-
cial ward for a period of two weeks (between 12 am of July 25, 2009 and 12am of
August 08, 2009). Each one of the 160 volunteers who participated in the study was equipped with a RFID device that detected the presence of another devices within a small distance (about one meter). Device identification was unique and was always associated with the same person. Every 30 seconds during the two-weeks period, each device registered the list of all devices (nodes) that were within its coverage area in order to establish the arrangement of the contacts among them (edges).

We have modeled the MOSAR dataset as a TVG $M = (V, E, T)$, as described in Section 3, where $V(M)$ is the set of all participants, $E(M)$ is the set of all in-person contacts, and $T(M)$ is the set of sequential time instants at a 30 seconds granularity. Thus, $|V(M)| = 160$ and $|T(M)| = 40320$. Contacts are represented by non-oriented edges, i.e., the edge between any two persons is represented by a directed edge and its reciprocal.

5.1 Analysis of Cover Time

We first present results for the cover time metric. As previously discussed, lower cover time values indicate more central time instants that can spread information in a shorter period of time. Figure 3 presents the cumulative distribution function (CDF) of the cover time for different values of the fraction $\tau$ of nodes to be covered in the TVG. Six different scenarios are considered with $\tau = \{0.1, 0.2, \ldots, 0.6\}$. Results compare the first 33000 time instants, so there is room for the diffusion to spread through the 40320 time instants of the studied TVG.

In Figure 3, each value in a curve represents the fraction of the studied time instants that achieved the corresponding $\tau$ fraction of nodes in at most the corresponding number of time steps, i.e. the cover time $t$. For instance, to cover 60% of the TVG ($\tau = 0.6$), 20% of the time instants take less than 2000 time steps. In other words, information diffusion starting at 20% of the time instants takes less than 16 hours and 40 minutes to cover 60% of the nodes in the TVG only spreading opportunistically through the in-person contact dynamic network.

From Figure 3 we clearly note that starting the diffusion at a few central time instants reduces significantly the speed of the spreading to reach a certain portion of the nodes in the TVG. Thereby, we also analyze the impact on diffusion efficiency provided by the most central time instants. In Figure 4, we compare the performance of the top 10 most central time instants according to the cover time metric against 10 randomly chosen time instants to start spreading a diffusion. Taking the median result as a reference, to cover 10% of the nodes in the studied TVG, a diffusion starting at the top 10 most central time takes 3 times less time steps than a diffusion starting at the random set of time instants.
5.2 Analysis of Time-Constrained Coverage

The time-constrained coverage is also able to select few time instants that present distinct diffusion performance. Figure 5 presents the complementary CDF of the time-constrained coverage of the TVG for different values of $\phi = \{500, 1000, 2000\}$ time steps. Likewise in cover time, the first 33000 time instants are compared on each case.

In Figure 5, each value in a curve represents the fraction of the studied time instants that achieved at least the corresponding time-constrained coverage $c$ of the TVG in $\phi$ time steps. For example, with a limit of $\phi = 2000$ time steps diffusion starting at 80% of the time instants reach at least 28% of the nodes in the TVG, noting that the maximum reachable coverage is 58%.

As expected, the larger is the maximum number $\phi$ of allowed time steps, the larger is the overall network coverage. Importantly, however, in all cases a very limited number of time instants—actually, the most central ones—provide superior efficiency in covering more nodes in the TVG with a constrained number of time steps when diffusion starts at these most central time instants. In Figure 6, we compare the performance of the top 10 most central time instants according to the time-constrained coverage metric against 10 randomly chosen time instants to start spreading a diffusion. Indeed, starting a diffusion on any of the top 10 most central time instants would reach at least 56% of network
coverage, whereas in the remaining randomly chosen time instants, a diffusion process would reach no more than 33% of the TVG nodes, considering a limit of $\phi = 2000$ time steps.

6 Summary and Outlook

In this work, we introduce the notion of time centrality in dynamic complex networks. Time centrality assesses the relative importance of a given time instant within a time-varying graph (TVG). In this context, we present two time centrality metrics focused on diffusion processes and evaluate them using a real-world dataset representing a dynamic in-person contact network. Our results show that starting a diffusion at the most central time instants, according to our centrality metrics, can make diffusion processes in TVGs faster and more efficient.

Considering the notion of time centrality in dynamic complex networks opens several perspectives for further research. As future work, we plan to further analyze the particular case of considering time centrality conditioned upon the seed node that holds the information to be diffused. Furthermore, we also intend to evaluate the trade-off on the diffusion performance in TVGs between waiting for the most central time instant to spread information or starting the diffusion
before the best moment with better performance expectations considering the total time of both retaining and spreading of information.

Moreover, we also intend to conceive as future work prediction models based on time centrality for complex systems that can be represented by TVGs. In that sense, strategies for the possible early identification of central time instants might be based on discovering evidence in the recent past or the present moment of the TVG evolution indicating that a given time instant is relatively better (i.e. more central) with respect to a set of others.

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Figure 6: TVG coverage in $\phi = 2000$ time steps.

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