Nonperturbative interaction in $q\bar{q}$ bound states

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Abstract

We report about recent progress in the treatment of bound states in QCD.

1 Introduction

One of the most difficult and less understood problems in Quantum Field Theory is the treatment of bound state systems. Even in QED where the interaction is simply given in perturbation theory, the practical calculation of the bound state properties is tricky. The complication comes from the mixing of the many characteristic scales of the bound state, the mass of the particles, the momentum and the energy of the state.

A way to handle this problem is provided by Non Relativistic QED (NRQED) which is an effective theory equivalent to QED and obtained from QED by integrating out the hard energy scale $m$. The ultraviolet regime of QED (at energy scale $m$) is perturbatively encoded order by order in the coupling constant $\alpha$ in the matching coefficients that appear in front of the operators in the effective Lagrangian. Each term in the effective Lagrangian has a definite power counting in $\alpha$ and then a disentangling of the various scales is achieved. A similar simplification is obtained using Non Relativistic QCD (NRQCD) in heavy quark systems. These systems are characterized by a dynamical dimensionless parameter, the quark velocity $v$, which is small and allows a classification of the energy scales of the problem in hard ($\sim m$), soft ($\sim mv$) and ultrasoft ($\sim mv^2$). This provides a power counting scheme. Here, however, a further and conceptual difficulty arises in connection with the nonperturbative nature of low-energy QCD. The relation between $v$ and the QCD parameters is unknown but certainly $v$ includes both perturbative and nonperturbative effects. Therefore, the evaluation of the bound state properties is ultimately done with a lattice simulation. In the case of bound systems with at least one light quark simplifications of this type do not hold. On one hand no small expansion parameter exists, on the other hand the light quark mass is generated via chiral symmetry breaking. Hence, the interplay between confinement and chiral symmetry breaking has to be considered.

In this talk we report about recent progress in the treatment of bound states in QCD. We show that it is possible to obtain a model-independent and gauge-invariant result for the heavy quark interaction at order $v^4$ of the systematic expansion in $v$. The interaction turns out to be simply given in terms of a generalized (distorted) Wilson loop. The result is suitable for lattice evaluation as well as for analytic
evaluation once a QCD vacuum model is considered. We show that the results for
the heavy quark dynamics are substantially under control and are given in terms of
two nonperturbative parameters \( T_g \), the gluon correlation length, and \( G_2 \), the gluon
condensate. Adopting the same framework in order to study the heavy-light bound
states in the non-recoil limit, spontaneous chiral symmetry breaking and a confining
chiral non-invariant interaction emerge quite naturally. We discuss this last case
with more details.

2 The heavy quark interaction

The heavy quark interaction can be obtained analytically at the order \( v^4 \) of the
systematic expansion of the interaction in \( v \). Here, we report only the main steps of
the derivation referring to [1, 2] for further details.

- **Step 1.** Set up the NRQCD Lagrangian. The NRQCD Lagrangian [3] is
  obtained from the QCD Lagrangian via a Foldy-Wouthysen transformation.
  At order \( O(v^4) \) the NRQCD Lagrangian describing a bound state between a
  quark of mass \( m_1 \) and an antiquark of mass \( m_2 \) is [4]

  \[
  L = Q_1^\dagger \left( iD_0 + c_{2} \frac{D^2}{2m_1} + c_{4} (iD^4 - \frac{g \cdot g}{2m_1} + c_{s} \frac{g \cdot D \cdot g}{8m_1^2} \right) \]

  \[+ i c_{s} \frac{g \cdot (D \times E - E \times D)}{8m_1^2} \right) Q_1 + \text{ antiquark terms (1} \leftrightarrow 2) + \frac{d_1}{m_1 m_2} Q_1^\dagger Q_2 Q_2^\dagger Q_1 \]

  \[+ \frac{d_2}{m_1 m_2} Q_1^\dagger \sigma Q_2 Q_2^\dagger \sigma Q_1 + \frac{d_3}{m_1 m_2} Q_1^\dagger T^a Q_2 Q_2^\dagger T^a Q_1 + \frac{d_4}{m_1 m_2} Q_1^\dagger T^a \sigma Q_2 Q_2^\dagger T^a \sigma Q_1, \]

  where \( Q_j \) are the heavy quark fields. Reparameterization invariance [4] fixes
  \( c_2 = c_4 = 1 \). The coefficients \( c_F, c_D, ... \) are evaluated at the matching scale \( \mu \)
  for a particle of mass \( m_j \). They encode the ultraviolet regime of QCD order by
  order in \( \alpha_s \). The explicit expressions and a numerical discussion can be found
  in [5]. The power counting rules for the operators of Eq. (1) are \( Q \sim (mv)^{3/2}, \)
  \( D \sim mv, gA_0 \sim mv^2, gA \sim mv^3, gE \sim m^2 v^3 \) and \( gB \sim m^2 v^4 \). Four
  quark operators which are apparently of order \( v^3 \) are actually suppressed by
  additional powers in \( \alpha_s \) in the matching coefficients and the octet contributions
  by an additional power in \( v^2 \) on singlet states. Therefore in the following we
  will neglect these contributions with the exception of a term which mixes
  under RG transformation with the chromomagnetic operator contribution to
  the spin-spin potential [4]. We will call the corresponding matching coefficient
  \( d \).

- **Step 2.** Write down the quark-antiquark gauge-invariant Green function using
  a path integral representation. The 4-point gauge invariant Green function \( G \)
associated with the Lagrangian \( \mathcal{L} \) is defined as \[7, 8\]

\[ G(x_1, y_1, x_2, y_2) = \langle 0 | Q_2^\dagger(x_2)U(x_2, x_1)Q_1(x_1)Q_1^\dagger(y_1)U(y_1, y_2)Q_2(y_2) | 0 \rangle, \]

where \( U(x_2, x_1) \equiv \exp \left\{ -ig \int_0^1 ds \, (x_2 - x_1)^\mu A_\mu(x_1 + s(x_2 - x_1)) \right\} \) is a Schwinger line added to ensure gauge invariance. After integrating out the heavy quark fields, \( G \) can be expressed as a quantum-mechanical path integral over the quark trajectories:

\[
G(x_1, y_1, x_2, y_2) = \int_{y_1}^{x_1} \mathcal{D}z_1 \mathcal{D}p_1 \int_{y_2}^{x_2} \mathcal{D}z_2 \mathcal{D}p_2 \exp \left\{ i \int_{-T/2}^{T/2} dt \sum_{j=1}^2 p_j \cdot z_j - m_j - \frac{p_j^2}{2m_j} \right. \\
+ \frac{p_j^4}{8m_j^2} \left\} \frac{1}{N_c} \left\{ \text{Tr} \mathcal{P} T_s \exp \left\{ -ig \int_\Gamma dz^\mu A_\mu(z) + i \int_{-T/2}^{T/2} dz_0 c_F^{(j)} \sigma \cdot \mathbf{B} \right. \\
+ c_F^{(j)} g \frac{\mathbf{D} \cdot \mathbf{E} - \mathbf{E} \cdot \mathbf{D}}{8m_j^2} + i c_S^{(j)} g \frac{\sigma (\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D})}{8m_j^2} \right\} \right. \\
\times \exp \left\{ \frac{i}{m_1 m_2} \int_{-T/2}^{T/2} dt \, g^2 dT^{(1)} \sigma^{(1)} T^{(2)} \sigma^{(2)} \delta^3(z_1 - z_2) \right\} \left. \right\} \\
\equiv \int_{y_1}^{x_1} \mathcal{D}z_1 \mathcal{D}p_1 \int_{y_2}^{x_2} \mathcal{D}z_2 \mathcal{D}p_2 \exp \left\{ i \int_{-T/2}^{T/2} dt \sum_{j=1}^2 p_j \cdot z_j - m_j - \frac{p_j^2}{2m_j} + \frac{p_j^4}{8m_j^2} - i \int_{-T/2}^{T/2} dt \mathcal{L} \right\},
\]

where the bracket means the Yang–Mills average over the gauge fields, \( \Gamma \) is the Wilson loop made up by the quark trajectories \( z_1 \) and \( z_2 \) and the endpoints Schwinger strings and \( y_2^0 = y_1^0 \equiv -T/2, \, x_2^0 = x_1^0 \equiv T/2, \) see Fig. \[\]

- **Step 3.** Extract the form of the heavy quark interaction from Eq. \( (3) \). Assuming that the limit exists, we define the heavy quark-antiquark potential \( V \) as

\[
V(r) = \lim_{T \to \infty} \frac{i \log(W(\Gamma))}{T} + \left( \frac{S^{(1)} \cdot L^{(1)}}{m_1^2} + \frac{S^{(2)} \cdot L^{(2)}}{m_2^2} \right) 2c_F V_1(r) + c_S V_0'(r) + \frac{S^{(1)} \cdot L^{(2)} + S^{(2)} \cdot L^{(1)}}{m_1 m_2} c_F V_2'(r)
\]

where \( S \) and \( L \) are the spinor and color matrices.

Figure 1: Distorted Wilson loop.
\[ + \left( \frac{S^{(1)} \cdot L^{(1)}}{m_1^2} - \frac{S^{(2)} \cdot L^{(2)}}{m_2^2} \right) \frac{2c_S V_{0}(r) + c_\pi V'_{0}(r)}{2r} + \frac{S^{(1)} \cdot L^{(2)} - S^{(2)} \cdot L^{(1)}}{m_1 m_2} \frac{-c_\pi V'_{0}(r)}{r} \]

\[ + \frac{1}{8} \left( \frac{c_{p_1}^{(1)}}{m_1} + \frac{c_{p_2}^{(2)}}{m_2} \right) (\Delta V_{0}(r) + \Delta V^E_{a}(r)) + \frac{1}{8} \left( \frac{c_{F}^{(1)}}{m_1} + \frac{c_{F}^{(2)}}{m_2} \right) \Delta V^B_{a}(r) + \frac{c_{F}^{(1)} c_{F}^{(2)}}{m_1 m_2} \times \]

\[ \left( \frac{S^{(1)} \cdot r S^{(2)} \cdot r}{r^2} - \frac{S^{(1)} \cdot S^{(2)}}{3} \right) V_3(r) + \frac{S^{(1)} \cdot S^{(2)}}{3m_1 m_2} \left( c_{F}^{(1)} c_{F}^{(2)} V_4(r) - 48\pi \alpha_s C_F d \delta^3(r) \right) \]

where \( W(\Gamma) \equiv \langle \exp \left\{ -ig \int_{\Gamma} d\mu A_\mu(z) \right\} \rangle \) is the averaged value of the deformed Wilson loop, see Fig. 1. The expansion of it around the static Wilson loop \( W(\Gamma_0) \) \( (\Gamma_0 \text{ is a } r \times T \text{ rectangle}) \) gives the static potential \( V_0 = \lim_{T \to \infty} i \log(W(\Gamma_0))/T \) plus velocity (non-spin) dependent terms \([1, 9]\) which are controlled by four scale-independent potentials \( V_0(r) i = b, c, d, e \). \( S^{(j)} \) and \( L^{(j)} \) are the spin and orbital angular momentum operators of the particle \( j \). The matching coefficients are defined as \( 2c_{F,S}^{(1)} \equiv c_{F,S}^{(1)} \pm c_{F,S}^{(2)} \). The spin-dependent potentials agree with the ones obtained in refs. \([8]\) with the exception of the matching coefficients that were introduced in \([6]\).

All the potentials \( V_1 - V_4, V_a, V_d \) are obtained as functions of the average value of the distorted Wilson loop \( \langle W(\Gamma) \rangle \) and insertions of one or two field strengths in this average, \( \langle F_{\mu\nu} W(\Gamma) \rangle \) or \( \langle F_{\mu\nu} F_{\rho\sigma} W(\Gamma) \rangle \). Via deformation of the quark or (antiquark) trajectory, such v.e.v. of field strength insertion can be calculated via functional derivatives of the Wilson loop. We conclude that to obtain the complete quark-antiquark order \( O(\nu^4) \) interaction (quenched) no other assumptions are needed than the behavior of \( \langle W(\Gamma) \rangle \): given \( \langle W(\Gamma) \rangle \) everything is analytically calculable. On the other side, expanding the average of the distorted Wilson loop \( \Gamma \) in terms of the static Wilson loop \( \Gamma_0 \) we get expressions for the potentials suitable for lattice evaluations \([10, 11]\). In this way, we can compare unambiguously the predictions for the heavy quark interaction obtained in various QCD vacuum models and the lattice measures as well as the phenomenological data.

### 3 QCD vacuum models

Models of the QCD vacuum are needed in order to describe the nonperturbative behavior of the Wilson loop average. To this aim one wants to exploit all the available lattice information on the mechanism of confinement and all the measurements of the Wilson loop. Let us consider the v.e.v. of the Wilson loop. It pays to expand this average in terms of field strength expectation values, by using the non-Abelian Stokes theorem \([12]\).

\[ \langle W(\Gamma) \rangle \equiv \langle \exp \{ ig \int_{\Gamma} d\mu A_\mu(z) \} \rangle = \langle P \exp i g \int_{S(\Gamma)} dS_{\mu\nu}(1) U(0,1) F_{\mu\nu}(1) U(1,0) \rangle = \]

4
The Lorentz decomposition is general and the dynamics is contained in the form factors $D$ and $D_1$. The function $D$ is responsible for the area law and confinement (indeed in QED, due to the Bianchi identity, we have $D = 0$). For $D$ and $D_1$ the lattice calculations $[13, 14]$ give an exponential (in Euclidean space) long-range decreasing behavior $\simeq G_2\exp\left\{-|x|/T_g\right\}$, where $G_2 \equiv \langle \alpha_s F^2(0) \rangle / \pi$ is the gluon condensate and $T_g \simeq 0.15 \div 0.2$ fm is the gluon correlation length (quenched).

In ref. $[15]$, the QCD two–point field strength correlator (5) has been related to the dual field propagator of the effective Abelian Higgs model describing infrared QCD. In this way the Gaussian dominance in the Wilson loop average is understood as following from the classical approximation in the dual theory. Moreover, it is possible to relate the QCD parameter $T_g$ and $G_2$ to the dual parameters. In the London limit $T_g$ is identified with the dual gluon mass $M$, without the London limit the relation is more involved but still $T_g$ is expressed in terms of the dual theory parameters.

From the calculation of the heavy quark interaction in various model of the QCD vacuum (minimal area law model $[9]$, stochastic vacuum model $[12, 16]$, dual QCD $[17, 16, 14]$, Isgur and Paton model $[18]$) we can state that:

- All these models give the same result for the nonperturbative heavy quark interaction not only in the long range regime but also in the transition region.
- Two nonperturbative parameters, that can be related to $T_g$ and $G_2$, control the nonperturbative interaction.

1Phenomenological calculation in high energy scattering indicates $T_g \equiv 0.3 \div 0.35$ fm
• All these models predict the Eichten-Feinberg nonperturbative spin interaction (pure Thomas precession) in the limit $T_g/r \to 0$. In the transition region there is a subleading correction to the spin-interaction coming from the magnetic interaction.

• All these models predict the nonperturbative velocity dependent corrections to be proportional to the flux tube angular momentum. This prediction is definitively different from the result obtained with the semi-relativistic reduction of Bethe-Salpeter kernels of the type $1/Q^4$, ($Q$ being the momentum transfer) with any Lorentz structure, see [1].

The conclusion is that we need two parameters $T_g$ and $G_2$ to describe the heavy quark dynamics and indeed they are necessary to control the structure of the flux tube. Had we only one parameter, like the string tension $\sigma$, we could encode the information of a constant energy density in the flux tube. However, the whole structure is important, and also the information about the width of the flux tube has to be considered. In the limit of very large inter-quark distances and in particular dynamical regimes, we can store the relevant information in one parameter, the string tension. For instance, from Eqs. (4) and (5), the confining part of the static potential is

$$V_0(r) \simeq G_2 \int_0^\infty d\tau \int_0^r d\lambda (r - \lambda) D(\tau^2 + \lambda^2)$$

(6)

and the string tension $\sigma$ emerges as an integral on the $D$ function $\sigma \simeq G_2 \int_0^\infty d\tau \int_0^\infty d\lambda \, D(\tau^2 + \lambda^2)$ in the limit $T_g/r \to 0$.

4 Heavy-light systems

We study the heavy-light bound state system in the non-recoil limit. We start from the gauge-invariant quark-antiquark Green function in the Feynman-Schwinger representation [19]:

$$G(x, u, y, v) = \frac{1}{4} \langle \text{Tr} P \left( i \frac{\partial}{\partial y} + m_1 \right) \int_0^\infty dT_1 \int_y^x Dz_1 e^{-i \int_0^{T_1} dt_1 \frac{m^2 + \dot{z}_1^2}{2}} \times \int_0^\infty dT_2 \int_v^u Dz_2 e^{-i \int_0^{T_2} dt_2 \frac{m^2 + \dot{z}_2^2}{2}} e^{ig \oint_{\Gamma} dz^\mu A_\mu(z)} \rangle$$

$$\times \left( \frac{i}{4} \sigma^{(1)}_{\mu\nu} F^{\mu\nu}(z_1) e^{i \int_0^{T_1} dt_1 \frac{g}{4} \sigma^{(1)}_{\mu\nu} F^{\mu\nu}(z_1)} \right),$$

$$\times \left( \frac{i}{4} \sigma^{(2)}_{\mu\nu} F^{\mu\nu}(z_2) e^{i \int_0^{T_2} dt_2 \frac{g}{4} \sigma^{(2)}_{\mu\nu} F^{\mu\nu}(z_2)} \right).$$

(7)

Again the dynamics is contained in the Wilson loop, that now looks like Fig. 2.

We can exploit the symmetry of the situation, taking the modified coordinate gauge

\footnote{The flux tube distribution between a static quark-antiquark pair is measured on the lattice, see [1].}
Figure 2: The Wilson loop in the static limit of the heavy quark and the interaction kernel $K$.

\[ A_\mu(x_0, 0) = 0, \ x^j A_j(x_0, x) = 0. \]

Notice that this gauge choice is possible due to the gauge-invariance of the formalism. Within this gauge is possible to express the gauge field in terms of the field strength tensor $A_\mu(x) = \int_0^1 d\alpha x^k F_{k\mu}(x_0, \alpha x)$ where $n(0) = 0, n(i) = 1$. Then, the only non-vanishing contribution to the Wilson loop is

\[ W(\Gamma) = \text{Tr} P \exp \left\{ ig \int_x^y dz^\mu A_\mu(z) \right\}. \]  

At this point, at variance from the heavy quark case, we have to make a model dependent assumption, i.e. we consider still valid the dominance of the bilocal correlator. Indeed, this should be a property of the vacuum. Then, we have

\[ \langle W(\Gamma) \rangle \approx \exp \left\{ -\frac{g^2}{2} \int_x^y dx^\mu \int_x^y dy^\nu D_{\mu\nu}(x', y') \right\}, \]  

\[ D_{\mu\nu}(x, y) \equiv x^k y^l \int_0^1 d\alpha x^{n(\mu)} \int_0^1 d\beta y^{n(\nu)} \langle F_{k\mu}(x_0, \alpha x) F_{l\nu}(y_0, \beta y) \rangle \]

and inserting Eq. (8) in (7) and expanding the exponential we obtain the following expression for the propagator $S_D$ of the light quark in the static heavy quark field:

\[ S_D = S_0 + S_0 K S_0 + S_0 K S_0 K S_0 + \cdots, \]  

$S_0$ being the free fermion propagator. Taking into account only the first planar graph (since we are interested only in contributions proportional to the gluon condensate), we have $K(y', x') = \gamma^\nu S_0(y', x') \gamma^\mu D_{\mu\nu}(x', y')$. A graphical representation of $K$ is given in Fig. 2. Eq. (10) can be written in closed form as $S_D = S_0 + S_0 K S_D$ (or in terms of the wave-function, $(\hat{p} - m - iK)\psi = 0; \ m \equiv m_1$). Therefore, $K$ can be interpreted as the interaction kernel of the Dirac equation associated with the motion of a quark in the field generated by an infinitely heavy antiquark.

Notice that \[\Box\]: 1) $K$ is not a translational invariant quantity. The coordinate gauge breaks explicitly this symmetry in the propagator. Physically this is due to the presence of the heavy quark. 2) The kernel depends on $D_{\mu\nu}$ which in turns is given in terms of the two-point correlator (8). Then, the heavy-light dynamics is
controlled by the same two parameters controlling the heavy-heavy dynamics, $T_g$ and $G_2$. 3) The problem has many relevant scales: the light mass $m$, the correlation length $T_g \sim \Lambda_{QCD}$, the characteristic energy and momentum of the bound state. We have different dynamical regimes in dependence on the relative values of these scales. Let us study the various situations. In the following we consider only the nonperturbative dynamics [19].

- **Potential Case:** $m > 1/T_g > p_0 - m, p, p - q$. We neglect the negative energy states and expand the kernel $K$ in $m$. We obtain:

$$V(r) \sim G_2 \left\{ \int_{-\infty}^{+\infty} d\tau \int_0^r d\lambda (r - \lambda) D(\tau^2 + \lambda^2) + \frac{\sigma \cdot L}{4m^2r} \int_{-\infty}^{+\infty} d\tau \int_0^r d\lambda \left( \frac{2\lambda}{r} - 1 \right) D(\tau^2 + \lambda^2) \right\}$$

which coincides in the limit of large $r$ with the Eichten-Feinberg potential [8]

$$V(r) = \sigma r - \frac{\sigma L \cdot r}{4m^2r}, \text{ with } \sigma \text{ defined as in Sec. 3.}$$

We emphasize that the Lorentz structure which gives origin to the negative sign in front of the spin-orbit potential (hence to the pure Thomas precession term) is in our case *not simply a scalar* ($K \simeq \sigma r$).

- **Sum Rules case:** $(1/T_g < p_0 - m, 1/T_g < m)$. We get the well-known Shifman-Vainshtein-Zakharov result for the heavy quark condensate

$$\langle \bar{Q} Q \rangle = - \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \text{Tr} \left\{ S_0(q) K(p,q) S_0(p) \right\} = - \frac{1}{12} \frac{\langle \alpha F^2(0) \rangle}{\pi m}.$$ 

- **$D_s$ and $B_s$ case:** $1/T_g > m$. For $D_s$ and $B_s$ one can probably still assume that the propagator inside the kernel is free and solve the equation to get the spectrum.

- **$D$ and $B$ case:** $m \ll 1/T_g$. We observe that in the zero mass limit the kernel $K$ gives a chirally symmetric interaction (while a purely scalar interaction breaks chiral symmetry at any mass scale). This means on one side that our interaction keeps the main feature of QCD i.e. in the zero mass limit chiral symmetry is broken only spontaneously. On the other side this seems to suggest that for very light quarks the quark propagator should be taken from the chiral broken solution, i.e. the nonlinear equation [20, 21]

$$S_D = S_0 + S_0 K(S_D) S_D$$

has to be solved with Schwinger–Dyson like techniques.

In conclusion our approach to the heavy-light systems contains the sum rules and the potential results still allowing for a chiral symmetric interaction. In the next section we discuss the interplay between confinement and chiral symmetry breaking that emerges in this picture.
5 Confinement and chiral symmetry breaking

We address the problem of solving Eq. (11). We will see how chiral symmetry breaking emerges in a heavy-light bound state and how it leads to a non-chiral invariant confining interaction. In place of (9) we consider the simplified interaction

$$g^2 D_{\mu\nu}(x, y) \simeq -i \frac{\delta_{\mu\nu}}{24} T_g (g^2 E^2(0)) \mathbf{x} \cdot \mathbf{y} \delta(x_0 - y_0)$$

where we have considered the leading contribution as given by the electric fields only depending on time and we have approximated the exponential fall off in time (with correlation length $T_g$) with an instantaneous delta-type interaction $^3$.

We find convenient to work in the Hamiltonian approach. The effective Hamiltonian corresponding to Eqs. (11) and (12) is:

$$H = \int d^3x \, q^\dagger(x) (-i \alpha \cdot \nabla + m \beta) q(x) - \frac{1}{2} \int d^3x \int d^3y V_0^3 r^2 q^\dagger(x) T^a q(x) q^\dagger(y) T^a q(y) + 2 \int d^3x \int d^3y V_0^3 R^2 q^\dagger(x) T^a q(x) q^\dagger(y) T^a q(y),$$

with $V_0^3 = -T_g (g^2 E^2(0)) / 96 = T_g \pi^2 G_2 / 96$ and $R = x/2 + y/2$ and $r = x - y$. From Eq. (13) it is clear the role played by the approximation (12). It allows to disentangle trivially in the effective Hamiltonian the self interacting part (function of $r$) from the external source interacting term (function of $R$). With a “realistic” lattice parameterization of the non-local gluon condensate these terms might be mixed up in a very complicate way.

By means of the Bogoliubov–Valatin variational method we select the chiral broken vacuum. The quark fields are expanded on a trial basis of spinors: $q(x) = \sum_s \int \frac{d^3k}{(2\pi)^3} e^{ik \cdot x} \left[ u_s(k) b_s(k) + v_s(k) d_s^\dagger(-k) \right]$, with trial spinors $u_s$ and $v_s$, $u_s(k) = \frac{1}{\sqrt{2}} \left[ \sqrt{1 + \sin \phi(k)} + \sqrt{1 - \sin \phi(k)} \alpha \cdot \mathbf{k} \right] u_0^0$, $v_s(k) = \frac{1}{\sqrt{2}} \left[ \sqrt{1 + \sin \phi(k)} - \sqrt{1 - \sin \phi(k)} \alpha \cdot \mathbf{k} \right] v_0^0$, where $u_0^0$ and $v_0^0$ are the usual rest-frame spinors on the chiral invariant vacuum. In the limiting case $\phi = 0$ the trial spinors reduce to the massless free one, while for $\phi = \pi/2$ they reduce to infinitely massive sources.

Expanding the Hamiltonian (13) on the trial basis we get $H = \mathcal{E} + H_2' + H_3 + H_4$, where

$$\mathcal{E} = \mathcal{V} \left\{ 3 \int \frac{d^3k}{(2\pi)^3} \text{Tr} \left[ (\alpha \cdot \mathbf{k} + m \beta) \Lambda_-(k) \right] - 2V_0^3 \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3} \text{Tr} \left[ \Lambda_-(k) \Lambda_+(k') \right] \int d^3r e^{-i(k - k') \cdot r} r^2 \right\},$$

$$H_2' = \int d^3x : q^\dagger(x) (-i \alpha \cdot \nabla + m \beta) q(x) :$$

$^3$This can be done since for light quarks the energy scale $T_g^{-1}$ is expected to be bigger than the other scales of the problem.
\[-\frac{2}{3} V_0^3 \int d^3 R \int d^3 r \int \frac{d^3 k}{(2\pi)^3} r^2 e^{i k \cdot r} : q^\dagger(x) [\Lambda_+(k) - \Lambda_-(k)] q(y) : \]

\[H_2^R = \frac{8}{3} V_0^3 \int d^3 R \int \frac{d^3 k}{(2\pi)^3} R^2 e^{i k \cdot r} : q^\dagger(x) [\Lambda_+(k) - \Lambda_-(k)] q(y) : \]

\[H_4 = -\frac{1}{2} V_0^3 \int d^3 R \int d^3 r (r^2 - 4R^2) : q^\dagger(x) T^a q(x) q(y) T^a q(y) : \]

where \( : \) is the normal ordering operator and \( V \) is the volume of the space. \( E \) is the vacuum energy, \( H_2^R \) is the light quark kinetic energy on the physical vacuum, \( H_2^R \) is the binding interaction (as far as the bare \( q\bar{q} \) mass is concerned we do not need to evaluate \( H_4 \) matrix elements).

- **The gap equation** \( \delta \mathcal{E}(\phi) = 0 \). Explicitly it reads

\[
m \cos \phi(k) - k \sin \phi(k) + \frac{2}{3} V_0^3 \left( \phi(k)'' + \frac{2}{k} \phi(k)' + \frac{2}{k^2} \cos \phi(k) \sin \phi(k) \right) = 0.
\]

The light quark condensate can be calculated and gives:

\[
\langle 0 | \bar{q} q | 0 \rangle = -\frac{3}{8} \int_0^\infty dk k^2 \sin \phi(k).
\]

On the solution of Eq. \((15)\) we get for \( m = 0 \)

\[
\langle 0 | \bar{q} q | 0 \rangle = -\frac{1}{24} T_g G_2 \times 0.3722 \tag{16}
\]

The result \((14)\) is appealing. It establishes a connection between the gluon condensate and the light quark condensate. The connection is possible since the non-local gluon condensate has introduced into the game a finite correlation length \( T_g \).

- **The bound state equation.** Taking the matrix element of \( H_2^R \) between a one-particle state of momentum \( p \) and a one-particle state of momentum \( q \), we have

\[
H_2^{R}\langle p, q \rangle_{ss'} \equiv \langle 0 | b_s(p) H_2^R b_{s'}(q) | 0 \rangle =
\]

\[
\frac{8}{3} V_0^3 u_s(p) \left\{ \beta \sin \left[ \phi \left( \frac{p + q}{2} \right) \right] + \frac{\alpha \cdot \hat{p} + \alpha \cdot \hat{q}}{2} \cos \left[ \phi \left( \frac{p + q}{2} \right) \right] \right\} u_{s'}(q)
\]

\[
\times \left[ -\Delta (2\pi)^3 \delta^3(p - q) \right].
\]

As expected the binding interaction would be chiral invariant \((\sim \alpha \cdot \hat{p} + \alpha \cdot \hat{q})\) for a massless particle on the perturbative vacuum \((\phi = 0)\). While for a infinitely massive particle \((\phi = \pi/2)\) chiral invariance would be maximally broken. In our case the solution of the gap equation \((15)\) gives rise to a binding interaction which contains two pieces. One is chiral invariant and the other, proportional to \( \beta \), breaks explicitly chiral invariance. The existence of such a term is suggested by the spin-orbit structure of the heavy quarkonium potential whose relativistic origin may be traced back to a scalar confining Bethe–Salpeter kernel. In a Hamiltonian language this would just correspond to an interaction
proportional to $\beta$. On the contrary, here we obtain an interaction not only proportional to $\beta$. It manifests, also under the strong simplifying assumption (12), a more complicate structure which interpolates between a chiral invariant vector interaction and a scalar interaction.

Summing up the contributions coming from the pieces $H^R_2$ and $H^R_2$ of the Hamiltonian, the bound state equation on the physical vacuum reads

$$\left\{ E(p) + \frac{8}{3} V_0^3 \left( \frac{1}{2p^2} [1 - \sin \phi(p)]^2 + \frac{2}{p^2} [1 - \sin \phi(p)] \mathbf{S} \cdot \mathbf{L} - \Delta \right) \right\} \Phi(p) = \bar{\Lambda} \Phi(p),$$

where we have introduced the spin operator $\mathbf{S} = \sigma/2$ and the orbital angular momentum operator $\mathbf{L} = \mathbf{r} \times \mathbf{p}$. The eigenvalues $\bar{\Lambda}$ of the equation are the energy levels of the bound state in the non-recoil limit, i.e. the difference between the mass of the considered heavy-light meson and the mass of the corresponding heavy quark.

6 Conclusions

We have described the heavy quark and the heavy-light quark bound state system using the same gauge-invariant approach and in terms of the two nonperturbative parameters $T_g$ and $G_2$. Chiral symmetry breaking and a chiral non-invariant binding interaction emerge quite naturally in our approach and a link is established between chiral symmetry breaking properties and confining interaction. In particular with Eq. (16) we establish a relation between the order parameter of chiral symmetry (the quark condensate $\langle 0 | \bar{q} q | 0 \rangle$) and that one which in our framework describes confinement (the gluon correlation length $T_g$). The actual calculations were performed under the rough approximation (12). This is unrealistic since it gives in the heavy quark limit a confining potential which is not linear. Moreover all magnetic contributions were not considered. Nevertheless we expect that the main features presented will still hold using a realistic parameterization of the bilocal gluon condensate.

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