Rectification of displacement currents in an adiabatic electron pump

P. W. Brouwer

Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, NY 14853-2501

(January 31, 2022)

Rectification of ac displacement currents generated by periodic variation of two independent gate voltages of a quantum dot can lead to a dc voltage linear in the frequency $\omega$. The presence of this rectified displacement current could account for the magnetic field symmetry observed in a recent measurement on an adiabatic quantum electron pump by Switkes et al. [Science 283, 1905 (1999)].

PACS numbers: 72.10.Bg, 73.23.-b, 73.40.Ei

In a recent publication, Switkes et al. reported measurements on an electron pump that uses periodic changes of the electronic wavefunction as the pumping mechanism. This “adiabatic quantum electron pump” consists of a quantum dot coupled to two electron reservoirs via ballistic point contacts. The shape of the quantum dot can be controlled by two independent gate voltages $X_1$ and $X_2$. The shape changes are small enough that they only affect the quantum mechanical wavefunction of electrons in the quantum dot, not their classical trajectories. Out-of-phase variation of the gate voltages generates a dc voltage $\bar{V}$ across the dot or a dc current $\bar{I}$ through the dot, depending on the measurement setup.

While some features of the observed dc voltage $\bar{V}$ agree well with the theoretical models of an adiabatic quantum electron pump, three remarkable observations of the experiment could not be explained. (Following Ref. 1, I focus on the voltage measurement; the situation of a current measurement, which was considered in the theories of Refs. 2–4 is discussed later.) First, for small driving amplitudes $\delta X_{1,2}$, where $\bar{V}$ is proportional to $\delta X_1 \delta X_2$, the measured dc voltage is a factor $\sim 20$ higher than the maximum voltage allowed by the theory for an adiabatic quantum pump in the small-amplitude regime. Second, the measured voltage $\bar{V}$ is symmetric under reversal of the magnetic field $B$, while the theory predicts no symmetry relation between $\bar{V}(B)$ and $\bar{V}(-B)$. (Though implicit in the theory of Ref. 2, this was not noted until after publication of the experimental findings.) Furthermore, in the experiment, the magnitude of the mesoscopic fluctuations of the dc voltage decreases by a factor two upon application of a time-reversal symmetry breaking magnetic field, while the theory has no difference between $B = 0$ and $B \neq 0$.

In this note, I show that the experimental observations of Ref. 1 can be explained if the device of Ref. 1 also serves as a rectifier for an ac bias current generated by a (parasitic) capacitive coupling of the gate voltage to the electron reservoirs. This scenario does not rule out the possibility that the device of Ref. 1 also pumps electrons. However, to explain the experimental data it is sufficient that the rectification voltage obscures the true pumped voltage.

An equivalent electrical circuit depicting the experimental setup for a voltage measurement and including the stray capacitances $C_1$ and $C_2$ between the gates and the electron reservoirs, is shown in Fig. 1a. At low frequencies, the ac gates voltages $X_1$ and $X_2$ generate the current

$$ I(t) = C_1 \frac{dX_1}{dt} + C_2 \frac{dX_2}{dt} \tag{1} $$

through the quantum dot. For slow variations of the gate voltages $X_1$ and $X_2$, the voltage $V(t)$ across the quantum dot is $V(t) = I(t)/G(t)$, where $G(t)$ is the dc conductance of the quantum dot and the time-dependence of $G$ arises from the time-dependence of the gate-voltages $X_1$ and $X_2$ controlling the shape of the quantum dot. The dc component $V_{\text{rect}}$ of the voltage is found by averaging over one period $\tau = 2\pi/\omega$,

$$ V_{\text{rect}} = \frac{1}{\tau} \int_0^\tau dt V(t), \tag{2} $$

which can be rewritten as an integral over the area $S$ enclosed by the contour in $(X_1, X_2)$ space traced out by the gate voltages $X_1$ and $X_2$ in one cycle (see Fig. 1b),

$$ V_{\text{rect}} = \frac{\omega}{2\pi} \int_S dX_1 dX_2 \frac{1}{G^2} \left( C_1 \frac{\partial G}{\partial X_2} - C_2 \frac{\partial G}{\partial X_1} \right). \tag{3} $$

Note that variation of only one gate voltage does not produce a dc current to first order in the frequency $\omega$.

If the experimental setup is such, that current is measured, rather than voltage, the ac gate voltages $X_1$ and $X_2$ serve as an ac voltage source across the quantum dot, $V(t) = RC_1 \frac{dX_1}{dt} + RC_2 \frac{dX_2}{dt}$, where $R$ is the resistance of the circuit path containing the current meter (assumed to be much smaller than the resistance of the quantum dot). Averaging over the period $\tau$, one finds the rectified current

$$ I_{\text{rect}} = \frac{\omega}{2\pi} R \int_S dX_1 dX_2 \left( C_2 \frac{\partial G}{\partial X_1} - C_1 \frac{\partial G}{\partial X_2} \right). \tag{4} $$

This expression has to be compared with the expression for the pumped current in the case of adiabatic pumping, which has the same integration domain as Eq. (3), but a different integrand.

For a quantum dot with an irregular shape (a chaotic quantum dot), as in Ref. 1, the statistical distribution...
and the magnetic field dependence of the conductance $G$ and its derivatives $\partial G/\partial X_1$, $\partial G/\partial X_2$ are known in the literature. The reader is referred to Ref. [8] for a review. Here we limit ourselves to a summary of the key qualitative properties of $V_{\text{rect}}$ that can be compared to the experimental observations reported in Ref. [1]. (The current $I_{\text{rect}}$ has the same qualitative properties.)

1. For small driving amplitudes $\delta X_1 = \delta X_2 \equiv \delta X$, one has $V_{\text{rect}} \propto (\delta X)^2$; for large $\delta X$, one has $V_{\text{rect}} \propto (\delta X)^{1/2}$. The crossover between small-amplitude driving and large-amplitude driving occurs at amplitude $\delta X \sim (e^2/h)\sigma(\partial G/\partial X)$, corresponding to $V_{\text{rect}} \sim (e^2/h)^2 G^{-2} C^2 \omega^2 \sigma(\partial G/\partial X)$, where $\sigma(\ldots)$ denotes standard deviation with respect to mesoscopic fluctuations.

2. The ensemble average ($\langle V_{\text{rect}} \rangle$) $= 0$.

3. The dc voltage $V_{\text{rect}}$ is symmetric under reversal of the magnetic field, $V_{\text{rect}}(B) = V_{\text{rect}}(-B)$.

4. For multichannel point contacts, the variance of $V_{\text{rect}}$ is decreased by a factor two when time-reversal symmetry is broken by a magnetic field. (The experiment is for two-channel point contacts.)

All of the properties 1 – 4 were observed by Switkes et al. Properties 1 and 2 also apply to the voltage generated by an adiabatic quantum pump, except for the magnitude of the voltage at the crossover between small-amplitude and large-amplitude driving, which is $V_{\text{pump}} \sim \omega e G$ for an adiabatic quantum pump, corresponding to a current of $\sim 1$ electron per cycle. The dc voltage observed in Ref. [1] corresponds to $\sim 20$ electrons per cycle at the crossover between small-amplitude and large-amplitude driving. Properties 3 and 4 do not hold for an adiabatic quantum pump. In view of the magnetic field symmetry and the magnitude of the observed dc voltage, it cannot be excluded that the voltage $V$ measured in Ref. [1] is dominated by the rectification voltage $V_{\text{rect}}$.

How can the rectification voltage $\langle V \rangle$ be distinguished from the voltage generated by a true quantum pump if, in principle, both are present in the same experiment? First, when the rectification voltage is dominant, Eq. [3] should provide a verifiable relation between the measured dc voltage $\bar{V}$ and the conductance $G$ and its derivatives $\partial G/\partial X_{1,2}$, which can be measured independently of $\bar{I}$. Second, the two voltages are easily distinguished by their magnetic field symmetry: the rectification voltage is magnetic-field symmetric, while the voltage generated by a quantum pump is not. It may be easier to measure pumping in a current measurement geometry with low-resistance contacts, as, in that case, the rectification current scales with the resistance $R$ of the circuit path containing the current meter.

I would like to thank C. M. Marcus, D. C. Ralph, and M. G. Vavilov for very helpful discussions. This work was supported by the NSF under grant no. DMR 0086509 and by the Sloan foundation.

1. M. Switkes, C. M. Marcus, K. Campman, and A. C. Gossard, Science 283, 1905 (1999).
2. P. W. Brouwer, Phys. Rev. B, 58, 10135 (1998).
3. F. Zhou, B. Spivak, B. Altshuler, Phys. Rev. Lett. 82, 608 (1999).
4. T. A. Shutenko, I. L. Aleiner, and B. L. Altshuler Phys. Rev. B 61, 10366 (2000).
5. I. L. Aleiner, B. L. Altshuler, and A. Kamenev, Phys. Rev. B 62, 10373 (2000).
6. A magnetic-field symmetric voltage, but not current, is expected for an electron pump with single-channel point contacts, see M. L. Polianski and P. W. Brouwer, in preparation. (That same conclusion was reached independently by I. Smolyarenko.) Magnetic-field symmetry is expected for both pumped current and voltage if the pump has a spatial symmetry, see Ref. [7].
7. Rectification of an ac bias generated by capacitive coupling of the gates and the electron reservoirs was previously proposed by Switkes to explain the presence of a residual dc current when the two gate voltages $X_1$ and $X_2$ were varied in phase. See M. Switkes, Ph. D. thesis (Stanford, 1999).
8. C. W. J. Beenakker, Rev. Mod. Phys. 69, 731 (1997).
9. In Ref. [1], the measured voltage at the crossover between small-amplitude driving and large-amplitude driving is 0.5 $\mu$V at driving frequency 10 MHz. The amplitude of the gate voltages at that point is 80 $\mu$V. For comparison, the dc voltage corresponding to one electron per cycle at a driving frequency of 10 MHz is 0.02 $\mu$V.