Quantum decoherence and gravitational waves

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The quite different behaviors exhibited by microscopic and macroscopic systems with respect to quantum interferences suggest the existence of a borderline beyond which quantum systems lose their coherences and can be described classically. Gravitational waves, generated within our galaxy or during the cosmic expansion, constitute a universal environment susceptible to lead to such a quantum decoherence mechanism. We assess this idea by studying the quantum decoherence due to gravitational waves on typical microscopic and macroscopic systems, namely an atom interferometer (HYPER) and the Earth-Moon system. We show that quantum interferences remain unaffected in the former case and that they disappear extremely rapidly in the latter case. We obtain the relevant parameters which, besides the ratio of the system’s mass to Planck mass, characterize the loss of quantum coherences.

I. INTRODUCTION

Quantum decoherence is a universal phenomenon which affects all physical systems as soon as they are coupled to a fluctuating environment. This effect plays an important role in the transition between quantum and classical behaviors, by washing out quantum coherences and thus justifying a purely classical description \([1, 2, 3, 4, 5]\). This implies that quantum decoherence should be very efficient for macroscopic systems, while remaining inefficient for microscopic ones. The quantum/classical transition would then introduce a borderline between microscopic and macroscopic systems.

Existing experimental observations of quantum decoherence confirm these intuitions. Decoherence has only been seen on ‘mesoscopic’ systems for which the decoherence time is neither too long nor too short, such as microwave photons stored in a high-Q cavity \([6]\) or trapped ions \([7]\). In such model systems, the environmental fluctuations are particularly well mastered and the quantum/classical transition has been shown to fit the predictions of decoherence theory \([8, 9]\).

It has also been early remarked that Planck mass, that is the mass scale which can be built up on Planck constant \(\hbar\), light velocity \(c\) and Newton gravitation \(G\), lies at the borderline between microscopic and macroscopic masses

\[
m_P = \sqrt{\frac{\hbar c}{G}} \sim 22\mu g
\]

That is to say, one may define microscopic and macroscopic values of a mass \(m\) by comparing the associated Compton length \(\ell_C\) to the Planck length \(\ell_P\)

\[
m \leq m_P \Leftrightarrow \ell_P = \sqrt{\frac{\hbar G}{c^3}} \leq \ell_C = \frac{\hbar}{mc}
\]

It is tempting to consider that this property is not just an accidental coincidence but rather reveals a general consequence of fundamental gravitational fluctuations \([11, 12, 13]\). Then, one is led to study the role that the fluctuating gravitational environment might play in the transition from quantum to classical behaviors.

Here, we briefly discuss the quantum decoherence due to our local gravitational environment, namely the stochastic background of gravitational waves surrounding the Earth. Details can be found in previously published work \([14, 15, 16, 17, 18]\). First, taking the example of the atomic interferometer HYPER, we show that gravitational waves do not lead to a significant decoherence at the microscopic level. We then show that, on the contrary, scattering of gravitational waves is the dominant decoherence mechanism, and an extremely efficient one, for macroscopic systems such as the Moon around the Earth. We also go beyond the simple scaling arguments just given above by providing estimates of gravitational quantum decoherence depending not only on the mass of the system, but also on its velocity, on its geometry and on the noise spectrum characterizing the gravitational fluctuations.
II. GRAVITATIONAL ENVIRONMENT

We first describe the fundamental fluctuations of space-time which originate from our gravitational environment and which are bound to play a crucial role in quantum decoherence. For current quantum systems which are only sensitive to frequencies lying far below Planck frequency, general relativity provides the appropriate description of gravitational phenomena [12], even if it may ultimately be replaced by a theory of quantum gravity. It follows that the relevant spacetime fluctuations which constitute our gravitational environment are simply the gravitational waves predicted by the linearized theory of gravity [20, 21, 22] and which are thoroughly studied in relation with the present development of gravitational wave detectors [23, 21, 25, 26].

Gravitational waves correspond to perturbations of the metric field and can be written in the transverse traceless (TT) gauge

\[ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad \eta_{\mu\nu} = \text{diag}(1, -1, -1, -1) \]

\[ h_{00} = h_{i0} = h_{i}^i = 0 \]  

(3)

Each Fourier component is a sum over the two circular polarizations \( h^{\pm} \), which are obtained as products of the polarization vectors \( \varepsilon^{\pm} \) well-known from electromagnetic theory. Gravitational waves correspond to wavevectors \( k \) lying on the light cone \( (k^2 = k_{\mu}k^{\mu} = 0) \), they are transverse with respect to this wavevector \( (k^{\mu}\varepsilon^{\pm}_{\mu} = 0) \) and the metric perturbation has a null trace \( (\varepsilon^{\pm})^2 = 0 \).

We consider for simplicity the case of stationary, unpolarized and isotropic backgrounds. Then, a given metric component, say \( h = h_{12} \), is a stochastic variable characterized by a noise spectrum \( S_h \)

\[ \langle h(t) h(0) \rangle = \int \frac{d\omega}{2\pi} S_h[\omega] e^{-i\omega t} \]

(5)

\( S_h \) is the spectral density of strain fluctuations considered in most papers on gravitational wave detectors (see for example [24]). It can be written in terms of the mean number \( n_{gw} \) of gravitons per mode or, equivalently, of a noise temperature \( T_{gw} \) with \( k_B \) the Boltzmann constant and \( G \) the Newton constant

\[ S_h = \frac{16G}{5c^5} h_{\omega} n_{gw} = \frac{16G}{5c^5} k_B T_{gw} \]

(6)

Knowledge on gravitational wave backgrounds comes from studies estimating the probability of events which might be observed by interferometric detectors of gravitational waves. An important component is constituted by the ‘binary confusion background’, that is the estimated level for the background of gravitational waves emitted by unresolved binary systems in the galaxy and its vicinity. This ‘binary confusion background’ leads to a nearly flat function \( S_h \), that is also to a nearly thermal spectrum, in the \( \mu \)Hz to \( 10 \)mHz frequency range [23]

\[ 10^{-6} \text{Hz} < \frac{\omega}{2\pi} < 10^{-4} \text{Hz} \]

\[ S_h \sim 10^{-34} \text{Hz}^{-1} \]  

(7)

With the conversion factors given above, this corresponds to an extremely large equivalent noise temperature \( T_{gw} \sim 10^{41} \) K. It is worth stressing that this is only an effective noise temperature. Such a value, larger than Planck temperature \( (\sim 10^{32} \) K), does not correspond to an equilibrium temperature and is allowed by the weakness of gravitational coupling.

Previous estimations correspond to the confusion background of gravitational waves emitted by binary systems in our Galaxy or its vicinity. Because of the large number of unresolved and independent sources, and as a consequence of the central limit theorem, they lead to a stochastic noise obeying gaussian statistics. There also exist predictions for gravitational backgrounds associated with a variety of cosmic processes [24], which are however model dependent and have a more speculative character. Associated temperatures vary rapidly with frequency and are dominated by the confusion binary background in the frequency range considered here.
III. QUANTUM DECOHERENCE OF ATOMIC INTERFEROMETERS

Atoms used in interferometry appear as particularly interesting microscopic systems for studying quantum decoherence, as it has recently been suggested that matter-wave interferometers could reveal the existence of intrinsic spacetime fluctuations, through an induced Brownian motion [27, 28]. Although it has not been possible to observe such an effect in existing matter-wave interferometers, instruments are now being designed, like the atomic interferometer HYPER for measuring the Lense-Thirring effect in space, which possess a very high sensitivity to gravitation fields [29]. It is thus important, in order to confirm the viability of such instruments, to obtain quantitative estimates of potential decoherence effects, in particular those associated with spacetime or gravitation fluctuations.

We shall consider the atomic field of the matter-wave interferometer HYPER as a typical example of a microscopic system affected by quantum decoherence (see for instance [30, 31, 32] for details on atomic interferometry). HYPER is an interferometer with a rhombic geometry which is used as a gyrometer, that is to say, its rotation with respect to inertial frames is measured through the observation of a Sagnac effect. The Sagnac dephasing $\Phi$ is proportional to the mass $m_{\text{at}}$ of the (non relativistic) atoms, to the area $A$ of the interferometer and to the rotation frequency $\Omega$

$$\Phi = \frac{1}{\hbar} \int p_i dx^i = \frac{2m_{\text{at}} A}{\hbar} \Omega, \quad p_\mu = g_{\mu \nu} m_{\text{at}} v_\nu, \quad A = v_{\text{at}}^2 \tau_\text{at} \sin \alpha$$  \hspace{1cm} (8)

$g_{\mu \nu}$ is the metric field in the frame of the rotating interferometer, $v_{\text{at}}$ is the atomic velocity, and the area $A$ is given by the length $v_{\text{at}} \tau$ of the rhomb side and the aperture angle $\alpha$ ($\tau_{\text{at}}$ is the time of flight on one rhomb side).

According to general relativity, a local inertial frame in the neighborhood of a rotating massive body differs from the celestial frame determined by the ‘fixed stars’ as a consequence of the dragging of inertial frames. This gravitomagnetic (Lense-Thirring) effect in the Earth neighborhood is measured by HYPER interferometer, by comparing the local celestial frame determined by the ‘fixed stars’ as a consequence of the dragging of inertial frames. This gravitomagnetic (Lense-Thirring) effect in the Earth neighborhood is measured by HYPER interferometer, by comparing the local inertial measurement performed by the atoms to the indication of a star tracker. A map of the Lense-Thirring effect around the Earth is obtained by recording the dephasings and building the corresponding interferogram for each position of the satellite on its orbit.

Gravitational waves, like other gravitational perturbations such as the Lense-Thirring effect, induce a dephasing of the matter waves within the two arms of the interferometer and thus affect the interference fringes [33].

$$\delta \Phi_{\text{gw}} = \frac{m_{\text{at}}}{2\hbar} \int h_{ij} v_{\text{at}}^i v_{\text{at}}^j d\tau = \frac{2m_{\text{at}} A}{\hbar} \delta \Omega_{\text{gw}}$$  \hspace{1cm} (9)

Metric components are evaluated in the TT (transverse traceless) gauge. Using the symmetry of the rhomb, this expression may be obtained from the derivative of the metric component $h_{12}$ lying in the spatial plane defined by the interferometer

$$\delta \Omega_{\text{gw}}(t) = \frac{1}{2} \frac{d \overline{h_{12}}}{dt}, \quad \overline{h_{12}}(t) = \int h_{12}(t - \tau) g(\tau) d\tau$$  \hspace{1cm} (10)

The linear filtering function $g$ has a triangular shape which reflects the distribution of the time of exposition of atoms to gravitational waves inside the rhombic interferometer. The square of its Fourier transform, which describes linear filtering in frequency space, is an apparatus function characterizing the interferometer [16]

$$|\tilde{g}[\omega]|^2 = \left( \frac{\sin \frac{\omega \tau_{\text{at}}}{2}}{\frac{\omega \tau_{\text{at}}}{2}} \right)^4$$  \hspace{1cm} (11)

We now consider the degradation of fringe contrast obtained by averaging over stochastic dephasings. This evaluation [16] can be shown to be equivalent to the other approaches to decoherence (see for example [34]). Stochastic gravitational waves with frequencies higher than the inverse of the averaging time identify with the unobserved degrees of freedom which are usually traced over in decoherence theory (see [8] and references therein). When $\delta \Phi_{\text{gw}}$ is a gaussian stochastic variable, the degraded fringe contrast is read as

$$\langle \exp(i \delta \Phi_{\text{gw}}) \rangle = \exp \left( -\frac{\Delta \Phi_{\text{gw}}^2}{2} \right), \quad \Delta \Phi_{\text{gw}}^2 = \langle \delta \Phi_{\text{gw}}^2 \rangle$$  \hspace{1cm} (12)

Using the expression of $\delta \Phi_{\text{gw}}$ in terms of the averaged time derivative of $h_{12}$ we write the variance $\Delta \Phi_{\text{gw}}^2$ as an integral over the noise spectrum $S_h$ [35]. Particularly interesting is the case of an approximately flat or thermal spectrum $S_h$ which, as discussed in previous section, is approximately realized by the binary confusion background on a significant
frequency range. With a white noise assumption, the variance is found to be proportional to the constant value of the noise spectrum $S_h$

$$\Delta \Phi_{gw}^2 = \left( \frac{2m_{at} v_{at}^2 \sin \alpha}{\hbar} \right)^2 S_h 2\tau_{at}$$  \hspace{1cm} (13)

After substitution of the numbers corresponding to HYPER [29], we deduce that the decoherence due to the scattering of gravitational waves is completely negligible

$$\Delta \Phi_{gw}^2 \sim 10^{-20} \ll 1$$  \hspace{1cm} (14)

We have discussed here the decoherence effect on atomic fields. In fact, it appears that the decoherence effect affecting the laser fields, involved in the stimulated Raman processes used for building up beam splitters and mirrors for matter waves, provides a larger contribution [16]. But this changes neither the mechanism of quantum decoherence which has been discussed here, nor its incidence on the instrument sensitivity. The phase noise induced by the scattering of gravitational waves remains completely negligible with respect to the phase noise induced by mechanical vibrations of the mirrors. In the real instrument, decoherence is expected to be induced by instrumental fluctuations rather than by fundamental fluctuations.

IV. QUANTUM DECOHERENCE OF PLANETARY SYSTEMS

After discussing the microscopic case on the example of atomic interferometers, we come to a case which lies at the opposite end, as it can be considered as extremely macroscopic, namely the planetary system built by the Moon orbiting around the Earth. The classicality of such a system may be expected to result from the strong efficiency of decoherence mechanisms acting on it, contrarily to the case of microscopic systems. Indeed, as we show, gravitational waves lead to an extremely rapid decrease of quantum coherences for such macroscopic systems. Moreover, although decoherence may usually be attributed to collisions of residual gas, to radiation pressure of solar radiation or, even, to the scattering of electromagnetic fluctuations in the cosmic microwave background, we show that, in the case of planetary motions, it is dominated by the scattering of stochastic gravitational waves.

The Earth and Moon constitute a binary system with a large quadrupole momentum, so that its internal motion is highly sensitive to gravitational waves. For the sake of simplicity, we shall describe the Earth-Moon system as a circular planetary orbit in the plane $x_1x_2$. The reduced mass $m$, defined from the masses of the two bodies, will be used, such as the radius $\rho$, that is the constant distance between the two masses, so that the orbital frequency $\Omega$, the normal acceleration $a$ on the circular orbit and the tangential velocity $v$ obey usual relations

$$a = \rho \Omega^2 = \frac{v^2}{\rho}$$  \hspace{1cm} (15)

Gravitational waves will be represented as metric perturbations $h_{\mu\nu}$, taken in the TT gauge [3], so that they will be related to Riemann curvature ($R_{0i0j} = \partial_i^2 h_{ij} \equiv \ddot{h}_{ij}$). The gravitational wave perturbation on the relative position $x^i$ in the binary system amounts to a tidal force $\delta F$ which may also be seen as a geodesic deviation

$$\delta \dot{p}_i(t) = \delta F_i(t) = mc^2 R_{0i0j} x^j(t)$$  \hspace{1cm} (16)

The stochastic background of gravitational waves then induces a Brownian motion on the relative position of the Moon, which may be characterized by a momentum diffusion with a variance varying linearly with the time of exposition $\tau$

$$< \delta p^2(t) > = 2D_{gw} \tau$$  \hspace{1cm} (17)

The momentum diffusion coefficient $D_{gw}$ is determined by the correlation function of gravitational waves [6,10,14]

$$D_{gw} = m \Gamma_{gw} k_B T_{gw}, \quad \Gamma_{gw} = \frac{32Gma^2_{eff}}{5c^3}$$  \hspace{1cm} (18)

$T_{gw}$ is the effective noise temperature of the gravitational background, evaluated at twice the orbital frequency, and $\Gamma_{gw}$ is the damping rate associated with the emission of gravitational waves. One recovers with equations (18) the fluctuation-dissipation relation on Brownian motion [35] and the quadrupole formula for gravitational wave emission [36] determined by Einstein. Although gravitational damping can be observed in the case of strongly bound binary systems [37], it appears to be extremely small for the Moon ($\Gamma_{gw} \approx 10^{-34} \text{ s}^{-1}$), with a negligible impact on its mean
motion. Moreover, it can be seen to be much smaller than the damping due to other environmental fluctuations, such as electromagnetic radiation pressure or Earth-Moon tides. The latter appear to give the dominant contribution to damping \[ \Gamma_{\text{em}} < \Gamma_{\text{tides}} \] (19)

However, as we show now, decoherence processes do not follow the same hierarchy.

Quantum decoherence may be evaluated by considering two neighbouring internal motions of the planetary system which correspond to the same spatial geometry but slightly different values of the epoch, the time of passage at a given space point. For simplicity, we measure this difference by the spatial distance \( \Delta x \) between the two motions, which is constant for uniform motion. The variation of momentum results in a perturbation of the quantum phase one may associate with the relative position in the binary system

\[ \delta \Phi_{\text{gw}}(t) = \frac{\delta p_i(t)}{\hbar} \Delta x^i \] (20)

The difference of phase between two neighboring motions then undergoes a Brownian motion, resulting in a random exponential factor \( e^{i \delta \Phi_{\text{gw}}} \). Averaging this quantity over the stochastic effect of gravitational waves, still supposed to obey gaussian statistics, one obtains a decoherence factor

\[ \langle e^{i \delta \Phi_{\text{gw}}} \rangle = \exp\left(-\frac{\Delta \Phi^2_{\text{gw}}}{2}\right) \] (21)

The decoherence factor may be expressed in terms of the variables characterizing the Brownian motion and the distance between the two motions \( \Delta x \)

\[ \Delta \Phi^2_{\text{gw}} = \frac{2D_{\text{gw}} \Delta x^2 \tau}{\hbar^2} \] (22)

Relation (22) agrees with the result expected from general discussions on decoherence: decoherence efficiency increases exponentially fast with \( \tau \) and \( \Delta x^2 \).

Relation (22) may be rewritten in terms of the gravitational waves spectrum and the geometric parameters of the binary system

\[ \Delta \Phi^2_{\text{gw}} = \left(\frac{2mv^2}{\hbar} \sin \alpha\right)^2 S_h 2\tau, \quad \sin \alpha = \frac{\Delta x}{2\rho} \] (23)

\( \frac{2mv^2}{\hbar} \sin \alpha \) is a frequency determined by the kinetic energy of the Moon and \( \sin \alpha \) is the aperture angle of the equivalent interferometer. In the case of the Earth-Moon system, one finds an extremely short decoherence time up to extremely short distances \( \Delta x \) (in the 10\( \mu \)s range for \( \Delta x \) of the order of the Planck length)

\[ D_{\text{gw}} \approx 10^{75} \text{s}^{-1} \text{m}^{-2} \] (24)

The gravitational contribution to decoherence appears to be much larger than the contributions associated with tide interactions and electromagnetic scattering

\[ D_{\text{gw}} \gg D_{\text{tides}} > D_{\text{em}} \] (25)

When compared with contributions to damping (19), decoherence contributions obey a modified hierarchy. This results from their further dependence on the level of noise induced by the environment and from the fact that gravitational waves constitute the environment with the largest effective noise temperature (7). To be precise, the ratio \( \frac{\Gamma_{\text{gw}}}{\Gamma_{\text{tides}}} \) of the damping constants associated with gravitational waves and tides is of the order of \( 10^{-16} \), while the ratio \( \frac{T_{\text{gw}}}{T_{\text{tides}}} \) is of the order of \( 10^{38} \). It follows that the ratio \( \frac{D_{\text{gw}}}{D_{\text{tides}}} \) remains very large and that the gravitational contribution to decoherence dominates the other ones.

The dominant mechanism leading to the classical behavior of very macroscopic systems appears to be due to gravitational waves, originating either from the confusion binary background in our galaxy or from extragalactic sources in a larger region of the universe. It is remarkable that the classicality and the ultimate fluctuations of very macroscopic systems appear to be determined by the classical gravitation theory which also explains their mean motion.
V. GRAVITATIONAL QUANTUM DECOHERENCE

The results obtained in the previous sections for gravitationally induced decoherence are reminiscent of the qualitative discussions of the Introduction. For microscopic probes, such as the atoms or photons involved in atomic interferometers, decoherence is so inefficient that it can be ignored with the consequence that quantum mechanics remains the appropriate description. For macroscopic bodies on the contrary, such as the Moon-Earth system, decoherence is extremely efficient with the consequence that potential quantum coherences between different positions can never be observed, leading to an appropriate purely classical description.

The scale arguments sketched in the Introduction may also be associated with precise expressions. In both the microscopic [13] and macroscopic [23] cases, the decoherence factor $e^{-\Delta \Phi^2_{gw}}$ induced by the gravitational environment takes a same form. It involves as an essential factor the gravitational spectral density $S_B$ [6], which may be expressed as an effective noise temperature, putting into evidence its dependence on Planck mass $m_P$:

$$S_B \simeq \Theta_{gw} t_P^2, \quad t_P = \frac{hG}{c^3} = \left( \frac{h}{m_P c^2} \right)^2, \quad \Theta_{gw} \simeq \frac{k_B T_{gw}}{\hbar} \simeq 10^{52} \text{s}^{-1}$$

(26)

$\Theta_{gw}$ is the temperature of the background measured as a frequency. Relations [13] and [23] may then be rewritten

$$\frac{\Delta \Phi^2_{gw}}{2} \simeq \left( \frac{2mv^2 \sin \alpha}{m_P c^2} \right)^2 \Theta_{gw} t$$

(27)

The ratio $\frac{m^2}{m_P}$ confirms the preliminary arguments of the Introduction, namely that the Planck mass effectively plays a role in the definition of a borderline between microscopic and macroscopic masses. However, other factors in the formula imply that the scaling argument on masses is not sufficient to obtain correct quantitative estimates. The ratio of the probe velocity over light velocity, the equivalent aperture angle $\alpha$ and the frequency $\Theta_{gw}$, measuring the gravitational noise level, enter the quantum decoherence time on an equal footing. In particular, the very large value of the gravitational noise level implies that the transition between quantum and classical behaviors could in principle be observed for masses smaller than Planck mass. Another interesting feature is that the parameter to be compared with Planck energy $m_P c^2$ is the kinetic energy $mv^2$ of the probe rather than its mass energy $mc^2$.

Finally, formula (27) provides a valuable insight into the way to design systems aiming at observing the quantum/classical transition induced by intrinsic gravitational fluctuations. The transition region $\Delta \Phi^2_{gw} \sim 1$ seems to be best approached by using heavy and fast particles in a matter-wave interferometer. At present, interference patterns have been observed on rather large molecules [39, 40]. But one checks that, in these experiments, the kinetic energy of the molecules, the area and aperture angle of the interferometer are such that the gravitational quantum decoherence remains negligible, as in HYPER. Increasing these sensitive parameters so that the transition could be approached appears as a formidable experimental challenge [18] (see [41, 42] for using fast molecules). Alternatively, one could consider using quantum condensates [43, 44], an approach however requiring further technological progress.

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