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Periodicity, Thermal Effects, and Vacuum Force: Rotation in Random Classical Zero-Point Radiation.

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Abstract

Thermal effects of acceleration through a vacuum have been investigated in the past from different perspectives, with both quantum and classical methods. However, the existence of the thermal effects associated with rotation in a flat vacuum requires a deeper analysis. In this work we show that for a detector rotating in a random classical zero-point electromagnetic or massless scalar radiation at zero temperature such thermal effects exist. Analysis and calculations are carried out in terms of correlation functions of random classical electromagnetic or massless scalar field in the rotating reference system. This system is constructed as an infinite set of Frenet-Serret tetrads $\mu_\tau$ defined so that the detector is at rest in a tetrad at each proper time $\tau$. Particularly, (1) correlation functions, more exactly their frequency spectrum, contain the Planck thermal factor $1/(\exp(\hbar \omega / k_B T_{rot}) - 1)$, and (2) the energy density the rotating detector observes is proportional to the sum of energy densities of Planck's spectrum at the temperature $T_{rot} = \frac{\hbar \Omega}{2 \pi k_B}$ and zero-point radiation. The proportionality factor is $\frac{2}{3}(4\gamma^2 - 1)$ for an electromagnetic field and $\frac{2}{9}(4\gamma^2 - 1)$ for a massless scalar field, where $\gamma = (1 - (\frac{\Omega}{c})^2)^{-1/2}$, and $r$ is a detector rotation radius. The origin of these thermal effects is the periodicity of the correlation functions and their discrete spectrum, both following rotation with angular velocity $\Omega$. The correlation functions without periodicity properties do not display thermal features. The thermal energy can also be interpreted as a source of a force, $f_{vac}$, applied to the rotating detector from the vacuum field, "vacuum force". The $f_{vac}$ depends on the size of neither the charge nor the mass, like the force in the Casimir model for a charged particle, but, contrary to the last one, it is directed to the center of the circular orbit. The $f_{vac}$ infinitely grows by magnitude when $r \to r_0 = c/\Omega$. Therefore the radius of circular orbits with a fixed $\Omega$ is bounded. The orbits with a radius greater than $r_0$ do not exist simply because the returning vacuum force becomes infinite. On the uttermost orbit with the radius $r_0$, a linear velocity of the rotating particle would have become $c$. The $f_{vac}$ becomes very small and proportional to $r$ when $r$ is small, $r \ll c/\Omega$. Such vacuum force dependence on radius, at large and small $r$, can be associated respectively with so called confinement and asymptotic freedom, known
in quantum chromodynamics, and provide a new explanation for them.

1 Introduction.

This work is focused on thermal effects hypothetically associated with rotation through a vacuum of a massless scalar or electromagnetic field in a flat, Minkowski, space, and performed in a classical approach.

Investigations of rotation are mostly based on the ideas developed for a linear acceleration through a vacuum [1] - [9]. For example, in [1], the authors write: “... in the Rindler case, a set of uniformly accelerated particle detectors ... will give zero response in the Rindler vacuum state, and will give a consistent thermal response to the Minkowski vacuum state.” And later on: “We might therefore expect a set of rotating detectors to similarly reveal the state of a rotating vacuum field”. This program for the rotation case was used, for example, in [3] and [10] and after that in [2]. Below we discuss some results of [2].

A rotating 4-space, with the Trocheries-Takeno (T) coordinates and a non-static non-diagonal metrics, with the associated quantum Fock space (referred below as T-F space) of a massless scalar field are considered in [2], along with Minkowski (M) space and its associated Fock (M-F) quantum space.

The T-coordinates \((t, r, \theta, z)\) are connected with M-coordinates \((\tilde{t}, \tilde{r}, \tilde{\theta}, \tilde{z})\) as:

\[
\begin{align*}
t &= \tilde{t} \cosh \Omega \tilde{r} - \tilde{r} \tilde{\theta} \sinh \Omega \tilde{r}, \\
r &= \tilde{r}, \\
\theta &= \tilde{\theta} \cosh \Omega \tilde{r} - \frac{\tilde{t}}{\tilde{r}} \sinh \Omega \tilde{r}, \\
z &= \tilde{z}.
\end{align*}
\]

The main motivation to use T-coordinates is that a particle at rest in T-space, with constant values of \((r, \theta, z)\), has a velocity \(v(\tilde{r}) = \tanh(\Omega \tilde{r})\) in the M-space, which is less than the speed of light for any \(\tilde{r}\).

The “rotating vacuum” of a massless scalar field in the T-F space is not the Minkowski vacuum, \(|0_M\rangle \neq |0_T\rangle\), because the Bogolubov coefficients of the transformation between creation-annihilation operators \((b_M^+, b_M)\) and \((a_T^+, a_T)\) of M-F and T-F quantum spaces respectively are not equal to zero.
Based on this fact, the response function, \( R(E) \), of a rotating Unruh - De Witt detector in a massless scalar field is obtained in [2]. It describes probability of excitation of the detector with energy \( E \) per unit proper time. The authors consider the \( R(E) \) for three different situations depending on the motion of the detector and the state in which the quantized field is prepared. 1. The response function, referred to as \( R_M^{(r)}(E, R_0) \) [2], for the field in the Minkowski vacuum state, \( |0_M⟩ \), and at the detector rotating \((r)\) in Minkowski space on a circular orbit with radius \( R_0 \). 2. The response function, \( R_T^{(i)}(E, R_0) \), for the field in the rotating vacuum state, \( |0_T⟩ \), of T-F quantum space, and the inertial \((i)\) detector, non rotating, at the distance \( R_0 \) from the field rotation center in M-space. 3. The response function, \( R_T^{(r)}(E, R_0) \), for the field in the rotating vacuum state, \( |0_T⟩ \), of T-F quantum space and the rotating \((r)\) detector in Minkowski space on an orbit with radius \( R_0 \).

The results obtained for the first two scenarios look self-consistent and meet the expectations based on the experience gained from the Rindler case of a uniformly accelerated detector, at least in part. They still do not reveal Planck’s thermal properties of a vacuum associated with rotation.

The third situation is less clear. This is what authors say about it [2]: “...we once again arrive at the same confrontation between canonical quantum field theory and the detector formalism, which was settled by Letaw and Pfautsch and Padmanabhan: how is it possible for the orbiting detector to be excited in the rotating vacuum”. The non-null excitation rate in the third scenario, the authors say in [2], can be attributed to two independent origins: 1. non-staticity of the Trocheries- Takeno metric, and 2. to the Unruh - De Witt detector model adapted in [2]. The Glauber model detector would not be excited in this situation.

The authors in [2] give this problem the following explanation and solution: “Because the rotating vacuum excites even a rotating detector, we consider this as a noise which will be measured by any other state of motion of the detector.” And: “This amounts to saying that the inertial detector will also measure this noise, and we normalize the rate in this situation by subtracting from it the value of \( R_T^{(r)}(E, R_0) \), resulting in a normalized excitation rate for the inertial detector in interaction with the field in the rotating vacuum.” So, instead of \( R_T^{(i)}(E, R_0) \), they use \( \tilde{R}_T^{(i)}(E, R_0) = R_T^{(i)}(E, R_0) - R_T^{(r)}(E, R_0) \).

But, even with this correction, there is one more problem associated with rotating vacuum which is not addressed in [2]. In the Minkowski space, none of the points of the rotating system considered as a “rotating vacuum” has an angular velocity \( \Omega \). Indeed, the angular velocity \( \Omega_M \), in the Minkowski
space, of a point with fixed spatial T coordinates \((r = R_0, \theta = \theta_0, z = z_0)\) is

\[
\Omega_M = \frac{d\theta}{dt} = \frac{1}{R_0} \tanh \Omega R_0 \neq \Omega.
\]  

(2)

So in Trocheres - Takeno coordinates formalism [2], \(\Omega\) is just a parameter without a clear physical sense, and the concept of “rotating vacuum” is ambiguous.

In this work we do not use the concept of “rotating vacuum”. The word “rotation” is associated with a detector moving on a circle only. Our approach to the problem is based on the concept of “measurements” made by a point-like detector, rotating in a scalar or electromagnetic vacuum. Bernard suggested in [11] “to represent measurements by an observable, without describing the detection process,” with a “transformation law which tells us how this observable is modified when the same detector is forced to move along some other world line”. This should be applicable to both quantum and classical theory. Nevertheless, analysis of local measurements in terms of local observables only, without any references to a detector features, turns out to have some restrictions. Indeed, the character of the motion of a detector implies some detector features and therefore determines a detection process. For example, in the frame of special relativity theory, a rotating detector should have a charge to be rotated and held on a circle. Therefore it behaves like a rotating oscillator and should be selective to frequencies. We will show that the angular velocity of the observer is a key parameter to describe the thermal properties of the rotation in random zero-point classical radiation.

Regarding the transformation law of an observable, mentioned above [11], to represent a measurement, both quantum and classical, the simplest assumption is that the observable is an invariant for all possible world lines and coordinate systems. Mathematically an observable with such properties can be described in a tetrad formalism, because tetrad components of vectors and tensors are invariants with respect to coordinate transformations [12] - [17].

A similar approach has been used in [4] for a uniformly accelerated observer, even though the tetrad formalism was not used explicitly. Inertial systems, local in terms of time and each defined at an observer proper time, were used in [4]. The tetrad formalism was used in [7] to describe interaction between two uniformly accelerated oscillators in a vacuum, located in a plane perpendicular to the motion direction.

In this work, the measurements made by the rotating detector are described in the rotating reference system consisting of an infinite number of instantaneous inertial reference frames and mathe-
matically defined as tetrads at each moment of the detector proper time. Along with such a reference system, the two-point correlation functions of the electromagnetic and scalar massless field and energy density of these fields are defined and analyzed for zero-point radiation.

The article is organized as follows.

Section 2 is dedicated to a detector motion through a random classical zero-point electromagnetic radiation. Subsection 2.1 Appendixes A and B The expressions for the components of the electromagnetic field measured at a Frennet-Seret tetrad are found in terms of the field components in the laboratory inertial coordinate system, and correlation functions of the electromagnetic field at a rotating detector are constructed. Subsection 2.2 The correlation function calculation scheme is described. Subsection 2.3 and Appendix C The final expressions of the correlation functions in terms of elementary functions are given. These correlation functions turned out not to display any thermal features. Subsection 2.4 An assumption about the existence of the periodicity of the correlation functions and a discrete spectrum associated with it is discussed and justified. To the best of our knowledge, this idea has not been discussed in the literature yet. Subsection 2.5 Appendix D New correlation functions with the discrete spectrum are constructed. Subsection 2.6 Appendix E An example of the correlation functions with discrete spectrum is calculated and discussed, with the use of the Abel-Plana formula. The temperature $T_{\text{rot}}$ associated with rotation is introduced. Section 3 Expression for energy density of the random classical zero-point electromagnetic field measured by a rotating detector is constructed. It is explicitly shown to display thermal features, following spectrum discreteness observed by the detector. Section 4 is dedicated to detector rotation in massless zero-point scalar field radiation. Subsection 4.1 A correlation function of the massless zero-point scalar field is calculated with the use of tetrad formalism. Subsection 4.2 The correlation function of the massless zero-point scalar field for a discrete spectrum following its periodicity is defined. Its spectrum contains the Planck’s factor. Subsection 4.3 Energy density of the massless scalar field measured by a rotating detector, and their thermal properties connected with the detector rotation and periodicity are obtained and discussed. Section 5 Conclusion and Perspectives.
2 Electromagnetic Field at a Rotating Detector Moving Through a Random Classical Zero - Point Radiation.

2.1 Local Measurements, Tetrads, and Correlation Functions.

Let the detector be a particle moving through an electromagnetic field in Minkowsky space-time and the detector measures it on the world line in a locally inertial reference frame. We assume that the field is classical and in a vacuum state. Mathematical definition of the vacuum field state is given in the next subsection.

The quantities associated with such local measurements can be described in 4-orthogonal tetrad (OT) formalism [14], [18]. Any vector or tensor may be resolved along 4 tetrad vectors $\mu_i^{(a)}$, $a = 1, 2, 3, \text{and } 4$. (The tetrad vectors are described in Appendix A). For example, a 4-vector velocity of a detector and the tensor of electromagnetic field are respectively

$$U_i = U_{(a)}(\mu) \mu_i^{(a)}$$

$$F_{ik} = \mu_i^{(a)} \mu_k^{(b)} F_{(ab)}(\mu).$$

The components

$$U_{(a)}(\mu) = \mu_i^{(a)} U_i$$

and

$$F_{(ab)}(\mu) = \mu_i^{(a)} \mu_k^{(b)} F_{ik}.$$ 

are invariants in the tensorial sense (i.e with respect to coordinate transformation) and defined in a local reference frame with locally lorentz-invariant metrics tensor $\eta_{ab} = \eta^{ab} = \text{diag}(1,1,1,-1)$ (Appendix A). Therefore they describe local observable quantities.

In this work OTs are defined as Frenet-Serret orthogonal tetrads associated with each point of the world line of the rotating detector with 4-vector velocity

$$U^i = c (-\beta \gamma \sin \alpha, \beta \gamma \cos \alpha, 0, \gamma),$$

where $\beta = v/c = \Omega a/c, \gamma = (1 - \beta^2)^{-1/2}, \alpha = \Omega \gamma \tau$, and $\Omega, a$ are angular velocity and circumference radius of the rotating detector respectively. 4-vectors of Frenet-Serret OT, solutions of the equations (78), have the form:

$$\mu_i^{(4)} = \frac{U_i}{c}.$$
\[ \mu_{(1)}^i = (\cos \alpha, \sin \alpha, 0, 0), \]
\[ \mu_{(2)}^i = (-\gamma \sin \alpha, \gamma \cos \alpha, 0, \beta \gamma). \]
\[ \mu_{(3)}^i = (0, 0, 1, 0). \] (7)

In local reference frames, defined by these tetrads, the detector is at rest:
\[ U_{(a)} = \mu_{(a)}^i U_i = \mu_{(a)}^i U^k g_{ik} = (0, 0, 0, -c). \] (8)

The 3-vector acceleration of the detector in it is constant in both magnitude and direction:
\[ \dot{U}_{(a)} = \mu_{(a)}^i \dot{U}_i = \mu_{(a)}^i \dot{U}^k g_{ik} = (-a\Omega^2 \gamma^2, 0, 0, 0), \quad g_{ik} = \text{diag}(1, 1, 1, -1), \] (9)
as it would be in the case of a uniformly accelerated detector. It is why we preferred to use Frenet-Serret tetrads, and not Fermi-Walker ones. Fermi-Walker tetrads do not have this feature (see Appendix A).

Following formulas (5) and (7), the electric \( E_k(\mu|\tau) \) and magnetic \( H_k(\mu|\tau) \) fields, which denote local observable quantities, in the Frenet-Serret reference frame \( \mu_\tau \) at the proper time \( \tau \) of the rotating detector can be given in terms of electric \( E_k \) and magnetic \( H_k \) fields in the inertial laboratory coordinate system:
\[ E_{(1)}(\mu|\tau) = F_{(41)}(\mu|\tau) = E_1 \gamma \cos \alpha + E_2 \gamma \sin \alpha - H_3 \beta \gamma; \]
\[ E_{(2)}(\mu|\tau) = F_{(42)}(\mu|\tau) = E_1 (-\sin \alpha) + E_2 \cos \alpha; \]
\[ E_{(3)}(\mu|\tau) = F_{(43)}(\mu|\tau) = E_3 \gamma + H_1 \beta \gamma \cos \alpha + H_2 \beta \gamma \sin \alpha; \]
\[ H_{(1)}(\mu|\tau) = F_{(23)}(\mu|\tau) = H_1 \gamma \cos \alpha + H_2 \gamma \sin \alpha + E_3 \beta \gamma; \]
\[ H_{(2)}(\mu|\tau) = F_{(31)}(\mu|\tau) = H_1 (-\sin \alpha) + H_2 \cos \alpha; \]
\[ H_{(3)}(\mu|\tau) = F_{(12)}(\mu|\tau) = H_3 \gamma + E_1 (-\beta \gamma \cos \alpha) + E_2 (-\beta \gamma \sin \alpha), \] (10)

where \( \alpha = \Omega \gamma \tau \).

A mathematical subject of this work is bilinear combinations of the local fields, which are taken in two tetrads, averaged over the field in a vacuum state defined in the laboratory coordinate system. Formulas (10) can be used to calculate the following two-field correlation functions (CF) of the electromagnetic field at the rotating detector:
\[ I_{(ab)}^{E} \equiv \langle E_{(a)}(\mu_1|\tau_1) E_{(b)}(\mu_2|\tau_2) \rangle, \quad I_{(ab)}^{EH} \equiv \langle E_{(a)}(\mu_1|\tau_1) H_{(b)}(\mu_2|\tau_2) \rangle, \quad I_{(ab)}^{H} \equiv \langle H_{(a)}(\mu_1|\tau_1) H_{(b)}(\mu_2|\tau_2) \rangle. \] (11)
where $a, b = 1, 2, 3$. In these expressions $\mu_1$ and $\mu_2$ are two reference frames (tetrads) on the circle of the rotating detector at the proper times $\tau_1$ and $\tau_2$ respectively. For example,

\[
I_{(11)}^E = < E_1(\tau_1)E_1(\tau_2) > \gamma^2 \cos \alpha_1 \cos \alpha_2 + < E_1(\tau_1)E_2(\tau_2) > \gamma^2 \cos \alpha_1 \sin \alpha_2 + \\
< E_2(\tau_1)E_1(\tau_2) > \gamma^2 \sin \alpha_1 \cos \alpha_2 + < E_1(\tau_1)H_3(\tau_2) > (-1)\beta\gamma^2 \cos \alpha_1 + \\
< H_3(\tau_1)E_1(\tau_2) > (-1)\beta\gamma^2 \cos \alpha_2 + < E_2(\tau_1)E_2(\tau_2) > \gamma^2 \sin \alpha_1 \sin \alpha_2 + \\
< E_2(\tau_1)H_3(\tau_2) > (-1)\beta\gamma^2 \sin \alpha_1 + < H_3(\tau_1)E_2(\tau_2) > (-1)\beta\gamma^2 \sin \alpha_2 + \\
(\beta\gamma)^2 < H_3(\tau_1)H_3(\tau_2) > . \tag{12}
\]

The expressions for some other CFs are given in Appendix [3]. They follow from [10]. When $\tau_1 \rightarrow \tau_2$ these expressions can be used to calculate expectation values for energy density. Here $\langle \rangle$ means averaging over a vacuum state of the electromagnetic field in the laboratory coordinate system. In the next section, we will consider averaging for a situation when a vacuum state of the electromagnetic field is a random classical zero point radiation.

### 2.2 Correlation Function Calculation Scheme: Example for $I_{(11)}^E$.

In the classical case, the electric and magnetic field components $E_k$ and $H_k$ in [10] and [12] represent the random zero-point radiation in the laboratory coordinate system [4] (47), (48) at a time-space position $(t, \bar{r})$ of the rotating detector:

\[
\bar{E}(\tau) = \sum_{\lambda=1}^{2} \int d^3k \hat{E}(k, \lambda) h_0(\omega) \cos[\bar{k}\bar{r}(\tau) - \omega\gamma\tau - \theta(\vec{k}, \lambda)], \\
\bar{H}(\tau) = \sum_{\lambda=1}^{2} \int d^3k \hat{E}(k, \lambda) h_0(\omega) \cos[\bar{k}\bar{r}(\tau) - \omega\gamma\tau - \theta(\vec{k}, \lambda)], \tag{13}
\]

where, in distinction from [4], the laboratory coordinates $\bar{r}(t)$ and time $t$ are taken in terms of proper time $\tau$ of the rotating observer:

\[
\bar{r}(\tau) = (a \cos \Omega\gamma\tau, a \sin \Omega\gamma\tau, 0), \quad t = \gamma\tau, \tag{14}
\]

the $\theta(\vec{k}, \lambda)$ describe random phases distributed uniformly on the interval $(0, 2\pi)$ and independently for each wave vector $\vec{k}$ and polarization $\lambda$ of of a plane wave, and

\[
\pi^2h_0^2(\omega) = (1/2)\hbar\omega. \tag{15}
\]
Averaging $\langle \rangle$ in (12) means averaging over random phases $\theta(k, \lambda)$. To illustrate a technique of CF calculation, we will compute the CF $I_{11}^E$ as an example. This technique is very similar to one in [4] developed for a uniformly accelerated case, as apposed to rotation, though the tetrad formalism is not used there.

The $<>$ expressions in (12), contain double integrals and double sums $\int d\vec{k} \int d\vec{k}' \sum_\lambda \sum_{\lambda'}$. Using the known $\theta$ - function properties [4]

\[ < \cos \theta(\vec{k}, \lambda) \cos \theta(\vec{k}', \lambda') > = < \sin \theta(\vec{k}, \lambda) \sin \theta(\vec{k}', \lambda') > = \frac{1}{2} \delta_{\lambda \lambda'} \delta^3 (\vec{k} - \vec{k}') \]

\[ < \cos \theta(\vec{k}, \lambda) \sin \theta(\vec{k}', \lambda') > = 0 \]  

(16)

and the sum over polarization

\[ \sum_{\lambda=1}^{2} \epsilon_i(\vec{k}, \lambda) \epsilon_i(\vec{k}', \lambda') = \delta_{ij} - k_i k_j / k^2 \equiv \delta_{ij} - \hat{k}_i \hat{k}_j, \]  

(17)

they can be reduced to an integral-sum of the type $\int d\vec{k} \sum_\lambda$. Then using variable change in the integrands, from $\vec{k}$ to $\vec{k}'$,

\[ \hat{k}_x \cos \alpha + \hat{k}_y \sin \alpha = \hat{k}'_x, \quad -\hat{k}_x \sin \alpha + \hat{k}_y \cos \alpha = \hat{k}'_y, \]  

(18)

with

\[ \alpha = \frac{\alpha_1 + \alpha_2}{2} = \frac{\Omega \gamma (\tau_2 + \tau_1)}{2}, \quad \hat{k}_i = k_i / k, \quad i = x, y, z, \]  

(19)

we come to the following expressions for the $<>$ terms in (12):

\[ < E_1(\tau_1)E_1(\tau_2) > = \int d^3 k R + (- \cos^2 \alpha) \int d^3 k \hat{k}^2_x R + (- \sin^2 \alpha) \int d^3 k \hat{k}^2_y R, \]

\[ < E_1(\tau_1)E_2(\tau_2) > = < E_2(\tau_1)E_1(\tau_2) > = - \sin 2\alpha \int d^3 k \hat{k}^2_x R + \sin 2\alpha \int d^3 k \hat{k}^2_y R, \]

\[ < E_1(\tau_1)H_3(\tau_2) > = < E_1(\tau_2)H_3(\tau_1) > = - \cos \alpha \int d^3 k \hat{k}_y R, \]

\[ < E_2(\tau_1)E_2(\tau_2) > = \int d^3 k R + (- \sin^2 \alpha) \int d^3 k \hat{k}^2_x R + (- \cos^2 \alpha) \int d^3 k \hat{k}^2_y R, \]

\[ < E_2(\tau_1)H_3(\tau_2) > = < E_2(\tau_2)H_3(\tau_1) > = (- \sin \alpha) \int d^3 k \hat{k}_y R, \]

\[ < H_3(\tau_1)H_3(\tau_2) > = \int d^3 k \hat{k}^2_x R + \int d^3 k \hat{k}^2_y R. \]  

(20)

In these expressions, the prime symbol of the “dummy” variable $k'$ is omitted for simplicity, and we use the following notations:

\[ R = \hbar^2 (\omega) \frac{1}{2} \cos kF, \quad F = c\gamma (\tau_2 - \tau_1) [1 - \hat{k}_y \frac{v \sin \delta/2}{c \delta/2}], \quad \delta = \alpha_2 - \alpha_1 = \Omega \gamma (\tau_2 - \tau_1). \]  

(21)
After some simplifications we come to the following expression for \( I_{(11)}^E \):

\[
I_{(11)}^E = \langle E(1)(\mu_1|\tau_1)E(1)(\mu_2|\tau_2) \rangle = \gamma^2 \cos \delta \int d^3 k \, \hat{h}_0^2(\omega) \frac{1}{2} \cos k F + 2\beta \gamma^2 \cos \frac{\delta}{2} \int d^3 k \, \hat{k}_y \hat{h}_0^2(\omega) \frac{1}{2} \cos k F + 
\gamma^2 [\beta^2 - \cos^2 \frac{\delta}{2}] \int d^3 k \, \hat{k}_x \hat{h}_0^2(\omega) \cos k F + \gamma^2 [\beta^2 + \sin^2 \frac{\delta}{2}] \int d^3 k \, \hat{k}_y \hat{h}_0^2(\omega) \frac{1}{2} \cos k F.
\]

(22)

This function clearly depends only on the proper time interval \( \tau_2 - \tau_1 \) and is not dependent on \( (\tau_1 + \tau_2)/2 \) that is

\[
I_{(11)}^E = I_{(11)}^E(\tau_2 - \tau_1).
\]

(23)

General expressions for other CFs can be found in Appendix B. They have the same properties and also depend only on the proper time interval \( \tau_2 - \tau_1 \).

2.3 The Correlation Function \( I_{(11)}^E \) in Terms of Elementary Functions.

The CF \( I_{(11)}^E \equiv \langle E(1)(\mu_1|\tau_1)E(1)(\mu_2|\tau_2) \rangle \) defined and discussed above can be represented in terms of elementary functions. After integration of (22) in spherical coordinates, over \( k \) and then over \( \phi \), we come to the expression:

\[
I_{(11)}^E = \frac{3hc}{2\pi^2 \hbar^2} \gamma^2 \{ \gamma^2 - \cos^2 \frac{\delta}{2} \} \int_0^\pi d\theta \frac{\sin \theta}{1 - k^2 \sin^2 \theta} \frac{(1 - k^2 \sin^2 \theta)^{7/2}}{1 - k^2 \sin^2 \theta} \sin \theta
\]

\[
+ \frac{3\pi k^2 \cos \delta - 2\pi \cos^2(\delta/2) + 2\pi \beta^2 - 8\pi \beta k \cos(\delta/2) + \pi}{1 - k^2 \sin^2 \theta} \int_0^\pi d\theta \frac{\sin^3 \theta}{1 - k^2 \sin^2 \theta} \frac{(1 - k^2 \sin^2 \theta)^{7/2}}{1 - k^2 \sin^2 \theta} \sin^3 \theta
\]

\[
+ \frac{-3\pi k^2 \cos^2(\delta/2) + 3\pi \beta^2 k^2 - 2\pi \beta k^3 \cos(\delta/2) + 4\pi k^2}{1 - k^2 \sin^2 \theta} \int_0^\pi d\theta \frac{\sin^5 \theta}{1 - k^2 \sin^2 \theta} \frac{(1 - k^2 \sin^2 \theta)^{7/2}}{1 - k^2 \sin^2 \theta} \sin^5 \theta
\}

(24)

(see Appendix C for details). The integrals over \( \theta \) in this expression are:

\[
\int_0^\pi \frac{\sin \theta}{(1 - k^2 \sin^2 \theta)^{7/2}} = \frac{2}{5(1 - k^2)^2} + \frac{8}{15(1 - k^2)^2} + \frac{16}{15(1 - k^2)^3},
\]

(25)

\[
\int_0^\pi \frac{\sin^3 \theta}{(1 - k^2 \sin^2 \theta)^{7/2}} = \frac{4}{15(1 - k^2)^2} + \frac{16}{15(1 - k^2)^3},
\]

(26)

\[
\int_0^\pi \frac{\sin^5 \theta}{(1 - k^2 \sin^2 \theta)^{7/2}} = \frac{16}{15(1 - k^2)^3}
\]

(27)

(See formulas [19], 1.5.23, 1.2.43. ) In these expressions, \( k \) is not a module of a wave vector \( \vec{k} \), but a constant for the CFs:

\[
k = -\frac{v \sin \delta/2}{c} \delta/2, \quad \delta = \Omega \gamma (\tau_2 - \tau_1).
\]

10
Other CFs can also be expressed in terms of elementary functions.

In this form the CFs do not display thermal features. In the next section we will investigate under what conditions they can display thermal properties. We will show that periodic CFs have thermal features.

2.4 Periodicity of Correlation Functions: Example for $I_{(11)}^E$.

We assume that CFs at a rotating detector should be periodic because CF measurements is one of the tools the detector can use to justify the periodicity of its motion. Mathematically it means that

$$I_{(11)}^E(t_2 - t_1) = I_{(11)}^E((t_2 - t_1) + \frac{2\pi}{\Omega} n)$$

or

$$I_{(11)}^E(\tau_2 - \tau_1) = I_{(11)}^E((\tau_2 - \tau_1) + \frac{2\pi}{\Omega\gamma} n)$$

Here $\Omega = \frac{2\pi}{T}$ is an angular velocity of the rotating detector and $n = 0, 1, 2, 3, \ldots$. Breaking down $\cos kF$ in (22) into odd and even powers of $k_y$ and taking into consideration that the odd part of the integrand gives zero after integration over $k_y$ it is easy to show that the CF is periodic if in its integrand

$$\omega = ck = \Omega n.$$  (30)

It means that the rotating detector observes not the entire random electromagnetic radiation spectrum but only a discrete part of it. We could also expect the same result based on the following consideration. Even though no assumptions about a structure of the rotating detector have been made so far, it should have some common features connected with the type of its motion. First of all it should have a charge simply because a neutral, not charged, detector cannot be used to observe electromagnetic field and can not be kept on a circular orbit. Then the charge of the rotating detector behaves as an oscillator with a frequency $\Omega$ and resonance frequencies $n\Omega$. Of course this discrete spectrum is the same as the radiation spectrum of a rotating electrical charge \[20\](39.29).

The expression (24) for $I_{(11)}^E$ cannot be used to analyze the periodicity consequences because the integration over entire continuous spectrum of $\omega$ has already been done in it. It is why we have used the expression (22), before the integration over $\omega$. 

Let us now consider the correlation function $I_{(11)}^E$, periodic over $\tau$, with the discrete spectrum. There are two ways to do this. The first one is simpler, just to modify the formula (22) for $I_{(11)}^E$ for the discrete spectrum. It will be described below in the next subsection. The second one is identical with the approach we have used above for the continuous spectrum but with the modified equations (13) and relationships (16) for the discrete spectrum. It is described in Appendix D.

2.5 Correlation Functions With the Discrete Spectrum: Example for $I_{(11)}^E$.

The integrals in (22) can be represented as

$$\int d^3k \left[ \frac{1}{2} h_0^2(\omega) \cos kF \right] = \frac{c \hbar k_0^4}{4\pi^2} \int dO \left[ \right] S,$$

where

$$S = \int d\kappa \kappa^3 \cos \kappa F_d, \quad dO = d\theta d\phi \sin \theta, \quad \kappa = \frac{k}{k_0}, \quad k_0 = \Omega/c,$$

and

$$F_d = k_0 F = \delta [1 - \hat{k_y} \frac{\nu \sin \delta / 2}{\delta / 2}],$$

The expressions in $[\ ]$ are 1, $\hat{k_y} = \frac{k_y}{\kappa}$, $\hat{k_x}^2 = (\frac{k_x}{\kappa})^2$, and $\hat{k_y}^2 = (\frac{k_y}{\kappa})^2$ do not depend on $\kappa$.

For the discrete spectrum case the integration in (31) over $\kappa$ should be changed to summation over $n$. So the the only term to be changed is $S \rightarrow S_d$. It becomes

$$S_d = \sum_{n=0}^{\infty} n^3 \cos nF_d.$$

Then the periodical CF, corresponding to (22), with the discrete spectrum can be defined in the form

$$I_{(11)d}^E \equiv \langle E_{(1)}(\mu_1|\tau_1) E_{(1)}(\mu_2|\tau_2) \rangle_d = \frac{c \hbar k_0^4}{4\pi^2} \{ \gamma^2 \cos \delta \int dO \ S_d + 2\beta \gamma^2 \cos \frac{\delta}{2} \int dO \ \hat{k_y} \ S_d + \gamma^2 [\beta^2 - \cos^2 \frac{\delta}{2}] \int dO \ \hat{k_x}^2 \ S_d + \gamma^2 [\beta^2 + \sin^2 \frac{\delta}{2}] \int dO \ \hat{k_y}^2 \ S_d \},$$

where integration is held on angular variables only, and $S_d$ is a series sum which is analyzed in the next section using the Abel-Plana formula.

2.6 The Abel-Plana Formula and Thermal Properties of Correlation Functions With the Discrete Spectrum: Example for $I_{(11)d}^E$.

Using Abel-Plana summation formula [21],[22], [23]

$$\sum_{n=0}^{\infty} f(n) = \int_0^\infty f(x) \ dx + \frac{f(0)}{2} + i \int_0^\infty dt \frac{f(it) - f(-it)}{e^{2\pi t} - 1},$$

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with
\[ f(n) = n^3 \cos n F_d \] (37)

we come to the following expression for \( S_d \) (34):
\[ \Omega^4 S_d = \int_0^\infty d\omega \omega^3 \cos(\omega \tilde{F}) + \int_0^\infty d\omega \frac{2\omega^3 \cosh(\omega \tilde{F})}{e^{2\pi\omega/\Omega} - 1}, \quad \tilde{F} = \frac{F_d}{\Omega}, \] (38)

and the CF (35) becomes
\[ I_{E(11)d} = \langle E_{(1)}(\mu_1|\tau_1)E_{(1)}(\mu_2|\tau_2) \rangle_d = \int dO K(\theta, \phi, \delta) \times \] \[ \frac{2}{3} \frac{\hbar}{\pi c^3} \left( \int_0^\infty d\omega \omega^3 \cos(\omega \tilde{F}) + \int_0^\infty d\omega \frac{2\omega^3 \cosh(\omega \tilde{F})}{e^{2\pi\omega/\Omega} - 1} \right), \] (39)

where
\[ K(\theta, \phi, \delta) = \frac{3}{8\pi} \left\{ \gamma^2 \cos \delta + 2\beta \gamma^2 \cos \frac{\delta}{2} \hat{k}_y + \gamma^2[\beta^2 - \cos^2 \frac{\delta}{2}] \hat{k}_x + \gamma^2[\beta^2 + \sin^2 \frac{\delta}{2}] \right\} \hat{k}_y. \] (40)

Expressions for \( S_d \) after integration over \( \omega \) are given in (96), Appendix E, and further discussion could have been made in terms of obtained elementary functions. But it is simpler to consider the structure of the integrand in the expression for \( S_d \) explicitly.

The CF \( I_{E(11)d} \) resembles the CF of the thermal radiation, with Planck’s spectrum and zero-point radiation included, observed by a detector at rest in an inertial frame [4](73):
\[ \langle E_{T_1}(0, s-t/2)E_{T_1}(0, s+t/2) \rangle = \frac{2}{3} \frac{\hbar}{\pi c^3} \left( \int_0^\infty d\omega \omega^3 \cos \omega t + \int_0^\infty d\omega \frac{2\omega^3 \cos \omega t}{e^{\hbar\omega/kT} - 1} \right) \] (41)

which corresponds to the spectral function
\[ \pi^2 h^2 T(\omega) = \frac{1}{2} \hbar \omega \coth \frac{\hbar \omega}{2kT} = \hbar \omega \left( \frac{1}{2} + \frac{1}{e^{\hbar\omega/kT} - 1} \right). \] (42)

Indeed, from (39) and (41) we can see that the integrands in both expressions have the Planck factor \( 1/(e^{\hbar\omega/kT} - 1) \) if, in (39), we define a new constant, a rotation temperature, \( T_{rot} \) as
\[ T_{rot} = \frac{\hbar \Omega}{2\pi k_B}. \] (43)

The Planck factor is an indication that some thermal effects accompany the detector rotation in the random classical zero-point electromagnetic radiation though there is also a significant distinction between them. In (39), \( \tilde{F} = t(1 - \hat{k}_y \frac{v \sin(\Omega t/2)}{\Omega t/2}) \) and \( \cosh \) are used instead of \( t \) and \( \cos \) respectively in
The coefficient $\tilde{F}$ depends on both $\theta$ and $\phi$ because $\hat{k}_y = \sin \theta \sin \phi$. Besides the expression (39), compared to (41), contains coefficient $K(\theta, \phi, \delta)$ and integration over $\theta$ and $\phi$.

So the CF $I_{(11)d}^F$ at a rotating detector explores some thermal properties but does not coincide with the CF $I_{(11)}^F$ at an inertial observer put in the radiation with Planck’s radiation. Partly it occurs because radiation, isotropic in the laboratory system, looks anisotropic for a rotating detector. Is there any situation when operands in (39) and (41) are identical? It is easy to see that in the limit $t \to 0$ and therefore $\tilde{F} \to 0$, when two observation points, $\tau_1$ and $\tau_2$ (or $t_1$ and $t_2$ in the laboratory system) coincide, both expressions are identical. This observation brings up the idea that the energy density (one-observation-point quantity and consisting of diagonal elements of the CF) of the random classical electromagnetic radiation measured by a detector, rotating through a zero point radiation, has the Planck spectrum at the temperature $T_{rot}$ (43). This issue will be discussed in the next section.

3 The Energy Density of Random Classical Electromagnetic Radiation Observed by a Rotating detector: Periodicity and Planck’s Spectrum.

In any reference frame $\mu_\tau$, with Minkowsky metrics $\eta_{(ab)}$, local lorentz coordinates can be introduced [13], section 9.6. The local reference frame, defined this way, is an inertial system, and all laws of Special Relativity should be true in this locally inertial reference frame. Then the energy density measured by the rotating observer at $\mu_\tau$ will be of the form:

$$w = \frac{1}{8\pi} \sum_{a=1}^{3} (\langle E^2_{(a)}(\mu|\tau) \rangle + \langle H^2_{(a)}(\mu|\tau) \rangle )$$

(44)

or, in terms of electric and magnetic fields measured in the laboratory coordinate system (10),

$$w = \frac{1}{4\pi} \{ [\langle E_1^2 \rangle + \langle E_3^2 \rangle ]\gamma^2 (1 + \beta^2) + \langle E_2^2 \rangle \} + \frac{1}{8\pi} 4 \gamma^2 \beta (\langle E_1 H_3 \rangle - \langle E_3 H_1 \rangle )$$

(45)

where as we will show below $\langle E_i^2 \rangle = \langle H_i^2 \rangle$, $i = 1, 2, 3$, and $w$ does not depend on the choice of a tetrad $\mu$.

We have already seen that the correlation functions with a periodicity have a discrete spectrum. Effectively, in calculations, it means that integral expressions for zero-point random radiation fields $E_i$ and $H_i$ in the laboratory coordinates should be modified and presented as series over frequencies.
Explicit expressions for the fields $E_i$ and $H_i$ with discrete spectrum are given in Appendix D and could be used in (45) to take into consideration periodicity. With the help of these formulas and using the technique for discrete spectrum described above we come to the following expressions

$$\langle E_1 H_3 \rangle - \langle E_3 H_1 \rangle = 0,$$

(46)

and

$$\langle E_i^2 \rangle = \langle H_i^2 \rangle = \frac{k_0^4 \hbar c}{2\pi^2} \int dO (1 - \hat{k}_i^2) \sum_{n=0}^{\infty} n^3, \quad k_0 = \Omega/c$$

(47)

for $i = 1, 2, 3$. Finally, after integration over $\theta$ and $\phi$, we have

$$w = \frac{4(\gamma^2 - 1)}{3} \left( w_{ZP} + w_T \right),$$

(49)

where

$$w_{ZP} = \frac{\hbar}{c^3 \pi^2} \int_0^{\infty} d\omega \frac{\omega^3}{2}, \quad w_T = \frac{\hbar}{c^3 \pi^2} \int_0^{\infty} d\omega \frac{\omega^3}{e^{\hbar \omega/k_B T\text{rot}} - 1} = \frac{\pi^2 k_B^4}{60(c\hbar)^3} T_{\text{rot}}^4 = \frac{4\sigma}{c} T_{\text{rot}}^4,$$

(50)

$k_B$ is the Boltzman constant, and $\sigma$ is the Stefan-Boltzman constant.

Thus, due to the periodicity of the motion, the detector rotating in the zero-point radiation under the temperature $T = 0$ observes not only original zero-point radiation, $w_{ZP}$, but also the radiation, $w_T$, with Planck’s spectrum if parameter $T_{\text{rot}}$ is interpreted as the temperature associated with the detector rotation. Expression $w_T$ is exactly the energy density of the black radiation at the temperature $T_{\text{rot}}$ [24], (60,14)]. The factor $\frac{2}{3}(4\gamma^2 - 1)$ comes from integration in (47) over angles due to anisotropy of the electromagnetic field measured by the rotating observer.

All this consideration is true when $\Omega r < c$. The first term of (49), corresponding to ZP radiation, is divergent for any $r$ and $\Omega$. The second one, describing the thermal properties, is convergent, though it is growing to infinity if $r \to c/\Omega$ for a fixed $\Omega$ or $\Omega \to c/r$ for a fixed $r$. 

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4 Random Classical Massless Zero-Point Scalar Field at a Rotating Detector.

4.1 Correlation Function.

The scalar field \( \psi_s(\mu_1 \mid \tau_1) \) in a tetrad \( \mu_\tau \) has the same form as in the laboratory coordinate system, \( \psi_s(\tau) \), taken in the location of the tetrad, because it is a scalar. Then the correlation function measured by an observer rotating through a random classical massless zero-point scalar field radiation has the form [4]:

\[
\langle \psi_s(\mu_1 \mid \tau_1) \psi_s(\mu_2 \mid \tau_2) \rangle = \langle \psi_s(\tau_1) \psi_s(\tau_2) \rangle,
\]

where

\[
\psi_s(\tau_i) = \int d^3k_i f(\omega_i) \cos \{ \vec{k}_i \vec{r}(\tau_i) - \omega_i \gamma \tau_i - \Theta(k_i) \},
\]

and (instead of \( \bar{h}_0(\omega) \) in (13) )

\[
f^2(\omega_i) = \frac{\hbar c^2}{2\pi^2 \omega_i}, \quad \omega_i = c k_0_i, \quad i = 1, 2.
\]

The \( \theta \)-functions, \( \vec{r}(\tau_i) \), and \( t(\tau_i) \) are defined in (14) and (16). Using these expressions and variable change (18) in the double-integral (51) we get the expression:

\[
\langle \psi_s(\mu_1 \mid \tau_1) \psi_s(\mu_2 \mid \tau_2) \rangle = \int d^3k f^2(\omega) \frac{1}{2} \cos kF,
\]

where \( F \) is defined in (21). Having integrated over \( k \), \( \phi \), and \( \theta \) we come to the expression for the CF of the random classical massless scalar field at the rotating detector moving through a zero point massless scalar radiation ( see details in Appendix F):

\[
\langle \psi_s(\mu_1 \mid \tau_1) \psi_s(\mu_2 \mid \tau_2) \rangle = -\frac{\hbar c}{\pi} \frac{1}{(\gamma(\tau_2 - \tau_1)c)^2 - 4r^2 \sin^2 \frac{\Omega(\tau_2 - \tau_1)}{2}}.
\]

This correlation function is also identical to the positive frequency Wightman function [1](3), up to a constant. This function does not expose thermal features. Nevertheless the situation changes if the CF periodicity is taken into consideration. In the scalar field, the CF can be considered periodical for the same reasons it is periodical in the electromagnetical fields. This issue is investigated below.
4.2 Periodicity of the Correlation Function, Abel-Plana Formula, and the Planck’s Factor.

To take into consideration the periodicity of the CF we have to use its expression (54) before integration over $\omega$. The equation (54) can be given in the form

$$\langle \psi_s(\mu_1|\tau_1) \psi_s(\mu_2|\tau_2) \rangle = \frac{\hbar c k^2}{4\pi^2} \int dO \int d\kappa \cos \kappa F_d, \quad dO = \sin \theta d\theta d\phi, \quad \kappa = \frac{k}{k_0}, \quad k_0 = \Omega/c, \quad F_d = k_0 F. \quad(56)$$

If this function of $\tau = \tau_2 - \tau_1$ is periodic then, as we saw for the CF $I_{(11)}^E$ above, $\kappa = \frac{ck}{ck_0} = n = 0, 1, 2, ..$ and the integral over $\kappa$ becomes an infinite series:

$$\langle \psi_s(\mu_1|\tau_1) \psi_s(\mu_2|\tau_2) \rangle_d = \frac{\hbar c k^2}{4\pi^2} \int dO \sum_{n=0}^{\infty} n \cos n F_d. \quad(57)$$

Expression (57) is a definition of a new correlation function, with periodicity, of the scalar massless field at the rotating detector.

The Abel-Plana summation formula in this case is

$$\sum_{n=0}^{\infty} n \cos n F_d = \int_0^\infty dt \cos t F_d - \int_0^\infty dt \frac{2t \cosh t F_d}{e^{2\pi t} - 1}. \quad(58)$$

or

$$\Omega^2 \sum_{n=0}^{\infty} n \cos n F_d = \int_0^\infty d\omega \cos \omega \tilde{F} - \int_0^\infty d\omega \frac{2\omega \cosh \omega \tilde{F}}{e^{\pi \omega / T_{rot}} - 1}. \quad(59)$$

where $\tilde{F} = F_d/\Omega$ and $T_{rot}$ is defined in (43).

Then

$$\langle \psi_s(\mu_1|\tau_1) \psi_s(\mu_2|\tau_2) \rangle_d = \frac{\hbar}{4\pi^2 c} \int dO \left\{ \int_0^\infty d\omega \cos \omega \tilde{F} - \int_0^\infty d\omega \frac{2\omega \cosh \omega \tilde{F}}{e^{\pi / T_{rot}} - 1} \right\}. \quad(60)$$

The expression in $\{ \}$ is similar to the right side of the expression [4], (27) for the correlation function of the scalar massless zero-point field at the detector at rest in Planck’s spectrum at the temperature $T$

$$\int_0^\infty d\omega \coth \frac{\hbar \omega}{2kT} \cos \omega t = \int_0^\infty d\omega \cos \omega t + \int_0^\infty d\omega \frac{2\omega \cos \omega t}{e^{\pi kT} - 1}. \quad (61)$$

The appearance of the Planck factor $(e^{kT_{rot}} - 1)^{-1}$ shows similarity between the radiation spectrum observed at the rotating detector in the massless scalar zero-point field and the radiation spectrum observed by an inertial observer placed in a thermostat filled with the radiation at the temperature
\[ T = T_{\text{rot}}. \] But there is also a difference between them. The \( \tilde{F} \) and cosh are used in the first expression whereas \( t \) and cos are used in the second expression respectively. The \( \tilde{F} \) is a function of \( \theta \) and \( \phi \). It means that a thermal radiation observed by the rotating detector moving in the massless scalar zero-point radiation is anisotropic.

The resemblance between both expressions becomes closer if \( t = 0 \) and \( \tilde{F} = 0 \) and two points of an observation agree. Both expressions are identical. But in the case of one-point observation which occurs when \( \tilde{F} = 0 \) it is better to consider the energy density of the scalar massless field, as is done in the next section.

### 4.3 The Energy Density and Planck’s Spectrum.

The energy density \( \langle T_{(44)} \rangle \) of the massless scalar field at the detector rotating through the zero-point massless scalar field can be expressed in terms of the tensor of energy-momentum \( T_{ik} \) at the location of the detector in the laboratory coordinate system [14] as

\[
\langle T_{(44)} \rangle = \mu^i_{(4)} \mu^k_{(4)} \langle T_{ik} \rangle
\]  

where \( \mu^i_{(4)} \) are tetrads. The energy-momentum tensor is [25](2.27)

\[
T_{ik} = \psi_i \psi_k - \frac{1}{2} \eta_{ik} \eta_{rs} \psi_r \psi_s, \quad \eta_{ik} = \eta_{ik} = \text{diag}(1, 1, 1, -1)
\]  

Using (52), (53), and Frenet-Serret tetrads it is easy to show that

\[
\langle T_{11} \rangle = \langle T_{22} \rangle = \langle T_{33} \rangle = \frac{1}{3} \langle T_{44} \rangle = \frac{\hbar c}{3\pi} \int dkk^3 = \frac{\hbar \Omega^4}{3\pi c^2} \int d\kappa \kappa^3
\]  

and

\[
\langle T_{(44)} \rangle = \frac{4\gamma^2 - 1}{3} \langle T_{44} \rangle = \frac{4\gamma^2 - 1}{3} \frac{\hbar \Omega^4}{\pi c^3} \int d\kappa \kappa^3
\]  

With periodical features taken into consideration this expression has the following form

\[
\langle T_{(44)} \rangle_d = \frac{4\gamma^2 - 1}{3} \frac{\hbar}{\pi c^3} \Omega^4 \sum_{n=0}^{\infty} n^3
\]  

( It has an additional factor \( n^2 \) compared with [57] because \( T_{ik} \) have derivatives of \( \psi \)-functions. ) or

\[
\langle T_{(44)} \rangle_d = \frac{4\gamma^2 - 1}{3} \frac{\hbar}{\pi c^3} 2 \left( \int_0^\infty d\omega \frac{1}{2} \omega^3 + \int_0^\infty d\omega \frac{\omega^3}{e^{\hbar \omega/kT_{\text{rot}}} - 1} \right).
\]  

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Let us compare this expression and the expression for the energy density of the massless scalar field with Planck’s spectrum of random thermal radiation at the temperature $T$, along with the zero-point radiation in an inertial reference frame,

$$
\langle T_{44}\rangle_T = \frac{1}{2} \left[ \left( \frac{\partial \psi_T}{\partial (ct)} \right)^2 + \left( \frac{\partial \psi_T}{\partial x} \right)^2 + \left( \frac{\partial \psi_T}{\partial y} \right)^2 + \left( \frac{\partial \psi_T}{\partial z} \right)^2 \right],
$$

(68)

where $\psi_T = \int d^3 k f_T(\omega) \cos \left[ \vec{k} \vec{r} - \omega t - \theta(\vec{k}) \right]$ 

(69)

and

$$
f_T^2(\omega) = \frac{c^2}{\pi^2} \frac{\hbar}{\omega} \left[ \frac{1}{2} + \frac{1}{\exp(\hbar \omega/kT) - 1} \right].
$$

(70)

It is easy to show that

$$
\langle T_{(44)} \rangle_d = \frac{2(4\gamma^2 - 1)}{9} \langle T_{44} \rangle_T = \langle T_{44} \rangle_{T=\text{rot}}.
$$

(71)

So, due to periodicity of the motion, an observer rotating through a zero point radiation of a massless random scalar field should see the same energy density as an inertial observer would see, moving in a thermal bath at the temperature $T_{\text{rot}} = \frac{M}{2\pi k}$, multiplied by the factor $\frac{2}{9}(4\gamma^2 - 1)$. This factor comes from integration over angles and is a consequence of anisotropy of the scalar field measured by an observer with angular velocity $\Omega$.

5 Conclusion and Perspectives.

The thermal effects of non inertial motion investigated in the past for uniform acceleration through classical random zero-point radiation of electromagnetic and massless scalar field are shown to exist in the case of rotation motion as well.

The rotating reference system $\{\mu_{\tau}\}$, along with the two-point correlation functions (CFs) and energy density, are defined and used as the basis for investigating effects observed by a detector rotating through random classical zero-point radiation. The reference system consists of Frenet - Serret orthogonal tetrads $\mu_{\tau}$. At each proper time $\tau$ the rotating detector is at rest and has a constant acceleration vector at the $\mu_{\tau}$.

The two-point CFs and the energy density at the rotating reference system should be periodic
with the period \(T = \frac{2\pi}{\Omega}\), where \(\Omega\) is an angular detector velocity, because CF and energy density measurements are one of the tools the detector can use to justify the periodicity of its motion. The CFs have been calculated for both electromagnetic and massless scalar fields in two cases, with and without taking this periodicity into consideration. It was found that only periodic CFs have some thermal features and particularly the Planck factor with the temperature \(T_{\text{rot}} = \frac{k\Omega}{2\pi k_B}\) \((k_B\) is the Boltzmann constant). Mathematically this property is connected with the discrete spectrum of the periodic CFs, and its interpretation is based on the Abel-Plana summation formula.

It is also shown that energy densities of the electromagnetic and massless scalar fields observed by the detector rotating through classical zero-point radiation at zero temperature are respectively

\[
    w = \frac{2}{3} \left(4\gamma^2 - 1\right) w_{\text{em}}(T_{\text{rot}})
\]

and

\[
    \langle T_{(44)} \rangle_d = \frac{2(4\gamma^2 - 1)}{9} \langle T_{44} \rangle_{T_{\text{rot}}},
\]

Each of them consists of two terms. The first term, corresponding to zero-point radiation energy density, is divergent, and the second one, describing the thermal effect, is convergent.

Let us discuss the convergent electromagnetic thermal energy density

\[
    w_{\text{em},T} = \frac{2}{3} \left(4\gamma^2 - 1\right) \times \frac{4\sigma c}{T_{\text{rot}}^4}
\]

\[
    \gamma^2 = \left(1 - \left(\frac{\Omega r}{c}\right)^2\right)^{-1}.
\]  

(72)

It includes factor \(\frac{2}{3}(4\gamma^2 - 1)\). Appearance of this factor is connected with the fact that rotation is defined by two parameters, angular velocity and the radius of rotation, in contrast with a uniformly accelerated linear motion which is defined by only one parameter, acceleration \(a\). If, for a fixed \(\Omega\), the radius of a circular orbit grows, \(r \to \frac{c}{\Omega}\), the second factor does not change but the first one grows. Such behaviour of the convergent term may have a mechanical interpretation.

Let several small particles with the same sign charge move through the vacuum field on a circular orbit. Let us further assume that repulsive interaction of the particles results in a shift of the particles to another circular orbit with slightly greater radius \(r\) but with the same angular velocity \(\Omega\). Then the thermal energy density \(w_{\text{em},T}\), observed locally by each of the particles, would increase. This increase demands an additional work against the vacuum field and therefore initiates the force, let us call it
the vacuum force, which acts on these particles from the vacuum field. The volume density of this force is given by

$$f_{\text{vac}} = -\frac{dw_{em,T}}{dr} = -\frac{8}{3} \frac{\Omega^2}{c^2} \times \frac{2r}{(1 - (\Omega r/c)^2)^2} \times \frac{4\sigma}{c} T_{rot}^4$$

The force $f_{\text{vac}}$ does not depend on the size of neither the charge nor the mass and originates from the thermal energy $w_{em,T}$, even though it is positive. These three features make $f_{\text{vac}}$ similar to the force, $f_{\text{cas}}$, in the Casimir model for a charged particle \cite{28, 29, 30}

$$E(a) = -C \frac{hc}{2a} \quad f_{\text{cas}} = -\frac{dE}{da} = -C \frac{hc}{2a^2}$$

where $a$ is a radius. This model was designed to explain a charged particle stability. The force $f_{\text{cas}}$ also does not depend on the size of neither the charge nor the mass, the energy $E(a)$ is positive (because $C \approx -0.09$) \cite{29}.

Nevertheless $f_{\text{vac}}$ and $f_{\text{cas}}$ are significantly different. Indeed,

1. The $f_{\text{vac}}$ is applied to the dynamical system of a particle (particles) moving on a circular orbit, not to a static one as $f_{\text{cas}}$ in the Casimir model.

2. The $f_{\text{vac}}$ is attractive one because it is directed from the location with greater positive energy density, $w_{em,T}$, to the location with smaller one, that is to the center of the circular orbit. Thus we could expect it might balance the repulsive force associated with interaction of the charged particles. In contrast, the $f_{\text{cas}}$ is known to be repulsive, directed from the center of the shell outward, and therefore can not balance repulsive electrical forces.

3. The $f_{\text{vac}}$ infinitely grows with $r \rightarrow r_0 = c/\Omega$ and works as a restoring spring force. Therefore the radius of circular orbits with a fixed $\Omega$ is bounded. The orbits with a radius greater than $r_0$ do not exist because the vacuum force becomes infinite. On the uttermost orbit with the radius $r_0$, a linear velocity of the rotating particle would have become $c$.

4. The $f_{\text{vac}}$ becomes very small and proportional to $r$ when $r$ is small, $r \ll c/\Omega$.

The last two features of the $f_{\text{vac}}$ mean that the further the rotating particle from the center is the more bounded it becomes or, in other words, confined. The closer to the center it is the freer it becomes. This reminds us of two significant concepts in quantum chromodynamics (QCD), the theory of strong interactions: asymptotic freedom and confinement.

Confinement theory of quarks and gluons is still a challenge for strong interaction physics \cite{31}. Therefore a concept of a newly introduced vacuum force can be useful for understanding a confinement.
phenomenon, even though the concept is introduced in the frame of the stochastic electrodynamics but confinement realizes in strong interactions. Moreover quarks, with strong interaction between them, do have an electrical charge, can interact with the electromagnetic field vacuum and experience the vacuum force. In Appendix G we make rough and preliminary estimates of the $f_{\text{vac}}$ and $T_{\text{rot}}$, just to understand what order of magnitude they could have in hadron.

This is only one of possible directions of the vacuum force $f_{\text{vac}}$ applications. More detailed discussion of the vacuum force will be given in a different publication.

The same consideration is true for the convergent thermal part of massless scalar field. Some of the results discussed in this paper have been obtained in [32].

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APPENDIX

A Orthogonal Tetrads.

An orthogonal tetrad (OT) is a set of four orthogonal and normalized 4-vectors $\mu^i_{(a)}$, labeled by $a=1,2,3,4$, so that

$$\mu^i_{(a)} \mu_{(b)i} = \eta_{(ab)}.$$  \hfill (75)

Co-vectors $\mu^i_{(b)}$ are defined as

$$\mu^{(a)i} = \eta^{(ab)} \mu^i_{(b)}; \quad \mu^i_{(a)} = \eta_{(ab)} \mu^i_{(b)}.$$  \hfill (76)

The $\eta_{(ab)}$ is a diagonal matrix

$$\eta_{(ab)} = \eta^{(ab)} = \text{diag}(1, 1, 1, -1).$$  \hfill (77)
Frenet-Serret OTs satisfy the formulas \[\text{[14]}(55)\]:

\[
D^{\mu}i^{(4)} = b^{\mu}i^{(1)}, \quad D^{\mu}i^{(1)} = \tilde{c}^{\mu}i^{(2)} + b^{\mu}i^{(4)}, \quad D^{\mu}i^{(2)} = d^{\mu}i^{(3)} - \tilde{c}^{\mu}i^{(1)}, \quad D^{\mu}i^{(3)} = -d^{\mu}i^{(2)},
\]

(78)

where \(D = \frac{d}{d\tau}\), \(\tau\) is a proper time of the detector, in the flat space-time with metric \(g_{ik} = \text{diag}(1,1,1,-1)\).

Solution of this system is given in \[\text{[17]}\], with

\[
b = -\beta\Omega\gamma^2, \quad \tilde{c} = \Omega\gamma^2, \quad d = 0.
\]

(79)

Fermi-Walker tetrad vectors are defined as \([\text{13]} (9.148, 4.139, \text{and} 4.167)\)

\[
\frac{de_{(a)k}}{d\tau} = (e_{(a)l}\dot{U}_l)/c^2 - (e_{(a)i}U_i)\dot{U}_k/c^2
\]

(80)

and can be given in the form \([\text{13]} 4.167\)

\[
e_{(1)k} = (\cos \alpha \cos \alpha \gamma + \gamma \sin \alpha \sin \alpha \gamma, \sin \alpha \cos \alpha \gamma - \gamma \cos \alpha \sin \alpha \gamma, 0, -i(v\gamma/c) \sin \alpha \gamma),
\]

\[
e_{(2)k} = (\cos \alpha \sin \alpha \gamma - \gamma \sin \alpha \cos \alpha \gamma, \sin \alpha \sin \alpha \gamma + \gamma \cos \alpha \cos \alpha \gamma, 0, +i(v\gamma/c) \cos \alpha \gamma),
\]

\[
e_{(3)k} = (0, 0, 1, 0),
\]

\[
e_{(4)k} = (i\gamma/v \cos \alpha, -i\gamma/v \cos \alpha, 0, \gamma).
\]

(81)

In \[\text{[30]}\], as in \([\text{13]} (4.167)\), the metric is chosen in the form \(g_{ik} = (1, 1, 1, 1)\).

We preferred to use Frenet-Serret tetrads, and not Fermi-Walker ones, because in a reference frame associated with a Fermi-Walker tetrad \(e_{(a)i}\) the 3-vector acceleration is not constant in both direction and magnitude

\[
\dot{U}_{(a)} = e_{(a)i}\dot{U}_i = (-a\Omega^2\gamma^2 \cos \alpha \gamma, -a\Omega^2\gamma^2 \sin \alpha \gamma, 0, 0),
\]

(82)

and the acceleration depends on proper time \(\tau\).

### B Some Correlation Functions of an Electromagnetic Field at a Rotating Detector as 3-Dimensional Integrals over( \(k, \theta, \phi\) ).

Two correlation functions mentioned at the end of Section 2.1 are the following:

\[
I^{E}_{(22)} = \langle E_{(2)}(\mu_1|\tau_1) E_{(2)}(\mu_2|\tau_2) \rangle = \langle E_1(\tau_1) E_1(\tau_2) \rangle \sin \alpha_1 \sin \alpha_2 + \langle E_1(\tau_1) E_2(\tau_2) \rangle (-1) \sin \alpha_1 \cos \alpha_2 +
\]
\( (E_2(\tau_1)E_1(\tau_2))(-1) \cos \alpha_1 \sin \alpha_2 + (E_2(\tau_1)E_1(\tau_2)) \cos \alpha_1 \cos \alpha_2, \)

\[ I_{(33)}^E = \langle E_{(3)}(\mu_1|\tau_1) E_{(3)}(\mu_2|\tau_2) \rangle = \gamma^2 \langle E_{(3)}(\tau_2)E_{(3)}(\tau_1) \rangle - \gamma^2 \frac{v}{c} \cos \alpha_1 \langle E_{(3)}(\tau_2)H_{(1)}(\tau_1) \rangle - \gamma^2 \frac{v}{c} \cos \alpha_2 \langle H_{(1)}(\tau_2)E_{(3)}(\tau_1) \rangle - \gamma^2 \frac{v}{c} \sin \alpha_1 \langle E_{(3)}(\tau_2)H_{(2)}(\tau_1) \rangle - \gamma^2 \frac{v}{c} \sin \alpha_2 \langle H_{(2)}(\tau_2)E_{(3)}(\tau_1) \rangle + \gamma^2 \left( \frac{v}{c} \right)^2 \cos \alpha_1 \langle H_{(1)}(\tau_2)H_{(1)}(\tau_1) \rangle + \gamma^2 \left( \frac{v}{c} \right)^2 \sin \alpha_1 \langle H_{(2)}(\tau_2)H_{(2)}(\tau_1) \rangle + \gamma^2 \left( \frac{v}{c} \right)^2 \cos \alpha_2 \sin \alpha_1 \langle H_{(1)}(\tau_2)H_{(2)}(\tau_1) \rangle + \gamma^2 \left( \frac{v}{c} \right)^2 \sin \alpha_2 \cos \alpha_1 \langle H_{(2)}(\tau_2)H_{(1)}(\tau_1) \rangle. \] (83)

Is is easy to show that they depend on the difference \( \delta = \alpha_2 - \alpha_1 \) only.

\[ I_{(22)}^E = \cos \delta \int d^3k \ R + \sin^2 \frac{\delta}{2} \int d^3k \hat{k}_x^2 \ R + (-1) \cos^2 \frac{\delta}{2} \int d^3k \hat{k}_y^2 \ R. \] (84)

\[ I_{(33)}^E = \gamma^2 \frac{v^2}{c^2} \cos \delta \int d^3k \ R + \gamma^2 \frac{v}{c} (-2) \cos \frac{\delta}{2} \int d^3k \hat{k}_y \ R + \gamma^2 \left[ 1 - \frac{v^2}{c^2} \sin^2 \frac{\delta}{2} \right] \int d^3k \hat{k}_x^2 \ R + \gamma^2 \left[ 1 + \frac{v^2}{c^2} \sin^2 \frac{\delta}{2} \right] \int d^3k \hat{k}_y^2 \ R. \] (85)

Expressions for \( R \) and \( \delta \) are given in (21).

The non diagonal components of the correlation function are zeroes:

\[ \langle E_{(1)}(\mu_1|\tau_1) E_{(2)}(\mu_2|\tau_2) \rangle = \langle E_{(1)}(\mu_2|\tau_2) E_{(2)}(\mu_1|\tau_1) \rangle = 0, \]
\[ \langle E_{(1)}(\mu_1|\tau_1) E_{(3)}(\mu_2|\tau_2) \rangle = \langle E_{(1)}(\mu_2|\tau_2) E_{(3)}(\mu_1|\tau_1) \rangle = 0, \]
\[ \langle E_{(2)}(\mu_1|\tau_1) E_{(3)}(\mu_2|\tau_2) \rangle = \langle E_{(2)}(\mu_2|\tau_2) E_{(3)}(\mu_1|\tau_1) \rangle = 0, \] (86)

Similar expressions have been received for the CF with magnetic field components. So all CFs can be given as 3-dimensional integrals over \((k, \theta, \phi)\).

C  Integral calculations: final expression for \( I_{(11)}^E \).

All non zero expressions for the CF in subsection (2.3) should be integrated over \( k, \theta, \phi \). The integral over \( k \) can be easily calculated:

\[ \int_0^\infty dk k^3 \cos \{k(2r \sin \frac{\delta}{2} \sin \theta \sin \phi - c(t_2 - t_1))\} = \frac{6}{\left[2r \sin \frac{\delta}{2} \sin \theta \sin \phi - c(t_2 - t_1)\right]^4} = \frac{6}{\left[\frac{1}{c(t_2 - t_1)}\right]^4 \left[1 - \frac{v^2}{c^2} \sin^2 \frac{\delta}{2} \sin \theta \sin \phi\right]^4}. \] (87)
The integrals over $\theta$ and $\phi$ can be represented in terms of elementary functions. Let us show it for $I_{(11)}^E \equiv \langle E_{(1)}(\mu_1|\tau_1)E_{(1)}(\mu_1|\tau_2) \rangle$:

$$
I_{(11)}^E = \frac{3hc}{2\pi^2|c(t_2-t_1)|^2} \int_0^\pi d\theta \times \left\{ \cos \delta \sin \theta + \left( -\cos^2 \frac{\delta}{2} + \frac{v^2}{c^2} \sin^3 \theta \right) \int_0^{2\pi} d\phi \frac{1}{(1+b\sin \phi)^4} 
+ \left( -\frac{v}{c} \cos \frac{\delta}{2} \right) \sin^2 \theta \int_0^{2\pi} d\phi \frac{\sin \phi}{(1+b\sin \phi)^4} + \sin^3 \theta \int_0^{2\pi} d\phi \frac{\sin^2 \phi}{(1+b\sin \phi)^4} \right\},
$$

(88)

We have taken into consideration here that

$$
\hat{k}_x = \sin \theta \cos \phi, \quad \hat{k}_y = \sin \theta \sin \phi, \quad \hat{k}_z = \cos \theta
$$

(89)

and used notations $b \equiv k \sin \theta$, $k \equiv -\frac{v}{c} \sin \delta/2$. So $k$ is a constant, not a wave vector.

The next step is to calculate the integral over $\phi$. Because (26),

$$
\int_0^{2\pi} d\phi \frac{1}{(1+b\sin \phi)^4} = \pi(2+3b^2) \left( \frac{1}{1-b^2} \right)^{7/2},
$$

(90)

$$
\int_0^{2\pi} d\phi \frac{\sin \phi}{(1+b\sin \phi)^4} = \frac{-b\pi(4+b^2)}{(1-b^2)^{7/2}},
$$

(91)

and

$$
\int_0^{2\pi} d\phi \frac{\sin^2 \phi}{(1+b\sin \phi)^4} = \frac{\pi(1+4b^2)}{(1-b^2)^{7/2}},
$$

(92)

the correlation function takes the form (24).

D Another Way to Receive the $I_{(11)}^E$ for the Discrete Spectrum.

In section (2.5) we have obtained the general expression for the CF $I_{(11)}^E d \equiv \langle E_{(1)}(\mu_1|\tau_1)E_{(1)}(\mu_2|\tau_2) \rangle_d$ with discrete spectrum, based on its periodicity. This also could be done directly using the following expressions for the fields $E_i$ and $H_i$, with discrete spectrum, instead of the equations (13):

$$
\vec{E}(\vec{r}, t) = a \sum_{n=0}^\infty \sum_{\lambda=1}^2 \int d\vec{k}_n \hat{e}(\vec{k}, \lambda) h_0(\omega_n) \cos[\vec{k}_n \cdot \vec{r} - \omega_n t - \Theta(\vec{k}_n, \lambda)],
$$

$$
\vec{H}(\vec{r}, t) = a \sum_{n=0}^\infty \sum_{\lambda=1}^2 \int d\vec{k}_n \hat{e}(\vec{k}, \lambda) h_0(\omega_n) \cos[\vec{k}_n \cdot \vec{r} - \omega_n t - \Theta(\vec{k}_n, \lambda)],
$$

$$
\vec{k}_n = k_n \hat{k}, \quad k_n = k_0 n, \quad k_0 = \frac{\Omega}{c}, \quad \omega_n = c k_n, \quad a = k_0.
$$

(93)
The unit vector \( \hat{k} \) defines a direction of the wave vector \( \vec{k} \) in a spherical momentum 3-space and does not depend on its value, \( n \).

The right side of the first equation in the relation (16) should be modified. We do this in two steps. First we rewrite them in a spherical momentum space [27], p.6 56 as :

\[
\langle \cos \theta (\vec{k}_1 \lambda_1) \cos \theta (\vec{k}_2 \lambda_2) \rangle = \langle \sin \theta (\vec{k}_1 \lambda_1) \sin \theta (\vec{k}_2 \lambda_2) \rangle = \frac{1}{2} \delta_{\lambda_1 \lambda_2} \frac{2}{k_1^2} \delta(k_1 - k_2) \delta(\hat{k}_1 - \hat{k}_2). \tag{94}
\]

And then, in the case of the discrete spectrum, it will be the following:

\[
\langle \cos \theta (\vec{k}_n^1 \lambda_1) \cos \theta (\vec{k}_n^2 \lambda_2) \rangle = \langle \sin \theta (\vec{k}_n^1 \lambda_1) \sin \theta (\vec{k}_n^2 \lambda_2) \rangle = \frac{1}{2} \delta_{\lambda_1 \lambda_2} \frac{1}{k_0 n_1 n_2} \delta(\hat{k}_1 - \hat{k}_2). \tag{95}
\]

The equation \( \sum_{\lambda=1}^{2} \epsilon_i (\vec{k} \lambda) \epsilon_j (\vec{k} \lambda) = \delta_{ij} - \hat{k}_i \hat{k}_j \) does not depend on \( n \). The correlation function finally takes the form (35,34).

### E Expression for \( S_d \) after Integration over \( \omega \).

The expressions for \( S_d \) in (38) mentioned in subsection 2.6, after integration over \( \omega \), are the following:

\[
S_d = \frac{6}{F_d^4} + \left[ 3 - 2 \sin^2(F_d/2) - \frac{6}{F_d^4} \right] \tag{96}
\]

or

\[
S_d = \frac{6}{F_d^4} + 6 \sum_{n=1}^{\infty} \frac{1}{(2\pi n)^4} \left[ \frac{1}{(1 + F_d/2\pi n)^4} + \frac{1}{(1 - F_d/2\pi n)^4} \right]. \tag{97}
\]

The first integral of \( S_d \) in (38) is divergent and the second one is convergent as can seen from (96).

### F Correlation Function Calculation for Random Zero-Point Radiation of a Scalar Massless Field

The expression (54) after integration over positive \( k = \omega/c \) becomes:

\[
\langle \psi_s(\mu_1|\tau_1)\psi_s(\mu_2|\tau_2) \rangle = -\frac{\hbar c}{4\pi^2} \int_0^\pi d\theta \sin \theta \int_0^{2\pi} d\phi \left[ E \sin \phi - B \right]^{-2}, \tag{98}
\]

where \( B = \gamma \tau c, E = 2a \sin \theta \sin \frac{\Omega \gamma \tau}{2}, \) and \( \tau = \tau_2 - \tau_1 \).

Because \( B - |E| = c\gamma \tau \{ 1 - \frac{v}{c} \sin \theta \frac{\sin \left( \pi (\gamma \tau)/T \right)}{\pi (\gamma \tau)/T} \} > c\gamma \tau (1 - v/c) > 0 \), and using (19) we obtain :

\[
\int_0^{2\pi} d\phi \frac{1}{|E \sin \phi - B|^2} = \frac{2\pi B}{(B^2 - E^2)^{3/2}}. \tag{99}
\]

Finally, integrating it over \( \theta \) we come to (55).
G  The Force $f_{\text{vac}}$ and Temperature $T_{\text{rot}}$ Estimations.

The only purpose of the following estimations is to figure out the order of a magnitude of the vacuum force $f_{\text{vac}}$ and rotation temperature $T_{\text{rot}}$ which can be associted with a proton size $r_0 \approx 10^{-15}$ m [33].

The radial component of the force acting on a spherical particle of the radius $a$ rotating through an electromagnetic zero-point field on a circular orbit with radius $r \approx r_0$ and with angular velocity $\Omega = c/r_0$ can be given in the form

$$F = f_{\text{vac}} \frac{4}{3} \pi a^3 = -\frac{x}{(1 - x^2)^2} \frac{4 \epsilon \hbar}{135 \pi r_0^3}, \ x = \frac{r}{r_0} \leq 1,$$

(100)

where $f_{\text{vac}}$ is given in (73). Just for estimation purposes, let us take $a \approx 10^{-18}$ m. Then

$$F \approx -\frac{x}{(1 - x^2)^2} \times (2.8) \times 10^{-7} \frac{J}{m} = -\frac{x}{(1 - x^2)^2} \times (1.75) \times 10^{-12} \frac{GeV}{Fermi}.$$  

(101)

For $1 - x \approx 10^{-6}$, $F \approx -(4.4) \times 10^{-1}$ $GeV/Fermi = .7 \times 10^5$ newtons in a good agreement with an order of magnitude of strong interaction forces.

Similarly,

$$T_{\text{rot}} = \frac{\hbar \Omega}{2 \pi k_B} = \frac{\hbar c}{2 \pi k_B} \frac{1}{r_0},$$

(102)

and, for distances $r_0 \approx 10^{-15}m$, corresponds to the temperature $T_{\text{rot}} \approx 3.4 \times 10^{11} K$, a little bit less then the temperature $(1.90 \pm 0.02) \times 10^{12} K$ needed for a quark-gluon plasma creation [34].

These estimations is a good motivation for further investigations.

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