Dependence of spin torque diode voltage on applied field direction

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The optimum condition of an applied field direction to maximize spin torque diode voltage was theoretically derived for a magnetic tunnel junction with a perpendicularly magnetized free layer and an in-plane magnetized pinned layer. We found that the diode voltage for a relatively small applied field is maximized when the projection of the applied field to the film-plane is parallel or anti-parallel to the magnetization of the pinned layer. However, by increasing the applied field magnitude, the optimum applied field direction shifts from the parallel or anti-parallel direction. These analytical predictions were confirmed by numerical simulations.

I. INTRODUCTION

Magnetization dynamics induced by spin torque in nano-structured ferromagnets have provided interesting phenomena, such as magnetization switching and oscillation. Many spintronics devices utilizing the spin torque have been proposed, such as, a magnetic random access memory (MRAM) based on magnetic tunnel junctions (MTJs) and a microwave oscillator. The spin torque diode effect is also an important phenomenon, in which an alternating current applied to an MTJ is rectified by synchronizing the resonant oscillation of tunnel magnetoresistance (TMR) by spin torque with the alternating current. The spin torque diode effect has been used to quantitatively evaluate the strength of the spin torque.

The spin torque diode effect is applicable to a magnetic sensor application, where a small magnetic field from a ferromagnetic or paramagnetic particle modulates the resonance condition of the spin torque diode. In such a sensor application, a large diode voltage is required to enhance sensitivity, defined as the ratio between the input power and the diode voltage. It should also be noted that the direction of the applied field, which is proportional to the spin of the particle, points in an arbitrary direction. Thus, it is important to clarify the relation between the spin torque diode voltage and the applied field direction, and to maximize the spin torque diode voltage.

In this paper, we derive the optimum condition of the applied field direction to maximize the spin torque diode voltage of an MTJ with a perpendicularly magnetized free layer and an in-plane magnetized pinned layer. This type of MTJ was recently developed in experiments, and is considered an ideal candidate for spin torque diode application because of its narrow linewidth and high diode voltage. We first derived the general formula of the spin torque diode voltage, and then, applied the formula to the system under consideration. The main result is Eq. (40), which represents the applied field direction at the maximized diode voltage. The diode voltage for a relatively small applied field is maximized when the projection of the applied field to the film-plane is parallel or anti-parallel to the magnetization of the pinned layer. However, the optimum applied field direction shifts from the parallel or anti-parallel direction by increasing the applied field magnitude. These results are confirmed numerically.

The paper is organized as follows. In Sec. II we derive the analytical solution to the linearized Landau-Lifshitz-Gilbert (LLG) equation of the free layer. In Sec. III we discuss the general formula of the spin torque diode voltage and its dependence on the magnetization alignment. Section IV is the main section in this paper, where we derive the optimum condition for the applied field direction in an MTJ with a perpendicularly magnetized free layer and an in-plane magnetized pinned layer. Section V is devoted to the conclusions.

II. SOLUTION TO THE LINEARIZED LLG EQUATION

In this section, we solve the linearized LLG equation for an arbitrary magnetization alignment. The system we consider is schematically shown in Fig. 1, where the MTJ consists of free and pinned layers separated by a thin nonmagnetic spacer. The \( x, y, \) and \( z \) axes are parallel to the uniaxial anisotropy axes of the free layer. The unit vectors pointing in the direction of the magnetizations of the free and the pinned layers are denoted as \( \mathbf{m} \) and \( \mathbf{p} = (\sin \theta_p \cos \varphi_p, \sin \theta_p \sin \varphi_p, \cos \theta_p) \), respectively, where the zenith and the azimuth angles of the magnetization of the pinned layer are denoted as \( \theta_p \) and \( \varphi_p \), respectively. We assume that the magnetization dynamics in the presence of the spin torque are well described by the macrospin LLG equation:

\[
\frac{d\mathbf{m}}{dt} = -\gamma \mathbf{m} \times \mathbf{H} - \gamma a \mathbf{m} \times (\mathbf{p} \times \mathbf{m}) + \gamma b \mathbf{p} \times \mathbf{m} + \alpha \mathbf{m} \times \frac{d\mathbf{m}}{dt},
\]

where \( \gamma \) and \( \alpha \) are the gyromagnetic ratio and the Gilbert damping constant, respectively. The magnetic field \( \mathbf{H} \) is defined as the derivative of the energy density \( E \) with
layers are denoted as \( \mathbf{m} \) and \( \mathbf{p} \), respectively. The positive current is defined as the electron flow from the pinned to the free layer.

respect to the magnetization, i.e.,

\[
\mathbf{H} = -\frac{1}{M} \frac{\partial E}{\partial \mathbf{m}},
\]

(2)

where \( M \) is the saturation magnetization. The energy density \( E \) is given by

\[
E = -MH_{\text{appl}} [\sin \theta_H \sin \phi \cos(\varphi - \varphi_H) + \cos \theta_H \cos \phi] + \sum_{\ell=x,y,z} 2\pi M^2 \tilde{N}_\ell m_\ell^2,
\]

(3)

where \( H_{\text{appl}} \), \( \theta_H \), and \( \varphi_H \) in the first term are the magnitude, the zenith angle, and the azimuth angle of the applied field, respectively. The second term of Eq. (3) describes the uniaxial anisotropy energy. The coefficient \( \tilde{N}_\ell \) (\( \ell = x, y, z \)) is defined as \( 4\pi M N_\ell = 4\pi M N_\ell - H_{\text{K}\ell} \), where \( 4\pi MN_\ell \) and \( H_{\text{K}\ell} \) are the shape anisotropy field (demagnetization field) and the crystalline anisotropy field along the \( \ell \)-axis, respectively. The demagnetization coefficients satisfy \( N_x + N_y + N_z = 1 \).

The two components of the spin torque in Eq. (1), the Slonczewski torque and the field like torque, are denoted as \( a_J \) and \( b_J \), respectively, whose explicit forms are given by

\[
a_J = \frac{h g I}{2eMV},
\]

(4)

and \( b_J = \beta a_J \). Here, \( I \) is the current and \( V \) is the volume of the free layer, respectively. The positive current corresponds to the electron flow from the free to the pinned layer. We assume that both the direct (dc) and alternating (ac) currents are applied to the MTJ, i.e., \( I = I_{dc} + I_{ac}(t) \). Thus, \( a_J \) and \( b_J \) are decomposed into the dc and the ac parts as \( a_J = a_{J,(dc)} + a_{J,(ac)} \) and \( b_J = b_{J,(dc)} + b_{J,(ac)} \), respectively. The magnitude of the direct current is on the order of 0.1-1.0 mA, while that of the alternating current is 0.1 mA. As shown below, the present formula is valid for \( |a_{J,(dc)}| < |a(H)| \), where the Gilbert damping constant is on the order of \( 10^{-2} \). The ratio of the Slonczewski torque to the field like torque, \( \beta \), is on the order of 0.1 for MTJs. The factor \( g \) characterizes the spin polarization of the current, and is given by

\[
g = \frac{\eta}{1 + \lambda \mathbf{m} \cdot \mathbf{p}}.
\]

(5)

The dimensionless parameters, \( \eta \) and \( \lambda \), characterize the magnitude of the spin polarization and the dependence of the spin torque on magnetization alignment, respectively. Although the relation among \( \eta \), \( \lambda \), and the material parameters depends on the theoretical models, the form of Eq. (5) is applicable to spin torque in both MTJs and giant-magnetoresponse system. For example, in the ballistic transport theory in MTJ, \( \eta \) is proportional to the spin polarization of the density of state of the free layer and \( \lambda = \eta^2 \). Below, we set \( \lambda = 0 \) for simplicity. The spin torque diode voltage and its optimum condition with finite \( \lambda \) are discussed in Appendix A.

The solution to the LLG equation is derived in an XYZ-coordinate in which the Z-axis is parallel to the steady state of the magnetization of the free layer. We denote \( \mathbf{m} \) at the steady state as \( \mathbf{m}^{(0)} \), where the condition \( d\mathbf{m}(t)/dt = 0 \) can be expressed in terms of the zenith and the azimuth angles, \( (\theta, \phi) \), as:

\[
H_{\text{appl}} \sin \theta_H \cos \theta (\varphi - \varphi_H) - \cos \theta_H \sin \theta]
\]

\[
-4\pi M \left( \tilde{N}_x \cos^2 \varphi + \tilde{N}_y \sin^2 \varphi - \tilde{N}_z \right) \sin \theta \cos \phi
\]

\[
- a_{J,(dc)} \sin \theta \sin (\varphi - \varphi_p)
\]

\[
+ b_{J,(dc)} \sin \theta \cos \phi \cos (\varphi - \varphi_p) - \cos \theta_p \sin \theta = 0,
\]

(6)

\[
H_{\text{appl}} \sin \theta_H (\varphi - \varphi_H) - 4\pi M \left( \tilde{N}_x \tilde{N}_y \sin \theta \sin \varphi \cos \phi \right)
\]

\[
- a_{J,(dc)} \sin \theta_p \cos \theta \cos (\varphi - \varphi_p) - \cos \theta_p \sin \theta
\]

\[
- b_{J,(dc)} \sin \theta_p \sin (\varphi - \varphi_p) = 0.
\]

(7)

In the absence of the direct current, \( (\theta, \phi) \) satisfying Eqs. (6) and (7) correspond to the equilibrium state, i.e., the minimum state of the energy density \( E \). The transformation from the \( xyz \)-coordinate to the \( XYZ \)-coordinate is performed by multiplying the following rotation matrix to Eq. (1):

\[
R = \begin{pmatrix}
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0 \\
\sin \theta & 0 & \cos \theta
\end{pmatrix}
\]

(8)

For example, the components of \( \mathbf{p} \) in the \( XYZ \)-coordinate can be expressed as

\[
\begin{pmatrix}
p_X \\
p_Y \\
p_Z
\end{pmatrix}
\]

\[
= \begin{pmatrix}
\cos \theta \sin \theta_p \cos (\varphi - \varphi_p) - \sin \theta \cos \theta_p \\
-\sin \theta_p \sin (\varphi - \varphi_p) \\
\sin \theta \sin \theta_p \cos (\varphi - \varphi_p) + \cos \theta \cos \theta_p
\end{pmatrix}.
\]

(9)

The alternating current exerts a small amplitude oscillation of the magnetization around the Z-axis. Then, the LLG equation can be linearized by assuming \( m_Z \simeq 1 \).
and |m_X|, |m_Y| ≪ 1, and is given by
\[
\frac{1}{\gamma} \frac{d}{dt} \begin{pmatrix} m_X \\ m_Y \end{pmatrix} = \begin{pmatrix} -H_{YX} + \alpha H_X & H_Y - \alpha H_{XY} \\ -H_{YX} - \alpha H_Y & H_Y + \alpha H_{XY} \end{pmatrix} \begin{pmatrix} m_X \\ m_Y \end{pmatrix}
\]
where we use the approximation that 1 + \alpha^2 ≈ 1 \cite{29}. The components of H are defined as
\[
H_X = H_X + b_{J(dc)} p_Z, \\
H_Y = H_Y + b_{J(dc)} p_Z, \\
H_{XY} = H_{XY} - a_{J(ac)} p_Z, \\
H_{YX} = H_{YX} + a_{J(ac)} p_Z.
\]
Here, H_X = H_{ZZ} - H_{XX}, H_Y = H_{ZZ} - H_{YY}. The field H_{ij} (i, j = X, Y, Z) are the i-components of the magnetic field in the XYZ-coordinate proportional to m_i. \[ H = (H_{XX} m_X + H_{XY} m_Y + H_{YX} m_X + H_{YY} m_Y, H_{ZZ} m_X + H_{XY} m_Y, H_{XX} m_X + H_{XY} m_Y) \] where the explicit forms of H_{ij} are given by
\[
H_{XX} = -4\pi M \left[ (\tilde{\eta}_x \cos^2 \varphi + \tilde{\eta}_y \sin^2 \varphi) \cos^2 \theta + \tilde{\eta}_z \sin^2 \theta \right], \\
H_{XY} = H_{YX} = 4\pi M \left( \tilde{\eta}_x - \tilde{\eta}_y \right) \cos \theta \sin \varphi \cos \varphi, \\
H_{YX} = H_{YY} = 4\pi M \left( \tilde{\eta}_x \sin^2 \varphi + \tilde{\eta}_y \cos^2 \varphi \right), \\
H_{ZZ} = 4\pi M \left( \tilde{\eta}_x \cos^2 \varphi + \tilde{\eta}_y \sin^2 \varphi - \tilde{\eta}_z \right) \sin \theta \cos \theta, \\
H_{XY} = H_{YX} = -4\pi M \left( \tilde{\eta}_y - \tilde{\eta}_x \right) \sin \theta \sin \varphi \cos \varphi, \\
H_{ZZ} = H_{appl} \left[ \sin \theta_H \sin \theta \cos(\varphi - \varphi_H) + \cos \theta_H \cos \theta \right] \\
- 4\pi M \left[ \tilde{\eta}_x \cos^2 \varphi + \tilde{\eta}_y \sin^2 \varphi \right] \sin^2 \theta + \tilde{\eta}_z \cos^2 \theta.
\]
By assuming that the alternating current is given by \[ I_{ac} \sin(2\pi ft) \], the solutions to (m_X, m_Y) in Eq. are, respectively, given by
\[
m_X \approx \text{Im} \left[ \frac{\tilde{\gamma}(if - \tilde{\gamma}H_{XY}) p_X - \tilde{\gamma}^2 H_{YX} p_Y}{f^2 - f_{res}^2 - if \Delta f} e^{2\pi if t} \right] a_{J(ac)}, \\
- \text{Im} \left[ \frac{\tilde{\gamma}(if - \tilde{\gamma}H_{XY}) p_X + \tilde{\gamma}^2 H_{YX} p_Y}{f^2 - f_{res}^2 - if \Delta f} e^{2\pi if t} \right] b_{J(ac)},
\]
where \[ \tilde{\gamma} = \gamma/(2\pi) \], and \[ a_{J(ac)} \) and \[ b_{J(ac)} \] are defined as \[ a_{J(ac)} = \tilde{a}_{J(ac)} \sin(2\pi ft) \] and \[ b_{J(ac)} = \tilde{b}_{J(ac)} \sin(2\pi ft) \], respectively. The resonance frequency \[ f_{res} \] and the linewidth \[ \Delta f \] are, respectively, given by
\[
f_{res} = \frac{\gamma}{2\pi} \sqrt{H_X H_Y - H_{XY} H_{YX}}, \\
\Delta f = \frac{\gamma}{2\pi} \left[ \alpha (H_X + H_Y) + H_{XY} - H_{YX} \right].
\]
In the absence of the direct current, Eq. is the ferromagnetic resonance (FMR) frequency, \[ f_{FMR} = \gamma \sqrt{H_X H_Y - H_{XY}/(2\pi)} \]. Since \[ \beta \] is on the order of 0.1 \cite{25}, and \[ a_{J(ac)} \) is on the order of a small parameter \[ \alpha, \beta \] in Eq. is negligible. Thus, \[ \Delta f \] can be approximated to
\[
\Delta f \approx \frac{\gamma}{2\pi} \left[ \alpha (H_X + H_Y) - 2a_{J(ac)} p_Z \right].
\]

### III. SPIN TORQUE DIODE VOLTAGE

The magnetoresistance of an MTJ is given by \[ R = R_p + (\Delta R/2)(1 - m \cdot p) \], where \[ \Delta R = R_{AP} - R_p \] is the difference in the resistances between the parallel (R_P) and the anti-parallel (R_AP) alignments of the magnetizations. The spin torque diode voltage is given by \[ V_{ac} = T^{-1} \int_0^T I(t)R(t)dt \], where \[ T = 1/f \] is the period of the alternating current. By using Eqs. and , the explicit form of the spin torque diode voltage is given by
\[
V_{ac} = \frac{\Delta R_{ac}}{4} \text{Re} \left[ \frac{-if(p_X^2 + p_Y^2) + \tilde{\gamma} \tilde{\mathcal{H}_a} \tilde{a}_{J(ac)} + \tilde{\gamma}^2 \tilde{\mathcal{H}_0} \tilde{b}_{J(ac)}}{f^2 - f_{res}^2 - if \Delta f} \right]
\]
\[
= \frac{\Delta R_{ac}}{4} \left[ \mathcal{L}(f) + \mathcal{A}(f) \right],
\]
where \[ \mathcal{H}_a \] and \[ \mathcal{H}_0 \] are, respectively, given by
\[
\mathcal{H}_a = \mathcal{H}_{XY} p_X^2 - \mathcal{H}_{YX} p_Y^2 + (\mathcal{H}_X - \mathcal{H}_Y) p_x p_y,
\]
\[
\mathcal{H}_0 = \mathcal{H}_{XY} p_X^2 + \mathcal{H}_{YX} p_Y^2 + (\mathcal{H}_{XX} + \mathcal{H}_{YY}) p_x p_y.
\]
The Lorentzian and the anti-Lorentzian parts, \[ \mathcal{L}(f) \] and \[ \mathcal{A}(f) \] are, respectively given by
\[
\mathcal{L}(f) = \frac{f^2 f_{res}^2 \tilde{\gamma} a_{J(ac)} (1 - p_Z^2)}{(f^2 - f_{res}^2)^2 + (f \Delta f)^2},
\]
\[
\mathcal{A}(f) = \frac{f^2 f_{res}^2 \tilde{\gamma} a_{J(ac)} (1 - p_Z^2)}{(f^2 - f_{res}^2)^2 + (f \Delta f)^2}.
\]
\[ \mathcal{A}(f) = -\frac{\alpha}{2} \left( H_c \tilde{a}_{j(ac)} - H_c \tilde{b}_{j(ac)} \right)^2 \left( f^2 - f^2_{\text{res}} \right) \left( f^2 - f^2_{\text{res}} + (\Delta f)^2 \right)^2. \]  

As shown, the Lorentzian part depends on the Slonczewski torque only while the anti-Lorentzian part depends on both the Slonczewski torque and the field like torque, in general.

The peak of the spin torque diode voltage appears around the resonance frequency, \( f_{\text{res}} \), where the Lorentzian part shows a peak while the anti-Lorentzian part is zero. At \( f = f_{\text{res}} \), the spin torque diode voltage is

\[ V_{dc}(f_{\text{res}}) = \frac{\Delta R I_{ac}}{4 I_{dc}} \frac{\tilde{a}_{j(ac)} \sin^2 \psi}{\alpha (H_c + H_f) - 2 \tilde{a}_{j(dc)} \cos \psi}, \]  

where \( \psi = \cos^{-1} \left( \frac{Z}{P} \right) = \cos^{-1} \left( \mathbf{m}^{(0)} \cdot \mathbf{p} \right) \) in the relative angle between the magnetizations of the free and the pinned layers. Equation (31) is maximized when the relative angle of the magnetizations is given by

\[ \psi_{\text{opt}} = \cos^{-1} \left( \frac{I_c}{I_{dc}} \pm \sqrt{\left( \frac{I_c}{I_{dc}} \right)^2 - 1} \right), \]  

where the double sign \( \pm \) means the upper (-) for \( I_{dc}/I_c > 0 \) and the lower (+) for \( I_{dc}/I_c < 0 \). The critical current of the spin torque induced magnetization dynamics in the case of \( \mathbf{m}^{(0)} \parallel \mathbf{p} \), \( I_c \), is given by

\[ I_c = \frac{2 e \alpha M V}{h \gamma} \left( \frac{H_c + H_f}{2} \right). \]  

Since \( \psi \) is a real number, the following condition should be satisfied:

\[ \left| \frac{I_c}{I_{dc}} \right| > 1. \]  

This condition means that the linear approximation cannot be applied to the LLG equation when the spin torque overcomes the damping. The maximized spin torque diode voltage is given by

\[ V_{dc}^{\text{opt}}(f_{\text{res}}) = \frac{\Delta R I_{ac}^2}{4 I_{dc}} \left( \frac{I_c}{I_{dc}} \pm \sqrt{\left( \frac{I_c}{I_{dc}} \right)^2 - 1} \right). \]  

Equations (32) and (34) are the main results in this section, and can be regarded as generalizations of the result in Ref. [32]. We emphasize that the optimum condition, Eq. (32), depends on not only the material (sample) parameters and the applied field but also the magnitude and direction of the direct current. It should be noted that Eq. (32) is \( 90^\circ \) for \( I_{dc} = 0 \), and shifts from this orthogonal alignment to a finite \( I_{dc} \).

Equation (32) is the optimum condition of the magnetization alignment to maximize the spin torque diode voltage. However, in experiments, the direction of the applied field is more easily controlled, than the magnetization alignment, because the direction of the magnetization of the pinned layer is fixed by the exchange bias from an anti-ferromagnetic layer. In the next section, by using Eq. (32), we derive the analytical formula of the optimum condition of the applied field direction to maximize the spin torque diode voltage in an MTJ with a perpendicularly magnetized free layer and an in-plane magnetized pinned layer.

**IV. OPTIMUM CONDITION OF APPLIED FIELD DIRECTION**

In an MTJ with a perpendicularly magnetized free layer and an in-plane magnetized pinned layer, \( (\theta_p, \varphi_p) \) in Fig. 1 are \( (90^\circ, 0^\circ) \). The free layer has uniaxial anisotropy along the easy axis which is normal to the film plane, and has a circular cross section. The components of the anisotropy field are \( 4 \pi M N_x = 4 \pi M N_y = 0 \), and \( 4 \pi M N_z = -H_K + 4 \pi M \), where the \( z \)-axis is parallel to the easy axis. Since we are interested in the perpendicularly magnetized free layer, the anisotropy field \( H_K \) should be larger than the demagnetization field \( 4 \pi M \). The \( x \)-axis is parallel to the magnetization of the pinned layer. We assume that the magnetic field is applied tilted from the \( z \)-axis with the angle \( \theta_H \). In the following, we investigate the optimum direction of the applied field in the film-plane, \( \varphi_H \), to maximize the diode voltage.

We assume that the steady state \( (\theta, \varphi) \) is determined by Eqs. (4) and (7) by neglecting the spin torque term, i.e., \( (\theta, \varphi) \) corresponds to the equilibrium state, because the spin torque term is on the order of a small parameter \( \alpha \). The equilibrium state of the free layer satisfies

\[ H_{\text{appl}} \sin (\theta - \theta_H) + (H_K - 4 \pi M) \sin \theta \cos \theta = 0, \]  

and \( \varphi = \varphi_H \). Then, the critical current

\[ I_c = \frac{2 e \alpha M V}{h \gamma} \left[ H_{\text{appl}} \cos (\theta - \theta_H) + H_K \cos^2 \theta + \cos 2 \theta \right], \]  

is independent of \( \varphi_H \). Equation (32) can be expressed as

\[ \sin \theta \cos \varphi = \frac{I_c}{I_{dc}} \mp \sqrt{\left( \frac{I_c}{I_{dc}} \right)^2 - 1}. \]  

As mentioned above, \( \varphi \) on the left-hand side of Eq. (38) can be replaced by \( \varphi_H \). When the condition

\[ \left| \frac{1}{\sin \theta} \left( \frac{I_c}{I_{dc}} \pm \sqrt{\left( \frac{I_c}{I_{dc}} \right)^2 - 1} \right) \right| < 1 \]  

is satisfied, the diode voltage is maximized at the field direction

\[ \varphi_H = \cos^{-1} \left\{ \frac{1}{\sin \theta} \left( \frac{I_c}{I_{dc}} \pm \sqrt{\left( \frac{I_c}{I_{dc}} \right)^2 - 1} \right) \right\}. \]  

Equation (40) is the main result in this paper. For a relatively small applied field magnitude, the equilibrium
state is close to the easy axis, i.e., $\theta \simeq \sin \theta \ll 1$, and Eq. (39) is not satisfied. Then, the diode voltage is maximized at $\varphi_H = 0$ or $\pi$, depending on the direction of the current. However, by increasing the applied field magnitude, the magnetization tilts from the easy axis, and Eq. (39) is satisfied. The optimum direction of the applied magnetic field then shifts from $\varphi_H = 0, \pi$ to $\varphi_H$ given by Eq. (40).

The physical meaning of the condition (39) is as follows. As mentioned after Eq. (35), the spin torque diode voltage is maximized near the orthogonal alignment of the magnetization. When the magnitude of the applied field is small, this condition is approximately satisfied. Then, the magnetization should oscillate in the $xz$-plane to obtain a large oscillation amplitude of TMR because $p$ points to the $x$-direction. Thus, Eq. (10) is 0 or $\pi$. However, the magnetization moves to the $xy$-plane for a relatively large applied field. To keep the relative angle of the magnetizations close to Eq. (32), the magnetization of the free layer should shift from the $x$-axis by changing the field direction. Thus, the optimum field direction shifts from $\varphi_H = 0, \pi$, according to Eq. (10).

The reason why the analytical solution of the optimum applied field direction, Eq. (10), can be obtained in this system is that, because of the axial symmetry, $\varphi$ in Eq. (38) can be replaced by $\varphi_H$. In the general system, both sides of Eq. (32) depend on the applied field direction ($\theta_H, \varphi_H$) through Eqs. (30) and (31). Consequently, an analytical expression of the optimum field direction cannot be obtained.

Let us quantitatively estimate the optimum direction, $\varphi_H$. Figure 2 shows the dependence of the spin torque diode voltage, $V_{dc}(f_{res})$, on the applied field direction, $\varphi_H$, for several values of $H_{appl}$ and $I_{dc}$. The values of the parameters are $M = 1313 \text{ emu/c.c.}$, $H_K = 17.9 \text{ kOe}$, $\theta_H = 60^\circ$, $V = \pi \times 50 \times 50 \times 2 \text{ nm}^3$, $\gamma = 17.32 \text{ MHz/Oe}$, $\alpha = 0.005$, $\eta = 0.33$, $\beta = 0.1$, $I_{dc} = 0.1 \text{ mA}$, and $\Delta R = 100 \Omega$, respectively. The values of $H_{appl}$ and $I_{dc}$ are (a) $(H_{appl}(\text{kOe}), I_{dc}(\text{mA})) = (1.0, 0.2)$, (b) $(1.0, -0.2)$, (c) $(5.0, 0.2)$, and (d) $(5.0, -0.2)$, respectively, where the value of $I_{dc}$ is chosen to observe the shift of the optimum $\varphi_H$ from 0 or $\pi$ to a cerating angle in a typical range of $H_{appl}$ in experiments. The current magnitude (0.2 mA) is also a typical value used in experiments (for example, Ref. [8]). The steady state of the magnetization of the free layer is $\theta = 26.3^\circ$ for $H_{appl} = 100 \text{ kOe}$. In this case, Eq. (39) is not satisfied, and thus, the diode voltage is maximized at $\varphi_H = 0$ for $I_{dc}/I_c > 0$ and at $\varphi_H = \pi$ for $I_{dc}/I_c < 0$, as shown in Figs. 2(a) and (b). On the other hand, the steady state is given by $\theta = 52.0^\circ$ for $H_{appl} = 5.0 \text{ kOe}$. The condition, Eq. (39), is satisfied, and the optimum direction of the applied field is given by $\varphi_H = 63.7^\circ$ and $296.3^\circ$ for $I_{dc} = 0.2 \text{ mA}$ and $73.9^\circ$ and $106.1^\circ$ for $-0.2 \text{ mA}$, respectively. The maximized voltage is estimated to be $272 \mu V$ while the diode voltages at $\varphi_H = 0$ and $\pi$ for $I_{dc} > 0$ are estimated to be $146$ and $73 \mu V$, respectively. Since the relative angle between the magnetizations decreases as the applied field magnitude increases, the maximized voltage for $H_{appl} = 5.0 \text{ kOe}$ is smaller than that for $H_{appl} = 1.0 \text{ kOe}$.

We perform numerical simulations to confirm the above analytical results. Figures 3(a) and (b) shows the dependences of the spin torque diode voltage at...
\[ \varphi_H = \varphi_H^{\text{opt}}, 0, \text{ and } \pi \text{ on the frequency of the alternating current with } I_{dc} = 0.2 \text{ mA and } -0.2 \text{ mA, respectively.} \] The magnitude of the applied magnetic field is \( H_{\text{appl}} = 5.0 \text{ kOe.} \) A sharp peak of the diode voltage appears near the FMR frequency, \( f_{\text{FMR}} \approx 13.8 \text{ GHz.} \) The magnitudes of the diode voltage at \( f = f_{\text{res}} \) agree well with the results shown in Fig. 2, demonstrating the validity of the above analytical formula.

**V. CONCLUSIONS**

In conclusion, we derive the optimum condition of the applied field direction to maximize the diode voltage of an MTJ with a perpendicularly magnetized free layer and an in-plane magnetized pinned layer, which was recently developed in experiments. For a relatively small applied field, the diode voltage is maximized when the projection of the applied field to the film-plane is parallel or anti-parallel to the magnetization of the pinned layer. However, the voltage is maximized at a certain direction shifted from the parallel or anti-parallel direction by increasing the applied field magnitude. These results are confirmed by numerically solving the Landau-Lifshitz-Gilbert equation.

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**Appendix A: Spin torque diode voltage and its maximized condition with finite \( \lambda \)**

In this appendix, the spin torque diode voltage with finite \( \lambda \) in Eq. (A6) is derived.

First, let us briefly describe the importance of \( \lambda \), which arises from the dependence of the tunneling probability on the magnetization alignment [10]. Since the magnitude of \( \lambda \) is small, for simplicity, we assume \( \lambda \) is zero in some cases [22]. However, when the magnetization alignment of the free and the pinned layers in equilibrium is orthogonal (\( \mathbf{m}^{(0)} \perp \mathbf{p} \)), a finite \( \lambda \) plays a key role in the spin torque induced magnetization dynamics. For example, when \( \lambda \) is neglected, the critical current for the magnetization dynamics, Eq. (A7) shown below, diverges for \( \mathbf{m}^{(0)} \perp \mathbf{p} \) (i.e., \( p_{Z} = 0 \)). This is because the work done by spin torque is zero at this alignment, and thus, the spin torque cannot overcome the damping. However, if \( \lambda \neq 0 \), the critical current remains finite because the work done by spin torque is also finite, and can be larger than the energy dissipation due to the damping [30].

Now we calculate the diode voltage with finite \( \lambda \). Let us redefine \( a, \beta \), and \( b, \beta \) as

\[ a = \frac{\hbar \eta I}{2eMV(1 + \lambda p_{Z})}, \quad b = \beta a, \quad \text{also, we introduce } A \text{ as} \]

\[ A = \frac{\lambda}{1 + \lambda p_{Z}}. \quad (A2) \]

Then, instead of Eqs. (11)-(14), we redefine \( \mathcal{H}_{x}, \mathcal{H}_{y}, \mathcal{H}_{XY}, \text{and } \mathcal{H}_{YX} \) as

\[ \mathcal{H}_{x} = H_{x} + J_{dc} p_{Z} + a J_{dc} p_{X} p_{Y} + b J_{dc} p_{X}^{2}, \quad (A3) \]

\[ \mathcal{H}_{y} = H_{y} + b J_{dc} p_{Z} - a J_{dc} p_{X} p_{Y} + b J_{dc} p_{Y}^{2}, \quad (A4) \]

\[ \mathcal{H}_{XY} = H_{XY} - a J_{dc} p_{Z} - a J_{dc} p_{X} p_{Y} - b J_{dc} p_{X} p_{Y}, \quad (A5) \]

\[ \mathcal{H}_{YX} = H_{YX} + a J_{dc} p_{Z} + a J_{dc} p_{X} p_{Y} - b J_{dc} p_{X} p_{Y}. \quad (A6) \]

By using these \( \mathcal{H} \), the resonance frequency, the linewidth, and \( \mathcal{K} \) are redefined according to Eqs. (23), (24), (27), and (28), respectively. Then, the diode voltage is given by Eqs. (29), (29), and (29). The critical current for the magnetization dynamics is given by

\[ \mathcal{K} = \frac{2\alpha eMV}{\hbar(1 - \alpha p_{Z}) + 1 - \lambda^{2}(1 - p_{Z}^{2})} \left( \frac{H_{x} + H_{y}}{2} \right). \quad (A7) \]

It is difficult for an arbitrary \( \lambda (-1 < \lambda < 1) \) to derive the optimum condition. However, the optimum condition for \( |\lambda| \ll 1 \) can be derived as follows. In this case, the diode voltage at \( f = f_{\text{res}} \) is given by Eq. (A1) in which \( a \) is replaced by Eq. (A1). Then, the diode voltage is maximized when the relative angle of \( \mathbf{m}^{(0)} \) and \( \mathbf{p} \) is given by

\[ \psi_{\text{opt}} = \cos^{-1} \left\{ \frac{(I_{c}/I_{dc}) \mp \sqrt{(I_{c}/I_{dc})^{2} - 1 - \lambda(I_{c}/I_{dc})^{2}}}{1 - \lambda(I_{c}/I_{dc})^{2}} \right\}. \quad (A8) \]

where \( I_{c} \) is defined by Eq. (31). Equation (A8) is identical to Eq. (32) in the limit of \( \lambda \to 0 \). The maximized voltage at \( f = f_{\text{res}} \) is given by

\[ V_{dc}^{\text{opt}} = \frac{\Delta R I_{dc}^{2}}{4 I_{dc}} \left\{ \frac{(I_{c}/I_{dc}) \mp \sqrt{(I_{c}/I_{dc})^{2} - 1 - \lambda(I_{c}/I_{dc})^{2}}}{1 - \lambda(I_{c}/I_{dc})^{2}} \right\}. \quad (A9) \]

The spin torque diode effect is useful for estimating the value of \( \lambda \) experimentally. For example, let us consider the spin torque diode effect of MTJ with the perpendicularly magnetized free layer and the in-plane magnetized pinned layer discussed in Sec. IV. The direct current is assumed to be zero. By fixing the magnitude (\( H_{\text{appl}} \)) and the tilted angle (\( \theta_{H} \)) of the applied field, the resonance frequencies and the linewidths at a certain \( \varphi_{H} \) and \( \pi - \varphi_{H} \) are identical. Then, the ratio of the diode voltages at \( \varphi_{H} \) and \( \pi - \varphi_{H} \) is given by

\[ \frac{V_{dc}(f = f_{\text{res}}, \varphi_{H})}{V_{dc}(f = f_{\text{res}}, \pi - \varphi_{H})} = \frac{1 - \lambda \sin \theta}{1 + \lambda \sin \theta.} \quad (A10) \]
where the factor $1 \mp \lambda \sin \theta$ appears from $\tilde{a} J_{(ac)} \propto 1/(1 + \lambda p Z)$ in the numerator of Eq. (31). Since the value of $\theta$ is determined by Eq. (36), the value of $\lambda$ can be estimated by this ratio. This method of estimating $\lambda$ is applicable to general system if there are at least two equilibrium states with identical resonance frequencies and linewidths and different relative angles with the magnetization of the pinned layer.

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