Determination of the Phase of $V_{ub}$
from Charmless Hadronic $B$ Decay Rates

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Abstract

We perform a model dependent fit to recent data on charmless hadronic $B$ decays and determine $\gamma$, the phase of $V_{ub}^*$, We find $\gamma = 114^{+25}_{-21}$ degrees, which disfavors the often quoted $\gamma \sim 60^\circ$ at the two standard deviation level. We also fit for the form factors $F_0^{B\pi}$ and $A_0^{B\rho}$, and the strange-quark mass. They are consistent with theoretical expectations, although $m_s$ is somewhat low. Such agreement and the good $\chi^2$ for the fit may be interpreted as a confirmation of the adequacy of our model assumptions.

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The measurement of a surprisingly large $\varepsilon'/\varepsilon$ value in 1999 is an exasperating reminder of how little we really know about $CP$ violation in Nature. Within the Standard Model (SM) with 3 quark generations, however, there is a unique phase in the Kobayashi-Maskawa (KM) matrix $V$, often defined as $\gamma = \arg(V_{ub}^*)$ in the usual phase convention. At present, there is no evidence that this phase fails to account for $CP$ violation phenomena.

Two $B$ factories, built to study $CP$ violation in the $B$ system, have just been completed. By comparing the time dependence of tagged $B^0$ vs. $\bar{B}^0 \to J/\psi K_S$ decays, one can cleanly measure the $CP$ phase in $B^0$-$\bar{B}^0$ mixing, which, in the SM, gives $\sin 2\beta$ where $\beta = \arg(V_{td}^*)$. Together with the demonstrated capabilities of collider detectors at the Tevatron, a precise measurement of $\sin 2\beta$ is assured within a year or two. The unitarity phase $\alpha$ can be measured via $\pi^+\pi^-$ or $\pi^+\pi^-\pi^0$ modes but now appears to be more challenging because the $\pi^+\pi^-$ rate is smaller than expected, which in turn implies larger “penguin pollution”.

However, it is the phase $\gamma$ that is usually viewed as the most difficult. All suggestions so far require very high statistics or various technical challenges. In this Letter we exploit the emerging rare $B$ decay data from CLEO and perform a fit that, though model dependent, allows extraction of $\gamma$ with just $10^7 \ B$ mesons using only $CP$-averaged rates. The goodness of fit and reasonableness of other fit parameters serve as checks on the adequacy of our model assumptions. Since neither vertexing nor tagging is required, this method will benefit from the improved statistics soon available from the CLEO upgrade as well.

The measurement of $\sin 2\beta$ is often compared to a double-slit interference experiment, the two slits being $B^0$ and $\bar{B}^0 \to J/\psi K_S$ decays. Charmless rare $B$ decay rates, even when CP averaged, can also be viewed as double-slit experiments that in principle probe the phase $\gamma$. The present observed pattern that $\bar{B} \to K\pi$ rates are larger than $\pi\pi$ but comparable to $\rho^0\pi^-$, $\rho^\mp\pi^\pm$ and $\omega\pi^-$ implies that both tree (T) and penguin (P) amplitudes contribute to these rates, hence the double-slit analogy. Unfortunately, hadronization effects such as final state interactions (FSI) could dilute such interference effects.

The Fleischer-Mannel bound on $\gamma$ is no longer effective since one now has $R \equiv \Gamma(B^- \to K^-\pi^+)/\Gamma(\bar{B}^0 \to K^0\pi^+) = 1.0 \pm 0.3$, where $\Gamma$ denotes the average of $B$ and $\bar{B}$...
widths. A more promising method \cite{5} is based on \( R_s = \frac{\Gamma(B^- \to \overline{K}^0\pi^-)}{2\Gamma(B^- \to K^-\pi^0)} \), with some reference to \( \pi\pi \) for control of model dependence. But with \( R_s = 0.75 \pm 0.30 \) at present, one cannot set a useful bound on \( \gamma \). More than an order of magnitude increase in data is needed for a restrictive measurement. In this Letter we take a more global view and perform fits which trade model independence for exhaustive use of available data. We also use only \( CP \)-averaged rates, since there is no sign of significant \( CP \) asymmetries \cite{6} and the errors are large. Asymmetries in addition are more sensitive to FSI than averaged rates.

We shall assume that naive factorization \((N_c = 3)\) is a good approximation, and use effective-theory matrix elements cross-checked by two groups \cite{7,8}, ignoring annihilation type diagrams. We make a \( \chi^2 \) fit of data to \( \gamma \) and four other parameters. Factorization in two body rare \( B \) decays may be heuristically justified by the large energy release \cite{9}: final state mesons move away from each other so fast that they do not interact. Recent theoretical work suggests that factorization may be derivable from QCD in certain limits \cite{10}. In our view, factorization provides a simple framework to describe hadronic \( B \) decays that is rich in predictions with a limited set of free parameters. It is therefore reasonable to use this framework when attempting a first global fit to the large number of results on charmless hadronic \( B \) decays now available. This work is motivated by Ref. \cite{11}, which pointed out that recent CLEO rare \( B \) data supports factorization if \( \cos\gamma < 0 \) is taken. Indirect fits to the unitarity triangle find a 95\% C.L. range for \( \gamma \) of 44° \(-\) 75° \cite{12}, 44° \(-\) 93° \cite{13}, 41° \(-\) 97° \cite{14}, and 36° \(-\) 97° \cite{15}, depending in part on how conservatively the theoretical errors are handled.

Let us illustrate the parameters that enter with \( \overline{B}^0 \to K^-\pi^+ \), which is a \( b \to s\overline{u}u \) transition under factorization. Ignoring annihilation terms, one has \cite{4,8}

\[
\mathcal{A}_{K^-\pi^+} \propto f_K F_0^{B\pi} \left( m_B^2 - m_\pi^2 \right) \left[ V_{ub}^* V_{ts} a_1 - V_{ts}^* V_{ub} \left[ a_4 + a_{10} + (a_6 + a_8) R_{su} \right] \right]. \tag{1}
\]

We are free to fix \(|V_{ts}| \equiv |V_{cb}| = 0.0381 \) \cite{2} since any uncertainty can be absorbed in form factors. The two relevant fundamental parameters are therefore \(|V_{ub}/V_{cb}| \) and \( \gamma \), and the latter clearly controls the interference between tree and penguin terms. The parameters
are related to short distance Wilson coefficients (WC) and evaluated within a QCD framework. They also depend on the scale parameter \( \mu_f \) where factorization is operative. The values of \( a_i \) in the literature are still evolving as issues of scale, scheme and gauge dependence are addressed. We use two sets of \( a_i \) from Refs. [7] (AKL) and [8] (CCTY). The dominant strong penguin coefficients are \( a_4 \) and \( a_6 \) (\(-0.04 \) to \(-0.06 \)), while the dominant electroweak penguin coefficient is \( a_9 \simeq -0.009 \) coming from the \( Z \) penguin. We use the \( a_i \) for \( b \to s \) since the difference for \( b \to d \) is small.

In Eq. (1) one also has the factor

\[
R_{su} = \frac{2m_B^2}{m_b - m_u}(m_s + m_u) .
\]  

A similar factor \( R_{sd} \), which enters \( A_{\bar{K}^0\pi^-} \), is taken to be equal to \( R_{su} \). This factor can be better understood as a product of two pieces: the factor \( 1/(m_b - m_u) \approx 1/m_b \) balances against \( m_B^2 - m_{\pi^+}^2 \); and \( m_{K^-}^2/(m_s + m_u) \) is nothing but the nonperturbative part of the pseudo-Goldstone boson mass formula, which is well defined within QCD but not yet very well determined. Although \( R_{su} \) is technically related to an \( m_s \)-independent hadronic matrix element, in the form of Eq. (2), it becomes a sensitive probe of \( m_s \) in a way that is analogous to \( K \to \pi\pi \) decay and \( \epsilon'/\epsilon \).

The factors \( f_K \) and \( F_0^{B\pi} \) arise from evaluating hadronic matrix elements of four quark operators under factorization: The former comes from forming \( K^- \) out of the vacuum via the \( \bar{s}\gamma_\mu\gamma_5u \) current, the latter arises from the transition \( \bar{B}^0 \to \pi^+ \) via the \( \bar{u}\gamma_\mu b \) current. While form factors are well defined, it is the reliance on models that causes us to lose track of the factorization scale \( \mu_f \). Popular form factor models are the BSW model and light-cone sum rule (LCSR) evaluations. A recent compilation of models can be found in [16], but we shall treat form factors as fit parameters.

The criteria for choosing the decay modes to include in the fit are as follows. First, a central value branching ratio (BR) with statistical and systematic errors must be available. Second, we exclude \( VV \) modes such as \( \omega K^* \) since there is insufficient data to constrain the extra form factors that enter. Third, we require that the experimental sensitivity (a few
times $10^{-6}$ at present) to be comparable to the range of factorization predictions. Only the \( \omega \pi \) final state, with a predicted BR below $10^{-7}$, is removed with this criterion. Since this and other suppressed decays such as \( \rho \pi \), \( \pi \pi \), \( \phi \pi \), \( K K \) and \( K \ast K \) may well be dominated by FSI from other charmless final states, factorization is less likely to work well. We therefore propose to exclude these modes even when suitable measurements become available. The only exceptions to these rules are final states involving \( \eta \) and \( \eta' \). We prefer to apply the fit to predict their BRs rather than use them in the fit because the \( q\bar{q} \) content and other issues of these mesons have recently been questioned. We note that the newly measured \( \eta K \ast \) modes, like \( \eta' K \), are larger than previous theoretical predictions \cite{7,8}.

We give the 14 measured BRs (averaged over \( B \) and \( \overline{B} \)) that enter our fit in Table I, where we also give the fitted output. To limit the number of fit parameters, we use approximate relations as follows. We assume KM unitarity hence 
\[
-V_{td}^* V_{tb} = V_{cd} V_{cb} + V_{ud} V_{ub} \approx -|V_{cb}| (\lambda - e^{-i\gamma} |V_{ub}/V_{cb}|),
\]
where \( \lambda = |V_{us}| \). Since \( \lambda - |V_{ub}/V_{cb}| \cos \gamma > 0 \) always, as noticed in Ref. \cite{11}, T-P interference is opposite in sign for P-dominated and T-dominated modes such as \( K^- \pi^+ \) and \( \pi^- \pi^+ \), leading to enhanced \( K^- \pi^+ \), and suppressed \( \pi^- \pi^+ \) for \( \cos \gamma < 0 \), in better agreement with data. The chiral relation
\[
m_{K^-}/m_{\pi^-} \approx (m_s + m_u)/(m_d + m_u)
\]
and the fact that \( m_s \gg m_{d,u} \) give \( R_{su} \approx R_{sd} \approx R_{da} \), while \( Q_{ij} = -R_{ij} \) for \( VP \) modes such as \( \rho \pi \), \( \omega \pi \) and \( \omega K \). We use form factors at \( q^2 = 0 \) and \( F_{BP}^1 = F_{BP}^0; F_{BK}^0/F_{B\pi}^0 = 1.13 \) which is consistent with both BSW and LCSR models; \( A_{BP}^\omega = A_{BP}^\rho \); and \( A_{BK}^\ast = 1.26 A_{BP}^\rho \) (used for predictions only). Surveying the amplitudes for modes in Table I, we find that just five parameters suffice for the fit: \( \gamma \), \( |V_{ub}/V_{cb}| \), \( R_{su} \), \( F_{B\pi}^0 \) and \( A_{BP}^\rho \).

The function minimized by the fit is

\[
\chi^2 = \sum_i ((\text{BR}_{i \text{meas}} - \text{BR}_{i \text{pred}})/\sigma_{i \text{meas}})^2 + ((0.08 - |V_{ub}/V_{cb}|_{\text{pred}})/0.02)^2 ,
\]

where we sum over the modes in Table I. The predicted BRs are calculated from formulas like Eq. \cite{11} taken from Refs. \cite{7,8}. We have checked that we confirm the \( N_c = 3 \) results of AKL and CCTY with the same input parameters. We take into account the full (asymmetric) experimental errors and correlations in \( K^- \pi^+ / \pi^- \pi^+ \), \( K^- \pi^0 / \pi^- \pi^0 \) and \( \omega K^- / \omega \pi^- \).
measurements, where the correlation coefficients are $-0.15$, $-0.29$ and $-0.17$, respectively. The fit is able to nearly optimally use the information for each of these modes individually, though $K/\pi$ separation improvements in the next round of experiments will help in this regard. For simplicity, we assume that systematic errors have the same correlation coefficient as statistical errors, i.e. we apply the correlation coefficient to the total error with all errors combined in quadrature. If the best fit value is below the experimental central value, the low-side experimental error is used, and conversely the high-side.

To understand the behavior of the $\chi^2$ function in the 5D fit parameter space, its dependence on various fixed parameters or the exclusion of certain experimental measurements, we have explored many variants of our nominal fit. In all cases we find $\gamma > 100^\circ$. Our nominal fit results, with CCTY $a_i$ values, are given in Table II. The $\chi^2$ per degree of freedom (DOF) in the last column indicates the good quality of the fit. We choose CCTY rather than AKL as nominal only because of their claim of improved gauge dependence of the $a_i$. The fit values for AKL (see Table II) differ only slightly from our nominal, and mostly because of the larger $|a_{4,6}|$ found by these authors. We note that $R_{su}$ for AKL input should be smaller than the CCTY case since quark masses are defined at $\mu = 2.5$ GeV rather than $m_b$. We have also checked that the strong phases of $a_{4,6}$ have little impact on our fitted $\gamma$ value.

The $\chi^2$ vs. $\gamma$ curves for the nominal fit with CCTY input are shown in Fig. 1. We note that our $\gamma$ value has a two-fold ambiguity since the fit is sensitive to $\cos \gamma$ rather than $\gamma$. From the contributions from individual modes given in Fig. 1(b), we see that the main discriminator for favoring large $\gamma$ comes from $K^-\pi$ and $\pi^-\pi$ modes, and, somewhat surprisingly, the $\omega K^-$ and $\phi K^-$ modes. The situation for $\phi K^-$ is a result of the procedure of minimizing $\chi^2$ for each $\gamma$ value. This induces an apparent sensitivity, due to changes in the other parameters, where there is no direct dependence.

The error on $|V_{ub}/V_{cb}|$ returned by the fit is only marginally better than the conservative range $|V_{ub}/V_{cb}| = 0.08 \pm 0.02$ [2] used as an additional term in Eq. (3). Sensitivity to $|V_{ub}/V_{cb}|$ largely comes from $\overline{B} \rightarrow \rho^0\pi^-$, $\rho^\pm\pi^\mp$ and $\omega\pi^-$, all of which depend on $A_0^{B^0}$. Removing the constraint on $|V_{ub}/V_{cb}|$ from the fit gives higher $|V_{ub}/V_{cb}|$ with large errors and strongly
correlated with $A_0^{B\rho}$ (and between $R_{su}$ and $F_0^{B\pi}$ as well) but $\gamma = 121^{+31}_{-24}$ degrees remains close to nominal (see Table II).

The fit favors large $R_{su}$ ($R_{du}$) since it facilitates the enhancement (suppression) of $K^-\pi^{+,0}$ ($\pi^-\pi^+$) modes. Furthermore, under factorization, the BRs for $\omega K$ modes are enhanced only for large $R_{su}$ such that $a_4$ and $a_6$ penguin contributions do not cancel fully. We have checked that when the $\omega K^0$ mode is removed from the fit, there is no significant change in $\gamma$, though $R_{su}$ drops to 1.69. Our nominal $R_{su}$ fit value implies $m_s = 58^{+14}_{-11}, 67^{+16}_{-13}$ MeV at $m_b$ and 2 GeV scale, respectively. This is lower than what is commonly used in most previous calculations, but consistent with recent unquenched lattice results which give $m_s(2 \text{ GeV}) = 84 \pm 7$ MeV \[18\]. In addition recent experimental results for $\varepsilon'/\varepsilon$ can be reconciled better with theoretical predictions if a smaller value of $m_s$ is used \[18\]. For comparison, the result for fixing $R_{su} = 1.21$ ($m_s(m_b) = 90$ MeV) is given in Table II. Note that $R_{su}$ is anti-correlated with $a_6$ since only the product appears in the amplitudes. As for the form factors, our fitted $F_0^{B\pi}$ ($A_0^{B\rho}$) is lower (higher) than but consistent with the LCSR result of $0.305 \pm 0.046 (0.372 \pm 0.074)$.

Predictions from our fit for some selected modes are given in Table III. The agreement with the newly measured \[17\] $\eta\bar{K}^*$ modes are rather striking. An enhancement factor of 1.7 comes from $A_0^{BK^*} \approx 0.60$ compared to LCSR value of $A_0^{BK^*} \approx 0.47$, the rest coming from our low $m_s$. The $\eta'\bar{K}^*$ modes are comparable in size to the observed $\eta\bar{K}^*$ modes. Since we can account for less than half the rate of $\eta\bar{K}$ modes and the missing contribution may well be specific to the $\eta'$ decay modes, our predicted $\eta\bar{K}^*$ rates should be viewed with some caution. The $\rho^-\pi^0$ mode is suppressed by $\cos \gamma < 0$, smaller $F_1^{B\pi}$ (which also suppresses $\bar{K}^{-}\pi$ modes) plus destructive interference between two terms because of low $m_s$. The $\rho K$ modes are enhanced by the low value of $m_s$ and larger $A_0^{B\rho}$ form factor, except for $\rho^0 K^-$, which is suppressed compared to $\rho^+ K^-$ by destructive interference between the strong $a_6$ and electroweak $a_9$ penguin terms (a similar effect suppresses $\bar{K}^{(*)0}\pi^0$ modes).

As an aside, we give the “penguin pollution” as determined from our fit. Defining $T (P)$
as the amplitude arising from $a_{1-2}$ ($a_{3-10}$), we find the ratio $|P/T|$ in $\pi^+\pi^- (\rho^0 \pi^\pm)$ to be $0.37 \pm 0.04$ ($0.20 \pm 0.04$) for our nominal fit. For comparison, the CCTY result for $N_c = 3$, $\gamma \simeq 65^\circ$, $|V_{ub}/V_{cd}| = 0.090$, $m_s(m_b) = 90$ MeV using LCSR form factors gives 0.20 (0.10).

How do we reconcile with the usual fit to $B$ and $K$ data other than charmless rare $B$ decays, which give a 95% C.L. range that excludes $\cos \gamma < 0$ [12]? The removal of the second quadrant in these fits results mostly from combining recent bounds on $B_s$ mixing from LEP, CDF and SLD, with lattice QCD results that relate $B_d$ and $B_s$ mixing parameters [13]. Thus, our results suggest that $B_s$ mixing could be very close to the present limit.

It should be stressed that the goodness of our fit (see Table II) suggests that corrections to factorization may be small compared with the present experimental precision. It is reassuring that the hadronic parameters from our fit are not at variance with theoretical expectations. We note that $\gamma$ is the most stable parameter in the fit, with $\cos \gamma < 0$ for all variations we have considered. This is because $\gamma$ directly controls the “double-slit” interference, while other parameters enter indirectly. We note that our larger value of $\gamma$ tends to reduce the value of $\sin 2\beta$.

In conclusion, we have made a model-dependent determination of $\gamma = 114^{+25}_{-21}$ degrees. It will be interesting to see if future, more precise measurements will confirm this result and the predictions in our tables.

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REFERENCES

[1] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).

[2] Particle Data Group, C. Caso et al., Eur. Phys. J. C 3, 1 (1998).

[3] Preliminary results are given in CLEO Collaboration, Y. Kwon et al., CLEO-CONF 99-14 and hep-ex/9908039.

[4] R. Fleischer and T. Mannel, Phys. Rev. D. 57, 2752 (1998).

[5] M. Neubert and J. Rosner, Phys. Lett. B441, 403 (1998); M. Neubert, JHEP 9902, 14 (1999).

[6] CLEO Collaboration, T.E. Coan et al., CLEO-CONF 99-16 and hep-ex/9908023.

[7] A. Ali, G. Kramer and C.D. Lü, Phys. Rev. D. 58, 094009 (1998).

[8] Y.H. Chen, H.Y. Cheng, B. Tseng and K.C. Yang, Phys. Rev. D. 60, 094014 (1999).

[9] J.D. Bjorken, Nucl. Phys. (Proc. Suppl.) B11, 325 (1989).

[10] T.W. Yeh and H.n. Li, Phys. Rev. D. 56, 1615 (1997); M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, Phys. Rev. Lett. 83, 1914 (1999).

[11] X.G. He, W.S. Hou and K.C. Yang, Phys. Rev. Lett. 83, 1100 (1999). The large $\gamma$ possibility was first stressed by N.G. Deshpande, X.G. He, W.S. Hou and S. Pakvasa Phys. Rev. Lett. 82, 2240 (1999).

[12] F. Parodi, P. Roudeau and A. Stocchi, hep-ph/9802289 and hep-ex/9903063.

[13] S. Mele, Phys. Rev. D 59, 113011 (1999).

[14] S. Plaszczynski, in Proceedings of the Conference on HF8, Southampton, 1999 (to be published).

[15] A. Ali, D. London, Eur. Phys. J. C9 (1999) 687.
[16] A. Deandrea, R. Gatto, G. Nardulli and A.D. Polosa, hep-ph/9907225, and references therein.

[17] CLEO Collaboration, S.J. Richichi et al., CLEO-CONF 99-12 and hep-ex/9908019; M. Bishai et al., CLEO-CONF 99-13 and hep-ex/9908018.

[18] S. Aoki, in Proceedings of the Symposium on LP99, Stanford, 1999 (to be published).
Tables

Table I. Measured BR (in $10^{-6}$ units) entering the fit and output BR from our fit, and for comparison (in parenthesis) the $N_c = 3$ values from Ref. [8] for $\gamma = 65^\circ$ and LCSR form factors.

| Mode | Measured BR | BR from Fit | Mode | Measured BR | BR from Fit |
|------|-------------|-------------|------|-------------|-------------|
| $K^-\pi^+$ | $18.8^{+2.8}_{-2.6} \pm 0.7 \pm 1.1$ | 20.9 (12.9) | $\pi^-\pi^+$ | $4.7^{+1.8}_{-1.5} \pm 0.5 \pm 0.3$ | 4.4 (10.7) |
| $K^-\pi^0$ | $12.1^{+3.0}_{-2.8}+1.9 \pm 0.8$ | 11.8 (8.8) | $\pi^-\pi^0$ | $5.4^{+2.1}_{-2.0} \pm 1.5 \pm 0.3$ | 3.4 (5.7) |
| $K^0\pi^-$ | $12.8^{+4.6}_{-4.0} \pm 0.9 \pm 1.3$ | 19.5 (15.7) | $K^0\pi^0$ | $14.8^{+5.9}_{-5.1}^{+2.4}_{-3.3} \pm 2.5$ | 8.0 (5.6) |
| $\rho^0\pi^-$ | $15 \pm 5 \pm 4$ | 14.0 (8.1) | $\rho^\mp\pi^\pm$ | $35^{+11}_{-10} \pm 5$ | 39.0 (41.3) |
| $\omega\pi^-$ | $11.3^{+3.3}_{-2.9} \pm 1.0$ | 11.5 (8.2) | $K^+\pi^0$ | $22^{+8}_{-6}^{+2.4}_{-6} \pm 5$ | 5.4 (4.3) |
| $\omega K^-$ | $3.1^{+2.4}_{-1.9} \pm 0.8$ | 3.7 (1.4) | $\phi K^-$ | $1.6^{+1.9}_{-1.2} \pm 0.2$ | 3.0 (5.0) |
| $\omega K^0$ | $10.0^{+5.4}_{-4.2} \pm 1.5$ | 4.3 (0.4) | $\phi K^0$ | $10.7^{+7.8}_{-5.7} \pm 1.1$ | 2.8 (4.6) |

Table II. Results for fitted parameters. CCTY (AKL) implies use of $a_i$’s from Ref. [8] (Ref. [7]).

| Input $a_i$ | $\gamma$ (degrees) | $|V_{ub}/V_{cb}|$ | $R_{su}$ | $F_0^{B\pi}$ | $A_0^{B\rho}$ | $\chi^2_{\text{min}}/\text{DOF}$ |
|-------------|---------------------|-----------------|----------|-------------|--------------|------------------|
| CCTY | $114^{+25}_{-21}$ | $0.087 \pm 0.016$ | $1.89^{+0.43}_{-0.36}$ | $0.26 \pm 0.04$ | $0.48^{+0.13}_{-0.10}$ | 11.3/10 |
| AKL | $105^{+23}_{-21}$ | $0.092 \pm 0.016$ | $1.70^{+0.34}_{-0.28}$ | $0.23 \pm 0.03$ | $0.45^{+0.12}_{-0.09}$ | 12.2/10 |
| CCTY | $124^{+56}_{-29}$ | $0.076 \pm 0.014$ | $1.21$ (fixed) | $0.32 \pm 0.03$ | $0.58^{+0.15}_{-0.11}$ | 15.0/11 |
| CCTY | $121^{+31}_{-24}$ | $0.105^{+0.063}_{-0.032}$ (free) | $2.21^{+1.30}_{-0.61}$ | $0.23^{+0.06}_{-0.07}$ | $0.48^{+0.13}_{-0.10}$ | 11.0/9 |
Table III. Predicted BR (in $10^{-6}$ units) with CCTY $a_i$s for some modes not used in the fit, and (in parenthesis) from CCTY for $N_c = 3$, $\gamma = 65^\circ$, $|V_{ub}/V_{cb}| = 0.090$, $m_s(m_b) = 90$ MeV using LCSR form factors. For $\eta'$, $\eta$ modes, we have used $f_{\eta'}^u$, $f_{\eta'}^s = 64, 141$ MeV, $f_{\eta}^u$, $f_{\eta}^s = 78, -113$ MeV, and $F_{B\eta}^{0}$, $F_{B\eta}^{-0} = 0.108, 0.121$, respectively [7,8]. The $f_{c}^{c(0)}$ effects are small.

| Mode     | Measured BR [17] | BR$_{pred}$ | Mode     | BR$_{pred}$ | Mode     | BR$_{pred}$ | Mode     | BR$_{pred}$ |
|----------|------------------|-------------|----------|-------------|----------|-------------|----------|-------------|
| $\eta'K^-$ | $80^{+10}_{-9} \pm 8$ | 32.4 (21.9) | $\eta K^-$ | 1.9 (1.7)   | $\rho^-\pi^0$ | 6.5 (15.0) | $\rho^+K^-$ | 6.1 (2.0)   |
| $\eta K^0$ | $88^{+18}_{-16} \pm 9$ | 28.1 (21.7) | $\bar{\eta}K^0$ | 1.9 (1.0)   | $K^{*-}\pi^0$ | 5.1 (3.3)  | $\rho^0K^-$ | 1.8 (0.5)   |
| $\eta K^{*-}$ | $27.3^{+9.6}_{-8.2} \pm 5.0$ | 16.2 (3.9)   | $\eta'K^{*-}$ | 14.4 (2.0)  | $K^{*0}\pi^-$ | 3.5 (5.5)  | $\rho^-\bar{K}^0$ | 8.0 (0.4)   |
| $\eta K^{*-0}$ | $13.8^{+5.5}_{-4.4} \pm 1.7$ | 13.2 (4.3)   | $\eta'K^{*-0}$ | 13.8 (1.0)  | $K^{*0}\pi^0$ | 0.6 (1.3)  | $\rho^0\bar{K}^0$ | 6.2 (1.1)   |
FIG. 1. (a) $\chi^2 - \chi^2_{\text{min}}$ vs. $\gamma$ from nominal fit, where $\chi^2_{\text{min}} = 11.3$ for 10 DOF; (b) Major $\chi^2$ contributions from individual measurements.