Non-Abelian Self-Dual String Solutions

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ABSTRACT: We consider the equations of motion of the non-abelian 5-branes theory recently constructed in [1] and find exact string solutions both for uncompactified and compactified spacetime. Although one does not have the full supersymmetric construction of the non-abelian (2,0) theory, by combining knowledge of conformal symmetry and R-symmetry one can argue for the form of the 1/2 BPS equations in the case when only one scalar field is turned on. We solve this system and show that our string solutions could be lifted to become solutions of the non-abelian (2,0) theory with self-dual electric and magnetic charges, with the scalar field describing a M2-brane spike emerging out of the multiple M5-branes worldvolume.

KEYWORDS: M-Theory, D-branes, M-branes, Gauge Symmetry.
1. Introduction

The low energy theory of $N$ coincident M5-branes is given by an interacting (2,0) superconformal theory in 6 dimensions [2]. On the M5-brane worldvolume there are self-dual strings. For a single M5-brane, the low energy theory is known [3–7]. The self-dual string soliton has also been constructed [4, 8]. Much less is known about the theory of multiple M5-branes, as well as the properties of multiple self-dual strings.

Recently, a theory of non-abelian chiral 2-form in 6-dimensions was constructed [1]. The construction was motivated by the analysis in [9,10] and a set of 5d Yang-Mills gauge fields was introduced in order to incorporate non-trivial interactions among the 2-form potential. The theory admits a self-duality equation on the field strength as the equation of motion. It has a modified 6d Lorentz symmetry. On dimensional reduction on a circle, the action gives the standard 5d Yang-Mills action plus higher order corrections. Based on these properties, it was proposed that the theory describes the gauge sector of multiple M5-branes in flat space. An important feature of this theory is that the self-interaction of the two-form gauge field is mediated by a set of five-dimensional Yang-Mills gauge field $A_{\mu}, \mu = 0, 1, 2, 3, 4$). The Yang-Mills gauge field is auxiliary and is constrained non-trivially to be given in terms of the non-abelian tensor gauge field and does not contain
any propagating degrees of freedom. In the Abelian case, the 1-form gauge field is free
and simply decouple. See also [11], [12, 13], [14], [15], [16], [17, 18], [10, 19–21], for some
other more relevant recent developments.

In this paper we give a further support of this proposal by constructing the non-
abelian self-dual strings to the equation of motion of the non-abelian theory [9]. Without
loss of generality, we consider a $SU(2)$ gauge group which corresponds to a system of two
M5-branes. A crucial observation in our construction is that the Perry-Schwarz solution
is supported by a Dirac monopole $A_a, a = 0, 1, 2, 3$). As the solution is translational
invariant along the direction (say $x^4$) of the string, this gauge field can be thought of as
a five dimensional one with $A_4 = 0$ and be interpreted as the auxiliary 1-form gauge fields
in the theory of [1]. This interpretation suggests that the non-abelian self-dual string
solution may be constructed by taking the auxiliary Yang-Mills gauge field to be given
by a non-abelian monopole. Quite remarkably this is indeed correct and we are able
to construct a self-dual string solution both for uncompactified six dimensions as well
as with one dimension compactified. Our solution is obtained by replacing the Dirac
monopole in the Perry-Schwarz string, in the uncompactified case to the non-abelian
Wu-Yang monopole; and in the compactified case to the ’t Hooft-Polyakov monopole.

The plan of the paper is as follows. In section 2, we review the non-abelian 5-
brane theory of [1]. In section 3, after reviewing the original Perry-Schwarz self-dual
string solution, we present a new abelian self-dual string solution which is orientated in
a different direction. The existence of the latter solution is guaranteed by the Lorentz
symmetry of the Perry-Schwarz theory. Then we solve the non-abelian equation of
motion of [1] and obtain an exact solution describing a string. We then discuss how this
solution can be lifted as a solution of the (2,0) supersymmetric theory. The resulting
solution describes a non-abelian string with self-dual charges. In section 4, we consider
the compactified case and construct the corresponding self-dual string solution. The
paper is concluded with some further comments and discussions in section 5.

2. Review of the Non-Abelian Multiple 5-brane Theory

In [1], an action for non-abelian chiral 2-form in 6-dimensions was constructed as a
generalization of the linear theory of Perry-Schwarz. As in Perry-Schwarz, manifest
6d Lorentz symmetry was given up and the self-dual tensor gauge field is represented
by a $5 \times 5$ antisymmetric field $B_{\mu \nu}, \mu, \nu = 0, .., 4$. Throughout the paper we use
the convention that the 5d and 6d coordinates are denoted by $x^\mu = (x^0, x^1, \cdots, x^4)$ and
$x^M = (x^\mu, x^5)$. We use $\eta^{MN} = (++, +++, ++)$ for the metric and $\epsilon^{01234} = -\epsilon_{01234} = 1,$
$\epsilon^{012345} = -\epsilon_{012345} = 1$ for the antisymmetric tensors. The Hodge dual of a 3-form $G_{MNP}$
is defined by

$$\tilde{G}_{MNP} := -\frac{1}{6} \epsilon_{MNPQRS} G^{QRS}.$$  \hspace{1cm} (2.1)
Motivated by the consideration in [10], a set of 5d 1-form gauge fields $A^a_\mu$ was introduced for a gauge group $G$. The proposed action is

$$S = S_0 + S_E$$  \hspace{1cm} (2.2)

with $S_0$ a non-abelian generalization of the Perry-Schwarz action,

$$S_0 = \frac{1}{2} \int d^6x \, \text{tr} \left( -\dot{H}^{\mu\nu} \dot{H}_{\mu\nu} + \ddot{H}^{\mu\nu} \partial_5 B_{\mu\nu} \right)$$  \hspace{1cm} (2.3)

where $H_{\mu\nu\lambda} = D_{[\mu} B_{\nu\lambda]} = [\partial_{\mu} + A_{[\mu}, B_{\nu\lambda]]}$; and with $S_E$

$$S_E = \int d^5x \, \text{tr} \left( (F_{\mu\nu} - c \int dx_5 \dot{H}_{\mu\nu}) E_{\mu\nu} \right),$$  \hspace{1cm} (2.4)

where $E_{\mu\nu}(x^\lambda)$ is a 5d auxiliary field, providing a constraint such that $A_\mu$ carries no extra degrees of freedom. Here $c$ is a constant and it was taken to be 1 in [1]. Actually one can take any nonzero value of $c$ and this makes no change to all the symmetries discusses in [1]. The only modification is the relation of the Yang-Mills coupling to the compactification radius, $g_{YM}^2 = \pi R c^2$. In the following we will show how the value of $c$ is fixed by the requirement of charge quantization of our self-dual string solution.

Besides the Yang-Mills gauge symmetry,

$$\delta A_\mu = \partial_\mu \Lambda + [A_\mu, \Lambda], \quad \delta B_{\mu\nu} = [B_{\mu\nu}, \Lambda], \quad \delta E_{\mu\nu} = [E_{\mu\nu}, \Lambda]$$  \hspace{1cm} (2.5)

for arbitrary $\Lambda = \Lambda(x^\lambda)$, the action has the tensor gauge symmetry

$$\delta_T A_\mu = 0, \quad \delta_T B_{\mu\nu} = \Sigma_{\mu\nu}, \quad \delta_T E_{\mu\nu} = 0,$$  \hspace{1cm} (2.6)

for $\Sigma_{\mu\nu}(x^M)$ satisfying $D_M \Sigma_{\mu\nu} = 0$. This form of symmetry first appears in [14]. As demonstrated in [1], the theory has manifest 5d Lorentz symmetry and a modified 6d Lorentz symmetry. To establish those symmetries of the action, we take the field configuration satisfying the boundary conditions:

$$D_\lambda B_{\mu\nu}, \partial_5 B_{\mu\nu} \to 0 \quad \text{as} \quad |x^M| \to \infty$$  \hspace{1cm} (2.7)

With an appropriate fixing of this tensor gauge symmetry, one can turn the equation of motion of $B_{\mu\nu}$ into a first order self-duality condition:

$$\ddot{H}_{\mu\nu} = \partial_5 B_{\mu\nu}.$$  \hspace{1cm} (2.8)

The gauge field is auxiliary and is determined by the equation:

$$F_{\mu\nu} = c \int dx_5 \dot{H}_{\mu\nu}$$  \hspace{1cm} (2.9)
This constraint was inspired from the analysis of the dimensional reduction, in which one gets multiple D4-branes plus higher derivative correction terms. Notice that, on mass-shell, the constraint (2.9) simply says that $F_{\mu\nu}$ is given by the boundary values of $B_{\mu\nu}$ for the uncompactified case:

$$F_{\mu\nu} = c(B_{\mu\nu}(x_5 = \infty) - B_{\mu\nu}(x_5 = -\infty))$$  \hspace{1cm} (2.10)

and

$$F_{\mu\nu} = 2\pi Rc\tilde{H}^{(0)}_{\mu\nu},$$  \hspace{1cm} (2.11)

when $x^5$ is compactified on a circle of radius $R$. Here $\tilde{H}^{(0)}_{\mu\nu}$ is the zero mode part of the field strength.

3. Non-Abelian Self-Dual String Solution: Uncompactified Case

In this section, we construct self-dual string solution that satisfies both (2.8) and (2.10). As mentioned above, a direct observation on the constraint (2.10) shows that the solution cannot be aligned in the $x^5$ direction since this would imply $F_{\mu\nu} = 0$ which is trivial. This does not imply the non-existence of a string solution in other directions, because the self-duality equation (2.8) has only 5d Lorentz symmetry as it’s a gauge fixed equation of motion [1]. Therefore, as a preparation to constructing the more general non-abelian self-dual string solution, we will first construct an abelian self-dual string solution aligning in the $x^4$ direction and we will start by reviewing the original abelian self-dual string solution of Perry and Schwarz.

3.1 Self-dual string solution in the Perry-Schwarz Theory

In [4], a nonlinear theory of chiral 2-form gauge field which results in the Born-Infeld action for a $U(1)$ gauge field when reduced to 5 dimensions was constructed. The Perry-Schwarz non-linear field equation is given by

$$\tilde{H}_{\mu\nu} = \frac{(1 - y_1)H_{\mu\nu 5} + H_{\mu\rho 5}H^{\rho\sigma 5}H_{\sigma\nu 5}}{\sqrt{1 - y_1 + \frac{3}{2}y_1^2 - y_2}},$$  \hspace{1cm} (3.1)

where

$$y_1 := -\frac{1}{2}H_{\mu\nu 5}H^{\mu\nu 5}, \quad y_2 := \frac{1}{4}H_{\mu\nu 5}H^{\rho\sigma 5}H_{\rho\sigma 5}H^{\alpha\mu 5}.$$  \hspace{1cm} (3.2)

As they demonstrated, the equation of motion (3.1) admits a solution describing a self-dual string soliton with finite tension aligning in the direction $x^5$. Since (3.1) is (non-manifest) 6d Lorentz covariant, it means there must also exist self-dual string solution aligned in other directions. In the following, we review their construction in section 3.1.1. Then we construct new self-dual string solution aligned in a different direction in section 3.1.2.
3.1.1 Self-dual string in the $x^5$ direction

The ansatz Perry and Schwarz considered for their self-dual string solution is

$$B = \alpha(\rho) dt dx^5 + \frac{\beta}{8}(\pm 1 - \cos \tilde{\theta}) d\tilde{\phi} d\tilde{\psi}, \tag{3.3}$$

where the 6d metric is

$$ds^2 = -dt^2 + (dx^5)^2 + d\rho^2 + \rho^2 d\Omega_3^2, \tag{3.4}$$

with the three-sphere given in Euler coordinates

$$d\Omega_3^2 = \frac{1}{4}[(d\tilde{\psi} + \cos \tilde{\theta} d\tilde{\phi})^2 + (d\tilde{\theta}^2 + \sin^2 \tilde{\theta} d\tilde{\phi}^2)], \tag{3.5}$$

where $0 \leq \tilde{\theta} \leq \pi, 0 \leq \tilde{\phi} \leq 2\pi, 0 \leq \tilde{\psi} \leq 4\pi$. For this ansatz, it is $y_1 = \alpha'^2, y_2 = \alpha'^4/2$ and the non-linear field equation (3.1) reads

$$\alpha'(\rho) = \frac{\beta}{\sqrt{\beta^2 + \rho^6}}, \tag{3.6}$$

This can be solved easily in terms of a hyper-geometric function. The solution is regular everywhere where $\alpha \sim \rho$ as $\rho \to 0$, while $\alpha \sim -\frac{\beta}{2\rho^2} + \text{const.}$ as $\rho \to \infty$. Note that the same ansatz also solves the linear self-duality equation, where in this case we have,

$$\alpha'(\rho) = \frac{\beta}{\rho^3}, \tag{3.7}$$

and the solution is singular at $\rho = 0$. In other words, the non-linear terms in the field equation has smoothen out the singularity at $\rho = 0$.

The magnetic charge $P$ and electric charge $Q$ per unit length of the string are given by

$$P = \int_{S^3} H, \quad Q = \int_{S^3} *H, \tag{3.8}$$

where $*$ denotes the Hodge dual operation and $S^3$ is a three sphere surrounding the string. It is straightforward to obtain that

$$P = 2\pi^2 \beta, \quad \text{and} \quad Q = 2\pi^2 \rho^3 \alpha'(\rho)|_{\rho \to \infty} = 2\pi^2 \beta, \tag{3.9}$$

hence the string is self-dual. This holds for both the nonlinear and the linear cases. Note that our answer is $1/8$ of those in [4] as we have introduced the factor of $1/4$ into the metric (3.5) in order to reproduce the correct volume $2\pi^2$ for a unit three sphere.

The charge quantization condition [22,23]

$$PQ + QP \in 2\pi \mathbb{Z} \tag{3.10}$$
for the self-dual string gives
\[ \beta = \pm \sqrt{\frac{n}{4\pi^3}}, \]  
(3.11)
i.e.
\[ P = Q = \pm \sqrt{n\pi}, \]  
(3.12)
where \( n \) is a positive integer. Note that the charge quantization condition we used is different from the Dirac-Teitelboim-Nepomechie charge quantization condition [24–26] Perry and Schwarz used. The condition (3.10) is obtained with a self-dual string probing another self-dual string and the positive sign in the charge quantization condition is appropriate for dyonic branes in \( D = 4k + 2 \) spacetime dimensions [22, 23].

Perry and Schwarz have also computed the tension of their string solution. Since the solution is static, the energy can be identified with the Lagrangian and the energy per unit length is found to be
\[ T = \tilde{c}\beta^{4/3}, \]  
(3.13)
where \( \tilde{c} \) is a numerical coefficient. We remark that for the self-dual string solution of the linearized theory, the tension is
\[ T = 0 \]  
(3.14)
since obviously the action vanishes on-shell. Since the charges and tension are well defined, it appears that the singularity at \( \rho = 0 \) is not harmful.

We also remark that the Perry-Schwarz self-dual string solution is non-BPS as there is no other matter field turned on to cancel the tensor field force. In the literature, there is also the 1/2 BPS self-dual string of Howe, Lambert and West [8]. In fact the Perry-Schwarz self-duality equation of motion can be embedded in the fully supersymmetric five-brane equation of motion of [3] by setting all the matter fields to zero and hence the Perry-Schwarz self-dual string solution can be lifted to be a solution of the full five-brane equation of motion, albeit a nonsupersymmetric one. Unlike the nonlinear Perry-Schwarz self-dual string solution, the Howe-Lambert-West self-dual string solution is singular at the location of the string. In fact \( B \sim 1/\rho^2 \) near the string, which is exactly as in linearized Perry-Schwarz self-dual string solution.

3.1.2 Self-dual string soliton in the \( x^4 \) direction
The Perry-Schwarz solution is translationally invariant along \( x^5 \). One may want to generalize this solution directly and construct a non-Abelian self-dual string solution which is translationally invariant along \( x^5 \) but this is not possible. As reviewed above, the gauge field strength in the non-abelian theory is given on-shell by the boundary value of \( B \)-field as (2.10) Therefore, if the non-Abelian solution is translationally invariant along \( x^5 \), then \( F_{\mu\nu} = 0 \) which is trivial.
To get a non-trivial solution, we need to base our construction on Perry-Schwarz solitons which are translationally invariant along other direction, say $x^4$. Such a solution can be easily obtained by rotating the original Perry-Schwarz solution as Perry and Schwarz has proved that their theory and the non-linear equation (3.1) respect Lorentz symmetry. Therefore, a simple Lorentz transformation which swap $(x_4, x_5) \to (-x_5, x_4)$ can be applied on the original Perry-Schwarz solution (the minus sign is needed to preserve the orientation of spacetime) to obtain the desired solution.

To facilitate the discussion, it is more convenient to use the spherical polar coordinates which is related to the Euler coordinates by the change of coordinates
\[
\tilde{\theta} = 2\theta, \quad \tilde{\phi} = \psi - \phi, \quad \tilde{\psi} = \psi + \phi.
\] (3.15)

With this coordinates, the three-sphere metric is given by
\[
d\Omega_3^2 = d\theta^2 + \sin^2 \theta d\phi^2 + \cos^2 \theta d\psi^2
\] (3.16)
with the ranges $0 \leq \theta \leq \pi/2$, $0 \leq \phi, \psi \leq 2\pi$, and the Perry-Schwarz ansatz (3.3) becomes
\[
B = \alpha(\rho) dt dx^5 + \beta \left( \frac{1}{4} \pm \frac{1}{4} - \frac{1}{2} \cos^2 \theta \right) d\phi d\psi.
\] (3.17)

Next change to Cartesian coordinates
\[
x = \rho \sin \theta \cos \phi, \quad y = \rho \sin \theta \sin \phi, \quad z = \rho \cos \theta \cos \psi, \quad w = \rho \cos \theta \sin \psi,
\] (3.18)
where we have denoted $(x^1, x^2, x^3, x^4) = (x, y, z, w)$. The metric becomes
\[
ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 + dw^2 + d(x^5)^2,
\] (3.19)
and the Perry-Schwarz ansatz reads
\[
B = \alpha(\rho) dt dx^5 + \beta \left( \frac{1}{4} \pm \frac{1}{4} - \frac{1}{2} \frac{w^2 + z^2}{\rho^2} \right) (xz dy dw - xwdydz - yz dx dw + yw dx dz).\] (3.20)

Keeping the orientation, we swap $(x_4, x_5) \to (-x_5, x_4)$ and obtain our ansatz for a string solution along the $x^4$ direction,
\[
B = \alpha(\rho) dt dw - \beta \left( \frac{1}{4} \pm \frac{1}{4} - \frac{1}{2} \frac{(x^5)^2 + z^2}{\rho^2} \right) (xz dy dx^5 - xz dx^5 dy dz - yz dx^5 dx^5 + yx^5 dx dz d).\] (3.21)

where now
\[
\rho = \sqrt{(x^5)^2 + r^2}, \quad r := \sqrt{x^2 + y^2 + z^2}.
\] (3.22)
It follows that
\[
H = \frac{\alpha'}{\rho} dt dw (xdx + ydy + zdz + x^5 dx^5) + \frac{\beta}{\rho^4} (x^5 dx dy dz - xdy dz dx^5 + zdy dx^5 x^5 - ydz dx^5).
\] (3.23)
\[ *H = \frac{\alpha'}{\rho} (x^5 dx dy dz - x dy dz dx^5 + z dy dx^5 - y dz dx^5) + \frac{\beta}{\rho^4} dtdw (xdx + ydy + zdz + x^5 dx^5), \]

and
\[ y_1 = \frac{(\alpha')^2 (x^5)^2}{\rho^2}, \quad y_2 = \frac{\beta^4 r^4}{2 \rho^8} + \frac{(\alpha')^4 (x^5)^4}{2 \rho^4}. \]

Then the field equation (3.1) gives
\[ \frac{\beta}{\rho^4} x^5 dtdw + \frac{\alpha'}{\rho} (-x dy dz + z dy dx - y dz dx) = \frac{\alpha' x^5}{\rho} G dtdw + \frac{1}{G} \frac{\beta}{\rho^4} (-x dy dz + z dy dx - y dz dx), \]

where
\[ G = \sqrt{\frac{1 + \beta^2 r^2 \rho^{-8}}{1 - \alpha'^2 (x^5)^2 \rho^{-2}}}. \]

The equation (3.26) is equivalent to
\[ \alpha' = \frac{\beta}{\sqrt{\beta^2 + \rho^8}}, \]

which is the same equation as before. As a consistency check, we integrate over the \( S^3 \) transverses to \( x^4 \) and obtain the same charges
\[ P = Q = 2\pi^2 \beta. \]

For the linearized case, \( \alpha' = \beta/\rho^3 \).

3.1.3 Self-dual string soliton in the \( x^4 \) direction in the \( B_{\mu 5} = 0 \) gauge

The potential \( B_{MN} \) in the solution (3.20) or (3.21) does not satisfy the condition \( B_{\mu 5} = 0 \) as needed in [1, 4]. However this is not a problem as they are indeed gauge equivalent to one which does. Instead of giving the gauge transformation, it is more instructive to construct directly the linearized self-dual string soliton in the \( x^4 \) direction in this gauge.

The starting point is (3.23) with \( \alpha' = \beta/\rho^3 \). Our strategy is to integrate the self-duality equation of motion
\[ H_{\mu 5} = \partial_5 B_{\mu \nu} \]

to get \( B_{\mu \nu} \). Then we use \( B_{\mu \nu} \) to compute the whole \( H_{MNP} \) and check its consistency with our ansatz. The components of \( H \) are
\[
H_{twi} = \frac{\beta x^i}{\rho^4}, \quad H_{ijk} = \frac{\epsilon_{ijk} \beta x^5}{\rho^4}, \quad H_{tw5} = \frac{\beta x^5}{\rho^4}, \quad H_{ij5} = -\frac{\epsilon_{ijk} \beta x^k}{\rho^4}.
\]
Integrating (3.32), we get the following components of $B_{\mu\nu}$:

$$B_{ij} = -\frac{1}{2} \beta \epsilon_{ijk} \frac{x_k}{r^3} \left( \frac{x^5 r}{\beta^2} + \tan^{-1}(x^5/r) \right), \quad B_{tw} = -\frac{\beta}{2\beta^2}. \quad (3.33)$$

In principle, $x^5$ independent constants of integration can be added but we will not need them. It is now easy to check a consistent solution is obtained by setting all the other independent components of $B_{\mu\nu}$ to be zero.

Two remarks are in order:

1. We remark that if we apply the condition (2.9) to the Perry-Schwarz self-dual string solution, we obtain

$$F_{ij} = -c\beta \pi \epsilon_{ijk} \frac{x_k}{r^3}, \quad F_{tw} = 0 \quad (3.34)$$

for the auxiliary gauge field. Certainly this $U(1)$ field decouples and play no role in the abelian case. However it is interesting to note that this is precisely the field strength of a Dirac monopole in the $(x, y, z)$ subspace! The presence of a Dirac monopole was already apparent in the original solution of [4]. Here, we reveal that the same monopole configuration also appears as the auxiliary gauge field. It turns out the use of an non-abelian monopole in place of the Dirac monopole is precisely what is needed to construct the non-abelian self-dual string solution.

2. The solution in the form (3.33) will be our basis for the construction of the non-abelian self-dual string in the next subsection. We remark that it is also quite interesting that this form of the solution provides a link between linearized Perry-Schwarz self-dual string and Howe-Lambert-West self-dual string [8]. To explain this, let us first give a brief review on the key construction of Howe-Lambert-West self-dual string. In the (2,0) supersymmetric theory, there are two non-linearly related 3-form field strengths which are called $H$ and $h$. The 3-form $H$ is exact but not necessarily self-dual while the 3-form $h$ is self-dual but not necessarily exact. When constructing self-dual string, one of the scalar fields is also turned on. The equation of motion is non-linear. However, with an appropriate ansatz, it is possible to impose a BPS condition which eventually gives a linear differential relation between $H$ and the scalar field. Writing in our notation, the BPS equations of motion read

$$H_{twi} = \partial_i \phi, \quad H_{tw5} = \partial_5 \phi, \quad (3.35)$$

$$H_{ijk} = \epsilon_{ijk} \partial_5 \phi, \quad H_{ij5} = -\epsilon_{ijk} \partial_k \phi, \quad (3.36)$$

where we have rescaled the scalar to absorb an inessential numerical factor. These conditions ensure the self-duality of $H$. Furthermore, they agree precisely with
the Perry-Schwarz’s equations of motion (3.30) if one identifies $B_{tw} = \phi$. In other words, the linearized Perry-Schwarz self-dual string solution could be lifted to a 1/2 BPS solution in the (2,0) supersymmetric theory by adding a scalar field that satisfies the ‘BPS’ condition (3.36) (due to self-duality, the condition (3.35) is not needed).

3.2 Non-abelian Wu-Yang string solution

Now we are ready for the non-abelian case. As noted above of the roles played by the Dirac monopole in the abelian Perry-Schwarz solution, it is natural to consider the non-abelian generalizations of the Dirac monopole in the construction of the non-abelian self-dual strings. Here we have two candidates: the Wu-Yang monopole and the ’t Hooft-Polyakov monopole where the latter involves a Higgs scalar field while the former does not. See, for example, [27] for a review of these solutions. We will use these non-abelian configurations to construct non-abelian self-dual string solutions for both the uncompactified case (where the Wu-Yang solution will be used) and compactified case (where the ’t Hooft-Polyakov monopole will be used).

Let us first briefly review the non-abelian Wu-Yang monopole. Without loss of generality, we will consider $SU(2)$ gauge group with Hermitian generators $T^a = \frac{\sigma^a}{2}$ satisfying

$$[T^a, T^b] = i\epsilon^{abc}T^c, \quad a, b, c = 1, 2, 3.$$  \hspace{1cm} (3.37)

This corresponds to the relative gauge symmetry of a system of two five-branes. Our convention for the Lie algebra valued fields are: $F_{\mu\nu} = iF^a_{\mu\nu}T^a$, $A_\mu = iA^a_\mu T^a$ and $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - \epsilon^{abc}A^b_\mu A^c_\nu$.

The non-abelian Wu-Yang monopole is given by

$$A^a_i = -\epsilon_{ak}x_k^2, \quad F^a_{ij} = \epsilon_{ijm}x_m^a x_a^2.$$ \hspace{1cm} (3.38)

Note that the field strength for the Wu-Yang solution is related to the field strength $F_{ij}^{(\text{Dirac})} = \epsilon_{ijm}x_m/r^3$ of the Dirac monopole by a simple relation:

$$F^a_{ij} = F_{ij}^{(\text{Dirac})} \frac{x^a}{r}.$$ \hspace{1cm} (3.39)

In fact by performing a (singular) gauge transformation

$$U = e^{i\sigma_3\phi/2}e^{i\sigma_2\theta/2}e^{-i\sigma_3\phi/2},$$ \hspace{1cm} (3.40)

one can go to an Abelian gauge where only the 3rd component of the gauge field survives. In this gauge

$$A^a_i = \delta^a_3 A^{(\text{Dirac})}_i.$$ \hspace{1cm} (3.41)
Despite its close connection with the Dirac monopole, the Wu-Yang solution is not a monopole since it does not source the non-abelian magnetic field. In fact the color magnetic charge vanishes

$$\int_{S^2} F^a = 0. \quad (3.42)$$

Nevertheless the Wu-Yang solution is a useful prototype for constructing a non-abelian monopole and we will follow the common practice of the literature to refer to it as the Wu-Yang monopole. In particular, a magnetic charge can be defined if there is also in presence a Higgs scalar field as in the 't Hooft-Polyakov monopole.

Inspired by the relation (3.39) of the Wu-Yang solution, we will try to solve the non-abelian self-duality equation (2.8) by adopting the following ansatz for the field strength,

$$H_{\mu\nu\lambda} = H^{(PS)}_{\mu\nu\lambda} \frac{x^a}{r} \quad (3.43)$$

Here \( r = \sqrt{x^2 + y^2 + z^2} \) and

$$H^{(PS)} := \frac{\beta}{\rho^4} \left[ dt dw(x dx + y dy + z dz + x^5 dx^5) + x^5 dx dy dz - z dx dy dx^5 - y dz dx dx^5 - x dy dz dx^5 \right] \quad (3.44)$$

is the field strength for the linearized Perry-Schwarz solution in the \( x^4 \) direction (3.23). The self-duality of (3.43) follows immediately from the self-duality of the Perry-Schwarz solution. For the moment, we will allow \( \beta \) to be a free parameter.

Our strategy is again to integrate \( H_{\mu\nu5} = \partial_5 B_{\mu\nu} \) to get \( B_{\mu\nu} \). Then we obtain \( F_{\mu\nu} \) and \( A_{\mu} \) from the boundary value of \( B_{\mu\nu} \). Finally, we use \( B_{\mu\nu} \) and \( A_{\mu} \) to compute the whole \( H_{MNP} \) and check its consistency with our ansatz. Now the components of our ansatz are:

$$H_{twi}^a = \frac{\beta x^i x^a}{r \rho^4}, \quad H_{ijk}^a = \frac{\epsilon_{ijk} \beta x^5 x^a}{r \rho^4}, \quad (3.45)$$

$$H_{tw5}^a = \frac{\beta x^5 x^a}{r \rho^4}, \quad H_{ij5}^a = -\frac{\epsilon_{ijk} \beta x^k x^a}{r \rho^4}. \quad (3.46)$$

Integrating (3.46), we get the following components of \( B_{\mu\nu} \):

$$B_{\mu\nu}^a = B_{\mu\nu}^{(PS)} \frac{x^a}{r}, \quad \mu\nu = ij \text{ or } tw, \quad (3.47)$$

where \( B_{ij}^{(PS)} \), \( B_{tw}^{(PS)} \) are the \( B \)-field components (3.33) for the Perry-Schwarz solution. In principle, \( x^5 \) independent constants of integration can be added but we will not need them.
A consistent solution can be obtained by setting all the other independent components of $B_{\mu\nu}$ to be zero. To see this, let us compute $F_{\mu\nu}$ from (2.10). It is remarkable that

$$F^a_{ij} = -\frac{c\beta\pi}{2} \epsilon_{ijm}x_mx_a, \quad F^a_{tw} = 0,$$

which is precisely the form (3.38) of the Wu-Yang monopole if we take

$$c\beta = -\frac{2}{\pi}. \quad (3.49)$$

As a result, the non-vanishing component of the gauge field is given by

$$A^a_i = -\epsilon_{aik}\frac{x_k}{r^2}. \quad (3.50)$$

So far we have used only the field strength components $H_{ij5}, H_{tw5}$ of (3.46). However since $D_\mu(x^aT^a/r) = 0$ for the Wu-Yang gauge field, therefore (3.45) is reproduced immediately and (3.43) is indeed satisfied.

Like the Wu-Yang monopole, the color magnetic charge of our Wu-Yang string solution vanishes. This is not a problem as we should not forget about the scalar fields as our ultimate aim is to construct the non-abelian self-dual string solution in the multiple M5-branes theory and so the inclusion of scalar fields is natural from the point of view of (2,0) supersymmetry. Although we do not have the full (2,0) supersymmetric theory, one can argue that the self-duality equation of motion (2.8) is not modified by the presence of the scalar fields. This can be seen by a simple dimensional analysis since the dimension of a canonically normalized scalar field is two, and there is no local polynomial term one can write down which is consistent with conformal symmetry. That the self-duality equation is not modified by the scalar fields is also the case in the other proposed constructions [13, 19]. As for the scalar field, first it is clear that due to R-symmetry, the self-interacting potential vanishes if there is only one scalar field turned on. As a result, the equation of motion of the scalar field is

$$D^2_M\phi = 0. \quad (3.51)$$

This is the general situation but for special cases, for example when a BPS condition is satisfied, the second order equation could be reduced to a first order equation. A reasonable form of the BPS equation is the non-abelian generalization of the BPS equation (3.35), (3.36)

$$H_{ijk} = \epsilon_{ijk}\partial_5\phi, \quad H_{ij5} = -\epsilon_{ijk}D_k\phi. \quad (3.52)$$

We conjecture that (3.52) is indeed a BPS equation of the non-abelian (2,0) theory since first of all it implies the equation of motion (3.51). Moreover (3.52) would follow immediately from the supersymmetry transformation ($\Gamma^{012345}\epsilon = \epsilon, \quad \Gamma^{012345}\psi = -\psi$)

$$\delta\psi = (\Gamma^M\Gamma^I D_M\phi^I + \frac{1}{3!2}\Gamma^{MNP}H_{MNP})\epsilon \quad (3.53)$$
(which is the most natural non-abelian generalization of the abelian (2,0) supersymmetry transformation) and the 1/2 BPS condition

\[ \Gamma^{046} \epsilon = -\epsilon, \quad (3.54) \]

together with the condition that \( \phi^6 := \phi = \phi(x^a), a = 1, 2, 3, 5. \)

We note that (3.52) is compatible with the self-duality equation if the scalar field is equal to the \( B_{tw} \) component:

\[ \phi^a = B^a_{tw} = -\frac{\beta}{2\rho^2} \frac{x^a}{r}, \quad (3.55) \]

or more generally,

\[ \phi^a = -\left( u + \frac{\beta}{2\rho^2} \right) \frac{x^a}{r}, \quad (3.56) \]

where \( u \) is a constant. To see the physical meaning of this solution, let us consider the transverse distance \( |\phi| \) defined by \( |\phi|^2 = \phi^a \phi^a \). This gives

\[ |\phi| = |u + \frac{\beta}{2\rho^2}|. \quad (3.57) \]

We will choose the constant \( u \) to be of the same sign as \( \beta \) so that \( |\phi| \) is never zero. This describes a system of M5-branes with a spike at \( \rho = 0 \) and level off to \( u \) as \( \rho \to \infty \). Hence the physical interpretation of our self-dual string is that two M5-branes are separating by a distance \( u \) and with an M2-brane ending on them. With this interpretation, there is a symmetry breaking and one can identify an \( U(1) \) \( B \)-field at the large distance \( \rho \):

\[ B_{\mu\nu} \equiv \hat{\phi}^a B^a_{\mu\nu} = \pm B^{(PS)}_{\mu\nu} \quad (3.58) \]

where \( \hat{\phi}^a := \phi^a / |\phi| \) and the \( + \) (\( - \)) sign in the second equation above corresponds to the case \( c > 0 \) (\( c < 0 \)). Since the field configuration approaches that of the abelian self-dual string at large distance, we immediately obtain the charges

\[ P = Q = -2\pi^2 |\beta| = -\frac{4\pi}{|c|} \quad (3.59) \]

and charge quantization determines that

\[ \beta = \mp \sqrt{\frac{n}{4\pi^3}}, \quad c = \pm 4\sqrt{\frac{\pi}{n}} \quad (3.60) \]
and \( P = Q = -\sqrt{n\pi} \). We require that the theory should admit solution with the minimal unit of charge and so the possible values of the constant \( c \) in the non-abelian action (2.4) is:

\[
c = \pm 4\sqrt{\pi}
\]  

and the charges of our solution are \( P = Q = -\sqrt{\pi} \).

Just as in the abelian case, the action for the gauge fields vanish on shell. Therefore the string gets its tension solely from the scalar field. In general, the kinetic term of scalar field is proportional to

\[
\text{tr}(D_M \phi D^M \phi).
\]

Since the scalar field satisfies

\[
D_M \phi \to 0, \quad \rho \to \infty,
\]

we see that at large distance \( \rho \to \infty \) from the string, the kinetic term vanishes. However the singularity at the origin leads to an infinite tension. This is the same as the Howe-Lambert-West self-dual string solution [8].

4. Non-Abelian Self-Dual String Solution: Compactified Case

In this section, we consider the theory with \( x^5 \) compactified on a circle with radius \( R \) and construct the self-dual string solution. The constraint that the gauge field has to satisfy is now (2.11). Without loss of generality, let us assume that the string aligns in the \( w = x^4 \) direction.

In the compactified theory, the field strength can be expanded in terms of Fourier modes,

\[
H_{MNP} = \sum_n e^{inx^5/R} H^{(n)}_{MNP}(r).
\]  

The gauge field \( B_{\mu\nu} \) can be then obtained by integrating over the equation of motion \( H_{\mu\nu5} = \partial_5 B_{\mu\nu} \). It is

\[
B_{\mu\nu} = \frac{x^5}{2\pi Rc} F_{\mu\nu}(r) + \sum_{n=-\infty}^{\infty} e^{inx^5/R} B^{(n)}_{\mu\nu}(r),
\]  

where we have used the boundary condition (2.11) to determine the first term and \( B^{(0)}_{\mu\nu}(r) \) is an integration constant. The higher modes \( B^{(n\neq0)}_{\mu\nu}(r) \) are given by:

\[
H^{(n\neq0)}_{\mu\nu5}(r) = \frac{in}{R} B^{(n\neq0)}_{\mu\nu}(r).
\]

Notice that the first term on the right hand side has no contribution to \( H_{\mu\nu\lambda} \) because of Bianchi identity and hence

\[
H^{(n)}_{\mu\nu\lambda} = D_{[\lambda} B^{(n)}_{\mu\nu]}.
\]
for all $n$.

Let us consider an ansatz with the only nonzero components of gauge potential being $B_{tw}$ and $B_{ij}$. The self-duality condition reads

$$H_{ijk} = \epsilon_{ijk} H_{t5}, \quad H_{twk} = -\frac{1}{2} \epsilon_{ijk} H_{ij5}, \quad (4.5)$$

or, written in terms of modes,

$$D_i B^{(0)}_{jk} = \epsilon_{ijk} \frac{F_{tw}}{2\pi Rc}, \quad D_k B^{(0)}_{tw} = -\frac{f_k}{2\pi Rc} \quad (4.6)$$

$$D_k b^{(n)}_k = i n \frac{B^{(n)}_{tw}}{R}, \quad D_k B^{(n)}_{tw} = -b^{(n)}_k \frac{in}{R}, \quad n \neq 0, \quad (4.7)$$

where we have denoted

$$f_k(r) := \frac{1}{2} \epsilon_{ijk} F_{ij} \quad \text{and} \quad b^{(n)}_k(r) := \frac{1}{2} \epsilon_{ijk} B^{(n)}_{ij} \quad \text{for} \quad n \neq 0. \quad (4.8)$$

Notice that the 2nd equation of (4.6) takes exactly the same form as the BPS equation for the 't Hooft-Polyakov magnetic monopole if we identify $-2\pi Rc B^{(0)}_{tw}$ as the scalar field there. Indeed in the BPS limit, the equation of motion for the 't Hooft-Polyakov monopole reads

$$\frac{1}{2} \epsilon_{ijk} F_{ij} = D_k \phi, \quad (4.9)$$

where $\phi$ is an adjoint Higgs scalar field. The solution is given by

$$A^a_i = -\epsilon_{aik} \frac{x^k}{r^2} (1 - k_v(r)), \quad \phi^a = \frac{vx^a}{r} h_v(r), \quad (4.10)$$

where

$$k_v(r) := \frac{vr}{\sinh(vr)}, \quad h_v(r) := \coth(vr) - \frac{1}{vr}. \quad (4.11)$$

Asymptotically $r \to \infty$, we have

$$A^a_i \to -\epsilon_{aik} \frac{x^k}{r^2}, \quad \phi^a \to \frac{|v|x^a}{r} := \phi_\infty, \quad (4.12)$$

which coincides with Wu-Yang monopole. Note that the gauge symmetry is broken at infinity to $U(1)$, the little group of $\phi_\infty$. This may be identified as the electromagnetic gauge group and one could use this to define the magnetic monopole charge $[28,29]$. The electromagnetic field strength can be defined as

$$F_{ij} = F_{ij}^a \frac{\phi^a}{|v|} = \epsilon_{ijk} \frac{x^k}{r^3}, \quad \text{for large} \quad r. \quad (4.13)$$
The magnetic charge is given by \( p = \int_{\mathcal{C}} \mathcal{F} = 4\pi \), which corresponds to a magnetic monopole of unit charge. Note that at the core \( r \to 0 \), we have

\[
A_i \to 0, \quad \phi \to 0
\]

and hence the \( SU(2) \) symmetry is unbroken at the monopole core.

The resemblance of our equation with the BPS equation of the 't Hooft-Polyakov monopole motivates us to take for \( A_i \) the same ansatz as in the 't Hooft-Polyakov monopole,

\[
A^a_i = -\epsilon_{aik} \frac{x^k}{r^2} (1 - k_v(r)),
\]

This implies \( F_{tw} = 0 \) and hence the 1st equation of (4.6) can be solved with

\[
B^{(0)}_{ij} = c_0 F_{ij},
\]

where \( c_0 \) is an arbitrary constant. On the other hand, (4.7) gives

\[
D_k D_k B^{(n\neq 0)}_{tw} = \frac{n^2}{R^2} B^{(n\neq 0)}_{tw}.
\]

For zero mode, we have \( D_k D_k B^{(0)}_{tw} = 0 \), combine them together we can write

\[
D_k D_k B^{(n)}_{tw} = \frac{n^2}{R^2} B^{(n)}_{tw}.
\]

We take the ansatz for \( B^{(n)}_{tw} \) as

\[
B^{(n)}_a = a_n(r) \frac{v x^a}{r}
\]

then the equation (4.18) is equivalent to

\[
\frac{\partial_r (r^2 \partial_r a_n(r))}{r^2} - \frac{2k_v(r)^2}{r^2} a_n(r) = \frac{n^2}{R^2} a_n(r).
\]

The well-behaved physical solution is

\[
a_0 = a_0 h_v(r),
\]

\[
a_{n\neq 0}(r) = \alpha_n e^{-|n| r/R} \frac{v R}{|n| \coth(vr)} \left( 1 + \frac{v R}{|n|} \coth(vr) \right),
\]

where \( \alpha_n \) are arbitrary constants. Here we have dropped the independent solutions which are exponentially increasing at large distance and hence not physical. As a result, we obtain for the gauge fields

\[
B^a_{tw} = \frac{h_v(r) v x^a}{2\pi R c} r + \sum_{n\neq 0} \alpha_n e^{in \phi / R} e^{-|n| r / R} \frac{v R}{|n| \coth(vr)} \left( 1 + \frac{v R}{|n|} \coth(vr) \right) \frac{v x^a}{r},
\]

\[ -16 - \]
\[ B_{ij}^a = \frac{x^5}{2\pi Rc} F_{ij}^a(r) + c_0 F_{ij}^a(r) + \sum_{n \neq 0} e^{inx^5/R} B_{ij}^{a(n)}(r). \] (4.24)

where
\[ b_k^{(n) a} = -v^3 \frac{R}{in} (ra_n - k_v(r)a_n) \frac{x^k x^a}{r} - \delta_k^a \frac{vr}{in} a_n k_v(r) \frac{1}{r}, \quad n \neq 0. \] (4.25)

The proportionality factor for \( a_0 \) is determined by recalling that \(-2\pi Rc B_{tw}^{(0)}\) is the scalar of the ’t Hooft-Polyakov monopole, while \( \alpha_{n \neq 0} \) are left undetermined. Physically this corresponds to different excitations over the fundamental solution with all \( \alpha_{n \neq 0} = 0 \). Note that there is a “winding mode” in \( B_{ij} \), while there is no such mode in \( B_{tw} \) because \( F_{tw} = 0 \). Although this has no effect classically, we expect that this is observable quantum mechanically like the Berry phase. See, for example, [30] for a discussion of Berry phase associated with branes in string theory.

Next let us include a (2,0) scalar field \( \phi \). As above we assume that it satisfies the BPS equation (3.36), then the BPS equation is satisfied automatically if we identify \( \phi^{(0)} = B_{tw}^{(0)} \). As a result, we have
\[ \phi^{(0) a} = -u \left( \coth(vr) - \frac{1}{vr} \right) \frac{x^a}{r}. \] (4.26)

where
\[ u := \frac{v}{2\pi Rc}. \] (4.27)

set the scale of the vev of \( \phi^{(0)} \) at large \( r \) since we can say \( \phi^{(0)} \rightarrow -\frac{|v|}{2\pi Rc} x^a T^a \) as \( r \rightarrow \infty \). In addition, one can define a \( U(1) \) projection onto \( \phi^{(0)} \). This allows us to define the charges
\[ P = Q = \int_{S^1 \times S^2} H^a \hat{\phi}^a \]
\[ = \pm \int dx^5 dS^k \frac{1}{2} \epsilon_{ijk} \left( \frac{1}{2\pi Rc} F_{ij}^a \frac{x^a}{r} + (KK) \right) \] (4.28)

where the \(- (+) \) sign in the second equation above corresponds to the case \( c > 0 \) \((c < 0)\); and the term \((KK)\) stands for the KK modes and their contribution to the charges is zero. Substituting (3.61), we find that the solution is self-dual and carries the charges \( P = Q = -\sqrt{\pi} \). Physically one can identified this self-dual string with the uncompactified one obtained in the previous section and so they carry the same charges.

The scalar profile of (4.26) is plotted in figure 2, for two compactification radius \( R = 1 \) and \( R = 4 \) and a fixed vev \( u = -0.5 \). One may compare our results to the scalar profile in [31]. In this work, a modified Nahm’s equation for the scalar field was
conjectured. However unlike the ordinary Nahm’s equation where one can obtain the non-abelian Yang-Mills gauge field at the same time, it is not clear how one might obtain the corresponding non-abelian tensor gauge field from the modified Nahm’s equation and the proposal still needed to be completed. Nevertheless, qualitatively their scalar profile is similar to ours.

![Figure 2: Scalar Profile. The red curve corresponds to $R = 4$ and the blue one to $R = 1$](image)

5. Discussions

In this paper we have constructed the non-abelian string solutions of the non-abelian 5-brane theory constructed in [1], for both uncompactified and compactified spacetime. The string solution in non-compact spacetime is supported by a non-abelian Wu-Yang monopole, while the string solution in compact spacetime is supported by a non-abelian ’t Hooft-Polyakov monopole. We showed how these solutions can be embedded in the (2,0) supersymmetric theory by including a single scalar field obeying a first order BPS equation. Although we don’t have the full (2,0) supersymmetric construction yet, we argued that it is the correct BPS equation of the (2,0) theory since it solves the equation of motion, and moreover it can be derived from the most natural form of the supersymmetry transformation law in the non-abelian (2,0) theory. These string solutions carry self-dual charges and has infinite tension arising from the scalar profile which corresponds to having a M2-brane spike on the M5-branes system. These properties are consistent with what one expects for the non-abelian self-dual strings living on a system of two M5-branes. Hence the results we obtained provide further support that the non-abelian theory constructed in [1] describes the gauge sector of a system of multiple M5-branes. Needless to say, it is of utmost importance to obtain the supersymmetric completion of the bosonic theory [1]. This is under investigation.
We have constructed a non-abelian self-dual string solution with unit charge. In the M-theory picture, it is possible to have non-abelian self-dual strings with higher charges. It will be interesting to construct them as well. The employment of multi-monopole seems appropriate, see for example [27,32,33] for a review. It would also be interesting to explore the possible loop space or twistor interpretation [34] of our self-dual string solution.

It is also hoped that the self-dual string solution constructed here could provide further insights into the understanding of the $N^3$ entropy growth of the multiple M5-branes system [35]. Recent progress on this problem has been achieved in [17,18].

As advocated in [9,15], just as in the D-branes case where Lie bracket which define the gauge symmetries for multiple D-branes captures the noncommutative geometry of a single D-brane in the presence of a large NSNS $B$-field, it is possible that the gauge symmetry for multiple M5-branes could also capture the structure of the quantum geometry of a single M5-branes in the presence of a large $C$-field. Given the dynamical evidence we presented in this paper, we believe that the non-abelian tensor gauge theory of [1] does describe the gauge sector of multiple M5-branes. It is thus interesting to try to understand how the gauge symmetry of the non-abelian theory [1] could describe the quantum Nambu geometry derived in [15] for a M5-brane in a large $C$-field. An encouraging sign is that both are described in terms of ordinary commutator.

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