$S$, $P$ and $D$-wave resonance contributions to $B_{(s)} \to \eta_c (1S, 2S) K \pi$ decays in the perturbative QCD approach

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In this work, we analyse the three-body $B_{(s)} \to \eta_c (1S, 2S) K \pi$ decays within the framework of the perturbative QCD approach (PQCD) under the quasi-two-body approximation, where the kaon-pion QCD approach (PQCD) under the quasi-two-body approximation, where the kaon-pion invariant mass spectra are dominated by the $K^*_0 (1430) ^0, K^*_0 (1950) ^0, K^* (892) ^0, K^* (1410) ^0, K^* (1680) ^0$ and $K^*_2 (1430) ^0$ resonances. The time-like form factors are adopted to parameterize the corresponding $S$, $P$, $D$-wave kaon-pion distribution amplitudes for the concerned decay modes, which describe the final state interactions between the kaon and pion in the resonant region. The $K \pi S$-wave component at low $K \pi$ mass is described by the LASS line shape, while the time-like form factors of other resonances are modeled by the relativistic Breit-Wigner (BW) function. We found the following main points: (a) the PQCD predictions of the branching ratios for most considered $B \to \eta_c (1S) (K^{*0} \to K^+ \pi^-)$ decays agree well with the currently available data within errors; (b) for $B (B^0 \to \eta_c (K^*_0 (1430) \to K^+ \pi^-))$ and $B (B^0 \to \eta_c K^+ \pi^- (NR))$, our predictions of the branching ratios are a bit smaller than the measured ones, while the PQCD prediction for the ratio $R_1 = B (B^0 \to \eta_c (K^*_0 (1430) \to K^+ \pi^-))/B (B^0 \to \eta_c K^+ \pi^- (NR))$ is consistent with the data; and (c) the PQCD results for the $D$-wave contributions considered in this work will be tested by the precise data from the future LHCb and Belle-II experiments.

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I. INTRODUCTION

The $B_{(s)}$ meson decays into charmonia and a kaon-pion pair are of great interest since only a few color-suppressed modes in hadronic $B$ decays have been measured so far. Some standard model (SM) parameters can be extracted from the $b \to c \bar{c}s$ transitions, while the studies

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of these decay channels can also provide us an ideal place to find the signal for the physics beyond the SM. The meson $\eta_c$ and $J/\psi$ have same quark content but with different spin angular momentum. As expected, the $B$ meson decays involving the $\eta_c$ will trigger considerable experimental attentions with the development of the experiments. In recent years, significant improvements for understanding the heavy quarkonium production mechanism have been achieved [1].

The $B^0 \rightarrow \eta_c(K^*(892)^0 \rightarrow K \pi)$ decay was observed by Belle [2] and BABAR [3, 4] collaborations in early years. Very recently, the measurement of the $B^0 \rightarrow \eta_c K^+ \pi^-$ decay is performed for the first time by the LHCb collaboration [5], with the $\eta_c$ meson reconstructed using the $p \bar{p}$ decay mode. This decay is expected to proceed through $K^{*0} \rightarrow K \pi$ intermediate states as well as the nonresonant S-wave component, where the $K^{*0}$ refers to various partial wave resonances, such as $K^{*0}_0(1430)^0$, $K^{*0}_0(1950)^0$, $K^{*0}(892)^0$, $K^{*0}(1410)^0$, $K^{*0}(1680)^0$ and $K^{*0}_2(1430)^0$. The P-wave $K^{*}(892)^0$ is the largest component: $\sim 50\%$, while the $K^{*}(1410)^0$, $K^{*}(1680)^0$ and D-wave $K^{*}_2(1430)^0$ states amount only up to a few percent.

The theoretical study and the experimental measurement for the three-body $B$ meson decays is still at its early stage. On the theoretical side, compared with those two-body decay modes, these three-body $B$ decays are more intractable due to the entangled resonant and nonresonant contributions, as well as the possible final-state interactions (FSIs) [6-8]. An important break through in the theory of three-body $B$ meson decays is the confirmation of the validity of factorization. We restrict ourselves to the specific kinematical configurations in which the three mesons are quasi aligned in the rest frame of the $B$ meson. This condition is particularly natural in the low effective $K \pi$ mass region of the Dalitz plot, where most of the $K \pi$ resonant structures are seen. When the two particles among the three final state mesons move collinearly and generate a small invariant mass recoiling against the third one, the three-body interactions are expected to be suppressed. Then it seems reasonable to assume the validity of the factorization for these quasi-two-body $B$ decays. In the quasi-two-body mechanism, the two-body scattering and all possible interactions between the two involved particles are included but the interactions between the third particle and the pair of mesons are ignored. In recent years, several different theoretical frameworks based on the factorization theorems and symmetry principles have been proposed to deal with the three-body $B$ meson decays. The QCD-improved factorization (QCDF) approach [9-12] has been widely used in the study of the three-body charmless hadronic $B$ meson decays [13-23]. The U-spin and the flavor SU(3) symmetries were also adopted in Refs. [24-29].

It has been known that the collinear factorization of the charmed and charmless two-body $B$ meson decays suffer the end-point singularities. The PQCD factorization approach relying on the $k_T$ factorization theorem [30, 31] was proposed in Refs. [32-34], which has been shown to be infrared-finite, gauge-invariant, and consistent with the factorization assumption in the heavy-quark limit [35-38]. The operator-level definition of the transverse-momentum-dependent (TMD) hadronic wave functions is highly nontrivial in order to avoid the potential light-cone divergence and the rapidity singularity [39, 40]. The Sudakov factors from the $k_T$ resummation have been included to suppress the long-distance contributions from the large $b$ region with $b$ being a variable conjugate to $k_T$. Therefore, the PQCD approach is a self-consistent framework and has a good predictive power. Based on the PQCD approach, we have studied the quasi-two-body $B$ meson decays in Refs. [41-51] by introducing the two-meson distribution amplitudes (DAs) [52-58], which catch the dynamics associated with the pair of mesons.

In this paper, we continue to study the quasi-two-body decays $B \rightarrow \eta_c K^{*0} \rightarrow \eta_c K \pi$ involving the $S$, $P$ and $D$-wave kaon-pion pairs as shown in Fig. 1, within the framework of the PQCD factorization approach. In literature, some works have been done for $B \rightarrow \eta_c K^{*}$ decays in the two-body framework [59-61]. From [41, 43, 45], we know that the width of the resonant state
FIG. 1: Typical Feynman diagrams for the quasi-two-body decays $B \rightarrow \eta_c (1S, 2S) (K^{*0} \rightarrow K\pi)$, and the symbol $\bullet$ denotes the weak vertex. $K^{*0}$ represents various partial wave intermediate states.

and the interactions between the final state meson pair will show their effects on the branching ratios especially on the direct $CP$ violations of the quasi-two-body decays. Thus, it seems more appropriate to treat the $K^{*0}$ as an intermediate resonance. As addressed before, this process is dominated by a series of resonances in $S, P$ and $D$ waves. The $S$-wave kaon-pion DAs for the resonance $K^*_0(1430)^0$ have been studied in Ref. [62], we will further investigate the dependence of the branching ratios in different scalar scenarios as proposed in Refs. [63, 64]. Besides, we have roughly determined the possible range of the first odd Gegenbauer moment $B_1$ for the $K^*_0(1950)^0$ resonance by fitting to the existing data, which demands to be tested in the future. We intend to adopt the same fitted parameters as those of the longitudinal kaon-pion DAs in Ref. [51], where the SU(3) flavor symmetry breaking effect has been considered and plays an important role in the longitudinal branching fractions. The $D$-wave resonance $K^{*2}_2(1430)^0$ is investigated for the first time in our work. Due to the limited studies on the tensor resonant states, we treat the $D$-wave DAs of the $K^{*2}_2(1430)^0$ in the same way as those of $f_2(1270)$ [45].

This paper is organized as follows. In Sec. II, we give a brief introduction for the theoretical framework. The numerical values, some discussions and the conclusions will be given in last two sections. The explicit PQCD factorization formulas for all the decay amplitudes are collected in the Appendix.

II. FRAMEWORK

In the framework of the PQCD factorization approach, the nonperturbative dynamics associated with the pair of the mesons can be absorbed into two-meson DAs, then the relevant decay amplitude $A$ for the quasi-two-body decays $B \rightarrow \eta_c K^{*0} \rightarrow \eta_c K\pi$ can be described in the following form:

$$A = \Phi_B \otimes H \otimes \Phi_{K\pi} \otimes \Phi_{\eta_c},$$

where $\Phi_B$ and $\Phi_{\eta_c}$ are the $B$ meson and charmonium DAs, respectively. The kaon-pion DA $\Phi_{K\pi}$ absorbs the nonperturbative dynamics in the $K\pi$ hadronization process. The hard kernel $H$ contains only one hard gluon and describes the dynamics of the strong and electroweak interactions in the three-body hadronic decays as in the formalism for the corresponding two-body decays.

In the light-cone coordinates, we make the kaon-pion pair and the final-state $\eta_c$ move along the direction of $n = (1, 0, 0_T)$ and $v = (0, 1, 0_T)$, respectively. The $B$ meson momentum $p_B$, the total momentum of the kaon-pion pair, $p = p_1 + p_2$, the final-state $\eta_c$ momentum $p_3$ and the quark
momentum \( k_i \) in each meson are in the form of

\[
p_B = \frac{m_B}{\sqrt{2}} (1, 1, 0_T), \quad k_B = \left( 0, x_B \frac{m_B}{\sqrt{2}}, k_{BT} \right),
\]

\[
p = \frac{m_B}{\sqrt{2}} (1 - r^2, \eta, 0_T), \quad k = \left( z(1 - r^2) \frac{m_B}{\sqrt{2}}, 0, k_T \right),
\]

\[
p_3 = \frac{m_B}{\sqrt{2}} (r^2, 1 - \eta, 0_T), \quad k_3 = \left( r^2 x_3 \frac{m_B}{\sqrt{2}}, (1 - \eta) x_3 \frac{m_B}{\sqrt{2}}, k_{3T} \right),
\]

with \( m_B \) is the mass of \( B \) meson, \( \eta = \frac{\omega^2}{m_B^2 (1 - r^2)} \) with \( r = m_{\eta c}/m_B, m_{\eta c} \) is the mass of charmonia, and the invariant mass squared \( \omega^2 = p^2 \). The momentum fractions \( x_B, z \) and \( x_3 \) run from zero to unity, respectively.

We also define the momentum \( p_1 \) and \( p_2 \) in the kaon-pion pair as

\[
p_1 = (\zeta p^+, (1 - \zeta) \eta p^+, p_{1T}), \quad p_2 = ((1 - \zeta) p^+, \zeta \eta p^+, p_{2T}),
\]

with \( \zeta = p_1^+ / P^+ \) characterizing the distribution of the longitudinal momentum of kaon and \( p_{2T}^2 = \zeta (1 - \zeta) \omega^2 \).

The \( B_{(s)} \) meson wave function and the charmonium DAs are the same as widely adopted in the PQCD approach [42, 44, 50, 65]. Below, we briefly describe the \( S, P \) and \( D \)-wave kaon-pion DAs and the corresponding time-like form factors. The \( S \)-wave kaon-pion DAs are introduced in analogous to the case of two-pion ones [66],

\[
\Phi_S = \frac{1}{\sqrt{2N_c}} [\phi^0_S(z, \zeta, \omega^2) + \omega \phi^*_S(z, \zeta, \omega^2) + \omega (\eta \phi - 1) \phi^*_S(z, \zeta, \omega^2)].
\]

In what follows the subscripts \( S, P, \) and \( D \) always associate with the corresponding partial waves.

We will use the asymptotic forms for the twist-3 ones, but no knowledge on the twist-2 is available at present. We shall adopt similar formulas as the ones for a scalar meson [63, 67], bearing in mind large uncertainties that may be introduced by this approximation. The detailed expressions of various twists DAs are described below,

\[
\phi^0_S(z, \zeta, \omega^2) = \frac{6}{2\sqrt{2N_c}} F_S(\omega^2) z(1 - z) \left[ \frac{1}{\mu_S} + B_1 C^{3/2}_1(t) + B_3 C^{3/2}_3(t) \right],
\]

\[
\phi^*_S(z, \zeta, \omega^2) = \frac{1}{2\sqrt{2N_c}} F_S(\omega^2),
\]

\[
\phi'_S(z, \zeta, \omega^2) = \frac{1}{2\sqrt{2N_c}} F_S(\omega^2)(1 - 2z),
\]

where the Gegenbauer polynomials \( C^{3/2}_1(t) = 3t, C^{3/2}_3(t) = \frac{2}{3} t(7t^2 - 3) \) with \( t = 1 - 2z \) and \( \mu_S = \omega/(m_2 - m_1) \). The Gegenbauer moments \( B_{1,3} \) and the related running current quark masses can be referred to Refs. [63, 64, 68]. It should be stressed that there is less information about the scalar resonance \( K^*_0(1950)^0 \), we only test the sensitivity of the branching ratios on the first odd Gegenbauer moment \( B_3 \) in our work.

If there are overlapping resonances or there is a significant interference with a nonresonant (NR) component both in the same partial wave, the relativistic Breit-Wigner (RBW) function leads to unitarity violation within the isobar model [69]. This is the case for the \( K^0 \) \( S \)-wave at low \( K^0 \pi \) mass, where the \( K^0(1430)^0 \) resonance interferes strongly with a slowly varying NR \( S \)-wave
component. In this work, the time-like scalar form factor $F_S(\omega^2)$ for the $S$-wave $K\pi$ system is parameterized by using a modified LASS line shape [70] for the $S$-wave resonance $K^*_0(1430)^0$, which has been widely used in the experimental analysis [5],

$$F_S(\omega^2) = \frac{\omega}{|\vec{p}_1| \cot(\delta_B) - i} + e^{2i\delta_B} \frac{m_0^2\Gamma_0/|\vec{p}_0|}{m_0^2 - \omega^2 - im_0^2\omega/|\vec{p}_0|},$$

with

$$\cot(\delta_B) = \frac{1}{a|\vec{p}_1|^2} + \frac{r|\vec{p}_1|^2}{2},$$

where the first part in Eq. (8) is an empirical term from inelastic scattering and the second term is the resonant contribution with a phase factor to retain unitarity. Here $m_0$ and $\Gamma_0$ are the pole mass and width of the $K^*_0(1430)$ state. The $|\vec{p}_1|$ is the momentum vector of the resonance decay product measured in the resonance rest frame, and $|\vec{p}_0|$ is the value of $|\vec{p}_1|$ when $\omega = m_{K^*}$. The parameters $a = (3.1 \pm 1.0) \text{GeV}^{-1}$ and $r = (7.0 \pm 2.4) \text{GeV}^{-1}$ are the scattering length and the effective range [5] respectively, which depend on the production mechanism. The slowly varying part (the first term of the Eq. (8)) is not well modelled at high masses and it is set to be zero for $m(K\pi)$ values above 1.7 GeV [5]. While for the $K^*_0(1950)^0$, we use the relativistic BW line shape to parameterize the time-like form factors $F_S(\omega^2)$, which is adopted in the experimental data analyses [5].

The $P$-wave kaon-pion DAs related to both longitudinal and transverse polarizations have been studied in Ref. [51]. In this work, we only consider the longitudinal contributions, the explicit expressions are listed as follows,

$$\Phi_P = \frac{1}{\sqrt{2N_c}} \left[ \phi^0_P(z, \zeta, \omega^2) + \omega \phi^s_P(z, \zeta, \omega^2) + \frac{\eta_1 \eta_2}{\omega(2\zeta - 1)} \phi^t_P(z, \zeta, \omega^2) \right],$$

The various twists DAs in Eq. (10) can be expanded in terms of the Gegenbauer polynomials:

\begin{align*}
\phi^0_P(z, \zeta, \omega^2) &= \frac{3F_P^0(\omega^2)}{2\sqrt{2N_c}} (1 - z) \left[ 1 + a_{1K^*}^\parallel 3t + a_{2K^*}^\parallel \frac{3}{2} (5t^2 - 1) \right] (2\zeta - 1 - \alpha), \\
\phi^s_P(z, \zeta, \omega^2) &= \frac{3F_P^s(\omega^2)}{2\sqrt{2N_c}} \left\{ t \left[ 1 + a_{1s}^t 3t \right] - a_{1s}^t 2z(1 - z) \right\} P_1(2\zeta - 1), \\
\phi^t_P(z, \zeta, \omega^2) &= \frac{3F_P^t(\omega^2)}{2\sqrt{2N_c}} t \left[ t + a_{1s}^t (3t^2 - 1) \right] P_1(2\zeta - 1),
\end{align*}

where the Legendre polynomial $P_1(2\zeta - 1) = 2\zeta - 1$ and the factor $\alpha = (m_K^2 - m_{\pi}^2)/\omega^2$ is treated as the SU(3) asymmetry factor.

The Gegenbauer moments $a_{1K^*}^\parallel, a_{2K^*}^\parallel, a_{1s}^t, a_{1t}^t$ and the parameter $\alpha$ are adopted the same values as those determined in Ref. [51]:

$$a_{1K^*}^\parallel = 0.2, \quad a_{2K^*}^\parallel = 0.6, \quad a_{1s}^t = -0.2, \quad a_{1t} = 0.3, \quad \alpha = 0.2.$$  

The relativistic BW line shape is adopted for the $P$-wave resonance $K^*(892), K^*(1410)$ and $K^*(1680)$ to parameterize the time-like form factors $F_P^\parallel(\omega^2)$. The explicit expressions are in the
following form [5],

\[
F_P^\parallel(\omega^2) = \frac{c_1 m_{K^*(892)}^2}{m_{K^*(892)}^2 - \omega^2 - im_{K^*(892)} \Gamma_1(\omega^2)} + \frac{c_2 m_{K^*(1410)}^2}{m_{K^*(1410)}^2 - \omega^2 - im_{K^*(1410)} \Gamma_2(\omega^2)} + \frac{c_3 m_{K^*(1680)}^2}{m_{K^*(1680)}^2 - \omega^2 - im_{K^*(1680)} \Gamma_3(\omega^2)},
\]

(15)

where the three terms describe the contributions from \(K^*(892)\), \(K^*(1410)\) and \(K^*(1680)\) respectively, while the weight coefficients \(c_i\) are the same as those being determined previously [51].

The mass-dependent width \(\Gamma_i(\omega^2)\) is defined by:

\[
\Gamma_i(\omega^2) = \Gamma_i \left( \frac{m_i}{\omega} \right) \left( \frac{|\vec{p}|}{|\vec{p}_0|} \right)^{2L_R+1},
\]

(16)

The \(m_i\) and \(\Gamma_i\) are the pole mass and width of the corresponding resonance, where \(i = 1, 2, 3\) represents the resonance \(K^*(892), K^*(1410)\) and \(K^*(1680)\), respectively. \(L_R\) is the orbital angular momentum in the \(K^+\pi^-\) decay and \(L_R = 0, 1, 2, ...\) corresponds to the \(S, P, D, ...\) partial wave resonances. Following Ref. [41], we also assume that

\[
F_P^\perp(\omega^2)/F_P^\parallel(\omega^2) \approx (f_{K^*}/f_{K^*}),
\]

(17)

with \(f_{K^*} = 0.217 \pm 0.005\)GeV, \(f_{K^*}^T = 0.185 \pm 0.010\)GeV [71]. Due to the limited studies on the decay constants of \(K^*(1410)\) and \(K^*(1680)\), we use the two decay constants of \(K^*(892)\) to determine the ratio \(F_P^\perp(\omega^2)/F_P^\parallel(\omega^2)\).

The \(D\)-wave kaon-pion DAs are introduced in analogous to the case of two-pion ones [45],

\[
\Phi_D = \frac{1}{\sqrt{2N_c}} \left[ \phi_D^0(z, \zeta, \omega^2) + \omega \phi_D^0(z, \zeta, \omega^2) + \frac{\eta_1 \eta_2 - \eta_2 \eta_1}{\omega(2\zeta - 1)} \phi_D^t(z, \zeta, \omega^2) \right].
\]

(18)

The \(D\)-wave \(K\pi\) system has similar asymptotic DAs as the ones for a tensor meson [72–74], but replacing the tensor decay constants with the time-like form factor:

\[
\phi_D^0(z, \zeta, \omega^2) = \frac{6F_D^\parallel(\omega^2)}{2\sqrt{2N_c}} z(1-z) \left[ 3a_1^0(2z-1) \right] P_2(2\zeta - 1),
\]

(19)

\[
\phi_D^t(z, \zeta, \omega^2) = -\frac{9F_D^\perp(\omega^2)}{4\sqrt{2N_c}} \left[ a_1^0(1-6z+6z^2) \right] P_2(2\zeta - 1),
\]

(20)

\[
\phi_D^t(z, \zeta, \omega^2) = \frac{9F_D^\perp(\omega^2)}{4\sqrt{2N_c}} \left[ a_1^0(1-6z+6z^2)(2z-1) \right] P_2(2\zeta - 1).
\]

(21)

where the Legendre polynomial \(P_2(2\zeta - 1) = 1 - 6\zeta(1 - \zeta)\) and the Gegenbauer moment \(a_1^0 = 0.4 \pm 0.1\) [45]. The time-like form factor \(F_D^\parallel(\omega^2)\) for the \(D\)-wave \(K\pi\) resonance is also described by the relativistic BW function as given in Eq. (15). Besides, the approximate relation \(F_D^\perp(\omega^2)/F_D^\parallel(\omega^2) \approx f_{K^*_2(1430)}^T/f_{K^*_2(1430)}\) is used in the following calculation with \(f_{K^*_2(1430)} = 0.118 \pm 0.005\)GeV, \(f_{K^*_2(1430)}^T = 0.077 \pm 0.014\)GeV [72].
III. NUMERICAL RESULTS AND DISCUSSIONS

In numerical calculations, besides the quantities specified before, the following input parameters (the masses and decay constants are in units of GeV) will be used [75],

\[ m_{B^0} = 5.28, \quad m_{B_s^0} = 5.367, \quad m_b = 4.8, \quad m_c = 1.275 \pm 0.025, \]
\[ m_{\pi^\pm} = 0.140, \quad m_{K^\pm} = 0.494, \quad m_{\eta_c(1S)} = 2.9834, \quad m_{\eta_c(2S)} = 3.6392, \]
\[ f_{B^0} = 0.19, \quad f_{B_s} = 0.23, \quad f_{\eta_c} = 0.42, \quad f_{\eta_c(2S)} = 0.243, \]
\[ \tau_{B^0} = 1.519 \text{ ps}, \quad \tau_{B_s} = 1.512 \text{ ps}. \]  

The pole mass and width of various partial wave resonances are summarized in Table I, while the values of the Wolfenstein parameters are adopted as given in Ref. [75]: \( A = 0.836 \pm 0.015, \lambda = 0.22453 \pm 0.00044, \rho = 0.122^{+0.018}_{-0.017}, \bar{\eta} = 0.355^{+0.012}_{-0.011}. \)

| Resonance          | Mass [Mev]    | Width [Mev] | \( J^P \) | Model  |
|--------------------|--------------|-------------|-----------|--------|
| \( K^*(892)^0 \)   | 895.55 ± 0.20| 47.3 ± 0.5  | 1\(^-\)   | RBW    |
| \( K^*(1410)^0 \)  | 1414 ± 15    | 232 ± 21    | 1\(^-\)   | RBW    |
| \( K_0^*(1430)^0 \)| 1425 ± 50    | 270 ± 80    | 0\(^+\)   | LASS   |
| \( K_2^*(1430)^0 \)| 1432.4 ± 1.3 | 109 ± 5     | 2\(^+\)   | RBW    |
| \( K^*(1680)^0 \)  | 1717 ± 27    | 322 ± 110   | 1\(^-\)   | RBW    |
| \( K_0^*(1950)^0 \)| 1945 ± 22    | 201 ± 90    | 0\(^+\)   | RBW    |

By using the Eqs. (23-24), the decay amplitudes in the Appendix and all the input quantities, the resultant branching ratios \( \mathcal{B} \) and the available experimental measurements for the considered \( B_{(s)}^0 \rightarrow \eta_c K \pi \) decays involved the \( S \)-wave resonances are summarized in Table II, while those for \( P \) and \( D \)-wave resonances are shown in Table III and IV. Since the charged \( B \) meson decays differ from the neutral ones only in the lifetimes and the isospin factor in our formalism, we can derive the branching ratios for the \( B^+ \) decay modes via multiplying those for the \( B^0 \) decay modes by the ratio \( \tau_{B^+}/\tau_{B^0} \).

In our calculations for the various partial wave resonances, the first uncertainty is induced by the Gegenbauer moments in the \( S, P \) and \( D \)-wave kaon-pion DAs as aforementioned. The second error comes from the variations of the shape parameter \( \omega_B(s) \) of the \( B(s) \) meson DA. We adopt the value \( \omega_B = 0.40 \pm 0.04 \text{ GeV} \) or \( \omega_{B_s} = 0.50 \pm 0.05 \text{ GeV} \) and vary its value with a 10\% range, which is supported by intensive PQCD studies [33, 34]. The last one is caused by the variation of the hard scale \( t \) from 0.75\( t \) to 1.25\( t \) (without changing \( 1/b_i \)), which characterizes the effect of...
TABLE II: PQCD results for the branching ratios of the $S$-wave resonances in the $B_s^0 \rightarrow \eta_c(1S, 2S)K^+\pi^-$ decays in scenario I and scenario II together with experimental data \cite{5}. The theoretical errors are attributed to the variation of the Gegenbauer moments $B_1$ and $B_3$, the shape parameters $\omega_{B_s}$ in the wave function of $B_s$ meson and the hard scale $t$, respectively.

| Modes                              | Quasi-two-body $B$ (in $10^{-5}$) | Exp $^a$ (in $10^{-5}$) |
|------------------------------------|-----------------------------------|--------------------------|
| $B^0 \rightarrow \eta_c K^+\pi^-$ (NR) | $0.85^{+0.34+0.32+0.01}_{-0.41-0.26-0.15}$ | $1.85^{+0.94+0.38+0.06}_{-0.59-0.24-0.31}$ | $5.90^{+1.25}_{-1.65}$ [5] |
| $B^0 \rightarrow \eta_c(K^0_s(1430)^0 \rightarrow K^+\pi^-)$ | $1.69^{+0.71+0.22+0.42}_{-0.69-0.17-0.24}$ | $4.75^{+2.10+1.17+1.05}_{-1.95-1.10-0.68}$ | $14.50^{+3.36}_{-3.14}$ [5] |
| $B^0_s \rightarrow \eta_c K^-\pi^+$ (NR) | $0.03^{+0.01+0.01+0.00}_{-0.01-0.01-0.00}$ | $0.09^{+0.04+0.03+0.02}_{-0.04-0.02-0.02}$ | \cdots |
| $B^0_s \rightarrow \eta_c(\bar{K}_0^*(1430)^0 \rightarrow K^-\pi^+)$ | $0.08^{+0.03+0.02+0.01}_{-0.02-0.01-0.01}$ | $0.21^{+0.11+0.07+0.05}_{-0.08-0.05-0.03}$ | \cdots |
| $B^0 \rightarrow \eta_c(2S)K^+\pi^-$ (NR) | $0.17^{+0.07+0.03+0.04}_{-0.06-0.02-0.05}$ | $0.41^{+0.22+0.12+0.11}_{-0.11-0.09-0.06}$ | \cdots |
| $B^0 \rightarrow \eta_c(2S)(\bar{K}_0^*(1430)^0 \rightarrow K^+\pi^-)$ | $0.56^{+0.28+0.14+0.12}_{-0.25-0.24-0.15}$ | $0.79^{+0.41+0.24+0.17}_{-0.19-0.13-0.12}$ | \cdots |
| $B^0 \rightarrow \eta_c(2S)K^-\pi^+$ (NR) | $0.007^{+0.004+0.003+0.001}_{-0.002-0.001-0.002}$ | $0.02^{+0.01+0.01+0.01}_{-0.01-0.01-0.00}$ | \cdots |
| $B^0_s \rightarrow \eta_c(2S)(\bar{K}_0^*(1430)^0 \rightarrow K^-\pi^+)$ | $0.02^{+0.01+0.01+0.00}_{-0.01-0.00-0.00}$ | $0.04^{+0.02+0.01+0.01}_{-0.02-0.01-0.01}$ | \cdots |

$^a$The experimental results are obtained by multiplying the relevant measured two-body branching ratios according to the Eq. (30).

TABLE III: PQCD results for the branching ratios of the $P$-wave resonances in the $B_s^0 \rightarrow \eta_c(1S, 2S)K^+\pi^-$ decays together with experimental data \cite{5, 75}. The theoretical errors are attributed to the variation of the Gegenbauer moments $(c_1^\perp, a_2^\perp)$ and $(a_1^\perp, a_{1s}^\perp)$, the shape parameters $\omega_{B_s}$ in the wave function of $B_s$ meson and the hard scale $t$, respectively.

| Modes                              | Quasi-two-body $B$ (in $10^{-5}$) | Exp $^a$ (in $10^{-5}$) |
|------------------------------------|-----------------------------------|--------------------------|
| $B^0 \rightarrow \eta_c(K^*(892)^0 \rightarrow K^+\pi^-)$ | $50.62^{+10.79+10.33+9.29}_{-16.72-9.85-6.08}$ | $34.67 \pm 5.33$ [75] |
| $B^0 \rightarrow \eta_c(K^*(1410)^0 \rightarrow K^+\pi^-)$ | $1.55^{+0.51+0.26+0.35}_{-0.56-0.27-0.29}$ | $1.20 \pm 0.90$ [5] |
| $B^0 \rightarrow \eta_c(K^*(1680)^0 \rightarrow K^+\pi^-)$ | $1.65^{+0.78+0.38+0.52}_{-0.63-0.30-0.30}$ | $1.26^{+1.44}_{-1.51}$ [5] |
| $B^0_s \rightarrow \eta_c(K^*(892)^0 \rightarrow K^-\pi^+)$ | $2.22^{+0.95+0.69+0.47}_{-0.81-0.57-0.35}$ | \cdots |
| $B^0_s \rightarrow \eta_c(K^*(1410)^0 \rightarrow K^-\pi^+)$ | $0.08^{+0.03+0.02+0.01}_{-0.03-0.02-0.01}$ | \cdots |
| $B^0_s \rightarrow \eta_c(K^*(1680)^0 \rightarrow K^-\pi^+)$ | $0.09^{+0.04+0.03+0.02}_{-0.03-0.02-0.01}$ | \cdots |
| $B^0 \rightarrow \eta_c(2S)(K^*(892)^0 \rightarrow K^+\pi^-)$ | $14.19^{+4.64+2.93+2.34}_{-4.51-2.17-2.22}$ | $< 26.00$ [75] |
| $B^0 \rightarrow \eta_c(2S)(K^*(1410)^0 \rightarrow K^+\pi^-)$ | $0.27^{+0.11+0.07+0.08}_{-0.09-0.05-0.06}$ | \cdots |
| $B^0_s \rightarrow \eta_c(2S)(K^*(892)^0 \rightarrow K^-\pi^+)$ | $0.67^{+0.23+0.21+0.13}_{-0.21-0.16-0.08}$ | \cdots |
| $B^0_s \rightarrow \eta_c(2S)(K^*(1410)^0 \rightarrow K^-\pi^+)$ | $0.02^{+0.01+0.01+0.00}_{-0.01-0.01-0.00}$ | \cdots |

$^a$The experimental results are obtained by multiplying the relevant measured two-body branching ratios according to the Eq. (30).

next-to-leading-order (NLO) in the PQCD approach. In Table II, III and IV, it is shown that the main uncertainties in our approach come from the Gegenbauer moments, which can reach about 60% in magnitude totally. The scale-dependent uncertainty is less than 25% due to the inclusion of the NLO vertex corrections. The other possible errors from the uncertainties of $m_c$ and CKM matrix elements are actually very small and can be neglected safely.
TABLE IV: PQCD results for the branching ratios of the $D$-wave resonances in the $B_{(s)}^0 \rightarrow \eta_c(1S, 2S)K^+\pi^-$ decays together with experimental data [5]. The theoretical errors are attributed to the variation of the Gegenbauer moment $a_0^1$, the shape parameters $\omega_B(s)$ in the wave function of $B_{(s)}$ meson and the hard scale $t$, respectively.

| Modes | Quasi-two-body $B$ (in $10^{-5}$) | Exp $^a$ (in $10^{-5}$) |
|-------|----------------------------------|--------------------------|
| $B^0 \rightarrow \eta_c(K_2^*(1430)^0 \rightarrow ) K^+\pi^-$ | $3.98^{+1.24+0.59+0.11}_{-1.74-0.55-0.04}$ | $2.35^{+1.08}_{-1.29}$ [5] |
| $B_s^0 \rightarrow \eta_c(K_2^*(1430)^0 \rightarrow ) K^-\pi^+$ | $0.23^{+0.13+0.04+0.01}_{-0.10-0.05-0.01}$ | ... |
| $B^0 \rightarrow \eta_c(2S)(K_2^*(1430)^0 \rightarrow ) K^+\pi^-$ | $0.55^{+0.31+0.12+0.02}_{-0.24-0.09-0.01}$ | ... |
| $B_s^0 \rightarrow \eta_c(2S)(K_2^*(1430)^0 \rightarrow ) K^-\pi^+$ | $0.04^{+0.02+0.01+0.01}_{-0.02-0.01-0.01}$ | ... |

$^a$The experimental results are obtained by multiplying the relevant measured two-body branching ratios according to the Eq. (30).

For the $S$-wave resonance $K^*_0(1430)^0$, it should be mentioned that there are two scenarios proposed to describe the scalar mesons above 1 GeV in the QCD sum rule method [63, 64]. In scenario I, the $K^*_0(1430)^0$ is treated as the first excited state, while $a_0(980)$ and $f_0(980)$ are regarded as the lowest lying states. In scenario II, we assume that $K^*_0(1430)^0$ is the lowest lying resonances and the corresponding first excited states lie in between $(2.0 - 2.3)$ GeV. Scenario II corresponds to the case that light scalar mesons are four-quark bound states. The Gegenbauer moments $B_1 = 0.58 \pm 0.07$ and $B_3 = -1.20 \pm 0.08$ are adopted in Scenario I, while $B_1 = -0.57 \pm 0.13$ and $B_3 = -0.42 \pm 0.22$ in Scenario II [64]. In this work, we consider two scenarios for the $S$-wave components and list the relevant results in Table II. One can see that the predicted $B$ in scenario I are always smaller than those in scenario II. This phenomena is mainly caused by the different signs of the Gegenbauer moment $B_1$ under different scenarios, which indicates that there is a large cancelation in scenario I.

In Table II, our predictions of the branching ratios for the $K^*_0(1430)^0$ resonance and NR components in scenario II are $B_{K^*_0(1430)^0} = (4.75^{+2.62}_{-2.34}) \times 10^{-5}$ and $B_{NR} = (1.85^{+1.20}_{-0.71}) \times 10^{-5}$, respectively, which are a bit smaller than the experimental measurements within errors. Anyway, as is well known that in contrast to the vector and tensor mesons, the identification of scalar mesons is a long standing puzzle. It is difficult to deal with scalar resonances since some of them have large decay widths, which cause a strong overlap between resonances and background. Furthermore, the underlying structure of scalar mesons is not well established theoretically (for a review, see [75]). We hope the situation can be improved using nonperturbative QCD tools including Lattice QCD simulations. None the less, we define PQCD prediction of the corresponding ratio for a more direct comparison with this available experimental data,

$$R_1^{PQCD} = \frac{B(B^0 \rightarrow \eta_c(K_0^*(1430)^0 \rightarrow ) K^+\pi^-)}{B(B^0 \rightarrow \eta_c K^+\pi^- (NR))} = 2.56^{+1.98}_{-1.71},$$ (25)

where the branching fraction of the $B^0 \rightarrow \eta_c(K_0^*(1430)^0 \rightarrow ) K^+\pi^-$ decay is measured relative to that of the corresponding NR contributions by the LHCb collaboration [5]:

$$R_1^{LHCb} = \frac{B(B^0 \rightarrow \eta_c(K_0^*(1430)^0 \rightarrow ) K^+\pi^-)}{B(B^0 \rightarrow \eta_c K^+\pi^- (NR))} = 2.45^{+0.81}_{-0.68}.$$ (26)

Our prediction is consistent with the LHCb measurement quite well. The combined analyses from the LHCb and Belle-II measurements for these decays in the near future would help us to further study the scalar resonances.
In Fig. 2 that the main portion of the branching ratios lies in the region around the resonance as expected

For the phenomenological study of the scalar meson $K_0^*(1950)^0$, we still lack the distribution amplitudes of the $K_0^*(1950)^0$ state at present. Fortunately, we are allowed to single out the $K_0^*(1950)^0$ component according to the kaon-pion DAs. In Fig. 2, we plot the variation of the branching fraction with the first odd Gegenbauer moment $B_1$ for the $B^0 \rightarrow \eta_c(K^*_0(1950)^0 \rightarrow )K^+\pi^-$ decay mode, as well as the experimental data $B_{\exp} = (2.18^{+1.32}_{-1.79}) \times 10^{-5}$ [5]. One can see that the theoretical prediction of the branching ratio shows strong dependence on the variation of the $B_1$. Combined with the available data, we can roughly determine the possible range of first odd Gegenbauer moment: from $-0.15$ to 0.05 or from 0.10 to 0.25, which should be examined both on the theory and experiment in the future.

In Fig. 3, we show the $\omega$-dependence of the differential decay ratio $dB(B^0 \rightarrow \eta_c K^+\pi^-)/d\omega$ (the solid curve) and $dB(B^0 \rightarrow \eta_c(1S,2S)K\pi)$ decays, the dynamical limit on the value of invariance mass $\omega$ is $(m_K + m_\pi) \leq \omega \leq (m_{B(\phi)} - m_{\eta_c(1S,2S)})$. However, $m_{K^*(1680)^0} > (m_B - m_{\eta_c(2S)})$, which means the resonance $K^*(1680)^0$ can not contribute to the $B^0 \rightarrow \eta_c(2S)K^+\pi^-$ decay. It is shown that the main portion of the branching ratios lies in the region around the resonance as expected in Fig. 3. For $B^0 \rightarrow \eta_c(1S)(K^*(892)^0 \rightarrow )K^+\pi^-$ decay, the central values of the branching ratio $\mathcal{B}$ are $26.24 \times 10^{-5}$ and $37.17 \times 10^{-5}$ when the integration over $\omega$ is limited in the range of $\omega = [m_{K^*} - 0.5 \Gamma_{K^*}, m_{K^*} + 0.5 \Gamma_{K^*}]$ or $\omega = [m_{K^*} - \Gamma_{K^*}, m_{K^*} + \Gamma_{K^*}]$ respectively, which amount to 52% and 73% of the total branching ratio $\mathcal{B} = 50.62 \times 10^{-5}$ as listed in Table III.

From Table III, for the $B^0 \rightarrow \eta_c(1S)K^*(892)^0 \rightarrow \eta_c K^+\pi^-$ decay, our PQCD prediction for its branching ratio is $\mathcal{B} = (50.62^{+21.50}_{-20.34}) \times 10^{-5}$, the central value of which is a little larger than the PDG2018: $(34.67 \pm 5.33) \times 10^{-5}$ [75]. The measurements from Belle [2], BABAR [3, 4] and
FIG. 3: Differential branching ratios for the $B^0 \rightarrow \eta_c(1S, 2S)[K^*(892)^0 \rightarrow K^+\pi^-]$ decays.

LHCb [5] collaborations, as well as the average value from HFLAV [76] are the following:

$$B(B^0 \rightarrow \eta_c(K^*(892)^0 \rightarrow K^+\pi^-)) = \begin{cases} (108^{+42}_{-46}) \times 10^{-5}, & \text{Belle [2]} \\ (53^{+28}_{-19}) \times 10^{-5}, & \text{BABAR [3]} \\ (38 \pm 7) \times 10^{-5}, & \text{BABAR [4]} \\ (29.5^{+3.6}_{-4.7}) \times 10^{-5}, & \text{LHCb [5]} \\ (41.3 \pm 6.6) \times 10^{-5}, & \text{HFLAV [76]} \end{cases} \quad (27)$$

One can see that the central value of the measured branching ratio for $K^*(892)^0$ resonance from different experiments vary in a wide range $(29 - 108) \times 10^{-5}$, the HFLAV world average of the measured ones from Belle and BABAR collaborations [2–4] leads to $(41.3 \pm 6.6) \times 10^{-5}$, in good agreement with our prediction.

Combined with the Clebsch-Gordan Coefficients, we can describe the relation

$$|K\pi\rangle = \frac{1}{\sqrt{3}}|K^0\pi^0\rangle - \frac{2}{\sqrt{3}}|K^+\pi^-\rangle. \quad (28)$$

The isospin conservation is assumed for the strong decays of an $I = 1/2$ resonance $K^{*0}$ to $K\pi$ when we compute the branching fraction of the quasi-two-body process $B \rightarrow \eta_c K^{*0} \rightarrow \eta_c K^+\pi^-$, namely,

$$\frac{\Gamma(K^{*0} \rightarrow K^+\pi^-)}{\Gamma(K^{*0} \rightarrow K\pi)} = 2/3, \quad \frac{\Gamma(K^{*0} \rightarrow K^0\pi^0)}{\Gamma(K^{*0} \rightarrow K\pi)} = 1/3. \quad (29)$$

Therefore, the corresponding two-body branching fraction $B(B \rightarrow \eta_c K^{*0})$ can be extracted directly from the quasi-two-body decay modes in Table III under the narrow width approximation relation:

$$B(B \rightarrow \eta_c K^{*0} \rightarrow \eta_c K^+\pi^-) = B(B \rightarrow \eta_c K^{*0}) \cdot B(K^{*0} \rightarrow K\pi) \cdot \frac{2}{3}, \quad (30)$$
There already exist some results for \( B^0 \rightarrow \eta_cK^+(892)^0 \) in the two-body framework \([59–61]\). One can see that the branching ratios of the quasi-two-body decay modes match well with those two-body analyses as presented in Ref. \([60]\) in PQCD approach. These results suggest that the PQCD factorization approach is suitable for describing the quasi-two-body \( B \) meson decays through analysing various resonances by reconstructing \( K\pi \) final states and reproducing the invariant mass spectra of Dalitz plots.

For the \( B^0 \rightarrow \eta_c(2S)(K^*(892)^0 \rightarrow)K^+\pi^- \) decay, the BABAR collaboration has measured an upper limit on the branching ratio \( \mathcal{B}(B^0 \rightarrow \eta_c(2S)(K^*(892)^0 \rightarrow)K^+\pi^-) < 26.00 \times 10^{-5} \) at 90\% confidence level \([4]\). The PQCD prediction of the \( \mathcal{B}(B^0 \rightarrow \eta_c(2S)(K^*(892)^0 \rightarrow)K^+\pi^-) = (14.19^{+5.67}_{-5.48}) \times 10^{-5} \) agrees with the limit. Meanwhile, one can find that the branching fractions of \( B^0 \rightarrow \eta_c(2S)(K^* \rightarrow)K^+\pi^- \) decays is always smaller than that for \( B^0 \rightarrow \eta_c(1S)(K^* \rightarrow)K^+\pi^- \) decays in Fig. 3, which is mainly induced by the difference between the DAs of \( \eta_c(2S) \) and \( \eta_c(1S) \) mesons: the tighter phase space and the smaller decay constant of \( \eta_c(2S) \) meson result in the suppression.

From the numerical results as given in Table III, we obtain the relative ratio \( R_2 \) between the branching ratio of \( B \) meson decays involving \( \eta_c(2S) \) and \( \eta_c(1S) \) for the resonance \( K^*(892)^0 \):

\[
R_2(K^*(892)^0) = \frac{\mathcal{B}(B^0 \rightarrow \eta_c(2S)(K^*(892)^0 \rightarrow)K^+\pi^-)}{\mathcal{B}(B^0 \rightarrow \eta_c(1S)(K^*(892)^0 \rightarrow)K^+\pi^-)} = 0.28^{+0.18}_{-0.14} \tag{31}
\]

which can be tested by the forthcoming LHCb and Belle-II experiments.

The branching ratios of the considered \( D \)-wave resonance are presented in Table IV. We emphasized that our predictions of these decay channels are only rough estimates in magnitude. Although there is no enough data at present, the calculated value \( \mathcal{B}(B^0 \rightarrow \eta_c(1S)(K^*_S(1430)^0 \rightarrow)K^+\pi^-) = (3.98^{+1.24}_{-1.74}) \times 10^{-5} \) is compatible with the measurement \((2.35^{+1.08}_{-1.29}) \times 10^{-5} \) \([5]\) within the large errors. Future experimental measurements with high precision will provide us a deep understanding of the properties of these tensor resonances.

For all the \( B^0 \rightarrow \eta_c(1S,2S)K\pi \) decays, such decay modes can be theoretically related to the counterpart \( B^0 \) decays since they have identical topologies and similar kinematic properties in the limit of \( SU(3) \) flavor symmetry. At the quark level, the \( B^0 \) and \( B^0_s \) decays correspond to \( b \rightarrow c\bar{c}s \) and \( b \rightarrow c\bar{c}d \) transition, respectively. The relative ratios of the branching fractions for \( B^0_s \) and \( B^0 \) decay modes are dominated by the Cabibbo suppression factor of \( |V_{cs}|^2/|V_{cd}|^2 \sim \lambda^2 \) under the naive factorization approximation. It is reasonable to see that the branching fractions of the \( B^0_s \) decays are smaller than those of the corresponding \( B^0 \) decays. Though the \( B^0_s \) channels have relatively small branching ratios, some of which will be potentially measurable at future experiments.

### IV. CONCLUSION

In this work, by introducing the kaon-pion DAs, we studied the quasi-two-body decays \( B^0 \rightarrow \eta_c(1S,2S)(K^{*0} \rightarrow)K\pi \) in the PQCD approach, in which the kaon-pion invariant mass spectra are dominated by the \( K^*_0(1430)^0, K^*_1(1950)^0, K^*(892)^0, K^*(1410)^0, K^*(1680)^0 \) and \( K^*_2(1430)^0 \) resonances. These six resonances fall into three partial waves according to their spins, namely \( S, P \), and \( D \)-wave states. The contributions from each partial wave can be parameterized into the corresponding time-like form factors involved in the kaon-pion DAs. The \( K\pi 
\text{S-wave component at low mass is described by the LASS line shape, while the time-like form factors of other resonances are modeled by the relativistic BW function.}
It has been shown that our predictions of the branching ratios for most of the considered $B^0 \rightarrow \eta_c(1S)(K^{*0} \rightarrow K^+\pi^-)$ decays are in good agreement with the existing data within the errors. For the $B^0 \rightarrow \eta_c(1S)(K_s^*(1430)^0 \rightarrow K^{*+}\pi^-)$ decay, although there exists a clear difference between the central value of the PQCD calculation for $B_K(1430)^0$, $B_{\text{NR}}$ and the measured ones, they are still consistent within three standard deviations due to the large experimental errors, which should be examined by the forthcoming experiments. The new ratio $R_2(K^*(892)^0)$ among the branching ratios of the considered decay modes has been defined and will be confronted with the future measurements.

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Appendix A: Decay amplitudes

The total decay amplitudes for the considered decay modes $B^0_{(s)} \rightarrow \eta_c K \pi$ in this work are given as follows:

$$
A(B^0_{(s)} \rightarrow \eta_c K \pi) = \frac{G_F}{\sqrt{2}} \left\{ V_{cb}^* V_{cs} \left[ (C_1 + \frac{1}{3} C_2) F^{LL} + C_2 M^{LL} \right] - V_{tb} V_{ts} \left[ (C_3 + \frac{1}{3} C_4 + C_9 + \frac{1}{3} C_{10}) F^{LL} + (C_5 + \frac{1}{3} C_6 + C_7 + \frac{1}{3} C_8) F^{LR} + (C_4 + C_{10}) M^{LL} + (C_6 + C_8) M^{SP} \right] \right\},
$$

(A1)

where $G_F = 1.16639 \times 10^{-5} \text{ GeV}^{-2}$ is the Fermi coupling constant and $V_{ij}$’s are the Cabibbo-Kobayashi-Maskawa matrix elements. The superscript $LL$, $LR$, and $SP$ refers to the contributions from $(V - A) \otimes (V - A)$, $(V - A) \otimes (V + A)$ and $(S - P) \otimes (S + P)$ operators, respectively. The explicit amplitudes $F(M)$ from the factorizable (nonfactorizable) diagrams in Fig. 1 can be obtained straightforwardly just by replacing the twist-2 or twist-3 DAs of the $\pi \pi$ and $KK$ system with the corresponding twists of the $K \pi$ ones in Eqs. (5)-(7), (11)-(13) and (19)-(21), since the kaon-pion distribution amplitudes considered in this work (Eqs. (4), (10) and (18)) have the same Lorentz structure as that of two-pion (kaon) ones in Refs. [42, 50].

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