Improvised deterministic and stochastic models for simulating partial discharges in non-conducting trees inside solid dielectrics

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Abstract

Electrical trees are defects originated and driven by partial discharges (PD) activity and this is the reason why their modelling and simulation are fundamental for the ageing mechanisms comprehension and diagnosis analyses of insulation systems compound by solid dielectrics. This study presents a brief review of the different models implemented to simulate PD in electrical trees inside solid dielectrics. In addition, an improved deterministic model as well as a stochastic model are presented, which allow predicting adequately the PD pulses distribution, in magnitudes and phase, for non-conducting electrical trees of different shape and length. Two case studies were simulated and their results exhibit good agreement when compared with measurements presented in the literature. It is concluded that the main parameters that state the PD behaviour in trees are the inception and extinction voltage magnitudes, including their probability distributions, and the tree geometry. The models can be used for prognosis analysis based on on-line measurements.

1 INTRODUCTION

Electrical trees are the result of damage accumulation caused by partial discharges (PD) activity in the gas channels that make the tree structure up [1,2]. They exhibit well known geometrical shapes: bush, branch or bush/branch, depending on the applied voltage, the electrodes and test arrangement, and physical properties of the materials [3].

When the electrical equipment is in use, detailed and exhaustive inspections are not feasible and non-intrusive techniques such as PD measurements are employed [4]. PD measurements and the characterization of their \( q - \varphi - n \) patterns allows inferring the existence of insulation defects and potentially dangerous conditions for the equipment life [5,6]. In electrical trees, the behaviour in phase, \( q_{\varphi} \), of those patterns are not dependent on the material [3]. However, they change depending on the tree shape and its propagation time [7–10], the voltage amplitude and wave shape [11] and the coexistence of various defects [12]. For the aforementioned reasons, the analysis of \( q - \varphi - n \) patterns together with statistical calculations
allows inferring the tree shape as well as predicting the remaining insulation lifetime [13,14].

In this study, two improved models for simulating PD in electrical trees are presented. Simulation results for 2 case studies are compared with experimental measurements found in the literature. A good agreement was found between simulated and measured q-p-n patterns and further improvements are also suggested. This study is organized as follows: first, a brief review of the capacitive, avalanche, self-consistent and artificial channel models for PD simulation in trees is presented in Section 2. Second, the improved deterministic and stochastic models (SMs) are explained in detail in Section 3. Third, the cases of study and the simulation results using the improved models are shown in Section 4. Finally, some conclusions are depicted in Section 5.

2 | MODELS FOR SIMULATING PD IN ELECTRICAL TREES INSIDE SOLID DIELECTRICS

Although the accurate simulation of PD in electrical trees requires a multi-physical approach, their modelling and simulation can be implemented on an approximated and reliable way just considering the dynamics of charge on the gas–solid boundary [15]. This is because of its time constant depends on process such as gas diffusion, conduction and trapping on the surface of the solid dielectric, and is much greater than the related to other physical variables such as the temperature and pressure [16].

2.1 | Capacitive model

In [17], it was presented a capacitive model for simulating PD in electrical trees based on experimental measurements of PD during tree propagation in low-density polyethylene, ethylene-vinyl acetate and ethylene-acrylic acid. The model is shown in Figure 1.

In Figure 1, \( C_g \) (F) represents the capacitance of a single tree channel, \( C_b \) (F) is the capacitance of the rest of the solid dielectric bulk, \( C_a \) (F) is the coupling capacitance and \( U_s \) (V) is the high voltage (HV) source. They found that the PD magnitude is proportional to the instantaneous voltage magnitude and the phase separation between consecutive PD is dependent on the time derivative of the applied voltage. For that reason, they introduced, although without a physical explanation, the series resistance \( R \) (\( \Omega \)). The PD occurrence is controlled by the voltage magnitude in the resistance and can be approximately calculated as follows:

\[
U_R(t) = RC \frac{dU_s(t)}{dt}
\]

where \( U_s(t) \) (V) is the applied voltage and \( C = C_gC_b/(C_g + C_b) \) (F). The stochastic behaviour of the PD phenomenon is modelled using the following probability density function:

\[
p_R(t) = \begin{cases} 
  k_p(U_R(t) - U_{Rth}) & U_R(t) > U_{Rth} > 0 \\
  0 & U_{Rth} > U_R(t) > 0
\end{cases}
\]

where \( k_p \) (V\(^{-1}\)) is a constant and \( U_{Rth} \) (V) is a critical threshold value.

Comparison between simulated and measured values exhibit good agreement. However, there is not a physical explanation for the inclusion of the series resistance and the equipotential conditions required for the capacitances calculations, are not explicitly analysed and explained. On the other hand, it is assumed that just a PD pulse can occur at each half cycle and, since the capacitances are constants, the PD magnitude in each branch is constant.

2.2 | Avalanche model

An electrostatic model, which simulates the PD activity as the result of one or more local electron avalanches was presented in [18]. This model is able to consider trees with different geometries that are constructed using linear segments in a uniform lattice. The size of the cells in the lattice is equal to the pin tip radius \( b \) (m). Each segment in the lattice belonging to the tree structure corresponds to an electric dipole, which defines the maximum length span of each avalanche. An avalanche will appear in a tree segment when the electric potential difference along the segment \( U_{av} \) (V) is greater than the inception value, \( U_{in} \) (V). Due to the local avalanches, charges are discretely added to the segment ends, simulated as dielectric spheres of radius \( b/2 \) (m) with uniform density, until \( U_{av} \) (V) is lower than or equal to an extinction magnitude, \( U_{e} \) (V). \( U_{in} \) (V) and \( U_{e} \) (V) depend on the characteristics of the gas in the tree tubules. A detailed explanation of this model is presented in Section 3.

Comparisons with measured values in epoxy resins allowed to conclude that the model adequately predicts the pulses distribution in phase with similar magnitudes and PD rate in each half cycle [18]. It was found that the changes in the q-p-n patterns during the tree propagation can be controlled through
the relative values of $U_{on}$ (V) and $U_{off}$ (V) comparing with the peak value of the applied voltage.

In [19], this avalanche model was improved including the channel conductivity as a parameter for simulating PD in conducting and non-conducting trees. For each segment in the tree structure a resistance value, $R_{seg}$ ($\Omega$), is assigned and the change in the charge distribution along the tree for each time step is calculated as follows:

$$\Delta Q_{seg} = \frac{U_{seg}}{R_{seg}} \Delta t_s$$

where $\Delta t_s$ (s) corresponds to the time step magnitude during the charge conduction calculations.

Comparison with measurements in flexible resins (non-conductive), $R_{seg} = 1 \times 10^{12}$ ($\Omega$) and glassy resins (conductive), $R_{seg} = 1 \times 10^6$ ($\Omega$) showed good agreement, which allowed to conclude that the charge transport mechanisms in glassy resins are dominated by conduction. On the other hand, the simulated $q$-$\phi$-$n$ patterns in non-conductive trees are symmetric. However, the measured ones are asymmetric [20], this is due to the deterministic nature of the simulation model.

### 2.3 Self-consistent model

In [21], it was presented a tree propagation model that explicitly simulates the PD activity during the tree advance. It is considered that the accumulated damage is proportional to the energy dissipated in the channels during PD activity [22]. Similarly to [18], it is considered that the tree structure is composed by linear segments of finite length with dielectric spheres of uniform charge density at their ends. A PD acting in a segment is simulated as an increase in its conductivity when the local electric field strength is greater than the inception magnitude $E_{inc}$ (V·m$^{-1}$). Due to the conductivity increment, charges are transferred between consecutive segments until the electric field strength magnitude is equal to or lower than a residual magnitude, $E_{res}$ (V·m$^{-1}$). If $\Delta q_i$ (C) is the electric charge transferred through the $i$-th channel segment during a PD activity, the increment in the damage in that segment for the instant $t$ (s) is:

$$\Delta W_i^t = k_W E_i \Delta q_i$$

where $k_W$ is the portion of the energy dissipated during the PD activity causing material damage and $E_i$ (V·m$^{-1}$) is the local electric field strength along the channel calculated from the difference of potential at the segment ends.

A new channel is added to the existing structure when the energy and the electric field strength are equal to or greater than the critical values $W_c$ (J) and $E_c$ (V·m$^{-1}$). The PD charge magnitude is calculated from the change in the charge, $q_i$ (C), on the dielectric spheres with:

$$Q = \sum_i \left( q_i^{\text{step}} - q_i^{\text{step-1}} \right)$$

where $t_{\text{step}}$ is the current time step and the summation is made over all the tree segments.

It is considered that after and before each PD event the tree segments are non-conductive. The magnitude and distribution of simulated $q$-$\phi$-$n$ patterns for alternating current (AC) sinusoidal and triangular voltages are qualitatively compared with measurements and a reasonable agreement was found [23]. An improvement to this model was presented in [24] for including the charge transport process using the Ohm’s law, which allows to obtain the individual PD pulses of current along the tree structure. This powerful model is too general and does not consider specific changes in the propagation modes due to physical-chemical variations in the media and the dependence on the polarity of the parameters of media [16].

### 2.4 Artificial channel model

It has been demonstrated that the $q$-$\phi$-$n$ patterns produced by PD in straight artificial channels are similar to those found in trees of different size [25]. In [26], it was presented a physical model, which is in fact an improvement to the capacitive model [17], for simulating PD in artificial channels. It is considered that PD will occur when the electric field strength at the pin tip is greater than the inception magnitude $E_{inc}$ (V·m$^{-1}$) and will propagate along the channel until the average electric field is lower than or equal to a residual field $E_{res}$ (V·m$^{-1}$). The residual field is assumed as uniform and is calculated as $E_{res} = 0.2 + 0.14 N_r$ (kV·mm$^{-1}$) where $N_r$ is a random number between zero and one. The above expression for $E_{res}$ allows to consider the stochastic behaviour of PD pulse magnitudes found experimentally [17].

Figure 2 shows a representation of the electric field distribution along the channel length before and after the $i$-th PD pulse is acting, respectively, $E_1$ (V·m$^{-1}$) and $E_2$ (V·m$^{-1}$). The field distribution after a PD is mainly dependent on the charge distribution on the channel walls, because of that charges can be trapped and will migrate slowly when the channel conductivity is low.

For calculating the induced charge due to the $i$-th PD, which propagates to the distance $r_i$ (m) along the channel, this is subdivided into various segments of length $r_s$ (m), with $0 < r_s < r_i$. Using the superposition principle, the induced charge by the $i$-th PD is calculated as follows:

$$Q_i = \int_{r_s}^{r_i} C(r) (E_{r,0} - E_{res}) dr$$

where $E_{r,0}$ (V·m$^{-1}$) is the electric field strength in the segment $r$, when the PD propagate to that segment and $C_r$ (F) is its equivalent capacitance.
the normal distribution with a mean value of 1 and a standard deviation of 0.2.

An artificial channel of 120 μm long was simulated in a pin-plane arrangement with AC applied voltages in the 6–10 kV range. It was found that the model adequately reproduces the main characteristics, distribution and magnitude of the q-q-n patterns and, for this reason, its quantitative results are reliable.

Table A1 in Appendix summarizes the main advantages and disadvantages of existing current models for simulating PD in electrical trees.

3 | IMPROVED MODELS FOR SIMULATING PD IN ELECTRICAL TREES INSIDE SOLID DIELECTRICS

The improved models proposed in this study are based on the electrostatic-deterministic model presented by Champion and Dodd [18]. According to what was concluded in [27], PD in electrical trees are controlled by five key parameters:

i. Geometry of the trees
ii. Applied voltage
iii. Inception voltage
iv. Extinction voltage
v. Residual voltage

The electrostatic-deterministic model allows defining and controlling these parameters. In addition, it is able to consider tree structures with fractal dimensions similar to those found experimentally.

The model is based on electrostatic theory. The following are the assumptions of the model: there is not tree growth during PD simulations, each PD event consists of one or more local electron avalanches within the gas tubules of the tree, avalanches are instantaneous and there is no charge transportation by conduction during PD.

The tree structure and electrodes arrangement are reconstructed using a square lattice as it is shown in Figure 3. The tree structure is constructed using linear segments of length \( b \) (m), where its magnitude is equal to the pin tip diameter and states the maximum distance of each avalanche. After an avalanche, a dipole is established over that distance. It is assumed that the permittivity of the tree structure is the same as the solid dielectric. Although this model can be used for calculating PD in conducting trees [19], we are going to consider the tree structure as non-conducting in order to compare with measured values reported by other authors.

The electrical tree structure must be reproduced in a 2D square lattice to be simulated, so the only limitation to the geometry of the tree structure that can be simulated lies in the possibility of representing it using the 2D square lattice and for this reason, only trees with fractal dimension below 2 [11] can be modelled.

The electric field strength and electric potential distribution along the tree structure in a pin-plane arrangement are

2.5 | Improved avalanche model

In [27], it was proposed an SM based on the deterministic model presented by Champion and Dodd [18] and using experimental measurements of inception and extinction voltages obtained from Pulse Sequence Analysis (PSA) [28,29]. It was found that the residual electric field strength inside the channels is lower than the calculated from the PSA residual voltage [30]. For this reason, it is assumed that the PD extinction in the segments adjacent to the pin tip is controlled by an extinction voltage magnitude, \( U_{off} \) (V), while inside the tree channels is controlled by a residual voltage \( U_{res} \) (V), where \( U_{res} < U_{off} \). The analysis of PSA patterns allowed inferring that the voltage difference between consecutive pulses exhibits a normal distribution, which can be simulated through the inception voltage as:

\[
U_{in} = \text{rand}(1,0.2)(U_{on} - U_{off}) + U_{off}
\]  

where \( U_{on} \) (V) is the inception voltage used during the simulations, \( U_{on} \) (V) is the measured inception voltage and \( \text{rand}(1,0.2) \) is a random numbers generation function from

![Electric field distribution](image)

**Figure 2** Electric field distribution along the channel length. \( E_1 \) (V m\(^{-1}\)) is the electric field before the i-th PD and \( E_2 \) (V m\(^{-1}\)) is the electric field distribution after the PD activity. PD, partial discharges.
where $\varepsilon$ (F·m$^{-1}$) is the permittivity of media, $r_{\text{pin}}$ (m) is the position vector of pin tip and $r_{\text{pin}}$ (m) the position vector of pin tip image with respect to the plane electrode. $U(t) = U_0 \sin(2\pi f t)$ (V) is the applied AC voltage to the pin electrode with frequency $f$ (Hz).

Using Equation (8), the potential at any location, $r$ (m), within the tree structure or the solid dielectric, can be calculated as:

$$U_{\text{app}}(r, t) = \frac{Q_{\text{app}}(t)}{4\pi \varepsilon b} \left( \frac{1}{|r - r_{\text{pin}}|} - \frac{1}{|r - r_{\text{pin}}|} \right)$$  \hspace{1cm} (9)

The charge distribution in the tree structure is modelled using dielectric spheres with uniform charge density and radius $b/2$ (m) at the segments ends as it is shown in Figure 4, where $\pm Q$ (C) is the charge added to the tree segment ends by a local avalanche, $r_{+Q}$ (m) and $r_{-Q}$ (m) are the position vectors of the tree segment ends and $\Delta U = U_{\text{seg}}$ (V) is the potential difference along the tree segment. In Figure 4, it can be seen that each tree segment corresponds to an electric dipole of length $b = |r_{+Q} - r_{-Q}|$ (m).

The potential at the pin tip corresponds to $U(r_{\text{pin}}, t) = U_{\text{app}}(r_{\text{pin}}, t) + U_{\text{im}}(r_{\text{pin}}, t) + U_{Q}(r_{\text{pin}}, t)$ (V) where $U_{Q}(r_{\text{pin}}, t)$ (V) is the potential at the pin tip due to the charges within the tree structure and $U_{\text{im}}(r_{\text{pin}}, t)$ (V) is the potential of the induced image charge on the pin tip for maintaining the boundary condition $U(r_{\text{pin}}, t) = U_{\text{app}}(r_{\text{pin}}, t)$. The induced image charge is calculated from Equation (8) as:

$$Q_{\text{im}}(t) = \frac{4\pi \varepsilon b}{(3 - \frac{1}{|r_{-Q} - r_{\text{pin}}|})} U_{\text{im}}(r_{\text{pin}}, t)$$  \hspace{1cm} (10)

Taking into account that the image charge and the applied charge occupies the same location, the potential at any location $r$ (m) and time $t$ (s) is:

$$U(r, t) = U_{\text{app}}(r, t) + U_{\text{im}}(r, t) + U_{Q}(r, t)$$  \hspace{1cm} (11)

$U_{\text{im}}(r, t)$ (V) is calculated using Equation (10) in Equation (9), and $U_{Q}(r, t)$ (V) from the dipoles and their images [18].

The electric field strength at the centre of each segment of the tree structure can be calculated using the potential difference between its ends and is considered as uniform along the tree segment length. For ease of computation, the analysis and calculations are made in function of the potential difference instead of the electric field strength, both related through $U = Eb$ (V). A PD occurs when in a segment is met:

$$U_{\text{seg}} \geq U_{\text{on}}$$  \hspace{1cm} (12)
where \( U_{\text{seg}} \) (V) is the difference of potential between the segment ends and \( U_{\text{on}} \) (V) is the inception voltage which is associated with the minimum magnitude necessary for the separation of electrons from the gas or at the surface, where could have been left by previous PD.

Due to the local avalanches, charges are added to the dielectric spheres at the segment ends. Those charges produce an electric field that opposes to the applied Laplacian field and the electric potential difference is reduced. In those segments where Equation (12) is met, charge dipoles will be added discretely until the following is fulfilled in all segments of the tree:

\[
U_{\text{seg}} \leq U_{\text{off}} + U_{\text{err}} \tag{13}
\]

where \( U_{\text{off}} \) (V) is the extinction voltage below which avalanches can no longer be sustained, because the ionization coefficient is equal to the electron trapping coefficient. \( U_{\text{err}} \) (V) is a small tolerable error (<10 V) which permits to control the convergence time. \( U_{\text{on}} \) (V) and \( U_{\text{off}} \) (V) depend on the characteristics of the gas in the tree tubules and the surface at the channel walls, their magnitudes can be directly calculated from experimental measurements [30].

It is assumed that all the tree segments have identical \( U_{\text{on}} \) (V), \( U_{\text{off}} \) (V) and \( U_{\text{err}} \) (V) magnitudes. For each time step, \( t + \Delta t \) (s), the change in the induced charge is calculated as \( \Delta Q_{\text{im}}(t + \Delta t) = Q_{\text{im}}(t + \Delta t) - Q_{\text{im}}(t) \) (C) and the change in the charge distribution along the tree is calculated using Equation (3). Equations (3) and (8)–(13) define the electrostatic-deterministic model, however, it has been shown that the behaviour of PD in electrical trees, as well as the propagation of these in solid dielectrics, is a chaotic-deterministic phenomenon [10]. In [20], it was shown that this purely deterministic model does not allow obtaining \( \varphi \varphi \varphi \varphi \varphi \) patterns with the asymmetries found in the experimental measurements. From the above, the following stochastic variations for the inception voltages are introduced in the model:

\[
U_{\text{on}}^+ = U_{\text{on}} \tag{14}
\]

\[
U_{\text{on}}^- = \text{rand}_\text{norm}(1, 0.2)(U_{\text{on}} - U_{\text{off}}) + U_{\text{off}} \tag{15}
\]

where \( U_{\text{on}}^- \) (V) and \( U_{\text{on}}^+ \) (V) are the inception voltages of the positive and negative PD, respectively, used during the simulation, \( U_{\text{on}} \) (V) and \( U_{\text{off}} \) (V) are the inception and extinction voltages experimentally measured [30]. \( U_{\text{on}}^- \) (V) is introduced to take into account that the inception voltage of negative PD is lower than the inception voltage of positive PD [27]. \( \text{rand}_\text{norm}(1, 0.2) \) is a function that generates random numbers with a mean of 1 from the normal distribution with a standard deviation of 0.2. This distribution and its standard deviation have been used since the measured and simulated distributions presented in [20], visually have the appearance of this type of distribution and the simulation results in [27], which use this same distribution, are consistent with experimental measurements.

The stochastic variation for the inception voltage of negative PD is based on the conclusions of the analyses of PD time sequences during tree propagation carried out by Kaneiwa et al. [8]. The model described by Equation (3) and Equations (8)–(13) corresponds to the proposed improved SM. The second improved model corresponds to a deterministic model in which the inception voltages are defined as follows:

\[
U_{\text{on}}^+ = U_{\text{on}} \tag{16}
\]

\[
U_{\text{on}}^- = U_{\text{on}} ' \tag{17}
\]

with \( U_{\text{on}}/U_{\text{on}} ' = 0.9 \) [27]. This same relationship applies to Equation (15). The model described by Equations (3), (8)–(13) and Equations (16) and (17) corresponds to the improved deterministic model (IDM).

Both models were implemented in Matlab. Figure 5 shows a flowchart of that implementation.

For calculating the electric field strength produced by the source equivalent charge, the Laplace field, and by the charge distribution due to the PD in the tree structure, the charge simulation and images methods were used [31]. The incoming parameters of the IDM and SM are:

i. The tree structure geometry. The tree structure \( t \) must be modelled using a 2D squared lattice \( b \) be simulated. As a first approximation, the lattice size is considered as the pin-tip diameter. However, it can be reduced as necessary in order to improve the modelling accuracy

ii. The relative permittivity of the dielectric, \( \varepsilon_r \). It is assumed that the tree structure and the solid dielectric have the same permittivity

iii. The peak value and frequency of the applied voltage, \( U_0 \) (V), \( f \) (Hz). Besides, different frequencies and non-sinusoidal voltage wave-shapes can be modelled. However, experimental validation is required

iv. The inception and extinction voltages, \( U_{\text{on}} (V) \) and \( U_{\text{off}} (V) \). They can be determined from PD PSA measurements [30].

v. The electric resistance of the tree segments, \( R_{\text{seg}} \) (Ω). Different values of resistance can be assigned to the main tree channel and the tree tips for modelling the PD behaviour in conducting trees.

### 4 CASE STUDIES AND SIMULATION RESULTS FOR PD IN NON-CONDUCTIVE ELECTRICAL TREES

Two case studies, which were experimentally measured by other authors, were simulated. The first case study corresponds to a non-conducting electrical tree in bisphenol-A flexible epoxy resin CY1311 whose growing in a pin-plane...
A pin-plane arrangement under 10 kV was previously analysed using a light emission and PD activity correlation approach [20]. The distance between the pin-tip and the plane is 2 mm, the applied voltage to pin electrode is AC 50 Hz, 10 kV peak and the plane is grounded. This first case study was included in this study in order to demonstrate the efficacy of the proposed models for predicting the PD characteristics in magnitude and phase.

On the other hand, a second case study was included in this study in order to analyse in detail the asymmetry in the induced PD charge magnitude during the positive and negative half cycles as well as the effect of the inception and extinction voltage magnitudes on the PD activity. The second case study corresponds to a non-conductive electrical tree in a flexible epoxy resin of relative permittivity 2.1 used for simulating the PD activity in non-conducting trees in polyethylene and for analysing the effect of relative permittivity on the PD behaviour in different materials [18]. A pin-plane arrangement is used applying a 14.14 kV peak, 50 Hz voltage to pin electrode, while the plane is grounded. The distance between the pin tip and the plane is 2 mm.

Comparisons between simulated and measured results allow evaluating the validity, applicability and capabilities of the proposed models. Each case study was simulated using the original deterministic model (ODM), Equation (3) and Equations (8)–(13), as well as the SM and IDM models.

**Figure 5** Proposed improved stochastic and deterministic models flowchart

**Figure 6** Case 1: tree geometry. Non-conductive tree, 45 × 45 lattice, pin tip at (23,5) in lattice units, $h = 50 \mu m$
4.1 | Case study 1

Figure 6, shows the geometry of the first case study, Case 1, taken from [18,20]. The tree structure is non-conductive, \( R_{\text{seg}} = 1 \times 10^{12} \, \Omega \), the fractal dimension is about 1.5 and the pin tip-plane separation is 2 mm. A 45 × 45 uniform lattice with \( h = 50 \, \mu m \) is used for modelling the geometry and the pin tip is at (23,5) in lattice coordinates. Table 1, summarizes the model parameters.

In Table 1, \( \Delta t_2 \) (μs) is the time step used during charge conduction simulations, \( \Delta t_2 = \Delta t_1(s) \). Simulation results for Case 1 during 50 cycles of the applied voltage are shown in Figure 7. As can be seen in Figure 7d, g the ODM model does not allow the \( q^{+} \cdot \phi \cdot n \) patterns to be adequately simulated since there is a symmetry between the positive and negative PD distributions that does not appear in the experimentally measured patterns.

The simulation results using the IDM model are shown in the second column of Figure 7. It can be seen in Figure 7c, h that this model yields \( q^{+} \cdot \phi \cdot n \) patterns with asymmetry in the positive and negative PD. Because of the inception voltage for negative PD is lower than for positive PD, there are more negative PD than positive in addition, these occur at a lower phase than with the ODM. On the other hand, the greater number of negative PD increases the charge left in the tree channels, which produces an increase in magnitude and the number of subsequent positive PD because of the intensification of the electric field.

Finally, it is observed in Figure 7e, i that, with the SM model, the \( q^{+} \cdot \phi \cdot n \) patterns exhibit a distribution similar to that obtained in the experimental measurements. On the other hand, the maximum values of the positive and negative PD are close to that experimentally measured for this case study in [20], Table 2. The values between parentheses in Table 2 correspond to the % error with respect to the measured values reported in [20].

The improvement to the deterministic model proposed here (IDM), using \( U^{\text{on}}_m \) (V) for negative PD, reduces the error and allows obtaining asymmetric patterns, however, the proposed SM not only reduces the error even further, but also, the distributions of the \( q^{+} \cdot \phi \cdot n \) patterns are similar to those experimentally measured in which it can be seen that the positive PD are of greater magnitude than the negatives and those are concentrated in clouds around the phase values corresponding to the voltage increments when the resultant voltage within the tree structure is equal to or greater than \( U^{\text{on}}_m \) (V), while the negative discharges have an approximate normal distribution.

In spite of the reasonable agreement for the maximum PD magnitudes, the accuracy of the models can be improved adjusting the inception voltage distribution using PSA experimental measurements.

4.2 | Case study 2

The second case of study is included here for analysing the time behaviour of the induced charge on the pin electrode for verifying if the asymmetries in the PD pulses are also reflected in the induced PD charge. On the other hand, the permittivity of media is reduced to 2.1, similar to polyethylene, which allows to evaluate the effect on the PD behaviour of the solid material parameters.

The Case 2, corresponds to the same geometry shown in Figure 6. For this case study, a 41 × 44 uniform lattice with \( h = 50 \, \mu m \) is used for reproducing the tree geometry. The pin tip is at (21,4) in lattice coordinates. The parameters of the model are presented in the third column of Table 1. Figure 8, shows the simulation results for the Case 2 during 50 cycles of the applied voltage.

In Figure 8, \( q^{+} \) (C) is the induced PD charge on the pin electrode. For this case, the \( q^{+} \cdot \phi \cdot n \) distributions obtained using each model have similar characteristics to that shown in Figure 7. However, the number of PD is greater due to the applied voltage is greater and the inception voltage is lower than in the Case 1. On the other hand, in despite of the increased applied Laplacian field to the same geometry, the PD magnitudes, maximum and minimum values, are lower than in Case 1. This is because the extinction voltage is greater and the voltage difference between inception and extinction is lower than in the Case 1, see Table 1. Table 3 summarizes the simulation results for the Case 2.

In Table 3, Max \( N_{PD} \) is the maximum number of PD for any phase in the \( q^{+} \cdot \phi \cdot n \) pattern. It can be seen that when the

| Parameter | Case 1 | Case 2 |
|-----------|-------|-------|
| \( U_{0} \) (kV) | 10 | 14.142 |
| \( U_{m} \) (kV) | 3.5 | 2.2 |
| \( U_{\text{off}} \) (kV) | 1 | 1.5 |
| \( R_{\text{seg}} \) (\( \Omega \)) | \( 1 \times 10^{12} \) | \( 1 \times 10^{12} \) |
| \( h \) (\( \mu m \)) | 4.8 | 2.1 |
| \( U_{\text{on}}^{\text{on}}/U_{m} \) | 0.9 | 0.9 |
| \( \Delta t_1 \) (\( \mu s \)) | 5.6 | 5.6 |
| \( \Delta t_2 \) (\( \mu s \)) | 0.28 | 0.28 |

**Table 2** Summary of simulation results for Case 1

| Variable/Model | ODM | IDM | SM | Measured [20] |
|----------------|-----|-----|----|---------------|
| Max(\( q^{+}_{PD} \)) (pC) | 156.9 (60.8) | 311 (22.3) | 449.7 (12.4) | 400 |
| Max(\( q^{-1/2}_{PD} \)) (pC) | 161.8 (59.6) | 266.6 (33.4) | 284.6 (28.9) | 200 |
| Min(\( q^{+1/2}_{PD} \)) (pC) | 17 | 15.1 | 3.4 | - |

Abbreviations: IDM, improved deterministic model; ODM, original deterministic model; SM, stochastic model.
permittivity of media is reduced the inception voltage decreases and the extinction voltage increases. Due to this, PD exhibit a phase advance, it is, they appear earlier in phase. This behaviour is similar to that found during periodic bursts when electrical trees propagate in conductive materials. From that it can be concluded that periodic bursts [16] can be modelled changing the inception and extinction voltages. On the other hand, the magnitude of induced charge decrease by a factor of almost 2, when the permittivity is reduced from 4.8 to 2.1 [18].

Additionally, Figure 9 shows the induced charge on the pin for the first five cycles of applied voltage using the ODM, IDM and SM. It can be appreciated that the induced charge on the pin electrode also exhibits an asymmetry similar to PD pulses and the proposed models are able to predict this behaviour with reasonable accuracy. The ODM model always presents symmetry while the other models present an asymmetry similar to that found in the experimental measurements in [18]. During the real acquisitions, the maximum value of the induced charge on the electrode for the positive half cycle of the AC signal is 40 pC while in the negative half cycle it is 60 pC, which implies an asymmetry of two-thirds in [18]. Using the results of the SM model, the maximum value in the positive half cycle is 60 pC, while in the negative half cycle it is 80 pC, which implies an asymmetry of three-fourths. Regarding the measurements, the SM model presents a difference of 12.5% in the asymmetry and 33.33% in the maximum values of the induced charge. This same error was found using the ODM model in [18] and it was associated with the inception and extinction voltages magnitudes used during the simulations, Table 1. Those voltage magnitudes can be adjusted in order to reduce the error. However, that task is out of the scope of this study. This allows us to demonstrate that the proposed models, not only calculate adequately the magnitude of the PD, but also their distribution as a function of phase and time.

The proposed improved models present an adequate performance and permits predicting with minor errors to the
ODM, the distributions of the $q\varphi n$ diagrams and the magnitudes of the PD pulses.

It should be borne in mind that the model only considers charge transport by conduction at the interface between the gaseous and dielectric channels and does not take into account other transport processes, such as in the gas or charge trapping at the surface, which can affect the distribution of the electric field strength. However, the results are considered to be reasonably accurate.

The changes on the PD behaviour during tree propagation depends on the changes on the tree shape and length [13] and the spatial and temporal variations in the tree channels conductivity [16]. In non-conductive trees the PD pattern during the tree propagation evolves from turtle to wing-like distribution and the maximum PD increases proportional to the tree length [27]. On the other hand, in conductive trees, the PD behaviour during tree propagation is characterized by low PD magnitudes, practically undetectable due to noise levels, at a very high rate with some periodic bursts when the PD pulse magnitude increases, those bursts are related to new branch formation. The variations in the PD behaviour in conductive trees can be modelled using the IDM and SM models assigning a lower resistance to the main tree channel than at the tree tips, in addition, the bursts can be modelled reducing the inception and extinction voltages which allows both, to increase the PD

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**TABLE 3** Summary of simulation results for Case 2

| Variable/Model | ODM | IDM | SM |
|----------------|-----|-----|----|
| $\max(q_{PD})$ (pC) | 137.9 | 158.4 | 178.3 |
| $\max(q_{PD})$ (pC) | 162.1 | 117.3 | 78.3 |
| $\min(q_{PD})$ (pC) | 17 | 15.1 | 3.4 |
| $\max N_{PD}$ | 19 | 10 | 12 |

Abbreviations: IDM, improved deterministic model; ODM, original deterministic model; SM, stochastic mode.
magnitude at low voltage phases and to increase the PD propagation length.

Detailed images and figures of the measurement results can be seen in [18,20].

Table A2 in Appendix, summarizes the advantages and disadvantages of the IDM and SM models.

5 CONCLUSIONS

The state-of-the-art of models for simulating PD in electrical trees was reviewed and it was found that the electrostatic-deterministic model appears to be the most adequate for simulating PD in trees mainly because it allows considering trees of different shape and length. The theoretical background of the models was briefly presented and their advantages and disadvantages were discussed.

Two novel models, IDM and SM, were proposed which can be considered as improvements to the ODM that serves as a basis since they allow obtaining $q \phi n$ patterns with distributions and magnitudes approximate to those found experimentally for PD in electrical trees and with smaller errors than those obtained with the ODM. The main advantages of the proposed model can be summarized as follows:

i. Electrical trees of different shape can be modelled

ii. The $q \phi n$ patterns obtained have magnitudes and phase distributions very close to those found experimentally

iii. The models are able to predict the asymmetric behaviour of the induced charge

iv. The models can be used for considering different frequencies and voltage wave shapes

The parameters that affect the most the PD behaviour are, the geometry, the magnitude of the inception and extinction voltages and the probability density function for the inception voltage of the negative PD.

Other improvements to the proposed models can be made by adjusting the inception voltages and the probability density function using distributions experimentally determined.

The simulation results through the improved models and their similarity to the experimental measurements, corroborate the hypothesis of other researchers that PD in trees are chaotic-deterministic phenomena.

The models can also be used for simulating PD in conducting trees. However, the validation analysis is quite difficult due to the high rate of PD pulses and their very low magnitude [19].

This model will be used in future works for correlating electrical tree characteristics, tree length and shape, with PD behaviour as a prognosis tool for evaluating the useful life of solid dielectrics.

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APPENDIX

| TABLE A1 | Advantages and disadvantages of models for simulating PD in electrical trees inside solid dielectrics |
|----------|--------------------------------------------------------------------------------------------------|
| Model    | Advantages                                                                                      | Disadvantages                                                                 |
| Capacitive [17] | Its calculation algorithm can be easily implemented. The \( q \phi n \) patterns present distributions similar to those measured experimentally. The stochastic behaviour of the PD is simulated by varying the extinction voltage. | The PD in the gaseous channels is controlled by the voltage in a resistance that does not have a physical explanation. Only one PD can occur in each half cycle. The channel capacitances are constant, so the PD will be of constant magnitude. |
| Avalanche [18,19] | It is an electrostatic model whose parameters and variables are physically justified and can be measured experimentally. Trees of different geometries can be simulated. Both conductive and non-conductive trees can be simulated. | It is a purely deterministic model, so the distributions of the \( q \phi n \) patterns do not present the asymmetries found in the experimental measurements. |
| Self-consistent PD + propagation [21,22] | It is a physical model that allows the simultaneous simulation of PD and the propagation of trees. The \( q \phi n \) patterns are similar to those measured experimentally with sinusoidal and triangular AC signals. The model uses the charge simulation method. Quite different geometries can be simulated since a predefined lattice is not used for the propagation of the trees. | It is a fairly general model and does not take into account changes in tree propagation modes due to physical and chemical variations in materials caused by PD. Dependence of polarity on media and model parameters is not considered. For the simulation of the PD it can be considered as equivalent to the avalanche model. |

(Continues)
**Model** | **Advantages** | **Disadvantages** |
---|---|---|
Artificial channel [26] | It is an improvement to the original model of capacitors. Multiple PD can be considered in a half cycle of a sinusoidal AC signal. The model adequately reproduces the experimentally measured $q\varphi_n$ patterns in artificial channels when the applied voltage or length is changed. The dependence of polarity on the model parameters is considered. | The capacitance between the channel tip and the plane electrode is constant so sudden changes in PD cannot be modelled by increasing the radial length of the tree structure. Only artificial, straight channels can be modelled. |
Improved avalanche [27] | The distributions and magnitudes of the simulated $q\varphi_n$ patterns are quite close to those measured experimentally, so the quantitative results are reliable. | The model does not consider the dependence on the polarity of the model parameters, so the $q\varphi_n$ patterns are symmetric. |

**Abbreviations:** AC, alternating current; PD, partial discharges.

### TABLE A2 Summary of advantages and disadvantages of proposed models for simulating PD in electrical trees

| Model | Advantages | Disadvantages |
|---|---|---|
| Original deterministic model (ODM) | It is an electrostatic model whose parameters and variables are physically justified and can be measured experimentally. Tree of different geometries can be simulated. Allows modelling and simulating conductive and non-conductive trees. | It is a purely deterministic model, so the distributions of the $q\varphi_n$ patterns do not present the asymmetries found in the experimental measurements. The trees must be represented using a 2D square lattice to be simulated. |
| Improved deterministic model (IDM) | The same as aforementioned for ODM. Besides, allows considering a lower inception voltage magnitude for the negative PD. | In spite of the different voltage inception magnitude for positive and negative PD, the $q\varphi_n$ patterns exhibit a symmetric wing-like distribution in both, negative and positive half cycles. The trees must be represented using a 2D square lattice to be simulated. It is only considered charge transport by conduction at the gas-solid interface. |
| Stochastic model (SM) | The same as aforementioned for ODM. Additionally, includes a stochastic variation in the inception voltage for negative PD with a distribution based on experimental measurements. The $q\varphi_n$ diagrams show an asymmetric behaviour similar to measured ones. The PD behaviour simulated using this approach allows to explain the chaotic-deterministic conduct of tree propagation. | The trees must be represented using a 2D square lattice to be simulated, for this reason only trees with fractal dimension $<2$ can be modelled. It is only considered charge transport by conduction at the gas-solid interface. |

**Abbreviation:** PD, partial discharges.