GK Bootis and AE Fornacis: two low-mass eclipsing binaries with dwarf companions\textsuperscript{*},\textsuperscript{**}

P. Zasche\textsuperscript{1}, P. Svoboda\textsuperscript{2}, and R. Uhlář\textsuperscript{3}

\textsuperscript{1} Astronomical Institute, Faculty of Mathematics and Physics, Charles University Prague, CZ-180 00 Praha 8, V Holešovičkách 2, Czech Republic
e-mail: zasche@sirrah.troja.mff.cuni.cz
\textsuperscript{2} Private observatory, Výpustky 5, Brno 614 00, Czech Republic
\textsuperscript{3} Private Observatory, Pohôň 71, 25401 Jílové u Prahy, Czech Republic

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ABSTRACT

Context. A study of late-type low-mass eclipsing binaries provides us with important information about the most common stars in the Universe.

Aims. We obtain the first light curves and perform period analyses of two neglected eclipsing binaries GK Boo and AE For to reveal their basic physical properties.

Methods. We performed both a period analysis of the times of the minima and a $BVR$ light curve analysis. Many new times of minima for both the systems were derived and collected from the data obtained by automatic and robotic telescopes. This allowed us to study the long-term period changes in these systems for the first time. From the light curve analysis, we derived the first rough estimates of the physical properties of these systems.

Results. We find that the analyzed systems are somewhat similar to each other. Both contain low-mass components of similar types, both are close to the Sun, both have short orbital period, and both contain another low-mass companions on longer orbits of a few years. In the case of GK Boo, both components are probably of K3 spectral type, while the distant companion is probably a late M star. The light curve of GK Boo is asymmetric, which probably causes the shift in the secondary minima in the O–C diagram. System AE For comprises two K7 stars, and the third body is a possible brown dwarf with a minimal mass of only about $47 M_{\text{Jup}}$.

Conclusions. We succeed in completing period and light curve analyses of both systems, although a more detailed spectroscopic analysis is needed to confirm the physical parameters of the components to a higher accuracy.

Key words. binaries: eclipsing – stars: fundamental parameters – stars: individual: GK Boo – stars: individual: AE For

1. Introduction

Low-mass stars are the most common stars in our Galaxy (e.g. Kroupa 2002). However, owing to their low luminosity, only these close to the Sun have been studied in detail and many of them have never been analyzed. Hence, we focused on two rather neglected low-mass eclipsing binary systems: GK Boo and AE For. Their light curves as well as their period modulation had never been studied. Some studies indicate that most late-type stars are single (e.g. Lada 2006), but the number of papers studying the multiplicity of the late-type systems is still rather limited. Therefore, the incidence of multiples in late-type stars remains unexplored.

The study of eclipsing binaries provide us with important information about the physical properties of both of their components – their radii, masses, and evolutionary status. However, when considering only with the light curve, several assumptions have to be made. For the analysis presented in this paper we also used the photometric data obtained by automatic and robotic telescopes (such as ASAS, Pi of the sky, and SWASP). Thanks to these huge databases of observations, the long-term evolution of these systems can be studied for the first time.

\textsuperscript{*} This paper uses observations made at the South African Astronomical Observatory (SAAO).

\textsuperscript{**} Tables 8 and 9 are available in electronic form at http://www.aanda.org

2. GK Boo

2.1. Introduction

The system GK Boo (=BD+37 2556, $V_{\text{max}} = 10.86$ mag) is an Algol-type eclipsing binary with an orbital period of about 0.48 day. It is also a primary component of a visual double designated WDS J14384+3632 in the Washington Double Star Catalog (WDS\textsuperscript{1}, Mason et al. 2001). The secondary component of this double star is about 14” distant, and is probably gravitationally bound to GK Boo itself. It is about 0.4 mag fainter, but since its discovery in 1933 there has been no detectable mutual motion of the pair, hence the orbital period is of about thousands of years (rough estimation from the Kepler’s law).

The star is too faint, thus was not observed by Hipparcos satellite, and its distance is therefore rather uncertain. Kharchenko (2001) introduced the parallax 30.29 mas, which is however only an estimate. Its spectral type is also unknown, but the $B−V$ index derived from the Tycho catalogue (Høg et al. 2000), $B−V = 0.89$ mag indicates a spectral type of about K1. On the other hand, the 2MASS infrared photometry (Cutri et al. 2003) gives $J−H = 0.527$ mag (therefore a spectral type of K3). Finally, Ammons et al. (2006) introduced a temperature corresponding to a spectral type of about K2-3. All these rough spectral estimates were taken from Popper (1980) and Cox (2000).

\textsuperscript{1} http://ad.usno.navy.mil/wds/
2.2. Light curve

The star was observed by the SuperWASP (Pollacco et al. 2006) project and its complete light curve (hereafter LC) is available. However, we did not use these data for the LC analysis because these were not measured in any standard photometric filter. These data were only used to derive the minima times (see below). We observed the target at the Ondřejov observatory in the Czech Republic with the 65-cm telescope equipped with the CCD camera. For the light curve analysis, only the data from two nights in May 2011 were used (see Tables 8 and 9). The remaining observations were used for the minima time derivation and to analyze the period changes in the system (see below Sect. 2.3). The observations were obtained in standard B, V, and R filters according to the specification of Bessell (1990).

At first, the complete LC was analyzed using the program PHOEBE (Prša & Zwitter 2005), which is based on the Wilson-Devinney algorithm (WD, Wilson & Devinney 1971). The derived quantities are as follows: the secondary temperature $T_2$, the inclination $i$, the luminosities $L_i$, the gravity darkening coefficients $g_i$, the albedo coefficients $A_i$, and the synchronicity parameters $F_i$. The limb darkening was approximated using a linear law, and the values of $x_i$ were interpolated from the van Hamme’s tables, given in van Hamme (1993).

At the beginning of the fitting process, we fixed the temperature of the primary component at $T_1 = 4700$ K (corresponding to spectral type K3, Cox 2000). In the absence of spectroscopy, the mass ratio was derived via a so-called “q-search method”. This means that we tried different values of mass ratio in the range 1.5–0.5 in steps of 0.1 and tried to find the best LC fit according to the lowest value of rms. Finally, we found that the best-fit solution was reached with the value $q = M_2/M_1 = 0.9$, which agrees with both eclipses having almost equal depths. For a given mass ratio, the semi-major axis was fixed to an appropriate value for the primary mass to be equal to a typical mass of a particular spectral type (e.g. Popper 1980; Harmanec 1988; or Andersen 1991). With this approach, we were able to estimate the masses, in addition to the radii of both components in absolute units.

However, during the LC fitting process we found that the LC of GK Boo is asymmetric. In particular, the part of the LC near the secondary minimum is distorted in all BVR filters. The brightness just after the ascent from the secondary minimum (near the phase 0.6) is higher than the brightness just before the descent (phase 0.4). The difference is about 0.022 mag in $B$, 0.018 mag in $V$, and 0.017 mag in $R$ filter, respectively.

With the PHOEBE code, we tried to fix the values of $A_i$ and $g_i$ to their appropriate values of 0.5 and 0.32, respectively. However, after then we also allowed these parameters to be fitted, because the fit is tighter (rms). However, probably owing to the asymmetry of the LC these quantities converged to the rather improbable values given in Table 1, and the shape of the observed LC could not be fitted properly. For the asymmetry of the curve, we also tried to introduce a star spot on either of the components. However, no acceptable solution with spot(s) was found to describe the shape of the light curve more accurately in the PHOEBE program. The parameters of the LC fit are given in Table 1, but these cannot sufficiently describe the shape of the LC.

We therefore tried a different code, called ROCHE, developed by Theo Pribulla (Pribulla 2004), which is also based on the WD code but has for instance also some other computing methods and different controlling of the calculation process. With this program, we used two star spots and similar input parameters as described above. At the beginning of the fitting, the values of $A_i$ and $g_i$ were fixed to the appropriate values of 0.5 and 0.32, respectively. However, to achieve a tighter fit both $A_i$ and $g_i$ values were also varied across the range from 0 to 1 in steps of 0.05 for both components. The synchronicity parameters $F_i$ converged to much more reliable values. The value of mass ratio was fixed to $q = 1.0$ and then also fitted as a free parameter. This was possible because there is a clear distortion of the LC outside the minima (see e.g. Terrell & Wilson 2005). For the fitting process, the two different limb darkening laws were also tried, namely a linear and logarithmic. The latter one provides a much tighter fit to our data. All of the resulting LC parameters are also given in Table 1 (together with parameters of two cooler spots located on the primary component – longitude, latitude, radius and temperature factor). As one can see, the two solutions clearly differ even outside their respective error bars for some of the parameters.

The individual errors in the parameters were not taken from the WD code, but derived in the following way. We computed a range of solutions for GK Boo, which were then used for its error estimation. All solutions with $\chi^2$ value close to the minimal one (5% from our final solution) were taken and the resultant values of parameters were used to compute the differences between the parameters. The errors in the individual parameters were then computed as a maximum difference and their individual WD errors, given by $\max(|a_i - \Delta a_i| + 0.1\, a_i + 0.1\Delta a_i)$.

This solution obtained with the ROCHE program provides a much closer fit to the observed data and is the fit plotted in Fig. 1. The value of the eccentricity was fixed at 0 (for a discussion about possible eccentricity see below). Our resultant parameters indicate that both the components are still located on

| Parameter | PHOEBE | ROCHE |
|-----------|--------|-------|
| $T_1$ [K] | 4700$^\dagger$ | 4700$^\dagger$ |
| $T_2$ [K] | 4540 ± 50 | 4615 ± 63 |
| $q (= M_2/M_1)$ | 0.9 ± 0.1 | 0.95 ± 0.12 |
| $e$ | 0$^\dagger$ | 0$^\dagger$ |
| $i$ [deg] | 89.83 ± 0.57 | 89.28 ± 0.37 |
| $g_1$ | 0.00 ± 0.04 | 0.35 ± 0.05 |
| $g_2$ | 0.00 ± 0.03 | 0.35 ± 0.05 |
| $A_1$ | 0.00 ± 0.08 | 0.80 ± 0.05 |
| $A_2$ | 1.00 ± 0.08 | 0.80 ± 0.05 |
| $F_1$ | 1.892 ± 0.107 | 1.131 ± 0.096 |
| $F_2$ | 1.866 ± 0.116 | 1.295 ± 0.108 |
| $L_1 (B) \%$ | 54.8 ± 1.9 | 52.4 ± 1.1 |
| $L_2 (B) \%$ | 45.2 ± 1.8 | 47.6 ± 1.0 |
| $L_1 (V) \%$ | 53.5 ± 1.5 | 51.6 ± 1.2 |
| $L_2 (V) \%$ | 46.5 ± 1.3 | 48.4 ± 1.1 |
| $L_1 (R) \%$ | 52.1 ± 1.4 | 51.1 ± 1.1 |
| $L_2 (R) \%$ | 47.9 ± 1.3 | 48.9 ± 1.0 |

| Spots: |
| $l_1$ [deg] | – | 287.2 ± 7.9 |
| $b_1$ [deg] | – | 60.5 ± 3.2 |
| $r_1$ [deg] | – | 37.9 ± 2.0 |
| $k_1$ | – | 0.75 ± 0.04 |
| $l_2$ [deg] | – | 63.3 ± 7.4 |
| $b_2$ [deg] | – | 47.4 ± 12.8 |
| $r_2$ [deg] | – | 28.7 ± 4.1 |
| $k_2$ | – | 0.76 ± 0.04 |

**Notes.**$^\dagger$ Fixed.

Table 1. Light curve parameters of GK Boo.

The individual errors in the parameters were not taken from the WD code, but derived in the following way. We computed a range of solutions for GK Boo, which were then used for its error estimation. All solutions with $\chi^2$ value close to the minimal one (5% from our final solution) were taken and the resultant values of parameters were used to compute the differences between the parameters. The errors in the individual parameters were then computed as a maximum difference and their individual WD errors, given by $\max(|a_i - \Delta a_i| + 0.1\, a_i + 0.1\Delta a_i)$.

This solution obtained with the ROCHE program provides a much closer fit to the observed data and is the fit plotted in Fig. 1. The value of the eccentricity was fixed at 0 (for a discussion about possible eccentricity see below). Our resultant parameters indicate that both the components are still located on
the main sequence, (as required because the age of the Universe does not allow low-mass stars to have evolved from the main sequence). If we follow the assumption of a K3V primary, then the secondary is also of K3V spectral type. These are consistent with the photometric indices presented above, as well as with the individual masses and radii for these types of stars (e.g. Harmanec 1988). An undetectable value of the third light was also resulted derived by this analysis. The presence of photospheric spots on both components of such a late spectral type star is also foreseeable.

2.3. Period analysis

To monitor the detailed long-term evolution of the system or its short-period modulation, we collected all available published minima observations. Photometry from the SWASP (Pollacco et al. 2006), ASAS (Pojmanski 2002), and PiOItTheSky (Burd et al. 2005) projects were used to derive many new minima times for GK Boo. All of these data are given in Table 8. The method of Kwee & van Woerden (1956) was used. Some of the data were of poor quality, but most were accurate enough to perform a detailed period analysis of the system. The range of these data is about 12 years.

We used these data to analyze the period modulation and found some interesting results. Applying the hypothesis of a third body in the system (the so-called Light-Time Effect, hereafter LITE, described e.g. by Irwin 1959), we found a weak period modulation with a period of about four years. The final fit of the modulation is clearly visible. Table 2 also provides the mass predicted mass. If we assume the parallax of GK Boo as given by Kharchenko (2001), π = 30.29 mas, we are also able to compute a more interesting finding is that of a period of about 4 years. Applying the LITE hypothesis, we obtained a final set of parameters given in Table 2, namely the period of the third body \( p_3 \), the semi-amplitude of the effect \( A \), the time of periastron passage \( T_0 \), the argument of periastron \( \omega \), and the eccentricity \( e \). Despite the low amplitude (about only 1.8 min) of the LITE, most of the observed minima times are of higher precision and the modulation is clearly visible. Table 2 also provides the mass function of the third body \( f(M_3) \), which helps us to estimate its predicted mass.

Having no information about the inclination between the orbits of the eclipsing pair and the hypothetical third body, we plotted Fig. 3, where a plot mass versus inclination is shown. Assuming the coplanar orbits (i.e. \( i_3 = 90^\circ \rightarrow M_3 = M_{3\min} \)), the resulted minimum mass of the third body is only about 0.116 \( M_\odot \), which places this body at the lower end of stellar masses, hence we can rule out the hypothesis of a brown dwarf or even an exoplanet. Despite of there being no upper limit to this mass (it goes to infinity with \( i_3 \rightarrow 0^\circ \)), we can estimate a lower limit to the mass. Taking into account that no third light is detected in the LC solution, e.g. \( L_3/(L_1 + L_2) < 0.01 \) and assuming a main-sequence star, we can estimate its mass to be lower than 0.22 \( M_\odot \), which is shown in Fig. 3 as a gray area. Further observations are still needed to confirm this hypothesis with higher conclusiveness.

If we assume the parallax of GK Boo as given by Kharchenko (2001), \( \pi = 30.29 \) mas, we are also able to compute

\[
\begin{array}{l|l}
\text{Parameter} & \text{Value} \\
\hline
HJD_0 & 2454305.4570 \pm 0.0006 \\
P \ [\text{day}] & 0.47777174 \pm 0.00000022 \\
p_1 \ [\text{day}] & 1472.7 \pm 170.0 \\
p_2 \ [\text{yr}] & 4.032 \pm 0.450 \\
A \ [\text{day}] & 0.0126 \pm 0.0012 \\
T_0 & 2454263.3 \pm 1108.3 \\
\omega \ [\text{deg}] & 56.54 \mp 15.0 \\
e & 0.084 \pm 0.267 \\
Q \ [\times 10^{-10}] & -1.071 \pm 0.206 \\
f(M_3) \ [M_\odot] & 0.000633 \pm 0.000002 \\
M_{3\min} \ [M_\odot] & 0.115 \pm 0.001 \\
M_{360} \ [M_\odot] & 0.134 \pm 0.002 \\
M_{330} \ [M_\odot] & 0.242 \pm 0.005 \\
\alpha_1 \sin i \ [\text{AU}] & 0.217 \pm 0.108 \\
\alpha_1 \ [\text{mas}] & 88.7 \pm 9.8 \\
\end{array}
\]
the angular distance of a hypothetical body to be about 89 mas. This separation of components is well above the limit for modern stellar interferometers. However, there is a problem with the brightness of the third component, which was found to be about more than five magnitudes fainter than the eclipsing pair itself. With the brightness of about 11 mag for the system, this makes a detection impossible. The magnitude difference of the third body with respect to the close pair also clarify why no third light was detected in the LC solution.

Another interesting result was a detection of displaced secondaries. This can be clearly seen in more precise data points (SWASP and our new observations). That secondary minima occur at a different phase of $\phi_2 \neq 0.5$ from the primary usually indicates that the system is on an eccentric orbit. GK Boo is a well-detached system, so the eccentric orbit cannot be ruled-out easily. Therefore, we assumed an apsidal motion hypothesis for our data set of minima times. We followed a procedure described by e.g. Giménez & García-Pelayo (1983) or Giménez & Bastero (1995) and obtained a set of apsidal motion parameters. The plot of residuals (after subtraction of the LITE fit) with the apsidal motion fit is shown in Fig. 4. It is obviously very slow because the position of secondaries versus primaries changes only very slowly. The resultant values of apsidal motion parameters are given in Table 3.

However, we have to rule out this hypothesis because it lead to unacceptable results. With some information about the physical parameters of both components, we can use the apsidal motion parameters to estimate the internal structure constant. The theoretical $\log k_2,\text{phot}$ value taken from Claret (2004) should range from $-1.35$ to $-1.65$. However, the mean value of $\log k_2$ of both components that can be derived from our solution is very different, even when $k_2 < 0$, which is unacceptable. Thus, the system is very probably on a circular orbit.

We may ask why the secondary minima deviate from the 0.5 phase. We published a finding that the displaced secondary minima can also be present in contact binaries where no eccentric orbit is possible (Zasche 2011), so one cannot perform an apsidal motion analysis based only on the minima times of a particular system. Some studies found that the secondary minimum is displaced because of the distortion of the LC, thus any standard routine for deriving the time of minimum (e.g. Kwee-van Woerden, bisector chord method or polynomial fitting) cannot be used properly because these consider symmetric minima only. When using these methods to determine minima where both ascending and descending branches have different slopes, we recover only a “false eccentricity”.

One can also ask about a significance of the fits presented in Figs. 2 and 4. For this comparison, we summarized different approaches in Table 4. In addition to the rms values, we also provide the values of BIC (Bayesian Information Criterion, see e.g. Liddle 2007), which show the significance of the fit. According to this method, the smaller the rms value, the tighter the fit. To conclude, our final fit provides the smallest rms, but its significance is low and still highly speculative. This is also caused by the poor data coverage, and large scatter in the minima and their low accuracy. Determinations of more precise minima are therefore needed to confirm or exclude this hypothesis.

3. AE For

3.1. Introduction

The Algol-type system AE For (=HIP 14568, $V_{\text{max}} = 10.22$ mag) is also a poorly studied binary. Its published spectral types range from K4 to M0, with the most probable one being K7V as derived by (Torres et al. 2006). The system was presented as a wide double with the star HD 19632 based on their similar parallaxes and proper motions (see Poveda et al. 1994).

Neither the light curve nor the radial velocity curve of AE For have been studied. The star was observed by the Hipparcos satellite and a few times also for the minima observations. It was also continuously monitored with automatic photometric systems such as PiOfTheSky and ASAS. However, the quality of these data do not allow us to use them for a LC analysis. The distance to the system was derived from the Hipparcos data to be $d = 31.5$ pc.

3.2. Light curve

We observed the star from the South African Astronomical Observatory (SAAO) in 2010, using the classical one-channel photoelectric photometer mounted on the 50-cm telescope. All
measurements were carefully reduced to the Cousins E-region standard system (Menzies et al. 1989) and corrected for differential extinction. Thanks to its orbital period close to one day, its complete light curve was observed once in standard BVR filters, with some overlapping points (about 170 data points in each filter were obtained). Unfortunately, the quality of the data acquired for several nights was not very good, hence the scatter in the curve is affected by these conditions. Two secondary and one primary minima were observed (see below).

We analyzed our data using the same computational procedure as for GK Boo. The primary temperature was fixed to the appropriate value of 4100 K (sp K7V), the eccentricity was fixed to 0, the values of gravity darkening coefficients were fixed at 0.32, and the albedo coefficients to 0.5 (as recommended for stars with convective envelopes), while the limb darkening coefficients were interpolated from values given in van Hamme (1993). The computational approach was different for the mass ratio $q$, which was fixed to $q = 1.0$ because of the weak outside-eclipse ellipsoidal variations and its detached configuration. In addition, the synchronicity parameters $F_j$ were set to values of 1.0 for both components. The program roche was used and the resulting LC parameters are given in Table 5, while the final solution is presented in Fig. 5.

One can see that the secondary temperature $T_2$ is close to the value of $T_1$, indicating that the components are similar. Thus, the estimated spectral types of both stars are probably K7V + K7V. Both components are still located on the main sequence and their properties are in agreement with the typical values of K7V stars (as presented by e.g. Harmanec 1988). The third light was also not detected here in any filter. In contrast to GK Boo, the LC of AE For seems to be symmetric.

### 3.3. Period analysis

Similarly to GK Boo, we tried to perform the period analysis of all available minima. The collection of minima is much smaller, but thanks to the first observation by Hipparcos (Perryman et al. 1997) these cover a longer time span than for GK Boo. Several new minima were derived based on our new observations from SAAO as well as those from the ASAS and PiOfTheSky surveys. The same hypothesis as for GK Boo was applied to the data points here. All of the minima times used for the analysis are summarized in Table 9. As one can see from Fig. 6, there is a clear variation in the minima times. We used the same third-body hypothesis (LITE) as for GK Boo, deriving a final fit to the data given by the parameters in Table 6. The LITE hypothesis resulted in a rather eccentric orbit, although the result is affected by a relatively large error, hence maybe the $e_1$ value should be lower. Only additional observations would help us confirm or refute this hypothesis, refine the period, and possibly detect some long-term evolution of the period similar to that in GK Boo, because the first observation from Hipparcos deviates significantly from the fit. With the same procedure as for GK Boo, we computed the significance of the fits according to the BIC criterion (see Table

### Table 5. Light curve parameters of AE For.

| Parameter | Value |
|-----------|-------|
| $T_1$ [K] | 4100$^\circ$ |
| $T_2$ [K] | 4065 ± 48 |
| $q (=M_2/M_1)$ | 1.0$^\circ$ |
| $e$ | 0$^\circ$ |
| $i$ [deg] | 86.51 ± 0.31 |
| $g_1 = g_2$ | 0.32$^\circ$ |
| $A_1 = A_2$ | 0.50$^\circ$ |
| $F_1 = F_2$ | 1.000$^\circ$ |
| $L_1$ (B) [%] | 63.2 ± 1.3 |
| $L_2$ (B) [%] | 36.8 ± 1.0 |
| $L_1$ (V) [%] | 63.1 ± 1.2 |
| $L_2$ (V) [%] | 36.9 ± 1.0 |
| $L_1$ (R) [%] | 62.6 ± 1.4 |
| $L_2$ (R) [%] | 37.4 ± 1.0 |

### Notes.

$^\circ$ Fixed.

### Fig. 5. Light curve in BVR filters for AE For, the solid line represents the final fit. The curves are shifted along the y-axis for greater clarity.

### Fig. 6. O–C diagram of AE For. up: with linear ephemeris. Bottom: with respect to the phase. The data points are fitted with the curve representing the third body hypothesis (see the text for details).
stars of very similar types (K3 interesting and similar features. Since both of them are low-mass GK Boo and AE For, which we have found to have several in-
analyses of the poorly studied Algol-type eclipsing binaries
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4. Discussion and conclusions

We have derived preliminary light-curve solutions and period
analyses of the poorly studied Algol-type eclipsing binaries
GK Boo and AE For, which we have found to have several in-
teresting and similar features. Since both of them are low-mass
stars of very similar types (K3+K3 for GK Boo, and K7+K7 for
AE For), both of them have short orbital periods. Moreover, both
are relatively close to the Sun and also appear to contain third
bodies in their systems, which cause a periodic modulation of
the orbital periods of both systems. Assuming a coplanar orbit,
for AE For this third body appears to be a brown dwarf, which
makes this system even more interesting. However, more photo-
metric and spectroscopic observations are needed to confirm or
refute this hypothesis.

The system GK Boo has an asymmetric light curve, which is
the probably accounts for the shift in the secondary minimum
in phase with the primary one. The apsidal motion hypothesis
cannot explain this discrepancy.

In general, if the third body hypothesis as proposed based
on the period analysis is found to be the correct one, here we
have considered quite curious examples of hierarchical quadru-
ple systems of low masses. As far as we know, there are only
a few similar multiple late-type systems for which one of the
components is an eclipsing binary (e.g. BB Scl or MR Del).

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Data System Bibliographic Services.

Table 6. Final parameters of the long orbit for AE For.

| Parameter                  | Value                        |
|----------------------------|------------------------------|
| HJD0                      | 2452605.97070 ± 0.00035      |
| P  [day]                  | 0.91820943 ± 0.00000012      |
| P1  [day]                 | 252.46 ± 149.6              |
| P2  [yr]                  | 6.912 ± 0.409               |
| A  [day]                  | 0.00083 ± 0.00032           |
| T0                         | 2453548.8 ± 413.1           |
| ø1  [deg]                 | 146.2 ± 57.8               |
| e1                         | 0.601 ± 0.414              |
| f(M3) [M_⊙]               | 0.000098 ± 0.0000001        |
| M3sin{i} [M_⊙]            | 0.047 ± 0.001              |
| M3,F20 [M_⊙]              | 0.055 ± 0.001              |
| M3,30 [M_⊙]               | 0.098 ± 0.003              |
| a12 sin{i} [AU]           | 0.167 ± 0.064              |
| a3 [mas]                  | 117.2 ± 8.3                |

7). As one can see, the fit is still very poor and highly specula-
tive. However, using only the linear ephemeris, there remains a
clear quasi-sinusoidal variation, which needs some physical ex-
planation.

From the LITE parameters, we were able to calculate the
minimal mass of the third body (i.e. coplanar orbits), which
we found to be only about 47 M_jup, which is even lower than
the limit of stellar masses. Therefore, if the orbits were copla-
nar (which only would be our assumption, because the pro-
cess of tidal coplanarization is very slow), the third body would
very probably be a brown dwarf (exoplanets have masses about
one half of this value). With such a body, we reach minimal
masses that can be detected by this method, because the ampli-
tude of LITE is comparable to the typical precision of individual
minima-time measurements. Whatever applies to the possible in-
terferometric detection of GK Boo companion also applies here,
because its luminosity is too low.

Table 7. Methods of minima fitting for AE For.

| Method of minima fitting | rms       | BIC       |
|-------------------------|-----------|-----------|
| Linear ephemeris        | 0.000235  | 24.4      |
| LITE and linear ephemeris| 0.00163   | 44.8      |

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Data System Bibliographic Services.
| HJD - 2 400 000 | Error | Type  | Filter | Reference |
|-----------------|-------|-------|--------|-----------|
| 54161.64928     | 0.0028 | prim  | SWASP  |           |
| 54160.69347     | 0.0018 | prim  | SWASP  |           |
| 54159.73799     | 0.0019 | prim  | SWASP  |           |
| 54156.63161     | 0.0053 | sec   | SWASP  |           |
| 54154.71989     | 0.0026 | sec   | SWASP  |           |
| 54153.76487     | 0.0019 | sec   | SWASP  |           |
| 54150.65998     | 0.0028 | prim  | SWASP  |           |
| 54149.70458     | 0.0021 | prim  | SWASP  |           |
| 54140.62657     | 0.0028 | prim  | SWASP  |           |
| 54131.50553     | 0.0017 | prim  | SWASP  |           |
| 54130.46171     | 0.0013 | prim  | SWASP  |           |
| 54132.41842     | 0.0017 | prim  | SWASP  |           |
| 54129.46492     | 0.0008 | sec   | SWASP  |           |
| 54126.42177     | 0.0031 | sec   | SWASP  |           |
| 54125.52507     | 0.0022 | sec   | SWASP  |           |
| 54118.48289     | 0.0047 | prim  | SWASP  |           |
| 54118.50426     | 0.0022 | sec   | SWASP  |           |
| 54113.49833     | 0.0017 | sec   | SWASP  |           |
| 54112.45376     | 0.0018 | sec   | SWASP  |           |
| 54111.41099     | 0.0076 | sec   | SWASP  |           |
| 54109.51287     | 0.0080 | sec   | SWASP  |           |
| 54109.46378     | 0.0031 | sec   | SWASP  |           |
| 54109.44381     | 0.0026 | sec   | SWASP  |           |
| 54108.95805     | 0.0005 | sec   | SWASP  |           |
| 54107.50383     | 0.0000 | sec   | SWASP  |           |
| 54106.46046     | 0.0001 | sec   | SWASP  |           |
| 54105.41623     | 0.0006 | sec   | SWASP  |           |
| 54103.47632     | 0.0029 | sec   | SWASP  |           |
| 54102.45798     | 0.0047 | sec   | SWASP  |           |
| 54102.42110     | 0.0040 | sec   | SWASP  |           |
| 54102.43588     | 0.0083 | prim  | SWASP  |           |
| 54101.39917     | 0.0039 | sec   | SWASP  |           |
| 54100.40796     | 0.0058 | sec   | SWASP  |           |
| 54100.42630     | 0.0045 | prim  | SWASP  |           |
| 54099.38275     | 0.0032 | prim  | SWASP  |           |
| 54097.39732     | 0.0033 | sec   | SWASP  |           |
| 54097.37030     | 0.0082 | prim  | SWASP  |           |
| 54096.38171     | 0.0000 | sec   | SWASP  |           |
| 54095.86367     | 0.0078 | sec   | SWASP  |           |
| 54087.68574     | 0.0005 | prim  | SWASP  |           |
| 54082.59740     | 0.0008 | prim  | SWASP  |           |
| 54080.52201     | 0.0000 | prim  | SWASP  |           |
| 54081.50669     | 0.0001 | prim  | SWASP  |           |
| 54078.46270     | 0.0015 | prim  | SWASP  |           |
| 54078.70138     | 0.0024 | prim  | SWASP  |           |
| 54073.65681     | 0.0047 | sec   | SWASP  |           |
| 54073.47683     | 0.0029 | sec   | SWASP  |           |
| 54071.57005     | 0.0105 | prim  | SWASP  |           |
| 54068.52690     | 0.0049 | prim  | SWASP  |           |
| 54053.48213     | 0.0027 | prim  | SWASP  |           |
| 54044.43199     | 0.0032 | prim  | SWASP  |           |
| 54035.39912     | 0.0000 | prim  | SWASP  |           |
| 54035.63171     | 0.0037 | sec   | SWASP  |           |
| 54036.56985     | 0.0051 | sec   | SWASP  |           |
| 54037.40780     | 0.0024 | prim  | SWASP  |           |
| 54030.50285     | 0.0008 | sec   | SWASP  |           |
| 54010.62657     | 0.0022 | prim  | SWASP  |           |
| 54009.70485     | 0.0020 | prim  | SWASP  |           |
| 54015.78056     | 0.0012 | sec   | SWASP  |           |
| 54013.74876     | 0.0019 | sec   | SWASP  |           |
| 54014.71989     | 0.0026 | sec   | SWASP  |           |
| 54015.675781    | 0.0061 | sec   | SWASP  |           |
| 54016.58616     | 0.0053 | sec   | SWASP  |           |
| 54017.58667     | 0.0065 | sec   | SWASP  |           |
| 54015.73999     | 0.0009 | sec   | SWASP  |           |
| 54016.69347     | 0.0018 | prim  | SWASP  |           |
| 54161.64922     | 0.0028 | prim  | SWASP  |           |

Table 8. Heliocentric minima of GK Boo used for the analysis. Continued.
Table 8. continued.

| HJD - 2 400 000 | Error   | Type | Filter | Reference |
|-----------------|---------|------|--------|-----------|
| 55364.4367      | 0.0001  | sec  | BVR    | IBVS 5963 |
| 55385.45891     | 0.0013  | sec  | BVR    | PS       |
| 55386.41381     | 0.0014  | sec  | BVR    | PS       |
| 55391.43075     | 0.00097 | prim | BVR    | PS       |
| 55392.38658     | 0.00113 | prim | BVR    | PS       |
| 55590.66335     | 0.00036 | prim | -      | RU       |
| 55599.50146     | 0.00018 | sec  | -      | RU       |
| 55616.46275     | 0.00005 | prim | B      | PZ       |
| 55616.46266     | 0.00012 | prim | R      | RU       |
| 55619.56796     | 0.00010 | sec  | R      | RU       |
| 55634.61898     | 0.00016 | prim | R      | RU       |
| 55640.58947     | 0.00019 | sec  | BVR    | PS       |
| 55644.41169     | 0.00005 | sec  | B      | PZ       |
| 55650.62303     | 0.00014 | sec  | R      | RU       |
| 55651.34031     | 0.00113 | prim | B      | PZ       |
| 55662.56706     | 0.00013 | sec  | R      | RU       |
| 55671.40664     | 0.00007 | prim | VR     | PZ       |
| 55685.50021     | 0.00012 | sec  | R      | RU       |
| 55687.41150     | 0.00011 | sec  | BVR    | PZ       |
| 55692.42853     | 0.00004 | prim | BVR    | PZ       |
| 55700.55071     | 0.00004 | prim | BVR    | PZ       |
| 55707.47805     | 0.00008 | sec  | BVR    | PZ       |

Table 9. Heliocentric minima of AE For used for the analysis.

| HJD - 2 400 000 | Error   | Type | Filter | Reference |
|-----------------|---------|------|--------|-----------|
| 48500.6591      | 0.001   | prim | Hp     | Hipparcos |
| 51180.9089      | 0.0002  | prim | R      | VSOLJ 37  |
| 51180.9090      | 0.0001  | prim | R      | VSOLJ 47  |
| 51191.0099      | 0.0002  | prim | R      | VSOLJ 37  |
| 51191.0100      | 0.0002  | prim | R      | VSOLJ 47  |
| 51191.9279      | 0.0001  | prim | V      | VSOLJ 37  |
| 51191.9280      | 0.0001  | prim | V      | VSOLJ 47  |
| 51196.9770      | 0.0001  | sec  | R      | VSOLJ 37  |
| 51504.11866     | 0.0002  | prim | I      | VSOLJ 37  |
| 51504.1190      | 0.0002  | prim | I      | VSOLJ 47  |
| 52065.14178     | 0.00121 | prim | V      | ASAS      |
| 52065.60355     | 0.0025  | sec  | V      | ASAS      |
| 52235.0410      | 0.0002  | prim | I      | VSOLJ 39  |
| 52240.9825      | 0.0002  | sec  | I      | VSOLJ 39  |
| 52258.8860      | 0.0002  | prim | R      | VSOLJ 39  |
| 52605.9710      | 0.00077 | prim | V      | ASAS      |
| 52618.07823     | 0.00165 | sec  | V      | ASAS      |
| 52901.1754      | 0.00005 | sec  | I      | VSOLJ 42  |
| 52929.1801      | 0.00005 | sec  | I      | VSOLJ 42  |
| 52936.0674      | 0.00005 | sec  | I      | VSOLJ 42  |
| 52957.1865      | 0.00005 | sec  | V      | VSOLJ 42  |
| 52970.0410      | 0.00005 | sec  | I      | VSOLJ 42  |
| 52976.0097      | 0.00005 | prim | I      | VSOLJ 42  |
| 52987.0267      | 0.00005 | prim | V      | VSOLJ 42  |
| 52987.9466      | 0.00005 | prim | I      | VSOLJ 42  |
| 53300.1379      | 0.00005 | prim | V      | VSOLJ 43  |
| 53333.0309      | 0.00005 | prim | V      | VSOLJ 43  |
| 53340.0796      | 0.00005 | sec  | V      | VSOLJ 43  |
| 53705.0677      | 0.0002  | prim | V      | VSOLJ 44  |
| 53728.02219     | 0.00093 | prim | V      | ASAS      |
| 53728.48316     | 0.00194 | sec  | V      | ASAS      |
| 54030.75501     | 0.00027 | sec  | -      | PiOTheSky |
| 54040.67301     | 0.00045 | sec  | -      | PiOTheSky |
| 54047.1000      | 0.0001  | sec  | V      | VSOLJ 45  |
| 54052.68040     | 0.00036 | prim | -      | PiOTheSky |
| 54448.61542     | 0.00131 | sec  | V      | ASAS      |
| 54726.57421     | 0.00036 | sec  | V      | ASAS      |
| 54862.9297      | 0.00005 | prim | V      | VSOLJ 50  |
| 55559.0139      | 0.00005 | prim | V      | VSOLJ 51  |
| 55570.41065     | 0.00072 | sec  | BVR    | PZ - SAAO |
| 55571.32812     | 0.00015 | sec  | BVR    | PZ - SAAO |
| 55576.37847     | 0.00005 | prim | BVR    | PZ - SAAO |