Partitioned coupling framework to combine a kinematic hardening plasticity model and a creep model for structures in a high-temperature environment

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Abstract
The present study proposes a partitioned coupling framework for the implementation of a combined kinematic hardening plasticity and creep model. In the present framework, plastic and creep implicit stress integrations are performed alternately and iteratively until convergence in a partitioned coupling manner. This approach enables us to tractably implement a constitutive model that combines a plasticity model and a creep model in the program. The present framework was applied to the combination of the Ohno–Wang plasticity model and the Norton–Bailey creep model for numerical examples. The numerical examples demonstrated the capability of the present framework in analyzing realistic structures at high temperature with the consideration of both plasticity and creep.

KEYWORDS
creep, cyclic plasticity, high temperature, nonlinear finite element method, structural analysis

1 | INTRODUCTION

As technology advances, various structures, such as gasifiers, rocket nozzles, and internal combustion engines, are operated at very high temperatures for high energy conversion. As a result, materials are being adversely affected by severe high-temperature environments. Furthermore, such structural components are generally operated in an alternating manner, that is, between in and out of service, which may cause ratcheting. In this situation, materials would exhibit creep behavior as well as plastic behavior. In particular, cyclic plasticity with a kinematic hardening law should be considered. For example, Ohno et al.1 and Kobayashi et al.2 analyzed the ratcheting characteristics of the 316F steel at high
temperature using the Ohno–Wang kinematic hardening model. Abdel-Karim and Ohno proposed a kinematic hardening model to model the steady state in ratcheting of the modified 9Cr-1Mo steel at 550°C and IN738LC alloy at 850°C. This kinematic hardening model is in the framework of strain hardening and dynamic recovery, which is similar to the Ohno–Wang model. Ohguchi et al. and Sasaki and Ohguchi analyzed soldering problems by a combined model of the Prager kinematic hardening model and the Norton creep model. These studies also provided experimental results for solder alloys that exhibited both plastic and creep behaviors. Kobayashi et al. and Akamatsu et al. also analyzed soldering problems by a nonunified constitutive model, in which both transient and steady state strain rates were considered. Using this model, Nishimoto et al. simulated the combustion chamber of a liquid rocket engine in a high-temperature environment. Chen et al. added the consideration of viscoplastcity and creep in the model of Kobayashi et al. to model a nickel-based super alloy and 40Sn-60Pb solder at high temperature. Panteghini and Genna used the model of Kobayashi et al. in conjunction with the Garofalo hyperbolic-sine creep law and the viscoplasticity model of Perzyna to calculate the residual stress of wire drawing processes. Hosseini et al. analyzed a compact tension specimen of the 1%CrMoV steel at 550°C using five different creep models, including the Norton, Norton–Bailey, Bartsch, and hyperbolic-sine models. The studies mentioned in this paragraph indicate that a combined plasticity–creep model is necessary in some situations rather than a single plasticity or creep model.

The most classical and simplest kinematic hardening model would be the Prager model, in which the back stress rate is proportional to the plastic strain rate. Frederick and Armstrong added a dynamic recovery term to the evolution equation of the Prager model. Then, Chaboche introduced the approach of multiple back stresses by superposing the Armstrong–Frederick model. After that, Ohno and Wang proposed an elastic–plastic constitutive model with a kinematic hardening model in which strain hardening and dynamic recovery with a critical state are considered. The Ohno–Wang model has been used for high-temperature mechanical problems, such as ratcheting at high temperature, soldering, a liquid rocket engine, and wire drawing.

Creep is frequently modeled by an evolution equation of the equivalent creep strain. The Norton creep model, which is also known as the power law creep model, is commonly used to represent secondary creep. The Norton creep model is a stress hardening model in which the equivalent creep strain rate is proportional to the equivalent stress to the power of a material parameter. Furthermore, for primary creep, the Bailey creep model, which is a time hardening model, is popular. Therefore, the Norton–Bailey model is appropriate for modeling both the primary and secondary creep. In the Norton–Bailey model, the equivalent creep strain rate is proportional to the equivalent stress to the power of a parameter multiplied by the time to the power of another parameter. The Norton or Norton–Bailey model has been applied to high-temperature problems such as soldering, a nickel-based super alloy and 40Sn-60Pb solder at high temperature, and 1%CrMoV steel at 550°C. Furthermore, for detailed modeling of transitions of mechanism depending on temperature and stress, a creep deformation mechanism map, such as the Ashby map, is considered. For creep crack growth, a creep damage model, such as the Nikbin–Smith–Webster (NSW) model, is used.

For the combination of plasticity and creep, several constitutive models have been proposed. These models can be categorized into unified and nonunified models. In unified models, plastic and creep strains are considered to arise from a single physical mechanism. However, unified models have difficulty in determining the material parameters. In contrast, plastic and creep material parameters can be measured independently in nonunified models, where the plastic and creep strains are assumed to arise from two independent constitutive laws. Nonunified models have been applied to problems at high temperature, such as solder joints, a liquid rocket engine, and wire drawing.

In the implementation of a nonunified model, a nonlinear system of equations consisting of plastic and creep stress integrations is solved monolithically for each element integration point. This procedure requires an algorithm and a program that are specific to the combined plasticity–creep model used in the analysis.

To overcome this issue, the present study proposes a framework for the implementation of nonunified models. The present framework is based on a partitioned coupling technique, which can be seen in many fluid–structure interaction analyses. Two existing plastic and creep models are combined in the framework of coupling iteration. This approach enables us to tractably reuse existing programs of a single plasticity model and a single creep model. The present framework is the generalization of the authors’ previous study presented at a conference proceeding. In the present study, numerical examples using a combined Ohno–Wang and Norton–Bailey model are presented. These numerical examples demonstrate the capability of the proposed partitioned coupling technique.

In the present article, a general description of the constitutive equations of the plasticity model and the creep model is first introduced. Then, we present partitioned stress integration algorithms and their consistent tangent moduli of the
single plasticity model, the single creep model, and the combined plasticity–creep model. After that, the algorithm and the implementation of the partitioned coupling framework with the combined plasticity–creep model are explained. In particular, the advantage of the partitioned coupling framework in separate implementation of the plasticity and creep models is emphasized. Using the partitioned coupling framework, we introduce specific equations of a combined plasticity–creep model, namely, the combined Ohno–Wang and Norton–Bailey model. Finally, the capability of the proposed partitioned coupling technique with the combined Ohno–Wang and Norton–Bailey model is demonstrated by numerical examples.

2 | CONSTITUTIVE MODELS

In this section, constitutive models, such as a general kinematic hardening plasticity model, a general creep model, and a combined plasticity–creep model, are introduced. First, vector representation of stress and strain tensors is explained, followed by operators to describe constitutive models. Then, a single kinematic hardening plasticity model and a single creep model are presented. Finally, a combined kinematic hardening plasticity and creep model is explained. Although a number of assumptions, such as the von Mises criterion and the associated flow rule, are introduced in the present study, the evolution equations of the back stresses and the equivalent creep strain are described in general forms.

2.1 | Vector representation and operators

In the present study, vector representation of stress and strain tensors is used to describe constitutive models. A stress, such as the stress, \( \sigma \), the deviatoric stress, \( s \), the back stress, \( \beta \), or the relative stress, \( \sigma^{\text{kin}} \), is represented by a vector having six components, that is,

\[
\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{yy} & \sigma_{zz} & \sigma_{xy} & \sigma_{yz} & \sigma_{zx} \end{bmatrix}^T. \tag{1}
\]

In this representation, symmetry is assumed, for example, \( \sigma_{xy} = \sigma_{yx} \). A strain such as the strain, \( \varepsilon \), the elastic strain, \( \varepsilon^e \), the plastic strain, \( \varepsilon^p \), or the creep strain, \( \varepsilon^{cr} \), is represented by

\[
\varepsilon = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{yy} & \varepsilon_{zz} & 2\varepsilon_{xy} & 2\varepsilon_{yz} & 2\varepsilon_{zx} \end{bmatrix}^T. \tag{2}
\]

Shear components of a strain vector are doubled for convenience.

In order to double the shear components of a stress vector, a linear operator,

\[
Q = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 \\
\end{bmatrix}, \tag{3}
\]

is introduced. Using \( Q \), a norm operator,

\[
\| \sigma \| = \sqrt{\sigma : Q \sigma}
\]

\[
= \sqrt{\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2 + 2 \left( \sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{zx}^2 \right)}
\]

\[
= \sqrt{\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2 + \sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{zx}^2 + \sigma_{yx}^2 + \sigma_{zy}^2 + \sigma_{xz}^2 + \sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{zx}^2}, \tag{4}
\]

is defined.
A volumetric operator, $T_{\text{vol}}$, and a deviatoric operator, $T_{\text{dev}}$, are introduced:

$$T_{\text{vol}} = \frac{1}{3} \begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}, \quad T_{\text{dev}} = \frac{1}{3} \begin{bmatrix}
2 & -1 & -1 & 0 & 0 & 0 \\
-1 & 2 & -1 & 0 & 0 & 0 \\
-1 & -1 & 2 & 0 & 0 & 0 \\
0 & 0 & 0 & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & 0 & 0 & 3
\end{bmatrix}. \quad (5)$$

Note that $T_{\text{vol}} + T_{\text{dev}} = I$.

### 2.2 General kinematic hardening plasticity model

First, the strain, $\varepsilon$, is introduced. The strain is decomposed additively into elastic and plastic parts, that is,

$$\varepsilon = \varepsilon^e + \varepsilon^p. \quad (6)$$

Here, $\varepsilon^e$ and $\varepsilon^p$ denote elastic and plastic strains, respectively. Then, the constitutive equation of a plasticity model is

$$\sigma = D^e \varepsilon^e = D^e (\varepsilon - \varepsilon^p), \quad (7)$$

where $\sigma$ denotes the stress. Here, $D^e$ is the elasticity matrix, which can be represented by

$$D^e = 3KT_{\text{vol}} + 2GQ^{-1}T_{\text{dev}}, \quad (8)$$

where $K$ and $G$ are the bulk modulus and the shear modulus, respectively.

In the present study, the plastic strain, $\varepsilon^p$, is assumed to evolve by the associated flow rule, as

$$\dot{\varepsilon}^p = \dot{\lambda} \frac{\partial g}{\partial \sigma^{\text{kin}}}, \quad (9)$$

where $\dot{\lambda}$ is the plastic multiplier, and $\sigma^{\text{kin}}$ denotes the relative stress, which is the stress, $\sigma$, minus the back stress, $\beta$, that is,

$$\sigma^{\text{kin}} = \sigma - \beta. \quad (10)$$

In addition, $g$ is the yield function, for which the von Mises yield criterion,

$$g = \sigma^{\text{kin}} - \sigma_Y = 0, \quad (11)$$

is adopted. Here, $\sigma^{\text{kin}}$ is the von Mises equivalent stress with respect to the deviatoric part of the relative stress, $\sigma^{\text{kin}}$, that is,

$$\sigma^{\text{kin}} = \sqrt{\frac{3}{2} \| \mathbf{s} - \beta \|} = \sqrt{\frac{3}{2} (\mathbf{s} - \beta) \cdot Q (\mathbf{s} - \beta)}, \quad (12)$$

where $\mathbf{s}$ is the deviatoric part of the stress, that is, $\mathbf{s} = T_{\text{dev}} \sigma$, and $\beta$ is the back stress. In Equation (11), $\sigma_Y$ is the yield stress, which is a function of the equivalent plastic strain, $\bar{\varepsilon}^p$, that is,

$$\sigma_Y = \sigma_Y (\bar{\varepsilon}^p). \quad (13)$$

Here, the equivalent plastic strain, $\bar{\varepsilon}^p$, is defined as

$$\bar{\varepsilon}^p = \int_0^t \dot{\varepsilon}^p \, dt, \quad (14)$$
where
\[ \dot{\varepsilon}^p = \sqrt{\frac{2}{3}} \dot{\varepsilon}^p \cdot Q^{-1} \varepsilon^p. \]  
(15)

Using Equations (9), (11), and (15), we have
\[ \dot{\varepsilon}^p = \frac{3}{2} \dot{\varepsilon}^p \frac{s - \beta}{\sigma_{\text{kin}}}. \]  
(16)

Based on the generalized Armstrong–Frederick model by Chaboche,\textsuperscript{17} the back stress, \( \beta \), can be decomposed into \( M \) parts, that is,
\[ \beta = \sum_{i=1}^{M} \beta_i. \]  
(17)

For the evolution of each part of the back stress, \( \beta_i \), the contribution of the plastic strain rate, \( \dot{\varepsilon}^p \), and that of the back stress component, \( \beta_i \), are considered:
\[ \dot{\beta}_i = \frac{2}{3} h_i Q^{-1} \dot{\varepsilon}^p + \dot{\gamma}_i \beta_i, \]  
(18)

where \( h_i \) is a material parameter, and \( \dot{\gamma}_i \) is a rate-type scalar variable, the expression of which is determined by the selection of a kinematic hardening model. The term \( \dot{\gamma}_i \) may depend on both material parameters and state variables. A nonlinear hardening can be represented by replacing the constant, \( h_i \), with a function of \( \varepsilon^p \).

### 2.3 General creep model

Similar to the previous subsection, the strain, \( \varepsilon \), is decomposed additively into elastic and creep parts, that is,
\[ \varepsilon = \varepsilon^e + \varepsilon^{\text{cr}}, \]  
(19)

where \( \varepsilon^e \) and \( \varepsilon^{\text{cr}} \) denote elastic and creep strains, respectively. Then, the constitutive equation of a creep model is
\[ \sigma = D^e \varepsilon^e = D^e (\varepsilon - \varepsilon^{\text{cr}}), \]  
(20)

where \( \sigma \) and \( D^e \) denote the stress and the elasticity matrix, respectively.

The creep strain, \( \varepsilon^{\text{cr}} \), is assumed to evolve in a similar manner to the plastic strain (Equation 16), as
\[ \dot{\varepsilon}^{\text{cr}} = \frac{3}{2} \dot{\varepsilon}^{\text{cr}} \frac{s}{\bar{\sigma}}, \]  
(21)

where the von Mises equivalent stress, \( \bar{\sigma} \), and the equivalent creep strain rate, \( \dot{\varepsilon}^{\text{cr}} \), are
\[ \bar{\sigma} = \sqrt{\frac{3}{2} ||s||} = \sqrt{\frac{3}{2} s \cdot Q s}, \]  
(22)
\[ \dot{\varepsilon}^{\text{cr}} = \sqrt{\frac{2}{3} \varepsilon^{\text{cr}} \cdot Q^{-1} \varepsilon^{\text{cr}}}. \]  
(23)

Note that the creep strain evolves in the direction of the deviatoric stress, \( s \), whereas the plastic strain evolves in the direction of the relative deviatoric stress, \( s - \beta \). Then, the equivalent creep strain rate, \( \dot{\varepsilon}^{\text{cr}} \), in Equation (21) is described in a general form, as
\[ \dot{\varepsilon}^{\text{cr}} = \dot{\varepsilon}^{\text{cr}} \left( \bar{\sigma}, t \right). \]  
(24)
2.4 Combined kinematic hardening plasticity and creep model

Similar to Sections 2.2 and 2.3, the strain, $\varepsilon$, is decomposed additively into elastic, plastic, and creep parts, that is,

$$\varepsilon = \varepsilon^e + \varepsilon^p + \varepsilon^c, \quad \tag{25}$$

where $\varepsilon^e$, $\varepsilon^p$, and $\varepsilon^c$ denote elastic, plastic, and creep strains, respectively. Then, the constitutive equation is

$$\sigma = D^e \varepsilon = D^e \left( \varepsilon - \varepsilon^p - \varepsilon^c \right), \quad \tag{26}$$

where $\sigma$ and $D^e$ denote the stress and the elasticity matrix, respectively. The evolution equations of the plastic strain, $\varepsilon^p$, and the creep strain, $\varepsilon^c$, are the same as the strains presented in Sections 2.2 and 2.3, respectively.

3 IMPLICIT STRESS INTEGRATION

In this section, stress integration procedures in the time interval from $t$ to $t + \Delta t$ are derived. First, stress integration of a single kinematic hardening plasticity model is presented, followed by that of a single creep model. Finally, partitioned stress integration of a combined kinematic hardening plasticity and creep model is explained.

3.1 Implicit stress integration of a single kinematic hardening plasticity model

The stress at $t + \Delta t$, $t^{+\Delta t} \sigma$, can be decomposed into volumetric and deviatoric parts:

$$t^{+\Delta t} \sigma = (T_{\text{vol}} + T_{\text{dev}})^{t^{+\Delta t} \sigma} = t^{+\Delta t} p + t^{+\Delta t} s. \quad \tag{27}$$

The inelastic deformation is assumed to be incompressible. In this case, the volumetric part (hydrostatic stress), $t^{+\Delta t} p$, is purely elastic, that is,

$$t^{+\Delta t} p = p + 3K T_{\text{vol}} \Delta \varepsilon, \quad \tag{28}$$

where $\Delta \varepsilon$ is the strain increment. On the other hand, the deviatoric part, $t^{+\Delta t} s$, is calculated simultaneously with the unknown plastic strain increment, $\Delta \varepsilon^p$, which is also related to $t^{+\Delta t} s$ and $t^{+\Delta t} \beta$. The backward Euler integration method combined with the elastic predictor and radial return method is used for the stress integration to calculate stress and strain increments. The deviatoric stress, $t^{+\Delta t} s$, can be calculated as

$$t^{+\Delta t} s = t^{+\Delta t} s^* - 2 G Q^{-1} \Delta \varepsilon^p, \quad \tag{29}$$

where $t^{+\Delta t} s^*$ is an elastic predictor that is calculated by

$$t^{+\Delta t} s^* = s + 2 G Q^{-1} T_{\text{dev}} \Delta \varepsilon. \quad \tag{30}$$

If $t^{+\Delta t} s^*$ is placed inside the yield surface ($g < 0$), then $t^{+\Delta t} s^*$ is in an elastic state and $t^{+\Delta t} s = t^{+\Delta t} s^* \Delta \varepsilon^p = 0$. If $t^{+\Delta t} s^*$ is outside the yield surface ($g \geq 0$), then

$$\Delta \varepsilon^p = \frac{3}{2} \Delta \varepsilon^p \frac{t^{+\Delta t} s - t^{+\Delta t} \beta}{t^{+\Delta t} \sigma^\text{kin}}. \quad \tag{31}$$

This equation was derived from Equation (16) using the backward Euler integration method. Here, $\Delta \varepsilon^p$ is the equivalent plastic increment. In Equation (31), $t^{+\Delta t} \beta$ and $t^{+\Delta t} \sigma^\text{kin}$ should be determined implicitly. Integrating Equation (18) yields the ith back stress increment:

$$\Delta \beta_i = t^{+\Delta t} \beta_i - t \beta_i = \frac{2}{3} h_i Q^{-1} \Delta \varepsilon^p + \Delta \gamma_i t^{+\Delta t} \beta_i. \quad \tag{32}$$
From this equation, $t^+\Delta t\beta$ in Equation (31) can be determined by

$$t^+\Delta t\beta = \sum_{i=1}^{M} t^+\Delta t\beta_i,$$

(33)

where

$$t^+\Delta t\beta_i = t^+\Delta t\beta_i^+,$$

(34)

$$t^+\Delta t\beta_i^+ = t\beta_i + \frac{2}{3} h_i Q^{-1} \Delta \varepsilon^p,$$

(35)

$$t^+\Delta t\theta_i = \frac{1}{1 - \Delta \gamma_i}.$$ 

(36)

For determining $\Delta \varepsilon^p$ in Equation (31), a scalar equation will be derived. From Equations (29), (31), and (33)–(35), we have

$$t^+\Delta t s - t^+\Delta t \beta = \sum_{i=1}^{M} t^+\Delta t\beta_i \left( t\beta_i + h_i Q^{-1} \frac{t^+\Delta t s - t^+\Delta t \beta}{t^+\Delta \sigma^{\text{kin}}} \right).$$

(37)

Then, moving $t^+\Delta t s - t^+\Delta t \beta$ to the left-hand side yields

$$\left( 1 + \left( 3G + \sum_{i=1}^{M} t^+\Delta t\theta_i h_i \right) \frac{\Delta \varepsilon^p}{t^+\Delta \sigma^{\text{kin}}} \right) (t^+\Delta t s - t^+\Delta t \beta) = \sum_{i=1}^{M} t^+\Delta t\beta_i t\beta_i.$$ 

(38)

Finally, Equation (38) is applied to Equation (12), resulting in

$$\sqrt{\frac{3}{2}} \left\| t^+\Delta t \beta_i \right\| - \Delta \varepsilon^{cr} - \left( 3G + \sum_{i=1}^{M} t^+\Delta t\theta_i h_i \right) \frac{\Delta \varepsilon^p}{t^+\Delta \sigma^{\text{kin}}} = 0.$$ 

(39)

The solution of this nonlinear scalar equation is $\Delta \varepsilon^p$.

### 3.2 Implicit stress integration of a single creep model

The stress can be decomposed into the hydrostatic and deviatoric stresses in the manner described in the previous subsection (Equation 27). Moreover, the hydrostatic stress follows Equation (28). The deviatoric stress, $t^+\Delta t s$, can be calculated by

$$t^+\Delta t s = t^+\Delta t s^e - 2GQ^{-1} \Delta \varepsilon^{cr},$$

(40)

where $t^+\Delta t s^e$ is an elastic predictor that is given by Equation (30), and $\Delta \varepsilon^{cr}$ is the creep strain increment, which is

$$\Delta \varepsilon^{cr} = \frac{3}{2} \Delta \varepsilon^{cr} \frac{t^+\Delta t s}{t^+\Delta \sigma}.$$ 

(41)

Here, $\Delta \varepsilon^{cr}$ is the equivalent creep strain increment, which should be determined implicitly. Equation (41) was derived from Equation (21) using the backward Euler integration method.

For determining $\Delta \varepsilon^p$ in Equation (41), a scalar equation will be derived. From Equations (40) and (41), we have

$$t^+\Delta t s = t^+\Delta t s^e - 3G \frac{\Delta \varepsilon^{cr} t^+\Delta t s}{t^+\Delta \sigma}.$$ 

(42)
Then, moving \( t + \Delta t \mathbf{s} \) to the left-hand side yields

\[
\left( 1 + 3G \frac{\Delta \varepsilon^{cr}}{t + \Delta t \sigma} \right)^{t + \Delta t \mathbf{s}} = t + \Delta t \mathbf{s}^*.
\] (43)

Finally, Equation (43) is applied to Equation (22), resulting in

\[
\sqrt{\frac{3}{2}} \left\| t + \Delta t \mathbf{s}^* \right\| - \frac{t + \Delta t \sigma}{3G} - 3G \Delta \varepsilon^{cr} = 0.
\] (44)

The solution of this nonlinear scalar equation is \( \bar{\sigma} \). The equivalent creep strain increment, \( \Delta \varepsilon^{cr} \), is the integration of Equation (24), that is,

\[
\Delta \varepsilon^{cr} = \Delta \varepsilon^{cr} \left( t + \Delta t, t + \Delta t \right) = \int_t^{t + \Delta t} \frac{\bar{\varepsilon}}{\bar{\sigma}} \, dt.
\] (45)

### 3.3 Partitioned implicit stress integration of a combined kinematic hardening plasticity and creep model

The stress can be decomposed into the hydrostatic and deviatoric stresses by Equation (27). Moreover, the hydrostatic stress and plasticity and creep model 3.3

\[
\text{Equations (31), (33)–(35), (41), and (46), we have}
\]

\[
(t + \Delta t \mathbf{s})^* - (t + \Delta t \mathbf{s}) = 3G \Delta \varepsilon^{p} \frac{(t + \Delta t \mathbf{s})^*}{t + \Delta t \sigma} - \sum_{i=1}^{M} (t + \Delta t \beta_i) \left( p_i h_i \Delta \varepsilon^{p} \frac{(t + \Delta t \mathbf{s})^* - (t + \Delta t \mathbf{s})}{t + \Delta t \sigma} \right),
\] (47)

\[
(t + \Delta t \mathbf{s}) - (t + \Delta t \mathbf{s})^* = 3G \Delta \varepsilon^{cr} \frac{(t + \Delta t \mathbf{s})^* - (t + \Delta t \mathbf{s})}{t + \Delta t \sigma} - 3G \Delta \varepsilon^{p} \frac{(t + \Delta t \mathbf{s})^*}{t + \Delta t \sigma}
\] (48)

Then, moving \( t + \Delta t \mathbf{s}^* - (t + \Delta t \mathbf{s}) \) in the former equation and \( t + \Delta t \mathbf{s} \) in the latter equation to the left-hand sides yields

\[
(t + \Delta t \mathbf{s})^* - (t + \Delta t \mathbf{s}) = \frac{3G \Delta \varepsilon^{p} \frac{(t + \Delta t \mathbf{s})^* - (t + \Delta t \mathbf{s})}{t + \Delta t \sigma} \sum_{i=1}^{M} (t + \Delta t \theta_i) h_i}{t + \Delta t \mathbf{x}^{cr} + \left( 3G + \frac{(t + \Delta t \mathbf{x}^{cr}) \sum_{i=1}^{M} (t + \Delta t \theta_i) h_i}{t + \Delta t \sigma^{kin}} \right) \frac{\Delta \varepsilon^{p}}{t + \Delta t \sigma^{kin}}},
\] (49)

\[
(t + \Delta t \mathbf{s}) - (t + \Delta t \mathbf{s})^* = \frac{(t + \Delta t \mathbf{s}) + (t + \Delta t \mathbf{x}^{p} \sum_{i=1}^{M} (t + \Delta t \theta_i) h_i)}{1 + (t + \Delta t \mathbf{x}^{p} + 3G \frac{\Delta \varepsilon^{p}}{t + \Delta t \sigma^{kin}}},
\] (50)

where

\[
(t + \Delta t \mathbf{x}^{p}) = \frac{3G \frac{\Delta \varepsilon^{p}}{t + \Delta t \sigma^{kin}}}{1 + \frac{\Delta \varepsilon^{p}}{t + \Delta t \sigma^{kin}} \sum_{i=1}^{M} (t + \Delta t \theta_i) h_i},
\] (51)

\[
(t + \Delta t \mathbf{x}^{cr}) = 1 + 3G \frac{\Delta \varepsilon^{cr}}{t + \Delta t \sigma}.
\] (52)
Finally, Equations (49) and (50) are applied to Equations (12) and (22), respectively, resulting in

\[
\sqrt{\frac{3}{2}} \left\| s^{t+\Delta t} - s^{t+\Delta t} \right\|_2 \leq \left( 3G + \chi^{t+\Delta t} \sum_{i=1}^M \psi_i b_i \right) \Delta \varepsilon^{p} = 0,
\]

(53)

\[
\sqrt{\frac{3}{2}} \left\| s^{t+\Delta t} + \chi^{t+\Delta t} \sum_{i=1}^M \psi_i b_i \right\|_2 - \left( 1 + \chi^{t+\Delta t} \right) \Delta \varepsilon^{c} - 3G \Delta \varepsilon^{cr} = 0.
\]

(54)

Although the nonlinear system of Equations (53) and (54) can be solved monolithically by the high-dimensional Newton–Raphson method, one cannot tractably reuse the stress integration algorithm of a single constitutive model. Moreover, when not yielding, the combined constitutive model becomes a single creep model. In this case, solving Equation (54) alone would be the simplest way. Therefore, we adopted a partitioned coupling approach in which Equations (53) and (54) are solved in a partitioned manner under an iterative method, such as the successive substitution method. First, Equation (54) is solved for \( \varepsilon^{p} \) using the Newton–Raphson method. The convergence at the \( j \)th iteration step is checked by

\[
\frac{\left| \Delta \varepsilon^{p(j+1)} - \Delta \varepsilon^{p(j)} \right|}{\Delta \varepsilon^{p(0)}} \leq \tau,
\]

(55)

where \( \tau \) is the tolerance. Then, after calculating several variables from the solution of Equation (54), the yield function, \( g \) in Equation (11), is evaluated. If \( \varepsilon^{t+\Delta t} \geq 0 \), then Equation (53) is solved for \( \Delta \varepsilon^{p} \) using the Newton–Raphson method. The convergence is checked by

\[
\frac{\left| \Delta \varepsilon^{p(j+1)} - \Delta \varepsilon^{p(j)} \right|}{\Delta \varepsilon^{p(0)}} \leq \tau.
\]

(56)

These procedures are repeated until both Equations (55) and (56) are satisfied.

4 | CONSISTENT TANGENT MODULI

In this section, tangent moduli that are consistent with the stress integration procedures in the previous section are derived. First, a consistent tangent modulus of a single kinematic hardening plasticity model is presented, followed by that of a single creep model. Finally, a consistent tangent modulus of a combined kinematic hardening plasticity and creep model is explained.

4.1 | Consistent tangent modulus of a single kinematic hardening plasticity model

The consistent tangent modulus, \( d^{t+\Delta t} \sigma / d^{t+\Delta t} \varepsilon \), of a single kinematic hardening plasticity model is derived in this section. Here, \( d^{t+\Delta t} \sigma \) can be decomposed into volumetric and deviatoric parts:

\[
d^{t+\Delta t} \sigma = (T_{vol} + T_{dev}) d^{t+\Delta t} \sigma = d^{t+\Delta t} p + d^{t+\Delta t} s.
\]

(57)

The volumetric part is assumed to be purely elastic, that is,

\[
d^{t+\Delta t} p = 3K T_{vol} d \Delta \varepsilon.
\]

(58)

For the deviatoric part, differentiating Equation (29) yields

\[
d^{t+\Delta t} s = 2GQ^{-1} \left( T_{dev} d \Delta \varepsilon - d \Delta \varepsilon^p \right).
\]

(59)
Then, differentiation of Equation \((31)\) gives
\[
d\Delta \varepsilon^p = \sqrt{\frac{3}{2} d\Delta \varepsilon^p Q^{i+\Delta t} n^{\text{kin}}} + \sqrt{\frac{3}{2} \Delta \varepsilon^p Q d^{i+\Delta t} n^{\text{kin}}},
\]  \hspace{1cm} (60)

where
\[
t^{+\Delta t} n^{\text{kin}} = \sqrt{\frac{3}{2} \frac{t^{+\Delta t} \Delta s - t^{+\Delta t} \beta}{t^{+\Delta t} \sigma^{\text{kin}}}}.
\]  \hspace{1cm} (61)

With \(t^{+\Delta t} n^{\text{kin}} \cdot Q d^{i+\Delta t} n^{\text{kin}} = 0\) that is derived from \(t^{+\Delta t} n^{\text{kin}} \cdot Q^{i+\Delta t} n^{\text{kin}} = 1\), multiplying both sides of Equation \((60)\) by \(t^{+\Delta t} n^{\text{kin}}\) yields
\[
d\Delta \varepsilon^p = \sqrt{\frac{2}{3} t^{+\Delta t} n^{\text{kin}}} \cdot d\Delta \varepsilon^p.
\]  \hspace{1cm} (62)

In addition, differentiation of Equation \((61)\) leads to
\[
d^{i+\Delta t} n^{\text{kin}} = \sqrt{\frac{3}{2} \frac{d^{i+\Delta t} s - d^{i+\Delta t} \beta}{t^{+\Delta t} \sigma^{\text{kin}}}} + \frac{t^{+\Delta t} n^{\text{kin}}}{t^{+\Delta t} \sigma^{\text{kin}}} t^{+\Delta t} H \cdot d\Delta \varepsilon^p.
\]  \hspace{1cm} (63)

where
\[
t^{+\Delta t} H = \frac{d^{i+\Delta t} \sigma^{\text{kin}}}{d\Delta \varepsilon^p} = \frac{d^{i+\Delta t} \sigma_Y}{d\Delta \varepsilon^p} = \frac{d^{i+\Delta t} \sigma_Y}{d^{i+\Delta \varepsilon^p}}.
\]  \hspace{1cm} (64)

Substituting Equations \((59)\) and \((62)\) into Equation \((63)\), we obtain
\[
d^{i+\Delta t} n^{\text{kin}} = \sqrt{\frac{3}{2} \frac{2GQ^{-1}}{t^{+\Delta t} \sigma^{\text{kin}}} (T_{dev} d\Delta \varepsilon - d\Delta \varepsilon^p) - \frac{d^{i+\Delta t} \beta}{t^{+\Delta t} \sigma^{\text{kin}}} t^{+\Delta t} H \Delta \varepsilon^p}.
\]  \hspace{1cm} (65)

Then, replacing \(d^{i+\Delta t} n^{\text{kin}}\) in Equation \((60)\) with Equation \((65)\), we have
\[
\left( \left( \frac{2}{3} \frac{t^{+\Delta t} \sigma^{\text{kin}}}{\Delta \varepsilon^p} + 2G \right) I + Q \sum_{i=1}^{M} t^{+\Delta t} H_i + \frac{2}{3} \left( t^{+\Delta t} H - \frac{t^{+\Delta t} \sigma^{\text{kin}}}{\Delta \varepsilon^p} \right) Q^{i+\Delta t} n^{\text{kin}} \otimes t^{+\Delta t} n^{\text{kin}} \right) d\Delta \varepsilon^p = 2G T_{dev} d\Delta \varepsilon,
\]  \hspace{1cm} (66)

where \(I\) is a 6 \times 6 unit matrix. Here, \(t^{+\Delta t} H_i\) is the constitutive matrix obtained by differentiating \(t^{+\Delta t} \beta_i\) with respect to \(\Delta \varepsilon^p\), that is, \(t^{+\Delta t} H_i = \partial^{i+\Delta t} \beta_i / \partial \Delta \varepsilon^p\). The detailed derivation of \(t^{+\Delta t} H_i\) for the Ohno–Wang model, as an example, is given by Kobayashi and Ohno.\(^{28}\) Equation \((66)\) establishes the relation between \(d\Delta \varepsilon^p\) and \(d\Delta \varepsilon\), which can be written as
\[
t^{+\Delta t} L^p d\Delta \varepsilon^p = 2G T_{dev} d\Delta \varepsilon,
\]  \hspace{1cm} (67)

where
\[
t^{+\Delta t} L^p = \left( \frac{2}{3} \frac{t^{+\Delta t} \sigma^{\text{kin}}}{\Delta \varepsilon^p} + 2G \right) I + Q \sum_{i=1}^{M} t^{+\Delta t} H_i + \frac{2}{3} \left( t^{+\Delta t} H - \frac{t^{+\Delta t} \sigma^{\text{kin}}}{\Delta \varepsilon^p} \right) Q^{i+\Delta t} n^{\text{kin}} \otimes t^{+\Delta t} n^{\text{kin}}.
\]  \hspace{1cm} (68)

Solving Equation \((67)\) yields
\[
d\Delta \varepsilon^p = 2G t^{+\Delta t} L^{p T} T_{dev} d\Delta \varepsilon.
\]  \hspace{1cm} (69)

By adding the volumetric and deviatoric parts, the consistent tangent modulus of a single kinematic hardening plasticity model becomes
\[
d^{i+\Delta t} \sigma = d^{i+\Delta t} p + d^{i+\Delta t} s = 3K T_{vol} d\Delta \varepsilon + 2GQ^{-1} \left( T_{dev} d\Delta \varepsilon - d\Delta \varepsilon^p \right),
\]  \hspace{1cm} (70)
Algorithm 1. Nonlinear finite element analysis

1: Initialize the nodal displacement: $^0u \leftarrow 0$
2: for $t \leftarrow 0$ to $t_{\text{max}}$ do
3: Initialize the nodal displacement increment: $\Delta u \leftarrow 0$
4: Generate the external force, $^t\Delta f^{\text{ext}}$, from boundary conditions
5: repeat
6: Calculate the strain increment: $\Delta \varepsilon \leftarrow B\Delta u$, where $B$ is the strain–nodal-displacement matrix
7: Calculate the stress, $^t\Delta \sigma$, by the implicit stress integration (Section 3)
8: Calculate the consistent tangent modulus; $^t\Delta K \leftarrow \int_V B^t \Delta \sigma dV$ (Section 4)
9: Update the displacement increment by solving a linear system of equations: $\Delta u \leftarrow \Delta u + ^{t+\Delta t}K^{-1}(^{t+\Delta t}f^{\text{ext}} - ^{t+\Delta t}f^{\text{int}})$
10: Update the time: $t \leftarrow t + \Delta t$
11: until convergence
12: Update the nodal displacement: $^{t+\Delta t}u \leftarrow ^tu + \Delta u$
13: Update the nodal displacement increment: $^t\Delta u \leftarrow ^t\Delta u$
14: until convergence
15: endfor

4.2 Consistent tangent modulus of a single creep model

Differentiating Equation (40) yields

$$d^{t+\Delta t}s = 2GQ^{-1}(T_{\text{dev}} d\Delta \varepsilon - d\Delta \varepsilon^{\text{cr}}).$$

(71)

Then, differentiation of Equation (41) gives

$$d\Delta \varepsilon^{\text{cr}} = \sqrt{\frac{3}{2} d\Delta \varepsilon^{\text{cr}}}Q^{t+\Delta t}n + \sqrt{\frac{3}{2} \Delta \varepsilon^{\text{cr}}}Q d^{t+\Delta t}n.$$  

(72)

where

$$^{t+\Delta t}n = \sqrt{\frac{3}{2} d^{t+\Delta t}s Q^{t+\Delta t}n}.$$  

(73)

With $^{t+\Delta t}n \cdot Q d^{t+\Delta t}n = 0$ that is derived from $^{t+\Delta t}n \cdot Q^{t+\Delta t}n = 1$, multiplying both sides of Equation (72) by $^{t+\Delta t}n$ gives

$$d\Delta \varepsilon^{\text{cr}} = \sqrt{\frac{2}{3} ^{t+\Delta t}n \cdot d\Delta \varepsilon^{\text{cr}}}.$$  

(74)

In addition, differentiation of Equation (73) leads to

$$d^{t+\Delta t}n = \sqrt{\frac{3}{2} d^{t+\Delta t}s} - ^{t+\Delta t}n \frac{1}{^{t+\Delta t}s} \left( \frac{d\sigma}{d\Delta \varepsilon^{\text{cr}}} \right) d\Delta \varepsilon^{\text{cr}}.$$  

(75)

Substituting Equations (71) and (74) into Equation (75), we obtain

$$d^{t+\Delta t}n = \sqrt{\frac{2}{3} 2GQ^{-1}(T_{\text{dev}} d\Delta \varepsilon - d\Delta \varepsilon^{\text{cr}})} - ^{t+\Delta t}n \frac{1}{^{t+\Delta t}s} \left( \frac{d\sigma}{d\Delta \varepsilon^{\text{cr}}} \right) \sqrt{\frac{2}{3} ^{t+\Delta t}n \cdot d\Delta \varepsilon^{\text{cr}}}.$$  

(76)

Then, replacing $d^{t+\Delta t}n$ in Equation (72) with Equation (76), we have

$$^{t+\Delta t}L^{\text{cr}} d\Delta \varepsilon^{\text{cr}} = 2GT_{\text{dev}} d\Delta \varepsilon,$$

(77)
Algorithm 2. Partitioned implicit stress integration

1: Input $\Delta \varepsilon, k, \beta, \bar{\varepsilon}^{p}, \bar{\varepsilon}^{cr}$
2: Initialize variables: $\Delta \bar{\varepsilon}^{p} \leftarrow 0, \Delta \bar{\varepsilon}^{cr} \leftarrow 0, t^{+\Delta t} \theta_{i} \leftarrow 1, t^{+\Delta t} \chi^{p} \leftarrow 0, t^{+\Delta t} \chi^{cr} \leftarrow 1$
3: Calculate the yield stress, $t^{+\Delta t} \sigma_y$ (Equation 13)
4: Decompose the stress, $t^{\varepsilon}$, into the hydrostatic and deviatoric stresses, $t^{p}$ and $t^{s}$ (Equation 27)
5: Calculate the hydrostatic stress, $t^{t+\Delta t} p$ (Equation 28)
6: Calculate the elastic predictor of the deviatoric stress, $t^{+\Delta t}s^{*}$ (Equation 30)
7: repeat
8: repeat
9: Solve Equation (54) for the equivalent stress, $t^{+\Delta t} \bar{\sigma}$, by fixing $t^{+\Delta t} \chi^{p}$ and $t^{+\Delta t} \theta_{i}$
10: Calculate the deviatoric stress, $t^{+\Delta t}s$ (Equation 50)
11: until Equation (55) is satisfied
12: Calculate $t^{+\Delta t} \chi^{cr}$ (Equation 52)
13: repeat
14: Solve Equation (53) for the equivalent plastic strain increment, $t^{+\Delta t} \bar{\varepsilon}^{p}$, by fixing $t^{+\Delta t} \chi^{cr}$
15: Calculate the yield stress, $t^{+\Delta t} \sigma_y$ (Equation 13)
16: Calculate the relative deviatoric stress, $t^{+\Delta t}s - t^{+\Delta t}\beta$ (Equation 49)
17: Calculate $t^{+\Delta t}\theta_{i}$ (Equation 36)
18: Calculate the back stress components, $t^{+\Delta t}\beta_{i}$ (Equation 34)
19: until Equation (56) is satisfied
20: Calculate $t^{+\Delta t} \chi^{p}$ (Equation 51)
21: until both Equations (55) and (56) are satisfied
22: Update the equivalent plastic strain: $t^{+\Delta t} \bar{\varepsilon}^{p} \leftarrow t^{+\Delta t} \bar{\varepsilon}^{p} + t^{+\Delta t} \bar{\varepsilon}^{cr}$
23: Update the equivalent creep strain: $t^{+\Delta t} \bar{\varepsilon}^{cr} \leftarrow t^{+\Delta t} \bar{\varepsilon}^{cr} + \Delta \bar{\varepsilon}^{cr}$
24: Incorporate the hydrostatic and deviatoric stresses, $t^{+\Delta t} \bar{\varepsilon}^{p}$ and $t^{+\Delta t} \bar{\varepsilon}^{cr}$, into the stress, $t^{+\Delta t} \sigma$ (Equation 27)
25: return $t^{+\Delta t} \sigma, t^{+\Delta t} \beta, t^{+\Delta t} \bar{\varepsilon}^{p}, t^{+\Delta t} \bar{\varepsilon}^{cr}$

where

$$
t^{+\Delta t}L^{cr} = \left( \frac{2}{3} \frac{t^{+\Delta t} \bar{\varepsilon}^{cr}}{\Delta \bar{\varepsilon}^{cr}} + 2G \right) I + \frac{2}{3} \left( t^{+\Delta t} \left( \frac{d\bar{\varepsilon}}{d\Delta \bar{\varepsilon}^{cr}} \right) - \frac{t^{+\Delta t} \bar{\varepsilon}^{cr}}{\Delta \bar{\varepsilon}^{cr}} \right) Q^{+\Delta t} n \otimes t^{+\Delta t} n. \tag{78}$$

Solving Equation (77) yields

$$
\frac{d\Delta \varepsilon^{cr}}{T_{dev}} = 2G^{+\Delta t}L^{cr-1} T_{dev} d\Delta \varepsilon. \tag{79}
$$

Finally, the consistent tangent modulus of a single creep model becomes

$$
d^{+\Delta t} \sigma = d^{+\Delta t} p + d^{+\Delta t}s
= 3K T_{vol} d\Delta \varepsilon + 2G Q^{-1} \left( T_{dev} d\Delta \varepsilon - d\Delta \varepsilon^{cr} \right)
= \left( D^{e} - 4G^{2}Q^{-1} t^{+\Delta t}L^{cr-1} T_{dev} \right) d\Delta \varepsilon. \tag{80}
$$

4.3 Consistent tangent modulus of a combined kinematic hardening plasticity and creep model

Differentiating Equation (46) yields

$$
d^{+\Delta t}s = 2G Q^{-1} \left( T_{dev} d\Delta \varepsilon - d\Delta \varepsilon^{p} - d\Delta \varepsilon^{cr} \right). \tag{81}
$$
Substituting Equations (81) and (62) into Equation (63), we obtain

$$d^{t+\Delta_t n_{\text{kin}}} = \sqrt{\frac{2}{3}} G Q^{-1} (T_{\text{dev}} d\Delta \varepsilon - d\Delta \varepsilon^p - d\Delta \varepsilon^c) - d^{t+\Delta_t \beta} \frac{d^{t+\Delta_t n_{\text{kin}}}}{t^{t+\Delta_t \beta}} - \frac{t^{t+\Delta_t H}}{2} \frac{d^{t+\Delta_t n_{\text{kin}}}}{t^{t+\Delta_t H}} \cdot d\Delta \varepsilon^p.$$  

(82)

Then, replacing $d^{t+\Delta_t n_{\text{kin}}}$ in Equation (60) with Equation (82), we have

$$t^{t+\Delta_t L^p} d\Delta \varepsilon^p + 2GI d\Delta \varepsilon^c = 2GT_{\text{dev}} d\Delta \varepsilon.$$  

(83)

where $I$ is a 6×6 unit matrix.

Following a similar derivation, the following creep-related equations can be obtained:

$$t^{t+\Delta_t L^c} d\Delta \varepsilon^c + 2GI d\Delta \varepsilon^p = 2GT_{\text{dev}} d\Delta \varepsilon.$$  

(84)

Equations (83) and (84) establish the relationship among $d\Delta \varepsilon^p$, $d\Delta \varepsilon^c$, and $d\Delta \varepsilon$, which can be written as

$$\begin{bmatrix} t^{t+\Delta_t L^p} & 2GI \\ 2GI & t^{t+\Delta_t L^c} \end{bmatrix} \begin{bmatrix} d\Delta \varepsilon^p \\ d\Delta \varepsilon^c \end{bmatrix} \begin{bmatrix} 2GT_{\text{dev}} d\Delta \varepsilon \\ 2GT_{\text{dev}} d\Delta \varepsilon \end{bmatrix} = I,$$

(85)

where $d\Delta \varepsilon^p + d\Delta \varepsilon^c$ can explicitly be expressed in terms of $d\Delta \varepsilon$ by solving Equation (85), that is,

$$d\Delta \varepsilon^p + d\Delta \varepsilon^c = 2G \begin{bmatrix} I & I \end{bmatrix} \begin{bmatrix} t^{t+\Delta_t L^p} & 2GI \\ 2GI & t^{t+\Delta_t L^c} \end{bmatrix}^{-1} \begin{bmatrix} I \\ I \end{bmatrix} T_{\text{dev}} d\Delta \varepsilon.$$  

(86)

Finally, the consistent tangent modulus of a combined kinematic hardening plasticity and creep model becomes

$$d^{t+\Delta_t \sigma} = d^{t+\Delta_t p} + d^{t+\Delta_t s}$$

$$= 3KT_{\text{vol}} d\Delta \varepsilon + 2GQ^{-1} \left( T_{\text{dev}} d\Delta \varepsilon - d\Delta \varepsilon^p - d\Delta \varepsilon^c \right)$$

$$= \left( D^p - 4G^2Q^{-1} \begin{bmatrix} I & I \end{bmatrix} \begin{bmatrix} t^{t+\Delta_t L^p} & 2GI \\ 2GI & t^{t+\Delta_t L^c} \end{bmatrix}^{-1} \begin{bmatrix} I \\ I \end{bmatrix} T_{\text{dev}} \right) d\Delta \varepsilon.$$  

(87)

5. Algorithm and Implementation of Partitioned Implicit Stress Integration and Consistent Tangent Modulus

First, the algorithm of a nonlinear finite element analysis that is based on the Newton–Raphson method is briefly explained in Algorithm 1. Then, we focus on the algorithm of the implicit stress integration in the seventh line of Algorithm 1. In particular, that of the partitioned implicit stress integration (Section 3.3) is listed in Algorithm 2.

In the implementation using the partitioned coupling technique, existing programs for the stress integration of plastic constitutive models and creep constitutive models can be reused with a few modifications because these programs are used separately in the partitioned coupling technique. In addition, an integrated program of material models can be developed easily. For instance, by using macros of the C preprocessor, functions of stress integration for a single plasticity model, a single creep model, and a combined plasticity–creep model can be generated from a single program.

In the stress integration procedures, the plastic part (lines 8–12 of Algorithm 2) can be implemented separately from the creep part (lines 14–20 of Algorithm 2). If the stress is placed inside the yield surface, then only the creep part is executed. Furthermore, the plasticity model or the creep model can be changed tractably by calling another function of the program. Each function is dedicated to a specific plasticity or creep model, such as the Ohno–Wang model or the Norton–Bailey model. A function for a specific plasticity–creep model, such as the combined Ohno–Wang and Norton–Bailey model, is not needed.

Although it is also possible to calculate consistent tangent moduli in a partitioned manner, a monolithic block matrix is adopted in the present study (see Equation 85). However, the advantage of separate implementation remains, even in the monolithic tangent modulus system. The plastic submatrix, $t^{t+\Delta_t L^p}$, and the creep submatrix, $t^{t+\Delta_t L^c}$, can be generated
separately for each plasticity or creep model. If the element integration point does not yield, ignoring the plastic submatrix, \( t^{+\Delta t}L^p \), reduces the monolithic 6×6 matrix to a 3×3 matrix, namely, \( t^{+\Delta t}L^c_t \). Furthermore, when another plasticity or creep model is to be used, the plastic submatrix, \( t^{+\Delta t}L^p \), or the creep submatrix, \( t^{+\Delta t}L^c_t \), can be replaced by that of the model to be used.

6 | COMBINED OHNO–WANG AND NORTON–BAILEY MODEL USING THE PARTITIONED COUPLING FRAMEWORK

For the evolution law of the \( i \)th back stress in Equation (18), the most classical choice would be the Prager model, in which

\[
\dot{\beta}_i = \frac{2}{3} h_i Q^{-1} \dot{\varepsilon}^p,
\]

where \( h_i \) is a material parameter. Note that \( M = 1 \) for the classical Prager model, whereas \( M \geq 2 \) for the generalized Prager model with the approach of multiple back stresses. This equation indicates that \( \dot{\gamma}_i \) in Equation (18) is zero. The next choice would be the Armstrong–Frederick model. In this model, a dynamic recovery term is considered in addition to the strain-hardening term (Equation (88)), as

\[
\dot{\beta}_i = \frac{2}{3} h_i Q^{-1} \dot{\varepsilon}^p - \xi_i \dot{\varepsilon}^p \beta_i, \tag{89}
\]

where \( h_i \) and \( \xi_i \) are material parameters for the strain hardening and the dynamic recovery, respectively. Note that \( M = 1 \) for the classical Armstrong–Frederick model, whereas \( M \geq 2 \) for the generalized Armstrong–Frederick model. In this equation, \( \dot{\gamma}_i = -\xi_i \dot{\varepsilon}^p \). Here, we adopted the Ohno–Wang model. This model replaces \( \dot{\varepsilon}^p \) in Equation (89) with a more general form that corresponds to the dynamic recovery of each \( \beta_i \) separately, that is,

\[
\dot{\beta}_i = \frac{2}{3} h_i Q^{-1} \dot{\varepsilon}^p - \xi_i \dot{\varepsilon}^p \beta_i, \tag{90}
\]

where

\[
\dot{\varepsilon}^p = H(f_i) \left\langle \frac{\beta_i}{r_i} \cdot \dot{\varepsilon}^p \right\rangle. \tag{91}
\]

Here, \( H \) denotes the Heaviside step function:

\[
H(x) = \begin{cases} 
1, & x \geq 0, \\
0, & x < 0,
\end{cases} \tag{92}
\]

where \( \langle \rangle \) are the Macaulay brackets, that is, \( \langle x \rangle = xH(x) \), and \( r_i \) is a material parameter given as \( r_i = h_i/\xi_i \). Moreover, \( f_i \) is defined as

\[
f_i = \frac{3}{2} \beta_i \cdot Q \beta_i - r_i^2. \tag{93}
\]

In the Ohno–Wang model, \( \dot{\gamma}_i \) in Equation (18) equals \( -\xi_i H(f_i) \left\langle \beta_i/ r_i \cdot \dot{\varepsilon}^p \right\rangle \). Furthermore, Equation (36) becomes

\[
t^{+\Delta t} \theta_i = \frac{1}{1 - \Delta \gamma_i} = \frac{1}{1 + \xi_i H(\dot{\gamma}_i) \left\langle \frac{\beta_i}{r_i} \cdot \dot{\varepsilon}^p \right\rangle}. \tag{94}
\]

Another expression of \( t^{+\Delta t} \theta_i \) is also available in the Ohno–Wang model:

\[
t^{+\Delta t} \theta_i = \begin{cases} 
\frac{r_i}{\Delta \theta_i}, & f_i \geq 0, \\
1, & f_i < 0,
\end{cases} \tag{95}
\]

where \( t^{+\Delta t} \beta_i = \sqrt{\frac{3}{2} \left\| t^{+\Delta t} \beta_i^c \right\|^2} \). The latter expression is used in computation.
For the equivalent creep strain rate, $\dot{\varepsilon}^{cr}$, in Equation (24), we have various choices, such as the Norton law,

$$\dot{\varepsilon}^{cr} = A\sigma^n,$$

(96)

the Bailey law,

$$\dot{\varepsilon}^{cr} = At^m,$$

(97)

the Norton–Bailey law,

$$\dot{\varepsilon}^{cr} = A\sigma^n t^m,$$

(98)

and other creep models that use an exponential or hyperbolic function. In these equations, $A$, $n$, and $m$ are material parameters. Among them, we used the Norton–Bailey law. The Norton–Bailey law considers both stress-hardening and time-hardening effects, enabling us to model both the primary and secondary creep. In particular, the time-hardening effect has not been considered in the previous time-dependent constitutive models with the Ohno–Wang model.8,9,12,13 The equivalent creep strain increment, $\Delta\dot{\varepsilon}^{cr}$, in Equation (45) can be calculated by integration of Equation (98) using the backward Euler integration method:

$$\Delta\dot{\varepsilon}^{cr} = \frac{1}{m+1}A^{t+\Delta t}\sigma^n ((t + \Delta t)^{m+1} - t^{m+1}).$$

(99)

### 7 | NUMERICAL EXAMPLES

The proposed partitioned coupling approach with the Ohno–Wang model and the Norton–Bailey model was implemented in an open-source parallel finite element solver, ADVENTURE_Solid.29,30 ADVENTURE_Solid is based on the hierarchical domain decomposition method31 and has an advantage in solving large-scale structural problems. Due to the symmetry of the consistent tangent modulus of the combined Ohno–Wang and Norton–Bailey model, 28 the conjugate gradient (CG) method was used to solve the linear system of equations in the hierarchical domain decomposition method. The Ohno–Wang model in the developed program was verified by numerical comparisons with a commercial finite element solver, ADVENTURECluster, which has the function of the Ohno–Wang model. The Norton–Bailey model was verified using a commercial finite element solver, ABAQUS. Although ADVENTURECluster has the function of a rate-dependent nonlinear kinematic hardening model, 9 it is not the same model as the combined Ohno–Wang and Norton–Bailey model in the present program. Therefore, the combined model could not be verified by numerical comparison with other solvers. The partitioned coupling framework was verified by checking the convergence criteria of Equations (55) and (56). It was also verified that the scalar equations (53) and (54) are satisfied when the convergence is attained. Using the developed program, first, a cube problem was solved to show the basic behavior of the present constitutive model. Then, a thruster problem was analyzed to demonstrate the capability of the present constitutive model and algorithm for a realistic structure in a high-temperature environment. In addition, the convergence performances of the Newton–Raphson method, the CG method, and the partitioned stress integration were investigated.

#### 7.1 | Cube problem

A cube model consisting of one hexahedral element was analyzed in order to investigate the basic behavior of the present constitutive model, such as the differences among the Ohno–Wang model, the Norton–Bailey model, and the combined model. The dimensions and the boundary conditions of the cube are illustrated in Figure 1A. The bottom surface of the cube was totally constrained, whereas the top surface was subjected to a cyclic load, the transition of which is plotted in Figure 1B. Time increment, $\Delta t$, was set to 1 s. The adopted material parameters of the Ohno–Wang and Norton–Bailey models are listed in Table 1. To demonstrate the differences among the three models, these parameters were calibrated so
that both plasticity and creep appear simultaneously. Note that, in the present numerical example, we adopted the linear isotropic hardening rule:

$$\sigma_Y = \sigma_{Y0} + \overline{H} \epsilon^p,$$  \hspace{1cm} (100)

where $\sigma_{Y0}$ and $\overline{H}$ are the initial yield stress and the isotropic hardening coefficient, respectively.

The computed relationships between stress $\sigma_z$ and strain $\epsilon_z$ of the Ohno–Wang model, the Norton–Bailey model, and the combined model are plotted in Figure 2A. The horizontal and vertical axes represent strain $\epsilon_z$ and stress $\sigma_z$, respectively. The dashed blue line, the dotted red line, and the solid black line denote the results of the Ohno–Wang model, the Norton–Bailey model, and the combined model, respectively. Plotted values in the figure are averages of values at eight integration points. The Ohno–Wang model produced a stable hysteresis loop after the first tensile load. In contrast, the Norton–Bailey model and the combined model gave shifted hysteresis loops due to the contribution of creep strain evolution. Finally, the transition of strain $\epsilon_z$ is shown in Figure 2B. The horizontal and vertical axes represent the time and strain $\epsilon_z$, respectively. The Ohno–Wang and combined models exhibited much larger strain than the Norton–Bailey model. Moreover, the results for the combined model differed gradually from those of the Ohno–Wang model.

### 7.2 Thruster problem

We analyzed a thruster model considering heat conduction and thermal expansion. The objectives of this numerical example are to show the importance of considering both a kinematic hardening plasticity model and a creep model in...
Figure 2 (A) Relationship between stress $\sigma_{zz}$ and strain $\varepsilon_{zz}$; (B) transition of strain $\varepsilon_{zz}$ in the analysis of the cube problem.

Figure 3 (A) Dimensions of the thruster model; (B) sectional view of the mesh of the thruster model; (C) hierarchical domain decomposition of the mesh; (D) boundary conditions applied to the thruster model.

Comparison with a single plasticity model and a single creep model and to demonstrate the capability of the present partitioned coupling framework in simulating a realistic structure at high temperature. A finite element heat conduction analysis was performed by ADVENTURE_Thermal, followed by a finite element structural analysis by ADVENTURE_Solid in a one-way coupling manner. ADVENTURE_Thermal is a heat conduction solver that is also based on the hierarchical domain decomposition method. A single mesh with its hierarchical domain decomposition was used in both the heat conduction and structural analyses. The temperature at each time step computed by the heat conduction analysis was used for the input of the structural analysis via files. Each time step was divided by linear interpolation. In the structural analysis, thermal strain, $\varepsilon^{th}$, was computed from the temperature, $T$, by

$$\varepsilon^{th} = \alpha (T - T_{\text{ref}}),$$

where $\alpha$ and $T_{\text{ref}}$ are the coefficient of thermal expansion and the reference temperature, respectively. Moreover, we assumed

$$\varepsilon = \varepsilon^\varepsilon + \varepsilon^p + \varepsilon^{cr} + \varepsilon^{th}$$

instead of Equation (25).
The dimensions of the thruster model are described in Figure 3A. Based on these dimensions, we created a three-dimensional solid mesh, the sectional view of which is depicted in Figure 3B. This mesh consisted of quadratic tetrahedral finite elements, each of which had 10 nodes. The numbers of elements and nodes were 1,608,768 and 2,445,696, respectively. For parallel computing, this mesh was decomposed hierarchically into 16 parts with 1250 subdomains for each part, as visualized in Figure 3C. In this figure, the parts are placed separately, whereas the subdomains are colored. The numbers of message passing interface (MPI) processes and OpenMP threads for each MPI process were set to 16 and 24, respectively. Eight computing nodes of the supercomputer Fugaku were used.

The boundary conditions of the heat conduction analysis are illustrated in Figure 3D. They consisted of a heating process of 50 s and a cooling process of 550 s. In the heating process, a fixed temperature was prescribed on the inner surface of the combustion chamber. The fixed temperature on this surface was 800°C, whereas the initial temperature at other locations was 20°C. This fixed-temperature boundary condition was a simplified model to simulate the contribution of combustion. The temperature in the vicinity of the inner surface was assumed to increase in a very short time. After that, the fixed temperature remained 800°C during the heating process (50 s). At the end of the heating process, this boundary became adiabatic. In the heating process, temperature is expected to rise, leading to high thermal stress and large plastic and creep deformation. In the cooling process, zero or very small plastic deformation and small creep deformation are expected. The time increment was set to 5 s in both the heating and cooling processes. Hence, the number of time steps was 120. In addition, the heat-transfer boundary condition was applied to the outer surface of the thruster throughout the analysis. The heat transfer coefficient and the ambient temperature were assumed to be $1.00 \times 10^{-2}$ W/(mm² K) and 20°C, respectively. Furthermore, the left-hand side of the combustion chamber was constrained in the structural analysis.

As the material, Ti-6Al-4V was assumed. The adopted material parameters are listed in Table 2. For the heat conduction analysis, a single set of material parameters was used. In contrast, multiple sets of material parameters were tested for the structural analysis to demonstrate possible behaviors. Moreover, we tested the Ohno–Wang model, the Norton–Bailey model, and the combined model to clarify the differences among the three models. For the Ohno–Wang model, a single

| TABLE 2 | Material parameters of the thruster problem |
|---------|-------------------------------------------|
| Thermal conductivity, $\kappa$ (W/(mm K)) | $6.10 \times 10^{-3}$ |
| Specific heat, $c$ (J/(kg K)) | $5.45 \times 10^{2}$ |
| Density, $\rho$ (kg/mm³) | $4.43 \times 10^{-6}$ |
| Young’s modulus, $E$ (MPa) | $110,000$ |
| Poisson’s ratio, $\nu$ | $0.32$ |
| Coefficient of thermal expansion, $\alpha$ (1/K) | $9.90 \times 10^{-6}$ |
| Reference temperature, $T_{\text{ref}}$ (°C) | $20$ |
| Initial yield stress, $\sigma_{Y_0}$ (MPa) | $880$ |
| Isotropic hardening coefficient, $\overline{H}$ (MPa) | $200$ |
| Ohno–Wang parameters, $\xi_i$ ($i = 1, 2, 3, 4$) | $1540, 553, 100, \text{ and } 26$ |
| Ohno–Wang parameters, $r_i$ ($i = 1, 2, 3, 4$) (MPa) | $58.44, 112.1, 32.10, \text{ and } 380.8$ |
| Norton–Bailey parameter, $A$, of material #1 | $2.221 \times 10^{-15}$ |
| Norton–Bailey parameter, $n$, of material #1 | $3.270$ |
| Norton–Bailey parameter, $m$, of material #1 | $-0.7338$ |
| Norton–Bailey parameter, $A$, of material #2 | $2.000 \times 10^{-22}$ |
| Norton–Bailey parameter, $n$, of material #2 | $6.000$ |
| Norton–Bailey parameter, $m$, of material #2 | $-0.7338$ |
| Norton–Bailey parameter, $A$, of material #3 | $1.000 \times 10^{-32}$ |
| Norton–Bailey parameter, $n$, of material #3 | $10.00$ |
| Norton–Bailey parameter, $m$, of material #3 | $-0.7338$ |
The empirical equation is from Ankem et al. Then, for the Norton–Bailey model, we adopted three sets of material parameters. Hence, the combined model involved three sets of material parameters. The three sets are referred to as materials #1, #2, and #3. The parameters for material #1 were calibrated to approximately reproduce the history of the equivalent creep strain of Ti-6Al-4V under a constant equivalent stress of 593.5 MPa for 0 h to 200 h. Although the difference between the curves obtained by material #1 and the experimental result was small for 593.5 MPa, the difference became large when the magnitude of the equivalent stress was changed. This is a limitation of the Norton–Bailey model, and the combined model is not fully validated for Ti-6Al-4V. A creep model that is more suitable for Ti-6Al-4V can be used for Equation (24) and is easily implemented using the proposed framework. Materials #2 and #3 are modified versions of material #1. Figure 4A shows histories of the equivalent creep strain for materials #1, #2, and #3, and those obtained by the empirical equation, which was fitted to the experimental results. Note that the histories are for 0 s to 600 s. The lines for materials #1, #2, and #3 follow the integral of Equation (98). The differences among materials #1, #2, and #3 are the effects of stress hardening. If the equivalent stress is low, the equivalent creep strain rate of material #3 is the smallest among them. If the equivalent stress is high, then it is the largest. Similarly, Figure 4B shows histories of the stress under constant strain. In this example, material #3 exhibits the largest amount of stress relaxation.

Figure 5 visualizes the distributions of the computed temperature at the end of the heating process (50 s) and the cooling process (600 s). Moreover, the transition of the temperature is plotted in Figure 6. The horizontal and vertical axes represent the time and the temperature, respectively. The solid and dashed lines denote the temperature at points A and B, respectively, in Figure 3D. The temperature of the combustion chamber part was very high in the heating process and decreased in the cooling process.
The results of the heat conduction analysis were used in the structural analysis. To set the initial temperature to 20°C, one time step was added to the beginning of the 120 time steps. Hence, the number of time steps of the structural analysis was 121. The time increment of the first step was assumed to be $1 \times 10^{-3}$ s.

Figures 7–9 show the distributions of the equivalent stress of the Ohno–Wang model, the Norton–Bailey model, and the combined model, respectively, at the final time step. In all of the models, the equivalent stress at the fixed end, which is plotted in Figure 10, was high. The Ohno–Wang model, the Norton–Bailey model with material #1, and the combined model with material #1 exhibited similar distributions of the equivalent stress, although the Norton–Bailey model showed higher equivalent stress at the fixed end. However, in the Norton–Bailey model and the combined model, the equivalent stress of material #2 was lower than that of material #1, due to stress relaxation. Moreover, that of material #3 was much lower. Figures 11 and 12 depict the distributions of the equivalent plastic strain of the Ohno–Wang model and the combined model, respectively. The equivalent plastic strain at the fixed end is plotted in Figure 13. Similar to the equivalent stress, the distributions of the equivalent plastic strain of the Ohno–Wang model and the combined model with material #1 were similar. However, those with materials #2 and #3 were smaller. Figures 14 and 15 visualize the distributions of the equivalent creep strain of the Norton–Bailey model and the combined model, respectively. The equivalent creep strain at the fixed end is plotted in Figure 16. In both the Norton–Bailey model and the combined model, material #1 exhibited very small equivalent creep strain. Those of materials #2 and #3 were larger. In summary, material #1 exhibited a plasticity-dominant behavior, whereas material #3 exhibited a creep-dominant behavior. Material #2 showed both plasticity and creep behaviors, in which the equivalent creep strain was slightly
FIGURE 8  Distributions of the equivalent stress at the final time step in the analysis of the thruster problem using the Norton–Bailey model with (A) material #1, (B) material #2, and (C) material #3

FIGURE 9  Distributions of the equivalent stress at the final time step in the analysis of the thruster problem using the combined Ohno–Wang and Norton–Bailey model with (A) material #1, (B) material #2, and (C) material #3

FIGURE 10  Equivalent stress at the final time step on the line from point C to point A

larger than the equivalent plastic strain. The combined model is able to represent these three behaviors in a unified manner.

The results for material #2 were investigated in detail. The transitions of the equivalent stress, the equivalent plastic strain, and the equivalent creep strain at point A in Figure 3D are plotted in Figure 17A–C, respectively. In the three figures, the horizontal and vertical axes represent the time step and the values at point A, respectively. The dashed blue lines, the dotted red lines, and the solid black lines denote the results of the Ohno–Wang model, the Norton–Bailey model, and the combined model, respectively. In the first 10 time steps, the combined model exhibited similar equivalent stress
FIGURE 11  Distribution of the equivalent plastic strain at the final time step in the analysis of the thruster problem using the Ohno–Wang model

FIGURE 12  Distributions of the equivalent plastic strain at the final time step in the analysis of the thruster problem using the combined Ohno–Wang and Norton–Bailey model with (A) material #1, (B) material #2, and (C) material #3

FIGURE 13  Equivalent plastic strain at the final time step on the line from point C to point A to the Ohno–Wang model due to plasticity, whereas the Norton–Bailey model produced higher equivalent stress. After that, the combined model differed from the Ohno–Wang model due to creep. Moreover, the combined model exhibited slightly smaller equivalent plastic strain than the Ohno–Wang model and much smaller equivalent creep strain than the Norton–Bailey model.

The convergence performances of the Newton–Raphson method, the CG method, and the partitioned stress integration for the combined model with material #2 were investigated. The numbers of Newton–Raphson iteration steps for all of the time steps are plotted in Figure 18A. The horizontal and vertical axes represent the time step and the number...
FIGURE 14  Distributions of the equivalent creep strain at the final time step in the analysis of the thruster problem using the Norton–Bailey model with (A) material #1, (B) material #2, and (C) material #3

FIGURE 15  Distributions of the equivalent creep strain at the final time step in the analysis of the thruster problem using the combined Ohno–Wang and Norton–Bailey model with (A) material #1, (B) material #2, and (C) material #3

FIGURE 16  Equivalent creep strain at the final time step on the line from point C to point A of Newton–Raphson iteration steps, respectively. The Newton–Raphson iteration converged within four iteration steps. Newton–Raphson and CG residuals for the combined model at the fourth time step, which involved the largest number of Newton–Raphson iteration steps, are shown in Figure 18B. The horizontal and vertical axes represent the CG iteration step and the residual, respectively. The solid line with the circle symbols denotes the Newton–Raphson residuals, whereas the dotted lines denote the CG residuals. As the preconditioner of the CG method, the balancing domain decomposition with diagonal scaling (BDD-DIAG) method was used. The tolerances for the convergence checks of the Newton–Raphson iteration and the CG iteration were set to $1.5 \times 10^{-6}$ and $1.0 \times 10^{-8}$, respectively. Both the Newton–Raphson iteration and
FIGURE 17  Transitions of (A) the equivalent stress, (B) the equivalent plastic strain, and (C) the equivalent creep strain at point A

FIGURE 18  (A) Numbers of Newton–Raphson iteration steps for time steps; (B) Newton–Raphson and CG residuals at the fourth time step in the analysis of the thruster problem

the CG iteration converged successfully. Then, the numbers of partitioned stress integration iteration steps for all of the time steps are plotted in Figure 19. The horizontal and vertical axes represent the time step and the number, respectively. For each time step, the number of partitioned stress integration iteration steps at the final Newton–Raphson iteration step was stored for all of the integration points. Then, the largest number of partitioned stress integration iteration steps was selected. The partitioned stress integration iteration converged within 27 iteration steps. After the 13th time step, the number of iteration steps remained at one because creep was dominant. Finally, the computational times of the Ohno–Wang model, the Norton–Bailey model with material #2, and the combined model with material #2 were 9356, 12,233, and 11,735 s, respectively. The combined model with the present implementation needed longer computational time than the single Ohno–Wang model, but shorter computational time than the single Norton–Bailey model. This indicates that the additional computational time of the partitioned stress integration is negligible.
CONCLUSIONS

We proposed a partitioned coupling framework to combine a kinematic hardening plasticity model and a creep model. General descriptions of the formulation and the algorithm of a single plasticity model, a single creep model, and a combined plasticity–creep model were presented. In the present framework, plastic and creep implicit stress integrations are performed alternately and iteratively in a partitioned coupling manner. The converged solution of the iteration is the stress state of the combined model. The present framework enables us to tractably develop a program of a combined plasticity–creep model from existing programs of a single plasticity model and a single creep model. The present framework was applied to the combination of the Ohno–Wang plasticity model and the Norton–Bailey creep model for numerical examples. The numerical example of the cube problem demonstrated the basic behavior of the present combined model. The combined model showed behavior other than either the single Ohno–Wang model or the single Norton–Bailey model. Then, in the numerical example of the thruster problem, three sets of material parameters were considered to simulate possible plasticity and creep behaviors. The first and third sets produced plasticity-dominant and creep-dominant results, respectively, whereas the second set exhibited both plasticity and creep behaviors. Such plasticity and creep phenomena could be simulated in a unified manner. The present results of the thruster problem showed the importance of considering both a kinematic hardening plasticity model and a creep model. The proposed framework can be a powerful tool to analyze structures in a high-temperature environment.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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FIGURE 19  Numbers of partitioned stress integration iteration steps for time steps in the analysis of the thruster problem
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