Local Distance Constrained Bribery in Voting

Palash Dey
Indian Institute of Technology, Kharagpur
Email: palash.dey@cse.iitkgp.ac.in

Abstract

Studying complexity of various bribery problems has been one of the main research focus in computational social choice. In all the models of bribery studied so far, the briber has to pay every voter some amount of money depending on what the briber wants the voter to report and the briber has some budget at her disposal. Although these models successfully capture many real world applications, in many other scenarios, the voters may be unwilling to deviate too much from their true preferences. In this paper, we study the computational complexity of the problem of finding a preference profile which is as close to the true preference profile as possible and still achieves the briber’s goal subject to budget constraints. We call this problem LOCAL DISTANCE CONSTRAINED BRIBERY. We consider three important measures of distances, namely, swap distance, footrule distance, and maximum displacement distance, and resolve the complexity of the optimal bribery problem for many common voting rules. We show that the problem is polynomial time solvable for the plurality and veto voting rules for all the three measures of distance. On the other hand, we prove that the problem is NP-complete for a class of scoring rules which includes the Borda voting rule, maximin, Copeland$^\alpha$ for any $\alpha \in [0, 1]$, and Bucklin voting rules for all the three measures of distance even when the distance allowed per voter is 1 for the swap and maximum displacement distances and 2 for the footrule distance even without the budget constraints (which corresponds to having an infinite budget). For the $k$-approval voting rule for any constant $k > 1$ and the simplified Bucklin voting rule, we show that the problem is NP-complete for the swap distance even when the distance allowed is 2 and for the footrule distance even when the distance allowed is 4 even without the budget constraints. We complement these hardness results by showing that the problem for the $k$-approval and simplified Bucklin voting rules is polynomial time solvable for the swap distance if the distance allowed is 1 and for the footrule distance if the distance allowed is at most 3. For the $k$-approval voting rule for the maximum displacement distance for any constant $k > 1$, and for the simplified Bucklin voting rule for the maximum displacement distance, we show that the problem is NP-complete (with the budget constraints) and, without the budget constraints, they are polynomial time solvable.

1 Introduction

Aggregating preferences of a set of agents over a set of alternatives is a fundamental problem in many applications both in real life and artificial intelligence. Voting has often served as a natural tool for aggregating preferences in such applications. Pioneering use of voting in key applications of artificial intelligence includes spam detection [CSS99], collaborative filtering [PHG00], etc. A typical voting setting consists of a set of alternatives, a set of agents each having a preference which is a complete order over the alternatives, and a voting rule which declares one or more alternatives as the winner(s) of the election.

However any such election scenario is susceptible to control attacks of various kinds – internal or external agents may try to influence the election system in someone’s favor. One such attack which has been studied extensively in computational social choice is bribery. In every model of bribery studied so far (see [FR16]), we have the preferences of a set of voters, an external agent called briber with some budget, a bribing model which dictates how much one has to bribe any voter to persuade her to cast a vote of briber’s choice, and the computational problem is to check whether it is possible to bribe the voters subject to the budget constraint so that some alternative of briber’s choice becomes the winner. This models not only serve as
Maximum displacement BTT89

Indeed, in the context of bribery, there can be situations where a voter may be bribed to report some preference which “resembles” her true preference but a voter is simply unwilling to report any preference which is far from her true preference. We remark that existing models of bribery do not capture the above constraint since, intuitively speaking, the budget feasibility constraint in these models restricts the total money spent (which is a global constraint) whereas the situations above demand (local) constraints per voter. For example, let us think of a voter with preference \( a > b > c \). Suppose the voter can be persuaded to make at most two swaps and the cost of persuading her does not depend on the number of swaps she performs in her preference. This could be the situation when she is happy to change her preference as briber advises (simply because she trusts the briber that her change will finally ensure a better social outcome) but does not wish to deviate from her own preference too much to avoid social embarrassment. One can see that the classical model of bribery (SWAP BRIbery for example) fails to capture the intricacies of this situation (for example, making the cost per swap to be 0 fails because the voter is not willing to cast \( c > b > a \)). In this paper, we fill this research gap by proposing a bribery model which directly addresses these scenarios.

More specifically, we study the computational complexity of the following problem which we call LOCAL DISTANCE CONSTRAINED $b$Bribery. Given preferences \( \mathcal{P} = (\succ_i)_{i \in [n]} \) of a set of agents, non-negative integers \( (\delta_i)_{i \in [n]} \) denoting the distance change allowed for corresponding agents, non-negative integers \( (p_i)_{i \in [n]} \) where \( p_i \) denotes the amount one has to pay the voter \( i \) to change his or her preference, a non-negative integer budget \( B \), and an alternative \( c \), compute if the preferences can be changed subject to the "price,
distance, and budget constraints” so that \( c \) is a winner in the resulting election for some voting rule. We also study an interesting special case of the Local Distance Constrained $\text{Bribery}$ problem where \( \delta_i = \delta \) for some non-negative integer \( \delta \) and \( p_i = 0 \) for every \( i \) and \( B = 0 \); we call the latter problem Local Distance Constrained Bribery. In this paper, we study the following commonly used distance functions on the set of all possible preferences (permutations on the set of alternatives): (i) swap distance \([\text{Ken}38]\), (ii) footrule distance \([\text{Spe}04]\), and (iii) maximum displacement distance \([\text{OE}12a, \text{OE}12b]\). The swap distance (aka Kendall Tau distance, bubble sort distance, etc.) between two preferences is the number of pairs of alternatives which are ranked in different order in these two preferences. Whereas the footrule distance (maximum displacement distance respectively) between two preferences is the sum (maximum respectively) of the absolute value of the differences of the positions of every alternative in two preferences. We refer to Section 2 for formal definitions of the problems and the distance functions above.

1.1 Contribution

We study the computational complexity of the Local Distance Constrained $\text{Bribery}$ and Local Distance Constrained Bribery problems for the plurality, veto, k-approval, a class of scoring rules which includes the Borda voting rule \([\text{Theorem 12}]\), maximin, Copeland\(^\alpha\) for any \( \alpha \in [0, 1] \), Bucklin, and simplified Bucklin voting rules for the swap, footrule, and maximum displacement distance. We summarize our results in Table 1. We highlight that all our results are tight in the sense that we even find the exact value of \( \delta \) till which the Local Distance Constrained Bribery problem for some particular voting rule is polynomial time solvable and beyond which it is NP-complete. As can be observed in Table 1, most of our NP-completeness results (except Theorems 10 and 17 for which the corresponding Local Distance Constrained Bribery problems are polynomial time solvable) hold even for the Local Distance Constrained Bribery problem (and thus for the Local Distance Constrained $\text{Bribery}$ problem too) and even for small constant values for \( \delta \). On the other hand, most of our polynomial time algorithms (except Theorems 4 and 7 for which the corresponding Local Distance Constrained $\text{Bribery}$ problems are NP-complete) work for the general Local Distance Constrained $\text{Bribery}$ problem (and thus for the Local Distance Constrained Bribery problem too). We would like to highlight a curious case – for the maximum displacement distance, the Local Distance Constrained Bribery problem is polynomial time solvable for the simplified Bucklin voting rule (for any \( \delta \)) and NP-complete for the Bucklin voting rule even for \( \delta = 1 \). To the best of our knowledge, this is the first instance where a natural problem is polynomial time solvable for the simplified Bucklin voting rule and NP-complete for the Bucklin voting rule. We also observe that, unlike the optimal manipulation problem in \([\text{OE}12a, \text{OE}12b]\), the complexity of the Local Distance Constrained Bribery problem for some common voting rule (k-approval with \( k > 1 \) and simplified Bucklin voting rules for example) can depend significantly on the distance function under consideration.

1.2 Related Work

Faliszewski et al. \([\text{FHH06}]\) propose the first bribery problem where the briber’s goal is to change a minimum number of preferences to make some candidates win the election. Then they extend their basic model to more sophisticated models of Shift Bribery and $\text{Bribery}$ \([\text{FHH09, FHHR09}]\). Elkind et al. \([\text{EFS09}]\) extend this model further and study the Swap Bribery problem where there is a cost associated with every swap of alternatives. Dey et al. \([\text{DMN17}]\) show that the bribery problem remains intractable for many common voting rules for an interesting special case which they call Frugal Bribery. The bribery problem has also been studied in various other preference models, for example, truncated ballots \([\text{BFLR12}]\), soft constraints \([\text{PRV13}]\), approval ballots \([\text{SFE17}]\), campaigning in societies \([\text{FGKT18}]\), CP-nets \([\text{DK16}]\), combinatorial domains \([\text{MPVR12}]\), iterative elections \([\text{MNRS18}]\), committee selection \([\text{BFNT16}]\), probabilistic lobbying \([\text{BEF} + 14]\), etc. Erdelyi et al. \([\text{EHH14}]\) study the bribery problem under voting rule uncertainty. Faliszewski et al. \([\text{FRRS14}]\) study bribery for the simplified Bucklin and the Fallback voting rules. Xia \([\text{Xia12}]\), and Kaczmarczyk and Faliszewski \([\text{KF16}]\) study the destructive variant of bribery. Dorn and Schlotter \([\text{DS12}]\) and Bredereck et al. \([\text{BCF} + 14]\) explore parameterized complexity of various bribery problems. Chen et
al. [CXX+18] provide novel mechanisms to protect elections from bribery. Knop et al. [KKM18] provide a uniform framework for various control problems. Although most of the bribery problems are intractable, few of them, Shift bribery for example, have polynomial time approximation algorithms [EF10, KHH18].

Manipulation, a specialization of bribery, is another fundamental attack on election [CW16]. In the manipulation problem, a set of voters (called manipulators) wants to cast their preferences in such a way that (when tallied with the preferences of other preferences) makes some alternative win the election. Obraztsova and Elkind [OE12a, OE12b] initiate the study of optimal manipulation in that context.

The rest of the paper has been organized as follows. We introduce our notation and formally define our computational problems in Section 2; then we present our polynomial time algorithms and NP-completeness results in respectively Section 3 and Section 4; we finally conclude with future research directions in Section 5. A short version of our work will appear in [Dey19].

2 Preliminaries

2.1 Voting and Voting Rules

For a positive integer $k$, we denote the set $\{1, 2, \ldots, k\}$ by $[k]$. Let $A = \{a_i : i \in [m]\}$ be a set of $m$ alternatives. A complete order over the set $A$ of alternatives is called a preference. We denote the set of all possible preferences over $A$ by $\mathcal{L}(A)$. A tuple $(\triangleright_i)_{i \in [n]} \in \mathcal{L}(A)^n$ of $n$ preferences is called a profile. We say that an alternative $a \in A$ is placed at the $\ell$-th position (from left or from top) in a preference $\triangleright \in \mathcal{L}(A)$ for some positive integer $\ell$ (and denote it by $\text{pos}(a, \triangleright)$) if $|\{b \in A : b \triangleright a\}| = \ell - 1$. We say that the distance of two alternatives $a, b \in A$ in a preference $\triangleright \in \mathcal{L}(A)$ is some positive integer $k$ if there exists a positive integer $\ell$ such that the positions of $a$ and $b$ in $\triangleright$ are either $\ell$ and $\ell + k$ respectively or $\ell + k$ and $\ell$ respectively. An election $E$ is a tuple $(\triangleright)_{\triangleright \in A}$ where $\triangleright$ is a profile over a set $A$ of alternatives. If not mentioned otherwise, we denote the number of alternatives and the number of preferences by $m$ and $n$ respectively. A map $\tau : \mathcal{L}_n \rightarrow 2^A \backslash \{\emptyset\}$ is called a voting rule. Given an election $E$, we can construct from $E$ a directed weighted graph $G_E$ which is called the weighted majority graph of $E$. The set of vertices in $G_E$ is the set of alternatives in $E$. For any two alternatives $x$ and $y$, the weight of the edge $(x, y)$ is $D_E(x, y) = N_E(x, y) - N_E(y, x)$, where $N_E(a, b)$ is the number of preferences where the alternative $a$ is preferred over the alternative $b$ for $a, b \in A, a \neq b$. Examples of some common voting rules are as follows.

- **Positional scoring rules**: An $m$-dimensional vector $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_m) \in \mathbb{N}^m$ with $\alpha_1 \geq \alpha_2 \geq \ldots \geq \alpha_m$ and $\alpha_1 > \alpha_m$ for every $m \in \mathbb{N}$ naturally defines a voting rule — an alternative gets score $\alpha_i$ from a preference if it is placed at the $i$-th position, and the score of an alternative is the sum of the scores it receives from all the preferences. The winners are the alternatives with the maximum score. Scoring rules remain unchanged if we multiply every $\alpha_i$ by any constant $\lambda > 0$ and/or add any constant $\mu$. Hence, we can assume without loss of generality that for any score vector $\alpha$, we have $\gcd((\alpha_i)_{i \in [m]}) = 1$ and there exists a $j < m$ such that $\alpha_i = 0$ for all $\ell > j$. We call such an $\alpha$ a normalized score vector. For some $k \in [m-1]$, if $\alpha_i$ is 1 for $i \in [k]$ and 0 otherwise, then, we get the $k$-approval voting rule. The $k$-approval voting rule is also called the $(m-k)$-veto voting rule. The 1-approval voting rule is called the plurality voting rule and the $(m-1)$-approval voting rule is called the veto voting rule. If $\alpha_i = m - 1$ for every $i \in [m]$, then we get the Borda voting rule.

- **Maximin**: The maximin score of an alternative $x$ is $\min_{y \neq x} D_E(x, y)$. The winners are the alternatives with the maximum maximin score.

- **Copeland**: Given $\alpha \in [0, 1]$, the Copeland$^\alpha$ score of an alternative $x$ is $|\{y \neq x : D_E(x, y) > 0\}| + \alpha|\{y \neq x : D_E(x, y) = 0\}|$. The winners are the alternatives with the maximum Copeland$^\alpha$ score.

- **Simplified Bucklin and Bucklin**: The simplified Bucklin score of an alternative $x$ is the minimum number $\ell$ such that $x$ is placed within the first $\ell$ positions in more than half of the preferences. The winners are the alternatives with the lowest simplified Bucklin score. Let $k$ be the minimum simplified Bucklin score of any alternative. Then the Bucklin winners are the alternatives who appear the maximum number of times within the first $k$ positions.
2.2 Distance Function

A distance function $d$ takes two preferences on a set $A$ of alternatives as input and outputs a non-negative number which satisfies the following properties: (i) $d(\succ_1, \succ_2) = 0$ for $\succ_1, \succ_2 \in \mathcal{L}(A)$ if and only if $\succ_1 = \succ_2$, (ii) $d(\succ_1, \succ_2) = d(\succ_2, \succ_1)$ for every $\succ_1, \succ_2 \in \mathcal{L}(A)$, (iii) $d(\succ_1, \succ_2) \leq d(\succ_1, \succ_3) + d(\succ_3, \succ_2)$ for every $\succ_1, \succ_2, \succ_3 \in \mathcal{L}(A)$. We will consider the following distance functions in this paper.

- **Swap distance:**
  $$d_{\text{swap}}(\succ_1, \succ_2) = |\{ (a, b) \subset A : a \succ_1 b, b \succ_2 a \}|$$

- **Footrule distance:**
  $$d_{\text{footrule}}(\succ_1, \succ_2) = \sum_{a \in A} |\text{pos}(a, \succ_1) - \text{pos}(a, \succ_2)|$$

- **Maximum displacement distance:**
  $$d_{\text{max displacement}}(\succ_1, \succ_2) = \max_{a \in A} |\text{pos}(a, \succ_1) - \text{pos}(a, \succ_2)|$$

It is well known that, for any two preferences $\succ_1, \succ_2 \in \mathcal{L}(A)$, we have $d_{\text{swap}}(\succ_1, \succ_2) \leq d_{\text{footrule}}(\succ_1, \succ_2) \leq 2d_{\text{swap}}(\succ_1, \succ_2)$ [DG77]. Let $r$ be any voting rule and $d$ be any distance function on $\mathcal{L}(A)$. We now define our computational problem formally for any distance function $d$ on $\mathcal{L}(A)$.

**Definition 1** (d-Local Distance Constrained Bribery). Given a set $A$ of alternatives, a profile $\succ = (\succ_i)_{i \in [n]} \in \mathcal{L}(A)^n$ of $n$ preferences, a positive integer $\delta$, and an alternative $c \in A$, compute if there exists a profile $\succ' = (\succ'_i)_{i \in [n]} \in \mathcal{L}(A)^n$ such that

(i) $d(\succ_i, \succ'_i) \leq \delta$ for every $i \in [n]$

(ii) $r(\succ') = \{c\}$

We denote an instance of **Local Distance constrained bribery** by $(A, \mathcal{P}, c, \delta)$.

**Definition 2** (d-Local Distance Constrained $\mathcal{P}$Bribery). Given a set $A$ of alternatives, a profile $\succ = (\succ_i)_{i \in [n]} \in \mathcal{L}(A)^n$ of $n$ preferences, positive integers $(\delta_i)_{i \in [n]}$ denoting distances allowed for every preference, non-negative integers $(p_i)_{i \in [n]}$ denoting the prices of every preference, a non-negative integer $\mathcal{P}$ denoting the budget of the Briber, and an alternative $c \in A$, compute if there exists a subset $J \subseteq [n]$ and a profile $\succ' = (\succ'_i)_{i \in \mathcal{L}(A)^n}$ such that

(i) $\sum_{i \in J} p_i \leq \mathcal{P}$

(ii) $d(\succ_i, \succ'_i) \leq \delta_i$ for every $i \in J$

(iii) $r((\succ'_i)_{i \in J}, (\succ_i)_{i \in [n]\setminus J}) = \{c\}$

We denote an instance of **Local Distance constrained $\mathcal{P}$Bribery** by $(A, \mathcal{P}, c, (\delta_i)_{i \in [n]}, (p_i)_{i \in [n]})$.

We remark that the optimal bribery problem, as described in Definition 1, demands the alternative $c$ to win uniquely. It is equally motivating to demand that $c$ is a co-winner. As far as the optimal bribery problem is concerned, we can easily verify that all our results, both algorithmic and hardness, extend easily to the co-winner case. However, we note that it need not always be the case in general (see Section 1.1 in [XC11] for example).

3 Polynomial Time Algorithms

In this section, we present our polynomial time algorithms. All our polynomial time algorithms are obtained by reducing our problem to the maximum flow problem which is quite common in computational social
choice (see for example [Fal08]). In the maximum flow problem, the input is a directed graph with two special vertices s and t and (positive) capacity for every edge, and the goal is to compute the value of maximum flow that can be sent from s to t subject to the capacity constraints of every edge. This problem is known to have polynomial time algorithms. Moreover, it is also known that, if the capacity of every edge is positive integer, then the value of maximum flow is also a positive integer, the flow in every edge is also a non-negative integer, and such a maximum flow can be found in polynomial time [Cor09].

**Theorem 3.** The **Local Distance Constrained $\$Bribery** problem is polynomial time solvable for the plurality and veto voting rules for the swap, footrule, and maximum displacement distance.

**Proof.** Let us first prove the result for the plurality voting rule. Let $P = \{x_{i} \in [m]\}$ be the input profile, and $x$ the distinguished alternative. Let the plurality score of any alternative $a \in A$ in $P$ be $s(a)$. We first guess the final score $\ell_x$ of $x$ in the range $s(x)$ to $n$. Let $Q$ be the sub-profile of $P$ consisting of preferences which do not place $x$ at their first position. For any preference $\succ_i \in Q$, we compute the set of alternatives $A_i \subseteq A$ which can be placed at the first position keeping the distance from $\succ_i$ at most $\delta_i$. We observe that, for the swap and maximum displacement distances, $A_i$ is the set of alternatives which appear within the first $\delta_i + 1$ positions in $\succ_i$; for the footrule distance, $A_i$ is the set of alternatives which appear within the first $\lfloor \ell_i/2 \rfloor$ positions in $\succ_i$. We now create the following minimum cost flow network $\mathcal{G}$ with demand on edges. It is well known that a minimum cost flow (of certain flow value) satisfying demands of all the edges can be found in polynomial time (see for example [GTT90]).

\[
\mathcal{G} = (V(G), E(G), \ell_x, \ell_x - s(x))
\]

From the construction it follows that the input **Local Distance Constrained $\$Bribery** instance has a successful bribery where the final plurality score of the alternative $x$ is at least $\ell_x$ if and only if there is an $s - t$ flow in $\mathcal{G}$ of flow value $|Q|$ with cost at most $B$. Since there are at most $n$ possible values that $\ell_x$ can take and we try all of them, the **Local Distance Constrained $\$Bribery** problem for the plurality voting rule for the three distance functions is polynomial time solvable.

We now prove the result for the veto voting rule. We first guess the final veto score $\ell_x$ of $x$ in the range 0 to $n$. For any preference $\succ_i \in P$, we compute the set of alternatives $A_i \subseteq A$ which can be placed at the last position keeping the distance from $\succ_i$ at most $\delta_i$. We observe that, for all the three the distance functions under consideration, $A_i$ is the set of all alternatives which appear within the last $\delta_i$ positions in $\succ_i$. We now create the following minimum cost flow network $\mathcal{G}$ with demand on edges.
\[
\forall [g] = \{ u_i : i \in P \} \cup \{ v_a : a \in A \} \cup \{ s, t \}
\]
\[
E[g] = \{ (s, u_1) \geq \epsilon \in P \} \cup \{ (v_a, t) : a \in A \}
\cup \{ (u_i, v_a) : \geq \epsilon \in P, a \in A_i \}
\]
\[
\text{capacity}(v_x, t) = \epsilon; \quad \text{capacity}(v_a, t) = n \forall a \in A \setminus \{ x \};
\]
\[
\text{capacity of every other edge is 1}
\]
\[
\text{demand}(v_a, t) = \epsilon \forall a \in A \setminus \{ x \}; \quad \text{demand of every other edge is 0}
\]
\[
\text{cost}(u_i, v_a) = p_i \forall \geq \epsilon \in P, a \in A_i, a \text{ does not appear at the last position in } \geq \epsilon; \quad \text{cost of other edges is 0}
\]

From the construction it follows that the input LOCAL DISTANCE CONSTRAINED $\$Bribery$ instance has a successful bribery where the final veto score of the alternative x is at most $\ell_x$, if and only if there is an $s - t$ flow in $\mathcal{G}$ of flow value $|P|$ with cost at most $B$. Since there are at most $n$ possible values that $\ell_x$ can take and we try all of them, the LOCAL DISTANCE CONSTRAINED $\$Bribery$ problem for the veto voting rule for the three distance functions is polynomial time solvable.

We prove the following results by reducing to the maximum flow problem.

**Theorem 4.**

**Proof.** The proof for the swap and maximum displacement distance is completely analogous to the proof of Theorem 3. The second part of the statement follows from the following claim: for any two preferences $\geq_1, \geq_2 \in \ell(A)$, if $d_{\text{footrule}}(\geq_1, \geq_2) \leq 3$, then $d_{\text{swap}}(\geq_1, \geq_2) \leq 1$. If $d_{\text{footrule}}(\geq_1, \geq_2) = 2$, then clearly $d_{\text{swap}}(\geq_1, \geq_2) = 1$. Now if, $d_{\text{footrule}}(\geq_1, \geq_2) = 3$, then $d_{\text{swap}}(\geq_1, \geq_2) \geq 1$. However, if $d_{\text{swap}}(\geq_1, \geq_2) = 2$, then, by analyzing all the cases, $d_{\text{footrule}}(\geq_1, \geq_2) = 4$ which proves the claims.

**Theorem 5.** The LOCAL DISTANCE CONSTRAINED bribery problem is polynomial time solvable for the k-approval voting rule for the maximum displacement distance for any k.

**Proof.** Let $P$ be the input profile and x the distinguished alternative. In every preference in $P$, if it is possible to place x within the first k position, which happens exactly when x appears within the first $k + \delta$ positions in a preference, we place x at the first position and keep the relative ordering of every other alternative the same. Let $P' \subseteq P$ be the set of such preferences. Then the score of x in the resulting profile is $|P'|$; let it be $\ell_x$. For any preference $\geq \in P$, we compute the set of alternatives $A_\geq \subseteq A$ for whom there exist two preferences $\geq_1, \geq_2 \in \ell(A)$ with distance at most $\delta$ each from $\geq$ such that the alternative appears within the first k positions in $\geq_1$ and it does not appear within the first k positions in $\geq_2$. We observe that $A_\geq$ is precisely the set of alternatives which appear in positions from $\max\{1, k - \delta\}$ to $\min\{k + \delta, m\}$. Let $\delta' = \min\{k, \delta\}$. For any alternative $y \in A \setminus \{x\}$, let $\ell_y$ be the number of preferences in $P$ where y appears within the first $k - \delta - 1$ positions. If there exists an alternative $y \in A \setminus \{x\}$ such that $\ell_y \geq \ell_x$, then we output NO. Otherwise we create the following flow network $\mathcal{G} = (V, E, c, s, t)$: $V = \{ u_\geq : \geq \in P \setminus P' \} \cup \{ v_y : y \in A \setminus \{x\} \} \cup \{ s, t \}$, $E = \{ (s, u_\geq) : \geq \in P \setminus P' \} \cup \{ (v_y, t) : y \in A \setminus \{x\} \} \cup \{ (u_\geq, v_y) : \geq \in P \setminus P', y \in A \setminus \{x\}, c((v_y, t)) = \ell_y - 1 - \ell_y v_y, t \in E, c((s, u_\geq)) = \delta' - 1 \forall (s, u_\geq) \in E, \geq \in P, c((s, u_\geq)) = \delta' \forall (s, u_\geq) \in E, \geq \in P \setminus P', c(e) = 1$ for every other $e \in E$. From the construction it is clear that the input instance is a YES instance if and only if there is an $s - t$ flow in $\mathcal{G}$ of value $\sum_{\geq \in P} c((s, u_\geq))$. Hence the LOCAL DISTANCE CONSTRAINED bribery problem for the k-approval voting rule for the maximum displacement distance reduces to the maximum flow problem which proves the result.

**Theorem 6.**

**Proof.** The proof of the statement uses arguments analogous to the proof of Theorem 4.

**Theorem 7.** The LOCAL DISTANCE CONSTRAINED bribery problem is polynomial time solvable for the simplified Bucklin voting rule for the maximum displacement distance.
4 Hardness Results

In this section, we present our hardness results. We use the following \((3, B2)\)-SAT problem to prove our hardness results which is known to be NP-complete [BKS03].

Definition 8 \(((3, B2)\)-SAT\). Given a set \(X = \{x_i : i \in [n]\}\) of variables and a set \(\{C_j : j \in [m]\}\) of \(m\) \(3\)-CNF clauses on \(X\) such that, for every \(i \in [n]\), \(x_i\) and \(\bar{x}_i\) each appear in exactly 2 clauses, compute if there exists any Boolean assignment to the variables which satisfy all the \(m\) clauses simultaneously.

We now present our hardness result for the \(k\)-approval voting rule for the swap and footrule distances. In this section, \(\delta\) always denotes the distance under consideration.

Theorem 9. The Local Distance constrained bribery problem is NP-complete for the \(k\)-approval voting rule for any constant \(k \geq 2\) for the swap distance even when \(\delta = 2\). Hence, the Local Distance constrained bribery problem is NP-complete for the \(k\)-approval voting rule for any constant \(k \geq 2\) for the footrule distance even when \(\delta = 4\).

Proof. Let us present the proof for the \(2\)-approval voting rule first. The Local Distance constrained bribery problem for the \(2\)-approval voting rule for the swap distance is clearly in \(NP\). To prove NP-hardness, we reduce from \((3, B2)\)-SAT to Local Distance constrained bribery. Let \((X = \{x_i : i \in [n]\}, \mathcal{C} = \{C_j : j \in [m]\})\) be an arbitrary instance of \((3, B2)\)-SAT. Let us assume without loss of generality that both \(n\) and \(m\) are even integers; if not, we take disjoint union of the instance with itself which doubles both the number of variables and the number of clauses. Let us consider the following instance \((A, \mathcal{P}, c, \delta = 2)\) of Local Distance constrained bribery.

\[
A = \{a(x_i, 0), a(x_i, 1), a(\bar{x}_i, 0), a(\bar{x}_i, 1) : i \in [n]\} \cup \{c, u\} \\
\cup \{w_i, z_i : i \in [n]\} \cup \{y_j, d_j, d'_j : j \in [m]\}
\]

We construct the profile \(\mathcal{P}\) using the following function \(f\). The function \(f\) takes a literal and a clause as input, and outputs a value in \((0, 1, -)\). For each literal \(l\), let \(C_l\) and \(C_{\bar{l}}\) with \(1 \leq l < j \leq m\) be the two clauses where \(l\) appears. We define \(f(l, C_{l}) = 0, f(l, C_{\bar{l}}) = 1, \text{ and } f(l, C_{k}) = -\) for every \(k \in [m] \setminus \{i, j\}\). This finishes the description of the function \(f\). We now describe \(\mathcal{P}\). Let \(\Delta = 100m^2n^2\).

- (I) For every \(i \in [n]\), we have
  - \(w_i > a(x_i, 0) > a(x_i, 1) > z_i > \text{others}\)
  - \(w_i > a(\bar{x}_i, 0) > a(\bar{x}_i, 1) > z_i > \text{others}\)

- (II) For every \(C_j = (l_1 \lor l_2 \lor l_3), j \in [m]\), we have
  - \(y_j > d_1 > a(l_1, f(l_1, C_j)) > d'_1 > \text{others}\)
  - \(y_j > d_1 > a(l_2, f(l_2, C_j)) > d'_1 > \text{others}\)
  - \(y_j > d_1 > a(l_3, f(l_3, C_j)) > d'_1 > \text{others}\)

- (III) \(c > d_1 > d_2 > d_3 > \text{others}\)

- (IV) \(\Delta + 2\) copies: \(u > d_1 > d_2 > c > \text{others}\)

- (V) For every \(1 \leq i \leq n/2\), we have
  - \(\Delta + 1\) copies: \(u > w_{2i-1} > w_{2i} > d_1 > \text{others}\)

- (VI) For every \(i \in [n]\)
  - \(\Delta + 1\) copies: \(u > a(x_i, 1) > a(\bar{x}_i, 1) > d_1 > \text{others}\)
  - \(\Delta + 2\) copies: \(u > a(x_i, 0) > a(\bar{x}_i, 0) > d_1 > \text{others}\)
(VII) For every \(1 \leq i \leq m/2\), we have

\[ \Delta \text{ copies: } u \succ y_{2i-1} \succ y_{2i} \succ d_1 \succ \text{others} \]

We claim that the two instances are equivalent. In one direction, let the \((3, B2)\)-SAT instance be a YES instance with a satisfying assignment \(g : X \rightarrow \{0, 1\}\). Let us consider the following profile \(\mathcal{Q}\) where the swap distance of every preference is at most 2 from its corresponding preference in \(\mathcal{P}\).

(I) For every \(i \in [n]\), we have

\[ \Delta \text{ copies: } u \succ y_{2i-1} \succ y_{2i} \succ d_1 \succ \text{others} \]

(II) For every \(j \in [m]\), if \(C_j = (l_1 \lor l_2 \lor l_3)\) and \(g\) makes \(l_1 = 1\) (we can assume by renaming), then we have

\[ \text{d}_j \succ a(l_1, f(l_1, C_j)) \succ y_j \succ d_j' \succ \text{others} \]
\[ y_j \succ d_j \succ a(l_2, f(l_2, C_j)) \succ d_j' \succ \text{others} \]
\[ y_j \succ d_j \succ a(l_3, f(l_3, C_j)) \succ d_j' \succ \text{others} \]

(III) \(c \succ d_1 \succ d_2 \succ d_3 \succ \text{others} \)

(IV) \(\Delta + 2\) copies: \(u \succ c \succ d_1 \succ d_2 \succ \text{others} \)

(V) For every \(1 \leq i \leq n/2\), we have

\[ \Delta + 1 \text{ copies: } w_{2i-1} \succ w_{2i} \succ u \succ d_1 \succ \text{others} \]

(VI) For every \(i \in [n]\)

\[ \Delta + 1 \text{ copies: } a(x_i, 1) \succ a(x_i, 1) \succ u \succ d_1 \succ \text{others} \]
\[ \Delta + 2 \text{ copies: } a(x_i, 0) \succ a(x_i, 0) \succ u \succ d_1 \succ \text{others} \]

(VII) For every \(1 \leq i \leq m/2\), we have

\[ \Delta \text{ copies: } y_{2j-1} \succ y_{2j} \succ u \succ d_1 \succ \text{others} \]

The 2-approval score of every alternative from \(\mathcal{Q}\) is given in Table 2. Hence, \(c\) wins uniquely in \(\mathcal{Q}\) and thus the local distance constrained bribery instance is a YES instance.

| Alternatives | Score |
|--------------|-------|
| \(c\)        | \(\Delta + 3\) |
| \(w_i, i \in [n]\), \(u, y_j, j \in [m]\) | \(\Delta + 2\) |
| \(a(x_i, 0), a(x_i, 1), a(x_i, 0), a(x_i, 1), i \in [n]\) | \(\leq \Delta + 2\) |
| \(z_i, i \in [n]\), \(d_i, d_i', j \in [m]\) | \(< \Delta\) |

Table 2: Summary of scores from \(\mathcal{Q}\)

In the other direction, let us assume that there exists a profile \(\mathcal{Q}\) such that the swap distance of every preference in \(\mathcal{Q}\) is at most 2 from its corresponding preference in \(\mathcal{P}\) and \(c\) wins uniquely in \(\mathcal{Q}\). We can
assume without loss of generality that $c$ appears within the first 2 positions in every preference in $\mathcal{Q}$ which corresponds to the preferences in Items (III) and (IV) in $\mathcal{P}$. We now observe that, irrespective of other preferences in $\mathcal{Q}$, the score of $c$ in $\mathcal{Q}$ is $\Delta + 3$. Also the score of $u$ from the preferences corresponding to Item (IV) in $\mathcal{Q}$ is already $\Delta + 2$. Hence, for $c$ to win uniquely in $\mathcal{Q}$, $u$ must not appear within the first two positions in any preference in $\mathcal{Q}$ which corresponds to the preferences in Items (V) to (VII) in $\mathcal{P}$. This makes the score of $w_i, i \in [n]$ $\Delta + 1$, the score of $a(l,0) \Delta + 2$ and the score of $a(l,1) \Delta + 1$ for every literal $l$, and the score of $y_{ij}, j \in [m]$ $\Delta$ from the preferences corresponding to Items (III) to (VII) in $\mathcal{Q}$. We now consider the following assignment $g : X \rightarrow \{0, 1\}$ - for every $i \in [n]$, $g(x_i) = 1$ if the preference corresponding to $w_i \succ a(x_i,0) \succ a(x_i,1) \succ z_i \succ \text{others}$ in $\mathcal{Q}$ keeps $w_i$ within the first 2 positions; if the preference corresponding to $w_i \succ a(x_i,0) \succ a(x_i,1) \succ z_i \succ \text{others}$ in $\mathcal{Q}$ keeps $w_i$ within the first 2 positions, then we define $g(x_i) = 0$. We observe that the function $g$ is well defined since $w_i$ must be pushed out of the first 2 positions at least once in the preferences corresponding to $w_i \succ a(x_i,0) \succ a(x_i,1) \succ z_i \succ \text{others}$ and $w_i \succ a(x_i,0) \succ a(x_i,1) \succ z_i \succ \text{others}$; otherwise the score of $w_i$ is the same as the score of $c$ which is a contradiction. We claim that $g$ is a satisfying assignment for the $(3, B2)$-SAT instance. Suppose not, then let us assume that $g$ does not satisfy $C_j = (l_1 \lor l_2 \lor l_3)$ for some $j \in [m]$. It follows from the definition of $g$ that the score of every alternative in $\{a(l_1,f(l_i,C_j)) : i \in [3]\}$ from the preferences corresponding to Items (I) and (III) to (VII) in $\mathcal{Q}$ is already $\Delta + 2$. Hence $y_j$ can never be pushed out of the first 2 positions in the preferences corresponding to Item (II) in $\mathcal{Q}$. However this makes the score of $y_j \Delta + 3$ in $\mathcal{Q}$. This contradicts our assumption that $c$ is the unique winner in $\mathcal{Q}$. Hence $g$ is a satisfying assignment and thus the $(3, B2)$-SAT instance is a YES instance.

Generalizations to $k$-approval for any constant $k \geq 3$ can be done by using $10m^2n^8$ dummy candidates and ensuring that any dummy alternative appears at most once within the first $k+3$ positions in the profile $\mathcal{P}$. The proof for the footrule distance follows from the observation that, for any two preferences $\succ_1, \succ_2 \in \mathcal{L}(\mathcal{A})$, if we have $d_{\text{footrule}}(\succ_1, \succ_2) = 4$, then we have $d_{\text{swap}}(\succ_1, \succ_2) = 2$.

Theorem 10. The LOCAL DISTANCE CONSTRAINED $\$BRIBERY$ problem is NP-complete for the $k$-approval voting rule for the maximum displacement distances even when $\delta_l = 2$ for every preference $l$ for any constant $k > 1$.

Proof. Let us present the proof for the 2-approval voting rule first. The LOCAL DISTANCE CONSTRAINED $\$BRIBERY$ problem for the 2-approval voting rule for the maximum displacement distance is clearly in NP. To prove NP-hardness, we reduce from $(3, B2)$-SAT to LOCAL DISTANCE CONSTRAINED $\$BRIBERY$. Let $(X = \{x_i : i \in [n]\}, \mathcal{C} = \{C_j : j \in [m]\})$ be an arbitrary instance of $(3, B2)$-SAT. Let us consider the following instance of LOCAL DISTANCE CONSTRAINED $\$BRIBERY$. The set of alternatives $\mathcal{A}$ is $\{a_i, a_{i+1}, b_i, b_{i+1}, w_i, w'_i : i \in [n]\} \cup \{y_j : j \in [m]\} \cup \mathcal{D}$, where $|\mathcal{D}| = 300m^2n^8$. We now describe the preference profile $\mathcal{P}$ along with their corresponding prices. The profile $\mathcal{P}$ is a disjoint union of two profiles, namely, $\mathcal{P}_1$ and $\mathcal{P}_2$. While describing the preferences of $\mathcal{P}$ below, whenever we say ‘others’ or ‘for some alternatives in $\mathcal{D}$’ or ‘for some subset of $\mathcal{D}$’, the unspecified alternatives are assumed to be arranged in such a way that, for every unspecified alternative $a \in \mathcal{A} \setminus \mathcal{D}$, at least 10 alternatives from $\mathcal{D}$ appear immediately before $a$. We also ensure that any alternative in $\mathcal{D}$ appears within top $k + 10$ positions at most once in $\mathcal{P}$ whereas every alternative in $\mathcal{A} \setminus \mathcal{D}$ appears within top 10m position in every preference in $\mathcal{P}$. This is always possible because $|\mathcal{D}| = 300m^2n^8$ whereas $|\mathcal{A} \setminus \mathcal{D}| = 6n + m + 1$ and $|\mathcal{D}| \leq 100(m + n)$. Let $f$ and $g$ be functions defined on the set of literals as $f(x_i) = a_i, g(x_i) = b_i$, and $f(x_i) = a_i, g(x_i) = b_i$ for every $i \in [n]$. We also define a function $h$ as $h(C_l, 1) = f(1)$ if the literal $l$ appears in the clause $C_l$ but $l$ does not appear in any clause $C_r$ for any $1 \leq r < l$, $h(C_l, 1) = g(l)$ if the literal $l$ appears in the clause $C_l$ but $l$ does not appear in any clause $C_r$ for any $l < r \leq m$, and $h(C_l, 1) = a_i$ otherwise. We first describe $\mathcal{P}_1$ below. The price of every preference in $\mathcal{P}_1$ is 1.

(I) For every $i \in [n]$, we have the following preferences.

$$\triangleright D_{k-2} \succ w_i \succ w'_i \succ a_i \succ b_i \succ \text{others for some } D_{k-2} \subseteq \mathcal{D} \text{ with } |D_{k-2}| = k - 2$$

$$\triangleright D_{k-2} \succ w_i \succ w'_i \succ a_i \succ b_i \succ \text{others for some } D_{k-2} \subseteq \mathcal{D} \text{ with } |D_{k-2}| = k - 2$$

(II) For every $C_j = (l_1 \lor l_2 \lor l_3)$ with $j \in [m]$, we have the following preferences.
\( \mathcal{D}_{k-2} > \ d > y_j > h(C_j, l_1) > d' > \text{others} \), for some \( d, d' \in \mathcal{D} \) for some \( \mathcal{D}_{k-2} \subseteq \mathcal{D} \) with \( |\mathcal{D}_{k-2}| = k - 2 \)

\( \mathcal{D}_{k-2} > \ d > y_j > h(C_j, l_2) > d' > \text{others} \), for some \( d, d' \in \mathcal{D} \) for some \( \mathcal{D}_{k-2} \subseteq \mathcal{D} \) with \( |\mathcal{D}_{k-2}| = k - 2 \)

\( \mathcal{D}_{k-2} > \ d > y_j > h(C_j, l_3) > d' > \text{others} \), for some \( d, d' \in \mathcal{D} \) for some \( \mathcal{D}_{k-2} \subseteq \mathcal{D} \) with \( |\mathcal{D}_{k-2}| = k - 2 \)

We now describe the preferences in \( \mathcal{P}_2 \). The price of every preference in \( \mathcal{P}_1 \) is 10\( mn \).

(I) \( \mathcal{D}_{k-2} > c > d > \text{others} \), for some \( d \in \mathcal{D}, \mathcal{D}_{k-2} \subseteq \mathcal{D} \) with \( |\mathcal{D}_{k-2}| = k - 2 \)

(II) For every \( x \in \{a_i, \bar{a}_i, b_i, \bar{b}_i, w_i, w'_i : i \in [n]\} \), we have 8 copies of the following preference.

\( \mathcal{D}_{k-2} > x > d > \text{others} \), for some \( d \in \mathcal{D}, \mathcal{D}_{k-2} \subseteq \mathcal{D} \) with \( |\mathcal{D}_{k-2}| = k - 2 \)

(III) For every \( x \in \{y_j : j \in [m]\} \), we have 7 copies of the following preference.

\( \mathcal{D}_{k-2} > \mathcal{D}_{k-2} > x > d > \text{others} \), for some \( d \in \mathcal{D}, \mathcal{D}_{k-2} \subseteq \mathcal{D} \) with \( |\mathcal{D}_{k-2}| = k - 2 \)

The budget of the briber is \( (m + n) \). The value of \( \delta_i \) for every voter \( i \) is 2. This finishes the description of the \((3, B2)\)-SAT instance. We now claim that the two instances are equivalent. In one direction, let the \((3, B2)\)-SAT instance be a \textit{YES} instance with a satisfying assignment \( \gamma : \{x_i : i \in [n]\} \rightarrow \{0, 1\} \). Let us consider the following profile \( \Omega \) where the maximum displacement distance of every preference is at most 2 from its corresponding preference in \( \mathcal{P} \).

(I) For every \( i \in [n] \), if \( \gamma(x_i) = 1 \), then we have the following preferences.

\( w_i > w'_i > a_i > b_i > \text{others} \)

\( \bar{a}_i > \bar{b}_i > w_i > w'_i > \text{others} \)

(II) For every \( i \in [n] \), if \( \gamma(x_i) = 0 \), then we have the following preferences.

\( a_i > b_i > w_i > w'_i > \text{others} \)

\( w_i > w'_i > a_i > b_i > \text{others} \)

(III) For every \( C_j = (l_1 \lor l_2 \lor l_3) \) with \( j \in [m] \) and \( \gamma \) makes \( l_1 = 1 \) (we can assume by renaming), then we have the following preferences.

\( d > h(C_j, l_1) > y_j > d' > \text{others} \), for some \( d, d' \in \mathcal{D} \)

\( d > y_j > h(C_j, l_2) > d' > \text{others} \), for some \( d, d' \in \mathcal{D} \)

\( d > y_j > h(C_j, l_3) > d' > \text{others} \), for some \( d, d' \in \mathcal{D} \)

The preferences in \( \mathcal{P}_2 \) remain same in \( \Omega \). Also since exactly \( (n + m) \) preferences in \( \mathcal{P}_1 \) change in \( \Omega \), the total prices of the preferences which are changed is \( B \). Hence the \textit{Local Distance constrained $\mathcal{B}$ribery} instance is a \textit{YES} instance.

In the other direction, let us assume that there exists a profile \( \Omega \) such that the maximum displacement distance of every preference in \( \Omega \) is at most 2 from its corresponding preference in \( \mathcal{P} \). Let the unique Borda winner in \( \Omega \), and the sum of the prices of the preferences which differ in \( \mathcal{P} \) and \( \Omega \) is at most \( (m + n) \). We first observe that the score of \( c \) in \( \Omega \) is the same as in \( \mathcal{P} \) which is 10. Also since every alternative in \( \mathcal{D} \) appears within the first \( k + 10 \) positions at most once in \( \mathcal{P} \), no alternative in \( \mathcal{D} \) can win in \( \Omega \). Also since the budget is \( (m + n) \) and the price of every preference in \( \mathcal{P}_2 \) is more than \( (m + n) \), no preference in \( \mathcal{P}_2 \) change in \( \Omega \). Let the preferences in \( \Omega \) which correspond to \( \mathcal{P}_1 \) be \( \Omega_1 \). Hence, for \( c \) to win uniquely in \( \Omega \), every alternative in \( \{a_i, \bar{a}_i, b_i, \bar{b}_i, w_i, w'_i : i \in [n]\} \) can appear within the first \( k \) positions at most once in \( \Omega_1 \). Also every alternative in \( \{y_j : j \in [m]\} \) can appear within the first \( k \) positions at most twice in \( \Omega_1 \) for \( c \) to win uniquely. Since the budget is \( (m + n) \) and the price of every preference in \( \mathcal{P}_1 \) is 1, for every \( i \in [n] \), there will be exactly
one preference in $Q_1$ where both $w_i$ and $w'_i$ appear within the first $k$ positions and there will be exactly two preferences in $Q_1$ where $y_j$ appears within the first $k$ positions for every $j \in [m]$. Let us now consider the following assignment $\gamma : \{x_i : i \in [n]\} \rightarrow \{0, 1\}$ defined as: $\gamma(x_i) = 0$ if the preference in $Q_1$ where both $w_i$ and $w'_i$ appear within the first $k$ positions has $a_i$ at the $(k+1)$-th position; otherwise $\gamma(x_i) = 1$. We claim that $\gamma$ is a satisfying assignment for the $(3, B2)$-SAT instance. Suppose not, then let us assume that $\gamma$ does not satisfy $C_1 = (l_1 \lor l_2 \lor l_3)$ for some $j \in [m]$. We have already observed that, for $c$ to become the unique k-approval winner in $Q$, the alternative $y_j$ must move to its right by one position in at least one of the 3 preferences in $P_1$ where it appears at the first position. However, it follows from the definition of $\gamma$ that this would make the k-approval score of at least one alternative in $\{f(l_1), f(l_2), f(l_3)\}$ same as the k-approval score of $c$ which contradicts our assumption that $c$ is the unique winner in $Q$. Hence $\gamma$ is a satisfying assignment and thus the $(3, B2)$-SAT instance is a YES instance.

We next present our result for the Borda voting rule.

**Theorem 11.** The Local Distance Constrained Bribery problem is NP-complete for the Borda voting rule for the swap and the maximum displacement distances even when $\delta = 1$. Hence, the Local Distance Constrained Bribery problem is NP-complete for the Borda voting rule for the footrule distance even when $\delta = 2$.

**Proof.** Let us prove the result for the maximum displacement distance first. The Local Distance Constrained Bribery problem for the Borda voting rule for the maximum displacement distance is clearly in NP. To prove NP-hardness, we reduce from $(3, B2)$-SAT to Local Distance Constrained Bribery. Let $(X = \{x_i : i \in [n]\}, C = (C_j : j \in [m]))$ be an arbitrary instance of $(3, B2)$-SAT. Let us consider the following instance $(A, P, c, \delta = 1)$ of Local Distance Constrained Bribery.

$A = \{a_i, \bar{a}_i, z_i : i \in [n]\}$
$\cup \{c\} \cup \{y_j : j \in [m]\}$
$\cup D$, where $|D| = 10m^7n^7$

We construct the profile $P$ which is a disjoint union of two profiles, namely, $P_1$ and $P_2$. We first describe $P_1$ below. While describing the preferences below, whenever we say ‘others’ or ‘for some alternative in $D$’ or ‘for some subset of $D$’, the unspecified alternatives are assumed to be arranged in such a way that, for every unspecified alternative $a \in A \setminus D$, at least 10 alternatives from $D$ appear immediately before $a$. We also ensure that any alternative in $D$ appears within top $10m^7n^7$ positions at most once in $P_1$ whereas every alternative in $A \setminus D$ appears within top $10m$ positions in every preference in $P_1$. This is always possible because $|D| = 10m^7n^7$ whereas $|A \setminus D| = 3n + m + 1$ and $|P_1| = 2n + 3m$. Let $f$ be a function defined on the set of literals as $f(x_i) = a_i$ and $f(x_i) = \bar{a}_i$ for every $i \in [n]$.

(I) For every $i \in [n]$, we have the following preferences.

$\triangleright z_i > a_i > d > d' > c >$ others, for some $d, d' \in D$

$\triangleright z_i > \bar{a}_i > d > d' > c >$ others, for some $d, d' \in D$

(II) For every $C_j = (l_1 \lor l_2 \lor l_3)$ with $j \in [m]$, we have the following preferences.

$\triangleright y_j > f(l_1) > d > c >$ others, for some $d \in D$

$\triangleright y_j > f(l_2) > d > c >$ others, for some $d \in D$

$\triangleright y_j > f(l_3) > d > c >$ others, for some $d \in D$

This finishes the description of $P_1$. Let the function $s_1(\cdot)$ maps every alternative in $A$ to the Borda score it receives from $P_1$. We observe that $s_1$ is integer valued and $s_1(c)$ is the unique maximum of $s_1$ for large enough $n$ and $m$. We now describe the profile $P_2$. Let $N_1$ be the number of preferences in $P_1$. While describing the preferences in $P_2$, we will leave the choice of alternatives in $D$ unspecified. However, we
assume that those choice are made in such a way that any alternative in \( \mathcal{D} \) appears within the first 10mn positions at most once in \( \mathcal{P}_2 \). This is always possible since \(|\mathcal{D}| = 10m^7n^7\) and \(|\mathcal{P}_2| \leq 10m^3n^3\). Let \( M = |\mathcal{A} \setminus \mathcal{D}| \); we have \( M = 4n + m + 1 \).

(I) For every \( i \in [n] \), we have the following preferences.

\[
\begin{align*}
\triangleright s_1(c) - s_1(z_i) + 2N_1 - 2 & \text{ preferences of type } d_0 \succ d_1 \succ c \succ b_1 \succ d_2 \succ b_2 \succ d_3 \succ \cdots \succ b_{M-2} \succ d_{M-1} \succ z_i \succ \text{others}, \text{ where } \mathcal{A} \setminus \mathcal{D} = \{c, z_i\} \cup \{b_k : k \in [M-2]\} \text{ and } d_{\ell} \in \mathcal{D} \text{ for } 0 \leq \ell \leq M-1. \\
\triangleright s_1(c) - s_1(z_i) + 2N_1 - 2 & \text{ preferences of type } z_i \succ d_0 \succ d_1 \succ b_{M-2} \succ d_2 \succ \cdots \succ b_1 \succ d_{M-1} \succ d_M \succ d_{M+1} \succ c \succ \text{others}, \text{ where } \mathcal{A} \setminus \mathcal{D} = \{c, z_i\} \cup \{b_k : k \in [M-2]\} \text{ and } d_{\ell} \in \mathcal{D} \text{ for } 0 \leq \ell \leq M+1. \\
\triangleright s_1(c) - s_1(a_i) + 2N_1 - 5 & \text{ preferences of type } d_0 \succ d_1 \succ c \succ b_1 \succ d_2 \succ b_2 \succ d_3 \succ \cdots \succ b_{M-2} \succ d_{M-1} \succ a_i \succ \text{others}, \text{ where } \mathcal{A} \setminus \mathcal{D} = \{c, a_i\} \cup \{b_k : k \in [M-2]\} \text{ and } d_{\ell} \in \mathcal{D} \text{ for } 0 \leq \ell \leq M-1. \\
\triangleright s_1(c) - s_1(a_i) + 2N_1 - 5 & \text{ preferences of type } a_i \succ d_0 \succ d_1 \succ b_{M-2} \succ d_2 \succ \cdots \succ b_1 \succ d_{M-1} \succ d_M \succ d_{M+1} \succ c \succ \text{others}, \text{ where } \mathcal{A} \setminus \mathcal{D} = \{c, a_i\} \cup \{b_k : k \in [M-2]\} \text{ and } d_{\ell} \in \mathcal{D} \text{ for } 0 \leq \ell \leq M+1. \\
\end{align*}
\]

(II) For every \( j \in [m] \), we have the following preferences.

\[
\begin{align*}
\triangleright s_1(c) - s_1(y_j) + 2N_1 - 2 & \text{ preferences of type } d_0 \succ d_1 \succ c \succ b_1 \succ d_2 \succ b_2 \succ d_3 \succ \cdots \succ b_{M-2} \succ d_{M-1} \succ y_j \succ \text{others}, \text{ where } \mathcal{A} \setminus \mathcal{D} = \{c, y_j\} \cup \{b_k : k \in [M-2]\} \text{ and } d_{\ell} \in \mathcal{D} \text{ for } 0 \leq \ell \leq M-1. \\
\triangleright s_1(c) - s_1(y_j) + 2N_1 - 2 & \text{ preferences of type } y_j \succ d_0 \succ d_1 \succ b_{M-2} \succ d_2 \succ \cdots \succ b_1 \succ d_{M-1} \succ d_M \succ d_{M+1} \succ c \succ \text{others}, \text{ where } \mathcal{A} \setminus \mathcal{D} = \{c, y_j\} \cup \{b_k : k \in [M-2]\} \text{ and } d_{\ell} \in \mathcal{D} \text{ for } 0 \leq \ell \leq M+1.
\end{align*}
\]

This finishes the description of \( \mathcal{P}_2 \) and thus of \( \mathcal{P} \). Since \( s_1(c) \) is upper bounded by \( 10m^7n^7 \), \( \mathcal{P} \) has only polynomially (in \( m, n \)) many preferences. Let \( N_1 = |\mathcal{P}_2| \) and \( N = |\mathcal{P}| = N_1 + N_2 \). We summarize the Borda scores of every alternative from \( \mathcal{P} \) in Table 3. Let the function \( s(\cdot) \) maps every alternative in \( \mathcal{A} \) to the Borda score it receives from \( \mathcal{P} \). We now claim that the two instance are equivalent.

| Alternatives | Borda scores from \( \mathcal{P} \) |
|--------------|----------------------------------|
| \( a_i, a_i \forall i \in [n] \) | \( s(c) + 2N - 5 \) |
| \( z_i, \forall i \in [n] \) | \( s(c) + 2N - 2 \) |
| \( y_j, \forall j \in [m] \) | \( s(c) + 2N - 2 \) |
| \( d \in \mathcal{D} \) | \( < s(c) - 10m^2n^2 \) |

Table 3: Borda scores of the alternatives in \( \mathcal{A} \) from \( \mathcal{P} \).

In one direction, let us assume that the \((3, B2)\)-SAT instance is a \textbf{YES} instance. Let \( g : X \rightarrow \{0, 1\} \) be a satisfying assignment for the \((3, B2)\)-SAT instance. Let us consider the following profile \( \Omega \rightarrow \{0, 1\} \) where the maximum displacement distance of every preference in \( \Omega \) from its corresponding preference in \( \mathcal{P} \) is at most 1. The preferences in \( \Omega \) which corresponds to \( \mathcal{P}_1 \) are as follows.

(I) For every \( i \in [n] \), we have the following preferences.

\[
\begin{align*}
\triangleright & \text{ If } g(x_i) = 1, \text{ then } \\
& - z_i \succ d \succ a_i \succ c \succ d' \succ \text{others}, \text{ for some } d, d' \in \mathcal{D} \\
& - a_i \succ z_i \succ d \succ c \succ d' \succ \text{others}, \text{ for some } d, d' \in \mathcal{D} \\
\triangleright & \text{ Else } \\
& - a_i \succ z_i \succ d \succ c \succ d' \succ \text{others}, \text{ for some } d, d' \in \mathcal{D}
\end{align*}
\]
Let us now consider the following assignment $g: \{x_i : i \in [n]\} \rightarrow \{0, 1\}$ defined as: $g(x_i) = 0$ if $a_i$ appears at the first position in at least one of the two preferences in $\mathcal{P}_1$ where $z_i$ appears at the first position; we define $g(z_i) = 1$ otherwise. We claim that $g$ is a satisfying assignment for the $(3, B_2)$-SAT instance. Suppose not, then let us assume that $g$ does not satisfy $C_j = (l_1 \lor l_2 \lor l_3)$ for some $j \in [m]$. We observe that, for $c$ to become the unique Borda winner, the alternative $y_j$ must move to its right by one position in at least one of the 3 preferences in $\mathcal{P}_1$ where it appears at the first position (which are exactly the preferences in Item (I)). However, this makes either $a_i$ or $a_i$, $i \in [n]$ to satisfy the condition that its maximum displacement distance from $P$ is at most 1, no alternative from $\mathcal{D}$ wins in $\Omega$ under the Borda voting rule. We can assume without loss of generality that, in every preference in $\Omega$, the alternative $c$ is moved to its left by 1 position since $c$ never appears at the first position in any preference in $\mathcal{P}$. We can also assume without loss of generality that every alternative in $A \setminus (\mathcal{D} \cup \{c\})$ moves to its right by 1 position in every preference corresponding to $\mathcal{P}_2$ since no alternative in $A \setminus (\mathcal{D} \cup \{c\})$ appears at the last position in any preference in $\mathcal{P}_2$ and every alternative in $A \setminus (\mathcal{D} \cup \{c\})$ appears at the last position in any preference in $\mathcal{P}_2$. We now observe that, for $c$ to become the unique Borda winner in $\Omega$, every alternative $z_i, i \in [n]$ must move to its right by one position in at least one of the two preferences in $\mathcal{P}_1$ where $z_i$ appears at the first position (which are exactly the preferences in Item (I)). However, this makes either $a_i$ or $a_i$, $i \in [n]$ to appear at the first position in at least one of the two preferences in $\mathcal{P}_1$ where $z_i$ appears at the first position. Let us now consider the following assignment $g: \{x_i : i \in [n]\} \rightarrow \{0, 1\}$ defined as: $g(x_i) = 0$ if $a_i$ appears at the first position in at least one of the two preferences in $\mathcal{P}_1$ where $z_i$ appears at the first position; we define $g(z_i) = 1$ otherwise. We claim that $g$ is a satisfying assignment for the $(3, B_2)$-SAT instance. Suppose not, then let us assume that $g$ does not satisfy $C_j = (l_1 \lor l_2 \lor l_3)$ for some $j \in [m]$. We observe that, for $c$ to become the unique Borda winner, the alternative $y_j$ must move to its right by one position in at least one of the 3 preferences in $\mathcal{P}_1$ where it appears at the first position (which are exactly the preferences in Item (I)). However, it follows from the definition of $g$ that this would make the Borda score of at least one alternative in $\{f(l_1), f(l_2), f(l_3)\}$ same as the Borda score of $c$ which contradicts our assumption that $c$ is the unique Borda winner in $\Omega$. Hence $g$ is a satisfying assignment and thus the $(3, B_2)$-SAT instance is a $\text{YES}$ instance.

For proving the result for the swap and footrule distances, we change the preferences in $\mathcal{P}_2$ by shifting every alternative in $A \setminus (\mathcal{D} \cup \{c\})$ to their right by 1 position. Then analogous argument proves the result. $
$ 

The proof of Theorem 11 can be adapted to prove the following result.
Theorem 12. Let \( \alpha = (\alpha_i)_{i \in [m]} \) be a score vector with \( \alpha_i - \alpha_{i+1} = \alpha_{i+1} - \alpha_{i+2} > 0 \) for some \( j \in [m/2] \). The LOCAL DISTANCE constrained bribery problem is NP-complete for the scoring rule with score vector \( \alpha \) for the swap and the maximum displacement distances even when \( \delta = 1 \). Hence, the LOCAL DISTANCE constrained bribery problem is NP-complete for the scoring rule with score vector \( \alpha \) for the footrule distance even when \( \delta = 2 \).

We will use the following structural lemma in our proofs for the maximin and Copeland voting rules.

Lemma 13. Let \( A = B \cup C \) be a set of alternatives, \( Z_{(a,b)} \), \( a, b \in B \), \( a \neq b \), non-negative integers which are either all even or all odd, and \( Z_{(a,b)} = -Z_{(b,a)} \) for every \( a, b \in B, a \neq b \). Let \( |C| > 10K^2|B|^2 \sum_{a,b \in B, a \neq b} |Z_{(a,b)}| \).

Then there exists a profile \( \mathcal{P} \) such that

(i) \( \mathcal{D}_\gamma(a,b) = Z_{(a,b)} \) for every \( a, b \in B, a \neq b, \mathcal{D}_\gamma(b',c') > 0 \) for every \( b' \in B, c' \in C \).

(ii) for every alternative \( b \in B \), there are at least \( \kappa/2 \) alternatives from \( C \) in the immediate \( \kappa/2 \) positions on both left and right of \( a \).

(iii) For every \( c \in C \), there exists at most 1 preference in \( \mathcal{P} \) where the distance of \( c \) from any alternative in \( B \) is less than \( \kappa/2 \).

Moreover, \( \mathcal{P} \) contains \( \text{poly}(\sum_{a,b \in B, a \neq b} |Z_{(a,b)}|) \) many preferences, and there is an algorithm for constructing \( \mathcal{P} \) which runs in time polynomial in \( \sum_{a,b \in B, a \neq b} |Z_{(a,b)}| + m \).

Proof. Let \( |B| = \ell, |C| = \sum_{a,b \in B, a \neq b} c_{(a,b)} \) where \( c_{(a,b)} \)s are pairwise disjoint and \( |c_{(a,b)}| \geq 4K\ell (|Z_{(a,b)}| + |Z_{(b,a)}|) \) for every \( a, b \in B, a \neq b \).

First let us prove the result when \( Z_{(a,b)}, \ a, b \in B, a \neq b \), are non-negative integers which are either all even.

For \( a, b \in B, a \neq b \), let \( c_{(a,b)} = \sum_{i=1}^{|Z_{(a,b)}|+|Z_{(b,a)}|} C_{(a,b)} \) where \( C_{(a,b)} \)s are pairwise disjoint and \( |C_{(a,b)}| \geq 2K\ell \). For \( a, b \in B, a \neq b \) such that \( Z_{(a,b)} > 0 \), we have the following preferences.

Let \( B = \{b_i : i \in [\ell - 2]\} \cup \{a, b\} \).

\( \triangleright (Z_{(a,b)})/2 \) copies of: \( a > e_{1}^{(a,b)} > b > e_{2}^{(a,b)} > b > e_{3}^{(a,b)} > \ldots > b_{\ell-2} > e_{\ell}^{(a,b)} \cdots \)

\( \triangleright (Z_{(a,b)})/2 \) copies of: \( b_{\ell-2} > e_{\ell-1}^{(a,b)} > b_{\ell-3} > e_{\ell-2}^{(a,b)} > \ldots > e_{2\ell-1}^{(a,b)} > a > e_{2\ell}^{(a,b)} > b > \ldots \)

Let \( \mathcal{P} \) be the resulting profile. It is immediate that \( \mathcal{P} \) satisfies all the conditions in the statement. This proves the statement when \( Z_{(a,b)}, \ a, b \in B, a \neq b \), are non-negative integers which are either all even.

From the proof above, it follows that the statement holds even when \( |C| = 8K^2|B|^2 \sum_{a,b \in B, a \neq b} |Z_{(a,b)}| \).

Let \( B = \{b_i : i \in [\ell]\} \). Let \( C = c' \cup (\cup_{i=1}^{\ell} c_i) \) where \( |c_i| = 5K \) for every \( i \in [\ell] \). Let \( \mathcal{C} \) be the following preference.

\( \triangleright = b_1 > e_1 > b_2 > e_2 > \ldots > e_{\ell-1} > b_{\ell} > e_{\ell} > \ldots \)

Let us define \( Z' : B \times B \rightarrow Z \) as follows.

\[ Z'(b_i, b_j) = \begin{cases} Z(b_i, b_j) + 1 & \text{if } i > j \\ Z(b_i, b_j) - 1 & \text{otherwise} \end{cases} \]

Let \( \mathcal{R} \) be the profile which satisfies the conditions in the statement for the set of alternatives \( B \cup C' \) and the integers \( Z'_{(a,b)}, \ a, b \in B, a \neq b \). We append the alternatives in \( \cup_{i=1}^{\ell} c_i \) at the bottom of every preference in \( \mathcal{R} \); let the resulting profile be \( \mathcal{Q} \). Clearly, \( (\mathcal{Q}, \triangleright) \) satisfies all the conditions in the statement.

We now present our result for the maximin voting rule.

Theorem 14. The LOCAL DISTANCE constrained bribery problem is NP-complete for the maximin voting rule for the swap distance and the maximum displacement distance even when \( \delta = 1 \). Hence, the LOCAL DISTANCE constrained bribery problem is NP-complete for the maximin voting rule for the footrule distance even when \( \delta = 2 \).
| Edges in weighted majority graph | weight in $\mathcal{P}$ | weight in $\mathcal{Q}$ | Maximin score of $c$ is $-10$ in $\mathcal{P}$ and $\mathcal{Q}$ |
|---------------------------------|--------------------------|--------------------------|-------------------------------------------------|
| $(b, c)$                        | 10                       | 10                       |                                                 |
| $(a_i, z_i), (w_i, a_i) \forall i \in [n]$ | 8                        | 12                       | Maximin scores of $a_i, a_i, i \in [n]$ are $-8$ in $\mathcal{P}$ and $\mathcal{Q}$. Maximin scores of $a_i, a_i, i \in [n]$ are $-12$ in $\mathcal{Q}$. Maximin scores of $z_i, i \in [n]$ are $-8$ in $\mathcal{P}$. Maximin scores of $z_i, i \in [n]$ are $-12$ in $\mathcal{Q}$. |
| $(a_i, z_i), (a_i, z_i) \forall i \in [n]$ | 8                        | $-12$                    |                                                 |
| If $g(x_i) = 1$, then $(a_i, z_i)$; else $(a_i, z_i), i \in [n]$ | $-12$                    | $-12$                    | Maximin score of $z_i$ is $-12$ in $\mathcal{Q}$. |
| $\forall j \in [m]$ if $C_j = (l_1 \vee l_2 \vee l_3), (f(l_k), y_j) \forall k \in [|\bar{C}|]$ | 10                       | $-12$                    | Maximin score of $y_j$ is $-10$ in $\mathcal{P}$. Maximin score of $y_j$ is $-12$ in $\mathcal{Q}$. |
| $\forall j \in [m]$ if $C_j = (l_1 \vee l_2 \vee l_3), (f(l_k), y_j) \text{ for some } k \in [|\bar{C}|]$ | $-12$                    | $-12$                    | Maximin score of $y_j$ is $-12$ in $\mathcal{Q}$. |
| $(y_i, b), (b, w_i), \forall i \in [n]$ | 12                       | 12                       | Maximin scores of $b, w_i, i \in [n]$ are $-12$ in $\mathcal{P}$ and $\mathcal{Q}$. |
| $(a, d) \forall a \in A \setminus D, d \in D$ | $10mn$                   | $10mn$                   | Maximin score of $d$ is $-10mn$ in $\mathcal{P}$ and $\mathcal{Q}$. |
| Any edge not mentioned above    | $0$                      | $-12$                    | Weights are same in both $\mathcal{P}$ and $\mathcal{Q}$. |

Table 5: Weighted majority graph for $\mathcal{P}$ in Theorem 14.

**Proof.** Let us first prove the result for the swap distance. The **Local Distance Constrained Bribery** problem for the maximin voting rule for the swap distance is clearly in NP. To prove NP-hardness, we reduce from $(3, B2)$-SAT to **Local Distance Constrained Bribery**. Let $(X = \{x_i : i \in [n]\}, \mathcal{C} = \{C_j : j \in [m]\})$ be an arbitrary instance of $(3, B2)$-SAT. Let us consider the following instance $(A, \mathcal{P}, c, \delta = 1)$ of **Local Distance Constrained Bribery**.

$$A = \{a_i, \bar{a}_i, w_i, z_i : i \in [n]\} \cup \{y_j : j \in [m]\} \cup \{c, b\} \cup D,$$

where $|D| = 10m^5n^5$.

We construct the profile $\mathcal{P}$ which is a disjoint union of two profiles, namely, $\mathcal{P}_1$ and $\mathcal{P}_2$. We first describe $\mathcal{P}_1$ below. While describing the preferences below, whenever we say ‘others’, the unspecified alternatives are assumed to be arranged in such a way that, for every unspecified alternative $a \in A \setminus D$, there are at least 10 alternatives from $D$ in the immediate 10 positions on both left and right of $a$. We also ensure that any alternative in $D$ appears within top 10mn positions at most once in $\mathcal{P}_1$, whereas every alternative in $A \setminus D$ appears within top 10mn position in every preference in $\mathcal{P}_1$. This is possible because $|D| = 10m^5n^5$ and $|A \setminus D| = 4n + m + 2$. Let $f$ be a function defined on the set of literals as $f(x_i) = a_i$ and $f(\bar{x}_i) = \bar{a}_i$ for every $i \in [n]$.

(I) For every $i \in [n]$, we have the following preferences.

- 2 copies of $z_i \succ a_i \succ w_i \succ$ others
- 2 copies of $z_i \succ \bar{a}_i \succ w_i \succ$ others

(II) For every $C_j = (l_1 \vee l_2 \vee l_3), j \in [m]$, we have the following preferences. Let $h$ be a function defined on the set of literals as $h(x_i) = w_i$ and $h(\bar{x}_i) = \bar{w}_i$ for every $i \in [n]$.

- $y_j \succ f(l_1) \succ h(l_1) \succ$ others
- $y_j \succ f(l_2) \succ h(l_2) \succ$ others
- $y_j \succ f(l_3) \succ h(l_3) \succ$ others

Due to Lemma 13 (using $K = 30$ in Lemma 13), there exists a profile $\mathcal{P}_2$ consisting of poly($m, n$) preferences such that the weighted majority graph of the profile $\mathcal{P} = \mathcal{P}_1 \cup \mathcal{P}_2$ is as described in Table 5. Moreover, due to Lemma 13, in every preference in $\mathcal{P}_2$, for every alternative $a \in A \setminus D$, there are at least 10 alternatives from $D$ in the immediate 10 positions on both left and right of $a$, and the distance of every $d \in D$ from every $a \in A \setminus D$ is less than 10 at most once in $\mathcal{P}_2$. This finishes the description of $\mathcal{P}$. We now claim that the two instances are equivalent.
In one direction, let us assume that the \((3, B2)\)-SAT instance is a \textsc{yes} instance with a satisfying assignment \(g : \{x_i\}_i \rightarrow \{0, 1\}\). Let us consider the following profile \(Q\) where every preference is obtained from the corresponding preference in \(P\) by performing at most 1 swap. The preferences in \(Q\) which corresponds to \(P_1\) are as follows.

(I) For every \(i \in [n]\), we have the following preferences.

\[\begin{align*}
&\text{If } g(x_i) = 1, \text{ then } \\
&\quad \begin{cases} 
2 \text{ copies of } z_i > w_i > a_i > \text{others} \\
2 \text{ copies of } \bar{a}_i > z_i > w_i > \text{others}
\end{cases} \\
&\text{Else} \\
&\quad \begin{cases} 
2 \text{ copies of } a_i > z_i > w_i > \text{others} \\
2 \text{ copies of } z_i > w_i > \bar{a}_i > \text{others}
\end{cases}
\end{align*}\]

(II) For every \(j \in [m]\), if \(C_j = (l_1 \lor l_2 \lor l_3)\) and \(g\) makes \(l_1 = 1\) (we can assume by renaming), then we have

\[\begin{align*}
&\text{if } f(l_1) > y_j > h(l_1) > \text{others} \\
&\text{if } y_j > f(l_2) > h(l_2) > \text{others} \\
&\text{if } y_j > h(l_3) > f(l_3) > \text{others}
\end{align*}\]

The preferences in \(P_2\) remain unchanged in \(P\) and \(Q\). It can be verified that \(c\) is the unique maximin winner in \(Q\). We describe the weighted majority graph for \(Q\) in Table 5 which shows that \(c\) is the unique maximin winner in \(Q\).

In the other direction, let \(Q = Q_1 \cup Q_2\) be a profile where \(c\) is the unique maximin winner, the sub-profile \(Q_k\) corresponds to \(P_k\) for \(k \in [2]\), and every preference in \(Q\) is at most one swap away from its corresponding preference in \(P\). We begin with the observation that, since in every preference in \(Q\), for every alternative \(a \in A \setminus D\), there are at least 10 alternatives from \(D\) in the immediate 10 positions on both left and right of \(a\) and, for every alternative \(d \in D\), there is at most two preferences in \(P_2\) where the distance of \(d\) from any alternative in \(A \setminus D\) is at most 10, performing any one swap in any preference in \(P_2\) leaves the maximin score of every alternative in \(A \setminus D\) unchanged. Also, it is clear that any alternative in \(D\) can never win by performing at most one swap per preference in \(P\). So, let us assume without loss of generality, that \(Q_2 = P_2\). Hence, the weighted majority graph of \(P_1 \cup Q_2\) is also as described in the second column of Table 5. Since, there are at least 10 alternatives from \(D\) in the immediate 10 positions on both left and right of \(c\) in every preference in \(P_1\), the maximin score of \(c\) is \(-10\) in \(Q\). Hence, for \(c\) to win uniquely, \(z_i\) must be preferred over either \(a_i\) or \(\bar{a}_i\) in at least twice of the 4 preferences in \(P_1\) where \(z_i\) appears at the first position. Let us now consider the following assignment \(g : \{x_i : i \in [n]\} \rightarrow \{0, 1\}\) defined as: \(g(x_i) = 0\) if \(a_i\) is preferred over \(z_i\) twice among the preferences in \(Q_1\) which correspond to the 4 preferences in \(P_1\) where \(z_i\) appears at the first position; otherwise \(g(x_i) = 1\). We claim that \(g\) is a satisfying assignment for the \((3, B2)\)-SAT instance. Suppose not, then let us assume that \(g\) does not satisfy \(C_j = (l_1 \lor l_2 \lor l_3)\) for some \(j \in [m]\). We first observe that, since \(C_j\) is not satisfied, from the definition of \(g\), for every \(k \in [3]\), the maximin score of \(f(l_k)\) is \(-8\) in the profile \(Q_k^\dagger \cup Q_2\), where \(Q_k^\dagger\) contains every preference in \(Q_1\) except the 2 preferences corresponding to the preferences \(>_{-1}, >_{-2}\) in \(P_1\) where \(f(l_k)\) appears immediately after \(y_{t_k}\) for some \(t \in [m]\) and contains \(>_{-1}\) and \(>_{-2}\). Now, for \(c\) to win uniquely, there must exist a \(k \in [3]\) such that \(f(l_k)\) is preferred over \(y_j\) in at least one of the three preferences in \(Q_1\) which corresponds to the 3 preferences corresponding to \(j\). Let us assume without loss of generality that \(f(l_1)\) is preferred over \(y_j\) in one of the three preferences in \(Q_1\) which corresponds to the 3 preferences corresponding to \(j\). Then the maximin score of \(f(l_1)\) in \(Q\) is at least \(-10\) which contradicts our assumption that \(c\) is the unique maximin winner in \(Q\). Hence, \(g\) is a satisfying assignment for the \((3, B2)\)-SAT instance and thus the \((3, B2)\)-SAT instance is a \textsc{yes} instance.

The exact same reduction and analogous proof proves the result for the footrule distance and the maximum displacement distance. \[\square\]
We now prove the result for the Copeland\(^\alpha\) voting rule for any \(\alpha \in [0, 1]\).

**Theorem 15.** Let \(\alpha \in [0, 1]\). Then the Local Distance constrained bribery problem is NP-complete for the Copeland\(^\alpha\) voting rule for the swap distance and the maximum displacement distance even when \(\delta = 1\). Hence, the Local Distance constrained bribery problem is NP-complete for the Copeland\(^\alpha\) voting rule for the footrule distance even when \(\delta = 2\).

**Proof.** Let us first prove the result for the swap distance. The Local Distance constrained bribery problem for the Copeland\(^\alpha\) voting rule for the swap distance is clearly in NP. To prove NP-hardness, we reduce from \((3, B2)\)-SAT to Local Distance constrained bribery. Let \((X = \{x_i : i \in [n]\}, C = \{C_j : j \in [m]\})\) be an arbitrary instance of \((3, B2)\)-SAT. Let us consider the following instance \((A, \mathcal{P}, c, \delta = 1)\) of Local Distance constrained bribery.

\[
\begin{align*}
\mathcal{A} = \{a_i, \bar{a}_i, z_i : i \in [n]\} \\
\cup \{c, w\} \cup \{y_j : j \in [m]\} \cup \mathcal{D}, \text{ where } |\mathcal{D}| = 10m^8n^8
\end{align*}
\]

We construct the profile \(\mathcal{P}\) which is a disjoint union of two profiles, namely, \(\mathcal{P}_1\) and \(\mathcal{P}_2\). We first describe \(\mathcal{P}_1\) below. While describing the preferences in \(\mathcal{P}_1\), whenever we say ‘others’ or ‘for some alternative in \(\mathcal{D}\)’ or ‘for some subset of \(\mathcal{D}\)’, the unspecified alternatives are assumed to be arranged in such a way that, for every unspecified alternative \(a \in A \setminus \mathcal{D}\), there are at least 10 alternatives from \(\mathcal{D}\) in the immediate 10 positions on both left and right of \(a\). We also ensure that any alternative in \(\mathcal{D}\) appears within top 10mn positions at most once in \(\mathcal{P}_1\) whereas every alternative in \(\mathcal{A}\) appears within top 10mn position in every preference in \(\mathcal{P}_1\). This is possible \(|\mathcal{D}| = 10m^8n^8, |A \setminus \mathcal{D}| = 3n + m + 2, \text{ and } |\mathcal{P}_1| = 2n + 3m\). Let \(f\) be a function defined on the set of literals as \(f(x_i) = a_i\) and \(f(\bar{x}_i) = \bar{a}_i\) for every \(i \in [n]\).

(i) For every \(i \in [n]\), we have the following preferences.

- \(z_i \succ a_i \succ w \succ \text{others}\)
- \(z_i \succ \bar{a}_i \succ w \succ \text{others}\)

(ii) For every \(j \in [m]\) if \(C_j = (l_1 \lor l_2 \lor l_3)\), we have the following preferences.

- \(y_j \succ f(l_1) \succ \text{others}\)
- \(y_j \succ f(l_2) \succ \text{others}\)
- \(y_j \succ f(l_3) \succ \text{others}\)

Due to Lemma 13, there exists a profile \(\mathcal{P}_2\) consisting of \(\text{poly}(m, n)\) preferences such that the weighted majority graph \(G\) of the profile \(\mathcal{P} = \mathcal{P}_1 \cup \mathcal{P}_2\) has the following properties. There is no tie in \(G\). There exists a positive integer \(N < |\mathcal{A}|\) such that,

(i) For every \(i \in [n]\), \(a_i\) and \(\bar{a}_i\) defeat exactly \(N - 2\) alternatives including \(w\) and lose against the remaining alternatives.

(ii) For every \(C_j = (l_1 \lor l_2 \lor l_3), j \in [m]\), then \(y_j\) defeats exactly \(N\) alternatives including the alternatives \(f(l_1), f(l_2), f(l_3)\) and loses against the remaining alternatives.

(iii) For every \(i \in [n]\), \(z_i\) defeats exactly \(N\) alternatives including the alternatives \(a_i\) and \(\bar{a}_i\) for every \(i \in [n]\) and loses against the remaining alternatives.

(iv) The alternative \(c\) defeats exactly \(N\) alternatives and loses against the remaining alternatives.

(v) The alternative \(w\) and every alternative in \(\mathcal{D}\) defeat at most \(N - 1\) alternatives and lose against the remaining alternatives.

18
(vi) The weight of all the edges described above is $10m^3n^3 + 1$ except the following. The weight of the edges $(z_i, a_i), (z_i, \bar{a}_i), (a_i, \bar{w}), (\bar{a}_i, \bar{w})$ are 1 for every $i \in [n]$. The weight of the edges $(y_j, f(l_1)), (y_j, f(l_2)), (y_j, f(l_3))$ are 1 for every $j \in [m]$ where $C_j = (l_1 \lor l_2 \lor l_3)$.

Moreover, due to Lemma 13 (using $K = 30$ in Lemma 13), in every preference in $\mathcal{P}_2$, for every alternative $a \in \mathcal{A} \setminus \mathcal{D}$, there are at least 10 alternatives from $\mathcal{D}$ in the immediate 10 positions on both left and right of $a$, and the distance of every $d \in \mathcal{D}$ from every $a \in \mathcal{A} \setminus \mathcal{D}$ is less than 10 at most once in $\mathcal{P}_2$. This finishes the description of $\mathcal{P}$. We now claim that the two instances are equivalent.

In one direction, let us assume that the $(3, B_2)$-SAT instance is a $\text{YES}$ instance with satisfying assignment $g : \mathcal{X} \rightarrow \{0, 1\}$. Let us consider the following profile $\Omega$ where every preference is obtained from the corresponding preference in $\mathcal{P}$ by performing at most 1 swap. The preferences in $\Omega$ which corresponds to $\mathcal{P}_1$ are as follows.

(I) For every $i \in [n]$, we have the following preferences.

- If $g(x_i) = 1$, then
  - $z_i \succ w \succ a_i \succ \text{others}$
  - $a_i \succ z_i \succ w \succ \text{others}$
- Else
  - $a_i \succ z_i \succ w \succ \text{others}$
  - $z_i \succ w \succ a_i \succ \text{others}$

(II) For every $j \in [m]$, if $C_j = (l_1 \lor l_2 \lor l_3)$ and $g$ makes $l_1 = 1$ (we can assume by renaming), then we have

- $f(l_1) \succ y_j \succ \text{others}$
- $y_j \succ h(l_2) \succ \text{others}$
- $y_j \succ h(l_3) \succ \text{others}$

The preferences in $\mathcal{P}_2$ remain unchanged in $\mathcal{P}$ and $\Omega$. We observe that the Copeland$^\alpha$ score of $c$ is $N$ whereas the Copeland$^\alpha$ score of every alternative is at most $N - 1$. Thus $c$ is the unique Copeland$^\alpha$ winner in $\Omega$.

In the other direction, let $\Omega = \Omega_1 \cup \Omega_2$ be a profile where $c$ is the unique Copeland$^\alpha$ winner, the sub-profile $\Omega_k$ for $k \in [2]$, and every preference in $\Omega$ is at most 1 swap away from its corresponding preference in $\mathcal{P}$. We begin with the observation that, due to the structure of $\mathcal{P}_1$ and $\mathcal{P}_2$ (from Lemma 13), any change in any preference in $\mathcal{P}_2$ up to 1 swap, irrespective of $\Omega_1$, leaves the Copeland$^\alpha$ score of every alternative in $\mathcal{A} \setminus \mathcal{D}$ unchanged. So we can assume without loss of generality that $\Omega_2 = \mathcal{P}_2$. Since, for every alternative $a \in \mathcal{A} \setminus \mathcal{D}$, there are at least 10 alternatives from $\mathcal{D}$ in the immediate 10 positions on both left and right of $a$ in every preference in $\mathcal{P}_1$, the weight of any edge incident on $s$ in the weighted majority graph is $10m^3n^3 + 1$, and any alternative in $\mathcal{A} \setminus \{c\}$ follows immediately $c$ in at most one preference in $\mathcal{P}$, the Copeland$^\alpha$ score of $c$ is $N$ in $\Omega$. Hence, for $c$ to win uniquely, $z_i$ must be preferred over either $a_i$ or $\bar{a}_i$ in at least 1 of the 2 preferences in $\mathcal{P}_1$ where $z_i$ appears at the first position. Let us now consider the following assignment $g : \{x_i : i \in [n]\} \rightarrow \{0, 1\}$ defined as: $g(x_i) = 1$ if $\bar{a}_i$ is preferred over $z_i$ once among the preferences in $\Omega_1$ which corresponds to the 2 preferences in $\mathcal{P}_1$ where $z_i$ appears at the first position; otherwise $g(x_i) = 0$. We claim that $g$ is a satisfying assignment for the $(3, B_2)$-SAT instance. Suppose not, then let us assume that $g$ does not satisfy $C_j = (l_1 \lor l_2 \lor l_3)$ for some $j \in [m]$. Let $h$ be a function defined on the set of literals as $h(x_i) = z_i$ and $h(x_i) = \bar{z}_i$ for every $i \in [n]$. We first observe that, since $C_j$ is not satisfied, from the definition of $g$, for every $k \in [3]$, the alternative $f(l_k)$ defeats $h(l_k)$ in $\Omega$. Hence, for $c$ to become the unique Copeland$^\alpha$ winner in $\Omega$, $f(l_k)$ must lose to $y_j$ for every $k \in [3]$. However, this makes the Copeland$^\alpha$ score of $y_j$ in $\Omega$ which contradicts our assumption that $c$ is the unique Copeland$^\alpha$ winner in $\Omega$. Hence, $g$ is a satisfying assignment for the $(3, B_2)$-SAT instance and thus the $(3, B_2)$-SAT instance is a $\text{YES}$ instance.

The exact same reduction and analogous proof proves the result for the footrule distance and the maximum displacement distance.
Theorem 16. The Local Distance constrained bribery problem is NP-complete for the simplified Bucklin voting rule for the swap distance even when $\delta = 2$. Hence the Local Distance constrained bribery problem is NP-complete for the simplified Bucklin voting rule for the footrule distance even when $\delta = 4$.

Proof. The Local Distance constrained bribery problem for the simplified Bucklin voting rule for the swap distance is clearly in NP. To prove NP-hardness, we reduce from $(3,B2)$-SAT to Local Distance constrained bribery. Let $\{X = \{x_i : i \in [n]\}, C = \{C_j : j \in [m]\}\}$ be an arbitrary instance of $(3,B2)$-SAT. Let us consider the following instance $(A, P, c, \delta = 2)$ of Local Distance constrained bribery.

$$A = \{a(x_i, 0), a(x_i, 1), a(\hat{x}_i, 0), a(\hat{x}_i, 1) : i \in [n]\} \cup \{c, u\}$$

$$\cup \{w_i : i \in [n]\} \cup \{y_j : j \in [m]\} \cup D, \text{ where } |D| = 10m^8n^8$$

We construct the profile $P$ using the following function $f$. The function f takes a literal and a clause as input, and outputs a value in $\{0, 1, -\}$. For each literal $l$, let $C_i$ and $C_j$ with $1 \leq i < j \leq m$ be the two clauses where $l$ appears. We define $f(l, C_i) = 0$, $f(l, C_j) = 1$, and $f(l, C_k) = -$ for every $k \in [m] \setminus \{i, j\}$. This finishes the description of the function $f$. We are now ready to describe $P$. While describing the preferences below, whenever we say ‘others’ or ‘for some alternative in $D$’ or ‘for some subset of $D’$, the unspecified alternatives are assumed to be arranged in such a way that, for every unspecified alternative $a \in A \setminus D$, there are at least 10 alternatives from $D$ in the immediate 10 positions on both left and right of $a$. We also ensure that any alternative in $D$ appears within top $10m^2n^2$ positions at most once in $P$ whereas every alternative in $A$ appears within top $10m^2n^2$ position in every preference in $P$. This is possible because $|D| = 10m^8n^8, |A \setminus D| = 5n + m + 2$, and $|P| = 4n + 6m - 1$. Let $A_0 = \{a(x_i, 0), a(x_i, 0) : i \in [n]\}, A_1 = \{a(x_i, 1), a(\hat{x}_i, 1) : i \in [n]\}, W = \{w_i : i \in [n]\}$.

(I) For every $i \in [n]$, we have

$$\triangleright c \succ D_{mn-3} \succ w_i \succ a(x_i, 0) \succ a(\hat{x}_i, 1) \succ d \succ \text{ others, for some } d \in D \text{ and } D_{mn-3} \subset D \text{ with } |D_{mn-3}| = mn - 3$$

$$\triangleright c \succ D_{mn-3} \succ w_i \succ a(\hat{x}_i, 0) \succ a(\hat{x}_i, 1) \succ d \succ \text{ others, for some } d \in D \text{ and } D_{mn-3} \subset D \text{ with } |D_{mn-3}| = mn - 3$$

(II) For every $j \in [m]$, if $C_j = (l_1 \lor l_2 \lor l_3)$, then we have

$$\triangleright c \succ D_{mn-3} \succ y_j \succ d \succ a(l_1, f(l_1, C_j)) \succ d' \succ \text{ others, for some } d, d' \in D \text{ and } D_{mn-3} \subset D \text{ with } |D_{mn-3}| = mn - 3$$

$$\triangleright c \succ D_{mn-3} \succ y_j \succ d \succ a(l_2, f(l_2, C_j)) \succ d' \succ \text{ others, for some } d, d' \in D \text{ and } D_{mn-3} \subset D \text{ with } |D_{mn-3}| = mn - 3$$

$$\triangleright c \succ D_{mn-3} \succ y_j \succ d \succ a(l_3, f(l_3, C_j)) \succ d' \succ \text{ others, for some } d, d' \in D \text{ and } D_{mn-3} \subset D \text{ with } |D_{mn-3}| = mn - 3$$

(III) $A_0 \succ D_{mn-2n+1} \succ c \succ \text{ others, for some } D_{mn-2n+1} \subset D \text{ with } |D_{mn-2n+1}| = mn - 2n + 1$

(IV) $(W \cup A_0 \cup A_1) \succ D_{mn} \succ \text{ others, for some } D_{mn} \subset D \text{ with } |D_{mn}| = mn$

(V) $2n + 3m - 3 \text{ copies: } A \succ D \succ c$

We now claim that the two instances are equivalent. In one direction, let us assume that the $(3,B2)$-SAT instance is a yes instance with satisfying assignment $g : X \longrightarrow \{0, 1\}$. Let us consider the following profile $\Omega$ where every preference is obtained from the corresponding preference in $P$ by performing at most 2 swaps.

(I) For every $i \in [n]$, we have

$$\triangleright \text{If } g(x_i) = 1, \text{ then } \cdots c \succ D_{mn-3} \succ w_i \succ d \succ a(x_i, 0) \succ a(x_i, 1) \succ \text{ others, for some } d \in D \text{ and } D_{mn-3} \subset D \text{ with } |D_{mn-3}| = mn - 3$$
- $c \succ \mathcal{D}_{mn-3} \succ a(x_i, 0) \succ a(x_i, 1) \succ w_i \succ d \succ$ others for some $d \in \mathcal{D}$ and $\mathcal{D}_{mn-3} \subseteq \mathcal{D}$ with $|\mathcal{D}_{mn-3}| = mn - 3$

\[ \triangleright \text{Else} \]

- $c \succ \mathcal{D}_{mn-3} \succ a(x_i, 0) \succ a(x_i, 1) \succ w_i \succ d \succ$ others for some $d \in \mathcal{D}$ and $\mathcal{D}_{mn-3} \subseteq \mathcal{D}$ with $|\mathcal{D}_{mn-3}| = mn - 3$
- $c \succ \mathcal{D}_{mn-3} \succ w_i \succ d \succ a(\bar{x}_i, 0) \succ a(\bar{x}_i, 1) \succ$ others for some $d \in \mathcal{D}$ and $\mathcal{D}_{mn-3} \subseteq \mathcal{D}$ with $|\mathcal{D}_{mn-3}| = mn - 3$

(II) For every $j \in [m]$, if $C_j = (l_1 \lor l_2 \lor l_3)$ and $g$ makes $l_1 = 1$ (we can assume by renaming), then we have

\[ \triangleright c \succ \mathcal{D}_{mn-3} \succ d \succ a(l_1, f(l_1, C_j)) \succ y_j \succ d' \succ$ others for some $d, d' \in \mathcal{D}$ and $\mathcal{D}_{mn-3} \subseteq \mathcal{D}$ with $|\mathcal{D}_{mn-3}| = mn - 3$

\[ \triangleright c \succ \mathcal{D}_{mn-3} \succ y_j \succ d \succ a(l_2, f(l_2, C_j)) \succ d' \succ$ others for some $d, d' \in \mathcal{D}$ and $\mathcal{D}_{mn-3} \subseteq \mathcal{D}$ with $|\mathcal{D}_{mn-3}| = mn - 3$

\[ \triangleright c \succ \mathcal{D}_{mn-3} \succ y_j \succ d \succ a(l_3, f(l_3, C_j)) \succ d' \succ$ others for some $d, d' \in \mathcal{D}$ and $\mathcal{D}_{mn-3} \subseteq \mathcal{D}$ with $|\mathcal{D}_{mn-3}| = mn - 3$

(III) $A_0 \succ \mathcal{D}_{mn-2n-1} \succ c \succ$ others for some $\mathcal{D}_{mn-2n+1} \subseteq \mathcal{D}$ with $|\mathcal{D}_{mn-2n-1}| = mn - 2n - 1$

(IV) $(\mathcal{W} \cup A_0 \cup A_1) \succ \mathcal{D}_{mn} \cdots$ for some $\mathcal{D}_{mn} \subseteq \mathcal{D}$ with $|\mathcal{D}_{mn}| = mn$

(V) $2n + 3m - 3$ copies: $A \succ \mathcal{D} \succ c$

Only the alternative $c$ appears a majority number of times within the first $mn$ positions in $\mathcal{Q}$. Hence $c$ is the unique simplified Bucklin winner in $\mathcal{Q}$ and thus the Local Distance Constrained Bribery instance is a YES instance.

In the other direction, let $\mathcal{Q}$ be a profile where $c$ is the unique simplified Bucklin winner and every preference in $\mathcal{Q}$ is at most 2 swaps away from its corresponding preference in $\mathcal{P}$. We observe that in the preference in $\mathcal{Q}$ which corresponds to Item (III), the alternative $c$ moves to its left by 2 positions since otherwise, irrespective of $\mathcal{Q}$ (subject to the condition that the swap distance of every preference in $\mathcal{Q}$ is at most 2 from its corresponding preference in $\mathcal{P}$), $c$ does not appear within the first $mn$ positions in $\mathcal{Q}$ whereas every alternative in $\mathcal{W}$ appears majority within the first $mn + 1$ positions contradicting our assumption that $c$ is the unique simplified Bucklin winner in $\mathcal{Q}$. Hence, $c$ appears a $(2n + 3m)$ (which is a majority) number of times within the first $mn$ positions in $\mathcal{Q}$ and does not appear a majority number of times within the first $mn - 1$ positions in $\mathcal{Q}$. Now, since $c$ is the unique simplified Bucklin winner in $\mathcal{Q}$, no alternative other than $c$ appears a majority number of times within the first $mn$ positions in $\mathcal{Q}$. Since, every alternative $w_i \in \mathcal{W}$ appears $(2n + 3m)$ number of times within the first $mn - 1$ positions in $\mathcal{P}$, $w_i$ must be moved in $\mathcal{Q}$ to its right by 2 positions in at least one of the two preferences in $\mathcal{P}$ where $w_i$ appears at position $(mn - 1)$. We now consider the following assignment $g : \{0, 1\} \rightarrow \{0, 1\}$ for every $i \in [n]$, $g(x_i) = 0$ if the preference in $\mathcal{Q}$ corresponding to the preference $c \succ \mathcal{D}_{mn-3} \succ w_i \succ a(x_i, 0) \succ a(x_i, 1) \succ d \succ$ others in $\mathcal{P}$ moves $w_i$ to its right by two positions; otherwise $g(x_i) = 1$. We claim that $g$ is a satisfying assignment for the $(3, B2)$-SAT instance. Suppose not, then let us assume that $g$ does not satisfy $C_j = (l_1 \lor l_2 \lor l_3)$ for some $j \in [m]$. We first observe that $y_j$ also appears $(2n + 3m)$ number of times within the first $mn - 1$ positions in $\mathcal{P}$. Hence, $y_j$ must be moved in $\mathcal{Q}$ to its right by 2 positions in at least one of the two preferences in $\mathcal{P}$ where $y_j$ appears at position $(mn - 1)$. However, this implies that at least one alternative in $\{a(l_i, f(l_i, C_j)) : i \in [3]\}$ appears at least $(2n + 3m)$ times within the first $mn$ positions in $\mathcal{Q}$ due to the definition of $g$. This contradicts our assumption that $c$ is the unique simplified Bucklin winner in $\mathcal{Q}$. Hence $g$ is a satisfying assignment and thus the $(3, B2)$-SAT instance is a YES instance.

The proof for the footrule distance follows from the observation that, for any two preferences $\succ_1, \succ_2 \in \mathcal{L}(A)$, if we have $d_{\text{footrule}}(\succ_1, \succ_2) = 4$, then we have $d_{\text{swap}}(\succ_1, \succ_2) = 2$. \[\square\]
Theorem 17. The Local Distance Constrained Bribery problem is NP-complete for the simplified Bucklin voting rule for the maximum displacement distance even when $\delta_1 = 2$ for every preference $i$.

Proof. The proof is analogous to the proof of Theorem 10. □

Theorem 18. The Local Distance Constrained Bribery problem is NP-complete for the Bucklin voting rule for the swap distance and maximum displacement distance even when $\delta = 1$. Hence, The Local Distance Constrained Bribery problem is NP-complete for the Bucklin voting rule for the footrule distance even when $\delta = 2$.

Proof. Let us first prove the result for the maximum displacement distance. The Local Distance Constrained Bribery problem for the Bucklin voting rule for the maximum displacement distance is clearly in NP. To prove NP-hardness, we reduce from $(3, B_2)$-SAT to Local Distance Constrained Bribery. Let $(X = \{x_i : i \in [n]\}, C = \{c_j : j \in [m]\})$ be any arbitrary instance of $(3, B_2)$-SAT. Let us consider the following instance $(A, P, c, \delta = 1)$ of Local Distance Constrained Bribery.

$$A = \{a_i, \bar{a}_i, z_i : i \in [n]\} \cup \{c\}$$
$$\cup \{y_j, e_j : j \in [m]\} \cup D,$$

where $|D| = 10m^8n^8$

We construct the profile $P$ using the following function $f$. The function $f$ takes a literal and a clause as input, and outputs a value in $\{0, 1, \ldots, 3\}$. For each literal $l$, let $C_i$ and $C_j$ with $1 \leq i < j \leq m$ be the two clauses where $l$ appears. We define $f(l, C_i) = 0$, $f(l, C_j) = 1$, and $f(l, C_k) = \overline{0}$ for every $k \in [m] \setminus \{i, j\}$. This finishes the description of the function $f$. We are now ready to describe $P$. While describing the preferences below, whenever we say ‘others’ or ‘for some alternative in $D$’ or ‘for some subset of $D$’, the unspecified alternatives are assumed to be arranged in such a way that, for every unspecified alternative $a \in A \setminus D$, there are at least 10 alternatives from $D$ in the immediate 10 positions on both left and right of $a$. We also ensure that any alternative in $D$ appears within top $10mn$ positions at most once in $P$ whereas every alternative in $A$ appears within top $10m^2n^2$ position in every preference in $P$. This is possible because $|D| = 10m^8n^8, |A \setminus D| = 3n + 2m + 2$, and $|P| \leq 10m^3n^3$. Let $h$ be a function defined on the set of literals as $h(x_i) = a_i$ and $h(x_i) = \bar{a}_i$ for every $i \in [n]$. Let $k = 10(m + n)$ and $N = 2n + 5m$. For any integer $s$ with $1 \leq s \leq 4$, let $Y_s = \{y_j : j \in [m]\}$ and if $C_j = (l_1 \lor l_2 \lor l_3)$ and $f(l_r, C_j) = 0$ for exactly $s − 1$ many $r \in \{1, 2, 3\}$. The profile $P$ is the disjoint union of two profiles $P_1$ and $P_2$. We first describe $P_1$ below.

(I) For every $i \in [n]$, we have

$\triangleright$ $c \triangleright D_{k-3} \triangleright z_i \triangleright a_i \triangleright$ others, for some $D_{k-3} \subset D$ with $|D_{k-3}| = k - 3$

$\triangleright$ $c \triangleright D_{k-3} \triangleright z_i \triangleright \bar{a}_i \triangleright$ others, for some $D_{k-3} \subset D$ with $|D_{k-3}| = k - 3$

(II) For every $j \in [m]$, if $C_j = (l_1 \lor l_2 \lor l_3)$, then we have

$\triangleright$ $c \triangleright D_{k-3} \triangleright y_j \triangleright e_j \triangleright$ others, for some $D_{k-3} \subset D$ with $|D_{k-3}| = k - 3$

$\triangleright$ $c \triangleright D_{k-2} \triangleright y_j \triangleright e_j \triangleright$ others, for some $D_{k-2} \subset D$ with $|D_{k-2}| = k - 2$

$\triangleright$ For every $r \in \{3\}$

If $f(l_r, C_j) = 0$, then

$\triangleright$ $c \triangleright D_{k-3} \triangleright y_j \triangleright h(l_r) \triangleright$ others, for some $D_{k-3} \subset D$ with $|D_{k-3}| = k - 3$

Otherwise

$\triangleright$ $c \triangleright D_{k-2} \triangleright y_j \triangleright h(l_r) \triangleright$ others, for some $D_{k-2} \subset D$ with $|D_{k-2}| = k - 2$

(III) 5 copies: $D_k \triangleright c \triangleright$ others, for some $D_k \subset D$ with $|D_k| = k$

(IV) For every $i \in [n]$

$\triangleright$ $N' - 1$ copies: $D_{k-3} \triangleright z_i \triangleright$ others, for some $D_{k-3} \subset D$ with $|D_{k-3}| = k - 3$

$\triangleright$ $N' - 1$ copies: $D_{k-3} \triangleright a_i \triangleright$ others, for some $D_{k-3} \subset D$ with $|D_{k-3}| = k - 3$
\[ (I) \text{ For every } i \in [n], \text{ we have} \]
\[ \diamond \quad \text{If } g(x_i) = 0, \text{ then} \]
\[ - \quad c \succ D_{k-3} \succ a_i \succ z_i \succ \text{ others}, \text{ for some } D_{k-3} \subset D \text{ with } |D_{k-3}| = k - 3 \]
\[ - \quad c \succ D_{k-3} \succ z_i \succ d \succ \tilde{a}_i \succ \text{ others}, \text{ for some } d \in D, D_{k-3} \subset D \text{ with } |D_{k-3}| = k - 3 \]
\[ \diamond \quad \text{If } g(x_i) = 1, \text{ then} \]
\[ - \quad c \succ D_{k-3} \succ z_i \succ d \succ a_i \succ \text{ others}, \text{ for some } d \in D, D_{k-3} \subset D \text{ with } |D_{k-3}| = k - 3 \]
\[ - \quad c \succ D_{k-3} \succ \tilde{a}_i \succ z_i \succ \text{ others}, \text{ for some } \tilde{a}_i \in D, D_{k-3} \subset D \text{ with } |D_{k-3}| = k - 3 \]

\[ (II) \text{ For every } j \in [m], \text{ if } C_j = (l_1 \lor l_2 \lor l_3), \text{ then we have} \]
\[ \diamond \quad \text{If there exists } l \in [3] \text{ with } f(l_1, C_j) = 0 \text{ and } g(l_1) = 1, \text{ then} \]
\[ - \quad c \succ D_{k-3} \succ y_j \succ d \succ e_j \succ \text{ others}, \text{ for some } d \in D, D_{k-3} \subset D \text{ with } |D_{k-3}| = k - 3 \]
\[ - \quad c \succ D_{k-2} \succ e_j \succ y_j \succ \text{ others}, \text{ for some } D_{k-2} \subset D \text{ with } |D_{k-2}| = k - 2 \]
\[ - \quad c \succ D_{k-3} \succ h(l_1) \succ y_j \succ \text{ others}, \text{ for some } D_{k-3} \subset D \text{ with } |D_{k-3}| = k - 3 \]
- For every \( r \in \{3\}, r \neq t \), if \( f(l_r, C_i) = 0 \), then
  \[ c \succ D_{k-3} \succ y_j \succ h(l_r) \succ \text{others}, \text{for some } D_{k-3} \subset D \text{ with } |D_{k-3}| = k-3 \]
  Otherwise
  \[ c \succ D_{k-2} \succ y_j \succ h(l_r) \succ \text{others}, \text{for some } D_{k-2} \subset D \text{ with } |D_{k-2}| = k-2 \]
- If there exists \( t \in \{3\} \) with \( f(l_t, C_i) = 1 \) and \( g(l_t) = 1 \), then
  - \( c \succ D_{k-3} \succ e_j \succ y_j \succ \text{others}, \text{for some } D_{k-3} \subset D \text{ with } |D_{k-3}| = k-3 \)
  - \( c \succ D_{k-2} \succ y_j \succ e_j \succ \text{others}, \text{for some } D_{k-2} \subset D \text{ with } |D_{k-2}| = k-2 \)
  - \( c \succ D_{k-2} \succ h(l_t) \succ y_j \succ \text{others}, \text{for some } D_{k-2} \subset D \text{ with } |D_{k-2}| = k-2 \)
  - For every \( r \in \{3\}, r \neq t \)
    If \( f(l_r, C_i) = 0 \), then
    \[ c \succ D_{k-3} \succ y_j \succ h(l_r) \succ \text{others}, \text{for some } D_{k-3} \subset D \text{ with } |D_{k-3}| = k-3 \]
    Otherwise
    \[ c \succ D_{k-2} \succ y_j \succ h(l_r) \succ \text{others}, \text{for some } D_{k-2} \subset D \text{ with } |D_{k-2}| = k-2 \]

(III) 5 copies: \( D_{k-1} \succ c \succ \text{others}, \text{for some } D_{k-1} \subset D \text{ with } |D_{k-1}| = k-1 \)

(IV) For every \( i \in [n] \)
  - \( N' - 1 \) copies: \( D_{k-2} \succ z_i \succ \text{others}, \text{for some } D_{k-2} \subset D \text{ with } |D_{k-2}| = k-2 \)
  - \( N' - 1 \) copies: \( D_{k-2} \succ a_i \succ \text{others}, \text{for some } D_{k-2} \subset D \text{ with } |D_{k-2}| = k-2 \)
  - \( N' - 1 \) copies: \( D_{k-2} \succ \bar{a}_i \succ \text{others}, \text{for some } D_{k-2} \subset D \text{ with } |D_{k-2}| = k-2 \)
  - 3 copies: \( D_{k-1} \succ a_i \succ \text{others}, \text{for some } D_{k-1} \subset D \text{ with } |D_{k-1}| = k-1 \)
  - 3 copies: \( D_{k-1} \succ \bar{a}_i \succ \text{others}, \text{for some } D_{k-1} \subset D \text{ with } |D_{k-1}| = k-1 \)

(V) For every \( j \in [m] \)
  - \( N' + 3 \) copies: \( D_{k-1} \succ e_j \succ \text{others}, \text{for some } D_{k-1} \subset D \text{ with } |D_{k-1}| = k-1 \)

(VI) For every \( s \in [4] \), for every \( j \in Y_s \)
  - \( N' - s + 1 \) copies: \( D_{k-2} \succ y_j \succ \text{others}, \text{for some } D_{k-2} \subset D \text{ with } |D_{k-2}| = k-2 \)
  - \( s - 1 \) copies: \( D_{k-1} \succ y_j \succ \text{others}, \text{for some } D_{k-1} \subset D \text{ with } |D_{k-1}| = k-1 \)

Table 7 shows the number of times every alternative appears within the first \( k-1 \) and \( k \) times in \( Q \) which proves that \( c \) is the unique Bucklin winner in \( Q \) and thus the Local Distance Constrained Bribery instance is a YES instance.

| Alternatives | Number of times it appears within first k – 1 positions | k positions |
|--------------|-------------------------------------------------------|-------------|
| \( c \)      | \( \lceil N/2 \rceil \)                              | \( \lceil N/2 \rceil + 5 \) |
| \( z_i, i \in [n] \) | \( \lceil N/2 \rceil \)                              | \( \lceil N/2 \rceil + 1 \) |
| \( a_i, \bar{a}_i, i \in [n] \) | \( \leq \lceil N/2 \rceil \)                       | \( \leq \lceil N/2 \rceil + 4 \) |
| \( y_j, j \in [m] \) | \( \lceil N/2 \rceil \)                              | \( \lceil N/2 \rceil + 4 \) |
| \( e_j, j \in [m] \) | \( \leq 1 \)                                       | \( \leq (N/2) + 4 \) |
| \( d \in D \) | \( \leq 1 \)                                       | \( \leq 1 \) |

Table 7: Number of times every alternative appears within first \( k-1 \) and \( k \) times in \( Q \) in the forward direction of the proof of Theorem 18.

In the other direction, let us assume that there exists a profile \( Q \) such that the maximum displacement distance of every preference in \( Q \) is at most 1 from its corresponding preference in \( P \) and \( c \) is the unique
Bucklin winner in $\Omega$. We first observe that, irrespective of $\Omega$ (subject to the condition that its maximum displacement distance from $\mathcal{P}$ is at most 1), any alternative in $\mathcal{D}$ does not appear a majority number of times within the first $2k$ positions in $\Omega$. Next we can assume without loss of generality that in the preference in $\Omega$ which corresponds to Item (III), the alternative $c$ moves to its left by 1 position since any alternative to the left of $c$ in Item (III) belongs to $\mathcal{D}$. Since in any preference in $\Omega$ other than Item (III), $c$ never appears in any position in $\{t \in \mathbb{N} : k - 3 \leq t \leq k + 3\}$, $c$ does not appear a majority number of times within the first $k - 1$ positions (it actually appears $(N/2)$ times) in $\Omega$. However, $c$ appears within the first $k$ positions $(N/2) + 5$ times in $\Omega$. Since, in every preference in Items (IV) to (VI), every alternative in $\mathcal{A} \setminus \mathcal{D}$ has at least 10 alternatives from $\mathcal{D}$ in its immediate left and right, we can assume without loss of generality that every alternative in $\mathcal{A} \setminus (\mathcal{D} \cup \{c\})$ moves to its right by 1 position in every preference in $\Omega$ corresponding to the preferences in Items (IV) to (VI). Now the alternative $z_i, i \in [n]$ appears $(N/2) + 1$ times within the first $k - 1$ positions, $y_j, j \in [m]$ appears $(N/2) + 1$ and $(N/2) + 5$ times within the first $k - 1$ and $k$ positions respectively, $\alpha_i, i \in [n]$ appear $(N/2) - 1$ and $(N/2) + 4$ times within the first $k - 1$ and $k$ positions respectively, and $e_j, j \in [m]$ appears 1 and $(N/2) + 4$ times within the first $k - 1$ and $k$ positions. We now observe that there are exactly two preferences in $\mathcal{P}$ where $z_i$ appears at the $(k - 1)$-th position. Hence, for $c$ to be the unique Bucklin winner in $\Omega$, $z_i$ must move to its right by 1 position in at least one preference among the two preferences in $\mathcal{P}$ where it appears at $(k - 1)$-th position. We now consider the following assignment $g : \mathcal{X} \rightarrow \{0, 1\}$ – for every $i \in [n]$, $g(x_i) = 0$ if there exists a preference in $\Omega$ where $z_i$ appears at the $k$-th position and $\alpha_i$ appears at the $(k - 1)$-th position; otherwise we define $g(x_i) = 1$. We claim that $g$ is a satisfying assignment for the $(3, B2)$-SAT instance. Suppose not, then let us assume that $g$ does not satisfy $C_1 = (l_1 \lor l_2 \lor l_3)$ for some $j \in [m]$. Suppose $y_j \in \mathcal{Y}_s$ for some $s \in \{1, 2, 3, 4\}$. Then, among the 5 preferences in Item (II) corresponding to $y_j$ (let us call them $Q_j$ and $P_j$ respectively in $\Omega$ and in $\mathcal{P}$), the alternative $y_j$ appears $s$ times at the $(k - 1)$-th position and $5 - s$ times at the $k$-th position. Now it follows from the definition of $g$ that every alternative in $\{h(l_1), h(l_2), h(l_3)\}$ appears $(N/2)$ and $(N/2) + 4$ times within the first $k - 1$ and $k$ positions in $(\Omega \setminus Q_j) \cup \mathcal{P}_j$ and thus cannot move to their left in any preference in $\mathcal{P}_j$. Now, to make $c$ the unique Bucklin winner of $\Omega$, the alternative $y_j$ must move to its right by one position each in at least one preferences where it appears at the $(k - 1)$-th position and where it appears at the $k$-th position. Hence, $e_j$ must move to its left in both the preferences in $\mathcal{P}_j$ where it appears at the immediate right of $y_j$. However, this makes $e_j$ appear $(N/2) + 5$ times within the first $k$ preferences in $\Omega$ which contradicts our assumption that $c$ is the unique Bucklin winner in $\Omega$. Hence $g$ is a satisfying assignment and thus the $(3, B2)$-SAT instance is a YES instance.

Now the result for the swap and footrule distances also follow from the analogous reductions from $(3, B2)$-SAT.

\section{Conclusion and Future Direction}

In this paper, we have proposed a new model of bribery. We have argued that the bribery models studied so far in computational social choice may fail to capture intricacies in certain situations, for example, where there is a fear of information leakage and voters care about social reputation. We have discussed how our \textsc{Local Distance Constrained $\$bribery} and \textsc{Local Distance Constrained bribery} problems can suitably model those scenarios. We have then shown that the \textsc{Local Distance Constrained $\$bribery} problem is polynomial time solvable for the plurality and veto voting rules for the swap, footrule, and maximum displacement distances, and for the k-approval voting rule for the swap distance if the distance allowed is 1 (and thus for the footrule distance, it is 3). For the k-approval and simplified Bucklin voting rules for the maximum displacement distance, we have shown that the \textsc{Local Distance Constrained bribery} problem is polynomial time solvable. We have then proved that the \textsc{Local Distance Constrained bribery} problem (and thus the \textsc{Local Distance Constrained $\$bribery} problem) is NP-complete for the k-approval and simplified Bucklin voting rules for the swap distance even if the distance allowed is 2 (and thus for the footrule distance, it is 4), for a class of scoring rules which includes the Borda voting rule, maximin, Copeland$^\alpha$ for any $\alpha \in [0, 1]$, and Bucklin voting rules for the swap and maximum displacement distances even when the distance allowed is 1 (and thus for the footrule distance, it is 3). In particular, we have proved tight (in terms of $\delta$) computational complexity results for the \textsc{Local Distance Constrained bribery}
problem for the swap, footrule, and maximum displacement distances. Our results show that the notion of optimality makes bribery much richer than optimal manipulation in the sense that the complexity of the Local Distance Constrained Bribery problem for some commonly used voting rule (k-approval for example) can change drastically if we change the measure of distance under consideration.

It would be interesting to find approximation algorithms for the Local Distance Constrained Bribery problem where it is NP-complete (our hardness proofs already show APX-hardness). The bribery problem in this paper can be extended by introducing a pricing model and a (global) budget for the briber which the briber needs to respect. In any setting where our Local Distance Constrained Bribery problem is NP-complete, hardness in such sophisticated models in the corresponding setting will immediately follow. However, it would be interesting to extend our polynomial time algorithms to those sophisticated models.

References

[BCF+14] Robert Bredereck, Jiehua Chen, Piotr Faliszewski, André Nichterlein, and Rolf Niedermeier. Prices matter for the parameterized complexity of shift bribery. In Proc. 28th AAAI Conference on Artificial Intelligence (AAAI), pages 1398–1404, 2014.

[BEF+14] Daniel Binkele-Raible, Gábor Erdélyi, Henning Fernau, Judy Goldsmith, Nicholas Mattei, and Jörg Rothe. The complexity of probabilistic lobbying. In Algorithmic Decision Theory, volume 11, pages 1–21. Discrete Optimization, 2014.

[BFLR12] Dorothea Baumeister, Piotr Faliszewski, Jérôme Lang, and Jörg Rothe. Campaigns for lazy voters: truncated ballots. In Proc. 11th International Conference on Autonomous Agents and Multiagent Systems, AAMAS 2012, Valencia, Spain, June 4-8, 2012 (3 Volumes), pages 577–584, 2012.

[BFNT16] Robert Bredereck, Piotr Faliszewski, Rolf Niedermeier, and Nimrod Talmon. Complexity of shift bribery in committee elections. In Proc. 30th AAAI Conference on Artificial Intelligence (AAAI), pages 2452–2458, 2016.

[BKS03] Piotr Berman, Marek Karpinski, and Alex D. Scott. Approximation hardness and satisfiability of bounded occurrence instances of SAT. Electronic Colloquium on Computational Complexity (ECCC), 10(022), 2003.

[BTT89] J.J. Bartholdi, C.A. Tovey, and M.A. Trick. The computational difficulty of manipulating an election. Soc. Choice Welf., 6(3):227–241, 1989.

[Cor09] Thomas H Cormen. Introduction to algorithms. MIT press, 2009.

[CSL07] V. Conitzer, T. Sandholm, and J. Lang. When are elections with few candidates hard to manipulate? J. ACM, 54(3):14, 2007.

[CSS99] William W. Cohen, Robert E. Schapire, and Yoram Singer. Learning to order things. J. Artif. Int. Res., 10(1):243–270, May 1999.

[CW16] Vincent Conitzer and Toby Walsh. Barriers to manipulation in voting. In Handbook of Computational Social Choice, pages 127–145. 2016.

[CXX+18] Lin Chen, Lei Xu, Shouhua Xi, Zhimin Gao, Nolan Shah, Yang Lu, and Weidong Shi. Protecting election from bribery: New approach and computational complexity characterization. In Proc. 17th International Conference on Autonomous Agents and MultiAgent Systems (AAMAS), pages 1894–1896, 2018.

[Dey19] Palash Dey. Local distance constrained bribery in voting. In to appear in Proc. International Conference on Autonomous Agents and Multiagent Systems (AAMAS), Montreal, Canada, May 13-17, 2019.
[DG77] Persi Diaconis and Ronald L. Graham. Spearman’s footrule as a measure of disarray. *J. Royal Stat. Soc. Series B (Methodological)*, pages 262–268, 1977.

[DK16] Britta Dorn and Dominikus Krüger. On the hardness of bribery variants in voting with cp-nets. *Ann. Math. Artif. Intell.*, 77(3-4):251–279, 2016.

[DMN17] Palash Dey, Neeldhara Misra, and Y. Narahari. Frugal bribery in voting. *Theor. Comput. Sci.*, 676:15–32, 2017.

[DS12] Britta Dorn and Ildikó Schlotter. Multivariate complexity analysis of swap bribery. *Algorithmica*, 64(1):126–151, 2012.

[EF10] Edith Elkind and Piotr Faliszewski. Approximation algorithms for campaign management. In *International Workshop on Internet and Network Economics (WINE)*, pages 473–482. Springer, 2010.

[EFS09] Edith Elkind, Piotr Faliszewski, and Arkadii Slinko. Swap bribery. In *Proc. 2nd International Symposium on Algorithmic Game Theory (SAGT 2009)*, pages 299–310. Springer, 2009.

[EHH14] Gabor Erdelyi, Edith Hemaspaandra, and Lane A Hamaszpaandra. Bribery and voter control under voting-rule uncertainty. In *Proc. 13th International Conference on Autonomous Agents and Multiagent Systems (AAMAS)*, pages 61–68, 2014.

[Fal08] Piotr Faliszewski. Nonuniform bribery. In *Proc. 7th International Conference on Autonomous Agents and Multiagent Systems (AAMAS)*, pages 1569–1572. International Foundation for Autonomous Agents and Multiagent Systems, 2008.

[FGKT18] Piotr Faliszewski, Rica Gonen, Martin Koutecký, and Nimrod Talmon. Opinion diffusion and campaigning on society graphs. In *Proc. 27th International Joint Conference on Artificial Intelligence, IJCAI*, pages 219–225, 2018.

[FHH06] Piotr Faliszewski, Edith Hemaspaandra, and Lane A Hamaszpaandra. The complexity of bribery in elections. In *Proc. 21st AAAI Conference on Artificial Intelligence, AAAI*, volume 6, pages 641–646, 2006.

[FHH09] Piotr Faliszewski, Edith Hemaspaandra, and Lane A. Hemaspaandra. How hard is bribery in elections? *J. Artif. Int. Res.*, 35(1):485–532, July 2009.

[FHR09] Piotr Faliszewski, Edith Hemaspaandra, Lane A Hamaszpaandra, and Jörg Rothe. Llull and Copeland voting computationally resist bribery and constructive control. *J. Artif. Intell. Res.*, 35(1):275, 2009.

[FR16] Piotr Faliszewski and Jörg Rothe. Control and bribery in voting. In *Handbook of Computational Social Choice*, pages 146–168. 2016.

[FRRS14] Piotr Faliszewski, Yannick Reisch, Jörg Rothe, and Lena Schend. Complexity of manipulation, bribery, and campaign management in bucklin and fallback voting. In *Proc. 13th International Conference on Autonomous Agents and Multiagent Systems (AAMAS)*, pages 1357–1358, 2014.

[GTT90] Andrew V. Goldberg, Éva Tardos, and Robert E. Tarjan. Network flow algorithms. In Bernhard Korte, László Lovász, Hans Jürgen Prömel, and Alexander Schrijver, editors, *Paths, Flows, and VLSI-Layout*, pages 101–164, 1990.

[Ken38] Maurice G. Kendall. A new measure of rank correlation. *Biometrika*, 30(1/2):81–93, 1938.

[KF16] Andrzej Kaczmarczyk and Piotr Faliszewski. Algorithms for destructive shift bribery. In *Proc. 15th International Conference on Autonomous Agents & Multiagent Systems (AAMAS)*, pages 305–313, 2016.
[KHH18] Orgad Keller, Avinatan Hassidim, and Noam Hazon. Approximating bribery in scoring rules. In Proc. 32nd International Conference on Artificial Intelligence (AAAI), pages 1121–1129, 2018.

[KKM18] Dušan Knop, Martin Koutecký, and Matthias Mnich. A unifying framework for manipulation problems. In Proc. 17th International Conference on Autonomous Agents and MultiAgent Systems (AAMAS), pages 256–264, 2018.

[MNRS18] Cynthia Maushagen, Marc Neveling, Jörg Rothe, and Ann-Kathrin Selker. Complexity of shift bribery in iterative elections. In Proc. 17th International Conference on Autonomous Agents and MultiAgent Systems (AAMAS), pages 1567–1575, 2018.

[MPVR12] Nicholas Mattei, Maria Silvia Pini, K Brent Venable, and Francesca Rossi. Bribery in voting over combinatorial domains is easy. In Proc. 11th International Conference on Autonomous Agents and Multiagent Systems (AAMAS), pages 1407–1408, 2012.

[OE12a] Svetlana Obraztsova and Edith Elkind. Optimal manipulation of voting rules. In Proc. 26th AAAI Conference on Artificial Intelligence (AAAI), pages 2141–2147, 2012.

[OE12b] Svetlana Obraztsova and Edith Elkind. Optimal manipulation of voting rules. In Proc. 11th International Conference on Autonomous Agents and Multiagent Systems (AAMAS), pages 619–626, 2012.

[PHG00] David M. Pennock, Eric Horvitz, and C. Lee Giles. Social choice theory and recommender systems: Analysis of the axiomatic foundations of collaborative filtering. In Proc. 17th National Conference on Artificial Intelligence and 12th Conference on on Innovative Applications of Artificial Intelligence, July 30 - August 3, 2000, Austin, Texas, USA., pages 729–734, 2000.

[PRV13] Maria Silvia Pini, Francesca Rossi, and Kristen Brent Venable. Bribery in voting with soft constraints. In Proc. 27th AAAI Conference on Artificial Intelligence (AAAI), pages 803–809, 2013.

[SFE17] Ildikó Schlotter, Piotr Faliszewski, and Edith Elkind. Campaign management under approval-driven voting rules. Algorithmica, 77(1):84–115, 2017.

[Spe04] Charles Spearman. The proof and measurement of association between two things. Am. J. Psychol., 15(1):72–101, 1904.

[XC11] Lirong Xia and Vincent Conitzer. Determining possible and necessary winners given partial orders. J. Artif. Intell. Res., 41:25–67, 2011.

[Xia12] Lirong Xia. Computing the margin of victory for various voting rules. In Proc. 13th ACM Conference on Electronic Commerce (EC), pages 982–999. ACM, 2012.