New phenomenon in exotic neutron-rich Sn isotopes: role of 3-body force

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Abstract. The excitation energies of the first 2⁺ states (E(2⁺1)) of even-even ¹³⁴–¹⁴⁰Sn have been calculated using shell model with empirical SMPN and realistic CWG interactions in the (gdsh) × ν(hfpi) valence space above ¹³²Sn core. The CWG predicts nearly constant E(2⁺1) for ¹³⁴–¹³⁸Sn isotopes, normally expected for singly-magic nuclei, and a weak shell closure at ¹⁴⁰Sn. The prediction of SMPN differs dramatically, decreasing E(2⁺1) with increasing valence neutron number for ¹³⁴–¹³⁸Sn and a strong shell closure at ¹⁴⁰Sn. This variation predicted by SMPN has striking similarity with the variation of experimental E(2⁺1) of even-even ¹⁸–²²O and ⁴²–⁴⁸Ca, clearly showing that the N = 84–88 spectra exhibit the effect of gradually filling up the (2f⁷/₂) orbital which finally culminates in a new shell closure at ¹⁴⁰Sn (N = 90). Spin-tensor decomposition of SMPN and CWG and the variation of their components with valence neutron number reveals that the origin of the decreasing E(2⁺1) and shell closure at ¹⁴⁰Sn might lie in the three-body effects. Calculations with CWG3M interaction, which is obtained by including a simple 3-body monopole term in the CWG predict decreasing E(2⁺1) for ¹³⁴–¹³⁸Sn and a shell closure at ¹⁴⁰Sn.

1. Introduction
Several experimental and theoretical investigations in the close-to-neutron-drip-line region above the doubly magic ¹³²Sn nucleus have recently revealed many intriguing issues concerning some newer aspects of nuclear structure and N-N interaction in such exotic environments. As the nuclei with 50 ≤ Z ≤ 56 and 82 ≤ N ≤ 88 lie on or close to the path of the astrophysical r-process flow, extensive knowledge about their structure, particularly the binding energies, low-lying excited states, and beta - decay rates of these nuclei at finite temperatures, are important ingredients for astrophysical calculations. Sn isotopes are of particular importance in this respect. Even Sn isotopes, viz. ¹³⁶Sn, is known to be the classical ”waiting point” nucleus in A=130 solar system abundance peak under typical r-process condition [1]. However, many of these important nuclei are yet to be studied experimentally. Some spectroscopic information exists only for ¹³⁴Sn. Recently, the beta -decay half-lives via delayed neutron emission have been measured for ¹³⁵Sn to ¹³⁸Sn nuclei. Their small production rates and lifetimes are the main causes of the severe limitations in acquiring spectroscopic information on them. Therefore, reliable theoretical calculations and their comparisons with the hitherto available experimental data are not only important for extending our knowledge of N-N interaction and nuclear structure in the exotic neutron-rich environment, but they also provide very useful ingredients for reviewing the
problems related to the nuclear astrophysics in general.

Until recently the nuclear magic numbers at N, Z = 2, 8, 20, 28, 50, 82 and 126 were considered to be valid throughout the entire nuclear chart. However, in the last two decades advent of radioactive ion beams and advancement of detection techniques have made it possible to study the evolution of the shells far from stability. New results for light nuclei showed some unexpected changes in the nuclear shell structure as a function of proton and neutron numbers. At least four doubly magic oxygen isotopes have been observed [2, 3, 4]. They are $^{14}O$, $^{16}O$, $^{22}O$ and $^{24}O$. Brown and Richter [4] have framed a generalised new rule for magic numbers, valid specially for lighter nuclei. Several theoretical investigations have been pursued to interpret these observations. New predictions [5] were made that some magic numbers will disappear and new shell gaps will appear in certain regions of the nuclear chart.

In this background, we present in this theoretical work, some new features and shell evolution in the neutron-rich semi - magic Sn isotopes above the doubly magic $^{132}Sn$ core. Our analysis of the Hamiltonians used for studying nuclear structure in the shell model formalism also points to the fact that the 3-body force might have a role in the shell evolution in this region too.

2. Formalism: model space and modified Hamiltonian

We have generated a new empirical interaction SMPN [6] by modifying CW5082 [7, 8] nuclear Hamiltonian derived from Kuo-Herling interaction of the $^{208}Pb$ region in the light of the recent data for this region. We assume $^{132}Sn$ as the inert core. The valence space consists of $\pi(1g_{7/2}, 2d_{3/2}, 2d_{5/2}, 3s_{1/2}, 1h_{11/2})$ and $\nu(1h_{9/2}, 2f_{7/2}, 2f_{5/2}, 3p_{3/2}, 3p_{1/2}, 1i_{13/2})$ orbitals. Few-valence-particle nuclei are above the strong shell closure at $^{132}Sn$ furnish very useful information on the N-N interaction in the n-rich environment. We have used available recent experimental information on the binding energy of $^{132}Sn$ and binding energies and level spectra of only A=134 isobars of Sn, Sb, Te. These A=134 isobars of Sn, Sb and Te provide information on the nn, np and pp two - body matrix elements, respectively. Motivation for construction of this new empirical Hamiltonian, SMPN, and the details of changes made are given in detail in Ref. [6] and the changed tbmes have been tabulated in Ref. [9].

In the present work, CWG, the realistic interaction obtained starting with a G matrix derived from the CD-Bonn nucleon-nucleon potential using the Q-box method [10] and the empirical SMPN [6] interactions have been used. The Hamiltonians have the same set of single-particle energies [11] of the valence orbitals but different sets of two-body interaction matrix elements (tbmes). Both these interactions predict excitation spectra of neutron - rich nuclei like $^{138}Te$ remarkably well [11] along with those of other A=138 isobars like $^{138}Xe$ and $^{138}Ba$, for which experimental data are available. The shell model codes OXBASH and NUSHELL@MSU have been used [12] in the present work.

3. Results and Discussions

3.1. $^{136,138}Sn$ and a probable new feature above the $^{132}Sn$ core

The binding energies predicted by these two calculations for the Sn isotopes till N=88 are compared in Ref. [6, 11]. It is remarkable that for $^{136,138}Sn$, where the experimental level schemes are not known, the excitation spectra predicted by two interactions differ dramatically [11]. Calculated energies for the first $2^+$ levels [$E(2^+_1)$] in $^{136,138}Sn$, using SMPN, fit nicely to the respective systematics of isotones for N >82 (Fig. 1). The $E(2^+_1)$ values calculated for Sn isotopes above N = 82 decreases with increasing neutron number. But predicted $E(2^+_1)$ values from the calculations using the CWG Hamiltonian deviate sharply from the systematics of isotones for N >82. For CWG, the $E(2^+_1)$ excitation energies for all the even-even Sn isotopes with N > 84 are almost constant, normally expected for singly - magic nuclei, similar to those below N = 82, for Sn isotopes with A=102-130. The only difference is that the bunching of $E(2^+_1)$ values now occurs at around 750 keV for N>84 instead of 1200 keV for N<82. The $2^+_1$, $4^+_1$, and $6^+_1$ states of
gradual filling up the \( 132 \) show striking similarity with the theoretical predictions with SMPN in the experimental E(2\(^+\))\(_{N=84,86} \) as a function of valence neutron numbers above \( 20 \) core. Similarly, the variation of the experimental E(2\(^+\))\(_{N=84,86} \) energies of even \( 20 \)\(_{N=84,86} \) with theoretical results for \( 136 \)\(_{N=84,86} \) is compared showing the differences of E(2\(^+\))\(_{N=84,86} \) for \( 136 \)\(_{N=84,86} \) with SMPN.

The SMPN results for \( 136 \)\(_{N=84,86} \) can reproduce the binding energy and spectra of \( 134 \)\(_{N=84} \) quite well \[14\]. Casten and Sherill \[13\] have pointed out that, although \( [E(2^+ \text{Sn} ) - E(2^+ \text{Te})] = 400 \) keV for a given neutron number over most of the \( N = 50-82 \) shell, the difference \( [E(2^+ \text{Sn} ) - E(2^+ \text{Te})] \) is only 119 keV for \( N = 84 \) [as, E(2\(^+\))\(_{Te} \) = 606 keV for \( 130 \text{Te} \)]. It is indeed remarkable that the difference between the calculated values of [E(2\(^+\))\(_{136 \text{Sn} }\) - E(2\(^+\))\(_{138 \text{Sn} }\)] for \( N = 86 \) is 108 keV with SMPN (Fig.2). It is consistent with the trend discussed by Casten and Sherill. For CWG, this difference is 733 - 356 = 377 keV for \( N = 86 \), which deviates from the trend. Thus the systematic behaviour of E(2\(^+\))\(_{136 \text{Sn} }\) and the E(2\(^+\))\(_{138 \text{Sn} }\) differences between isotones of Sn and Te are reproduced well by SMPN. Therefore the decreasing trend of E(2\(^+\))\(_{134 \text{Sn} }\) for \( 134 \text{Sn} \) obtained with SMPN might be a strong possibility and constitutes a new feature for semi-magic nuclei in this mass region. This is similar to that observed in even -even (e-e) Ca isotopes in the pf shell and that observed very recently in the c-e O isotopes in the sd shell (Fig.3).

### 3.2. The new shell closure at \( N=90 \) and comparison with other neutron-rich domains

The SMPN results for \( 136,138 \text{Sn} \) show that for increasing neutron number (>82), the \( Z = 50 \) shell closure weakens and \( 130 \text{Sn} \) isotopes behave very similar to the other \( Z = 52-60 \) nuclei (Fig.1). The E(2\(^+\)) energies of isotopes of Sn for \( A=132-140 \) have been shown in Fig. 3 as a function of valence neutrons above \( 132 \text{Sn} \) core. The same figure contains variation of the experimental E(2\(^+\)) energies of even \( 20 \text{Ca} \) isotopes from \( A=40 \) to 48 as a function of valence neutrons above \( 40 \text{Ca} \) core. Similarly, the variation of the experimental E(2\(^+\)) energies of even \( 8 \text{O} \) isotopes from \( A=16 \) to 24 as a function of valence neutron numbers above \( 16 \text{O} \) core are also shown. The variations of experimental E(2\(^+\)) with the valence neutron number for two different mass regions and shells, show striking similarity with the theoretical predictions with SMPN in the \( 132 \text{Sn} \) region. The gradual filling up the \( \nu(2f_{7/2}) \) orbital by neutrons is very distinctively shown by the variation of...
E(2\textsuperscript{+}) from \textsuperscript{134-140}Sn. The E(2\textsuperscript{+}) for \textsuperscript{140}Sn is 1949 keV showing a sudden increase for N=90, indicating a closed shell structure for \textsuperscript{140}Sn.

The trend is very similar to that observed for neutron - rich isotopes of Ca while filling up the \nu(1\textit{f}\textsubscript{7/2}) orbital and that shown by neutron - rich oxygen isotopes while filling up the \nu(1\textit{d}\textsubscript{5/2}) single particle orbital (Fig.3). The shell closure at N=14 for \textsuperscript{22}O with 6 neutrons in \nu(1\textit{d}\textsubscript{5/2}) is indicated by the fact that \textit{2}\textsuperscript{+} state energy (\textapprox;3.199 MeV) in it is almost twice of that in the adjacent N=10 and 12 isotopes of oxygen. With filling up the \nu(2\textit{s}\textsubscript{1/2}) orbital, the increase in \textit{2}\textsuperscript{+} energy (E(\textit{2}\textsuperscript{+}) = 4.72 MeV) in \textsuperscript{24}O is much more dramatic indicating another new shell gap at N=16 for Z=8 which is not present for Z=6,10 and 12. It has been discussed [3] that the mechanism behind the appearance of these new shell gaps for the oxygen isotopes is the tensor force. With protons present in the \pi(1\textit{d}\textsubscript{5/2}) orbital in Ne and Mg isotopes, the \nu(1\textit{d}\textsubscript{5/2}) orbital is drawn down in energy via the isospin component of the tensor force, thereby reducing the N = 16 shell gap. Similar situation also prevails around \textsuperscript{140}Sn. It has been observed experimentally that N=90 is suitable for onset of deformation for nuclei above Sn (like Xe, Ba etc.) [2]. But for N=90 isotope of Sn nucleus, a distinct shell closure is predicted by the SMPN results, while CWG denotes it to be a weak one. The presence of valence protons above the inert core is essential [15, 16] for onset of collectivity. The SMPN interaction which indicate the features of a shell closure for Z=50 at N=90, also indicate the possibility of onset of deformation at N=90 with increasing Z. This has been discussed in Ref. [17].

3.3. The effective single particle energies and the spin-tensor decomposition
In order to understand the shell closure at \textsuperscript{140}Sn more precisely, the variation of the effective single-particle energies (ESPE) [18] of the neutron orbitals as functions of valence neutron numbers for the two Hamiltonians have been compared. The ESPE is defined as bare single particle energy (spe) added with the monopole part of the diagonal two body matrix elements (tbme). The bare spe is originated from the interaction of a valence nucleon with the doubly closed core. The monopole interaction contribution is the (2\textit{J} + 1) weighted average of the diagonal tbme, which arises from the interaction of a valence nucleon with the other valence nucleons. The ESPE for the configurations \nu(2\textit{f}\textsubscript{7/2})\textsuperscript{n} in \textsuperscript{132-140}Sn with neutron number varying from 82-90 (valence neutron number n varying from 0 to 8) are shown in Fig. 4. For both SMPN and CWG, the energy gap between \nu(2\textit{f}\textsubscript{7/2}) and \nu(3\textit{p}\textsubscript{3/2}) single particle orbitals is 854 keV for \textsuperscript{132}Sn core. But the gap between the corresponding ESPEs increases to 2246 MeV at N=90 with SMPN. This gap is sufficient to make \textsuperscript{140}Sn a doubly- closed shell nucleus. For CWG this gap does not show any increase but instead decreases slightly to 826 keV. To analyse the origin of this new shell closure, the important physical aspects of both the residual interactions are extracted by a spin-tensor decomposition [19] of the two body matrix elements (tbmes) in to central, antisymmetric spin-orbit (\textit{als}), spin-orbit (\textit{ls}) and tensor components. Fig.5 shows this decomposition. From this figure, for SMPN results, one can find that the variation in \textit{als} part.

![Figure 3. Comparison of the variation of calculated E(2\textsuperscript{+}) values of Sn isotopes with the experimental E(2\textsuperscript{+}) values of neutron rich isotopes of O and Ca as functions of valence neutron numbers with respect to \textsuperscript{132}Sn, \textsuperscript{16}O and \textsuperscript{40}Ca cores, respectively.](image-url)
is primarily responsible for this observed shell gap at N=90. The als component in the tbmes for an empirical interaction usually arises from inadequate constraint by the data indicating important contributions from higher order renormalisation or many body forces to the effective interactions. In empirical SMPN such many - body effects might have been included in some way through the modification of important tbmes. Fig. 6 shows the spin tensor decomposition of the USD interaction which reproduces the experimentally observed new shell closures for oxygen isotopes. It is quite revealing that the decomposition of the empirical USD interaction also indicates that the variation in als part is also responsible for the observed shell gap at N=14. Thus it seems natural to conjecture that the realistic CWG differs from SMPN owing to the absence of any many (three) - body effects in the CWG tbmes.

3.4. The three-body effects
At this point it is important to note that shell model calculations using two-body realistic interactions derived from the free nucleon-nucleon force fail to reproduce some shell closures [20]. It is now rather well established that increase of the $1d_{5/2} - 2s_{1/2}$ gap for Z=8 and $1f_{7/2} - 2p_{3/2}$ gap for Z=20 (as a function of neutron number), required to explain empirical data are not obtained in the calculations with these interactions. It has been shown that the three-body forces have to be taken into account to reproduce these shell gaps [20, 21, 22]. As a next
step therefore, we have incorporated a simple three-body monopole term in CWG to construct CWG3M as prescribed in Ref.[21] for KB3 interaction. However as the mass regions considered in KB3 (A>40) and the present case (A> 132) are quite far off, we have included the effect of mass scaling by a factor of (40/132)\(^{(1/3)}\). It should be noted that the correction factor will be effective for nuclei for which the valence neutron number n \geq 3. A shell gap for N=90 now appears with CWG3M which is very close to that with SMPN. The \(E(2^+_1)\) energies of \(^{136,138}\)Sn are 0.639 and 0.633 MeV, respectively. The \(E(2^+_1)\) energy of \(^{140}\)Sn predicted by CWG3M (1.889 MeV) is also close to that predicted by SMPN (1.949 MeV) (Fig.3).

4. Conclusions
The SMPN results for \(^{136,138}\)Sn show that for increasing neutron number (>82), the Z = 50 shell closure weakens and Sn isotopes behave very similar to the other Z = 52-60 nuclei. However the new result on \(^{140}\)Sn, its high \(2^+_1\) energy and its comparison with examples from other neutron-rich domains clearly show that N=84-88 spectra with SMPN show the effect of gradual filling up of \(2f_7/2\) orbital which finally culminates in a new shell closure at N=90. A large contribution of \als\ term in the ESPE of \(2f_7/2\) in empirical interaction SMPN has been found to be responsible for the shell gap similar to that in sd shell. A simple three-body monopole term has been included in CWG to get CWG3M, which predicts a shell gap at N=90 for Sn isotopes as well as decreasing \(2^+_1\) energies for \(^{136,138}\)Sn, similar to that from SMPN. This indicates that three body effect plays an important role for shell evolution in neutron rich Sn isotopes above \(^{132}\)Sn, as also observed in sd and pf shells. The anomalously depressed \(2^+_1\) states in Sn isotopes having N=84-88, and the new shell closure for N=90, might have interesting consequences for the r-process nucleosynthesis.

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