Algorithms for modelling the load of teeth from pinion cutter tool used at worms machining

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Abstract. The turning of worms with tools of pinion cutter type means to generate helical surfaces by rolling. In this case, the process of detaching the chip by tool successive teeth is characterized by uneven variation of chip total area, calculated as sum of the areas corresponding to the chips simultaneously detached by all tool teeth in contact at a time with the machined part. As consequence, the cutting forces that load the tool cutting edges are also uneven which negatively impacts the machining process: uneven wear of tool teeth and uneven elastic deformation of technological system with bad effects onto the precision of generated worm flanks. This paper proposes the analytical modelling for both area and shape of the chips detached, during the machining process, by the successive teeth of the pinion cutter that generates a cylindrical worm. The tool tooth involute flanks and the arc from tooth head are modelled, thus delimiting the region formed by the successive active teeth of the tool. The model of the chip detached during the rolling between tool and part centred is determined and, on this base, the area of the chip detached for an increment of tool rotation is found. Starting from proposed model, a graphical application in CATIA environment was developed in order to calculate the cumulated area of chips simultaneously detached by tool teeth, which is further needed in order to reduce the cutting force unevenness.

1. Introduction

The cutting schemes modelling by rolling method is a special problem, especially in what concerns the tools generating by enwrapping and having more teeth, even more so when performed in order to find the shape and the area of detached chips. However, this modelling is necessary when aiming to smooth the energetic load of teeth cutting edges, which has positive consequences regarding cutting edges wear and cutting torque dynamics.

The tools as the rack tool, the pinion cutter, the rotating cutter, the worm tool, in particular when used at machining the teeth of involute gears, raise apart issues regarding the analysis of the shape and dimensions of the successively detached chips, due to the complexity of teeth machining process.

They were performed researches aiming the improvement of worm tool behaviour from smoothness point of view, during the machining of gears teeth [1-3]. They also can be mentioned researches with similar purpose and concerning the machining with pinion cutters of the globoid worms (on tooining machines) or the turning with rotating cutters of the threads with high values of the pitch (on specialized lathes) [3, 4]. All such approaches start from the kinematics of the process of generating by enwrapping (the rolling method) and have as mathematical support the Theory of
Enwrapped Surfaces [5]. The specific of the tools generating by rolling method is also an important issue [4, 6, 7].

The development of graphical design environments enables an easier approach of the problem, due to the facilities offered by the graphical methods. Thus, the profile of the tools generating by enwrapping can be rigorously determined, no matter if they are used for machining gear teeth or helical surfaces. At the same time, the graphical methods have potential for being used in order to analyze the generating processes in what concerns the finding of the shape and of the area of detached chips. Solutions regarding the modification of tools construction aiming to reduce the maximum values of cutting force and torque, when generating gears teeth with rack tool, pinion cutter or hob mill are presented in [6].

In this paper, it is proposed an algorithm for finding the area of the chips detached by the pinion cutter when generating cylindrical worms by turning. The algorithm was implemented in CATIA. Tool teeth profile was considered as involute. Regarding paper structure, next section presents the analytical model of tool tooth. The third section refers to the analytical modelling of tool successive teeth and the fourth deals with the model of the worm generating process. The fifth section presents the graphical solution for finding the detached chips area, while the last is for conclusions.

2. The analytical model of tool tooth

The pinion cutter tooth is considered as belonging to an ordinate whirl of involute profiles – the gear with straight teeth. The figure 1 presents the involute profile of tooth left flank, referred to the local system of $i$-th tool tooth, $X_iY_i$.

![Figure 1. Tooth left flank profile.](image)

According to involute definition, the arc $ST$ from the base circle of $R_b$ radius, and the segment $PT$ must have equal lengths, hereby we have:

$$R_b (\varphi + \psi) = R_b \tan \varphi,$$

which is equivalent to

$$\psi = \tan \varphi - \varphi = \text{inv} \varphi.$$

The coordinates of $P$ current point from left involute flank of the tooth, expressed into $X_iY_i$ system are:

$$\begin{align*}
X_i &= R_b \sin (\varphi - \kappa) - R_b (\varphi + \psi) \cos (\varphi - \kappa); \\
Y_i &= R_b \cos (\varphi - \kappa) + R_b (\varphi + \psi) \sin (\varphi - \kappa).
\end{align*}$$

The variation limits of $\varphi$ angular parameter result after imposing to $P$ the condition of belonging to the circles corresponding to the foot and to the head of the tooth, and having the radii $R_i$ and $R_e$, respectively. Hereby,
\[ \varphi_i \leq \varphi \leq \varphi_e \], where \( \varphi_i = \sqrt{\left(R_i^2 - R_b^2\right)/R_b^2 - \gamma} \) and \( \varphi_e = \sqrt{\left(R_e^2 - R_b^2\right)/R_b^2 - \gamma} \).  

(4)

In the equations of the tooth involute flank, the following notations were introduced:

- \( \frac{1}{2} \delta_b \), meaning half of the arc of circle corresponding onto basis circle to tool tooth

\[ \frac{1}{2} \delta_b = m[\pi/2 + z \cdot \text{inv}(20^\circ)], \text{ hence } \kappa = s_b/(2 \cdot R_b) - \gamma; \]  

(5)

- \( z \) – the plane gear number of teeth;

- \( m \) – the normal module of the gear (at straight gears being identical to frontal module);

- \( \text{inv}(20^\circ) = \tan(20^\circ) - 20^\circ/180^\circ; \)  

(6)

- \( \varphi \) – angular variable measuring the tool rotation around plane gear axis.

The equations of tooth right flank have a form similar to (2), with the difference that the angular parameters are defined in mirror relative to the picture from figure 1:

\[
\begin{align*}
X_i &= -R_b \sin(\varphi - \kappa) + R_b(\varphi + \gamma)\cos(\varphi - \kappa); \\
Y_i &= R_b \cos(\varphi - \kappa) + R_b(\varphi + \gamma)\sin(\varphi - \kappa).
\end{align*}
\]  

(7)

The figure 2 presents the tooth head, which is the arc of circle comprised between \( (H_L) \) and \( (H_R) \), points. Their coordinates can be found with the help of equations (3) and (7), respectively. For example, in the case of \( (H_L) \) by replacing \( \varphi \) with \( \varphi_e \) in (3), we have:

\[
\begin{align*}
X_{Hel} &= R_b \sin(\varphi_e - \kappa) - R_b(\varphi_e + \gamma)\cos(\varphi_e - \kappa); \\
Y_{Hel} &= R_b \cos(\varphi_e - \kappa) + R_b(\varphi_e + \gamma)\sin(\varphi_e - \kappa).
\end{align*}
\]  

(8)

The equations of tooth head profile are:

\[
\begin{align*}
X_i &= R_e \sin \theta; \\
Y_i &= R_e \cos \theta,
\end{align*}
\]  

(9)

where \( \theta \) means a parameter varying between \( \theta_l \) and \( \theta_r \), these limits being calculated with:

\[ \theta_l = X_{Hel}/Y_{Hel}, \text{ respective } \theta_r = X_{Her}/Y_{Her}. \]  

(10)

3. The analytical modelling of tool successive teeth

If the tool has \( z \) teeth, then the angle \( \delta \) meaning the gear angular step (the angle between the axis of two consecutive teeth, see figure 3) is

\[ \delta = 2\pi/z. \]  

(11)

If \( XY \) means the local reference system associated to the pinion cutter, then the coordinates transform (12) can be written between the systems \( X_iY_i \) and \( XY \). It should be noticed that their relative position remains unchanged, all \( X_iY_i \) systems, \( i = 1K \) \( z \) moving together with the tool.

\[
\begin{pmatrix}
X \\
Y
\end{pmatrix} = \begin{pmatrix}
\cos(i-1)\delta & -\sin(i-1)\delta \\
\sin(i-1)\delta & \cos(i-1)\delta
\end{pmatrix} \begin{pmatrix}
X_i \\
Y_i
\end{pmatrix}.
\]  

(12)

In relation (12), the system \( XY \) is considered as overlapped to \( X_iY_i \), while the vector \( X_iY_i \) may be formed by coordinates of the points from the left flank, the right flank or the tooth head.

In principle, the reference relative position between the tool and the worked worm is when the \( i \)-th tool tooth is in the middle of the gearing zone, whilst other tool teeth (from \( (i-k) \)-th to \( (i+k) \)-th) are also in contact with the worked worm. The actual value of \( k \) depends on the geometry of both tool and worked worm.
Thus, the analytical model of teeth that are simultaneously detaching chips can be written an ensemble of equations:

\[
\begin{align*}
X &= X_i \cos(\delta k) - Y_i \sin(\delta k), \\
Y &= X_i \sin(\delta k) + Y_i \cos(\delta k),
\end{align*}
\]

(\(\delta = -2, -1, 0, 1, 2 \ldots\)),

where \(X_i\) and \(Y_i\) are successively replaced with the expressions given by (3), (7) and (9) in order to describe teeth left flanks, right flanks and heads.

### 4. The analytical model of worm generating process

The worm generating process [8] involves the rolling motion between the centrodes of the generated worm (the straight line \(C_1\)) and of the pinion cutter (the circle \(C_2\) of \(R_d\) radius), see figure 4. If \(\Phi_1\) and \(\Phi_2\) mean the angular parameters of tool and worm motions around their axis, respectively, then the rolling condition between the two centrodes has the expression:

\[
\Phi_1 = \left[ \frac{p_{ax}}{2 \pi R_d} \right] \Phi_2,
\]

(14)

where \(p_{ax}\) is the axial pitch of the generated worm (with a single thread) and \(R_d\) – the radius of tool pitch circle:

\[
p_{ax} = m \cdot \pi, \quad R_d = m \cdot z / 2.
\]

(15)

Let us consider the transversal advance of the pinion cutter as proportional with tool rotation angle by \(\lambda\) ratio. As consequence, if by \(A_{12-0}\) we mean the initial distance between the generated worm axis and the tool axis, then at a given moment the actual distance between the two axes becomes:

\[
A_{12} = A_{12-0} - \lambda \cdot \Phi_1.
\]

(16)

The \(A_{12-0}\) distance corresponds to the tangency between the exterior cylinder of the generated worm (of \(r_e\) radius) and the exterior circle of the tool (of \(R_e\) radius), hence:

\[
A_{12-0} = r_e + R_e.
\]

(17)

In order to express the kinematics of worm generating process, the following reference systems are needed (figure 4): \(xy\), which is the tool global system, having the origin on the initial position of tool axis, \(x_0y_0\) – the worm global system, having the origin on worm axis, \(XY\) – local mobile system, attached to the tool, and \(X_0Y_0\) – local mobile system, attached to the generated worm and having the \(X_0\) axis coincident to worm axis.

The absolute motions equations are:
\( x = \omega_3^T (\phi_1) \cdot X \), (tool motion),
\( x_0 = X_0 + \left( \begin{array}{c} -R_d \cdot \phi_1 \\ 0 \end{array} \right) \), (worm motion).

In relation (18), \( \omega_3 \) means the well known matrix of rotation around \( XY \) system origin. The relative position between the two global systems is described by the relation:
\[ x_0 = x - \begin{pmatrix} 0 \\ A_{12-0} - \lambda(\phi_1) \end{pmatrix}. \]

The matrix equation expressing the generating motion (the relative motion between the two mobile systems) results from relations (18) – (20) as:
\[ X_0 = \omega_3^T (\phi_1) \cdot X - \begin{pmatrix} -R_d \cdot \phi_1 \\ A_{12-0} - \lambda(\phi_1) \end{pmatrix}. \]

The equation (21) gives the positions of pinion cutter teeth relative to the worked part. After developing, the following couple of equations results:
\[
\begin{align*}
X_0 &= X \cos \phi_1 - Y \sin \phi_1 + R_d \cdot \phi_1; \\
Y_0 &= X \sin \phi_1 + Y \cos \phi_1 - A_{12-0} + \lambda(\phi_1).
\end{align*}
\]

In these equations, \( X \) and \( Y \) have the meaning from (12); hereby, \( 2k+1 \) couples of forms can be used, corresponding to the \( 2k+1 \) tool teeth concomitantly in contact with the worked part.

Remarks

- Despite the motions of both tool and worked part are continuous, the calculus of detached chips cumulated area requires to look to them in discrete form. Thus, the tool rotation is considered as incremental, having the step \( \Delta \Phi_1 \):
\[ \Delta \Phi_1 = 2\pi / N, \quad \phi_j = j \cdot \Delta \Phi_1, \quad j = 1, 2, K \]

Here \( N \) means the number of angular steps corresponding to a tool complete rotation, its value being adopted after modelling targeted accuracy. The \( \Delta \Phi_2 \) angular step of part rotation, corresponding to \( \Delta \Phi_1 \), can be calculated by having in view the dependence (14).

- The number of tool teeth that are concomitantly in contact, at a given moment, with the worked part (in its axial plane) can be found from the condition of intersection between tool exterior circle (having the radius \( R_e \) and the centre in the current position according to (16)) and worked part axial section (rectangle with \( 2r_e \) width).

- The tool feed motion towards worked part axis should also be considered in discrete form, having \( \Delta \lambda \) step. Thus it can be written:
\[ \Delta \lambda = (r_e - r_i) / n, \quad \lambda(\phi_1) = j \cdot \Delta \lambda, \quad j = 1, 2, K \]

Here \( r_e \) and \( r_i \) mean the radii of worm head respective foot cylinders, while \( n \) is a parameter whose value is set from technological reasons. When the current value of \( A_{12} \) reaches the value corresponding to tangency between the centrodes \( C_1 \) and \( C_2 \), the tool radial feed stops and the value of \( A_{12} \) remains constant.

5. The graphical solution for finding detached chips area

A graphical solution for finding the detached chips cumulated area has been developed in CATIA environment, as alternative to the analytical solution, which requires complicated numerical calculus. The graphical solution is further introduced with the help of a numerical simulation, performed in the case of machining a worm having one thread, involute profile \((m = 5 \text{ mm})\) and the exterior diameter
\( r_e = 17 \text{ mm}, \) with a pinion gear having \( z = 20 \) teeth. The actions supposed to be accomplished in order to implement the graphical solution are presented below.

- The models of both worked part and tool are drawn. Worked part model is a cylinder. Pinion cutter model is generated at first in Sketcher module, the coordinates of the points from tooth profile being calculated with the relations (3), (7) and (9), see figure 5. Then, the sketch is extruded with Pad tool from Sketch Based Features toolbar and a mono-tooth circular disc results. Finally, the tooth is multiplied on foot circle circumference by Circular pattern tool (figure 6).

![Figure 5. Tool tooth sketch.](image1)

![Figure 6. Pinion cutter model.](image2)

- The two models are brought into contact, after rotating the tool model such as the symmetry axis of a tooth becomes normal onto part model surface, figure 7. Eight axial sections of the part are considered, each one making a 45° angle with the next, for measuring the detached chips area.

![Figure 7. Tool – worked part initial positioning.](image3)

![Figure 8. Detached chips shape (12-th part rotation).](image4)

- The tool and part motions are simulated by successively considering the eight sections of the part and the corresponding positions of the tool – for a 45° rotation of the part, the tool model should rotate with \( \Delta \Phi_1 = \frac{2\pi}{8z} = 2.25^\circ \) and translate towards part axis with \( \lambda \cdot \Delta \Phi_1 = 0.0625 \text{ mm} \).

- For each tool / part relative position, the two models are overlapped, the intersection areas are removed from part section (by Remove command) and the remaining surface is measured. The shape and the area of the detached chips result by comparing two successive remaining surfaces from the same axial section (corresponding to two consecutive rotations of the part (figures 8, 9).

In the case of the addressed simulation, the complete generation of worm teeth profile requires 23 complete rotations of the part, if a radial feed of 0.5 mm/rot is considered. The numerical results that have been obtained are sampled in Table 1. The variation of detached chips cumulated area against the current number of tool / part relative position from Table 1 is represented in figure 10.
**Figure 9.** Tool consecutive cuts in the same axial section.

**Table 1.** Numerical results (excerpt)

| Crt. Part no. rot. | Section number | Total radial feed [mm] | Part rotation angle [°] | Tool rotation angle [°] | Chips cumulated area [mm²] |
|-------------------|----------------|------------------------|------------------------|------------------------|---------------------------|
| 1                 | 1              | 0.0625                 | 45                     | 2.25                   | 0.05                      |
| 2                 | 1              | 0.125                  | 90                     | 4.5                    | 0.008                     |
| 3                 | 1              | 0.1875                 | 135                    | 6.75                   | 0                         |
| 4                 | 1              | 0.25                   | 180                    | 9                      | 0                         |
| 5                 | 1              | 0.3125                 | 225                    | 11.25                  | 0.013                     |
| 6                 | 1              | 0.375                  | 270                    | 13.5                   | 0.49                      |
| 7                 | 1              | 0.4375                 | 315                    | 15.75                  | 0.997                     |
| 8                 | 1              | 0.5                    | 360                    | 18                     | 1.295                     |
| 9                 | 2              | 0.625                  | 405                    | 20.25                  | 1.304                     |
| 10                | 2              | 0.625                  | 450                    | 22.5                   | 1.161                     |
| 11                | 2              | 0.6875                 | 495                    | 24.75                  | 0.756                     |
| 12                | 2              | 0.75                   | 540                    | 27                     | 0.406                     |
| 13                | 2              | 0.8125                 | 585                    | 29.25                  | 1.09                      |
| 14                | 2              | 0.875                  | 630                    | 31.5                   | 1.432                     |
| 15                | 2              | 0.9375                 | 675                    | 33.75                  | 1.539                     |

**Figure 10.** Variation of detached chips cumulated area.
6. Conclusions
The subject addressed in this paper is the modelling for both area and shape of the chips detached, during the machining process, by the successive teeth of the pinion cutter that generates a cylindrical worm. In this purpose, the tool tooth involute flanks and the arc from tooth head have been modelled, thus delimiting the region formed by the successive active teeth of the tool. The model of the chip detached during the rolling between tool and part centrodes has been determined and, on this base, the area of the chip detached for an increment of tool rotation can be found. Starting from proposed model, a graphical application in CATIA environment has been developed in order to calculate the cumulated area of chips simultaneously detached by tool teeth.

The results obtained within a numerical simulation performed in an actual case of worm generation enable to draw some conclusions:

- The CATIA graphical application is feasible in finding the shape and the detached chips cumulated area,
- The detached chips cumulated area steadily increases during tool radial feed motion (from 0 up to 17 \text{mm}^2 in the performed simulation),
- Besides the general increasing trend, the detached chips cumulated area presents a second, irregular variation pattern during each rotation of the worked part, due to the successive teeth of the tool that are coming in and out from the contact with the part, and
- It is possible to find a simple variation law of the radial feed speed, such as the variation of detached chips cumulated area is significantly reduced, at least during the tool radial motion. The main target should be the annulling of general increasing trend, because the other variation pattern is much less important.

The target of future researches is to design an experimental stand for actually validating the researches concerning the diminishing of the cutting force unevenness during teeth cutting processes.

References
[1] Antoniadis A, Vidakis N and Bilalis N 2002 Fracture Fatigue Investigation of Cemented Carbide Tools \textit{J Manuf Sci Eng}, \textbf{124}, 784-791.
[2] Georgescu V, Oancea N and Constantin E 1981 Le chargement des tranchants de point des dents de la fraise helicoidal (translation: The loading of teeth cutting edges at the helical mills) \textit{Buletinul Universitatii din Galați}, V, 29-34.
[3] Gimpert D 1994 Gear Hobbing Process \textit{Gear Technology} \textbf{11}, 38-44.
[4] Dima M, Oancea N and Teodor V G 2007 Modelarea schemelor de aşchiere la danturare (translation: Modelling of tootthing cutting schemes) CERMI Publishing House Iaşi.
[5] Litvin F L 1989 \textit{Theory of gearing} NASA, Scientific and Technical information Division, Washington DC.
[6] Oancea N 2004 \textit{Surface Generation by Enwrapping, Vol. II, Complementary theorems „Dunărea de Jos” Publishing House, Galați.}
[7] Dima M, Rusu C and Parvu G 2001 A Geometric Model regarding the Engendering Generalisation of Some Polyform Surfaces, \textit{The Annals of “Dunărea de Jos” University of Galați}, V 38-44.
[8] Maros D, Kilimann V and Rohanyi V 1966 \textit{Angrenaje melcate} (translation: Worm gears) Editura tehnică, București.