Band Splitting Permutations for Spatially Coupled LDPC Codes Enhancing Burst Erasure Immunity

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Abstract—It is well known that spatially coupled (SC) codes with erasure-BP decoding have powerful error correcting capability over memoryless erasure channels. However, the decoding performance of SC-codes significantly degrades when they are used over burst erasure channels. In this paper, we propose band splitting permutations (BSP) suitable for \((l, r, L)\) SC-codes. The BSP splits a diagonal band in a base matrix into multiple bands in order to enhance the span of the stopping sets in the base matrix. As theoretical performance guarantees, lower and upper bounds on the maximal burst correctable length of the permuted \((l, r, L)\) SC-codes are presented. Those bounds indicate that the maximal correctable burst ratio of the permuted SC-codes is given by \(\lambda_{\text{max}} \simeq 1/k\) where \(k = r/l\). This implies the asymptotic optimality of the permuted SC-codes in terms of burst erasure correction.

I. INTRODUCTION

Low-Density Parity-Check (LDPC) codes that are linear codes defined by extremely sparse parity check matrices were developed by Gallager in 1963 [1]. The combination of LDPC codes and belief propagation provides remarkable error correcting performance with reasonable time complexity. In recent days, it is easy to find practical applications of LDPC codes in wireless/wired communication systems and storage systems. Not only a practical importance but also recent theoretical advancement produces renewed interests in this field. Kudekar et al. [2] proposed a new class of LDPC codes, that is called spatially coupled codes (SC-codes) and they provided theoretical arguments on threshold saturation of SC-codes [2]. The origin of SC-codes is LDPC-convolutional codes that date back to the work due to Felstrom and Zigangirov [3]. Lentmaier et. al [4] showed an ensemble of an LDPC-convolutional code can have a higher threshold than that of a component LDPC code ensemble. From these works on SC-codes, it is unveiled that well-designed SC-codes have capacity achieving performance over symmetric memoryless channels.

A burst erasure means a consecutive erased symbols. In many practical situations, we can observe occurrences of burst erasures due to slow fading in mobile wireless communication, buffer overflow at a congested router in a packet based network, and media flaw in a magnetic recording system. A strong erasure correcting code should have high erasure correcting capability not only for memoryless random erasures but also for burst erasures. Ohashi et. al [5] pointed out that SC-codes are not immune to burst erasures compared with conventional LDPC codes such as regular LDPC codes. In a typical decoding process of SC-codes, reliabilities of bit estimation gradually improves from both side into inside as a domino toppling. Since a burst erasure interferes the propagation of a wave of such reliable estimations, it causes severe degradation on decoding performance. In order to overcome this difficulties, they proposed a new class of multidimensional SC-codes that shows higher immunity against burst erasures.

It is known that the burst erasure correcting capability of LDPC codes depends on a column order of parity check matrices of LDPC codes [12]. This is because the minimum length of stopping sets determining the burst correcting capability depends on the column order of a parity check matrix. In order to enhance the burst erasure correctability, several heuristic algorithms to improve the column order have been presented by Wadayama [9], Paolini and Chiani [15], Hosoya et. al. [3]. Of course, the column order of a parity check matrix does not affect the decoding performance over memoryless erasure channels.

In this paper, we will propose a class of column permutations that is called band splitting permutations suitable for \((l, r, L)\) SC-codes. A band splitting permutation is applied to the base matrix of \((l, r, L)\) SC-codes having a single diagonal band and it results in a column-permuted base matrix with several diagonal bands. By lifting up the permuted base matrix, we can obtain a parity check matrix of a permuted \((l, r, L)\) SC-codes. It will be proved that an appropriate band splitting permutation produces permuted \((l, r, L)\) SC-codes that have near optimal minimum length of stopping sets. The permuted SC-codes constructed in such a way have burst erasure correcting superior to those of conventional SC-codes. Upper and lower bounds on the minimum length of stopping sets to be proved in this paper can provide theoretical performance guarantees for burst erasure correcting capability of permuted SC-codes.

The outline of this paper is as follows. Section 2 provides notion and fundamental definitions required throughout this paper. Section 3 presents several theorems regarding stopping sets in a base matrix. The band splitting permutations will be defined and analyzed in Section 4. Results on computer experiments will be shown in Section 5.

II. PRELIMINARIES

A. \((l, r, L)\) SC-codes

In this subsection, the definition \((l, r, L)\) SC-codes proposed by Kudekar et al. [2] is reviewed. The \((l, r, L)\) SC-codes belong to the class of protograph LDPC codes and its parity check matrix can be obtained by lifting up the base matrix \(B(l, r, L)\). The base matrix \(B(l, r, L)\) is a binary \((L+l-1) \times kL\) matrix \((k = r/l)\) and its structure is illustrated...
in Fig. 1 The parameters \( l \) and \( r \) represents the column weight and maximal row weight of \( B(l, r, L) \), respectively. We assume that the ratio \( k = r/l \) is integer throughout the paper. The parameter \( L \) denotes the number of sections.

A parity check matrix of an \((l, r, L)\) SC-code can be obtained by lifting up the base matrix \( B(l, r, L) \). A lift-up process is summarized as follows: For each element one in \( B(l, r, L) \), we can replace it with any binary \( M \times M \) permutation matrix. The zeros in \( B(l, r, L) \) should be replaced with a binary \( M \times M \) zero matrix. Let a parity check matrix obtained by the above process be \( H \). The binary linear code defined by \( H \) is called an \((l, r, L)\) SC-code. The size of the permutation matrices, \( M \), is said to be the lift up factor. The number of rows of \( H \) is \( M(L + l - 1) \) and the number of columns is \( MkL \). The design rate of \((l, r, L)\) SC-codes, \( R(l, r, L) \), is thus given by

\[
R(l, r, L) = 1 - \frac{1}{k} - \frac{l - 1}{kL}.
\]

### B. Maximal correctable burst length

Yang and Ryan [12] introduced a measure for burst erasure correcting capability of LDPC codes that is called the maximal correctable burst length. Let \( H \) be a parity check matrix that defines an LDPC code. The maximal correctable burst length of this code is denoted by \( W_{\text{max}}(H) \). The meaning of \( W_{\text{max}}(H) \) is the following. A burst erasure is a sequence of consecutive erasures occurred on an erasure channel. In this paper, we assume that only single burst erasure occurs in a code block. If the length of a single burst erasure is less than or equal to \( W_{\text{max}}(H) \), it can be perfectly corrected by belief propagation (BP) decoding for erasure channels. On the other hand, there exists a single burst erasure of length \( W_{\text{max}}(H) + 1 \) that cannot be corrected with erasure-BP. Namely, \( W_{\text{max}}(H) \) represents the maximum guaranteed correctable length for any single burst erasure. As a related measure for burst erasure correcting capability, we here introduce the maximal correctable burst ratio defined by

\[
\lambda_{\text{max}} = \frac{W_{\text{max}}(H)}{n},
\]

where \( n \) is the code length. This quantity is useful for studying asymptotic behavior of the burst correcting capability.

### C. Stopping sets and maximal correctable burst length

#### 1) Stopping sets: Let \( H = (h_1, h_2, \ldots, h_n) \in \mathbb{F}_2^{m \times n} \) be a parity check matrix. The vector \( h_i \) is the \( i \)-th column vector of \( H \). A sub-matrix of \( H \) consists of a subset of column vectors in \( H \); namely a sub-matrix of \( H \) has the form:

\[
H_{\{i_1, \ldots, i_u\}} = (h_{i_1}, h_{i_2}, \ldots, h_{i_u}) \in \mathbb{F}_2^{m \times u}.
\]

The subscript in \( H_{\{i_1, \ldots, i_u\}} \) represents the column indices of \( H \) corresponding to the column vectors in the sub-matrix.

#### Definition 1 (Stopping sets [13])

Let \( H \) be a parity check matrix and \( S = \{i_1, i_2, \ldots, i_u\} \subseteq [1, n] \) be an index set. The notation \([a, b]\) denotes the set of consecutive integers from \( a \) to \( b \). If the sub-matrix \( H_S \) has no rows with weight one, the index set \( S \) is said to be a stopping set.

It is well known that stopping sets are closely related to correctability of erasure patterns if we exploit erasure-BP. Assume that a transmitted word is a codeword of the code defined by \( H \) and that some symbol erasures happen over the channel. Let \( E = \{e_1, e_2, \ldots, e_w\} \subseteq [1, n] \) be the indices corresponding to the symbol erasures. This erasure pattern cannot be corrected with erasure-BP if there exists a non-empty stopping set \( S \) satisfying \( S \subseteq E \). This fact indicates that the set of stopping set in \( H \) determine \( W_{\text{max}}(H) \) [14].

Assume that \( H = (h_1, h_2, \ldots, h_n) \in \mathbb{F}_2^{m \times n} \) is given and an index set \( S = \{i_1, i_2, \ldots, i_u\} \subseteq [1, n] \) is given as well. The length of \( S \), that is denoted by \( \text{Len}(S) \), is defined by

\[
\text{Len}(S) = 1 + \max_{a, b \in S} |a - b|.
\]

Let us denote the set of non-empty stopping sets of \( H \) by \( \mathcal{Q}(H) \). The span of \( H \), \( \text{Span}(H) \), is defined by

\[
\text{Span}(H) = \min_{S \subseteq \mathcal{Q}(H)} \text{Len}(S).
\]

It is clear that a burst erasure of length shorter than \( \text{Span}(H) \) cannot cover any non-empty stopping set in \( H \). This means that we have

\[
W_{\text{max}}(H) = \text{Span}(H) - 1.
\]

Note that the quantity \( W_{\text{max}}(H) \) can be evaluated efficiently by using erasure-BP [13]. From the definition, we can see that \( \text{Span}(H) \) strongly depends on the order of the column vectors in \( H \). It has been shown that an appropriate rearrangement of column order can increase the span of LDPC codes [8], [9].

#### 2) Irreducible stopping sets:

We provide the definition of irreducible stopping sets that will be required in the next section.

#### Definition 2 (Irreducible stopping sets)

Let \( S \subseteq [1, n] \) be a non-empty stopping set of \( H \). If removing any subset of elements from \( S \) yields an index set that is not a stopping set, then \( S \) is said to be irreducible stopping set.

From the above definition of irreducible stopping sets, it is straightforward to see that the inequality \( \text{Len}(S') \leq \text{Len}(S) \) holds for a pair of nested stopping sets where \( S \) is a stopping set and \( S' \subseteq S \) is an irreducible stopping set in \( S \). From this inequality, we have

\[
\text{Span}(H) = \min_{S \subseteq \mathcal{Q}(H)} \text{Len}(S'),
\]

where \( \mathcal{Q}(H) \) is the set of irreducible stopping set of \( H \). This means that we only need to focus on the set of irreducible stopping sets when we discuss the span of \( H \).
III. IRREDUCIBLE STOPPING SETS IN BASE MATRIX

In this section, we will prepare several theorems regarding the maximal correctable burst length that are required for the argument in Section IV.

A. Maximal correctable burst length of base matrices

Sridharan et. al [16] studied the maximal correctable burst length of protograph LDPC codes. They showed a tight relationship between $W_{max}(B)$ and $W_{max}(H)$ where $H$ is a parity check matrix obtained by lifting up a base matrix $B$. The next theorem states this relationship.

**Theorem 1 (Maximal correctable burst length (16))**

Assume that a base matrix $B \in \mathbb{F}_2^{m \times n}$ is given. Let $H$ be a parity check matrix obtained by lifting up $B$. The following inequalities hold:

$$W_{max}(B) - 1)M < W_{max}(H) < (W_{max}(B) + 1)M.$$ (7)

Theorem 1 indicates that the maximal correctable burst length of a protograph LDPC code is nearly determined by $W_{max}(B)$. This means that an appropriate column permutation for a base matrix $B$ might be able to improve the maximal correctable burst length of a resulting photograph code. Of course, $(l,r,L)$ SC-codes belong to the class of protograph LDPC codes. It is reasonable to devise an appropriate column permutation for $B(l,r,L)$, which will be discussed in the next section.

B. Irreducible stopping sets in $B(l,r,L)$

The maximal correctable burst length of the base matrix $B(l,r,L)$ is determined by the set of irreducible stopping sets in $B(l,r,L)$. In this subsection, we will show a structural property on the set of irreducible stopping sets in $B(l,r,L)$.

Let us denote the base matrix of the $(l,r,L)$ SC-codes as $B(l,r,L) = (b_1, b_2, \ldots, b_{kL}) \in \mathbb{F}_2^{m \times kL}$.

A block $T_i (i \in [1,L])$ that is a subset of indices is defined by

$$T_i = \{(i-1)k+1, (i-1)k+2, \ldots, (i-1)k+k\}.$$ (8)

From the structure of $B(l,r,L)$ (i.e., Fig. 1), it is easy to see that $b_\alpha = b_\beta$ holds if and only if $\alpha, \beta \in T_i$. The next theorem characterizes the structure of irreducible stopping sets in $B(l,r,L)$.

**Theorem 2 (Irreducible stopping sets of base matrix)**

The set of irreducible stopping sets in the base matrix $B(l,r,L)$ is given by

$$Q'(B(l,r,L)) = \{ (\alpha, \beta) | \alpha, \beta \in T_i, i \in [1,L] \}.$$ (9)

The theorem states that an irreducible stopping set consists of two column indices belonging to the same block. (Proof) Suppose an ordered index set $S = \{ j_1, \ldots, j_u \} \subseteq [1,n]$ is given where $j_1 < j_2 < \cdots < j_u$. The sub-matrix corresponding to $S$ is written as $B(l,r,L)_{(j_1,\ldots,j_u)} = (b_{j_1}, b_{j_2}, \ldots, b_{j_u})$.

We will first show a sufficient condition that $S$ is not a stopping set. Assume that $b_\alpha \neq b_\beta$ holds for any $\alpha, \beta \in S (\alpha \neq \beta)$. Let us focus on the first nonzero element of the first column of the sub-matrix $B(l,r,L)_{(j_1,\ldots,j_u)}$. Due to the assumption that $b_\alpha \neq b_\beta$ and the definition of $B(l,r,L)$, it is evident that the row corresponding to the first nonzero element has Hamming weight 1. This means that $S$ cannot be a stopping set in this case.

By using this sufficient condition, we can immediately show that any stopping set of $B(l,r,L)$ contains two different indices which belong to the same block. In other words, any stopping set must contain $\alpha, \beta$ satisfying $b_\alpha = b_\beta (\alpha \neq \beta)$. If a stopping set without such a pair exists, it contradicts the sufficient condition shown above.

It is clear that a pair of indices $(\alpha, \beta)$ $(\alpha, \beta \in [1,kL], \alpha \neq \beta)$ is an irreducible stopping set if both indices $\alpha$ and $\beta$ belong to the same block. The last job is to show that there are no irreducible stopping sets with size larger than 2. Suppose that $S$ is an irreducible stopping set with size larger than 2. From the above argument, $S$ must contain at least a pair of two elements that belong to the same block. Since such a pair constitutes an irreducible stopping set, it contradicts the assumption that $S$ is an irreducible stopping set. This completes the characterization of the set of irreducible stopping sets of $B(l,r,L)$.

C. Burst erasure correcting capability of $(l,r,L)$ SC-codes

An immediate application of Theorem 2 is to analyze the burst erasure correcting capability of $(l,r,L)$ SC-codes. The size of irreducible stopping set is two and the minimal length of the stopping set is thus two; we have $\text{Span}(B(l,r,L)) = 2$. This gives $W_{max}(B(l,r,L)) = 1$ and we can use Theorem 1 to obtain lower and upper bounds on maximal correctable burst length of $(l,r,L)$ SC-codes:

$$0 < W_{max}(H) < 2M,$$ (10)

where $H$ represents a parity check matrix of $(l,r,L)$ SC-codes. By dividing both sides in (10) by the code length $kLM$, we have inequalities for the maximal correctable burst ratio:

$$0 < \lambda_{max} < \frac{2}{kL}.$$ (11)

It is clear that $\lambda_{max}$ converges to zero when $L$ goes to infinity. This inequality presents that the conventional $(l,r,L)$ SC-codes have poor burst erasure correcting capability in the asymptotic regime when $L \to \infty$. This result justifies the observation made by Ohashi et. al [7].

IV. BAND SPLITTING PERMUTATIONS

In this section, we will propose band splitting permutations (BSP) for the base matrix $B(l,r,L)$. The BSP is designed to improve the span of $B(l,r,L)$.

A. Definition

When a BSP $\sigma_{k,L}$ is applied to a base matrix $B(l,r,L)$, we have permuted base matrix with multiple bands as shown in Fig. 2. The formal definition of BSP $\sigma_{k,L}$ is given as follows: According to Cauchy’s two-line notation on a permutation, the permutation $\sigma_{k,L}$ is described as

$$\sigma_{k,L} = \left( \begin{array}{cccc} 1 & 2 & \cdots & kL \\ f(1) & f(2) & \cdots & f(kL) \end{array} \right).$$ (12)
Let $H$ be a parity check matrix obtained by lifting up $B^*(l, r, L)$ with the lift up factor $M$. The maximal correctable burst length $W_{\text{max}}(H)$ of the permuted SC-code satisfies the following inequalities:

$$ (L - 1)M < W_{\text{max}}(H) < (L + 1)M. \quad (19) $$

**(Proof)** Assume that $S = \{i_1, i_2, \ldots, i_u\} \subseteq [1, n]$ is a stopping set of $B(l, r, L)$. The BSP maps $S$ to $S^* = \{f^{-1}(i_1), f^{-1}(i_2), \ldots, f^{-1}(i_u)\}$.

Note that $S^*$ is also a stopping set of $B^*(l, r, L)$ because $B^*(l, r, L)\{f^{-1}(i_1), f^{-1}(i_2), \ldots, f^{-1}(i_u)\}$ contains a row of weight 1 as well. This means that there is one-to-one correspondence between stopping sets in $B(l, r, L)$ and those in $B^*(l, r, L)$. Theorem 2 indicates that a non-empty irreducible stopping set consists of two indices in the same block. Assume that a pair $\alpha, \beta \in [1, kL]$ is such a pair of indices. From a definition of the span (6), it is clear that $\alpha$ and $\beta$ belong to the same column in $A$. The definition of $f$ in (13) thus leads to the inequality $|f^{-1}(\alpha) - f^{-1}(\beta)| \geq L$ that implies the length of irreducible stopping sets in $B^*(l, r, L)$ is larger than or equal to $L + 1$. Note that the equality holds when $\alpha$ and $\beta$ are consecutive. From the definition of the span (6), we thus have $\text{Span}(B^*(l, r, L)) = L + 1$ and this implies $W_{\text{max}}(B^*(l, r, L)) = L$. By using Theorem 1 the claim of this theorem is obtained.

The inequalities of Theorem 3 indicates that the maximal correctable burst length of the permuted $(l, r, L)$ SC-codes is proportional to the number of sections $L$. The inequality (19) indicates that the maximal correctable burst length does not depend on $L$ for the case of the conventional $(l, r, L)$ SC-codes. This result clearly shows the advantage of the permuted SC-codes over the conventional $(i.e.,$ non-permuted) SC-codes with respect to the maximal correctable burst length.

### C. Maximal correctable burst ratio

In this subsection, we focus on the maximal correctable burst ratio $\lambda_{\text{max}}$ of the permuted $(l, r, L)$ SC-codes.

By dividing both sides in (19) by the code length $kLM$, we can obtain following inequalities for the maximal correctable burst ratio:

$$ \frac{L - 1}{kL} < \frac{1}{kL} \leq \lambda_{\text{max}} < \frac{L + 1}{kL}. \quad (20) $$

From (20), it is clear that $\lambda_{\text{max}}$ converges to $1/k$ when $L \to \infty$. On the other hand, the design rate $R(l, r, L)$ of the $(l, r, L)$ SC-codes converges to $1 - 1/k$ as $L$ goes to infinity. From these results, we have

$$ \lim_{L \to \infty} \left( \lambda_{\text{max}} + R(l, r, L) \right) = 1 \quad (21) $$

that indicates asymptotic optimality of permuted $(l, r, L)$ SC-codes in terms of burst erasure correction with erasure-BP. Note that no binary linear code of length $n$ with design rate $r$ can correct burst erasures of length larger than $n(1 - r)$.

### V. Numerical results

We have seen that the maximal correctable burst ratio of permuted SC-codes can be approximated by $\lambda_{\text{max}} \approx 1/k$.

The second row of two line notation, i.e., the bijective function $f$ on $[1, kL]$, is defined by

$$ (f(1) \ f(2) \cdots f(kL)) = (a_1 \ a_2 \cdots a_k), \quad (13) $$

where $a_i (i \in [1, k])$ is given by

$$ a_1 = (1 \ 1+k \ 1+2k \cdots 1+(L-1)k) $$

$$ a_k = (k \ k+k \ k+2k \cdots k+(L-1)k). \quad (14) $$

The permutation $\sigma_{k,L}$ can be seen as a block interleaver of interleaving depth $k$. Applying $\sigma_{k,L}$ to the base matrix $B(l, r, L) = (b_1, \ldots, b_{kL})$, we obtain a column permuted version of a base matrix $B^*(l, r, L) = (b_{f(1)}, \ldots, b_{f(kL)})$.

For example, when $k = 2, L = 3$, we have

$$ a_1 = (1 \ 3 \ 5), \ a_2 = (2 \ 4 \ 6) \quad (15) $$

$$ \sigma_{2,3} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \ 1 & 3 & 5 & 2 & 4 & 6 \end{pmatrix}. \quad (16) $$

Applying $\sigma_{2,3}$ to $B(3, 6, 3)$, the permuted base matrix is obtained as

$$ B^*(3, 6, 3) = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \ 1 & 1 & 0 & 1 & 1 & 0 \ 1 & 1 & 1 & 1 & 1 & 1 \ 0 & 1 & 1 & 0 & 1 & 1 \ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}. \quad (17) $$

Let us define $k \times L$ matrix $A$ by

$$ A = \begin{pmatrix} a_1 \\ \vdots \\ a_k \end{pmatrix}. \quad (18) $$

It is easy to see that $i$-th column of $A$ corresponds the block $T_i$ which is defined by (8). This implies that column vectors in $B(l, r, L)$ belonging to the same block are rearranged in $B^*(l, r, L)$ as apart as possible. This property enhances the span of the base matrix.

### B. Bounds on maximal correctable burst length

By lifting up the permuted base matrix $B^*(l, r, L)$, we can obtain a parity check matrix of permuted $(l, r, L)$ SC-codes. The next theorem provides upper and lower bounds on the maximal correctable burst length of permuted $(l, r, L)$ SC-codes. This is the main contribution of this work.

**Theorem 3 (Bounds on maximal correctable burst length)**

Let $B^*(l, r, L)$ be the permuted base matrix defined above.
when $L$ is large enough. We will here show the relationship between $\lambda_{\text{max}}$ and $L$ when $L$ is finite. Figure 3 presents the bounds on $\lambda_{\text{max}}$ and the BP threshold of $(3,6,L)$ SC-codes. The horizontal axis represents the number of sections $L$ and the vertical axis is the maximal correctable burst ratio $\lambda_{\text{max}}$. It can be observed that $\lambda_{\text{max}}$ of the conventional $(3,6,L)$ SC-codes is decreasing as $L$ increases. On the other hand, we can see that $\lambda_{\text{max}}$ of the permuted $(3,6,L)$ SC-codes converges to $1/2$. It should be remarked that $\lambda_{\text{max}}$ of the permuted SC-codes is higher than the BP threshold $\theta(3,6,L)$ when $L \geq 80$. For example, when $L = 128$, $\lambda_{\text{max}}$ of the permuted SC-codes is 0.496 but the BP threshold $\theta(3,6,128)$ is 0.488. Assume the case where the lift up factor $M \to \infty$. A combination of the conventional $(l,r,L)$ SC-codes and an ideal symbol interleaver that can convert a single burst erasure into a memoryless random erasures may achieve the $\lambda_{\text{max}} = \theta(3,6,L)$. Thus, the permuted SC-codes yields better asymptotic burst erasure correcting performance when $L$ is large enough.

Figure 4 presents histograms of $\lambda_{\text{max}}$ of randomly permuted $(3,6,32)$ SC-codes and permuted $(3,6,32)$ SC-codes (i.e., proposed codes), which are obtained by computer experiments with 1000 samples for each. The lift up factor is assumed to be $M = 40$. A randomly permuted SC-code is generated as follows: a parity check matrix of conventional SC-codes is at first produced and a uniformly random column permutation is then applied to it. It can be observed that $\lambda_{\text{max}}$‘s of randomly permuted SC-codes are far less than those of the proposed SC-codes. This result suggests that it is not trivial to find a superior permutation which provides better burst erasure correcting capability than that of a systematically designed BSP.

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