On decays of $X(3872)$ to $\chi_{cJ}\pi^0$ and $J/\psi\pi^+\pi^-$

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By describing the $X(3872)$ using the extended Friedrichs scheme, in which $D\bar{D}^*$ is the dominant component, we calculate the decay rates of the $X(3872)$ to $\pi^0$ and a $P$-wave charmonium $\chi_{cJ}$ state with $J = 0, 1, 2$, and its decays to $J/\psi\pi^+\pi^-$ where $\pi^+\pi^-$ are assumed to be produced via an intermediate $\rho$ state. The decay widths of $X(3872) \rightarrow \chi_{cJ}\pi^0$ for $J = 0, 1, 2$ are of the same order. However, this model calculation exhibits that the decay rate of $X(3872)$ to $\chi_{c1}\pi^0$ is one order of magnitude smaller than its decay rate to $J/\psi\pi^+\pi^-$. 
INTRODUCTION

Discovery of the narrow hadron state $X(3872)$, first observed by the Belle collaboration in 2003 \[1\] and soon confirmed by the CDF, BABAR, and D0 collaborations \[2,3\], challenges the prediction of quark model and arouses enormous experimental explorations and theoretical studies, as reviewed by Ref. \[3–5\]. Recently, the BESIII collaboration searched in $e^+e^- \rightarrow \gamma \chi_{cJ}\pi^0$ for $J = 0, 1, 2$ and reported an observation of $X(3872) \rightarrow \chi_{c1}\pi^0$ with the branching fraction $3$

$$\frac{B(X(3872) \rightarrow \chi_{c1}\pi^0)}{B(X(3872) \rightarrow J/\psi\pi^+\pi^-)} = 0.88^{+0.33}_{-0.27} \pm 0.10.$$  \hbox{(1)}

They also set 90\% confidence level upper limits on the corresponding ratios for the decays to $\chi_{c0}\pi^0$ and $\chi_{c2}\pi^0$ as 19 and 1.1, respectively. Soon after, the Belle collaboration made a search for $X(3872)$ in $B^+ \rightarrow \chi_{c1}\pi^0K^+$ but did not find a significant signal of $X(3872) \rightarrow \chi_{c1}\pi^0$. They reported an upper limit$^2$

$$\frac{B(X(3872) \rightarrow \chi_{c1}\pi^0)}{B(X(3872) \rightarrow J/\psi\pi^+\pi^-)} < 0.97$$  \hbox{(2)}

at 90\% confidence level.

The ratio of $X(3872)$ to $\chi_{c1}\pi^0$ with $J = 0, 1, 2$ is suggested to be sensitive to the internal structure of $X(3872)$ in Ref. \[10\], and the ratios of decay rates are estimated to be $\Gamma_0 : \Gamma_1 : \Gamma_2 = 0 : 2.7 : 1$ when assuming the $X(3872)$ as a traditional charmonium state or $\Gamma_0 : \Gamma_1 : \Gamma_2 = 2.88 : 0.97 : 1$ as a four-quark state. Several other calculations in a similar spirit are also carried out in Refs. \[11–14\] based on the effective field theory (EFT) approach. However, the calculations for the ratio of the decay rate of $\chi_{cJ}\pi^0$ to $J/\psi\pi^+\pi^-$ has not been found in the literature, which is directly related to the experimental observable.

In this paper we would undertake a new calculation from the constituent quark point of view. In principle, calculations at the constituent quark level have proved to be successful in understanding the mass spectrum of most meson states and the model parameters have been determined to high accuracy, such as in Godfrey-Isgur (GI) model \[15\]. Furthermore, the constituent quark models use the wave functions of the meson states to represent the dynamical structure of the state rather than regard them as point-like state, which also naturally suppress the divergences in the large momentum region.

The theoretical basis of this work is that $X(3872)$ state automatically presents in the extended Friedrichs scheme (EFS) and can be expressed as the combination of $c\bar{c}$ components and continuum components such as $DD^*$, in which the $DD^*$ component is dominant \[16\]. This picture has proved successful in obtaining the mass and width and the isospin-breaking effects of the $X(3872)$ decays \[17\], and another calculation with the similar spirit also indicate the reasonability of this scheme \[18\]. This approach can be extended to discuss the decays to $\chi_{cJ}\pi^0$ processes by considering one of the final states being a $P$-wave state. Since the dominant continuum components is $DD^*$, and pure $c\bar{c}$ contribution is OZI-suppressed, we consider only the contribution from $DD^*$ component of $X(3872)$ to the decay. Since $DD^*$ component could be separated into $S$-wave and $D$-wave parts, we need to calculate the amplitude of these different angular momentum components to the $P$-wave final $\chi_{cJ}\pi^0$. This can be achieved by the Barnes-Swanson (BS) model \[19–22\]. This model has been used in studying the heavy meson scattering \[23–24\]. With these partial wave amplitude, the decay rates of $X(3872)$ to $\chi_{cJ}\pi^0$, $J/\psi\pi$, and $J/\psi\omega$ could be calculated by combining the previous result from the Friedrichs model scheme, and thus the branch ratios could be obtained. In this calculation, there is no free parameters introduced since all the parameters are the input of GI model or have been determined by obtaining the correct $X(3872)$ pole \[16\]. However, since this calculation has some model dependence, we would not expect this approach to give precise result of decay width, but just an order of magnitude guidance. Nevertheless, we found that in this calculation the decay rates of $X(3872)$ to the $\chi_{cJ}\pi^0$ are one order of magnitude smaller than its decays to $J/\psi\pi^+\pi^-$. 

THE MODEL

In the extend Friedrichs scheme \[25–26\], the $X(3872)$ state is dynamically generated by the coupling between the bare discrete $\chi_{c1}(2P)$ state and the continuous $DD^* + h.c.$ and $D^*D^*$ states \[16\], and its wave function could be explicitly written down as

$$|X\rangle = N_B \left(|c\bar{c}\rangle + \int_{M_{D^*}}^\infty dE \sum_{l,s} f_{ls}^{00}(E) \langle l(E)_{1s}D^*\bar{D}^{**} + h.c. \rangle + \int_{M_{D^*}}^\infty dE \sum_{l,s} f_{ls}^{+-}(E) \langle l(E)_{1s}D^*D^{**} + h.c. \rangle + \cdots \right),$$ \hbox{(3)}
where \( |\bar{c}c\rangle \) denotes the bare \( \chi_c(2P) \) state and \( |E_i^0\rangle = \sqrt{\mu_k} |k, j, \sigma, ls\rangle \) denotes the two-particle “n” state (“n” denotes the species of the continuous state) with the reduced mass \( \mu \) of two particle, the magnitude of one particle three-momentum \( k \) in their c.m. frame, total spin \( s \), relative orbital angular momentum \( l \), total angular momentum \( j \), and its third component \( \sigma \). The coupling form factors \( f_{ls}^{00} \) and \( f_{ls}^{++} \) could also be written down explicitly by using the quark pair creation (QPC) model \( 13, 27 \) and the wave functions from the quark potential models, such as the Godfrey-Isgur (GI) model \( 15 \). \( M_{00} \) and \( M_{++} \) in the integral limits are the threshold energies of \( D^0 \tilde{D}^{**} \) and \( D^+ \tilde{D}^{**} \) respectively. \( z_X \) is the \( X(3872) \) pole position, the zero point of the resolvent \( \eta(z) \), and \( N_B = \eta'(z_X)^{-1/2} \) is the normalization factor, where the resolvent is defined as \( \eta(z) = z - E_0 - \sum_{n,l,s} \int_{-\infty}^{\infty} dE \frac{|\bar{E} (E)|^2}{z-E} \). The \( \cdots \) represents other continuous states as \( D^* \tilde{D}^* \), but the compositeness of \( D^* \tilde{D}^* \) continua is about 0.4 percent so that their contribution to this calculation is tiny and could be omitted.

In general, the transition rate for a single-particle state \( \alpha \) decaying into a two-particle state \( \beta \) (including particle \( \beta_1 \) and particle \( \beta_2 \)) could be represented as \( \Gamma(\alpha \to \beta) = 2\pi |M_{\alpha\beta}|^2 \delta^4(p_{\beta_1} + p_{\beta_2} - p_\alpha) d^3p_{\beta_1} d^3p_{\beta_2} \), where \( M_{\alpha\beta} \) is the transition amplitude. In a non-relativistic approximation, the partial decay width can be represented as

\[
\Gamma(\alpha \to \beta) = \sum_{l',s'} 2\pi |M_{l's'}|^2 \mu' k' = \sum_{l',s'} 2\pi |F_{l's'}|^2
\]

(4)

where \( M_{l's'} \) is the partial-wave decay width, \( \mu' \) is the reduced mass of two-particle state \( \beta \), \( k' \) is the magnitude of three-momentum of one particle in their c.m. frame, and \( F_{l's'} \) is the decay amplitude with the phase space factor \( \sqrt{\mu'k'} \) absorbed in.

To calculate the hadronic decays of the \( X(3872) \), e.g. \( \chi_c J^P = 0^+ \) for \( J = 0, 1, 2 \), the partial-wave amplitude reads

\[
\begin{align*}
F_{l's'} &= \langle \chi_{cJ^P} |H| \chi_{cJ^P}\rangle = N_B \left( \chi_{cJ^P} \langle E'|H| \chi_{cJ^P} \rangle \right) |\bar{c}c\rangle \\
&+ \int_{-\infty}^{\infty} dE \sum_{l,s} \frac{f_{ls}^{00}(E)}{z_X - E} \langle \chi_{cJ^P} |E'|H|E_s\rangle d^0 \bar{E}^{*s} + h.c. \\
&+ \int_{-\infty}^{\infty} dE \sum_{l,s} \frac{f_{ls}^{++}(E)}{z_X - E} \langle \chi_{cJ^P} |E'|H|E_s\rangle d^+ \bar{E}^{**} + h.c. \\
&+ \cdots
\end{align*}
\]

(5)

Once the matrix elements such as \( \chi_{cJ^P} \langle E'|H|E_s\rangle d^0 \bar{E}^{*s} \), which is the one for \( D^* \tilde{D}^* \rightarrow \chi_{cJ^P} \) with total angular momentum \( j = 1 \), are obtained, the partial decay widths and branch ratios could be obtained directly. In general, the hadron-hadron interaction matrix element of \( AB \rightarrow CD \) is expressed as

\[
\begin{align*}
\bar{v}_{l's'}^i \langle E'|H|E_s\rangle^i_l &= \delta(E' - E) \mathcal{M}_{l's'n',lsn} \end{align*}
\]

(6)

and the partial-wave amplitude reads

\[
\begin{align*}
\mathcal{M}_{l's'n',lsn} &= \frac{\sqrt{\mu k' k''}}{s} \sum_{\nu'\nu''\mu'} \langle \nu'\nu''|\sigma_A \sigma_B \sigma_C \sigma_D \rangle \\
&\times \langle j_A s_A j_B s_B |\nu\rangle \langle s_A m_A j_A |\nu'\rangle \langle j_B s_B m_B |\nu''\rangle \\
&\times \int d\Omega_k \int d\Omega_k' \mathcal{M}_{E_{\nu''\nu},k' - \bar{E}_{\nu'\nu'},k} \bar{Y}_l^{(k)} Y_{l'}^{m}(k')
\end{align*}
\]

(7)

where \( \nu \) is the third-component of total spin \( s \). The symbols with primes represent the ones for the final states.

A simple model for calculating the scattering amplitude \( \mathcal{M}_{E_{\nu''\nu},k' - \bar{E}_{\nu'\nu'},k} \) is the Barnes-Swanson model \( 19, 22 \), which evaluates the lowest (Born) order \( T \)-matrix element between two-meson scattering states by considering the interaction between the quarks or anti-quarks inside the scattering mesons. In the \( \bar{q}_a(\bar{q}_a) + \bar{q}_b(\bar{q}_b) \rightarrow \bar{q}_c(\bar{q}_c) + \bar{q}_d(\bar{q}_d) \) quark(antiquark) transitions, the initial and final momenta are denoted as \( \bar{a} \rightarrow \bar{a} \bar{b} \). It is convenient to define \( \bar{q} = \bar{a} - \bar{a}, \bar{p}_1 = (\bar{a} + \bar{a})/2, \bar{p}_2 = (\bar{b} + \bar{b})/2 \).

In general, six kinds of interactions, the spin-spin, color Coulomb, linear, one gluon exchange (OGE) spin-orbit, linear spin-orbit, and tensor interactions, are considered, which is similar to the interaction potential terms in obtaining the mass spectrum and the meson wave functions in the GI model. Thus, they are consistent with the calculations of the EFS to determine the wave function of the \( X(3872) \).
Four kinds of diagrams are considered, among which the quark-antiquark interactions are denoted as Capture$_1$, Capture$_2$, and the quark-quark(antiquark-antiquark) interactions are denoted as Transfer$_1$, and Transfer$_2$. To reduce the so-called “prior-poster” ambiguity, the four “poster” diagrams are considered similarly and averaged to obtain the final result. For more details of the calculation of the model, the readers are referred to the original papers [19, 21, 22].

By standard derivation, one could obtain the partial wave scattering amplitudes for each of the four diagrams with only meson $C$ being a $P$-wave state using

$$
M^i_{jC,iB} = \sqrt{\mu k_2 k_1} \sum_{m'm'C} \langle j_B - mlm|10 \rangle \\
\times \langle j_C - m'l'm'|10 \rangle \langle C_{LC,C} s_C (-m' - mlC)|j_C - m' \rangle \\
\times \langle \psi_{14}\psi_{32} \psi_{12}\psi_{34} \rangle \omega_{14}\omega_{32}|H_C|\omega_{12}\omega_{34} \\
\times \int d\Omega_k \int d\Omega_\ell \langle \chi_{C\ell D}|I^m_{\text{Space}}(k, \ell)|\chi_{A\ell B}\rangle Y^m(k)Y^{m'*}(k')
$$

(8)

where $\langle \psi_{14}\psi_{32} \psi_{12}\psi_{34} \rangle$ is the flavor factor, and $\langle \omega_{14}\omega_{32}|H_C|\omega_{12}\omega_{34} \rangle$ the color factor, which is $-4/9$ and 4/9 for interactions of $q\bar{q}$ and $qq$ respectively. $\chi_A$ represents the spin wave function of meson $A$. The space integral

$$
I^m_{\text{Space}}(k, \ell) = \int d^3p \int d^3q \psi^A_{000}(\bar{p}_A)\psi^B_{000}(\bar{p}_B)\psi^{C*}_{01m_C}(\bar{q}_C)\psi^D_{000}(\bar{q}_D)T_{f_1}(\bar{q}, \bar{p}_1, \bar{p}_2)
$$

(9)

where $\psi_{n,Lm_L}(\vec{r}_r)$ is the wave function for the bare meson state, with $n_r$ being the radial quantum number, $L$ the relative angular momentum of the quark and anti-quark, $m_L$ its third component, $\vec{p}_r$ the relative momentum of quark and antiquark in the meson.

The quark interactions involved in this calculation are

$$
T_{f_1}(\bar{q}, \bar{p}_1, \bar{p}_2) = \\
\left\{ \begin{array}{ll}
-\frac{8\pi\alpha_s}{3\mu_1 \mu_2} \frac{[\vec{S}_1 \cdot \vec{S}_2]}{q^2} & \text{Spin – spin} \\
\frac{4\pi\alpha_s}{\mu_1 \mu_2} \frac{[\vec{S}_1 \cdot (\vec{q} \times (\vec{p}_1 - \vec{p}_2) - \vec{p}_1 \times \vec{S}_2)]}{q^2} & \text{Coulomb} \\
\frac{4\pi\alpha_s}{\mu_1 \mu_2} \frac{[\vec{S}_1 \cdot (\vec{q} \times \vec{p}_1) - \vec{S}_2 \cdot (\vec{q} \times \vec{p}_2)]}{q^2} & \text{Linear OGE spin – orbit} \\
\frac{4\pi\alpha_s}{\mu_1 \mu_2} \frac{[\vec{S}_1 \cdot (\vec{q} \times \vec{p}_1) - \vec{S}_2 \cdot (\vec{q} \times \vec{p}_2)]}{q^2} & \text{Linear spin – orbit} \\
\frac{4\pi\alpha_s}{\mu_1 \mu_2} \frac{[\vec{S}_1 \cdot \vec{q} \vec{S}_2 \cdot \vec{q} - \frac{q^2}{2} \vec{S}_1 \cdot \vec{S}_2]}{q^2} & \text{OGE tensor} \\
\end{array} \right.
$$

(10)

where $\alpha_s = \sum_k \alpha_k e^{-\gamma k q^2}$ as the parametrization form in the GI model. $m_1$ and $m_2$ are the masses of the two interacting quarks.

Similarly, one could obtain the decay amplitude of $X(3872) \rightarrow J/\psi \rho$ and $J/\psi \omega$, which is simpler because there is only S-wave states involved in the scattering amplitudes $M^i_{f's'n'lsn}$.

**NUMERICAL CALCULATIONS**

As analyzing the properties of $X(3872)$, we use the famous GI model as input. The wave functions of all the bare meson states have been determined in the GI model. Furthermore, the Barnes-Swanson model do not adopt any new parameter since the quark-quark interaction terms share the same form as the GI model. The whole calculation has only one free parameter, the quark pair creation strength $\gamma$, which is determined by requiring $z_{X(3872)} = 3.8716$GeV.

The running coupling constant is chosen as $\alpha_s(q^2) = 0.25 e^{-q^2} + 0.15 e^{-2q^2} + 0.20 e^{-5q^2}$, and the quark masses are $m_u = 0.2175$GeV, $m_d = 0.2225$GeV, $m_c = 1.628$GeV, $b = 0.18$, $\gamma \approx 4.0$. There is a technical difficulty in the numerical calculation. To obtain the partial wave scattering amplitude, one encounters a ten-dimensional integration with six for the momentum variables and four for the partial wave decomposition, which is not able to be calculated accurately by the programme. To get around this difficulty, we make an approximation by using the simple harmonic oscillator (SHO) wave function to represent the four involved mesons with their effective radii equal to be the r.m.s radii calculated from the wavefunctions of GI model. In such a simplification, the space overlap function of Eq. 9 could be integrated out analytically. With the help of

$$
\int d^3 q \frac{1}{q^2} e^{-\mu(q^2 - \phi_0^2)} = \frac{2\pi^{3/2}}{\sqrt{\mu}} e^{-\mu\phi_0^2} F_1 \left( \frac{1}{2}, \frac{3}{2}, \mu\phi_0^2 \right)
$$

(11)
\[ \frac{d^3q}{q^4} e^{-\mu(q^2)} = -4\pi \sqrt{q^2} e^{-\mu q^2} F_1 \left( \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, \mu q^2 \right), \]  

(12)

and \( \frac{d}{dz} F_1(a, b, z) = \frac{a}{b} F_1(a + 1, b + 1, z) \), the momentum integration of Eq. (9) could be represented by the confluent hypergeometric functions \( \text{\cite{20, 22}} \). Then, the partial-wave integration is only four-dimensional and can be evaluated numerically.

The wave function of \( X(3872) \) has the \( S \)-wave and \( D \)-wave \( DD^* \) components as shown in Fig. 1, both of which could, in principle, transit to the final \( \chi_{c1}\pi^0 \) state in \( P \)-wave. However, the \( S \)-wave components contribute dominantly, and their partial wave scattering amplitudes to \( \chi_{cJ}\pi^0 \) states of \( P \)-wave are shown in Fig. 2.

Because the \( X(3872) \) is very close to the \( D^0\bar{D}^{0*} \) threshold, the \( 1/(z_X - E) \) term will greatly enhance the contributions of \( f_{ls}M_{\ell's',ls} \) near the \( D^0\bar{D}^{0*} \) threshold, and it also leads to extreme suppression of the contributions of the \( D \)-wave \( DD^* \) components. As an example, \( f_{ls}M_{\ell's',ls}^{(\chi_{cJ}\pi^0)} \) for \( S \)-wave \( D^0\bar{D}^{0*} \) or \( D^+\bar{D}^{-*} \) to \( P \)-wave \( \chi_{cJ}\pi^0 \) is plotted in Fig. 3. Since the flavor wave functions of \( \pi^0 \) is \( (\bar{q}u - \bar{d}l)/\sqrt{2} \), the cancellation naturally happens between the neutral charmed states \( D^0\bar{D}^{0*} \) and the charged \( D^+\bar{D}^{-*} \) components, which is similar to that of \( X(3872) \to J/\psi\rho \). One could find that the contributions of \( D^0\bar{D}^{0*} \) and \( D^+\bar{D}^{-*} \) in the large momentum region will cancel each other and the contribution near the \( D^0\bar{D}^{0*} \) threshold will contribute dominantly.

In this calculation, the decay rates of \( X(3872) \to \chi_{cJ}\pi^0 \) for \( J = 0, 1, 2 \) are quite small, of the order of \( 10^{-7} \) GeV, with a ratio \( \Gamma_0 : \Gamma_1 : \Gamma_2 = 1.5 : 1.3 : 1.0 \). This ratio is comparable with the EFT calculations in Refs. \( \text{\cite{10, 11}} \). Our calculation also suggests that the magnitude of the decay rates \( \chi_{cJ}\pi^0 \) might not be large even if the \( D^0\bar{D}^{0*} \) component is dominant. In Refs. \( \text{\cite{10, 11}} \) a factor determined by the internal dynamics can not be present, so they did not present the magnitudes of such decay rates.

At the same time, we could also calculate the decay rates to \( J/\psi\pi^+\pi^- \) and \( J/\psi\pi^+\pi^-\pi^0 \) by assuming the final states \( \pi^+\pi^- \) and \( \pi^+\pi^-\pi^0 \) produced via \( \rho \) and \( \omega \) resonances, respectively. The contributions of neutral and charged \( DD^* \) components in \( X(3872) \to J/\psi\rho \) are cancelled, while they are added in \( X(3872) \to J/\psi\omega \) because \( \omega = (\bar{q}u + \bar{d}l)/\sqrt{2} \).
For simplicity, we describe the $\rho$ and $\omega$ resonances by their Breit-Wigner distribution functions \[ \mathcal{F} \delta_{i,j} \] and then obtain

\begin{align}
\Gamma(X \to J/\psi\rho(\pi^+\pi^-)) &= \int_{2m_\pi}^{m_X-m_{J/\psi}} \sum_{l,s} |F_{l,s}(X \to J/\psi\rho)|^2 \Gamma_{\rho} \left( E - m_\rho \right)^2 + \Gamma_{\rho}^2 / 4 \, dE, \\
\Gamma(X \to J/\psi\omega(\pi^+\pi^-\pi^0)) &= \int_{3m_\pi}^{m_X-m_{J/\psi\omega}} \sum_{l,s} |F_{l,s}(X \to J/\psi\omega)|^2 \Gamma_{\omega} \left( E - m_\omega \right)^2 + \Gamma_{\omega}^2 / 4 \, dE, \tag{13}
\end{align}

in which the lower limits of the integration are chosen at the experiment cutoffs as in Ref. [30, 31].

The obtained $J/\psi\pi^+\pi^-$ is at the order of keV, and the ratio of decay rates to $X(3872) \to \chi_{c0}\pi^0$, $\chi_{c1}\pi^0$, $\chi_{c2}\pi^0$, $J/\psi\pi^+\pi^-$, and $J/\psi\pi^+\pi^0\pi^0$ is about $1.5 : 1.3 : 1.0 : 16 : 26$. This is a calculation based on the Barnes-Swanson model, there exist several model dependence in this calculation: (1) The meson wave functions are approximated by the SHO wave functions for computing the space overlap factor, and such an approximation might cause the "prior-poster" ambiguity. (2) The scheme is non-relativistic, and all the momentum-energy relations are non-relativistic. Thus we would expect that the absolute magnitude of the decay width is just a rough estimate and only provides an order of magnitude guidance. In this calculation the result of $\chi_{cJ} \to \chi_{cJ}\pi^0$ states could only appear in $P$-wave, while the $J/\psi\rho$ states could appear in $S$-wave. Usually, the higher partial waves will be suppressed. Furthermore, the phase space of $\rho \to \pi^+\pi^-$ will enlarge the decay width of $X(3872) \to J/\psi\pi^+\pi^-$. The ratio $\frac{\Gamma(X(3872) \to J/\psi\pi^+\pi^-)}{\Gamma(X(3872) \to J/\psi\pi^+\pi^0)}$ is about 16, which is comparable with the measured result $1.0 \pm 0.4 \pm 0.3$ by Belle [30], $0.8 \pm 0.3$ by BaBar [31] and $1.6^{+0.4}_{-0.3} \pm 0.2$ BESIII [32].

**SUMMARY**

In summary, in the extended Friedrichs scheme we make a model calculation of the decay rates of $X(3872) \to \chi_{c0}\pi^0$, $\chi_{c1}\pi^0$, $\chi_{c2}\pi^0$, $J/\psi\pi^+\pi^-$, and $J/\psi\pi^+\pi^0\pi^0$ in the same scheme, and find that the relative ratio will be about $1.5 : 1.3 : 1.0 : 16 : 26$. The decay rate of $\chi_{c1}\pi^0$ is one order of magnitude smaller than that of $J/\psi\pi^+\pi^-$ in this calculation. Our result is smaller than the central value measured by BESIII [3], but we noticed that the result of BESIII has sizable uncertainty, so more data are needed to increase the statistics and decrease the error bar. In Belle’s experiment the $X(3872)$ signal is not observed in $B^+ \to \chi_{c1}\pi^0 K^+$ [9], but its upper limit of $\frac{\Gamma(X(3872) \to \chi_{c1}\pi^0)}{\Gamma(X(3872) \to J/\psi\pi^+\pi^-)}$ does not contradict with BESIII’s result. Recently, the BelleII has started to accumulate data with higher statistics and it is expected that more accurate measurements could be obtained in the future.

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