Low-Lying States Properties of the Even-Even $^{78}\text{Se}$ and $^{80}\text{Kr}$ Isotopes

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Abstract. The interacting boson model has been applied to calculate the low-lying bands in Se and Kr nuclei with neutron number (N=44) and proton numbers Z=34 and 36. Reasonable agreement with available energies and B(E2) transition rates. The potential energy surfaces (PESs) to the IBM Hamiltonian have been obtained using the intrinsic coherent state.

Keywords: Interacting Boson Model; Energy levels; B(E2), PES.

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1. Introduction

Arima and Iachello (1979) [1] have developed the interacting boson model (IBM), which is based on the well-known shell model and on geometrical collective model of the atomic nucleus. The simplest form of the interacting boson model (IBM) describes even-even nuclei in term of an inner core plus the valence particles outside the nearest closed shells considered as boson. The number of boson is equal to half of the total number of valence protons and neutrons. The ingredients of the IBM are monopole s and quadrupole d bosons, which correspond respectively to collective nucleon-pairs with angular momenta $J^\pi = 0^+$ and $2^+$. Different reductions of the unitary group U(6) give three dynamical symmetry limits, known as harmonic oscillator, deformed rotor and asymmetric deformed rotor, which are labeled by U(5), SU(3), and O(6), respectively [3–5]. Another phenomenological study indicated that nuclei might have an intermediate structure of the U(5)–SU(3), U(5)–O(6) and SU(3)–O(6) limits [6–8]. IBM-1 is a successful model for describing structures of the low excited states in the medium and heavy even-even nuclei [9, 10].

The neutron-proton interaction is known to play a dominant role in quadrupole correlations in nuclei. As a consequence, the excitation energies of collective quadrupole excitations in nuclei near a closed shell are strongly dependent on the number of nucleons outside the closed shell. The nuclei $^{78}\text{Se}$ and $^{80}\text{Kr}$, have atomic number Z = 34 and 36, and same neutron number N = 44, these nuclei with Z > 28, N < 50 protons and neutrons are allowed to occupy $g_{9/2}$, $p_{1/2}$, $p_{3/2}$ and $f_{5/2}$ orbitals. In the past few years, low-lying energy states and electromagnetic transition properties in several isotopes like Se and Kr have been measured for the first time or re-measured with higher precision in the A ≈ 80 mass region [11–18]. Recently, in the same region of O(6) symmetry, some nuclei studied like the even–even Pt isotopes for A=190 to 196 [19], some even $^{120–126}\text{Xe}$ isotopes [20], even-even Pd isotopes for $A=102$ to 106 [21] and the low-lying positive parity yrast bands in $^{190–196}\text{Hg}$ nuclei[22].
The aim of the present work by application of IBM-1 to predict the low-lying levels, reduced transition probabilities and PES to understand the type of dynamical symmetry which exist in Se and Kr nuclei for neutron \(N = 44\).

2. Method of Calculations

The IBM-1 Hamiltonian can be expressed as \([1, 23, 24]\):
\[
H = \epsilon_s (s^\dagger, s) + \epsilon_d (d^\dagger, d)
\]
\[
+ \frac{1}{2} \sum_{L=0}^{2} \left \{ \frac{L(L+1)}{2} C_c \left \{ L d^{\dagger} \times d^{\dagger} \right \} \times \left \{ L d \times d \right \} \right \}^{(L)}
\]
\[
+ \frac{1}{\sqrt{2}} v_2 \left \{ \left \{ L d^{\dagger} \times d^{\dagger} \right \} \times \left \{ L d \times d \right \} \right \}^{(2)}
\]
\[
+ \frac{1}{2} u_0 \left \{ \left \{ L d^{\dagger} \times d^{\dagger} \right \} \times \left \{ L s \times s \right \} \right \}^{(0)}
\]
\[
+ \frac{1}{2} u_0 \left \{ \left \{ L s \times s \right \} \times \left \{ L d \times d \right \} \right \}^{(0)}
\]
\[
+ u_2 \left \{ \left \{ L d^{\dagger} \times d^{\dagger} \right \} \times \left \{ L d \times d \right \} \right \}^{(0)}
\]
\[
(1)
\]

This Hamiltonian is specified by nine parameters, two appearing in the one body terms \((\epsilon_s, \epsilon_d))\), and seven in the two body terms \([c_c, v_2, v_1, u_2, u_1])\), where \(\epsilon_s\) and \(\epsilon_d\) are the single-boson energies. However, the total number of boson \(N_b\) (pairs) is conserved, \(N_b = n_+ + n_- [24].\)

Then the IBM-1 Hamiltonian in equation (1) can be written in general form as \([24, 25]\):
\[
\hat{H} = \hat{n}_d + \alpha_1 \hat{P} \hat{L} + \alpha_2 \hat{Q} \hat{T}_r + \alpha_3 \hat{F}_3 + \alpha_4 \hat{F}_4 + \alpha_5 \hat{F}_5 + \alpha_6 \hat{F}_6 + \alpha_7 \hat{F}_7 + \alpha_8 \hat{F}_8
\]
\[
(2)
\]
where \(\epsilon\) is the boson energy, and the operators are:
\[
\hat{n}_d = (d^\dagger, d)
\]
\[
\hat{P} = \frac{1}{2} \left \{ (d^\dagger, d) - (s, s) \right \}
\]
\[
\hat{L} = \sqrt{10} (d^\dagger \times d)^{(2)}
\]
\[
\hat{Q} = \chi (d^\dagger \times 3d)^{(2)}
\]
\[
\hat{T}_r = \chi (d^\dagger \times d)^{(2)}
\]
\[
(3)
\]
Here, \(\hat{n}_d\) is the total number of d boson operator, \(\hat{P}\) is the pairing operator, \(\hat{L}\) is the angular momentum operator, \(\hat{Q}\) is the quadrupole operator \((\chi\) is the quadrupole structure parameter take the values 0 and \(\pm \sqrt{\frac{3}{2}} [4, 26, 27])\) and \(\hat{T}_r\) is the octupole \((r = 3)\) and hexadecapole \((r = 4)\) operator. The phenomenological parameters \(\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4\) represent the strengths of the pairing, angular momentum, quadrupole, octupole and hexadecapole interaction between the bosons, respectively. In that case, one says that the Hamiltonian \((H)\) has a dynamical symmetry. These symmetries are called U(5) vibrational, SU(3) rotational and O(6) \(\gamma\)-unstable [24].

The eigenvalues for these three limits are given by \([24]\):
\[
E = \epsilon n (n d + \beta n d (n d + 4) + 2 \gamma \nu (\nu + 3) + 2 \delta \lambda (L + 1))
\]
\[
E = \frac{\alpha_0}{2} \left \{ (2 \lambda^2 + \mu^2 + \lambda \mu + 3 (\lambda^2 + \mu \mu)) + (\alpha_1 - \frac{3 \alpha_2}{8}) \right \} L (L + 1)
\]
\[
E = \frac{\alpha_0}{4} (N - \sigma)(N + \sigma + 4) + \frac{\alpha_3}{2} \tau (\tau + 3) + (\alpha_1 - \frac{3 \alpha_2}{10}) L (L + 1) + O(6)
\]
\[
(4)
\]
According to the Hamiltonian \([1, 24]\), we can be discussed the calculated results separately by plotting the potential energy surface \((\langle N, \beta, \gamma \rangle)\), which gives a final shape to the nucleus. The technique described by Dieperink et al. [28] allows one also to give an algebraic description of the nature of the transition between one phase and another. Simpler expressions, which display the essential dependence on \(\beta\) and \(\gamma\), have been given \([4, 24, 29]\):
\[
E(N_b, \beta, \gamma) = \epsilon N_b [\beta^2 / (1 + \beta^2)], \ldots U(5)
\]
\[
E(N_b, \beta, \gamma) = \frac{\alpha_0}{2} N_b (N_b - 1) ([1 + 3/4\beta^4 (\cos 3\gamma) / (1 + \beta^2)^3], \ldots SU(3)
\]
\[
(5)
\]
\[
(6)
\]
$$E(N_b, \beta, \gamma) = a_0 N_b (N_b - 1) \{(1 - \beta^2) y (1 + \beta^2)\} \ldots O(6), \quad (7)$$

where $\beta$ and $\gamma$ are the intrinsic deformation parameters which determine the geometrical shape of the nucleus. These expression give (for large $N_b$) $\beta_{\text{min}} = 0, \sqrt{2},$ and 1 for U(5), SU(3), and O(6), respectively.

3. Results and discussion

Se and Kr isotopes have neutron number $N = 44$ and have an even atomic number $Z = 34$ and 36 (Z values near mid shell and N value near closed shell would suggest structure the O(6) symmetry and O(6) – U(5) transition). The degree (and type) of collectivity can be expressed in terms of the energy ratio $R_{4/2} = E_{4^+}/E_{2^+}$, which used as a starting point and is a good indicator of the shape deformation of the nucleus and its value is 10/3 for the well-deformed nuclei SU(3), 2.5 for O(6) or $\gamma$-unstable nuclei and 2 for vibrational U(5), 2.2 for the analytically solvable symmetry E(5) on the U(5) - O(6) path and 2.9 for the approximate X(5) symmetry on the U(5)-SU(3) path[24, 30-34].

The experimental values of $R = E_{4^+}/E_{2^+}$ of low-lying energy levels of $^{78}$Se and $^{80}$Kr nuclei are shown in Table 1. From this Table, $R_{4/2}$ attains the O(6) value of ~ 2.5 in $^{78}$Se at Z= 34 and in Z= 36 $^{80}$Kr lie close to the E(5) symmetry.

| Nucleus | $^{78}$Se | $^{80}$Kr |
|---------|---------|---------|
| $R_{4/2}$ | 2.44 | 2.32 |

3.1. Table 1. The ratio $R_{4/2} = E_{4^+}/E_{2^+}$ for $^{78}$Se and $^{80}$Kr nuclei [35-37].

The calculations have been performed using IBM with PHINT code [38] and, hence, no distinction made between neutron and proton bosons which calculated from the sum of the proton bosons and the neutron bosons of the close shells (28 and 50). The number of bosons and the parameters of the IBM-1 Hamiltonian (2) which give the best fitting between theoretical and experimental energy levels [35-37] of the above isotopes are shown in Table 2.

| Isotopes | $N_b$ | $\varepsilon$ | PAIR | ELL | OCT |
|----------|------|-------------|------|-----|-----|
| $^{78}$Se | 6 | -- | 0.107 | 0.080 | 0.053 |
| $^{80}$Kr | 7 | 0.1 | 0.083 | 0.079 | 0.054 |

(Ell = 2$a_1$ and QQ = 2$a_2$, CHQ = $\sqrt{5}$)[24].

Figure 1 show that the calculated and experimental values of ground (GSB), $\beta$- and $\gamma$- bands are plotted for even-even Se and Kr isotopes. In the Figure 1, the calculated energy levels are in good agreement with the experimental [35-37] ones for Se and Kr isotopes. Levels with "( )" in g, $\gamma$ and $\beta$ states correspond to cases for which the spin and/or parity of the corresponding states are not well established experimentally.
Now, we discuss other information on the structure of nuclei. The transition strength between the excited states and can be expressed in terms of the reduced E2 matrix element which must be a Hermitian tensor of rank two when $N_b$ must be conserved. The $B(E2)$ strength for the $E2$ transitions is given by $B(E2)$

$$T_{E2}^{SE} = \alpha_2 \left[ d \bar{s} s + s \bar{d} d \right]^{(2)} + \beta_2 \left[ d \bar{d} \right]^{(2)} = e_B \hat{Q}$$

(8),

where $(s', d')$ and $(s, d)$ are creation and annihilation operators for $s$ and $d$ bosons, respectively, while $\alpha_2$ and $\beta_2$ are two parameters, and $\left( \beta_2 = \chi \alpha_2, \alpha_2 = e_B (\text{effective charge of boson}) \right)$. The reduced transition probability for the U(5) and O(6) limits are given by:

$$U(5) \quad B(E2; L \rightarrow L - 2) = e_B^2 (n_d + 1)(N - n_d)$$

$$O(6) \quad B(E2; L \rightarrow L - 2) = e_B^2 (N - \tau)(N + \tau + 4) \frac{\tau + 1}{2\tau + 5}$$

(9) (10)

where $L$ is the angular momentum. From the given experimental value $B(E2)$ of transition $(2^+_1 \rightarrow 0^+_1)$, one can calculate the value of the parameter $e_B = \alpha_2$ for each isotope. This value is used to calculate the reduced transition probabilities $B(E2; L \rightarrow L - 2)$. Table 3 shows the values of the $e_B$ parameter, which was obtained in the present calculations. Table 4 shows the $B(E2)$ in the low lying states for Se and Kr nuclei with the neutron number $N=44$ and compared with the experimental data [35-37]. The reduced transition probabilities $B(E2)$ are increase as proton number increases and most of the calculated results in IBM-1 reasonably consistent with the available experimental data, except for few cases that deviate from the experimental data [35-37] in Se and Kr isotopes.

| Table 3. Parameter (in eb) used to reproduce B(E2) values for $^{78}$Se and $^{80}$Kr isotopes. |
|---------|---------|---------|
| A       | $N_b$   | $e_B$   |
| $^{78}$Se | 6       | 0.074   |
| $^{80}$Kr | 7       | 0.070   |
Table 4. The IBM-1 and Experimental [35-37] values of B(E2) for $^{78}$Se and $^{80}$Kr isotopes (in e$^2$ b$^2$).

| $L_i \rightarrow L_f$ | $^{78}$Se | $^{80}$Kr |
|----------------------|----------|----------|
|                      | EXP. | IBM | EXP. | IBM |
| $2^+_1 \rightarrow 0^+_1$ | 0.066 | 0.0670 | 0.0760 | 0.0751 |
| $2^+_1 \rightarrow 2^+_1$ | 0.0439 | 0.0868 | 0.051 | 0.1003 |
| $2^+_1 \rightarrow 2^+_1$ | 0.0089 | 0.0000 | -- | 0.0005 |
| $2^+_2 \rightarrow 0^+_2$ | 0.0198 | 0.0203 | -- | 0.0645 |
| $3^+_1 \rightarrow 2^+_1$ | 0.0015 | 0.0023 | 0.0010 | 0.0090 |
| $3^+_1 \rightarrow 2^+_2$ | 0.0495 | 0.0631 | 0.0690 | 0.0755 |
| $3^+_1 \rightarrow 4^+_1$ | -- | 0.0006 | 0.1020 | 0.0235 |
| $4^+_1 \rightarrow 2^+_1$ | 0.0975 | 0.0868 | 0.1420 | 0.1003 |
| $4^+_2 \rightarrow 4^+_1$ | -- | 0.0765 | 0.1020 | 0.0854 |
| $5^+_1 \rightarrow 4^+_1$ | -- | 0.0025 | 0.0020 | 0.0061 |

In Figure 4, the contour plot of the potential energy surfaces, E(N,β,γ), show that $^{78}$Se is a deformed and has γ-unstable-like characters ($γ \approx \frac{5}{6}$), while $^{80}$Kr, the shape phase transition from γ-unstable O(6) to vibrational U(5) symmetry.

![Figure 2](image-url) (Color online) the potential energy surfaces for Se and Kr nuclei.

4. Conclusions
The low-lying bands are calculated using IBM-1 for $^{78}$Se and $^{80}$Kr nuclei with neutron number N=44. The result shows good agreement with published experimental data. The reduced transition probabilities B(E2) values have been calculated using Interacting Boson Model (IBM). A good agreement is obtained for all the observed able studied. The contour plot of PES show, that $^{78}$Se is a deformed and has γ-unstable-like characters, while $^{80}$Kr lie close to the E(5) symmetry.
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