Generic strange-metal behaviour of overdoped cuprates

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Abstract. We present an analysis of the temperature dependence of the zero-field in-plane resistivity $\rho_{ab}(T)$ of overdoped Tl$_2$Ba$_2$CuO$_6$+$_\delta$ (Tl2201). Taking our cue from earlier resistivity and angle-dependent magnetoresistance studies of Tl2201, as well as high-field measurements on La$_{2-x}$Sr$_x$CuO$_4$ (LSCO), we delineate $\rho_{ab}(T)$ into two $T$-dependent components below 200 K, one linear in temperature, the other quadratic. As found in LSCO, the $T$-linear component $\alpha_1(0)$ is finite for all superconducting samples, its magnitude scaling with the transition temperature $T_c$. By contrast, the $T^2$ coefficient $\alpha_2(0)$ is essentially doping independent. Such an extended regime (in doping) of $T$-linear resistivity at low $T$ is at odds with conventional quantum critical scenarios involving the collapse of an ordered phase, possibly associated with the normal state pseudogap, to 0 K at a critical doping level. Its confirmation in Tl2201, whose electronic state (as revealed by quantum oscillation experiments) is highly homogeneous over hundreds of unit cells, appears to rule out phase separation or electronic inhomogeneity as the origin of this extended critical behaviour.

1. Introduction

The phrase “strange metal” has long been associated with the high-$T_c$ cuprate superconductors. It was coined originally to describe the funnel-shaped region of the phase diagram above the superconducting dome in which the in-plane resistivity $\rho_{ab}(T)$ varies linearly with temperature and the inverse Hall angle $\cot\delta_H(T)$ varies quadratically with temperature [1]. In certain optimally doped cuprates, the $T$-linearity of the resistivity can extend up to extraordinarily high temperatures of order 1000 K [2]. This V-shaped region is defined by two temperature scales, $T^*$ or the pseudogap temperature on the low-doping side and the coherence temperature $T_{coh}$ on the other, both of which appear to merge at a critical doping level $p_{crit} \sim 0.19$ [3], close to optimal doping at which the superconducting transition temperature is a maximum. Its similarity to the temperature-doping phase diagram of some quantum critical metals and expectations from conventional quantum critical theory has prompted many in the field to associate the closing of the pseudogap at $p = p_{crit}$ to the existence of a zero-temperature quantum phase transition, possibly associated with the reconstruction of the Fermi surface [4].

Beyond this enigmatic region, it has been largely presumed that the electronic state of the cuprates evolves into a more conventional Fermi-liquid (FL) behaviour in the overdoped region. This presumption, however, is highly misleading. Only at very large doping, i.e. beyond the superconducting dome on the overdoped side, does the low-$T$ resistivity follow a strictly $T^2$ dependence characteristic of a canonical FL [5,6]. Between optimal doping and the edge of the
superconducting dome, there is a large swathe of the phase diagram in which $\rho_{ab}(T)$ remains anomalous and non-FL like. In this region, $\rho_{ab}(T)$ has been described either in terms of a single anomalous power law with an exponent $n$ between 1 and 2 [7], or as a combination of two components, one linear-in-temperature, the other quadratic [8].

The latter description has gained experimental support from a combination of pulsed and persistent high magnetic field measurements carried out on two single-layer hole-doped cuprate families, LSCO and Tl2201 respectively [9-13]. In LSCO, pulsed fields were used to suppress superconductivity and to track the evolution of the normal state resistivity down to low temperatures (of order 1.5 K) [8]. These measurements revealed the persistence of a $T$-linear component $\alpha_1(0)T$ from optimal doping ($x = 0.17$) all the way out to $x = 0.29$, i.e. beyond the superconducting dome of LSCO. This unusual so-called ‘extended criticality’ contrasted markedly with expectations from conventional quantum critical scenarios in which non-FL-like resistivity survives only at a singular point in the phase diagram - the quantum critical point (QCP) - as seen, for example, in YbRh$_2$Si$_2$ [14] where the relevant tuning parameter is the strength of the magnetic field.

The origin of this extended $T$-linearity in LSCO is unknown at present. One possible explanation for the distinct behaviour of $\rho_{ab}(T)$ in LSCO is microscopic electronic phase separation into hole-rich (normal) and hole-poor (superconducting) regions, a phenomenon that has been well documented in the LSCO family [15]. In this scenario, the $T$-linear component is due to regions within the sample having a hole concentration close to optimal while the quadratic component originates from regions with a carrier density closer to 30 %, i.e. beyond the superconducting dome. This scenario is consistent with the observation that the coefficient $\alpha_1(0)$ scales with $T_c$ reaching a maximum value of around 1 $\mu\Omega$cm/K at $p = p_{crit}$ [8]. However, a similar phenomenology (i.e. coexistence of $T$ and $T^2$ components in the resistivity with the $T$-linear term persisting over a wide region of the phase diagram) has also been seen in the Bechgaard salt (TMTSF)$_2$PF$_6$ [16]. Here $\alpha_1(0)$ and $T_c$ display the same intimate correlation (in this case, $T_c$ is varied through the application of hydrostatic pressure) yet as far as we are aware, there is no corresponding evidence for phase separation in these quasi-one-dimensional organic conductors and they have a well defined chemical stoichiometry [16].

An alternative scenario is one in which the two components in the resistivity arise from two independent scattering mechanisms being present in a homogeneous electronic state. Support for this picture comes from angle-dependent magnetoresistance (ADMR) measurements on overdoped TI2201 where two distinct scattering rates were uncovered [10-14]; one that is isotropic over the Fermi surface and follows a $T^2$ dependence at low $T$ and a $T$-linear term that is strongly anisotropic, peaking at the zone boundary and vanishing along the zone diagonal. The identification of a non-FL $T$-linear term in overdoped TI2201 at low $T$ is significant in that quantum oscillations (QO), the most definitive signature of fermionic quasiparticles, have been observed in crystals of similar doping [17-19]. Moreover, the observation of QO implies that the electronic state is highly homogeneous in overdoped TI2201, with the carrier number changing by less than 0.2% over a distance equivalent to a cyclotron radius of order 1-2000 $\AA$ [18].

The observation of QO in TI2201 would appear to rule out microscopic phase separation as the origin of the two $T$-dependent components in $\rho_{ab}(T)$ in overdoped cuprates. However, detailed ADMR measurements have only been carried out to date on samples with comparable $T_c$ values (~ 15 K) and therefore similar carrier densities. Thus it has not been possible to confirm whether the low-$T$ $T$-linear term in the scattering rate or resistivity persists over a wide doping range in TI2201 and whether this extended critical behaviour is a generic feature of the high-$T_c$ cuprate superconductors.
In this contribution, we reveal that the evolution of $\rho_{ab}(T)$ in Tl2201 with doping exhibits a remarkably similar phenomenology to that first reported in LSCO, at least in the normal state above $T_c$. This finding affirms that this strange-metal behaviour is indeed a generic property of overdoped cuprates, one that is intrinsic to the electronic state of these fascinating materials and one tied to the strong correlation physics inherent in their proximity to a Mott insulating state.

2. In-plane resistivity of Tl2201

The left panel of figure 1 shows $\rho_{ab}(T)$ data taken by Manako and co-workers on a non-superconducting (non-SC) Tl2201 single crystal [5] (labelled hereafter Tl-0K). As reported by Manako et al., $\rho_{ab}(T)$ varies quadratically below approximately 50 K down to the lowest $T$ measured (4.2 K), in a similar fashion to that found in overdoped non-superconducting LSCO ($x = 0.33$) [6]. As $T$ increases, $\rho_{ab}(T)$ shows progressively less curvature and becomes almost $T$-linear near 300 K.

A simple but elegant way to model the evolution of $\rho_{ab}(T)$ over the entire temperature range is to fit the data to the parallel resistor formula (PRF) first introduced by Weismann et al. to model the behaviour of saturating metals, i.e. metals whose resistivity tends to a constant (the so-called Mott-Ioffe-Regel or MIR limit) at high $T$ once the mean free path $\ell$ has diminished to a value of order the lattice spacing $a$ [20]. Using this fitting protocol, the two components in the PRF for both Tl-0K and non-SC LSCO33, reproduced in the left panel of figure 1, are the low-$T$ component $\alpha_0 + \alpha_2 T^2$ and a maximum resistivity $\rho_{\text{max}}$, which surprisingly, has a value consistent (to within a factor of 2) with the condition $\ell = a$ in both cases. Recall that for a quasi-2D metal, the Drude conductivity at the MIR limit
corresponds to $\sigma_{\text{MIR}} = 1/\rho_{\text{max}} = (e^2/2\pi \hbar d)k_Fa$. Thus for quasi-2D metals with comparable carrier densities and Fermi wave vectors $k_F$, $\rho_{\text{max}}$ ought to scale with the interlayer spacing $d$. For Tl2201, $d = 11.6$ Å, while for LSCO, $d = 6.4$ Å.

The excellence of the two fits is surprising since it is now well established that in all cuprates, $\rho_{ab}(T)$ does not saturate at high $T$, but rather continues to increase, quasi-linearly with temperature, attaining values well in excess of the MIR limit. It has been shown however, both experimentally [21, 22] and theoretically [23], that once the dc resistivity of a non-saturating (bad) metal exceeds the MIR limit, the optical response becomes non-Drude like, with low-energy spectral weight being transferred to much higher energies of order the on-site Coulomb repulsion $U$. This transfer of spectral weight, a manifestation of strong electron correlation [23], implies that the escalation of the dc resistivity cannot be associated with an ever-increasing scattering rate (which would simply broaden the Drude response). Moreover, it has been argued previously [22] that below the MIR limit, the dc resistivity of bad metals like the cuprates may still evolve in a manner similar to that of saturating metals and thus the PRF is still an appropriate means of modelling the temperature evolution of $\rho_{ab}(T)$.

Figure 2. In-plane resistivity $\rho_{ab}(T)$ data (solid curves) for a series of Tl2201 crystals, taken from Ref. [5, 22,23]. The dashed lines are fit to equation (1) between $T_c$ and 200 K.

With this mind, we proceeded to model the $\rho_{ab}(T)$ data of a series of superconducting Tl2201 single crystals taken from the literature [5, 24, 25], having $T_c$ values that varies between 0 K and 63 K, using the PRF. For all data sets below 200 K, the data could only be fitted satisfactorily with the inclusion of an additional $T$-linear term $\alpha_1 T$ such that the PRF reads $1/\rho_{\text{eff}} = 1/\rho_{\text{max}} + 1/\rho_{\text{max}} + 1/\rho_{\text{max}} + 1/\rho_{\text{max}}$. Figure 2 shows the $\rho_{ab}(T)$ data together with the corresponding fits for a selected number of samples across the doping range. Note that for all fits, $\rho_{\text{max}}$ is fixed at 1800 $\mu$Ωcm. Note too that the residual resistivities are comparable across the range.
Figure 3. The temperature derivative $d\rho_{ab}/dT$ for a Tl-30K crystal [25] showing the various components of the $\rho_{ab}(T)$ curve, in particular the low-$T$ $T$-linear term $\alpha_{1}(0)$, the low-$T$ $T^2$ term $\alpha_{2}(0)$ and the high-$T$ $T$-linear component $\alpha_{1}(\infty)$. The dashed lines are guides to the eye.

An alternative way to examine the $T$-dependence of $\rho_{ab}(T)$ is to plot the derivative $d\rho_{ab}/dT$. A representative example, for Tl-30K [25], is shown in figure 3. Above around 240 K, the derivative is essentially constant, illustrating the quasi-linear dependence of the high-$T$ resistivity that was discussed above. As done previously [3], we label the coefficient of the high-$T$ linear resistivity $\alpha_{1}(\infty)$. Between 40 K (i.e. above the fluctuation regime) and 150 K, $d\rho_{ab}/dT$ has a well-defined linear slope that extrapolates to a finite zero-temperature offset. These two features in the temperature derivative correspond respectively to the $T^2$ and $T$-linear terms included in the PRF fits of figure 2. The derivative plots therefore provide estimates of the two coefficients $\alpha_{2}(0)$ and $\alpha_{1}(0)$ that are independent of both the PRF and the residual resistivity $\alpha_{0}$.

3. Discussion and conclusions

The resultant doping dependences of $\alpha_{1}(0)$ and $\alpha_{2}(0)$ are plotted in figures 4 and 5 respectively and compared directly with the results obtained from similar analysis carried out on the LSCO family [8]. The $p$ values for TI2201 were obtained from the recently revised $T_c$ versus $p$ diagram extracted from quantum oscillation experiments [18]. Apart from a factor of 2 difference in the magnitude of $\alpha_{1}(0)$ (note the different scales on the ordinate axes), the evolution of the linear and quadratic components of $\rho_{ab}(T)$ in LSCO and TI2201 are almost identical. The key features to note here are the scaling of $\alpha_{1}(0)$ with $T_c$ in both systems, the persistence of the $T$-linear component across the entire overdoped region of the phase diagram and the near constancy of the coefficient of the $T^2$ term as one approaches $p_{crit} \sim 0.19$ (marked by a vertical dashed line in figure 5). As mentioned in the Introduction, these two latter features are difficult to reconcile with conventional quantum critical scenarios in which the low $T$ region of the Fermi–liquid-like resistivity would vanish on approaching the QCP. Indeed, the region over which $d\rho_{ab}/dT$ is linear, i.e. over which the $T^2$ term is robust, only decreases by of order 10 % (from 150 K to 135 K) between $p = 0.31$ and $p = 0.22$. 

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The confirmation of a near-identical phenomenology in the in-plane resistivity of two distinct cuprate families with different Fermi surface topologies, electronic anisotropies, disorder levels and electronic homogeneities provides strong evidence that this strange-metal behaviour is a generic property of the overdoped cuprates. The c-axis warping of highly overdoped Tl2201, as determined by both ADMR and QO experiments [18, 26], has a symmetry that is only consistent with the Fermi surface being located around the body-centred-tetragonal zone corners, implying that its Fermi surface topology remains hole-like up to $p = 0.30$. In LSCO, by contrast, the Fermi surface passes through a van Hove singularity at a doping level close to $p_{\text{crit}} = 0.19$ and becomes electron-like for $p > 0.19$ [27]. Thus, the phenomenology observed in $\rho_{ab}(T)$ in both systems beyond this doping level must be independent of Fermi-surface topology. The normal state resistive anisotropies of Tl2201 and LSCO differ by a factor of 10 or more, while their superconducting states are best described as two-dimensional (i.e. Josephson-coupled) and anisotropic but three-dimensional respectively [28]. The differences in the disorder levels of the two systems are self-evident in their respective residual resistivities.

Finally, but perhaps most significantly, the phenomenology summarized in figure 4 is observed in Tl2201 whose electronic state, as revealed by QO measurements [17-19], is extremely homogeneous, with the carrier number changing by less than 0.2% over a distance equivalent to a cyclotron radius of order 1-2000 Å [18]. It is therefore highly unlikely that this two-component resistive behaviour is due to microscopic phase separation. In the phase separation scenario one would expect similar $T$ and doping dependence of the resistivity and Hall angle, but Hall angle shows, in contrast to the resistivity, $T^2$ dependence in the whole overdoped regime. This further suggests that the phase separation is not sufficient to describe transport properties in the overdoped regime, unless additional assumptions, e.g., anisotropic elastic scattering [29], are used. It also argues against overdoped LSCO being intrinsically inhomogeneous since it is, in effect, displaying the same transport behaviour as the highly homogeneous Ti2201 family. Indeed, a careful neutron diffraction study has shown that beyond optimal doping, overdoped LSCO is in fact much more homogenous and less susceptible to microscopic phase separation than its underdoped counterparts [30].

Figure 4. Left panel: Doping dependence of $\alpha_1(0)$, the coefficient of the $T$-linear component of the in-plane resistivity, in both LSCO (left ordinate axis) and Tl2201 (right ordinate axis). Note the left and right vertical scales differ by a factor of two. Right panel: Corresponding plot of the doping dependence of $\alpha_2(0)$, the coefficient of the $T^2$ component of $\rho_{ab}(T)$. 
The presence of a $T$-linear term in $\rho_{ab}(T)$ of overdoped Tl2201 is consistent with the identification of an anisotropic $T$-linear term in the quasiparticle scattering rate by ADMR [10-13]. The scaling of $\alpha_1(0)$ with $T_c$ is also consistent with the report of a correlation between the strength of anisotropic scattering (at a fixed temperature of 40 K) and $T_c$ [11] and appears to be consistent with a connection between the low-$T$ $T$-linear resistivity and the existence of a strongly $k$-dependent scattering process that is maximal at the zone boundaries and vanishes along the zone diagonals as well as the link between this scattering and superconductivity itself. The combined plot of $\alpha_1(0)(p)$, shown in the left panel of figure 4, demonstrates much more clearly that the $T$-linear component vanishes in both cuprate families at a critical doping level of $p_0 = 0.31$. This value of $p_0$ coincides precisely with the edge of the superconducting dome in Tl2201 and provides further strong support that the anisotropic ($d$-wave-like) $T$-linear scattering rate is linked to the pairing mechanism of high-$T_c$ superconductivity.

In a recent set of papers [31, 32], we showed how the combination of an isotropic FL self-energy and an anisotropic marginal Fermi-liquid (AMFL) self-energy (having a $d$-wave form factor and a $T$- and $\omega$-linear energy dependence) could explain not only the ADMR results, but also the dc resistivity, the Hall coefficient, the transverse magnetoresistance, the optical conductivity, the evolution of the quasiparticle mass and the specific heat coefficient of Tl2201 with overdoping. Thus, the form of the scattering would appear to reconcile a wide range of physical properties in the single-layer thallium system and given the similarity in the dc response of Tl2201 and LSCO, it would appear natural to apply the same phenomenology to LSCO and other cuprate families.

The AMFL model self energy [31, 32] includes a doping independent FL part which is capable of describing the weak doping dependence of $\alpha_1(0)$ shown in Figure 4 (right) and an anisotropic marginal Fermi liquid part whose strength increases with underdoping and captures the behavior of $\alpha_1(0)$ shown in Figure 4 (left). The success of the model again shows that the overdoped cuprates are not simple Fermi liquids. Specifically, the anisotropy over the Fermi surface, resembling the anisotropy of the underdoped (pseudogap) phase, starts to develop already in the overdoped side of the phase diagram. The isotropic scattering arises due to some local physics, presumably due to strong onsite Coulomb repulsion, and can be captured with, e.g., dynamical mean field theory (DMFT) or a Brinkman-Rice picture. The anisotropic part is a nonlocal effect, which needs to be described with more sophisticated methods, e.g., cluster DMFT [33]. Its origin is unclear and different fluctuations (antiferromagnetic, superconducting, and $d$-density wave), Fermi surface nesting and proximity to a QCP have been suggested to be relevant [32]. Considering the similarity between LSCO and Tl2201 discussed here the latter two seems unlikely. Furthermore, the success of the AMFL model shows, that there is no need to evoke more exotic theories, like two types of quasiparticles [e.g. spinons and holons with different scattering rates [34] or scattering mechanism connected to the charge conjugation properties of different currents [35], to simultaneously describe $T$-linear resistivity and $T$-quadratic Hall angle, at least in the overdoped regime.

In conclusion, the in-plane resistivity of both overdoped Tl2201 and LSCO have been shown to be composed of two $T$-dependent components below 200 K, one linear in temperature, the other quadratic. The $T$-linear component $\alpha_1(0)$ is finite for all superconducting samples with a magnitude that scales with $T_c$. This extended regime (in doping) of $T$-linear resistivity at low $T$ is at odds with conventional quantum critical scenarios associating the pseudogap with the collapse of an ordered phase to 0 K at a critical doping level $p_{\text{crit}} \approx 0.19$. Its observation in Tl2201 appears to rule out phase separation or electronic inhomogeneity as the origin of this extended critical behaviour. A similar $T$-linear resistivity, with a magnitude scaling with $T_c$, has also been reported recently in the electron-doped cuprates [36]. The growing universality would appear to suggest that this strange-metal behaviour is both intrinsic and generic to all overdoped high-$T_c$ cuprates.
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