A Note on Fine–Tuning in Mirage Mediation

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Recent progress in string theory moduli stabilization has motivated a mixed modulus–anomaly mediated supersymmetry breaking scenario, also dubbed ‘mirage mediation’. This scenario has a number of phenomenologically attractive features, in particular with respect to the cosmological gravitino/moduli problem. In this note, we investigate the issues of fine–tuning associated with obtaining the correct electroweak symmetry breaking scale in the mirage mediation scenario. We find that, due to lighter gluinos, the fine–tuning is smaller than that in other mediation mechanisms.

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1. Introduction

The minimal supersymmetric extension of the standard model (SM), the MSSM, enjoys high popularity. One of the main theoretical reasons for it is that a supersymmetry (SUSY) breaking scale close to the electroweak (EW) scale would allow one to understand stability of the electroweak scale against radiative corrections. In addition, SUSY models with TeV–scale soft masses offer the most attractive scenario for perturbative gauge coupling unification.

On the other hand, it is precisely the Higgs sector that casts some shadow on this scheme. The reason is the following. At tree-level, one has an upper bound on the mass of the lightest Higgs [1,2],

\[ m_{h^0} < m_Z |\cos 2\beta| , \]

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where, as usual, \( \tan \beta = v_u/v_d \) denotes the ratio of the two Higgs expectation values. This is in conflict with the current experimental lower limit on the Higgs mass,

\[
m_{\text{Higgs}} \gtrsim 114 \text{ GeV}.
\]  (2)

Luckily, this does not rule out the MSSM because there are sizable radiative corrections to the Higgs mass [3–5], the most important one being

\[
\Delta (m^2_{h_0}) \simeq \frac{3g^2 m_t^4}{8\pi^2 m_W^2} \ln \left( \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \right),
\]  (3)

where \( m_{\tilde{t}_1,2} \) denote the masses of the scalar top quarks (‘stops’). To lift the Higgs mass above the lower experimental bound (2), one needs sizable superpartner masses, \( m_{\tilde{t}_1,2} \gtrsim \text{TeV} \).

On the other hand, electroweak symmetry breaking requires

\[
\frac{m_Z^2}{2} = \mu^2 + m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta \frac{\tan^2 \beta - 1}{\tan^2 \beta - 1},
\]  (4)

where \( m_{H_d,u} \) are the soft Higgs mass parameters and \( \mu \) is the supersymmetric \( \mu \)-parameter, both evaluated at the electroweak scale. Thus one naturally expects the SUSY mass parameters (or at least \( m_{H_u} \) and \( \mu \)) to be of order 100 GeV, otherwise a large cancellation between \textit{a priori} independent terms would be required. The problem is, however, that the GUT scale parameters get renormalized, in particular

\[
\frac{d}{dt} m_{H_u}^2 \supset \frac{3y_t^2}{8\pi^2} \left( m_{\tilde{t}_1}^2 + m_{\tilde{q}_3}^2 \right),
\]  (5)

where \( t \) is a scale variable, and \( m_{\tilde{t}_1}^2 \) and \( m_{\tilde{q}_3}^2 \) denote the soft mass parameters for the right- and left-handed \( \tilde{t} \), respectively. Thus, one has a radiative correction

\[
\delta_{\mu G} m_{H_u}^2 \simeq - \frac{3y_t^2}{8\pi^2} \left( m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2 \right) \ln \left( \frac{\Lambda}{m_{\tilde{t}}} \right),
\]  (6)

with \( m_{\tilde{t}} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \) and \( \Lambda \) being the high energy scale at which the boundary conditions are specified. Taking \( \Lambda = M_{\text{GUT}} \simeq 2 \times 10^{16} \text{ GeV} \), one expects \( |\delta m_{H_u}^2| \) to be of order \( m_{\tilde{t}}^2 \).

The former is constrained by the Higgs mass bound to be in the \((\text{TeV})^2\) range, indicating that large cancellations in Eq. (6) are required. This is the essence of the supersymmetric ‘little hierarchy’ problem [6, 7].

There are two obvious ways to evade this conclusion:

1. low \( \Lambda \)

2. cancellations
The first possibility has been studied rather extensively (see, e.g., [8]). Although the fine-tuning can be superficially reduced, there is a price one has to pay. Namely, one loses the apparent gauge coupling unification and other appealing features involving high scales such as the see-saw mechanism (see, e.g., [9]).

The second possibility gained some popularity more recently. It has been realized that in the framework of flux compactifications of string theory, the soft masses [10, 11] have certain features that may help ameliorate the SUSY fine-tuning problem. In particular, in the scheme of ‘mirage mediation’\(^1\), some cancellations between the input value of \(m^2_{H_u}\) and its RG corrections are possible [13, 14]. In what follows, we focus on this possibility.

2. Mirage Mediation and Mirage Unification

We start by reviewing main features of the ‘mirage mediation’ scheme. It is motivated by Calabi–Yau compactifications of string theory with fluxes. A particular realization is given by the model of Kachru, Kallosh, Linde and Trivedi (KKLT) [15]. In this scenario, all the moduli are fixed and the cosmological constant is close to zero. One of the phenomenologically attractive features of this setup is a hierarchy among the MSSM soft masses, the gravitino and moduli masses [10, 11],

\[
m_{\text{MSSM}} \ll m_{3/2} \ll m_{\text{moduli}},
\]

such that the gravitino and the moduli can be made heavy so as to avoid cosmological problems associated with late decays of these particles.

Another interesting feature is that the MSSM soft terms receive comparable contributions from gravity (modulus) mediated and anomaly mediated [16,17] SUSY breaking. Specifically, for the MSSM on D7 branes we have [11]\(^2\)

\[
\begin{align*}
M_a &= M_s \left[ \alpha + b_a g_a^2 \right], \\
m_i^2 &= M_s^2 \left[ \alpha^2 - \dot{\gamma}_i + 2\alpha \left( T + \bar{T} \right) \partial_T \gamma_i \right], \\
A_{ijk} &= M_s \left[ 3\alpha - \gamma_i - \gamma_j - \gamma_k \right].
\end{align*}
\]

Here \(M_s = m_{3/2}/(16\pi^2)\) is the scale of the soft terms, \(\alpha\) measures the balance between the anomaly and the \(T\)-modulus mediated contributions and typically lies in the range \(0 < \alpha \leq 10\), \(b_i\) are the beta function coefficients for the gauge couplings \(g_a\), \(\gamma_i\) is the anomalous dimension and \(\dot{\gamma}_i = 8\pi^2 \frac{\partial \gamma_i}{\partial \log \mu}\). The modulus \(T\) is responsible for the standard model gauge couplings and is fixed at the value \(\text{Re} \, T = 1/g^2_{\text{GUT}} \simeq 2\).

\(^1\) This name was coined in [12].
\(^2\) We follow the conventions of [18]. Here we choose the ‘effective modular weights’ \(n_i = 0\) for the matter fields, whereas other choices are also possible (see e.g. [13, 14]). We also assume that the Yukawa couplings are independent of the \(T\)-modulus.
Fig. 1: The ‘mirage’ and ‘real’ unification of the gaugino masses (a), and the gauge coupling unification (b). The solid (red), dashed (green), dash-dotted (blue) curves in Plot (a) show the evolution of $M_3$, $M_2$, $M_1$. Plot (b) displays the evolution of $\alpha_i = g_i^2/4\pi$. Above the unification scale $M_{\text{GUT}} \simeq 2 \times 10^{16}$ GeV, we use an SU(5) theory with 3 generations of $10 \oplus \overline{5}$, one pair $5 + \overline{5}$ as well as an adjoint SU(5) Higgs.

It has been observed that the above gaugino masses unify at an intermediate, the so-called ‘mirage unification’ scale [19, 20],

$$\mu_{\text{mir}} = M_{\text{GUT}} e^{-8\pi^2/\alpha}.$$  

This is a feature of the boundary conditions in the mixed anomaly–modulus mediated scenario. At the mirage scale no ‘new’ physics appears, hence the name ‘mirage’. We note that large $\alpha$ correspond to modulus domination and the mirage scale is the GUT scale, whereas in the anomaly dominated limit $\alpha \to 0$ the mirage scale approaches zero.

One might now ask the question what happens if we start with a really unified theory such as an SU(5) GUT. Clearly, at energies above the unification scale, one has only one gaugino and there is only one gaugino mass. Just below the GUT scale we have
non–universal gaugino masses due to the anomaly contribution. These two limits are reconciled via the threshold corrections at the GUT scale (cf. e.g. [21]). We are therefore led to the picture where there is ‘real unification’ of gaugino masses in the realm of the unified theory, and ‘mirage unification’ at an intermediate scale. We illustrate these effects in Fig. 1.

An important feature of the setup is that for typical $\alpha$ the gluino is the lightest gaugino at the GUT scale. This is because the SU(3)$_c$ beta function is the largest one and it is being subtracted from the universal modulus contribution. Clearly, this feature is specific to ‘mirage mediation’ and does not hold in its limiting cases $\alpha \rightarrow 0$, $\alpha \rightarrow \infty$. Light gluinos are desirable regarding the fine–tuning problem since gluinos control to a large extent the RG running of the squark and Higgs masses. In the next section, we analyze this issue in detail.

3. Fine-Tuning in Mirage Mediation

As discussed in the Introduction, a certain degree of fine–tuning is required in SUSY models to obtain the electroweak scale from the scale of the soft masses. This can be understood qualitatively by using approximate analytic solutions to the RG equations. One can rewrite Eq. (4) in terms of the input SUSY parameters at the GUT scale as [22]

$$m_Z^2 \simeq -1.8 \mu^2 + 5.9 M_3^2 - 0.4 M_2^2 - 1.2 m_{H_u}^2 + 0.9 m_{Q_i(s)}^2 + 0.7 m_{U_i(s)}^2 - 0.6 A_t M_3 + 0.4 M_2 M_3 + \ldots$$

(10)

where we have taken $\tan \beta = 5$ and neglected terms with smaller numerical coefficients. This equation shows sensitivity of $m_Z$ to various input parameters. If all the parameters are about 100 GeV, no significant fine–tuning is needed. However, as we have argued the lightest Higgs mass bound requires the stops of about 1 TeV (at the EW scale), such that 100 GeV input parameters are typically inconsistent with experiment. Then, in order to get the right $m_Z$, some cancellations in Eq. (10) are needed.

3.1. Cancellations and Tachyons

Clearly, $m_Z$ is most sensitive to the input value of the gluino mass. Thus reduction of $M_3$ is welcome from the fine–tuning perspective, as it occurs in ‘mirage mediation’. Then given a larger $m_{H_u}^2$ at the GUT scale, significant cancellations in Eq. (10) are possible to achieve. In other words, $m_{H_u}^2(m_Z)$ in Eq. (4) can be made of order 100 GeV by cancelling its GUT input value by the RG evolution. However, implementation of this mechanism in simple versions of ‘mirage mediation’ requires tachyons at the GUT scale and is strongly constrained by the Higgs mass bound.

An example of the cancellation effect is shown in Fig. 2. There the GUT boundary conditions are chosen such that $M_i = A_i = \sqrt{2} m_{sferm.} \sim$ TeV and $m_{H_u,d} \ll$ TeV at
Fig. 2: RG evolution of the soft masses for $\mu_{\text{mir}} \simeq 1\, \text{TeV}$ and $\tan \beta = 5$. The GUT boundary conditions are chosen such that $M_i = A_i = \sqrt{2}m_{\text{sferm}} \sim \text{TeV}$ and $m_{H_u,d} \ll \text{TeV}$ at the TeV scale.

the TeV scale [13,14]. Clearly, this requires tachyonic squarks and sleptons at the GUT scale signalling that the boundary conditions are not well defined. One may ignore this
problem by saying that what matters is the masses at the ‘mirage scale’, but as we have discussed there is no new physics appearing at this scale and the ‘true’ boundary conditions should be defined at the GUT scale, above which a new theory sets in. The presence of tachyons appears as too high a price for ameliorating fine–tuning.

On the other hand, in general, large cancellations in Eq. (10) can be achieved without tachyons in the spectrum. What matters for the fine–tuning problem is the stop masses and these are non–tachyonic (Fig. 2). The other mass squareds can be made positive by choosing appropriate ‘effective modular weights’ for them, subject to the FCNC constraints. This will lead to a more complicated set of boundary conditions reminiscent of the general MSSM. Given enough freedom one can make \( m_{H_u}(m_Z) \) arbitrarily small, as it happens in the MSSM.

### 3.2. Fine–Tuning in Simple Versions of ‘Mirage Mediation’

One may now ask what is the degree of fine–tuning typical to ‘mirage mediation’. As a representative example, let us consider ‘minimal mirage mediation’ where the soft mass parameters are given by Eq. (8). To quantify the fine–tuning, we will need a proper measure. A reasonable measure of fine–tuning is given by a variation in the \( Z\)–mass upon a small change in the SUSY GUT parameters. Specifically, we define the sensitivity \( \Delta \xi_i \) of \( m_Z \) to the input parameters \( \xi_i \) of the theory by [23]:

\[
\Delta \xi_i = \frac{\delta m_Z^2}{m_Z^2} \delta \xi_i \tag{11}
\]

In our setup, the relevant parameters are \( \alpha \) and \( m_3/2 \equiv 16\pi^2 M_s \). Then, the question is how much \( m_Z \) changes if we perturb these parameters while keeping \( \tan \beta, \mu \) and \( B\mu \) fixed. From Eq. (11) we have

\[
\Delta \xi = \frac{2 \xi}{m_Z^2} \frac{d}{d\xi} \left( m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta \right) \frac{\tan^2 \beta - 1}{\tan^2 \beta - 1} \tag{12}
\]

with \( \xi \) being \( \alpha \) or \( m_{3/2} \), and \( m_{H_d,u}^2 \) evaluated at the electroweak scale. For our purposes, it is convenient to define a mean sensitivity \( \Delta \),

\[
\Delta = \sqrt{\Delta^2 + \Delta_{m_{3/2}}^2} \tag{13}
\]

\( \Delta \) is calculated by taking numerical derivatives of \( m_{H_u}^2 \) and \( m_{H_d}^2 \) with respect to \( \alpha \) and \( m_{3/2} \) at one loop. Our results are presented in Fig. 3.

These results can be understood as follows. Expressing the mass parameters in Eq. (10) in terms of \( \alpha \) and \( M_s \) (Eq. (5)), we get

\[
m_Z^2 \sim -1.8 \mu^2 + 4.5 M_s^2 (\alpha^2 - 3.7 \alpha + 3.1) \tag{14}
\]
Now, the degree of fine-tuning can be estimated analytically. We find that $\Delta_\alpha = 0$ at $\alpha \simeq 2$ and $\Delta_m^{3/2} = 0$ at $\alpha \simeq 2.4$ and 1.3. For the mean sensitivity, we have

$$\Delta \to \min \quad \text{at} \quad \alpha \simeq 2,$$

in which case $\Delta$ is $\leq 1$ for $\sim 100$ GeV soft masses. This is also evident from Fig. 3. On the other hand, at $\alpha \simeq 2$ there is no electroweak symmetry breaking, i.e. Eq. (14) cannot be satisfied for any $\mu$ and $M_s$. Furthermore, the squarks and the sleptons are tachyonic at the GUT scale so the boundary conditions are simply ill-defined. An even more restrictive bound comes from the Higgs mass limit which excludes large portions of the parameter space.

\[ \text{Fig. 3: Fine-tuning $\Delta$ as a function of $\alpha$ and $m_{3/2}$ at $\tan \beta = 5$ and $m_t = 174$ GeV. The darker shaded area shows the presence of tachyons, while the lighter shaded area is excluded by the LEP constraint on the Higgs mass. The electroweak symmetry breaking and the LEP chargino mass constraints (cf. [18]) are not shown.} \]

The main features of $\Delta(\alpha, m_{3/2})$ are quite transparent. Fine-tuning increases rapidly with $m_{3/2}$ since the gravitino mass sets the scale of the soft masses. It also increases with $\alpha$ for two reasons. First, larger $\alpha$ correspond to gravity dominated SUSY breaking and thus larger gluino masses, and second, they increase the scale of the soft masses. The sharp decrease in the fine-tuning around $\alpha = 2$ is a special property of the ‘mirage
mediation’ soft terms. However, the area with $\Delta < 100 - 1000$ (depending on $\tan \beta$) is excluded by the Higgs mass bound.

It appears that although ‘mirage mediation’ has a nice qualitative feature that the gluino mass is suppressed, a combination of the EW symmetry breaking, Higgs mass bound and absence of tachyons constraints typically requires a relatively large degree of fine-tuning, similar to that of mSUGRA.

3.3. Towards a Solution of the Fine-Tuning Problem

In its simple incarnation, ‘mirage mediation’ somewhat ameliorates the MSSM fine-tuning problem, yet does not solve it. There exist regions in the parameter space where the fine-tuning is small, but these are problematic for various reasons, in particular, tachyonic boundary conditions at the GUT scale.

The above problems can perhaps be circumvented in more general versions of ‘mirage mediation’. In particular, given enough freedom in ‘effective modular weights’, one can arrange for significant cancellations in Eq. (10) consistently with other constraints. This is a model-dependent issue and can be studied only within particular semirealistic models realizing the MSSM on D–branes.

What is clear, however, is that ‘mirage mediation’ has a robust feature that $M_3 < M_2$ at the GUT scale and thus the fine-tuning is reduced [24–26]. In particular, one can study the fine-tuning with respect to (to a large extent) model independent parameters such as the gaugino masses. In ‘mirage mediation’, the gaugino contribution to the $Z$–mass is given by (cf. Eq. (10))

$$\delta m_Z^2 |_{\text{gaugino}} \simeq 5.9 M_3^2 - 0.4 M_2^2 \simeq 5.5 M_s^2 (\alpha - 1.1) (\alpha - 2.1).$$

(16)

In the gravity mediation limit $\alpha \to \infty$, with $\alpha M_s$ fixed, the gaugino contribution to the $Z$–mass is at least of the order of the soft masses $\alpha M_s$. On the other hand, in ‘mirage mediation’ this contribution is reduced and can even be zero. That means that a generic prediction of this scenario is that the usual MSSM fine-tuning is reduced.

4. Summary

The scheme of ‘mirage mediation’ has a number of phenomenologically desirable features. Most notably, the usual conflict between supergravity theories and nucleosynthesis, known as the ‘gravitino/moduli problem’, is resolved since the gravitino and moduli are sufficiently heavy to decay before nucleosynthesis.

In this note, we have discussed the issue of fine-tuning associated with obtaining the correct electroweak breaking scale in the ‘mirage mediation’ scenario. We find that there exist regions in the parameter space where the sensitivity of $m_Z$ to the input parameters...
$(\alpha, m_{3/2})$ is considerably reduced. However, in simple versions of ‘mirage mediation’, these regions are problematic since the corresponding GUT boundary conditions are tachyonic. This appears to be a model–dependent feature. Presumably, with a more sophisticated choice of ‘effective modular weights’ one can avoid tachyons as well as satisfy the electroweak symmetry breaking conditions and the Higgs mass bound.

An important model–independent feature of ‘mirage mediation’ is that it predicts $M_2 > M_3$ at the high energy scale consistently with grand unification. Therefore, the ‘mirage mediation’ scheme ameliorates to some extent the notorious MSSM fine–tuning.

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