In-medium $\eta N$ interactions and $\eta$ nuclear bound states

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Abstract

The in-medium $\eta N$ interaction near and below threshold is constructed from a free-space chirally-inspired meson-baryon coupled-channel model that captures the physics of the $N^*(1535)$ baryon resonance. Nucleon Pauli blocking and hadron self-energies are accounted for. The resulting energy dependent in-medium interaction is used in self-consistent dynamical calculations of $\eta$ nuclear bound states. Narrow states of width $\Gamma_\eta \lesssim 2$ MeV are found across the periodic table, beginning with $A \geq 10$, for this in-medium coupled-channel interaction model. The binding energy of the $1s_\eta$ state increases with $A$, reaching a value of $B_{1s}(\eta) \approx 15$ MeV. The implications of our self-consistency procedure are discussed with respect to procedures used in other works.

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$N^*(1535)$ resonance, meson–baryon interactions, mesons in nuclear matter, mesic nuclei
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1. Introduction

Haider and Liu realized in 1986 that a moderately attractive $\eta N$ interaction, with scattering length estimated as $a_{\eta N} = 0.27 + i0.22$ fm, may lead to a robust pattern of $\eta$-nuclear bound states across the periodic table beginning with $^{12}$C$^1$. Their pioneering work has been followed by numerous studies of the $\eta N$ interaction within various theoretical models that yielded a wide range of values for $\text{Re } a_{\eta N}$ from 0.2 fm$^2$ to about 1.0 fm$^3$, as summarized in 2005 by Arndt et al. $^4$. Among the very recent works demonstrating this large variation we mention the $\pi N-\eta N-K\Lambda-K\Sigma$ coupled-channel chiral model of Mai, Bruns, Meißner$^5$ with values of $\text{Re } a_{\eta N} = 0.22$ fm and
0.38 fm in its two versions, and the $K$-matrix analysis involving additional channels by Shklyar, Lenske, Mosel [6] with $\text{Re } a_{\eta N} = 1.0 \text{ fm}$. This wide range of values introduces considerable uncertainty into the evaluation of $\eta$-nuclear spectra, as shown very recently by Friedman, Gal, Mareš (FGM) [7] for $1s_\eta$ nuclear bound states. Calculated $1s_\eta$ binding energies in $^{208}$Pb, for example, range approximately between 10 and 30 MeV. Generally and as naively expected, the larger and hence more attractive $\text{Re } a_{\eta N}$ is, the larger is the calculated binding energy of a given $1s_\eta$ nuclear state. In particular, the $\eta$-nuclear interaction generated from the Green-Wycech (GW) [3] $\text{Re } a_{\eta N} \sim 1.0 \text{ fm}$ amplitude is sufficiently strong to bind additional single-particle $\eta$ states in heavy nuclei, as shown in Sect. 4 of the present work. Regarding $\text{Im } a_{\eta N}$, most analyses result in a much narrower interval of values between 0.2 to 0.3 fm. Therefore, one might think that calculated widths of $\eta$-nuclear states should exhibit little model dependence. However, this expectation is not borne out in the very recent calculations by FGM that find widths of the $1s_\eta$ state in $^{208}$Pb ranging from a few MeV to about 25 MeV, depending on the assumed $\eta N$ interaction model.

An important lesson of the FGM work is that the in-medium $\eta N$ scattering amplitudes that serve input in the calculation of $\eta$-nuclear bound states cannot be determined in terms of threshold $\eta N$ scattering amplitudes alone, be it free-space or in-medium threshold amplitudes. It was shown that $\eta N$ scattering amplitudes down to about 50 MeV below threshold are involved in $\eta$-nuclear bound state calculations [7]. In the coupled-channel studies of the $N^*(1535)$ resonance region cited above, the extrapolation from $\sqrt{s} \sim 1535 \text{ MeV}$ to the $\eta N$ threshold at $\sqrt{s_{\eta N}} = 1487 \text{ MeV}$ and further down to the $\eta N$ subthreshold region introduces appreciable model dependence which is reflected in the large span of reported values for $a_{\eta N}$, particularly for its real part. The only model-independent property shared by such studies is that both real and imaginary parts of the $\eta N$ scattering amplitude decrease steadily as one goes below the $\eta N$ threshold. This is demonstrated in Fig. 11 where the real and imaginary parts of the $\eta N$ center-of-mass (cm) scattering amplitude $F_{\eta N}(\sqrt{s})$ are plotted as a function of $\sqrt{s}$ for five different $s$-wave interaction models. The position of the $N^*(1535)$ resonance is closely related to the maximum of $\text{Im } F_{\eta N}(\sqrt{s})$ on the right panel. We note that no simple relationship emerges between the hierarchy of $\eta N$ scattering amplitudes shown here, neither for $\text{Re } F_{\eta N}$ nor for $\text{Im } F_{\eta N}$, and the gross properties of the $N^*(1535)$ resonance such as its peak position or width.

In Ref. [7], in-medium $\eta N$ scattering amplitudes $F_{\eta N}(\sqrt{s}, \rho)$ that satisfy
Figure 1: Real (left panel) and imaginary (right panel) parts of the $\eta N$ cm scattering amplitude $F_{\eta N}(\sqrt{s})$ as a function of the total cm energy $\sqrt{s}$ from five meson-baryon coupled-channel interaction models, in decreasing order of Re $a_{\eta N}$: Dot-dashed curves: GW [3]; solid: CS [8]; dotted: KSW [9]; long-dashed: M2 [5]; short-dashed: IOV [10]. The thin vertical line denotes the $\eta N$ threshold.

the low-density requirement $F_{\eta N}(\sqrt{s}, \rho) \to F_{\eta N}(\sqrt{s})$ upon density $\rho \to 0$ were formed from the corresponding free-space $\eta N$ scattering amplitudes $F_{\eta N}(\sqrt{s})$ using the Ericson-Ericson multiple-scattering reformulation given in Ref. [11] (and employed recently also in Ref. [12]):

$$F_{\eta N}(\sqrt{s}, \rho) = \frac{F_{\eta N}(\sqrt{s})}{1 + \xi(\rho)(\sqrt{s}/E_N)F_{\eta N}(\sqrt{s})\rho},$$

where

$$\xi(\rho) = \frac{9\pi}{4p_F^2} I(\kappa), \quad I(\kappa) = 4 \int_0^\infty \frac{dt}{t} \exp(-\kappa t) J_1^2(t).$$

Here $E_N$ is the nucleon energy, often approximated by its mass $m_N$, $p_F$ is the local Fermi momentum corresponding to density $\rho = 2p_F^3/(3\pi^2)$ and $\xi(\rho)$ accounts for Pauli blocking. At threshold, $\kappa = 0$, $I(\kappa) = 1$ and $\xi(\rho) = 9\pi/(4p_F^2)$. For subthreshold energies represented by $\eta$ nuclear complex binding energies $B_\eta + i\Gamma/2$, $\kappa = \sqrt{2m_\eta(B_\eta + i\Gamma/2)/p_F}$, $I(\kappa)$ remains dominantly real but with magnitude less than one, typically 0.5. We have used expression (2) to revise the FGM bound-state calculations done for $\kappa = 0$. This leads to a moderate increase of the $\eta$-nuclear attraction and, consequently, to binding energies that are somewhat larger, by a few MeV at most, than those reported by FGM [7]. The calculated widths hardly change.
The primary aim of the present work is to extend the FGM analysis by using in-medium \( \eta N \) interactions constructed here within the recent chirally-inspired meson-baryon coupled-channel model by Cieplý and Smejkal (CS) [8] in which self-energy insertions, disregarded by FGM [7], are now included. This construction follows a similar one by these authors for in-medium \( s \)-wave \( \bar{K}N \) interactions in Ref. [13] where several \( \bar{K} \)-nuclear applications are reviewed [14, 15, 16, 17, 18, 19]. The corresponding in-medium \( \eta N \) scattering amplitudes \( F_{\eta N}(\sqrt{s}, \rho) \) are applied in the present work directly within a comprehensive study of \( \eta \)-nuclear bound states, without having to approximate in-medium amplitudes by using the multiple-scattering expressions (1) and (2) (which nevertheless are found to provide a very good approximation to within few percent). Comparison is also made with another study done within a different coupled-channel approach [10] and its in-medium implementation [20], and with a different procedure of handling the energy dependence of in-medium \( \eta N \) scattering amplitudes in \( \eta \)-nuclear bound-state calculations [21, 22].

For a given \( \eta N \) interaction model, the calculation of \( \eta \)-nuclear bound states in the present work follows the same steps introduced by FGM [7]. Thus, one solves the Klein-Gordon (KG) equation

\[
\left[ \nabla^2 + \tilde{\omega}_\eta^2 - m_\eta^2 - \Pi_{\eta}(\omega_\eta, \rho) \right] \psi = 0 ,
\]

where \( \tilde{\omega}_\eta = \omega_\eta - i \Gamma_\eta/2 \) and \( \omega_\eta = m_\eta - B_\eta \), with \( B_\eta \) and \( \Gamma_\eta \) the binding energy and the width of the \( \eta \)-nuclear bound state, respectively. The self-energy operator \( \Pi_{\eta}(\omega_\eta, \rho) \) is given by

\[
\Pi_{\eta}(\omega_\eta, \rho) \equiv 2\omega_\eta V_\eta = -4\pi \frac{\sqrt{s}}{E_N} F_{\eta N}(\sqrt{s}, \rho) \rho ,
\]

where \( \rho \) is the nuclear density (normalized to the number of nucleons \( A \)) and

\[
s = (\omega_\eta + E_N)^2 - (p_\eta + p_N)^2
\]

is the Lorentz invariant Mandelstam variable. The factor \( (\sqrt{s}/E_N) \) transforms the in-medium cm scattering amplitude \( F_{\eta N}(\sqrt{s}, \rho) \) to the corresponding laboratory (lab) amplitude which for \( A \gg 1 \) is the one relevant in \( \eta \)-nuclear calculations. In the lab system, in distinction from the \( \eta N \) two-body cm system where \( p_\eta + p_N = 0 \), the two momenta are determined separately by the nuclear medium and their combined contribution is well approximated
by a non-zero \((p_{\eta}^2 + p_N^2)\) term. Including this negative-definite contribution to \(s\) in the evaluation of \(F_{\eta N}(\sqrt{s}, \rho)\) weakens both real and imaginary parts of the \(\Pi_{\eta}(\omega, \rho)\) self-energy input to the KG equation, reducing thereby the calculated \(\eta\)-nuclear binding energy and width with respect to all other calculations that preceded FGM. The actual binding energy and width calculation requires a self-consistent procedure, since each of the four kinematical variables \(\omega, E_N, p_{\eta}^2\) and \(p_N^2\) of which \(\sqrt{s}\) consists depends in the nuclear medium on the nuclear density \(\rho\). In particular, as discussed below in Sect. 3, \(p_{\eta}^2\) depends on \(\rho\) primarily through the self-energy \(\Pi_{\eta}(\omega, \rho)\), so even the input self-energy operator requires iterative cycles to become uniquely determined. Previous \(\eta\)-nuclear calculations were only concerned with the dependence of the input \(\sqrt{s}\) (through the \(\omega\) term) on the output binding energy \(B_{\eta}\) [21, 22].

Another improvement offered here, as well as in the recent FGM calculation, consists of solving RMF equations of motion for the in-medium nucleons in sequence and self-consistently [23] with the \(\eta\)-nuclear KG equation, thereby allowing the \(\eta\) meson to polarize the nuclear core. However, the core polarization effect on \(B_{\eta}\) and \(\Gamma_{\eta}\) was found in these dynamical calculations to be smaller than 1 MeV, thus justifying the use of static nuclear densities at the present state of the art in \(\eta\)-nuclear calculations.

The paper is organized as follows. In Sect. 2 we derive the in-medium \(\eta N\) scattering amplitudes from the free-space coupled-channel chirally-inspired separable-interaction NLO30\(\eta\) model due to CS [8]. Our methodology of treating energy dependence, density dependence and combining these together self-consistently as in Ref. [7] is outlined in Sect. 3 while results of dynamical bound-state calculations of \(\eta\)-nuclear states are reported and discussed in Sect. 4. The paper ends with summary and outlook in Sect. 5.

2. In-medium chirally-inspired coupled-channel \(\eta N\) interaction

It was made clear in the Introduction that in-medium \(\eta N\) scattering amplitudes \(F_{\eta N}(\sqrt{s}, \rho)\) are needed for constructing the \(\eta\) self-energy operator \(\Pi_{\eta}(\omega, \rho)\), or equivalently the \(\eta\)-nuclear potential [11]. In close relationship to our recent works on \(\bar{K}\)-nuclear interaction [15, 16] we employ chirally motivated meson-baryon \(s\)-wave scattering amplitudes \(F_{ij}\), given in the two-body cm system by a separable form

\[
F_{ij}(k, k'; \sqrt{s}) = g_i(k^2) f_{ij}(\sqrt{s}) g_j(k'^2),
\]  

(6)
with off-shell form factors chosen as
\[ g_j(k) = \frac{1}{1 + (k/\alpha_j)^2}, \]

superposed on the purely energy-dependent reduced amplitudes \( f_{ij} \). The indices \( i \) and \( j \) label meson-baryon coupled channels: \( \pi N, \eta N, K\Lambda \) and \( K\Sigma \), in order of their threshold energies. The meson-baryon cm momenta in the initial (final) state are denoted \( k \) (\( k' \)), \( \sqrt{s} \) stands for the total energy in the two-body cm system, and the inverse-range parameters \( \alpha \) characterize the interaction range in the specified meson-baryon channels. The scattering amplitudes \( F_{ij} \) solve the coupled-channels Lippmann-Schwinger (LS) equation
\[ F = V + VGF, \]
where \( G \) stands for the intermediate-state Green’s function and the coupled-channels potential matrix \( V \) is given in separable form
\[ V_{ij}(k,k';\sqrt{s}) = g_i(k^2) v_{ij}(\sqrt{s}) g_j(k'^2), \]
with the same form factors \( g_j(k^2) \) as in (6), here given by Eq. (7). The energy-dependent \( v_{ij}(\sqrt{s}) \) elements of the potential matrix are determined by matching to SU(3) chiral amplitudes derived to a given order of the chiral expansion. While the basic features of the \( K\Lambda \) coupled channel interactions are satisfactorily described already by the leading order (LO) Tomozawa-Weinberg (TW) term, a good reproduction of the \( \pi N \) and \( \eta N \) experimental data requires next-to-leading-order (NLO) contributions. In the present work we use model NLO30_{\eta} from the recent work of Ciepły and Smejkal [8].

The intermediate-state meson-baryon Green’s function \( G \) is diagonal in the channel indices \( i,j \). It follows then that the LS equations (8) allow for algebraic solution, with reduced amplitudes \( f_{ij}(\sqrt{s}) \) given by
\[ f_{ij}(\sqrt{s},\rho) = [(1 - v(\sqrt{s}) G(\sqrt{s},\rho))^{-1} \cdot v(\sqrt{s})]_{ij}. \]
Marked in this equation explicitly, in addition to energy dependence, is also a density dependence of the reduced amplitudes implied by the density dependence that the Green’s function acquires in the nuclear medium owing to (i) Pauli blocking and to (ii) self-energy insertions. The Green’s function in channel \( n \) is expressed as
\[ G_n(\sqrt{s},\rho) = -4\pi \int_{\Omega_n(\rho)} \frac{d^3p}{(2\pi)^3 k_n^2 - p^2 - \Pi_n(\sqrt{s},\rho) + i0}, \]
6
where the integration over the intermediate meson-baryon momenta is restricted to a region $\Omega_n(\rho)$ ensuring that in channels involving nucleons the intermediate nucleon energy is above the Fermi level (see Ref. [24] for details). The self-energy $\Pi^{(n)}(\sqrt{s}, \rho)$ stands for the sum of hadron self-energies $\Pi_h^{(n)}(\sqrt{s}, \rho)$ in channel $n$, given in terms of hadron-nucleus potentials $V_h(\sqrt{s}, \rho)$:

$$\Pi_h^{(n)}(\sqrt{s}, \rho) = 2\mu_n(\sqrt{s})V_h(\sqrt{s}, \rho). \tag{12}$$

Here $\mu_n(\sqrt{s})$ is the meson-baryon relativistic reduced energy and the potential $V_h$ is chosen for simplicity linear in the density:

$$V_h(\sqrt{s}, \rho) = v_h(\sqrt{s})\rho / \rho_0, \tag{13}$$

except for $V_\eta$ which is determined self-consistently as explained below. The value $\rho_0 = 0.17$ fm$^{-3}$ is used for nuclear-matter density. In Table 1 we list the self-energy (SE) input hadron potential depths $v_h$ at the $\eta N$ threshold. The baryon nuclear-matter potentials are the same ones used in our earlier work [25], with energy dependence disregarded. The meson potential depths are determined as itemized below.

Table 1: Potential depths $v_h$ (in MeV), Eq. (13) at the $\eta N$ threshold $\sqrt{s_{\eta N}} = 1487$ MeV, providing input to self-energies in Eqs. (11) and (12), with values from Ref. [25] for baryons and values discussed in the text for mesons.

|     | $\pi$ | $K$  | $N$  | $\Lambda$ | $\Sigma$ |
|-----|-------|------|------|-----------|----------|
|     | 20−i40 | 31.6 | -(60+i10) | -(30+i10) | 30−i10 |

- For pions we derived empirical $\pi N$ scattering amplitudes from SAID [26] including several partial waves beyond $s$ waves. With on-shell pion momenta of over 400 MeV/c at the $\eta N$ threshold region, Pauli blocking is negligible and the $\pi N$ free-space amplitude $F_{\pi N}(\sqrt{s})$ should approximate well the $\pi N$ in-medium amplitude. The corresponding $\pi N$ free-space forward scattering amplitude was then substituted in

$$2\mu_{\pi N}(\sqrt{s})v_{\pi}(\sqrt{s}) = -4\pi F_{\pi N}(\sqrt{s}; 0^\circ)\rho_0 \tag{14}$$

to estimate the pion-nuclear potential depth. The resulting pion potential and SE (with $v_\pi$ listed in Table 1) are weakly repulsive but
substantially absorptive, with little energy dependence around the $\eta N$ threshold, exercising negligible effect on the outcome in-medium $\eta N$ scattering amplitude in the subthreshold region of interest. Thus, reversing the sign of the real part of the pion SE or setting it to zero affects marginally the resulting $\eta N$ amplitudes. Reducing the imaginary part also has marginal effect. It appears that the pion SE is insignificant owing to the considerably larger kinetic energy of pions at the $\eta N$ threshold region. Furthermore, we recall that the $\pi N$ and $\eta N$ systems are decoupled at LO, both communicating with each other through the attractive kaon-hyperon systems which generate the two major $(1/2)^- N^*(1535)$ and $N^*(1650)$ resonances above the $\eta N$ threshold.

- For kaons we used a value of $v_K = 30$ MeV at the $KN$ threshold $\sqrt{s_{KN}} = 1433$ MeV, averaging over two recent phenomenological derivations from GSI experiments: (i) $(20 \pm 5)$ MeV from the in-medium $K^0$ inclusive cross sections in $\pi^-$-induced reactions on several nuclear targets at $1.15$ GeV/c [27], and (ii) about $40$ MeV from transverse momentum spectra and rapidity distributions of $K^0_s$ in $Ar$+KCl reactions at a beam kinetic energy of $1.756$ $Au$ GeV [28]. This value of $v_K$ is very close to the value $v_K \approx 32.1$ MeV derived from the energy-independent SE employed by Inoue and Oset [20]. To account for energy dependence we multiplied the $KN$-threshold value of $v_K$ by the ratio of reduced energies $\mu_{KN}(\sqrt{s})/\mu_{KN}(\sqrt{s_{KN}})$ which provides a good approximation away from the $KN$ threshold, resulting in the tabulated value. The kaon potential and SE are moderately repulsive in the $\eta N$ threshold, exercising a nonnegligible effect on the resulting in-medium $\eta N$ scattering amplitude in the subthreshold region of interest, as shown below.

- The self-consistently derived $\Pi_\eta$ is largely independent of whatever input $v_\eta$ value is used. For a representative value, consider implementing multi-channel Pauli blocking without introducing simultaneously any SE. This gives $v_\eta = -(42.1 + i 16.1)$ MeV at the $\eta N$ threshold in model NLO30$_\eta$ of CS [8]. The LS equations were then solved iteratively to achieve convergence for the in-medium SE $\Pi_\eta$ of Eqs. (3) and (4). For completeness we note that $\Pi_\eta$ is related to the $\eta N$-channel SE $\Pi_{\eta N}^{(\eta N)}$ by $\Pi_\eta = (\sqrt{s}/E_N)\Pi_{\eta N}^{(\eta N)}$. We recall that $\Pi_\eta$ is the only SE constrained by a self-consistency requirement in our iterative solution of the LS equations.
With self-energies accounted for, and considering the energy and density dependence of $\Pi_\eta$, the coupled-channels LS equations are solved iteratively to achieve self-consistency, see Ref. [16] for details. No more than 5–7 iterations are normally needed to achieve the required precision.

The nuclear medium effect on the energy dependence of the $\eta N$ scattering amplitude is demonstrated in Fig. 2. With inverse-range parameter $\alpha_{\eta N} = 1635$ MeV/c [8], any explicit momentum dependence is negligible in this model, and there is practically no difference between the amplitudes $F_{\eta N}$ and their respective reduced parts $f_{\eta N}$ [6]; hence, the self-energy $\Pi_\eta$ [4] has no explicit momentum dependence beyond the implicit one arising from the dependence of $s$, Eq. (5), on $p_\eta$. We note that the peak structure observed in the figure for $\text{Im } F_{\eta N}$ may be ascribed to the $N^*(1535)$ resonance generated dynamically in this coupled-channel model. In-medium Pauli blocking (dot-dashed curves) shifts the resonance to higher energies, making it more pronounced. Implementing hadron self-energies (solid curves) spreads the resonance structure over a broad interval of energies, practically dissolving it in the nuclear medium. This behavior is different from that observed for the $\bar{K}N$ system where the hadron self-energies compensate to large extent for the effect of Pauli blocking and bring the peak structure back below the $\bar{K}$ threshold, resulting in strong in-medium attraction with little energy depen-
dence at subthreshold energies relevant for kaonic atoms and for $K^-$-nuclear bound states [16]. For the $\eta N$ system, in contrast, the in-medium amplitudes decrease rapidly in going to the subthreshold energies relevant for $\eta$-nuclear bound states and are weaker than the respective free-space amplitudes. In particular, the relatively large value of the free-space $\text{Re} a_{\eta N}$ is almost halved for nuclear matter density.

![Figure 3](image-url)

Figure 3: Effects of introducing self-energies on the real (left panel) and imaginary (right panel) parts of the Pauli-blocked $\eta N$ cm scattering amplitude generated in the NLO30$_\eta$ model of CS [8] for $\rho_0 = 0.17$ fm$^{-3}$. Dot-dashed curves: before adding self-energies. Self-energies are added sequentially to nucleons (dotted), to hyperons (short-dashed), to pions and kaons (long-dashed), and self-consistently to the $\eta$ meson (solid).

The sensitivity of the in-medium $\eta N$ cm scattering amplitude $F_{\eta N}$ to various SE insertions is demonstrated in Fig. 3. Dot-dashed curves show the in-medium Pauli-blocked amplitude, without any self-energy insertion. We then introduce successively self-energies due to nucleons, hyperons, mesons (excluding $\eta$), and finally in solid curves also the $\eta$ SE self-consistently. In all the cases displayed here the amplitudes decrease monotonically in going deeper below the $\eta N$ threshold. The effect of adding self-energies to the Pauli blocked amplitude is seen to be moderate at best. The resulting $\text{Re} F_{\eta N}$ is somewhat weaker than its Pauli-blocked counterpart. This is not the case for $\text{Im} F_{\eta N}$ around the $\eta N$ threshold, but its two versions (with and without self-energies) become equally weak about 20 MeV below threshold.
3. In-medium energy and density dependence

The methodology of calculating self-consistently binding energies and widths of $\eta$-nuclear states has been presented recently by FGM [7]. Two novelties of this study are the derivation of the $\eta$-nuclear potential from in-medium $\eta$-nucleon scattering amplitudes at subthreshold energies, and the introduction of Relativistic Mean Field (RMF) equations for nucleons that are solved dynamically along with the KG equation (3) for the $\eta$ meson, allowing thus for polarization of the nucleus by the bound meson. This approach was applied beforehand in analyses of kaonic atoms data [12] and in calculations of strongly bound $K^-$-nuclear states [18]. Here we outline the methodology of handling self-consistently in-medium $\eta N$ subthreshold scattering amplitudes $F_{\eta N}(\sqrt{s},\rho)$ for use in $\eta$-nuclear bound-state calculations.

We recall that the meson-baryon Mandelstam variable $s$ is given by

$$s = (m_\eta + m_N - B_N - B_\eta)^2 - (\vec{p}_\eta + \vec{p}_N)^2,$$

including a non-zero in-medium momentum dependent term that provides additional downward energy shift to that arising from the sum of binding energies ($B_\eta + B_N$). Near threshold, to leading order in binding and kinetic energies with respect to masses, one may approximate $\sqrt{s}$ by [7, 15, 16]

$$\sqrt{s} \approx m_\eta + m_N - B_N - B_\eta - \xi_N \frac{p_N^2}{2m_N} - \xi_\eta \frac{p_\eta^2}{2m_\eta},$$  \hspace{1cm} (15)

where $\xi_{N(\eta)} \equiv m_{N(\eta)}/(m_N + m_\eta)$. To transform momentum dependence into density dependence, the nucleon kinetic energy $p_N^2/(2m_N)$ is approximated within the Fermi gas model by $T_N(\rho/\rho_0)^{2/3}$, with average bound-nucleon kinetic energy $T_N = 23.0$ MeV at nuclear-matter density $\rho_0$. Furthermore, the $\eta$ kinetic energy $p_\eta^2/(2m_\eta)$ is substituted within the local density approximation by $-B_\eta - \text{Re} V_\eta(\sqrt{s},\rho)$. Hence, the in-medium $\sqrt{s} = m_\eta + m_N + \delta \sqrt{s}$ energy argument of $F_{\eta N}(\sqrt{s},\rho)$ in expression (4) for the self-energy exhibits explicit density dependence, with a form adjusted to respect the low-density limit, $\delta \sqrt{s} \to 0$ upon $\rho \to 0$, as used recently in $K^-$-atom studies [12]:

$$\delta \sqrt{s} \approx -B_N \frac{\rho}{\bar{\rho}} - \xi_N B_\eta \frac{\rho}{\rho_0} - \xi_N T_N \left(\frac{\rho}{\rho_0}\right)^{2/3} + \xi_\eta \text{Re} V_\eta(\sqrt{s},\rho).$$  \hspace{1cm} (16)

Here $B_N \approx 8.5$ MeV is an average nucleon binding energy and $\bar{\rho}$ is the average nuclear density. We note that in contrast with the assumption $p_\eta = 0$ made normally in nuclear matter calculations, $p_\eta \neq 0$ in finite nuclei which explains the origin of $V_\eta$ in expression (16). Furthermore, for an attractive $V_\eta$ and
as long as $\rho \neq 0$, the shift of the two-body energy away from threshold is negative definite, $\delta \sqrt{s} < 0$, even as $B_\eta \rightarrow 0$.

It is clear from Eq. (16) that $\sqrt{s}$ depends on Re $V_\eta(\sqrt{s}, \rho)$ which by Eq. (14) depends on $\sqrt{s}$. Therefore, for a given value of $B_\eta$, $V_\eta$ is determined self-consistently by iterating Eq. (16) with input from Eq. (14). Up to six iterations suffice for convergence. This is done at each radial point where $\rho$ is given and for each $B_\eta$ value during the calculation of bound states.

Figure 4: Downward energy shift as a function of the nuclear density, obtained self-consistently using models GW [3] and CS [8] for in-medium $\eta N$ scattering amplitudes in $1s_\eta$ bound state dynamical calculations for Ca (left panel) and comparison with static calculations of the $1s_\eta$ and $1p_\eta$ bound states in Ca within model GW (right panel).

The downward subthreshold energy shift $\delta \sqrt{s} \equiv E - E_{\text{th}}$ is plotted in Fig. 4 as a function of the nuclear density $\rho$ in Ca, evaluated self-consistently according to Eq. (16) for several different calculations. In the left panel of the figure we compare the correlation between $\delta \sqrt{s}$ and $\rho$ obtained by using the in-medium GW amplitudes $F_{\eta N}(\sqrt{s}, \rho)$ with that for two versions of the in-medium CS amplitudes, all for the $1s_\eta$ state in Ca in dynamical calculations. The GW curve was calculated using Eqs. (1) and (2), modifying thereby the FGM calculation that used $\kappa = 0$ in Eq. (2) and thus yielding up to 10 MeV larger energy shifts than shown in Fig. 2 of FGM [7]. The two CS versions account for Pauli blocking within a coupled-channel calculation, one version (Pauli) excludes and the other one (Pauli+SE) includes self-energies. It is seen that downward energy shifts ranging within $55 \pm 10$ MeV are correlated with nuclear central densities, and that the shift for the GW
model exceeds that for the CS model, reflecting the stronger real part of the free-space amplitude $F_{\eta N}(\sqrt{s})$ in the GW model. Among the two CS versions, somewhat larger values of the downward energy shift are obtained in the version without self-energies. This reflects the larger values of $\text{Re } F_{\eta N}(\sqrt{s}, \rho)$ generated, on average, at subthreshold energies and finite densities upon suppressing the hadron self-energies.

In the right panel of Fig. 4 we show similar results obtained for the GW in-medium amplitude within (i) a dynamical calculation of the $1s_\eta$ state in Ca (lowest curve, identical with the GW curve in the left panel) and within (ii) a static-density calculations of the $1s_\eta$ and $1p_\eta$ states in Ca. One observes that somewhat larger downward energy shifts are reached in the dynamical calculation, and that among the two static-density calculations larger downward energy shifts are obtained for the deeper bound state, $1s_\eta$. The difference of almost 10 MeV between the energy shifts at $\rho_0$ for these two GW static-density calculations is correlated with the slightly larger binding energy difference between the $1s_\eta$ and $1p_\eta$ states in Ca as demonstrated for dynamical calculations in Fig. 8 of the next section.

4. Results and discussion

![Figure 5: $1s_\eta$ binding energies in nuclei, calculated self-consistently and dynamically using in-medium $\eta N$ subthreshold scattering amplitudes constructed with (Pauli+SE) and without (Pauli) self energies in model NLO30$_\eta$ of CS [8]. Pauli blocking is included within full coupled-channel calculations.](image)
We have used the in-medium scattering amplitudes $F_{\eta N}(\sqrt{s}, \rho)$ evaluated in model NLO30$_{\eta}$, as outlined in Sect. \ref{sect:methodology} within dynamical calculations of $\eta$-nuclear binding energies and widths in several nuclei across the periodic table, as described in Sect. \ref{sect:results}. Calculated binding energies in this model, marked CS, are shown in Fig. \ref{fig:binding} for $1s_{\eta}$ nuclear states. The effect of including hadron self-energies (Pauli+SE) is demonstrated, resulting in 2–3 MeV lower binding energies than those calculated with Pauli blocking only. The present procedure of treating Pauli blocking within in-medium coupled channels gives binding energies larger by 0.5–0.7 MeV than those calculated using the multiple-scattering approach specified by Eqs. (1) and (2). Not shown in the figure are the remarkably small widths of about 2 MeV calculated for $1s_{\eta}$ nuclear states in model CS (these widths are shown below in Fig. \ref{fig:widths}).

![Figure 6: Real (left panel) and imaginary (right panel) parts of free-space and in-medium scattering amplitudes at the $\eta N$ subthreshold region in model NLO30$_{\eta}$ of CS \cite{8} and as used in the $\eta$-nuclear bound-state calculation of GR \cite{21}. Free-space amplitudes are marked by 0 within brackets, nuclear-matter amplitudes accounting for Pauli-blocking and self-energies are marked by $\rho_0$ within brackets.](image)

The only other available calculations of $\eta$-nuclear bound states in nuclei using a coupled-channel in-medium model that accounts for Pauli blocking and for self-energies are due to García-Recio (GR) et al. \cite{21}. These calculations are based on a coupled channels approach developed in Ref. \cite{10} and its in-medium implementation in Ref. \cite{20}. The GR underlying free-space and in-medium $\eta N$ subthreshold scattering amplitudes with Pauli blocking and self-energies are shown in Fig. \ref{fig:amp} compared to those of CS that are used in
the present work. We note that the GR in-medium real part is about 70% higher at $\rho_0$ than the free-space real part \cite{20}, contrary to what is found by us for medium modifications based on model NLO30$_{\eta}$ of CS \cite{8}. We have no explanation for this disagreement between the two approaches except to recall that they differ appreciably in some of the self-energies input, notably for pions and $\Sigma$ hyperons. A partial resolution is offered by reversing Re $V_0^\Sigma$ in the CS in-medium evaluation from the value $+30$ MeV listed in Table I to the unrealistic value $-30$ MeV assumed by Inoue and Oset \cite{20}. This increases appreciably Re $F^{CS}_{\eta N}(\sqrt{s}, \rho_0)$ so that it almost reaches the level of its free-space counterpart in the subthreshold region. Im $F^{CS}_{\eta N}(\sqrt{s}, \rho_0)$ too increases appreciably, exceeding substantially its free-space counterpart and reaching the level of Im $F^{GR}_{\eta N}(\sqrt{s}, \rho_0)$ as constructed in Ref. \cite{20}. (The case for repulsive $ReV_0^\Sigma$ is reviewed in Ref. \cite{29}.) We note furthermore that these differences in going from free-space amplitudes to in-medium amplitudes between GR and CS appear already at the level of imposing Pauli blocking without recourse to SEs. In the GR calculations Pauli blocking increases the $\eta N$ free-space attraction at threshold according to \cite{20}, whereas in our CS-based model calculations it decreases this attraction in remarkable agreement with applying the multiple-scattering modification of Eqs. (1) and (2).

Figure 7: Binding energies (left panel) and widths (right panel) of $1s_{\eta}$ bound states in nuclei calculated using the GR in-medium amplitudes \cite{21} with different procedures for handling self-consistently the subthreshold energy shift: $\delta\sqrt{s}$ stands for Eq. (16) and $-B_{\eta}$ stands for the procedure applied originally by GR. Shown also are results using NLO30$_{\eta}$ in-medium amplitudes marked CS \cite{8} with the present $\delta\sqrt{s}$ procedure Eq. (16).
In spite of the differences in the underlying models, it is instructive to apply our self-consistency scheme of calculating \( \eta \)-nuclear bound states, based on \( \delta \sqrt{s} \) of Eq. (16), to the GR in-medium energy-dependent and density-dependent \( \eta N \) interaction, and to compare the results with those obtained by GR using a density-independent \( \delta \sqrt{s} = -B_\eta \) self-consistency requirement. This comparison is made in Fig. 7 where the in-medium NLO30\( _\eta \) model results are included (denoted CS) using Eq. (16) for subthreshold energy values (marked \( \delta \sqrt{s} \) in the figure). The left and right panels exhibit \( 1s_\eta \)-nuclear binding energies and widths, respectively. All calculations include self-energies and coupled-channels evaluation of Pauli blocking.

Comparing binding-energy and width results obtained by applying different self-consistency procedures, as presented in Fig. 7, one sees that our \( \delta \sqrt{s} \) Eq. (16) procedure reduces considerably the GR binding energies and widths with respect to the original calculations that used a \( \delta \sqrt{s} = -B_\eta \) procedure. However, even the reduced GR widths are still quite high, 20 MeV and over, suggesting that \( \eta \)-nuclear states will be prohibitively difficult to resolve if the GR model is the physically correct one.

Considering the CS results one notes the remarkable smallness of the calculated widths shown on the right panel of Fig. 7, with values about 2 MeV. These very small widths do not include contributions from two-nucleon processes which are estimated to add a few MeV. We therefore anticipate that \( 1s_\eta \) and, wherever bound, also \( 1p_\eta \) nuclear states could in principle be observed if model NLO30\( _\eta \) turns out to prove a realistic model.

Finally, in Fig. 8 we compare \( \eta \)-nuclear single-particle spectra across the periodic table evaluated self-consistently using two in-medium models, GW [3] (left panel) and NLO30\( _\eta \) due to CS [8] (right panel). The free-space \( \eta N \) amplitudes for these models may be viewed in Fig. 1. These dynamical calculations include Pauli blocking, using Eqs. (11) and (2) for GW, and the coupled-channel approach discussed in Sect. 2 for CS. The latter model also incorporates in-medium hadron self-energies, resulting in 2–3 MeV lower binding energies (see Fig. 5). The widths calculated in both models are remarkably small, as shown in Fig. 7 for CS. For these two in-medium models \( \eta \)-nuclear single-particle bound states stand a chance of being observed, provided a suitable production/formation reaction is found. Other models studied by us produce either prohibitively large widths or are too weak to generate \( \eta \)-nuclear bound states over a substantial range of the periodic table.
Figure 8: Spectra of $\eta$-nuclear single-particle bound states across the periodic table, cal-
culated self-consistently using in-medium models of the $\eta N$ subthreshold scattering am-
plitude, are shown in the left panel for the GW model and in the right panel for the
NLO30$_{\eta}$ model of CS. Pauli blocking is included for both in-medium models, whereas
hadron self-energies are accounted for only in the CS-based calculations.

5. Summary and Outlook

We have extended in the present work the self-consistent calculations of $\eta$-
nuclear bound states reported recently in Ref. [7] by using $\eta N$ scattering am-
plitudes that follow from the chirally-inspired meson-baryon coupled-channel
model NLO30$_{\eta}$ [8]. Pauli blocking and hadron self-energies are accounted for
in the construction of in-medium amplitudes that serve as self-energy in-
put to the $\eta$-nuclear KG equation for bound states. These amplitudes are
both energy- and density-dependent, decreasing as one goes deeper into sub-
threshold for fixed density, as shown in Fig. 2. The in-medium subthreshold
amplitudes encountered in $\eta$-nuclear bound-state calculations are substan-
tially weaker both in their real part as well as in their imaginary part than
the $\eta N$ scattering length. We have displayed in Fig. 4 the correlation in
models CS [8] and GW [3] between the subthreshold energy downward shift
$\delta \sqrt{s}$ and the nuclear density $\rho$ implied by satisfying Eq. (16). The resulting
energy shifts of $-(55 \pm 10)$ MeV for central nuclear densities surpass consid-
erably the shift $\delta \sqrt{s} = -B_\eta$ used in other works [21, 22]. This is reflected
in our calculated bound-state energies and widths which are smaller than
those calculated in comparable models using $\delta \sqrt{s} = -B_\eta$, as shown in Fig. 7
here. It is safe to conclude that irrespective of the underlying two-body $\eta N$
interaction model, a self-consistent treatment which couples together the energy dependence of the in-medium $\eta N$ scattering amplitude below threshold and the density dependence of the $\eta$-nuclear self-energy is mandatory in any future calculation of $\eta$-nuclear bound states.

The small $\eta$-nuclear widths of 2–3 MeV calculated in the CS in-medium model, and also in the GW in-medium model, might encourage further experimental activity seeking to produce and identify $\eta$-nuclear bound states. These small widths, however, are model dependent, as evidenced by the substantially larger widths calculated in other models as displayed in Fig. 7. Additional width contributions disregarded in our in-medium model are due to two-pion production $\eta N \to \pi \pi N$ and two-nucleon absorption $\eta NN \to NN$. These contributions are estimated to add a few MeV to $\eta$-nuclear widths evaluated in meson-baryon coupled-channels approaches, so we feel it is safe to assume that the total $\eta$-nuclear widths in model NLO30$_{\eta}$ do not exceed 5–10 MeV. To appreciate the smallness of these estimated widths, we recall the semiclassical estimate $\Gamma_{\eta}^{QF} \approx v_{\eta} \sigma_{\eta N}^{abs} \rho_0$, where $v_{\eta} = p_{\eta}/E_{\eta}$ and $\sigma_{\eta N}^{abs} = (30 \pm 2.5_{\text{stat}} \pm 6_{\text{syst}})$ mb is the $\eta$-meson absorption cross section in nuclear matter as determined from near threshold photoproduction of quasi-free (QF) $\eta$ mesons on complex nuclei at MAMI [30]. Using the lowest $\eta$-meson kinetic energy, $T_{\eta} \approx 25$ MeV, at which this determination of $\sigma_{\eta N}^{abs}$ appears stable, one gets $\Gamma_{\eta}^{QF} \approx (29 \pm 6)$ MeV. Of course, given the rapid fall-off of $\text{Im} F_{\eta N} (\sqrt{s})$ as $\sqrt{s} \to \sqrt{s_{\eta N}}$, this semiclassical QF estimate cannot be reliably extrapolated down to threshold and would certainly break down in the $\eta$-nuclear subthreshold region where the calculations presented here are performed. We recall, furthermore, that only a single claim of observing $\eta$-nuclear bound states has been made to date, in the reaction $p + ^{27}\text{Al} \to ^{3}\text{He} + ^{25}\eta\text{Mg} \to ^{3}\text{He} + p + \pi^- + X$ as reported recently by the COSY-GEM collaboration [31]. The width extracted for the claimed peak is $\Gamma_{\eta}^{(25}\text{Mg}) = (10 \pm 3)$ MeV. For updated overview of past, present and future in $\eta$-nuclear experiments and theory, we refer to the recent symposium on Mesic Nuclei at Cracow, Sept. 2014 [32].

The subthreshold behavior of s-wave meson-baryon scattering amplitudes and its consequences for meson-nuclear bound states has been dealt by us extensively for $K^-$ mesons, see Refs. [33, 34] for recent reviews. Another meson-baryon system of interest is $\eta'(958)N$. The QCD connection between $\eta$- and $\eta'$-nuclear bound states has been highlighted recently by Bass and Thomas, emphasizing predictions of the QMC model [35]. Experimentally, a value of $\Gamma_{\eta'}(\rho_0) = (20 \pm 5)$ MeV for the in-medium $\eta'$-meson width was
derived from measured transparency ratios in $\eta'$ photoproduction on nuclei [36]. Very recently, a value of $V_{\eta'}(\rho_0)=-(37\pm10_{\text{stat}}\pm10_{\text{syst}})$ MeV for the real part of the $\eta'$-nuclear potential depth has been determined by measuring the $\eta'$-meson excitation function and momentum distribution in photoproduction on $^{12}$C [37]. It is worth noting however that the high-momentum $\eta'$ mesons produced in these photoproduction experiments, with $p_{\eta'} \sim 1\text{ GeV}/c$, are kinematically far away from the low-momentum range expected for meson-nuclear bound-state systems. Furthermore, the rather strong in-medium $\eta'N$ attraction and absorption derived from these experiments is at odds with the value $|a_{\eta'N}| \approx 0.1\text{ fm}$ derived from the near-threshold $pp \rightarrow pp\eta'$ reaction [38]. Dedicated experiments are planned to search for $\eta'$-nuclear bound states in $(\pi^+,p)$ or $(p,d)$ reactions [39, 40]. Yet, the issue of energy dependence of the $\eta'N$ scattering amplitude and its relevance for the calculation of $\eta'$-nuclear bound states has not been considered, and in view of the results reported here for $\eta$-nuclear bound states it is of considerable interest to follow in near-future work.

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