Analysis of linguistic terms of variables representing the wave of arterial diameter variation in radial arteries using fuzzy entropies

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Abstract. In this work we use 53 Arterial Diameter Variation (ADV) waves extracted from radial artery of normotense males, along with the values of variables that represent the ADV wave, obtained by means of multivariate analysis. Then, we specify the linguistic variables and the linguistic terms. The variables are fuzzified using triangular and trapezoidal fuzzy numbers. We analyze the fuzziness of the linguistic terms by applying discrete and continuous fuzzy entropies. Finally, we infer which variable presents the greatest disorder associated to the loss of arterial elasticity in radial artery.

1. Introduction
Illnesses with loss of arterial elasticity, such as atherosclerosis and blood hypertension, are the main cause of people deaths and disability. One of their characteristics is their asymptomatic nature during their onset. On these accounts, we recognize the importance of counting with a method to determine and evaluate in what stage of peoples’ life, the largest disorder in the state of arterial walls takes place. To approach this issue, we define seven variables that represent the ADV wave, obtained by means of multivariate analysis [1], while developing step-by-step a succession of basic definitions to relate the linguistic terms and the fuzzy numbers in order to fuzzify the variables that represent the ADV wave.

We also define the properties and the most important functions $g(t)$ in the fuzzy entropy. Starting from these premises, we calculate the fuzziness values of the intersection and union of the fuzzy numbers for each one of the ADV variables, by applying the discrete and continuous fuzzy entropies. Then, we interpret the results and determine which ADV variable has the largest fuzziness value.

2. Material and Methods
Using electronic equipment described in [2], we have obtained 53 ADV waves from normotense non-medicated patients as follows: 20 youths aged 14 to 18, 12 adults aged 41 to 59 and 21 male seniors aged 70 to 81. Each wave results from averaging eight waves whose amplitude is normalized on the ordinate axis to the maximum value of the wave, whereas time is referred on the abscissa.

Figure1 shows the different ordered pairs stated as normalized amplitude and time. We have defined fifty variables that completely identify the ADV waves [1].
The techniques of Principal Component Analysis and Multiple Regression [3] [4] [5] have been applied to the values of the ADV variables, obtaining the seven variables described below:

**TO.E:** Numeric variable \([t_e - t_o]\). Time period from the beginning \(t_0\), until the beginning of the descent slope of the systolic wave (DSSW).

**TD.E:** Numeric variable \([t_e - t_d]\). Time period from the ascent slope of the systolic wave (ASSW) changes until the beginning of DSSW.

**TO.P1OD:** Numeric variable \([t_{P1OD} - t_o]\). Time period from the beginning \(t_0\) until the end of DSSW.

**TO.V1OD:** Numeric variable \([t_{V1OD} - t_o]\). Time period from the beginning \(t_0\) until the time variable \(t_{V1OD}\).

**TO.P2OD:** Numeric variable \([t_{P2OD} - t_o]\). Time period time from the beginning \(t_0\) until the foot of the second diastolic wave \(t_{P2OD}\).

**TD.V20D:** Numeric variable \([t_{V20D} - t_d]\). Time difference between the point where the ascending slope of the systolic wave (ASSW) changes and the time variable \(t_{V20D}\).

**TE.F2ODP:** Percentage variable \([t_{CA} - t_{t_C} \%]\). It is the time variable \(TE.F2OD\) regarding the active cycle \(t_{CA}\).

![Graphical representation of ADV wave variables](image_url)

**Figure 1.** Graphical representation of the ADV wave variables.

Table 1 describes the minimum, medium and maximum values [1] for each variable of the ADV waves in youths, adults and seniors.
Table 1. Minimum medium and maximum values of the ADV wave variables.

| ADV Variables | \( a_1 \) | \( b_1 \) | \( c_1 \) | \( a_2 \) | \( b_2 \) | \( c_2 \) | \( a_3 \) | \( b_3 \) | \( c_3 \) |
|----------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
|                | msg min | msg med | msg max | msg min | msg med | msg max | msg min | msg med | msg max |
| TO.E           | 90     | 130    | 195    | 120    | 150    | 203    | 130    | 218    | 450    |
| TD.E           | 20     | 34     | 65     | 15     | 38     | 70     | 35     | 102    | 290    |
| TO.P1OD        | 0      | 33     | 200    | 0      | 127    | 235    | 170    | 226    | 380    |
| TO.V1OD        | 0      | 43     | 285    | 0      | 174    | 295    | 220    | 305    | 580    |
| TO.P2OD        | 0      | 312    | 355    | 205    | 32     | 375    | 245    | 372    | 680    |
| TD.V2OD        | 0      | 289    | 360    | 205    | 324    | 380    | 245    | 360    | 680    |
| TE.F2ODP       | 18     | 58     | 78     | 55     | 74     | 95     | 60     | 76     | 95     |

2.1. Basic definitions
Let \( X \) be a finite universe of objects represented by the symbols \( x_1, x_2, \ldots, x_n \). A fuzzy set \( A \) in \( X \) is a set of orderly pairs:

\[
A = \{ (x, \mu_A(x)) | x \in X \},
\]

\( \mu_A(x) \) is called the membership function or the membership grade of \( x \) in \( A \), and it applies \( X \) to the membership set \([0,1]\):

\[
\mu_A : X \to [0,1],
\]

The family of all the fuzzy and crisp sets in the universal set \( X \) is called the power set and is represented by \( F_m(X) \) and \( \mathcal{P}_m(X) \) [6],[7],[8],[9],[10].

Definition 1. Let \( \alpha \in [0,1] \). An \( \alpha \)-cuts of a fuzzy set \( A \) is a crisp set \( A_\alpha \) such that:

\[
A_\alpha = \{ x \in X | \mu_A(x) \geq \alpha \},
\]

which contains all the elements of the universal set \( X \), and \( \mu_A(x) \) has a membership grade which is equal or larger than \( \alpha \).

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Definition 2. A fuzzy set $A$ is convex, if and only if the sets $\alpha$-cuts ($A_{\alpha}$) are convex. That is to say:

$$\mu_A(\lambda x_1 + (1 - \lambda)x_2) \geq \min(\mu_A(x_1), \mu_A(x_2)), \quad x_1, x_2 \in X, \lambda \in [0,1]$$

Definition 3. The height of a fuzzy set $A$ is defined by the expression:

$$\text{height}(A) = \sup_{x \in X} \mu_A(x)$$

A fuzzy set is normalized if $\exists x \in X$ such that $\mu_A(x) = 1$.

2.1.1. Operations on fuzzy sets. Let there be two fuzzy sets $A, B$ in the universe of the discourse $X$, $\forall x \in X$; we define the following operations between fuzzy sets:

$$\mu_{A \cap B}(x) = \mu_A(x) \land \mu_B(x) \quad \land = \min$$

$$\mu_{A \cup B}(x) = \mu_A(x) \lor \mu_B(x) \quad \lor = \max$$

2.1.2. Fuzzy numbers. A convex and normalized fuzzy set defined on the real line $\mathbb{R}$ whose membership function is continuous and each $\alpha$-cut is a closed interval is called fuzzy number. That is, a fuzzy number $A$ is a fuzzy set that fulfills the following properties:

$$\mu_A(\lambda x_1 + (1 - \lambda)x_2) \geq \min(\mu_A(x_1), \mu_A(x_2)), \quad x_1, x_2 \in \mathbb{R}, \lambda \in [0,1]$$

$$\mu_A(x) = 1, \quad \text{for } x \in \mathbb{R}.$$  

$A_{\alpha} = \{x \in \mathbb{R} / \mu_A(x) \geq \alpha\}$ is a closed set.

The fuzzy sets that we use are: triangular fuzzy number (2) and the trapezoidal fuzzy numbers expressed by functions (1) and (3).

2.1.3. Linguistic variables. A linguistic variable is a variable whose values are words or sentences in the artificial or natural language. In this way, we define a linguistic variable $\nu$ characterized by a quintuple, $(\nu, T(\nu), X, G, M)$ where $\nu$ is the name of the linguistic variable; $T(\nu)$ is the set of terms of the linguistic values of $\nu$; $X$ is the universe of the discourse and $x$ is the base variable whose numeric value is the value of the linguistic variable; $G$ is a syntactic rule that generates the terms $T(\nu)$, and $M$ is a semantic rule that associates each value of the linguistic variable with its meaning and is a fuzzy set of $X$.

2.1.4. Fuzzification of the ADV wave variables. In order to fuzzyfy each one of the ADV wave variables, we use the linguistic variable "Time", and its linguistic terms are: small, medium, big. The quantification of this linguistic variable "Time" is carried out in each one of the universes of the discourse $X_1,...,X_7$. Consequently we use the fuzzy numbers: trapezoidal for the linguistic terms "small", "big" and the triangular fuzzy number for the linguistic term "medium" and we express them in the following way:

$$\mu_{\text{SMALL}}(x) = \begin{cases} 
0 & 0 \leq x \leq b_1 \\
\frac{c_1 - x}{c_1 - b_1} & b_1 \leq x \leq c_1 \\
0 & c_1 \leq x 
\end{cases}$$

(1)
When replacing the values of the variables of table 1 in the fuzzy numbers "small" "medium" and "big" we obtain the fuzzyfication of each one of the variables of the ADV wave.

2.2. Analysis of the fuzzy entropy for triangular and trapezoidal membership functions.
We divide the analysis of the fuzzy entropies in two parts: in the first part we specify the properties of the discrete entropy and in the second part we apply the Theorem of Existence [6] to the continuous functions that verify the properties defined in the first part.

The difference in our analyses when applying each one of the fuzzy entropies is as follows:

a) The values of the discrete entropy when fuzzifying each one of the variables of the ADV wave depend on the wave number that we use.

b) When applying the continuous entropies under the same conditions that the point a), we obtain the maximum values of the fuzzy entropies.

2.2.1. Discrete entropy, properties. To analyze the discrete entropies and their properties, we define that X is a finite universe and represented as 

\[ X = \{x_1, x_2, \ldots, x_m\} \]

with \( \log \), the natural logarithm, as it is well-known.

Definition 4. A real function \( H : F_m(X) \rightarrow \mathbb{R}^+ \) is called entropy, if it fulfills the following properties:

HP1: \( H(D) = 0 \) \( \forall D \in \phi_m(X) \)

HP2: \( H([0, 0]) = \max_{A \in F_m(X)} H(A) \)

HP3: If \( A^\alpha \) is a sharper version of \( A \), then \( H(A^\alpha) \leq H(A) \).

HP4: \( H(A^\cap) = H(A) \) \( \forall A \in F_m(X) \)

HP5: \( H(A \cap B) + H(A \cup B) = H(A) + H(B) \)

The fuzzy entropy \( H \) has the form \( H(A) = \sum_{i=1}^{m} g(\mu_A(x_i)) \). The most important \( g(t) \) functions and the properties they verify [9][10][11] are detailed as follows:

a) De Luca y Termini. The function \( g(t) = -(t \log t + (1-t) \log (1-t)) \), \( \forall t \in [0, 1] \), fulfills the properties HP1-HP5.

b) Jumarie. The function \( g(t) = -t \log t \), \( \forall t \in [0, 1] \) does not fulfill the properties HP2-HP4.

c) Pal-Pal. The function \( g(t) = -t e^{-t} + (1-t)e^t \), \( \forall t \in [0, 1] \) does not fulfill the property HP1.

d) Bhandari y Pal. The function \( g(t) = \frac{1}{1-\alpha} \log \left( t^\alpha + (1-t)^\alpha \right) \), \( \alpha < 1 \), \( \forall t \in [0, 1] \), fulfills the properties HP1-HP4.
e) The Pal-Bezdek multiplicative function, \( g(t) = -t (1-t) \), \( \forall t \in [0,1] \) fulfills the properties HP1-HP4.

f) The Pal-Bezdek additive function, \( g(t) = -4 t (1-t) \), \( \forall t \in [0,1] \), fulfills the properties HP1-HP4.

2.2.2. The continuous entropy. In the analysis of the continuous entropy for triangular membership functions we use the theorem developed by Knopchamer [6] for continuous entropy. The expression that measures the continuous entropy is as follows:

\[
H(A) = \frac{1}{\pi(X)} \int g(A(x)) \, d\pi(x), \quad \mu_A : X \rightarrow [0,1] \text{ and } g : [0,1] \rightarrow [0,1], \quad A \in F(X)
\]  

(4)

where \( F(X) \) is the set of all fuzzy sets and \( \pi(X) \) is the measure on the universe of the discourse \( X \). In this case, the universe of the discourse \( X \) that we use is in the form \( X = [0,h] \) and, thus, we take it as the measure \( \pi(X) = h \).

When selecting the continuous functions \( g(t), \forall t \in [0,1] \) we use the properties HP1-HP4. Now we express the properties for different \( g(t) \) continuous functions that we have used to obtain the discrete entropy.

The only continuous functions that fulfill the properties HP1-HP4 are a, d, e, f, and, by the Theorem of the Existence of Knopfmacher [6], the corresponding continuous entropy fulfills the properties HP1-HP5 which is our requirement for continuous entropy. When solving such integrals using the different expressions of \( g(t) \), we can prove that the results, when using the cases d, e, f, cannot be easily expressed, so we just use the De Luca y Termini function of \( g(t) \). Then, letting \( A, B \in F(X) \), we first obtain the continuous entropy of the fuzzy sets \( H(A), H(B) \) and, second, the continuous entropy of the intersection of two triangular fuzzy sets \( H(A \cap B) \); finally, the continuous entropy of the union of two triangular fuzzy sets \( H(A \cup B) \) using the property HP5. Thus, we define the universe of discourse and the fuzzy sets \( A, B \) as well as their intersection \( A \cap B \).

Let there be an universe of discourse \( X = [0,h] \) and the triangular fuzzy set \( A \in F(X), A \subset X \) as follows:

\[
A = \left\{ x \leq a, \frac{x-a}{b_1-a_1} ; x \in [a_1,b_1] \bigg| \frac{c_1-x}{c_1-b_1} ; x \in [b_1,c_1], 0 : h \geq c_1 \right\}
\]

\[
B = \left\{ x \leq a_2, \frac{x-a_2}{b_2-a_2} ; x \in [a_2,b_2] \bigg| \frac{c_2-x}{c_2-b_2} ; x \in [b_2,c_2], 0 : x \geq c_2 \right\}
\]

(5)

with \( k = \max\{\min(\mu_A(x), \mu_B(x))\} \).

The continuous entropy of the fuzzy sets \( A,B \) is:

\[
H(A) = \frac{1}{\pi(X)} \int g(A) \, d\pi(x) = \frac{1}{2h \log(2)} (c_1 - a_1) \quad \text{and} \quad H(B) = \frac{1}{\pi(X)} \int g(B) \, d\pi(x) = \frac{1}{2h \log(2)} (c_2 - a_2)
\]

(6)

(7)
We obtain the continuous entropy of the intersection of two triangular fuzzy sets as follows:

\[
H(A \cap B) = \frac{1}{\pi(X)} \int g(A \cap B) \, d\pi(x) = \frac{K}{2 \log(2)} (c_1 - a_2) \tag{8}
\]

being \( K = \left( k \log(1-k) + 1-k \log k - 2 \log(1-k) + \frac{1}{k} \log(1-k) \right) \), when using the property HP5, we obtain the fuzzy entropy of the two fuzzy numbers union in the following way:

\[
H(A \cup B) + H(A \cap B) = H(A) + H(B),
\]

\[
H(A \cup B) = H(A) + H(B) - H(A \cap B) \tag{9}
\]

When replacing (6),(7) and (8) in (9), we have:

\[
H(A \cup B) = \frac{1}{2 \log(2)} (c_1 - a_1) + \frac{1}{2 \log(2)} (c_2 - a_2) - \frac{K}{2 \log(2)} (c_1 - a_2)
\]

\[
H(A \cup B) = \frac{1}{2 \log(2)} (c_1 - a_2)(1-K) + (c_2 - a_1) \tag{10}
\]

To obtain the fuzzy entropy of the fuzzy numbers \( \{A \cup C\} \) and \( \{B \cup C\} \) we apply the expressions (6)-(10), with the changes of corresponding parameters; see table 1.

3. Results and Conclusions

We express only the values of discrete and continuous fuzzy entropy for different conditions of the De Luca and Termini function \( g(t) \) that fulfills the properties HP1-HP5, so as to reduce the number of tables and to ease their analysis.

Being the linguistic terms: small, medium and big represented by the fuzzy sets \( A, B \) and \( C \), we obtain the following values for the discrete fuzzy entropy: the fuzzy entropy of the intersection and union between the fuzzy sets \( \{A \cup B\}, \{A \cup C\}, \{B \cup C\} \), for each variable of the ADV wave.

Using the continuous entropy we obtain the values of the entropy of the fuzzy sets \( A, B \) and \( C \), the values of the constant \( K \) and the intersection and union between the fuzzy sets \( \{A \cup B\}, \{A \cup C\}, \{B \cup C\} \) for each one of the variables of the ADV wave; see table 2.

| Variables | \( H(A \cap B) \) | \( H(A \cup B) \) | \( H(A \cap C) \) | \( H(A \cup C) \) | \( H(B \cap C) \) | \( H(B \cup C) \) |
|-----------|----------------|
| TO.E      | 0.1318         | 0.1062         | 0.0969         | 0.1461         | 0.1221         | 0.1529         |
| TD.E      | 0.1263         | 0.0787         | 0.0560         | 0.1780         | 0.0713         | 0.2197         |
| TO.P1OD   | 0.3924         | 0.3296         | 0.0251         | 0.3794         | 0.1055         | 0.4165         |
| TO.V1OD   | 0.3735         | 0.2705         | 0.0472         | 0.3443         | 0.0778         | 0.3782         |
| TO.P2OD   | 0.1634         | 0.0546         | 0.1223         | 0.0517         | 0.1470         | 0.1570         |
| TD.V2OD   | 0.1747         | 0.0783         | 0.1266         | 0.0644         | 0.1527         | 0.1403         |
| TE.F2ODP  | 0.1729         | 0.2531         | 0.1278         | 0.1272         | 0.2600         | 0.1370         |

Table 3 expresses numerically the values for continuous entropy of fuzzy sets \( A, B \) and \( C \):
Table 3. Continuous fuzzy entropy of fuzzy sets A, B and C.

| Variables | $H(A)$ | $H(B)$ | $H(C)$ |
|-----------|--------|--------|--------|
| TO.E      | 0.1042 | 0.2080 | 0.1410 |
| TD.E      | 0.0771 | 0.1368 | 0.1666 |
| TO.P1OD   | 0.3170 | 0.4461 | 0.1063 |
| TO.V1OD   | 0.3000 | 0.3669 | 0.1057 |
| TO.P2OD   | 0.0456 | 0.1803 | 0.1347 |
| TD.V2OD   | 0.0753 | 0.1856 | 0.1220 |
| TE.F2ODP  | 0.1518 | 0.3037 | 0.1214 |

The values for $K$ detailed in table 4 below are obtained through the integral of the continuous entropy of the intersections between fuzzy sets A, B and C.

Table 4. The values of $K$.

| Variables | $H(A \cap B) / K_{12} / 2\log(2)$ | $H(A \cap C) / K_{13} / 2\log(2)$ | $H(B \cap C) / K_{23} / 2\log(2)$ |
|-----------|---------------------------------|---------------------------------|---------------------------------|
| TO.E      | 0.7932                          | 0.6699                          | 0.7335                          |
| TD.E      | 0.7667                          | 0.5601                          | 0.6056                          |
| TO.P1OD   | 0.7863                          | 0.3308                          | 0.6485                          |
| TO.V1OD   | 0.7875                          | 0.4361                          | 0.6236                          |
| TO.P2OD   | 0.7653                          | 0.7796                          | 0.7930                          |
| TD.V2OD   | 0.7905                          | 0.7719                          | 0.7932                          |
| TE.F2ODP  | 0.7586                          | 0.7214                          | 0.7576                          |

The values obtained for continuous entropy of the intersection and the union between the fuzzy sets A, B and C are detailed in the following table:

Table 5. De Luca y Termini continuous fuzzy entropy

| Variables | $H(A \cap B)$ | $H(A \cup B)$ | $H(A \cap C)$ | $H(A \cup C)$ | $H(B \cap C)$ | $H(B \cup C)$ |
|-----------|---------------|---------------|---------------|---------------|---------------|---------------|
| TO.E      | 0.1322        | 0.1804        | 0.0969        | 0.1493        | 0.1956        | 0.1538        |
| TD.E      | 0.1321        | 0.0817        | 0.0588        | 0.1858        | 0.0739        | 0.2303        |
| TO.P1OD   | 0.4138        | 0.3493        | 0.0264        | 0.3972        | 0.1109        | 0.4415        |
| TO.V1OD   | 0.3869        | 0.2899        | 0.0488        | 0.3578        | 0.0806        | 0.3920        |
| TO.P2OD   | 0.1688        | 0.0572        | 0.1261        | 0.0542        | 0.1516        | 0.1666        |
| TD.V2OD   | 0.1812        | 0.0797        | 0.1305        | 0.0667        | 0.1578        | 0.1501        |
| TE.F2ODP  | 0.1729        | 0.2719        | 0.1366        | 0.1366        | 0.2791        | 0.1461        |
As it is well known, the heart-rate period has an average value of 1 seg. The values of the variables that represent the ADV wave in youths, adults and seniors are included in the heart-rate period.

In this way, we have analyzed the fuzziness of the seven variables in two steps: in the first one, we have calculated the fuzziness of this variable using discrete entropy, thus obtaining the values of entropy of the ADV waves used in the work. In the second step, we have applied the continuous entropy, as an approach to the universal group of normotense patients. The only function that could be expressed in a simple way is the De Luca and Termini function. We have presented only the table corresponding to the De Luca y Termini function in its two forms and, thus, we have been able to compare both results.

The values of the discrete entropy of the union and the intersection of the fuzzy numbers: small, medium and big are smaller than the values obtained using the continuous entropy, as it was expected and pointed out in points a) and b), respectively.

In the variable time \( TO.PIOD \), we have the biggest value in the fuzzy entropy of the union between the fuzzy numbers B and C, \( H(B \cup C) \), being "medium" and "big" their linguistic terms. This value of fuzzy entropy does not contribute to any conclusion to evaluate arterial elasticity.

When analyzing the fuzzy entropy of the intersection between the fuzzy numbers in all the variables, we have found that the variable time \( TO.PIOD \) has the biggest fuzziness expressed by the fuzzy entropy, being the fuzzy numbers A and B, the quantification of the small-medium linguistic terms that correspond to the group of seniors-adults patients. This result means that, as the age of patients increases, the variable \( TO.PIOD \) decreases its value. Therefore, the systolic wave gets wider, due to the increase of pulse wave propagation speed [5].

We conclude that the group of seniors-adults patients is the one that presents the greatest change of arterial elasticity in radial artery.

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