Shapiro step at nonequilibrium conditions

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Abstract - Detailed numerical simulations of intrinsic Josephson junctions of high-temperature superconductors under external electromagnetic radiation are performed taking into account a charge imbalance effect. We demonstrate that the charge imbalance is responsible for a slope in the Shapiro step in the IV-characteristic. The value of slope increases with a nonequilibrium parameter. Coupling between junctions leads to the distribution of the slope’s values along the stack. The nonperiodic boundary conditions shift the Shapiro step from the canonical position determined by $V_{ss} = \hbar f / (2e)$, where $f$ is a frequency of external radiation. This fact makes the interpretation of the experimentally found Shapiro step shift by the charge imbalance effect ambiguous.

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The nonequilibrium effects created by stationary current injection in layered superconducting materials have been studied very intensively in recent years [1–7]. Actually, due to the fact that charge does not screen in the superconducting layers a formed system of intrinsic Josephson junctions (IJJ) in high-temperature superconductors cannot be in the equilibrium state at any value of the electrical current [8,9]. The influence of charge coupling on Josephson plasma oscillations was stressed in refs. [6,8]. However, the charge imbalance in the systematic perturbation theory is considered only indirectly as far as it is induced by fluctuations of the scalar potential [1,2,5]. In ref. [10], it is taken into account as an independent degree of freedom and, therefore, the results are different from those of earlier treatments.

Clear experimental evidence of the nonequilibrium effects in IJJ, which were explained by the charge imbalance in the superconducting layers produced by the quasiparticle current, was observed in ref. [11]. The experiments were based on the idea that a bias current generates charge accumulation on the layers between resistive and superconducting junctions. The current through a resistive junction is carried mostly by quasiparticles, while the current through a barrier in the superconducting state is carried by Cooper pairs. This leads to charge fluctuations of the superconducting condensate in S-layers, which can be expressed by a shift of the chemical potential of the condensate and the charge imbalance between electron-like and hole-like quasiparticles. The authors of ref. [11] observed experimentally a shift of the Shapiro step (SS) voltage from the canonical value $V_{ss} = \hbar f / (2e)$ for the single mesa structures. They also detected an influence of the current through one mesa on the voltage measured on the other one in the double mesa structures. The results were also explained by the charge imbalance effect.

The answer to the question how strong are the nonequilibrium effects in the real system, is very important for different applications. Here we suggest a way to answer it. A study of the nonequilibrium effects created by current injections in the coupled system of Josephson junctions was presented in ref. [12] and it was shown how the disequilibrium parameter affects the branch structure of IV-characteristic at different values of coupling and Mc-Cumber parameters. However, the effect of external radiation on the charge imbalance was not taken into account. In the present paper, we study the nonequilibrium effects created by current injection in a stack of IJJ under external electromagnetic radiation and demonstrate the specific behavior of the Shapiro step. The IV-characteristics of IJJ are numerically calculated using the resistively and
Fig. 1: (Color online) Layered system of $N+1$ superconducting layers forms a stack of Josephson junctions. Since the 0-th and $N$-th layers are in contact with normal metal, their thicknesses $d_0^N$ and $d_{N+1}^N$ are different from the thickness of the other S-layers $d_s$ inside of the stack due to the proximity effect.

capacitively shunted junction model. The model takes into account the coupling between the layers and the quasiparticle charge imbalance effect [8,12]. We solve numerically a full set of equations which include the first-order differential equations for phase differences, generalized Josephson relations and the kinetic equations. A slope of SS is demonstrated. Its value depends on the value of the nonequilibrium parameter $\eta$. The effect of boundary condition based on the proximity effect on the SS is discussed.

We obtain the branch structure of IV-characteristics and investigate the SS at different boundary conditions and nonequilibrium conditions. A system of $N+1$ superconducting layers (S-layers) presented in fig. 1 is characterized by the order parameter $\Delta_i(t) = s|\psi(\theta_i(t))|$ and time-dependent phase $\theta_i(t)$, where $l = 0, 1, 2, \ldots, N$ is a number of IJJ in the stack. The phase dynamics of the stack is described by a gauge-invariant phase difference between the S-layers $\phi_i(t) \equiv \phi_{l+1,l}(t) = \theta_l(t) - \theta_{l+1}(t) - \frac{2\pi l}{N} \int_{l-1}^{l} dz A_s(z, t)$, where $A_s(z, t)$ is the vector potential in the barrier [10]. The thickness of the S-layer is comparable with the Debye screening length that leads to the generalized Josephson relation

$$\frac{d\psi_i(t)}{dt} = \frac{2e}{\hbar} \left( V_i(t) + \Phi_i(t) - \Phi_{i-1}(t) \right) \quad (1)$$

with a voltage $V_i$ between the layers $l-1$ and $l$, $V_i(t) \equiv V_{l,l-1}(t) = \int_{l-1}^{l} dz E_s(z, t)$ and the gauge-invariant scalar potential $\Phi_i(t)$ of the S-layer $\Phi_i(t) = \phi_i(t) - \frac{2\pi i}{N} \theta_i$, where $\phi_i(t)$ is the electrical scalar potential and the dot indicates the time derivative of the variable. In contrast with the usual Josephson relation, the frequency of the Josephson oscillations is determined by $V_i$ and $\Phi_i - \Phi_{i-1}$. So, in the nonequilibrium case the total energy $\hbar \omega(t)$ required to transfer a Cooper pair from the $l-1$ to $l$ is different from the equilibrium case by $\Phi_i - \Phi_{i-1}$ [10].

The nonperiodic boundary conditions (BCs) are characterized by the parameter $\gamma$ and, as we see below, the equations for the first and the last S-layers are different from the equation for the middle S-layer [8,13]. The total current density $J_{l-1,l} \equiv J_l$ through each S-layer is given as a sum of displacement, superconducting, quasiparticle and diffusion terms:

$$J_l = C \frac{dV_l}{dt} + J_c \sin \phi_i + \frac{\hbar}{2eR} \dot{\phi}_i + \frac{\Psi_{i-1} - \Psi_i}{R}, \quad (2)$$

where $C$ is the capacitance, $J_c$ is the critical current density, and $R$ is the junction resistance. This equation together with (1) and kinetic equations for $\Psi_i$

$$\frac{\partial \Psi_i}{\partial t} = \frac{4\pi^2}{d_s^2} (J_{l+1}^p - J_{l-1}^p) - \frac{\dot{\psi}_i}{\tau_{np}} \quad (3)$$

describe the physics of IJJs in HTSC. In formula (3), $\tau_{np}$ is the Debye length, $d_s^l$ is the thickness of the $S$-layers, and $\tau_{np}$ is the quasiparticle relaxation time.

In the normalized form the system of equations is

$$\dot{\psi}_l = \left[ I - \sin \phi_l - \beta \dot{\phi}_l + A \sin \omega t + I_{noise} \right] + \psi_l - \psi_{l-1}, \quad (4)$$

$$\dot{\phi}_l = (1 + 2\alpha)v_l - \alpha (\phi_{l-1} - \phi_{l+1}) + \frac{\psi_l - \psi_{l-1}}{\beta}, \quad (5)$$

$$\dot{\psi}_N = v_N - \alpha (\psi_{N-1} - (1 + \gamma) \psi_N) + \frac{\psi_N - \psi_{N-1}}{\beta}, \quad (6)$$

$$\zeta_0 \dot{\psi}_0 = \eta_0 (I - \beta \phi_{01} + \psi_0 - \psi_0), \quad (7)$$

$$\zeta_0 \dot{\psi}_l = \eta_l (\beta [\phi_{l+1,l} - \phi_{l+1,l}] + \psi_{l+1} - \psi_{l+1} - 2\psi_l - \psi_l), \quad (8)$$

$$\zeta_N \dot{\psi}_N = \eta_N (-I + \beta \phi_{N-1,N} + \psi_{N-1} - \psi_N), \quad (9)$$

where the dot shows a derivative with respect to $\tau = \omega \tau_p$, $I = J/J_c$ is the dimensionless current density, $\omega_p = \sqrt{2e\gamma} \sqrt{\hbar^c}$ is plasma frequency and $\alpha = \epsilon \bar{\epsilon}/2\epsilon^2 N(0)\bar{\epsilon}$ is the coupling parameter, $\epsilon$ is the dielectric constant, $\bar{\epsilon}$ is the vacuum permittivity, $d$ is the distance between the superconducting layers and $N(0)$ is the density of states. Other dimensionless parameters are the dissipation parameter $\beta = \frac{\bar{\epsilon} \omega}{2eR}$, the normalized quasiparticle relaxation time $\zeta = \omega \tau_{np}$ and the nonequilibrium parameter $\eta = \frac{4\pi^2}{d_s^2} \tau_{np}$. The parameter of the nonperiodic boundary conditions $\gamma$ is $\gamma = \frac{d_{N+1}}{d_s} = \frac{d_{0}}{d_s}$. The term $A \sin \omega t$ introduces the effect of external radiation with amplitude $A$ and frequency $\omega$, which are normalized to $J_c$ and $\omega_p$, respectively. To reflect the experimental situation, we have added the noise $I_{noise}$ in the bias current with the amplitude $\sim 10^{-8}$ which is produced by a random number generator and its amplitude is normalized to the critical current density value $J_c$. The effect of the noise is important for branching the IV-characteristics of the coupled Josephson junctions. The noise in current helps create the longitudinal plasma wave in the stack. However, for described features of the Shapiro step in the outermost branch its effect is not essential and the particular value of the noise is not very important.

This system of equations is solved numerically using the fourth-order Runge-Kutta method. We assume here that the nonequilibrium parameters for all the S-layers are the
same (i.e., \( \eta_0 = \eta = \eta_N = \eta \)). We consider the under-damped case with the McCumber parameter \( \beta_c = 25 \) or \( \beta = 0.2 \). In our simulations, we use the external radiation frequency \( \omega = 6 \) to have SS on the outermost branch where all JJJs are in the rotating state, and we put the amplitude \( A = 1.6 \) for a clear manifestation of the SSs. The method of simulations is described in detail in ref. [14].

The coupled Josephson junctions at the nonequilibrium conditions are described by IV-characteristic with intensive branching near the critical current and in the hysteresis region, related to the transitions between the rotating and oscillating states of junctions in the stack [13,15,16]. External radiation leads to the appearance of the Shapiro steps in the IV-curve and a decrease in the hysteresis. In our study, we choose the values of radiation frequency \( \omega = 6 \) and its amplitude \( A = 1.6 \) to have a clear manifestation of the SS on the outermost branch in the middle of the hysteresis region. We consider here the under-damped junctions with the dissipation parameter \( \beta_c = 0.2 \) and use the nonperiodic boundary conditions to reflect a proximity effect on the boundary between the normal electrodes and superconducting layers. The nonperiodic boundary conditions are determined by the parameter \( \gamma \) which demonstrates an effective change of the superconducting layer thickness near the electrode. The simulated IV-characteristics of JJ stacks in the case without the charge imbalance \( \eta = 0 \) (solid line) and at \( \eta = 0.6 \) (dashed line) are presented in fig. 2. The simulations have been made for the stacks with five JJJs, coupling parameter \( \alpha = 0.5 \) and \( \gamma = 0.5 \). The IV-curve without the charge imbalance at periodic boundary conditions are shown as well. We see that the position of the SS (dashed line) corresponds to the canonical value of the SS voltage \( V = 30 \) in agreement with the value of external frequency \( \omega = 6 \) and a number of junctions in the stack \( N = 5 \). The nonperiodic boundary conditions with \( \gamma \neq 0 \) shift the outermost branch relatively to the curve of CCJJ+DC model (capacitively coupled Josephson junction model with diffusion current), leading to the corresponding shift of the Shapiro steps. The charge imbalance manifests itself as appearance of the slope in the Shapiro step, which is clearly demonstrated in the inset for the case \( \eta = 0.6 \). Figure 3(a) shows that the slope of the Shapiro step \( \delta = \Delta V / \Delta I \) increases with \( \eta \), while it is absent in the case of \( \eta = 0 \). The slope as a function of \( \eta \) shown in fig. 3(b) demonstrates a monotonic dependence. Fitting of the simulated data (solid line) gives \( \delta = 8.18664 \eta - 1.93047 \eta^2 \). Using the fact that the Dabey screening length in high-temperature superconductors like BSCCO is comparable to the thickness of the S-layer \( (\eta_D \approx d_N) \) we find \( \eta = 4 \eta D T_0 / R \). It makes possible to estimate the relaxation time for the quasiparticles based on the fitting results.

There is another interesting feature of the Shapiro step at the nonequilibrium conditions: the SS slope in the IV-characteristics of each JJ of the stack can have a different value. The enlarged parts of the IV-characteristics with the SS for all JJJs in the stack for the case \( N = 5 \) and \( \eta = 0.6 \) are shown in fig. 4(a). Additionally to the corresponding shift due to the nonperiodic BCs, we see here that the SS of JJ in the middle of the stack has a minimal slope. This feature is related to the fact that the charge imbalance potential at the boundary of the stack has the highest value. The distribution of the SS slope along the stacks with 5 and 10 JJJs is demonstrated in fig. 4(b). It
shows that with an increase in the number of junctions in the stack, the effect of the charge imbalance on the SS of the middle junctions is getting weaker. Particularly, the slope of the second and third JJs in the stack with ten junctions is smaller than in case of $N = 5$. Based on this result, it is natural to expect that in the stack with a large number of JJs the middle ones have practically no slope.

Actually, a finite slope of the Shapiro steps in the IV-characteristics of intrinsic JJs is manifested in some experimental results. Particularly, in ref. [17] the authors explained it as a manifestation of the phase-diffusion effect. According to our presented results, the slope of the SS might be related to the charge imbalance effect. We note also that the width of the SS increases with the nonequilibrium parameter, as we can see also in the upper inset to fig. 2.

Let us now discuss shortly an experimental testing of the predicted charge imbalance manifestations in the intrinsic Josephson junctions. To estimate the corresponding values of the parameters, we present the existing experimental data [18–21] in table 1. Particularly, we can estimate the tunneling frequency $\nu$ for the quasiparticles [18] based on the formula $\nu = \frac{I_c^{(0)}}{2\pi e\Delta N^{(0)} d}$, which gives $\nu \approx 1.24 \times 10^9 \text{s}^{-1}$. It allows one to find the nonequilibrium parameter $\eta = \nu \tau_{qp} = 0.375$. In principal, the nonequilibrium parameter can be larger at the corresponding choice of the parameter presented in the table. For the normalized relaxation time we have obtained $\xi = \omega_p \tau_{qp} \sim 0.3$ at $\omega_p \sim 1 \text{GHz}$. So, we expect that the experimental IV-characteristics would clearly demonstrate a slope and the shift of the Shapiro step.

The effect of the charge imbalance on the Shapiro step in the first branch of the IV-characteristic was experimentally studied in ref. [11]. The observed Shapiro step shift from the canonical value was explained by the charge imbalance in the superconducting layer close to the normal electrode, where the corresponding JJ was in the resistive state. Taking into account the contact voltage between the normal electrode and the first S-layer [11,22], the authors have determined a new position $V_{ss}^*$ of the SS by

$$V_{ss}^* = \frac{\hbar \omega_p}{2e} - \delta V,$$

where $\delta V = J \tau_{qp}/(2e^2 N(0))$. The theory of the stationary charge imbalance effect was used for the explanation of the experimental results when the charge imbalance potential on the S-layer was determined by

$$\Psi_n = \frac{\tau_{qp}}{2e^2 N^{(0)}} (\tau_{qp} - J^n_l),$$

(12)

We note that based on the value of the SS shift, we can determine the relaxation time for the quasiparticles as

$$\tau_{qp} = \delta V 2e^2 N^{(0)} / I,$$

(13)

where $A$ is the area of the mesa and $I$ is the biased current.

In general, the intrinsic JJs are in the nonstationary state at any value of the bias current [9], and the effect of the charge imbalance on the SS in this case is not investigated yet. The dynamics of the quasiparticle potential is now determined by the kinetic eqs. (7)–(10) instead of eq. (12). As we have demonstrated, the nonstationary charge imbalance leads to a slope in the SS. The slope and the width of the SS depend on the value of the nonequilibrium parameter. Probably, the slope of SSs is manifested in the experimental results of ref. [17], but the authors explained it as a result of phase diffusion. Also, we can see a small slanting in the results of ref. [11] (see fig. 3 there). The answer to the question how strong the nonequilibrium effects are in the system can be obtained by measurements of the Shapiro steps slope.

We note the importance of the role of boundary and proximity effects in intrinsic Josephson junctions causing the nonperiodic boundary conditions which are not investigated carefully yet. As we demonstrated in the present paper, the nonperiodic boundary conditions can be a reason for the SS shift in the experimental IV-characteristics.

As summary, we have investigated the effect of the charge imbalance on the Shapiro step in the outermost branch at the nonequilibrium conditions. Two important features for the Shapiro step are predicted. First, the Shapiro step demonstrates a shift of its position from the canonical value $N \omega$, where $N$ is the number of junctions in the stack and $\omega$ is the frequency of the external radiation. The value of this shift depends on the boundary conditions and coupling between Josephson junctions. Due to the coupling, the effect of the boundary conditions is extended to the neighboring junctions. Second, the Shapiro step demonstrates a finite slope in the IV-characteristics of a stack of coupled junctions. The value of the slope depends on the value of the nonequilibrium parameter. The origin of the slope is related to the charge imbalance in the superconducting layers because it is absent in the resistively and capacitively shunted junction models.

An important question that should be tested concerns how the nonequilibrium leading to the slope of the Shapiro step can be controlled experimentally and what physical parameters may affect it. As it was discussed above, the nonequilibrium parameter is determined by the Debye screening length, the thickness of the superconducting layer, junction resistance in the normal state and the

| Parameter                      | Value   | For estimate |
|-------------------------------|---------|--------------|
| $N(0)$, states/eV cm$^3$      | $10^{22}$ | $10^{22}$   |
| $d$, Å                       | 3–5     | 4            |
| $\Delta(T)$, meV             | from 30 to 0 at $T = T_c$ | 20          |
| $I_c$, A/cm$^2$              | $10^2$–$10^5$ | $10^4$      |
| $\tau_{qp}$, ps             | 1–1000 at $T = 4.2$ K | 300        |
quasiparticle relaxation time. So we might expect the influence of the critical current variation and, correspondingly, temperature effects on the nonequilibrium in the system and on the slope of the Shapiro steps. Since the nonequilibrium in the system appears due to the diffusion current, the interface transparency and the diffusivity of the layers can affect the charge imbalance too. The analysis of the experimental situation in the artificial layered structures based on aluminum and niobium the highly nonequilibrium regime can be realized. The parameter \( \nu \tau_c \), where \( \tau_c \) is the inelastic relaxation time, in such systems can be much larger than unity, \( \nu \tau_c \gg 1 \). This can be achieved by means of an increase in the value of the barrier transmissivity \( \varepsilon \), since \( \nu = v_F \varepsilon / 4d_0 \), where \( v_F \) is the Fermi velocity. Therefore, an enhancement in the value of the nonequilibrium parameter with increasing barrier transmissivity is quite possible (at least in principle). Thus, the nonequilibrium parameter in such experiments can be controlled by the resistance \( R \) of the Josephson junction and diffusion parameter \( D \), which affect the relaxation time of quasiparticles \( \tau_{qp} = l_{qp} / \sqrt{D} \).

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