The Anderson Model out of equilibrium: Time dependent perturbations

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Abstract

The influence of high-frequency fields on quantum transport through a quantum dot is studied in the low-temperature regime. We generalize the non crossing approximation for the infinite-U Anderson model to the time-dependent case. The dc spectral density shows asymmetric Kondo side peaks due to photon-assisted resonant tunneling. As a consequence we predict an electron-photon pump at zero bias which is purely based on the Kondo effect. In contrast to the resonant level model and the time-independent case we observe asymmetric peak amplitudes in the Coulomb oscillations and the differential conductance versus bias voltage shows resonant side peaks with a width much smaller than the tunneling rate. All the effects might be used to clarify the question whether quantum dots indeed show the Kondo effect.

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The influence of time-dependent external fields on quantum transport through mesoscopic devices has attracted a lot of interest in the last few years. In the noninteracting case ac response of resonant tunneling devices has been studied both experimentally and theoretically by many authors [1–5]. Recently Coulomb interactions have been included by Büttiker et.al. [6] by a self-consistent potential method to account correctly for the displacement currents. In addition, for ultrasmall metallic tunnel junctions and quantum dots in the nanoscale regime, Coulomb blockade phenomena are important. They have been studied in the presence of time-dependent fields either for low [7–9] or high frequencies [10, 12]. Interesting effects like single-electron pumps [7], turnstiles [8,9] and electron-photon pumps [11] have been predicted. Furthermore, the possibility of photon-assisted tunneling in connection with single-electron effects gives rise to a multitude of new resonant features in the Coulomb oscillations and the Coulomb staircase [11,12]. Some of these effects have been observed in a recent experiment by Kouwenhoven et.al. [12] by applying microwave irradiation of 19GHz to a split-gate quantum dot device. However, all these papers deal with the high-temperature regime where the classical picture of sequential tunneling applies.

In this letter we will investigate for the first time the influence of high-frequency fields on single-electron effects for very low temperatures where coherent resonant tunneling processes become important. To display the physical effects occurring in such a case most clearly we concentrate here on a quantum dot with two degenerate energy levels subject to an explicit time-dependent external field. For large Coulomb-interaction we can represent this system by the infinite-U Anderson model with spin degeneracy \( N = 2 \). The time dependent modulation can either be applied to the electrons in the leads or to the two levels on the quantum dot. In the time-independent case this model has already attracted much interest since the occurrence of a Kondo resonance at low temperatures increases the conductance of the system considerably [13,14]. Furthermore the differential conductance versus bias voltage shows a sharp maximum for zero bias due to a splitting of the Kondo resonance [13,14]. The research in this direction is further motivated by a recent experiment of Ralph & Buhrman [17]. At very low temperatures they found singularities in the differential resistance of metal
point contacts containing defects which can be modeled by a two level system or a two channel Anderson model \[18\]. The experimental data seem to be in agreement with theoretical investigations, either using equilibrium conformal field theory \[19\] or nonequilibrium Green functions \[18\].

In the presence of time-dependent perturbations we will combine in this work the picture of photon-assisted tunneling according to Tien & Gordon \[20\] with resonant tunneling phenomena and the Kondo physics. The condition for the Kondo resonance to occur is a very low temperature to enhance the sharpness of the Fermi surface which gives rise to a sharp peak of the spectral density near the Fermi energy. If we apply the external fields only to the leads, the electrons there will absorb or emit multiples of the photon energy $\hbar \Omega$, where $\Omega$ is the applied external frequency. As a consequence, there will be a whole set of sharp Fermi surfaces in the reservoirs. Thus in the low temperature regime the Kondo peak will split up into a whole set of Kondo peaks, separated by the external frequency and weighted approximately by $J_n^2(\Delta \Omega)$, where $J_n$ denotes the Bessel function of order $n$ and $\Delta$ is the power of the applied external field. This leads to a drastic change of the spectral density of the dot due to ac-voltages in the leads which can not be observed in the case of a single resonant level. In the latter case the only effect is photon-assisted tunneling due to excitations of the electrons in the leads but the spectral density in the dot remains completely unchanged. We will calculate and analyze these new features by generalizing the Non Crossing Approximation (NCA) for nonequilibrium systems to the present case. Our detailed numerical analysis shows that the distribution of the Kondo peak amplitudes is significantly asymmetric and depends on the value of the gate voltage. For an asymmetric coupling of the external fields to the two leads this gives rise to a pump effect at zero bias which is solely due to the Kondo physics. Furthermore we predict asymmetric side peaks in the Coulomb oscillations and find resonant peaks in the differential conductance versus bias voltage which are due to overlaps of Kondo peaks arising either from the finite voltage or the external field. All the effects can not be observed in the time-independent situation or for the $N = 1$ case with only one resonant level. Thus our results may help to find further
experimental evidences for the occurrence of Kondo phenomena in mesoscopic devices.

The system under consideration is an Anderson Hamiltonian of a particle on a dot level with infinite strong Coulomb repulsion coupled to two large electron reservoirs. In slave boson representation \([21]\), our Hamiltonian reads

\[
H = \sum_{p,\sigma,\alpha} \left( \epsilon_p + \mu_\alpha + \Delta^\alpha(t) \right) c^\dagger_{p\sigma} c^\alpha_{p\sigma} + \left( \epsilon_d + \Delta(t) \right) \sum_\sigma f^\dagger_\sigma f_\sigma \\
+ \sum_{p,\sigma,\alpha} T_\alpha \left( f^\dagger_\sigma b^\alpha_{p\sigma} + h.c. \right)
\]

where the first term describes the noninteracting electrons of the left and right lead with different time dependent chemical potentials \(\mu_\alpha + \Delta^\alpha(t)\), \(\alpha = L, R\) (we take \(\mu_\alpha\) to be +/- \(V/2\), respectively, for the finite bias case). The second and third terms describe the time dependent dot level \(\epsilon_d + \Delta(t)\) and the tunneling terms, respectively, where \(f\) and \(b\) are the slave fermion and slave boson operators. The physical particle operator on the dot is represented by \(d^\dagger_\sigma = f^\dagger_\sigma b\), supplemented by the constraint \(\sum_\sigma f^\dagger_\sigma f_\sigma + b^\dagger b = 1\). With a standard canonical transformation we can move all explicit time dependence to the tunneling matrix elements \(T_\alpha \to T_\alpha(t)\). We have \(T_\alpha(t) = T_\alpha \exp \left( i \int_{-\infty}^t d\tau [\Delta^\alpha(\tau) - \Delta(\tau)] \right)\), where \(\Delta^\alpha(\tau), \Delta(\tau)\) are the time variations of the energy levels in the leads and the dot, respectively (throughout this work we set \(\hbar = 1\). From this we can see that only the time dependent motion of the leads relative to the dot is important. For numerical convenience, we assume that \(\Delta^\alpha(\tau) - \Delta(\tau)\) takes a simple form, namely \(\Delta^\alpha(\tau) - \Delta(\tau) = \Delta_\alpha \sin \Omega \tau\), that is, a harmonic oscillation with (outer) frequency \(\Omega\) and amplitude \(\Delta_\alpha\).

We compute the differential conductance within the Non Crossing Approximation (NCA) for the Anderson model in the Kondo limit \([22, 24]\). The NCA has been very successful in describing the equilibrium Kondo problem except for the appearance of spurious nonanalytic behavior at a temperature far below the Kondo temperature \(T_K\). These spurious low-T properties are due to the fact that the NCA neglects vertex corrections responsible for restoring the low \(T\) Fermi liquid behavior \([25]\). However, there is a wide range of temperature where \(T\) is much higher than the energy scale of these spurious effects and still well below \(T_K\). For quantum dots, we are interested in temperatures of about \(T_K\), since smaller temperatures
are very hard to reach experimentally. Therefore, the NCA should give a reliable qualitative understanding of the physics in typical quantum dots.

In order to calculate the conductance in nonequilibrium, the NCA must be generalized using nonequilibrium Green functions [26]. One solves for both the retarded Green functions for these operators, \( G^r \) (fermions) and \( D^r \) (boson), and for the ‘lesser’ Green functions \( G^< \) and \( D^< \), which contain information about the nonequilibrium distribution function. The derivation of the integral equations for these four functions follows the work of Meir, Wingreen, and Lee [16], but has to be generalized to the case of time dependent perturbations (for the adiabatic case see Ref. [27]). One obtains complicated equations for the self-energies \( \Sigma \) (fermion) and \( \Pi \) (boson) (lesser, retarded) which now (aside from the usual frequency dependence) have an additional (discrete) index \( n \) referring to the harmonics due to the explicit time dependence of the hamiltonian. The equations (which we can not show due to restricted space) are very hard to solve in their full generality, since they involve three constrained sums over the new index \( n \). However, in the nonadiabatic regime, where the outer frequency \( \Omega \) is much larger than the Kondo temperature \( T_K \) (or the tunneling rates \( \Gamma_\alpha(\omega) = 2\pi T_\alpha^2 N_\alpha(\omega) = \Gamma_\alpha N_\alpha(\omega) \) in the mixed valence regime \( N_\alpha(\omega) \) is the density of states in reservoir \( \alpha \) ) , we can restrict ourselves to the dc - components of the Green functions and the self energies, since the ac–components are suppressed by a factor \( nT_K/\Omega \) (\( n\Gamma_\alpha/\Omega \) in the mixed valence regime). Then the equations reduce to

\[
\Sigma_o^< (\omega) = \frac{1}{2\pi} \int d\omega' K^+(\omega - \omega') D_o^< (\omega') \ , \quad (2a)
\]

\[
\Pi_o^< (\omega) = \frac{N}{2\pi} \int d\omega' K^- (\omega' - \omega) G_o^< (\omega') \quad (2b)
\]

with the integration kernels given by \( K^\pm = \sum_\alpha K^\pm_\alpha \) and \( K^\pm_\alpha (\omega) = \int d\omega' \gamma^\pm_\alpha (\omega') P_\alpha (\omega - \omega') \). Here \( \gamma^+_\alpha (\omega) = \Gamma_\alpha (\omega) f_\alpha (\omega) \) and \( \gamma^-_\alpha (\omega) = \Gamma_\alpha (\omega)(1 - f_\alpha (\omega)) \) are the transition rates in lowest order for an electron to enter (leave) the quantum dot in the absence of time-dependent perturbations \( f_\alpha (\omega) = (\exp \beta (\omega - \mu_\alpha) + 1)^{-1} \) is the Fermi distribution function for the lead \( \alpha \) ). Furthermore,
\[ P_\alpha(\omega) = \sum_n J_n^2(\frac{\Delta_\alpha}{\Omega})\delta(\omega - n\Omega) \]  

(3)

denotes the probability for an electron in reservoir \( \alpha \) to emit or absorb the energy \( \omega \). Since we are considering microwave irradiation this energy has to be identical to multiples of \( \Omega \). This reflects the physical effect that electrons in the leads can now absorb or emit energy quanta \( n\Omega \) which gives rise to a shift of the whole Fermi sea.

In principle, the sum over \( n \) in Eq. (3) runs from \(-\infty\) to \( \infty \). However, the higher Bessel functions have negligible values (compared to the \( n = 0, 1, 2, 3 \) Bessel functions) for arguments \( \Delta_\alpha/\Omega < 3 \), especially since they enter the integration kernels squared. Therefore, if we restrict ourselves to this range of argument, we can in good approximation cut off the sum at \( n = 2 \).

Furthermore we find the relation

\[ G_o^<(\omega) = \Sigma_o^<(\omega) |G_o^r(\omega)|^2, \quad D_o^<(\omega) = \Pi_o^<(\omega) |D_o^r(\omega)|^2. \]  

(4)

Thus, we need the retarded Green functions as input in order to solve the equations for the lesser Green functions. They can be found from \( G_0^r(\omega) = (\omega - \epsilon_d - \Sigma_0^r(\omega))^{-1} \), \( D_0^r(\omega) = (\omega - \Pi_0^r(\omega))^{-1} \) and

\[ Im\Sigma_o^r(\omega) = \frac{1}{2\pi} \int d\omega' K^-((\omega - \omega')ImD_o^r(\omega')) , \]  

(5a)

\[ Im\Pi_o^r(\omega) = \frac{N}{2\pi} \int d\omega' K^+(\omega' - \omega)ImG_o^r(\omega') . \]  

(5b)

The real parts of the retarded self energies can be found from the Kramers–Kronig relation.

We have solved Eqs. (2), (4) and (5) numerically by iteration. The code is designed to sacrifice memory economy for higher speed. We are able to go to temperatures orders of magnitude below \( T_K \), although for the cases of interest here it is enough to go to \( T_K/5 \) (where the Kondo peaks have essentially reached their low temperature form). The validity of the numerical solution is established by checking the sum rules. They are fulfilled to within 0.5% even in the worst cases (lowest \( T \), large \( n \), etc.). The constraint is enforced after each iteration, so that no unphysical solution can emerge.
Aside from the density of states of the lead electrons, all the integration kernels can be interpreted as modified distribution functions with steps heights (at low temperatures) determined mostly by the Bessel functions. Since the dot spectral function tends to have a peak at energies close to these steps (if they are pronounced enough), we anticipate a splitting of the equilibrium Kondo peak into a variety of smaller peaks at positions determined by $\pm n\Omega \pm V/2$.

The dc-dot spectral function, $A_d$, is computed from the slave Green functions via the convolution [18] (using the conventions of Müller–Hartmann [22] for the spectral functions, $A(\omega) = -\text{Im}G^r_o(\omega)/\pi$, $B(\omega) = -\text{Im}D^r_o(\omega)/\pi$, and the ‘lesser’ Green functions, $a(\omega) = \text{Im}G^<_o(\omega)/2\pi$ and $b(\omega) = \text{Im}G^<_o(\omega)/2\pi$),

$$A_d(\omega) = \int \frac{d\epsilon}{\pi} \left[ a(\epsilon) B(\epsilon - \omega) + A(\epsilon) b(\epsilon - \omega) \right].$$

(6)

Generalizing the known formulas [15,28] we can calculate the dc current from $A_d(\omega)$ (if the couplings to the left and right leads are the same),

$$I = \frac{eN}{2} \int d\omega A_d(\omega) \left[ K^+_L(\omega) - K^-_R(\omega) \right]$$

(7)

and from the current the conductance $G(V) = dI(V)/dV$ via numerical derivative. We can also calculate the magnetic susceptibility and other transport properties. Additionally, it is possible to apply this method to bulk Kondo systems in the presence of time dependent fields. Results on that and a more detailed discussion of the full NCA equations, approximation and the numerics will be published elsewhere.

Since the Anderson model in the presence of a static potential (finite bias) alone has been discussed in [16] and for the two channel case in [18] we will start the discussion of results for the zero bias case with an oscillating potential with frequency $\Omega$ and amplitude $\Delta_L = \Delta_R = \Delta$ ($\Delta/\Omega = 1.4$).

Taking the Kondo limit with $\epsilon_d$ large and negative ($\epsilon_d = -1340T_K$, except for Fig. 3) we obtain a resonance (Kondo peak) with a width at half maximum (up to a factor of the order of unity) given by $T_K$ [24]. Applying now an oscillating potential of frequency $\Omega = 16T_K$ the
Kondo peak splits, creating side peaks at +/- $n\Omega$ shifted frequencies, see the top curve of Fig. 1. For $\Omega$ much below that value our approximation might not be valid. If we nevertheless look at the behavior of the Kondo peak for these smaller values we find that the original peak first broadens and finally splits into three peaks (in analogy to the case of a static potential where it splits into two peaks [16,18]). Observe the asymmetry of the side peaks, the one at positive frequencies is higher than the one at negative frequencies as long as the negative one is not merging with the broad hump (with width $N\Gamma = N(\Gamma_L + \Gamma_R)$, $\Gamma = 300T_K$ except for Fig. 3) of the bare dot level (not shown, since far below the Fermi level).

This asymmetry can be used to built a 'Kondo pump'. Suppose the right lead is kept at a fixed potential (Fermi energy) whereas the left one has the oscillation with frequency $\Omega$ applied, that is $\Delta_L = \text{finite}$, $\Delta_R = 0$. In the left lead electrons in an energy range up to $\Omega$ below the Fermi energy can be excited and move to the right lead via the states at the dot with energies above the Fermi level. On the other hand electrons in the right can move to the left lead using states of the dot with energies below the Fermi level. Due to the asymmetry of the dot spectral function (which is enhanced upon what is seen in Fig. 1 with symmetric coupling) these currents will not be equal. Therefore, one should observe a dc current through the dot, even if no finite bias $V$ is applied. This current will be directed from left (right) to right (left) if $\Delta_L > (<) \Delta_R$. In Fig. 2 we show a graph of this 'pump' current in dependence of $\Delta_R$ for a fixed value of $\Delta_L$. The current is quite large (equivalent to a bias $V$ of about $1/10 T_K$ if just a static potential is applied with symmetric couplings) due to peaks in the dot spectral function at the 'right' frequencies. At these frequencies the lorentzian spectral function of the corresponding noninteracting resonant level model would be unspectacular flat and very small for the given set of parameters of dot level energy $\epsilon_d$ and width $\Gamma$. This current enhancement is strong temperature dependent and one might naively expect that it vanishes as $T$ becomes larger than $T_K$. However, as long as $T$ is well below $\Omega$ the effect remains, since the equilibrium $T_K$ is no longer the relevant width of the Kondo peaks [16,18].

Another effect partly due to the asymmetry of $A_d$ is an asymmetry of the zero bias
conductance (now again \( \Delta_L = \Delta_R = \Delta \), with \( \Delta/\Omega = 1.4 \), so that only the zeroth and first Bessel function have relevant effects on \( A_d \)) as function of the gate voltage (Coulomb-oscillations). We increase the level energy \( \epsilon_d \) from the Kondo limit through the mixed valence regime into the empty dot regime (where \( \epsilon_d \) is now large and positive). Taking \( \Omega > \Gamma \) \((T << \Gamma)\), we obtain a peak in the conductance for \( \epsilon_d \) slightly below \(-\Omega\), the usual, higher peak just below the Fermi energy, and another, even higher peak slightly below \(+\Omega\), all with the bare level width \( \Gamma \) as shown in Fig. 3. This asymmetry in the heights of the conductivity peaks can be easily understood by considering the increase in total spectral weight \( \int d\omega A_d(\omega) \). Having a value of just below 1/2 (per spin) in the Kondo limit (\( \epsilon_d \) large and negative), it increases monotonically and approaches unity in the empty dot limit. The increase in total weight is rising the value of \( A_d \) at all frequencies, leading to an increased conductivity peak height for the \(+\Omega\) shifted peak. This is again in contrast to the resonant level case, where the total weight of the lorentzian spectral function remains constant (equal unity) which leads therefore to equal heights for corresponding side peaks.

We can also study the influence of the outer frequency \( \Omega \) on the temperature dependence of the zero bias conductance. In equilibrium, the zero bias conductance saturates for \( T < T_K \) and drops logarithmically for \( T >> T_K \), see Fig. 4. However, for \( \Omega = 10T_K \) the main drop now occurs somewhat below \( \Omega \), with \( T_K \) playing no significant role anymore. The low \( T \) zero bias conductance depends roughly logarithmically on \( \Omega \) in the nonadiabatic regime \( \Omega > 10T_K \). At the lowest \( \Omega \) for which we can trust our approximation the zero bias conductance is already suppressed by about one order of magnitude.

In addition, we now apply a finite bias \( V \) to the dot. If \( V \) and \( \Omega \) are large enough (and \( T \) low enough) to allow a clear separation of all features, the applied bias splits all peaks into two (see Fig. 4), as it did in the static case \((\Delta = 0)\) with the single Kondo peak [16,18]. If \( V \) is now increased until it is close to \( \Omega \), the peaks positioned at \(-\Omega + V/2\) and \(-V/2\) as well as the peaks at \(+V/2\) and \(+\Omega - V/2\) merge. By further increasing \( V \) until it is close to \( 2\Omega \), we reach the point where the peaks positioned at \(-\Omega + V/2\) and \(+\Omega - V/2\) merge into one peak at about the Fermi energy. It seems obvious that the current should increase rapidly around
these special values of $V$, leading to strongly enhanced conductances. However, due to broadening while increasing $V$, the overall values of $A_d$ monotonically decrease, and spectral weight is just shifted from the valleys to the peaks when these mergings of peaks occur. The final say in this matter has the numerical evaluation of the conductance. Fig. 3 shows the conductance $G(V)$ for a low temperature $T = T_K/5$. Indeed there are strong features in the conductance at values of $V$ slightly below the naively expected values. Especially the peak at $2\Omega$ is quite distinguished (it rises 25–30 % above its background) and the width is in rough agreement with the width of the peaks of $A_d$ at this bias. The first feature is less pronounced and wider, which we can trace back to uncomplete separation before the merging of the peaks at this lower value of $V$. Again, this feature is strongly $T$– dependent, but, as long as $T$ is low enough to allow the separation and merging of peaks without washing them out altogether (roughly $T \sim 1/10\Omega$), similar, though wider and smaller conductance anomalies should be observable.

In conclusion, we performed numerical calculations which constitute the first theoretical work on the Anderson model with infinite strong Coulomb repulsion at low temperatures in the presence of time dependent perturbations. We compute the conductance in the linear and nonlinear response regime. Both show peaks in accordance to theoretical arguments. We find the effect of a 'Kondo pump' which creates a dc current in the presence of an ac potential alone, if the amplitudes of potential oscillations in the leads are different from each other. All these features may be used to experimentally clarify the question of the presence of Kondo physics in quantum dots. The temperature dependence of these effects are strong, but observability is not limited to temperatures below the equilibrium Kondo temperature $T_K$.

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FIGURES

FIG. 1. Spectral function $A_d$ for $\Omega = 16T_K$ vs. frequency for various applied bias $V$ (curves are offset). The topmost curve shows the splitting of the Kondo peak into three peaks at $-\Omega, 0 (= \epsilon_F)$ and $+\Omega$ at zero bias. The other curves illustrate the merging of peaks at the voltages $V = \Omega$ and $V = 2\Omega$. Also observe the general broadening of the peaks upon increase of bias $V$.

FIG. 2. 'Pump'–Current $I(\Delta R)$ vs. the amplitude of oscillations $\Delta R$ in the right lead. For fixed left amplitude $\Delta L$ the current is nonzero for all values of $\Delta R \neq \Delta L$ even though no bias $V$ is applied. $I(0)$ denotes $I(\Delta R = 0)$. This 'Kondo pump' effect can be explained with the asymmetry of the split Kondo peaks about the Fermi level.

FIG. 3. Zero bias conductance $G(\epsilon_d)$ vs. dot–level energy $\epsilon_d$. The three peaks slightly below $-\Omega, 0$ and $+\Omega$ show an increasing height mostly due to the increasing spectral weight when $\epsilon_d$ moves from the Kondo limit to the empty dot limit. $G(0)$ denotes $G(\epsilon_d = 0)$.

FIG. 4. Zero bias conductance $G(T)$ vs. temperature $T$ on double–logarithmic scale. Shown are data for the equilibrium $\Omega = 0$ and for $\Omega = 10T_K$. $G(0)$ denotes $G(T = 0)$ for the case ($\Omega = 0$). In equilibrium the zero bias conductance starts to drop already below $T_K$. With an outer frequency $\Omega$ applied the main drop starts just below the corresponding temperature, the equilibrium $T_K$ playing only a minor role. At very low temperatures $G(T)$ is reduced by almost one order of magnitude for this value of $\Omega$. At high $T$ all features due to $\Omega$ are washed out and $G(T)$ is independent of $\Omega$.

FIG. 5. Nonlinear conductance $G(V)$ vs. bias $V$ for an outer frequency $\Omega = 16T_K$. $G(0)$ denotes the zero bias conductance $G(V = 0, T)$. Aside from the usual zero bias peak (not fully resolved) there are peaks just below multiples of $\Omega$. The first peak is lower relative to the background than the second peak. The peaks are due to merging of side peaks at the corresponding bias $V$ (see Fig. 4). These conductance peaks are present even though $T$ is larger $T_K$ as long as $T << \Omega << \Gamma$. 

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\[ \frac{G(\varepsilon_d)}{G(0)} \]

\[ \frac{\Delta}{\Omega} = 1.4 \]
\[ \Gamma = \frac{1}{5} \Omega \]
\[ T = \frac{1}{200} \Omega \]

Level Energy \( \varepsilon_d/\Omega \)
Current $I(\Delta_R)/I(0)$

For $\Omega = 20\ T_K$,

$\Delta_L/\Omega = 1.4$

$T = 1/5\ T_K$

Right Amplitude $\Delta_R/\Omega$
$\Omega = 16 \ T_K \ \ T = 1/5 \ T_K$

$\Delta/\Omega = 1.4$

Spectral Function $A_d(\omega)$

- $V = 0$
- $V = \Omega/2$
- $V = \Omega$
- $V = 3\Omega/2$
- $V = 2\Omega$

Frequency $\omega/T_K$ from $-40$ to $40$
\[ \Omega = 16 \, T_K \]
\[ \Delta / \Omega = 1.4 \]
\[ T = \frac{1}{5} \, T_K \]