Z-number-based conjoint analysis method for analyzing decision-makers’ preference levels in attribute ratings

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Abstract. Fuzzy Conjoint Analysis Method (FCAM) is widely used in multi-attribute decision making areas particularly in analyzing decision makers’ preferences towards attributes. Discrete fuzzy sets are commonly used to define the membership function of linguistic values. Nevertheless, the use of discrete fuzzy sets in representing human judgment and preference levels may not be sufficient in describing human preference defined on continuous scale. In FCAM, discrete fuzzy sets or fuzzy numbers are commonly used to represent the linguistic terms that merely describe ratings or levels of satisfaction on attributes based on decision makers’ opinions or preferences. The certainty elements in terms of confidence level associated with decision makers’ judgment and preferences are seldom considered in FCAM. Thus, in this study, Z-numbers which are composed of ratings with certainty components (in the form of confidence level) are integrated in the existing FCAM procedure. In the proposed procedure of Z-number based Conjoint Analysis Method (Z-CAM), preferences described in the form of ratings with confidence levels are expressed in the form Z-numbers. The existing FCAM procedure is slightly modified so as to cater the Z-number input data. Z-CAM has an additional feature in comparison to the existing FCAM whereby in addition to ranking the attributes, the former could also produce overall ratings supported with confidence levels.

1. Introduction
Conjoint analysis is basically a type of statistical analysis popularly used in determining consumers’ preferences towards components of services or products through surveys [1]. The respondents to the survey normally give their preferences levels by means of Likert scales. In Likert scales, the linguistic terms strongly agree to strongly disagree are commonly used to represent the level of agreement of respondents towards statements related to attitude towards product or services [2]. Each term is associated with a numerical value upon which the level of agreement can be measured. Considering the element of fuzziness inherited in linguistic terms, fuzzy sets have also been popularly used by researchers to mathematically define these terms. Ref. [3] proposed a discrete fuzzy set-based conjoint model known as Fuzzy Preference Model that integrates discrete fuzzy sets in representing the linguistic terms associated with consumers’ preferences. Nevertheless, the use of discrete fuzzy sets may not be adequate in expressing the continuous nature of human preference and judgement. For this reason, the discrete fuzzy set-based conjoint analysis method is further extended to fuzzy number-based conjoint analysis method [4,14].
Most existing procedures in discrete fuzzy set-based or fuzzy number-based conjoint analysis models (FCAM) \[3,4,5,6,14\] mainly describe ratings or levels of satisfaction on attributes based on decision makers’ opinions or preferences per se. The certainty element that describes how certain the decision maker is in assigning values to a linguistic variable representing an attribute is seldom taken into account in existing procedure of FCAM. In dealing with this issue, the concept of Z-number can be employed and integrated into existing FCAM. A Z-number, mathematically denoted by the pair \((\tilde{A}, \tilde{B})\) is composed of two elements namely the restriction component represented as \(\tilde{A}\), and the certainty component, \(\tilde{B}\). The certainty component may represent the level of sureness, confidence, reliability, possibility, strength of belief or other similar concepts \[7,8\].

In this paper, a Z-number based Conjoint Analysis Method (Z-CAM) is proposed whereby the decision-makers’ preference levels are described in the form of Z-numbers. Specifically, the Z-numbers representing the decision makers’ preference levels in the Z-CAM procedure are composed of the rating component and the certainty component in the form of confidence level. The applicability of the Z-CAM procedure is illustrated by implementing the procedure on a hypothetical case study on analysis of factors influencing postgraduates’ program selection in a university.

2. Preliminaries

In this section, basic theories and operations related to fuzzy numbers and Z-numbers are presented.

**Definition 1** [9]

Given a collection of objects, \(X\), a fuzzy set \(\tilde{A}\) of a universe of discourse \(X\) is an ordered pair \(\tilde{A} = (x, \mu_{\tilde{A}}(x) | x \in X)\) such that \(\mu_{\tilde{A}}(x)\) represent the membership function that characterizes the degree of membership of an element \(x\) in \(\tilde{A}\).

**Definition 2** [10]

A triangular fuzzy number (TFN) denoted as \(\tilde{A} = (a_1, a_2, a_3)\) is a fuzzy number defined by the membership function \(\mu_{\tilde{A}}(x)\) such that

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{(x-a_1)}{(a_2-a_1)}, & a_1 \leq x \leq a_2 \\
\frac{(x-a_2)}{(a_3-a_2)}, & a_2 \leq x \leq a_3 \\
0, & \text{otherwise}.
\end{cases}
\]

**Definition 3** [10]

The arithmetic operation of triangular fuzzy numbers \(\tilde{A} = (a_1, a_2, a_3)\) and \(\tilde{B} = (b_1, b_2, b_3)\) are given as follows:

(i) Addition: \(\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)\)

(ii) Multiplication: \(\tilde{A} \otimes \tilde{B} = (a_1b_1, a_2b_2, a_3b_3)\).

(iii) Scalar multiplication: \(k \otimes \tilde{A} = (ka_1, ka_2, ka_3)\), \(k > 0\).

**Definition 4** [11]

The similarity degree between fuzzy numbers \(\tilde{A} = (a_1, a_2, a_3)\) and \(\tilde{B} = (b_1, b_2, b_3)\) can be calculated as

\[
S(\tilde{A}, \tilde{B}) = \frac{1}{1 + d(\tilde{A}, \tilde{B})}
\]

where \(d(\tilde{A}, \tilde{B}) = |P(\tilde{A}) - P(\tilde{B})|\) with \(P(\tilde{A}) = \frac{a_1 + 4a_2 + a_3}{6}\) and \(P(\tilde{B}) = \frac{b_1 + 4b_2 + b_3}{6}\).
Definition 5 [8]
A Z-number is an ordered pair of fuzzy numbers denoted as \( Z = (\tilde{A}, \tilde{B}) \) where \( \tilde{A} = (a_1, a_2, a_3) \) and \( \tilde{B} = (b_1, b_2, b_3) \). The first component, \( \tilde{A} \), is a fuzzy number that describe the restriction (constraint) on the values that a real-valued uncertain variable \( X \) are allowed to take. On the other hand, the second component, \( \tilde{B} \), measures the certainty of the first component.

The certainty component may indicate reliability, sureness, confidence level, or possibility measure that suits one’s perception of \( \tilde{A} \) [7,8]. Note that \( \tilde{A} \) and \( \tilde{B} \) can be represented by triangular fuzzy numbers, trapezoidal fuzzy numbers, or any other appropriate fuzzy number representations. The following examples illustrate how Z-numbers can be used in describing an uncertain situation:

Suppose someone is asking about the “time taken for Amina to reach home from work”. Possible answers to the questions and the corresponding Z-numbers could be given as follows:

\( A1: \) Usually, Amina takes about 45 minutes to reach home.
Z-number: \( (about\ 45\ minutes,\ usually) \)

\( A2: \) Sometimes, Amina takes more than 1 hour to reach home.
Z-number: \( (more\ than\ 1\ hour,\ sometimes) \)

Here, the linguistic terms \( \{about\ 45\ minutes\} \) and \( \{more\ than\ 1\ hour\} \) represent the restrictions on the values that the variable “time taken for Amina to reach home from work” can take. Meanwhile, the terms \( \{usually\} \) and \( \{sometimes\} \) indicate the level of certainty towards the assigned restriction \( \{about\ 45\ minutes\} \) and \( \{more\ than\ 1\ hour\} \), respectively.

3. Z-Number-Based Conjoint Analysis Method (Z-CAM)

In this section, a general procedure of the proposed Z-number-based Conjoint Analysis Method (Z-CAM) is laid out. The Z-CAM procedure is an extension of the triangular fuzzy number-based conjoint analysis method (TFN-CAM) introduced in [14].

3.1 General Procedure of Z-Number-Based Conjoint Analysis Method (Z-CAM)

The general procedure of Z-CAM is presented as follows.

Phase 1: Generating regular fuzzy numbers from pre-defined Z-numbers for rating of attributes
In this phase, several steps are involved in generating regular fuzzy numbers (FNs) for attribute ratings based on Z-numbers. The steps are as follows:

Step 1: Identify the set of attributes as \( F = \{F_i\}, i = 1,...,m \).

Step 2: Set the predefined Z-numbers for attribute ratings, \( Z_{ip} = (\tilde{L}_k, \tilde{R}_p) \) where \( \tilde{L}_k \) is the predefined linguistic ratings such that \( \tilde{L}_k = (a_{k1}, a_{k2}, a_{k3}), k = 1,...,t \) and \( \tilde{R}_p \) is the predefined confidence level where \( \tilde{R}_p = (r_{p1}, r_{p2}, r_{p3}), p = 1,...,h \).

Step 3: Convert the Z-numbers \( Z_{ip} = (\tilde{L}_k, \tilde{R}_p) \) into regular fuzzy numbers \( \tilde{Z}_{ip} \) which represent the predefined linguistic ratings with confidence level. The following conversion steps adopted from [13] are as follows:

Step 3(a): Calculate the weight, \( \alpha \), of the certainty component (confidence level), \( \tilde{R}_p \) where

\[
\alpha = \frac{\int x\mu_{\tilde{R}_p}(x)dx}{\int \mu_{\tilde{R}_p}(x)dx}
\]

such that \( \mu_{\tilde{R}_p}(x), p = 1,...,h, \) is a triangular membership function with \( x \in X \).

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Step 3(b): Estimate the weighted Z-number $Z_{kp}^\alpha$ by integrating the weight $\alpha$ into the restriction component, $\tilde{L}_k$, such that

$$Z_{kp}^\alpha = \{<x, \mu_{\tilde{L}_k}^\alpha(x)>| \mu_{\tilde{L}_k}^\alpha(x) = \alpha \mu_{L_k}(x), x \in X\}.$$ 

Step 3(c): Convert the weighted Z-number $Z_{kp}^\alpha$ into regular fuzzy number $\tilde{Z}_{kp}$ where

$$\tilde{Z}_{kp} = \{<x, \mu_{\tilde{Z}_{kp}}(x)>| \mu_{\tilde{Z}_{kp}}(x) = \mu_{L_k}\left(\frac{x}{\sqrt{\alpha}}\right), x \in \sqrt{\alpha}X\}$$

such that $\mu_{\tilde{Z}_{kp}}(x)$ is the membership function of $\tilde{Z}_{kp}$ as illustrated in Figure 1.

![Figure 1. Regular fuzzy number $\tilde{Z}_{kp}$ for Z-number $Z_{kp}$](image)

Step 4: Collect the linguistic ratings with confidence levels from the decision makers (respondents) in the form of Z-number ($Z_{ikp}$) such that $Z_{ikp} \in \{Z_{kp}, k=1,2,...,t \ , p=1,2,...,h\}$. The corresponding regular FNs of $Z_{ikp}$ are $\tilde{Z}_{ikp} \in \{\tilde{Z}_{kp}, k=1,2,...,t \ , p=1,2,...,h\}$, $i=1,...,m$.

**Phase 2: Aggregating Linguistic Ratings with Confidence levels**

The levels of preference of decision makers (respondents) on attributes $i$ are presented in the form of Z-numbers, $Z_{ikp}$ such that $Z_{ikp} \in \{Z_{kp}, k=1,2,...,t \ , p=1,2,...,h\}$. The aggregations of the respondents’ ratings on each attribute are performed by aggregating the Z-numbers’ associated regular fuzzy numbers, $\tilde{Z}_{ikp}$, as in the following steps:

Step 5: Obtain the number of respondents $\lambda_{ikp}$ who rate the $i$-th attribute as $Z_{ikp}$.

Step 6: Calculate the aggregated regular fuzzy numbers, $\tilde{Z}_i$, representing linguistic ratings with confidence levels, with respect to the $i$-th attributes by using the formula:

$$\tilde{Z}_i = \sum_{p} \sum_{k=1}^{h} w_{ikp} \tilde{Z}_{ikp} = (a'_1, a'_2, a'_3)$$

where

$$w_{ikp} = \frac{\lambda_{ikp}}{\sum_{p=1}^{h} \sum_{k=1}^{t} \lambda_{ikp}}, \ i=1,2,...,m$$

such that $\lambda_{ikp}$ is the number of responses associated with $Z_{ikp}$, and $\tilde{Z}_{ikp}$ is the corresponding regular fuzzy number of $Z_{ikp}$.
Phase 3: Identifying Overall Preference Levels and Ranking of Attributes

This phase represents the steps of identifying the preference levels, and also ranking of the \( i \)-th attributes.

Step 7: Calculate the degree of similarity \( S_{lp}(\tilde{Z}_k, \tilde{Z}_{lp}) \) based on [11] between the aggregated FNs

\[
\tilde{Z}_k = (a_{1k}^{i}, a_{2k}^{i}, a_{3k}^{i}) \quad \text{and} \quad \tilde{Z}_{lp} = (a_{1l}^{kp}, a_{2l}^{kp}, a_{3l}^{kp})
\]

by using the formula:

\[
S_{lp}(\tilde{Z}_k, \tilde{Z}_{lp}) = \frac{1}{1+|P(\tilde{Z}_k) - P(\tilde{Z}_{lp})|}
\]

where

\[
P(\tilde{Z}_k) = \frac{a_{1i}^{i} + 4a_{2i}^{i} + a_{3i}^{i}}{6}, \quad P(\tilde{Z}_{lp}) = \frac{a_{1l}^{kp} + 4a_{2l}^{kp} + a_{3l}^{kp}}{6}.
\]

Step 8: Identify the output Z-numbers representing the overall preference levels in terms of linguistic ratings and their confidence levels, with respect to the \( i \)-th attribute, \( F_i \), \( i = 1, \ldots, m \) based on the highest degree of similarity calculated in Step 7.

Step 9: Rank the attributes in descending order based on their overall linguistic ratings, confidence levels and degrees of similarities.

3.2 Illustrative example

Applicability of the proposed procedure presented in Section 3.1 is illustrated in this section. A hypothetical case study on factors influencing selection of a postgraduate program is constructed for illustrative purposes. The input data in terms of ratings of factors are expressed in the form of Z-numbers. Suppose 51 students undertaking Program \( Y \) are asked to rate the factors (attributes), \( F_i \), \( i = 1, 2, \ldots, 10 \) that had influenced them in selecting the program. Let the factors be Academic background \( (F_1) \), University’s facilities \( (F_2) \), Program reputation \( (F_3) \), Teaching quality \( (F_4) \), Peer influence \( (F_5) \), Tuition fees \( (F_6) \), Family support \( (F_7) \), Financial support \( (F_8) \), Interest towards program \( (F_9) \) and Career opportunity \( (F_{10}) \). In applying the Z-CAM procedure, attributes under consideration are represented as factors. Linguistic terms for ratings, confidence levels, and their corresponding triangular fuzzy numbers (TFN) characterizing the predefined Z-numbers are as displayed in table 1.

**Table 1.** Linguistic terms and triangular fuzzy numbers for ratings and confidence levels

| \( \tilde{L}_k \) | Linguistic terms | TFN |
|---|---|---|
| Strongly Disagree (SD) | \( (0, 0, 0.2) \) |
| Disagree (D) | \( (0, 0.2, 0.4) \) |
| Moderately Agree (MA) | \( (0.3, 0.5, 0.7) \) |
| Agree (A) | \( (0.6, 0.8, 1) \) |
| Strongly Agree (SA) | \( (0.8, 1, 1) \) |
| Low (L) | \( (0, 0.2, 0.6) \) |
| Medium (M) | \( (0.2, 0.6, 0.8) \) |
| High (H) | \( (0.6, 1, 1) \) |

The predefined Z-numbers i.e. \( Z_{kp} = (\tilde{L}_k, \tilde{R}_p) \) presented in table 2 are used in representing the respondents’ preferences towards factors. The certainty components \( (\tilde{R}_p) \) in the Z-numbers indicate the confidence levels of the respondents for assigning the factor rating as \( \tilde{L}_k \). Suppose a respondent
assigned a Z-number \((SA,H)\) in response to the influencing factor \textit{Family Support}. This implies that the respondent \textit{Strongly Agree (SA)} with \textit{High (H)} level of confidence that \textit{Family Support} influenced his/her selection of the program. Other Z-numbers listed in table 2 can be interpreted in similar manner.

| Table 2. Predefined Z-numbers for rating of factors with confidence levels |
|-------------------|-------------------|-------------------|
| \(Z_{kp}\) | Z-Number \(Z_{kp} = (L_L, R_R)\) | \(Z_{kp}\) | Z-Number \(Z_{kp} = (L_L, R_R)\) |
| \((SD,L)\) | \((0, 0, 0.2), (0, 0, 0.6)\) | \((MA,H)\) | \((0.3, 0.5, 0.7), (0.6, 1, 1)\) |
| \((SD,M)\) | \((0, 0, 0.2), (0.2, 0.6, 0.8)\) | \((A,L)\) | \((0.6, 0.8, 1), (0, 0, 0.6)\) |
| \((SD,H)\) | \((0, 0, 0.2), (0.6, 1, 1)\) | \((A,M)\) | \((0.6, 0.8, 1), (0.2, 0.6, 0.8)\) |
| \((D,L)\) | \((0, 0.2, 0.4), (0, 0, 0.6)\) | \((A,H)\) | \((0.6, 0.8, 1), (0.6, 1, 1)\) |
| \((D,M)\) | \((0, 0.2, 0.4), (0.2, 0.6, 0.8)\) | \((SA,L)\) | \((0, 8.1, 1), (0, 0, 0.6)\) |
| \((D,H)\) | \((0, 0.2, 0.4), (0.6, 1, 1)\) | \((SA,M)\) | \((0.8, 1, 1), (0.2, 0.6, 0.8)\) |
| \((MA,L)\) | \((0.3, 0.5, 0.7), (0, 0, 0.6)\) | \((SA,H)\) | \((0, 8.1, 1), (0.6, 1, 1)\) |
| \((MA,M)\) | \((0.3, 0.5, 0.7), (0.2, 0.6, 0.8)\) |

Predefined Z-numbers are converted into regular fuzzy numbers using conversion method \([12,13]\) as presented in Step 3(a)-(c) in the Z-CAM procedure. The corresponding weighted Z-numbers \((Z^a)\) and regular fuzzy numbers \((\tilde{Z}_{kp})\) of the predefined Z-numbers \((Z_{kp})\) are depicted in table 3.

| Table 3. Weighted Z-numbers and the associated regular fuzzy numbers |
|-------------------|-------------------|-------------------|
| Z-number, \(Z_{kp} = (L_L, R_R)\) | Weighted Z-Number, \(Z^a\) | Regular FN, \(\tilde{Z}_{kp}\) |
| \(Z_{11} = (SD,L)\) | \((0, 0, 0.2 ; 0.2)\) | \(\tilde{Z}_{11} = (0, 0, 0.0894)\) |
| \(Z_{12} = (SD,M)\) | \((0, 0, 0.2 ; 0.4)\) | \(\tilde{Z}_{12} = (0, 0, 0.1265)\) |
| \(Z_{13} = (SD,H)\) | \((0, 0, 0.2 ; 0.9)\) | \(\tilde{Z}_{13} = (0, 0, 0.1897)\) |
| \(Z_{21} = (D,L)\) | \((0, 0.2, 0.4 ; 0.2)\) | \(\tilde{Z}_{21} = (0.0, 0.0894, 0.1789)\) |
| \(Z_{22} = (D,M)\) | \((0, 0.2, 0.4 ; 0.4)\) | \(\tilde{Z}_{22} = (0, 0.1265, 0.2530)\) |
| \(Z_{23} = (D,H)\) | \((0, 0.2, 0.4 ; 0.9)\) | \(\tilde{Z}_{23} = (0, 0.1897, 0.3795)\) |
| \(Z_{31} = (MA,L)\) | \((0.3, 0.5, 0.7 ; 0.2)\) | \(\tilde{Z}_{31} = (0.1342, 0.2236, 0.3130)\) |
| \(Z_{32} = (MA,M)\) | \((0.3, 0.5, 0.7 ; 0.4)\) | \(\tilde{Z}_{32} = (0.1897, 0.3162, 0.4427)\) |
| \(Z_{33} = (MA,H)\) | \((0.3, 0.5, 0.7 ; 0.9)\) | \(\tilde{Z}_{33} = (0.33, 0.4743, 0.6641)\) |
| \(Z_{41} = (A,L)\) | \((0.6, 0.8, 1 ; 0.2)\) | \(\tilde{Z}_{41} = (0.2683, 0.3578, 0.4472)\) |
| \(Z_{42} = (A,M)\) | \((0.6, 0.8, 1 ; 0.4)\) | \(\tilde{Z}_{42} = (0.3795, 0.5060, 0.6325)\) |
| \(Z_{43} = (A,H)\) | \((0.6, 0.8, 1 ; 0.9)\) | \(\tilde{Z}_{43} = (0.5692, 0.7589, 0.9487)\) |
| \(Z_{51} = (SA,L)\) | \((0.8, 1, 1 ; 0.2)\) | \(\tilde{Z}_{51} = (0.3578, 0.4472, 0.4472)\) |
| \(Z_{52} = (SA,M)\) | \((0.8, 1, 1 ; 0.4)\) | \(\tilde{Z}_{52} = (0.5060, 0.6325, 0.6325)\) |
| \(Z_{53} = (SA,H)\) | \((0.8, 1, 1 ; 0.9)\) | \(\tilde{Z}_{53} = (0.7589, 0.9487, 0.9487)\) |

As an example, for Z-number \(Z_{11} = (SD,L)\), the weight \((a)\) of the certainty term \textit{Low (L)} can be calculated by performing Step 3(a) of Phase 1. We have
As such, the weighted Z-number for (SD,L) can be written as (0,0,0.2;0.2), and performing Step 3(c), the associated regular fuzzy number is constructed as $\bar{Z}_{11} = \sqrt{\alpha(a_1^k,a_2^k,a_3^k)} = \sqrt{0.2}(0,0,0.2) = (0,0,0.0894)$

Suppose the number of responses with respect to each Z-number representing the respondents’ ratings with confidence level is as displayed in Table 4.

**Table 4. Number of respondents with respect to factors and predefined Z-numbers**

| Factor, $F_i$ | $Z_{k^p}$, $Z_{k^p} = (L_k, R_p)$ | TOTAL |
|---------------|---------------------------------|-------|
| $F_1$         | $Z_{11}$, $Z_{12}$              | 51    |
| $F_2$         | $Z_{21}$, $Z_{22}$              | 51    |
| $F_3$         | $Z_{31}$, $Z_{32}$              | 51    |
| $F_4$         | $Z_{41}$, $Z_{42}$              | 51    |
| $F_5$         | $Z_{51}$, $Z_{52}$              | 51    |
| $F_6$         | $Z_{61}$, $Z_{62}$              | 51    |
| $F_7$         | $Z_{71}$, $Z_{72}$              | 51    |
| $F_8$         | $Z_{81}$, $Z_{82}$              | 51    |
| $F_9$         | $Z_{91}$, $Z_{92}$              | 51    |
| $F_{10}$      | $Z_{101}$, $Z_{102}$            | 51    |

Based on Table 4, the corresponding aggregated regular fuzzy number $\bar{Z}_{F_i}$ with respect to the $i$-th factor can be obtained by performing Step 6 in Z-CAM procedure. The overall aggregated regular fuzzy numbers are presented in Table 5. Specifically for $F_1$,

$$\bar{Z}_{F_1} = \frac{1}{51} \bar{Z}_{22} + \frac{1}{51} \bar{Z}_{23} + \ldots + \frac{17}{51} \bar{Z}_{52} + \frac{6}{51} \bar{Z}_{53} = (0.4370,0.5785,0.6537).$$

**Table 5. Aggregated regular fuzzy numbers**

| Factor, $F_i$ | Aggregated Regular FN, $\bar{Z}_{F_i}$ | Factor, $F_i$ | Aggregated Regular FN, $\bar{Z}_{F_i}$ |
|---------------|----------------------------------------|---------------|----------------------------------------|
| $F_1$         | $\bar{Z}_{F_1} = (0.4370,0.5785,0.6537)$| $F_6$         | $\bar{Z}_{F_6} = (0.3326,0.4736,0.5761)$|
| $F_2$         | $\bar{Z}_{F_2} = (0.3611,0.4000,0.5962)$| $F_7$         | $\bar{Z}_{F_7} = (0.1194,0.2320,0.3784)$|
| $F_3$         | $\bar{Z}_{F_3} = (0.2835,0.4292,0.5650)$| $F_8$         | $\bar{Z}_{F_8} = (0.3630,0.5079,0.5952)$|
| $F_4$         | $\bar{Z}_{F_4} = (0.2768,0.4280,0.5606)$| $F_9$         | $\bar{Z}_{F_9} = (0.2093,0.3483,0.4910)$|
| $F_5$         | $\bar{Z}_{F_5} = (0.3583,0.5078,0.6265)$| $F_{10}$      | $\bar{Z}_{F_{10}} = (0.3324,0.4673,0.5733)$|

Similarity degrees between the aggregated regular FNs ($\bar{Z}_{F_i}$) of the $i$-th factors, $F_1, i=1,2,\ldots,10$ across the 15 regular FNs ($\bar{Z}_{k^p}$), $k=1,2,\ldots,5$, $p=1,2,3$ of the predefined Z-numbers presented in Table 3 need to be calculated. This step is carried out for the purpose of identifying the best Z-number that would
represent the respondents’ overall preference level for each factor. The similarity degrees of the pairs are derived by performing Step 7 in the Z-CAM procedure, and the calculated values are displayed in table 6.

**Table 6. Similarity degrees between \( \tilde{Z}_{F_i} \) and \( \tilde{Z}_{kp} \)**

| Aggregated FN for Factor, \( F_i \) | \( \tilde{Z}_{F_1} \) | \( \tilde{Z}_{F_2} \) | \( \tilde{Z}_{F_3} \) | \( \tilde{Z}_{F_4} \) | \( \tilde{Z}_{F_5} \) | \( \tilde{Z}_{F_6} \) |
|------------------------------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Similarity Degree, \( S_{kp}(\tilde{Z}_{F_i}, \tilde{Z}_{kp}) \) | \( \tilde{Z}_{11} \) | \( \tilde{Z}_{12} \) | \( \tilde{Z}_{13} \) | \( \tilde{Z}_{21} \) | \( \tilde{Z}_{22} \) | \( \tilde{Z}_{23} \) | \( \tilde{Z}_{31} \) | \( \tilde{Z}_{32} \) | \( \tilde{Z}_{33} \) |
| \( \tilde{Z}_{F_1} \) | 0.6441 | 0.6467 | 0.6511 | 0.6766 | 0.6940 | 0.7259 | 0.7442 | 0.7992 | 0.9149 |
| \( \tilde{Z}_{F_2} \) | 0.6797 | 0.6825 | 0.6875 | 0.7159 | 0.7355 | 0.7713 | 0.7920 | 0.8547 | 0.9883 |
| \( \tilde{Z}_{F_3} \) | 0.7079 | 0.7110 | 0.7164 | 0.7473 | 0.7686 | 0.8079 | 0.8306 | 0.8998 | 0.9821 |
| \( \tilde{Z}_{F_4} \) | 0.7092 | 0.7123 | 0.7177 | 0.7488 | 0.7702 | 0.8096 | 0.8324 | 0.9020 | 0.9752 |
| \( \tilde{Z}_{F_5} \) | 0.6722 | 0.6750 | 0.6798 | 0.7076 | 0.7267 | 0.7617 | 0.7818 | 0.8429 | 0.9725 |
| \( \tilde{Z}_{F_6} \) | 0.6886 | 0.6915 | 0.6966 | 0.7258 | 0.7459 | 0.7828 | 0.8041 | 0.8689 | 0.9930 |
| \( \tilde{Z}_{F_7} \) | 0.8179 | 0.8220 | 0.8292 | 0.8709 | 0.9000 | 0.9543 | 0.9028 | 0.9391 | 0.9862 |
| \( \tilde{Z}_{F_8} \) | 0.6741 | 0.6769 | 0.6818 | 0.7098 | 0.7290 | 0.7642 | 0.7845 | 0.8460 | 0.9766 |
| \( \tilde{Z}_{F_9} \) | 0.7496 | 0.7531 | 0.7591 | 0.7940 | 0.8181 | 0.8627 | 0.8887 | 0.9684 | 0.9156 |

| Aggregated FN for Factor, \( F_i \) | \( \tilde{Z}_{F_1} \) | \( \tilde{Z}_{F_2} \) | \( \tilde{Z}_{F_3} \) | \( \tilde{Z}_{F_4} \) | \( \tilde{Z}_{F_5} \) | Output \( Z \)-Number |
|------------------------------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Similarity Degree, \( S(\tilde{Z}_{F_i}, \tilde{Z}_{kp}) \) | \( \tilde{Z}_{A1} \) | \( \tilde{Z}_{A2} \) | \( \tilde{Z}_{A3} \) | \( \tilde{Z}_{A4} \) | \( \tilde{Z}_{A5} \) | \( \tilde{Z}_{A6} \) |
| \( \tilde{Z}_{F_1} \) | 0.8267 | **0.9421** | 0.8918 | 0.8810 | 0.9180 | 0.7872 | \( Z_{A2} \) |
| \( \tilde{Z}_{F_2} \) | 0.8862 | 0.9468 | 0.8292 | 0.9489 | 0.8518 | 0.7380 | \( Z_{A3} \) |
| \( \tilde{Z}_{F_3} \) | 0.9348 | 0.8797 | 0.7773 | **0.9967** | 0.7971 | 0.6966 | \( Z_{A4} \) |
| \( \tilde{Z}_{F_4} \) | 0.9371 | 0.8742 | 0.7730 | **0.9896** | 0.7926 | 0.6931 | \( Z_{A5} \) |
| \( \tilde{Z}_{F_5} \) | 0.8735 | 0.9377 | 0.8222 | 0.9343 | 0.8445 | 0.7324 | \( Z_{A6} \) |
| \( \tilde{Z}_{F_6} \) | 0.9014 | 0.9117 | 0.8021 | 0.9663 | 0.8232 | 0.7164 | \( Z_{A7} \) |
| \( \tilde{Z}_{F_7} \) | 0.8994 | 0.7577 | 0.6805 | 0.8429 | 0.6956 | 0.6178 | \( Z_{A8} \) |
| \( \tilde{Z}_{F_8} \) | 0.8768 | 0.9438 | 0.8269 | 0.9381 | 0.8493 | 0.7361 | \( Z_{A9} \) |
| \( \tilde{Z}_{F_9} \) | **0.9972** | 0.8260 | 0.7350 | 0.9282 | 0.7527 | 0.6624 | \( Z_{A10} \) |
| \( \tilde{Z}_{F_{10}} \) | 0.9052 | 0.9169 | 0.8062 | 0.9707 | 0.8275 | 0.7197 | \( Z_{A11} \) |

The similarity degree between factors’ aggregated FNs (\( \tilde{Z}_{F_i} \)) and all the 15 predefined FNs (\( \tilde{Z}_{kp} \)) associated with the \( Z \)-numbers are compared. The highest values will determine the output \( Z \)-numbers which represent the factors’ overall final linguistic ratings and confidence levels. As an illustration, based on table 6, for Factor \( F_1 \), the highest similarity degree of 0.9421 belongs to the pair of \( \tilde{Z}_{F_1} \) and \( \tilde{Z}_{A2} \). Referring to table 2, the corresponding output \( Z \)-number for \( \tilde{Z}_{A2} \) is \( Z_{A2}=(A,M) \). Hence, it can be
interpreted that the respondents Agree (A) with Medium (M) confidence level that Factor $F_1$ influenced their selection on selecting a postgraduate program. Similar interpretations are applied to the rest of the factors. The overall output Z-numbers, linguistic ratings, confidence levels as well as the ranking positions of factors are displayed in table 7.

| Factor                        | Output Z-Number | Linguistic Rating | Confidence Level | Ranking position |
|-------------------------------|-----------------|-------------------|------------------|------------------|
| Academic background ($F_1$)   | ($A,M$)         | Agree (A)         | Medium (M)       | 3                |
| University’s facilities ($F_2$) | ($MA,H$)       | Moderately Agree (MA) | High (H)         | 6                |
| Program reputation ($F_3$)    | ($SA,L$)        | Strongly Agree (SA) | Low (L)          | 1                |
| Teaching quality ($F_4$)      | ($SA,L$)        | Strongly Agree (SA) | Low (L)          | 2                |
| Peer influence ($F_5$)        | ($MA,H$)        | Moderately Agree (MA) | High (H)         | 9                |
| Tuition fees ($F_6$)          | ($MA,H$)        | Moderately Agree (MA) | High (H)         | 5                |
| Family support ($F_7$)        | ($MA,L$)        | Moderately Agree (MA) | Low (L)          | 10               |
| Financial support ($F_8$)     | ($MA,H$)        | Moderately Agree (MA) | High (H)         | 8                |
| Interest towards program ($F_9$) | ($A,L$)     | Agree (A)         | Low (L)          | 4                |
| Career opportunity ($F_{10}$) | ($MA,H$)        | Moderately Agree (MA) | High (H)         | 6                |

Table 7. Overall output Z-numbers, and ranking of factors

Note that for each factor, a confidence level is associated for each output linguistic rating that describes how confident the respondents are in assigning the ratings. Based on the output linguistic ratings and confidence levels, the factors could also be ranked. Nevertheless, should a tie occur between two or more factors whereby these factors have similar output Z-numbers, the degrees of similarity can be used to determine the ranking positions of the associated factors. As an example, based on table 7, similar output Z-number i.e. (SA,L) belongs to factors ($F_3$) and ($F_4$). Table 6 shows that the similarity degrees between the corresponding aggregated Z-numbers of each $F_3$ and $F_4$ with the predefined Z-numbers $\tilde{Z}_{51}$ are $S(\tilde{Z}_{F_3},\tilde{Z}_{51}) = 0.9967$ and $S(\tilde{Z}_{F_4},\tilde{Z}_{51}) = 0.9896$, respectively. Hence, $F_3$ is ranked higher than $F_4$. However, factors $F_2$ and $F_{10}$ are equally ranked as they produce similar degree of similarity i.e. $S(\tilde{Z}_{F_2},\tilde{Z}_{33}) = S(\tilde{Z}_{F_{10}},\tilde{Z}_{33}) = 0.9883$. Overall, the ordering of factors is $F_5 \succ F_4 \succ F_1 \succ F_9 \succ F_6 \succ F_7 \approx F_{10} \succ F_8 \succ F_3$ with $\succ$ and $\approx$ indicating higher ranking position and equal ranking position, respectively.

4. Conclusion
A conjoint analysis procedure integrating the Z-numbers is proposed in this paper. The procedure known as Z-number-based conjoint analysis method (Z-CAM) has an advantage over existing fuzzy conjoint analysis method (FCAM) in the sense that the former integrates the element of certainty in terms of confidence levels in rating the attributes or factors. In other words, the extension of the FCAM into Z-number environment, allows the ratings to be supported or strengthened by the decision makers’ or respondents’ confidence levels. In addition, ranking of attributes or factors can also be performed based on the output Z-numbers and the corresponding similarity degrees of the associated regular fuzzy numbers. For future works, other variants of similarity functions could be used to measure the degrees of similarity between compared regular fuzzy numbers of the associated Z-numbers. Comparisons of attribute rating and rankings by Z-CAM and other Z-number-based methods could also be done to further investigate the performance and robustness of the proposed procedure.
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