Q^2 evolution of parton distributions at small x
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We investigate the Q^2 evolution of parton distributions at small x values, obtained in the case of flat initial conditions. The results are in excellent agreement with deep inelastic scattering experimental data from HERA.

1. Introduction

The measurements of the deep-inelastic scattering structure function F_2 in HERA \cite{1} have permitted the access to a very interesting kinematical range for testing the theoretical ideas on the behavior of quarks and gluons carrying a very low fraction of momentum of the proton, the so-called small x region. In this limit one expects that non-perturbative effects may give essential contributions. However, the reasonable agreement between HERA data and the next-to-leading order (NLO) approximation of perturbative QCD that has been observed for Q^2 > 1 GeV^2 (see the recent review in \cite{2}) indicates that perturbative QCD could describe the evolution of structure functions up to very low Q^2 values, traditionally explained by soft processes. It is of fundamental importance to find out the kinematical region where the well-established perturbative QCD formalism can be safely applied at small x.

The standard program to study the small x behavior of quarks and gluons is carried out by comparison of data with the numerical solution of the DGLAP equations by fitting the parameters of the x profile of partons at some initial Q^2_0 and the QCD energy scale \Lambda (see, for example, \cite{3,4}). However, if one is interested in analyzing exclusively the small x region (x \leq 0.01), there is the alternative of doing a simpler analysis by using some of the existing analytical solutions of DGLAP in the small x limit (see \cite{5} for review). This was done so in Ref. \cite{6}-\cite{8} where it was pointed out that the HERA small x data can be interpreted in terms of the so called doubled asymptotic scaling phenomenon related to the asymptotic behavior of the DGLAP evolution discovered in \cite{8} many years ago.

Here we illustrate results obtained recently in \cite{6}. These results are the extension to the NLO QCD approximation of previous leading order (LO) studies \cite{5,7}. The main ingredients are:

1. Both, the gluon and quark singlet densities are presented in terms of two components (\'+\' and \'-\') which are obtained from the analytical Q^2 dependent expressions of the corresponding (\'+\' and \'-\') parton distributions moments.
2. The \'-\' component is constant at small x, whereas the \'+\' component grows at Q^2 \geq Q^2_0 as \sim \exp(\sigma), where
\[
\sigma = 2\sqrt{(\hat{d}_s + \hat{D}_p)\ln x},
\]
and the LO term \hat{d}_+ = -12/\beta_0 and the NLO one \hat{D}_\pm = \hat{d}_{\pm\pm} + \hat{d}_\pm \beta_1/\beta_0 with \hat{d}_{\pm\pm} = 412f/27\beta_0. Here the coupling constant a_s = \alpha_s/(4\pi), s = \ln(\alpha(Q^2_0)/\alpha(Q^2)) and p = \alpha(Q^2_0) - \alpha(Q^2), \beta_0 and \beta_1 are the first two coefficients of QCD \beta\text{-function and } f \text{ is the number of active flavors.}

2. Basical formulae

Thus, our purpose is to demonstrate the small x asymptotic form of parton distributions in the framework of the DGLAP equation starting at
some \( Q_0^2 \) with the flat function:

\[
f_a(Q_0^2) = A_a \quad (a = q, g),
\]

where \( f_a \) are the parton distributions multiplied by \( x \) and \( A_a \) are unknown parameters that have to be determined from data. Through this work at small \( x \) we neglect the non-singlet quark component.

In \cite{5} an effective method to reproduce the \( x \)-dependence of parton distributions has been developed. It is based on a separation of the singular and regular parts of the exact solution for the moments of parton distributions and on the method to replace Mellin convolution by usual products \cite{4}. The method allows in simplest way to reproduce the LO results \cite{5} and to construct the \( x \)-dependence of parton densities at NLO \cite{4}:

\[
\begin{align*}
\sigma & = \left[ A_q^+(Q^2, Q_0^2) + A_g^+(Q^2, Q_0^2) \right] - A_q^-(Q^2, Q_0^2) - A_g^-(Q^2, Q_0^2), \\
\lambda_q & = 2 \rho K_1(\rho) + 20 \rho Q^2 I_0(\rho), \\
\lambda_g & = f \left( f_q(x, Q^2) + \frac{2}{3} f(2 \rho Q^2) f_g(x, Q^2) \right),
\end{align*}
\]

where \( I_n(\sigma) \) are modified Bessel functions, which have \( \nu \)-independent limit \( \exp(\sigma \nu) \) at \( \sigma \to \infty \), \( e = (\sum_i \epsilon_i^2)/f \) is the average charge square for \( f \) active quarks, \( \rho = \sigma/2\ln(1/x) \) and the magnitudes

\[
\begin{align*}
A_q^+(Q^2, Q_0^2) & = A_q^-(Q^2, Q_0^2) = \left[ 1 - \frac{80}{81} f(2 \rho Q^2) \right] A_q, \\
A_g^+(Q^2, Q_0^2) & = A_g^-(Q^2, Q_0^2) = A_g - A_g^-(Q^2, Q_0^2), \\
A_q^+ & = \frac{f}{9} \left( A_q + \frac{4}{9} A_g \right), \\
A_q^- & = A_q - 20 \rho(Q^2) A_q^+.
\end{align*}
\]

The regular parts of the terms \( d_\pm \) and \( D_\pm \) have the form \cite{5}

\[
\begin{align*}
\overline{d}_+(1) & = 1 + \frac{4}{3\beta_0} f, \\
\overline{d}_-(1) & = \frac{16}{27 \beta_0} f, \\
\overline{d}_+(1) & = \frac{8}{\beta_0} \left( 36 \zeta_3 + 33 \zeta_2 - \frac{1643}{12} \right), \\
& + \frac{2}{9} f \left[ \frac{68}{9} - 4 \zeta_2 - \frac{13}{243} f \right], \\
d_- & = \frac{16}{9\beta_0} \left( 2 \zeta_3 - \zeta_2 + \frac{13}{4} \right), \\
& + f \left[ 4 \zeta_2 - \frac{23}{18} + \frac{13}{243} f \right],
\end{align*}
\]

where \( \zeta_n \) are Euler \( \zeta \)-functions.

3. Slopes

The behaviour of eqs. \cite{5} can mimic a power law shape over a limited region of \( x, Q^2 \):

\[
\begin{align*}
\sigma_f(x, Q^2) & \sim x^{-\lambda_q^{eff}(x, Q^2)} \quad \text{and} \\
F_2(x, Q^2) & \sim x^{-\lambda_g^{eff}(x, Q^2)}
\end{align*}
\]

The quark and gluon effective slopes \( \lambda^{eff}_q \) are reduced by the NLO terms that leads to the decreasing of the gluon distribution at small \( x \). For the quark case it is not the case, because the normalization factor \( A_q^{eff} \) of the \( \text{"+" component} \) produces an additional contribution undampening as \( \sim (\ln x)^{-1/2} \).

The gluon effective slope \( \lambda_g^{eff} \) is larger than the quark slope \( \lambda_q^{eff} \), that is in excellent agreement with a recent MRS and GRV analyses \cite{5}.

Indeed, the effective slopes have the asymptotical values (at large \( Q^2 \)):

\[
\begin{align*}
\lambda_{g, a}^{eff, as}(x, Q^2) & \approx \rho - \frac{1}{4 \ln(1/x)}, \\
\lambda_{g, a}^{eff, as}(x, Q^2) & \approx \rho - \frac{3}{4 \ln(1/x)}, \quad (5)
\end{align*}
\]

where symbol \( \approx \) marks approximations obtained by expansions of modified Bessel functions \( I_n(\sigma) \). The slope \( \lambda_{F2}^{eff, as}(x, Q^2) \) lies between

\( \frac{3 \alpha(Q^2)}{\ln(1/x)} \).
4. Results of the fits

With the help of the results obtained in the previous section we have analyzed $F_2$ HERA data at small $x$ from the H1 collaboration (first article in [1]). In order to keep the analysis as simple as possible we have fixed $\Lambda_{\overline{MS}}(n_f = 4) = 250$ MeV which is a reasonable value extracted from the traditional (higher $x$) experiments. The initial scale of the parton densities was also fixed into the fits to $Q^2_0 = 1$ GeV$^2$, although later it was released to study the sensitivity of the fit to the variation of this parameter. The analyzed data region was restricted to $x < 0.01$ to remain within the kinematical range where our results are accurate. Finally, the number of active flavors was fixed to $f=4$.

Fig. 1 shows $F_2$ calculated from the fit with $Q^2 > 1$ GeV$^2$ in comparison with H1 data. Only the lower $Q^2$ bins are shown. One can observe that the NLO result (dot-dashed line) lies closer to the data than the LO curve (dashed line). The lack of agreement between data and lines observed at the lowest $x$ and $Q^2$ bins suggests that the flat behavior should occur at $Q^2$ lower than 1 GeV$^2$. In order to study this point we have done the analysis considering $Q^2_0$ as a free parameter. Comparing the results of the fits (see [6]) one can notice the better agreement with the experiment of the NLO curve at fitted $Q^2_0 = 0.55$ GeV$^2$ (solid curve) is apparent at the lowest kinematical bins.

Finally with the help of Eqs. (5) we have estimated the $F_2$ effective slope using the value of the parameters extracted from NLO fits to data. For H1 data we found $0.05 < \lambda^{eff,as}_{F_2} < 0.30 - 0.37$. The lower (upper) limit corresponds to $Q^2 = 1.5$ GeV$^2$ ($Q^2 = 400$ GeV$^2$). The dispersion in some of the limits is due to the $x$ dependence. Fig. 2 shows that the three types of asymptotical slopes have similar values, which are in very good agreement with H1 data (presented also in Fig. 2). The NLO values of $\lambda^{eff,as}_{F_2}$ lie between the quark and the gluon ones but closer to the quark slope $\lambda^{eff,as}_q$. These results are in excellent agreement with those obtained by others (see the review [2] and references therein).

5. Conclusions

We have shown that the results developed recently in [6] have quite simple form and reproduce
many properties of parton distributions at small $x$, that have been known from global fits.

We found very good agreement between our approach based on QCD at NLO approximation and HERA data, as it has been observed earlier with other approaches (see the review [2]). Thus, the nonperturbative contributions as shadowing effects, higher twist effects and others seems to be quite small or seems to be canceled between them and/or with $\ln(1/x)$ terms containing by higher orders of perturbative theory (see discussion also in [10]). In our opinion, this very good agreement between approaches based on perturbative QCD and HERA data may be explained also by the fact that at low $x$ values the real effective scale of coupling constant is like $Q^2/x^c$, where $1/2 \leq c \leq 1$ (see [11]). To clear up the correct contributions of nonperturbative dynamics and higher orders containing large $\ln(1/x)$ terms, it is necessary more precise data and further efforts in developing of theoretical approaches.

Acknowledgments. One of the authors (A.V. K.) would like to express his sincerely thanks to the Organizing Committee and especially to R. Fiore and A. Pappa for the kind invitation and the financial support at such remarkable Conference, and V.S. Fadin, L.L. Jenkovszky and L.N Lipatov for fruitful discussions. A.V.K. and G.P. were supported in part, respectively, by Alexander von Humboldt fellowship and RFBR (98-02-16923) and by Xunta de Galicia (PPI20615PR) and CICYT (AEN99-0589-C02-02).

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