Discovery of Four Space Dimensions in a Sphere for Electronic Orbitals in a Neon Shell

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Abstract. For the first time we discover four space dimensions in a sphere, namely two periphery points, two semicircular arcs, two hemispherical surfaces, and two spherical solids. When these four geometric elements are compared with electronic orbitals in a neon shell, we draw a conclusion that while 2s orbitals are spherical, three types of 2p orbitals in the X, Y, and Z directions are different in the number of space dimensions. These anisotropic 2p orbitals prompt us to modify Schrödinger’s equation to reflect these fine space gradients. Within the atomic sphere, electrons are transforming from one dimension to another following the rule of calculus implemented by circular functions rather than position movement. An electron may exist in the physical shape of a point, an arc, a surface, or a solid. Eight electrons in the neon shell are undergoing various harmonic oscillations that form a continuous octet cycle. This motion feature is derived from a four-dimensional space harmonic oscillation equation coupled with a one-dimensional time harmonic oscillation equation in synthesis of the modified Schrödinger’s equation. Substantial amount of evidence can be found in organic chemistry where a carbon atom has an anisotropic electron configuration of 2s2p_x2p_y2p_z. By furnishing a fresh and fundamental spacetime worldview, the quaternion theory may change the direction of quantum mechanics from discrete linear algebra toward continuous dynamic calculus in the future.

1. Introduction
The perception of space and time dimensions is of paramount important in natural sciences because it sets the stage for any fundamental research work. For space dimensions, we tend to follow the Cartesian coordinate system that there are three dimensions: X-dimension, Y-dimension, and Z-dimension. For time dimensions, we tend to regard Newtonian uniform time as one-dimensional. Therefore it is traditionally believed that space is three-dimensional while time is one-dimensional, both coupled into a four-dimensional spacetime by events. Other perceptions of spacetime seem out of the question. Even though we are taught in linear algebra about n-dimensional vector space, geometrical shapes of more than three space dimensions have never been depicted satisfactorily. Here we demonstrate that the crucial step to delineating four-dimensional space is to define four geometric elements corresponding to four dimensions, namely points, arcs, surfaces, and solids in a full sphere. Accurate definitions of space and time help us understand the world entities better [1]. Among other things, the discovery of four space dimensions provides us a sound scientific foundation for defining electronic orbitals in a four-dimensional neon shell.
2. Four Static Space Dimensions

We shall stand at the center of a circle and look around the circle. A circle is composed of two lines and two points (figure 1). For each point \( A \) in the lines, there is a point \( a \) in the circle corresponding to it; and for each point \( b \) in the cycle except points \( P_1 \) and \( P_2 \), there is a point \( B \) in the lines corresponding to it. Hence we find point-to-point projective correspondences between the circle and two lines plus two points. If we treat points \( P_1 \) and \( P_2 \) as a geometric dimension and lines \( L_1 \) and \( L_2 \) as another, then the circle possesses two dimensions. In the same way, a spherical surface corresponds to two flat planes, two straight lines, and two points. Hence a spherical surface has three space dimensions in stairs: one-dimensional points \( P_1 \) and \( P_2 \), two-dimensional lines \( L_1 \) and \( L_2 \), and three-dimensional planes \( S_1 \) and \( S_2 \).
inner one within an even larger sphere; and an inner sphere may become an outer one as we shift our focus to it. This gives scalability of the four dimensions for constructing multiple spherical layers.

3. Four Dynamic Space Dimensions

What is a dimension? Here we shall establish the concept in a way that is slightly different from that in linear algebra. Traditionally, we take for granted that length, width, and height of an object constitute three space dimensions or orientations. Three Cartesian coordinate axes represent three dimensions best. In this way, a real number axis along the X direction typically denotes a dimension. For a spherical structure, if we can establish that the geometric variable has a point-to-point correspondence with a real number axis, then the structure would qualify as a dimension. It is easy to relate a circle Q to an axis line. If we stand at point S and look at the real number axis tangential to the circle, it is obvious that there is a point-to-point correspondence between the circle except point S and the real number axis (figure 3). Hence a circle, a uniform circular motion, or a harmonic oscillation cycle constitutes a dimension strictly without conflicting with linear algebra.

![Figure 3](image)

**Figure 3.** Perspective correspondence between any point A on the circle and a point A’ on the real number axis by the observer standing on a point S.

Quaternity space means four space dimensions of harmonic oscillations in a full sphere. The first dimension is the displacement of two particles from center O along the radial directions (figure 4). At any specific moment, a particle is moving harmonically from center O to M, which is the vertical projection of point G in uniform circular motion.

![Figure 4](image)

**Figure 4.** Quaternity space in a solid sphere defined as (a) one-dimensional moving points, (b) two-dimensional stretching arcs, (c) three-dimensional extending surfaces, and (d) four-dimensional imploding solids. Each downstream dimension always starts with the final state of the upstream one and extends in the perpendicular direction.
Another particle \( N \) travels from spherical center \( O \) towards \( B \) along the radial line \( OB \) following the harmonic oscillation principle corresponding to uniform motion along semicircular arc \( OHB \). As shown in dashed arcs in figure 4(a), both arcs \( OGA \) and \( OHB \) may combine into a close loop or a space dimension. The first dimension explains both \( 2p_x \) electrons displacing off the nuclear center towards periphery points, increasing their potential energy from the \( 2s \) electron at the origin.

The second dimension is the stretching of both particles \( A \) and \( B \) into both arcs uniformly, one stretching along semicircle \( ACB \) and another along \( BDA \). They finally form a full circle. The circle is the actual matter shape as well as the path of harmonic oscillation front points \( C \) and \( D \) (figure 4b). This harmonic oscillation is slightly different from the conventional sense because of its stretching action instead of point displacement along the path.

The third dimension involves the rotation of circular arc \( ACBD \) around \( AB \) axis for \( \pi \) radian angle uniformly so that both semicircular arcs smear evenly over both shaded ribbons and over both hemispherical surfaces in the end (figure 4c). When applied to the explanation of electronic orbitals, this dimension transforms two electrons from both semicircular arcs into both hemispherical surfaces.

The fourth dimension is the implosion of both hemispherical surfaces \( AEB \) and \( BFA \) to fill the entire sphere. This process also follows the general harmonic oscillation principle where the imploding front \( S \) proceeds smoothly along the hypothetical semicircular arc \( ESO \) while \( T \) along \( FTO \) synchronously (figure 4d). The state front points \( S \) and \( T \) define the thickness of the hollow ball, which finally becomes a full solid one when both points meet. These four space dimensions are mutually exclusive, perpendicular, but continuous in a sphere.

By the way, the time dimension is never independent of any evolving space dimensions. Dynamic transformation of any space dimension manifests a steady electron flow with order that features the time property. Without a time dimension, space dimensions are only a snapshot like those shown in figure 2. Newtonian uniform time is useful because it establishes a universal clock for comparing the speed of various space dimensions in transformation.

Quaternity space is harmonic oscillation waves of four continuous dimensions within a sphere. It follows immediately that this four-dimensional space model might be used to characterize the structure of atoms and cells in nature [2, 3]. Specifically, the division of four space dimensions in a sphere can be applied to the characterization of four pairs of electrons \( 2s^22p_x^22p_y^22p_z^2 \) in a neon shell, each pair colonizing a dimension. These four pairs of electrons correspond to the four space dimensions in a sphere (figures 2 and 4). We assume that an electron may exist in the form of a point, an arc, a surface, or a solid even though it features a particle when physically detected by traditional experimental instruments. Although the use of the wave-particle duality has worked well in physics, the wave and particle relation or conversion mechanism has not been satisfactorily resolved [4]. The assumption of diverse electronic shapes accounts for it perfectly. This geometrical model of electronic orbitals is a complement to quantum mechanical description of electronic orbitals.

By adopting the Laplacian operator, \( \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \), quantum mechanics treats electronic orbitals isotropic in the \( X \), \( Y \), and \( Z \) directions. However, this treatment is questionable because it is inadvertently framed by the Cartesian coordinates. A theater ticket to a great auditorium may illustrate the dimensional difference between the \( X \), \( Y \), and \( Z \) axes (figure 5). An \( X \) reading indicates a seat, many seats form a tier of a \( Y \) reading, and many tiers form a floor of a \( Z \) reading. In the same manner, three types of \( 2p \) electronic orbitals are different in geometrical shapes of points, arcs, and surfaces corresponding to the first three dimensions of figure 2, the fourth dimension being spherical \( 2s \) orbitals [1]. We carefully distinguish the dimensional differences of three \( 2p \) electron orbitals in space. In formulating multi-dimensional wave equations for electrons in a neon shell, our quaternity theory climbs to a higher rank of derivatives than the second derivative and embodies four space dimensions in \( \frac{\partial}{\partial t}, \frac{\partial^2}{\partial t^2}, \frac{\partial^3}{\partial t^3}, \frac{\partial^4}{\partial t^4} \), and \( \frac{\partial}{\partial t} \) forms rather than using the isotropic Laplacian operator. The relationship between adjacent types of electronic orbitals is a calculus operation instead of linear algebra. This is the law of nature for the motion of dimensions.
4. The Modified Schrödinger’s Equation

Four dimensions in a sphere may be applied to the characterization of the structure of spherical entities such as a neon atomic shell. The traditional three-dimensional Schrödinger’s equation (1) is:

$$i\hbar \frac{\partial \Phi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Phi + i \Phi,$$

where $\hbar$ is the rationalized Planck’s constant, $m$ is the mass of electron, $\Phi$ is the wave function and $V$ is the potential energy of the particle. Recognizing dimension gradients of three 2p orbitals in the X, Y, and Z directions, we have a modified Schrödinger’s equation (2):

$$i\hbar \frac{\partial \Phi}{\partial t} = -\frac{\hbar^2}{2m\psi^2} \left( \frac{\partial}{\partial l} + \frac{\partial^2}{\partial l^2} + \frac{\partial^3}{\partial l^3} \right) \Phi + i \Phi,$$

where three terms in the parentheses correspond to 2p$_x$, 2p$_y$, and 2p$_z$ electronic orbitals in the neon shell and $i \Phi$ corresponds to the inner spherical 2s orbital or the potential energy term in equation (3):

$$i \Phi = -\frac{\hbar^2}{2m\psi^2} \frac{\partial^4 \Phi}{\partial l^4}.$$

This modified Schrödinger’s equation is stricter than the conventional one because it distinguishes the fine levels of space dimensions among X, Y, and Z directions. The anisotropic 2p orbitals in carbon atoms can explain the structure of many organic molecules perfectly [1]. The modified Schrödinger’s equation is a four-dimensional equation governing four types of electronic orbitals, 2s2p$_x$2p$_y$2p$_z$, in their respective dimensions individually and simultaneously. The solutions to this equation require the introduction of spherical quantities in dynamic calculus, which provide complementary accounts to physical quantities with linear algebra.

5. Dynamic Calculus of Spherical Quantities

The harmonic oscillations described in figure 4 are not particles in movement except the first dimension. They represent dimensional shift from one dimension to another that involves unit transformation rather than magnitude change. The electrons are undergoing harmonic oscillations because their front points are tracing the semicircular arcs uniformly in figure 4. During the process, their space quantities at any moment are measured by their subtended chords sinusoidally. Therefore, we invent spherical quantities in dynamic calculus implemented by circular functions instead of discrete physical quantities. A spherical quantity has a fixed magnitude and a continuously varying dimension, which is complementary to a physical quantity that has a fixed dimension and a variable magnitude [3]. The operation on a spherical quantity is by the rule of calculus rather than linear algebra. A spherical quantity characterizes electronic motion by sinusoidal dimension transformation...
instead of position displacement. However, a spherical quantity at a whole dimension position corresponds to a physical quantity of that integer unit dimension.

Assuming wave function $\Phi$ is composed of time spherical quantity $\Theta$ and space spherical quantity $\theta$, we have equation (4):

$$\Phi = \Theta \cdot \theta.$$  

The modified Schrödinger’s equation (2) can be rewritten into

$$i\hbar \frac{1}{\Theta} \frac{\partial \Theta}{\partial t} = - \frac{\hbar^2}{2m\psi^2\Theta} \left( \frac{d}{dl} + \frac{d^2}{dl^2} + \frac{d^3}{dl^3} \right) \Theta - \frac{\hbar^2}{2m\psi^2\Theta} \cdot \frac{d^4\Theta}{dl^4},$$  

which can be separated into time (equation 6) and space (equation 7) components by assigning both sides of equation (5) into a constant energy $E$ following the practice of quantum mechanism:

$$i\hbar \frac{1}{\Theta} \frac{\partial \Theta}{\partial t} = E,$$  

$$- \frac{\hbar^2}{2m\psi^2\Theta} \left( \frac{d}{dl} + \frac{d^2}{dl^2} + \frac{d^3}{dl^3} \right) \Theta - \frac{\hbar^2}{2m\psi^2\Theta} \cdot \frac{d^4\Theta}{dl^4} = E.$$  

Because the spherical quantity for space $\theta$ in its various dimensions is always varying sinusoidally, we may adopt a circular function to represents its transformation process in equations (8) and (9):

$$\theta = A \cos \psi,$$  

$$\psi' = - \frac{d\psi}{dl}.$$  

where $A$ is a dimension constant. Orbital radius $\frac{1}{\psi'}$ has some basic relationships with energy $E$ and momentum $p$ for a 2s2p electron in equations (10) and (11):

$$- \frac{1}{\psi'} = \frac{\hbar}{p},$$  

$$- E = \frac{p^2}{2m},$$  

whence we obtain equation (12):

$$- \frac{\hbar^2}{2mE} = \frac{1}{\psi'^4}.$$  

Substituting this equation into equation (7) yields equation (13):

$$\frac{1}{\psi'^4} \left( \frac{d^4\Theta}{dl^4} + \frac{d^3\Theta}{dl^3} + \frac{d^2\Theta}{dl^2} + \frac{d\Theta}{dl} \right) = \Theta.$$  

This is a quaternity equation for space in a neon shell [1], which indicates that performing differential operations four times consecutively upon $\theta$ with respect to space dimension $l$ returns it to the same orbital type in the neon shell. The data structure of spherical quantities in dynamic calculus is like a
dimensional array in computer programming languages. From equations (8) and (9), we get equations (14) to (17):

\[
\frac{d\theta}{dl} = -\sin \psi \cdot \frac{dy}{dl} = \psi' \sin \psi, \quad (14)
\]

\[
\frac{d^2\theta}{dl^2} = \psi' \cdot \cos \psi \cdot \frac{dy}{dl} = -\psi'^2 \cos \psi, \quad (15)
\]

\[
\frac{d^3\theta}{dl^3} = -\psi'^2 \cdot (-\sin \psi) \cdot \frac{dy}{dl} = -\psi'^3 \sin \psi, \quad (16)
\]

\[
\frac{d^4\theta}{dl^4} = -\psi'^3 \cdot \cos \psi \cdot \frac{dy}{dl} = \psi'^4 \cos \psi. \quad (17)
\]

These calculations amount to equation (18):

\[
\frac{1}{\psi'^4} \frac{d^4\theta}{dl^4} = \theta, \quad (18)
\]

for the relationship between both 2s orbitals with three intervening dimensions \(d\theta/dl\), \(d^2\theta/dl^2\), and \(d^3\theta/dl^3\) serving as the 2p, 2p, and 2p, electronic orbitals respectively. A sphere reduced four dimensions in consecutive differential operations produces the same shape of a different size. Four reduced space dimensions are a radius, both surfaces, both arcs, and both points in sequence, and the final result is a smaller sphere denoted by an inner radius (figure 6). These geometric elements correspond to 2s2p2p2p, 2s, 2s2p, 2s, 2p, 2p, 2p, 2s orbital shapes that are contiguous in atomic space. While equation (13) describes the calculus relationship between spherical quantities of four orbital types, figure 6 is another geometrical expression of the calculus relationship between four geometric elements in a neon sphere besides figures 2 and 4. Four electrons oscillate by differential operations (equations 14 to 17) while another quartet oscillates by integral operations, the reverse operations of equations (14) to (17).

![Figure 6](image)

**Figure 6.** Differential operations upon a sphere four times producing a smaller sphere with a spherical surface, a circle, and two symmetric points in between.

On the other hand, the time component of wave function \(\Phi\) may be treated as a one-dimensional harmonic oscillation in spherical quantities, which is in synchrony with the space component. The
general solution to one-dimensional harmonic oscillation is a circular complex function [1] in equations (19) and (20):

$$\Theta = B (\cos \Psi - i \sin \Psi),$$  \hspace{1cm} \text{(19)}

$$\dot{\Psi} = - \frac{\partial \Psi}{\partial t},$$  \hspace{1cm} \text{(20)}

where $B$ is a dimension constant, $\dot{\Psi}$ is the angular velocity of 2s2p electrons, and $i$ is an imaginary number equal to the characteristic property of the system $\Psi$. The time component oscillates back and forth between both terms of equation (19) by negative differential operations. Note that we assume the previously reported four-dimensional time component $\Theta$ as a one-dimensional spherical quantity for brevity here [2]. There are some important relationships in equations (21) and (22).

$$i\Theta = B(i \cos \Psi + \sin \Psi) = - \frac{\partial \Theta}{\partial \Psi},$$  \hspace{1cm} \text{(22)}

so that we obtain equation (23):

$$\dot{\Psi} i\Theta = \dot{\Psi} \cdot (- \frac{\partial \Theta}{\partial \Psi}) = - \frac{\partial \Psi}{\partial t} \cdot (- \frac{\partial \Theta}{\partial \Psi}) = \frac{\partial \Theta}{\partial t},$$  \hspace{1cm} \text{(23)}

Substituting equations (21) and (23) into equation (6) yields equation (24):

$$i \frac{1}{\Psi} - \frac{\Theta}{\Theta} = E,$$  \hspace{1cm} \text{(24)}

$$\frac{1}{\Psi \dot{\Theta}} \frac{d^2 \Theta}{dt^2} = -1.$$

\hspace{1cm} \text{(25)}

Equation (25) is a well-known one-dimensional harmonic oscillation equation that fulfills its solution in equation (19). Hence we have characterized the spherical quantity $\Theta$ in harmonic oscillation coherently. The time and space components are antiparallel, i.e., time dimension decreases while space increases, and vice versa. The imaginary number $i$ on the right hand side of equation (2) indicates this anti-parallelism.

\textbf{Figure 7.} Dimension diagrams of time component coupled with space component for a $2p_z$ electronic orbital as an example with reference to equations (14) and (19).
For time component, the negative differential operation upon $-B \sin \Psi$ with respect to time $t$ yields $B \cos \Psi$ as radian angle $\Psi$ decreases from $\pi/2$ to 0; and for space component, the differential operation upon $A \cos \varphi$ with respect to space $l$ yields $A \varphi' \sin \varphi$ as radian angle $\varphi$ increases from 0 to $\pi/2$ (figure 7). As long as radian angles $\Psi$ and $\varphi$ keep complementary, the sinusoidal time component coupled with the sinusoidal space components gives 2s2p electronic orbitals smooth harmonic oscillations in four various dimensions as delineated in figure 4. The time component of the electrons effectively gives each electron a dynamic dimension featuring harmonic oscillation for its moving front point (figure 4). Since the moving front point travels uniformly along eight semicircular arcs that connect into a large cycle, eight electrons 2s2p in a neon shell form an octet cycle ($2s \rightarrow 2p_x \rightarrow 2p_y \rightarrow 2p_z \rightarrow 2s \rightarrow 2p_x \rightarrow 2p_y \rightarrow 2p_z \rightarrow 2s$) of general harmonic oscillations.

6. Conclusions
We have discovered four space dimensions in a sphere characteristic of a neon shell. These four geometric elements have dimensional gradients so that the transformation from one element to another must be accounted for by calculus instead of linear point displacement. Applying the spherical structures to the characterization of 2s2p electronic orbitals, we have formulated the modified Schrödinger’s equation with anisotropic 2p orbitals in the X, Y, and Z directions. These three types of orbitals take the shapes of two points, two semicircular arcs, and two hemispherical surfaces. They appear to be the intervening terms in the process of four consecutive dynamic calculus operations upon spherical 2s orbitals. Because the modified Schrödinger’s equation can be separated into a four-dimensional space harmonic oscillation equation and a one-dimensional time harmonic oscillation equation, it governs four space dimensions with one time dimensions individually and simultaneously. Both space and time spherical quantities are synchronized in phase through the complementary radian angles ($\Psi + \varphi = \pi/2$) during dynamic calculus in their respectively wave functions. The modified Schrödinger’s equation delineates vividly eight electrons within a neon shell in general harmonic oscillations in four various space dimensions. Undoubtedly, this original description of four-dimensional space for atomic structure is interesting and revolutionary in quantum mechanics.

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