Computational Photography: High Dynamic Range and Light Fields

Saghi Hajisharif
COMPUTATIONAL PHOTOGRAPHY: HIGH DYNAMIC RANGE AND LIGHT FIELDS

Saghi Hajisharif
Description of the cover image

The cover of this thesis represents computational photography for light field imaging and HDR imaging. The light manifold illustrates the light field and the RGB band is a combination of different color filter arrays (CFAs) with spatially interlaced multi-exposure patterns that have been used for the HDR imaging research presented in this thesis.

Computational Photography: High Dynamic Range and Light Fields

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Abstract

The introduction and recent advancements of computational photography have revolutionized the imaging industry. Computational photography is a combination of imaging techniques at the intersection of various fields such as optics, computer vision, and computer graphics. These methods enhance the capabilities of traditional digital photography by applying computational techniques both during and after the capturing process. This thesis targets two major subjects in this field: High Dynamic Range (HDR) image reconstruction and Light Field (LF) compressive capturing, compression, and real-time rendering.

The first part of the thesis focuses on the HDR images that concurrently contain detailed information from the very dark shadows to the brightest areas in the scenes. One of the main contributions presented in this thesis is the development of a unified reconstruction algorithm for spatially variant exposures in a single image. This method is based on a camera noise model, and it simultaneously resamples, reconstructs, denoises, and demosaics the image while extending its dynamic range. Furthermore, the HDR reconstruction algorithm is extended to adapt to the local features of the image, as well as the noise statistics, to preserve the high-frequency edges during reconstruction.

In the second part of this thesis, the research focus shifts to the acquisition, encoding, reconstruction, and rendering of light field images and videos in a real-time setting. Unlike traditional integral photography, a light field captures the information of the dynamic environment from all angles, all points in space, and all spectral wavelength and time. This thesis employs sparse representation to provide an end-to-end solution to the problem of encoding, real-time reconstruction, and rendering of high dimensional light field video data sets. These solutions are applied on various types of data sets, such as light fields captured with multi-camera systems or hand-held cameras equipped with micro-lens arrays, and spherical light fields. Finally, sparse representation of light fields was utilized for developing a single sensor light field video camera equipped with a color-coded mask. A new compressive sensing model is presented that is suitable for dynamic scenes with temporal coherency and is capable of reconstructing high-resolution light field videos.
Beräkningsfotografi (från engelskans computational photography) kombinerar optik, bildsensorer och beräkning för att utöka eller skapa helt nya möjligheter för kamerabaserad avbildning eller mätning av både helt vardagliga eller mycket specifika scener eller objekt. Forskningsområdet som ligger i skärningspunkten mellan forskningsområdena optik, datorseende och datorgrafik har under det senaste decenniet genererat ett antal nya tillämpningar och forskningsfrågeställningar som kommer ett forma hur vi i framtiden avbildar en scen med en kamera. Två av dessa, vilka också ligger i fokus för den här avhandlingen, är rekonstruktion av bilder med hög dynamiskt omfång (HDR, från engelskans High Dynamic Range) samt komprimering och syntetisering (rendering) av ljusfält (från engelskans light field) för både still- och rörlig bild.

Bilder med högt dynamiskt omfång kan användas inom många olika områden, såsom filmspelnling, datorseende och rendering av syntetiska bilder med fotorealistisk ljussättning för att nämnna några. De innehåller detaljerad information om både mörka och ljusa områden i scener samtidigt, något som bilder tagna med en vanlig kamera inte kan. Tekniken för avbildning i HDR har de senaste åren mognat, och används idag i nästan alla mobiltelefonkameror för att förbättra bildkvalitén. Den första delen av avhandlingen presenterar algoritmer för att rekonstruera HDR-bilder från multipla exponeringar av en bild. Dessa metoder, vilka är baserade på modeller av sensors brusegenskaper, rekonstruerar och reducerar brus i bilden samtidigt som den utökar dess dynamiska omfång.

Den andra delen av avhandlingen handlar om att fånga in och rekonstruera ljusfältsbilder och video i realtid. Till skillnad från traditionell fotografering fångar ljusfält informationen om miljön från alla vinklar och punkter i rymden, inte bara i 2D. Den här informationen kan sedan användas, till exempel för att ändra fokus i den tagna bilden eller för skattning av avstånd eller rekonstruktion av 3D-modeller med hjälp av datorseende-algoritmer. Ytterligare exempel är att använda den här informationen för 3D-visualisering av infångade objekt och miljöer, till exempel för tillämpning inom virtuell verklighet (VR). Ljusfält är mycket hög-dimensionell data, vilket innebär utmaningar för insamling, lagring och visualisering av datan. Den här avhandlingen presenterar en rad algoritmer och metoder som löser dessa utmaningar dels genom att reducera datamängden vid insamling och lagring, samt genom effektiva rekonstruktionsalgoritmer specifikt designade för visualisering av stora mängder visuell data i realtid.
“For my part I know nothing with any certainty, but the sight of the stars makes me dream.”

Vincent Van Gogh
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on the HDR imaging project and learned a lot from his passion for this field. Nothing would have been possible without Per Larsson who was behind designing and implementing our capturing devices for light fields and HDR. I would also like to extend my thanks to my former and current colleagues at VCL: Andrew Gardner, Erik Olsson, Gabriel Baravdish, Karin Stacke and Mildi Poceviciute. Every day spent with you was a fun day of research, and I enjoyed our interesting conversations! It was great to be working with you guys!

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Saghi Hajisharif
Norrköping, January 2020
This thesis includes research that is based on the publications listed below:

- S. Hajisharif, J. Kronander, and J. Unger, “HDR Reconstruction for Alternating Gain (ISO) Sensor Readout,” in *Eurographics 2014 - Short Papers*, E. Galin and M. Wand, Eds. The Eurographics Association, 2014

- S. Hajisharif, J. Kronander, and J. Unger, “Adaptive DualISO HDR Reconstruction,” *EURASIP Journal on Image and Video Processing*, vol. 2015, no. 41, Dec 2015

- E. Miandji, S. Hajisharif, and J. Unger, “A Unified Framework for Compression and Compressed Sensing of Light Fields and Light Field Videos,” *ACM Trans. Graph.*, vol. 38, no. 3, pp. 23:1–23:18, May 2019

- S. Hajisharif, E. Miandji, P. Larsson, K. Tran, and J. Unger, “Light field video compression and real time rendering,” *Computer Graphics Forum*, vol. 38, no. 7, pp. 265–276, 2019

- S. Hajisharif, E. Miandji, G. Baravadish, and J. Unger, “Compression and Real-Time Rendering of Inward Looking Spherical Light Fields,” in *Eurographics 2020 - Short Papers*, U. Assarsson and D. Panozzo, Eds. The Eurographics Association, (Submitted)

- S. Hajisharif, E. Miandji, C. Guillemot, and J. Unger, “Single Sensor Compressive Light Field Video Camera,” in *Eurographics 2020*, U. Assarsson and D. Panozzo, Eds. The Eurographics Association, (Conditionally Accepted)

Other publications by the author that are relevant to this thesis but are not included are a conference paper on HDR image based lighting and a chapter of a book:

- S. Hajisharif, J. Kronander, E. Miandji, and J. Unger, “Real-time Image Based Lighting with Streaming HDR Light Probe Sequences,” in *SIGRAD 2012*. Sweden: Linköping University Electronic Press, 2012

- J. Unger, J. Kronander, and S. Hajisharif, “Unified Reconstruction of Raw HDR Video Data,” in *High Dynamic Range Video: From Acquisition, to Display and Applications*, 1st ed., F. Dufaux, P. L. Callet, R. K. Mantiuk, and M. Mrak, Eds. Elsevier, 2016, ch. 2, pp. 62–83
Contributions

The contributions made in this thesis are in the area of computational photography ranging from High Dynamic Range (HDR) reconstruction to Light Field (LF) imaging. The thesis is divided into two main components: HDR and LFs. The HDR reconstruction is the main focus of the first part of the thesis (Paper A and Paper B), while LF acquisition, compression, and reconstruction is the centerpiece of the second part (Paper C, Paper D, Paper E, and Paper F). In what follows, the publications included in this thesis are listed with a short description and the author’s contributions.

Paper A: HDR Reconstruction for Alternating Gain (ISO) Sensor Read-out

S. Hajisharif, J. Kronander, and J. Unger, “HDR Reconstruction for Alternating Gain (ISO) Sensor Readout,” in *Eurographics 2014 - Short Papers*, E. Galin and M. Wand, Eds. The Eurographics Association, 2014

The main idea comes from a unified framework for the reconstruction of HDR video from multiple exposure images [9]. The framework was modified and applied on a spatially multiplexed image captured with multiple gain settings on a conventional camera which extended the dynamic range of the captured image by $2 - 3$ f-stops. The result was presented at Eurographics 2014 as a short paper by the author of this thesis. The author was the main contributor behind the design and implementation of the method as well as capturing and providing the data and the majority of the written manuscript. This work has been also published in a book chapter [8].

Paper B: Adaptive DualISO HDR Reconstruction

S. Hajisharif, J. Kronander, and J. Unger, “Adaptive DualISO HDR Reconstruction,” *EURASIP Journal on Image and Video Processing*, vol. 2015, no. 41, dec 2015

This paper presents a novel HDR reconstruction where the size of the filter kernel adjusts to the statistical features of the camera noise model and the image structure, preserving the edges and important features of the scene. This work is a continuation of the work that is presented in Paper A. The result was published as a journal article and the author was responsible for the design and the implementation of the method as well as its written presentation.
Paper C: A Unified Framework for Compression and Compressed Sensing of Light Fields and Light Field Videos

E. Miandji, S. Hajisharif, and J. Unger, “A Unified Framework for Compression and Compressed Sensing of Light Fields and Light Field Videos,” ACM Trans. Graph., vol. 38, no. 3, pp. 23:1–23:18, may 2019

The paper presents a novel compressed sensing and compression framework for multidimensional visual data. Two applications are presented for light field images and light field videos. The author of this thesis has made close contributions on the developments of the framework and the design of the real-time rendering and reconstruction of light fields and light field videos. The author also assisted in writing and editing the manuscript and providing results in terms of images and videos.

Paper D: Light Field Video Compression and Real-Time Rendering

S. Hajisharif, E. Miandji, P. Larsson, K. Tran, and J. Unger, “Light field video compression and real time rendering,” Computer Graphics Forum, vol. 38, no. 7, pp. 265–276, 2019

The paper presents a framework for compression and real-time rendering of light field videos. This is an extension of Paper C where an in-depth study of the effect of image noise on the compression efficiency is carried out. Additionally, a novel method is proposed for improving the efficiency of the compression by pruning the trained ensemble of dictionaries. Real-time reconstruction and novel view generation of the compressed data has also been implemented. An application for this framework was also presented for heart surgery documentation using light field videos. The author of this thesis was the main contributor to the paper that closely collaborated with E. Miandji and J. Unger and contributed to the majority of the implementations and written presentation. The results of this project was presented at the Pacific Graphics 2019 conference by the author.

Paper E: Compression and Real-time Rendering of Inward Spherical Light Fields

S. Hajisharif, E. Miandji, G. Baravadish, and J. Unger, “Compression and Real-Time Rendering of Inward Looking Spherical Light Fields,” in Eurographics 2020 - Short Papers, U. Assarsson and D. Panozzo, Eds. The Eurographics Association, (Submitted)
As a continuation of the previous light field compression and rendering methods in Paper D and Paper E, we applied a similar method to a new light field format, where the cameras are mounted on a moving arm rotating and capturing the object at the center. We explored the effect of entropy coding of the sparse coefficients on the compression ratio and reconstruction quality. The author of this thesis was the main contributor of this project as well as the written manuscript.

**Paper F: Single Sensor Compressive Light Field Video**

S. Hajisharif, E. Miandji, C. Guillemot, and J. Unger, “Single Sensor Compressive Light Field Video Camera,” in *Eurographics 2020*, U. Assarsson and D. Panozzo, Eds. The Eurographics Association, (Conditionally Accepted)

This work is based on the idea of using compressed sensing for capturing high dimensional visual data. The paper proposes a novel reconstruction solution for a camera design where a mask is placed between the aperture and the sensor. The proposed reconstruction algorithm takes advantage of the correlations in the consecutive frames of a light field video to enable accurate reconstruction of the data in the temporal as well as spatial and angular domains. The author is responsible for the idea, design, and implementation of the method in addition to the written presentation. This paper is submitted to Eurographics 2020, and conditionally accepted.
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Digital cameras have been an omnipresent technology, from mobile phones and DSLR cameras to security systems and industrial quality control applications. Traditional imaging relies mainly on hardware systems for capturing a scene. Although the hardware of digital cameras has evolved very fast during the past decades, they are still physically limited in many aspects, particularly the sensor’s capabilities to capture challenging scenes. Computational photography is an emerging field that intersects with computer vision, computer graphics, and applied optics, and provides solutions to the limitations of digital cameras through new optical designs, and mathematical and computational techniques. These algorithms have already been incorporated into many digital camera software systems to improve the capturing process and assist the user. As an example, face detection is commonly used to identify people in a scene and adjust the focus, exposure, and camera settings to obtain the best image of each person. De-blurring and denoising algorithms have long been parts of the capturing system. Algorithms for capturing higher dynamic range are now available in most consumer cameras, where typically multiple shots are captured and fused to improve the final image. Tone mapping is used to map this high dynamic range image to the dynamic range of the display used to view the image.

Plenoptic imaging is a subset of computational photography aiming to acquire all or some of the dimensions of the plenoptic function, such as the spatial, angular, spectral, and temporal domains using both hardware modulations and computational algorithms. These algorithms can be utilized in various applications from medical imaging [10], saliency detection [11], scene flow estimation [12], increasing
Figure 1.1: Digital camera pipeline: Camera optics, color filter array (CFA), sensor, gain control, analogue-to-digital (A/D) converter, camera processing unit consisting of different stages to covert the raw digital input to an 8-bit final image. The green box indicates the analog signal and the blue box contains the processes that are applied on the digital signal.

This chapter provides a brief introduction to the topics of computational photography and focuses on plenoptic imaging, which is most relevant to the research conducted in this thesis. It starts with an introduction to traditional digital imaging, HDR imaging, and light field (LF) imaging, and concludes with the objectives of the thesis and a list of contributions by the author of this thesis towards these objectives.

1.1 Digital Imaging

The traditional digital imaging pipeline, as shown in Figure 1.1, consists of an optical unit such as lens and aperture, color filter array (CFA), a sensor, gain amplifier, analogue-to-digital (A/D) converter, and a digital image processing unit. The incoming luminance passes through the optics of the camera and the CFA and is projected onto the sensor. The sensor converts the incoming photons to
electric charges and stores them in the photosites or capacitors while the shutter is open. Too much light might lead to the overflow of the pixel capacitors and saturate the image at the corresponding pixels. The gain control unit, which accommodates the ISO settings on modern cameras, amplifies the analog signal before it is quantized into the digital signal. Amplification of the analog signal is necessary to bring the voltage to a range that is required to get a desired digital output in the A/D converter stage. In the digital image processing unit, the raw digital pixel values are adjusted for the black level. The black level is usually calculated from a dark frame in the raw data measuring the sensor noise. The white balancing step transfers the image to a new space to mimic the chromatic adaptation of the eye so that the colors are represented correctly. Additionally, the image is debayered (or demosaiced) [17] to reconstruct all color channels for each pixel. Finally, the camera response function curve is applied on the color image along with other enhancement algorithms, and the image is compressed to an 8-bit JPEG format.

One of the main shortcomings of the traditional 2D imaging is the constraint of capturing a single point in space at a given time. It is prevalent that the artists and directors would like to change the focus point or the camera’s viewpoint to make a scene more attractive according to artistic intentions. Usually, they are required to re-shoot the scene, which is time-consuming, costly, and in some cases, not possible. Another limitation is the incapability of sensors to capture all the luminance coming from the scene, limiting the dynamic range of the acquired image. As an example, it is incredibly challenging to capture an indoor scene simultaneously with what is seen through a window.

In order to extend the capabilities of digital imaging, new optical designs, new sensors, sampling methods, and algorithms are required to handle data and re-create the real scene as close to the original as possible. Computational photography provides the platform for developing optical designs and algorithms for recovering the original signal and transforming it into a proper format for displaying on a suitable device. The optical design modifies the camera by adding optical elements such as ND-filters [18], beam-splitter [19, 20], microlens arrays [21], and color-masks [22, 23] to the optical path of the camera to enhance the acquisition process. Figure 1.2 illustrates a simplified pipeline for computational photography. Please note that the elements showed in this figure, such as microlens array and spatially interlaced exposures, are not necessarily built together. The raw data from A/D converter is passed to a unit consisting of different computational methods that reconstruct an estimate of the original signal. Unlike the traditional camera pipeline that compresses the image using JPEG format, computational methods find the proper encoder and decoder based on the context of the image, the display technology, and the application at hand.
Figure 1.2: A simplified pipeline for computational photography. The optical designs modify the camera architecture by adding optical elements such as ND-filters, microlens arrays, and modifying the gain control per pixel as shown in the optical and hardware modification unit (purple box). The CFA pattern can be adapted to the features of the scene, and the gain settings can be controlled to enable multi-exposure capturing. The digital raw data is then processed for a specific task and then compressed for storage. To display the captured data, the image is reconstructed and visualized on a suitable device.

By designing suitable algorithms, computational photography enables us to process the captured data efficiently and recover as much information as possible for a given optical design. This thesis contributes to the development of these algorithms for retrieving and storing high dimensional information such as light fields from a real environment. The following sections will explain a few examples of computational photography, which is most related to the topics covered by this thesis.

1.1.1 High Dynamic Range Imaging

One of the areas in which digital cameras rely heavily on computational methods is capturing a scene with a high dynamic range, or enhancing the dynamic range of the acquired image in a post-processing phase. High dynamic range imaging is a technique used to digitally capture a wide range of luminance in the scene, including the visible light from direct sunlight to the darkest shadows. The actual luminance values of the world can be measured in an HDR image and used to illuminate 3D objects through a family of techniques known as Image Based Lighting (IBL) [24]. IBL is used in applications such as photo-realistic rendering, relighting for virtual and augmented reality, entertainment, cinematography and gaming industry [7, 25, 26, 27, 28, 29]. There are many other applications of HDR imaging, including, but not limited to, astrophotography [30, 31], medical imaging [32, 33, 34, 35], and display systems [36, 37].
What we describe as HDR photography today, was first introduced by Gustave Le Gray in 1850, who combined two negatives into a single positive print to capture the extreme dynamic range of the sky and the sea, Figure 1.3 (a). The principles of high dynamic range using differently exposed pictures were inspired by the pioneering work of Charles Wyckoff [38] with the invention of "extended response film", which consisted of three layers of different light sensitive films. The final image is a composition of these layers. Figure 1.3 (b) shows an example where this technique was used to photograph a nuclear explosion. To translate this design into digital cameras, we need to temporally capture multiple exposures of the scene where the final image is created by fusing these images using some weighting filter based on the noise characteristics or nonlinearities in the captured images. Despite the prevalence of this method, it is mostly suitable for static environments, as any movements in the scene will result in a blurry image that requires tedious image registration techniques [39]. Therefore, in most cases, these methods fail to capture real scenes with moving objects accurately and are not suitable for dynamic environments.

One of the main challenges in HDR imaging is to acquire the HDR image in a single capture setting to avoid the ghosting artifacts introduced by temporal multi-exposure techniques [40, 41]. An HDR image can be captured in a single shot by combining a multi-sensor setup with a beam splitter and Neural Density (ND) filters to change the exposure on each sensor[19]. Nevertheless, the required types of equipment for this method are usually costly and not suitable for hand-held consumer cameras.
Single HDR image acquisition is also possible by spatially multiplexing the exposure by placing an ND-filter in front of the sensor or lens. However, ND-filters can cause color shifts, and since they limit the effective light throughput, they lead to image noise in the final result. Another way is to utilize multiple gain settings in each capture where gain values amplify the analog signal before passing to the A/D converter unit. However, in this way, the noise is also amplified, which results in a grainy image. There are a variety of algorithms proposed for reconstructing an HDR image from spatially interlaced exposure input, which mainly focuses on recovering each exposure setting separately and using some weighting function to fuse them to achieve an HDR image. However, these techniques will result in low spatial resolution in the final reconstructed image since they do not employ the information from other exposure settings in the reconstruction process. In this thesis, the focus is on the capture and reconstruction of HDR images using spatially multiplexing by varying (dual) ISO settings through a single image capture. Our proposed method recovers a full-resolution HDR image in a single-step procedure from a dual-ISO image. More details are explained in Chapter 3.

1.1.2 Light Field Imaging

Light field imaging is a powerful technique striving to capture the spatial, as well as the angular, radiance information of a scene, while enabling sophisticated digital processing methods for this high dimensional data. Light field imaging has a history of more than 100 years with the first lenticular parallax stereogram introduced by Fredric Ives [42] in 1903, followed by the first practical design for a light field camera from the Nobel prize-winner Gabriel Lippman [43]. Lippman proposed a model to place a lenticular multi-array lens in front of the primary lens to capture light rays from different directions. The term "light field" was first coined by Gershun [44] in 1936. The computer vision community was introduced to the concept in 1992 by Adelson and Bergen [45] with the plenoptic camera design, which was later on implemented and improved by Ng et al. [46] as a hand-held plenoptic camera. Only in the past decade, there have been significant advancements in light field imaging concerning capturing, compression, and rendering.

Multiple designs have been suggested to sample the incoming illumination, including multi-camera systems [47], microlens arrays placed between the primary lens and the sensor [46], and gantry systems [48]. Each design is suitable for a specific application and scene characteristics (e.g. static or dynamic). For instance, multiple camera systems are usually suitable for dynamic scenes, while the gantry system can capture high angular resolution of a static scene. Figure 1.4 shows a summary of advancements in light field imaging. An overview of the field can be found in Chapter 5.
One of the main challenges in light field imaging is that capturing high angular resolution light fields requires a lot of data, meaning that the capturing, processing and storage of the data becomes very challenging. New light field camera designs can reduce the required number of samples in the capturing phase using e.g. compressive sensing to alleviate the excessive bandwidth requirement of light fields. Additionally, effective compression techniques can encode these data even further in a way that they can be transferred over a network or fit on the GPU for real-time rendering. This thesis proposes a set of novel techniques to obtain a framework for capturing, compression, and rendering of 4D-6D light field data sets with significant improvements over the state-of-the-art methods.

1.1.3 Applications

Digital images assist humans and machines in a variety of applications. Image-based rendering can be considered as the first application of digital imaging. IBR techniques are a compelling alternative to geometry-based approaches in image synthesis. Figure 1.5 shows a continuum of the image-based representations that depend on the available geometric information versus the number of input images. Pure image-based techniques rely only on the photos or videos captured from the scene. However, depending on the environment and its complexity, additional information is sometimes required for seamless renderings, such as implicit geometry information like depth or disparity map. In the rightmost end of this continuum is rendering with available explicit geometry and scene details like material properties, and illumination information. This type of image synthesis is known as physically based rendering, an accurate but computationally expensive technique.

HDR rendering or lighting is an IBR technique where the lighting information is stored in an HDR domain. Utilizing this lighting domain in rendering allows us to preserve the details of the scene that might be lost due to limiting contrast ratios. HDR images of the environment illumination, often called radiance maps, are used to illuminate virtual 3D objects, a practice with many advantages in the
photography and film industry [28], as well as in image-based lighting [49] and material editing [50]. Creating a virtual world using an HDR rendering is a more realistic and compelling solution compared to modeled lighting due to the accurate simulation of the physics of light. Figure 1.6 (a) shows an HDR video camera and light probe, where the HDR camera can capture up to 24 f-stops of the scene. The HDR environment map sequence from this camera is employed to illuminate a virtual object placed in a real environment, as seen in Figure 1.6 (b).

HDR imaging has been used for photography of scenes with complex illumination as well as in computer vision and image processing techniques to better find features in the captured images. Moreover, with the development of HDR displays, HDR content is necessary to bring entertainment experiences closer to reality. HDR imaging has also been employed in the automotive industry for advertisement, where a new car model is rendered with real-scene illumination before massive production. The vehicle can also be equipped with an HDR camera to improve the safety of autonomous driving in all lighting conditions.

The recent advancements of light field imaging have affected various disciplines such as computer vision, computer graphics, medicine, and biology, and demonstrated more promising results when compared to traditional imaging techniques. Light field imaging in computer graphics is a crucial tool for rendering, as it is a process independent of the geometry, and complex material representations [51, 52]. From a different perspective, one can employ geometrical information such as depth map or optical flow to reconstruct novel views and extend the angular resolution of a light field. Refocusing an image is yet another application of light field imaging where the depth of field is changed in a post-capturing process [21].

The light field embeds a significant amount of information (e.g. depth), which can be explored as an epipolar plane image (EPI) to increase the angular resolution by generating novel views [53]. Additionally, in the field of computer vision, light field imaging can be employed to estimate scene flow and material identification more...
robustly, and regardless of illumination variations [54]. The material properties can also be approximated in terms of spatially varying bidirectional reflectance distribution function (SVBRDF) using light field images [55]. In addition, many other fields can benefit from light field imaging techniques. For instance, microscopy [56], vision-based robot control [57], and bio-medicine tools such as otoscope [58].

### 1.2 Objectives and Contributions

The principal research objective of this thesis is to advance the capabilities of ordinary cameras to be able to capture more enhanced information about the scene. This includes, but is not limited to, higher dynamic range and higher dimensional data such as light fields. Advancing the state-of-the-art in these areas requires solving the following problems:

- A statistical noise model suitable for reconstructing an HDR image from a spatially multiplexed image with dual gain settings.
- A framework for jointly solving multiple problems such as denoising, demosaicing, and HDR fusion in a single step for spatially multiplexed images.
- Designing a framework for encoding and real-time reconstruction and rendering of light fields and light field videos that is robust to the noise.
- A design for capturing light field video in the compressive setting.

![HDR Video camera](image1.png)

**Figure 1.6:** (a) HDR Video camera developed in collaboration between SpheronVR and Linköping University that enables panoramic lighting measurements [9]. (b) Rendering virtual objects illuminated with the footage from the HDR video camera [7].
The research carried out by the author of this thesis has led to the following contributions:

**Increasing the dynamic range of consumer cameras.** A unified kernel regression algorithm based on the statistical noise model of the camera is introduced for the reconstruction of interlaced multiexposure images (Paper A) where the processes of denoising, reconstruction, and demosaicing are performed jointly in a unified manner. The adaptive kernel preserves the structure of the image while it instantaneously removes noise according to image statistics and camera noise model (Paper B).

**Compression and rendering of light field images and videos.** A robust framework is designed to compress high resolution light field data sets such that the encoded coefficients can fit easily on the GPU memory. The framework is based on learning a multidimensional dictionary ensemble (MDE) to exploit the sparsity of the natural light field data sets. To improve the computational complexity of the training stage, and the sparsity, a novel pre-clustering algorithm based on an $\ell_0$ pseudo norm is applied to the training set. The obtained MDEs from each pre-cluster, are collected into an aggregated multidimensional dictionary ensemble (AMDE) that is used to compress the data (Paper C). To improve the compression performance, the AMDE is pruned with a novel algorithm that finds the most distinct dictionaries in the AMDE (Paper D). A real-time reconstruction algorithm is introduced in Paper C, where each element of the light field data points are reconstructed independently. The real-time rendering method is extended in Paper D with novel view synthesis. The effect of camera noise on the compression algorithm is extensively studied and demonstrated that even with slight denoising before encoding, the compression ratio improves significantly. Finally, in Paper E, a new design for capturing spherical light field is introduced, and we show that the AMDE framework is suitable for compression and real-time rendering of such massive amounts of data. The quantization and entropy coding of sparse coefficients obtained from the AMDE framework have also been studied for spherical light field data.

**Compressive light field video camera.** A novel reconstruction algorithm is presented to recover the light field video data captured with a coded-aperture design (Paper F). A color-coded mask is placed between the sensor and the aperture plane that convolves the incoming light rays into a single 2D image. The high-resolution light field video is reconstructed by employing a novel compressive sensing model using a dictionary-learning algorithm that considers spatial, angular, spectral, and temporal domains.
1.3 Thesis Outline

The rest of this thesis is structured as follows: Chapter 2 provides a brief and necessary background, as well as technical information, about the fundamentals of the camera and its properties. Chapter 3 presents details of the camera noise model and the HDR reconstruction algorithm for spatially multiplexed data. The sparse representation of the signal is explained in Chapter 4 that covers the basic concepts of dictionary learning and compressed sensing. Chapter 5 details our framework, known as AMDE, for the compression and reconstruction of light fields and light field videos. The main ideas of the compressive light field video camera are explained in Chapter 6. Finally, Chapter 7 provides a conclusion and a summary of the topics presented in the thesis, as well as a discussion of future directions for the research.
Fundamental Camera Concepts

Photography is about seeing the world through a camera lens, by capturing the light arriving at the camera sensor. The photographer controls the amount of light reaching the sensor by tuning a set of parameters such as exposure time, aperture, and ISO. Understanding the fundamental concepts of digital cameras is necessary in order to develop computational methods that enhance the capturing process. This chapter describes the basic computational photography concepts that have been used throughout this thesis. These include an explanation of the digital camera pipeline, various camera parameters, and noise characteristics in the capturing process. Furthermore, the dynamic range of the human visual system and digital cameras are compared.

2.1 In-Camera Processing

There are primarily two technologies used for the camera sensor to acquire a digital image: charge-coupled devices (CCD) and complementary metal-oxide-semiconductors (CMOS). Although the operational principles of these two sensor designs are different, their acquisition models are similar in the sense that both are converting the incoming light photons into electric voltage values. In this section, the acquisition process is explained in details, including the camera parameter description, as well as various sources of uncertainty that can be introduced at each stage of the acquisition.
Figure 2.1: RGB color filter array (CFA) on a digital sensor camera with RGGB pattern. Each element of the CFA passes through the light rays in the same spectrum as the filter resulting in an RGB image.

2.1.1 Color Filter Array

In conventional cameras, a color filter mosaic, generally called Color Filter Array (CFA), is placed on top of the imaging sensor, resulting in a mosaiced image. The most common CFA is with RGB colors, which is called Bayer pattern [59]. Color filter arrays such as the Bayer pattern are based on the sensitivity of human vision to the green wavelength rather than other colors, and it is designed in a quad of pixels that follows a pattern of 25% red, 25% blue and 50% green. Each photosite only samples one color. Figure 2.1 illustrates a CFA with an RGGB pattern placed in front of the sensor where only photons with the same wavelength as CFA pass through and are converted to electrons. The acquired image is spatially undersampled in color channels, and requires reconstruction through an interpolation process called demosaicing or debayering [17].

Other types of CFAs also exist, such as CMY (Cyan, Magenta, Yellow) CFA, RGBE (Red, Green, Blue, Emerald) CFA, and CYGM (Cyan, Yellow, Green, Magenta) CFA. Different designs for the CFA pattern have been proposed, such as RGBW (Red, Green, Blue, White) CFA [60], a perceptually based design with a random pattern [61], designs robust to noise [62] and many more. Throughout this thesis, the assumption is that the camera is equipped with a CFA with a Bayer pattern.

Demosaicing algorithms aim to recover the full-color image from the spatially incomplete color samples of the image sensor overlaid with a CFA. Demosaicing algorithms can be categorized as following: interpolation-based methods and learning-based methods. Simple interpolation of color channels results in color artifacts, especially around the high-frequency edges. Considering the local features of the image and utilizing the correlations among color channels leads to better results [63, 64, 65, 66]. However, these methods fail to recover the signal without artifacts in complicated structures.

Learning-based methods, on the other hand, learn the local distribution of the image patches and provide better color fidelity in the reconstructed image [67, 68]. Deep convolutional neural networks (CNNs) have recently shown promising results for joint denoising and demosaicing of the sensor output, where the visual artifacts are reduced compared to traditional methods [69]. For a comprehensive overview
of these methods see [70].

2.1.2 Camera Parameters

There are a number of parameters on the camera that constrain the way an image is captured. These parameters control the brightness of the image, the depth of focus, sharpness, and noise. A correct combination of them allows capturing a properly exposed image. However, each of these elements can also introduce some side effects that should be taken into account during the imaging process. This section explains a few of the essential parameters that are used in the computational algorithms.

Shutter speed  Shutter speed or exposure time is the length of time that the sensor is exposed to incoming light. The amount of light arriving at the sensor is proportional to the exposure time, meaning that more photons reach the sensor by increasing the exposure time.

Aperture  The optical system of the digital cameras consists of lens components and an aperture that controls the amount of light that arrives at the imaging sensor. The aperture and the focal length determine the bundle of rays that are in focus on the image plane. The aperture controls the brightness and the depth of field of the captured image. A large depth of field means that most of the image is in focus while the shallow depth of field means that only a small part of the image is in focus. Opening the aperture increases the brightness of the image while creating a shallow depth of field.

ISO/Gain  Gain is proportional to the ISO sensitivity on modern cameras, which allows the signal to be amplified before analog-to-digital (A/D) conversion. ISO is comparable to film sensitivity in analog cameras. Increasing ISO will amplify both the signal and the noise in the signal, which can result in a grainy image.

Exposure  There are different ways to adjust the exposure: one method relies on modifying the shutter speed or the integration time. Short integration times capture bright parts in the scene well, while longer integration times are suited for capturing dark regions in the scene. The latter can lead to motion blur. Another approach is to change the aperture, which affects the image focal length, a not always desirable effect. ND-filters can be used to change the exposure in all or parts of the image. Even though they are effective in capturing dynamic scenes, blocking the incoming luminance can increase the noise in the dark areas of the scene. Another approach is to change the gain or ISO setting, which can have the benefits of avoiding motion blur and improving the focal length.
2.1.3 Camera Noise Sources

When a sensor measures light, there are several sources of noise that can corrupt the signal in different stages of capturing. Understanding these sources can help in estimating the underlying signal more accurately. Figure 2.2 shows different sources of noise that appear in different stages of digital imaging. The sensor noise can be stationary fixed-pattern noise, or vary over time. Fixed-pattern noise occurs due to irregularities during the manufacturing of the sensor that introduces unwanted spatially varying noise and remains the same regardless of the signal or the capturing time.

**Photo-response non-uniformity (PRNU)** Theoretically, each pixel should collect precisely the same amount of photons when uniform light falls on the sensor. However, due to variations in the substrate material, and pixel geometry (e.g. pixel size), the output values are slightly varied over the sensor pixels. The difference between the expected response from uniform light and the actual output value from the sensor is called PRNU. Removing PRNU is almost impossible since it is caused by the physical properties of the sensor. However, the effects of this noise can be reduced by using a lookup table (LUT) made of correction factors for each pixel that is calculated by exposing the camera sensor to a uniform light source.

**Dark signal non-uniformity (DSNU)** This type of fixed-pattern noise occurs because of variations in the sensitivity of each pixel for collecting photons due to fabrication errors. This noise can be estimated by averaging multiple photographs taken with fixed lens cap. Some cameras have built-in functionality to remove this noise by taking a second exposure with the shutter closed and subtracting it from the real captured image.

**Photon shot noise** The number of photons reaching the sensor changes for every capturing, and it results in fluctuations in the captured signal known as photon shot noise. The time of arrival of each photon is random, and it follows a
Poisson distribution indicating that if the number of photons increases, the variance in the signal also increases by the same amount [72].

**Dark current noise** This noise is introduced due to the thermal electron vibrations on the sensor, especially in low-light photography, such as in astronomy, when a longer exposure time is required. The effect of it can be reduced by cooling down the sensor, and in most applications, it can be neglected for exposure times less than one second\(^1\). Dark current noise or *dark shot noise* is independent of the number of photoelectrons generated, which means it is an additive noise and a Poisson distribution can model it well [73].

**Reset noise** Before capturing an image, the photosite wells need to reset from any charge or electrons. However, if the required time for resetting each well is longer than the clock speed of the camera, some wells might not be reset when the next capturing begins, meaning that some wells might contain electrons from previous capturing. This residual charge creates a spatially varying signal, which is known as reset noise [74]. This noise, similarly to DSNU, can be accounted for by capturing a bias frame where after each pixel is reset, the remaining signal is read. The noise is removed by subtracting the bias frame from each captured image. The bias frame is acquired by taking an image with the cap on, resulting in the offset values for the pixels of the image. The reset noise can be accurately modeled as a Gaussian distribution [75].

**Readout noise** During the readout stage of the camera pipeline, the electron charges gathered in each photosite are read. The readout noise is created between the photoreceptors and the A/D converter circuitry and is thermally generated, which is modeled as Gaussian noise [76]. In the CMOS sensors, the readout phase is done by reading the pixels line by line, which introduces a patterned noise that can be disturbing to the human eye [76].

**ADC noise** In the quantization process of the A/D converter, where the analog voltage measures are converted to digital values, a uniformly distributed quantization error is introduced in an additive fashion. Compared to other sources of noise, ADC noise can be neglected [77].

Removing the noise introduced during the capturing process requires solving an inverse problem with an infinite number of solutions. Since noise, edges, and textures are high-frequency features, it is difficult to distinguish between them for removing the noise. As mentioned above, some sources of noise are signal-dependent, i.e. multiplicative noise, while other sources are signal-independent or additive.

\(^1\) [http://theory.uchicago.edu/~ejm/pix/20d/tests/noise](http://theory.uchicago.edu/~ejm/pix/20d/tests/noise)
Figure 2.3: Real world luminance and sensitivity of the human visual system (HVS), digital cameras and HDR exposure bracketing.

The major challenges of image denoising algorithms are to keep the homogeneous regions as smooth as possible, while the structure of the edges remains intact, and no new artifacts and blurs are introduced.

2.2 Dynamic Range

The human visual system (HVS) is capable of perceiving illumination in a range of approximately $10^{14}$, from $10^{-6}$ to $10^8 \text{ cd/m}^2$ by transforming the incident light on the eye into nerve impulses using photo-receptors. There are two categories of photo-receptors in the human visual system: Rods, which are more sensitive to the light, enabling vision in dark environments with illumination ranging from $10^{-6}$ to $10 \text{ cd/m}^2$, e.g. night scenes. This is called scotopic range when only rods are active.

Cones are another category of photo-receptors that are less sensitive to the light, and they are active during daylight conditions, forming the photopic vision. Rods have multiple receptors that are sensitive to long, medium, and short wavelengths, while cones can only register monochrome information. The rods are effectively saturated in photopic vision, ranging from about 0.03 to $10^8 \text{ cd/m}^2$. The range that both rods and cones are active is referred to as mesopic vision which extends from about 0.03 to $3 \text{ cd/m}^2$ [28]. Figure 2.3 depicts the sensitivity of the human visual system with scotopic, mesopic, and photopic vision at different light levels. Photopic vision corresponds to normal light levels where rods saturate, and cones dominate. The scotopic vision resembles low light levels where rods dominate due to the lack of sensitivity of the cones.
2.3 Quality Metrics

This extensive dynamic range of illumination allows humans to see objects under sunlight as well moonlight. However, the HVS cannot operate over such an extensive range simultaneously. This significant variation in sensing luminance is accomplished by brightness adaptation. Therefore the simultaneous range that humans can sense is rather small and in the range of $10^3 \text{ cd/m}^2$ at any particular state of adaptation.

On the digital cameras, the dynamic range is dependent on the bit-depth of the sensor, which means that for a camera with 14 bits per pixel, it can measure up to $2^{14}$ photons. However, in reality, this range is lower, and the dynamic range is dependent on the size of each photosite or pixel capacitor and its sensitivity to light. Each photosite transforms photons that hit the corresponding pixel on the sensor to electrons (charge), and it overflows and saturates if its well capacity is reached. Furthermore, the sensitivity of the photosite further determines the darkest measurable light intensity. Therefore the dynamic range of a digital camera is usually calculated as a ratio between the highest number of photons that each photosite can contain and the darkest measurable light intensity. The most common unit for measuring the dynamic range of the camera is the f-stop. For example, a contrast ratio of 1024:1 is translated into 10 f-stops ($2^{10}$).

2.3 Quality Metrics

The quality of a reconstruction algorithm for image processing tasks such as denoising, demosaicing, needs to be evaluated by a well-established quality metric. Although the best way to assess the quality of an image is by a combination of objective and subjective quality metrics, this thesis considers only the objective metrics for evaluation as they provide accurate measures for the stated problems.

2.3.1 PSNR

The objective measure that is most commonly used in image and video processing is Peak-Signal-to-Noise Ratio (PSNR), which is defined for an image $I_r$ and its approximation $I_t$ as follows:

$$\text{PSNR}(dB) = 10 \log_{10} \frac{L_{\text{max}}^2}{\text{MSE}}, \quad (2.1)$$

where, $L_{\text{max}}$ is the maximum possible pixel value of the image and MSE is the mean square error calculated as follows:

$$\text{MSE} = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} (I_r(i,j) - I_t(i,j))^2. \quad (2.2)$$
2.3.2 SSIM

Another metric is the structural similarity index (SSIM) [78] which is a method for measuring the similarity between two images. The quality of this metric is reportedly corresponds better to perceived differences than MSE and PSNR [79]. The SSIM is calculated between two windows of size $N \times N$ pixels within the original signal $x$ and the reconstructed signal $y$ as follows:

$$SSIM = \frac{\left(2\mu_x\mu_y + c_1\right)\left(2\sigma_{xy} + c_2\right)}{\left(\mu_x^2 + \mu_y^2 + c_1\right)\left(\sigma_x^2 + \sigma_y^2 + c_2\right)},$$

(2.3)

where $\mu_x$ and $\mu_y$ are the mean of $x$ and $y$, respectively, and $\sigma_x^2$ and $\sigma_y^2$ denote their corresponding variances. The covariance between two matrices is denoted as $\sigma_{xy}$. The two variables $c_1$ and $c_2$ are constants that prevent the numerical instability and are defined as:

$$c_1 = (k_1L)^2$$

$$c_2 = (k_2L)^2,$$

(2.4)

where $L$ is the dynamic range of the image and $k_1 = 0.01$ and $k_2 = 0.03$.

2.3.3 HDR Quality Metrics

High dynamic range imaging algorithms can be evaluated objectively, or subjectively. A human observer can detect the visual differences between the results of two algorithms and choose the one that most resembles the reference image. However, subjective evaluation is not always practical and requires an extensive amount of experiments. Therefore, it is necessary to find a quality metric that is suitable for these computational tasks.

High dynamic range pixel values correspond proportionally to the physical luminance. In contrast, the low dynamic range pixel values are usually gamma-corrected and relate nonlinearly to the luminance of the scene. The gamma curve approximates the response of the human eye to luminance. The HVS is more sensitive to the luminance ratios than absolute values (Weber-Fechner law). Consequently, the direct comparison of HDR pixel values does not represent what is perceived by the human eye. Quality metrics for HDR algorithms should compensate for this effect to provide a reliable evaluation. One approach is to transform the HDR values into a domain representing an HDR or LDR display, such as perceptually uniform algorithms [80]. Other methods convert HDR pixel values into a domain that mimics the human perception of luminance [81].

A common method is to use nonlinear luminance metrics where the HDR pixel values are encoded to a domain linear to the HVS, such as the logarithmic domain.
The LDR quality metrics such as PSNR or SSIM can then be applied to the logarithmic values [82]:

$$\log\text{PSNR} = 10\log_{10}\frac{\log_{10}(L_{\text{max}})^2}{\text{MSE}}$$

(2.5)

$$\text{MSE} = \frac{1}{N} \sum_{i=1}^{N} [\log_{10}(\hat{I}_t(i)) - \log_{10}(\hat{I}_r(i))]^2,$$

(2.6)

where,

$$\hat{I}_t(i) = \max(I_t(i), L_{\text{min}}), \quad \hat{I}_r(i) = \max(I_r(i), L_{\text{min}}),$$

(2.7)

where $L_{\text{max}}$ and $L_{\text{min}}$ are the peak luminance level, and minimum luminance above the noise level, respectively. The distorted image is denoted as $I_t$ and the reference image as $I_r$. Similarly, SSIM can be estimated in the logarithmic domain. One can also normalize the log-transformed HDR pixel values before calculating the SSIM or PSNR[83].

The HDR pixel values can also be encoded by applying a transformation into a domain where encoded pixel values correspond to a perceptually uniform (PU) representation [80]. The encoding is derived from the contrast sensitivity function (CSF), which describes the capacity of the HVS to predict luminance and chrominance differences as a function of contrast and spatial frequency. The transformation is further constrained to imitate sRGB nonlinearity. The encoded values are then passed to an LDR metric such as SSIM, see Figure 2.4. The resulting quality metric is known as $\text{PU-SSIM}$.

A more comprehensive encoding model of the contrast visibility that takes into account luminance dependent effects like intraocular light scattering is HDR-VDP-2 [81]. This quality metric is also derived from a measured CSF and predicts the visual differences at all range of luminances. HDR-VDP-2 calculates the probability of detecting a difference between two HDR images, as well as the perceived level of distortion.
High Dynamic Range Imaging

In a two-dimensional imaging system, the photons are integrated from multiple angles during a specified time (exposure) and absorbed by the camera sensor’s capacitors. The limitations of the capacitors indicate the dynamic range of the sensor. If the number of photons exceeds the limitation, it overflows (the pixel saturates), and the information in that region becomes unreliable. The dynamic range is defined as the ratio of the maximum and the minimum number of photons a sensor can gather at each pixel. A scene’s dynamic range is represented by its contrast ratio from the darkest shadows to the very well lit areas. A typical scene may exhibit a linear dynamic range in the order of 10,000,000:1. In contrast, the maximum dynamic range a camera can measure nowadays is up to 16.5 f-stops, which corresponds to a contrast ratio of around 92,000:1, for cameras such as RED Helium 8K and ARRI Alexa\(^1\). Depending on the ISO sensitivity of the sensor, clamping, and noise, the useful range is far less in reality.

Consequently, even the high-end cameras are incapable of capturing the full dynamic range and radiance distribution of a scene. Unfortunately, with the current development of the sensors, it is not possible to obtain the human dynamic range in a single shot. As a result, there is a demand for computational methods together with new system designs to overcome these limitations. This chapter presents an overview of the computational techniques for capturing and reconstructing HDR images. Furthermore, it describes details of the statistical algorithms for single-shot spatially interlaced HDR image reconstruction, as presented in Paper A and Paper B.

\(^1\) www.dxomark.com
3.1 Outline and Contributions

This chapter presents two algorithms for extending the dynamic range of conventional cameras in a unified framework for reconstruction, demosaicing, and denoising based on the statistical information acquired from the camera noise model. The main contributions of the research presented in this section are related to the statistical reconstruction [9] of interlaced data with multiple gain settings, using Local Polynomial Approximation (LPA) [84]. As the first contribution, Paper A, a filter kernel based on LPA is applied to a spatially interlaced image with two different gain values, where the dynamic range of the reconstructed image is extended by about 2-3 f-stops. In the second contribution, Paper B extends this approach by adapting the filter kernel to the local image statistics, the sensor noise, as well as the underlying signal structure present in the image. The adaptive filter kernel increases the image quality in various examples presented here.

This chapter is organized as follows: Section 3.2 provides an overview of the acquisition methods, where three categories of techniques are discussed. Section 3.3 explains various HDR reconstruction approaches, which is followed by Section 3.4, where the details of the two statistical HDR reconstruction frameworks are explained. Finally, Section 3.5 concludes the chapter by a summary of the presented methods and the possible future venues for research.

3.2 HDR Acquisition Overview

High dynamic range imaging (HDRI) has been a very active and well-studied research topic in computational photography. HDR image acquisition has many applications from digital photography, image-based lighting [49], digital cinema, computer games, virtual reality, and HDR display [85]. For an extensive overview of HDR algorithms for capturing and reconstruction see [28]. Various approaches for capturing HDR images can be broken into the following categories: the first category is multiple exposure acquisition using a single sensor, the second one is single capturing with multiple sensors, and finally single shot with a single sensor. Acquiring HDR videos using conventional cameras and mobile phones is still an ongoing research problem. This section presents an overview of the state-of-the-art methods for capturing the HDR content of natural scenes.

3.2.1 Multi Shot - Single Sensor

Temporal multi-exposure capturing, known as Exposure Bracketing, is a very well-researched and well-established method that is available on most consumer cameras. The idea was reintroduced to the digital photography field by Madden [86] in 1993. In this approach, multiple low dynamic range (LDR) photographs
Figure 3.1: Exposure bracketing for HDR capturing. The highlights are captured with low exposure while the dark regions of the scene are captured with high exposure image.

are sequentially captured with different exposure times. These LDR images are fused into a single, high contrast HDR image as a weighted average of the pixel values across the different exposures, by taking the camera response function into account [40, 41, 87, 88]. A very wide dynamic range can be captured with this approach since one can take many images of the scene, where each image is exposed such that a different luminance range is acquired. This technique is now available on most consumer products such as smartphone cameras and is known as HDR mode. The HDR mode usually works by blending several images taken at different exposures, and its blending weights are based on measures of quality such as color distribution or local contrast [89]. An example of exposure bracketing is shown in Figure 3.1, where three exposures are combined to create a full dynamic range of the scene. The short exposures, i.e. the top right image, captures the highlights of the sky while the long exposure image, i.e. the bottom right, is capable of recoding darker areas of the scene.

Despite its popularity, exposure bracketing is not capable of capturing dynamic scenes and motions. Any movements in the scene will result in blurring and ghosting artifacts, which requires advanced methods to recover a sharp image. Several approaches have been proposed to overcome these issues by aligning the images from a stack of multiple exposures to a reference image and eliminating any misalignments before reconstruction. These methods are usually dependent on correspondences at pixel-level [90, 91, 92] or patch level [93] and optical flow [94], while others take into account per-pixel values as a blending parameter.
Nevertheless, finding correspondences between the images is computationally expensive and an error-prone task, especially when there is a large shift between the corresponding points.

Deghosting the HDR images using rank minimization [95, 96] is another approach for removing the artifacts from the reconstructed image. A patch-based method with energy-minimization can potentially align multiple exposures and reconstruct the HDR image using a joint optimization model. Despite all the advances, these methods often fail to robustly handle movements in a general scene due to parallax, occlusion, and deformable motions. For an overview of these methods see [97].

Recently, deep learning methods have tried to overcome the dependency on the correspondences for alignment by hallucinating the details of an HDR image in the absence of information caused by e.g. occlusion, saturation, and under-exposed regions [98]. Nonetheless, these methods are unable to remove the ghosting artifacts in the final reconstruction altogether.

### 3.2.2 Single Shot - Multiple Sensors

Another method for capturing HDR content is using beam-splitter and multiple sensors, which has shown to be useful for dynamic scenes and also capturing HDR video [19]. Aggarwal et al. [99] suggested a design where the aperture is split into multiple parts, and the incoming ray is redirected into different beams. Each beam moves towards a sensor where the exposure is either controlled by setting parameters or by splitting the incoming ray unevenly.

Spheron VR and Linköping University employed a similar method to develop an HDR video camera that is capable of capturing 24 f-stops at 30 frames per second [9], see Figure 3.2 for the prototype design. To control the amount of light reaching each sensor, they utilized ND-filters with different sensitivity for each sensor. The

![Diagram of HDR video camera](image-url)
significant advantage of these systems over temporal exposure fusion HDR methods is that the motion in the scene is handled very robustly as all the sensors have the same exposure time. However, using these systems, the light efficiency decreases due to ND-filters blocking light rays, which introduces noise in dark regions of the image. Tocci et al. [19] suggested an optimized design using a beam splitter setup that increases the light efficiency, as shown in the right image of Figure 3.2.

Nevertheless, another disadvantage of these methods is that they require high precision in the optical design and accurate geometrical alignment of different sensors [9, 99, 100]. Furthermore, the hardware and the prototype setup are generally expensive and challenging to make available to consumers. The dynamic range captured by these systems is limited to the number of sensors that can be used. Any additional sensor complicates the system’s optical light splitting design and reduces the adequate light reaching each sensor.

3.2.3 Single Shot - Single Sensor

The final acquisition category relies on using only a single sensor, where spatial multiplexing is employed to capture an HDR image. The research presented in this thesis falls into this category of techniques. There are two main factors to consider for designing a spatially varying system. Firstly, the dynamic range is dependent on the number of different exposures that are captured in a single shot. A higher number of spatially varying exposures increases the dynamic range while reducing the spatial resolution of the reconstructed image. Secondly, the spatial distribution of these varying per-pixel exposures, i.e. regular or random, affects the reconstruction differently. Regular patterns introduce an aliasing effect, especially around high-frequency features of the image, e.g. edges. Although a random pattern can alleviate these artifacts, they are harder to implement in practice. One way to vary the exposure in a single image is to place a spatially varying ND-filter in front of the sensor with regular [18, 101] or irregular patterns [102, 103]. However, these methods lead to low signal-to-noise ratio due to the light inefficiency of the ND-filters.

A theoretical design called Gradient Camera was introduced in 2005 by Tumblin et al. [104], which is based on capturing the gradients of the image rather than the actual pixel intensity and later applying an expensive Poisson solver to reconstruct the intensity values from it. In Programmable Imaging [105, 106], an aligned spatial light modulator, such as Digital Micromirror Device (DMD), was used instead of ND-filters and provided adaptive control over the exposure of each pixel. This method’s main disadvantage is the pixel-level alignment of the DMD with the sensor, which is a laborious task [107]. Nayar and Branzoi [108] proposed an alternative implementation of the spatial exposure variation, called Adaptive Dynamic Range Imaging (ADRI). They mounted a transmissive spatial light modulator near the
aperture plane of the camera to simulate an adaptive optical density mask instead of a fixed-pattern element. However, this method only modulates the low spatial frequencies in the image.

The coded rolling shutter photography \[48, 109, 110\] controls the readout timing and the exposure time for each row of the CMOS sensors to enhance the captured image by deblurring, skew correction and extending the dynamic range. Reconstruction of HDR image requires the calculation of optical flow, cubic interpolation of each exposure time, and fusing them, which is computationally costly. Another method is to use a spatial light modulator to create a random mask pattern in front of the sensor to encode the light rays arriving at the sensor. Afterward, a sparse convolutional coding method can be applied to the modulated image to reconstruct the HDR image \[111\].

More recently, with the advancements in the CNNs, algorithms have been proposed that map an LDR image to its corresponding HDR one by recovering information from the saturated regions \[112\] and extending the dynamic range. Similarly, CNNs can be used to train a single image contrast enhancer where the training set is created from multiple exposure images \[113\]. These algorithms are mainly focused on recovering saturated information, which limits their capabilities.

An alternative solution is to use all incident light using imaging sensors where the pixel gain is varied over the sensor \[48, 109, 114\], such that the low gain setting captures the bright regions of the image, by avoiding saturation, while the high gain setting has a better signal-to-noise ratio in the dark areas in the image. This method can be applied to existing off-the-shelf digital cameras equipped with a customized firmware update (e.g., Magic Lantern firmware for Canon cameras). These camera sensors are then capable of using two ISO settings simultaneously by varying the gain for every other row, covering the RGGB Bayer pattern for each exposure setting, as depicted in Figure 3.3. Some sensors such as Aptina AR1331CP, Sony IMX135, and Apertus Axiom are also capable of recording two exposure settings for every other row (every two rows of the raw Bayer image covers all the color samples). This chapter introduces methods for reconstructing an HDR image using the output of such capturing systems.

## 3.3 HDR Reconstruction Overview

Early HDR reconstruction techniques focused on fusing differently exposed LDR images using the response curve of the sensor acquired from a calibration process \[41, 87, 88\]. Later on, some methods consider weighting the pixels by exploiting the noise characteristics in the linear domain to minimize the variance of estimation \[115\]. Granados et al. \[116\] takes into account different sources of sensor noise,
Figure 3.3: Gain patterns for Dual-ISO setting. The left image shows a unified reconstruction kernel applied on the green color channel for this gain pattern.

i.e. temporal (photon, dark current shot noise, readout noise) and spatial (Photo-response and dark current non-uniformity) as a function of irradiance reaching the sensor. Gallo et al. [117] analyze the image histogram, and adaptively select a minimal number of exposures to capture the scene with an optimal signal-to-noise ratio. By taking into account different sources of sensor noise, Hasinoff et al. [71] introduced a method to minimize the variance of the estimation.

The emphasis of most HDR acquisition methods is on fixing the ISO/gain settings to a low value to reduce the noise. However, it has been shown that sometimes high ISO is desirable and can lead to a better signal-to-noise ratio of the final image [118, 119]. Higher gain value and lower exposure time have been used to eliminate and reduce the ghosting artifacts of the time-sequential multi-exposure methods [120].

Reconstructing high dynamic range images and videos demands solving various tasks such as reconstruction of full color from CFA data, alignment of the images for multi-sensor algorithms, and HDR assembly, where low dynamic range (LDR) images are fused to produce the HDR content, and finally a denoising step to reduce the excessive noise introduced in the capturing process.

Traditionally, HDR reconstruction algorithms [121] adopt a pipeline where each of these tasks are solved independently from others, as shown in the top image of Figure 3.4. Splitting the processing unit into individual tasks poses several issues and shortcomings. Each process requires prior assumptions for its solution and introduces an error which is accumulated throughout the pipeline. More importantly, all of these problems are correlated, and there is no natural order in solving them.

As an example, some reconstruction methods propose to solve demosaicing before HDR assembly [41], which causes artifacts, especially in saturated regions where there are not enough samples. Furthermore, some color channels saturate at different
Traditionally HDR reconstruction methods proposed a reconstruction pipeline consisting of independent processes such as demosaicing, resampling, HDR assembly, and denoising. A unified algorithm combines all these steps into one simultaneous task.

Solving multiple tasks in a joint approach has shown to be more accurate and efficient when compared to the traditional pipelines. An example is joint demosaicing and denoising [123, 124], which results in a sharper image, especially in low light conditions. Other examples include joint demosaicing and deblurring [125], joint denoising and deblurring [126], joint demosaicing and HDR reconstruction [101, 122], where all report reduction in artifacts.

Heide et al. [127] proposed an adaptive framework for multiple joint computational photography tasks such as denoising, color interpolation, and HDR reconstruction. Their method solved an inverse problem by combining different global natural image priors, such as block-matching and 3D filtering (BM3D), and total variational (TV). Although their framework was able to adapt for different tasks, including reconstruction of the interlaced HDR image, it is highly dependent on priors, such that if an image violates the priors, it can result in overly smoothed or sharpened edges. Furthermore, it does not utilize a heterogeneous camera noise model, and their method requires solving a global optimization problem, which is computationally expensive.

Kronander et al. [20] introduced a unified framework for the reconstruction of multi-exposure raw images captured with an HDR video camera shown in Figure 3.2. Their reconstruction method incorporates an accurate statistical noise model of the camera sensor, which was suggested earlier by [76]. This thesis proposes HDR reconstruction methods based on [20], where the reconstruction window is adapted for spatially multiplexed imaging systems. In the following section we describe the details of our novel HDR reconstruction algorithm.
3.4 Dual-ISO Spatial Multiplexing

*Kernel regression* (KR) is an effective tool for the super-resolution of an image from single or multiple frames, as well as reducing the noise that is introduced by the imaging pipeline [128]. Locally adaptive filtering has been used in a variety of image processing operations, such as normalized convolution [129], and bilateral filtering [130]. A deep connection between these methods and traditional non-parametric statistical methods [131] have been shown in [128, 132]. As discussed earlier, a unified framework that solves multiple tasks simultaneously can reduce the error of reconstruction. This section explains a reconstruction method that adopts a unified statistical model similar to [20] on a spatially interlaced image with two gain values. The final HDR image is accurately reconstructed in a joint process of resampling, color interpolation, and denoising, as shown in Figure 3.4. The accuracy of the reconstruction is based on including the statistical noise properties of the sensor in the reconstruction model. Different gain settings have different effects on the noise characteristics that must be considered in the statistical model.

The input samples in the local neighborhood of a reconstruction point are interpolated using local polynomial approximation in the method proposed in this section. The LPA combined with *maximum likelihood* is used for designing a nonlinear filter by fitting a polynomial in a sliding window. A desirable filter is defined by the window, its size, and the order of the polynomial. In this method, we employed a Gaussian kernel to interpolate the observed pixel values. The interpolation weights are calculated using the estimated variance of each sample point to ensure that reliable samples are weighted higher than noisy, and unreliable ones. The effect of the Gaussian filter is to guarantee that samples further away from the reconstruction point have lower weights compared to neighboring points. The size of the filtering kernel can be adjusted iteratively to the statistical characteristics of the signal and its underlying variance to preserve the edges and important features of the image. In what follows, the details of this algorithm are explained.

3.4.1 Sensor Noise Model

The number of photoelectrons, \( e_i \), accumulated at each pixel, indexed by \( i \), during the exposure time, \( t \), follows a Poisson distribution where the expected value and variance are:

\[
E[e_i] = t(a_i f_i) \\
Var[e_i] = E[e_i],
\]

(3.1)

where \( a_i \) is pixel non-uniformity (see 2.1.2). Each pixel value \( y_i \), corresponds to the number of photo-induced electrons collected on the sensor per unit time, known as incident radiant power, \( f_i \). The pixel values, \( y_i \), consist of the incident radiant
power, $f_i$, and the measurement noise that is dependent on sensor characteristics such as the gain, $g$, and the signal independent readout noise:

$$y_i = g(e_i) + r_i(g),$$

(3.2)

where $r_i(g)$ is the readout noise that is dependent on the gain settings of the sensor. For simplicity, exposure settings longer than one second are not considered in this thesis. Therefore, since the effect of dark current noise is negligible for short exposure times, it is not included in the noise model, see 2.1.2. Previous methods consider a simple parametric model for the readout noise dependence on the gain [20, 71]. This model is not suitable for multiple gain settings found on modern cameras, and instead, the readout noise, $r_i(g)$, is estimated carefully for each gain/ISO setting through a calibration process, see Section 3.4.3.

Recovering the full HDR image requires us to transform the noisy digital pixel values, $y_i$, to an estimation of incident radiant power, $\hat{f}_i$, such that the pixel values are in the same radiometric space. The sensor gain varies for every pixel of the sensor, but the exposure scaling coefficient, exposure time, and other parameters are similar over the sensor. The pixel values are transformed using a radiometric model [9, 20, 133], where non-saturated pixels follow a random variable distribution as follows:

$$y_i \sim N(g_i a_i t f_i + \mu_R, g_i^2 a_i t f_i + \sigma_R^2(g_i)),$$

(3.3)

where $g_i$ is the per-pixel gain, $a_i$ is the pixel non-uniformity, $t$ is the exposure time, and $\mu_R$ and $\sigma_R^2$ are the mean and the variance of the readout noise. The variance of the radiant power can be estimated through a calibrated radiometric model that will be explained in Section 3.4.3. We assume the readout noise follows a Gaussian distribution similar to [20, 116].

The incident radiant power from the noisy digital input sample values $y_i$ is estimated using the following:

$$\hat{f}_i = \frac{y_i - b_i}{g_i t a_i},$$

(3.4)

where $b_i$ is the bias frame or black image computed as the average of a large set of black images captured carefully without any light rays reaching the sensor.

Other sensor noise models that include the effects of saturation on the variance of the pixels, $\text{Var}[y_i]$, can also be applied [134]. These models, however, are impractical and complicated for the local polynomial approximation that is used in this thesis since they consider $y_i$ to follow a clipped normal distribution.

### 3.4.2 Variance Estimate

To calculate the variance of the estimated radiant power, $\hat{f}_i$, the assumption is that $\hat{f}_i$ has a normal distribution with mean $f_i$ and standard deviation $\sigma_{\hat{f}}$. A normal
distribution can approximate the photon shot noise with a Poisson distribution in the bright regions of the image. In low light environments or darker regions of the image, the photon shot noise is dominated by signal-independent readout noise that can as well be approximated by a normal distribution. Assuming that the pixels are non-saturated, the variance of \( \hat{f}_i \) can be found by:

\[
Var[\hat{f}_i] = \sigma_j^2 = \frac{g_i^2 (Var[e_i]) + Var[r_i(g)]}{g_i^2 t^2 a_i^2}.
\] (3.5)

To estimate \( Var[e_i] \), we need to know the actual value of \( f_i \) due to the existence of photon shot noise. However, in practice, the noisy estimate of the incident radiant power, \( \hat{f}_i \), obtained from Equation (3.4), is used instead. Therefore, the variance of the estimated incident radiant power is similarly approximated as:

\[
\hat{\sigma}^2_{\hat{f}_i} = \frac{g_i^2 t a_i \hat{f}_i + \hat{\sigma}_{R}^2(g_i)}{g_i^2 t^2 a_i^2},
\] (3.6)

where \( g_i \), \( a_i \) and \( \hat{\sigma}_{R}^2(g_i) \) are calculated through a calibration process. The estimated variances are assumed to be independent of each other, and the effect of pixel cross-talk is not considered here.

The assumption so far has been that there are no saturated pixels in the captured image. Nonetheless, in practice, some pixels might get saturated due to excessive amounts of light, e.g., direct sunlight. These saturated pixels are unreliable and should be excluded from the computations using a saturation frame to avoid further errors. The saturation frame indicates at what level each pixel saturates. The pixels are compared with this frame and excluded if they are saturated.

### 3.4.3 Camera Parameters Calibration

The noise characteristics of the camera are estimated through a calibration process that is performed once for each camera with the same optical elements (e.g., lens). The pixel non-uniformity, \( a_i \), can be estimated from a set of flat-field images. For simplicity, we consider that \( a_i \) is constant for all pixels. The readout noise parameters, e.g., mean, \( \mu_R \), and the variance, \( \sigma_R^2 \), are estimated by capturing a set of dark images, \( \{b_i\} \), using the same camera settings, (i.e. gain, \( g \), and exposure time, \( t \)), with the lens cap on, such that no light rays arrive at the sensor. Per-pixel sensor gain \( g_i \) is estimated with the following relation:

\[
\frac{Var[y_i] - Var[b_i]}{E[y_i] - E[b_i]} = \frac{g_i^2 Var[e_i]}{g_i E[e_i]} = g_i.
\] (3.7)

As the number of collected photoelectrons, \( e_i \), follows the Poisson distribution with expected variance \( Var[e_i] = E[e_i] \), (see Equation (3.1)), the second equality
is simplified to \( g_i \). Additionally, \( E[y_i] \) is estimated from the flat-field images. Moreover, \( E[b_i] \) and \( \text{Var}[b_i] \) can be computed using the dark images, as explained above. Figure 3.5 illustrates the mean standard deviation of the readout noise when the ISO parameter is changed for two camera models: Canon 5D Mark II and Canon 5D Mark III.

### 3.4.4 Local Polynomial Approximation

To approximate an HDR value, \( \tilde{f}_j \), at output HDR pixel location \( X_j = [x_1, x_2] \), the observation samples of the incident radiant power are fitted to a polynomial in its local neighborhood using an LPA [84] or kernel regression [128]. Since the radiant power \( f_j \) can be considered as a smooth function in a local neighborhood around the output location \( X_j \), we employ an \( M \)-th order Taylor series expansion to approximate the radiant power at an observation location \( X_k \) near the output location \( X_j \), as follows:

\[
\tilde{f}(X_k) = C_0 + C_1(X_k - X_j) + C_2 \text{tril} \{(X_k - X_j)(X_k - X_j)^T\} + \ldots, \tag{3.8}
\]
where \( \text{tril} \) lexicographically vectorizes the lower triangular part of a symmetric matrix \(^2\) and \( C_{1:M} \) are the fitted polynomial coefficients defined as:

\[
C_0 = f(X_j) \tag{3.9}
\]

\[
C_1 = \nabla f(X_j) = \begin{bmatrix}
\frac{\partial f(X_j)}{\partial x_1}, & \frac{\partial f(X_j)}{\partial x_2}
\end{bmatrix} \tag{3.10}
\]

\[
C_2 = \frac{1}{2} \begin{bmatrix}
\frac{\partial^2 f(X_j)}{\partial x_1^2}, & \frac{\partial^2 f(X_j)}{\partial x_1 \partial x_2}, & \frac{\partial^2 f(X_j)}{\partial x_2^2}
\end{bmatrix} \tag{3.11}
\]

Therefore, the HDR pixel value, at the output location \( X_j \) is estimated by \( C_0 = f(X_j) \). Moreover, \( C_1 \) and \( C_2 \) are the first and second order gradients, respectively.

### 3.4.5 Maximum Localized Likelihood Fitting

The polynomial coefficients, \( C_{1:M} \), are estimated by maximizing the localized likelihood function \([135]\) that is defined by a smoothing window centered at \( X_j \):

\[
\mathcal{W}_H(X_j) = \frac{1}{\text{det}(H)\mathcal{W}(H^{-1}X_j)} \tag{3.12}
\]

where \( H \) is a \( 2 \times 2 \) smoothing matrix that controls the shape and size of the filtering window. One choice to determine the smoothing window is to define it as a truncated Gaussian window. As a result, Equation (3.12) using a Gaussian window becomes:

\[
\mathcal{W}_H(X_j) = \frac{1}{2\pi H^2} \exp\left\{ -\frac{(X_k - X_j)^T(X_k - X_j)}{H} \right\}. \tag{3.13}
\]

The smoothing matrix, \( H \), affects the shape of the window function. Some approaches define a complex shape that adapts to the signal by rotation, elongation, and scaling of the window \([128]\). The simplest choice is to set \( H = hI \), where \( I \) is the identity matrix and \( h \) is the global scale parameter that determines the size of the kernel. Setting this parameter to a fixed user-defined value will create a globally fixed-size sliding window, as presented in Paper A. However, this parameter can be set for each reconstructed pixel such that the filter kernel adapts to the statistical features of the image as will be explained in Section 3.4.6.

The observed pixel samples in the local neighborhood of pixel location \( X_j \), are denoted as \( \hat{f}_k \) with linear index \( k = 1 \ldots K \). By assuming that the radiant power estimates, \( \hat{f}_k \), obtained by Equation (3.4), have a normal distribution, we maximize the localized likelihood function to approximate the polynomial coefficients \( \tilde{C} \) by the weighted least squares estimate:

\[\text{tril}(\begin{bmatrix} a & b & c \\ b & e & f \\ c & f & i \end{bmatrix}) = [a,b,c,e,f,i]^T\]

\(^2\) i.e. \( \text{tril} \)
Figure 3.6: Unified HDR reconstruction results of real data from Canon 5D MarkIII with alternating ISO readout. Our method with dual-ISO settings: 100-800, 100-1600 and Magic Lantern (ML) DUALISO method [114] with dual-ISO settings: 100-800.

\[
\tilde{C} = (\Phi^T W \Phi)^{-1} \Phi^T W \tilde{f},
\]

(3.14)

where

\[
\tilde{f} = [\tilde{f}_1, \tilde{f}_2, \ldots, \tilde{f}_K]^T
\]

\[
W = diag\left[ \frac{W_h(X_1)}{\sigma^2_{\tilde{f}_1}}, \frac{W_h(X_2)}{\sigma^2_{\tilde{f}_2}}, \ldots, \frac{W_h(X_K)}{\sigma^2_{\tilde{f}_K}} \right]
\]

\[
\Phi = \begin{bmatrix}
1 (X_1 - X_j) & \text{tril}^T\{(X_1 - X_j)(X_1 - X_j)^T\} & \ldots \\
1 (X_2 - X_j) & \text{tril}^T\{(X_2 - X_j)(X_2 - X_j)^T\} & \ldots \\
\vdots & \vdots & \ddots & \vdots \\
1 (X_K - X_j) & \text{tril}^T\{(X_K - X_j)(X_K - X_j)^T\} & \ldots 
\end{bmatrix}
\]

(3.15)

This approach allows us to estimate the polynomial coefficients \( C_{1:M} \) effectively for an \( M \)-th order polynomial and to compute the HDR pixel value \( \tilde{f}_j \) for a given \( h \) value. The smoothing parameter, \( h \), determines the trade-off between the bias and the variance of the estimation, which corresponds to the image sharpness vs. noise reduction. Figure 3.6 illustrates the result of applying the kernel regression with a globally fixed scaling parameter, \( h \), using the proposed method. The figure shows a comparison of our approach with ISO 100-800 and ISO 100-1600 and Magic Lantern DUALISO method [114] with ISO 100-800.
3.4.6 Adaptive Kernel Regression

As mentioned earlier, the size of the kernel function determines a trade-off between the variance and the bias of the reconstructed image. Large window size reduces the noise, and hence the variance, while increasing the bias, e.g. by smoothing out sharp edges. The window size, ideally, should be adjusted to the underlying signal to preserve important features of the image, such as edges, while removing noise in homogeneous regions.

The smoothing parameter, $h$, determines the size of the smoothing kernel. The effect of the smoothing parameter is illustrated in Figure 3.7, where the signal value, the black point in the figure, is estimated by a kernel with three sizes: $0 < h_0 < h_1 < h_2$. When incrementing the kernel size from $h_0$ to $h_1$, the variance of the estimated value can still be explained by the variance of the underlying signal. However, increasing the window size too much as in $h_2$ forces the kernel to cross the edge, where the estimated point is no longer explained by the signal variance, resulting in over-smoothness. The size of the filter kernel can be adapted iteratively to increase gradually until a certain criterion is achieved. Selecting the filter size adaptively is a necessary element of a successful local approximation [84].

Algorithm 1 explains each step that is required to reconstruct an output HDR pixel $f_j$ by adapting the smoothing parameter $h_l$. In each iteration of the algorithm, the HDR signal and its variance are estimated for the current smoothing parameter. Afterwards, an update rule, explained in the next section, is applied that determines whether the current parameters satisfy the estimation or not. The iteration continues until no further update is valid, or the maximum value of the smoothing parameter, $h_{max}$ (defined by the user), is reached. We introduce two updating rules in the next two sections and explain how the variance of the estimated signal is computed.
Input: Bayer dual-ISO image, $y_i, b_i, h_{\text{min}}, h_{\text{max}}$.  
Output: Reconstructed HDR image

1. for each color channel in R,G,B do
   2. for each HDR pixel estimate $\tilde{f}_j$ do
      3. $h_l \leftarrow h_{\text{min}}$;
      4. Compute $\tilde{f}_{j,h_l}$ using LPA with degree $M$;
      5. $\tilde{f}_j \leftarrow \tilde{f}_{j,h_l}$;
      6. while $h_l < h_{\text{max}}$ do
         7. $h_{l-1} \leftarrow h_l$ and $h_l \leftarrow h_{l-1} + h_{\text{inc}}$;
         8. Estimate $\tilde{f}_{j,h_l}$ using LPA with degree $M$;
         9. Estimate $\tilde{\sigma}_{j,h_l}$ and the reconstruction error $\epsilon_l$ (for EVS);
         10. Apply update rule (ICI or EVS) based on $\tilde{f}_{j,h_l}$;
         11. if variation in $\tilde{f}_{j,h_l}$ can be explained by $\tilde{\sigma}_{j,h_l}$ then
            12. $\tilde{f}_j \leftarrow \tilde{f}_{j,h_l}$;
            end
         14. else
            15. Break;
         16. end
      17. end
   18. end
19. end

Algorithm 1: Adaptive HDR reconstruction.

3.4.7 Update Rule 1: Error of Estimation Versus Standard Deviation (EVS)

This updating rule is built upon the intuition to examine whether the polynomial model is a good fit for the underlying signal or not. A good fit is found when the difference between the weighted mean reconstruction error is explained by the expected signal variations, i.e. the weighted mean and standard deviation. The noise can affect the capabilities of the polynomial model in representing the data. Therefore, to reduce this effect, in each iteration, $l$, of Algorithm 1, the smoothing parameter, $h_l$, is increased with an offset, $h_{\text{inc}}$, that is controlled by the user. The EVS update rule estimates the weighted reconstruction error, $\epsilon_l$ as follows:

$$
\epsilon_l = \sqrt{\sum_{k=1}^{K} W^2(k,k)(\hat{f}(X_k) - \hat{f}_k)^2}.
$$

(3.16)
where $K$ is the number of pixels in the neighborhood, and $\mathbf{W}$ is the weight matrix considering both the variance of the original pixels and the spatial Gaussian kernel as described in Equation (3.15). The error of reconstruction, $\epsilon_l$, is compared to the weighted standard deviation of the HDR pixel estimate that is obtained from the covariance matrix $\tilde{\mathbf{M}}_C$ for the fitted polynomial coefficients, $\tilde{C}$, given by:

$$\tilde{\mathbf{M}}_C = (\Phi^T \mathbf{W} \Phi)^{-1} \Phi^T \mathbf{W} \Sigma \mathbf{W}^T \Phi (\Phi^T \mathbf{W}^T \Phi)^{-1}, \quad (3.17)$$

where $\Sigma = \text{diag}[\sigma_1^2, \sigma_2^2, ..., \sigma_k^2]$ is the variance of the observations. In each iteration, the smoothing parameter, $h_l$, is updated to $h_l = h_{l-1} + h_{\text{inc}}$ as long as the reconstruction error is smaller than the standard deviation, $\epsilon_l < \Gamma \tilde{\sigma}_{\tilde{z}_l, h_l}$, where $\Gamma$ is a constant defined by the user to control the trade-off between the level of denoising that is applied by the kernel function and the reconstruction quality.

Figure 3.8 illustrates the EVS update rule and how it fits the structure of the image and preserves the edges by keeping the kernel size small around those areas. The effect of the algorithm on different color channels is also shown in this figure.
Figure 3.9: Reconstruction of a simulated dual-ISO scene with different methods for comparison: (a) LPA, $M = 2$, from left to right: $h = 0.6, 1.4, \text{ and } 5.0$; (b) SKR [128], $M = 2$ from left to right $h = 0.6, 1.4, \text{ and } 5.0$; (c) our method with ICI, $M = 2$ from left to right $\Gamma = 0.6, 1.0, \text{ and } 1.4$; (d) our method with EVS, $M = 2$ from left to right $\Gamma = 0.6, 1.0, \text{ and } 1.4$.

| Algorithm | PU-SSIM | Log-Normalized-SSIM | HDR-VDP 2.2 |
|-----------|---------|---------------------|-------------|
| LPA       | $h = 0.6$ | 0.9737               | 0.9883       | 54.3042     |
|           | $h = 1.4$ | 0.9893               | 0.9903       | 56.8246     |
|           | $h = 5.0$ | 0.9683               | 0.9828       | 53.5847     |
| SKR       | $h = 0.6$ | 0.9655               | 0.9766       | 51.0470     |
|           | $h = 1.4$ | 0.9912               | 0.9924       | 56.9265     |
|           | $h = 5.0$ | 0.9826               | 0.9897       | 55.3868     |
| ICI       | $\Gamma = 0.6$ | 0.9893             | 0.9929       | 57.5614     |
|           | $\Gamma = 1.0$ | 0.9924             | 0.9937       | 59.3245     |
|           | $\Gamma = 1.4$ | 0.9928             | 0.9937       | 59.8197     |
| EVS       | $\Gamma = 0.6$ | 0.9772             | 0.9893       | 54.4240     |
|           | $\Gamma = 1.0$ | 0.9930             | 0.9935       | 58.8865     |
|           | $\Gamma = 1.4$ | **0.9945**          | **0.9943**   | 58.4647     |

Table 3.1: Quantitative comparisons of Figure 3.9.
3.4.8 Update Rule 2: Intersection of Confidence Intervals (ICI)

Another way to adapt the smoothing parameter to the signal is based on the ICI algorithm [84]. This algorithm ensures that the largest scaling parameter, \( h_{\text{min}} \leq h_l \leq h_{\text{max}} \), is obtained in a local neighborhood of the estimation point such that the polynomial model would stay as a possible fit to the signal. For each iteration \( l \) of Algorithm 1, the ICI rule indicates a confidence interval, \( D_l = [L_l, U_l] \), based on the estimated radiant power value, \( \tilde{f}_{j,h_l}(x) \) for the current scaling parameter,
where, $\bar{\sigma}_{z,j,h_i}$ is the weighted standard deviation of this estimation calculated using Equation 3.17. The user-defined parameter, $\Gamma$, controls the length of the intersection interval. During each iteration, $h_l$ is updated to $h_l = h_{l-1} + h_{inc}$ if there is an overlap between the confidence intervals, $D_{l-1}$ and $D_l$. In practice, $\Gamma$ creates a trade-off between the noise removal and the sharpness of the image (to read more about ICI rule see [136]).

A comparison of the proposed fixed and adaptive kernel methods with the two updating rules and the steering kernel regression (SKR) method [128] are demonstrated in Figure 3.9 and Table 3.1. The reconstruction is evaluated using different parameters that are mentioned in this section. Both proposed updating rules outperform the other methods in denoising and image sharpness. The adaptive window support can reduce the color artifacts around the sharp edges. Although SKR adapts the shape of the kernel to the features of the image, it fails to recover areas where both gain settings are saturated. Increasing the size of the kernel improves this problem but leads to over-blurring the image. On the contrary, our proposed methods with both ICI and EVS updating rules reconstruct the saturated regions with minimum artifacts. As shown in Table 3.1, ICI is less dependent on the $\Gamma$ coefficient value compared to EVS update rule.

Figure 3.10 shows the result of reconstructing a real dual-ISO image taken by Canon 5D Mark III with dual-ISO settings $100 - 1600$, $(g = 0.3 - 4.8)$, using the ICI update rule where a high-quality HDR image is achieved while preserving the details.

### 3.4.9 Spatially Interlaced Gain Patterns

As explained earlier, different patterns affect the reconstructed HDR image. Regular patterns such as the row pattern of dual-ISO mode on Canon Cameras (with Magic Lantern firmware), introduce aliasing and color artifacts, particularly at high-frequency edges. We evaluated our method on two other spatially multiplexed dual gain patterns using simulation, as shown in Figure 3.11. The results of these patterns together with row pattern of the Magic Lantern dual-ISO are shown at the left side of Figure 3.11, where the aliasing artifacts are significantly improved by the diagonal or block pattern. This is confirmed in Table 3.2 which shows that the diagonal pattern results in a better reconstruction quality in both PU-SSIM [80] and HDR-VDP-2 [137] quality metrics.
Figure 3.11: Comparison of different gain patterns: Block, diagonal, and row patterns. EVS adaptation rule with $M = 2, \Gamma = 1.0$.

| Gain Pattern | Diagonal Pattern | Block Pattern | Row Pattern |
|--------------|------------------|---------------|-------------|
| PU-SSIM      | 0.9967           | 0.9966        | 0.9960      |
| HDR VDP v2.2 | 66.21            | 66.09         | 62.62       |

Table 3.2: Quantitative comparisons for gain patterns: diagonal, block, row, as shown in Figure 3.11.

3.5 Summary and Future Work

This chapter presented a novel unified HDR reconstruction algorithm based on the statistical model of the sensor noise for the spatially interlaced images with variable exposures. The local polynomial model, based on the non-parametric regression, estimates the HDR pixel value in a noisy neighborhood around the pixel by applying a localized maximum likelihood approach. Two updating rules for adapting the filter size to the features of the signal and noise were presented. Adaptive kernel windows show improvements in image denoising and reconstruction quality while reducing color artifacts. The proposed framework is applied to both synthetically created data and real images captured by a conventional camera with spatially varying gain settings.

The future research direction involves developing the reconstruction framework for the HDR videos captured with multiple gain settings. A three-dimensional kernel window must be considered that can adapt to both spatial and temporal domains to include observation points within each frame and across multiple frames of the video, similar to the method of [138]. Another intriguing research direction is to consider cross channel correlation in the reconstruction model to improve the
accuracy of demosaicing and reducing the color artifacts.

Our proposed reconstruction method adapts the kernel size to the image content and noise model. Combining our approach with the idea from steering kernel regression (SKR) [128] where the shape of the kernel is changed based on the important features in the image can further improve the result.
This chapter describes the basic concepts used in the second part of this thesis that relate to the sparse light field imaging techniques. The covered topics include concepts such as sparsity and sampling, dictionary learning, overcomplete representation and redundancy, compressed sensing, and sparse representations. This chapter provides the theoretical background for compressive light field imaging, light field sparse representation, encoding, and rendering that are presented in chapters 5 and 6.

4.1 Sparse Signal Representation

The field of signal processing has seen significant advancements on efficient modeling and sampling of signals during the last two decades with the introduction of sparse representation of signals. Many applications in this field benefit from sparsity, such as restoration, feature extraction, and compression. The attractiveness of sparse representation techniques comes from their capacity to represent the data compactly. A signal $x$ is considered sparse if most of its entries are zero. A $\tau$-sparse signal is defined as $\|x\|_0 < \tau$ where the $\ell_0$ pseudo-norm defines the number of nonzero elements in the signal $x$. A non-sparse signal can be sparsified if it is represented in a suitable transform domain. Signal $x \in \mathbb{R}^N$ can be represented as a superposition of elements of a dictionary $D \in \mathbb{R}^{N \times T}$ using a vector of coefficients
The elements of a dictionary are called atoms. A few examples of these atoms include wavelets, monomials, and sinusoids. Linear superposition of multiple atoms in a building block can create a complex waveform. Given a dictionary, through an *encoding* process, each signal $\mathbf{x}$ is projected onto the dictionary to obtain a vector of coefficients as: $\mathbf{s} = \mathbf{D}^T \mathbf{x}$. The *decoding* phase, on the other hand, reconstructs the signal by superposing atoms using the coefficient vector, Equation (4.1).

An overcomplete or redundant dictionary has more columns than rows, $T > N$. Similar to dictionaries in natural language processing, the richer and larger the dictionary, the better it can represent the signal. An overcomplete dictionary with high redundancy enhances the sparse representation in the signal and improves tasks such as pattern recognition, compression, and compressive sensing [139]. On the other hand, there is no guarantee for obtaining a unique solution from the sparse coefficient vector associated with a dictionary that exhibits high redundancy. This is due to the fact that the linear system to be solved, i.e. Equation 4.1, is under-determined.

To obtain a suitable representation, first, it is required to decompose the signal into its primary components, i.e. dictionaries and coefficients. Secondly, such decompositions need to adapt to the signal properties. Wavelets are suitable for representing isotropic features, while overcomplete dictionaries such as ridgelet [140] and curvelets [141] lead to sparser representation for anisotropic and curvilinear structures. For periodic and stationary signals, the Fourier basis is well-suited [142]. Overcomplete dictionaries are typically preferable if there is a fast algorithm to transform the signal to and from the dictionary space. Choosing one transform rather than another requires prior information on the structure of the data. When such information is not available, one can learn the dictionary.

Finding a suitable dictionary that admits sparse representation of the data is a cumbersome task. To solve this problem, *Dictionary Learning* (DL) has emerged to create the possibility of adapting a dictionary $\mathbf{D}$ to the data through a set of examples that represent such data, known as the training set. Research has shown that data-driven dictionaries lead to higher performance when compared to pre-designed analytical dictionaries in applications such as inpainting [143], denoising [14], and other types of inverse problems [144]. The next section explains the dictionary learning for sparse representation of data.
4.2 Sparse Dictionary Learning

To represent a signal using the sparse domain of a learned dictionary requires solving the following:

$$\min_{s} \|s\|_0 \quad s.t. \quad \|y - Ds\|_2^2 \leq \epsilon,$$  \hspace{1cm} (4.2)

where $\epsilon$ can be set as an estimation for noise. If we have an estimate for the sparsity value, $\tau$, we can set the constraint to $\|s\|_0 \leq \tau$. However, in the absence of these estimates, Equation (4.2) can be calculated as an unconstrained problem known as the basis pursuit denoising (BPDN) [145]:

$$\min_{s} \lambda \|s\|_0 + \frac{1}{2} \|y - Ds\|_2^2.$$ \hspace{1cm} (4.3)

This problem is NP-hard due to the non-convexity of the $\ell_0$ norm, which can be replaced by an $\ell_1$ norm to create a convex optimization problem with a global optima. For more insights into the geometrical interpretation of the sparse recovery problem, see [146, 147, 148].

There are different categories of approaches for estimating a sparse signal. For instance, there exists heuristic methods using iterative approaches such as Matching Pursuit (MP) [149] or Orthogonal Matching Pursuit (OMP) [150]. Other methods based on optimization, and relaxation of the $\ell_0$ norm with $\ell_1$ norm, attempt to solve a convex problem using linear programming [151, 152, 153, 154], see [155] for a comprehensive survey. Although there is a convergence guarantee for these methods, they have a higher computational complexity compared to greedy algorithms. Deriving theoretical guarantees for the heuristic methods often requires stringent conditions compared to $\ell_1$ methods [156].

Another category of algorithms is called hybrid methods that try to solve the non-convex $\ell_0$ sparse recovery problem by using surrogate functions, such as smoothly clipped absolute deviation (SCAD) norm [157], see also [158, 159]. Another example is the SLO algorithm [160], which is, in some cases, two to three orders of magnitude faster than other linear programming methods while maintaining the accuracy.

So far, we have considered one-dimensional signals. The sparse representation can also be applied to multidimensional signals. When the dimensionality of data increases, the self-similarities or coherence within it also grows [161]. Representing the signal in a sparse domain where it can be described with a few coefficients can lead to faster processing and efficient storage of the data. Reducing the dimensionality of a signal has been studied through low-rank approximation techniques [162] and nonlinear methods known as manifold learning [163, 164] based on the assumption that the high dimensional data lies on a low dimensional manifold.
Representing the data in the sparse domain is not always feasible. One method to enforce sparsity is to decompose the signal into a dictionary and a set of coefficients, followed by the nullification of small or non-significant coefficients. A popular method for decomposing a matrix is singular value decomposition (SVD) that factorizes a matrix $\mathbf{M} \in \mathbb{R}^{m \times n}$ to:

$$ \mathbf{M} = \mathbf{U}\Sigma\mathbf{V}^T, \quad (4.4) $$

where $\mathbf{U} \in \mathbb{R}^{m \times m}$ and $\mathbf{V} \in \mathbb{R}^{n \times n}$ are orthogonal matrices containing singular vectors and $\Sigma \in \mathbb{R}^{m \times n}$ is a diagonal matrix containing singular values in a descending order. The rank of the matrix is defined by the number of nonzero singular values from SVD. Moreover, one can approximate the matrix $\mathbf{M}$ by setting the smallest singular values to zero. Such a decomposition is known as the truncated-SVD.

Similar to one-dimensional signals, a high dimensional signal, also known as tensor, can also be decomposed using dictionaries constructed by Fourier, wavelets, or learning-based basis functions [165]. For instance one of the most commonly used tensor decomposition methods is higher-order SVD (HOSVD) [166] defined for a high dimensional signal as:

$$ \mathbf{L} = \mathbf{S} \times_1 \mathbf{U}^{(1)} \times_2 \mathbf{U}^{(2)} \cdots \times_n \mathbf{U}^{(n)}, \quad (4.5) $$

where $\{\mathbf{U}^{(i)}\}_{i=1}^n$ are orthogonal or orthonormal matrices and $\mathbf{S}$ is the core tensor. It is possible to compress $\mathbf{L}$ by setting the small values in $\mathbf{S}$ to zero, as well as discarding the columns of $\{\mathbf{U}^{(i)}\}_{i=1}^n$ that correspond to the nullified elements; this algorithm is known as truncated-HOSVD. Note that unlike truncated-SVD, the approximation obtained from the truncated-HOSVD is not necessarily optimal in an $\ell_2$ sense.

### 4.2.1 Patch-Based Learning

Training a dictionary on a large number of exemplars is numerically intractable. Natural images exhibit nonlocal self-similarities, where blocks or patches from the image share similar content. Therefore, instead of the whole image, we can employ image patches for learning a dictionary [14, 143, 144]. Since the size of the dictionary atoms is equal to the signal size, using image patches reduces the size of the atoms, and hence the computational complexity. The other benefit of patch-based learning is that one can obtain from a limited set of images, or other types of data, a large number of exemplars for training.

### 4.2.2 Examples of Dictionary Training Algorithms

**K-SVD** is a learning-based algorithm that generalizes the k-means clustering method. The method is based on updating the coefficient vectors by sparse coding
using the current dictionary and fixing the coefficients while altering the atoms of the dictionary for a better fit to the data [165, 167]. The name K-SVD comes from the Singular Value Decomposing (SVD) algorithm that forms the core of the dictionary update step. The process is repeated until all atoms are updated. K-SVD ensures that the elements of the input data are encoded by a linear combination of $\tau$ elements:

$$\arg\min_{D,s} \|x - Ds\|_2^2 \quad s.t \quad \|s\|_0 \leq \tau, \quad (4.6)$$

where $\tau$ denotes the sparsity value.

K-SVD has applications in many research areas such as image and audio processing, document analysis, and biology. As many algorithms, K-SVD has a few disadvantages; for instance, getting stuck at local minima due to the non-convexity of the problem. The computational complexity of training a K-SVD dictionary is high, which makes this method only suitable for representing small signals or small patches from the data.

**Online dictionary learning for sparse coding** Most algorithms assume that all training data is available at the time of learning the dictionary. However, in some cases, it is not possible to fit large amounts of data into memory, or it is required that the input data is streamed gradually to the algorithm. To resolve these issues, a series of algorithms have been developed to iteratively update the sparse model as soon as new data is available [143]. In these methods, first, the dictionary is initialized by random values. Then with a fixed dictionary, sparse coefficient vectors are calculated by solving:

$$s_t = \arg\min_s \frac{1}{2} \|x_t - D_{t-1}s\|_2^2 + \lambda \|s\|_1, \quad (4.7)$$

Once the coefficient vector, $s$, is calculated, the dictionaries are updated by solving:

$$D_t = \arg\min_D \frac{1}{t} \sum_{i=1}^{t} \frac{1}{2} \|x_i - Ds_i\|_2^2 + \lambda \|s_i\|_1, \quad (4.8)$$

where $t$ is the $t$-th input sample. Experiments show that using online dictionary learning for large training sets leads to faster dictionary learning when compared to the other methods.

### 4.3 Compressive Sensing

Many problems in science and technology require reconstructing a signal from its measurements. Assuming a linear sampling process, this corresponds to solving:

$$y = \Phi x, \quad (4.9)$$
where $y \in \mathbb{R}^m$ is the measured signal, $\Phi \in \mathbb{R}^{m \times N}$ is a matrix that models the linear measurement process, and $x \in \mathbb{R}^N$ is the original signal we would like to reconstruct. Since $m < N$, the linear system is under-determined, and there exist infinitely many solutions to it, which means it is impossible to recover the signal without additional information. The Shannon-Nyquist sampling theorem states in order to ensure full recovery of the signal, the sampling rate of a continuous-time signal must be twice its highest frequency. The introduction of Compressive Sensing (CS) [168] has shown that under certain assumptions, it is actually possible to reconstruct the signal from $m$ incoherent measurements where $m$ is much smaller than the signal length $N$. The number of required samples is related to the sparsity level of the signal. Incoherency means that the data information is spread out in the domain that represents it. A signal can be recovered perfectly from $O(\tau \log N)$ incoherent measurements, which are much less than the original signal.

Many compression algorithms such as JPEG, MPEG [169] are developed based on the assumption of the sparsity of the signal. JPEG, for instance, employs discrete cosine transform (DCT) or wavelet basis, and by setting small coefficients to zero, it induces sparsity on the signal for compression. Figure 4.1 shows the sparsity that a natural image exhibits when transformed to a Haar wavelet basis. The reconstructed image is achieved by setting 90% of the Haar coefficients to zero, and as it is shown, the difference between the reconstructed and the original image is negligible. The main idea of compressive sensing is that since most signals are compressible under a certain basis function, far fewer samples are required in the acquisition process rather than applying the compression after sampling. In other words:

“We would like to compressively sense a compressible signal! This constitutes the basis goal of compressive sensing”[170].

Figure 4.1: (a) Original image (b) Reconstruction using 10% of the largest absolute Haar wavelet coefficients where the rest of the coefficients are set to zero.
Two components should be considered to achieve compressive sampling: The design of the measurement matrix $\Phi$ and an algorithm to find a solution to Equation (4.9). It has been shown that using random matrices with normal or Bernoulli i.i.d. distribution can uniquely recover $x$ in Equation (4.9) [171, 172, 173]. It is also possible to find the optimal $\Phi$ using learning-based methods, which is an ongoing research problem [174, 175, 176].

As explained earlier, a sparse signal can be represented by linear combinations of dictionary atoms, such that $x = Ds$. Therefore, by substituting the signal $x$ with its sparse representation in Equation (4.9), the sensing model can be rewritten as:

$$y = \Phi Ds. \quad (4.10)$$

Consequently, recovering a sparse signal requires solving the following:

$$\hat{s} = \arg\min_s \|s\|_0 \quad \text{s.t.} \quad \|y - \Phi Ds\|_2^2 \leq \epsilon. \quad (4.11)$$

However, as we saw in the previous section, $\ell_0$-minimization is NP-hard, usually basis pursuit or $\ell_1$-minimization is used to solve the problem as:

$$\hat{s} = \arg\min_s \|s\|_1 \quad \text{s.t.} \quad \|y - \Phi Ds\|_2^2 \leq \epsilon, \quad (4.12)$$

where the sensing matrix $\Phi \in \mathbb{R}^{p \times m}$ maps the signal of dimension $m$ to a vector of dimension $p$. The dictionary $D \in \mathbb{R}^{m \times k}$ enables the sparse representation of an unknown signal $x$ with assumed sparsity $\tau$. The number of measurements, $p$, are usually set by the user. The signal $x$ can be recovered by computing $\hat{x} = D\hat{s}$. All the algorithms discussed in Section 4.2, can be used to solve Equations (4.11) and (4.12).

Compressed sensing has applications in several fields, such as computational photography, computer vision, signal processing, medical imaging, and biology. An
early example of compressed sensing in imaging is the development of *Single-Pixel* cameras [178] that comprises a single point detector coupled with a spatial light modulator, i.e. DMD and a random number generator that creates a random pattern for DMD. The light reflected off the DMD is integrated into a single-pixel sensor. A reasonable quality image is achieved by capturing about 1300 measurements from the scene. The required amount of measurements is still much lower than the Nyquist rate (about 50 times). A diagram demonstrating this camera is shown in Figure 4.2. Other imaging systems that are formed with the help of compressed sensing include compressive light field camera [22, 23, 179], lensless imaging [180], and compressive video sensing [181, 182].

Besides imaging of visible light, CS has applications in areas where the invisible light needs to be captured and highly expensive sensor technologies are required. A few examples of such areas include capturing infrared light [183], magnetic resonance imaging (MRI) [184, 185], Computed Tomography (CT) imaging [186], bio-sensing for DNA-based identification [187], and compressive imaging [188].

In the next two chapters, the theoretical backgrounds presented here are employed to develop a framework for efficient light field encoding and rendering in Chapter 5 and designing a sensing model for a light field video camera in Chapter 6.
In conventional imaging systems, the sensor integrates the incoming light rays from various angles towards a pixel, or photo-sensor, while the shutter is open. Therefore, the angular information of the incoming light is lost in this process. Light field imaging has emerged to capture both the spatial and angular information of light rays emanating from a scene towards the camera. During the past few decades, there have been several approaches to record the scene’s illumination from various viewing angles and positions in space. The proposed techniques range from handheld cameras to bulky and expensive multi-camera systems. Light field imaging faces three main challenges: capturing a large amount of data corresponding to angular and spatial visual information of the scene (in real-time), efficiently storing the massive amounts of acquired information, and displaying the light field in real-time and according to the requirements of the application at hand. The focus of this chapter is to develop a framework to solve these challenges for light field images and videos.

5.1 Outline and Contributions

This chapter introduces a framework for sparse representation of light field images and videos. The framework develops a compression method based on the concept of sparse signal representation (introduced in Chapter 4) to efficiently and effectively encode such high dimensional data. The proposed framework provides a unified
solution by exploring the redundancies that exist within the light field data. A dictionary learning method is introduced that considers multi-dimensionality and redundancy of light fields by training a Multidimensional Dictionary Ensemble (MDE). The sparsity of the representation is increased by a novel pre-clustering method that improves the training time and reconstruction quality and leads to the construction of an Aggregated Multidimensional Dictionary Ensemble (AMDE). The compressed data with AMDE can be decoded and displayed efficiently since the method provides local access to the elements of the light field data. Paper C presents the full details and results of the framework that can be applied to any high dimensional visual data.

The trained AMDE is further improved in Paper D by a novel dictionary pruning algorithm based on pairwise mutual coherence of dictionaries in the ensemble. The pruned dictionary improves the encoding time significantly. Moreover, an extensive analysis of the effect of image noise on AMDE training and testing has been carried out, which shows that with a slight amount of denoising, the compression efficiency improves significantly. The sparse coefficients obtained from encoding, are used to reconstruct the light field in real-time with novel view generation.

The sparse coefficients can be further compressed by quantization and entropy coding, as presented in Paper E. An AMDE is trained for spherical inward-looking light field data, and the obtained sparse coefficients are quantized and encoded using Huffman coding. Our results show about three times improvements in the compression ratio while preserving the reconstruction quality.

In what follows, we describe the contributions mentioned above in more details. Specifically, in Section 5.2, we describe the plenoptic function and its relation to light fields. In Section 5.3, we describe how the light field is patched into small data points to be used for dictionary training. A framework for compression and rendering using MDE is presented with details in Section 5.4, followed by Section
5.2 Plenoptic Function

Light rays in a scene are described by a 7-dimensional (7D) function called the plenoptic function [45] that models a 3D dynamic environment at every point in space towards every possible direction over any range of wavelengths and at any given time:

\[ l(x, y, z, \theta, \phi, \lambda, t), \]

where spatial coordinates are parameterized by \((x, y, z)\), angular coordinates by \((\theta, \phi)\), spectral domain by \(\lambda\), and the time domain by \(t\). The plenoptic term was derived from plenus meaning complete and optus meaning view. A sketch of the plenoptic function is shown in Figure 5.1. One can assume that light travels constantly through the air to simplify Equation (5.1) to a 6D function. By considering only a static scene, this function can be simplified to 5D, and by restricting the wavelength to RGB channels, and limiting light rays to the ones leaving the convex hull of the bounding box of the object, it is reduced to a 4D function [189, 190].

The plenoptic function can be described by any arbitrary surface that surrounds the environment. Thus, it can be parameterized in different ways, such as the two plane parameterizations, \(l(x, y, s, t)\), where \((x, y)\) defines the ‘focal plane’ and \((s, t)\) defines the ‘aperture plane’, see Figure 5.2. Other parameterizations of light fields include spherical parameterization [191, 192], sphere-plane parameterization [192], and direction and point parameterization [193]; see [24] for a review of different parameterization methods.

Figure 5.2: The two plane parameterization of the light fields.

5.5, which explains the formation of the AMDE for designing a unified compression and rendering framework. Section 5.6 demonstrates the capabilities of the AMDE framework for compressing and rendering spherical light field data sets. Finally, a summary and possible future research directions are presented in Section 5.7.
Adopting the two plane parameterization for 6D light field video representation leads to the following formulation of the plenoptic function, Equation (5.1):

\[ l(x, y, \theta, \phi, \lambda, t). \]  

(5.2)

### 5.3 Light Field Data Points

Since processing a light field sequence is computationally infeasible, it is divided into smaller elements, called *data points* with the dimensionality \( m_1 \times m_2 \times m_3 \times m_4 \times m_5 \times m_6 \), where \( m_1 \times m_2 \) is the resolution of the spatial domain in the data point, \( m_3 \times m_4 \) is the resolution of the angular domain, and the spectral and temporal domain resolutions are shown as \( m_5 \) and \( m_6 \), respectively. The size of data points depends on the structure of the data set, and the application at hand. For angular dimension, \( m_3 \times m_4 \), and spectral dimension, \( m_5 \), usually, the intention is to include all the available information to avoid inconsistencies among different views while rendering the light field. However, for high resolution angular or spectral data sets, the data is divided into smaller patches to exploit the coherency in those domains as well. The coherency of the temporal domain, \( m_6 \), is utilized by including a few consecutive frames of the light field video in the data points. The light field data points can be constructed by a non-overlapping sliding window in the 6D light field video data set. The overlapping sliding window has a negative effect on the compression ratio, but it achieves significant improvements for the compressed sensing applications [14, 194]. The framework proposed in this chapter for sparse representation of light field data is not limited to the dimensionality of the data and can handle \( n \)-D data points.

### 5.4 Light Field Compression and Rendering

This section explains the main contributions of this thesis for the compression and rendering of visual data. Specifically, a novel framework is proposed for the compression and real-time rendering of light field and light field video data sets by exploiting the large amounts of correlation that exist within a light field data set. The framework is based on a highly efficient multidimensional dictionary learning approach. Figure 5.3 illustrates the framework in different steps.

The first step of the framework creates small \( n \)-D data points, where their size depends on the application. Usually, for light field images, the dimensionality of the data points is 5D, and for light field videos, it is 6D, as explained in Section 5.3. The training data points are used for one-time training of a multidimensional dictionary ensemble (MDE). Each dictionary element is a basis function that represents the light field data point along different dimensions. The main purpose of MDE is to create a high degree of sparsity when data points are represented in the transform.
domain of the dictionary. The sparsity is a crucial factor for efficient compression as shown by [139, 195, 196] and also in compressed sensing [171, 197].

The framework includes a novel nonlocal pre-clustering step before training the MDEs that increases the sparsity exhibited in the data set. The pre-clustering algorithm minimizes the reconstruction error and the training time, see Section 5.5.1. The effectiveness of pre-clustering has been explored in computer graphics before; for instance, an iterative variant of K-Means [198] has been utilized for clustering light transport matrices [199] and surface light fields [194].

After pre-clustering the light field data, MDE is trained for each pre-cluster. Combining all MDEs creates an aggregated multidimensional dictionary ensemble or AMDE. The memory footprint of AMDE is negligible, and encoding the data points with it provides fast access to a random location inside each data point, which is useful in a real-time reconstruction algorithm.

Every data set contains high-frequency variations added to the original signal during the acquisition process. To decrease the effect of noise on the compression performance and improve the sparsity, applying a denoising operator before encoding is therefore recommended. An in-depth analysis for motivating this step is explained in Section 5.5.5. The sparse coefficients obtained from encoding using AMDE is further compressed by quantization and entropy coding, see Section 5.5.7.

In the reconstruction and rendering algorithm, see Section 5.5.6, the encoded coefficients are decoded and reconstructed in real-time on GPU, from a specific vantage point, and at a given frame. To increase the angular resolution of the light field data set, the reconstructed viewpoints are interpolated during rendering using auxiliary information such as depth. The following sections provide the details and motivations for each step of the proposed unified framework.
5.4.1 Multidimensional Dictionary Ensemble

A multidimensional dictionary ensemble is designed such that the represented data is sparse in its transform domain. In addition to sparsity, a suitable learning-based representation should have the following properties:

- **Small dictionary size**: One of the objectives of the proposed framework is to transfer the data to a decoder for fast reconstruction. Therefore, the size of the learned dictionary should be small enough to fulfill this purpose. This is unlike previous methods where the constructed dictionary size is relatively large [165, 200, 201].

- **Nonlocal clustering**: Using a smaller number of dictionaries compared to the number of data points to represent the data, leads to nonlocal clustering. The reason is that multiple data points share one dictionary, which results in clustering. Since the training algorithm is based on $\ell_0$ norm minimization, the clustering is nonlocal. This feature has shown to promote sparsity [194, 202, 203, 204].

- **Representative power**: The ensemble of dictionaries should be able to represent any unobserved data set of the same dimensionality without the need for retraining.

- **Arbitrary dimensionality**: The framework should not be dependent on the dimensionality of the data set. It should adapt to any form of data such as bidirectional texture function (BTF), Bidirectional reflectance distribution function (BRDF), light field images, light field video. We have left the application of AMDE to these data sets for future work.

- **Local reconstruction**: Fast access to independent elements of each light field data point is required for real-time rendering.

Our proposed framework in Paper C satisfies the above criteria, which are elaborated on in what follows.

Similar to other learning-based algorithms for sparse representation of a data set, we also require a training set and a testing set. The training set consists of data points with similar dimensionality and characteristics of the testing set that we wish to compress. The training set denoted as $\{\mathcal{L}^{(i)}\}_{i=1}^{N_l}$, where $\mathcal{L}^{(i)} \in \mathbb{R}^{m_1 \times m_2 \times \cdots \times m_n}$ for $nD$ dimensional data consisting of $N_l$ data points.
Our objective is to train an ensemble of $K \ll N_l$ multidimensional dictionaries, such that each data point $\mathcal{L}^{(i)}$ is representative by only one dictionary:

$$\mathcal{L}^{(i)} = \mathbf{S}^{(i)} \times_1 \mathbf{U}^{(1,k)} \times_2 \cdots \times_n \mathbf{U}^{(n,k)} = \mathbf{S}^{(i)} \prod_{j=1}^{n} \mathbf{U}^{(j,k)},$$

where $\mathbf{S}^{(i)} \in \mathbb{R}^{m_1 \times m_2 \times \cdots \times m_n}$ is a sparse coefficient tensor, and $\mathbf{U}^{(j,k)} \in \mathbb{R}^{m_j \times m_j}$ is the $j$-th element of the $k$-th dictionary corresponding to the dimension $j \in \{1, \ldots, n\}$. The symbol $\times_j$ defines the tensor-matrix multiplication along the $j$-th mode.

Learning an ensemble of $n$-D orthonormal dictionaries for sparse representation of data can be formulated as:

$$\min_{\mathbf{U}^{(j,k)}, \mathbf{S}^{(i,k)}, \mathbf{M}_{i,k}} \sum_{i=1}^{N_l} \sum_{k=1}^{K} \mathbf{M}_{i,k} \left\| \mathcal{L}^{(i)} - \mathbf{S}^{(i,k)} \prod_{j=1}^{n} \mathbf{U}^{(j,k)} \right\|_F^2,$$

subject to

$$\left( \mathbf{U}^{(j,k)} \right)^T \mathbf{U}^{(j,k)} = \mathbf{I}, \quad \forall k = 1, \ldots, K, \quad \forall j = 1, \ldots, n,$$

$$\| \mathbf{S}^{(i,k)} \|_0 \leq \tau_l,$$

$$\sum_{k=1}^{K} \mathbf{M}_{i,k} = 1, \quad \forall i = 1, \ldots, N_l,$$

where $\mathbf{S}^{(i,k)}$ are the coefficients of the $i$-th data point when projected onto the $k$-th dictionary in the ensemble, $K$ denotes the number of dictionaries in the ensemble, and each dictionary consists of $n$ matrices operating along the data point dimensions. Equation (5.4b) ensures the orthogonality of each dictionary. The sparsity parameter $\tau_l$ is defined by the user for the training. $\mathbf{M} \in \mathbb{R}^{N_l \times K}$ is a binary matrix called membership matrix that assigns each training data point $\mathcal{L}^{(i)}$ to a dictionary $\{\mathbf{U}^{(1,k)}, \ldots, \mathbf{U}^{(n,k)}\}$. Equation (5.4d) guarantees that each data point is only represented by one dictionary and not more. The membership matrix can also be interpreted as a clustering matrix that defines which cluster each data point belongs to. Matching the dimensionality of a dictionary with the data set has shown superior performance and lower representation error when compared to the case when the dictionary dimension is lower than the data dimension, e.g. by vectorizing the high dimensional data [165, 205].

The training process for producing MDE, i.e. $\{\mathbf{U}^{(1,k)}, \ldots, \mathbf{U}^{(n,k)}\}_{k=1}^{K}$ by solving Equation (5.4) is denoted as:

$$\{\mathbf{U}^{(1,k)}, \ldots, \mathbf{U}^{(n,k)}\}_{k=1}^{K} = \text{Train} \left( \{\mathcal{L}^{(i)}\}_{i=1}^{N_l}, K, \tau_l \right).$$

This function takes the following as the input arguments: the training set, number of dictionaries in the ensemble, and the training sparsity. The dictionaries are
initialized with random orthonormal matrices and the membership matrix $M$ is initialized with $1/K$. To solve this function, we iteratively update the dictionary matrices and the coefficient tensor using the following update rules respectively:

$$U^{(k,j)} = Z^{(j,k)} \left( (Z^{(j,k)})^T Z^{(j,k)} \right)^{-1/2},$$  \hspace{1cm} (5.6)$$

$$S^{(i,k)} = \mathcal{L}^{(i)} \times_1 (U^{(1,k)})^T \times_n (U^{(n,k)})^T,$$  \hspace{1cm} (5.7)$$

where

$$Z^{(j,k)} = \sum_{i=1}^{N_l} M_{i,k} \mathcal{L}_{[j]}^{(i)} (U^{(n,k)} \otimes \ldots \otimes U^{(j+1,k)} \otimes \ldots \otimes U^{(1,k)})(S_{[j]}^{(i,k)})^T,$$  \hspace{1cm} (5.8)$$

and the membership matrix is updated using:

$$M_{i,k} = \left( \sum_{b=1}^{K} e^{\beta (\delta_{ijkl} - \delta_{ijkl})} \right)^{-1},$$  \hspace{1cm} (5.9)$$

where

$$\delta_{ijkl} = \| \mathcal{L}^{(i)} - S^{(i,k)} \times_1 U^{(1,k)} \ldots \times_n U^{(n,k)} \|_F^2.$$  \hspace{1cm} (5.10)$$

To reduce the dimensionality of the data and increase the sparsity, we nullify $(\prod_{j=1}^{n} m_j) - \tau_l$ elements of $S^{(i,k)}$ that have the smallest absolute value each time after updating the coefficient tensor, $S^{(i,k)}$. The temperature parameter, $\beta$, in Equation (5.9) is initialized with a small value. For a fixed value of $\beta$, the MDE matrices, $\{U^{(1,k)}, \ldots, U^{(n,k)}\}_{k=1}^{K}$, are sequentially updated until the changes are minimal. Then the temperature parameter increases and the next updating iteration is applied. MDE converges once the membership matrix is binary or near binary, i.e. satisfying the following criteria:

$$\|M - \lfloor M \rfloor\|_2 < \epsilon,$$  \hspace{1cm} (5.11)$$

where $\epsilon$ is a small nonnegative user-defined value. The operator $\lfloor \cdot \rfloor$ rounds its argument to the closest integer value. The detailed description of this algorithm can be found in the Paper $C$ and its supplementary material.

### 5.5 Aggregated Multidimensional Dictionary Ensemble

Training a multidimensional dictionary is dependent on the size of the training set, $N_l$, and the number of dictionaries, $K$, such that more training data and
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Input: \( \{ \mathcal{L}^{(i)} \}_{i=1}^{N_l}, C, N_r, p, \epsilon \)

Result: \( c = \text{Precluster} \left( \{ \mathcal{L}^{(i)} \}_{i=1}^{N_l}, C, N_r, p, \epsilon \right) \)

Init.: Rearrange \( \{ \mathcal{L}^{(i)} \}_{i=1}^{N_l} \) into \( F \in \mathbb{R}^{N_l \times m_1 m_2 \ldots m_n} \)

1. Compute \( F = A B^T \) using CoP with \( p \) principal components
2. Apply K-Means to rows of \( A \) with \( N_r \) clusters
3. The representative set \( \{ \bar{\mathcal{L}}^{(i)} \}_{i=1}^{N_r} \) is computed as the centroids of \( \{ \mathcal{L}^{(i)} \}_{i=1}^{N_l} \) using cluster indices returned by K-Means.
4. Calculate the ensemble \( \{ \bar{U}^{(1,c)}, \ldots, \bar{U}^{(n,c)} \}_{c=1}^{C} = \text{Train} \left( \{ \bar{\mathcal{L}}^{(i)} \}_{i=1}^{N_r}, C, \tau_l, \epsilon \right) \)
5. Compute the pre-clustering membership \( c = \text{Test} \left( \{ \mathcal{L}^{(i)} \}_{i=1}^{N_l}, \{ \bar{U}^{(1,c)}, \ldots, \bar{U}^{(n,c)} \}_{c=1}^{C}, \tau_l, \epsilon \right) \)

Algorithm 2: The pre-clustering algorithm implementing \( c = \text{Precluster} \left( \{ \mathcal{L}^{(i)} \}_{i=1}^{N_l}, C, N_r, p, \epsilon \right) \)

dictionaries lead to a more representative dictionary ensemble. Due to the high nonlinearity presented in the data, more training sets and dictionaries often do not lead to over-fitting. However, increasing \( N_l \) and \( K \) results in higher computational cost, which is undesirable. Therefore, to accelerate the training process and improve the reconstruction quality, a pre-clustering step is added into the framework for a predefined number of training data and dictionaries. In the following, we explain the pre-clustering step that leads to creating AMDE, followed by AMDE pruning and utilizing the resulted dictionary to encode and decode the light field data.

5.5.1 Pre-Clustering

The training set, \( \{ \mathcal{L}^{(i)} \}_{i=1}^{N_l} \) consisting of \( N_l \) data points, is divided into pre-clusters using our proposed algorithm that ensures each pre-cluster contains data points that are similar in their sparse representation. As mentioned earlier, MDE trains an ensemble of dictionaries by increasing the sparsity and reducing the representation error where each data point is assigned to only one dictionary. This results in MDE to indirectly perform clustering on the input training data as well. Therefore, if we pre-cluster the data before training the dictionaries, a fewer number of iterations will be required for finding suitable dictionaries for each pre-cluster, making the training stage faster. The clustering effect of MDE on each pre-cluster means that there are less variations of sparsity in each cluster. Therefore, each pre-cluster can be represented with a fewer number of basis or dictionaries, which considerably reduces the computational cost of training and encoding stages.
One clustering algorithm that has been used a lot for dimensionality reduction is K-Means and its variants [198]. However, since the K-Means approach is based on the $\ell_2$ norm distance metric rather than $\ell_0$ norm that promotes sparsity, it cannot improve the computational time of the training algorithm.

For pre-clustering, in the first step, we perform dimensionality reduction of the training set by decomposing it as $\mathbf{F} = \mathbf{A}\mathbf{B}^T$, where $\mathbf{F} \in \mathbb{R}^{N_l \times m_1 \cdot m_2 \cdot \ldots \cdot m_n}$ is a matrix with rows storing the vectorized elements of a data point tensor with size $m_1 \times m_2 \times \cdots \times m_n$, $\mathbf{A} \in \mathbb{R}^{N_l \times p}$ is the coefficient matrix, and $\mathbf{B} \in \mathbb{R}^{p \times m_1 \cdot m_2 \cdot \ldots \cdot m_n}$ contains the basis vectors in its columns, and $p \ll \prod_{j=1}^{n} m_j$. To calculate the decomposition of $\mathbf{F}$, we employ a factorization method called Coherence Pursuit (CoP) [206]. CoP is a fast and simple extension of PCA that is very robust to outliers and noise common in visual data captured with camera sensors.

As the next step in the pre-clustering algorithm, we employ K-Means to find a small subset of the training data points that best depicts the data. We apply K-Means with $N_r \ll N_l$ clusters to group the $p$-dimensional rows of the coefficient matrix, $\mathbf{A}$, obtained from CoP method. Using the calculated cluster indices, the centroids of the training set $\{\mathcal{L}^{(i)}\}_{i=1}^{N_l}$ are then estimated. The centroids are called representative set and are denoted as $\{\bar{\mathcal{L}}^{(i)}\}_{i=1}^{N_r}$.

We train an MDE with $C$ dictionaries on the representative set, where $C$ denotes a user-defined constant for the number of pre-clusters. Afterwards, in the final step, the trained MDE, together with the entire light field data points, are used to create membership vector $c \in \mathbb{R}^{N_l}$ that assigns each data point to a pre-cluster. This step is similar to the encoding process that will be explained in Section 5.5.2.

The pre-clustering process is shown as the following function:

$$ c = \text{Precluster} \left( \{\mathcal{L}^{(i)}\}_{i=1}^{N_l}, C, N_r, p, \epsilon \right), $$

in Algorithm 2.

The most important parameters during training are the number of pre-clusters, $C$, and the number of dictionaries, $K$, as shown in the evaluation of the parameters in Paper C. In general, the value of $C$ and $K$, are proportional to the reconstruction quality, but it comes as a trade-off with computational time. Table 5.1 shows the result for different scenarios: no pre-clustering, pre-clustering with our proposed method, pre-clustering with our proposed method while the number of dictionaries and pre-clusters are doubled, and using K-Means for pre-clustering. Our proposed method reduces computational time significantly while keeping the reconstruction quality almost intact. Increasing $K$ and $C$ results in better PSNR; however, the training time also increases proportionally. The pre-clustering step has a little computational cost compared to the training time.
**5.5 Aggregated Multidimensional Dictionary Ensemble**

**Input:** The training set \( \{ \mathcal{L}^{(i)} \}_{i=1}^{N_l} \), number of clusters \( C \), number of dictionaries per cluster \( K \), training sparsity \( \tau_l \), number of representatives \( N_r \), number of PCA coefficients \( p \), and an error threshold \( \epsilon \).

**Result:** A dictionary ensemble \( \{ \mathbf{U}^{(1,k)}_c, \ldots, \mathbf{U}^{(n,k)}_c \}_{k=1}^{CK} \)

**Init.:** Create data points from input training light field videos, apply pre-clustering on data points.

1. for \( c = 1 \ldots C \) do
2. Apply MDE on each pre-cluster \( c \)
3. end
4. Compute aggregated ensemble \( \Psi \) using Equation (5.13)

**Algorithm 3:** The training algorithm that is performed only once, \( c = \text{Train} \left( \{ \mathcal{L}^{(i)} \}_{i=1}^{N_l}, C, K, \tau_l, N_r, p, \epsilon \right) \)

| C | K | PSNR | pre-cluster | train |
|---|---|------|--------------|-------|
| No pre-clustering | 1 | 32 | 65.00 | - | 62:31 |
| pre-clustering (Algorithm 2) | 8 | 4 | 64.65 | 01:24 | 6:51 |
| pre-clustering, \( 2C, 2K \) (Algorithm 2) | 16 | 8 | 66.66 | 02:28 | 14:12 |
| K-Means for pre-clustering ([196]) | 16 | 8 | 63.75 | 00:32 | 16:27 |

**Table 5.1:** The effect of pre-clustering for a training set of 2D data points. For Algorithm 2 we use \( N_r = 2048 \), \( p = 8 \), \( \tau_l = \tau_t = 8 \), and \( \epsilon = 10^{-5} \).

Aggregating all MDE dictionaries of all pre-clusters, creates an aggregated multidimensional dictionary ensemble (AMDE) formulated as:

\[
\Psi = \bigcup_{c=1}^{C} \{ \mathbf{U}^{(1,k,c)}_c, \ldots, \mathbf{U}^{(n,k,c)}_c \}_{k=1}^{K} = \{ \mathbf{U}^{(1,k)}_c, \ldots, \mathbf{U}^{(n,k)}_c \}_{k=1}^{CK}.
\]  

where, \( CK \) is the total number of dictionaries. A summary of the AMDE training stage is given in Algorithm 3. AMDE provides flexibility where we can train the dictionaries only once and re-use it on other data sets with the same dimensionality. One can also combine AMDEs trained on different data sets to create a new set of dictionaries without the need for retraining. The AMDE can also be pruned to represent a data set more efficiently, which is described in Section 5.5.4.
Figure 5.4: Visual quality comparison for Painter [207], a natural light field video data set. See the supplementary video of Paper C for real-time playback of the this data set and Table 5.2 below for quantitative evaluations.

| Uncompressed size: 20604MB |
|---------------------------|
| size | PSNR | SSIM | $t_e$ | $t_d$ |
| AMDE | 941MB | 38.25 | 0.9286 | 12436 | 572 |
| K-SVD | 942MB | 38.12 | 0.9281 | 67235 | 1465 |
| 6D DCT | 942MB | 36.91 | 0.9189 | 489 | 532 |
| HOSVD | 941MB | 36.79 | 0.9147 | 792 | 555 |
| CDF 9/7 | 941MB | 31.69 | 0.8222 | 974 | 371 |

Table 5.2: Results for the natural light field video data set, Painter [207]. Encoding and decoding times are denoted $t_e$ and $t_d$, respectively, and measured in seconds.

5.5.2 AMDE Encoding

The acquired AMDE from the training step is used to encode the test set denoted as $\{T^{(i)}\}_{i=1}^{N_t}$, containing $N_t$ data points. The data points are projected onto the AMDE using Equation (5.7) and a dictionary that best represent each data point is chosen. The selected dictionary should lead to the most sparse coefficients with the least reconstruction error. This procedure is represented as the following function:

$$\left[\{S^{(i)}\}_{i=1}^{N_t}, m, z\right] = \text{Test}(\{T^{(i)}\}_{i=1}^{N_t}, \Psi, \tau_t, \epsilon), \quad (5.14)$$
where the inputs to the function are the data points of the testing set, AMDE, and thresholds for sparsity and error, respectively. We set the upper bound of the testing sparsity by $\tau_t$, i.e. the maximum number of nonzero coefficients for each testing data point. Usually the sparsity is lower than this upper bound. The function $\text{Test}(.)$ outputs a collection of sparse coefficient tensors, $\{S^{(i)}\}_{i=1}^{N_t}$, and the membership vector, $m \in \mathbb{R}^{N_t}$, associating the data points with the dictionaries in AMDE. Each element of the membership vector is an index to the best dictionary that describes the corresponding data point. The number of nonzero coefficients in each data point is stored in $z \in \mathbb{R}^{N_t}$.

5.5.3 AMDE Decoding

To reconstruct a light field after the encoding procedure, we need the set of sparse coefficient tensors, as well as the AMDE. Each data point is estimated as $\hat{T}^{(i)}$ by simply multiplying the sparse coefficients, $S^{(i)}$, with its corresponding dictionary, $m_i$, in the ensemble $U^{(1,1)}, \ldots, U^{(n,CK)}$ as follows:

$$
\hat{T}^{(i)} = S^{(i)} \times_1 U^{(1,m_i)} \times \cdots \times_n U^{(n,m_i)}.
$$

(5.15)

We will explain later how this equation can be modified to reconstruct a single element of the light field video data set in Section 5.5.6.

Figure 5.4 and Table 5.2 present the result of compressing natural light field videos using AMDE framework (without pruning the AMDEs). The result from AMDE is compared with analytically-driven dictionaries such as 6D Discrete Cosine Transform (DCT), and CDF 9/7 wavelets [208] used in standard JPEG2000 image compression standard [209], and learning-based dictionaries such as KSVD [165], and HOSVD. As mentioned earlier, AMDE has a small memory footprint compared to other methods; as an example, the dictionary size for AMDE is 0.0627MB, while the dictionary size for K-SVD is 432MB for the results presented in Table 5.2.

5.5.4 Dictionary Ensemble Pruning

As discussed earlier, increasing the number of dictionaries, i.e., $CK$, in AMDE can improve the reconstruction quality and compression ratio. However, it also proportionally increases the computational complexity of encoding unobserved data. To solve this problem, a pruning phase is added to the framework to select the most distinct dictionaries within AMDE as the representative dictionaries. To do so, a clustering algorithm is applied on AMDE to group dictionaries that are similar to each other without affecting the reconstruction quality and compression ratio. Intuitively, we would like to choose the most diverse dictionaries in the ensemble that represent the essence of the data set. One such metric for clustering is mutual
coherence (MC) [210], which defines the maximum cross-correlation between the columns of a matrix $X \in \mathbb{R}^{n \times n}$ as:

$$
\mu(X) = \max_{1 \leq u \neq v \leq n} \left| X_u^T X_v \right|,
$$  

(5.16)

where $X_u$ is the $u$th column of $X$. For AMDE, the mutual coherence of each dictionary element is zero as it is an orthogonal matrix. Mutual coherence on a pair of matrices is defined as:

$$
\mu(X, Y) = \max_{1 \leq u, v \leq n} \left| X_u^T Y_v \right|.
$$  

(5.17)

To prune the AMDE dictionaries, one can use average mutual coherence (AMC) [212] which is defined as follows:

$$
\mu_{\text{avg}}(X, Y) = n^{-2} \sum_{u, v=1, u \neq v}^{n} \left| X_u^T Y_v \right|.
$$  

(5.18)
However, this metric is suitable for pair of matrices while each dictionary in AMDE consist of 6 matrices for a 6D light field video: \( \{ U^{(1,k)}, \ldots, U^{(6,k)} \}_{k=1}^K \). Therefore, a pair-wise distance for each dimension \( j \) of dictionaries can be computed as following:

\[
\mu_{\text{avg}}(U^{(j,k)}, U^{(j,k)}) = \frac{1}{n_j} \sum_{u,v=1}^{n_j} \left\| (U^{(j,k)})^T U^{(j,k)} \right\|_{u,v} \forall k \in \{1, \ldots, K\}. \tag{5.19}
\]

To find the most diverse dictionaries based on the metric defined in (5.19), we use K-Means++ \([213]\). To initialize the cluster centers for K-Means++, dictionaries in AMDE are selected randomly to start the calculation of the pairwise distances. However, we found that random initialization creates fluctuations in the reconstruction quality based on the choice of the first dictionary. To alleviate this problem, the membership matrix, \( M \) defined in (5.4d) is used to select the most frequently used dictionaries in AMDE as the starting point of the K-Means++ algorithm.

The result of dictionary pruning is shown in Table 5.3 where the encoding time is improved by a factor of 4 for the data sets Chess \([211]\) and Heart shown in Figure 5.5.

### 5.5.5 Denoising Prior to Compression

Light fields contain high-frequency variations that can affect and deteriorate the sparse representation of the data and lower the compression performance. The effect of noise on the efficiency of the sparse representation and the reconstruction quality has been thoroughly studied from a theoretical standpoint \([156, 210]\).

To see the effect of noise on the performance of the training and testing stages of AMDE, we performed a study by applying a denoising operator with different levels of denoising on both the training and test sets. We compare the training and testing performance of the denoised data sets with the original noisy data sets. For choosing a denoising operator, there is a large number of algorithms that can be employed in this study. Our first criteria for selecting a denoiser was finding a method that keeps the main underlying structure of the signal intact while removing the high-frequency noise. Another criterion was the ability to process large data sets, such as light field videos. As a result, a deep learning method called FFDNet \([214]\) was chosen as the representative state-of-the-art denoiser that is suitable for our task.

To evaluate the effect of noise, we carried out the following experiments: setting denoising strength parameter to \( \sigma = \{3, 5, 10, 15\} \) for both training and testing set, where \( \sigma = 3 \) is the lowest, and \( \sigma = 15 \) is the highest amount of denoising. In the AMDE training algorithm we set the following parameters: 12 frames of light field video, 32 dictionaries with parameters, \( C = 4, K = 8, \) and sparsity \( \tau_l = 64 \). In the testing procedure, 50 unseen frames of the same data sets were used for evaluation.
Figure 5.6: (a)-(b) The effect of image noise on training. The plots show that the compression efficiency stays relatively flat for all denoising levels on the training set, and that the denoising of the testing data leads to a higher degree of sparsity and better PSNR. For denoising we used FFDNet [214] with the parameter $\sigma = 3, 5, 7, 10, 15$. (c) Shows the effect of image noise on testing (compression) of Painter and Chess data sets shown in Figure 5.5. The plot shows that the PSNR increases along with $\sigma$.

The testing sparsity and threshold are set to $\tau_t = 256$ and $\delta_t = 1e^{-4}$ for Chess [211] and $\tau_t = 512$, $\delta_t = 5e^{-5}$ for Painter [207]. The compression efficiency is then measured as PSNR / file size, i.e. taking into account the sparsity achieved by each configuration.

The plots illustrated in Figure 5.6(a)-(b), indicate that denoising the training set has a negligible impact on the compression efficiency of the AMDE algorithm, meaning that the training algorithm is robust to the effect of noise. However, when the evaluation concerning the testing set is performed, we see that the compression efficiency or PSNR / file size substantially increases, as shown in Figure 5.6 (a)-(b) and in Figure 5.6 (c) for the PSNR. More experimental results are shown in
Another interesting observation is that the effect of noise on encoding the testing set reduces significantly if the light field data set is intrinsically sparse such as Chess data set. The Chess data set has a higher overlap between the neighboring views compared to the Painter data set with disparities up to 180 pixels.

5.5.6 Real-Time Decoding and Rendering of Light Field Video

Angular super-resolution of light fields requires the utilization of the depth information in the light field, either by calculating the depth while rendering or using pre-computed or captured depth. The auxiliary information that accompanies light fields, e.g. depth, can be compressed alongside the light field images in a single process. When creating the data points, one can include the depth as a color channel; for instance, the Painter data set consisting of depth maps and images, is divided into data points with size $10 \times 10 \times 4 \times 4 \times 4 \times 4$, that includes $10 \times 10$ samples of the spatial domain, $4 \times 4$ of the angular domain, 4 color channels (depth as the 4th channel), and 4 frames along the temporal domain.

A light field video data set can be displayed in real-time for a virtual point of view, which corresponds to a 2D slice through the data for a given viewing direction and time. As mentioned earlier, AMDE dictionary has a very small memory footprint which can be easily uploaded on the GPU memory together with the sparse coefficients corresponding to frame $t$ of the light field video. Using a pixel buffer object (PBO), the frames are streamed to the GPU for reconstruction and rendering. Once the data is uploaded on the GPU, each $i$th data point, $\mathbf{T}^{(i)}$, can be reconstructed by a simple multiplication of the coefficients, $\mathbf{S}^{(i)}$, with its corresponding dictionary as explained in Equation (5.15). For a 6D light field video, Equation (5.15) is simplified to:

$$
\hat{\mathbf{T}}^{(i)} = \mathbf{S}^{(i)} \prod_{j=1}^{6} \mathbf{U}^{(j,m_i)}.
$$

(5.20)

This equation reconstructs the entire data point that contains several thousands of elements, which is not suitable for real-time reconstruction and rendering. Recall that each data point contains all the angular information of a light field video frame. However, during rendering, we only require one light field view for a specified frame. Therefore, reconstructing only a fraction of $T$ is a requirement for real-time rendering. In order to reconstruct only one element of the data point with arbitrary
location $x_1, \ldots, x_6$, we propose the following formulation:

$$
\mathbf{T}_{x_1, \ldots, x_6} = \frac{\tau_j}{l_{j1}, \ldots, l_{j6}} \prod_{m=1}^{6} U_{x_1, l_{1m}}^{(1, m)} \cdots \prod_{m=1}^{6} U_{x_6, l_{6m}}^{(6, m)},
$$

(5.21)

where $l_{j1}, \ldots, l_{j6}$ describe the location of $j$th nonzero element in the sparse tensor $S^{(i)}$. Equation (5.21) enables fast access to each individual elements of the light field video which is used for fast reconstruction and rendering of each frame.

**View Interpolation**  The angular resolution of the acquired light field images and videos are usually very low. Low angular resolution creates gaps between the sampling views, which results in sudden shifts when the virtual camera moves from
Figure 5.8: The histogram of the nonzero coefficients of a spherical light field data set called TOY (shown on the left) with the angular resolution of $13 \times 360$ and spatial resolution of $800 \times 600$.

One light field view to another. Creating novel views for a seamless rendering of light field data sets is an active research topic [53, 215, 216, 217, 218]. However, the majority of these techniques do not allow real-time playback of light field videos. Here, in order to evaluate the performance of our decoding method, we implemented a bilinear interpolation between the four nearest sample points.

We place a proxy geometry in the middle of the scene to guide the rendering process and act as a surface on which the real scene is placed. Any simple geometry such as plane, sphere, cube, or cylinder can be used as the proxy geometry. Since we typically encode the depth map together with the light field data set, we can employ the reconstructed depth map on the GPU to correct the proxy geometry for better forward and backward projection of each point into the real-world and the camera sensors. Please note that the depth map is also reconstructed in real-time on the GPU using Equation (5.21).

The closest cameras to the virtual view are found using the calibration matrices from the sampling points, see Figure 5.7(a). Using the four closest cameras, $C_A, C_B, C_C, C_D$, and their corresponding distance to the virtual camera, as shown in Figure 5.7, we apply a bilinear interpolation to reconstruct a novel view between the sampled view points using a plane as a proxy geometry:

$$
I = (1 - \beta) \ast (\alpha \ast I_{C_A}(\hat{x}_{C_A}) + (1 - \alpha) \ast I_{C_B}(\hat{x}_{C_B})) + \\
\beta \ast (\alpha \ast I_{C_C}(\hat{x}_{C_C}) + (1 - \alpha) \ast I_{C_D}(\hat{x}_{C_D})), \quad (5.22)
$$

where parameters, $\alpha$ and $\beta$ determine the distance of the virtual camera from $C_A$ and $C_C$, respectively. Moreover, $\hat{x}_A, \hat{x}_B, \hat{x}_C$, and $\hat{x}_D$ are spatial positions of each
Figure 5.9: Spherical light field data set with depth map from left to right: POT, STOOL, VASE, LAMP, and TOY.

| Data set       | POT   | STOOL | VASE  | LAMP  | TOY   |
|----------------|-------|-------|-------|-------|-------|
| AMDE Comp. Ratio | 1022:1 | 58:1  | 129:1 | 59:1  | 148:1 |
| PSNR(dB)      | 53.11 | 39.72 | 47.23 | 35.57 | 44.71 |
| AMDE + Quant. + Huff. Comp. Ratio | 3054:1 | 130:1 | 294:1 | 128:1 | 329:1 |
| PSNR(dB)      | 52.41 | 39.18 | 45.96 | 35.32 | 44.15 |

Table 5.4: The comparison of AMDE results with and without quantization and entropy coding of the sparse coefficients using the Huffman coding. The data sets include depth information, see Figure 5.9. All data sets are with spatial resolution of 800 x 600 except for POT which is 4000 x 3000.

By sorting the sparse coefficients in descending order before uploading them to the GPU, the user can interactively control the number of coefficients that are employed during the real-time reconstruction. This provides a trade off between reconstruction quality and performance of rendering.

5.5.7 Quantization and Entropy Coding

The sparse coefficients obtained by applying the AMDE encoding algorithm (4.5), can be further compressed through a quantization and entropy coding procedure. Figure 5.8 illustrates the distribution of the nonzero coefficients for the TOY data set (spherical light field, see Section 5.6), where a large number of coefficients are close to zero. This motivates the entropy coding of the coefficients. As the coefficients are in floating points precision, it is required to quantize them before applying any entropy coding algorithm on them.
We employ the Fisher-Jenks classification algorithm [219] that classifies the features of a 1D vector by finding the natural breaks in the data values by minimizing the sum of the squares of the deviations from the class means. The user defines the number of cluster centroids. It should be noted that besides the nonzero coefficients, we also store other information such as the locations of the nonzero elements in each light field data point. After quantization of the nonzero coefficients to 8 bits per coefficient, the Huffman algorithm is applied to the quantized coefficients together with the corresponding locations. The location of a nonzero coefficient is a tuple of integers. For instance, for a 4D light field, the location of a nonzero coefficient consumes e.g. 32-bits, i.e. 8-bits per dimension.

The Huffman coded coefficients are stored on the disk and decoded once they are loaded into the memory. The compression ratio after applying quantization and entropy coding improves about three times while preserving the quality of the reconstruction, as shown in Table 5.4 and Figure 5.10.

5.6 Inward Looking Spherical Light Fields

To capture light fields with larger baselines, one method is to employ many cameras positioned around an object [47] or to use a single camera that rotates around the object [190]. These two methods can also be combined to create a light field acquisition device where multiple cameras are placed on a robotic arm that rotates with a high-precision motor and captures the scene as illustrated in Figure 5.11. We simulated the proposed design using synthetic light fields that were generated...
by placing virtual cameras in Maya and rendering multiple scenes, including objects with various materials. The light fields captured by this method has a very large memory footprint. If 13 cameras are attached to the rotating arm, and the camera movements are one degree apart in the azimuthal direction, then the angular resolution of the captured data is $13 \times 360$. We use AMDE for compression and rendering of these data sets.

Since the angular resolution of these types of light fields is much higher than other data sets, we cannot create data points that include all angular information in it. For this specific data set, the angular resolution along the azimuth is higher than the elevation. Hence, we create data points that include samples for each elevation ring along the azimuth coordinate. We include every $3 - 5$ consecutive sampled views on the azimuth coordinate in the data points depending on the structure of the captured scene.

Table 5.4 shows the result for compression of spherical light field data sets using the AMDE framework. The compression ratio, without quantization and entropy coding, varies from 58:1 to 1022:1 depending on the resolution of the data set and its characteristics. The light field is also rendered in real-time with view interpolation using the method described in Section 5.5.6.

### 5.7 Summary and Future Work

This chapter presented a pipeline for sparse representation and compression of light field data sets. The proposed framework provides efficient and effective compression of a variety of light field data types using AMDE. With GPU implementations, light fields can be displayed in real-time from any vantage point, which can be used for interactive applications such as virtual reality (VR). Novel view generation
is possible due to the fast access to the individual elements of the light field in real-time, which was shown with a bilinear interpolation method.

The novel pruning method improves the computational complexity of the AMDE. We showed that the AMDE is very robust to the effect of camera noise in the training phase, which can be explained with the novel pre-clustering step that increases the sparsity of the representation. On the other hand, reducing the noise in the testing phase can improve the compression ratio significantly.

In the future, it would be interesting to perform novel view generation in the sparse domain of the dictionaries to improve the reconstruction performance. Adding the dictionary similarity as a constraint on the AMDE optimization is another direction for future work. Accurate estimation of the noise parameters for each data set can be used together with the sparsity condition to develop a new sparse recovery algorithm, similar to [156, 220, 221, 222]. The AMDE method performs best when there is considerable overlap between the neighboring views. Therefore, adapting the data points shape to the features of the data set by employing information such as optical flow or disparity map is expected to improve the efficiency of the algorithm.
Compressive Light Field Imaging

Capturing high-quality light fields with a single sensor is a challenging task. Various optical designs have been proposed in the past two decades to record a light field of a scene. One of these designs is plenoptic camera, also known as integral imager in the optics community. While these optical architectures are capable of sampling the plenoptic function in one snapshot using a single sensor, the resulting measurement is usually a low-dimensional representation with a trade-off between the spatial and angular resolutions. Another design that utilizes the redundancies presented in the light field of a natural scene is called coded-aperture photography, where the light field is measured compressively as a two-dimensional projection of light rays using a known mask.

Compressive light field imaging is an attempt to employ an on-sensor compression approach to measure just enough samples to be able to reconstruct a full-resolution light field. This chapter explores different design concepts for sampling the plenoptic function and introduces a compressive sensing model for designing a light field video camera.

6.1 Outline and Contributions

The basic concepts of compressed sensing have been explained in detail in Chapter 4. This chapter utilizes the idea of compressive sampling for designing a single sensor light field video camera with a new sensing model. The proposed sensing model presented here is formulated to increase the sparsity and incoherence in the measurements, in order to improve the reconstruction quality. Our novel sensing
model is tested with two camera configurations, where a color-coded mask is placed in front of either a monochrome sensor or a sensor equipped with the CFA. We utilized multiple frames of 2D raw images from either of these two configurations to reconstruct a full resolution light field video. The contributions presented in this chapter are included in Paper F.

This chapter is organized as follows: Section 6.2 presents an overview on the light field acquisition methods, Section 6.3 provides details for designing a compressive light field video camera, and finally a summary and possible future extensions of this method are explained in Section 6.4.

### 6.2 Light Field Acquisition Techniques

The history of light field imaging is more than a hundred years old, where the first attempts were in the area of acquisition. Various methods are proposed, from massive camera array systems to hand-held light field cameras. Capturing high resolution light field and light field videos requires an excessively large amount of bandwidth, which has been the main barrier for light field imaging. Light fields and light field videos have varieties of applications such as refocusing [21], interactive 3D video [223], free-viewpoint television (FTV) [224], light field display [225], auto-stereoscopic 3D display [226], depth estimation and occlusion modeling [227, 228, 229], scene flow estimation [230], light field segmentation [231], stitching [232], material recognition [233], compositing light field video [217], and photo-real digital actors [234].

In the past two decades, there has been an extensive amount of research for developing methods and technologies to capture different dimensions of the plenoptic function. Each method is designed to sample the light fields based on different parameterizations. Light fields can be sampled uniformly or non-uniformly (unstructured). In this thesis, we consider only uniform sampling.

Designing a light field acquisition system depends on a few important factors, such as the baseline between the neighboring views, spatial and angular resolutions, and the ability to capture a light field video. These factors make the capturing design suitable for a specific application. As an example capturing wide-baseline is suitable for applications such as 3D Television (3DTV), Free-view Television (FTV) [224], and augmented reality (AR) applications. The ideal capturing device would sample densely over all viewpoints. However, the large volume of data and high capturing costs are the main limitations of this process. Light field capturing systems can be classified into two main categories as single-sensor and multi-sensor systems, which are explained in the following.
6.2.1 Multi Sensor LF Acquisition

Light field acquisition with multi-sensor design was first developed by Wilburn et al. [47], see Figure 6.1 (b), and Zhang et al. [236]. Capturing concentric mosaic with an array of cameras placed in front of the scene in a circular or planar arrangement was introduced long before, and was used in movies such as Matrix [51]. Marton et al. [237] explored a multi-viewing system for real-time capturing and reconstruction of light fields and displaying it with full horizontal parallax on a multi-projector light field display. Camera arrays have been used on mobile phones as well, such as Pelican [238], where the phone is equipped with an array of small cameras, as shown in Figure 6.1(e). Multi-camera system has been used to create life-size digital humans by capturing real people in a light stage environment and displaying them as a hologram using an automultiscopic projector array [239]. Recently, Overbeck
et al. [240] presented a system for acquiring, processing, and rendering panoramic light field stills. Their capturing system consisted of two designs with multiple cameras for capturing spherical light field images.

The design introduced in Section 5.6 makes it possible to capture spherical light fields of large scale objects and artifacts. A prototype of this design is shown in Figure 6.1 (c), where the cameras are attached on a motorized circular arm. Multi-camera systems have also been used for capturing light field video of heart surgery, as shown in Figure 6.1(d) and presented as an application in Paper D.

Multi-sensor designs are capable of recording a dynamic scene in high spatial resolution and create contents that provide plausible and immersive experiences [235, 241]. These systems provide a very large baseline in the captured data, which increases the accuracy of the estimated depth layers, and is suitable for recovering the 3D structure and dense 3D motions of the dynamic nonrigid scenes [242]. However, the disadvantage of these systems is that they are often very bulky and expensive and require careful calibration and maintenance, see Figure 6.1(a) as an example where 100 cameras are arranged to record the scene.

### 6.2.2 Single Sensor LFs

Light field capturing with a single sensor is dependent on the optical elements and hardwares that are used in designing the capturing system. A few examples of these additional components include micro-lens arrays, attenuation mask, and a robotic arm. In what follows, a few examples of these designs are explained.

**Lenslet cameras** Lippman was the first to introduce the idea of placing micro-lenses array (MLA) in front of a sensor in 1908 [43], as shown in Figure 6.2 (a). Adelson and Wang [243] proposed a similar and improved design called *plenoptic camera*, where an MLA was placed on the focal point between the objective lens and the sensor, enabling hand-held light field capture. This design was later modified and implemented by Ng et al. [46], leading to the commercial production of Lytro and Raytrix cameras. Utilizing an array of lens-prism pairs in front of the main lens can turn a conventional high-resolution camera into a light field camera [244]. However, the lens-prisms must be well-corrected for accurate capturing. Micro-lenses have also been used in light field microscopy, where it allows for oblique views and 3D reconstruction and refocusing [10]. A few of these cameras are shown in Figure 6.2(b)-(e).

Since the sensor size is limited, lenslet design suffers from a trade-off between angular and spatial resolution [244]. Another problem associated with lenslet cameras is the narrow baseline where the distance between the neighboring views is minimum, leading to sub-pixel disparities, which reduces the accuracy of the
6.2 • Light Field Acquisition Techniques

(a) Lippman

(b) Lytro  (c) Raytrix  (d) Adobe Lens Prism  (e) LF Microscopy

Figure 6.2: (a) Lippman’s 1908 Integral Imaging. Spatially-multiplexed light field capture using lenslet (Fly’s Eye lenslet). (b)-(e) Single sensor light field camera with micro-lens array (MLA) design.

depth layers. This design is mainly suitable for recovering a shallow depth map and post-capture focusing.

Coded Aperture Cameras In coded-aperture photography, a known mask is placed between the sensor and the aperture plane. Coded-aperture imaging has been used in many fields such as computational photography [22, 23, 179, 180, 245], astronomy, and medicine [246, 247, 248].

For light field imaging, Veeraraghavan et al. [249] utilized frequency domain and a heterodyne method to derive a simple attenuating mask for designing a light field camera. Higher reconstruction quality can be achieved by using Liquid Crystal Array (LCA) to change the mask pattern to capture multiple shots of the scene and increase the number of light field measurements [250]. The spatial resolution of the light field image can improve by employing two masks in the light field camera [251] at the cost of reducing the light efficiency due to the masks.

Another coded-aperture design was proposed by Babacan et al. [252], where a randomly generated mask was placed at the aperture plane, and the final light field image was reconstructed using a Bayesian framework. A compressive sensing framework for capturing light fields as a 2D projection on the sensor was proposed by Marwah et al. [22] where they built a prototype camera using their proposed design. This design was improved later by Miandji et al. [23] by suggesting to use a color-coded mask instead of a monochrome mask and training 5D dictionaries.
as priors for the reconstruction of the light field image. In contrast to lenslet cameras that suffer from spatio-angular trade-off, coded aperture cameras enable higher spatio-angular resolutions at the cost of higher computational complexity. A coded-aperture solution for light field video acquisition is explained in Section 6.3, where a color-coded mask is placed in front of the sensor. By exploring the temporal correlations between the light field frames, a desirable sparse representative model is developed for recovering the light field frames.

**Light Field Gantry** Another way to capture light fields is by using a single camera that is pinned to a motorized device that moves in a specified path and captures the scene. Since the camera can only capture one instance at a time, this design is only suitable for static scenes and cannot record dynamic scenes. Levoy et al. [189] attached a camera on a robotic arm to rotate it around a small object to acquire images from different angles, see Figure 6.3 (a). Unger et al. [253] introduced a system for capturing incident light field as a volume by moving the camera and acquiring the spatially varying illumination, see Figure 6.3 (c). Other gantry systems include Lego Mindstorms Gantry\(^1\), Figure 6.3 (b), and marker-less capture [52]. An interactive capturing system that guides the user for sampling light fields using mobile phones was introduced by Mildenhall et al. [216]. They used multi-plane images for synthesizing the intermediate views.

### 6.3 Compressive Light Field Video Camera

A 6D light field video described with two-plane parameterization is denoted as \( L(s, t, u, v, \lambda, f) \), where \((s, t)\) is the spatial domain, \((u, v)\) describes the angular

\(^1\) [http://andrew.adams.pub/](http://andrew.adams.pub/)
domain, $\lambda$ spectral dimension, and $f$ is the time domain. A 2D image is captured by integrating light rays over the angular domain and projecting onto the camera sensor:

$$y(s,t,\lambda) = \int_{u,v} L(s,t,u,v,\lambda) \cos^4 \alpha \, du \, dv,$$

(6.1)

where $\alpha$ is the angle between incoming light ray and the sensor, and $\cos^4 \alpha$ is the vignetting effect [254], which we ignore in our model for simplicity. A monochrome coded-mask, $\Phi$, at a small distance, $d_m$, in front of the sensor, modulates the light rays and projects it onto the sensor [22]. This design changes the integral photography Equation (6.1) into the following:

$$y(s,t,\lambda) = \int_{u,v} L(s,t,u,v,\lambda) a(s + \sigma(u - s), t + \sigma(v - t)) \, du \, dv,$$

(6.2)

where the function $a(\cdot)$ is the attenuation mask and $\sigma = d_m/d_a$ defines the shear of the mask pattern. This equation can be written in discrete form as a matrix-vector multiplication:

$$y = \left[ \Phi^1 \Phi^2 \ldots \Phi^\nu \right] \left[ \begin{array}{c} x^1 \\ x^2 \\ \vdots \\ x^\nu \end{array} \right],$$

(6.3)

where $\nu = |u||v|$ denotes the angular resolution and $x^i \in \mathbb{R}^\omega$ contains the vectorized light field view images, with $\omega = |s||t|$ as the spatial resolution. The mask model in Equation (6.2), is represented as $\Phi = \{\Phi^i\}_{i=1}^\nu$, with diagonal matrices $\Phi^i \in \mathbb{R}^{\omega \times \omega}$.

A sensing matrix corresponding to a monochrome mask should be applied on each color channel independently, which is shown to increase the coherency of the measurements, and in turn leading to a lower reconstruction quality [23]. In contrast, a sensing matrix for a color-coded mask is applied to all color channels of a light field simultaneously.
6.3.1 Sensing Matrix Design

For designing a sensing matrix that is suitable for a compressive light field video camera, we considered two camera configurations: First, a sensor, equipped with a color filter array (CFA) and a color-coded mask that is placed between the aperture and the sensor at a predefined distance $d_m$ as illustrated in Figure 6.4 (a). Second, a monochrome sensor with a color-coded mask placed similarly as the first configuration. The raw input of each configuration is shown in Figure 6.4 (b) and (c). In what follows, we will explain the sensing matrix for each configuration. These matrices are used to reconstruct a full resolution light field video.

Sensor with CFA and color mask  Placing a random color mask at a small distance in front of the sensor of a camera with overlaid CFA for compressive imaging was first introduced by Miandji et al. [23]. Their method assumes that the scene is static or nearly static in order to capture multiple snapshots, and thereby increasing the incoherency of the measurements, which leads to the reconstruction of a high-quality light field image. However, this method is not suitable for recording the light field video of a dynamic scene, as capturing multiple shots for each frame is not feasible. The consecutive frames of a light field video, however, contain significant correlations that can be used in the reconstruction algorithm, which is explained in Section 6.3.3. The mask can be changed using a piezo motor to create a unique random mask pattern for each light field frame. The randomness of the mask also increases the incoherency of the measured light field video, which is suitable for compressive light field recovery.

The color-coded mask may contain any wavelength in the spectrum, however, without loss of generality, we assume Red, Green, Blue (RGB) as color channels to simplify the notations. The sensing matrix for each frame $j \in \{1, \ldots, N\}$ of a light field video with $N$ frames, using a color-coded mask and a CFA sensor is similar to the definition of Miandji et al. [23]:

$$
\Psi^j = \begin{bmatrix}
\Phi^{1,R,j} & \ldots & \Phi^{\nu,R,j} & 0 & 0 \\
0 & \Phi^{1,G,j} & \ldots & \Phi^{\nu,G,j} & 0 \\
0 & 0 & \ldots & \Phi^{1,B,j} & \ldots & \Phi^{\nu,B,j}
\end{bmatrix}. \tag{6.4}
$$

For light field video, we propose to stack vertically $\beta$ consecutive frame measurement matrices, denoted as $\Psi^j$, $i \in \{1, \ldots, \beta\}$, as follows:

$$
\Phi_{(I)} = \begin{bmatrix}
\Psi^1 \\
\vdots \\
\Psi^\beta
\end{bmatrix} \in \mathbb{R}^{\beta \omega \lambda \times \omega \nu \lambda}. \tag{6.5}
$$

The sensing matrices can alternatively be stacked horizontally as:

$$
\Phi_{(II)} = \begin{bmatrix}
\Psi^1 & \ldots & \Psi^\beta
\end{bmatrix} \in \mathbb{R}^{\omega \lambda \times \beta \omega \nu \lambda}. \tag{6.6}
$$
We will see in the next section how horizontal arrangement reduces incoherency of the measurements. This arrangement is equivalent to linear combination of the input frames with a convolution filter over their angular domain into $y \in \mathbb{R}^{\omega \lambda}$. On the other hand, the sensing matrix $\Phi_{(I)}$ (6.5) contains $\beta$ times more samples, and as it will be confirmed later, is superior to $\Phi_{(II)}$ (6.6).

**Monochrome sensor with color mask** Another design for capturing compressive light field video, similar to [179, 255], is to place a color-coded mask in front of a monochrome sensor. In this design, the spectral domain is compressed during capturing, leading to a higher compression ratio when compared to the design with a CFA sensor. This feature is suitable for the fast transmission of the captured data, and in practice, it is easier to place the mask in the desired location without the CFA on the way. Another benefit of the monochrome sensor design is that the light efficiency is higher when compared to CFA equipped sensor.

The sensing matrix for a single frame of a monochrome sensor with an RGB color-coding mask is:

$$
\Lambda_j = \begin{bmatrix}
\Phi_1^{R,j} & \ldots & \Phi_\nu^{R,j} \\
\Phi_1^{G,j} & \ldots & \Phi_\nu^{G,j} \\
\Phi_1^{B,j} & \ldots & \Phi_\nu^{B,j}
\end{bmatrix}.
$$

The concatenation of the sensing matrices vertically, $\Phi_{(III)}$, and horizontally, $\Phi_{(IV)}$, is defined as follows:

$$
\Phi_{(III)} = \begin{bmatrix}
\Lambda_1 \\
\vdots \\
\Lambda_\beta
\end{bmatrix} \in \mathbb{R}^{\beta \omega \times \omega \nu \lambda}.
$$

$$
\Phi_{(IV)} = \begin{bmatrix}
\Lambda_1 & \ldots & \Lambda_\beta
\end{bmatrix} \in \mathbb{R}^{\omega \times \beta \omega \nu \lambda},
$$

It should be noted that these sensing matrices do not require custom hardware implementations and are formulated after the capturing process and only affect the reconstruction algorithm. We assume that only the encoded frames captured with either of the design configurations and their corresponding masks are available to the reconstruction method explained in Section 6.3.3.

### 6.3.2 Dictionary Training

The inverse problem of light field reconstruction from compressive measurements, Equation (6.3), requires a proper basis function or dictionary $D$ to represent the natural light field data sparsely. The proposed approach for compressive light field video acquisition trains a dictionary that admits the sparse representation of the data while including the temporal domain in the atoms of the dictionary. Doing so will increase the sparsity due to the correlation between the neighboring frames.
Assuming we have a training set \( Z = \{ z^1 \ldots z^t \} \) consisting of \( t \) light field video frames, the dictionary \( D \) is trained using an online dictionary learning algorithm [143] by solving:

\[
\min_D \frac{1}{t} \sum_{i=1}^{t} \min_{h^i} \frac{1}{2} \| z^i - Dh^i \|_2^2 + \lambda \| h^i \|_0, \tag{6.10}
\]

where \( h^i \) is the latent representation for each training data \( z^i \). The non-negative coefficient \( \lambda \) in (6.10) defines a trade-off between the reconstruction error and sparsity.

To solve Equation (6.10), we create small data points from the light field video data set. The dimensionality of the data points defines the quality of the dictionary and how well it represents the data. Marwah et al. [22] show that using 4D spatio-angular data points increases angular coherency in the reconstructed light field. Including the color information in the data points and expanding them to 5D has shown to improve the reconstruction quality even further compared to 4D patches [23]. We propose to include the temporal dimension and expand the data points to 6D such that it spans spatial, angular, spectral, and temporal domains concurrently. Adding temporal information is expected to increase the sparsity, improve the reconstruction quality, and increase the temporal coherency of the reconstructed light field video. The dimensionality of the data points is then denoted as \( n = s \times t \times u \times v \times \lambda \times \beta \) corresponding to the spatial, angular, spectral, and temporal resolution of the data point, respectively.

We considered two options for training the dictionaries: Single-frame dictionary where the dictionary is trained on 5D data points that are extracted from each individual frame. The atoms of this dictionary is a basis function representing spatial, angular and spectral domains. The second option is to extract 6D data points as explained above to include \( \beta \)-consecutive frames and train a multi-frame dictionary with the following structure:

\[
D = \begin{bmatrix}
D_1 \\
D_2 \\
\vdots \\
D_\beta
\end{bmatrix},
\tag{6.11}
\]

where \( D \in \mathbb{R}^{\beta \lambda \nu \omega \times \rho \beta \lambda \nu \omega} \) and \( \rho \) is the over-completeness factor. The multi-frame dictionary contains the temporal information which improves the sparsity when compared to the single-frame dictionary.

### 6.3.3 Sparse Reconstruction

To recover a light field video from the sparse measurements, the reconstruction algorithm or the sensing model needs to take into account the sensing matrices.
6.3 • Compressive Light Field Video Camera

Figure 6.5: Comparison of proposed sensing models of the Boxer data set [211] on a monochrome sensor with color-coded mask.

| Models         | Boxer | Chess |
|----------------|-------|-------|
| Miandji et al. [23] | 0.8426 | 0.8832 |
| SM1            | 0.9023 | 0.9201 |
| SM2            | 0.8135 | 0.8620 |
| SM3            | 0.9500 | 0.9619 |

Table 6.1: Comparison of the proposed sensing models; data sets used are Boxer and Chess [211] with non-overlapping patches, each with 5 frames for testing. Moreover, we set $\beta = 3$, $n = 7 \times 7 \times 5 \times 5 \times 3 \times 3$, image size: [540,960], and we used 10 frames for training (distinct from the testing set).

-described in Section 6.3.1 and the dictionaries described in Section 6.3.2. As explained in Chapter 4, a signal is recovered from sparse measurements by solving Equation (4.11). For the sake of clarity we revisit the equation here:

$$\arg\min_{\theta} \|\theta\|_0 \text{ s.t. } \|y - \Phi D\theta\|_2^2 \leq \epsilon,$$

(6.12)

We considered three possible sensing models for the reconstruction of the full resolution light field video, and each model will be explained in detail below. These sensing models are applicable to both camera designs that were explained above. However, for simplicity, we only explain the sensing model for a monochrome sensor.

Sensing Model 1 (SM1) In the first sensing model, we append vertically the sensed 2D raw image of each frame, $y^i$, into a vector $y \in \mathbb{R}^{3\omega}$ and their corresponding
sensing matrices $\Lambda^i$ are also stacked vertically into matrix $\Phi(III) \in \mathbb{R}^{\beta \omega \times \omega \nu \lambda}$ as explained in Equation (6.8). We train a single-frame dictionary for this sensing model on 5D training data points, to create a dictionary $D \in \mathbb{R}^{\omega \nu \lambda \times \rho \omega \nu \lambda}$. Utilizing $\beta$-consecutive light field frames $\{x_1, \ldots, x_\beta\}$ in the reconstruction, where $x_i \in \mathbb{R}^{\omega \nu \lambda}$, leads to formulating the following minimization problem:

$$\arg\min_{\theta} \|\theta\|_0 \text{ s.t. } \left\| \begin{bmatrix} \Lambda^1 x^1 \\ \vdots \\ \Lambda^\beta x^\beta \end{bmatrix} - \begin{bmatrix} \Lambda^1 \\ \vdots \\ \Lambda^\beta \end{bmatrix} D \theta \right\|_2^2 \leq \epsilon,$$  

(6.13)

where $\epsilon$ is the error of the reconstruction that is defined by the user. Arranging the matrices in the vertical form increases the number of incoherent samples which leads to improvement of the reconstruction quality. The result of using $SM1$ is illustrated in Figure 6.5, where as it is visible, the sensing model performs quite well in the recovery of the stationary objects in the background but it reconstructs the moving objects with artifacts along the edges due to the lack of temporal information in the single-frame dictionary.

**Sensing Model 2 (SM2)** To include the temporal information in our sensing model, we train a multi-frame dictionary on 6D data points that spans the temporal domain in $SM2$. However, unlike $SM1$, in this model the elements of the sensing matrix are organized horizontally as Equation (6.9). Therefore the reconstruction depends on solving the following problem:

$$\arg\min_{\theta} \|\theta\|_0 \text{ s.t. } \left\| \begin{bmatrix} \Lambda^1 x^1 \\ \vdots \\ \Lambda^\beta x^\beta \end{bmatrix} - \begin{bmatrix} \Lambda^1 \\ \vdots \\ \Lambda^\beta \end{bmatrix} \begin{bmatrix} D_1 \\ \vdots \\ D_\beta \end{bmatrix} \theta \right\|_2^2 \leq \epsilon.$$  

(6.14)

The multi-frame dictionary that is trained on $\beta$ consecutive frames enables $SM2$ to reconstruct the light field frames with less temporal artifacts compared to $SM1$. The quantitative and visual comparison of the two models can be seen in Figure 6.5 and Table 6.1. As expected, horizontal arrangement of the sensing matrix decreases the number of incoherent samples that are used in solving the BPDN problem (6.12). The optimization problem cannot find the suitable coefficients for the reconstruction despite the fact that the dictionary atoms encode multi-dimensional information. As an example, in Figure 6.5, the light field frame is slightly blurry and the colors are not recovered correctly.

**Sensing Model 3 (SM3)** The advantages and disadvantages presented in $SM1$ and $SM2$ led us to design a third sensing model to maximize the incoherency of the measurements as well as the sparsity. Therefore, we combine the sensing matrix of $SM1$ to have $\beta$ times more incoherent samples, with the multi-frame dictionary of
SM2 to benefit from the temporal correlations in our optimization problem. Since each light field video frame is captured independently from the other frames, we re-arrange the matrix multiplication of the sensing matrix and the dictionary to achieve the following minimization problem:

$$\arg\min_\theta \|\theta\|_0 \text{ s.t. } \left\| \begin{bmatrix} \Lambda^1 x^1 \\ \vdots \\ \Lambda^\beta x^\beta \end{bmatrix} - \begin{bmatrix} \Lambda^1 D_1 \\ \vdots \\ \Lambda^\beta D_\beta \end{bmatrix} \theta \right\|_2^2 \leq \epsilon, \quad (6.15)$$

where $D_i \in \mathbb{R}^{\lambda \omega \lambda \omega \times \rho \beta \lambda \nu \omega}$, $i \in \{1, \ldots, \beta\}$, are sub-matrices of the multi-frame dictionary $D \in \mathbb{R}^{\beta \lambda \omega \lambda \omega \times \rho \beta \lambda \nu \omega}$, defined in (6.11), corresponding to frame $i$ of the captured light field video. The recovered signal from solving the optimization problem of SM3 is $\hat{x} \in \mathbb{R}^{\beta \omega \nu \lambda}$, which means that for each frame of the light field, we recover $\beta$ frames. By applying a simple averaging method over the $\beta$ recovered light field frames, we further improve the reconstruction quality, see Figure 6.6 for an illustration of temporal averaging of the estimated light field frames. The result is expected to be improved with a more sophisticated motion-aware algorithm, such as [256]. The superiority of the SM3 over other sensing models is confirmed in Figure 6.5 and Table 6.1.

To solve the optimization problem presented in the three sensing models, i.e. (6.13), (6.14), and (6.15), we applied the Smoothed-$\ell_0$ (SL0) algorithm [160] as it has shown a better trade-off between the reconstruction quality and the speed compared to other sparse recovery methods such as [150, 153, 257, 258, 259].

To evaluate the performance of SM3, we simulated the compressive capturing of both sensing configurations of a monochrome sensor and CFA equipped sensor with a color-coded mask. Three light field video data sets were used for this purpose: Boxer-Gladiator-Irish, Chess, and Chess-moving from [211]. We compared the result of reconstruction using SM3 with temporal window size of 3 ($\beta = 3$), with respect to the method of Marwah et al. [22], Miandji et al. [23], and the deep

![Figure 6.6: A window of size $\beta = 3$ for reconstruction is chosen so that the current frame is placed at the center of the window (except for corner cases). For each frame of the original light field video (frame 2 in this example), we reconstruct three light field sequences (three rows shown with dashed lines). Therefore, we can combine three reconstructed frames (shown with a red box) to obtain a single frame corresponding to frame 2 in the original light field video.](image)
Table 6.2: Dictionary type comparison; Boxer and Chess data sets; non-overlapping data points with size: \( n = s \times t \times u \times v \times \lambda \times \beta = 7 \times 7 \times 5 \times 5 \times 3 \times 3 \), number of frames for testing: 5, number of frames for training: 10, image size = [540, 960], reconstruction time is for all frames. The comparison is with Marwah et al. [22], Miandji et al. [23], and Inagaki et al. [176].

learning method of Inagaki et al. [176]. The dictionary \( D \) is constructed by training on a set consisting of 10 frames of the Boxer-Gladiator-Irish, and 10 frames of Chess. The evaluation is done on unseen frames from all three data sets. The result is summarized in Table 6.2. The reconstruction quality of the monochrome sensor compared to the CFA equipped sensor is lower as expected since the CFA sensor contains three times more samples. For more details about our algorithm and the results, see Paper F.

The effect of temporal window size is shown in Figure 6.7. Although temporal window size, \( \beta \), is context-dependent, the reconstruction quality improves by including more samples from the neighboring frames. However, this comes at the cost of a larger dictionary and higher computational complexity.

6.4 Summary and Future Work

This chapter presented a compressive solution for designing a single sensor light field video camera. Similar to previous compressive coded-aperture imaging, a random color mask is placed between the sensor and the aperture plane. To enforce the redundancy to the measured data for recovering the light field video frames, we stack a few consecutive measured frames to create a sensing model that explores the temporal correlations of the video. Since the mask is random for each captured frame, stacking the sensing matrix and measurement also improves the incoherency
Temporal window size affects the reconstruction quality for both configurations of color-coded mask with monochrome sensor and sensor equipped with CFA. The result is for Boxer-Gladiator-Irish data set [211].

of the measurements. By training a multidimensional dictionary that includes temporal information in its atoms, we were able to reconstruct a full resolution light field video with minimum artifacts. The result confirms that the proposed sensing model can handle movements in the scene, even in challenging scenarios, when the camera moves rapidly within the scene.

Since the representativeness of the trained dictionary is dependent on the motions in the scene, incorporating a motion aware clustering (i.e. disparity or optical flow) into the training and reconstruction step can benefit the algorithm. Interpolation between the reconstructed frame using optical flow is also another interesting research direction. As we showed in Figure 6.7, the temporal window size affects the quality of reconstruction, and it is highly dependent on the motion in the scene. Therefore, adapting the window size to the context of the frames is a natural direction for future research.
This dissertation provided algorithms that improve computational photography techniques for high dynamic range imaging and light field photography. This final chapter gives a summary of the research contributions of this thesis and presents discussions for future research directions.

7.1 Summary of Contributions

We discussed the challenges for accurate imaging of natural scenes using conventional cameras. Specifically, we identified problems for reconstructing a high dynamic range image from a spatially multiplexed image with multiple exposures. For light-field imaging, we described the current obstacles and provided solutions for capturing, compression, and rendering of a variety of light field data sets. We summarize the contributions of this thesis below:

- We showed that capturing high dynamic range images with a single sensor is feasible using an accurate and robust noise-aware HDR reconstruction framework that performs operations such as re-sampling, color interpolation, denoising, and HDR reconstruction in a single unified kernel function that adapts to the features of the image.

- Related to light-field imaging, we provided a solution for the challenging task of acquiring light field videos due to the excessive bandwidth requirements of such data. A robust compressive sensing model was introduced for two
camera designs with monochrome sensors and sensors equipped with CFA, where an attenuation color-coded mask is placed between the aperture plane of the camera and the sensor, with a user-defined distance from the sensor.

- We demonstrated how our proposed compression framework using an ensemble of multidimensional dictionaries outperforms prior work for compressing high resolution light field and light field video data sets. The presented framework provides random access to the memory, which makes it suitable for real-time reconstruction, novel view synthesis, and rendering.

- Finally, we presented various applications of our compression method from large scale 360° inward-looking light field acquisition devices to light fields captured with hand-held and multi-camera systems.

7.2 Future Work

The computational methods introduced in this dissertation provide a guideline for future research. The field of HDR imaging has matured during the last decade, leading to commercial realization of HDR televisions, monitors, and cameras. Despite these advancements, there are still many challenges for capturing high dynamic range videos on conventional cameras. There have been a few attempts to acquire HDR images of dynamic scenes such as [260, 261], but these solutions are not suitable for capturing HDR video on consumer cameras. One possible research direction to solve this challenge is to employ the spatially multiplexed images in a sequence with various exposure settings on a single sensor, e.g. by utilizing the dual-ISO technique, and reconstruct the interlaced frames using our unified framework for HDR reconstruction with three-dimensional kernels including both spatial and temporal information. The sparse representation can also be utilized for the reconstruction of HDR video from dual-ISO frames, similar to [262]. Another interesting research direction for the HDR reconstruction method is to explore the similarity of nonlocal regions of the image and incorporate that in the reconstruction algorithm. This is similar to modern denoising algorithms, and might also solve the problem with regions where both exposures are saturated.

The compressive light field video camera can potentially be extended for capturing higher dynamic range light field videos. One approach can be to modify the compressive sensing model to take into account the interlaced multi-exposure frames, similar to the dual-ISO video, in both capturing and reconstruction steps. We explored the proposed sensing model for two camera designs, where a color-coded mask is placed in front of the sensor of both configurations. An interesting extension of this method is to place a random color-coded mask [61] in front of the sensor to act as the CFA. More interestingly, one can optimize this random CFA
over a large data set of light fields to improve the incoherence and reconstruction quality. Indeed we can also move the CFA using a piezo motor for enhanced incoherence in the measurements. We showed in Chapter 6, that the temporal window size affects the quality of reconstruction. However, that also depends on the scale of motion in the scene. Therefore, it would be natural to adapt the temporal window size to the movements in the scene (or the camera) by considering the optical flow into our model for training a dictionary and reconstructing the frames.

Light field data sets encode a large amount of hidden information that can be explored to derive explicit representations such as depth map, optical flow, and BRDF. There have been a few articles on estimating SVBRDF from light field data sets [55]. This can be a very interesting future research direction.

As computational photography is mainly intended for our visual system, visual perception must be considered when designing a computational framework. As an example, we are utilizing a denoiser before applying AMDE encoding on the light field data set. The denoising can be applied based on the human visual system such that noise with visual significance in the data are removed, similar to [263].

The proposed compression framework, AMDE, can be further improved by enforcing the dictionary similarity metric in the training step and optimizing the number of dictionaries in the AMDE.

Since the introduction of mobile cameras in 2008, there has been a massive surge

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7.2 • Future Work

Figure 7.1: Total number of film, DSLR, mirrorless and smartphone cameras sold globally. Source: CIPA

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1 http://www.cipa.jp/stats/documents/common/cr1000.pdf
in mobile photography, as shown in Figure 7.1. This has led to a decrease in the sales of professional digital cameras. The landscape of imaging is shifting towards mobile photography, which demands to be in the center of attention in the future research directions. Enabling onboard processing is a necessity for modern mobile phones, where the user will be able to capture professional images with minimum effort. Currently, there is a competition on the number of cameras installed on a smartphone, and as Figure 7.2 illustrates, the penetration rate for multi-camera smartphones, particularly dual-camera ones, is rapidly increasing. This creates an excellent opportunity to explore computational imaging on mobile cameras, and develop algorithms for light field capture, compression, and rendering that are computationally efficient for the low-powered processors of the smartphones.

Figure 7.2: There has been a huge surge worldwide in the number of smartphones with dual, triple, and multi cameras. Note the significant increase in the number of dual camera smartphones since 2016. Source: Statista © 2019

https://www.statista.com
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