On the identification of bearings of active interference sources

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Abstract. The features of bearing information processing in a two-position radar system are considered. It is shown that in these systems, due to the absence of redundant information, the identification of bearings is a multi-alternative problem of statistical selection of hypotheses, the solution of which is especially important when locating a significant number of sources of active interference. The analysis of the matrix of coordinates of intersections of azimuthal bearings is carried out, a rule for identifying possible hypotheses of identification is formulated. To reduce the a priori uncertainty in the identification of bearings, it is proposed to use the energy differences of interference signals.

1. Introduction

Increasing the noise immunity of radar systems is achieved by a set of measures that can be combined into two groups. The first of them includes methods aimed at suppressing interfering radiation, and the second includes passive location methods that ensure the detection and measurement of the coordinates of radiation sources. The most important class of radiation sources are sources of active jamming signal (AJS), i.e. airborne objects carrying one or more interference transmitters.

To determine the spatial coordinates of the AJS, as is known, radar systems are used that contain at least two geographically separated receiving points and implement one of three methods for processing interference signals. Currently, the most widely used triangulation (direction finding) method for measuring the spatial coordinates of the AJS. This is due to the fact that the existing active radar systems allow the direction finding of the AJS, and the data transmission systems provide the point-to-point exchange of information about the direction finding results [1-3].

One of the main tasks in the joint processing of bearings is their identification. The solution to this problem becomes significantly more complicated with an increase in the number of AJS located in the detection zone of the radar system. The most effective of the known identification methods is based on the analysis of bearing information coming from three or more (t) receiving points, according to the logic “three out of three” or “t out of t”, respectively [4, 5]. However, the use of a logical method for identifying bearings imposes certain requirements on the structure of the radar system, as well as on the location of its receiving points. In this regard, of practical interest are ways of identifying azimuth bearings in a two-position passive location system. These methods, along with the known ones (for example, using the analysis of elevation bearings), will improve the quality of bearing identification, which is especially important in conditions of a large number of AJS.
2. Materials and methods
Bearing identification is a multi-alternative problem of statistical choice of hypotheses. If there are two sources of interference in the detection zone of a two-position passive locating system (figure 1), then two intersections of bearings represent the coordinates of the AJS, and the other two represent the coordinates of fictitious radiation sources, i.e. When solving the problem of identifying the bearings of two AJSs, two hypotheses are possible:

- hypothesis $H_1^*$, asserting that bearings $\beta_{11}^*$, $\beta_{21}^*$ refer to one AJS, and bearings $\beta_{12}^*$, $\beta_{22}^*$ – to another AJS (here the first index corresponds to the number of the receiving point, the second index is the bearing number assigned to him at the appropriate collection point, and the asterisk denotes the evaluation symbol);

- hypothesis $H_2^*$, asserting that bearings $\beta_{11}^*$, $\beta_{22}^*$ refer to one AJS, and bearings $\beta_{12}^*$, $\beta_{21}^*$ – to another AJS.

![Figure 1. Two-position passive locating system.](image)

In the case of bearing $N$ interferers, the number of bearing crossings increases $N^2$. Obviously, the number of possible hypotheses for identifying bearings will also increase. In this case, not only the number of hypotheses is of significant interest, but also specific combinations of identified bearings for each hypothesis. To solve this problem, it is enough to analyze the variant of direction finding of three sources of interference and extend it to the case of locating an arbitrary number of AJSs.

If there are three jammers in the detection zone of a two-position passive location system, then nine bearings crossings are subject to analysis. Using azimuthal values of bearings $\{\beta_{11}^*, \beta_{12}^*, \beta_{13}^*\}$ and $\{\beta_{21}^*, \beta_{22}^*, \beta_{23}^*\}$, the block matrix of coordinates of their intersections is calculated.

$$ R_{xy}^* = [XY_{ks}^*] = \begin{bmatrix} XY_{11}^* & XY_{12}^* & XY_{13}^* \\ XY_{21}^* & XY_{22}^* & XY_{23}^* \\ XY_{31}^* & XY_{32}^* & XY_{33}^* \end{bmatrix} $$

(1)

In a block matrix (1) the elements $XY_{ks}^*$ are matrices of size $1 \times 2$, i.e. $XY_{ks}^* = [X_{ks}^* \ Y_{ks}^*]$. The elements of the latter represent the coordinates of the intersection of the $k$-th and $s$-th bearings, calculated by the formulas [4]:

$$ X_{ks}^* = \frac{X_{n2}^* \sin \beta_{2s}^* - Y_{n2}^* \cos \beta_{2s}^*}{\sin (\beta_{2s}^* - \beta_{1k}^*)} \cos \beta_{1k}^* \quad \text{and} \quad Y_{ks}^* = \frac{X_{n2}^* \sin \beta_{2s}^* - Y_{n2}^* \cos \beta_{2s}^*}{\sin (\beta_{2s}^* - \beta_{1k}^*)} \sin \beta_{1k}^*. $$

(2)
It is not difficult to identify all possible identification hypotheses using the enumeration method. It should be borne in mind that the same bearing for a given hypothesis can be used only once. There are six possible hypotheses for the case under consideration: $H_1^*, H_2^*, H_3^*, H_4^*, H_5^*, H_6^*$, which are represented by mnemonic matrices $R_{xy}^*$ in figure 2.

![Figure 2. Mnemonic representation of the $R_{xy}^*$ matrix, illustrating possible hypotheses for identifying bearings.](image)

In figure 2, asterisks mark the elements of the $R_{xy}^*$ matrix, which represent the coordinates of the AJS for the corresponding hypothesis. Analyzing the given mnemonic matrices, it is easy to see the following pattern. A possible hypothesis of bearing identification corresponds to the coordinates $X_{ks}^*$, $Y_{ks}^*$, which belong to different columns and rows of the $R_{xy}^*$ matrix. These elements are sometimes called independent in linear programming theory. It is easy to verify that any other combination of the coordinates of the points of intersection of bearings refers to an unrealistic hypothesis.

Obviously, the number of possible identification hypotheses is determined by the size of the matrix $R_{xy}^*$. For a $2 \times 2$ matrix $R_{xy}^*$ the number of possible hypotheses is two. For a $3 \times 3$ matrix $R_{xy}^*$, this number increases to six. It is easy to show that for a matrix $R_{xy}^*$ of arbitrary size, the number of possible hypotheses is determined by the factorial of its size.

It should be noted that the formulated rule for identifying possible hypotheses is essentially the initial stage of the algorithm for identifying bearings. The implementation of this algorithm makes it possible to reduce the number of selection combinations (equal to the number of combinations from $N^2$ to $N$) to a significantly smaller value (equal to $N!$).

The triangulation method for measuring AJS coordinates is based on the analysis of the energy characteristics of the received interference signals. In particular, at each point of the passive location system, the value of the correlation integral is calculated:

$$ z = \int_0^T |y(t)|^2 dt, \quad (3) $$

where $y(t)$ – is the implementation of the interference signal observed on the interval $T$.

The calculated value of the correlation integral (3) is used in the interests of solving problems of detecting an interference signal and measuring the azimuth bearing of the AJS. As is known, detection is reduced to comparing the value of $z$ with a given threshold $z_0$ [5, 6]. When the condition $z > z_0$ is satisfied, the fact of detection is recorded, and the azimuthal bearing value is simultaneously estimated.

Along with the listed operations of intra-point processing, it is of interest to evaluate the distinctive features of interference signals. The set of such features, tied to the corresponding azimuth bearings, can be used at the stage of tertiary (joint) processing of bearing information and, in particular, when solving the problem of identifying bearings. The distinguishing features of interference signals in triangulation location systems, first of all, include energy parameters, for example, the value of the correlation integral. The value of the latter is proportional to the average power of the received interference signal, since
\[
P = \frac{1}{T} \int_0^T |y(t)|^2 \, dt = \frac{z}{T}.
\] (4)

In the general case, the average power of an interference signal received at points \(L_1\) and \(L_2\), is mainly determined by the power of the interference transmitter and the length of the propagation path of this signal [7, 8]:

\[
P_{ij} = \frac{P_j D_j A_i G_i}{4\pi r_{ij}^2},
\] (5)

where \(P_{ij}\) is the average signal power at the \(i\)-th point when locating the \(j\)-th AJS; \(P_j\) is the average power of the jammer of the \(j\)-th AJS; \(D_j\) – coefficient of directional action of the antenna of the \(j\)-th AJS; \(A_i\) – effective antenna area of the \(i\)-th receiving point; \(G_i\) – is the antenna gain of the \(i\)-th receiving point; \(r_{ij}\) is the distance between the \(i\)-th receiving point and the \(j\)-th AJS.

Assuming that weakly directional antennas are used in jamming transmitters (with a radiation pattern width of the order of \(45^\circ\) … \(60^\circ\)), and identical sets of equipment are located at reception points, relation (5) is reduced to the form:

\[
P_{ij} = g \frac{P_j}{r_{ij}^2},
\] (6)

where \(g\) is some coefficient of proportionality.

Using relations (4) - (6), we find the expression for the correlation integral calculated at the \(i\)-th point when the \(j\)-th AJS is located:

\[
Z_{ij} = gT \frac{P_j}{r_{ij}^2} = k \frac{P_j}{r_{ij}^2},
\] (7)

where \(k = gT\) is the proportionality coefficient.

Thus, as the initial data for solving the problem of identifying bearings when locating \(N\) sources of active interference, it is proposed to use the matrices of bearings and correlation integrals:

\[
\beta^* = \begin{bmatrix}
\beta_{11}^* & \beta_{12}^* & \ldots & \beta_{1N}^*
\beta_{21}^* & \beta_{22}^* & \ldots & \beta_{2N}^*
\end{bmatrix}
\] (8)

\[
z = \begin{bmatrix}
z_{11} & z_{12} & \ldots & z_{1N}
z_{21} & z_{22} & \ldots & z_{2N}
\end{bmatrix}
\] (9)

3. Results and discussion

The solution to the multi-alternative problem of identification of bearings, the distinguishing features of which are the elements of matrix (9), is based on the analysis of the functional relationship between the measured values of bearings \(\{\beta_{11}^*, \beta_{12}^*, \ldots, \beta_{1N}^*; \beta_{21}^*, \beta_{22}^*, \ldots, \beta_{2N}^*\}\) and correlation integrals \(\{z_{11}, z_{12}, \ldots, z_{1N}; z_{21}, z_{22}, \ldots, z_{2N}\}\). Let us reveal this dependence for the simplest case (figure 1), when there are two AJSs in the detection zone of the passive location system. For the situation under consideration, matrices (8), (9) have size 2x2:

\[
\beta^* = \begin{bmatrix}
\beta_{11}^* & \beta_{12}^* \\
\beta_{21}^* & \beta_{22}^*
\end{bmatrix}; \quad z = \begin{bmatrix}
z_{11} & z_{12} \\
z_{21} & z_{22}
\end{bmatrix}.
\] (10)
Using the values of the elements of the matrix $\mathbf{B}^*$, we calculate according to expressions (2) the matrix of coordinates of intersections of bearings:

$$
\mathbf{R}_{xy}^* = \begin{bmatrix}
X_{11}^* & X_{12}^* \\
Y_{11}^* & Y_{22}^*
\end{bmatrix}
$$

Depending on the accepted hypothesis ($H_1^*$ или $H_2^*$) the coordinates are selected from the $\mathbf{R}_{xy}^*$ matrix:

- $X_{11}^*, Y_{11}^*, X_{22}^*, Y_{22}^*$ if hypothesis $H_1^*$ is considered correct;
- $X_{12}^*, Y_{12}^*, X_{21}^*, Y_{21}^*$ if hypothesis $H_2^*$ is considered correct.

Let us formulate the principle of choosing the coordinates of the AJS from the $\mathbf{R}_{xy}^*$ matrix according to the distinctive features of bearings. For this purpose, we first analyze the intersection of the bearings of the same AJS, and then - the intersection of the bearings of different AJSs.

Consider, for example, the intersection of bearings $\beta_{11}^*$, $\beta_{21}^*$, which, according to hypothesis $H_1^*$ determine the coordinates $X_{11}^*, Y_{11}^*$ of the jammer $J_1$. Bearings $\beta_{11}^*$ and $\beta_{21}^*$ under consideration are characterized by energy parameters $z_{11}$ and $z_{21}$ respectively:

$$
z_{11} = k \frac{P_1}{r_{11}^2}, \quad z_{21} = k \frac{P_1}{r_{21}^2}.
$$

In the general case, the lengths of the paths of propagation of interference signals $r_{11}$, $r_{21}$ differ from each other. Therefore, the values of the correlation integrals $z_{11}$ and $z_{21}$ turn out to be unequal. However, if $z_{11}$ is multiplied by $r_{11}^2$, and $z_{21}$ is multiplied by $r_{21}^2$, then we get the identity for any spatial position $J_1$:

$$
z_{11} \times r_{11}^2 = z_{21} \times r_{21}^2.
$$

From a physical point of view, this operation is equivalent to recalculating the energy parameters $z_{11}$ and $z_{21}$ to the points of emission of interference signals. Since the bearings under consideration are due to the same source ($J_1$), the result of the recalculation leads to identity (13). This regularity can be used as the basis for the principle of bearing identification.

Verification of fulfillment of condition (13) requires knowledge of the true distances from the receiving points to the source of active interference $J_1$. However, the true values $r_{11}$, $r_{21}$ are unknown. Therefore, as parameters $r_{11}$, $r_{21}$ their estimates $r_{11}^*$ and $r_{21}^*$, calculated from the values of the elements of the matrix $\mathbf{R}_{xy}^*$ can be used:

$$
r_{11}^* = \sqrt{(X_{11}^*)^2 + (Y_{11}^*)^2}, \quad r_{21}^* = \sqrt{(X_{21}^* - X_{11}^*)^2 + (Y_{21}^* - Y_{11}^*)^2}
$$

Replacing the true path lengths $r_{11}$, $r_{21}$ in identity (13) with their estimates $r_{11}^*$, $r_{21}^*$ we obtain an expression whose left and right sides are approximately equal to each other:

$$
z_{11} \times (r_{11}^*)^2 \approx z_{21} \times (r_{21}^*)^2
$$

The sign of approximate equality in expression (15) takes into account the anisotropy of radiation, errors in the estimation of distances $r_{11}$, $r_{21}$, as well as possible differences in the parameters of the equipment sets of the receiving points, which affect the estimation of the values of $z_{11}$ and $z_{21}$. 

Obviously, the obtained functional dependence is also valid for bearings $\beta_{12^*}$, $\beta_{22^*}$, which determine the coordinates $X_{22^*}$, $Y_{22^*}$ of the source of active interference $J_2$. Therefore, relations (14) - (15) can be extended to the case of an arbitrary number of sources of active interference:

$$z_{ik} \times (r^*_{1 \text{ks}})^2 \approx z_{2s} \times (r^*_{2 \text{ks}})^2;$$  

$$r^*_{1 \text{ks}} = \sqrt{(X^*_k)^2 + (Y^*_k)^2}, \quad r^*_{2 \text{ks}} = \sqrt{(X^*_k - X^*_{L_1})^2 + (Y^*_k - Y^*_{L_1})^2}.$$  

Solving the problem of identifying bearings involves analyzing each intersection of bearings. Therefore, the matrix of coordinates of points of intersection of bearings $R^*_{xy}$ is calculated in advance.

The values of the matrix elements $R^*_{xy}$ are used to calculate the squared distances from each radiation source (both true and fictitious) to the first and second receiving points. Based on the calculation results, matrices are compiled:

$$R^*_{1k} = \begin{bmatrix} (r^*_{111})^2 & (r^*_{112})^2 & \cdots & (r^*_{11N})^2 \\ (r^*_{121})^2 & (r^*_{122})^2 & \cdots & (r^*_{12N})^2 \\ \vdots & \vdots & \ddots & \vdots \\ (r^*_{1N1})^2 & (r^*_{1N2})^2 & \cdots & (r^*_{1NN})^2 \end{bmatrix}, \quad R^*_{2k} = \begin{bmatrix} (r^*_{211})^2 & (r^*_{212})^2 & \cdots & (r^*_{21N})^2 \\ (r^*_{221})^2 & (r^*_{222})^2 & \cdots & (r^*_{22N})^2 \\ \vdots & \vdots & \ddots & \vdots \\ (r^*_{2N1})^2 & (r^*_{2N2})^2 & \cdots & (r^*_{2NN})^2 \end{bmatrix}.$$  

Further, to check the fulfillment of condition (16), two matrices ($V_1$, $V_2$) are compiled, the first of which represents the values of the left side, and the second represents the values of the right side of expression (16), i.e.:

$$V_1 = \begin{bmatrix} Z_{11} (r^*_{111})^2 & Z_{11} (r^*_{112})^2 & \cdots & Z_{11} (r^*_{11N})^2 \\ Z_{12} (r^*_{121})^2 & Z_{12} (r^*_{122})^2 & \cdots & Z_{12} (r^*_{12N})^2 \\ \vdots & \vdots & \ddots & \vdots \\ Z_{1N} (r^*_{1N1})^2 & Z_{1N} (r^*_{1N2})^2 & \cdots & Z_{1N} (r^*_{1NN})^2 \end{bmatrix}, \quad V_2 = \begin{bmatrix} Z_{21} (r^*_{211})^2 & Z_{21} (r^*_{212})^2 & \cdots & Z_{21} (r^*_{21N})^2 \\ Z_{22} (r^*_{221})^2 & Z_{22} (r^*_{222})^2 & \cdots & Z_{22} (r^*_{22N})^2 \\ \vdots & \vdots & \ddots & \vdots \\ Z_{2N} (r^*_{2N1})^2 & Z_{2N} (r^*_{2N2})^2 & \cdots & Z_{2N} (r^*_{2NN})^2 \end{bmatrix}.$$  

These matrices characterize the values of the energy parameters of interference signals, recalculated to the points of intersection of bearings. In this case, the matrix $V_1$ characterizes the energy parameters for the first, and the matrix $V_2$ – for the second receiving point. Comparing the matrices $V_1$ and $V_2$, one should select the elements with the same indices $ks$, for which two conditions are fulfilled: the elements represent one of the hypotheses for identifying bearings, i.e. belong to different columns and rows; condition (16) is satisfied for the selected elements.

The final stage in the identification of bearings is the selection of elements $X_{ks^*}$ and $Y_{ks^*}$ from the matrix $R^*_xy$, which have the same indices as the selected elements of the matrices $V_1$ and $V_2$. The selected elements $X_{ks^*}$, $Y_{ks^*}$ according to condition (16) are the sought coordinates of the source of active interference.

4. Conclusion

Note that the central role in solving the bearing identification problem, which is based on taking into account the energy differences of interference signals, is assigned to the matrices $V_1$ and $V_2$. Therefore, in what follows, these matrices will be called decisive.

Thus, the considered principle of bearing identification is associated with the compilation of decision matrices and the finding of their independent elements that satisfy condition (16). When developing an algorithm for identifying bearing information, these operations should be presented in a form that allows their formalization. In this regard, it is of interest, firstly, to substantiate the structure of the decision
matrix that optimizes the algorithm for identifying bearings and, secondly, to search for methods that make it easy to identify the elements of the decision matrix when choosing the identification hypothesis.

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