ON FLOW PAST AN EXPONENTIALLY ACCELERATED VERTICAL PLATE WITH HEAT AND MASS TRANSFER IN THE PRESENCE OF HALL EFFECT AND ROTATION

K.Muthuracku Alias Prema*, R.Muthucumaraswamy

*1, 2Department of Applied Mathematics, Sri Venkateswara College of Engineering, Pennalur, Sriperumbudur Taluk- 602117, India

DOI: https://doi.org/10.29121/granthaalayah.v5.i4.2017.1794

Abstract

This paper analyzes the thermal radiation effects on unsteady free convective flow of a viscous incompressible flow past an exponentially accelerated infinite vertical plate with variable temperature and mass diffusion. At time the plate is linearly accelerated with a velocity exp in its own plane. And at the same time, plate temperature and concentration levels near the plate raised linearly with time t. The system of equations such as equation of momentum, energy, mass diffusion has been transformed by usual transformation into a non-dimensional form. An exact solution to the dimensionless governing equations has been obtained by the Laplace transform method in terms of exponential function and complementary error function. All the numerical calculations are done with respect to air (Pr=0.71). The temperature, the concentration, the primary and the secondary velocity profiles are studied for different parameters such as rotation parameter, Hall parameter, Hartmann number, Schmidt number, radiation parameter thermal Grashof number and mass Grashof number, accelerated parameter and time and presented through graphs.

Keywords: Hall Effect; Rotation; Flow Past; Vertical Plate; Heat Transfer; Mass Transfer.

Cite This Article: K.Muthuracku Alias Prema, and R.Muthucumaraswamy. (2017). “ON FLOW PAST AN EXPONENTIALLY ACCELERATED VERTICAL PLATE WITH HEAT AND MASS TRANSFER IN THE PRESENCE OF HALL EFFECT AND ROTATION.” International Journal of Research - Granthaalayah, 5(4), 51-61. https://doi.org/10.29121/granthaalayah.v5.i4.2017.1794.

1. Introduction

Study of flow with heat and mass transfer play an important role in engineering sciences. Effect of heat and mass transfer plays vital role, in space craft design, in the cooling of liquid metal of nuclear reactors, pollution of environment etc. Radiative heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power
plants, gas turbines and various propulsion devices for aircraft, combustion and furnace design. Ahmmed et al. [1] have analyzed radiation and mass transfer effects on MHD free convection flow past a vertical plate with variable temperature and concentration. Amit and Srivastava Saraswat [2] studied heat and mass transfer on flow past an exponentially accelerated infinite vertical plate with variable temperature and mass diffusion through a porous medium.

Kaprawi [3] explains analysis of transient natural convection flow past an accelerated infinite vertical plate. Kumar and Verma [4] discussed radiation effects on MHD flow past an impulsively started exponentially accelerated vertical plate with variable temperature in the presence of heat generation. Muthucumarasamy and Visalakshi [5] investigated radiative flow past an exponentially accelerated vertical plate with variable temperature and mass diffusion in the presence of magnetic field. Okejide and Lamidi [6] have considered analytical solution of mass transfer effects unsteady flow past an accelerated vertical porous plate with suction. Sathapathan and Muthucumaraswamy [9] rendered radiation effects on exponentially accelerated vertical plate with uniform mass diffusion. Rajput and Sahu [7] discussed effects of rotation and magnetic field on the flow past an exponentially vertical plate with constant temperature. Santan Das et al. [8] investigated radiation effects on free convection MHD couette flow started exponentially with variable wall temperature in presence of heat generation. Thamizhsudar and Pandurangan [10] explains combined effects of radiation and Hall current on MHD flow past an exponentially accelerated vertical plate in the presence of rotation. Uwanta and Sarki [11] presented heat and mass transfer with variable temperature and exponential mass diffusion.

2. Formulation of the Problem

Consider the unsteady flow of an incompressible viscous fluid past an exponentially accelerated infinite vertical plate with variable temperature and mass diffusion has been studied. A transverse magnetic field of uniform strength $B_0$ is applied transversely to the plate. Initially, the plate and the fluid are at the same temperature $T_0'$ in the stationary condition with concentration level $C_0'$ at all the points. At time $t' \leq 0$, the plate and fluid are at the same temperature $T_0'$ and concentration $C_0'$. At time $t' > 0$, the plate is exponentially accelerated with a prescribed velocity $u = u_0 \exp (a't')$, in its own plane and at the same time the temperature of the fluid near the plate is raised linearly with time and concentration level near the plate is also increased linearly with time. Since the plate is of an infinite extent in $x'$ and $y'$ directions and is electrically non-conducting, all physical quantities except pressure, depend on $z'$ and $t'$ only. Then under the usual Boussinesq’s approximation the unsteady flow equations are momentum equation, energy equation, and diffusion equation respectively.

Equation of Momentum:

\[
\frac{\partial u}{\partial t'} - 2 \Omega' v = \frac{\partial^2 u}{\partial z'^2} - \frac{1}{\rho} \frac{\partial p}{\partial x'} + g \frac{B_0}{\rho} j_y
\]
\[
\frac{\partial v}{\partial t'} + 2 \Omega' u = \frac{\partial^2 v}{\partial z'^2} - \frac{B_0}{\rho} j_x
\]
Equation of Energy:

\[ \rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial z^2} - \frac{\partial q_r}{\partial z} \]

(3)

Equation of diffusion:

\[ \frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial z^2} \]

(4)

Since there is no large velocity gradient here, the viscous term in Equation (1) vanishes for small and hence for the outer flow, beside there is no magnetic field along x-direction gradient, so we have

\[ 0 = -\frac{\partial p}{\partial x} - \rho \_\infty g \]

(5)

By eliminating the pressure term from Equations (1) and (5), we obtain

\[ \frac{\partial u}{\partial t'} - 2\Omega'v = \varrho \frac{\partial^2 u}{\partial z^2} + (\rho \_\infty - \rho) g + \frac{B_0}{\rho} j_y \]

(6)

The Boussinesq approximation gives

\[ \rho \_\infty - \rho = \rho \_\infty \beta (T' - T'_\infty) + \rho \_\infty \beta^* (C' - C'\_\infty) \]

(7)

On using (7) in the equation (6) and noting that \( \rho \_\infty \) is approximately equal to 1, the momentum equation reduces to

\[ \frac{\partial u}{\partial t'} - 2\Omega'v = \varrho \frac{\partial^2 u}{\partial z^2} + \frac{B_0}{\rho} j_y + g \beta (T' - T'_\infty) + g \beta^* (C' - C'\_\infty) \]

(8)

The generalized Ohm’s law, on taking Hall currents into account and neglecting ion-slip and thermo-electric effect, is

\[ \vec{j} + \frac{\omega_e \tau_e}{B_0} (\vec{j} \times \vec{B}) = \sigma (\vec{E} + \vec{q} \times \vec{B}) \]

(9)

The equation (9) gives

\[ j_x - m j_y = \sigma_v B_0 \]

(10)

\[ j_y + m j_x = -\sigma_u B_0 \]

(11)

where \( m = \omega_e \tau_e \) is the Hall parameter. Solving (10) and (11) for \( j_x \) and \( j_y \), we have

\[ j_x = \frac{\sigma B_0}{1 + m^2} (v - mu) \]

(12)

\[ j_y = \frac{\sigma B_0}{1 + m^2} (u + mv) \]

(13)

On the use of (12) and (13), the momentum equations (8) and (2) become
\[
\frac{\partial u}{\partial t'} = \frac{\partial^2 u}{\partial z^2} + 2\Omega' v - \frac{\sigma B^2_0 (u + mv)}{\rho(1 + m^2)} + g\beta (T' - T'_w) + g\beta' (C' - C'_w)
\] (14)

\[
\frac{\partial v}{\partial t'} = \frac{\partial^2 v}{\partial z^2} - 2\Omega' u + \frac{\sigma B^2_0 (mu - v)}{\rho(1 + m^2)}
\] (15)

The initial and boundary conditions are given by

\[
\begin{align*}
&u = 0, v = 0, T' = T'_w, C' = C'_w, t' \leq 0 \quad \forall z, \\
t' > 0, &\quad u = u_0 \exp(\alpha t'), \quad v = 0, \quad T' = T'_w + (T'_w - T'_w)A t', C' = C'_w + (C'_w - C'_w)A t' \quad \text{at} \quad z = 0 \\
&u \to 0, v \to 0, \quad T' \to T'_w, C' \to C'_w \quad \text{as} \quad z \to \infty
\end{align*}
\] (16)

where \( A = \frac{u_0^2}{\gamma} \)

The local radiant for the case of an optically thin gray gas is expressed by

\[
\frac{\partial q_r}{\partial z} = -4a^* \sigma (T'^4_w - T'^4)
\] (17)

It is assumed that the temperature differences within the flow are sufficiently small such that \( T'^4 \) may be expressed as a linear function of the temperature. This is accomplished by expanding \( T'^4 \) in a Taylor series about \( T'_w \) and neglecting higher-order terms, thus

\[
T'^4 \approx 4T'_w^3 T' - 3T'_w^4
\] (18)

By using equations (17) and (18), equation (3) reduces to

\[
\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial z^2} + 16a^* \sigma T'_w^3 (T'_w - T')
\] (19)

Let us introducing the following non-dimensional quantities

\[
U = \frac{u}{u_0}, \quad V = \frac{v}{u_0}, \quad Z = \frac{zu_0}{\gamma}, \quad t = \frac{t' u_0^2}{\gamma}, \quad \Omega = \frac{\Omega' \gamma}{u_0}, \quad M^2 = \frac{\Omega \sigma B^2_0 \gamma}{2 \rho \mu u_0^2}, \quad \theta = \frac{T' - T'_w}{T'_w - T'_w},
\]

\[
C = \frac{C' - C'_w}{C'_w - C'_w}, \quad Pr = \rho C_p k, \quad Sc = \frac{v}{D}, \quad Gr = \frac{g\beta'(T'_w - T'_w)}{u_0^3}, \quad Gc = \frac{g\beta' \gamma (C'_w - C'_w)}{u_0^3},
\]

\[
R = \frac{16a^* \gamma^2 T'_w^3}{ku_0^2}, \quad a = \frac{a^* \gamma}{u_0^2}
\]

Using these boundary conditions in above equations, we obtain the following dimensionless form of the governing equations:

\[
\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial Z^2} + 2\Omega V - \frac{2M^2(U + mV)}{1 + m^2} + Gr \theta + Gc C
\] (20)

\[
\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial Z^2} - 2\Omega U + \frac{2M^2(mU - V)}{1 + m^2}
\] (21)
\[
\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Z^2} - \frac{R}{Pr} \theta
\]

(22)

\[
\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Z^2}
\]

(23)

The boundary conditions for corresponding order are

\[
\begin{align*}
U &= 0, V = 0, \theta = 0, C = 0 \text{ at } t \leq 0 \text{ for all } Z \\
t > 0, U &= \exp(\alpha t), V = 0, \theta = t, C = t \text{ at } Z = 0 \\
U &\to 0, V \to 0, \theta \to 0 \text{ as } Z \to \infty
\end{align*}
\]

(24)

Now equations (20) & (21) and boundary conditions (24) can be combined to give

\[
\frac{\partial F}{\partial t} = \frac{\partial^2 F}{\partial Z^2} - wF + Gr\theta + GcC \quad \text{Where} \quad w = \frac{2M^2}{1+m^2} + 2i(\Omega - \frac{M^2}{1+m^2})
\]

(25)

\[
\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Z^2} - \frac{R}{Pr} \theta
\]

(26)

\[
\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Z^2}
\]

(27)

The initial and boundary conditions in non-dimensional quantities are

\[
\begin{align*}
F &= 0, \theta = 0, C = 0 \text{ for all } Z, t \leq 0 \\
t > 0, F &= \exp(\alpha t), \theta = t, C = t \quad \text{at } Z = 0 \\
F &\to 0, \theta \to 0 \text{ as } Z \to \infty
\end{align*}
\]

(28)

Exact solution for the fluid temperature and concentration of (26), (27) is expressed in the following form by taking inverse Laplace transform of solution as

\[
C(Z,t) = t \left[ (1 + 2\eta^2 Sc) \text{erfc}(\eta \sqrt{Sc}) - \frac{2\eta \sqrt{Sc}}{\sqrt{\pi}} \exp(-\eta^2 Sc) \right]
\]

(29)

\[
\theta(Z,t) = \frac{t}{2} \left[ \exp(2\eta \sqrt{Pr t}) \text{erfc}(\eta \sqrt{Pr + \sqrt{bt}}) + \exp(-2\eta \sqrt{Pr t}) \text{erfc}(\eta \sqrt{Pr - \sqrt{bt}}) \right] - \frac{\eta \sqrt{Pr \sqrt{t}}}{2\sqrt{b}} \left[ \exp(-2\eta \sqrt{Pr t}) \text{erfc}(\eta \sqrt{Pr - \sqrt{bt}}) - \exp(2\eta \sqrt{Pr t}) \text{erfc}(\eta \sqrt{Pr + \sqrt{bt}}) \right]
\]

(30)

The equations (25), (26), (27), subject to the boundary conditions (28), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

\[
\begin{align*}
F(Z,t) &= g_1(\exp(2\eta \sqrt{(w+a)t}) \text{erfc}(\eta + \sqrt{(w+a)t}) + \exp(-2\eta \sqrt{(w+a)t}) \text{erfc}(\eta - \sqrt{(w+a)t})) + (A + B)(\frac{1}{2})(\exp(2\eta \sqrt{wt}) \text{erfc}(\eta + \sqrt{wt}) + \exp(-2\eta \sqrt{wt}) \text{erfc}(\eta - \sqrt{wt})) + \\
\end{align*}
\]
In order to get the physical insight into the problem the numerical values of the velocity, temperature and concentration fields are studied for different physical parameters such as Rotation parameter, Hall parameter, Hartmann number, Schmidt number, radiation parameter thermal Grashof number and mass Grashof number, accelerated parameter and time upon the nature of flow and transport. The value of the Schmidt number Sc is taken to be 0.6 which corresponds to water-vapor, also the value of Prandtl number Pr are chosen such that they represent air (Pr=0.71). The effect of Prandtl number is important in temperature profiles. The effect of heat transfer is more in the presence of air than in water. Numerical evaluation of the analytical results is in terms of an exponential and complementary error function and a representative a set of results is reported graphically. Fig 1 represents the effect of concentration profiles at time t=0.2 for different Schmidt number (Sc=0.16, 0.3, 0.6, 2.01). It is observed that the wall concentration increases with decreasing values of the Schmidt number. Fig 2 shows that the temperature profiles for different values of the thermal radiation parameter (R=0.2, 0.2, 2.0,
It is observed that the temperature increases with decreasing radiation parameter and the temperature increases with increasing values of the time t. The primary and the secondary velocity profiles of rotation parameter are noticed in Fig 3 and 4. It is observed that the primary velocity decreases as \( \Omega \) increases from 0.1 to 0.8, while the secondary velocity increases when Sc=0.6, Pr=0.71, a=0.1, t=0.2, M=0.5, m=0.5, Gr=5, Gc=5, R=5. Fig 5 and 6 reveal that the primary and the secondary velocity profiles for various values of M. Due to an increase in the Hartmann number M, the transient primary velocity decreases while the secondary velocity increases when Sc=0.6, Pr=0.71, a=0.1, t=0.2, \( \Omega = 0.1 \), m=0.5, Gr=5, Gc=5, R=5. Fig 7 and 8 represent the primary and the secondary velocity profiles for various values of \( m \). It is seen that due to an increase in the \( m \) value there is rise in both velocity components when Sc=0.6, Pr=0.71, a=0.1, t=0.2, \( \Omega = 0.1 \), M=0.5, Gr=5, Gc=5, R=5. Fig 9 and 10 shows the influence of thermal buoyancy force parameter Gr and the modified buoyancy force parameter Gc on the primary and secondary velocity while all other parameters are kept at some fixed values. It is clear from Fig 9 and 10 that the primary velocity and the secondary velocity increase with increasing thermal Grashof number and mass Grashof number. Fig 11 and 12 represent primary and secondary velocity profiles due to variations in radiation parameter. From these figures it is clear that the primary velocity increases with decreasing values of the radiation parameter and due to an increase in the radiation parameter, the secondary velocity increases. Fig 13 and 14 the primary and secondary velocity profiles are shown for different values of the Schmidt number (Sc) for aiding flows in the presence of foreign mass and constant mass flux respectively. It is observed that the primary velocity increases with decreasing Schmidt number and the secondary velocity increases with increasing values of Sc when Pr=0.71, R=5, t=0.2, \( \Omega = 0.1 \), M=0.5, m=0.5, Gr=5, Gc=5. The variation of primary and secondary velocity for different values of dimensionless time (t = 0.3, 0.4, 0.6), Pr=0.71, R=5, Sc=0.6, \( \Omega = 0.1 \), M=0.5, m=0.5, Gr=5, Gc=5, a=0.1 is shown in Fig 15 and 16. It is noticed that the primary velocity increases and that the secondary velocity increases with increasing values of t near the plate and then decays to zero asymptotically. Effects of accelerating parameter a is studied through Fig 17 and 18 on both the velocities primary and as well as secondary and taking other parameters Pr=0.71, R=5, t=0.2, \( \Omega = 0.1 \), M=0.5, m=0.5, Gr=5, Gc=5, Sc=0.6 fixed. From these figures it is observed that the primary and the secondary velocity increase with increasing values of accelerating parameter.
Figure 3: Primary velocity profiles for different $\Omega$

Figure 4: Secondary velocity profiles for different $\Omega$

Figure 5: Primary velocity profiles for different $M$

Figure 6: Secondary velocity profiles for different $M$

Figure 7: Primary velocity profiles for different $m$

Figure 8: Secondary velocity profiles for different $m$
Figure 9: Primary velocity profiles for different Gr and Gc

Figure 10: Secondary velocity profiles for different Gr and Gc

Figure 11: Primary velocity profiles for different R

Figure 12: Secondary velocity profiles for different R

Figure 13: Primary velocity profiles for different Sc

Figure 14: Secondary velocity profiles for different Sc
4. Conclusion

We have considered the theoretical solution of flow past an exponentially accelerated isothermal vertical plate in the presence of variable temperature and mass diffusion. While the temperature of the plate is constant, the concentration at the plate is considered to be a linear function with respect to time \( t \). The plate is assumed to be exponentially accelerated with a prescribed velocity against the gravitational field. Exact solutions of the governing equations were found using Laplace transforms. The effects of different parameters such as rotation parameter \( \Omega \), Hartmann number \( M \), Hall parameter \( m \), radiation parameter \( R \), thermal Grashof number \( Gr \), mass Grashof number \( Gc \), Schmidt number \( Sc \), accelerating parameter \( a \) and time \( t \) are studied. In the analysis of the flow the following conclusions are made. The concentration increases with decreasing values of the Schmidt number. The temperature increases with decreasing radiation parameter. The primary velocity rises due to increasing value of the Hall parameter, accelerating parameter, thermal Grashof number and mass Grashof number and time. The primary velocity \( U \)
falls when $\Omega$ are increased, the velocity increases with decreasing values of the Hartmann number, the radiation parameter. Secondary velocity increases as $\Omega$ increases, due to an increase in the Hartmann number $M$, the Hall parameter, $m$, accelerating parameter, the radiation parameter $R$, $Gr$, $Gc$, $Sc$ and $t$.

References

[1] Ahmmed, Parvin and Morshed. (2013). “RADIATION AND MASS TRANSFER EFFECTS ON MHD FREE CONVECTION FLOW PAST A VERTICAL PLATE WITH VARIABLE TEMPERATURE AND CONCENTRATION.” International Journal of Physics and Mathematical Sciences, 3, 54-61.

[2] Amit, Srivastava and Saraswat. (2013). “HEAT AND MASS TRANSFER ON FLOW PAST AN EXPONENTIALLY ACCELERATED INFINITE VERTICAL PLATE WITH VARIABLE TEMPERATURE AND MASS DIFFUSION THROUGH A POROUS MEDIUM.” International Journal of Engineering, Business and Enterprise Applications, 4, 79-87.

[3] Kapravi. (2015). “ANALYSIS OF TRANSIENT NATURAL CONVECTION FLOW PAST AN ACCELERATED INFINITE VERTICAL PLATE.” International Journal of Engineering Research, 4, 47 – 50.

[4] Kumar and Verma. (2011). “RADIATION EFFECTS ON MHD FLOW PAST AN IMPULSIVELY STARTED EXPONENTIALLY ACCELERATED VERTICAL PLATE WITH VARIABLE TEMPERATURE IN THE PRESENCE OF HEAT GENERATION. International Journal of Engineering Science and Technology, 3, 2897-2909.

[5] Muthucumarasamy and Visalakshi. (2012). “RADIATIVE FLOW PAST AN EXPONENTIALLY ACCELERATED VERTICAL PLATE WITH VARIABLE TEMPERATURE AND MASS DIFFUSION IN THE PRESENCE OF MAGNETIC FIELD.” International Journal of Mathematical Archive, 3, 2225-2233.

[6] Okedoye and Lamidi. “ANALYTICAL SOLUTION OF MASS TRANSFER EFFECTS UNSTEADY FLOW PAST AN ACCELERATED VERTICAL POROUS PLATE WITH SUCTION.” Journal of Nigeria Association Mathematical Physics,15, 501-51.

[7] Rajput and Sahu. “EFFECTS OF ROTATION AND MAGNETIC FIELD ON THE FLOW PAST AN EXPONENTIALLY VERTICAL PLATE WITH CONSTANT TEMPERATURE.” International Journal of Mathematical Archive, 2, 2831-2843.

[8] Sanatan Das, Bhaskar Chandra Sarkar and Rabindra Nath Jana. (2012). “RADIATION EFFECTS ON FREE CONVECTION MHD COUETTE FLOW STARTED EXPONENTIALLY WITH VARIABLE WALL TEMPERATURE IN PRESENCE OF HEAT GENERATION.” Open Journal of Fluid Dynamics, 2, 14-27.

[9] Sathappan and Muthucumaraswamy.(2011).“RADIATION EFFECTS ON EXPONENTIALLY ACCELERATED VERTICAL PLATE WITH UNIFORM MASS DIFFUSSION.” International Journal of Automotive and Mechanical Engineering, 3, 341-349.

[10] Thamizhhsudar and Pandurangan.(2014). “COMBINED EFFECTS OF RADIATION AND HALL CURRENT ON MHD FLOW PAST AN EXPONENTIALLY ACCELERATED VERTICAL PLATE IN THE PRESENCE OF ROTATION.” International Journal of Innovative Research in Computer and Communication Engineering, 2, 7498-7509.

[11] Uwanta and Sarki.(2012). “HEAT AND MASS TRANSFER WITH VARIABLE TEMPERATURE AND EXPONENTIAL MASS DIFFUSION.” International Journal of Computational Engineering Research, 5, 1487-1494.

*Corresponding author.
E-mail address: kmprema66@yahoo.co.in