Dota Underlords game is NP-complete

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Abstract
In this paper, we demonstrate how the problem of the optimal team choice in the popular computer game Dota Underlords can be reduced to the problem of linear integer programming. We propose a model and solve it for the real data. We also prove that this problem belongs to the NP-complete class and show that it reduces to the maximum edge weighted clique problem.

1 Introduction
People love to play games. Many games and puzzles that people play are interesting by its complexity: you need to be smart enough to solve it. In many cases, such complexity can be expressed as computation complexity depending on input size. For example, it has been shown [1] that the chess game belongs to the EXPTIME class complexity; decision problem of player in legendary video game “Tetris” is NP-hard [2]. It was shown that the puzzle “Sokoban” has polynomially solvable [3].

The special place in theoretical computer science has NP-complete computational class. It was found in [4] that “Minesweeper” belongs to the NP-complete class. The problem of finding a minimal number of chip movements in a generalized version of 15-puzzle for the board of size $N \times N$ belongs to the NP-class also [5].

In some sense, every NP-complete problem is a puzzle, and vice-verse, many puzzles are NP-complete. For a deeper study of the topic of the computational complexity of puzzles and games, we refer the reader to the review [6].

In this paper, we consider a popular video game Dota Underlords. It is one of the so-called auto-chess games. It turns out that this problem can be represented as a combinatorial optimization problem, which belongs to the class NP-complete.

The article organized as follows. In section 2 Dota Underlords gameplay is described. We present the formulation of the Dota Underlords problem as a linear integer programming problem in section 3. In the 4 section, we show the NP-completeness of this task and reduce the problem to the maximum edge-weighted clique problem. The solution of the integer programming model for the real data is published in section 5. Traditionally, we summarize in the section 6 “Conclusion”.

2 Dota Underlords game play description
During the game, eight players build a team of “heroes” – creatures that can fight each other on the game map. Each of the heroes has basic parameters: health, damage, attack speed, and others, as well as a special ability that determines its role in the game. Each hero belongs to two or more “alliances” – sets that unite several heroes. For example, the hero Enchantress belongs simultaneously to the alliance “druids” and to the alliance “predators”. When there are several heroes in the team who belong to the same alliance (for each alliance this number is individual), the player receives a bonus consisting of improving the characteristics of his heroes or worsening the characteristics of his enemy’s heroes.
Also during the game, you can strengthen your heroes by upgrading them to higher levels or by purchasing in-game items. In this work, these aspects will not be taken into account. Thus, the strength of a players team is determined by:

1. The power of selected heroes
2. Bonuses from the alliances which they are belong

### 3 Problem reformulation to the linear integer programming language

#### 3.1 The simplest problem statement

We formalize the problem as follows. We assume that in total we have \( n \) heroes to choose from. We assume that the strength (power) of some \( i \)-th hero is presented by some non-negative value \( s_i \). As \( x_i \) we denote the belonging of hero \( i \) to the team. Let \( x_i = 1 \), if the \( i \)-th hero belongs to the players team and \( x_i = 0 \) otherwise. The condition that there is no more than \( m \) heroes in a team can be written as \( \sum_{i=1}^{n} x_i \leq m \). Then in the simplest form this problem can be expressed as follows:

\[
\max \sum_{i=1}^{n} x_i s_i \\
\sum_{i=1}^{n} x_i \leq m \\
x_i \in \{0, 1\} - \text{decision variable}
\]

\( n, m, s_i \) – constants

In this statement, the problem is solved elementarily - the solution is to take \( n \) elements with the largest weights.

#### 3.2 Problem statement with alliances

As mentioned, in “Dota Underlords” each hero belongs to two or more “alliances” — in turn, each alliance includes several heroes. When a team has several heroes from the same alliance (for each alliance this number is individual), the player receives a bonus, which is expressed in strengthening all the heroes from the alliance, strengthening all his heroes, or weakening all the heroes of the opponent. The last can be interpreted as a relative strengthening of the player’s heroes, and therefore only the first two cases will be considered throughout the work. It should be noted that for one alliance, there can be several bonuses that are unlocked by the different numbers of heroes of the corresponding alliance. These bonuses can also be of various types.

We propose to model this situation by introducing a 3-index tensor \( e_{ijk} \in \mathbb{R} \) which represents a bonus to the hero \( i \) from the alliance \( j \), in which there are at least \( k \) heroes of the alliance \( j \). In other words, \( e_{ijk} \) is the \( k \)-th bonus of the alliance \( j \) for the hero \( i \).

Using the tensor \( e_{ijk} \), we support both types of alliances – those that give bonuses to their members and those that give bonuses to all the heroes of the player. Moreover, the alliances of the considered types differ only in one thing. In the alliances that give a bonus to their members, the value of \( e_{ijk} \) is zero if and only if the \( i \)-th hero does not belong to the \( j \)-th alliance. In the general case, this is not necessarily true. Note that the tensor \( e_{ijk} \) is sparse for the real instances of auto-chess games since the alliances from which bonuses go to all the heroes are few.

We propose to control the occurrence of the bonus \( e_{ijk} \) in the total strength of the team using the control binary variable \( I_{ijk} \). So we can write down the objective function as the following sum \( \sum_{i=1}^{n} x_i s_i + \sum_{i=1}^{n} \sum_{j=1}^{l} \sum_{k=1}^{q} e_{ijk} I_{ijk} \). The connection between the variables \( x_i \) and \( I_{ijk} \) is given by the inequalities \( \forall i, j, k : \sum_{i'=1}^{n} a_{i'j} x_{i'} - k \geq M(I_{ijk} - 1) \).

These inequalities do not allow the binary variable \( I_{ijk} \) to take the value 1 if the solution includes less than \( k \) heroes from the alliance \( j \). When the solution contains less than \( m \) heroes from the alliance \( j \), the left side of this inequality is negative, so for the inequalities to be observed, the right side should be even smaller. It is possible only when the binary variable \( I_{ijk} \) is zero. In this case, the right-hand side is \( -M \), where \( M \) is a big constant known to be larger than \( k \), that is, larger than the maximum size of the alliance \( q \).
We require that the bonus for the hero $i$ can be activated ($I_{ijk} = 1$) only if the hero $i$ belongs to the solution. This is given by the inequalities $\forall i, j, k : I_{ijk} \leq x_i$. We also want the bonus $e_{ijk}$ to be activated only if the character $i$ belongs to the alliance $j$. For this, we include in the model inequalities $\forall i, j, k : I_{ijk} \leq a_{ij}$.

Thus, after introducing the alliances into the model, the system of equations can be written as the following:

**Objective function**

$$\max \sum_{i=1}^{n} x_i s_i + \sum_{i=1}^{n} \sum_{j=1}^{t} \sum_{k=1}^{q} e_{ijk} I_{ijk}$$

**Constraints for the input data**

$$\forall j : \sum_{i=1}^{n} a_{ij} \leq q$$

**Constraints for the decision variables**

$$\forall i, j, k : \sum_{i'=1}^{n} a_{i'j} x_{i'} - k \geq M(I_{ijk} - 1)$$

$$\sum_{i=1}^{n} x_i \leq m$$

$$\forall i, j, k : I_{ijk} \leq x_i$$

(2)

**Decision variables**

$I_{ijk} \in \{0, 1\}$, 1 – if for the hero $i$, the $k$-th bonus is activated for $j$-th alliance, $x_i \in \{0, 1\}$, 1 – if hero $i$ belongs to solution

**Constants**

$n \in \mathbb{N}$ – number of heroes,

$m \in \mathbb{N}$ – maximum size of the team

$t \in \mathbb{N}$ – the total number of alliances

$q \in \mathbb{N}$ – maximum size of an alliance,

$s_i \in \mathbb{R}$ — the strength of the hero $i$,

$e_{ijk} \in \mathbb{R}$ — the bonus for the hero $i$, if $k$-th bonus is activated for the $j$-th alliance

$a_{ij} \in \{0, 1\}$ – indicates if hero $i$ belongs to the alliance $j$

4 Proof of an NP-completeness of the Dota Underlords problem

To prove that a problem is NP-complete, it is necessary to show that it is both an NP-hard problem and that it belongs to the NP class. Let’s us prove both statements.

4.1 Reduction maximum density sub-graph problem to the Dota Underlords problem

**Theorem 1.** The problem of finding the maximum dense subgraph of $k$ vertices reduces to the Underlords problem.

**Proof.** Consider its special case — let all the alliances have a size equal to two, and the power of all the heroes is the same. Consider a special case of the Dota Underlords problem with the following restrictions:

1. The power of all heroes is the same ($\forall i, j s_i = s_j$)
2. Alliances can give bonuses only to the heroes that belong to the corresponding alliance. ($\forall i, j, k a_{ij} = 0 \implies e_{ijk} = 0$)
3. All alliances have the same size equaling two ($\forall j \sum_i a_{ij} = 2$)
4. All alliances give a bonus if and only if both heroes are present in the team ($\forall i, j e_{ij1} = 0$)
5. Bonuses from all alliances are the same (\(\forall i, j, j', a_{ij} = 1, a_{i'j'} = 1 \implies e_{ij} = e_{i'j'}\))

Then the data can be represented in the form of a graph \(G(V, E)\), where the set of vertices \(V\) corresponds to the heroes, and the set of edges \(E\) corresponds to the active alliances. You may notice that in this case, the optimal team of size \(k\) corresponds to the densest subgraph \(G' \subset G\) with \(k\) vertices. Density in this formulation can be understood as the value \(\frac{G'(E)}{G'(V)}\). Indeed, under these restrictions, the total strength of the team linearly depends on the number of active alliances, which corresponds to \(G'(E)\). Since \(k\) is invariable, with the increasing density of the graph \(G'\) the total strength of the team grows.

It was shown in [7] that the problem of finding a subgraph with a fixed size and maximum density is NP-complete. We have shown that it is a special case of the Dota Underlords problem, so it is reducible to Dota Underlords. Therefore the Dota Underlords problem is no less difficult than the well-known NP-complete problem. Thus the Dota Underlords problem is NP-hard.

4.2 Dota Underlords belongs to NP-class

The decision version of Dota Underlords problem (problem with the answer “yes” or “no”) can be formulated as follows: “Is there a team with at most \(m\) heroes and with a total power greater than some given constant?” Then, we are ready to state the following theorem.

**Theorem 2.** The decision version of Dota Underlords problem belongs to the NP class.

**Proof.** By the definition of the NP-class, the problem belongs to the NP class, when the presented solution can be checked in polynomial time. In our case, the solution is a set of \(m\) heroes. To verify the solution, we need to calculate the total power of the team.

So, we just need to calculate the objective function. In turn, to do that, at first we need to find out what alliances are formed. This means that we need to calculate the number of non-zero elements for each column in the matrix \(a_{ij}\), taking into account only rows corresponding to heroes from the team i.e we mean submatrix \(\{a_{ij} : j \in 1, m, i \in \{i' \in 1, n : x_{i'} = 1\}\}\). It is can be done for \(O(nt)\) operations.

After that, the objective function can be calculated in a straightforward way by \(O(ntq)\) number of operations. Thus, we need can check the solution for the polynomial time on the input size.

4.3 NP-completeness of Dota Underlords problem

**Theorem 3.** The Dota Underlords problem defined by the system of inequalities [3] belongs to the class of NP-complete problems.

**Proof.** Theorem 1 states that there exists a polynomial reduction of an NP-complete problem to DU. At the same time, by the theorem 2 we showed that the problem DU belongs to the NP class. Thus, the DU problem is NP-hard, and at the same time, it lies in the NP class. Therefore the decision version of Dota Underlords problem is NP-complete.

4.4 Reduction from the Dota Underlords problem to the maximum edge-weighted clique problem

While working on the paper, we also found a reduction from the Dota Underlords problem to the well-known problem — Maximum Edge Weighted Clique (MEWC). Thus, when solving individual instances of the Dota Underlords problem, anyone can use the already developed algorithms for the MEWC problem such as [8] or efficient quadratic formulations from [9].

In this reduction, we will consider a problem with the maximum size of the alliance bounded by some constant \(q\). The reduction will be proposed through the series of theorems, where each theorem describes the reduction from a less and less simplified version of DU to the maximum edge-weighted clique problem.

**Theorem 4.** The Dota Underlords problem without alliances is reduced to the maximum edge-weighted clique problem.

**Proof.** We construct a graph \(G\) with weighted edges such that the solution of the problem DU (Dota Underlords) follows from the solution of the problem MEWC (Maximum Edge-Weighted Clique). Moreover, the size of the MEWC problem is limited by a polynomial on the size of the DU problem. We construct the set \(V^2\) of \(n\) vertices corresponding
to the set of heroes in the DU problem. To each vertex we assign one of the heroes from the DU problem – or, in other words, we name each vertex in honor of one of the heroes of the DU problem. We enumerate these vertices according to the order of the heroes $v_1^1, v_2^1, ..., v_n^1$. We additionally construct $m - 1$ sets of vertices $V^2, V^3, \ldots$ up to $V^m$, in each we also name one vertex in honor of one of the heroes of the DU problem. Similarly to the first set, we enumerate the vertices in the set $V^i$ as $v_1^i, v_2^i, ..., v_n^i$. Denote the family of these sets as $\mathcal{F}$. Thus, we get $m$ sets of $n$ vertices, where each set has one vertex corresponding to one of the heroes. We build edges in the graph as follows – between the vertices $v_a^i$ and $v_{a'}^{i'}$, an edge is drawn if both of the following conditions are true:

- The vertices $v_a^i$ and $v_{a'}^{i'}$ correspond to different heroes ($a \neq a'$)
- The vertices $v_a^i$ and $v_{a'}^{i'}$ lie in different sets from the family $\mathcal{F}$ ($i \neq i'$)

Consider all the maximum clique in this graph. Obviously, in any such clique, there is exactly one vertex from each set $V^i$ – total $m$ vertices. Also, all these vertices correspond to different heroes. Thus, each clique sets a team of heroes in the DU task. It should be noted that each team can correspond to several cliques.

Now we introduce the heroes’ power. For this, we assign the weight $\frac{s_a}{m-1} + \frac{s_{a'}}{m-1}$ to the edge connecting the vertices $s_a^i$ and $s_{a'}^{i'}$. We show that the sum of the weights of the edges in a click corresponding to a certain team is exactly the strength of this team. Indeed, in a clique for each of its vertices, there is exactly $m - 1$ edge incident to it. Then each term $\frac{s_i}{m-1}$ corresponding to a vertex with a subscript $i$ is included in the sum exactly $m - 1$ times. It follows that the sum of all the weights of the edges in a clique is the sum of all the values $s_i$ corresponding to the numbers of the vertices that form this clique.

**Theorem 5.** The Dota Underlords problem with alliances of the size 2 is reduced to the maximum edge-weighted clique.

![Graph](image-url)
We also build all edges between all the vertices from the sets $F$ and $F_2$. We assign a weight of 0 to all these new edges. Evidently, any maximal clique contains exactly one vertex from each of the sets $V$ and $W$ of the families $F$ and $F_2$.

Assign to each edge $w_{a,b}$ some big constant weight $N$. These edges connect a vertex from the family $F$ to a certain hero with a vertex from the family $F_2$ corresponding to a pair of heroes where this hero belongs. See the figure for example of graph $G'$.

Now we are going to show that any maximal clique in graph $G'$ containing a set of vertices from the family $F$ corresponding to some set of heroes also contains the set of vertices from the family $F_2$ corresponding to all pairs of said heroes from this team. In this clique, the edges connecting the vertices from the families $F$ and $F_2$ make a total contribution to the weight equals to $2\binom{n}{2}N$, because maximum clique includes exactly $2\binom{n}{2}$ edges with additional big weight $N$ – two incident to each vertex from $F_2$.

It is easy to see that if a vertex in a clique belongs to the family $F_2$ and does not correspond to a pair of associated vertices of $F$ that are in the clique, then the clique will contain at least one edge that has an additional big constant weight $N$ less. Thus, the click will not have the maximum weight. Thus, the statement is proved.

Note that adding weights on the edges that are small compared to $N$ preserves the truth of the statement. Since $N$ is chosen arbitrarily, we can assume that all values of power and bonuses are small compared to $N$. Therefore we add bonuses that an alliance of a pair of heroes with numbers $a$ and $b$ gives the hero with number $c$ to the weights of the edges $(v_a^i, w_{a,b}^{i,j})$, connecting the vertices from the sets $F$ and $F_2$. If the selected team has the heroes $a$, $b$, and $c$, then this bonus will be included in the weight of the clique. Since the same number of edges with an additional weight $N$ are included in all the cliques under consideration, the maximum clique will be the one where the sum of the heroes’ strengths (the sum of the edges’ weights between the vertices of the $F$ family) and bonuses (the edges’ weights between the vertices of the families $F$ and $F_2$ without taking into account the constants $N$) is the maximum. Thus, the weight of the clique corresponds to the total bonus from the team, from which point the reduction is clear.

**Theorem 6.** Dora Underlords problem with alliances of size $q$ reduces to the maximum edge-weighted clique.

The proof will be constructed similarly to the proof of the theorem. We will create additional vertices associated with all possible combinations of $q$ heroes. The bonuses from the formation of the corresponding alliances will be the same as in the theorem on the edges. The main difference will be that for the vertices we will use $q$ indices instead of two indices. The formal considerations are given below.

**Proof.** We construct a graph $G'$ with weighted edges in such a way that the solution of the problem DU follows from the solution of the MEWC problem. Moreover, the size of the MEWC problem is limited by a polynomial on the size of the DU problem. We take a graph $G$ from the theorem as the base. We construct a set $W^{1,2}$ with $\binom{n}{2}$ vertices, where each vertex corresponds to an unordered set of $q$ heroes. We enumerate these vertices in the lexicographic order corresponding to the order of combinations of $\binom{n}{q}$ elements of $w_{1,2,\ldots,q}$.

It is important that each of these elements has exactly $q$ indices.
Within the simplified model, we accept the following:

We apply this model to analyze the real Dota Underlords problem. Note that our result should not be considered as some objective assessment of the quality of the team of heroes. The reason is the inevitable simplification of the heroes’ power as well as the influence that the alliances have. Each hero in Underlords has a certain ability, which is activated when various conditions satisfied, and in addition, the ability has some recharge time. Alliance abilities and bonuses are also very diverse in their influence on the game—they can cause damage, heal allies, prevent enemies from using their abilities, and more. Fortunately, the game has a system of five “tiers”, arranged so that the characters inside the tier are approximately equal in strength.

Within the simplified model, we accept the following:

5 Model application for real data

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Within the simplified model, we accept the following:
# Heroes Power Alliances

| # | Heroes               | Power          | Alliances                                      |
|---|---------------------|----------------|-----------------------------------------------|
| 0 | tusk                | 1              | savage, warrior                               |
| 1 | venomancer          | 1              | scaled, summoner                              |
| 2 | shadow demon        | 1              | demon, heartless                              |
| 3 | drow ranger         | 1              | heartless, hunter, vigilant                   |
| 4 | bloodseeker         | 1              | blood-bound, deadeye                          |
| 5 | nyx assassin        | 1              | assassin, insect                              |
| 6 | crystal maiden      | 1              | human, mage                                   |
| 7 | tiny                | 1              | primordial, warrior                          |
| 8 | haider              | 1              | knight, troll                                 |
| 9 | magnus              | 1              | duid, savage                                  |
| 10 | snapfire            | 1              | brawny, dragon                                |
| 11 | arc warden          | 1              | primordial, summoner                         |
| 12 | razor               | 1              | mage, primordial                              |
| 13 | weaver              | 1              | hunter, insect                                |
| 14 | warlock             | 1              | blood-bound, healer, warlock                  |
| 15 | dazzle              | 2              | healer, troll                                 |
| 16 | earth spirit        | 2              | spirit, warrior                               |
| 17 | storm spirit        | 2              | mage, spirit                                  |
| 18 | witch doctor        | 2              | troll, warlock                                |
| 19 | bristleback         | 2              | brawny, savage                                |
| 20 | legion commander    | 2              | champion, human                              |
| 21 | queen of pain       | 2              | assassin, demon                               |
| 22 | nature's prophet    | 2              | druid, summoner                              |
| 23 | luna                | 2              | knight, vigilant                              |
| 24 | windranger          | 2              | hunter, vigilant                              |
| 25 | ogre magi           | 2              | blood-bound, brute, mage                      |
| 26 | pudge               | 2              | heartless, warrior                           |
| 27 | beastmaster         | 2              | brawny, hunter                                |
| 28 | chaos knight        | 2              | demon, knight                                |
| 29 | slardar             | 2              | scaled, warrior                               |
| 30 | abaddon             | 3              | heartless, knight                            |
| 31 | viper               | 3              | assassin, dragon                             |
| 32 | juggernaut          | 3              | brawny, warrior                              |
| 33 | ember spirit        | 3              | assassin, spirit                             |
| 34 | io                  | 3              | druid, primordial                            |
| 35 | shadow fiend        | 3              | demon, warlock                               |
| 36 | lycan               | 3              | human, savage, summoner                      |
| 37 | broodmother         | 3              | insect, warlock                              |
| 38 | morphling           | 3              | mage, primordial                             |
| 39 | lifestealer         | 3              | brute, heartless                             |
| 40 | omniknight          | 3              | healer, human, knight                        |
| 41 | terrorblade         | 3              | demon, hunter                                |
| 42 | shadow shaman       | 3              | summoner, troll                              |
| 43 | enigma              | 3              | primordial, void                             |
| 44 | treant protector    | 3              | brute, druid                                 |
| 45 | doom                | 4              | brute, demon                                 |
| 46 | disraptor           | 4              | brawny, warlock                              |
| 47 | void spirit         | 4              | spirit, void                                 |
| 48 | mirana              | 4              | hunter, vigilant                             |
| 49 | tidehunter          | 4              | scaled, warrior                              |
| 50 | necrophos           | 4              | healer, heartless, warlock                   |
| 51 | lone druid          | 4              | druid, savage, summoner                     |
| 52 | sven                | 4              | human, knight, scaled                        |
| 53 | slark               | 4              | assassin, scaled                             |
| 54 | templar assassin    | 4              | assassin, vigilant, void                     |
| 55 | keeper of the light | 4              | human, mage                                  |
| 56 | axe                 | 5              | brawny, brute                                |
| 57 | faceless void       | 5              | assassin, void                               |
| 58 | sand king           | 5              | insect, savage                               |
| 59 | lich                | 5              | heartless, mage                              |
| 60 | medusa              | 5              | hunter, scaled                               |
| 61 | dragon knight       | 5              | dragon, human, knight                        |
| 62 | troll warlord       | 5              | troll, warrior                               |

**Table 1:** Heroes power and alliances structure

1. The forces of all the heroes of the first tier are equal to 1, the second – 2, the third – 3, the fourth – 4, the fifth – 5;

2. Alliances give the same percentage bonus to everyone they equally influence;

3. The alliance bonus is approximately 10-30 percent of the hero’s power.

Information about the strength of the heroes and the structure of alliances is given in the table [1]. A complete table defining a matrix of bonuses from the alliances $e_{ijk}$ can be found in our repository [10].

We provide a solution of the linear integer programming model defined by the system of inequalities [2], as a table [2]
### Table 2: Optimal team structure for the Dota Underlords game with all the active bonuses

| Hero            | Alliance contribution | Hero power | Sum |
|-----------------|-----------------------|------------|-----|
| broodmother     | heartless 2 +0.3      | human 2 +0.3 | insect 2 +0.3 | troll 2 +0.3 | warlock 2 +0.6 | warlock 4 | 3.0 | 3.0 | 6.0 |
| disruptor       | heartless 2 +0.4      | human 2 +0.4 | insect 2 +0.8 | troll 2 +0.4 | warlock 2 +0.8 | warlock 4 | 4.0 | 4.0 | 8.0 |
| dragon knight   | heartless 2 +0.5      | human 2 +0.5 | insect 2 +1.0 | troll 2 +1.0 | warlock 2 +0.5 | warlock 4 | 5.0 | 5.0 | 10.0 |
| lich            | heartless 2 +0.5      | human 2 +0.5 | insect 2 +1.0 | troll 2 +1.0 | warlock 2 +0.5 | warlock 4 | 4.0 | 5.0 | 9.0 |
| medusa          | heartless 2 +0.5      | human 2 +0.5 | insect 2 +1.0 | troll 2 +1.0 | warlock 2 +0.5 | warlock 4 | 4.0 | 4.0 | 8.0 |
| necrophos       | heartless 2 +0.4      | human 2 +0.4 | insect 2 +0.8 | troll 2 +0.4 | warlock 2 +0.8 | warlock 4 | 4.0 | 5.0 | 9.0 |
| sand king       | heartless 2 +0.5      | human 2 +0.5 | insect 2 +1.0 | troll 2 +1.0 | warlock 2 +0.5 | warlock 4 | 4.0 | 5.0 | 9.0 |
| sven            | heartless 2 +0.4      | human 2 +0.4 | insect 2 +0.8 | troll 2 +0.4 | warlock 2 +0.4 | warlock 4 | 4.0 | 4.0 | 8.0 |
| troll warlord   | heartless 2 +0.5      | human 2 +0.5 | insect 2 +1.0 | troll 2 +1.0 | warlock 2 +0.5 | warlock 4 | 4.5 | 5.0 | 9.5 |
| witch doctor    | heartless 2 +0.2      | human 2 +0.2 | insect 2 +0.4 | troll 2 +0.4 | warlock 2 +0.4 | warlock 4 | 2.2 | 2.0 | 4.2 |

### 6 Conclusion

In this paper, we demonstrated how a key to winning in a video game can lie in using linear integer math programming. The initial data and the results of solving the model in the form of a Jupyter-notebook can be found in our open repository [10]. We hope that this article will help to attract the attention of young minds to integer programming, discrete optimization methods, and also to the millennium problem \( P \neq NP \). It is important that the mathematical formulation of the problem given by the set of inequalities (2) can be considered by itself, abstracting from the domain. And in this paper, it is shown that the seemingly NP-hard task, in the “yes” or “no” version, is NP-complete. Thus, this work contributes to the study of NP-complete problems.

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