The text describes the spatial distribution of protons and neutrons in heavy nuclei, and the asymmetry in the production of positive and negative pions at CERN SPS energies. This difference leads to an impact parameter dependence of the pion ratio in heavy ion collisions. A recent experiment at CERN confirms these predictions, suggesting a possibility for determining neutron density distribution in nuclei.

**Keywords**: heavy ion collisions; inclusive pion production; neutron skin.

### 1. Introduction

Electron scattering off nuclei provides direct information about charge distribution, which is closely related to the spatial distribution of protons. The information about the neutron spatial distribution is not accessible directly.

The radiochemical method applied to antiprotonic atoms allows one to measure the neutron-to-proton ratio at the peripheries of nuclei. Another method is based on the analysis of X-ray spectra of antiprotonic atoms.

On the theoretical side, the difference between the proton and neutron distributions can be obtained in the framework of Hartree-Fock (HF) method (see for example) or Hartree-Fock-Bogoliubov (HFB) method (see for example).

Stable heavy nuclei exhibit an excess of neutrons over protons. A recent experiment at CERN for charged pion production in the $^{208}Pb + ^{208}Pb$ collision has observed an interesting dependence of the ratio $R_{+/-} = \frac{d\sigma\pi^+}{dxF} / \frac{d\sigma\pi^-}{dxF}$ on the Feynman variable $x_F$. Surprisingly, the ratio for central and peripheral collisions differs significantly. This presentation is based on where more details are given.
2. Estimate of the effect

We assume that pions are produced in elementary N-N collisions. Then

$$\frac{d\sigma_{A_1 A_2 \to \pi^\pm}}{dx_F}(b, x_F; W) = \sum_{\alpha \beta = p, n} N_{\alpha \beta \to \pi^\pm}(b; W) \frac{d\sigma_{\alpha\beta \to \pi^\pm}}{dx_F}(x_F; W),$$

where $W$ is energy per binary N-N collision and $N_{\alpha\beta \to \pi^\pm}$ are numbers of collisions of a given type. Only elementary cross sections for the $pp \to \pi^\pm$ processes are known experimentally. Therefore in the following we shall use $\frac{d\sigma_{\alpha\beta \to \pi^\pm}}{dx_F}(x_F; W)$ calculated in the HIJING model.

It is often assumed that the dynamics of nuclear collisions is governed by the number of binary collisions. The number of binary N-N collisions at a given impact parameter is proportional to the nucleus-nucleus thickness $T_{A_1, A_2}(\vec{b}) = \int d^2 s_1 T_{A_1}(s_1) T_{A_2}(s_1 - \vec{b}) = \int d^2 s_2 T_{A_1}(s_2 - \vec{b}) T_{A_2}(s_2).$

In Eq.(2) we introduced $T_{A_i}(\vec{b}) = \int dz_i \rho_{A_i}(\vec{b}, z_i)$, where $\rho_{A_i}$ is the density function of the nucleus $A_i$ normalized to the number of nucleons. Analogously the number of binary collisions of a given type in Eq.(1) can be written as

$$N_{\alpha \beta \to \pi^\pm}(b; W) = T_{A_1 A_2}^{\alpha \beta}(\vec{b}) \cdot \sigma_{ine}(W),$$

where

$$T_{A_1 A_2}^{\alpha \beta}(\vec{b}) = \int d^2 s_1 T_{\alpha}^{A_1}(s_1) T_{\beta}^{A_2}(s_1 - \vec{b}) = \int d^2 s_2 T_{\alpha}^{A_1}(s_2 - \vec{b}) T_{\beta}^{A_2}(s_2),$$

where now $T_{\alpha}^{A_1}$ and $T_{\alpha}^{A_2}$ are nucleus thicknesses of protons and neutrons, respectively. We have assumed one universal inelastic cross section $\sigma_{ine}(W) = \sigma_{ine}^{NN}(W)$.

As a second limiting case we consider the wounded nucleon model. We assume that the production of particles is proportional to the number of wounded nucleons. Then the cross section for the nuclear collision can be written as

$$\frac{d\sigma_{A_1 A_2 \to \pi^\pm}}{dx_F}(b, x_F; W) \propto \sum_{\alpha, \beta = p, n} \left[ N_{\alpha\beta \to \pi^\pm}^{wou}_{\alpha/A_1}(b, W) w_2^{\beta}(b) \frac{d\sigma_{\alpha\beta \to \pi^\pm}}{dx_F}(x_F; W) \right. + \left. N_{\alpha\beta \to \pi^\pm}^{wou}_{\alpha/A_2}(b, W) w_1^{\beta}(b) \frac{d\sigma_{\alpha\beta \to \pi^\pm}}{dx_F}(x_F; W) \right].$$

In the formula above $N_{\alpha\beta \to \pi^\pm}^{wou}_{\alpha/A_1}$ is the number of wounded $\alpha$ (p or n) in nucleus $A_1$ or $A_2$ and $w_1^{\beta}$ is the probability that the wounded $\alpha$ interacted with $\beta$ (p or n) from nucleus $A_2$ or $A_1$, respectively. Eq.(5) is equivalent to Eq.(1) with

$$N_{\alpha \beta \to \pi^\pm} = N_{\alpha\beta \to \pi^\pm}^{wou}_{\alpha/A_1}(b, W) w_2^{\beta}(b) + N_{\alpha\beta \to \pi^\pm}^{wou}_{\alpha/A_2}(b, W) w_1^{\beta}(b).$$

By construction, in our approach the numbers of wounded protons and neutrons reproduce the well known formula from HIJING for the number of wounded nucleons

$$N_{\alpha\beta \to \pi^\pm}^{wou}_{\alpha/A_1}(b) = N_{p/A_1}(b) + N_{n/A_1}(b).$$
Our construction requires also
\[ w_p^p(b) + w_n^n(b) = 1. \]  
(8)
The fractions \( w_i^\alpha \) were estimated as
\[ w_i^\alpha(b) = \frac{N_{wou}^{\alpha/A_i}(b)}{N_{wou}^{p/A_i}(b) + N_{wou}^{n/A_i}(b)}, \]  
(9)
which by construction fulfils (8). We use proton and neutron densities calculated in the HFB method with Skyrme interaction SLy4.

**Fig. 1.** The ratio \( R_+/− \) as a function of \( x_F \) for the model with binary collision (left panel) and for the model with the scaling with number of wounded nucleons (right panel) for selected values of impact parameters. Elementary cross sections are taken from HIJING model (solid lines) or from our fit to NA49 experimental \( p+p \) and \( p+n \) data (dashed lines).

In Fig. 1 we present the ratio \( R_+/− \) as a function of \( x_F \) for different values of \( b \) in the two models considered. For comparison we show preliminary experimental data for "central collisions" (solid circles) and "peripheral collisions" (open circles) from Ref. 7. The notion of central and peripheral collisions was not specified in Ref. 7. Therefore the data can be used only as an indication of the effect. Our approach explains the experimental data provided they are extremely peripheral.

Our results are reliable up to \( x_F \approx 0.2 \). It is known that the HIJING code does not describe the \( \pi^+−\pi^- \) asymmetry in elementary collisions in the region of large \( x_F \). In principle, instead of model calculations of the elementary cross sections one could use directly experimental data. Using isospin symmetry and approximate relations between elementary cross sections one can write for \( x_F >0.1 \):
\[ R_+/−(x_F) = \frac{N_{pp}(b) + N_{pn} + r(x_F)[N_{np}(b) + N_{nn}(b)]}{r(x_F)[N_{pp}(b) + N_{pn}(b)] + N_{np}(b) + N_{nn}(b)}, \]  
(10)
where
\[ r(x_F) = \frac{d\sigma_{pp→π^−}}{d\sigma_{pp→π^+}}. \]  
(11)
We fit the NA49 data shown in Fig.2a of Ref. 7 with the simple form \( r(x_F) = c_r(1−x_F)^{α_r} \). The corresponding results for \( R_+/−(x_F) \) are shown in Fig. 1 by the.
The ratio $R_+/−$ as a function of impact parameter for the model with binary collision (left panel) and for the model with the scaling with number of wounded nucleons (right panel) for selected values of $x_F$. Dashed lines. We find a good agreement with the calculations based on HIJING below $x_F ≈ 0.2$. Above $x_F ≈ 0.2$ the results based directly on experimental data lay below those based on HIJING.

For completeness in Fig. 2 we present the ratio $R_+/−$ as a function of the impact parameter $b$ for different values of $x_F$.

3. Summary

Surprisingly the binary collision picture gives very similar results to the predictions of the wounded nucleon model. This suggests that a detailed comparison of model results with the well defined ($x_F, b$) experimental data could open a new possibility to study the neutron density profile. We expect that the NA49 collaboration at CERN will be able to gather the corresponding experimental data in the near future. Can it provide a method competitive to that offered by proton-nucleus elastic scattering, antiprotonic atoms or parity violating electron scattering? Of course results of these methods must finally converge. Therefore one may hope that together they will provide more reliable information on neutron distribution in nuclei.

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