Kappa distributions: Thermodynamic origin and Generation in space plasmas

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Abstract. The paper reviews the new developments on the thermodynamic origin of kappa distributions and their generation in space plasmas. We present the correct formulation of kappa distributions that is consistent with thermodynamics. Then, we present the characterization of mechanisms that generate kappa distributions. There are three fundamental plasma conditions for the kappa distributions to be generated in space plasmas. Several applications of kappa distributions in space plasmas are presented and discussed. Finally, the thermodynamic validity of kappa distributions is presented. According to the zeroth law of thermodynamics, the most generalized form of particle distribution assigned with a temperature, is given by the kappa distributions, where temperature and kappa are two independent parameters spanning the 2-D abstract space of thermodynamics.

1. Introduction
In the last ~60 years, numerous analyses have established the theory of kappa distributions and provided a plethora of different applications in space and astrophysical plasmas exhibiting non-Maxwellian behavior (c.f., Figure 1 in [1], Chapter 1).

Classical collisional particle systems residing in thermal equilibrium have their particle velocity/energy distribution function stabilized into a Maxwell-Boltzmann distribution. On the contrary, space and astrophysical plasmas are collisionless particle systems residing in stationary states characterized by the so-called kappa distribution function. Kappa distributions have become increasingly widespread across the physics of space plasma processes, describing particles in the heliosphere, from the solar wind and planetary magnetospheres to the heliosheath and beyond, the interstellar and intergalactic plasmas (Tables 1 &2).

Kappa distributions were introduced to describe magnetospheric electron data, by Olbert and its Ph.D. students and colleagues [2-4]; Binsack (1966) [2] was the first to publish the usage of kappa distributions, but he acknowledged that the kappa function was actually “introduced by Prof. Olbert of MIT in his studies of IMP-1.”

Single types of kappa distributions are usually sufficient to model the space plasma populations (e.g., [5-7]); however, more complicated models of kappa distributions superposition have been employed to describe rare features (e.g., anisotropy) (e.g., [8-10]). A kappa distribution is primarily formulated to describe a Hamiltonian, i.e., the particle kinetic and potential energy ([11,12]; [1], Chapter 3). In several cases, though, the potential energy of a particle is small compared to its kinetic energy and can often be ignored, and thus, the system’s statistical description is reduced to the kappa distribution of the particle velocities.
Table 1. Examples of Applications of Kappa Distributions in Space and Astrophysical Plasmas

| Space/Astrophysical Plasma | References |
|---------------------------|------------|
| Solar Atmosphere          | [13]       |
| Solar Spectra             | [14,15]    |
| Solar Corona              | [16-20]    |
| Solar Flares              | [21-24]    |
| Solar Radio Bursts        | [25,26]    |
| Solar Wind                | [8,10,27-44] |
| Solar Energetic Particles | [45,46]    |
| Corotating Interaction Regions | [47]   |
| Planetary Magnetospheres | (See Table 2) |
| Cometary Magnetospheres   | [48]       |
| Inner Heliosheath         | [6,36,49-58] |
| HII Regions               | [60]       |
| Planetary Nebulae         | [61,62]    |
| Supernovae Magnetospheres | [63]       |
| Cosmic Rays               | [64,65]    |
| Cosmology Scales          | [66]       |

(Note: See also [1], Chapters 10-17)

Table 2. Examples of Applications of Kappa Distributions in Planetary Magnetospheric Plasmas

| Magnetospheric Plasma   | References |
|-------------------------|------------|
| Terrestrial Magnetosphere |            |
| Magnetosheath           | [4,67,68]  |
| Magnetopause            | [69]       |
| Magnetotail             | [70]       |
| Ring Current            | [71]       |
| Plasma Sheet            | [72-74]    |
| Magnetospheric Substorms| [24,75,76] |
| Aurora                  | [77]       |
| Magnetoospheres of Giant Planets |        |
| Jovian                  | [78,79]    |
| Saturnian               | [5,80-82]  |
| Uranian                 | [83]       |
| Neptunian               | [84]       |
| Magnetoospheres of Planetary Moons |       |
| Io                      | [85]       |
| Enceladus               | [86]       |

The purpose of this paper is to review the new developments on the thermodynamic origin of kappa distributions and their generation in space plasmas. In Section 2, we present the formulation of kappa distributions that is consistent with thermodynamics. In Section 3, we present what the observations and measurements of kappa distributions have revealed regarding the trends between the kappa index and other plasma parameters. These trends can be used as indications to detect the mechanisms generating the observed kappa distributions. In section 4, we briefly picture the theory of mechanisms responsible for generating kappa distributions in space plasmas, and discuss three conditions that affect the generation of these distributions. In Section 5, we examine the thermodynamic validity of kappa distributions. The zeroth law of thermodynamics can be used to derive the most generalized form of particle velocity/energy distributions assigned with a temperature, where temperature and kappa are two independent parameters spanning the 2-D abstract space of thermodynamics. Finally, in Section 6, we summarize the main conclusions of the paper.
2. Formulation of Kappa distributions
A breakthrough in the field came with the connection of kappa distributions with the solid background of non-extensive statistical mechanics. Understanding the statistical origin of kappa distributions was a cornerstone of further theoretical developments and applications. The origin of the kappa distribution in non-extensive statistical mechanics has already been examined by several authors [87-90].

Historically, the first evidence of a connection between the theory of kappa distributions and the framework of non-extensive statistical mechanics was mentioned by Treumann (1997) [87] in the appendix of his investigation regarding the superdiffusion near the magnetopause. A detailed derivation of the kappa distribution through non-extensive statistical mechanics was referred to in the analysis of Milovanov & Zelenyi (2000) [88] and Leubner (2002) [89]. They showed the connection of kappa distributions with the canonical probability distribution that maximizes the Tsallis entropy under the constraints of canonical ensemble (e.g., [91]).

Livadiotis & McComas [10,34,90] completed the connection of kappa distributions with the modern non-extensive statistical mechanics using the concept of physical temperature that unifies the thermodynamic and kinetic definitions of temperature (see also [7], [1], Chapter 1). The developed kappa distribution formulation is the only one that includes correctly the notion of temperature:

\[ P(\varepsilon) \propto \left[ 1 + \frac{1}{\kappa} \left( \frac{\varepsilon - U}{k_b T} \right)^{\frac{1}{\kappa} - 1} \right], \quad (1) \]

where \( U = \langle \varepsilon \rangle \) stands for the mean kinetic energy.

Thereafter, the connection of kappa distributions with non-extensive statistical mechanics revealed the robust physical meaning of temperature, thermal pressure, and other thermodynamic parameters. The properties of kappa distributions and the proven tools of non-extensive statistical mechanics have been successfully applied to a variety of space plasmas throughout the heliosphere. These analyses led to the determination of the thermodynamic variables of these plasmas, as well as, the understanding of the underpinning plasma processes.

The standard 3-dimensional kappa distribution function of particle velocities is given by

\[ P(\vec{u}; \theta; \kappa) = \pi^{-3} \cdot A(\kappa) \cdot \theta^{-3} \cdot \left[ 1 + \frac{1}{k - \frac{\theta}{2}} \left( \frac{\vec{u} - \vec{u}_b}{\theta} \right)^2 \right]^{-\kappa^{-1} - 3}, \quad A(\kappa) = \frac{(k - \frac{\theta}{2})^{-3/2}}{\Gamma(k - \frac{\theta}{2})}, \quad (2) \]

which is parametrized by the 3-dimensional kappa index \( \kappa \) (in general depends on the dimensionality), and the thermal speed \( \theta = \sqrt{2k_b T / m} \); the latter is defined in order to have no dependence on the kappa index but only on the temperature (it can be seen as the temperature in speed units). The bulk velocity \( \vec{u}_b \) gives the average velocity of the particles, that is, the flow velocity.

The classical thermal equilibrium corresponds to \( \kappa \to \infty \), where we have \( A(\kappa) \to 1 \) and \( \left[ 1 + x / (k - \frac{\theta}{2}) \right]^{-1} \to e^{-x} \), thus, the distribution recovers the Maxwellian formulation:

\[ P(\vec{u}; \theta; \kappa \to \infty) = \pi^{-3} \cdot \theta^{-3} \cdot \exp \left\{ -\left( \frac{\vec{u} - \vec{u}_b}{\theta} \right)^2 \right\}. \quad (3) \]

As mentioned above, the kappa index \( \kappa \) depends on the degrees of freedom with the linear relation:

\[ \kappa(f) = \kappa_0 + \frac{1}{f}, \quad f = d \cdot N, \quad (4) \]

where \( f \) are the total kinetic degrees of freedom from all the \( N \) particles of \( d \) degrees each. The kappa index, \( \kappa_0 \), called “invariant kappa index”, arranges the different stationary states along the kappa index, independently of the dimensionality, the degrees of freedom, or the number of particles (e.g., [7,54]).

Then, using the invariant kappa index we can formulate the \( d \)-dimensional kappa distribution:
\[ P(\tilde{u}; \theta, \kappa) = \pi^{-\frac{d}{2}} A_j(\kappa_0) \cdot \theta^{-d} \cdot \left[ 1 + \frac{1}{\kappa_0} \frac{(\tilde{u} - \tilde{u}_0)^2}{\theta^2} \right]^{-\kappa_0 - \frac{1}{2}}, \quad A_j(\kappa_0) = \frac{\Gamma(\kappa_0 + 1 + \frac{1}{2})}{\Gamma(\kappa_0 + 1)}. \] 

For example, if using a 3D kappa distribution to describe a plasma population we find \( \kappa \approx 3 \), while using a 1D kappa distribution we find \( \kappa \approx 2 \), then there should be no conflict between those two results; both lead to the same (invariant) kappa index: \( \kappa(3) = \kappa_0 + \frac{1}{2} \approx 3 \) and \( \kappa(1) = \kappa_0 + \frac{1}{2} \approx 2 \) lead to \( \kappa_0 \approx \frac{1}{2} \).

### 3. Observations

Recently, we have shown that the values of the kappa index (the parameter that labels and governs kappa distributions) are related to other solar wind plasma parameters (e.g., density \( n \), temperature \( T \)) [7]. These trends reveal relationships that vary with location, time, and the type of space plasmas. In Figure 1, we illustrate the representative values of density \( n \), temperature \( T \), and kappa index \( \kappa \) of ~40 different space plasmas. We also plot the quantity \( M = 1/(\kappa - 0.5) \), which provides a measure of how far the system resides from thermal equilibrium [34,35]. (a) The parameters are plotted in a \((n, T)\) diagram, using a color-map to illustrate the values of \( \kappa \) and \( M \). Panel (b) shows that \( M \) is negatively correlated (or, \( \kappa \) is positively correlated) with the temperature and density. More precisely, \( M \) is linearly decreasing as

\[ M \approx 2.27 - 0.19 \cdot \log(0.19 + 2.27 \cdot nT^{0.6}). \]

**Figure 1.** (a) Color-map of the 3D kappa indices of various space plasmas with different values of density \( n \) and temperature \( T \). The kappa index \( \kappa \) or the measure \( M = 1/(\kappa - 1/2) \) spans the whole interval from \( \kappa \to \infty \) or \( M \to 0 \) (thermal equilibrium) to \( \kappa \to 1.5 \) or \( M \to 1 \) (Anti-equilibrium, i.e., the furthest state from thermal equilibrium). (b) The measure \( M \) is negatively correlated (or, the kappa index is positively correlated) with the temperature and density. (Adapted from [7])

Figure 1 shows that, on average, the kappa index is positively correlated with temperature and density. Similar cases where the kappa index is positively correlated with temperature have also been observed in individual space plasmas, e.g., the magnetosheath [68] or the inner heliosheath [6,56,57], as shown in Figure 2.
Figure 2. Examples of empirical relations between temperature and kappa index characterizing (a) inner heliosheath and (b) magnetosheath. (Adapted from [6] and [68])

The kappa index is mostly positively correlated with temperature, while temperature and density can be either correlated or anti-correlated, depending on the polytropic index (if it is $a>1$ or $a<1$, respectively). A polytropic relation connects the values of thermal variables (such as, density $n$, temperature $T$, thermal pressure $P$, the kappa index, $\kappa$, etc.) along a certain streamline of the plasma flow. A polytrope is a certain thermodynamic process characterized by such a relation. Typically, this is a power-law between two thermodynamic variables,

$$[P(\tilde{r})/P(\tilde{r}_i)]=[n(\tilde{r})/n(\tilde{r}_i)]^a, \text{ or } [n(\tilde{r})/n(\tilde{r}_i)]=[T(\tilde{r})/T(\tilde{r}_i)]^\nu, \text{ with } \nu \equiv 1/(a-1),$$

where the exponent $a$ denotes the typical polytropic index; $\nu$ is another way of expressing the polytropic index, which corresponds to the effective degrees of freedom.

While the most of space plasmas exhibit positive correlations between density and temperature, there are several plasmas with negative correlations, consistent with constant or quasi-constant thermal pressure, were found in heliosheath [6,38] and planetary magnetosheaths (e.g., the low latitude boundary layer at the terrestrial magnetosheath [92]; in the terrestrial central plasma sheet [93,94]; and in the Jovian magnetosheath, [95]).

| Space Plasma          | Datasets                | Resulting $a$ | References |
|-----------------------|-------------------------|---------------|------------|
| Bow Shock             | ISEE-1                  | 2             | [96]       |
| Bow Shock             | ISEE-1                  | 1.6-1.7       | [97]       |
| Venus bow Shock       | Pioneer Venus Orbiter   | 1.85          | [98]       |
| Plasma Sheet          | AMPTE/IRM               | 1.67 ± 0.5, 1.4 | [99]   |
| Plasma Sheet          | Cluster                 | -0.15, 0-1    | [93,94]    |
| Magnetosheath         | ISEE-1 & -2             | ~0            | [92]       |
| Jovian Mag/sheath     | New Horizons            | ~0            | [95]       |
| Solar Wind            | Helios-1                | 1.46, 1.58    | [100]      |
| Solar Wind            | Pioneer Venus Orbiter   | 5/3, 2        | [101]      |
| Solar Wind            | Wind                    | 0.5-2.5       | [102]      |
| Solar Wind            | OMNI                    | 1.8 ± 2.4     | [103]      |
| Solar Wind            | Wind                    | ~1.66         | [104]      |
| Solar Wind            | Wind                    | ~1.15, ~1.65  | [43]       |
| Magnetic Clouds       | IMP-8                   | 0.5, 0.6      | [105]      |
| CMEs                  | Ulysses/ CMEs           | 0.73, 0.78    | [106]      |
| Inner Heliosheath     | IBEX                    | -0.04 ± 0.07  | [38]       |
| Inner Heliosheath     | IBEX                    | ~0            | [107]      |
Aside from the temperature and density, the kappa index was also found to be connected with other plasma parameters, e.g., polytropic index (e.g., [43,108]; [1], Chapter 5), thermal pressure (e.g., [6]), potential energy (e.g., [43,77]), and magnetic field (e.g., [44]). The latter has significant interplay in the generation of kappa distributions in space plasmas, where the kappa index is found to decrease as the magnetic field strength increases (e.g., [68,109,110]).

The observed trends between the kappa index and other space plasma parameters can be caused by mechanisms responsible for generating kappa distributions.

4. Generation of kappa distributions

There are various mechanisms mentioned in the literature that can generate kappa distributions, and thus, produce relationships among the kappa index and other plasma parameters. Some examples are studied in the book of kappa distributions [1]: Connection with polytropes ([43,107]; [1], Chapter 5); strong Debye shielding and magnetic coupling ([44]; [1], Chapter 5); superstatistics ([111,112]; [1], Chapter 6); presence of weak turbulence ([40]; [1], Chapter 8); effect of pickup ions especially in the inner heliosheath ([37]; [1], Chapter 10); turbulence related to diffusion inverse proportional to velocity ([22]; [1], Chapter 13); pump acceleration mechanism ([113]; [1], Chapter 15); and, effect of shock waves ([114]; [1], Chapter 16).

Systems with no correlations between their particles are stabilized into the classical thermal equilibrium – the state where the particle distribution function of velocities/energies is described by a Maxwell-Boltzmann distribution. Systems with particle correlations, however, reside in stationary states out of this classical thermal equilibrium, described by kappa distributions. In fact, the kappa index is inversely proportional to the correlation coefficient $\rho$ between the energies of any two particles, $\kappa=(3/2)/\rho$ [115,54,12]. The largest kappa index is infinity, corresponding to the system residing at thermal equilibrium, where the particles are characterized by zero correlation ($\rho=0$). The smallest kappa index is 3/2 (for 3D distributions), corresponding to the furthest state from thermal equilibrium, called anti-equilibrium, where the particles have the highest correlation ($\rho=1$). All other cases reside between these extremes, with kappa indices between 3/2 and $\infty$, i.e., with correlation $\rho$ between 0 and 1 (Figure 3). Therefore, local correlations among the velocities or energies of particles are interwoven with kappa distributions.

![Figure 3](image.png)

**Figure 3.** Arrangement of stationary states depending on the kappa index $\kappa$ or the correlation $\rho$ between particle energies. The two extreme stationary states correspond to the classical thermal equilibrium ($\kappa\to\infty$) and the recently studied state of anti-equilibrium ($\kappa\to0$). (Taken from [10])
A mechanism that generates kappa distributions involves plasma processes that induce local correlations among particles. Livadiotis et al. (2018) [44] identified three fundamental conditions that can generate kappa distributions in space plasmas, noted as: (i) Debye shielding, (ii) magnetic field coupling, and (iii) temperature fluctuations, each one prevailing in different scales of solar wind plasma and magnetic field properties (Figure 4):

(i) **Debye shielding**: Debye shielding produces a natural ordering of the particle correlations in plasmas (e.g., [116]). Inside a Debye sphere, particles are highly correlated with each other and act together through their electromagnetic interactions. In contrast, at distances greater than a Debye length \( \lambda_D \), particles are largely uncorrelated owing to the Debye shielding of the closer particles. Therefore, each Debye sphere represents a cluster of correlated particles (ions and electrons), which is essentially uncorrelated to the more distant particles and their Debye spheres. Weakly coupled plasmas have small coupling parameter \( \Gamma \), or equivalently, a large number of particles within their Debye spheres, \( N_D \) (Debye number), because of the relation \( \Gamma \equiv 0.1 \cdot N_D^{-2/3} \). Local correlations among particles within the Debye spheres can persist for a substantial length of time without being destroyed by collisions. The larger the Debye number \( N_D \), the more effective the collective behavior in plasma, which inclines the system even further from thermal equilibrium; thus, the smaller the kappa index becomes. Hence, \( N_D \) is negatively correlated with \( \kappa \).

(ii) **Magnetic field binding**: The magnetic field’s long-range interactions induce correlations among particles, turning the system away from thermal equilibrium, i.e., the kappa index decreases. Unfortunately, we do not know the exact functional relationship between \( B \) and \( \kappa \), but we expect an inverse correspondence (negative correlation): the largest the magnetic field, the more intense the induced correlations (strongest “binding”), and the lower the kappa index; thus, \( B \) is negatively correlated with \( \kappa \). While the magnetic energy “binds” the particles together, the thermal energy, that is a measure of collisions, disorders the particles and destroys their correlations, thus competes with the magnetic binding, leading to a positive correlation between \( T \) and \( \kappa \). Then, the densities of magnetic and thermal energy have an opposite effect on the kappa index, that is, negative vs. positive correlation, respectively. The competition between magnetic and thermal pressure can be represented by the plasma beta \( \beta \) (the ratio of thermal over magnetic pressure). As \( \beta \) increases, the magnetic over thermal pressure decreases, and thus, long-range interactions due to the magnetic field become weaker and the kappa index increases. Hence, \( \beta \) is positively correlated with \( \kappa \).

(iii) **Temperature fluctuations**: Complex driven nonequilibrium systems, such as space plasmas exhibit temperature fluctuations on a large spatio-temporal scale compared to local relaxation times. There is a relatively fast dynamics, given by the local particle distributions and their statistical mechanics, and also a slow one, given by a spatio-temporally inhomogeneous parameter that varies globally with the environment. The two effects produce a superposition of two statistics, called “superstatistics” [111]. According to the concept of superstatistics, local fluctuations of temperature shift the system into stationary states described by kappa distributions, where the stabilized kappa index is linearly related to the inverse of the variance of temperature fluctuations.

![Figure 4](image_url)

**Figure 4.** We observe three regions with different coupling and dynamics, corresponding to three processes that can generate kappa distributions: magnetic field coupling, Debye shielding, and
temperature fluctuations, respectively. The three regions are noted by the parameters beta plasma $\beta$, number of particles in a Debye sphere $N_D$, and standard deviation of temperature fluctuations $\delta T$. (Taken from [44])

The general scheme that characterizes these mechanisms is composed of the following two parts: (1) Generation of local correlations among particles, and (2) “Thermalization”, that is, the re-stabilization of the particle system into a non-equilibrium stationary state assigned by a temperature, and thus, described by kappa distributions (Figure 5).

![Figure 5. General scheme of mechanisms generating kappa distributions: Correlation and Thermalization. The input can be systems described either by Maxwell-Boltzmann or kappa distributions. (Taken from [44])](image)

While there are several mechanisms capable of generating kappa distributions in space plasmas, none of these mechanisms can explain why the kappa distributions are allowed by the laws of thermodynamics. Fortunately, it has been shown that this is the case; namely, the zeroth law of thermodynamics is consistent with kappa distributions [117].

5. Thermodynamics of kappa distributions
One of the most frequent misinterpretations of temperature is that it is assumed to be dependent on the kappa index, instead of being a fundamental independent parameter.

The temperature is a well-understood concept in physics when it comes to particle systems at thermal equilibrium. This is based on the equivalence of the two fundamental definitions of temperature, that is (i) the kinetic definition of Maxwell (1866) [118] and (ii) the thermodynamic definition of Clausius (1862) [119]. The same equivalence is shown to exist for particle systems in stationary states out of the classical thermal equilibrium, where the particle distribution functions of velocities/energies cannot be described by the Maxwell-Boltzmann behavior. Indeed, when the system resides in any arbitrary stationary state assigned with a temperature, then its particle distribution function can be generally described by kappa distributions and combinations thereof.

According to the zeroth law of thermodynamics, if two thermodynamic systems are each in thermal equilibrium with a third, then they are in thermal equilibrium with each other. This is equivalent to have two systems in thermal equilibrium to each other. Indeed, two systems are in thermal equilibrium [120], if (a) they are linked by a wall permeable only to energy transfer (heat), and (b) they do not change over time. “Thermalization” is the characterization of a system of particles residing in any stationary state assigned by a temperature; in such a state, the particle velocities or energies can be stabilized into a Maxwell-Boltzmann distribution, or a kappa distribution, in general.

The mean particle kinetic energy is given by $U = \frac{3}{2} d k_B T$. Then, the kinetic definition is given by $T_{\text{kin}} = \frac{2}{(d k_B)} U$. On the other hand, the thermodynamic definition of Clausius is given by the
The notion of “physical temperature”, \( T_{\text{phys}} = (\partial S / \partial U)^{-1} \cdot [1 - \frac{1}{k} \cdot S / k_B] \), which is consistent with the zeroth law of thermodynamics; \( S \) is the entropy and \( U \) is the internal mean energy of the system. The physical temperature \( T_{\text{phys}} \) is obtained in accordance with the generalized zeroth law (e.g., [117,121,122]), while serves also the role of the kinetic definition of temperature within the framework of non-extensive Statistical Mechanics [90]. Therefore, all the advantages of a kinetically defined temperature, in contrast to other configurational definitions [123], can be ascribed to \( T_{\text{phys}} \). Several inconsistencies concerning the kinetic definition in regards to the zeroth law of thermodynamics [124,125] are fully recovered, since the origin of \( T_{\text{phys}} \) establishes the generalized zeroth law. (See also: [7,10,34,54,90,116]; [1], Chapter 1).

The outcome of mixing two plasmas that reside in two different stationary states (different kappa indices) is similar to the mixing of classical gases, which obey simple calorimetry rules; e.g., if we mix two plasmas with different temperatures and densities, the result will be a mass-weighted average temperature of the combined plasma (Figure 6). This was resolved by demonstrating the equivalence of the two basic temperature definitions, (1) kinetic and (2) thermodynamic, for kappa distributions of particles out of equilibrium [90,34].

![Figure 6](image_url)

**Figure 6.** Space and laboratory plasmas can be understood under the same unique thermodynamics. The temperature of solar corona can be calculated using the kinetic definition, while the temperature of a laboratory plasma can be measured simply a thermometer (that makes use of the thermodynamic definition). Thus, the resulting temperature follows from simple calorimetry rules. (Taken from [7])

Having understood the independence of temperature on other thermodynamic parameters, we now need to interpret correctly the observations of correlations between temperature and the kappa index. A similar example can be retrieved from plasmas at classical thermal equilibrium: The plasma particles are described by Maxwell distributions, dependent only on temperature \( T \) and density \( n \). While these are two independent parameters, along the streamlines of a plasma flow, we frequently observe various relationships between \( n \) and \( T \), typically, described by a polytropic relation, \( T = A \cdot n^{-\alpha} \). However, each streamline has different constant \( A \), while the polytropic index \( \alpha \) characterizes the specific streamline ([103,104,126,127]; [1], Chapter 5).

There are other descriptions of generalized polytropic relations, \( T = f(n) \) [107]. Generalized polytropic relations may also include the kappa index for space plasmas out of thermal equilibrium, i.e., \( T = f(n, \kappa) \). In fact, very often the dependence on the density is ignored, dealing only with the more interesting relation of \( \kappa - T \). For example, see the positively correlated kappa index and temperature values in the inner heliosheath [6], in the magnetospheres of Earth [69], Jupiter [78], and Saturn [5].

The observed relationships between the temperature and the kappa index by no means imply any universal dependence between those two thermodynamic parameters, in the same way that the observed relations between the temperature and the density do not imply any universal dependence between them. Care should be shown to distinguish the local relations that depend on certain plasma streamlines or the whole plasma, and not misinterpret these with any universal behavior that indicates to the foundations of physical laws.

Therefore, the stationary state at the classical thermal equilibrium is neither unique nor has any special properties that can be identified by measuring the internal energy \( U \). All stationary states can equivalently describe system’s internal energy, leading to the *Relativity Principle for Statistical Mechanics*: “A particle system is identified with the same internal energy from any of the stationary states that can attain” ([1], Chapter 1).
6. Conclusions
In this paper, we presented new developments on the thermodynamic origin of kappa distributions and their generation in space plasmas. We presented the formulation of kappa distributions that is consistent with thermodynamics, as well as a brief synopsis of the observations and measurements of kappa distributions. We discussed the observed trends between the kappa index and other plasma parameters, their origin and importance, as they are indications of the mechanisms generating kappa distributions. We then pictured the theory of mechanisms generating kappa distributions in space plasmas, and discussed three conditions that affect the generation of these distributions. We finally showed the thermodynamic validity of kappa distributions: The zeroth law of thermodynamics can be used to derive the most generalized form of particle distributions which are assigned with a temperature, where temperature and kappa are two independent parameters spanning the 2-D abstract space of thermodynamics.

In summary, we presented the following topics:

- The physical meaning of the temperature for stationary states out of thermal equilibrium was established, showing the equivalence of the thermodynamic and kinetic definitions of temperature. The concept of temperature was exclusively known for systems residing at the classical thermal equilibrium – the stationary state described by a Maxwellian distribution of velocities and the BG statistical mechanics.
- Therefore, the stationary state at the classical thermal equilibrium is neither unique nor has any special properties that can be identified by measuring the internal energy $U$. All stationary states can equivalently describe system’s internal energy, leading to the Relativity Principle for Statistical Mechanics: “A particle system is identified with the same internal energy from any of the stationary states that can attain”.
- Resolution of misinterpretations of temperature. The temperature is a fundamental independent parameter, and should not be considered dependent on the kappa index.
- The outcome of mixing two plasmas that reside in two different stationary states (different kappa indices) is similar to the mixing of classical gases, which obey simple calorimetry rules.
- The physical meaning of the kappa index is interwoven with the statistical correlation between the energy of the particles. The kappa index classifies the different stationary states independently of the dimensionality, the degrees of freedom, or the number of particles.
- The kappa index depends on the particle degrees of freedom $f$, with $\kappa(f) = \kappa_0 + \frac{1}{2} f$. The invariant kappa index, $\kappa_0$, classifies the different stationary states independently of the dimensionality, the degrees of freedom, or the number of particles, and allows the construction of multi-dimensional kappa distributions.
- The general scheme that characterizes these mechanisms is composed of the following two parts: (1) Generation of local correlations among particles, and (2) “Thermalization”, that is, the re-stabilization of the particle system into a non-equilibrium stationary state assigned by a temperature, and thus, described by kappa distributions.
- There are three fundamental plasma conditions that need to be fulfilled for the kappa distributions to be generated in space plasmas, noted as: (i) Debye shielding, (ii) magnetic field coupling, and (iii) temperature fluctuations, each one prevailing in different scales of solar wind plasma and magnetic field properties.

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