Robust path-following control of underactuated AUVs with multiple uncertainties and state constraints

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Abstract—This paper proposes a novel robust controller for horizontal path-following problem of an underactuated AUV subject to multiple uncertainties and state constraints. Firstly, four reduced-order extended state observes (ESOs) are designed to estimate the multiple uncertainties, and the estimated values are adopted in the design of kinematic and dynamic controller. Secondly, to address the state constraints, the barrier Lyapunov function is incorporated with the kinematic controller. To resolve the problem of input saturation, the auxiliary design system is utilized in the dynamic controller. To address the problem of “explosion of complexity” inherent in the conventional back-stepping method, a nonlinear tracking differentiator is utilized to obtain the derivative of the desired yaw speed. Finally, the results of numerical simulation are performed to demonstrate the effectiveness of the proposed controller.

1. Introduction

In recent years, autonomous underwater vehicle (AUV) has been widely utilized in all kinds of underwater applications, such as deep sea exploration, underwater target tracking and ocean sampling [1, 2]. In order to better fulfill the above-mentioned tasks, it is essential to achieve accurate control for an AUV. However, due to the existence of model nonlinearity, model parameters perturbations and unknown environment disturbances, it is a difficult task to precise control the AUV [3].

Path-following is main technical basis of an AUV to accomplish various complex underwater missions, and it has been attracted widespread attention from researchers. In [4], a compound controller, which is based on reduced-order ESOs, was proposed for the path-following of the underactuated AUV subject to multiple uncertainties, where the reduced-order ESOs were designed to estimate the multiple uncertainties. However, the problem of input saturation is not taken into consideration. In [5], a novel controller was presented for the path-following of an underactuated unmanned surface vehicle (USV) with disturbances. A disturbance observer was utilized to compensate the unknown dynamics, and an auxiliary system was adopted to eliminate the effect of input saturation. However, the state constraints are not considered. In [6], a trajectory tracking controller, which was incorporated with barrier Lyapunov function, was proposed for an USV with input saturation and full-state constraints. The barrier Lyapunov function was employed to address the full-state constraints, and an anti-windup compensator was utilized to compensate the input saturation.

Inspired by the afore-mentioned considerations, this paper proposes a robust controller for horizontal path-following of an underactuated AUV subject to multiple uncertainties and state constraints. The
multiple uncertainties, including the model parameters perturbances, the environment disturbances, and unmodeled dynamics, are estimated by the constructed reduced-order ESOs, the state constraints are addressed by using the barrier Lyapunov function, and the input saturation is compensated by the anti-windup compensator.

2. Preliminaries

2.1 Underactuated AUV model

Based on the previous study [7], the kinematic model is described as:

\[
\begin{align*}
\dot{x} &= u \cos \psi - v \sin \psi \\
\dot{y} &= u \sin \psi + v \cos \psi \\
\dot{\psi} &= r
\end{align*}
\]  

(1)

Referring the previous study [8], the dynamics of an underactuated AUV in the horizontal plane is modified as:

\[
\begin{align*}
\dot{u} &= \frac{m_{22}}{m_{11}} v r - \frac{X_u}{m_{11}} u - \frac{X_v}{m_{11}} v + \frac{\tau_u}{m_{11}} + d_u \\
\dot{v} &= -\frac{m_{11}}{m_{22}} u r - \frac{Y_u}{m_{22}} u - \frac{Y_v}{m_{22}} v + \frac{\tau_v}{m_{22}} + d_v \\
\dot{r} &= \frac{m_{11} - m_{22}}{m_{33}} u v - \frac{N_u}{m_{33}} u r - \frac{N_v}{m_{33}} v r + \frac{\tau_r}{m_{33}} + d_r
\end{align*}
\]  

(2)

Taking into account the effect of input saturation, the saturated control inputs are defined as:

\[
\begin{align*}
\tau_i &= \begin{cases} 
\tau_{i \text{max}}, & \tau_{i0} > \tau_{i \text{max}} \\
\tau_{i0}, & \tau_{i \text{min}} \leq \tau_{i0} \leq \tau_{i \text{max}}, \ i = u, r \\
\tau_{i \text{min}}, & \tau_{i0} < \tau_{i \text{min}}
\end{cases}
\end{align*}
\]  

(3)

2.2 Path-following error dynamics

Referring the previous study [8], the path-following error dynamics is represented as:

\[
\begin{align*}
\dot{\psi}_e &= -\dot{s} \left(1 - c_c \dot{\psi}_e\right) + v_c \cos \psi_e \\
\dot{\psi}_e &= -c_c \dot{s} \dot{\psi}_e + v_c \sin \psi_e \\
\psi_e &= r + \beta - c_c \dot{s}
\end{align*}
\]  

(4)

In order to overcome the dependency on an accurate model, the unknown time-varying side-slip angular velocity in the path-following error dynamics is regarded as the kinematic uncertainty, and the reduced-order ESO is utilized to estimate it. Therefore, the third equation of the error dynamics is modified as:

\[
\hat{\psi}_e = r + d_{\psi} - c_c \dot{s}
\]  

(5)

The line-of sight (LOS) law is given by:

\[
\psi_{los} = -\tan^{-1}\left(\frac{\psi_e}{\Delta}\right)
\]  

(6)
2.3 Problem formulation
The horizontal path-following problem can be described as follows:
Considering an underactuated AUV governed by Eqs. (1) and (2), design a robust controller for the control inputs, so that the underactuated AUV can follow a predefined path under the given desired speed.

3. Path-following controller design
3.1 The reduced-order ESO design
In this section, the reduced-order ESOs are designed to estimate the kinematic and dynamic uncertainties. Four reduced-order ESOs are given by:

\[
\begin{align*}
\dot{p}_1 &= -\omega_1 p_1 - \omega_1^2 u - \omega_1 \left[ \frac{m_{22}}{m_{11}} u r - \frac{X_u}{m_{11}} u |u| + \tau_u \right] \\
\dot{d}_u &= \omega_1 u + p_1, \quad \omega_1 > 0 \\
\dot{p}_2 &= -\omega_2 p_2 - \omega_2^2 v - \omega_2 \left( \frac{m_{22}}{m_{22}} u r - \frac{Y_v}{m_{22}} v |v| \right) \\
\dot{d}_v &= \omega_2 v + p_2, \quad \omega_2 > 0 \\
\dot{p}_3 &= -\omega_3 p_3 - \omega_3^2 r - \omega_3 \left[ \frac{m_{22}}{m_{33}} u v - \frac{N_r}{m_{33}} r |r| + \tau_r \right] \\
\dot{d}_q &= \omega_3 q + p_3, \quad \omega_3 > 0 \\
\dot{p}_4 &= -\omega_4 p_4 - \omega_4^2 \psi_e - \omega_4 \left( r + c_r s \right) \\
\dot{d}_\psi &= \omega_4 \psi_e + p_4, \quad \omega_4 > 0
\end{align*}
\]
3.2 Kinematic controller design

3.2.1 Attitude control law design
Consider the following Lyapunov function candidate:
\[ V_1 = \frac{1}{2} (\psi_e - \psi_{los})^2 \]  
(8)

The derivative of Eq. (8) is given by:
\[ \dot{V}_1 = (\psi_e - \psi_{los})(r + \beta - c_e(s)\dot{s} - \psi_{los}) \]  
(9)

With the estimated value of kinematic uncertainty \( \hat{d}_e \), the desired yaw velocity is designed as:
\[ r_d = c_e(s)\dot{s} - \dot{\psi}_e + \psi_{los} - k_1(\psi_e - \psi_{los}) \]  
(10)

3.2.2 The virtual target movement control law design
To address the state constraints, choose the following barrier Lyapunov function:
\[ V_{xy} = \frac{1}{2}((c_0 - c_2)x_v^2 + \sin x_v^2 - (c_1 - c_2)x_v^2y_v) \]  
(11)

Differentiating the Eq. (11), we have:
\[ \dot{V}_{xy} = -x_v(c_1\dot{s} - \dot{\psi}_e \cos \psi_e + c_1x_v c_0 y_v - c_2x_v c_0 y_v) + c_1y_v \dot{\psi}_e \sin \psi_e \]  
(12)

The virtual target movement control law is designed as:
\[ \dot{s} = \frac{k_2}{c_1} x_v + \dot{\psi}_e \cos \psi_e - (c_1 - c_2)x_v c_0 y_v \]  
(13)

where \( c_1 = (\frac{1}{k_{c1} - x_v^2} + 1) \), \( c_2 = (\frac{1}{k_{c2} - y_v^2} + 1) \).

3.3 Dynamic controller design

3.3.1 Yaw angular velocity control law design
Choose the following Lyapunov function candidate:
\[ V_3 = k_4 V_1 + \frac{1}{2} (r - r_d)^2 + \frac{1}{2} X_1^2 \]  
(14)

In order to address the effect of the input saturation, an anti-windup compensator is employed as:
\[ \dot{X}_1 = \begin{cases} -k_{X_1} X_1 + \frac{[r - r_d] \Delta \tau_r}{X_1^2} + 0.5 \gamma^2 \tau_r^2 X_1 + \gamma_1 \Delta \tau_r, & |X_1| > \overline{X} \\ 0, & |X_1| \leq \overline{X} \end{cases} \]  
(15)

Differentiating Eq. (15), we have:
\[ \dot{V}_3 = k_4 \left[ (\psi_e - \psi_{los})(r + r_d + \beta - c_e(s)\dot{s} - \psi_{los}) \right] + \frac{[m_{11} - m_{12} N_r - m_{13} - m_{14}]}{m_{33} N_r} [r_{c} + \Delta \tau_r + d_r - \tau_{r0}] \]  
\[ + X_1 \left[ -r_{c0} X_1 + 0.5 \gamma^2 \tau_r^2 X_1 + \gamma_1 \Delta \tau_r \right] \]  
(16)

With the estimated value of dynamic uncertainty \( \hat{d}_r \), design the control moment \( r_{r0} \) as:
\[ r_{r0} = m_r \left[ \hat{d}_r - k_3 (r - r_d) - k_4 (\psi_e - \psi_{los}) + k_3 X_1 \right] \left[ \frac{m_{11} - m_{12} N_r}{m_{33}} + N_r \frac{N_r}{m_{33}} - \frac{N_r}{m_{33}} \right] - \hat{d}_r \]  
(17)

In order to overcome the problem of “explosion of complexity” inherent in the traditional backstepping method, the following NTD is utilized to obtain the derivative of the desired yaw speed [9]:
\[
\begin{align*}
fh &= \tanh(r_c(k) - r_d(k), \dot{r}_c(k), R, h) \\
\dot{r}_c(k+1) &= r_c(k) + T\cdot \dot{r}_c(k) \\
\dot{r}_c(k+1) &= \dot{r}_c(k) + T\cdot fh
\end{align*}
\]

Therefore, the control moment \( \tau_{r_0} \) is modified as:

\[
\tau_{r_0} = m_r \left[ \dot{r}_c - k_3(r - r_c) - k_4(\psi_e - \psi_{\text{los}}) - k_3 \dot{x}_1 - \frac{m_{11} - m_{22}}{m_{33}} u v - \frac{N_r}{m_{33}} + \frac{N \rho l_r}{m_{33}} p + \dot{d}_e \right]
\]

3.3.2 Surge velocity control law design

Consider the following Lyapunov function candidate:

\[
V_4 = \frac{1}{2}(u - u_d)^2 + \frac{1}{2} \Delta^2
\]

Similar to the design of yaw angular velocity control law, an anti-windup compensator is employed as:

\[
\dot{x}_2 = \begin{cases} 
-k_x, & x_2 > \bar{X} \\
0, & |x_2| \leq \bar{X} \\
-k_x, & x_2 < -\bar{X}
\end{cases}
\]

Differentiating Eq. (21), we have:

\[
\dot{V}_4 = m_r \left[ \dot{u}_d - k_5 (u - u_d) + \frac{X_u}{m_{11}} u + \frac{X_u}{m_{11}} u_d + \frac{X_u}{m_{11}} u + \frac{X_u}{m_{11}} u_d - \dot{d}_u \right]
\]

4. Simulation results

In this section, numerical simulations are performed to validate the effectiveness of the proposed controller, and the parameters of the underactuated AUV are the same as in [8]. The parameters of the proposed controller are chosen as: \( k_1 = 10 \), \( k_2 = 5 \), \( k_3 = 5 \), \( k_4 = 2 \), \( k_5 = 3 \). The desired path is parameterized by \( x_F(\mu) = \sum_{i=0}^{4} a_i \mu^i \) and \( y_F(\mu) = \sum_{i=0}^{4} b_i \mu^i \), and the corresponding path parameters are shown in [10]. The multiple uncertainties are generated as: \( d_u = 0.1 \sin(0.1 t) \), \( d_v = 0.1 \sin(0.05 t) \), \( d_e = 0.1 \cos(0.1 t) \).

The path-following results are shown in Fig. 2(a)-(d). In Fig. 2(a), the controller can force the underactuated AUV to accurately converge to the desired path under the multiple uncertainties. The velocity evolutions and control inputs are drawn in Fig. 2(b) and Fig. 2(c), respectively. It is obvious that the surge speed can converge to the given desired speed, and the control inputs are limited in a valid rage. The path-following errors are shown in Fig. 2(d). It is evident that the tracking errors quickly tend to narrow field near zero.
Fig. 2. (a) path-following performance. (b) velocity evolutions. (c) control inputs. (d) path-following errors.

5. Conclusion
In this paper, a novel controller is proposed to address the path-following problem of an underactuated AUV subject to multiple uncertainties and state constraints. Four reduced-order ESOs are utilized to estimate the kinematic and dynamic uncertainties, the barrier Lyapunov function is developed to address state constraints, and the auxiliary design system is adopted to eliminate the effect of input saturation.

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