Mixed Symmetry States for even-even $^{218-224}$Ra Isotopes by using (IBM-2)

Mohammed Abdul Kadhim Hadi Al-Sadi$^1$, Ali Hussein F.Alnasraui$^2$
Dhay Alaa Albttbiey$^3$

$^1$Environment Pollution Department, Collage of Environment, Al-Qasim Green University
$^2$College of Biotechnology, Al-Qasim Green University
Email: Mohammed1986@environ.uoqasim.edu.iq
Email: Ali.hussein@biotech.uoqasim.edu.iq

Abstract
In present investigation we employ IBM-2 parameters in Hamiltonian operator and applying the NPBOS bundle to determine mixed symmetry states for even-even $^{218-224}$Ra isotopes, in addition to extract some theoretical nuclear properties, one of which the excited energy levels, which gave best match between practical data and theoretical results specially for the ground state bands, and acceptable agreement for semi β bands and semi γ bands. The (MSS) property which gets a speedy reaction while changing majorana term $\delta^2$ due to mixture between wave function for protons and neutrons was diagnosed for levels Energy ratios were calculated between and which shows these isotopes have a varied dynamic stretch starting from their pure vibrational state at $A = 218$, transition state U (5) -O (6) at $A = 220$ and finally transition state O (6)-SU (3), when $A = 222$ And 224, respectively. Finally, determined the likelihood for electric quadruple transition $B(E2)$, magnetic dipole $B(M1)$, which All express a decent agreement with the practical data.

Keywords: Hamiltonian operator, mixed symmetry states, IBM-2, electric quadruple transition $B(E2)$, magnetic dipole $B(M1)$

1. Introduction
Heavy nuclei are unstable nuclei with low nuclear binding energy because they contain large numbers of nucleons. This instability makes them distorted nuclei, irregular in shape, lacking the spherical shape[1]. The majority of known nuclei are unstable, since the analysis of the radiation spectra emitted by the nucleus gave a large amount of information about the nucleus of the atom. The evolution of the properties of the nucleus and its proton and neutron numbers is an important question in nuclear physics and its study is the most important objectives of modern physics in the study of nuclear structure. In the absence of a comprehensive theory of nuclear structure, many attempts have been made to understand the nature of structure by a number of nuclear models that have been proposed to describe the interaction between nucleons within the nucleus. These nuclear models
include the liquid drop model, the shell model and the collective model. According to the concept of liquid drop (liquid drop) when the nucleus is exerted by external pressure in addition to that the excitation of the nucleus can be seen as surface vibrations or rotational movement and as a result the surface can make vibrations (Vibrations)[2]. The shell model succeeded in explaining the phenomenon of magic numbers, in which the nucleus is highly stable, as well as determining the angular momentum of the energy levels of the nucleus. However, this model could not explain the quadruple Moments of some nuclei that contain nucleons outside the enclosed shell or even marital-even nuclei in which the terrestrial nuclear spin is always zero. In the collective nuclear model, the fundamental probability is that nucleons within the nucleus exert a centrifugal pressure on the surface of the nucleus. As a result, the nucleus can be deformed and take a non-spherical shape. Therefore, the researchers (Arima and Iachello) proposed a new nuclear model called the model of reactive bosons and one of its objectives is to describe the vibrational and rotational spectra and the mean states of the nuclei based on group symmetries (Group Symmetries) instead of the geometric description of the variables of the shape of the nucleus. The first interacting bosons model IBM-2 does not distinguish between the proton wave function and the neutron wave function, A second model IBM-2, which distinguishes between protons bosons and neutrons bosons, has been released. It contains more information about the properties of energy levels and determining the shape of the nucleus. This gives him special practical significance. Its use makes it possible to know the unknown nuclear spectrum, that will be adopted in our current work [3]. In our current work we present a computational study in the field of nuclear structure of Radium isotope heavy with proton boson number Nπ=3 and a varying neutron bosons Nv between (2-5). The concept of symmetry is one of the most important concepts in the nuclear structure, which should be determined precisely because the shape of the nucleus has a fundamental relationship in the determination of nuclear properties such as low-lying of the excited energy levels, the probability of electromagnetic transition, mixed symmetry states, which is the goal of our current study.

2. The Interacting Proton and Neutron Boson Model

The Hamiltonian in IBM-2 is define as:

\[ H = \sum \text{energy of d-bosons} + \text{number of d-bosons} \times \text{quadruple-quadruple interaction between proton and neutron} \times \text{coefficient expresses the strength interaction between them} \]

Where \( \rho \) correspond to \( \pi \) (proton) or \( \nu \) (neutron) bosons, is the quadruple - quadruple interaction between proton and neutron and is a coefficient expresses the strength interaction between them. And the quadruple operator is defined as:[4]

\[ -\sum \]

The third and fourth term which correspond to the interaction between like boson, Which define as:[5]

\[ -\sum \]

A standout amongst those characteristics from claiming degrees of freedom, may be on investigation the blended symmetry states, when mixture the middle of those wave capacities for Proton Also neutron, to IBM-2 those majorana haul incorporates three parameters are \( \xi_1, \xi_2 \) Furthermore \( \xi_3 \) used to consider blended symmetry states, be composed as: [6]

\[ -\sum \]

3. Results and Discussion

Isotope behaviour is determined in our current work, based on the values of practical energy levels and compared with the theoretical nuclear spectra calculated (ideal) by the IBM2 program, and a comparison of low-lying excited energy levels, (positive parity) after calculating the energy ratios between and Show that these isotopes have a varied dynamic stretch starting from
their pure vibrational state at $A = 218$, transition state $U(5)$ - $O(6)$ at $A = 220$ and finally transition state $O(6)$ - $SU(3)$, when $A = 222$ and $224$, respectively. As shown in Figure 1.

**Figure 1.** Comparing theoretical results with practical values of the energy ratios between and states.

To think about the energy levels of $^{218-224}$Ra isotopes, we have to gauge the parameters utilized in (IBM-2), by applying the NPBOS bundle, the fitted estimations of these parameters are recorded in

| Isotopes | $N_v$ | $N_\pi$ | $\epsilon$ | $\kappa$ | $\chi_\pi$ | $\chi_\nu$ | $C_{l_v=0,2,4}$ | $C_{l_\pi=0,2,4}$ | $\zeta_2$ | $\zeta_1=3$ |
|----------|-------|---------|------------|----------|------------|------------|----------------|----------------|----------|------------|
| $^{218}$Ra | 2     |         | 0.390     | -0.010   | -0.800     | 0.0,0      | 0.0,0         | -0.100          |          | -0.100     |
| $^{220}$Ra | 3     |         | 0.260     | -0.022   | -0.900     | 0.0,0      | 0.0,0         | -0.120          |          | -0.100     |
| $^{222}$Ra | 4     | 3       | 0.215     | -0.099   | -0.950     | 0.0,0      | 0.0,0         | -0.140          |          | -0.180     |
| $^{224}$Ra | 5     |         | 0.200     | -0.091   | -0.990     | 0.0,0      | 0.0,0         | -0.180          |          | -0.180     |

**Table 1.** IBM-2 Parameter for even-even $^{218-224}$Ra isotopes

It was seen that the energy of bosons ($\epsilon$) diminishes with the quantity of neutrons, on the grounds that the bosons are affected by a close shell. The parameter diminishes relatively with the expansion in the quantity of neutrons relying upon the response power of the electric quadrant of protons and neutrons, and the worth (expanding) consistently increments directly as the quantity of neutrons increments. Majorana parameter wording esteems ($\delta_1=\delta_3$) were selected for specific values and adjustable with the study of mixed symmetry (MSS) for excited energy levels. The estimations of $\chi_\pi = -1.20$ and $\delta_1=\delta_3=-0.100$ are fixed for all isotopes of $^{2018-224}$Ra. Additionally, normality in the estimation of isotope parameters is considered by the development of the main energy level of every isotope, and this is obvious from the striking similarity between the energy of the boson and the principal excited energy level, where both decreases as the number of $N$ bosons decreases as shown in Figure 2.
Figure 2. The relationship between energy level, practical and theoretical with energy of bosons

The best match between experimental data and theoretical results was achieved for the ground state bands, while the semi β bands, semi γ bands it was found to have acceptable agreement. Figures from 1 along with 4 shows an comparison between the experimental data energy level and hypothetical outcome in IBM-2.

Figure 3. Comparison between energy level outcome in IBM-2 and experimental data for all studied Radium isotopes. (218Ra = A, 220Ra = B, 222Ra = C, 224Ra = D)
4. Mixed symmetry states (MSS)

The Majorana expression assumes a great role when study the mixed symmetry states (MSS) of the some excited energy level. MSS happen when the movements of the proton and neutrons are not in stage in the quantum state [9]. These states were made by blend of two wave function, one for proton and other for neutron [13]. The MSS dictated by the F-spin formalism, where the F-spin formalism is closely resembling the isospin quantum number of the nucleons. Proton bosons and neutron bosons have $F = 1/2$ and $z$-projection, where $F_z = +1/2$, -1/2 for protons and neutrons individually. For a framework comprise of $N_p$ proton boson and $N_n$ neutron boson, the most extreme F-turn is $F = F_{\text{max}} = (N_p + N_n)/2$, while the blended balance states described by diminishing F-turn esteem ($F = F_{\text{max}}$, $F = F_{\text{max}} - 1$, $F = F_{\text{max}} - 2$, ..., $F = F_{\text{min}} = |N_p - N_n|/2$). A state created by $N_p$ proton bosons and $N_n$ neutron bosons with F-turn quantum number $F = F_{\text{max}}$ can be changed by the progressive activity of the F-turn raising administrator $F^+$ into an express that comprises of proton bosons as it were. This state has still an all-out F-turn quantum number $F = F_{\text{max}}$ since the raising administrator doesn't change the all-out F-turn quantum number. This new state has just proton bosons and clearly remains unaltered under a pairwise trade of proton and neutron names. In this way, IBM-2 states with $F = F_{\text{max}}$ are called Full Symmetry States (FSS). These states relate really to the IBM-1 states which are largely symmetric. All others states with F-turn quantum numbers $F < F_{\text{max}}$ contain sets (in any event one) of proton and neutron bosons that are hostile to symmetric under a pairwise trade of protons and neutrons marks. They are called Mixed-Symmetry States(MSS) [14]. The general F-turn choice guideline is $\Delta F = 0, \pm 1$. Other than, the M1 change gather between two absolutely symmetric states ($F = F_{\text{max}}$) are prohibited, therefor M1 progress happen between low-lying aggregate states, these states must contain segments with blended evenness state, when ($F < F_{\text{max}}$), this permits to utilize the quality of M1 change between low-lying aggregate state and F-spin blending [14-15]. At the point when study the impact of Majorana parameter ($\delta_1,3, \delta_2$) on the determined excitation vitality level for 218-224Ra isotopes, we fixed the estimation of $\delta_1,3 = -0.100$ for all isotopes and fluctuate the $\delta_2$, around the best-fitted. It is discovered that vitality estimations of the state are reacted quickly to the progressions of the $\delta_2$ parameters in certain isotopes just, in this way these states are checked the principal property of the Mixed balance state (MSS). Figure:2 clarify the vitality variety of these state as an element of the Majorana parameter $\delta_2$. 

![Energy levels comparison](image-url)
5. Electromagnetic transition

5.1 Electric quadruple

The E2 transition operator is given by [10,11].

\[ \langle \text{transition} \rangle = e_\pi N_\rho e_\nu \]

where \( e_\pi \) and \( e_\nu \) are boson effective charges, contingent upon the boson number \( N_\rho \), those parameters are free and can take any an incentive to suit the exploratory information. In this work, the viable charge of proton \( e_\pi = 0.45 \) (eb) and neutron \( e_\nu = 0.22 \) (eb) got from test information of.

The determined diminished electric quadrupole probabilities \( B(E2) \) of \( ^{218-224}\text{Ra} \) isotopes are demonstrated inside the table:2. The IBM-2 result are appropriate in comparison to the available experimental values and they appearance to have a decent precise.

| Transition       | \(^{218}\text{Ra}\) Exp. | \(^{218}\text{Ra}\) IBM-2 | \(^{220}\text{Ra}\) Exp. | \(^{220}\text{Ra}\) IBM-2 | \(^{222}\text{Ra}\) Exp. | \(^{222}\text{Ra}\) IBM-2 | \(^{224}\text{Ra}\) Exp. | \(^{224}\text{Ra}\) IBM-2 |
|------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| \( 2_1^+ \rightarrow 0_1^+ \) | 0.1989 | 0.1932 | 0.9466 | 0.9220 | | | | |
| \( 0_2^+ \rightarrow 2_1^+ \) | --- | 0.0015 | 0.8828 | 0.9137 | | | | |
| \( 2_2^+ \rightarrow 2_1^+ \) | --- | 1.1666 | --- | 0.9121 | | | | |
| \( 2_2^+ \rightarrow 2_3^+ \) | --- | 0.0053 | --- | 0.2904 | | | | |
| \( 2_2^+ \rightarrow 2_1^+ \) | --- | 0.2215 | --- | 0.0134 | | | | |
| \( 2_1^+ \rightarrow 2_5^+ \) | --- | 0.0100 | --- | 0.0101 | | | | |
| \( 3_1^+ \rightarrow 2_2^+ \) | --- | 0.2146 | --- | 0.9782 | | | | |
| \( 3_2^+ \rightarrow 2_1^+ \) | --- | 0.0556 | --- | 0.0001 | | | | |
| \( 4_1^+ \rightarrow 2_1^+ \) | 0.4919 | 0.4985 | --- | 1.4337 | | | | |
| \( 5_1^+ \rightarrow 3_1^+ \) | --- | 0.5414 | --- | 0.7654 | | | | |
| \( 6_1^+ \rightarrow 4_1^+ \) | 0.3593 | 0.3884 | --- | 1.6357 | | | | |
| \( 8_1^+ \rightarrow 6_1^+ \) | 0.7037 | 0.7872 | --- | 1.5554 | | | | |
| \( 10_1^+ \rightarrow 8_1^+ \) | 0.4456 | 0.4153 | --- | 1.5305 | | | | |
5.2 Magnetic dipole

To find M1 transition probability, one has to appraise the effective g-factor for proton and neutron. These transactions are obtained through the relationship of Sambataro defined

where are -factors for proton and neutron respectively. g factor related with magnetic momentum, is \( \mu = 2g \), and in this work we utilize practical data of magnetic momentum for the first excites state to gauge the g factor, where seen that the foreseen qualities are \( g_\pi = 0.752(\mu\text{N}) \) and \( g_\nu = 0.236(\mu\text{N}) \), and \( (g_\pi - g_\nu) = 0.516(\mu\text{N}) \). The M1 operator is gotten by making \( l = 1 \) in the single boson operator of the IBM-2 and it is written as follows [17]:

The determined qualities for B(M1) is suitable to some extent comparing with practical data which are very few in practical nuclear data. The B(M1) transition of \( ^{218-224}\text{Ra} \) isotopes was determined and introduced in Table (3).

| Transition | \( ^{222}\text{Ra} \) | \( ^{224}\text{Ra} \) |
|------------|------------------|------------------|
| \( 2^+_1 \rightarrow 0^+_1 \) | 0.8623 | 0.8935 |
| \( 0^+_2 \rightarrow 2^+_1 \) | 0.4917 | 0.4313 |
| \( 2^+_2 \rightarrow 2^+_1 \) | 0.3427 | 0.0049 |
| \( 2^+_2 \rightarrow 2^+_1 \) | 0.1594 | 0.0006 |
| \( 3^+_1 \rightarrow 2^+_2 \) | 0.6209 | 1.3971 |
| \( 3^+_2 \rightarrow 2^+_1 \) | 0.2620 | 0.0013 |
| \( 4^+_1 \rightarrow 2^+_1 \) | 0.0958 | 0.0996 |
| \( 5^+_1 \rightarrow 3^+_1 \) | 1.2226 | 1.1184 |
| \( 6^+_1 \rightarrow 4^+_1 \) | 2.1721 | 2.4480 |
| \( 8^+_1 \rightarrow 6^+_1 \) | 2.0260 | 0.5060 |

| Transition | \( ^{222}\text{Ra} \) | \( ^{224}\text{Ra} \) |
|------------|------------------|------------------|
| \( 2^+_1 \rightarrow 0^+_1 \) | 0.13752 | 0.14236 |
| \( 2^+_2 \rightarrow 2^+_1 \) | 0.00001 | 0.00007 |
| \( 2^+_2 \rightarrow 2^+_1 \) | 0.00025 | 0.00106 |
| \( 2^+_2 \rightarrow 2^+_4 \) | 0.00014 | 0.00010 |
| \( 2^+_2 \rightarrow 2^+_2 \) | 0.00183 | 0.00009 |
| \( 3^+_1 \rightarrow 2^+_2 \) | 0.00051 | 0.00065 |
| \( 5^+_1 \rightarrow 4^+_2 \) | 0.00067 | 0.00079 |
6. Conclusion
In present work we have determine the mixed symmetry states, where, it is found that the (MSS) property which gets a speedy reaction while changing Majorana term δ2 for states ( . Also the ratios for the ground state bands were calculated between ( and ), observed that these isotopes have a varied dynamic stretch starting from their pure vibrational state at A = 218, transition state U (5) - O (6) at A = 220 and finally transition state O (6)-SU (3), when A = 222 And 224, respectively. Finally, determined the likelihood for electric quadruple transition B(E2), magnetic dipole B(M1), which All express a decent agreement with the practical data

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