A gravitational mechanism which gives mass to the vectorial bosons

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(Dated: April 20, 2010)

A mechanism in which gravity is the responsible for the generation of the mass of scalar bosons and fermions was proposed recently [1]. The purpose of the present paper is to extend this scheme to vector bosons. The main characteristic of this mechanism is that the gravitational field acts really as a catalyzer, once the final expression of the mass we obtained depends neither on the intensity of the gravitational field nor on the value of Newton’s constant.

PACS numbers:

I. INTRODUCTION

In the realm of high-energy physics, the Higgs model produced an efficient scenario for generating mass to the vector bosons [2]. At its origin appears a process relating the transformation of a scalar field \( \varphi \) that appears as the vehicle which provides mass to the gauge vector field \( W_\mu \). A particular form of self-interaction of \( \varphi \) allows the existence of a constant value \( V(\varphi_0) \) that is directly related to the mass of \( W_\mu \). For the mass to be a real and constant value (a different value for each field) the scalar field must be in a minimum state of its potential. This fundamental state of the self-interacting scalar field has an energy distribution described as

\[
T_{\mu \nu} = V(\varphi_0) g_{\mu \nu}
\]

defining a cosmological constant of dimension \( \text{length}^{-2} \) given by \( \Lambda = \kappa V(\varphi_0) \) that produces a particular configuration of the geometry of space-time through the equations of general relativity.

However, in the Higgs mechanism, this fact is not further analyzed, once at that level, gravity is ignored. The reason for this is attributed to the smallness of Newton’s constant \( \kappa = 8\pi G_N / c^4 \). In the present paper, we shall argue in the opposite way and will present a mechanism in which gravity is the true responsible for the generation of the mass. The final expression of the mass depends neither on the intensity of the gravitational field nor on the value of Newton’s constant.

In our model we use a slight modification of Mach principle. We start by recalling Mach’s idea as the statement according to which the inertial properties of a body \( A \) are determined by the energy-momentum throughout all space. The simplest way to implement this idea is to consider the state that takes into account the whole contribution of the rest-of-the-universe onto \( A \) as the most homogeneous one. Thus it is natural to relate it to what Einstein attributed to the cosmological constant or, in modern language, the vacuum of all remaining bodies. This means to describe the energy-momentum distribution of all complementary bodies of \( A \) in the Universe

\[
T_{\mu \nu}^A = \frac{\lambda}{\kappa} g_{\mu \nu}
\]

where \( \lambda \) is a constant of dimensionality \( \text{length}^{-2} \). In this work the body \( A \) will be identified with a vector field interacting only gravitationally. Although we borrow the idea of the rest-of-the-universe from Mach’s principle, and use its correspondent form of energy-momentum distribution characterized by a constant \( \lambda \), as in [1], one should not identify naively this \( \lambda \) with the cosmological constant obtained from actual observations on cosmology. In our scenario the rest-of-the-universe should not be identified with an unique structure, but instead it designates the environment of \( A \) or, in other words, all the remaining bodies in the universe that can have an influence on \( A \). We synthesize the overall effect of this structure on \( A \) as an attribute of \( \lambda \).

II. THE VECTOR FIELD

We start with a scenario in which there are only three ingredients: a vector field, the gravitational field and an homogeneous distribution of energy - that is identified with the vacuum. The theory is specified by the Lagrangian (we use units \( \hbar = c = 1 \))

\[
L = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{\kappa} R - \frac{\lambda}{\kappa}
\]

The corresponding equations of motion are

\[
F_{\mu \nu} ; \nu = 0
\]

and

\[
\alpha_0 ( R_{\mu \nu} - \frac{1}{2} R g_{\mu \nu} ) = - T_{\mu \nu}
\]
where \( F_{\mu \nu} = \nabla_\nu W_\mu - \nabla_\mu W_\nu \) and, for graphical simplicity, we set \( \alpha_0 = 2/\kappa \). In this theory, the vacuum \( \lambda \) is invisible for \( W_\mu \). The energy distribution represented by \( \lambda \) interacts with the vector field only indirectly once it modifies the geometry of space-time. In HM this vacuum is associated to a fundamental state of a scalar field \( \phi \) and it is transformed in a mass term for \( W_\mu \). The field \( \phi \) has not only this intermediary role. Its function goes far beyond this: it provides the vacuum energy. Indeed, in HM, one identifies \( \lambda \) with the value of the potential \( V(\phi) \) in its homogeneous state. To be able to realize this, we modify the above Lagrangian to enterprize a real scalar field must couple non-minimally with the vector field.

So much for a well-known scheme. Let us turn now to a new one. The first step is to answer the following question: is it really necessary to invoke a new field \( \phi \) to act as a bridge between the rest-of-the-universe vacuum and the vector field? We claim that this is not necessary and we shall demonstrate this using the universal character of gravitational interaction to generates mass for \( W_\mu \) without introducing any extra field.

The point of departure is the recognition that gravity may be the real responsible for breaking the gauge symmetry. For this, we modify the above Lagrangian to represent non-minimal coupling of the field \( W_\mu \) and gravity in order to explicitly break such invariance. There are only two possible ways for this \(^3\) and the total Lagrangian must be of the form

\[
\mathcal{L} = - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{\kappa} R + \frac{\sigma}{6} R \Phi + \sigma \mu_\nu W^\mu W^\nu - \frac{\lambda}{\kappa}
\]

where we set

\[
\Phi \equiv W_\mu W^\mu.
\]

The first two terms of \( \mathcal{L} \) represents the free part of the vector and the gravitational fields. The second line represents the non-minimal coupling interaction of the vector field with gravity. The parameter \( \sigma \) is dimensionless. The vacuum – represented by \( \lambda \) – is added by the reasons presented above and it must be understood as the definition of the expression “the influence of the rest-of-the-universe on \( W_\mu \)”. We will not make any further hypothesis on this \(^3\).

In the present proposed mechanism, such \( \lambda \) is the real responsible to provide mass for the vector field. This means that if we set \( \lambda = 0 \) the mass of \( W_\mu \) will vanish.

Independent variation of \( W_\mu \) and \( g_{\mu \nu} \) yields

\[
F_{\mu \nu} + \frac{\sigma}{3} R W^\nu + 2\sigma R_{\mu \nu} W^\nu = 0 \tag{4}
\]

\[
\alpha_0 (R_{\mu \nu} - \frac{1}{2} R g_{\mu \nu}) = - T_{\mu \nu} \tag{5}
\]

The energy-momentum tensor defined by

\[
T_{\mu \nu} = \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} L)}{\delta g^{\mu \nu}}
\]

is given by

\[
T_{\mu \nu} = E_{\mu \nu} + \frac{\sigma}{3} \nabla_\mu \nabla_\nu \Phi - \frac{\sigma}{3} \Box \Phi g_{\mu \nu} + \frac{\sigma}{3} \Phi (R_{\mu \nu} - \frac{1}{2} R g_{\mu \nu})
\]

\[
+ \frac{\sigma}{3} R W_\mu W_\nu + 2\sigma R_{\mu \nu} W_\lambda W^\lambda + 2\sigma R_{\mu \nu} W_\lambda W^\lambda
\]

\[
- \sigma R_{\alpha \beta} W^\alpha W^\beta g_{\mu \nu} - \sigma \nabla_\alpha \nabla_\beta (W^\alpha W^\beta) g_{\mu \nu}
\]

\[
+ \sigma \nabla_\mu \nabla_\beta (W_\mu W^\beta) + \sigma \nabla_\mu \nabla_\beta (W_{\nu} W^\beta)
\]

\[
+ \sigma \Box (W_\mu W_\nu) + \frac{1}{\kappa} \lambda g_{\mu \nu} \tag{6}
\]

Taking the trace of equation (5) we obtain

\[
R = 2 \lambda - \kappa \sigma \nabla_\alpha \nabla_\beta (W^\alpha W^\beta) \tag{7}
\]

Then, using this result back into equation (4) it follows

\[
F^{\mu \nu} + \frac{2 \sigma \lambda}{3} W^\mu
\]

\[- \frac{\kappa \sigma^2}{3} \nabla_\alpha \nabla_\beta (W^\alpha W^\beta) W^\mu
\]

\[+ 2 \sigma R_{\mu \nu} W^\nu = 0 \tag{8}
\]

The non-minimal coupling with gravity yields an effective self-interaction of the vector field and a term that represents its direct interaction with the curvature of space-time. Besides, as a result of this process the vector field acquires a mass \( \mu \) that depends on the constant \( \sigma \) and on the existence of \( \lambda \) given by

\[
\mu^2 = \frac{2}{3} \sigma \lambda \tag{9}
\]

Note that the Newton’s constant does not appear in our formula for the mass. If \( \lambda \) vanishes then the mass of the field vanishes. This is precisely what we envisaged to obtain: the net effect of the non-minimal coupling of gravity with \( W_\mu \) corresponds to a specific self-interaction of the vector field. The mass of the field appears only if we take into account the existence of the rest-of-the-universe — represented by \( \lambda \) — in the state in which this environment is on the corresponding vacuum. The values of different masses for different fields are contemplated in the parameter \( \sigma \).

### III. ACKNOWLEDGEMENTS

I would like to thank FINEP, CNPq and Faperj for financial support. I would like also to thank J. M. Salim for many enthusiastic conversations on the subject of this paper and A. Dolgov for a critical comment in a previous paper concerning the value of \( \lambda \).
[1] M. Novello: *A mechanism to generate mass: the case of fermions* (arXiv [astro-ph.CO]: 1003.5126v1).

[2] Francis Halzen and Alan D. Martin *Quarks and Leptons: An introductory course in Modern Particle Physics, John Wiley and Sons, 1963* and references therein.

[3] M. Novello and S. E. P. Bergliaffa: *Bouncing Cosmologies, Physics Report* vol 463, n 4 (2008).

[4] There is not any compelling reason to identify this constant with the actual cosmological constant or the value of the critical density $10^{-48} GeV^4$ provided by cosmology.