Multi-area Generation Dispatching Distributed Coordination and Optimization Algorithm

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Abstract. As the fast expansion and interconnection of power system, it makes higher requirements for Optimal Power Flow (OPF) calculation in modern power grid. A distributed coordination and optimization method is presented in this paper. Injection power equivalent of sub-system boundary bus is utilized to solve multi-area dispatching problem. Based on regional decomposition model, it decouples the integrated optimization and avoids complex computation of system parameters equivalent in conventional methods. Besides, it reduces the communication between different regions. The nonlinear primal-dual interior point method is used for optimization in each sub-system, outer coordination of boundary bus's voltage correction is conducted to compute the optimized dispatching in the whole interconnected power system. Numerical simulations are taken in IEEE test systems. The results show that the proposed method is validated, effective and suitable for online application.

Keywords: Generation Dispatching, Distributed Power Flow Computation, Coordination and Optimization, Equivalent Injection Power, Decomposition model.

1. Introduction
With the increasing demand of electricity, the network structure became more and more complicated and the optimization problem multiplied as well. Meanwhile, as the wide area distributed system established, it is difficult for the collection, integration and maintenance of heterogeneous data and parameters. In electric market environment, the information barriers between each dispatching institution further hindered data transferring and sharing especially. Based on decomposition and coordination, the distributed Optimal Power Flow (OPF) can solve the entire network unified simulation and calculation while preserving each dispatch center's calculation independence.

The aim of interconnected power grid distributed OPF is to decompose large scale system optimization problem into a series of small, distributed and coordinated ones [1]. Through boundary bus data exchange of adjacent areas, reference [2] decomposed the overall optimization problem into regions based on part duality principle in DC-OPF model. Reference [3] utilized genetics algorithm (GA) to realize parallel calculation of reactive power optimization, but the inherent disadvantages of GA still exist, like slow searching speed and high dependence of initial population selection. Reference [4] used Lagrangian algorithm to deal with the transmission congestion problem in electric market by relaxing regional coupling constraints into the objective. Auxiliary Problem Principle (APP) is introduced in [5-6] to reduce the computing time in distributed reactive parallel optimization. However, the APP function...
is relatively complicated to realize. The network equivalent technique is applied to multi-area power system reactive power parallel optimization in [7]. For local optimization in sub-system, the operation parameters of adjacent regions are in need, therefore it is not feasible in electric market environment.

Based on the research above, a distributed coordination optimization method is presented in this paper. Power injection equivalent of subnet boundary bus is introduced for multi-area generation dispatching. With a few data exchange of boundary bus between adjacent areas, the proposed method can realize the optimization of whole network. Based on substitution principle for simplified modeling in [8], nonlinear primal dual interior point method is utilized in the optimization of subnet independently. Voltage weighted modification is conducted in outer iteration to update equivalent power injection of subnet boundary bus. The optimization results of generation scheduling can be obtained finally in multi-area interconnected power system.

2. Economic Dispatching Model in Multi-area Interconnected Power System

Power system interconnection is beneficial to optimal allocation of power resources. Dispatching institutions are set up based on the principle of certain generation managements all over the world. Figure 1 shows the system structure of centralized dispatching model.

In this mode, the subnets acquire system operation parameters of its jurisdictional area and upload them to the superior power grid control center for centralized processing (solid arrow lines in Figure 1). After the entire network data integration and optimization calculation, the central control center will send the control commands to regional control centers to complete operation instruction (dotted arrow lines in Figure 1).

The minimum fuel cost of generation scheduling is selected as the optimization objective in the paper. The entire network model can be expressed as follows.

\[
\begin{align*}
\min & \quad \sum_{i=1}^{N} f_i(x_i) \\
\text{s.t.} & \quad h(x) = 0 \\
& \quad g(x) \leq 0
\end{align*}
\]

Where \( f_i(x_i) \) is the objective function of each sub-system. \( h(x) \) is the equality constraint of power flow equations. \( g(x) \) is the inequality constraint including generator active and reactive power, bus voltage amplitude and branch active power.

For the manager and director of power generation, more and more complicated tasks in power grid scheduling institution are undertaken, as the fast expansion of the power system scale. In addition, certain rules of power market become the information barriers between dispatching institutions. That further hinders the internal data sharing. Therefore, a new form of generation scheduling mode is in need nowadays. It is crucial to make it suitable for modern power grid.
3. Multi-area Control Center Decomposition and Coordination

As an important aim of the distributed coordination and optimization method, unified entire network optimization results can be obtained while the calculation in regional control center is independent. In another word, the regional dispatcher can effectively participate in generation scheduling. Therefore, the method proposed in this paper is expected to be effective means for integrated simulation and calculation in large scale interconnected system.

![Multi-area Control Center Decomposition and Coordination](image)

**Figure 2.** Two-area interconnected system.

A two-area interconnected system is shown in Figure 2 to elaborate power system decomposition principle. Sub-systems $S_1$ and $S_2$ are connected by tie lines $l_{ij}$ and $l_{mn}$. As the two terminal endpoints of tie lines, define the bus near inner grid and the one close to outer grid as internal and external boundary bus respectively.

The key point of whole network optimization based on decomposition is to determine the coupling restraints between adjacent regions. The known tie line flows reflect the transmission power between regions. If the calculated tie line power equals to the true value, the interconnected power system can be decomposed by repeated modeling of tie lines in each area. The decomposition model is shown in Figure 3.

![Decomposition model of two-area system](image)

**Figure 3.** Decomposition model of two-area system

Take the tie line power $\bar{S}_{ij}$ for example. According to equivalence principle of area decomposition, it is required that the regional calculated $\bar{S}_{ij}(1)$ and $\bar{S}_{ij}(2)$ must be equal. Obviously, $\bar{S}_{ij}(1)$ is equal to the equivalent injection power of bus/in subnet $S_i$, so as $-\bar{S}_{ij}(2)$ and $\bar{S}_i$ relatively. Namely, while the equivalent injection power of boundary bus equals to its true value, the whole net optimization problem can be decomposed. The regional optimization model of $S_1$ after decomposition can be expressed as followed:

\[
\begin{align*}
\min & \quad f_1(x_1) \\
\text{s.t.} & \quad h(x_1) = 0 \\
& \quad g(x_1) \leq 0
\end{align*}
\]
As the injection power of subnet boundary bus is the function of voltage vector, the active and reactive power of $S_j$ can be written as follows:

$$P_j = P_{ij}(V_i, \theta_i, V_j, \theta_j) \quad (7)$$

$$Q_j = Q_{ij}(V_i, \theta_i, V_j, \theta_j) \quad (8)$$

Therefore, the coupling constraint that the tie line power calculated by adjacent areas equal can be further equivalent to boundary bus voltage vector matching.

The unbalance value between $P_j, Q_j$ and their calculated one can be used to solve the state variables of bus $j$ in sub-system $S_1$, and the state variables of other boundary bus can be obtained similarly as above. Thereafter, the voltage vector as well as the equivalent injection power of boundary bus can be modified in outer iteration.

4. Distributed Coordination and Optimization of Generation Scheduling

4.1. Regional independent optimization

The minimum fuel cost is chosen as the optimization objective. The mathematical model of subnet $S_1$ is derived as follows.

$$\min f(P_{G(1)}, a_{G(1)}) \quad (9)$$

s.t. $\lambda(x_{(1)}) = 0 \quad (10)$

$$P_{G(1)} \leq P_{G(1)} \leq \bar{P}_{G(1)} \quad (11)$$

$$Q_{G(1)} \leq Q_{G(1)} \leq \bar{Q}_{G(1)} \quad (12)$$

$$V_{(1)} \leq V_{(1)} \leq \bar{V}_{(1)} \quad (13)$$

$$P_{l(1)} \leq P_{l(1)} \leq \bar{P}_{l(1)} \quad (14)$$

Where $P_{G(1)}$ is the active power of generator bus in subnet $S_1$ and $a_{G(1)}$ is the corresponding parameter of fuel cost; $x_{(1)}$ is the correlated parameter of equality constraint; $Q_{G(1)}$ is the reactive power of generator bus; $V_{(1)}$ is the voltage amplitude of each bus; $P_{l(1)}$ is the active power of each branch.

The above equality constraint includes power flow equation of internal bus and injection power correction equation of external boundary bus. Take $S_j$ in Figure 3 as an example, its equivalent injection power correction equation can be expressed as follows.

$$\Delta P_j^{(n)} = P_j^{(k)} - P_{ij}(V_i^{(n)}, \theta_i^{(n)}, V_j^{(n)}, \theta_j^{(n)}) = 0 \quad (15)$$

$$\Delta Q_j^{(n)} = Q_j^{(k)} - Q_{ij}(V_i^{(n)}, \theta_i^{(n)}, V_j^{(n)}, \theta_j^{(n)}) = 0 \quad (16)$$

Where $P_j^{(k)}$ and $Q_j^{(k)}$ are calculated by modified boundary bus voltage vector in the $k$ time outer iteration of coordination; $V_i^{(n)}, \theta_i^{(n)}$, $V_j^{(n)}$ and $\theta_j^{(n)}$ are the corresponding parameter in the $n$ time iteration of inner regional optimization.
According to the models above, the system equation can be obtained for nonlinear primal dual interior point method. The coefficient matrix and constant term can be computed to conduct regional optimization. Meanwhile, the voltage vector and active/reactive power of each bus will be updated.

4.2. Voltage weighted correction

Take the boundary bus $i$ in Figure 3 for example. If the voltage vector $V^{(i)}(1)$ and $V^{(i)}(2)$ calculated by each subnet are equal, the optimization calculation ends. Otherwise, voltage weighted modification must be taken. In the early iteration of distributed OPF calculation, the same boundary bus voltage vectors obtained by different subnets are generally unequal. Therefore, outer coordination iteration should be introduced. It is utilized to update boundary bus voltage vector as well as its equivalent power injection which could accurately reflect the change of adjacent area power flow.

In the $k$ time outer iteration, the voltage vector of boundary bus $i$ can be weighted by the following formulas:

$$
V_i^{(k)} = \lambda_i V_i^{(1)} + (1 - \lambda_i) V_i^{(2)} \quad (17)
$$

$$
\theta_i^{(k)} = \lambda_i \theta_i^{(1)} + (1 - \lambda_i) \theta_i^{(2)} \quad (18)
$$

Where $\lambda_i$ and $\lambda_i'$ are the weight coefficients of voltage amplitude and phase angle respectively. The weight coefficient can be obtained from the active and reactive power Thvenin equivalent network in reference [9]. The diagonal elements of nodal admittance matrix are necessary for the formulation of voltage weighted coefficients. They are obtained from boundary data exchange in the preparatory stage.

Reference bus is need in every subnet calculation. The original balance bus is still taken as reference bus in its subnet. As for other sub-systems, one of its $PV$ buses is selected as reference bus in this paper. Due to the differences of reference bus selection, modification must be taken before regional voltage weighted correction. For example, in subnet $S_1$, the voltage phase angle $\theta_{(i),iw}^{(1)}$ transferred from $S_2$ needs to be modified as follows:

$$
\theta_{(i),iw}^{(2)} = \theta_{(i),iw}^{(1)} + \frac{1}{n_{B_{12}}} \sum_{i \in B_{12}} (\theta_{(i),iw}^{(1)} - \theta_{(i),iw}^{(2)}) \quad (19)
$$

Where $B_{12}$ is the same boundary bus set of $S_1$ and $S_2$, $n_{B_{12}}$ is the number of the set; $\theta_{(i),iw}^{(k)}$ is the modified phase angle that can be used for voltage weighted correction in $S_1$.

Through the outer voltage weighted iteration, boundary bus voltages are updated constantly, and the equivalent injection power in (7) and (8) also approach to the true value unceasingly. Finally, the proposed multi-area generation dispatching can be realized.

5. Calculation Steps

The process of multi-area distributed coordination and optimization method can be summarized as follows:

1. Preparation stage: Based on the substitution principle, simplified the system into subnets. Exchange the nodal admittance matrix diagonal elements between adjacent areas to calculate voltage weighted coefficients.

2. Inner iteration stage: Nonlinear primal dual interior point method is taken in each sub-system. The optimization result of regional fuel cost independently, and the state variables of each bus are renewed.
(3) Convergence discrimination: Calculate the voltage difference $\Delta V$ and $\Delta \theta$ of the same boundary bus between adjacent areas. According to the convergence criterion, if the voltage differences are all sufficiently small, then the whole iteration ends, otherwise it turns to step (4).

(4) Outer iteration stage: Modify boundary bus voltage according to equation (17) and (18) and update its equivalent injection power according to equation (7) and (8), and then return to step (1).

6. Case Study
In order to validate the effectiveness of the proposed algorithm, multiple IEEE test systems are tested in this section. Single area generation scheduling optimization and coordination mechanism are analyzed as well in order to research the coordination and cooperation between regional power dispatching system and whole network power generation scheduling support system. The convergence criterions of duality gap and boundary bus voltage deviation are $10^{-5} \text{(p.u.)}$ and $10^{-4} \text{(p.u.)}$ respectively.

6.1. Distributed coordination and optimization results
Execute system decomposition and compare the obtained distributed generation dispatching results with the centralized one. The results from the test runs are summarized in Table 1.

| Test system | Tie-lines                      | Iterations | Fuel cost($) | Distributed | Centralized |
|-------------|--------------------------------|------------|--------------|-------------|-------------|
| IEEE 14     | 4-7, 4-9, 5-6                 | 7          | 940.67       | 931.13      |
| IEEE 30     | 4-12, 6-10, 6-9, 28-27         | 7          | 809.25       | 802.55      |
| IEEE 118    | 15-33, 19-34, 30-38, 23-24     | 9          | 14552.63     | 14524.67    |

Compared with the centralized generation scheduling optimization results, the distributed algorithm can obtain reasonable and relatively accurate fuel cost while reduce the scale of calculation notably. Each sub-system can finish regional optimization without adjacent area's operation data, and thereby reflecting the regional dispatcher's participating in whole network generation scheduling.

Figure 4 illustrates the boundary bus maximum voltage deviation of IEEE 118 test system in each iteration step. Although the voltage deviation may increase slightly in the initial iteration step, but its general trend is decreased especially after three times iterations. It conforms that the outer coordination process can make subnet boundary bus equivalent power injection modified effectively and ensures the equivalence property of power system decomposition. Few iteration times reflects good convergence performance of the proposed distributed algorithm as well.

6.2. Regional generation scheduling optimization
As for multi-area interconnected power system generation scheduling distributed optimization, each
area is subject to regional optimization calculation and realize the optimization of whole system through the outer coordination process. When it turns to subsystem single area optimization, considering the information barriers of power market environment, if each sub-system can obtain the boundary bus PF results of adjacent area, further analysis of its coordination mechanism is in need.

![Figure 5. The partition of IEEE 14 bus system](image)

The tie lines of IEEE 14 two-area interconnected system are 5-6, 4-7 and 4-9 as shown in Figure 5. Bus 1 and bus 8 are the reference node of $S_1$ and $S_2$ respectively. Define three computing modes:

- **Mode 1**: $S_1$ execute OPF calculation while $S_2$ conduct PF computing.
- **Mode 2**: $S_2$ conduct PF computing while $S_1$ execute OPF calculation.
- **Mode 3**: both $S_1$ and $S_2$ conduct OPF calculation.

Un-optimized fuel cost and the results obtained from the proposed calculation modes are summarized in Table 2.

| Computing mode | Un-optimized | Mode 1 | Mode 2 | Mode 3 |
|----------------|--------------|--------|--------|--------|
| $S_1$ fuel cost ($) | 1046.77 | 604.02 | 982.26 | 602.03 |
| $S_2$ fuel cost ($) | 348.69 | 349.53 | 338.46 | 338.63 |

Table 2. Comparison of Regional Generation Optimization Results in Each Mode

In un-optimized area generator node active output was set in its initial value, so the entire network fuel cost of mode 1/2 is higher than that of mode 3. The fuel cost of $S_1$ in mode 1 is 604.02 ($), which almost equal to 602.03 ($) calculated in mode 3. It certifies that the sub-system single area optimization results correspond with that of distributed coordinated optimization.

7. **Conclusions**

A distributed coordinated optimization method is proposed in the paper. It is utilized to solve generation dispatching problem in multi-area interconnected system. A wide range of numerical simulations are taken in IEEE test systems. It shows that the proposed method has good applicability and the following characters.

Without external network equivalence, the presented method avoids tedious calculating and updating of equivalent parameters. Therefore, it is suitable for large-scale interconnected power system.

The demand of every power grid is independent. Different optimization can be conducted in each sub-system with flexibility according to its practical need.

Small amount of boundary bus voltage must be exchanged between sub-systems. The proposed distributed method is easy to be implemented.
The sum of optimization results in sub-systems matches with whole network central optimized fuel cost. The suggested method can be adapted to the subsections and headquarters integration mode for generation dispatching.

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