Spur Gear Design using Axiomatic Design Approach

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Abstract: The researchers had been deliberative the number of theoretical and practical approach related to the axiomatic design (AD) principles from many times. Straight off a day taking the various instrument and machine is vital for any technology. In this research, two fundamental Axioms (Information Axiom & Independence Axiom) of Axiomatic Design principles are applied for Gear material selection and improve existing Spur Gear Design. There are wide variety of gear material are available today for the gear design such as Steel Cast iron, Aluminum, Bronze, Brass, Hardened steel, Case hardened steel and any others. So, final selection is based on understanding of material properties and application requirement. Some of the considerations in the choice of material include allowable bending and contact stress, ultimate tensile strength, deflection, size, weight, hardness, cost etc. In AD, Fundamental Axioms are useful for designers and practicing engineers for best selection and improve existing design of Spur gear. A step-by-step procedure, graph and tables are presented to demonstrate the concept and effectiveness of suggested design methodology.

Keywords: Axiomatic Design, AD, Design Principles, Optimization, Spur Gears, Case Study

I. INTRODUCTION

In today’s highly competitive manufacturing world, companies are forced to develop and deliver high-quality products manufactured at low costs with shorter cycle. Since unsatisfactory design results in a great number of process iterations, the effectiveness of design can be of great value for improving performance and quality of manufacturing and of service to customers. To reduce the probability of design failures, systematic approaches have been proposed in recent decades (Ulrich and Eppinger, 1995). As one of the representative methodologies, axiomatic design (AD) has been suggested to methodically design products and systems and has gained wide recognition in both the research and industrial communities. Developed by N.P. Suh and his associates, AD offers a systematic approach to manage interactions between elements of the design and functions the design must fulfil (Suh, 1990; Suh, 2001; Suh and Lee, 2006). Based on two axioms that provide a solid scientific foundation for design, AD theory helps to overcome the shortcomings of trial-and-error approach to product design and development. These axioms convey the basic idea that the specification of more functional requirements (FRs) than necessary can lead to over-design and attendant costs, while specification of fewer FRs than necessary to achieve design objectives can lead to unacceptable solutions. Many AD applications have appeared in the literature, including process and product development (Cha and Cho, 1999; Gebala and Suh, 1992; Gonçalves-Coelho and Mourão, 2007; Gunasekera and Ali, 1995; Park et al., 1996; Suh, 2001; Sung and Gyung, 2001; Tseng and Jiao, 1997), manufacturing systems (Suh et al., 1998), structural design in civil engineering structures (Albano and Suh, 1992) and design for environment (Chen, 2001; Wallace and Suh, 1993). Cochran et al., (2000) convert a complex production system into small, flexible and decentralised production segments, using lean principles in conjunction with segmentation and AD principles. Chen et al. (2000) propose a knowledge-based decision support system using the independence axiom in order to improve cell performance. Houshmand and Jamshidnezhad (2002) also propose a lean manufacturing production system design model based upon the organisational capabilities, technological capabilities and value stream analysis. Kulak et al., (2005) provide a framework and a road map for transforming a production system from process orientation to cellular orientation, based on AD principles. In addition, many studies in the last decade have persuasively shown the benefits of AD in solving a variety of design problems. AD principles have been used to software and quality system design (Kim et al., 1991; Chen, 1998; Suh, 1995). Durmusoglu and Kulak (in press) develop a methodology for designing an efficient office operation and Lo and Helander (2006) formalise AD principles into a framework for analysing the complexity of human – machine systems. Xue et al. (2006) introduce a systematic approach for configuration design considering the impact of design changes on downstream processes by using AD matrices to model the relationships between design configurations and their implementation processes. As a measure for evaluating safety in nuclear power plants, Heo and Lee (2007) propose an AD based methodology to examine the design process of emergency core cooling systems.
Often simple things or products are not given the attention they should, compared with the seriousness of the problem they could solve. AD is a valuable methodology for designing complex systems. Actually, it is a general design framework, rather than a design theory. In addition to the axiomatic approach to design, there are many other methods that are based on an algorithmic approach to design such as systematic design (VDI), Taguchi method, design for assembly or disassembly (DFA, DFD) etc. In algorithmic design, the design process is identified or prescribed so that it leads the designer to a specific solution that satisfies the design goals. However, AD may be of great value even for simple products. In this spirit, the present paper illustrates the use of the axiomatic approach in the design and analysis of an innovative product from the field of applied engineering (IPO, 2006). The case concerns an industrial door hinge with four degrees of freedom. The current design is described in detail and attention is made to analysis the mechanical basis of the product's design. Since one of the difficulties in learning the axiomatic approach has been the process of applying the axioms in actual design activities (Gebala and Suh 1992), we hope also that this case may be useful in teaching AD.

II. LITERATURE REVIEW

A. Dynamic optimization of spur gears
B. Spur gear optimization by using genetic algorithm
C. Axiomatic Design approach for marine design problem
D. Design and Analysis of a spur gear in different geomantic condition
E. Extension of axiomatic design under fuzzy environment
F. An axiomatic design approach for motorcycle steering damper

III. METHODOLOGY

Material Selection Method by Information Axiom:

1) \textit{Step 1:} Collect possible gear material with necessary decision criteria and its data. Let M1, M2, M3, M4 are selected gear material and C1, C2, C3, C4, C5 are decision criteria.

| Materials | Attributes |
|-----------|------------|
|           | C1 | C2 | C3 | C4 | C5 |
| M1        |    |    |    |    |    |
| M2        |    |    |    |    |    |
| M3        |    |    |    |    |    |
| M4        |    |    |    |    |    |
| M5        |    |    |    |    |    |

2) \textit{Step 2:} Functional requirements are selected as per requirements of users such as hardness, surface fatigue limit, bending fatigue limit, ultimate tensile strength and cost. Select criteria as per requirement (Functional Requirement-FR) is called Design Range. eg, Say hardness of material M1 is first functional requirement ($FR_1$) and user select range of $FR_1$ is some A to B as per requirement. Same as other functional requirements can be defined such as FR. FR. FR.........

3) \textit{Step 3:} Find out System Range of each alternative System range is found from selected decision criteria of different alternatives. It is found from Step 1.

4) \textit{Step 4:} Find out Common Range of each alternative from system range and design range using graphical method. Common range gives original required criteria and it is found from overlapping of system range and design range.. c.g., To find common range of attribute C1 of alternative M1, System range = 300 - 700. Design Range 500 – 1000

5) \textit{Step 5:} Find out Information Content (1) of each alternative using Equation . Information Content of each alternative can be found by following the procedure of step 4.

6) \textit{Step 6:} Finally Select best alternative based on total information content by using the result of each alternatives which is found in step 5. Best alternative is selected on the bases of minimum total information content.

So we can obtain common range easily from graph is 500-700 Using this system range and common range, Information content of attribute C1 of alternative M1 can be obtained.
Optimization Of Spur Gears by Independence Axiom

From the outcome of literature review and industrial requirement, selected objective functions are minimizing weight, minimizing tooth deflection and minimizing centre distance. Design variables are module, face width and number of teeth on pinion. Design constraints are taken to optimize design such as bending stress, compressive stress etc.

A. Objective Functions

Minimizing Centre Distance

\[ a = 0.5 \left( d_1 + mT_2 \right) \]

Minimizing Tooth Deflection

\[ \delta_T = \frac{15.12}{T_2 b d_1 E} \left\{ \left( \frac{0.785 - \tan \Psi}{1.25 \tan \Psi + 0.7854} \right) \left( \frac{0.785 - \tan \Psi}{1.25 \tan \Psi + 0.7854} - 3 \right) \left( \frac{0.785 - \tan \Psi}{1.25 \tan \Psi + 0.7854} - 1 \right) + 2 \log \right\} \]

Minimizing Weight of meshing gear set

\[ W = \frac{\pi}{4} m^2 b p g \left( T_1^2 + T_2^2 \right) \]

B. Design Variables

Module

\[ m \geq 1.26 \left\{ \frac{M_4}{y_{np}^2 \rho m r_1} \right\}^{\frac{1}{2}} \]

Face width

\[ 8m \leq b \leq 12 m \]

Number of Teeth on Pinion

\[ 8m \leq b \leq 12 m \]
C. Design Constraints

Bending Stress

\[
\frac{1.8 + 1}{112.92} \times \frac{1.8 + 1}{1.8 \times 36} \times 210 \times 10^3 \times [662.99] \times 10^3 \leq [\sigma_c]
\]

Compressive Stress

\[
\frac{i+1}{\alpha \beta} [M_{\text{max}}] \leq [\sigma_b]
\]

IV. CASE STUDY OF AN INDUSTRIAL ROTARY TILLER MACHINE

A. Selection of Gear Material by using Information Axiom

| Sr. No. | Material          | Hardness | SFL (MPA) | BFL (MPA) | UTS (MPA) |
|---------|-------------------|----------|-----------|-----------|-----------|
|         |                   | Surface  | Core      |           |           |
| 1       | DIN 1691          | 200 - 278| 200 - 278| 200 - 330| 60 - 100  |
| 2       | DIN 1692          | 165 - 300| 165 - 300| 300 - 460| 200 - 360 |
| 3       | BS 3100           | 220 - 320| 220 - 320| 560 - 700| 420 - 450 |
| 4       | 15NiCr1Mo15      | 519 - 565| 192 - 265| 800 - 1240| 550 - 680 |
| 5       | 14CrMoV6-9       | 647 - 738| 256 - 337| 1000 - 1250| 550 - 680 |
| 6       | St 50             | 220 - 320| 220 - 320| 600 - 740 | 500 - 580 |
| 7       | 15NiCr1Mo15      | 560 - 580| 270 - 350| 1150 - 1380| 600 - 870 |
| 8       | 20 MnCR5          | 610 - 700| 260 - 340| 1000 - 1420| 610 - 910  |

Table Various Gear Material and Data

1) Design Range

a) FR₁ = Core Hardness of Material is required between 230 to 310 BHN
b) FR₂ = Surface Hardness of Material is required between 550 to 660 BHN
c) FR₃ = Surface Fatigue Limit (SFL) of Material is required between 690 to 1200 MPa
d) FR₄ = Bending Fatigue Limit (BFL) of Material is required between 570 to 860 MPa
e) FR₅ = Ultimate Tensile Strength (UTS) of Material is required between 970 to 1410 MPa

2) System Range
From the information of system range and design range we can define common range from graph shown below.

Here, common range calculation for material M4 of core hardness attribute is shown in graph.

3) Common Range

| Sr. No | Material       | Hardness          | SFL (MPA) | BFL (MPA) | UTS (MPA) |
|--------|----------------|-------------------|-----------|-----------|-----------|
|        |                | Surface           | Core      |           |           |
| 1      | DIN 1691       | 0                 | 230 - 278 | 0         | 0         | 0         |
| 2      | DIN 1692       | 0                 | 230 - 300 | 0         | 0         | 0         |
| 3      | BS 3100        | 0                 | 220 - 310 | 690 - 700 | 0420 - 450| 970 - 1100|
| 4      | 34CrMo4        | 550 - 565         | 230 - 265 | 800 - 1200| 570 - 680 | 1240 - 1410|
| 5      | DIN 1692       | 0                 | 230 - 300 | 0         | 0         | 970 - 1100|
| 6      | St 50          | 0                 | 230 - 310 | 690 - 740 | 570 - 580 | 970 - 1410|
| 7      | 14CrMoV6-9     | 647 - 660         | 256 - 310 | 1000 - 1200| 590 - 860 | 1000 - 1250|
| 8      | 34CrMo4        | 0                 | 0         | 0         | 0         | 0         |
| 9      | 15NiCr1Mo15    | 560 - 580         | 270 - 310 | 1150 - 1200| 600 - 860 | 970 - 1250|
| 10     | 20 MnCR5       | 610 - 660         | 260 - 310 | 1000 - 1200| 610 - 860 | 1000 - 1300|

By using system range and common range we can compute the information content of each criteria. Here, information content of material M4 of core hardness is shown below.
\[ I = \log_2 (1/p_i) \]
\[ = \log_2 (\text{System Range/Common Range}) \]
\[ = \log (\text{System Range/Common Range}) / \log 2 \]
\[ = \log (73/35) / \log 2 \]
\[ = 1.060 \]

Information Content

From the table, the Information content of material M1, M2, M3, M5, M8 have infinite hence it cannot satisfy \( FR_1, FR_2, FR_3, FR_4 \).

From the table, the Information content of material M4, M7, M9 and M10 having less information so it can satisfy the functional requirement.

Since Material M10 Carburized Steels (20MnCr5) has minimum total information content, it is thus selected as best alternative.
B. Optimization using Independence Axiom

For optimize spur gear design, need to find out design variable and design constraint must be satisfied. First check that constraint are satisfied by given input data or violate. input data are shown below.

| Power (P) | 29 kw | Face width (b) | 54 mm |
|----------|-------|----------------|-------|
| Speed \(N_1\) | 543 rpm | Density of material \((\delta)\) | 7.85 g/cm³ |
| Gear Ratio \((i)\) | 1.8 | \(\psi_m\) | 10 |
| Centre distance \((a)\) | 168 mm | \(\Psi\) | 0.3 |
| Module \((m)\) | 6 mm | Modulus of Elasticity | 210 GPa |
| Tooth on Pinion \((T_1)\) | 20 | Form Factor | 0.377 |

1) Check for Centre Distance

\[
a \geq (i + 1) \left[ \frac{0.74}{\left[ \frac{\sigma_c}{\Psi} \right]} \right]^{\frac{1}{1.8}} \left[ \frac{210 \times 10^3 [662.99 \times 10^2]}{1.8 \times 0.3} \right]^{\frac{1}{1.8}}
\]

\[
a \geq (1.8 + 1) \left[ \frac{0.74}{931.95} \right]^{\frac{1}{1.8}} \left[ \frac{210 \times 10^3 [662.99 \times 10^2]}{1.8 \times 0.3} \right]^{\frac{1}{1.8}}
\]

\[a \geq 152.81\]

Now take centre distance as 152.81 mm and using find out module for optimum design.

\[a = 0.5 \left( d_1 + mT_2 \right)\]

\[152.81 = 0.5 \left( d_1 + 36m \right)\]

\[m = 5.45 \text{ mm}\]

\[m \approx 5.5 \text{ mm}\]

Take standard value of module is \(m = 5.5 \text{ mm}\)
2) Check for module

\[ m \geq 1.26 \left( \frac{M_t}{y \sigma_p m T_1} \right)^\frac{1}{3} \]

\[ m \geq 1.26 \left[ \frac{662.99 \times 10^3}{0.377 \times 313.92 \times 10 \times 20} \right]^\frac{1}{3} \]

\[ m \geq 3.83 \]

From value of module should be more than 3.83 mm and our standard module is 5.5 mm

3) Check for Face Width

\[ 8m \leq b \leq 12 \ m \]

From equation by taking face width of 8m, 9m it violates given constraint. So that for constraint satisfy by,

\[ b = 10m \]

\[ = 10(3) \]

\[ b = 55 \text{ mm} \]

Check constraint satisfaction by taking new value of module, face width and centre distance.

\[ g_1(x) = \frac{i+1}{amb} [M_{\text{max}}] \leq 313.92 \]

\[ [\sigma_p] = 313.92 \text{ Mpa} \]

\[ M_{\text{max}} = M_t k_d k_1 \]

\[ M_t = \frac{P}{2 \pi N_1} \]

\[ = \frac{29000 \times 60}{2 \pi \times 543} \]

\[ = 509.9 \times 10^3 \text{ Nmm} \]

\[ M_{\text{max}} = 662.99 \times 10^3 \text{ Nmm} \]

\[ g_1(x) = \frac{1.8 + 662.99 \times 10^3}{152.81 \times 5.5 \times 5.5 \times 0.377} \leq 313.92 \]

\[ g_1(x) = 106.52 \leq 313.92 \]

\[ g_2(x) = 0.74 \frac{i+1}{a} \sqrt{\frac{i+1}{10} E[M_t]} \leq 931.95 \]

\[ g_2(x) = 0.74 \frac{1.8 + 1}{11.29} \sqrt{\frac{1.8 + 1}{1.8 \times 36} 210 \times 10^3 \times [662.99 \times 10^3]} \leq 931.95 \]

\[ g_2(x) = 850.86 \leq 931.95 \]

\[ g_3(x) = T_1 + T_2 \geq 24 \]

\[ = 20 + 36 \geq 24 \]

\[ g_3(x) = 56 \geq 24 \]
C. **Objective Functions**

**Minimizing Centre Distance**

\[
a = 0.5 \left( d_1 + mT_2 \right)
\]

\[
a = 0.5 \left( 110 + 5.5 \times 36 \right)
\]

\[
= 154 \text{ mm}
\]

**Minimizing Tooth Deflection**

\[
\delta_T = \frac{15.12P}{T_2b d_1 E} \left\{ \frac{0.785 - \tan \psi}{1.25 \tan \psi + 0.785} - \frac{3}{125 \tan \psi + 0.785} - 1 \right\} + 2 \log \left( \frac{0.785 - \tan \psi}{1.25 \tan \psi + 0.785} \right)
\]

\[
= \frac{15.12 \times 29000}{36 \times 55 \times 110 \times 210 \times 10^3} \left\{ \frac{0.785 - 0.3}{125 \tan \psi + 0.785} - \frac{3}{125 \tan \psi + 0.785} - 1 \right\} + 2 \log \left( \frac{0.785 - 0.3}{125 \tan \psi + 0.785} \right)
\]

\[
\delta_T = 1.082 \times 10^{-6}
\]

**Minimizing Weight of meshing gear set**

\[
W = \frac{\pi}{4} m^2 b p g \left( T_1^2 + T_2^2 \right)
\]

\[
= \frac{\pi}{4} (5.5)^2 \times 55 \times 7.85 \times 10^{-6} \times 9.81 \times (20^2 + 36^2)
\]

\[
W = 170.66 \text{ N}
\]

V. **RESULT AND DISCUSSION**

A. **Gear Material Selection Result**

| Sr. No. | Material      | Hardness | SFL (MPA) | BFL (MPA) | UTS (MPA) | TOTAL |
|---------|---------------|----------|-----------|-----------|-----------|-------|
|         |               | Surface  | Core      |           |           |       |
| 1       | DIN 1691      | INF.     | 0.700     | INF.      | INF.      | INF.  |
| 2       | DIN 1692      | INF.     | 0.948     | INF.      | 0.000     | INF.  |
| 3       | BS 3100       | INF.     | 0.322     | 3.807     | 0.000     | INF.  |
| 4       | 34CrMo4       | 1.616    | 1.060     | 0.138     | 0.241     | 4.989 |
| 5       | DIN 1692      | INF.     | 0.778     | INF.      | 0.000     | INF.  |
| 6       | St 50         | INF.     | 0.322     | 1.485     | 3.000     | 0.396 |
| 7       | 14CrMoV6-9    | 2.807    | 0.584     | 0.322     | 0.200     | 1.066 |
| 8       | 34CrMo4       | INF.     | INF.      | INF.      | 0.096     | INF.  |
| 9       | 15NiCr1Mo15   | INF.     | 0.000     | 2.200     | 0.300     | 0.579 |
| 10      | 20 MnCR5      | 0.000    | 0.848     | 1.070     | 0.000     | 2.859 |

Table: Total Information Content
Design Axiom (Information Axiom) is applied on problem taken from industry to check whether any change require in material selection for spur gear which is used in rotary tiller machinery. By taking same nine alternatives of Case Study 1 and one industrial material (20MnCr5) applied Information Axiom to select best alternative out of ten alternatives. Result shown below in Table.

From the Table, The Information content of material M1, M2, M3, M5, M6, M8 have infinite since it cannot satisfy FR1, FR2, FR3, FRA, FRs, i.e., the design range and the system range do not overlap at all.

From the Table, The Information content of material M4, M7, M9 and M10 having less information so it can satisfy the functional requirement.

Since Material M10 Carburised Steel (20MnCr5) has minimum total information content, it is thus selected as best alternative.

### B. Spur Gear Design Optimization Result

| Variables          | Conventional Method | Independence Axiom |
|--------------------|---------------------|--------------------|
| Module (m)         | 6                   | 5.5                |
| Face width (b)     | 54                  | 55                 |
| Tooth on Pinion ($T_1$) | 20                | 20                 |
| Weight (W)         | 199.4               | 170.66             |
| Tooth Deflection ($\delta_T$) | $1.06 \times 10^{-6}$ | $1.082 \times 10^{-6}$ |
| Centre Distance (a) | 168                | 154                |

Table: Result Comparison

Independence Axiom is applied on problem taken from industry of selected spur gear material (20MnCr5) using information axiom to get optimum spur gear design. Here, comparison of result of Conventional method and Independence axiom shown below in Table.

Above Table shows the comparison between the conventional method and independence axiom calculation for the 20MnCr5 Carburised steel material.

Whereas independence axiom gives optimum result which satisfies given design constraints from found design variables. Also gives the optimal weight, centre distance and tooth deflection.

The performance of designed gear set using independence axiom is evaluated and compared with conventional method and gives optimal solution.

### VI. CONCLUSION

In the Axiomatic Design, Information Axiom is related to the probability of achieving the given requirements, can be used as a criterion for the selection of best solution among the selected alternatives. The information axiom assists to select the best result by quantifying specific design attributes.

Here successfully applied information axiom for selecting the gear material. Best alternative is selected from the calculated minimum total information content. Information Axiom assists to select the best result by quantifying specific design attributes.

Independence Axiom also gives more efficient solution for optimization of complex system such as gears made up of various materials.

Independence Axiom is useful especially to determine the shape of product by keeping the independence between functional requirements in conceptual design.

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