Distinguishing (Dirac or Majorana) neutrinos in purely leptonic decays of leptons

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Abstract

We investigate purely leptonic decays of leptons $l'' \rightarrow l' \bar{\nu}_l \nu_l$ to distinguish Dirac or Majorana neutrinos. We derive the differences of the decay width (and associated quantities) between the two neutrino hypotheses by the effect of identical particles. Evidence from experimental data makes the hypothesis of Majorana neutrinos very unlikely and our results strongly favor the hypothesis of Dirac neutrinos (in the standard three-neutrino theory). Moreover, our exploration for decay widths can be extended to the polarization angles of leptons, which will inspire new information for experimental and theoretical studies.

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I. INTRODUCTION

The question of whether neutrinos are Dirac [1] or Majorana [2] particles lies at the heart of particle physics. Experimentally, neutrinoless double beta decay [3] is the most popular access to test the hypothesis that neutrinos are Majorana particles. Unfortunately, it only keeps setting higher precision limits so far [4], and whether neutrinos being Majorana particles or not are not concluded yet. This leads to an urgent need to find another way to identify neutrinos (Dirac or Majorana). Fortunately, we have found that purely leptonic decays of leptons provides an ideal opportunity to identify neutrinos considering the precise experimental results of lepton decays.

In this work, we investigate purely leptonic decays of leptons \( l'^- \rightarrow l^- \bar{\nu}_l \nu_l \), where \( l' = \tau, l = e, \mu \) or \( l' = \mu, l = e \). According to neutrino oscillations [5, 6], \( l'^- \rightarrow l^- \bar{\nu}_l \nu_l \) is the superposition of several \( l'^- \rightarrow l^- \bar{\nu}_i \nu_j \), where \( \bar{\nu}_i \) and \( \nu_j \) indicate the mass eigenstates of neutrinos. It makes no different between Dirac and Majorana neutrino hypotheses to investigate purely leptonic decays of leptons in their flavor eigenstates, i.e. \( l'^- \rightarrow l^- \bar{\nu}_i \nu_j \) \((i \neq j)\), but it has obvious distinction between the two hypotheses to investigate in neutrinos’ mass eigenstates, i.e. \( l'^- \rightarrow l^- \bar{\nu}_i \nu_i \) when \((i = j)\). \( \bar{\nu}_i, \nu_i \) are not identical particles in Dirac neutrinos, while identical in Majorana neutrinos. Analogous to what mentioned in [7, 8], the branching ratio of \( D^+ \rightarrow \pi^+ \pi^- \pi^+ \) must include the influence of two identical \( \pi^- \)'s. Identical neutrinos bring an extra interference term than non-identical neutrinos do, which causes the decay width of purely leptonic decays of leptons under Majorana neutrino hypothesis always narrower than that under Dirac neutrino hypothesis. The key reason is Fermi-Dirac statistics for identical neutrinos (details will be shown in Formalism).

Besides having a pair of neutrino and anti-neutrino, which could be identical, in the final states, the purely leptonic decays of leptons \( l'^- \rightarrow l^- \bar{\nu}_l \nu_l \) can play as gold decay channels to distinguish Dirac or Majorana neutrinos due to two other reasons. Firstly, the neutrino oscillations have a substantial flavour mixing angle, which means different flavour neutrinos seriously overlap in the neutrino mass eigenstate space, and then cause a large proportion of identical particles under Majorana neutrino hypothesis. This will make a significant difference of conclusions between the two neutrino hypotheses. Secondly, the purely leptonic decays of leptons have no strong or electromagnetic interaction, which makes them can be calculated very accurately and precisely even in four-fermion interactions. The
small theoretical uncertainty leaves no ambiguity to draw out the difference of the two neutrinos hypotheses.

At last, although we investigate only the decay width (and associated quantities), polarization angles of $l'^{-}$ and $l^{-}$ are also excellent places to distinguish Dirac or Majorana neutrinos, which will inspire new information for experimental or theoretical studies. Investigating polarization angles in purely leptonic decays of leptons is the next step of our work plan.

II. FORMALISM

The decay widths of the $l'^{-} \rightarrow l^{-} \bar{\nu}_l \nu_l$ decay, considering neutrino oscillation in the standard three-neutrino theory, is given by

$$\Gamma(l'^{-} \rightarrow l^{-} \bar{\nu}_l \nu_l) = \sum_{i=1}^{3} \sum_{j=1}^{3} \Gamma(l'^{-} \rightarrow l^{-} \bar{\nu}_i \nu_j). \quad (1)$$

At first, we assume neutrinos are Dirac particles, the amplitude of $l'^{-} \rightarrow l^{-} \bar{\nu}_l \nu_l$ can be written as

$$M_D(l'^{-} \rightarrow l^{-} \bar{\nu}_l \nu_l) = \frac{G_F}{\sqrt{2}} U_{li} U_{l'j} \bar{u}_l \gamma^\mu (1 - \gamma^5) \nu_i \bar{\nu}_j \gamma_\mu (1 - \gamma^5) u_{l'} \quad (2)$$

where $U_{li}$ and $U_{l'j}$ are the matrix elements of PMNS matrix [6]. In the $l'$ rest frame, the partial decay width reads

$$\Gamma_D(l'^{-} \rightarrow l^{-} \bar{\nu}_l \nu_l) = \frac{1}{(2\pi)^3} \frac{1}{32m_{l'}^3} \int |M_D|^2 dm_{12}^2 dm_{23}^2 \quad (3)$$

with $m_{12}^2 = (p_l + p_{\nu_l})^2$, $m_{23}^2 = (p_l + p_{\nu_l})^2$, $K_{l'} = 1 - 8y_{l'\nu} + 8y_{l'\nu}^2 - y_{l'\nu} - 12y_{l'\nu} \ln y_{l'\nu}$, and $y_{l'\nu} = m_l^2/m_{l'}^2$. With knowing of PMNS matrix is unitary, we can write the partial decay width as [9]

$$\Gamma_D(l'^{-} \rightarrow l^{-} \bar{\nu}_l \nu_l) = \frac{G_F^2 m_{l'}^5}{24(2\pi)^3} K_{l'} \quad (4)$$

Then we reach the decay width $\Gamma(\tau^{-} \rightarrow \mu^{-} \bar{\nu}_\mu \nu_\mu)$ versus the decay width $\Gamma(\tau^{-} \rightarrow e^{-} \bar{\nu}_e \nu_e)$:

$$R_D = \frac{\Gamma_D(\tau^{-} \rightarrow \mu^{-} \bar{\nu}_\mu \nu_\mu)}{\Gamma_D(\tau^{-} \rightarrow e^{-} \bar{\nu}_e \nu_e)} = \frac{K_{\mu\tau}}{K_{e\tau}} \quad (5)$$
On the other side, using the same way in the assumption of Dirac neutrinos, the partial decay width with the assumption of Majorana neutrinos $\Gamma_M(l'^- \rightarrow l'\bar{\nu}_i\nu_j)$ in the case of $(i \neq j)$ is given by

$$\Gamma_M(l'^- \rightarrow l'\bar{\nu}_i\nu_j) = \Gamma_D(l'^- \rightarrow l'\bar{\nu}_i\nu_j), \ (i \neq j)$$

(6)

but it is a quiet different story for $\Gamma_M(l'^- \rightarrow l'\bar{\nu}_i\nu_i)$ in the case that $\bar{\nu}_i, \nu_i$ are identical, where the effects of Fermi-Dirac statistics must be taken into account. As similar way in Ref. [7, 8], (in which the effects of Bose-Einstein statistics is taken into account), the amplitude of $l'^- \rightarrow l'\bar{\nu}_i\nu_i$ is given by

$$\mathcal{M}_M(l'^- \rightarrow l'\bar{\nu}_i\nu_i) = \frac{G_F}{\sqrt{2}} U_{l'i} U_{l'j}^* \{ \bar{u}_i(p_3)\gamma^\mu(1 - \gamma^5)\nu_i(p_1)\bar{u}_i(p_2)\gamma_\mu(1 - \gamma^5)u_r(p_0)
\-
\bar{u}_i(p_3)\gamma^\mu(1 - \gamma^5)\nu_i(p_2)\bar{u}_i(p_1)\gamma_\mu(1 - \gamma^5)u_r(p_0)\}\}.$$

(7)

The partial decay width then reads

$$\Gamma_M(l'^- \rightarrow l'\bar{\nu}_i\nu_i) = \frac{1}{2} \frac{1}{(2\pi)^3} \frac{1}{32m_\nu^2} \int |\mathcal{M}_M(l'^- \rightarrow l'\bar{\nu}_i\nu_i)|^2 dm_{12}^2 dm_{23}^2, \tag{8}$$

where the factor of 1/2 accounts for the effect of Fermi-Dirac statistics. With the helping of Eqs. (3) and (8), we can know the non-interference term of $\Gamma_M(l'^- \rightarrow l'\bar{\nu}_i\nu_i)$ equals to $\Gamma_D(l'^- \rightarrow l'\bar{\nu}_i\nu_i)$. Consequently, the partial decay width with the assumption of Majorana neutrinos can be written as

$$\Gamma_M(l'^- \rightarrow l'\bar{\nu}_i\nu_i) = \Gamma_D(l'^- \rightarrow l'\bar{\nu}_i\nu_i) - \sum_{i=1}^{i=3} \Gamma_{M}^{\text{int}}(l'^- \rightarrow l'\bar{\nu}_i\nu_i), \tag{9}$$

where the $\Gamma_{M}^{\text{int}}(l'^- \rightarrow l'\bar{\nu}_i\nu_i)$ is the interference term of $\Gamma_M(l'^- \rightarrow l'\bar{\nu}_i\nu_i)$ and is always positive. With

$$\Gamma_M(l'^- \rightarrow l'\bar{\nu}_i\nu_i) < \Gamma_D(l'^- \rightarrow l'\bar{\nu}_i\nu_i), \tag{10}$$

this interference term can be written as

$$\sum_{i=1}^{i=3} \Gamma_{M}^{\text{int}}(l'^- \rightarrow l'\bar{\nu}_i\nu_i) = \sum_{i=1}^{i=3} |U_{l'i}|^2 |U_{l'j}^*|^2 \frac{G_F^2}{2(2\pi)^3} \frac{1}{32m_\nu^2} \int |\Gamma_{M}^{\text{int}}|^2 dm_{12}^2 dm_{23}^2$$

(11)

and

$$|\Gamma_{M}^{\text{int}}| = \frac{1}{2} Tr[\gamma^\nu(p_3 + m_3)\gamma^\mu(1 - \gamma^5)v(p_1)\bar{v}(p_2)(1 + \gamma^5)]
\-
Tr[\gamma_\mu(p_0 + m_0)\gamma_\nu(1 - \gamma^5)u(p_1)\bar{u}(p_2)(1 + \gamma^5)], \tag{12}$$
where

\[ (1 - \gamma^5)v(p_1)\bar{v}(p_2)(1 + \gamma^5) = (1 - \gamma^5)p_1^\gamma p_2^0(1 + \gamma^5)/\sqrt{2E_1}\sqrt{2E_2} \]
\[ (1 - \gamma^5)u(p_1)\bar{u}(p_2)(1 + \gamma^5) = (1 - \gamma^5)p_1^\gamma p_2^0(1 + \gamma^5)/\sqrt{2E_1}\sqrt{2E_2}. \]  

(13)

Here, we have summed over the spin of \( v(p_1)\bar{v}(p_2) \) and \( u(p_1)\bar{u}(p_2) \), and \( E_1 \) and \( E_2 \) are energies of neutrinos in the \( l' \) rest frame. Using Eqs. (11), (12) and (13), we can get

\[ \sum_{i=1}^{i=3} \Gamma_{\mathcal{M}}^{\text{int}}(l'^{-} \to l'^{-}\bar{\nu}_i\nu_i) = \frac{G_F^2 m_i^5}{24(2\pi)^3} Y_{ll} \]  

(14)

and

\[ Y_{ll} = \sum_{i=1}^{i=3} |U_{ll}|^2 |U_{l'i}^*|^2 \left\{ (1 - y_{ll})^2 \ln y_{ll} - (1 - y_{ll}) - 6(1 - 3y_{ll}) \ln y_{ll} - 12(1 - y_{ll})^2 \ln y_{ll} + 2\pi^2 (1 - y_{ll})^2 + \right. 
\]
\[ \left. (1 - y_{ll})^3 \frac{y_{ll}^3 - 11y_{ll}^2 + 61y_{ll} - 39}{2} \right\} \]  

(15)

where \( \gamma_2 \) is dilogarithm function (Spence’s function). Eventually, the ratio of the partial decay width \( \Gamma_{\mathcal{M}}(\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau) \) to \( \Gamma_{\mathcal{M}}(\tau^- \to e^-\bar{\nu}_e\nu_\tau) \) is determined to be

\[ \mathcal{R}_{\mathcal{M}} = \frac{\Gamma_{\mathcal{M}}(\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau)}{\Gamma_{\mathcal{M}}(\tau^- \to e^-\bar{\nu}_e\nu_\tau)} = \frac{K_{\mu\tau} - Y_{\mu\tau}}{K_{e\tau} - Y_{e\tau}} \]  

(16)

III. NUMERICAL RESULTS

To perform the numerical analysis, the PMNS matrix elements as the theoretical inputs are given by \((\nu_{e1}, \nu_{e2}, \nu_{e3}) = (\cos \theta_{12} \cos \theta_{13}, \sin \theta_{12} \cos \theta_{13}, \sin \theta_{13} e^{i\delta}), (\nu_{\mu1}, \nu_{\mu2}, \nu_{\mu3}) = (-\sin \theta_{12} \cos \theta_{23} - \cos \theta_{12} \sin \theta_{23} e^{i\delta}, \cos \theta_{12} \cos \theta_{23} - \sin \theta_{12} \sin \theta_{23} e^{i\delta}, \sin \theta_{23} \cos \theta_{13}), \) and \((\nu_{\tau1}, \nu_{\tau2}, \nu_{\tau3}) = (\sin \theta_{12} \sin \theta_{23} - \cos \theta_{12} \cos \theta_{23} e^{i\delta}, -\cos \theta_{12} \sin \theta_{23} - \sin \theta_{12} \cos \theta_{23} \sin \theta_{13} e^{i\delta}, \cos \theta_{23} \cos \theta_{13}) \) with \((\sin^2 \theta_{12}, \sin^2 \theta_{23}, \sin^2 \theta_{13}, \delta) = (0.307 \pm 0.013, 0.546 \pm 0.021, 0.0220 \pm 0.0007, (1.36^{+0.20}_{-0.16})\pi) \) [9] and \( (0 < \theta_{12}, \theta_{23}, \theta_{13} < \pi/2) \) [11]. (We have ignored two CP-violating phases that are physically meaningful only if neutrinos are Majorana particles, which have no effect to all conclusions in this paper.) Next, the data inputs are given by

\[ \sum_{i=1}^{i=3} |U_{e1i}|^2 |U_{e1i}^*|^2 = 0.271 \pm 0.039, \quad K_{e\tau} = 1.0000 \pm 0.0000, \quad Y_{e\tau} = 0.065 \pm 0.009 \]
\[ \sum_{i=1}^{i=3} |U_{\mu1i}|^2 |U_{\mu1i}^*|^2 = 0.384 \pm 0.005, \quad K_{\mu\tau} = 0.9726 \pm 0.0000, \quad Y_{\mu\tau} = 0.090 \pm 0.001 \]
\[ \sum_{i=1}^{i=3} |U_{e1i}|^2 |U_{e1i}^*|^2 = 0.196 \pm 0.017, \quad K_{e\mu} = 0.9998 \pm 0.0000, \quad Y_{e\mu} = 0.047 \pm 0.004 \]  

(17)
and lifetime $t_\tau = (2.903 \pm 0.005) \times 10^{-13}$ s and $t_\mu = (2.197 \pm 0.000) \times 10^{-6}$ s. Since Fermi constant $G_f$ comes from measurements of $\mu$ lepton lifetime from the decay channel $\mu^- \to e^- \bar{\nu}_e \nu_\mu$, the data input of Fermi constant $G_f^{\exp} = 1.1664 \times 10^{-5}$ GeV$^{-2}$ from PDG can only be used for Dirac neutrino hypothesis ($G_f^D = G_f^{\exp}$). Fermi constant for Majorana neutrino hypothesis are $G_f^M = G_f^{\exp} \sqrt{K_{e\mu} Y_{e\mu}} = (1.1948 \pm 0.0025) \times 10^{-5}$ GeV$^{-2}$.

All the calculated contrasts to experimental measurements are summarized in table I.

### IV. DISCUSSIONS AND CONCLUSIONS

As summarized in Table I, all the results under Dirac neutrino hypothesis agrees with experimental results better than that of Majorana neutrino hypothesis (except for the decay width $\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu)$, which is one of the inputs). In particular, the ratio $R$ of the decay width $\Gamma(\tau^- \to \mu^- \bar{\nu}_e \nu_\mu)$ to the decay width $\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau)$, which is related only to lepton masses in Dirac neutrino hypothesis, while is involved with lepton masses and neutrino mixing matrix parameters in Majorana neutrino hypothesis. The uncertainties of theoretical calculations is comparable to that of experimental measurements. The calculated result of $R$ under Majorana neutrino hypothesis deviates from the experimental measurements more than 3$\sigma$, which is a strong support to rule out Majorana neutrino hypothesis. However, the $R$ value under Dirac neutrino hypothesis is consistent with experimental measurements within 1.3$\sigma$, as well as the two branching fractions of $\tau^-$ decays.

All the discussion in this paper is based on the standard three-neutrino theory, but even if we extend the standard three-neutrino model to other models, the effect of identical particles will still contribute strong constraints.

| $l'^- \to l^- \bar{\nu}_l \nu_l$ | Majorana neutrinos | Dirac neutrinos | experiment [10] |
|-------------------------------|-------------------|----------------|-----------------|
| $\mathcal{B}(\tau^- \to \mu^- \bar{\nu}_e \nu_\mu) \times 10^2$ | 16.53 $\pm$ 0.08 | 17.36 $\pm$ 0.03 | 17.39 $\pm$ 0.04 |
| $\mathcal{B}(\tau^- \to e^- \bar{\nu}_e \nu_\mu) \times 10^2$ | 17.51 $\pm$ 0.19 | 17.85 $\pm$ 0.03 | 17.82 $\pm$ 0.04 |
| $\mathcal{B}(\mu^- \to e^- \bar{\nu}_e \nu_\mu) \times 10^2$ | 100.15 $\pm$ 0.00 | 100.15 $\pm$ 0.00 | $\approx$ 100 |
| $R = \frac{\Gamma(\tau^- \to \mu^- \bar{\nu}_e \nu_\mu)}{\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau)}$ | 0.9440 $\pm$ 0.0091 | 0.9726 $\pm$ 0.0000 | 0.9762 $\pm$ 0.0028 |
V. SUMMARY AND OUTLOOK

In summary, we have investigated $l^- \rightarrow l^- \bar{\nu} \nu$ decays. According to the effect of identical particles, we have found the difference between Dirac and Majorana neutrino hypotheses and presented the comparisons to experimental measurements, indicating a badly dislike to the hypothesis that neutrinos are Majorana particles and a strong favor to the hypothesis that neutrinos are Dirac particles in the standard three-neutrino theory.

This work can be extended to study the angular distribution of polarization in purely leptonic decays of leptons, which could give more crucial information to distinguish Dirac neutrinos from Majorana neutrinos. This is a focus of our next work and should also be the focus of future experimental observations. For example, large and clean data sets of $e^+e^- \rightarrow \tau^+\tau^-$ have been and will be collected at BESIII [12], and further study about polarization angles of leptons in $\tau^-$ decay can be done.

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