A new recipe for $\Lambda$CDM

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It is well known that a canonical scalar field is able to describe either dark matter or dark energy but not both. We demonstrate that a non-canonical scalar field can describe both dark matter and dark energy within a unified setting. We consider the simplest extension of the canonical Lagrangian $\mathcal{L} \propto X^\alpha - V(\phi)$ where $\alpha \geq 1$ and $V$ is a sufficiently flat potential. In this case the kinetic term in the Lagrangian behaves just like a perfect fluid, whereas the potential term mimicks the cosmological constant. For very large values, $\alpha \gg 1$, the equation of state of the kinetic term drops to zero and the universe expands as $\Lambda$CDM. The velocity of sound in this model, and the associated gravitational clustering, is sensitive to the value of $\alpha$. For very large values of $\alpha$ the clustering properties of our model resemble those of cold dark matter (CDM). But for smaller values of $\alpha$, gravitational clustering on small scales is suppressed, and our model has properties resembling those of warm dark matter (WDM). Therefore our non-canonical model has an interesting new property: while the background universe expands like $\Lambda$CDM, its clustering properties can resemble those of either cold or warm dark matter.

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I. INTRODUCTION

Ever since the discovery that high redshift type Ia supernovae supported an accelerating universe, concordance cosmology or ΛCDM, has come to dominate popular thinking. Although issues relating to the smallness of Λ have given rise to several rival models of cosmic acceleration [1, 2] there is no doubt that, despite some recent evidence to the contrary [3–6], ΛCDM agrees well with a large set of cosmological observations [7].

As its name suggests, ΛCDM consists of two components: the cosmological constant, Λ, and cold dark matter (CDM). Despite its enormous success in explaining observations, the origin of Λ is not known. It may simply be a residual vacuum fluctuation, although quantum field theory usually predicts much larger values, and the cosmological constant problem remains unresolved. As concerns dark matter, mainstream thinking usually assumes it to be a non-baryonic relic of the big bang [8, 9] but other explanations can also be found in the literature [10–16]. Furthermore, since 96% of the content of the universe is of unknown origin, attempts have been made to describe both dark matter and dark energy within a unified setting. The Chaplygin gas (and its subsequent generalization) belongs to this category of models, since its equation of state (EOS) behaves like pressureless dust at early times and like a Λ-term at late times [17]; also see [18]. Unfortunately the Chaplygin gas has problems with gravitational clustering and so falls short of describing the real universe [19–23].

In this paper we show that a unified description of dark matter and dark energy can emerge from non-canonical scalar fields; see [26, 29, 30] for earlier work in this direction. These fields possess an additional degree of freedom (encoded in the parameter α) which allows a scalar field rolling along a flat potential to behave like a two component fluid consisting of an almost pressureless kinetic component (dark matter) and a cosmological constant. For large values of α the equation of state of the kinetic component drops to zero and the universe expands like ΛCDM. Non-canonical scalars cluster on small scales, thereby providing us with a realistic model of an accelerating universe consisting of dark matter and dark energy. For very large values of α the kinetic component clusters like cold dark matter, whereas for smaller α values, clustering in our model resembles warm dark matter.
II. NON-CANONICAL SCALARS AND $\Lambda$CDM

Perhaps the simplest generalisation of the canonical scalar field Lagrangian density

$$\mathcal{L}(X, \phi) = X - V(\phi), \quad X = \frac{1}{2} \dot{\phi}^2$$

which preserves the second order nature of the field equations, is the non-canonical Lagrangian [25, 29, 31–33]

$$\mathcal{L}(X, \phi) = X \left( \frac{X}{M^4} \right)^{\alpha - 1} - V(\phi),$$

where $M$ has dimensions of mass while $\alpha$ is dimensionless. When $\alpha = 1$ the k-essence Lagrangian (2) reduces to (1).

We shall be working in the spatially flat Friedmann-Robertson-Walker (FRW) universe

$$ds^2 = dt^2 - a^2(t) \left[ dx^2 + dy^2 + dz^2 \right],$$

for which the energy-momentum tensor has the form

$$T^\mu_{\nu} = \text{diag} (\rho_\phi, -p_\phi, -p_\phi, -p_\phi),$$

where the energy density, $\rho_\phi$, and pressure, $p_\phi$, are given by

$$\rho_\phi = \left( \frac{\partial \mathcal{L}}{\partial \dot{X}} \right) (2X) - \mathcal{L},$$

$$p_\phi = \mathcal{L}. $$

Substituting for $\mathcal{L}$ from (2) into (5) and (6) one gets

$$\rho_\phi = (2\alpha - 1) X \left( \frac{X}{M^4} \right)^{\alpha - 1} + V(\phi),$$

$$p_\phi = X \left( \frac{X}{M^4} \right)^{\alpha - 1} - V(\phi),$$

which reduces to the canonical form $\rho_\phi = X + V, \quad p_\phi = X - V$ when $\alpha = 1$. The two Friedmann equations are

$$H^2 = \frac{8\pi G}{3} \left[ (2\alpha - 1) X \left( \frac{X}{M^4} \right)^{\alpha - 1} + V(\phi) \right],$$

$$\frac{\ddot{a}}{a} = \frac{8\pi G}{3} \left[ (\alpha + 1) X \left( \frac{X}{M^4} \right)^{\alpha - 1} - V(\phi) \right],$$
where $\phi(t)$ satisfies the equation of motion

$$\ddot{\phi} + \frac{3 H \dot{\phi}}{2\alpha - 1} + \left(\frac{V' (\phi)}{\alpha (2\alpha - 1)}\right) \left(\frac{2M^4}{\phi^2}\right)^{\alpha - 1} = 0,$$

(10)

which reduces to the standard canonical form $\ddot{\phi} + 3H\dot{\phi} + V' (\phi) = 0$ when $\alpha = 1$.

Consider the equation of motion (10) for the simplest case when $V (\phi) = \Lambda / 8\pi G$. Setting $V' = 0$ in (10) one finds

$$\ddot{\phi} = \frac{-3 H \dot{\phi}}{2\alpha - 1},$$

(11)

which is easily integrated to give

$$\dot{\phi} \propto a^{-\frac{3}{2\alpha - 1}},$$

(12)

and which reduces to the canonical result, $\dot{\phi} \propto a^{-3}$, when $\alpha = 1$. Substituting for $X \equiv \dot{\phi}^2 / 2$ from (12) into (7) one readily finds (we have set $8\pi G = 1$ for simplicity)

$$\rho_\phi = \rho_X + \Lambda$$

(13)

$$p_\phi = p_X - \Lambda,$$

(14)

with

$$\rho_X = (2\alpha - 1) X \left(\frac{X}{M^4}\right)^{\alpha - 1} \equiv \rho_{0X} a^{-3(1+w)}$$

and

$$p_X = w \rho_X,$$

(15)

is the equation of state (EOS) of the kinetic component of the scalar field. From (13) & (14) it follows that the non-canonical scalar field behaves like a mixture of two non-interacting perfect fluids: $\rho_X$ and $\Lambda$, where the equation of state of $\rho_X$ is given by (15). Substituting (13) into (8) one finds

$$H(z) = H_0 \left[ \Omega_{0X} (1+z)^{3(1+w)} + \Omega_\Lambda \right]^{1/2},$$

(16)

where $\Omega_{0X} = \frac{8\pi G \rho_{0X}}{3H_0^2}$ and $w$ is described by (15).

From (15) and (16) we find that the expansion history is very sensitive to the value of $\alpha$. When $\alpha = 1$ the scalar field behaves like a mixture of ‘$\Lambda$ + stiff matter’. For $\alpha = 2$ the expansion history mimicks ‘$\Lambda$ + radiation’. For $\alpha \gg 1$, $w \to 0$ and $\rho_X \propto a^{-3}$, consequently (16) describes $\Lambda$CDM in this limit:

$$H(z) \simeq H_0 [\Omega_{0m} (1+z)^3 + \Omega_\Lambda]^{1/2}, \quad \text{where} \quad \Omega_{0m} \equiv \Omega_{0X}.$$

(17)
FIG. 1. $\Delta H$ is plotted against the expansion factor, $a$, for $\alpha = 50, 100, 200, 1000$ (top to bottom). Here $\Delta H = (H - H_{\Lambda CDM})/H_{\Lambda CDM}$ and $a = 1$ corresponds to the present epoch.

The equation of state of the scalar field, $w_\phi = p_\phi/\rho_\phi$, is given by

$$1 + w_\phi(z) = (1 + w) \left[ 1 + \frac{\Omega_\Lambda}{\Omega_{0X}(1 + z)^{1+w}} \right]^{-1}$$

(18)

where $w$ is described by (15). We find that $w_\phi \simeq w$ when $z \gg 1$, while its current value is $w_{\phi,0} = (1 + w) \left[ 1 + \frac{\Omega_\Lambda}{\Omega_{0X}} \right]^{-1} - 1$. From (15) and (18) one finds that for $\alpha \gg 1$, $w_\phi \simeq 0$ at $z \gg 1$, and $w_{\phi,0} \simeq -\Omega_\Lambda$ at $z = 0$. Thus for large values of $\alpha$, the EOS of the scalar field smoothly interpolates between dust-like behaviour at high redshifts and a negative value at present.\(^1\) In figure 1 we plot the fractional difference between the Hubble parameter $H(z)$ in our model from $\Lambda$CDM for different values of the parameter $\alpha$ (but identical values of the matter density). One can see for $\alpha \geq 10^3$ the deviation is less than 1%. Hence for such large values of $\alpha$ our model will be virtually indistinguishable from $\Lambda$CDM model by observables measuring background cosmology alone.

We have assumed thus far that $V(\phi)$ is a constant. This however need not necessarily be the case. It is important to note that $\Lambda$CDM-like expansion can also arise in the case of other potentials which are flat. The reason for this is simple. Our treatment above was based on the assumption that the last term in (10) was negligibly small compared to the remaining two terms, allowing the former to be neglected. This feature is shared by several flat potentials some of which are described below.

\(^1\) In the limit when $\alpha \to \infty$ our model has properties resembling those of [29].
• $V(\phi) = \frac{1}{2}m^2\phi^2$. In order to study this power-law potential, we first form an autonomous system of equations for our model. For Lagrangians of the form $\mathcal{L} = F(X) - V(\phi)$, De-Santiago et al [24] have already constructed an autonomous system of equations involving the dimensionless variables

$$
x = \frac{\sqrt{2XF_x - F}}{\sqrt{3m_{pl}H}},
$$

$$
y = \frac{\sqrt{V}}{\sqrt{3m_{pl}H}},
$$

$$
w_k = \frac{F}{2XF_x - F},
$$

$$
\sigma = -\frac{m_{pl}}{\sqrt{3|\rho_k|}} \frac{d\log V}{dt}.
$$

(19)

Applying these variables to our Lagrangian given by (2), we get

$$
w_k = \frac{1}{\alpha - 1},
$$

$$
\Omega_\phi = x^2 + y^2 = 1,
$$

$$
\gamma = 1 + w_\phi = x^2(1 + w_k).
$$

(20)

For $\alpha >> 1$, $w_k \sim 0$ and one can safely approximate $\gamma \sim x^2$. Using the formulation prescribed in De-Santiago et al [24], we can now form an autonomous system of equations for $\gamma$ and $\sigma$:

$$
\gamma' = 3\sigma(1 - \gamma)\sqrt{\gamma} - 3\gamma(1 - \gamma),
$$

$$
\sigma' = -3\sigma^2 \sqrt{\gamma}(\Gamma - 1) + \frac{3}{2}\sigma(1 - \sigma(1 - \gamma)\sqrt{\gamma})).
$$

(21)

Here ‘prime’ denotes derivative w.r.t log $a$ and $\Gamma = \frac{V''(\phi)}{V'(\phi)^2}$, so that $\Gamma = \frac{1}{2}$ for $V = \frac{1}{2}m^2\phi^2$. In order to solve this autonomous system, one requires initial conditions for $\gamma$ and $\sigma$. We set these at decoupling, $a \sim 10^{-3}$, assuming that initially the scalar field kinetic energy dominates over its potential energy, so that $w_\phi \sim \frac{1}{2a - 1}$. With $\alpha >> 1$, we have $\gamma_i \sim 1$. Similarly one finds $\sigma_i \sim 0$ at decoupling. (One should note that $w_\phi$ is not exactly equal to zero initially, due to the large but finite value of $\alpha$.) With these initial conditions, the autonomous system of equations in (20) is evolved from decoupling until today, and the resultant behaviour of the equation of state for the scalar field, $w_\phi$, is shown in figure 2. We find that the behaviour of $w_\phi$ in our model is very similar to that described by (18) for $\Lambda$CDM.
FIG. 2. The equation of state for the scalar field \( w_\phi \) as a function of scale factor for \( V(\phi) \sim \phi^2 \) (Solid line). The dashed line is for equation of state described by (18) for \( \Omega_\Lambda = 0.7 \) and \( \Omega_0 X = 0.3 \).

- Finally, an interesting example of a piece-wise flat potential is the step potential

\[
V(\phi) = A + B \tanh \beta \phi
\]  

(22)

where \( A + B = V_{\text{initial}} \) and \( A - B = V_{\text{final}} \). For \( V_{\text{initial}} \simeq 10^{64}\text{GeV}^4 \), \( V_{\text{final}} \simeq 10^{-47}\text{GeV}^4 \) this potential would interpolate between inflation at early times, and dark energy at late times, and therefore might describe a model of Quintessentital-Inflation. We shall examine this possibility in greater detail in a companion paper.

We therefore find that a single non-canonical scalar field can play the dual role of dark matter and dark energy viz-a-viz the expansion history of the universe. However in order to deepen the parallel between scalar field dynamics and ΛCDM we also need to demonstrate that the field \( \phi \) can cluster. In order to do this we first note that linearized scalar perturbations in a spatially flat FRW universe are described by the line element [34–36]

\[
ds^2 = (1 + 2 A) \, dt^2 - 2 a(t) (\partial_i B) \, dt \, dx^i \\
- a^2(t) \left[ (1 - 2 \psi) \, \delta_{ij} + 2 (\partial_i \partial_j E) \right] \, dx^i \, dx^j.
\]

The linearized Einstein’s equation \( \delta G_{\mu\nu} = \kappa \delta T_{\mu\nu} \) together with the perturbation equation for the scalar field give

\[
R''_k + 2 \left( \frac{\dot{z}'}{z} \right) R'_k + c_s^2 k^2 R_k = 0 ,
\]

(23)
where
\[ z \equiv a \left( \rho_\phi + p_\phi \right)^{1/2} / c_s H , \tag{24} \]
\( R \) is the curvature perturbation
\[ R \equiv \psi + \left( \frac{H}{\dot{\phi}} \right) \delta \phi , \tag{25} \]
and \( \psi, \delta \phi \) correspond to the metric perturbation and the scalar field perturbation, respectively. The derivative in (23) is taken with respect to the conformal time, \( \eta = \int dt/a(t) \) and \( c_s \) is the effective sound speed of perturbations in the scalar field [37]
\[ c_s^2 \equiv \left[ \frac{\partial L / \partial X}{(\partial L / \partial X) + (2X) (\partial^2 L / \partial X^2)} \right] . \tag{26} \]
Rewriting (23) in terms of the Mukhanov-Sasaki variable \( u_k \equiv z R_k \), one gets
\[ u_k'' + \left( c_s^2 k^2 - \frac{z''}{z} \right) u_k = 0 \] (27)
The key to our understanding of gravitational clustering is provided by the sound speed. Substituting (2) into (26) we get
\[ c_s^2 = \frac{1}{2 \alpha - 1} . \tag{28} \]
We therefore find that the sound speed is a constant, and that for \( \alpha \gg 1, \ c_s^2 \to 0 \). In other words, when the value of the non-canonical parameter \( \alpha \) is large, the sound speed vanishes, and the scalar field begins to behave like a pressureless fluid.

An important property of our model follows from (17) and (28), namely, when \( \alpha \gg 1 \), the background universe expands like \( \Lambda \)CDM, while its clustering properties could resemble those of cold dark matter or even warm dark matter. The non-canonical scalar therefore provides a unified prescription for dark matter and dark energy since both components are sourced by the same non-canonical scalar field. We elaborate on this issue below.

The evolution equation for the linear density contrast of the X-fluid in (13), namely \( \delta = \frac{\rho_X - \bar{\rho}_X}{\bar{\rho}_X} \), evaluated on sub-horizon scales \(|k| \gg H/c\), is given by [20]
\[ \delta_k'' = -[2 + A - 3(2w - c_s^2)] \delta_k' \]
\[ + \frac{3}{2} \Omega_X (1 - 6c_s^2 + 8w - 3w^2) \delta_k - \left( \frac{k c_s}{a H} \right)^2 \delta_k \] (29)
FIG. 3. The scale-dependence of linear gravitational clustering is illustrated. The linear density contrast, $\delta$, is shown as a function of the expansion factor, $a$, for the non-canonical scalar field with $\alpha = 5 \times 10^4$, $10^5$, $5 \times 10^5$ and also for ΛCDM (bottom to top with ΛCDM at the top). Two scales are considered: $k = 0.01 h/Mpc$ (left) and $k = 0.1 h/Mpc$ (right).

where $w$, the equation of state of the X-fluid, is described by (15), $\Omega_X = \frac{8\pi G\rho_X}{3H^2}$, $A = \frac{\left(\frac{H^2}{2H^2}\right)^{\prime}}{2H^2}$ and $\dot{\epsilon} = \frac{d}{d\log a}$. We evolve this equation from the decoupling epoch ($a = 10^{-3}$) when it is reasonable to assume $\delta_k \sim a$ and $\frac{\delta_k}{a} \sim 1$. Our results are shown in figure 3 for two different scales, $k = 0.01h/Mpc$, $0.1h/Mpc$. We find that gravitational clustering in this model is scale-dependent. On very large scales $k \leq 0.01h/Mpc$, scalar field models with large values of $\alpha \geq 10^4$ display clustering identical to ΛCDM. However on smaller scales $k \geq 0.1h/Mpc$, the density contrast in our model is suppressed relative to ΛCDM even for $\alpha$ values as large as $10^5$, for which the background expansion is indistinguishable from ΛCDM, as demonstrated in figure 1.

We have thus demonstrated that our model is capable of mimicking the behaviour of a dark matter + vacuum energy model both with respect to cosmological expansion and gravitational clustering. Another possibility provided by our model is that the non-canonical scalar comes as an add-on to dark matter (instead of replacing it). This is the usual procedure adopted by models such as quintessence, in which the matter part of the Lagrangian remains unchanged while dark energy is sourced by a potential such as $V \propto \phi^{-\alpha}$. It is easy to see that the expansion history in such a model (consisting of conventional dark matter and a
The clustering properties of the non-canonical scalar are once more given by (28). The new model (30) therefore describes a universe filled with the cosmological constant and two kinds of dark matter: the first being the usual dark matter whereas, depending upon the value of \( w \equiv c_s^2 \), the second component, \( \Omega_{0X} \), can behave like a hot, warm or cold dark matter component. This could have interesting cosmological consequences. For instance, as recently demonstrated in [38], a model with \( \Omega_{0X} \ll \Omega_{0m} \) could help alleviate the tension faced by \( \Lambda \)CDM in simultaneously fitting CMB and weak lensing data. A subdominant component of dark matter, like the one discussed in this paper, could also seed early black hole formation, as discussed in [39].

III. DISCUSSION

In this paper we have demonstrated that a single non-canonical scalar field can play the dual role of describing both dark matter and dark energy. To summarize, a non-canonical scalar field rolling along a flat potential has a kinetic energy which decreases rapidly with time and a potential energy which decreases much more slowly. For large values of the non-canonical parameter \( \alpha \) in (2), the kinetic energy can play the role of dark matter while the potential energy behaves like a cosmological constant. The expansion history of this model therefore mimics \( \Lambda \)CDM.

On its own this result, while surprising, is not unique. It is well known that for a given expansion history, \( a(t) \), it is always possible to reconstruct the canonical scalar field potential \( V(\phi) \) which will reproduce the expansion history precisely [40]. Therefore, in principle, it is possible to obtain a potential which reproduces the \( \Lambda \)CDM expansion rate \( a(t) \propto \left( \sinh \frac{3}{2} \sqrt{\frac{1}{3} \Lambda t} \right)^{2/3} \). However the fact that (non-oscillating) canonical scalar fields do not cluster on sub-horizon scales, prevents this potential from providing a realistic portrayal of \( \Lambda \)CDM; also see [21].

The big advantage of non-canonical scalars arises from the fact that, for large values of the non-canonical parameter \( \alpha \), the sound speed in (28) drops to zero. Therefore the
non-canonical scalar field can cluster, in contrast to canonical models in which clustering is absent.

It is necessary to point out that properties similar to those possessed by our model have also appeared in other discussions of unification (of dark matter and dark energy). For instance in [26] Scherrer proposed a non-canonical model which had an expansion rate exactly like ΛCDM. Our model differs from [26] in two main respects:

(i) The purely kinetic Lagrangian $\mathcal{L}(X)$ in [26] possesses an extremum in $X$ about which it is expanded in a Taylor series. Our Lagrangian, on the other hand, has a power law kinetic term with no extremum.

(ii) The sound velocity in [26] drops off as $a^{-3}$ whereas in our model the sound velocity is a constant and is given by (28).

We therefore conclude that whereas the expansion history in our model and in [26] is identical (corresponding to ΛCDM), the nature of gravitational clustering in these two models is rather different. Indeed, gravitational clustering in our model is scale-dependent, and is sensitive to the choice of $\alpha$.

![FIG. 4. Perturbations, $\delta_k$, in the non-canonical scalar field model are shown at the present epoch. $\delta_k$ is plotted against $k$ for $\alpha = 2 \times 10^7$, $7 \times 10^7$, $5 \times 10^8$ (bottom to top, solid lines). Perturbations in warm dark matter consisting of a sterile neutrino with mass = 0.5, 1, 3 KeV are also shown (bottom to top, dashed lines). The top most solid line corresponds to ΛCDM.]

In this context one should note that the value of $\alpha$ can never be infinitely large. Consider two models characterized by $\alpha_1$ and $\alpha_2$ where $1 \ll \alpha_1 \ll \alpha_2$. Since the Jeans length in our
model is
\[ \lambda_J \sim c_s / \sqrt{G \rho}, \text{ where } c_s = \frac{1}{\sqrt{2 \alpha - 1}}, \]  
(31)
it follows that the clustering properties of our field will be sensitive to the value of \( \alpha \). Clearly gravitational clustering in the model with \( \alpha_1 \) will be inhibited on small scales relative to the model with \( \alpha_2 \). This property was illustrated by figure 3. It is interesting that a similar situation arises when dark matter is sourced by an oscillating massive canonical scalar field with mass \( m \) \cite{12–15}. In this case, as shown in \cite{14, 16}, the Jeans length depends upon the scalar field mass as \( \lambda_J \sim (G \rho)^{-1/4} m^{-1/2} \). Ultra-light scalars are therefore able to suppress clustering on small scales, thereby providing a resolution to the substructure and cuspy core problems which plague standard cold dark matter.\(^2\) One expects that a similar mechanism will operate in our model as well, with \( \alpha \) playing the role of \( m \). (A key distinction between the two models is that whereas the canonical scalar field needs to oscillate in order to describe dark matter, the non-canonical field does not oscillate but simply rolls along its flat potential.)

A useful analogy can also be drawn between clustering in our model and that in particle dark matter. Consider the case when a relic particle of mass \( m \) (such as a neutralino or a sterile neutrino) plays the role of dark matter. In this case perturbations on scales smaller than the free-streaming distance \cite{8}
\[ \lambda_{fs} \sim 40 \text{ Mpc} \left( \frac{m}{30 \text{ eV}} \right)^{-1} \]  
(32)
are effectively erased during the relativistic motion of the particle.

A larger value of \( m \) in (32) leads to a smaller value of \( \lambda_{fs} \). Comparing (32) with (31) we find that the role of mass, in particle dark matter models, is played by the parameter \( \alpha \) in our model. In other words, whereas very large values of \( \alpha \) will make clustering in our model resemble cold dark matter, smaller values of \( \alpha \) will make our model closer to warm dark matter. We demonstrate the similarity of our model with the sterile neutrino model for warm dark matter in figure 4. In this figure we show \( \delta_k \) for our model, obtained by solving equation (25), and compare it with the density contrast in the warm dark matter model, as described in \cite{41}; also see \cite{42}. This figure clearly demonstrates that, for suitable values of \( \alpha \),

\(^2\) The substructure problem relates to the observation that CDM predicts an order of magnitude more faint galaxies than are observed. The cuspy core problem refers to the tension between simulations of CDM, which predict a density profile steeper than \( \rho \sim 1/r \) for dark matter halos, and the much shallower ‘cored’ profiles observed in individual galaxies; see \cite{10} and references therein.
clustering in our model can be like cold or warm dark matter, even as its expansion history mimicks ΛCDM. The cosmological properties of our model will be examined in greater detail in a companion paper.

Finally, we would like to draw attention to the following interesting property of the non-canonical scalar which follows from (12). If the value of $\alpha$ is large, then the velocity of the scalar field freezes to an almost constant value, i.e. for $\alpha \gg 1$, $X \to$ constant. This is completely unlike the behaviour of a canonical scalar ($\alpha = 1$) for which $\dot{\phi} \propto a^{-3}$ if $V = \text{constant}$. In the canonical case the universe inflates once $V > X$, at which point since $a \propto e^{\int \sqrt{V} dt}$ and $\dot{\phi} \propto a^{-3}$, the value of $\dot{\phi}$ rapidly drops to zero and the motion of the scalar field comes to an abrupt halt. Not so for the non-canonical scalar which can continue to roll along a flat potential even as the universe inflates.

This may have interesting consequences. It has been postulated [1, 43] that the vacuum energy is not unchanging but a dynamical quantity whose value changes abruptly during the course of a phase transition. Such behaviour is mimicked by the step-like potential (22). Since this potential is piece-wise flat, it can, in principle, describe a universe which inflates twice – once at early times, and then again at the current cosmological epoch. But for this to occur, the scalar field must continue to roll along $V$ for a protracted time interval (corresponding to the age of the universe) and not stop in between (as in the canonical case). In the non-canonical context, for large values of $\alpha$, the kinetic term is almost a constant, which should allow the scalar field to roll along its potential as the latter cascades to lower values. Such a scenario may provide us with a model of Quintessential-Inflation and this possibility will be examined in greater detail in a companion paper.

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3 While we have drawn attention to the close similarity between k-essence models and cold/warm dark matter, this analogy has been drawn at the linearized level. At the nonlinear level these two approaches may give rise to distinct scenario's of structure formation, as noted in [39]. This issue deserves more scrutiny.
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