q-Rung Orthopair Fuzzy N-Soft Aggregation Operators and Corresponding Applications to Multiple-Attribute Group Decision Making

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q-Rung Orthopair Fuzzy N-Soft Aggregation Operators and Corresponding Applications to Multiple-Attribute Group Decision Making

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Abstract

In this paper, by integrating the q-rung orthopair fuzzy set (q-ROFS) with the N-soft set (NSS), we first propose a q-rung orthopair fuzzy N-soft set (q-ROFNSS). Based on the q-ROFNSS, then we explore the q-rung orthopair fuzzy N-soft weighted average (q-ROFNSWA) operator and q-rung orthopair fuzzy N-soft weighted geometric (q-ROFNSWG) operator, and investigate some properties of the q-ROFNSWG operator and q-ROFNSWG operator including idempotency, monotonicity and boundedness. Finally, two kinds of multiple-attribute group decision making (MAGDM) methods based on q-rung orthopair fuzzy N-soft aggregation operators are established. In addition, a practical example is provided to illustrate the effectiveness and correctness of the new decision-making approaches. Through comparison with existing methods, the advantages of our method are elaborated.

Key words: q-ROFS, q-ROFNSS, q-ROFNSWA operator, q-ROFNSWG operator, MAGDM methods

1 Introduction

In order to solve various types of uncertainties and complex MAGDM problems, the theory of fuzzy sets is proposed by Zadeh [30]. Later on, Atanassov [1] introduced the intuitionistic fuzzy set (IFS) theory to extend the concept of fuzzy set. Because decision makers consider both membership degree and non-membership degree in decision making process, IFS theory is more accurate to deal with the uncertainties and MAGDM problems than fuzzy set. However, this theory needs to satisfy the restricted condition that the sum

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of the degrees of membership and the degrees of non-membership is less than or equal to 1. Under this restricted condition, the range of applications of IFSs is very narrow. Therefore, Yager [29] proposed the Pythagorean fuzzy set (PFS) in which the restricted condition is that the square sum of the membership degree and non-membership degree is less than or equal to 1. However, both IFS and PFS have limitations to describe uncertainty and fuzziness problems. For example, when decision makers adopt 0.6 and 0.9 as membership degree and non-membership degree to express their opinions, it is obvious beyond the range of applications of IFSs or PFSs. For this, Yager [27] again proposed a new theory which is referred to as the q-ROFS, in which the restricted condition is that the sum of the qth power of membership degree and non-membership degree is less than or equal to 1. Obviously, the range of its application is more accurate and sufficient than IFSs and PFSs. Since then, many scholars have studied the q-ROFS in many fields including the aggregation operators [4, 11, 13, 20, 24] and the combination with other theories [9, 10, 22]. On one hand, in terms of aggregation operators there exist plenty of research results. For example, Liu and Wang [11] gave the concept of q-rung orthopair fuzzy numbers (q-ROFNs), proposed the q-rung orthopair fuzzy weighted average (q-ROFWA) operator and applied it to MAGDM problems. In [12], the authors introduced the classic Bonferroni average operator into the q-ROFS, and proposed a cluster of q-rung orthopair fuzzy Bonferroni means (q-ROFBM) operators. Wang [21] presented a series of q-rung orthopair fuzzy Muirhead means (q-ROFMM) operators and applied them to decision making problems. Xing [23] defined a q-rung orthopair fuzzy point weighted aggregation operator and explored its application to MAGDM problems. In [15], Liu discussed the MAGDM problems based on q-rung orthopair fuzzy Heronian mean operators. Peng [18] proposed the exponential operation of q-ROFNs, and also studied the exponential aggregation operator of q-ROFSs. Liu and Wang [14] gave a new q-ROFBM operator based on Archimedean t-norm and t-conorm, and explored its applications to MAGDM problems. On the other hand, in terms of the combination with other theories, a series of fusion models have been produced in order to extend q-ROFSs. For example, by combining q-ROFSs and rough sets, Hussain [8] proposed q-rung orthopair fuzzy rough sets, and studied their applications to MAGDM problems based on the classic decision-making method TOPSIS. In [9], the authors defined the concept of q-rung orthopair hesitant fuzzy sets through integrating hesitant fuzzy sets with q-ROFSs. Hussain [7] presented a q-ROFSS model by combining soft sets and q-ROFSs. On the basis of the model, he gave the q-rung orthopair fuzzy soft average aggregation operators and considered their applications. In addition, Joshi [10] proposed the interval-value q-ROFSs and some related concepts. Wang [19] initiated a generalized interval-valued orthopair fuzzy set and applied it to decision making problems.

In real life, there are a lot of ambiguities. In order to solve such problems, many uncertain theories have been produced, such as the fuzzy set theory [30], IFS theory [1], PFS theory [29] and other theories based on soft set [16, 17, 25, 26, 28]. It is noted that most of
the works related on the theories focus on binary estimation (either 0 or 1), or else real numbers between 0 and 1. However, we often find that the data structure is not all binary evaluation structure in practical life, such as evaluation systems, ranking or voting situations. In reality, we usually use the number of points and stars to represent the ranking of evaluation objects. For example, one point means bad; one star means better; two stars mean good; three stars mean well; four stars mean best. To solve this issue, Fatimah [5] extended the concept of soft set theory, proposed a new model called NSS, and explained the importance of ordered grades in practical problems under non-binary evaluation environment. Afterwards, by combining fuzzy sets and NSSs, Akram [2] proposed the fuzzy N-soft set (FNSS). It is noted that the FNSS only considers the membership degree of the parameterized objects without considering non-membership degree. In order to remedy this defect, Akram [3] presented the intuitionistic fuzzy N-soft set (IFNSS) theory by integrating IFSs with NSSs. However, in practical problems, when the decision maker evaluates the decision object from two perspectives of negative and positive, the sum of membership degree and non-membership degree provided by the decision maker may be greater than 1. In other words, IFNSSs cannot fully express decision information, which will result in the loss of decision information. In light of that, through combining PFS and NSS, Zhang et al. [31] initiated the theory of Pythagorean fuzzy N-soft set and applied it to MAGDM problems. However, as previously stated, both IFS and PFS have limitations to describe uncertainty and fuzziness problems. Therefore, whether it is IFNSSs or Pythagorean fuzzy N-soft set, they have fatal flaws in handling uncertain information. As a result, considering that membership degree and non-membership degree provided by the decision maker may express knowledge and information in a narrow range, it is natural to extend IFNSSs and Pythagorean fuzzy N-soft set into generalized orthopair fuzzy environment. In this paper, we attempt to establish the q-ROFNSSs through extending IFNSSs and Pythagorean fuzzy N-soft set into generalized orthopair fuzzy environment. In addition, we focus on the weighted average (WA) operators and weighted geometric (WG) operators to handle MAGDM problems based on the q-ROFNSSs.

The structure of this paper is arranged as follows. The next section reviews some basic definitions on q-ROFSs and NSSs. Section 3 gives the concept of q-ROFNSS. In Section 4, we propose the q-ROFNSWA operator and q-ROFNSWG operator based on the q-ROFNSS, and explore some properties of the q-ROFNSWG operator and q-ROFNSWG operator, such as idempotency, monotonicity and boundedness. Section 5 establishes two algorithms related to the q-ROFNSWA operator and q-ROFNSWG operator to handle MAGDM problems. In Section 6, a practical example is provided to illustrate the effectiveness and practicality of our decision making method. Comparative analysis with the other methods is also conducted. We conclude in Section 7.
2 Preliminaries

In this section, we recall some fundamental notions concerning q-ROFSs and NSSs that are useful for discussions in the next sections.

Definition 2.1 ([27]) Let $C$ be an universal set, then a q-ROFS $Z$ on $C$ is expressed as follows:

$$Z = \{(c, \mu_Z(c), \eta_Z(c))| c \in C\},$$

where $\mu_Z(c)$ and $\eta_Z(c)$ denote the membership degree and non-membership degree, respectively, with the condition that for all $\mu_Z(c), \eta_Z(c) \in [0, 1]$ and $q \geq 1$, $0 \leq \mu_Z(c)^q + \eta_Z(c)^q \leq 1$.

We call $(\mu_Z(c), \eta_Z(c))$ a q-ROFN, which can be written as $\xi = (\mu, \eta)$.

Definition 2.2 ([11]) Given that $\xi = (\mu, \eta)$ and $\xi' = (\mu', \eta')$ are two q-ROFNs, and $\rho$ is a positive real number. Then the basic operations is defined as follows:

$$\xi \oplus \xi' = ((\mu^q + \mu'^q - \mu^q \mu'^q)^{1/q}, \eta'),$$  

$$\xi \otimes \xi' = (\mu \mu', (\eta^q + \eta'^q - \eta^q \eta'^q)^{1/q}),$$  

$$\rho \xi = ((1 - (1 - \mu^q)^{1/q} \rho, \eta)^q),$$  

$$\xi^\rho = (\mu^\rho, (1 - (1 - \eta^q)^{1/q} \rho).$$

Definition 2.3 ([11]) Let $\xi = (\mu, \eta)$ and $\xi' = (\mu', \eta')$ be two q-ROFNs. The score function and the accuracy function of $\xi$ can be defined as $S(\xi) = \mu^q - \eta^q$ and $H(\xi) = \mu^q + \eta^q$, respectively. For $\xi = (\mu, \eta)$ and $\xi' = (\mu', \eta')$, then

(1) if $S(\xi) > S(\xi')$, then $\xi > \xi'$.

(2) if $S(\xi) = S(\xi')$, then

if $H(\xi) > H(\xi')$, then $\xi > \xi'$; if $H(\xi) = H(\xi')$, then $\xi = \xi'$.

In the following, Fatimah et al. [5] introduced the concept of N-soft sets.

Definition 2.4 ([5]) Suppose that $S$ is an universal set of objects and $A$ is a set of attributes, $L \subseteq A$. Given that $R = \{0, 1, 2, ..., N - 1\}$ is a family of ordered grades with $N = \{2, 3, ...\}$. A triple $(Q, L, N)$ is referred to as an NSS over $S$ if $Q$ is a mapping $Q : L \rightarrow 2^{S \times R}$, where for each $l \in L$ and $s \in S$, there exists a $(s, r_l) \in S \times R$ such that $(s, r_l) \in Q(l)$, $s \in S, r_l \in R$.

3 q-Rung orthopair fuzzy N-soft set

In this current section, we shall propose the concept of q-ROFNSSs, and an example is provided to elaborate the concept.
Definition 3.1 Suppose that $S$ is an universe set of objects and $A$ is a collection of parameters. Given that $R = \{0, 1, 2, \ldots, N - 1\}$ is a set of ordered grades where $N = \{2, 3, \ldots\}$. For $K \subseteq A$, a triple $(f_q, K, N)$ is referred to as a $q$-ROFNSS on $S$, if $f_q$ is a mapping such that $f_q : K \to 2^{S \times R} \times q\text{-ROFN}$, in which for each $s \in S$ and $k \in K$, there exists a $(s, r(k)) \in S \times R$ such that $r(k) \in R$ and $q\text{-ROFN} = (\mu(k), \eta(k))$, i.e., $(f_q, K, N)$ can be expressed as

$$f_q(k) = ((s, r(k)), (\mu(k), \eta(k))),$$

where $r(k)$ denotes the level of the element attribute; $\mu(k)$ and $\eta(k)$ denote the membership and non-membership degrees, respectively, satisfying the condition that $0 \leq \mu^q(k) + \eta^q(k) \leq 1$, for all $k \in K$ and $q \geq 1$.

Example 3.2 Assume that $S = \{s_1, s_2, s_3, s_4, s_5\}$ is a collection of five electronic products and $K = \{k_1 = \text{Shape}, k_2 = \text{Performance}, k_3 = \text{Price}, k_4 = \text{Lifetime}\}$ is a set of parameters. The evaluation information provided by experts can be expressed as Table 1, where

- ‘◦’ means ‘bad’,
- ‘⋆’ means ‘better’,
- ‘⋆⋆’ means ‘good’,
- ‘⋆⋆⋆’ means ‘well’,
- ‘⋆⋆⋆⋆’ means ‘best’.

Table 1: Evaluation information provided by experts

| $S$ | $k_1$ | $k_2$ | $k_3$ | $k_4$ |
|-----|-------|-------|-------|-------|
| $s_1$ | ⋆⋆⋆⋆ | ⋆⋆⋆⋆ | ⋆⋆⋆ | ⋆⋆ |
| $s_2$ | ⋆⋆ | ⋆ | ⋆⋆ | ⋆⋆ |
| $s_3$ | ⋆ | ⋆ | ⋆ | ⋆ |
| $s_4$ | ⋆ | ⋆ | ⋆ | ⋆ |
| $s_5$ | ⋆ | ⋆ | ⋆ | ⋆ |

For being convenient to study, we use numbers to replace the symbols in Table 1. Therefore, the evaluation data of Table 1 can be converted into a 5-soft set which is shown in Table 2, where

- 0 stands for ‘◦’,
- 1 stands for ‘⋆’,
- 2 stands for ‘⋆⋆’,
- 3 stands for ‘⋆⋆⋆’,
- 4 stands for ‘⋆⋆⋆⋆’.

Due to the ambiguity and complexity of the data information, the evaluation data for these electronic products are characterized by data in the range of 0 to 1. Therefore, when the expert evaluates an electronic product to determine its grade, we generally determine
by experts meet the following requirements: for all \( k \) ROFNSSs to establish how grades are scaled. The grade standard of the evaluation provided by the degree of membership. Therefore, we introduce q-ROFNSSs to establish how grades are scaled. The grade standard of the evaluation provided by experts meet the following requirements: for all \( k \in K \), \( 0 \leq \mu(k) \leq 1 \) and \( q \geq 1 \) with \( 0 \leq \mu(k) + \eta(k) \leq 1 \),
\[
\begin{align*}
& r(k) = 0, \text{ if } 0 \leq \mu(k) < 0.2, \\
& r(k) = 1, \text{ if } 0.2 \leq \mu(k) < 0.4, \\
& r(k) = 2, \text{ if } 0.4 \leq \mu(k) < 0.6, \\
& r(k) = 3, \text{ if } 0.6 \leq \mu(k) < 0.8, \\
& r(k) = 4, \text{ if } 0.8 \leq \mu(k) < 1.
\end{align*}
\]

Now take \( q = 3 \). By Definition 3.1, the q-rung orthopair fuzzy \( 5 \)-soft set can be expressed as follows:

\[
(f_q, K, 5) = \begin{cases} 
  f_q(k_1) = \{((s_1, 3), (0.71, 0.32)), ((s_2, 2), (0.47, 0.42)), ((s_3, 2), (0.50, 0.92)), \\
  ((s_4, 0), (0.13, 0.93)), ((s_5, 3), (0.66, 0.87))\}, \\
  f_q(k_2) = \{((s_1, 4), (0.91, 0.53)), ((s_2, 1), (0.27, 0.73)), ((s_3, 2), (0.47, 0.54)), \\
  ((s_4, 1), (0.28, 0.79)), ((s_5, 2), (0.45, 0.55))\}, \\
  f_q(k_3) = \{((s_1, 3), (0.68, 0.73)), ((s_2, 3), (0.67, 0.32)), ((s_3, 1), (0.32, 0.73)), \\
  ((s_4, 2), (0.49, 0.89)), ((s_5, 1), (0.23, 0.79))\}, \\
  f_q(k_4) = \{((s_1, 2), (0.45, 0.56)), ((s_2, 2), (0.47, 0.89)), ((s_3, 3), (0.77, 0.25)), \\
  ((s_4, 1), (0.25, 0.73)), ((s_5, 2), (0.47, 0.58))\}.
\end{cases}
\]

### 4 q-Rung orthopair fuzzy N-soft aggregation operators

In this section, we will introduce the q-ROFNSWG operator and q-ROFNSWA operator.

**Definition 4.1** Suppose that \( \xi_i = (\mu_i, \eta_i) (i = 1, 2, 3, \ldots, n) \) is a collection of q-ROFNs. Given a mapping \( q - \text{ROFNSWG} : H^n \to H \) such that

\[
q - \text{ROFNSWG}(\xi_1, \xi_2, \ldots, \xi_n) = \xi_1^{r(k_1)e_1} \otimes \xi_2^{r(k_2)e_2} \otimes \cdots \otimes \xi_n^{r(k_n)e_n},
\]

where \( H \) is the set of all q-ROFNs, \( r(k) = (r(k_1), r(k_2), \ldots, r(k_n)) \) denotes the level of the
element attribute, and $E = \{e_1, e_2, ..., e_n\}$ is weight vector of $\xi_i = (\mu_i, \eta_i)(i = 1, 2, 3, ..., n)$ with $e_i \in [0, 1]$ and $\sum_{i=1}^{n} e_i = 1$. Then the mapping $q$-ROFNSWG is referred to as a $q$-ROFNSWG operator.

**Theorem 4.2** Let $\xi_i = (\mu_i, \eta_i)(i = 1, 2, 3, ..., n)$ be a collection of $q$-ROFNs. Then the aggregation result obtained by Eq. (1) is still a $q$-ROFN, and has

$$q - \text{ROFNSWG}(\xi_1, \xi_2, ..., \xi_n) = \left( \prod_{i=1}^{n} \mu_i^{r(k_i)e_i}, \left(1 - \prod_{i=1}^{n} (1 - \eta_i^q)^{r(k_i)e_i}\right)^{1/q}\right)$$

(2)

**Proof.** Firstly, this theorem can be proved by using the mathematical inductive method.

(1) When $n = 2$, we have

$$\xi_1^{r(k_1)e_1} = \left(\mu_1^{r(k_1)e_1}, \left(1 - \eta_1^q\right)^{r(k_1)e_1}\right)^{1/q},$$

$$\xi_2^{r(k_2)e_2} = \left(\mu_2^{r(k_2)e_2}, \left(1 - \eta_2^q\right)^{r(k_2)e_2}\right)^{1/q}.$$

Then,

$$q - \text{ROFNSWG}(\xi_1, \xi_2) = \xi_1^{r(k_1)e_1} \otimes \xi_2^{r(k_2)e_2}$$

$$= \left(\mu_1^{r(k_1)e_1}, \mu_2^{r(k_2)e_2}, \left(\left(1 - \eta_1^q\right)^{r(k_1)e_1}\right)^{1/q} + \left(1 - \eta_2^q\right)^{r(k_2)e_2}\right)^{1/q}$$

$$- \left(1 - \eta_2^q\right)^{r(k_1)e_1} \left(1 - \eta_1^q\right)^{r(k_2)e_2}$$

$$= \left(\mu_1^{r(k_1)e_1}, \mu_2^{r(k_2)e_2}, \left(1 - \eta_1^q\right)^{r(k_1)e_1} + 1 - \eta_2^q\right)^{1/q}$$

$$- \left(1 - \eta_1^q\right)^{r(k_1)e_1} \left(1 - \eta_2^q\right)^{r(k_2)e_2}\right)^{1/q}$$

$$= \left(\mu_1^{r(k_1)e_1}, \mu_2^{r(k_2)e_2}, \left(1 - \eta_1^q\right)^{r(k_1)e_1} (1 - \eta_2^q\right)^{r(k_2)e_2}\right)^{1/q}$$

$$= \left(\prod_{i=1}^{2} \mu_i^{r(k_i)e_i}, \left(1 - \prod_{i=1}^{2} (1 - \eta_i^q)^{r(k_i)e_i}\right)^{1/q}\right).$$

(2) When $n = j$, we suppose that the equation is valid, and it is shown as follows

$$q - \text{ROFNSWG}(\xi_1, \xi_2, ..., \xi_j) = \left(\prod_{i=1}^{j} \mu_i^{r(k_i)e_i}, \left(1 - \prod_{i=1}^{j} (1 - \eta_i^q)^{r(k_i)e_i}\right)^{1/q}\right).$$
Then when $n = j + 1$, we have

$$q - ROFNSWG(\xi_1, \xi_2, \ldots, \xi_{j+1}) = \xi_1^{r(k_1)e_1} \otimes \ldots \otimes \xi_j^{r(k_j)e_j} \otimes \xi_{j+1}^{r(k_{j+1})e_{j+1}}$$

$$= \left( \prod_{i=1}^{j} \mu_i^{r(k_i)e_i}, \left( 1 - \prod_{i=1}^{j} (1 - \eta_i^q)^{r(k_i)e_i} \right)^{1/q} \right) \otimes \left( \mu_{j+1}^{r(k_{j+1})e_{j+1}}, \left( 1 - (1 - \eta_{j+1}^q)^{r(k_{j+1})e_{j+1}} \right)^{1/q} \right)$$

$$= \left( \prod_{i=1}^{j} \mu_i^{r(k_i)e_i}, \mu_{j+1}^{r(k_{j+1})e_{j+1}}, \left( 1 - \prod_{i=1}^{j} (1 - \eta_i^q)^{r(k_i)e_i} \right)^{1/q} q \right)$$

$$+ \left( 1 - (1 - \eta_{j+1}^q)^{r(k_{j+1})e_{j+1}} \right)^{1/q} q - \left( 1 - \prod_{i=1}^{j} (1 - \eta_i^q)^{r(k_i)e_i} \right)^{1/q} q$$

$$= \left( 1 - (1 - \eta_{j+1}^q)^{r(k_{j+1})e_{j+1}} \right)^{1/q} q$$

$$= \left( 1 - (1 - \eta_{j+1}^q)^{r(k_{j+1})e_{j+1}} \right)^{1/q} q$$

Therefore, when $n = j + 1$, the equation is valid.

(3) According to the steps (1) and (2), we can obtain the conclusion that Eq. (2) holds for any $i$.

Secondly, we will prove that the aggregation result obtained by Eq. (2) is still a $q$-ROFN. Due to $\mu^q + \eta^q \leq 1$, we have $\eta^q \leq 1 - \mu^q$. So

$$\left( \prod_{i=1}^{n} \mu_i^{r(k_i)e_i} \right)^q + \left( 1 - \prod_{i=1}^{n} (1 - \eta_i^q)^{r(k_i)e_i} \right)^{1/q} q \leq \left( \prod_{i=1}^{n} \mu_i^{r(k_i)e_i} \right)^q + \left( 1 - \prod_{i=1}^{n} \mu_i^{r(k_i)e_i} \right)^q = 1.$$}

Hence, the aggregation result obtained by Eq. (2) is still a $q$-ROFN. \[\square\]

In the following, we shall explore some properties of the $q - ROFNSWG$ operator, like idempotency, monotonicity and boundedness.

**Theorem 4.3 (Idempotency)** Let $\xi_i = (\mu_i, \eta_i)(i = 1, 2, 3, \ldots, n)$ be a family of $q$-ROFNs. If
\[ \xi = (\mu, \eta), \quad r(k_i) = r(k) = 1, \text{ then} \]
\[ q - ROFNSWG(\xi_1, \xi_2, ..., \xi_n) = \xi. \]

**Proof.** From Eq. (2), we have

\[ q - ROFNSWG(\xi_1, \xi_2, ..., \xi_n) = \left( \prod_{i=1}^{n} \mu_i r(k_i) e_i, \left( 1 - \prod_{i=1}^{n} (1 - \eta_i^q) r(k_i) e_i \right)^{1/q} \right) \]
\[ = \left( \prod_{i=1}^{n} \mu_i r(k) e_i, \left( 1 - \prod_{i=1}^{n} (1 - \eta_i^q) r(k) e_i \right)^{1/q} \right) \]
\[ = \left( \sum_{i=1}^{n} r(k) e_i, \left( 1 - (1 - \eta_i^q) \sum_{i=1}^{n} r(k) e_i \right)^{1/q} \right) = (\mu, \eta) = \xi. \]

Therefore, the \( q - ROFNSWG \) operator is idempotency. \( \square \)

**Theorem 4.4 (Monotonicity)** Let \( \xi_i = (\mu_i, \eta_i) \) and \( \xi'_i = (\mu'_i, \eta'_i)(i = 1, 2, 3, ..., n) \) be two \( q \)-ROFNs. For any \( i \), if \( \mu_i \geq \mu'_i, \eta_i \leq \eta'_i \), then

\[ q - ROFNSWG(\xi_1, \xi_2, ..., \xi_n) \geq q - ROFNSWG(\xi'_1, \xi'_2, ..., \xi'_n). \]

**Proof.** Since \( \mu_i \geq \mu'_i, \eta_i \leq \eta'_i \) for all \( i \), we have \( \prod_{i=1}^{n} \mu_i r(k_i) e_i \geq \prod_{i=1}^{n} \mu'_i r(k_i) e_i \),

\[ 1 - \eta_i^q \geq 1 - \eta'_i^q \Rightarrow \left( 1 - \prod_{i=1}^{n} (1 - \eta_i^q) r(k_i) e_i \right)^{1/q} \leq \left( 1 - \prod_{i=1}^{n} (1 - \eta_i'^q) r(k_i) e_i \right)^{1/q}. \]

Thus, when \( \xi = q - ROFNSWG(\xi_1, \xi_2, ..., \xi_n) \) and \( \xi' = q - ROFNSWG(\xi'_1, \xi'_2, ..., \xi'_n) \), we obtain

\[ S(\xi) = \left( \prod_{i=1}^{n} \mu_i r(k_i) e_i \right)^q - \left( \prod_{i=1}^{n} (1 - \eta_i^q) r(k_i) e_i \right)^{q} \]
\[ \geq \left( \prod_{i=1}^{n} \mu_i r(k_i) e_i \right)^q - \left( \prod_{i=1}^{n} (1 - \eta_i'^q) r(k_i) e_i \right)^{q} = S(\xi'). \]

(1) When \( S(\xi) > S(\xi') \), by Definition 2.3 we have

\[ q - ROFNSWG(\xi_1, \xi_2, ..., \xi_n) > q - ROFNSWG(\xi'_1, \xi'_2, ..., \xi'_n). \]

(2) When \( S(\xi) = S(\xi') \), we have

\[ \prod_{i=1}^{n} \mu_i r(k_i) e_i = \prod_{i=1}^{n} \mu'_i r(k_i) e_i, \quad \left( 1 - \prod_{i=1}^{n} (1 - \eta_i^q) r(k_i) e_i \right)^{1/q} = \left( 1 - \prod_{i=1}^{n} (1 - \eta_i'^q) r(k_i) e_i \right)^{1/q}. \]
In the light of Definition 2.3, we obtain

\[ H(\xi) = \left( \prod_{i=1}^{n} \mu_i^{r(k_i)e_i} \right)^q + \left( \frac{1}{q} \prod_{i=1}^{n} (1 - \eta_i^{q} r(k_i)e_i)^{1/q} \right)^q \]

\[ = \left( \prod_{i=1}^{n} \mu_i^{r(k_i)e_i} \right)^q + \left( \frac{1}{q} \prod_{i=1}^{n} (1 - \eta_i^{q} r(k_i)e_i)^{1/q} \right)^q = H(\xi'). \]

Therefore, based on (1) and (2) we make the conclusion that

\[ q - ROFNSWG(\xi_1, \xi_2, ..., \xi_n) \geq q - ROFNSWG(\xi'_1, \xi'_2, ..., \xi'_n). \]

\[ \square \]

**Theorem 4.5** (Boundedness) Given that \( \xi_i = (\mu_i, \eta_i)(i = 1, 2, 3, ..., n) \) is a q-ROFN. If \( \xi^- = \left( \bigwedge_{i=1}^{n} \mu_i, \bigvee_{i=1}^{n} \eta_i \right) \), \( \xi^+ = \left( \bigvee_{i=1}^{n} \mu_i, \bigwedge_{i=1}^{n} \eta_i \right) \), then

\[ \xi^- \leq q - ROFNSWG(\xi_1, \xi_2, ..., \xi_n) \leq \xi^+. \]

**Proof.** By Theorem 4.3, we have

\[ q - ROFNSWG(\xi^-, \xi^-, ..., \xi^-) = \xi^- , q - ROFNSWG(\xi^+, \xi^+, ..., \xi^+) = \xi^+. \]

In the light of Theorem 4.4, we obtain

\[ q - ROFNSWG(\xi^-, \xi^-, ..., \xi^-) \leq q - ROFNSWG(\xi_1, \xi_2, ..., \xi_n), \]

\[ q - ROFNSWG(\xi^+, \xi^+, ..., \xi^+) \geq q - ROFNSWG(\xi_1, \xi_2, ..., \xi_n). \]

Thus, it is easy to obtain

\[ \xi^- \leq q - ROFNSWG(\xi_1, \xi_2, ..., \xi_n) \leq \xi^+. \]

\[ \square \]

**Remark 4.6** (1) If \( q = 1 \), then

\[ q - ROFNSWG(\xi_1, \xi_2, ..., \xi_n) = \left( \prod_{i=1}^{n} \mu_i^{r(k_i)e_i}, \left( 1 - \prod_{i=1}^{n} (1 - \eta_i^{q} r(k_i)e_i) \right)^{1/q} \right). \]

In this context, the q-ROFNSWG operator degrades into the intuitionistic fuzzy N-soft weighted geometric (IFNSWG) operator.

(2) If \( q = 2 \), then

\[ q - ROFNSWG(\xi_1, \xi_2, ..., \xi_n) = \left( \prod_{i=1}^{n} \mu_i^{r(k_i)e_i}, \left( 1 - \prod_{i=1}^{n} (1 - \eta_i^{q} r(k_i)e_i) \right)^{1/2} \right). \]
In this context, the q-ROFNSWG operator degrades into the Pythagorean fuzzy N-soft weighted geometric (PFNSWG) operator.

In what follows, a q-ROFNSWA operator will be established, and we shall investigate some of interesting properties concerning the q-ROFNSWA operator.

Definition 4.7 Let \( \xi_i = (\mu_i, \eta_i)(i = 1, 2, 3, ..., n) \) be a family of q-ROFNs. Given a mapping q – ROFNSWA : \( H^n \rightarrow H \) such that

\[
q - \text{ROFNSWA}(\xi_1, \xi_2, ..., \xi_n) = r(k_1)e_1\xi_1 \oplus r(k_2)e_2\xi_2 \oplus ..., \oplus r(k_n)e_n\xi_n
\]

where \( H \) is the set of all q-ROFNs, \( r(k) = (r(k_1), r(k_2), ..., r(k_n)) \) denotes the level of the element attribute, and \( E = \{e_1, e_2, ..., e_n\} \) is weight vector of \( \xi_i = (\mu_i, \eta_i)(i = 1, 2, 3, ..., n) \) with \( e_i \in [0, 1] \) and \( \sum_{i=1}^{n} e_i = 1 \). Then, the mapping q-ROFNSWA is referred to as a q-ROFNSWA operator.

Theorem 4.8 Let \( \xi_i = (\mu_i, \eta_i)(i = 1, 2, 3, ..., n) \) be a family of q-ROFNs. Then the aggregation result obtained by Eq. (3) is still a q-ROFN, and has

\[
q - \text{ROFNSWA}(\xi_1, \xi_2, ..., \xi_n) = \left( 1 - \prod_{i=1}^{n} (1 - \mu_i^q)^{r(k_i)e_i} \right)^{1/q} \prod_{i=1}^{n} \eta_i^{r(k_i)e_i}.
\]

Proof. It is similar to the proof of Theorem 4.2. \( \square \)

Theorem 4.9 (Idempotency) Let \( \xi_i = (\mu_i, \eta_i)(i = 1, 2, 3, ..., n) \) be a family of q-ROFNs. If \( \xi_i = \xi = (\mu, \eta), r(k_i) = r(k) = 1 \), then

\[
q - \text{ROFNSWA}(\xi_1, \xi_2, ..., \xi_n) = \xi.
\]

Proof. It is similar to the proof of Theorem 4.3. \( \square \)

Theorem 4.10 (Monotonicity) Let \( \xi_i = (\mu_i, \eta_i), \xi'_i = (\mu'_i, \eta'_i)(i = 1, 2, 3, ..., n) \) be two q-ROFNs. For any \( i \), if \( \mu_i \geq \mu'_i, \eta_i \leq \eta'_i \), then

\[
q - \text{ROFNSWA}(\xi_1, \xi_2, ..., \xi_n) \geq q - \text{ROFNSWA}(\xi'_1, \xi'_2, ..., \xi'_n).
\]

Proof. It is similar to the proof of Theorem 4.4. \( \square \)

Theorem 4.11 (Boundedness) Given that \( \xi_i = (\mu_i, \eta_i)(i = 1, 2, 3, ..., n) \) is a q-ROFN. If \( \xi^- = \left( \bigwedge_{i=1}^{n} \mu_i, \bigvee_{i=1}^{n} \eta_i \right), \xi^+ = \left( \bigvee_{i=1}^{n} \mu_i, \bigwedge_{i=1}^{n} \eta_i \right) \), then

\[
\xi^- \leq q - \text{ROFNSWA}(\xi_1, \xi_2, ..., \xi_n) \leq \xi^+.
\]
Proof. It is similar to the proof of Theorem 4.5. □

Remark 4.12 (1) If $q = 1$, then

$$q - \text{ROFNSWA} (\xi_1, \xi_2, \ldots, \xi_n) = \left( 1 - \prod_{i=1}^{n} (1 - \mu_i)^{r(k_i)e_i}, \prod_{i=1}^{n} \eta_i^{r(k_i)e_i} \right).$$

In this context, the $q$-ROFNSWA operator degrades into the intuitionistic fuzzy N-soft weighted average (IFNSWA) operator.

(2) If $q = 2$, then

$$q - \text{ROFNSWA} (\xi_1, \xi_2, \ldots, \xi_n) = \left( 1 - \prod_{i=1}^{n} (1 - \mu_i^2)^{r(k_i)e_i} \right)^{1/2}, \prod_{i=1}^{n} \eta_i^{r(k_i)e_i}. $$

In this context, the $q$-ROFNSWA operator degrades into the Pythagorean fuzzy N-soft weighted average (PFNSWA) operator.

5 The decision making methods based on $q$-rung orthopair fuzzy N-soft aggregation operators

Let $S = \{s_1, s_2, \ldots, s_m\}$ be a family set of $m$ alternatives and $A = \{a_1, a_2, \ldots, a_n\}$ be a set of $n$ attributes. Given that $E = \{e_1, e_2, \ldots, e_n\}$ is weight vector of $A$ with $e_j \in [0, 1]$ and $\sum_{j=1}^{n} e_j = 1$. Suppose that $E = (M_{ij})_{m \times n} = (r(k_{ij}), (\tilde{\mu}_{ij}, \tilde{\eta}_{ij}))_{m \times n}$ is the $q$-rung orthopair fuzzy N-soft decision matrix, where $r(k_{ij})$ denotes the level of the attribute $a_j (1 \leq j \leq n)$; $\tilde{\mu}_{ij}$ and $\tilde{\eta}_{ij}$ is the membership degree and non-membership degree of alternative $s_i (1 \leq i \leq m)$ to attribute $a_j$, respectively.

Then the methods for MAGDM based on q-ROFNSWG operator or q-ROFNSWA operator are established as follows:

Step 1. Normalize the decision matrix. In practical MAGDM problems, there are two basic types of attribute: one is benefit type; the other is cost type. Therefore, cost type should be converted into benefit type by using the following expression:

$$M_{ij} = (r(k_{ij}), (\tilde{\mu}_{ij}, \tilde{\eta}_{ij})) = \begin{cases} (r(k_{ij}), (\mu_{ij}, \eta_{ij})), & \text{for benefit attribute } a_j, \\ (r(k_{ij}), (\eta_{ij}, \mu_{ij})), & \text{for cost attribute } a_j. \end{cases}$$

Step 2. Aggregate all evaluation value of alternative $s_i$ by virtue of the q-ROFNSWG operator or q-ROFNSWA operator to obtain the comprehensive value $m_i$,

$$m_i = q\text{-ROFNSWA} = (M_{i1}, M_{i2}, \ldots, M_{in}) \text{ or } m_i = q\text{-ROFNSWG} = (M_{i1}, M_{i2}, \ldots, M_{in}).$$

Step 3. Rank $m_i$ using the score function.

Step 4. Choose the best $s_i$ based on the sorting results. The higher the value $m_i$ is, the better the alternative $s_i$ is.
6 Application example

With the improvement of people's living standards, sports are getting more and more attention. Therefore, a large number of sports clubs have been produced, such as basketball clubs, football clubs and volleyball clubs. Now, suppose that a certain sport club needs to recruit an outstanding athlete to improve the strength of the club. After investigation, the evaluation experts from the club decides to choose the best one from five candidates $s_1, s_2, s_3, s_4$ and $s_5$. Assume that the indicators evaluated by the evaluation experts are mainly five attributes: explosiveness ($a_1$), patience ($a_2$), concentration ($a_3$), development potential ($a_4$) and personal quality ($a_5$). The weight vector of attributes is $e = \{0.25, 0.15, 0.12, 0.27, 0.21\}$. Now the experts evaluate the attributes for the candidates by using q-ROFNs. Furthermore, a q-rung orthopair fuzzy 5-soft decision matrix is shown in Table 3 as follows.

| $S$  | $a_1$       | $a_2$       | $a_3$       | $a_4$       | $a_5$       |
|------|-------------|-------------|-------------|-------------|-------------|
| $s_1$| (2,(0.55,0.65)) | (4,(0.91,0.13)) | (2,(0.58,0.53)) | (2,(0.45,0.56)) | (3,(0.69,0.39)) |
| $s_2$| (3,(0.77,0.38)) | (4,(0.95,0.13)) | (3,(0.67,0.32)) | (2,(0.47,0.49)) | (1,(0.35,0.73)) |
| $s_3$| (2,(0.50,0.52)) | (2,(0.45,0.59)) | (1,(0.32,0.73)) | (3,(0.77,0.25)) | (2,(0.47,0.59)) |
| $s_4$| (0,(0.13,0.93)) | (1,(0.28,0.79)) | (2,(0.49,0.55)) | (1,(0.25,0.73)) | (2,(0.42,0.58)) |
| $s_5$| (3,(0.66,0.27)) | (3,(0.68,0.39)) | (1,(0.23,0.79)) | (2,(0.47,0.58)) | (2,(0.45,0.55)) |

6.1 Decision-making steps

Firstly, the decision making steps will be performed by using the q-ROFNSWA operator.

Step 1. Because all the attributes we give are benefit types, our decision matrix is a normalized decision matrix. The result is shown in Table 3 above.

Step 2. Aggregate all evaluation value of alternative $s_i$ by virtue of the q-ROFNSWA operator to obtain the following the comprehensive value $m_i (1 \leq i \leq 5)$ (assume $q = 3$):

$m_1 = (0.8978, 0.0822), m_2 = (0.9430, 0.0601), m_3 = (0.7788, 0.1545), m_4 = (0.4073, 0.6110), m_5 = (0.7419, 0.1382).$

Step 3. Calculate the score value for all $m_i$ by the score function,

$S(m_1) = 0.7231, S(m_2) = 0.8383, S(m_3) = 0.4686, S(m_4) = -0.1605, S(m_5) = 0.4057.$

So we have $m_2 > m_1 > m_3 > m_5 > m_4$.

Step 4. Now, we make the conclusion that $s_2$ is the best candidate.

Secondly, the decision making steps will be conducted by using the q-ROFNSWG operator.
Step 1. A normalized decision matrix is shown in Table 3 above.

Step 2. Aggregate all evaluation value of alternative $s_i$ by virtue of the q-ROFNSWG operator to obtain the following the comprehensive value $m_i (1 \leq i \leq 5)$ (assume $q = 3$):

\[ m_1 = (0.3163, 0.6624), \quad m_2 = (0.3682, 0.5876), \quad m_3 = (0.2860, 0.6455), \]
\[ m_4 = (0.3326, 0.6762), \quad m_5 = (0.2455, 0.6480). \]

Step 3. Calculate the score value for all $m_i$ by the score function,

\[ S(m_1) = -0.2590, \quad S(m_2) = -0.1530, \quad S(m_3) = -0.2456, \quad S(m_4) = -0.2724, \quad S(m_5) = -0.2573. \]

So we have $m_2 > m_3 > m_5 > m_1 > m_4$.

Step 4. Now, we make the conclusion that $s_2$ is the best candidate.

### 6.2 Comparative analysis with the other methods

To expand on the advantages of the developed methods, we compare them with the existing methods by solving the same example, such as the Pythagorean fuzzy weighted geometric (PFWG) [6] operator, the q-ROFWA [11] operator and the q-ROFMM [21] operator. Applying the above mentioned methods, we obtain the comparison results shown in Table 4.

| Methods       | The score function | ranking          |
|---------------|--------------------|------------------|
| PFWG [6]      | $S(m_1) = 0.0810, S(m_2) = 0.0853,$ | $s_2 > s_1 > s_3 > s_5 > s_4$ |
|               | $S(m_3) = -0.0217, S(m_4) = -0.5573,$ |                     |
|               | $S(m_5) = 0.0516$ |                  |
| q-ROFWA [11]  | $S(m_1) = 0.1151, S(m_2) = 0.2145,$ | $s_2 > s_1 > s_3 > s_5 > s_4$ |
|               | $S(m_3) = 0.0744, S(m_4) = 0.0208,$ |                     |
|               | $S(m_5) = 0.0561$ |                  |
| q-ROFMM [21]  | $S(m_1) = 0.3056, S(m_2) = 0.4138,$ | $s_2 > s_1 > s_3 > s_5 > s_4$ |
|               | $S(m_3) = 0.0490, S(m_4) = 0.2994,$ |                     |
|               | $S(m_5) = 0.0571$ |                  |
| q-ROFNSWA     | $S(m_1) = 0.7231, S(m_2) = 0.8383,$ | $s_2 > s_1 > s_3 > s_5 > s_4$ |
|               | $S(m_3) = 0.4687, S(m_4) = 0.1606,$ |                     |
|               | $S(m_5) = 0.4057$ |                  |
| q-ROFNSWG     | $S(m_1) = -0.2590, S(m_2) = -0.1529,$ | $s_2 > s_1 > s_3 > s_5 > s_4$ |
|               | $S(m_3) = -0.2455, S(m_4) = -0.2723,$ |                     |
|               | $S(m_5) = -0.2573$ |                  |

From Table 4, we observe that the optimal ranking results are essentially the same in the different methods, even though the score functions are different in different methods. Therefore, the novel methods proposed by us are reasonable and valid. Moreover,
compared the proposed q-ROFNSWA method with the existing methods, there is little
difference between the score functions obtained from the existing methods. So the effect
of applying the existing methods is not obvious. In addition, these methods only focus on
the degree of membership and non-membership without considering the evaluation level
of attributes, which leads us to make decisions only in the binary evaluation system.
Unfortunately, the data structure is not all binary evaluation structure in real life, such
as evaluation systems, ranking and voting situations. As a result, the existing methods
still remain in many insufficiencies and flaw. As for the limitations, the novel method-
s proposed by us can avoid those defects, whose the significant advantage is that they
not only focus on the degree of membership and non-membership, but also consider the
evaluation level. Therefore, decision-making processing based on the novel methods
can be performed in a non-binary evaluation system, which implies that the propose methods
have greater advantages than the existing methods.

In order to further analyze the flexibility and sensitivity of the parameters $q$, we set
the different values $q$ to rank the alternatives $s_i$ and the ranking results are shown in Table
5 and Fig. 1.

Figure 1: Decision results by q-ROFNSWA and q-ROFNSWG operator with respect to $q$.

From Table 5 and Fig. 1, we observe that when applying the q-ROFNSWA operator,
the optimal ranking results are always the same with different values $q$. When the value of
the parameter $q$ is relatively small, the score function is relatively large. When the value
of the parameter $q$ is large, the score function is relatively small. It is well known that the
value of parameter $q$ stands for the pessimistic or optimistic attitude of the decision maker
towards the decision plan. When the decision maker is pessimistic about the decision plan,
he or she can assign a larger value to the parameter $q$ in the q-ROFNSWA operator.
Conversely, when the decision maker is more optimistic about the decision plan, a smaller
value can be assigned to the parameter $q$ in the q-ROFNSWA operator. So this is for
the q-ROFNSWG operator. Meanwhile, from Table 5 and Fig. 1, we observe that the
optimal ranking results are different with different values $q$ in the q-ROFNSWG operator.
| Methods     | q   | The score function ranking          |  |
|-------------|-----|-------------------------------------|---|
| IFNSWA      | q = 1 | Cannot be calculated | No |
| PFNSWA      | q = 2 | $S(m_1)=0.8383, S(m_2)=0.9099, S(m_5)=0.5921$ | $s_2 > s_1 > s_3 > s_5 > s_4$ |
|             | q = 3 | $S(m_1)=0.7231, S(m_2)=0.8383, S(m_3)=0.4687, S(m_4)=-0.1606, S(m_5)=0.4057$ | $s_2 > s_1 > s_3 > s_5 > s_4$ |
|             | q = 5 | $S(m_1)=0.5259, S(m_2)=0.6971, S(m_3)=0.2496, S(m_4)=-0.0723, S(m_5)=0.1742$ | $s_2 > s_1 > s_3 > s_5 > s_4$ |
| q-ROFNSWA   | q = 10 | $S(m_1)=0.2696, S(m_2)=-0.4577, S(m_3)=0.0605, S(m_4)=-0.0070, S(m_5)=0.0217$ | $s_2 > s_1 > s_3 > s_5 > s_4$ |
|             | q = 15 | $S(m_1)=0.1560, S(m_2)=0.3225, S(m_3)=0.0161, S(m_4)=0.0029, S(m_5)=0.0029$ | $s_2 > s_1 > s_3 > s_5 > s_4$ |
|             | q = 20 | $S(m_1)=0.0943, S(m_2)=-0.2370, S(m_3)=0.0044, S(m_4)=0.0001, S(m_5)=0.0004$ | $s_2 > s_1 > s_3 > s_5 > s_4$ |
| IFNSWG      | q = 1 | Cannot be calculated | No |
| PFNSWG      | q = 2 | $S(m_1)=-0.3888, S(m_2)=-0.2421, S(m_3)=-0.3745, S(m_4)=-0.3466, S(m_5)=-0.4024$ | $s_2 > s_4 > s_3 > s_1 > s_5$ |
|             | q = 3 | $S(m_1)=-0.2590, S(m_2)=-0.1529, S(m_3)=-0.2455, S(m_4)=-0.2723, S(m_5)=-0.2573$ | $s_2 > s_3 > s_5 > s_1 > s_4$ |
|             | q = 5 | $S(m_1)=-0.0994, S(m_2)=-0.0622, S(m_3)=-0.0946, S(m_4)=-0.1428, S(m_5)=-0.1012$ | $s_2 > s_3 > s_1 > s_5 > s_4$ |
| q-ROFNSWG   | q = 10 | $S(m_1)=-0.0088, S(m_2)=-0.0096, S(m_3)=-0.0096, S(m_4)=-0.0288, S(m_5)=-0.0153$ | $s_1 > s_3 > s_2 > s_5 > s_4$ |
|             | q = 15 | $S(m_1)=-0.0009, S(m_2)=-0.0019, S(m_3)=-0.0014, S(m_4)=-0.0070, S(m_5)=-0.0037$ | $s_1 > s_3 > s_2 > s_5 > s_4$ |
|             | q = 20 | $S(m_1)=-0.0001, S(m_2)=-0.0004, S(m_3)=-0.0002, S(m_4)=-0.0019, S(m_5)=-0.0011$ | $s_1 > s_3 > s_2 > s_5 > s_4$ |
Concretely, in the q-ROFNSWG operator the optimal ranking results change from $s_2$ to $s_1$ as the parameter $q$ increases. In light of that, the method based on q-ROFNSWA operator is relatively more stable than the method based on q-ROFNSWG operator.

In view of the above analysis, the methods proposed in this paper are more flexible and reasonable than the existing methods including the PFWG, the q-ROFWA and the q-ROFMM. Therefore, they are more suitable for solving MAGDM problems.

7 Conclusion

In this study, we introduce a q-ROFNSS model, and investigate two operators including the WA operator and WG operator to handle MAGDM problems based on q-ROFNSS. Then two algorithms are established based on the q-ROFNSWA operator and q-ROFNSWG operator. A practical example is provided to illustrate the effectiveness of two algorithms, and we also compare the novel decision making methods with the existing methods. Through comparison and analysis, we make the conclusion that the method based on q-ROFNSWA operator possesses the more advantages than other methods to deal with MAGDM problems. In future research, the combination of the q-ROFNSS and other aggregation operators will be explored. Furthermore, the model will be applied to MAGDM problems.

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Compliance with ethical standards

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Conflict of interest

The authors declare that there is no conflict of interests.
Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Authorship contributions

Haidong Zhang contributed to the manuscript preparation and the conception of the study, and made important revisions to the paper; TaiBen Nan wrote the manuscript; Yanping He performed the experiment and the data analysis.

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Figures

Figure 1

Decision results by q-ROFNSWA and q-ROFNSWG operator with respect to $q$. 