Numerical analysis of micropolar hybrid nanofluid in the presence of non-Fourier flux model and thermal radiation

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Abstract
The influence of various influential factors on the flow field, temperature, and concentration variations are observed throughout the study of thermo-physical properties. The transfer of heat in fluids and thermal instability/stability are fascinating areas of study because of their vast range of applications, and physical significance in many engineering systems. This research aims to investigate and evaluate the flow characteristic, heat and concentration variations of hybrid nanofluids containing MHD natural convection flow of micropolar CuO-Ag/water in porous media across a vertically positioned plate. The flow model is treated with suction/injection at the plate’s surface, thermal radiation, heat generation and absorption, Joule heating, and viscous dissipation. The non-Fourier theory for the heat flux model is used to diminish the thermal instability. Mathematical system for the proposed model having some physical aspects results in a system of PDEs which is restricted the boundary layer approximation is used. The PDEs model is then converted into an ODEs system using the suitable transformations. Numerical scheme RK-4 in collaboration shooting technique is used to find the best approximate results. For the validation of the employed technique, a comparison is offered from literature to confirm the dependability of the produced solution. Physical characteristics of the given solution have been studied and demonstrated against various associated influential factors. In the case of hybrid nano-structures, thermal growth is accelerated rather than in the event of nanofluid. The momentum layer thickness is more essential in hybrid nanoparticles than in nanoparticles. It’s also being looked at how crucial flow parameters affect heat transmission and skin friction.

Keywords
Thermal radiation, hybrid nanofluid, Joule heating, micropolar, suction/injection, viscous dissipation

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**Introduction**

Every sector demands heat transfer characteristics in a period because of the rising need for thermal energy. Due to its various uses in a wide variety of sectors, such as biomedicine and engineering, culminating in the incredible ability of nano-science to speed up the rate of thermal convection from adjacent sources. It piqued the interest of scientists and investigators. Heat efficiency refinement might give a leg up on other requests, such as plasma research. Computers, nuclear reactors, and other microelectronic devices use microchips, space cooling, energy production, and so much more. Choi and Eastman\(^1\) gave the concept of melting nanometer sized particles in conventional fluids. Xuan and Li\(^2\) developed preparatory procedures for a variety of nanoparticle samples and investigated thermos-physical parameters such as shape, quantity, quality, and dimensions using a rectilinear Riga plate. Rawat et al.\(^3\) studied the flow of silver and copper nanofluids. Because the Riga bowl’s height was considered to be infinite, they used the speed of the top wall. Many different types of research\(^4–9\) have been conducted on nanofluids to investigate its heat transmission properties. A nanomaterial type may not have all the desired properties for an application there may be physical or spiritual factors. Despite the many serious applications, hybrid nanofluids are ready to have different properties, so they are very important.

A hybrid nanofluid is a novel form of nanofluid made up of two or more nanoparticles dispersed in a base fluid. By balancing the advantages and disadvantages of single suspensions, hybrid nanofluids can improve heat transmission, high efficiency thermal conductivity, and stability. Since nanoparticles have a larger aspect ratio, a better thermal network and are widely used in nuclear power sectors such as generator cooling, electronic cooling, coolant in machineries, and coolant for freezing towers, it is because of the complex formation of nanomaterials. The ability of hybrid nanofluids to improve thermal conductivity allows them to be considered the thermal energy in real-world concerns. Waini et al.\(^10\) investigated hybrid nanofluids moving on permeable surfaces with a fixed volume percentage of 0.1% nanoparticles of alumina and copper nanoparticles. To hybridize nanofluids in aqueous aggregates, some of the previously introduced nanoparticles were added. Ashwinkumar et al.\(^11\) investigated a vertical cone and plate using thermal non-linear radiation are used to circulate a CuO-Al\(_2\)O\(_3\)/water hybrid nanofluid. Heat transfer owing to the stretched surface dusty nanofluid flow and dusty hybrid nanofluid flow was examined by Samrat et al.\(^12\) Refs\(^13–19\) are some examples of foundation investigations in the area of hybrid nanofluids. The micro-polar liquid theory is a boundary layer theory that considers nanoparticle micro-rotation. Eringen\(^20\) investigated micropolar fluid can be thought of as an improvement to the Navier–Stokes state. They are a sort of microfluidics because they take into account the fluid’s microstructure as well as the substrate’s inertial characteristics, allowing for experimentation. The transport of heat from the sheet stretched across micropolar fluid was studied by Hassani and Gorla.\(^21\) Subhani and Nadeem\(^22\) explored flow in a permeable material that is time-dependent, concentrating on two-dimensional micropolar MHD fluid flow, after the model was mathematically solved. The Hybrid nanofluid containing Copper and Alumina Alloy nanoparticles in base fluid water was taken for MHD boundary layer flow through a movable slandering needle, and various parameters for nanoparticles and hybrid nanoparticles were investigated.\(^23\) In another study, the hybrid nanofluid comprising the Copper and Magnesium Sulfate nanoparticles in base fluid water is studied for the non-Newtonian pseudo plastic Williamson flow under the effects of suction/injection on the stretched surface, heat generation, and thermal conduction to explore the fluid momentum and energy characteristics.\(^24\) References\(^25–28\) include further studies on micropolar fluids and nanofluids.

In recent years, the problems of employing the vertical plate with nanofluids have been examined by numerous researchers. Khalid et al.\(^29\) and Hussanan et al.\(^30\) employed a vertical oscillating plate with a Newtonian heating and a constant temperature at the wall to solve Casson fluid flow. Ali et al.\(^31\) looked at how a Brinkman-type nanofluidic flowed on a perpendicular plate sheet in water through four distinct nanoparticle morphologies. Second-grade fluid over an infinite perpendicular smooth plate in a time-dependent way is investigated by Imran et al.\(^32\) They used the Laplace transform method to solve the system of differential equations. The influence of silver nanoparticles on the MHD free convection flow of Jeffrey fluid across an oscillating vertical plate embedded in a porous medium was examined by Mohd Zin et al.\(^33\) The Atangana–Baleanu and Caputo–Fabrizio fractional derivatives for the generalized Cason fluid model with heat generation and chemical reaction were compared and analyzed by Sheikh et al.\(^34\) Application of Caputo–Fabrizio derivatives to MHD free convection flow of the generalized Walters-B fluid model was studied by Ali et al.\(^35\) Casson fluid convection was studied using atangana baleanu and caputo–fabrizio fractional derivatives by Sheikh et al.\(^36\) Ali Shah et al.\(^37\) studied magneto-hydrodynamic free convection flows in porous media with thermal memory across a moving vertical plate. Ghalambaz et al.\(^38\) investigated the flow and heat transport of hybrid nanofluids across a vertical plate using mixed convection and stability analysis. In
the presence of hall current, nonlinear convection, and heat absorption, Amala and Mahanthesh\textsuperscript{39} investigated hybrid nanofluidic flow across a perpendicular rotating plate. Khashi et al.\textsuperscript{40} used a hybrid Cu-Al\textsubscript{2}O\textsubscript{3}/water nanofluid to study mixed convective stagnation point flow toward a vertical Riga plate. In water, kerosene, and motor oil, Hussananet al.\textsuperscript{41} investigated convective thermal transportation in micropolar-nanofluids containing copper oxide nanoparticles. Khalidet al.\textsuperscript{42} looked at the close form solution for the free convective flow of nanofluidic along a ramping wall temperature.

The term MHD refers to a fluid that is subjected to magnetic and electromagnetic forces. Solar panels, highly conductive-boilers, and the polymeric industry all employ the MHD. Researchers have done a wide range of work in this field. The idea is to protect nanofluids from being affected by electromagnetic forces. The squeezing of micro liquid flow in a magnetic field-affected medium was examined by Ghadikolaei et al.\textsuperscript{43} The Keller box approach was used by Ullah et al.\textsuperscript{44} to statistically study a non-Newtonian fluid across a stretched sheet in the presence of a magnetic field and Newtonian heating. The heat transmission of a ferro-fluid down a vertical conduit in the presence of a magnetic field was explored by Gulet et al.\textsuperscript{45} In nanofluids, hybrid nanofluids, and micropolar fluids/nanofluids, magnetic and electromagnetic forces are studied, for instance the references\textsuperscript{46–54} respectively, presented the significance literature about the said study.

The impacts of numerous operational parameters, For example, Reynolds number, nanoparticle volume percentage, and slip effects are only a few examples have been established in several earlier investigations, both solo and in combination. As the author knows, the effects of multiple slip in hydro-magnetic mixed convection are a micropolar nanofluid in turbulent flow with radiation as well as a heat-slip source in the presence of a non-Fourier flow were not studied first. In certain uncommon instances, numerical solutions are studied, and graphs are utilized to discuss the broad physical meaning of different parameters. Elattar et al.\textsuperscript{55} examined hybrid nanofluid flow over a slender stretching.

For the heat flux, the well-known law about flow was initially presented by Fourier and Darboux.\textsuperscript{56} The evolution of parabolic energy expression is constrained by this rule. Using the energy expression, it is evident that the disruption can be seen initially by the flow medium. This is known as the paradox of heat conduction. To prevent this, Fourier’s law of thermal conduction is altered in a variety of means and situations. Cattaneo\textsuperscript{57} provides an improved version of Fourier’s equation of heat conduction, which incorporates the thermal relaxation time component. He found that the existence of thermal relaxation time causes the hyperbolic energy expression to arise. Christov\textsuperscript{58} is the source of the Cattaneo hypothesis modification. In the Cattaneo–Christov heat flow model, he used the upper-convective Oldroyd derivative. Other relevant literature of non-Fourier flux is found in Refs.\textsuperscript{59–61}

In this article, the non-Fourier theory is employed on the nanofluid model through vertical sheets. For simulation purpose, using proper similarity transformations, the governing nonlinear PDEs are converted into a group of nonlinear system, which are then numerically solved using the Rk-4 and shooting approaches. The numerical solutions generated in the graphical structures explained the flow system in detail.

**Description of fluid flow and its mathematical formulation**

Considered the two-dimensional incompressible combine convective micropolar hybrid nanofluid flow that is past a vertical sheet. Hybrid nanofluids comprise the copper oxide and silver nanoparticles suspended in the host fluid water (CuO-Ag/water). The flow is assumed under the effects of suction-injection, nonlinear thermal radiative flux, viscous dissipation, and heat generation-absorption. Non Fourier theory named Cattaneo-Christov heat model is employed on flow in order to stable the heat transfer throughout the boundary layers. As displayed in Figure 1, the magnetic field B\textsubscript{0} is applied to the surface in an orthogonal manner. The temperature value at the surface is T\textsubscript{w}(x) = T\textsubscript{w} + bx in which T\textsubscript{w} is ambient temperature. The ambient velocity is calculated as U\textsubscript{w}(x) = cx. The mass flux velocity is considered as v\textsubscript{w}. The procedure is supposed to be subjected to the effects of radiation of heat and to be
immersed in a porous material in the existence of non-Fourier theory which come from the main focus of the work, “thermal flow transportation through a vertical sheet.”

Under the above assumptions, the governing Navor’s Stokes system is as follow\(^{62}\):

\[
\begin{align*}
\frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial y} &= 0 \\
\hat{u} \frac{\partial \hat{u}}{\partial x} + \hat{v} \frac{\partial \hat{u}}{\partial y} &= \hat{U}_c \frac{d\hat{U}_c}{dx} + \frac{1}{\rho_{	ext{hbf}}} \left( \mu_{\text{hbf}} + k_1 \right) \frac{\partial^2 \hat{u}}{\partial y^2} + \frac{\kappa_1}{\rho_{\text{hbf}}} \frac{\partial \hat{N}}{\partial y} - \frac{\sigma_{\text{hbf}}}{\rho_{\text{hbf}}} B_0^2 (\hat{u} - \hat{U}_c) \\
&+ \frac{\hat{g} (\rho \beta)_{\text{hbf}}}{\rho_{\text{hbf}}} \left( \hat{T} - \hat{T}_\infty \right)
\end{align*}
\]

(1)

![Equation 1](image1)

![Equation 2](image2)

The equation that controls fluid flow has the following boundary conditions\(^{63}\):

\[
\begin{align*}
\hat{u} &= \hat{L} \frac{\partial \hat{u}}{\partial y}, \hat{v} = \hat{v}_w, \hat{N} = -\hat{n} \frac{\partial \hat{u}}{\partial y}, \hat{T}_w (\hat{x}) \at \hat{y} = 0 \\
\hat{N} \to 0, \hat{T} \to \hat{T}_\infty, \hat{u} \to \hat{u}_c \as \hat{y} \to \infty
\end{align*}
\]

(9)

Where \(\hat{u}, \hat{v}\) are the velocity components in \(\hat{x}, \hat{y}\) directions, each, \(\rho_{\text{hbf}}\) is the density of hybrid fluid, \(\mu_{\text{hbf}}\) is the dynamic viscosity of hybrid fluid, \(k_1\) is viscosity of vertex, \(\sigma_{\text{hbf}}\) is electrical conductivity of fluid, \(B_0\) is the strength of the magnetic field, \(j\) is the micro inertia density, \(\gamma_{\text{hbf}} = \mu_{\text{hbf}} \frac{(b_1)}{2}\) is the gradient of spin viscosity, \(g\) is acceleration due to gravity, \(\beta_{\text{hbf}}\) is ferro-fluid thermal expansion. \(\hat{N}\) is the vector of micro-rotation, \(\hat{T}\) is temperature, \(\alpha_{\text{hbf}}\) is thermal diffusivity, \(L\) is the length of slip, \(n\) the value of the micro-gyration constraint varies in \((0, 1)\), and \(Q_0\) is coefficient of heat absorption/generation.

### Thermo-physical properties of hybrid nanofluid

In order to maximize the efficiency of an energy conversion system, thermal performance must be optimized. The thermo-physical properties of the applied fluids utilized in energy conversion systems greatly influence this kind of thermal performance. The parameters of thermal conductivity, specific heat, and dynamic viscosity is greatly affected by thermal transportation. As potential heat transfer fluids, hybrid nanofluids are affected by a variety of factors, including volume fraction, the size of solid particles, and the temperature. This research focus a hybrid nanofluid flow model composed of a host fluid water and the CuO and Ag nanoparticles have been employed to demonstrate its properties. The volume fractions of CuO nanoparticles and Ag nanoparticles are denoted by the numbers \(\mathcal{O}_1\) and \(\mathcal{O}_2\), respectively, and these fractions were changed from 1% to 8%. Here, Table 1 displays the correlations that were employed to create the hybrid nanofluid.
Table 1. Thermophysical characteristics of hybrid nanofluid.

| Properties             | Hybrid nanofluid |
|------------------------|------------------|
| Dynamic viscosity      | \( \mu_{hbf} \)  |
| Density                | \( \rho_{hbf} \) |
| Thermal Conductivity   | \( k_{hbf} \)   |
| Electrical conductivity| \( \sigma_{hbf} \) |
| Heat capacitance       | \( C_{p,hbf} \) |

Table 2. The values of nanoparticles and host fluid.

| Properties | Constituent | CuO  | Ag  | Water |
|------------|-------------|------|-----|-------|
| \( \rho \) (kg/m\(^3\)) | 18  | 429  | 9.7 |
| \( \mu \) (W/mK)     | 540 | 250  | 4179|
| \( \alpha \) (1/m\(^\circ\)) | 0.05| 2.09 \times 10^4 | 0.05|
| \( \beta \) (1/K)     | 58  | 1.89  | 1.3 |

Table 3. Comparison table of Skin friction \( f''(0) \) with existing literature for \( \phi_1 = \phi_2 = 0 \), \( M = R_f = Ec = Q = S = \delta = 0 \).

| Pr | Amanat al. \( ^{50} \) | Zaid et al. \( ^{52} \) | Present |
|----|------------------------|------------------------|---------|
| 0.7 | 1.7063                 | 1.7063                 | 1.70582 |
| 0.1 | 1.6754                 | 1.6754                 | 1.67121 |
| 0.7 | 1.5179                 | 1.5179                 | 1.51241 |
| 10  | 1.4928                 | 1.4928                 | 1.48124 |
| 20  | 1.4485                 | 1.4485                 | 1.41390 |
| 50  | 1.3989                 | 1.3989                 | 1.39065 |

Table 2 gives the numerical values of the characteristics of the nanoparticles (CuO and Ag) and the host fluid. Table 3 gives the comparison of the skin friction with the previous studies and the present findings.

**Suitable similarity transformations**

The purpose of suitable similarity transformation is to decrease the intricacy of fluid flow system in order to compute the quick and easy way to find the solution of the problem. The appropriate transform variable are,

\[
\begin{align*}
\bar{u} &= \varepsilon \psi f'(\eta), \quad \bar{v} = -\sqrt{\psi_j \varepsilon} \quad f(\eta), \quad \bar{\psi}(\eta) = \varepsilon \sqrt{\varepsilon} \quad \psi_2^2(\eta), \\
\theta(\eta) &= \frac{T - T_\infty}{T_w - T_\infty}, \quad \text{where} \quad \eta = \frac{\varepsilon \sqrt{\varepsilon}}{v_f}
\end{align*}
\]

(10)

Expressions in equation (10) into equations (1) to (3) and (8) give the following non-dimensional system of momentum, micropolar fluid, and non-Fourier temperature distributions.

\[
\begin{align*}
\frac{1 + d_1 K}{d_1}f''(\eta) + d_2 (1 - f'^2(\eta)) + f(\eta)f'(\eta) &\quad - M d_3 (f'(\eta) - 1) + \lambda d_4 \psi(\eta) = 0, \\
\frac{1}{d_1} + \frac{K}{2}P''(\eta) - \frac{d_1 (g(\eta)f'(\eta) - g'(\eta)f(\eta))}{2} &\quad - K(f''(\eta) + 2g(\eta)) = 0, \\
\frac{k_{hbf}}{k_f} + \frac{4 R d}{3} \theta''(\eta) + \frac{\Pr Ec}{d_1} \left( f''(\eta) + d_1 d_3 M f''(\eta) \right) &\quad + \Pr \theta(\eta) + \Pr d_5 (f(\eta)) \theta'(\eta) - \theta(\eta) f''(\eta)) + \Pr \gamma \left( f(\eta)f''(\eta) \phi(\eta) - f'(\eta) \phi''(\eta) \right) = 0,
\end{align*}
\]

(11) (12) (13)

with corresponding boundary conditions,

\[
\begin{align*}
f'(0) &= \delta f''(0), \\
f(0) &= S, \\
g(0) &= -\eta f''(0), \quad \text{as} \quad \eta = 0, \\
\theta(0) &= 1, \quad \text{as} \quad \eta \to \infty
\end{align*}
\]

(14)
Here the non-dimensional quantities are defined as,
\[ K = \frac{k_c}{\rho_f} \] is micropolar parameter, \( \lambda = \frac{\sigma_m}{\rho_f} \) is combined convective parameter (ratio of Grashof number and Reynold number), \( Rd = \frac{\alpha \beta^2}{k_f} \) is thermal radiation parameter, \( Pr = \frac{v_f}{\alpha f} \) is Prandtl number, \( Ec = \frac{\nu_f^2}{(T_w - T_\infty)(\nu)} \) is Eckert number, \( Q = \frac{Q_0}{L^2} \) is thermal generation and absorption quantity, \( \delta = L \sqrt{\frac{\sigma_m}{\rho_f}} \) denoted the slip parameter, \( Re_s = \frac{u_0}{v_f} \) is Reynold number. \( \gamma = \lambda_1 \eta \) denote the thermal time relaxation or non-Fourier flux quantity, and \( M = \frac{\sigma_m R^2}{\rho_f} \) is magnetic force parameter.

In the above equations, the dimensionless constant has the value of

\[
\begin{align*}
d_1 &= (1 - \mathcal{Q}_1)^{2.5} (1 - \mathcal{Q}_2)^{2.5} \\
d_2 &= (1 - \mathcal{Q}_2) \left\{ (1 - \mathcal{Q}_1) + \frac{\rho_1 \mathcal{Q}_1}{\rho_f} \right\} + \mathcal{Q}_2 \left( \frac{\rho_2}{\rho_f} \right) \\
d_3 &= \sigma_f \left\{ \frac{3(s - 1) \left( \frac{\alpha_f}{\alpha} - 1 \right) \mathcal{Q}_1}{\left( \frac{\alpha_f}{\alpha} + 2 \right) - (s - 1) \left( \frac{\alpha_f}{\alpha} - 1 \right) \mathcal{Q}_1} \right\} \\
&\quad \times \left\{ \frac{3(s - 1) \left( \frac{\alpha_f}{\alpha} - 1 \right) \mathcal{Q}_2}{\left( \frac{\alpha_f}{\alpha} + 2 \right) - (s - 1) \left( \frac{\alpha_f}{\alpha} - 1 \right) \mathcal{Q}_2} \right\} \\
d_4 &= (1 - \mathcal{Q}_2) \left\{ (1 - \mathcal{Q}_1) + \left( \frac{\rho \beta}{\rho_f} \right) \mathcal{Q}_1 \right\} + \left( \frac{\rho \beta}{\rho_f} \right) \mathcal{Q}_2 \\
d_5 &= \frac{k_f + k_f (s - 1) - (k_f - k_s) (s - 1) \mathcal{Q}_1}{k_f + k_f (s - 1) + (k_f - k_s) \mathcal{Q}_1} \\
&\quad \times \frac{k_f + k_f (s - 1) - (s - 1) \mathcal{Q}_2 (k_f - k_s)}{k_f + k_f (s - 1) + \mathcal{Q}_2 (k_f - k_s)} \\
&= \frac{k_f + k_f (s - 1) - (k_f - k_s) (s - 1) \mathcal{Q}_1}{k_f + k_f (s - 1) + (k_f - k_s) \mathcal{Q}_1} \\
&\quad \times \frac{k_f + k_f (s - 1) - (s - 1) \mathcal{Q}_2 (k_f - k_s)}{k_f + k_f (s - 1) + \mathcal{Q}_2 (k_f - k_s)}
\end{align*}
\]

### Solution process of the hybrid nanofluidic system

Digital computers and advance methods are used in computational fluid dynamics (CFD), which uses conservation laws (such as the conservation of mass, momentum, and energy) to make quantitative predictions about fluid movement. CFD has grown in relevance and accuracy, yet no forecast is ever 100% accurate. However, a sophisticated CFD solver is able to effectively solve these complicated physical system numerically by transforming these rules into differential equations. We use Shooting method and RK4 as CFD solver in order to find best estimation results and predict the dimensionless quantities.

### Basic principles of the shooting technique

The transport equations are quite nonlinear and can be affected by boundary circumstances. The R-K-F fourth–fifth-order structure and shooting approach may be used to compute this numerically. This method has a small margin of error. Converting the resultant differential equations into first-order equations is the first step in solving the framed model of a hybrid nanofluid flow problem. The essential replacements for the aforementioned step are as follows,

\[
(f, f', f'', g, g', \theta, \theta')^T = \left( \hat{x}_1, \hat{x}_1' = \hat{x}_2, \hat{x}_2' = \hat{x}_3, \hat{x}_4' = \hat{x}_5, \hat{x}_5' = \hat{x}_6, \hat{x}_6' = \hat{x}_7 \right)^T
\]

Few assumptions are taken to solve the fluidic model:

- A relevant number \( \eta \) for representing the distant field is taken 10 for far field.
- Conditions in the far-field w.r.t $\eta \to \infty$ are $f'(\eta) \to 1$, $\theta(\eta) \to 0$, $G(\eta) \to 0$.
- The scale of convergence is $10^{-5}$, and
- For calculations, the step size is recorded as $\eta = 0.025$.

Equations (11) to (13) can be written as

$$
\begin{pmatrix}
\dot{x}_2 \\
\dot{x}_3 \\
\dot{d}_1 \\
\dot{x}_5 \\
\dot{g}' \\
\dot{g}'' \\
\dot{\theta}' \\
\dot{\theta}''
\end{pmatrix} =
\begin{pmatrix}
1 + \frac{1}{d_1} i d_2 - K x_1 + M d_1 (d_2 - 1) + \lambda d_4 \dot{x}_4 \\
2 d_1 \dot{x}_3 - K \dot{x}_5 + 2 i d_4 \\
\frac{1}{d_1} i d_2 \dot{x}_3 - K \dot{x}_5 + 2 i d_4 \\
-2 d_1 \dot{x}_5 - d_2 (\dot{x}_3 - 3 \dot{x}_5) - K (\dot{x}_5 + 2 i d_4) \\
\frac{1}{d_1} i d_2 \dot{x}_3 - K \dot{x}_5 + 2 i d_4 \\
\frac{1}{d_1} i d_2 \dot{x}_3 - K \dot{x}_5 + 2 i d_4 \\
\frac{1}{d_1} i d_2 \dot{x}_3 - K \dot{x}_5 + 2 i d_4 \\
\frac{1}{d_1} i d_2 \dot{x}_3 - K \dot{x}_5 + 2 i d_4
\end{pmatrix}
$$

(18)

Corresponding conditions according to the variable are as,

$$
\begin{cases}
\dot{x}_1(0) = S, \dot{x}_2(0) = \delta b_1, \dot{x}_3(0) = b_1, \dot{x}_4(0) = -n \dot{x}_3(0) \\
\dot{x}_5(\infty) = b_2, \dot{x}_6(0) = 1, \dot{x}_7(0) = b_3
\end{cases}
$$

Where $b_1 = f''(0)$, $b_2 = g'(0)$, and $b_3 = \theta'(0)$, which can be determined via shooting method and the first order system in equations (11) to (13) is integrated via RK4– scheme.

**Numerical results and discussion**

This article examines micropolar hybrid nanofluidic flow across a perpendicular plate. Using the shooting and RK4 approaches, the governing equations and associated boundary conditions are expressed and resolved numerically, subsequent in ODEs. Graphics are used to show the effects of different settings on the hybrid nanofluid flow and its characteristics. Relevant parameters are suction-injection (S), heat generation/absorption ($Q$), Prandtl number $Pr$, thermal relaxation ($\gamma$), Eckert number ($Ec$), magnetic parameter ($M$), micropolar parameter ($K$), and thermal radiation $I$ are represented in Figures 2 to 10. Figure 2(a) and (b) demonstrates the velocity profile decreases with rising value of magnetic parameter in the presence of suction or injection levels. The finding is that when suction is effective at the border zone, velocity is lower than when injection is effective. Figure 3(a) shows that micropolar parameter cause to decrease the velocity profile. Micropolar parameter relates to the fluids which are non-Newtonian and comprise a mixture of tiny particles fluids and colloidal components like big dumbbell molecules. The microstructure and internal motion of fluid components are taken into consideration in the theory of micropolar fluids. Due to the suspension of tiny body, flow motion of the fluid disturb and hence as larger the micropolar parameter, slower the flow motion. Figure 3(b) shows the flow sudden enhancing as the values of Eckert number increase. The Eckert number depicts a correlation between kinetic energy and the enthalpy imbalance of boundary layer. Boundary layer flux increases with increasing Eckert number. As a result, extra heat is generated near to the sheet, and the velocity of the particles rises as a result of the greater kinetic energy of the molecules. Heat generation/absorption impact over velocity field is displayed in Figure 3(c). Figure 4(a) to (c) shows the variations in the micro-rotational profile as a function of the suction-injection parameter, magnetic parameter, and micropolar quantity. Suction-injection parameter and micropolar quantity rapidly increase the micro-rotational flow due to the feature of controlling the flow at boundary layers. Suction/injection is used exothermic reactions of Arrhenius kinetic to regulate the flow of fluid in the stream as dispatched in Figure 4(a). Magnetic field has resistive effect on rotational flow of nanofluid as displayed in Figure 4(b). When the quantity of micropolar parameter increases, obviously it enhances the micro-rotation field (Figure 4(c)). Figure 5(a) and (b) depicts the temperature profile changes as a function of the Eckert number ($Ec$) and thermal relaxation parameters. The Eckert number ($Ec$) is used to show the impact of fluid self-heating due to dissipation effects. With an increase in the Eckert number and thermal relaxation, it is thought that the temperature rises. Figure 6(a) and (b) shows the temperature profile as a function of thermal radiation and Prandtl number. Thermal radiation increases with temperature and the Prandtl number $Pr$ is decreasing. Thermal radiative flux is electromagnetic radiation released from a substance as a result of the material’s heat, and its properties are dependent on its temperature whereas the ratio of momentum swirling diffusivity to heat transmission eddy diffusivity defines a non-
Figure 2. Influence of (a) magnetic parameter and (b) suction-injection parameter on velocity profile.

Figure 3. Influence of (a) micropolar parameter, (b) Eckert number, and (c) heat generation/absorption parameter on velocity profile.
Figure 4. (a) Suction-injection parameter, (b) magnetic parameter, and (c) micropolar quantity on micropolar field.

Figure 5. Influence of (a) Eckert number and (b) thermal relaxation parameters on temperature distribution.
dimensional Prandtl number. That’s why increasing range of thermal radiation has positive impacts on temperature distribution and the inverse relation of thermal transportation diffusivity provoke the temperature to enhance. Figure 7 demonstrates the increase volume fraction increase the thermal distribution.

**Influence of skin friction and local Nusselt number**

Figures 8 to 10 exhibit the skin friction and local Nusselt number variations against various parameters. In Figure 8(a), the magnetic field in collaboration heat suction injection observations rapidly elevate the drag force field. Figure 8(b) displays the thermal stratification versus heat suction. Figure 8(c) displays the rising effect of magnetic parameter and heat suction injection quantity. However, beyond a specific range of magnetic parameters, the skin friction factor suddenly rises. Figure 8 indicates that increasing the magnetic parameter increases the heat transfer coefficient, whereas the radiation parameter ($R_d$) has the opposite effect. However, increasing the magnetic value causes the heat transfer coefficient to increase exponentially. Figures 9 and 10 shows the increasing variations between the Prandtl number and thermal relaxation against thermal radiation in the local Nusselt number field and decreasing variations between the heat suction absorption and the magnetic parameter of the local Nusselt number field. Figures 11 and 12 show the contour effects of drag force and local Nusselt number. Contour study is significant for the irregular surface outline of any shape at boundary. The nano-type tiny particles shapes are usually irregulars. The contour plots of skin friction and local Nusselt numbers for various physical parameters dispatch here. Figure 11(a) shows the decreasing effect in Skin friction variation against the increasing range of suction/injection and magnetic parameter. As skin friction is directly proportionate to the square of the velocity and is directly proportional to the area in contact with the fluid thus the presence of Lorentz force on the plate with suction/injection effect disturb the drag force at boundary layer. Figure 11(b) and (c) shows that the particular values of Prandtl number increases the effect of drag force monotonically. Figure 12(a) to (c) displays the local Nusselt number dropping effect against magnetic effect with suction injection parameter and Prandtl number near the boundary and far the boundary as well. It can be noted that the
Figure 8. Skin friction quantity for (a) magnetic parameter against heat suction absorption, (b) thermal stratification against heat suction absorption, and (c) suction injection parameter.

Figure 9. (a) Prandtl number and (b) thermal relaxation against thermal radiation in local Nusselt number field.
Figure 10. Heat suction absorption against magnetic parameter in local Nusselt number field.

Figure 11. Contour plotting of skin friction coefficients against (a) Suction-injection parameter, (b) Magnetic parameter, and (c) Prandtl number.
Nusselt number proportional to the heat transfer rate is decline by the rising Lorentz force.

Conclusions
This work investigates the suction/injection effect in the flow of a micropolar hybrid nanofluid composed of cooper oxide and silver nanoparticles in the host fluid water (CuO-Ag/water) via a vertically positioned plate. The flow of a hybrid nanofluid is predicted using correlations based on the volume fraction of nanoparticles (CuO and Ag). The similarity technique was used to extract ODEs from the physical system of partial differential equations. Using the shooting method in collaboration Rk-4 numerical technique, the nonlinear reckonings of the flow model solved successfully. Key points from the results are as follows:

- The flow field and energy distribution is much significant in the case of injective flux rather the suction flux.
- Velocity profile decrease with the increasing range of magnetic, micropolar, and suction/injection parameters, where as it elevates due to the increasing values of Eckert number and heat generation absorption quantities.
- Micropolar flow field is positive correlated with the suction injection fluxes but resistive against magnetic force.

Figure 12. Contour plotting of local Nusselt number against (a) Suction-injection, (b) Prandtl number, and (c) Magnetic parameter.
• Eckert number, thermal time relaxation parameter, and thermal radiation have highly favorable influential toward the energy distribution near the surface.
• Surface drag force enhance due to the magnetic effect and suction-injection fluxes.

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References

1. Choi SUS and Eastman JA. Enhancing thermal conductivity of fluids with nanoparticles. Am Soc Mech Eng Fluids Eng Div FED 1995; 231: 99–105.
2. Xuan Y and Li Q. Heat transfer enhancement of nanofluids. Int J Heat Fluid Flow 2000; 21: 58–64.
3. Rawat SK, Mishra A and Kumar M. Numerical study of thermal radiation and suction effects on copper and silver water nanofluids past a vertical Riga plate. Multidiscip Model Mater Struct 2019; 15: 714–736.
4. Aaiza G, Khan I and Shafie S. Energy transfer in mixed convection MHD flow of nanofluid containing different shapes of nanoparticles in a channel filled with saturated porous medium. Nanoscale Res Lett 2015; 10: 490.
5. Rawat SK and Kumar M. Cattaneo–Christov heat flux model in flow of copper water nanofluid through a stretching/shrinking sheet on stagnation point in presence of heat generation/absorption and activation energy. Int J Appl Comput Math 2020; 6: 1–26.
6. Rawat SK, Pandey AK and Kumar M. Effects of chemical reaction and slip in the boundary layer of MHD nanofluid flow through a semi-infinite stretching sheet with thermophoresis and Brownian motion: the Lie group analysis. Nanoscale Technol 2018; 9: 47–68.
7. Rawat SK, Upreti H and Kumar M. Thermally stratified nanofluid flow over porous surface cone with Cattaneo–Christov heat flux approach and heat generation (or) absorption. SN Appl Sci 2020; 2: 1–18.
8. Rawat SK, Upreti H and Kumar M. Numerical study of activation energy and thermal radiation effects on Oldroyd-B nanofluid flow using the Cattaneo–Christov double diffusion model over a convectively heated stretching sheet. Heat Transf 2021; 50: 5304–5331.
9. Sheikholeslami M, Arabkoohsar A, Khan I, et al. Impact of Lorentz forces on fe3o4-water ferrofluid entropy and exerytment treatment within a permeable semi annulus. J Clean Prod 2019; 221: 885–898.
10. Waini I, Ishak A and Pop I. Hybrid nanofluid flow and heat transfer past a vertical thin needle with prescribed surface heat flux. Int J Numer Methods Heat Flow 2019; 29: 4875–4894.
11. Ashwinkumar GP, Samrat SP and Sandeep N. Convective heat transfer in MHD hybrid nanofluid flow over two different geometries. Int Commun Heat Mass Transf 2021; 127: 105563.
12. Samrat S, Ashwinkumar G and Sandeep N. Simultaneous solutions for convective heat transfer in dusty-nano- and dusty-hybrid nanoliquids. Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering 2021; 236(2): 473–479.
13. Yaseen M, Kumar M and Rawat SK. Assisting and opposing flow of a MHD hybrid nanofluid flow past a permeable moving surface with heat source/sink and thermal radiation. Partial Differ Equations Appl Math 2021; 4: 100168.
14. Sulochana C, Aparna SR and Sandeep N. Magnetohydrodynamic MgO/CuO-water hybrid nanofluid flow driven by two distinct geometries. Heat Transf 2020; 49: 3663–3682.
15. Acharya N. Spectral quasi linearization simulation on the hydrothermal behavior of hybrid nanofluid spraying on an inclined spinning disk. Partial Differ Equations Appl Math 2021; 4: 100094.
16. Sulochana C, Aparna SR and Sandeep N. Impact of linear/nonlinear radiation on incessantly moving thin needle in MHD quiescent Al-Cu/methanol hybrid nanofluid. Int J Ambient Energy 2022; 43: 2694–2700.
17. Manohar GR, Venkatesh P, Giresha BJ, et al. Dynamics of hybrid nanofluid through a semi spherical porous fin with internal heat generation. Partial Differ Equations Appl Math 2021; 4: 100150.
18. Yaseen M, Rawat SK and Kumar M. Hybrid nanofluid (MoS2–SiO2/water) flow with viscous dissipation and ohmic heating on an irregular variably thick convex/concave-shaped sheet in a porous medium. Heat Transf 2022; 51: 789–817.
19. Garia R, Rawat SK, Kumar M, et al. Hybrid nanofluid flow over two different geometries with Cattaneo–Christov heat flux model and heat generation: a model with correlation coefficient and probable error. Chin J Phys 2021; 74: 421–439.
20. Eringen AC. Theory of micropolar fluids. J Math Mech 1996; 16: 1–18.
21. Hassanien IA and Gorla RSR. Heat transfer to a micropolar fluid from a non-isothermal stretching sheet with suction and blowing. Acta Mech 1990; 84: 191–199.
22. Subhani M and Nadeem S. Numerical analysis of micropolar hybrid nanofluid. Appl Nanosci 2019; 9: 447–459.
23. Nagendarmma V, Rao BM, Sivakumar N, et al. Energy dissipative MHD Cu-AA7072/water-based hybrid nanofluid flow over a perpetually moving slender needle. Waves Random Complex Media. Epub ahead of print 14 June 2022. DOI: 10.1080/17455030.2022.2083721.
24. Kavya S, Nagendramma V, Ahammad NA, et al. Magnetic-hybrid nanoparticles with stretching/shrinking cylinder in a suspension of MoS2 and copper nanoparticles. *Int Commun Heat Mass Transf* 2022; 136: 106150.

25. Ahmad S, Ashraf M and Ali K. Simulation of thermal radiation in a micropolar fluid flow through a porous medium between channel walls. *J Therm Anal Calorim* 2021; 144: 941–953.

26. Alzallami SAM, Zahir H, Muhammad T, et al. Numerical simulation of Marangoni Maxwell nanofluid flow with Arrhenius activation energy and entropy anamorization over a rotating disk. *Waves Random Complex Media*. Epub ahead of print 9 March 2022. DOI: 10.1080/17455030.2022.2045385.

27. Shankar Goud B. Heat generation/absorption influence on steady stretched permeable surface on MHD flow of a micropolar fluid through a porous medium in the presence of variable suction/injection. *Int J Thermofluids* 2020; 7–8: 100044.

28. Abo-Dahab SM and Hatem A. Solution of a free convection effect on oscillatory flow of an electrically conducting micropolar concentration fluid with thermal relaxation within porous medium. *Alex Eng J* 2020; 59: 1243–1257.

29. Khalid A, Khan I, Khan A, et al. Unsteady MHD free convection flow of casson fluid past an oscillating vertical plate embedded in a porous medium. *Eng Sci Technol* 2015; 18: 309–317.

30. Hussanan A, Zuki Salleh M, Tahar RM, et al. Unsteady boundary layer flow and heat transfer of a Casson fluid past an oscillating vertical plate with Newtonian heating. *PLoS One* 2014; 9: e108763.

31. Ali F, Gohar M and Khan I. MHD flow of water-based brinckman type nanofluid over a vertical plate embedded in a porous medium with variable surface velocity, temperature and concentration. *J Mol Liq* 2016; 223: 412–419.

32. Imran MA, Khan I, Ahmad M, et al. Heat and mass transport of differential type fluid with non-integer order time-fractional Caputo derivatives. *J Mol Liq* 2017; 229: 67–75.

33. Mohd Zin NA, Khan I and Shafie S. The impact silver nanoparticles on MHD free convection flow of Jeffrey fluid over an oscillating vertical plate embedded in a porous medium. *J Mol Liq* 2016; 222: 138–150.

34. Sheikh NA, Ali F, Saqib M, et al. Comparison and analysis of the Atangana–Bailean and Caputo–Fabrizio fractional derivatives for generalized Casson fluid model with heat generation and chemical reaction. *Results Phys* 2017; 7: 789–800.

35. Ali F, Saqib M, Khan I, et al. Application of Caputo-Fabrizio derivatives to MHD free convection flow of generalized Walters'-B fluid model. *Eur Phys J Plus* 2016; 131: 1–10.

36. Sheikh NA, Ali F, Saqib M, et al. A comparative study of Atangana-Baleanu and Caputo-Fabrizio fractional derivatives to the convective flow of a generalized Casson fluid. *Eur Phys J Plus* 2017; 132: 1–14.

37. Ali Shah N, Ahmed N, Elaqaeb T, et al. Magnetohydrodynamic free convection flows with thermal memory over a moving vertical plate in porous medium. *J Appl Comput Mech* 2019; 5: 150–161.

38. Ghalambaz M, Roşca NC, Roşca AV, et al. Mixed convection and stability analysis of stagnation-point boundary layer flow and heat transfer of hybrid nanofluids over a vertical plate. *Int J Numer Methods Heat Fluid Flow* 2019; 30: 3737–3754.

39. Amala S and Mahanthesh B. Hybrid nanofluid flow over a vertical rotating plate in the presence of holl current, nonlinear convection and heat absorption. *J Nanofluids* 2018; 7: 1138–1148.

40. Khashiie NS, Md Arifin N and Pop I. Mixed convective stagnation point flow towards a vertical Riga plate in hybrid Cu-Al2O3/water nanofluid. *Mathematics* 2020; 8: 912.

41. Hussanan A, Salleh MZ, Khan I, et al. Convection heat transfer in micropolar nanofluids with oxide nanoparticles in water, kerosene and engine oil. *J Mol Liq* 2017; 229: 482–488.

42. Khalid A, Khan I and Shafie S. Exact solutions for free convection flow of nanofluids with ramped wall temperature. *Eur Phys J Plus* 2015; 130: 1–14.

43. Ghadiolaei SS, Yassari M, Sadeghi H, et al. Investigation on thermophysical properties of TiO2–Cu/H2O hybrid nanofluid transport dependent on shape factor in MHD stagnation point flow. *Powder Technol* 2017; 322: 428–438.

44. Ullah I, Shafie S and Khan I. Effects of slip condition and Newtonian heating on MHD flow of Casson fluid over a nonlinearily stretching sheet saturated in a porous medium. *J King Saud Univ Sci* 2017; 29: 250–259.

45. Gul A, Khan I, Shafie S, et al. Heat transfer in MHD mixed convection flow of a ferrofluid along a vertical channel. *PLoS One* 2015; 10: e0141213.

46. Shuaib M, Ali A, Khan MA, et al. Numerical investigation of an unsteady nanofluid flow with magnetic and suction effects to the moving upper plate. *Adv Mech Eng* 2020; 12: 168784020903588.

47. Al-Muhaddel FS, Allehiany FM, Nofal TA, et al. Rheological model for generalized energy and mass transfer through hybrid nanofluid flow comprised of magnetized cobalt ferrite nanoparticles. *J Nanomatter* 2022; 2022: 1–11.

48. Khan LA, Raza M, Mir NA, et al. Effects of different shapes of nanoparticles on peristaltic flow of MHD nanofluids filled in an asymmetric channel. *J Therm Anal Calorim* 2020; 140: 879–890.

49. Alhowaity A, Hamam H, Bilal M, et al. Numerical study of Williamson hybrid nanofluid flow with thermal characteristics past over an extending surface. *Heat Transf*. Epub ahead of print 25 May 2022. DOI: 10.1002/htj.22616.

50. Aman S, Zokri SM, Ismail Z, et al. Effect of MHD and porosity on exact solutions and flow of a hybrid Casson-nanofluid. *J Adv Res Fluid Mech Therm Sci* 2018; 44: 131–139.

51. Ali A, Ahmammad NA, Tag Eldin E, et al. MHD Williamson nanofluid flow in the rheology of thermal radiation, Joule heating, and chemical reaction using the Levenberg–Marquardt neural network algorithm. *Front Energy Res* 2022; 10: 1175.

52. Zia A, Khan U, Shah Z, et al. Optimization of entropy generation in flow of micropolar mixed convective magnetic (Fe3O4) ferroparticle over a vertical plate. *Alex Eng J* 2019; 58: 1461–1470.

53. Ramadevi B, Anantha Kumar K, Sugunamma V, et al. Magnetohydrodynamic mixed convective flow of
micropolar fluid past a stretching surface using modified Fourier’s heat flux model. *J Therm Anal Calorim* 2020; 139: 1379–1393.

54. Dawar A, Shah Z, Kumam P, et al. Chemically reactive MHD micropolar nanofluid flow with velocity slips and variable heat source/sink. *Sci Rep* 2020; 10: 20926.

55. Elattar S, Helmi MM, Elkotb MA, et al. Computational assessment of hybrid nanofluid flow with the influence of hall current and chemical reaction over a slender stretching surface. *Alex Eng J* 2022; 61: 10319–10331.

56. Fourier JBJ and Darboux G. *Théorie analytique de la chaleur*. Paris: Didot, 1822.

57. Cattaneo C. Sulla conduzione del calore. *Atti Sem Mat Fis Univ Modena* 1948; 3: 83–101.

58. Christov CI. On frame indifferent formulation of the Maxwell–Cattaneo model of finite-speed heat conduction. *Mech Res Commun* 2009; 36: 481–486.

59. Ali B, Ali L, Abdal S, et al. Significance of Brownian motion and thermophoresis influence on dynamics of Reiner-Rivlin fluid over a disk with non-Fourier heat flux theory and gyrotactic microorganisms: a numerical approach. *Phys Scr* 2021; 96: 094001.

60. Khan MJ, Duraisamy B, Zuhra S, et al. Numerical solution of Catteno-Christov heat flux model over stretching/shrinking hybrid nanofluid by new iterative method. *Case Stud Therm Eng* 2021; 28: 101673.

61. Alhowaity A, Bilal M, Hamam H, et al. Non-Fourier energy transmission in power-law hybrid nanofluid flow over a moving sheet. *Sci Rep* 2022; 12: 1–12.

62. Hosseinzadeh K, Asadi A, Mogharrebi AR, et al. Investigation of mixture fluid suspended by hybrid nanoparticles over vertical cylinder by considering shape factor effect. *J Therm Anal Calorim* 2021; 143: 1081–1095.

63. Gumber P, Yaseen M, Rawat SK, et al. Heat transfer in micropolar hybrid nanofluid flow past a vertical plate in the presence of thermal radiation and suction/injection effects. *Partial Differential Equations in Applied Mathematics* 2022; 5: 100240.