Optimizing the mean and variance of bead geometry in the wire + arc additive manufacturing using a desirability function method

Jin-Soo Cho1 · Dong-Hee Lee2 · Gi-Jeong Seo3 · Duck-Bong Kim3 · Seung-Jun Shin1

Received: 3 November 2021 / Accepted: 17 April 2022 / Published online: 4 May 2022
© The Author(s), under exclusive licence to Springer-Verlag London Ltd., part of Springer Nature 2022

Abstract
Wire + arc additive manufacturing is an arc welding process that uses non-consumable tungsten electrodes to produce the weld. The material used in this study is a titanium, carbon, zirconium, and molybdenum alloy that is physically and chemically stable and has good performance for use as a welding and high-temperature heating element. In this study, welding experiments are designed based on a central composite design, and single-layer wire + arc additive manufacturing is performed using the titanium, carbon, zirconium, and molybdenum alloy. Consequently, 17 beads are obtained and the height, width, left and right toe angles, which represent the geometry of the beads are measured. Based on the measured geometry, response surface models for mean and standard deviation of the four geometries are fitted. Mean absolute percentage error of the four response surface models is 16.6% on average which implies that the models are reasonably well fitted. Based on the response surface models, the optimal settings for the Wire + arc additive manufacturing parameters are obtained by using a desirability function method. At the optimal setting, the desirability function value shows 0.85 on average which is close to ideal value of 1.00. This result indicates that valid optimal settings for the process parameters can be obtained via the proposed method.

Keywords Wire + arc additive manufacturing process · TZM · Bead geometry · Desirability function method · Response surface methodology

1 Introduction
Welding is a typical manufacturing process whereby metals are joined by using high heat to melt the metals together and allowed to cool thereafter. Wire + Arc additive manufacturing (WAAM) is a typical welding process that uses non-consumable tungsten electrodes. Notably, WAAM is used in various industries, including aerospace, construction, auto repair, art, nuclear, auto renovation, and shipping, because of its ability to produce large-sized metallic parts with a high deposition rate, low equipment cost, and high material utilization [1].

In this study, WAAM is applied to Titanium-Zirconium-Molybdenum (TZM) which is one of the most widely used Molybdenum (Mo)-based alloys. In general, it has the composition of 0.5 wt.% Titanium (Ti), 0.08 wt.% Zirconium (Zr), and 0.02 wt.% Carbon (C) [2]. The physical properties and chemical composition of TZM is summarized in Tables 1 and 2, respectively [3]. The raw TZM alloy is transformed into a weld bead through the WAAM process. Figure 1 shows examples of TZM beads. TZM has a physically stable
Table 1 Physical properties of TZM

| Density (g/cm³) | Melting point (°C) | Thermal conductivity (W/m·K) | Coefficient of linear expansion (10⁻⁶/K) |
|----------------|-------------------|------------------------------|----------------------------------------|
| 10.22          | 2620              | 126                          | 5.3                                    |

performance with 1400° of recrystallization and good performance for welding and high-temperature heating, making it widely used in the real world for several applications, including nozzles, valve bodies, gas piping, gated tube molds, and hot electric heat sinks [4–8]. However, TZM is an expensive metal; thus, TZM should be carefully conducted to prevent defective TZM beads [9]. It is also important for the bead to have good bead geometry, where the height, length, and angle to the surface are desirable. Compared to the lower image in Fig. 1, the upper image in Fig. 1 shows a desirable bead geometry.

In the WAAM process, the bead geometry is affected by certain process parameters, such as the current, welding speed, and feed rate. To achieve good bead geometry, these process parameters should be maintained at predetermined optimal settings during the WAAM process. Response surface methodology (RSM) can be used for this purpose. RSM is a bundle of statistical and optimization techniques that empirically studies the relationship between a response variable and the input variables that affect it. The ultimate goal of RSM is to determine the optimal settings of the input variables to optimize the response variable [10]. In terms of RSM, the WAAM process parameters correspond to the input variables, and the bead geometry corresponds to the response variable. The geometry can be represented by height, length, and left and right angles.

Several studies have been conducted on bead geometry and welding process parameters. These studies have all aimed at determining the optimal settings for the welding process parameters. Kim et al. proposed a method to optimize the welding process parameters of a gas metal arc process using genetic algorithms and RSM [11]. In this method, the optimal settings for the wire feed rate and welding voltage were obtained to optimize the height, width, and penetration of the bead. Dey et al. suggested the use of a genetic algorithm and penalty function to obtain the optimal settings for voltage, current, and welding speed to optimize the height, width, and penetration of the bead [12]. Geng et al. suggested a method for optimizing wire feeding in a WAAM process [13]. They improved the deposition accuracy in WAAM when the wire was fed in a sidewise direction. A mathematical model was developed to calculate the wire flying distance in the arc zone, according to which the displacement compensation was designed to ensure size accuracy. Benyounis et al. investigated the effect of three welding process parameters—laser power, welding speed, and focal point position—on three responses—penetration, welded zone width, and heat-affected zone width—using RSM [14]. They built regression models to explain the effects and obtained the optimal settings of the three process parameters whereas the three responses were desirable. Gunaraj and Murugan conducted a study to predict the weld bead quality in submerged arc welding and determine the essential parameters for the desired quality and process optimization using RSM [15].

Most of the above-mentioned methods attempted to optimize multiple response variables. They are typical examples of multiple response problems in the welding process field. Multiple response surface optimization (MRSO) is a research field for solving the multiple response problem based on the RSM framework [16]. In MRSO, multiple responses are often in conflict. That is, in many multiresponse problems, improving one response is likely to worsen one or more other responses [17]. In such cases, it is important to consider the tradeoffs between the multiple response variables and find an optimal setting of the input variables whereas the multiple response variables are optimally compromised. To find the optimally compromised setting, it is necessary to reflect a process engineer’s preference information regarding the multiple responses. Most of the existing methods have a disadvantage in that they do not systematically reflect the process engineer’s preference information regarding the multiple process. In addition, they did not consider the variability of the bead geometry, which is critical to bead quality.

The desirability function method, suggested by Derringer and Suich, is a representative approach for obtaining a compromised optimal setting by systematically reflecting the process engineer’s preference information [18]. The desirability function approach converts each response variable to an individual desirability function. The individual desirability function can be viewed as a DM’s utility function, and it ranges from 0 to 1. When a response variable achieves a target value, the desirability function value becomes 1, and the desirability function value decreases as the response variable deviates from the target value. The value becomes 0 when the response value reaches lower or upper specification limits that the process

Table 2 Elemental composition (wt.%) of TZM wire used in this study

| Alloying elements | Mo | Ti | Zr | C  | O  | N  | Fe  | Ni  | Si  |
|-------------------|----|----|----|----|----|----|-----|-----|-----|
| Composition (wt.%)| Balance | 0.40–0.55 | 0.06–0.12 | 0.01–0.04 | < 0.03 | < 0.002 | < 0.01 | < 0.005 | < 0.005 |
engineer defined. Once the desirability functions for individual response variables are defined, these functions are aggregated into a single measure, called an overall desirability function. A geometric mean or weighted geometric mean of the individual desirability functions can be used as the overall desirability function.

One of the challenging issues of the desirability function approach is the need to consider the variability of multiple responses. Most of the existing desirability function-based methods assume that the variability of response variables is stable; thus, they focus mainly on the optimization of the mean of multiple responses [17]. However, this stable variability assumption often does not apply in most of practical situations; thus, the bead quality can be severely damaged due to the high variability.

Taguchi method, called as robust design, is a representative method for reducing the variability of response variable. Lunani et al. suggested graphical methods for robust design with dynamic characteristics [19]. The aim of the graphical methods is to optimize both mean and variance of a response variable simultaneously. In addition to this graphical method, there have been several methods for optimizing mean and variance of a single response, called dual response surface optimization (DRSO) [20–24]. They build separate response surface models for mean and standard deviation of a response variable and optimize these two functions simultaneously. DRSO methods are advantageous in optimizing both mean and variance. However, they focus on optimization of single response; thus, it has limited applicability in optimizing multiple responses.

Loss function methods consider both mean and variance of multiple responses in MRSO. In these methods, multiple responses are aggregated into a single measure called loss (or expected loss) and then the optimal setting of input variables is obtained by minimizing the (expected) loss [25–28]. It is desired that the mean of multiple responses be close to their targets, and the variances be small at the optimal setting. In the aggregated measure, covariance matrix among multiple responses is used for reducing the variance of the multiple responses and cost matrix is used for reflecting the process engineer’s preference information regarding the multiple responses.

Compared to the desirability function methods, loss function methods are advantageous in that they attempt to reduce the variance of the multiple responses. However, loss function methods are generally difficult to use and less popular because they are not intuitive compared to the desirability function methods. The squared loss value is difficult to interpret. The cost matrix used for constructing the loss function represents penalty for deviation of the response value from the target and process economics [28]. It plays an important role in obtaining the optimal setting of the input variables; however, the choice of the cost matrix is difficult in general [29]. In contrast, the optimization result of the desirability function approach is easy to interpret since the desirability function represents the process engineer’s degree of satisfaction as the desirability function value from 0 to 1 regarding the obtained solution. For this reason, the desirability function methods are more popular than the loss function methods. Most statistical software, including Minitab, SAS, Design Expert, and R, provides the desirability function analysis module for MRSO, but not a loss function analysis module.

This paper suggests using a desirability function method to obtain an optimal setting of input variables where the four response variables (i.e., height, length, and left and right angles) are optimized. First, experiments are conducted for the WAAM process based on RSM to investigate the relationship between the input variables and the mean of each response. In addition, the relationship between the input variables and the standard deviation of each response is investigated. Based on the experimental data, regression models are built for the mean and standard deviation for each response variable. Thereafter, individual desirability functions are constructed for the mean and standard deviation of the four response variables and the individual desirability functions are aggregated into an overall desirability function. Finally, the optimal settings of the input variables are obtained by maximizing the overall desirability function. At the optimal settings, the four response variables are close to their target values. Additionally, the variability of the four responses across the bead is stable, which means that the variances of the four response variables are low.

The proposed method is advantageous in that it systematically reflects the process engineer’s preference information to obtain the optimal settings where the four response variables are compromised. As previously mentioned, most of the desirability function method assumes that variability of the
multiple responses is stable or even constant. The proposed
does not rely on this unrealistic assumption and attempts
to reduce the variability of multiple responses represent-
ing bead geometry in the optimization through the desir-
ability functions. In addition, it has a significant industrial
contribution, as it is the first approach toward optimizing
the TZM bead geometry in the WAAM process. The TZM
bead geometry is difficult to measure in practice because
the bead should be cut to measure, and it is an expensive
metal such that cutting requires high cost and effort. To cope
with the difficulty, the TZM bead geometry are measured
using a coordinate measuring machine (CMM). Based on
the measured data, the proposed method establishes the rela-
tionship between the bead geometry and the WAAM process
parameters. Ultimately, the optimal settings of the WAAM
process parameters are obtained using the desirability func-
tion method.

The remainder of this paper is organized as follows: The
WAAM process optimization using the desirability function
method is explained in Sect. 2. The concluding remarks are
provided in Sect. 3.

2 Optimization of the WAAM process

2.1 Overview of the WAAM process

In this section, the input and response variables of the
WAAM process for welding the TZM alloy are intro-
duced. Thereafter, the overall procedure for optimizing
the WAAM process is presented. Three parameters for the
input variables are considered: current, welding speed,
and feed rate. Figure 2 shows a schematic diagram of the
three input variables in the WAAM process. The current
in welding (ampere) is the welding parameter representing
the flow of electrons that crosses the arc gap between the
electrode end and the metal being welded. The greater the
electrical resistance, the greater the heat generated by the
arc, and the higher the temperature. The current affects
weld penetration. As the weld current increases, both the
deposition speed and penetration depth increase. Exces-
sive current can cause electrode wires to overheat, thereby
causing arc instability, poor welding quality, disconnec-
tion, or bowling. The current value must be controlled to
an appropriate level to maintain the stability of the arc.

The welding speed is the speed at which the electrode moves
relative to the junction. In this study, it is expressed in
centimeters per minute (cm/min). Usually, thick materi-
als use slow speed, and thin materials use fast speed. The
welding speed adversely affects the weld bead size. A fast
(slow) welding speed reduces (increases) the weld size. A
very low welding speed can cause the arc to collide with a
thick layer of molten metal, thereby reducing penetration.

The feed rate is the speed at which the material in the
wire is transferred to the weld. It is also expressed in cm/
min. The feed rate is directly related to the weld current
and current growth as the wire feed speed increases. The
faster the wire is supplied, the better the contacts are deliv-
ered, and more amps pass through the wire to increase the
heat strength. If the wire speed is too high, the arc may
start incorrectly, and a wide bead shape may appear.

Four response variables representing the bead geometry
(i.e., height, width, left toe angle, and right toe angle) are
considered. In this study, the optimal setting of the WAAM
process parameters is determined using the desirability func-
tion method, which considers not only the mean but also the
standard deviation of the four response variables. Figure 3
shows the overall flow of the study. It consists of seven steps,
and they can be mapped into the three RSM stages (i.e., data
collection, model building, and optimization). Steps 1 and
2 can be mapped to data collection. Step 3 involves model
building. Steps 4–7 are used for optimization.

2.2 Step-by-step procedure of WAAM optimization

2.2.1 Step 1: conducting experiments

Welding experiments are conducted using WAAM to gener-
ate TZM beads, as described in Fig. 4. The experiments are
1. Conducting experiments

2. Measuring the geometry of the bead

3. Modeling the geometry of the bead

4. Specifying the target values for the bead geometry

5. Constructing desirability functions

6. Maximizing the desirability function

7. Validating the result of the optimization

Table 3 Factors and levels for the center composite design

| Level | Current (A) | Welding speed (cm/min) | Feed rate (cm/min) |
|-------|-------------|------------------------|-------------------|
| –1    | 275         | 10                     | 100               |
| 0     | 300         | 15                     | 150               |
| +1    | 325         | 20                     | 200               |

Fig. 3 Overall flow of this study

conducted as “bead on plate tests,” which generate TZM beads on a plate according to experimental conditions.

In addition to the three input variables, the other process parameters are determined as conventional values. The gas used for welding was a mixture of 70% argon and 30% helium. The substrate used as a test plate for welding was based on a base metal and was 100 mm × 100 mm in size. A 5-mm-diameter wire was used. To minimize external factors in the experiment and ensure uniform settings, the temperature of the substrate was maintained between 50–55 °C during each experiment.

The experimental factors include the process parameters, current, welding speed, and feed rate of welding. According to central composite design (CCD), three levels were set [−1, 0, 1], and the actual experimental values were set using empirical data from experts in the welding field. Table 3 presents the actual experimental values of the three factors for the three coded values [−1, 0, 1]. The design point of Level 0 in Table 3 (i.e., current = 275 A; welding speed = 15 cm/min, feed rate = 150 cm/min) is common condition where WAAM operates in stable; thus, this point is designated as a center point. To investigate the effects of these three parameters, three levels of current: 250, 275, and 300 A; welding speed: 250, 275, and 300 cm/min; and feeding rate: 144, 180, and 216 cm/min are determined in the experiment. According to CCD, eight experimental points are set as factorial points, six as axial points, and three as center points. Thus, a total of 17 experiments were conducted according to the design points listed in Table 4.

The experiment was conducted using single-layer welding. An image of the upper surface of the bead with the completed welding experiments is presented in Table 5. It can be confirmed that the bead geometry varies depending on the process parameters of the welding experiments. Since the photos of beads in Table 5 are taken from the top, bead widths can be compared. In Table 5, beads 1, 3, 5, 7, 9 are much narrower than beads 2, 4, 6, 8, 10. This means that current affects the bead widths. It is expected that other geometries are also affected by:

2.2.2 Step 2: measuring the geometry of the bead

In step 1, 17 TZM beads were obtained. In this step, the four response variables representing the bead geometry were

![Experimental facility](image)
measured at 100 evenly divided points on each bead. There exist 17 beads, 100 points on each bead, and 4 response variables; thus, a total of 6,800 (i.e., $17 \times 100 \times 4$) values for the response variables should be measured. It is almost impossible to manually measure 6,800 values; thus, an automatic measurement program using the MATLAB 2019a software are developed. The details of the program development are presented as follows:

First, the start and end parts of the beads are excluded. When TZM bead welding is performed, there is an instability that does not reliably reflect the welding settings selected by the user at the beginning and end of the welding. This is due to external factors, such as arc, temperature, and wire supply changes. The start and end areas of the beads are sensitive to these external factors. Thus, these areas are excluded during geometry measurement. The excluded area is approximately less than 10% of the total bead.

Second, approximately 90% of the bead is divided into 100 points, and the 4 response values are measured at each point. For this purpose, CMM is used to obtain a cross-sectional image at each point and convert this image into coordinate information. The CMM scans the beads to obtain a cross-sectional image and converts it into coordinate information. When scanning beads using CMM, a matte spray is sprayed on top of a single-sided bead to prevent measurement errors and noise, followed by matte treatment. After scanning, the scanned cross-sectional image is converted into XYZ coordinate information. The x coordinate indicates the location where the cross-sectional image is scanned at the bead. Once x is given, the y and z coordinates are generated from the scanned cross-sectional image at x. Figure 5 shows an example of the XYZ coordinate information obtained from the cross-sectional image.

Third, based on the coordinate information, interpolation is conducted on the coordinate information. One of the measurement-related difficulties is that coordinate points are not densely present, as shown in Fig. 5. Each point is fitted with tertiary spline interpolation to form a curve. Tertiary spline interpolation is a method of obtaining an n-order polynomial past that n-point, given a discontinuous n-point. In other words, the sub-intercept between each point is connected in a tertiary polynomial, and all the segments are combined to make differentiation possible. The third spline interpolation method, which exists as a built-in function in the MATLAB 2019a software, is applied. The left-hand figure in Fig. 6 is the image during the third spline interpolation process, and the right-hand figure is the image connected in a curved form after applying the interpolation method.

Fourth, after applying the spline interpolation method, the height, width, right toe angle, and left toe angle of the bead are measured as shown in the interpolated images. It is easy to measure the height and width of the beads from the interpolated images. However, toe angles cannot be measured directly because they show curvature. To measure the toe angles, the point in the curve data that maximizes the curvature value is identified first. Based on this point, the toe angle is measured using a tangent line at the part corresponding to the center of the horizontal line, curve, and bead height. As shown in Fig. 7, the curvature is the maximum part of the area that appears red. Based on this point, the left and right toe angles are measured using tangents. Overall, the height, width, left toe angle, and right toe angle

| Experiment Number | Coded value | Real value |
|-------------------|-------------|------------|
|                   | Current     | Welding speed | Feed rate |
|                   | Current     | Welding speed | Feed rate |
|                   | Real value  |            | |

| Experiment Number | Coded value | Real value |
|-------------------|-------------|------------|
|                   | Current     | Welding speed | Feed rate |
|                   | Current     | Welding speed | Feed rate |
|                   | Real value  |            | |

| Experiment Number | Coded value | Real value |
|-------------------|-------------|------------|
|                   | Current     | Welding speed | Feed rate |
|                   | Current     | Welding speed | Feed rate |
|                   | Real value  |            | |

| Experiment Number | Coded value | Real value |
|-------------------|-------------|------------|
|                   | Current     | Welding speed | Feed rate |
|                   | Current     | Welding speed | Feed rate |
|                   | Real value  |            | |
| Experiment Number (Bead Number) | Bead Image |
|-------------------------------|------------|
| 1                             | ![Image 1](image1) |
| 2                             | ![Image 2](image2) |
| 3                             | ![Image 3](image3) |
| 4                             | ![Image 4](image4) |
| 5                             | ![Image 5](image5) |
| 6                             | ![Image 6](image6) |
| 7                             | ![Image 7](image7) |
| 8                             | ![Image 8](image8) |
| 9                             | ![Image 9](image9) |
| 10                            | ![Image 10](image10) |
| 11                            | ![Image 11](image11) |
| 12                            | ![Image 12](image12) |
| 13                            | ![Image 13](image13) |
| 14                            | ![Image 14](image14) |
| 15                            | ![Image 15](image15) |
| 16                            | ![Image 16](image16) |
| 17                            | ![Image 17](image17) |
are measured at 100 points on each bead using the automatic measurement program.

2.2.3 Step 3: modeling the geometry of the bead

In step 2, 100 values for each response variable and bead were obtained. Based on these data, regression models are built for the four response variables. The average and standard deviation values of each response at each bead are calculated, and the regression models are built for the mean and standard deviation for each response variable. Therefore, a total of eight regression models are built as shown in the following expressions: \( \hat{Y}_{\mu1}, \hat{Y}_{\mu2}, \hat{Y}_{\mu3}, \) and \( \hat{Y}_{\mu4} \) represent the fitted models for the mean of height, width, right toe angle, and left toe angle, respectively, while \( \hat{Y}_{\sigma1}, \hat{Y}_{\sigma2}, \hat{Y}_{\sigma3}, \) and \( \hat{Y}_{\sigma4} \) represent the fitted models for the standard deviations of the height, width, right toe angle, and left toe angle, respectively, and \( x_1, x_2, \) and \( x_3 \) represent the coded current, welding speed, and feed rate, respectively.

\[
\hat{Y}_{\mu1} = 1.460 - 0.1315 x_1 - 0.1020 x_2 + 0.4428 x_3 \\
- 0.0885 x_1^2 - 0.1177 x_2^2 - 0.0441 x_3^2 \\
+ 0.0886 x_1 x_2 + 0.0139 x_1 x_3 - 0.0053 x_2 x_3
\]

\[
\hat{Y}_{\mu2} = 6.399 + 0.709 x_1 - 0.277 x_2 + 0.247 x_3 \\
- 0.448 x_1^2 - 0.335 x_2^2 - 0.269 x_3^2 \\
+ 0.024 x_1 x_2 + 0.127 x_1 x_3 + 0.006 x_2 x_3
\]

\[
\hat{Y}_{\mu3} = 152.99 + 6.614 x_1 + 4.503 x_2 - 9.177 x_3 \\
- 3.551 x_1^2 - 2.55 x_2^2 - 0.864 x_3^2 \\
- 0.80 x_1 x_2 + 1.52 x_1 x_3 - 0.44 x_2 x_3
\]

\[
\hat{Y}_{\mu4} = 151.89 + 7.18 x_1 + 4.86 x_2 - 8.57 x_3 \\
- 3.24 x_1^2 - 2.14 x_2^2 - 0.44 x_3^2 \\
- 0.39 x_1 x_2 + 1.99 x_1 x_3 - 0.98 x_2 x_3
\]

\[
\hat{Y}_{\sigma1} = 0.1484 - 0.0250 x_1 - 0.0046 x_2 + 0.0201 x_3 \\
+ 0.0024 x_1^2 - 0.0169 x_2^2 - 0.0240 x_3^2 \\
+ 0.0030 x_1 x_2 - 0.0153 x_1 x_3 + 0.0124 x_2 x_3
\]

\[
\hat{Y}_{\sigma2} = 0.4170 + 0.0714 x_1 + 0.0112 x_2 - 0.1617 x_3 \\
- 0.0281 x_1^2 + 0.0187 x_2^2 + 0.0812 x_3^2 \\
- 0.0181 x_1 x_2 - 0.0749 x_1 x_3 - 0.0285 x_2 x_3
\]

\[
\hat{Y}_{\sigma3} = 3.39 - 0.718 x_1 - 0.403 x_2 + 0.084 x_3 \\
+ 0.371 x_1^2 + 0.374 x_2^2 - 0.082 x_3^2 \\
- 0.089 x_1 x_2 - 0.725 x_1 x_3 + 0.281 x_2 x_3
\]

\[
\hat{Y}_{\sigma4} = 3.210 - 0.123 x_1 - 0.584 x_2 - 0.045 x_3 \\
- 0.094 x_1^2 + 0.556 x_2^2 - 0.303 x_3^2 \\
- 0.277 x_1 x_2 - 0.506 x_1 x_3 + 0.180 x_2 x_3
\]

2.2.4 Step 4: specifying the target values for the bead geometry

For optimization, the target values for the bead geometry should be specified in advance. The target values are specified by process engineers considering the purpose of the WAAM process. For example, flat and wide beads can be preferred over high and narrow beads, or vice versa. In this
regard, this step assumes three types of ideal beads: beads 1, 2, and 3, as shown in Fig. 8. Bead 1 is assumed to be the flattest and widest bead, while bead 3 is the narrowest one. Thereafter, the target geometry values are set for the three ideal beads by interviewing field experts in charge of the WAAM process. The target geometry values of the three ideal beads are listed in Table 6.

2.2.5 Step 5: constructing the desirability functions

In this step, desirability functions are constructed for the mean and standard deviation of each response variable. To construct the desirability functions, $y_{\min}$, $T$, and $y_{\max}$ should be predetermined. Table 7 reports the $y_{\min}$, $T$, and $y_{\max}$ values for the eight response surface models fitted in step 3. The four mean response surface models (i.e., $\hat{y}_{\mu 1}$, $\hat{y}_{\mu 2}$, $\hat{y}_{\mu 3}$, and $\hat{y}_{\mu 4}$) are nominal-the-best types. The four desirability functions, $d_{\mu j}(x)$ for $j = 1, 2, 3, 4$, are constructed based on Derringer and Suich [18]. They are shown in Fig. 9 and represented as follows:

$$d_{\mu j}(x) = \begin{cases} 0, & \text{if } \hat{y}_{\mu j}(x) \leq y_{\min} \mu j \text{ or } \hat{y}_{\mu j}(x) \geq y_{\max} \mu j \\ \left( \frac{\hat{y}_{\mu j}(x) - y_{\min} \mu j}{T_{\mu j} - y_{\min} \mu j} \right)^{s_{\mu j}}, & \text{if } y_{\min} \mu j < \hat{y}_{\mu j}(x) < T_{\mu j} \\ \left( \frac{\hat{y}_{\mu j}(x) - y_{\max} \mu j}{y_{\max} \mu j - T_{\mu j}} \right)^{t_{\mu j}}, & \text{if } T_{\mu j} \leq \hat{y}_{\mu j}(x) < y_{\max} \mu j \end{cases}$$

$T_{\mu j}$ is the target value of $y_{\mu j}$, $y_{\min}$ and $y_{\max}$ are the lower and upper limits of $y_{\mu j}$, respectively, and $s_{\mu j}$ and $t_{\mu j}$ determine the shapes of $d_{\mu j}$. These values are specified as shown in Table 7. It should be noted that the $T_{\mu j}$ have already been determined in step 4; thus, the target values given in Table 7 are the same as those in Table 6. For simplicity, this step assumes linear functions for the desirability functions (i.e., $s_{\mu j} = t_{\mu j} = 0$, for $j = 1, 2, 3, 4$).

Conversely, the four standard deviation response surface models (i.e., $\hat{y}_{\sigma 1}$, $\hat{y}_{\sigma 2}$, $\hat{y}_{\sigma 3}$, and $\hat{y}_{\sigma 4}$) are smaller-the-better types because it is desirable to have small variability for the bead geometry. Thus, the values for only the target and upper limit are specified as shown in Table 7. Similar to step 4, these values are determined by liaising with the field
experts in charge of the WAAM process. Like the four mean of response variables, this step assumes linear functions for the desirability functions (i.e., \( t_j = 0 \), for \( j = 1, 2, 3, 4 \)). The four desirability functions, \( d_j(x) \) for \( j = 1, 2, 3, 4 \), are also constructed based on Derringer and Suich [18]. They are shown in Fig. 9 and represented as follows:

\[
d_j(x) = \begin{cases} 
1, & \text{if } \hat{y}_j(x) \leq y_j^{\min} \\
\left( \frac{y_j^{\max} - y_j(x)}{y_j^{\max} - y_j^{\min}} \right)^{t_j}, & \text{if } y_j^{\min} < \hat{y}_j(x) < y_j^{\max} \\
0, & \text{if } \hat{y}_j(x) \geq y_j^{\max}
\end{cases}
\]

2.2.6 Step 6: maximizing the desirability function

The eight individual desirability functions given in Fig. 9 are aggregated into an overall desirability function, and then the optimal setting of the input variable is obtained by maximizing the overall desirability function. This step assumes that the eight desirability functions are equally important; thus, they are aggregated as a geometric mean as follows:

\[
D(x) = \left( \prod_{j=1}^{8} d_j(x) \right)^{\frac{1}{8}}
\]

Table 6 reports the result of maximizing the overall desirability function. It is carried out using the Minitab 19 software. The maximization is carried out for each bead; thus, three sets of optimal settings of the input variables are obtained: \( D \) is highest when bead 1 is used as the target bead geometry, and \( D \) is close to 1, which means that it is capable to obtain bead 1 by setting the input variable values into the optimal setting. Conversely, it is relatively difficult to obtain Bead 3 according to the optimization results.

2.2.7 Step 7: validating the result of the optimization

In step 7, the optimal settings obtained in step 6 are validated. Usually, additional experiments are conducted by maintaining the input variables at the optimal settings. Then, the measured response values from the additional experiments are compared to the predicted response values. If they are similar, it is believed that the obtained optimal setting

| Table 6 | Target geometry values for beads 1, 2, and 3 |
|---|---|
| Geometry | Unit | Bead 1 | Bead 2 | Bead 3 |
| Height | mm | 1.1 | 2.0 | 1.2 |
| Width | mm | 6.6 | 7.3 | 4.5 |
| Left toe angle | Degree | 159.4 | 148.2 | 132.7 |
| Right toe angle | Degree | 158.9 | 148.6 | 135.7 |

| Table 7 | Parameter values for individual desirability functions |
|---|---|---|---|---|
| Response | Model | Type | Bead | \( \gamma^{\min} \) | \( T \) | \( \gamma^{\max} \) |
| Height | \( \hat{y}_{\mu_1} \) | NTB | 1 | 0.4 | 1.1 | 1.5 |
| | | | 2 | 1.4 | 2.0 | 2.3 |
| | | | 3 | 0.9 | 1.2 | 1.4 |
| | | STB | 1 | – | 0.2 | 0.5 |
| | | | 2 | – | 0.1 | 0.5 |
| | | | 3 | – | 0.1 | 0.2 |
| | \( \hat{y}_{\mu_2} \) | NTB | 1 | 1.7 | 6.6 | 7.9 |
| | | | 2 | 6.5 | 7.3 | 8.2 |
| | | | 3 | 4.0 | 4.5 | 5.0 |
| | | STB | 1 | – | 1.0 | 3.1 |
| | | | 2 | – | 0.4 | 0.9 |
| | | | 3 | – | 0.2 | 0.5 |
| Left toe angle | \( \hat{y}_{\mu_3} \) | NTB | 1 | 144.9 | 159.4 | 170.1 |
| | | | 2 | 139.1 | 148.2 | 157.4 |
| | | | 3 | 124.5 | 132.7 | 141.3 |
| | | STB | 1 | – | 3.3 | 12.6 |
| | | | 2 | – | 2.7 | 9.1 |
| | | | 3 | – | 3.2 | 8.4 |
| Right toe angle | \( \hat{y}_{\mu_4} \) | NTB | 1 | 129.5 | 158.9 | 165.0 |
| | | | 2 | 137.5 | 148.6 | 152.9 |
| | | | 3 | 126.6 | 135.7 | 146.6 |
| | | STB | 1 | – | 3.7 | 17.8 |
| | | | 2 | – | 2.8 | 8.7 |
| | | | 3 | – | 3.3 | 10.0 |
Fig. 9 The eight individual desirability functions

Table 8 Optimization results

| Bead | Optimal setting | Predicted response value at the optimal setting | D |
|------|----------------|-----------------------------------------------|---|
|      | Coded | Real |                                            |   |
| 1 | 1.01 0.84 -0.09 | 323 18.80 146 | 1.10 0.11 6.17 0.49 | 158.08 2.93 158.75 2.69 0.98 |
| 2 | 0.64 -0.20 0.75 | 315 14.11 185 | 1.66 0.13 6.80 0.34 | 148.30 2.81 148.57 2.79 0.82 |
| 3 | -1.27 -1.23 -0.83 | 271 9.42 112 | 1.18 0.13 4.40 0.35 | 136.38 5.22 135.60 3.83 0.74 |

Table 9 Experimental conditions and results

| Experiment number | Experimental conditions | “Predicted” mean and standard deviation of bead geometry | “Actual” mean and standard deviation of bead geometry |
|-------------------|-------------------------|------------------------------------------------------|----------------------------------------------------|
|                   |                         | \( \bar{y}_{\mu 1} \), \( \bar{y}_{\mu 2} \), \( \bar{y}_{\mu 3} \), \( \bar{y}_{\mu 4} \), \( \bar{y}_{\mu 1} \), \( \bar{y}_{\mu 2} \), \( \bar{y}_{\mu 3} \), \( \bar{y}_{\mu 4} \), \( \bar{y}_{\mu 4} \), \( \bar{y}_{\mu 4} \), \( \bar{y}_{\mu 4} \) | \( y_{\mu 1} \), \( y_{\mu 1} \), \( y_{\mu 2} \), \( y_{\mu 2} \), \( y_{\mu 3} \), \( y_{\mu 3} \), \( y_{\mu 4} \), \( y_{\mu 4} \), \( y_{\mu 4} \), \( y_{\mu 4} \), \( y_{\mu 4} \), \( y_{\mu 4} \) |
| 1 | 275 15 150 | 1.50 0.18 5.11 0.31 142 4.60 140 3.23 | 1.76 0.20 5.85 0.80 124 3.74 129 3.50 |
| 2 | 325 15 150 | 1.21 0.22 6.64 0.46 156 3.05 156 2.97 | 1.50 0.26 7.16 0.57 135 5.40 139 4.15 |
| 3 | 300 10 150 | 1.43 0.23 6.30 0.43 145 4.29 144 4.53 | 2.05 0.30 7.81 0.59 114 4.02 126 4.43 |
| 4 | 300 20 150 | 1.20 0.22 5.69 0.45 155 3.40 155 3.24 | 1.40 0.25 5.90 0.58 137 4.86 141 3.10 |
| 5 | 300 15 100 | 0.93 0.20 5.82 0.69 162 3.20 161 2.90 | 1.05 0.22 6.49 0.54 145 3.27 148 3.55 |
| 6 | 300 15 200 | 1.89 0.14 6.35 0.34 142 3.39 142 2.81 | 2.08 0.21 8.04 1.13 132 4.55 133 2.56 |

Table 10 MAPE of the eight response surface models

| \( \bar{y}_{\mu 1} \), \( \bar{y}_{\mu 2} \), \( \bar{y}_{\mu 3} \), \( \bar{y}_{\mu 4} \), \( \bar{y}_{\mu 1} \), \( \bar{y}_{\mu 2} \), \( \bar{y}_{\mu 3} \), \( \bar{y}_{\mu 4} \), \( \bar{y}_{\mu 4} \), \( \bar{y}_{\mu 4} \), \( \bar{y}_{\mu 4} \) | MAPE |
|-------------------------------------------------|------|
| \( \bar{y}_{\mu 1} \), \( \bar{y}_{\mu 2} \), \( \bar{y}_{\mu 3} \), \( \bar{y}_{\mu 4} \), \( \bar{y}_{\mu 1} \), \( \bar{y}_{\mu 2} \), \( \bar{y}_{\mu 3} \), \( \bar{y}_{\mu 4} \), \( \bar{y}_{\mu 4} \), \( \bar{y}_{\mu 4} \), \( \bar{y}_{\mu 4} \) | 16.4 17.2 12.4 27.9 14.9 21.8 10.2 11.9 |

\( \text{Springer} \)
is valid. However, conducting additional experiments by keeping the input variables at the optimal settings were not available because they were difficult to precisely control. Therefore, the optimal settings were validated indirectly by validating the eight response surface models.

It is important to validate the eight response surface models because the optimal setting of the input variables is obtained based on the eight response surface models, assuming that these models are fitted reasonably well. More specifically, the eight response surface models are transformed into eight individual desirability functions, which are aggregated into the overall desirability function. The optimal setting is obtained by maximizing the overall desirability function. Thus, when the assumption is not satisfied, the obtained optimal setting is far from the true optimal setting [30].

To validate the eight response surface models, six additional experiments are conducted, and six beads are obtained thereof. Thereafter, the height, width, and left and right angles at 100 points on each bead are measured, as in step 2. The rightmost eight columns in Table 9 present the mean and standard deviations of the measured values for the four geometries. For validation, the predicted mean and standard deviation of the bead geometry from the eight response surface models are calculated and compared to actual data from the experiments.

Based on the predicted and actual values, the mean absolute percentage error (MAPE) is calculated to evaluate the performance of the response surface models. This is an indicator of the amount of error that constitutes the predicted value. The MAPE has values ranging from 0 to 100, and the closer it is to zero, the better the performance of the response surface model. When $n$ samples are present, $A_i$ is the actual value and $F_i$ is the predicted value. In this example, six samples exist (i.e., $n = 6$) because six additional experiments were conducted. Table 10 lists the MAPE values of the eight response surface models. The eight models show generally low MAPE values, which means that the response surface models are reasonably well-fitted.

$$\text{MAPE} = \frac{100}{n} \sum_{i=1}^{n} \left| \frac{A_i - F_i}{A_i} \right|$$

3 Conclusions

In this study, a method for determining the optimal process parameters is presented for the ideal beads defined by users in welding using TZM materials. The tradeoffs between the eight responses vis-à-vis the bead geometry is systematically considered by employing the desirability function method. Furthermore, TZM single-layer welding is performed under the designed settings (i.e., CCD), the bead geometry is measured, the response surface model is established, and the individual and overall desirability functions are established to optimize the optimal process parameters for each ideal bead. This study establishes optimal parameters that minimize variability in beads, considering not only the mean of the bead geometry but also the standard deviation, in the process of determining optimal process parameters. At the optimal parameter setting, the desirability function value shows 0.85 on average which is close to ideal value of 1.00. This result indicates that valid optimal settings for the process parameters can be obtained via the proposed method. The plausibility of the study is proven by validating the eight response surface models. MAPE for the eight response surface models is 16.6% on average which implies that the models are reasonably well-fitted.

Usually, in MRSO, multiple responses are often in conflict; thus, tradeoffs between the responses should be carefully analyzed. In our case, the individual desirability functions are aggregated by geometric mean with equal weights. In this simple aggregation approach, it is difficult to understand the tradeoffs between the responses and to obtain a satisfactory compromised solution.

Aggregating the individual desirability functions into a single measure, called overall desirability, is a typical multi-objective optimization problem or group decision making problem. Thus, various multi-objective or group decision making methods can be employed for a more systematic aggregation. Ji et al. suggested a bi-objective optimization model to aggregate individual probability distribution functions representing expert opinions [31]. This method develops two objective functions representing objectivity and reliability and aggregates expert opinions to improve the two objective functions. Similarly, the proposed method may develop new objective functions and aggregate the individual desirability functions to improve the objective functions.

On the other hand, the functions can be aggregated through an iteration algorithm such as Cai et al. method [32]. The iteration algorithm usually includes an iterative procedure. For example, an analyst (software) generates overall desirability function and obtains an initial solution by maximizing the overall desirability function. Then, the process engineer evaluates the initial solution and adjusts parameters used for aggregating the overall desirability function. Weights are representative parameters for this purpose. Based on the adjusted parameters, new overall desirability function is generated, and new solution is generated accordingly. The process engineer evaluates again the new solution. This iterative process continues until a satisfactory solution is obtained. Through the iterative procedure, the process engineer can have several chances to articulate his/her preference on the responses which is helpful for understanding the tradeoffs between the responses.

Funding This research was supported by the MSIT (Ministry of Science, ICT), Korea, under the High-Potential Individuals Global
Training Program) (No. 2020–0–01539) supervised by the IITP (Institute for Information & Communications Technology Planning & Evaluation).

**Declarations**

**Ethics approval** The submitted work is original, complete, and has not been submitted/published elsewhere in any form or language.

**Consent to participate** Not applicable.

**Consent for publication** The authors provide their consent to publish the manuscript in the International Journal of Advanced Manufacturing Technology.

**Competing interests** The authors declare no competing interests.

**References**

1. Wu B, Pan Z, Ding D, Cuiuri D, Li H, Xu J, Norrish J (2008) A review of the wire arc additive manufacturing of metals: properties, defects and quality improvement. J Manuf Process 35:127–139. https://doi.org/10.1016/j.jmapro.2018.08.001
2. Danisman CB, Yavas B, Yucel O, Sahin F, Goller G (2016) Processing and characterization of spark plasma sintered TZM alloy. J Alloys Compd 685:860–868. https://doi.org/10.1016/j.jallcom.2016.06.161
3. Yu B, Wang T, Lv Y, Jiang S, Yang J, Feng J (2021) Interfacial strengthening mechanism of electron beam welding-brazed TZM/30CrMoSiA joint with a vanadium interlayer. Mater Sci Eng, A 817:141369. https://doi.org/10.1016/j.msea.2021.141369
4. Moskal G, Grabowski A, Lisiecki A (2015) Laser remelting of silicide coatings on Mo and TZM alloy. Solid State Phenom 226:121–126. https://doi.org/10.4028/www.scientific.net/SSP.226.121
5. Chakraborty SP, Krishnamurthy N (2014) Fabrication of a Mo based high temperature TZM alloy by non-consumable arc melting technique. Proc Int Symp Disch Electr Insul Vac ISDEIV 2014:749–752. https://doi.org/10.1109/DEIV.2014.6961791
6. Sharma IG, Chakraborty SP, Suri AK (2005) Preparation of TZM alloy by aluminothermic smelting and its characterization. J Alloys Compd 393:122–127. https://doi.org/10.1016/j.jallcom.2004.09.055
7. Ghazali MAM, Harimon MA, Mustapa MS (2020) Mechanical behavior and microstructural analysis of Molybdenum-TZM alloy subjected to different annealing temperature. JSE J Sci Eng, A 817:141369. https://doi.org/10.1016/j. ijse.2020.30650
8. Fan J, Lu M, Cheng H, Tian J, Huang B (2009) Effect of alloying elements Ti, Zr on the property and microstructure of molybdenum. Int J Refract Met Hard Mater 27:78–82. https://doi.org/10.1016/j.ijrmhm.2008.03.006
9. Kaserer L et al (2020) Microstructure and mechanical properties of molybdenum-titanium-zirconium-carbon alloy TZM processed via laser powder-bed fusion. Int J Refract Met Hard Mater. https://doi.org/10.1016/j.ijrmhm.2020.105369
10. Myers RH, Montgomery DC, Anderson-Cook CM (2016) Response surface methodology: process and product optimization using designed experiments. John Wiley & Sons
11. Kim D, Rhee S, Park H (2002) Modelling and optimization of a GMA welding process by genetic algorithm and response surface methodology. Int J Prod Res 40:1699–1711. https://doi.org/10.1080/00207540110119964
12. Dey V, Pratihar DK, Datta GL, Jha MN, Saha TK, Bapat AV (2009) Optimization of bead geometry in electron beam welding using a genetic algorithm. J Mater Process Technol 209:1151–1157. https://doi.org/10.1016/j.jmatprotec.2008.03.019
13. Geng H, Li J, Xiong J, Lin X, Zhang F (2017) Optimization of wire feed for WAAM based additive manufacturing. J Mater Process Technol 243:40–47. https://doi.org/10.1016/j.jmatprotec.2016.11.027
14. Benyounis KY, Olabi AG, Hashmi MSJ (2005) Effect of laser welding parameters on the heat input and weld-bead profile. J Mater Process Technol 164:978–985. https://doi.org/10.1016/j.jmatprotec.2005.02.060
15. Gunaraj V, Muruga N (1999) Application of response surface methodology for predicting weld bead quality in submerged arc welding of pipes. J Mater Process Technol 88:266–275. https://doi.org/10.1016/S0924-0136(98)00405-1
16. Lee DH, Kim SH, Byun JH (2020) A method of steepest ascent for multiresponse surface optimization using a desirability function method. Qual Reliab Eng Int 36:1931–1948. https://doi.org/10.1002/qre.2666
17. Lee DH, Jeong JJ, Kim KJ (2018) A desirability function method for optimizing mean and variability of multiple responses using a posterior preference articulation approach. Qual Reliab Eng Int 34:360–376. https://doi.org/10.1002/qre.2258
18. Derringer G, Suich R (1980) Simultaneous optimization of several response variables. J Qual Technol 12:214–219. https://doi.org/10.1080/00224065.1980.11989068
19. Lunani M, Nair VN, Wasserman GS (1997) Graphical methods for robust design with dynamic characteristics. J Qual Technol 29:327–338
20. Vining GG, Myers RH (1990) Combining Taguchi and response surface philosophies: a dual response approach. J Qual Technol 22:38–45
21. Lin DK, Tu W (1995) Dual response surface optimization. J Qual Technol 27:34–39
22. Kim KJ, Lin DK (1998) Dual response surface optimization: a fuzzy modeling approach. J Qual Technol 30:1–10
23. Copeland KA, Nelson PR (1996) Dual response optimization via direct function minimization. J Qual Technol 28:331–336
24. Lee DH, Jeong JJ, Kim KJ (2009) A posterior preference articulation approach to dual-response-surface optimization. IIE Trans 42:161–171
25. Ames A, Mattucci N, McDonald S, Szonyi G, Hawkins D (1997) Quality loss function for optimization across multiple response surfaces. J Qual Technol 29:339–346
26. Pignatiello J (1993) Strategies for robustmultiresponse quality engineering. IIE Trans 25:5–15
27. Vining G (1998) A compromise approach to multiresponse optimization. J Qual Technol 30:309–313
28. Ko Y, Kim K, Jun C (2005) A new loss function-based method for multiresponse optimization. J Qual Technol 37:50–59
29. Lee D, Kim K, Köksalan M (2012) An interactive method to multiresponse surface optimization based on pairwise comparisons. IIE Trans 44:13–26
30. Xu D, Albin SL (2003) Robust optimization of experimentally derived objective functions. IIE Trans 35:793–802. https://doi.org/10.1016/j.ijqre.200408170304408
31. Ji C, Lu X, Zhang W (2020) A biobjective optimization model for expert opinions aggregation and its application in group decision making. IEEE Syst J 15:2834–2844
32. Cai M, Lin Y, Han B, Liu C, Zhang W (2016) On a simple and efficient approach to probability distribution function aggregation. IEEE Trans Syst Man Cybern Syst 47:2444–2453

**Publisher’s note** Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.