\textit{R}^2 \text{ Corrections to Black Ring Entropy}

Iosif Bena and Per Kraus

\textit{Department of Physics and Astronomy, UCLA, Los Angeles, CA 90095-1547, USA}

Recently, Guică, Huang, Li, and Strominger considered an \textit{R}^2 correction to the entropy of a black ring, and found a mismatch between supergravity and the CFT. However, such a comparison should take into account the subtle distinction between the asymptotic charges of the black ring and the charges entering the CFT description. We show that using the correct charges yields perfect agreement.
1. Introduction

There has been a recent surge of interest in higher derivative corrections to black hole entropy in string theory (an incomplete list of references is [1,2,3,4,5,6]). In certain cases it is possible to demonstrate precise agreement between the supergravity and CFT corrections. A recent paper by Guica, Huang, Li, and Strominger [7] considered this in the context of five dimensional black rings [8,9,10,11]. On the supergravity side they included an $R^2$ term known to be present in M-theory on CY$_3$, and used Wald’s formula [12] to obtain the entropy correction

$$\Delta S_{BR} = \frac{\pi}{6} c_2 \cdot p \sqrt{\frac{\hat{q}_0}{D}} .$$

(1.1)

On the CFT side they took into account the correction to the central charge, and obtained

$$\Delta S_{BR} = \frac{\pi}{6} c_2 \cdot p \sqrt{\frac{\hat{q}_0}{D}} + \frac{\pi}{24} c_2 \cdot p \sqrt{\frac{D}{\hat{q}_0}} + \ldots .$$

(1.2)

The second term represents a mismatch. However, in performing such comparisons one has to be careful to correctly translate the charges of the black ring into parameters appearing in the CFT. The correct microscopic assignment appears in [13] and differs from the one that was discussed in [14] and was used in [7] to obtain equation (1.2). When the difference between the two is taken into account, the second term in (1.2) is absent, and the two sides agree, as we now demonstrate.

2. Two Candidates for the Microscopic Description of the Black Ring

There exist two distinct proposals [13,14] to count the entropy of the 5D BPS black ring by relating it to the $(4,0)$ CFT of the 4D BPS black hole [15]. The CFT entropy is

$$S_{BR} = 2\pi \sqrt{\frac{c_L \hat{q}_0}{6}}$$

(2.1)

and the leading contribution to the central charge is the product of the dipole charges:

$$c_L = 6D = 6D_{ABC} p^A p^B p^C .$$

(2.2)

However, the expression for $\hat{q}_0$ is different. In [14] is was argued that

$$\hat{q}_0 = -J_\psi + \frac{1}{12} D^{AB} q_A q_B - \frac{c_L}{24} .$$

(2.3)

\footnote{To facilitate easy comparison, we follow the notation of [14,7]. To translate, use that $(Q_A, q_A)$ of [13] are identified with $(q_A, p^A)$ of [14,7]. There is also a flip in the sign of $J_\psi$.}
where $J_\psi$ is the black ring angular momentum in the plane of the ring, and the $q_A$ are the conserved charges of the black ring as measured at infinity.

Earlier, in [13] it had been argued (see pp. 5-6 of that paper) that the black ring entropy is exactly that of a 4D black hole with charges given by the seven parameters $p^1, p^2, p^3, \overline{q}_1, \overline{q}_2, \overline{q}_3$ and $J_{\text{tube}} \equiv J_\psi - J_\phi$. The microscopic entropy computation for this 4D black hole was performed quite some time ago [14,16]. The entropy and central charge are given by (2.1) and (2.2), except that $\hat{q}_0$ is given by

$$\hat{q}_0' = -(J_\psi - J_\phi) + \frac{1}{12} D^{AB} \overline{q}_A \overline{q}_B .$$

(2.4)

Written in this form, the black ring entropy is manifestly $E_7(7)$ invariant [13], as should be the case for a 4D black hole entropy. By contrast, the $\frac{c_L}{24}$ shift in (2.3) obscures this invariance. Refs. [9,13] argued that $J_{\text{tube}}$ and $\overline{q}_A$ are the momentum and charge living on the ring itself, which differ from $J_\psi$ and $q_A$. The relation between the charges is

$$\overline{q}_A = q_A - \frac{6D_{ABC}}{2} p^B p^C .$$

(2.5)

As one can easily check, $\hat{q}_0$ and $\hat{q}_0'$ are equal, so they both yield agreement at leading order with the entropy calculated in gravity [9,10].

This situation resulted in some confusion concerning which charges should appear in the CFT. Note that both [13] and [14] were trying to replace the circular black ring with a straight black string, which is a discontinuous process and so inherently ambiguous.

Fortunately, it is possible to resolve this puzzle unambiguously by putting the black ring in Taub-NUT, implementing the philosophy suggested and used in [17]. In [18] it was shown that by adjusting the moduli it is possible to move the black ring arbitrarily far from the Taub-NUT center, and the black ring solution then reduces to the 4D black hole/5D black string solution constructed in [19]. One finds that the momentum and charges of this 4D black hole are $J_{\text{tube}}$ and $\overline{q}_A$. In this way one derives the 4D black hole description rather than having to assume it, and one has no choice in the charge assignments. This interpolation thus proves that the correct microscopic description of the black ring is (2.4) and not (2.3).

3. $R^2$ corrections

We now proceed to show that by using the microscopic description (2.4) we resolve the mismatch [7] between the $R^2$ corrections to the gravitational entropy and the corresponding corrections in the CFT.

The corrections on the CFT side are obtained by considering the first correction to $c_L$:

$$c_L \rightarrow c_L + \Delta c_L$$

(3.1)
where $\Delta c_L = c_2 \cdot p$. If one uses the microscopic description (2.3), a correction to $c_L$ affects both the explicit $c_L$ in (2.1) as well as $\hat{q}_0$. The change in the microscopic entropy is given by equation (11) in [7]:

$$\Delta S_{BR} = \frac{\pi}{6} c_2 \cdot p \sqrt{\frac{\hat{q}_0}{D}} + \frac{\pi}{24} c_2 \cdot p \sqrt{\frac{D}{\hat{q}_0}} + ...$$

(3.2)

However, if one uses the microscopic description (2.4), the correction to $c_L$ does not affect $\hat{q}_0'$, and the change in the entropy formula is

$$\Delta' S_{BR} = \frac{\pi}{6} c_2 \cdot p \sqrt{\frac{\hat{q}_0'}{D}} + ...$$

(3.3)

This agrees exactly with the correction coming from the $R^2$ terms in the gravity description (1.1) (equation (9) in [7]). Thus in the microscopic description (2.4) the second term in (3.2) simply does not exist, and the agreement with gravity is perfect. This gives further confirmation for the correctness of the microscopic description (2.4).

**Acknowledgements:**

The work of IB and PK is supported in part by NSF grant 0099590.

---

2 One might try to argue that the second term in equation (3.2) is subleading and therefore not captured by the Cardy formula. However, this argument could equally well be applied to the terms of the order $(p^1 p^2 p^3)^2$ in the uncorrected entropy. But omission of such terms would destroy the match between the supergravity and CFT entropies at the leading order.
References

[1] G. L. Cardoso, B. de Wit, J. Kappeli and T. Mohaupt, “Supersymmetric black hole solutions with R² interactions”, [arXiv:hep-th/0003157]; G. Lopes Cardoso, B. de Wit and T. Mohaupt, “Area law corrections from state counting and supergravity”, Class. Quant. Grav. 17, 1007 (2000) [arXiv:hep-th/9910179]; “Macroscopic entropy formulae and non-holomorphic corrections for supersymmetric black holes”, Nucl. Phys. B 567, 87 (2000) [arXiv:hep-th/9906094]; “Deviations from the area law for supersymmetric black holes”, Fortsch. Phys. 48, 49 (2000) [arXiv:hep-th/9904005]; “Corrections to macroscopic supersymmetric black-hole entropy”, Phys. Lett. B 451, 309 (1999) [arXiv:hep-th/9812082].

[2] H. Ooguri, A. Strominger and C. Vafa, “Black hole attractors and the topological string,” Phys. Rev. D 70, 106007 (2004) [arXiv:hep-th/0405146].

[3] A. Dabholkar, “Exact counting of black hole microstates”, [arXiv:hep-th/0409148].

[4] A. Sen, “How does a fundamental string stretch its horizon?,” [arXiv:hep-th/0411255].

[5] V. Hubeny, A. Maloney and M. Rangamani, “String-corrected black holes,” [arXiv:hep-th/0411272].

[6] A. Dabholkar, F. Denef, G. W. Moore and B. Pioline, “Exact and asymptotic degeneracies of small black holes,” [arXiv:hep-th/0502157].

[7] M. Guica, L. Huang, W. Li and A. Strominger, “R² Corrections for 5D Black Holes and Rings,” [arXiv:hep-th/0505188].

[8] H. Elvang, R. Emparan, D. Mateos and H. S. Reall, “A supersymmetric black ring,” Phys. Rev. Lett. 93, 211302 (2004) [arXiv:hep-th/0407065].

[9] I. Bena and N. P. Warner, “One ring to rule them all ... and in the darkness bind them?,” [arXiv:hep-th/0408106].

[10] H. Elvang, R. Emparan, D. Mateos and H. S. Reall, “Supersymmetric black rings and three-charge supertubes,” Phys. Rev. D 71, 024033 (2005) [arXiv:hep-th/0408120].

[11] J. P. Gauntlett and J. B. Gutowski, “General concentric black rings,” Phys. Rev. D 71, 045002 (2005) [arXiv:hep-th/0408122].

[12] R. M. Wald, “Black hole entropy is Noether charge,” Phys. Rev. D 48, 3427 (1993) [arXiv:gr-qc/9307038].

[13] I. Bena and P. Kraus, “Microscopic description of black rings in AdS/CFT,” JHEP 0412, 070 (2004) [arXiv:hep-th/0408180].

[14] M. Cyrier, M. Guica, D. Mateos and A. Strominger, “Microscopic entropy of the black ring,” [arXiv:hep-th/0411187].

[15] J. M. Maldacena, A. Strominger and E. Witten, “Black hole entropy in M-theory,” JHEP 9712, 002 (1997) [arXiv:hep-th/9711053].
[16] M. Bertolini and M. Trigiante, “Microscopic entropy of the most general four-dimensional BPS black hole,” JHEP 0010, 002 (2000) [arXiv:hep-th/0008201]. “Regular BPS black holes: Macroscopic and microscopic description of the generating solution,” Nucl. Phys. B 582, 393 (2000) [arXiv:hep-th/0002191].

[17] D. Gaiotto, A. Strominger and X. Yin, “5D Black Rings and 4D Black Holes,” arXiv:hep-th/0504126.

[18] I. Bena, P. Kraus and N. P. Warner, “Black Rings in Taub-NUT,” arXiv:hep-th/0504142.

[19] I. Bena, “Splitting hairs of the three charge black hole,” Phys. Rev. D 70, 105018 (2004) [arXiv:hep-th/0404073].