Bulk viscosity of the gluon plasma in a holographic approach

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Abstract

A gravity-scalar model in 5-dim. Riemann space is adjusted to the thermodynamics of SU(3) gauge field theory in the temperature range 1 - 10 $T/T_c$ to calculate holographically the bulk viscosity in 4-dim. Minkowski space. Various settings are compared, and it is argued that, upon an adjustment of the scalar potential to reproduce exactly the lattice data within a restricted temperature interval above $T_c$, rather robust values of the bulk viscosity to entropy density ratio are obtained.

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I. INTRODUCTION

The duality of $\mathcal{N} = 4$ supersymmetric Yang-Mills theory in 4-dim. Minkowki space with type IIB superstring theory on $AdS_5 \times S^5$ [1] has initiated a wealth of investigations aimed at exploiting the AdS/CFT correspondence to relate mutually properties of the gravitation sector (which is anti-de Sitter (AdS) in 5-dim. Riemann space) with conformal field theories (CFT). Such techniques look particularly useful for 4-dim. strongly coupled theories, where real-time processes are difficult to access. This in turn applies especially to strongly interacting systems, as subjects to QCD, created in the course of relativistic heavy-ion collisions, i.e. the quark-gluon plasma (QGP). Here, holographic techniques, based on the AdS/CFT correspondence, allow to calculate from suitable gravity duals the wanted observables quantifying properties of the QGP. Among the important quantities is the bulk viscosity which has a potentially strong impact on the analysis of the flow pattern in relativistic heavy-ion collisions [2] and may help to solve the photon-$\nu_2$ puzzle [3].

The gravity dual of QCD, even in the pure Yang-Mills sector, is not known. Moreover, QCD is not a CFT since, due to dimensional transmutation, an inherent energy scale is emergent which steers the running coupling. In such a situation and with a lacking top-down approach from string theory, it looks promising to utilize a bottom-up approach which incorporates a selected set of properties one is going to calculate after an appropriate adjustment of the 5-dim. Einstein gravity theory which emerges, strictly speaking, only in the large-$N_c$ limit and at large ’t Hooft coupling. A famous example is the gravity-scalar set-up, where a real scalar field is consistently coupled to gravity. The scalar $\phi$, dual to an operator $\mathcal{O}_\phi$, breaks conformal invariance of AdS space, simulating the corresponding breaking in Yang-Mills theory, the latter being expressed by the trace anomaly relation $T^\mu_\mu = \beta(\alpha)/(8\pi\alpha^2)\text{Tr} F^2$ of the Yang-Mills energy-momentum tensor $T_{\mu\nu}$, $\beta$ function, running coupling $\alpha$ and trace of the field strength tensor squared $\text{Tr} F^2$. Being interested in thermodynamic properties of the gluon plasma one embeds in the asymptotically AdS space a black brane which introduces a temperature via Hawking temperature and an entropy via Bekenstein-Hawking entropy. Besides the equilibrium thermodynamics, encoded in the gravity metric as dual of the gauge theory energy-momentum tensor, near-to-equilibrium quantities are accessible as correlators based on the energy-momentum tensor. For a medium without conserved charges these are the shear and bulk viscosities as first-order transport coefficients in a gradient expansion.
II. GRAVITY-SCALAR HOLOGRAPHIC MODELS

The class of gravity-scalar duals is defined by the action

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left( R - \frac{1}{2} (\partial \phi)^2 - V(\phi) \right) + \mathcal{L}_{GH}$$

(1)

where $\mathcal{L}_{GH}$ is the Gibbons-Hawking surface term, irrelevant or our purposes, and $G_5$ denotes the 5-dim. gravity constant. The "potential" $V(\phi)$ determines the self-interaction of the scalar $\phi$; it contains the constant term $V_0 = -12/L^2$ ensuring asymptotic AdS behavior with $L$ being the curvature scale set by the negative cosmological constant. The Riemann space is accordingly specified by extending the conformally flat 4-dim. space-time by the bulk variable $u$ resulting in the ansatz for the infinitesimal line element squared

$$ds^2 = \exp\{2A(u)\} \left( dx^2 - f(u) dt^2 + \frac{1}{f(u)} du^2 \right),$$

(2)

where (in conformal coordinates) $\lim_{u \to 0} f(u) = 1$ and $\lim_{u \to 0} A = \log(L/u)$ ensure the AdS property at the boundary $u \to 0$ and the simple zero of $f(u_H)$ defines the horizon at $u_H > 0$.

The scalar is supposed to have a radial profile $\phi(u)$ which for potentials such as $V(\phi) = V_0 + \frac{1}{2} m^2 \phi^2 + \cdots$ is constrained by the equation of motion to $\phi(u) = \phi(4-\Delta) u^{4-\Delta} + \phi_{\Delta} u^\Delta + \cdots$ near the boundary of AdS, where $\phi(4-\Delta)$ implies an additional term $\propto \int d^4x \phi(4-\Delta) O_\phi$ as deformation of the original CFT and $\langle O_\phi \rangle \propto \phi_{\Delta}$, i.e. $\phi$ is holographically dual to the operator $O_\phi$ with conformal dimension $\Delta_{\phi}$. For $\Delta_{\phi} = 4$, the dual operator is exactly marginal and the scalar field is massless, while for $\Delta_{\phi} \neq 4$ the source $\phi(4-\Delta)$ introduces a mass scale $\Lambda = \phi_{(4-\Delta)}^{1/(4-\Delta)}$ which explicitly breaks conformal invariance. The mass $m$ and the conformal dimension $\Delta_{\phi}$ are related by $m^2 L^2 = \Delta_{\phi}(\Delta_{\phi} - 4)$, which must satisfy $m^2 L^2 \geq -4$ to fulfill the Breitenlohner-Freedman bound. Renormalizability on the gauge theory side requires $\Delta_{\phi} \leq 4$, i.e. $m^2 L^2 \leq 0$. While an extension to $1 \leq \Delta_{\phi} \leq 2$ is possible [4], we restrict our attention to the upper branch of the mass-dimension relation and relevant operators, i.e. $2 < \Delta < 4$. This is already a special setting which follows, e.g., [5, 7, 8] and serves as outline of our analysis below. The improved holographic QCD (IHQCD) model [12], in contrast, is based on different potential asymptotics $V(\phi) - V_0 \propto e^\phi + \cdots$ which encodes the running ’t Hooft coupling $\lambda \propto e^\phi$ close to the boundary (here at $\phi \to -\infty$) and results in the marginal case $\Delta_{\phi} = 4$, while, for large ’t Hooft coupling, $V(\phi)$ is constructed to accomodate confinement and a linear glueball spectrum, cf. [5, 9].
III. THERMODYNAMICS

The two basic AdS/CFT thermodynamic relations

\[ T = -\frac{1}{4\pi} \left. \frac{df}{du} \right|_{u_H}, \quad s = \frac{1}{4G_5} \exp\{3A\} \left|_{u_H} \right. \]

(3)
determine the thermodynamics, e.g. by \( s(T)/T^3 \) for parametrically given temperature \( T(u_H) \) and entropy density \( s(u_H) \). Here, \( u_H \) is the horizon position in the bulk. Einstein’s equations determine, via the above conditions at the boundary, the metric coefficients at \( u_H \). To be specific we utilize

\[ V(\phi)L^2 = -12 \cosh \gamma \phi + b\phi^2 + \sum_{n=2}^{5} c_{2n} \phi^{2n} \]

(4)

with \( b = 6\gamma^2 + \Delta(\Delta - 4)/2 \) from [7, 8], but use solely the matching condition to lattice data of the SU(3) Yang-Mills equation of state in a finite temperature interval above \( T_c \). That is we ignore an \textit{a priori} scale setting at a certain energy and leave thus \( 2 < \Delta < 4, \gamma \) and \( c_{2n} \) as free parameters.

Without further integration constant, the velocity of sound squared, \( v_s^2 = d \log s/d \log T \), is given, while the pressure \( p = p_c + \int_{T_c}^{T} dT' s(T') \), energy density \( e = -p + sT \) and interaction measure \( I = e - 3p \) need one additional constant. A possibility is to employ the lattice input with \( p_c = p(T_c) \), which needs a definition of \( T_c \). The IHQCD model has a clear definition of \( T_c \); other options could be to choose \( T_c = T_{\min} \), where \( T_{\min} \) is the minimum of the temperature \( T \) as a function of \( u_H \) or \( s/T^3 \); in the latter case, the inflection point \( T_{ip} \) can be utilized to define \( T_c \) in cases where \( T \) as a function of \( s/T^3 \) does not have a minimum. If one refrains to catch Yang-Mills features at zero temperature (e.g. a linear glue ball spectrum w.r.t. a radial quantum number) and the latent heat in the deconfinement phase transition as in IHQCD [12] one can adjust the value of \( T_c \) arbitrarily; also, \( G_5 \) can be chosen without other constraints than the optimum reproduction of a given data set in a restricted temperature interval above \( T_c \). Here, we choose \( LT_c = (LT_{\min}, LT_{ip}) \) and adjust \( \gamma, \Delta, c_{2n} \) and \( G_5/L^3 \) by minimizing

\[ \chi^2_{s/T^3} = \log \left( \frac{1}{N} \sum_{i=1}^{N} \left[ \sigma(x_i) - y(x_i T_c L) \right]^2 \right), \]

(5)

where \( \sigma \equiv s(T)/T^3 \) refers to the lattice data at \( N \) mesh points \( x_i \equiv T_i/T_c \) and \( y \equiv G_5 s(T L)/(T L)^3 \) to the holographically calculated scaled entropy density.
IV. BULK VISCOSITY

The class of gravity-scalar models considered here belongs to so-called two-derivative models which provide the normalized shear viscosity $\eta/s = 1/(4\pi)$, irrespectively of a specific form of $V(\phi)$, at variance with the asymptotic behavior of weakly coupled QCD [13] and the expected minimum near $T_c$. Higher-order gravity models [10] abandon such a temperature independence. Nevertheless, in the strongly coupled region, $\eta/s = 1/(4\pi)$ represents an intriguingly important result which got popular since the analysis of flow observables in relativistic heavy-ion collisions at RHIC and LHC appeared consistent with that.

The bulk viscosity $\zeta$ follows within the present set-up from

$$\frac{\zeta}{\eta} = \left( \frac{d \log V}{d \phi} \right)^2 \left| p_{11} \right|_{\phi_H}^2$$

where (using the profile of the scalar field as bulk coordinate) the horizon value of the perturbation $p_{11}$ of the $x_1x_1$-metric component is determined by solving a linearized Einstein equation [11].

A. Optimum adjustment to lattice data

As shown in [15], a perfect matching to lattice data is accomplished by the potential (4) for $\Delta = 3.7650$ and $\gamma = 0.6580$ when including the polynomial distortions $c_{2n}$; omitting the latter ones (with $\Delta = 3.5976$ and $\gamma = 0.6938$) the match is near-perfect, see left panel in Fig. 1. The bulk to shear viscosity ratio (cf. right panel in Fig. 1) displays a linear section, where $\zeta/\eta = \pi C \Delta v_s^2$ with $C \approx 1.2$, thus fulfilling the Buchel bound $\zeta/\eta \geq 2\Delta v_s^2$ [16]. Such a linear relation $\zeta/\eta \propto \Delta v_s^2 = 1/3 - v_s^2$ is considered in [6] as interesting but as unclear whether it is a generic result of $Dp$ brane gauge theories. With the results of the next subsection we argue that it is generic for the gravity-scalar set-up only for perfect matching to SU(3) Yang-Mills theory. We emphasize that a quasi-particle model [17] obeys quantitatively a similar proportionality in the strong coupling regime, also with the perfect matching of SU(3) Yang-Mills thermodynamics as a prerequisite.
FIG. 1: Left: Scaled interaction measure as a function of $T/T_c$. The solid (dashed) curve is for the potential $\Phi$ with (without) the polynomial distortions $c_{2n}\phi^{2n}$. Other thermodynamic quantities (e.g. $v^2_s$, $e/T^4$, $p/T^4$ and $s/T^3$) agree perfectly (cf. [15]) with the lattice data (symbols, from [14]).

Right: Bulk to shear viscosity ratio as a function of the non-conformality measure. The blue dot-dashed line is a linear fit $\zeta/\eta = 1.2\pi \Delta v^2_s - 0.03$, while the dotted line depicts the Buchel bound $\zeta/\eta = 2\Delta v^2_s$ [16].

B. Dependence of bulk viscosity on potential parameters

We demonstrate now the sensitivity of the bulk viscosity on the parameters of the potential $\Phi$ with $c_{2n} = 0$. The analysis is restricted to $3 \leq \Delta \leq 3.9$. The numbers in Fig. 2 indicate selected loci at which we calculate the equation of state and the bulk viscosity exhibited in Fig. 3 below. The deviation measure $\chi^2_{v^2_s} = \frac{1}{N} \sum_{i=1}^{N} [v^2_s(x_i) - v^2_{s,L}(x_i T_c L)]^2$ indicates already the (in)accuracy of matching the velocity of sound squared, $v^2_s$, from lattice QCD. Hereby, $v^2_s$ and $v^2_{s,L}$ are obtained from the holographic calculation and the lattice data; $x_i$ and $LT_c$ are as in (5). We emphasize the corridor, in which the points 2, 7 and 12 are localized, which deliver an equally good, though not perfect, reproduction of the lattice data (cf. left column of Fig. 3), due to the individual adjustments of $G_5$. The values of $\zeta/T^3$ spread out by a factor of three for $T > T_c$ when comparing the results for all considered loci 1 - 12 (cf. middle column of Fig. 3). In contrast, $\zeta/\eta$ as a function of the non-conformality measure $\Delta v^2_s$ looks very much the same for loci 2, 7 and 12, while for the other loci significant variations of $\zeta/\eta$ can be observed, in particular for $\Delta v^2_s \rightarrow 1/3$, i.e. for $T \rightarrow T_c$. This observation lets us
argue that a perfect matching of the equation of state may lead to a robust result for $\zeta/\eta$.

V. SUMMARY

Despite of a lacking gravity dual to thermal SU(3) gauge theory, a gravity-scalar model with an appropriate ansatz for the potential allows for perfectly matching of thermodynamics in the temperature region $(1 - 10)T_c$. Note that no additional constraints are required, e.g. on scale settings or on the confined low-temperature phase or on the asymptotic behavior. The matching condition forces the bulk to shear viscosity ratio to $\zeta/\eta = C\pi \Delta v_s^2$ with $C \approx 1.2$ for $\Delta v_s^2 < 0.25$, in agreement with a previously employed quasi-particle model [17] and the IHQCD model [12]. Without matching, the considered class of potentials exhibits significant variations of both $s/T^3$ and $\zeta/\eta$; deviations from the linear relation $\zeta/\eta \propto \Delta v_s^2$ may occur over a larger range of $\Delta v_s^2$. The increase of $\zeta/\eta$ as a function of the temperature toward $T_c$, however, seems to be a generic feature. It is always less pronounced than the behavior.
FIG. 3: Equation of state $I/T^4$ as a function of temperature (left column), scaled bulk viscosity $\zeta/T^3$ as a function of temperature (middle column) and bulk to shear viscosity ratio as a function of non-conformality measure (right column). The numbers in the left panels refer to the loci in the $\gamma$ vs. $\Delta$ plane in Fig. 2.

Our considerations ignore potentially strong curvature effects beyond the classical gravity scenario, the reference to large ’t Hooft coupling as well as a direct link to the QCD $\beta$ function. In so far, we present an exploratory study of a restricted set of observables in a special bottom-up set-up leaving a systematic relation to the $ad hoc$ employed AdS/CFT correspondence with controlled deformation to accommodate the non-conformality for further studies.

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