Design and analysis of frequency-independent reflectionless single-layer metafilms

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We develop a theory for realizing frequency-independent, reflectionless, single-layer metafilms based on the Brewster effect. A metafilm designed based on the theory is numerically analyzed using a finite-difference time-domain method. The numerical analysis demonstrates that the reflectance of the metafilm vanishes independent of the frequency and that the metafilm behaves like an all-pass filter (with finite loss). An analysis based on an electrical circuit model of the reflectionless metafilm reveals that the energy of the suppressed reflection wave is not stored in the metafilm but is radiated to the transmission direction.

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In recent years there has been considerable interest in controlling electromagnetic waves using metamaterials. In particular, metafilms that are composed of two-dimensionally arranged meta-atoms have attracted much attention, because they can be fabricated more easily than three-dimensional metamaterials and enable the realization of a wide variety of elements for controlling electromagnetic waves. For example, metafilms have been applied to polarization control [1-5], harmonic generation enhancement [6-9], and slow group velocity propagation [10-13]. In these studies, metafilms with planar structures, which are usually referred to as metasurfaces, were used. Although metasurfaces are very useful for controlling electromagnetic waves, arbitrary optical properties cannot be obtained using single-layer metasurfaces with negligible thicknesses. This is because only electric dipoles can be induced in the plane of metasurfaces, which sets some restrictions on the optical properties of single-layer metasurfaces. For example, the absorbance cannot exceed 0.5 [14,15], and the amplitude and phase of the transmitted (reflected) wave cannot be controlled independently [16].

The above restrictions can be removed in multilayer metasurfaces and metafilms with finite thicknesses. Perfect absorption [17] and independent control of the reflection amplitude and phase [18] have been realized using ground planes. In addition, control of the transmission phase with nearly unity amplitude [16, 19-23] and ground-plane-less perfect absorbers [24, 25] have been realized in metasurfaces (metafilms) composed of more than two-layer structures [16, 19, 20], Huygens’ metasurfaces [21-24], and omega metafilms [25]. Although suppression of the transmission/reflection is essential in these examples, it is much more difficult to suppress the reflection independent of the frequency than the transmission. Thus far, the frequency-independent suppression of reflection in metasurfaces (metafilms) has been realized only in Huygens’ metasurfaces [23, 24] and omega metafilms [25], where the amplitudes of radiated waves from the induced electric and magnetic dipoles are designed to be equal over a broad frequency range.

In this study, we propose and numerically verify a method for suppressing reflection independent of frequency in single-layer metafilms composed of metaatoms with only an electric response. The frequency-independent reflectionless metafilm is designed based on the Brewster effect in metafilms [26] and the optical properties of the designed metafilm are numerically analyzed using a finite-difference time-domain method [27]. The reflectionless metafilm is modeled as an electrical circuit using the results of the numerical simulation, and the mechanism of the electromagnetic response of the metafilm is clarified using the electrical circuit model.

First, we briefly review the Brewster effect, the physical origin of which is the key of the present study. When an electromagnetic wave with certain polarization is incident on a planar interface between two distinct media at a particular angle, called the Brewster angle, the wave is transmitted with zero reflection [28]. The physical origin of the Brewster effect can be understood by considering radiations from the electric and magnetic dipoles in-
duced in the media [29]. Here, an electromagnetic wave is assumed to be incident from vacuum onto a purely dielectric medium for simplicity. In this case, the reflected wave consists of radiation from the electric dipoles induced in the dielectric medium. Because these electric dipoles cannot radiate electromagnetic waves in the direction of the oscillation, the reflected wave vanishes if the propagation direction of the reflected wave coincides with the oscillation direction of the electric dipoles.

We develop a method for suppressing the reflection from single-layer metafilms independent of the incident frequency based on the Brewster effect. Let us assume that the oscillation direction of the electric dipoles in a metafilm is in the $y$ direction and that meta-atoms are arranged at angle $\theta$ with respect to the $x$ axis, as shown in Fig. 1. (The system is assumed to be periodic in the $z$ direction.) The incident and reflection angles are denoted by $\phi$. A geometric calculation shows that the reflected wave propagates along the $y$ direction when $\theta = \phi$ is satisfied. Thus, the reflected wave vanishes if the arrangement of the meta-atoms and the propagation direction of the incident wave are chosen such that $\theta = \phi$ is satisfied. The reflection can be suppressed as long as the electric dipoles oscillate along the $y$ direction; therefore, the suppression of the reflection can be achieved independent of the incident frequency.

We design a frequency-independent, reflectionless, single-layer metafilm based on the above theory. In this study, an electric-field-coupled inductive-capacitive resonator (ELC) [30] shown in Fig. 2(a) is used as the meta-atom. (Other structures with a purely dielectric response can also be used as the meta-atom.) This structure is one of the commonly used meta-atoms and exhibits a purely dielectric response for electromagnetic waves that propagate along the $x$ direction. When a $y$-polarized electromagnetic wave is incident on the ELC, an electric dipole is induced in the $y$ direction. The frequency dependence of the ELC response is Lorentz-type with a resonance frequency that is determined by the inductance of the ring and the capacitance of the central gap. The metafilm is constructed by periodically arranging ELCs in the $z$ direction and in the direction of $\theta = \phi = \pi/4$, for which the interaction between the incident wave and the metafilm becomes strongest because the propagation direction of the incident wave coincides with the direction in which the radiation from each electric dipole is largest. Such a structure can be fabricated using microelectromechanical systems-based technologies [31].

We numerically analyzed the transmission and reflection properties of the designed metafilm using a finite-difference time-domain method. Figure 2(b) shows the simulation system. The dimension of the simulation space was $6 \text{ mm} \times 6 \text{ mm} \times 75 \mu\text{m}$. Periodic boundary conditions were applied to the $z$ direction and perfectly matched layer boundary conditions were applied to the $x$ and $y$ directions. The unit cell of the metafilm, shown in Fig. 2(a), was periodically arranged in the direction of $\theta = \pi/4$, and the number of the arranged ELCs was 56. A $y$-polarized Gaussian beam with a focal spot width of 1.2 mm and a Rayleigh range of 3.8 mm was incident from the $-x$ direction onto the metafilm at the incident angle of $\phi = \pi/4$. The transmittance (reflectance) of the metafilm is calculated as the ratio of the amplitude of the transmitted (reflected) wave to the amplitude of the incident wave in the far-field region.

Figure 3 shows the transmission and reflection spectra of the metafilm when the conductivity $\sigma$ of the ELC is equal to $1.0 \times 10^7 \text{ S/m}$, $1.0 \times 10^8 \text{ S/m}$, $2.0 \times 10^4 \text{ S/m}$, and $5.0 \times 10^4 \text{ S/m}$. To allow for a detailed discussion of the electromagnetic response of the metafilm in the following
paragraphs, the optical properties are calculated for several values of $\sigma$. The reflectance vanishes independent of the incident frequency in every case. Therefore, the theory for realizing frequency-independent reflectionless single-layer metafilms is confirmed, and hereafter, we focus on the transmission properties of the reflectionless metafilms. The amplitude transmittance is almost unity independent of the frequency for $\sigma = 1.0 \times 10^7$ S/m. This observation can be understood from energy conservation. The absorption in the metafilm is negligible because the ELC is made of an almost perfect electric conductor. Both the reflection and absorption vanish in the metafilm, resulting in perfect transmission. The transmittance at the resonance frequency, 0.935 THz, decreases as the conductivity decreases, and it vanishes for $\sigma = 2.0 \times 10^5$ S/m, which implies that perfect absorption takes place. The transmittance increases with further decreasing the conductivity. The transmission phase monotonically increases with the incident frequency and ranges from $-\pi$ to $\pi$ for $\sigma > 2 \times 10^5$ S/m. On the other hand, for $\sigma < 2 \times 10^5$ S/m, the transmission phase decreases with increasing the incident frequency at around the resonance frequency and takes on a narrower range of values compared with the case of large conductivity. Although the reflectionless metafilm is composed of a commonly used Lorentz-type meta-atom, the dependence of the metafilm transmittance on the frequency and the loss in meta-atoms is quite different from that of usual Lorentz-type metamaterials.

Next, we create and analyze an electrical circuit model of the frequency-independent reflectionless metafilm to understand its electromagnetic response mechanism. Since the metafilm behaves like an all-pass filter when the ELC is made of a perfect electric conductor, we consider the electrical circuit shown in Fig. 4 where $L$, $C$, $R_r$, and $R_{nr}$ represent the inductance of the ring, capacitance at the central gap, radiation loss, and non-radiation loss of the ELC, respectively. The voltage source $V_{in}$ corresponds to the incident wave. Applying Kirchhoff's voltage law to the electrical circuit yields $V_{out} = -\{[R_r - R_{nr} + i\omega L + (i\omega C)^{-1}]/(R_r + R_{nr} - i\omega L - (i\omega C)^{-1})\}V_{in}/2$. In the case of $R_{nr} = 0$,
$|V_{\text{out}}/V_{\text{in}}| = 1/2$ is satisfied independent of the source frequency and $\arg(V_{\text{out}}/V_{\text{in}})$ ranges from $-\pi$ to $\pi$. Assuming that $V_{\text{out}} = 2V_{\text{out}}$ is the output and $V_{\text{in}}$ is the input of the electrical circuit, the electrical circuit can be regarded as an all-pass filter with unity gain. Figure 3 shows the frequency dependence of $V_{\text{out}}'/V_{\text{in}}$ for four different values of $R_{\text{nr}}/R_{\text{r}}$, which correspond to the four conditions in the numerical simulation. The frequency dependences of the absolute value and argument of $V_{\text{out}}'/V_{\text{in}}$ agree well with the transmission properties of the reflectionless metafilm. Therefore, the electrical circuit shown in Fig. 4 can safely be regarded as a suitable electrical circuit model of the reflectionless metafilm. It is important to notice that $V_{\text{out}}' = V_{\text{in}} - 2R_{\text{r}}I$ is satisfied in this electrical circuit. The mechanism of the electromagnetic response of the metafilm can be understood based on this equation.

To understand the meaning of the above equation, we consider the electromagnetic response of the metafilm with $\theta = 90^\circ$ (metasurface) for normal incidence. In this case, the amplitudes of the radiated waves from the metasurface propagating in the transmission and reflection directions are the same. Although the configuration of the electrical circuit model of the metasurface is identical to that of the electrical circuit shown in Fig. 4, the transmittance does not correspond to $V_{\text{out}}'/V_{\text{in}}$ in this case. The amplitude of the transmitted wave corresponds to $V_{\text{in}} - R_{\text{r}}I$. This is because the transmitted wave is composed of a superposition of the incident wave and the radiation from the metasurface, which is proportional to the current $I$ and the radiation loss $R_{\text{r}}$ in the meta-atom, and the transmittance of the Lorentz-type metasurface at the resonance frequency vanishes in the case of $R_{\text{nr}} = 0$ [4, 32]. Therefore, the amplitude of the radiated wave from the metasurface propagating in the transmission direction corresponds to $R_{\text{r}}I$.

Now we discuss the meaning of the equation, $V_{\text{out}}' = V_{\text{in}} - 2R_{\text{r}}I$. Based on the above analysis and equation, the amplitude of the radiated wave from the reflectionless metafilm propagating in the transmission direction is found to be twice as large as that of the radiated wave from the metasurface propagating in the transmission direction. This implies that the energy of the suppressed reflection wave is not stored in the reflectionless metafilm but is radiated to the transmission direction.

For further understanding the physical mechanism of the electromagnetic response, we consider the transmittance of the metafilm at the resonance frequency for two special cases, $R_{\text{nr}} = 0$ and $R_{\text{nr}} = R_{\text{r}}$, using the result of the above discussion. When $R_{\text{nr}} = 0$, the metafilm radiates an electromagnetic wave with twice the amplitude than and opposite phase to the incident wave. This implies that the absolute value and argument of the transmittance are unity and $\pi$, respectively. When $R_{\text{nr}} = R_{\text{r}}$, the current $I$ that flows in the meta-atom is equal to half of that in the case of $R_{\text{nr}} = 0$. Thus, the metafilm radiates an electromagnetic wave with the same amplitude as and opposite phase to the incident wave, and the transmittance vanishes. Here, the radiated power from the metafilm is equal to the absorbed power in the metafilm because $R_{\text{nr}} = R_{\text{r}}$ is satisfied. (This condition is known as critical coupling [33].) Therefore, perfect absorption occurs.

Note that the macroscopic optical property of the reflectionless metafilm is similar to that of a Huygens’ metasurface with spectrally overlapping electric and magnetic resonances [34]. Although only electric dipoles are induced in the reflectionless metafilm, the suppression of the reflection causes the equivalent effect for the radiation from magnetic dipoles, which has the same amplitude as that from the induced electric dipoles.

In conclusion, we developed a theory for realizing frequency-independent, reflectionless, single-layer metafilms and analyzed the optical properties of one such metafilm, which was designed based on the theory. The frequency-independent suppression of the reflection is realized by arranging the meta-atoms so that the oscillation direction of the electric dipole induced in the meta-atom coincides with the propagation direction of the reflected wave. The designed metafilm was numerically analyzed using a finite-difference time-domain method. The reflectance of the metafilm was confirmed to vanish independent of the incident frequency and of the loss in the meta-atom. An electrical circuit model of the reflectionless metafilm was created and the mechanism of the electromagnetic response of the metafilm was analyzed based on the model. The analysis revealed that the energy of the suppressed reflection wave is not stored in the metafilm but is radiated to the transmission direction.

The frequency-independent reflectionless metafilm behaves as an all-pass filter when the non-radiative loss $R_{\text{nr}}$ is much smaller than the radiation loss $R_{\text{r}}$. All-pass filters are important elements for applications such as signal processing [35, 36]. When $R_{\text{nr}} = R_{\text{r}}$ is satisfied, perfect absorption can be realized using the single-layer metafilm developed here. The suppression of the reflection in our method is realized as long as the induced electric dipole oscillates along the $y$ direction. Thus, transmission-type spatial phase modulators and multiband perfect absorbers may be realized using our method. The concept of suppressing reflection by considering the radiation pattern of meta-atoms and the propagation direction of the reflected wave may yield various useful devices for controlling electromagnetic waves.

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