Time-series analysis of trial-to-trial variability of MEG power spectrum during rest state, unattended listening, and frequency-modulated tones classification

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ABSTRACT

Background: The nonstationarity of EEG/MEG signals is important for understanding the functioning of the human brain. From our previous research we know that short, 250–500-ms MEG signals are variance-nonstationary. The covariance of a stochastic process is mathematically associated with its spectral density, therefore we investigate how the spectrum of such nonstationary signals varies in time.

New method: We analyse data from 148-channel MEG, which represent rest state, unattended listening, and frequency-modulated tones classification. We transform short-time MEG signals to the frequency domain and for the dominant frequencies of 8–12 Hz we prepare the time series representing their trial-to-trial variability. Then, we test them for level- and trend-stationarity, unit root, heteroscedasticity, and gaussianity, and propose ARMA-modelling for their description.

Results: The analysed time series have weak-stationarity properties independently of the functional state of the brain and channel localization. Only a small percentage of them, mostly related to the cognitive task, reveal nonstationarity. The obtained mathematical models show that the spectral density of the analysed signals depends on only two to three previous trials.

Comparison with existing methods: The presented method has limitations related to FFT resolution and univariate models, but it is computationally simple and allows obtaining a low-complex stochastic model of the EEG/MEG spectrum variability.

Conclusions: Although physiological short-time MEG signals are in principle nonstationary in time, their power spectrum at the dominant (alpha) frequencies varies as a weakly stationary process. The proposed methodology has possible applications in prediction of EEG/MEG spectral properties in theoretical and clinical neuroscience.

1. Introduction

Time series modelling plays an important role in the description of complex biological systems and has numerous scientific applications. With regard to the electro- (EEG) or magnetoencephalography (MEG), proposing an adequate model of neural activity is crucial for understanding the functioning of neuronal connections in the brain. Especially in the context of the brain’s response to stimulation with external stimuli, proper modelling of perception and cognitive functions is very useful in numerous fields of neuroscience and medicine. This includes areas such as source modelling, seizure detection prediction, memory and attention research, brain-computer interfaces, and a large number of clinical observations in neurology, psychiatry, and other medical disciplines.

Many different methods are used in the literature to describe EEG/MEG signals. Traditional evaluation of spontaneous (rest state) brain activity is based on a description of the waveform morphology in the time domain, while in the frequency domain methods based on the Fourier transform are usually used. In turn, the clinically accepted technique for obtaining event-related responses is the averaging method. A significant methodological limitation in such studies is the fact that EEG/MEG signals are nonstationary, i.e. their properties change over time (Kaplan et al., 2005; Kipiński et al., 2011), and there exist correlations between the spontaneous and evoked activity (Sayers...
Therefore, more mathematically advanced methods of analysis had to find their place in neuroscience (Kipiński and Maciejowski, 2010, and references therein). Plenty of them are time-frequency analysis, with the use of tools such as the wavelet transform or the matching pursuit (Başar et al., 2001; Baumgartner et al., 2013; Durka, 2007; Herrmann et al., 2005; Jörn et al., 2011; Saavedra et al.; Sielużycki et al., 2009). Some of them are fractals or entropy based (Racz et al., 2020; Trujillo, 2019). Others use the independent component analysis and similar techniques (Jung et al., 2001; Anemüller et al., 2003; Piazza et al., 2020; Hsu et al., 2018). Also, many algorithms based on statistical methods were developed, mostly devoted to the estimation of evoked responses (de Munck, 2004; Sielużycki and Kordowski, 2014; Georgiadis et al., 2005; Kipiński, 2007; König et al., 2015; Wang and Davila, 2019). Among the latter, attention should be paid to the stochastic time series models, dedicated to the univariate (Matsuda and Komaki, 2017a) or multivariate (Galka et al., 2011; Matsuda and Komaki, 2017b) description of EEG/MEG data. These methods can be applied for predicting changes in signal parameters over time (Shakeel et al., 2020).

Moreover, it is not entirely clear how the human brain processes external information. Traditional view on brain evoked-responses generation assumes the so-called additive model, in which stimulus-induced activity is hidden in the noise associated with the spontaneous EEG and potentials resulting from the processing of accompanying stimuli, including cognitive processes and artifacts (König et al., 2015). However, the evoked activity is correlated in the frequency domain with the ongoing background (Savers et al., 1974). A more recent approach is based on stimulus-related changes in the ongoing EEG/MEG oscillations, where the evoked responses reflect rather the synchronization of such oscillations during stimuli processing in neural networks (Başar et al., 1999; Pfurtscheller, 1992; Pfurtscheller and Lopes da Silva, 2004; Żygierewicz et al., 2008). It is worth noting that the data description method is usually selected to match the previously assumed functional model, which may bias the obtained results (Yeung et al., 2004). Therefore, it is vital to search for methods of analysis that are free from heavy assumptions regarding the functional model and are instead based on the real properties of acquired EEG/MEG signals, in hope this will lead to a better understanding of their nature.

In this paper, we have devoted attention to examining some aspects of the variability of MEG activity. For this purpose we applied statistical test to infer about stochastic properties of encephalographic data, similarly as in the work of Kipiński et al. (2011)). As mentioned above, the bioelectrical activity of the brain is considered a nonstationary process, as it is well known that it is strongly time-varying, especially its spectral power. Therefore, we analysed the trial-to-trial spectrum variability, obtained in a way similar to those in the works of Kawabata (1976) and McEwen and Anderson (1975). In contrast to the work of Kipiński et al. (2011), where the variance-nonstationarity of brain signals is demonstrated and which corresponds to the time-varying MEG spectrum, the analysis presented in this article concerns the verification of what is responsible for this variability and which mathematical models are representative for its description. The alpha frequency band is analysed because it dominates the spectrum of the tested MEG signals and the dominant frequencies exert the greatest influence on the nature of the signal.

2. Aim

Even very short (0.25–0.50 s) fragments of MEG signals are proved to be nonstationary due to the time-dependent variance (Kipiński et al., 2011). However, the covariance matrix is mathematically associated with signal energy and the spectral density. Therefore, the variance-nonstationarity is equivalent to the time-variability of spectral power of MEG signals and it is widely described in the literature for both spontaneous and event-related EEG/MEG data, and has important neurophysiological consequences. Since changes in the spectrum are responsible for the nonstationarity of these signals, our goal was to find out how the spectral structure of short-term MEG fragments changes. We chose MEG data recorded at rest state, during passive listening to sounds, and related to the recognition and classification of frequency-modulated tones, so that our calculations could be used to compare whether the indicated spectrum variability is related to auditory information processing and, if so, at what level.

Presented results are a continuation of the research published in (Kipiński et al., 2011). Like in Section 4.3 of that work, modern tests for (non)stationarity were applied to infer about time-varying statistical properties of MEG power spectra. Moreover, the same real data set was used in both analysis. Yet, in contrast to the previous work of Kipiński et al. (2011), where only six channels were taken into account (four above the auditory cortices of the two hemispheres, and two—frontal and occipital sensors—as “reference” channels), here a multichannel analysis is performed. The new approach in the present work is the statistical analysis performed for series of the trial-to-trial variability of MEG power spectra and the estimation of stochastic time series models of this variability. In this way, we learn how to describe and parametrize the variability of the spectral power of MEG signals, and thus to check what lies at the basis of variance-nonstationarity demonstrated in previous studies.

3. MEG experiment

The MEG experiment was performed at the Special Lab for Non-Invasive Brain Imaging at the Leibniz Institute for Neurobiology in Magdeburg, Germany. A whole-head MEG device (Magne5 2500 WH, 4-D Neuroimaging, San Diego, USA) containing 148 magnetometer sensors coupled to DC SQUIDs was used to acquire the data. The measured signals were corrected for environmental noise by means of a weighted subtraction of reference signals detected by additional sensors located in close proximity to the field detectors. In order to avoid any noise contribution from the liquid helium, all pick-up and reference detectors, as well as the 148 SQUID’s, are located in vacuum. The MEG apparatus was located in a magnetically shielded room. Data were sampled at 1017.25 Hz and bandpass filtered during acquisition between 0.1 Hz and 100 Hz.

The experiment was intended to acquire MEG signals representing various functional states of the human brain. For this purpose, it was decided to measure: 1. spontaneous brain activity; 2. signals registered during the repeated tonal stimulation (evoked fields); 3. MEG waveforms containing cognitive event-related components connected with perception of frequency-modulated (FM) tones (with their classification task during one session). In each FM sweep the instantaneous frequency changes linearly in time, either upward or downward. A set of 32 FM was used, 16 for each of the two categories, presented in random order. The frequency range of each sweep covered one octave. The initial frequencies for the rising FM’s varied from 0.5 Hz to 2 kHz in steps of 100 Hz, those for falling FM’s from 4 kHz to 1 kHz in steps of 200 Hz. Each FM sweep had duration of 500 ms including 10 ms linear ramps both at the beginning and at the end.

The acoustic stimulation with FM tones was chosen because they play a fundamental role in the human speech as well as in the animal vocalizations. Studies using FM tones provide a possibility to gain insight into the processing of prosodies independent of any speech-specific mechanisms. The FM tones can be easily separated into two natural categories with respect to the underlying direction of frequency modulation, upward or downward. Thus, they also serve as an ideal tool for comparing the auditory cortex response to a passive condition, that is, mere listening to a sequence of FM tones, with an active condition (cognitive task), that is, the directional categorization of exactly the same FM tones. Different activation patterns for the two conditions provide insight into the impact of cognitive aspects (related to FM tone discrimination) on the processing of FM tones in the auditory cortex (Żygierewicz et al., 2008). More details about the FM stimuli are given in

et al., 1974).
the paper of Konig et al. (2008).

There were two stages of the experiment when the FM tones were used. In the first one, the exposure condition (FM/E), the subject had to listen passively to the sequence of FM tones. In the second one, the task condition (FM/T), he was instructed to discriminate the direction of frequency modulation of the same FM tones and to categorize them by pressing one of two response keys. The subject was instructed to signal rising FM sweeps with his left index finger and falling FM sweeps with his right index finger as soon as he identifies the direction of frequency modulation. Within an experimental session, the presentation of the FM tones was binaural and randomized.

Two sessions in which the spontaneous activity was registered (i.e. during the period when the subject was not exposed to any stimulation) were introduced to serve as the natural reference level for the sessions involving stimulation.

Within each session involving stimuli, the stimulus was presented 96 times. For the 1-kHz stimulation, each stimulus was exactly the same, whereas the FM tones were presented randomly, each sweep three times, resulting in total number of 96 stimuli (32 × 3). The measurement was continuous, thus every single trial (epoch) corresponding to a single stimulus presentation was equivalent to 2 s of registration, where \( t = 0 \) corresponds to the stimulus onset for each trial. Thus, the stimulus onset interval was equal to 2 s and the inter-stimulus interval was 1.5 s (given the 0.5-s long stimulus). The spontaneous brain activity was also measured continuously over the period of time, whose length was equal to 96 × 2 s. Therefore, each experimental session lasted 3 min and 12 s.

Timing diagram of a single trial is given in Fig. 1.

During the experiment acoustic stimuli were generated outside the MEG chamber and delivered to the subject’s ears via two approximately 6-m long plastic tubes. These tubes ended with special ear-moulds which were individually chosen to adapt to each subject’s pinnae. The time delay caused by the signal conduction along the plastic tubes was measured as 20 ms, and was taken into account in the data analysis. At the beginning of each measurement, the sound pressure level of the acoustic stimulus was adjusted to 90 dB SPL using a 1-kHz sinusoidal tone as reference signal and the subject rated this sound intensity as comfortable.

As the result of measurements from a healthy volunteer, multi-channel MEG signals of high quality were obtained for all sessions. All were evaluated by a neurologist experienced in description of that kind of data as physiological, with 1) the spontaneous MEG activity being typical for resting state in humans and free of any seizure phenomena or sleep waveforms, 2) evoked fields being typical early cortical auditory responses with proper amplitudes and latencies of main components after averaging, as well as expected over-head topography, and 3) with the event-related fields containing proper cognitive long-latency waveforms after averaging, with physiological topography over the head. Moreover, the results obtained for the sessions related to acoustic stimulation show no signs of habituation.

4. Methods

4.1. Data pre-processing

No artifact correction was performed on the measured data sets so as not to lose any trials and to maintain the original information of the individual trials. Our approach requires MEG signal segmentation to \( T \) equal-length fragments, estimation of the spectral power for each segment separately, and finally the statistical analysis of time-dependent properties of the series of trial-segregated Fourier coefficients calculated for given frequencies. A similar, yet not equivalent algorithm was applied in the studies of McEwen and Anderson (1975) and Kawabata (1976). The method of defragmentation of the measurement signal and determination of the Fourier spectrum for short-time MEG fragments as well as the creation of a time series of Fourier coefficients are similar. The difference is that in this paper we focus on the 8–12-Hz alpha band, which is dominant in the spectrum of the analysed signals, and our method of analysing the obtained time series is completely different: instead of simply testing the time series for randomness (the sign test) and for the regression slope in a data set (the trend test) we applied more modern tools for testing for stationarity as well as the time series modelling methods, which is described in detail in Section 4.2.

To evaluate possible connection of the trial-to-trial variability of MEG power spectrum with the experimental condition (tonal stimulation, stimulation by FM tones with or without categorization task, spontaneous activity) the registered signals were divided into segments time-locked to the stimulus (or to the beginning of the examination in the case of no stimuli) and separated from other segments by the same time interval. This interval is the difference between the fixed length of each trial (2 s) and the length of the segments, like in the earlier work (Kipiński et al., 2011—see Fig. 2a). For consecutive trials, the spectral power was calculated as the discrete Fourier transform using the fast Fourier transform (FFT) algorithm (see, for example, Burrus, 2012). Furthermore, sequences of selected coefficients of these spectra, \( z_n, t = 1, \ldots, T \), were derived for some fixed frequency \( f \). Note that only one spectral coefficient for each segment was selected at a time—see Fig. 2.

Taking into account the frequency resolution and the nonstationary character of the data it was decided to perform analysis for two lengths of the aforementioned segments. It was important to use rather short segments in order to diminish the time error of the Fourier estimation and to maximize the signal-to-noise ratio of the Fourier transformed data (this is because the stimulus-related activity was assumed to last relatively shortly after the stimulus onset), yet, we also tried to preserve a reasonable frequency resolution, which increases with increasing the length of the analysed signal to be transformed. Therefore, the signals of both 250-ms and 500-ms duration were analysed. The 250-ms signals contain the evoked components but not the waveforms related to endogenous (e.g., cognitive) activity. The 500-ms signals enable investigation of all event-related waveforms.

The time series analysis of a trial-to-trial power spectrum variability was performed for frequencies from the dominant range of 8–12-Hz, which is the frequency band of the alpha rhythm. This limitation was introduced due to the fact that the dominant frequencies have the greatest impact on the nature of the analysed signals, transferring the largest part of their energy, and due to the number of analyses needed to obtain results for each frequency of the FFT spectrum separately. We obtained 148 (the number of MEG channels) series for each kind of stimulation and each of the selected signal durations. For the 500-ms signals, the FFT spectra revealed three significant peaks, at 8 Hz, 10 Hz, and 12 Hz, whereas the spectra of the 250-ms signals demonstrated two clear peaks—at 8 Hz and 12 Hz. This is because of the inherent frequency resolution of the fast Fourier transform, which for slightly with the period of stimulus presentation B, as the subject responded already before the end of the stimulus in some trials. Time is given in seconds.

![Fig. 1. Timing of a single trial: A—final part of a previous trial; B—the period of stimulus presentation; C—response time for classification in the task condition FM/T (absent in the FM/E condition); D—post-stimulus period for recovery of the MEG signal. B is absent in the session without stimulation. In the FM/T session, C may occasionally overlap with D. C overlaps

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The page contains a diagram labeled Fig. 1. It illustrates the timing of a single trial with labeled segments A to D, each representing different parts of the trial: A—final part of a previous trial; B—the period of stimulus presentation; C—response time for classification in the task condition FM/T (absent in the FM/E condition); D—post-stimulus period for recovery of the MEG signal. B is absent in the session without stimulation. In the FM/T session, C may occasionally overlap with D.
selected to compute their spectra. b) The single-trial MEG selected in step (a) pointed by arrows and the marked segments (500-ms ones in this example) are MEG power spectrum: a) The original MEG signal segmentation according to L. Kipi transformed with the use of FFT to the frequency domain; for a fixed frequency (8 Hz in this case) the time series of the consecutive Fourier coefficients is separately, and for all fixed frequencies from the dominant alpha range (and 12 Hz (taking into account the spectral resolution) separately.

The spectral density of a stationary process \[ Z_t \] is defined as

\[ \gamma(h) = \int_{-\pi}^{\pi} e^{ih\lambda} f(\lambda) d\lambda, \quad (3) \]

for \( h = 0, \pm 1, \pm 2, \ldots \), \( e^i = \cos(\lambda) + i\sin(\lambda) \), and \( i = \sqrt{-1} \). Equation 3 is the inverse transform of the spectral density of \( Z_t \) defined as

\[ f(\lambda) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} e^{-ih\lambda} \gamma(h), \quad (4) \]

where \(-\pi \leq \lambda \leq \pi\). The spectral density is an analogue to the probability density function; for all \( \lambda \in (-\pi, \pi) \), \( f(\lambda) \geq 0 \) and it is even, i.e., \( f(\lambda) = f(-\lambda) \).

To transform one function into another that is the frequency-domain representation of the original function, we use the Fourier transforms (FTs). Discrete Fourier transform (DFT) is given by

\[ F(o_i) = \frac{1}{\sqrt{n}} \sum_{t=1}^{n} Z_t e^{-i\omega_0 t}, \quad \omega_0 = \frac{2\pi}{n} \]

where \( n \) is the number of observations of a time series \( Z \), \( o_k = \frac{2k\pi}{n} \), and \( k = -[n/2], \ldots, [n/2] \), where \([y]\) denotes the largest integer smaller than or equal to \( y \). Then, each component of an observable time series \( Z \) can be written as

\[ Z_t = \sum_{k=-[\pi/2]}^{[\pi/2]} \left| a_k \right| F(o_k) \left( \cos(o_k t) + i\sin(o_k t) \right), \quad (6) \]

where \( t = 1, \ldots, n \), which shows that \( Z_t \) can be represented as a linear combination of sine waves with frequencies \( o_k \).

After the procedure described in Section 4.1, the obtained sequence of the selected coefficients of the single-trial power spectra for a fixed frequency will now be the time series \( z_t \) being a realization of the stochastic process \( Z_t \) representing time-varying brain activity in the frequency domain.

Let us now present the procedure that we propose to infer the properties of \( z_t \) and to fit an adequate time-series model to it. This procedure is given in a diagram form in Fig. 3.

In order to identify the stochastic nature of \( Z_t \), it is crucial to verify its stationarity. Like in the previous work (Kipiński et al., 2011), we applied: the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test for level-stationarity and trend-stationarity (Kwiatkowski et al., 1992), the Phillips-Perron (PP) test for unit root (Phillips and Perron, 1988), and the White test for heteroscedasticity (variance-nonstationarity) (White, 1980). We used these tests at the beginning of time series analysis and if the trend-nonstationarity was detected we made detrending to eliminate it, as it is shown in the upper part of the decision tree from Fig. 3. Besides, the Jarque–Bera test (Jarque and Bera, 1980) to verify the hypothesis about normality of the analysed series of spectral coefficients was also applied. Whenever gaussianity was found, an appropriate model was fitted from the class of autoregressive moving-average (ARMA) models of the orders \( p, q \in [0,10] \), which was justified by the obtained form of the sample autocorrelation function (ACF) and the sample partial autocorrelation function (PACF) plots. The time series \( \{Z_t\} \) is an (ARMA(\( p,q \))) process:

\[ Z_t = \sum_{i=1}^{q} \varphi_i Z_{t-i} + a_t + \sum_{j=1}^{p} \theta_j a_{t-j}, \quad (7) \]

with some constant coefficients \( \varphi_i \) and \( \theta_j \), where \( \{a_t\} \) is a white noise (Box et al., 2016).

Fitting the time series model is a complex, iterative procedure and cannot be fully described here, yet we introduce it shortly in a lower part of the diagram in Fig. 3. In this work, the methods explained in detail in the book of Brockwell and Davis (2016) were applied. For a preliminary estimation (Fig. 3 (1)) of the parameters of ARMA models we applied:
the Yule–Walker estimation and the Burg’s algorithm for pure autoregressive (AR) processes, the innovations algorithm and the Hannan–Rissanen algorithm for pure moving-average (MA) models, and the last-mentioned method for mixed (ARMA) models. Estimators of the model parameters were obtained by maximizing the gaussian likelihood of the ARMA process. The major criterion for order selection of the estimated model was minimization of the bias-corrected Akaike criterion (AICC) value. At the end, we checked the goodness of fit of a statistical model to a set of data (Fig. 3 (2)). We applied it by testing the residuals of the model, which should have white noise properties if the model is appropriately fitted. Therefore, we applied various tests for randomness (the Ljung–Box test, the McLeod–Li test, the turning points test, the difference-sign test, the rank test, and the Jarque–Bera test), as suggested by Brockwell and Davis (2016).

Finally, let us remind that the procedure described above, leading to the adequate time series model of the trial-to-trial variability of the MEG power spectrum, was performed for each of the experimental conditions, each single MEG channel, each segment length, and each of the selected frequencies separately.

5. Results

5.1. Inference about stationarity

The stationarity of the series of Fourier coefficients was tested independently for all channels and measurement sessions. This verified the presumptions that the character of the power-spectrum trial-to-trial variability is related to the functional state of the brain. Such correlation should be observable in the predominant frequency component of the MEG signals—the alpha band in this case. All the results of hypotheses testing are shown in Table 1.

The highest percentage of rejecting the null hypothesis $H_0$ was observed for the Phillips–Perron (PP) test, and the percentage of its acceptance is not fully comparable with the results obtained with the adequate version of the Kwiatkowski–Phillips–Schmidt–Schin (KPSS) test. The PP test is, in a sense, opposite to the KPSS test for level-stationarity in that $H_0$ rejection by PP corresponds to its acceptance by KPSS. The different power of the two tests is responsible for the differences in the obtained results. The KPSS test is a test of high power, and was proposed as an alternative for the unit-root tests due to the fact that those tests were known to fail to reject the null hypothesis of a unit root for many time series (Kwiatkowski et al., 1992). Therefore, the results of the KPSS test are more objective and we used them as the main factor of decision rule in our analysis. An example interpretation of the results of the PP test and the corresponding set of results for the KPSS test for the FM/T condition is shown in Fig. 4.

Comparing the results obtained by both version of the KPSS test and the PP test across the different experimental conditions it is difficult to find clear relationships related to experimental conditions. In particular, the two measurement sessions in which spontaneous MEG activity was recorded (without stimulation) differed in the obtained acceptance/rejection ratios of the stationarity tests. For 250-ms signals and both analysed frequencies, in the second session without stimulation, the percentage of results indicating nonstationarity of trial-to-trial

![Fig. 3. The statistical inference as the decision tree and the modelling procedure. Explanations of decision boxes: (a) Is $Z_t$ stationary in variance? (b) Is $Z_t$ stationary in trend? (c) Is $Z_t$ gaussian? (d) Is $Z_t$ gaussian? More detailed explanations of boxes (1) and (2) are given in the text.](image-url)
variability is slightly higher than in the first session. There is no such evident difference for 500-ms signals. For measurements with acoustic stimulation, the percentage of rejection of the null hypothesis of the KPSS test for level-stationarity increases above 10% at: 10 Hz for 500-ms segments in the tonal stimulation, 12 Hz for 250-ms segments in FM exposure condition, as well as at 8 Hz for 250-ms segments and 12 Hz for 500-ms segments in FM task condition. In turn, for the PP test, a similar result was obtained only at 8 Hz for 500-ms segments in the FM exposure condition, as well as at 8 Hz for both segment lengths in FM task condition. These results do not allow drawing consistent conclusions on the impact of stimulation on the stationarity of the MEG spectrum variability.

Having compared the results obtained with the two versions of the KPSS test, the presence of trend-nonstationarity in the analysed data was found only in a very small number of channels. Nor could we find any differences between the results for the two segment lengths. No correlations with the specific localization (i.e. auditory or occipital cortex, taking into account the kind of stimulation and analysed frequency band) were found—see Fig. 5.

For signals recorded without stimulation (in both sessions), the percentage of series with a detected trend is very small (max. 5.4% at 12 Hz for 500-ms segments). A similarly low percentage of the occurrence of trends characterizes measurements with tonal stimulation. The percentage of trends increased slightly for FM exposure condition, reaching 7.4% at 12 Hz for 250-ms segments and 8.1% at 8 Hz for 500-ms segments. On the other hand, for FM task condition it again drops below 5% for all analysed frequencies and both segment lengths.

The existence of variance-nonstationarity was generally not proved either. However, the results of the White test obtained for the FM task condition and 500-ms segments differ from both those for the 250-ms segments of the same experimental condition and those for all other experimental conditions. Here, the percentage of rejection of the null hypothesis of the test was obtained in 10.1% of cases at 8 Hz and 14.9% at 10 Hz, whereas for all other cases this percentage exceeded 5% only once (5.4% at 10 Hz for 500-ms segments in the FM exposure condition). No clear channel-dependent properties were found in this case either.

The obtained results show that the trial-to-trial variability of the selected power spectrum coefficients is in the overwhelming majority stationary in the wide sense. This property is independent of the length of the analysed signals, yet, the influence of the frequency resolution remains unknown.

5.2. ARMA modelling

The results for normality testing and time series modelling of the trial-to-trial variability of the MEG power spectrum are presented in Table 2.

Table 1

| Experimental condition | Segment duration [ms] | Frequency [Hz] | Percentage of rejection of the null hypothesis of the test (see Fig. 5.) |
|------------------------|----------------------|----------------|-------------------------------------------------------------------------|
| No stimuli (session 1)  | 250                  | 8              | 3.4 0.0 95.9 0.0                                                        |
|                        | 12                   | 0.7 0.7 98.0 0.7 |
|                        | 8                    | 5.4 1.4 97.3 0.0 |
|                        | 10                   | 1.4 0.0 95.9 0.7 |
|                        | 500                  | 8              | 8.1 5.4 91.9 3.4                                                        |
|                        | 12                   | 8.6 2.0 91.2 4.7 |
| 1-kHz sound stimulation| 250                  | 8              | 4.7 2.7 92.6 2.0                                                        |
|                        | 12                   | 0.0 0.0 100.0 1.4 |
|                        | 8                    | 14.9 3.4 96.6 0.0 |
|                        | 500                  | 10             | 2.0 0.0 98.0 0.7                                                        |
|                        | 12                   | 0.0 0.0 94.6 2.0 |
| No stimuli (session 2)  | 250                  | 8              | 1.4 1.4 97.3 0.0                                                        |
|                        | 12                   | 3.4 2.7 97.3 3.4 |
|                        | 500                  | 10             | 6.8 1.4 93.2 0.0                                                        |
|                        | 12                   | 0.0 0.0 94.6 2.0 |
| FM/E (exposure condition) | 250            | 8              | 2.0 0.7 99.3 0.0                                                        |
|                        | 12                   | 12.2 7.4 93.2 2.7 |
|                        | 8                    | 8.1 8.1 79.0 0.0 |
|                        | 500                  | 10             | 2.7 2.0 94.6 5.4                                                        |
|                        | 12                   | 7.4 1.4 93.2 1.4 |
| FM/T (task condition)  | 250                  | 8              | 10.1 3.4 88.5 2.0                                                        |
|                        | 12                   | 4.7 2.0 94.6 0.0 |
|                        | 8                    | 0.7 0.0 84.5 10.1 |
|                        | 500                  | 10             | 4.1 2.7 95.3 14.9                                                       |
|                        | 12                   | 14.2 4.1 94.0 0.0 |

The obtained results show that the trial-to-trial variability of the selected power spectrum coefficients is in the overwhelming majority stationary in the wide sense. This property is independent of the length of the analysed signals, yet, the influence of the frequency resolution remains unknown.
related components. For these data, this percentage reached the maximum of 17.6%.

Unfortunately, no correlation between a particular time series model characteristics and the MEG-channel location was found. In Fig. 5, the spatial distribution of the time series models obtained for the task condition (FM/T) is shown in a form of 2D stereographic projection of the MEG channels onto the plane. Although a clustering tendency can be observed for trend-nonstationarity, any informative conclusions concerning those distributions would be difficult to draw. For the other experimental conditions, suchlike ambiguities hold as well. The lack of dependence between the properties of time series models and the location of the MEG channel proves that even in an auditory experiment the stationarity of the trial-to-trial variability of the alpha-band spectrum in channels above the auditory cortex is similar as in the occipital cortex, where strongest alpha activity is observed, and other brain areas.

Moreover, the influence of the segment duration on the actual results is clear, which can be attributed to the nonstationarity of MEG signals in the time domain, but it is primarily due to the limited resolution of the Fourier transform.

6. Discussion

Although the physiological MEG signals recorded during the presented complex acoustic experiment are in principle nonstationary in the time domain, the weakly stationary character of their power spectra exists at several frequencies of the alpha band. Our results suggest that such weak stationarity is the common property for this kind of signals, rather independent from the functional state of the brain, i.e., recognition and classification of cognitive stimuli (reflected in event-related responses), processing of audio-sensory information (auditory evoked
The temporal resolution of the Fourier transform is limited by the length of a time window to be transformed. The frequency resolution is limited too, as the lowest frequency to be observed is the inverse of a time-window length. Therefore, the intrinsic limitations of the Fourier transform might have biased our results.

For this reason, the practical application of the presented method of analysis should be approached with caution. The presented algorithm is not computationally complicated and allows to obtain a low-complexity stochastic model of the EEG/MEG spectrum variability. Therefore, it could be useful in theoretical neuroscience. For example, when applied to data revealing the habituation effect, we expect to obtain ARMA-plus-trend nonstationary models of trial-to-trial spectrum variability, which are also easy to estimate. The proposed method of analysis may also find applications in the research where prediction of spectral changes plays an important role. Recently, machine learning has been used to estimate the required statistics of encephalographic data modelled by a complex network and a lot of research has been devoted to this issue (Craik et al., 2019; Nguyen et al., 2020). It would be interesting to check how deep learning methods solve the problem of parametrizing the trial-to-trial variability—in the presented work this procedure required making decisions in an iterative modelling algorithm (see Fig. 3). Prediction of EEG characteristics is helpful in various neurostimulation applications, such as the brain-machine interface based techniques (Abiri et al., 2019; Cha et al., 2020; Binias et al., 2020) or other EEG-incorporating methods like electrical brain stimulation (Mansouri et al., 2017). It also has a great clinical utility, especially for patients in coma (Rosenthal, 2012) or in epileptic seizure prediction (Williamson et al., 2012; Hasan et al., 2017). In this place let us notice that AR time series models are already used in forecasting of the alpha oscillatory phase of EEG signals (Shakeel et al., 2020) and the real-time epileptic seizure prediction (Chisci et al., 2010). We hope that our research may support the above mentioned and similar studies.

Unfortunately, the influence of the segment duration on the actual results undermines the possibility of an application of such models to the categorization and classification of encephalographic data. Therefore, the problem of describing the trial-to-trial variability of the EEG/MEG spectrum requires improving the methodology of such a time series modelling.
Earlier we noted that the spectral density estimation method (FFT in our case) may bias the results. One can consider the use of wavelets or the matching pursuit (MP) algorithm for this purpose, the latter used in an earlier work of Kipiński (2011), but then approximations of the nonstationary spectrum would be achieved and the ARMA model would serve only to describe the residual noise. Another improvement would be to use multivariate space-space models instead of their univariate version, similarly like in the work of Galka et al. (2011) and Matsuda and Komaki (2017b). Such models address the characteristics of the signals from all EEG leads or MEG channels at once, taking into account the cross-channel correlations.

7. Conclusions

The nonstationarity of MEG is important for understanding the brain functioning on each level of the brain’s neuronal network. In this paper we investigated short MEG signals with proven variance-nonstationary character and representing rest state in comparison to the auditory information processing at a few levels: from passive listening to simple tonal and frequency modulated stimuli to active listening and categorization. We investigated the trial-to-trial variability of their frequency spectra at the dominant alpha frequencies (8–12 Hz) and we found similarities independently of the functional state of the brain or channel location. The weakly stationary character was not found in only a small percentage of the trial-to-trial series of power-spectrum coefficients, mostly related to the classification of the direction of the frequency modulation of the FM stimuli. This suggests that nonstationarity may be the result of a cognitive process underlying the generation of an auditory evoked response.

We conclude that some information about event-related neural processing can exist in the trial-to-trial spectrum variability and it is possible to parametrize it indirectly by rather simple stochastic models. We have proposed univariate ARMA-class stationary time series models of a low order for their description and showed that the spectral density of analysed signals depends on only two to three previous trials. Of nonstationarity of MEG is important for understanding the brain processing can exist in the trial-to-trial spectrum variability and it is possible to parametrize it indirectly by rather simple stochastic models. We have proposed univariate ARMA-class stationary time series models (EEG) classification tasks: a review. J. Neural Eng. 16 (3), 1–28. Article no. 031001.

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