CP Phases in Supersymmetric Tri–lepton Signals at the Tevatron

S.Y. Choi
Korea Institute for Advanced Study, 207–43, Cheongryangri–dong Dongdaemun–gu, Seoul 130–012, Korea

M. Guchait
Deutsches Elektronen–Synchrotron (DESY), D–22603 Hamburg, Germany

H.S. Song and W.Y. Song
Center for Theoretical Physics and Department of Physics Seoul National University, Seoul 151-742, Korea

Abstract

We have analyzed the supersymmetric tri–lepton signals for sparticle searches at the Tevatron in the minimal supersymmetric standard model with general CP phases without generational mixing. The CP phases may affect very strongly the chargino and neutralino mass spectrums and $\sigma(pp \rightarrow \tilde{\chi}_-^1 \tilde{\chi}_0^2)$ as well as $B(\tilde{\chi}_-^1 \rightarrow \tilde{\chi}_1^0 \ell^- \nu)$ and $B(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 \ell^+ \ell^-)$. Even under the stringent constraints from the electron electric dipole moment the CP phases can lead to a minimum of the tri–lepton event rate for their non–trivial values.

PACS number(s): 14.80.Ly, 12.60.Jv, 13.85.Qk
The low–energy minimal supersymmetric standard model (MSSM) [1] in general involves a large number of CP–violating phases. Nevertheless, their presence has largely been ignored in phenomenological analyses because of the complexity due to the introduction of many new independent parameters and because of severe constraints on individual phases by the experimental limits for electron and neutron electric dipole moments (EDM) obtained by fixing other phases to zero [2]. However, many recent works [3–5] have shown that these constraints could be evaded without suppressing the CP–violating phases of the theory. One option is to make the first two generations of scalar fermions rather heavy so that one–loop EDM constraints are automatically evaded. As a matter of fact one can consider so–called effective SUSY models [4] where de–couplings of the first and second generation sfermions are invoked to solve the SUSY FCNC and CP problems without spoiling the naturalness condition. Another possibility is to arrange for partial cancellations among various contributions to the electron and neutron EDM’s [5]. Following the suggestions that the phases do not have to be suppressed, many important works on the effects due to the phases have been already reported; the effects are very significant in extracting the parameters in the SUSY Lagrangian from experimental data [3], estimating dark matter densities and scattering cross sections and Higgs boson limits [7–9], CP violation in the $B$ and $K$ systems [10], and so on.

The reaction $p\bar{p} \rightarrow \tilde{\chi}_{\pm 1} \tilde{\chi}_{0 2}$ is one of the cleanest SUSY processes at the Tevatron. The subsequent leptonic decays of charginos and neutralinos can lead to clean, i.e. jet–free tri–lepton plus $E_T$ events which have very low standard model backgrounds. Since charginos and neutralinos are expected to be lighter than gluinos in models where gaugino masses unify near the GUT scale, the clean tri–lepton signals potentially offer the largest reach and the leptonic decays of $\tilde{\chi}_{\pm 1}$ and $\tilde{\chi}_{0 2}$ are not suppressed. These promising aspects of the SUSY tri–lepton signals have led to a lot of detailed theoretical investigations [11] and they have been confirmed by several simulation works [12]. However, most of the works have been done under the assumption that all the couplings are related at the grand unification or Planck scale and they are real. In this letter, in light of the possibility of large CP phases in the general MSSM, we re–visit the SUSY tri–lepton signals at the Tevatron in the framework of MSSM assuming CP phases which are also severely constrained by EDM’s.

The SUSY parameter set of the electroweak gaugino sector in the MSSM is $\{|M_1|, \Phi_1, M_2, |\mu|, \Phi_\mu, \tan \beta\}$. Note that the phase of the SU(2) gaugino mass $M_2$ is rotated away by field redefinitions so that $M_2$ is assumed real and positive without loss of any generality while the phases $\Phi_1$ and $\Phi_\mu$ of the U(1) gaugino mass $M_1$ and higgsino mass parameter $\mu$ remain as physical phases [13]. For the sfermion sector, we assume the flavor–diagonal sfermion mass matrices, a universal soft–breaking mass $m_{\tilde{f}_{L,R}}$ for sfermions of each chirality, and a universal trilinear term $A$. The universality condition enables us to have a controllable number of parameters while the effects due to its violation can be kept small by taking rather large sfermion masses and trilinear couplings in the general case with more complicated sfermion sectors.

Since the CP phases might give a significant contribution to the electron and neutron EDM’s, it is incumbent to take into account the constraints on the CP phases of the EDM’s, in particular, the electron EDM, which are known to be stronger than those of the neutron EDM [2]. Related with the electron EDM constraints ($|d_e| \leq 4.3 \times 10^{-27} e \cdot cm$ at 95% confidence level [14]), we investigate two distinct scenarios for the higgsino mass parameter
\(|\mu|\) and the slepton and squark masses:

\begin{align*}
S1 : \ |\mu| = .2 \text{ TeV}, \ m_{\tilde{\chi}_{L,R}^0} = 10 \text{ TeV}, \ m_{\tilde{q}_{L,R}} = 10 \text{ TeV}, \\
S2 : \ |\mu| = .7 \text{ TeV}, \ m_{\tilde{\chi}_{L,R}^0} = .2 \text{ TeV}, \ m_{\tilde{q}_{L,R}} = .5 \text{ TeV}.
\end{align*}

In addition, we take a small value of \(\tan \beta = 3\) and \(M_2 = 100 \text{ GeV}\), assume the gaugino mass unification \(|M_1| = \frac{5}{4} \tan^2 \theta_W M_2 \approx 0.5 M_2\) only for the magnitude, and take \(A = 1 \text{ TeV}\) for the universal trilinear term while its phase is left arbitrary. Two CP phases \(\Phi_\mu\) and \(\Phi_1\) relevant for the tri–lepton signal are strongly restricted in \(S2\) by the electron EDM\(^5\) while they are not in \(S1\) with very heavy first and second–generation sfermions suggested by the effective SUSY models, unless \(\tan \beta\) is very large and at the same time the third–generation sfermions are light\(^{13}\). A larger value of \(|\mu| = .7 \text{ TeV}\) in \(S2\) than in \(S1\) is chosen so that a relatively large space is allowed for the CP phases against the electron EDM constraints\(^3\). Squark masses are taken to be larger than slepton masses in \(S2\), which is compatible with the sfermion spectrum in the minimal supergravity framework. The constraints imposed on the phases by the electron EDM data are for some particular values of soft parameters with relatively light mass spectra in \(S2\). Nevertheless, we expect that the results will exhibit general features typical for similar choices.

Although the off–diagonal elements in the sfermion mass matrices are proportional to small Yukawa couplings, the trilinear terms are very crucial for the electron and neutron EDM’s because the contributions of their phases to the EDM’s require a chirality flip leading to dipole moments proportional to the relevant mass. Therefore, it is necessary to maintain the sfermion left–right mixing in evaluating the EDM’s. On the other hand, a relatively small \(\tan \beta = 3\) along with large sfermion masses tends to degrade the importance of sfermion left–right mixing effects except the effects from the stop sector in the tri–lepton process. Without generational mixing, the parameters related to the third–generation particles are not directly involved in the tri–lepton process although they affect the decay branching fractions indirectly as well as the electron EDM through two–loop diagrams\(^{15}\). Therefore, we can safely neglect the sfermion left–right mixing in evaluating tri–lepton event rates\(^{16}\) and concentrate mainly on the impact of two CP phases \(\{\Phi_1, \Phi_\mu\}\) and the SU(2) gaugino and higgsino parameters \(M_2\) and \(|\mu|\) on the tri–lepton signals at the Tevatron with only the \(e\) and \(\mu\) as the final–state leptons\(^{17}\). We emphasize in passing that the formidable hadronic backgrounds at the Tevatron experiments prevent one from using the hadronic decays of charginos and neutralinos as useful search modes unlike at clean \(e^+e^–\) collider experiments.

Figure 1 shows the mass spectrum of the lightest chargino \(\tilde{\chi}^\pm_1\) and the neutralinos \(\tilde{\chi}^0_{1,2}\) in the two scenarios; \(S1\) and \(S2\) in Eq. (1). The upper three figures are for \(S1\) with large sfermion masses of 10 TeV and the lower three figures for \(S2\) where the shadowed areas denote the region excluded by the electron EDM constraints. Except for the region of \(\Phi_\mu = 0, 2\pi\) in \(S1\), \(m_{\tilde{\chi}^\pm_1}\) and \(m_{\tilde{\chi}^0_2}\) are very similar in size and independent of \(\Phi_1\) in both \(S1\) and \(S2\) while \(m_{\tilde{\chi}^0_1}\) exhibits a very strongly correlated dependence on the phases. The chargino mass \(m_{\tilde{\chi}^\pm_1}\) increases as \(\Phi_\mu\) approaches \(\pi\), while \(m_{\tilde{\chi}^0_1}\) becomes maximal at non–trivial values of \(\Phi_\mu\) and \(\Phi_1\) in \(S1\) with \(|\mu| = 200 \text{ GeV}\). This implies that \(m_{\tilde{\chi}^0_1}\) is strongly affected by a small value of \(|\mu|\), while \(m_{\tilde{\chi}^\pm_1}\) and \(m_{\tilde{\chi}^0_2}\) are essentially determined by the SU(2) gaugino mass \(M_2\). Furthermore, a small \(|\mu|\) tends to reduce \(m_{\tilde{\chi}^0_{1,2}}\) and \(m_{\tilde{\chi}^\pm_1}\) on the whole.
The decay patterns for the charginos and neutralinos are very much parameter–dependent as well. The chargino decay $\tilde{\chi}_1^{-} \rightarrow \tilde{\chi}_1^{0} \ell^{-} \nu_{\ell}$ occurs though the $W$–exchange, slepton and sneutrino exchanges. For the sneutrino and slepton much heavier than the chargino, the $W$–exchange contribution dominates, and the decay branching ratios among the leptonic modes are determined by those of the on–shell $W$ boson. Similarly, the 3–body decay $\tilde{\chi}_2^{0} \rightarrow \tilde{\chi}_1^{0} \ell^{+} \ell^{-}$ occurs through virtual $Z$ bosons and sleptons. For the sleptons much heavier than the neutralino, the neutralino decays proceed through $Z^*$ with branching ratios similar to those of the on–shell $Z$ boson. We calculate the semileptonic branching fractions fully incorporating all the possible decay modes of the chargino and neutralino and find that the branching fractions are extremely sensitive to $\Phi$.

We calculate the semileptonic branches incorporating all the possible decay modes of the chargino and neutralino and find that the branching fractions are extremely sensitive to $\Phi_{\mu}$ and $\Phi_{1}$, especially in the scenario $S2$ with a large value of $|\mu|$. Figure 2 shows $\mathcal{B}(\tilde{\chi}_1^{-} \rightarrow \tilde{\chi}_1^{0} \ell^{-} \nu_{\ell})$ and $\mathcal{B}(\tilde{\chi}_2^{0} \rightarrow \tilde{\chi}_1^{0} \ell^{+} \ell^{-})$ for $\ell = e$ or $\mu$ in $S1$ (two upper figures) and in $S2$ (two lower figures). $\mathcal{B}(\tilde{\chi}_1^{-} \rightarrow \tilde{\chi}_1^{0} \ell^{-} \nu_{\ell})$ is almost constant over the whole space of the phases in $S1$, while $\mathcal{B}(\tilde{\chi}_2^{0} \rightarrow \tilde{\chi}_1^{0} \ell^{+} \ell^{-})$ is very sensitive to $\Phi_{\mu}$ around $\Phi_{\mu} = 0, 2\pi$. On the other hand, both branching fractions strongly depend on $\Phi_{\mu}$ and $\Phi_{1}$ in $S2$. Remarkably $\mathcal{B}(\tilde{\chi}_1^{-} \rightarrow \tilde{\chi}_1^{0} \ell^{-} \nu_{\ell})$ is minimal for non–trivial phases. $\mathcal{B}(\tilde{\chi}_2^{0} \rightarrow \tilde{\chi}_1^{0} \ell^{+} \ell^{-})$ is enhanced in $S2$ because the slepton–exchange contributions due to mainly the gaugino components of the neutralinos become dominant due to the small slepton masses and the large value of $|\mu|$. On the other hand, $\mathcal{B}(\tilde{\chi}_1^{-} \rightarrow \tilde{\chi}_1^{0} \ell^{-} \nu_{\ell})$ does not change so much in size, but the dependence of the branching fractions on the phases becomes very different in $S2$.

The parton–level production process $d\bar{u} \rightarrow \tilde{\chi}_1^{+} \tilde{\chi}_2^{0}$ is generated by the s–channel $W$–exchange, t–channel $\bar{d}$ exchange and u–channel $\bar{u}^{*}$ exchange. We need to convolute an effective parton distribution with the cross section of the parton–level process to obtain the total production cross section in $p\bar{p}$ collisions, $\sigma(p\bar{p} \rightarrow \tilde{\chi}_1^{+} \tilde{\chi}_2^{0} + X)$. For our analysis we use the CTEQ4m parton distribution function [19] with the QCD scale of the c.m. energy of the parton–level process and with the dominant QCD radiative corrections included by taking the value of the enhancement factor $\kappa = 1.3$ [20]. The production cross section $\sigma(p\bar{p} \rightarrow \tilde{\chi}_1^{+} \tilde{\chi}_2^{0} + X)$ for the positive chargino and neutralino pair in the CP self–conjugate $p\bar{p}$ collisions is the same as its charge–conjugate one. The two upper figures in Fig. 3 shows the dependence of $\sigma(p\bar{p} \rightarrow \tilde{\chi}_1^{+} \tilde{\chi}_2^{0} + X)$ on $\Phi_{\mu}$ and $\Phi_{1}$ in (a) $S1$ and (b) $S2$. Note that except for the region of $\Phi_{\mu} = 0, 2\pi$ in $S1$, the cross section is almost independent of the phase $\Phi_{1}$. The large $|\mu|$ and small squark masses in $S2$ reduce the production cross section due to a destructive interference between the $W$–exchange and the squark–exchange diagrams. We find that in both cases $\sigma(p\bar{p} \rightarrow \tilde{\chi}_1^{+} \tilde{\chi}_2^{0} + X)$ decreases as $\Phi_{\mu}$ approaches $\pi$.

A realistic analysis for the tri–lepton signal demands a numerical simulation fully incorporating all the background processes, for which one needs to make a quite considerable investigation. We defer such a detailed analysis to our next work [21] and present the total event rate of the tri–lepton signal without any experimental cuts against possible backgrounds. The lower two figures in Fig. 3 show the dependence of the total cross section $\sigma(p\bar{p} \rightarrow 3\ell + X)$ on $\Phi_{\mu}$ and $\Phi_{1}$ in (a) $S1$ and (b) $S2$, which can be obtained by multiplying $\sigma(p\bar{p} \rightarrow \tilde{\chi}_1^{+} \tilde{\chi}_2^{0})$ with $\mathcal{B}(\tilde{\chi}_1^{-} \rightarrow \tilde{\chi}_1^{0} \ell^{-} \nu_{\ell})$ and $\mathcal{B}(\tilde{\chi}_2^{0} \rightarrow \tilde{\chi}_1^{0} \ell^{+} \ell^{-})$. Since $\mathcal{B}(\tilde{\chi}_1^{-} \rightarrow \tilde{\chi}_1^{0} \ell^{-} \nu_{\ell})$ remains almost constant in $S1$ as shown in Fig. 2, the total cross section is mainly affected by $\mathcal{B}(\tilde{\chi}_2^{0} \rightarrow \tilde{\chi}_1^{0} \ell^{+} \ell^{-})$ and so it strongly depends on $\Phi_{1}$ around $\Phi_{1} = 0, 2\pi$. Note that the total tri–lepton cross section is $\mathcal{O}(10 \text{fb})$, too small to be seen with the present accumulated luminosity at the Tevatron of $\mathcal{O}(0.1 \text{fb}^{-1})$. However, the future Tevatron experiments with its upgraded luminosity of $2 \text{fb}^{-1}$ may exclude the region around $\Phi_{\mu} = 0, 2\pi$ and $\Phi_{1} = \pi$. 

4
if SUSY is not discovered. In $S_2$, the total cross section depends very strongly on the CP phases, takes its minimum value for non-trivial CP phases, and is larger than that in $S_1$ due to the largely enhanced neutralino branching fraction as shown in Fig. 2 which surpasses the reduction due to the destructive interference in the production cross section. Depending on the integrated luminosity, therefore, the very existence of the minimum event rate and the simultaneous small mass splitting for non-trivial phases as can be checked in Fig. 1 reflect that the range of the chargino and neutralino masses, which could be ruled out at the Tevatron, might be much smaller than that ruled out in the context of SUGRA and GUT inspired SUSY models.

To summarize, we have investigated the impact of the phases $\Phi_\mu$ and $\Phi_1$ on the SUSY tri-lepton signals at the Tevatron in the MSSM with general CP phases without generational mixing under the constraints on the phases by the electron EDM data. For the sake of generality, we have considered two exemplary scenarios for the relevant SUSY parameters; $S_1$ with very heavy first- and second-generation sfermions and $S_2$ with relatively light sfermions but a large $|\mu|$. We have found that in both scenarios the CP phases significantly affect the production cross section and especially the partial leptonic branching fractions of the chargino $\tilde{\chi}_1^\pm$ and neutralino $\tilde{\chi}_2^0$. As a result, there may lead to a minimum rate of the tri-lepton signal for non-trivial CP phases. This implies that one should be careful when interpreting the chargino and neutralino mass limits derived under the assumption of vanishing phases, since the worst case is not (always) covered by just flipping the sign of $\mu$; rather it can occur from some non-trivial phases in between.

We are grateful to Manuel Drees and Peter Zerwas for valuable comments and helpful discussions. This work was supported in part by the Korea Science and Engineering Foundation (KOSEF) through the KOSEF–DFG large collaboration project, Project No. 96–0702–01–01–2, and in part by the Center for Theoretical Physics. MG acknowledges Alexander von Humboldt Stiftung foundation for financial help and also KOSEF for funding during his stay in Yonsei University, Seoul, where this work was initiated.
REFERENCES

[1] For reviews, see H. Nilles, Phys. Rep. 110, 1 (1984); H.E. Haber and G.L. Kane, Phys. Rep. 117, 75 (1985); S. Martin, in Perspectives on Supersymmetry, edited by G.L. Kane, (World Scientific, Singapore, 1998).

[2] A. Masiero and L. Silvetrini, in Perspectives on Supersymmetry, edited by G.L. Kane, (World Scientific, Singapore, 1998); J. Ellis, S. Ferrara and D.V. Nanopoulos, Phys. Lett. B114, 231 (1982); W. Buchmüller and D. Wyler, ibid. B121, 321 (1983); J. Polchinsky and M.B. Wise, ibid. B125, 393 (1983); J.M. Gerard et al., Nucl. Phys. B253, 93 (1985); P. Nath, Phys. Rev. Lett. 66, 2565 (1991); R. Garisto, Nucl. Phys. B419, 279 (1994).

[3] Y. Kizukuri and N. Oshimo, Phys. Rev. D D45, 1806 (1992); 46, 3025 (1992).

[4] S. Dimopoulos and G.F. Giudice, Phy. Lett. B 357, 573 (1995); A. Cohen, D.B. Kaplan and A.E. Nelson, ibid. B 388, 599 (1996); A. Pomarol and D. Tommasini, Nucl. Phys. B488, 3 (1996).

[5] T. Ibrahim and P. Nath, Phys. Rev. D 57, 478 (1998); M. Brhlik, G.J. Good and G.L. Kane, ibid. D 59, 115004-1 (1999); S. Pokorski, J. Rosiek and C.A. Savoy, hep-ph/9906200.

[6] S.Y. Choi et al., Eur. Phys. J. C 7, 123 (1999); G. Moortgat–Pick and H. Fraas, Phys. Rev. D 59, 015016-1 (1998); hep-ph/9903220.

[7] M. Brhlik and G.L. Kane, Phys. Lett. B 437, 331 (1998); S.Y. Choi, J.S. Shim, H.S. Song and W.Y. Song, ibid. B 449, 207 (1999).

[8] T. Falk and K.A. Olive, hep-ph/9806236; T. Falk, A. Ferstl and K.A. Olive, hep-ph/9806413; T. Falk and K.A. Olive, Phys. Lett. B 375, 196 (1996); T. Falk, K.A. Olive and M. Srednicki, ibid. B 354, 99 (1995).

[9] A. Pilaftsis and C.E.M. Wagner, hep-ph/9902371; D.A. Demir, hep-ph/9901389; B. Grzadkowski, J.F. Gunion and J. Kalinowski, hep-ph/9902308.

[10] G.C. Branco, G.C. Cho, Y. Kizukuri and N. Oshimo, Phys. Lett. B 337, 316 (1994); Nucl. Phys. B 449, 483 (1995); D.A. Demir, A. Masiero and O. Vives, Phys. Rev. Lett. 82, 2447 (1999); S.W. Baek and P. Ko, hep-ph/9812221 (to appear in Phys. Rev. Lett.).

[11] P. Nath and R. Arnowitt, Mod. Phys. Lett. A 2, 331 (1987); H. Baer, K. Hagiwara and X. Tata, Phys. Rev. D 35, 1598 (1987); R. Barbieri et al., Nucl. Phys. B367, 28 (1991); H. Baer and X. Tata, Phys. Rev. D 47, 2739 (1993); H. Baer, C. Kao and X. Tata, ibid. D 48, 5175 (1993); T. Kamon, J. Lopez, P. McIntyre and J.T. White, ibid. D 50, 5676 (1994); H. Baer, C-H. Chen, F. Paige and X. Tata, ibid. D 54, 5866 (1996); S. Mrenna, G. Kane, G.D. Kribs and J.D. Wells, ibid. D 53, 1168 (1996).

[12] See, for example, F. Paige and S. Protopopescu, in Supercollider Physics, edited by D. Soper (World Scientific, Singapore, 1986), p. 41; H. Baer et al., in Proceedings of the Workshop on Physics at Current Accelerators and Supercolliders, edited by J. Hewett, A. White and D. Zeppenfeld (Argonne National Laboratory, Argonne, IL, 1993); S. Mrenna, Comput. Phys. Commun. 101, 232 (1997); S.Katsanevas and P. Morawitz, Comput. Phys. Commun. 112, 227 (1998).

[13] In a general phase convention, $\Phi_1$ and $\Phi_\mu$ should be replaced by two re–phasing invariant combinations, $\Phi_1 - \Phi_2$ and $\Phi_\mu + \Phi_2$. 
[14] E.D. Commins, S.B. Ross, D. DeMille, and B.S. Regan, Phys. Rev. A 50, 2960 (1994); K. Abdullah et al., Phys. Rev. Lett. 65, 2347 (1990).
[15] D. Chang, W.-Y. Keung and A. Pilaftsis, Phys. Rev. Lett. 82, 900 (1999).
[16] Certainly, for the large tan $\beta$ case, we need to incorporate all the possible left–right mixing contributions, especially, from the stau and sbottom sectors.
[17] In principle, we can include the $\tau$ lepton modes which become very important for a large tan $\beta$.
[18] H. Baer, C-H. Chen, M. Drees, F. Page and X. Tata, Phys. Rev. Lett. 79, 968 (1997); Phys. Rev. D 58, 075008 (1998).
[19] CTEQ Collaboration, H.L. Lai et al., Phys. Rev. D 51, 4763 (1995).
[20] M. Spira, hep-ph/9812407.
[21] S.Y. Choi et al., in preparation.
[22] M. Carena et al., in Perspectives on Supersymmetry, edited by G.L. Kane, (World Scientific, Singapore, 1998); D0 Collaboration, B. Abbott et al., Phys. Rev. Lett. 80, 1591 (1998); CDF Collaboration, F. Abe et al., ibid. 80, 5275 (1998).
FIG. 1. (a) $\chi^\rightarrow_1$ Mass [GeV], (b) $\chi^0_1$ Mass [GeV] and (c) $\chi^0_2$ Mass [GeV] on the $\{\Phi_\mu, \Phi_1\}$ plane in $S1$ (upper part) and $S2$ (lower part).
FIG. 2. \(B(\tilde{\chi}_1^0 \rightarrow \chi_1^0 l^+ l^-)\) and \(B(\tilde{\chi}_2^0 \rightarrow \chi_1^0 l^+ l^-)\) for \(l = e\) or \(\mu\) on the \(\{\Phi_\mu, \Phi_1\}\) plane in the scenarios \(S1\) (upper part) and \(S2\) (lower part).
FIG. 3. $\sigma(p\bar{p} \to \tilde{\chi}_1^- \tilde{\chi}_2^0 + X)$ (upper part) and $\sigma(p\bar{p} \to 3\ell + X)$ (lower part) on the $\{\Phi_\mu, \Phi_1\}$ plane in (a) $S1$ and (b) $S2$. 