New developments on hot and dense QCD in effective field theories are reviewed. Recent investigations in lattice gauge theories for the low-lying Dirac eigenmodes suggest survival hadrons in restored phase of chiral symmetry. We discuss expected properties of those bound states in a medium using chiral approaches. The role of higher-lying hadrons near chiral symmetry restoration is also argued from the conventional and the holographic point of view.

Keywords: Chiral symmetry restoration, Deconfinement, Trace anomaly

1. Introduction

The interplay between dynamical chiral symmetry breaking and color confinement in a hot/dense medium has not been sufficiently understood, and remains one of the central subjects in QCD [1, 2]. The chiral symmetry breaking and its restoration are well characterized by the quark-antiquark (chiral) condensate, whereas no reliable order parameter for the confinement-deconfinement phase transition is known. The Polyakov-loop expectation value, which plays the role of the order parameter in pure Yang-Mills (YM) theory, is disturbed seriously by dynamical quarks. Hence, even though the expectation value exhibits an inflection point at a certain temperature, it is not manifest that the system undergoes a transition from hadrons to quarks and gluons. A constructive way to identify the deconfined phase is to explore various fluctuations associated with conserved charges. In particular, the kurtosis of net-quark number fluctuations measures clearly the onset of deconfinement [3].

Recently, other fluctuations more addressing the gluon sector have been calculated in lattice gauge theory with light quarks [4, 5], where two ratios, \( R_T = \chi_I/\chi_R \) and \( R_A = \chi_A/\chi_R \), are considered in terms of the susceptibilities associated with the modulus, real and imaginary parts of the Polyakov loop. Asymptotic values of those ratios are properly quantified within a Z(3)-symmetric model when there are no dynamical quarks (see Fig. 1). Once the light flavored quarks are introduced, the \( R_T \) becomes much broadened, similarly to the Polyakov loop expectation value. On the other hand, the \( R_A \) retains the underlying center symmetry fairly well even in full QCD with the physical pion mass. Also, ambiguities of the renormalization prescription can be avoided to large extent in the ratio. The \( R_A \) thus serves as a better pseudo-order parameter than the Polyakov loop by itself. In Fig. 2, the \( R_A \) is compared with the kurtosis of the quark number fluctuations. The quark liberation takes place evidently together with a qualitative changeover in \( R_A \). Those abrupt changes in the Polyakov loop and quark number fluctuations appear in a narrow range of temperature lying on the pseudo-critical temperature of chiral symmetry restoration. Therefore, at vanishing chemical potential, \( T_{\text{deconf}} \approx T_{\text{chiral}} \) is concluded.
Figure 1. Lattice results of the ratios of the Polyakov loop susceptibilities, $R_T = \chi_I/\chi_R$ and $R_A = \chi_A/\chi_R$ for pure YM and $N_f = 2 + 1$ QCD at vanishing chemical potential [5]. The temperature is normalized by the critical temperature in pure YM theory, and by the pseudo-critical temperature for the chiral symmetry restoration in full QCD.

Figure 2. The ratio of the Polyakov loop susceptibilities $R_A = \chi_A/\chi_R$ and the kurtosis of net quark number fluctuations. Lattice data points are taken from [5, 6].
From the field theoretical point of view, it remains incomplete to capture the interplay of such non-perturbative dynamics in a form of an effective theory. In this contribution, we will briefly review recent progress in QCD thermodynamics and address the issues to be disentangled.

2. Low-lying Dirac eigenmodes and confinement

Spontaneous chiral symmetry breaking is locked to a non-vanishing spectral density with the zero eigenvalues of the Dirac operator, known as the Banks-Casher relation \[7\]. An intriguing question is whether confinement would also be lost if the Dirac zero modes are artificially removed from a system. In \[8, 9, 10, 11, 12\], a relation of the Dirac eigenmodes to the Polyakov loop has been formulated on a lattice, and their dynamical correlations have been investigated in SU(3) lattice gauge theory. The expression in a gauge-invariant formalism is found as \[11, 12\]

\[
\langle L \rangle = \frac{(2\beta)^{N_f-1}}{12V} \sum_n \lambda_n^{N_f-1} \langle n|\hat{U}_4|n\rangle.
\] (1)

Those simulations revealed that there are no particular modes which crucially affect confinement. In fact, the string tension extracted from the potential between static quarks is unchanged even when the low-lying Dirac modes are eliminated, as shown in Fig. 3. This apparently indicates that the disappearance of the chiral symmetry breaking does not dictate deconfinement of quarks. The derived analytic relation (1) tells manifestly that the Dirac zero modes has no role in the Polyakov loop. However, the matrix elements of the link variable must be affected by the light quarks, so that the Wilson loop and the quark potential may be significantly modified. The simulations carried out so far are limited to the quenched theories. Therefore, a conclusive statement should be postponed until computations with the dynamical quarks are made in future.

A conventional picture for color confinement is based on the dual superconductor \[13, 14\]. With a particular gauge fixing for the QCD Lagrangian, so-called Maximum Abelian Gauge \[15, 16\], magnetic monopoles naturally emerge and get condensed. As a result, a linear confinement potential and dynamical chiral symmetry breaking are generated. In order to accommodate the relation between the Polyakov loop and Dirac zero eigenmodes into this scenario, one needs to somehow link the Dirac eigenmodes to (a part of) the monopoles.
3. Hadrons near chiral symmetry restoration

Another implication of the chiral symmetry restoration with confinement is found in the hadron mass spectra with a systematic removal of the low-lying Dirac modes on a lattice \[17\]. The masses of several baryons and mesons with positive and negative parity are summarized in Fig. 4. As increasing the truncation level of the Dirac modes, the masses of parity partners approach and eventually become degenerate. What is remarkable is that those hadrons remain quite massive, around 1 GeV for the lowest nucleon and \( m_\rho \) for the lowest vector meson. Furthermore, universal scaling — 2\( m \) for mesons and 3\( m \) for baryons — is not observed. Hence, it is considerably suggestive that those hadrons keep their particle identities and survive in the chiral restored phase.

Given the lattice observation that \( T_{\text{deconf}} \approx T_{\text{chiral}} \) at vanishing chemical potential, the chiral restored phase with confinement might appear at high density. A large hadron mass needs to be saturated by certain condensates of chirally even operators. A good candidate is gluon condensates. Not only in matter-free space but also in a medium, the QCD trace anomaly exists and this is accompanied by a non-vanishing expectation value of a dilaton field, which is identified with a scalar glueball \[18\]. The in-medium gluon dynamics in the context of scale symmetry breaking is accommodated in a chiral effective field theory. In \[19\], the \( \rho \) and \( \omega \) mesons are shown to interact with a nucleon differently: the \( \rho NN \) coupling runs, whereas the \( \omega NN \) coupling walks in density. An immediate consequence is that the in-medium nucleon mass reaches a constant around the saturation density, and stays in higher density, as given in Fig. 5. Note that the original Lagrangian does not have an explicit “bare” mass. Nevertheless, due to the dynamics...
in dense matter, a chirally-invariant mass for the nucleon emerges. The above density dependences encoded in the renormalization group equations are governed by a non-trivial IR fixed point, dilaton-limit fixed point [20, 21]. These features remind us of the modern technicolor models for the Higgs physics beyond the Standard Model [22].

Higher-dimension operators can also be condensed and contribute to the hadronic quantities in dense matter. In particular, tetra-quark states play a crucial role to construct reliable equations of state for nuclear matter [23] and near the chiral phase transition via a mixing to a bilinear quark condensate [24]. Also, a novel phase with chiral symmetry breaking on top of the vanishing chiral condensate is an interesting theoretical option at finite density [25], where the tetra-quark condensate saturates the pion decay constant and could yield more critical point(s) in the phase diagram.

4. Role of higher-lying hadrons

The in-medium vector spectrum $\rho_V$ is more or less established both in theory and in dilepton measurements [26]. Yet, it has not been clarified how the observed modifications are linked to the (partial) chiral symmetry restoration. Instead of measuring the in-medium axial-vector spectrum $\rho_A$ in experiments, which is hopeless, a method to construct $\rho_A$ using a phenomenologically accepted $\rho_V$ via QCD and Weinberg sum rules has been proposed [27]. Fig. 5 summarizes the obtained thermal evolution of the spectra. The $a_1$ meson mass smoothly approaches the $\rho$ mass, and the two spectral functions become almost on top at a high temperature, indicating chiral restoration. Not only the lowest vector states, $\rho$ and $a_1$, but also the second lowest states, $\rho'$ and $a_1'$, are shown to contribute to the $\rho_{VA}$ rather significantly as approaching the restoration temperature. It is intuitively understood since more hadronic states must be populated toward the chiral phase transition that cannot be achieved within any conventional perturbative treatment including just a few numbers of mesonic states.

At non-vanishing chemical potential, it is more involved since charge conjugation invariance is lost, which leads to a mixing between transverse $\rho$ and $a_1$ states at tree level:

$$L_{mix} = 2C e^{\ln k} \text{tr} [\partial_\nu V_A \cdot A_\rho + \partial_\nu A_A \cdot V_\rho].$$

Their dispersion relations are modified and the spectral functions do not follow a simple Breit-Wigner distribution [28] (see Fig 7). Its relevance on observables crucially relies on a mixing strength $C$ which intrinsically depends on density. There are two available numbers: $C = 1$ GeV at the saturation density $n_0$ from an AdS/QCD model [29], and $C = 0.1$ GeV at $n_0$ from the gauged Wess-Zumino-Witten action in four dimensions as well as a mean field approximation [28].
Figure 6. Vector and axial-vector spectral functions at various temperatures [27].

The former yields vector meson condensation slightly above $n_0$, which is odd. Therefore, this is most likely an artifact of the large $N_c$ approximation employed for the gauge/gravity duality conjecture.

One conceivable reason for this huge difference in C’s is that all the Kaluza-Klein (KK) modes, corresponding to all the vector mesons, contribute to the dynamics in holographic QCD models. Heavier states can be integrated out, whereas a naive truncation may provide a different result since truncated modes carrying the information about the underlying physics are artificially omitted. In fact, the nuclear potential as a function of a distance $r$ exhibits a $1/r^2$ dependence when all the KK modes are considered [30], while it follows a $1/r$ behavior when a truncation is made.

As increasing temperature and density, more hadrons are activated and eventually change the ground state. It is not straightforward to deal with many (or all the) hadrons. In holographic QCD approach, although infinite KK modes are naturally accommodated, a systematic technique to include $1/N_c$ corrections in a medium is not established yet. In the standard effective theories in four dimensions, interactions with the higher-lying hadrons are not completely known. Integrating the heavier modes out at finite temperature and density is not an easy task either. Those effective interactions near the phase transition may be to some extent captured by use of more microscopic computations, e.g. lattice simulations, Dyson-Schwinger equations and functional approach [31].

5. Conclusions

Various fluctuations of conserved changes [32] as well as the ratio of the Polyakov-loop susceptibilities $R_A$ in lattice QCD consistently indicate that deconfinement takes place in the chiral crossover region at vanishing chemical potential. Although the kurtosis of net quark number fluctuations is reasonably quantified in a class of chiral models with the Polyakov loop [33], modifications in $R_A$ and $R_T$ by the dynamical quarks cannot be explained in the same framework. Since those models do not posses the dynamical mechanism for quark confinement, the properties of gluon-oriented quantities are supposed to be less captured. An effective theory that can better handle the confinement nature of non-abelian gauge theories is indispensable to reveal the Polyakov loop fluctuations in the presence of light quarks. Also, it is vital to bridge the gap between the Dirac zero eigenmodes, responsible for dynamical chiral symmetry breaking, and the magnetic monopoles.

The role of higher-lying hadrons has been found in low-energy constants and spectral functions near the QCD phase transition. On a practical level, it is not yet established to fully accommodate them to effective theories. Several attempts constrained by the relevant global symmetries lead to certain non-trivial medium effects. More elaborated and systematic prescription certainly requires a novel scheme. Functional approaches in terms of quarks and gluons may provide some benefits in this context.

Heavy-light hadrons, such as charmed mesons, are also good probes for the quark-gluon dynamics. In dilute nuclear matter, in-medium modifications of the color-electric and color-magnetic gluons are extracted from the D
and B meson dynamics\cite{34}. In increasing density/temperature, those heavy-light mesons will change their chiral properties, as expected from the chiral doubling scenario\cite{35,36,37}. The mass gap between the chiral partners is around 350 MeV, and this is much bigger than a mass difference between the charged D mesons in dense matter, ~50 MeV. Further theoretical investigations, along with the lattice input\cite{38}, will supply more reliable understanding of heavy-flavor transport properties.

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Figure 8. The chiral susceptibility calculated in lattice QCD for $N_f = 2$ and $N_f = 2 + 1 + 1$.

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