Primordial black holes from monopoles connected by strings

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Abstract

Primordial black holes (PBHs) are known to be produced from collapsing cosmic defects such as domain walls and strings. In this paper we show how PBHs are produced in monopole-string networks.
1 Introduction

S.W.Hawking has discussed how cosmic string loops that shrink by a factor of order $1/G\mu$ will form black holes\cite{1}. In this case, a tiny fraction $f \ll 1$ of order $(G\mu)^{2x-4}$, where $x$ is the ratio of the loop length to the correlation length, loops will form black holes.\cite{2} The result obtained by Hawking is cosmologically important because the emission of $\gamma$-rays from little black holes is significant\cite{3}. Numerical simulation of loop fragmentation and evolution was studied later by Caldwell and Casper\cite{4}, where the authors obtained the value of the fraction:

$$f = 10^{4.9\pm0.2}(G\mu)^{4.1\pm0.1}. \tag{1.1}$$

Black holes created by these collisions are so small that they lose their energy due to the Hawking evaporation process. The fraction of PBHs today in the critical density of the Universe is discussed by MacGibbon et al\cite{5}, where the authors calculated the fraction of black hole remnants

$$\Omega_{PBH}(t_0) = \frac{f}{\rho_{crit}(t_0)} \int_{t_*}^{t_0} dt \frac{dn_{BH}}{dt} m(t, t_0), \tag{1.2}$$

where $t_0$ is the present age of the Universe, and $t_*$ is the time when PBHs with initial mass $M_* \simeq 4.4 \times 10^{14} g$ were formed and which are expiring today. $m(t, t_0)$ is the present mass of PBHs created at time $t$. The approximate form of the mass function $m(t, t_0)$ is given by

$$m(t, t_0) \simeq \alpha t. \tag{1.3}$$

The extragalactic $\gamma$-ray flux observed at 100MeV is commonly accepted as providing a strong constraint on the population of black holes today. According to Carr and MacGibbon\cite{6}, the limit implied by the EGRET experiment is

$$\Omega_{PBH} < 10^{-9}. \tag{1.4}$$

The scaling solution of the conventional string network suggests that the rate of the formation of PBHs is

$$\frac{dn_{BH}}{dt} = f \frac{\rho_{loop}}{dt} \sim \alpha^{-1} ft^{-4}. \tag{1.5}$$

\footnote{Other possible origins and applications are discussed in ref.\cite{2}.}

\footnote{See the first picture in Fig.1}
Using the above results one can obtain an upper bound\[^5\]

\[ G\mu < 10^{-6}, \]  

which is close to the constraint obtained from the normalization of the cosmic string model to the CMB.

In addition to the simplest mechanism that we have stated above, it is always important to find a new mechanism for PBH formation, especially when PBHs produced by the new mechanism have a distinguishable property. Based on the above arguments we will consider less simplified networks of hybrid defects. The networks we consider in this paper are:

1. Pairs of monopole-antimonopole that are connected by strings.

2. Tangled networks of monopoles and strings where \( n > 2 \) strings are attached to each monopole.

We will show how PBHs are formed in the monopole-string networks. There are qualitative and quantitative differences between our new mechanisms and the conventional ones.

Before discussing the collision of the monopoles, it is important to note that we are considering heavy PBHs that can survive until late and may affect our present Universe. This means that the separation must be large enough (i.e. \( t_{\text{in}} \) must be late enough) so that the pairs can form “heavy” PBHs. In this case, unlike the conventional scenario of the monopole-antimonopole “annihilation”, the strings connecting the pairs do not have to dissipate their energy before the gravitational collapse, since the monopoles that go into the Schwarzschild radius cannot come back. One might think that if there are monopoles that have unconfined charges other than the magnetic charge, which might happen in natural setups, there could be a monopole-antimonopole “scattering” occurring before they go into the Schwarzschild radius and prevents the gravitational collapse. To get a rough understanding of the interactions mediated by a massless gauge boson, it is helpful to remember a famous result that the cross section for a scattering of the relativistic particles with significant momentum is given by\[^8\]

\[ \sigma \sim \frac{e^2}{T^2}, \]  

(1.7)
where $T$ denotes the typical particle momentum. From this equation people might think that the scattering must prevent the gravitational collapse. However, the monopole-antimonopole scattering cross section should not be estimated by using the typical momentum of the particles in the thermal plasma, but by the specific momentum of the colliding objects. In our model, the typical momentum of the monopoles that are about to collide is obviously very large. Scattering is not important in this case, since the cross section is much smaller than the Schwarzschild radius.

2 Monopole-antimonopole connected by a string

First, we will consider a simple toy model. Here we consider a model proposed by Langacker and P"{i}[7], in which the Universe goes through a phase transition with the $U(1)$ symmetry of electromagnetism spontaneously broken. The most obvious consequence of this additional phase transition is that during this phase monopoles and antimonopoles are connected by strings, due to the superconductivity of the vacuum. Let us first examine whether PBHs are formed in the original Langacker and P"{i} scenario. The separation between monopoles at the time of the string formation ($d(t_s)$) is bounded by[8]

$$d(t_s) < (t_s t_M)^{1/2},$$

(2.1)

where $t_M$ is the time when monopoles are produced. Strings are formed later at $t_s > t_M$, when charges are confined and monopoles are connected by the strings. The total energy of a pair is about $\sim \mu d(t_s)$. The Schwarzschild radius for this mass is given by

$$R_g \sim G\mu d.$$ (2.2)

Here we can ignore the frictional forces acting on this system, since they do not alter the above result[8]. In this case, black holes are formed if the Schwarzschild radius is larger than the width of the strings $\sim \eta^{-1} \sim \mu^{-1/2}$. As was discussed by Hawking in ref.[1], topological defects that can shrink instantly to their Schwarzschild radius $R_g$ turn into black holes. In our present model, the size of the monopoles is smaller than the width of

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4Besides the pairs that follow the conventional bound (2.1) in Ref.[8], there will be a network of longer strings that has the scaling solution. This network will remain after the conventional annihilation and will lead to another kind of PBH formation. See also Section 3.
the strings, and they are connected by almost straight strings. In this case, the minimum size of the defect when they shrink is larger than the size of the monopoles but is about the same order as the width of the strings. The condition for the black hole formation is therefore given by

\[ R_g > \eta^{-1}. \]  

(2.3)

Unfortunately, the bound contradicts the criteria given in eq. (2.3). In order to increase the typical mass of monopole-string-antimonopole, we will consider a less simplified model in which monopoles are diluted but not completely inflated away during the period of an (additional) inflationary expansion. The basic idea of the model is shown schematically in Fig.2. As we have discussed in the previous section, black holes will be formed when the monopole-antimonopole pair comes into the horizon at \( t = t_{in} \), where \( t_{in} \) is determined by the dilution mechanism. The production probability of PBHs is expected to be \( O(1) \) for long and straight strings that has monopoles at their endpoints.

The number distribution of the mass of the PBHs has a sharp peak. One may find a similar characteristic in the conventional mechanism of the PBH formation in models of hybrid inflation\[^9\]. The typical mass of such PBHs is given by\[^9\]

\[ M_{pbh} \sim \frac{M_p^2 e^{2N_c}}{H_I}, \]  

(2.4)

where the Universe is assumed to be inflated \( e^{N_c} \) times after the phase transition. \( M_{pbh} \) in eq. (2.4) is about the same order as the total mass that is contained in the horizon. To show explicitly the difference between the new model and the conventional one, let us evaluate the typical mass (and the size) of PBHs produced in the monopole-string networks. At the end of inflation, the physical distance between the diluted monopoles is about \( H_I^{-1} e^{N_c} \), where \( H_I \) is the Hubble constant during inflation. Then, the scale factor of the Universe develops as \( (tH_I)^{1/2} \), which is nothing but the usual evolution of the radiation-dominated Universe.\[^6\] Then the typical distance between the diluted monopoles is given by

\[ H_I^{-1} e^{N_c} \times (tH_I)^{1/2}, \]  

(2.5)

\[^5\]See \[^5\] for conventional reviews of cosmic strings and monopoles. Keep in mind that we are considering a very common situation.

\[^6\]We assume for simplicity that the equation of state becomes \( p = \rho/3 \) (ultrarelativistic gas) soon after inflation.
which becomes comparable to the particle horizon at \( t_{in} \sim H_I^{-1} e^{2N_c} \). At this time, the mass of a monopole-antimonopole pair connected by a string is

\[
M_{pbh} \simeq \frac{\mu}{H_I} e^{2N_c},
\]

where \( \mu \) is the tension of the string.

### 3. Tangled networks of monopoles and strings

In the previous section we showed that PBHs are produced in the monopole-string networks if inflation dilutes the monopoles. Therefore, it will be very interesting to consider a more complicated model of the tangled monopole-string networks, where monopoles are connected to \( n > 2 \) strings. According to the previous work in this field, the network is characterized by a single length scale, \( d(t) \sim \gamma t \).

Let us consider the networks of \( Z_n \)-strings\[10, 11, 12\]. The first stage of symmetry breaking occurs at a scale \( \eta_m \), when monopoles are produced. Then the second symmetry breaking produces a string network at a scale \( \eta \), where the symmetry breaking is given by

\[
G \rightarrow K \times U(1) \rightarrow K \times Z_n.
\]

The monopole mass and the string tension are given approximately by \( m \sim 4\pi \eta_m/e \) and \( \mu \sim \eta^2 \) with gauge coupling \( e \). The evolution of the string-monopole network has been studied by Vachaspati and Vilenkin[11]. These authors showed that the networks exhibit scaling behavior

\[
d(t) \sim \gamma t,
\]

where \( \gamma \) was taken from Berezinsky et al[10]. Assuming that radiation of gauge quanta is the dominant energy loss mechanism of the monopole-string networks, the value of \( \gamma \) is calculated[8, 10], and is given by

\[
\gamma \sim 4\pi \mu/e^2 m^2.
\]

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7We are assuming that the initial separation of the monopole and antimonopole is larger than the Hubble radius, which is due to the inflationary expansion after the phase transition. We also assume that the distance will expand at constant comoving distance until it enters the Hubble radius.

8In spite of the similarity between (2.6) and (2.4), the origin of the factor \( e^{2N_c} \) is rather different. See ref.[9] for more details.
In this model, the energy loss mechanism is important in determining this parameter. In ref. [10], the authors discussed how the ultrarelativistic motion of the monopoles may produce ultra-high energy gamma rays. Let us explain why monopoles in the networks of $Z_n$-strings may reach huge kinetic energy. In monopole-string networks, with $n$ strings attached to each monopole, the proper acceleration of a monopole should be determined by the vector sum of the tension forces exerted by the $n$ strings, which is given by $a \sim \mu/m$ by order of magnitude. Therefore, considering the result (3.2), one can obtain the typical energy of a monopole:

$$E_m \sim \mu d \sim \mu \gamma t.$$  

Now it is clear that the monopoles have huge kinetic energy proportional to the separation distance and this distance grows with time. What we consider in this paper is the gravitational collapse of such monopoles. Of course, the number of the collisions of monopoles per unit time is very small because of their small number density. However, once PBHs are formed, they can be cosmologically important even if their number density is very small. Therefore, in the typical collision of such monopoles, one must not disregard the black hole formation. In this case, the typical mass of the black hole is given by

$$m_{BH} \sim \mu \gamma t.$$  

One may think that the above result looks similar to result (1.3) obtained for the simplest string networks. However, remember that the networks that we are considering in this section are quite different from the conventional string networks. We are not considering the PBH formation from string loops, but the ones that come about from the collision of the energetic monopoles that come closer, within their Schwarzschild radius, $R_g \sim G\mu\gamma t$.

Let us calculate the number density of PBHs and see if we can put bounds on the tension of the strings (or on the mass of the monopoles). The number density of the monopoles is given by $n_m \sim d^{-3}$. Due to the random motion of the monopoles, the nucleation rate of the black holes is given by the conventional formula which is

$$\frac{dn_{BH}}{dt} \sim n_m^2 \pi R_g^2 \sim \frac{\pi \mu^2}{M_{pl}^3 \gamma^4 t^4},$$  

where the velocity of the energetic monopoles is $v_m \simeq 1$. Now it is easy to calculate the number distribution of the PBHs, $dn_{PBH}/dM$. Neglecting the mass loss of the black holes...
holes with the initial mass greater than $M_*$, we obtained the present value of $dn_{PBH}/dM$ by redshifting the distribution\[^5\]. Following the conventional calculation\[^5\], the result is given in a straightforward manner by

$$\frac{dn_{BH}}{dM} \propto M^{-2.5}. \quad (3.7)$$

The number distribution of the PBHs obtained above looks similar to the one obtained in Hawking’s scenario. However, there is a crucial discrepancy in the formation probability of the PBHs, which is usually denoted by “$f$” in Hawking’s scenario. In the conventional scenario, the value of $f$ is given by eq.(1.1), which was obtained in ref.[3]. The value of $f$ is $f \sim 10^{-20}$ for $G\mu \sim 10^{-6}$, which is of course quite tiny and characterizes Hawking’s mechanism. In our case, however, “$f$” must be different from the “$f$” in Hawking’s scenario because the situation of the PBH formation is qualitatively different. PBHs are formed whenever the energetic monopoles come closer than their Schwarzschild radius. The Schwarzschild radius is much larger than the size of the monopoles in that we are considering large PBHs with mass $M_{pbh} > M_*$. Therefore, the production probability of the colliding monopoles is $f \sim 1$ for the heavy PBHs that can survive today, and thus, $f$ in our scenario is much larger than the one obtained in Hawking’s scenario. On the other hand, the nucleation rate of PBHs is not about $10^{20}$ times as large as the conventional value, since there is a small factor in eq.(3.6), as we have discussed above. Eq.(3.6) may correspond to the production ratio of the closed string loops in the conventional scenario.

Comparing our result with (1.5) and using the result obtained by MacGibbon et al\[^5\], we obtained the constraint

$$(G\mu) < 10^{-10} \left[ \frac{M_*}{4.4 \times 10^{14} g} \right]^{1/7} \left[ \frac{\gamma}{10^{-2}} \right]^{5/7} \left[ \frac{t_{eq}}{3.2 \times 10^{10} s} \right]^{-1/7}, \quad (3.8)$$

where the calculation is straightforward. The result obtained here puts a new bound on the tension of the strings. However, the constraint is not important here. What is important in this paper is that we have found a novel mechanism for PBH formation, which is both qualitatively and quantitatively different from the old ones.
4 Conclusions and discussions

For usual cosmic strings, we know that only a tiny fraction of string loops can collapse to form black holes. Although the fraction of the production probability is very tiny for the conventional strings, this unique mechanism for PBH formation is cosmologically very important. In this paper, we considered two scenarios for the PBH formation in the monopole-string networks. The typical mass and the number distributions of the PBHs obtained in our model are distinctive.

First, we considered a model in which monopoles are diluted (weakly inflated away) but not completely inflated away. We found a narrow mass range and obtained $M_{\text{pbh}} \simeq \frac{\mu}{H_I} e^{2N_c}$. Qualitatively, PBHs formed in our model look similar to the ones produced during hybrid inflation. However, there is a hierarchical discrepancy in the typical mass. The difference is due to the crucial differences between the two mechanisms.

Our second model is the monopole-string networks with $n > 2$ strings attached to each monopole. We examined another mechanism of the PBH formation and found that the number density distribution of the PBHs of mass $M$ is proportional to $M^{-2.5}$. It is known that there is a similar distribution in the conventional string networks. However, in the Hawking’s scenario there is always a tiny probability $f$, which is absent in our model. On the other hand, there is another small factor in our result. The difference is due to the qualitative differences between the two distinctive models, as we have discussed. It may be important to note that our mechanisms can work in a hidden sector.

In a previous paper[13] we have considered cosmic necklaces and discussed another important implications of the defect-induced PBH formations, focusing our attention to brane inflation in the brane Universe. Although it has long been believed that “only strings are produced in the brane Universe”, it is not difficult to show explicitly how defects other than strings are produced in the brane Universe. For example, monopoles, necklaces and domain walls are discussed in ref.[14], and Q-balls are discussed in ref.[15]. A natural solution to the domain wall problem in a typical supergravity model is discussed in ref.[17], where the required magnitude of the gap in the quasi-degenerated vacua is induced by $W_0$ in the superpotential. The mechanism discussed in ref.[17] is natural since the constant term $W_0$ in the superpotential is necessary so as to cancel the cosmological
constant. Cosmic strings and other defects are important because they are produced after various kinds of brane-motivated inflationary models. They could be used to distinguish the brane models from the conventional ones. Moreover, if the fundamental scale of the brane world is very low, one needs to construct mechanisms of inflation and baryogenesis that may work in the low-scale models. The ideas of the low-scale inflationary models and baryogenesis are discussed in ref.[18] and ref.[19], where defects play crucial roles.

5 Acknowledgment

We wish to thank K.Shima for encouragement, and our colleagues at Tokyo University for their kind hospitality.

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Figure 1: The figure in the first line shows an example of the PBH formation in Hawking’s scenario. We show the simplest example of a perfect circle. The figure in the second line shows the PBH formation from a pair of monopoles. The figures in the third and the last line show examples of the network of $Z_3$ strings. Our model of the PBH formation is a natural extension of the original scenario in the sense that the kinetic energy of the defect at the collision plays an important role.
Figure 2: This picture shows an interesting possibility that may arise in the context of inflationary scenario\[8\]. Monopoles are formed during inflation but are not completely inflated away. Strings are formed by the succeeding phase transition that induces confinement. Strings can either be formed later during inflation or in the post-inflationary epoch. A string that connects a pair is initially much longer than the Hubble radius. Therefore, the strings connecting monopoles have Brownian shapes, as is shown on the left. During the evolution, the correlation length of the strings grows faster than the monopole separation due to the small loop production and the damping force acting on the strings. Finally, the correlation length of the strings becomes comparable to the monopole separation, and thus, one is left with a pair of monopoles connected by more or less straight strings, as is shown on the right.