Unified Control Strategy of Heterogeneous Thermostatically Controlled Loads with Market-based Mechanism

Yao Yao and Peichao Zhang

Abstract—This paper studies the coordination of heterogeneous thermostatically controlled loads (TCLs) to provide the real-time ancillary services. A market-based control framework is adopted for its advantages. The first advantage is that the demand curve-oriented approach makes it possible to form a unified control scheme for heterogeneous loads without identifying their different characteristics. The second one is that the broadcast price signal helps simplify the downlink control and reduce the implementation cost. Then, the separate demand curve construction strategies based on a virtual price for different types of TCLs are presented. The flexibility of each TCL is reflected through the curve, and its practical constraints, i.e., comfort requirements of users and operation constraints of devices, are satisfied explicitly. To ensure the control fairness and full utilization for the regulation ability of TCL cluster, a comfort-level-based price signal helps simplify the downlink control and reduce the implementation cost. Simulations are carried out to verify the effectiveness of the proposed method in providing frequency regulation services, for which a regulation capacity estimation method is developed. Finally, a series of case studies are conducted considering the practical situations, e.g., model errors, imperfect communication and sudden load change after the end of services.

Index Terms—Thermostatically controlled load (TCL), market-based control, coordination control, frequency regulation.

I. INTRODUCTION

The integration of large-scale renewable energy resources has imposed great threat on power system operation and makes it difficult and inefficient to maintain the power balance only depending on traditional generators [1]. Therefore, more regulation resources have to be exploited to guarantee the stability of power systems. Recent advances in information technologies have brought great attention to the demand-side resources as an alternative to provide the grid services [2].

Thermostatically controlled load (TCL) has been regarded as one of the most promising demand-side resources for its massive power consumption and thermal storage ability. Researchers have proposed effective strategies that enable a TCL cluster to provide the ancillary services. The state-queueing model proposed in [3] is adopted by [4] to provide the tie-line smoothing services. In [5], a priority-stack-based method is introduced to track the automatic generation control (AGC) signal. The algorithm in [6] solves the optimal setpoint change for target tracking. The evolution dynamics of aggregate temperature state is modeled in [7] to manage power imbalance.

However, the above control strategies are applicable only to the ON/OFF controlled TCLs, e.g., the fixed-frequency air-conditioning loads (FFAs). Besides the FFAs, the inverter air-conditioning loads (IVAs) which have continuous power regulation ability are also prevalent for their higher efficiency and better comfort [1], [8]. However, compared with FFAs, the coordination control strategy for IVAs has not been thoroughly discussed. Recent literature proposes several modeling and control methods for IVAs. For example, an aggregated thermal battery model for IVAs is presented in [1] and its optimal dispatch performance is demonstrated.

Unfortunately, most of the existing control methods are applicable either to FFAs or IVAs, but not to both, since these two types of TCLs are significantly different in operation characteristics. A load aggregator (LA) has to identify the TCL type, then either aggregates only one type of TCLs, or deploys a different control strategy according to the TCL type, leading to the insufficient utilization of the flexibility of TCLs or a higher integration cost.

To coordinate the heterogeneous TCLs, several literature has put forward some unified control strategies. For example, model predictive control scheme in [9] can deal with both continuous and discontinuous power, making it possible to aggregate both FFAs and IVAs. In [10], a generalized battery model is established to characterize the flexibility of commercial high voltage alternating current (HVAC), residential FFAs and energy storage. However, these methods require the private information from all TCLs, thus may cause privacy issues. Besides, all operation constraints are included in a single optimal problem, and the operation state of each TCL has to be specified in each control cycle, leading to higher computational burden and implementation cost.

In order to address the above problems, this paper applies the market-based control (MBC) method [11], [12] to pro-
pose a unified control framework for heterogeneous TCLs. The MBC method has attracted much attention in recent years as a new type of demand response coordination method [11]. Specific coordination schemes have been proposed using this method for some flexible loads with single type such as electric vehicles [13] and FFAs [14]. Compared with the existing studies, this paper highlights the following contributions:

1) For an LA, a unified framework for coordinating heterogeneous TCLs including both FFAs and IVAs is developed based on the market mechanism, which facilitates the aggregation and disaggregation of large-scale TCLs in a location-oriented manner rather than a load type-oriented one.

2) For end users, a demand curve construction method based on a virtual price concept is proposed to express the flexibility of heterogeneous TCLs in a unified way, while ensuring device security, user preferences and privacy concerns. Furthermore, the proposed method can ensure the control fairness in terms of comfort level regardless of the size and type of loads.

3) A regulation capacity estimation method that considers both physical limitations and customer experiences is proposed for a TCL cluster to provide the frequency regulation services.

II. PHYSICAL MODELS OF TCLs

A. Thermal Dynamic Model

This paper adopts the second-order equivalent thermal parameter (ETP) model in [15] as in (1). Note that this paper focuses on the cooling-mode air-conditioners.

\[
\begin{align*}
C_s \frac{dT_s}{dt} &= Q_s - G_s (T_s - T_o) - G_a (T_a - T_s) \\
C_n \frac{dT_n}{dt} &= Q_n - G_n (T_n - T_o) \\
Q_a &= -Q_{ac} + (1 - q_1)Q_i + (1 - q_2)Q_s \\
Q_s &= g_1Q_i + g_2Q_s
\end{align*}
\]

where \(T_s, T_n, T_o\) are the indoor air temperature, mass temperature and outdoor temperature, respectively; \(Q_{ac}, Q_s, Q_a\) are the heat rates from TCL, internal gain and solar gain, respectively; \(Q_s, Q_a\) are the heat rates to house air and house mass, respectively; \(C_s\) and \(C_n\) are the air heat capacity and mass heat capacity, respectively; \(G_s, G_a\) are the external and internal loss coefficients, respectively; and \(g_1, g_2\) are the distribution coefficients of heat rate and are both assigned to be 0.5.

B. Electrical Model

Electrical model describes the relationship between the heat rate and electric power of a TCL. The electrical model of an FFA can be described as:

\[
P_{ac,FFA} = \begin{cases} 
Q_{ac,FFA} \times COP & \text{ON state} \\
0 & \text{OFF state}
\end{cases}
\]

where \(P_{ac,FFA}\) is the rated electric power of an FFA; \(Q_{ac,FFA}\) is the heat rate of an FFA; and \(COP\) is the coefficient of performance.

Unlike FFAs, the power consumption and heat rate of IVAs can be adjusted continuously by controlling the frequency of the compressor. The electric power and heat rate can be expressed as a function of the frequency. In this paper, the simplified linear model in [1], [8] is adopted as:

\[
\begin{align*}
P_{ac,IVA} &= h_p(f) = k_{p1}f + k_{p2} \\
Q_{ac,IVA} &= h_q(f) = k_{q1}f + k_{q2}
\end{align*}
\]

where \(P_{ac,IVA}\) is the electric power of an IVA; \(Q_{ac,IVA}\) is the heat rate; \(f\) is the frequency of compressor; \(k_{p1}, k_{p2}, k_{q1}, k_{q2}\) are the coefficients of the linear model; and \(h_p(f)\) and \(h_q(f)\) mean that \(P_{ac,IVA}\) and \(Q_{ac,IVA}\) are the functions of frequency \(f\), respectively.

C. Coupling Model

By combining the thermal and electrical models, the following relation can be derived for a TCL:

\[
\hat{P}_w = V_f(T_d, t_p)
\]

where \(\hat{P}_w\) is the electric power required to render the air temperature to the designated value \(T_d\) from its current value after a certain time period of \(t_p\). The derivation of function \(V_f(T_d, t_p)\) is detailed in Appendix A. It will be used in demand curve construction and regulation capacity estimation, which will be discussed in the following sections.

III. MARKET-BASED COORDINATION FRAMEWORK

An MBC framework is proposed in this paper to coordinate heterogeneous TCLs. A virtual market [14], which functions as a traditional market while aiming at target power tracking, is established in each LA. The cyber-physical modeling [16] is employed to improve TCL autonomy in the framework. Each TCL locally determines its demand curve based on its physical model to reflect the urgency and flexibility of the demand, and properly responds to control signals. The following design principles of demand curves are considered: ① satisfying users’ diversified comfort preferences; ② satisfying TCL operation constraints, e.g., lockout time, power limit and ramp rate; ③ allocating the target power fairly to each TCL in terms of users’ comfort level.

The detailed construction method will be introduced in the next section.

With the required demand curves, TCLs in a cluster could be coordinated according to the following three stages in each control cycle, as illustrated in Fig. 1(a).

A. Bidding Stage

Each TCL constructs its demand curve. Let \(d(\lambda)\) denote the demand curve of TCL \(i\), where \(\lambda\) is the virtual price as it is dimensionless and only used as a control signal. To facility control, the range of virtual price is limited within \([-1, 1]\).

B. Aggregating and Clearing Stage

The virtual market built up by the LA collects the demand curves from all controlled TCLs and forms the aggregated demand curve as:

\[
\hat{P}_w = V_f(T_d, t_p)
\]
where $N$ is the number of controlled TCLs.

Denote the target power of the TCL cluster as $P_{AC}^*$. The clearing price $\lambda'$ can then be solved at the intersection of $P_{AC}^*$ and the aggregate demand curve, calculated as:

$$\lambda' = D^{-1}(P_{AC}^*)$$

(5)

prove the autonomy and mask sensitive data such as preferences and model parameters.

3) The unified control framework and the uniform demand curves facilitate a hierarchical aggregation of TCLs in a wider region. As illustrated in Fig. 1(b), $d_{ij}$ is the demand curve of the $i^{th}$ TCL in the $j^{th}$ LC, $d_{LC}$ is the aggregated demand curve of the $j^{th}$ LC. Several local concentrator (LC) layers can be engaged in the control framework. Each LC, usually deployed in a building or on a campus, would pre-aggregate the demand curves from lower level.

IV. DEMAND CURVE OF A SINGLE TCL

Following the design principles mentioned in the previous section, this section proposes the construction method of demand curves based on a comfort level index.

A. Comfort Level

Considering the diversity of users’ comfort preferences, a normalized index is introduced to measure users’ comfort level, which is called the state of air temperature (SOA). SOA at time $k$ is defined as:

$$SOA(k) = \begin{cases} \frac{T_a(k) - T_{des}}{T_{max} - T_{min}} & T_a(k) \geq T_{des} \\ \frac{T_{des} - T_a(k)}{T_{max} - T_{min}} & T_a(k) < T_{des} \end{cases}$$

(7)

where $T_a(k)$ is the indoor air temperature at time $k$; $T_{des}$ is the desired air temperature; and $T_{max}$ and $T_{min}$ are the upper and lower limits of air temperature, respectively.

According to the definition, SOA should be kept within $[-1, 1]$ to satisfy users’ comfort requirements, and it equals to 0 when $T_a(k) = T_{des}$.

B. Demand Curve of FFAs

FFAs are ON/OFF controlled loads with two operation power, i.e., $P_{min}$ equals to 0 and $P_{max}$ equals to the rated power $P_{ac}$. They usually have a minimum ON/OFF time called lockout time [9], which is an important physical constraint. Figure 2(a) illustrates the state machine diagram of an FFA including lockout states. According to the current state of an FFA, its demand curve has three possible shapes, as shown in Fig. 2(b)-(d).

C. Disaggregating and Responding Stage

The virtual market broadcasts $\lambda'$ to each TCL. TCL $i$ calculates its response power by $P_{ac,i} = d_i(\lambda')$. The target power can be accurately disaggregated among individual TCLs in each cycle.

The proposed coordination method is beneficial to both LAs and end users:

1) For an LA, the control process is greatly simplified and the communication traffic is significantly relieved, as it does not need to identify each TCL type, and only needs to broadcast a single control signal, rather than specifying the operation state of each TCL as required in [9], [10].

2) For end users, the demand curves can effectively im-
1) **ON/OFF State**

In these two states, an FFA demand curve is stair-shaped, and the inflection point is \((P_{ac}\_SOA'})\) where \(SOA'\) is derived from \(SOA\) by:

\[
SOA' = \begin{cases} 
\frac{SOA + 1}{2} & \text{ON state} \\
\frac{SOA - 1}{2} & \text{OFF state} 
\end{cases}
\]  

(8)

2) **LockON and LockOFF States**

In these two states, an FFA must keep its state unchanged until the lockout time, i.e., \(t_{lockon}\) or \(t_{lockoff}\) expires. Thus, its bid demand curve is:

\[
d(\lambda) = \begin{cases} 
P_{ac} \_r & \forall \lambda, \text{LockON state} \\
0 & \forall \lambda, \text{LockOFF state} 
\end{cases}
\]  

(9)

The above bidding strategy ensures that: ① for FFAs in the same state, those with a larger \(SOA\) value have higher priority to keep/switch ON, thus the users’ comfort can be better maintained; ② FFAs in the ON state always have higher priority to keep ON than those in the OFF state to switch ON, thus the switching frequency can be as low as possible; ③ the lockout time constraint can be explicitly satisfied.

**Remark 1** The demand curve construction method ensures that the average \(SOA\) value of an FFA cluster denoted as \(SOA\), is approximately equal to the clearing price \(\lambda'\). It is referred to as \(SOA-\lambda'\)-equality control in this paper.

For the purpose of explanation, \(\lambda'\) is assumed to be constant over a period of time. The case in which \(\lambda' > 0\) is considered. According to (8), an ON state FFA will switch off when \(SOA < 2\lambda' - 1\), then its \(SOA\) increases, while an OFF state FFA will switch on when \(SOA \geq 1\), then its \(SOA\) decreases. Thus, \(SOAs\) of all FFAs vary within \([2\lambda' - 1, 1]\) in the steady state. Similarly, if \(\lambda' < 0\), \(SOAs\) of all FFAs vary within \([-1, 2\lambda' + 1]\). In both cases, if \(SOA\) is assumed to be uniformly distributed in the cluster, it can be derived that \(SOA = \lambda'\).

**C. Demand Curve of IVAs**

To construct the demand curve of IVAs, some characteristic powers are defined as follows [8].

1) **\(SOA\)-equality power \(P_{SOA}\)**

\(P_{SOA}\) is defined as the power required by an IVA to maintain its current \(SOA\) value. According to (4), \(P_{SOA}\) at time \(k\) can be obtained by:

\[
P_{SOA} = V_{r}(T_{c}(k), t_{c})
\]  

(10)

where \(t_{c}\) is the control cycle.

\(P_{SOA}\) is used to achieve the \(SOA\)-equality control in the IVA cluster. The mechanism will be detailed later.

2) **Comfort-constraint power \(P_{CFT}\)**

Define \(P_{CFT\_min}\) as the minimum power required to keep \(SOA\) within the upper limit in a certain period of time \(t_{c}\). In this paper, \(t_{c}\) is assigned to be 5 min. According to (4), \(P_{CFT\_min}\) can be obtained by:

\[
P_{CFT\_min} = V_{r}(T_{mc}, t_{c})
\]  

(11)

Similarly, \(P_{CFT\_max}\) which is the maximum power required to keep \(SOA\) within the lower limit in \(t_{c}\) is:

\[
P_{CFT\_max} = V_{r}(T_{min}, t_{c})
\]  

(12)

3) **Operation-constraint power \(P_{OPT}\)**

The power adjustment range of an IVA in a control cycle \(t_{c}\) is mainly restricted by the frequency range and ramp rate of its compressor. \(P_{OPT}\) can be calculated by:

\[
\begin{align*}
P_{OPT\_min} &= h_{p}(\min(\lambda\_f + f, \lambda_{t}), t_{c}) \\
P_{OPT\_max} &= h_{p}(\min(\lambda\_f + f, \lambda_{t}), t_{c})
\end{align*}
\]  

(13)

where \(f\) and \(f_{c}\) are the current frequency and ramp rate, respectively; and \(\lambda_{min}\) and \(\lambda_{max}\) are the minimum and maximum frequencies, respectively. The minimum and maximum power can be calculated as \(P_{min}=h_{p}(\lambda_{min})\) and \(P_{max}=h_{p}(\lambda_{max})\).

Each IVA calculates the above characteristic power value, and then constructs its demand curve in each control cycle. As shown in Fig. 3, the demand curve includes the following critical points:

1) Point A \((P_{SOA\_SOA})\)

This point means that if the clearing price \(\lambda'\) is equal to the current \(SOA\), IVA’s response power will be \(P_{SOA}\) as defined in (10).

2) Point B \((P_{CFT\_min} + 1)\) and Point C \((P_{CFT\_max} - 1)\)

These two points ensure that IVA’s response power to any price \(\lambda'\) is within \([P_{CFT\_min}, P_{CFT\_max}]\), thus the users’ comfort constraints can be satisfied. These two points could ensure users’ comfort, while they do not guarantee the operation constraints. Thus, Points D and E are further introduced.

3) Points D and E

These two points lie on lines AB and AC, respectively. Their \(x\)-axis values equal to \(P_{OPT\_min}\) and \(P_{OPT\_max}\) respectively, ensuring that the response power will not exceed the physical constraints.

![Fig. 3. Demand curve of an IVA.](image-url)
eral advantages:
1) The target power can be allocated to different types of TCLs fairly in terms of comfort level. Furthermore, this control feature helps avoid the control failure caused by some of TCLs reaching the temperature limit first.
2) An LA can easily estimate SOA, which reflects the energy level and potential regulation capacity of a TCL cluster based on λ. 
3) It is helpful to derive an aggregate dynamic model of a TCL cluster, which will be detailed in our following research paper.

V. APPLICATION

In this paper, TCLs are aggregated to provide the frequency regulation services. Let \( R_{\text{reg}} \) denote the normalized regulation signal, which ranges within \([-1,1]\). In response to \( R_{\text{reg}} \), a TCL cluster has to deviate from its baseline load. Thus the aggregate target power is:

\[
P_{\text{AC}}^* = P_{\text{base}} + R_{\text{reg}} C
\]

(14)

where \( C \) is the contracted regulation capacity; and \( P_{\text{base}} \) is the baseline load of the cluster. \( R_{\text{reg}} \) is the traditional regulation signal retrieved from the PJM website [17].

An LA should determine the regulation capacity \( C \) of the TCL cluster at each hour. Physical limitations of TCL and users’ comfort requirements are considered as the restriction factors of regulation capacity.

The maximum capacity limited by the physical constraints can be calculated as:

\[
C_{\text{physic}} = \min \{ P_{\text{MAX}} - P_{\text{base}}, P_{\text{base}} - P_{\text{MIN}} \}
\]

(15)

where \( P_{\text{MIN}} = \sum_{i=1}^{N} P_{\text{MIN},i} \) and \( P_{\text{MAX}} = \sum_{i=1}^{N} P_{\text{MAX},i} \) are the minimum and maximum power that a TCL cluster with \( N \) TCLs can provide, respectively.

To ensure users’ comfort, the air temperature of each TCL has to be kept within \([T_{\text{min}}, T_{\text{max}}]\). For each TCL, the electric power required to make the air temperature reach the upper/lower limit in 1 hour can be calculated according to (4) as:

\[
\begin{align*}
\bar{P}_{\text{ac}} &= V_{\text{r}}(T_{\text{max}}, 1) \\
P_{\text{ac}} &= V_{\text{r}}(T_{\text{min}}, 1)
\end{align*}
\]

(16)

The aggregate power of a cluster that makes SOA of all TCLs reach the limits in 1 hour is estimated by:

\[
\begin{align*}
\bar{P}_{\text{AC}} &= \sum_{i=1}^{N} \bar{P}_{\text{ac},i} \\
P_{\text{AC}} &= \sum_{i=1}^{N} P_{\text{ac},i}
\end{align*}
\]

(17)

Therefore, if the target power is in the range of \([\bar{P}_{\text{AC}}, P_{\text{AC}}]\), SOA value could be kept within \([-1,1]\). Since the normalized regulation signal is within \([-1,1]\), the maximum regulation capacity limited by comfort requirements can be conservatively calculated by:

\[
C_{\text{conf}} = \min \{ P_{\text{AC}} - P_{\text{base}}, P_{\text{base}} - \bar{P}_{\text{AC}} \}
\]

(18)

To conclude, considering TCL physical constraints and comfort constraints, the cluster’s regulation capacity can be estimated as:

\[
C = \min \{ C_{\text{physic}}, C_{\text{conf}} \}
\]

(19)

VI. SIMULATION

The case studies are conducted on a typical three-level control system as shown in Fig. 1(b), which consists of 600 TCLs in total. Five LCs are responsible for pre-aggregating local TCLs, and the number of FFAs and IVAs of each LC is shown in Table I.

| Index of LC | No. of total TCLs | No. of FFAs | No. of IVAs |
|------------|------------------|-------------|-------------|
| 1          | 80               | 80          | 0           |
| 2          | 160              | 120         | 40          |
| 3          | 180              | 90          | 90          |
| 4          | 120              | 30          | 90          |
| 5          | 60               | 0           | 60          |

The thermal parameter settings are taken from [14], and the suitable rated cooling capacity \( Q_{\text{ac},r} \) is chosen according to the thermal parameters. The rated power can be calculated as \( P_{\text{ac},r} = Q_{\text{ac},r}/\text{COP} \), where COP conforms to the uniform distribution \( U(3, 4) \). Users’ preferences and IVAs’ parameters are shown in Table II, and coefficients of the linear electrical model in (3) are calculated by the parameters. The outdoor temperature and solar radiation are shown in Fig. 4. In all cases, the simulation time step is 10 s and \( t_{\varepsilon} \) is 1 min.

| Parameter | Value |
|-----------|-------|
| \( T_r \) (°C) | \( U(2,3) \) |
| \( T_{\text{des}} \) (°C) | \( N(25,1) \) |
| \( f_{\text{min}} \) (Hz) | 30 |
| \( P_{\text{rate}, \text{min}} \) | \( U(0.08,0.12) \) |
| \( Q_{\text{rate}, \text{min}} \) | \( U(0.08,0.12) \) |
| \( T_r \) (°C) | \( U(2,3) \) |
| \( f_r \) (Hz/s) | 1 |
| \( f_{\text{max}} \) (Hz) | 130 |
| \( P_{\text{rate}, \text{max}} \) | \( U(1.3,1.7) \) |
| \( Q_{\text{rate}, \text{max}} \) | \( U(1.0,1.3) \) |

Note: \( T_{\varepsilon} = T_{\text{max}} - T_{\text{des}} \); \( T_{\varepsilon} = T_{\text{des}} - T_{\text{min}} \); \( P_{\text{min}} = P_{\text{max}, \text{min}}, P_{\text{ac},r} \); \( P_{\text{max}} = P_{\text{rate}, \text{max}}, P_{\text{ac},r} \); \( Q_{\text{min}} = Q_{\text{rate}, \text{min}}, Q_{\text{ac},r} \); \( Q_{\text{max}} = Q_{\text{rate}, \text{max}}, Q_{\text{ac},r} \); \( U(a, b) \) denotes the uniform distribution within \([a, b] \); \( N(\cdot) \) denotes the normal distribution.

A. Validation of Capacity Estimation Method

To test the capacity estimation method in Section V, \( P_{\text{AC}}^* \) is set to be \( P_{\text{ac},r} \) during 07:00-08:00 and \( \bar{P}_{\text{AC}} \) during 15:00-16:00. \( P_{\text{AC}}^* \) in the rest of the day is set to be \( P_{\text{base}} \). We can see from Fig. 5 that SOAs value of all TCLs approach the lower/upper limit at the end of the selected hour. The above result demonstrates that the calculation method in (17) could obtain the
expected value of $\bar{P}_{ac}$ and $P_{ac*}$ which are the lower/upper power limits that keep SOA value in $[-1, 1]$. It also demonstrates the effectiveness of the capacity estimation method.

B. Response to Regulation Signal

Figure 6 shows the tracking performance of TCL cluster, in which the required regulation power $P_{reg} = R_{reg} C$ and the response power $P_{res} = P_{ac} - P_{base}$ where $P_{ac} = \sum_{i} P_{ac,i}$. The grey area denotes the regulation capacity $C$. As we can see, the proposed coordination method can provide the accurate power tracking services in ideal situations.

Figure 7 illustrates the clearing price $\lambda^*$ and SOA of individual TCLs in different LCs. We can find that regardless of the proportion of FFAs and IVAs, TCLs in all LCs can be maintained at a basically same comfort level. For IVAs, SOA gradually becomes equal after the transient process at the beginning of control and then tracks the dynamic change of $\lambda^*$ well. For FFAs, SOA generally follows $\lambda^*$ and stays close to the SOA of IVAs. The simulation results demonstrate the effectiveness of SOA-$\lambda^*$-equality control for IVAs and SOA-$\lambda^*$-equality control for FFAs.

In Fig. 6, the regulation capacity restricted by physical limitation and comfort requirements are also plotted. The grey area denotes the contracted regulation capacity $C$. During 00:00-06:00, regulation capacity is determined by $C_{\text{physical}}$ due to the low baseline load and relatively high $C_{\text{conf}}$. During the rest periods, the regulation capacity equals to $C_{\text{conf}}$. The value of $C_{\text{conf}}$ is highly correlated with SOA of the TCL cluster: when SOA approaches zero, $C_{\text{conf}}$ increases; when SOA gets close to ±1, $C_{\text{conf}}$ decreases. Figure 8 shows the regulation capacity $C$ that is normalized based on $P_{\text{MAX}}$. We can see that the aggregated TCLs are able to provide the regulation capacity that accounts for 10%-35% of $P_{\text{MAX}}$. 

![Fig. 4. Outdoor temperature and solar radiation.](image)

![Fig. 5. Tracking performance and SOA trajectory when target power $P_{ac*}$ is set to be $\bar{P}_{ac}$ and $P_{ac*}$ (a) Tracking performance. (b) SOA and $\lambda^*$.](image)

![Fig. 6. Tracking performance.](image)

![Fig. 7. SOA of FFAs and IVAs in different LCs. (a) LC1. (b) LC2. (c) LC3. (d) LC4. (e) LC5.](image)

![Fig. 8. Normalized regulation capacity.](image)
C. Performance When SOA Reaches Limits

In this section, \( C_{\text{conf}} \) is not considered in order to evaluate the response behavior when the target power exceeds the regulation ability of TCL cluster.

As shown in Fig. 9(b), during 06:00-09:00, SOA reaches the lower limit \(-1\); during 15:00-18:00, SOA reaches the upper limit \(+1\). During the above period, though FFAs switch frequently, the lockout constraints are satisfied. While IVAs adjust their power consumption adaptively to avoid exceeding the comfort limit. Therefore, the proposed method can ensure users’ comfort. However, as shown in Fig. 9(a), this comfort-first principle can lead to a poor tracking performance when SOA reaches \pm 1. The advantage of the proposed method under such condition is that poorer tracking performance would only occur when all the TCLs reach their comfort limits. Compared with the method in [1], the proposed method in this paper ensures a better utilization of the TCL cluster’s regulation ability.

![Fig. 9. Tracking performance and SOA trajectory with different errors. (a) Error is within \([-0.15, 0.15]\). (b) Error is within \([-0.3, 0.3]\).](image)

As we can see, model error enlarges the deviation of SOA from the clearing price \( \lambda^* \) and leads to a worse SOA-\( \lambda^* \)-equality control performance of IVAs. When the characteristic power error is 30%, the largest SOA difference is about 0.3. However, SOAs of IVAs still follow the trajectory of clearing price in general. In contrast, since FFA demand curve is irrelevant to physical model, the model error has almost no impact on FFAs’ SOA trajectory.

E. Effect of Imperfect Communication

The above simulations are implemented on a perfect communication system. However, since our MBC method is heavily dependent on communication systems [11], communication problems, e.g., package delay and package loss, might influence the response performance. As listed in Table III, four cases with different communication problems are studied in this section.

| Case | Specific scenario setting |
|------|---------------------------|
| Ideal case | With perfect communication system |
| Case 1 | Downlink delay. The latency is uniformly distributed among 0, 10 and 20 s |
| Case 2 | Downlink package loss. The loss rate is set to be 10% |
| Case 3 | Uplink package loss. The loss rate is set to be 3% |
| Case 4 | Uplink package loss. The loss rate is set to be 3%. The lost demand curves are filled by the curves bidding into the last market period |

The tracking performance during 12:15-12:30 in each case is shown in Fig. 11. In addition, the precision score \( S_p \) defined by PJM [18] is illustrated in Fig. 12.

In Case 1, the downlink delay makes the response power unable to track the target power immediately. In Case 2, TCLs that fail to receive the clearing price maintain their current operation states, thus enlarging the tracking error when the ramp rate of \( P_{\text{inc}} \) is large. The values of \( S_p \) in Case
1 and Case 2 are both slightly smaller than that of the ideal case. Considering that broadcasting signal scarcely presents such a high latency and loss rate as set in Case 1 and Case 2, the downlink communication problems have little impact on tracking performance.

Fig. 11. Tracking performance of imperfect communication cases. (a) Ideal. (b) Case 1. (c) Case 2. (d) Case 3. (e) Case 4.

Fig. 12. Precision score of imperfect communication cases.

Compared with the downlink communication problems, uplink package loss would lead to a poorer response performance. In Case 3, due to the lost demand curves, $\lambda^*$ would be lower than its ideal value, making response load higher than the target power. The result of Case 3 shows that even a relatively small uplink loss rate of 3% would lead to a large tracking error, especially when the target power $P_{ac}^*$ is high. Since uplink communication burden is much heavier than downlink burden in our method, solutions are desired to address the adverse effect. One possible method is to replace the lost curves with those in the previous market [11], and the simulation results of Case 4 have demonstrated the effectiveness of this strategy: the tracking performance is significantly improved, and $S_p^*$ gets very close to that of the ideal case.

F. Recovery Strategy

In practical situations, LA has to release the control of TCLs after several hours of service provision. A peak load usually occurs in this process [19]. In this case, TCL cluster responds to the regulation signal during 07:00-17:00 and thermostat deadband in natural operation mode is distributed in $U(0.6,1.4)$ ˚C.

As shown in Fig. 13, almost all TCLs increase their power consumption simultaneously after the end of service provision, causing the aggregate power to soar to the highest level in a short time as circled in Fig. 13(a). Such load peak may impose the threat on system operation. Therefore, recovery strategies should be designed to enable TCLs to turn back to normal operation state in a smoother manner.

The $SOA-\lambda^*$-equality control feature can serve as an effective tool to mitigate load peak because the LA can control the $SOA$ trajectory of TCLs by adjusting the clearing price. In this paper, the clearing price varies linearly from the value at the end of the service to zero and the recovery time is set to 1 hour. Once air temperature reaches the desired value, each TCL turns back to natural operation mode. The performance of this smooth strategy is shown in Fig. 14. Compared with the direct recovery case, the load peak which is circled in Fig. 14(a) is mitigated significantly.

Fig. 14. Recovery performance with smooth strategy. (a) Aggregated power. (b) $SOA$ and clearing price trajectory.

VII. CONCLUSION

This paper develops a unified coordination method of heterogeneous TCLs which adopts the market equilibrium principle. The demand curves of two typical TCL types, i.e., FFAs and IVAs are constructed, which could fully reflect the flexibility of each TCL and realize comfort-level-equality control. The control framework enables an LA to coordinate TCLs without identifying TCL type or collecting private information. To provide the frequency regulation services, a
conservative regulation capacity estimation method is also put forward.

Simulation results indicate that the proposed method can accurately track the regulation signal. Model error mainly affects the SOA-$\xi$-equality control of IVAs, while has little impact on FFAs. It is also found that uplink communication problems may lead to the tracking error, but such problems can be improved with proper solutions. In the end, a recovery strategy is proposed and proved to be effective in mitigating load peak after the service provision.

Future work will extend the method to more flexible resources, e.g., energy storage, electric vehicles. Aggregate dynamic model will also be developed based on the comfort-level-equality feature to facilitate new control strategies for more services, e.g., energy optimization.

APPENDIX A

The derivation of the function $V_r(T_d, t_p)$ is as follows.

Based on the first two equations in (1), $T_a$ can be eliminated and a second-order linear differential equation of $T_a$ can be obtained as:

$$a \frac{d^2 T_a}{d t^2} + b \frac{d T_a}{d t} + c T_a = d$$  \hspace{1cm} (A1)

where $a = C_s; \ b = C_s(G_s + G_m)/G_a + C_s; \ c = G_s; \ \ \text{and} \ \ d = Q_m + Q_s + G_s T_d$.

The solution of the above equation has the following form:

$$T_a(t) = A_1 e^{(r_1)t} + A_2 e^{(r_2)t} + \frac{d}{c}$$ \hspace{1cm} (A2)

where $r_1$ and $r_2$ are determined by thermal parameters; and $A_1$ and $A_2$ are determined by thermal parameters as well as $Q_s$ and $T_\infty$.

Let $t = t_p, \ T_a(t_p) = T_d$ in (A2), and given the internal gain $Q$ and solar gain $Q_s$ in (A1), the required heat rate $Q_w$ can be derived. According to (A2) and (A3), the required power can be obtained by:

$$P_a = V_r(T_d, t_p) = \begin{cases} \frac{Q_w}{COP} & \text{FFA} \\ \frac{(\bar{Q}_w - k_{8i})}{k_{81}} + k_{82} & \text{IVA} \end{cases}$$ \hspace{1cm} (A3)

REFERENCES

[1] M. Song, C. Gao, H. Yan et al., “Thermal battery modeling of inverter air conditioning for demand response,” IEEE Transactions on Smart Grid, vol. 9, no. 6, pp. 5522-5534, Nov. 2018.

[2] M. Wang, Y. Mu, T. Jiang et al., “Load curve smoothing strategy based on unified state model of different demand side resources,” Journal of Modern Power Systems and Clean Energy, vol. 6, no. 3, pp. 540-554, May 2018.

[3] N. Lu, “An evaluation of the HVAC load potential for providing load balancing service,” IEEE Transactions on Smart Grid, vol. 3, no. 3, pp. 1263-1270, Sept 2012.

[4] D. Wang, S. Ge, H. Jia et al., “A demand response and battery storage coordination algorithm for providing microgrid tie-line smoothing services,” IEEE Transactions on Sustainable Energy, vol. 5, no. 2, pp. 476-486, Apr. 2014.

[5] H. He, B. M. Sanandaji, K. Poolla et al., “Aggregate flexibility of thermostatically controlled loads,” IEEE Transactions on Power Systems, vol. 30, no. 1, pp. 189-198, Jan. 2015.

[6] W. Wei, D. Wang, H. Jia et al., “Hierarchical and distributed demand response control strategy for thermostatically controlled appliances in smart grid,” Journal of Modern Power Systems and Clean Energy, vol. 5, no. 1, pp. 30-42, Jan. 2017.

[7] J. L. Mathieu, S. Koch, and D. S. Callaway, “State estimation and control of electric loads to manage real-time energy imbalance,” IEEE Transactions on Power Systems, vol. 28, no. 1, pp. 430-440, Feb. 2013.

[8] Y. Yao and P. Zhang, “Coordinated control method for ancillary services of power system with participation of large-scale inverter air-conditioner,” Automation of Electric Power Systems, vol. 42, no. 22, pp. 183-198, Nov. 2018.

[9] M. Liu and Y. Shi, “Model predictive control for thermostatically controlled appliances providing balancing service,” IEEE Transactions on Control Systems Technology, vol. 24, no. 6, pp. 2082-2093, Nov. 2016.

[10] H. He, D. Wu, J. Lian et al., “Optimal coordination of building loads and energy storage for power grid and end user services,” IEEE Transactions on Smart Grid, vol. 9, no. 5, pp. 4335-4345, Sept. 2018.

[11] J. Lian, H. Ren, Y. Sun et al., “Transactive system Part II: analysis of two pilot transactive systems using foundational theory and metrics,” Pacific Northwest National Lab. [Online]. Available: https://www.pnnl.gov/main/publications/external/technical_reports/PNNL-27235Part2.pdf.

[12] M. Ji, P. Zhang, Y. Yao et al., “Distributed energy optimization method for microgrid using market-based control,” Automation of Electric Power Systems, vol. 41, no. 15, pp. 34-41, Aug. 2017.

[13] S. Vandal, B. Claessens, M. Hommelberg et al., “A scalable three-step approach for demand side management of plug-in hybrid vehicles,” IEEE Transactions on Smart Grid, vol. 4, no. 2, pp. 720-728, Jun. 2013.

[14] Y. Yao, Y. Cheng, and P. Zhang, “Market-based control of air-conditioning loads with switching constraints for providing ancillary services,” in Proceedings of IEEE PES General Meeting, Portland, USA, Aug. 2018, pp. 1-5.

[15] K. P. Schneider, J. C. Fuller, and D. P. Chassin, “Multi-state load models for distribution system analysis,” IEEE Transactions on Power Systems, vol. 26, no. 4, pp. 2425-2433, Nov. 2011.

[16] L. Zeng, Y. Sun, X. Zhou et al., “Markov chain model and transition matrix for demand response,” Proceedings of International Workshop on Large-scale Integration of Wind Power into Power Systems as well as on Transmission Networks for Offshore Wind Power Plants, Berlin, Germany, Nov. 2014, pp. 1-6.

[17] L. Zeng, Y. Sun, X. Zhou et al., “Markov chain model and transition matrix for demand response,” Proceedings of International Workshop on Large-scale Integration of Wind Power into Power Systems as well as on Transmission Networks for Offshore Wind Power Plants, Berlin, Germany, Nov. 2014, pp. 1-6.

[18] Y. Yao, Y. Cheng, and P. Zhang, “Market-based control of air-conditioning loads with switching constraints for providing ancillary services,” in Proceedings of IEEE PES General Meeting, Portland, USA, Aug. 2018, pp. 1-5.

[19] P. Schneider, J. C. Fuller, and D. P. Chassin, “Multi-state load models for distribution system analysis,” IEEE Transactions on Power Systems, vol. 26, no. 4, pp. 2425-2433, Nov. 2011.

[20] L. Zeng, Y. Sun, X. Zhou et al., “Markov chain model and transition matrix for demand response,” Proceedings of International Workshop on Large-scale Integration of Wind Power into Power Systems as well as on Transmission Networks for Offshore Wind Power Plants, Berlin, Germany, Nov. 2014, pp. 1-6.

[21] R. Ahsikari, M. Pipattanasomporn, and S. Rahman, “An algorithm for optimal management of aggregated HVAC power demand using smart thermostats,” Applied Energy, vol. 207, pp. 166-177, May 2018.

Yao Yao received her B.S. degree in electrical engineering from Shanghai Jiao Tong University, Shanghai, China, in 2012, where she is currently pursuing the M.S. degree. Her research interests include the control of thermostatically controlled loads and Energy Internet.

Peichao Zhang received his M.S. and Ph.D. degrees from Shanghai Jiao Tong University, Shanghai, China, in 2016 and 2018, respectively. He is an associate professor in Shanghai Jiao Tong University, Shanghai, China. His research interests include distributed energy resource integration, application of optimal control theory on active distribution grid operation and management, and transactive energy.