THE PARTONIC CONTENT OF $h_1(x)$ AND $h_2(x)$

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Abstract

A light-cone wavefunction interpretation is presented for the polarized distribution functions $h_1(x)$ and $h_2(x)$. All matrix elements for moments of these distributions are given in terms of overlap integrals between Fock state amplitudes of the target state. In a suitable spinor basis, $h_1(x)$ involves only diagonal matrix elements so can be interpreted as a density. Matrix elements of $h_2(x)$ connect Fock states differing by one gluon so that $h_2(x)$ has no simple interpretation as a density. Nevertheless, in the wavefunction decomposition, $h_2(x)$ is described through a compact set of elementary quark-gluon processes which are averaged over the target wavefunction.

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In a theoretical study of the polarized Drell-Yan process, Ralston and Soper [1] showed that the cross section for polarized $p - \bar{p}$ scattering can be expressed in terms of nine structure functions. From their analysis, they derived a formula in polarized Drell-Yan production between transversely polarized $pp$ pairs for the spin asymmetry defined as,

$$A_{\lambda A \lambda B} = \frac{\sigma(\lambda A, \lambda B) - \sigma(\lambda A, -\lambda B)}{\sigma(\lambda A, \lambda B) + \sigma(\lambda A, -\lambda B)}$$  \hspace{1cm} (1-1)$$

where $\lambda_i$ is the polarization of particle $i$ and $-\lambda_i$ means the opposite polarization for the same particle. Their relation was expressed in terms of the distribution $h_1(x)$, often referred to as the transversity distribution, to be,

$$A_{TT} = \sin^2 \theta \cos 2\phi \frac{\sum_a e_a^2 h_1^a(x)}{1 + \cos^2 \theta} \frac{h_1^a(y)}{\sum_a e_a^2 f_1^a(x)}$$ \hspace{1cm} (1-2)$$

where $\phi$ is the azimuthal angle and $x$ and $y$ are the longitudinal momentum fractions carried by the quarks in the two impinging hadrons. In following these lines, Jaffe and Ji [2] obtained the spin asymmetry for longitudinal-transverse collisions in terms of $h_2(x)$ to be,

$$A_{LT} = \frac{2 \sin 2\theta \cos \phi}{M} \frac{\sum_a e_a^2 [g_1^a(x) y g_1^a(y) + x h_1^a(x) h_1^a(y)]}{\sum_a e_a^2 f_1^a(x) f_1^a(y)}$$ \hspace{1cm} (1-3)$$

where,

$$h_L(x) \equiv h_1(x) + \frac{h_2(x)}{2}.$$ Since their introduction, for the most part these distributions have remained relatively unmentioned within the standard lines. However with the advent of a polarized proton-proton beam at Argonne and future hopes of higher energy polarized proton beams, these distributions are reaching a stage of tangibility. On the theoretical side, there has been little theoretical development of them. A significant amount of clarification was given to them by Jaffe and Ji [2] who illuminated their operator product tensor structure. To reach a closer physical connection, what remains is still to analyze the parton wavefunction interpretation for them. It is the purpose of this paper to do this.

In this paper we will study the light-cone wavefunction interpretation of $h_1(x)$ and $h_2(x)$. We will derive explicit formulas for the moments of these distributions in terms of parton correlations. For $h_1(x)$ such an undertaking is less significant since it has an interpretation as a quark-antiquark density in a suitable basis. On the other hand, $h_2(x)$ has no such simple interpretation. For this reason its physical meaning has remained somewhat obscure However, as we shall see, its parton content can be understood through only six types of matrix elements as opposed to two for $h_1(x)$.

The paper is organized as follows. In section two we review the operator-product-expansions for the moments of $h_1(x)$ and $h_2(x)$. In section three we discuss the light-cone wavefunction formalism in a manner suitable for our further developments. In section four we give our light-cone wavefunction analysis of $h_1(x)$ and $h_2(x)$. Finally we give some closing comments and discuss future directions in the conclusion. There is also an appendix which tabulates various light-cone spinors as a reference since we often found them to be useful in the analysis.

**Section 2**

In this section we will review the formal properties of $h_1$ and $h_2$ based on the work of Ralston and Soper [1] and Jaffe and Ji [2]. These distributions are defined through matrix elements between
the target proton state of a specific quark bilinear operator as,

\[
\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|\bar{\psi}(0)\sigma_{\mu\nu}i\gamma_5\psi(\lambda n)|PS\rangle \equiv 2 \left[ h_1(x)(S_{\perp\mu}p_\nu - S_{\nu\perp\mu})/M 
\right.
\]

\[
+ (h_1(x) + \frac{h_2(x)}{2}) (p_\mu n_\nu - p_\nu n_\mu)(S \cdot n) 
\]

\[
+ h_3(x) (S_{\perp\mu} n_\nu - S_{\nu\perp\mu}) \right] .
\]

(2 - 1)

where \( n^2 = n^+ = 0 \). By a suitable choice of external spin and momenta, the desired densities can be isolated as,

\[
h_1(x) = \frac{1}{\sqrt{2}p^+} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS_\perp|\bar{\psi}_+^\dagger(0)\gamma_5\psi_+(\lambda n)|PS_\perp\rangle .
\]

(2 - 2)

which is the twist two component and,

\[
(h_1(x) + \frac{h_2(x)}{2}) \equiv h_L(x) = \frac{1}{2M} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS_z|\bar{\psi}_+^\dagger(0)\gamma_0\gamma_5\psi_+(\lambda n) - \bar{\psi}_+^\dagger(0)\gamma_0\gamma_5\psi_-(\lambda n)|PS_z\rangle .
\]

(2 - 2b)

which contains a mixture of both twist two and three components. Recall that in the light-cone formalism of [3], the twist is directly related to the number of good and bad fields in the quark bilinear with twist two have no bad components and twist three having one.

To obtain the operator product expansion for the matrix elements above we must perform a covariant Taylor expansion of the quark field about the origin on the left hand side in (2-1) and compare like terms to the right hand side in order to extract the appropriate bilinear operator expansion terms. For \( h_1(x) \) the relevant operator bilinear one obtains is [4].

\[
\theta^{\mu_1 \mu_2 \cdots \mu_n} \equiv S_n\bar{\psi}_i\gamma_5\sigma^{\mu_1}iD^{\mu_2} \cdots iD^{\mu_n}\psi .
\]

(2 - 3)

Computing its matrix element between the proton state one gets,

\[
\langle PS|\theta^{\mu_1 \mu_2 \cdots \mu_n}|PS\rangle = 2a_nS_n(S^\nu P^{\mu_1} - S^{\mu_1} P^\nu)P^{\mu_2} \cdots P^{\mu_n}/M + \text{ terms involving } g^{\mu_i \mu_j} .
\]

(2 - 4)

where \( a_n \) is related to \( h_1(x) \) by the moments relation,

\[
\int_{-\infty}^{\infty} dx \ x^{n-1}h_1(x) = \int_0^1 dx \ x^{n-1}(h_1(x) - (-1)^{n-1}h_1(x)) = a_n .
\]

(2 - 5)

and \( h_1(x) \) is defined like (2-2) except for antiquarks.

For \( h_2(x) \) we consider the Jaffe-Ji equivalent to the Wilczek-Wadzuda decomposition of \( g_2(x) \) and write,

\[
h_L(x) = 2x \int_x^1 \frac{h_1(y)}{y^2} dy + \frac{m}{M} \left[ g_1(x) - x \int_0^x \frac{g_1(y)}{y^2} dy \right] + h_3^3(x) .
\]

(2 - 6)

where the moments of \( h_3^3_L(x) \) are,

\[
\mathcal{M}_n[h_3^3_L] = - \sum_{l=2}^{[(n+1)/2]} \left(1 - \frac{2l}{n+2}\right) b_{n,l} .
\]

(2 - 7)

with,

\[
b_{n,l} \equiv c_{n,n-l} - c_{n,l} .
\]

(2 - 8)
here $c_{n,l}$ is obtained from the matrix element,
\[
\langle PS | \theta_{l}^{\mu_1 \cdots \mu_n} | PS \rangle \equiv 2c_{nl}M S_n S_{\mu_1} P^{\mu_2} \cdots P^{\mu_n} \tag{2-9}
\]
with
\[
\theta_{l}^{\mu_1 \cdots \mu_n} = \frac{1}{2} S_n \bar{\psi} \sigma^{\alpha \mu_1} i \gamma_5 i D^{\mu_2} \cdots i g F_{\alpha}^{\mu_1} \cdots i D^{\mu_n} \psi - \text{traces}. \tag{2-10}
\]

Section 3

In this section we will review salient features of light-cone field theory that will be relevant for our calculation. The reader is urged to also examine reference [4] which supplements the discussion in this section. Our purpose here is to explicate necessary quantities used in our work for the convenience of the reader.

We imagine a particle which in its rest frame has its spin quantization axis defined along the $z$-direction. The particle is now observed from a frame moving at the idealized limit of the speed of light in the $-z$-direction. We now desire to describe the kinematic properties of this particle, be it quarks or gluons, in terms of light-cone coordinates of this boosted frame and in the light-cone gauge. The light-cone coordinates (no subscript) are defined with respect to the spacetime coordinates, $x_s$, as,
\[
x^+ = x^- = \frac{x_s^0 + x_s^3}{\sqrt{2}},
\]
\[
x^+ = x^- = \frac{x_s^0 + x_s^3}{\sqrt{2}}.
\]
where $x^+$ is taken as the light-cone time and the scalar product of two four-vectors $v_1$ and $v_2$ is $v_1 \cdot v_2 = v_1^+ v_2^- + v_1^- v_2^+ - v_1^+ \cdot v_2^.$

We introduce the fields and definitions needed in our work through the QCD Lagrangian,
\[
\mathcal{L} = -\frac{1}{2} Tr (F^{\mu \nu} F_{\mu \nu}) + \bar{\psi} (i D^{\mu} - m) \psi \tag{3-2}
\]
where the covariant derivative $i D^{\mu} = i \partial^{\mu} - g A^{\mu}$ and field strength $F^{\mu \nu} = \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu}$. The quark field $\psi$ is a color triplet spinor where only one flavor is written above. The gluon gauge field $A^{\mu} = \sum_a A_a^{\mu} T^a$ is a traceless 3x3 color matrix. The independent dynamical fields at a given light-cone time, say $x^+=0$, are $\psi_+ = \Lambda_+ \psi$ and $A_1$ (i=1,2), where the projection operators $\Lambda_{\pm} \equiv \frac{1}{2} (\gamma^0 \gamma^\pm + \gamma^\pm)$ with $\gamma^\pm \equiv \gamma^0 \pm \gamma^3$ and $\partial^\pm = \partial^0 = -\partial^3$. From the equations of motion, the remaining fields can be obtained. With $\psi = \psi_+ + \psi_-$, one can express $\psi_-$ in terms of $\psi_+$ as,
\[
\psi_- \equiv \Lambda_- \psi = \frac{1}{i \partial^+} [i \bar{\psi} (i \partial^+ - g A^{\mu}) \psi] = \bar{\psi}_- - \frac{1}{i \partial^+} g A_1 \cdot \hat{\alpha} \psi_+. \tag{3-3}
\]
Turning to the gauge field in the light-cone gauge one has,
\[
A^+ = 0 \tag{3-4a}
\]
\[
A^- = \frac{2}{i \partial^+} i \bar{\psi}_- \cdot \hat{\alpha} + \frac{2g}{(\partial^+)^2} \{ [i \partial^+ A_1^+, A_1^+] + 2 \psi_+ T^a \psi_+ T^a \} \tag{3-4b}
\]
with $\beta = \gamma^0$ and $\alpha_\perp = \gamma^0 \gamma_\perp$. Upon quantization, the independent dynamical coordinates can be decomposed into creation and annihilation operators at a given light-cone time, say $x^+ = 0$ again, as,

$$\psi_+(x) = \int_{k^+ > 0} \frac{dk^+ d^2k}{k^+ 16\pi^3} \sum_{\lambda} \left\{ b(k^+, \vec{k}, \lambda) u_+(k^+, \vec{k}, \lambda) e^{-ik\cdot x} + a^\dagger(k^+, \vec{k}, \lambda) v_+(k^+, \vec{k}, \lambda) e^{ik\cdot x} \right\}_{\tau = x^+ = 0}$$

$$A^\dagger_\perp(x) = \int_{k^+ > 0} \frac{dk^+ d^2k}{k^+ 16\pi^3} \sum_{\lambda} \left\{ a(k^+, \vec{k}, \lambda) e^{i\lambda} e^{-ik\cdot x} + c.c. \right\}_{\tau = x^+ = 0} ,$$

with commutation relations,

$$\{b(k^+, \vec{k}, \lambda), b^\dagger(p^+, \vec{p}, \lambda')\} = \{d(k^+, \vec{k}_\perp, \lambda), d^\dagger(p^+, \vec{p}_\perp, \lambda')\}$$

$$= [a(k^+, \vec{k}_\perp, \lambda), a^\dagger(p^+, \vec{p}_\perp, \lambda)]$$

$$= 16\pi^3 k^+ \delta(k^+ - p^+) \delta(\vec{k}_\perp - \vec{p}_\perp) \delta\lambda\lambda'$$

$$\{b, b\} = \{d, d\} = \cdots = 0 .$$

(3 - 6a)  \hspace{1cm} (3 - 6b)

In calculation, it is often more convenient to treat the fermion field not separately in terms of its plus and minus components but rather to write $\psi_-(x)$ as,

$$\psi_-(x) = \Phi_-(x) - g \frac{i\partial^\perp}{\partial \partial^\perp} (\alpha_\perp A_\perp \Psi_\perp) ,$$

(3 - 7)

where

$$\Phi_-(x) = \left( \frac{1}{i\partial^\perp} \right) \left[ (-\imath\alpha_\perp \partial_\perp + \beta m) \psi_+ \right] .$$

(3 - 8)

Next defining $\psi(x) = \psi_-(x) + \psi_+(x)$, its expansion at $x^+ = 0$ is,

$$\psi(x) = \int_{k^+ > 0} \frac{dk^+ d^2k}{16\pi^3 k^+} \sum_{\lambda = \pm 1/2} [b(k^+, k_\perp, \lambda) u(k^+, k_\perp, \lambda) e^{-ik\cdot x} + a^\dagger(k^+, k_\perp, \lambda) v(k^+, k_\perp, \lambda) e^{ik\cdot x}]_{x^+ = 0}$$

(3 - 9)

Here $u, v$ are light-cone spinors which are explicitly given in the Appendix, Eqs (A-1).

Having defined the field theory, let us now turn to the Fock space representation of the wavefunctions which we will sometimes refer to as hadronic wavefunctions. For our purpose, we can consider the wavefunction as a expansion in the Fock space at some renormalization scale $\mu$. We will not be addressing issues regarding zero modes or ground state properties nor matters of renormalization. Our concern is only to examine the Fock space correlations contained in the distributions $h_1(x)$ and $h_2(x)$.

The light-cone hadron wavefunction can be thought of as an infinite dimensional column vector representation for Fock space. The Fock space of interest here is of the many body quark and gluon system. Each Fock space component is a product of quark and gluon quanta with specified quantum numbers. Premultiplying this component is a complex valued factor which specifies the probability amplitude to find that state in the normalized hadronic wavefunction. A qualitative remark about the light-cone wavefunction formalism is that scattering processes have static versus dynamic description in that it is more natural here to imagine constructing the bound state wavefunction...
first, with in particular its high energy tail, and then coupling it to a scattering process. This contrasts the spacetime picture in which the "building" of the wavefunction is often incorporated within the Feynman diagram representing the scattering process. A useful example is the Alterilli Parisi evolution equations for deep-inelastic-scattering which are typically interpreted as a radiative bremsstrahlung by the parton before scattering. Alternatively, if one thinks of the parton as part of a bound state system, which it is, this same process of radiation is actually generating the high energy Fock space components of the wavefunction. Mathematically, of course, both methods are equivalent, however each has its advantages in analyzing physical processes. The light-cone wavefunction approach provides a useful language for the analysis of partonic correlations in hadrons and so extends naturally for use in modeling low energy hadronic structure.

Turning to the representation of the light-cone wavefunction we write it as,

\[ |PS\rangle = \sum_{n=0}^{\infty} \sum |f_1,\ldots,f_n\rangle \sum |\lambda_1,\ldots,\lambda_n\rangle \int [dx]_n [d^2 k_\perp]_n \psi_S(k_\perp, x, \lambda) a^\dagger(f_1, p_1^+, p_{\perp 1}, \lambda_1) \cdots a^\dagger(f_n, p_n^+, p_{\perp n}, \lambda_n)|\Omega\rangle \]

where

\[ [dx]_n = \prod_{i=1}^{n} dx_i \delta \left( \sum_{i=1}^{n} x_i - 1 \right) \],

\[ [d^2 k_\perp]_n = \prod_{i=1}^{n} \frac{d^2 k_{\perp i}}{16\pi^3} \delta(2) \sum_{i=1}^{n} k_{\perp i} \].

Section 4

Up to now, everything we have said is well known. Now we will apply this methodology to compute light-cone wavefunction matrix elements of \( h_1(x) \) and \( h_2(x) \). For any non-singlet SU(3)-flavor channel, one must insert the appropriate Gell-Mann \( \lambda \)-matrix between the fermion fields. In the singlet-channel, the quark and gluon fields mix under renormalization. This requires further consideration in order to make a partonic interpretation. We will not pursue this matter in this work. This section is divided into two parts, In the first part, we will examine \( h_1(x) \), which entails no complications since in an appropriate basis its matrix elements are all diagonal. In the second part we will examine \( h_2(x) \), for which a detailed analysis is required of the relevant fermion bilinear operator and the diagonal and nondiagonal matrix elements it yields. Nevertheless, once the dust settles, we find that the final answer is reasonably simple.

For a general operator, \( \theta^{[\mu]} \), we are interested in its matrix element between the hadron wavefunction. In general this matrix element decomposes into a sum of terms, each one connecting a particular bra Fock space state to one of the ket states. Our task is to evaluate all the individual contributions arising from the Fock space decomposition for the matrix elements of (2-2a) and (2-2b). It is convenient to factor each of the individual terms into two pieces, which we refer to as the momentum and spin factors. The momentum factor, \( f_q(q, q'; \{g\}_k, \{g\}_k') \), contains the momentum dependent contributions of the covariant derivatives when acting on the two fermions, \( q \) and \( q' \), and \( n-2 \) gluons, \( g \), for twist \( n \) that are being connected by the operator. The spin factor \( O^s(q, q'; \{g\}_k, \{g\}_k') \), on the other hand, contains the contribution from the fermion bilinear operator and gluon polarization operators when acting on the connected states. It depends on the two momentum and spin states of the connected fermions and the momentum and polarization states of the connected gluons.
With this decomposition, we can write the general form of the matrix element as,
\[
\langle PS|\theta^{[\mu]|PS\rangle = \sum_{q \in \{f,f\}_n} \sum_{\{g\}_k \in \{g\}_m} \sum_{\{g'\}_k' \in \{g'\}_m'} \int [dx_q d^2 k_{\perp q} dx_{q'} d^2 k_{\perp q'} (d^3 k_q)_k (d^3 k_{q'})_{k'} (d^3 k_{q''})_{k''}]
\]
\[
f_p(q,q',\{g\}_k,\{g\}_k') \Psi^m_{s\mu} (x_{q_1}, k_{\perp q_1}, \lambda_{q_1}, \ldots, x_{q_n}, k_{\perp q_n}, \lambda_{q_n}; \{g\}_m)
\]
\[
O_s(q,q',\{g\}_k,\{g\}_k') \Psi^{m'}_{s\mu'} (x_{q'_{1}}, k_{\perp q'_{1}}, \lambda_{q'_{1}}, \ldots, x_{q'_n}, k_{\perp q'_n}, \lambda_{q'_n}; \{g'\}_m')
\]
\]

where $\theta^{[\mu]}$ in general is a quark bilinear operator with tensor indices $[\mu]$. Above, the double square brackets around all the momentum differentials are to imply insertion of a single $\delta$-function factor for each the bra and ket states of the form,
\[
\delta \left[ \sum_{i=1}^{n+m} x_i - 1 \right] \delta^{(2)} \left[ \sum_{i=1}^{n+m} k_{\perp i} \right] \delta \left[ \sum_{i=1}^{n'+m'} x'_i - 1 \right] \delta^{(2)} \left[ \sum_{i=1}^{n'+m'} k'_{\perp i} \right]
\]

Also for notational brevity,
\[
(d^3 k)_n \equiv \prod_{i=1}^{n} \frac{dx_i d^2 k_{\perp i}}{16 \pi^3},
\]

$h_1(x)$

To compute the coefficient $a_n$ in (2-4) for $h_1(x)$, it is sufficient to evaluate the operator $\theta^{[\mu]}$, with all the symmetrized indices $\mu_1\mu_2-\mu_n$ carrying Lorentz index + and the remain index $\nu$ carrying a transverse component which we take as x. For the proton state, it is convenient to observe it in its center of mass frame where $P_{cm}^+=P_{cm}^x=\frac{M}{2}$. Boost invariance in the light-cone allows us to make such a choice. For the polarization state of the proton, consistent with our above choice we take it polarized in the +x-direction so that $S=(0,0,M,0)$. The coefficient $a_n$ can then be obtained from
\[
M_{+x} \equiv \langle PS_{+x} | -\bar{\psi} \gamma^x \gamma^+ \gamma^n (i\partial^+)^{n-1} \psi | PS_{+x} \rangle = 2a_n S^x (P^+)^n / M \tag{4-2}
\]

Defining as in [3],
\[
Q^i_{\pm} \equiv \frac{1}{2}(1 \mp \gamma^5 \gamma^i) \tag{4-3}
\]

with i=1,2 being the transverse coordinates x and y respectively, we find,
\[
Q^i_{\pm} u^i_+(k, \lambda = \pm) = u^i_+(k, \lambda = \pm)
\]
\[
Q^i_{\pm} v^i_+(k, \lambda = \pm) = v^i_+(k, \lambda = \pm)
\]
\[
Q^i_{\pm} u^i_+(k, \lambda = \mp) = Q^i_{\mp} v^i_+(k, \lambda = \mp) = 0 \tag{4-4}
\]
where the "transverse spinors" $u^i_{\pm}$, $v^i_{\pm}$ are given in the appendix eqs (A-2). Reexpressing (4-2) in terms of (4-3) we obtain,

$$2a_n(P^+)^n = \sum_{m=qq\ldots\lambda_m} \sum_{s,s'\epsilon(f)} \sum_{p=0}^\infty \left\{ \sum_{s,s'\epsilon(f)} \int [d^3k_{(f,f)}] [d^3k_g] (m+p)(P^+)^{n-1} \right. $$

$$\times \psi_{S,s}^m(x_1, k_\perp, \lambda_1, \ldots, x_{s'}, k_{\perp s'}, \lambda_{s'} \ldots x_m, k_{\perp m}, \lambda_m; \{g\}_p) $$

$$\left. \times \left[ x_s^m \frac{\bar{u}^1_\perp(\lambda')}{\sqrt{x_s'}} (Q_1^+ - Q_1^-) \left( \frac{u^1_\perp(\lambda)}{\sqrt{x_s}} \right) \psi_{S,s'}^m(x_1, k_{\perp 1}, \lambda_1, \ldots, x_s, k_{\perp s}, \lambda_s \ldots x_m, k_{\perp m}, \lambda_m; \{g\}_p) \right. $$

$$+ \sum_{s,s'\epsilon(f)} \int [d^3k_{(f,f)}] [d^3k_g] (m+p)(P^+)^{-n-1}(-1)^n$$

$$\times \psi_{S,s}^m(x_1, k_{\perp 1}, \lambda_1, \ldots, x_{s'}, k_{\perp s'}, \lambda_{s'} \ldots x_m, k_{\perp m}, \lambda_m; \{g\}_p) $$

$$\left. \times \left[ x_s^m \frac{\bar{v}^1_\perp(\lambda')}{\sqrt{x_s'}} (Q_1^+ - Q_1^-) \left( \frac{v^1_\perp(\lambda)}{\sqrt{x_s}} \right) \psi_{S,s'}^m(x_1, k_{\perp 1}, \lambda_1, \ldots, x_s, k_{\perp s}, \lambda_s \ldots x_m, k_{\perp m}, \lambda_m) \right] \right\}$$

(4-5)

where the sum on m is to imply both quarks and antiquarks and one factor of -1 in the second term is due to the ordering of antiquarks. Recall also that a Fock space amplitude coefficient for n-quantum, $\psi_n(\{k\})$, has mass dimensions $-(n-1)$. One can check by inspection above that $a_n$ is dimensionless. All the contributions to the matrix element for $a_n$ in this basis, where the quark spins are along the x-direction, are now between the same Fock space state with no mixing between states. In terms of the general form (3-10) we can identify $f_p(q, q'; \{g\}_k, \{g\}_{k'})$ as,

$$f_p(q, q'; \{g\}_k, \{g\}_{k'}) = (P^+_x)^n = \left( \frac{M}{\sqrt{x^2}} x_q \right)^n$$

(4-6)

To calculate the spin function $O_s(q, q'; \{g\}_k, \{g\}_{k'})$, we evaluate,

$$\frac{\bar{u}^1_\perp(\lambda')}{\sqrt{x_s'}} (Q_1^+ - Q_1^-) \left( \frac{u^1_\perp(\lambda)}{\sqrt{x_s}} \right)$$

(4-7a)

for quarks, and

$$(-1)^n \frac{\bar{v}^1_\perp(\lambda')}{\sqrt{x_s'}} (Q_1^+ - Q_1^-) \left( \frac{v^1_\perp(\lambda)}{\sqrt{x_s}} \right)$$

(4-7b)

for antiquarks, which leads to the expectation values,

$$P^+ \langle \delta_{\lambda_{s'}^\perp \lambda_{s'}^{\perp}} - \delta_{\lambda_{s'}^\perp \lambda_{s'}^{\perp}} \rangle$$

(4-8a)

and

$$P^+ \langle \delta_{\lambda_{s'}^\perp \lambda_{s'}^{\perp}} - \delta_{\lambda_{s'}^\perp \lambda_{s'}^{\perp}} \rangle$$

(4-8b)

respectively. We have now given the explicit form of all matrix elements of $h_1(x)$. In regards to our earlier discussion, in this basis we can readily identify $h_1(x)$ as a density since the nonvanishing contributions to the matrix elements needed to compute its moments do not mix Fock space components. Explicitly we can express the moments of $h_1(x)$ as,

$$\int_0^1 dx x^{n-1} (h_1(x) - (-1)^{n-1} h_1(x)) = a_n = \int_0^1 x^{(n-2)} [T_q(x) - (-1)^{(n-1)} T_q(x)] \quad n = 1, 2, \ldots$$

(4-9)
where

\[
\delta^{(2)}(P_\perp - P_\perp')T_{q(\bar{q})}(x) = \int \frac{d^2k_\perp}{16\pi^2} \langle PS_\perp | b^+_q(xP, k_\perp, \uparrow) | b_q(\bar{x}P, k_\perp, \uparrow) \rangle
\]

\[
- \langle b^+_q(xP, k_\perp, \downarrow) | b_q(\bar{x}P, k_\perp, \downarrow) | P'S_\perp \rangle
\]

(4 - 10a)

which gives,

\[
T_{q(\bar{q})}(x) = \sum_{m=0}^{\infty} \sum_{n=3}^{\infty} \int \int \Sigma |[\Psi_{nm}^m(x_1, k_{1,\perp}, \lambda_1, \ldots, x_{n_q}, k_{n_q,\perp}, \lambda_{n_q}, \ldots, x_{n}, k_{n,\perp}, \lambda_n; \{g\})|^2|
\]

\[
(4 - 10b)
\]

where,

\[
\int \Sigma = \sum_{\{f_1, \ldots, f_{n_q}, f_{n_q+1}, f_{n_q+1}\}} \sum_{\{\lambda_1, \ldots, \lambda_n\}} \sum_{\{g\}} \int [d^3k_1(f_1) d^3k_2]_{n+m}
\]

is the sum over all Fock states with \(n\)-fermions where at least one quark (antiquark) carries longitudinal momentum fraction \(x\). In eqs. (4-9) for \(n=1\), eqs. (4-10) gives the light-cone wavefunction representation of the sum rule that was defined as the tensor charge in \([2]^*\).

\(h_2(x)\)

We will now extract the coefficients \(c_{n,l}\) from the matrix element (2-9) for \(h_2(x)\). As a suitable choice for the proton state we take for the momenta \(P = (P^+, M/2P^+, 0, 0)\) with polarization along the +z-direction so that \(S = (P^+, 0, 0, 0)\). From (2-9) we see that \(c_{n,l}\) can be identified from

\[
c_{n,l} = \frac{\langle PS|\theta|^+|PS \rangle}{2M(P^+)^n}
\]

(4 - 11)

In terms of the general form (4-1), the spin and momentum terms \(O_s(q, q'; \{g\}_k, \{g\}_k')\) and \(f_p(q, q', \{g\}_k, \{g\}_k')\) that are nonvanishing are given in table 1. The nonvanishing Fock space components that contribute are given in table 2-4 and will be further discussed below. Each entry in these tables corresponds to the spin and polarization state shown as well as the one where they are all flipped. The diagrams in the tables indicate how the parton distribution operator contracts with the partons in the wavefunction. The lines to the left (right) contract with the bra (ket) states. Note that Fock state amplitude coefficients that differ by one gluon also differ dimensionally by one mass unit. In the overlap integrals for matrix elements of \(h_2(x)\) this difference is balanced on the left-hand side of eqs. (2-9) by the factor \(M\).

All nonvanishing matrix elements connect Fock states which differ by one gluon. Furthermore, in tables 2 and 3, the connected Fock states have the same number of \(q\) and \(\bar{q}\) with only one fermion in the two respective states differing in momenta. In Table 4, the connected Fock states differ by a single additional \(q\bar{q}\) pair in one state relative to the other. Among the eight \(q\bar{q}g\)-operator combinations from (4-11) the two missing in tables 2-4 are the case where in table 4 one swaps places
between $\epsilon$ and $\epsilon^*$. These two cases lead to contractions which were denoted as "disconnected" in \cite{4}. As argued by these authors, these cases can be ignored as the states so formed after contraction decouple.

Tables (2-4) give the full partonic interpretation of $h_L(x)$ and thus also $h_2(x)$. To further clarify the tensor structure we note the identity,

$$
\bar{\psi}\sigma^j i\gamma^5\vec{V}_j^\perp \psi = \bar{\psi}\gamma^+ [\vec{V}_j^\perp \times \vec{\gamma}_j^\perp + i\gamma^5 \vec{\gamma}_j^\perp \cdot \vec{V}_j^\perp] \psi
$$

where $V_j^\perp$ is an arbitrary vector with nonvanishing components only in the xy-plane. We can therefore express the matrix elements as,

$$
c_{n,l} = \frac{1}{4M(P+)^n} < PS | \bar{\psi} \gamma^+(i\partial^+)l^{-2} \{ \partial^+ \vec{A}^\perp \times \vec{\gamma}_j^\perp - i\gamma^5 \vec{\gamma}_j^\perp \cdot \partial^+ \vec{A}^\perp \} \psi | PS >
$$

(4 – 12)

Conclusion

Our analysis has given the Fock space light-cone wavefunction decomposition of $h_1(x)$ and $h_2(x)$. This unifies the interpretation of these quantities with the more common structure functions $f_1(x)$ and $g_1(x)$ as well as $g_2(x)$ from the work in \cite{4}. It is more difficult to describe $h_2(x)$ compared to $f_1(x), g_1(x)$ or $h_1(x)$, as it is not readily interpreted as a density. However, we have see that its interpretation has a natural explanation within the light-cone wavefunction formalism.

Our analysis shows that like $g_2(x)$, $h_2(x)$ contains information about the correlation between Fock space components differing by one gluon and one quark-antiquark pair. Furthermore, it has a nonperturbative dependence on the QCD coupling constant. By this we mean, the coupling constant that enters in the matrix elements tabulated in tables 2-4 is evaluated at its low energy scale. Experimental information combined with perturbative QCD scaling formulas, would therefore allow one to deduce nonperturbative information about low energy quark-glue correlations. In the past, low energy constituent quark model descriptions of the hadron wavefunction have gained considerable insight from experimental information on $f_1(x)$. Analogously to gain empirical insight about gluon correlations which in turn could be used for guidance in forming low energy models, experimental data on $g_2(x)$ and $h_2(x)$ would provide helpful information.

In this paper we have presented the formal building blocks for the analysis of $h_1(x)$ and $h_2(x)$ in the light-cone wavefunction formalism. These results have been of a technical nature, but are a necessary first step in understanding the partonic content of these distributions. We have not addressed the more pragmatic question of how to compute them from models. Let us therefore briefly turn to this issue. For $h_1(x)$ it would be interesting to test SU(6)-breaking quark model wavefunctions such as those in \cite{5} and their extensions along the lines of \cite{4}. Turning to $h_2(x)$, one possibility in a low-energy model would be to expand the constituent quark model hilbert space by the inclusion of just one additional gluon component. Such an approach has already be tried by Lipkin \cite{7} with some success in explaining the nucleon longitudinal spin content. Further insight on modeling "valence gluons" could be gained from the flux-tube model of Isgur and Paton \cite{8}.

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* The expressions for the quark densities in section 3 of [2] are dimensionally inconsistent but can be easily fixed by insertion of $\delta$-function factors similar to what is done in eq. (4-10a).

Appendix

In all our work, we used the Dirac representation of the $\gamma$-matrices. It is often useful in calculations of higher twist distributions to have readily available, explicit expressions for the spinors. For convenience, we have reproduced below the spinors relevant to our work. The light-cone Dirac spinors are,

$$u_{\uparrow}(k) = \frac{1}{\sqrt{2k^+}} \begin{pmatrix} k^+ + m \\ k_x + ik_y \\ k^+ - m \\ k_x + ik_y \end{pmatrix} \quad (A - 1a)$$

$$u_{\downarrow}(k) = \frac{1}{\sqrt{2k^+}} \begin{pmatrix} -k_x + ik_y \\ k^+ + m \\ k_x - ik_y \\ -k^+ + m \end{pmatrix} \quad (A - 1b)$$

$$u_{\uparrow}(k) = \frac{1}{\sqrt{2k^+}} \begin{pmatrix} -k_x + ik_y \\ k^+ - m \\ k_x - ik_y \\ -k^+ - m \end{pmatrix} \quad (A - 1c)$$

$$u_{\downarrow}(k) = \frac{1}{\sqrt{2k^+}} \begin{pmatrix} k^+ - m \\ k_x + ik_y \\ k^+ + m \\ k_x + ik_y \end{pmatrix} \quad (A - 1d)$$

For the light-cone spinors in the eigenstate basis of $Q_i^{\pm}$ we have, The transverse light cone spinors, $u^i_{\uparrow}(k, \lambda), v^i_{\downarrow}(k, \lambda)$ which satisfy (4-4) are, where below $u_{\uparrow}(k, \lambda), v_{\downarrow}(k, \lambda)$ are the good components
of the light-cone spinors,

\[
\begin{align*}
\mathbf{u}_+^x(k, \lambda = +) &= \mathbf{u}_+(k, \lambda = \uparrow) - \mathbf{u}_+(k, \lambda = \downarrow) \\
&= \sqrt{\frac{k^+}{2}} \begin{pmatrix} 1 \\ -1 \\ 1 \\ 1 \end{pmatrix} \\
(A - 2a) \\
\mathbf{v}_+^x(k, \lambda = +) &= \mathbf{v}_+(k, \lambda = \downarrow) - \mathbf{v}_+(k, \lambda = \uparrow) \\
&= \sqrt{\frac{k^+}{2}} \begin{pmatrix} 1 \\ -1 \\ 1 \\ 1 \end{pmatrix} \\
(A - 2b) \\
\mathbf{u}_+^z(k, \lambda = -) &= \mathbf{u}_+(k, \lambda = \uparrow) - \mathbf{u}_+(k, \lambda = \downarrow) \\
&= \sqrt{\frac{k^+}{2}} \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \\
(A - 2c) \\
\mathbf{v}_+^z(k, \lambda = +) &= \mathbf{v}_+(k, \lambda = \downarrow) - \mathbf{v}_+(k, \lambda = \uparrow) \\
&= \sqrt{\frac{k^+}{2}} \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \\
(A - 2d) \\
\mathbf{u}_+^y(k, \lambda = +) &= \mathbf{u}_+(k, \lambda = +) - i\mathbf{u}_+(k, \lambda = \downarrow) \\
&= \sqrt{\frac{k^+}{2}} \begin{pmatrix} 1 \\ i \\ 1 \\ -i \end{pmatrix} \\
(A - 2e) \\
\mathbf{v}_+^y(k, \lambda = +) &= \mathbf{v}_+(k, \lambda = \downarrow) - i\mathbf{v}_+(k, \lambda = \uparrow) \\
&= \sqrt{\frac{k^+}{2}} \begin{pmatrix} 1 \\ i \\ 1 \\ -i \end{pmatrix}
\end{align*}
\]
\[ u_\lambda^\nu (k, \lambda = -) = u_\lambda (k, \lambda = +) - i u_\lambda (k, \lambda = \downarrow) \]
\[ = \sqrt{\frac{k^+}{2}} \begin{pmatrix} 1 \\ -i \\ 1 \\ i \end{pmatrix} \]

\[ v_\lambda^\nu (k, \lambda = -) = v_\lambda (k, \lambda = \downarrow) - iv_\lambda (k, \lambda = \uparrow) \]
\[ = \sqrt{\frac{k^+}{2}} \begin{pmatrix} 1 \\ -i \\ 1 \\ i \end{pmatrix} \]
Table Captions

Table 1: Momentum and spin terms arising in the Fock space matrix elements of $h_2(x)$.

Table 2: Evaluation of spin factor for the gluon annihilation matrix elements to $h_2(x)$. Table gives expressions for the spin states shown and the case when all spins are flipped.

Table 3: Evaluation of spin factor for the gluon creation matrix elements to $h_2(x)$. Table gives expressions for the spin states shown and the case when all spins are flipped.

Table 4: Evaluation of spin factor for the quark pair creation and annihilation matrix elements to $h_2(x)$. Table gives expressions for the spin states shown and the case when all spins are flipped.
Table 1

| $O^{n,l}(q, q', s, s', \{g\})$ | $f_{p}^{n,l}(q, q', \{g\})/M(P^{+})^{n-1}$ | Process |
|--------------------------------|----------------------------------|---------|
| $\frac{u(k_{g})}{\sqrt{x_{q}}} + \frac{5g}{(k_{g})} \frac{u(k_{g}=k_{q}-k_{g})}{\sqrt{x_{q}'}}$ | $(x_{q})^{l-2}x_{g}(x_{q} - x_{g})^{n-1}$ | $\overline{u}$ $(k_{q}) \sqrt{x_{q}} \gamma + \gamma_{5}^{5} \sqrt{x_{q}}$ |
| $\frac{u(k_{g})}{\sqrt{x_{q}}} + \frac{5g}{(k_{g})} \frac{v(k_{g}=k_{q}-k_{g})}{\sqrt{x_{q}'}}$ | $(x_{q})^{l-2}x_{g}(x_{q} - x_{g})^{n-1}$ | $\overline{u}$ $(k_{q}) \sqrt{x_{q}} \gamma + \gamma_{5}^{5} \sqrt{x_{q}}$ |
| $\frac{u(k_{g})}{\sqrt{x_{q}}} + \frac{5g}{(k_{g})} \frac{v(k_{g}=k_{q}+k_{g})}{\sqrt{x_{q}'}}$ | $(x_{q})^{l-2}x_{g}(x_{q} + x_{g})^{n-1}$ | $\overline{u}$ $(k_{q}) \sqrt{x_{q}} \gamma + \gamma_{5}^{5} \sqrt{x_{q}}$ |
| $\frac{v(k_{g})}{\sqrt{x_{q}}} + \frac{5g}{(k_{g})} \frac{u(k_{g}=k_{q}+k_{g})}{\sqrt{x_{q}'}}$ | $(x_{q})^{l-2}x_{g}(x_{q} + x_{g})^{n-1}$ | $\overline{u}$ $(k_{q}) \sqrt{x_{q}} \gamma + \gamma_{5}^{5} \sqrt{x_{q}}$ |
| $\frac{v(k_{g})}{\sqrt{x_{q}}} + \frac{5g}{(k_{g})} \frac{v(k_{g}=k_{q}-k_{g})}{\sqrt{x_{q}'}}$ | $(x_{q})^{l-2}x_{g}(x_{q} - x_{g})^{n-1}$ | $\overline{v}$ $(k_{q}) \sqrt{x_{q}} \gamma + \gamma_{5}^{5} \sqrt{x_{q}}$ |
| $\frac{v(k_{g})}{\sqrt{x_{q}}} + \frac{5g}{(k_{g})} \frac{u(k_{g}=k_{q}-k_{g})}{\sqrt{x_{q}'}}$ | $(x_{q})^{l-2}x_{g}(x_{g} - x_{q})^{n-1}$ | $\overline{v}$ $(k_{q}) \sqrt{x_{q}} \gamma + \gamma_{5}^{5} \sqrt{x_{q}}$ |
| $\frac{v(k_{g})}{\sqrt{x_{q}}} + \frac{5g}{(k_{g})} \frac{v(k_{g}=k_{q}-k_{g})}{\sqrt{x_{q}'}}$ | $(x_{q})^{l-2}x_{g}(x_{g} - x_{q})^{n-1}$ | $\overline{v}$ $(k_{q}) \sqrt{x_{q}} \gamma + \gamma_{5}^{5} \sqrt{x_{q}}$ |
### Table 2

| Process | \( k_q' \) | \( k_q \) | \( k_q' \) | \( k_q \) |
|---------|----------------|----------------|----------------|----------------|
| \( \frac{1}{p} u_{\lambda q}(k_q') \gamma + \gamma \frac{5}{\sqrt{x_q q'}} \frac{u_{\lambda q}(k_q)}{\sqrt{x_q q'}} \) | \( \frac{1}{p} v_{\lambda q}(k_q) \gamma + \gamma \frac{5}{\sqrt{x_q q'}} \frac{v_{\lambda q}(k_q)}{\sqrt{x_q q'}} \) |
| \( \uparrow \downarrow +1 \) | \( -2 \sqrt{\frac{2}{x_q' - x_q}} \) | \( 0 \) |
| \( \uparrow \downarrow -1 \) | \( 0 \) | \( -2 \sqrt{\frac{2}{x_q' - x_q}} \) |

### Table 3

| Process | \( k_q' \) | \( k_q \) | \( k_q' \) | \( k_q \) |
|---------|----------------|----------------|----------------|----------------|
| \( \frac{1}{p} u_{\lambda q}(k_q') \gamma + \gamma \frac{5}{\sqrt{x_q q'}} \frac{u_{\lambda q}(k_q)}{\sqrt{x_q q'}} \) | \( \frac{1}{p} v_{\lambda q}(k_q) \gamma + \gamma \frac{5}{\sqrt{x_q q'}} \frac{v_{\lambda q}(k_q)}{\sqrt{x_q q'}} \) |
| \( \uparrow \downarrow -1 \) | \( -2 \sqrt{\frac{2}{x_q' - x_q}} \) | \( 0 \) |
| \( \uparrow \downarrow +1 \) | \( 0 \) | \( -2 \sqrt{\frac{2}{x_q' - x_q}} \) |

### Table 4

| Process | \( k_q' \) | \( k_q \) | \( k_q' \) | \( k_q \) |
|---------|----------------|----------------|----------------|----------------|
| \( \frac{1}{p} u_{\lambda q}(k_q') \gamma + \gamma \frac{5}{\sqrt{x_q q'}} \frac{u_{\lambda q}(k_q)}{\sqrt{x_q q'}} \) | \( \frac{1}{p} v_{\lambda q}(k_q) \gamma + \gamma \frac{5}{\sqrt{x_q q'}} \frac{v_{\lambda q}(k_q)}{\sqrt{x_q q'}} \) |
| \( \uparrow \downarrow -1 \) | \( -2 \sqrt{\frac{2}{x_q' - x_q}} \) | \( -2 \sqrt{\frac{2}{x_q' - x_q}} \) |