An auto-test method based on acoustic theory of judging inception cavitation of runner blade of model Francis-turbine

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Abstract. The vibration and noise of large Francis-turbine units tends to increase, the efficiency decreases and the units operate unstably if the units operate under the cavitation condition. Hence it must be eradicated completely that the units run under the cavitation condition. Nowadays, the judgment of inception cavitation of model Francis-turbine runner blades remains stagnant at the stage of artificial observation so that the results lacks of objectivity and consistency. An auto-test method based on acoustics theory and the principle of cavitation is presented. When cavitation happens, the acoustic energy radiated by bubbles while collapsing is different at different stages of cavitation, for the size and number of bubbles are different. And therefore the distribution of cavitation noise energy in the frequency domain varies along with the severe degree of cavitation, according to which the inception cavitation can be determined exactly by acquiring the acoustics signals in the water with hydrophone mounted on the draft tube cone and then abstracting and analyzing the characteristics parameters of cavitation noise in the frequency domain when the model Francis-turbine operates. The rationality and accuracy have been proved by model tests, and it is significant to determine the inception cavition of model Francis-turbine runner blades by automatic way instead of artificial way.

1. Introduction
Nowadays, a requirement at the higher level for the stability of the units has been made while the hydroelectric facility tends to develop with the feature of upsizing and complication. Along with the gradual large-sized trends of the Francis turbine units, the capacity of the single unit correspondently increases, and consequently, there emerges a great batch of Francis turbine units with 500~700MW single unit capacity, among them, the Three Gorges is representative. Recently, the Francis turbine units with 1000 MW single unit capacity will successively be applied in Baihetan hydro power plants etc. Cavitation, a kind of inevitable sabotage in the operation process of Francis turbine, aggrandizes noise, causes vibration and eventually destroy the operation stability and results in the declining of the efficiency of large Francis turbine units, In order to guarantee the running safety and stability from being destroyed by the influencing factor derived from cavitation, and to achieve the goal of “few or none on duty”, and to actualize the automatic operation level of the entire hydro power station, it must be prevented large Francis turbine units from operating under the cavitation condition.
Currently, on the question that the prototype runs under the cavitation condition, people’s research stands still on the stage of development, and the prime method is based on the symptom of cavitation, such as, draft tube cone pressure fluctuation, the vibration of the units, and the noise. At present, the best way to predict the cavitation characteristics of Francis turbine is model test. As the development of hydraulic techniques, improvement of people’s knowledge and the advances of test method, people are no longer satisfied with critical cavitation coefficient in the research of the cavitation of the Francis turbine because when the sigma becomes the critical cavitation coefficient; the efficiency of the turbine has already dropped by 1% and cavitation has been severe. For the sake of ensuring the Francis turbine runs under the condition of non-cavitation, and more precisely confirming the operation range of non-cavitation, therefore people’s focus has been shifted to the study of inception sigma, that is, the cavitation coefficient under which the bubbles firstly appears on the runner blades of Francis turbine.

Since the inception sigma directly determines the operation range of non-cavitation, the definition of it assumes great importance. At present, the inception cavitation of the runner blades of model Francis turbine is mainly depends on visual observation, a method to modulate the frequency of the stroboscope approximately to rotational speed of model Francis turbine, and then to observe by eyes through the transparent draft tube cone if the inception cavitation happens on the runner blades of the model Francis turbine. Once it is ascertained that the attached bubbles on the model runner blades appears, the cavitation coefficient is defined as inception sigma under this condition. The observed result of the visual observation is sometimes confined by the test condition and equipments, for example, the transparency of the glass draft tube cone and the quality of the water altogether influence the accuracy of the results, therefore, the judgment of the inception cavitation of the runner blades of model Francis turbine often relies on monitoring the sound emitted when the bubbles collapses.

It is easy to find that, compared with the routine quantifiable critical cavitation coefficient; the methods introduced above all stands still on the stage of artificial way. The results gained through these methods not only takes time but also presents subjectivity, therefore, it is essential to propose an method which devoid of the subjectivity of the experimenters and judge completely by the rules of objective test data the inception cavitation of the runner blades of the model Francis turbine.

In this article, an auto-test method based on acoustics theory is presented by analyzing the cavitation noise signals of runner blades of model Francis and taking advantage of the regularity of signals changing with sigma to determine when inception cavitation happens. The method upgrades the way of ascertaining the occurrence of the inception cavitation of the runner blades of the model Francis turbine from the stage of artificial discrimination to that of quantifiable judgment so that the objectivity and veracity of the experimental results of the Francis turbine runner blades inception cavitation is guaranteed.

2. The basic principle
Theoretically, as the pressure draws down, there separates out from the draft tube of turbine the air in the form of dissociative bubbles. As the pressure further declines, the bubbles also appear on the runner blades of the model Francis turbine. The state that the attached bubbles emerge on the runner blades of the model Francis turbine is just the inception cavitation\(^1\). Along with the continuous pressure drawdown, cavitation is severe; the bubbles attached to the runner blades of model Francis turbine and dissociative ones in the water both proceeds to increase. The formation of the bubbles described above is a dynamic process which constantly repeats occurrence, development, and collapse. The course of bubbles’ occurrence, growth, and collapse is accompanied with the radiation of energy in the form of sound, light, and heat to all directions. Among the three forms, the energy of sound takes up the overwhelming majority, and so the sound energy predominates in the total amount of energy produced in the course of bubble collapse, and thus it makes possible the way of judging the inception cavitation of the runner blades of the model Francis turbine. The research shows that the noise spectrum is composed of a series of small random impulses which is created when the collapse of the bubbles; therefore, spectrum of cavitation noise signal is continuous. Because the collapse sound of the bubble attached to the runner blades of the turbine is much greater than the energy of the
collapse dissociative bubbles, the sound energy of the inception cavitation of the runner blades of the model Francis turbine is greater than that before inception cavitation.

The background noise of hydraulic turbine is featured with large amplitude and low frequency. The test result proves that the frequency of background noise of hydraulic turbine is usually below 200Hz, and in the power spectrum density, the amplitude of low frequency is relatively high [2]. As the pressure draws down, there separates out dissociative bubbles which are few small sized ones. Meanwhile, the inception cavitation has not yet happened to model Francis turbine. To take the background noise into consideration, the signal in the water consists of the energy of high and low frequency, and the distribution of the two parts of energy possesses clear boundary point. Because the bubbles are less in numbers, the low frequency is dominant. As the pressure further declines, the number of bubbles separated out in the water increases, there appear attached bubbles on the runner blades. According to the theory of the hydro-acoustics, the frequency depends on the size of bubbles: the bigger bubbles to low frequency while the smaller ones to high ones. At the moment, the size of bubbles attached to the runner blades is small, so the boundary point of low and high frequency moves to the area of high frequency in pace with the decline of the pressure in the water. When cavitation is severe, the number in the water increases, and smallest bubbles merge and become bigger ones, therefore, the boundary point of frequency moves back to the low frequency area. Hence, in this process there must be an extreme of boundary point frequency, and the extreme should be the maximum of boundary point frequency.

3. Test system

Based on the acoustics characteristic of cavitation noise mentioned above, a test system was designed which employed hydrophone as acoustics sensor and sound card as data acquisition card. It took advantage of computer and software to monitor and analyze the inception cavitation of runner blade of model Francis turbine in real-time. The test system framework is shown as figure 1.

![Figure 1. Test system framework](image)

3.1. Hydrophone and signal conditioning module

The type of hydrophone is 8103 produced by B&K Company. Type 8103 hydrophone is suitable for industrial use and particularly for the acoustic study of marine animals or for cavitation measurements [3]. Its frequency range is from 0.1Hz to 180kHz with the sensitivity of -15dB, and this range is able to cover all the frequency bands of cavitation noise researched in this article. The signal conditioning module used in the test system is type 2653 charge amplifier produced by B&K Company as well. The hydrophone is installed on the draft tube cone with one side immersed into the water, so that signal can be captured by hydrophone thorough the flow field. Because the amplitude of signal is small, the charge amplifier is used to amplify amplitude of voltage of signal so as to be acquired by DAQ device.

3.2. DAQ device

A civil sound card is employed to acquire the signal amplified by 2653, and its highest sample rate is 44.1 kHz. Because the frequency spectrum of cavitation noise is continuous, and the frequency spectrum distribution is wide, lower sampling frequency could cause the frequency aliasing and missing information of original signals. According to experience, the sound emitted by bubbles when
they collapse can be heard by observers, so 44.1 KHz sample rate is high enough to avoid the phenomenon mentioned above, meanwhile the resolution is 16 bit.

3.3. Test system software

The test system software is programmed with LabVIEW developed by National Instrument. This software is composed of data acquisition and display in real-time module, data analysis in frequency – domain module, data replay module and saving original data module etc. The sketch of test system is shown as figure 2.

LabVIEW is not only a kind of graphical programming language, but also a develop environment. Its drivers are compatible with many data acquisition device and are able to call windows drivers to configure sound card. Its signal analysis tool set contains plants of spectral decomposition functions which are convenient to analyzing data in frequency-domain.

![Figure 2. The sketch of test system software](image)

4. Spectral decomposition and mathematical model of signal analysis

As mentioned in section 2, the spectrum of cavitation noise signal of runner blade of model Francis turbine is continuous as shown in figure 3 in which the curves are power spectrum density of cavitation noise signals. In Figure 3, Y axis represents the logarithm of voltage of signals acquired, and the trend of power spectrum density changes notably with the declining sigma. It can be found obviously that the whole power spectrum density curve can be divided into two parts: the lower frequency part and the higher frequency part, and the two parts are divided clearly by a boundary point in frequency, this point is defined a “Breakpoint” and the corresponding frequency is called Breakpoint frequency, as $f_{BP}$.

![Figure 3. Power spectrum density of cavitation noise signals](image)
According to the analysis in section 2, $f_{BP}$ increases monotonously before inception cavitation and decreases monotonously after inception cavitation with the declining sigma. Shown as figure 3, in this process, in which $f_{BP}$ verifies sigma, there must be a certain max $f_{BP}$ and the sigma corresponding to the maximum is inception sigma shown in figure 4 as well.

![Figure 4. The curve of $f_{BP}$ against sigma](image)

Moreover, another interesting regularity must be paid attention to. With sigma going down, the trend of distribution of low frequency energy and high frequency energy is varied. The ratio of distribution of low frequency part and high frequency part decreases monotonously when inception cavitation does not occur and increases when cavitation is severe shown as figure 4. There must be a certain minimum extreme in this process; meanwhile the sigma corresponding to the minimum extreme is inception sigma shown as figure 4, too.

![Figure 5. The curve of ratio of distribution trend against sigma](image)

For locating $f_{BP}$ exactly from the power spectrum density curve and finding a quantified expression of ratio of distribution of low frequency energy and high frequency energy of cavitation noise signals, mathematics model must be use. The least square method is employed to fit the power spectrum density curve with piecewise linear function $\bar{y}$ as shown in figure 3, then $\bar{y}$

$$\bar{y} = \begin{cases} k_0 x + b_0 , & x \leq x_m \\ k_1 x + b_1 , & x > x_m \end{cases}$$

Where $x_m$ represents the breakpoint frequency namely $f_{BP}$. For each $(x_i, y_i)$, residual is $V_i$, then $V_i = y_i - \bar{y}_i$, that is
\[ V_i = \begin{cases} y_i - k_0 x_i - b_0, & x_i \leq x_m \\ y_i - k_i x_i - b_i, & x_i > x_m \end{cases} \] (2)

It is supposed that the frequency point number is \( n_1 \) when \( x_i \leq x_m \), \( n_2 \) when \( x_i > x_m \), and \( n_1 + n_2 = N \), where N represents the total frequency point number of power spectrum density curve of cavitation noise signal after Fourier transform. Then the residual sum of squares \( Q_i \) is

\[ Q_i = \sum V_i^2 = \sum_{i=1}^{n_1} V_i^2 + \sum_{i=n_1+1}^{n_1+n_2} V_i^2 \] (3)

It is supposed that \( \frac{\partial Q_i}{\partial k_0} = 0, \frac{\partial Q_i}{\partial b_0} = 0, \frac{\partial Q_i}{\partial k_1} = 0, \frac{\partial Q_i}{\partial b_1} = 0 \), then two equations sets can be obtained as followed:

\[
\begin{align*}
  a_1 k_0 + a_1 b_0 &= c_1 \\
  a_2 k_0 + a_1 b_0 &= c_2 \\
  a_1 k_1 + a_2 b_1 &= c_1' \\
  a_2 k_1 + a_1 b_1 &= c_2'
\end{align*}
\] (4-5)

The solutions of the equations sets above are

\[
\begin{align*}
  a_1' &= \sum_{i=1}^{n_1} x_i^2 \\
  a_2' &= \sum_{i=1}^{n_1} x_i \\
  a_3' &= \sum_{i=1}^{n_1} y_i \\
  c_1' &= \sum_{i=1}^{n_1} y_i x_i \\
  c_2' &= \sum_{i=1}^{n_1} y_i
\end{align*}
\] (6)

For locating the breakpoint frequency \( f_{BP} \), the optimum fitted curve must be found. \( x_m \) increases as the way of \( x_m = x_0 + ih \) (\( i = 1, 2, \ldots, N-1 \). \( h \) is frequency resolution). When the residual sum of square \( Q_i \) is minimum, the corresponding \( x_m \) is the breakpoint frequency \( f_{BP} \), in other word, in certain operation condition, with the declining sigma, the sigma corresponding to the maximum \( f_{BP} \) is the inception sigma of runner blade of model Francis turbine in this condition.

Taking \( f_{BP} \) as the breakpoint frequency, the power spectrum density curve is divided into low frequency part and high frequency part accurately. The slopes of piecewise linear function \( k_0 \) and \( k_1 \) are used to denote the trend of distribution of two frequency parts. So, according to equations (4-6), \( k_0 \) and \( k_1 \) are respectively

\[
k_0 = \frac{\sum_{i=1}^{n_1} y_i x_i - \sum_{i=1}^{n_1} y_i x_1 x_i}{(\sum_{i=1}^{n_1} x_i)^2 - \sum_{i=1}^{n_1} x_i^2 x_i} \] (7)
\[ k_i = \frac{\sum y_i \times \sum x_i - \sum y_i \times x_i \times \sum}{(\sum x_i)^2 - \sum x_i \times \sum} \]  

(8)

And the quantified ratio \(k_0/k_i\) of trend of distribution is

\[ k_0/k_i = \frac{\left(\sum y_i \times \sum x_i - \sum y_i \times x_i \times \sum\right)}{\left(\sum x_i\right)^2 - \sum x_i \times \sum} \]  

(9)

So in certain operation condition, with the declining sigma, the sigma corresponding to the minimum ratio \(k_0/k_i\) is the inception sigma of runner blade of model Francis turbine in this condition.

The inception sigma of model Francis turbine, based on mathematics method in frequency-domain, should be consistent with the result obtained by conventional observation under the same operation condition.

5. The model test and result comparison

The runner blade inception cavitation test of Francis turbine was performed on the test stand I of HEC with acoustics method. The main parameters of model Francis turbine runner used in the test are shown in table 1, and this runner was used for acceptance tests of some domestic hydro-project. The same test has been performed on the same stand in acceptance test with visual observation.

| Table 1. The main parameters of model Francis turbine |
|------------------------------------------------------|
| Runner diameter D | Number of runner blade Z₀ | Number of guide vane | Guide vane pitch circle diameter D₀ | Height of guide vane B₀ |
| 0.420m | 16 | 24 | 487.2mm | 76.68mm |

Two operation conditions were chosen to perform inception cavitation test in this article, and the operation condition parameters are shown as table 2.

| Table 2. Test operation conditions of inception cavitation test |
|---------------------------------------------------------------|
| Test operation condition 1 | 25 | 25 | 579.1 |
| Test operation condition 2 | 24 | 25 | 575.6 |

The test result of operation condition 1 is shown as figure 6. The inception sigma is 0.089 with visual observation. As shown in Figure 6, with sigma going down, the breakpoint frequency \(f_{BP}\) increases monotonically while \(k_0/k_i\) decreases monotonically with sigma goes down and before the sigma declines to 0.089. By contraries, \(f_{BP}\) decreases monotonically while \(k_0/k_i\) increases monotonically with after the sigma is smaller than 0.089. Both \(f_{BP}\) and \(k_0/k_i\) reach the extreme points respectively at the same time. And the sigma corresponding to the extreme is 0.089 which is inception sigma consistent with the result obtained by visual observation.
The test result of operation condition 1 is shown as figure 7. The inception sigma is 0.08 with visual observation. As shown in figure 7, with the sigma declining, the breakpoint frequency $f_{BP}$ increases monotonically while $k_0/k_1$ decreases monotonically with sigma going down and before the sigma declines to 0.080. By contraries, $f_{BP}$ decreases monotonically while $k_0/k_1$ increases monotonically with after the sigma is smaller than 0.080. Both $f_{BP}$ and $k_0/k_1$ reach the extreme points respectively at the same time. And the sigma corresponding to the extreme is 0.080 which is inception sigma consistent with the result obtained by visual observation.

The trends of $f_{BP}$ and $k_0/k_1$ against sigma correspond with the theoretical analysis stated in section 2. Some other operation conditions were chosen to performed inception cavitation test and range of discharge covered both small and large discharges. Because the discharge affects cavitation directly and large discharge could cause cavitation easily, and the discharges of the two test operation conditions presented in this article are large, hence, they are more persuasive and representative. Figure 8 is the hill chart of the model Francis turbine used in this model test. According to the acceptance test report, the test results are 0.089 and 0.082, and the test results with acoustics method are 0.089 and 0.080. Both the results of acceptance test and test results with acoustics method were drawn in the same figure. It can be seen clearly that both results coincide perfectly.
Figure 8. The hill chart of model Francis turbine.

6. Conclusion
The essence of acoustics method presented in this article is to research the regularity of distribution of low and high frequency energy of cavitation noise, and it provides a new way to explore the inception cavitation of model Francis turbine runner blade.

According to the model test results, the rationality and accuracy have been proved, which upgrades the test method of inception cavitation from artificial manner to automatic way.

If acoustics method to determine inception cavitation is applied to prototype Francis turbine, it will be able to avoid the units operating under cavitation condition.

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