Living systems as coherent anharmonic oscillators

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Abstract. A model of living systems considered as coherent, time-dependent anharmonic oscillators is presented. It is based on the concept of space-like coherent states minimizing the time-energy uncertainty relation, adapted to the case of biological systems whose growth is described by the Gompertz or West-Brown-Enquist functions. The coherent states of biological growth evolve coherently in space being localized along the classical time trajectory; hence, the growth is predicted to be coherent in space. It is proven that the Gompertz function is a special solution of the space-like Horodecki-Feinberg equation for the time-dependent Morse oscillator in the dissociation state. Its eigenvalue represents the momentum of biological growth, associated with a space-like component whose properties resemble those attributed by vitalists to the life momentum or vital impulse. The physical characteristics of the life energy and momentum and their connection with the concept of zero-point momentum of vacuum are presented.

1. Introduction

In 1968 Herbert Fröhlich [1] discovered the effect of the long-range bio-coherence appearing in living systems treated as coupled oscillators in a heat bath with energy supplied at a constant rate. When the rate exceeds a certain mean one, the oscillators condense into one giant dipole whose sub-elements are spatially interrelated to each other like constituents of the low-temperature macroscopic quantum Bose-Einstein condensates. The Fröhlich model has been confirmed by Molski and Konarski [2] who investigated the biological growth described by the Gompertz function [3]. They found that this function represents the ground coherent state of the Gompertzian growth, which belongs to the class of the non-local coherent states minimizing the time-energy uncertainty relation. Such states evolve coherently in space being localized along the classical time-trajectory; hence the Gompertzian growth has been predicted to be coherent in space [4]. One of the most remarkable achievements presented in [2,4] was derivation of the second-order growth equation which contains a time-dependent Morse potential [5], whose space-dependent form is widely used to describe anharmonic vibrations of diatomic molecules. The model of non-local bio-coherence based on the Gompertz function has been generalized [6] to include the West-Brown-Enquist [7] growth function, which satisfies the second-
order differential equation for the oscillator characterized by the Wei Hua potential [8]. The results obtained reveal that the living systems can be considered as the time-dependent oscillators whose growth is governed by the anharmonic potentials. The main purpose of the present study is to investigate the consequences of the oscillatory model for our understanding of the essence of life and nature of biological growth. In particular, it will be proved that oscillatory model of living systems permits revitalization of the old concept of vitalism completely removed from the area of modern biology, medicine and physics. We shall also be concerned with generalization of the Leggett’s classification [9] of the macroscopic quantum phenomena to include a new class of the quasi-quantum phenomena: macroscopic quantization, uncertainty, coherence and non-locality appearing in the biological systems whose growth is characterized by the Gompertz or West-Brown-Enquist functions.

2. The basic concepts of the model

The oscillatory model of the living systems is based on the well-known biological and physical concepts: the biological growth and the generalized quantum theory including non-local (space-like) quantum states and systems. The most important basic concepts of the model are presented below.

2.1. The biological growth

In 1825 Benjamin Gompertz [3] introduced in the framework of actuarial mathematics the sigmoidal (S-shaped) function

\[ G(t) = G_0 \left( \frac{b}{a} \left[ 1 - \exp(-at) \right] \right) \]  

(1)

which permits calculating the force of mortality. Nowadays it is employed in biology and medicine to describe the growth of living systems (organ, tissue, organism, population of organisms, bacterial colony, tumour). Here \( G_0 \) stands for the initial mass, volume, radius or number of cells or entities, whereas the constants \( a \) and \( b \) (\( a, b > 0 \)) characterize the dynamics of growth (regression) according to the first-order Gompertzian kinetics

\[ \frac{d}{dt} G(t) = b \exp(-at) G(t) \quad \frac{d}{dt} G(t)^\dagger = -b \exp(-at) G(t)^\dagger \]  

(2)

Here \( G(t)^\dagger = G(t, b \to -b) \) defines the Gompertzian system in the regression (decay) phase. One may prove that function (1) is also a solution of the differential equation

\[ -\frac{d^2 G(t)}{dt^2} + \frac{a^2}{4} \left[ 1 - \exp[-a(t-t_e)] \right]^2 G(t) = \frac{a^2}{4} G(t) \]  

(3)

in which the second term represents (with accuracy to the constant) the time-dependent Morse potential

\[ V(t) = D_e \left[ 1 - \exp[-a(t-t_e)] \right]^2 \]  

(4)

Here, \( D_e \) is the dissociation energy of the Morse oscillator, \( a \) is the so-called range parameter, whereas

\[ t_e = \frac{1}{a} \ln \left( \frac{2b}{a} \right) \]  

(5)

is an equilibrium time at which the potential (4) attains a minimum \( V(t=t_e)=0 \). One may prove that the growth function (1) and differential equation (3) expressed in the dimensionless variable

\[ \tau = \frac{a(t-t_e)}{\sqrt{2}} \]  

(6)

can be specified in the form
\[ G(\tau) = G_0 \exp \left[ -\frac{1}{2} \exp(-\sqrt{2\tau}) \right] \quad G_\tau = G_0 \exp \left( \frac{b}{a} \right) \]  
(7)

\[ -\frac{1}{2} \frac{d^2 G(\tau)}{d\tau^2} + \frac{1}{4} \left[ 1 - \exp(-\sqrt{2\tau}) \right]^2 G(\tau) = \frac{1}{4} G(\tau) \]  
(8)

2.2. The extended quantum theory

In 2006 Molski [4] proved that equation (8) is a special case of the more general non-local Horodecki-Feinberg equation [10,11]

\[ -\frac{\hbar^2}{2mc^2} \frac{d^2\Psi}{dt^2} + V(\tau)\Psi = Pc\Psi \]  
(9)

for a space-like object of mass \( m \) moving in the field of the Morse vector potential (4). Here, \( \Psi \) is a non-local matter wave associated with the superluminal particle of momentum \( P \), \( \hbar=1,05457266\cdot10^{-34} \) J∙s is the Planck constant divided by \( 2\pi \), \( c \) is the light velocity. Equation (9) represents the non-relativistic version of the relativistic Feinberg equation for a particle of mass \( m \) moving at a velocity \( v \) close to infinity and endowed with the relativistic energy and momentum [11]

\[ E = \frac{mc^2}{\sqrt{v^2 - 1}} \quad \text{as} \quad v \to \infty \]  
\[ P = \frac{mv}{\sqrt{v^2 - 1}} \quad \text{as} \quad v \to \infty \]  
(10)

In the case of infinite speed, the particle has zero-energy \( E=0 \) and non-zero momentum \( P=mc \) and is called the transcendental tachyon [12]. The Horodecki-Feinberg equation (9) with the potential (4)

\[ \left\{ -\frac{1}{2} \frac{d^2}{d\tau^2} + \frac{1}{4x_e} \left[ 1 - \exp(-\sqrt{2x_e}\tau) \right]^2 \left[ \left( v + \frac{1}{2} \right) + x_e \left( v + \frac{1}{2} \right)^2 \right] \right\} \Psi_v = 0 \]  
(11)

in which

\[ \tau = \frac{a(t-t_0)}{\sqrt{2x_e}} \]  
\[ x_e = \frac{\hbar a}{c \sqrt{8mD_e}} \]  
(12)

Here \( \tau \) is a dimensionless variable whereas \( x_e \) is the anharmonic parameter characterizing the Morse oscillator [4]. The eigenvalues of the quantum equation (11)

\[ P_v = \frac{\hbar \omega}{c} \left[ \left( v + \frac{1}{2} \right) - x_e \left( v + \frac{1}{2} \right) \right] \]  
\[ \omega=\frac{a}{c} \sqrt{\frac{2D_e}{m}} \]  
(13)

represent the quantized momentum, which means that it takes only discrete values according to the quantum number \( v=0,1,2,\ldots \) whereas the wave function of the ground states (\( v=0 \)) reads [4]

\[ \Psi_0 = N_0 \exp \left[ -\frac{1}{2x_e} \exp(-\sqrt{2x_e}\tau) \right] \exp \left[ -\frac{1}{\sqrt{2x_e}} (1-x_e) \tau \right] \]  
(14)

2.3. The second-order micro-macro correspondences

The quantum Horodecki-Feinberg equation (11) and its ground state solution (14) can be transformed to the macroscopic growth equation (8) and its solution (7), upon the substitutions \( x_e=1, \ v=0 \), which characterize the dissociation ground state of the Morse oscillator [2,4]
They evidently prove that the macroscopic second-order equation (8) is a special case of the more general quantum non-local Feinberg-Horodecki equation (11), whose ground state eigenfunction \( v=0 \) for \( x_e=1 \) is reduced to the Gompertz function (7). Equation (8) does not contain the Planck’s constant but on the other hand it can be derived from the quantum equation (11). Hence, it is neither classical nor quantum formula. It belongs to the class of quasi-quantum equations whereas the Gompertzian growth is a quasi-quantum phenomenon [6].

2.4. The non-local coherent states of the Morse oscillator

The Horodecki-Feinberg equation (11) can be factorized using the creation-annihilation operators [4]

\[
\hat{A}^\dagger \hat{A} \Psi_v = P_v - P_0 \quad \Rightarrow \quad \hat{A}^\dagger \hat{A} \Psi_0 = 0
\]

\[
\hat{A}^\dagger = \frac{1}{\sqrt{2}} \left[ -\frac{d}{d\tau} + \frac{1 - \exp(-\sqrt{2}x_c\tau)}{\sqrt{2}x_c} - \sqrt{\frac{x_c}{2}} \right] \quad \hat{A} = \frac{1}{\sqrt{2}} \left[ \frac{d}{d\tau} + \frac{1 - \exp(-\sqrt{2}x_c\tau)}{\sqrt{2}x_c} - \sqrt{\frac{x_c}{2}} \right]
\]

(17)

The non-local coherent states of the time-dependent Morse oscillator can be defined as eigenstates of the annihilation operator [4]

\[
\hat{A} |\alpha\rangle = \alpha |\alpha\rangle \quad |\alpha\rangle = \exp\left[-\frac{1}{2x_c}\exp(-\sqrt{2}x_c\tau)\right] \exp\left[-\frac{1}{\sqrt{2}x_c}(1 - x_c)\tau\right] \exp[\alpha\sqrt{2}\tau]
\]

(18)

Such states minimize the time-energy uncertainty relation expressed in the Morse variable \( T(\tau) \)[4]

\[
\Delta T(\tau)^2 \Delta E^2 = \frac{\hbar^2}{4} \langle \alpha | \exp(-\sqrt{2}x_c\tau) |\alpha\rangle^2 \quad T(\tau) = \frac{1 - \exp\left(-\sqrt{2}x_c\tau\right)}{\sqrt{2x_c}} - \frac{x_c}{\sqrt{2}}
\]

(19)

hence they represent the non-local minimum-uncertainty coherent state. The ordinary local (time-like) coherent states minimize the position-momentum uncertainty relation and propagate coherently in time on the well-defined space trajectory. The non-local (space-like) coherent states propagate coherently in space along the well-defined time trajectory [4].

2.5. The first-order micro-macro correspondences

The micro-macro correspondences for Gompertzian growth and quantum coherence can be obtained from equations (17) and (18) by introducing \( x_e=1 \) into the quantum annihilation-creation operators and the ground coherent state \( \alpha=\theta \). Then one gets the macroscopic equations (2) describing biological growth and regression [4]
Here, $|0\rangle$ and $\langle 0|$ denote the ground state eigenfunctions of the annihilation and creation operators identified with the Gompertzian growth and regression states of the biological system [2]. The results obtained reveal another quasi-quantum effect in biological systems i.e. non-local macro-coherence.

2.6. The macro-uncertainty

The biological growth according to the Gompertz growth function (7) is characterized also by the macroscopic version of the quantum time-energy uncertainty relation to be obtained by substituting $x_e=1$ in (19). Hence, the macroscopic function (7) can be shown to minimize the microscopic time-energy uncertainty relation [4]

$$
\Delta T(\tau)^2 \Delta E^2 \geq -\frac{1}{4} \left\langle 0 \left| [T(\tau), \hat{E}] \right| 0 \right\rangle^2 \quad [T(\tau), \hat{E}] = i\hbar \exp(-\sqrt{2}\tau)
$$

Here, $T(\tau)$ is the Morse variable for $x_e=1$, $\langle 0|$ and $|0\rangle$ denote the Gompertz functions of regression and growth respectively, whereas the remaining quantities are defined as follows

$$
T(\tau) = -\frac{1}{\sqrt{2}} \exp(-\sqrt{2}\tau) \quad \hat{E} = i\hbar \frac{d}{d\tau}
$$

$$
\Delta T^2 = \langle 0 | T(\tau)^2 | 0 \rangle - \langle 0 | T(\tau) | 0 \rangle^2 \quad \Delta E^2 = \langle 0 | \hat{E}^2 | 0 \rangle - \langle 0 | \hat{E} | 0 \rangle^2
$$

$$
\Delta T^2 \Delta E^2 = \frac{\hbar^2}{4} \langle 0 | \exp(-\sqrt{2}\tau) | 0 \rangle^2
$$

This result indicates that the quantum uncertainty formalism can be directly applied to the macroscopic function (1) describing biological growth and regression. It also proves that the Gompertz growth function (7) represents the so-called intelligent coherent states [13]; which not only minimize the time-energy uncertainty relation but also maintain this relation in space due to its spatial stability. Such non-local states differ from the ordinary time-like coherent states, which maintain the position-momentum Heisenberg relation in time due to its temporal stability [4]. Consequently, the model presented predicts that formation of the specific growth patterns during biological growth is a non-local and spatially coherent phenomenon. It is a result of the long-range cooperation between the micro-level represented by the singular cell and the macro-level i.e. the bio-system as a whole. This conclusion is consistent with the Fröhlich [1] model of the long-range bio-coherence appearing in living systems treated as coupled oscillators.

2.7. Generalization of the model
The model presented can be extended to include the anharmonic Wei Hua oscillator [9] described by the time-dependent potential

\[ V(t) = D_e \left\{ \frac{1 - \exp[-c_i(t - t_e)]}{1 + s \exp[-c_i(t - t_e)]} \right\}^2 \]  

(24)

which is a generalization of the Morse function (4). Then the quantum Horodecki-Feinberg equation (9) for the time-dependent Wei-Hua oscillator

\[ \left\{ -\frac{1}{2} \frac{d^2}{dt^2} + D \left[ 1 - \exp(-\tau) \right]^2 \right\} \Psi_{0} = 0 \]  

(25)

expressed in the dimensionless form with

\[ \tau = c_i(t - t_e) \quad D = D_e \frac{mc^2}{\hbar^2 c_i^2} \quad P = \frac{P_0 mc^3}{\hbar^2 c_i^2} \]  

(26)

has the ground state solution [6]

\[ \Psi_{0} = M \left[ 1 - c_0 \exp(-\tau) \right]^2 \left[ c \exp[-\tau] \right]^{c_2} \xrightarrow{c^2 = 0, \quad P \to 0} M \left[ 1 - c_0 \exp(-\tau) \right]^2 \]  

(27)

In the dissociation state \( c_i = D - P = 0 \), it reduces to the West-Brown-Enquist function [7]

\[ m(t) = M \left[ 1 - c_0 \exp(-c_i t) \right]^\frac{1}{2} \]  

(28)

\[ c_0 = 1 - \left( \frac{m_0}{M} \right)^\frac{1}{2}, \quad c_i = -\frac{a_0}{4M^{1/2}}, \quad c_2 = \frac{1}{4}, \quad m(t = 0) = m_0, \quad m(t = \infty) = M \]  

(29)

describing the ontogenic growth. The function (28) has been derived by West et al. [7] on the basis of the first principles including the conservation of metabolic energy, the allometric scaling of metabolic rate, and energetic costs of producing and maintaining biomass. It fits very well the data for a variety of different species from protozoa to mammalians. In (29) \( m_0 \) denotes the initial mass of the system, \( M \) is the maximum body size reached whereas \( a_0 \) is the metabolic parameter. The West-Brown-Enquist function (28) better describes the biological growth than the Gompertz function (1) as the Wei Hua potential (24) for \( s = 0 \) reduces to the Morse function (4) being a special case of the former.

3. The quasi-quantum vitalism

The nature of life is commonly attributed to its functional and structural aspects. The contrasting viewpoint called vitalism assumes [14-18] that all living systems possess some unique component, which endows them with the special quality – life. This component is said to be the source of life and is commonly associated with power to growth and vital force. Despite many attempts of revitalization [19-22], vitalism is still treated as a metaphysical and non-falsifiable concept completely removed from the area of modern biology, medicine and physics. According to Daniel Denet [23]: Vitalism - the view that living things contain some special physical but equally mysterious stuff - élan vital - has been relegated to the trash heap of history. The presented oscillatory model of living systems permits revitalization of the relegated concept of vitalism.

3.1. The momentum of life

The eigenvalues of the Horodecki-Feinberg equation (11) characterize the momentum of the time-dependent Morse oscillator. In the limiting case \( v = 0 \) and \( x_e = 1 \) they are reduced to the momentum
of growing Gompertzian systems. The existence of the non-vanishing momentum associated with the biological growth is the first theoretical evidence of the momentum of life (élan vital), characterizing all biological systems growing according to the Gompertz function (1). Because this quantity is a purely mechanical concept it is possible to associate with it a mass and energy by making use of the anharmonic parameter (12), which in the Gompertzian systems is equal to one

\[ x_e = \frac{h a}{c \sqrt{8 m D_e}} = 1 \Rightarrow D_e = \frac{h^2 a^2}{8 m c^2} \]  

(31)

Consequently, we can calculate the dissociation energy \( D_e \) appearing in the Morse potential (4). On the other hand, from (30) one can determine the ground-state momentum \( P_0 \) associated with the growing system. By analogy to the zero-point energy of the harmonic oscillator, equation (30) describes the so-called zero-point momentum \( P_0 \) [24, 25] of the anharmonic Morse oscillator. Combining equations (30) with (31) and using the definition of the vibrational frequency of the Morse oscillator [4] appearing in (30)

\[ \omega \approx \frac{a}{c} \sqrt{\frac{2 D_e}{m}} \]  

(32)

one may calculate the approximate momentum associated with the Gompertzian growth

\[ P_0 = \frac{D_e}{c} \approx \frac{h^2 a^2}{8 m c^2} \approx mc \]  

(33)

The approximation (33) is possible as the Horodecki-Feinberg equation (9) describes an object moving almost instantaneously [10]. In such circumstances \( P_0 \) is approximately equal to the momentum of a transcendent tachyon (10). Consequently, equation (33) permits estimation of the mass of the particle being a carrier of the biological growth momentum

\[ m = \frac{h a}{\sqrt{8 c^2}} \]  

(34)

It depends on the value of \( a \)-parameter appearing in the Gompertz function (1). The combination of fundamental constants appearing in (34)

\[ \frac{h}{c^2} = 1.17336901 \cdot 10^{-51} \text{ Kg} \cdot \text{s} \]  

(35)

is extremely small, hence the mass (34) associated with momentum of life takes values much lower than the electron neutrino mass measured in MINOS experiment \( m_\nu = 7.132 \cdot 10^{-38} \text{ Kg} \) (0.04 eV). Additionally, the Gompertzian \( a \)-parameter is usually expressed in hours, days or years, so the calculation of mass (34) requires application of a suitable factor conversion to seconds which decreases the value (34) by a few orders. For example, the Gompertz function (1) fits the data for the mean weight of a male body in the period (0,18) \([\text{year}]\) [26] providing the parameters \( G_0 = 7.58(65) \text{[kg]}, b=0.2005(208) \text{[year}^{-1}]) \) and \( a=0.0638(101) \text{[year}^{-1}]). Hence, the mass (34) takes the value \( m = 8.3927 \cdot 10^{-61} \text{[kg]} \).

3.2. The life expectancy and time-energy uncertainty

In the Gompertz model, the life expectancy at birth \( L_0 \) can be calculated from the population’s survival function \( S(t) \) [27]

\[ L_0 = \int_0^\infty S(t)dt \quad S(t) = \left[ 1 - \frac{b}{a \lambda} (e^{-at} - 1) \right]^{-k} \]  

(36)
in which \( \lambda \) is approximated by the relation \( \lambda = k \). Integration (36) provides the life expectancy formula

\[
L_0 = \frac{1}{a \lambda} \, _2F_1 \left( k; 1; k + 1; 1 - \frac{b}{a \lambda} \right) \approx \frac{1}{a}
\]  

(37)
in which \( _2F_1 \) denotes the Gaussian hypergeometric function [27]. The obtained result indicates that life expectancy is inversely proportional to Gompertzian parameter \( a \), whereas parameter \( b \) has no significant influence on \( L_0 \) [27]. So, determining the \( a \)-parameter by the fitting to the biological growth data one can approximately calculate the life expectancy of a biological system. For instance, for \( a = 0.0638 \) [year\(^{-1}\)] determined from the data for male body of mean weight in the period \((0,18)\) [year] [26] one gets

\[
L_0 \approx \frac{1}{a} = 15.67 \pm 2.48 \text{ [year]}
\]  

(38)
which very well approximates the upper time-limit (18 years) of the data employed in evaluation of the \( a \)-parameter. The error \( \sigma = \pm 2.48 \) [year] propagates from the standard error \( \sigma = \pm 0.0101 \) [year] of the \( a \)-parameter. Taking into account the momentum-mass relationship (33) given in an alternative form

\[
\left( \frac{1}{a} \right)(D_a) = \frac{\hbar}{2 \sqrt{2}} \quad \text{or} \quad \left( \frac{1}{a} \right)(E) = \frac{\hbar}{2}
\]  

(39)
in which

\[
E = \frac{\hbar a}{2} = D_a \sqrt{2}
\]  

(40)
is the life energy defined at the equilibrium time by equations (A2) and (A3). We conclude that (39) has identical form as the minimum time-energy uncertainty relation

\[
\Delta t \Delta E = \frac{\hbar}{2} \quad \Delta t = \frac{1}{a} \quad \Delta E = \frac{\hbar a}{2}
\]  

(41)
which links the life energy and approximate life expectancy of a growing biological system. The proof of the correctness of relations (41) is presented in Appendix. It is noteworthy that from (32) and (33) one can calculate the frequency of growth

\[
\omega = \frac{a}{c} \sqrt{\frac{2D_a}{m}} = a \sqrt{2}
\]  

(42)
which reveals that each growing biological system according to the Gompertz function (1) can be considered as a time-dependent Morse oscillator endowed with a characteristic frequency (42).

3.3. The vital force

In the oscillatory model, the Gompertzian growth of biological systems takes place under the influence of the Morse potential energy (4) including the dissociation term to be calculated from (40)

\[
V(t) = \frac{\hbar a}{\sqrt{8}} \left\{ 1 - \exp\left[-a(t - t_c)\right] \right\}^2
\]  

(43)
Consequently, biological growth according to (1) should be endowed with a growth (vital) force or using the Winsor’s notion power to growth to be calculated in the same manner as the electric force is calculated in electrodynamics: it is a negative temporal derivative of the vector potential (43)

\[
F(t) = -\frac{\hbar a^2}{\sqrt{2}} \left\{ 1 - \exp\left[-a(t - t_c)\right]\right\} \exp\left[-a(t - t_c)\right]
\]  

(44)
In the case of dimensionless potential form appearing in (8) the vital force takes the form

\[
F(\tau) = -\frac{1}{\sqrt{2}} \left\{ 1 - \exp\left[-\sqrt{2}\tau\right]\right\} \exp\left[-\sqrt{2}\tau\right]
\]  

(45)
It is well-known that the Gompertz function (1) describes exponential growth, which then is exponentially retarded and saturated as time continues. According to equation (44), the Gompertzian growth is governed by the force (44) which stimulates growth in the period \((t, t_e)\) and constrains it in the period \((t_e, \infty)\) leading to the saturation. The plots of the Gompertz function \(G(\tau)\), Morse potential \(V(\tau)\) and associated vital force \(F(\tau)\) are presented in Fig. 1.

![Plots of the Gompertz function, Morse potential, and vital force](image)

Fig.1 Plots of the dimensionless Gompertz function \(G(\tau)\), Morse potential \(V(\tau)\) and vital force \(F(\tau)\). The straight line represents the momentum of growth \(P_0\) for quantum number \(v=0\); \(\tau_i\) is inflection point of the Gompertz function; \(\tau_e\) stands for the equilibrium point at which potential \(V(\tau)\) attains minimum, whereas force \(F(\tau)\) passes from accelerating to decelerating stage of growth.

### 4. Conclusions

The Gompertz function (1) is a ground state eigenfunction of the Horodecki-Feinberg equation for the time-dependent Morse oscillator with the anharmonicity parameter \(x_e = 1\). This solution corresponds to the dissociation state, so the growth of biological systems is consistent with the arrow of time (is not oscillating type) and its dynamics is governed by the time-dependent Morse potential. The oscillatory model of growth permits generalization of the Leggett’s classification [9] of the macroscopic quantum phenomena to include a new class of the quasi-quantum phenomena: macroscopic quantization, uncertainty, coherence and non-locality appearing in the biological systems whose growth is characterized by the Gompertz or West-Brown-Enquist functions. For the latter case the coherent states of growth and macro time-energy uncertainty relationship have been derived in [6]. The eigenvalue of the Horodecki-Feinberg equation represents the momentum, which, in the Gompertzian systems, can be interpreted as the momentum of growth or vital impulse. It takes a constant value, which does not alter during the growth, in contradistinction to the life energy, which changes according to equation (A2). The quasi-quantum interpretation of vitalism proposed has interesting philosophical consequences. According to the Bergson’s theory [15] the only way to account for the origin and functioning of life is to posit a non-material component being a carrier of momentum of life - \(\text{élan vital}\), which cannot be reduced to the material (chemical, physical) constitution of biological bodies. The results reported here demonstrate that \(\text{élan vital}\) can be interpreted in an exactly materialistic manner employing the concept of space-like quantum states and objects in the...
framework of the Horodecki-Feinberg theory. In particular the carrier of élan vital is a superluminal particle endowed with non-vanishing momentum, which moves in the space-time along the spatial world-line at the velocity close to infinity. Such an infinite-speed particle is called the transcendental tachyon [12]. According to Corben [31,32], there is a possibility of interaction and coupling of such space-like objects (tachyons) with ordinary subluminal time-like objects (bradyons) leading to formation of composite systems built up of both time- and space-like components, which trap each other in a relativistically invariant way. It can be proven that absorption of a tachyon by a bradyon at rest changes the kinematic state of the latter – this effect is called tachyro-kinematic [33,34]. In view of this the Corben’s theory predicts that the carrier of momentum of life can couple with somatic matter causing its motion (growth). This property is consistent with the vitalistic concept of anima, which is assumed to be the constituent of the living system endowing them with motion [34]. As it has been proved recently [2], the Gompertz function (1) represents also the ground eigenstate of the annihilation (growth) operator, which minimizes the time-energy uncertainty relation; hence it describes the biological systems that evolve coherently in the space. Consequently, the Gompertz function (1) represents the minimum-uncertainty coherent state of the Gompertzian growth. The uncertainty relation (22) can be given in the approximate form (41) valid for the time-dependent harmonic oscillator. Consequently, the time uncertainty can be expressed by a-parameter appearing in the function (1) and it is possible to connect the microscopic time-energy uncertainty with the macroscopic a-parameter characterizing the Gompertzian growth. Since equation (41) can be specified in the equivalent form

\[
\left( \frac{1}{a} \right) m = \frac{\hbar}{\sqrt{8c^2}}
\]

(46)

there is a direct link between the mass carrier of the life momentum and the approximate life expectancy of a growing Gompertzian system. Because mass m, according to (35), is extremely small, a hypothesis can be put forward that it can be created from vacuum as a virtual particle existing in the period approximately equal to the life expectancy 1/a. This conclusion, although controversial, is in full agreement with the time-energy uncertainty relation permitting the creation of virtual particle-antiparticle pairs from vacuum. Considering the space-like particle, its antiparticle moves in the opposite direction [12]. Virtual particles are real and have measurable effects (e.g. Casimir and Unruh effects, Lamb shift) although they cannot be directly observed. According to the Heisenberg's uncertainty principle "something" can arise from "nothing" provided that the "something" annihilates to the "nothing" after a period of time characterized by the time-energy uncertainty relation. Hence, the "empty" space-time should be considered as [seething] with an infinite number of particle-antiparticle pairs that come into being and then annihilate each other in a suitable time-interval. Virtual particles are manifestations of the fluctuations of vacuum endowed with the so-called zero-point energy. Recent investigation carried out by Feigel [24] has revealed that the vacuum fluctuations contribute to motion of dielectrics in crossed electric and magnetic fields. They induce a flow in the dielectric liquids endowed with velocity 50 [nm/s] predicted theoretically for electric field 105 [V/m] and magnetic 17 [T] [24]. The possibility of creation of the zero-point momentum from vacuum (momentum from nothing [25]) is an important evidence supporting the aforementioned hypothesis on the origin of life momentum. It is worth mentioning that Bergson [15] considered evolution as a tension between virtuality and actuality, which in the framework of the presented quasi-quantum model of vitalism can be taken literally.

**Appendix**

The quantum Horodecki-Feinberg operator appearing in the wave equation (5) can be derived by quantization of the classical formula [10]

\[
\frac{E^2}{2mc^2} + V(t) = P_c \quad E \rightarrow \hat{E} = \imath \hbar \frac{d}{dt}
\]

(A1)
in which $\hat{E}$ denotes the energy operator and $i$ is the imaginary constant. Introducing the Morse potential (4) to (A1) and employing relations (33), (42) one can calculate the energy of the growing Gompertzian system

$$E = \frac{\hbar a}{2} \sqrt{\exp[-a(t-t_e)] [2 - \exp[-a(t-t_e)]]} \quad (A2)$$

Equation (A2) has a quasi-quantum form: on the one hand it contains Planck’s constant, while on the other it does not contain quantum operators. One can easily demonstrate that (A2) attains a maximum at the equilibrium point

$$E(t_e) = \frac{\hbar a}{2} = \Delta E \quad (A3)$$

equal to the energy uncertainty in the vicinity of the equilibrium time $t_e$. To prove this thesis, we expand the Morse potential (4) in the Taylor series in the vicinity of the equilibrium time

$$V(t) = V(t_e) + \left[ \frac{dV(t)}{dt} \right]_{t_e} (t-t_e) + \frac{1}{2} \left[ \frac{d^2V(t)}{dt^2} \right]_{t_e} (t-t_e)^2 + \ldots \quad (A4)$$

The first two terms in A4 are equal to zero, so neglecting the high-order terms one can approximate anharmonic potential (4) by the harmonic one

$$V(t) = \frac{1}{2} mc^2 \omega^2 (t-t_e)^2 \left[ \frac{d^2V(t)}{dt^2} \right]_{t_e} = 2D_e a^2 \quad (A5)$$

in which $\omega$ is the vibrational frequency defined by (42). Consequently, a classical counterpart of the quantal Horodecki-Feinberg equation for the time-dependent harmonic oscillator can be specified in the form

$$E \rightarrow \Delta E \quad t - t_e \rightarrow \Delta t \quad \Delta E \Delta t = \frac{\hbar}{2} \quad (A6)$$

If we replace in (A6) the time and energy by their uncertainties and assume that they minimize the Heisenberg relation

$$(\Delta E)^2 + \frac{1}{2} mc^2 \omega^2 (\Delta t)^2 = P(\Delta E, \Delta t) \quad \Delta E = \frac{\hbar}{2\Delta t} \quad \Delta t = \frac{\hbar}{2\Delta E} \quad (A7)$$

then, consequently, (A6) takes the form

$$\frac{(\Delta E)^2}{2mc^3} + \frac{1}{2} mc^2 \omega^2 (\Delta t)^2 = P(\Delta E, \Delta t)$$

From the mathematical point of view the momentum in (A8) is a function of uncertainties, so assuming that it attains a minimum one can calculate values of uncertainties at assistance (A7). In the case of the time uncertainty, we have

$$\frac{\hbar^2}{8mc^3 (\Delta t)^2} + \frac{1}{2} mc^2 \omega^2 (\Delta t)^2 = P(\Delta t) \quad (A9)$$

$$- \frac{\hbar^2}{4mc^3 (\Delta t)^3} + mc^2 \omega^2 \Delta t = \frac{dP(\Delta t)}{d\Delta t} = 0 \Rightarrow \Delta t = \sqrt{\frac{\hbar}{2mc^3 \omega}} = \frac{1}{a} \quad (A10)$$

whereas for the energy uncertainty, similar calculations provide

$$\frac{(\Delta E)^2}{2mc^3} + \frac{\hbar^2 mc^2 \omega^2}{8(\Delta E)^2} = P(\Delta E) \quad (A11)$$
Now it is apparent, that uncertainties (A10) and (A12) satisfy the Heisenberg relation (A7), whereas (A3) represents the energy uncertainty in the vicinity of the equilibrium time $t_e$.

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