Light deflection by photonic crystals

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When propagating through periodically structured media, i.e., photonic crystals, optical waves will be modulated with the periodicity. As a result, the dispersion of waves will no longer behave as in a free space, and so-called frequency band structures appear. Under certain conditions, waves may be prohibited from propagation in certain or all directions, corresponding to partial and complete bandgaps respectively. Here we report a new fascinating phenomenon associated with the partial gaps, that is, deflection of optical waves. This phenomenon will render novel applications in manipulating light flows.

The band phenomenon was first investigated for electrons in solids. The well-known Bloch theorem has been proposed and led to successful explanation of some important properties of solids such as conducting, semi-conducting, and insulating states. Applying these concepts to electromagnetic waves started and flourished with the seminal papers published in 1987. Not only all the phenomena previously observed or discussed for electronic systems are successfully transplanted to classical systems, but many more significant and novel ideas, and applications have well gone beyond expectation, and are so far reaching that a fruitful new field called photonic crystals has come into existence, signified by the establishment of a comprehensive webpage.

Photonic crystals (PCs) provide a new possibility in designing optoelectronic devices. Up to date, most applications have mainly relied on the existence of complete bandgaps. This includes, for example, cavity laser, optical fibers, and optical waveguides. Here we demonstrate a new property of PCs, which is associated with partial bandgaps, referring to the situation in which waves are prohibited from propagation along certain directions.

We show that partial bandgaps can give rise to a fascinating phenomenon: deflection of optical waves. This phenomenon would allow for better optical steering, bending and beam collimating. It may also help explain some recent mysteries about the negative refraction or left-handed-material (LHM) behavior and the amphoteric refraction revealed by PCs.

The systems considered here are two dimensional photonic crystals made of arrays of parallel dielectric cylinders placed in a uniform medium, which we assume to be air. Such systems are common in both theoretical simulations or experimental measurements of two dimensional PCs. For brevity, we only consider the E-polarized waves, that is, the electric field is kept parallel to the cylinders. The following parameters are used in the simulation. (1) The dielectric constant of the cylinders is 14, and the cylinders are arranged to form a square lattice. (2) The lattice constant is a and the radius of the cylinders is 0.3a; in the computation, all lengths are scaled by the lattice constant. (3) The unit for frequency is 2πc/a.

The band structure is plotted in Fig. 1, and the qualitative features are similar to that obtained for a square array of alumina rods in air. A complete band gap is shown between frequencies of 0.22 and 0.28. Just below the complete gap, there is a regime, sandwiched by two horizontal lines, of partial band gap in which waves are not allowed to propagate along certain directions.

We wish to examine the properties of the energy flow of the eigenmodes which correspond to the frequency bands shown in Fig. 1. Differing from the common approaches which mainly rely on the curvatures of frequency bands to infer the energy flow, we consider the genuine definition of the energy flow. By Bloch’s theorem, the eigenmodes corresponding to the frequency bands of PCs can be expressed as $E_K(\vec{r}) = e^{i\vec{K} \vec{r}} u_K(\vec{r})$, where $\vec{K}$ is the Bloch vector, as the wave vector, and $u_K(\vec{r})$ is a periodic function with the periodicity of the lattice. When expressing $E_K(\vec{r})$ as $|E_K(\vec{r})| e^{i\theta_K(\vec{r})}$, the corresponding energy flow is derived as $J_K(\vec{r}) \propto |E_K(\vec{r})|^2 \nabla \theta_K(\vec{r})$; clearly $\theta_K$ combines the phase from the term $e^{i\vec{K} \vec{r}}$ and the phase from the function $u_K(\vec{r})$, making the determination of the phase and group velocities in PCs less obvious, or even perhaps not so useful. To explore the characteristics of the partial bandgap, we have computed the energy flow $E_K(\vec{r})$ and also the energy flow of the eigenmodes. The results are shown in Fig. 2.

A significant discovery from Fig. 2 is that the energy flow of eigenmodes does not always follow the direction of the Bloch vectors. As evidenced by Fig. 2 (b2), the energy tends to flow into the direction of $\Gamma M$, i.e., the [11] direction, while the Bloch vector points to an angle of $22.5^\circ$ that lies exactly between $\Gamma X$ and $\Gamma M$. This feature, however, cannot be inferred from the field pattern in (b1), i.e., the main wave front is more or less perpendicular to the direction of the Bloch vector rather than that of the energy flow, implying that it is not appropriate to determine the energy flow purely from the field pattern. For the other two Bloch vectors along $\Gamma X$ and $\Gamma M$ respectively, the net energy flows and the normals of the wave fronts follow the directions of the Bloch vectors, as shown by Figs. 2(a)
From (a1), (b1) and (c1), we see that the eigenmodes have also periodic structures perpendicular to the direction of the propagation. The feature in (b2) can be explained in the context of partial bandgaps. In Fig. 2 (b), the frequency 0.185 lies within the partial bandgap in the $\Gamma X$ direction. Since waves are not allowed to propagate along $\Gamma X$ and the directions perpendicular to $\Gamma X$, we may expect from the symmetry consideration that the energy flow will be tilted towards $\Gamma M$ for waves whose Bloch vectors make an angle to $\Gamma X$. Note here that in Fig. 2, the frequency corresponding to the Bloch vector along $\Gamma X$ is outside the partial gap.

The deviation of the energy flow from the direction of the Bloch vector, serving as the wave vector, suggests an intriguing phenomenon, that is, the light deflection. Consider a plane wave propagation in a uniform medium. When it encounters an perpendicular interface of PCs with the normal of the interface being along the direction exactly between $\Gamma X$ and $\Gamma M$, what can be expected to happen? In the conventional thought, the wave would keep going straight, and the wave vectors on both sides of the interface will point to the same direction, as there are no components in the tangent direction. However, according to Fig. 2, the flow of the energy will be deflected into the direction of $\Gamma M$. This is a surprising finding. To confirm this, we have carried out further simulations. For example, first, when we chose frequencies that lie outside the partial gap region, the deflection phenomenon disappears.

We have also studied the transmission properties. In Fig. 3, we plot the transmission of EM waves across two slabs of photonic crystals with two different lattice orientations: one is along the diagonal direction $\Gamma M$, and the other is along the direction making an equal angle to the $\Gamma X$ and $\Gamma M$ directions. The incidence is normal to the slab interface. In the simulation, we have employed the standard multiple scattering method\textsuperscript{9} to compute the transmitted intensity, which is usually the quantity to be measured in experiments. The frequencies chosen correspond to those in Fig. 2. We have used three types of sources in the simulations: (1) plane waves; (2) collimated waves by guiding the wave propagation through a window before incidence on the slabs; (3) a line source. The results from these three scenarios are similar. Here, we see that when the incidence is along the $\Gamma M$ direction, the transmission follows a straight path inside the slab. For the second case in Fig. 3, the transmission direction inside the slab is deflected towards the $\Gamma M$ direction. Fig. 3 clearly indicates that there are some favorable directions for waves to travel, fully in agreement with the above light deflection picture and in consistency with the results in Fig. 2. We also note that there is relatively small transmission along the $[1-1]$ direction; by symmetry, this direction is also a probable passing path for the waves.

The light deflection in the presence of partial bandgaps can be important in a number of occasions. First, it may help designing novel optoelectronic devices in controlling optical flows. Compared to applications based on complete bandgaps, there is no reflection loss because the light deflection occurs in the passing bands, allowing for efficient wave transmission. Second, the mechanism of the light deflection discussed here may help understand the recent puzzles about the negative refraction\textsuperscript{4} or LHM behavior\textsuperscript{6} and the amphoteric refraction revealed by PCs\textsuperscript{7}. To elaborate, we consider the slab shown in Fig. 3 (b). Assume that a plane wave is incident on the slab with a tilt angle. We can expect from the above results that as long as the tilt angle is not too large, the energy flow will be deflected towards the direction of $\Gamma M$, no matter the incidence is positive or negative. Consequently, an amphoteric refraction may prevail and is conceptually illustrated by Fig. 4.

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Figure Captions

Figure 1 The band structure of a square lattice of dielectric cylinders. The lattice constant is $a$ and the radius of the cylinders is $0.3a$. $\Gamma M$ and $\Gamma X$ denote the [11] and [10] directions respectively. A partial gap is between the two horizontal lines.

Figure 2 Left panel: the field pattern of eigenmodes. Right panel: the energy flow of the eigenmodes. The eigenmodes along three directions are considered: (a) $\vec{K} = (0.9\pi/a, 0)$, i.e. along $\Gamma X$; the corresponding frequency is 0.175; (b) $\vec{K} = (0.9\pi/a, 0.37\pi/a)$, i.e. along an angle of 22.5° exactly between $\Gamma X$ and $\Gamma M$ directions; the corresponding frequency is 0.185; (c) $\vec{K} = (0.7\pi/a, 0.7\pi/a)$, i.e. along $\Gamma M$; the corresponding frequency is 0.192. The direction of the Bloch vectors are denoted by the blue arrows, while the red arrows denote the local energy flow including the direction and the magnitude. The circles refer to the cylinders. Both frequencies in (b) and (c) lie within the partial gap. Due to the periodicity, we only plot the energy flow within one unit cell. Note that although the features shown by (b) also hold for other Bloch vectors for which the corresponding frequencies lie within the partial gap regime, we only plot here for the case of 22.5°

Figure 3 The imaging fields for slabs of photonic crystal structure. Two lattice orientations are considered: (a) The slab measures as about $56 \times 10$ and the incidence is along the [11] direction; (2) the slab measures as $50 \times 13$ and the incidence is along the direction that makes an equal angle to the [11] and [10] directions. The main lobes in the transmitted intensities are shown. There is a gap along the [10] direction.

Figure 4 Conceptual illustration of amphoteric refraction due to partial bandgaps, or perhaps better termed as amphoteric deflection. Depending on the incident angle, the deflection may not exactly follow the direction of [11].
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