Non-equilibrium dynamics of an ultracold Bose gas under a multi-pulsed quantum quench in interaction

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We investigate the nonequilibrium dynamical properties of a weakly-interacting Bose gas at zero temperature under the multi-pulsed quantum quench in interaction by calculating one-body, two-body correlation functions and Tan’s contact of the model system. The multi-pulsed quench is represented as follows: first suddenly quenching the interatomic interaction from $g_i$ to $g_f$ at time $t = 0$, holding time $t$, and then suddenly quenching interaction from $g_f$ back to $g_i$, holding the time $t$ sequence $n$ times. In particular, two typical kinds of quenching parameters are chosen, corresponding to $(g_i/g_f > 1)$ and $(g_i/g_f < 1)$ respectively. We find that the more the quenching times of $n$ are, the more the excitations are excited, which suggests that the multi-pulsed QQ is more powerful way of studying the non-equilibrium dynamics of many-body quantum system than the ‘one-off’ quantum quench. Finally, we discuss the ultra-short-range properties of the two-body correlation function after the $n$th quenching, which can be used to probe the ‘Tan’s contact’ in experiments. All our calculations can be tested in current cold atom experiments.

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I. INTRODUCTION

Quantum quench (QQ) [1], referred to as how a prepared state based on the initial Hamiltonian of $H_i$ can evolve with another Hamiltonian of $H_f$, provides a powerful tool for investigating nonequilibrium dynamics of quantum many-body systems. Efforts along this line have been driven, on the one hand, by the remarkable progress in series of recent experiments [2–4] with ultracold atomic gases, on the other hand, by the desire to understand the basic questions in nonequilibrium physics, ranging from thermalization and equilibration [1, 5–10] and their relation to integrability [11–15], to the introduction of new concept of dynamical phase transition [10, 16–21]. It can be said, in particular, that quenching ultracold atomic gases has becoming the forefront for studying the nonequilibrium physics.

Up to now, typical scenarios for a QQ consist of a sudden change in interatomic interaction due to Feshbach resonance or in the strength of confining potential [22–28], characterized by the transition from $H_i$ to $H_f$ happening over a time scale shorter than any other time scale in the problem. The key ingredient of a QQ is that the final state arrived after a QQ has more excitations than the corresponding equilibrium state, which in turn measures the abilities of QQ driving the model system out of equilibrium. Based on such a understanding, so far, most of the previous theoretical studies [29–31] have focused on the ‘one-off’ QQ defined by a QQ happens only once and subsequently investigated how the key quantities of the model system, typical of one-body or two-body correlation functions, can relax. However, the ‘one-off’ QQ sets a fundamental limit for the ability of driving the model system out of equilibrium, which can be best understood in terms of the Loschmidt echo (LE) $L(t) = |\langle \Psi_0(t)|\Psi(t)\rangle|^2$. Here, the LE $L(t)$ is defined as the overlap of two wave functions of $|\Psi_0(t)\rangle$ and $|\Psi(t)\rangle$ evolved from the same initial state, but with different Hamiltonian $H_i$ and $H_f$. Remarkably, Refs. [32, 33] have proved that the $L_{sq}(t \to \infty)$ under the sudden quench is square of the adiabatic counterpart $L_{ad}(t \to \infty)$, which is hold in general. This gives the maximum value of a ‘one-off’ QQ driving system out of equilibrium to $L_{sq}(t) = L_{sq}^{mp}(t)$. Therefore, how to further improve the power of a QQ inducing the out-of-equilibrium dynamics, which is of fundamental interest to studying nonequilibrium physics in a regime not accessible to a ‘one-off’ QQ, has becoming a real challenge.

Here, we propose and analyze a novel kind of QQ denoted to the multi-pulsed quench, wherein the Hamiltonian is quenched many times, in order to further enhance the out-of-equilibrium dynamics. As shown in Fig. 1, the typical protocol of a multi-pulsed QQ studied in this work consists of as follows: at time $t = 0$, suddenly quenching the interatomic interaction from $g_i$ to $g_f$, holding time $t$, and then suddenly quenching interaction from $g_f$ back to $g_i$, holding the time $t$ sequence $n$ times. The power of our approach can be illustrated in terms of general physical arguments based on the LE $L_{mp}(t)$ corresponding to a multi-pulsed QQ. It can be proved that $L_{mp}(2nt)$ after $n$-time multi-pulsed quench is the $2n$-th power of the adiabatic counterpart $L_{ad}(t)$, i.e. $L_{mp}(2nt) = L_{sq}^{mp}(t) = L_{ad}^{2n}(t)$ [32, 33]. The fact that the exponent is enhanced by a factor of $n$ compared to the ‘one-off’ quench shows that more far away equilibrium regime can be reached. Moreover, Our proposal hints at the possibility to induce the non-equilibrium dynamics in a highly controllable way.

In this work, motivated by the recent development of quenching ultra-cold atom systems in both the experimental and the theoretical sides, we have launched systematic studies on the effects of the multi-pulsed QQ on two kinds of correlation functions of a three-dimensional (3D) ultracold Bose gas, i.e. one-body, two-body correlation functions and Tan’s contact [34–40] of the model system respectively. The reasons are two-fold: first, these two (one- and two-body) correlation functions provide direct insight into the dynamical properties of both equilibrium and nonequilibrium quantum many-body system; second, the one-body correlation function is directly connected with the noncondensated fraction which can be measured in time-of-flight experiments [25]. The two-body correlation function denotes the correlation between two particles in different spatial positions at the same time and can be detected using Bragg spectroscopy [23, 38, 41], noise correlations [42, 43], and even by in situ measurements [24, 26]. Moreover, we also discuss the ultra-short-range properties of the two-body correlation function, which is related to the internal energy via the ‘Tan’s contact’ proposed by Tan [34] in the system of a two-component Fermi gas interacting with a contact interaction. In more details, we use the time-dependent Bogoliubov approximation to study the non-equilibrium dynamics of a 3D Bose gas after a multi-pulsed quench. In particular, by changing quench times $n$, we try to collect information about how correlation functions evolve with time. We have found that the model system can produce more elementary excitations with increasing the quench times $n$ as expected. Although the two-body correlation function oscillates fast in short-time-range, these two kinds of correlation functions tend to a constant value eventually even we increase the quenching times to 12. Finally, we also find that the Tan’s contact changes slightly with time, which almost is independent of the quench times.

II. 3D BOSE GAS UNDER A MULTI-PULSED QQ IN INTERACTION

The model system considered in this work is composed of a 3D Bose gas of $N$ ultracold bosonic atoms. In addition, the interatomic interaction is assumed to be weak and can be well described by the contact description. In such, the
many-body Hamiltonian under a multi-pulsed QQ in the interaction reads as follows

\begin{equation}
H = \sum_{k} (\epsilon_k - \mu) a_k^{\dagger} a_k + \frac{g(t)}{2V} \sum_{k_1, k_2, q} a_{k_1}^{\dagger} a_{-k_1 + q} a_{k_2} a_{-k_2 + q},
\end{equation}

with \( a_k^{\dagger} \) and \( a_k \) being the bosonic creation and annihilation operators, \( \epsilon_k = \hbar^2 k^2 / (2m) \) labeling the single-particle dispersion relation and \( \mu \) and \( V \) being the chemical potential and the volume of the system respectively. The \( g(t) \) in Hamiltonian (2.1) describes the multi-pulsed quench protocol. To be specific, as shown in Fig. 1, we consider the case: (i) the system is initially prepared at the ground state \( |\Psi_0(t)\rangle \) of Hamiltonian (2.1) with \( g(t) = g_i \), labeled by \( H_i \); (ii) then, at \( t = 0 \), the interaction strength is suddenly switched to \( g(t) = g_f \) such that the time evolution from \( t > 0 \) is governed by the final Hamiltonian (2.1) of \( H_f \); (iii) after holding the time \( t \), the interaction suddenly quenches form \( g_f \) back to \( g_i \), holding the time \( t \) sequence \( n \) times.

We focus on the regime of weak interatomic interaction, in which Hamiltonian (2.1) can be well described by the standard Bogoliubov approximation. As a standard fashion, we proceed to transform the Hamiltonian (2.1) into the effective Hamiltonian: the zeroth-order term is found by substituting all creation and annihilation operator by \( \epsilon_k = \hbar^2 k^2 / (2m) \) labeling the single-particle dispersion relation and \( \mu \) and \( V \) being the chemical potential and the volume of the system respectively. The \( g(t) \) in Hamiltonian (2.1) describes the multi-pulsed quench protocol. To be specific, as shown in Fig. 1, we consider the case: (i) the system is initially prepared at the ground state \( |\Psi_0(t)\rangle \) of Hamiltonian (2.1) with \( g(t) = g_i \), labeled by \( H_i \); (ii) then, at \( t = 0 \), the interaction strength is suddenly switched to \( g(t) = g_f \) such that the time evolution from \( t > 0 \) is governed by the final Hamiltonian (2.1) of \( H_f \); (iii) after holding the time \( t \), the interaction suddenly quenches form \( g_f \) back to \( g_i \), holding the time \( t \) sequence \( n \) times.

Collecting all terms calculated above, we arrive at the effective Hamiltonian:

\begin{equation}
H_{\text{eff}} = \frac{1}{2} g(t) n_0 N_0 + \sum_{k \neq 0} (\epsilon_k + g(t) n_0) a_k^{\dagger} a_k + \frac{1}{2} g(t) n_0 \sum_{k \neq 0} (a_k a_{-k} + a_k^{\dagger} a_{-k}^{\dagger}),
\end{equation}

with \( n_0 = N_0 / V \) being the condensed density. Next, the effective Hamiltonian (2.2) can be diagonalized by a standard Bogoliubov variational ansatz, where we write the bosonic creation \( a_k^{\dagger} \) and annihilation \( a_k \) operators in new defined operators \( b_k^{\dagger} \) and \( b_k \),

\begin{align}
& a_{k \neq 0}(t) = u_k(t) b_k + v_k^*(t) b_{-k}^{\dagger}, \\
& a_{-k \neq 0}(t) = v_k(t) b_k + u_k^*(t) b_{-k}^{\dagger},
\end{align}

with \( b_k^{\dagger} \) and \( b_k \) denoting the bosonic creation and annihilation operators for noncondensed atoms, respectively. These two operators have no time dependence and always are treated as small quantities. To assure operators \( b_k^{\dagger} \) and \( b_k \) still comply with the standard commutation relations for bosonic creation and annihilation operators, we have double-checked the relation \(|u_k(t)|^2 - |v_k(t)|^2 = 1\), which is always satisfied. Substituting Eq. (2.3) into Eq. (2.2), we can obtain the diagonalized Hamiltonian expressed by operators \( b_k^{\dagger} \) and \( b_k \). Furthermore, at time \( t = 0 \), we find that

![FIG. 1. (Color online) Schematics of the multi-pulsed quantum quench where only three repeating sequences are visible. The upper (blue) one refers to quenching from \( g_i \) to \( g_f \) with the ratio \( g_i / g_f \geq 1 \); The lower (green) one denotes the opposite case with the ratio \( g_i / g_f < 1 \). The arrows show the time instant corresponding to the \( n \)th order quench.](image.png)
coherence factors $u_k$ and $v_k$ must be solutions of the following equations:

$$gn_0 (u_k^2 + v_k^2) + 2 (\epsilon_k + gn_0) u_kv_k = 0,$$
$$g_0 (u_k^2 + v_k^2) + gn_0 (v_k^* u_k + u_k^* v_k) = \hbar \omega_k.$$  (2.4)

Combined with the normalization $|u_k|^2 - |v_k|^2 = 1$, we can easily find the solutions as follows

$$u_k (t = 0) = \sqrt{[(\epsilon_k + g_0 n_0) / E_k^i + 1] / 2},$$
$$v_k (t = 0) = -\sqrt{[(\epsilon_k + g_0 n_0) / E_k^i - 1] / 2}$$  (2.5)

with $\hbar \omega_k = E_k^i = \sqrt{\epsilon_k (\epsilon_k + 2g_0 n_0)}$. Finally, the diagonalized Hamiltonian reads as:

$$H_{\text{eff}} = -\frac{1}{2} g_0 n_0 + \frac{1}{2} \sum_{k \neq 0} [\hbar \omega_k - (\epsilon_k + g_0 n_0)] + \sum_{k \neq 0} \hbar \omega_k b_k^* b_k.$$  (2.6)

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**FIG. 2.** (Color online) Quasimomentum distribution (one-body matrix) via the time $t$ with different values of $n$ in the multi-pulsed quantum quench. The left panel refers to quenching to a smaller interaction with $g_i/g_f = 1.1$, and the right panel denotes the opposite case with $g_i/g_f = 0.8$. The fraction of noncondensed atoms in both cases are all normalized to the initial excitation fraction. The characteristic relaxation time is set by $\tau_{MF} = \hbar / g_f n_0$. The quenching times are selected to $n = 1, 2, 6, 12$. 

**FIG. 3.** (Color online) Plotted are nonequilibrium dynamics of density-density correlations (two-body correlation function) via the time $t$. Density-density correlations $g^{(2)}(t) - n_0^2$ is normalized to the asymptotic value at long times. Time and length scale are measured in terms of the $\tau_{MF} = \hbar / g_f n_0$ and $\zeta = \hbar / (m g_f n_0)$ respectively. The dimensionless distance between two different spatial positions is chosen to be $\delta / \zeta = 4$. The quenching times in multi-pulsed quantum quench are selected to $n = 1, 2, 6, 12$. 

Both the one-body and the two-body correlation functions can be expressed as a function of $u_k(t)$ and $v_k(t)$. Hence how to get these two factors is the core of issue. We need to work in the Heisenberg representation and make use of the equations of motion for $a_k(t)$ as follows,

$$i\hbar \partial_t a_k(t) = \left[ a_k(t), H^{\text{eff}} \right]$$  \hspace{1cm} (2.7)

The solving process of $u_k(t)$ and $v_k(t)$ is straightforward but somewhat complicated, we only give a number of important steps. Substituting Eq. (2.3) into Eq. (2.7), we can get differential equations for $u_k(t)$ and $v_k(t)$:

$$i\hbar \partial_t \begin{pmatrix} u_k(t) \\ v_k(t) \end{pmatrix} = \begin{pmatrix} \epsilon_k + g_{n_0} & -g_{n_0} \\ -g_{n_0} & -\epsilon_k + g_{n_0} \end{pmatrix} \begin{pmatrix} u_k(t) \\ v_k(t) \end{pmatrix}$$  \hspace{1cm} (2.8)

We can solve Eqs. (2.8) based on the initial conditions $u_k(t = 0)$ and $v_k(t = 0)$, reading,

$$\begin{pmatrix} u_k(t) \\ v_k(t) \end{pmatrix} = U_{i\rightarrow f}(t) \begin{pmatrix} u_k(0) \\ v_k(0) \end{pmatrix}$$  \hspace{1cm} (2.9)

where the time evolution operator $U_{i\rightarrow f}(t)$ is defined as Eq. (2.10)

$$U_{i\rightarrow f}(t) = \begin{pmatrix} \cos \left( \frac{E_k t}{\hbar} \right) - i \frac{\gamma_{n_0} + \gamma_{n_0}}{E_k^2} \sin \left( \frac{E_k t}{\hbar} \right) & -i \frac{\gamma_{n_0}}{E_k^2} \sin \left( \frac{E_k t}{\hbar} \right) \\ i \frac{\gamma_{n_0}}{E_k^2} \sin \left( \frac{E_k t}{\hbar} \right) & \cos \left( \frac{E_k t}{\hbar} \right) + i \frac{\gamma_{n_0} + \gamma_{n_0}}{E_k^2} \sin \left( \frac{E_k t}{\hbar} \right) \end{pmatrix}.$$  \hspace{1cm} (2.10)

In the derivation of the expressions of the time-dependent coherence factors $u_k(t)$ and $v_k(t)$, we limit ourselves into the regime where the time dependence of $n_0$ can be safely neglected as shown in Refs. [29, 44]. Moreover, the condition of $n_{ex} \ll n$ can be easily obtained in the typical ultra-cold atomic experiments.

### III. NONEQUILIBRIUM DYNAMICS OF ONE- AND TWO-BODY CORRELATION FUNCTIONS

Using $u_k(t)$ and $v_k(t)$, the multiple quench scenario (see in Fig. (1)) can be expressed as follows. The Bogoliubov coefficients of $u_k^{(n)}(t)$ and $v_k^{(n)}(t)$ at time $t$ after the $n$-th quench can be determined as follows,

$$\begin{pmatrix} u_k^{(n)}(t) \\ v_k^{(n)}(t) \end{pmatrix} = [U_{f\rightarrow i}(t) U_{i\rightarrow f}(t)]^n \begin{pmatrix} u_k(0) \\ v_k(0) \end{pmatrix}$$  \hspace{1cm} (3.1)

Then the one- and two-body correlation functions for $n$th quench protocol, according to their definitions $n_{ex}(t) = \sum_{k\neq 0} a_k(t) a_k(t)$ and $g^{(2)}(\mathbf{r} - \mathbf{r}')(t) = \sum_{\mathbf{q}} e^{i\mathbf{q} \cdot (\mathbf{r} - \mathbf{r}')} \langle \rho_{\mathbf{q}}(t) \rho_{-\mathbf{q}}(t) \rangle$ with $\rho_{\mathbf{q}}(t) = \sum_k a_k^{\dagger}(t) a_k(t)$, can be expressed as follows

$$n_{ex}^{(n)}(t) = \sum_k \left| v_k^{(n)}(t) \right|^2,$$  \hspace{1cm} (3.2)

and

$$g^{(2)(n)}(\delta)(t) = n_0^2 + n_0 \sum_k e^{i\mathbf{k} \cdot \delta} \left( u_k^{(n)*}(t) v_k^{(n)}(t) + u_k^{(n)}(t) v_k^{(n)*}(t) + 2 \left| v_k^{(n)}(t) \right|^2 \right)$$  \hspace{1cm} (3.3)

with $\delta = \mathbf{r} - \mathbf{r}'$. We point out that the terms quartic in $u_k$’s and $v_k$’s arising from correlations between the noncondensed atoms are ignored because these correlations become unimportant at long distances $\delta > a \sim 50$ nm; in contrast, the short-distance structure of the two-body correlation will become very important. Therefore, we also derive the concrete expression for Tan’s contact as follows,

$$C(\delta) = 16\pi^2 \lim_{\delta \to 0} \delta^2 \left[ \left( \sum_k e^{-i\mathbf{k} \cdot \delta} \left| v_k^{(n)}(t) \right|^2 \right)^2 + \sum_k e^{-i\mathbf{k} \cdot \delta} (u_k^{(n)}(t) v_k^{(n)*}(t))^2 \right].$$  \hspace{1cm} (3.4)
Throughout this paper we consider two typical kinds of the multi-pulsed QQ: quench to a smaller interaction $g_i/g_f > 1$ or quench to a bigger interaction $g_i/g_f < 1$. In both cases, the noncondensed fractions, and the equal-time density-density correlation functions, and the Tan’s contacts all have analytical expressions, however, they are a bit complicated, so we do not list them here. In what follows, we focus on the influence of the multi-pulsed QQ on two kinds of correlation functions as shown in Figs. 2, 3 and 4, which are plotted on the basis of their analytic expressions. In this end, we devise two scenarios: first, we refer to $n = 1, 2, 6, 12$ as the times of quench in the protocol and then study how the correlation functions evolve with time after the quench with the different values of $n$; second, we choose the parameter regimes of both $g_i/g_f > 1$ and $g_i/g_f < 1$ and consider how the detailed changes of interaction affect the non-equilibrium dynamics of correlation functions.

In the first scenario, the results are plotted in Figs. 2, 3 and 4. As shown in Fig. 2, it is evident that the multi-pulsed QQ can produce more noncondensed fraction with increasing the quench times of $n$ as expected, which suggests that more far away equilibrium regimes can be achieved. Moreover, both the rapid relaxation of the one-body matrix in Fig. 2 and two-body correlation in Fig. 3 suggest that, after the 3D Bose gas is brought out of equilibrium by a QQ, it can relaxes to a steady state on a time-scale within the experimental reach. In Fig. 4, the fact that the quench times of $n$ has the relative small effects on the Tan’s relation can be understood as follows: a QQ usually induces the low-energy fluctuations compared to the energy scale which can contribute to Tan’s relation.

In the second scenario, as shown in the left and right panels of Figs. 2 and 3, we compare the effects of the different choices of $g_i/g_f > 1$ and $g_i/g_f < 1$ on correlation functions. It’s clear that the more bigger is the final interaction $g_f$, the more excitations a QQ can induce as shown in Fig. 2. As for two-body correlation functions in Fig. 3, they will all develop the rapid oscillation matter in short-time range, and evolve to a final equilibrium state as long as the time is long enough. Moreover, the oscillate amplitude decrease with increasing the quench times when the ratio $g_i/g_f > 1$, and will increase when the ratio $g_i/g_f < 1$ (see Fig. 4).

![Fig. 4](image_url)

**FIG. 4.** (Color online) Plotted are the ‘Tan’s relation’ $C(\delta) = \delta^2 g^{(2)}(\delta)$ via $\delta$. At $t < 0$, the model system is noninteracting with $C(\delta) = 0$. Immediately after the quench, the zero-distance correlations respond instantaneously. Length and time scales are measured in terms of the condensate healing length ($\zeta = \hbar/\sqrt{mg_f n_0}$) and mean-field time $\tau_{\text{MF}} = \hbar/g_f n_0$ in the final state respectively. The colored curves correspond to different quench times (bottom to top): $n = 1$ (black), $n = 2$ (red), $n = 6$ (green), and $n = 12$ (blue). The parameters read $t/\tau_{\text{MF}} = 1$, $g_i/g_f = 1.1$ and $a = 0.5\zeta$.  

**IV. CONCLUSIONS**

In summary, we first investigate the non-equilibrium dynamics of one- and two-body correlation functions in a 3D homogeneous Bose gas at zero temperature following the multi-pulsed QQ in interaction. Our results of one- and two-body correlation functions show that the multi-pulsed QQ can bring the model system far more away equilibrium regime than the ‘one-off’ QQ, which suggests that the multi-pulsed QQ is more powerful way of studying the non-equilibrium dynamics of many-body quantum system.
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