Relaxed constraints on neutrino oscillation parameters

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Abstract. We study the cosmological constraints on active-sterile neutrino oscillations $\nu_e \leftrightarrow \nu_s$ for the case when $\nu_s$ is partially filled initially, i.e. $0 < \delta N_s < 1$. We provide numerical analysis of the cosmological production of $^4\text{He}$, in the presence of $\nu_e \leftrightarrow \nu_s$ oscillations, effective after neutrino decoupling, accounting for all known oscillations effects on cosmological nucleosynthesis. Cosmological constraints on oscillation parameters corresponding to higher than 5% $^4\text{He}$ overproduction and different non-zero initial populations of the sterile state $\delta N_s < 1$ are calculated. These generalized cosmological constraints corresponding to $\delta N_s > 0$ are relaxed in comparison with the $\delta N_s = 0$ case and the relaxation is proportional to $\delta N_s$.

Keywords: BBN constraints, neutrino oscillations

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1. Introduction

Atmospheric, solar and terrestrial neutrino oscillations experiments provided evidence for neutrino oscillations. The atmospheric neutrino data analysis confirmed the $\nu_\mu \leftrightarrow \nu_\tau$ channel as the dominant atmospheric oscillation solution. The results of the terrestrial experiment KamLAND and the results of SNO salt phase prefer flavor oscillation LMA solutions to the $\nu_e \leftrightarrow \nu_s$ one for the solar neutrino problem. Active-sterile neutrino oscillations are discussed as a supplementary sub-dominant channel. Even after the confirmation of LMA solution as a dominant solution of the solar neutrino problem, the oscillation parameters are not precisely known - the completeness of LMA solution was questioned and the scope for some possible sub-dominant transitions was explored \[1, 2, 3, 4, 5, 6, 7, 8\]. The available results of the neutrino oscillations from the solar experiments, including the later SNO ES, CC and NC data, slightly favor the existence of a small sterile oscillating sector. Thus, it is still interesting to explore the cosmological influence of sterile oscillating neutrinos.

Neutrino $\nu_e \leftrightarrow \nu_s$ oscillations affect the expansion rate and the neutrino involved processes in the early Universe and in particular, influence Big Bang Nucleosynthesis (BBN) \[9, 10\]. This allows to put stringent constraints on neutrino $\nu_e \leftrightarrow \nu_s$ oscillations from BBN considerations. The whole LMA and partially LOW $\nu_e \leftrightarrow \nu_s$ solar oscillation solutions and atmospheric $\nu_e \leftrightarrow \nu_s$ solution were excluded by cosmological considerations \[11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24\].

Detail reviews discussing neutrino role in cosmology and neutrino masses and mixings may be found in refs. \[23, 24, 25, 26\].

Cosmological constraints were obtained usually assuming zero sterile neutrino state population before neutrino oscillations epoch, $\delta N_s = \rho_{\nu_s}/\rho_{eq_{\nu_e}} = 0$, where $\rho_{eq_{\nu_e}}$ is the equilibrium electron neutrino energy density. However, sterile neutrinos production is predicted by different types of models, like GUT models, models with large extra dimensions, many-fold Universe models, mirror matter models, neutrino oscillations models, etc., and may be present at the onset of BBN epoch. Hence, the degree of population of $\nu_s$ and its initial energy spectrum distribution depends on the $\nu_s$ production model, and in general $\delta N_s \geq 0$.

The general case of initially (before oscillations) non-zero sterile neutrino population was examined and its influence on the oscillation effects at BBN epoch was discussed in reference \[27\]. It was found that the kinetic effect of oscillations is sensitive to the initial population of $\nu_s$. BBN constraints on neutrino oscillations parameters in the general case of sterile neutrino state, partially occupied initially, were discussed in reference \[28\] for oscillations effective after electron neutrino decoupling. The case of 3% $^4\text{He}$ overproduction was considered. The presence of non-empty sterile neutrino $0 < \delta N_s < 0.54$ before oscillations was shown to lead to strengthening of the cosmological constraints proportional to the $\delta N_s$ value.

Recent studies of the systematic uncertainties of primordial $^4\text{He}$ abundance suggest as a more reliable value of $^4\text{He}$ observational uncertainty $\delta Y_p/Y_p > 5\%$. In what
follows we derive 5.2% cosmological constraints, corresponding to $\delta Y_p/Y_p > 5\%$ $^4\text{He}$ overproduction on $\nu_e \leftrightarrow \nu_s$ oscillation parameters for the same type of non-equilibrium neutrino oscillations. We show that in contrast to 3% $^4\text{He}$ constraints, in the $\delta Y_p/Y_p > 5\%$ case the non-zero initial sterile population leads to a relaxation of the constraints on oscillation parameters proportional to the value of $\delta N_s$, and finally to their alleviation for $\delta N_s = 1$.

In the next section, we discuss briefly the primordial $^4\text{He}$ production in the case of non-zero sterile population at the start of oscillations. In the last section we present the cosmological constraints corresponding to $^4\text{He}$-overproduction $\delta Y_p/Y_p > 5\%$ and different levels of initial population of the sterile state and compare them to $\delta N_s = 0$ case.

2. Production of primordial $^4\text{He}$ and neutrino oscillations

2.1. Primordial $^4\text{He}$

$^4\text{He}$ is the most abundantly produced, most precisely measured and calculated element among the primordially formed light elements. It has also a simple post-BBN chemical evolution - it is only produced in stars. Therefore, it is the preferred light element used for obtaining limits on nonstandard physics. Primordially produced $^4\text{He}$ is calculated with great precision [29, 30, 31, 32, 33] $Y_p = 0.2485 \pm 0.0005$. The predicted $^4\text{He}$ value is in agreement with the observational data for $^4\text{He}$ inferred from astrophysical observations [34, 35, 36, 37, 38, 39, 40].

Currently there are significantly different observational determinations of the primordial $^4\text{He}$ abundance, which point to the existence of greater systematic errors than assumed before. Izotov and Thuan have found $Y_p \approx 0.242 \pm 0.009$ for 82 HII regions and $Y_p = 0.2421 \pm 0.0021$ for 7 preferred HII regions, and $Y_p = 0.2429 \pm 0.0009$ for an extended data set of 89 HII regions [35, 36]. Olive and Skillman have re-analyzed their data and gave $Y_p \approx 0.245 \pm 0.013$ for all 82 HII regions and $Y_p = 0.2491 \pm 0.0091$ for the 7 preferred targets [37]. The real uncertainty on the primordial $^4\text{He}$ abundance may be larger than previously assumed 3%. Present determinations of primordial $^4\text{He}$ abundance from observations of extragalactic HII regions, indicate a significantly greater uncertainty for the $^4\text{He}$ mass fraction [36, 37, 39, 40] namely $dY/Y \sim 5\%$. ||

Thus a derivation of cosmological constraints on oscillation parameters corresponding to about 5% $^4\text{He}$ overproduction seems relevant.

The primordial $^4\text{He}$ abundance essentially depends on the freeze-out of nucleons, which occurs when in the process of expansion the weak processes rates $\Gamma_w$, governing the neutron to proton transitions, become comparable to the expansion rate $H(t)$. So, the primordially produced mass fraction of $^4\text{He}$ $Y_p \sim 2(n/p)/(1 + n/p)$, is a strong function of relativistic degrees of freedom at BBN epoch, which enter through $H(t)$, || See also the pioneer papers discussing the possibility for higher systematic errors in the determination of primordial $^4\text{He}$ abundance, which was believed to be much smaller than at present [41, 42, 43].
where $H(t) \sim \sqrt{G_N g_{\text{eff}}} T^2$. § $Y_p$ also depends on the electron neutrino characteristics, namely neutrino energy spectrum, number densities $N_{\nu}$ and the neutrino-antineutrino asymmetry, entering through $\Gamma_w, \Gamma_w \sim G_F^2 E_\nu^2 N_{\nu}$.

2.2. Oscillations effects

Non-empty initial sterile neutrino population, present at the start of oscillations, influences BBN by (i) increasing the expansion rate and (ii) suppressing the kinetic effects of $\nu_e \leftrightarrow \nu_s$ on BBN.

(i) The presence of $\delta N_s$ means increased effective number of neutrino species in equilibrium during BBN, and leads to faster expansion of the Universe, $H(t) \sim g_{\text{eff}}^{1/2}$ causing earlier $n/p$-freezing, $T_f \sim (g_{\text{eff}})^{1/6}$, hence, $^4\text{He}$ is overproduced. This dynamical effect is parametrized through $\delta Y_d \sim 0.013 \delta N_s$.

(ii) In case of oscillations between non-equilibrium sterile neutrino state $0 < \delta N_s < 1$ and electron neutrino, proceeding after $\nu$ decoupling, $\nu_e \leftrightarrow \nu_s$ oscillations lead to considerable and continuous deviations from the equilibrium $\nu_e$ spectrum (spectrum distortion) and production of neutrino-antineutrino asymmetry.|| This effects nucleons kinetics in the pre-BBN epoch [10, 41, 27]. This kinetic effect may be parametrized in terms of additional neutrino $\delta N_{\text{kin}}$ and will be denoted further $\delta Y_{\text{kin}} \sim 0.013 \delta N_{\text{kin}}$. Non-zero $\delta N_s$ suppresses the kinetic effect of oscillations: $\delta N_{\text{kin}}$ decreases with the increase of $\delta N_s$, hence $\delta N_s$ leads to a decrease of the overproduction of $^4\text{He}$.

For a wide range of oscillation parameters the kinetic effect of oscillations is large during the period of freezing of the nucleons and, therefore, affects BBN. This effect is strong even when there is a considerable population of the sterile neutrino state at the onset of the electron–sterile oscillations. The $\nu_e$ energy spectrum distortion is the greatest, if the sterile state is empty at the start of oscillations, $\delta N_s = 0$ and decreases with the increase of the sterile state degree of population [27]. The same behavior is to be expected for the kinetic effect of oscillations.

The total effect of (i) and (ii) can be approximately described by $\delta Y_p \sim 0.013 \delta N$

$\delta N = \delta N_s + \delta N_{\text{kin}}$, where $\delta N_{\text{kin}} = \delta N_{\text{kin}}^{\max} (1 - \delta N_s)$ and $\delta N_{\text{kin}}^{\max}$ is the kinetic oscillations effect, corresponding to $\delta N_s = 0$. The expression presents a good approximation to the numerically calculated dependence of the kinetic effect on the initial population of $\nu_s$, derived in reference [27]:

§ Due to its strong dependence on $g_{\text{eff}}$, $^4\text{He}$ abundance was used to put constraints on the number of the relativistic particles during BBN epoch [15, 46, 17], parametrized by $\delta N_{\nu} = N_{\nu} - 3$. Cosmological data (except BBN) from CMB, LSS, SN, are not very restrictive on the extra light particles [38, 49, 50, 51]. || Neutrino-antineutrino asymmetry is generated during the $\nu_e \leftrightarrow \nu_s$ resonant oscillations [14]. This dynamically produced asymmetry suppresses oscillations at small mixing angles. Therefore, it leads to underproduction of $^4\text{He}$ and alleviates BBN constraints on oscillation parameters compared to the case without the account of asymmetry growth. We have accounted in this work for its effect as well, however the effect is sub-dominant and will not be discussed further on.
Relaxed constraints on neutrino oscillation parameters

There is an interesting interplay between the different effects which \( \delta N_s \neq 0 \) exerts on oscillations and on BBN with oscillations and hence, the \( \delta Y_p \) production and the cosmological constraints on oscillations parameters for the case \( \delta N_s \neq 0 \) differ from the ones derived in references [16, 17, 52] for \( \delta N_s = 0 \).

As found in [28], for \( \delta Y_p/Y_p > 5\% \) corresponding to \( \delta N_{\text{kin}}^{\text{max}} > 1 \), the suppression effect (ii) dominates over the dynamical effect (i) of \( \delta N_s \neq 0 \). Hence, the total effect is a decreasing function of \( \delta N_s \), i.e. \( ^4\text{He} \) overproduction decreases with \( \delta N_s \) (in comparison with the case \( \delta N_s = 0 \)) and correspondingly the BBN constraints on oscillation parameters relax. In the opposite case \( \delta Y_p/Y_p < 5\% \), corresponding to \( \delta N_{\text{kin}}^{\text{max}} < 1 \), the dynamical effect (i) dominates and the total effect is increasing with \( \delta N_s \). I.e. \( ^4\text{He} \) overproduction increases and the BBN constraints on oscillations strengthen in comparison with the case \( \delta N_s = 0 \). In the case \( \delta Y_p/Y_p = 5\% \) the constraints for \( \delta N_s \neq 0 \) coincide with the ones for \( \delta N_s = 0 \), due to the cancellation of the two effects (i) and (ii).

For illustration of \( \delta N_{\text{kin}}^{\text{max}} > 1 \) case, which we consider further on, we present in Figure 1 the effects (i) and (ii) on \( ^4\text{He} \) overproduction at \( \delta m = 10^{-7} \text{ eV}^2 \) and \( \sin^2 2\theta = 1 \). We have studied numerically the contribution of these effects on neutrons to nucleons freezing ratio \( X_n = n_n^f/n_{nuc} \) for different \( \delta N_s \). The primordial yield of helium to a good approximation is expressed through \( X_n: Y_p \sim X_n \exp(-t/\tau_n) \), where \( \tau_n \) is the neutron lifetime. For the chosen set of parameters the \( ^4\text{He} \) overproduction decreases with the increase of \( \delta N_s \). The suppression effect (ii) of \( \delta N_s \) dominates and a relaxation of the cosmological constraints compared to \( \delta N_s = 0 \) must be expected.
In the next section we present and discuss the calculated cosmological constraints on oscillation parameters corresponding to higher than 5% $^4\text{He}$ overproduction and $0 < \delta N_s < 1$.

3. Cosmological constraints on oscillation parameters

BBN constraints corresponding to $\delta Y_p/Y_p = 3\%$ overproduction of $^4\text{He}$ and non-zero initial population of the sterile neutrino $\delta N_s < 0.54$ were calculated recently \[28\]. As far as $\delta Y_p/Y_p = 3\%$ corresponds to $\delta N_{\text{kin}}^{\max} < 1$, the constraints strengthen with the increase of the $\delta N_s$ value. They increase the BBN 3% $^4\text{He}$ exclusion region for oscillation parameters corresponding to $\delta N_s = 0$ towards smaller $\delta m^2$.

Having in mind the existence of a large systematic error of $^4\text{He}$ measurements, in this work we provide a numerical calculation of cosmological constraints corresponding to $\delta Y_p/Y_p > 5\%$ $^4\text{He}$ overproduction ($\delta N_{\text{kin}}^{\max} > 1$) and different initial degrees of sterile neutrino population in the range $0 \leq \delta N_s < 1$. We have chosen $\delta Y_p/Y_p = 5.2\%$, i.e. a value slightly higher than the critical one 5%, inorder to illustrate the possibility for relaxation of the cosmological constraints on oscillations for $\delta N_s \neq 0$.

Our numerical analysis has shown that cosmological constraints corresponding to 5.2% $^4\text{He}$ overproduction relax with the increase of $\delta N_s$ values. Up to $\delta N_s = 0.5$ the cosmological constraints are slightly relaxed in comparison with the case $\delta N_s = 0$, however, for higher $\delta N_s$ values, the constraints relax noticeably. The reason for that is the predominance of (ii) effect (the suppression of the oscillations kinetic effects) over the dynamical effect (i) for the given uncertainty of $^4\text{He}$. All cosmological constraints corresponding to $\delta Y_p/Y_p > 5\%$ will have such behaviour, namely, they will be relaxed in comparison to the constraints for $\delta N_s = 0$. (Vice versa, the constraints corresponding to lower than 5% $^4\text{He}$ uncertainty will be more stringent than the ones for initially empty sterile neutrino state.)

In Figure 2 $\delta Y_p/Y_p = 5.2\%$ BBN constraints are presented for different values of the initial population of the sterile state, namely the lowest dashed contour corresponds to a zero population, the solid curve corresponds to $\delta N_s = 0.5$, the dotted and the dot dashed contours to $\delta N_s = 0.7$ and to $\delta N_s = 0.9$, respectively.

Another interesting result is that there are considerable constraints even for a very high $\delta N_s$ values for that really high $^4\text{He}$ uncertainty. I.e. the constraints on neutrino mixing parameter vanish only when the sterile state is in equilibrium before oscillations, when the kinetic effect due to neutrino spectrum distortion disappears, i.e. $\delta N_s = 1$.

4. Conclusions

We have studied BBN constraints on neutrino $\nu_e \leftrightarrow \nu_s$ oscillations for the specific case when the sterile neutrino is partially filled initially $0 < \delta N_s < 1$. Non-zero $\delta N_s$ has two-fold effect on BBN with neutrino oscillations: a dynamical effect leading to overproduction of He-4 and a kinetic effect, leading to underproduction of He-4 in
Relaxed constraints on neutrino oscillation parameters

Figure 2. The dashed contour presents $\delta Y_p/Y_p = 5.2\%$ BBN constraints for $\delta N_s = 0$, the solid curve corresponds to $\delta N_s = 0.5$, the dotted and the dot dashed contours — $\delta N_s = 0.7$ and to $\delta N_s = 0.9$, respectively. The resonant oscillations case corresponds to l.h.s of the figure, the non-resonant one to the r.h.s.

comparison with the case of $\delta N_s = 0$. So, depending on the interplay between these opposite effects, the cosmological constraints may be either relaxed or strengthened.

We have provided detail numerical analysis of the BBN production of $^4$He, $Y_p$, in the presence of $\nu_e \leftrightarrow \nu_s$ neutrino oscillations, effective after electron neutrino decoupling. We have calculated and discussed cosmological constraints on oscillation parameters, corresponding to higher than 5% uncertainty of helium-4, for non-zero initial population of the sterile state $\delta N_s < 1$. It was found that the cosmological constraints on oscillation parameters relax with the increase of $\delta N_s$. The cosmological constraints for the cases $\delta N_s \leq 0.5$ are slightly changed in comparison to $\delta N_s = 0$ case, however, for bigger $\delta N_s$ the constraints are relaxed considerably and for $\delta N_s = 1$ they are alleviated.

Resuming the results of the works discussing $\delta N_s$ effect on BBN: Cosmological constraints corresponding to higher than 5% uncertainty of helium-4, relax with the increase of the initial population of the sterile state, while the constraints corresponding to lower than 5% uncertainty of helium-4 strengthen with $\delta N_s$.

It is remarkable, that in case of BBN with non-equilibrium oscillations between electron and sterile neutrinos, it is possible to obtain cosmological constraints on oscillation parameters even in the case when the $^4$He abundance is known with uncertainty greater than 5% (Actually, it is possible to derive constraints on neutrino oscillation parameters for He-4 uncertainty up to 32% in the resonant oscillations case, and up to 14% uncertainty in the nonresonant oscillations case, as far as these are the maximal possible helium overproduction values [53]). The cosmological constraints persist while initially the sterile state is non equilibrium. When $\delta N_s = 1$ initially,
Relaxed constraints on neutrino oscillation parameters

the kinetic effect of oscillations is zero and it is not possible to obtain constraints on oscillation parameters in the discussed BBN models with non-equilibrium electron-sterile neutrino oscillations, as well as in BBN models with equilibrium electron-sterile neutrino oscillations.

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Relaxed constraints on neutrino oscillation parameters

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