Chiral vortical effect in pionic superfluid vs spin alignment of baryons

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We consider chiral fluids, with (nearly) massless fermionic constituents, in the confining phase. Chiral vortical effect (CVE) is the flow of axial current along the axis of rotation of the fluid while the spin alignment is a non-vanishing correlation of polarizations of baryons with the axis of rotation. As the theoretical framework we use the model of pionic superfluidity induced by a non-vanishing isotopic chemical potential. We note that the average value of spin of virtual baryons reproduces the CVE. The role of defects, or vortices is crucial. The model does not apply directly to the quark-gluon plasma but might indicate existence of a mechanism to produce baryons with relatively large polarization in heavy-ion collisions.

Introduction

Observation of non-vanishing polarization of \( \Lambda(\bar{\Lambda}) \)-hyperons in heavy ion collisions by the STAR Collaboration [1] is considered to be one of most remarkable experimental findings of recent years. The common interpretation nowadays is that spin of the hyperons is aligned with the rotation of plasma produced in peripheral ion-ion collisions. Moreover, such a prediction was made prior to the experimental discovery, (see, e.g. [2, 3] and references therein) and explored in some detail after the data appeared [4, 5, 6].

“Magnetization by rotation” was introduced first about a century ago [6]. The origin of this phenomenon is readily understandable. Indeed, with account of chemical potential \( \mu_\mu \), external magnetic field \( \hat{B} \) and of rotation with angular velocity \( \hat{\omega} \), the density operator \( \hat{\rho} \) takes the form:

\[
\hat{\rho} = \frac{1}{Z} \exp \left( -\hat{H}/T + \mu_\mu \hat{Q}_i / T + \hat{\omega} \cdot \hat{J}/T + \mu_\mu \hat{B} / T \right),
\]

where \( \hat{H} \) is the Hamiltonian, \( T \) is temperature, \( \hat{Q}_i \) are charges conjugated with \( \mu_\mu \), \( \hat{J} \) is the total angular momentum, \( \mu_\mu \) is the magnetic moment. Thus, correlation of the total angular momentum \( \hat{J} \) with \( \hat{\omega} \) is quite obvious. Spin \( \hat{S} \) is a part of the total angular momentum. However, separation of the spin from the orbital momentum is, generically, not a simple problem. Note, that the appearance of combination \( \hat{L} + \hat{S} \) in the Dirac equation [7] for the rotating frames may be related [8] to the equivalence principle for Dirac fermions [9, 10, 11].

In case of equilibrium of weakly interacting particles evaluations with the density operator [10] are quite straightforward, for recent examples and references see, e.g., [3]. An exception is, probably, the massless case when one has to switch from the spin variables to chirality. On the other hand, the limit of massless spin particles is of special interest since there are new, chiral symmetries arising in this limit. Alignment of chiralities of massless fermions with the magnetic field \( \hat{B} \) results in the so called magnetic chiral effect (CME) [11] which is a flow of electric current along the magnetic field. In case of a single charged particle:

\[
\hat{j}_{el} = \frac{e\mu_5}{2\pi^2} \hat{B},
\]

where \( \mu_5 \) is the chiral chemical potential, \( \mu_5 = 1/2(\mu_R - \mu_L) \). Another effect [12] is the flow of axial current along the axis of rotation, or \( \hat{\omega} \):

\[
\hat{j}_{A} = \frac{\mu^2 + \mu_5^2}{2\pi^2} \hat{\omega},
\]

where \( \mu \) is the chemical potential associated with the electric charge and we neglected temperature effects for simplicity. It is actually the chiral (axial) vortical effect [13] that occupies our attention mostly.

In recent years, the chiral effects attracted a lot of attention, see, e.g., the volume of review articles [13]. The progress was made in direction of creating theory of systems much more complicated than free fermions and include, in particular, condensed-matter systems. Also, the chiral effects were recognized as macroscopic manifestations of the anomalous triangle graphs. Instead of trying to compose a comprehensive list of references let us just mention a breakthrough paper [14] where it was shown that Eq. (1) holds, in a slightly modified form, for a generic liquid in hydrodynamic approximation. Namely, Eq. (1) becomes

\[
\hat{j}_{A}^\mu = \frac{\mu^2 + \mu_5^2}{2\pi^2} \omega^\mu, \quad \omega^\mu = e^{\mu_\rho \sigma} u_\nu \partial^\rho u_\sigma,
\]

where \( u_\mu \) is the 4-velocity of an element of liquid.

Despite of all the progress made, it is not clear, how to apply theory to confining phases. And that is what is needed to appreciate the results on hyperon polarization mentioned above. A crucial question is whether the spin orientation of quarks survives the transition to confinement. A heuristic estimate of polarization of the \( \Lambda \)-hyperon was suggested in Ref. [13] in terms of vacuum
expectation values of chirality-changing (C-parity-odd) operators:
\[ < \sigma_z > \simeq \frac{< \overline{\Psi} \sigma_{xy} \gamma_5 \Psi >}{< \Psi \Psi >} \]  
(5)
where \( < \overline{\Psi} \Psi > \) is the quark condensate, and \( < \overline{\Psi} \sigma_{xy} \gamma_5 \Psi > \) is induced in the vacuum if external magnetic field or rotation is applied (for details see [15]).

We are utilizing the model of pionic superfluidity induced by a chemical potential \( \mu_3 \) breaking isotopic invariance, see [16] and references therein. It is not a realistic model for the quark-gluon plasma but this is a rare example when the effects of confinement can be accounted for. We will demonstrate that the average density of spins (polarizations) of baryons matches the density of axial charges by non-vanishing chemical potentials, \( \mu_3 \). Note that vortices have been considered in many papers. Our treatment of vortices is close to that of papers [17–19].

A crucial role in derivation of (6) is played by defects, or vortices. Formally, in case of superfluid the vortices are infinitely thin. Baryons regularize the vortices at short distances. Note that vortices have been considered in papers [17, 19].

In the next sections we give details of derivation of (6).

**Pionic superfluidity**

As is well known, at low energies, \( E \ll \Lambda_{QCD} \) pions are representing the light, or physical degrees of freedom. One can construct effective field theory describing the pion interactions which respects the symmetries of the original theory (QCD). Moreover, the effective action is expandable in derivatives, in the spirit of hydrodynamics. To make the problem tractable, one introduces non-vanishing chemical potentials, \( \mu_V, \mu_A \) which violate isotopic symmetry and are associated with the conserved charges \( Q^3, Q^3_\pi \), where the superscript “3” stands for the third component of isotopic vector. The temperature is kept, for simplicity, zero, \( T = 0 \). It was demonstrated, see [16] and references therein, that using the general framework of effective field theories, one can show that the pionic liquid is superfluid. Let us recall the basic steps of the derivation [16] emphasizing the points crucial for our considerations.

The effective Lagrangian is constructed in terms of unitary matrices \( U \):
\[ U = \exp (i \lambda^a \pi^a / f_\pi) \]  
(7)
where \( \pi^a \) are massless Goldstone fields [31] associated with the spontaneous breaking of the chiral symmetry, \( \lambda^a \) are Hermitian matrices. The effective Lagrangian incorporating the chemical potential \( \mu_V, \mu_A \) looks as [16, 20]:
\[ L_{\text{chiral}} = \frac{f_\pi^2}{4} \left( D_\mu U D^\mu U^\dagger \right) , \]  
(8)
where the covariant derivatives are defined as:
\[ D_\mu U = \partial_\mu U - i \delta_\mu a (\bar{\mu}_L U - U \bar{\mu}_R) \]  
(9)
and we will consider only \( \bar{\mu}_L, \bar{\mu}_R \equiv \mu_L \sigma_3, \mu_R \sigma_3 \), where \( \bar{\mu}_L, \bar{\mu}_R \) are matrices and \( \mu_L, \mu_R \) are numbers. The next step is to minimize the potential energy \( V_{\text{chiral}} \):
\[ V_{\text{chiral}} = \frac{f^2}{3} ((\mu^2 - \mu_3^2) Tr(U \sigma_3 U^\dagger \sigma_3) - (\mu_1^2 + \mu_2^2) Tr I) \]  
(10)
One can readily see that in case of \( \mu_V = 0, \mu_A \neq 0 \) the minimum is reached on matrices \( U \) of the form:
\[ U_{\text{min}} = I \cos \phi + \sigma_3 \sin \phi , \]  
(11)
where \( \phi \) is an angle and independence of the energy on \( \phi \) signals presence of a Goldstone boson. In other words, we have a trivial vacuum in this case:
\[ U_{\text{min}} = I , \; \text{ if } \mu_A \neq 0, \mu_V = 0 . \]  
(12)
Similarly,
\[ U_{\text{min}} = \frac{1}{\sqrt{2}} (\sigma_1 + i \sigma_2) , \; \text{ if } \mu_V \neq 0, \mu_A = 0 , \]  
(13)
where \( \sigma_{1,2} \) are Pauli matrices.

Although the cases (12) and (13) look different they can be reduced to each other by a change in notations, or by global chiral rotations of matrices \( U_{\text{min}} \). The point is explained in detail in Ref. [20], see in particular Fig. 2 of this paper. In particular, the \( U(1) \) subgroup which rotates \( \pi_1 \) and \( \pi_2 \) into each other is replaced by the \( U(1) \) subgroup that rotates scalar \( \sigma \) and \( \sigma_3 \) into each other. We will concentrate on the case \( \mu_A \neq 0, \mu_V = 0 \).

So far we tacitly assumed that the kinetic terms vanish identically, since we considered minimum of energy. However the covariant derivative \( (D_\mu U) \) does not vanish if the matrix \( U \) does not depend on time. Instead, we should rather look for a solution of equations of motion. To be more specific, turn to the case (12) and consider the matrix \( U_{\text{solution}} \) explicitly depending on time \( t \) [32]:
\[ U_{\text{solution}} = \exp (i \sigma_3 \mu_A \cdot t) , \]  
(14)
where \( \mu_A \) is now a number. Clearly,
\[ D_0 U_{\text{solution}} = 0 , \]  
(15)
It is also important that the matrix (13) is constructed entirely on the Cartan subgroup, i.e. matrices \( I, \sigma_3 \).

Note that the proportionality to time of the phase of the condensate is a well known signature of superfluidity, see, e.g., [21]. Indeed, the criterion of superfluidity is a specific form of the correlator of \( T_{0i} \) components of the energy-momentum tensor:
\[ < T_{0i} T_{0k} >_{\text{superfluidity}} = (\text{const}) \frac{q_i q_k}{q^2} . \]  
(16)
The solution (14) implies that the \( π_0 \)-field looks as,

\[
\frac{π_0}{f_π} = μ \cdot t + ϕ(x_i),
\]

where the notation “\( π_0 \)” is chosen to make connection with the matrices \( U \) (7) explicit, and the field \( ϕ(x_i) \) satisfies equation \( Δϕ = 0 \). It is then obvious that the criterion (10) is fulfilled. Indeed the \( T_{0i} \) component for a scalar field contains term \( T_{0i} \sim ∂_i φ∂_i φ \) and in our case \( T_{0i} \sim µ∂_i φ \) where \( φ \) is a 3d massless field.

In conclusion of this section let us emphasize that we consider the case of a “small” chemical potential. Namely, introduction of a small chemical potential does not change the absolute value of the vacuum condensate but only rotates it. In other words, one actually assumes that the potential energy contains a term like

\[
with\quad π \gg μ.
\]

\( \Delta \) is the criterion (16) is fulfilled. Indeed the \( 5 \) criterion is explicit, and the field \( ϕ(x) \) enters Eq. (17):

\[
V \sim M \left( UU^\dagger - I \right),
\]

with \( M \gg μ_L, R \). The density \( n \) of particles in the condensate is calculable by taking derivative from the energy with respect to the chemical potential;

\[
n_{\text{v}, A} \sim \frac{∂}{∂μ_{\text{v}, A}} V_{\text{chiral}} = 2f_π^2μ_{\text{v}, A}.
\]

That is, the density disappears with \( μ_{\text{v}, A} \rightarrow 0 \).

Vortices

Vortices in superfluid represent, probably, the earliest example of “defects”, see, e.g., textbooks [22, 23]. General formalism was adapted to the case of relativistic superfluidity in many papers as well, see, e.g., [19, 21, 24, 25].

Let us first recollect most general properties of the vortices [22]. The velocity \( v_i \) of the superfluid component is related to the gradient of the field \( ϕ \) entering Eq. (17):

\[
∂_i ϕ = μv_i .
\]

Note that mostly we are using non-relativistic notations. Fully relativistic equations are always possible to introduce in terms of \( u_μ \).

Because of Eq. (20), naively, rotation of the superfluid is not allowed. However the angular momentum is still transferred to the liquid through vortices which are singular on the (z) axis. Namely, near the axis:

\[
π_0/f_π = μt + nθ
\]

where \( θ \) is the polar angle in plane orthogonal to the axis, \( 0 \leq θ \leq 2π \) and \( n \) is integer. Clearly, the angle \( θ \) is not defined on the axis. This is reflected in a singularity:

\[
(∂_x∂_y - ∂_y∂_x)θ = 2πδ(x, y) ,
\]

which is responsible for the transfer of rotation to the superfluid.

Let us also remind the reader, how the current (4) is related to the chiral anomaly. The point is that introduction of the chemical potential \( μ \) assumes replacing the original Hamiltonian \( H_0 \) by \( H_0 - μQ \), where \( Q \) is the charge associated with the potential \( μ \). In case we are considering

\[
δH = -μ \int d^3x \bar{Ψ}γ_0Ψ - μ5 \int d^3x \bar{Ψ}γ_5γ_0Ψ ,
\]

where \( Ψ \) stands for a generic massless spin 1/2 field.

Generalization, of (23) to the case of liquid is achieved, as usual, by using the 4-vector \( u_μ \). In terms of the density of the Lagrangian we then have:

\[
δL = μu_α \bar{Ψ}γ^αΨ + μ5u_α \bar{Ψ}γ_5γ^αΨ .
\]

Note the similarity of the novel, specific for thermodynamics interaction (24) with the ordinary electromagnetic interaction, \( δL_{el} = eA_α \bar{Ψ}γ^αΨ \). This similarity implies that there is an extension of the standard chiral triangle anomaly which can be generated [23] by the substitution:

\[
eA_α \rightarrow eA_α + μu_α + μ5u_α .
\]

Keeping, for simplicity, \( μ = 0, μ_5 \neq 0 \) we come to the anomalous current \( j_5^α = (μ_5^2/2π^2)ω^α \), where \( ω^α \) is defined in the Eq. (1).

After these preliminary remarks, we are in position to evaluate the anomalous current (4) in our case of pionic superfluidity. This is in three simple steps. First, the generalization of the triangle graph produces the current:

\[
j_5^μ = \frac{1}{4π^2} f_π^2 μ^{μνρσ}(∂_ρπ^σ)(∂_σ∂_νπ^ρ)
\]

The \( π_0 \) field near the axis of a vortex looks as given in Eq. (21). As a result the current induced by the given vortex is:

\[
j_5^i = \frac{μm}{2π} δ(x_{i+1}, x_{i+2}) \quad (i = 1, 2, 3)
\]

At first sight, this result is in variance with Eq. (4) since the current (27) is linear in the chemical potential \( μ \) while the current (4) is quadratic in \( μ \). However, to check the dependence on \( μ \) we have to express the current (27) in terms of the same angular velocity \( ω_5 \) as used in Eq. (4) and we will see in a moment that this removes the apparent discrepancy between (27) and (4).

The fact that \( n \) is integer manifests quantization of the angular momentum carried by the vortex. Indeed, from uniqueness of the phase of the wave function one concludes

\[
∫ \bar{Ψ}α dx_i = 2πn.
\]
and the corresponding value of $n$ (assuming again that vorticity is directed along the 3rd axis) is given by

$$n = \frac{1}{2\pi} \int dx_1 \partial_1 \varphi = \frac{\mu}{\pi} \int d^3 x \omega^3,$$  \hspace{1cm} (29)

where we used $\text{curl} \vec{v} = 2\vec{\omega}$. Substituting [24] into (27) and taking into account that vorticity is non-zero only on the vortices we find out that Eq. (27) matches (3) exactly.

Note that the different sign of topological number $n$ for the antiparticles, when sign of chemical potential changes, leads to the same sign of the axial current, in accordance to its positive C-parity. This is compatible to the same signs of polarization of $\Lambda$ and $\Lambda$ hyperons observed experimentally [1].

Note that rewriting the current (27) in the form similar to (3) is reasonable in case of a particular physical set up. Namely, it is assumed that a bucket with superfluid is rotated with angular velocity $\Omega$ which is kept constant. As is mentioned above, naively, the rotation is not transferred to the fluid because of the condition $\vec{v} = \vec{V} \varphi$. However, this naive picture does not hold because of the singularity [22] and there is a net of vortices in the fluid which carry the same angular momentum as if the liquid were rotated as a whole. Each particular vortex has $n = 1$ since such a configuration minimized the energy [22, 23]. The density of vortices per unit surface is then fixed by the condition that the whole of the angular momentum is carried by the vortices and can be determined, say, from Eq. (29).

### Chiral effects in superfluid as radiative corrections

As is mentioned above, chiral effects in superfluid have been considered in many papers. For our purposes and interpretations it is instructive to view the chiral effects as radiative corrections to the Born approximation which is nothing else but the superfluidity itself. This viewpoint was emphasized recently in [20].

Superfluidity induced by the chemical potential $\mu_A$ (see [16] and a brief summary above) can be described as existence of the current $(j_A^5)_s$:

$$(j_A^5)_s = 2f_\pi^2 \mu_A (u_\alpha)_s \equiv n_A (u_\alpha)_s,$$  \hspace{1cm} (30)

where we utilize Eqs (11) and, for the sake of definiteness, consider the case $\mu_A \neq 0$, $\mu_V = 0$. Moreover, $(u_\alpha)_s$ is the 4-velocity of the superfluid component. Note that the current [23] looks as a standard hydrodynamic current, $j_\alpha = n \cdot u_\alpha$.

Then the chiral magnetic effect is readily recognizable as a one-loop quantum correction to (30) induced by electromagnetic interaction:

$$j_\alpha^{\text{el}} \sim f_\pi^2 \mu_A \frac{e}{4\pi^2} B_\alpha = \frac{f_\pi^2}{4\pi^2} B_\alpha,$$  \hspace{1cm} (31)

where $B_\alpha$ is the magnetic field, $B_\alpha = \epsilon_{\alpha\beta\gamma} \delta^\beta F^\gamma$, and the factor $(4\pi^2)^{-1}$ represents a “typical” numerical factor associated with a loop graph. On the technical side, the precise form of Eq. (31) can be obtained by using the Goldstone-Wilczek current [28] (see also [23]).

It is worth emphasizing that the radiative correction (31) comes from short distances. And the current (31) is to be considered as a polynomial. Indeed, the light particles are represented entirely by pions. The Born-approximation current (20) is due to the flow of pions. On the other hand, the current (31) cannot be supported by pions alone. Indeed, electromagnetic current of pions is a component of an isotopic vector. While to get a non-vanishing contribution (31) one needs an isoscalar component of the electromagnetic current as well. This isoscalar component is manifested in the electromagnetic current associated with baryons, like proton and neutron. Within the framework considered $m_\pi/m_B \to 0$ and (31) is a polynomial.

Now, we come to the central point. The same as in the preceding example, the correction (30) is associated with effective interaction $\mu_A u_\alpha$, see discussion after Eq. (29). The radiative correction induces an axial-vector current with density:

$$\delta j_5 \sim f_\pi^2 \mu_A \frac{\Lambda^2 \omega^5}{4\pi^2 f_\pi},$$  \hspace{1cm} (32)

see Eqs (27), (29). Alternatively, and as discussed above the radiative correction on average generates density of angular momentum $\mathcal{M}$:

$$\mathcal{M} \sim \delta j_5.$$

Now, we come to a central point. The same as in the preceding example, the correction (30) is associated with short distances. Because of this, the current (32) can be identified as the current of heavy particles, or baryons. Moreover, the spatial component of the axial current reduces to the density of spin of baryons, $\delta j_5 \sim <\sigma_\uparrow>$. This conclusion fits nicely [33]. In other words, the core of the vortices is built on spins of heavy particles. For a similar construction see [17, 18].

Note that vortices in the pionic superfluid were considered, in particular, in Refs. [19, 30]. It was assumed that there are massless quark modes propagating along the axes of the vortices. In other words, the confining vacuum is destroyed and perturbative vacuum is seen on the rotation axes. However, as is emphasized in the discussion concluding section 2, destroying the pion condensate does not mean that we reach deconfining, or perturbative vacuum.

For simplicity, we limited ourselves to the pionic fluid with the respective isotopic chemical potential. Because of the relation of $A$ polarization to that of strange quarks [2], the octet contribution to chemical potential may also play a role, although it is not obvious how realistic can be such more complicated model anyway.
In conclusion of this section, let us discuss fixation of the overall coefficients in expressions for the chiral effects. The point is that the Goldstone-Wilczek currents, which are central to evaluate the chiral effects, are finite. It is expressed in terms of Goldstone particles which interact with heavy particles. Superficially, the overall coefficient could depend on how many species of heavy particles, or baryons are kept. However, the overall coefficients are determined from the matching to the triangle anomalies which depend on quantum numbers of quarks in the underlying field theory. In particular, the vortex-related baryonic current is given by (see, e.g., [22]):

\[
(j^5_B)_\alpha = \frac{N_c}{36\pi^2 f_\pi^2} \epsilon_{\alpha\beta\gamma\delta} (\partial^\beta \pi^0)(\partial^\gamma \partial^\delta \pi^0),
\]

where \(N_c\) is the number of colors.

**Discussion**

Recent measurement of \(\Lambda(\bar{\Lambda})\) polarization motivate further theoretical studies of the chiral vortical effect in the confining phase and of the role played in it by heavy particles. We considered the model of pionic superfluidity induced by isotopic-symmetry violating chemical potentials, see [16] and references therein. This is not a realistic model of the quark-gluon plasma but it allows for a remarkably simple and convincing description of a hadronic medium.

Apriori, the impression is that this model is not suited at all to address the issue of evaluation of the polarization of baryons since the model *per se* does not mention heavy particles. However, a more detailed consideration does reveal that heavy particles do play a role as an ultraviolet cut off in radiative corrections. The radiative corrections to superfluidity, in turn, describe the chiral magnetic and vortical effects. In hydrodynamic terms, the radiative corrections correspond to higher orders in derivatives. Introduction of the ultraviolet cut off is crucial to regularize the singularity on the axes of vortices.

Thus, one starts with a model which includes only light, Goldstone particles and comes to the conclusion that the model is inconsistent without invoking heavy particles, or baryons as an ultraviolet cut off. Moreover, the whole of the chiral vortical effect is associated with the spins of heavy particles. In this sense and qualitatively, the polarization of baryons turns to be large.

Thus, the vortex in superfluid looks as follows, see also [17]. On the periphery of the vortex there are light degrees of freedom, circling the core. The core is built on the polarization of heavy particles. This picture is a kind of complementary to the vortex considered by Callan and Harvey [29]. In the latter case the core is circled by spins of heavy particles, with electric current flowing towards the core. The core itself is occupied by light (massless) degrees of freedom. Finally, in case of magnetic vortices the magnetic moment is curling the core on the periphery and look along the core in the middle.

There are many reservations to applying the picture emerging to the realistic plasma. First of all, the heavy particles in case of pionic superfluidity are rather virtual, not real. Then, it is also important that the baryonic current [31] is an isotopic scalar and its consideration is somewhat ambiguous because of the gluonic anomaly. Nevertheless, one might hope that vortices in pionic superfluid medium provide us with an example of mechanism of transformation of rotation of plasma to polarization of baryons. Also, vortices might be relevant to the quark-quark gluon plasma since the viscosity of the plasma is known to be low.

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[31] Note that we keep pion mass vanishing. In fact, the effect of small masses could be iso included, see [16, 20].

[32] We follow here [26].