The wood composite ribbed panels on mechanical joints

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Abstract. Structures in the form of ribbed panels with sheathings from sheet materials are used in construction of buildings and structures of various purposes: civil, industrial, agricultural, etc. Panels are universal enclosures and supporting structures, they can be used as coverings, floors, walls. Ribbed coating slabs simultaneously perform the functions of roof purline, floorboards, sheathing, provide heat and sound protection of the building. They are characterized by unification, high level of factory readiness; the ability to control the strength, sound and heat-insulating properties, and low density of materials used, which allows significantly reducing the cost of laying the foundation of the building. The paper presents the results of investigations of the influence of various parameters on the stress-strain state of panels (type of joints, coefficient of stiffness of the bonded joints, the magnitude of the span, the width of the panel, and the material of anisotropic sheathing). To solve this problem, the well-known solution of the plane stress problem is used (Faylon method), in which the constants of integration are determined as the Fourier series expansion coefficient of contour loads. As a result of numerical researches, reduction coefficients for bendable ribbed wood-composite panels with mechanical joints were obtained.

1 Introduction

The degree of influence of the sheathings in the work of the ribs is ensured by the joints of the shearing forces applied along the edges of the longitudinal ribs. The normal stresses across the width of the section will be unevenly distributed. This phenomenon is called the "shear lag". The accuracy of the calculation of ribbed panel structures largely depends on the correctness of the sheathing calculation involved in the overall work. The extent to which the siding is included in the overall work is estimated by the reduction coefficient. Reduction coefficient being known, it is possible to calculate such structures with sufficient for practical purposes accuracy. The method of calculating panels on a wooden frame, presented in domestic and foreign design standards [1, 2], regulates the procedure for calculating only panels with plywood sheathing, which is connected to the ribs with glue.

Despite the fact that traditionally a rigid glue joint is used at the border between the rib and the sheathing [3, 4], a number of experimental – theoretical studies [5-8] have shown that it is reasonable to take sheathing into account when fastening it to the ribs, modern bonded

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joints being used. The use of joints with increased resistance to longitudinal shear [9] allows their use as an alternative to adhesive joints [10, 11]. It is also worth noting that conventional panels with sheathings made of wood-based materials, such as DSP [12], OSB [13], etc.

The use of bonded joints at the border between the rib and the sheathing changes the nature of the distribution of tangential stresses, and for materials such as plywood and OSB there is a significant difference in elastic and anisotropic properties compared to CSBs. These factors will undoubtedly influence the sheathing VAT, and should also be taken into account when determining its width. The purpose of the work is to use the modified solution of the plane stress problem (Faylon method of a thin anisotropic plate [14, 15] to study the effect of various design parameters (type of joints, rigidity of bonded joints, material of sheathing) on the distribution of normal compressive stresses in the cross section of sheathing made of bendable panels, to get the coefficients of the reduced width, compare them with the data of the calculation according to current standards.

The floor panel can be considered as a spatial system consisting of a thin slab and longitudinal ribs supporting it. A significant contribution to the improvement of the design of the panels and the calculation method was made by A. B. Gubenko, V.F. Ivanov, K.P. Kashkarov, M.E. Kagan, G.G. Carlsen, M.F. Kovalchuk, I.M. Linkov, et al. The research of such scientists as Abovskiy, P.A. Dmitriev, V.I. Zhadanov, A.A. Zhuravlev, I.S. Inzhutov, S.A. Korzon, V.G. Lenov, B.V. Labudin, B.K. Mikhailov, A.Ya. Naichuk, R.B. Orlovich, K.P. Pyatikrestovsky, E.N. Serov, V.Ya. Terentyev and others is devoted to the problems of development and improvement of the theory of lamellar structures.

2 Materials and Methods

Calculation of ribbed panels on a wooden frame can be produced according to the beam scheme, as elements of T-shaped or I-sections, the sheathing of which act as shelves. Normal stresses directed along the axis of the panel, appearing in the shelves, will have a maximum value at the ribs, and decrease with distance from them. In the middle of the sheathing the stresses will be minimal. Therefore, while making calculation reduced width of sheathing should be used. The reduced width is determined taking into account that the values of the greatest longitudinal stresses for the actual and reduced section are equal [4].

When calculating panels [1, 2], the actual cross section is replaced with the one shown (figure 2). The reduced width of the panel sheathing section is calculated by multiplying the actual width of the \( b_{act} \) by the coefficient of reduction the sheathing (reduction ratio) \( k_{red} \).

![Fig. 1. Structure of wood composite ribbed panel with upper sheathing: a - panel with bonded joints; b – panel with hard adhesive joint of sheathing.](https://doi.org/10.1051/e3sconf/20199102021)
where \( \sigma_x^m \) – is the average normal stress across the entire width of the sheathing in the cross section of the panel, calculated by the formula (2); \( \sigma_{x}^{\text{max}} \) – maximum normal stress.

\[
\sigma_x^m = \frac{1}{b_{sh}} \int_a^{b_{sh}} \sigma_x dy \approx \frac{1}{b_{sh}} \sum_{i=1}^{n} \sigma_i \cdot \Delta y_i ,
\]

where \( \sigma_i \) – is the value of the normal stress at each point across the width of the cross section; \( \Delta y_i \) – is infinitely small width of the area between the points of stress measurement; \( b_{fact} \) – dimensional width of the panel.

The reduced width of the panel section is calculated by multiplying the actual width \( b_{fact} \) coefficient of reduction \( k_{red} \).

\[
b_{calc} = k_{red} \cdot b_{fact} ,
\]

where \( k_{red} \) – coefficient of reduction of the estimated width of the sheathing.

Neglecting the thickness and bending of the sheathing, and assuming that the forces are transferred from ribs to the sheathing in its middle plane, the stress distribution in the sheathing can be considered as 2-dimensional. In this case, the differential equation of a flat orthotropic plate will look like [16]:

\[
\frac{1}{E_1} \frac{\partial^2 \varphi}{\partial y^2} + \left( \frac{1}{G} - \frac{2\mu_2}{E_1} \right) \frac{\partial^2 \varphi}{\partial x^2} + \frac{1}{E_2} \frac{\partial^2 \varphi}{\partial x \partial y} = 0,
\]

where \( \varphi = \varphi (x; y) \) – Erie stress function; \( E_1 \) – the elastic modulus of sheathing in the direction of the longitudinal ribs; \( E_2 \) – the elastic modulus of sheathing in the perpendicular direction; Poisson's ratio; \( G \) – shear modulus.

Fig. 2. Scheme of the distribution of normal compressive stresses in the sheathing:

a) - cross-section of the panel and the distribution of stresses in the sheathing for panels with rigid
connection; b) - cross-section of the panel and the distribution of stresses in the sheathing for panels with bonded joints; c) scheme of the calculated cross-section and the diagram of stresses.

In modified solution of the Faylon type, the function \( \varphi(x, y) \) has the form:

\[
\varphi(x, y) = \sum_{n=1}^{\infty} \left\{ -\frac{1}{\alpha^2} \left[ a_n \psi_1(y) + b_n \psi_2(y) + c_n \psi_3(y) + g_n \psi_4(y) \right] \right\} \sin \alpha x,
\]

where \( \alpha = n\pi / l \), \( n=1,2,3,\ldots \); \( l \) - panel length; \( a_n; b_n \) - the coefficients of decompositions into trigonometric rows of the contour, normal to the longitudinal edges of the loads, respectively \( y = \pm \frac{b}{2} \), \( b \) - panel width; \( c_n; g_n \) - the same tangents; \( \psi_i(y) \) - functions depending on the roots of the characteristic equation:

\[
r^4 - m_1 \alpha^2 r^2 + m_2 \alpha^4 = 0,
\]

where \( m_1 = E_1 / G - 2 \nu_1 \); \( m_2 = E_1 / E_2 \).

The distribution function of normal stresses \( \sigma \) at initial loads applied only along the long edges of the sheathing:

\[
\sigma(x, y) = \frac{\partial^2 \varphi}{\partial y^2} = \sum_{n=1}^{\infty} -\frac{1}{\alpha^2} \left[ c_n \psi_3(y) + g_n \psi_4(y) \right] \sin \alpha x
\]

\[\text{Fig. 3. Diagram of the action of tangential loads on the longitudinal edges of the skin.}\]

For complex roots of equation (6) of the form \( \pm s \pm ki \), then the functions \( \psi_i \) have the form [15]:

\[
\psi_3(y) = \varphi_s ch(sa y) \cdot \cos(k\alpha y) - \varphi_s sh(sa y) \cdot \cos(k\alpha y) + \varphi_s ch(sa y) \cdot \sin(k\alpha y) - \varphi_s sh(sa y) \cdot \sin(k\alpha y),
\]

\[
\psi_4(y) = -\varphi_s ch(sa y) \cdot \cos(k\alpha y) - \varphi_s sh(sa y) \cdot \cos(k\alpha y) + \varphi_s ch(sa y) \cdot \sin(k\alpha y) + \varphi_s sh(sa y) \cdot \sin(k\alpha y).
\]
and the coefficients $\phi_i$ are determined by the expressions: $\phi_3 = A \cdot \text{sh}(s \cdot u_n)\cos(k \cdot u_n)$; $\phi_4 = A \cdot \text{ch}(s \cdot u_n)\sin(k \cdot u_n)$; $\phi_5 = B \cdot \text{sh}(s \cdot u_n)\sin(k \cdot u_n)$; $\phi_6 = B \cdot \text{ch}(s \cdot u_n)\cos(k \cdot u_n)$; $u_n = n\pi b/l$, the coefficients $A$ and $B$ according to the formulas (10):

$$A = \frac{1}{k \cdot \text{sh}(2su_n) - s \cdot \sin(2ku_n)}, \quad B = \frac{1}{k \cdot \text{sh}(2su_n) + s \cdot \sin(2ku_n)}.$$  \hfill (10)

With real roots of equation (6) of the form $s_1, s_2$:

$$\psi_3(y) = 0.5 \cdot (-\phi_2 \text{ch}(s_1 \alpha y) + \phi_4 \text{ch}(s_2 \alpha y) - \phi_3 \text{sh}(s_1 \alpha y) + \phi_6 \text{sh}(s_2 \alpha y)), \quad \psi_4(y) = 0.5 \cdot (\phi_2 \text{ch}(s_1 \alpha y) - \phi_4 \text{ch}(s_2 \alpha y) - \phi_3 \text{sh}(s_1 \alpha y) + \phi_6 \text{sh}(s_2 \alpha y)).$$  \hfill (11)

The coefficients $\phi_i$ are determined by the expressions: $\phi_3 = A \cdot \text{ch}(s_2 \cdot u_n)$; $\phi_4 = A \cdot \text{sh}(s_2 \cdot u_n)$; $\phi_5 = B \cdot \text{sh}(s_1 \cdot u_n)$; $\phi_6 = B \cdot \text{ch}(s_1 \cdot u_n)$ the coefficients $A$ and $B$ according to the formulas (13):

$$A = \frac{1}{s_2 \text{ch}(s_2 u_n) \text{sh}(s_2 u_n) - s_2 \text{sh}(s_2 u_n) \text{ch}(s_2 u_n)}; \quad B = \frac{1}{s_2 \text{ch}(s_2 u_n) \text{sh}(s_2 u_n) - s_1 \text{sh}(s_2 u_n) \text{ch}(s_2 u_n)},$$  \hfill (13)

The boundary conditions in the case of a panel with two longitudinal edges: $\sigma_3(-b/2) = \sigma_3(b/2) = 0$.

Coefficients $a_n = b_n = 0$, decomposition coefficients $c_n = -g_n$ determined by Fourier formulas:

$$c_n = \frac{2}{l} \int_0^l \tau(x) \cos(\alpha x) dx; \quad g_n = -\frac{2}{l} \int_0^l \tau(x) \cos(\alpha x) dx.$$  \hfill (14)

For a panel with bonded joints loaded with a uniformly distributed load, the shear stress at the edge $\tau(x)$ according to [17] bonds is determined by relationship (12).

$$\tau(x) = \frac{\xi c q \left( \lambda^2 l - 2 \lambda^2 x + 2 \lambda \sinh(\lambda x) - \lambda^2 l \cosh(\lambda x) \right)}{2 \sum EI \cdot \lambda^3} + \frac{\xi c q \cdot \cosh(\lambda x) \cdot \left( \lambda l \sinh(\lambda l) - 2 \cosh(\lambda l) + 2 \right)}{2 \sum EI \cdot \lambda^3 \sinh(\lambda l)},$$  \hfill (15)

where $x$ is the coordinate measured from the left support of the panel; $q$ - uniformly distributed load, reduced to the heat one edge; $l$ - span panel; $\xi$ is the stiffness coefficient of bonded joints, for a panel with a rigid glue connection $\xi = \infty$; $\lambda$ - coefficient determined by the formula:

$$\lambda = \sqrt{\xi \cdot \gamma},$$  \hfill (16)

$$\gamma = \frac{1}{E_s A_{sh}} + \frac{1}{E_v A_v} + \frac{c^2}{\sum EI},$$  \hfill (17)
where $E_{sh}$, $E_r$ are the elastic moduli of the sheathing and the rib, respectively (in the direction of the panel span); $A_{sh}$, $A_r$ is the cross-sectional area of the sheathing and rib, respectively; $\sum EI$ is the sum of the bending stiffness of the sheathing and the rib.

The coefficient $\xi$ is determined by the formula (18) and depends on the shear stiffness of the compounds used, and on the step of their placement [10].

$$\xi = \frac{n \cdot c_c}{S_c},$$

where $c_c$ is the stiffness coefficient of 1 compound; $S_c$ is the joining step, $n$ is the number of ribs.

The values of the maximum and middle stresses are determined by formulas (19) and (20), respectively:

$$\sigma_{s,\max} = \frac{1}{t_{sh}} \cdot \sum_{n=1}^{\infty} \frac{1}{\alpha} c_n \left[ \psi_n'(-b/2) - \psi_n'(-b/2) \right] \sin \alpha x,$$

$$\sigma_{s,m} = \frac{1}{t_{sh} \cdot b} \int 2t(x) \cdot 1 \cdot dx = \frac{2}{t_{sh} \cdot b} \int c_n \cos \alpha x \cdot dx. \quad (20)$$

where $t_{sh}$ is the thickness of the sheathing.

After calculating the maximum and middle values of stresses, the coefficient of reduction of the covering $k_{sh}$ is determined by the formula (1).

### 3 Results

The presented solution will be considered on a concrete example: a wood composite panel with dimensions of $1.0 \times 3.0$ m with two longitudinal ribs (pine / spruce, grade II), hinged and supported at the ends to the wall piping. The thickness of the sheathing is 18 mm, the cross-section of the ribs is $50 \times 150$ mm. The sheathing is connected to the ribs by flexible joints on the screws (the pitch of the screws for fastening the upper casing is 100 mm, the bottom one is 200 mm). Each edge of the panel is affected by its own weight of elements, filing and payload ($\sum g = 3$ kN / m). 3 options for sheathings are considered: plywood FK according to GOST 3916.1−96; OSB / 3 EN 300 plates; cement – particleboard DSP GOST 28816−2016. The stiffness coefficient of the connections $\xi$ is considered in the range of 5000 ... 35000 kN / m², this range of values can vary due to the use of various types of ties (nails, screws, brackets, combined compounds), as well as changes in the spacing of their arrangement. The results of the calculations are summarized in Table 1. The dependences of the maximum $|\sigma_{s,\max}|$ and average $|\sigma_{s,m}|$ Stresses from sheathing material and bond stiffness are shown in Fig. 4.

| Coefficient of bond stiffness $\xi$, kN/m² | Plywood | OSB | CSB |
|------------------------------------------|---------|-----|-----|
| $|\sigma_{s,m}|$ | $|\sigma_{s,\max}|$ | $k_{red}$ | $|\sigma_{s,m}|$ | $|\sigma_{s,\max}|$ | $k_{red}$ | $|\sigma_{s,m}|$ | $|\sigma_{s,\max}|$ | $k_{red}$ |
| 5000 | 776 | 1548 | 0.501 | 762 | 977 | 0.780 | 769 | 1173 | 0.656 |
| 10000 | 1253 | 2503 | 0.501 | 1205 | 1546 | 0.779 | 1225 | 1869 | 0.655 |
| 15000 | 1576 | 3150 | 0.500 | 1493 | 1918 | 0.778 | 1525 | 2329 | 0.655 |
| 20000 | 1809 | 3619 | 0.500 | 1869 | 2180 | 0.778 | 1738 | 2657 | 0.654 |
| 25000 | 1985 | 3973 | 0.500 | 1846 | 2375 | 0.777 | 1897 | 2902 | 0.654 |
| 30000 | 2122 | 4250 | 0.499 | 1961 | 2526 | 0.776 | 2020 | 3092 | 0.653 |
| 35000 | 2231 | 4473 | 0.499 | 2053 | 2645 | 0.776 | 2117 | 3244 | 0.653 |
| $\xi=\infty$ (hard connection) | 2788 | 5624 | 0.496 | 2502 | 3246 | 0.771 | 2601 | 4012 | 0.648 |

Table 1. The results of the calculation of stress in sheathing.
4 Discussion

According to the data obtained (Table 1), the coefficient of reduction depends to a large extent on the material of the sheathing, and, to a lesser extent, on the rigidity of shear bonds. The smallest values of the coefficient \( k_{\text{red}} \) correspond to plywood lining, and are 0.496 \ldots 0.501. For CSB, the indicators of this coefficient have intermediate values, and make up 0.648 \ldots 0.656. The highest rates of reduction coefficient correspond to OSB-sheathing, and make up 0.771 \ldots 0.78. It has been established that the calculations by Russian standard and EN \[1\] give greatly underestimated values of the coefficient of the reduced width of the plywood siding \((k_{\text{red}} = 0.16 \text{ in the calculation according to and 0.3 - according to [1] taking into account the delay of the shift})\). These standards can not be applied for plating of other materials.

5 Conclusions

1. The sheathing material and its physic mechanical characteristics significantly influence the reduction coefficient of sheathing \( k_{\text{red}} \). For OSB sheathings, the reduction factors were 1.55 times higher, for CSBs - 1.31 times higher than for plywood sheathing. So the type of sheathing material should be taken into account in the calculations when determining the reduced width of the calculated section shelf.

2. With an increase in the stiffness coefficient of shear joints, an insignificant decrease in the reduction coefficient \( k_{\text{red}} \) is observed for all the sheathings considered, however, the revealed differences in the coefficients are insignificant (up to 1% for plywood, 1.2% for OSB and CSB) and therefore they may not be taken into account in calculations.
3. The method of determination is recommended to use in the calculations of wood composite panels on mechanical joints, as well as to clarify the existing methodology in the regulatory documents.

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