Chiral Symmetry and Neutrino Pion Production off the Nucleon

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Abstract. The neutrino pion production off the nucleon is traditionally described in the literature by means of the weak excitation of the $\Delta(1232)$ resonance and its subsequent decay into $N\pi$. Here, we present results from a model that includes also some background terms required by chiral symmetry. We show that the contribution of these terms is sizeable and leads to significant effects in total and partially integrated pion production cross sections at intermediate energies of interest for neutrino oscillation experiments. Finally, we discuss parity-violating contributions to the pion angular differential cross section induced by the interference of these non-resonant terms with the $\Delta$ piece.

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INTRODUCTION

The interest of studying pion production off the nucleon or off some nuclei like oxygen or carbon is twofold. First, it helps to unravel some aspects of the hadronic structure which are not accessible to electromagnetic probes. Second, these processes play an important role in the analysis of the present neutrino oscillation experiments, where they constitute a major source of uncertainty in the identification of electron and muon events. In this talk, we focus on the neutrino and antineutrino pion production off the nucleon driven both by Charged and Neutral Currents (CC and NC) at intermediate energies. The model presented here will allow us to extend the results of Refs. [1] for NC and Ref. [2] for CC driven neutrino-nucleus reactions in the quasielastic region, to higher nuclear excitation energies up to the $\Delta(1232)$ resonance region.

There have been several studies of the weak pion production off the nucleon at intermediate energies [3–7]. Most of them are based on the dominance of the weak excitation of the $\Delta(1232)$ resonance and its subsequent decay into $N\pi$, neglecting thus any contribution arising from non-resonant mechanisms. Here in addition, we also consider some background terms required by the pattern of spontaneous chiral symmetry breaking of QCD. Their contributions [8] are sizeable and lead to significant effects in total and partially integrated pion production cross sections even at the $\Delta(1232)$—resonance peak, and they are dominant near pion threshold. As a consequence, when we compare the predictions of the model with the ANL bubble chamber cross section data [8], we need to use an axial $\Delta W$ strength substantially smaller than that traditionally assumed in the literature and deduced from the off diagonal Goldberger-Treiman relation (GTR). Corrections to GTR turn out to be smaller if the BNL data [9] were considered. Some more details and extended results can be found in Ref. [10].

THE $W^+ + N \rightarrow N'\pi$ REACTION

We will focus on the $\nu_l(k) + N(p) \rightarrow l^-(k') + N(p') + \pi(k_\pi)$ reaction, though the generalization to antineutrino induced reactions and/or NC driven processes is straightforward [10]. The unpolarized LAB fifth differential cross section (kinematics is sketched in Fig 1) is determined by the contraction of the leptonic $(L)$ and hadronic $(W)$ tensors,

$$d^5\sigma_{el} = \frac{d\Omega(k')dE'd\Omega(k_\pi)}{4\pi^2 G^2} \left|\hat{k}\right|^2 \int_0^{\infty} d[k_\pi][k_\pi]^2 L_{\mu\sigma}^{(3)}(W_\mu\sigma)_{\pi}^{(3)}$$

with $L_{\mu\sigma}^{(3)} = k_\mu' k_\sigma + k_\sigma' k_\mu - k' \cdot k g_{\mu\sigma} + i\epsilon_{\mu\sigma\alpha\beta} k'^\alpha k_\beta$, while the hadronic tensor is written in terms of the matrix

\footnotetext{1}{Some non-resonant terms were also studied in Refs. [4] and [6]. In the latter reference, the chiral counting was broken to account explicitly for $\rho$ and $\omega$ exchanges in the $t$—channel, while the first work is not consistent with the chiral counting either and it uses a rather small axial mass ($\approx 0.65$ GeV), as well.
elements of the quark CC $j_{cc+}^\mu$,
\[
(W_{\mu\alpha}^{CC})^{(v)} = \sum_{\text{spin}} \int \frac{d^3 p'}{(2\pi)^3} \frac{\delta^4(p' + k_\pi - q - p)}{8M_E H} \times \langle N' | j_{cc+}^\mu | N \rangle \langle 0 | N' | 0 \rangle \langle N | \rangle^*(2)
\]
with $q_\mu = k_\mu - k_{\mu}'$, $M$ the nucleon mass. Lorentz invariance restricts the dependence of the cross section on $\phi_\pi$ (pion azimuthal angle),
\[
\frac{d^3\sigma_{\nu l}}{d\Omega(k')dE'd\Omega(k_\pi)} = \frac{\tilde{k}' | G^2 |}{k | 4\pi^2} \left\{ A + B\cos\phi_\pi + C\cos2\phi_\pi + D\sin\phi_\pi + E\sin2\phi_\pi \right\}
\]
with $A, B, C, D$ and $E$ real, structure functions, which depend on $q^2$, $p \cdot q$, $k_\pi \cdot q$ and $k_\pi \cdot p$. To compute the background contributions to $\langle N' | j_{cc+}^\mu | 0 | N \rangle$, we start from a chiral SU(2) non-linear $\sigma$ model involving pions and nucleons. We derive the corresponding Noether's vector and axial currents and those determine, up to some form-factors (constrained by CVC and PCAC and the experimental data on the $q^2$ dependence of the $WNN$ vertex), the contribution of the chiral non-resonant terms. To include the $\Delta$ resonance, we parametrize the $W^+N \rightarrow \Delta$ hadron matrix element as in [3] with the set of form-factors used in [7]. In particular, for the dominant $C_3^A(q^2)$ axial form–factor, we initially take
\[
C_3^A(q^2) = \frac{1.2}{(1 - q^2/M_{AA}^2)^2} \times \frac{1}{1 - q^2/5M_{AA}^2}
\]
with $M_{AA} = 1.05$ GeV and $C_3^A(0)$ is fix to the prediction of the off-diagonal GTR. The model consists of seven diagrams (right panel of Fig.1) and it is an extension of that developed in Ref. [11] for the $eN \rightarrow e'N\pi$ reaction.

**DISCUSSION**

In Fig.2(left panel) we present the flux averaged $q^2$ differential $\nu_\mu p \rightarrow \mu^- p \pi^+$ cross sections from the ANL experiment, which incorporates a $\pi N$ invariant mass cut $W \leq 1.4$ GeV. The agreement with the ANL data is certainly worsened when the non-resonant terms, required by chiral symmetry, are considered (dashed-dotted line). This strongly suggests the re-adjustment of the dominant $C_3^A(q^2)$ form–factor. A $\chi^2$–fit leads to
\[
C_3^A(0) = 0.867 \pm 0.075, \quad M_{AA} = 0.985 \pm 0.082\text{GeV},
\]
We observe a correction of the order of 30% to the off diagonal GTR. Results for total and differential neutrino, antineutrino and NC cross sections for different isospin channels can be found in Ref. [10]. In general, the inclusion of the chiral symmetry background terms brings in an overall improved description of data as compared to the case where only the $\Delta$ pole mechanism is considered (see for instance the right panel of Fig.2).

The interference between the $\Delta$ pole and the background terms produces parity-violating contributions [10] (structure functions $D$ and $E$ in Eq. (3)) to $d^3\sigma_{\nu l}/d\Omega(k')dE'd\Omega(k_\pi)$, as shown in Fig. 3. Such dependences on $\phi_\pi$ might be used to constrain the axial form factor $C_2^\mu$. On the other hand, the NC cross section dependence on $\phi_\pi$ constitutes a potential tool to distinguish $\tau_\nu$– from $\nu_\tau$, below the $\tau$–production threshold, but above the pion production one [12].

We have recently extended the present study to describe two pion production processes near threshold [13].

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FIGURE 1. Left: Definition of the different kinematical variables used through this work. Right: Model for the $W^+N \rightarrow N'\pi$ reaction. It consists of seven diagrams: Direct and crossed $\Delta(1232)$ (first row) and nucleon (second row) pole terms, contact and pion pole contribution (third row) and finally the pion-in-flight term. The circle in the diagrams stands for the weak transition vertex.

FIGURE 2. Theoretical results and ANL data for $\nu\mu p \rightarrow \mu^- p\pi^+$. Left: Flux averaged $q^2$–differential cross section $\int_{M_{\Delta} - M_{\pi}}^{M_{\Delta} + M_{\pi}} dW \frac{d^3\sigma}{dq^2 dW}$. Right: Flux averaged $\pi N$ invariant mass distribution of events. Dashed lines stand for the contribution of the $\Delta$ pole term with $C_A^{\Delta}(0) = 1.2$ and $M_A^{\Delta} = 1.05$ GeV. Dashed–dotted and central solid lines are obtained when the full model of Fig. 1 is considered with $C_A^{\Delta}(0) = 1.2$, $M_A^{\Delta} = 1.05$ GeV (dashed-dotted) and with our best fit parameters $C_A^{\Delta}(0) = 0.867$, $M_A^{\Delta} = 0.985$ GeV (solid). We also show the 68% CL bands from Eq. (5).

FIGURE 3. $D$ and $E$ structure functions (Eq. (3)) for the $\nu p n \rightarrow \mu^- p \pi^0$ and $\bar{\nu}_p p \rightarrow \mu^+ n\pi^0$ reactions, as a function of the pion polar angle (in the CM $\pi N$ frame). The neutrino incoming energy is $E = 1.5$ GeV, $q^2 = -0.5$ GeV$^2$, $W = M_A$ GeV. Dashed lines stand for the contribution of the excitation of the $\Delta^0$ resonance and its subsequent decay with $C_A^{\Delta}(0) = 1.2$ and $M_A^{\Delta} = 1.05$ GeV. Dashed–dotted and solid lines are obtained when the full model of Fig. 1 is considered with $C_A^{\Delta}(0) = 1.2$, $M_A^{\Delta} = 1.05$ (dashed-dotted) and with our best fit parameters $C_A^{\Delta}(0) = 0.867$, $M_A^{\Delta} = 0.985$ GeV (solid).