Imperfection of the convergent cross-mapping method

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Abstract. In 2012, the Convergent Cross Mapping method for finding a causal relationship between system variables from their time series was published. This method is widely used in the study of systems of various nature - from assessing the effect of cosmic radiation on the climate, to the study of cerebral activity. The relevance and prospects of using this method prompted us to master it and to apply it to identifying causal relationships between the variables of the "biosphere-climate" system. The obtained results seemed suspicious and forced us to check carefully the adequacy of the CCM conclusions in relation to various model systems. This paper presents examples of model situations when the method gives false estimates of the presence of causal relationships and offers a possible explanation for the causes of false estimates appearance.

1. Introduction
The main goal of scientific research is to search for patterns of interaction between elements of the biological system under study and determine the influencing external factors to identify the "mechanisms" that determine its behavior. There are known attempts to construct a clear theoretical and experimental procedure for identifying cause-and-effect relationships of the system under study [1, 2]. In these works, sophisticated procedures are proposed based on the conjugate analysis of stationary values of the measured parameters and the non-stationary response of the system to an external disturbance.

Unfortunately, it is not always possible to fully control the state of all dynamic variables of the system under study and/or to have a trial effect on them. Therefore, the identification of causality by time series is important in biology, climatology, epidemiology, financial regulation, industry, etc.

Since 1969, the so-called "Granger causality" (GC) assessment used in economic analysis [3]. According to this approach, a variable $X$ is a Granger cause of $Y$ (in some idealized model) if the predictability of $Y$ falls after removing $X$ from the set of all possible influencing variables. A key requirement of the GC is the "separability" of indicators, namely, when information about the influencing factor is independent of other variables. The requirement of separability means that the GC is not correctly applied to interdependent variables. For example, when evaluating the impact of a predator on a given type of prey, it is necessary to assume that the number of preys does not affect the number of predators, which is possible in quite rare cases. In other words, separability implies that systems can be understood piecemeal rather than as a whole.

In [4], a method of convergent cross-mapping (CCM) was proposed for detecting cause-and-effect relationships in nonlinear dynamical systems, which is free from the limitations of the Granger method. One of the fundamental ideas of this work is that if causality is unidirectional, i.e. $X \Rightarrow Y$ (X affects Y),
then one can evaluate $X$ by $Y$, but not $Y$ by $X$. This is counterintuitive and for examplesuggests that if the weather affects the fish population, then we can use fish to reconstruct the weather, but not vice versa. The paper [4] provides an illustration, and in a sense, proof, of the correctness of this idea.

In addition to the description of the CCM paper [4] presents the approbation of this method on test examples, as well as on two examples of real ecosystems. The first example was the application of CCM to time series obtained in a classic experiment conducted for the first time by the biologist G. F. Gause who used the slipper infusoria (Paramecium caudatum) as a victim, and the Didinium nasutum infusoria as a predator. The second example included time series of numbers of Pacific sardines, Northern anchovies, and sea surface temperature. Later, this method was applied in the work of Tsonis et al. [5], where it was based on the conclusion that galactic cosmic rays have a moderate effect on the change in the average annual global temperature of the Earth's surface. Some examples of CCM application are presented in resent papers [6-9].

However, a thorough analysis of the applicability of CCM to identify cause-and-effect relationships in various test cases showed that the results obtained by this method cannot serve as evidence of these relationships. Given paper provides a brief description of the CCM and provides some examples of nonlinear dynamical systems for which this method gives erroneous causality estimates.

2. Convergent Cross Mapping method

The interaction between variables of a dynamical system can be represented by some manifold. Then, if the first variable depends on the second one it gives possibility to get an estimate of this second variable by the value of the first one. This statement is well illustrated in [4]. The formal presentation of this possibility and the actual conclusion of this quantitative assessment are, in short, the essence of the CCM.

To illustrate, consider a manifold representing the well-known Lorentz attractor $M(t) = \{X(t), Y(t), Z(t)\}$. Consider two time series of length $L$, $\{X\} = \{X(1), X(2), ..., X(L)\}$ and $\{Y\} = \{Y(1), Y(2), ..., Y(L)\}$, corresponding to the two variables $X$ and $Y$. Let's form a set of vectors constructed from lagging variables

$$X^*(t) = \{X(t), X(t-\tau), X(t-2\tau), ..., X(t-(E-1)\tau)\}$$

where $\tau$ varies from $1+(E-1)\tau$ to $t=L$, and where $E$ is the dimension of the variable space of the system (for example 3). This set of vectors for the variable $X$ is called a shadow manifold and is denoted by $M_s$ [10]. In a similar way the shadow manifold $M_f$ is formed for the variable $Y$.

To create a cross-mapped estimate of $Y(t)$, denoted by $\hat{Y}(t)|M_s$, we find the nearest neighbors in the shadow manifold $X^*(t) E+1$. Next, we denote the time indices (from the nearest to the most recent) $E+1$ of the nearest neighbors $X^*(t)$ $t_1, ..., t_{E+1}$. To estimate $\hat{Y}(t)$ from locally weighted average $E+1$ values of $Y(t_i)$, the formula is used

$$\hat{Y}(t)|M_s = \sum_{i=1}^{E+1} w_i Y(t_i)$$

where $w_i$ is the weighting factor that depends on the distance between $X^*(t)$ and the $i$-th nearest neighbor on $M_s$. $Y(t_i)$ are the corresponding $Y$ values. Weight coefficients are determined by the following dependencies:

$$w_i = \frac{u_i}{\sum_{j=1}^{E+1} u_j}$$

where $u_i = \exp[-d(X^*(t),X^*(t_i))]/d(X^*(t),X^*(t_j))$, and $d(X^*(s),X^*(t))$ is Euclidean distance between two vectors. The cross-mapping from $Y$ to $X$ is defined similarly.

If $X$ and $Y$ are dynamically linked, the nearest neighbors of $M_s$ must determine the time values corresponding to the nearest neighbors on $M_f$. As $L$ increases the attractor fills the manifold and the distance between $E+1$ nearest neighbors decrease. Therefore, $\hat{Y}(t)|M_s$ must converge to $Y(t)$, and $\hat{X}(t)|M_f$ must converge to $X(t)$. Thus, the key parameter for determining cause-and-effect relationships in this method is the convergence denoted by $\rho$, which is the Pearson correlation determined between the
measured $Y(t)$ and the predicted $\hat{Y}(t)|M_x$ values and increases with increasing time-series length $L$ to converge to a plateau value.

The criterion $\rho$ actually states that if there is a convergence between $M_x$ and $M_y$, then there is some common manifold $M$ corresponding to some dynamical system, otherwise, if there is no convergence, then there is no such system, and therefore there are no causal relationships between the variables under consideration.

Here is one of the examples from the article [4], demonstrating the ability of the method to detect the presence of cause-effect relationships set by the cross coefficients of the model (1). This example shows that in the absence of external similarity of the time series (figure 1a), resulting in a low value of the correlation coefficient (0.056), CCM confidently indicates the presence of relationships between variables, and moreover, allows us to estimate their intensity (in this example, the effect of $X$ on $Y$ is significantly more than reverse effect that displayed the curves of increasing $\rho$ criterion (figure 1b)).

$$\begin{align*}
X(t+1) = & \ X(t)[3.8 - 3.8X(t) - 0.02Y(t)] \\
Y(t+1) = & \ Y(t)[3.5 - 3.5Y(t) - 0.1X(t)]
\end{align*}$$

(1)

![Figure 1](image_url)

**Figure 1.** Convergence of the Sugihara’s test example. Subfigure a shows model (Lotka-Volterra type) time series $X$ and $Y$. Subfigure b shows dependency between length of tested time series ($L$) and correlation coefficient between real variable and variable predicted with CCM. Increasing of correlation with increasing $L$ is the sign of cause-effect relationships. Region with negative correlation excluded from bottom figure.

### 3. Test examples

Unfortunately, as mentioned above, the CCM does not always work so brilliantly. Let’s present a few examples that show that the results obtained by CCM may be incorrect. The first group of examples demonstrates cases when the CCM shows unidirectional causality, at its complete absence, and the second group demonstrates bidirectional causality, in the presence of only unidirectional one.

Consider two independent deterministic time series generated by:
\[
\begin{align*}
X(t) &= \lambda_1 X(t-1)[1-X(t-1)] \\
X^{\text{trend}}(t) &= \alpha X(t) + \beta t \\
Y(t+1) &= \lambda_2 Y(t)[1-Y(t)]
\end{align*}
\]

where \( \lambda_1 = 3.82, \alpha = 0.015, \beta = 0.0003, \lambda_2 = 3.74 \). These time series are shown on figure 2.

**Figure 2.** Time series generated by equations (2).

The convergence resulting from the application of the CCM to detect causality between \( X^{\text{trend}} \) and \( Y \) is shown in figure 3. As can be seen, the CCM has shown a very strong one-way causality between these time series, when causality is obviously absent.

**Figure 3.** Convergence of the \( X^{\text{trend}} \) and \( Y \) time series described by equations (2).

To show that this example is not exceptional, consider another model system in which one time series \( X \) contains an additive random component and does not depend on the second series \( Y \) (figure 4):

\[
\begin{align*}
X(t) &= \lambda_1 + \beta t/(2L+t) + \gamma \cdot \text{Rnd} \\
Y(t+1) &= \lambda_2 \cdot Y(t)(1-Y(t))
\end{align*}
\]

where \( \lambda_1 = 2, \beta = 0.3, L = 250, \gamma = 0.1, \text{Rnd} \) is a random evenly distributed number within \([0,1)\), \( \lambda_2 = 3.4 \). The result of the causality study between these time series using CCM is shown in figure 5. As can be seen from figure 5, in this case, the CCM shows a false two-way connection.
Finally, we will conduct a direct test of the CCM operation by evaluating the presence of a causal relationship between the real time series of cosmic rays CR (data from [5]) and the model time series Y(t) (see figure 6), described by the equation

$$Y(t+1) = \lambda Y(t)(1 - Y(t))$$  \hspace{1cm} (4)

where $\lambda = 3.74$. In this case, the CCM shows a fairly strong one-way causality between CR and Y (figure 7), but between these time series, as in the previous example, there is obviously no causality.

There is a question about the causes of CCM errors. It should be noted that in all examples considered, the time series have a certain form – one of the series is a periodic function of time, and the second series has a time trend. According to the CCM, if there is a cause-and-effect relationship, the close points of one shadow manifold should be mapped to the close points of another.
Figure 7. Convergence of time series $CR$ and $Y(t)$ obtained by equation (3).

Figure 8 shows what happens when comparing a periodic time series with a monotonically increasing variable. It can be seen that points from the periodic manifold ($L_1$) that are close in value are mapped to sufficiently close points of the "monotone manifold" ($L_2$). This implies the influence of a monotonically increasing variable on the periodic variable. Conversely close points of a monotone variable ($L_3$) are mapped to far points of a periodic variable ($L_4$), which corresponds to a weak influence of the periodic variable on the monotone. This means that the CCM may give a false conclusion about the existence of a relationship when studying periodic and smoothly changing time series.

Another example demonstrates the presence of a false causal relationship at its absence for periodic series, similar to those used to illustrate causality in the real predator-prey system [4].

Let's consider a test example of an ecosystem described by differential equations:

\[
\begin{align*}
\frac{dX}{dt} &= V_0 \sin(t) + 1)(X_0 - X)(X - X^2) \\
\frac{dY}{dt} &= V_D X - \frac{V_m Y}{K_m + Y}
\end{align*}
\]

Formally, the periodicity is set by an external influence (a kind of imitation of seasonality) and the use of nonlinearities simply complicates the dynamics, and removes suspicion in a special selection of simple linear equations. Noise simulating measurement errors was added additively to the obtained...
dynamics, and in addition the results of system integration were thinned out to simulate the sampling moments and to better resemble the real curves given in the article [4]. As can be seen from the equations, the variable $X$ does not depend on $Y$, while $Y$ depends on $X$. The solution of this system is shown in figure 9.

![Figure 9. Solution of the system (5).](image)

Applying CCM to the solution of equations (5) shows that there is a two-way relationship between the variables $X$ and $Y$ (figure 10).

![Figure 10. Convergence of time series obtained by solving the system (5).](image)

This example shows that the restriction on the application of CCM to systems with a rigid functional relationship between variables (as mentioned in article [4]) when the CCM indicates a two-way causal relationship for an obvious one-way functional relationship, should be extended to dynamic systems. This is expected in principle since the solution of a system of ordinary differential equations is a set of certain functions that describe the dynamics of variables over time. When considering these functions in pairs, it is always possible to exclude time and get a functional relationship between the corresponding variables. This also means that the case of a functional relationship between two variables will be indistinguishable for the CCM from the case when both variables do not affect each other, but depend on some third reason, which further limits the resolving capabilities of the method.

Therefore, in the case when there is no mutual influence between time series, the values of correlation and convergence in the CCM as shown by given examples can take large values (close to one). Therefore, it is impossible to say that there is a relationship between time series in this case, but it can be assumed. Correlation is not suitable for determining the direction of mutual influence between time series, and the CCM as shown in the examples can only be used to establish its possible presence. If there is a two-way relationship between time series, the correlation can take values close to zero, for example, for equations (1), while the convergence of the CCM always indicates its presence.
4. Conclusions
Presented examples demonstrate the imperfection of the CCM cause-and-effect relationships detection method, which is proposed as an alternative to correlation analysis and the Granger method [3]. Certainly data preprocessing such as detrending or removing seasonal components can remove artifacts described above. However we can lose that features of time series, which are essential for CCM and can get new artifacts. So the question on preprocessing data for CCM requires additional studies.

The conducted research leads to the conclusion that the results of the CCM can be relied on in one case, when it indicates the absence of causal relationships between the considered dynamic variables, i.e. it acts as a necessary, but not sufficient condition for the presence of these relationships in the system under study. It seems that, despite the identified significant limitations, the method of convergent cross-mapping can be used in the analysis of the structures of biological objects and their communities.

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