Reparameterization of Single Difference and Undifferenced Kinematic GPS Positioning Models

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1 Introduction

The double difference (DD) model is widely employed in GPS data processing, since it can eliminate receiver and satellite clock errors totally, and ionospheric and tropospheric errors to a great extent. This makes the number of parameters be estimated smaller and computing burden less than that for the single difference (SD) and undifferenced (ZD) models. However, the SD and ZD models also have their own advantages: it is not necessary to constitute the DD observations, which is not trivial, and no mathematical correlation is introduced into the stochastic models for the single baseline, which makes the quality control and quality assessment more specific. The SD and ZD models, secondly, result in SD and ZD residuals, which are more convenient for analyzing the spatial correlation between individual satellites and satellite elevation dependent effects. The situation that less information loses in the SD model and no information loses in the ZD model makes the further research on receiver or satellite clock information possible. For the reasons above, research on the SD and ZD models is necessary. However, not all parameters in the SD and ZD models can be estimated, because the models are rank defect.

Reparameterization provides a method for resolving this rank defect problem by estimating some combinations of the unknowns rather than the unknowns themselves. In this paper, we introduce the reparameterization of the SD and ZD models according to the following criterions:

1) Reparameterization does not change anything in the observations space, but affects the parameter space.
2) The parameters of interest, e.g., the baseline vector, will keep intact.

3) The carrier ambiguities will be expressed in terms of double difference ambiguities, since they can be resolved and kept fixed to their integer values.

4) New receiver clock bias parameters should be the same for all satellites at one epoch, but different for each type of observations.

5) New satellite clock bias parameters should be the same for all receivers at one epoch, but different for each type of observations.

In this paper, we first give the ZD and SD observation equations for code and carrier phase. Next, the reparameterization of SD and ZD observation equations is discussed in detail, followed by the introduction of single epoch SD and ZD functional and stochastic models. Finally, a theoretical proof of the equivalence of undifferenced and difference models and the relationship between SD and DD residuals are given.

1 ZD and SD observation equations

Considering single epoch, the linearized ZD observation equations for code and carrier phase are:

\[ \mathbf{P}'_{i,p_i} = - (\mathbf{u}_i)^T \Delta \mathbf{r}_i + \epsilon \mathbf{d}_{i,p_i} - \epsilon \mathbf{d}_{i,p_i} \]
\[ + T_i + I_i + \epsilon \mathbf{e}_{i,p_i} \]
\[ \Phi_{i,p_i} = - (\mathbf{u}_i)^T \Delta \mathbf{r}_i + \epsilon \mathbf{d}_{i,p_i} - \epsilon \mathbf{d}_{i,p_i} \]
\[ + T_i + I_i + \lambda_i N_{i,p_i} + \epsilon \mathbf{e}_{i,p_i} \] (1)

where \( \mathbf{P}'_{i,p_i} \) and \( \Phi_{i,p_i} \) are the observed minus computed ZD code and the carrier observations of satellite at receiver \( i \), respectively, in meter; \( P_i \) and \( L_i \) denote the code and the carrier phase on \( L_i \) band, respectively; \( \Delta \mathbf{r}_i \) is a three-vector containing the increment to the initial station coordinates \( r_{i0} \) and \( \mathbf{u}_i \) is a three-vector containing the partial derivatives of the unknown satellite-receiver range with respect to the unknown coordinates, which capture the geometry of the receiver-satellite configuration. And \( \Delta \mathbf{r}_i \) and \( \epsilon \mathbf{d}_{i,p_i} \) are increments to clock errors of the receiver \( i \) and the satellite \( s \), respectively. \( T_i \) is the tropospheric delay and \( I_i \) the \( L_i \) ionospheric delay; \( I_i \) should be multiplied by \( \gamma = f_1^2 / f_2^2 \) to obtain the ionospheric delay on \( L_2 \) if the \( L_2 \) observations are also considered in the functional model, where \( f_1 \) and \( f_2 \) are the frequencies of \( L_1 \) and \( L_2 \), respectively. \( N_i \) are the ZD real-valued sum of carrier ambiguities and hardware delays; \( \lambda_i \) is the wavelength of \( L_i \)-carrier and the observation noise and unmodeled effects of code and carrier, which includes multipath. And \( c \) is the speed of light in vacuum.

The SD observations are obtained by subtracting ZD observations of one receiver from the other, and the satellite clock errors are completely eliminated from the resulting equations. Here we consider only short baselines, which means that two receivers are close enough to each other to assure that ionospheric and tropospheric effects are completely absent in the observation equations. Then the linearized SD observation equations for code and carrier phase can be written as

\[ \Delta \mathbf{P}'_{s,p_i} = - [ (\mathbf{u}_i)^T \Delta \mathbf{r}_i - (\mathbf{u}_j)^T \Delta \mathbf{r}_j ] + c \Delta \mathbf{e}_{s,p_i} \]
\[ (3) \]

\[ \Delta \Phi_{s,p_i} = - [ (\mathbf{u}_i)^T \Delta \mathbf{r}_i - (\mathbf{u}_j)^T \Delta \mathbf{r}_j ] + c \Delta \mathbf{e}_{s,p_i} \]
\[ + \lambda_i \Delta N_{s,p_i} + \Delta \mathbf{e}_{s,p_i} \] (4)

where \( \Delta \) denotes SD except for \( \Delta \mathbf{r}_j \); \( j \) is the second receiver, and \( \Delta \mathbf{e} \) the SD observation noise and unmodelled effects of code and carrier phase.

The DD observations are obtained by subtracting the SD observations of a reference satellite from the other ones, and the receiver clock errors are completely eliminated in the equations. Then the linearized DD observation equations for code and carrier phase can be written as

\[ \nabla \Delta \mathbf{P}'_{s,p_i} = - [ (\mathbf{u}_i)^T \Delta \mathbf{r}_i - (\mathbf{u}_j)^T \Delta \mathbf{r}_j ] + \nabla \Delta \mathbf{e}_{s,p_i} \]
\[ (5) \]
where \( \nabla \Delta \) denotes DD, \( q \) is the reference satellite. The ambiguities now are a linear combination of four integer ambiguities, and the hardware delays have been completely eliminated.

\[
\begin{align*}
\nabla \Delta \tilde{\phi}_s^q_{i,m} &= \left[ \left( (u_i^q)^T \Delta r_i - (u_i^q)^T \Delta r_j \right) \\
&- \left( (u_i^q)^T \Delta r_i - (u_i^q)^T \Delta r_j \right) \right] \\
&+ \lambda_1 \nabla \Delta N_{i,1}^s + \nabla \Delta \varepsilon_{i,1}
\end{align*}
\]  

(6)

where \( \nabla \Delta \) denotes DD, \( q \) is the reference satellite. The ambiguities now are a linear combination of four integer ambiguities, and the hardware delays have been completely eliminated.

\section{Reparameterization of SD and ZD observation equations}

Before the reparameterization of the SD and ZD observation equations, we give the DD functional model for further comparison with the SD and ZD models. Consider two receivers 1 and 2, observe \( m \) satellites and choose satellite 1 as a reference satellite. In kinematic relative positioning, the antenna of receiver 1 is assumed to be at a known station, \( \Delta r = 0 \), and the antenna of receiver 2 is moving. The coordinates of receiver 2 are estimated with respect to receiver 1, so we just need to estimate \( \Delta r_2 \). It is easy to obtain the complete and compact linearized DD functional model for a single epoch form Eqs. (5) and (6) when considering dual-frequency observations as

\[
\begin{align*}
E\left( \begin{bmatrix}
\nabla \Delta \tilde{\phi}_s^1_{1,1} \\
\nabla \Delta \tilde{\phi}_s^2_{1,1} \\
\nabla \Delta \tilde{\phi}_s^3_{1,1} \\
\nabla \Delta \tilde{\phi}_s^4_{1,1} \\
\end{bmatrix} \right) &= \begin{bmatrix}
\Delta G & \lambda_1 I_{m-1} & \begin{bmatrix}
\Delta r_2^1 \\
\Delta N_{i,1}^s \\
\Delta N_{i,2}^s \\
\end{bmatrix} \\
\end{bmatrix} \\
\frac{\nabla \Delta N_{i,1}^s}{\nabla \Delta N_{i,2}^s}
\end{align*}
\]  

(7)

where \( \nabla \Delta \tilde{\phi}_s^q_{1,1} = \nabla \Delta \tilde{\phi}_s^{12,1,1} \nabla \Delta \tilde{\phi}_s^{13,1,1} \cdots \nabla \Delta \tilde{\phi}_s^{m,1,1} \right)^T \). And other abbreviations of DD observations and ambiguities can be obtained by using the same defining method as for \( \nabla \Delta \tilde{\phi}_s^q_{1,1} \); \( I_{m-1} \) is the \( (m-1) \times (m-1) \) unit matrix and

\[
\begin{align*}
\Delta G &= \begin{bmatrix}
- (u_1^s - u_2^s)^T \\
- (u_1^s - u_2^s)^T \\
\cdots \\
- (u_1^s - u_2^s)^T
\end{bmatrix}^T
\end{align*}
\]

In order to resolve the unknown integer ambiguities, the least squares ambiguity decorrelation adjustment (LAMBDA) method, developed at Delft University of Technology, is used in this paper. This method is based on a decorrelation of original ambiguities and a search procedure\(^{[5-6]}\).

Although the parameters of DD functional model are fewer since the clock errors are absent, and the design matrix is of full rank, it is not trivial to constitute the DD observations before constructing the functional model. In addition, it is impossible to find out the impact of unmodeled errors of one observation on DD residuals for some special purposes, e.g. research on the stochastic characteristics of GPS observations, since the DD residuals are constructed from four observations.

In the case of SD model, there are two reasons for reformulating Eq. (4). One is that this model cannot be used directly, for it is singular. The other is that we prefer to express the carrier ambiguities in terms of double differences. If we choose satellite 1 as the reference satellite, on the basis of the parameterization criterions proposed in the introduction, the SD carrier phase observation can be written as\(^{[7]}\)

\[
\begin{align*}
\Delta \tilde{\phi}_s^{12,1,1} &= \left( u_i^0 \right)^T \Delta r_i + c \Delta \delta t_{12,1,1} \\
&+ \lambda_i \left( \Delta N_{i,1}^{1,1} - \Delta N_{i,1}^{1,2} + \Delta N_{i,2}^{1,1} \right) + \Delta \varepsilon_{1,1} \\
&= \left( u_i^0 \right)^T \Delta r_i + c \Delta \delta t_{12,1,1} + \lambda_i \Delta \tilde{\phi}_s^{12,1,1} \\
&+ \lambda_i \left( \Delta N_{i,1}^{1,1} - \Delta N_{i,1}^{1,2} \right) + \Delta \varepsilon_{1,1}
\end{align*}
\]

where \( \Delta \tilde{\phi}_s^{12,1,1} \) is the DD ambiguity between satellites 1 and 1, and the DD ambiguity of the reference satellite \( \Delta N_{i,1}^{1,1} \) is zero. For consistency, \( c \Delta \delta t_{12,1,1} \) in Eq. (3) is written as \( \Delta t_{12,1,1} \), which is assumed to absorb the receiver delay. And \( \Delta t_{12,1,1} \) and \( \Delta N_{i,1}^{1,1} \) are considered as receiver clock biases, which are the same for all satellites at an epoch, but different for each frequency.

After the reparameterization of the SD obser-
vation equations, it is very easy to obtain the complete and compact linearized SD functional model for a single epoch when considering dual-frequency observations:

\[
E(\Delta \Phi_i, \Delta \Phi_j, \Delta \beta_{r1}, \Delta \beta_{r2}) = \begin{bmatrix}
\Delta \beta_{r1} \\
\Delta \beta_{r2} \\
\Delta \beta_{r1} \\
\Delta \beta_{r2}
\end{bmatrix}
\]

\[
\begin{bmatrix}
G \lambda_1 E \\
G \lambda_2 E \\
G \\
G
\end{bmatrix}
\begin{bmatrix}
e_m \\
e_m \\
e_m \\
e_m
\end{bmatrix}
\]

where \( \Delta \Phi_i \) and other abbreviations of SD observations can be obtained by using the same defining method as for \( \Delta \Phi_i \). The design matrix of the SD model is of full rank after the reparameterization in the parameter space. Four receiver clock biases appear in the parameter space, which increase the computational burden in kinematic positioning, but it gives the opportunity for analyzing the variation characteristics of receiver clock. The SD residuals are more suitable for analyzing the spatial correlation between individual satellite and satellite elevation dependent effect than the DD residuals.

The ZD model should be reformulated for the same reasons, moreover, tropospheric and ionospheric delays are not absent, which makes the reparameterization more complex than in the SD model. Considering a short baseline, we have \( T = T_1 = T_2 \) and \( I = I_1 = I_2 \). On the basis of the parameterization criteria proposed in the introduction, the code observation at receiver 2 can be written as

\[
P_{2,P_1} = -\left(\mathbf{u}_2^T\Delta r_2 + \mathbf{c} \delta t_{2,P_1} - \mathbf{c} \delta t_{1,P_1}\right) + T + I + \epsilon_{P_1}
\]

\[
= -\left(\mathbf{u}_2^T\Delta r_2 + (\mathbf{c} \delta t_{2,P_1} - \mathbf{c} \delta t_{1,P_1})\right) - (\mathbf{c} \delta t_{1,P_1} - T - I - \mathbf{c} \delta t_{1,P_1}) + \epsilon_{P_1}
\]

\[
= -\left(\mathbf{u}_2^T\Delta r_2 + \delta t_{2,P_1} - \delta t_{1,P_1} + \epsilon_{P_1}\right)
\]

(10)

Note that a negative and a positive \( \delta t_{1,P_1} \) are appended in the parameter space and combined with receiver and satellite clock bias parameters, respectively. Considered as a receiver clock bias parameter, \( \delta t_{1,P_1} \) is the same for all satellites at an epoch, but different for each frequency. And \( \delta t_{1,P_1} \) combined with tropospheric and ionospheric delays is considered as satellite clock bias parameter, which is the same for two receivers at one epoch, but different for each type of observations of each satellite.

The carrier phase observation at receiver 2 can be written and reparameterized by using the same method as

\[
\Phi_{2,P_1} = -\left(\mathbf{u}_2^T\Delta \Phi_2 \right) + \mathbf{c} \delta \Phi_{2,P_1} + \mathbf{c} \delta \Phi_{1,P_1} + T + I
\]

\[
+ \lambda_1 \mathbf{N}_{2,t_1} + \epsilon_{t_1} = -\left(\mathbf{u}_2^T\Delta \Phi_2\right) + (\mathbf{c} \delta \Phi_{2,P_1} - \mathbf{c} \delta \Phi_{1,P_1} + \lambda_1 \mathbf{N}_{2,t_1} - \lambda_1 \mathbf{N}_{1,t_1} - \lambda_1 \mathbf{N}_{2,t_1} - \lambda_1 \mathbf{N}_{1,t_1})
\]

\[
- (\mathbf{c} \delta \Phi_{1,t_1} - T - I - \mathbf{c} \delta \Phi_{1,t_1} - \lambda_1 \mathbf{N}_{2,t_1})
\]

\[
+ \lambda_1 (\mathbf{N}_{1,t_1} - \mathbf{N}_{2,t_1} - \mathbf{N}_{1,t_1} + \mathbf{N}_{1,t_1}) + \epsilon_{t_1}
\]

\[
= -\left(\mathbf{u}_2^T\Delta \Phi_2 + \delta \Phi_{2,P_1} - \delta \Phi_{1,P_1} + \epsilon_{P_1}\right)
\]

\[
+ \lambda_1 \mathbf{N}_{2,t_1} + \epsilon_{t_1}
\]

(11)

Note that a negative and a positive \( \lambda_1 \mathbf{N}_{1,t_1} \), \( \lambda_1 \mathbf{N}_{1,t_1} \), \( \lambda_1 \mathbf{N}_{1,t_1} \), and \( \delta \Phi_{1,t_1} \) are appended in the parameter space and combined with receiver and satellite clock bias parameters and DD ambiguities, respectively.

Because \( \Delta r_1 = 0 \), the code observation at receiver 1 can be re-written as

\[
P_{1,P_1} = 0 + \mathbf{c} \delta \Phi_{1,P_1} + \mathbf{c} \delta \Phi_{1,P_1} + T + I + \epsilon_{P_1}
\]

\[
= -\left(\mathbf{c} \delta \Phi_{1,P_1} - T - I - \mathbf{c} \delta \Phi_{1,P_1} + \epsilon_{P_1}\right)
\]

\[
= -\mathbf{d} \epsilon_{P_1} + \epsilon_{P_1}
\]

(12)
and the carrier phase observation at receiver 1 as
\[ \Phi_{1,t_1} = 0 + \epsilon \Delta t_{1,t_1} - \epsilon \Delta t_{2,t_1} + T' - I' + \lambda_1 N_{1,t_1} + \epsilon_{t_1} \]
\[ = (\epsilon \Delta t_{1,t_1} - T' + I' - \epsilon \Delta t_{2,t_1} - \lambda_1 N_{1,t_1}) + \epsilon_{t_1} = -\epsilon \Delta t_{1,t_1} + \epsilon_{t_1} \]
(13)

After the reparameterization of the ZD observation equations, we obtain the complete and compact linearized ZD functional model for a single epoch when considering the following dual-frequency observations:

\[
E(\Phi_{1,t_1}, \Phi_{2,t_2}, P_{1,P_1}, P_{2,P_2}) = \begin{bmatrix}
G & \lambda_1 E & e_m \\
G & \lambda_2 E & e_m \\
G & e_m & -I_m \\
G & e_m & -I_m \\
G & e_m & -I_m \\
G & e_m & -I_m \\
\end{bmatrix} \cdot \begin{bmatrix}
\Delta r_1 \\
\nabla \Delta N_{1,t_1} \\
\nabla \Delta N_{2,t_2} \\
dt_{12,t_1} \\
dt_{12,t_2} \\
dt_{12,t_1} \\
dt_{12,t_2} \\
dt_{12,t_1} \\
dt_{12,t_2} \\
dt_{12,t_1} \\
dt_{12,t_2} \\
dt_{12,t_1} \\
dt_{12,t_2} \\
\end{bmatrix}
\]
(14)

where \( \Phi_{1,t_1} = [\Phi_{1,t_1} \Phi_{2,t_2} \cdots \Phi_{m,t_1}]^T \), \( dt^t = [dt_{1,t_1} dt_{2,t_2} \cdots dt_{m,t_1}] \), and other abbreviations of the ZD observations and satellite clock biases can be obtained using the same defining method as for \( \Phi_{1,t_1} \) and \( dt^t \).

The design matrix of the ZD model is of full rank after reparameterization in the parameter space. The \( 4 \times m \) satellite clock biases appear in the parameter space, which increase the computational burden significantly in kinematic positioning. Although it seems to give the opportunity to analyze the stochastic characteristics of one-way observation between satellite and receiver, this is actually impossible; the relationship between SD and ZD residuals will be given later.

### 3 ZD, SD and DD stochastic model

According to the widely used stochastic model we assume that the observations of the same style (code and carrier phase) from all satellites have the same precision and all GPS observations are independent. In general, the stochastic model of ZD observations for all satellites can be written as

\[
Q_s = \text{diag}(Q_{s_1}, Q_{s_2})
\]
\[
= \text{diag}(Q_{s_{1,t_1}}, Q_{s_{1,t_2}}, Q_{s_{1,t_1}}, Q_{s_{1,t_2}}, Q_{s_{2,t_1}}, Q_{s_{2,t_2}}, Q_{s_{2,t_1}}, Q_{s_{2,t_2}},
Q_{s_{1,t_1}}, Q_{s_{1,t_2}}, Q_{s_{1,t_1}}, Q_{s_{1,t_2}}, Q_{s_{2,t_1}}, Q_{s_{2,t_2}}, Q_{s_{2,t_1}}, Q_{s_{2,t_2}})
\]
(15)

where \( Q_{s_{1,t_1}} = \text{diag}(\sigma_{t_1}^2, \sigma_{t_2}^2, \cdots, \sigma_{t_m}^2) \), \( \sigma_{t_i}^2 = \sigma_t^2 \) and \( Q_{s_{1,t_1}} = \text{diag}(\sigma_h^2, \sigma_{h_2}^2, \cdots, \sigma_{h_n}^2) \), \( \sigma_h^2 = \sigma_h^2 \) and \( Q_{s_1} = Q_{s_2} \); \( y_1 \) and \( y_2 \) refer to the observations at the first and second stations, respectively.

The application of the error propagation law to diagonal matrix Eq. (15) yields the SD stochastic model:
\[
Q_{s_{ab}} = \text{diag}(Q_{s_{ab_1}}, Q_{s_{ab_2}}, Q_{s_{ab_1}}, Q_{s_{ab_2}})
\]
(16)

The application of the error propagation law to diagonal matrix Eq. (16) yields the DD stochastic model:
\[
Q_{s_{ab}} = \text{diag}(Q_{s_{ab_1}}, Q_{s_{ab_2}}, Q_{s_{ab_1}}, Q_{s_{ab_2}})
\]
(17)

where \( Q_{s_{ab_1}} = \begin{bmatrix}
4\sigma_t^2 & 2\sigma_t^2 & \cdots & 2\sigma_t^2 \\
2\sigma_t^2 & 4\sigma_t^2 & \cdots & 2\sigma_t^2 \\
\cdots & \cdots & \cdots & \cdots \\
2\sigma_t^2 & 2\sigma_t^2 & \cdots & 4\sigma_t^2 \\
\end{bmatrix} \)

and \( Q_{s_{ab_1}}, Q_{s_{ab_2}}, Q_{s_{ab_1}}, Q_{s_{ab_2}} \) have the same definition as \( Q_{s_{ab_1}} \) for individual observation.
It is clear that the mathematical correlation does not exist in the ZD and SD stochastical models, but does in the DD stochastic model.

4 Equivalence of ZD and SD models, and the relationship between ZD and SD residuals

Without loss of generality, we define the observations of two receivers, respectively.

\[ y_1 = [\Phi_{t_1}^T, \Phi_{t_2}^T, P_{t_1}, P_{t_2}]^T \quad \text{and} \quad y_2 = [\Phi_{t_1}^T, \Phi_{t_2}^T, P_{t_1}, P_{t_2}]^T \]

\[ A = \begin{bmatrix} G \lambda E & e_m \\ G \lambda_2 E & e_m \\ G & e_m \\ I & e_m \end{bmatrix} \]

\[ I = \text{diag}(I_m, I_m, I_m, I_m) \]

\[ x = [\Delta t_1, \Delta N_t_1, \Delta N_t_1, dt_{t_1}, dt_{t_2}] \]

\[ x' = [dt_{t_1}, dt_{t_2}, dt_{t_1}, dt_{t_2}] \]

\[ \text{and thus, the ZD model can be written as} \]

\[ E[y_1, y_2] = (A - I) x = Q \]

\[ \text{and the SD model as} \]

\[ E[y_1 - y_2] = Ax, Q_{ax} = 2Q \]

The normal equations of the ZD model are:

\[ A^T Q^{-1} A x = A^T Q^{-1} y \]

The least squares solution for \( x \) reads:

\[ x = (A^T Q^{-1} A)^{-1} A^T Q^{-1} y \]

\[ x' = (2Q)^{-1} (y_1 + y_2) \]

Thus, the following knowledge can be concluded from the above description.

1) The ZD and SD model are equivalent from the point of view of estimation of the parameters \( x \).

2) The ZD residuals at receiver 1 and 2 are half the size of the SD residuals, and they have opposite sign when the stochastic models are the same for both the receivers.

3) Analyzing the ZD residuals is not of any additional advantages over the SD residuals.

5 Conclusions

In order to resolve the rank defect problems
The Half of L1 Carrier Phase SD Residual

The Half of L2 Carrier Phase SD Residual

The Half of C1 Code SD Residual

The Half of P2 Code SD Residual

Fig. 1 Some experimental results

which appear in the SD and ZD models a simple reparameterization has been proposed in this paper, which brings the advantages of ZD and SD models playing well in GPS data processing, especially research on the stochastic characteristics of GPS observations. Some interesting particularities of reparameterization can be summarized.

1) It keeps the parameters of interest intact, e.g. the baseline vector, and estimates some combinations of the unknowns rather than the unknowns themselves.

2) The new satellite and receiver clock bias parameters, resulting from the reparameterization, have the same properties as the original ones. The receiver clock biases are the same for all satellites at one epoch but different for each observation type.

3) The carrier ambiguities are still expressed in terms of DD ambiguities, no matter which model is chosen in data processing, since only DD ambiguities can be resolved and kept fixed to their integer ambiguities.

And the ZD residuals at two receivers of a single baseline are half of the SD residuals, and have opposite signs.

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