Method for Parametric Shaping Architectural Free Forms Roofed with Transformed Shell Sheeting

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Abstract. An innovative method for shaping attractive architectural free forms of buildings is proposed. Consistency of shell roofs and plane-walled oblique elevations of the building free forms is preserved due to utilization of specific geometrical tetrahedrons controlling general forms of entire buildings. The method proposed enables shaping roofs as warped shell forms made up of plane steel sheets folded in one direction and connected to each other along their longitudinal edges to obtain a plane strip. Next, the strip is elastically transformed into a shell shape so that a freedom of the width increments of each shell fold would be ensured. Such effective sheet shape transformations make it possible to limit the negative influence of these initial fold’s shape changes on the strength and stability of the designed roof shell. The method also allows to shape oblique plane elevation walls almost freely both individual buildings and their complex structures.

1. Introduction
Unified nominally plane steel sheets folded in one direction and connected to each other into one strip can be spread over skew roof directrices to obtain a corrugated warped shell roof [1]. The attractiveness of such shell roofs is the really important advantage of the designed architectural free forms [2], see figure 1.

![Figure 1. Free form shell roof](image)

Each folded steel strip can easily be transformed into many diversified straight doubly-curved shell sheeting in a continuous way and a relatively big range [3]. The diversity results from diversification
of curvatures and mutual positions of two skew directrices supporting the strip at its crosswise ends [4], see figure 2.

![Figure 2. Elastic shape transformations executed on the nominally flat strip of three folded steel sheets](image2)

In addition, in order to increase the diversity and attractiveness of the created free form buildings covered with such shell sheeting, a possibility of using oblique plane-walled elevations is allowed [5], see figure 3. Therefore a general method for geometrical and structural shaping such innovative free form shell roofs and their structural systems seems to be needed [6, 7].

![Figure 3. Visualization of a building free form](image3)

2. State of the art

In the author’s opinion, Adam Reichhart is the precursor of shaping the free form shell roofs made up of initially deformed steel sheets connected to each other into warped strips, figure 4 [8]. He has elaborated approximated algorithms for a method of geometrical and strength shaping such free form shell roofs made up of freely deformed singly-corrugated steel sheeting [2]. His algorithms are very reduced, because all shell folds are modelled with very simple central sectors of right hyperbolic paraboloids [4]. So, the number of independent variables describing possible shapes of the proposed roof sheeting is insufficient [2]. He has formulated no condition related to the fact that the contraction of each shell fold has to pass through the half on the fold’s length, which is needed to obtain the really free deformation of the folded sheets.

Jacek Abramczyk has elaborated a more accurate innovative geometrical method for shaping the transformed folded steel shells [9]. Diversified types of stiffness of the sheets influencing on orthotropic properties of the considered shell sheeting are the basis for the method [10, 11]. He has extended the method to shape architectural free forms of entire buildings [8, 12, 13], see figure 3, and their structures [6, 14], figure 5.
Figure 4. A free form roofed with corrugated shell sheeting used as an internal layer

Figure 5. A free form roofed with shell structure

If a freedom of transverse width increments of each shell fold in a sheeting is ensured, then its shape transformation, is effective and its longitudinal axis is identical with twist axis [8]. It is straight during and after the transformation, so it is said to be the neutral axis of fold. Such shell sheeting can be modelled with ruled surfaces characterized by lines of striction passing through the halves of the shell folds on their lengths [15, 16], see figure 6.

Figure 6. A warped shell roof free form

The modified method exploits tetrahedrons of a specific type that are called control compositions $\mathcal{I}$; see figure 7 [17]. Each control composition has got: a) four vertices $H_i$; b) four sides $P_iB_{i-1}P_iB_i$; c) two axes $o_i$. It contain a free form $\Sigma$ covered with a shell sector $\Omega$. It models shell roof and plane-walled elevations as well as enables to achieve fine integration of their shapes.
3. The aim of the method

The aim of the paper is to present an innovative method for geometrical shaping of parametric building free forms. It is especially required such a version of the method that allows intuitively to search for innovative and attractive free forms. Thus, its algorithm exploits a relatively small set of parameters and their chosen proportions to shape intuitively the expected free form shapes of entire buildings. The method can easily be aided with graphical computer programs, for example the author employs AutoCAD and its programming language Auto LISP.

The important feature of the method is that it allows to obtain a fine integration of shapes of a warped shell roof and plane-walled oblique elevation walls. Its usage to search for attractive and consistent architectural free forms [18] is described. Specific structural systems for such roof shells are employed [19, 20] and specific methods for presenting the proposed forms can be used [21, 22].

4. The method of shaping architectural free forms

Each complete free form is created with the help of a specific kind of tetrahedrons called the control compositions. Each control composition $\Gamma = \langle E_2F_2E_{13}F_{13}F_{16}E_{15}E_{16} \rangle$ enables controlling general shape of a free form, see figure 8. Positions of its eight vertices $E_2, F_2, E_{15}, F_{15}, F_8, F_{11}, E_5, E_{16}$ are defined on the basis of a plane reference polygon $P_\gamma = C_{15}C_2 \cup C_2C_8 \cup C_8C_9 \cup C_9C_8 \cup C_9C_11 \cup C_11C_9 \cup C_9C_{16} \cup C_{11}C_{16} \cup C_{16}C_{15}$ as the points of the intersection of straight lines perpendicular to the plane of $P_\gamma$ and two planes $\gamma_1$ and $\gamma_3$. The roof forms are corrugated shell sheetings modelled by regular warped surfaces $\Omega$. 

Figure 7. A model $\Sigma$ for a building free form whose shell roof is determined by two straight directrices $B_1B_2$ and $B_3B_4$.
Figure 8. Creation of a control compositions with reference polygons $\Delta$ and $P_r$

The reference polygon $P_r$ is created by means of chosen vertices of a plane figure $\Delta$ being the sum of four rectangles $\Delta_1 = \langle C_1C_2C_3C_6 \rangle$, $\Delta_2 = \langle C_4C_5C_6C_7 \rangle$, $\Delta_3 = \langle C_7C_8C_11C_{10} \rangle$ and $\Delta_4 = \langle C_1C_6C_{18}C_{15} \rangle$. The specific feature of the reference polygon $\Delta$ is as follows. Its chosen edges are contained in the $z$-axis, where $\Delta = \Delta_1 \cup \Delta_2 \cup \Delta_3 \cup \Delta_4$, see figures 8 and 9.

Figure 9. First configuration of the reference polygons $\Delta$ and $P_r$

The rectangles $\Delta_1$ and $\Delta_4$ have a vertex $C_1$ in common that is the origin of the local co-ordinate system $[x, z]$ of $\Delta$. The rectangles $\Delta_1$, $\Delta_2$, $\Delta_3$ and $\Delta_4$ have a common vertex $C_6$. Four independent variables $a$, $b$, $c$ and $d$ prescribing their edge lengths should be adopted for the above four rectangles, see figure 9. Four additional parameters prescribing two opposite gable wall planes: $\gamma_1(E_2, E_5, E_{15})$ and $\gamma_2(F_2, F_8, F_{11})$ ought to allow to determine the positions of the vertices of $\Gamma$. In this way, a longitudinal shape of the free form is clearly defined. The obtained shape of $\Gamma$ determines a general shape of the entire free form. The figures $\Delta$ and $P_r$ shown in figures 8 and 9 correspond to the following proportions: $b = 2a$, $c = 2.5a$ and $d = 1.5a$, where $h = b + d$ is the length of one of two edges of $\Delta_1$. 


Approximate relations between edges and areas of the free form at the relevant observation direction are expressed as proportions between edges and areas of the above rectangles $\Delta_i$. The inclination of the roof rulings and edge lines as well as elevations to the vertical resulting from the proportions between the above four parameters affect the attractiveness of the free forms.

To obtain values of the dependent variables $e$ and $f$ describing the positions of $C_8$ and $C_{16}$, the equations presented below should be taken into account. At first, from the homology of geometrical properties of the triangles: $<C_4C_8C_{20}>$ and $<C_4C_{10}C_{11}>$, see figure 9, we obtain

$$\frac{e}{f} = \frac{d}{2c} \quad (1)$$

From the homology of geometrical properties of the triangles: $<C_2C_8C_{13}>$ and $<C_2C_3C_{14}>$ we get

$$\frac{c-a}{b+d} = \frac{g}{b+e} \quad (2)$$

In addition, we can stated that

$$g + f = c - a \quad (3)$$

After some we can calculate values of dependent variable $e$ from

$$e = \frac{(c-a) \cdot d}{2 \cdot c \cdot (b+d) + c - a} \quad (4)$$

Finally, the dependent variable $f$ may be expressed as

$$f = \frac{2 \cdot c \cdot e}{d} \quad (5)$$

The rectangles $\Delta_1$, $\Delta_2$, $\Delta_3$ and $\Delta_4$ creating $\Delta$, shown in figure 10, have not a common vertex. Here, $\Delta$ and $Pr$ are unsymmetrical about the z-axis. For the above four rectangles, eight independent variables $a_1$, $b_1$, $c_1$, $d_1$, $a_2$, $b_2$, $c_2$ and $d_2$ prescribing their edge lengths should be adopted. Parameters mentioned above and describing these rectangles make it possible to intuitively define many attractive transverse shapes of the designed free forms. To achieve a really great diversification and attractiveness of the plane and spatial figures presented earlier, including $\Gamma$, $\Delta$, $\Omega$ and $Pr$, the sets of 14 parameters – independent variables ought to be adopted with the method’s algorithm in a relevant way.
Figure 10. Another configuration of the reference polygons $\Delta$ and $P_r$.

Roof directrices $e$ and $f$ can be straight segments as it is shown in figures 8, 10 and 11. Here, $e$ and $f$ determine a hyperbolic paraboloid sector $\Omega$ representing a warped shell roof and limited by a spatial quadrangle.

Figure 11. Free form of the first type with straight directrices.

However, roof directrices can be shaped as skew curved lines, see figure 12. They can be adopted as circle arcs $e$ and $f$ contained in the planes of the control composition $\Gamma$. Two additional parameters should be used to prescribe the ridges of the above directrices. The values of these ridges should be conditioned to selected transverse dimensions of the free form to obtain some proportions leading to its attractive shape.
Figure 12. Free form of the first type with curved directrices

The second configurations of $\Delta$ and $P_r$, related to the proportions $b = 0.5a$, $c = 0.75a$ and $d = a$ are proposed in figure 13. We should notice that $c / a < 1$ for the present case. The conditions (1) to (5) are analogous in this configuration of $P_r$.

A control composition $\Gamma$ of this second type created on the basis of the above reference polygons $\Delta_i$ is shown in figure 14. It is symmetrical about $z$-axis and its shape contracts in the top direction.

A visualization of an architectural free form related to the above mentioned control composition is presented in figure 15. Fine integration of the forms of its roof and elevations can be noticeable.
Figure 14. Free form $\Gamma$ of the second type with straight directrices $E_5E_{16}$ and $F_8F_{11}$

Figure 15. Free form of the second type with curved directrices

5. Conclusions
The method for creating innovative architectural free forms of buildings being characterized by corrugated warped shell roofs and plane-walled oblique elevations is proposed. The process of shaping these free forms is very intuitive and the number of independent variables is as small as possible. It enables to assess the attractiveness of these forms and the integration of roof and elevation shapes.

The method’s algorithm proposed can easily be implemented into graphical computer applications. It is still developed to achieve more innovative and attractive free forms and their coherent folded shell structures. The considered geometrical models for the shell roof sheeting are the base for elaborating their mechanical and computational counterparts that are used for strength and stability analyses of the corrugated shell roofs. In addition, a relationship between the effort of the mentioned roofs and their supporting conditions related to the curvature and mutual position of roof directrices is examined by the author with the help of these models.

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