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Abstract: The authors analyzed CFD calculations of flow in vaneless diffusers with relative width in range from 0.014 to 0.100 at inlet flow angles in range from $10^0$ to $45^0$ with different inlet velocity coefficients, Reynolds numbers and surface roughness. The aim is to simulate calculated performances by simple algebraic equations. The friction coefficient that represents head losses as friction losses is proposed for simulation. The friction coefficient and loss coefficient are directly connected by simple equation. The advantage is that friction coefficient changes comparatively little in range of studied parameters. Simple equations for this coefficient are proposed by the authors. The simulation accuracy is sufficient for practical calculations. To create the complete algebraic model of the vaneless diffuser the authors plan to widen this method of modeling to diffusers with different relative length and for wider range of Reynolds numbers.

Nomenclature

$b$ - width of diffuser;
$c_2$ - inlet velocity;
$c_3$ - outlet velocity;
$D_2$ - inlet diameter;
$D_3$ - outlet diameter;
h_w - lost head;
k - isentropic coefficient;
k_{rg} - surface roughness;
p_2$ - static pressure at VLD inlet;
p_3$ - static pressure at VLD outlet;
$R$ – gas constant;
$Re_{D_2}$ - Reynolds number;
$T_2$ - static temperature at VLD inlet;
$T_3$ - static temperature at VLD outlet;
$T_{t2}$ - total temperature at VLD inlet;
u_2$ - blade velocity;
$\nu_2$ - kinetic viscosity at VLD inlet;
$\alpha_2$ - flow angle at VLD inlet;
$\eta$ - efficiency;
$\Delta\eta$ - loss of efficiency;
\[ \lambda_{c2} - \text{velocity coefficient;} \]
\[ \lambda_{fr} - \text{friction coefficient;} \]
\[ \xi - \text{recovery coefficient;} \]
\[ \psi_{fr} - \text{loading factor;} \]
\[ \zeta - \text{loss coefficient;} \]

**Abbreviation**

VLD – vaneless diffuser.

1. **INTRODUCTION**

Engineering methods of gas dynamic design and centrifugal compressor performances calculation are developing in many countries in spite of CFD methods wide application. There is no contradiction as engineering methods are and will be necessary as instruments of optimization of flow path main sizes and of quick overall analysis. Math models applied to calculate head losses in flow path elements step by step. The Universal modeling [1] principle is to identify models of all elements by comparison of calculated and measured efficiency of stages. This method is successfully applied in design practice since 1990\textsuperscript{th} and is improved constantly [2, 3, 4, 5]. The authors propose simple algebraic equations instead of not the simple math model of a vaneless diffuser. The database for the equations is CFD calculations presented in [6, 7, 8, 9]. The authors of these works have compared different nets and turbulence models. Good correlation of CFD and math model performance curves of a stage stator parts (VLD included) is presented in [10]. This may be treated as an indirect proof of CFD calculation validity. The stage math model with the proposed equations will be identified by measured performances of stages. It will lead to necessary corrections.

2. **BASIC INFORMATION ON VANELESS DIFFUSERS**

Vaneless diffuser is the simplest element of a stage flow path. It is disposed between control planes \( D_2 - D_3 \) at Figure 1. But for a period of time the flow behavior in it was interpreted not properly. The empirical character of its performances is misleading [1]. It demonstrates Figure 2.

Loss coefficient has minimum if it is calculated on a base of pressures measured at a control plane disposed close to an impeller. The measurements at diameter 1.05 were traditional for Russian researchers [11]. The analogy with channel diffuser and its optimum divergence angle was the model to explain measured performances at Figure 2. Separation losses are absent in accordance with this model at low flow angles. The opposite situation must be if flow angles are big.

The study of flow structure, equilibrium equation analysis [12], experiments with diffusers separately of an impeller [13] demonstrated the real situation. The bigger is flow angle the lower are friction losses and less is flow separation possibility. The structure of flow based on measurements in an experimental stage is presented at Figure 3.

The difference of measured and calculated loss coefficients at Figure 2 is explained by flow structure at an impeller exit. Jet-wake mixing losses are result of impeller imperfectness. Mixing process is shown at Figure 4.

Mixing process ends at a distance 20% of an impeller diameter. It coincides with experimental data [12]. Diffuser performances calculated by measurements between \( 1,2D_2 - D_3 \) do not depend of an impeller. It is also proven experimentally [12]. At Figure 2 mixing losses is the difference of measured and calculated values of the VLD loss coefficient.

The aim of this work is to propose simple algebraic model for loss coefficient of VLD. It could be used in computer programs of the Universal modeling method [4, 5, 11]. In the Method all impeller losses are calculated as if they start and end inside of it. Therefore the model of VLD loss coefficient must be simulated as it is shown at Figure 2 by solid line.
Figure 1. Vaneless diffuser in an industrial centrifugal compressor stage

Figure 2. Measured and calculated vaneless diffuser loss coefficient performance curves. Stroke line – measurement, solid line – correct calculation [1]

Figure 3. Flow separation in VLD at low flow angle [12]
The objects for algebraic model creation were VLD with relative width in range \( b/D_2 = 0.014 - 0.10 \) and relative length \( D_3/D_2 = 1.60 \). The regime parameter is an inlet flow angle in range \( \alpha = 10 - 45^0 \). Similarity criteria were varied in range of the velocity coefficient \( \lambda_{c_2} = c_2 / \left( \frac{2k}{k+1} \frac{RT_{c_2}}{c} \right)^{0.5} \) = 0.23 – 0.82 and of Reynolds number \( \text{Re}_{D_2} = \frac{c_2 D_2}{\nu_2} \) = 3.8E6 – 10.3E6. Isentropic coefficient \( k = 1.4 \).

Surfaces were hydraulically smooth. The surface roughness influence was studied by additional calculations in range of \( \kappa_{rg} / 2b = 0 - 2.2 \times 10^{-3} \).

The program ANSYS CFX 14.0 and the net generator ICEM, H-type structured nets with 225216 elements, \( k-\varepsilon \) turbulence model were applied. The information about flow behavior in these objects is presented in [6, 7, 8]. Some key facts are:

- Flow separation takes place in diffusers with \( b/D_2 \geq 0.029 - 0.10 \) at flow angles \( < 10 - 25^0 \). The bigger width and velocity coefficient are, the bigger the critical angle is;
- There is no separation at any conditions in diffusers with relative width less than 0.029. High level of shear stresses due to walls’ closeness prevents separation but leads to lower efficiency;
- Non viscous core disappears in all studied VLD. The smaller relative width and flow inlet angle are, the earlier non viscous core disappears;

The equations for correct calculation of VLD efficiency and loss coefficient in a base of calculated flow parameters are:

\[
\eta = \frac{\ln \left( \frac{p_1}{p_2} \right)}{k - 1 \cdot \ln \left( \frac{T_3}{T_2} \right)}
\]
4. DIFFUSER PERFORMANCE MODELING – SMOOTH SURFACE

In principle the performance \( \zeta = f(\alpha^2) \) depends on:

- non-dimensional sizes \( b/D_2, D_3/D_2 \) and relative surface roughness \( k_{rg}/2b \),
- flow inlet angle \( \alpha^2 \),
- similarity criteria \( \lambda_{c2}, \text{Re}_D, k \).

Therefore:

\[
\zeta = f \left( b/D_2, D_3/D_2, k_{rg}/2b, \alpha^2, \lambda_{c2}, \text{Re}_D, k \right) \tag{3}
\]

The aim is to simulate CFD calculations by means of algebraic equations. The simplified analysis of VLD performance curves is presented in [12]. It is based on assumption that the constant value of a friction coefficient can be applied to estimate loss coefficient for diffusers of different configurations operating at different flow angles. The loss coefficient and the friction coefficient are connected by the equation:

\[
\zeta = \lambda_{fr}^2 = \frac{1 - \frac{1}{D_1/D_2} \sin \alpha^2}{4 \frac{b}{D_2}} \quad \text{or} \quad \lambda_{fr}^2 = \frac{4 \frac{b}{D_2} \sin \alpha^2}{1 - \frac{1}{D_1/D_2}} \tag{4}
\]

In fact, the friction coefficient cannot be of constant value. The coefficient depends on the same parameters presented as arguments in eq. (3). From the other side, estimated in [12] diffuser performance curves are rather similar to expected ones. It shows that values of \( \lambda_{fr} \) are surely not constant but differ little. The result of simulation on the base of \( \lambda_{fr} \) is presented for the series of diffusers with most usual relative length \( D_1/D_2 = 1,60 \). Surfaces are hydraulically smooth. The graphic representation of \( \lambda_{fr}^2 \) recalculation by eq. (4a) is presented at Figure 5 for \( \lambda_{c2} = 0.82, \text{Re}_D = 10.3E6 \). The results for \( \lambda_{c2} = 0.64, \text{Re}_D = 8.04E6 \) and \( \lambda_{c2} = 0.39, \text{Re}_D = 4.9E6 \) are the same by character and differs not too much in Figures.

There are some facts that deserve special attention. At flow angle 45° the loss coefficients \( \lambda_{fr45°} \) are constant practically for VLD with different relative width at given \( \text{Re}_D \). The values of \( \lambda_{fr45°} \) for different \( \text{Re}_D \) and \( \lambda_{c2} \) are presented in the Table 1.

| \( \text{Re}_D \) | 4.9E6 | 8.04E6 | 10.3E6 |
|-----------------|--------|--------|--------|
| \( \lambda_{c2} \) | 0.39   | 0.64   | 0.82   |
| \( \lambda_{fr45°} \) | 0.0238 | 0.0215 | 0.0209 |
Figure 5. Friction coefficient $\lambda_{fr}$ as function of inlet flow angle for series of VLD with $b/D_2 = 0.014 - 0.10$. $D_3/D_2 = 1.60$, $\lambda_{c2} = 0.82$, $Re_{D2} = 10.3E6$

Many authors use Reynolds criteria $Re_b = \frac{2b \cdot c_2}{v_2}$ for vaneless diffusers. Evidently it does not work for the friction coefficient $\lambda_{fr,45^\circ}$. Data at Figure 5 show that the friction coefficient $\lambda_{fr,45^\circ}$ is independent of $b/D_2$, i.e. of $Re_b$. From the other side data in the Table 1 shows that the bigger are $\lambda_{c2}$ and $Re_{D2}$ the lower is $\lambda_{fr,45^\circ}$. It points on the fact that positive influence of higher $Re_{D2}$ exists and prevails on negative influence of $\lambda_{c2}$.

The smaller is $\alpha_2$ the lower is $\lambda_{fr}$ for diffusers with $b/D_2 = 0.014 - 0.038$. Separation losses are absent or small in these diffusers. In diffusers with relative width $b/D_2 \geq 0.057$ higher friction coefficient value corresponds to inlet angles. Mixing losses prevail in wide diffusers.

The diffusers with $b/D_2 = 0.043 - 0.050$ have practically constant values of the friction coefficient. Not monotonous part of the function $\zeta = f(\alpha_2)$ corresponds to regimes with developed separation zones. Real stages don’t operate at these regimes due to dangerous unsteady processes.

Data analysis has shown that at inlet flow angle $10^\circ$ friction coefficient $\lambda_{fr,10^\circ}$ is a linear function of a diffuser relative width. The values are independent of $\lambda_{c2}$ and $Re_{D2}$.

The above mentioned facts have made possible to offer simple set of Eq. (5-7) to simulate results of numerical experiment:

$$\lambda_{fr} = \lambda_{fr,45^\circ} + \left(\lambda_{fr,10^\circ} - \lambda_{fr,45^\circ}\right) \left(\frac{\alpha_{2,\text{max}} - \alpha}{\alpha_{2,\text{max}} - \alpha_{2,\text{min}}}\right)^{x_i} = \lambda_{fr,45^\circ} + \left(\lambda_{fr,10^\circ} - \lambda_{fr,45^\circ}\right) \left(\frac{45^\circ - \alpha_{2}}{35^\circ}\right)^{x_i} \quad (5)$$
\[ \lambda_{\text{conv45}} = X_2 \frac{0.316}{\text{Re}_{D_2}^{0.25}} \]  

(6)

\[ \lambda_{fr10^\circ} = \lambda_{fr10^\circ \, b_{\text{min}}} \left( \frac{b_{\text{max}}}{D_2} - \frac{b_{\text{min}}}{D_2} \right) \left( b / D_2 - b_{\text{min}} / D_2 \right) = 0.012 \times \left( 0.014 - 0.001 \right). \]  

(7)

Coefficients \( X_1, X_2 \) are empirical coefficients proposed by the authors. Eq. (6) looks as known Blasius formula for a friction coefficient of a pipe. Reynolds number in the eq. (6) is proportional to an inlet diameter of a diffuser. Reynolds number in Blasius formula is proportional to a diameter of a pipe. Therefore the value of \( X_2 \) is far from 1.

Graphics at Figure 6 demonstrate results of \( \lambda_{fr} \) simulation for several diffusers.

Simulation doesn’t match CFD results in zones of flow separation in wide diffusers. It is not important practically. Average error for all VLD at all three \( \lambda_{v2} \) and \( \text{Re}_{D_2} \) is less than 1.2% - separation zones included. Average error for non-separated zones is 0.5%. The error of stage efficiency estimation is much less. Loss of efficiency in a diffuser is \[ \Delta \eta = \frac{c_2}{\psi_r} \left( \frac{c_2}{u_2} \right)^2. \]  

(8)

Calculated by CFD and simulated performances \( \zeta = f \left( \alpha_2 \right) \) are presented at Figure 7.

**Figure 6.** Friction coefficient \( \lambda_{fr} \) as function of inlet flow angle for series of VLD with \( b / D_2 = 0.014 - 0.10, \ D_3 / D_2 = 1.60, \ \lambda_{v2} = 0.64, \ \text{Re}_{D_2} = 9.2 \times 10^6 \)  Solid – CFD calculation, stroke – simulation by eq. (5 -7)
Figure 7. Loss coefficient as function of inlet flow angle for series of VLD with $b / D_2 = 0.014 – 0.10$. $D_3 / D_2 = 1.60$, $\lambda_{c2} = 0.64 \text{ and } Re_{D2} = 9.2 \cdot 10^6$. Solid – CFD calculation, stroke – simulation by eq. (5-7).

5. SURFACE ROUGHNESS INFLUENCE

The sample of surface roughness influence is presented at Figure 8. The range of relative roughness covers the zone of practical interest. For instance, in case of an impeller with diameter 0.85m the width of the VLD that is presented at Figure 8 is 0,8 \cdot 0.057 \approx 0,046 \text{ m}. The maximum absolute roughness is $(0.046 \cdot 2 \cdot 2 / 10^3) \cdot 10^6 \approx 200 \mu m$, i.e. 0.2 mm. Accordingly, the minimal relative roughness 0.055E-3 corresponds to 5 $\mu m$.

Calculation analysis has shown that friction coefficient of rough surfaces $\lambda_{fr \text{rg}} = f(\alpha_2)$ has the same character as in case of hydraulically smooth surface, i.e. the eq. (5) can be applied in principle. Therefore for the “rough” friction coefficient simulation is sufficient to find function

$$\lambda_{fr \text{rg}45^\circ} = f\left(\frac{k_{rg}}{2b}\right) \text{ and } \lambda_{fr \text{rg}10^\circ} = f\left(\frac{k_{rg}}{2b}\right).$$

The ratios $\left(\lambda_{fr \text{rg}} / \lambda_{fr}\right)_{45^\circ}$ and $\left(\lambda_{fr \text{rg}} / \lambda_{fr}\right)_{10^\circ}$ appeared to be the linear functions of the relative roughness logarithm presented as it is shown at Figure 9.

$$\left(\frac{\lambda_{fr \text{rg}}}{\lambda_{fr}}\right)_{45^\circ} = 1 + 4.37 \cdot \left\{\frac{1}{-\lg\left(\frac{k_{rg}}{2b}\right)} - \frac{1}{-\lg\left(0.031 \cdot 10^{-3}\right)}\right\}. \quad (9)$$
\[
\left( \frac{\lambda_{fr,rg}}{\lambda_{fr}} \right)_{10^\theta} = 1 + 2.62 \cdot \left[ \frac{1}{-\lg \left( \frac{k_{rg}}{2b} \right)} - \frac{1}{-\lg \left( 0.031 \cdot 10^{-3} \right)} \right].
\] 
(10)

\[Figure 8. \] VLD performance with different relative roughness. \( b / D_2 = 0.057, \lambda_{c,2} = 0.64, \text{Re}_{D_2} = 8.04 \cdot 10^6, \kappa_{rg} / 2b = 0.055 \cdot 10^{-3}, 0.11 \cdot 10^{-3}, 0.55 \cdot 10^{-3}, 1.1 \cdot 10^{-3}, 2.2 \cdot 10^{-3} \]

\[Figure 9. \] Relation of rough and smooth surfaces friction coefficient. The linear function are presented by eq. (9) and (10)

The ratio \( \frac{\lambda_{fr,rg}}{\lambda_{fr}} \) is equal to one when the relative roughness is \( k_{rg} / 2b = 0.031 \cdot 10^{-3} \). It means that the surface is hydraulically smooth when \( k_{rg} / 2b \leq 0.031 \cdot 10^{-3} \). In case of rough surface a friction coefficient can be calculated by Eq. (5, 6, 7) meaning that:

\[
\lambda_{fr,rg}^{45^0} = \lambda_{fr}^{45^0} \left( \frac{\lambda_{fr,rg}}{\lambda_{fr}} \right)_{45^0}, \text{ } \lambda_{fr,rg}^{10^0} = \lambda_{fr}^{10^0} \left( \frac{\lambda_{fr,rg}}{\lambda_{fr}} \right)_{10^0}.
\]

(11)

**CONCLUSION**

The presented way to simulate CFD – calculated VLD loss coefficient performance by algebraic equations is simple and precise. The analysis of VLD performances with different relative length and
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