Study of structures and dynamical decay mechanisms for multiquark systems

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The inner structures of the multiquark states are an interesting subject in hadron physics, because it is obviously beyond the simple $qq$ or $qqq$ texture which was established by Gell-Mann more than half century ago. The molecular structures of the multiquark systems have been studied by many authors, in comparison the tetraquark states have much less been investigated so far. Generally, the tetraquark is suggested to be composed of a diquark and an antidiquark which reside in color antitriplet and triplet, respectively [1–3]. Because of the different color structures, the dynamics for molecular states and tetraquark would be very different. In fact, in molecular states, the two constituents are bound by exchanging color-singlet mesons or baryons which can be described by the chiral Lagrangian. Instead, the two constituents in tetraquarks are bound by direct gluon exchange. As for the quarkonium, the interaction between quark and antiquark is realized by exchanging gluons, a single-gluon exchange composes the leading contribution which results in a Coulomb-type effective potential, however, for the hadron formation, the energy scale is below $\Lambda_{QCD}$, therefore the nonperturbative QCD effects would play an important role which induces a confinement piece in the potential. Similarly, for the tetraquark case, the diquark and antidiquark interact by exchanging gluons and definitely the single gluon exchange exists and induce a Coulomb-like potential whereas the nonperturbative QCD effects should also be introduced. It is generally believed that such interaction may be described by the color-flux model [2]. Following this scenario, we will study the dynamics which not only results in the different structures of the two configurations, but also determines their different decay patterns. Because the newly observed $Y(4630)$ is very likely to be a tetraquark, it would be an ideal place for carrying research on the tetraquark and molecular states via their decay behaviors.

Y(4630) has been observed in the invariant mass spectra of the $e^+e^- \to \Lambda_c\bar{\Lambda}_c$ channel [4], and it is identified as a $J^{PC} = 1^{-+}$ resonance with mass $M = 4634^{+9}_{-11}$ MeV and width $\Gamma = 92^{+41}_{-32}$ MeV.

There are many alternative interpretations for the observed peak [5, 6], for example, the authors of Ref. [7] consider that a strong attraction between $\Lambda_c$ and $\bar{\Lambda}_c$ bind them together, so that $Y(4630)$ may be interpreted as a baryon-antibaryon molecule. Instead, in Refs. [8, 9] $Y(4630)$ is interpreted as a $5^3S_1$ charmonium state. Also, the $Y(4630)$ is considered to be induced by a threshold effect instead of being a genuine resonance [10].

Among those proposals, the suggestion that $Y(4630)$ is a tetraquark state is more favorable [11–13]. In Ref. [11], $Y(4630)$ is identified as the ground state with its orbital angular momentum $L = 1$. By reanalyzing the $\Lambda_c\bar{\Lambda}_c$ and $\psi(2S)\pi^+\pi^-$ spectra, Cotugno et al. suggested that $Y(4630)$ and $Y(4660)$ [14, 15] could be the same tetraquark state, and is the first radial excitation of the $Y(4360)$ with $L = 1$ [12]. In Ref. [13], the authors studied the open-charm decay $Y(4630) \to \Lambda_c\bar{\Lambda}_c$ by assuming that $Y(4630)$ is a radially excited state of the diquark-antidiquark bound state with hidden charm. By another theoretical physics group [16] $Y(4630)$ is interpreted as a molecular state made of $\psi(2S)$ and $f_0(980)$.

As is well known, in general the multiquark states may be in tetraquark states which are composed of colored constituents, or in molecular states which are composed of two color singlets, or their mixtures. Therefore, the mechanisms which bind the components in a unique system and induce the multiquark states to decay would be different in those cases. In this work, using the quantum mechanics we analyze the dynamical mechanisms inducing decays of the tetraquarks where $Y(4630)$ stands as an example for the study, we also comment on the molecular states without making numerical computations.

The paper is organized as follows: after this introduction, we study the decay of $Y(4630)$ with the tetraquark and molecular interpretations in Secs II and III respectively, then Sec IV is devoted to our discussion and conclusion.

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I. INTRODUCTION
\section*{II. TETRAQUARK PICTURE}

Inspired by the fact that the $Y(4630)$ decays into charmed baryon pair, one is tempted to conjecture this resonance as a tetraquark which is made of the diquark-antidiquark $[cq][\bar{c}\bar{q}]$, where $q$ is a light quark either $u$ or $d$, $[cq]$ resides in a color antitriplet whereas $[\bar{c}\bar{q}]$ is in a color triplet.

Here we take the diquark-antidiquark picture proposed by Brodsky \textit{et al.} \cite{2} that a diquark and an antidiquark are bound together by a gluon-flux-tube into a color singlet tetraquark where the constituents are separated by a substantial distance once they are created. The interaction for the system can be well described by a generalized Cornell potential \cite{17} since the constituents (diquark and antidiquark) are treated as two pointlike color sources in analog to the configuration for an ordinary $Q\bar{Q}$ quarkonium.

In terms of the quark pair creation (QPC) model where a quark-antiquark pair is excited out from vacuum, we calculated the decay width of $Y(4630) \rightarrow \Lambda_c\Lambda_c$. \cite{13}. In that picture, the QCD vacuum is excited and a quark-antiquark pair is created. The quark-antiquark pair would "tear" apart the diquark and antidiquark due to strong QCD interaction between quark and antiquark (quark and diquark). Then joining the diquark, the created quark becomes a constituent of the charmed baryon $\Lambda_c$, and as well for the $\bar{c}\Lambda_c$.

In parallel, let us consider an alternative way to discuss the production of $\Lambda_c\bar{\Lambda}_c$ in the framework of quantum mechanics.

In fact, the color flux-tube results in a potential barrier to forbid the inner constituents (either the diquark or the antiquark) to escape from the bound state. In terms of a modified flux-tube-induced potential, the decay of $Y(4630)$ may occur as the diquark-antidiquark bound system falling apart via tunneling through the effective potential barrier. After tunneling out the barrier, the diquark (antidiquark) would immediately attract a quark (antiquark) from the vacuum to compose a color singlet $\Lambda_c$ ($\bar{\Lambda}_c$) and the hadronization process is somehow similar to the picture frequently used to study the multiparticle production at high energy collision. Then the transition probability can be calculated in terms of the WKB (Wentzel-Kramers-Brillouin) approximation.

\subsection*{A. Potential model}

First, we employ a nonrelativistic potential model with a Cornell-like potential where some free parameters are obtained by fitting the mass spectrum of heavy quarkonia and generalized to the case for tetraquark, then we get the wave function of $Y(4630)$ by solving the Schrödinger equation.

The general Hamiltonian of a diquark-antidiquark system (i.e. a quarkonium-like system) can be written as

$$H = \frac{p_1^2}{2m_1} + m_1 + \frac{p_2^2}{2m_2} + m_2 + V(r),$$

where $m_1(p_1)$ and $m_2(p_2)$ are the masses(3-momenta) of the diquark $cq$ and antidiquark $\bar{c}\bar{q}$ respectively, we take $m_1 = m_2 = m$ in this work. The interaction potential is

$$V(r) = V_{oge}(r) + V_{conf}(r),$$

where $r$ is the distance between the diquark and the antidiquark. The one-gluon-exchange (oge) term $V_{oge}(r)$, which plays the main role at short distances, is \cite{18}

$$V_{oge}(r) = -\frac{4\alpha_s}{3} + \frac{32\pi\alpha_s}{9m^2} S_1 \cdot S_2 \delta(r),$$

and the confinement part $V_{conf}(r)$ takes the linear form\cite{19}

$$V_{conf}(r) = br + c,$$

where $S_{1(2)}$ and $-4/3$ are, respectively, the spin operators and the color factor specific to $3-\bar{3}$ attraction. $b$ is the string tension and $c$ is a global zero-point energy. $\alpha_s$ is the phenomenological strong coupling constant.

The $\delta$-function in Eq.(3) is replaced by a Gaussian smearing function \cite{20} with a fitted parameter $\sigma$

$$\delta(r) \rightarrow \left(\frac{\sigma}{\sqrt{\pi}}\right)^3 e^{-\sigma r^2}.$$

The spin wave functions for the $Y(J^{PC} = 1^{--})$ state with $L = 1$ is taken as $Y_1 = |0,0,0,1\rangle_1$ in the basis of $|S_{qc},S_{\bar{c}\bar{q}},S_{\text{total}},L\rangle_{J=1}$ \cite{11}.

Here we take the values from Ref. \cite{18}: $\alpha_s = 0.5461, b = 0.1425$ GeV$^2, c = 0, \sigma_1 = 1.0946$ GeV.

By adopting the suggestion given by authors of Refs. \cite{12, 13} that $Y(4630)$ is a radial excitation state of P-wave ($L=1$), the (anti)diquark mass is determined to be 1.878 GeV which is close to the $D$ meson mass. This value is also consistent with that computed by using QCD sum rules in Ref. \cite{21} where it is $1.86 \pm 0.05$ GeV.

The fitted spectra are presented in Fig. 1, and the charmonium spectra calculated by authors of Ref. \cite{18} are also shown in the figure for a clear comparison. In this framework, we fit the ground state to be 4235 MeV, and such a state is consistent with the observed $Y(4230)$ resonance \cite{22} and/or $Y(4220)$ resonance \cite{23, 24} which is also considered as a tetraquark \cite{25}. The radial wave function of $Y(4630)$ is plotted in Fig. 2.

\subsection*{B. Decay of $Y(4630)$ as a tetraquark}

The interaction between the constituents in the tetraquark cannot simply be derived from the quantum field theory yet and phenomenologically the dynamics of the nonperturbative QCD effects which determines the confinement behavior may be described by the flux-tube model. Moreover, as is well known, when the tension of the flux-tube goes beyond a certain bound, namely, the distance between the diquark and antidiquark gets long enough, the flux-tube will break into two
strings and at the new ends a quark-antiquark pair is created \([27, 28]\). One can use a step function to describe the breaking effect as

\[
1 - \theta(r - r_0) \times V(r),
\]

where \(r_0\) is a parameter corresponding to the strengthening limit of the string at where the probability of the string fragmentation reaches maximum. A typical scale for nonperturbative QCD is \(\Lambda_{\text{QCD}}\), therefore it is natural to consider \(r_0\) should be of order of \(\sim 1/\Lambda_{\text{QCD}}\). Just as smearing the delta function, we need also to smear the step function. In fact

\[
1 - \theta(r - r_0) = \lim_{\epsilon \to 0} \frac{1}{e^{-\frac{(r-r_0)}{\epsilon}} + 1},
\]

so smearing the step function implies that we keep \(\epsilon\) as a nonzero free parameter to be determined. In fact, if we do not consider breaking of the flux tube, the effective potential is the same as \(V(r)\) given in Eq. (2). Therefore, taking into account of the “breaking” effect does not affect the computation on the \(Y(4630)\) spectrum.

Here the interaction between the diquark and the antiquark at a relatively large distance is described by a modified potential as \([29]\).

\[
V'(r) = V(r) \frac{1}{e^{-\frac{(r-r_0)}{\epsilon}} + 1}.
\]

In this scenario, we translate the flux-tube induced confinement into the potential barrier, and breaking the tube corresponds to tunneling through the barrier. The diquark-antidiquark bound system falls apart by tunneling through this effective potential barrier, then is hadronized into a \(\Lambda_c \bar{\Lambda}_c\) pair. The process is graphically shown in Fig. 3. By means of the WKB approximation, the transition probability of the tunneling process can be calculated.

Under this assignment, the transition probability is given by

\[
T = \exp\left[-\frac{2}{\hbar} \int_a^b \sqrt{2\mu(E - V'(r))} \, dr\right],
\]

where \(\mu = m_1 m_2 / (m_1 + m_2) = m/2\) is the reduced mass of the \([cq]-[\bar{c}\bar{q}]\) system, \(a\) and \(b\) are the turning points, as shown in Fig. 3.

One can obtain the effective velocity \(v\) of the motion of a particle with the reduced mass inside the system from the average kinetic energy \(\langle \psi(r)| \frac{\hbar^2}{2m} | \psi(r) \rangle\) where \(\psi(r)\) is the wave function obtained in terms of the pure Cornell potential without the flux tube breaking correction. Since the breaking effect only affects the long distance behavior, it does not change the wave function \(\psi(r)\) and the decay width \(\Gamma_Y = \frac{\hbar}{2m} T\) is deduced.

Assuming the \(Y(4630)\) as the first radial excited state, we compute the partial decay width of the channel \(Y(4630) \to \Lambda_c \bar{\Lambda}_c\). In Fig. 4, dependence of the calculated partial width \(\Gamma_Y\) on the free parameter \(\epsilon\) is plotted. The purple, blue and cyan curves correspond to the cases of \(r_0 = 1.5, 1.6, 1.7\) fm, respectively. As \(\epsilon\) increases, the predicted decay width increases as...
shown in Fig. 4, where each curve in the figure corresponds to a special \(r_0\) value, and it is noted that each curve stops at some value of \(\epsilon\), because at that \(\epsilon\) value, the diquark and antidiquark are no longer bound by the modified potential. It sets a constraint on \(\epsilon\). The dashed black line and the green band correspond to the central value and error for the total width of \(Y(4630)\) measured by the Belle collaboration (\(\Gamma = 92^{+41}_{-32} \text{ MeV}\) [4]). The purple, blue and cyan curves correspond to the three different \(r_0\) assignments respectively.

It is noticed that the calculated partial decay width changes with \(r_0\), but for any specific value of \(r_0\), there exists a range of \(\epsilon\) which allows the predicted width to be consistent with the experimental data, namely, falls within the error tolerance region set by present measurements, which is rather wide. Indeed, even though the calculated decay width of \(Y(4630) \to \Lambda_c \bar{\Lambda}_c\) is smaller than the central value of the measured total width, one still cannot conclude that \(Y(4630)\) is not a pure tetraquark yet.

Our numerical results indicate that the tetraquark picture is able to predict the correct decay width of \(Y(4630)\), even though not completely confident, we believe that its decay mechanism could be considered as the diquark-antidiquark system falling apart via tunneling through an effective potential barrier, and then diquark and antidiquark are respectively hadronized into color singlet hadrons, and the \(\Lambda_c \bar{\Lambda}_c\) pair should be the main product. This conclusion is consistent with that of Ref. [13], in which a similar result is obtained within the QPC framework.

It is noteworthy that if the future measurement indeed confirms a rather large total width which is larger than our prediction based on the pure tetraquark structure, a possible mixture between tetraquark and molecular state should be taken into account and other decay modes such as \(Y(4630) \to \psi(2S) \pi^+ \pi^-\) may occur with non-negligible fraction.

### III. MOLECULAR PICTURE

As is mentioned in the introduction and the above discussion, the hadronic molecular picture \(\psi(980)\) is another possible choice for the inner structure of \(Y(4630)\) which was proposed by Guo et al. [16]. It is noted that the mass and width of \(Y(4630)\) are consistent with those for the \(Y(4660)\) state (\(M = 4652 \pm 10 \pm 8 \text{ MeV}, \Gamma = 68 \pm 11 \pm 1 \text{ MeV}\) [15]) within error tolerance. By taking into account the \(\Lambda_c \bar{\Lambda}_c\) final state interaction, it is found that the \(Y(4630)\) may be the same state as \(Y(4660)\), and the resonance can be a \(\psi(980)\) molecular bound state.

Now let us study the mechanism which may induce the decay of the molecular state. Because the two constituents in the molecular state are color singlets, they do not interact by directly exchanging gluons, but only via exchanging color-singlet mesons, and the leading contribution is coming from the \(\sigma(f_0(600))\) exchange between \(\psi(2S)\) and \(f_0(980)\) which does not induce a potential barrier as for the tetraquark case.

Instead, the interaction provides a potential well and the constituents are confined in the well, as shown in Fig. 5. Thus we should have \(M_{Y(4630)} = M_{\psi(2S)} + M_{f_0(980)} + \Delta E\) where \(\Delta E\) is the binding energy of the molecular state and roughly is \(-30 \text{ MeV}\). In the traditional framework of quantum mechanics it is a stable structure, i.e. \(Y(4630)\) cannot dissolve into on-shell \(\psi(2S)\) and \(f_0(980)\), however, due to the quantum fluctuation, \(f_0(980)\) can jump out the potential well to become an off-shell virtual particle. If the virtual \(f_0(980)\) does not transit into two real pions, it would fall back to its original state inside the molecule. The duration of it being virtual particle can be estimated by the uncertainty principle as \(\Delta t \cdot \Delta E \sim \frac{\hbar}{\Delta E}\) in the natural unit system, thus the virtuality time \(\Delta t\) is proportional to \(\frac{1}{\Delta E}\) where \(\Delta E\) is the binding energy. Obviously the decay amplitude should be proportional to \(\Delta t\), namely the larger the binding energy \(\Delta E\) is, the shorter \(\Delta t\) is, and then the smaller the decay probability would be. By this principle, we can write
out an effective Lagrangian which induces the decay of the molecule $Y(4630) \rightarrow \psi(2S) + f_0^*(980) \rightarrow \psi(2S) + \pi^+\pi^-$ where the superscript $\ast$ denotes that $f_0(980)$ is an off-shell virtual meson which later transits into two pions. The effective vertex at $Y(4630) - \psi(2S) f_0(980)$ is

$$L = \frac{g}{|\Delta E|} A_{\mu}^{\ast} \partial_{\alpha} B_{\nu}^{\ast} \partial_{\alpha} \phi,$$

(9)

where $A_{\mu}$, $B_{\nu}$ and $\phi$ correspond to $\psi(2S)$, $Y(4630)$, $f_0(980)$ and $g$ is a dimensionless universal coupling constant. By the equation of motion it is easy to be reduced into $L'$ which reads as

$$L' = \frac{g(m_g^2 - m_{\psi}^2)}{|\Delta E|} A_{\mu}^{\ast} B_{\nu}^{\ast} \phi.$$ (10)

The effective coupling given by authors of Ref. [16] is $\frac{\alpha^2}{4\pi} = 4(M_{\psi} + m_{f_0(980)})^2 \sqrt{2|\Delta E|}/\mu'$ where $\mu'$ is the reduced mass of the $\psi'$ and $f_0(980)$, $g'$ has an energy dimension. Thus we can identify the relation

$$g(m_{\psi}^2 - m_{f_0(980)}^2) = g'.$$ (11)

With this assumption another decay mode of $Y(4630)$ could be $Y(4630) \rightarrow \psi(2S) \pi^+\pi^-$ for the off-shell $f_0(980)$ mainly decaying into $\pi^+\pi^-$, as shown in Fig. 6.

With the given Lagrangian, the decay width was calculated in Ref. [16] as $\Gamma(Y \rightarrow \psi(2S) \pi^+\pi^-) = 8$ MeV, here we do not repeat it and advise readers to refer to that paper.

![Diagram illustrating a possible decay channel of the $Y(4630)$ in the molecular picture which is $Y(4630) \rightarrow \psi(2S) \pi^+\pi^-$.](image)

**FIG. 6:** Diagram illustrating a possible decay channel of the $Y(4630)$ in the molecular picture which is $Y(4630) \rightarrow \psi(2S) \pi^+\pi^-$. [Diagram image]

### IV. CONCLUSION AND DISCUSSION

The hadronic decay is closely associated with the nonperturbative QCD, and a lot of phenomenological models are proposed to account for its effects. For example, the QPC model, flux-tube model, QCD sum rules and lattice QCD, etc. have been successfully used to estimate decay rates, even though, with the exception of the lattice calculation, none of them can be directly derived from quantum field theory so far.

For the $Q\bar{Q}$ systems, the physics picture is clear, even though a phenomenological model must be embedded to reflect the nonperturbative QCD effects and the computation schemes are mature. However, for the four-quark states, the inner structure and dynamics which leads the binding and decay of the state are still not well understood and there are various proposals for them. In this work we study the decay mechanisms of $Y(4630)$ in both tetraquark and molecule pictures in the framework of quantum mechanics. Namely, we use the WKB approximation to calculate the decay width of $Y(4630)$ as it is assumed to be a tetraquark state, and then qualitatively discuss its decay mechanism as it is postulated as a molecular state where $f_0(980)$ jumps out the potential well due to a quantum fluctuation and becomes a virtual particle and later transits into two real pions.

Definitely, all of the assignments to the observed resonance at 4630 MeV should be tested in the future by more precise measurements. In our other works [13, 31], we study the case that if $Y(4630)$ is a tetraquark, its favorable decay mode should be $Y(4630) \rightarrow \Lambda_c \Lambda_c$ which would overwhelmingly dominate its width, but due to the inelastic rescattering processes between $\Lambda_c$ and $\bar{\Lambda}_c$, some other final states, such as $p\bar{p}$, $n\bar{n}$, $D^{(*)} \bar{D}^{(*)}$ and $\pi\pi$, $K\bar{K}$ might be produced with measurable rates, whereas, if $Y(4630)$ is a molecular state, its dominant decay mode would be $\psi(2S)\pi\pi$ and due to the decay of $\psi(2S)$ and final state interaction, the pattern of the decay products which will be experimentally measured would be completely different from the tetraquark case. Thus the measurements would provide more information about the assignment of $Y(4630)$. We are lying hope on the future experiments which will be carried out at the BELLEII, BESIII and even LHCb in the coming years.

Moreover, we suspect if there is a mixing between the tetraquark and molecular states which results in $Y(4630)$ and $Y(4660)$, it would be an interesting picture.

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[1] L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, “Diquark-antidiquarks with hidden or open charm and the nature of \( \Lambda(3872) \),” Phys. Rev. D 71, 014028 (2005) [hep-ph/0412098].

[2] S. J. Brodsky, D. S. Hwang and R. F. Lebed, “Dynamical Picture for the Formation and Decay of the Exotic XYZ Mesons,” Phys. Rev. Lett. 113, 112001 (2014) [arXiv:1406.7281 [hep-ph]].

[3] L. Montanet, G. C. Rossi and G. Veneziano, “Baryonium Physics,” Phys. Rept. 63, 149 (1980).

[4] G. Pakhlova et al. [Belle Collaboration], “Observation of a near-threshold enhancement in the \( e^+e^- \rightarrow A_0^\prime A_0^\prime \) cross section using initial-state radiation,” Phys. Rev. Lett. 101, 172001 (2008) [arXiv:0807.4458 [hep-ex]].

[5] H. X. Chen, W. Chen, X. Liu and S. L. Zhu, “The hidden-charm pentaquark and tetraquark states,” arXiv:1601.02092 [hep-ph].

[6] A. Esposito, A. L. Guerrieri, F. Piccinini, A. Pilloni and A. D. Polosa, “Four-Quark Hadrons: an Updated Review,” Int. J. Mod. Phys. A 30, 1530002 (2015) [arXiv:1411.5977 [hep-ph]].

[7] N. Lee, Z. G. Luo, X. L. Chen and S. L. Zhu, “Possible Deuteron-like Molecular States Composed of Heavy Baryons,” Phys. Rev. D 84, 014031 (2011) [arXiv:1104.4257 [hep-ph]].

[8] A. M. Badalian, B. L. G. Bakker and I. V. Danilkin, “The S-D mixing and di-electron widths of higher charmonium \( 1^- \) states,” Phys. Atom. Nucl. 72, 638 (2009) [arXiv:0805.2291 [hep-ph]].

[9] J. Segovia, D. R. Entem and F. Fernandez, “Charm spectroscopy beyond the constituent quark model,” arXiv:0810.2875 [hep-ph].

[10] E. van Beveren, X. Liu, R. Coimbra and G. Rupp, “Possible \( \psi(5S), \psi(4D), \psi(6S) \) and \( \psi(5D) \) signals in \( \Lambda_c^+ \),” Europhys. Lett. 85, 61002 (2009) [arXiv:0809.1151 [hep-ph]].

[11] L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, “The \( Z(4430) \) and a New Paradigm for Spin Interactions in Tetraquarks,” Phys. Rev. D 89, 114010 (2014) [arXiv:1405.1551 [hep-ph]].

[12] G. Cotugno, R. Faccini, A. D. Polosa and C. Sabelli, “Charmed Baryonium,” Phys. Rev. Lett. 104, 132005 (2010) [arXiv:0911.2178 [hep-ph]].

[13] X. Liu, H. W. Ke, X. Liu and X. Q. Li, “Exploring open-charm decay mode \( \Lambda_c \), of charmonium-like state \( Y(4630) \),” arXiv:1601.00762 [hep-ph].

[14] X. L. Wang et al. [Belle Collaboration], “Observation of Two Resonant Structures in \( e^+e^- \rightarrow \pi^+\pi^- \psi(2S) \) via Initial State Radiation at Belle,” Phys. Rev. Lett. 99, 142002 (2007) [arXiv:0707.3699 [hep-ex]].

[15] X. L. Wang et al. [Belle Collaboration], “Measurement of \( e^+e^- \rightarrow \pi^+\pi^- \psi(2S) \) via Initial State Radiation at Belle,” Phys. Rev. D 91, 112007 (2015) [arXiv:1410.7641 [hep-ex]].

[16] F. K. Guo, J. Haidenbauer, C. Hanhart and U. G. Meissner, “Reconciling the \( Y(4630) \) with the \( Y(4660) \),” Phys. Rev. D 82, 094008 (2010) [arXiv:1005.2055 [hep-ph]].

[17] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane and T. M. Yan, “Charmonium: The Model,” Phys. Rev. D 17, 3090 (1978) [Phys. Rev. D 21, 313 (1980)].

[18] T. Barnes, S. Godfrey and E. S. Swanson, ”Higher charmonia,” Phys. Rev. D 72, 054026 (2005) [hep-ph/0505002].

[19] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane and T. Yan, Phys. Rev. D 17, 3090 (1978); ibid, D 21, 203 (1980).

[20] J. D. Weinstein and N. Isgur, “The \( q\bar{q}q\bar{q} \) System in a Potential Model,” Phys. Rev. D 27, 588 (1983).

[21] M. Ablikim et al. [BESIII Collaboration], “Study of \( e^+e^- \rightarrow \omega K^+\pi^- \) at center-of-mass energies from 4.21 to 4.42 GeV,” Phys. Rev. Lett. 114, 092003 (2015) [arXiv:1410.6538 [hep-ex]].

[22] C. Z. Yuan, “Evidence for resonant structures in \( e^+e^- \rightarrow \pi^+\pi^- h^- \),” Chin. Phys. C 38, 043001 (2014) [arXiv:1312.6399 [hep-ex]].

[23] C. Z. Yuan, “Recent progress on the study of the charmoniumlike states,” Int. J. Mod. Phys. A 29, 1430046 (2014) [arXiv:1404.7768 [hep-ex]].

[24] R. Faccini, G. Filaci, A. L. Guerrieri, A. Pilloni and A. D. Polosa, “Note on the newly observed \( Y(4220) \) resonance,” Phys. Rev. D 91, 117501 (2015) [arXiv:1412.7196 [hep-ph]].

[25] K. A. Olive et al. [Particle Data Group Collaboration], “Review of Particle Physics,” Chin. Phys. C 38, 090001 (2014).

[26] R. Kokoski and N. Isgur, “Meson Decays by Flux Tube Breaking,” Phys. Rev. D 35, 907 (1987).

[27] S. Kumano and V. R. Pandharipande, “Decay of Mesons in Flux Tube Quark Model,” Phys. Rev. D 38, 146 (1988).

[28] X. W. Liu, H. W. Ke, Y. B. Ding and X. Q. Li, “Study of the structures of four-quark states in terms of the Born-Oppenheimer approximation,” Chin. Phys. C 39, 083103 (2015) [arXiv:1409.5939 [hep-ph]].

[29] X. W. Liu, Z. G. Luo, Y. R. Liu and S. L. Zhu, “\( \chi(3872) \) and Other Possible Heavy Molecular States,” Eur. Phys. J. C 61, 411 (2009) [arXiv:0808.0073 [hep-ph]].

[30] X. D. Guo, D. Y. Chen, H. W. Ke, X. Liu and X. Q. Li, “Study on the rare decays of \( Y(4630) \) induced by final state interactions,” Phys. Rev. D 93, 054009 (2016) [arXiv:1602.02222 [hep-ph]].