Masses of Heavy Quarkonium states in magnetized matter
- effects of PV mixing and (inverse) magnetic catalysis

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Abstract

We study the in-medium masses of the heavy quarkonium (charmonium and bottomonium) states in isospin asymmetric nuclear matter in presence of an external magnetic field. The mass modifications of the heavy quarkonia are obtained from the medium modifications of a scalar dilaton field, $\chi$, calculated within a chiral effective model. The dilaton field is introduced in the model through a scale invariance breaking logarithmic potential, and, simulates the gluon condensates of QCD. Within the chiral effective model, the values of the dilaton field along with the scalar (isoscalar, $\sigma(\sim \langle \bar{u}u \rangle + \langle \bar{d}d \rangle)$, isoscalar $\zeta(\sim \langle \bar{s}s \rangle)$ and isovector $\delta(\sim \langle \bar{u}u \rangle - \langle \bar{d}d \rangle)$) fields, are solved from their coupled equations of motion. These are solved accounting for the effects of the Dirac sea (DS) as well as anomalous magnetic moments (AMMs) of the nucleons. When AMMs are neglected, both at zero density and at nuclear matter saturation density, $\rho_0$, the Dirac sea contributions are observed to lead to enhancement of the quark condensates (through $\sigma$ and $\zeta$ fields) with increase in magnetic field, an effect called the magnetic catalysis (MC). However, the inclusion of AMMs is observed to lead to the opposite trend of inverse magnetic catalysis (IMC) for $\rho_B = \rho_0$. The magnetic field effects on the masses of the heavy quarkonia include the mixing of the pseudoscalar (spin 0) and vector (spin 1) states (PV mixing), as well as, the effects from (inverse) magnetic catalysis. These effects are observed to be significant for large values of the magnetic field. This should have observable consequences on the production of the heavy quarkonia and open heavy flavour mesons, resulting from ultra-relativistic peripheral heavy ion collision experiments, where the created magnetic field can be huge.

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I. INTRODUCTION

The study of the in-medium properties of the heavy quarkonia is a topic of intense research work due to its relevance in heavy ion collision experiments\cite{1}. The magnetic fields created in peripheral ultra-relativistic heavy ion collision experiments, e.g., at LHC, CERN and RHIC, BNL are estimated to be huge\cite{2}. This has initiated a lot of work in the study of hadrons, in particular, of the heavy quarkonia as well as open heavy flavour mesons in the presence of magnetic fields, due to the reason that these heavy mesons are created at the early stage when the magnetic field can still be large. The heavy quarkonium (charmonium and bottomonium) states have been investigated in the literature using the potential models\cite{3 13}, the QCD sum rule approach\cite{14 30}, the coupled channel approach\cite{31 37}, the quark meson coupling (QMC) model\cite{38 46}, heavy quark symmetry and interaction of these mesons with nucleons via pion exchange\cite{47}, heavy meson effective theory\cite{48}, studying the heavy flavour meson as an impurity in nuclear matter\cite{49}.

Using leading order QCD formula\cite{50 52}, the mass modifications of the charmonium states were calculated in a linear density approximation in Ref.\cite{53}, due to the medium change of the scalar gluon condensate. The study showed much larger mass shifts for the excited states, $\psi(2S)$ and $\psi(1D)$, as compared to the mass shift of $J/\psi$. Within a chiral effective model\cite{54 56}, generalized to include the interactions of the charm and bottom flavoured hadrons, the in-medium heavy quarkonium (charmonium and bottomonium) masses are obtained from the medium changes of a scalar dilaton field, which mimics the gluon condensates of QCD\cite{57 59}. The mass modifications of the open heavy flavour (charm and bottom) mesons within the chiral effective model have also been studied from their interactions with the baryons and scalar mesons in the hadronic medium\cite{57 58 60 64}. The chiral effective model, in the original version with three flavours of quarks (SU(3) model), has been used extensively in the literature, for the study of finite nuclei\cite{55}, strange hadronic matter\cite{56}, light vector mesons\cite{65}, strange pseudoscalar mesons, e.g. the kaons and antikaons\cite{60 69} in isospin asymmetric hadronic matter, as well as for the study of bulk matter of neutron stars\cite{70}. Using the medium changes of the light quark condensates and gluon condensates calculated within the chiral SU(3) model, the light vector mesons ($\omega$, $\rho$ and $\phi$) in (magnetized) hadronic matter have been studied within the framework of QCD sum rule approach\cite{71 72}. The kaons and antikaons have been recently studied in the presence of strong...
magnetic fields using this model \[73\]. The model has been used to study the partial decay widths of the heavy quarkonium states to the open heavy flavour mesons, in the hadronic medium \[58\] using a light quark creation model \[74\], namely the \(^3P_0\) model \[75, 78\] as well as using a field theoretical model for composite hadrons \[79, 80\]. Recently, the effects of magnetic field on the charmonium partial decay widths to \(D\bar{D}\) mesons have been studied using the \(^3P_0\) model \[81\] and, charmonium (bottomonium) decay widths to \(D\bar{D}\) (\(B\bar{B}\)) using the field theoretic model of composite hadrons \[82, 83\].

In the present work, we study the modifications of the masses of the charmonium (\(J/\psi\), \(\psi(2S)\) and \(\psi(1D)\)) and the bottomonium (\(\Upsilon(1S), \Upsilon(2S), \Upsilon(3S)\) and \(\Upsilon(4S)\)) states in magnetized (nuclear) matter, within a chiral effective model. The mass modifications of these states arise due to the medium change of a dilaton field, which is incorporated into the model to simulate the gluon condensates of QCD. Using the approximation that the scalar fields are treated as classical fields, the value of the dilaton field, \(\chi\), is obtained from solution of the coupled equations of motion of the scalar (nonstrange isoscalar \(\sigma\), strange isoscalar \(\zeta\) and nonstrange isovector \(\delta\)) fields and \(\chi\). The Dirac sea contributions of the nucleons are also taken into consideration for obtaining the nucleon self-energy, by summing over the tadpole diagrams. The effects of the anomalous magnetic moments (AMMs) of the nucleons are observed to lead to important modifications to the self energy of the nucleons. Within the chiral effective model, for zero density as well as for the baryon density, \(\rho_B = \rho_0\), the nuclear matter saturation density, when the AMMs are not taken into account, both for symmetric and asymmetric nuclear matter, the Dirac sea contributions are observed to lead to an enhancement of the quark condensates (through the scalar \(\sigma \sim (\langle \bar{u}u + \bar{d}d \rangle)\) and \(\zeta(\sim \langle \bar{s}s \rangle)\) fields), with increase in the magnetic field, an effect called ‘magnetic catalysis (MC)’. However, with inclusion of the AMMS of the nucleons, for \(\rho_B = \rho_0\), the opposite trend, i.e., the inverse magnetic catalysis (IMC) is observed within the chiral model. The increase of the light quark condensates with magnetic field, has been studied in a large extent on the quark matter sector using the Nambu-Jona-Lasinio (NJL) model \[84, 85\]. In Ref. \[86\], the effect of magnetic catalysis has been studied for the nuclear matter using the Walecka model and an extended linear sigma model. In Ref. \[87\], the effects of magnetic field have been studied in the Walecka model by using a weak field approximation of the fermion propagator. The effect of the anomalous magnetic moment of the nucleons are seen to enhance the catalysis effect at zero temperature and zero baryon density \[87\]. However, at finite temperature, the
critical temperature for the vacuum to nuclear matter phase transition for nonzero anomalous magnetic moment of the nucleons, is seen to rise with increasing magnetic field, implying inverse magnetic catalysis \[88\], whereas for vanishing AMMs of the nucleons, the behavior is opposite, indicating the magnetic catalysis. Thus, the effect of the anomalous magnetic moments of the nucleons are important to study the contributions from the Dirac sea in presence of finite magnetic field. In the literature there are very few works on the magnetic catalysis effect in the nuclear matter. In the present work, we have incorporated the effects of the Dirac sea through summation of nucleonic tadpole diagrams within a chiral effective model. In the present study of the heavy quarkonia masses, we also consider the mixing of the pseudoscalar and vector meson (PV mixing) \[82, 83, 89, 90\].

The outline of the paper is as follows. In section II, we describe briefly the computaiton of the mass modificaitons of the heavy quarkonium states in magnetized (nuclear) matter using a chiral effective model. These masses are calculated within the model from the modification of the scalar dilaton, which mimics the scale symmetry breaking of QCD. The magnetic field effects considered are the pseudoscalar - vector meson (PV) mixing and the contributions of the Dirac sea of the nucleons. The PV mixing corresponds to the mixing of the pseudoscalar (spin 0) and the vector (spin 1) mesons in the presence of a magnetic field. The latter leads to the (inverse) magnetic catalysis effect. In section III, the results of the medium modifications of the charmonium and bottomonium masses in magnetized matter are discussed and section IV summarizes the findings of this work.

II. MASS MODIFICATIONS OF HEAVY QUARKONIUM STATES IN MAGNETIZED MATTER

The in-medium masses of the charmonium (\(J/\psi\), \(\psi(2S)\) and \(\psi(1D)\)) and bottomonium states (\(\Upsilon(1S)\), \(\Upsilon(2S)\), \(\Upsilon(3S)\) and \(\Upsilon(4S)\)) are studied in magnetized (nuclear) matter. The effects of pseudoscalar - vector meson (PV) mixing \((J/\psi - \eta_c, \psi(2S) - \eta_c(2S), \psi(1D) - \eta_c(2S)\) for charmonium states and \(\Upsilon(1S) - \eta_b, \Upsilon(2S) - \eta_b(2S), \Upsilon(3S) - \eta_b(3S), \Upsilon(4S) - \eta_b(4S)\) for the bottomonium states), and the Dirac sea contributions for the nucleons are considered in the present study of mass modifications of these heavy mesons in the presence of a magnetic field.

The mass shift of the heavy quarkonium state is proportional to the change in the gluon...
FIG. 1: Masses (MeV) of $J/\psi$ and $\eta_c$ are plotted as functions of $eB/m_\pi^2$ for $\rho_B = \rho_0$ and $\eta = 0$, with and without the effects of PV mixing. These are shown in (a) and (c), when the Dirac sea contributions of nucleons (resulting in (inverse) magnetic catalysis) are not included and in (b) and (d), while these effects are included. In (b) and (c), the masses are also shown for $\rho_B = 0$ due to the Dirac sea contributions. The results are for cases when the AMMs are included, which are compared with the cases when AMMs are not considered (shown as dotted lines).

condensate in the medium. This is the leading order result of a study of the heavy quarkonium state in a gluon field, assuming the distance between the heavy quark and antiquark (bound by a Coulomb potential) to be small as compared to the scale of gluonic fluctuations [50–52]. The chiral effective model as used in the present work, is based on a non-linear
realization of chiral symmetry. The model also incorporates the broken scale invariance of QCD through a scalar dilaton field. The dilaton field \( \chi \) of the scale breaking term \( L_{\text{scalebreak}} \) in the chiral effective model is related to the scalar gluon condensate of QCD and this relation is obtained by equating the trace of the energy momentum tensor in the chiral effective model and in QCD \([57-59]\). The mass shift of the charmonium (bottomonium) state in the magnetized nuclear matter is hence computed from the medium change of the dilaton field from vacuum value, calculated within the chiral effective model, and is given as \([57-59]\)

\[
\Delta m = \frac{4}{81} (1 - d) \int d|k|^2 \langle |\frac{\partial \psi(k)}{\partial k}|^2 \rangle \frac{|k|}{|k|^2/m_{c(b)} + \epsilon (\chi^4 - \chi_0^4)},
\]

FIG. 2: Same as Figure 1 but with \( \eta=0.5 \).
FIG. 3: Masses (MeV) of $\psi(2S)$ and $\eta_c(2S)$ are plotted as functions of $eB/m^2_\pi$ for $\rho_B = \rho_0$ and $\eta = 0$, with and without the effects of PV mixing. These are shown in (a) and (c), when the Dirac sea contributions of nucleons (resulting in (inverse) magnetic catalysis) are not included and in (b) and (d), while these effects are included. In (b) and (c), the masses are also shown for $\rho_B = 0$ due to the Dirac sea contributions. The results are for cases when the AMMs are included, which are compared with the cases when AMMs are not considered (shown as dotted lines).

where

$$\langle |\frac{\partial \psi(k)}{\partial k}|^2 \rangle = \frac{1}{4\pi} \int |\frac{\partial \psi(k)}{\partial k}|^2 d\Omega.$$

(2)

In equation (1), $d$ is a parameter introduced in the scale breaking term in the Lagrangian, $\chi$ and $\chi_0$ are the values of the dilaton field in the magnetized medium and in vacuum re-
spectively. The wave functions of the charmonium (bottomonium) states, \(\psi(k)\) are assumed to be harmonic oscillator wave functions, \(m_{c(b)}\) is the mass of the charm (bottom) quark, \(\epsilon = 2m_{c(b)} - m\) is the binding energy of the charmonium (bottomonium) state of mass, \(m\). The mass shifts of the heavy quarkonium states are thus obtained from the values of the dilaton field, \(\chi\), (using equation (1)). For given values of the baryon density, \(\rho_B\), the isospin asymmetry parameter, \(\eta = (\rho_n - \rho_p)/(2\rho_B)\) (with \(\rho_n\) and \(\rho_p\) as the neutron and proton number densities), the magnetic field, \(B\) (chosen to be along z-direction), the values of the fields \(\chi, \sigma, \zeta\) and \(\delta\) are solved from their coupled equations of motion. The AMMs of the nucleons are considered in the present study \(\bar{91}-\bar{93}\). In the ‘no sea’ approximation, for the matter part of the densities and scalar densities of the nucleons, there are contributions of
FIG. 5: Masses (MeV) of $\psi(1D)$ and $\eta_c(2S)$ are plotted as functions of $eB/m_\pi^2$ for $\rho_B = \rho_0$ and $\eta = 0$, with and without the effects of PV mixing. These are shown in (a) and (c), when the Dirac sea contributions of nucleons (resulting in (inverse) magnetic catalysis) are not included and in (b) and (d), while these effects are included. In (b) and (c), the masses are also shown for $\rho_B = 0$ due to the Dirac sea contributions. The results are for cases when the AMMs are included, which are compared with the cases when AMMs are not considered (shown as dotted lines).

the Landau levels for the charged baryon, i.e., proton, whereas the neutron interacts with the magnetic field, due to its anomalous magnetic moment. In the present study, the contributions of the Dirac sea are also taken into account for the self energies of the nucleons, including the effects of AMMs of the nucleons.
A. Pseudoscalar meson-Vector meson (PV) mixing

In the presence of a magnetic field, there is mixing between the pseudoscalar meson and vector mesons, which modifies the masses of these mesons [82, 89, 90, 94–98]. The PV mixing leads to a drop (rise) in the mass of the pseudoscalar (longitudinal component of the vector meson). The mass modifications have been studied using an effective Lagrangian density of the form [95, 98]

$$\mathcal{L}_{PV} = \frac{g_{PV}}{m_{av}} \bar{e} \tilde{F}_{\mu\nu} (\partial^\mu P)^V \nu,$$

for the heavy quarkonia [82, 95], the open charm mesons [89] and strange (K and \( \bar{K} \)) mesons [90]. In equation (3), \( m_{av} = (m_V + m_P)/2 \), \( m_P \) and \( m_V \) are the masses for the pseudoscalar...
FIG. 7: Masses (MeV) of Υ(1S) and η_b are plotted as functions of eB/m^2 for ρ_B = ρ_0 and η = 0, with and without the effects of PV mixing. These are shown in (a) and (c), when the Dirac sea contributions of nucleons (resulting in (inverse) magnetic catalysis) are not included and in (b) and (d), while these effects are included. In (b) and (c), the masses are also shown for ρ_B = 0 due to the Dirac sea contributions. The results are for cases when the AMMs are included, which are compared with the cases when AMMs are not considered (shown as dotted lines).

and vector charmonium states, \( \tilde{F}_{\mu \nu} \) is the dual electromagnetic field. In equation (3), the coupling parameter \( g_{PV} \) is fitted from the observed value of the radiative decay width, \( \Gamma(V \to P + \gamma) \) given as

\[
\Gamma(V \to P \gamma) = \frac{e^2 g_{PV}^2 P_{cm}^3}{12 \pi m_{av}^2},
\]  

(4)
where, $p_{cm} = (m_V^2 - m_P^2)/(2m_V)$ is the magnitude of the center of mass momentum in the final state. The masses of the pseudoscalar and the longitudinal component of the vector mesons including the mixing effects are given by

$$m^2_{P,V \parallel} = \frac{1}{2} \left( M_+^2 + \frac{c_{PV}^2}{m_{av}^2} \mp \sqrt{M_-^4 + \frac{2c_{PV}^2M_+^2}{m_{av}^2} + \frac{c_{PV}^4}{m_{av}^4}} \right),$$

where $M_+^2 = m_P^2 + m_V^2$, $M_-^2 = m_V^2 - m_P^2$ and $c_{PV} = g_{PV}eB$. In Ref. [82], the effective Lagrangian term given by equation (3) has been observed to lead to appreciable mass modifications of the pseudoscalar and the longitudinal component of the vector charmonium states, due to the PV mixings ($J/\psi - \eta_c$, $\psi(2S) - \eta_c(2S)$ and $\psi(1D) - \eta_c(2S)$). This is observed to lead to substantial modification of the partial decay width of $\psi(1D) \rightarrow D\bar{D}$.
FIG. 9: Masses (MeV) of $\Upsilon(2S)$ and $\eta_b(2S)$ are plotted as functions of $eB/m^2_\pi$ for $\rho_B = \rho_0$ and $\eta = 0$, with and without the effects of PV mixing. These are shown in (a) and (c), when the Dirac sea contributions of nucleons (resulting in (inverse) magnetic catalysis) are not included and in (b) and (d), while these effects are included. In (b) and (c), the masses are also shown for $\rho_B = 0$ due to the Dirac sea contributions. The results are for cases when the AMMs are included, which are compared with the cases when AMMs are not considered (shown as dotted lines).

Due to $\psi(1D) - \eta_c(2S)$ mixing, as well as, due to $D(\bar{D}) - D^*(\bar{D}^*)$ mixing effects [89]. These mixing effects have been considered on the masses calculated using the chiral effective model. In the present work, the Dirac sea contributions are taken into account to compute the masses of the heavy quarkonium states, additionally, the PV mixing effects are consid-
FIG. 10: Same as Figure [9] but with $\eta=0.5$.

Considered for the masses of these mesons. The mixing parameter $g_{PV}$ is determined from the observed decay widths of $V \rightarrow P\gamma$ for the open and hidden charm sector. However, due to lack of data (radiative decay) for the bottomonium states, we estimate the modifications to the masses of the bottomonium pseudoscalar and vector mesons [83] due to mixing of these states in the presence of a magnetic field, using the Hamiltonian [97, 98]

$$H_{\text{spin-mixing}} = -\sum_{i=1}^{2} \mu_i \cdot B,$$

which describes the interaction of the magnetic moments of the quark (antiquark) with the external magnetic field. In the above, $\mu_i = g|e|q_iS_i/(2m_i)$ is the magnetic moment of the $i$-th particle, $g$ is the Lande g-factor (taken to be $2(-2)$ for the quark(antiquark)), $q_i$, $S_i$, ...
FIG. 11: Masses (MeV) of Υ(3S) and η_b(3S) are plotted as functions of $eB/m^2_\pi$ for $\rho_B = \rho_0$ and $\eta = 0$, with and without the effects of PV mixing. These are shown in (a) and (c), when the Dirac sea contributions of nucleons (resulting in (inverse) magnetic catalysis) are not included and in (b) and (d), while these effects are included. In (b) and (c), the masses are also shown for $\rho_B = 0$ due to the Dirac sea contributions. The results are for cases when the AMMs are included, which are compared with the cases when AMMs are not considered (shown as dotted lines).

$m_i$ are the electric charge (in units of the magnitude of the electronic charge, $|e|$), spin and mass of the $i$-th particle [95, 98]. This interaction leads to a drop (increase) of the mass of the pseudoscalar (longitudinal component of the vector meson) given as [97]

$$\Delta M^{PV} = \frac{\Delta E}{2} \left( 1 + \Delta^2 \right)^{1/2} - 1,$$  \hspace{1cm} (7)
where $\Delta = 2g|eB|((q_1/m_1) - (q_2/m_2))/\Delta E$, $\Delta E = m_V - m_P$ is the difference in the masses of the pseudoscalar and vector mesons. Here, the masses $m_V$ and $m_P$ refer to those calculated from the medium change of the dilaton field within the chiral effective model, using equation (1). These masses are calculated considering the Dirac sea contributions. We study the PV mixing effects and consequently, the modifications to the masses of the pseudoscalar and the longitudinal component of the vector bottomonium states, arising due to the $\Upsilon(1S)^\parallel - \eta_b$, $\Upsilon(2S)^\parallel - \eta_b(2S)$, $\Upsilon(3S)^\parallel - \eta_b(3S)$, and $\Upsilon(4S)^\parallel - \eta_b(4S)$ mixing effects.
FIG. 13: Masses (MeV) of $\Upsilon(4S)$ and $\eta_b(4S)$ are plotted as functions of $eB/m^2_\pi$ for $\rho_B = \rho_0$ and $\eta = 0$, with and without the effects of PV mixing. These are shown in (a) and (c), when the Dirac sea contributions of nucleons (resulting in (inverse) magnetic catalysis) are not included and in (b) and (d), while these effects are included. In (b) and (c), the masses are also shown for $\rho_B = 0$ due to the Dirac sea contributions. The results are for cases when the AMMs are included, which are compared with the cases when AMMs are not considered (shown as dotted lines).

III. RESULTS AND DISCUSSIONS

We discuss the results obtained due to the effects of Dirac sea contributions for the nucleons, as well as, PV mixing on the masses of the charmonium and bottomonium states in magnetized isospin asymmetric nuclear matter including the effects of the Dirac sea
of nucleons. These are obtained, using equation (1), from the values of the dilaton field which are solved from the coupled equations of motion of the scalar fields ($\sigma$, $\zeta$ and $\delta$) and the dilaton field within the chiral effective model, for given values of the baryon density, $\rho_B$, isospin asymmetry parameter, $\eta$ and the magnetic field, $B$. The heavy quarkonium masses are studied considering the AMMs of the nucleons and compared to the cases when AMMs are not taken into account. There is observed to be enhancement of the quark condensates (through scalar fields $\sigma$ and $\zeta$) due to Dirac sea contributions for zero density and at $\rho_B = \rho_0$, when the AMMs are not taken into account. The trend persists, when the AMMs are considered, for $\rho_B = 0$, whereas, for $\rho_B = \rho_0$, there is observed to be the opposite trend of inverse magnetic catalysis. The solution of $\chi$ is observed to be a rise (drop) when
the AMMs are neglected (included) for $\rho_B = \rho_0$. This leads to a rise (drop) of the heavy quarkonium masses when magnetic field, without (with) the AMMs.

In figures 1 and 2, the masses of $J/\psi$ and $\eta_c$ are plotted for isospin symmetric nuclear matter ($\eta=0$) and asymmetric nuclear matter (with $\eta=0.5$) with and without PV ($J/\psi - \eta_c$) mixing effects. The masses are plotted considering the AMMs of the nucleons and compared with the cases when the AMMs are not taken into account (shown as dotted lines). These are shown in (a) and (c), when the Dirac sea contributions are not considered. In the absence of PV mixing, there is observed to be almost no change in the masses of $J/\psi$ and $\eta_c$ mesons. The PV mixing is observed to lead to substantial rise (drop) in the mass of $J/\psi(\eta_c)$ meson. The effects of the Dirac sea contributions are shown in (b) and (d) respectively. There is observed to be an increase (drop) in the mass of $J/\psi$ with increase in the magnetic field, without (with) accounting for the AMMs at $\rho_B = \rho_0$, when the PV mixing is not taken into account. However, with PV mixing, there is an increase in the mass of $J/\psi(\eta_c)$, which is observed to lead to a rise of the mass, when the the Dirac sea, as well as, PV mixing are taken into account, however, the modification is much larger at high magnetic field values, when AMMs are not considered. For $\eta_c$, there is observed to be drop in the mass with increase in magnetic field and the drop is larger with AMMs as compared to when the AMMs are ignored. The effect due to PV mixing is observed to be much more dominant as compared to the contributions due to the Dirac sea effects for the masses of both $J\psi$ and $\eta_c$ mesons. In (b) and (c), the results for masses are compared with the case of zero baryon density. The plots for $\eta = 0.5$ shown in figure 2 show that the masses of $J/\psi$ and $\eta_c$ have very small dependence on the isospin asymmetry of the nuclear matter.

Figures 3 and 4 show the plots of masses of $\psi(2S)$ and $\eta_c(2S)$ with and without the PV mixing effects, for $\eta = 0$ and $\eta = 0.5$ respectively. The contribution from (inverse) magnetic catalysis is observed to lead to appreciable (drop) increase in the masses of these mesons. The modifications due to the AMMs are observed to be quite significant. The PV effect leads to a rise (drop) in the mass of $\psi(2S)(\eta_c(2S))$ meson. The effect of Dirac sea contributions is observed to dominate over the PV mixing contributions. The modifications due to the isospin asymmetry of the medium are observed to be marginal as compared to the effects of (inverse) magnetic catalysis and PV mixing.

In figures 5 and 6, we plot the masses of $\psi(1D)$ and $\eta_c(2S)$ for $\eta = 0$ and $\eta = 0.5$ for $\rho_B = \rho_0$. The mixing effect for $\psi(1D) - \eta_c(2S)$, along with the Dirac sea contributions, is
observed to be lead to significant modifications to the mass of $\psi(1D)$, which should have observable effects on the production of the charmonium states and open charm mesons, due to modification of the decay width of $\psi(1D) \rightarrow D\bar{D}$.

In figures 7 and 8 the effects of the PV mixing and Dirac sea on the masses of the ground states $\Upsilon(1S)$ and $\eta_b$ are shown at $\rho_B = \rho_0$ and for $\eta = 0$ and $\eta = 0.5$ respectively. There is observed to be a rise (drop) in the mass of $\Upsilon(1S)^{||}(\eta_b)$, when the effects of DS and PV mixing are both considered, and, the AMMs of nucleons are taken into account. When only the DS effects are considered, the masses of $\Upsilon(1S)$ and $\eta_b$ are observed to drop (rise) with increase in magnetic field, with (without) AMMs of the nucleons. For the excited bottomonium states, there is observed to be drop in masses of both the pseudoscalar and the vector mesons, when only DS effects are taken into account. The effect of isospin asymmetry is observed to lead to smaller modifications of these excited states, as might be seen from the figures 9, 11 and 13 for $\eta = 0$, and, from the figures 10, 12 and 14 for $\eta = 0.5$. The effect of PV mixing leads to a positive (negative) contribution to the mass of $\Upsilon(NS)^{||}(\eta_b(NS))$, $N = 1, 2, 3, 4$, with magnetic field, the mass shifts found from the equation 7. However, the PV mixing is observed to have the opposite trend for the $\Upsilon(4S) - \eta_c(4S)$ mixing at higher values of magnetic field ($eB > 7m^2_\pi$) for $\rho_B = \rho_0$ in symmetric nuclear matter ($\eta = 0$), as can be seen from figure 13. This is because the effective mass of the $\Upsilon(4S)$ turns out to be smaller than that of $\eta_b(4S)$, as calculated within the chiral effective model, which makes $\Delta E$ (and hence $\Delta M^{PV}$) of equation (7) to be negative. However, it is observed that the PV mixing has much smaller contribution as compared to the Dirac sea contributions for the excited bottomonium states, contrary to the charm sector, where both PV mixing and the DS contributions are observed to be important. The (inverse) magnetic catalysis, along with PV mixing can thus modify the heavy quarkonium decay widths to the open heavy flavour mesons, and hence can affect the production of the charm and bottom mesons in non-central ultra-relativistic heavy ion collision experiments, due to existence of strong magnetic fields.

IV. SUMMARY

To summarize, we have studied the masses of the heavy quarkonium states in magnetized nuclear matter. The masses are calculated within a chiral effective model from the medium change of a scalar dilaton field, which mimics the gluon condensates of QCD. The effects
if Dirac sea of nucleons are taken into consideration, which lead to increase (decrease) of the magnitudes of the scalar fields (thus leading to increase in the light quark condensates), with rise in the magnetic field, an effect called (inverse) magnetic catalysis. For zero density, one observes the magnetic catalysis, when the anomalous magnetic moments (AMMs) are (not) considered. However, at $\rho_B = \rho_0$, there is observed to be (inverse) magnetic catalysis, when the AMMs are (included) ignored. In the presence of the magnetic field, there are further modifications to the masses of the charmonium and bottomonium states due to PV mixing. The effects from isospin asymmetry on the in-medium masses of heavy quarkonia are observed to be small and the dominant contributions due to magnetic field effects are observed to be arising from the effects of PV mixing as well as Dirac sea contributions. These should have observable consequences on the production of heavy quarkonium states and open heavy flavour mesons, as these are created at the early stage of the non-central ultra-relativistic heavy ion collision experiments, when the magnetic field can be large.

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