Optimal error-tolerant design of universal multiport interferometers

S.A. Fldzhyan, M.Yu. Saygin,* and S.P. Kulik
Quantum Technologies Center and Faculty of Physics,
M.V.Lomonosov Moscow State University, GSP-1,
Leninskie gory, Moscow 119991 Russian Federation

Photonic information processing often demands programmable multiport interferometers capable of implementing arbitrary transfer matrices, for which planar meshes of tunable Mach-Zehnder interferometers (MZIs) are usually exploited. However, these MZI-based interferometers require balanced static beam-splitter (BSs) that make them sensitive to manufacturing errors. Here, we abandon the error-sensitive MZI and propose an alternative design that uses a single static BS and a variable phase shift as the building block of the interferometer mesh. Our BS-based design has been shown to possess superior resilience to manufacturing errors, which is achieved without addition of extra elements into the schemes. Namely, the power transmissivities of the static BSs constituent the interferometers can take arbitrary values in the range from $\approx 1/2$ to $\approx 4/5$.

I. INTRODUCTION

Photonic information processing often demands programmable multiport interferometers capable of implementing arbitrary transfer matrices, for which planar meshes of tunable Mach-Zehnder interferometers (MZIs) are usually exploited. However, these MZI-based interferometers require balanced static beam-splitter (BSs) that make them sensitive to manufacturing errors. Here, we abandon the error-sensitive MZI and propose an alternative design that uses a single static BS and a variable phase shift as the building block of the interferometer mesh. Our BS-based design has been shown to possess superior resilience to manufacturing errors, which is achieved without addition of extra elements into the schemes. Namely, the power transmissivities of the static BSs constituent the interferometers can take arbitrary values in the range from $\approx 1/2$ to $\approx 4/5$.

Universal interferometers can be reprogrammed to implement an arbitrary linear transformation defined by a specific transfer matrix. To construct these interferometers, decomposition methods are used that represent unitary matrices as products of simpler building blocks

unitary matrices into planar meshes of variable beam-splitters (BSs), having triangular and rectangular forms, respectively. In these schemes, each variable BS is conveniently realized by a standard element of the Mach-Zehnder interferometer (MZI). It is made up of two static balanced BSs with the required variability provided by two phase shifts. Thus, the overall scheme turns out to be reprogrammable by setting the phase shifts of the whole device [21, 22].

For these multiport schemes to be universal, it is crucial that the static BSs should necessarily be balanced. However, this condition can not be fully satisfied because of the errors that occur at realization, limiting the scheme’s universality [23, 24]. The negative effect of implementation errors progresses as the interferometer size scales up, effectively imposing stringent requirements on the fabrication tolerances and making realization of large-scale universal interferometers a challenging task.

Methods exist that can restore the universal capabilities of the MZI-based interferometers at the cost of adding extra elements into their optical schemes [24, 25]. However, the common drawback of these methods is the burden of auxiliary control needed to manipulate the additional MZIs and the increased real estate occupied by the scheme on the integrated chip. Therefore, developing more efficient designs of universal error-tolerant interferometers is highly demanded nowadays.

In this work, we propose a new design of universal planar interferometers. Its main difference from the MZI-based one is that it uses a single static BS and a tunable phase shift as a building block. The paper is organized as follows. In Sec. II we describe the conventional MZI-based interferometer design [18], to compare our BS-based design with. Then, in Sec. III, we present the proposed novel BS-based design together with the results of conducted error-tolerance analysis. We draw a conclusion in Sec. IV.

* saygin@physics.msu.ru
II. THE MZI-BASED AND BS-BASED INTERFEROMETERS

An \(N\) - port interferometer may be described by an \(N \times N\) transfer matrix acting on vectors of field amplitudes according to the relation: \(a^{(\text{out})} = U a^{(\text{in})}\), where \(a^{(\text{in})}\) and \(a^{(\text{out})}\) are the input and output vectors, respectively. In the following we assume lossless interferometers so that their transfer matrices \(U\) are unitary.

We first describe interferometers constructed with the canonical MZI-based design by Clements et al \[18\]. The optical scheme of the MZI-based design is depicted in Fig. 1a. It is formed by \(N\) successive layers consisting of multiple MZIs, each acting locally only on two neighboring channels. In this scheme, the overall number of MZIs is equal to \((N(N-1))/2\). Accounting for the \(N-1\) phase shifts at the output, the total number of phase shifts in the interferometer is \(N^2-1\), which is exactly the number of independent real parameters that parametrize an arbitrary unitary \(N \times N\) matrix. Previously, this universal design was considered optimal, as it is balanced with respect to losses as a result of the rectangular arrangement of elements.

\[ a_{\text{out}} = \Phi V^{(m)}_{\text{MZI}} \cdots V^{(2)}_{\text{MZI}} V^{(1)}_{\text{MZI}} a_{\text{in}} \]  

where \(V^{(m)}_{\text{MZI}}\) is the transfer matrix of the \(m\) - th layer, \(\Phi = \text{diag}(e^{i\varphi_1}, \ldots, e^{i\varphi_{N-1}}, 1)\) is the diagonal matrix with \(\varphi_j^{(\text{out})}\) being the phase-shifts introduced in the end. In (1), layer transfer matrices \(V^{(m)}_{\text{MZI}}\) are of the form:

\[ V^{(m)}_{\text{MZI}} = \prod_{j \in \Omega^{m}_{\text{MZI}}} T^{(m)}_{\text{MZI},j}(\varphi_{2j-1}, \varphi_{2j}) \]  

where

\[ T^{(m)}_{\text{MZI},j}(\varphi_1, \varphi_2) = \begin{pmatrix} 1 & \cdots & \cdots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & 1 \end{pmatrix} \]

is the block matrix of single MZI placed in the \(m\) - th layer between channels \(j\) and \(j+1\), \(\Omega^{m}_{\text{MZI}}\) denotes the ordered sequence of MZIs in the layer with index \(m\). Block matrix (3) has all diagonal elements 1 except those labeled \(B_{1,1}^{(m)} = e^{i(\varphi_1^{(m)} - \varphi_2^{(m)})} \alpha_1^{(m)}\) and \(B_{2,2}^{(m)} = -e^{i\varphi_2^{(m)}} \alpha_2^{(m)}\), and all off-diagonal elements equal to 0 except those labeled \(B_{1,2}^{(m)} = e^{i\varphi_1^{(m)}} \alpha_1^{(m)}\) and \(B_{2,1}^{(m)} = e^{i(\varphi_1^{(m)} + \varphi_2^{(m)})} \alpha_2^{(m)}\), where we introduced the shorthand notations: \(a = \sin \varphi_1^{(m)} \cos(\alpha_1^{(m)} - \alpha_2^{(m)}) + i \cos \varphi_1^{(m)} \sin(\alpha_1^{(m)} + \alpha_2^{(m)})\) and \(b = \cos \varphi_1^{(m)} \cos(\alpha_1^{(m)} + \alpha_2^{(m)}) + i \sin \varphi_1^{(m)} \sin(\alpha_1^{(m)} - \alpha_2^{(m)})\).

The variable phase shifts have required ranges \(0 \leq \varphi_j^{(m)} \leq 2\pi\) and \(0 \leq \varphi_j^{(m)} \leq 2\pi\), which are used to reconfigure the interferometer. Parameters \(\alpha_j^{(m)}\) describe errors caused by the imbalances of the static BSs resulted from a non-ideal realization of the scheme. In particular, in integrated photonics schemes fabricated by lithography, the occurrence of non-zero \(\alpha_j^{(m)}\) may be due to the variations of the wafer thickness.

In the ideal case when \(\alpha_j^{(m)} = 0\) the MZI-based interferometer is capable of implementing an arbitrary unitary transfer matrix, however, imbalances \(\alpha\)'s render it universal only to a certain degree. To illustrate the negative effect of imbalances, a trivial example of rerouting port 1 into port \(N\) of an \(N\)-port interferometer is considered in Fig. 2a for \(N = 6\). Obviously, to attain this transformation all diagonal MZIs (colored in red in Fig. 2a) should be in the cross state. However, calculating the transmissivity of an MZI: \(|b|^2 = \cos^2 \varphi_1 \cos^2(\alpha_1 + \alpha_2) + \sin^2 \varphi_1 \sin^2(\alpha_1 - \alpha_2)\), we note that the relevant extreme state with \(\tau = 1\) can be obtained only for perfectly balanced BSs with \(\alpha_1 = \alpha_2 = 0\). More thorough study of the effect of errors on the transformation quality of the MZI-based interferometers will be conducted below.

We now introduce our design of universal interferometers, which has superior error-tolerance as compared to the MZI-based ones. The schematic representation of the design is depicted in Fig. 1b. Similar to the MZI-based design described above, our design has rectangular placement of building blocks, however, each building block...
is a single static BS and single tunable phase shifter. Hence, the name BS-based for our design. In order to obtain the number of variable phase shifts that enough to parametrize arbitrary unitary matrices, the scheme of the $N$-port interferometer should consist of $2N$ layers, rather than $N$ layers in the case of the MZI-based interferometer. Therefore, the interferometer transfer matrix takes the form:

$$U_{\text{BS}} = \Phi V_{\text{BS}}^{(2N)} \cdots V_{\text{BS}}^{(2)} V_{\text{BS}}^{(1)},$$

in which layer transfer matrices read:

$$V_{\text{BS}}^{(m)} = \prod_{j \in \mathbb{D}_{\text{BS}}} T_{\text{BS},j}^{(m)}(\varphi_j^{(m)}),$$

where the BS-block matrix situated in the layer with index $m$ and that transform channels $j$ and $j + 1$ reads

$$T_{\text{BS},j}^{(m)}(\varphi) =
\begin{pmatrix}
1 & \cdots & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
0 & \cdots & \cdots & 1
\end{pmatrix},$$

where the angle $\theta_0$ quantifies the transmission of the static BSs, which specific value will be given below.

In (6), the parameter $\alpha$ quantifies the imbalance of the static BS that can be caused by non-ideal implementation of the scheme. However, in the next section we will show that the BS-based design can tolerate large values of $\alpha$’s without loosing universality, so that the strict requirements imposed on the static elements of the MZI-based interferometers do not apply to the interferometers of the BS-based design. Similarly to the case of the MZI-based interferometer, we consider the rerouting operation performed by the BS-based interferometer depicted in Fig. 2b. As the figure suggest, because the signals can transverse the scheme faster compared to the traditional MZI-based scheme, multiple possibilities exist to accomplish the transformation that utilize large number of elements. In turn, more tunable phase shifts can be used to correct for non-zero $\alpha_j$.

It should be noted that the depths of both MZI- and BS-based interferometers, as quantified by the maximum static BS elements crossed by the signals, are equal. Therefore, both designs should have the same loss level, as losses inevitably occur in practical devices. Also, the realization complexity of the interferometers remains the same.

III. UNIVERSALITY AND TOLERANCE TO ERRORS OF THE BS-BASED INTERFEROMETERS

We consider errors as $\alpha_j \neq 0$ that tune the splitting ratios of the static BSs off required values. To study the effect of errors on a multiport transformation, an error model – a particular distribution of the biases $\alpha_j$ across the optical scheme – should be specified. Firstly, we consider the coherent type of errors at which all static BSs experience equal biases $\alpha_j = \alpha$, i.e. the errors are fully correlated. For the schemes manufactured by planar lithography techniques this type of errors is linked to the variations of waveguide’s material and geometry, which is dominated as their spatial scale is usually large compared with the area occupied by the scheme [26]. These arguments can also be applied to interferometers manufactured by other methods, for example, femtosecond direct laser writing [22], as well as alternative implementation approaches exploiting repetitively few optical elements to obtain the desired transformation between multiple modes [27]. Furthermore, the study of this type of errors in the first place is justified since transformation quality is typically more sensitive to coherent errors than to the stochastic one [19], which we will investigate below.

We evaluate the performance of multiport interferometers by calculating the fidelity, defined as:

$$F = \frac{1}{N^2} \left| \text{Tr}(U^\dagger U_0) \right|^2,$$

which compares the target unitary matrix $U_0$ and the actual transfer matrix $U$ realized by the interferometer, where $N$ is the size of the matrices. Provided that the matrices $U$ and $U_0$ are equal up to a global phase, the fidelity (7) gets its maximum value of $F = 1$.

Generally, no analytical solution is known to derive phase shifts that maximize the fidelity (7), except for the case of error-free MZI-based interferometers (i.e. at $\alpha = 0$), for which an analytical procedure is provided in [18].
FIG. 3. Infidelity $1 - F$ as a function of error parameter $\alpha$ for the MZI-based and BS-based interferometers at $\theta_0 = \pi/4$ and a) $N = 5$ and b) $N = 10$. For each value of $\alpha$ the infidelity distribution were obtained numerically using a set of 300 unitary matrices drawn randomly from uniform distribution. The solid curves correspond to the average over all samples size; the lower and upper boundaries of the shaded regions are averages for 10 infidelities with the lowest and highest values, respectively.

by the constructive universality proof. Unfortunately, we could not found analogous procedure for the error-free BS-based interferometers. Therefore, in our analysis we used a numerical optimization algorithm.

Our optimization was based on the basinhopping algorithm implemented in the SciPy python library. Given a unitary matrix $U_0$, the algorithm was searching for a global minimum of infidelity $1 - F$ over the space of phase-shifts. To decrease the chance of falling into local minima, we used multiple runs of the optimization with random initial values of the phases. Each numerical experiment involved optimization over a series of 300 matrices $U_0$, drawn from the Haar random distribution using the method based on the QR-decomposition of random matrices [28]. With this algorithm it took $\sim 1$ day to find optimal phase shifts for a single $10 \times 10$ transfer matrix, so that a multi-core cluster has been utilized to derive required dependencies. We understand that more efficient numerical algorithms can be developed for this specific task [29], however, this is beyond the scope of the present work.

The obtained infidelity $1 - F$ as a function of the error parameter $\alpha$ is plotted in Fig. 3. Notice that finite accuracy of the numerical algorithm sets the minimal infidelity value of $\sim 10^{-10}$, which could not be overcome neither for the MZI-based interferometer with $\alpha = 0$ where exact zero was expected. However, due to the small inaccuracy value, we consider the corresponding fidelity values to be of ideal case. The figures suggest that the MZI-based interferometers are equally sensitive to both positive and negative values of the error parameter $\alpha$, for which the acceptable range of errors is of the order of several degrees. At the same time, infidelity

FIG. 4. Normalized histogram of infidelity $1 - F$ for the BS-based (a) and MZI-based (b) 10-port interferometers at random errors. The error angles $\alpha_j$ were drawn from the Gaussian probability distribution (8) with $\Delta = 10$ degrees. For the BS-based interferometer the parameter of the static BSs $\theta_0 = 55$ degrees, roughly corresponding to the center of the high-fidelity plateau, depicted in Fig. 3b. The histograms is the result of the optimization performed over a set of 300 randomly sampled unitary matrices.
angle $\alpha$ relevant geometry parameters of the DC; b) the imbalance.

FIG. 5. The effect of wavelength dispersion of the standard silicon-on-insulator directional coupler (DC) operating in the telecom range and balanced at 1.5 um wavelength. a) the relevant geometry parameters of the DC; b) the imbalance angle $\alpha$ as a function of wavelength obtained numerically for the DC element. The shaded regions correspond to the parameter ranges, in which the average infidelity of the 10-port interferometers depicted in Fig. 3b do not exceed $10^{-5}$.

behaves radically different for the interferometers constructed with the BS-based design: while at $\alpha < 0$ the performance of the two are comparable, at $\alpha > 0$ the BS-based interferometers provide perfect fidelity for $\alpha$ as large as $\sim 20$ degrees — several times larger than for the MZI-based interferometers. Fig. 3 suggests that when the positive and negative values of $\alpha$ are equiprobable, the optimal choice of the static BSs defined at the design stage is such that $\theta_0 \approx 55$ degree, corresponding to the center of the high-fidelity plateau.

The observed superior performance of the BS-based interferometers at positive $\alpha$‘s can be attributed to the interplay of two competing properties. On the one hand, more transmissive BSs with $\alpha > 0$ enable more efficient travelling of the signal amplitudes across the scheme, which has been recognized as a prerequisite for robustness to errors in other universal interferometer architectures [19]. However, on the other hand, the increase of the transmissivity reduces the interference interaction between the signals, which is completely absent in the limiting case of $\alpha = \pi/4$.

Opposite to the coherent error model, we also consider the incoherent error model at which errors are distributed at random across the static BSs. Setting $\theta_0 = 55$ degree and considering the case of $N = 10$, we sampled angles $\alpha_j$ independently from the Gaussian probability distribution: 

$$p(\alpha) = \frac{1}{\sqrt{2\pi}\Delta} \exp\left(-\frac{\alpha^2}{2\Delta^2}\right),$$

where $\Delta$ is the distribution width that quantifies the level of incoherent errors. The obtained histogram of infidelities is shown in Fig. 4a and Fig. 4b for the BS-based and MZI-based interferometer, respectively. One can clearly see from the figures, our BS-based interferometer design is ultimately tolerant to the incoherent errors.

Aside from errors caused by manufacturing, wavelength dispersion can also render the multiport interferometer non-universal, when the signal wavelength is different from the one the device has been designed for. To measure the advantage of our BS-based design in terms of the width of acceptable wavelength range, we consider the specific example of the silicon-on-insulator directional coupler (DC) often used as a constituent element of photonic integrated circuits. The layout of the element is shown in Fig. 5a. To obtain the dispersion characteristics of the DC element, the 3D FDTD numerical method implemented in the Lumerical package has been used. Fig. 5b shows the DC imbalance parameter $\alpha$ as a function of operating wavelength. The geometry of the DC was chosen in such a way that it is balanced at 1.5 um wavelength. Using the results of Fig. 3b obtained for 10-port interferometers and taking the acceptable average infidelity to be $10^{-5}$, the imbalance $\alpha$ is mapped into wavelength ranges, as marked by the shaded regions in Fig. 5b. For the MZI-based interferometer, the narrow range of acceptable imbalances $\alpha$ is translated into the wavelength range of $\approx 20$ nm width, while the wavelength range corresponding to the BS-based interferometer is several times larger and amounts to $\approx 120$ nm.

IV. CONCLUSION

We proposed a novel design of universal multiport interferometers, which exhibit superior tolerance to errors than the previously known counterparts, while maintaining the same realization complexity and it does not require redundant elements. In fact, since the MZI elements are not necessary in our design, the optical schemes can be considered even less complex than the counterparts, making possible their implementation by specific experimental platforms, which MZI-based ones do not fit well. For example, the static interferometers that have been used recently in quantum experiments on boson sampling [14, 30] have suitable placement of the passive BS elements, yet they lack programmability.

In conclusion, by this work, we have demonstrated that there is room for improvement of the quality and scaling
capabilities of universal programmable photonic circuits solely by choosing more optimal architectures.

**FUNDING**

Grant of The Russian Federation Ministry of Education and Science.

[1] A. Annoni, E. Guglielmi, M. Carminati, G. Ferrari, M. Sampietro, D. A. Miller, A. Melloni, and F. Morichetti, *Light: Science & Applications* **6**, e17110 (2017).

[2] Y. Shen, N. C. Harris, S. Skirlo, M. Prabhu, T. Baehr-Jones, M. Hochberg, X. Sun, S. Zhao, H. Larochelle, D. Englund, and M. Soljacic, *Nature Photonics* **11**, 441 (2017).

[3] H. Bagherian, S. Skirlo, Y. Shen, H. Meng, V. Ceperic, and M. Soljacic, arXiv e-prints, arXiv:1808.03303 (2018), arXiv:1808.03303 [cs.ET].

[4] T. W. Hughes, M. Minkov, Y. Shi, and S. Fan, *Optica* **5**, 864 (2018).

[5] T. Rudolph, *APL Photonics* **2**, 030901 (2017).

[6] N. C. Harris, J. Carolan, D. Bunandar, M. Prabhu, M. Hochberg, T. Baehr-Jones, M. L. Fanto, A. M. Smith, C. C. Tison, P. M. Alsing, and D. Englund, *Optica* **5**, 1623 (2018).

[7] F. Lenzini, J. Janousek, O. Thearle, M. Villa, B. Haylock, S. Kasture, L. Cui, H.-P. Phan, D. V. Dao, H. Yonezawa, P. K. Lam, E. H. Huntington, and M. Lobino, *Science Advances* **4** (2018), 10.1126/sciadv.aat9331, https://advances.sciencemag.org/content/4/12/eaat9331.full.pdf.

[8] A. P. Lund, S. Rahimi-Keshari, and T. C. Ralph, *Phys. Rev. A* **96**, 022301 (2017).

[9] A. Politi, J. C. F. Matthews, and J. L. O’Brien, *Science* **325**, 1224 (2009).

[10] X.-Q. Zhou, P. Kalasuwan, T. C. Ralph, and J. L. O’Brien, *Nature Photonics* **7**, 223 (2013).

[11] J. B. Spring, B. J. Metcalf, P. C. Humphreys, W. S. Kolthammer, X.-M. Jin, M. Barbieri, A. Datta, N. Thomas-Peter, N. K. Langford, N. K. Langford, X.-M. Jin, M. Barbieri, A. Datta, N. Thomas-Peter, N. K. Langford, M. Barbieri, A. Datta, N. Thomas-Peter, N. K. Langford, W. S. Kolthammer, X.-M. Jin, M. Barbieri, A. Datta, N. Thomas-Peter, N. K. Langford, M. Barbieri, A. Datta, N. Thomas-Peter, N. K. Langford, W. S. Kolthammer, and J. C. Gates, B. J. Metcalf, P. C. Humphreys, W. S. Kolthammer, X.-M. Jin, M. Barbieri, A. Datta, N. Thomas-Peter, N. K. Langford, D. Kundys, J. C. Gates, B. J. Smith, P. G. R. Smith, and I. A. Walmsley, *Science* **339**, 798 (2012).

[12] M. A. Broome, A. Fedrizzi, S. Rahimi-Keshari, J. Dove, S. Aaronson, T. C. Ralph, and A. G. White, *Science* **339**, 792 (2012).

[13] A. Crespi, R. Osellame, R. Ramponi, D. J. Brod, E. F. Galvão, N. Spagnolo, C. Vitelli, M. E. Maiorino, P. Mataloni, and F. Sciarrino, *Nature Photonics* **7**, 545 (2013).

[14] H.-S. Zhong, Y. Li, W. Li, L.-C. Peng, Z.-E. Su, Y. Hu, Y.-M. He, X. Ding, W. Zhang, H. Li, L. Zhang, Z. Wang, L. You, X.-L. Wang, X. Jiang, L. Li, Y.-A. Chen, N.-L. Liu, C.-Y. Lu, and J.-W. Pan, *Phys. Rev. Lett.* **121**, 250505 (2018).

[15] G. R. Steinbrecher, J. P. Olson, D. Englund, and J. Carolan, *npj Quantum Information* **5**, 60 (2019).

[16] N. Killoran, T. R. Bromley, J. M. Arrazola, M. Schuld, N. Quesada, and S. Lloyd, arXiv e-prints, arXiv:1806.06871 (2018), arXiv:1806.06871 [quant-ph].

[17] M. Reck, A. Zeilinger, H. J. Bernstein, and P. Bertani, *Physical Review Letters* **73**, 58 (1994).

[18] W. R. Clements, P. C. Humphreys, B. J. Metcalf, W. S. Kolthammer, and I. A. Walmsley, *Optica* **3**, 1460 (2016).

[19] M. Saygin, I. Kondragyte, I. Dyakonov, S. Mironov, S. Straupe, and S. Kulik, arXiv e-prints, arXiv:1906.06748 (2019), arXiv:1906.06748 [quant-ph].

[20] N. C. Harris, G. R. Steinbrecher, M. Prabhu, Y. Lahini, J. Mower, D. Bunandar, C. Chen, F. N. C. Wong, T. Baehr-Jones, M. Hochberg, S. Lloyd, and D. Englund, *Nature Photonics* **11**, 447 (2017).

[21] C. Taballione, T. A. W. Wolterink, J. Lugani, A. Eckstein, B. A. Bell, R. Grootsjans, I. Visscher, J. J. Renema, D. Geskus, C. G. H. Roeloffzen, I. A. Walmsley, P. W. H. Pinkse, and K.-J. Boller, in *Frontiers in Optics / Laser Science* (Optical Society of America, 2018) p. JTu3A.58.

[22] I. V. Dyakonov, I. A. Pogorelov, I. B. Bobrov, A. A. Kalinkin, S. S. Straupe, S. P. Kulik, P. V. Dyakonov, and S. A. Evtushkin, *Physical Review Applied* **10**, 044048 (2018).

[23] J. Mower, N. C. Harris, G. R. Steinbrecher, Y. Lahini, J. Mower, D. Bunandar, C. Chen, F. N. C. Wong, T. Baehr-Jones, M. Hochberg, S. Lloyd, and D. Englund, *Nature Photonics* **11**, 447 (2017).

[24] R. Burgwal, W. R. Clements, D. H. Smith, J. C. Gates, W. S. Kolthammer, J. J. Renema, and I. A. Walmsley, *Opt. Express* **25**, 28236 (2017).

[25] D. A. B. Miller, *Optica* **2**, 747 (2015).

[26] W. Bogaerts and L. Chrostowski, *Laser & Photonics Reviews* **12**, 1700237 (2018), https://onlinelibrary.wiley.com/doi/pdf/10.1002/lpor.201700237.

[27] K. R. Motes, A. Gilchrist, J. P. Dowling, and P. P. Rohde, *Phys. Rev. Lett.* **113**, 120501 (2014).

[28] F. Mezzadri, arXiv e-prints, math-ph/0609050 (2006), arXiv:math-ph/0609050 [math-ph].

[29] S. Pai, B. Bartlett, O. Solgaard, and D. A. B. Miller, *Phys. Rev. Applied* **11**, 064044 (2019).

[30] A. Crespi, R. Osellame, R. Ramponi, V. Giovannetti, R. Fazio, L. Sansoni, F. De Nicola, F. Sciarrino, and P. Mataloni, *Nature Photonics* **7**, 322 (2013).