Manifestation of aging in giant magnetoresistance of the Co/Cu/Co nanostructure

V V Prudnikov, P V Prudnikov, M V Mamonova, M M Firstova and A A Samoshilova

Omsk State University, Pr. Mira 55A, Omsk 644077, Russia

E-mail: prudnikv@univer.omsk.su

Keywords: magnetic multilayer structures, Monte Carlo methods, giant magnetoresistance

Abstract

A Monte Carlo simulation of the non-equilibrium behavior of multilayer magnetic nanostructure Co/Cu(100)/Co consisting of alternating magnetic and nonmagnetic nanolayers is carried out. Analysis of calculated two-time autocorrelation function for structure relaxing from both high-temperature and low-temperature initial states reveals aging characterized by a slowing down of correlation characteristics with increase of the waiting time. It is shown that, in contrast to bulk magnetic systems, the aging effects in nanostructure arise not only at the ferromagnetic ordering temperature \( T_c \) but also within a wide temperature range at \( T \leq T_c \). For evolution from high-temperature initial state, the study of dependence of aging characteristics on thickness \( N \) of cobalt films reveals a weakening of the aging with increasing \( N \) at the critical temperatures \( T_s(N) \) and an opposite tendency at temperatures \( T < T_s(N) \) with strengthening of aging with increasing \( N \) for considered \( N \leq 9 \) ML. This phenomenon is connected with increasing correlation and relaxation times in nanostructures when temperature is decreased. For case of the low-temperature initial state, it is shown that correlation times are two-three orders of magnitude smaller than those in the evolution from a high-temperature initial state at the same \( t_w \) values. In this case, time behavior of the autocorrelation function doesn’t depend considerably on temperature for \( T_s \leq T_c \) and thickness \( N \) of cobalt films. Simulation of transport properties in Co/Cu(100)/Co structure permitted to calculate temperature dependence of its equilibrium magnetoresistance values. For the first time, it was revealed influence of non-equilibrium behavior on the magnetoresistance with demonstration of nontrivial aging effects. It has been shown that the magnetoresistance reaches plateau in asymptotic long-time regime with values \( \delta^\infty(N, T) \), which depend on type of initial state, thickness of cobalt films, and temperature.

1. Introduction

The physics of ultrathin magnetic films and nanostructures on their basis becomes important direction of intensive research over the past two decades [1]. This heightened interest of scientists is caused by quite a number of unique properties of these films with thicknesses from one and two to several tens of monolayers considerably different from properties of bulk materials.

Significant achievements in the development of technology give possibility now to receive a high-quality ultrathin films and multilayer coatings on the basis of the magnetic transition metals Fe, Co, Ni, and their alloys [1, 2]. Investigation of the nature of magnetism in ultrathin films has a large fundamental interest because the dimensional dependence of the magnetic characteristics, which demonstrate the transition from the specific bulk values for films with thicknesses of several tens monolayers \((d \geq 10 \text{ nm})\) to the two-dimensional values in films with thicknesses less than \(4-6\) monolayers \((d \leq 1 - 2 \text{ nm})\) [3, 4]. All that defines an importance of these new objects both for development of fundamental physics of magnetism, surface physics and their practical applications [5, 6]. Thus, the ultrathin films of magnetic metals and alloys are used as the building blocks for magnetic multilayer superstructures, which are characterized by wide range of applications in devices based on
the phenomena of the giant magnetoresistance (GMR) [7–9] and the tunneling magnetoresistance (TMR) [10–12]. The using of the multilayer antiferromagnetic superstructures in magnetic random access memory (MRAM) [13] can reduce the critical current [14] and switching time [15].

The nanoscale periodicity in magnetic multilayer structures gives rise to the mesoscopic effects of the strong spatial spin correlation with the slow relaxation dynamics of magnetization accompanying the quenching of the system in the non-equilibrium state. In contrast to the bulk magnetic systems, where the slow dynamics and aging effects manifest themselves near the critical point [16], magnetic superstructures with nanoscale periodicity allow increasing the relaxation time owing to the effects related to the larger characteristic spin–spin correlation length. That is why the aging and nonergodicity effects can be experimentally observed in the multilayer Co/Cr magnetic structure [17] within a wider temperature range as compared to that for the bulk magnetic systems. We have performed in the paper [18] a numerical Monte Carlo simulation of the non-equilibrium behavior of the multilayer Co/Cr/Co magnetic structure with the thickness of Co films equals to $N = 3$ monolayers. Calculations of the two-time correlation functions and the staggered magnetization allow us to reveal the aging effects, which are characterized by slowing down of correlation and relaxation processes with an increase in the system’s ‘age’ $t_m$ as the time between a sample preparation and the beginning of measurement of its characteristics. The revealed aging effects for our model of multilayer structure are in good agreement with results of experimental observations in the paper [17]. It was shown that aging in the non-equilibrium critical behavior of multilayer magnetic structure is occurred not only at $T_c$ but also within a wide range of temperatures at $T \ll T_c$. Therefore, the existence of these non-equilibrium features should surely be taken into account in any applications of the multilayer magnetic structures for spintronic devices based on the giant magnetoresistance phenomena.

At the present time, the statistical Monte Carlo methods have proved to be successful methods for simulation of behavior and describing the physical properties of various magnetic systems with different dimensionalities, identifying their features during phase transitions [19–22]. The Monte Carlo study of dimensional crossover effects in the critical properties of multilayer Heisenberg films has been performed in papers [23, 24]. The authors have taken into account for films with different thicknesses the effect of anisotropy generated by the crystal field of the substrate. The values of calculated critical exponents in papers [23, 24] clearly demonstrate the dimensional crossover from the two-dimensional to three-dimensional properties of the films with an increase in the number of monolayers. We have applied in work [18] the methodology and results of the multilayer Heisenberg film studies from papers [23, 24] to Monte Carlo simulations and calculation of magnetic properties in magnetic multilayer structure Co/Cr/Co in which ferromagnetic layers of the cobalt separated by nonmagnetic layer of the chromium.

In this paper we perform a Monte Carlo study of the non-equilibrium behavior of a multilayer magnetic superstructure Co/Cu/Co, which is used extensively in active elements of spintronic devices, with different thicknesses $N$ of the magnetic films and within a wide range of temperatures at $T \ll T_c(N)$. This makes it possible to reveal the dependence of aging effects in this structure on thickness $N$ of the magnetic films and temperature $T$ in magnetic-ordered phase. In addition, we plan to calculate the temperature dependence of equilibrium values of the magnetoresistance for Co/Cu/Co structure with different thicknesses $N$ of the magnetic films with the use of the methodology which has been developed in papers [25, 26] for determination of the magnetoresistance by the Monte Carlo method. Thereafter, the study of influence of aging effects on the magnetoresistance of Co/Cu/Co structure is realized and obtained two-time dependences of the magnetoresistance are compared in long-time regime with equilibrium values of the magnetoresistance.

2. Simulation of equilibrium properties of the multilayer magnetic structure Co/Cu/Co

The observed thickness-dependent magnetic properties of ultrathin films Fe, Co, and Ni on nonmagnetic metal substrates [1–4] are determined generally by interfacial interaction films with substrate and magnetic anisotropy generated by the crystal field of the substrate as well as their changes with increasing of thicknesses $N$ of the magnetic films. Among a wide number of numerical and theoretical investigations of equilibrium and dynamical properties of low-dimensional magnets using several model Hamiltonians, Heisenberg-like models are one of the most widely used to describe a real magnetic materials. For the statistical Monte Carlo description of ferromagnetic Co films contacting with Cu film with surface plane orientation (100), we apply the anisotropic Heisenberg model with the Hamiltonian of the spin system in the form

$$ H = -\frac{J_1}{\hbar} \sum_{i,j} [(S_i^x S_j^x + S_i^y S_j^y) + (1 - \Delta(N)) S_i^z S_j^z] $$

(1)

where $\vec{S}_i = (S_i^x, S_i^y, S_i^z)$ is a three-dimensional unit vector, fixed in cite of the Co fcc-lattice, $J_1 > 0$ is the exchange integral describing the interaction between the neighboring spins in film, $\Delta(N)$ is the parameter characterizing the effective influence of anisotropy generated by the crystal field of the substrate on magnetic
properties of film subject to its thickness $N$ in terms of monoatomic layers (ML). $\Delta = 0$ corresponds to the isotropic Heisenberg model, $\Delta = 1$ to the XY-model. The Hamiltonian (1) describes magnetic properties of systems with anisotropy of ‘easy’ magnetic plane type with magnetization oriented in XY plane. The presence of anisotropy leads to the appearance of long-range magnetic order in two-dimensional Heisenberg systems with the critical temperature $T_0 \neq 0$.

The dependence of the anisotropy parameter $\Delta(N)$ in the Hamiltonian (1) on the film thickness $N$ was calculated on basis of experimental data for the critical temperatures in films Ni/Cu(100) and Co/Cu(100) [4] as quantity determined by relative change of the ferromagnetic transition temperature $T_c(N)$ in films of cobalt (nickel) to $T_c(\infty)$ in the bulk sample of cobalt (nickel). The resulting dependence of $\Delta(N)$ is given in figure 1. The anisotropy parameter $\Delta(N)$ is characterized by decrease with thickness $N$ growth of the ferromagnetic film because of the dimensional dependence of the magnetic characteristics demonstrating the transition from the two-dimensional values in films with thicknesses less than 4–6 monolayers to the bulk values for films with thicknesses of several tens monolayers, which is accompanied by weakening of anisotropy influence and demonstration of isotropic magnetic properties in Co films [3, 4].

In this work we consider the multilayer magnetic structure Co/Cu(100)/Co, consisting of two ferromagnetic Co films separated by the nonmagnetic Cu film (figure 2). The thickness of this nonmagnetic metal layer is selected for obtaining GMR effects in such a way that the long-range and oscillatory RKKY interlayer exchange interaction between spins of the ferromagnetic layers has an effective antiferromagnetic character. Through this interaction, the magnetizations of the adjacent ferromagnetic layers are oriented opposite to each other. When this structure is placed in an external magnetic field, the magnetizations of layers

**Figure 1.** Dependence of the anisotropy parameter $\Delta (N)$ versus the thickness of the film $N$. The dotes correspond to experimental data for Ni/Cu(001) and Co/Cu(001) [4]. The solid curve is obtained by approximation of experimental data.

**Figure 2.** The model of the multilayer structure consisting of two ferromagnetic films separated by a nonmagnetic metal film. $L$ and $N$ are the linear sizes of the films; $J_1$ and $J_2$ are the exchange integrals.
begin to orient parallel that leads to a significant change in the electrical resistance. The Hamiltonian for simulation of magnetic properties of Co/Cu(100)/Co structure is taken in the form

\[ H = -J_1 \sum_{i,j \in \{1,2\}} \left[ S_i^x S_j^x + S_i^y S_j^y + S_i^z S_j^z \right] + J_2 \sum_{i \in N_1, j \in N_2} \left[ S_i^x S_j^x + S_i^y S_j^y + S_i^z S_j^z \right] + (1 - \Delta(N)) S_i^z S_j^z, \]

where the exchange integral \( J_1 \) defines the interaction between the neighboring spins in the same film, whereas \( J_2 = -0.3J_1 \) determines the interlayer interaction between spins of \( N_1 \) monolayer of first cobalt film and spins of \( N_2 \) monolayer of second cobalt film (figure 2). The temperature \( T \) of the system is measured in units of the exchange integral \( J_1 / k_B \). The simulations were performed for fcc-Co films with linear sizes \( L \times L \times N \) with applied periodic boundary conditions in the plane of the film. \( N \) gives the number of monolayers in the thin Co film.

At the beginning, we calculated the equilibrium characteristics of the multilayer magnetic structure with the aim to determine the critical temperatures \( T_c(N) \) of the ferromagnetic phase transition in films with different thicknesses \( N \). For a more accurate determination of the critical temperatures, we considered the structures with films of different linear sizes \( L = 20, 32, \) and 40. Metropolis algorithm was used for updating spin configurations. During simulation, \( 10^5 \) MCS/s were discarded for equilibration of spin system, and then measured equilibrium quantities are averaged over \( 10^5 \) MCS/s with 500 runs.

We calculated such characteristics as the staggered magnetization \( m_{\text{stg}} = m_1 - m_2 \), where \( m_1 \) and \( m_2 \) are the magnetizations of the films

\[ m_n = |m_n| = \frac{1}{N_s} \left[ \sum_{i \in \{x,y,z\} \in N_1 \cup N_2} \left( \sum_{i \in N_1} S_i^n \right) \right]^{1/2}, \quad n = 1, 2, \]

the staggered susceptibility (figure 3)

\[ \chi_{\text{stg}} = \frac{1}{T} (\langle m_{\text{stg}}^2 \rangle - \langle m_{\text{stg}} \rangle^2), \]

where \( N_s = NL^2 \) is the total number of spins in the film and angular brackets denote the statistical averaging.

The susceptibility peaks in figure 3 show a displacement towards higher temperature values when the film thickness increases that is connected with dimensional increasing dependence of the critical temperature \( T_c(N) \) owing to increase of average number of nearest neighbors in interaction of magnetic ions.

The most accurate values of the critical temperatures \( T_c(N) \) for structures with different film thicknesses \( N \) were obtained by the method of intersection of curves for temperature dependencies of the Binder cumulant \( U_4(N, T, L) \) for systems with different linear sizes \( L \):

\[ U_4(N, T, L) = 1/2 \left( 3 - \frac{\langle m_{\text{stg}}^4 \rangle}{\langle m_{\text{stg}}^2 \rangle^2} \right), \]

Figure 3. Temperature dependence of the staggered susceptibility \( \chi_{\text{stg}}(T) \) for structures with \( L = 40 \) and different layer thicknesses \( N = 3, 5, 7, \) and 9 ML.
The scaling dependence of the cumulant  

\[ U_4(N, T, L) = u(L^{1/\nu}(T - T_c)) \]  

makes it possible to determine the critical temperature \( T_c(N) \) from the coordinate of the intersection points of the curves specifying the temperature dependence \( U_4(N, T, L) \) for different \( L \) (figures 4 and 5). Consequently, we determined for magnetic structures with film thicknesses \( N = 3, 5, 7, 9 \) ML the following values of magnetic ordering temperatures:

\[ T_c(N = 3) = 2.541 \pm 0.008, \quad T_c(N = 5) = 2.903 \pm 0.012, \quad T_c(N = 7) = 3.035 \pm 0.016, \quad T_c(N = 9) = 3.101 \pm 0.011. \]

The results of the present study can be compared with results of our calculations of equilibrium critical characteristics for Co/Cr/Co structure with cobalt film thickness \( N = 3 \) [18]. We have determined the temperature \( T_c(N = 3) = 0.78(3) \) of the ferromagnetic phase transition in ferromagnetic films and the temperature \( T_N = 0.19(3) \) characterizing the antiferromagnetic configuration of the film magnetizations arising owing to the negative sign of \( J_2 \). We simulated simple cubic lattice of cobalt films contacting with chromium film in Co/Cr/Co structure while cobalt films contacting with copper film in Co/Cu/Co structure are considered with fcc-lattice. Therefore, the obtained value of magnetic ordering temperature \( T_c(N = 3) = 2.541 \pm 0.008 \) in Co/Cu/Co structure is higher than \( T_c(N = 3) = 0.78(3) \) in Co/Cr/Co structure. Though, an universal critical characteristics such as critical exponents must be equal for these structures with different cubic lattices for ferromagnetic films.
3. Non-equilibrium behavior of the magnetic structure Co/Cu/Co. Study of aging effects

As it was shown in works [17, 18], the effects of slow dynamics appear in magnetic superstructures within a wide range of temperatures at $T < T_c$.

One of important features arisen in non-equilibrium behavior of systems with slow dynamics [16] is dependence on an initial states. The non-equilibrium behavior of a system is realized during transition at the starting instant $t = 0$ from the initial state at temperature $T_0$ to the state with temperature $T$, differing from $T_0$. The accompanying equilibration process is characterized by relaxation time $t_{rel}(T)$, and equilibrium corresponding to temperature $T_0$ is reached in times $t \gg t_{rel}(T)$, while the system dynamics prove stationary and invariant with respect to time reversal. However, in times $t \ll t_{rel}(T)$, the evolution of the system depends on its initial state. In this connection, the non-equilibrium behavior of the system depends on whether it evolves from a high-temperature $T_0 > T_c$ or a low-temperature $T_0 < T_c$ initial state. The high-temperature initial state for magnetic systems is created at $T_0 > T_c$ and characterized by initial magnetization $m_0 = 0$, while the low-temperature initial states with $T_0 < T_c$ are characterized by $m_0 \neq 0$. The non-equilibrium behavior of a systems with slow-dynamics demonstrates the breakdown of translational invariance in time due to the long-time influence of non-equilibrium initial states. It manifests itself through two-time characteristics of the system, such as the autocorrelation functions

$$C(t, t_w) = \frac{1}{V} \int d^d x \left[ \langle S(x, t) S(0, t_w) \rangle - \langle S(x, t) \rangle \langle S(0, t_w) \rangle \right],$$

and response functions

$$R(t, t_w) = \frac{1}{V} \int d^d x \frac{\delta \langle S(x, t) \rangle}{\delta h(x, t_w)} \bigg|_{h=0},$$

where the waiting time $t_w$ is the time between a sample preparation and the beginning of measurement of its characteristics and $t > t_w$ is the time of observation with $t > t_w$, $t_w \ll t_{rel}$.

Also, such quantity as dynamic susceptibility (thermostatic susceptibility) can be introduced for description two-time dependent phenomena which is an integral characteristic related to the response function by the expression [16]:

$$\chi_{TR}(t, t_w) = \int_0^{t_w} dt' R(t, t').$$

Description of the non-equilibrium critical behavior of a number of model statistical systems such as three-dimensional Ising model, the two-dimensional XY model, and multilayer magnetic structure Co/Cr/Co given in review [16] shows that the two-time dependent quantities (7)–(9) demonstrate so-called aging. This phenomenon is characterized by both translation symmetry breaking in time and a slow-down of relaxation and correlation processes with increasing "age" $t_w$ of the sample. The aging is displayed most pronouncedly in two-time dependence of the autocorrelation function (7) during evolution of system from the high-temperature initial state. So, it was shown in work [18] for Co/Cr/Co magnetic structure with ferromagnetic films thickness $N = 3$ that correlation times in the course of system’s evolution from a high-temperature initial state are two-three orders of magnitude more than those in the evolution from a low-temperature initial state at the same $t_w$ values. Therefore, for clearing-up the aging effects in Co/Cu(100)/Co magnetic structure dependence on ferromagnetic film thicknesses $N$ in a wide quenching temperature range $T_i \leq T_c$ we concentrate our efforts uppermost on study of autocorrelation function two-time behavior with evolution from the high-temperature initial state with $T_0 > T_c$, and $m_0 = 0$.

We computed the two time-dependent autocorrelation function taken in the form

$$C(t, t_w) = \left\{ \frac{1}{N_i} \sum_{i=1}^{N} S_i(t) S_i(t_w) \right\}$$

$$- \left\{ \frac{1}{N_i} \sum_{i=1}^{N} S_i(t) \right\} \left\{ \frac{1}{N_i} \sum_{i=1}^{N} S_i(t_w) \right\}.$$  

The simulated structures were characterized by the linear size of the films $L = 64$ and thicknesses $N = 3, 5, 7, 9$ ML. The quenching temperatures $T_f(N)$ were taken equal and also fractional to critical temperatures, i.e., $T_f = T_c(N)$, $(5/6)T_c(N)$, $(2/3)T_c(N)$, and $T_c(N)/2$. We used the waiting times $t_w = 10, 30, 50, 100$ MCS/s in the study of the two-time dependence of $C(t, t_w)$. Statistical averaging of the autocorrelation function $C(t, t_w)$ was carried out on 500 MC runs for every $t_w$.

As an example, the obtained time dependence of the autocorrelation function $C(t, t_w)$ from observation time $t - t_w$ for waiting time values $t_w = 30, 50, 100$ MCS/s is presented in figure 6 for structure with cobalt films thickness $N = 9$ ML at quenching temperatures $T_i = T_c = 3.1014$, $(5/6)T_c$, and $(1/2)T_c$. 

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The curves of $C(t, t_w)$ clearly demonstrate the aging effects in this magnetic structure, i.e., the non-exponential slowing down of time correlations with increasing system age $t_w$. This time behavior of $C(t, t_w)$ is connected with power-law time dependence of the correlation length $\xi(t) \sim t^{-a}$ with the exponent $a$ equals the dynamic critical exponent $z$ at the critical temperature $T_c$. The dynamic critical exponent $z$ characterizes a critical relaxation rate of spin excitation $\omega_{rel} \sim k^z$ in long-wave approximation. We must note that the aging in the multilayer structures arises not only at $T_s = T_c$ as in bulk systems, but also in the low-temperature phase at $T_s < T_c$. Furthermore, the comparison of curves in figure 6 at temperatures $T_s = T_c (5/6) T_c$ and $(1/2) T_c$ shows that the aging effects becomes stronger with decrease of quenching temperature relative to the critical temperature, i.e. the falling of time correlations at $T_s < T_c$ with the same waiting times $t_w$ become more slow than at the critical temperature. Reason for this behavior is connected with XY-type anisotropy, which is realized in Co/Cu(100)/Co structures, and with extremely slow dynamics in two-dimensional XY-model characterized by aging not only near the temperature of the Berezinskii-Kosterlitz-Thouless phase transition $T_{BKT}$, but also in the entire range of the existence of the low-temperature phase [27–30].

For calculation of statistical errors in the characteristics of the non-equilibrium critical behavior obtained in our study, we used the technique developed in papers [31–33]. Let us consider its realization in determining the errors in the values of autocorrelation function $C(t, t_w)$ at the non-equilibrium stage of its variation for magnetic structure with $N = 9$ ML at $T_0 = T_c = 3.1014$, $(5/6) T_c$, and $(1/2) T_c$. The total set of statistical data for 500 MC runs was divided into $n = 4$ statistically independent groups. The statistical error for each instant was determined as the standard deviation

$$\Delta C(t, t_w) = \left[ \sum_{i=1}^{n} \Delta_i^2 / n(n-1) \right]^{1/2},$$

where $\Delta_i$ is the deviation of the mean value of the autocorrelation function $C_i(t, t_w)$ over the $i$th group of runs from the mean value of function $C(t, t_w)$ over the total number of MC runs.

Figure 7 shows the calculated time dependencies $\Delta C(t, t_w)$ averaged over total number 550 runs, as well as over 120 and 240 runs. It can be seen that in the case of averaging over 500 runs, the mean value of the error $\Delta C(t, t_w)$ is small (0.002) and does not exceed 0.004. Therefore, the obtained time dependence of the mean square error $\Delta C(t, t_w)$ confirms the reliability of our results for all values of the observation time. In the case of averaging over 120 runs, the error does not exceed 0.014 and 0.006 in the case of averaging over 240 runs. In averaging over more than 300 runs, the error reaches the asymptotic level corresponding to the results of averaging over 500 runs.

We note that statistical errors $\Delta C(t, t_w)$ of the autocorrelation function calculated at the critical temperature $T_s = T_c$ include contribution of fluctuating effects, when the fluctuating contribution is absent in $\Delta C(t, t_w)$ for other investigated temperatures $T_s < T_c$. Therefore, errors $\Delta C(t, t_w)$ for these temperatures are turned out less than at the critical temperature.

The obtained two–time dependencies of the autocorrelation function $C(t, t_w)$ on the observation time $t - t_w$ and waiting times $t_w = 50$ and 100 MCS/s, which are presented in figure 8, characterize the non-equilibrium behavior of magnetic structure Co/Cu(100)/Co with different thicknesses $N$ of Co films at temperatures $T_s/T_c(N) = 1; 5/6; 2/3; 1/2$. The curves also demonstrate realization of the aging in these
structures at all temperatures $T_s \leq T_c(N)$. It can be seen that at the critical temperatures $T_s = T_c(N)$ (figure 8(d)), the aging effects in behavior of the autocorrelation function $C(t, t_w)$ become weaker with increasing thickness $N$ of cobalt films because curves $C(t, t_w)$ for structures with greater $N$ are situated lower than curves with smaller $N$ for the same values of waiting time $t_w$. This phenomenon is connected with decay of critical correlation during dimensional crossover in magnetic superstructures from films with quasi-two-dimensional properties to films with three-dimensional properties.

The curves $C(t, t_w)$ in case of temperatures $T_f(N) < T_c(N)$ in the low-temperature phase (figures 8(a)–(c)) demonstrate the aging with more complicated dependence on thicknesses $N$ of cobalt films. So, for lowest from considered temperatures with $T_s = T_c(N)/2$, an opposite tendency is observed as compared to case with $T_s = T_c(N)$, which is characterized by strengthening aging effects with increasing thickness $N$ of cobalt films (in any event for considered $N \leq 9$ ML). This phenomenon in defined nanostructures is connected with increasing characteristic correlation length for transverse spin-spin correlations when the temperature is decreased that leads to increasing correlation and relaxation times in these systems.

It is well known that the two-time autocorrelation function is characterized in the aging regime at times $t - t_w \sim t_w \gg 1$ by the scaling form [34]

$$C(t, t_w) \sim t_w^{-b} F_C(t/t_w), \quad (12)$$

where the exponent $b$ at the critical temperature $T_s = T_c$ is expressed in terms of the critical exponents as $b = 2\beta/\nu$. The scaling function $F_C(t/t_w)$ in (12) is a homogeneous function of its argument $t/t_w$, which decay at an evolution stage with long time separations $t - t_w \gg t_w$ as a power law:

Figure 7. Time dependences of the mean square error $\Delta C(t, t_w)$ of the autocorrelation function, which are averaged over 120, 240, and 500 MC runs for structure with Co films thickness $N = 9$ ML at $T_s = T_c = 3.1014$ with the waiting time $t_w = 50$ MCS/s.

Figure 8. Dependence of the autocorrelation function $C(t, t_w)$ on the observation time $(t - t_w)$ and waiting times $t_w = 30$ and 100 MCS/s for structure with Co films thicknesses $N = 3, 5, 7, 9$ ML at temperatures $T_f(N)/T_c(N)$ equal: 1/2 (a), 2/3 (b), 5/6 (c), 1 (d) with evolution from the high-temperature initial state.
The exponent $c_\nu$ in (13) is determined at $T_s = T_c$ by relation $c_\nu = d/z - \theta'$ for evolution from the high-temperature initial state, where $\beta, \nu, z$, and $\theta'$ are well-known static and dynamic critical exponents, $d$ is spatial dimension of system. The exponents $\delta$ and $c_\nu$ in (12)–(13) at temperatures $T_s = T_c$ are not connected already with the critical exponents of this system.

To check the scaling predictions for $C(t, t_w)$ given by relation (12), we plot the dependencies of $t_w^0C(t, t_w) = F_C(t/t_w)$ versus $t/t_w$ in figure 9 for different Co film thicknesses $N$ at the critical temperatures $T_s = T_c(N)$ with the use of exponent $b_2 = 2\beta/\nu z$ chosen to fit the data for different $t_w$ values to a single curve at $t/t_w \gg 1$. The exponent $b_2 = 2\beta/\nu z$ obtained from this fit is characterized by following values: $2\beta/\nu z = 0.318$ for $N = 3$, $2\beta/\nu z = 0.414$ for $N = 5$, $2\beta/\nu z = 0.436$ for $N = 7$, and $2\beta/\nu z = 0.456$ for $N = 9$. Growth of exponent $2\beta/\nu z$ values with increasing Co film thickness $N$ demonstrates a dimensional crossover from quasi-two-dimensional for $N = 3$ ML to quasi-three-dimensional for $N = 9$ ML systems. Experimental [1, 3, 4] and Monte Carlo numerical [23, 24] investigations of critical behavior of ultrathin magnetic films show that critical exponents take on two-dimensional values for films with thicknesses $N \leq 6$ ML and three-dimensional values for films with thicknesses $N \geq 20$ ML.

In witness of an assertion that correlation times for case of system’s evolution from a high-temperature initial state are much greater than those in the evolution from a low-temperature initial state at the same $t_w$ values, we present in figures 10 and 11 the graphs of two-time dependence of the autocorrelation function on the observation time $t - t_w$ and waiting times $t_w = 30, 50,$ and $100$ MCS/s for evolution of nanostructures with different thicknesses $N = 3 \div 9$ ML of cobalt films from an ordered initial state with $m_0 = 1$ at temperatures $T_s(N) = T_c(N)$ and $T_s(N) = T_c(N)/2$. We can see that the autocorrelation function demonstrates aging and takes on a small values about $10^{-3}$ already for times $10-20$ MCS/s. This time behavior of the autocorrelation function doesn’t depend considerably on temperature for $T_s \leq T_c$ and thickness $N$ of cobalt films (see in figures 11(c) and (d)). As it was shown in our paper [35] by giving an example of non-equilibrium critical dynamics of 3D Ising model with evolution from an ordered initial state, time dependence of two parts of the autocorrelation function in (10): $C_{uu}(t, t_w) \sim \langle S(t)S(t_w) \rangle$ and $C_{mn}(t, t_w) \sim \langle S(t)S(t_w) \rangle$ begins to coincide at the observation times $t - t_w \gg t_w$ that leads to their mutual compensation in the total autocorrelation function. These temporal properties of two parts of the autocorrelation function $C_{uu}(t, t_w)$ and $C_{mn}(t, t_w)$ are presented in figure 12 for structure with thickness $N = 5$ ML of cobalt films.

4. Calculation of magnetoresistance in magnetic structure Co/Cu/Co

We calculated the magnetoresistance for a multilayer structure Co/Cu(001)/Co which is introduced by the relation

$$\delta = \frac{R_{AP} - R_P}{R_P},$$

where $R_{AP}$ is the resistance of the structure when the magnetizations of adjacent ferromagnetic layers are aligned antiparallel, and $R_P$ is the resistance of the structure for parallel orientation of the magnetization of ferromagnetic layers. For the structure (figure 2) with antiferromagnetic exchange coupling of adjacent ferromagnetic layers, $R_{AP}$ characterizes the resistance of the structure without an external magnetic field, and $R_P$ is the resistance in magnetic field above the saturation field for ferromagnetic layer.
Note that the multilayer structure can connect two ways in the current line for measurements of the resistance. For measurements of the current in plane (CIP), the current conduction is realized along layers and the electrodes are situated on one side of the structure. For measurements of the current perpendicular to plane (CPP), the current conduction is realized perpendicular to the layers of the superstructure and the electrodes are situated on both sides of the structure. It was shown in works [36–38] that the CPP magnetoresistance is characterized by larger values than the CIP magnetoresistance, approximately twice as high. Regardless of the fact that the CPP method of measurement is complicated for technical realization, this method now has great practical interest because CPP magnetoresistive sensors demonstrate more sensitivity than CIP-MR sensors [5].

In the present paper we present the calculations of the CPP magnetoresistance in Co/Cu/Co structure.

At the first stage of this study, we realize calculation of temperature dependence of the magnetoresistance as equilibrium characteristic. To this effect, the simulated structure was reduced to equilibrium state for times much greater than relaxation time. We have used in papers [25, 26] for calculation of the CPP magnetoresistance the two-current Mott model to describe the resistance of different conduction channels [39]. It was introduced.
the resistance of a ferromagnetic film for two groups of electrons with spins up \( R_\uparrow \) and spin down \( R_\downarrow \). As a result, the magnetoresistance of the multilayer structure is determined by the relation:

\[
\delta = \left( \frac{R_\uparrow - R_\downarrow}{4R_\uparrow R_\downarrow} \right) = \left( \frac{J_\uparrow - J_\downarrow}{4J_\uparrow J_\downarrow} \right),
\]

where \( J_{\uparrow,\downarrow} = e n_{\uparrow,\downarrow} \langle V_{\uparrow,\downarrow} \rangle \) is the current density. Here, \( n_{\uparrow,\downarrow} \) is the density of electrons with \( x \) components of spin moment equal to \( +1/2 \) and \(-1/2 \) (axis \( x \) is the quantization axis determined by orientation of magnetization in plane of films for Co/Cu/Co structure), \( n = n_\uparrow + n_\downarrow \) is the total electron density and \( \langle V_{\uparrow,\downarrow} \rangle \) are the averaged velocities of electrons with corresponding spin projections. The electron densities with spin up and down can be expressed through the magnetization of the film \( m_{\uparrow,\downarrow}/n = (1 \pm m)/2 \) determined in the process of the Monte Carlo simulation of magnetic properties of the structure. The averaged electron velocity \( \langle V_{\uparrow,\downarrow} \rangle \) can be expressed through the electron mobility and the external electric field intensity \( E \), and after that through the probability of electron displacement in unit time (corresponding to one Monte Carlo step per spin) \( v \) and the unit cell 


time dependence of two parts of the autocorrelation function: \( C_{\text{nn}}(t, t_w) \sim \langle S(t)S(t_w) \rangle \) (solid lines) and \( C_{\text{mm}}(t, t_w) \sim \langle S(t)S(t_w) \rangle \) (dashed lines) for structure with \( N = 5 \) ML with evolution from the low-temperature initial state.

Graphs of the calculated temperature dependence of the CPP magnetoresistance \( \delta(T, N) \) are presented in figure 13 for the multilayer Co/Cu(100)/Co structure with different thicknesses \( N \) of the cobalt films. The temperature scale in figure 13 is defined through the value of the exchange integral \( J_1 = 4.4 \cdot 10^{-14} \) erg corresponding to the bulk cobalt. This value of \( J_1 \) was calculated with the use of the well known mean-field relation connecting the exchange constant \( J \) with the critical temperature \( T_c = 2 \pi S(S + 1)/3k_B \) in a nearest neighbour interaction approach, where \( z = 12 \) is the coordination number for a fcc lattice. Graphs of \( \delta(T, N) \) demonstrate close to linear law growth of the magnetoresistance values with decrease of temperature and consecutive growth of the magnetoresistance with increase of thickness \( N \) of the cobalt films. The last phenomenon is determined by increasing the critical temperature \( T_c(N) \) of magnetic ordering in Co/Cu/Co structure with increase of Co film thickness.

We carried out the comparison of the calculated and experimentally measured in the paper [36] temperature dependence of the CPP-MR for structure Co/Cu(100)/Co with the thickness of the cobalt films equal to 1.2 nm, corresponding to \( N = 9 \) ML. One can see that the calculated temperature dependence of the magnetoresistance agrees very well with experimental data. Analysis of errors \( \Delta \delta(T, N) \) for calculated magnetoresistance values as results of 40 runs shows that errors are less than 1%. We must note that calculation of equilibrium values of the magnetoresistance was realized after system’s relaxation from an ordered initial state with \( m_0 = 1 \) during 20 000 MCS/s and averaged after that and on 20 000 MCS/s.

At the next stage of this work, we study of influence of non-equilibrium behavior of the multilayer magnetic structure on its magnetoresistance with realization of evolution from both high-temperature and low-temperature initial states with the films magnetization \( m_0 = 0 \) and \( m_0 = 1 \), correspondingly.

We calculate two-time dependence of the magnetoresistance \( \delta(t, t_w) \) on observation time \( t - t_w \) and waiting time \( t_w \). The waiting time \( t_w \) characterizes the time between a sample preparation in non-equilibrium initial state and the beginning of measurement of its magnetoresistance. During the time period \( t - t_w \ll t_\text{rel}, \) where \( t_\text{rel} \) is the system’s relaxation time, the temporal behavior of the system is influenced by its initial state. As an example, we present in figure 14 calculated time dependence of the magnetoresistance \( \delta(t, t_w) \) in Co/Cu (100)/Co structure with the cobalt film thicknesses \( N = 5, 7, \) and 9 ML on observation time \( t - t_w \) with the

Figure 12. Time dependence of two parts of the autocorrelation function: \( C_{\text{nn}}(t, t_w) \) \( \sim \langle S(t)S(t_w) \rangle \) (solid lines) and \( C_{\text{mm}}(t, t_w) \) \( \sim \langle S(t)S(t_w) \rangle \) (dashed lines) for structure with \( N = 5 \) ML with evolution from the low-temperature initial state.
duration at 10 000 MCS/s for waiting times $t_w = 50, 100, 200, 400$ MCS/s with evolution from the high-temperature initial state (figure 14(a)) and the low-temperature initial state (figure 14(b)) at temperatures $T_s = T_s(N)/4$ (for $N = 5$ $T_s \simeq 231.5$ K, for $N = 7$ $T_s \simeq 242.1$ K, and for $N = 9$ $T_s \simeq 247.3$ K). Values of the

Figure 13. The dependence of the CPP magnetoresistance $\delta(T, N)$ in Co/Cu(100)/Co structure on the temperature for different thicknesses $N$ of the cobalt films. The comparison of the calculated and experimentally measured in paper [36] temperature dependence of $\delta(T)$ was made for structure with the thickness of the Co film equal to 1.2 nm ($N = 9$ ML). Errors of $\delta(T, N)$ values are represented by vertical bars which don’t exceed the size of the symbols for some sequence of data.

Figure 14. Time dependence of the magnetoresistance $\delta$ in Co/Cu(100)/Co with the thicknesses $N = 5 \div 9$ ML of the cobalt films at temperatures $T_s = T_s(N)/4$ for different waiting times $t_w = 50, 100, 200,$ and $400$ MCS/s with evolution from (a) the high-temperature and (b) low-temperature initial states.
magnetoresistance $\delta(t, t_w)$ given in figure 14 were averaged over 250 runs for $N = 5$ ML and 500 runs for $N = 7$ ML and 9 ML.

One can see that the magnetoresistance demonstrates dependence on waiting time $t_w$ as general criterion of aging and only for times about 6 000–10 000 MCS/s values of the magnetoresistance $\delta(t, t_w)$ reach a plateau with values $\delta^\infty(N, T)$, which depend on type of initial state, thickness of cobalt films and temperature. We present in table 1 calculated values of $\delta^\infty(N, T)$ for structures with thicknesses of cobalt films $N = 3, 5, 7, 9$ ML and temperatures $T = T(N)/4$ for both types of initial states. One can see that values of $\delta^\infty(N, T)$ obtained for case of evolution of system from the low-temperature initial state agree very well with equilibrium values of the magnetoresistance $\delta^{eq}(N, T)$ (figure 13) in the limits of statistical errors. But values of $\delta^\infty(N, T)$ obtained for case of evolution from the high-temperature initial state differ from equilibrium values of the magnetoresistance and lower these values on 1.3 % for films with $N = 3$ ML and 2.5 % for $N = 9$ ML. We connect these differences with influence of the effective temperature $T_{eff} = T/X^\infty$, where $X^\infty$ is the asymptotic value of the fluctuation-dissipation ratio [40]. Non-equilibrium critical dynamics of the most statistical model systems is characterized by $X^\infty < 1$ [16]. Values of $X^\infty$ in the multilayer magnetic structures are unknown for temperatures $T_c < T$, but we can use information about temperature dependence of the fluctuation-dissipation ratio with $X^\infty(T) < 1$ and $T_{eff}(T) > T$ obtained in paper [29] for the 2D XY model with $T_c < T_{BKT}$ ($T_{BKT}$ is the Berezinskii-Kosterlitz-Thouless (BKT) phase transition temperature) for case of evolution from the high-temperature initial state. Some community of non-equilibrium properties of the 2D XY model and the multilayer Co/Cu(100)/Co structure permits to declare that $T_{eff}(T) > T_c$ and, consequently, values of the magnetoresistance on plateau $\delta^\infty(N, T_{eff})$ must be less than equilibrium value of the magnetoresistance for $T < T_{eff}$.

Comparison of aging manifestation in time dependence of the autocorrelation function $C(t, t_w)$ (figure 8) and the magnetoresistance $\delta(t, t_w)$ (figure 14) shows that the magnetoresistance $\delta(t, t_w)$ faster reaches a values close to equilibrium with increase of waiting time $t_w$, while the autocorrelation function demonstrates slowing-down of correlation processes with increasing $t_w$. The zero-field cooled susceptibility is characterized by similar properties to the magnetoresistance. The zero-field cooled susceptibility is an integral characteristic related to the response function by the expression [41]

$$\chi_{ZFC}(t, t_w) = \int_{t_w}^{t} dt' R(t, t').$$

(16)

In contrast to the thermostatic susceptibility given by relation (9), the procedure of $\chi_{ZFC}$ determination is characterized by conditions when the quench of non-equilibrium system from initial state at temperature $T_0$ to state at temperature $T_c$ and subsequent evolution are performed without a magnetic field which is only turned on after the waiting time $t_w$ and affects during the observation time $t - t_w$. Determination of time dependence of the magnetoresistance corresponds to these conditions, therefore the magnetoresistance is characterized by similar properties to the zero-field cooled susceptibility $\chi_{ZFC}$. Two-time dependent properties of such dynamic susceptibility have been studied in our paper [42] devoted to simulation of non-equilibrium critical behavior of the three-dimensional site-diluted Ising model.

Thus realized study of non-equilibrium behavior of magnetic nanostructure, characterizing by slow dynamics in magnetically ordered phase, reveals aging in two-time dependence of the magnetoresistance, i.e., the dependance of magnetoresistance values on beginning of measurement of its characteristics. It was shown by giving an example of Co/Cu(100)/Co structure with the thicknesses $N = 3 \div 9$ ML of the cobalt films that the magnetoresistance reaches plateau with values $\delta^\infty(N, T)$, which depend on type of initial state, thickness of cobalt films, and temperature.

### Table 1. Values of the magnetoresistance on plateau $\delta^\infty(N, T)$ for Co/Cu(100)/Co structure with thicknesses of cobalt films $N = 3, 5, 7, 9$ ML for quenching temperatures $T = T(N)/4$ with evolution from the high-temperature (HT) and low-temperature (LT) initial states in comparison with equilibrium magnetoresistance $\delta^{eq}(N, T)$.

| $N$ (ML) | $T_c$ (K) | $\delta^\infty$, % (HT) | $\delta^\infty$, % (LT) | $\delta^{eq}$, % |
|---------|-----------|--------------------------|--------------------------|-----------------|
| 3       | 202.7     | 6.1(1)                   | 7.4(1)                   | 7.39(1)         |
| 5       | 231.5     | 36.4(1)                  | 38.3(1)                  | 38.38(1)        |
| 7       | 242.1     | 49.6(1)                  | 51.7(1)                  | 51.68(1)        |
| 9       | 247.3     | 62.2(1)                  | 64.7(1)                  | 64.72(1)        |
5. Conclusions

In the present paper a Monte Carlo simulation of the non-equilibrium behavior of multilayer Co/Cu(100)/Co nanostructure, used extensively in active elements of spintronics devices, is carried out with consideration of different thicknesses \( N \) of the cobalt films and variation temperature in wide range with \( T \leq T_c(N) \). Two-time dependencies of the autocorrelation function, calculated for both cases of system evolution from high-temperature and low-temperature initial states, demonstrate aging effects in magnetic structure for all temperatures in low-temperature phase. This phenomenon is characterized by slowing-down of correlation processes with increasing the waiting time \( t_w \). For case of the high-temperature initial state, the strengthening of aging effects is revealed with decrease of quenching temperature \( T_q \) relative to the critical temperature \( T_c \), which is characterized by greater slowing-down of correlation processes with increasing \( t_w \) for lower temperatures \( T_q < T_c(N) \). For case of the low-temperature initial state, it is shown that correlation times are two-three orders of magnitude smaller than those in the evolution from a high-temperature initial state at the same \( t_w \) values. In this case, time behavior of the autocorrelation function doesn’t depend considerably on temperature for \( T_q \leq T_c \) and thickness \( N \) of cobalt films.

However, for case of the high-temperature initial state, analysis of two-time dependence of the autocorrelation function \( C(t, t_w) \) calculated at the critical temperatures \( T_c(N) = T_c(N) \) shows a weakening of the aging effects with increasing thickness \( N \) of cobalt films. This phenomenon is connected with decay of critical correlation during dimensional crossover in magnetic superstructures from films with quasi-two-dimensional properties to films with three-dimensional properties. However, the autocorrelation function \( C(t, t_w) \) demonstrates the aging with more complicated dependence on thickness \( N \) of cobalt films at temperatures \( T(N) < T_c(N) \) in the low-temperature phase. So, we have revealed for temperatures with \( T(N) = T_c(N)/2 \) an opposite tendency as compared to case with \( T(N) = T_c(N) \) with strengthening of aging effects with increasing thickness of cobalt films for considered \( N \leq 9 \) ML. This phenomenon in given nanostructures is connected with increasing correlation length for transverse spin-spin correlations when the temperature is decreased that leads to increasing correlation and relaxation times in these systems.

In this work we have carried out calculations of temperature dependence of the CPP magnetoresistance \( \delta(T, N) \) in Co/Cu(100)/Co structure for different thicknesses of cobalt films. Graphs of \( \delta(T, N) \) demonstrate close to linear law growth of the magnetoresistance values with decrease of temperature and consecutive growth of the magnetoresistance with increase of thickness of the cobalt films. We show that the calculated temperature dependence of the CPP magnetoresistance is in good agreement with the experimental results obtained in work [36] for the Co/Cu(100)/Co multilayer structure with the thickness of the cobalt films equal to 1.2 nm, corresponding to \( N = 9 \) ML.

We have realized after that the study of influence of non-equilibrium behavior in Co/Cu(100)/Co structure on its magnetoresistance. It is revealed aging effects in time dependence of the magnetoresistance \( \delta(t, t_w) \), which is characterized by dependence of its values on waiting time \( t_w \) as the beginning of magnetoresistance measurement. It has been shown by giving an example of structures with thicknesses \( N = 3 \div 9 \) ML for Co films that the magnetoresistance reaches plateau in asymptotic long-time regime with values \( \delta^\infty(N, T) \), which depend on type of initial state, thickness of cobalt films, and temperature. Only values of \( \delta^\infty(N, T) \) obtained for case of evolution of system from the low-temperature initial state agree with equilibrium values of the magnetoresistance, but values of \( \delta^\infty(N, T) \) obtained for case of evolution from the high-temperature initial state differ from equilibrium values of the magnetoresistance and less these values. The existence of these non-equilibrium effects should surely be taken into account in any applications of the multilayer magnetic structures for spintronic devices based on the giant magnetoresistance effect.

Acknowledgments

This study was supported by the Russian Foundation for Basic Research, projects no. 17-02-00279, 18-42-550003 and grant MD-6 868.2018.2 of the President of the Russian Federation. Calculations were performed using the resources provided by the Shared Facility Center ‘Data Center of FEB RAS’ (Khabarovsk), the Supercomputing Center of Lomonosov Moscow State University, Moscow Joint Supercomputer Center and St Petersburg Supercomputer Center of the Russian Academy of Sciences.

ORCID iDs

V V Prudnikov @ https://orcid.org/0000-0001-9158-5320
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