Impressionism and Surrealism in Multiparticle Dynamics

R. M. Weiner†

Physics Department, University of Marburg, Wieselacker 8, 35041 Marburg, Germany and
Laboratoire de Physique Théorique et Hautes Énergies, Univ. Paris-Sud, 177 rue de Lourmel, 75015 Paris, France

Peter Carruthers and multiparticle dynamics; a story of common interests

Peter Carruthers, the scientist, was a man of remarkable mathematical and physical culture. He had also something which is less common now a days, namely a profound physical intuition which guided him to approach in an original way the most interesting problems at a given moment and in this way he became a guide also for others.

Multiparticle dynamics being by definition a many-body problem, it has to use methods specific for many body-physics. A physicist like Peter Carruthers, who had brought important and well known contributions to solid state physics, was therefore best qualified to enter the rapidly developing field of multiparticle dynamics. It is however less known that he was the first one to postulate the existence of a new state of quark matter, which he called “quarkium” and which he assumed to be a “bizarre Fermi liquid”. Note that this was three years before the now accepted concept of deconfined quark matter was proposed.

Two of the methods used in many-body physics are of particular importance for multiparticle dynamics, hydrodynamics and quantum statistics and Pete brought lasting contributions to both of these. His scientific achievements would certainly have been even more numerous, hadn’t he have to spend much of his time in the seventies and eighties with administrative duties. (He was the founding director of the theory division of the Los Alamos National Laboratory and later chairman of the Physics Department of the University of Arizona.)

Local equilibrium and hydrodynamics In the early seventies, when multiparticle production transcended cosmic rays and entered accelerator physics, it became clear to some physicists that “conventional” theoretical methods like Reggeology or current algebra were inadequate for the handling of the complex nature of problems one was facing. When

*Invited talk at the CORR98 meeting on Correlations, Mátraháza, June 1998, to appear in the proceedings of the meeting in World Scientific. This talk is dedicated to the memory of Peter Carruthers.
†E.Mail: weiner@mailer.uni-marburg.de
I had expressed this point of view at the Batavia conference in 1972, I felt rather isolated. I learned later from Pete, who apparently had participated at that session, but whom I do not remember to have met in Batavia, that he had shared from the beginning this insatisfaction. We were in good, but “restricted” company. I personally had had the opportunity of many interesting discussions with Hagedorn during my stay at CERN in 1969-1970, whose theory showed clearly how far “non-conventional” approaches could lead. I was also aware of course that E. L. Feinberg was a distinguished founding member of this “club”, but this was (almost) it. In 1972-1974 appeared the papers by Pete and Minh on the Landau hydrodynamical model where they showed that many empirical observations made in, at that time, “high energy” p-p reactions could be explained by this model. Most instrumental was however the now famous paper by Pete “Heretical Models of Particle Production” [2] in which he resumed the successes, up to that date, of the Landau model in multiparticle dynamics. The very title of the paper is self-explanatory.

The superiority of the Landau and the Hagedorn models as compared with all other approaches to multiparticle dynamics consists, among other things, in the fact that these are the only models which can explain the limitation of transverse momenta of secondaries, which is the most characteristic phenomenological property of strong interaction physics. Other models just postulate this property (in the Regge model e.g. this is reflected in the empirical $\beta$ function). Furthermore, the Landau model predicted the violation of Feynman scaling at high energies and Pete and collaborators had a tough time in convincing adepts of “conventional” physics that the data showed that Landau was right and that they were wrong. This story had a follow-up in heavy ion physics (cf. the end of this section).

On the other hand there is apparently a high price to be paid for the possibility to apply a classical method like hydrodynamics to particle physics. It is the assumption of local (thermodynamical) equilibrium (LE) and the constraint that correlations between positions and momenta of particles should not exceed the limits imposed by the Heisenberg incertitude relations. The justification of LE has preoccupied us for quite a time and lead to the organization of the series of meetings LESIP starting in 1984. It has found apparentely an explanation in the realization that the formation of hadronic matter is most probably preceded by the phase of quark matter.

The classical versus quantum issue stimulated Pete to write with Zachariasen a review paper [4] on the applicability of the Wigner function to multiparticle dynamics, this function being at the border between classical and quantum physics. (The topic of the Wigner function will be discussed below also in another context.)

Correlations Classical fields play also a major role in the modern developments of particle physics through spontaneously broken symmetries and the Goldstone-Higgs-Kibble mechanism. This lead Fowler and myself to investigate the “Effects of Classical Fields in Meson Correlations” [5] out of which resulted a long series of studies on correlations and multiplicity distributions.

On the issue of correlations Pete’s and my interests again met, since Pete (with

---

1The remarkable contribution of Frautschi to the Hagedorn bootstrap idea should also be mentioned.

2LESIP IV was devoted almost exclusively to the topic of correlations and got its title CAMP (Correlations and Multiparticle Production) from this subject. It is gratifying to see that the series of meetings on fluctuations started by Pete in Aspen in 1986, and which has continued under various names up to the present one, has adopted now the title CORRELATIONS. This choice is quite appropiate as fluctuations are essentially based on correlations.
Nieto) had contributed with important papers to the subject of coherent states. This also explains how Pete was among the first to understand why we thought that Bose-Einstein correlations were at the center of interest of multiparticle dynamics, at a time when practitioners in this field were mostly concerned with multiplicity distributions and had not yet realized that multiplicity distributions and correlations were complementary subjects. This explains also our collaboration in \[6\] which was the first paper to suggest that the so called intermittency effect was essentially a mere consequence of BEC. \[3\]

A follow-up of Pete’s work on multiplicity distributions was his interest in complexity studies. He was probably the first one to give a course on this subject, a copy of which I was happy to get from him in the early nineties.

Peter Carruthers was not only an outstanding scientist, but also a very talented poet and painter and some of his poems appear published for the first time in this volume. He was a true Renaissance man interested in all what is human and an admirer of European culture. However his artistic preoccupations never interfered destructively with his scientific work. He always distinguished between rigour in science and artistic freedom.

While (some of) his paintings were impressionistic or surrealist, his physics papers met always highest scientific standards, despite the fact that they were very imaginative. Unfortunately, in some of the recent literature on multiparticle dynamics and in particular in high energy heavy ion physics, these standards are not always respected \[4\]. An example is the erroneous “impression” of a large part of the heavy ion community about the existence of the rapidity plateau. In the discussions preceding the design of the ALICE detector for heavy ion reactions at the CERN LHC it took great efforts to convince some people to extend the accessible rapidity range beyond one rapidity unit, as planned initially. The idealization of boost invariance made such an extension appear superfluous. Other examples of “impressionism” and “surrealism” in physics, in particular in BEC will be given below \[5\].

Impressionism in Bose-Einstein correlations

Impressionism:“... a theory and practice of presenting the most immediate ...aspects ...with ...little study of ... realistic detail” (Webster’s Comprehensive Dictionary)

Uses and ab-uses of the Wigner function in BEC The experimental observation of the fact that the two particle correlation function depends not only on the difference of momenta \(q = k_1 - k_2\) but also on the sum \(k_1 + k_2\) lead to the introduction \[7\] of a “source” function within the well known Wigner function formalism of quantum mechanics. (For its relativistic generalization and application in hydrodynamics cf. \[8\]). \[6\] While it turned out later \[11\] that this property of the correlation function can be derived within the current formalism without the approximations involved by the Wigner formalism, this formalism

\[3\]This idea was presented by Erwin Friedlander at LESIP III.

\[4\]That is why Pete and others avoided lately “Quark Matter” meetings.

\[5\]For an introduction to the subject of BEC cf. a forthcoming textbook by the author to be published by John Wiley & Sons in 1999. This will be quoted in the following as \[I\].

\[6\]An attempt \[9\] to consider the correlation between coordinates and momentum within the ordinary wave function formalism was shown in \[10\] to have pathological features as it leads in some cases to a violation of the lower bounds of the correlation function.
is still useful within a hydrodynamical context. That is why we will describe this formalism in the following.

The Wigner function called also source function, \( g(x, k) \), may be regarded as the quantum analogue of the density of particles of momentum \( k \) at space-time point \( x \) in classical statistical physics. It is defined within the wave function formalism as

\[
g(x, k, t) = \int d^3x' \psi^*(x + \frac{1}{2}x', t) \psi(x - \frac{1}{2}x', t) e^{ikx'}
\]

\[
= \int d^3k' \psi^*(k + \frac{1}{2}k', t) \psi(k - \frac{1}{2}k', t) e^{-ikx}
\]  

and is related to the coordinate and momentum densities by the equations

\[
n(x, t) = \int d^3k g(x, k, t)
\]

and

\[
n(k, t) = \int d^3x g(x, k, t)
\]

respectively. To be able to use it in the context of hydrodynamics which is a classical theory and where it should approximate as far as possible the Boltzmann distribution function, one has to make sure that the quantum effects (i.e. the weight of the domain where \( g(x, k) \) takes negative values) are small. This is achieved by expanding the wave function in powers of Planck’s constant \( \hbar \). It turns out that the semi-classical approximation is valid as long as the relation

\[
\frac{\Lambda}{4\pi} \left| \frac{dk}{dx} \right| \ll k.
\]

is satisfied. Here \( \Lambda \) is the de Broglie wave length of the particle. In the case of BEC we deal with correlations of particles which originate from the entire source. Eq.(1) then implies that the quantum effects are small provided \( k \) does not vary very much across a region of the order of \( L \) where \( L \) is a typical length characterizing the source. Using an analogy, one might express this condition by saying that the gradients of temperature across the “fireball” should be small. However even this condition is not sufficient if one considers particles which are produced from the same space-time point. This implies that the “surprising” effects, in particular particle-antiparticle correlations [12] cannot be treated with the conventional Wigner function approach presented above.

Furthermore it turns out that the Wigner function is useful for BEC only if a more stringent condition is fulfilled, namely that the difference of momenta \( q \) of the pair is small. It is thus clear that its applicability is more restricted than that of the classical current approach, where only the “no recoil” condition, i.e. small total momentum of produced particles must be respected. This circumstance is often overlooked when comparing theoretical predictions based on the Wigner approach with experimental data. In particular herefrom also follows that the application of the Wigner formalism to data has necessarily to take into account from the beginning resonances which dominate the small \( q \) region. Heuristically the use of the Wigner function for BEC is justified only in special cases as e.g. when a coherent hydrodynamical study is performed, i.e. when the observables are related to an equation of state and when simultaneously single and higher order inclusive distributions are investigated. Unfortunately only very few papers where the Wigner function formalism is used are bona fide hydrodynamical studies. The majority of “theoretical” papers in this context are “pseudo-hydrodynamical” in the sense
that the source function $g$ is expressed in terms of effective physical variables like effective temperature or effective velocity, which are not related by an equation of state. The choice of the form of $g$ is up to the impression of the practitioner. In this procedure the application of the Wigner approach is a luxury. This is a fortiori true given the fact that the Wigner approach is mathematically not simpler that the classical current approach, of which it is a particular case.

In second quantization $g(x, k)$ is defined in terms of the correlator $<a^\dagger(k_i)a(k_j)>$ by the relation

$$<a^\dagger(k_i)a(k_j)> = \int d^4x \exp[-ix_\mu(k_i^\mu - k_j^\mu)] \cdot g[x, \frac{1}{2}(k_i + k_j)]$$

(5)

This is a natural generalization of eq.(3) to which it reduces in the limit $k_i = k_j$.

The relation between the Wigner approach and the classical current approach is established by expressing the rhs of eq.(5) in terms of the currents. One has

$$g(x, k) = \frac{1}{2\sqrt{E_iE_j}(2\pi)^3} \int d^4z <J (x + \frac{z}{2})J (x - \frac{z}{2})> \exp[-ik^\mu z_\mu]$$

(6)

The derivation of the Wigner formalism from the classical current formalism has the important advantage that it avoids violations of quantum mechanical bounds as those mentioned previously.

¿From the above considerations follows that a parameter in the space-time approach, such as the correlation length $L$, can be linked to parameters that enter the Wigner function, e.g., the temperature $T$ if the system is in thermal equilibrium. This particular case of a relation between $L$ and $T$ was derived in [15].

As mentioned already, the use of the Wigner formalism is worthwhile within a true hydrodynamical approach when the relation with the equation of state is exploited. In this case the probability to produce a particle of momentum $k$ from the space-time point $x$ then depends on the fluid velocity, $u^\mu(x)$ and the temperature $T(x)$ at this point, and one has

$$\sqrt{E_iE_j} <a^\dagger(k_i)a(k_j)> = \frac{1}{(2\pi)^3} \int_\Sigma \frac{\frac{1}{2}(k_i^\mu + k_j^\mu)d\sigma(x_\mu)}{\exp\left[\frac{\frac{1}{2}(k_i^\mu + k_j^\mu)u_\mu(x_\mu)}{T_f(x_\mu)}\right] - 1} \cdot \exp[-ix_\mu(k_i^\mu - k_j^\mu)]$$

(7)

Here, $d\sigma^\mu$ is the volume element on the freeze-out hypersurface $\Sigma$ where the final state particles are produced. Despite the fact that the use of the Wigner formalism can be defended only if combined with bona-fide hydrodynamics and an equation of state$^7$, with the exception of a few real, albeit numerical, hydrodynamical calculations, most phenomenological papers on BEC in heavy ion reactions have used the Wigner formalism without a proper hydrodynamical treatment, i.e. without solving the equations of hydrodynamics; hydrodynamical concepts like velocity and temperature were used just to parametrize the Wigner source function. While such a procedure may be acceptable as a theoretical exercise, it is certainly no substitute for a professional analysis of heavy ion reactions.

$^7$That full-fledged hydrodynamics is sometimes indispensable was illustrated e.g. in the study of the role of the transverse and longitudinal expansion on the effective radii$^3$. 

5
This is particularly true when real data have to be interpreted.

As examplified in [8] such a procedure is unsatisfactory, among other things because it can lead to wrong results.

The use of this “pseudo-hydrodynamical” approach is even more surprising if one realizes that the Wigner formalism not only is not simpler than the more general current formalism but it is also less economical. The number of independent parameters necessary to characterize the BEC within the Wigner formalism is (cf. e.g. ref. [14]) 10, i.e. it is as large as that in the current formalism [15]. However the 10 parameters of [14] describe a very particular source as compared with that of the current formalism: besides the fact that the second order correlation function \( C_2 \) in the Wigner approach is assumed to be Gaussian, it is completely chaotic and it can provide only a correlation length \( L \). In the current formalism on the other hand, with 10 parameters the correlation function is not restricted to the Gaussian form, one describes also coherence and one distinguishes between the correlation length \( L \) and the geometrical radius \( R \). For the search of quark gluon plasma, \( R \) is the relevant quantity, because the energy density is defined in terms of \( R \) and the use of \( L \) instead of \( R \) leads to an overestimate of this energy density.

Furthermore the physical significance of the parameters of the Wigner source is unclear if the Gaussian assumption does not hold. Not only is there no apriori reason for a Gaussian form, but on the contrary, both in particle physics and in quantum optics, there exists experimental evidence that in many cases an exponential function in \(|q|\) is at small \( q \) a better approximation for \( C_2 \) than a Gaussian. Furthermore, it is known [16] that in the presence of coherence, no single simple analytical function, and in particular no single Gaussian is expected to describe \( C_2 \). This is a straightforward consequence of quantum statistics. Last but not least, in heavy ion reactions it has been shown that resonances deform any Gaussian form into a non-Gaussian one.

Given the fact that good experimental BEC data are expensive both in terms of accelerator running time and man-power, the use of inappropriate theoretical tools, when more appropriate ones are available, is a waste which has to be avoided.

---

8 A recent experimental paper [13] where such an analysis is performed is a good illustration of the limits of pseudo-hydrodynamical models. Despite the fact that the statistics are so rich that “the statistical errors on the correlation functions are negligible”, the outcome of the analysis is merely the resolution of the ambiguity between temperature and transverse expansion velocity of the source. It is clear that such an ambiguity is specific to pseudo-hydrodynamics and is from the beginning absent in a correct hydrodynamical treatment. Moreover even this result is questionable given some doubtful assumptions which underly this analysis. To quote just two: (i) The assumption of boost invariance made in [13] decouples the longitudinal expansion from the transverse one. This not only affects the conclusions drawn in this analysis but prevents the (simultaneous) interpretation of the experimental rapidity distribution. (ii) The neglect of long lived resonances which strongly influences the \( \lambda \) factor (cf. eq. (9) below) and thus also the extracted radii. Of course, despite the claimed richness of the data, no attempt to relate the observations to an equation of state can be made within this naive phenomenological approach.

9 To consider such an approach as “model independent” as has become customary in the heavy ion literature is misleading.
Higher order correlations

The modern treatment of BEC is based on the density matrix $\rho$ and field theory. In quantum optics (QO) one writes $\rho$ in terms of the coherent state representation and the most frequently form for this expansion is the Gaussian one.10

Besides its mathematical convenience this form follows from the central limit theorem for an infinite number of independent fields. One most remarkable consequence of the Gaussian form of the density matrix is the fact that all higher order correlation functions are determined just by the first two correlation functions. However this does not imply at all that higher order correlation measurements are unnecessary, once the first two correlation functions are determined. Indeed there are at least three reasons why the measurement of higher order correlations is important:

(i) In the absence of a theory which determines from first principles the first two correlation functions, models for these quantities are used, which are only approximations. The errors introduced by these phenomenological parametrizations manifest themselves differently in each order and thus violate the central limit theorem.

(ii) In experiments, because of limited statistics and sometimes also because of theoretical biases not all physical observables are determined, but rather averages over certain variables are performed, which again introduces errors which propagate (and are amplified) from lower to higher correlations.

(iii) The conditions of the applicability of the above theorem and in particular the postulate that the number of fields (sources) is infinite and that they act independently can never be fulfilled exactly.

Conversely, from the comparison of correlation functions of different order one can test the applicability of the central limit theorem and pin down more precisely the parameters which determine the first two correlation functions, which is essentially the purpose of particle interferometry.

Initially theoretical calculations for higher order BEC were based on this QO formalism. After a slight confusion related to the use of an incorrect formula for the comparison of the various orders of correlation functions with data 17, it was found 18 that in a first approximation a simplified formalism of this type using a single BEC variable, the invariant momentum difference $Q$, could account for the experimental measurements. For a discussion of these issues cf. also 19.

In the mean time further theoretical and experimental developments took place.

On the experimental side a new technique for the study of higher order correlations was developed (this method was developed in great part by collaborators of Peter Carruthers), the method of correlation integrals which was applied 20 to a subset of the same UA1 data in order to test the above quoted QO formalism. The fits were restricted to second and third order cumulants only. As in 17 it was found that by extracting the effective parameters (chaoticity and radius) from the second order data, the “predicted” third order correlation, this time by using a correct QO formula, differed significantly from the measured one. If confirmed, such a result could indicate that the QO formalism provides only a rough description of the data and that higher precision data demand also more more realistic theoretical tools. 11 Such tools are the quantum statistical (QS)

10 The Gaussian form of this representation must not be confused with the form of the correlations function.

11 A further, but more remote possibility would be to look for deviations from the Gaussian form of the density matrix. However, a more mundane reason for the result of ref. 20 will be mentioned below.
space-time approach to BEC developed in the nineties in \cite{15} and mentioned above. This approach based on the classical current formalism is more appropriate to particle physics than the quantum optical approach and is a generalization of the former. The fields are replaced by currents and the parametrization of the space-time distribution of the source, which is introduced in the QO formalism essentially only through dimensional considerations, gets a more rigorous foundation. Moreover, and most importantly, it turns out that there are two different length scales in BEC, one related to the space-time distribution of the source and another associated with the correlator and that $Q$ is not a natural variable for BEC if one wants to obtain from it information about the geometry of the source, the correlation length, and the chaoticity. To get this type of information from measurements made in the variable $Q$ a complex projection involving integrations over unobserved physical quantities has to be performed \cite{12}.

However this new approach, although more advanced, does not make redundant the determination of higher order correlations as the considerations (i)-(iii) continue to be valid.

The space-time formalism has been used recently in \cite{23} to study higher order correlations up to the fourth order in the variable $Q$ and the calculated results were compared with the Na22 data \cite{24}. It was found that the data could be fitted without difficulties with quite reasonable space-time parameters. At the same time it was found that a possible reason for the negative results obtained in \cite{20} within the simpler QO approach was the questionable procedure used for testing the relation between second and third order correlations. Indeed in \cite{20} one did not perform a simultaneous fit of second and third order data to check the QO formalism. Such a simultaneous fit is necessary before drawing conclusions, because as mentioned above (cf.(i) and (ii)), the errors involved in “guessing” the form of the correlator, and the fact that the variable $Q$ does not characterize completely the two-particle correlation, limit the applicability of the theorem which reduces higher order correlations to first and second ones. As a matter of fact, it was found \cite{23} that the second order correlation data are quite insensitive to the values of the parameters which enter the correlator, while once higher order data are used in a simultaneous fit, a strong delimitation of the acceptable parameter values results. Thus there are several possible solutions if one restricts the fit to the second order correlation and the correct one among these can be found only by fitting simultaneously all correlations. If by accident one chooses in a lower correlation the wrong parameter set, then the higher correlations cannot be fitted anymore.

**Photon versus hadron interferometry**

Photons and mesons are both bosons and therefore satisfy the same Bose-Einstein statistics. This leads to similarities in the corresponding Bose-Einstein correlations which underly photon and hadron intensity interferometry. However there are also differences between the two effects and some of these will be analyzed in the following.

Photon interferometry in particles physics is from a certain point of view superior to hadron interferometry, because photons are weakly interacting particles, while hadrons

\footnote{This topic was discussed in detail in \cite{21} in connection with the issue of intermittency, after it had been suggested by Bialas \cite{22} that the source itself may not have a fixed size, but rather a size which fluctuates from event to event with a power distribution. In \cite{21} it was proven that, by starting from a space-time correlator with a fixed correlation length and a source distribution with a fixed radius, one gets after integrations over the unobserved variables, a correlation function which mimics power-behaviour.}
interact strongly. This has two important consequences in photon BEC: (i) there is (up to higher order corrections) no final state interaction between photons, so that the BEC effect is “clean”; (ii) in a high energy reaction, hadrons are produced only at the end of the reaction (at freeze-out), while photons are produced from the beginning, so that photons can provide unique information about the initial state. For the search of quark-gluon plasma this is essential, because if such a state of matter is formed, then this happens only in the early stages of the reaction. This is also important in lower energy heavy ion reactions where the dynamics of the reaction as well as its space-time geometry are studied in this way.

These advantages of photon interferometry have stimulated theoretical and experimental studies, despite the technical difficulties due to the small rates of photon production and the background due to \( \pi^0 \) decays.

Besides the difference in the coupling constant, photons and hadrons (for the sake of concreteness we shall refer in the following to pions) have also other distinguishing properties like spin, isospin, and mass which manifest themselves in the corresponding BEC and which sometimes are overlooked. The role of spin will be discussed below\(^{13}\).

**Photon spin and bounds of BEC.** In refs. \([25]\) and \([26]\) it was found that while for (pseudo-)scalar pions the intercept of the second order correlation function \( C_2(k,k) \) is a constant, even for unpolarized photons the intercept is a function of \( k \). One thus finds that, while for a system of charged pions (i.e. a mixture of 50% positive and 50% negative) the maximum value of the intercept \( \text{Max} C_2(k,k) \) is 1.5, for photons \( \text{Max} C_2(k,k) \) exceeds this value and this excess reflects the space-time properties of the source, the degree of (an)isotropy of the source, and the supplementary degree of freedom represented by the photon spin. The fact that the differences between charged pions and photons are enhanced for soft photons reminds us of a similar effect found with neutral pions (cf. ref. \([15]\)). Neutral pions are in general more bunched than identically charged ones and this difference is more pronounced for soft pions. This similarity is not accidental, because photons as well as \( \pi^0 \) particles are neutral and this circumstance has quantum field theoretical implications which will be mentioned also below.

The results quoted above, in particular those obtained by Neuhauser \([25]\) were challenged by Slotta and Heinz \([27]\). Among other things, these authors claim that for photon correlations due to a chaotic source “the only change relative to 2-pion interferometry is a statistical factor \( \frac{1}{2} \) for the overall strength of the correlation which results from the experimental averaging over the photon spin”. In \([27]\) a constant intercept \( \frac{3}{2} \) is derived which is in contradiction with the results presented above.

We would like to point out here that the reason for the difference between the results of \([25]\), \([26]\) on the one hand and those of ref. \([27]\) on the other is mainly due to the fact that in \([27]\) a formalism was used which is less general than that used in \([25]\) and \([26]\) and which is inadequate for the present problem. This implies among other things that unpolarized photons cannot be treated in the naive way proposed in \([27]\) and that the results of \([25]\) and \([26]\) are correct, while the results of \([27]\) are not.

In \([27]\) the following formula for the second order correlation function is used:

\[
C_2(k_1,k_2) = 1 + \frac{\tilde{g}_{\mu\nu}(q,K)\tilde{g}^\mu\nu(-q,K)}{\tilde{g}_{\mu\nu}(0,k_1)\tilde{g}^\mu\nu(0,k_2)}
\]

\(^{13}\)For a more detailed comparison of photon and hadron interferometry cf.\([I]\).
Here $\tilde{g}$ is the Fourier transform of a source function $(g(x, K))$ and $q = k_1 - k_2$, $K = \frac{1}{2}(k_1 + k_2)$.

This formula is a particular case of a more general formula for the second order correlation function derived by Shuryak [28] using a model of uncorrelated sources, when emission of particles from the same space-time point is negligible.

As is clear from this derivation there exists also a third term, neglected in eq.(8) and which corresponds to the simultaneous emission of two particles from the same point (cf. [15]). While for massive particles this term is in general suppressed, this is not true for massless particles and in particular for soft photons. Indeed in [23] and [24] this additional term had not been neglected as it was done subsequently in [27] and therefore it is not surprising that ref. [27] could not recover the results of refs. [25] and [26]. The neglect of the term corresponding to emission of two particles from the same space-time point is not permitted in the present case. Emission of particles from the same space-time point corresponds in a first approximation to particle-antiparticle correlations and this type of effect leads also to the difference between BEC for identical charged pions and the BEC for neutral pions. This is so because neutral particles coincide with the corresponding antiparticles. (As a consequence of this circumstance e.g. while for charged pions the maximum of the intercept is 2, for neutral pions it is 3 (cf. [15]). Photons being neutral particles, similar effects like those observed for $\pi^0$-s are expected and indeed found.

This misapplication of the current formalism invalidates completely the conclusions of ref. [27].

Intuitively the fact that for unpolarized photons $\text{Max}C_2(k, k)$ is 2 and not 1.5 as stated in [27], can be explained as follows: a system of unpolarized photons consists on the average of 50% photons with the same helicities and 50% photons with opposite helicities. The first ones contribute to the maximum intercept with a factor of 3 and the last ones with a factor of 1 (corresponding to unidentical particles).

For further details of the topics discussed here cf. [29].

**Surrealism and pion condensates; pasers?**

Surrealism: “A movement ...characterized by the incongruous and startling arrangement and presentation of subject matter”

(Webster’s Comprehensive Dictionary)

One of the most important effects of quantum optics which is based on coherence is the phenomenon of *lasing*. Lasers are Bose condensates and it has been speculated that such condensates, in particular pion condensates, may exist also in nuclei (cf. e.g. [30]) or be created in heavy ion reactions (cf. e.g. [31], [32]).

However there exist important differences between photon condensates i.e. lasers and pion condensates. Furthermore there are different theoretical approaches to the problem of pion condensates and some confusing statements as to how pion condensates are produced. In the following we shall discuss briefly these issues.

**The multiplicity dependence of BEC** BEC for inclusive processes, which constitute by far the most interesting and most studied reactions both with hadrons and photons, have to be treated by quantum field theory, which is the appropriate formalism when the number of particles is not conserved. For certain purposes however, sometimes one is interested in considering events with a fixed number of particles. Thus the number of
particles in a given event can help selecting central collisions with small impact parameter. Theoretically this situation can be handled within field theory, using the methods of quantum statistics \[33\]. On the other hand for the construction of event generators wave functions appear so far to be a convenient tool and therefore, and also for historical reasons, some theorists have continued to use the “traditional” method of wave function (wf). This implies the explicit symmetrization of the products of single particle wf, while in field theory the symmetrization (of amplitudes) is automatically achieved through the commutation relations of the field operators. When the multiplicities are large, the symmetrization of the wf becomes tedious. This lead Zajc \[34\] to use numerical Monte Carlo techniques for estimating \(n\) particle symmetrized probabilities, which he then applied to calculate two-particle BEC. He was thus able also to study the question of the dependence of BEC parameters on the multiplicity \(n\). Using as input a second order BEC function parametrized in the form

\[ C_2 \sim 1 + \lambda \exp(-q^2R^2), \]  

(9)

where \(q\) is the momentum transfer and \(R\) the radius, Zajc found, and this was confirmed in \[33\], that the “incoherence” parameter \(\lambda\) decreased with increasing \(n\). \[14\]

However Zajc did not consider that this effect means that events with higher pion multiplicities are denser and more coherent. On the contrary he warned against such an interpretation and concluded that his results have to be used in order to eliminate the bias introduced by this effect into experimental observations. \[15\]

This warning apparently did not deter the authors of \[31\] and \[35\] to do just that. Ref.\[31\] went even so far to derive the possible existence of pionic lasers (pasers) from considerations of this type.

In a concrete example Pratt considers a non-relativistic source distribution \(g\) in the absence of symmetrization effects:

\[ g(k,x) = \frac{1}{(2\pi R^2 m T)^{3/2}} \exp \left( \frac{k_0 T}{2} - \frac{x^2}{2R^2} \right) \delta(x_0) \]  

(10)

where

\[ k_0/T = k^2/2\Delta^2 \]  

(11)

Here \(T\) is an effective temperature, \(R\) an effective radius, \(m\) the pion mass, and \(\Delta\) a constant with dimensions of momentum.

Let \(\eta_0\) and \(\eta\) be the number densities before and after symmetrization, respectively. In terms of \(g(k,x)\) we have

\[ \eta_0 = \int g(k,x) d^4kd^4x \]  

(12)

and a corresponding expression for \(\eta\) with \(g\) replaced by the source function after symmetrization.

Then one finds \[31\] that \(\eta\) increases with \(\eta_0\) and above a certain critical density \(\eta_0^{\text{crit}}\), \(\eta\) diverges. This is interpreted by Pratt as pasing.

The reader may be rightly puzzled by the fact that while \(\eta\) has a clear physical meaning the number density \(\eta_0\) and a fortiori its critical value have no physical meaning, because in

\[14\] In \[34\] the clumping in phase space due to Bose symmetry was also illustrated.

\[15\] The same interpretation of the multiplicity dependence of BEC was given in \[33\]. In this reference the nature of the “fake” coherence induced by fixing the multiplicity is even clearer, as one studies there explicitly partial coherence in a consistent, quantum statistical formalism.
nature there does not exist a system of bosons the \( \psi \) of which is not symmetrized. That is why we call this approach “surrealistic”. The physical facts which induce condensation are, for systems in (local) thermal and chemical equilibrium, pressure and temperature and the symmetrization is contained automatically in the form of the distribution function

\[
f = \frac{1}{\exp[(E - \mu)/T] - 1}
\]

where \( E \) is the energy and \( \mu \) the chemical potential.

To realize what is going on it is useful to observe that the increase of \( \eta_0 \) can be achieved by decreasing \( R \) and/or \( T \). Thus \( \eta_0 \) can be substituted by one of these two physical quantities. Then the blow-up of the number density \( \eta \) can be thought of as occurring due to a decrease of \( T \) and/or \( R \). However this is nothing but the well known Bose-Einstein condensation phenomenon.

While from a purely mathematical point of view the condensation effect can be achieved also by starting with a non-symmetrized \( \psi \) and symmetrizing it afterwards “by hand” , the causal i.e. physical relationship is different: one starts with a bosonic i.e symmetrized system and obtains condensation by decreasing the temperature or by increasing the density of this *bosonic* system. To obtain a pion condensate e.g., the chemical potential has to equate the pion mass.

A scenario for such an effect in heavy ion reactions has been proposed in [32].

To conclude the “paser” topic, one must correct another confusing interpretation which relates to the observation made also in [34] that the symmetrization produces a broadening of the multiplicity distribution (MD). In particular starting with a Poisson MD for the non-symmetrized \( \psi \) one ends up after symmetrization with a negative binomial. While Zajc correctly considers this as a simple consequence of Bose statistics, ref. [31] goes further and associates this with the so called pasing effect. That such an interpretation is incorrect is obvious from the fact that for true lasers the opposite effect takes place. Before “condensing” i.e. below threshold their MD is in general broad and of negative binomial form corresponding to a chaotic (thermal) distribution, while above threshold the laser condensate is produced and as such corresponds to a coherent state and therefore is characterized by a Poisson MD.

References

[1] P. Carruthers, Collective Phenomena, 1 (1973) 147-161.

[2] P. Carruthers, Annals of The New York Academy of Sciences, 229 (1974) 91-123.

[3] Local Equilibrium in Strong Interaction Physics, editors D. Scott and R. Weiner, Bad-Honnef 1984, World Scientific 1985; Hadronic Matter in Collision, (LESIP II) editors P. Carruthers and D. Strottman, Santa-Fe 1986, World Scientific 1986; Hadronic Matter in Collision 1988 (LESIP III) , editors P. Carruthers and J. Rafelski, Tucson 1988, World Scientific 1989; Correlations and Multiparticle Production-CAMP (LESIP IV), editors M. Flümer, S. Raha, and R. M. Weiner, Marburg 1990, World Scientific 1991.

\[\text{16For lasers the determining dynamical factor is among other things the inversion of the occupation of atomic levels.}\]
[4] P. Carruthers and F. Zachariasen, Rev. Mod. Phys. 55 (1983) 245.

[5] G.N. Fowler and R.M. Weiner, Phys. Rev. D 17 (1978) 3118.

[6] P. Carruthers, E. M. Friedlander, C. C. Shih, and R. M. Weiner, Phys. Lett. 222 (1989) 487.

[7] S. Pratt, Phys. Rev. Lett. 53 (1984) 1219.

[8] B. R. Schlei et al., Phys. Lett. B 293 (1992) 275.

[9] F. B. Yano and S. E. Koonin, Phys. Lett. 78 B (1978) 556.

[10] A. Timmermann et al., Phys. Rev. C 50 (1994) 3060.

[11] I. V. Andreev and R. M. Weiner, Phys. Lett. B 253 (1991) 416.

[12] I. Andreev, M. Plümer, R. Weiner, Phys. Rev. Lett. 67 (1991) 3475.

[13] H. Appelshäuser et al., Na49 collaboration, Eur. Phys. J C 2 (1998) 661

[14] S. Chapman, J. Nix and U. Heinz, Phys. Rev. C 52 (1995) 2694.

[15] I. V. Andreev, M. Plümer, and R. M. Weiner, Int. J. Mod. Phys. A 8 (1993) 4577.

[16] R. M. Weiner, Phys. Lett. B 232 (1989) 278; Phys. Lett. B 242 (1990) 547.

[17] N. Neumeister et al. (UA1 Collaboration), Phys. Lett. B 275 (1992) 186.

[18] M. Plümer, L. V. Razumov, and R. M. Weiner, Phys. Lett. B 286 (1992) 335.

[19] Bose-Einstein Correlations in Particle and Nuclear Physics, A collection of reprints, edited by R. M. Weiner, John Wiley and Sons, Chichester, New York, 1997.

[20] H. C. Eggers, P. Lipa, and B. Buschbeck, Phys. Rev. Lett. 79 (1997) 197.

[21] I. V. Andreev et al., Phys. Rev. D 49 (1994) 1217.

[22] A. Bialas, Nucl. Phys. A 545 (1992) 285c; Acta Phys. Pol. B 23 (1992) 561.

[23] N. Arbex, M. Plümer and R. M. Weiner, Phys. Lett. B 438 (1998) 193.

[24] N. M. Agababyan et al., Z. Phys. C 68 (1995) 229.

[25] D. Neuhauser, Phys. Lett. B 182 (1986) 289.

[26] L. V. Razumov and R. M. Weiner, Phys. Lett. B 319 (1993) 431.

[27] C. Slotta, U. Heinz, Phys. Lett. B 391 (1997) 469.

[28] E. V. Shuryak, Sov. J. Nucl. Phys. 18 (1974) 667.

[29] R. M. Weiner, Physics Reports, to be published.

[30] A. B. Migdal, Rev. Mod. Phys. 50 (1978) 107.

[31] S. Pratt, Phys. Lett. B 301 (1993) 159.
[32] U. Ornik, M. Plümer and D. Strottman; Phys.Lett. B314 (1993) 401; U. Ornik et al., Phys.Rev. C 56 (1997) 412.

[33] G.N. Fowler et al., Phys.Lett. B 253 (1991) 421.

[34] W.A. Zajc, Phys.Rev. D 35 (1987) 3396.

[35] W.Q. Chao, C.S. Gao, and Q.H. Zhang, J.Phys. G 21 (1994) 847.