Improved radiation shielding analysis considering vector calculus

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Summary
The future of nuclear energy in the energy mix faces a permanent scrutiny of safety aspects in conciliation with bridled costs, either fission- or fusion-based. This affects to all the exciting milestones pursued in XXIst. To name few in the field of fission, the deployment of the IVth generation reactors is expected or the definitive solution to the radioactive wastes is sought. A mention apart is made to fusion technology, with ITER as the flagship project. It seeks a virtually infinite energy source, intrinsically safe and with reduced radioactive waste production with respect to fission the first commercial reactor. All these, and many other endeavors, share the operation of sophisticated devices in the presence of intense ionizing radiation fields. Humans and electronics must be protected to ensure safe and reliable performance, while shielding normally represents a large fraction of the budget. This involves nuclear analysis in the design phase to forecast the radiation conditions. The complexity of the simulation of 3D radiation fields that is computationally affordable nowadays is unprecedented. While sophistication in geometries and source definitions has become routine, the resulting complexity of these scalar fields makes their analysis increasingly difficult. The need of enhancement of the analysis techniques is evident today. Vector calculus is proposed following a physical interpretation of the field lines that boosts the analysis capabilities. It identifies the trajectories around which shielding is weakest in an automated errorless and effortless approach. Its power is illustrated with an example relevant to the ITER reactor.

KEYWORDS
ITER, nuclear analysis, nuclear fusion, radiation shielding

1 | INTRODUCTION

Power production in XXIst century envisages exciting milestones in the field of nuclear technology. For fission technology, that includes the deployment of the IVth generation reactors,1 where private initiatives are playing a leading role with multiple technological alternatives,2,3 the definitive solution to the radioactive wastes in...
geological disposals,\textsuperscript{4,5} with support of reprocessing or in transmutation systems,\textsuperscript{6} and the settlement of the long-term management of the Chernobyl and Fukushima accidents. And of course, maintaining all the activities, from mining to reprocessing, of the currently growing fleet of reactors.

For fusion technology, it includes the demonstration of a high energy gain factor, the design and construction of the first demonstrator reactors,\textsuperscript{7} and a strong R&D supporting the mainstream path,\textsuperscript{8} and also permanently trying higher-risk/higher benefit alternative schemes. Current major projects in operation are the NIF,\textsuperscript{9} LMJ\textsuperscript{10} and JET\textsuperscript{11} and W7X.\textsuperscript{12} Among many others. ITER\textsuperscript{13} (Figure 1), currently in construction, is likely the most relevant project, and it expects energy gain $Q = 10$ starting DT operation by 2035. Other projects currently in conceptual design phase include DEMO\textsuperscript{14} or IFMIF-DONES,\textsuperscript{15} and HiPER,\textsuperscript{16,17} just to name the most ambitious international initiatives.

All these endeavors share the operation of sophisticated devices in the presence of intense ionizing radiation fields. Humans and electronics must be protected to ensure safe and reliable performance in the context of a growing scrutiny to safety in conciliation with briddled costs. This involves more demanding nuclear analysis in the design phase to forecast the radiation conditions. Thus, nuclear analysis is a fundamental discipline with growing interest for the future of nuclear energy, but not only. Relevant non-energetic applications such as space travels\textsuperscript{18,19} and medicine\textsuperscript{20} also demand enhanced nuclear analysis.

Protection against intense radiation fields normally means intricate and expensive shielding engineering, one of the subsidiary challenges to the development of these important endeavors. Because shields are heavy and voluminous, their presence often contravenes other devices design aspects, and the reality of engineering ruins the implementation of a promising conceptual design. Meeting such important challenges relies in a proper shielding design. The design shall maintain the engineering viability, contain costs, and respect the radiation limits. In other words, radiation shielding design is a key technology for the future.

The methods and high-performance computing capabilities developed in the last 20 years have unlocked the simulation of radiation fields of unprecedented complexity. The construction of research infrastructures like LHC\textsuperscript{21} at CERN or SLAC\textsuperscript{22} at Stanford have been at the vanguard of these technologies. However, the development of the nuclear fusion has played a prominent leading role. The construction of major facilities, like NIF,\textsuperscript{9} LMJ,\textsuperscript{10} and ITER\textsuperscript{13} (see Figure 1) has demanded a significant step forward in the field, needing to demonstrate prominent safety cases. Thus, radiation fields of prodigious complexity can be computed nowadays routinely. Consequently, and, as we shall show, understanding such fields to conduct a proper shielding analysis is becoming increasingly difficult.

We propose the use of vector analysis to analyze the radiation fields. It represents a powerful technique to boost the shielding analysis to complement the powerful computational capabilities and methods available...
nowadays. Adapting this set of mathematical tools used before in other branches of physics requires elaboration: a physical interpretation of the field lines is needed.

Its use in nuclear analysis for shielding design shows promising perspectives with an unprecedented efficiency of application in many areas of relevance for the science of the coming decades. Being an analysis technique, it is agnostic of the software used to obtain the radiation field. It is considered as postprocessing applicable to fields obtained in any discipline: particle accelerators, fusion reactors, fourth generation fission reactors, neutron spallation sources, radioactive waste disposal facilities satellites, spacecrafts, and stations and bases out of the terrestrial atmosphere among others.

We illustrate its application using an ITER-related dummy example with a complexity of relevance for the mentioned scientific challenges.

2 | COMPLEXITY OF THE GEOMETRIES AND RADIATION SOURCES IN SIMULATION NOWADAYS

We focus on ITER as the current cutting-edge and most prominent facility for nuclear fusion as an energy source worldwide. The radiation fields in discussion in this work are determined using ITER standards, and specifically, MCNP code.23

One driver of the increment in the complexity captured in the modern simulation of radiation fields is the use of tools to support the simulation with 3D Computer Aided Design (CAD) models, preserving relevant aspects with a high degree of detail.24-27 Examples of this increased complexity are the so-called ITER reference MCNP models: C-model,28 the Tokamak Complex29 (TC) or the E-lite30 (360° model of the ITER Tokamak).

In the example considered here, the ITER TC MCNP model of the buildings, reflecting the baseline design of 2016 is used. It represents a facility with dimensions of 120 m × 80 m × 80 m. Over 5500 penetrations in walls and floors were modeled explicitly. In combination with concrete walls of different thickness (some over 2-m thick), this model represents an epitome of complexity in terms of preferential paths for radiation to leak in all directions. Identifying the critical aspects of the geometry requiring shielding enhancement presents many obstacles. A combination of the TC and the ITER neutral-beam (NB) cell31 MCNP models is shown in Figures 2 to 5. This is the ITER-related dummy example selected to illustrate the power of vector calculus for nuclear analysis. Aspects of relevance for later discussion are highlighted. Note the NB lines, which consist of four particle accelerators constructed from thick steel and polyethylene walls, connected with high voltage (HV) lines enclosed in thick steel sleeves.

The modeling of radiation sources for ITER has followed an evolution as intense as the geometry treatment, with diverse new methods and tools to deal with them, from the decay of activated components32-34 to Activated Corrosion Products.35 The modeling of the primary radiation source (neutrons from the plasma) is given here most of the attention. A relevant development
FIGURE 4 Vertical cross-section view of the ITER Tokamak Complex and NB cell MCNP models along central plane [Colour figure can be viewed at wileyonlinelibrary.com]

FIGURE 5 Region of interest to illustrate vector analysis

to address the radiation mapping beyond the bio-shield was achieved in 2016 (SRC-UNED36). The ITER Tokamak is one of the most sophisticated machines in the world. The plasma radiation field gets complex in terms of energy spectrum, and its spatial and angular distribution as it leaves the machine and spreads throughout the facility (see Figure 1). The heterogeneity of the radiation field impinging on the inner face of the ITER bio-shield is modeled using the so-called mosaic source approach. It allows its reconstruction for further simulations with an extraordinary level of detail and information. This radiation source fed with the information computed with E-lite30 is considered here to illustrate the need and power of the vector calculus in the analysis of complex radiation fields. It is shown in Figure 14.

Despite the unparalleled complexity captured in models and sources for ITER, radiation fields are regularly obtained nowadays. Identifying weak aspects of the shielding when considering such complexity geometries is not always straightforward, or even feasible with current analysis techniques.

3 | VECTOR ANALYSIS OF RADIATION FIELDS

The most advanced techniques of nuclear analysis for shielding design in ITER-like facilities nowadays target the calculation of the 3D spatial distribution of the radiation field or its adjoint function. Let \( \phi(r) \) be the stationary total flux scalar field under consideration, or whichever related field, such as dose rate. For the last few decades, visual inspection of the field \( \phi(r) \) or the adjoint \( \phi^*(r) \) and iso-value lines or surfaces has been the basis for the analysis to identify shielding deficiencies or to check requirements compliance. Starting in the region of interest, the analyst must identify visually the maximum rate of spatial variation of the field up to the source. That is the trajectory around which the shielding deficiencies are sought. The radiation shielding of the suspiciously weak areas is reinforced in iterative computations to check their actual influence in the field \( \phi(r) \). This is a trial-and-error approach. It is time-consuming, and often misleading and an optimum solution could be overlooked. To make the approach more efficient, we use one element of the vector calculus, interpreted physically: the field lines. Its use expands the nuclear analysis capabilities of the radiation scalar fields noticeably, as they help judging which aspects of the field \( \phi(r) \) deserve more attention for shielding improvement.

A vector field \( \mathbf{F}(r) \) can be obtained by computing the gradient, \( \mathbf{F}(r) = \nabla \phi(r) \). This field indicates, in every point, the maximum increment of the scalar field and its direction. The gradient is perpendicular to the iso-surfaces of \( \phi(r) \). Starting at any given point \( r \) of the field \( \mathbf{F}(r) \), one can trace a unique curve that is always tangent to the vectors of the field \( \mathbf{F}(r) \), and perpendicular to the iso-surfaces of the field \( \phi(r) \). It follows the gradient at every point, and in the positive direction it indicates the steepest ascent. These curves are used in other disciplines, like fluid-dynamics, being \( \mathbf{F}(r) \) the velocity field (the so-called “streamlines”). In this context we propose to call them “leakage lines.”

Followed in the positive sense, a leakage line represents the path with the maximum spatial rate of increment in the scalar quantity \( \phi(r) \). Leakage lines start at any point of interest and pursue the maximum scalar value around, until they reach a maximum of the function, either global (summit) or local (shoulder). The summit will be the most intense radiation source, while shoulders will be less intense sources or regions of intense scattering.
In maximizing the increment in every point, a leakage line will avoid the situations that attenuate the scalar quantity to the extent possible. Another way to interpret a leakage line is, then, as the path with the least attenuation of the scalar quantity that links a point of interest \( \mathbf{r} \) to the source. In fact, it traces the trajectory that the analyst looks for when conducting visual inspection of the scalar field \( \phi(\mathbf{r}) \). Thus, it supports the analyst to interpret the evolution of the field \( \phi(\mathbf{r}) \) in 3D in an automated errorless and effortless approach. It identifies cleanly critical aspects of the radiation field influencing at a position \( \mathbf{r} \), like the important penetrations producing streaming. Because radiation does not behave like a fluid, the leakage lines must be used with caveats to avoid misleading interpretations. The radiation particles do not follow the leakage line from the source to a given position \( \mathbf{r} \). Leakage lines must be judged together with other analysis tools, like the contour lines and gradient field \( \mathbf{F}(\mathbf{r}) \) and the scalar field \( \phi(\mathbf{r}) \). In combination, they help to deduce the positions at which blocking the radiation transmission will be the most effective shielding measure to reduce the scalar quantity in the position \( \mathbf{r} \).

The proposed physical interpretation for the leakage lines represents a powerful tool expanding the nuclear analysis capabilities for shielding design. Vector calculus results in a novel approach necessitated by the increasing complexity of the radiation field simulations that is affordable nowadays. In the next section an illustrative example is given.

4 | AN APPLICATION TO ITER-LIKE DEVICES

Let us consider the combination of the TC\textsuperscript{29} including the NB cell\textsuperscript{31} MCNP models, as shown in Figures 2 to 4. Note these models are obsolete by today, and are also missing hundreds of tons of equipment, so the results are not usable to elaborate on the ITER design. However, they are useful to provide a meaningful example. The mosaic source\textsuperscript{36} fed with information obtained with E-lite\textsuperscript{30} was considered. Further details on the methods to obtain the fields under discussion here are given in section 5.

A dummy meaningless scalar field \( \phi(\mathbf{r}) \) proportional to the neutron flux, expressed in arbitrary units was obtained. Let the region of interest be the wall shown in Figure 5. Visual inspection of the field shown in Figures 6 and 7 leads to two sorts of intuitive predictions and shielding proposal. With respect to Heating Neutral Beams (HNB) and Diagnostics Neutral Beam (DNB) apertures, the scalar quantity seems to have entered the NB lines, scattered inside and traveled along the busbar and the HV lines to the apertures in the wall. With respect to Cargo lift accesses, a relatively straight radiation propagation from the source to the accessed within each of the levels is the most plausible path, provided levels are separated by thick concrete slabs (between 0.8 and 1 m). According to this understanding, should the quantity be reduced in the regions of interest, shielding would be enhanced inside the NB lines as well as in the cargo lift walls at every level. This shielding scheme will be referred to as the scalar-based proposal.

In Figures 8 and 9 vector calculus supported analysis is shown. The gradient field and the leakage lines followed in the positive sense from the region of interest are shown. They indicate an intricate and counterintuitive interpretation. With respect to the HNB and DNB apertures in the North Wall, the leakage lines cross the wall to the HV deck, then they cross the slab to penetrate the NB cell. From that point, most pass through the empty balcony, while others enter the NB lines through the busbars. In relation to the cargo lift accesses in the wall, the leakage lines from all the levels enter the cargo lift and they all cross the cargo lift door at the HV deck.

**FIGURE 6** Spatial distribution of the dummy quantity in a transversal vertical plane. The yellow arrows indicated the suggested intuitive path of radiation leakage, while the scalar-based shielding proposal is shown in red [Colour figure can be viewed at wileyonlinelibrary.com]
(see Figure 3). They then cross the slab to the NB cell and finally pass through the empty balcony. In view of the leakage lines, shielding the balcony can be advantageous for all the regions of interest, while shielding the busbars and the cargo lift door can be of importance (major reductions) for the HNB and DNB apertures and the cargo lift accesses respectively. This shielding scheme (shown in Figure 10) will be referred to as the vector-based proposal. Note that both scalar-based and vector-based shielding proposals are noticeably different.

Both shielding proposals are analyzed in Table 1, where the shielding factor (i.e., the scalar quantity is reduced to this level compared to the unshielded value) of the quantity in the regions of interest after blocking those regions is shown. The scalar-based proposal leads to reductions from 47% to 64% for the cargo lift accesses and from 35% to 41% in the HNB and DNB apertures.

**Figure 7** Spatial distribution of a dummy quantity in different levels of the TC. The yellow arrows indicate the suggested intuitive path of radiation leakage, while the scalar-based shielding proposal is shown in red [Colour figure can be viewed at wileyonlinelibrary.com]

**Figure 8** Leakage lines starting in the HNB and DNB apertures and cargo lift accesses in the North Wall [Colour figure can be viewed at wileyonlinelibrary.com]

**Figure 9** In picture A, the gradient field $F(R)$ is shown in the central plane of the NB cell and HV deck. In picture B, the 3D leakage lines reaching HNB2 and isolines of the field $\phi(r)$ in the central plane. Note leakage lines are not computed in the plane but in the 3D field [Colour figure can be viewed at wileyonlinelibrary.com]
However, the vector-based proposal leads to reductions from 76% to 98% for the cargo lift accesses and from 75% to 80% in the HNB and DNB apertures, markedly better.

This example illustrates the portentous advantage in applying vector calculus to shielding analysis. Reaching the vector-based shielding proposal based only in the analysis of the scalar field $\phi(r)$ in its full extension, if possible, would have required expensive iterative runs blocking the suspicious aspects. However, vector calculus lead to a straightforward understanding of the weak shielding aspects immediately regardless of the complexity of the field, either in terms of geometry or source. Note that vector calculus is applied directly in the scalar field $\phi(r)$, agnostic of the methodology considered to compute the field and the nature of the radiation. Thus, it results of application in all the fields that involve the presence of radiation requiring mitigation.

5 | PROGRAMMABLE REALIZATION OF THE METHODOLOGY PRINCIPLE

All the simulations shown in this work have been conducted with D1S-UNED v3.1.3 code, an extension of MCNP5 with relevant enhancements to facilitate the simulation of geometries and sources as complex as those related to ITER.

The geometry in consideration in this work was produced by integrating the MCNP models of the ITER Tokamak Complex and the NB cell. They were assembled by nesting the NB cell in a dedicated universe inside the Tokamak Complex, letting the concrete of the TC model to prevail in any case. The resulting model is shown in Figures 2 to 4 of the main body.

The radiation source considered makes use of the so-called mosaic-source approach of SRC-UNED. A WSSA file computed with D1S-UNED v3.1.3 using an obsolete E-lite model. It corresponds to a version of the model with an obsolete NBI 80° sector in-bio-shield model, consistent with the geometry in consideration out-bio-shield and some other out-of-date representations of port plugs. It is shown in Figures 11 to 13. Nonetheless, it is still of full relevance for this work since we are not evaluating any particular ITER design.

In Figure 14 we show the neutron flux impinging in the cylindrical surface $R = 1355$ cm and $R = 1470$ cm, obtained in the same run as the WSSA file as a record. The corresponding statistical error is shown in Figure 15. The WSSA file was then processed to obtain Piecewise Probability Distributions with SRC-UNED which constitute the mosaic source.

D1SUNED v3.1.3 was used then to obtain the radiation field $\phi(r)$ shown in Figures 6 to 10 with this MCNP model and radiation source. The global variance reduction (GVR) technique was used, computing the weight

![Figure 10](https://example.com/figure10.jpg)  
**FIGURE 10** Leakage lines in the TC and shielding and vector-based shielding proposal shown in red [Colour figure can be viewed at wileyonlineibrary.com]

| Region of interest | Scalar-based shielding proposal | Vector-based shielding proposal |
|--------------------|---------------------------------|-------------------------------|
| B2 access          | 0.36                            | 0.24                          |
| B1 access          | 0.39                            | 0.22                          |
| L1 access          | 0.38                            | 0.14                          |
| L3 access          | 0.53                            | 0.02                          |
| DNB aperture       | 0.60                            | 0.23                          |
| HNB1 aperture      | 0.59                            | 0.25                          |
| HNB2 aperture      | 0.64                            | 0.22                          |
| HNB3 aperture      | 0.65                            | 0.20                          |
FIGURE 11  Neutron flux at surface \( R = 1355 \text{ cm} \) for B2 level and \( R = 1470 \text{ cm} \) for B1 to L3 levels, where the WSSA file was recorded [Colour figure can be viewed at wileyonlinelibrary.com]

FIGURE 12  Spatial distribution of the relative error in the neutron flux at surface \( R = 1355 \text{ cm} \) for B2 level and \( R = 1470 \text{ cm} \) for B1 to L3 levels were the WSSA file was recorded [Colour figure can be viewed at wileyonlinelibrary.com]

FIGURE 13  Cross-section view at plane \( PZ = -450 \text{ cm} \) of the 360° model of the ITER tokamak considered to record the WSSA [Colour figure can be viewed at wileyonlinelibrary.com]

FIGURE 14  Cross-section view at plane \( PZ = 62 \text{ cm} \) of the 360° model of the ITER tokamak considered to record the WSSA [Colour figure can be viewed at wileyonlinelibrary.com]
windows distribution using an MCNP model with densities diluted to 1/3, to promote the penetration of the radiation. A mesh tally with 1x1x1 m³ voxels was defined covering the whole MCNP model.

For the shielded cases described in Figures 6, 7 and 10, specific MCNP cells were allocated importance equal to zero to block the radiation transmission through them. This is equivalent to a perfect shield. To produce the results shown in Tables 1, 4 and 5, dedicated cells with dimensions specified in Table 2 were introduced in the inputs and the corresponding F4 cell tallies were defined. The number of histories and computational loads of each of the simulations is shown in Table 3. For convergence of these cell tallies, these three runs were computed with no variance reduction to promote a number of histories (NPS) as high as possible. Note they all have passed the 10 statistical tests of MCNP for convergence.

The results of the rectangular cell tallies are shown in Table 4, and their statistical errors in Table 5. The mesh

![Cross-section view at plane PZ = 550 cm of the 360° model of the ITER tokamak considered to record the WSSA](Colour figure can be viewed at wileyonlinelibrary.com)

**Table 2** Coordinates of the regions of interest to illustrate the advantage of the use of leakage lines

| Region of interest | X (cm)       | Y (cm)       | Z (cm)       |
|--------------------|--------------|--------------|--------------|
| B2 access          | [1750, 2270] | [4580, 4680] | [−1370, −850] |
| B1 access          | [1750, 2270] | [4580, 4680] | [−840, −325] |
| L1 access          | [1750, 2270] | [4580, 4680] | [−210, 310]  |
| L3 access          | [1750, 2270] | [4580, 4680] | [1315, 1840] |
| DNB line           | [570, 1065]  | [4580, 4680] | [1025, 1515] |
| HNB1 line          | [−305, 155]  | [4580, 4680] | [1245, 1695] |
| HNB2 line          | [−1430, −970]| [4580, 4680] | [1245, 1695] |
| HNB3 line          | [−2420, −1960] | [4580, 4680] | [1245, 1695] |

**Table 3** Estimated computational loads of the cases considered

| Case                  | # histories (NPS) | Computational load |
|-----------------------|-------------------|--------------------|
| Initial               | $1.2 \times 10^{11}$ | 270 000 cpu-hr     |
| Initial – (with GVR)  | $4.5 \times 10^{10}$ | 364 500 cpu-hr     |
| Scalar-based proposal | $1.4 \times 10^{11}$ | 360 000 cpu-hr     |
| Vector-based proposal | $1.2 \times 10^{11}$ | 270 000 cpu-hr     |

**Table 4** Values of the scalar quantity expressed in arbitrary units in the regions of interest in the initial and shielded cases

| Region     | Initial case | Scalar-based proposal | Vector-based proposal |
|------------|--------------|-----------------------|-----------------------|
| B2 access  | $2.21 \times 10^{-14}$ | $8.05 \times 10^{-15}$ | $5.21 \times 10^{-15}$ |
| B1 access  | $3.29 \times 10^{-14}$ | $1.28 \times 10^{-14}$ | $7.28 \times 10^{-15}$ |
| L1 access  | $6.48 \times 10^{-14}$ | $2.45 \times 10^{-14}$ | $9.26 \times 10^{-15}$ |
| L3 access  | $1.25 \times 10^{-13}$ | $6.63 \times 10^{-14}$ | $2.29 \times 10^{-15}$ |
| DNB line   | $2.28 \times 10^{-14}$ | $1.38 \times 10^{-14}$ | $5.15 \times 10^{-15}$ |
| HNB1 line  | $3.97 \times 10^{-13}$ | $2.33 \times 10^{-13}$ | $9.76 \times 10^{-14}$ |
| HNB2 line  | $3.61 \times 10^{-13}$ | $2.30 \times 10^{-13}$ | $8.11 \times 10^{-14}$ |
| HNB3 line  | $1.93 \times 10^{-13}$ | $1.25 \times 10^{-13}$ | $3.95 \times 10^{-14}$ |

**Table 5** Statistical error of the scalar quantity in the regions of interest in the initial and shielded cases

| Region     | Initial case | Scalar-based proposal | Vector-based proposal |
|------------|--------------|-----------------------|-----------------------|
| B2 access  | 0.038        | 0.063                 | 0.083                 |
| B1 access  | 0.032        | 0.050                 | 0.078                 |
| L1 access  | 0.021        | 0.032                 | 0.062                 |
| L3 access  | 0.016        | 0.020                 | 0.117                 |
| DNB line   | 0.050        | 0.060                 | 0.099                 |
| HNB1 line  | 0.010        | 0.013                 | 0.021                 |
| HNB2 line  | 0.011        | 0.013                 | 0.023                 |
| HNB3 line  | 0.015        | 0.018                 | 0.033                 |
tally was converted to vtr format and the statistical errors related to Figures 6 and 7 are shown in Figures 16 and 17. The vtr file contains the 3D spatial distribution of the radiation field $\varphi(r)$ with values averaged over voxels (cell data). They have been opened with Paraview V5.6.0 to produce all the plots shown in this work. With this program they were converted to point data, and the gradient field $F(r)$ and the leakage lines were computed.

The gradients field and the leakage lines shown in Figures 8 and 9 have been computed with Paraview as streamlines for 500 positions randomly sampled in a region with values of $\varphi(r)$ ranging from $1.5 \times 10^{-14}$ to $7.71 \times 10^{-13}$ and errors lower than 0.3, in the plane $Y = 4700$ cm. Parameters for the streamlines are indicated in Table 6.

### Table 6: Details considered to compute the streamlines

| Parameter                  | Value          |
|----------------------------|----------------|
| Integration direction      | Forward        |
| Integrator type            | Runge–Kutta 4–5|
| Integration step unit      | Cell length    |
| Initial step length        | 0.2            |
| Minimum step length        | 0.01           |
| Maximum step length        | 0.5            |
| Maximum steps              | 21 280         |
| Maximum streamline length  | 21 280         |
| Terminal speed             | $10^{-20}$     |
| Maximum error              | $10^{-6}$      |
6 | CONCLUSIONS

Modern techniques for nuclear analysis deal with unprecedented complexities for geometry and source modeling. Thanks to evolving High Performance Computing infrastructures, heavy simulations have become affordable. Thus, increasingly complex radiation fields are regularly obtained for shielding analysis in support to the design of nuclear fusion facilities. However, analysis techniques of the radiation fields have not evolved consequently.

Shielding analysis of complex radiation fields like those found in ITER nowadays is mostly based on visual inspection of scalar fields, on expensive iterative computational calculations with trial and error approach, commonly leading only to partial conclusions.

Vector calculus is proposed in this work as an improvement of the current nuclear analysis capabilities. In particular, a physical interpretation of the streamlines in the gradient vector field of a radiation scalar field allows identifying the paths with the least opposition to the propagation of the scalar quantity from the radiation source to any region of interest. These can be understood as paths around which shielding weakness must be sought. This means an automated and errorless interpretation of the complex 3D radiation field that can save human and computational resources.

The power of this technique is illustrated with an ITER relevant example. An MCNP model representing the Tokamak Complex and the NB cell, including over 5500 penetrations and multiple walls with diverse thickness is considered together with one of the most complex radiation sources ever considered for ITER nuclear analyses. It refers to the radiation impinging on the inner face of the bio-shield with full spatial resolution in 360°. A dummy scalar field proportional to the neutron flux is obtained to illustrate the proposal. Shielding analysis based on visual inspection and based on vector analysis have been proposed and analyzed. The proposal based on vector analysis showed to be straightforward to deduce and more efficient in reducing the radiation in the areas of interest.

Vector calculus is applicable to scalar radiation fields regardless of the methodology and the aim with which they were obtained. It provides the required evolution in the analysis capabilities to deal with the current sophistication in the determination of radiation fields.

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CONFLICT OF INTEREST

The authors declare no conflicts of interest.

AUTHOR CONTRIBUTIONS

This work was conceived by Rafael Juarez. Michael J. Loughlin, Antonio J. Lopez-Revelles, Patrick Sauvan and Javier Sanz have been involved in the understanding and the critical challenge of the leakage lines concept. Gabriel Pedroche and Aljaz Kolsek have been in charge of the computational aspects and simulations of the results illustrated. All authors worked together to interpret the results and to write the paper. Rafael Juarez was responsible for overall project direction, and Michael J. Loughlin conceived the ITER-related example to show the vector analysis relevance.

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