Numerical Study of Inflationary Preheating with Arbitrary Power-law Potential and a Realization of Curvaton Mechanism

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Abstract

During inflationary preheating, the energy stored in the inflaton field is rapidly converted into excitations of other entropy fields. This stage is characterized by exponential particle production due to parametric resonance and is notoriously difficult to analyze using analytic methods. We develop a detailed numerical simulation to investigate inflationary preheating when the potential of the inflaton is a power-law function with arbitrary power index. To achieve a successful graceful exit from a primordial inflationary phase to a smooth, oscillatory phase during reheating, we assume the inflaton potential reduces to a quadratic function in the infrared regime, which may be regarded as a mass term at low-energy scales. With this simplification, our numerical method may be applied to unconventional chaotic inflation models. To demonstrate its validity, we numerically analyze the preheating stage of axion-monodromy inflation, which is inspired by string theory. By performing perturbation analyses, our result shows that the power spectrum of primordial curvature perturbation can be dominated by fluctuations of entropy field rather than those of inflaton, which can be regarded as a particular realization of the curvaton mechanism through a preheating process.

Key words: cosmological parameters – cosmology: theory – inflation

1. Introduction

Reheating is a hypothetical mechanism used to generate a thermal phase of hot big bang expansion from an initially cold and empty primordial universe. During the reheating phase, energy is quickly transferred from a primordial matter field, namely the inflaton scalar field, to other matter fields through interaction terms in the Lagrangian. For inflationary cosmology, this process occurs after the stage of slow-roll inflation, but earlier than the time of thermal equilibrium. The energy transfer from the inflaton to matter field was initially analyzed using first-order perturbation theory and discussed in terms of decaying inflaton quanta into standard model particles (Abbott et al. 1982; Albrecht et al. 1982; Dolgov & Linde 1982).

It was realized that the coherence of the inflaton condensates at the beginning of the reheating phase can play an important role in the analyses with nonlinear effects (Traschen & Brandenberger 1990). Matter generation in the inflaton condensate was studied in a semi-classical analysis involving non-perturbative effects in Traschen & Brandenberger (1990). It was found that a parametric resonance instability plays a crucial role. Parametric resonance after inflation was further studied in Kofman et al. (1994), Shtanov et al. (1995), and the particle production process was analyzed in Kofman et al. (1997). In recent years, this topic has been extensively studied in the literature in the framework of inflationary cosmology (Boyanovsky et al. 1995; Baucke et al. 1997; Greene et al. 1997; Felder et al. 2001; Cormier et al. 2002; Duaux et al. 2006; Shuhmaher & Brandenberger 2006; Abolhasani et al. 2010) as well as bounce cosmology (Cai et al. 2011a, 2013; Quintin et al. 2014; de Haro & Cai 2015). We refer to Bassett et al. (2006), Allahverdi et al. (2010), and Amin et al. (2014) for comprehensive reviews.

While the reheating process can provide a dynamical mechanism for the origin of entropy and particles observed in our universe, details remain somewhat heuristic due to the existence of various uncertainties. For instance, the evolution of the equation-of-state (EoS) parameter strongly depends on the specific construction (Podolsky et al. 2006). Consequently, one way of probing information about reheating is to track the expansion of the universe between the moment the cosmic microwave background (CMB) scales cross the Hubble radius during inflation and the time they re-enter, which connects the energy scale of inflation with the reheating temperature directly. This method was extensively studied in the literature; see, for instance, (Liddle & Leach 2003; Martin & Ringeval 2006, 2010; Adshew et al. 2011; Mielczarek 2011; Easther & Petris 2012; Dai et al. 2014; Cai et al. 2015; Cook et al. 2015; Domcke & Heisig 2015; Martin et al. 2015; Drewes 2016; Lozanov & Amin 2017).

Nevertheless, such constraints on the reheating temperature strongly rely on measurements of primordial curvature perturbations and relic gravitational waves, from various high-precision experiments of CMB photons and their polarizations (Ade et al. 2016b; Aghanim et al. 2018a). So far these studies were based on the theoretical predictions of particularly chosen inflation models (Ade et al. 2016a; Akrami et al. 2018b). Moreover, it was observed that some nonlinear processes, namely parametric resonance effects, which could yield a potential amplification on the amplitudes of primordial density perturbations (Taruya & Nambu 1998; Bassett et al. 1999a, 1999b; Bassett & Vineigra 2000; Finelli & Brandenberger 2000; Tsujikawa & Bassett 2002; Brandenberger et al. 2008; Chambers & Rajantie 2008; Bond et al. 2009; Bethke et al. 2013; Bazrafshan Moghaddam et al. 2015, 2017;
McDonough et al. 2016; Svendsen et al. 2016; Graef et al. 2017; Gu & Brandenberger 2018), have not yet been taken into account in a self-consistent way. In particular, if these amplifications have an impact on curvature perturbations due to a conversion from isocurvature modes, the theoretical predictions for inflation models may be strongly altered or even completely destroyed (Finelli & Brandenberger 2000; Brandenberger et al. 2008; Bazrafshan Moghadam et al. 2015; McDonough et al. 2016). Therefore, it is necessary to numerically examine whether these amplifications occur in specific inflationary models and to revisit the corresponding predictions for CMB observations.

For those lattice codes of preheating such as LATTICEASY (Felder & Tkachev 2008), DEFROST (Frolov 2008), PSpectRE (Easther et al. 2010), and HLattice (Huang 2011), the cosmological system has been treated as a fixed comoving box. However, in the present study we would like to trace the evolutions of cosmological perturbations at super-Hubble scales so that we could examine whether these modes would continue the evolutions after the Hubble radius crossing. Accordingly, in this article we will use the C++ computer language to develop a new program code to perform the numerical simulation of the inflationary preheating phase, in which the potential of the inflaton is taken to be a power-law function with an arbitrary power index.

To ensure that the oscillatory behavior of the inflaton field around the bottom is continuous and smooth, we introduce a cutoff for the potential in the infrared regime, which behaves as a quadratic potential at low energy scales. The entire background evolution then becomes numerically trackable, even if the underlying inflation model has some unconventional potential forms with singular field derivatives at high energy scales. This treatment is particularly appropriate for a class of the so-called axion-monodromy inflation models inspired by string theory. Here we consider the ordinary chaotic inflation models with quadratic and quartic potentials, and then study particle production in models of power-law potentials with power index being 1, 2/3 and 1/2, respectively.

This article is organized as follows. In Section 2, we introduce the construction of the inflaton potential and introduce the background equations applied in the numerical simulations. In Section 3, we give a detailed description of the numerical method we developed to calculate the preheating process. In Section 4, we apply the code to several inflationary models to demonstrate the efficiency of the numerical method. In the same Section we first consider well known cases with a power index of the potential $p = 2$, to demonstrate the validity of the method; and then investigate some nonconventional models with $p = 1, 2/3, 1/2$ and $1/3$, which are inspired by models of axion-monodromy inflation. Finally, we summarize our results with a discussion in Section 6.

Throughout the article we adopt a metric signature $(−, +, +, +)$, use natural units with $c = \hbar = 1$, and use the reduced Planck mass $M_p = 1/\sqrt{8\pi G}$.

### 2. Model Construction

We begin with a homogeneous and isotropic universe, described by the (observationally favored, spatially flat) Friedmann–Lemaître–Robertson–Walker (FLRW) metric:

$$ds^2 = -dt^2 + a^2(dx^2),$$

where $a$ is the scale factor. Accordingly, the field equations of general relativity yield the Friedmann equations for the background:

$$H^2 = \frac{1}{3M_p^2}\rho, \quad \frac{\ddot{a}}{a} = -\frac{1}{6M_p^2}(\rho + 3p),$$

where $H \equiv \dot{a}/a$ is the Hubble parameter characterizing the expansion rate of the universe, and the dot represents differentiation with respect to cosmic time $t$. Here $\rho$ and $p$ denote energy density and pressure, respectively.

As matter sources, we take two canonical scalar fields, one responsible for driving a period of slow-roll inflation at a high energy scale, the other as an entropy field. The Lagrangian is

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - V(\phi, \chi),$$

with potential including interaction term

$$V(\phi, \chi) = \lambda M_p^4 \rho [(\phi^2 + \mu^2)^{p/2} - \mu^p] + \frac{1}{2}g^2\phi^2\chi^2.$$  

In the above, $\phi$ denotes the inflaton field and $\chi$ is an entropy field, which is assumed to be much lighter than the inflaton, and only excited during the preheating phase. The coefficients $\lambda$ and $g$ are the coupling constants. Here, $p$ is the model-dependent power-law index we discuss in later sections (see also Figure 1 for a sketch). In addition, we have introduced a very small mass term $\mu$ in order to ensure the potential behaves smoothly in the infrared regime (in particular, for circumstances when the values of $p$ are smaller than unity).

The background energy density and pressure are

$$\rho = \frac{1}{2} \sum_i \dot{\varphi}_i^2 + V, \quad p = \frac{1}{2} \sum_i \dot{\varphi}_i^2 - V,$$

Figure 1. Potential as a function of $\phi$. The blue line is the real inflaton potential $V(\phi)$. The green line plots $\phi^2$ and the orange one plots $\phi^3$. As shown in the figure, when $\phi > 10\mu$, $V \propto \phi^2$; while $\phi < 0.1\mu$, $V \propto \phi^3$. We adopt $p = 1/3$ as an example for illustration.
where we have ignored spatial gradient terms, and $\varphi_t$ represents the inflaton field $\phi$ and the entropy field $\chi$, respectively. The equations of motion for two fields can be derived by varying the Lagrangian with respect to the field variables, respectively:

$$
\ddot{\phi} + 3H\dot{\phi} - \frac{\nabla^2}{a^2}\phi + V_{,\phi} = 0,
$$

$$
\ddot{\chi} + 3H\dot{\chi} - \frac{\nabla^2}{a^2}\chi + V_{,\chi} = 0.
$$

(6)

The field perturbation in a spatially flat gauge is the Mukhanov-Sasaki variable

$$
Q_I = \delta\varphi_I + \frac{\varphi_I}{H}\psi,
$$

(7)

which is used here for analyzing the perturbation, including the metric perturbation $\psi$, where $H$ is the comoving Hubble parameter. The equations of motion of the perturbation fields are

$$
\ddot{Q}_I + 3H\dot{Q}_I - \frac{\nabla^2}{a^2}Q_I
+ \sum_J \left[ V_{,IJ} - \frac{1}{M^2}\frac{d}{dt}\left(\frac{a^3}{H}\dot{\varphi}_I\varphi_J\right)\right]Q_J = 0,
$$

(8)

where the comma denotes the derivative with respect to the field $\phi$ or $\chi$. After operating the Fourier transform to Equation (8), the partial derivative with respect to space coordinate, $-\nabla^2$, changes to the comoving wavenumber $k^2$ in Fourier space. The partial differential equations then become ordinary differential equations (ODE), simplifying computations.

During inflation, the contribution of the $\chi$ field is negligible because the $\chi$ energy density is completely diluted by the exponential expansion of the universe without fine-tuning parameters. Consequently, it is natural to take $\chi_I = \chi_I = 0$ as the initial conditions at the beginning of inflation. Following the traditional slow-roll paradigm, we introduce the slow-roll parameter

$$
\epsilon = \frac{M^2}{2}\left[ \frac{V(\phi, 0)^2}{V(\phi, 0)} \right],
$$

(9)

and inflation ends when $\epsilon = 1$. We set the initial conditions of the perturbations to be the Bunch-Davies vacuum. As the power index $p$ of the inflaton potential changes, the initial conditions of $\phi$ and $\dot{\phi}$ must change in order to achieve a sufficient number of e-foldings, $N \approx 50 \sim 60$. Furthermore, in order to ensure the potential behaves like $\phi^p$

$$
V \approx \lambda M_p^{4-p}[\phi^p - \mu^p],
$$

(10)

during the inflation, we require that $|\phi| \gg 10\mu$, restricting

$$
\dot{\phi}_I = 1 \gg 10\mu.
$$

(11)

Thus, for different $p$, we can yield different upper limits for $\mu$. Note that, during inflation, the effective mass of the inflaton is given by

$$
m_{\phi_{\text{eff}}}^2 = 2\lambda M_p^{4-p}\phi^p - 2.
$$

(12)

After inflation, the universe is dynamically driven by the oscillations of $\phi$ about the vacuum, which is called preheating. The energy transfers from the inflaton to the matter field, exciting relativistic particles. For models of axion-monodromy inflation with $p < 1$ as inspired by string theory, the first and second derivatives of the potential with respect to the inflaton would encounter a singularity at $\phi = 0$, this the treatment of these models would become tricky in detailed analyses during preheating. However, this technical trouble can be avoided in our model due to a smooth cutoff of the newly introduced $\mu$ term. In our model, when $|\phi|$ is smaller than 0.1$\mu$, the potential $V$ becomes

$$
V(\phi, \chi) = \lambda p p^{p-2} + \phi^2 + O(\phi^4).
$$

(13)

The effective mass of inflaton is

$$
m_{\phi_{\text{eff}}}^2 = p\lambda M_p^{4-p}p^{p-2},
$$

(14)

is constant, and can avoid the singularity at $\phi = 0$ when $p < 1$. The graceful exit from $\phi^p$ to $\phi^2$ results in the constant effective mass of the $\phi$ field, and efficient particle production during the preheating phase. During this time, the EoS parameter has the same value it has during matter domination, $s/\varepsilon \approx t^{2/3}$, $H = \frac{\pi}{\varepsilon} = \frac{2}{3t}$, and the equation of motion of the inflaton becomes

$$
\ddot{\phi} + \frac{2}{t}\dot{\phi} + m_{\phi_{\text{eff}}}^2 \phi = 0.
$$

(15)

This equation of motion is solved by

$$
\phi = \frac{\Phi_0}{m_{\phi_{\text{eff}}}t}\sin(m_{\phi_{\text{eff}}}t),
$$

(16)

where $\Phi_0$ is the coefficient when the inflaton enters the $V \sim \phi^2$ regime.

Since the potential is proportional to $\phi^2$ during preheating, the condition for parametric resonance is quite similar to that of pioneer work on $\phi^2$ inflation (Kofman et al. 1997). We rescale the matter field $\chi_k$ with $X_k = a^{1/2}\chi_k$, and rewrite the equation of motion:

$$
\ddot{X}_k + \omega_k^2 X_k = 0,
$$

(17)

where $\omega_k^2 = \frac{k^2}{a^2} + g^2\phi^2 - \frac{9}{4}H^2 - \frac{3}{2}\dot{H}$, the last two terms vanish as the EoS parameter approaches zero, and

$$
\omega_k^2 = \frac{k^2}{a^2} + \frac{g^2\phi_0^2}{m_{\phi_{\text{eff}}}^2 t^2} - \frac{g^2\phi_0^2}{m_{\phi_{\text{eff}}}^2 t^2}\cos(2m_{\phi_{\text{eff}}}t).\n$$

(18)

In the standard Mathieu function from McLachlan (1947) we have

$$
X_k'' + [A_k - 2g\cos(2\varepsilon)]X_k = 0,
$$

(19)

where $z = m_{\phi_{\text{eff}}}t$, a prime denotes the derivate with respect to $z$, and $A_k = \frac{k^2}{m_{\phi_{\text{eff}}}^2 a^2} + \frac{g^2\phi_0^2}{m_{\phi_{\text{eff}}}^2 a^2}q = g^2\phi_0^2 q_{\text{eff}}/2$. We use $q = g^2\phi_0^2 > 1$ to determine the broad resonance, where $\Phi$ is the initial amplitude. We see that when $\mu$ decreases, the effective resonance area decreases, and thus the lower limit for $g$ (i.e., $g > 2m_{\phi_{\text{eff}}}q/\Phi$) increases, indicating stronger coupling.

In order to numerically investigate the evolution of the matter field $\chi$ in this phase, we introduce a small, homogeneous
and isotropic perturbation $\delta \chi$ as the initial value for $\chi$ and $d\chi$ at the beginning of preheating. The background homogeneous part of the $\chi$ field, though very small, can be interpreted as the condensate matter field. The inhomogeneous part, $\chi_k$, can, in turn, be interpreted as the matter particles with different comoving wavenumbers excited by the scattering with the inflaton field. It is important to distinguish $\chi_k$ and the spatially flat field perturbation, the Mukhanov-Sasaki variable $Q_\chi$, which includes the metric perturbation in our code. The occupying number $n_k$ of the $\chi$ field is defined by $X_k = a^{3/2}\chi_k$ instead of $Q_\chi$,

$$n_k = \frac{\omega_k}{2} \left( \frac{|X_k|^2}{\omega_k^2} + |X_k|^2 \right) - \frac{1}{2},$$

and the comoving curvature perturbation

$$\mathcal{R} = H \frac{Q_\phi \dot{\phi} + Q_\chi \dot{\chi}}{\dot{\phi}^2 + \dot{\chi}^2},$$

where the curvature perturbation mainly depends on $Q_\phi$ and $Q_\chi$. Previously, people did not include the metric perturbation (namely, see DEFROST (Frolov 2008) and LATTICE EASY (Felder & Tkachev 2008)) when they considered the change of curvature perturbation caused by the matter field. Instead, they introduced the backreaction afterward. However, due to the improving ability to calculate, we are able to consider the metric perturbation at the beginning, which highly improves the precision.

3. Description of the Numerical Method

In this section, we present a detailed description of the numerical method we apply to the preheating analysis. Previous codes only considered the field fluctuations instead of including metric perturbations, which leads to an ambiguous definition between the curvature perturbation and the occupation number. In our code, we separate the field perturbation and the Mukhanov-Sasaki variable, making the definition clear. Before presenting the numerical method, we would like to mention that the linear perturbation theory captures the effects of particle production on the curvature perturbation only under the condition of the linear approximation, and when the linear perturbation theory breaks down, the lattice simulations are essential. In our concrete analysis, as the amplifications upon field fluctuations are expected to be under control, we limited our study within the linear approximation throughout the background evolution.

We write an MPI-version to run the evolution for different wavevectors in parallelization and get the power spectrum all at once. We write the code for inflationary preheating in the C++ computer language and hope to revisit the inflationary predictions of the $(n_s - r)$ contour plot. The numerical algorithm adopted in our code is based on the fourth-order Runge–Kutta approach, which is comparably precise and fast.

We introduce the application of the fourth-order Runge–Kutta method in Section 3.1 and discuss the structure of the code together with the initial values in Section 3.2.

3.1. The Fourth-order Runge–Kutta Method

Here we give a brief introduction to the fourth-order Runge–Kutta method for our ODE set, (see Equations (2), (6), and (8)). For a differential equation $\frac{dy}{dx} = f(x, y)$, the next step of $y_{n+1}$ is

$$y_{n+1} = y_n + \frac{h}{6} [K_1 + 2K_2 + 2K_3 + K_4],$$

$$K_1 = f(x_n, y_n),$$

$$K_2 = f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}hK_1),$$

$$K_3 = f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}hK_2),$$

$$K_4 = f(x_n + h, y_n + hK_3),$$

where $h$ is the time step. Using this method the numerical computation can converge very fast and remain high-precision.

3.2. The Structure of the Code

We first calculate the inflationary part. The matter field $\chi$ has almost diluted away, so it is appropriate to set $\chi = \chi_0 = 0$ during inflation. $\phi_f$ and $\dot{\phi}_f$ is determined by $N_{\text{inf}}$, the e-folding number, to ensure a theoretically viable phase of inflation. The Mukhanov-Sasaki variable $Q_\phi$ and $Q_\chi$ are introduced at the beginning of the inflation, where the Bunch-Davies vacuum gives the initial values for these two variables. We refer to Appendix B for a detailed description.

With the strong assumption that cosmological perturbations are not altered throughout the whole preheating phase, one can directly restrict the value of the model parameter $\lambda$ by applying observational data from CMB experiments. Namely, one can (roughly) set the e-folding number and the amplitude of the power spectrum of primordial curvature perturbations to be

$$N_{\text{inf}} \equiv \ln a_{\text{end}} - \ln a_1 \in [50, 60],$$

$$P_R = \frac{H^2}{8\pi^2\epsilon M_p^2} \approx 2 \times 10^{-9},$$

where $\epsilon$ is the slow-roll parameter. When $\epsilon \sim \mathcal{O}(1)$, inflation ends, and $\phi_{\text{end}}$ is used for determining $\mu$ by setting the upper limit $h_{\text{max}} = 0.1\phi_{\text{end}}$.

After inflation, we take the ending values of

$$(\phi, \dot{\phi}, Q_\phi, \dot{Q}_\phi, Q_\chi, \dot{Q}_\chi)_{\text{end}}$$

as the initial conditions of the preheating phase $((\phi, \dot{\phi}, Q_\phi, \dot{Q}_\phi, Q_\chi, \dot{Q}_\chi)_{\text{p}})$. For the $\chi$ field, we introduce the Bunch-Davies Vacuum as the initial value $\chi_{0}(k)$ during the preheating period (see Appendix A). The reason we do not let $\chi$ evolve through the entire process is due to its complete dilution due to the hierarchy between $\chi$ and $\phi$ during inflation. We determine the coupling constant $g$ with the semi-analytical method.

During preheating, the inflaton $\phi$ oscillates near $\phi = 0$ with the potential $V \sim \phi^2$, thus we can approximate $\phi \sim \Phi \sin(m_{\phi\text{eff}}t)$, where $\Phi$ is the amplitude and $m_{\phi\text{eff}} = \sqrt{pM_p^4 - \mu^2}$ is the effective mass during preheating. We take $\Phi \approx 0.1\mu$ for simplicity, where $0.1\mu$ represents the resonance area. The equation of motion of $X_k(t) = a^{3/2}(t)\chi_k(t)$ is

$$X_k + \left( \frac{k^2}{a^2} + g^2\Phi^2(t)\sin^2m_{\phi\text{eff}}t \right)X_k = 0,$$
where we have neglected the $(-\frac{2}{3}H^2 - \frac{2}{3}H)$ term because the background is matter-dominated. One can straightforwardly rewrite Equation (23) in the form of the Mathieu equation as follows:

$$X_k'' + (A_k - 2q \cos 2z)X_k = 0,$$

$$A_k = \frac{k^2/a^2}{m_{\phi_{\text{fl}}}^2} + 2q, \quad q = \frac{g^2\delta^2}{4m_{\phi_{\text{fl}}}^2} \approx \frac{g^2\mu^{4-p}}{400\lambda M_p^{4-p}}, \quad (24)$$

where the prime denotes the derivative with respect to $z = m_{\phi_{\text{fl}}}t$. The parameter $q$ is often used to distinguish the narrow resonance ($q \ll 1$) and the broad one ($q \gg 1$). The narrow resonance is inefficient and only suitable for the narrow band near $k \approx m_{\phi_{\text{fl}}}$. In the present article, we analyze $q \sim 1.5$, which induces broad resonance for all long-wavelength modes. These parameters are capable of efficiently producing matter particles, changing $P_R$, the power spectrum of curvature perturbation, by adding entropy perturbation and also restricting $P_R$ within the observational limit.

We would like to emphasize that our numerical analysis on cosmological perturbations shall be valid under the linear approximation. To be specific, in our detailed study the energy densities carried by cosmological perturbations are characterized by $\delta\rho_\phi$ and $\delta\rho_\chi$, which are always much smaller than the total energy density of the universe throughout the preheating process. Moreover, one may estimate that $\delta\rho_\phi/\rho_0 \lesssim 10^{-5}$ at the end of inflation due to the limit of CMB observations. Although during preheating the field fluctuation of the $\chi$ field could be amplified via parametric resonance as shown in the following analyses, the approximation of linear perturbation can be kept if the amplitudes of field fluctuations are not amplified by a factor larger than $10^5$. In the present study we attempt to illustrate that, even if cosmological perturbations remain under the linear approximation throughout the preheating process, their predictions from the standard paradigm of inflationary cosmology, which have been widely applied to confront CMB observations, would be severely altered due to the existence of parametric resonance upon the field fluctuations during preheating.

Because we focus on the linear perturbation, we use MPI to run parallel computation for different comoving wavenumbers. Choosing $10^{-4} \text{ Mpc}^{-1} < k < 10^{-1} \text{ Mpc}^{-1}$ and converting them into Planck units, we get the range for comoving wavenumber as follows:

$$k \in [1 \times 10^{-60}, 1 \times 10^{-57}] L_p^{-1}. \quad (25)$$

However, the observational range of $k$ is based on the fact that our current scale factor $a_0 = 1$. In our code, we set $a_0 = 1$ at the beginning of inflation for calculation convenience. To make $k/a$ in the same order with the observation, we need to adjust the range of $k$ in our code. Supposing the preheating temperature $T_{\text{pre}} \approx 10^{13} \text{ GeV}$ and from the relation

$$\frac{T_{\text{pre}}}{T_0} = \left(\frac{43}{13}\right)^{1/3}\frac{a_0}{a_{\text{pre}}} \quad \text{(Dai et al. 2014)},$$

we can estimate the ratio of $a_0$ to $a_{\text{pre}}$ by $10^{13} \text{ GeV}/10^{-4} \text{ eV} \sim 10^{26}$. The initial condition should be chosen to ensure $k/aH \gg 1$ at the beginning of inflation so that every wavevector is inside the Hubble radius. That is,

$$\frac{k_{\text{min}}}{e^{-N_i} e^{-N_{\text{re}}}} \gg H_{\text{ini}}, \quad (26)$$

where $N_{\text{re}}$ is the e-folding number during inflation, $N_{\text{re}}$ is the e-folding number from the start of preheating to today, and $H_{\text{ini}}$ is the initial Hubble parameter. Because we have already have $N_{\text{re}} = \ln(10^{26})$, we can use the above conditions to get the right range of wavenumbers for different cases of $p$.

Generally speaking, the scale factor would bump $e^{70} \sim 10^{30}$ during inflation no matter which $p$ we choose. Together with the additional 26 orders of magnitude after the inflation, the total amplification on the magnitude of the scale factor is about of order $O(10^{55})$. It is proper to choose the wavenumber range for our code to be $(10^{-3} L_p^{-1}, 10^{-2} L_p^{-1})$. In the following sections, we show the pivot scale, $k = 10^{-3} L_p^{-1}$, for illustration.

To test the precision, we use two different scale factor evolution methods. One is $\dot{a}/a = H = \sqrt{\rho/(3M_p^2)}$, and the other $\dot{a}/a = -(\rho + 3p)/(6M_p^2)$. They give equivalent results for the following sections.

4. Applications to Specific Examples of the Background Field Evolution

We now apply the numerical method introduced above to our potentials with different polynomial power laws. We begin with the classic preheating model (e.g., $p = 2$), which has been analyzed thoroughly (Kofman et al. 1997). From the plot, we will show that our method agrees with previous results. We then consider other cases (e.g., $p = 1/2, 1/3, 1/2, 1/3$) to analyze the process of preheating for small field inflation.

4.1. Regular Chaotic Inflation with $p = 2$

Following (Bassett et al. 2006), we plot the production of particles in for $p = 2$. When choosing the proper initial conditions, the preheating period is efficient, producing enough matter particles. Our result matches the work of Kofman et al. (1997), while they chose the mode $k = 0$, which can be understood as producing the condensate $\chi$ particles.

From Figure 2, we see that for the case of $p = 2$, the $\chi$ field can experience the resonance excitation in a chosen parameter band, and the physical particle number increases when $\chi$ is resonating. However, potentials with $\phi^2$ or $\phi^4$ are now significantly disfavored by the CMB data measured by the Planck satellite (Akrami et al. 2018b), motivating consideration...
of potentials with $p < 1$. The key point is to see whether there will be enough particle production and whether the curvature perturbation predictions are significantly changed.

Note that, from Figure 3 one can read that the ratio of the energy densities of two fields, $\rho_\chi / \rho_\phi$, would eventually approach unity but is still slightly less than unity. This is because, without taking into account backreaction effects, as well as other nonlinear effects, such as turbulence, the energy densities of two fields would evolve parallel to each other due to the fact that both fields would oscillate around their own vacuum points at the end of cosmological evolution.

4.2. Nonconventional Models with $p = 1, 2/3, 1/2, 1/3$

Now we apply the code to analyze the preheating process when $p < 1$. As has been discussed previously, we ensure a smooth transition from $\phi^p$ to $\phi^3$ by fixing the upper limit of $\mu$. As a result, we can make the approximation $\phi \sim \sin n_{\phi} \mu t$, which simplifies the analytical calculation of $\chi$ fields. It can be seen in the evolutions of EoS, which are shown in Figure 4. Different potentials ultimately reduce to the case of $(p = 2, w = 0)$ at the end.

It is well known that for $\phi^2$, the particle production is sufficient. As our potentials always come back to $\phi^2$, the particle production should also be sufficient. Indeed, our numerical results indicate that the $p < 1$ cases efficiently produce particles.

Figure 5 shows the $\chi$ field and occupying number in the case of $p = 1/2$. We can see that at the early time, the comoving particle number density increases while $\chi$ is resonating, and then remains unchanged in the comoving universe. The analysis recovers the results found in the $\phi^3$ case, as expected.

5. Applications to Specific Examples of the Perturbation and Power Spectrum

One of the most powerful observational results of the CMB data is the $(n_s - r)$ plot, which almost rules out $\phi^5$ and $\phi^3$ models. However, it is unfair to avoid considering the significant preheating process when approximating the spectral index $n_s$. The effect of the $\chi$ field, acting as an entropy field to unfreeze the curvature perturbation on super-Hubble scales, cannot be neglected. In this section, we analyze how the second field influences the curvature perturbation and how it may correct the predictions on the power spectrum of the single-field inflation.

5.1. Regular Chaotic Inflation with $p = 2$

We begin with an analysis of the standard quadratic model, $V \sim \phi^2$.

Figure 6 shows that the power spectrum of the double-field curvature perturbation is also nearly scale-invariant, like that of the single-field curvature perturbation; however, due to particle production, the magnitude of the curvature perturbation rises by nearly 3 orders of magnitude. In other parameter bands, this amplification will be even more significant and uncontrolled. This provides a new way to constrain the parameter space. Moreover, the analysis demonstrates that the curvature perturbation is not frozen after exiting the Hubble horizon and will keep evolving due to entropy perturbations. It is therefore inappropriate to use the single-field curvature perturbation when extra matter fields are introduced to induce efficient particle production during preheating. One ought to consider a combination of both fields when analyzing the CMB information from inflationary predictions, which will be addressed in a follow-up project.

It is interesting that, for a sizeable parameter space, as illustrated in the above numerical estimate, the power spectrum of primordial curvature perturbation remains nearly scale-invariant, but the amplitude can be altered significantly due to
the contribution of the fluctuations of the entropy field. This scenario coincides with the so-called curvaton mechanism (Mollerach 1990; Linde & Mukhanov 1997; Moroi & Takahashi 2001; Enqvist & Sloth 2002; Lyth & Wands 2002), which requires at least two primordial fields, one being the inflaton and the other being the curvaton field. When the curvaton is subdominant it only provides entropy perturbations during inflation (Lyth et al. 2003), and afterward these entropy perturbations can be converted to curvature perturbations through various dynamical processes (Sasaki et al. 2006; Easson et al. 2008; Huang 2008; Huang & Wang 2008; Cai & Xia 2009; Cai & Xue 2009; Kobayashi & Mukohyama 2009; Li et al. 2009; Cai & Wang 2010; Gong et al. 2010; Zhang et al. 2010; Cai et al. 2011b; Alexander et al. 2015; Addazi et al. 2018), so that the curvaton field can start to dominate the universe. For traditional curvaton models, the universe has to enter the standard thermal history after the curvaton decays. However, in our scenario, the curvaton mechanism is achieved by the preheating process.

5.2. Nonconventional Models with \( p = 1, 2/3, 1/2, 1/3 \)

Figure 7 shows that the double-field curvature perturbation is not frozen after exiting the Hubble horizon, unlike the single-field curvature perturbation. The peaks correspond to the particle production. The curvature perturbation has an overall increase on average.

Figure 8 is the case of \( p = 2/3 \) in which the second field makes such a significant contribution to lift the power spectrum that it cannot simply be neglected. At the late time of preheating, \( \delta \phi^2 \) dominates, so the power spectrum has results similar to those of the \( p = 2 \) model. The matter-field perturbation contributes significantly to the overall curvature perturbation. In general, the power spectrum of the double-field curvature perturbations for \( p = 1, 2/3, 1/2 \) and \( 1/3 \) all have an amplification of several magnitudes compared to that of single-field inflationary models. In this regard, our present study provides a concrete realization of the curvaton mechanism via the preheating process for a generic inflation model with an arbitrary power-law potential.

Our results demonstrate the significance of possible matter-field contributions to the curvature perturbation. The traditional \( (n_s - r) \) plot used to constrain inflationary models does not take into account the preheating process (largely due to uncertainties in the details of preheating mechanics). However, our analysis demonstrates that preheating may strongly affect the curvature perturbation, radically altering the predictions of single-field inflation models.

6. Conclusion and Discussions

In the present article, we developed a code based on the C++ computer language and MPI parallelization, aiming to generate a numerical simulation of the preheating process after inflation. Users can easily change the potential for the inflaton \( \phi \) and matter \( \chi \) fields. Here we have mainly focused on the non-perturbative particle producing process and its influences on the curvature perturbation.
We showed that the case of $p = 2$ corresponds to the $\phi^2$ model studied in the literature, as a viability test for the code. We then extended the analysis to the small $p$ case, where the extra term $\mu$ in our potential smooths the singularity. We find these $\phi^p$ models ultimately transition to the familiar $\phi^2$ model so that after choosing the proper parameters, particle production remains efficient. Because of this, our method can be used to study preheating in axion-monodromy inflationary models, by eliminating the singularity at the minimum of the potential, providing a smooth transition from the axion-monodromy like potential inflation to an ordinary $\phi^2$ potential, making the preheating process as efficient as the quadratic case.

Taking into account the effect of the matter-field fluctuations, the curvature perturbation can keep evolving at super-Hubble scales, which conflicts with the single-field curvature prediction. The extra field can introduce an entropy perturbation that acts as a source of the curvature perturbation, amplifying the power spectrum. The amplification can be large, leading to additional constraints on the parameter space. This can be regarded as a particular realization of the curvaton mechanism, in which the primordial power spectrum is dominated by the fluctuations of the curvaton field instead of those of the inflaton field. However, unlike the traditional curvaton paradigm, which requires a process of curvaton decay, in our model, the preheating phase itself serves this purpose. Our study demonstrates that the ordinary predictions for the $(n_s - r)$ plot, which are based on the single-field inflationary phase, may fail to capture crucial information due to the unavoidable preheating phase. We conclude that the preheating phase needs to ultimately be taken into account when we confront various inflationary models with cosmological observations at high accuracy. This fact calls for significant improvement in our fundamental understanding of the production of particles in the early universe. To gather more information connecting inflationary cosmology with cosmological observations, a detailed investigation on primordial gravitational waves during preheating process must be included, which has been extensively studied in the literature (Khlebnikov & Tkachev 1997; Easther & Lim 2006; Dufaux et al. 2007; Easther et al. 2007, 2008; Garcia-Bellido & Figueroa 2007; Garcia-Bellido et al. 2008; Price & Siemens 2008; Martin et al. 2014). We leave the extension of this analysis based on our numerical code for a follow-up study.

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### Appendix A

**Initial Condition for the Matter Field $\chi$**

We introduce the initial condition for the matter field $\chi$ at the end of the inflationary period, when $\epsilon = 1$. We choose the Bunch-Davies vacuum to be the initial condition for cosmological perturbations.

We rewrite the equation of motion of the matter field by eliminating the first-order derivative

$$X_k + \omega_k^2 X_k = 0,$$

where $X_k(t) = a^3(t) \chi_k(t)$; the $\omega_k$ is

$$\omega_k^2 = k^2 + g^2 \phi^2 - \frac{9}{4} H^2 - \frac{3}{2} H,$$  

$$= \frac{k^2}{a^2} + g^2 \phi^2 - \frac{3}{8 M_p^2} \dot{\phi}^2 - \frac{3}{4} V(\phi).$$

Thus, the Bunch-Davies vacuum in the rescaled coordinate is

$$X_k = \frac{1}{\sqrt{2 \omega_k}} e^{-i \omega_k t} = \frac{1}{\sqrt{2 \omega_k}} (\cos \omega_k t - i \sin \omega_k t),$$

and the derivative is

$$\dot{X}_k = \frac{-i \omega_k}{\sqrt{2 \omega_k}} e^{-i \omega_k t} = \left( \frac{-i \omega_k}{\sqrt{2 \omega_k}} \right) (\cos \omega_k t - i \sin \omega_k t)$$

$$= -\omega_k \sin \omega_k t - i \omega_k \cos \omega_k t,$$

so in the physical coordinate (real part) the corresponding vacuum is

$$\chi_k = a^{-3/2} \frac{1}{\sqrt{2 \omega_k}} e^{-i \omega_k t} = a^{-3/2} \frac{1}{\sqrt{2 \omega_k}} \cos \omega_k t,$$

where the derivative is

$$\dot{X}_k = \frac{-3}{2} a^{-5/2} \dot{a} \frac{1}{\sqrt{2 \omega_k}} e^{-i \omega_k t} + a^{-3/2} \frac{-i \omega_k}{\sqrt{2 \omega_k}} e^{-i \omega_k t}$$

$$= \frac{-3}{2} a^{-3/2} \dot{H} \frac{1}{\sqrt{2 \omega_k}} \cos \omega_k t - a^{-3/2} \frac{\omega_k}{\sqrt{2 \omega_k}} \sin \omega_k t$$

$$= -a^{-3/2} \frac{1}{\sqrt{2 \omega_k}} \left( \frac{3}{2} \dot{H} \cos \omega_k t + \omega_k \sin \omega_k t \right).$$

### Appendix B

**Initial Conditions for Cosmological Perturbations $Q_\phi$ and $Q_\chi$**

Similar to the former subsection, we rescale the equation of cosmological perturbations in comoving time

$$\dot{Q}_\phi + 3 H \dot{Q}_\phi + \frac{k^2}{a^2} Q_\phi + M_{\phi \phi} Q_\phi + M_{\phi \chi} Q_\chi = 0,$$

$$\Rightarrow Q''_\phi + 2 H Q'_\phi + k^2 Q_\phi + a^2 M_{\phi \phi} Q_\phi + a^2 M_{\phi \chi} Q_\chi = 0,$$

where the prime denotes the derivative with respect to the comoving time, $H$ is the Hubble parameter in comoving time, $M_{\phi \phi}$ is given by $V_{,\phi \phi} - \frac{8 \pi G}{a^4} \frac{\ddot{a}}{a} (\dot{a}^2 \dot{\phi}^2)$, and $M_{\phi \chi}$ is
\[ V_{\phi} \chi = - \frac{1}{2} \rho_{\phi} \frac{d}{dt} \left( \frac{a^2}{H} \phi \dot{\chi} \right). \]

Initially the matter field \( \chi \) is absent, hence \( M_{\phi \chi} = 0 \).

We introduce the canonical variable \( \nu_\phi = a Q_\phi \), and then derive the equation without a first-order derivative

\[ \nu''_\phi - 3 H \nu'_\phi - \omega^2 \nu_\phi + k^2 \nu_\phi + M_{\phi \nu_\phi} \nu_\phi = 0, \]

thus the Bunch-Davies vacuum for the perturbation of the Mukhanov-Sasaki variable takes \( \nu_\phi = a Q_\phi \rightarrow e^{-i \omega k / a} \), where

\[ \omega_k = k - \chi' - \chi^2 + M_{\phi \phi}, \]

and the derivative is

\[ \nu'_\phi = (a Q_\phi)' \rightarrow -i \sqrt{\frac{\omega_k}{2}} e^{-i \omega k/\chi}, \]

\[ = a' Q_\phi + a Q'_\phi = a^2 H Q_\phi + a^2 Q_\phi \rightarrow -i \sqrt{\frac{\omega_k}{2}} e^{-i \omega k/\chi}. \]

(36)

Accordingly, in physical scale, the initial condition for the perturbation takes

\[ Q_{\phi}^{\text{ini}} \rightarrow \frac{1}{a} e^{-i \omega k/\chi}, \]

and the associated derivative term is expressed as

\[ \dot{Q}_{\phi}^{\text{ini}} \rightarrow -i \frac{\omega_k}{a} e^{-i \omega k/\chi} - H_{\text{ini}} Q_{\phi}^{\text{ini}} \]

\[ = -i \frac{\omega_k}{a} e^{-i \omega k/\chi} - H_{\text{ini}} - \frac{1}{a} \sqrt{2 \omega_k} e^{-i \omega k/\chi} \]

\[ = -i \frac{\omega_k}{a} e^{-i \omega k/\chi} \left( -\frac{i \omega k}{a} - H_{\text{ini}} \right). \]

(38)

Both the real and imaginary parts should be taken into consideration.

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