Two Low Computational Complexity Improved Multiband-Structured Subband Adaptive Filter Algorithms

Mohammad Shams Esfand Abadi, John Håkon Husøy, and Mohammad Javad Ahmadi

Abstract

The improved multiband-structured subband adaptive filter (IMSAF) applies the input regressors at each subband to speed up the convergence rate of MSAF. When the projection order is increased, the convergence rate of the IMSAF algorithm improves at the cost of increased complexity. The present research introduces two new IMSAF algorithms with low computational complexity feature. In the first algorithm, the selective partial update approach (SPU) is extended to IMSAF algorithms and SPU-IMSAF is established. In SPU-IMSAF, the filter coefficients are partially updated at each subband for every adaptation. In the second algorithm, the set-membership (SM) strategy is utilized in IMSAF and SM-IMSAF is established. The SM-IMSAF has fast convergence rate, low steady-state error and low computational complexity features at the same time. Also, by combining SM and SPU methods, the SM-SPU-IMSAF is introduced. Simulation results demonstrate the good performance of the proposed algorithms.

Index Terms

Improved multiband-structured subband adaptive filter (IMSAF), selective partial update (SPU), set-membership (SM), convergence rate, computational complexity.

I. INTRODUCTION

Adaptive filters are utilized in many applications such as system identification, system inversion, signal prediction, and multisensor interference cancellation [1], [2], [3]. In these applications, the generated signals are processed

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to identify the characteristics of the unknown system. This aim is successfully achieved by using adaptive filters. The adaptive filters apply a recursive algorithm to design itself. The algorithm updates the weight coefficients through successive iterations and finally converges to the optimal Wiener-Hopf solution when signals are statistically stationary. The performance of an adaptive filtering algorithm is evaluated by the rate of convergence, misadjustment, and computational complexity features. The conventional least mean squares (LMS) adaptive filter algorithm has the advantage of being very simple; is easy to implement; and it has a very low computational complexity. However, when the input signal is highly colored, the LMS convergence slows down [3], [4].

To improve the convergence behavior of LMS, various adaptive algorithms such as the affine projection algorithm (APA) and multiband-structured subband adaptive filter (MSAF) were proposed [5], [6], [7], [8], [9], [10]. The APA is one of the important family of adaptive filter algorithm. Since the interplay between the computational complexity and the performance of adaptive signal processing systems is important [5], several types of affine projection algorithms have been proposed. For example, in selective partial update APA (SPU-APA), the filter coefficients are partially updated at each time iteration [11], [12], [13], [14], [15]. This algorithm has close performance to conventional APA. Also, the SPU-MSAF was proposed in [16], [17]. In SPU-MSAF algorithm, the filter coefficients were partially updated rather than the entire filter at every adaptation. Recently, the sign-regressor MSAF (SR-MSAF) and sign-error MSAF were introduced in [18] and [19]. In [18], the sign of input regressors is applied in update equation. The SR-MSAF was successfully extended to adaptive distributed network in [20].

There is another class of adaptive filter algorithms which have high convergence speed, low computational complexity, and low steady state error at the same time. These algorithms are established based on set-membership (SM) approach [21]. The SM normalized LMS (SM-NLMS) was introduced in [22]. The SM-APA and SM-MSAF were derived in [23] and [16], respectively.

To increase the convergence speed of MSAF, the improved MSAF (IMSAF) was developed [24], [25], [26]. This algorithm utilizes multiple input regressors in each subband during the adaptation. Therefore, the computational complexity of IMSAF increases. To reduce the computational complexity of IMSAF, two approaches were introduced in [27]. In the proposed algorithms, the input regressors were optimally selected at each subband during the adaptation. In selective regressor IMSAF (SR-IMSAF), this selection was fix and in dynamic SR-IMSAF (DSR-IMSAF), this selection was dynamic. This paper proposes two new solutions to reduce the computational load of the IMSAF algorithm. In the first approach, the SPU method is extended to IMSAF algorithm. In SPU-IMSAF, the filter coefficients are partially updated at each subband for every adaptation. The SPU-IMSAF has close performance to IMSAF. To have fast convergence speed, low steady-state error, and low computational complexity at the same time, the set-membership method is utilized in IMSAF and SM-IMSAF is proposed. Finally, by combination of
SM and SPU approaches, the SM-SPU-IMSAF is established.

This paper is organized as follows. In Section II, the NLMS, SPU-NLMS, and SM-NLMS algorithms are reviewed. Section III reviews the MSAF and IMSAF algorithms. Section IV introduces the SPU-IMSAF algorithm. The SM-IMSAF and SM-SPU-IMSAF algorithms are presented in Section V. The computational complexity of the proposed algorithms is discussed in Section VI. Finally, before concluding the paper, we demonstrate the usefulness of the introduced algorithms by presenting several experimental results.

Throughout the paper, the following notations are used:

- $|\cdot|$ Norm of a scalar.
- $\|\cdot\|^2$ Squared Euclidean norm of a vector.
- $\text{Tr}(\cdot)$ Trace of a matrix.
- $(\cdot)^T$ Transpose of a vector or a matrix.
- $E\{\cdot\}$ Expectation operator.

II. BACKGROUND ON NLMS, SPU-NLMS, AND SM-NLMS ALGORITHMS

Consider a linear data model for $d(n)$:

$$d(n) = x^T(n)w^o + v(n),$$

where $w^o$ is an unknown $M$-dimensional vector that we aim to estimate, $v(n)$ is the measurement noise with variance $\sigma_v^2$, and $x(n) = [x(n), x(n-1), \ldots, x(n-M+1)]^T$ denotes an $M$-dimensional input (regressor) vector. It is assumed that $v(n)$ is zero mean, white, Gaussian, and independent of $x(n)$. It is well known that the NLMS algorithm can be derived from the solution of the following optimization problem:

$$\min \|w(n+1) - w(n)\|^2$$

subject to

$$d(n) = x^T(n)w(n+1),$$

where, $w(n) = [w_0(n), w_1(n), \ldots, w_{M-1}(n)]^T$ is the vector of adaptive filter coefficients. Using the method of Lagrange multipliers to solve this optimization problem, the following update equation for NLMS algorithm is given by

$$w(n+1) = w(n) + \mu \frac{x(n)}{\|x(n)\|^2} e(n),$$

where $e(n) = d(n) - x^T(n)w(n)$ and $\mu$ is the step-size.
Now, partition the input signal vector and the vector of filter coefficients into \( B \) blocks each of length \( L \) (\( B = M/L \) and is an integer), which are defined as

\[
x(n) = [x_1^T(n), x_2^T(n), \ldots, x_B^T(n)]^T
\]

\[
w(n) = [w_1^T(n), w_2^T(n), \ldots, w_B^T(n)]^T.
\]

The SPU-NLMS algorithm for a single block update at every iteration minimizes the following optimization problem

\[
\min \|w_j(n+1) - w_j(n)\|^2,
\]

subject to (3), where \( j \) denotes the index of the block that should be updated. Again, by using the method of Lagrange multipliers, the update equation for SPU-NLMS is established as

\[
w_j(n+1) = w_j(n) + \mu \frac{x_j(n)}{\|x_j(n)\|^2} e(n),
\]

where \( j = \arg \max \|x_i(n)\|^2 \) for \( 1 \leq i \leq B \) [11].

The SM-NLMS algorithm minimizes (2) subject to \( w(n + 1) \in \Psi(n) \), where

\[
\Psi(n) = \{ w \in \mathbb{R}^M : |d(n) - x(n)^T w| \leq \gamma \}.
\]

This aim is achieved by an orthogonal projection of the previous estimate of \( w \) onto the closest boundary of \( \Psi(n) \). Doing this, the recursion for the SM-NLMS is obtained by [22]

\[
w(n + 1) = w(n) + \alpha(n) \frac{x(n)}{\|x(n)\|^2} e(n),
\]

where

\[
\alpha(n) = \begin{cases} 
1 - \frac{\gamma}{|e(n)|} & \text{if } |e(n)| > \gamma \\
0 & \text{otherwise} 
\end{cases}
\]

III. **BACKGROUND ON MSAF AND IMSAF ALGORITHMS**

Fig. 1 shows the structure of the MSAF [7]. In this figure, \( f_0, f_1, \ldots, f_{N-1} \) and \( g_0, g_1, \ldots, g_{N-1} \), are analysis and synthesis filter unit pulse responses of an \( N \) channel orthogonal perfect reconstruction critically sampled filter bank system. \( x_i(n) \) and \( d_i(n) \) are nondecimated subband signals. It is important to note that \( n \) refers to the index of the original sequences and \( k \) denotes the index of the decimated sequences (\( k=\text{floor}(n/N) \)). The decimated output signal is defined as \( y_{i,D}(k) = x_i^T(k)w(k) \) where \( x_i(k) = [x_i(kN), x_i(kN - 1), \ldots, x_i(kN - M + 1)]^T \) and \( w(k) = [w_0(k), w_1(k), \ldots, w_{M-1}(k)]^T \). Also, the decimated subband error signal is expressed as \( e_{i,D}(k) = d_{i,D}(k) - x_i^T(k)w(k) \). The filter update equation for MSAF can be established through the following cost function:

\[
\min \|w(k + 1) - w(k)\|^2,
\]

\(^1\)The set \( \Psi(n) \) is referred to as the constraint set, and its boundaries are hyperplanes. Also, \( \gamma \) is the magnitude of the error bound.
subject to $d_{i,D}(k) = x^T_i(k)w(k+1)$. Using Lagrange multipliers approach to solve this optimization problem leads to the filter coefficients update equation for MSAF as

$$w(k+1) = w(k) + \mu \sum_{i=0}^{N-1} \frac{x_i(k)}{||x_i(k)||^2} e_{i,D}(k).$$  (13)

The IMSAF minimizes (12), subject to $d_{i,D}(k) = X^T_i(k)w(k+1)$, where

$$X_i(k) = [x_i(k), x_i(k-1), \ldots, x_i(k-P+1)],$$  (14)

and

$$d_{i,D}(k) = [d_{i,D}(k), \ldots, d_{i,D}(k-P+1)]^T.$$  (15)

The parameter $P$ is the number of recent regressors. The IMSAF algorithm is derived from the solution of the following constraint minimization problem:

$$\Theta(k) = ||w(k+1) - w(k)||^2 + \sum_{i=0}^{N-1} \Lambda_i[d_{i,D}(k) - X^T_i(k)w(k+1)],$$  (16)

where $\Lambda_i = [\lambda_{i,1}, \lambda_{i,2}, \ldots, \lambda_{i,P}]$ is the Lagrange multipliers vector with length $P$. Using $\frac{\partial \Theta(k)}{\partial w(k+1)} = 0$ and $\frac{\partial \Theta(k)}{\partial \Lambda_i} = 0$, we get

$$w(k+1) = w(k) + \frac{1}{2} \sum_{i=0}^{N-1} X_i(k)\Lambda_i^T,$$  (17)

where

$$\Lambda_i^T = 2[X^T_i(k)x_i(k)]^{-1}e_{i,D}(k),$$  (18)

and

$$e_{i,D}(k) = d_{i,D}(k) - X^T_i(k)w(k).$$  (19)

Therefore, the update equation for IMSAF becomes

$$w(k+1) = w(k) + \mu \sum_{i=0}^{N-1} X_i(k)[X^T_i(k)x_i(k)]^{-1}e_{i,D}(k).$$  (20)

To take care of the possibility that $[X^T_i(k)x_i(k)]$ may be close to singular, it is replaced by $[\epsilon I + X^T_i(k)x_i(k)]$, where $\epsilon$ is the regularization parameter. Note that for $P = 1$, the conventional MSAF is established. Also, it is important to note that the relation in (18) is established when the cross-correlation between different sub-bands, $[X^T_i(k)x_j(k)]$, is ignored. This algorithm is called simplified IMSAF (SIMSAF) algorithm. In the following, we use IMSAF algorithm based on this assumption [27].
IV. THE SPU-IMSAF ALGORITHM

In this section, the SPU-IMSAF algorithm is introduced. In the SPU-IMSAF, the filter coefficients are partially updated rather than the entire filter at each subband for every adaptation. This strategy leads to the reduction in computational complexity. We establish the SPU-IMSAF algorithm for single and multiple blocks.

A. Single Block

In the SPU-IMSAF, the filter coefficients and the input signal matrix are partitioned into the $B$ blocks each of length $L$ ($B = M/L$ and is an integer) as

$$w(k) = [w_{1}^{T}(k), w_{2}^{T}(k), \ldots, w_{B}^{T}(k)]^{T},$$

and

$$X_{i}(k) = \begin{bmatrix} X_{i,1}(k) \\ X_{i,2}(k) \\ \vdots \\ X_{i,B}(k) \end{bmatrix}$$

where the $L \times P$ matrices $X_{i,b}(k)$ for $b = 1, 2, \ldots, B$ are given by

$$X_{i,b}(k) = \begin{pmatrix} x_{i}(kN - (b-1)L) & x_{i}(kN - (b-1)L - 1) & \cdots & x_{i}(kN - (b-1)L - P + 1) \\ x_{i}(kN - (b-1)L - 1) & x_{i}(kN - (b-1)L - 2) & \cdots & x_{i}(kN - (b-1)L - P) \\ \vdots & \vdots & \ddots & \vdots \\ x_{i}(kN - (b-1)L - L + 1) & x_{i}(kN - (b-1)L - L) & \cdots & x_{i}(kN - (b-1)L - L - P + 2) \end{pmatrix}.$$  

The SPU-IMSAF minimizes the following cost function

$$\Upsilon(k) = \min \|w_{b}(k + 1) - w_{b}(k)\|^{2},$$

subject to $X_{i}^{T}(k)w(k + 1) = d_{i,D}(k)$. Therefore, the cost function for the SPU-IMSAF is obtained as

$$\Upsilon(k) = \|w_{b}(k + 1) - w_{b}(k)\|^{2} + \sum_{i=0}^{N-1} \Lambda_{i}[d_{i,D}(k) - X_{i}^{T}(k)w(k + 1)],$$

where $\Lambda_{i} = [\lambda_{i,1}, \lambda_{i,2}, \ldots, \lambda_{i,P}]$ is the Lagrange multipliers vector with length $P$. Using $\frac{\partial \Upsilon(k)}{\partial w_{b}(k+1)} = 0$ and $\frac{\partial \Upsilon(k)}{\partial \Lambda_{i}} = 0$, we get

$$w_{b}(k + 1) = w_{b}(k) + \frac{1}{2} \sum_{i=0}^{N-1} X_{i,b}(k)\Lambda_{i}^{T},$$

(26)
and
\[ A_i^T = 2[X_{i,b}^T(k)X_{i,b}(k)]^{-1}e_{i,D}(k). \]  

(27)

By substituting (27) into (26), we obtain the SPU-IMSAF algorithm as
\[ w_{b}(k+1) = w_{b}(k) + \mu \sum_{i=0}^{N-1} X_{i,b}(k)[eI + X_{i,b}^T(k)X_{i,b}(k)]^{-1}e_{i,D}(k). \]  

(28)

It is important to note that the relation in (27) is established when the cross-correlation between different subbands, \([X_{i,b}^T(k)X_{j,b}(k)]\), is ignored. This phenomena is achieved for the filter banks with good band partitioning in different subbands. Now we turn our attention to determination of the block to be updated at each subband for every iteration. Based on (24), the selection of the block to be updated should be made by determining the block with the smallest squared Euclidean norm update. Therefore, by using (28), the block to be updated at each subband for every iteration is given by
\[ b = \arg\min_{\mathcal{B}} \|w_{b}(k+1) - w_{b}(k)\|^2 = \arg\min_{\mathcal{B}} \sum_{i=0}^{N-1} e_{i,D}^T(k)[X_{i,b}^T(k)X_{i,b}(k)]^{-1}e_{i,D}(k). \]  

(29)

Since, the computational complexity of (29) is high, the simplified approach to find the index of the block at each subband for every adaptation is proposed as [8], [11], [16]

1) Compute the following values for \(1 \leq b \leq B\)
\[ \sum_{i=0}^{N-1} \text{Tr}[X_{i,b}^T(k)X_{i,b}(k)]. \]  

(30)

2) The index \(b\) at each subband corresponds to the largest value of (30).

B. Extension to Multiple Blocks

Suppose we wish to update \(Q\) blocks out of \(B\) at each subband for every iteration. Let \(G_Q = [b_1, b_2, \ldots, b_Q]\) denote a \(Q\)-subset (subset with \(Q\) members) of the set \(\{1, 2, \ldots, B\}\). The SPU-IMSAF minimizes the following cost function
\[ \Upsilon_{G_Q}(k) = \min\|w_{G_Q}(k+1) - w_{G_Q}(k)\|^2, \]  

(31)

subject to \(X_{i,b}^T(k)w(k+1) = d_{i,D}(k)\), where
\[ w_{G_Q}(k) = [w_{b_1}(k), w_{b_2}(k), \ldots, w_{b_Q}(k)]^T. \]  

(32)

Therefore, the cost function is defined as
\[ \Upsilon_{G_Q}(k) = \|w_{G_Q}(k+1) - w_{G_Q}(k)\|^2 + \sum_{i=0}^{N-1} \Lambda_i[d_{i,D}(k) - X_{i,b}^T(k)w(k+1)]. \]  

(33)
Using $\frac{\partial \mathbf{Y}_{GQ}(k)}{\partial \mathbf{w}_{GQ}(k+1)} = 0$ and $\frac{\partial \mathbf{Y}_{GQ}(k)}{\partial \mathbf{A}_i} = 0$ lead to the following update equation

$$\mathbf{w}_{GQ}(k+1) = \mathbf{w}_{GQ}(k) + \mu \sum_{i=0}^{N-1} \mathbf{X}_{i,GQ}(k) [\mathbf{e} + \mathbf{X}_{i,GQ}^T(k) \mathbf{X}_{i,GQ}(k)]^{-1} \mathbf{e}_{i,D}(k),$$

(34)

where

$$\mathbf{X}_{i,GQ}(k) = \begin{bmatrix}
\mathbf{X}_{i,b_1}(k) \\
\mathbf{X}_{i,b_2}(k) \\
\vdots \\
\mathbf{X}_{i,b_Q}(k)
\end{bmatrix}.$$ 

(35)

The indices of the blocks are obtained according to the following condition

$$G_Q = \arg\min \| \mathbf{w}_{GQ}(k+1) - \mathbf{w}_{GQ}(k) \|^2 = \arg\min \sum_{i=0}^{N-1} \mathbf{e}_{i,D}^T(k) [\mathbf{X}_{i,GQ}^T(k) \mathbf{X}_{i,GQ}(k)]^{-1} \mathbf{e}_{i,D}(k).$$

(36)

Due to the high computational complexity of (36), the simplified approach to select the indices of $G_Q$ is proposed as [8], [11], [16]

1) Compute the following values for $1 \leq b \leq B$

$$\sum_{i=0}^{N-1} \text{Tr}[\mathbf{X}_{i,b}^T(k) \mathbf{X}_{i,b}(k)].$$

(37)

2) The indices of $G_Q$ at each subband correspond to the $Q$ largest values of (37).

Table I summarizes the SPU-IMSAF algorithm.

V. THE SM-IMSAF ALGORITHM

The SM-NLMS algorithm was introduced in [22] which had fast convergence speed, low steady-state error, and low computational complexity features. This idea was extended to APA and MSAF in [23] and [16]. To improve the performance of the IMSAF, we extend the SM method to IMSAF algorithm. First, we define for $\mathbf{x}_{i,D}(k)$ and $d_{i,D}(k)$ at time instant $k$, the constraint set $\mathcal{H}_i(k)$ containing all vectors $\mathbf{w}$ with estimation errors upper bounded in magnitude by $\gamma$ as

$$\mathcal{H}_i(k) = \{ \mathbf{w} \in \mathcal{R}^M : |d_{i,D}(k) - \mathbf{x}_{i,D}^T(k) \mathbf{w}| \leq \gamma \}. $$

(38)

The membership set $\Psi_i(k)$ is defined as

$$\Psi_i(k) = \mathcal{H}_i(1) \cap \mathcal{H}_i(2) \ldots \cap \mathcal{H}_i(k).$$

(39)

Since $\Psi_i(k)$ in (39) is not easily computed, adaptive approaches are needed. For example, in SM-MSAF, the information is provided by the constraint set $\mathcal{H}_i(k)$ [16]. The update equation for SM-MSAF algorithm was
introduced as
\[
\mathbf{w}(k+1) = \mathbf{w}(k) + \sum_{i=0}^{N-1} \alpha_i(k) \frac{x_i(k)}{||x_i(k)||^2} e_{i,D}(k),
\]  
(40)
where
\[
\alpha_i(k) = \begin{cases} 
1 - \frac{\gamma}{|e_{i,D}(k)|} & \text{if } |e_{i,D}(k)| > \gamma \\
0 & \text{otherwise}
\end{cases}.
\]  
(41)

In SM-MSAF, the filter coefficients are updated according to the condition in (41). If the condition was satisfied, the filter coefficients are updated. Therefore, the number of filter coefficients in update will be reduced. In the following, we derive the SM-IMSAF whose updates belong to a set formed by \(P\) constraint sets. Let define \(\Psi^P_i(k)\) which is the intersection of the last constraint sets as [23]
\[
\Psi^P_i(k) = \mathcal{H}_i(k - P + 1) \cap \ldots \mathcal{H}_i(k).
\]  
(42)
The objective is to derive an algorithm whose filter coefficients update belong to the last \(P\) constraint sets, \(\mathbf{w}(k+1) \in \Psi^P_i(k)\). The SM-IMSAF minimizes (12) subject to
\[
\mathbf{d}_{i,D}(k) - \mathbf{X}_i^T(k)\mathbf{w}(k+1) = \mathbf{g}_i(k),
\]  
(43)
where
\[
\mathbf{g}_i(k) = [g_i(k), g_i(k-1), \ldots, g_i(k - P + 1)]^T,
\]  
(44)
specifies the point in \(\Psi^P_i(k)\). All choices for \(\mathbf{g}_i(k)\) satisfying the bound constraint are valid. By using the method of Lagrange multipliers, the following cost function is introduced as
\[
\Omega(k) = \|\mathbf{w}(k + 1) - \mathbf{w}(k)\|^2 + \sum_{i=0}^{N-1} \Lambda_i[\mathbf{d}_{i,D}(k) - \mathbf{X}_i^T(k)\mathbf{w}(k+1) - \mathbf{g}_i(k)].
\]  
(45)
Solving this optimization problem leads to
\[
\mathbf{w}(k + 1) = \mathbf{w}(k) + \sum_{i=0}^{N-1} \mathbf{X}_i(k)[\mathbf{X}_i^T(k)\mathbf{X}_i(k)]^{-1}[\mathbf{e}_{i,D}(k) - \mathbf{g}_i(k)].
\]  
(46)
Eq. 46 is performed when \(|e_{i,D}(k)| > \gamma\) and otherwise, the filter coefficients don’t change. There are several choices for \(\mathbf{g}_i(k)\). A simplest choice is \(\mathbf{g}_i(k) = 0\). This approach leads to a considerable reduction in complexity in comparison with conventional IMSAF. Another choice is \(g_i(k) = \gamma \text{sgn}(e_{i,D}(k))\) [23]. In this case, the update equation is given by
\[
\mathbf{w}(k + 1) = \mathbf{w}(k) + \sum_{i=0}^{N-1} \mathbf{X}_i(k)[\mathbf{e} + \mathbf{X}_i^T(k)\mathbf{X}_i(k)]^{-1} \alpha_i(k) e_{i,D}(k) \mathbf{u}_1,
\]  
(47)
where \( \mathbf{u}_1 = [1, 0, 0, \ldots, 0]^T \) is \( P \times 1 \) vector and \( \alpha_i(k) \) is obtained by (41). Since (47) is related to the first element of the error vector at each subband, the following update equation for SM-IMSAF is introduced as

\[
\mathbf{w}(k+1) = \mathbf{w}(k) + \mu \sum_{i=0}^{N-1} \alpha_i(k) \mathbf{X}_i(k)[\mathbf{e} + \mathbf{X}_i^T(k)\mathbf{X}_i(k)]^{-1}\mathbf{e}_{i,D}(k),
\]  

where

\[
\alpha_i(k) = \begin{cases} 
1 - \frac{\gamma}{\|\mathbf{e}_{i,D}(k)\|_2} & \text{if } \|\mathbf{e}_{i,D}(k)\|_2 > \gamma \\
0 & \text{otherwise}
\end{cases}
\]  

(49)

In (48), the step-size controls the stability of the algorithm. Since \( \mathbf{e}_{i,D}(k) \) is the vector, we use an Euclidean norm operator (\( L_2 \)-norm) in (49). We compare the learning curves based on \( L_2 \)-norm and \( L_1 \)-norm of \( \mathbf{e}_{i,D}(k) \) in the simulation results section. Table II summarizes the SM-IMSAF algorithm. By combining the SPU and SM approaches, the SM-SPU-IMSAF can be established. The update equation for SM-SPU-IMSAF is proposed as

\[
\mathbf{w}_{GQ}(k+1) = \mathbf{w}_{GQ}(k) + \mu \sum_{i=0}^{N-1} \alpha_i(k) \mathbf{X}_{i,GQ}(k)[\mathbf{e} + \mathbf{X}_{i,GQ}^T(k)\mathbf{X}_{i,GQ}(k)]^{-1}\mathbf{e}_{i,D}(k).
\]  

(50)

In this algorithm, the filter coefficients are partially updated and the subbands are selected according to (49) at each iteration. Combining these strategies significantly reduces the computational complexity.

VI. COMPUTATIONAL COMPLEXITY

Table III compares the computational complexity of the IMSAF, SM-IMSAF, SPU-IMSAF, and SM-SPU-IMSAF algorithms in terms of the number of multiplications per iteration for real data. In this table, \( M \) is the filter length, \( N \) is the number of subbands, \( P \) is the number of input regressors, \( L \) is the length of channel filters, \( B \) is the number of blocks, \( Q \) is the number of selected blocks, and \( N(k) \) is the number of selected subbands according to (49) at iteration \( k \). As we can see, the number of multiplications in SPU-IMSAF is lower than IMSAF especially for large values of \( M \). In SM-IMSAF, the number of multiplications at each iteration is depend on the condition in (49). The parameter \( \alpha_i(k) \) determines that which subband is incorporated in update equation. In the worst case, the computational complexity of SM-IMSAF is the same as IMSAF. It means that all subbands will be selected at each iteration. In the simulation results section, we show that the computational complexity of SM-IMSAF is significantly lower than IMSAF. We also observe that SM-SPU-IMSAF has lower computational complexity than SM-IMSAF due to SM and SPU strategies.

VII. SIMULATION RESULTS

We demonstrate the performance of the proposed algorithm by several computer simulations in acoustic echo cancellation (AEC) setup. The impulse response of the car echo path with 256 taps (\( M = 256 \)) has been used as
an unknown system in the experiment (Fig. 2) [11]. The input signal is an AR(1) signal generated by passing a zero-mean white Gaussian noise with unit variance through a first-order system \( H(z) = \frac{1}{1 - 0.95 z^{-1}} \) and the value of \( \sigma_v^2 \) was set to \( 10^{-2} \). The filter bank used in the simulations was the extended lapped transform (ELT) [16], [28]. In all simulations, we show the normalized mean square deviation (NMSD), \( E\left[ \frac{\|w - w(k)\|^2_2}{\|w\|^2_2} \right] \), which is evaluated by ensemble averaging over 50 independent trials.

Since the exact IMSAF algorithm has large computational complexity [24], the simplified version of this algorithm is applied in the literature [27]. Therefore, we firstly compare the performance of IMSAF and simplified IMSAF (SIMSAF) algorithms in Fig. 3. The parameter \( P \) is set to 4 and two values for \( N \) are selected. The step-size is set to 0.5 for both values of \( N \). We observe that the SIMSAF has close performance to IMSAF algorithm. In the following, we use the same name (IMSAF) for both algorithms. Fig. 4 compares the NMSD learning curves of SPU-IMSAF algorithm (single block) based on Eqs. 29 and 30. The step-size is set 0.05 and the parameter \( N \) is set to 2, 4, and 8. Also, the parameters \( P \) and \( B \) are set to 4. We observe that the approximation relation (Eq. 30) has close performance to exact relation (Eq. 29). Fig. 5 presents the same results for multiple blocks situation with \( Q = 3 \). In this case, the step-size is set to 0.3. Again, close performance between the learning curves can be seen based on Eqs. 36 and 37 for different values of \( N \).

Fig. 6 presents the NMSD learning curves of IMSAF and SPU-IMSAF algorithms with \( N = 4 \). The parameters \( N \) and \( P \) are set to 4. In SPU-IMSAF, the number of blocks \( (B) \) is set to 4 and different values for \( Q \) are chosen. Also, the step-size in IMSAF is set to 0.5 and to make the comparison fair, the step-size for SPU-IMSAF algorithm is chosen to get approximately the same steady-state NMSD as IMSAF. The results show that by increasing the parameter \( Q \), the performance of SPU-IMSAF will be close to the IMSAF. For \( Q = 3 \), the similar performance between IMSAF and SPU-IMSAF is observed. It is important to note that for SPU-IMSAF with \( \frac{Q}{B} = 3 \), 128 and 192 coefficients out of 256 are updated.

In Figs. 7 and 8, we set the number of blocks \( (B) \) to 16. Fig. 7 shows the learning curve for different values of \( Q \). In this simulation, the step-size is set to 0.5. By increasing the parameter \( Q \), the performance of SPU-IMSAF will be close to IMSAF. The NMSD learning curves for the same steady-state error have been shown in Fig. 8. The step-size in IMSAF is set to 0.5 and to make the comparison fair, the step-size for SPU-IMSAF algorithm is chosen to get approximately the same steady-state NMSD as IMSAF. The values of the step-size for \( Q = 10, 12, 14, \) and \( 16 \) are 0.26, 0.33, 0.4, and 0.5, respectively. The results show that the performances of SPU-IMSAF algorithm for \( Q = 10, 12, \) and \( 14 \) are close to the conventional IMSAF algorithm. Fig. 9 compares the performance of the SPU-IMSAF algorithm for different values of \( B \) and \( Q \) where the ratio, \( \frac{Q}{B} \), is fix. In this case, the number of filter coefficients which is update at each iteration is 192. We observe close performance for all algorithms.
The performance of SPU-IMSAF for $B = 16$ and $Q = 12$ is slightly better than other curves. Fig. 10 presents the steady-state NMSD values versus $Q$ for different values of $N$. The parameters $B$ and $P$ were set to 4. Also, the step-size was set 0.1. As we can see, by increasing the parameter $Q$, the steady-state NMSD decreases. This observation can be seen for $B = 16$ in Fig. 11.

Fig. 12 shows the NMSD learning curves of SM-IMSAF algorithm based on $L_1$ and $L_2$ norms of $e_{i,D}(k)$. The parameters $P$ and $N$ are set to 4 and the step-size is set 0.5. We observe that the performance of SM-IMSAF based on $L_2$-norm of $e_{i,D}(k)$ is better than $L_1$-norm. This advantage can be seen in Fig. 13 for $N = 8$. Therefore, in the following we use $L_2$-norm of $e_{i,D}(k)$. Fig. 14 compares the performance of IMSAF and SM-IMSAF for $N = 4$ and 8. For IMSAF, two values for the step-size were chosen (0.1 and 0.5). In SM-IMSAF, the value for $\gamma$ was set to $\sqrt{5\sigma_e^2}$ [16], [23]. The results show that SM-IMSAF has better performance than IMSAF. Fig. 15 shows that when the filter coefficients in each subband ($i = 0, 1, 2, 3$) will be updated during the adaptation. This figure shows that in different iterations, we don’t need to update the filter coefficients which leads to the reduction in computational complexity. Fig. 16 shows these selections in the limited range of iterations. We clearly observe that the adaptation is not performed in different subbands.

Fig. 17 compares the performance of IMSAF with all proposed algorithms. This figure shows that the SPU-IMSAF has close performance to IMSAF. Also, the SM-IMSAF and SM-SPU-IMSAF algorithms have faster convergence speed and lower steady-state error than conventional IMSAF. Fig. 18 compares the performance of the proposed algorithms with SR-IMSAF and DSR-IMSAF algorithms in [27] for the same steady-state error. We observe that the SM-IMSAF has better convergence speed than other algorithms. Also, SM-SPU-IMSAF has better performance than IMSAF and close to the DSR-IMSAF algorithm. Table IV shows the total number of multiplications, the processing time, and the number of iterations until convergence based on Fig. 18. As we see, the computational complexity of the proposed algorithms is lower than IMSAF. The SM-IMSAF needs $3.88 \times 10^6$ multiplications. In the worst case, the number of multiplications of SM-IMSAF is the same as IMSAF. But, the performance of SM-IMSAF is better than other algorithms in both convergence speed and steady-state error features.

Figs. 19-21 show the NMSD learning curves of the proposed algorithms for different levels of SNR. The parameter $B$ is set to 4. The values of $N$ and $P$ are set to 2 and $Q$ is set to 3. Also, the step-size is set to 0.5. Fig. 19 presents the results for IMSAF and SPU-IMSAF. We observe that by decreasing the level of SNR, the steady-state error increases. The NMSD learning curves for IMSAF and SM-IMSAF have been shown in Fig. 20. Fig. 21 shows the results for IMSAF and SM-SPU-IMSAF. The same performance as Fig. 19 can be seen for both figures. Fig. 22 presents the tracking performance of IMSAF, SPU-IMSAF, SM-IMSAF, and SM-SPU-IMSAF. For tracking performance analysis, we consider a system to identify the two unknown filters with $M = 256$, whose z-domain
transfer functions are given by,

\[ W_1(z) = \sum_{n=0}^{127} z^{-n} - \sum_{n=128}^{M-1} z^{-n} \quad (51) \]

and

\[ W_2(z) = -\sum_{n=0}^{M-1} z^{-n}, \quad (52) \]

where the transfer function of optimum filter coefficients will be \( W_1(z) \) for \( n \leq 1700 \), and the transfer function of optimum filter coefficients will be \( W_2(z) \) for \( 1700 < n \leq 3400 \). The parameters \( N \) and \( P \) were set to 8 and 4, respectively. The NMSD learning curves show that the SM-IMSAF and SM-SPU-IMSAF algorithms have better tracking performance than IMSAF. Also, the SPU-IMSAF has close performance to conventional IMSAF. In Fig. 23, we presented the results for real speech input signal. The parameters \( N \) and \( P \) were set to 8 and the step-size was set to 0.5. Again, the SM-IMSAF and SM-SPU-IMSAF have faster convergence rate than IMSAF. Figs. 24 and 25 show when the adaptation is performed in SM-IMSAF and SM-SPU-IMSAF algorithms. These figures have been presented in the limited range of iterations for all subbands. Therefore, the computational complexity of the SM-IMSAF and SM-SPU-IMSAF is lower than IMSAF. Again, in the worst case, the computational complexity of SM-IMSAF is the same as IMSAF. But, the performance of SM-IMSAF will be better than conventional IMSAF algorithm.

Figs. 26 and 27 evaluate the stability bounds of the proposed algorithms. Fig. 26 shows the steady-state NMSD versus the step-size for SPU-IMSAF algorithm with \( B = 4 \). Different values for \( Q \) have been selected. For low values of \( Q \), the stability bounds are low. By increasing the parameter \( Q \), the stability bounds increase. In Fig. 27, the steady-state NMSD values versus the step-size for IMSAF, SPU-IMSAF, SM-IMSAF and SM-SPU-IMSAF algorithms have been presented. As we see, the SM-IMSAF has higher stability bound than the other algorithms. Table V shows the maximum values for the step-size (\( \mu_{max} \)) to guarantee the stability of the algorithms.

**VIII. CONCLUSION**

This paper proposed two new adaptive filter algorithms with low computational complexity feature. These algorithms utilized the SPU and SM approaches in IMSAF algorithm. In SPU-IMSAF, a subset of filter coefficients are optimally selected and updated at each subband for every iteration. The SM-IMSAF had fast convergence speed, low steady-state error, and low computational complexity features at the same time. Also, by combining SM and SPU approaches, the SM-SPU-IMSAF was introduced. We demonstrated the good performance of the proposed algorithms through several experiments.
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TABLE I
THE SPU-IMS AF ALGORITHM

1. Initialization the parameters
\[ \mu, \epsilon, N, P, B, Q, w(-1) = 0 \]
For \( k = 0, 1, \ldots \)

For \( i = 0, 1, \ldots, N - 1 \)
\[ w(k) = [w_1^T(k), w_2^T(k), \ldots, w_B^T(k)]^T \]
\[ X_i(k) = [X_{i,1}^T(k), X_{i,2}^T(k), \ldots, X_{i,B}^T(k)]^T \]
\[ d_{i,D}(k) = [d_{i,D}(k), d_{i,D}(k-1), \ldots, d_{i,D}(k-P+1)]^T \]
\[ e_{i,D}(k) = d_{i,D}(k) - X_i^T(k)w(k) \]

2. Determining the indices of \( G_Q = [b_1, b_2, \ldots, b_Q] \) according to \( Q \) largest values of (⋆)
For \( 1 \leq b \leq B \)
\[ \sum_{i=0}^{N-1} \text{Tr}[X_{i,b}^T(k)X_{i,b}(k)] \quad (⋆) \]
end

3. Update the input signal matrix according to the selected blocks
\[ X_{i,G_Q}(k) = [X_{i,b_1}^T(k), \ldots, X_{i,b_Q}^T(k)]^T \]

4. Select the blocks of filter coefficients according to the selected blocks
\[ w_{G_Q} = [w_{b_1}^T(k), w_{b_2}^T(k), \ldots, w_{b_Q}^T(k)]^T \]

5. Update the filter coefficients
\[ w_{G_Q}(k + 1) = w_{G_Q}(k) + \mu \sum_{i=0}^{N-1} X_{i,G_Q}(k)[\epsilon I + X_{i,G_Q}^T(k)X_{i,G_Q}(k)]^{-1}e_{i,D}(k) \]
end
TABLE II
THE SM-IMSAF ALGORITHM

1. Initialization the parameters
\[ \mu, \epsilon, N, P, \gamma, w(-1) = 0 \]

For \( k = 0, 1, \ldots \)
For \( i = 0, 1, \ldots, N - 1 \)
\[ X_i(k) = [x_i(k), x_i(k-1), \ldots, x_i(k-P+1)]^T \]
\[ d_{i,D}(k) = [d_{i,D}(k), d_{i,D}(k-1), \ldots, d_{i,D}(k-P+1)]^T \]
\[ e_{i,D}(k) = d_{i,D}(k) - X_i^T(k)w(k) \]

2. Determining the coefficients \( \alpha_i(k) \)
If \( \|e_{i,D}(k)\|_2 > \gamma \)
\[ \alpha_i(k) = 1 - \frac{\gamma}{\|e_{i,D}(k)\|_2} \]
Else
\[ \alpha_i(k) = 0 \]
End

3. Update the filter coefficients
\[ w(k+1) = w(k) + \mu \sum_{i=0}^{N-1} \alpha_i(k)X_i(k)[\epsilon I + X_i^T(k)X_i(k)]^{-1}e_{i,D}(k) \]
End

End

TABLE III
COMPUTATIONAL COMPLEXITY OF IMSAF, SM-IMSAF, SPU-IMSAF, AND SM-SPU-IMSAF ALGORITHMS PER ITERATION

| Algorithm         | Number of Multiplications                                      |
|-------------------|----------------------------------------------------------------|
| IMSAF             | \((P^2 + 2P)M + P^3 + P^2 + 3NL\)                             |
| SM-IMSAF          | \(\frac{N(k)}{N}((P^2 + 2P)M + P^3 + P^2 + 1) + P + 3NL\)     |
| SPU-IMSAF         | \(MP + (P^2 + 2P)QL + P^3 + P^2 + 3NL\)                      |
| SM-SPU-IMSAF      | \(P(M + 1) + \frac{N(k)}{N}((P^2 + 2P)QL + P^3 + P^2 + 1) + 3NL\) |
TABLE IV
TOTAL NUMBER OF MULTIPLICATIONS, PROCESSING TIME, AND THE NUMBER OF ITERATIONS UNTIL CONVERGENCE FOR IMSAF, SR-IMSAF, DSR-IMSAF, SPU-IMSAF, SM-IMSAF, AND SM-SPU-IMSAF ALGORITHMS UNTIL CONVERGENCE

| Algorithm                  | Number of multiplications | Processing time (sec) | Iterations until convergence |
|----------------------------|---------------------------|-----------------------|-----------------------------|
| IMSAF, $N = 4, P = 4, \mu = 0.1$ | $3.84 \times 10^7$        | 60.51                 | 1500                        |
| SR-IMSAF, $N = 4, P = 4, \mu = 0.1$ | $3.11 \times 10^7$        | 49                    | 1800                        |
| DSR-IMSAF, $N = 4, P = 4, \mu = 0.15$ | $1.44 \times 10^7$        | 22.69                 | 1300                        |
| SM-IMSAF, $N = 4, P = 4, \mu = 0.5$ | $3.88 \times 10^6$        | 6.11                  | 1000                        |
| SPU-IMSAF, $N = 4, P = 4, B = 4, Q = 3, \mu = 0.08$ | $3.7 \times 10^7$        | 58.3                  | 1900                        |
| SM-SPU-IMSAF, $N = 4, P = 4, B = 4, Q = 3, \mu = 0.34$ | $5.48 \times 10^6$        | 8.63                  | 1700                        |

TABLE V
STABILITY BOUNDS OF IMSAF, SPU-IMSAF, SM-SPU-IMSAF, AND SM-IMSAF ALGORITHMS

| Step-size | IMSAF | SPU-IMSAF, $Q = 1$ | SPU-IMSAF, $Q = 2$ | SPU-IMSAF, $Q = 3$ | SM-SPU-IMSAF, $Q = 3$ | SM-IMSAF |
|-----------|-------|-------------------|-------------------|-------------------|----------------------|-----------|
| $\mu_{max}$ | 0.873 | 0.12              | 0.39              | 0.857             | 0.868                | 1.14      |

Fig. 1. Structure of the MSAF algorithm.
Fig. 2. Impulse responses of car echo paths.

Fig. 3. The NMSD learning curves of IMSAF and simplified IMSAF (SIMSAF) for $N = 2$ and 4 ($M = 256, P = 4, \mu = 0.5$).
Fig. 4. The NMSD learning curves of SPU-IMSAF based on Eqs. 29 and 30.

Fig. 5. The NMSD learning curves of SPU-IMSAF based on Eqs. 36 and 37.
Fig. 6. The NMSD learning curves of IMSAF and proposed SPU-IMSAF algorithms for the same steady-state error ($M = 256, N = 4, B = 4$).

Fig. 7. The NMSD learning curves of IMSAF and proposed SPU-IMSAF algorithms ($M = 256, N = 4, B = 16, \mu = 0.5$).
Fig. 8. The NMSD learning curves of IMSAF and proposed SPU-IMSAF algorithms for the same steady-state error ($M = 256, N = 4, B = 16$).

Fig. 9. The NMSD learning curves of SPU-IMSAF with different values of $B$ and $Q$. 
Fig. 10. The steady-state NMSD versus $Q$ with different values of $N$ for SPU-IMSAF ($M = 256, B = 4, \mu = 0.1$).

Fig. 11. The steady-state NMSD versus $Q$ with different values of $N$ for SPU-IMSAF ($M = 256, B = 16, \mu = 0.1$).
Fig. 12. The NMSD learning curves of SM-IMSAF based on $L_1$-norm and $L_2$-norm of $e_{i,D}(k)$ ($M = 256, N = 4, \mu = 0.5$).

Fig. 13. The NMSD learning curves of SM-IMSAF based on $L_1$-norm and $L_2$-norm of $e_{i,D}(k)$ ($M = 256, N = 8, \mu = 0.5$).
Fig. 14. The NMSD learning curves of IMSAF and proposed SM-IMSAF algorithms ($M = 256, P = 4$ and $N = 4, 8$).

Fig. 15. Filter coefficients in update for SM-IMSAF in different subbands ($M = 256, N = 4, P = 4$).
Fig. 16. Filter coefficients in update for SM-IMSAF in different subbands in limited range of iterations ($M = 256, N = 4, P = 4$).

Fig. 17. The NMSD learning curves of IMSAF and proposed SPU-IMSAF, SM-IMSAF, and SM-SPU-IMSAF algorithms ($M = 256, N = 4, P = 4, B = 4, Q = 3$).
Fig. 18. The NMSD learning curves of IMSAF, SR-IMSAF, DSR-IMSAF, and proposed SPU-IMSAF, SM-IMSAF, and SM-SPU-IMSAF algorithms for the same steady-state error ($M = 256, N = 4, P = 4, B = 4, Q = 3$).

Fig. 19. The NMSD learning curves of IMSAF and SPU-IMSAF for different values of SNR.
Fig. 20. The NMSD learning curves of IMSAF and SM-IMSAF for different values of SNR.

Fig. 21. The NMSD learning curves of IMSAF and SM-SPU-IMSAF for different values of SNR.
Fig. 22. Tracking performance of IMSAF and proposed SPU-IMSAF, SM-IMSAF, and SM-SPU-IMSAF algorithms ($M = 256, N = 8, P = 4, B = 4, Q = 3$).

Fig. 23. NMSD learning curves of IMSAF and proposed SPU-IMSAF, SM-IMSAF, and SM-SPU-IMSAF algorithms for real speech input signal ($M = 256, N = 8, P = 8, B = 4, Q = 3$).
Fig. 24. Filter coefficients in update for SM-IMSAF in different subbands ($M = 256, P = 8, N = 8$).

Fig. 25. Filter coefficients in update for SM-SPU-IMSAF in different subbands ($M = 256, P = 8, N = 8$).
Fig. 26. Steady-state NMSD versus the step-size for SPU-IMSAF algorithm with different values of $Q$.

Fig. 27. Steady-state NMSD versus the step-size for IMSAF, SM-IMSAF, SPU-IMSAF, and SM-SPU-IMSAF algorithms.
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