Neutrino Oscillations as a Lepton-Flavor-Violating Interaction

W-Y. Pauchy Hwang

Asia Pacific Organization for Cosmology and Particle Astrophysics, Institute of Astrophysics, Center for Theoretical Sciences, and Department of Physics, National Taiwan University, Taipei 106, Taiwan

(24 July 2012; revised: August 15, 2013)

Abstract

To describe neutrino oscillations in the sense of quantum mechanics and quantum field theory, we propose to use an off-diagonal neutrino-Higgs (mass) interaction, as discussed originally in a family gauge theory and in the extended Standard Model. For neutrino oscillations which take place presumably between point-like Dirac particles, the proposed description would be unique in the quantum mechanics sense. This may help us to resolve a few outstanding puzzles - the question of why there are only three generations, the question of why the masses of neutrinos are so tiny, the question of why neutrinos oscillate, and the question of why the dark-matter world is so huge (25%) as compared to the visible ordinary-matter world (5%).

PACS Indices: 12.60.-i (Models beyond the standard model); 98.80.Bp (Origin and formation of the Universe); 12.10.-g (Unified field theories and models).

1 Why are neutrinos so interesting?

Neutrinos have masses, the tiny masses far below the range of the masses of the quarks and charged leptons. Maybe due to the non-zero masses, neutrinos oscillate among themselves, giving rise to a lepton-flavor violation (LFV). Neutrino masses and neutrino oscillations may be regarded as one of the most important experimental facts over the last thirty years [1].

Certain LFV processes such as $\mu \to e + \gamma$ [1] and $\mu + A \to A^* + e$ are closely related to the most cited picture of neutrino oscillations so far [1] - they also occur, however tiny, if neutrinos oscillate. In this note, I wish to point out that the cross-generation or off-diagonal neutrino-Higgs interaction may serve as the detailed mechanism of neutrino oscillations, with some vacuum expectation value (VEV) of the new Higgs field(s).

Presumably just like other building blocks of matter such as quarks and charged leptons, we could treat these neutrinos as point-like Dirac particles. Then, neutrino oscillations are fundamental and deep, certainly deeper than oscillations in other composite systems - such as oscillations in the $K^0 - \bar{K}^0$ system. Thus, it would be natural to describe the reaction as $i\eta \Psi \times \Psi \cdot \Phi$ with some VEV for the family Higgs field $\Phi$, where $\Psi$ and $\bar{\Psi}$ are family-triplet Dirac fields. Here the curl-dot product is to be explained later. The existence of this unique story for neutrino oscillations is amazing.

1 Correspondence Author; Email: wyhwang@phys.ntu.edu.tw; The second version of arXiv:1207.6443v1 [hep-ph] 27 Jul 2012.
For over half a century, we have the outstanding question why there are three generations in the minimal Standard Model \[2\]. And, for the last decade, another outstanding puzzle emerges that the dark-matter world is about five times the visual ordinary-matter world (the latter described by the minimal Standard Model). Besides the role in the minimal Standard Model, neutrinos may be able to tell us something in the dark-matter world which our neutrinos are capable of talking to (or interacting with).

Indeed, there is room left for something very interesting. Remember that the right-handed neutrinos never enter in the construction of the minimal Standard Model \[2\]. The message that the right-handed neutrinos seem to be "unwanted" could be telling us something. Now, the fact that neutrinos have tiny masses suggests that "more naturally" they would be four-component Dirac particles, and unlikely to be the two-component Majorana particles.

The room left for the right-handed neutrinos is that they are "unwanted" in the minimal Standard Model and that they could form some multiplet(s) under a new (dark-matter) gauge group besides the minimal Standard Model. We have some candidate from the symmetries - the family symmetry that there are three generations in the building blocks of (ordinary) matter, and so far only three. The puzzle so well-known that we no longer question ourselves why or why not! We have seen this fact, but we don’t know why - let’s speculate that it could be the story associated with the dark-matter world.

It arises naturally the so-called family gauge theory \[3\]. Note that the right-handed neutrinos do not appear in the minimal Standard Model. So, we could make a massive $SU_f(3)$ gauge theory completely independent of the minimal Standard Model, including the particle content. We could treat $(\nu_\tau R, \nu_\mu R, \nu_e R)$ as a triplet under this $SU_f(3)$ - so to give rise to a family gauge theory. This completes the derivation of the family gauge theory \[3\]. The $SU_f(3)$ is by definition the massive gauge theory - all the involved particles, except the neutrinos, are massive dark-matter particles.

2 Neutrino Oscillations as a Lepton-Flavor-Violating Interaction

So, the question becomes: Can we construct the overall consistent theory based on the group $SU_r(3) \times SU_L(2) \times U(1) \times SU_f(3)$, i.e., to add an extra $SU_f(3)$ to the minimal Standard Model?

The answer is an amazing "yes". The first step is to decide what our "basic units" (out of the building blocks of matter) are and how many they are. For instance, the right-handed neutrino triplet $(\nu_\tau R, \nu_\mu R, \nu_e R)$ (\equiv $\Psi_R(3,1)$ - $SU_f(3)$ triplet and $SU_L(2)$ singlet) would be a "basic unit". In Hwang and Yan \[4\], we assign three $SU_f(3)$ fermion triplets - $\Psi_L(3, 2)$, $\Psi_R(3, 1)$, and $\Psi_{CR}(3, 1)$ (charged). All quarks are singlets under $SU_f(3)$. As the major second step, we have to check whether the complicated Higgs mechanisms would work out. This is the "amazing" part of the story. In the extended Standard Model \[5\], we have three scalar/Higgs fields: the Standard-Model Higgs $\Phi(1, 2)$, the family Higgs triplet $\Phi(3, 1)$, and the mixed family-triplet and $SU_L(2)$-doublet scalar/Higgs $\Phi(3, 2)$. In the U-gauge, the Standard-Model Higgs picks out the neutral component (one degree of freedom), which in turn projects out the neutral components in $\Phi(3, 2)$ such that the neutral part
has the spontaneous symmetry breaking (i.e. the "projected-out Higgs mechanism") but
the charged part remains to be massive. The neutral part of Φ(3, 2) and the family Higgs
Φ(3, 1) make the eight gauge bosons very massive, leaving four real family Higgs.

Using this language \[5\], we can write the mass term of the neutrinos:

\[ i\eta\bar{\Psi}_L(3, 2) \times \Phi(3, 2) \cdot \Psi_R(3, 1) + h.c., \]

which is an off-diagonal matrix (in \(SU_f(3)\)). Although it is trivial, the operation does not
belong to the mathematics of the matrix. That is, \(\nu_e\) would transform into \(\nu_\mu\) or into \(\nu_\tau\),
\(\nu_\mu\) would into \(\nu_\tau\) or \(\nu_e\), and so on. This is interesting in view of neutrino oscillations, since
it could be regarded as the underlying interaction (mechanism) for neutrino oscillations
(which we are talking about \[1\]). An oscillation occurs in a way similar to the decay by way
of creating a new species plus the vacuum expectation value (or, changing the vacuum). In
quantum mechanics, this may be so far the only way how an oscillation can occur.

To illustrate the point further, we calculate the golden lepton-flavor-violating decay
\(\mu \rightarrow e + \gamma\) as the celebrated example. We show in Figures 1(a), 1(b), and 1(c) three leading
basic Feynman diagrams. Here the conversion of \(\nu_\mu\) into \(\nu_e\) is marked by a cross sign and it
is a term from our off-diagonal interaction given above with the Higgs vacuum expectation
values \(u_+\) and \(u_-\). Here the Higgs masses are assumed to be very large, i.e., greater than a
few \(TeV\), as in \(SU_f(3)\). The only small number (coupling) is \(\eta\), explaining the tiny masses
of neutrinos.

![Feynman diagrams](attachment:image.png)

Figure 1: The leading diagrams for \(\mu \rightarrow e + \gamma\).

Using Feynman rules from Wu and Hwang \[2\], we write, for Fig. 1(a),

\[
\frac{1}{(2\pi)^4} \int d^4q \cdot \bar{u}(p', s') \cdot i \gamma_\lambda (1 + \gamma_5) \\
\frac{1}{i m_2^2 + q^2 - i\epsilon} \cdot \frac{1}{2\sqrt{2} \sin\theta_W} \cdot i\eta \cdot \bar{\nu}_\mu \rightarrow \nu_e \cdot \frac{m_1 - i\gamma q}{i m_1^2 + q^2 - i\epsilon} \\
\cdot i \gamma_\lambda (1 + \gamma_5) \cdot u(p, s) \\
\]

\[
\frac{1}{2\sqrt{2} \sin\theta_W} \cdot i\eta \cdot \bar{\nu}_\mu \rightarrow \nu_e \cdot \frac{m_1 - i\gamma q}{i m_1^2 + q^2 - i\epsilon} \\
\cdot i \gamma_\lambda (1 + \gamma_5) \cdot u(p, s) \\
\]

3
and 1(c) in the treatment of the decays $\mu \rightarrow e + \gamma$, $\mu \rightarrow 3e$, and $\mu \rightarrow A + A^*$. Thus, one may ignore some important cancelation, if any.

Comparing this to the dominant mode $\mu \rightarrow e\bar{\nu}_e \nu_\mu$ [2], we could obtain the branching ratio. The decay rate for $\mu \rightarrow e + \gamma$, as would be obtained here, would be of the order $(m_{\text{neutrino}} \cdot m_\mu) / M_W^2$, which is extremely small.

The off-diagonal mass matrix would be modified by the self-energy diagram since the neutrinos form a triplet under $SU_f(3)$. It is presumed that these self-energy diagrams, after the ultraviolet divergences get subtracted, lead to masses of the right order.

The four family Higgs have to belong to two triplets - the neutral part of $\Phi(3,2)$ and the purely family Higgs $\Phi(3,1)$. If it is two-two divided such as that addressed in [3], then the situation would be as follows: If the off-diagonal mass matrix is diagonalized alone, the
three roots would be two negative and one positive, adding up to zero. This seems like one ordering in the masses of neutrinos - one up and two downs. Of course, it could be three-one divided as well.

Besides the golden decay $\mu \rightarrow e + \gamma$ (much too small) and neutrino oscillations (already observed), violation of the $\tau - \mu - e$ universality is also anticipated and might be observed. As the baryon-antibaryon asymmetry is sometime attributed to the lepton-antilepton asymmetry, the current scenario for neutrino oscillations [11] seems inadequate for this purpose. If we take the hints from neutrinos rather seriously, there are so much to discover, even though the minimal Standard Model for the ordinary-matter world would, by and large, remain to be intact.

Of course, the Standard-Model Higgs has now been discovered. The direct search for the family Higgs and the massive family bosons in the $TeV$ range would be too costly. So, the searches for some lepton-flavor-violating decays and for violation of the $\tau - \mu - e$ universality would be alternative for the moment.

To sum up, if we treat neutrinos as "point-like" Dirac particles, the curl-dot product as in Eq. (1) would be the way to go. It is the way to connect neutrino oscillations with the lepton-flavor-violating decays or reactions. The curl-dot products are not the matrix operations (in the mathematics sense); it represents a new way to introduce renormalizable interactions and so expands the horizon of quantum field theory.

3 Further Thoughts

We believe that, in the dark-matter world, the dark-matter particles are also species in the extended Standard Model. Most of reactions happening among dark-matter particles, even involving neutrinos, cannot be detected in the ordinary-matter world. It is clear that the minimum extended Standard Model would be the extended Standard Model to be based on the group $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$, since this model is rather unique (and economical). The issue is whether our Standard Model would close up our world - that is, all particles in our world are accounted for.

In a slightly different context [6], I propose to work with two working rules: "Dirac similarity principle", based on eighty years of experience, and "minimum Higgs hypothesis", from the last forty years of experience. Using these two working rules, the extended model mentioned above becomes rather unique - so, it is so much easier to check it against the experiments. The close-up question of our world may have to be answered, provided that the two working rules, or similar, are assumed.

We would be curious about how the dark-matter world looks like, though it is difficult to verify experimentally. The first question would be: The dark-matter world, 25 % of the current Universe (in comparison, only 5 % in the ordinary matter), would clusterize to form the dark-matter galaxies, maybe even before the ordinary-matter galaxies. The dark-matter galaxies would then play the hosts of (visible) ordinary-matter galaxies, like our own galaxy, the Milky Way. Note that a dark-matter galaxy is by our definition a galaxy that does not possess any ordinary strong and electromagnetic interactions (with our visible ordinary-matter world). This fundamental question deserves some thoughts, for the structural formation of our Universe.
Of course, we should remind ourselves that, in our ordinary-matter world, those quarks can aggregate in no time, to hadrons, including nuclei, and the electrons serve to neutralize the charges also in no time. Then atoms, molecules, complex molecules, and so on. These serve as the seeds for the clusters, and then stars, and then galaxies, maybe in a time span of 1 Gyr (i.e., the age of our young Universe). The aggregation caused by strong and electromagnetic forces is fast enough to help giving rise to galaxies in a time span of 1 Gyr. On the other hand, the seeded clusterings might proceed with abundance of extra-heavy dark-matter particles such as familons and family Higgs, all greater than a few TeV and with relatively long lifetimes (owing to very limited decay channels). So, further simulations on galactic formation and evolution may yield clues on our problem.

Finally, coming back to the fronts of particle physics, neutrinos might couple to the dark-matter particles. Any further investigation along this direction would be of utmost importance. It may shed light on the nature of the dark-matter world and, eventually, we would be able to close up our world.

This research is supported in part by National Science Council project (NSC 99-2112-M-002-009-MY3). We wish to thank the authors of the following books [2] for thorough reviews of the minimal Standard Model.

References

[1] Particle Data Group, "Review of Particle Physics", J. Phys. G: Nucl. Part. Phys. 37, 1 (2010).

[2] Ta-You Wu and W-Y. Pauchy Hwang, "Relativistic Quantum Mechanics and Quantum Fields" (World Scientific 1991); Francis Halzen and Alan D. Martin, "Quarks and Leptons" (John Wiley and Sons, Inc. 1984); E.D. Commins and P.H. Bucksbaum, "Weak Interactions of Leptons and Quarks" (Cambridge University Press 1983). This was "my" early list of the textbooks on the "Standard Model".

[3] W-Y. Pauchy Hwang, Nucl. Phys. A844, 40c (2010); W-Y. Pauchy Hwang, International J. Mod. Phys. A24, 3366 (2009); the idea first appeared in hep-ph, arXiv:0808.2091; talk presented at 2008 CosPA Symposium (Pohang, Korea, October 2008), Intern. J. Mod. Phys. Conf. Series 1, 5 (2011); plenary talk at the 3rd International Meeting on Frontiers of Physics, 12-16 January 2009, Kuala Lumpur, Malaysia, published in American Institute of Physics 978-0-7354-0687-2/09, pp. 25-30 (2009).

[4] W-Y. Pauchy Hwang and Tung-Mow Yan, arXiv:1212.4944v1 [hep-ph] 20 Dec 2012 and the slightly modified version.

[5] W-Y. Pauchy Hwang, arXiv:1304.4705v1 [hep-ph] 17 Apr 2013.

[6] W-Y. P. Hwang, arXiv:11070156v1 (hep-ph, 1 Jul 2011), Plenary talk given at the 10th International Conference on Low Energy Antiproton Physics (Vancouver, Canada, April 27 - May 1, 2011), unpublished.