On the structure of cosmic ray electron-positron fluxes at the GeV-TeV energy range

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Abstract

We reveal that the energy spectra of electrons-positrons in primary cosmic rays measured at atmosphere top have double structures: an excess component \( \Phi^+_{e^+} (E) = \Phi^-_{e^-} (E) \) around 400 GeV, which origins from a strong \( e^+ e^- \)-source and the distorted background \( \Phi_{e^-}^0 (E) \). We proved that the difference between AMS-CALET and Fermi-LAT-DAMPE data origins from the energy loss of the fluxes due to the anomalous bremsstrahlung effect. Their evolutions satisfy an improved electromagnetic cascade equation. The above spectra are parameterized and they can regard as the subjects exploring new physical signals. We suggest to re-examine the previous applications of the BH formula in the study of the propagation of electrons in interstellar medium.

**keywords:** Cosmic rays; Anomalous bremsstrahlung effect; New physical signals
The precise measurements of cosmic ray electron-positron fluxes by Alpha Magnetic Spectrometer (AMS) [1,2], Fermi Large Area Telescope (Fermi-LAT) [3], DArk Matter Particle Explorer (DAMPE) [4] and Calorimetric Electron Telescope (CALET) [5] present a complex structure of the energy spectra at the GeV-TeV energy range. In particular, AMS reports that the positron flux has a significant excess peaked at $\sim 300 \text{ GeV}$ (or $\sim 400 \text{ GeV}$ if subtracting the contribution of the diffuse background), while the electron flux exhibits a single power law on the same energy range. On the other hand, DAMPE observed a break of the total electron and positron spectrum at $\sim 700 \text{ GeV}$. A confusing problem is that the data of AMS and CALET are noticeably lower than that of DAMPE and Fermi-LAT in the above mentioned energy band (Fig. 1). Obviously, if the cause of this difference is not clarified, any research based on these data will lose their credibility.

Both AMS and CALET set on the international space station at $\sim 400 \text{ km}$ height, while Fermi-LAT and DAMPE are orbiting the Earth at $500 \sim 560 \text{ km}$ altitude. Charged particles may interact with atoms in atmosphere and form the electromagnetic shower. In the normal condition the radiation length $\lambda \simeq 37 \text{ g/cm}^2$, it corresponds to a real distance $X_0 = \frac{\lambda}{\rho} \simeq 4 \times 10^{15} \text{ cm}$ since the atmospheric density $\sim 10^{-14} \text{ g/cm}^3$ at $400 \sim 500 \text{ km}$ altitude. The altitude difference $100 \text{ km}$ is only $1/100000000 X_0$. Therefore, according to the traditional electromagnetic radiation theory, the difference of the above energy spectra can’t be attributed to the electromagnetic shower.

One of the basis for the electromagnetic shower theory is bremsstrahlung. When charged particles scatter off electric field of proton or nucleus, they can emit real photons. This is bremsstrahlung (braking radiation). A quantum-mechanical description of the bremsstrahlung emission is the famous Bethe-Heitler (BH) formula, which is proposed in 1934 [6] and it has been widely applied in many branches of physics and astrophysics. In the previous works, one of us (WZ) pointed out that the BH formula should be modified
Figure 1: Cosmic electron-positron spectra multiplied by $E^3$ as a function of energy. Data are taken from [1-5]. A and B indicate the spectra at height $\sim 500$ and $\sim 400$ km. The thin solid curves are the source fluxes, the dashed curves are the background and the thick solid curves are their sum. All curves from A to B satisfy the improved cascade equation with 0.13 radiation length.
as a new form in thin ionosphere, where the integrated bremsstrahlung cross section has an unexpected big increment, which is large enough to cause significant reduction of the electron and positron fluxes in the range of 100 km [7]. Based on this discovery, an improved cascade equation for electromagnetic shower is established in Ref. [7].

In this letter we use these results to analyze the measured data of electron-positron fluxes. We find that there is a same excess source in both electron- and positron-fluxes, which is peaked at $\sim 400 \text{ GeV}$. On the other hand, the background of electrons has the broken power form: from $10 \text{ GeV}$ to $400 \text{ GeV}$ the spectrum obeys a single power law, after $400 \text{ GeV}$ the spectrum is hardened somewhat, then it again becomes steep (see the dashed curves in Fig. 1). The evolutions of the above fluxes from set A to set B satisfy quantitatively the prediction of the improved cascade equation.

We consider the electromagnetic cascades of electrons-positrons specifically in the atmosphere of the Earth from height $\sim 500 \text{ km} - 400 \text{ km}$, where the oxygen atoms are not only completely ionized, but the gas density is extremely thin. The nuclear Coulomb potential may penetrate into such a broader space, where the bremsstrahlung cross section will restore the big geometric cross section. Thus, one can get a large enough increment of the cross section, and this result can be used to explain the difference of the spectra in Fig. 1.

We divide the measured electron-positron spectra $\Phi_{e^+e^-}$ in Fig. 1 into two sets: A (DAPLE-Fermi-LAT) and B (AMS-CALET). We begin from the AMS data about positron spectrum. The background spectrum $\Phi_{e^+}^0(E)$ of positrons is described at the lower energy range in the diffuse model. Therefore, we can extract the background $\Phi_{e^+}^{0,B}(E)$ (or $\Phi_{e^+e^-}^{0,B}(E)$) using the difference between the measured spectra $\Phi_{e^+e^-}^s(B)$ and $\Phi_{e^+}^s(B)$.

As same as Ref. [2] we define the source as the predictions of dark matter model [8] or gluon condensation (GC) model [9], where $\Phi_{e^+}^s = \Phi_{e^-}^s$; all remaining components are
regarded as the background $\Phi_{e^+}^0$ and $\Phi_{e^-}^0$. According to this classification, the asymptotic forms of $\Phi^s(E)$ at $E \to 0$ and $\infty$ have a same power index, while $\Phi^0(E)$ has not such symmetry, i.e, $\Phi_{e^+}^0 \neq \Phi_{e^-}^0$.

AMS has parameterized the positron flux as [2]

$$\Phi^{s,B}_{e^+}(E) = \frac{E^2}{E^2} \times 6.8 \times 10^{-5}(\hat{E}/60)^{-2.58} \exp(-\hat{E}/813),$$

(1)

where $\hat{E} = E + 1.1$, (the units see Ref. [2]). Its shape is described by the thin curve B in Fig. 1. The background $\Phi^{0,B}_{e^++e^-}(E)$ is obtained by subtracting $2\Phi^{s,B}_{e^+}(E)$ from the data of $\Phi^{B}_{e^++e^-}(E)$. We find that $\Phi^{0,B}_{e^++e^-}(E)$ presents a single power law $\sim E^{-0.283}$ till $E \sim 300 - 400$ GeV, then it deviates from this form at more higher energy. However, the large errors in the data of $\Phi^{B}_{e^++e^-}(E)$ at $E > 400$ GeV hinder the extraction of the information. We noticed that the DAMPE data for $\Phi^{A}_{e^++e^-}(E)$ have a break around $E \sim 700$ GeV. If set A and set B are connected by the electromagnetic cascade, the flux $\Phi^{0,B}_{e^++e^-}(E)$ will roughly keep the shape in a short cascade length ($t \ll 1$). Therefore, we have the following form

$$\Phi^{0,B}_{e^++e^-}(E) = \begin{cases} 
431E^{-0.283} & E < 400 \text{ GeV} \\
108E^{-0.053} & 400 \text{ GeV} \leq E \leq 700 \text{ GeV} \\
12500E^{-0.783} & E > 700 \text{ GeV}
\end{cases}$$

(2)

A similar "knee"-structure was appeared in energy spectra of cosmic rays of all particles at more higher energy range [10]. One can find that $\Phi^{B}_{e^++e^-}(E) = 2\Phi^{s,B}_{e^+}(E) + \Phi^{0,B}_{e^++e^-}(E)$ (the thick curve in Fig. 1) is consistent with the data. This result shows that $\Phi^{B}_{e^++e^-}(E)$ has a double structure: a new source creating high-energy electron-positron pair and the electron background, the later is distorted in propagation.

Now we derive the structure of electron and positron fluxes at $\sim 500$ km using the improved cascade equation. The elemental high-energy processes that make up electromagnetic cascade are pair production and bremsstrahlung [10]. We concern the energy
spectra of electrons and positrons at high energy ($E > 10$ GeV) in this work. Therefore, the contributions of $\gamma \rightarrow e^+ + e^-$ can be neglected. We denote $X$ and $\lambda$ as the depth and the radiation length in unity $g/cm^2$. The contributions of $\gamma \rightarrow e^+ + e^-$ to the electromagnetic showers at the high energy band can be neglected. The flux of electron-positron $\Phi_e (e = e^+, e^-)$ satisfies the following simplified cascade equation

$$
\frac{d\Phi_e(E, t)}{dt} = \int_{E}^{\infty} \frac{dE'}{E'} P_{e \rightarrow e} \left( \frac{E}{E'} \right) \Phi_e(E', t) - \Phi_e(E, t) \int_{0}^{1} dz P_{e \rightarrow e}(z).
$$

(3)

Where $P_{e \rightarrow e}(z) dt dz$ is the probability for an electron/positron of energy $E'$ reduces to energy $zE'$ after radiation in traversing $dt = dX/\lambda$ [7],

$$
P_{e \rightarrow e}(z) = \frac{3}{4} \frac{1 + z^2}{1 - z^2}.
$$

(4)

The cascade processes have not the reverse solutions. We take a series of input spectra, which like $\Phi_{e^+}^{s,B}(E)$ and $\Phi_{e^+ e^-}^{0,B}(E)$ but with different parameters. Then comparing the solutions at a given cascade step $t$ with known fluxes (1) and (2). Note that the anomalous bremsstrahlung effect is happened at high energy range, while the BH formula is valid at lower energy, where the anomalous bremsstrahlung effect disappears. We use a simple function $\chi(E) = 0.5 \log(E/1 \text{ GeV}) - 0.5$ at $(10 \text{ GeV} < E < 1000 \text{ GeV})$ to connect the solutions between low and high energy ranges, i.e.,

$$
\Phi_{e^+}^{s,B}(E) \equiv \Phi_{e^+}^{s,A}(E) - \chi(E)[\Phi_{e^+}^{s,A}(E) - \Phi_{e^+}^{s,B}(E)],
$$

(5)

and

$$
\Phi_{e^+ e^-}^{0,B}(E) \equiv \Phi_{e^+ e^-}^{0,A}(E) - \chi(E)[\Phi_{e^+ e^-}^{0,A}(E) - \Phi_{e^+ e^-}^{0,B}(E)],
$$

(6)

where $\Phi_{e^+}^{s,B}(E)$ and $\Phi_{e^+ e^-}^{0,B}(E)$ are the solutions of the cascade equation. Finally, we find following forms (Fig. 1)
\[ \Phi_{e^+,A}^s(E) = \frac{E^2}{E^2} 7.4 \times 10^{-5} (\hat{E}/60)^{-2.54} \exp(-\hat{E}/813), \quad (7) \]

and

\[ \Phi_{e^+,e^-}^{0,A}(E) = \begin{cases} 
389E^{-0.238} & E < 400 \text{ GeV} \\
125 E^{-0.050} & 400 \text{ GeV} \leq E \leq 700 \text{ GeV} \\
1369 E^{-0.693} & E > 700 \text{ GeV}
\end{cases}, \quad (8) \]

All cascade steps from curves A to B take \( t = 0.13 \). We point out that the applications of the anomalous bremsstrahlung effect can’t extend to infinite space since the restriction of detector resolution. According to \( t = 0.13 \) we estimate that the limit scale \( r_{\text{max}} \sim 10^{-5} \text{cm} \). Comparing with \( r \sim 10^{-3} \text{cm} \), which is an average scale per atom in the complete ionosphere about 400 ~ 500 km height, we conclude that where the anomalous effect of bremsstrahlung and pair creation has been saturated.

Note that a modified formula \( \chi(E) \) can improve the agreement between the curve A and the experimental data at \( E < 100 \text{ GeV} \). Although \( \chi(E) \) is a phenomenological formula, the improved cascade equation dominates the spectrum structure at the key energy range \( 100 \text{ GeV} < E < 1 \text{ TeV} \). The \( \chi^2 \) test comparing the described (or predicted) and observed electron+positron flux is given by

\[ \chi^2 = \sum_i \frac{(\Phi_{i}^{\text{des/pre}} - \Phi_{i}^{\text{obs}})^2}{\sigma_{\text{sys},i}^2 + \sigma_{\text{stat},i}^2}, \quad (9) \]

where \( i \) runs over the energy range \( 100 \text{ GeV} < E < 1 \text{ TeV} \). As an input we have \( \chi^2 = 3.1 \) for the described flux in set B by AMS-02. We obtain \( \chi^2 = 20.6 \) (or \( \chi^2/d.o.f. = 20.6/16 \), note there is only one free parameter \( t = 0.13 \) in the solution of Eq. (3)) if comparing the predicted and observed fluxes in set A, the later is reported by DAMPE.

Thus, we recognize that the data of set A and set B reflect a same double structure. The parameterized formulas (7) and (8) are closer to the real signals and they can be regarded as the objects exploring new physics. We predict that the anomalous effect
of bremsstrahlung and pair creation may arise a big enhancement of the increasing cross section at the thin ionized gas. If this conclusion is correct, the energy spectrum of cosmic-ray electrons above $\sim 600 \text{ km}$ altitude will be obviously higher than the Fermi-LAT and DAMPE data, although the effect will be reduced with decreasing gas density since it has been saturated at $r \sim 10^{-5}\text{cm}$.

In summary, primary cosmic ray electrons and positrons enter to the top of atmosphere, their energy spectra may changed by an anomalous bremsstrahlung effect. This leads to a complex structure of the measured spectra. Using an improved electromagnetic cascade equation we proved the existence of the anomalous effect in bremsstrahlung and pair creation, and find that the resulting spectra appear a double structure. All explorations based on AMS-CALET-Fermi-LAT-DAMPE data should take into account this double structure. We also suggest to re-examine the previous applications of the BH formula in the study of the propagation of electrons in interstellar medium.

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