Minimal cut-off vacuum state constraints from CMB bispectrum statistics

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In this short note we translate the best available observational bounds on the CMB bispectrum amplitudes into constraints on a specific scale-invariant New Physics Hypersurface (NPH) model of vacuum state modifications, as first proposed by Danielsson, in general models of single-field inflation. As compared to the power spectrum the bispectrum constraints are less ambiguous and provide an interesting upper bound on the cut-off scale in general models of single-field inflation with a small speed of sound. This upper bound is incompatible with the power spectrum constraint for most of the parameter domain, leaving very little room for minimal cut-off vacuum state modifications in general single-field models with a small speed of sound.

INTRODUCTION

Bounds on higher-order non-Gaussian contributions to the observed and approximately Gaussian temperature anisotropy map of the Cosmic Microwave Background are rapidly entering a phase where they can be used to constrain and sometimes even rule out certain (exotic) models of inflation. For instance, bounds on the amplitude of the CMB three-point function, or bispectrum, have already narrowed down the parameter range of string inspired Dirac-Born Infeld models of inflation.

In a separate development it was recently shown that modifications to the inflationary vacuum state can also result in significantly enhanced non-Gaussian signals \cite{1-4}. In particular the bispectrum can reveal enhanced features that essentially arise due to interactions between excited quanta in the modified vacuum. The power spectrum is also sensitive to deviations from the Bunch-Davies vacuum, but there the corrections are suppressed by the Bogoliubov parameter, whose magnitude is small and typically related to the inflationary Hubble parameter divided by the scale of new physics, which is already constrained to be smaller than $10^{-2}$ \cite{5}. For the bispectrum one instead encounters an additional enhancement factor, independent of the Bogoliubov parameter, that is in fact proportional to the scale where effective field theory is supposed to break down. The appearance of this enhancement factor can be traced back to the New Physics Hypersurface (NPH) used to define the modified vacuum state. The higher the cut-off scale representing new physics, the more time the fields have before they cross the horizon and freeze, and the more time the interactions effectively have to generate a significant non-Gaussian contribution. This also implies that the (decoupling) limit of taking the cut-off scale to infinity in a modified vacuum is in general ill-defined, producing an infinite bispectrum, whereas the power spectrum yields the standard result in this decoupling limit (assuming the Bogoliubov parameter is suppressed by one over the cut-off scale).

In this work the most recent bispectrum constraints will be applied to a particular and arguably the simplest model of vacuum state corrections, which was first considered in \cite{6}, but constructed and applied more generally by Danielsson \cite{7}. We will derive to what extent this vacuum state model can be ruled out by the bispectrum data alone in two particular scenarios; a slow-roll single field inflationary model with a higher derivative operator representing the interactions and a non-canonical single field model with a small speed of sound. The bispectrum prediction for a general (NPH) vacuum state modification in these single-field models was computed in \cite{3,4}. As an important and hopefully illuminating illustration we will slightly extend and improve the analysis, and apply it to the specific vacuum state proposal of Danielsson.

This paper is organized as follows. We will start with a quick review and small generalization of the Danielsson vacuum state proposal, followed by a summary of the results for the bispectrum in the context of a vacuum state modification in general single-field models of inflation. We then explain how these results can be turned into an upper bound on the scale of new physics and to what extent this vacuum state proposal is ruled out or not. We end with some conclusions and prospects for future improvement of the bounds.
Let us begin with a short review of Danielsson’s original proposal for a modified vacuum state, under the natural assumption that a high energy cut-off scale $L_c$ exists beyond which effective field theory breaks down \[^7\]. Solving for the mode functions in an inflating (pure de Sitter) background one writes down solutions for the field and conjugate momentum operators in terms of creation and annihilation operators in the standard way. Now identify a (conformal) time $\eta_0$ serving as the initial time where one would like to define the vacuum state. This initial time $\eta_0$ is related to the cut-off $\Lambda_c$ by demanding that the physical momentum at $\eta_0$ equals the cut-off scale, i.e. $|k\eta_0| = \frac{\Lambda_c}{2}$. This ensures that the description is always within the effective field theory regime. Note that since we are imposing a cut-off on the physical momentum the initial time $\eta_0$ is necessarily a function of the comoving momentum $k$. With respect to this initial time $\eta_0$ the creation operators (and subsequently the annihilation operators) can then be expressed as

$$ a_k(\eta) = u_k(\eta) a_k(\eta_0) + v_k(\eta) a_{-k}^\dagger(\eta_0), $$

(1)

the description is always within the effective field theory regime. Note that since we are imposing a cut-off on the physical momentum the initial time $\eta_0$ is necessarily a function of the comoving momentum $k$. With respect to this initial time $\eta_0$ the creation operators (and subsequently the annihilation operators) can then be expressed as

$$ a_k(\eta) = u_k(\eta) a_k(\eta_0) + v_k(\eta) a_{-k}^\dagger(\eta_0), $$

(2)

 describes nothing else but the mixing of creation and annihilation operators as time progresses. To ensure the time-independent commutation relations for the creation- and annihilation operators requires that $|u_k|^2 - |v_k|^2 = 1$. Let us now define a natural candidate for a vacuum state at some initial conformal time $\eta_0$

$$ a_k(\eta_0) |0, \eta_0\rangle = 0. $$

(3)

Obviously this choice requires that $v_k(\eta)$ vanishes at $\eta = \eta_0$ and as a direct consequence this candidate vacuum corresponds to a minimal uncertainty state at $\eta = \eta_0$ \[^7\]. This choice of vacuum can be understood as selecting the ‘local’ empty state at the time the physical momentum equals the high-energy cut-off $\Lambda_c$. This time will be different for different comoving momenta and defines a so-called ‘New Physics Hypersurface’. It can therefore be viewed as the vacuum state that is closest to the Bunch-Davies state in the presence of a high-energy cut-off and can in that sense be regarded as a minimal modification.

To understand the relation of this choice of vacuum to the standard Bunch-Davies state, consider the field operator $f_k(\eta)$ in terms of mode-function solutions to the equations of motion $f_k(\eta)$ this implies that

$$ f_k(\eta) = N_k (u_k(\eta) + v_k^*(\eta)), $$

(4)

where the overall (real) normalization $N_k$ is fixed by the Klein-Gordon normalization condition. Similarly, the expression for the canonical momentum operator defines a function $g_k(\eta)$, which is proportional to the difference between $u_k(\eta)$ and $v_k(\eta)$.

$$ g_k(\eta) = \tilde{N}_k (u_k(\eta) - v_k^*(\eta)). $$

Using these relations one can derive the following expression for the function $v_k(\eta)$ in terms of $f_k$ and $g_k$

$$ v_k^*(\eta) = \frac{1}{2} \left[ N_k^{-1} f_k(\eta) - \tilde{N}_k^{-1} g_k(\eta) \right]. $$

(5)

Defining a vacuum state $a_k(\eta_0) |0, \eta_0\rangle = 0$ is now explicitly seen to be equivalent to picking a mode-function solution $f_k$ (and its canonically conjugate function $g_k$) such that $v_k(\eta)$ vanishes at some $\eta = \eta_0$. Bunch-Davies mode-functions have the property that $v_k(\eta)$ only vanishes in the limit $\eta \to -\infty$, i.e. the Bunch-Davies state in this class of vacua corresponds to the minimal uncertainty vacuum state in the infinite past.

Now let us instead consider the general solution, which can be constructed from the Bunch-Davies mode-function and its complex conjugate

$$ f_k(\eta) = A_k f_k^{BD}(\eta) + B_k f_k^{BD*}(\eta), $$

(6)

where $A_k$ and $B_k$ are complex coefficients satisfying $|A_k|^2 - |B_k|^2 = 1$. Similarly, the momentum mode-function $g_k(\eta)$ can be expressed in terms of the Bunch-Davies momentum mode-functions

$$ g_k(\eta) = A_k g_k^{BD}(\eta) - B_k g_k^{BD*}(\eta). $$

(7)

To find the natural candidate vacua at some fixed time $\eta_0$ one demands that $v_k(\eta_0) = 0$. Using (5) this gives the
following expression for the Bogoliubov rotation parameter $\beta_k \equiv \frac{v_k}{u_k}$ in terms of the Bunch-Davies field and momentum mode-functions

$$\beta_k = \frac{N_k}{N_k} \frac{g_k^{BD}(\eta_0) - f_k^{BD}(\eta_0)}{g_k^{BD*}(\eta_0) + f_k^{BD*}(\eta_0)}.$$  \hspace{1cm} (8)

In terms of the (Bunch-Davies) functions $u_k^{BD}(\eta)$ and $v_k^{BD}(\eta)$ that appear in the expression for the annihilation and creation operators, the result simply reads

$$\beta_k = \frac{v_k^{BD}(\eta_0)}{u_k^{BD}(\eta_0)}.$$  \hspace{1cm} (9)

A couple of comments are in order. Since the (late time) power spectrum of fluctuations is proportional to $|f_k|^2$, the Bogoliubov rotation parameter $\beta_k$ denotes the leading order correction to the power spectrum due to a vacuum state modification. Scale invariant vacuum states of course require that $\beta_k$ is independent of the comoving momentum $k$, which will be guaranteed by the relation $k\eta_0 = \frac{\Lambda}{H}$, i.e. $\eta_0$ is $k$-dependent. Also note that this minimal cut-off vacuum construction has so far been completely general, not depending on any details of the inflationary Lagrangian. Whatever inflationary model one is interested in, to construct these states and determine the Bogoliubov parameter $\beta_k$ one simply plugs in the relevant (mode-) functions in the inflationary case of interest.

The best known (and originally discussed) example is that of the massless scalar field, that is associated to standard slow-roll inflation. The Bunch-Davies field and momentum mode-function in that case read

$$f_k^{BD}(\eta) = \frac{1}{\sqrt{2k}} e^{-ik\eta} \left(1 - \frac{i}{k\eta}\right), \quad g_k^{BD}(\eta) = \sqrt{\frac{k}{2}} e^{-ik\eta}.$$  \hspace{1cm} (10)

Using the expression (8) or (9) one then derives the following expression for the Bogoliubov parameter $\beta_k$

$$\beta_k = \frac{i}{2k\eta_0 + i} e^{-2ik\eta_0}.$$  \hspace{1cm} (11)

Since $|k\eta_0| = \frac{\Lambda}{H} \gg 1$ this Bogoliubov parameter is indeed scale invariant and approximates to $\beta_k \approx \frac{H}{2\Lambda} e^{i(\frac{3}{2}\pi - \frac{3\eta_0}{H})}$. As we have emphasized, the construction of these minimal cut-off states is completely general and can just as well be applied in the context of small speed of sound models of inflation, which includes DBI inflation as a special class. One might suggest that in non-canonical models of inflation the introduction of a cut-off scale is not required, since an infinite number of higher order corrections have been resummed leading to the non-canonical kinetic term. However, these corrections are only a subset of all the possible higher-dimensional operators that are expected to contribute as the string- (or cut-off) scale is approached, motivating the general introduction of a cut-off scale\(^1\). So the main, and for our purposes only, difference between small speed of sound and slow-roll models of inflation is the introduction of a reduced (and assumed constant) speed of sound $c_s < 1$ of the inflaton fluctuations. For the Bunch-Davies functions $u_k^{BD}$ and $v_k^{BD}$ one can check that the only effect is the appearance of a single factor of the speed of sound $c_s$ in the $k\eta_0$ terms as compared to the standard slow-roll case. The end-result for the Bogoliubov parameter therefore reads

$$\beta_k = \frac{i}{2kc_s\eta_0 + i} e^{-2i k c_s \eta_0},$$  \hspace{1cm} (12)

which translates into the scale-invariant approximate expression $\beta_k \approx \frac{H/\Lambda_c}{2c_s \Lambda_c} e^{i(\frac{3}{2}\pi - \frac{3\eta_0}{H})}$. One observes that the ratio $H/\Lambda_c$ governing the magnitude of the corrections in standard slow-roll is modified to $H/c_s\Lambda_c$ in these models, which could be orders of magnitude bigger. Note that the length scale $1/H^* \equiv c_s/H < 1/H$ corresponds to the sound horizon in small speed of sound models, corresponding to the length scale at which the behavior of the mode-functions turns non-adiabatic. The appearance of the factor $kc_s \eta_0$ is a general feature in these models, which was for instance also observed for the enhancement factors in the three-point function that appear whenever one introduces an arbitrary modified initial state at some high-energy cut-off. We conclude that the same is true for the Bogoliubov parameter in

\(^1\) More specifically, in [8] it was shown that in DBI models of inflation under certain conditions the standard procedure for defining the Bunch-Davies vacuum can fail.
cut-off modified initial state proposals: the magnitude (and phase) depends on the combination $k c_s \eta_0$ which results in larger absolute values of the Bogoliubov parameter for the same value of the cut-off and inflationary Hubble parameter, as compared to standard slow-roll models.

This ends our short summary of minimal cut-off initial states. The results for the three-point function, as reported in [3, 4], were model-independent in the sense that they only relied on the presence of a physical momentum cut-off. The Bogoliubov parameter was left as a free parameter and general bounds were derived on its magnitude for different inflationary Lagrangians. In this work we would like to specifically constrain the minimal cut-off modified vacuum state models for which we will need the expressions for the Bogoliubov parameters derived above.

**GENERAL BISPECTRUM PREDICTIONS**

As already referred to, in our previous work we calculated the results for the three-point function under the assumption of an arbitrary scale-invariant initial state modification in the NPH framework. The presence of a physical high-energy momentum cut-off was assumed, which identified an initial time for each comoving momentum mode where the initial state was defined by introducing a $k$-independent, but undetermined, Bogoliubov parameter. The presence of such a cut-off alone already has major consequences for the three-point function, also known as the bispectrum in Fourier space. Whenever the Bogoliubov parameter is non-zero, particles are injected at the initial (cut-off) time, which allows potential (self-) interactions to generate a large bispectrum amplitude at the time of horizon crossing. In the calculations these effects show up as large enhancement factors that depend on powers of $\Lambda_c / H \gg 1$. Depending on the details of the inflationary model under consideration the power of the enhancement factor could be as large as three, resulting in a huge bispectrum amplitude. The only reason why these large non-Gaussian signals have not yet been detected or ruled out is their specific (oscillatory) shape, which is orthogonal to any of the non-Gaussian templates used in analyzing the CMB data so far. Proposals for better adapted templates and improved methods to detect or constrain these oscillatory non-Gaussian signals have recently been put forward [9, 10], but have not yet been applied to the available data, forcing us to concentrate on the standard non-Gaussian shapes that have been constrained. Projecting the large bispectrum onto any one of the available templates still allows one to derive reasonable constraints in general. In fact, in some cases the constraints are already stronger than those available from the power spectrum. Before discussing the results of the projections to the observational templates it should be pointed out that in principle the results only apply to the three-dimensional bispectrum. However, in general the changes resulting from the reduction to the two-dimensional sphere are minimal and can (partially) be taken into account by introducing a weight function in the three-dimensional analysis [11].

For models with a small speed of sound ($c_s \ll 1$) the projections to the local and orthogonal templates [12], including the phase, were computed analytically [1]. To derive these results the assumption was made that the modifications to the BD state were strictly oriented in the $k_1$ direction, introducing a constant $k_1 \eta_0$ parameter, breaking the symmetry between the different momenta in the triangle. Properly maintaining the symmetry would instead require introducing three constant parameters $k_i \eta_0$, where $k_i$ is the direction in which one has perturbed the BD vacuum state via a Bogolyubov transformation. Consequently, one can factor out the constant $k_1 \eta_0$ for each perturbed state $\beta_k$. As expected, the results of this more complete analysis that preserves the symmetry between triangle momenta differ only slightly. Moreover, we have ameliorated our (numerical) integration methods and used the improved definition of the inner product between shapes as proposed by [11], which introduces a weight function to simulate projection effects onto the 2D CMB sky. This increases the correlation between smooth and oscillating shapes as oscillations are slightly damped by the weight function.

General single field models are defined by a single (Lagrangian) function $P(X, \phi)$, where $X = g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$. From this function $P(X, \phi)$ expressions can be derived for the slow-roll parameters, the speed of sound and two other variables $\Sigma$ and $\lambda$, whose ratio $\Sigma / \lambda$ will appear in the three-point function [11, 13]. Introducing a modified NPH vacuum state the final enhancement factor in the projections turns out to be at best quadratic in $p \equiv k c_s \eta_0 = \Lambda_c c_s / H \gg 1$. The results, for an undetermined Bogoliubov parameter $\beta = |\beta| e^{i \delta}$, in the limit that $1 / c_s^2 \gg 2 \Sigma / \lambda$ and $p \gg 1$ is

\begin{align}
\Delta f_{NL}^\text{local} &\simeq 4 \times 10^{-3} \frac{p^2 |\beta|}{c_s^2} \cos(\delta), \\
\Delta f_{NL}^\text{ort} &\simeq -6 \times 10^{-2} \frac{p^2 |\beta|}{c_s^2} \cos(\delta).
\end{align}

The additional terms that have been neglected are subleading in $p$ and are out of phase in the sense that the phase parameter $\delta$ is shifted with $\pi / 2$. Also note that these results do not reduce to the standard slow-roll results in
the \( c_s = 1 \) limit. Subdominant terms that have been neglected in the \( c_s \ll 1 \) limit will turn in to the dominant contributions to the bispectrum in the \( c_s = 1 \) limit, explaining why the \( c_s = 1 \) limit of the above result does not reproduce the slow-roll bispectrum. It is also important to stress that the DBI models are a special class of small sound speed models for which \( 2 \lambda / \Sigma = 1 - 1/c_s^2 \) and as a result the leading contributions to the bispectrum cancel exactly [13]. The remaining contributions to the projections of the DBI modified NPH initial state bispectrum are only linearly enhanced in the small \( c_s \) limit and the details of the projections in the DBI case can only be evaluated semi-analytically [4]. This leads to DBI constraints that are far less interesting as compared to general small sound speed models. We will therefore concentrate on the general models and not discuss the DBI results in detail.

In the case of standard canonical single-field slow-roll inflation with a specific dimension 8 higher-derivative term \([14]\) the enhancement factor appearing in the bispectrum amplitude was found to be quadratic in \( \Lambda_c / H \) \([3]\). However, in the projection to the available observational templates \( f_{NL}^{\text{eq}} = \Delta(O,T) f_{NL}^{\text{local}} \), where \( \Delta(T,O) \) is the projection factor, one loses one power of the enhancement, leaving a linear enhancement in \( \Lambda_c / H \) as far as the projections are concerned. In addition the amplitude of the enhancement is further reduced by slow-roll (via \( \epsilon \)) and the dimensionful coupling of the higher-derivative term. In these results the effects of a (cut-off dependent) phase in the Bogoliubov parameter was neglected and for our purposes here we would like to remedy this situation. To incorporate the phase we had to rely on numerical methods to determine the projection factors. For the projection onto local non-Gaussianities we obtain (in the limit of large \( p \equiv \Lambda_c / H \))

\[
\Delta f_{NL}^{\text{local}} \approx \frac{5}{6} p \lambda |\beta| \left( \frac{M_p^2 H^2}{\Lambda^4} \right) \cos(\delta),
\]

which, not surprisingly, is very similar to equation (6.11) in \([3]\), except for the appearance of a phase dependence.

It is important to realize that the inclusion of the perturbative effect of the dimension 8 higher-derivative operator is only consistent when \( \phi / \Lambda_c^2 \lesssim 1 \) \([14]\), which translates into \( \epsilon \geq \Lambda_c^2 / M_p^2 H^2 \). The power spectrum observations tell us that \( \epsilon = \frac{10^{-6}}{8 \pi} \frac{H^2}{M_p^2} \), which implies that \( H/\Lambda_c < 10^{-2} \) to ensure a consistent higher-derivative expansion. As a consequence the non-Gaussianities due to the presence of the dimension 8 higher-derivative term reach a maximal magnitude for \( H/\Lambda_c \sim 10^{-2} \), yielding an equilateral amplitude of at most \( f_{NL}^{\text{eq}} \sim \mathcal{O}(1) \). On the other hand, modifying the initial state leads to an additional enhancement factor that is proportional to the ratio between the UV cutoff \( \Lambda_c \) and the scale of inflation \( H \). The required consistency of the higher-derivative expansion, being inversely proportional to the cut-off scale \( \Lambda_c \), reduces this enhancement and as a consequence weakens the constraints on \( \beta \). In fact, as we will see in the next section, for the minimal cut-off vacuum state proposal assumed in this paper the constraint becomes practically meaningless. This should not come as much of a surprise, since in order to boost non-Gaussianities from higher-derivative operators the cut-off scale needs to be relatively close to the inflationary scale, whereas the non-Gaussian amplitude as a consequence of initial state modifications grows as the hierarchy between the cut-off and the Hubble scale increases. For completeness, let us remark that the correlation between the orthogonal template and the standard single-field higher-derivative bispectrum, being very similar, yields comparable constraints.

After this very short review and slight improvement of what is known about bispectrum projections due to a generic NPH initial state, let us now derive the best available constraints on the parameters of minimal cut-off initial states.

**BISPECTRUM CONSTRAINTS ON MINIMAL CUT-OFF VACUUM STATES**

Having summarized the results for minimal cut-off initial states and the general projected non-Gaussian amplitudes due to arbitrary cut-off (or NPH) modified vacua, we would now like to apply the most recent observational bounds to derive constraints on the parameters of the minimal cut-off initial state proposal.

To proceed we will make use of the following recent (2 \( \sigma \)) constraints on \( f_{NL}^{\text{local}} \) and \( f_{NL}^{\text{ort}} \) \([15, 16]\)

\begin{align*}
-10 & \leq f_{NL}^{\text{local}} \leq 74, \\
-410 & \leq f_{NL}^{\text{ort}} \leq 6.
\end{align*}

The observational constraints on the equilateral amplitude \( f_{NL}^{\text{eq}} \) can be ignored, because the projection to the equilateral template turns out to be orders of magnitude smaller. One should keep in mind that the power spectrum constraint on \( |\beta| \) is roughly \( 10^{-2} \), which is based on the lack of evidence for oscillatory behavior with a larger amplitude in the power spectrum \([5]\).

A notable feature of the results for the projections is that they oscillate as a function of the phase \( \delta \) of the Bogoliubov
Different values for the phase can therefore result in constraints that deviate considerably. Indeed, for special values of the phase the projection vanishes and the constraints disappear altogether. In minimal cut-off models the phase of the Bogoliubov parameter is actually a function of the cut-off scale $\Lambda_c$ and as a consequence for special values of $\Lambda_c$ the projected amplitudes would be significantly reduced, as can be seen in figure 1. Since the precise value of the cut-off $\Lambda_c$ is unknown, the constraints will be derived under the reasonable assumption that a typical value of $\Lambda_c$ is expected to be closer to an (absolute value) maximum in the projection than to the special point where the projection vanishes. To implement this we will derive the constraints based on an expression that is related to the absolute value average over a single oscillatory domain in the parameter $\Lambda_c$.

Let us first analyze general single field models with a small speed of sound. The minimal cut-off vacuum state proposal predicts a Bogoliubov parameter with an absolute value approximately equal to $|\beta_k| \approx \frac{H^2}{2c_s\Lambda_c}$ and a phase $\delta = i(\frac{3}{2}\pi - \frac{2c_s\Lambda_c}{H})$. In the limit $p \equiv \Lambda_c c_s / H \gg 1$ we derive, to leading order in $p$,

$$\Delta f_{\text{local}} \sim -2 \times 10^{-3} \frac{P}{c_s^2} \sin(2p),$$

$$\Delta f_{\text{ort}} \sim 3 \times 10^{-2} \frac{P}{c_s^2} \sin(2p),$$

The absolute value of the Bogoliubov coefficient has removed one enhancement factor, resulting in a linear $p$ dependence of the prefactor in front of the oscillatory $\sin(2p)$ term. As an example, the maximum contribution to the orthogonal non-Gaussian template from initial state modifications has been plotted in figure 1. As already alluded to, the oscillations in the projection factor force us to work with the average over a single oscillatory domain in order to derive a constraint on the ratio $\Lambda_c / H$. Simply replacing $\sin(2p)$ with the average $2/\pi$ the derived bounds would only apply to about 60 percent of the $p$ domain. If instead demanding that at least 90 percent of the domain is included within the constraints, we need to replace $\sin(2p)$ with $\sim 0.16$.

The ambiguous sign due to the oscillatory behavior also prompts us to consider the bound on the (largest) absolute value of the non-Gaussian amplitude, which for the local amplitude is $|f_{\text{local}}| \leq 74$ and for the orthogonal type $|f_{\text{ort}}| \leq 410$. Using these largest absolute values for the local and orthogonal amplitude and insisting on a 90 percent

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2 To be more precise, for those values one can no longer neglect the subleading terms in $p$, which are in effect $\pi/2$ out of phase and therefore maximize exactly when the leading terms minimize. Nevertheless, the fact that they are subleading in $p$ implies that the constraints for those values of $p$ become practically meaningless.
coverage, an upper bound on the parameter \( \frac{\Lambda_c}{H} \) can be derived from local and orthogonal type non-Gaussianities

\[
\text{local : } \frac{\Lambda_c}{H} \leq 2.5 \times 10^5 \, c_s \quad \text{orthogonal : } \frac{\Lambda_c}{H} \leq 8.5 \times 10^4 \, c_s
\] (18)

Clearly the strongest result is obtained from the orthogonal bound, becoming stronger as \( c_s \) is smaller. In fact, the speed of sound \( c_s \) cannot be much smaller than \( 10^{-2} \) because that would produce a leading order equilateral non-Gaussian result in conflict with observations. Applying that minimal value of \( c_s \) the upper bound on \( \frac{\Lambda_c}{H} \) roughly equals

\[
\frac{\Lambda_c}{H} \leq 10^3,
\] (19)

Again we would like to stress that this is an upper bound. The cut-off scale should be close to the inflationary Hubble scale to avoid too large an enhancement of the (projected) non-Gaussian signal, with the exception of special higher values of \( \Lambda_c \) for which the phase \( \delta \) is close to an integer number times \( \pi \), corresponding to 10 percent of the parameter domain. This constraint is weaker than initially anticipated in [4], mainly because the phase in this case is a function of \( \Lambda_c \), producing oscillations in the non-Gaussian signal, forcing us to introduce a factor related to the average of the absolute non-Gaussian signal and the most conservative (upper) bound on the absolute value of the observed constraints.

Incorporating the, admittedly crude, lower bound on \( \frac{\Lambda_c}{H} > 10^2 \) from analysis of the power spectrum, for a general speed of sound one arrives at

\[
\frac{10^2}{c_s} \leq \frac{\Lambda_c}{H} \leq 8.5 \times 10^4 \, c_s
\] (20)

There is a small window remaining for \( c_s \sim 10^{-1} \), but for small speed of sound models with \( c_s \sim 10^{-2} \) one can conclude that minimal initial state modifications are practically ruled out, except for those special values of the phase at which the contribution to the orthogonal non-Gaussian template (nearly) vanishes, corresponding to about 10 percent of the parameter domain.

To briefly illustrate the impact of the oscillatory behavior in this analysis, consider neglecting the oscillating behavior, choosing \( \sin 2\rho = 1 \) for local non-Gaussianities and \( \sin 2\rho = -1 \) for orthogonal non-Gaussianities. Again this would yield a strongest upper bound from the orthogonal constraints that is roughly equal to \( \frac{\Lambda_c}{H} \leq 2 \times 10^2 \, c_s \), which for \( c_s \sim 10^{-2} \) can essentially be ruled out from the non-Gaussian data alone and would in general be inconsistent with the power spectrum data that shows no evidence of an oscillatory effect with an amplitude that large.

As already commented on, for DBI models the constraints weaken considerably, because the enhancement of the non-Gaussian signal is reduced by a factor of \( p \). This results in no enhancement at all in the projection to the local (or orthogonal) template, removing all dependence on the cut-off scale. As a consequence the observational constraints do not result in (interesting) bounds on the cut-off scale \( \Lambda_c \), which parameterizes the initial state modification.

Finally, let us briefly consider standard slow-roll inflation \( (c_s = 1) \), including a dimension 8 higher derivative term that, together with the initial state modification, is responsible for a large \( p^2 \) enhanced non-Gaussian signal. Inserting \(|\beta|\) and the phase into eq. (14) one obtains

\[
f_{NL}^{\text{local}} \approx -\frac{5}{12} \sin(2\rho)
\] (21)

All enhancement in \( p \equiv \Lambda_c/H \) is lost and one is left with a maximum contribution to the local non-Gaussian template that is of (less than) order 1, which is significantly below set constraints on local type non-Gaussianities. As a consequence minimal initial state modifications in this particular canonical single field inflationary model are not constrained by the local non-Gaussian projection. In order to become sensitive to the intrinsically large non-Gaussian signal set by the ratio \( \Lambda_c/H \), templates better adapted to the theoretical non-Gaussian shape should be designed and compared to the available data.

**CONCLUSIONS**

To summarize, we have applied the currently best available constraints on local, orthogonal and equilateral shape non-Gaussianities to derive bounds on the cut-off scale parameterizing a (slightly generalized) minimal model of initial
state modifications. The results strongly depend on the model of inflation under consideration. Interesting bounds can be derived in the context of general, non-DBI, single field small speed of sound models of inflation. In that case, non-Gaussian constraints from the orthogonal template alone lead to the following upper bound on the cut-off scale

\[ \frac{\Lambda_c}{H} \leq 8.5 \times 10^4 c_s. \]  

(22)

Combined with results from the equilateral template and the power spectrum leaves only a very small window \((c_s \sim 10^{-1}, \Lambda_c/H \sim 10^3)\), and minimal initial state modifications in general single field models with a small speed of sound can almost be ruled out. To be more precise, the above upper bound is valid for 90 percent of the cut-off parameter domain, excluding small isolated regions covering in total 10 percent of the parameter domain where the projection to the orthogonal template nearly vanishes as a consequence of the oscillating nature of the projection.

For DBI and a canonical single field model with a dimension 8 higher derivative operator the analysis does not lead to interesting non-Gaussian constraints. The reason is the complete lack of sensitivity of the available templates to the large non-Gaussian signal of minimal initial state modifications. Consequently, more suitable templates have to be developed to efficiently probe (minimal) initial state modifications in these models, perhaps along the lines of [9, 10]. Work in this direction is ongoing and we hope to report on more effective non-Gaussian templates and observational strategies in the near future.

An important property that we have emphasized is that (minimal) initial state modifications lead to a non-Gaussian signal that increases as the separation between the cut-off scale and the inflationary Hubble parameter increases. As a consequence non-Gaussian constraints can only give rise to an upper bound on the cut-off scale parameterizing the initial state modifications arising due to new physics. Together with lower bounds on the cut-off scale from other considerations (including the power spectrum data or generic effective field theory arguments) this carries a strong potential to rule out these types of initial state modifications in the future as the non-Gaussian analysis and corresponding constraints improve.

At the same time the interesting feature of an upper bound on the cut-off scale poses somewhat of a theoretical conundrum, since it is obviously in conflict with the idea of decoupling. Perhaps this should be interpreted as signalling a fundamental flaw of these type of initial state modifications that might be responsible for inconsistencies in the perturbative expansion of the quantum field theory, conceivably ruling out these vacuum states. We hope to come back to this important issue in future work.

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