COMPACT DUSTY CLOUDS IN A COSMIC ENVIRONMENT

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ABSTRACT

A novel mechanism of the formation of compact dusty clouds in astrophysical environments is discussed. It is shown that the balance of collective forces operating in space dusty plasmas can result in the effect of dust self-confinement, generating equilibrium spherical clusters. The distribution of dust and plasma density inside such objects and their stability are investigated. Spherical dusty clouds can be formed in a broad range of plasma parameters, suggesting that this process of dust self-organization might be a generic phenomenon occurring in different astrophysical media. We argue that compact dusty clouds can represent condensation seeds for a population of small-scale, cold, gaseous clumps in the diffuse interstellar medium. They could play an important role in regulating its small-scale structure and its thermodynamical evolution.

Key words: dust, extinction – ISM: clouds – ISM: structure – plasmas

1. INTRODUCTION

Self-organization of dusty (complex) plasmas has been observed in numerous experiments. Different types of structures formed in dusty plasmas under microgravity conditions (in experiments performed on the International Space Station) as well as on the ground include compact clusters, voids surrounded by dust shells, vortices, etc. (Tsytovich 1997; Fortov et al. 2005; Tsytovich et al. 2008; Bonitz et al. 2010). The mechanisms governing such phenomena are associated with plasma fluxes generated due to electron and ion absorption on grains. Plasma fluxes exert the forces that can result in the effect of dust self-confinement.

It was shown theoretically that homogeneous dusty plasmas are intrinsically unstable (Morfill & Tsytovich 2000; Bingham & Tsytovich 2001; Tsytovich & Watanabe 2003). For small, dusty clouds, the long-range (∝r−1) attraction between grains can be caused by ion “shadowing” forces, induced due to plasma absorption on the grain surfaces (Tsytovich et al. 2008). The theory of such gravitational-like instabilities was developed for laboratory conditions and the analogy with the Jeans instability was pointed out. The important differences between the laboratory and space conditions are (Tsytovich 1997; Whittet 2003; Draine 2009) (1) the volume ionization in astrophysical environments is much less important than in laboratory plasmas, (2) the grain screening in space is described (to a very good accuracy) by a simple linear Debye screening, while in laboratory conditions it is normally highly nonlinear, and (3) the ratio of ion-to-electron temperatures in space is close to unity, while in laboratory experiments it is typically ∝−2. These distinctions require new theoretical and numerical treatment of self-organization in space dusty plasmas.

The principal aim of this paper is to point out the importance and possibility of dust self-organization in cosmic environments like the diffuse interstellar medium (ISM). We present and solve the basic set of self-consistent equations describing equilibrium dusty clouds, identify necessary conditions for such clouds to exist, and analyze their stability. Unlike stars, where the equilibrium is primarily governed by gravity and pressure, the dusty clouds are formed due to the balance of the ion drag force (associated with the self-consistent plasma flux) and the electrostatic force on charged grains.

We show that cosmic dust can form stable spherical clouds, with typical sizes of the order of 10–100 AU or less and with total masses of the order of 10−3 Earth masses or below. Although the dust density inside the clouds can exceed the ambient density by many orders of magnitude—we therefore call them “compact dusty clouds”—they remain optically thin. We predict that this process can occur in a broad range of plasma parameters, which indicates that such self-organization might be a generic phenomenon operating in different astrophysical media.

One particularly interesting application is the formation of compact dusty clouds in the diffuse ISM. Ultra-high resolution observation of interstellar absorption (Braun & Kanekar 2005; Smith 2013; Cordiner et al. 2013) provide evidence that the diffuse ISM is structured on scales below 1 pc, indicating the presence of tiny, distinct, dense, and cold H1 cloudlets, the origin of which had not been understood until now. This gas component might play an important role in regulating the thermodynamical state of the ISM and in triggering phase transitions.

The paper is organized as follows. In Section 2, we summarize principal simplifying assumptions relevant to interstellar environment. In Section 3, we discuss the generic mechanism resulting in formation of compact dusty clouds and we also introduce proper normalization of variables. In Section 4, we assume negatively charged dust and formulate self-consistent equations describing equilibrium spherical clouds in this case. In Section 5, we solve the equations numerically, to present distributions of parameters inside the clouds and estimate their major characteristics (such as size, mass, and dust density) for some idealized ISM phases. In Section 6, we analyze the stability of the obtained equilibrium clouds. In Section 7, we consider a simplified model for positively charged dust and show that equilibrium compact clouds can be formed in this case as well. In Section 8, we discuss the effect of compact dusty clouds on the small-scale structure of the diffuse ISM and in Section 9 we summarize the results and discuss possibilities to observe compact dusty clouds.

2. IDEALIZED ASTROPHYSICAL ENVIRONMENTS AND SIMPLIFYING ASSUMPTIONS

Table 1 summarizes physical parameters for some idealized ISM phases (Whittet 2003; Draine & Lazarian 1998; Yan et al.
clouds have a fixed effective size. The principal parameters relevant for the further analysis are the gas temperature $T$, the atomic hydrogen density $n_H$, the molecular hydrogen density $n_{H_2}$, and the density of hydrogen ions $n_{H^+}$. (Below, unless explicitly specified, we employ the notation $n$ for the ion density and $n_e$ for the total density of neutrals.) For these conditions, we can make the following simplifying assumptions.

1. **Dust grains have the same size.** Dust in astrophysical environments is extremely polydisperse. In the range between several nm to a few tenths of a μm, the dust size distribution can normally be approximated by the Mathis, Rumpl, & Nordsieck (MRN) dependence (Mathis et al. 1977), $dn/da \propto a^{-3.5}$. As we will show in the next section, the two major forces—ion drag and electrostatic—whose balance results in the formation of equilibrium dusty clouds are dominated by the small-size part of the distribution. Therefore, we can restrict our analysis to some effective size (the radius of the effective small-size cutoff). In the normalized (dimensionless) form, the governing equations used for the analysis do not contain the size, so that general results obtained below remain unaffected by this assumption.

2. **Linear regime of dust screening.** The criterion of linearity of the plasma–dust interaction is very well satisfied for astrophysical conditions, which allows us to assume a linear Debye screening of dust (Tsytovich et al. 2008). For a grain of charge $eZ$ and radius $a$, this criterion requires the potential energy of the ion–grain interaction at the Debye length $\lambda_D$ to be much smaller than the plasma temperature, i.e., $|Z|e^2/\kappa_B k_B T \sim a/\lambda_D \ll 1$. This assumption significantly simplifies the force balance equation.

3. **Fluid description of ions.** We assume that the ion collisions with neutrals as well as with charged grains are frequent enough so that the resulting mean free path of ions is much smaller than any macroscopic length scale of the problem. This allows us to introduce the local friction force on ions, which is proportional to their local flow velocity. Furthermore, we assume the flow velocity to be much smaller than the ion thermal velocity (which is confirmed a posteriori by the obtained solutions).

4. **External forces and background inhomogeneities are negligible.** The analysis below shows that typical sizes of dusty clouds are relatively small ($\lesssim 30$ AU). Therefore, neglecting the centrifugal forces and inhomogeneities due to the magnetohydrodynamic (MHD) turbulence (Yan et al. 2004; Yan & Lazarian 2003) is a reasonable approximation. The condition to neglect tidal forces is discussed in Appendix A.

5. **Evolution of dust grains is negligible.** We completely neglect processes contributing to the dust growth (e.g., due to continuous plasma absorption) or destruction (e.g., due to sputtering erosion). A complicated analysis of all these processes (whose balance determines the overall evolution of dust in a particular ISM phase) is beyond the scope of this paper and therefore here we assume that grains comprising dusty clouds have a fixed effective size.

### Table 1

| Parameter | RN | CNM | WNM |
|-----------|----|-----|-----|
| $T$ (K)   | 100| 100 | 6000|
| $n_0$ (cm$^{-3}$) | $10^3$ | 30 | 0.4 |
| $2n_{H_2}/n_H$ | $10^{-2}$ | 0 | 0 |
| $n_{H^+}/n_H$ | $10^{-3}$ | $10^{-3}$ | $10^{-1}$ |

These assumptions are only aimed to make the physical picture as transparent as possible and demonstrate that the plasma absorption on dust grains results in a universal self-confinement effect that can lead to the formation of stable compact dusty clouds in a cosmic environment.

3. PHYSICAL PROCESSES IMPORTANT FOR CLOUD FORMATION

Figure 1 illustrates the principal role of plasma fluxes for dusty clouds.

The absorption of electrons and ions by a single grain leads to its charging. In a plasma where external ultraviolet (UV) radiation does not play a dominant role, the resulting charge is negative, because electrons are much faster than ions. Then the equilibrium charge $-eZ$ of a grain of radius $a$, as presented in the dimensionless form (Fortov et al. 2005)

$$ z = \frac{e^2 Z}{a \kappa_B T} $$

is normally about a few (for isothermal hydrogen plasmas $z \approx 2.5$). In this paper, we mostly discuss this charging regime. The case when UV radiation dominates and grains are charged positively is considered in Section 7.

For a large dusty cloud, in addition to the local electron and ion fluxes in a close proximity of each grain, there is also a global plasma flux pointed inward. The total flux $J$ fluctuates around its equilibrium value, $J = \langle J \rangle + \delta J$, both due to fluctuations of plasma fluxes on individual particles and fluctuations of the particle density (high-frequency fluctuations in electron–ion plasmas, not associated with dust and hence irrelevant to our problem, are neglected). The plasma flux is inevitably accompanied by the self-consistent electric field, $E = \langle E \rangle + \delta E$. The fluctuating field (associated with the fluctuating flux and/or grain charges) can result in a variety of interesting effects, such as stochastic particle acceleration, also seen in astrophysical environments (Ivlev et al. 2010; Hoang & Lazarian 2012).

In this paper, we neglect the fluctuations and focus on the effect of the regular components (so that the notation $\langle \ldots \rangle$ is omitted below). According to Figure 1, the radial flux (due to absorption in a spherical dusty cloud) is always pointed toward the cloud center. The radial electric field generated in this case is directed inward when grains are charged negatively (while for positively charged grains it is pointed outward; see Section 7).
Thus, the average flux provides the confining effect via the ion drag force and the average electric field tends to compensate for it via the electrostatic force. Let us elaborate on this mechanism.

The electric field force acting on a single charged grain is

\[ F_{el} = -eZE. \]  

(2)

As for the ion drag force on the grain, \( F_{dr} \), it consists of two parts arising due to the ion absorption and ion scattering (Tsytovich 1997; Fortov et al. 2005). For astrophysical conditions, the latter provides the major contribution due to a large value of the Coulomb logarithm, \( L \equiv \ln(\lambda_D/\ell) \geq 1 \), which yields

\[ F_{dr} \approx \frac{2\sqrt{2\pi}}{3} m_i \nu_i T \left( \frac{e^2 Z}{k_B T} \right)^2 u_i, \]  

(3)

where \( n_i \) and \( u_i \) are the local ion density and flow velocity, respectively, and \( \nu_i = \sqrt{k_B T/m_i} \) is the ion thermal velocity. Furthermore, when considering a force on dust per unit volume, \( \propto \int d\alpha (dn_\alpha/da) F(a) \), one should take into account that \( F_{el} \propto a \) and \( F_{dr} \propto a^2 \) (for given \( \mathbf{E} \) and \( u_i \)). Thus, both forces are dominated by the small-size part of the MRN distribution (Mathis et al. 1977), i.e., the balance per unit volume is equivalent to the balance of forces on a single grain of some effective size (near the small-size cutoff).

The intrinsic length scale to be used further for the normalization is the ion mean free path (for a fluid description of ions and neutrals is constant, we choose the ion mean free path due to collisions with neutral hydrogen, \( \ell = (\sigma_D n_n)^{-1} \), as the natural length scale for the normalization of coordinates and forces (as well as of the electric field):

\[ R = \frac{r}{\ell}, \quad \tilde{F} = \frac{\ell F}{k_B T}, \quad \tilde{E} = \frac{\ell E}{k_B T}. \]  

(4)

In the normalized form, the electrostatic and drag forces are

\[ F_{el} = -Z\tilde{E}, \quad F_{dr} = Zz\tilde{E}NU, \]  

(5)

where

\[ U = \frac{u_i}{\sqrt{2\nu_i}}, \quad N = \frac{n_i}{n_{eff}}, \]  

(6)

and

\[ n_{eff} = \frac{3k_B T \sigma_{in}}{4\sqrt{\pi} LE^2 a} n_n. \]  

(7)

Inside the dust region, the balance of averaged forces per grain yields

\[ \tilde{E} = z\tilde{E}NU. \]  

(8)

Thus, we derived the equilibrium equation relating the ion flow velocity and the associated electric field. The field can be found without solving the Poisson equation, since the quasineutrality condition,

\[ P = N - n, \]  

(9)

is satisfied with very good accuracy \( \sim \ell/\lambda_D \). Here, the electron density \( n_e \) is normalized by \( n_{eff} \) while for the dust density \( n_d \) the modified Havnes parameter \( P \) is implemented:

\[ n = \frac{n_e}{n_{eff}}, \quad P = \frac{Zn_d}{n_{eff}}. \]  

(10)

Note that \( P \) is different from the conventionally used Havnes parameter \( P_H = Zn_d/n_e \) (Fortov et al. 2005)—the latter also depends on varying plasma density and therefore is not convenient for the dimensionless analysis.

4. MASTER EQUATIONS FOR EQUILIBRIUM SPHERICAL CLOUDS: NEGATIVELY CHARGED DUST

Let us derive self-consistent conditions of equilibrium for spherically symmetric dusty clouds. The force balance for ions should include the momentum loss due to collisions with dust. Using Equation (8), we express the latter as \(-n_d/n_i F_{dr} = -(P/N)\tilde{E} \) (force per ion). Neglecting the ram pressure force \( U dU/\mathbf{R} \) (since \( U \ll 1 \)) and taking into account the ion pressure, we obtain the ion force balance equation:

\[ \left( 1 - \frac{P}{N} \right) \tilde{E} - \frac{1}{N} \frac{dN}{dR} = 0. \]  

(11)

Here, \( \tilde{E} \) is the radial component of the electric field and the term \(-U\) describes the ion friction on neutrals (the ion mobility equals unity in the employed normalization). The force balance for electrons is governed by Boltzmann equilibrium:

\[ \tilde{E} + \frac{1}{n} \frac{dn}{dR} = 0. \]  

(12)

The ion flux \( NU \) is determined by the ion absorption on dust grains (found from a simple orbital motion limited (OML) charging theory, see Fortov et al. 2005). For negatively charged dust, we obtain

\[ \frac{1}{R^2} \frac{d}{dR} R^2 NU = - \left( \frac{1 + \frac{1}{z} }{ } \right) \frac{3PN}{2L}. \]  

(13)

The change of the normalized charge \( z \) with distance \( R \) can be found by differentiating the OML charging equation (Fortov et al. 2005):

\[ \sqrt{\mu} e^{-z} n = \left[ z + 1 - \left( z - 1 \right) \frac{U^2}{3} \right] N, \]  

(14)

where \( \mu = m_i/m_e \) (1840) is the ion-to-electron mass ratio (for a hydrogen plasma). Here, we took into account that the electron flux is thermal, while in the ion flux the lowest-order (quadratic) velocity corrections are included. The latter is necessary since all terms in \( dz/dR \) are proportional to \( U \).

By combining Equations (11)–(13) with the quasineutrality condition (Equation (9)) and differentiating Equation (14), we obtain the following set of master equations for the case of negatively charged dust:

\[ \frac{dN}{dR} = (zn - 1)NU, \]  

(15)

\[ \frac{dn}{dR} = -znNU, \]  

(16)

\[ \frac{dU}{dR} = - \frac{2U}{R} - \left( \frac{1 + \frac{1}{z} }{ } \right) \frac{3P}{2L}; \]  

(17)

\[ \frac{dz}{dR} = - \left( \frac{z + 1}{z + 2} \right) \left( zN + zn - 1 \right) \]  

\[ + \left( \frac{z + 1}{z + 2} \right) \left( \frac{P}{L} + \frac{4U}{3R} \right) U. \]  

(18)

The equations are written in the form that does not contain the dust size explicitly (except for the Coulomb logarithm \( L \),...
which can be considered constant for relevant astrophysical environments). Note that the ionization is assumed to be small and therefore is neglected (the role of ionization is briefly discussed in the end of Section 5).

To solve the master equations, it is sufficient to know the values \( N_0 \equiv N(0) \) and \( P_0 \equiv P(0) \) at the center, since \( n_0 \equiv n(0) \) can be found from the quasineutrality condition. The dust charge at the center \( z_0 \) is found from Equation (14):

\[
\frac{e^{-z_0}}{z_0 + 1} \left( 1 - \frac{P_0}{N_0} \right) = \frac{1}{\sqrt{\mu}}. \tag{19}
\]

The flux at the center vanishes as \( U(R) \to U_0^+ R \) for \( R \to 0 \). According to Equation (17),

\[
U_0^+ = - \left( 1 + \frac{1}{z_0} \right) \frac{P_0}{2L} < 0, \tag{20}
\]

since \( P_0 > 0 \), i.e., the plasma flux is naturally directed toward the center. Thus, the only restriction in the formation of dusty clouds is

\[
N_0 - P_0 > 0. \tag{21}
\]

In what follows, we limit ourselves to the consideration of spherically symmetric clouds.

5. NUMERICAL RESULTS AND ESTIMATES FOR THE ISM

We carried out calculations for various values of \( N_0 \) and \( P_0 \) limited by Equation (21) and did not find any solutions other than compact dusty clouds with a size \( R_{cl} \) determined by \( P(R_{cl}) = 0 \). A qualitative change in the spatial distribution of the cloud parameters was only found when \( P_0 \) approaches \( N_0 \). We also checked the applicability of the hydrodynamic approach, defined by the condition that the characteristic inhomogeneity length should be much larger than the mean free path of ions, and found that all clouds with \( N_0 \lesssim 10 \) satisfy the condition. The calculations indicate that \( R_{cl} \) increases with decreasing \( N_0 \), implying an improved applicability of the hydrodynamic approach. We note that the cloud size increases approximately proportionally to the Coulomb logarithm \( L \). All calculations shown below were performed for \( L = 20 \).

The results are illustrated in Figures 2 and 3. One can see qualitative changes occurring in the radial density distributions as \( N_0 \) decreases: while for \( 1.5 \lesssim N_0 \lesssim 10 \) both dust and ion densities decrease monotonically toward the edge (Figure 2), at \( N_0 \approx 1.5 \) an ion density hump appears near the edge (Figure 3). The dust number density \( P(z) \) remains monotonic in the latter case, but the dust charge density \( P \) exhibits a hump as well. The hump grows as \( N_0 \) decreases, so that charges mainly concentrate at the periphery of the cloud, resulting in the steepening of the density gradients as \( R \to R_{cl} \).

Let us discuss the implications for the ISM. From Table 1, we see that the background ionization fraction, \( n_{i,b} / n_n \), varies in different environments in the range from \( \sim 10^{-1} \) for the WNM to \( \sim 10^{-3} \) for RNe and the CNM. For the ion density inside the dust clouds, we employ Equation (7):

\[
\frac{n_i}{n_n} = \frac{3}{4\sqrt{\pi L}} \frac{N Z \sigma_{in}}{\bar{a}^2}, \tag{22}
\]

with the momentum transfer cross section given by \( \sigma_{in} \approx 10^{-14} \text{ cm}^2 \) (it slightly decreases with \( T \) and is about the same for collisions with atomic and molecular hydrogen; the used value corresponds to \( k_B T \approx 0.1 \text{ eV} \)). Since the ion density at the cloud edge is practically equal to the background density \( n_{i,b} \) (with a slight deviation due to the finite ion velocity), from Equation (22) we conclude that Figures 2 and 3 represent typical dusty clouds in the WNM with a \( \sim 3 \times 10^{-7} \) cm and in RNe with a \( \sim 3 \times 10^{-6} \) cm. The cloud size \( R_{cl} \approx R_{cl} \ell \), being of the order of a few ion–neutral mean free paths \( \ell = (\sigma_{in} n_n)^{-1} \), varies in the range between \( \sim 30 \) AU for the WNM and \( \sim 10^{-5} \) AU for RNe.

Regarding the dust density inside such clouds, it can be estimated from the relation

\[
\frac{n_d}{n_n} = \frac{3}{4\sqrt{\pi L}} \frac{P \sigma_{in}}{\bar{a}^2}. \tag{23}
\]

Using the MRN size distribution (Mathis et al. 1977), we obtain that the background number density of dust with size \( \geq a \) follows the scaling \( n_{d,b}(a)/n_n \approx 0.3 A_{\text{MRN}} a^{-2.5} \), where \( A_{\text{MRN}} \approx 10^{-23} \text{ cm}^{-2.5} \) is the scaling constant (Draine & Lee 1984). For \( a \approx 10^{-6} \text{ cm} \), from Equation (23) we get \( n_d/n_n \approx 3 \times 10^{-4} \) at the center, whereas the corresponding background density is \( n_{d,b}/n_n \approx 3 \times 10^{-11} \). Hence, the dust density inside the clouds can be increased by a factor of \( \sim 10^7 \) with respect to the background level (note that this factor scales as \( a^{0.5} \)). Therefore, the total mass of the cloud \( m_{cl} \) is dominated by
dust and can be roughly estimated as \( m_{cl} \sim (4/3)\pi a^3 \rho_d \), where \( m_{cl} \) is the mass of a dust grain expressed via the effective grain density \( \rho_d \). Assuming \( \rho_d \sim 1 \text{ g cm}^{-3} \), for the WNM we get \( m_{cl} \sim 10^{-3} \text{ Earth masses} \). The cloud mass approximately scales as \( m_{cl} \propto a/n_r^2 \), so that the presented estimate for the WNM should be considered as the upper bound.

The optical depth of dusty clouds is expected to be determined by Rayleigh scattering. Using the scattering cross section (Landau & Lifshitz 1960) \( \sigma_r = (128/3)\pi a^4/\lambda^3 \) (where \( \lambda \) is the optical wavelength) and employing Equation (23), we conclude that the optical depth, \( \sim \sigma_r n_a n_d \sim (a/\lambda)^4 \), has a very strong dependence on the dust size and is not (explicitly) related to \( n_r \). For \( a \sim 3 \times 10^{-6} \text{ cm} \), the optical depth can reach values of \( \sim 10^{-2} \), i.e., dusty clouds are expected to be optically thin for the discussed astrophysical environments.

We conclude this section with two notes concerning the applicability of our model.

1. The volume ionization was neglected in the above consideration. We investigated the effect of this process inside the cloud by adding the source term \( \propto n \) to the right-hand side of Equation (13). The analysis showed that as long as the plasma loss on dust dominates (i.e., the source term is much smaller than the absorption term), the overall distributions, as well as the size of the cloud, remain practically unchanged.

2. For the applicability of the fluid approach, the mean free path of ions (which is determined by collisions with neutrals and dust) must be much smaller than the characteristic length of the density inhomogeneity. This requirement results in the condition

\[
\frac{d \ln N}{dR} \ll \frac{zP}{\sqrt{2}},
\]

which is always satisfied in the central region but can be violated near the cloud edge. For the examples shown in Figures 2 and 3, the right-hand side of Equation (24) at \( R = R_d \) is 5–10 times larger than the left-hand side, and hence the use of the fluid approach is well justified.

6. STABILITY OF CLOUDS: SPHERICAL PERTURBATIONS

As we pointed out in Section 5, the fluid approach is applicable for calculating the clouds illustrated in Figures 2 and 3. Now, the principal question is under which conditions the clouds are stable, in particular, whether the clouds with non-monotonic charge density distributions shown in Figure 3 can be unstable.

To address this question, let us consider eigenmodes of the equilibrium clouds. We shall assume small (linear) deviations from the steady state and restrict ourselves to the analysis of spherical perturbations (which are presumably the most “dangerous” for the stability). In the dimensional form, we consider perturbations of the electron and ion densities, \( \delta n_e(r) \), ion velocity \( \delta u_i(r) \), dust charge \( \delta Z(r) \), and dust velocity \( u_d(r) \) (we do not write \( \delta \) in front of \( u_d \) since it is zero in equilibrium), all proportional to \( e^{-i\omega t} \).

The new variable \( u_d \) as well as the frequency \( \omega \) and mass of a dust grain \( m_d \) are normalized in such a way that both the dust inertia force, \( -i\omega m_d u_d \), and the friction force due to dust interaction with the ambient gas (Fortov et al. 2005; Purcell 1969), \( (8\sqrt{2\pi/3})a^2 m_n n_r v_{Ti} u_d \), are written in the simplest dimensionless form. This yields

\[
V \equiv \frac{u_d}{u_{eff}}, \quad \Omega \equiv \frac{\epsilon \omega}{u_{eff}}, \quad M \equiv \frac{e^2}{ak_B T} \frac{m_d u_{eff}^2}{k_B T},
\]

where

\[
u_{eff} = \frac{3k_B T \sigma_{in}}{8\sqrt{2\pi} e^2 a} v_{Ti}.
\]

Here, \( v_{Ti} = \sqrt{k_B T/m_n} \) (\( \approx v_{Te} \)) is the thermal velocity of neutrals.

The analysis of spherically symmetric perturbations is relatively straightforward. In the steady state, the balance of drag and electric field forces has the form of Equation (8). By taking into account the friction and inertial forces in the momentum equation for dust, we obtain the field perturbation:

\[
\tilde{\delta E} = \delta(zNU) - \frac{1}{z}(1 - iM\Omega) V,
\]

whereas the perturbation of the dust charge is found from the charging equation (Equation (14)):

\[
\delta Z = -\frac{z + 1}{z + 2} \left( \frac{\delta N}{N} - \frac{\delta n}{n} \right) + \frac{2}{3} \left( \frac{z - 1}{z + 2} \right) U \delta U.
\]

These relations are used in equations for perturbations of the ion and electron densities derived from Equations (11) and (12):

\[
\frac{d \delta n}{dR} = \delta(\tilde{E}n - NU),
\]
The equation for the ion velocity perturbation follows from Equation (17):
\[
\frac{d\delta U}{dR} = -\frac{2}{R} \delta U - \frac{3}{2\mathcal{L}} \left( 1 + \frac{1}{z} \right) \delta P + \frac{3P}{2\mathcal{L}z^2} \delta z,
\]
while for the dust velocity perturbation we use the continuity equation:
\[
\frac{dV}{dR} = i\Omega \left( \frac{\delta P}{P + \delta P} - \frac{\delta z}{z} \right) - \left( \frac{2}{R} \frac{dP/dR}{P + \delta P} - \frac{dz/dR}{z} \right) V,
\]
where \( P \) is the equilibrium value of the Havnes parameter and \( \delta P \) is its perturbation:
\[
\delta P = \delta N - \delta n.
\]

Generally, Equation (32) requires special consideration near the cloud edge, since it becomes essentially nonlinear when \( R \to R_d \). On the other hand, Figures 2 and 3 show that \( P(R) \) is a rather steep function near the edge, so that the nonlinearity is only important in close proximity to \( R_d \). Thus, to a first approximation, one can neglect \( \delta P \) in the denominator of Equation (32) for the analysis of eigenmodes and then Equations (29)–(32) can be presented in the following matrix form:
\[
\frac{d\mathbf{X}}{dR} = \mathbf{A} \cdot \mathbf{X},
\]
where \( \mathbf{X} = (\delta N, \delta n, \delta U, V)^T \) is the perturbation vector and the elements of matrix \( \mathbf{A} \) are given in Appendix B.

Equation (34) requires four boundary conditions. The first two obvious conditions are \( \delta U = 0 \) and \( V = 0 \) at the center. By substituting \( \delta U(R) \to \delta U_0 R \) in Equation (31) and \( V(R) \to V_0 R \) in Equation (32) for \( R \to 0 \), we find
\[
\begin{align*}
\delta U'_0 &= -\frac{1}{2\mathcal{L}} \left[ \left( 1 + \frac{1}{z_0} \right) \delta P_0 + \frac{z_0 + 1}{z_0^2(z_0 + 2)} \frac{\delta N_0}{N_0 - n_0} \right], \\
V'_0 &= i\frac{\Omega}{3} \left[ \delta P_0 + \frac{z_0 + 1}{z_0(z_0 + 2)} \frac{\delta N_0}{N_0 - n_0} \right],
\end{align*}
\]
where \( \delta P_0 = \delta N_0 - \delta n_0 \). The other two conditions follow from the assumptions that (1) the number of grains is conserved in perturbations:
\[
\int_0^{R_d} dR R^2 \frac{\delta P}{z} \left( \frac{P}{z} \delta z \right) = 0;
\]
and (2) the external plasma flux at the cloud edge is not affected by perturbations:
\[
(U \delta N + N \delta U)_{R_d} = 0.
\]

## 6.1. Examples of Eigenmodes

The numerical solution of Equation (34) was carried out by separating the real and imaginary parts and representing the eigenmodes in the form \( \Omega = \Omega_{re} + i\Omega_{im} \). The most important conclusion of our analysis is that the eigenmodes remain stable for all combinations of parameters studied in Section 5. For a given pair of \( P_0 \) and \( N_0 \), we obtain a finite number of eigenmodes with increasing \( \Omega_{re} \) and practically constant \( \Omega_{im} \) (the latter is primarily determined by the gas friction). The modes also depend on the dimensionless mass of a dust grain, which can be presented in the following form:
\[
M = \frac{9}{128\pi} \frac{Z m_d}{m_n} \left( \frac{\sigma_m}{a^2} \right)^2.
\]

Since \( Z \) is roughly proportional to the size \( a \) and \( m_d \propto a^3 \), the dimensionless mass is practically independent of \( a \) and varies in the range \( M = 100–1000 \) for different ISMs.

Let us illustrate the eigenmodes for a spherical cloud corresponding to Figure 2. By setting the dust density \( P_0 = 2.28 \) (solid line) and mass \( M = 100 \), we obtain a discrete series with slowly increasing \( \Omega_{re} \) (varying in the range between 0.107 and 0.633 for the first six modes) and constant \( \Omega_{im} = -0.005 \). This represents weakly damped dust-acoustic wave modes sustained in a cloud with an inhomogeneous distribution of dust and ions. One can roughly estimate the frequency in dimensional units as \( \omega_{re} \sim 2\pi \mathcal{C}_{DA}/R_d \), where \( \mathcal{C}_{DA} = Z \sqrt{(n_d/n_i)k_B T/m_d} \) is the dust-acoustic speed (Fortov et al. 2005). In dimensionless units, this yields \( \Omega_{re} \sim (2\pi/R_d)\sqrt{zP/MN} \), which is indeed close to the lowest value of \( \Omega_{re} \) from the calculated series.

## 7. DUSTY CLOUDS FORMED BY POSITIVELY CHARGED GRAINS

Generally, dust charges in a cosmic environment are determined by a balance of plasma currents absorbed on grains (resulting in negative charges) and the photoemission currents induced by cosmic UV radiation (tending to charge grains positively; Draine 2004, 2011). The analysis of dusty clouds becomes rather complicated in this case. Instead of solving the problem self-consistently, here we study the regime when the UV radiation dominates and dust acquires large positive charges \( eZ \). Such a situation occurs when the electron photoemission current from the grain exceeds the current of the surrounding electrons on the grain (Khramtsov et al. 1999), i.e., when \( \gamma J_{UV} \gg n_e \sqrt{k_B T/m_e} \), where \( J_{UV} \) is the UV photon flux and \( \gamma \) is the yield of photoelectrons (for simplicity, we assume the photoelectron temperature to be equal to the plasma temperature \( T \)). In this regime, the normalized grain charge \( z \), written in the form of Equation (1), has a logarithmic dependence on \( \gamma J_{UV} \) and therefore depends on external conditions (in contrast to the case of negatively charged grains considered above).

In order to analyze master equations for equilibrium clouds formed by positively charged dust, let us for simplicity fix the value of the normalized charge \( z(\pm 2.5) \). Then, we can replace \( z \to -z \) in the above equations and expect that the electric field changes sign as well, \( E \to -E \). In this case, Equation (8), which determines the force balance on dust, is still valid. Furthermore, we replace \( P \to -P \), so that the quasi-neutrality condition, Equation (9), is modified to
\[
P = n - N.
\]

Finally, using the OML ion current on positively charged grains (Fortov et al. 2005), from Equations (15)–(17) we obtain the
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The results are for \( n_0 = 5 \) and four different combinations of \( P_0 \) and \( N_0 \) (related by \( n_0 = P_0 + N_0 \), represented by different lines). The normalized charge \( z = 2.5 \) is fixed. The size of the cloud \( R_{cl} \) is determined by the condition \( P(R_{cl}) = 0 \).

Figure 4. Radial distribution of parameters in a compact cloud of positively charged dust: the normalized dust number density \( P/z \), electric field \( \tilde{E} \), ion density \( \tilde{N} \), and electron density \( \tilde{n} \) versus the distance \( R \) from the center. The size of the cloud \( R_{cl} \) is determined by the condition \( P(R_{cl}) = 0 \).

The following set of master equations:

\[
\frac{dN}{dR} = -(z n + 1) N U,
\]

\[
\frac{dn}{dR} = z n N U,
\]

\[
\frac{dU}{dR} = \frac{2U}{R} - \frac{3Pe^{-z}}{2Lz}.
\]

The numerical solution of the equations, obtained for the parameters similar to those in Figure 2, is presented in Figure 4. One can see that clouds of positively charged dust become several times larger, while the dust density profile keeps the same form. The electric field reverses sign (and also becomes weaker), which ensures the force balance in Equation (8) and modifies the distributions of the ion and electron densities, in such a way that the form of \( n(R) \) resembles that of \( N(R) \) in Figure 2 and vice versa. The ion velocity profile (not shown) remains similar to that in Figure 2.

Note that in order to get a universal dependence on \( z \), one can simply re-scale variables in Equations (38)–(41). For instance, the solution for arbitrary \( z \) is obtained from that for \( z = 1 \) by employing the re-scaling \( z^{-1}/n, N, P \rightarrow n, N, P \), \( a^{-1}U \rightarrow U \) and \( aR \rightarrow R \), where \( a = ze^{(1/2)(z-1)} \).

Thus, we conclude that positively charged dust can also form compact clouds. This suggests that the mechanism of self-confinement associated with plasma absorption on grains can indeed be generic and independent of the dominant charging process.

8. COMPACT DUSTY CLOUDS AND THE SMALL-SCALE STRUCTURE OF THE DIFFUSE ISM

So far, in our consideration, we neglected feedback to the surrounding medium. In particular, we assumed that properties of the neutral gas remain unaffected in the presence of dusty clouds. However, the dust density inside the clouds is increased by many orders of magnitude. Therefore, the clouds may in fact play the role of “condensation seeds” for the gas: concentrated dusty cores may effectively cool down the surrounding gas due to efficient graybody radiation, and hence may cause the gas density to increase due to compression by the hotter surroundings. Thus, one might expect the formation of compact gaseous “nuggets” whose equilibrium is determined by the balance of the total dust radiation and the diffusive heat flux from the outside.

To demonstrate the feasibility of this process, let us first evaluate how effective the thermal coupling between dusty clouds and the surrounding gas would be. One can expect that the gas temperature inside the cloud is close to the dust temperature provided that the mean free path of atoms/molecules due to collisions with dust, \( \ell_{nd} = (\pi a^2 n_d)^{-1} \), does not exceed the cloud size \( r_{cl} \). As we see from Figures 2–4, the latter varies between a few and a few dozen times the mean free path \( \ell = (\sigma n n_h)^{-1} \).

Taking this into account and also using Equation (23), we obtain that the ratio \( r_{cl}/\ell_{nd} \) is practically a universal constant:

\[
\frac{r_{cl}}{\ell_{nd}} = \frac{3\sqrt{\pi}}{4Lz} P R_{cl},
\]

which is of the order of unity for any astrophysical environment. Thus, the gas temperature inside dusty clouds should always be close to the dust temperature.

Next, we estimate the efficiency of the radiative dust cooling. The cooling time \( \tau_{rad} \) for an individual particle can be roughly obtained from the following balance: \( 4\pi a^2 \sigma_{SB} T_d^4 \sim (4/3)\pi a^4 \rho u c_b k_B T_d / \tau_{rad} \), where \( \sigma_{SB} \) is the Stefan–Boltzmann constant, \( c_b \) is the specific heat of a grain material (whose effective mass density is \( \rho_d \sim 1 \text{ g cm}^{-3} \)), and \( T_d \) is the (effective) dust temperature. Assuming \( T_d \sim 30 \text{ K} \) and \( a \sim 10^{-6} \text{ cm} \), we get \( \tau_{rad} \sim 10 \text{ s} \). This timescale should be compared with the characteristic heat diffusion time \( \tau_{rad} \sim r_{cl}^2 / (\nu_T \ell_{nd}) \), where \( \ell_{nd} = (\sigma n n_h)^{-1} \) is the neutral mean free path. Taking into account that for hydrogen \( \sigma_{na} \sim 0.1 \text{ cm}^2 \), we get \( \tau_{rad} \sim r_{cl} / \nu_T \sim 1 \text{ yr} \) for \( r_{cl} \sim 1 \text{ AU} \). We conclude that the radiation is a very efficient cooling mechanism and, hence, \( T_d \) can be much smaller than the ambient temperature \( T \).

Finally, we obtain the temperature profile of the gas around a dusty cloud, \( T_n(r) \), by employing the conservation of the total diffusive heat flux, \( n_h v_T dT_n/dr \), and the boundary conditions \( T_n(r_{cl}) = T_d \) and \( T_n(\infty) = T \) under the assumption of a constant pressure, \( n_h(r) T_n(r) = \text{const} \), we get that the size of the “cold atmospheres” around dusty clouds is always about a few \( r_{cl} \). For example, the size of gaseous nuggets created by dusty clouds in the WNM can reach a few hundred AU, while the gas density inside such objects can be increased by a factor of \( \sim T/T_d \sim 100 \) (for \( T_d \sim 30 \text{ K} \)).

9. DISCUSSION AND CONCLUSIONS

We showed that dust grains in various ISMs can self-organize themselves into equilibrium compact spherical clouds. The formation of such clouds is caused by the ion “shadowing” forces (Morfill & Tsytovich 2000; Bingham & Tsytovich 2001; Tsytovich & Watanabe 2003). This triggers the “shadowing” instability that resembles the Jeans instability and originates from the generic mechanism of plasma absorption on dust grains. The confinement stabilizing equilibrium clouds is generated by the inward plasma flux which, in turn, is created due to the plasma
absorption on dust. Let us summarize the major characteristics of compact dusty clouds.

1. After proper normalization, the cloud is characterized by a combination of two global parameters—the normalized ion and dust densities at the center. The former is determined by the background plasma density in a given astrophysical environment and the latter is a free parameter whose \( \delta > 0 \) is a free parameter whose variation is only limited by Equation (21).

2. For typical astrophysical conditions, the size of compact clouds is below \( \sim 30 \text{ AU} \). The dust density inside the clouds can exceed the background density by about seven orders of magnitude and their total mass can be as large as \( \sim 10^{-3} \text{ Earth masses} \). At the same time, the optical depth of the clouds (determined by the Rayleigh scattering) remains much smaller than unity.

There have been a few reports on the existence of small-scale objects observed in different astrophysical environments, whose origins remain unclear. In particular, ultra-high resolution observations of interstellar absorption (Braun & Kanekar 2005; Lauroesch 2007; Smith 2013; Cordiner et al. 2013) have provided convincing evidence that the diffuse ISM is structured on scales below \( 1 \text{ pc} \) down to dozens of AU, indicating the origin of which has not been understood up until now. As sketched in Section 8, compact, dusty clouds might trigger the formation of such cloudlets. However, the details of their growth in the turbulent diffuse ISM are certainly more complex than outlined by the simple hydrostatic model of Section 8. We plan to investigate this interesting process in detail in a subsequent paper. Also, the motion of compact clouds near the Galactic center has been recently detected (Gillessen et al. 2012). Some properties of these clouds (e.g., small optical depth, size \( \sim 100 \text{ AU} \), and mass less than an Earth mass) are similar to those predicted by our model. The UV radiation near the Galactic center is very strong, so that dust is positively charged and characteristics of the resulting clouds should be similar to those presented in Section 7. On the other hand, the tidal forces exerted on clouds in this region should be so strong (see Appendix A) that their equilibrium is highly questionable and requires further analysis.

In conclusion, we would like to note that dust in ISM is extremely polydisperse and, hence, different clouds are expected to contain grains of different sizes. Understanding the size distribution of dusty clouds and their morphology would be a very important problem to address. Another very interesting topic would be to investigate the coupling of dusty clouds (and the surrounding atmosphere) to the dynamics of the ambient ISM. Since the characteristic size of dusty clouds is expected to be much smaller that the cutoff scale of the MHD turbulence, one can use the results of this paper as the input for such analysis.

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APPENDIX A

CONDITION TO NEGLECT TIDAL FORCES

The formation of dusty clouds in close proximity to stars or black holes requires a more careful analysis due to the presence of gravitational forces neglected in this paper. Below, we estimate the condition when the resulting tidal force is small in comparison with the ion drag force (Equation (3)).

For a cloud of radius \( r_{cl} \), comprised of dust grains of size \( a \) and located at the distance \( d_{star} \) from a star/black hole of the mass \( m_{star} \), the tidal force is negligible when

\[
\left( \frac{r_{cl}}{1 \text{ AU}} \right) \lesssim \mathcal{L} \left( \frac{m_{cl}}{m_{star}} \right)^{3/2} \left( \frac{d_{star}}{100 \text{ AU}} \right)^{3} \left( \frac{1 \mu m}{a} \right) \times \left( \frac{1 \text{ g cm}^{-3}}{\rho_{d}} \right) \left( \frac{n_{i}}{10 \text{ cm}^{-3}} \right) \left( \frac{k_{B} T}{1 \text{ eV}} \right).
\]

For instance, assuming \( m_{star} = 3 \times 10^{6} m_{O} \) (corresponding to the massive black hole in our Galaxy), \( r_{cl} = 3 \text{ AU} \), \( a = 10^{-6} \text{ cm} \), \( \rho_{d} = 1 \text{ g cm}^{-3} \), \( n_{i} = 0.3 \text{ cm}^{-3} \), and \( k_{B} T = 0.1 \text{ eV} \), we conclude that the gravity can be neglected when \( d_{star} \gtrsim 10^{4} \text{ AU} \).

APPENDIX B

ELEMENTS OF THE DYNAMICAL MATRIX

The dynamical matrix \( A \) in Equation (34) has the following elements:

\[
A_{\delta N, \delta N} = \left( z^{2} + z - 1 \right) U, \quad A_{\delta N, \delta n} = \frac{z^{2} + 3z + 1}{z + 2} N U,
\]

\[
A_{\delta N, \delta U} = (zn - 1) N, \quad A_{\delta n, \delta N} = -\frac{1}{z}(1 - i \Omega) N,
\]

\[
A_{\delta n, \delta n} = - \frac{z^{2} + z - 1}{z + 2} n U, \quad A_{\delta n, \delta U} = - \frac{z^{2} + 3z + 1}{z + 2} N U,
\]

\[
A_{\delta U, \delta n} = \frac{3}{2} \frac{(1 + \frac{1}{z})}{\frac{P}{N} + 1}, \quad A_{\delta U, \delta U} = 2 \frac{z - 1}{R} \frac{P U}{\frac{z^{2} + 2}{R}}, \quad A_{\delta U, \delta U} = 0,
\]

\[
A_{\delta V, \delta N} = i \Omega \left[ \frac{1}{P} + z + 1 \frac{1}{z(z + 2) N} \right],
\]

\[
A_{\delta V, \delta n} = - i \Omega \left[ \frac{1}{P} + z + 1 \frac{1}{z(z + 2) n} \right],
\]

\[
A_{\delta V, \delta U} = \frac{2(\Omega - i \Omega / 3)}{z^{2} + 2} U,
\]

\[
A_{\delta V, \delta N} = - \frac{1}{R} \frac{dz}{P dR} + \frac{1}{z} dR.
\]

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