Probing the Gravitational Scale via Running Gauge Couplings

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Abstract

According to a recent paper by Robinson and Wilczek, the leading gravitational corrections to the running of gauge couplings tend to reduce the values of the couplings at energies below the gravitational scale, defined to be the energy above which gravity becomes strongly interacting. If the physical gravitational scale is sufficiently low, as conjectured in certain extra-dimension models, this behavior of the gauge couplings can be measured in future high energy experiments, providing a way to determine where the gravitational scale lies. We estimate that measurements of the fine structure constant at the Large Hadron Collider and the proposed International Linear Collider energies can probe the gravitational scale up to several hundred TeV, which will be the most stringent test that can be obtained in the conceivable future.

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In a recent paper [1], Robinson and Wilczek have examined the modification to the running of gauge couplings due to gravitational corrections [2]. At the one-loop level, they found that the effective gravitational corrections amount to a negative term in the beta function of the gauge coupling $g$:

$$\beta(g, E) = \frac{dg}{d \ln E} = -\frac{b_0}{(4\pi)^2} g^3 - \frac{3}{\pi} \left( \frac{E}{\Lambda_G} \right)^2 g,$$

where $E$ is the energy, $\Lambda_G$ is the gravitational scale, the energy at which gravitational interactions become strong, and $b_0$ is the usual coefficient of the $\beta$ function in the absence of gravity. The gravitational correction is the same for all gauge couplings because gravitons carry no gauge charge. If one assumes that in a consistent quantum theory of gravitational interactions, which is yet to be discovered, the above one-loop contributions represent the leading corrections, then all gauge couplings will be driven toward zero at energies around $\Lambda_G$. Thus, gravitational interactions have the tendency to asymptotically unify all gauge couplings. While this perturbative result is not reliable at $\Lambda_G$, it should be a reasonable approximation at energies sufficiently below $\Lambda_G$. And if this negative running of the gauge couplings causes sizable corrections at energies sufficiently below $\Lambda_G$, it may be detectable in future high energy experiments, depending on what the value of $\Lambda_G$ is.

While it is usually assumed that $\Lambda_G$ is of order the Planck energy, $M_P \equiv \sqrt{\hbar c/G_N} = 1.2 \times 10^{19}$ GeV, where $\hbar$ is the Planck constant divided by $2\pi$, $c$ is the speed of light in vacuum, and $G_N$ is the Newton gravitational constant, one cannot be certain until a consistent quantum theory of gravity is established. There are proposals that involve the assumed existence of extra spatial dimensions [3], in which the physical gravitational scale may be much lower than $M_P$. For instance, with $n$ extra dimensions compactified on a torus of radius $R$, the gravitational scale is given by [3]

$$\Lambda_{(ADD)}^{n+2} = \frac{\hbar c}{G_N} \left( \frac{\hbar}{c} \right)^n \frac{1}{(2\pi R)^n}$$

where $(2\pi R)^n$ is the volume of the $n$-torus. Thus, the gravitational scale can be sufficiently low if $R$ and/or $n$ is sufficiently large. The current experimental upper bound on $R$ is $0.13$ mm [4], for $n = 2$. This corresponds to a lower bound on $\Lambda_G$ of order 1 TeV.

If $\Lambda_G$ is indeed many orders of magnitude smaller than $M_P$, the gravitational effects on the running of gauge couplings may be measurable in future collider experiments and
be used to determine the value of $\Lambda_G$. This is what we will examine below. What we are primarily interested in is to examine the feasibility of determining $\Lambda_G$ by this method and to estimate the potential reach of the Large Hadron Collider (LHC), rather than to test specific models. We will therefore use the result (1) as our testing ground and neglect additional terms that may arise in specific models (e.g., there may be additional corrections from the KK gravitons in extra-dimension models). In other words, we use the result (1), which was obtained from Einstein gravity, as a generic description of the running of the gauge couplings even though it may not be an accurate description of the running in a specific model. And, instead of assuming $\Lambda_G$ to be the Planck energy, as was done in Ref. 1, we let it be a free parameter and examine what limits on $\Lambda_G$ may be accessible from precision measurements of the gauge couplings in future colliders such as the LHC.

Figure 1 compares the running of the gauge couplings in the Standard Model with and without the gravitational corrections for various values of $\Lambda_G$. Here $\alpha_i = g_i^2/(4\pi)$ ($i = 1, 2, 3$), where $g_1$, $g_2$, and $g_3$ are the couplings of the gauge groups $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$, respectively. The evolution of the couplings are obtained by integrating Eq. (1) from the initial energy of $M_Z$, the mass of the $Z$ boson, with the initial data [5]:

$$\alpha^{-1}(M_Z) = 128.91 \pm 0.02,$$
$$\sin^2 \theta_W(M_Z) = 0.23120 \pm 0.00015,$$
$$\alpha_3(M_Z) = 0.1182 \pm 0.0027,$$

where $\alpha$ is the fine structure constant and $\theta_W$ is the weak mixing angle. Since we are only interested in an estimate of the size of the one-loop gravitational corrections, we take only the one-loop contributions to the coefficient $b_0$ in Eq. (1), with $b_0 = 7$ for $SU(3)_C$, $b_0 = 19/6$ for $SU(2)_L$, and $b_0 = -41/6$ for $U(1)_Y$, assuming three generations of quarks and leptons. In all cases, we see that the gravitational corrections become significant at energies which are more than one order of magnitude below $\Lambda_G$, energies for which the effective gravitational corrections from (1) may be considered reliable. This behavior persists for values of $\Lambda_G$ larger than those illustrated in Figure 1.

Instead of $\alpha_1$ and $\alpha_2$, it may be more useful to examine $\alpha$ and $\sin^2 \theta_W$, which have been measured to high precisions. The running of these two quantities are shown in

\[3\]
FIG. 1: Running of Standard Model gauge couplings with (solid curves) and without (dashed curves) gravitational corrections, for (a) $\Lambda_G = 35$ TeV and (b) $\Lambda_G = 10^3$ TeV. For comparison, the running of the gauge couplings in the minimal supersymmetric standard model is also shown (dotted curves).

FIG. 2: Running of the fine structure constant with (solid) and without (dashed) gravitational corrections, for (a) $\Lambda_G = 35$ TeV and (b) $\Lambda_G = 10^3$ TeV.

Figures 2 and 3. Because $\alpha$ is more precisely known, and hence can provide the more stringent constraint, it is an ideal probe for the gravitational scale. As an illustration, if we require that the fractional change in $\alpha^{-1}$ due to the gravitational corrections at
FIG. 3: Running of $\sin^2 \theta_W$ of the Standard Model with (solid) and without (dashed) gravitational corrections, for (a) $\Lambda_G = 35$ TeV and (b) $\Lambda_G = 10^3$ TeV.

$E = 200$ GeV (see Table 1 below) cannot exceed twice the fractional error obtained from (3), we find that $\Lambda_G$ must be at least 35 TeV. If we assume the same error as in (3) will be achieved in measuring $\alpha$ at $E = 200$ GeV, the lower bound on $\Lambda_G$ will be raised to about 60 TeV.

| E       | $100 \times |\Delta \alpha^{-1}|/\alpha^{-1}$ | $100 \times |\Delta \sin^2 \theta_W|/\sin^2 \theta_W$ | $100 \times |\Delta \alpha_3|/\alpha_3$ |
|---------|-------------------------------------|-------------------------------------|-------------------------------------|
| 2 TeV   | 3.997                               | 0.03949                             | 3.662                               |
| 1 TeV   | 0.978                               | 0.00969                             | 0.921                               |
| 500 GeV | 0.237                               | 0.00219                             | 0.225                               |
| 200 GeV | 0.031                               | 0.00019                             | 0.030                               |

TABLE I: Gravitational corrections to the running of gauge couplings for $\Lambda_G = 35$ TeV.

For $\Lambda_G = 35$ TeV, we show in Table 1 the fractional changes in $\alpha^{-1}$, $\sin^2 \theta_W$, and $\alpha_3$ arising from the gravitational corrections at various energies. For each quantity $x$, the displayed $|\Delta x|_x \equiv |(x \text{ with gravitational corrections} - x \text{ without gravitational corrections})|/x$ divided by $x$ without gravitational corrections. Because the changes increase with ene-
gies, future high energy experiments at the LHC and beyond, e.g., the proposed International Linear Collider (ILC), will be able to probe larger values of $\Lambda_G$, depending on the precisions that can be achieved in measuring these quantities. For example, assuming the error in measuring $\alpha^{-1}$ at 1.5 TeV, which is a representative energy for the LHC and the ILC, will be twice the error given in (3), we find that $\Lambda_G$ must be at least 300 TeV in order that the gravitational correction does not exceed this assumed experimental error. Obviously, a smaller $\Lambda_G$ will be allowed by a larger experimental error. As an illustration, if the error in measuring $\alpha^{-1}$ at 1.5 TeV were five times the current error in (3), the lower bound on $\Lambda_G$ would become 200 TeV.

In comparison, the range of $\Lambda_G$ that can be probed by the running coupling of the strong interaction will be lower. For instance, assuming the error in measuring $\alpha_3$ at 1.5 TeV will be two (five) times the error given in (3), we find that $\Lambda_G$ must be at least 27 TeV (18 TeV) in order that the gravitational correction does not exceed this assumed error.

We highlight the very important feature in the numerical examples above that limits on $\Lambda_G$ are derived from the corresponding gravitational corrections at energies which are at least two orders of magnitude below the cutoff scale $\Lambda_G$, thus making them reliable predictions.

We have also considered the case of the minimal supersymmetric standard model (MSSM), for which the running of the gauge couplings differs from the Standard Model. We assume for simplicity that the effective supersymmetry breaking scale is $M_Z$ [6]. We compare in Figure 1 the MSSM running with the gravitationally corrected Standard Model running. The two cases can clearly be distinguished. In particular, the gravitational corrections affect all gauge couplings the same way, which is not the case for MSSM. There may be other extensions of the Standard Model that would affect the running of the gauge couplings. However, it is unlikely any of these extensions will share the same characteristics as the gravitational correction, which is to reduce every gauge coupling in a similar manner. It will therefore be relatively straightforward to identify the gravitational effects.

In conclusion, we have shown that the effective gravitational corrections to the gauge coupling running can be detected at energies significantly below the gravitational scale $\Lambda_G$. If $\Lambda_G$ is no more than several hundred TeV, such effects may be measurable at the
LHC or the ILC, signaled by values of the gauge couplings which are smaller than the Standard Model expectations. It is very interesting that a precise measurement of the running gauge couplings can provide information about the nature of the gravitational interactions and yield the most stringent limit on the scale of quantum gravity attainable in the conceivable future.

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[1] S. P. Robinson and F. Wilczek, Phys. Rev. Lett. 96, 231601 (2006). See also S. P. Robinson, M.I.T. Ph.D. Thesis (2005).

[2] Modifications of the running of gauge couplings due to interactions from the conformal sector of quantum gravity have been studied in S. Odintsov and R. Percacci, Mod. Phys. Lett. A 9, 2041 (1994). The resulting corrections to the beta functions are different from that in Ref. 1, but we will not consider them here.

[3] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B 429, 263 (1998); Phys. Rev. D 59, 086004 (1999); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B 436, 257 (1998).

[4] C. D. Hoyle et al, Phys. Rev. D 70, 042004 (2004). For a review of earlier results, see E. G. Adelberger, B. R. Heckel and A. E. Nelson, Ann. Rev. Nucl. Part. Sci. 53, 77 (2003).

[5] S. Eidelman et al. [Particle Data Group Collaboration], Phys. Lett. B 592, 1 (2004).

[6] P. Langacker and N. Polonsky, Phys. Rev. D 47, 4028 (1993); M. Carena, S. Pokorski and C. E. M. Wagner, Nucl. Phys. B 406, 59 (1993).