Hydrogen Clouds and the MACHO/EROS Events

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ABSTRACT

We propose that the recently reported MACHO/EROS events correspond to gravitational amplification by dark clouds rather than compact objects. These clouds must be very dense with \( M \sim 0.1 \, M_\odot \) and \( R \lesssim 10^{14} \, \text{cm} \). In all likelihood, the clouds will be members of a family of objects with different sizes and masses. We therefore expect events of longer duration than the ones reported by the MACHO and EROS groups but with light curves very different from the ones derived assuming point mass lenses. We suggest that one such event has already been observed in radio measurements of the quasar 1502+106. The abundances of free electrons, metals, complex molecules, and dust grains are constrained to be very small suggesting that the clouds are formed from a primordial mixture of hydrogen and helium. Cosmic rays and background UV radiation ionize a halo around the cloud. Radio waves from distant sources will be scattered by the electrons in this halo, an effect which may have already been observed in quasars such as 1502+106. We argue that dark clouds are a viable alternative to compact objects for baryonic dark matter in the halo.
I. Introduction

Recently, the MACHO (Alcock 1993) and EROS (Aubourg 1993) collaborations announced that gravitational amplification or microlensing of light from stars in the Large Magellanic Cloud may have been observed. Three ‘events’ were reported where an event corresponds to a transient, time-symmetric, achromatic brightening of an LMC star (Paczynski 1986). The interpretation of these events is that a compact object has passed very close to the line of sight to the star. The masses inferred from the three events are all in the range $0.03 - 0.3 \, M_\odot$. This is consistent with the lenses being brown dwarfs or low-mass stellar remnants though more exotic objects such as black holes can also explain the observations. Microlensing events are extremely rare and some five million stars were observed in order to find just these three events. Still, the observations suggest that some, and perhaps a significant fraction of dark matter in our galaxy is in the form of nonluminous compact objects and therefore, in all likelihood, dark baryons. This conclusion is consistent with results from primordial nucleosynthesis, which indicate that the density in baryons is greater than the density in luminous matter (Walker et al. 1991). In any case, the MACHO/EROS discoveries, if confirmed, will have important implications for star formation, galaxy evolution, and cosmology.

In this work, we suggest that the lenses are dense clouds rather than compact objects. While brown dwarfs or stellar remnants are appealing dark matter candidates (they are after all, the only dark matter candidates known to exist) the discovery that stellar-type compact objects make up a significant fraction of the halo dark matter would be surprising from the standpoint of both theory and observation (see, for example, Liebert and Probst 1987). Dark baryon clouds represent an alternative way of hiding large amounts of baryons (Pfenniger, Combes, and Martinet 1993 (PCM), Pfenniger and Combes 1993 (PC)).

In Section II we derive light curves for a number of extended lens models illustrating that a variety of clouds can explain the MACHO and EROS events. In Section III we discuss observational and theoretical evidence for baryonic clouds. Our focus from the observational side is to connect the lensing events with ‘Extreme Scattering Events’ observed in radio measurements of quasars (Fiedler et al. 1987 (F1), Fiedler et al. 1993 (F2)). Our theoretical discussion attempts to make plausible the idea that dense baryon clouds are a long-lived, if not final phase in the evolution of primordial baryonic matter. It also indicates that the clouds will be distributed in mass and size suggesting observations which can test the theory.
## II. Lensing by Extended Objects

Consider a compact gravitational lens of mass $M$ at distance $l$ from the observer and a source at distance $L$. Significant amplification occurs when the lens passes within a distance $\xi_0 \equiv \sqrt{4GMD/c^2}$ of the line of sight to the source where $D \equiv l(L - l)/L$. $\xi_0 = 4 \times 10^{13} \text{ cm} (M/M_\odot)^{1/2} (D/\text{kpc})^{1/2}$ is often referred to as the Einstein radius. For a source in the LMC and a 0.1 $M_\odot$ lens in the halo of our galaxy, $D \lesssim 13 \text{ kpc}$ and $\xi_0$ corresponds to roughly $10^{-4}$ arcsec on the sky. An extended object with a characteristic size $R \ll \xi_0$ will behave much like a point mass lens while an object with $R \gg \xi_0$ will not cause significant amplification. It is the intermediate regime that interests us here.

Now consider a few simple examples of extended lenses (see, for example, Schneider et al., 1992, and references therein). The lenses are assumed to be transparent at the frequencies where lensing is observed. Furthermore, for simplicity, only lenses that are axially symmetric about the line connecting the center of the lens to the observer are considered. Let $\Sigma(\xi) = \Sigma_{cr} \kappa(x)$ be the surface density where $\xi = \xi_0 x$ is the distance from the center of the lens in the lens plane and $\Sigma_{cr} \equiv c^2/4\pi GD$ defines the critical surface density. The lens equation,

$$y = x - \frac{m(x)}{x},$$  \hspace{2cm} (1)

determines where the light ray from the source crosses the lens plane. Here $m(x) = 2 \int_0^x x'dx' \kappa(x')$ is the (dimensionless) mass within a circle of radius $x$ and $y = \eta/\eta_0$ is the (dimensionless) position of the source in the source plane where $\eta_0 = \xi_0 L/l$. In general, there can be more than one solution to Eq. (1) corresponding to multiple images of the source. For each image $i$, the amplification is given by

$$A_i = \left(\frac{y \ dy}{x \ dx}\right)_i^{-1}.$$  \hspace{2cm} (2)

Microlensing refers to the case where the images cannot be resolved. What is measured is then the total amplification, $A \equiv \Sigma_i A_i$.

For the point mass lens, $m(x) = 1$, there are two images, and

$$A = \frac{y^2 + 2}{y\sqrt{y^2 + 4}}.$$  \hspace{2cm} (3)

Light curves are constructed by setting $y = (y_0^2 + \tau^2)^{1/2}$ where $\tau = vt/\xi_0$, $t$ is time, and $v$ is the transverse velocity of the lens assuming source and observer are fixed (Paczynski
1986). (See also Griest (1991) for a more general discussion where the velocities of observer, lens, and source are taken into account.)

A particularly simple model for an extended lens is the singular isothermal sphere (SIS). Here, \( \rho(r) = \sigma^2/2\pi Gr^2 \), \( \Sigma(\xi) = \sigma^2/2G\xi \) and \( \xi_0 = 4\pi(\sigma/c)^2D \) where \( \sigma \) is the line-of-sight velocity dispersion. Of course real clouds have a large radius cut-off so that the total mass is finite, and an inner core radius which removes the singularity at \( r = 0 \). Still, the SIS works fine as a model for realistic isothermal spheres so long as the light ray from the source crosses the lens plane outside the core radius and inside the cut-off radius. There are either one or two images depending on the value of \( y \) and the total amplification is

\[
A = \begin{cases} 
2/y & 0 \leq y \leq 1 \\
(1 + y)/y & 1 \leq y
\end{cases}.
\]

Again, light curves are found by setting \( y = (y_0^2 + \tau^2)^{1/2} \). In Fig. 1a we compare light curves for the point mass and SIS lenses. The curves are chosen to match the MACHO group event (Alcock 1993). In particular, \( y_0 \) is chosen for each model so that the peak amplitude is \( A_{\text{max}} = 6.86 \). The horizontal scale is set so that the peak occurs at \( \tau = 0 \) and \( A = 2.0 \) at \( \tau = \pm 1 \). For the MACHO event, \( A = 2 \) occurs at \( t \simeq \pm 9 \) days from the peak. Using this we find that \( M = 0.06 M_\odot D_{10}^{-1} v_{220}^{-2} \) for the point mass lens and \( \sigma = 2 \text{ km/sec } D_{10}^{-1} v_{220}^{-2} \) for the SIS lens where \( D = D_{10} \text{ kpc and } v = v_{220} \text{ km/sec} \).

A second model is a spheroid of constant density \( \rho_0 \), and radius \( R = (3M/4\pi\rho_0)^{1/3} \equiv \xi_0 x_0 \). In this case, there are one, two, or three images depending on the values of \( y \) and \( x_0 \). The lens equation is solved numerically and one can once again generate light curves to match the observations. Fig. 1b for example, compares theoretical light curves for a spheroid lens with \( x_0 = 0.6 \) and \( M = 0.04 M_\odot D_{10}^{-1} v_{220}^{-2} \) and the point mass lens of Fig. 1a. The cusps occur where the number of images changes from 1 to 3 and the amplification formally becomes infinite though not if we take the finite extent of the source into account.

Our finally example is a disc of radius \( R = \xi_0 x_0 \) with constant surface density \( \Sigma(x) = M/\pi R^2 = \Sigma_{\text{cr}}/x_0^2 \). The model is meant to correspond to an object flattened due to rotation and viewed face on. Again, there are one, two, or three images depending on the values of \( y \) and \( x_0 \). In order to fit the MACHO event we require that \( x_0 < 1 \). For
this case

\[
A = \begin{cases} 
\frac{y^2+2}{y\sqrt{y^2+4}} + \left(\frac{x_0^2}{1-x_0^2}\right)^2 & \text{for } 0 \leq y \leq \frac{1-x_0^2}{x_0} \\
\frac{y^2+2+y\sqrt{y^2+4}}{2y\sqrt{y^2+4}} & \text{for } \frac{1-x_0^2}{x_0} \leq y
\end{cases}
\]  

(5)

Fig. 1c compares the disc model \((x_0 = 0.6, M = 0.05 M_\odot D_{10}^{-1} v_{220}^2)\) and the point mass lens discussed above.

One should keep in mind that the data for the MACHO and EROS events consists of 10-20 measurements for each light curve with error bars varying from point to point and typically in the range \(\Delta A \sim 0.1 - 1.0\). Clearly, these measurements are not detailed enough to distinguish among the models; the error bars are too large to rule out, say the SIS and the sampling rate is not high enough to pick up the cusps which occur, for example, in the spherical model. However, a single event with better sampling and smaller error bars would be able to confirm or severely constrain the extended lens hypothesis.

III. Baryonic Clouds : Observation and Theory

Clearly extended objects can cause gravitational amplification consistent with the MACHO and EROS events. But what are these objects? One possibility is that they are virialized clouds of collisionless, exotic particles such as axions or supersymmetric particles. While some small-scale clumpiness in collisionless particles is likely to occur, indications are that the objects which form are either too low in density (Hogan and Rees 1987, Silk and Stebbins 1993) or too small (Kolb and Tkachev 1993).

Less speculative but more interesting is the possibility that the clouds are composed of baryonic matter. An object of mass 0.05 \(M_\odot\) and radius \(2 \times 10^{13}\) cm has a density \(\rho \simeq 3 \times 10^{-9}\) g/cm\(^3\), and velocities \(\simeq 2\) km/sec. For atomic hydrogen, this would correspond to a number density \(n_H \simeq 2 \times 10^{15}\) cm\(^{-3}\) and a column density \(N_H \simeq 10^{29}\) cm\(^{-2}\). Charged particles, complex molecules, and dust grains can all absorb light in the visible, obscuring the background source, and so we have strict limits on the ionization fraction and molecular and dust densities in the clouds. For example, the ionization fraction must be less than about \(10^{-6}\). The density of 5000 Å dust grains must be \(\lesssim 10^{-6}\) cm\(^{-3}\).

The above arguments suggest clouds formed from a primordial mixture of hydrogen and helium. We now discuss both the observational evidence for their existence and theoretical evidence that they represent a long-lived phase in the history of primordial matter. Direct observation of the baryonic clouds will be difficult because they are
fairly compact. For example, a cloud $1 \text{ AU} \simeq 1.5 \times 10^{13} \text{ cm}$ in size and $1 \text{ kpc}$ away subtends $10^{-3}$ arcsec on the sky. Take, for example, the $21 \text{ cm}$ line. An optically thick cloud at $500^\circ \text{K}$ (corresponding to a characteristic velocity of $2 \text{ km/s}$) produces a flux of only $\simeq 2 \times 10^{-9} \text{ Jy}$. Even a VLA beam of $1$ arcsec, which would have something like $100 - 1000$ objects in the beam (depending on the distribution of objects with size), would receive a background signal at the level of $\mu \text{Jy}$, well below current detection levels. However, the clouds may produce other effects. Indeed, related observations may well be the “Extreme Scattering Events” (ESEs) seen in radio measurements of quasars (F1, F2). At $3 \text{ GHz}$, ESEs are characterized by a flat-bottomed flux minimum bracketted by flux maxima. In most, but not all cases, the $8 \text{ GHz}$ flux does not show any unusual behaviour. So far, ten ESEs have been identified with timescales ranging from $0.2 - 1 \text{ yr}$. An ESE may have also been observed in radio observations of the millisecond pulsar PSR 1937+21 (Cognard et al. 1993). Here, an unusual event is seen in both the flux density and timing measurements of the pulsar’s signal. F1 argue that ESEs are due to occultations by localized regions where the electron density is $4 \times 10^3 \text{ cm}^{-3}$. These regions are typically $10 \text{ AU}$ in size and $1 \text{ kpc}$ away. Scattering of a given ray of light can be by many electron clumps, as in the statistical approach of F2, or by a single localized region with a smoothly varying electron density, as in Romani et al. (1987). Similarly, Cognard et al. (1993) argue that their event is due to the passage of a fairly small ($R \simeq 0.05 \text{ AU}$) and slow moving ($v \simeq 15 \text{ km/sec}$) cloud with an electron density of $n_e \simeq 250 \text{ cm}^{-3}$.

ESEs tell us something about the electron densities in the intervening cloud. To determine the mass of the cloud, we need to know the ionization fraction. Using an ionization fraction for dense clouds of $10^{-7}$ as estimated, for example by Genzel (1992, p. 342), PCM and PC find a hydrogen density of $10^{10} \text{ cm}^{-3}$, consistent with their picture of hydrogen clouds as galactic dark matter (see below). These densities are too small for gravitational lensing (at least for a $10 \text{ AU}$ object). Here, we imagine a much denser cloud ($n_H = 10^{15} \text{ cm}^{-3}$) with a low ionization fraction in the core. Electron densities of $4 \times 10^3 \text{ cm}^{-3}$ might be indicative of a more highly ionized halo. Simple calculations show that the UV background, for example, will only ionize the halo of an otherwise pure atomic hydrogen cloud with electron densities in the halo consistent with what is required for an ESE.

Clearly, the simultaneous observation of a gravitational lensing event and an ESE would be powerful evidence in favour of our picture. Remarkably, inspection of the data for 1502+106 (F2 and Figs. 2a, 2b) shows a candidate event. F2 interpret the
variability of this source between ∼ 1985.0 and ∼ 1987.0 as due to an ESE superimposed on a flare intrinsic to the source. The necessary coincidence is explained in terms of the abrupt appearance of a point-like component to the source associated with the flare which enhances the likelihood of an ESE. Our interpretation is that a super-dense cloud having a core density of $n_H \simeq 10^{15}$ cm$^{-3}$ and mean ionization fraction of $\sim 10^{-11} - 10^{-12}$ has passed close to the line-of-sight to the source. The light curve will be affected by gravitational amplification, electron scattering, and the finite extent of the source. In general, the emitting region of the source is larger at longer wavelengths. This, together with the fact that electron scattering is also greater at longer wavelengths implies that the 8 GHz light curve will be closer to a pure gravitational lensing event. We fit the 8 GHz light curve taking as a model lens a constant surface density disc with $R = 2.5 \xi_0 = 6 \times 10^{14}$ cm and $M \simeq 3 M_\odot D_{10}^{-1} \nu_{220}^2$. The predicted light curve is shown by the dashed lines in Figs. 2a and 2b. As predicted, the fit is much better in the shorter wavelength channel.

F2 estimate that there are roughly 250 – 450 clouds per arcsec$^2$. This is comparable to but somewhat higher than the number density of gravitational microlenses that would be inferred from the MACHO and EROS results. Our suggestion is therefore that ESEs and the MACHO and EROS events are caused by related objects but not that all partially ionized clouds capable of causing an ESE are dense enough to cause gravitational amplification.

The question remains as to whether dark baryonic clouds are theoretically plausible and, more to the point, whether such objects are preferred over low mass stars and brown dwarfs in the evolution from primordial densities. New physics, namely different stellar mass functions for the disc and halo, would be required to explain a large population of brown dwarfs or low mass stars in the halo. This may not be so difficult to imagine as halo material is lower in metals and dust which are important for cooling. Moreover, magnetic fields, important for removing angular momentum from a rotating and collapsing cloud, may be absent. The conjecture that baryons in the halo end up in clouds rather than compact objects takes this one step further by suggesting that star formation, at least in its final phases, is far less efficient in the halo than in the disc.

Consider first the simple case of an isolated cloud. Recall that our prototype lens has $M \simeq 0.1 M_\odot$, $R \simeq 10^{14}$ cm and $\sigma \simeq 2$ km/sec. For an isothermal sphere, the corresponding temperature is $T \simeq 500^\circ K$ and the mass of such an object is roughly the Jeans mass $M_J = (\pi kT/Gm_p\mu)^{3/2} \rho^{-1/2}$ where $m_p$ is the mass of the proton and $\mu$ is the mean molecular weight. Objects more massive than the Jeans mass might persist
if they are supported in two dimensions by rotation provided that the Toomre stability condition \( Q \equiv v_s \kappa / \pi G \Sigma \gtrsim 3 \) (Binney and Tremaine, 1987, Chpt. 6) is satisfied. In this expression, \( v_s \) is the sound speed and \( \kappa \) is the epicyclic frequency. Suppose we have an isothermal disc whose characteristic thickness \( z_0 \) is much smaller than its radius. It is straightforward to show that \( \sigma_z^2 = 2\pi z_0 G \Sigma \) where \( \sigma_z \) is the velocity dispersion perpendicular to the plane of the disc (Binney and Tremaine 1987, p.282 and Spitzer 1942). Assuming \( \Sigma = \Sigma_{cr} \), \( v_s = \sigma_z \), and \( \kappa = v_c/R \) where \( v_c \) is the circular velocity at the edge of the disc, the Toomre stability criterion becomes

\[
\frac{v_c}{c} \left( \frac{z_0}{R} \right)^{1/2} \gtrsim 3 \times 10^{-5}
\] (6)

where we have set \( D = 10 \text{ kpc} \) and \( R = 2.5 \times 10^{13} \text{ cm} \). The above condition can be easily satisfied though one should keep in mind that there is no guarantee such an object will be stable to very high order, nonaxially-symmetric instabilities if the Jeans mass becomes very small with respect to the total mass.

A Jeans mass object which radiates a significant fraction of its energy will of course contract. For a pure hydrogen and helium cloud, radiation is due mainly to molecular hydrogen transitions. Palla, Salpeter, and Stahler (1983) study the evolution of just such a cloud. They find that the high density phase of the cloud’s evolution is roughly independent of the starting conditions and is greatly affected by the formation of \( \text{H}_2 \) through 3-body reactions. Their Fig. 3 shows that the temperature, density, and Jeans mass all pass through typical values for the clouds we require. Nevertheless, by their calculations, the clouds continue to evolve through this region on the free-fall time-scale due to the overlap in density of the molecular cooling regime with the regime of collisional ionization and hence of bremsstrahlung cooling. The calculation is however dynamically naive (rotation, inhomogeneity and magnetic fields are neglected) and it does not seem impossible for this overlap to be reduced leaving a phase with very low fractional levels of both \( \text{H}_2 \) abundance and ionization. The largely \( \text{HI} \) cloud that results cools very inefficiently and might be quasi-stable.

Another, perhaps more interesting possibility is that there is an ensemble of clouds having a supersonic velocity dispersion. If the collision time is shorter than the cooling time, shock heating can dissociate the \( \text{H}_2 \) well before the occurrence of collisional ionization (Palla, Salpeter, and Stahler 1983).

The picture of an ensemble of clouds is similar, in some respects, to the ideas proposed by PCM and PC. These authors suggest that dark matter around spiral galax-
ies is in the form of cold $H_2$ gas in a fractal structure. The gas is assumed to be in thermal equilibrium with the microwave background and therefore has a temperature of $3^\circ K$. The smallest indivisible elements in the fractal distribution, called clumpuscules, are set by equating the free-fall time with the Kelvin-Helmholtz timescale (Rees 1976) and have a mass $M \simeq 4 \times 10^{-3} M_\odot (T/\degree K)^{1/4} \mu^{-9/4} \simeq 10^{-3} M_\odot$ and radius $R \simeq 2.3 \times 10^{15} \text{ cm} (T/\degree K)^{-3/4} \mu^{-5/4} \simeq 10^{15} \text{ cm}$. The largest structures in the fractal are of the scale of molecular clouds with $M = 10^6 M_\odot$ and $R \simeq 30 \text{ pc}$. PC therefore find a fractal dimension $d \equiv \log (M/M_0)/\log (R/R_0) \simeq 1.7$. They argue that the structures along the fractal will be in statistical equilibrium with coalescence, fragmentation, and evaporation being the main processes. Collisions occur often enough to disrupt the clumpuscules and prevent collapse and the formation of brown dwarfs or low mass stars. We imagine a similar picture but with a fragmentation temperature of $500^\circ K$ and $d \simeq 1.1$. The resulting fractal structure would then contain objects capable of explaining the MACHO/EROS results. Such a fractal structure would not have a very large sky covering factor. PC (Fig. 8) show for example, that half of the mass in the ensemble would be in $0.3\%$ of the area on the sky for $d = 1.5$ assuming that elements of the distribution are truncated isothermal spheres. The covering factor is even less at $d = 1.1$

Another type of fractal distribution which has a mixture of sub-clouds and diffuse gas on each scale was introduced as a model for molecular clouds by Henriksen and Turner (1984), Henriksen (1986) and reviewed by Henriksen (1991). In this model, a statistical mechanism for solving the well-known angular momentum problem is proposed. In this hierarchically virialized, collisional ensemble, angular momentum is transported from a fraction of the clouds on small scales to larger scales principally by the action of tidal torques during collisions of the small-scale clouds. There is also a fraction of the small-scale clouds that are spun up by the collisional interactions, and consequently one measure of the star formation efficiency would be the fraction that the ‘spun-down’ clouds are of the total. Roughly speaking, star formation is shut off if the collision time of small-scale clouds, $t_c$, is small compared to the time $t_J$ for these clouds to become Jeans unstable due to cooling and angular momentum loss. For in this case, a given cloud will be spun up and heated to virial temperatures by collisions before collapsing. The model leads to a dynamical population of virialized clouds ‘rebounding’ from the small scale $R$ where $t_c(R) \ll t_J(R)$. The smallest clouds in the ensemble may well be the objects responsible for the MACHO and EROS events.

The fractal structures described above are an example of a simple, but very specific
way in which the clouds are distributed in mass and radius, namely one in which there is a one-to-one relationship between mass and radius with $M \propto R^d$. More generally, there will be a distribution of radii for each mass. In any case, the implication is that there will be lensing events of different duration times. For distributions in which $M \propto R^d$ with $d < 2$ (which is usually the case), $R/\xi_0 \propto M^{2-d}$ so that above some characteristic mass, $R \gg \xi_0$ and gravitational amplification becomes negligible. It is in the transition region, where $R = \text{few} \times \xi_0$ and lensing occurs but is easily distinguished from lensing by a compact object, that we have the best hopes of confirming our hypothesis. This makes the 1502+106 light curve all the more intriguing. It also suggests that the MACHO and EROS groups may want to reexamine their selection criteria. In particular, events might be selected whose light curves are roughly independent of frequency but do not fit theoretical light curves calculated assuming point mass lenses. For example, clouds may cause lensing events that are not time-symmetric. Indeed, by selecting candidate events based on light curves derived assuming point mass lenses, we may be significantly underestimating the density of dark baryons in the galaxy.

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FIGURE CAPTIONS

**Figure 1:** Light curves for various extended lens models as compared with the light curve derived assuming a point mass lens. In each figure, the solid line refers to the extended lens and the dashed line refers to the point mass lens. Figs. 1a, 1b, and 1c are for a singular isothermal sphere, a constant density sphere, and a constant surface density disc respectively. The curves are fit to give an amplification factor of 6.86 at the peak (time = 0) and can therefore be used to fit the single event reported by the MACHO collaboration (Alcock et al. 1993). The horizontal scale is normalized so that $A = 1$ at time $= \pm T$. For the MACHO event, $T \approx 9$ days.

**Figure 2:** Radio frequency light curves for the quasar 1502+106 at 8.1 and 2.7 GHz. time = 0 roughly corresponds to November 1985. The dashed line is a theoretical light curve derived assuming a constant surface density disc with radius 2.5 times the Einstein radius (see text). The model was chosen to fit the 8.1 GHz data where scattering by electrons is expected to be negligible and where the quasar can be modeled as a point source.
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