Spontaneous breaking of symmetry in Moyal spacetime with twisted Poincaré symmetry

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Abstract. After briefly reviewing the gauge symmetry in Moyal spacetimes, we analyse aspects of symmetry breaking within a quantisation program preserving the twisted Poincaré symmetry. We develop the LSZ approach for Moyal spacetimes and derive a mapping for scattering amplitudes on these spacetimes from the corresponding ones on the commutative spacetime. This map applies in the presence of spontaneous breakdown of symmetries as well. We also derive Goldstone’s theorem on Moyal spacetime. The formalism developed here can be directly applied to the twisted standard model 1.

Keywords: Gauge fields, Non-Commutative Geometry, twisted Poincaré symmetry
PACS: 11.10.Nx, 11.30.Cp

1. INTRODUCTION

It has been pointed out by many in this Symposium the role of noncommutative geometries when we incorporate quantum gravity[1] Noncommutative geometry is expected to play a role in near horizon geometry of a blackhole[2] as well as near big bang singularity and cosmological constant[3]. The need to go beyond the conventional notions of geometry was anticipated by none other than Riemann himself, given lack of knowledge of physics at infinitesimal length scales. As pointed out by Riemann “the metric relations of space in the infinitely small do not conform to hypotheses of geometry; and we ought in fact to suppose it, if we can thereby obtain a simpler explanation of phenomena....”[4]. Fuzzy physics as provided by the coadjoint orbits of Lie groups [5, 6, 7, 8] and κ space-time geometry[9, 10] are classic examples used in physics literature. One can even consider deformations of Lie algebra leading to topology change in the commutative limit[11]. Here we consider the Moyal space-time which is the most popular example for noncommutative geometry and look at the way it can change our expectations in fundamental physics.

Moyal space-times are defined by the following algebra:

\[ [\hat{x}_\mu, \hat{x}_\nu] = i\theta_{\mu\nu}\mathbb{1} \]  

(1)

It is well known we can obtain the same through the star product rule in the algebra of functions on \( \mathbb{R}^4 \).

\[ f \ast g = m_0(f \otimes g) = m_0(F_\theta(f \otimes g)) \]  

(2)

1 Work done in collaboration with A P Balachandran and Sachin Vaidya
where $F_{\theta} = e^{-\frac{i}{2}(-i\partial_{\mu})g^{\mu\nu}(-i\partial_{\nu})}$. In commutative spacetime we have pointwise multiplication.

### 1.1. Drinfeld twist

As pointed out by Balachandran[1, 12, 13] we have to find a twisted coproduct $\Delta_{\theta}$ compatible with the multiplication map:

$$m[(\rho \otimes \rho) \Delta(g)(a \otimes b)] = \rho(g) m(a \otimes b)$$

where $a, b \in \mathcal{A}_{\theta}(\mathbb{R}^4)$. The above can be shown as commutative diagram (Fig 1).

Indeed such a twisted coproduct for Moyal space is:

$$\Delta_{\theta}(g) = \hat{F}_{\theta}^{-1}(g \otimes g)\hat{F}_{\theta}$$

where $\hat{F}_{\theta} = e^{-\frac{i}{2} p_{\mu} \otimes \theta^{\mu\nu} P_{\nu}}$, $P_{\mu}$ is the generator of translations. It is easy to check that the coproduct is compatible with the multiplication map.

$$m_{\theta}(\rho \otimes \rho) \Delta_{\theta}(g)(a \otimes b) = m_{0} [F_{\theta}F_{\theta}^{-1} \rho(g) \otimes \rho(g) F_{\theta}a \otimes b]$$

### 1.2. Twisted Poincaré invariance and statistics

For $\theta^{\mu\nu} = 0$ statistics is imposed on the two-particle sector by working with the symmetrized or anti-symmetrized tensor product $\mathcal{A}_{0}(\mathbb{R}^4) \otimes_{s,a} \mathcal{A}_{0}(\mathbb{R}^4)$. It has for example

$$v \otimes_{s,a} w = \frac{1}{2}[v \otimes w \pm w \otimes v], \quad v, w \in \mathcal{A}_{0}(\mathbb{R}^4).$$

But the twisted coproduct does not preserve (anti)symmetrization:

$$\Delta_{\theta}(v \otimes_{s,a} w) \notin \mathcal{A}_{0}(\mathbb{R}^4) \otimes_{s,a} \mathcal{A}_{0}(\mathbb{R}^4)$$

We are forced to twist statistics. To achieve this consider $\tau_{0}$ to be the flip map:

$$\tau_{0}(v \otimes w) = w \otimes v.$$  

Then

$$\tau_{\theta} := F_{\theta}^{-1} \tau_{0} F_{\theta} = F_{\theta}^{-2} \tau_{0}$$

**FIGURE 1.** commutative diagram
commutes with $\Delta_\theta$. The product $\mathcal{A}_\theta(\mathbb{R}^4) \otimes_{s_\theta,a_\theta} \mathcal{A}_\theta(\mathbb{R}^4)$ with twisted (anti-) symmetrization is:
\[
v \otimes_{s_\theta,a_\theta} w = \frac{1}{2}[I \pm \tau_\theta](v \otimes w)
\] (10)
Imposing this change on the quantum fields leads to the commutation relations[13]:
\[
a(p)a(q) = e^{ip \wedge q}a(q)a(p), \text{ and } a(p)a^+(q) = e^{-ip \wedge q}a^+(q)a(p) + 2p_0 \delta^3(p-q).
\] (11)

2. DIFFEOMORPHISM AND GAUGE INVARIANCE IN MOYAL SPACE-TIMES

To define diffeomorphisms and gauge symmetries in this framework is very complicated. We will adopt a novel way[14]. Consider $x^c_{\mu} = \frac{1}{2}(x^L_{\mu} + x^R_{\mu})$ where $x^L_{\mu} \alpha = x_\mu \ast \alpha$ and $x^R_{\mu} \alpha = \alpha \ast x_\mu$. It is easy to see
\[
[x^c_{\mu}, x^c_{\nu}] = 0.
\] (12)
This simply means $x^c_{\mu}$ form a basis for commutative algebra $\mathcal{A}_0(\mathbb{R}^4)$. One can define Poincaré group of generators using $x^c_{\mu}$ as
\[
M_{\mu\nu} = x^c_{\mu} p_\nu - x^c_{\nu} p_\mu, p_\mu = -i\partial_\mu
\] (13)
We get modified Leibnitz rule:
\[
M_{\mu\nu}(\alpha \ast \beta) = M_{\mu\nu} \alpha \ast \beta + \alpha \ast M_{\mu\nu} \beta - \frac{1}{2}[(p, \theta)_\mu \alpha \ast p_\nu \beta - (p_\nu \alpha \ast (p, \theta)_\mu \beta - \mu \leftrightarrow \nu]
\] (14)
This is exactly the same as what we get from twisted coproduct $\Delta_\theta$. In the above $M_{\mu\nu}$ in Eq.(13) is a particular vector field. This can be extended to general vector fields $\nu = v^\mu(x^c)\partial_\mu$. These generate the diffeomorphisms on the Moyal spacetime. If we assume the framefields $e^\mu_\nu$ are dependent only on $x^c$ then pure gravity without matter can be treated as in commutative spacetimes. Gauge fields $A_\lambda$ transform as one-forms under diffeomorphisms for $\theta^{\mu\nu} = 0$. For $\theta^{\mu\nu} \neq 0$, the vector fields $v^\mu$ generating diffeomorphisms depend on $x^c$. If a diffeomorphism acts on $A_\lambda$ in a conventional way with $A_\lambda$ and $\delta A_\lambda$ are to depend on just one combination of noncommutative coordinates, then $A_\lambda$ can depend only on $x^c$. In the standard approach to gauge symmetry in noncommutative geometry the covariant derivatives act with the $\ast$-product and this imposes severe constraints. It is possible to have only particular representations of $U(N)$ gauge groups in the theory. We cannot impose standard model group consistently. There is no such limitation now in our novel way of introducing gauge transformations. This approach was inspired by quantum Hall effect, where guiding center coordinates act as a model of Moyal space-times[15]. Here the algebra of observables is $\mathcal{A}_\theta(\mathbb{R}^2) \otimes \mathcal{A}_\theta(\mathbb{R}^2)$. Here too
covariant derivatives of the $U(1)$ electromagnetism do act in the way we have described above and not with a $\ast$ product. But the twisted coproduct on the “global” group $\mathcal{G}$ is,

$$\Delta\theta(g(x^\prime)) = F^{-1}_\theta[g(x^\prime) \otimes g(x^\prime)]F_\theta,$$

and is compatible with the $\ast$-multiplication.

### 2.1. Dressing transformation

The creation/annihilation operators $a(p), a^\dagger(p)$ for the twisted fields can be realized in terms of untwisted Fock space operators $c(p), c^\dagger(p)$ by the “dressing transformation” [16]

$$a(p) = c(p)e^{-\frac{i}{2}p^\dagger P}, \quad a^\dagger(p) = c^\dagger(q)e^{\frac{i}{2}p^\dagger P},$$

where

$$P_\mu = \int d\mu(q)q_\mu[a^\dagger(q)a(q)] = \text{total momentum operator}$$

and $\theta = \int d\mu(q)q_\mu[a^\dagger(q)a(q)] = \text{total momentum operator}$.

If $\phi_1, \phi_2, \cdots \phi_n$ are quantum fields and $\phi_i(x) = \phi_i^c e^{\frac{i}{2}x^\dagger \partial^\wedge P}(x)$, then

$$(\phi_1 \ast \phi_2 \ast \cdots \ast \phi_n)(x) = (\phi_1^c \phi_2^c \cdots \phi_n^c)e^{\frac{i}{2}x^\dagger \partial^\wedge P}(x)$$

For example the interaction Hamiltonian density is:

$$\mathcal{H}_I = \mathcal{H}_I^0 e^{\frac{i}{2}x^\dagger \partial^\wedge P}$$

The covariant derivative should transport consistently with the statistics as well as gauge transformations and hence the natural choice is:

$$D_\mu \phi = ((D_\mu)^c \phi^c)e^{\frac{i}{2}x^\dagger \partial^\wedge P}$$

where $(D_\mu)^c \equiv \partial_\mu + (A_\mu)^c$ and $(A_\mu)^c$ is the commutative gauge field, a function only of $x^\prime$. It is easy to check:

$$[D_\mu, D_\nu] \phi = \left([D_\mu^c, D_\nu^c]\phi^c\right)e^{\frac{i}{2}x^\dagger \partial^\wedge P} = \left(F^c_{\mu\nu} \phi^c\right)e^{\frac{i}{2}x^\dagger \partial^\wedge P}.$$
2.2. Gauge theory on Moyal space-time

Having set up the necessary formalism, we can now write down the interaction Hamiltonian density for pure gauge fields as:

\[ \mathcal{H}^G_{I0} = \mathcal{H}^G_{0}. \]

But when both matter and gauge fields are present, the interaction Hamiltonian density is:

\[ \mathcal{H}^I_{\theta} = \mathcal{H}^{M,G}_{I0} + \mathcal{H}^G_{I0}, \]

where

\[ \mathcal{H}^{M,G}_{I0} = \mathcal{H}^{M,G}_{0} e^{i\theta \partial \wedge P} \]

In QED\(_\theta\), we have \( \mathcal{H}^G_{I0} = 0 \).

\[ S^{QED}_{\theta} = S^{QED}_{0}. \]

On the other hand in QCD\(_\theta\), we have \( \mathcal{H}^{SU(3)}_{I0} \neq 0 \), so that

\[ S^{M,SU(3)}_{\theta} \neq S^{M,SU(3)}_{0}. \]

Lastly we can also look for Standard model\(_\theta\) after discussing spontaneous symmetry breakdown. But first we will develop Lehmann, Symanzik, Zimmerman (LSZ) formalism suitable for Moyal spacetime which will facilitate computations in scattering theory.

3. LSZ ON MOYAL SPACETIME

In standard scattering theory, the Hamiltonian \( H \) is split into a “free” Hamiltonian \( H_0 \) and an “interaction” piece \( H_I \): \( H_0 \) is used to define the states in the infinite past and future. Then the states at \( t = 0 \) which in the infinite past (future) become states by evolving \( H_0 \) as the in(out) states.

\[ e^{-iHT} |\psi, \text{in(out)} \rangle \xrightarrow{T \rightarrow \pm \infty} e^{-iH_0T} |\psi, F \rangle, \quad F \equiv \text{free} \]

Hence

\[ |\psi, \text{in(out)} \rangle = \Omega_\pm |\psi, F \rangle, \Omega_\pm \equiv e^{iHT_\pm} e^{-iH_0T_\mp}, \quad \text{as } T_\pm \rightarrow \pm \infty, \]

Here \( \Omega_\pm \) are the Moller operators. We have:

\[ |\psi, \text{out} \rangle = \Omega_- \Omega_+ |\psi, \text{in} \rangle \]

If the incoming(outgoing) state is \( |k_1, k_2, \cdots k_N, F \rangle \), it follows that

\[ |k_1, k_2, \cdots k_N, \text{in(out)} \rangle = \Omega_\pm |k_1, k_2, \cdots k_N, F \rangle \]

has eigenvalue \( \sum k_{i0} \) for the total Hamiltonian \( H \). The scattering amplitude is:

\[ \langle \psi, \text{out} | \psi, \text{in} \rangle = \langle \psi, \text{in} | \Omega_+ \Omega_-^\dagger |\psi, \text{in} \rangle \]
The LSZ $S$-matrix is
\[ S = \Omega_+ \Omega_+^\dagger, \quad |\psi, \text{out}\rangle = S^\dagger |\psi, \text{in}\rangle \] 
\hspace{1cm} (32)

Between the “free” states, the $S$-operator is different:
\[ \langle \psi, \text{out}| \phi, \text{in}\rangle = \langle \psi, F| \Omega_+^\dagger \Omega_+ |\phi, F\rangle \] 
\hspace{1cm} (33)

The LSZ formalism works exclusively with in- and out-states, as Haag’s theorem shows that $\Omega_\pm$ do not exist for quantum field theories. The operators $a_k^{\text{in(out)}}$, $a_k^{\text{in(out)}}$ are introduced to create states $|k_1, k_2, \cdots k_N, \text{in(out)}\rangle$ from the vacuum. The in- and out-fields $\phi_{\text{in(out)}}$ are then defined using superposition. They look like free fields, but are not, since for the total four-momentum $P_\mu$, we have
\[ P_\mu |k_1, k_2, \cdots k_N, \text{in(out)}\rangle = (\sum_i k_i \mu) |k_1, k_2, \cdots k_N, \text{in(out)}\rangle. \] 
\hspace{1cm} (34)

We also assume:

1. The vacuum and single particle states are unique.
2. There is only one vacuum $|0\rangle$, $\langle 0|0\rangle = 1$, and $S|0\rangle = |0\rangle$ 
\hspace{1cm} (35)
3. There exists an interpolating field $\phi$ between in- and out- states in weak topology:
\[ \phi - \phi_{\text{in,out}} \to 0 \quad \text{as} \quad \tau \to \pm \infty \] 
\hspace{1cm} (36)

Then LSZ show that
\[ \langle k'_1, \cdots k'_N, \text{out}| k_1, \cdots k_N \text{in}\rangle = \mathcal{S} \ G_N(x_1, x'_1, \cdots, x_N, x'_N) \] 
\hspace{1cm} (37)

where
\[ \mathcal{S} = \int \prod d^4 x'_i \prod d^4 x_j \ e^{i(k'_i \cdot x'_i - k_j \cdot x_j)} i(\partial_i^2 + m^2) \cdot i(\partial_j^2 + m^2) \] 
\hspace{1cm} (38)

and
\[ G_N \equiv \langle 0|T(\phi(x_1)\phi(x'_1)\cdots\phi(x_N)\phi(x'_N))|0\rangle \] 
\hspace{1cm} (39)

We have argued that the noncommutative field theory comes from the commutative one by the replacement:
\[ \phi_\theta = \phi_0 e^{\frac{i}{2} \tilde{\theta} \wedge \mathcal{P}}. \] 
\hspace{1cm} (40)

This consistently twists the in- and out- fields:
\[ \phi_\theta^{\text{in(out)}} = \phi_{\text{in(out)}} e^{\frac{i}{2} \tilde{\theta} \wedge \mathcal{P}}, \quad \phi_0 \to \phi_\theta^{\text{in(out)}} \quad \text{as} \quad t \to \pm \infty \] 
\hspace{1cm} (41)

LSZ holds for scattering amplitude with $G_N$ changed to $G_N^\theta$:
\[ G_N^\theta(x_1, \cdots x_N) = T e^{\frac{i}{2} \sum_{i<j} \hat{\partial}_i \wedge \hat{\partial}_j \mathcal{W}_N^0(x_1, \cdots x_N)} \] 
\hspace{1cm} (42)
and $W_0^N$ are the standard Wightman functions for untwisted fields:

$$W_0^N(x_1, \cdots x_N) = \langle 0 | \phi_0(x_1) \cdots \phi_0(x_N) | 0 \rangle. \quad (43)$$

It is important that because of translational invariance, the $W_N$ (and hence the $G_N$) depend only on coordinate differences. For simplicity, we have included only matter fields, and that too of one type only, in (39). Gauge fields can also be included, but they are not acted on by the twist exponential in (42).

### 3.1. Gell-Mann-Low formula for Moyal spacetime

We assume Heisenberg fields $\phi$ obey the same canonical algebra as free fields $\phi_F$ at $t = 0$. The interaction representation Hamiltonian is

$$H_I(t) = e^{itH_0}H_I(0)e^{-itH_0}, \quad H_I(0) = H_I \quad (44)$$

The time evolution operator is:

$$U(t_1, t_2) = T \exp \left( -i \int_{t_1}^{t_2} dt H_I(t) \right), \quad (45)$$

Then Gell-Mann and Low (G L) formula show:

$$G_N(x_1, x_2, \cdots x_N) = \frac{\langle 0, F | T \left( \phi_F(x_1) \cdots \phi_F(x_N)e^{i \int d^4x \gamma^\mu \partial_\mu} \right) | 0, F \rangle}{\langle 0, F | e^{i \int d^4x \gamma^\mu \partial_\mu} | 0, F \rangle} \quad (46)$$

G L formula applies to $G_0^N$. We can rewrite the result in terms of the Wightman functions $W_N(x_1, \cdots x_N)$:

$$W_N(x_1, \cdots x_N) = \langle 0 | \phi(x_1) \cdots \phi(x_N) | 0 \rangle \quad (47)$$

Then,

$$G_0^N(x_1, \cdots x_N) = Te^{i \Sigma_{<I} \partial_I \otimes \partial_J} W_N(x_1, \cdots x_N) \quad (48)$$

This results in shifting of each $x_I^0$ to

$$x_I^0 + \delta x_I^0, \quad \delta x_I^0 = \delta x_I^0(k_1, \cdots k_N). \quad (49)$$

where the $\delta x_I^0$ actually depend on the ordering on $x_I^0$.

We emphasize couple of important observations.

1. Firstly, (37) involves only the $\theta^{\mu\nu} = 0$ fields in $W_N^0$. So it can be used to map any commutative theory to noncommutative one, including the standard model. But special care is needed to treat gauge fields. Gauge fields are not twisted unlike matter fields. As explained elsewhere, this means the Yang-Mills tensor is not twisted, $F_0^{\mu\nu} = F_0^{\mu\nu}$. But covariant derivatives of matter fields $\phi_\theta$ are twisted:

$$\langle D_\mu \phi_\theta \rangle = \langle D_\mu \phi_0 \rangle e^{\frac{i}{2} \theta^{\mu\nu} \gamma_5 \partial_\nu} \gamma^\lambda \gamma^\rho. \quad \text{where } \langle D_\mu \phi_0 \rangle \text{ is the untwisted covariant derivative of the untwisted } \phi_0. \text{ Thus in correlators } W_N^\theta, \text{ we must use } \langle D_\mu \phi_\theta \rangle \text{ for matter fields, } F_0^{\mu\nu} \text{ for Yang-Mills tensor.}$$

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2. There are ambiguities in formulating scattering theory. At this moment, lacking a rigorous scattering theory, we do not know the correct answer. In this connection, we must mention the important work of Buchholz and Summers [17] which rigorously develops the wedge localisation ideas of Grosse and Lechner [18] to establish a scattering theory for two incoming and two outgoing particles.

For calculating (37), we need a formalism for doing perturbation theory to compute Wightman functions. Once we have that, we can calculate the time-ordered product by writing it in terms of Wightman functions and twist factors. The details of these can be found in our paper [14].

3.2. Golstones’ theorem on Moyal spacetime

We will now take up the question of the spectrum in Moyal spacetimes. In answering this question spontaneous breakdown of symmetry and Goldstones’ theorem plays an important role. The noncommutative currents \( \tilde{J}_{a,\mu}^{\theta}(x) = J_0^{a,\mu}(x)e^{\frac{i}{2} \tilde{\partial} \wedge P} \) are conserved:

\[
\partial_{\mu} \tilde{J}_{a,\mu}^{\theta}(x) = \partial_{\mu} J_0^{a,\mu}(x)e^{\frac{i}{2} \tilde{\partial} \wedge P} = (\partial_{\mu} J_0^{a,\mu}(x))e^{\frac{i}{2} \tilde{\partial} \wedge P} = 0.
\]

The vacuum expectation value of the currents \( \tilde{J}_{a,\mu}^{\theta}(y) \) and the quantum field \( \phi_i,\theta(x) \):

\[
\langle 0|\left[e^{\frac{i}{2} \tilde{\partial} \wedge P}J_0^{a,\mu}(y), \phi_i,0(x)e^{\frac{i}{2} \tilde{\partial} \wedge P}\right]|0\rangle = \langle 0|\left[J_0^{a,\mu}(y), \phi_i,0(x)\right]|0\rangle
\]

This commutator is the same as the one for the corresponding commutative case. Using spectral density and twisted Lorentz invariance we can infer the existence of massless bosons.

4. HIGGS\(_{\theta}\) MECHANISM

Consider a set of scalar fields coupled to a gauge field with a local symmetry group \( G \). This dynamics is described by:

\[
\mathcal{L} = \text{Tr} \left( -\frac{1}{4} F_{\mu\nu}^2 + |D_\mu \phi|^2 - V(\phi) \right)
\]

The Higgs potential

\[
V(\phi) = \lambda (\phi^\dagger \phi - a^2)^2 \]

\[
= \lambda (\phi_c^\dagger \phi_c - a^2) e^{\frac{i}{2} \tilde{\partial} \wedge P}
\]

We assume the breaking \( G \rightarrow H \). In vacuum

\[
\langle \phi_c \rangle = \phi^0, \quad \phi^0 \wedge \phi \wedge \phi = a^2, \quad h \phi^0 = \phi^0, h \in H
\]
4.1. Mass of the gauge boson

The vacuum manifold is:

\[ \phi = g \phi^0, \quad g \in G, \text{ and } (gh) \phi^0 = g \phi^0 \]  

(56)

The gauge field acquires mass and is given by the term:

\[ M = (D_\mu \phi)^\dagger (D^\mu \phi) = [(D_\mu^c \phi_c)^\dagger (D^{\mu c} \phi_c)] e^{\frac{i}{2} \mathcal{A} \cdot \mathcal{P}} \]  

(57)

If \( V(\alpha), S(i) \) are basis of orthonormal generators of Lie algebra \( G \) of \( G \), then:

\[ V(\alpha) \phi^0 = 0 \]  

(58)

If a gauge transformation is performed from \( A^c_\mu \rightarrow B^c_\mu \) where \( B^c_\mu = g^\dagger D^c_\mu g \), then,

\[ M = \phi^{c\dagger \alpha}(B^c_\mu B^{\mu c})_{\alpha \beta} \phi^c_\beta \]  

(59)

As usual we write:

\[ B^c_\mu = B^c_\mu^\alpha V_\alpha + B^c_\mu^i S_i \]  

(60)

Then we get:

\[ M = (D^c_\mu \phi^c)^\dagger (D^{\mu c} \phi^c) = \phi^{0\dagger S_i B^i_\mu B^{\mu j} S_j \phi^0 + \cdots} \]  

(61)

This shows gauge fields in the direction of \( V_\alpha \) do not acquire mass and only those in the direction of \( S_i \) do. \( B^c_\mu \) is the gauge transformation of \( D^c_\mu \). This preserves the pure gauge Hamiltonian \( H_{10} = H_{10} \). After gauge fixing the Hamiltonian with the mass term is:

\[ H_0 = \int \left\{ \partial \cdot B^c + (\partial^0 B^i - \partial^i B_0)^2 + \cdots + M \right\} \]  

(62)

The Hamiltonian \( P_0 = H \) and the spatial translation generator for the twisted standard model are the same as for the case \( \theta^{\mu \nu} = 0 \) except for the twist factors. What is changed in addition in our LSZ approach are the in and out fields which are twisted as discussed. Hence the scattering calculations can be based on appropriately modified Wightman functions. Detailed calculations for specific processes will be considered in our future papers.

ACKNOWLEDGMENTS

This work is based on the joint work with A P Balchandran(APB) and Sachin Vaidya(SV). It was supported by the Department of Science and Technology, India program CPSTIO. I thank APB and SV for discussions.
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