A Macroscopic Gravity Wave Effect

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Abstract

Gravitational waves, although generally associated with extremely microscopic effects, can displace by hundreds of kilometers the pulsar interstellar scintillation patterns that bathe the Earth. The combination of the pulsar and the interstellar medium acts as a kiloparsec-long, nature-provided gravity wave amplifier. We show how an effective scheme for the detection of periodic gravity waves can be constructed based on this effect. This approach to detection does not require the development of new, ad hoc technology, but the optimization of existing observational techniques in a few different fields of astronomy. Part of the scheme is a new, purely numerical detection technique that can also be used in the data processing of other projects of periodic gravity wave detection.
It was recently realized that gravity waves can interfere with interstellar scintillation in a potentially observable way [1]: Pulsar radio waves are scattered by electron density inhomogeneities in the interstellar medium. This results in a pattern of intensity maxima and minima of typical size $\bar{S}$ in the vicinity of the Earth. As the latter moves through the pattern with a relative velocity $V_r$, the average pulse intensity appears to fluctuate on a timescale $\bar{t}_s = \bar{S}/V_r$. This is interstellar scintillation. (For an excellent review and references, see [2,3].) When a foreground gravity wave source such as a binary star lies close enough to the pulsar line-of-sight, the steering effect of the gravity waves [4,5] moves the scintillation pattern quasi-rigidly and periodically by a distance $D = \alpha_{gw} L$, where $\alpha_{gw}$ is the deflection angle due to the close encounter of the radio waves with the gravity wave source, and $L$ is the distance from that source to the Earth.

This amounts to a natural amplification of the gravity wave deflection effect by the gigantic lever arm $L$. A preliminary search by C.F. Quist [6] has identified several actual cases of pulsar-binary star alignment for which the displacement $D$ could reach hundreds of kilometers (examples below.) Hence the perhaps unexpected possibility that gravity waves could be responsible for natural periodic phenomena that are macroscopic.

Several approaches to the detection of this effect have been envisaged, two of which are discussed here (schemes 1 and 2), the second of the two being experimentally the most advantageous. In both schemes, part of the detection is done purely numerically in a manner that is akin to stochastic resonance. Under certain conditions, the noise in a nonlinear dynamic system, instead of being a hindrance to signal detection, can be used to reveal the presence of a weak (weaker than the noise) periodic signal. This is stochastic resonance [7]. Paradoxically, the signal in some such systems would not be detectable at all in the absence of noise. In our case, the raw data is the noisy pulsar intensity time series $I(t)$, which is a random distribution reflecting the passage of the Earth through the irregular scintillation pattern (fig.1). In the following, we turn the stochastic resonance argument around to obtain a method for detecting weak periodic gravity wave signals. This method is independent of (and often more efficient than) the usual period folding of data strings. The method can also be adapted to almost any other scheme of periodic gravity wave detection.

However, before stochastic resonance ideas can be used to detect the gravity wave effect above, two problems need to be solved: First, gravity
waves modulate the phase of the intensity fluctuations ($\tilde{I}(t) = I(t + \delta t)$, where $\tilde{I}(t)$ is the intensity in the presence of gravity waves) whereas a priori stochastic resonance ideas only apply to amplitude modulations (problem 1.) Second, stochastic resonance is usually shown to arise from a nonlinearity in the system’s dynamics, but no such dynamics is at play in our case (problem 2.) We shall study two different solutions to problem 1, each leading to a different detection scheme. Problem 2 is then addressed separately within each scheme.

Scheme 1.- Our first approach to solving problem 1 draws on some old and well established methods developed in connection with threshold systems such as electric relays [8,9,10]. We set a pulsar intensity threshold $I_0$ and decide that a peak of intensity is defined by $\tilde{I}(t) > I_0$ (fig.1). This introduces a new stochastic variable: the peak duration $\tau(t)$. It can be shown (e.g. [10]) that this new variable has well defined statistical properties that are derivable from those of the original variable $\tilde{I}(t)$, provided $I_0$ is chosen to be sufficiently large. Since, as seen earlier, the gravity wave induced displacement of the pattern modulates the time spent by the observer inside any particular intensity feature, it directly modulates the amplitude of $\tau(t)$. Thus, the threshold mechanism has turned the phase modulations of $\tilde{I}(t)$ into amplitude modulations of $\tau(t)$, which solves problem 1.

To build a detection scheme around this particular solution to problem 1, we still need to address problem 2: the absence of nonlinear dynamics. The threshold mechanism again provides an answer. This time it is applied to $\tau(t)$ itself. Since $\tau(t)$ is modulated in amplitude, one can devise a purely numerical procedure (based on a chosen threshold $\tau_0$) that is completely analogous to a physical trigger mechanism. Trigger mechanisms being obviously highly nonlinear, this has the potential of solving problem 2.

Let us then arrange that our computer code “fires” (i.e. registers a pulse, see [9,10]) whenever $\tau(t) > \tau_0$. The result is yet another stochastic distribution: a random pulse train $p(t)$. The pulses are square functions with an arbitrary fixed height and an arbitrary fixed small width (fig.1.)

The argument now is that this nonlinear pseudo-dynamic system does exhibit stochastic resonance. That is, if the irregularities in the raw intensity data $\tilde{I}(t)$ are very weakly but coherently phase modulated by gravity waves, this modulation eventually shows up as a periodicity in the pulse train $p(t)$, which is the output of the double threshold mechanism just described (applying thresholds $I_0$ then $\tau_0$; see fig.1.)
That this is indeed the case can be shown analytically by applying the results in [8,9,10] to the case of a weak cosine signal of amplitude $s$ and angular frequency $\omega_{gw}$ buried in Gaussian white noise, which is a good approximation to the physical case at hand [3]. The statistics of the pulse train are obtained from the correlation function $\psi(t) \equiv \langle \tau(0)\tau(t) \rangle$. The rate at which $\tau(t)$ crosses the threshold $\tau_0$ is [9,10]

$$R(\tau_0) = \frac{1}{2\pi} \left( -\frac{\psi''(0)}{\psi(0)} e^{-\tau_0^2/\psi(0)} \right)^{1/2},$$

where primes indicate time derivatives. The power spectrum $P(\omega)$ of $p(t)$ can be calculated either directly or by Fourier transforming the correlation function $<p(0)p(t)>$ (i.e. using the Wiener-Khintchine theorem.)

The signal-to-noise ratio ($SNR$) can then be obtained by deviding the power at the signal frequency $P(\omega_{gw})$ by the power of the output background noise interpolated at $\omega_{gw}$. We thus obtain

$$SNR = \frac{1}{\sqrt{3}} \Delta t \frac{s^2 \tau_0^2}{t_c \sigma^2 \sigma^2} \exp \left( -\frac{\tau_0^2}{2\sigma^2} \right),$$

where $\sigma$ is the standard deviation of the random noise, $\Delta t$ the total duration of the experiment and $t_c$ the average duration of a single measurement. The $\sqrt{3} t_c$ term comes from the Rice threshold crossing rate for Gaussian white noise [9]. It can easily be verified that the results are qualitatively the same for other types of noise, so long as the correlation time of the noise is much smaller than the period of the deterministic signal (which is the case in our observational situation.) In the case of scheme 1, $t_c$ is of the order of the correlation time of the $I(t)$ sequence. Eq.(2) was verified numerically (see below.)

Hence, since $\Delta t \gg t_c$, the mere tuning of the threshold $\tau_0$ (a trivial and purely numerical procedure) can make $SNR > 1$ even if the noise is much louder than the periodic signal ($\sigma >> s$). In stochastic resonance, it is usually the noise level that is adjusted in order to achieve detection. In our case, where the noise level $\sigma$ is a fixed observational constraint, we maximize $SNR$ by adjusting the threshold $\tau_0$ ($\tau_0 \approx \sigma$.) In this way, one can a priori detect any periodic signal with an amplitude exceeding $s_{min} \approx \sigma \sqrt{t_c/\Delta t}$. For very weak signals, some fine tuning of $\tau_0$ is necessary when confirming this numerically.
To maximize the action of the interstellar medium, which is the masterpiece of the “natural” detector, one should observe using the lowest possible electromagnetic frequencies. This means observing at about 50 MHz, which corresponds to $t_c \sim 1\text{min}$ for several pulsars [2,3]. On the other hand, a realistic value for the total duration of the experiment is $\Delta t \approx 10^8 \text{sec}$. These numbers result in $s_{\text{min}} \approx \sigma/10^3$. Thus, this mechanism is in principle capable of detecting signals that are 1000 times weaker than the noise.

In practice, this sensitivity will surely be reduced by the variance of the periodograms $P(\omega)$ which are actually calculated and which approximate the power spectrum $P(\omega)$. We shall return to this point further below when we apply similar techniques to the more sensitive scheme 2. However, two remarks should be made here. (1) The detection procedure described above is independent of the gravitational wavelength: it can be applied to detect gravity waves with periods $T_{gw}$ anywhere between $T_{gw} = t_c$ and $T_{gw} = \Delta t$. (2) The above procedure is unrelated to the usually smaller sensitivity increase (about $\sqrt{\Delta t/T_{gw}}$) that can be achieved by folding the data over the gravity wave period $T_{gw}$. For binary stars, $\sqrt{\Delta t/T_{gw}}$ is usually smaller than 10, while numerical simulations show that the above detection technique is sensitive in practice at the $\sigma/s \approx 100$ level, even before any periodogram variance reduction. For the fastest binaries ($T_{gw} \sim 1 \text{hour}$), the theoretical sensitivity increase from period folding $\sqrt{\Delta t/T_{gw}}$ is still about one order of magnitude smaller than the theoretical sensitivity increase from the above method $\sqrt{\Delta t/t_c}$. In fact, folding the data first and then using the new method represents a double sensitivity enhancing procedure (operating in the time domain) that is an alternative to applying the new method first as was done above, and then performing a periodogram variance reduction procedure (thus operating in the frequency domain.)

We have learned recently that the strength of this gravity wave numerical detection method is in fact a manifestation of a broader phenomenon (dubbed non-dynamic stochastic resonance) discovered lately by F. Moss [11] quite independently from gravity wave considerations. Note however that the signal-to-noise ratio formula eq.(2) above is somewhat different from its counterpart in [11]: (1) it is dimensionless and (2) it increases with the total observation time $\Delta t$. Moreover, motivated by specifically gravity wave constraints, our numerical code is designed to detect periodic signals that are much weaker with respect to the noise, and also, it does not take advantage
of periodogram averaging. (More details below.)

*Scheme 2.*- We now go back to the problem that gravity waves modulate the phase rather than the amplitude of the pulsar intensity data (problem 1). The detection scheme above (scheme 1) was based on solving problem 1 by deriving from $I(t)$ the peak duration variable $\tau(t)$ which is amplitude modulated. In the following we investigate a different solution to problem 1, which turns out to generate a more sensitive detection scheme (scheme 2). One more astronomical fact will now be exploited, albeit a trivial one: the fact that the ground based observer has access to a finite space of the order of the Earth diameter $D_E$.

A perhaps remarkable coincidence makes this fact not so benign: $D_E$ is of the same order of magnitude as the typical size of intensity features in the interstellar scintillation pattern (at the electromagnetic frequency of 50 MHz chosen here.) This makes it fruitful to register the same intensity feature more than once, as it passes over two or more widely spaced radio telescopes. The relative velocity of the scintillation pattern can then be deduced directly from the cross-correlation of the intensity records collected at the different stations (see [2] and references therein.) When gravity waves are at work, the pattern velocity becomes $\tilde{V}(t) = V(t) + n(t) + v_{gw}(t)$, where $v_{gw}(t)$ is the gravity wave contribution and $n(t)$ is the noise from all other sources. Hence, the gravity wave effect generates here an amplitude modulation at the outset, which solves problem 1.

The virtual trigger mechanism applied to $\tau(t)$ in scheme 1 can now be applied to $(\tilde{V}(t) - V(t))$ (after normalization by subtraction of the mean), thus solving problem 2. Here, the pulse width could be made variable and set equal successively to the time intervals during which $(\tilde{V}(t) - V(t))$ is above the chosen threshold. However, numerical simulations show that such a change in the pulse shape does not affect the results qualitatively; i.e., the key information here is contained in the separation between pulses.

The great advantage of scheme 2 over scheme 1 is that the noise in scheme 2 (the uncertainty in the normalized $(\tilde{V}(t) - V(t))$) can be substantially reduced by various precision techniques of scintillation pattern velocity measurements, whereas the noise in scheme 1 (the variation in the peak duration $\tau$ from one feature to the other) was mostly due to the stochastic nature of the interstellar scintillation, about which little can be done. With the specific aim of gravity wave detection in mind, the precision in pattern velocity measurements can be substantially improved over what it has been so
far in interstellar scintillation experiments, which had rather different priorities and objectives. For example, experiments such as the Penticton-Jodrell Bank investigation of PSR 0329+54 were often interested in the exceptional cases of pulsars with large proper velocities or lines-of-sight with high levels of interstellar medium instability [2,3]. This is just the opposite of the slow and quiet conditions that would be favorable to the present gravity wave detection schemes.

Let us recapitulate the steps that are involved in detecting periodic gravity waves according to scheme 2. Some of these steps will demand the optimization of existing astronomical techniques, but none will demand the development of completely new ad hoc technology.

First, one must investigate the set of known pulsars (which are rapidly approaching 1000) for alignment with foreground binary stars. C.F. Quist has conducted a very preliminary run of 500 pulsars against only catalogued stars [6]. 106 objects were found at less than 2′ arc of pulsar lines-of-sight. About half of these objects are expected to be binary stars. Several were immediately identified as catalogued binary stars, including three X-ray binaries (at J2000 coordinates [+05 29 34 ; -66 53 02], [+15 13 44 ; -59 07 25] and [+18 16 59 ; -36 16 26]) which are potential candidates for strong gravity wave production. A binary star (HD 121454) was found at only 46″ arc from PSR 1553-62. Several alignments fell close to the 10′ arc range, although the precise nature of the corresponding objects still remains to be determined. These findings indicate that a thorough search of binary stars along several hundred pulsar lines-of-sight should reveal a number of interesting alignments in the 10′ arc range. Recall that for our purposes only one good alignment need be found.

From [1,12] one can show that the amplitude of the modulation $v_{gw}(t)$ is

$$v_{gw} \approx 1 \text{m/sec} \left( \frac{5′ \text{arc}}{\phi} \right) \left( \frac{f_{gw}}{10^{-3} \text{Hz}} \right)^{5/3} \frac{\mu}{M_\odot} \left( \frac{M}{M_\odot} \right)^{2/3},$$

where $\phi$ is the angular separation between pulsar and binary star and $f_{gw}$ the gravity wave frequency (twice the orbital frequency). $\mu$ and $M$ are the reduced mass and the total mass of the binary, and $M_\odot$ is the mass of the Sun.

A desirable candidate (i.e. a combination of pulsar, interstellar medium and binary star) is also one that involves a stable interstellar medium (which
is in fact the generic case) and a moderate value of $V(t)$ (order of magnitude of $10\,km/sec.)$ Under such circumstances, the uncertainty in the measurement of $\tilde{V}$ can be reduced to 1% in an optimized and updated version of the Penticton-Jodrell Bank experiment, observing in a bandwidth that is broad enough for several independent features and possibly using more than two radio antennae. To achieve this precision without making each measurement last more than about $10^3\,sec$, one must observe at the longest possible radio wavelengths, since the scintillation features are then smaller and their crossing by the Earth proportionally faster [2].

The $10^8\,sec$ long experiment should then yield a time series of about $10^5$ measurements (normalized $[\tilde{V}(t_i) - V(t_i)] = n(t_i) + v_{gw}(t_i)$ $v_{gw} << n$) that looks like random noise with a standard deviation $\sigma \approx 100m/sec$ (1% of $10km/sec.$) According to eq(2), a gravity wave signal of amplitude $v_{gw} > 100/10^{2.5} \approx 0.3m/sec$ could then be detected by the virtual trigger mechanism. It was confirmed numerically that gravity wave modulations of $1m/sec$ (see eq.(3)) or more are detectable (fig.2) and efforts to approach the $0.3m/sec$ limit are underway.

The numerical simulation starts with the generation of a random signal $n(t)$ with correlation time $t_c$ and standard deviation $\sigma$ that is derived from the delta correlated, zero-mean Gaussian white noise $G(t)$ through the low-pass filter

$$\frac{dn(t)}{dt} = -\frac{n(t)}{t_c} + \frac{\sigma}{\sqrt{t_c}}G(t) . \quad (4)$$

A pulse train $p(t)$ is then generated as prescribed in the earlier discussion of scheme 1. In evaluating the power spectrum $P(\omega)$ of $p(t)$ the difficulty consists in writing a code that (1) generates a periodogram $P(\omega)$ with a small enough variance, and (2) can accommodate occurrences of missing data due to observational constraints.

As can be seen in fig.(2), the code was able to detect a simulated gravity wave signal $v_{gw}$ buried in a noise $n(t)$ that is one hundred times louder ($\sigma = 100v_{gw}$). No data folding or periodogram averaging were used. Hence, there is room for a further increase of the sensitivity ratio $\sigma/v_{gw}$ beyond the 100 mark by refining this numerical phase of the detection. Current attempts in this direction are underway. The indication so far is that an additional half
an order of magnitude in sensitivity could be achieved, modulo the dedication of nontrivial computing resources.

Finally, it is clear that the virtual trigger mechanism described above could be used as a basis for new data processing approaches in other periodic gravity wave detection projects such as LISA and partially VIRGO, and more recent proposals based on new applications of pulsar timing [13] or on astrometry [14,15,16]. It should also be noted that the sensitivity enhancement implied by eq.(2) can alternatively be reached by more conventional filter matching techniques, according to well established data analysis theorems. Hence, the detectability of the interstellar medium gravity wave effect is not conditional to the ultimate success of the new numerical approach.

To conclude, it appears that through the optimization of existing techniques in a few different fields of astronomy and the dedication of sufficient computing power, one can design an effective project for a timely detection of periodic gravity waves. The underlying idea in this approach is to make new combinations of astronomical and numerical phenomena do a substantial part of the detection work.

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Figure captions

Figure 1.- Blowup of the double trigger mechanism of scheme 1 leading from the phase modulated intensity $\tilde{I}(t)$ to the amplitude modulated intensity-peak duration $\tau(t)$ to the pulse train $p(t)$, which is to be spectrally analyzed.

Figure 2.- As predicted by eq.(2), the detection (through scheme 2) of a gravity wave signal of amplitude $v_{gw}$ and angular frequency $\omega_{gw}$ buried in random noise of loudness $\sigma$ is successful all the way up to the realistic range $\sigma \sim 100v_{gw}$. No period folding or periodgram variance reduction were used.
Figure 1

\[ I(t) \]

\[ \tau(t) \]

\[ p(t) \]
Figure 2

$P(\omega)$

Noise = 100 x Signal

$\omega_{gw}$

$P(\omega)$

Noise = 50 x Signal

$\omega_{gw}$

$P(\omega)$

Noise = 10 x Signal

$\omega_{gw}$