Magnetic Feshbach resonances in $^7$Li–$^{133}$Cs mixtures

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Abstract

Motivated by the prospect of observing Efimov and Bose polaron physics in ultracold mixtures of bosonic atoms with large mass imbalance, this work investigates the magnetic Feshbach resonances between $^7$Li and $^{133}$Cs. The resonances are predicted at the 1 gauss level using the model of Ref. [1] obtained from experimental observations of resonances between $^6$Li and $^{133}$Cs. It is found that a few resonances in a practical range of magnetic field intensity could be used to tune the scattering length between $^7$Li and $^{133}$Cs atoms. Opportunities for observing Efimov and Bose polaron physics are discussed.

1 Introduction

The Efimov effect has been studied experimentally for several years with ultracold atoms. The effect is enhanced for systems of two heavy and one light particles, near the two-body resonance between a heavy and a light particle. This has been observed in mixtures of atoms with a large mass imbalance such as $^6$Li and $^{133}$Cs, for which the universal scaling factor between subsequent three-body states is 4.88 instead of 22.7 for three particles of identical mass, enabling the observation of up to three Efimov three-body bound states [2, 3]. The experiments are performed at sufficiently low density to assume in first approximation that the three-body observables are unaffected by the surrounding medium. However, it was found theoretically that the medium can make significant changes to the Efimov three-body spectrum and can even lead to an interesting interplay between Efimov and polaron physics. In particular, the case of heavy impurities immersed in a Bose-Einstein condensate of light atoms leads to a crossover between a phonon-mediated Yukawa force and particle-mediated Efimov attraction [4], leading to the prospect of observing bipolarons and tripolaron Physics in the crossover region where the mediated interaction becomes resonant.

One of the candidate species to observe this physics are $^{133}$Cs atoms immersed in a Bose-Einstein condensate of $^7$Li atoms. To this purpose, the use of a two-body resonance between these two species is needed. While the magnetic Feshbach resonances between $^7$Li and $^{133}$Cs have not been observed yet, the resonances of $^6$Li and $^{133}$Cs have been precisely measured and theoretically modelled [5, 6, 1]. The purpose of this work is to use this theoretical model to obtain predictions of the Feshbach resonances of $^7$Li and $^{133}$Cs, as a guide for experimental investigation.

2 Model

The model used in this work is based on the standard two-atom Hamiltonian,

$$\hat{H} = \hat{T} + \hat{V} + \hat{H}_A + \hat{H}_B ,$$

(1)

where $\hat{T} = -\hbar^2 \nabla_R^2 / (2\mu)$ is the relative kinetic energy of the two atoms, with a reduced mass $\mu$, and $\hat{V}$ is the interaction potential between the two atoms, assumed to depend only on the total electronic spin and relative distance $R$ of the two atoms. The Hamiltonians $\hat{H}_A$ and $\hat{H}_B$ of each separated atom $A$ and $B$, consist of hyperfine and Zeeman terms and read (for $k = A, B$),

$$H_k = A_k \hat{s}_k \cdot \hat{a}_k + (g_{i,k} \hat{\sigma}_k + g_{i,k} \cdot \hat{i}_k) \cdot \mu_B \vec{B}$$

(2)

where $\hat{s}_k$ and $\hat{i}_k$ are respectively the electronic and nuclear spins of atom $k$, $A_k$ is its hyperfine structure constant, and $g_{i,k}$ and $g_{i,k}$ are its electronic and nuclear gyromagnetic factors. Here, we have neglected the electronic spin-spin interaction and the effective $R$-dependence of the hyperfine structure constants $A_k$. These effects, which were included in the model of Ref. [1], are expected to affect the positions of resonances by about a gauss or less. Indeed, solving the above model for $A = ^6$Li and $B = ^{133}$Cs with the singlet and triplet potentials given in Ref. [1], yields all the resonance positions of that system within one gauss of the reported experimental and theoretical values, as shown in Table 1.

In the absence of experimental data for $A = ^7$Li and $B = ^{133}$Cs, one can assume that the same singlet and triplet potentials as those for $^6$Li and $^{133}$Cs can be used. This approximation justifies to limit the accuracy of our model to the one gauss level. Due to the isotopic mass difference, the singlet and triplet scattering lengths $a_s$ and $a_t$ change from $(a_s, a_t) = (30.15, -34.24) a_0$ for $^6$Li–$^{133}$Cs, to $(a_s, a_t) = (45.47, 908.2) a_0$ for $^7$Li–$^{133}$Cs, as al-
| Entrance channel | Experiment [6] | Theory [6] | Experiment [3] | Theory [1] | This work |
|------------------|----------------|-----------|----------------|-----------|-----------|
| $^6$Li$|1/2, +1/2⟩⊕ ^{133}$Cs$|3, +3⟩$ | 843.4(2) | 843.1(2) | 843.5(4) | 842.99 | 843.80 |
| | | 892.9(2) | 893.0(2) | 892.87(7) | 892.98 | 893.98 |
| $^6$Li$|1/2, −1/2⟩⊕ ^{133}$Cs$|3, +3⟩$ | 816.1(2) | 816.4(2) | 816.24(2) | 816.36 | 817.05 |
| | | 889.0(2) | 888.8(2) | 889.2(2) | 888.74 | 889.56 |
| | | 943.4(2) | 943.4(2) | 943.26(3) | 943.38 | 944.39 |

Table 1: Positions (magnetic field in gauss) of resonances between $^6$Li and $^{133}$Cs, for different hyperfine entrance channels labelled by the hyperfine state $|f, m_f⟩$ of each atom.

Figure 1: Scattering length and near-threshold bound states in the $aa$ entrance channel. The right panel is a close-up near the first resonance. In each panel, the red dashed curve represents the resonance formula Eq. (3) with its parameters given in Table 2. The green dot-dashed line in the left panel represents the linear fit: $a = 0.8382(B - 460.237)$.

| Entrance channel | $B_0$ (G) | $Δ$ (G) | $a_{bg}$ ($a_0$) |
|------------------|-----------|----------|-----------------|
| $^7$Li$|1, +1⟩⊕ ^{133}$Cs$|3, +3⟩$ (aa) | 538.43 | 4.89 | 43.85 |
| | | 3029.95 | 918.74 | 838.95 |
| $^7$Li$|1, 0⟩⊕ ^{133}$Cs$|3, +3⟩$ (ba) | 618.31 | 13.29 | 39.25 |
| | | 735.76 | 6.37 | 132.36 |
| | | 3149.85 | 874.77 | 840.645 |

Table 2: Predicted resonances between $^7$Li and $^{133}$Cs, for different hyperfine entrance channels labelled by the hyperfine state $|f, m_f⟩$ of each atom.
The values of $a_s$ and $a_l$ obtained from the potentials of Ref. [1] are slightly different from the values stated in that reference and Ref. [3], namely $(30.252(100),-34.259(200))$ and $(45.477(150),908.6(100))$, although within the theoretical uncertainty. They are also close to the values $(30.2(1),-34.5(1))$ reported in Ref. [6].

This situation is attributed to the anomalously large triplet scattering length, resulting in a large negative background scattering length. It is likely that a virtual state very close to the threshold of the open channel shifts and broadens the resonance. A similar situation occurs in $^6$Li-$^6$Li scattering [10]. It can be checked that artificially changing the triplet scattering length to a more nominal value (on the order of $\bar{a}$) results in a much narrower resonance at smaller magnetic field, very close to the first resonance in Fig. 1 around 538 G.

The results for the $ab$ channel are shown in Fig. 2 in the range of magnetic field where collisions are elastic. It turns out that no resonance appears in this range. The scattering length is negative and varies from -100 $a_0$ to zero. For larger magnetic fields, in the $ba$ channel (Fig. 3), a situation similar to the $aa$ channel occurs. Two resonances are found around 618 and 735 G, in addition to an extremely broad resonance occurring near $B = 3000$ G, resulting again from the anomalously large triplet scattering length. The broad resonance creates again a wide range of magnetic field where the scattering length varies linearly.

Each resonance can be fitted by the usual formula,

$$a = a_{bg} - \frac{\Delta}{B - B_0}$$

where $a_{bg}$ is the background scattering length, $B_0$ is the resonance position, and $\Delta$ is the magnetic width of the resonance. The resonances and their parameters are summarised in Table 2.

4 Discussion

In light of these results, we can look into whether the predicted resonances could be used to investigate Efimov and polaron physics. One aspect to consider is the interaction between atoms of the same species. In particular, for lithium-7 to form a stable condensate, the Li-Li scattering length must positive and not too large. A negative scattering implies that the condensate of lithium-7 would be either unstable, or limited to a very small number of atoms.

In the lithium-7 $a$ state, the Li-Li scattering length is positive only at magnetic fields $B < 140$ G and in the window $540$ G $< B < 740$ G [8]. In the first region, the Li-Cs scattering length is nearly constant and negative, around $-150 a_0$. Although this could not be used to study resonant physics, it could be used to study attractive Bose polarons (i.e. impurities interacting attractively with a condensate). In the second region, shown in the left panel of Fig. 4, the Li-Cs scattering length grows nearly linearly from nearly 0 to about $200 a_0$. Again, this region cannot be used to study resonant physics, but repulsive Bose polarons may be produced (i.e. impurities interacting repulsively with a condensate). Although the direct interaction between caesium atoms is relatively strong (with a scattering length around $3000 a_0$), their scattering length is affected by the interaction induced by lithium-7 atoms. The resulting effective scattering

![Graph](image-url)
Figure 3: Scattering length and near-threshold bound states in the $ba$ entrance channel. The right panel is a close-up near the first resonances. In both panels, the red dashed curve represents the fit formula: $a = 0.833636(B - 580.992) - \Delta^{(1)}/(B - B_0^{(1)}) - \Delta^{(2)}/(B - B_0^{(2)})$ where the resonance parameters $\Delta$ and $B_0$ are given in Table 2.

Figure 4: Scattering lengths of a mixture of lithium-7 and caesium-133 atoms as a function of magnetic field, with lithium-7 in the ground hyperfine state $a$ (left panel) and the first excited hyperfine state $b$ (right panel). The Cs-Cs scattering length is taken from Ref. [7], while the Li-Li scattering length is taken from Ref. [8] (left) and Ref. [9] (right). The dashed curve represents the effective scattering length of caesium impurities immersed the lithium-7 condensate, obtained from the formula Eq. (4).
length can be estimated from the simple formula [11],
\[ \tilde{a}_{\text{Cs-Cs}} = a_{\text{Cs-Cs}} - \frac{M m a_{\text{Li-Cs}}^2}{4 \mu^2 a_{\text{Li-Li}}} \]  
(4)
where \( M \) is the mass of caesium-133, \( m \) is the mass of lithium-7, and \( \mu = (1/m + 1/M)^{-1} \approx m \) is their reduced mass. This effective scattering length is shown as a dashed curve in Fig. 4, where it can be seen to vanish around 650 G.

In the lithium-7 \( b \) state, the Li-Li scattering length is positive in larger regions of magnetic field, namely \( B < 400 \) G and \( 600 \) G < \( B < 800 \) G. The first region cannot be used, since the lithium-caesium \( ba \) channel is inelastic in that region. The second region is more promising for resonant physics, as it includes the two \( ba \) resonances at 618 G and 735 G. However, both resonances are relatively narrow and require a precise stabilisation of the magnetic field. Moreover, the Cs-Cs scattering length around these resonances is relatively large (around \( 4000 \) \( a_0 \)) which may cause unwanted losses if the density of caesium atoms is too large. We note that the effective Cs-Cs scattering length may again be reduced near 650 G. Alternatively, one could work around 870 G where the direct Cs-Cs scattering length vanishes, however in that region the Li-Cs scattering length may not be varied by much and the Li-Li scattering length in turn becomes near-resonant.

5 Conclusion

It was found that the bosonic lithium-caesium mixtures exhibit a few interspecies resonances accessible to experiments. While these resonances may not be ideal for studying Efimov physics, the lithium-caesium mixtures nevertheless present some interesting opportunities for studying Bose polaron physics.

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