Radial oscillations of charged strange stars

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Abstract. The radial oscillations of charged strange quark stars is investigated. It is considered that the fluid pressure follows the MIT bag model equation of state and the charge density to be proportional to the energy density, \( \rho_e = \alpha \rho \) (where \( \alpha \) is proportionality constant). The modified equations of radial oscillations to the introduction of the electric charge are integrated to determine the fundamental mode. It is found that the stability of the charged object decreases with the increment of the central energy density and with the growth of the charge fraction.

1. Introduction
Strange quark stars or strange stars (for short) are objects that contain approximately equal number of up, down and strange quarks together with smaller number of electrons. In strange stars the electrons are distributed on its surface, forming and electron surface width of several fermis. With the aim to known as the electric charge affects the equilibrium configurations of the strange stars, in [1, 2] Negreiros and collaborators studied the charged strange quark stars considering a star composed by strange matter where radial pressure follows the MIT bag model equation of state and a Gaussian distribution of the electric charge on the surface of the star. The authors found that the distribution of charge on the surface of the object produce significant changes in the structure of the object.

An important point that has not been analyzed in [1, 2] was the stability of charged strange stars. The stability of the charged stars are studied by solving the equation of radial oscillations for charged stars [3].

In this work, it is studied the stability of charged strange stars. We consider that the fluid pressure follows the MIT bag model equation of state and the charge density to be a function of the energy density of the form \( \rho_e = \alpha \rho \) (where \( \alpha \) is proportionality constant) [4]. We use the units \( c = 1 = G \).

2. Radial oscillation equations for charged stars
To the study of radial oscillations of charged strange stars we use the equations presented in Brillante and Mishustin [3]

\[
\frac{d\xi}{dr} = \frac{\xi}{2} \frac{d\nu}{dr} - \frac{1}{r} \left( 3\xi + \frac{\Delta p}{p\Gamma} \right),
\] (1)
\[
\frac{d\Delta p}{dr} = e^\lambda \nu (p + \rho) \omega^2 \xi r + \left( \frac{d\nu}{dr} \right)^2 \frac{(p + \rho) \xi r}{4} - 4\xi \left( \frac{dp}{dr} \right) - 8\pi (p + \rho) \xi r e^\lambda \left( p + \frac{q^2}{8\pi r^4} \right) - \left( \frac{1}{2} \frac{d\nu}{dr} + 4\pi r e^\lambda (p + \rho) \right) \Delta p,
\]

being \( \omega \) the eigenfrequency. The quantities \( \xi \equiv \frac{\Delta r}{r} \) and \( \Delta p \) are assumed to have time dependence \( e^{i\omega t} \). The parameter \( \Gamma = (1 + \rho/p) \left( \frac{dp}{d\rho} \right) \).

In order to solve Eqs. (1) and (2), in the center of the star \( r = 0 \) is required that for \( r \to 0 \) the coefficient of the \( 1/r \) in Eq. (1) must vanish. In other words, we require \( (\Delta p)_{\text{center}} = -3(\xi \Gamma p)_{\text{center}} \). The surface of the star surface \( r = R \) is determined by the condition \( p \to 0 \). This implies \( (\Delta p)_{\text{surface}} = 0 \).

To solve the radial oscillation equations we use the shooting method. As a first step we integrate the hydrostatic equilibrium equation [5, 6, 7] for different values of \( \alpha \) and \( \rho_c \). After obtaining the coefficient of the radial oscillations for a given \( \alpha \) and \( \rho_c \), the pulsation equations are integrated from the center to the surface of the star. This process starts considering a trial value of \( \omega^2 \). If after the integration the boundary condition \( (\Delta p)_{\text{surface}} = 0 \) is not satisfied, the trial value \( \omega^2 \) is corrected in order to match this condition at the surface of the object.

### 3. Results and final remarks

In Figs. 1 is plotted the total mass normalized in solar masses as a function of the central energy density for four values of \( \alpha \). In this figure we can note that the mass of the spherical object grows with the charge fraction. The increase of the mass is due to the charge grows with the increment of the charge fraction. From this it can be understood that the charge acts as an affective pressure and helps the radial pressure to counteract more mass against the collapse.

![Figure 1](image1.png)

**Figure 1.** Total mass of the star, in solar masses, against the central energy density for some values of \( \alpha \).

![Figure 2](image2.png)

**Figure 2.** The eigenfrequency of oscillation of the fundamental mode as a function of the central energy density for some values of \( \alpha \).

The behavior of the eigenfrequency of the fundamental mode with the central energy density, for some values of \( \alpha \), it is shown in Fig. 2. Note that the eigenfrequency of oscillation decreases with the increment of the central energy density indicating that the stability of the object decreases with the increment of \( \rho_c \). We can observe also that the increment of the charge fraction affects the stability of the charged star. The stability of the charged object decreases with the increment of \( \alpha \).

Observing Figs. 1 and 2 we can note that the central energy density used to find the maximum mass points are not the same central energy densities used to find the zero eigenfrequencies.
This means that, when fixing the charge fraction, the condition $dM/d\rho_c > 0$ ($dM/d\rho_c < 0$) is a necessary but not a sufficient condition to determine regions made of stable (unstable) equilibrium configurations.

A detailed analysis about radial oscillations of charged strange quark stars will be reported in a future work [14].

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