A Study of Swing-By Trajectories in the Galilean Satellites of Jupiter

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Abstract. The Swing-By maneuver is a technique that has been under use in space missions for many years now, with the goal of reducing the fuel consumption of a spacecraft travelling near a massive body and that requires a modification in its orbit. Several well-known missions used this approach, like Voyager, Galileo, Cassini, Ulysses, and many others. The present research has the goal of studying and classifying Swing-by maneuvers around the Galilean satellites of Jupiter. The approach to solve this problem is to simulate a large variety of initial conditions and then to perform a classification of the orbits according to the effects that the close approach made in the orbit of the spacecraft, in particular looking for captures and escapes that resulted from the close approach.

1. Introduction
The Swing-By is a very important tool used by several missions to save fuel consumption in orbital maneuvers. Several papers studied this problem, as can be seen in references [1] to [21]. In particular, examples of real missions that used this technique can be found in Minovich [6], that studied the trajectories for the Voyager mission, Farquhar and Dunham [4], that obtained trajectories to explore the Earth’s geomagnetic tail; Farquhar et. al. [5], that considered the orbital maneuvers for the ISEE-3/ICE comet mission; Dowling et. al. [3], that described several applications studied by JPL/NASA during the early years of 1962-1963; Weinstein [10], that studied a mission to Pluto and Swenson [9] that calculated trajectories to Neptune. Some other studies considered the Swing-by maneuvers under different aspects and models, like including an impulse during the passage [7]; considering elliptic orbits for the primaries [8]; comparing different dynamical models [12]; studying the close approach of a cloud of particles [13], [14], [15]; optimizing conditions for the maneuvers [16], [17]; obtaining orbits with successive passages [18], [19]; including the presence of an atmosphere in the planet [20] or finding trajectories to approach the Sun using the inner planets [21].

It is possible to use many different sets of initial conditions to identify one swing-by trajectory. In the present paper the variables used in Broucke [1] and Broucke and Prado ([2], [11]) are also used. They are: 1) $J$, the Jacobian constant of the spacecraft, that is an integral of the Restricted Three-Body Problem; 2) $\psi$, that is called "angle of approach" and is defined as the angle between the line of the periapsis of the orbit of the spacecraft around the Galilean satellite and the line connecting the primaries (Jupiter-Satellite) and; 3) $R_p$, the periapsis distance of the orbit of the spacecraft around the satellite.
So, by varying those variables, a numerical integration of the equations of motion is performed backward and forward in time, until the spacecraft is at a distance that can be considered far enough from the Galilean satellite, and its effects can be neglected. At those points, two-body celestial mechanics can be used to obtain the energy and the angular momentum before and after the Swing-By. Then, following Broucke [1] and Prado and Broucke [2], sixteen classes of orbits are defined according to the changes of energy and angular momentum made by the Swing-By and the orbits are mapped.

2. Definition of the Problem
The system is composed by three bodies, all of them assumed to be points of mass: Jupiter, the Galilean satellite and the spacecraft, assumed to have a negligible mass. Jupiter and the Galilean satellite are considered to be in circular orbits around their center of mass.

Then, the problem is to study the variation of energy and angular momentum of the spacecraft with respect to Jupiter due to the Swing-By. After obtaining the results it is possible to classify the orbits. For this classification, the orbits are divided into four categories: elliptic direct (positive angular momentum and negative energy), elliptic retrograde (negative angular momentum and energy), hyperbolic direct (positive angular momentum and energy) and hyperbolic retrograde (negative angular momentum and positive energy).

To have a better understanding of this maneuver, Fig. 1 explains the geometry of the Swing-By. The spacecraft comes from the point A, crosses the horizontal axis (Jupiter-Galilean Satellite line), arrives at the point P (periapsis of the orbit) and then goes up to the point B. Points A and B are chosen such that they are far enough from the Galilean Satellite and the system Jupiter-Spacecraft can be considered a two-body problem. Two of the initial conditions are marked in this figure: \( R_p \) (periapsis distance) and \( \psi \) (angle of approach). The Jacobian constant \( (J) \) of the spacecraft is the third initial condition.

![Fig. 1 - Geometry of the Swing-By.](image)

3. Mathematical Model and Algorithm
The model used to describe the motion of the spacecraft is the planar restricted circular three-body problem. So, the equations of motion, in the usual canonical system of units for the spacecraft are:
\[ \dot{x} - 2\dot{y} = x - \frac{\partial V}{\partial x} = \frac{\partial \Omega}{\partial x} \]
\[ \dot{y} + 2\dot{x} = y - \frac{\partial V}{\partial y} = \frac{\partial \Omega}{\partial y} \]  
(1-2)

\[ \Omega = \frac{1}{2}(x^2 + y^2) + \frac{(1 - \mu)}{r_1} + \frac{\mu}{r_2} \]  
(3)

To obtain the energy and the angular momentum of the spacecraft it is used [2]:

\[ E = \frac{(x + \dot{y})^2 + (\dot{x} - y)^2}{2} - \frac{1 - \mu}{r_1} + \frac{\mu}{r_2}, \quad C = x^2 + y^2 + xy - y\dot{x} \]  
(4-5)

To increase the accuracy in the numerical integrations, in particular when \( r_1 \) or \( r_2 \) are small, the Lemaître regularization is used. It consists in a substitution of the physical variables \( (x, y, t) \) by another group of variables \( (\omega_1, \omega_2, \tau) \), in such way that the singularities are eliminated. To perform this transformation, it is first defined a new complex variable \( q \) \( (q_1 + iq_2) \), with \( q_1 \) and \( q_2 \) given by:

\[ q_1 = x + 1/2 - \mu \]
\[ q_2 = y \]  
(6) (7)

After that, the transformation made is:

\[ q = f(\omega) = \frac{1}{4}\left(\omega^2 + \frac{1}{\omega^2}\right) \]  
(8)

for the position variables \( (x, y) \) and:

\[ \frac{\partial t}{\partial \tau} = \left| f'(\omega) \right|^2 = \frac{|\omega^4 - 1|^2}{4|\omega|^6} \]  
(9)

for the time, where \( f'(\omega) \) denotes \( \frac{\partial f}{\partial \omega} \).

The equations of motion then become:

\[ \omega'' + 2i|f'(\omega)|^2\omega' = \text{grad}_\omega \Omega^* \]  
(10)

with \( \omega = \omega_1 + i\omega_2 \) is the new complex variable for position, \( \omega' \) and \( \omega'' \) are the first and second derivatives of \( \omega \) with respect to the regularized time \( \tau \); \( \text{grad}_\omega \Omega^* \) represents \( \frac{\partial \Omega^*}{\partial \omega_1} + i\frac{\partial \Omega^*}{\partial \omega_2} \) and \( \Omega^* \) is the transformed pseudo-potential given by:

\[ \Omega^* = \left( \Omega - \frac{C}{2}\right)|f'(\omega)|^2 \]  
(11)
where $C = \mu (1-\mu) - 2J$.

Then, a numerical algorithm is built to solve the problem. It has the following steps:

i) Numerical values are attributed to the three variables $R_p$, $J$, $\psi$.

ii) The initial position is obtained from $X_i = R_p \cos(\psi) + (1-\mu)$, $Y_i = R_p \sin(\psi)$ and the initial velocity from $V_{xi} = -V \sin(\psi)$, $V_{yi} = +V \cos(\psi)$, with $V = \sqrt{x^2 + y^2}$.

iii) A numerical integration of the equations of motion is performed forward in time until the spacecraft is far from the Galilean satellite. Then the energy ($E+$) and angular momentum ($C+$) are calculated;

iv) The initial conditions go back to the point P and the equations of motion are integrated backward in time. Energy ($E-$) and angular momentum ($C-$) are then obtained;

v) The variation in energy ($E+ - E-$) and angular momentum ($C+ - C-$) are then calculated.

4. Results

The results are shown in letter-plots, with one letter describing the outcome of the close approach. The figures have the Jacobian constant in the vertical axis and $\psi$ in the horizontal axis. A figure is made for a fixed value of the parameter $R_p$. The letters A to P are defined in Table 1 and they are used for the classification of the orbits. Letter Z represents an orbit where the spacecraft stayed around the Galilean satellite for a long time.

**Table 1 - Definition of the orbits**

| Before:          | After:          | Direct Ellipse | Retrograde Ellipse | Direct Hyperbola | Retrograde Hyperbola |
|------------------|-----------------|----------------|-------------------|------------------|---------------------|
| Direct Ellipse   | A               | E              | I                 | M                |
| Retrograde Ellipse| B               | F              | J                 | N                |
| Direct Hyperbola | C               | G              | K                 | O                |
| Retrog. Hyperbola| D               | H              | L                 | P                |

The interval used here for the variables is: $180^\circ \leq \psi \leq 360^\circ$ and $-1.35 \leq J \leq 1.55$. The interval $0^\circ \leq \psi \leq 180^\circ$ is not used, because the figure is symmetric, since an orbit with $\psi = \theta$ differs from an orbit with $\psi = \theta + 180^\circ$ by a time reversal. So, there is a correspondence: I$\leftrightarrow$C, J$\leftrightarrow$G, L$\leftrightarrow$O, B$\leftrightarrow$E, N$\leftrightarrow$H, M$\leftrightarrow$D. Orbits A, F, K and P are not changed.

For the periapsis distance the value 1.1 radius of the Galilean satellite is used, since it gives strong effects. Fig. 2 shows the results.

The numerical values for the limits involved in this research are:

1. Distance from the Galilean Satellite to the points A and B: 0.5 canonical units;
2. Distance for the spacecraft to be considered too far from Jupiter: 2.0 canonical units;
3. Mass of Jupiter = $1.89813 \times 10^{27}$ kilograms;
4. Distance from Jupiter to the Sun: 778,330,001 km.

**Table 2 - Data for the Galilean satellites in canonical units**

| Satellite | Mass      | Radius   |
|-----------|-----------|----------|
| Io        | 0.00004725| 0.004318 |
| Europa    | 0.00002539| 0.002326 |
| Ganimedes | 0.00007797| 0.002458 |
| Calisto   | 0.00005690| 0.001280 |
Ganimedes: $180^\circ \leq \psi \leq 360^\circ$

Io: $180^\circ \leq \psi \leq 360^\circ$

| $\psi$ | J | $\psi$ | J |
|-------|---|-------|---|
| 1.55  | 1.55 |
| 1.45  | 1.45 |
| 1.35  | 1.35 |
| 1.25  | 1.25 |
| 1.15  | 1.15 |
| 1.05  | 1.05 |
| 0.95  | 0.95 |
| 0.85  | 0.85 |
| 0.75  | 0.75 |
| 0.65  | 0.65 |
| 0.55  | 0.55 |
| 0.45  | 0.45 |
| 0.35  | 0.35 |
| 0.25  | 0.25 |
| 0.15  | 0.15 |
| 0.05  | 0.05 |
| -0.05 | -0.05 |
| -0.15 | -0.15 |
| -0.25 | -0.25 |
| -0.35 | -0.35 |
| -0.45 | -0.45 |
| -0.55 | -0.55 |
| -0.65 | -0.65 |
| -0.75 | -0.75 |
| -0.85 | -0.85 |
| -0.95 | -0.95 |
| -1.05 | -1.05 |
| -1.25 | -1.25 |
| -1.35 | -1.35 |

Calisto: $180^\circ \leq \psi \leq 360^\circ$

| $\psi$ | J | $\psi$ | J |
|-------|---|-------|---|
| 1.55  | 1.55 |
| 1.45  | 1.45 |
| 1.35  | 1.35 |
| 1.25  | 1.25 |
| 1.15  | 1.15 |
| 1.05  | 1.05 |
| 0.95  | 0.95 |
| 0.85  | 0.85 |
| 0.75  | 0.75 |
| 0.65  | 0.65 |
| 0.55  | 0.55 |
| 0.45  | 0.45 |
| 0.35  | 0.35 |
| 0.25  | 0.25 |
| 0.15  | 0.15 |
| 0.05  | 0.05 |
| -0.05 | -0.05 |
| -0.15 | -0.15 |
| -0.25 | -0.25 |
| -0.35 | -0.35 |
| -0.45 | -0.45 |
| -0.55 | -0.55 |
| -0.65 | -0.65 |
| -0.75 | -0.75 |
| -0.85 | -0.85 |
| -0.95 | -0.95 |
| -1.05 | -1.05 |
| -1.25 | -1.25 |
| -1.35 | -1.35 |

Ganymedes: $180^\circ \leq \psi \leq 360^\circ$

| $\psi$ | J | $\psi$ | J |
|-------|---|-------|---|
| 1.55  | 1.55 |
| 1.45  | 1.45 |
| 1.35  | 1.35 |
| 1.25  | 1.25 |
| 1.15  | 1.15 |
| 1.05  | 1.05 |
| 0.95  | 0.95 |
| 0.85  | 0.85 |
| 0.75  | 0.75 |
| 0.65  | 0.65 |
| 0.55  | 0.55 |
| 0.45  | 0.45 |
| 0.35  | 0.35 |
| 0.25  | 0.25 |
| 0.15  | 0.15 |
| 0.05  | 0.05 |
| -0.05 | -0.05 |
| -0.15 | -0.15 |
| -0.25 | -0.25 |
| -0.35 | -0.35 |
| -0.45 | -0.45 |
| -0.55 | -0.55 |
| -0.65 | -0.65 |
| -0.75 | -0.75 |
| -0.85 | -0.85 |
| -0.95 | -0.95 |
| -1.05 | -1.05 |
| -1.25 | -1.25 |
| -1.35 | -1.35 |

Fig. 2 - Results for $R_P = 1.1$. 

Io: $180^\circ \leq \psi \leq 360^\circ$

Calisto: $180^\circ \leq \psi \leq 360^\circ$
5. Conclusions

A numerical algorithm to calculate the effects of a close approach with the Galilean satellites of Jupiter in the trajectory of a spacecraft is developed. By varying the initial conditions of the trajectories, the orbits can be classified according to the changes in energy and angular momentum, to identify captures, escapes and situations where the sense of the orbit is reversed. It is shown that, although the Galilean Satellites have masses much smaller when compared to the giant planets, all of them can provide small regions of escape and capture trajectories from the Jupiter system, as well as some orbits that reverse the sense of the motion. A closer look at the plots made here reveals that all of them have similar characteristics. Those trajectories can be used to insert a spacecraft coming from the Earth in orbit around Jupiter, or to help a spacecraft to leave the Jupiter system.

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