Analysis of the Stirling engine in the Schmidt approximation

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Abstract. Exploration, oil and gas well development, and oil and gas production facilities are often located in remote, remote locations away from electrical networks and highways. Autonomous power supply of these facilities is currently carried out by means of diesel power plants, which is associated with the necessary supply of fuel and associated high overhead costs. In this regard, the use of renewable energy sources (RES) is of particular relevance. As such, wind energy, geothermal heat, solar radiation energy, associated petroleum gas energy are known. The same group should include local energy sources such as peat, wood, coal, coal shale, etc. with the exception of the first of these sources, the rest require the conversion of thermal energy into mechanical energy, and then, by means of an electric generator into electrical energy. The only known universal converter is the Stirling generator - a machine in which the generator is combined with the Stirling engine. Stirling engine (DS) – refers to external combustion engines, more precisely external heat supply. For the operation of the DS, it is only required to bring the temperature difference to its heat exchangers, and both the increased temperature of the heater and the reduced temperature of the refrigerator are important. As a heater, you can use the heat of the burner, steam or water heated in geothermal or solar collectors. It is possible to supply directly solar energy from solar concentrators. For these cases, a special design of the DS with a quartz head is provided, through which concentrated solar radiation enters directly into the expansion chamber. Geothermal water supplied directly from the well or associated petroleum gas, which can be highly corrosive, can also be used for these engines. As a refrigerator, you can use the environment, chilled water, ice or snow. DS works in a closed cycle, which allows you to completely seal the body and use helium or hydrogen as a working fluid. This ensures high working fluid pressure, which increases the specific power (power per unit volume or weight of the engine). Currently, these engines have not received mass application, which is explained by both economic reasons (high cost of development, competition from the ice) and commercial (patent restrictions, trade secrets).

1. The operating principle of the Stirling Engine. The classic Stirling cycle
The Stirling engine [1-3] is a closed – loop heat engine, according to which the working fluid expands at a higher temperature and contracts at a lower temperature, which provides a gain in operation at the forward and reverse stroke of the piston. Expansion occurs at a nearly constant temperature of $T_1$ in the expansion chamber and compression occurs at $T_2$ in the compression chamber. The gas transition from one temperature to another is carried out at almost constant volume by moving the body from one chamber to another. This movement is carried out with the help of a special displacer, which, depending on the design, can either fit snugly to the cylinder or have a gap. The work of the expanding working fluid is carried out by means of the working piston. The engine also provides a regenerator (economizer), which receives part of the heat from the heated working fluid when it is cooled and gives it when heated.
The prototype of the real cycle is an ideal Stirling cycle consisting of two isotherms and two isochores. The Stirling machine can work both in a direct cycle (in this case it is an engine) and in the reverse (in this case, it is a refrigerating machine).

![Figure 1. Thermodynamics of Stirling cycle.](image)

where: \( Q_1 \) – the heat received from the heater, J; \( Q_2 \) – the heat given to the refrigerator, J; \( q_1 \) – the heat returned from the regenerator to the working body at isochoric heating, J; \( q_2 \) – the heat taken away from the working body by the regenerator at its isochoric cooling, J.

The area within a cycle in p-V coordinates is a useful work per cycle. The area in the t-S coordinates is equal to the heat converted to work during the cycle.

We obtain an expression for the efficiency of the cycle. The working fluid receives heat during isochoric heating and isothermal expansion \( Q_n \), J:

\[
Q_n = q_1 + Q_1 = vC_V(T_1 - T_2) + vRT_1\ln\frac{V_2}{V_1},
\]

where: \( v \) – is the amount of substance in the heated (cooling) sample, mol; \( C_V \) is the molar heat capacity at a constant volume, J / mol·K; \( T_1 \) is the temperature of the heater, K; \( T_2 \) is the temperature of the refrigerator, K; \( R \) is the universal gas constant, J / mol·K.

And gives during isochoric cooling and isothermal compression:

\[
Q_x = q_2 + Q_2 = vC_V(T_1 - T_2) + vRT_2\ln\frac{V_2}{V_1}.
\]

Work, perfect for the cycle a, J is:

\[
A = Q_1 - Q_2 = vR,
\]

The thermal efficiency is equal to

\[
\eta = \frac{A}{Q_n} = \frac{vR\ln\frac{V_2}{V_1}(T_1 - T_2)}{vRT_2\ln\frac{V_2}{V_1} + q_1},
\]

Stirling proposed to receive heat \( q_1 \), not from the heater, but from the regenerator, into which this heat comes as a result of heat exchange during isochoric cooling of the gas. In this case, \( q_1 \) in the denominator disappears and the efficiency is found from the following expression:

\[
\eta = \frac{T_1 - T_2}{T_1},
\]

i.e., coincides with the efficiency of a perfect engine.

Figure 2 shows a diagram of such a cycle for a two-piston engine, called an \( \alpha \) - type engine in engineering practice.

2. Cycle Schmidt

The classic cycle of Stirling, if it can be carried out in a real machine, even with some approximation, it can not be considered technological. Really, the pistons should turn - to move, to stand still, to sharply change direction. Technically, this is very difficult to implement. Moreover, at the time of a sudden change in speed, the entire system is experiencing a mechanical shock, which will inevitably lead to its failure. More acceptable from a practical point of view is the sinusoidal movement of the pistons in combination with the rotational movement of the transmission.
Figure 2. Diagram of the Stirling engine and chart changes in the volume of the working fluid where: \( V_E \) - expansion volume; \( V_C \) - compression volume; \( H \) - heater; \( C \) - refrigerator; \( R \) - regenerator; 1 - isothermal compression; 2 - isochoric compression; 3 - isothermal expansion; 4 - isochoric expansion.

A characteristic difference of the Schmidt cycle is that here it is impossible to distinguish separate stages, as in the classical Stirling cycle. At any time, part of the gas is in the expansion chamber, the other part – in the compression chamber, the ratio between these parts depends on the phase difference \( \delta \) between the expansion and compression oscillations. Therefore, we can only talk about mainly one or another stage of the process.

We proceed to the conclusion of the mathematical model of the Schmidt cycle.

The gas, being ideal, obeys the equation of Men de leev - Clapeyron \( PV = VRT \), from where we obtain an expression for the number of moles of substance \( \nu = \frac{PV}{RT} \) (mol). We assume that the entire gas is distributed over three volumes: the expansion volume \( V_E \) (m\(^3\)) at \( T_1 \) (K), the compression volume \( V_C \) (m\(^3\)), at \( T_2 \) (K), and the dead volume \( V_D \) (m\(^3\)) at \( T_D \) (K). The total amount of substance is

\[
\nu = \nu_1 + \nu_2 + \nu_3 = \frac{P}{R} \left( \frac{V_E}{T_1} + \frac{V_C}{T_2} + \frac{V_D}{T_D} \right),
\]

where do we get the expression for pressure, \( PA \):

\[
P = \frac{\nu R}{\frac{V_E}{T_1} + \frac{V_C}{T_2} + \frac{V_D}{T_D}}.
\]

In accordance with Schmidt’s assumption about the harmonic motion of the pistons, we assume [5-8]:

\[
V_E = \frac{V_0}{2} (1 + \cos \varphi); \quad V_C = \frac{k V_0}{2} (1 + \cos(\varphi - \delta)),
\]

where: \( \varphi \) is the rotation angle of the crankshaft (phase process), \( \delta \) is the phase lag of the compression piston from the piston extension \( V_0 \) – the volume of the compression chamber, \( V_{20} \) – the volume of the expansion chamber.

Put also: \( \tau = \frac{T_1}{T_2} \); \( V_D = XV_0 \); \( T_D = \frac{1}{2} (T_1 + T_2) \). The expression for pressure takes the following form, \( PA \):

\[
P = \frac{\nu R T_2}{V_0} \frac{2}{\frac{1}{2} (1 + \cos \varphi) + k(1 + \cos(\varphi - \delta)) + \frac{4X}{(1 + \tau)}}.
\]

The work in the expansion of the gas is, \( J \):

\[
A_E = \oint P_e dV_E = -\nu RT_2 \oint \frac{\sin \varphi d\varphi}{\frac{1}{2} (1 + \cos \varphi) + k(1 + \cos(\varphi - \delta)) + \frac{4X}{(1 + \tau)}}.
\]

The work is under compression, \( J \):

\[
A_C = \oint P_c dV_C = -\nu RT_2 \oint \frac{k \sin(\varphi - \delta) d\varphi}{\frac{1}{2} (1 + \cos \varphi) + k(1 + \cos(\varphi - \delta)) + \frac{4X}{(1 + \tau)}}.
\]
Total work per cycle is equal to

\[ A = A_E + A_C \]  \hspace{1cm} (11)

Here the symbol \( \oint \) means cycle-by-cycle integration: \( \oint = \int_{\phi=0}^{2\pi} \).

Performing numerical integration in the above formulas, it is possible to analyze the operation of the Stirling engine in the Schmidt approximation. The purpose of the analysis is to identify the dependence of dimensionless parameters characterizing the cycle performance \( (P_0, V_1, V_2, A, Q_1, Q_2) \) on dimensionless parameters characterizing the design features of the process, its mode, or state \( (\tau = \frac{T_1}{T_2}, k = \frac{V_{20}}{V_0}, X = \frac{V_D}{V_0}, \delta \) — the lag phase of contraction from expansion).

Conclusion:

1. A comparison of the Stirling cycle with the reference Carnot cycle is given, it is shown that with ideal regeneration and the absence of thermal losses, the efficiency of the DS coincides with the Carnot cycle, which is the maximum possible, and the work per cycle is proportional to the amount of the working fluid substance, and hence its pressure.

2. Proposed circuit device DS \( \alpha \) type and charts the movements of his pistons. The physical possibility of the classical Stirling cycle is shown. Based on the analysis of graphs shows the similarity and difference between the classical cycle and the Schmidt cycle, which can be considered a Stirling cycle.

3. The conclusion of the mathematical model of Schmidt cycle is proposed, which allows to obtain pressure and volume of expansion and compression chambers for different stages of the cycle, as well as expansion and compression work for the entire cycle by four input parameters characterizing the design features of the DS.

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