A Direction Finding Method with A 3-D Array Based on Aperture Synthesis

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Abstract. Direction finding for electronic warfare application should provide a wider field of view as possible. But the maximum unambiguous field of view for conventional direction finding methods is a hemisphere. It cannot distinguish the direction of arrival of the signals from the back lobe of the array. In this paper, a full 3-D direction finding method based on aperture synthesis radiometry is proposed. The model of the direction finding system is illustrated, and the fundamentals are presented. The relationship between the outputs of the measurements of a 3-D array and the 3-D power distribution of the point sources can be represented by a 3-D Fourier transform, and then the 3-D power distribution of the point sources can be reconstructed by an inverse 3-D Fourier transform. And in order to display the 3-D power distribution of the point sources conveniently, the whole spherical distribution is represented by two 2-D circular distribution images, one of which is for the upper hemisphere, and the other is for the lower hemisphere. Then a numeric simulation is designed and conducted to demonstrate the feasibility of the method. The results show that the method can estimate the arbitrary direction of arrival of the signals in the 3-D space correctly.

1. Introduction

In electronic warfare field, direction finding of the signals in the space is one of the most important aspects. Many direction finding methods have been developed, such as interferometer direction finding [1], amplitude comparison direction finding [2], TDOA-based direction finding [3], MUSIC based on the array signal processing [4, 5], direction based on sparsity [6], and so on.

For electronic warfare application, direction finding should provide the field of view as wider as possible. It’s very important for some special application. But conventional direction finding methods are based on a linear array or a planar array, which causes ambiguity in the practical application. When the minimum antenna spacing is half a wavelength, the methods can estimate the direction of arrivals of the signals in the hemisphere. Under this condition, they can obtain the maximum field of view, equal to 2π steradian, which is the solid angle of a hemisphere. But the signals exist in the 4π steradian (4π is the solid angle of the whole sphere), these methods cannot distinguish the direction of arrival of the signals from the back lobe of the antennas, i.e., there is ambiguity.

In this paper, a direction finding method based on aperture synthesis (DFAS) radiometry with a 3-D array is proposed. It utilizes a 3-D array to receive the signals, and an image including the spatial distribution of the sources is outputted. Firstly, the model of the direction finding system and the
fundamentals are presented. Then in order to demonstrate the feasibility of the method, numeric simulation is conducted and analyzed.

2. The fundamentals of the DFAS

The illustration of a DFAS is shown as figure 1. The system contains a 3-D array, receivers, and digital correlators. The array receives the signals emitted by the point sources. The signals pass through the receivers and are correlated between any pair of antennas in the digital correlators. The output of digital correlators is called visibility function. Then visibility function is processed by 3-D Fourier transform to obtain a 3-D image including the spatial distribution of the point sources. Thus, the direction of arrivals of these point sources can be obtained from the image

![Diagram of DFAS system](image)

Figure 1. The illustration of the model of a DFAS system

According to the principle of the aperture synthesis radiometry in astronomy or Earth remote sensing [7-9], the correlation between the signals received by any pair of antennas is called visibility function. For a pair of antennas $a_i$ and $a_j$ with the coordinates $(x_i, y_i, z_i)$ and $(x_k, y_k, z_k)$ normalized respect to the wavelength, the relationship between the visibility function and the distribution of point sources is expressed as follow:

$$ V(u, v, w) = \int \int P(\xi, \eta) e^{-j2\pi(u\xi + v\eta + w\cos\phi \cos\theta)} d\xi d\eta $$  \hspace{1cm} (1)

Where $(u, v, w) = (x_k - x_o, y_k - y_o, z_k - z_o)$ are the spatial frequencies, $(\xi, \eta) = (\sin \theta \cos \varphi, \sin \theta \sin \varphi)$ are the directional cosines. $P(\xi, \eta)$ is the power of the point source. For different pair of antennas, one obtains the different samples of the visibility function. And a 3-D array can provide a 3-D spatial frequency distribution and the corresponding 3-D samples of the visibility function.

It can not reconstruct $P(\xi, \eta)$ from the 3-D visibility samples directly. But (1) can be rewritten in the form of a 3-D Fourier transform involving the third direction cosine $\eta$ defined with respect to $w$ axis.

$$ V(u, v, w) = \int \int P(\xi, \eta, \sigma) \delta(\cos \theta - \sigma) e^{-j2\pi(u\xi + v\eta + w\sigma \cos \phi \cos \theta)} d\xi d\eta d\sigma $$ \hspace{1cm} (2)

The delta function $\delta(\cos \theta - \sigma)$ maintains the condition $\sigma = \cos \theta$. And the spatial distribution is rewritten as $P(\xi, \eta, \sigma)$. When $\sigma$ is greater than zero, it indicates the point source locates at the upper hemisphere. When $\sigma$ is smaller than zero, it indicates the point source locates at the lower hemisphere. Thus, it can distinguish the point source in the $4\pi$ space. The spatial distribution of the point sources can be reconstructed by 3-D inverse Fourier transform:

$$ P(\xi, \eta, \sigma = \cos \theta) = \int \int V(u, v, w) e^{j2\pi(u\xi + v\eta + w\sigma \cos \phi \cos \theta)} dudwd\sigma $$  \hspace{1cm} (3)

For $(\xi, \eta, \sigma)$, they satisfy the condition:

$$ \xi^2 + \eta^2 + \sigma^2 = 1 $$  \hspace{1cm} (4)
Obviously, it is the function of the sphere in the 3-D coordinates. For actual application, only some samples of the 3-D visibility function can be measured. Thus, the spatial distribution image of the point sources can be estimated by discrete Fourier transform:

\[ P(\xi, \eta, \sigma) = \cos \theta = \Delta u \Delta v \Delta w \sum_{j} \sum_{n} \sum_{m} V(u_j, v_n, w_m) e^{j 2\pi (u_j \xi + v_n \eta + w_m \sigma)} \] (5)

Generally speaking, this method provides a mapping from a 3-D spatial frequency domain to a sphere spatial domain. But it is not convenient to display a power distribution of the point sources in a 3-D viewpoint. It should be simplified further. According to (4), \( \sigma \) depends on \( \xi \) and \( \eta \). So the whole sphere can be projected to two unit circle. One of the circle with \( \sigma > 0 \) denotes the upper hemisphere, and the other with \( \sigma < 0 \) denotes the lower hemisphere. The illustration is shown in Figure 2. Then the direction of the arrivals of the point sources can be obtained from the reconstructed 2-D image.

**Figure 2.** The illustration of the whole sphere projected to two unit circles. (a) The whole sphere. (b) The unit circle with \( \sigma > 0 \). (c) The unit circle with \( \sigma < 0 \).

### 3. Numerical Simulation for DFAS

The distribution of the 3-D spatial frequencies depends on the 3-D array arrangement. Thus, how to design an optimal 3-D array is an important problem. Here, the antenna array is not referred, and the distribution of 3-D spatial frequencies is directly provided. Considering a set of spatial frequency samples \( \{(u_l, v_m, w_n)\mid -20 \leq l \leq 20, -20 \leq m \leq 20, -5 \leq w \leq 5\} \), The spatial frequency samples are uniformly distributed with the spacing of 0.5. Three point sources locate at \( (\theta_1, \phi_1) = (\pi/3, \pi/3) \), \( (\theta_2, \phi_2) = (2\pi/3, \pi/3) \) and \( (\theta_3, \phi_3) = (\pi/4, 3\pi/4) \) respectively. The corresponding directional cosines are
about \((\xi, \eta, \sigma) = (0.43, 0.75, 0.5)\), \((\xi, \eta, \sigma) = (0.43, 0.75, -0.5)\), and \((\xi, \eta, \sigma) = (-0.5, 0.5, 0.71)\) respectively. The first point source and the third one locate on the upper hemisphere, and the second one locate on the lower hemisphere.

For a 2-D array, the point source with \((\theta, \phi) = (\pi / 3, \pi / 3)\) and the point source with \((\theta, \phi) = (2\pi / 3, \pi / 3)\) can not be distinguished, and the point source with \((\theta, \phi) = (2\pi / 3, \pi / 3)\) will be estimated with the direction of the arrival of \((\theta, \phi) = (\pi / 3, \pi / 3)\). It is because the directional cosines of the two point sources for a 2-D array are the same, i.e., there is ambiguity.

For a 3-D array above, the reconstructed distribution image of the point sources are shown in figure 3. Figure 3(a) is the reconstructed 2-D image corresponding to the projection of the 3-D upper hemisphere, and figure 3(b) is the reconstructed 2-D image corresponding to the projection of the 3-D lower hemisphere. It can be found there are two point sources in figure 3(a), and the corresponding directional cosines are \((\xi, \eta) = (0.44, 0.73)\) and \((\xi, \eta) = (-0.49, 0.49)\) respectively, which is approximate to the actual directional cosines of the first point source and the third one. It means that the two points located on the upper hemisphere can be correctly reconstructed. In figure 3(b), there is only one point source, and the corresponding direction cosine is \((\xi, \eta) = (0.44, 0.73)\), which is approximate to the actual directional cosine of the second point source, which locates on the lower hemisphere. It means that DFAS can correctly reconstruct the distribution of the point sources locating on the whole sphere, and the direction of arrivals can be correctly estimated.

\[ \begin{align*}
\text{Figure 3. The simulation results of DFAS. (a) the reconstructed 2-D distribution image corresponding to the projection of the 3-D upper hemisphere. (b) the reconstructed 2-D distribution image corresponding to the projection of the 3-D lower hemisphere.} 
\end{align*} \]

4. Conclusion

The aperture synthesis radiometry in astronomy and Earth remote sensing is introduced to estimate the direction of arrivals from the whole sphere. The simulation results show that it can handle the ambiguity caused by the planar array. In this paper, there is only a brief presentation of DFAS, and more problems should be discussed further.

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References
[1] Lee J H and Woo J M 2015 IEEE Antennas Wireless Propag. Lett. 14 719
[2] Orduyilmaz A, Serin M and Yildirim A 2015 Proc. Signal Processing and Communications Applications Conference (IEEE) p 109
[3] Cui X, Yu K and Lu S 2015 IEEE Transactions on Instrumentation & Measurement 64 2347
[4] Guo Z, Wang X and Wei H 2017 IEEE Transactions on Wireless Communications 16 5384
[5] Ciunonzo D and Rossi P S 2017 IEEE Signal Processing Letters 24 397
[6] Liu Z M, Huang Z T, Zhou Y Y and Liu J 2012 IEEE Transactions on Aerospace & Electronic Systems 48 2690
[7] Zhang C, Liu H, Wu J, Zhang S, Yan J, Niu L, Sun W and Li H 2014 IEEE Transactions on Geoscience & Remote Sensing 53 207
[8] Ruf C S, Swift C T, Tanner A B and Vine D M L 1998 IEEE Transactions on Geoscience & Remote Sensing 26 597
[9] Thompson A R, Morgan J M and Swenson G W 2001 Interferometry and Synthesis in Radio Astronomy (New York: Wiley–Interscience)