Particle breakage and micromechanical characteristics of calcareous sand during shearing

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\textbf{Abstract:} Based on the discrete element method, the numerical simulation of the shear test for calcareous sand was carried out to explore the particle breakage and micromechanical mechanism of calcareous sand in the shear process. The results show that the particle breakage can reduce the peak strength of calcareous sand, and make the bulk deformation change from dilatancy to shrinkage. The particle breakage is concentrated in the shear band; with the increase of axial strain, the degree of particle breakage increases, which is also positively correlated with confining pressure and axial strain. The contact forces are redistributed when the particles are broken; the dominant direction of the contact forces in the strong force chain is consistent with $\sigma_1$, always in the vertical direction, while the dominant direction of the contact forces in the weak force chain is consistent with $\sigma_3$, in the horizontal direction, and it gradually deflects to the vertical direction with increasing axial strain; the anisotropy of the contact normal also decreases significantly after particle breakage. The magnitudes of the contact forces in the strong and weak force chains conform to the Weibull distribution. After particle crushing, the magnitudes of the contact forces decrease significantly, and the numbers of the contact forces increase significantly, and the percentage of the weak forces increases significantly, which leads to the decrease of the strength of the sample.

1. Introduction
Calcarenite sand, mainly comprised of fragments of bioclasts such as coral and shells, retains the pores of protozoan skeleton due to relatively short distance transportation in the deposition process and characterizes by high porosity, fragility and irregular shape, which makes its strength and deformation properties greatly different from ordinary terrigenous sand [1-3]. The discrete element method (DEM) proposed by Cundall et al. [4,5] has been widely used to investigate the particle breakage and related characteristics of calcareous sand in recent years. Thornton et al. [6,7] used the aggregates bonded by several two-dimensional disks to quantify the particle breakage and studied the effect of loading speed on particle breakage. Robertson [8] simulated broken particles with hexagonal stacked clusters and found that particle breaking strength caused by random movement of clusters complied with Weibull distribution. Zhang et al. [9] simulated the pile driving process in the calcareous sand foundation in which the calcareous sand was represented by the cluster that composed of several small balls using cohesive model method; the influence of particle breaking on soil deformation and stress of soil around the pile in the process of pile driving was analysed. Zhang et al. [10] carried out the compression tests on the siliceous sand, Sacramento River sand and calcareous sand based on DEM simulation, and...
analysed the effects of particle breaking on the deformation characteristics of the three materials and found that the decrease of anisotropy in the sample during particle breakage process. Kuang et al. [11] studied the influence of loading rate on the calcareous sand particle breakage by means of physical experiment and DEM simulation, and found that the particle breakage strength of calcareous sand obeyed Weibull distribution. In this study, the biaxial shear tests were carried out based on DEM numerical simulations considering the particle breakage, and the influence of particle breakage on the micromechanical properties of calcareous sand subjected to shearing was explored.

2. Model foundation of numerical simulation
The biaxial shear tests of calcareous sand were simulated based on the discrete element method. The sample with the size of 50×100 mm is shown in Fig. 1. The radius expansion method was used to achieve the targeted void ratio at 0.18 during sample preparation. The Hertz-Mindlin contact model was utilized for particle - particle and particle - wall contacts, and the shear modulus $G$ and Poisson’s ratio $\nu$ are 3GPa and 0.3, respectively. The main parameters of the numerical simulation are shown in Table 1. Confining pressure was 800 kPa while the axial loading was applied with the speed of 0.00035%/s to keep the quasistatic condition. In order to investigate the influence of particle breakage on the micromechanical characteristics of calcareous sand, two series of simulations were set up, that is the samples with and without particle breakage. Particle breakage criterion method was adopted to realize particle breakage [12]. When the stress conditions of the particle satisfies the breakage criterion, a series of sub-particles with the same material properties replace the original particle. The density of the sub-particles is set as 1.2 times of the density of the original particle to make up for the lack of sample mass and maintain the conservation of mass in the system.

| Property                  | Parameter     | Value     |
|---------------------------|---------------|-----------|
| Sample size               | Sample height $H$ (mm) | 100       |
|                           | Sample width $D$ (mm) | 50        |
| Particle properties       | Particle density $\rho_s$ (kg m$^{-3}$) | 2740      |
|                           | Particle size (mm) | 0.5 ~ 1   |

Figure 1. Diagram of biaxial test based on DEM simulation.

Table 1. Model parameter [10, 13].
### 3. Stress-strain and volumetric strain curves

Fig. 2 shows the stress-strain and volumetric strain response of calcareous sand in biaxial shear tests. With the increase of axial strain, the stress-strain curves for the samples with and without particle breakage during shearing have the same trend, that is, the samples was compressed until the strength reaches the peak \( \sigma_1/\sigma_3 \approx 2.65 \), then, stress dropped and the stress ratio of \( \sigma_1/\sigma_3 \) remains 1.7 to 1.8 in the critical state. The stress-strain and volumetric strain response in this study is consistent with the behaviour in the real dense calcareous sand during triaxial test[14], which can reflect the strength and deformation characteristics of calcareous sand before and after particle breakage. The contact force between particles increases with increasing axial strain. When the limit condition of particle breakage is met, particle breakage occurs. With the accumulation of the number of broken particles, the stress-strain and volumetric strain curves for the sample with the particle breakage gradually deviate from sample with no particle breakage. The peak strength of the sample is reduced by the particle breakage, and it soon drops to the critical state. The deformation of the sample changes from dilatation to contraction.

![Fig 2. Stress-strain and volumetric strain curves: (a) stress-strain curve;(b) volumetric strain curve.](image)

### 4. Particle breakage characteristics

To quantify the degree of particle breakage, the relative particle breakage \( B_r \) proposed by Hardin is introduced to quantitatively express the particle breakage [15]. \( B_r \) is the ratio of the total breakage generated in the test (the area enclosed by the grading curve after the test, the initial grading curve and the vertical line at particle diameter of 0.074mm in the particle size distribution curve) to the initial breakage potential (the area enclosed by the initial grading curve and the vertical line at a particle
diameter of 0.074mm in the particle size distribution curve). Fig. 3 shows the variation of $B_r$ with axial strain for the sample with particle broken. It can be seen that particle breakage mainly occurs in the shear zone. The number of particle breakage and $B_r$ increase with axial strain, and the degree of particle breakage is positively correlated with deviatoric stress.

5. Micromechanical properties of particles
The contact force between particles can be divided into the normal force and the tangential force. The external loading on the sample during shearing is mainly supported by the normal force, and the variation trend of the normal force is the same as the stress-strain curve [16]. Therefore, in this study, the contact normal force between particles was used to explore the micromechanics of the particle breakage and the related properties. Guided by Rdajai et al. [17], when the contact force is larger than the mean value, it belongs to the strong contact force chain, and vice versa. Through analysing the changes of strong and weak force during shearing, the variation of the internal structure and the force distribution is discussed. Three represented states, i.e., before peak stress, at peak stress point and after peak stress, corresponding to the axial stain at 3%, 4% and 15% for unbroken group, and the axial stain at 2%, 3% and 15% for broken group, were used to analyse the evolution of the micromechanical characteristics during shearing.

5.1 Contact normal
The contact normal is the direction of the contact between two particles. The contact normal distributions in strong and weak force chain are totally different, as shown in Fig. 4. From the inserted rose diagrams, it is clearly seen that dominant directions of the contact normal for the strong and weak force chain are orthogonal to each other, i.e., vertical and horizontal directions, respectively, which can be expressed by the principal orientation angle, that is the angle between the dominant direction and the horizontal direction. In the Fig. 4, S and W represent the strong and weak force, respectively. The principal orientation angle for the strong force is around 90° (the direction of $\sigma_1$), which also has little change with the increasing axial strain. For the weak force, the principal orientation angle for is around 0° (the direction of $\sigma_3$); particle breakage significantly increases the number of particle contacts in the weak force; moreover, after the peak stress, the principal direction of the contact normal in the weak force for the sample with particle breakage gradually deflects to the vertical direction, and the rose diagram tends to be round, which indicates the anisotropy decrease. However, the principal orientation angle is still in the horizontal direction (0°).
Fig 4. Principal orientation angle of the contact normal between particles.

5.2 Magnitude of contact force
The distributions of the magnitudes of the normal contact forces are shown in Fig. 5. The probability of the magnitude of the normal contact force distribution conforms to Weibull distribution with the confidence more than 95%. The related parameters of the fitting curves for Weibull distributions are shown in Table 2, where the shape parameter \(k\) reflects the basic shape of the probability function curve with the value usually between 1 and 7. When \(k\) is around 3.5, the Weibull distribution is almost the same to the normal distribution; while \(k\) is ~1, the Weibull distribution is approximately equal to the exponential distribution. The scale parameter \(\lambda\) could magnify or diminish the coordinates without affecting the shape of the curve [18-19], that is, the larger the \(\lambda\) is, the larger the data distribution range will be. Therefore, the scale parameter of the contact force distribution has a good correlation with its mean value. When the scale parameter is constant, the larger the shape parameter is, the peak point appears later.

From Fig. 5c and Table 2, it can be seen that the shape parameters of the contact forces for the samples without particle breakage are all greater than 1, which are the typical Weibull distributions. \(k\) decreases with increasing loading, which indicates that the distribution range of the contact forces decreases; the scale parameter, the mean and standard deviation increase first and then decrease. The distribution of the magnitude of the contact force for the sample with particle breakage is similar to that of without particle breakage before the peak strength point in which the particle breakage is not serious. After the peak stress, due to the effect of particle breakage, the internal contact force and its distribution range of the sample are greatly reduced, which can be indicated by the significantly decreasing shape and scale parameters; the shape parameter decreases to 1, and an exponential distribution pattern is obtained.
Fig 5. Distribution diagram of normal contact force between particles: (a) for the sample without particle breakage; (b) for the sample with particle breakage; (c) The probability function curves.

Table 2. Contact force size Weibull distribution fitting results.

| Group                         | Parameter       | Shape parameter | Scale parameter | Mean  | Standard deviation |
|-------------------------------|-----------------|-----------------|-----------------|-------|--------------------|
| Without particle breakage     | Before peak stress | 1.46            | 78600           | 71658 | 47140              |
|                               | Peak stress point | 1.37            | 84732           | 77968 | 54299              |
|                               | After peak stress | 1.36            | 67961           | 62560 | 44642              |
| With particle breakage        | Before peak stress | 1.68            | 68597           | 61714 | 36162              |
|                               | Peak stress point | 1.47            | 77611           | 70701 | 46186              |
|                               | After peak stress | 1.01            | 26625           | 26494 | 26812              |

5.3 Force chain change
Guided by Gao et al. [20], the percentage of weak force plays an important role in the strength of the whole sample. Fig 7. shows the variation of the percentage of weak force chains with respect to axial strain. It can be seen that the proportion of the number of weak forces is always more than 50%, which is larger than the number of strong forces, and it gradually increases with increasing axial strain, and then remains stable. When the particle breakage occurs, the number of samples in the strong and weak forces are significantly increased, and the proportion of weak force chain increased significantly, that is, particle breakage increases the proportion of the internal weak contact forces, and the larger the proportion of the number of weak forces, the lower the local strength of the sample, which weakens the structural stability [20]. As a result, particle breakage leads to the overall decrease of the strength of the sample.
6. Conclusions
In this study, a biaxial test was simulated on calcareous sands based on the discrete element method, and the influence of particle breakage on the internal micromechanical characteristics of calcareous sand was explored. The test results show that: (1) particle breakage usually occurs in the shear band and the degree of particle breakage gradually increases with the axial strain; particle breakage can reduce the peak stress of calcareous sand while the volumetric strain changes from dilatation to contraction; (2) the principal orientations of the strong and weak contact forces are the vertical and horizontal direction, respectively, which are the directions of the maximum and minimum principal stresses; (3) the distribution of the magnitude of normal contact forces obeys the Weibull distribution with little particle breakage while it changes to exponential distribution with relatively larger particle breakage; (4) particle breakage significantly reduces the magnitudes of strong and weak contact forces in the sample, and the percentage of weak contact forces increases significantly, which in turn leads to a decrease in the strength of the sample.

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