Signals of extra gauge bosons and exotic leptons in
SU(6)\textsubscript{L} \otimes U(1)\textsubscript{Y}

Ricardo Gaitán-Lozano\textsuperscript{1,2}, Albino Hernández-Galeana\textsuperscript{1,4},
Sergio A. Tomás\textsuperscript{1}, William A. Ponce\textsuperscript{1,3}
and Arnulfo Zepeda\textsuperscript{1}

1- Departamento de Física, Centro de Investigación y de Estudios Avanzados del IPN
A.P. 14–740, 07000 México D.F., México.
2- Departamento de Física, Universidad Surcolombiana, A. A. 385, Neiva, Colombia.
3- Departamento de Física, Universidad de Antioquia, A. A. 1226, Medellín, Colombia.
4-Escuela de Ciencias, UAEM. Unidad del Cerrillo, Piedras Blancas
Instituto Literario 100, C.P 50000, Toluca Edo. México.

January 20, 2022

January 19, 1995

Abstract

We study some of the consequences of the SU(6)\textsubscript{L} \otimes U(1)\textsubscript{Y} model of unification
of electroweak interactions and families with a horizontal gauge group SU(2)\textsubscript{H},
paying special attention to processes with flavor changing neutral currents. We
compute at tree level the decays \( K^+ \rightarrow \pi^+\mu^+\nu_\mu \), \( K^0_L \rightarrow \mu^+\nu_\mu \) and \( \mu^- \rightarrow e^-\bar{\nu}_e\nu_\mu \) from which we obtain lower bounds for the mass of the horizontal gauge
boson associated with FCNC. Finally we obtain limits on the mixing between
ordinary and exotic charged leptons.
1 Introduction

The precision measurements carried out in the last years have established that the Standard Model (SM) gives an excellent description of the phenomena of particle physics up to 100 GeV. Even though the SM describes with excellent accuracy the physics observed up to now, it does not answer some questions such as the number of families, CP violation and the fermion mass hierarchy problem. For these reasons physicists believe that the SM is not the ultimate theory in particle physics. There are many attempts which try to give an answer to the above questions named in a generic form as extensions of the standard model. All of these extensions imply the existence of new particles additional to those introduced in the SM (either fermions, gauge bosons or scalars). In some extensions the appearance of new exotic fermions is a necessary condition to free the model from anomalies, while the extra gauge bosons arise in a natural way due to additional generators of the gauge group. Exotic fermions manifest themselves either through direct production or by their mixing with ordinary quarks and leptons.

In the SM there are several processes which are strongly suppressed or forbidden; they are called rare decays. Of special interest are those due to the possible existence of flavor changing neutral currents (FCNC). In the SM the FCNC are suppressed by the Glashow-Iliopoulos-Maiani (GIM) mechanism in the quark sector and by the conservation of individual lepton numbers in the leptonic sector due to the masslessness of the neutrinos. The constraints on FCNC, and on rare decays in general, play an important roll in testing possible new physics beyond the SM.

Recently an extension of the SM, based on the SU(6) ⊗ U(1) has been introduced in order to account for some of the peculiarities of the fermion spectrum. In particular the model is capable of accounting for the fact that the top quark is much heavier than the rest of the ordinary fermions. In this article we study some of the new physics implied by this model; in particular we analyze the combined effects due to the appearance of new gauge bosons and exotic leptons. We concentrate on FCNC processes which arise in the model from three sources: horizontal interactions, mixing between exotic and ordinary leptons, and mixing of the standard neutral gauge boson with a horizontal one.

2 The model

The gauge group of the model is SU(6) ⊗ U(1)Y, where SU(6)L unifies the weak isospin SU(2)L of the SM with a horizontal gauge group G_H. SU(2)L ⊗ SU(3)_H is a maximal special subgroup of SU(6)_L. The thirty-five SU(6)_L generators in an SU(2)_L ⊗ SU(3)_H basis are

\[ \sigma_i \otimes 1_3, \quad 1_2 \otimes \lambda_\alpha, \quad \sigma_i \otimes \lambda_\alpha, \]  

(1)
where $\sigma_i$, $i = 1,2,3$ are the Pauli matrices, $T_i = \frac{1}{2}\sigma_i$ are the SU(2)$_L$ generators, $\lambda_\alpha$, $\alpha = 1, 2, ..., 8$, are the Gell-Mann matrices, $\frac{1}{2}\lambda_\alpha$ are the SU(3)$_H$ generators, and $1_3$ and $1_2$ are $3 \times 3$ and $2 \times 2$ unit matrices, respectively.

The fermionic content of the model and the requirement that the exotics which trigger the seesaw mechanism for neutrinos obtain mass at the scale where SU(6)$_L$ is broken demand that $G_H$ be identified with the special maximal subgroup SU(2)$_H$ of SU(3)$_H$ and not with SU(3)$_H$ itself. The special maximal embedding of SU(2)$_H$ into SU(3)$_H$ is achieved by using as generators of SU(2)$_H$ the set

$$(\lambda_1 + \lambda_6)/\sqrt{2}, (\lambda_2 + \lambda_7)/\sqrt{2}, (\lambda_3 + \sqrt{3}\lambda_8)/2.$$  

(2)

The three families are in the adjoint representations of SU(2)$_H$.

The model has 36 gauge bosons: 35 associated with the generators of SU(6)$_L$ and 1 associated to U(1)$_Y$. Besides the standard gauge bosons $W_3$, $W^\pm$ and $B$, we have 32 extra gauge bosons, which can be divided in four classes: 12 charged gauge bosons which produce transitions among families (Family Changing Charge Currents, FCCC); 4 charged gauge bosons which do not produce transitions among families but their couplings are family dependent (Non-Universal Family Diagonal Charged Currents, NUFDCC); 12 neutral gauge bosons which produce transitions among families (Flavor Changing Neutral Currents, FCNC) and 4 neutral gauge bosons which couple non-universally without changing flavor (Non-Universal Flavor Diagonal Neutral Currents, NUFDNC). The gauge bosons of SU(6)$_L$ and the associated generators are displayed in Table 1. The interactions mediated by the gauge fields associated with the generators of SU(2)$_L$ are universal, that is, family independent.

With the usual definition for the electric charge operator ($Q = T_z + Y/2$), the fermionic content of the model is given by the following set of irreducible representations (irreps) of SU(6)$_L \otimes U(1)_Y$ (one for each color in the case of quarks)

$\{6(1/3)\}_L = (u,d,c,s,t,b)_L \equiv \psi^\alpha(1/3)_L,$

$\{6(-1)\}_L = (e^-, \nu_e, \mu^-, \nu_\mu, \tau^-, \nu_\tau)_L \equiv \psi_\alpha(-1)_L,$

$\{1_I(-4/3)\}_L = q^I_-(4/3)_L \quad I = 1,2,3,$ for $u^c, c^c,$ and $t^c$ respectively,

$\{1_I(2/3)\}_L = q^I_+(2/3)_L \quad I = 1,2,3$ for $d^c, s^c,$ and $b^c$ respectively,

$\{1_I(2)\}_L = l^I_+(2)_L \quad I = 1,2,3$ for $e^c, \mu^c,$ and $\tau^c$ respectively,

$\{15(0)\}_L = \psi_{[\alpha\beta]}(0)_L,$

where $I$ is a generational index, $\alpha$ and $\beta$ are SU(6) tensor indices and $u, d, \ldots$ refers to the up quark, down quark, ... fields. The label L refers to left handed Weyl spinors and the upper c symbol indicates a charge-conjugated field. The number in parenthesis is the hypercharge and the symbol $[\alpha\beta]$ stands for antisymmetric ordering, $\psi_{[AB]} = (\psi_{AB} - \psi_{BA})/\sqrt{2}$.

The particle content of $\psi_{[\alpha\beta]}(0)_L$ is 3 exotic electrically charged leptons $E^{+}_{1L}, E^{+}_{2L}$ and $E^{+}_{3L}$ with their respective antiparticles $E^{-}_{1L}, E^{-}_{2L}$ and $E^{-}_{3L}$, and 9 neutral Weyl states $N_i$, 

3
Explicitly we write

\[
\psi^{L}_{[\alpha\beta]}(0) = \begin{pmatrix}
0 & N_1 & E_1^- & N_4 & E_2^- & N_6 \\
0 & N_5 & E_1^+ & N_7 & E_2^+ \\
0 & N_2 & E_3^- & N_8 \\
0 & N_9 & E_3^+ \\
0 & N_3 \\
0 & 0
\end{pmatrix}_L.
\]

With respect to SU(2)_L, the quantum numbers of the above exotic leptons are 3 triplets and 6 neutral singlets. The three triplets are

\[
\begin{pmatrix}
E_1^+ \\
(N_4 + N_5)/\sqrt{2}
\end{pmatrix}, \quad
\begin{pmatrix}
E_2^+ \\
(N_6 + N_7)/\sqrt{2}
\end{pmatrix}, \quad
\begin{pmatrix}
E_3^+ \\
(N_8 + N_9)/\sqrt{2}
\end{pmatrix},
\]

and the six neutral singlets are

\[N_1, N_2, N_3, (N_4 - N_5)/\sqrt{2}, (N_6 - N_7)/\sqrt{2}, (N_8 - N_9)/\sqrt{2}.\]

Possible deviations from the SM predictions could be either due to mixing of ordinary with exotic leptons or due to mixings between the standard gauge bosons and extra ones. Both effects modify the couplings of ordinary fermions to the standard gauge bosons.

### 3 Symmetry Breaking.

The symmetry breaking is realized in three stages: In the first stage

\[
\text{SU}(6)_L \otimes \text{U}(1)_Y \rightarrow \text{SU}(2)_L \otimes \text{SU}(2)_H \otimes \text{U}(1)_Y
\]

at the scale M_1 where the six SU(2)_L singlets of exotic leptons get mass of order M_1. This breaking is achieved with a Higgs scalar in the irrep \(\phi_1 = \{105(0)\}\) of SU(6)_L.

The next stage of symmetry breaking is

\[
\text{SU}(2)_L \otimes \text{SU}(2)_H \otimes \text{U}(1)_Y \rightarrow \text{SU}(2)_L \otimes \text{U}(1)_Y
\]

at the scale M_2 and it is implemented with a Higgs \(\phi_2 = \{15(0)\}\). At this stage the horizontal symmetry is completely broken and simultaneously the exotic leptons which transform as triplets of SU(2)_L get a mass of order M_2. The Vacuum Expectation Values (VEV’s) of \(\phi_1\) and \(\phi_2\) can be read from Ref.[4].
Charged ordinary fermion mass terms. The final stage of the symmetry breaking chain,

\[ \text{SU}(2)_L \otimes \text{U}(1)_Y \longrightarrow \text{U}(1)_{EM} , \] (8)

is achieved by using a Higgs \( \phi_3 = \bar{6}(1) \equiv \phi_{3a}(1) \) with VEV’s in the neutral components \( \langle \phi_{3a}(1) \rangle = v_\alpha \) for \( \alpha = 1, 3, 5 \), (which play the same role as the ordinary Higgs in the SM model). This guarantees that the only unbroken generator is \( Q \).

With \( \langle \phi_3 \rangle \) the following mass term for quarks may be written

\[ \sum_I \gamma_I q^T_L (-4/3) f^T_C \langle \phi_3(1) \rangle \psi^a_L (1/3) + h.c. , \] (9)

where \( \gamma_I \) are Yukawa couplings of order 1. The \( Q=2/3 \) quark mass matrix at tree level produced by eq.(9) is

\[
\begin{pmatrix}
\gamma_u v_1 & \gamma_u v_2 & \gamma_u v_3 \\
\gamma_c v_1 & \gamma_c v_2 & \gamma_c v_3 \\
\gamma_t v_1 & \gamma_t v_2 & \gamma_t v_3
\end{pmatrix}
\] (10)

whose only eigenvalue different from zero, \( m_t = \gamma v, \gamma = \sqrt{\gamma_u^2 + \gamma_c^2 + \gamma_t^2}, \) \( v = \sqrt{v_1^2 + v_2^2 + v_3^2} \), may be recognized as the bare top quark mass. Since we assume that the Yukawa couplings are of order 1, then \( m_t \sim M_W = gv/2 \). There are no \( Q = -1/3 \) quark mass term but \( \phi_3 \) allows also a mass term for the leptonic sector. This mass term would generate a very large mass for the \( \tau \) lepton. To avoid it we may either implement a universal seesaw mechanism through the exotic charged leptons, or we may introduce a \( Z_5 \) discrete symmetry that distinguishes between quarks and leptons as is explicitly done in Ref.[4].

The Higgses introduced in this section achieved the following tasks:

1. The VEV’s \( \langle \phi_1 \rangle \oplus \langle \phi_2 \rangle \oplus \langle \phi_3 \rangle \) break the symmetry \( \text{SU}(6)_L \otimes \text{U}(1)_Y \) down to \( \text{U}(1)_{EM} \).
2. They produce heavy masses to all the exotic leptons.
3. They give a mass of order \( 10^2 \) GeVs to the \( t \) quark. The rest of the known fermion masses are light, because they can be generated only as radiative corrections[6].

4 Mixing between exotic and ordinary fermions

To describe the mixing between ordinary and exotic fermions we follow the formalism given in Ref.[5]. Since \( \text{U}(1)_{EM} \) is unbroken, different eigenstates of weak interactions can mix only when they have the same electric charge.

In general the mixing among \( n \) ordinary fermions and \( m \) exotic fermions of a given charge can be described by a unitary matrix of order \( (n + m) \times (n + m) \). It is convenient
to introduce the vectors $\psi^o_L$ and $\psi^o_R$ which can be decomposed in ordinary and exotic sectors

$$\psi^o_L = \begin{pmatrix} \psi_{EL}^o \\ \psi_{OL}^o \end{pmatrix}, \quad \psi^o_R = \begin{pmatrix} \psi_{ER}^o \\ \psi_{OR}^o \end{pmatrix},$$

(11)

where $\psi_{OL}^o$ is a column vector formed by $n_L$ ordinary fields, while $\psi_{EL}^o$ contains $m_L$ exotic fields, and similarly, $\psi_{OR}^o$ contains $n_R$ ordinary fields, while $\psi_{ER}^o$ contains $m_R$ exotic fields. The superindex $o$ refers to the weak interaction base, that is, to the original fields in the lagrangean with well defined transformation properties under $SU(2)_L \otimes U(1)_Y$. For each helicity one has $n + m$ mass eigenstates, which can be arranged in vector columns as

$$\psi_L = \begin{pmatrix} \psi_{hL} \\ \psi_{lL} \end{pmatrix}, \quad \psi_R = \begin{pmatrix} \psi_{hR} \\ \psi_{lR} \end{pmatrix},$$

(12)

where $\psi_{lL}$ is a vector of $n_L$ light mass eigenstates, $\psi_{hL}$ represents $m_L$ heavy mass eigenstates, and similarly for $\psi_{lR}, \psi_{hR}$.

The gauge and mass eigenstates are related by the equations

$$\psi^o_L = U_L \psi_L, \quad \psi^o_R = U_R \psi_R,$$

(13)

where $U_L$ and $U_R$ are unitary matrices of dimension $(n + m) \times (n + m)$, which diagonalize the fermion mass matrices. $U_L$ and $U_R$ can be written in the block form

$$U_a = \begin{pmatrix} G_a & F_a \\ E_a & A_a \end{pmatrix}, \quad a = L, R.$$

(14)

Here $A_a$ is a mass matrix of dimension $n_a \times n_a$ which relates the ordinary weak states and the light mass eigenstates and $G_a$ is a matrix of dimension $m_a \times m_a$ which relates the exotic and heavy states. $E_a$ and $F_a$ are of dimension $n_a \times m_a$ and $m_a \times n_a$, respectively, and describe the mixing of the two sectors. The matrices $A_a$ and $G_a$ are not unitary, but from the unitarity of $U_a$ one obtains the relations

$$A_a^\dagger A_a + F_a^\dagger F_a = I_{n \times n},$$
$$A_a A_a^\dagger + E_a E_a^\dagger = I_{n \times n}, \quad a = L, R,$$
$$G_a^\dagger G_a + E_a^\dagger E_a = I_{m \times m},$$
$$G_a G_a^\dagger + F_a F_a^\dagger = I_{m \times m}.$$  

(15)

From these relations one can see that $A_a$ and $G_a$ violate unitarity by a small mixing between light and heavy states. Most of the physical effects of mixing are related to the no unitarity of $A_a$.

An arbitrary mixing between exotic and ordinary fermions will induce, in general, flavor changing neutral currents in the light sector of fermions in a rate larger than the experimental limits. These bounds show that for the charged sector these processes of FCNC are extremely suppressed. Contrary to this situation, for neutrinos there are no
experimental limits on possible FCNC. For this reason it is convenient to concentrate
on the charged fermions.

The present experimental limits\(^7\) on the transitions \(sd, cu, bd, bs, \mu e, \tau \mu\) and \(\tau e\) allow us to assume the non existence of FCNC involving light fermions. The absence of FCNC combined with the fact that the matrices \(A^\dagger_a A_a\) and \(F^\dagger_a F_a\) become diagonal, and that \(0 \leq (A^\dagger_a A_a)_{ii} \leq 1\) and \(0 \leq (F^\dagger_a F_a)_{ii} \leq 1\) allows one to write

\[
A^\dagger_a A_a = C^2_a, \quad F^\dagger_a F_a = S^2_a, \quad a = L, R.
\]

where

\[
C_a = \text{diag} \left( C^1_a, C^2_a, \ldots, C^n_a \right), \quad S_a = \text{diag} \left( S^1_a, S^2_a, \ldots, S^n_a \right), \quad a = L, R
\]

such that \(C^i_a \equiv \cos \theta^i_a\) and \(S^i_a \equiv \sin \theta^i_a\), where \(\theta^L_a\) and \(\theta^R_a\) are the light-heavy mixing angles.

5 Mass matrices

In the model under study the known charged fermions obtain mass only from radiative corrections except for the top quark, which obtains a mass at tree level. The mass matrices of fermions and gauge bosons obtained in the model have a complicated structure, making it difficult to find an analytic solution to the eigenvalue problems. To solve it we use an approximation method based on perturbation theory taking advantage of the fact that the components of the mass matrices depend on the three scales of SSB, \(M_1\), \(M_2\) and \(v\), with the hierarchy \(M_1 \gg M_2 \gg v\).

5.1 Mass for charged leptons

From reference \(4\) we can write the following mass matrix for the charged leptons in the base \((E^0_1, E^0_2, E^0_3, e^0, \mu^0, \tau^0)\)

\[
M^{(0)}_C = \begin{pmatrix}
M_2 & 0 & 0 & w & w & w \\
0 & M_2 & 0 & w & w & w \\
0 & 0 & M_2 & w & w & w \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

where \(w \ll v\), because this contribution comes from triplets of SU(2)_L. The rank of this matrix is three and as a consequence it does not give any mass to the electron, muon or
tau. Adding the contribution from radiative corrections, $M_C$ can be parameterized as

$$M_C = \begin{pmatrix} M_2 & 0 & 0 & w & w & w \\ 0 & M_2 & 0 & w & w & w \\ 0 & 0 & M_2 & w & w & w \\ 0 & 0 & 0 & V_1 & V_2 & V_3 \\ 0 & 0 & 0 & V'_1 & V'_2 & V'_3 \\ 0 & 0 & 0 & V''_1 & V''_2 & V''_3 \end{pmatrix}, \tag{19}$$

where $V_i, V'_i, V''_i, i = 1, 2, 3$, are real parameters of order of the electron, muon and tau masses, respectively (at this stage we are ignoring phases, so CP is conserved).

$M_C$ can be diagonalized by a biunitary transformation

$$U_{CL}^\dagger M_C U_{CR} = M_D, \tag{20}$$

where $M_D$ is a diagonal matrix and $U_{CL}, U_{CR}$ are unitary matrices. They satisfy

$$U_{CL}^\dagger M_C M_C^\dagger U_{CL} = U_{CR}^\dagger M_C^\dagger M_C U_{CR} = M_D^2. \tag{21}$$

To diagonalize $M_C$ we write first

$$M_C M_C^\dagger = (M_C M_C^\dagger)_0 + (M_C M_C^\dagger)_1, \tag{22}$$

where $(M_C M_C^\dagger)_0 = M_C^0 M_C^0$, while $(M_C M_C^\dagger)_1$ contains the parameters coming from radiative corrections and their mixings with $M_2$ and $w$. $M_C^{00}$ is easily diagonalized. Next we consider $(M_C M_C^\dagger)_1$ as a perturbation. In order to simplify the calculation we take $V''_1 = V''_2 = V''_3 \equiv V''$.

The weak eigenstates $e^0_L, \mu^0_L, \tau^0_L$ are related to the mass eigenstates $e_L, \mu_L, \tau_L$ through the relations

$$e^0_L = \sqrt{3} \frac{w}{M^2} (V_1 + V_2 + V_3) E_{1L} + \theta_1 e_L + \beta_1 \mu_L + \alpha_1 \tau_L,$$

$$\mu^0_L = \sqrt{3} \frac{w}{M^2} (V'_1 + V'_2 + V'_3) E_{1L} + \theta_2 e_L + \beta_2 \mu_L + \alpha_2 \tau_L, \tag{23}$$

$$\tau^0_L = \sqrt{3} \frac{w}{M^2} V'' E_{1L} + \theta_3 e_L + \beta_3 \mu_L + \alpha_3 \tau_L,$$

where the constants $\theta_i, \beta_i$ and $\alpha_i, i = 1, 2, 3$ are functions of the parameters $V_i, V'_i$ and $V''$. The explicit expressions for $\theta_i, \beta_i$ and $\alpha_i$ are given in the Appendix. The corresponding expressions for right handed charged leptons are not needed here.
5.2 Mass for neutral leptons

With respect to the neutral sector, the masses for ordinary neutrinos are generated through the see-saw mechanism. In the basis

\[
\begin{pmatrix}
N_1, N_2, N_3, \frac{(N_4 - N_5)}{\sqrt{2}}, \frac{(N_6 - N_7)}{\sqrt{2}}, \frac{(N_8 - N_9)}{\sqrt{2}}, \\
\frac{(N_4 + N_5)}{\sqrt{2}}, \frac{(N_6 + N_7)}{\sqrt{2}}, \frac{(N_8 + N_9)}{\sqrt{2}}, \nu_e^0, \nu_\mu^0, \nu_\tau^0
\end{pmatrix},
\]

the mass matrix of this sector has the form

\[
M_{N\nu} = \begin{pmatrix}
A_{6\times6} & 0_{6\times3} & B_{6\times3} \\
0_{3\times6} & C_{3\times3} & 0_{3\times3} \\
B^T_{3\times6} & 0_{3\times3} & 0_{3\times3}
\end{pmatrix},
\]

where \( A \) has entries of order \( M_1 \) and \( M_2 \), \( B \) of order \( v \), and \( C \) of order \( M_2 \). To diagonalize this matrix we use a double perturbation theory. After the algebra is done we get the following expressions for the ordinary neutrinos \( \nu_e^0 \) and \( \nu_\mu^0 \)

\[
\begin{align*}
\nu_e^0 &= -\frac{1}{\sqrt{2}}\nu_1 + \frac{1}{1.4668}\nu_2 + \frac{1}{5.3302}\nu_3 + O(\delta_2)N_i, \\
\nu_\mu^0 &= \frac{0.3842}{1.4668}\nu_2 - \frac{5.1397}{5.3302}\nu_3 + O(\delta_2)N_i,
\end{align*}
\]

where the parameter \( \delta_2 = v/M_2 \ll 1 \). These expressions are used later to compute the decay rate of the muon.

5.3 Neutral gauge bosons

For the neutral gauge bosons (see Table 1) in the base

\[
\begin{pmatrix}
Z_0, Y_{H(1)}, E_{13(1)}, Y_{3(1)}, E_{15(1)}, \Sigma_2, Z_4, \Sigma_1, Z_3, Y_{r(1)}, F_{13(1)} \\
Y_{3(2)}, E_{15(2)}, Y_{H(2)}, E_{13(2)}, Y_{r(2)}, F_{13(2)}
\end{pmatrix},
\]

we obtain the mass matrix

\[
\begin{pmatrix}
M_N^2 & 0 \\
0 & M_N^2
\end{pmatrix},
\]

where \( M_N^2 \) is a symmetric mass matrix of dimension 11 \( \times \) 11 containing terms of order \( M_1^2, M_2^2 \) and \( v^2 \); 0 are zero matrices and \( M_N^2 \) is a symmetric matrix of dimension 6 \( \times \) 6. \( Z_0 \) is the gauge boson of the SM,

\[
Z_0 = \frac{gW_3 - g'B}{\sqrt{g^2 + g'^2}},
\]
where $g$ and $g'$ are the coupling constants of SU(6)$_L$ and U(1)$_Y$ respectively.

The diagonalization of $M_N^2$ is achieved considering

$$M_N^2 = (M_N^2)_0 + (M_N^2)_1,$$  \hfill (29)

with $(M_N^2)_0$ being the matrix with components of order $M_1^2$ and $M_2^2$ and $(M_N^2)_1$ containing components of order $v^2$. In a similar way to the charged lepton sector we solve the eigenvalue problem using perturbation theory. The results show that $Z_0$ mixes essentially with the horizontal gauge boson $Y_{H(1)}$. This fact allows us to relate two mass eigenstates, $Z_{SM}$ and $Z'$, with the eigenstates of weak interactions through an orthogonal transformation with a mixing angle $\Theta$, in the form

$$\begin{pmatrix} Y_{H(1)} \\ Z_0 \end{pmatrix} = \begin{pmatrix} \cos \Theta & -\sin \Theta \\ \sin \Theta & \cos \Theta \end{pmatrix} \begin{pmatrix} Z' \\ Z_{SM} \end{pmatrix}$$ \hfill (30)

where

$$\cos \Theta = \frac{1}{k_3}, \quad \sin \Theta = \frac{1}{k_3} \left( 2 \frac{v^2}{3\sqrt{3} M_2^2} \sqrt{g^2 + g'^2} \right)$$ \hfill (31)

and

$$k_3 = \sqrt{1 + \left( 2 \frac{v^2}{3\sqrt{3} M_2^2} \sqrt{g^2 + g'^2} / g \right)^2}$$ \hfill (32)

The mass of the $Z_{SM}$ gauge boson is given by

$$M_Z^2 \simeq \frac{M_W^2}{\cos^2 \theta_W}$$ \hfill (33)

where $M_W$ is the mass of the charged boson $W$ and $\theta_W$ is the weak mixing angle given by

$$\tan \theta_W = \frac{g'}{g}.$$ \hfill (34)

The mass for $Z'$ is of order $M_2$, while the mass for the other extra gauge bosons are of order $M_1$ and $M_2$.

### 6 Calculation of decays

In this section we compute the decays $K^+ \rightarrow \pi^+ e^+\nu_e$, $K_L^0 \rightarrow \mu^+ e^-$ and $\mu \rightarrow e\bar{\nu}_e\nu_\mu$. The sensitivity of the experimental measurements allow us to give a lower bound for the mass of the horizontal gauge boson $Z'$. 

10
6.1 $K^+ \rightarrow \pi^+ \mu^+ e^-$

In this decay the only contribution to the amplitude arises from the hadronic vector current, because the kaon and the pion have the same parity.

The tree level diagram for this decay is given by Fig.(1). The decay amplitude is

$$\mathcal{M} = \frac{G_{\text{eff}}}{\sqrt{2}} \bar{e} \gamma^\mu (C - D \gamma_5) \mu A \left\{ f_+(t) \left[ (P + P')_\mu - \frac{m_K^2 - m_\pi^2}{t} (P - P')_\mu \right] \right.$$ \begin{align*}
+ & f_0(t) \frac{m_K^2 - m_\pi^2}{t} (P - P')_\mu \right\},
\end{align*}

where

$$A = \frac{1}{4} [V_{cs} V_{ud}^* + V_{ts} V_{cd}^*] (\cos \Theta + 1) + \frac{1}{4} [V_{us} V_{cd}^* + V_{cs} V_{td}^*] (\cos \Theta - 1)$$ \begin{align*}
& + \sin \Theta \frac{C_{Vq}}{\sqrt{3} \cos \theta_W} (V_{us} V_{ud}^* + V_{cs} V_{cd}^* + V_{ts} V_{td}^*)
\end{align*}

$$C = \sin \Theta \frac{\delta_3}{\sqrt{3} \cos \theta_W} C_V + \frac{\cos \Theta \Delta_1 + \Delta_2}{4},$$

$$D = \sin \Theta \frac{\delta_3}{\sqrt{3} \cos \theta_W} C_A + \frac{\cos \Theta \Delta_1 + \Delta_2}{4},$$

and

$$\frac{G_{\text{eff}}}{\sqrt{2}} = \frac{g^2}{8 m_Z^2}.$$ \hspace{1cm} (37)

$V_{ab}$ are the elements of the Kobayashi-Maskawa mixing matrix and

$$\Delta_1 \equiv (\theta_1 \beta_2 + \theta_2 \beta_1 + \theta_2 \beta_3 + \theta_3 \beta_2),$$

$$\Delta_2 \equiv (\theta_1 \beta_2 - \theta_2 \beta_1 + \theta_2 \beta_3 - \theta_3 \beta_2),$$

$$\delta_3 \equiv (\theta_1 \beta_1 + \theta_2 \beta_2 + \theta_3 \beta_3),$$ \hspace{1cm} (38)

with $C_A$, $C_V$ and $C_{Vq}$ being the constants

$$C_V = -1/2 + 2 \sin^2 \theta_W,$$

$$C_A = -1/2,$$

$$C_{Vq} = -1/2 + 2/3 \sin^2 \theta_W.$$ \hspace{1cm} (39)

$P, P', P_1$ and $P_2$ are the fourmomenta of $K^+, \pi^+, \mu^+$ and $e^-$, respectively, and $f_+(t)$ and $f_0(t)$ are form factors of order 1 (in fact, in our calculation they are taken equal to 1).
Obviously the momentum transfer \( t = (P - P')^2 \) is negligible compared with the square mass of the horizontal gauge boson \( Z' \), that is, \( t \ll m_{Z'}^2 \).

From the above amplitude we compute the partial decay rate

\[
d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{32m_K^3} \sum_{\text{pol}} |\mathcal{M}|^2 dt ds
\]

with \( s = (P - P_1)^2 \), and we find

\[
\Gamma = \frac{9G_F^2(C^2 + D^2)A^2}{32(2\pi)^3} \frac{M_W^4}{m_{Z'}^2 m_K^3} [2.464 \times 10^{20} \text{ MeV}^8]
\]

where \( G_{\text{eff}} \) and \( G_F \) (Fermi’s constant) are related by

\[
C_{\text{eff}}^2 = \frac{9M_W^4}{m_{Z'}^2} C_F^2.
\]

As we can see, \( \Gamma \) depends on several parameters of the model. Among these parameters we have \( \theta_i, \beta_i \) and \( \alpha_i \) (which are functions of \( V_i, V'_i \) and \( V''_i \) introduced from the radiative corrections generating the masses of charged leptons, eq.(19)), the angle \( \Theta \) (which describes the mixing between the gauge bosons \( Z_0 \) and \( Y_{H(1)} \)) and the mass \( M_{Z'} \) of the horizontal boson \( Z' \).

In order to compare the expression (41) with the experimental limits \( \text{[10]} \), we have taken

\[
\begin{align*}
V_i &\sim m_e, \\
v'_i &\sim m_\mu, \\
v''_i &\sim m_\tau.
\end{align*}
\]

With the above approximations the only non negligible parameters in the set \( (\theta_i, \beta_i, \alpha_i) \) are \( \theta_1, \beta_2 \) and \( \alpha_3 \). In particular

\[
(\theta_1/\beta_2)^2 \sim (1 + \frac{m_\mu^2}{m_\tau^2})^{-1}.
\]

In consequence we obtain the values

\[
\begin{align*}
C &= -0.4927, \\
D &= 0.4927, \\
A &= 0.2300.
\end{align*}
\]

Taking into account the values for \( m_\mu, m_K, m_\pi, M_W, G_F \) and the experimental bound for this decay \( \text{[10]} \)

\[
\Gamma_{\text{exp}} < 11.17327314 \times 10^{-24} \text{ Mev},
\]

we get the constraint

\[
m_{Z'} > 19.1767 \text{ TeV}.
\]
6.2 $K^0_L \rightarrow e^- \mu^+$

The state $K^0_L$ is a superposition of the states $K^0$ and $\bar{K}^0$, defined as

$$|K^0_L\rangle = \frac{[(1 + \epsilon)|K^0\rangle - (1 - \epsilon)|\bar{K}^0\rangle]}{[2(1 + |\epsilon|^2)]^{1/2}},$$

(47)

$\epsilon$ being a parameter that describes the CP violation mixing.

The amplitude for the leading diagram contributing to the process, Fig. (2), is given by

$$\mathcal{M} = -\frac{G_{\text{eff}}^2}{2} \bar{e} \gamma^\mu (C - D \gamma_5) \mu B f_K (P_1 + P_2) \mu,$$

(48)

where $C$ and $D$ are given by eq.(36) and

$$B = -\frac{1}{4} [V_{cs}^* V_{ud} + V_{ts}^* V_{td}] (\cos \Theta + 1) - \frac{1}{4} [V_{us}^* V_{cd} + V_{cs}^* V_{td}] (\cos \Theta - 1)$$

$$+ \sin \Theta \frac{1}{2\sqrt{3} \cos \theta_W} (V_{us}^* V_{ud} + V_{cs}^* V_{td} + V_{ts}^* V_{td})$$

(49)

$P_1$ and $P_2$ are the four momenta of the electron and the muon respectively, and $f_K$ is the decay constant of the kaon. In the above amplitude we have considered again that the momentum transfer is small compared with the square mass of the boson $Z'$. Hence we obtain the total decay rate

$$\Gamma = \frac{9B^2 f_K^2 M_W^4}{32\pi} G_F^2 (C^2 + D^2) \frac{m^2_{\mu}}{m^3_{K^0}} (m^2_{K^0} - m^2_{\mu})^2.$$ 

(50)

Now, taking into account the approximations given in Eq. (43) we find

$$B = -0.2300.$$ 

(51)

Using now the experimental data

$$f_K^+ = 160.6 \text{ MeV},$$

$$m_{K^0} = 497.676 \pm 0.030 \text{ MeV},$$

$$\Gamma^{\text{exp}} < 12.01397025 \times 10^{-25} \text{ MeV},$$

(52)

and taking for the parameters $C$ and $D$ the values given in eq. (45), we obtain the constraint

$$m_{Z'} > 34.7546 \text{ TeV}$$

(53)
6.3 Muon Decay $\mu \rightarrow e\bar{\nu}_e\nu_\mu$.

Another process that allows us to establish a lower bound for the mass of the horizontal gauge boson $Z'$ is the muon decay $\mu \rightarrow e\bar{\nu}_e\nu_\mu$. This process is mediated by the standard $W$ charged boson as well as by the horizontal boson $Z'$. The corresponding diagrams are shown in Figs. (3) and (4).

The total amplitude is

$$\mathcal{M} = \mathcal{M}_W + \mathcal{M}_{Z'}$$

(54)

where $\mathcal{M}_W$ ($\mathcal{M}_{Z'}$) is the amplitude for the process mediated by the $W$ ($Z'$). In order to get the couplings we must take into account that, unlike the charged sector, in the neutral sector we do not know the masses for the neutrinos. This fact gives as a result that, while the charged leptons are written in the lagrangean in terms of their mass eigenstates, the neutrinos are left in the weak interaction base.

The amplitudes $\mathcal{M}_{Z'}$ and $\mathcal{M}_W$ are

$$\mathcal{M}_W = \left[G_{\mu} \bar{\nu}_e \gamma_\mu \left(\frac{1 - \gamma^5}{2}\right) \nu_\mu \right] \frac{g_{\mu\nu}}{M_W^2} \left[G_e \bar{\nu}_e \gamma_\nu \left(\frac{1 - \gamma^5}{2}\right) \nu_\mu \right],$$

(55)

and

$$\mathcal{M}_{Z'} = - \left[G_{n} \bar{\nu}_e \gamma_\mu \left(\frac{1 - \gamma^5}{2}\right) \nu_\mu \right] \frac{g_{\mu\nu}}{M_{Z'}^2} \left[G_{ch} \bar{\nu}_e \gamma_\nu \left(C_V' - C_A' \gamma^5\right) \mu \right],$$

(56)

where

$$G_n \equiv \frac{g}{2\sqrt{2}} \cos \Theta + 1,$$

$$G_{ch} \equiv \frac{g}{2\sqrt{2}} \cos \Theta \Delta_1 + \Delta_2,$$

(57)

and

$$G_{\mu} \equiv \frac{g_{SM}}{\sqrt{2}} \beta_2, \quad G_e \equiv \frac{g_{SM}}{\sqrt{2}} \theta_1, \quad g_{SM} = g/\sqrt{3}.$$

(58)

We have defined

$$C_V' \equiv xC_V + 1,$$

$$C_A' \equiv xC_A + 1,$$

(59)

$$x \equiv \frac{4 \sin \Theta}{\cos \Theta \Delta_1 + \Delta_2 \sqrt{3} \cos \theta_W},$$

where we have used the relations given by (38) and (39) and we have considered that the transferred momentum in the propagator is small compared with $M_{Z'}^2$ ($M_W^2$).

To evaluate the nonstandard contribution to the amplitude, given by Eq. (56), we Fierz-transform it and compare its scalar and pseudoscalar part with the parameterization used in Ref. [11].

$$\mathcal{M}_{Z'}^{SP} = \frac{g^2}{2\sqrt{2}M_W^2} \bar{e}_R \nu_{eL} \bar{\nu}_{\mu L} \mu_R$$

(60)
and we obtain

\[ g^+ = \frac{G_F}{g^2} 8 \sqrt{2} r \frac{M^2_{Z'}}{M^2_{Z}} \sin^2 \Theta_W \]

\[ = \sqrt{6} (\cos \Theta + 1) \sin \Theta \delta_3 \sin^2 \theta_W \frac{M^2_W}{M^2_{Z'}}. \]  

(61)

Using now the bounds for \( g^+ \) obtained in Ref. 12, we get

\[ M^2_{Z'} > 10 \sqrt{6} M^2_W \sin^2 \theta_W (\cos \Theta + 1) \sin \Theta \delta_3 \]

(62)

Since \( \sin \Theta \) and \( \delta_3 \) are \( \ll 1 \) we conclude that this bound on \( M_{Z'} \) is weaker than those obtained above from the rare kaon decays.

7 Limits on the mixing angles between exotic and ordinary leptons.

Here we use the formalism given in section 4 to analyze the mixing between exotic and ordinary leptons. In particular, we estimate the square of the mixing angles.

From the discussion of Section 2, and from the matrices \( U_{\odot,\odot} \), the submatrices \( F_{L,(R)} \) obtained in the model are

\[ F_L = - \begin{pmatrix} \frac{3 w}{M^2_{Z}} V'' & \frac{w}{M^2_{Z}} (V'_1 + V'_2 + V'_3) & \frac{w}{M^2_{Z}} (V_1 + V_2 + V_3) \\ \frac{3 w}{M^2_{Z}} V'' & \frac{w}{M^2_{Z}} (V'_1 + V'_2 + V'_3) & \frac{w}{M^2_{Z}} (V_1 + V_2 + V_3) \\ \frac{3 w}{M^2_{Z}} V'' & \frac{w}{M^2_{Z}} (V'_1 + V'_2 + V'_3) & \frac{w}{M^2_{Z}} (V_1 + V_2 + V_3) \end{pmatrix} \]

(63)

and \( F_R = \)

\[ \begin{pmatrix} \frac{\sqrt{3} w}{M^2_{Z}} (V_1 V_3 + V'_1 V'_3 + V''_3) & \frac{\sqrt{3} w}{M^2_{Z}} (V_1 V_2 + V'_1 V'_2 + V''_2) & \frac{\sqrt{3} w}{M^2_{Z}} (V_1^2 + V'_1^2 + V''_2^2) \\ \frac{\sqrt{3} w}{M^2_{Z}} (V_1 V_3 + V'_1 V'_3 + V''_3) & \frac{\sqrt{3} w}{M^2_{Z}} (V_1 V_2 + V'_1 V'_2 + V''_2) & \frac{\sqrt{3} w}{M^2_{Z}} (V_1^2 + V'_1^2 + V''_2^2) \\ \frac{\sqrt{3} w}{M^2_{Z}} (V_1 V_3 + V'_1 V'_3 + V''_3) & \frac{\sqrt{3} w}{M^2_{Z}} (V_1 V_2 + V'_1 V'_2 + V''_2) & \frac{\sqrt{3} w}{M^2_{Z}} (V_1^2 + V'_1^2 + V''_2^2) \end{pmatrix} \]

(64)

From these equations we can compute \( F_{L,(R)}^+ F_{L,(R)} \). The limits found for the mass of the horizontal gauge boson which mediates FCNC, allow us to neglect the off diagonal terms, and by the formalism of section 4, we can reparameterize the terms in the diagonal; that is, we can write

\[ F_a^+ F_a = \begin{pmatrix} (S^1_a)^2 & 0 \\ 0 & (S^2_a)^2 \end{pmatrix}, \quad a = L, R, \]

(65)
where
\[
(S^3_L)^2 = \frac{3w^2}{M^2_2}(V_1 + V_2 + V_3)^2,
\]
\[
(S^2_L)^2 = \frac{3w^2}{M^2_2}(V'_1 + V'_2 + V'_3)^2,
\]
\[
(S^1_L)^2 = \frac{27w^2}{M^2_2}V''^2,
\]
and
\[
(S^1_R)^2 = (S^2_R)^2 = (S^3_R)^2 = 9 \left[ \frac{w}{M^2_2} + \frac{w}{M^2_2}(V_1V_3 + V'_1V'_3 + V''^2) \right]^2
+ 9 \left[ \frac{w}{M^2_2}(V_1V_2 + V'_1V'_2 + V''^2) \right]^2
+ 9 \left[ \frac{w}{M^2_2}(V_1^2 + V'_1^2 + V''^2) \right]^2.
\]

With the constraint \( M_{Z'} \sim M_2 \), and using \( V \sim m_e \), \( V' \sim m_\mu \) and \( V'' \sim m_\tau \), we get

\[
\begin{array}{|c|c|c|c|c|}
\hline
M_{Z'} & (S^1_R)^2 & (S^2_R)^2 & (S^3_R)^2 & (S^3_R)^2 \\
\hline
19.0363 \text{ TeV} & 654.443 \times 10^{-14} & 2.295 \times 10^{-14} & 5.369 \times 10^{-19} & 24.836 \times 10^{-5} \\
34.5 \text{ TeV} & 606.523 \times 10^{-14} & 2.127 \times 10^{-14} & 4.977 \times 10^{-20} & 7.561 \times 10^{-5} \\
\hline
\end{array}
\]

where in the last column \( i = 1, 2, 3 \).

\section*{8 Conclusions.}

We have analyzed some phenomenological implications of the model
\( \text{SU}(6)_L \otimes U(1)_Y \) of unification of families with the standard electroweak interactions. The model contains exotic leptons and extra gauge bosons. The work has been focused on FCNC and rare decays produced by three sources: the horizontal interactions, the mixing between exotic and ordinary leptons, and the mixing of the standard Z neutral gauge boson with a horizontal gauge boson of \( \text{SU}(2)_H \).

Given the complex structure of the mass matrices of fermions and gauge bosons, we implement a mechanism to diagonalize these matrices by using perturbation theory. To establish some limits on the scale of the breaking of the horizontal symmetry and limits for the mixing angles between exotic and ordinary charged leptons, we have computed the rare decays \( K^+ \rightarrow \pi^+ \mu^+ e^- \) and \( K^0 \rightarrow \mu^+ e^- \), as well as the muon decay \( \mu \rightarrow e \nu_e \nu_\mu \).

The results can be summarized as follows: a) The mass of the flavor changing gauge boson is limited to values higher than 34 TeV; b) the limits on \( S^2 \), the square of the sine of the mixing angle between exotic and ordinary charge leptons are stringent for the right sector \( (S^2_R \leq 7.5 \times 10^{-5}) \), and extremely stringent for the left sector \( (S^2_L \leq 6 \times 10^{-12}) \).

\textbf{Acknowledgements} This work was supported in part by CONACyT (Mexico). A.Z. benefitted from conversations with R. Shrock in the stimulating atmosphere of the Aspen Center for Physics. We thank G. López- Castro for fruitful conversations. R. G. acknowledges financial support from the O.E.A. and W.A.P. from Colciencias (Colombia).
References

[1] S. L. Glashow, Nucl. Phys. 22, 579(1961);  
S. Weinberg, Phys. Rev. Lett. 19, 1964(1967);  
A. Salam, in *Elementary Particle Theory, Relativistic groups and Analicity*, ed. by  
N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.

[2] P. Langacker, M. Luo and A. Mann, Rev. Mod. Phys. 64, 87(1992);  
P. Langacker, Phys. Rep. 72, 185(1981).

[3] P. Langacker and D. London, Phys. Rev. D38, 886(1988); *ibid* 907(1988).

[4] A. Hernández, W. A. Ponce and A. Zepeda, Z. Phys. C-Particles and Fields 55,  
423(1992).

[5] E. Nardi, E. Roulet and D. Tommasini, Phys.Rev. D46, 3040(1992);  
E. Nardi, Phys. Rev. D48, 1240(1993);  
E. Nardi, E. Roulet and D. Tommasini, Nucl. Phys. 386B, 239(1992);  
P. Langacker and D. London Ref[3].

[6] X-G He, R. R. Volkas and D-D. Wu, Phys. Rev. 41, 1630(1990);  
B.S. Balakrishna, Phys. Rev. Lett.60, 1602(1988); B.S. Balakrishna, A.L. Kagan  
and R.N. Mohapatra, Phys. Lett. 205B, 345(1988).

[7] Particle Data Group, M. Aguilar-Benitez et al., Phys. Rev. D45, 11(1992).

[8] See for example: W. A. Ponce, A Zepeda and R. Gaitán-Lozano, Phys Rev D49,  
4954(1994).

[9] R. Gaitán-Lozano, Ph.D. Thesis, Unpublished, CINVESTAV, 1993.

[10] T. Akagi et al., Phys. Rev. Lett. 67, 2614(1991);  
A. M. Lee et al., Phys. Rev. Lett. 64, 165(1990).

[11] P. Langacker and D. London, Phys. Rev. D39, 266(1989).

[12] W. Fetscher, H. J. Gerber, and K. F. Johnson, Phys. Lett. B173, 102(1986).
Table 1
Clasification of generators and gauge bosons of SU(6)_L. The numbers in the first column denote (SU(2)_L, SU(2)_H) dimensionality.

| Branching | Class   | Generators     | Gauge bosons |
|-----------|---------|----------------|--------------|
| (3, 1)    | standard| \(\sigma_1 \otimes I_3\) | \(W_1\)      |
|           |         | \(\sigma_2 \otimes I_3\) | \(W_2\)      |
|           |         | \(\sigma_3 \otimes I_3\) | \(W_3\)      |
| (1, 3)    | NUFDNC  | \(I_2 \otimes (\lambda_3 + \sqrt{3}\lambda_8)/2\) | \(\Sigma_1\) |
| FCNC      |         | \(I_2 \otimes (\lambda_1 + \lambda_6)/\sqrt{2}\) | \(Y_{H(1)}\) |
|           |         | \(I_2 \otimes (\lambda_2 + \lambda_7)/\sqrt{2}\) | \(Y_{H(2)}\) |
| (1, 5)    | NUFDNC  | \(I_2 \otimes (\sqrt{3}\lambda_3 - \lambda_8)/2\) | \(\Sigma_2\) |
| FCNC      |         | \(I_2 \otimes (\lambda_1 - \lambda_6)/\sqrt{2}\) | \(Y_{r(1)}\) |
|           |         | \(I_2 \otimes (\lambda_2 - \lambda_7)/\sqrt{2}\) | \(Y_{r(2)}\) |
|           |         | \(I_2 \otimes \lambda_4\) | \(Y_{3(1)}\) |
|           |         | \(I_2 \otimes \lambda_5\) | \(Y_{3(2)}\) |
| (3, 3)    | NUFDNC  | \(\sigma_3 \otimes (\lambda_3 + \sqrt{3}\lambda_8)/2\) | \(Z_3\)      |
| FCNC      |         | \(\sigma_3 \otimes (\lambda_1 + \lambda_6)/\sqrt{2}\) | \(E_{13(1)}\) |
|           |         | \(\sigma_3 \otimes (\lambda_2 + \lambda_7)/\sqrt{2}\) | \(E_{13(2)}\) |
| NUFDC 2   |         | \(\sigma_1 \otimes (\lambda_3 + \sqrt{3}\lambda_8)/2\) | \(E_{12(1)}\) |
|           |         | \(\sigma_2 \otimes (\lambda_3 + \sqrt{3}\lambda_8)/2\) | \(E_{12(2)}\) |
| FCCC      |         | \(\sigma_1 \otimes (\lambda_1 + \lambda_6)/\sqrt{2}\) | \(E_{14(1)}\) |
|           |         | \(\sigma_2 \otimes (\lambda_1 + \lambda_6)/\sqrt{2}\) | \(E_{14(2)}\) |
|           |         | \(\sigma_1 \otimes (\lambda_2 + \lambda_7)/\sqrt{2}\) | \(F_{14(2)}\) |
|           |         | \(\sigma_2 \otimes (\lambda_2 + \lambda_7)/\sqrt{2}\) | \(F_{14(1)}\) |
| (3, 5)    | NUFDNC  | \(\sigma_3 \otimes (\sqrt{3}\lambda_3 - \lambda_8)/2\) | \(Z_4\)      |
| FCNC      |         | \(\sigma_3 \otimes (\lambda_1 - \lambda_6)/\sqrt{2}\) | \(F_{13(1)}\) |
|           |         | \(\sigma_3 \otimes (\lambda_2 - \lambda_7)/\sqrt{2}\) | \(F_{13(2)}\) |
|           |         | \(\sigma_3 \otimes \lambda_4\) | \(E_{15(1)}\) |
|           |         | \(\sigma_3 \otimes \lambda_5\) | \(E_{15(2)}\) |
| NUFDC 2   |         | \(\sigma_1 \otimes (\sqrt{3}\lambda_3 - \lambda_8)/2\) | \(F_{12(1)}\) |
|           |         | \(\sigma_2 \otimes (\sqrt{3}\lambda_3 - \lambda_8)/2\) | \(F_{12(2)}\) |
| FCCC      |         | \(\sigma_1 \otimes (\lambda_1 - \lambda_6)/\sqrt{2}\) | \(G_{14(1)}\) |
|           |         | \(\sigma_2 \otimes (\lambda_1 - \lambda_6)/\sqrt{2}\) | \(G_{14(2)}\) |
|           |         | \(\sigma_1 \otimes (\lambda_2 - \lambda_7)/\sqrt{2}\) | \(H_{14(2)}\) |
|           |         | \(\sigma_2 \otimes (\lambda_2 - \lambda_7)/\sqrt{2}\) | \(H_{14(1)}\) |
|           |         | \(\sigma_1 \otimes \lambda_4\) | \(E_{16(1)}\) |
|           |         | \(\sigma_2 \otimes \lambda_4\) | \(E_{16(2)}\) |
|           |         | \(\sigma_1 \otimes \lambda_5\) | \(F_{16(2)}\) |
|           |         | \(\sigma_2 \otimes \lambda_5\) | \(F_{16(1)}\) |
Appendix

Parameters $\theta_i$, $\beta_i$ and $\alpha_i$:

$$\alpha_1 = \left(\frac{V_1 + V_2 + V_3}{V''_n}\right) / r^\frac{3}{2}$$

$$\alpha_2 = \left(\frac{V'_1 + V'_2 + V'_3}{V''_n}\right) / r^\frac{1}{2}$$

$$\alpha_3 = \frac{1}{r^\frac{1}{2}}$$

$$\beta_1 = \left(\frac{3(V_1 + V_2 + V_3)(V'_1 + V'_2 + V'_3) - (V_1 V'_1 + V_2 V'_2 + V_3 V'_3)}{(V'_1^2 + V'_2^2 + V'_3^2) - 3(V'_1 + V'_2 + V'_3)^2}\right) / r^\frac{1}{2}$$

$$\beta_2 = -\frac{1}{r^\frac{1}{2}}$$

$$\beta_3 = \alpha_2$$

$$\theta_1 = 1$$

$$\theta_2 = \frac{\beta_1}{r^\frac{1}{2}}$$

$$\theta_3 = -\left(\frac{\alpha_1}{r^\frac{1}{2}}\right) - \beta_1 \left(\frac{V'_1 + V'_2 + V'_3}{V''_n r^\frac{1}{2}}\right)$$

where

$$r = 1 + \frac{(V'_1 + V'_2 + V'_3)^2}{V''^2}$$
Fig. 1 Tree level diagram for $K^+ \rightarrow \pi^+ e^- \mu^+$ through a horizontal gauge boson $Z'$.

Fig. 2 Tree level diagram for $K^0_L \rightarrow e^- \mu^+$ through a horizontal gauge boson $Z'$.

Fig. 3 Diagram which contribute to the $\mu \rightarrow e\bar{\nu}_e \nu_\mu$ in SU(6)$_L$⊗U(1)$_Y$ through the weak gauge boson $W$.

Fig. 4 Diagram which contribute to the $\mu \rightarrow e\bar{\nu}_e \nu_\mu$ in SU(6)$_L$⊗U(1)$_Y$ through the horizontal gauge boson $Z'$. 
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9502220v1
This figure "fig1-3.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9502220v1