Abstract. The detection of a time variation of the angle between two distant sources would reveal an anisotropic expansion of the Universe. We study this effect of cosmic parallax within the ellipsoidal universe model, namely a particular homogeneous anisotropic cosmological model of Bianchi type I, whose attractive feature is the potentiality to account for the observed lack of power of the large-scale cosmic microwave background anisotropy. The preferred direction in the sky, singled out by the axis of symmetry inherent to planar symmetry of ellipsoidal universe, could in principle be constrained by future cosmic parallax data. However, that will be a real possibility if and when the experimental accuracy will be enhanced at least by two orders of magnitude.

I. INTRODUCTION

Observational cosmology is entering a precision era thanks to, just to cite a few, the high precision data collected by the Wilkinson Microwave Anisotropy Probe (WMAP) satellite [1], to the recently-launched PLANCK mission [2], and to planned space projects such as the Gaia astrometric mission [3].

Therefore, one may wonder if perhaps it is time to directly test the correctness of the assumptions which are at the base of the by-now-accepted standard cosmological model, the Lambda cold dark matter (ΛCDM) concordance model [4], namely homogeneity and isotropy of the large-scale structure of the Universe.

Indeed, some anomalous features in the seven-year WMAP data have already been interpreted as a hint that our Universe could be not isotropic [5] (see also Refs. [6–20]).

In particular, the analysis of inhomogeneities in the distribution of the excursion sets in the cosmic microwave background (CMB) maps, suggests the existence of an anomalous plane-mirroring symmetry on large angular scales [21, 22].

Moreover, the presence of a preferred direction in the sky, revealed by the alignment of quadrupole and octupole modes of CMB anisotropy spectrum, the so-called “Axis of Evil” (AE) [23], seems to indicate planar symmetry in the geometry of the Universe on large cosmic scales.

Finally, the same CMB data show a suppression of power at large angular scales, i.e. a quadrupole amplitude lower than that predicted by the ΛCDM model. This “anomalous” occurrence, refereed to as “quadrupole problem”, has been widely studied in the literature [24–44].

A possible solution to that problem has been given in Ref. [5] and rests on the idea that the Universe is expanding anisotropically with respect to a certain direction in the sky. The resulting cosmological model is essentially of Bianchi type I, where the free parameters are fixed by requiring that the quadrupole generated by anisotropic expansion lowers, down to the desired level, that caused by the standard inflationary mechanism. In the following, this particular cosmological model will be referred to as “ellipsoidal universe”.

It is only recently that the possibility has been put forward that an anisotropic expansion of the Universe could be directly revealed by high-precision astrometric measurements. As shown in the seminal paper by Quercellini, Quartin and Amendola [45], space missions like Gaia could measure tiny variations, over a period of a decade, of the relative angular position of two bright sources in the sky, namely an effect of “cosmic parallax”. If this will be the case, far reaching consequences there will be on our vision of the structure of the Universe.

The aim of this paper is to analyze such a cosmic effect of parallax in the context of the ellipsoidal universe.

The plan of the paper is then as follows. In section II, we point out all the essential properties of a universe filled with a nonstandard fluid with anisotropic equation of state and we discuss its relation to the ellipsoidal universe model. In section III, we discuss cosmic parallax emphasizing its dependence on the direction of the axis of symmetry introduced by the ellipsoidal universe model. In the Conclusions, we briefly comment on our results.
II. ELLIPSOIDAL UNIVERSE AND COSMIC SHEAR

The ellipsoidal universe proposal \cite{6} rests on the assumption that the large-scale geometry of the Universe is described by a Taub line element \cite{10}

\[
ds^2 = dt^2 - a^2(t)(dx^2 + dy^2) - b^2(t)dz^2,
\]

with two expansion parameters, \(a\) and \(b\), which we normalize as \(a(t_0) = b(t_0) = 1\) at the present cosmic time \(t_0\). The above metric is homogeneous but anisotropic and, according to Bianchi classification, is of type I. The resulting universe possesses (at large cosmological scales) planar symmetry, with axis of symmetry directed along the \(z\)-axis.

In such a type of universe, a quadrupole term develops in the CMB radiation \cite{5} which adds to that caused by the inflation-produced gravitational potential at the last scattering surface. By a suitable orientation of the two terms, it is possible to lower the overall quadrupole power to such an extent to match the “low” value of the observed quadrupole to that predicted by the standard cosmological model. Introducing the “eccentricity”

\[
e = \begin{cases} \sqrt{1 - (b/a)^2}, & a > b, \\ \sqrt{1 - (a/b)^2}, & a < b, \end{cases}
\]

it has been shown in Refs. \cite{6, 7} that this possibility is achieved if the eccentricities at decoupling is approximatively

\[
c_{\text{dec}} \approx \sqrt{\frac{1}{15} \left[3\sqrt{15} - 5\operatorname{sgn}(a-b)\right]} \frac{Q_I}{T_{\text{cmb}}},
\]

where \(\operatorname{sgn} x\) is the sign function, \(Q_I^2 \approx 1200 \mu K^2\) \cite{11} is the best-fit value of the quadrupole amplitude for the \(\Lambda\)CDM concordance model, and \(T_{\text{cmb}} \approx 2.73\) K is the actual (average) CMB temperature.

Cosmic anisotropy could be triggered, as discussed in Refs. \cite{6, 7}, by the presence in the Universe of a (almost) uniform magnetic field, or topological defects such as a domain wall or a cosmic string. According to Ref. \cite{11}, instead, a possible cause of anisotropization could be Lorentz symmetry violation during inflation which, in turn, generates magnetic fields possessing planar symmetry at large cosmological scales. Recently, it has been shown that a dark energy component with anisotropic equation of state has all the requirements to give rise an ellipsoidal universe \cite{6}.

Whatever is the source of anisotropy, its energy-momentum tensor has to be consistent with planar symmetry:

\[
T^\mu_\nu = \begin{pmatrix} \rho & -p_\parallel & -p_\parallel & -p_\perp \\ -p_\parallel & \rho_A & p_\parallel & p_\parallel \\ -p_\parallel & p_\parallel & \rho_A & p_\parallel \\ -p_\perp & p_\perp & p_\perp & \rho_A \end{pmatrix},
\]

where \(\rho\) is the energy density, while \(p_\parallel\) and \(p_\perp\) are the “longitudinal” and “normal” pressures.

In the following, we consider a universe filled with a generic and unspecified anisotropic component,

\[
(T_A)^\mu_\nu = \begin{pmatrix} \rho_A & -p_\parallel^A & -p_\parallel^A & -p_\perp^A \\ -p_\parallel^A & \rho_A & p_\parallel^A & p_\parallel^A \\ -p_\parallel^A & p_\parallel^A & \rho_A & p_\parallel^A \\ -p_\perp^A & p_\perp^A & p_\perp^A & \rho_A \end{pmatrix},
\]

which induces the planar symmetry and an isotropic contribution,

\[
(T_I)^\mu_\nu = \begin{pmatrix} \rho_I & -p_I & -p_I & -p_I \\ -p_I & \rho_I & p_I & p_I \\ -p_I & p_I & \rho_I & p_I \\ -p_I & p_I & p_I & \rho_I \end{pmatrix},
\]

made up of three different components: a radiation component \((r)\), a matter component \((m)\), and a cosmological constant component \((\Lambda)\),

\[
\rho_I = \rho_r + \rho_m + \rho_\Lambda,
\]

\[
p_I = \rho_r + \rho_m + \rho_\Lambda,
\]

with equations of state: \(p_r = \rho_r/3\), \(p_m = 0\), and \(p_\Lambda = -\rho_\Lambda\).

Let us introduce, in the usual way, the cosmic shear, \(\Sigma\), and the “mean Hubble parameter”, \(H\), as

\[
\Sigma \equiv (H_a - H) / H, \quad H \equiv \dot{A}/A,
\]

where \(H_a \equiv \dot{a}/a\) and \(A \equiv (a^2b)^{1/3}\) is the “mean expansion parameter”.

Taking into account the above definitions, the Einstein’s equations for the cosmological model at hand read

\[
(1 - \Sigma^2)H^2 = \frac{8\pi G}{3}(\rho_I + \rho_A),
\]

\[
(1 - \Sigma + \Sigma^2)H^2 + [(2 - \Sigma)H] = -\frac{8\pi G}{3}(p_I + p_A),
\]

\[
(1 + \Sigma)H^2 + 2[(1 + \Sigma)H] = -\frac{8\pi G}{3}(p_I + p_A),
\]
where a dot denotes a differentiation with respect to the cosmic time.

The source of anisotropy is proportional to the difference between the longitudinal and normal pressures of the anisotropic fluid, as it is evident if we subtract side by side Eqs. (11) and (12):

\[
(H\Sigma) + 3H^2\Sigma = \frac{8\pi G}{3} (p_\parallel - p_\perp). \tag{13}
\]

For the sake of simplicity, we assume that all components are noninteracting, so that they are separately conserved. For the isotropic components we have \((T_X)^{\mu\nu}_{\,\,\,\mu} = 0\), where \(X = r, m, \Lambda, A\), which gives

\[
\rho_r = \rho_r^{(0)} A^{-4}, \quad \rho_m = \rho_m^{(0)} A^{-3}, \quad \rho_\Lambda = \rho_\Lambda^{(0)}, \tag{14}
\]

where from now on an index “0” defines quantities evaluated at the actual time. The conservation of the anisotropic part of the energy-momentum tensor, \((T_A)^{\mu\nu}_{\,\,\,\,\mu} = 0\), gives instead

\[
\dot{\rho}_A + [3(1 + w) + 2\delta\Sigma] H\rho_A = 0, \tag{15}
\]

where we have introduced the “mean equation of state parameter” \(w\) and the “skewness” \(\delta\) as

\[
w = \frac{2p_\parallel + p_\perp}{3\rho_A}, \quad \delta = \frac{p_\parallel - p_\perp}{\rho_A}. \tag{16}
\]

Moreover, we assume that \(w\) and \(\delta\) are constants and that \(\Sigma\) is a small quantity (as we will verify \textit{a posteriori}). Therefore, we can neglect the second term in the square brackets of Eq. (15), which simply gives:

\[
\rho_A = \rho_A^{(0)} A^{-3(1+w)}. \tag{17}
\]

Introducing the “mean density parameters”

\[
\Omega_X = \frac{\rho_X^{(0)}}{\rho_c^{(0)}}, \quad \rho_c^{(0)} = \frac{3H_0^2}{8\pi G}, \tag{18}
\]

where \(X = r, m, \Lambda, A\), and taking into account Eqs. (14)-(18), the shear equation (13) can be solved to give

\[
\Sigma(A) = \frac{\Sigma_0 + (E - E_0) \Omega_A \delta}{A^3H/H_0}, \tag{19}
\]

where

\[
H(A)/H_0 = \sqrt{\Omega_r A^{-4} + \Omega_m A^{-3} + \Omega_\Lambda + \Omega_A A^{-3(1+w)}} \tag{20}
\]

and we have defined the function

\[
E(A) = \int_0^A \frac{dx}{x^{1+3w}H(x)/H_0}. \tag{21}
\]

Evaluating Eq. (20) at the present time gives

\[
\Omega_r + \Omega_m + \Omega_\Lambda + \Omega_A = 1, \tag{22}
\]

so that the density parameters are not all independent.

The standard (isotropic) ΛCDM concordance model fits, at a high level of accuracy, cosmological astrophysical data coming from very disparate phenomena taking place in various epoch of the Universe, such as Big Bang Nucleosynthesis and Large Scale Structure formation. For this reason, we only want to consider anisotropic cosmological models which are very close to it. We then require both that the energy density of the anisotropic component is negligibly small compared to the energy densities of the isotropic ones ("subdominance condition"),

\[
\forall A \in [0,1] : \quad \rho_A(A) \ll \max_{A\in[0,1]} [\rho_r + \rho_m + \rho_\Lambda], \tag{23}
\]

and that the Universe isotropize at early times ("isotropization condition"),

\[
\lim_{A\to0} \Sigma(A) = 0. \tag{24}
\]
Taking the limit $A \to 0$ in Eq. (19), we find that the isotropization condition is satisfied if and only if

$$\Sigma_0 = E_0 \Omega_A \delta, \quad w < 1/3. \quad (25)$$

When $w = 1/3$, condition (24) is never fulfilled. However, in this particular case, we find that at early times $\Sigma$ approaches the constant value

$$\lim_{A \to 0} \Sigma(A) = \frac{\Omega_A \delta}{\Omega}, \quad w = 1/3 \quad (26)$$

if Eq. (25) is satisfied. Even if the Universe does not isotropize for $A \to 0$, it becomes almost isotropic for, as we will show, the quantity $\Omega_A \delta$ is vanishingly small for $w = 1/3$ (see middle-right panel of Fig. 1).

Assuming that all the above conditions are fulfilled, we can write the shear as

$$\Sigma(A) = \Sigma_0 \frac{E/E_0}{A^3 H/H_0}, \quad (27)$$
together with its asymptotic expansion for $A \to 0$:

$$A \ll 1 : \Sigma(A) \simeq \frac{\delta}{2 - 3w} \frac{\Omega_A}{\Omega_r} A^{1 - 3w}. \quad (28)$$

In Fig. 1 (upper panel), we show the shear $\Sigma$ as a function of the mean expansion parameter $A$. The smallness of shear (which we will verify in a moment) assures that the values of mean parameters $\Omega_r$ and $\Omega_A$ are very close to the analogous ones for the isotropic standard cosmological model. For this reason, we take in this paper: $\Omega_r \simeq \Omega^{(\text{isotropic})}_r \simeq 0.83 \times 10^{-4}$ [47] and $\Omega_A \simeq \Omega^{(\text{isotropic})}_A \simeq 0.73$ [48].

We want now to connect the density parameter $\Omega_A$ to the eccentricity at decoupling, since the anisotropic component is supposed to be the cause of the anisotropization of the Universe at decoupling. To this end, we observe that the eccentricity and the shear are connected by the following relation:

$$e^2 = 6 \text{sgn}(a - b) \int_1^A \frac{dx}{x} \Sigma(x), \quad (29)$$

valid for small eccentricities, $e \ll 1$, and coming from definitions (2) and (9). By evaluating the above equation at the time of decoupling, and using Eqs. (25) and (27), we get

$$\text{sgn}(a - b) = -\text{sgn} \delta \quad (30)$$

together with

$$\Omega_A = \frac{c_{\Omega}(w)}{\delta} e_{\text{dec}}^2, \quad (31)$$

and we recast $\Sigma_0$ in the form

$$|\Sigma_0| = c_{\Sigma}(w) e_{\text{dec}}^2, \quad (32)$$

where

$$c_{\Omega} \equiv \left[ 6 \int_{A_{\text{dec}}}^{1} \frac{dx}{x^4} \frac{E(x)}{H(x)/H_0} \right]^{-1}, \quad (33)$$

$$c_{\Sigma} \equiv E_0 c_\Omega. \quad (34)$$

Here, $A_{\text{dec}} = A(t_{\text{dec}})$ is the mean expansion parameter evaluated at the time of decoupling and will be simply taken to be $A_{\text{dec}} = 1/(1 + z_{\text{dec}})$, with $z_{\text{dec}} \simeq 1090$ [48] the redshift at decoupling.

In Fig. 1 (middle panels), we show the actual shear $\Sigma_0$ and the density parameter $\Omega_A$ as a function of the mean equation of state parameter $w$, with $e_{\text{dec}}$ given by Eq. (3), together with the asymptotic expansions for large negative values of $w$:

$$w \ll -1 : |\Sigma_0| \simeq \frac{1}{2} \sqrt{\Omega_A} e_{\text{dec}}^2 |w|, \quad (35)$$

$$w \ll -1 : \Omega_A \simeq \frac{3}{2} \Omega_A e_{\text{dec}}^2 |\delta| \quad (36)$$

As it is clear from the upper panel and the left middle panel of Fig. 1, the cosmic shear is always much smaller than unity, as we have previously assumed. Moreover, the actual fraction of energy associated to the anisotropic component (see the right middle panel of Fig. 1) is negligible with respect to those of dark matter and cosmological constant if $\delta$ is not too small in absolute value. Very small values of $|\delta|$ are not allowed, however, by the subdominance condition (23), which taking into account Eq. (31) can be rewritten as

$$|\delta| \gg \delta_{\text{min}} = c_{\delta}(w) e_{\text{dec}}^2, \quad (37)$$

where

$$c_{\delta} \equiv \frac{c_{\Omega}}{\max_{A \in [0,1]} \left| A^{3(1+w)} H^2(A)/H_0^2 \right|}. \quad (38)$$
TABLE I: Some components with anisotropic equation of state which could give rise to an ellipsoidal universe: The skewness \( \delta \) is the deviation from isotropy of the equation of state, whose mean parameter is \( w \); the mean density parameter is \( \Omega_A \), while the actual amount of anisotropy in the geometry of the universe is the shear \( \Sigma_0 \).

| Component  | \( w \) | \( \delta \) | \( \Omega_A \)  | \( \Sigma_0 \) |
|------------|-------|-----|--------|--------|
| B (planar) | 1/3   | -1 | \( 1.3 \times 10^{-9} \) | \( 4.5 \times 10^{-9} \) |
| B (uniform)| 1/3   | 2  | \( 9.9 \times 10^{-10} \) | \( 6.7 \times 10^{-9} \) |
| String     | -1/3  | 1  | \( 9.2 \times 10^{-6} \)  | \( 5.0 \times 10^{-6} \) |
| Domain wall| -2/3  | -1 | \( 2.1 \times 10^{-5} \)  | \( 7.7 \times 10^{-6} \) |

In Fig. 1 (lower panels), we plot the minimum skewness \( \delta_{\text{min}} \) as a function of \( w \), together with the asymptotic expansion:

\[
w \ll -1 : \quad \delta_{\text{min}} \simeq \frac{3}{2} \Omega_A c_{\text{dec}}^2 w^2.\]

Finally, in Tab. 1, we show the results for some particular and physically relevant case of anisotropic component, such as planar magnetic field (discussed in Ref. [7]), uniform magnetic field, cosmic string, and domain wall.

Looking at the Table and again at Fig. 1, we get that (realistic) values of \( |w| \) of order unity give at most \( \Sigma_0 \) of order \( 10^{-4} \). This result will be important for the following discussion about the possibility to detect cosmic anisotropy by cosmic parallax effects.

### III. COSMIC PARALLAX AND A PREFERRED DIRECTION IN THE SKY

The angle \( \gamma \) between two sources in an ellipsoidal universe, as view by an observer centered at the origin of reference system, is given by [49]:

\[
\cos \gamma(t) = \frac{\sum_i a_i^{-2} \hat{p}_i \hat{q}_i}{(\sum_i a_i^{-2} \hat{p}_i^2)^{1/2} (\sum_i a_i^{-2} \hat{q}_i^2)^{1/2}},
\]

where \( a_1 \equiv a_2 \equiv a \) and \( a_3 \equiv b \) are the expansion parameters, while \( \hat{p} \) and \( \hat{q} \) are the direction cosines defining the angular position of the sources. If the Universe expands anisotropically, then the angle \( \gamma \) is time dependent and one can hope to detect its temporal variation, the so-called cosmic parallax \( \Delta \gamma \), looking at two different sources at two different times.

It is worth stressing that the above result applies to the case where the axis of symmetry is directed along the \( z \)-axis. We may, however, easily generalize this result to the case where the symmetry axis is directed along a particular direction \( (b, l) = (b_A, l_A) \) in the galactic coordinate system, where \( b \) and \( l \) are, respectively, the galactic latitude and galactic longitude. To this end, we perform a rotation

\[
R \equiv R_x(\pi/2 - b_A) R_x(\pi/2 + l_A)
\]

of the coordinate system \((x, y, z)\), where \( R_x(\pi/2 - b_A) \) and \( R_x(\pi/2 + l_A) \) are rotations of angles \( \pi/2 - b_A \) and \( \pi/2 + l_A \) about the \( x \) - and \( z \)-axis, respectively. In the galactic coordinate system the axis of symmetry is defined by the direction cosines

\[
\hat{n}_A \equiv (\cos b_A \cos l_A, \cos b_A \sin l_A, \sin b_A),
\]

while \( \hat{p} \) and \( \hat{q} \) are defined by the new direction cosines \( \hat{n} = R^{-1} \hat{p} \) and \( \hat{n'} = R^{-1} \hat{q} \), where

\[
\hat{n} \equiv (\cos \alpha \cos l, \cos \alpha \sin l, \sin \alpha),
\]

\[
\hat{n'} \equiv (\cos \alpha' \cos l', \cos \alpha' \sin l', \sin \alpha').
\]

In the galactic coordinate system, with the axis of symmetry defined by the unit vector \( \hat{n}_A \), the angle \( \gamma \) is

\[
\cos \gamma(t) = \frac{\cos \gamma_0 - e^2 (\cos \gamma_0 - \cos \alpha \cos \alpha')}{(1 - e^2 \sin^2 \alpha)^{1/2} (1 - e^2 \sin^2 \alpha')^{1/2}}.
\]

Finally, in Tab. 1, we show the results for some particular and physically relevant case of anisotropic component, such as planar magnetic field (discussed in Ref. [7]), uniform magnetic field, cosmic string, and domain wall.
when \( a > b \), or

\[
\cos \gamma(t) = \frac{\cos \gamma_0 - e^2 \cos \alpha \cos \alpha'}{(1 - e^2 \cos^2 \alpha)^{1/2} (1 - e^2 \cos^2 \alpha')^{1/2}},
\]

if \( a < b \), where we have introduced the eccentricity \( e \). The quantities \( \gamma_0 \), \( \alpha \) and \( \alpha' \) are the angles between, respectively, \( \hat{n} \) and \( \hat{n}' \), \( \hat{n} \) and \( \hat{n}_A \), and \( \hat{n}' \) and \( \hat{n}_A \):

\[
\cos \gamma_0 \equiv \hat{n} \cdot \hat{n}' = \sin b \sin b' + \cos b \cos b' \cos (l - l'),
\]

\[
\cos \alpha \equiv \hat{n} \cdot \hat{n}_A = \sin b \sin b_A + \cos b \cos b_A \cos (l - l_A),
\]

\[
\cos \alpha' \equiv \hat{n}' \cdot \hat{n}_A = \sin b' \sin b_A + \cos b' \cos b_A \cos (l' - l_A).
\]

Differentiating Eqs. \((45)\) and \((46)\), we straightforwardly obtain the expression for the cosmic parallax \( \Delta \gamma \) in a small time interval \( \Delta t \) centered around \( t_0 \) and caused by an anisotropic expansion of the Universe:

\[
\Delta \gamma \approx \left. \frac{d\gamma}{dt} \right|_{t=t_0} \Delta t = 3 \Upsilon H_0 \Sigma_0 \Delta t,
\]

where we introduced the “modulating function” \( \Upsilon \) as

\[
\Upsilon(b_A, l_A; b, l; b', l') \equiv \cot \gamma_0 (\cos^2 \alpha + \cos^2 \alpha') - 2 \csc \gamma_0 \cos \alpha \cos \alpha',
\]

whose values are in the interval \([-1, 1]\).

In the case where the axis of symmetry is directed along the \( z \)-axis \( (b_A = \pi/2) \), one easily recovers the results of Ref. \[50, 51\].

Equation \((49)\) is an involved expression of the coordinates of the axis of symmetry \( (b_A, l_A) \) and of the directions of the two sources \( (b, l) \) and \( (b', l') \).

There are, however, some interesting cases where it gives very simple results. For example, if one of the two sources is in the direction of the symmetry axis, let us say \( \hat{n}' \equiv \hat{n}_A \), and the other one has the same longitude of the symmetry axis, \( l = l_A \), we simply have

\[
\Upsilon(b_A, l_A; b, l; b_A, l_A) = \frac{1}{2} \sin 2(b - b_A).
\]

In Fig. 2, we plot the modulating function versus the latitude \( b \) of one source for different values of its longitude \( l \). We take the other source pointing in the direction of the symmetry axis which we assume to be the axis of evil \[52\]:

\[
(b_{AE}, l_{AE}) \simeq (20^\circ, 135^\circ).
\]

In principle, cosmic parallax data from planned astronomic missions, such as Gaia, could allow, other than constraining the cosmic shear [see Eq. \((45)\)], to determine the direction of the symmetry axis \textit{via} the analysis of the modulating

FIG. 2: The modulating function \( \Upsilon \) [see Eqs. \((47)-(49)\)] versus \( b \) for \((b', l') = (b_A, l_A) = (b_{AE}, l_{AE}) = (20^\circ, 135^\circ) \) for different values of \( l \). The thick dotted curve is Eq. \((50)\).
Indeed, the latter is very sensitive to the position of the axis since, for example, if we look at two sources, one along the \( z \)-axis and the other along the \( x \)-axis \([ (b', l') = (0, 0)]\), we find:

\[
\Upsilon(b_A, l_A; \pi/2, l; 0, 0) = \sin 2b_A \cos l_A.
\] (52)

The modulating function, in this case, is zero if the symmetry axis coincides with the \( z \)-axis (as supposed in Refs. \([49–51]\)) or, in general, when lies in the galactic plane \((b_A = 0)\) or in a plane perpendicular to it \((b_A = \pm \pi/2)\), maximal \((\Upsilon = 1)\) for \((b_A, l_A) = (45^\circ, 0^\circ)\) or \((-45^\circ, 180^\circ)\), and minimal \((\Upsilon = -1)\) in the directions \((b_A, l_A) = (45^\circ, 180^\circ)\) and \((-45^\circ, 0^\circ)\).

However, if one takes into account that the maximum precision of Gaia in measuring cosmic parallax is about \(6 \mu\text{as} [52]\), a large cosmic anisotropy of order \(|\Sigma_0| \sim 10^{-2}\) is required in order to extract information from the modulating function. In fact, assuming a capability to detect the angular position of two sources at two different times separated by \(\Delta t = 10\text{yr}\), we can conveniently rewrite Eq. (48) as

\[
\Delta \gamma \approx 4.4 \Upsilon \frac{h}{0.70} \frac{\Sigma_0}{10^{-2}} \frac{\Delta t}{10\text{yr}} \mu\text{as},
\] (53)

where the little-\( h \) constant, \(H_0 = 100h \text{km} / \text{sec} / \text{Mpc}\), is about \(h^{(\text{isotropic})} \approx 0.70 [48]\) in the isotropic standard cosmological model.

The ellipsoidal universe proposal, allowing at maximum cosmic shears of order \(|\Sigma_0| \sim 10^{-4}\) (see Fig. 1), is then not testable, for the time being, by cosmic parallax measurements.

**IV. CONCLUSIONS**

Homogeneity and isotropy of the Universe on large cosmological scales are the foundation of standard cosmology. Although they are simply assumed as the basis of observational and theoretical cosmology, experimental data seem, up today, to confirm and support the grounding of those hypotheses. In particular, two decades of observations of the cosmic microwave background radiation and analysis of magnitude-redshift data on type Ia supernovae, have firmly established that, if there are deviations from isotropy, they must be very tiny.

If on the one hand the predictions of the isotropic \(\Lambda\text{CDM}\) concordance model are in stunning agreement with a huge amount of observational data collected in the last years, on the other hand some tension, between theoretical and inferred value of the CMB quadrupole anisotropy, still persists in CMB data, even after the recent results of seven years WMAP observations.

The deficit of power on large angular scales, emerged since the first data of COBE back in 1992 and now known as “quadrupole problem”, could be a hint of anisotropization of the Universe at cosmic scales. Indeed, the simplest anisotropic cosmological model, namely a Bianchi type I characterized by a plane-symmetric line element, has been shown to be compatible with all CMB and supernovae data so far gathered and analyzed and, moreover, allow for a solution to the quadrupole problem.

In this paper, we have further investigated such a kind of nonstandard cosmological model, named “ellipsoidal universe”, turning our attention to a possible signature that could be detected in high-precision astrometric observations, that is a “cosmic parallax” effect. As already noticed in the literature, the relative angular position of two distant sources changes in time if the Universe expands anisotropically. Intuitively to understand but difficult to measure, this effect of parallax represents a unique possibility to directly test the hypothesis of isotropy/anisotropy of the Universe.

We have shown, indeed, that planned astrometric missions such as Gaia, have too low accuracies both to appreciate the small amount of anisotropy and to confirm the existence of a preferred direction in the sky predicted by the ellipsoidal universe model.

This, however, does not exclude the possibility that, in a not-too-far future, Gaia-like missions with enhanced sensitivity of about at least two order of magnitude could either see a signal of cosmic anisotropy or completely rule out the ellipsoidal universe model.
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