NUMERICAL SIMULATION OF HOT ACCRETION FLOWS. II. NATURE, ORIGIN, AND PROPERTIES OF OUTFLOWS AND THEIR POSSIBLE OBSERVATIONAL APPLICATIONS

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**ABSTRACT**

Hydrodynamical (HD) and magnetohydrodynamical (MHD) numerical simulations of hot accretion flows have indicated that the inflow accretion rate decreases inward. Two models have been proposed to explain this result. In the adiabatic inflow–outflow solution (ADIOS), this is because of the loss of gas in the outflow. In the alternative convection-dominated accretion flow model, it is thought that the flow is convectively unstable and gas is locked in convective eddies. We investigate the nature of the inward decrease of the accretion rate using HD and MHD simulations. We calculate various properties of the inflow and outflow such as temperature and rotational velocity. Systematic and significant differences are found. These results suggest that the inflow and outflow are not simply convective turbulence; instead, systematic inward and outward motion (i.e., real outflow) must exist. We have also analyzed the convective stability of MHD accretion flows and found that they are stable. These results favor the ADIOS scenario. We suggest that the mechanisms of producing outflow in HD and MHD flows are the buoyancy associated with the convection and the centrifugal force associated with the angular momentum transport mediated by the magnetic field, respectively. The latter is similar to the Blandford & Payne mechanism but no large-scale open magnetic field is required. We discuss some possible observational applications, including the Fermi bubble in the Galactic center and winds in active galactic nuclei and black hole X-ray binaries.

**Key words:** accretion, accretion disks – black hole physics – hydrodynamics

**Online-only material:** color figures

1. INTRODUCTION

There are two series of black hole accretion solutions, depending on the temperature of the accretion flow. The cold series includes the standard thin disk and the slim disk, bounded by the Eddington accretion rate $\dot{M}_{\text{Edd}} \equiv 10L_{\text{Edd}}/c^2$ (Shakura & Sunyaev 1973; Abramowicz et al. 1988). For the hot series, when the accretion rate is below $\sim \alpha^2 \dot{M}_{\text{Edd}}$, where $\alpha$ is the viscous parameter, we have advection-dominated accretion flow (ADAF; Narayan & Yi 1994, 1995; Abramowicz et al. 1995; see reviews by Narayan et al. 1999, 2001; Kato et al. 1998; F. Yuan & R. Narayan 2012, in preparation). The main application of ADAFs is in the dim black hole sources, including the supermassive black hole in our Galactic center, low-luminosity active galactic nuclei (AGNs), and quiescent and hard states of black hole X-ray binaries (see reviews by Narayan 2005; Yuan 2007; Narayan & McClintock 2008; Ho 2008). Above $\alpha^2 \dot{M}_{\text{Edd}}$, the solution is described by the luminous hot accretion flow (LHAF; Yuan 2001, 2003). LHAFs correspond to higher accretion rates and radiative efficiency therefore emitted luminosity is higher. This model has been applied to explain the origin of hard X-ray emission in relatively luminous sources such as the luminous hard states of black hole X-ray binaries and some AGNs (e.g., Yuan & Zdziarski 2004; Yuan et al. 2007).

In the early analytical studies of ADAFs, it was assumed that the mass accretion rate, or more precisely the inflow rate (inflow and outflow rates are defined as the mass flux of gas with negative and positive radial velocities; see Equations (12) and (13) for their definitions), is constant with radius. Correspondingly, the density follows a power-law distribution: $\rho(r) \propto r^{-3/2}$. Later many numerical simulations were performed to study the multi-dimensional dynamics of the hot accretion flow, including both hydrodynamical (HD) and magnetohydrodynamical (MHD) simulations (e.g., Stone et al. 1999, hereafter SPB99; Igumenshchev & Abramowicz 1999, 2000; Stone & Pringle 2001; Hawley et al. 2001; Machida et al. 2001; Hawley & Balbus 2002; Igumenshchev et al. 2003; Pen et al. 2003; De Villiers et al. 2003, 2005; Yuan & Bu 2010; Pang et al. 2011; McKinney et al. 2012; Narayan et al. 2012; Li et al. 2012; Yuan et al. 2012, hereafter Paper I). One of the most important findings of these simulations is that the inflow accretion rate decreases with decreasing radius. The radial profile of the inflow rate can be described by a power-law function of radius, $M(r) \propto r^s$ with $s \approx 0.5-1$. Correspondingly, the radial profile of density becomes flatter, $\rho(r) \propto r^{-p}$ with $p \approx 1.5 - s \approx 1-0.5$. Such a result not only has theoretical significance, but is also of great importance to observations. This is because it determines the emitted spectrum of the accretion flow, and also the strength of “mechanical AGN feedback,” if such a profile is caused by mass outflow as we will illustrate here.

Because of its importance, we first need to critically examine the reliability of this result. This is because, due to technical difficulties, all the numerical simulations mentioned above have a rather small radial dynamical range, which usually spans two orders of magnitude. In this case, the results may suffer from the boundary conditions and are thus suspect. To overcome this problem, in Paper I we presented a “two-zone” approach, successfully extending the dynamical range to four orders of magnitude. We found that in this case the profiles of accretion rate and density remain almost unchanged compared with previous works. In addition, we combined previous numerical simulation work on hot accretion flows, including both HD and MHD simulations, and found that the profiles of accretion rate and density in all these works are quite similar. Such
consistency is in agreement with the prediction of a recent work by Begelman (2012), although the slopes are different. What is more exciting is that these theoretical results have been confirmed by observations, e.g., of Sgr A* and NGC 3115 (Yuan et al. 2003; Wong et al. 2011; see Paper I for details).

The next question is then: what is the physical reason for the decrease of the accretion rate? Two main models have been proposed to answer this question. The first is the adiabatic inflow–outflow solution (ADIOS; Blandford & Begelman 1999, 2004; Begelman 2012; see also Becker et al. 2001; Xue & Wang 2005; Jiao & Wu 2011 for related works.). In this model there are inflowing and outflowing zones with almost equal but opposite mass fluxes. The net accretion rate is orders of magnitude smaller. The inward decrease of the mass accretion rate is caused by the mass loss in the outflow launched at every radius. In early versions of the model (Blandford & Begelman 1999, 2004), the origin of the outflow was assumed to be because of the positive sign of the Bernoulli parameter of the accretion flow. In the latest version of the ADIOS model the mechanism of producing outflow is not specified but left open (Begelman 2012).

The second model is the convection-dominated accretion flow (CDAF; Narayan et al. 2000; Quataert & Gruzinov 2000). This model is based on the assumption that the hot accretion flow, both HD and MHD, is convectively unstable. In this model, when $\alpha$ is small, the inward angular momentum transport by convection and outward transport by viscous stress almost cancel each other. A convective envelope solution was then constructed which can reproduce the simulated flat density profile. In this scenario, the inward decrease of the mass accretion rate is because, with accretion, more and more fluid is locked in convective eddies moving in circular motion.

The first aim of the present work is to investigate which of the two scenarios above is correct. For this purpose, we run some HD and MHD simulations, and calculate the respective inflow and outflow properties, including their radial and rotational velocities, temperature, and Bernoulli parameter. If the CDAF scenario is correct, i.e., the motion of the flow is dominated by convective turbulence, and inflow and outflow rates are simply due to turbulent fluctuation, we should expect that the properties of inflow and outflow are similar. As we will see, however, we find that the properties of inflow and outflow are systematically and significantly different.

The second question we want to address is the convective stability of MHD accretion flow. The HD hot accretion flow was predicted to be convectively unstable (Narayan & Yi 1994). Physically this is because the entropy increases with decreasing radius: entropy is produced by viscous dissipation and little entropy is lost in the radiation. Such a prediction has been well confirmed by HD simulations (SPB99; Igumenschev & Abramowicz 1999, 2000). The CDAF model further assumes that an MHD accretion flow is also convectively unstable and applies to an MHD flow as well. However, such applicability has always been questioned by some authors who think that the dynamics of MHD flow is controlled by magneto rotational instability (MRI; Balbus & Hawley 1991, 1998) rather than convection (Stone & Pringle 2001; Balbus & Hawley 2002). Narayan et al. (2002) performed a linear MHD stability analysis and argued that if the flow is unstable the long-wavelength modes of the instability are intrinsically “convection” so the CDAF model should be applicable to MHD accretion flows; moreover, the convective stability can be judged by the HD Høiland criterion. Here we use the simulation data to analyze the convective stability of MHD accretion flows. We find that they are convectively stable. Based on this result, combined with the fact that HD and MHD accretion flows have an almost identical inward decrease in the accretion rate, we conclude that the decrease of the accretion rate for both HD and MHD flows is not because of convection but is caused by systematic outflow.

Then two immediate questions arise. From the theoretical aspect we need to understand the physical origin of these outflows. For observational applications, the most important question concerns the main properties of outflows such as their terminal velocity and kinetic power. We investigate these two questions again by systematically comparing the properties of inflows and outflows. One potential complication of numerical simulations is the effect of the initial condition adopted in the simulation. Many simulations choose a rotation torus as the initial condition; in some other works the gas is injected into the computation domain, i.e., an “injection-type” initial condition. The effect of the initial condition has never, to our knowledge, been studied systematically. Our idea of the potential importance of the initial condition is stimulated by the one-dimensional steady global solution of ADAFs. Since the accretion equations are a set of differential equations, and since the differential terms in ADAFs (i.e., advection) are very important, we expect that the outer boundary condition should be important in determining the global dynamics of ADAFs. In other words, for given parameters, the solutions of ADAFs are not unique, but comprise a series of solutions corresponding to different outer boundary conditions. This expectation was fully confirmed by the detailed calculations of Yuan (1999). In this spirit, the initial condition should also play some role in multi-dimensional numerical simulations, i.e., the final solution should preserve some memory of the initial condition, although we do not know to what extent until the detailed calculations are performed. We expect that while the main properties of accretion flow should remain, some properties of outflow may depend on the initial conditions.

One particular example is the Bernoulli parameter ($Be$) of the accretion flow. This parameter is the sum of the kinetic energy, enthalpy, and potential energy (Equation (17)). As we will describe later, its value will determine whether the outflow can escape to infinity and the terminal velocity of the outflow. In the self-similar solution of ADAFs, it was found that $Be > 0$ (Narayan & Yi 1994). However, as later pointed out by several authors the positive sign of $Be$ is only a result of the self-similar solution. In the global solution, its sign should depend on the outer boundary condition (Nakamura 1998; Yuan 1999; Abramowicz et al. 2000). This was confirmed by the numerical calculations of Yuan (1999).

We assign the systematic study of the effects of the initial condition and boundary condition in numerical simulation of accretion flow to a separate paper (D. F. Bu et al. 2012, in preparation). Here, we consider three different initial conditions in our HD simulations, but only one initial condition for the MHD simulation. The paper is structured as follows. The four models, one MHD and three HD ones with different initial conditions, will be described in Section 2. The systematic comparison of properties between inflows and outflows is presented in Section 3. In Section 4, we compare our simulation results with previous ones. We find that some puzzling results in previous works can be understood as due to different initial conditions. In Section 5, we analyze the convective stability of MHD accretion flow and argue that the ADIOS scenario is favored by our simulations. The origin of outflows in both HD and MHD accretion flows is discussed in Section 6. In Section 7,
we discuss the observational application of our study. The last section (Section 8) is devoted to a summary.

2. MODELS

2.1. Equations

We perform both HD and MHD calculations of two-dimensional axisymmetric accretion flows around black holes. The equations we use are exactly the same as those of SPB99 and Stone & Pringle (2001). For the convenience of readers, we copy them here. The HD equations are

\[ \frac{d\rho}{dt} + \rho \nabla \cdot \mathbf{v} = 0, \]  
\[ \frac{d\mathbf{v}}{dt} = -\nabla p - \rho \nabla \psi + \nabla \cdot \mathbf{T}, \]  
\[ \frac{d(e/\rho)}{dt} = -\rho \nabla \cdot \mathbf{v} + \mathbf{T}^2/\rho. \]

The MHD equations are

\[ \frac{d\rho}{dt} + \rho \nabla \cdot \mathbf{v} = 0, \]  
\[ \frac{d\mathbf{v}}{dt} = -\nabla p - \rho \nabla \psi + \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B}, \]  
\[ \frac{d(e/\rho)}{dt} = -\rho \nabla \cdot \mathbf{v} + \eta \mathbf{J}^2, \]  
\[ \frac{d\mathbf{B}}{dt} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \mathbf{J}). \]

In the above equations, \( \rho, P, v, e, \mathbf{B}, \) and \( \mathbf{J} = (c/4\pi) \nabla \times \mathbf{B} \) are the mass density, pressure, velocity, internal energy, magnetic field, and the current density, respectively. We adopt an adiabatic equation of state \( P = (\gamma - 1)\rho v \) with \( \gamma = 5/3. \) \( \psi \) is the gravitational potential. We adopt the Paczyński & Wiita potential \( \psi = -GM/(r - r_s), \) where \( M \) is the central black hole mass, \( G \) is the gravitational constant, and \( r_s \equiv 2GM/c^2. \) The self-gravity of the accretion flow is neglected. We use the spherical coordinates \( (r, \theta, \phi) \) to solve these equations. We choose units such that \( c = M = G = 1. \)

In the HD equations, \( \mathbf{T} \) is the anomalous stress tensor. The final two terms in Equations (2) and (3) represent anomalous stress and its corresponding heating, respectively. Since we do not have a magnetic field, we add two terms to transfer the angular momentum and convert energy, so that we may mimic the real MHD case (see below). The viscosity coefficient \( \mu = \nu \rho \) and \( v \) is the kinematic viscosity coefficient. We adopt the form \( \nu = \alpha r^{1/2} \) because it corresponds to the usual "\( \alpha \)" description (refer to "Run K" in SPB99). We set \( \alpha = 0.01. \) Following SPB99, to better mimic the MHD case we assume that the only non-zero components of \( \mathbf{T} \) are the azimuthal components (see SPB99 for details). In the MHD equations, the final terms in Equations (6) and (7) are the magnetic heating and dissipation rate mediated by a finite resistivity \( \eta. \) Since the energy equation here is actually an internal energy equation, numerical reconnection inevitably results in loss of energy from the system. By adding the anomalous resistivity \( \eta, \) the energy loss can be captured in the form of heating in the current sheet (Stone & Pringle 2001).

The MHD simulation is, of course, more realistic than the HD one. Then what are the motivations for performing the HD simulation? The reasons are threefold. The first reason is mainly of academic interest. We hope to compare with and understand previous HD works, both analytical and numerical. The second is to study the effects of the initial conditions. Although we can use MHD simulations to accomplish this, it will obviously be much more expensive. More importantly, we believe that in terms of the effect of the initial condition, the results for HD and MHD simulations should be intrinsically the same. Lastly, by comparing the HD and MHD simulations we hope to be able to deepen our understanding of the production and properties of outflows.

2.2. Model Description

We investigate four models. Three models (models A, B, and C) are HD and one (model D) is MHD. The first three models have different initial conditions. The initial condition of model D is identical to that of model A, except that a magnetic field is included. Our intention is that the models should be as simple and as clean as possible. Based on this consideration, except for the two models that are devoted to studying the effect of initial conditions (i.e., models B and C), the other two (models A and D) are the most "popular" in the literature. Their details are as follows.

**Model A.** This model is almost identical to "Run K" in SPB99. The initial condition is a rotating torus with constant angular momentum, which is identical to that adopted in Stone et al. (1999). Readers are referred to SPB99 for the detailed description of the torus. In contrast to SPB99, where the Newtonian potential was adopted, we adopt the Paczyński & Wiita (1980) potential. This introduces the length scale, so we put the center of the density maximum of the torus at \( \sim 100r_s. \) We set the maximum density of the torus \( \rho_{\text{max}} = 1.0, \) the density of the surrounding medium \( \rho_0 = 10^{-4}, \) and pressure \( p_0 = \rho_0/r. \) The torus is a bound system; therefore the Bernoulli parameter \( Be < 0 \) (see the dotted line in Figure 5 for its value at the equatorial plane).

**Model B.** A rotating torus has no radial velocity while a realistic accretion flow has. In this sense, we think that perhaps a better initial condition is that the gas has both radial and rotational velocities. This is one of our motivations for the initial conditions in models B and C. In model B, to obtain the initial condition, we first determine the one-dimensional global solution of two-temperature ADAFs. This corresponds to solving the one-dimensional version of Equations (1)–(3), although the equations are for one-temperature accretion flow. We use the calculation results (e.g., density and temperature) for ions as the initial conditions in our simulation. We choose to solve the two-temperature rather than one-temperature case simply because we think the former is more realistic; however a one-temperature solution should not make any notable difference. The solution extends from the inner boundary to the outer boundary, and it has both rotation and radial velocities. The details of the calculations are described in Yuan (1999). Specifically, we choose the outer boundary condition so that \( Be < 0 \) in the solution, but larger than that in model A. We do not choose \( Be > 0 \) in model B in order to discriminate this model from model C, where \( Be > 0 \). We then expand this one-dimensional solution to two dimensions by assuming that the vertical density profile follows an exponential distribution but all other quantities such as temperature and velocity retain their original one-dimensional values. The thickness
of the accretion flow is determined by the scale height $H = c_s/Ω_k$.

Model C. We adopt the “injection-type” initial condition in this model. The gas is injected at the outer boundary. The properties of the injected gas, such as temperature, rotational and radial velocities, are completely determined by the self-similar solution of Narayan & Yi (1994). In this case, $Be > 0$ (Narayan & Yi 1994).

Model D. This is an MHD model. The model is fully identical to “Run F” in Stone & Pringle (2001). Namely, the initial condition is a rotating torus with the density maximum located at $r = 100 r_s$ exactly the same as model A. So the initial $Be$ is identical in the two models. The difference is that an initial magnetic field is added. The field is poloidal, confined to the interior of the torus. The loops are parallel to the density loops. Readers are referred to Stone & Pringle (2001) for details. One reason we adopt this model is because, as Hawley & Krolik (2002) argued, compared to the initial toroidal field configuration, the poloidal configuration is more realistic.

We will see that while the main results remain similar for the different initial conditions, some results do differ. In particular, as we will show in Section 3.3, the sign of $Be$ is largely dependent on the value of $Be$ in the initial condition. What initial condition is more realistic? This is a difficult question: the answer may depend on different environments. These environments include, among other things, the accretion flow formed in the Roche Lobe overflow or stellar wind in the case of X-ray binaries, the centers of galaxies in the case of various types of AGNs, the molecular cloud in the case of star formation, or even a rotation disk in the case of planetary formation. In terms of the Bernoulli parameter of the initial gas, if the radiative energy loss can be neglected during accretion, we expect $Be ≥ 0$ since it comes from infinity. This may be the case for most elliptical galaxies, such as M87, in which hot interstellar medium (ISM) fuels the black hole, and some spiral galaxies such as Sgr A.* In these cases, models B and C are likely more realistic than models A and D. But if radiation is important, as perhaps it is in the case of most spiral galaxies and black hole X-ray binaries where the gas accreted is cool and dense, we usually consider that at large radius the accretion flow is described by a standard thin disk. This disk may be truncated and replaced by a hot accretion flow within a transition radius if the accretion rate is not too large (see Section 7.1 for more details). In this case the sign of $Be$ of the inner hot accretion flow is unclear, although it is certain that $Be < 0$ for the outer truncated thin disk. This is because the physics of the transition from the outer thin disk to the inner hot accretion flow is still an unsolved problem. The two models proposed include evaporation and radial turbulent transport. There must be energy transfer either vertically (in the former model) or radially (in the latter model). Therefore the value of $Be$ must be increased after the transition. Similar uncertainty exists for the hot corona sandwiching the thin disk.

2.3. Approach

We use the ZEUS code (Stone & Norman 1992a, 1992b) in spherical geometry to solve the equations. The two modifications to the code are to include the shear stress terms (in the HD case) and the implementation of the Paczyński & Wiita (1980) potential. The computational grids extend from an inner boundary at $r = 1.2 r_s$ to $r = 500 r_s$. The inner boundary is small enough to make sure that our solution is transonic. The standard outflow boundary condition is adopted at both the inner and outer radial boundaries. In the angular direction, the boundary conditions are set by symmetry at the poles. To adequately resolve the flow, we adopt the same non-uniform grid in both the radial and angular directions as in SPB99 and Stone & Pringle (2001).

3. RESULTS: DIFFERENT PROPERTIES OF INFLOWS AND OUTFLOWS

3.1. General Results

For the aim of the present paper we are interested only in the steady state of the solution. We judge the radial range within which the steady solution is achieved by examining whether the net accretion rate (Equation (14)) is a constant of radius. The general descriptions of the simulation results of models A and D can be found in SPB99 and Stone & Pringle (2001). For model B, at the beginning of the simulation, we find a shock is formed at the inner boundary and it propagates outward. The flow achieves a steady state after 0.7 orbital times at the outer boundary. In model C, initially the whole computational domain is filled with a very low density ISM. Gas is steadily injected into the calculation domain from the equatorial plane at the outer boundary. The injected gas quickly moves inward and forms an accretion flow. After about nine orbital times at the outer boundary, the accretion flow achieves a quasi-steady state.

When we evaluate the radial distribution of physical quantities, such as $Be$, velocity, and temperature, we need to calculate the angle-integrated average. We adopt the mass flux-weighted value, which is defined as (for quantity $q$)

$$
\langle q(r) \rangle = \frac{2\pi r^2 \int_0^\pi \rho q \max(v_r, 0) \sin \theta d\theta}{2\pi r^2 \int_0^\pi \rho \max(v_r, 0) \sin \theta d\theta}
$$

for outflow and

$$
\langle q(r) \rangle = \frac{2\pi r^2 \int_0^\pi \rho q \min(v_r, 0) \sin \theta d\theta}{2\pi r^2 \int_0^\pi \rho \min(v_r, 0) \sin \theta d\theta}
$$

for inflow. We have also tried the density-weighted values, which are defined as

$$
\langle q_d(r) \rangle = \frac{2\pi r^2 \int_0^\pi \rho q \max(v_r, 0)/v_r \sin \theta d\theta}{2\pi r^2 \int_0^\pi \rho \max(v_r, 0)/v_r \sin \theta d\theta}
$$

and

$$
\langle q_d(r) \rangle = \frac{2\pi r^2 \int_0^\pi \rho q \min(v_r, 0)/v_r \sin \theta d\theta}{2\pi r^2 \int_0^\pi \rho \min(v_r, 0)/v_r \sin \theta d\theta}
$$

for outflow and inflow, respectively. We find the results are very similar. In the following, we compare the various properties of inflow and outflow.

3.2. Inflow and Outflow Rates

Following SPB99, the mass inflow and outflow rates, $\dot{M}_{in}$ and $\dot{M}_{out}$, are defined as the following time-averaged and angle-integrated quantities:

$$
\dot{M}_{in}(r) = 2\pi r^2 \int_0^\pi \rho \min(v_r, 0) \sin \theta d\theta,
$$

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3 The data we use in the present paper for model D were kindly sent to us by Jim Stone. We also performed the simulation ourselves, using higher resolution and a smaller inner boundary. The results show no notable differences.
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Figure 1. Radial distributions of inflow (thin solid), outflow (dashed), and net rates (dotted). The thick solid lines denote the rate of outflow with a positive $Be$. The top left, top right, bottom left, and bottom right plots are for models A, B, C, and D, respectively.

\[ \dot{M}_{\text{out}}(r) = 2\pi r^2 \int_0^\pi \rho \max(v_r, 0) \sin \theta \, d\theta. \]  

(13)

The net mass accretion rate is

\[ \dot{M}_{\text{net}}(r) = \dot{M}_{\text{in}}(r) - \dot{M}_{\text{out}}(r). \]  

(14)

The net rate is the accretion rate that finally falls onto the black hole. Note that the above rates are obtained by time-averaging the integrals rather than integrating the time averages.

Figure 1 shows the radial distributions of the inflow and outflow rates and net rate for the four models. The radial profile of the inflow rate in all four models can be described by $\dot{M}_{\text{in}}(r) \propto r^s$. Regarding the value of $s$, Paper I presents a more precise measurement since the dynamical range of simulations presented there is much larger than in the present work. The radial profile of density is also measured, and we find $\rho(r) \propto r^{-p}$. Moreover, the relationship $s = 1.5 - p$ is well satisfied, which indicates that the flattening of density is caused by the inward decrease of the accretion rate. Another notable result is that the accretion rate profiles for all four models are roughly similar. In Paper I, we discussed this consistency in a much more comprehensive and detailed way by combining all published HD and MHD simulation works. Also shown in the figure by the thick solid lines are the outflow rates with a positive $Be$. We will discuss this issue later.

We study the angular distribution of the mass rate by time-averaging the following outflow and inflow rates as a function of $\theta$:

\[ M_{\text{out}}(\theta) = 2\pi r^2 \rho \max(v_r, 0) \sin \theta \Delta\theta. \]  

(15)

\[ \dot{M}_{\text{in}}(\theta) = 2\pi r^2 \rho \min(v_r, 0) \sin \theta \Delta\theta. \]  

(16)

Figures 2 and 3 show the angular distributions of the inflow rate and the outflow rate, respectively. We can see that their angular distributions are in general quite broad; there is no angle where there is only inflow or outflow in the time-average sense. The distributions of models A and D are similar; they are nearly symmetric to the equatorial plane. The distributions of models B and C are similar, peaked at an angle $\theta$ away from the equatorial plane. The discrepancy between these two groups is obviously because of the initial conditions. Models A and D have the same initial condition, while models B and C are similar since in both models the gas has an initial radial velocity. Comparing these two figures, we can see that, for all four models, the angular distributions of inflow and outflow roughly match, in the sense that in the region where the inflow is strong the outflow is weak. For example, for model A, most of the inflow occurs within $\theta = 70^\circ - 90^\circ$, while the outflow is very weak in this region. Most of the outflow occurs at the surface of the accretion flow, peaking at $\theta = 50^\circ$ and $140^\circ$. Model D is similar to model A.

For models B and C, the distributions of both inflow and outflow rates are not symmetric to the equatorial plane and there is one peak, located at $\theta \sim 50^\circ$ and $130^\circ$ depending on the radius and model. But again outflow is strong in the region where the inflow is weak. For example, for model B, at $r = 50r_s$, most of the inflow occurs below the equatorial plane, around $\theta \sim (110-140)^\circ$ whereas most of the outflow occurs above the equatorial plane, around $\theta \sim (40-70)^\circ$. This suggests that the inflow and outflow rates are dominated by large-scale bulk motion rather than fluctuations, and the bulk of the inflowing gas is turned around and becomes outflow. As we will discuss later, the outflowing motion is caused by the buoyant force. We speculate that the reason for the difference in the angular distributions between models B and C and models A and D may be due to the difference values of $Be$ in the
Figure 2. Angular distributions of the inflow rate. The solid and dashed lines are for $r = 50r_s$ and $10r_s$, respectively. The top left, top right, bottom left, and bottom right plots are for models A, B, C, and D, respectively.

Figure 3. Angular distributions of the outflow rate. The solid and dashed lines are for $r = 50r_s$ and $10r_s$, respectively. The top left, top right, bottom left, and bottom right plots are for models A, B, C, and D, respectively.
initial condition. For models B and C, the Bernoulli parameter of the initial gas is significantly higher, which means that the gas is more “energetic.” This results in apparently more “violent” motion.

3.3. Bernoulli Parameter

The Bernoulli parameter is the sum of kinetic energy, enthalpy, and potential energy:

\[ Be = \frac{v^2}{2} + \frac{\gamma P}{(\gamma - 1)r} - \frac{GM}{r - r_s}. \]  

(17)

Here the magnetic field is not included. In the case of inviscid flow, the Bernoulli parameter of a fluid element along a streamline is conserved. A positive \( Be \) means that the accretion flow has sufficient energy to overcome the gravitational energy and does work to its surroundings to escape to infinity.

Figure 4 presents the two-dimensional contours of \( Be \) for the accretion flow (both inflow and outflow) for the four models. The red regions denote \( Be > 0 \). We can see that the results of the four models are quite different. For model A \( Be < 0 \) in most of the area, as stated in SPB99 and Yuan & Bu (2010). The latter found that at the outer boundary only about 1% of the outflow has a positive \( Be \). For model B, the region with a positive \( Be \) becomes larger. For model C, \( Be \) is positive in almost all of the area. The main reason for such differences is that the initial conditions are different, i.e., different models have different initial \( Be \). The value of \( Be \) along the equatorial
plane of the initial condition in the four models is shown by the dotted lines in Figure 5. For models A and D, \( Be \) is the smallest and the gas is strongly bound, i.e., \( Be < 0 \). In fact, we find that in the initial torus the temperature \( T \) of gas is about 10 times lower than the virial value. For model B, the value of \( Be \) of the initial condition is larger, but still less than zero. For model C, \( Be > 0 \) for all the injected gas. Since radiation is neglected in our simulation, the total energy of the gas in the computational domain should be roughly conserved (if we neglect energy loss through the outer boundary). Therefore it is natural to expect that the final value of the \( Be \) is a function of \( Be \) of the initial condition.

Although their initial condition is identical, the contours of models D and A have one significant difference. The value of \( Be \) close to the rotation axis in model D is positive; while it is negative in model A. Such a difference should be because of the magnetic effect. In model D, we find that a large-scale ordered poloidal magnetic field is present in the small-\( \theta \) region (and only in that region). Therefore, compared to model A, additional energy flux, coming from the rotational energy of the accretion flow, is transferred from small radius outward along the field line, which increases the value of \( Be \) there. This is the Blandford & Payne (1982) mechanism.

The thick solid lines in each plot of Figure 1 represent the radial profile of the outflow rate with a positive \( Be \). We can see that for models A and B a small fraction of the outflow has a positive \( Be \). For model C, \( Be > 0 \) for almost all of the outflow. For model D, a very tiny fraction of the outflow has \( Be > 0 \). The result of model D is consistent with Narayan et al. (2012). This is because the initial conditions in the two works are similar.

While the value of \( Be \) for different models is quite different as shown by Figure 4, we see from Figure 1 that the strength of the outflow is similar for the four models, independent of the value of \( Be \). This indicates that the outflow is not produced because \( Be > 0 \). Of course, for these outflows to reach infinity, \( Be > 0 \) is still required.

Figure 5 shows the radial distribution of the flux-weighted \( Be \) of the outflow (dashed line) and inflow (solid line). In all models \( Be \) increases inward, including the case of model C. The apparent violation of \( Be \) conservation (i.e., \( Be \) is not a constant of radius) is because of the energy flux from small to large radii associated with the viscous stress. The dotted lines in the figure show the value of \( Be \) in the initial condition. We can immediately find that the value of \( Be \) for the outflow is always larger than that of the inflow for models A, B, and C. Even the difference between inflow and outflow seems to be similar for these three models. For model D, however, the value of \( Be \) is almost equal for inflow and outflow. This again suggests that the mechanism of the production of outflow is different for HD and MHD flows.

We are especially interested in the value of \( Be \) for the outflow since it determines whether the outflow can escape to infinity and the terminal velocity of the outflows if it can (Section 3.5). Unfortunately from Figure 5 we can see that the results are quite diverse for the four models, depending on the initial conditions. For models A and D, the \( Be \) of the initial torus is the smallest; the value of \( Be \) for the outflow in these two models is also roughly the smallest.\footnote{The values of \( Be \) in models A and D shown in the figure are larger than their initial values. This is compensated by the smaller \( Be \) at \( r > 100 r_s \), which is not shown in the figure; thus the total \( Be \) between the initial condition and the steady state is roughly conserved.} For model C, \( Be > 0 \) for the initially injected gas and subsequently \( Be > 0 \) for both inflow and outflow. Unfortunately, as we have discussed in Section 2.2, it is difficult to determine what kind of initial condition is more realistic. But as we have argued there, in the case that the accretion flow...
starts from very large radius, it is likely that the initial condition with $Be$ close to or larger than zero will be picked up. In this case we expect that $Be > 0$ for the outflow. To illustrate this point, we have calculated the value of $Be$ for the outflow of model A in Paper I. In that model, the initial condition is the same rotating torus as in models A and D of the present paper, except that the torus is located at a much larger radius, $10^4 r_s$. Figure 6 shows the results. We see that $Be > 0$ for all the outflow. In another example, Li et al. (2012) started their hot accretion flow at 10 times the Bondi radius, so $Be > 0$ for the initial condition. They found $Be > 0$ for all their outflow (J. Ostriker 2012, private communication). Finally, as we shall see in Section 7, observations seem to indicate the existence of outflow from hot accretion flows. In the following discussions, we therefore assume that the outflow from a hot accretion flow can always escape to infinity, i.e., $Be > 0$.

We next present some speculations which further support the above assumption of $Be > 0$. Note that the importance of the initial condition does not mean that the final solution can be arbitrary. It must be restricted in a certain range. From Figure 5, we see that the diversity of the solution at the inner region of the accretion flow away from the outer boundary seems to become smaller compared to that of the initial condition. There seems to exist a “common” (but not unique) solution at the inner region, which can be very roughly described by

$$Be \approx (-0.1 \sim 0) v^2_k$$

for inflow and

$$Be \approx (0.1 \sim 0.2) v^2_k$$

for outflow. At large radii, the solutions strongly deviate from the above “common” solution because they are still “tied” to the initial condition there. Based on this result, we speculate that if the outer boundary is set at a very large radius and the location of the torus is at a large radius, the solution at the inner region may suffer less from the initial condition and becomes more “converged.” In other words, the degree of the dependence of the solution on the initial condition depends on how far the gas is away from the black hole. In almost all current simulations, including the present work, the outer boundary is rather small; thus it is hard for the accretion flow to fully evolve and deviate from the initial condition. Only at small radii is the accretion flow better evolved; thus solutions there can better approach the “genuine” solution. For model B, at both small and large radii, the simulated solution is close to the initial condition. This is likely because the initial condition of model B is “good,” i.e., close to the “genuine” solution. It is interesting to note that in this model the value of $Be/v^2_k$ is roughly constant, consistent with the self-similar solution (Narayan & Yi 1994). So this seems to again indicate that model B is more realistic than the other three models. Based on these considerations, when we calculate the terminal velocity in Section 3.5, we will assume Equation (19).

We emphasize again that the above consideration is rather speculative. It is crucial to explore it further by simulations with a much larger outer boundary. This will be our future work.

On the other hand, we believe that the value of $Be$ determined by Equation (19) should be correct in order of magnitude; then the calculated terminal velocity (Equation (21)) will be correct within a factor of three.

Comparing models A and D, we can see that the value of $Be$ in model D is significantly smaller than that in model A, although they have the same initial condition. This is because in model D some energy is converted into magnetic energy which is not included in our value of $Be$.

In the case that the outer boundary is small, or additional physics such as radiation is included, the value of $Be$ for the outflow should be determined mainly by the initial condition. While there is some deviation, this should still be limited because of energy conservation. If the value of $Be$ of the initial condition is far below zero, we expect that there will be no outflow with $Be > 0$, i.e., no jet. This result supplies an important clue to the following long-standing observational puzzle, namely why a relativistic jet can only be formed in a hot accretion flow and not in the standard thin disk. Black hole X-ray binaries usually come in various states (e.g., McClintock & Remillard 2006). The two most prominent states are hard and soft states, which are described by hot and cold accretion flows, respectively (e.g., see reviews by Zdziarski & Gierlinski 2004; Done et al. 2007). Radio observations clearly show that relativistic jets exist only in the hard state, not in the soft state. That is, a jet can only be formed from a hot accretion flow, not from the thin disk. We speculate that this may be because of the difference in the values of $Be$ for the two types of accretion flows. Because of strong radiative cooling, $Be$ in the case of the thin disk is obviously much lower than that in the hot accretion flow. Following this argument, we predict that for hot accretion flows the ratio of the mass-loss rate in the jet and the accretion rate should decrease with increasing accretion rate. When the accretion rate is high, radiative cooling is strong and $Be$ should be low. This is consistent with the modeling result of Yuan & Cui (2005).

Figure 7 shows the angular distribution of $Be$ at $r = 10r_s$ and $50r_s$ for the four models. At a given radius, part of the outflow has positive $Be$, but part has negative $Be$. For both models A and D, $Be$ is largest away from the equatorial plane and smallest around the equatorial plane. The difference between these two models is that in model D the contrast of $Be$ is much larger and the region of large $Be$ is much closer to the axis. This is because of the existence of the large-scale poloidal field close to the axis, as we have explained in the previous section. The distributions of models B and C are different. It is interesting to note that $Be$ is largest at the angles $\theta$ where the mass outflow rate is largest (Figure 3).

### 3.4. Temperature

Figure 8 shows the flux-weighted radial distributions of the temperature of the inflows and outflows for the four models.
Figure 7. Angular distributions of flux-weighted $Be$ in units of $v_k^2$ for outflows at different radii. The solid and dashed lines are for $r = 50r_s$ and $r = 10r_s$, respectively. The top left, top right, bottom left, and bottom right plots are for models A, B, C, and D, respectively.

Figure 8. Radial distributions of the flux-weighted temperature. The solid and dotted lines are for inflow and outflow, respectively. The top left, top right, bottom left, and bottom right plots are for models A, B, C, and D, respectively.
One distinct result is that for the three HD models, the outflow temperature is higher than the inflow temperature by nearly $T_{\text{vir}}$. Here the virial temperature is defined as

$$T_{\text{vir}} = \frac{GMm_p}{3kr}.$$ (20)

Since the temperatures of both the inflow and the outflow are nearly virial, the internal energy plays an important role in $Be$. This is why the value of $Be$ for the outflow is higher than that for the inflow (Figure 5). This is strong evidence for the convection origin of outflow in HD flows, i.e., the fluid elements with temperature higher than that of the surrounding medium will move outward due to buoyancy. For model D, however, the temperatures of the inflow and outflow are almost identical. This suggests that an MHD flow is not convectively unstable, as we will confirm by stability analysis. Correspondingly, the mechanism of producing outflow in model D is not convection.

### 3.5. Radial Velocity

Figure 9 shows the flux-weighted radial velocity of inflow and outflow for the four models. For the profiles of inflow, we see that models B and C are similar, as are models A and D are similar. The former can be described roughly by $v_r/v_k \sim \text{constant}$, or $v_r \propto r^{-0.5}$. This scaling is consistent with the self-similar solution of ADAF (Narayan & Yi 1994). For models A and D, the radial velocity scaling with radius is much steeper, i.e., the radial velocity increases faster inward compared to the Keplerian velocity. The discrepancy between the two groups of models is caused by the difference in the initial conditions. In models A and D, the radial velocity in the torus is initially zero; it is thus difficult for the gas at large radii ($\gtrsim 100r_s$) to achieve a large velocity because of the boundary effect. On the other hand, because of the very strong gravity, the radial velocity has to increase rapidly inward at $r \lesssim 10r_s$. These two effects make the profile steeper. If the initial torus in models A and D is located at a much larger radius, i.e., the dynamical range is much larger, the results for these two models will be similar to those for models B and C. This has actually been confirmed by our simulation in Paper I, where the torus is located at $10^4r_s$.

However, the profile of the radial velocity of the outflow is completely different. It is not so sensitive to the initial condition. The ratio of radial velocity to Keplerian velocity remains roughly constant with radius, and this ratio is similar for all four models, i.e., $v_r/v_k \approx (0.2-0.4)$, regardless of the differences in the initial conditions and whether the magnetic field is included or not. Given that the mechanism of producing outflows in HD and MHD flows is different, as we will discuss later, the reason for the same outflow radial velocities remains to be explored. We would like to emphasize that the systematic difference in the profile of radial velocity between inflow and outflow again strongly indicates that the outflow is not simply due to turbulent fluctuation or convective motion. There is a systematic outward flux of mass.

Figure 10 shows the angular distribution of the radial velocity, averaged for all accretion flows including both inflows and outflows. It is clear that, in the time-average sense, the angular distributions of the radial velocities for models A and D are quite symmetric to the equatorial plane. Taking $r = 50r_s$ (i.e., the solid lines in each plot) as an example, the inflow concentrates around the equatorial plane, $60^\circ \lesssim \theta \lesssim 120^\circ$ while the outflow occurs at $15^\circ \lesssim \theta \lesssim 60^\circ$ and $120^\circ \lesssim \theta \lesssim 165^\circ$. For models B and C, the distributions are not symmetric to the equatorial plane. When the inflow concentrates above (below) the plane, i.e., $\theta \lesssim 90^\circ$ ($\theta \gtrsim 90^\circ$), the outflow then concentrates below (above) the plane. We note that the angular distributions of radial
velocity of the inflow and the outflow shown in this figure are consistent with the angular distributions of inflow and outflow rates, shown in Figures 2 and 3. Another notable feature is that, for all the four models, close to the axis average the radial velocity is negative, i.e., it is inflow.

One important question is the terminal radial velocity of the outflow if it reaches infinity. We are interested in this question since outflow is potentially very important for AGN feedback. We suppose that, once launched, the viscous stress in the outflow and the mixing of mass, energy, and angular momentum between streamlines can be ignored. For such an adiabatic expanding inviscid flow, $Be$ is conserved. When $r$ is large enough, the last two terms in Equation (17) vanish. If the angular momentum is conserved, $v_\phi \approx 0$ so the velocity is mainly in the radial direction. Therefore the terminal velocity is

$$v_{\text{term}} \sim v_r \sim \sqrt{2Be(r_{\text{out}})}.$$  \hspace{1cm} (21)

Combining with Equation (19), we have

$$v_{\text{term}} \sim 0.5v_k(r_{\text{out}}).$$  \hspace{1cm} (22)

That is to say, the radial terminal velocity of the outflow is roughly half of the Keplerian velocity at the outer radius of the hot accretion flow where most of the outflow originates.

A positive sign of $Be$ is only a condition for the inviscid HD outflow to escape to infinity. When viscosity and a magnetic field are present, energy may be transferred from small to large radii and the magnetic force may also do work to the outflow. In this case, $Be$ defined by Equation (17) is not conserved, and it may be not necessary for the outflow to have $Be > 0$ to escape to infinity. The terminal velocity given by Equation (21) will be a lower limit. We do not consider these complications in the present work. The exact solution of the terminal velocity as a function of its initial properties in the presence of a magnetic field will be an interesting question to study.

### 3.6. Angular Momentum

It has long been speculated that the outflow can carry away energy and angular momentum (e.g., Shakura & Sunyaev 1973; Blandford & Begelman 1999). This requires that the specific energy and specific angular momentum of the outflow should be larger than those of the inflow. We have discussed the Bernoulli parameter in the previous section and found that the value of $Be$ for the outflow is systematically larger than that for the inflow for models A, B, and C, but for model D the values of $Be$ for the inflow and the outflow are the same. In this section we examine angular momentum transfer by the outflow.

Figure 11 shows the distributions of specific angular momentum for both the inflow and the outflow. The results for the three HD models are similar, but different from model D. For the three HD models, the angular momentum of the outflow is larger than that of the inflow at small radii, consistent with Stone et al. (1999). This result indicates that the outflow can transfer angular momentum at small radii. At large radii, however, the angular momentum of the outflow is smaller than that of the inflow. This means that the radial profile of the angular momentum of the outflow is flatter than that of the inflow, and the angular momentum of the outflow tends to be conserved during its outward propagation. For both inflow and outflow, the angular momentum is significantly lower than the Keplerian value at the equatorial plane.

Model D is completely different. We see from the figure that throughout the accretion flow the specific angular momentum of the outflow is very close to and even in some regions slightly
larger than the Keplerian value at the equatorial plane. This is significantly larger than that of the inflow, whose value is substantially sub-Keplerian. This indicates that in an MHD flow, the outflow can transfer angular momentum much more efficiently compared to the HD flows. This is related to the mechanism of producing outflow. We will return to this question in Section 6.

3.7. Momentum Flux and Kinetic Power of the Outflow

Based on the above results, we can estimate the radial momentum flux and kinetic power of the outflow. The terminal radial momentum flux is

\[ \dot{p}_w = M_{\text{out}}(r_{\text{out}}) v_{\text{term}} \sim 0.5 M_{\text{out}}(r_{\text{out}}) v_k(r_{\text{out}}). \]  

Here \( M_{\text{out}}(r_{\text{out}}) \) is the outflow rate at the outer boundary of the hot accretion flow \( r_{\text{out}} \).

It is interesting to compare this momentum flux with the radiation flux \( L/c \). Adopting the result from Paper I, the inflow rate at \( r = 10r_s \) is \( M_{\text{in}}(10r_s) = M_{\text{in}}(r_{\text{out}})(10r_s/r_{\text{out}})^{0.5} = M_{\text{out}}(r_{\text{out}})(10r_s/r_{\text{out}})^{0.5} \). The radiative flux is then

\[ L/c \sim \eta M_{\text{out}}(r_{\text{out}})(10r_s/r_{\text{out}})^{0.5} c. \]

Here \( \eta \) is the radiative efficiency of the hot accretion flow defined as \( \eta \equiv L_{\text{bol}}/M_{\text{in}}(10r_s)c^2 \). So the ratio of momentum flux of the outflow to the radiation flux is

\[ \frac{\dot{p}_w}{L/c} \approx \frac{0.1}{\eta}. \]

The efficiency \( \eta \) is mainly a function of accretion rate and the parameters \( \delta \) and \( \alpha \) (see Xie & Yuan 2012 for details). For \( \delta = 0.5 \), as indicated from the detailed modeling of Sgr A* (Yuan et al. 2003), and \( \alpha = 0.1 \), we have (Xie & Yuan 2012)

\[ \eta = 0.05(\dot{M}_{\text{in}}(10r_s)/4 \times 10^{-3} \dot{M}_{\text{Edd}})^{0.2} \]  

for \( \dot{M}_{\text{in}}(10r_s) = (4 \times 10^{-3} - 10^{-5})\dot{M}_{\text{Edd}} \) and

\[ \eta = 0.05(\dot{M}_{\text{in}}(10r_s)/4 \times 10^{-3} \dot{M}_{\text{Edd}})^{0.6} \]  

for \( \dot{M}_{\text{in}}(10r_s) = (4 \times 10^{-3} - 10^{-2})\dot{M}_{\text{Edd}} \). Above \( 10^{-2}\dot{M}_{\text{Edd}} \), the accretion flow is two-phase. The detailed study of the dynamics and radiation of this kind of accretion flow remains to be explored. The approximate calculation presented in Xie & Yuan (2012) gives \( \eta \approx 0.08 \). So in general we expect

\[ \dot{p}_w \gtrsim L/c. \]

The involvement of a jet is still controversial. Assuming it is dominated by normal plasma, we estimate the momentum flux of the jet:

\[ \dot{p}_{\text{jet}} = \Gamma_{\text{jet}}^2 \eta_{\text{jet}} (\frac{10r_s}{r_{\text{out}}})^{0.5} M_{\text{in}}(r_{\text{out}})c = 9 \Gamma_{\text{jet}}^2 \eta_{\text{jet}} \dot{p}_w, \]

with \( \eta_{\text{jet}} \) the ratio of the mass-loss rate in the jet to the mass accretion rate at \( 10r_s \), and \( \Gamma_{\text{jet}} \) the Lorentz factor of the jet. The value of \( \eta_{\text{jet}} \) is highly uncertain. From the detailed modeling of the black hole X-ray binary XTE J1118+480 (Yuan et al. 2005), we get \( \eta_{\text{jet}} \sim 1\% \). For comparison, Fender et al.

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\[ \text{Figure 11. Radial distributions of the specific angular momentum } l = v_\phi r \sin \theta \text{ for both inflow (solid line) and outflow (dotted line). The dashed lines denote the Keplerian angular momentum at the equatorial plane. The top left, top right, bottom left, and bottom right plots are for models A, B, C, and D, respectively. Note the sharp contrast between the HD and MHD models.} \]
based on $\eta$ and $\Gamma$. The Lorentz factors of jets are different for AGNs and black hole X-ray binaries, and reasonable values are a factor of three. The Lorentz factors of jets is consistent with the Yuan et al. (2005) value within the range of jet power obtained is $\sim 10^7 M \odot yr^{-1}$ from Di Matteo et al. (2003) and $\Gamma_{\text{jet}} \approx 6$ (Biretta et al. 1999), is $L_{\text{jet}} \approx 7 \times 10^{42} \text{erg s}^{-1}$. This value is completely consistent with the independent estimation of Reynolds et al. (1996). So the momentum flux of the outflow is comparable to that of a jet for AGNs, while it is a factor of 10 larger than that of a jet for black hole X-ray binaries.

The kinetic energy power of the outflow is estimated to be

$$\dot{E}_{\text{w}} = \frac{1}{2} \dot{M}_{\text{out}}(r_{\text{out}}) v_{\text{term}}^2 \approx 0.1 \dot{M}_{\text{out}}(r_{\text{out}}) v_{\text{jet}}^2(r_{\text{out}}).$$

(30)

Here we use the approximate flux-weighted radial velocity which is not precise but correct in order of magnitude. Since $v_{\text{jet}}^2(r_{\text{out}}) = GM/r_{\text{out}} = c^2/(2r_{\text{out}}/r_s)$, we have

$$\dot{E}_{\text{w}} = \frac{\dot{M}_{\text{out}}(r_{\text{out}}) c^2}{20 r_{\text{out}}/r_s}. \quad (31)$$

We now compare $\dot{E}_{\text{w}}$ with several other powers. The accretion power at $10 r_s$ is

$$P_{\text{acc}} = \left(\frac{10 r_s}{r_{\text{out}}}\right)^{0.5} \dot{M}_{\text{in}}(r_{\text{out}}) c^2 \approx 60 \dot{E}_{\text{w}} \left(\frac{r_{\text{out}}}{r_s}\right)^{0.5} \gg \dot{E}_{\text{w}}. \quad (32)$$

The jet power is

$$\dot{E}_{\text{jet}} = \Gamma_{\text{jet}}^2 \eta_{\text{jet}} \left(\frac{10 r_s}{r_{\text{out}}}\right)^{0.5} \dot{M}_{\text{in}}(r_{\text{out}}) c^2 \approx 60 \Gamma_{\text{jet}}^2 \eta_{\text{jet}} \dot{E}_{\text{w}} \left(\frac{r_{\text{out}}}{r_s}\right)^{0.5}. \quad (33)$$

The radiation power of the accretion flow is

$$\dot{E}_{\text{rad}} = \eta \left(\frac{10 r_s}{r_{\text{out}}}\right)^{0.5} \dot{M}_{\text{in}}(r_{\text{out}}) c^2 \approx 60 \eta \dot{E}_{\text{w}} \left(\frac{r_{\text{out}}}{r_s}\right)^{0.5}. \quad (34)$$

The outer boundary of the hot accretion flow $r_{\text{out}}$ in the cases of both AGNs and black hole X-ray binaries is uncertain. In the case of AGNs, one possibility is that the outer boundary is the Bondi radius. This may be the case if the accretion flow starts out hot, as is likely in elliptical galaxies. Another possibility is that the accretion flow starts out cool, so it is described at large radii by a standard thin disk. This may be the case for spiral galaxies. If the accretion rate is not too high, this thin disk will be truncated and replaced by a hot accretion flow at a transition radius (Yuan 2007). This transition radius is then the outer boundary of the hot accretion flow, $r_{\text{out}}$. Detailed modeling shows that it is a function of the mass accretion rate or, equivalently, the luminosity of the system (Yuan & Narayan 2004). When the luminosity is very low, say $L \lesssim 10^{-6} L_{\text{Edd}}$, as in the case of Sgr A*, $r_{\text{out}} \sim r_{\text{Bondi}} \sim 10^3 r_s$. If the luminosity is higher, $L \gtrsim 10^{-4} L_{\text{Edd}}$, $r_{\text{out}}$ may be in the range from several tens to several thousands of $r_s$.

From the above assumptions on parameters, we conclude that the power of the jet and radiation is usually much stronger than that of the outflow, $\dot{E}_{\text{jet}} \gg \dot{E}_{\text{rad}} \gg \dot{E}_{\text{w}}$. But note that this does not mean the mechanical feedback of the outflow is not important compared to the jet. Since the jet is well collimated, it just drills through the surrounding gas with little deposition of energy and momentum within the galaxy. Rather, the jet may act as the dominant feedback mechanism for galaxy clusters. For the evolution of a single galaxy, the solid angle of the outflow is very large so that it can fully interact with the ISM. It has been shown that the momentum feedback of the outflow dominates the growth of the central black hole (Ostriker et al. 2010). In Section 4, we will discuss two additional examples of such interactions.

4. COMPARISON WITH PREVIOUS WORKS

Igumenshchev & Abramowicz (2000) performed two-dimensional HD simulations. Similar to our model B, they adopted the “injection-type” initial condition. The gas was injected at the outer boundary, with a rotation velocity of $0.9 v_k$. The radial velocity was not given but assumes their value “automatically” after accretion begins, which means that the initial radial kinetic energy is zero. The temperature is not specified either, thus the value of $B_e$ of the initial gas is unknown. Interestingly, they also found nonsymmetric large-scale circular-like (but not closed) bulk motion in their low-$\alpha$ model (Model M; refer to the right plot of their Figure 15), which is similar to our models B and C (refer to the discussions about Figures 2 and 3 in Section 3.2). Regarding the outflow, they found that the value of $B_e$ of most of the outflow is negative, except in a narrow region (in the $\theta$-direction) where the outflow has $B_e > 0$. This is consistent with our result from model A. This suggests that the initial $B_e$ of their model is similar to that of our model A, but significantly smaller than our models B and C.

Compared to the HD simulations, there are more MHD simulation works on the hot accretion flow. Where outflow is concerned, most of these works focus mainly on the jet, i.e., the outflow close to the axis and with a positive $B_e$. Only some of the previous work discussed the outflow away from the axis (e.g., Stone & Pringle 2001; Hawley & Balbus 2002; De Villiers & Hawley 2003; Igumenshchev et al. 2003; De Villiers et al. 2005). These simulations often adopt a rotating torus as the initial condition, similar to our models A and D, and their results are also similar to our model D (e.g., Hawley & Balbus 2002; Narayan et al. 2012), as expected. For example, in Hawley & Balbus (2002) the torus is centered at $200 M$. Like us, they found that most of the outflow is bound, whereas close to the axis it is unbound.

De Villiers et al. (2005) carried out three-dimensional MHD simulations with initial toroidal and poloidal magnetic field configuration. The center of the initial torus was located at $25 M$. In the poloidal case, jets were found in the regions of the axial funnel and funnel wall. This is consistent with our model D, as shown by, e.g., the right bottom plots of Figure 4. They did not find jets in the toroidal case. They argued that this is perhaps because a poloidal magnetic field is not formed in the funnel region. In Hawley & Krolik (2002) the initial configuration of the magnetic field was very similar to De Villiers et al. (2005). The initial condition was also similar, i.e., a hydrostatic torus with its pressure maximum located at $20 M$. Hawley & Krolik (2002), however, did not find any jets since they found all gas is bound, even in the case of a poloidal initial magnetic field. De Villiers et al. (2005) speculated that the reason was the choice of a cylindrical grid in Hawley & Krolik (2002), which prevents the buildup of the poloidal field close to the funnel. However, Hawley & Balbus (2002) and Kato et al. (2004) also adopted a
cylindrical grid, similar to Hawley & Krolik (2002). In contrast to Hawley & Krolik (2002), both Hawley & Balbus (2002) and Kato et al. (2004) found an unbound jet in their simulations. What is the reason for the discrepancy?

Whether jets can be formed or not is ultimately determined by the sign of $Be$ of the outflow. Our simulations show that the value of $Be$ of the outflow is crucially determined by the initial condition of the simulation. If $Be$ is large in the initial condition, the value of $Be$ of the outflow will be large and thus a jet can be formed easily. Of course, the existence of a poloidal magnetic field can also help in the formation of jets, because the value of $Be$ of the outflow can be increased by the Lorentz force. In Hawley & Balbus (2002) and Kato et al. (2004), the initial torus was located at a relatively large radius, $\sim200M$ and $80M$, respectively. Therefore the value of $Be$ of the initial torus is likely large enough to produce unbound jets. Following this line of thought, the temperature of the initial torus also matters. This is perhaps the case in the work of De Villiers & Hawley (2003), who considered a hotter torus compared to De Villiers et al. (2005). Consequently, although the torus was closer to the hole, located at $\sim15M$, a higher fractional jet rate was still found.

5. ADIOS OR CDAF?

As we mentioned in Section 1, two different models have been proposed to explain the inward decrease of the accretion rate, namely ADIOS and CDAF. In the former, the inward decrease of the accretion rate is thought to be because of the systematic mass loss in the outflow which is launched continuously from almost all radii. The rates of inflow and outflow (Equations (12) and (13)) are dominated by the systematic fluxes of mass. In the CDAF model, however, the inward decrease of the accretion rate is thought to be because, with accretion, more and more fluid circulates in convective eddies and becomes locked in them. The inflow and outflow rates are assumed to be dominated by convective turbulence. In this section we will try to argue, based on our simulation results presented in Section 3, that the ADIOS scenario is more likely.

First, as pointed out by Blandford & Begelman (2004) and Begelman (2012), the CDAF model faces secular difficulties related to the global mass supply. The inward decrease of the mass accretion rate is explained by the fluid being locked in the convective eddies. In this scenario, most of the outflowing fluid (which contributes to the outflow rate) will eventually turn around and feed the inflow. Thus when we have continuous mass supply from an outer boundary, we would expect mass accumulation and thus a steady state cannot be achieved. In most previous numerical simulations, the initial condition was a torus with limited mass. In this case the simulation cannot run for too long a time and because the available mass is very limited it is hard to examine this point. In model C of the present paper, we continuously inject mass from an outer boundary which enables us to run the code for a sufficiently long time. We do not observe any mass accumulation, and the profiles of the accretion rate and density are no different to those of the other three models. The above argument also applies if the turbulence is caused by another instability, such as MRI, rather than convection. If there were no real systematic outflow, but the outflow rate we calculate were just the turbulent fluctuation, it would be hard to understand why the inflow rate decreases inward. Of course, as we will state below, turbulent fluctuations must contribute to the outflow rate, although they are not dominant.

The second issue with the CDAF model is its applicability to an MHD accretion flow. Stone & Pringle (2001), Hawley et al. (2001), and Balbus & Hawley (2002) argued that the dynamics of the MHD accretion flow should be dominated by MRI rather than convection, since the dominant mode of angular momentum transport in an MHD accretion flow is MRI, no matter whether or not destabilizing entropy gradients are present. Narayan et al. (2002) performed a linear MHD stability analysis of a rotating MHD accretion flow. They found that, in general, both “convective” and “MRI” modes exist. The stability criterion for the convective mode is the standard Høiland criterion of hydrodynamics (see Equation (35) below; but see Shcherbakov 2008). When the accretion flow is convectively stable according to this criterion, MRI is the only unstable mode. When the accretion flow is convectively unstable, however, the result is complicated and depends on the wavelength of the perturbation. The short-wavelength modes are still MRI. But the long-wavelength modes are found to be independent of the magnetic field and are intrinsically convective modes because the buoyancy force which drives convection is much stronger than the magnetic force which drives MRI. Since the saturated value of the magnetic field in an accretion flow is not very strong, $\beta > 1$, the long-wavelength fluctuation with $\lambda/H > \beta^{-1/2}$ can usually fit inside the disk.

It is therefore crucial to examine whether the MHD accretion flow is convectively stable or not. The Høiland criterion for the stability is (e.g., Tassoul 1978; Begelman & Meier 1982)

$$\nabla \cdot (g \cdot dR) - \frac{2\gamma v_{\phi}}{R^2} \left[ \nabla (v_R) \cdot dR \right] R < 0. \quad (35)$$

Here $R = r \sin \theta$ is the cylindrical radius, $dR = dr \hat{r} + r d\theta \hat{\theta}$ is the displacement vector, $s = \ln(p) - \gamma \ln(\rho)$ is $(\gamma - 1)$ times the entropy, $g = -\hat{r} v_K^2/r + \hat{\theta} v_{\phi}^2/R$ is the effective gravity, and $v_K$ and $v_{\phi}$ are the Keplerian and rotational velocities, respectively. For a non-rotating flow, this condition is equivalent to an inward increase of entropy, which is the well-known Schwarzschild criterion. We study convective stability based on the simulation data of model D. The results are shown in Figure 12. The red regions are convectively unstable. We see from the figure that most of the regions of the flow are not red, indicating that the MHD accretion flow is convectively stable. This implies that the CDAF model cannot be applied to MHD accretion flows. Given that the mass accretion rate in both MHD and HD accretion flows decreases inward, CDAF is likely not a feasible model.

The third problem of the CDAF model is its difficulty in explaining the systematic differences between the properties of the inflows and outflows. In the CDAF model, the inflow and outflow rates (Equations (12) and (13)) are assumed to be dominated by turbulent fluctuation. If this were true, we would expect that the properties of inflow and outflow should be roughly similar. To check this point, here we have performed numerical simulations and calculated the Bernoulli parameter $Be$, temperature, and radial and rotational velocities of the inflows and outflows. Our results indicate that the properties of the outflows and inflows are systematically different (see Figures 5–11 and Section 3 for details). Such differences are hard to explain if the inflow and outflow rates are dominated by turbulent fluctuations. Rather, they are strong evidence that the systematic fluxes of inflowing and outflowing mass must contribute at least significantly, if not dominantly, to the inflow and outflow rates. This systematic inflow and outflow motion can be observed in a movie based on the HD or MHD
Figure 12. Convective stability analysis of the MHD accretion flow (model D) according to Equation (35) based on the simulation data at $t = 3.5$ orbits at $R = 100r_s$. The red color denotes unstable regions. The result indicates that the MHD flow is convectively stable in most regions.

(A color version of this figure is available in the online journal.)

Figure 13. Snapshot of the velocity vectors (arrows) of model A in the steady state, overlaid with density. Both systematic inflowing and outflowing motion and convective turbulence are evident.

(A color version of this figure is available in the online journal.)

Figure 14. Time-averaged radial inflow and outflow rates for model A as a function of radius. The red solid line shows the inflow rate calculated by the “second” approach (Equations 12 and 13), while the black dotted line shows the net rate (Equation 12) at the black hole horizon. The inflow rate denoted by the red solid line is smaller than the net rate, indicating an underestimate of the systematic inflow rate. Beyond $r \sim 40r_s$, the inflow and outflow rates obtained by the two approaches differ by a factor of less than two. However, at such large radii we are not confident of the result because of the possible boundary condition effect. The overestimation of the contribution of turbulent fluctuation has the following two consequences. First from Figure 14 we can see that the inflow rate calculated by the “second” approach (red solid line) is even smaller than the net rate (black dotted line), which is the accretion rate at the black hole horizon or, to put it in another way, the inflow rate denoted by the red solid line decreases outward in some regions. Second, we find that the inflow and outflow rates calculated by the “second” approach depend on the time interval adopted in the time-averaged integration. The longer the interval, the smaller the rates are.

To further examine the contribution of turbulent fluctuation, we have performed two other tests. One is to calculate again the temperatures and radial velocities of the inflow and outflow, following Equations (8) and (9), but this time based on the

and thus underestimate the inflow and outflow rates. Therefore this approach gives an approximate lower limit to the inflow and outflow rates. The two red lines in the upper left plot of Figure 14 shows the results of inflow and outflow rates calculated by this “second” approach. In comparison with the black lines, which are based on our “first” approach, we can see that within $r \sim 30r_s$, the inflow and outflow rates are now significantly smaller, which is consistent with Narayan et al. (2012). Beyond $r \sim 40r_s$, the inflow and outflow rates obtained by the two approaches differ by a factor of less than two. However, at such large radii we are not confident of the result because of the possible boundary condition effect. The overestimation of the contribution of turbulent fluctuation has the following two consequences. First from Figure 14 we can see that the inflow rate calculated by the “second” approach (red solid line) is even smaller than the net rate (black dotted line), which is the accretion rate at the black hole horizon or, to put it in another way, the inflow rate denoted by the red solid line decreases outward in some regions. Second, we find that the inflow and outflow rates calculated by the “second” approach depend on the time interval adopted in the time-averaged integration. The longer the interval, the smaller the rates are.

To further examine the contribution of turbulent fluctuation, we have performed two other tests. One is to calculate again the temperatures and radial velocities of the inflow and outflow, following Equations (8) and (9), but this time based on the
Figure 14. Studying the contribution of turbulent fluctuation in the inflow and outflow rates (Equations (12) and (13)). All data are based on model A. Top left: black lines are exactly the same as those in the top left plot of Figure 2. The two red lines show the inflow (solid) and outflow (dashed) rates calculated by the “second” definition (i.e., integrating the time averages of fluxes over $\theta$). Top right: black lines are exactly the same as those in the top left plot of Figure 8. The red lines show the temperatures of inflow (solid) and outflow (dotted) based on the “second” definitions. Bottom left: black lines are exactly the same as those in the top left plot of Figure 9. The red lines show the radial velocities of inflow and outflow based on the “second” definitions. Bottom right: black and red lines show the ratio $v_r/v_\theta$ for inflow and outflow based on our (“first”) definitions, respectively. See the text for details.

(A color version of this figure is available in the online journal.)

“second” definition of inflow and outflow. The results are shown by the red lines in the top right and bottom left plots of Figure 14. The black lines show the previous result. We can see that there are few differences. This again indicates that turbulent fluctuation should not contribute significantly to the inflow and outflow rates defined by our “first” approach. Finally, we calculate the ratio $v_r/v_\theta$ at each radius, geometrically averaged$^7$ over all $\theta$ for inflow and outflows defined by our (“first”) approach. The result is shown by the bottom right plot of Figure 14. In our calculation, at each radius we exclude the grids with $v_r/v_\theta > 10$ to avoid numerical overflow. This is why the line is broken at some radii. So this result should be regarded as the lower limit. If the motion of fluid is dominated by turbulent fluctuation, we would expect that $v_r/v_\theta \sim 1$. However, we see from the figure that $v_r/v_\theta > 2$, i.e., the motion of both inflow and outflow is dominated by systematic radial motion. We would like to emphasize that all these tests give us only a qualitative idea about the contribution of turbulence. It is necessary in the future to perform more precise and quantitative calculations.

6. MECHANISMS OF PRODUCING OUTFLOWS IN HD AND MHD FLOWS: BUOYANCY AND THE “MICRO” BLANDFORD–PAYNE MECHANISM

In the early version of the ADIOS scenario (Blandford & Begelman 1999), the production of outflow was proposed to be due to the positive $Be$ of the accretion flow, a result obtained in the self-similar solution of ADAFs (Narayan & Yi 1994). However, previous HD and MHD numerical simulations have shown the existence of outflow when $Be < 0$ (e.g., Igumenshchev & Abramowicz 1999; SPB99; Yuan & Bu 2010), although in this case the outflow may not be able to escape to infinity. We propose that the mechanism of producing outflow is different for HD and MHD accretion flows. In the former case the mechanism is the buoyant force (i.e., convection) while in the latter case it is the magnetic centrifugal force.

In the case of an HD flow, it has been shown that the flow is convectively unstable. This means that if the temperature of a fluid element is perturbed to be higher than the surrounding medium, its density will become lower and the fluid element will feel the buoyancy force and escape outward. In contrast to the CDAF model, the flow will not turn around and circulate in convective eddies. Instead, it will keep flowing outward and form an outflow. This is similar to the motion of an upward running air bubble in water. This picture is supported by the systematically higher temperature of outflow compared with inflow, as shown in Figure 8.

However, we have shown that an MHD accretion flow is convectively stable and thus the scenario above does not apply. This is confirmed by the roughly equal temperatures between inflow and outflow as shown by the right bottom plot of Figure 8. Then what is the mechanism of producing outflow? From the right bottom plot of Figure 11, we see that the specific angular momentum of the outflow is nearly Keplerian, and much larger

\[ \langle q \rangle = (q_1 \times q_2 \ldots \times q_n)^{1/n}. \]

\(^7\) Here “geometrical average” means that $\langle q \rangle = (q_1 \times q_2 \ldots \times q_n)^{1/n}$. 
than that of the inflow. This feature provides a sharp contrast to the case of the three HD models where the angular momentum of the outflow is significantly sub-Keplerian and is overall very similar to that of the inflow. This strongly suggests that the outflow in an MHD accretion flow is driven by the centrifugal force. An interesting question is then: what causes the difference in angular momentum between the inflow and the outflow? This is explained in Figure 15. Consider two fluid elements located at two different radii in a differential rotating accretion flow. They are initially moving inward, connected by a magnetic field line. Magnetic stress will transport the angular momentum from the inner fluid element to the outer one since the former rotates faster. Once the angular momentum of the outer element reaches a nearly Keplerian value, the centrifugal force, combined with the gradient of the gas pressure, will be able to make the fluid element turn around and throw it outward. This mechanism is similar to the Blandford & Payne (1982) mechanism, in the sense that the outflow is produced by the centrifugal force mediated by magnetic field. But there are also differences. In the Blandford & Payne (1982) model, a large-scale open poloidal magnetic field is required, and the model works only in the coronal region where magnetic pressure is much larger than the gas pressure (thus fluid elements anchored in the same field line have the same angular velocity). In our case, both requirements are not necessary. In fact, in our model the magnetic field in both the accretion flow and the coronal region is tangled, thus its scale is not large. In addition, from the bottom right plot of Figure 11, we see that the specific angular momentum of the outflow is not super-Keplerian, as expected if the Blandford–Payne mechanism were operating. Therefore, this mechanism is different from the Blandford–Payne mechanism and we call it a “micro” Blandford–Payne (MBP hereafter) mechanism. It will be interesting to check whether this mechanism also works for the standard thin disk.

In the literature, two currently popular models of producing outflow from accretion disks are radiation pressure driving (e.g., Murray et al. 1995) and large-scale magnetic field acceleration (e.g., Blandford & Payne 1982). In terms of producing outflows from hot accretion flows, we propose that the MBP mechanism may be the dominant one in nature. This is based on the fact that the radial density profile of accretion flow obtained from MHD numerical simulations, which incorporate only this mechanism, is in good agreement with that obtained from observations (see Paper I for details). Theoretically, the optical depth of the outflow from a hot accretion flow may be too small for the radiation mechanism to work. The Blandford–Payne mechanism, as we mentioned before, requires the existence of a large-scale open poloidal magnetic field over a large range of radius of the accretion flow. However, the origin of this kind of field is still an open question. Two mechanisms have been considered. One is the MHD dynamo (e.g., Tout & Pringle 1996), and the other is the direct inward advection of a large-scale poloidal field by the accretion flow from large radii. However, current MHD simulations show that the dynamo mechanism seems to produce only a small-scale poloidal field on the order of the local disk thickness (e.g., De Villiers et al. 2005). Beckwith et al. (2009) considered the second possibility. They found that the large-scale poloidal field only exists close to the black hole. However, McKinney et al. (2012) show that if magnetic flux can be accumulated, the large-scale poloidal field can be formed even away from the black hole.

7. POSSIBLE OBSERVATIONAL APPLICATIONS

AGN outflow has been of great interest in recent years. One important reason is because it is believed that AGN feedback plays an important role in galaxy formation and evolution (e.g., Di Matteo et al. 2005; Hopkins et al. 2005; Fabian 2012). In terms of the forms of the output from the central AGN, the feedback can be caused by radiative output (e.g., Ciotti et al. 2010) and mass outflow (e.g., Ostriker et al. 2010). Compared to radiative feedback, feedback by mass outflow is more localized in the vicinity of the black hole and is thus more efficient in reducing the mass accretion rate and, further, the growth of the black hole. There are two types of mass outflows, i.e., jet and outflow (or winds). Their respective roles are different. The energy flux of the former is much larger than that of the latter. Perhaps for this reason, most works on AGN feedback concentrate only on jets. However, outflow has gradually received more and more attention for the following reasons. First, jets are well collimated and thus they simply pill through the galaxy without any significant interaction with the ISM. Therefore, jets can have an important role only in galaxy clusters. Second, nearly 90% of AGNs are radio-quiet and do not have strong jets. Third, the momentum flux of outflow is likely larger than that of jets and thus outflow plays a more important role in some cases. For example, Ostriker et al. (2010) have shown that the growth of the central supermassive black hole in the Galactic center is mainly determined by the mass and momentum feedback of the outflow. Outflow has been detected directly in AGNs such as broad absorption line quasars (e.g., Crenshaw et al. 2003; Tombesi et al. 2010, 2011a, 2012), but the origin and acceleration of this outflow are still unsolved problems. Widely suggested and studied mechanisms include radiation pressure (e.g., Murray et al. 1995; Murray & Chiang 1997; Proga et al. 2000), magnetic driving (e.g., Blandford & Payne 1982; Emmering et al. 1992; Romanova et al. 1997; Bottorff et al. 2000), and thermal driving (e.g., Begelman et al. 1983; Chelouche & Netzer 2005). While the radiation driving model is very popular, some problems have been found. For example, according to Murray & Chiang (1997), when explaining high ionization blueshifted lines, only when viewed at the pole can we see the blueshifted line and the lines must be very narrow. This is not consistent with observations (Wang et al. 2011). The detailed modeling of some individual AGN outflows also indicates that the radiation mechanism may not work, as we will state later in this section. The Blandford & Payne (1982) mechanism must confront the problem of the origin of the large-scale magnetic field (see the discussion in the last section). In this section, we discuss the possibility that
some AGN outflows can potentially be explained by the model studied in this work.

In addition to AGN outflows detected via absorption lines, bubbles around black holes are also possible evidence for their existence. This is perhaps the case for the “Fermi bubble” detected in the center of the Galaxy. In this section, we will also discuss this possibility. To illustrate that outflow from hot accretion flow may be relevant and promising in interpreting some observed AGN outflows, we need to introduce some background on the accretion models of AGNs. Finally, outflows have also been detected in black hole X-ray binaries. A comparison of AGNs and X-ray binaries is obviously both useful and important.

7.1. Accretion Models of AGNs and Black Hole X-ray Binaries

Outflows from AGNs are generally detected via absorption lines. If we hope to explain observations using the model proposed in this paper, we may ask if the hot accretion flow model can be applied to luminous AGNs and how the absorption line can be produced if the temperature of the flow is as hot as virial. Before we answer these questions, we first briefly introduce the accretion model for black hole binaries. An individual black hole X-ray binary comes in several states, mainly high, soft, and steep power-law (or very high) states. These three states are associated with continuous jets, no jets, and episodic jets, respectively (Fender 2006). The luminosity of the hard state spans a large range, from very low values up to $\sim 10\% L_{\text{Edd}}$ or even $30\% L_{\text{Edd}}$ (e.g., Zdziarski & Gierliński 2004). The soft state, on the other hand, only exists above $\sim 2\% L_{\text{Edd}}$. Sources with $2\% L_{\text{Edd}} \lesssim L \lesssim 10\% L_{\text{Edd}}$ are not certain because of the “hysteresis” (Zdziarski & Gierliński 2004). The soft state is described by a standard thin disk extending to the innermost stable circular orbit, while the hard state is described by an inner hot accretion flow, which contributes most of the bolometric luminosity of the hard state, and an outer truncated thin disk (e.g., Narayan 1996; Esin et al. 1997; Yuan et al. 2005; Yuan & Zdziarski 2004; Oda et al. 2012; see Narayan 2005; McClintock & Remillard 2006; Done et al. 2007 for reviews). The transition radius between the hot and cool accretion flows ($r_\text{tr}$) is a function of accretion rate or luminosity (Yuan & Narayan 2004). The model of the very high state is still unclear.

Compared to black hole X-ray binaries, the accretion model for AGNs is far less certain. Since the physics of accretion is irrelevant with black hole mass, by analogy to the black hole binaries, it is reasonable to assume that low-luminosity AGNs with $L$ up to $(2-10)\% L_{\text{Edd}}$ should correspond to the “hard state” and are powered by a hot accretion flow. This picture has received observational support (see Yuan 2007; Ho 2008 for reviews). The model of luminous AGNs is more complicated and may not be easily associated with the standard thin disk. This is because, on the one hand, some luminous AGNs may correspond to the very high state whose model is unknown; on the other hand, the observational evidence for the thin disk is not as good as in the soft state of black hole X-ray binaries. Moreover, it seems that the hard X-ray emission of luminous AGNs, which cannot be explained by a standard thin disk, is much stronger than that in the soft state of black hole X-ray binaries. This indicates that hot gas must exist. One suggestion is to invoke a corona sandwiching the thin disk (e.g., Galeev et al. 1979; Haardt & Maraschi 1991). If the dynamics of the corona can be described by a hot accretion flow, the study we present in this paper will be applicable.

Almost all works, including this one, assume that the hot accretion flow is one-phase. Hot accretion flow is likely thermally unstable under short-wavelength perturbations (Kato et al. 1997; Wu 1997; Yuan 2003). Yuan (2003) found that when the accretion rate of the accretion flow $M \gtrsim \alpha^2 M_{\text{Edd}}$, the timescale of growth of perturbation will be shorter than the accretion timescale; consequently, cold dense clumps may be formed within the hot phase. So we expect that when $M \gtrsim \alpha^2 M_{\text{Edd}}$, the accretion flow and outflow should be two-phase. In this case, the presence of an absorption line is expected. If $\alpha = 0.1$, the corresponding $M \sim 0.01 M_{\text{Edd}}$ and $L \sim 10^{-3} L_{\text{Edd}}$.

7.2. Bubbles Inflated by Outflow

7.2.1. The Fermi Bubble in the Galactic Center

The first example is the two “Fermi bubbles” revealed by Fermi-LAT, extending 50° above and below the Galactic center (Su et al. 2010). The bubbles have approximately uniform surface brightness with sharp edges and are symmetric about the Galactic plane. The bubbles are spatially correlated with the hard-spectrum microwave excess known as the Wilkinson Microwave Anisotropy Probe (WMAP) haze. The initial analysis in Su et al. (2010) indicates that the bubbles are most likely created by some large episode of energy injection in the Galactic center. The age and energy of the bubble have been roughly estimated by Su et al. (2010) to be $10^7$ yr and $E \sim (10^{54}-10^{55})$ erg, respectively.

Several models have been proposed to explain the origin of the Fermi bubble. They usually associate the bubble with recent activity of the supermassive black hole Sgr A* (e.g., Guo & Mathews 2012; Zubovas & Nayakshin 2012). Guo & Mathews (2012) proposed that the bubble was inflated by a past jet in Sgr A*. They performed some numerical simulations and roughly reproduced the morphology of the bubble. The problems with this scenario are that the jet is more likely an outflow since it is not collimated, and the required mass flux in the jet is super-Eddington, which may be unreasonable. In another model, Zubovas & Nayakshin (2012) required that the bolometric luminosity of Sgr A* in the past was very large, approaching $L_{\text{Edd}}$, i.e., Sgr A* was a very luminous quasar. In this case, a powerful outflow could have been produced by the radiation pressure, which then inflated the Fermi bubble. We note, however, that the required high luminosity of Sgr A* is in conflict with the study of Totani (2006), who argued based on other observations that Sgr A* in the past should still have been within the regime of ADAF, although the mass accretion rate was three to four orders of magnitude higher than the present value.

Our scenario is that the bubbles are inflated by the outflow from an ADAF. Shocks are formed when the outflow interacts with the ISM, and electrons are accelerated to relativistic energies in the shock front. These electrons emit synchrotron and inverse Compton radiation by scattering with cosmic microwave background photons. The former explains the WMAP haze while the latter is responsible for the Fermi bubbles. We now estimate whether the outflow can reasonably power the bubble. Taking $E \sim 10^{54}$ erg and $10^7$ yr as an example, the required bubble power is $3 \times 10^{39}$ erg s$^{-1}$. From Equation (31), the required accretion rate is

$$M_{\text{out}} \sim 10^{-5} r_{\text{tr}} M_{\text{Edd}}.$$  \hspace{1cm} (36)

Here $\dot{M}_{\text{Edd}} \equiv 10 L_{\text{Edd}} / c^2 \approx 9 \times 10^{-2} M_{\odot}$ yr$^{-1}$ is the Eddington accretion rate of Sgr A*. Note that according to Yuan & Narayan...
M 87 is a famous low-luminosity AGN powered by an ADAF (e.g., Di Matteo et al. 2003). The radio (90 cm) image of M 87 (Figure 1 in Owen et al. 2000) clearly shows two bubbles with a radius of ~20 kpc. The shape of the bubbles looks very similar to the Fermi bubbles in the Galactic center. The orientation of the two bubbles is perpendicular to the radio jet and it therefore seems unlikely that the bubbles are inflated by the jet; rather, they may be inflated by the outflow from the ADAF.

7.3. Outflow from AGNs with High and Low Luminosities

One popular model for producing AGN winds is radiation driving. However, detailed studies of individual sources often exhibit some problems. In the following we present several examples.

NGC 3783. This is a Seyfert 1 galaxy, with mass of the black hole \( (3 \pm 1) \times 10^7 \) M\(_\odot\), bolometric luminosity \( L_{\text{bol}} \sim 3 \times 10^{46} \) erg s\(^{-1}\) \( \sim 0.1L_{\text{Edd}}\). Chelouche & Netzer (2005) carefully calculated the physical properties of the highly ionized outflow and compared their theoretical calculation with the Chandra data. The main results they obtained are: (1) the highly ionized outflow in this source is not driven by the radiation pressure (they suggested a thermal pressure gradient); (2) the rate of outflow is \( \sim 0.01-0.1 \) M\(_\odot\) yr\(^{-1}\) \( \approx 0.02-0.2M_{\text{Edd}}\) (\( M_{\text{Edd}} \equiv 10L_{\text{Edd}}/c^2 \approx 0.5 M\odot \) yr\(^{-1}\)); (3) the velocity of the outflow is \( \sim 1000 \) km s\(^{-1}\); (4) the global covering factor of the flow is \( \sim 20\%\).

NGC 4151. Kraemer et al. (2005) presented a detailed analysis of the intrinsic X-ray absorption in this source using Chandra data and found the presence of a very highly ionized outflow in addition to lower ionization gas. The authors pointed out that the outflows are so highly ionized and the luminosity of these galaxies is only \( \sim 4\%L_{\text{Edd}}\) so they are unlikely to be accelerated by radiation pressure. They pointed out that this kind of high ionization outflow is very common among Seyfert galaxies and its existence is the key to understanding the origin of mass outflow.

3C 111. Tombesi et al. (2011b) detected blueshifted absorption lines in 3C 111 based on their Suzaku observations. The mass of the black hole is \( (2-30) \times 10^8 \) M\(_\odot\) and the bolometric luminosity is \( L_{\text{bol}} \approx 8 \times 10^{45} \) erg s\(^{-1}\) \( \sim (2-30)\%L_{\text{Edd}}\). Detailed modeling indicates that the location of absorption material is constrained at <0.006 pc \( (20-300)r_s\), and the velocity and mass outflow rate of the outflow are \( \sim 0.1c\) and \( M_{\text{out}} \sim 1M\odot\) yr\(^{-1}\), respectively. The latter is similar to the mass accretion rate \( M_{\text{acc}} \sim L_{\text{bol}}/\eta c^2\) (for \( \eta = 0.1\)). The momentum flux of the outflow is comparable to the momentum flux of radiation, \( M_{\text{out}} \sim L_{\text{bol}}/c\). However, the Thomson scattering optical depth of the outflow is found to be very small, \( \tau \sim 0.05\). Therefore, for the radiation mechanism to work, we have to look for other opacities. This seems to be quite difficult given the high ionization of the outflow, similar to NGC 4151. In addition, while the rather small upper limit on the location of the outflow indicates its accretion flow origin and rules out other models, this location is more than one order of magnitude smaller than that predicted in the radiation driving model of Murray et al. (1995).

Low-luminosity AGNs. Outflow can be more easily detected in luminous AGNs partly because these sources are suitable for high-resolution spectroscopy. Recently, outflows have also been detected in low-luminosity AGNs. Crenshaw & Kraemer (2012) investigated the outflow from a sample of nearby AGNs, focusing on determining their mass outflow rate and the kinetic luminosity. Although they have to restrict their sample to apparently luminous AGNs and they do not include ultrafast outflow, of the ten nearby Seyfert 1 galaxies in their sample, six sources still have their bolometric luminosities below 5\%\(L_{\text{Edd}}\). Their detailed study of the UV and X-ray absorbers clearly shows that a strong outflow exists in these sources. The bolometric luminosity of one source, NGC 4395, is even as low as \( 10^{-3}L_{\text{Edd}}\). These low-luminosity AGNs are described by an ADAF rather than a standard thin disk. Thus it is questionable whether the radiation driving model works for them. Crenshaw & Kraemer found that the mass outflow rate exceeds the mass accretion rate by a factor of 10–1000. The kinetic luminosities of the outflows are approximately 0.5%–5% of their bolometric luminosities for half of the moderate-luminosity AGNs in their sample.

Now we estimate whether the above examples can possibly be explained by our outflow model. We want to emphasize that such estimations are necessarily very crude.

NGC 3783. During the propagation of outflow, the mass flux may increase and the velocity may decrease because of the contamination by the ISM, but the momentum flux should be conserved. Assuming an X-ray luminosity \( L_x \sim f_s L_{\text{bol}} \sim 0.1f_s L_{\text{Edd}}\) and assuming that the hard X-ray emission is produced by a hot corona which can be modeled by a hot accretion flow, the required \( M(10r_s) \sim 0.1\eta^{-1}f_s M_{\text{Edd}}\) (here \( \eta \) is the radiative efficiency of the hot accretion flow). So \( M_{\text{out}}(r_{\text{out}}) = 0.1\eta^{-1}f_s M_{\text{Edd}}(r_{\text{out}}/10r_s)^{0.5}v_{\text{term}} \sim 0.5\eta^{-1}(r_{\text{out}}) \sim c/2\sqrt{2r_{\text{out}}/r_s}\). The momentum flux is then \( p_{\text{out}} = 0.02f_s\eta^{-1}M_{\text{Edd}} \sim 2 \times 10^{-7}M_{\text{Edd}}c\) for \( f_s = 10^{-2}\) and \( \eta = 0.1\). This is consistent with the observed \( p_{\text{out}} \sim (0.7-7) \times 10^{-3}M_{\text{Edd}}c\). In addition, assuming \( r_{\text{out}} \sim 100r_s\), the outflow rate is \( M_{\text{out}}(r_{\text{out}}) \sim 0.03M_{\text{Edd}}\), which also satisfies the requirement that the outflow rate produced from the hot accretion flow should be smaller than that observed because of contamination by the ISM.

3C 111. The location of the absorption material is very close to the black hole, so we expect little contamination of the outflow. First, there is obviously no problem with our model producing outflows from a radius as small as \( 100r_s\). Second, assuming \( r_{\text{out}} \sim 100r_s\) (since luminosity is relatively high), since \( (100r_s/10r_s)^{0.5} \sim 3\), this explains why \( M_{\text{out}}(100r_s)\) is not so different from the accretion rate at \( 10r_s\). Third, the velocity of outflow is \( \sim 0.5v_{\text{acc}}(100r_s) \sim 0.1c\), in good agreement with observations. Finally, the approximate equality of the momentum flux between radiation and outflow is exactly what we expect (Equation (28)).

Low-luminosity AGNs. First, for low-luminosity AGNs, a reasonable assumption is that \( r_{\text{out}} \sim 10^3-10^5r_s\), depending on the exact value of the accretion rate and the temperature of the fueling material. In this case, the ratio of the mass outflow rate and accretion rate (at \( 10r_s\)) is \( M_{\text{out}}(r_{\text{out}})/M_{\text{acc}}(10r_s) \sim (r_{\text{out}}/10r_s)^{0.5} \sim 10-100\). Given further the contamination by the ISM, this explains the observed ratio of the outflow rate and
accretion rate. Second, from Equation (34), for \( \eta \sim 0.05 \), we have \( \dot{E}_w \sim 1\% \dot{E}_{\text{rad}} \), again in good agreement with observations.

7.4. Outflow from Black Hole X-ray Binaries

A number of high ionized winds have been detected in black hole and neutron star X-ray binaries in recent years (see references in Neilsen & Homan 2012). GRO J1655-40 is one of the best examples. This source was observed by *Chandra* twice during its 2005 outburst. The second observation was made when the source was in the soft state. A series of absorption lines from a dense and highly ionized wind have been revealed. Detailed spectral analyses and theoretical studies have been performed by different groups and confirmed repeatedly that this outflow was driven not by radiation or thermal pressure, but likely by a magnetic mechanism (Miller et al. 2006, 2008; Kallman et al. 2009; Luketic et al. 2010). Another observation was made 20 days earlier than this one, when the source was just at the beginning of the transition from hard to soft state. This observation only detected one absorption line (Fe xxvi). Neilsen & Homan (2012) analyzed the data and concluded that the outflow in the hard state also cannot be driven by radiation pressure because the ionization parameter is too high. Their photoionization modeling indicates that the difference between the *Chandra* spectra in the two observations cannot possibly be explained by a change in the ionizing spectrum. Instead, the properties of the wind in the two states must be changed. This is reasonable since the accretion flow in the two states is different.

Most recently, King et al. (2012) have investigated the kinetic power of outflows and jets across the black hole mass scale. They found that both powers scale with the radiative power. More importantly, they found that the scaling relations are consistent with each other within error bars, although the normalization of the jet relation is higher than that of the outflow. They argued that such a formal consistency suggests a common origin mechanism for the outflow and jets. Since it is generally believed that jets are of magnetic origin, this would imply that outflow is also of magnetic origin.

8. SUMMARY

Numerical simulations of hot accretion flow, both HD and MHD, have revealed that the mass accretion rate decreases with decreasing radius. Consequently the radial density profile of the accretion flow becomes much flatter compared to the self-similar solution (Narayan & Yi 1994) which is based on the assumption of a radius-independent mass accretion rate. Denoting the profiles of accretion rate and density as \( \dot{M}(r) \propto r^p \) and \( \rho(r) \propto r^{-p} \), the HD simulation in Paper I, which has the largest radial dynamical range so far (from \( r_0 \) to over \( 10^3r_0 \)), gives \( s \sim 0.4-0.75 \) and \( p \sim 0.65-0.85 \), respectively. As a comparison, the self-similar solution of ADAFs gives \( s = 0 \) and \( p = 1.5 \). Such a flatter density profile has obtained strong observational support in Sgr A* and NGC 3115 (see Paper I for details). Two different models have been proposed to explain such a result, namely ADIOS and CDAF. In this paper we investigate the nature of the inward decrease of the accretion rate and other related questions.

With this aim, we have performed a series of HD and MHD numerical simulations, comparing the various properties of the inflow (the gas with a negative radial velocity) and the outflow (the gas with a positive radial velocity) such as radial and rotational velocities, temperature, and the Bernoulli parameter. We found systematic and significant differences (Section 3). Such differences are hard to understand if the inflow and outflow are simply the appearance of turbulent fluctuation, but strongly suggest that the motion of the accretion flow is dominated by systematic inward and outward fluxes of mass. We have also analyzed the convective stability of MHD flows. We found that they are stable (Section 5). This indicates that the CDAF model at least cannot be applied to MHD flows. Based on these results, together with other arguments, we conclude that the inward decrease of the accretion rate is because of mass loss via outflow (Section 5).

An immediate question is then the origin of the outflow. This is discussed in Section 6. The detailed comparison of properties between inflow and outflow again provides important information to answer this question. In the HD case, we found that the temperature of the outflow is systematically higher than that of the inflow (Figure 8). This suggests that the outflow is driven by the buoyancy which arises because of the convective instability of the HD accretion flow. In the case of the MHD flow, we found that the specific angular momentum of the outflow is very high, close to the Keplerian angular momentum at the equatorial plane, while the angular momentum of the inflow is much lower (Figure 11). This suggests that it is the centrifugal force that drives the production of outflow. The magnetic field in the flow efficiently transfers the angular momentum between fluid elements. Whenever one element gets sufficient angular momentum, it will turn around and be thrown outward. This mechanism is similar to the Blandford & Payne (1982) mechanism, except that no large-scale magnetic field is required here. We therefore call it the “micro” Blandford–Payne mechanism (Figure 15).

The properties of outflows are of great interest, in particular because of their potential important role in AGN feedback. We have calculated the mass flux, terminal radial velocity, momentum flux, and kinetic power of the outflow which should be useful in the comparison with observations of AGN winds and in the study of AGN feedback (Section 3). One of the most important properties is the Bernoulli parameter of the outflow since its sign determines whether the outflow can escape to infinity. We have run three HD models with different initial conditions. We found that while many properties of the accretion flow are not dependent on the initial condition, the sign of \( Be \) of the outflow is crucially determined by the value of \( Be \) in the initial condition (Figure 5). If \( Be \) is large in the initial condition, \( Be \) of the outflow tends to be positive. This is natural since the total energy should be conserved in the simulation. Unfortunately, it is uncertain what kind of initial condition is more realistic, and the answer may depend on circumstances. We think that the value of \( Be \) of the gas is likely to be positive at least in some cases such as when the radiative energy loss of the accretion flow is small. Even in the case that \( Be < 0 \), these outflows can still escape out of the outer boundary of the accretion flow, as shown by our simulations; thus they can still interact with the ISM and play a similar role to those outflows with \( Be > 0 \).

We have also discussed the possible applications of the outflow from hot accretion flow in explaining observations (Section 7). These include the formation of the “Fermi bubble” in the Galactic center, and the origin of observed winds from both AGNs and black hole X-ray binaries. These winds have been detected from sources with a variety of luminosities. Detailed analyses of some sources published in the literature have shown that it is very hard for the winds to be produced by radiation or thermal driving mechanisms. Our simple estimation
suggests that it is promising to explain their origin by our “micro” Blandford–Payne mechanism.

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REFERENCES

Abramowicz, M. A., Chen, X., Kato, S., Lasota, J.-P., & Regev, O. 1995, ApJ, 438, L37
Abramowicz, M. A., Czerny, B., Lasota, J.-P., & Szuszkiewicz, E. 1988, ApJ, 332, 646
Abramowicz, M. A., Lasota, J.-P., & Igumenshchev, I. V. 2000, MNRAS, 314, 775
Balbus, S. A., & Hawley, J. F. 1991, ApJ, 376, 214
Balbus, S. A., & Hawley, J. F. 1998, Rev. Mod. Phys., 70, 1
Balbus, S. A., & Hawley, J. F. 2002, ApJ, 573, 749
Becker, P. A., Subramanian, P., & Kazanas, D. 2001, ApJ, 552, 209
Beckwith, K., Hawley, J. F., & Krolik, J. H. 2009, ApJ, 707, 428
Begelman, M. 2012, MNRAS, 420, 2912
Begelman, M. C., & Meier, D. L. 1982, ApJ, 253, 873