Ginzburg-Landau micromagnetic model to study domain wall dynamics in thin ferromagnetic systems

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Fully understanding domain wall motion in ferromagnetic systems is considered to be essential for the design of new nano-electronic devices. Along with experiments, numerical simulations are therefore a key to gaining insight into the underlying mechanisms. However, simulating model systems at time and length scales comparable with those of experiments still represents a great challenge. Here, we present a simplified micromagnetic model—halfway between full micromagnetism and Ginzburg-Landau theory—to study the dynamics of domain walls in quasi two-dimensional ferromagnetic systems with perpendicular magnetic anisotropy (PMA). Our approach relies on the local parametrization of the in-plane magnetization in terms of its out-of-plane component. We show that our model quantitatively reproduces previous experimental velocity-field data in the archetypal PMA Pt/Co/Pt ultra-thin films in the three dynamical regimes of domain wall motion (creep, depinning, and flow). In addition, we present a statistical analysis of the domain wall width parameter, showing that our model can provide detailed nano-scale information while reaching length and time scales comparable to experiments.

I. INTRODUCTION

Elastic driven interfaces are ubiquitous in nature. They appear in systems as diverse as contact lines in wetting, earthquakes, vortices in type-II superconductors, and domain walls in ferroelectric and ferromagnetic materials. The latter, particularly, present very promising technological applications related to the possibility of tuning domain wall motion with controllable parameters such as electric currents or magnetic fields. The behavior of the velocity in these systems, however, depends not only on these external parameters but also on the relevant interactions between magnetic moments, the magnetic texture of the domain wall, the pinning disorder of the material, and thermal activation. A theoretical approach that accounts for the interplay of all these ingredients and allows for simulations at time and length scales comparable to experiments would be key for the design and engineering of new materials.

A general theory for the velocity-field response in magnetic systems was derived in the context of the elastic interface model, where the domain wall is modeled as an elastic interface without any internal structure. While in a perfectly-ordered system the velocity-field relation is simply linear, quenched disorder is responsible for a much more complex behavior. When considering an applied external magnetic field \( H \), and in the presence of quenched disorder, there is a field \( H_d \) called the depinning field below which no domain wall motion can exist at \( T = 0 \). At finite temperature, a thermally-activated creep regime appears at low fields following the law \( \ln(v) \propto H^{-\mu} \), where the universal creep exponent is given by \( \mu = (2\zeta_{eq} + d - 2)/(2 - \zeta_{eq}) \) with \( d \) the dimension of the interface and \( \zeta_{eq} \) the equilibrium roughness exponent. In a one-dimensional interface, then, the theoretical value \( \zeta_{eq} = 2/3 \) implies a creep exponent \( \mu = 1/4 \), which has been found experimentally as well.

In the depinning regime, just above \( H_d \), the velocity presents universal power-law behavior associated to the underlying \( T = 0 \) transition. Finally, \( v \propto H \) at higher fields, in what is known as the flow regime. The proportionality constant in this linear behavior is the mobility, and is the same than in the system without quenched disorder.

From the standpoint of statistical mechanics, a suitable model to study magnetic domains is given by Ginzburg-Landau theory. This theory was originally proposed as a mean-field approach to continuous phase transitions, relating the order parameter to the underlying symmetries of the system. Moreover, simple dissipative dynamics for the order parameter can be considered, allowing to study time-dependent phenomena. The observed domain wall dynamics using this scalar-field model presents...
the same non-linear response (creep, depinning and flow) for the velocity-field curve.\textsuperscript{28}

Models such as the aforementioned elastic line and Ginzburg-Landau theory have proven useful to understand the universal characteristics of domain wall dynamics.\textsuperscript{28} These models are very helpful on unveiling general mechanisms and providing universal features, but miss material-dependent characteristics, thus falling short in the connection to experiments. Micromagnetic theory, on its turn, provides important insight into the physical properties of magnetic materials, with particular care of material and experimental parameters.\textsuperscript{30–35} Moreover, this approach has shown the velocity-field dependence in a system without disorder to be much more complex, with two linear regimes (stationary and precessional) separated by what is known as the Walker breakdown.\textsuperscript{36} On the negative side, although important advances have been recently made,\textsuperscript{35} simulating long time and large scale behavior is still a demanding task, which sometimes challenges the physical interpretation of the observed phenomena.

In this work, we present a simplified micromagnetic model that bridges the gap between micromagnetism and Ginzburg-Landau theory, allowing us to exploit their best features. As shown in Fig. 1, we are able to simulate polar magneto-optical Kerr effect (PMOKE) experiments, reaching experimental length and time scales (we will return to Fig. 1 in Sec. \textit{III}). We use previous experimental velocity-field data in Pt/Co/Pt ultra-thin films\textsuperscript{22} to test our material-dependent model. Not only we find good agreement in the three regimes of domain wall motion, but we also recover universal features as the creep exponent. In addition, we report new results regarding domain wall width fluctuations in this system, which are typically not exposed by PMOKE experiments. In this way, our model shows great versatility and potentiality, and can be key to large-scale simulations of magnetic materials.

\section*{II. GINZBURG-LANDAU MICROMAGNETIC MODEL}

In the following, we shall present how the Ginzburg-Landau micromagnetic model is derived, starting from the micromagnetic description for the dynamics of a ferromagnetic model. A ferromagnet is a system that presents a net magnetization (that is, magnetic moment per unit volume) in the absence of external field. It can be thought of as composed by cells of a given volume which is large compared to the atomic scale, and containing a great number of atomic magnetic moments. Then, micromagnetism provides a way to model the evolution of the magnetization $\mathbf{M}$ in each cell by following the stochastic Landau-Lifshitz-Gilbert (SLLG) equation. In the Landau formulation (see Ref. 37 and references therein) it is written

$$\dot{\mathbf{M}} = -\frac{\gamma\mu_0}{1 + \eta_0} \mathbf{M} \times \left[ \mathbf{H}_{\text{eff}} + \mathbf{H}_{\text{th}} \right] + \frac{\eta_0}{M_S} \mathbf{M} \times \left[ \mathbf{H}_{\text{eff}} + \mathbf{H}_{\text{th}} \right], \quad (1)$$

where $\eta_0$ is the adimensional damping constant, $\mu_0$ is the vacuum permeability, $\gamma$ is the gyromagnetic ratio and $M_S$ is the saturation magnetization. While the effective field is derived from the free-energy Hamiltonian $\mathcal{H}$ of the system by the relation

$$\mu_0 \mathbf{H}_{\text{eff}} = -\frac{\delta \mathcal{H}}{\delta \mathbf{M}}; \quad (2)$$

$\mathbf{H}_{\text{th}} = f_x \hat{e}_x + f_y \hat{e}_y + f_z \hat{e}_z$ is a random vector field that represents thermal noise and satisfies

$$\langle f_{\rho \tau}(t) f_{\tau'}(t') \rangle = 2 D \delta_{\rho \tau} \delta_{\tau j} \delta(t - t') \quad (3)$$

for cells $\rho$ and $\tau$, Cartesian coordinates $i, j = x, y, z$, and

$$D = \frac{\eta_0 k_B T}{\gamma V M_S \mu_0^2}, \quad (5)$$

with $k_B$ the Boltzmann constant, $T$ the temperature of the system and $V$ the volume of the cell.

We are interested in studying the evolution of the out-of-plane component of the magnetization in materials with dominant perpendicular magnetic anisotropy, i.e. the $z$ component of the magnetization in a two-dimensional system. We therefore consider exchange interactions with stiffness $A$, effective easy-axis anisotropy with strength $K$, and Zeeman coupling to an out-of-plane

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig1}
\caption{Spatial distribution of the out-of-plane magnetization $m_z$ for a two-dimensional system with quenched Gaussian Voronoi disorder ($\epsilon = 0.19$ and $\ell = 30$ nm, see text) at $T = 300$ K before and after applying an out-of-plane magnetic-field pulse of (a) $\mu_0 H_z = 0.45$ T during $\delta t = 12$ ns, and (b) $\mu_0 H_z = 0.007$ T during $\delta t = 10$ $\mu$s. While the white strip corresponds to the initial relaxed domain with $m_z = +1$, gray represents its growth after the pulse. Black stands for $m_z = -1$.}
\end{figure}
magnetic field of intensity $H_z$. The free energy of the system is then given by
\[
\mathcal{H} = A \int |\nabla \mathbf{m}(\mathbf{r})|^2 \, d\mathbf{r} - K \int [\hat{e}_z \cdot \mathbf{m}(\mathbf{r})]^2 \, d\mathbf{r} - \mu_0 H_z M_S \int \hat{e}_z \cdot \mathbf{m}(\mathbf{r}) \, d\mathbf{r},
\]
(6)
where $\mathbf{m} = \mathbf{M}/M_S = m_x \hat{e}_x + m_y \hat{e}_y + m_z \hat{e}_z$ (with the norm constraint $|\mathbf{m}| = 1$). Now, following the presentation of Eq. (2) to calculate the effective field, Eq. (1) for the $z$ component of the magnetization can be written
\[
\frac{1 + \eta^2}{\gamma \mu_0 \eta_0} \partial_t m_z = a \nabla^2 m_z + (k m_z + H_z) (1 - m_z^2) + f_x (m_y/\eta_0 - m_x m_z) + f_y (m_x/\eta_0 - m_y m_z) + f_z (1 - m_z^2) + N_1 + N_2,
\]
(7)
where for simplicity we have defined $a = 2A/M_S$, $k = 2K/M_S$ (known as the anisotropy field), and
\[
N_1 = -a/\eta_0 \left( m_x \nabla^2 m_y - m_y \nabla^2 m_x \right) \quad \text{(8)}
\]
\[
N_2 = a m_z \left[ (\nabla m_x)^2 + (\nabla m_y)^2 + (\nabla m_z)^2 \right] \quad \text{(9)}
\]
The full micromagnetic description is comprised of the other two coupled equation for $m_x$ and $m_y$, analogous to Eq. (7), and the norm constraint. Our aim here is to obtain an effective description in terms of $m_z$ using one single evolution equation. Therefore, in order to uncouple Eq. (7) from the ones for the other components of $\mathbf{m}$, we propose to parameterize $\mathbf{m}_{xy} = m_x \hat{e}_x + m_y \hat{e}_y$ as a function of $m_z \equiv m_z(x,y)$. Following Fig. 2(a), we achieve this by locally fixing the direction of the in-plane magnetization and writing
\[
\mathbf{m}_{xy} = \sqrt{1 - m_z^2} \left[ \cos \theta \frac{\nabla m_z}{|\nabla m_z|} + \sin \theta \frac{\nabla \times (m_z \hat{e}_z)}{|\nabla \times (m_z \hat{e}_z)|} \right].
\]
(10)
In this way the in-plane part of $\mathbf{m}$ is decomposed in one component in the direction of the gradient of $m_z$ and one component orthogonal to it. Note that fixing the value of $\theta$ reduces the number of degrees of freedom so that only one evolution equation—indeed, that for $m_z$—is needed to describe the magnetic properties of the system. The angle $\theta$ is a constant that generally relates $\mathbf{m}_{xy}$ to $\nabla m_z$ in each cell. Indeed, although we used a curved domain wall for the diagram in Fig. 2(a), the proposed parametrization is actually independent of the existence of a domain wall in the system: any finite $\nabla m_z$ implies a finite $\mathbf{m}_{xy}$. In the case where a wall is present, the angle between its normal and the in-plane magnetization in the center of the wall (where $m_z = 0$) is equal to $\theta$. In this way, choosing the angle for the parametrization implies deciding a priori if the wall will be Bloch ($\theta = \pi/2, 3\pi/2$; see Fig. 2(b)), Néel ($\theta = 0, \pi$; see Fig. 2(c)) or anything in between, and its chirality.

Now that Eq. (10) provides us with a tool to write the in-plane components of $\mathbf{m}$ as functions of $m_z$, we assess the implications of the terms in Eqs. (8) and (9) by considering a system with a flat domain wall, i.e. $m_z \equiv m_z(x)$ with $m_z(x \to \pm\infty) = \pm 1$. In this simple case, the in-plane components can be written as $m_x = \cos \theta \sqrt{1 - m_z^2} \text{sgn}(\partial_x m_z)$ and $m_y = -\tan \theta m_x$, so Eqs. (8) and (9) reduce to
\[
N_1 = 0 \quad \text{(11)}
\]
\[
N_2 = a m_z \frac{(\partial_x m_z)^2}{1 - m_z^2} \quad \text{(12)}
\]
Notice that both quantities are independent of the choice of $\theta$. While the first term vanishes exactly, the second is relevant within the wall and can be related to the domain wall width. The effect of $N_2$ can be evaluated using Eq. (7) at zero temperature and imposing $m_z(x \to \pm\infty) = \pm 1$. In this case the system evolves to a single domain wall configuration with
\[
m_z(x) = \tanh \left( \frac{x - x_0}{\Delta} \right), \quad \text{(13)}
\]
where $x_0$ and $\Delta$ are the wall position and width parameter, respectively (the width itself is given by $\pi \Delta^{36}$). By simulating a system with values $K$ and $A$ for the anisotropy and stiffness constants, we obtain a width parameter consistent with the micromagnetic definition $\Delta_0 = \sqrt{A/K}$. Also, in this model, the surface tension of the wall is given by $4\sqrt{AK}$. Therefore, in this simple
case we recover the domain wall profile well known from micromagnetic theory.

Instead, if the $N_2$ term is set to zero the domain wall profile can still be described by Eq. (13) but with a larger domain wall width parameter $\sqrt{2A/K}$ and a smaller domain wall surface tension $(4/3)\sqrt{2AK}$. This can be recast by exploiting the relation of the present model with the Ginzburg-Landau model. If we associate $m_z \leftrightarrow \phi$ and ignore $N_2$ (as previously stated, $N_1$ vanishes), Eq. (7) has in fact the shape of the Langevin equation for the modified Ginzburg-Landau scalar-field model:

$$\langle 1/\Gamma \rangle \partial_t \phi = \beta \nabla^2 \phi + (\alpha \phi + h) \left(1 - \phi^2\right) + \xi,$$

where $\Gamma$ is a damping parameter, $\beta$ is a rigidity constant, $\alpha$ is the energy barrier in a two-well potential, and $h$ is an external field. This model—which includes temperature through an additive white noise $\xi$ instead of our multiplicative thermal-vector components $f_i$—yields domain walls with width parameter $\sqrt{2\beta/\alpha}$ and surface tension $(4/3)\sqrt{2\beta K}$. Therefore, the effect of the $N_2$ term is to effectively narrow the domain wall width of the Ginzburg-Landau model so as to obtain the domain wall width of the proper micromagnetic model. Explicitly including $N_2$ in Eq. (7) can then be avoided if two new parameters $K_{eff}$ and $A_{eff}$ are conveniently defined in order to provide the micromagnetic domain wall width parameter and surface tension in a scalar-field simulation at zero temperature. In other words, using these effective quantities will allow us not to compute the $N_2$ term, which can have a high computational cost. The effective anisotropy and stiffness constants can be found by simply solving the system of equations:

$$\begin{align*}
\sqrt{2A_{eff}/K_{eff}} &= \sqrt{A/K} \\
\frac{4}{3} \sqrt{2A_{eff}K_{eff}} &= 4\sqrt{AK}
\end{align*}$$

and thus obtaining $K_{eff} = 3K$ and $A_{eff} = 3A/2$. Taking this into account, Eq. (7) becomes

$$\frac{1 + \eta_0^2}{\gamma \mu_0 \eta_0} \partial_t m_z = a_{eff} \nabla^2 m_z + (k_{eff} m_z + H_z) \left(1 - m_z^2\right)$$

$$+ f_x \left(m_y(m_z)/\eta_0 - m_x(m_z) m_z\right)$$

$$+ f_y \left(m_z(m_z)/\eta_0 - m_y(m_z) m_z\right)$$

$$+ f_z \left(1 - m_z^2\right),$$

with $a_{eff} = 2A_{eff}/M_S$ and $k_{eff} = 2K_{eff}/M_S$. This evolution equation constitutes the Ginzburg-Landau micromagnetic model. It represents a considerable simplification of the micromagnetic model since it is written in terms of one single component of the magnetization, and has the shape of the well-known modified Ginzburg-Landau scalar-field model for phase transitions. It should be noticed that, since our model does not include in-plane interactions of any kind (e.g., in-plane magnetic field, Dzyaloshinskii-Moriya interactions or dipolar coupling), the magnetic moments have no preference for any particular value of the parametrization angle $\theta$. Indeed, this angle appears only in the temperature terms—which are basically random numbers—yielding identical results for both Néel and Bloch walls. In the following section, numerical simulations of this model are compared with experimental results measured by Metaxas and collaborators.22

### III. DOMAIN WALL DYNAMICS

We now compare velocity-field curves calculated with the proposed model and those obtained by PMOKE experiments. We solve Eq. (17) through a semi-implicit Euler scheme,28,39 and perform simulations following the typical PMOKE experimental protocol (see for example Refs. 9, 22, and 28). The $L \times L \times s$ system of side $L$ and thickness $s$ is initialized with a narrow $m_z = +1$ stripe (white regions in Figs. 1(a) and 1(b)) surrounded by $m_z = -1$, and is allowed to relax in the presence of temperature and quenched disorder. Then, a positive field $H_z$ is applied, favoring the growth of the +1 domain until a certain maximum area of the growing domain is reached. At that point, the field is removed and the system is again allowed to relax. We calculate the domain wall velocity as

$$v = \frac{1}{2L} \frac{\delta a}{\delta t},$$

with $\delta a$ the difference in area between the final and initial relaxed configurations (gray regions in Figs. 1(a) and 1(b)) and $\delta t$ the duration of the field pulse.

The material parameters are taken to be those of Pt(4.5 nm)/Co(0.5 nm)/Pt(3.5 nm) ultra-thin films at $T = 300 \text{K}$, as reported by Metaxas et al.:22 $A = 1.4 \times 10^{-11} \text{J/m}$, $K = 3.2 \times 10^5 \text{J/m}^2$ and $M_S = 9.1 \times 10^5 \text{A/m}$. We work on a system of side $L = 8.192 \mu\text{m}$ and thickness $s = 0.5 \text{nm}$, with simulation cells of volume $V = l \times l \times s$ with $l = 2 \text{nm}$. A simulation of a system without structural disorder allows us to determine a damping constant $\eta_0 = 0.255$ (Fig. 3(a)), i.e., within 6% difference from the experimental value.22 This quantity is compatible with the precessional flow suggested in Ref. 22 and recently confirmed in Ref. 40. It is surprising that our model with fixed $\theta$ is able to reproduce this regime. The reason may be that, since no in-plane contributions to the free energy are being considered, the Walker field is $H_W = 0$.36 In this limit where $H_z \gg H_W$ for any finite field, the model can be thought to effectively be in the precessional regime even though the angle between the domain wall and $\mathbf{m}_{xy}$ is a constant. This fact is quite reasonable given that—as explained at the end of Sec. II—Eq. (17) does not depend on $\theta$.

Afterwards, quenched disorder is included by means of a Voronoi tesselation with mean grain size $\ell$. The anisotropy is modified as $k_{eff} \to k_{eff}[1 + \epsilon \zeta(\mathbf{r})]$, where
\(\epsilon\) is the disorder intensity and \(\zeta(\mathbf{r})\) has a constant value for each grain obtained from a Gaussian distribution with zero mean and unit variance. \(^{28}\) To determine the disorder parameters of the system, we simulate diverse values of \(\epsilon\) and \(\ell\) (Fig. 4) for a field in the creep regime \((\mu_0 H_z = 0.01 \text{T})\). The resulting velocities are compared with the corresponding experimental value, and the best-fitting set of parameters is found. The best agreement with the experimental velocity is obtained with \(\epsilon = 0.19\) and \(\ell = 30 \text{ nm}\), at a temperature \(T = 300 \text{K}\). The full line is a fit of the experimental data in the linear region of \(\ln(v)\) vs. \((\mu_0 H_z)^{-1/4}\), while the dotted line shows the position of the depinning field \(\mu_0 H_z = 0.023 \text{T}\) at the creep plot.

Results presented in Fig. 3(b) validate the use of the Ginzburg-Landau micromagnetic model to obtain domain wall velocities. The model is capable of quantitatively reproducing experimental data in the three regimes: creep, depinning and flow. Quite remarkably, although the simulations using the Ginzburg-Landau micromagnetic model are material-dependent, universal features like the creep exponent \(\mu\) are clearly recovered. In spite of being a simplified version of the SLLG equations, our model is the first to quantitatively reproduce these experimental results. Indeed, a previous attempt to fit the same data can be found in Ref. 40, but full micromagnetic simulations were performed at \(T = 0\). \(^{41}\) In Ref. 42, on its turn, finite-temperature numerical simulations of a micromagnetic model were used to study the velocity-field response in a related family of materials (Pt/Co/Au/Pt\(_1-x\)). Although no direct comparison with the creep regime was made, results present good agreement in the depinning and flow regimes. It should be noticed, however, that our proposed model allows us to simulate a system more than two orders of magnitude bigger, and can be potentially used to study large-scale phenomena.

Finally, we discuss how fast the flow regime is reached in the Ginzburg-Landau micromagnetic model. It has been shown in numerical simulations using the elastic line model \(^{43}\) and the scalar-field approach \(^{28}\) that the crossover from the depinning regime to the flow regime is rather slow, as compared to the experimental case. \(^{18}\) The slow approach to the flow regime is also present in our numerical model, as shown in Fig. 5(a). As predicted in Ref. 44, the relative difference \(\delta v/v_0 = (v_0 - v)/v_0\) between the simulated velocity \(v\) and the expected value in the flow regime \(v_0 = m_f \mu_0 H_z\), where \(m_f\) is the mobility, vanishes asymptotically as \(1/(\mu_0 H_z)^2\) (Fig. 5(b)). Therefore, all these models seem to share the slow approach to the flow regime, thus still missing some ingredient of the experimental counterpart.

IV. DOMAIN WALL WIDTH

While it allows us to simulate systems comparable to experiments, our Ginzburg-Landau micromagnetic model provides at the same time access to small length scales beyond PMOKE capabilities. In this work we shall focus on the internal structure of the domain wall, which
can be seen in Fig. 6(a). To characterize it, the system is sliced in paths parallel to the $x$ axis, and the $m_z$ profiles are fitted with Eq. (13) to determine the position $x_0(y)$ and width parameter $\Delta(y)$ for each constant $y$ (Fig. 6(b)). The position $u(y)$ of the full domain wall is then given by the succession $\{x_0(y)\}$. The mean value of the width parameter measured along the $x$ axis is presented in Fig. 7(a) as a function of the out-of-plane magnetic field. It appears to have a slight dependence on the field, increasing within the creep regime.

There are many ways in which the domain wall width can be defined. Our previous approach, although widely used for modeling the wall as an elastic line $u(y)$, has the drawback of not taking into account its local tilting $\psi$, which can be significant in presence of temperature and structural disorder. This can explain, for example, the greater value of the width parameter at low fields as a consequence of the increased roughness of the wall (see Figs. 1(a) and 1(b), corresponding to fields in the flow and creep regimes, respectively). Another possibility is, then, to measure the width normal to the wall at each point. As shown in Fig. 6(c), $\Delta_n(y)$ for a given point $x_0(y)$ can be calculated from the width parameter measured along the $x$ axis as

$$\Delta_n(y) = \Delta(y) \sin \left( \arctan \left( \frac{2l}{x_0(y) - x_1(y)} \right) \right), \quad (19)$$

where we have defined $x_{-1} = x_0(y-l)$ and $x_1 = x_0(y+l)$ as the positions of the previous and next points in the wall, and $2l$ is their (constant) separation along the $y$ axis. Figure 7(a) shows the mean value of the normal-width parameter as a function of the field. Now, $\langle \Delta_n \rangle \approx \Delta_0 = \sqrt{A/K}$ in all the field range. Usually, the difference between the width along a given direction and normal to the wall is not taken into account when modeling domain walls as elastic lines in disordered media. Therefore, the results presented here are relevant for a proper derivation of an equation for the evolution of $u(y,t)$ from a model for $m_z(x,y,t)$.

Finally, it is worth mentioning that important fluctuations of the domain wall width parameter are present, as observed in Fig. 6(a). From the values of $\Delta(y)$ and $\Delta_n(y)$ we present in Figs. 7(b) and 7(c) the normalized histograms corresponding to two values of the field in the creep and flow regimes. Two ingredients can in principle be responsible for the width and asymmetry of the distributions: quenched disorder and temperature. Indeed, a $T = 0$, $\epsilon = 0$ simulation yields a planar domain wall with a probability distribution for both $\Delta$ and $\Delta_n$ consistent with a Dirac delta centered in $\Delta_0$. As mentioned in Sec. III, disorder is introduced as a Voronoi tesselation where each grain of mean size $\ell$ has an anisotropy constant given by a Gaussian distribution centered in $k_{eff}$ with variance $\epsilon^2$. Consequently, when $L \gg \ell \gg \Delta_0$ as in our case, finite values of $\epsilon$ imply a probability distribution.
that implied fixing the internal angle of the domain wall. This resulted in the Ginzburg-Landau micromagnetic model, which is a simplification of the full SLLG micromagnetic model and is closely related to a previous effective scalar-field approach. The presented model allowed us to make real-scale simulations of Pt/Co/Pt ultra-thin films. We obtained velocity-field curves that quantitatively reproduce experimental data in the three regimes (creep, depinning and flow), finding a good correspondence with experimental values for a field range greater than that of previous numerical results.

At the same time, we used our model to study phenomena at length scales out of experimental reach. We showed that care needs to be taken when the domain wall width is defined, in order to consider contributions of the local tilting. In particular, the theoretical value for the domain wall width parameter $\Delta_0 = \sqrt{\frac{A}{K}}$ was only recovered in mean when the width was measured perpendicularly to the wall. This fact is particularly relevant when deriving a one-dimensional model for the wall from a two-dimensional one for the magnetization. We also studied the distribution of the domain wall width parameter, which we found to be very wide and asymmetrical mostly due to thermal effects.

The presented Ginzburg-Landau micromagnetic model fills in the gap between models of the Ginzburg-Landau type, inspired in statistical mechanics, and the SLLG equation. The cost in the approximation is that the internal degree of freedom of the domain wall, parameterized by the angle $\theta$ in Eq. (10), is fixed. Despite that, the results properly account for the velocity-field characteristics in the precessional regime. In addition, the model allows to reach length and time scales comparable to PMOKE experiments. This model then represents a powerful numerical approach to domain wall dynamics in magnetic systems and related problems.

V. CONCLUSIONS

We proposed a local parametrization of the in-plane magnetization in terms of the out-of-plane component that implied fixing the internal angle of the domain wall. This resulted in the Ginzburg-Landau micromagnetic model, which is a simplification of the full SLLG micromagnetic model and is closely related to a previous effective scalar-field approach. The presented model allowed us to make real-scale simulations of Pt/Co/Pt ultra-thin films. We obtained velocity-field curves that quantitatively reproduce experimental data in the three regimes (creep, depinning and flow), finding a good correspondence with experimental values for a field range greater than that of previous numerical results.

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The presented Ginzburg-Landau micromagnetic model fills in the gap between models of the Ginzburg-Landau type, inspired in statistical mechanics, and the SLLG equation. The cost in the approximation is that the internal degree of freedom of the domain wall, parameterized by the angle $\theta$ in Eq. (10), is fixed. Despite that, the results properly account for the velocity-field characteristics in the precessional regime. In addition, the model allows to reach length and time scales comparable to PMOKE experiments. This model then represents a powerful numerical approach to domain wall dynamics in magnetic systems and related problems.
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A distribution \( P(k'_{\text{eff}}) \) for the anisotropy random variable \( k'_{\text{eff}} = k_{\text{eff}} (1 + \epsilon \zeta) \) defines a distribution for the normal-width parameter \( P(\Delta_n) = |d k'_{\text{eff}}/d \Delta_n| P(k'_{\text{eff}}) \) via the relation \( k'_{\text{eff}} = \frac{2 a_{\text{eff}}}{\Delta_n^2} \).