Bilateral Control System of Nonlinear Flexible Master-Slave Arms with Random Delay Affected by Contact Force with Obstacle

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Abstract
This paper investigates a networked bilateral control system affected by a contact force. The proposed system consists of rigid master and nonlinear flexible slave arms, and the network which causes a random delay connects both arms. The slave arm collides with an obstacle during its motion. An extended Kalman filter is designed to reduce the effect of the random delay. Numerical simulations are demonstrated to confirm the performance of the proposed system.

1 Introduction
IoT (Internet of Things) technology is widely introduced in human life. In the near future, anything and everything will be connected with network and human will have inextricable connection with network. If robots are introduced in general environments such as houses, offices and so on, there are also connected with network. In this case, teleoperation technologies are very important to send human skills in distant places and these technologies should be realized with existing networks such as local area networks (LAN), wide area networks (WAN), and wireless LANs to easily introduce. In this research, a bilateral control system with a network is discussed. The proposed system consists of a master arm and a slave arm and both arms are connected by a network. Signals of both arms interacted with each other become noisy because the network causes random delay. Thus, it is considered that the proposed system becomes instability due to the network.

Many researchers have studied teleoperation technologies and arms. Matsuno et al. derived a proportional derivative and strain (PDS) controller for a flexible beam [1]. Hoshino et al. studied a bilateral system which consists of a flexible master-slave manipulator [2]. Namerikawa suggested a control strategy for bilateral systems with networks which causes time-varying delay [3]. However, random delay was not considered in these studies.

A lot of researchers have studied networked control systems with random delay. Liu et al. proposed a model predictive control for networked control systems [4]. Guo et al. discussed a discrete-time system with random delay to prove the exponential stability [5]. Schenato studied a design of optimal estimators with random delay [6]. However, the random delay in these studies was not modeled by a stochastic model (e.g., Gaussian noise).

In our previous research, a networked bilateral control system was considered. The performance of the proposed system with time-varying delay was demonstrated and the stability and passivity were proved [7]. The proposed system with random delay was also studied and a Kalman filter was designed to reduce the effect of random delay [8]. Furthermore, when the slave arm of the proposed system collided with an obstacle during its motion, it was confirmed that the proposed system did not become instability [9]. However, in [9], the flexible slave arm was modeled as the linearized model.

A bilateral control system of nonlinear flexible master-slave arms with random delay affected by a contact force is discussed in this research. The proposed system consists of the rigid master arm and the flexible slave arm. Mathematical model of both arms are derived by using Hamilton’s principle. The flexible arm is modeled as a nonlinear system and its model includes a contact force. The random delay is defined as a sum of an average delay and a Gaussian noise. An extended Kalman filter is designed to estimate states. The PD controller and the PDS controller are designed to generate the reaction torque for the master arm and the reference signal for the slave arm, respectively. Numerical simulations are accomplished to confirm the performance of the proposed system.
2 Mathematical Models

A rigid master arm and a flexible slave arm construct the proposed bilateral control system in this study. The slave arm is driven by a high-geared servomotor and a communication network connects with these arms. Moreover, the network causes the random delay. In this section, the mathematical models of these factors are derived.

2.1 Rigid Master Arm

The rigid master arm shown in Fig. 2 consists of an uniform rigid rod and a hub. This arm is driven by a DC motor. $O_mX_mY_m$ is the inertial Cartesian coordinate system. $\ell_m$ and $m_m$ are the length and mass. A human operator added the operating force $F_h(t)$ to the end of the master arm. An operating torque $\tau_h(t)$ is generated by this force, i.e. $\tau_h(t) := F_h(t)\ell_m$. $\hat{\theta}_m(t)$ is the rotational angle; $\tau_m(t)$ is the reaction torque; $J_h$ and $J_m$ are the moments of inertia of the motor’s rotor and the master arm; and $\mu_m$ is the coefficient of friction of the motor shaft. The mathematical model of the rigid arm is represented as

$$ (J_h + J_m + \frac{1}{4}m_m\ell_m^2)\ddot{\theta}_m(t) + \mu_m\dot{\theta}_m(t) = \tau_h(t) + \tau_m(t) + g_m\gamma_m(t), \quad (1) $$

where $\gamma_m(t)$ is the system noise which is defined as a Gaussian noise process. $g_m$ is the coefficient of the system noise.

2.2 High-geared Servomotor

In this research, a high-geared servomotor drives the flexible slave arm. A high-geared DC motor and a PD-controller construct the servomotor and Fig. 3 depicts the block diagram of this servomotor. As seen in this figure, $\Theta_{com}(s)$ and $\Theta(s)$ are a reference angle and an output angle. These angles are the Laplace transform of $\theta_{com}(t)$ and $\theta(t)$, i.e. $\Theta_{com}(s) = \mathcal{L}[\theta_{com}(t)]$ and $\Theta(s) = \mathcal{L}[\theta(t)]$.

The control torque added to the flexible arm $\tau(t)$ can be derived by considering the mathematical model of the DC motor and applying the inverse Laplace transform \[10\]. Thus, $\tau(t)$ is expressed as

$$ \tau(t) = \frac{k_P}{R} \{ \theta_{com}(t) - \dot{\theta}(t) \} + \frac{k_D}{R} \{ \ddot{\theta}_{com}(t) - \ddot{\theta}(t) \} - k_v \dot{\theta}(t), \quad (2) $$
where $K_P$ and $K_D$ denote the proportional and derivative gains; $k_e$ and $k_r$ denote the back electromotive force constant and the torque constant; and $R$ is the internal resistance of the coil in the DC motor.

### 2.3 Flexible Slave Arm

The slave arm employs the parallel-structured single-link flexible arm (shown in Fig. 4). As seen in this figure, the flexible arm consists of a pair of uniform Euler-Bernoulli beams. An one end of each beams clamps a hub unit and the other end of each beams clamps a tip-mass. In this research, the parallel-structured single-link flexible arm is simplified as the single-link flexible arm which is depicted by Fig. 5 because the displacement of both beams can be considered equal and the centrifugal force is assumed to be sufficiently small [11]. The simplified flexible arm consists of an uniform Euler-Bernoulli beam and both ends are fixed on the unit hub and the tip-mass. Moreover, the boundary conditions of the simplified flexible arm is the same conditions as the original flexible arm.

$OXY$ denotes the inertial Cartesian coordinate system and $Oxyz$ denotes the rotating coordinate system. $l$ is the length; $\theta(t)$ is the rotational angle; and $u(t, x)$ is the transverse displacement of the flexible beam from the $x-$axis. The physical parameters are as follows: $S$ is the cross-sectional area; $\rho$ is the uniform mass density; $EI$ is the uniform flexible rigidity ($E$ and $I$ are Young’s modulus and the second moment of the cross-sectional area); $J_0$ is the inertial moment of the motor shaft; $c_D$ and $\mu$ are the coefficient of Kelvin-Voight-type damping and friction of the motor shaft; and $m$ is the mass of the tip-mass which is assumed as a point mass.

In this research, the flexible arm contact with a rigid obstacle at $x = x_0$ ($0 < x_0 < l$) and this obstacle is fixed at $x_0$. Because the obstacle is fixed, the energy of the flexible arm does not dissipate due to the collision. This interaction between the flexible arm and the obstacle does not considered. As seen in Fig. 5, $\phi_0$ is the rotational angle between the $OX-$axis and the line from $O$ to the collision point. The geometric constraint for the flexible arm and the obstacle defined as

$$\psi(t) = u(t, x_0) - x_0 \tan(\phi_0 - \theta(t)) = 0.$$  

(3)

The total kinetic energy $T(t)$ and the potential energy $U(t)$ are calculated to derive the mathematical model of the flexible arm.

$$T(t) = T_h(t) + T_b(t) + T_{im}(t)$$  

(4)

$$U(t) = \int_0^l \frac{1}{2} EI \{u''(t, x)\}^2 dx,$$  

$$T_h(t) = \int_0^l \frac{1}{2} J_0 \dot{\theta}(t)^2 dt$$  

(5)

$$T_b(t) = \int_0^l \frac{1}{2} \rho S \left[ \left\{ \dot{\theta}(t) + \dot{u}(t, x) \right\}^2 + u^2(t, x) \dot{\theta}(t)^2 \right] dx$$  

(6)

$$T_{im}(t) = \frac{1}{2} m \left\{ \left\{ \dot{\theta}(t) + \dot{u}(t) \right\}^2 + \left\{ \ddot{u}(t) \dot{\theta}(t) \right\}^2 \right\},$$  

(7)

where $\ddot{u}(t)$ is expressed as $u(t, \ell)$. $T_h(t), T_b(t)$ and $T_{im}(t)$ are the energies of the rotation of the unit-hub, the translation of the flexible beam, and the translation of the tip-mass, respectively. $U(t)$ is the bending strain energy of the flexible beam. The prime represented the derivative with respect to $x$, i.e., $\{\}' = \partial / \partial x$.

Hamilton’s principle is used to derive the mathematical model. Then, the following equation is obtained.

$$\int_{t_1}^{t_2} \left[ \delta T(t) - \delta U(t) + \delta W(t) + s \delta \psi(t) \right] dt = 0,$$  

(8)

where $\delta W(t)$ denotes the virtual work [8]. $s$ is the Lagrange multiplier and is considered a force which is given by colliding with the obstacle. Hence, this term is defined as the contact force $s(t)$. Finally, the mathematical model of the flexible arm including the contact force is expressed as

$$J_0 \ddot{\theta}(t) + \mu \dot{\theta}(t) + m \dddot{u}(t) \left\{ 2\dddot{u}(t) \dot{\theta}(t) + \dddot{u}(t) \dddot{\theta}(t) \right\}$$  

$$- c_D I \{u''(t, 0) - \dddot{u}(t, t)\} - EI \{u''(t, 0) - u''(t, t)\}$$  

$$+ \int_0^l \rho S \left\{ 2u(t, x)\ddot{u}(t, x)\dot{\theta}(t) + u^2(t, x)\dot{\theta}(t) \right\} dx$$

- 3 -
Therefore, the random delay is defined as
\[ \tau(t) = s(t) + \int_0^t g_s \gamma_s(t, x) \, dx \]
(9)
where \( \gamma_s(t, x) \) denotes the system noise and \( g_s \) denotes the coefficient of \( \gamma_s(t, x) \). This system noise is defined as a white Gaussian noise.

### 3 Modeling of Communication Network

The communication network connected with the master and slave arms, and this network causes the random delay. In this research, the sum of an average time delay \( T \) and a random value \( \gamma_0(t) \) approximates as the random delay. Thus, the random delay is expressed as \( \tau + \gamma_0(t) \) and it does not become a negative value. Therefore, the random delay is defined as
\[
\gamma_0(t) = \begin{cases} 
\gamma(t) & \text{for } \gamma_0(t) > -T \\
0 & \text{otherwise,} 
\end{cases}
\]
(11)
where \( \gamma(t) \) is the white Gaussian process. Its mean is \( \mathcal{E}\{\gamma(t)\} = 0 \) and its covariance is \( \mathcal{E}\{\gamma^2(t)\} = : R \). Assuming that the covariance of \( \gamma(t) \) behaves as \( R \ll T^2 \), \( \gamma_0(t) \) can be regarded as the white Gaussian process, i.e., \( \gamma(t) \cong \gamma_0(t) \). Finally, the random delay is defined as the sum of the average time delay and the white Gaussian process.

### 4 Synthesis of Extended Kalman Filter

A state estimator should be designed to reduce the effect of the random delay because interacted signals become noisy due to the random delay. In this paper, an extended Kalman filter is employed as a state estimator since the slave arm is the nonlinear system. The linearized state space model of the flexible arm should be derived to design this state estimator. To do this aim, the Taylor series expansion form is applied to the mathematical model of the flexible arm [8].

An ordinary time-invariant linear stochastic system with the random delay is considered to be the extended Kalman filter. The considered stochastic system is represented as
\[
\begin{align*}
\dot{x}(t) &= A x(t) dt + B u(t) dt + G dw_s(t) \\
\dot{y}(t) &= C x(t) dt - (T + \gamma(t)) dt
\end{align*}
\]
(12)
where \( x(t) \in \mathbb{R}^n \) and \( y(t) \in \mathbb{R}^p \) denote the state vector and the observation vector. \( u(t) \in \mathbb{R}^q \) denotes a control input. \( dw_s(t) \) denotes an increment of Wiener process with zero-mean and its covariance \( Q_s := \mathcal{E}\{dw_s^2(t)\} \). Moreover, \( dw_s(t) \) is defined as \( \dot{\gamma}_s(t) dt \). \( \gamma_s(t) \) is the system noise which is defined as a white Gaussian noise.

The Taylor series expansion form is applied to the state vector \( x(t - (T + \gamma(t))) \) around \( t_s := t - T \) in Eq. (13). The resultant function is substituted into Eq. (12). The observation system is written as
\[
\dot{y}(t) = C x(t) dt - C \left( A x(t) + B u(t) \right) dw_s(t),
\]
(14)
where \( dw_s(t) := \gamma(t) dt \) denotes an increment of Wiener process with zero-mean and its covariance \( R := \mathcal{E}\{dw_s^2(t)\} \). The multiplicative noise defined as the product of the state \( x(t_s) \) and the random variable \( \gamma(t) \) appears in Eq. (14).

The correlation function of the innovation process \( dv(t) \) is calculated and the orthogonal projection is applied. Finally, the state estimate \( \hat{x}(t_s) \) can be obtained [12]. The estimation error covariance \( P(t_s|t_s) \) and the autocorrelation function of the state \( \Pi(t_s) \) are given by the Riccati differential equation and the Lyapunov differential equation. Furthermore, the control input \( u(t_s) \) cannot be obtained. Eventually, these equations are expressed as
\[
\begin{align*}
\dot{x}(t_s|t_s) &= A \hat{x}(t_s|t_s) dt + B u(t - (T + \gamma(t))) dt \\
&\quad + P(t_s|t_s) C^T \left[ C \left\{ A \Pi(t_s) A^T + B Q_s B^T \right\} R C^T \right]^{-1} \{ dy(t) - C \hat{x}(t_s|t_s) dt \} \\
\dot{P}(t_s|t_s) &= A P(t_s|t_s) + P(t_s|t_s) A^T + Q_s \left[ C \left\{ A \Pi(t_s) A^T + B Q_s B^T \right\} R C^T \right]^{-1} C P(t_s|t_s) \\
&\quad - P(t_s|t_s) C^T \left[ C \left\{ A \Pi(t_s) A^T + B Q_s B^T \right\} R C^T \right]^{-1} C P(t_s|t_s) \\
\Pi(t_s) &= A \Pi(t_s) + A \Pi(t_s) A^T + Q_s \left[ C \left\{ A \Pi(t_s) A^T + B Q_s B^T \right\} R C^T \right]^{-1} C P(t_s|t_s)
\end{align*}
\]
(15-17)

### 5 Synthesis of Controllers

In our previous research, the PD and PDS controllers are employed to generate the reaction torque for the master arm and the reference angle for the slave arm [8]. These controllers are also employed in this research and represented as follow:
\[
\tau_m(t) = K_{P_m} \left( \theta_m(t_s) - \theta_m(t) \right) + K_{D_m} \left( \dot{\theta}_m(t_s) - \dot{\theta}_m(t) \right)
\]
(18)
\[
f(t) = \frac{R}{k_r K_{D_s}} \left[ K_{P_s} \left( \dot{\theta}_m(t_s) - \theta_m(t) \right) + K_{D_s} \left( \dot{\theta}_m(t_s) - \theta(t) \right) \\
+ K_S \left[ c_D I \left( \ddot{u}(t, 0) - \ddot{u}(t, \ell) \right) \right] \\
+ E I \left( \ddot{u}(t, 0) - \ddot{u}(t, \ell) \right) \right] \\
- \frac{k_r K_{P_s}}{R} \left( \theta_{com}(t) - \theta(t) \right)
\]
(19)
where $K_{Pm}$ and $K_{Dm}$ are the proportional and derivative gains for the master arm: $K_{Pc}$ and $K_{Dc}$ are the proportional and derivative gains for the slave arm: and $K_S$ is the strain gain. The control input for the slave arm $f(t)$ is defined as $\dot{\theta}_{com}(t)$. However, the input signal for the high-geared servomotor should be the angle. Thus, the reference angle $\theta_{com}(t)$ can be calculated by integrating $f(t)$.

### 6 Numerical Simulation

The numerical simulation is used to confirm the performance of the proposed bilateral control system. In this simulation, It is confirmed that the flexible slave arm tracks the master arm, the extended Kalman filter reduces the effect of the random delay and the flexible slave arm does not become instability due to the collision. Matlab and Simulink are employed to demonstrate the numerical simulation.

#### 6.1 Setup

The material of the flexible arm is phosphor bronze and its thickness and width are $1.0 \times 10^{-3}$[m] and $4.0 \times 10^{-2}$[m], respectively. The physical parameters of both arms and the high-geared servomotor are listed in Table 1 to Table 3.

The initial values are $\theta_m(0) = 0[\text{rad}], \dot{\theta}_m(0) = 0[\text{rad/s}], \theta(0) = 0[\text{rad}], \dot{\theta}(0) = 0[\text{rad/s}], u(0, x) = 0[\text{m}], \text{and } \dot{u}(0, x) = 0[\text{m/s}].$ The collision position $x_c$ and angle $\phi_0$ are set as $0.20[\text{m}]$ and $0.25[\text{rad}]$, respectively. The average time delay $T$ is set as $0.25[\text{s}].$

#### 6.2 Results

The numerical simulation results are shown in Fig. 6. Fig. 6(a) depicts the angles where the solid line is the angle of the master arm $\theta_m(t)$ and the dashed line is the estimated angle of the slave arm $\hat{\theta}(t_s[t_s])$. Fig. 6(b) depicts the tip displacement $u(t, t)$. Fig. 6(c) depicts the torques where the solid line is the reaction torque $\tau_m(t)$ and the dashed line is the torque due to the collision force $x_c s(t)$. Fig. 6(d) depicts the random delay.

As seen in Fig. 6(a) and (b), the motion of the flexible arm is constrained by the collision force and the estimated angle of the slave arm tracks the angle of the master arm well. Moreover, the tip displacement has the large value during the collision with the obstacle. However, it quickly converges to zero due to the PDS controller. On the other hand, as seen in Fig. 6(c), the contact force affects the reaction torque and the reaction torque has the large value because the differences of the state between the master arm and the slave arm are large at the starting operation. The operator may feel uncomfortable since each torques are affected by each system noise.

### 7 Conclusions

A bilateral control system for nonlinear flexible master-slave arms with random delay was discussed in this paper. The flexible slave arm collided with an obstacle during its motion. The mathematical model of both arms were derived by using Hamilton’s principle and the model of the flexible arm included the collision force. The model of the high-geared servomotor which driven the slave arm was derived based on its construction. The random delay was defined as the sum of the average delay and the white Gaussian noise. To design the extended Kalman filter, the Taylor series expansion form was applied to the observation system including the random delay. The PD and PDS controllers were designed to generate the reaction torque for the master arm and the reference signal for the slave arm.

The performance of the proposed system were confirmed by using numerical simulations. The Hertz contact model was used as the contact force from the obstacle. As seen in the simulation results, the effect of the random delay was reduced by the extended Kalman filter, and the reaction torque and the reference signal

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Table 1: Physical parameters of the rigid master arm.

| Symbol | Value |
|--------|-------|
| $J_h$  | 0.70  |
| $\mu_m$ | $3.03 \times 10^{-2}$ |
| $m_m$  | $29.05 \times 10^{-3}$ |
| $\ell_m$ | 0.30 |
| $J_m$  | $6.54 \times 10^{-4}$ |

Table 2: Physical parameters of the flexible slave arm.

| Symbol | Value |
|--------|-------|
| $J_0$  | 0.70  |
| $\mu$  | $3.03 \times 10^{-2}$ |
| $\ell$  | 0.30  |
| $\rho$  | $8.8 \times 10^3$ |
| $S$    | $4.0 \times 10^{-5}$ |
| $E$    | $1.1 \times 10^{11}$ |
| $I$    | $8.33 \times 10^{-13}$ |
| $c_D$  | $1.93 \times 10^9$ |
| $m$    | 0.245 |

Table 3: Physical parameters of the high-geared servomotor.

| Symbol | Value |
|--------|-------|
| $K_P$  | 32 |
| $K_D$  | $32 \text{[V/(rad/s)]}$ |
| $k_c$  | 2.6 |
| $k_r$  | 2.6 |
| $R$    | 1.73 |

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were generate well by the PD and PDS controllers. Fin-
ally, the flexible slave arm of the proposed bilateral
control system tracked the estimated angle of the rigid
master arm in spite of the contact force.

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