COUPLING NONLINEAR $\sigma$-MODELS TO RELAXED YANG-MILLS SUPERMULTIPLETS IN (2,0)-SUPERSPACE

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Abstract

Following a previous work on Abelian (2,0)-gauge theories, one reassesses here the task of coupling (2,0) relaxed Yang-Mills superpotentials to a (2,0)-nonlinear $\sigma$-model, by gauging the isotropy or the isometry group of the latter. One pays special attention to the extra “chiral-like” component-field gauge potential that comes out from the relaxation of superspace constraints.

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In a previous paper \[1\], one has investigated the dynamics and the couplings of a pair of Abelian vector potentials of a class of (2,0)-gauge super multiplets(2]-[8]) whose symmetry lies on a single U(1) group. Since a number of interesting features came out, it was a natural question to ask how these fields would behave if the non-Abelian version of the theory was to be considered.

We can see that some subtle changes indeed occur. As we wish to make a full comparison between the two aspects (Abelian and non-Abelian) of the same sort of theory, all the general set up of the original formulation was kept.

The fundamental non-Abelian matter superfields are the scalar and left-handed spinor superfields, both subject to the chirality constraint; their respective component-field expressions are given by:

\[
\begin{align*}
\Phi_i(x; \theta, \bar{\theta}) &= e^{i\theta\bar{\theta}\partial_{++}}(\phi_i \lambda^i), \\
\Psi^I(x; \theta, \bar{\theta}) &= e^{i\theta\bar{\theta}\partial_{++}}(\psi^I \sigma^I),
\end{align*}
\]

(1)

The fields \(\phi^i\) and \(\sigma^I\) are scalars, whereas \(\lambda^i\) and \(\psi^I\) stand respectively for right- and left-handed Weyl spinors. The indices \(i\) and \(I\) label the representations where the correspondent matter fields are set to transform under the Yang-Mills group.

We present below the gauge transformations of both \(\Phi\) and \(\Psi\), assuming that we are dealing with a compact and simple gauge group, \(G\), with generators \(G_a\) that fulfill the algebra \([G_a, G_b] = if_{abc}G_c\). The transformations read as below:

\[
\Phi^i = R(\Lambda)^i_j\Phi^j, \quad \Psi^I = S(\Lambda)^I_J\Psi^J,
\]

(2)
where $R$ and $S$ are matrices that respectively represent a gauge group element in the representations under which $\Phi$ and $\Psi$ transform. Taking into account the chiral constraints on $\Phi$ and $\Psi$, and bearing in mind the exponential representation for $R$ and $S$ in terms of the group generators, we find that the gauge parameter superfields, $\Lambda^a$, must satisfy the same sort of constraint, namely, they are chiralscalar superfields.

The kinetic action for $\Phi^i$ and $\Psi^I$ can be made invariant under the local transformations (2) by minimally coupling gauge potential superfields, $\Gamma_{a-}(x;\theta,\bar{\theta})$ and $V^a(x;\theta,\bar{\theta})$, according to the minimal coupling prescriptions:

$$S_{inv} = \int d^2x d\theta d\bar{\theta}\{i[\bar{\Phi}e^{hV}(\nabla_{-}\Phi) - (\nabla_{-}\bar{\Phi})e^{hV}\Phi] + \bar{\Psi}e^{hV}\Psi\}, \quad (3)$$

as it has already been done in ref. [1].

The infinitesimal gauge transformations for $V^a$ and $\Gamma_{a-}$ are given by

$$\delta V^a = \frac{i}{\hbar}(\bar{\Lambda} - \Lambda)^a - \frac{1}{2}f^{abc}(\bar{\Lambda} + \Lambda)_bV_c \quad (4)$$

and

$$\delta \Gamma_{a-} = -f^{abc}\Lambda_b\Gamma_{c-} + \frac{1}{g}\partial_{-}\Lambda^a. \quad (5)$$

No derivative acts on the $\Lambda^a$'s in eq.(4), which suggests the possibility of choosing a Wess-Zumino gauge for $V^a$. If such a choice is adopted and if the superfield $V$ is kept the same as in ref. [1], the same identifications done in the Abelian case hold but once the Wess-Zumino gauge is left behind, the superfield-strength is lost. In order to make it possible to write down a superfield-strength in any gauge chosen, it is necessary to redefine our superfield $V$ as follows:

$$V^a(x, \theta, \bar{\theta}) = C^a + \theta\xi - \bar{\theta}\bar{\xi} + \theta\bar{\theta}\hat{v}_{++}, \quad (6)$$

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where the $\hat{v}_{++}$ is given by

$$\hat{v}_{++} = v_{++}^a + \frac{ig}{2} f^{abc} \xi_b \bar{\xi}_c$$

(7)

is the real gauge field of the theory. The $\theta$-expansion for $\Gamma^a_{--}$ is the same as in ref. [1] and reads:

$$\Gamma^a_{--}(x; \theta, \bar{\theta}) = \frac{1}{2} (A^a_{--} + iB^a_{--}) + i\theta(\rho^a + i\eta^a)$$

$$+ i\bar{\theta}(\chi^a + i\omega^a) + \frac{1}{2} \bar{\theta} \bar{\theta}(M^a + iN^a).$$

(8)

$A^a_{--}, B^a_{--}$ and $v^a_{++}$ are the light-cone components of the gauge potential fields; $\rho^a, \eta^a, \chi^a$ and $\omega^a$ are left-handed Majorana spinors; $M^a, N^a$ and $C^a$ are real scalars and $\xi^a$ is a complex right-handed spinor.

The gauge transformations for the $\theta$-component fields read as below:

$$\delta C^a = -f^{a}_{bc}(\Re \alpha^b)C^c,$$

$$\delta \xi^a = -\frac{i}{g} \beta - f^{a}_{bc}(\Re \alpha^b)\xi^c - \frac{1}{2} f^{a}_{bc}\beta^b C^c,$$

$$\delta v_{++}^a = \frac{2}{g} \partial_{++} \Re \alpha^a - f^{a}_{bc}(\Re \alpha^b) v_{++}^c,$$

$$\delta A^a_{--} = \frac{2}{g} \partial_{--} (\Re \alpha^a) - f^{a}_{bc}(\Re \alpha^b) A^c_{--},$$

$$\delta B^a_{--} = -f^{a}_{bc}(\Re \alpha^b) B^c_{--},$$

$$\delta \eta^a = -f^{a}_{bc}(\Re \alpha^b) \eta^c + \frac{1}{2} f^{a}_{bc}(\Re \beta^b) A^c_{--}$$

$$- \frac{1}{2} f^{a}_{bc}(\Im \beta^b) B^c_{--} - \frac{1}{g} \partial_{--} \Re \beta^a,$$

$$\delta \rho^a = -f^{a}_{bc}(\Re \alpha^b) \rho^c - \frac{1}{2} f^{a}_{bc}(\Re \beta^b) B^c_{--},$$

$$\delta M^a = -f^{a}_{bc}(\Re \alpha^b) M^c + f^{a}_{bc}(\partial_{++} (\Re \alpha^b) B^c_{--} - 2f^{a}_{bc}(\Re \beta^b) \omega^c$$

$$\delta N^a = \frac{2}{g} \partial_{++} \partial_{--} \Re \alpha^a - f^{a}_{bc}(\Re \alpha^b) N^c - f^{a}_{bc}(\partial_{++} \Re \alpha^b) A^c_{--}$$
+ 2f_{bc}^{a} (\Re \beta^{b}) \chi^{c} - 2f_{bc}^{a} (\Re \beta^{b}) \omega^{c} - 2f_{bc}^{a} (\Im m \beta^{b}) \omega^{c} \\
\delta \chi^{a} = -f_{bc}^{a} \alpha^{b} \chi^{c}, \\
\delta \omega^{a} = -f_{bc}^{a} \alpha^{b} \omega^{c} 
\tag{9}.

The gauge variations suggest that the $v_{++}^{a}$-component could be identified as the light-cone partner of $A_{--}^{a},$

$$v_{++}^{a} \equiv A_{++}^{a}.$$  \tag{10}.

This procedure yields two component-field gauge potentials: $A^{\mu} \equiv (A^{0}, A^{1}) = (A^{++}; A^{--})$ and $B_{--};$ the latter without the $B_{++}$ partner just as it happened in the Abelian case.

To discuss the field-strength superfields, we start analysing the algebra of the gauge covariant derivatives.

After doing so we find out that the gauge field, $A_{\mu},$ has its field strength, $F_{\mu \nu},$ located at the $\theta$-component of the combination $\Omega \equiv W_{-} + \bar{U}_{-},$ where

$$[\nabla_{+}, \nabla_{--}] \equiv W_{+} = -igD_{+} \Gamma_{--} - \partial_{++} \Gamma_{+} - ig[\Gamma_{+}, \Gamma_{--}],$$

$$[\bar{\nabla}_{+}, \nabla_{--}] \equiv U_{-} = -ig\bar{D}_{+} \Gamma_{--}.$$ \tag{11}.

This suggests the following kinetic action for the Yang-Mills sector:

$$S_{YM} = \frac{1}{8g^{2}} \int d^{2}x d\theta d\bar{\theta} Tr \Omega \bar{\Omega}$$

$$= \int d^{2}x Tr \left[\frac{-1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{i}{8} \Sigma \bar{\partial}_{++} \Sigma + \frac{1}{8} M^{2} \right.$$ 

$$+ \frac{1}{8} (\partial_{++} \partial_{--} C)^{2} + \frac{1}{4} (\partial_{++} \partial_{--} C) M \right] + interactions, \tag{12}.$$ 

where $\Sigma = \rho + i\eta + \bar{\chi} - i\bar{\omega}$ and $A \bar{\partial} B \equiv (\partial A)B - A(\partial B).$
Choosing now a supersymmetry-covariant gauge-fixing, instead of the Wess-Zumino, we propose the following gauge-fixing term in superspace:

\[
S_{gf} = -\frac{1}{2\alpha} \int d^2 x \theta d\bar{\theta} Tr[\Pi \Pi]
\]  \hspace{1cm} (13)

where \( \Pi = -iD_+ \Gamma_{--} + \frac{1}{2} D_+ \partial_{--} V \). With this, the gauge-fixing Lagrangian became

\[
S_{gf} = -\frac{1}{2\alpha} \int d^2 x \{ ((\partial_\mu A^\mu)^2 + (\partial_\mu A^\mu)N + \frac{1}{4} N^2] + \frac{1}{4} [M^2 - 2M(\partial_{++} B_{--}) + (\partial_{++} B_{--})^2 + (\partial_{++} \partial_{--} C)^2
+ 2M(\partial_{++} \partial_{--} C)(\partial_{++} B_{--}) - 2M(\partial_{++} \partial_{--} C)]
- i(\rho + i\eta) \partial_{++} (\bar{\rho} - i\bar{\eta}) \} + \text{interactions.}  \hspace{1cm} (14)
\]

So, the total action reads as follows:

\[
S = \int d^2 x Tr\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\alpha} (\partial_\mu A^\mu)^2 - \frac{1}{2\alpha} (\partial_\mu A^\mu)N - \frac{1}{8\alpha} N^2 + \frac{1}{8} (1 - \frac{1}{\alpha}) M^2
+ \frac{1}{4\alpha} M(\partial_{++} B_{--}) - \frac{1}{8\alpha} (\partial_{++} B_{--})^2 + \frac{1}{8} \left(1 - \frac{1}{\alpha}\right)(\partial_{++} \partial_{--} C)^2
+ \frac{1}{4} \left(1 + \frac{1}{\alpha}\right) (\partial_{++} \partial_{--} C) M - \frac{1}{4\alpha} (\partial_{++} \partial_{--} C)(\partial_{++} B_{--})
- \frac{i}{2\alpha} (\rho + i\eta) \partial_{++} (\bar{\rho} - i\bar{\eta}) + \frac{i}{8} \Sigma \partial_{++} \Sigma \}
\]  \hspace{1cm} (15)

Using eq. (15), we are ready to write down the propagators for \( A^a, B^a_{--}, C^a, N^a, M^a, \rho^a, \eta^a, \chi^a \) and \( \omega^a \):

\[
\langle AA \rangle = -\frac{2i}{\Box} (\partial_{\mu\nu} + 2\alpha \omega_{\mu\nu}),
\]

\[
\langle AN \rangle = -\langle NA \rangle = -4i\alpha \partial_\mu,
\]

\[
\langle NN \rangle = 8i\alpha
\]
\[ \langle CC \rangle = \frac{32i\alpha}{3\alpha^2 + 4\alpha + 4 \Box^2}, \]
\[ \langle BC \rangle = -\langle CB \rangle = -\frac{32i\alpha(3\alpha + 2) \partial_+}{3\alpha^2 + 4\alpha + 4 \Box^2} \]
\[ \langle MC \rangle = \langle CM \rangle = -\frac{32i\alpha (\alpha + 1)}{3\alpha^2 + 4\alpha + 4 \Box} \]
\[ \langle BB \rangle = -\frac{16i\alpha(\alpha + 2)(3\alpha + 10) \partial_+^2}{3\alpha^2 + 4\alpha + 4 \Box^2}, \]
\[ \langle MM \rangle = \frac{8i\alpha (\alpha + 4)}{3\alpha^2 + 4\alpha + 4} \]
\[ \langle MB \rangle = -\langle BM \rangle = -\frac{48i\alpha (\alpha + 2) \partial_+}{3\alpha^2 + 4\alpha + 4 \Box} \]
\[ \langle (\rho + i\eta)(\bar{\rho} - i\bar{\eta}) \rangle = -4\alpha \frac{\partial_{++}}{\Box} \]
\[ \langle (\rho + i\eta)(\chi + i\omega) \rangle = 4\alpha \frac{\partial_{++}}{\Box} \]
\[ \langle (\bar{\chi} - i\bar{\omega})(\bar{\rho} - i\bar{\eta}) \rangle = 4\alpha \frac{\partial_{++}}{\Box} \]
\[ \langle (\bar{\chi} - i\bar{\omega})(\chi + i\omega) \rangle = 4(\alpha + 4) \frac{\partial_{++}}{\Box}. \]  

(16)

One immediately checks that the extra gauge field, \( B_{--} \), does not decouple from the matter sector. In the non-Abelian case, the extra gauge potential \( B_{--} \) also behaves as a second gauge field, exactly as it did in the Abelian case. It is very interesting to point out that, in the Abelian case, \( B_{--} \) showed the same behaviour as here: a massless pole of order two \([\Box]\); the difference is that there, it coupled only to \( C \) instead of \( C \) and \( M \), but these two fields show the same kind of behaviour: they are both compensating fields. Once again, this ensures us to state that \( B_{--} \) behaves as a physical gauge field: it has dynamics and couples both to matter and the gauge field \( A^\mu \). Its only remaining peculiarity regards the presence of a single component
in the light-cone coordinates.

Let us now turn to the coupling of the two gauge potentials, \( A_\mu \) and \( B_{-\cdot} \), to a non-linear \( \sigma \)-model always keeping a supersymmetric scenario. It is our main purpose henceforth to carry out the coupling of a \((2,0)\) \( \sigma \)-model to the relaxed gauge superfields of the ref. \cite{7}, and show that the extra vector degrees of freedom do not decouple from the matter fields (that is, the target space coordinates)\cite{9}-\cite{12}. To perform the coupling of the \( \sigma \)-model to the Yang-Mills fields we reason along the same considerations as i ref. \cite{1} and find out that:

\[
L_\xi = \partial_i[K(\Phi, \bar{\Phi}) - \xi(\Phi) - \tilde{\xi}(\bar{\Phi})] \nabla_{-\cdot} \Phi^i + \\
- \tilde{\partial}_i[K(\Phi, \bar{\Phi}) - \xi(\Phi) - \tilde{\xi}(\bar{\Phi})] \nabla_{-\cdot} \bar{\Phi}^i,
\]

where \( \xi(\Phi) \) and \( \tilde{\xi}(\bar{\Phi}) \) are a pair of chiral and antichiral superfields, \( \Phi_i \equiv \exp(iL_{V;k})\bar{\Phi}_i \) and \( \nabla_{-\cdot} \Phi^i \) and \( \nabla_{-\cdot} \bar{\Phi}^i \) are defined in perfect analogy to what is done in the case of the bosonic \( \sigma \)-model:

\[
\nabla_{-\cdot} \Phi_i \equiv \partial_{-\cdot} \Phi_i - g\Gamma^\alpha_{-\cdot}k^i_\alpha(\Phi)
\]

and

\[
\nabla_{-\cdot} \bar{\Phi}_i \equiv \partial_{-\cdot} \bar{\Phi}_i - g\Gamma^\alpha_{-\cdot}\bar{k}^i_\alpha(\bar{\Phi}).
\]

The interesting point we would like to stress is that the extra gauge degrees of freedom accommodated in the component-field \( B_{-\cdot}(x) \) of the superconnection \( \Gamma_{-\cdot} \) behave as a genuine gauge field that shares with \( A^\mu \) the feature of coupling to matter and to \( \sigma \)-model \cite{7}. This result can be explicitly read off from the component-field Lagrangian projected out from the
superfield Lagrangian $\mathcal{L}_\xi$. We therefore conclude that our less constrained $(2,0)$-gauge theory yields a pair of gauge potentials that naturally transform under the action of a single compact and simple gauge group and may be consistently coupled to matter fields as well as to the $(2,0)$ non-linear $\sigma$-models by means of the gauging of their isotropy and isometry groups.

Relaxing constraints in the $N = 1$- and $N = 2 - D = 3$ supersymmetric algebra of covariant derivatives may lead to a number of peculiar features of gauged nonlinear $\sigma$-model\[13\] in the presence of Born-Infeld terms for the pair of gauge potentials that show up from the relaxation of the superspace constraints\[14\].

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