New Superparticle Models Outside the HLS Supersymmetry Scheme

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Abstract

We consider the superparticle models invariant under the supersymmetries with tensorial central charges, which were not included in \(D = 4\) Haag-Lopuszanski-Sohnius (HLS) supersymmetry scheme.

We present firstly a generalization of \(D = 4\) Ferber-Shirafuji (FS) model with fundamental bosonic spinors and tensorial central charge coordinates. The model contains four fermionic coordinates and possesses three \(\kappa\)-symmetries thus providing the BPS configuration preserving \(3/4\) of the target space supersymmetries. We show that the physical degrees of freedom (8 real bosonic and 1 real Grassmann variable) of our model can be described by \(OSp(8|1)\) supertwistor. Then we propose a higher dimensional generalization of our model with one real fundamental bosonic spinor. \(D = 10\) model describes massless superparticle with composite tensorial central charges and in \(D = 11\) we obtain \(0\)-superbrane model with nonvanishing mass which is generated dynamically. The introduction of \(D = 11\) Lorentz harmonics provides the possibility to construct massless \(D = 11\) superparticle model which can be formulated in a way preserving \(1/2, 17/32, 18/32, \ldots, 31/32\) supersymmetries. In a special case we obtain the twistor-like formulation of the usual massless \(D = 11\) superparticle proposed recently by Bergshoeff and Townsend.

\(^\ast\) To be published in the Proceedings of 12-th Max Born Symposium: Theoretical Physics - Fin de Siecle, held 24-27.09.1998 in Wroclaw (Poland), ed. by A. Borowiec, B. Jancewicz, W. Karwowski and W. Cegla, Springer Verlag.

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1 Introduction

It is our great pleasure to contribute this article to the volume dedicated to Professor Jan Lopuszanski on his 75-th birthday. He is one of the founders of algebraic background for present supersymmetric theories. In seventies, when in 1975 he published fundamental paper with Haag and Sohnius (see [1]) it was however assumed that the relativistic superalgebra should contain in its bosonic sector a direct sum of space-time symmetry generators (Poincaré, de-Sitter, conformal) and internal symmetry generators, i.e. the space-time bosonic generators and internal bosonic generators should commute. As a consequence the internal Abelian generators, called also central charges, had to be scalar. Recently however this conclusion has been relaxed, and in present algebraic framework of SUSY appear generalized central charges - tensorial [2]–[6] or even spinorial [7, 8] ones. The best example can be provided by D=11 supersymmetry algebra, containing topological contributions from M2 and M5 superbranes:

\[
\{Q_\alpha, Q_\beta\} = P_m \Gamma^m_{\alpha\beta} + Z_{m_1 m_2} \Gamma^{m_1 m_2} + Z_{m_1...m_5} \Gamma^{m_1...m_5}.
\] (1.1)

In this lecture we shall consider the new superparticle models, invariant under SUSY with tensor charge generators. We shall formulate such a model following the ideas of supertwistor formulation by Ferber and Shirafuji [10, 11]. In Sect 2 we shall consider the D=4 model which is invariant under the following D=4 SUSY algebra

\[
\{Q_A, Q_B\} = Z_{AB}, \quad \{\bar{Q}_\dot{A}, \bar{Q}_\dot{B}\} = \bar{Z}_{\dot{A}\dot{B}},
\] (1.2)

where \((Q_A)^* = \bar{Q}_\dot{A}\), \((P_{AB})^* = P_{BA}\), \((Z_{AB})^* = \bar{Z}_{\dot{A}\dot{B}}\) and six real commuting central charges \(Z_{\mu\nu} = -Z_{\nu\mu}\) are related to the symmetric complex spin-tensor \(Z_{AB}\) by

\[
Z_{\mu\nu} = \frac{i}{2} \left( \bar{Z}_{\dot{A}\dot{B}} \sigma^{\dot{A}\dot{B}}_{\mu\nu} - Z_{AB} \sigma^{AB}_{\mu\nu} \right).
\] (1.3)

Thus the spin-tensors \(Z_{AB}\) and \(\bar{Z}_{\dot{A}\dot{B}}\)

\[
Z_{AB} = \frac{i}{4} Z_{\mu\nu} \sigma^{\mu\nu}_{AB}, \quad \bar{Z}_{\dot{A}\dot{B}} = -\frac{i}{4} Z_{\mu\nu} \sigma^{\mu\nu}_{\dot{A}\dot{B}}
\]

represent the self-dual and anti-self-dual parts of the central charge matrices. It should be stressed that the superalgebra (1.2-3) goes outside of the HLS scheme.

The D = 4 model considered in Section 2 can be reformulated in terms of two Weyl spinors \(\lambda_A, \mu_A\) and one real Grassmann variable \(\zeta\) expressed by the generalization of supersymmetric Penrose–Ferber relations [15, 16, 17] between supertwistor and superspace coordinates. Such reformulation is described by \(Osp(8|1)\) invariant free supertwistor model with the action

\[
S = -\frac{1}{2} \int d\tau Y_A G^{AB} \dot{Y}_B
\] (1.4)

\footnote{For two-component \(D = 4\) Weyl spinor formalism see e.g. [1]. We have \((\sigma_{\mu\nu})^B_A = \frac{1}{2\tau} \left( (\sigma_\mu)_{AB} \sigma^{BB}_\nu - (\sigma_\nu)_{AB} \sigma^{BB}_\mu \right) = -\frac{i}{2\tau} \epsilon_{\mu\nu\rho\lambda} (\sigma^{\rho\lambda})^B_A = [(\sigma_{\mu\nu})^B_A]^*).}
where \( Y_A = (y_1, \ldots, y_8; \zeta) \equiv (\lambda_\alpha, \mu^\alpha, \zeta) \) is the real \( SO(8|1) \) supertwistor (see e.g. \cite{14}) and
\[
G^{AB} = \begin{pmatrix} \omega^{(8)} & 0 \\ 0 & 2i \end{pmatrix} = \begin{pmatrix} 0_2 & I_2 & 0_2 \\ -I_2 & 0_2 & 0_2 \\ 0_2 & 0_2 & -I_2 \\ 0_2 & 0_2 & 0_2 \end{pmatrix}
\]
is the \( OSp(8|1) \) supersymplectic structure with bosonic \( Sp(8) \) symplectic metric \( \omega^{(8)} = -(\omega^{(8)})^T \). It should be mentioned therefore that due to the presence of tensorial central charges the standard \( SU(2,2|1) \) supertwistor description \cite{10, 11, 15, 16, 17, 18} of the Brink–Schwarz (BS) massless superparticle \cite{19} with one complex Grassmann coordinate is replaced by a model with \( OSp(8|1) \) invariance and one real Grassmann degree of freedom.

It should be stressed that by the use of spinor coordinates in the presence of tensorial central charges

- we do not increase the initial number of spinor degrees of freedom (four complex or eight real components) in comparison with the model without tensorial central charges;
- we keep the manifest Lorentz invariance despite the presence of tensorial central charges.

In fact, when we use our formulae (see Section 3)
\[
P_{AB} = \lambda_A \bar{\lambda}_B, \quad Z_{AB} = \lambda_A \lambda_B, \quad \bar{Z}_{\dot{A}\dot{B}} = \bar{\lambda}_{\dot{A}} \bar{\lambda}_{\dot{B}}
\]
we find that, in comparison with standard FS model \( P_{\dot{A}\dot{B}} = \lambda_{\dot{A}} \bar{\lambda}_{\dot{B}}, \ Z_{AB} = \bar{Z}_{\dot{A}\dot{B}} = 0 \), only the phase of spinor \( \lambda_A \) becomes an additional physical bosonic degree of freedom.

In Section 3 we shall consider the D=10 and D=11 models described by multidimensional extensions of FS model with one fundamental spinor coordinates. The D=11 model is invariant under the superalgebra \cite{14}. It appears that D=10 model is massless (due to the famous Fierz identities for D=10 gamma matrices) and D=11 is generally a massive one with a mass generated dynamically. In Section 4 we shall consider the large family of D=11 massless models with particular fundamental spinor coordinates described by Lorentz harmonics.

We would like to add that the results presented in Sections 2 and 3 can also be found in our recent article \cite{28}, but all the results from Section 4 are new.

## 2 Generalization of Ferber–Shirafuji superparticle model: spinor fundamental variables and central charges

We generalize the model presented in \cite{14} as follows
\[
S = \int d\tau \left( \lambda_A \bar{\lambda}_B \Pi_{\tau}^{AB} + \lambda_A \lambda_B \Pi_{\tau}^{AB} + \bar{\lambda}_{\dot{A}} \bar{\lambda}_{\dot{B}} \Pi_{\tau}^{AB} \right),
\]
where
\[
\begin{align*}
\Pi^{AB} & \equiv d\tau \pi^{AB} = dX^{AB} + i \left( d\Theta^A \bar{\Theta}^B - \Theta^A d\bar{\Theta}^B \right), \\
\Pi_{AB} & \equiv d\tau \pi_{AB} = d\zeta^{AB} - i \Theta^A d\Theta^B), \\
\Pi^{\hat{A}\hat{B}} & \equiv d\tau \pi^{\hat{A}\hat{B}} = d\zeta^{\hat{A}\hat{B}} - i \bar{\Theta}^{(\hat{A})} d\Theta^{(\hat{B})},
\end{align*}
\] (2.2)
are the supercovariant one–forms in \( D = 4, N = 1 \) generalized flat superspace
\[
M^{(4+6|4)} = \{ Y^{M} \} \equiv \{ (X^{A\hat{A}}, z^{AB}, \zeta^{\hat{A}\hat{B}}, \Theta^A, \bar{\Theta}^A) \},
\] (2.3)
with tensorial central charge coordinates \( z^{mn} = (z^{AB}, \zeta^{\hat{A}\hat{B}}) \) (see (1.3)). The complete configuration space of the model (2.1) contains additionally the complex-conjugate pair \((\lambda_A, \bar{\lambda}_A)\) of Weyl spinors
\[
\mathcal{M}^{(4+6+4|4)} = \{ q^{M} \} \equiv \{ (Y^{M}; \lambda^A, \bar{\lambda}^{\hat{A}}) \} \equiv \{ (X^{A\hat{A}}, z^{AB}, \zeta^{\hat{A}\hat{B}}, \lambda^A, \bar{\lambda}^{\hat{A}}; \Theta^A, \bar{\Theta}^A) \},
\] (2.4)
Calculating the canonical momenta
\[
\mathcal{P}_{\mathcal{M}} = \frac{\partial L}{\partial \dot{q}^{M}} = (P_{A\hat{A}}, Z_{AB}, \bar{Z}_{\hat{A}\hat{B}}, P^A, \bar{P}^{\hat{A}}, \pi^A, \bar{\pi}^{\hat{A}}),
\] (2.5)
we obtain the following set of the primary constraints
\[
\Phi_{AB} \equiv P_{AB} - \lambda_A \bar{\lambda}_B = 0,
\] (2.6)
\[
\Phi_{\hat{A}\hat{B}} \equiv Z_{\hat{A}\hat{B}} - \lambda_A \lambda_B = 0,
\] (2.7)
\[
\Phi_{A\hat{B}} \equiv \bar{Z}_{A\hat{B}} - \bar{\lambda}_A \lambda_B = 0,
\] (2.8)
\[
P_A = 0, \quad \bar{P}_{\hat{A}} = 0,
\] (2.9)
\[
D_A \equiv -\pi_A + i P_{A\hat{B}} \bar{\Theta}^B + i Z_{AB} \Theta^B = 0,
\] (2.10)
\[
\bar{D}_{\hat{A}} \equiv \bar{\pi}_{\hat{A}} - i \Theta^B P_{B\hat{A}} - i \bar{Z}_{A\hat{B}} \Theta^B = 0.
\] (2.11)
Because the action (2.1) is invariant under the world line reparametrization, the canonical Hamiltonian vanishes
\[
H \equiv \dot{q}^{M} \mathcal{P}_{\mathcal{M}} - L(q^{M}, \dot{q}^{M}) = 0
\] (2.12)
It can be deduced that the set (2.6)-(2.11) of 14 bosonic and 4 fermionic constraints contains 6 bosonic and 3 fermionic first class constraints
\[
B_1 = \lambda^A \bar{\lambda}^{\hat{B}} P_{AB} = 0,
\] (2.13)
\[
B_2 = \lambda^A \hat{\mu}^{\hat{B}} P_{A\hat{B}} - \lambda^A \hat{\mu}^{\hat{B}} Z_{AB} = 0,
\] (2.14)
\[
B_3 \equiv (B_2)^* = \hat{\mu}^{\hat{A}} \lambda^B P_{\hat{A}B} - \hat{\mu}^{\hat{A}} \lambda^B \bar{Z}_{\hat{A}\hat{B}} = 0,
\] (2.15)
\[
B_4 = 2 \hat{\mu}^{\hat{A}} \hat{\mu}^{\hat{B}} P_{A\hat{B}} - \hat{\mu}^{\hat{A}} \hat{\mu}^{\hat{B}} Z_{AB} - \hat{\mu}^{\hat{A}} \hat{\mu}^{\hat{B}} \bar{Z}_{A\hat{B}} = 0,
\] (2.16)
\[
B_5 = \lambda^A \bar{\lambda}^{\hat{B}} Z_{AB} = 0,
\] (2.17)
\[
B_6 \equiv (B_5)^* = \bar{\lambda}^{\hat{A}} \lambda^B \bar{Z}_{A\hat{B}} = 0,
\] (2.18)
\[ F_1 = \lambda^A D_A = 0, \]  
\[ F_2 \equiv (F_1)^* = \bar{\lambda}^\dot{A} \bar{D}_{\dot{A}} = 0, \]  
\[ F_3 = \hat{\mu}^A D_A + \hat{\bar{\mu}}^\dot{A} \bar{D}_{\dot{A}} = 0, \]  
where we assume that \( \lambda^A \mu_A \neq 0 \) and

\[ \hat{\mu}^A = \frac{\mu^A}{\lambda^B \mu_B}, \quad \hat{\bar{\mu}}^\dot{A} = \frac{\bar{\mu}^\dot{A}}{\bar{\lambda}^\dot{B} \bar{\mu}_B}, \]  
i.e. \( \lambda^A \hat{\mu}_A = \bar{\lambda}^\dot{A} \hat{\bar{\mu}} = 1 \). One can show \( ^2 \) that our first class constraints (2.13) - (2.21)
can be chosen for any particular form of the second spinor \( \mu_A \) as a function of canonical
variables \((q^M, p_M)\). Further we shall propose and motivate the choice for \( \mu^A, \bar{\mu}^\dot{A} \).

The remaining 8 bosonic and 1 fermionic constraints are the second class ones. They are

\[ \lambda^A \hat{\mu}^B P_{AB} + \lambda^A \mu^B Z_{AB} = 0, \quad \hat{\mu}^A \bar{\lambda}^\dot{B} P_{AB} + \bar{\lambda}^\dot{A} \bar{\mu}^\dot{B} \bar{Z}_{AB} = 0, \]  
\[ \hat{\mu}^A \mu^B Z_{AB} - 1 = 0, \quad \hat{\bar{\mu}}^\dot{A} \bar{\mu}^\dot{B} \bar{Z}_{AB} - 1 = 0, \]  
\[ P_A = 0, \quad \bar{P}_{\dot{A}} = 0, \]  
\[ S_F \equiv \hat{\mu}^A D_A - \hat{\bar{\mu}}^\dot{A} \bar{D}_{\dot{A}} = 0, \]  

We see that the number \( \# \) of on-shell phase space degrees of freedom in our model is

\[ \# = (28_B + 8_F) - 2 \times (6_B + 3_F) - (8_B + 1_F) = 8_B + 1_F \]  

in distinction with the standard massless superparticle model of Brink–Schwarz \([19]\) or Ferber-Shirafuji \([10, 11]\) containing \( 6_B + 2_F \) physical degrees of freedom.

In order to explain the difference in the number of fermionic constraints, let us write
down the matrices of Poisson brackets for the fermionic constraints (2.10), (2.11). In our
case it has the form

\[
C_{\alpha\beta} = \begin{pmatrix}
\{ D_A, D_B \}_P & \{ D_A, \bar{D}_{\dot{B}} \}_P \\
\{ \bar{D}_{\dot{A}}, D_B \}_P & \{ \bar{D}_{\dot{A}}, \bar{D}_{\dot{B}} \}_P
\end{pmatrix} = \begin{pmatrix}
\lambda_A \lambda_B & \lambda_A \bar{\lambda}_{\dot{B}} \\
\bar{\lambda}_A \lambda_B & \bar{\lambda}_A \bar{\lambda}_{\dot{B}}
\end{pmatrix}
\]  

while for the standard FS model \([10, 11]\) we obtain

\[
C^{FS}_{\alpha\beta} = \begin{pmatrix}
0 & \lambda_A \bar{\lambda}_{\dot{B}} \\
\bar{\lambda}_A \lambda_B & 0
\end{pmatrix}
\]  

Now it is evident that in our case the rank of the matrix \( C \) is one, while for FS model it
is equal to two

\[ \text{rank}(C) = 1, \quad \text{rank}(C^{FS}) = 2. \]  

Consequently, in our model there are three fermionic first class constraints generating
three \( \kappa \)–symmetries \([12]\), one more than in the FS model.

\[ ^2 \text{ We recall} [22] \text{ that the first class constraints are defined as those whose Poisson brackets with all}
\text{constraints weakly vanish. Then one can show} [22] \text{ that the first class constraints form the closed algebra.} \]
In order to clarify the meaning of the superparticle model (2.1) and present an explicit representation for its physical degrees of freedom, we shall demonstrate that it admits the supertwistor representation in terms of independent bosonic spinor $\lambda^A$, bosonic spinor $\mu^A$ being composed of $\lambda^A$ and superspace variables

$$\mu^A = (X^{AB} + i\theta^A\bar{\theta}^B)\bar{\lambda}_B + 2z^{AB}\lambda^B + i\theta_A(\Theta^B\lambda_B),$$  

(2.30)

$$\bar{\mu}^\dot{A} = (X^{\dot{A}B} - i\theta^B\bar{\theta}^A)\lambda_B + 2z^{\dot{A}B}\bar{\lambda}_B - i\bar{\theta}_A\bar{\Theta}^B\bar{\lambda}_B,$$  

(2.31)

and one real fermionic composite Grassmann variable

$$\zeta = \Theta^A\lambda_A + \bar{\Theta}^\dot{A}\bar{\lambda}_\dot{A},$$  

(2.32)

Eqs. (2.30) - (2.32) describe $OSp(8|1)$–supersymmetric generalization of the Penrose correspondence which is alternative to the previously known $SU(2,2|1)$ correspondence, firstly proposed by Ferber [10]. Performing integration by parts and neglecting boundary terms we can express our action (2.1) in terms of $OSp(8|1)$ supertwistor variables as follows:

$$S = -\int \left(\mu^A d\lambda_A + \bar{\mu}^\dot{A} d\bar{\lambda}_\dot{A} + id\zeta \zeta\right).$$  

(2.33)

Eq. (2.33) presents the free $OSp(8|1)$ supertwistor action. It can be rewritten in the form (1.4) with real coordinates $Y^A = (\mu^\alpha, \lambda^\alpha, \zeta)$ where real Majorana spinors $\mu^\alpha, \lambda^\alpha$ are obtained from the Weyl spinors $(\mu^A, \bar{\mu}^\dot{A}), (\lambda^A, \bar{\lambda}_\dot{A})$ by a linear transformation changing for the $D = 4$ Dirac matrices the complex Weyl to real Majorana representation.

The action (2.33) produces only the second class constraints

$$P^{(\lambda)}_A - \mu_A = 0, \quad P^{(\mu)}_A = 0,$$  

(2.34)

$$\bar{P}^{(\lambda)}_A - \bar{\mu}_A = 0, \quad \bar{P}^{(\mu)}_A = 0,$$  

(2.35)

$$\pi^{(\zeta)} = i\zeta,$$  

(2.36)

The Dirac brackets for the $OSp(8|1)$ supertwistor coordinates are

$$[\mu_A, \lambda^B]_D = \delta^B_A, \quad [\bar{\mu}_A, \bar{\lambda}^\dot{B}]_D = \delta^\dot{B}_A,$$  

(2.37)

$$\{\zeta, \zeta\}_D = -\frac{i}{2}.$$  

(2.38)

They can be also obtained after the analysis of the Hamiltonian system described by the original action (2.1). For this result one should firstly perform gauge fixing for all the gauge symmetries, arriving at the dynamical system which contains only second class constraints, and then pass to the Dirac brackets in a proper way (see [17] for corresponding analysis of the BS superparticle model). This means that the generalization of the Penrose correspondence (2.30), (2.31), (2.32) should be regarded as coming from the second class constraints (primary and obtained from the gauge fixing) of the original system and, thus, should be considered as a relations hold in the strong sense (i.e. as operator identities after quantization) [22]. Hence, after the quantization performed in the frame of supertwistor approach, the generalized Penrose relations (2.30), (2.31), (2.32) can be substituted into the wave function in order to obtain the $D = 4$ superspace description of our quantum system.
We shall discuss now the relation of Eq. (2.30), (2.31), (2.32), (2.33) with the known
$FSU(2,2|1)$ supertwistor description of the BS superparticle [10, 11, 13, 16, 17, 18].

The standard FS description is given by the action

$$S = - \int \left( \mu^A d\lambda_A + \bar{\mu}^A d\bar{\lambda}_A + i d\xi \bar{\xi} \right)$$

(2.39)

supplemented by the first class constraint

$$\mu^A \lambda_A - \bar{\mu}^A \bar{\lambda}_A + 2i\xi \bar{\xi} = 0$$

(2.40)

The $SU(2,2|1)$ supertwistor $(\lambda^A, \bar{\mu}_A, \xi)$, contains complex Grassmann variable $\xi$ and
the supersymmetric Penrose–Ferber correspondence is given by

$$\bar{\mu}^A = \left( X^{B\dot{A}} - i \Theta^B \bar{\Theta}^{\dot{A}} \right) \lambda_B$$

(2.41)

$$\xi = \Theta^A \lambda_A, \quad \bar{\xi} = \bar{\Theta}^{\dot{A}} \bar{\lambda}_{\dot{A}}.$$ 

(2.42)

Comparing Eqs. (2.39) – (2.42) with our $OSp(8|1)$ supertwistor description (2.30) –
(2.33) of the superparticle (2.1) with additional central charge coordinates, we note that

- Besides additional terms proportional to tensorial central charge coordinates $z^{AB}$,
  $\bar{z}^{\dot{A}\dot{B}}$, there is present in (2.31) the second term quadratic in Grassmann variables.
  This second term, however, does not contribute to the invariant $\mu^A \lambda_A$.

- In our model we get

$$\mu^A \lambda_A - \bar{\mu}^A \bar{\lambda}_A = 2\lambda_A \lambda_B z^{AB} - 2\bar{\lambda}_{\dot{A}} \bar{\lambda}_{\dot{B}} \bar{z}^{\dot{A}\dot{B}} + 2i\Theta^A \lambda_A \bar{\Theta}^{\dot{A}} \bar{\lambda}_{\dot{A}}$$

(2.43)

i.e. we do not have additional first class constraint generating $U(1)$ symmetry
(compare to (2.40) of the standard supertwistor formulation). Thus our action
(2.33) is not singular in distinction to (2.39), where the first class constraint (2.40)
should be taken into account, e.g. by introducing it into the action with Lagrange
multiplier [18].

- The complex Grassmann variable $\xi$ (2.42) of FS formalism is replaced in our case by
  the real one $\zeta$ (2.32). This difference implies that in our supertwistor formalism the
  limit $z^{AB} \rightarrow 0$, $\bar{z}^{\dot{A}\dot{B}} \rightarrow 0$ does not reproduce the standard $SU(2,2|1)$ supertwistor
  formalism. Indeed, this is not surprising if we take into account that, from algebraic
  point of view, $SU(2,2|1)$ is not a subsupergroup of $OSp(8|1)$.

The model (2.1) can be slightly generalized as follows

$$S = \int d\tau \left( \lambda_A \bar{\lambda}_B \Pi^A_{\tau} \Pi^B_{\tau} + Z \lambda_A \lambda_B \Pi^A_{\tau} \Pi^B_{\tau} + \bar{Z} \bar{\lambda}_A \bar{\lambda}_B \bar{\Pi}^{\dot{A}\dot{B}}_{\tau} \bar{\Pi}^{\dot{A}\dot{B}}_{\tau} \right),$$

(2.44)

where $Z$, $\bar{Z}$ are complex numerical constants. It appears that for all values of $Z \neq 1$
the model (2.44) will have only two $\kappa$-symmetries, and only for particular value $Z = 1$
we obtain three $\kappa$-symmetries. The quantization of the model (2.44) is now under
consideration [37].
3 \quad D = 10 \text{ and } D = 11 \text{ models with one fundamental spinor}

Recently the most general superparticle model associated with space–time superalgebra (1.1) was proposed by Rudychev and Sezgin [20]. Introducing generalized real superspace $(X^{\alpha\beta}, \Theta^\alpha)$ they consider the following action

$$S = \int d\tau L = \int d\tau \left( P_{\alpha\beta} \Pi^{\alpha\beta} + \frac{1}{2} e_{\alpha\beta} P^{\alpha\gamma} C_{\gamma\delta} P^{\delta\beta} \right),$$

(3.1)

where $\Pi_{\alpha\beta} = \dot{X}^{\alpha\beta} - \dot{\theta}^{(\alpha\beta)}$ ($\dot{a} \equiv \frac{da}{d\tau}$), $C$ is the charge conjugation matrix and $e_{\alpha\beta}$ is the set of Lagrange multipliers, generalizing einbein in the action for standard Brink-Schwarz massless superparticle [19].

Generalized mass shell condition, obtained by varying $e_{\alpha\beta}$ in (3.1), takes the form

$$P^{\alpha\gamma} C_{\gamma\delta} P^{\delta\beta} = 0.$$  

(3.2)

We shall look for $P_{\alpha\beta}$ expressing it as spinor bilinears and satisfying the generalized mass shell condition (3.2). Particular solution is provided by the following extension of our representation (1.6) to any dimension $D > 4$ with the use of one real $D$-dimensional Majorana spinor $\lambda_\alpha$ ($\alpha = 1, \ldots, 2^k$, $k = 4$ for $D = 10$, $k = 5$ for $D = 11$):

$$P_{\alpha\beta} = \lambda_\alpha \lambda_\beta, \quad (\lambda_\alpha)^* = \lambda_\alpha,$$

(3.3)

where (1.6) is obtained if $k = 2$. The expression (3.3) solves the BPS condition $\det P_{\alpha\beta} = 0$ as well as more strong Rudychev-Sezgin generalized mass shell constraint (3.2) valid in the model (3.1) with antisymmetric charge conjugation matrix $C$ ($C_{\alpha\beta} = -C_{\beta\alpha}$).

Using (3.3) we get the multidimensional generalization of our action (2.1) which reads

$$S = \int \mathcal{M} \lambda_\alpha \lambda_\beta \Pi^{\alpha\beta}$$

(3.4)

$$\Pi^{\alpha\beta} = dX^{\alpha\beta} - i d\Theta^{(\alpha\Theta^\beta)},$$

($\alpha = 1, \ldots, 2^k$)

and for $k = 2$ we get the action (2.1).

The case $k = 4$ can be treated as describing spinorial $D = 10$ massless superparticle model with 126 composite tensorial central charges $Z_{m_1 \ldots m_5}$ (cf. with [2, 3]). Indeed, using the basis of antisymmetric products of $D = 10$ sigma matrices we obtain

$$\lambda_\alpha \lambda_\beta \equiv P_{\alpha\beta} = P_m \sigma_{\alpha\beta}^m + Z_{m_1 \ldots m_5} \sigma_{\beta_1 \ldots \beta_5}^{m_1 \ldots m_5},$$

(3.5)

Contraction of this equation with $\sigma^{m\alpha\beta}$ produces the expression for momenta in terms of bosonic spinors

$$P_m = \frac{1}{16} \lambda_\alpha \sigma_{\alpha\beta}^m \lambda_\beta \quad \Rightarrow \quad P_m P^m = 0.$$  

(3.6)

The mass shell condition $P_m P^m = 0$ appears then as a result of the $D = 10$ identity $(\sigma_m)_{(\alpha\beta}(\sigma^m)_{\gamma\delta)} = 0$.

The action (3.4) for $k = 8$ can be treated as describing a 0–superbrane model in $D = 11$ superspace with 517 composite tensorial central charge described by 32 components of one
real Majorana $D = 11$ bosonic spinor. In distinction to the above case such model does not produce a massless superparticle\(^3\). Indeed, decomposing (3.3) in the basis of products of $D = 11$ gamma matrices, one gets

$$\lambda_\alpha \lambda_\beta = P_m \Gamma_m^{\alpha \beta} + Z_{m_1 m_2} \Gamma_{\beta \alpha}^{m_1 m_2} + Z_{m_1 \ldots m_5} \Gamma_{\beta \alpha}^{m_1 \ldots m_5},$$

(3.7)

The $D = 11$ energy-momentum vector is then given by

$$P_m = \frac{1}{32} \lambda_\alpha \Gamma_m^{\alpha \beta} \lambda_\beta$$

(3.8)

and the $D = 11$ mass-shell condition reads

$$M^2 = P_m P^m = \frac{1}{1024} (\lambda \Gamma^m \lambda) (\lambda \Gamma^m \lambda)$$

(3.9)

Using the $D = 11$ Fierz identities one can prove that the mass shell condition acquires the form

$$M^2 = P_m P^m = 2 Z_{mn} Z_{mn} - \frac{32 \cdot 5!}{32^2} Z_{m_1 \ldots m_5} Z_{m_1 \ldots m_5}$$

(3.10)

with $Z_{mn} = -\frac{1}{64} \lambda \Gamma_{mn} \lambda$, $Z_{m_1 \ldots m_5} = \frac{1}{32 \cdot 5!} \lambda \Gamma_{m_1 \ldots m_5} \lambda$.

If we take into consideration that the equations of motion for our model (3.4) imply that the bosonic spinor $\lambda_\alpha$ is constant ($d\lambda_\alpha = 0$), we have to conclude that (3.4) with $k = 8$ provides the $D = 11$ superparticle model with mass generated dynamically in a way similar to the tension generating mechanism, studied in superstring and higher branes in [21].

Performing the integration by parts we can rewrite the action (3.4) in the $OSp(1|2^k)$ (i.e. $OSp(1|16)$ for $D = 10$ and $OSp(1|32)$ for $D = 11$) supertwistor $Y^A = (\mu^\alpha, \zeta)$ components:

$$S = -\int (\mu^\alpha d\lambda_\alpha + i d\zeta \zeta), \quad \alpha = 1, \ldots, 2^k.$$ 

(3.11)

The generalized Penrose–Ferber correspondence between real supertwistors and real generalized superspace looks as follows

$$\mu^\alpha = X^{\alpha \beta} \lambda_\beta - i \Theta^\alpha (\Theta^\beta \lambda_\beta), \quad \zeta = \Theta^\alpha \lambda_\alpha.$$ 

(3.12)

4 A set of $D = 11$ massless superparticle models with conservation of more then $1/2$ target space supersymmetries

In order to formulate the model we need to describe $SO(1,10)/(SO(1,1) \otimes SO(9) \otimes K_9)$ Lorentz harmonic formalism.

\(^3\)Note, that the $D = 11$ Green–Schwarz superparticle model does exist and was presented in [26].
4.1 \( SO(1,10) \otimes SO(9) \otimes K_9 \) spinor moving frame

The \( SO(1,10) \) valued moving frame matrix \( u^a_m \) splits into two light–like and 9 space–like vectors [31]

\[
u^a_m = (u^+m, u^-m, u^I_m) \quad \in \quad SO(1,10)
\]

\[
\Leftrightarrow \quad u^m_a v^b_m = \eta_{ab} \Leftrightarrow \begin{cases} u^{++}m u^{++} = 0, \\ u^{-m} u^{-m} = 0, \\ u^{\pm \pm} u^{\pm \pm} = 0, \\ u^I_m u^I_m = -\delta^{Ij} \end{cases}
\]

where \( I = 1, ..., 9 \) is \( SO(9) \) vector index.

The \( Spin(1,10) \) valued spinor moving frame matrix \( v^\alpha_\mu \) representing the same Lorentz rotation

\[
u^a_m \Gamma^m_\mu = v^\alpha_\mu \Gamma^\alpha_\mu v^\beta_\nu, \quad (4.14)
\]

\[
v^a_m \Gamma^\alpha_\mu = v^\alpha_\mu \Gamma^\mu_\nu v^\beta_\nu, \quad (4.15)
\]

splits into two rectangular blocks

\[
\nu^\alpha_\mu = (v^+_\mu A, v^-_\mu A) \quad \in \quad Spin(1,10)
\]

where \( A = 1, ..., 16 \) is \( SO(9) \) spinor index and the sign superscripts denote the \( SO(1,1) \) weight of the vector and spinor harmonics.

As the \( Spin(1,10) \) transformations keep invariant not only the gamma matrices (4.14), but the \( D = 11 \) charge conjugation matrix as well

\[
v^\alpha_\mu C^\mu_\nu v^\beta_\nu = C^\alpha_\beta, \quad (4.17)
\]

the spinor harmonics (4.16) are normalized by

\[
v^+_\mu A v^-_\mu B = -v^-_\mu A v^+_\mu B = -i\delta_{AB}, \quad v^-_\mu A v^-_\mu B = 0, \quad v^+_\mu A v^+_\mu B = 0. \quad (4.18)
\]

Eqs. (4.18) is equivalent to the following decomposition of \( 32 \times 32 \) unity matrix \[1\]

\[
\delta^\mu_\nu = iv^+_\mu A v^-_\nu A - iv^+_\mu A v^+_\nu A \quad (4.19)
\]

In a suitable \( SO(1,1) \otimes SO(9) \otimes K_9 \) invariant representation for \( D = 11 \) gamma matrices the Eqs. (4.14) acquire the form

\[
u^{++}m \Gamma^{++}_m = 2v^+_\mu A v^+_\mu A, \quad u^{-m} \Gamma^{-m}_m = 2v^-_\mu A v^-_\mu A \quad u^I_m \Gamma^I_m = 2v^+_\mu A \Gamma^I_\mu A v^-_\mu A, \quad (4.20)
\]

(compare e.g., with \( D = 10 \) cases from Refs. [33, 32, 34, 30]). The decomposition of the relations (4.13) includes, in particular

\[
v^+-\mu A \Gamma^\mu_\nu v^--\nu B = 2\delta_{AB} u^-\mu A \quad (4.21)
\]

\[4\]The appearance of multiplier \( i \) in Eqs. (4.18), (4.19) is due to the fact that \( D = 11 \) charge conjugation matrix is imaginary for our choice of notations and signature \( \eta^{ab} = diag(+1, -1, \ldots, -1) \)
4.2 Action for $D = 11$ massless superparticle with tensorial central charge coordinates

The twistor-like action for $D = 11$ massless superparticle with tensorial central charge coordinates has the form

$$S = \int_{\mathcal{M}} P^{++}_{AB} v^{-}_{A\mu} v^{-}_{B\nu} \Pi^{\mu\nu}$$

(4.22)

with

$$\Pi^{\mu\nu} = dX^{\mu\nu} - id\Theta^{(\mu}{\Theta^{\nu)}}$$

and symmetric $SO(9)$ spin-tensor Lagrange multiplier $P^{++}_{AB}$.

The canonical momenta

$$P_{\mu\nu} = \frac{\partial L}{\partial \dot{X}^{\mu\nu}} = P^{++}_{AB} v^{-}_{A\mu} v^{-}_{B\nu}$$

(4.23)

evidently satisfy the BPS condition

$$\det(P_{\mu\nu}) = 0$$

as well as the more strong Rudychev-Sezgin generalized mass shell constraint

$$P_{\mu\rho} C^{\rho\sigma} P_{\sigma\nu} = 0.$$

The rank of the matrix $P_{\mu\nu}$ is less or equal to 16, equal in fact to the rank of the matrix $P^{++}_{AB}$. As we will demonstrate just this rank defines the number of preserved target space supersymmetries.

The variation of the action (4.22) with respect to the coordinate fields

$$\delta S = \int_{\mathcal{M}} P^{++}_{AB} v^{-}_{A\mu} v^{-}_{B\nu} (d\delta \Pi^{\mu\nu} - 2id\Theta^{(\mu}{\delta \Theta^{\nu)})$$

(4.24)

includes effectively the $\delta \Theta^{\mu}$ variation only in the combination

$$d\Theta^{\nu} v^{-}_{A\nu} P^{++}_{AB} \delta \Theta^{\mu} v^{-}_{A\mu}$$

Thus the half of $\Theta$ variations $\delta \Theta^{\mu} v^{-}_{A\mu}$ are not involved in the variation of action and, therefore, parameterize the 16 kappa symmetries.

When $\det(P^{++}_{AB}) \neq 0$, the rest 16 of the 32 Grassmann variations $\delta \Theta^{\mu} v^{-}_{A\mu}$ acts effectively and produce nontrivial equations of motion

$$d\Theta^{\nu} v^{-}_{A\nu} P^{++}_{AB} = 0, \quad \Rightarrow \quad d\Theta^{\nu} v^{-}_{A\nu} = 0.$$

We see that there are only 16 kappa symmetries in such dynamical system and so it describes the BPS state preserving 1/2 of the $D = 11$ target space supersymmetry.

We obtain an important particular case of the model (4.22) with $\det(P^{++}_{AB}) \neq 0$ when the Lagrange multiplier $P^{++}_{AB}$ is proportional to the unity matrix $P^{++}_{AB} = P^{++} \delta_{AB}$. Due to the properties (4.20) of the Lorentz harmonic, the product of spinor harmonics $v^{-}_{A\mu} v^{-}_{A\nu}$
is proportional to the gamma matrix $\Gamma_{m\mu\nu}$, hence it does not contain components proportional to $\Gamma_{m\mu\nu}^{\alpha\beta}$, $\Gamma_{\mu\nu}^{\alpha\beta\gamma\delta}$. Thus the central charge coordinates disappear from the action which in this case can be equivalently rewritten as

$$S = \frac{1}{32} \int_{\mathcal{M}} P_{-v_A v_B} \Gamma_{m}^{\mu\nu} \Pi^m \tag{4.25}$$

$$\Pi^m = dX^m - i d \Theta^\mu \Gamma_{m}^{\mu\nu} \Theta^\nu$$

The formula (4.25) provides the twistor-like formulation of the action for the 'standard' $D=11$ massless superparticle (without tensorial central charge coordinates), whose 'standard' (Brink–Schwarz type) action was proposed recently in Ref. [29].

The generic case of nondegenerate $P_{AB}^{++}$ matrix corresponds the model with central charge coordinates and half of $32$ space time supersymmetries conserved.

The case with the matrix $P_{AB}^{++}$ having the rank $1$ can be described by

$$P_{AB}^{++} = \lambda_A^+ \lambda_B^+$$

with one bosonic $SO(16)$ spinor $\lambda_A^+$. The action (4.22) in this case reduces to

$$S = \int_{\mathcal{M}} (\lambda_A^+ v_{A\mu}) (\lambda_A^+ v_{B\nu}) \Pi^{\mu\nu} \tag{4.26}$$

If one denotes $\lambda_A^+ v_{A\mu}^+ = \lambda_\mu$, one arrives to the expression $S = \int_{\mathcal{M}} \lambda_\mu \lambda_\nu \Pi^{\mu\nu}$ which formally coincides with the action proposed in [28]. But the composite nature of the bosonic spinor $\lambda_\mu$ in the action (4.26) results in the relation

$$32 P_m \equiv \lambda_\mu \Gamma_{\mu\nu} \lambda_\nu = (\lambda_A^+ v_{A\mu}^+) (\lambda_A^+ v_{B\nu}^-) \Gamma_{m}^{\mu\nu} = \lambda_A^+ \lambda_A^+ u_m^- , \tag{4.27}$$

where $u_m^-$ is a light-like harmonic vector $u_m^- u_m^- = 0$. Thus $P_m P^m = 0$ and we conclude that (4.26) describes a massless $D=11$ superparticle with central charge coordinate in distinction with the $D=11$ model described by (3.4) [28], where, in general, the particle is massive with mass generated dynamically [21].

Nevertheless both the models (3.4) and (4.26) describe BPS configurations with preservation of $31/32$ part of the $D=11$ target space supersymmetries.

Indeed the variation of the action (4.26) includes effectively only one Grassmann variation $\delta \Theta^\mu \lambda_\mu$ (with $\lambda_\mu$ composed from harmonic and $SO(16)$ spinor as in (4.26)), which remains the same for the action (3.4), where the $\lambda_\mu$ spinor is fundamental (see [28]).

The matrix $P_{AB}^{++}$ of the rank $r$, $1 < r < 8$ can be represented as

$$P_{AB}^{++} = \lambda_A^+ s \lambda_B^{+s} , \quad s = 1, \ldots, r , \quad 1 < r < 8 . \tag{4.28}$$

It is easy to see that such a model describes the BPS states preserving $\frac{(32-r)}{32}$ supersymmetries.
5 Final remarks

We would like to recall that in the ‘M-theoretic’ approach (see e.g. [27, 3, 13]) the tensorial central charges \(Z_{m_1 \ldots m_p}\) are considered as carried by p-branes. Following such treatment, one should interpret e.g. in \(D = 4\) central charges \(Z_{\mu \nu}\) as an indication of presence of \(D = 4\) supermembrane \((p = 2)\). The relation of our superparticle model with such \(D = 4\) membrane states is not clear now and can be regarded as an interesting subject for further study. Here we should only guess that there should be some singular point-like limit of supermembrane, which should keep the nontrivial topological charge and increase the number of preserved (realized linearly) \(D = 4\) target space supersymmetries. Similar limiting prescription should be possible e.g. for 5–branes in \(D = 10, 11\) leading to the \(D = 10\) and \(D = 11\) superparticle actions (3.3) with the relation (3.3) describing composite tensor charges.

At the end of the paper we proposed a generalized FS model for \(D > 4\). The straightforward generalization provides us with \(D = 10\) massless superparticle model preserving \(15/16\) supersymmetries and \(D = 11\) superparticle model with arbitrary, in general nonvanishing, mass generated dynamical \([21]\). The latter conserves \(31/32\) of the target space supersymmetries. Then we introduce spinor harmonics and formulate massless \(D = 11\) superparticle model preserving \(1/2, 17/32, 18/32, \ldots, 31/32\) supersymmetries dependent on the rank of the Lagrange multiplier matrix \(P^{++}_{AB}\). The case with \(1/2\) corresponds to nondegenerate matrix \(P^{++}_{AB} \neq 0\). For the choice \(P^{++}_{AB} = \propto \delta_{AB}\), the dependence on central charge coordinates disappears and we arrive at the twistor-like formulation of the usual massless \(D = 11\) superparticle proposed recently by Bergshoeff and Townsend.

It should be also mentioned that the superparticle model invariant under super-Poincare symmetries with central charges can be obtained as a contraction limit of superparticle model defined on the orthosymplectic supergroup manifolds. The \(D = 4\) case \((OSp(4|1)\) model) is now under consideration \([38]\).

Acknowledgements

The authors would like to thank D. Sorokin for useful discussions. The authors are grateful for the hospitality at the Departamento de Fisica Teorica and financial support of the Universidad de Valencia (J.L.) and at the Dipartimento di Fisica ”Galileo Galilei”, Universita Degli Study di Padova ed INFN, Sezione di Padova (I.B.), which permitted to complete the present paper.

The work was supported by the Austrian Science Foundation in the form of the Lise Meitner Fellowship under the Project M472–TPH, by the INTAS grants INTAS-96-308, INTAS-93-127-EXT and KBN grant 2P03B13012.

References

[1] R. Haag, J. Lopuszański and M. Sohnius, Nucl.Phys. B88 (1995) 257.
[2] J. van Holten and A. van Proyen, J.Phys. A15 (1982) 3763.
[3] P.K. Townsend, P-brane democracy, hep-th/9507048; Four lectures on M–theory, hep-th/9612121; M-theory from its superalgebra, hep-th/9712004.
[4] I. Bars, Phys. Lett. B373, 68 (1996); Phys.Rev. D54 (1996) 5203.

[5] Y. Eisenberg and S. Solomon, Phys.Lett B220 (1989) 562.

[6] S.F. Hewson and M.J. Perry, Nucl.Phys. B492 (1997) 249;
S.F. Hewson, Nucl.Phys. B501 (1997) 445; An approach to F–theory, hep-th/9712017.

[7] P. d’Auria and P. Fré, Nucl.Phys. B201 (1982) 101.

[8] E. Sezgin, Phys.Lett. B392 (1997) 323.

[9] E.M. Corson, Introduction to Tensors, Spinors and Relativistic Wave Equations, Blodue and Son Limited, London and Glasgow, 1953.

[10] A. Ferber, Nucl.Phys. B132 (1978) 55.

[11] T. Shirafuji, Progr.Theor.Phys. 70 (1983) 18.

[12] J.A. de Azcarraga and J. Lukierski, Phys.Lett. B113 (1982) 170;
W. Siegel, Phys.Lett. B128 (1983) 397.

[13] D. Sorokin and P.K. Townsend, Phys. Lett. B412 (1997) 265.

[14] J. Lukierski, Lett.Nuov.Cim. 24 (1979) 309;
W. Heidenreich and J. Lukierski, Mod.Phys.Lett. A5 (1990) 439.

[15] A. Bengtsson, I. Bengtsson, M. Cederwall and N. Linden, Phys.Rev. 36D (1987) 1766;
J. Lukierski and A. Nowicki, Phys.Lett. B211 (1988) 276.

[16] D. Sorokin, V. Tkach and D. Volkov, Mod.Phys.Lett. A4 (1989) 901;
D. Volkov and A. Zheltukhin, Lett.Math.Phys. 48 (1998) 61; Nucl.Phys. B335 (1990) 723;
D. Sorokin, V. Tkach, D. Volkov and A. Zheltukhin, Phys.Lett. 216B (1989) 302.

[17] A. Gumenchuk and D. Sorokin, Sov.J.Nucl.Phys. 51 (1990) 350.

[18] P.K. Townsend, Phys.Lett. B261 (1991) 65.

[19] L. Brink and J. Schwarz, Phys. Lett. 100B (1981) 310.

[20] E. Sezgin and I. Rudychev, Superparticles, p-form Coordinates and the BPS Condition, hep-th/9711128.

[21] P.K. Townsend, Phys.Lett. B277 (1992) 285;
E. Bergshoeff, J.A.J. London, P.K. Townsend, Class.Quantum Grav. 9 (1992) 2545;
P. K. Townsend, Phys.Lett. B409 (1997) 131-135, hep-th/9705160.
M. Cederwall and P.K. Townsend, J.High Energy Phys. 09 (1997) 003, hep-th/9709002.
M. Cederwall and A. Westenberg, J.High Energy Phys. 02 (1998) 004, hep-th/9710160.
E. Bergshoeff and P.K. Townsend, Super-Dp-branes revisited, hep-th/9804011.
[22] P.A.M. Dirac, *Lectures on Quantum Mechanics*, Academic Press, NY 1967.

[23] I. A. Bandos, Sov.J.Nucl.Phys. 51 (1990) 906; JETP.Lett. 52 (1990) 205; 
I.A. Bandos, A.A. Zheltukhin, Phys.Lett. B77 (1992) 77; Class.Quantum Grav. 12 
(1995) 609; 
I. Bandos, P. Pasti, D. Sorokin, M. Tonin and D.V. Volkov, Nucl.Phys. B446 (1995) 
79.

[24] B. Zumino, J. Math.Phys. 3 (1962) 1055-1057.

[25] S. Ferrara, C.A. Savoy and B. Zumino, Phys.Lett. B100 (1981) 393.

[26] E. Bergshoeff and P.K. Townsend, Nucl.Phys. B490 (1997) 145.

[27] J.A. de Azcarraga, J.P. Gauntlett, J.M. Izquierdo and P.K. Townsend, 
Phys.Rev.Lett. 63 (1989) 2443.

[28] I. Bandos and J. Lukierski, *Tensorial Central Charges and New Superparticle Models 
with Fundamental Spinor Coordinates*, [hep-th/9811022]

[29] E. Bergshoeff, P.K. Townsend, *Super-D-branes*, Nucl.Phys. B490 (1997) 145–162, 
[hep-th/9611173]

[30] I. Bandos, P. Pasti, D. Sorokin, M. Tonin and D. Volkov, Nucl.Phys. B446, 79 (1995).

[31] E. Sokatchev, Phys. Lett. B169, 209 (1987); Class. Quantum Grav. 4, 237 (1987) 
and refs. in [30]

[32] A.S. Galperin, P. Howe and K.S. Stelle, Nucl. Phys. B368 (1992) 248-280

[33] A.S. Galperin, F.Delduc and E. Sokatchev, Nucl. Phys. B368 (1992) 280

[34] I.A. Bandos and A. A. Zheltukhin, JETP. Lett. 54, 421 (1991); Phys. Lett. B288, 
77 (1992); Sov. J. Nucl. Phys. 56, 198 (1993); J. Elem. Part. Atom. Nucl. 25, 453 
(1994).

[35] I. A. Bandos and A. A. Zheltukhin , Class. Quantum Grav. 12, 609 (1995).

[36] A. Galperin, E. Ivanov, S. Kalitzin, V. Ogievetsky and E. Sokatchev, Class. Quantum 
Grav. 1, 498 (1984); Class. Quantum Grav. 2, 155 (1985).

[37] J.A. de Azcarraga, I. Bandos, J. Lukierski and D. Sorokin, (in preparation).

[38] I. Bandos, J. Lukierski and D. Sorokin, (in preparation).