About spin precession in electromagnetic wave
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Abstract

It is shown that two ways of the description of the 1/2–spin precession in a circularly polarized electromagnetic wave: on the basis of the Bargmann-Michel-Telegdi equation and on the basis of the Dirac equation with Pauli term give the same result in the second order on the wave’s field for arbitrary wave length.

1. Introduction. The spin rotation of a particle in an electromagnetic wave is a perspective method of research of the electromagnetic and weak interactions. In recent work [1] electron spin rotation was considered, when electron moves towards circularly polarized electromagnetic wave. It was shown, that because of quantum electrodynamite effects frequency of spin rotation is much more than it would follow from the Bargmann-Michel-Telegdi (BMT) equation. The account quantum electrodynamite effects means, that an electron is considered as not a dot particle but as a particle having some electromagnetic ”structure” due to absorption and radiation of the virtual \( \gamma \) quanta. Similar effects were discussed in [2] where the polarization rotation of \( \gamma \)–quanta in the medium with polarized electrons is considered. The condition, at which the effect becomes essential, looks like

\[
\frac{\varepsilon \omega}{m^2} \geq 1,
\]

where \( \omega \) is a frequency of the electromagnetic wave, \( m \) is electron mass (system of units \( \hbar = c = 1 \) is used). Under this condition energy of the photon emitted becomes comparable with the electron energy \( \varepsilon \), however, under the same condition quasi-classical description of the moving particle becomes inapplicable and the concept of a trajectory loses sense. The description used in [1] kept features of the quasi-classical description, typical for the BMT equation. If to look at the Dirac equation for a free electron, we find out, that free electron is not a dot particle too i.e. it has some ”structure”. Let’s following [3] consider wave packet with the average momentum equal to zero. If to require, that the wave packet is a superposition of the solutions only with positive energy, we shall notice, that we cannot localize it in the area about \( 1/m \) size. To locate electron in the range of the order \( 1/m \) it is necessary to add to wave function solutions with negative energy. Electron localized such a way becomes ”electron-positron”. Concept of a trajectory (and the BMT equation) cannot be used if in the electron rest system (where electron average momentum is equal to zero) localization length \( 1/m \) greater than the inverse electromagnetic wave frequency \( 1/\omega \) in the electron rest system. Passing to the laboratory system we shall receive for the relativistic electrons the inequality above within a factor of 2. That is Dirac structure of the electron becomes essential under the same condition, that electromagnetic one. The question arises about Dirac structure influence on the frequency of the spin rotation compared with the BMT equation. To learn it, we shall calculate frequency of rotation of the electron spin in a circularly polarized wave, proceeding from the BMT equation and from the Dirac equation with the anomalous magnetic moment. Running forward we should tell, that both methods give identical result in the second order on the wave field strength, i.e. BMT equation casually gives correct result even behind the area of the quasi-classical description applicability.

2. The description of a spin rotation in a frame of BMT equation.

Let us present field of the circularly-polarized wave propagating against axis \( z \) as:

\[
\begin{align*}
H_x &= H_0 \cos \xi , \\
H_y &= H_0 \sin \xi , \\
E &= e_z \times H \\
\xi &= k_\mu x^\mu \equiv \omega(t + z) , \\
A_\mu &= a_\mu e^{-i \xi} + a_\mu^* e^{i \xi}.
\end{align*}
\]
Here \( \mathbf{H}, \mathbf{E} \) are strength of the magnetic and electrical fields, \( A_\mu \) is 4-potential of the circularly polarized wave, \( \omega \) is wave frequency. The constant 4-vector \( a_\mu \) is: \( a_\mu = (0, a, ia, 0) \), where \( a = -H_0/2\omega \).

Consider electron motion in the circularly polarized wave. The solution, for which momentum component perpendicular to \( z \) axis is equal to zero in average, looks like [4]:

\[
p_{\perp} = -eA(\xi), \quad p_z = \text{const}, \quad \epsilon^2 = p_z^2 + e^2A^2 + m^2 = \text{const},
\]

\[
z = p_z t \epsilon, \quad \mathbf{v}_\perp = \frac{p_{\perp}}{\epsilon} = \frac{eH(\xi)}{\epsilon\omega}, \quad r_{\perp} = -\frac{e}{\omega(\epsilon + p_z)} \int A(\xi) d\xi.
\]

Substituting speed \( v \) to the BMT equation [5]

\[
\frac{d\zeta}{dt} = \frac{2\mu m + 2\mu'(\epsilon - m)}{\epsilon}(\zeta \times \mathbf{H}) + \frac{2\mu'}{\epsilon + m}(v \cdot \mathbf{H})(\mathbf{v} \times \zeta) + \frac{2\mu m + 2\mu'\epsilon}{\epsilon + m}\zeta \times (\mathbf{E} \times \mathbf{v}),
\]

we receive in the second order on the field strength:

\[
(v \cdot \mathbf{H})(\mathbf{v} \times \zeta) = (\mathbf{v}_\perp \cdot \mathbf{H})(\mathbf{v}_\parallel + \mathbf{v}_\perp) \times \zeta \approx \frac{ev_{\parallel}}{\epsilon\omega} H^2(e_z \times \zeta),
\]

\[
\zeta \times (\mathbf{E} \times \mathbf{v}) = \zeta \times (\mathbf{E} \times \mathbf{v}_\parallel) + \zeta \times (\mathbf{E} \times \mathbf{v}_\perp) = v_{\parallel}(\zeta \times \mathbf{H}) - (\zeta \times e_z)H^2\frac{e^2}{\epsilon\omega}.
\]

Having substituted (4) in (3) we have:

\[
\frac{d\zeta}{dt} = \left(2\mu' \left(1 + \frac{\sqrt{\epsilon^2 + m^2}}{\epsilon}\right) + \frac{e}{\epsilon} \left(1 + \frac{\sqrt{\epsilon - m}}{\epsilon + m}\right)\right) (\zeta \times \mathbf{H})
\]

\[
- \left(\frac{2\mu'\epsilon}{\epsilon\omega} \left(1 + \frac{\sqrt{\epsilon - m}}{\epsilon + m}\right) + \frac{e^2}{\epsilon(\epsilon + m)\omega}\right) H^2(\zeta \times e_z)
\]

The equation (3) contains at the right hand side sum of the oscillating and constant terms, however one cannot reject oscillating term simply. Let's share the vector \( \zeta \) into two parts [6]:

\[
\zeta(t) = \theta(t) + \varepsilon\omega(t),
\]

where \( <\varepsilon\omega(t)> = 0 \) (angular brackets mean averaging on interval of time much more exceeding the period of the electromagnetic wave). For \( \theta(t) \) and \( \varepsilon\omega(t) \) we may write

\[
\frac{d\theta}{dt} = \left(2\mu' \left(1 + \frac{\sqrt{\epsilon^2 + m^2}}{\epsilon}\right) + \frac{e}{\epsilon} \left(1 + \frac{\sqrt{\epsilon - m}}{\epsilon + m}\right)\right) <\varepsilon\omega \times \mathbf{H}>
\]

\[
- \left(\frac{2\mu'\epsilon}{\epsilon\omega} \left(1 + \frac{\sqrt{\epsilon - m}}{\epsilon + m}\right) + \frac{e^2}{\epsilon(\epsilon + m)\omega}\right) H^2(\theta \times e_z)
\]

\[
\frac{d\varepsilon\omega}{dt} = \left(2\mu' \left(1 + \frac{\sqrt{\epsilon^2 + m^2}}{\epsilon}\right) + \frac{e}{\epsilon} \left(1 + \frac{\sqrt{\epsilon - m}}{\epsilon + m}\right)\right) (\theta \times \mathbf{H})
\]

In the second equation the terms proportional to \( <\varepsilon\omega \times \mathbf{H}> <\varepsilon\omega \times \mathbf{H}> \) and \( H^2(\varepsilon\omega \times e_z) \) are rejected, because they give contribution higher than second order on the field strength to the equation for \( \frac{d\theta}{dt} \). Integrating the equation (8) we get

\[
\varepsilon\omega(t) = \frac{H_0\epsilon}{2\omega(\epsilon + p_z)} \left(2\mu' \left(1 + \frac{\sqrt{\epsilon^2 + m^2}}{\epsilon}\right) + \frac{e}{\epsilon} \left(1 + \frac{\sqrt{\epsilon - m}}{\epsilon + m}\right)\right) \theta \times (e_x \sin \xi - e_z \cos \xi).
\]
Having substituted (9) in (7) we arrive at
\[
\frac{d\theta}{dt} = 2\mu^2 \frac{\varepsilon + \sqrt{\varepsilon^2 + m^2}}{\varepsilon \omega} H^2(\theta \times e_z),
\]
\[
\frac{d\theta}{dz} = \frac{\varepsilon}{p_z} \frac{d\theta}{dt} = \frac{2\mu^2}{\omega} \frac{\varepsilon + \sqrt{\varepsilon^2 + m^2}}{\sqrt{\varepsilon^2 + m^2}} H^2(\theta \times e_z).
\]
(10)

It is necessary to note, that generalization of the BMT equation on a case of non-uniform fields is deduced in [7,8]. The equation [7,8] is mysterious, in the sense, that it fails to be deduced from the Dirac equation with the anomalous magnetic moment, using Foldi-Woithausen transformation [9,10], however it can be deduced from the Dirac equation applying the correspondence principle [11,12,13]. Frequency of spin rotation received from the equation [7,8] contains in comparison with (10) additional terms.

3. Description of the spin rotation in electromagnetic wave, proceeding from the Dirac equation. Let us proceed from the assumption, that in a field of the electromagnetic wave electron has some spin-dependent effective energy, which results to different refractive indexes for electrons with the different spin projections to the momentum direction. To describe it we receive the equation for the electron wave function averaged over phase of the electromagnetic wave. Averaged electron wave function is a plain wave with a wave 4-vector satisfying to the dispersion equation.

Write down equation for bispinor \(\Psi(x,b)\) of the electron in the electromagnetic wave as:
\[
\alpha_{x-b}\Psi(x,b) = 0,
\]
(11)
Where \(x = \{t, r\}\), and \(b\) is a constant 4-vector connected with the phase of the electromagnetic wave \(\phi_0\) as \(\phi_0 = k^\mu b_\mu\). Averaging of wave function \(<\Psi(x)\> = \frac{1}{\Omega} \int_{\Omega} \Psi(x,b) d^4b\) is equivalent to the averaging over phase of the wave. Here \(\Omega\) is some large 4-volume. Average wave function can be presented as:
\[
<\Psi(x)> = e^{-ip'x'}u(p') ,
\]
(12)
here \(u(p')\) is bispinor, and \(p'\) is 4-quasi-momentum required for which we are going to deduce dispersion equation. Let’s designate \(\overline{\alpha}(p') = \frac{1}{\Omega} \int_{\Omega} e^{-ip'x} \alpha_x e^{ip'x} dx^4\) and rewrite (11) as:
\[
\overline{\alpha}\Psi(x,b) = (\alpha - \alpha_{x-b})\Psi(x,b) = 0.
\]
(13)
The averaging of (13) on \(b\) gives
\[
\overline{\alpha} <\Psi(x)> = \frac{1}{\Omega} \int_{\Omega} (\overline{\alpha} - \alpha_{x-b})\Psi(x,b) d^4b = \frac{1}{\Omega} \int_{\Omega} (\overline{\alpha} - \alpha_{x-b})e^{-ip'b}\Psi(x-b,0) d^4b
\]
\[
= \exp(-ip'x) \int_{\Omega} e^{ip'x'}(\overline{\alpha} - \alpha_{x'})\Psi(x',0) d^4x'.
\]
(14)
Under derivation of (14) we take into account that by virtue of space - time uniformity \(\Psi(x,b)\) has the following transformation properties [14]: \(\Psi(x+b',b+b') = e^{-ip'b'}\Psi(x,b)\), where \(b'\) is any 4-vector. Presenting \(\Psi(x,0)\) as \(\Psi(x,0) = <\Psi(x) > + \phi(x)\) and substituting it into the (14) we find
\[
\tilde{\alpha} <\Psi(x)> = \exp(-ipx)Z(p') ,
\]
(15)
where \(Z(p') = \frac{1}{\Omega} \int_{\Omega} e^{ip'x'}(\tilde{\alpha} - \alpha_{x'})\phi(x') dx'^4\). Writing down (14) For \(b = 0\) and subtracting it from (15) we obtain
\[
(\tilde{\alpha} - \alpha_x) <\Psi(x)> - \alpha_x\phi(x) = e^{-ip'x}Z(p') .
\]
(16)
Let’s consider integral:

\[ Z(p') = \left( 1 - \frac{1}{\Omega} \int \limits_\Omega e^{ip'x'}(\vec{a} - \alpha x') \frac{1}{\alpha x'}e^{-ip'x'}d^4x' \right)^{-1} \]

\[ \times \frac{1}{\Omega} \int \limits_\Omega e^{ipx}(\vec{a} - \alpha x)(\vec{a} - \alpha x) < \Psi(x) > d^4x . \tag{17} \]

Substituting \( < \Psi(x) > \) in the form (12) to (17) and then substituting (17) in (12) the dispersion equation is received

\[ \left( \vec{\alpha}(p') - \left( 1 - \frac{1}{\Omega} \int \limits_\Omega e^{ip'x'}(\vec{a} - \alpha x') \frac{1}{\alpha x'}e^{-ip'x'}d^4x' \right)^{-1} \]

\[ \times \frac{1}{\Omega} \int \limits_\Omega e^{ipx}(\vec{a} - \alpha x)(\vec{a} - \alpha x)e^{-ipx}d^4x \right) u(p') = 0 . \tag{18} \]

For the Dirac equation with Pauli anomalous magnetic moment \( \mu' \) operator \( \alpha_x \) is

\[ \alpha_x = \gamma_{\mu}(p'^\mu - eA^\mu(x)) - m + \frac{i}{2}\mu'\sigma_{\mu\nu}F^{\mu\nu}(x) , \tag{19} \]

Where \( F^{\mu\nu}(x) = \partial^\mu A^\nu - \partial^\nu A^\mu ; \sigma_{\mu\nu} = \frac{1}{2}(\gamma_{\mu}\gamma_{\nu} - \gamma_{\nu}\gamma_{\mu}) ; p'^\mu = \{i\frac{\partial}{\partial t}, -i\vec{\nabla}\}. \]

In the second order on the field strength dispersion equation will be

\[ \left( \gamma^\mu p'_\mu - m - \frac{1}{\Omega} \int \limits_\Omega e^{ipx} \left( e\gamma_{\mu}A^\mu(x) - \frac{i}{2}\mu'\sigma_{\mu\nu}F^{\mu\nu}(x) \right) G_0(x,x') \]

\[ \times \left( e\gamma_{\mu}A^\mu(x') - \frac{i}{2}\mu'\sigma_{\mu\nu}F^{\mu\nu}(x') \right) e^{-ip'x'}d^4xd^4x' \right) u(p') = 0 . \tag{20} \]

Green function [5] of the free Dirac equation \( G_0(x,x') \) is determined as

\[ G_0(x,x') = \int \frac{G(p)}{(2\pi)^4}e^{-ip(x-x')}d^4p, \quad G(p) = \frac{\gamma_{\mu}p'^\mu + m}{p'^2 - m^2} . \]

Taking 4-potential \( A^\mu \) in the form \( (1) \) we find

\[ e\gamma_{\mu}A^\mu(x) - \frac{i}{2}\mu'\sigma_{\mu\nu}F^{\mu\nu}(x) = \left( e\gamma_{\mu}a^\mu - \frac{\mu'}{2}\sigma_{\mu\nu}(k^\mu a^\nu - k^\nu a^\mu) \right) e^{-i\xi} + \]

\[ \left( e\gamma_{\mu}a^\mu + \frac{\mu'}{2}\sigma_{\mu\nu}(k^\mu a^\nu - k^\nu a^\mu) \right) e^{i\xi} \equiv W^-e^{-i\xi} + W^+e^{i\xi} \tag{21} \]

Let’s consider integral:

\[ \frac{1}{\Omega} \int \limits_\Omega e^{ipx} \left( W^-e^{-i\xi} + W^+e^{i\xi} \right) G(p)e^{-ip(x-x')} \left( W^-e^{-i\xi} + W^+e^{i\xi} \right) e^{ip'x'}d^4xd^4x' \frac{d^4p}{(2\pi)^4} \]

\[ = \frac{(2\pi)^4}{\Omega} \int \limits_\Omega \left( W^-\delta^4(p + k - p') + W^+\delta(p - k + p') \right) G(p) \]

\[ \times \left( W^-\delta^4(p - k - p') + W^+\delta(p + k - p') \right) d^4p W^-G(p' - k)W^+ + W^+G(p' + k)W^- . \tag{22} \]

At derivation of (22) we use, that \( \left( \delta^4(p) \right)^2 = \frac{1}{(2\pi)^4} \int e^{ipx}d^4x\delta^4(p) = \frac{\Omega}{(2\pi)^4}\delta^4(p) \). This gives following matrix equation in the second order on a field strength

\[ \left( \gamma^\mu p'_\mu - m - W^-G(p - k)W^+ + W^+G(p + k)W^- \right) u(p') = 0 \tag{23} \]
The determinant of the equation \([23]\), if to take standard matrices representation \([5]\) is:

\[
\begin{vmatrix}
\varepsilon^+ - A_1(\omega) & 0 & -p' - B(\omega) & 0 \\
0 & \varepsilon^+ - A_1(-\omega) & 0 & p' + B(-\omega) \\
p' + B(\omega) & 0 & -\varepsilon^+ + A_2(\omega) & 0 \\
0 & -p' - B(-\omega) & 0 & -\varepsilon^+ - A_2(-\omega)
\end{vmatrix} .
\tag{24}
\]

Here we have designated: \(\varepsilon^\pm = \varepsilon \pm m\),

\[
A_1(\omega) = -\frac{2a^2}{\omega(\varepsilon + p)} \left( \varepsilon^2(\varepsilon - \omega - m) - 2e\mu'\omega(\varepsilon - m + p) + 2\mu'^2\omega^2(\varepsilon + p) \right),
\]

\[
A_2(\omega) = -\frac{2a^2}{\omega(\varepsilon + p)} \left( \varepsilon^2(-\varepsilon + \omega - m) - 2e\mu'\omega(\varepsilon + m + p) - 2\mu'^2\omega^2(\varepsilon + p) \right),
\]

\[
B(\omega) = -\frac{2a^2}{\omega(\varepsilon + p)} \left( -\varepsilon^2(p + \omega) + 2e\mu'\omega m + 2\mu'^2\omega^2(\varepsilon + p) \right).
\]

Evaluating the determinant \((24)\) and equating it to zero the dispersion equation is obtained

\[
\left( (\varepsilon^- - A_1(\omega))(\varepsilon^+ + A_2(\omega)) - (p' + B(\omega))^2 \right) 
\times \left( (\varepsilon^- - A_1(-\omega))(\varepsilon^+ + A_2(-\omega)) - (p' + B(-\omega))^2 \right) = 0 .
\tag{25}
\]

For the solutions with positive \(p'\) (we remind, that electron goes in a positive direction of \(z\) axis) we have:

\[
P_1' = -B(\omega) + \sqrt{(\varepsilon^- - A_1(\omega))(\varepsilon^+ + A_2(\omega))}
\]

\[
P_2' = -B(-\omega) + \sqrt{(\varepsilon^- - A_1(-\omega))(\varepsilon^+ + A_2(-\omega))}.
\]

Two different helicities correspond to the two different electron wave 3-vectors. By taking their difference \([15]\) we receive frequency of spin rotation \(\frac{d\theta}{dz}\), which in the second order on a field strength coincides with received from the formula \((10)\).

4. How quantum electrodynamics effects can be taken into account. First of all we note, that there is a way to find photon medium refractive index which take into account QED effects, with the help of photon-electron forward scattering amplitude \([16]\). On physical sense this way corresponds to the situation, when electron moves through non-coherent photon medium, while in our case photons are in a coherent state. Let’s show how to receive dispersion equation in our approach with the account of QED effects. Under deduction of the dispersion equation we proceeded from the equation:

\[
\alpha_x \Psi(x) = 0.
\tag{26}
\]

The equation it is possible to rewrite as:

\[
G_x^{-1} \Psi(x) = 0,
\tag{27}
\]

where \(G_x\) is a Green function of the operator \(\alpha_x\). To take into account QED effects, it is necessary, keeping form of the equation \((27)\) to take as \(G_x\) one-partial QED Green function of the electron in the field of the electromagnetic wave. As it is known \([5]\), \(G_x\) kernel looks like

\[
G(x, x') = -i \frac{\langle 0|T\psi(x)\bar{\psi}(x')S|0 \rangle}{\langle 0|S|0 \rangle},
\]

where \(T\) is the time-ordering operator.
where \( S = \text{Exp} \left( -ie \int \bar{\psi}(x)A^\mu(x)\gamma_\mu\psi(x)d^4x \right) \), \( \psi(x) = \sum_n a_n \psi_n^+(x) + b_n \psi_n^-(x) \),

\( \bar{\psi}(x) = \sum_n a_n \bar{\psi}_n^+(x) + b_n \bar{\psi}_n^-(x) \). Here \( A^\mu(x) \) is an operator of the electromagnetic field quantized.

The external field is taken into account in the operators \( \psi(x), \bar{\psi}(x) \), since \( \psi_n^+(x) \) and \( \psi_n^-(x) \) is exact electron and positron solutions of the Dirac equation in a field of an electromagnetic wave (the Volkov solution [5]). Substituting \( G_x^{-1} \) in (18) instead of the operator \( \alpha_x \) we find:

\[
\left( \gamma_\mu p'_\mu - m - \overline{M} - \left( 1 - \frac{1}{\Omega} \int_{\Omega} e^{ip'x'}(M_{x'} - \overline{M} + eA^\mu_\gamma \gamma^\mu)G_{x'}e^{-ip'x'}d^4x' \right) \right)^{-1} \\
\times \frac{1}{\Omega} \int_{\Omega} e^{ipx}(M_x - \overline{M} + eA^\mu_\gamma \gamma^\mu)G_x(M_x - \overline{M} + eA^\mu_\gamma \gamma^\mu)e^{-ipx}d^4x \right) u(p') = 0 .
\]

Here we have designated \( M_x = (G_x^\gamma)^{-1} - G_x^{-1} \), where \( G_x^\gamma \) Green function of the Dirac equation (without the anomalous magnetic moment) in a field of a wave and

\( \overline{M}(p') \equiv \frac{1}{\Omega} \int_{\Omega} e^{-ip'x}M_x e^{ip'x}d^4x \) . To carry out calculations with the help of the dispersion equation (28) it is possible to take the mass operator \( M_x \) of a particle in field of an electromagnetic wave calculated in works [17,18,19].

5. The conclusion. So, we have shown that two ways of calculation of the spin rotation frequency of a particle driven towards circularly polarized electromagnetic wave: Proceeding from the BMT equation and from the Dirac equation give identical results in the second order on a field strength. Let’s notice, that the equation BMT turns out from the Dirac equation at the following assumptions:

a) From Dirac hamiltonian acting in basis of electron and positron solutions carted hamiltonian is deduced acting in the basis only from the electron states. It is done to remove so-called ”zitterbewegung” [20].

b) From above mentioned hamiltonian operator equation of the spin motion is deduced. Then operators are replaced with the appropriate classical values, i.e. motion of a spin on classical trajectories is considered.

Under dispersion equation derivation it is not used any from those assumptions and coincidence of the results seems surprising.

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