A fast frequency estimation algorithm and its application in transformer terminal unit

Cheng Li¹, Jun Li and Zhiming Chen
Jiangsu Frontier Electric Power Technology Co., Ltd., Nanjing 211103, China

¹ E-mail: lc@js.sgcc.com.cn

Abstract. To accurately obtain the frequency of voltage and current signals and form the basis for the analysis of power quality in the Transformer Terminal Unit (TTU), a Hanning window and interpolation algorithm is proposed to estimate the signal frequency. The principle of the algorithm is introduced, and the computer program is written. At the same time, the voltage signals with different frequencies are numerically generated and the real voltage is measured, and the signal frequency is estimated by using the algorithm. The results reveal that the proposed method can accurately estimate the signal frequency for TTU, and the maximum error of frequency estimation is only 2 mHz when the signal frequency varies in the range of 49.8-50.2 Hz. However, the maximum error estimated by the FFT algorithm is up to 0.2 Hz. The computation time of the Hanning window and interpolation algorithm is only 1.29 times that of the Fast Fourier Transform algorithm. The work lays the foundation for accurate and rapid estimation of voltage and current signal frequency in TTU.

1. Introduction
Distribution transformer is a key electrical equipment in distribution network. To improve the reliability of power supply, it is necessary to improve the operation reliability of distribution transformer. Transformer terminal unit (TTU) [1-3] is an effective device to improve the reliability of distribution transformers. The important aspect of TTU is to measure the voltage, current, power, frequency and harmonics of distribution transformer, especially the frequency, which is an important indicator of power quality.

At present, the signal frequency estimation methods include the window and interpolation algorithm [4-6], the artificial neural network (ANN) method [7-8], and the least squares fitting method [9-10]. The ANN method uses various types of ANN to achieve the map from signal to harmonic component. With the help of the self-learning ability, the method can achieve a good performance. The least squares fitting method can estimate frequency with high accuracy, but it is relatively slow to compute. Therefore, it is particularly suitable for the occasions where the requirement on accuracy is high. The window and interpolation algorithm can decrease frequency estimation error by use of window and interpolation. Typical Hanning window and interpolation algorithm has been applied in the field of wind power and rolling bearing fault diagnosis [5-6], but there are few investigations on frequency estimation for voltage or current signals in TTU by use of the algorithm. It cannot be determined whether or not the accuracy and real-time performance of this algorithm can meet the requirements.

To fix this problem, the Hanning window and interpolation algorithm is implemented and is used to estimate frequency for the voltage signal in TTU. At the same time, the voltage signals with frequency
ranging from 49.8-50.2 Hz [11] are numerically generated and the real voltage signals are measured. The computer programs about the fast Fourier transform (FFT) algorithm and the Hanning window and interpolation algorithm are written. The latter is validated from the comparison of frequency accuracy and real-time performance between the two algorithms.

2. Hanning window and interpolation harmonic analysis algorithm

2.1. Spectral leakage and picket-fence effect
Discrete Fourier transformation (DFT) deals with discrete signals with short duration, and the result is the convolution of the DFT of the signal and the DFT of the rectangular window. In the case of asynchronous sampling, due to the influence of windowing, each harmonic component will be superimposed on each other, resulting in spectral leakage [12]. At the same time, the frequency resolution point obtained by DFT is not the frequency of the actual harmonic of the signal. Therefore, the actual harmonic component cannot be obtained directly according to the results obtained by DFT, which is the picket-fence effect. The schematic diagram of spectral leakage and picket-fence effect is shown in Figure 1.

![Figure 1. Discrete spectrum.](image)

2.2. Hanning window and interpolation method
The Hanning window can be expressed as follows:

\[
\omega(n) = \frac{1}{2} \left(1 - \cos \left(\frac{2\pi n}{N}\right)\right), \quad n=0, 1, ..., N-1
\]  

(1)

where \(N\) is the window size.

Assume that \(\Delta f\) is the frequency resolution. The fundamental frequency can be obtained by Equation (2).

\[
f = (k + \Delta k) \Delta f
\]  

(2)

where \(f\) is the fundamental frequency; \(k\) is an integer and \(y(n)\) reaches the maximum value at \(n=k\); \(Y=[y(1), y(2), ..., y(N)]\) is sequence obtained by DFT for the Hanning windowed signal; \(\Delta k\) is the frequency correction factor, \(-1<\Delta k<1\).

\(\Delta k\) can be calculated by Equation (3) [4].

\[
\Delta k = \begin{cases} 
2\left|y(k+1)\right| - \left|y(k)\right|, & \text{if } \left|y(k+1)\right| \geq \left|y(k-1)\right| \\
\left|y(k+1)\right| + \left|y(k)\right|, & \text{if } \left|y(k+1)\right| < \left|y(k-1)\right|
\end{cases}
\]  

(3)
3. **Validation of the algorithm**

3.1. **Numerically generated signal**

The voltage signals acquired by TTU are respectively simulated by using the signal containing only fundamental harmonic, the noise-free signal with harmonics and the signal with harmonics and noise.

(1) **Signal containing only fundamental harmonic**

Without loss of generality, the voltage signal is expressed as follows:

\[ x(t) = \sin(2\pi ft + \pi/3) \]  

(4)

where \( x \) is voltage in V; \( t \) is time in s; \( f \) is fundamental frequency in range 49.8-50.2 Hz \[11\] with a step size of 0.02 Hz. The sampling frequency is set to 5 kHz and the number of samples is 1200.

To clearly demonstrate the signal, a typical signal with a number of samples of 240 is displayed in Figure 2(a). The estimated frequency and the corresponding error are shown in Figure 2(b). It can be seen from Figure 2(b) that the estimated frequency is in good agreement with the exact value, and the maximum error is less than 20 μHz. The conventional FFT algorithm cannot accurately estimate signal frequency and the maximum frequency error is up to 0.2 Hz.

![Figure 2](image)

(a) Typical signal, \( f=50.2 \) Hz  
(b) Estimated frequency and corresponding error

**Figure 2.** Typical signal and calculation results for the signal containing only fundamental harmonic.

(2) **Noise-free signal with harmonics**

Without loss of generality, the voltage signal is expressed as follows. Relevant parameters remain unchanged.

\[ x(t) = \sin(2\pi ft + \pi/3) + 1.5 \times 10^{-2} \sin(6\pi ft + \pi/2) + 10^{-2} \sin(10\pi ft + \pi/5) + 10^{-2} \sin(14\pi ft + \pi/4) \]  

(5)

A typical signal with a number of samples of 240 is shown in Figure 3(a). The estimated frequency and the corresponding error are shown in Figure 3(b). Similar to the case without harmonics, the Hanning window and interpolation algorithm can estimate the signal frequency accurately, and the maximum error is less than 20 μHz.

(3) **Signal with harmonic and noise**

Without loss of generality, the voltage signal can be expressed as follows:

\[ x(t) = \sin(2\pi ft + \pi/3) + 1.5 \times 10^{-2} \sin(6\pi ft + \pi/2) + 10^{-2} \sin(10\pi ft + \pi/5) + 10^{-2} \sin(14\pi ft + \pi/4) + 0.01x \]  

(6)

where \( x \) is a Gaussian random variable with zero mean and standard deviation. The other parameters remain unchanged. To better measure the accuracy of the algorithm, for each \( f \) value, the signal is numerically generated 1000 times and the program is also performed 1000 times.

A typical signal with a number of samples of 240 is shown in Figure 4(a), and the estimated frequency and the corresponding error are shown in Figure 4(b). By comparing Figure 4(b) with Figures 2(b) and 3(b), it can be seen that noise results in an increase in the frequency error. However, the maximum error is less than 2 mHz and it is only one-hundredth of the maximum frequency deviation. Therefore, it can meet the requirement of signal frequency estimation in TTU.
The Hanning window and interpolation algorithm takes 5.92 \( \mu s \) for each run, and the FFT algorithm takes 7.62 \( \mu s \) for each run. In other words, the computation time of the Hanning window and interpolation algorithm is 1.29 times that of the FFT algorithm. Therefore, it does not significantly increase the computational burden in practical application.

![Figure 3](image1.png)
(a) Typical signal, \( f = 50.2 \) Hz  
(b) Estimated frequency and corresponding error

**Figure 3.** Typical signal and calculation results for the noise-free signal with harmonics.

![Figure 4](image2.png)
(a) Typical signal, \( f = 50.2 \) Hz  
(b) Estimated frequency and corresponding error

**Figure 4.** Typical signal and calculation results for signal with harmonics and noise.

### 3.2. Measured signal

An arbitrary waveform generator (model DG1022U, RIGOL Technologies Co., Ltd., China) is used to generate 4 V\(_{pp}\) voltage signals with a frequency range of 49.8-50.2 Hz and a step size of 0.02 Hz. A digital oscilloscope (model DS1104Z, RIGOL Technologies Co., Ltd., China) is used to acquire the signal with a sampling frequency of 5 kHz and a number of samples of 1200. Measured typical signal with a number of samples of 240 is shown in Figure 5(a). The estimated frequency and the corresponding error are shown in Figure 5(b). It can be seen from Figure 5(b) that, similar to the results in Section 2.1 for the numerically generated signal, the Hanning window and interpolation algorithm can accurately estimate the frequency for measured signal, and the maximum error is only slightly greater than 2 mHz.

To sum up, it can be seen in Section 2 that the Hanning window and interpolation algorithm can accurately estimate the signal frequency with high real-time performance and can meet the requirements on signal frequency estimation in TTU.
5. Typical measured signal and calculation results.

(a) Typical signal, \( f = 50.2 \) Hz
(b) Estimated frequency and corresponding errors

Figure 5. Typical measured signal and calculation results.

4. Conclusions
In this work, the Hanning window and interpolation algorithm is used to estimate the signal frequency in TTU. The calculation results of the simulated and measured signals show that the maximum frequency error is only about 2 mHz in the signal frequency range of 49.8-50.2 Hz. However, the maximum error estimated by the FFT algorithm is up to 0.2 Hz. And the computation time of this algorithm is only 1.29 times that of the FFT algorithm. It can meet the requirement of signal frequency estimation in TTU. This work provides a reference for accurate and rapid estimation of voltage and current frequency in TTU.

Acknowledgement
This work was financially supported by the Research Project of the Jiangsu Frontier Electric Power Technology Co., Ltd., under Grant KJ201916.

References
[1] Zheng G 2020 Research on design of monitoring terminal and load forecast of distribution transformer Yangzhou: Yangzhou University
[2] Zhang B, Wang Z, Wang W, Wang Z and Liu D 2020 Security assessment of intelligent distribution transformer terminal unit based on RBF-SVM 2020 IEEE 4th Conference on Energy Internet and Energy System Integration IEEE 4642-4346
[3] Yang Z, Shen Y, Yang F, Su L and Yang F 2019 Active fault analysis of distribution transformer service area based on intelligent transformer terminal unit 2019 IEEE Sustainable Power and Energy Conference IEEE 629-633
[4] Zeng H, Han F and Liu Y 2014 Development and current situation of Fourier analysis Modern Electronics Technique 37(3) 144
[5] Li X, Xie Z and Luo J 2018 Applications of windowed interpolation FFT algorithm in rolling bearing fault diagnosis China Mechanical Engineering 29(10) 1166-1172
[6] Qiu Y, Zhang X and Zhang C 2021 Study on sub/super-synchronous harmonic detection method in dense wind power areas Acta Energiae Solaris Sinica 42(1) 286-293
[7] Chen Z, Wang Y, Yu R, Hu C, Xue H and Wang Y 2018 Electric vehicle charging current harmonic analysis method based on trigonometric function neural network Journal of Shanghai University of Electric Power 34(6) 558-562, 566
[8] Zhang H, Wang Y, Zhang X and Wang T 2020 Double adaptive neural network and fast TLS-ESPRIT harmonic detection method and simulation Water Resources and Power 38(9) 203-205, 113
[9] Wang Y, Bai Y, Xu X, Wang J and Li L 2020 A method for high-accuracy spectrum analysis of structural response in sine-swept vibration test Space Craft Environment Engineering 37(6) 552-560

[10] Wang W, Li F and Tan K 2001 Higher-order sine fitting algorithm for dielectric loss measurement Journal of Tsinghua University (Sci & Tech) 41(9) 5-8

[11] General Administration of Quality Supervision, Inspection and Quarantine of the People's Republic of China, China National Standardizing Committee 2008 GB/T 15945-2008 Power quality-frequency deviation for power system Beijing: China Standard Press

[12] Marrero L M, Sadamori L, Dominiak S, Dersch U and Gómez J T 2019 Improving soft decoding by spectral leakage reduction in presence of narrow band interference in PLC IEEE Access 7 79491-79502