Gauging Away the Strong CP Problem

G. Aldazabal $^{1,2}$, L. E. Ibáñez $^1$ and A. M. Uranga $^1$

$^1$ Departamento de Física Teórica C-XI and Instituto de Física Teórica C-XVI, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain.

$^2$ Instituto Balseiro and Centro Atómico Bariloche, 8400 S.C. de Bariloche, (CNEA and CONICET), Argentina

Abstract: We propose a new solution to the strong-CP problem. It involves the existence of an unbroken gauged $U(1)_X$ symmetry whose gauge boson gets a Stuckelberg mass term by combining with a pseudoscalar field $\eta(x)$. The latter has axion-like couplings to $F_{\text{QCD}} \wedge F_{\text{QCD}}$. This system leads to mixed gauge anomalies and we argue that they are cancelled by the addition of an appropriate Wess-Zumino term, so that no SM fermions need to be charged under $U(1)_X$. In this setup the axion and $\theta$ parameter can be rotated away using the symmetries of the system. We discuss scenarios in which the above set of fields and couplings appear. The mechanism is quite generic, but a natural possibility is that the the $U(1)_X$ symmetry arises from bulk gauge bosons in theories with extra dimensions or string models. We show that in certain D-brane Type-II string models (with antisymmetric tensor field strength fluxes) higher dimensional Chern-Simons couplings give rise to the required $D = 4$ Wess-Zumino terms upon compactification. In one of the possible string realizations of the mechanism the $U(1)_X$ gauge boson comes from the Kaluza-Klein reduction of the eleven-dimensional metric in M-theory.

Keywords: Strong CP problem, axions, anomalies, Wess-Zumino terms.
1. The strong CP-problem

The strong-CP problem \cite{1, 2} is one of the oldest fine-tuning problems in particle physics. It is the statement that the QCD $\tilde{\theta}$-parameter appearing in the action

$$\frac{\tilde{\theta}}{32\pi^2} F_{\mu\nu}^{QCD} \tilde{F}_{\mu\nu}^{QCD} \quad (1.1)$$

is indeed a physically observable parameter. The presence of such a term (which explicitly breaks P and CP) is a consequence of the non-trivial structure of the QCD vacuum, and gives rise to computable contributions to the electric dipole moment of the neutron which are about ten orders of magnitude too large for $\tilde{\theta}$ of order one. Thus one should have $\tilde{\theta} \leq 10^{-10}$. This requires a fine-tuning which gives rise to the strong CP problem.

There are a number of proposals to solve the strong CP problem but perhaps the most elegant ones are the following two:

- **A massless quark.** It is known \cite{3, 1} that if one of the quarks is massless the $\tilde{\theta}$ phase becomes unobservable, unphysical. This is related to the fact that with a massless quark there is a global chiral $U(1)$ symmetry preserved by perturbative interactions and violated by the chiral anomaly. This is perhaps the simplest
solution and it indeed has been proposed that the u-quark mass could be zero \([4]\). However this has always been disfavoured by physicists working on effective chiral Lagrangians \([5]\). Recent lattice calculations seem also to disfavour the possibility of a massless u-quark \([6]\).

- **The axion solution**

In this solution \([7]\) the idea is to introduce a dynamical pseudoscalar field \(\eta^0\) with an axial coupling to the QCD field strength

\[
\eta^0 \frac{f_a}{f_a} F_{\mu \nu}^{QCD} \tilde{F}_{\mu \nu}^{QCD}
\]

where \(f_a\) is a mass parameter which measures the decay width of the axion \(\eta^0\). In this mechanism the pseudoscalar \(\eta^0\) (or rather \(\eta = \eta^0 + \bar{\theta}\)) becomes a dynamical ‘theta parameter’. Although the axion is perturbatively massless it acquires a periodic scalar potential at the non-perturbative level so that energy is minimized at \(\eta = 0\). Thus the system is relaxed at zero effective \(\theta\)-parameter and there is no strong CP violation. This is an attractive solution but direct searches and astrophysical and cosmological limits already rule out most of the parameter space for this model. Only a small window with \(f_a \propto 10^{10}\) GeV seems to be allowed \([2]\).

2. Gauging away the strong CP problem

2.1 The model

Our proposal has certain features from both solutions, as will become clear below. It also borrows some inspiration from string theory. The key idea is to introduce a \(U(1)\) gauge symmetry, under which ordinary quarks are neutral. More specifically, the proposal is to extend the SM with

- A pseudoscalar state \(\eta\) with axionic couplings to the QCD field strength, very much like in the axion solution.

- A \(U(1)_X\) gauge interaction whose gauge boson gets a Stuckelberg mass \(M\) by combining with the axion introduced above. This means we have a Lagrangian of the form:

\[
\mathcal{L} = \mathcal{L}_{QCD} + \eta F_{\mu \nu}^{QCD} \tilde{F}_{\mu \nu}^{QCD} - \frac{1}{4g_X^2} F_{\mu \nu}^{X} F_{\mu \nu}^{X} - \frac{M^2}{2}(A_{\mu}^{X} + \partial^{\mu} \eta)^2
\]

The mass term is gauge invariant under the transformation

\[
A_{\mu}^{X} \to A_{\mu}^{X} - \partial^{\mu} \Theta(x) ; \eta(x) \to \eta(x) + \Theta(x)
\]
Instead of a pseudoscalar $\eta$ one can equally consider its Hodge dual, a 2-index antisymmetric tensor $B_{\mu \nu}$, representing the same degrees of freedom. In this dual language one can write for the relevant Lagrangian:

$$\mathcal{L} = \mathcal{L}_{QCD} - \frac{1}{12} H^{\mu \nu \rho} H_{\mu \nu \rho} - \frac{1}{4 g_X^2} F^X_{\mu \nu} F^X_{\mu \nu} + \frac{M}{4} \epsilon^{\mu \nu \rho \sigma} B_{\mu \nu} F^X_{\rho \sigma},$$

(2.3)

where

$$H^{\mu \nu \rho} = \partial^\mu B^{\nu \rho} + \partial^\rho B^{\nu \mu} + \partial^\nu B^{\rho \mu}$$

(2.4)

and $F^X_{\mu \nu}$ is the field strength of the $U(1)_X$ gauge field. A duality transformation gives back the original Lagrangian in eq.(2.1).

As it stands this system looks problematic since the combined presence of the $U(1)_X$ transformation of the scalar $\eta(x)$ and the axionic coupling implies the presence of a mixed $U(1)_X \times SU(3)^2_{QCD}$ anomaly, as depicted in Fig.1-a.

An obvious way to cancel this anomaly is to assume the presence of chiral fermions which are coloured and charged under the $U(1)_X$. Their contribution to the chiral anomaly (Fig.1-b) may easily cancel the above anomalous term. This would be a standard $D = 4$ Green-Schwarz mechanism in which the axion gauge transformation cancels the mixed $U(1)_X \times SU(3)^2_{QCD}$ anomaly [8]. In the case of the SM this would require that at least some quark (i.e., the u-quark) is charged and chiral under $U(1)_X$. Since we need the $U(1)_X$ symmetry to be unbroken, this means that the u-quark will remain massless (zero ‘current’ mass). This we would like to avoid since we already mentioned that a massless u-quark is disfavoured by chiral Lagrangian analysis and recent lattice computations. There are additional reasons to try to avoid the physical quarks being charged under $U(1)_X$, as we will describe below.

Instead of that we propose that all quarks are neutral under $U(1)_X$ (so that they do not contribute to the mixed anomaly). We also propose that the anomaly generated by the axion $\eta(x)$ gauge transformation and the axionic coupling is cancelled by a Wess-Zumino term involving the $U(1)_X$ and QCD gauge boson fields (Fig.1-c).

Thus we are proposing a purely bosonic anomaly cancellation mechanism. Note that

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Contributions to $U(1)_X \times SU(3)^2$ anomalies: a) Green-Schwarz contribution from the exchange of the pseudoscalar $\eta$; b) Standard fermion triangle graph and c) Contribution from a Wess-Zumino term.}
\end{figure}
the \( U(1)_X \) gauge boson may be arbitrarily heavy. The only low-energy remnant arises from the existence of the Wess-Zumino term, as we discuss later on.

Let us be a bit more concrete about the required Wess-Zumino term. A Wess-Zumino term is an explicit non-gauge invariant interaction whose variation has the structure of a chiral gauge anomaly. Since an anomaly is a gauge variation which cannot be cancelled against a local counterterm, it is clear that a four-dimensional Wess-Zumino term is non-local (although its gauge variation is local) \(^1\).

The simplest way to write such terms (see e.g. \([9]\)) is as follows: Pick a five-dimensional manifold \( X_5 \) whose boundary is four-dimensional spacetime \( M_4 \). Next, extend the four-dimensional gauge field to \( X_5 \); that is, define a five-dimensional gauge field in \( X_5 \) such that it reduces to the four-dimensional one at the boundary \( M_4 \). The Wess-Zumino terms we need are of the form

\[
S_{WZ} = \int_{X_5} \left[ F_{U(1)_X} \, \text{tr} [F_{QCD}^2] \right]^{(0)}
\]

Here we are using differential forms (with wedge products implied) and the Wess-Zumino descent notation. Namely, for a closed gauge-invariant anomaly polynomial \( Y(F) \), we define \( Y = dY^{(0)} \), and \( \delta Y^{(0)} = dY^{(1)} \), where \( \delta \) denotes gauge variation.

The gauge variation of (2.5) gives

\[
\delta S_{WZ} = \int_{X_5} d \left[ F_{U(1)_X} \, \text{tr} [F_{QCD}^2] \right]^{(1)} = \int_{M_4} \left[ F_{U(1)_X} \, \text{tr} [F_{QCD}^2] \right]^{(1)}
\]

which is precisely of the form of a mixed gauge anomaly, and hence cancels against the Green-Schwarz contribution mediated by the axion.

In more pedestrian language, we may write (2.5) as

\[
S_{WZ} = \int_{X_5} A_X \, \text{tr} [F_{QCD}^2]
\]

so that its change under a \( U(1)_X \) gauge variation \( A_X \rightarrow A_X + d\lambda \) is clearly

\[
\delta S_{WZ} = \int_{X_5} d\lambda \, \text{tr} [F_{QCD}^2] = \int_{X_5} d(\lambda \, \text{tr} [F_{QCD}^2]) = \int_{M_4} \lambda \, \text{tr} [F_{QCD}^2]
\]

This is precisely the contribution required to cancel the gauge variation due to the axion shift.

In Section 3 we will present higher-dimensional setups containing Wess-Zumino terms in their effective actions. In their discussion the Wess-Zumino descent notation turns out to be a bit more convenient, so we will stick to it.

\(^1\)In theories with extra dimensions, however, non-local four-dimensional Wess-Zumino terms may arise from local higher-dimensional interactions, e.g. Chern-Simons terms. Hence Wess-Zumino terms of the kind discussed here are more natural in higher-dimensional setups, see Section 3.
2.2 Gauging away the strong CP problem

In this section we argue that the symmetries of the above system are such that the \( \theta \) parameter is unphysical.

Since the theta parameter is related to the vev of the axion, and the latter is shifted by the \( U(1)_X \) gauge symmetry, a naive proposal would be to use the symmetry to shift the axion, and hence its vev, to zero. However, this idea does not quite work, as is clear from the fact that there is no mixed \( U(1) \)-\( SU(3)^3 \) anomaly. Namely the action must be invariant under \( U(1)_X \) gauge transformations. In fact, although the \( U(1)_X \) transformation (2.2) shifts the theta parameter, a compensating shift arises from the change of the gauge potential in the Wess-Zumino term.

The system however has an additional symmetry which we have not exploited yet, and which does allow to rotate away the theta parameter. The symmetry is deeply rooted in the structure of the Wess-Zumino term. In defining it, we need to extend the 4d gauge field to a 5d gauge field on \( X_5 \); namely to define a 5d gauge field on \( X_5 \), four of whose components reduce to the physical 4d gauge field at the boundary \( M_4 \). This still leaves the freedom to choose freely the fifth component. In particular we are free to choose the constant value of the fifth component of the gauge field \( A_4 \) on the boundary. This is an additional \( U(1) \) global symmetry of the system, since this component does not appear anywhere else in the action. As is clear from (2.7), this arbitrary choice changes the effective value of the theta parameter, showing that it is indeed unphysical in the system.

More formally, this can be stated as follows. In the quantum theory, one should path integrate over the 5d gauge field. This implies that, for a fixed choice of 4d gauge field, we still path integrate over \( A_4 \) and in particular over its constant piece at the boundary. This implies that the quantum theory includes a path integral over the effective 4d theta parameter, so that its specific value is unphysical, it is not a parameter of the theory.

The above discussion can be mapped to a perhaps more familiar one by regarding a Wess-Zumino term as a piece of the (non-local) effective action arising from integrating out a chiral fermion in a theory. Specifically, the Wess-Zumino term contains the information concerning the anomaly properties of such chiral fermion, with respect to gauge and global anomalies. In our case, the Wess-Zumino term can be regarded as mimicking a chiral fermion charged under \( U(1)_X \) and \( SU(3)_{QCD} \). Indeed the \( U(1) \) global phase rotation symmetry of a chiral fermion corresponds to the global shift of the fifth component of the gauge field in the Wess-Zumino term. In particular, the anomaly of this global symmetry in the theory with the chiral fermion is encoded in the explicit change of the Wess-Zumino terms under a shift of \( A_4 \). Hence the fact that the theta parameter of a gauge theory can be removed by a phase rotation of a charged chiral fermion, corresponds to the statement that

\[ \text{We thank D. E. Kaplan for comments on this point} \]
the theta parameter can be removed by a shift of the extra component of the gauge field in the Wess-Zumino term. In other words, the anomaly of the chiral symmetry of a massless quarks turns theta into a dynamical variable (the fermion phase), over which one path integrates in the quantum theory, and whose value is therefore not a physical parameter of the theory. Analogously, the Wess-Zumino term turns theta into a dynamical variable (the gauge field component $A_4$), over which we path integrate in the quantum theory, and whose value is therefore not a physical parameter of the theory.

Hence, our mechanism to remove the theta parameter is very similar to having massless fermion, with the important different that we do not have such a dynamical fermion in the theory, but rather an explicit Wess-Zumino term which reproduces exactly the same anomaly properties. Notice that the complete structure of the theory is required for this mechanism to work. Consider starting with just the Standard Model, and add a Wess-Zumino term to eliminate the strong CP problem. In order to have a Wess-Zumino term of the appropriate kind, an additional $U(1)_X$ gauge symmetry is required. This term then generates mixed gauge anomalies; in order to cancel them without introducing fermions charged under the $U(1)_X$ symmetry, we need to implement a Green-Schwarz mechanism, namely introduce an axion coupling to QCD and mixing with the $U(1)_X$ gauge boson. Hence the model also shares some features of the axion solution to the strong CP problem. Happily the mixing of the axion with the $U(1)_X$ gauge boson allows to gauge it away and avoid inconsistency with experiment.

The key ingredient in the mechanism is the additional $U(1)_X$ gauge sector, which contains enough symmetries to set to zero both the axion field and the theta parameter by a combined gauge and global symmetry. We call this proposal gauging away the strong CP problem.

2.3 Discussion of other string models

Axion-like fields transforming under anomalous gauged $U(1)_X$ symmetries have appeared in the past in $D = 4$ string constructions [8, 10, 11]. However, one of the main differences with our present proposal is that, in the specific string models provided in the past, quarks and leptons were charged under $U(1)_X$. Thus the fermion contribution to gauge anomalies was cancelled by a Green-Schwarz mechanism. In our proposal here there are no triangle chiral anomalies: rather, the Green-Schwarz contribution cancels against an explicit Wess-Zumino term.

This proposal has a nice advantage. In previously considered string models, due to the quarks and leptons being charged, the $U(1)_X$ gauge symmetry is always eventually broken in one way or another. For example, in the heterotic $D = 4$ vacua [8, 10] such an axion is $\text{Im} S$, the pseudoscalar partner of the dilaton, present in Calabi-Yau or orbifold compactifications. In those models there is also a dilaton ($\text{Re} S$) dependent Fayet-Iliopoulos term associated to the (unique) anomalous $U(1)_X$,
which forces some scalars charged under it to get a vev \[10\]. Thus \(U(1)_X\) does not survive at low energies in heterotic models. A different kind of phenomenological string constructions is provided by the explicit D-brane models constructed in the last few years. There the situation is in principle slightly better \[11\]. There are in general more than one anomalous gauged \(U(1)\), and it is possible to construct D-brane systems in which the \(U(1)_X\) remains unbroken (e.g. D6-brane intersecting models with positive mass square for all scalars at intersections). However this is not sufficient: Since \(U(1)_X\) remains an unbroken symmetry and typically quarks are charged under it, either some quark remains massless (a disfavoured possibility as discussed above) or else the Higgs doublets are charged under the residual global \(U(1)_X\). In the latter case, \(U(1)_X\) gets broken in electroweak symmetry breaking, spoiling the solution to the CP problem; Moreover, a \(U(1)\) with a Stuckelberg mass remains as a global symmetry from the effective low-energy theory viewpoint, so its breaking would generate an (axion-like) goldstone boson, which is inconsistent with present experimental bounds.

The origin of these problems is the fact that in all these string models the quarks and leptons were generically charged under the anomalous \(U(1)_X\)’s. To avoid these complications the simplest possibility is to assume that quarks are neutral under the \(U(1)_X\) generator. This possibility was not considered before because it was not obvious how to cancel the mentioned mixed gauge anomalies. However recently, in the context of string compactifications with \(p\)-form field strength fluxes, it has been realized that cancellation of the anomaly may be achieved in a purely bosonic manner via a Wess-Zumino term. Those Wess-Zumino terms have been shown to appear in explicit D-brane configurations in \[12\].

One can consider our proposal to gauge away the strong-CP problem independently of any string theory or extra dimension argument. However natural candidates for \(U(1)_X\) bosons, in brane-world models with extra dimensions, are bulk gauge fields, which have no couplings to brane chiral fermions. In this way the usual quarks and leptons (which live on the D-braves) are neutral under \(U(1)_X\). In what follows we will describe how this structure may naturally appear in models with extra dimensions. In particular we will show how the required couplings and fields appear in explicit D-brane constructions.

3. Some examples from extra dimensions and string theory

3.1 Wess-Zumino terms from higher dimensions

It is easy to understand that Wess-Zumino terms of the kind needed above can easily appear in theories with extra dimensions. The reason for this is that non-local four-dimensional Wess-Zumino terms may arise from local operators in higher dimensions. For instance, five-dimensional Chern-Simons terms roughly of the form
$\int_{X_5} [\text{tr} F^3]^{(0)}$ have appeared in five-dimensional orbifold models \cite{13} in order to cancel the four-dimensional anomaly generated by chiral fermions at the fixed points of the orbifold (i.e. boundaries of the five-dimensional space $X_5$). From the perspective of the four-dimensional boundary such interactions behave as $D = 4$ Wess-Zumino terms.

Here we would like to show that higher dimensional Chern-Simons interactions (of a different kind) also lead to four-dimensional Wess-Zumino terms, in general compactifications of field theories with $p$-form field strength fluxes. Consider a $\left(p+4\right)$-dimensional theory with the QCD and $U(1)_X$ gauge bosons propagating in the bulk. Consider the space is compactified to four dimensions on a $p$-dimensional manifold $X_p$. Also introduce a $(p-1)$-index antisymmetric tensor field $C_{p-1}$, whose field strength $H_p$ has non-zero (quantized) flux over $X_p$,

$$\int_{X_p} H_p = k_{\text{flux}} \in \mathbb{Z} \quad (3.1)$$

and which interacts with the $U(1)_X$ and $SU(3)_c$ gauge bosons via a Chern-Simons coupling

$$S_{CS} = \int_{M_4 \times X_p} C_{p-1} A_X \text{tr} F^2_{QCD} \quad (3.2)$$

Here we are using differential form notation, with wedge products implied. Using Wess-Zumino descent notation, this may be written as

$$S_{CS} = \int_{M_4 \times X_p} C_{p-1} \left[ F_X \text{tr} F^2_{QCD} \right]^{(0)} \quad (3.3)$$

This term is not invariant under $U(1)_X$, $SU(3)_c$ gauge transformations, its variation being given by

$$\delta S_{CS} \simeq \int_{M_4 \times X_p} H_p \left[ F_X \text{tr} F^2_{QCD} \right]^{(1)} = k_{\text{flux}} \int_{M_4} \left[ F_X \text{tr} F^2_{QCD} \right]^{(1)} \quad (3.4)$$

Hence it behaves exactly as a four-dimensional Wess-Zumino term of the required kind. That is, it generates a four-dimensional gauge variation of precisely the form required to cancel the mixed $U(1)_X$-$SU(3)_c^2$ anomaly generated by the Green-Schwarz contribution.

The above ingredients, $p$-form field, interactions, etc, have been introduced in a rather ad hoc fashion. In the following we discuss that these ingredients, and this mechanism, are automatically present in large classes of string compactifications with D-branes and fluxes.
3.2 Wess-Zumino terms in string theory

In this section we show that Wess-Zumino terms of the kind discussed above are naturally present in large classes of type II string compactifications with $p$-form field strength fluxes. Such compactifications have recently received attention \cite{[14]} since they lead to other phenomenologically interesting properties, for instance they provide mechanisms of moduli stabilization.

On the other hand, we are interested in models with phenomenologically appealing spectra, hence including non-abelian gauge symmetries and chiral fermions. In order to achieve this, we consider compactifications with $D$-branes; specifically we consider type IIA compactification on a six-dimensional manifold $Y_6$, with $D6$-branes wrapped on 3-cycles on $Y_6$. Standard model gauge interactions propagate on the volumes of the different $D6$-brane stacks, while four-dimensional chiral fermions arise at intersections of the $D6$-branes \cite{[13]}. Phenomenological compactifications of this kind with $Y_6$ a six-torus (or orbifold/orientifold quotients thereof) have been studied in \cite{[16, 17, 18, 19, 20]}.

Hence we consider type IIA theory compactified on $Y_6$, with $K$ stacks of $N_a$ coincident $D6$-branes, $a = 1, \ldots, K$, wrapped on 3-cycles $[\Pi_a]$ in $Y_6$ \cite{[4]}. Quantization of the open string sectors leads to $U(N_a)$ gauge interactions propagating on the volume of the $D6_a$-branes, and chiral four-dimensional fermions in the bi-fundamental representation $(N_a, \overline{N_b})$ at the intersections of the 3-cycles $[\Pi_a], [\Pi_b]$ in $Y_6$. Such fermions arise with multiplicity given by the number of intersections $I_{ab} = [\Pi_a] \cdot [\Pi_b]$. The closed string sector contains several $p$-index antisymmetric tensor fields, the RR $p$-forms, $C_1, C_3, C_5, C_7$, which can lead to four-dimensional 1-forms upon compactification. These fields may easily play the role of the $U(1)_X$ gauge boson $A_X$ in our above mechanism. In the following we describe this in the case of the type IIA RR 1-form $A_X = C_1$.

First we need to identify the QCD axion in our setup. In Type IIA string theory the gauge fields on the $D6$-brane couple to the closed string RR modes via Chern-Simons couplings. Among them we have

$$\int_{D6_a} C_5 \, F_a ; \quad \int_{D6_a} C_3 \, \text{tr} \, F_a^2$$

where products in all equations are exterior products. From the four-dimensional

\footnote{See \cite{[21, 22]} for other phenomenological D-brane model building setups. In principle it should be possible (and interesting) to implement the gauging away of the CP problem in these alternative setups.}

\footnote{An explicit realization of these brane configurations in which $Y_6$ is a 6-torus is given in the appendix.}
perspective, defining $\eta_a = \int_{\Pi_a} C_3$, the second interaction becomes
\[
\int_{M_4} \eta_a \text{tr} F_a^2
\] (3.6)
So if QCD arises from a stack of D6-branes wrapped on a given cycle $[\Pi_a]$, then $\eta_{a0}$ is the QCD axion. For future use we notice that the $\eta_a$ degree of freedom may be also represented by its four-dimensional Hodge dual, given by $B_{a0}^2 = \int_{\tilde{\Pi}_a} C_5$, where $[\tilde{\Pi}_a]$ is the cycle dual to $[\Pi_a]$.

On the other hand, in compactifications with non-zero flux for the NS-NS 3-form field strength $H_{NS}$, the axions $\eta_a$ have Stuckelberg couplings with bulk gauge fields via the four-dimensional reduction of the type IIA ten-dimensional interaction
\[
\int_{M_4 \times Y_6} C_5 H_{NS} F_2
\] (3.7)
where $F_2$ is the field strength of the type IIA RR 1-form, $C_1$. Now, if we turn on a flux of $H_{NS}$ along $[\Pi_{a0}]$, $\int_{\Pi_{a0}} H_{NS} = k_{\text{flux}}$, the term in (3.7) gives rise to a coupling
\[
k_{\text{flux}} \int_{M_4} B_{a0}^2 F_2
\] (3.8)
Note that this coupling is analogous to the last term in (2.3). Thus, the bulk RR 1-form field plays the role of $U(1)_X$, and its gauge invariance makes the theta parameter for the QCD $U(N_{a0})$ group unobservable.

Note that in general the antisymmetric form $B_{a0}^2$ may also have similar $B_{a0}^2 \wedge F_{brane}^2$ couplings with the gauge bosons living on the D6-branes. Those appear after dimensional reduction from the first equation in (3.5), taking into account that $B_{a0}^2 = \int_{\Pi_{a0}} C_5$. These couplings are dangerous if we want the mechanism to solve the strong CP problem to work. The reason is that if both those couplings and the ones in (3.8) are present, the bulk $U(1)$ field will mix with the $U(1)$’s on the branes. Since the brane fermions (like e.g. quarks in a realistic model) are charged with respect to the brane $U(1)$’s, they will also acquire charges with respect to the bulk $U(1)$ (more correctly, the different BF couplings induce mixing among the different $U(1)$’s, so that in general $U(1)_X$ is a mixture of bulk and brane $U(1)$’s, under which the quarks are in general charged). As we discussed in previous sections, this is something we would like to avoid. As we will show in the specific example in the appendix, it is

\footnote{In general the fields $\eta_a$ are not linearly independent. In order to work with independent fields, we may choose a basis of 3-cycles $[\Sigma_i]$, decompose $[\Pi_a] = \sum_i n_{ai} [\Sigma_i]$, and define $\eta_i = \int_{\Sigma_i} C_3$. The interaction then reads $\sum_i n_{ai} \int \eta_i \text{tr} F_a^2$. We skip the subtlety for clarity.}

\footnote{More generally, the duals of $\eta_i$ are $B_i^2 = \int_{\Lambda_i} C_5$ with $\Lambda_j$ a basis of 3-cycles dual to $\Sigma_i$, namely $[\Sigma_i] \cdot [\Lambda_j] = \delta_{ij}$. In compactifications with general fluxes $\int_{\Sigma_i} H_{NS} = k_i$ there are, among others, induced couplings $k_i \int_{M_4} B_i^2 F_2$.}
easy to find D6-brane configurations in which this mixing of bulk and brane $U(1)$’s is absent.

As discussed, the combination of the couplings (3.6) and (3.8) gives rise to mixed $U(1)_X-U(N_a)^2$ anomalies. However we now show that they are cancelled by Wess-Zumino terms which are present in the theory. Indeed, the model contains Chern-Simons interactions which generate the adequate four-dimensional WZ terms. In fact, following [12] the interaction

$$\int_{D6} (C_1 + C_3 + \ldots) e^{F+B_{NS}}$$

on the D6-brane world-volume leads to a coupling

$$S_{CS} = \int_{D6_{a0}} B_{NS} C_1 \text{tr} F_{a0}^2 = \int_{D6_{a0}} B_{NS} [F_2 \text{tr} F_{a0}^2]^{(0)}$$

where $B_{NS}$ is the NS-NS 2-index antisymmetric tensor field. This is precisely of the form (3.2). Hence, its gauge variation is

$$\delta S_{CS} = \int_{D6_{a0}} H_{NS} [F_2 \text{tr} F_{a0}^2]^{(1)} = k_{\text{flux}} \int_{M_4} [F_2 \text{tr} F_{a0}^2]^{(1)}$$

and provides the required term to cancel Green-Schwarz contribution from eq.(3.6) and (3.8).

In the appendix we present concrete examples of this mechanism in explicit string constructions with standard model like spectrum.

It is possible to implement the above mechanism using four-dimensional bulk gauge modes arising from compactification of higher degree $p$-forms in IIA theory. It is also easy to describe the mechanism in type IIB compactifications with fluxes and D-branes.

An amusing feature of the particular realization we have described is the following. Note that the gauge boson $U(1)_X$ comes the Type-IIA RR 1-form, $C_1$. If we do the lift to M-theory such 1-form comes from the circle compactification of the mixed component of the eleven-dimensional metric, i.e. $C_\mu = g_{\mu 11}$. The $U(1)_X$ gauge invariance arises from local translation invariance of the circle of the 11-th dimension. Hence in the above setup the $\theta$-parameter has been gauged away using diffeomorphism invariance in M-theory.

4. Final comments

We have proposed a new solution to the strong-CP problem. The mechanism involves the gauging of a $U(1)_X$ symmetry whose boson gets a Stuckelberg mass by combining with an axion-like field $\eta(x)$. The latter has axionic couplings to $F_{QCD} \wedge F_{QCD}$. The mass of the combined system axion-gauge boson is arbitrary. The combined system
leads to mixed gauged anomalies $U(1)_X \cdot SU(3)^2_{QCD}$, cancelled by Wess-Zumino terms which should be provided by the underlying theory. The axion can be gauge away using the $U(1)_X$ gauge transformation, while the QCD theta parameter can be removed by a shift of the extra component of the gauge field in the Wess-Zumino term. Hence the strong CP problem is solved by the specific symmetries of the additional $U(1)_X$ gauge sector.

The required extra fields, a $U(1)_X$ anomalous gauge boson and a pseudoscalar which provides its longitudinal degrees of freedom, naturally appear in models with extra dimensions. This is also the case of the required Wess-Zumino terms cancelling anomalies. We have shown how certain higher dimensional Chern-Simons couplings provide us with the appropriate Wess-Zumino terms in the presence of certain antisymmetric tensor field fluxes. As an example we have shown that simple type II string compactifications in the presence of D-branes have all the ingredients for the mechanism to work if certain antisymmetric field fluxes are present. An explicit Type II toroidal example with a configuration of intersecting D6-branes yielding a semirealistic three generation model is provided in which the strong-CP problem is gauged away. It is interesting that in the string examples which we have discussed the $U(1)_X$ symmetry corresponds to the RR 1-form of Type IIA string theory. Thus this $U(1)_X$ gauge symmetry admits a geometrical interpretation as part of the reparametrization invariance of eleven-dimensional M-theory.

Acknowledgements

We thank D. Cremades, C. Fosco, F. Marchesano, J. F. García Cascales, F. Quevedo and R. Rabadán for useful discussions. A.M.U. thanks M. González for kind encouragement and support. This work has been partially supported by CI-CYT (Spain), the European Commission (grant ERBFMRX-CT96-0045), G.A work is partially supported by ANPCyT grant 03-03403. A.M.U. is supported by the Ministerio de Ciencia y Tecnología (Spain) via a Ramón y Cajal contract. G.A. thanks U.A.M for hospitality.
A. Appendix I: An explicit D-brane example

In this appendix we show how the above ideas can be implemented in a Type IIA string theory context with D6-branes at angles. We start by briefly summarizing some facts about such D6-brane models (see ref.[17, 18, 19] for details).

We consider type IIA theory compactified on a factorizable six-torus $T^6 = (T^2)_1 \times (T^2)_2 \times (T^2)_3$, product of three two-dimensional tori. Each such two-torus $(T^2)_I (I = 1, 2, 3)$, taken rectangular for simplicity, is obtained as a quotient of $\mathbb{R}^2$ by lattice translations generated by unit vectors $e_1^I = (1, 0)^I$ and $e_2^I = (0, 1)^I$.

We also introduce $K$ sets of $N_a (a = 1 \ldots , K)$ coincident D6-branes wrapped on 3-cycles of $T^6$, constructed as a factorized product of three one-cycles on each of the three two-tori $(T^2)_I$. Thus, each set of branes defines the wrapping numbers $(n^I_a, m^I_a)$ on each $(T^2)_I$, namely it spans a one-cycle in $(T^2)_I$ wrapping $n^I_a$ times around the $e_1^I$ direction and $m^I_a$ times around the $e_2^I$ direction. Therefore, the angle of these branes with the $e_1^I$ axis is given by

$$\tan \vartheta^I_a = \frac{m^I_a R^I_2}{n^I_a R^I_1} \tag{A.1}$$

where $R^I_1, R^I_2$ are the two-tori radii. Such considerations are easily generalizad to skewed two-tori.

Open strings stretching within the same set of $N_a$ D6-a-branes give rise to a $U(N_a)$ gauge group. The chiral spectrum comes from strings stretching between branes in different sets. Thus, the gauge group and chiral fermion spectrum read

$$\prod_{a=1}^K U(N_a) \overline{\sum_{a<b} I_{ab} (N_a, N_b)} \tag{A.2}$$

where $I_{ab}$

$$I_{ab} = [\Pi_a] \cdot [\Pi_b] = \prod_{i} (n^i_a m^i_b - m^i_a n^i_b) \tag{A.3}$$

counts the number of intersections.

Cancellation of RR tadpoles $\sum_a N_a[\Pi_a] = 0$ requires the wrapping numbers to satisfy the constraints

$$\begin{align*}
\sum_a N_a n^1_a n^2_a n^3_a &= 0 & \sum_a N_a n^1_a m^2_a m^3_a &= 0 \\
\sum_a N_a m^1_a n^2_a n^3_a &= 0 & \sum_a N_a m^1_a m^2_a m^3_a &= 0 \\
\sum_a N_a n^1_a m^2_a n^3_a &= 0 & \sum_a N_a m^1_a m^2_a n^3_a &= 0 \\
\sum_a N_a n^1_a m^2_a m^3_a &= 0 & \sum_a N_a m^1_a m^2_a m^3_a &= 0
\end{align*} \tag{A.4}$$

\footnote{Actually, if the wrapping numbers on a given two-torus $(n, m)$ are not coprime, the world-volume gauge group is $U(N_a/r)^r$ with $r = \gcd(n, m)$ the greatest common divisor [18].}
which ensure the cancellation of cubic non-abelian anomalies, which for the $SU(N_a)$ factor in (A.2) read

$$\sum_{b=1}^{K} I_{ab} N_b = 0 \quad (A.5)$$

Gauge fields from D6-branes, wrapped on 3-cycles, couple to RR (pseudo)- scalar $\eta_i$ fields and to their 2-form duals $B_i^{(2)}$ as discussed in section 3.2. Let us be more explicit for the $T^6$ case we are considering, and classify the different axion like fields by indicating which basis 3-cycle they arise from (see footnote 4). Namely,

$$\eta_{123} = \int_{e_1^1 \otimes e_1^2 \otimes e_1^3} C_3$$
$$\eta_I = \int_{e_1^1 \otimes e_2^I \otimes e_2^K} C_3$$
$$\eta_{IJ} = \int_{e_1^I \otimes e_1^J \otimes e_2^K} C_3$$
$$\eta = \int_{e_2^1 \otimes e_2^2 \otimes e_2^3} C_3 \quad (A.6)$$

where $I \neq J \neq K \neq I$ in the second and third rows.

The Hodge dual 2-forms are defined accordingly. For instance, the Hodge dual for $\eta_{123}$ is

$$B_{123}^{(2)} = \int_{e_2^1 \otimes e_2^2 \otimes e_2^3} C_5 \quad (A.7)$$

Thus, for D6$_a$-branes with wrappings numbers $(n_a^I, m_a^I)$ the following couplings between RR fields and brane gauge bosons are obtained

$$N_a m_a^I m_a^K \int_{M_4} B_{123}^{(2)} \wedge F_a \quad ; \quad n_b^I n_b^K \eta_{123} \wedge F_b \wedge F_b$$
$$N_a n_a^I n_a^K m_a^I \int_{M_4} B_{I}^{(2)} \wedge F_a \quad ; \quad n_b^I m_b^K m_b^K \int_{M_4} \eta_I \wedge F_b \wedge F_b$$
$$N_a n_a^I m_a^K m_a^I \int_{M_4} B_{IJ}^{(2)} \wedge F_a \quad ; \quad n_b^I n_b^J m_b^K \int_{M_4} \eta_{IJ} \wedge F_b \wedge F_b$$
$$N_a n_a^K n_a^J n_a^I \int_{M_4} B^{(2)} \wedge F_a \quad ; \quad m_b^K m_b^J m_b^I \int_{M_4} \eta \wedge F_b \wedge F_b$$

In order to achieve the mechanism for gauging away the $\theta$-parameter we must identify an axion-like field $\eta$ which couples to QCD through a term $\eta \text{tr} F_{QCD}^2$, and whose Hodge dual has BF couplings to a bulk RR 1-form field $U(1)_X$ as in section 3.2. As stressed above, couplings of the dual axion field with $U(1)$ gauge bosons from D6$_a$-branes must be avoided.

In what follows a Standard like model with these properties, obtained from D6-branes at angles, is presented. It contains six sets of D6-branes with $N_1 = 3$, leading to QCD group, $N_2 = 2$ and $N_3 = N_4 = N_5 = N_6 = 1$. Wrapping numbers are given in Table I and lead to the following, non-zero, intersection number.
\[ I_{12} = 3 = I_{56} = I_{25} \quad (A.8) \]
\[ I_{13} = I_{16} = I_{35} = -3 \]
\[ I_{24} = I_{46} = 6 = -I_{34} \]

By recalling eq. (A.2) we find the gauge group

\[ SU(3) \times SU(2) \times U(1)_Y \times U(1)'s \quad (A.9) \]

with the chiral fermion spectrum

\begin{align*}
3(3, 2, 1/6) & \to (1, -1, 0, 0, 0) + 3(3, 1, -2/3) \to (-1, 0, 1, 0, 0) + 3(3, 1, 1/3) \to (-1, 0, 0, 0, 1) + \\
3(1, 2, 1/2) & \to (0, 0, 0, 1, 0) + 3(1, 2, -1/2) \to (0, 1, 0, -1, 0, 0) + \\
3(1, 1, 1) & \to (0, 0, 1, 0, 0, -1) + 3(1, 1, -1) \to (0, 0, 0, -1, 0, 1) + \\
3(1, 1, 0) & \to (0, 0, 0, 0, -1, 0) + 3(1, 1, 0) \to (0, 0, 0, 1, 1, 0) 
\end{align*}

where undelining means permutation (notice multiwrapping on third torus for \( N_4 \)).

Hypercharge is defined as a linear combination of \( U(1) \) generators

\[ Y = -\left( \frac{Q_1}{3} + \frac{Q_2}{2} + Q_5 + Q_6 \right) \quad (A.10) \]

where \( Q_a \) is the \( U(1) \) generator in \( U(N_a) \).

From the wrapping numbers presented in Table 1, it follows that \( \eta_{123} \) has an axion coupling \(^8\) to QCD through the first term in (A.8)

\[ \int_{M_4} \eta_{123} \wedge \text{tr} \left( F_{QCD} \wedge F_{QCD} \right) \quad (A.11) \]

Moreover, the dual 2-form \( B_{123}^{(2)} \) has no BF coupling to brane gauge fields, since \( m^1_a m^2_a m^3_a = 0 \) for all \( a \), \( (a = 1, \ldots, 6) \).

Finally, by turning on \( k_{\text{flux}} \) units of flux \(^9\) for \( H_{NS} \) along the directions \( e_1^1, e_1^2, e_1^3 \) the dual 2-form has the required BF coupling with the bulk type IIA RR 1-form

\(^8\) The careful reader may notice that the QCD axion field is actually a linear combination of \( \eta_{123} \) and other RR fields; however, this is enough to gauge away the QCD \( \theta \)-parameter, since the gauge \( U(1)_Y \) symmetry used below shifts the total axion via its shift on \( \eta_{123} \).

\(^9\) The conditions on allowed field-strength \( p \)-form fluxes have been well studied in the literature \[^{14}\]. In general it is possible to complement the above choice of NS-NS flux with additional RR fluxes so that the complete set of fluxes satisfies the closed string equations of motion (at least for certain choices of geometric/dilaton moduli). We skip this discussion since it is irrelevant for our main point.
Therefore the above model is an explicit stringy example where the gauging away \( \theta \) mechanism proposed in the article is realized.

This is quite remarkable since the model we have discussed is indeed really close to the structure of the Standard Model. On one hand the spectrum contains the minimal Standard Model fermions, and a not too large set of extra fields, namely six extra doublets and some zero hypercharge singlets. This extra matter is expected from tadpole cancellation (see [23] for a discussion). Extra doublets and at least part of these singlets may become massive through a Higgs mechanism, and can be absent in other constructions. The gauge group is also close to the Standard Model, plus some \( U(1) \)'s. However, several of the \( U(1) \)'s become massive due to the BF couplings (A.8) [18, 23]. This makes some of the D6-brane \( U(1) \)'s massive, so they disappear from the low-energy physics. Fortunately, the linear combination (A.10) which we identified with hypercharge in the above model can be checked to be free of BF couplings, hence it remains massless and part of the Standard Model gauge group.
References

[1] C. Callan, R. Dashen and D. Gross, “The structure of the gauge theory vacuum”, Phys. Lett. B61(1976)334 ;
R. Jackiw and C. Rebbi, “Vacuum periodicity in a Yang-Mills quantum theory”, Phys.Rev.Lett.37(1976)172.

[2] For some reviews and references see e.g.:
J.E. Kim, “Light pseudoscalars, particle physics and cosmology”, Phys.Rep.150(1987)1;
R.P. Peccei, “ Reflections on the Strong CP Problem, hep-ph/9807514;
M. Dine, “ TASI Lectures on the Strong CP Problem”, hep-ph/0011376.

[3] G. ‘t Hooft, “Computation of the quantum effects due to a four-dimensional pseudoparticle”, Phys.Rev.D14(1976)3432.

[4] D. Kaplan and A. Manohar, “ Current mass ratios of the light quarks”, Phys.Rev.Lett.56(1986)2004;
A. Cohen, D. Kaplan and A. Nelson, “ Testing $m_u = 0$ on the lattice”, JHEP-9911(1999)027, hep-lat/9909091.

[5] H. Leutwyler, “ The ratios of the light quark masses”, Phys.Lett.B378(1996)313, hep-ph/9602366,

[6] UKQCD Collaboration:A. Irving, C. McNeile, C. Michael, K. Sharkey, “ Is the up-quark massless?”, Phys.Lett.B518(2001)243, hep-lat/0107023;
D. Nelson, G. Fleming and W. Kilcup, “ Is strong CP due to a massless up quark?”, hep-lat/0112029.

[7] R. Peccei and H. Quinn, “CP conservation in the presence of instantons”, Phys.Rev.Lett.38(1977)1440; “Constraints imposed by CP conservation in the presence of instantons”, Phys.Rev.D16(1977)1791;
S. Weinberg, “A new light boson?”, Phys.Rev.Lett.40(1978)223;
F. Wilczek, “ Problem of strong P and T invariance in the presence of instantons”, Phys.Rev.Lett.40(1978)279;
J. E. Kim, “Weak interaction singlet and strong CP invariance”, Phys.Rev.Lett.43(1979)103;
M. Shifman, A. Vainstein and V. Zakharov, “Can confinement ensure natural CP invariance of strong interactions?”, Nucl.Phys.B166(1980)493;
M. Dine, W. Fischler and M. Srednicki, “A simple solution to the strong CP problem with a harmless axion”, Phys.Lett.B104(1981)104.

[8] M. Dine, N. Seiberg and E. Witten, Nucl. Phys. B289 (1987) 585;
J. Atick, L. Dixon and A. Sen, Nucl. Phys. B292 (1987) 109;
M. Dine, I. Ichinose and N. Seiberg, Nucl. Phys. B293 (1987) 253.

[9] J. L. Mañes, Differential geometric construction of the gauged Wess-Zumino action”, Nucl. Phys. B250 (1985) 369.
[10] L.E. Ibáñez, H.P. Nilles and F. Quevedo, “Orbifolds and Wilson lines”,
Phys.Lett.B187(1987)187;
L.E. Ibáñez, J.E. Kim, H.P. Nilles and F. Quevedo, “Orbifold compactifications with	hree families of $SU(3) \times SU(2) \times U(1)^N$”, Phys.Lett.B191(1987)282;
J.A. Casas and C. Muñoz, “Three generation $SU(3) \times SU(2) \times U(1)_Y \times U(1)$
orbifold models through Fayet-Iliopoulos terms”, Phys.Lett.B209(1988)214;
A. Font, L.E. Ibáñez, H.P. Nilles and F. Quevedo, Yukawa couplings in degenerate
orbifolds: towards a realistic $SU(3) \times SU(2) \times U(1)$ superstring”,
Phys.Lett.210B(1988)101;
J.A. Casas, E.K. Katehou and C. Muñoz, “U(1) charges in orbifolds: anomaly
cancellation and phenomenological consequences”, Nucl.Phys.B317(1989)317;
I. Antoniadis, J. Ellis, J. Hagelin and D.V. Nanopoulos, “The flipped $SU(5) \times U(1)$
string model revamped”, Phys.Lett.B231(1989)65;
A. Faraggi, “A new standard-like model in the four-dimensional free fermionic string
formulation”, Phys.Lett.B278(1992)131;
L.E. Ibáñez, “Computing the weak mixing angle from anomaly cancellation”,
Phys.Lett.B303(1993)55.

[11] L. E. Ibáñez, R. Rabadán and A. M. Uranga, Anomalous U(1)’s in type I and type
IIb $D = 4, N = 1$ string vacua, Nucl. Phys. B 542 (1999) 112 hep-th/9808139;
E. Poppitz, On the one loop Fayet-Iliopoulos term in chiral four dimensional type I
orbifolds, Nucl. Phys. B 542 (1999) 31 hep-th/9810010;
I. Antoniadis, C. Bachas and E. Dudas, Gauge couplings in four-dimensional type I
string orbifolds, Nucl. Phys. B 560 (1999) 93 hep-th/9906039;
L. E. Ibáñez and F. Quevedo, Anomalous U(1)’s and proton stability in brane
models, JHEP 9910 (1999) 001 hep-ph/9908305;
M. Klein, Anomaly cancellation in $D = 4$, $N = 1$ orientifolds and linear/chiral
multiplet duality”, Nucl. Phys. B 569 (2000) 362 , hep-th/9910143.;
C. Scrucca and M. Serone, “Gauge and gravitational anomalies in D=4 N=1
orientifolds”, JHEP 9912(1999)024;
G. Aldazabal, L. E. Ibáñez, F. Quevedo and A. M. Uranga, D-branes at
singularities: A bottom-up approach to the string embedding of the standard model,
JHEP0008(2000)002, hep-th/0005067;
I. Antoniadis, E. Kiritsis and J. Rizos, Anomalous U(1)s in Type I superstring
vacua, hep-th/0204153.

[12] A. M. Uranga, ‘D-brane, fluxes and chirality’, JHEP 0204 (2002) 016,
hep-th/0201221.

[13] C. A. Scrucca, M. Serone, L. Silvestrini, F. Zwirner, ‘Anomalies in orbifold field
theories’, Phys. Lett. B525 (2002) 169, hep-th/0110073;
L. Pilo, A. Riotto, ‘On anomalies in orbifold theories’, hep-th/0202144;
R. Barbieri, R. Contino, P. Creminelli, R. Rattazzi, C.A. Scrucca, ‘Anomalies,
Fayet-Iliopoulos terms and the consistency of orbifold field theories’, hep-th/0203039;
S. Groot Nibbelink, H. P. Nilles, M. Olechowski, ‘Instabilities of bulk fields and anomalies on orbifolds’, hep-th/0205012.

[14] J. Polchinski, A. Strominger, ‘New vacua for type II string theory’, Phys. Lett. B388 (1996) 736, hep-th/9510227;
K. Becker, M. Becker, ‘M theory on eight manifolds’, Nucl. Phys. B477 (1996) 155, hep-th/9605053;
J. Michelson, ‘Compactifications of type IIB strings to four-dimensions with nontrivial classical potential’, Nucl. Phys. B495 (1997) 127, hep-th/9610151;
S. Gukov, C. Vafa, E. Witten, ‘CFT’s from Calabi-Yau four folds’, Nucl. Phys. B584 (2000) 69, Erratum-ibid. B608 (2001) 477, hep-th/9906070;
K. Dasgupta, G. Rajesh, S. Sethi, ‘M theory, orientifolds and G-flux’, JHEP 9908 (1999) 023, hep-th/9908088;
T. R. Taylor, C. Vafa, ‘R R flux on Calabi-Yau and partial supersymmetry breaking’, Phys. Lett. B474 (2000) 130, hep-th/9912152;
B. R. Greene, K. Schalm, G. Shiu, ‘Warped compactifications in M and F theory’, Nucl. Phys. B584 (2000) 480, hep-th/0004103;
G. Curio, A. Klemm, D. Lust, S. Theisen, ‘On the vacuum structure of type II string compactifications on Calabi-Yau spaces with H fluxes’, Nucl. Phys. B609 (2001) 3, hep-th/0012213;
M. Haack, J. Louis, ‘M theory compactified on Calabi-Yau fourfolds with background flux’, Phys. Lett. B507 (2001) 296, hep-th/0103068;
S. B. Giddings, S. Kachru, J. Polchinski, ‘Hierarchies from fluxes in string compactifications’, hep-th/0105097;
S. Kachru, M. Schulz, S. Trivedi, ‘Moduli stabilization from fluxes in a simple iib orientifield’, hep-th/0201028;
A. R. Frey, J. Polchinski, ‘N=3 warped compactifications’, hep-th/0201029;
J. Louis, A. Micu, ‘Type 2 theories compactified on Calabi-Yau threefolds in the presence of background fluxes’, hep-th/0202168;
K. Becker, M. Becker, M. Haack, J. Louis, ‘Supersymmetry breaking and alpha-prime corrections to flux induced potentials’, hep-th/0204254.

[15] M. Berkooz, M. R. Douglas, R. G. Leigh, ‘Branes intersecting at angles’, Nucl. Phys. B480 (1996) 265, hep-th/9606139.

[16] R. Blumenhagen, L. Gorlich, B. Kors, ‘Supersymmetric 4-D orientifolds of type IIA with D6-branes at angles’, JHEP 0001 (2000) 040, hep-th/9912204;
S. Forste, G. Honecker, R. Schreyer, ‘Supersymmetric Z(N) x Z(M) orientifolds in 4-D with D branes at angles’, Nucl. Phys. B593 (2001) 127, hep-th/0008250.

[17] R. Blumenhagen, L. Goerlich, B. Kors, D. Lust, ‘Noncommutative compactifications of type I strings on tori with magnetic background flux’, JHEP 0010 (2000) 006, hep-th/0007024.
[18] G. Aldazabal, S. Franco, L. E. Ibanez, R. Rabdan, A. M. Uranga, ‘D=4 chiral string compactifications from intersecting branes’, J. Math. Phys. 42 (2001) 3103, hep-th/0011073; ‘Intersecting brane worlds’, JHEP 0102 (2001) 047, hep-ph/0011132.

[19] R. Blumenhagen, B. Kors, D. Lust, ‘Type I strings with F flux and B flux’, JHEP 0102 (2001) 030, hep-th/0012156; L. E. Ibanez, F. Marchesano, R. Rabdan, ‘Getting just the standard model at intersecting branes’, JHEP 0111 (2001) 002, hep-th/0105155; S. Forste, G. Honecker, R. Schreyer, ‘Orientifolds with branes at angles’, JHEP 0106 (2001) 004, hep-th/0105208; R. Blumenhagen, B. Kors, D. Lust, T. Ott, ‘The standard model from stable intersecting brane world orbifolds’, Nucl. Phys. B616 (2001) 3, hep-th/0107138; D. Cremades, L. E. Ibanez, F. Marchesano, ‘SUSY Quivers, Intersecting Branes and the Modest Hierarchy Problem’, hep-th/0201205; ‘Intersecting brane models of particle physics and the Higgs mechanism’ hep-th/0203160; ‘Standard Model at Intersecting D5-branes: lowering the string scale’, hep-th/0205074; C. Kokorelis, ‘GUT model hierarchies from intersecting branes’, hep-th/0203187; ‘New standard model vacua from intersecting branes’, hep-th/0205147.

[20] M. Cvetic, G. Shiu, A. M. Uranga, ‘Chiral four-dimensional N=1 supersymmetric type 2A orientifolds from intersecting D6 branes’, Nucl. Phys. B615 (2001) 3, hep-th/0107166; ‘Three family supersymmetric standard - like models from intersecting brane worlds’, Phys. Rev. Lett. 87 (2001) 201801, hep-th/0107143.

[21] G. Aldazabal, L. E. Ibáñez, F. Quevedo, ‘Standard-like models with broken supersymmetry from type I string vacua’, JHEP 0001 (2000) 031, hep-th/9909172; ‘A D-brane alternative to the MSSM’, JHEP 0002 (2000) 015, hep-ph/0001083; G. Aldazabal, L. E. Ibáñez, F. Quevedo, A. M. Uranga, ‘D-branes at singularities: A Bottom up approach to the string embedding of the standard model’, JHEP 0008 (2000) 002, hep-th/0005067; M. Cvetic, A. M. Uranga, J. Wang, ‘Discrete Wilson lines in N=1 D = 4 type IIB orientifolds: A Systematic exploration for Z(6) orientifold’, Nucl. Phys. B595 (2001) 63, hep-th/0010091; D. Berenstein, V. Jejjala, R. G. Leigh, ‘The Standard model on a D-brane’, Phys. Rev. Lett. 88 (2002) 071602, hep-ph/0105042; D. Bailin, G. V. Kraniotis, A. Love, ‘Searching for string theories of the standard model’, hep-th/0108127; L. F. Alday, G. Aldazabal, ‘In quest of just the Standard Model on D-branes at a singularity’, JHEP 0205 (2002) 022, hep-th/0203129.

[22] I. Antoniadis, E. Kiritsis, T. N. Tomaras, ‘A D-brane alternative to unification’, Phys. Lett. B486 (2000) 186, hep-ph/0004214.

[23] L. E. Ibanez, F. Marchesano, R. Rabdan, as in [19].