Numerical Slug Flow Model of Curved Pipes with Experimental Validation

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ABSTRACT: The research for gas—liquid two-phase flow is very important for flow assurance and flow stability of chemical transportation. In terms of transportation pipelines, the curved section is a very significant part. Therefore, the present work proposes a transient slug flow model for the curve pipes, and we conducted some experiments to validate it. This slug flow model is a four-equation model that contains mass and momentum balances with the closure relations. The normal two-dimensional rectangular coordinate system is simplified to the one-dimensional polar coordinate system, which will make the simulation faster and easier. The common flow parameters, such as the pressure profile along the pipeline, real-time pressure, and liquid holdup, are calculated in this model. Three groups of experiments with three different pipe curvatures were carried out to validate this model; the experiments were under the same conditions as those of the model calculations. The transient pressure and liquid holdup were measured at the middle of the curved pipe. The experimental data are compared to the calculated results, and there are error analyses of pressure and liquid holdup that are made to review the model’s performance. The analyses show that a large proportion of the pressure errors is within 10%, and most of the liquid holdup errors are within 0.1. The comparisons between the model and experiments show good agreement.

1. INTRODUCTION

The gas—liquid two-phase flow occurs in the nuclear reactors, oil and gas transportation pipelines,1,2 chemical transportation pipelines, and heat exchangers. In the oil and chemical engineering field, the study of the gas—liquid two-phase flow is mainly on transportation of different chemical fluids. Almost all transportation pipelines have a curved section due to the complex structure of different terrains, which will cause some flow pattern changes in contrast with straight sections. Various two-phase flow patterns exist in the pipe, and the most unsteady one is the slug flow, which has influence on the efficiency of transportation and reduces the service life of pipes. In this flow pattern, the liquid phase exists intermittently in the form of a liquid slug and liquid film along the pipe, and the gas phase exists as an elongated bubble and tiny bubbles along the pipe. This complicated flow pattern could appear in all the horizontal, inclined, and curved pipelines, and it could cause fluctuations of liquid holdup and pressure. Therefore, the accurate prediction of the slug flow characteristics of the curved section is of great significance in chemical transportation.

The development of the gas—liquid two-phase flow has been an important research for many years. In the earlier periods, studies of two-phase flow in the vertical pipe were concentrated on the partition of different flow patterns by experiments, and then researchers obtained several steady-state models about pressure, density, and liquid holdup along the pipe.3−8 Later on, some researchers proposed different flow patterns, new flow pattern transition criteria, and new steady-state models to predict liquid holdup and pressure drop, and they conducted experiments to validate these models.9−15

As the number of requirements for the study on dynamic changes of slug flow increases, transient models were developed for its two different types, which are the slug tracking model and two-fluid model.

Barnea and Taitel16 came up with a slug tracking model to simulate the movement of the bubbles by replacing the unit cell with the bubble translational velocity. Then, Zheng et al.17 added the liquid mass balance on the kinematic model created by Barnea and Taitel;16 their model can describe all the simplified slug characteristics of a slug unit in a hilly pipe. Nydal and Banerjee18 came up with a Lagrangian slug tracking model. There are mass and momentum balances on each slug and bubble unit in their model, which can calculate the pressure and the slug velocities. However, the aeration of the slug was ignored in this model. Taitel and Barnea19 modified the slug tracking model. In their model, the momentum equations assumed a local equilibrium force balance for each cell. It can predict the slug growth and decay under transient conditions. Ujiang et al.20 obtained a new slug tracking model by applying the liquid slug and the elongated bubble as different objects, and a quasi-steady momentum equation was used to obtain the pressure in the model. The distribution of slug length at the inlet is highly sensitive in their model. Franca et al.21 developed a model that
can estimate the convection heat transfer in a horizontal slug flow. The average convective heat transfer is calculated based on the unit cell model. Kjeldby et al.\textsuperscript{22} proposed a slug tracking model that is called a slug capturing technique. Their model is mainly used in capturing the slug in horizontal and nearly horizontal pipes, which can present the properties of hydrodynamic slugs in the pipes. However, it is not applied to vertical flows. Medina et al.\textsuperscript{23} coupled heat transfer-governing equations to the slug tracking model with energy balance in developed, nonboiling horizontal two-phase slug flows.

The two-fluid model was first established by Ishii et al.,\textsuperscript{24} and then, it was incorporated in several industrial codes such as the OLGA model. This OLGA model was developed by Bendiksen et al.,\textsuperscript{25} and it can simulate transient pressure in a two-phase slug flow. Based on the two-fluid model, De Henau\textsuperscript{26} proposed a new method for the drag coefficient and the virtual mass force of the slug flow. What is more, simulations and experimental results were compared in three slug flow cases of horizontal pipes and inclined pipes. Lima and Yeung\textsuperscript{27} offered a two-fluid model to get the volumetric liquid holdup, pressure, and velocities of the gas and liquid phases. Issa and Kempf\textsuperscript{28} proposed the one-dimensional transient two-fluid model that can capture slug initiation. They made numerical simulations at the unsteady stratified flow in which instabilities could grow to form slugs. The numerical results showed that the slug flow can be simulated by their model, and continuous slugs can be generated automatically. Chang and Liu\textsuperscript{29} introduced additional dissipation terms into the numerical flux. This scheme, called AUSM +, can be applied to the liquid flow and gas flow. Danielson et al.\textsuperscript{30} came up with a transient three-phase 1D simulator that was developed for capturing transient slug flow. It is a multifluid model consisting of mass, momentum, and energy-conservative equations. Fullmer et al.\textsuperscript{31} found that the two-fluid model with their interfacial pressure model is unconditionally stable. Recently, Panicker et al.\textsuperscript{32} proposed a closure relation based on the research of Biesheuvel and Gorissen.\textsuperscript{33} This closure relation has a dispersion coefficient determined to ensure the hyperbolic attribute of the two-fluid model, which makes the convergence of the two-fluid model to be well achieved without adding another pressure term.

However, all these models were concentrated on the horizontal, vertical, and inclined situations, and only few research studies focused on the slug flow in the curved pipe, which almost exists in all transportation pipelines as shown in Figure 1. In the present work, a slug flow model is proposed for the curved sections, which can be applied to chemical transportation. It can predict the real-time pressure profile, transient pressure, and liquid holdup of the slug flow. The model is formed by mass and momentum equations for each phase with closure relations. A polar coordinate system is used instead of a rectangular coordinate system to simplify the model to a 1D geometry. Three groups of curved-pipe slug flow experiments were carried out in the Key Laboratory of Petroleum Engineering of China University of Petroleum-Beijing. Then, the calculation results are compared with experimental data in order to validate the model.

2. COMPUTATIONAL METHODS

The governing equations are for an isothermal transient slug flow; thus, the heat transfer between the liquid and gas has been ignored. Only four equations are required without energy balances, which are the mass and momentum balances of the gas phase and liquid phase. The gas mass balance equation is

\[
\frac{\partial \rho_{A} \alpha_{g}}{\partial t} + \frac{\partial \rho_{A} \alpha_{g} u_{g}}{\partial x} + \frac{\partial \rho_{A} \alpha_{g} u_{g}}{\partial y} = 0
\]  

(1)

The gas momentum balance equation is

\[
\frac{\partial \rho_{A} \alpha_{g} u_{g}}{\partial t} + \frac{\partial \rho_{A} \alpha_{g} u_{g}^2}{\partial x} + \frac{\partial \rho_{A} \alpha_{g} u_{g}^2}{\partial y} = -\alpha_{g} \frac{\partial p}{\partial x} - \alpha_{g} \frac{\partial p}{\partial y} - \frac{\tau_{S} S}{A} - \frac{\tau_{S} S}{A} + F_{D} - \alpha_{g} \rho_{g} g \sin \theta
\]  

(2)

The liquid mass balance equation is

\[
\frac{\partial \rho_{L} \alpha_{l}}{\partial t} + \frac{\partial \rho_{L} \alpha_{l} u_{l}}{\partial x} + \frac{\partial \rho_{L} \alpha_{l} u_{l}}{\partial y} = 0
\]  

(3)

The liquid momentum balance equation is

\[
\frac{\partial \rho_{L} \alpha_{l} u_{l}}{\partial t} + \frac{\partial \rho_{L} \alpha_{l} u_{l}^2}{\partial x} + \frac{\partial \rho_{L} \alpha_{l} u_{l}^2}{\partial y} = -\alpha_{l} \frac{\partial p}{\partial x} - \alpha_{l} \frac{\partial p}{\partial y} - \frac{\tau_{S} S}{A} + \frac{\tau_{S} S}{A} - F_{D} - \alpha_{l} \rho_{g} g \sin \theta - \alpha_{g} \rho_{g} \frac{\partial h}{\partial x} \cos \theta
\]  

(4)

The last term on the right side of eq 4 indicates the hydrostatic pressure in the liquid.

The slug model of the curved pipe is two-dimensional in a rectangular coordinate system. In order to make it easier to solve, it can be simplified to a one-dimensional model. The rectangular coordinate system is converted to a polar coordinate system by using the trait of the curve. The relations between the two coordinate systems are

\[
x = R \cos \theta
\]  

(5)

\[
y = R \sin \theta
\]  

(6)

Therefore, the one-dimensional governing equations are written as follows:

Gas mass balance:

\[
\frac{\partial \rho_{A} \alpha_{g}}{\partial t} + \frac{\partial \rho_{A} \alpha_{g} u_{g}}{\partial (b \cos \theta)} + \frac{\partial \rho_{A} \alpha_{g} u_{g}}{\partial (b \sin \theta)} = 0
\]  

(7)
The volume balance in two-phase flow models:

\[ \alpha_g + \alpha_l = 1 \]  

2.1. Closure Relations. 2.1.1. Shear Stress. As calculated by Issa and Kempf, the Taitel and Dukler relations and Spedding and Hand relations are more accurate in inclined pipes, and the Walls relation is more accurate in vertical pipes. In the present study, the curved pipe is divided into several segments by the angle \( \theta \), which are similar to the inclined segments, so the Taitel and Dukler relation is adopted. The liquid–wall, gas–wall, and gas–liquid interfacial shear stresses are

\[ \tau_w = \frac{f_g \rho_b u_g^2}{2}, \quad \tau_k = \frac{f_g \rho_b u_k^2}{2}, \quad \tau_l = \frac{f_g \rho_b (u_k - u_l)^2}{2} \]  

2.1.2. Friction Factor. The relations of friction factor \( f \) are shown in Table 1. In Table 1, the Reynolds numbers of the gas, liquid, and interface are defined as

\[ \text{Re}_g = \frac{d u_g \rho_b}{\mu_g}, \quad \text{Re}_l = \frac{d u_l \rho_l}{\mu_l}, \quad \text{Re}_i = \frac{d u_i}{\mu_o} \]  

The wetted perimeter of the gas and wall (as shown in Figure 2): 

\[ S_w = (2\pi - \beta) \frac{D}{2} \]  

The perimeter between the liquid and wall:

\[ A \frac{df}{dt} + B \frac{df}{dx} + C \frac{df}{dy} = A \frac{df}{dt} + B' \frac{df}{dy} = C \]  

Table 1. Relations of Friction Factor

| conditions | equations |
|------------|-----------|
| gas–wall | \( \text{Re}_g \leq 2100 \) | \( f_g = 16/\text{Re}_g \) |
| liquid–wall | \( \text{Re}_g \geq 2100 \) | \( f_g = 0.046(\text{Re}_g)^{-0.22} \) |
| gas–liquid | \( \text{Re}_l \leq 2100 \) | \( f_l = 24/\text{Re}_l \) |
| liquid–liquid | \( \text{Re}_l \geq 2100 \) | \( f_l = 0.0262(\alpha \text{Re}_l)^{-0.199} \) |

Figure 2. Cross-sectional area and wetted perimeters for the two-phase flow.

\[ S_l = \beta \frac{D}{2} \]  

The hydraulic diameters of the gas and liquid are

\[ d_k = \frac{4A_l}{S_k + S_l}, \quad d_l = \frac{4A_l}{S_l} \]  

2.1.3. Drag Force. The drag force of bubbles in the liquid slug is given by

\[ F_D = \frac{3 \rho_b C_D (u_k - u_l) u_k - u_l}{8 \tau_b} \]  

The drag coefficient \( C_D \) is different in the stratified flow and in the turbulent flow:

In the stratified flow:

\[ C_D = \frac{24}{\text{Re}_w} \]  

In the turbulent flow:

\[ C_D = \frac{24}{\text{Re}_w} (1 + 0.1 \text{Re}_w^{0.75}) \]  

As this work mainly focuses on the slug flow, the \( C_D \) for the turbulent flow is used in the model. \( \text{Re}_w \) is defined as

\[ \text{Re}_w = \frac{2 \pi \rho_b |u_k - u_l|}{\mu_m} \]  

In this equation, \( \tau_b \) and \( \mu_m \) are the radius of bubble and the viscosity of mixture:

\[ \tau_b = \left[ \frac{3}{32} \left( \frac{\alpha_b V}{\pi} \right)^{1/3} \right], \quad \mu_m = \mu_m (1 - \alpha_b)^{-2.5(\alpha_b + 0.4\mu_l)/\mu_l + \mu_i} \]  

3. NUMERICAL SOLUTION

The governing equations can be written in the matrix form:

\[ A \frac{df}{dt} + B \frac{df}{dx} + C \frac{df}{dy} = A \frac{df}{dt} + B' \frac{df}{dy} = C \]
B′ = \left(\frac{1}{-b \sin \theta} + \frac{1}{b \cos \theta}\right) B = \frac{\sin \theta - \cos \theta}{b \sin \theta \cos \theta} B \tag{24}

where the vector \( \phi \) is given as

\[ \phi = [p \, \alpha \, u_b \, u_l]^T \tag{25} \]

The matrices of \( A, B, B', \) and \( C \) are written respectively as

\[
A = \begin{bmatrix}
\frac{\alpha_g}{c^2} & \rho_b & 0 & 0 \\
0 & -\rho_b & 0 & 0 \\
0 & \rho_b \alpha_g u_b & \rho_b \alpha_g u_b & 0 \\
0 & 0 & -\rho_l u_l & 0 \\
0 & 0 & 0 & \rho_l \alpha_l
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
\frac{\alpha_g}{c^2} & \rho_b \alpha_g u_b & \rho_b \alpha_g u_b & 0 \\
0 & -\rho_b & 0 & 0 \\
0 & \rho_b \alpha_g u_b & \rho_b \alpha_g u_b & 0 \\
0 & 0 & -\rho_l u_l & 0 \\
0 & 0 & 0 & \rho_l \alpha_l
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
0 \\
0 \\
-\tau_{S_b} \frac{S}{A} + F_D - \alpha_{\theta} \rho_g \sin \theta \\
-\tau_{S_l} \frac{S}{A} + F_D - \alpha_{\theta} \rho_g \sin \theta
\end{bmatrix}
\]

\[
\begin{align*}
3.1. \text{Discretization.} & \quad \text{The partial differential equations can be solved by several numerical methods. The present study adopts a finite difference method, which is widely used in different situations, and it is easier and faster than other methods. Discretizing eqs 1–6 with an implicit first-order scheme, the discretizations are written as the following:} \\
\text{Gas mass balance discretization:} & \quad \frac{\alpha_g}{c^2} \left(\frac{u_{b,i+1} - u_{b,i}}{\Delta t} + u_p \frac{P_{g,i+1} - P_{g,i}}{b \sin \theta \Delta \theta} + u_g \frac{P_{g,i+1} - P_{g,i}}{b \cos \theta \Delta \theta} \right) \\
& \quad + \frac{\alpha_g}{c^2} \left(\frac{u_{b,i+1} - u_{b,i}}{\Delta t} + \rho_{g,i+1} - \rho_{g,i} \right) \\
& \quad + \rho_{g,i} \frac{u_{b,i+1} - u_{b,i}}{b \cos \theta \Delta \theta} + \rho_{g,i} \frac{u_{g,i+1} - u_{g,i}}{b \sin \theta \Delta \theta} \\
& \quad + \rho_{g,i} \frac{u_{b,i+1} - \rho_{g,i}}{b \cos \theta \Delta \theta} - \rho_{g,i} \frac{u_{b,i+1} - \rho_{g,i}}{b \cos \theta \Delta \theta} = 0 \tag{29}
\end{align*}
\]

\[
\begin{align*}
\text{Gas momentum balance discretization:} & \quad \rho_g \frac{u_{b,i+1} - u_{b,i}}{\Delta t} + u_p \frac{P_{g,i+1} - P_{g,i}}{b \sin \theta \Delta \theta} \\
& \quad + \rho_{g,i} \frac{u_{b,i+1} - u_{g,i}}{b \cos \theta \Delta \theta} = -\alpha_{\theta} \frac{P_{g,i+1} - P_{g,i}}{b \cos \theta \Delta \theta} \\
& \quad - \alpha_{\theta} \rho \frac{u_{g,i+1} - \rho_{g,i}}{b \cos \theta \Delta \theta} - \frac{\tau_{S_b}}{A} + \frac{\tau_{S_l}}{A} + F_D - \alpha_{\theta} \rho \sin \theta \tag{30}
\end{align*}
\]

Liquid mass balance discretization:

\[
\frac{\alpha_{l,i+1} - \alpha_{l,i}}{\Delta t} + \frac{u_{l,i+1} - u_{l,i}}{b \sin \theta \Delta \theta} = -\rho_{l,i} \frac{P_{l,i+1} - P_{l,i}}{b \cos \theta \Delta \theta} \\
+ \alpha_{\theta} \rho_{l,i} \frac{u_{l,i+1} - \rho_{l,i}}{b \cos \theta \Delta \theta} = 0 \tag{31}
\]

Liquid momentum balance discretization:

\[
\rho_{l,i} \frac{u_{l,i+1} - u_{l,i}}{\Delta t} + u_p \frac{P_{l,i+1} - P_{l,i}}{b \sin \theta \Delta \theta} + \alpha_{\theta} \rho_{l,i} \frac{u_{l,i+1} - u_{l,i}}{b \sin \theta \Delta \theta} \\
- \alpha_{\theta} \rho_{l,i} \frac{u_{l,i+1} - \rho_{l,i}}{b \cos \theta \Delta \theta} = \frac{\tau_{S_b}}{A} + \frac{\tau_{S_l}}{A} + F_D - \alpha_{\theta} \rho \sin \theta \tag{32}
\]

The discretizations use a first-order upwind scheme in space and use an implicit first-order scheme in time. The solution procedure of eqs 29–32 can be summarized as the following steps:

Step 1. Obtain the data of the initial conditions.
Step 2. The gas–wall, liquid–wall, and gas–liquid interfacial shear stresses, friction factors, and the drag force are calculated.
Step 3. Every element of the coefficients’ matrix and every independent vector are calculated using the initial conditions given in Table 2.

Table 2. Experimental Initial and Boundary Conditions

| Parameter                        | case 1 | case 2 | case 3 | unit |
|----------------------------------|--------|--------|--------|------|
| pipe diameter                    | 0.05   | 0.05   | 0.05   | m    |
| gas velocity                     | 4      | 4      | 4      | m/s  |
| liquid velocity                  | 0.2    | 0.2    | 0.2    | m/s  |
| temperature                      | 20     | 20     | 20     | °C   |
| curvature                        | 1      | 1.5    | 2      | m    |
| water density                    | 998.2  | 998.2  | 998.2  | kg/m³|
| gas density                      | 1.205  | 1.205  | 1.205  | kg/m³|
| surface tension                  | 0.075  | 0.075  | 0.075  | N/m  |
| sound speed                      | 340    | 340    | 340    | m/s  |

Step 4. Calculations are repeated from cell \( i = 2 \) to cell \( i = n \); use the variables calculated in cell \( i = 2 \) as the input variables to cell \( i = 3 \) and so on until the input variables for cell \( i = n \) are calculated.

Step 5. When the variables are calculated in all the cells, a new time \( t + \Delta t \) is assigned to \( t \), and the calculation is repeated from step 3 until the entire simulation is finished.

The iteration loop is stopped until the residual is smaller than \( 10^{-5} \).

4. EXPERIMENTAL SECTION

In this section, a total of three different groups of experiments were conducted to validate the slug model of the curved section.

4.1. Experiment Arrangement. Air and water are used as flow media in the experiments. The experimental system includes a horizontal pipe connected to a curved pipe at its...
downstream, and the outlet of the pipeline is connected to the air as shown in Figure 5. Measurements were taken at a nominal temperature of 20 °C. These pipes are made of transparent acrylic resin where the flow patterns can be observed easily. The inner diameter of the pipes is 50 mm. The length of the straight pipe is 1 m, and the curvature of the pipe can be changed by replacing the curved pipe (the curvature $r = 1, 1.5, \text{and } 2$ m). As shown in Figure 4, there is a pressure measuring point in the middle of the pipe.

The water is injected by a screw pump from the water tank to the mixer, which mixes water with air supplied from the gas tank. There are several flow meters installed on the pipeline, including a turbine flow meter for the water and a volume flow meter for the gas; the experiments use electromagnetic valves to control the gas flow rate within the range of 15–1000 L/min and liquid flow rate within the range of 0.1–1 m$^3$/h. The mixture flows through the curved section and is discharged into a tank to separate the gas from the liquid. The maximum uncertainties of liquid and gas flow rate measurements are 2%.

4.2. Experimental Parameter Measurement. As shown in Figure 5, three Rosemount pressure gauges with a measurement range of 0–500 kPa are installed on the experimental loop; the three gauges are placed at the entrance of the curved pipe, the middle of the curved pipe, and the outlet of the curved pipe. The turbine flow meter for the water has a measurement range of 0.1–1.5 m$^3$/h, and the gas volume flow meter has a measurement range of 10–2000 L/min. A conductivity measurement device for liquid holdup, proposed by Fossa, is installed at the outlet of the curved pipe. The device can detect the liquid holdup changes as the mixture flows through the curved pipe. Contrasted with other approaches, this method is rather simple and can measure the liquid holdup in real time without breaking the flow pattern. This device is shown in Figure 6. Experimental initial and boundary conditions are listed in Table 2, which are also used in the numerical scheme.

4.3. Experimental Procedure. First, we started the air compressor to inject air into the gas tank until it reached a stable
pressure state. Then, we injected water into the pipe at the rate of 0.2 m/s. When the curved pipe was full of water, we injected the gas into the mixer from the gas tank at the rate of 4 m/s. We recorded the pressures and liquid holdup for several seconds and the flow pattern in the curved pipe. Then, we replaced the curved pipe with different curvatures and repeated the above steps.

5. RESULTS AND DISCUSSION

The slug model of the curved pipe is also simulated for three groups. We changed the curvature, and kept the other conditions the same. Then, the results are compared with experimental results to validate the model.

First, initial and boundary conditions for the numerical simulations are given the same as the experimental conditions, which are summarized in Table 2. The pipeline is divided into a grid of 1000 cells with the same angle $\theta$. The time step size is calculated by the number of CFL, which is an important part for the transient simulation and is defined as

$$C = \frac{u_{g, \text{max}} \Delta t}{\Delta x} = \frac{u_{g, \text{max}} \Delta t}{b \Delta \theta}$$

(33)

where the $\Delta t$ and $\Delta \theta$ are the time step size and grid size (Figure 3), respectively. The constant value $C$ is set to 0.5.

5.1. Transient Pressure Profiles. Figures 7−9 are numerical results of transient pressure profiles of cases 1, 2, and 3, respectively. These three graphs show the pressure profiles along the pipeline as time elapses ($t = 0.25, 0.5, 0.75, 1, 1.25, \text{and } 1.5 \text{ s are chosen to be presented in the graphs}$), and the scattered points are experimental data. It can be seen that the pressure curve has a descending middle part, which basically corresponds to the curved pipe structure. In Figure 7, the maximum error between the model and experimental data is 18.6%, and the average error is 12.2%. In Figure 8, the maximum error between the model and experimental data is 13.7%, and the average error is 9.8%. In Figure 9, the maximum error between the model and experimental data is 19.9%, and the average error is 13.8%. It can be inferred that all the errors between model calculations and experimental data are in the range of 20%, which is relatively acceptable. The simulation pipeline is full of the gas−liquid mixture at approximately 1.0, 1.1, and 1.25 s for cases 1, 2, and 3, respectively. Figure 9 indicates that a larger pipe curvature has a higher pressure value. Because the air is introduced in the pipe gradually, pressure oscillations grow along the pipeline, and the pressure profile curve rapidly becomes the typical shape of a slug flow wave like the phenomenon of a slug flow observed in experiments (as shown in Figure 10 with pipe curvatures of 1 and 2 m).

Figure 7. Numerical transient pressure profile of case 1 at different times (the pipe curvature is 1 m).

Figure 8. Numerical transient pressure profile of case 2 at different times (the pipe curvature is 1.5 m).

Figure 9. Numerical transient pressure profile of case 3 at different times (the pipe curvature is 2 m).

Figure 10. Slug units flow through the curved section (the photos were taken by S.S.).
5.2. Transient Pressure and Error Analysis. The pressures of model prediction and experiments against time with a pipe curvature of 1 m (case 1) are plotted in Figure 11.

![Figure 11. Comparison between the numerical results of pressure and the experimental data for pressure of case 1.](image)

The maximum values of experimental data and predicted data at the measuring point are 19.38 and 19.10 kPa, respectively; the minimum values of experimental data and predicted data are 12.50 and 13.84 kPa, respectively. The liquid superficial velocity (0.2 m/s) and gas superficial velocity (4 m/s) are used as initial values of the model calculation, which are also used in the experiment. As can be seen from Figure 11, the experimental pressure curve (orange line) shows many tiny fluctuations because of the tiny gas bubbles in the slug and the vortex motion by the liquid film. The simulation of pressure (blue line) is performed relatively well from 2 to 4 s; from 0 to 2 s, the predicted curve is averagely ahead of the experimental curve, and after 4 s, the predicted curve lags behind the experimental curve. Figure 12 shows the error analysis of pressure data, and in order to make the error analysis more visual, only the peak and trough values of the experimental data points are compared with those of the predicted model curve. The horizontal error bar is set to a fixed value of 0.1 s, and the vertical error bar is set to 10%. It can be seen that three vertical errors are larger than 10%; the maximum vertical error is 13.1%, and the average pressure error between prediction and experiments is 7.03%. Eight horizontal errors are larger than 0.1 s; the maximum horizontal error is 0.16 s, and the average horizontal error is 0.08 s. In general, the model curve of case 1 is fitted well to the experimental results.

![Figure 12. Pressure error analysis of the curve model of case 1.](image)

Figure 13 shows the pressure curves of experimental data and predicted values of the model with the pipe curvature of 1.5 m (case 2). The maximum values of experimental pressure data and predicted data at the measuring point are 22.57 and 22.53 kPa, respectively; the minimum values of experimental data and predicted data at the measuring point are 13.23 and 14.15 kPa, respectively. From 2.6 to 6 s, the predicted curve has an obvious time deviation compared to the experimental curve, and the rest of the parts of the predicted curve perform relatively well. With regard to error analysis, it can be seen in Figure 14 that all vertical errors except four points are controlled within a margin of 10%; the maximum vertical error is 14.15%, and the average pressure error between the prediction and experiments is 7.74%. Horizontally, the maximum horizontal error is 0.14 s, and the average horizontal error is 0.07 s. Therefore, the model of case 2 is well fitted.

![Figure 13. Comparison between the numerical results of pressure and the experimental data for pressure of case 2.](image)

Figure 14. Pressure error analysis of the curve model of case 2.

Figure 15 shows the pressure curves of experimental data and predicted values of the model with the pipe curvature of 2 m (case 3). The maximum values of experimental pressure data and predicted data at the measuring point are 26.33 and 25.51 kPa, respectively; the minimum values of experimental data and predicted data at the measuring point are 18.98 and 19.71 kPa.

![Figure 15. Comparison between the numerical results of pressure and the experimental data for pressure of case 3.](image)
respectively. There are only two points that have obvious time deviations, and the rest of the parts of the predicted curve perform relatively well. With regard to error analysis, it can be seen in Figure 16 that all vertical errors are controlled within a margin of 10%; the maximum vertical error is 6.84%, and the average vertical error between predication and experiments is 3.02%. Horizontally, the maximum horizontal error is 0.14 s, and the average horizontal error is 0.06 s. It shows that the model of case 3 is fitted very well.

5.3. Transient Liquid Holdup and Error Analysis. The liquid holdups against time are plotted in Figure 17. In this figure, the predicted liquid holdup curve is compared to the experimental liquid holdup curve (case 1). The maximum values of experimental data and predicted data are 0.78 and 0.735, respectively; the minimum values of experimental data and predicted data are 0.09 and 0.13, respectively. It is shown that the liquid holdup curve of this model has severe oscillations, which indicate different slugs flow through the pipe corresponding to the phenomenon of a slug flow through the pipe as shown in Figure 10. It also can be seen in Figure 17 that the predicted liquid holdup values are fitted well to experimental data, but some predicted liquid holdup oscillations have time deviations from the experimental curve. Figure 18 is the error analysis of the liquid holdup of case 1, which is just like the pressure error analysis, and only values of the peak points and trough points are selected. The horizontal error bar is set to a fixed value of 0.1 s, and the vertical error bar is set to a fixed value of 0.1. It is found that all the vertical errors are controlled within the margin of 0.1 s; the maximum vertical error is 0.09, and the average vertical error is 0.04. Most of the horizontal errors are controlled within the margin of 0.1 s except for five points; the maximum horizontal error is 0.25 s, and the average horizontal error is 0.08 s. As a result, the error analysis is fairly ideal. However, there are many irregular liquid holdup fluctuations caused by the unstable churn flow at the slug tail and slug head that can be seen on the experimental curve; therefore, the simulated result cannot entirely reflect these irregular fluctuations in the present study. Figure 19 shows the comparison of liquid holdup curves between the experiments and model predictions (case 2). The maximum values of the experiment and prediction are 0.86 and 0.75, respectively; the minimum values of the experiment and prediction are 0.12 and 0.11, respectively. In case 2, oscillations of the liquid holdup curve are more severe than those of case 1. However, the time deviations of case 2 are less than those of case 1. Figure 20 shows the error analysis of liquid holdup of case 2. In terms of the vertical errors, there is one point that exceeds the margin of 0.1; the maximum liquid holdup error is 0.11, and the average liquid holdup error is 0.05. When it comes to the horizontal errors, there are five horizontal errors that are larger than 0.1 s; the maximum horizontal error is 0.14 s, and the average horizontal error is 0.07 s. Overall, the errors are fairly ideal.
acceptable, and the predicted curve is well matched to the experimental curve.

In Figure 21, the comparison between the model liquid holdup curve and experimental liquid holdup curve of case 3 is shown. The maximum values of the experiment and prediction are 0.90 and 0.75, respectively; the minimum values of experiment and prediction are 0.10 and 0.12, respectively. Figure 22 shows the error analysis of liquid holdup of case 3. In terms of the vertical errors, there are two points that exceed the margin of 0.1; the maximum vertical error is 0.16, and the average vertical error is 0.05. With regard to the horizontal errors, there are three horizontal errors that are larger than 0.1 s; the maximum horizontal error is 0.11 s, and the average horizontal error is 0.05 s. In general, the predicted curve is well matched to the experimental curve.

6. CONCLUSIONS

A transient slug flow model for curved sections is proposed in this study, which can closely reflect the characteristics of the slug flow in a curved pipe. The simulation results are compared with experimental results under same conditions. One of the main points of the model is that the two-dimensional rectangular coordinate system is simplified to a one-dimensional polar coordinate system of a curved section, which saves time vastly and makes the simulations more convenient.

Three groups of slug flow experiments with different pipe curvatures were conducted in the Key Laboratory of Petroleum Engineering of China University of Petroleum-Beijing. The slug flow in the curved pipe was observed in the visualized experiments. Subsequently, pressures were measured at different locations of the curved pipe, and the liquid holdup at the outlet of the curved pipe was also measured.

The transient pressure profiles along the pipe can well reflect the slug propagations in the pipeline at different times, which was observed in the experiments. The pipe is full of the gas–liquid mixture at approximately 1.0, 1.1, and 1.25 s for cases 1, 2, and 3, respectively. When comparing the experimental pressure data with the calculated pressure profile, the average errors are 12.2, 9.8, and 13.8%, respectively. The real-time pressure and liquid holdup, the most common standards in a multiphase flow field, are calculated to be compared with the experimental results. The comparison results and error analyses of cases 1, 2, and 3 show that the model calculations are well fitted to experimental results in general.

This work could provide good guidance for predicting the characteristics of the slug flow in the curved pipelines, which are important for the improvement of flow stability.

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Notes
The authors declare no competing financial interest.

The data used to support the findings of this study are available from the corresponding author upon request.

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■ NOMENCLATURE

\( p \) (Pa) pressure
\( x \) horizontal axial coordinate
\( y \) vertical axial coordinate
\( A \) (m\(^2\)) area
\( S \) (m) wetted perimeter and interfacial width
\( F_D \) (N) drag force of bubbles in the liquid slug
\( u \) (m/s) velocity
\( g \) (m/s\(^2\)) gravitational acceleration
\( h \) (m) height of liquid level
\( t \) (s) time
\( b \) (m) curvature radius of pipe
\( c \) (m/s) sound velocity in gas
\( d \) (m) hydraulic diameter
\( D \) (m) pipe diameter
\( f \) friction factor
\( Re \) Reynolds number
\( C_D \) drag coefficient
\( V \) (m\(^3\)) volume of each cell

Greek letters
\( \alpha \) liquid holdup
\( \beta \) (rad) liquid level angle
\( \rho \) (kg/m\(^3\)) density
\( \tau \) (N) shear stress
\( \theta \) (rad) angle corresponding to the arc length of the pipe
\( \mu \) (Pa·s) viscosity

Subscripts
I liquid phase
gas phase
i interface

REFERENCES

(1) Chi, Y.; Daraboina, N.; Sarica, C. Investigation of inhibitors efficacy in wax deposition mitigation using a laboratory scale flow loop. *AIChE J.* 2016, 62, 4131−4139.

(2) Chi, Y.; Daraboina, N.; Sarica, C. Effect of the flow field on the wax deposition and performance of wax inhibitors: cold finger and flow loop testing. *Energy Fuels* 2017, 31, 4915−4924.

(3) Poettman, F. H.; Carpenter, P. G. The multiphase flow of gas, oil, and water through vertical flow strings with application to the design of gas-lift installations. In *Drilling and Production Practice*; American Petroleum Institute, 1952, January.

(4) Hagedorn, A. R.; Brown, K. E. Experimental study of pressure gradients occurring during continuous two-phase flow in small-diameter vertical conduits. *J. Pet. Technol.* 1965, 17, 475−484.

(5) Orkiszewski, J. Predicting two-phase pressure drops in vertical pipe. *J. Pet. Technol.* 1967, 19, 829−838.

(6) Beggs, H. D.; Brill, J. P. Two-phase flow in pipes; University of Tulsa: Oklahoma, 1978.

(7) Dukler, A. E.; Wicks, M.; Cleveland, R. G. Frictional pressure drop in two-phase flow: B. An approach through similarity analysis. *AIChE J.* 1964, 10, 44−51.

(8) Aziz, K.; Govier, G. W. Pressure drop in wells producing oil and gas. *J. Can. Pet. Technol.* 1972, 11, 1−12.

(9) Taitel, Y.; Bornea, D.; Dukler, A. E. Modelling flow pattern transitions for steady upward gas-liquid flow in vertical tubes. *AIChE J.* 1980, 26, 345−354.

(10) Ansari, A. M.; Sylvester, N. D.; Shoham, O.; Brill, J. P. A Comprehensive Mechanistic Model for Upward Two-Phase Flow in Wellsbores. *SPE Prod. Facil.* 1990, 9, 143−151.

(11) Grolman, E.; Fortuin, J. M. H. Gas-liquid flow in slightly inclined pipes. *Chem. Eng. Sci.* 1997, 52, 4461−4471.

(12) Petalsa, N.; Aziz, K. A mechanistic model for multiphase flow in pipes. In *Annual Technical Meeting*; Petroleum Society of Canada, 1998, January.

(13) Im, H.; Park, J.; Lee, J. W. Prediction of Main Regime Transition with Variations of Gas and Liquid Phases in a Bubble Column. *ACS Omega* 2019, 4, 1329−1343.

(14) Chi, Y.; Zhou, S.; Daraboina, N.; Sarica, C. Experimental study of wax deposition under two-phase gas-oil stratified flow. In *11th North American Conference on Multiphase Production Technology*, BHR Group, 2018, December.

(15) Chi, Y.; Sarica, C.; Daraboina, N. Experimental investigation of two-phase gas-oil stratified flow wax deposition in pipeline. *Fuel* 2019, 247, 113−125.

(16) Barnea, D.; Taitel, Y. A model for slug length distribution in gas-liquid slug flow. *Int. J. Multiphase Flow* 1993, 19, 829−838.

(17) Zheng, G.; Brill, J. P.; Taitel, Y. Slug flow behavior in a hilly terrain pipeline. *Int. J. Multiphase Flow* 1994, 20, 63−79.

(18) Nydal, O. J.; Banerjee, S. Dynamic slug tracking simulations for gas-liquid flow in pipelines. *Chem. Eng. Commun.* 1996, 141-142, 13−39.

(19) Taitel, Y.; Barnea, D. Effect of gas compressibility on a slug tracking model. *Chem. Eng. Sci.* 1998, 53, 2089−2097.

(20) Ujjang, P. M.; Lawrence, C. J.; Hewitt, G. F. Conservative incompressible slug tracking model for gas-liquid flow in a pipe. In *5th BFHRG North American conference on multiphase technology*; Banff: Canada, 2006, May, 31.

(21) Franca, F. A.; Bannwart, A. C.; Camargo, R. M. T.; Gonçalves, M. A. L. Mechanistic modeling of the convective heat transfer coefficient in gas-liquid intermittent flows. *Heat Transfer Eng.* 2008, 29, 984−998.

(22) Kjeldby, T. K.; Henkes, R. A. W. M.; Nydal, O. J. Lagrangian slug flow modeling and sensitivity on hydrodynamic slug initiation methods in a severe slugging case. *Int. J. Multiphase Flow* 2013, 53, 29−39.

(23) Medina, C. D. P.; Bassani, C. L.; Cozin, C.; Barbuto, F. A. D. A.; Junqueira, S. L. M.; Morales, R. E. M. Numerical simulation of the heat transfer in fully developed horizontal two-phase slug flows using a slug tracking method. *Int. J. Therm. Sci.* 2015, 88, 258−266.

(24) Ishii, M.; Hibiki, T. *Thermo-fluid dynamics of two-phase flow*; Springer Science & Business Media, 2010.

(25) Bendiksken, K. H.; Maines, D.; Moe, R.; Nuland, S. The dynamic two-fluid model OLGA: Theory and application. *SPE Prod. Eng.* 1991, 6, 171−180.

(26) De Henau, V.; Raithby, G. D. A study of terrain-induced slugging in two-phase flow pipelines. *Int. J. Multiphase Flow* 1995, 21, 365−379.

(27) Lima, P. C. R.; Yeung, H. Modelling of transient two-phase flow operations and offshore pigging. In *SPE Annual Technical Conference and Exhibition*; Society of Petroleum Engineers, 1998, January.

(28) Issa, R. I.; Kempf, M. H. W. Simulation of slug flow in horizontal and nearly horizontal pipes with the two-fluid model. *Int. J. Multiphase Flow* 2003, 29, 69−95.

(29) Chang, C. H.; Liou, M. S. A robust and accurate approach to computing compressible multiphase flow: Stratified flow model and AUSM+-up scheme. *J. Comput. Phys.* 2007, 225, 840−873.

(30) Danielson, T. J.; Bansal, K. M.; Djoric, B.; Larrey, D.; Johansen, S. T.; De Leebeek, A.; Kjølaas, J. Simulation of slug flow in oil and gas pipelines using a new transient simulator. In *Offshore Technology Conference*; Offshore Technology Conference, 2012, April.

(31) Fullmer, W. D.; de Bertodano, M. A. L.; Chen, M.; Clause, A. Analysis of stability, verification and chaos with the Kreiss−Yström equations. *Appl. Math. Comput.* 2014, 248, 28−46.

(32) Panicker, N.; Passalacqua, A.; Fox, R. O. On the hyperbolicity of the two-fluid model for gas−liquid bubbly flows. *Appl. Math. Model.* 2018, 57, 432−447.

(33) Biesheuvel, A.; Gorissen, W. C. M. Void fraction disturbances in a uniform bubbly fluid. *Int. J. Multiphase Flow* 1990, 16, 211−231.

(34) Taitel, Y.; Dukler, A. E. A model for predicting flow regime transitions in horizontal and near horizontal gas-liquid flow. *AIChE J.* 1976, 22, 47−55.

(35) Spedding, P. L.; Hand, N. P. Prediction in stratified gas-liquid co-current flow in horizontal pipelines. *Int. J. Heat Mass Transfer* 1997, 40, 1923−1935.

(36) Wallis, G. B. *One-dimensional two-phase flow*; McGraw-Hill: New York, 1969.

(37) Lahey, R. T., Jr. *Boiling heat transfer: modern developments and advances*. Elsevier Science, 2013.

(38) Fossa, M. Design and performance of a conductance probe for measuring the liquid fraction in two-phase gas-liquid flows. *Flow Meas. Instrom.* 1998, 9, 103−109.