Cryptanalysis of Xinyu et al.'s NTRU-lattice based key exchange protocol

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Abstract

Xinyu et al. proposed a public key exchange protocol, which is based on the NTRU-lattice based cryptography. In this paper, we show that how Xinyu et al.'s NTRU-KE: A lattice based key exchange protocol can be broken, under the assumption that a man-in-the-middle attack is used for extracting private keys of users who participated in the key exchange protocol.

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1. Introduction

Key exchange protocol is one of the public key cryptographic primitives that provides a platform to negotiate keys among a group of parties. In a key exchange protocol, private keys can be exchanged among the group of parties over public insecure communication networks and agree upon a common session key, which can be used for later secure communication among them. A security goal of the key exchange protocol is that private keys are to be shared among the group of parties without compromising their secrecy. Key exchange protocol works as one of the basic building blocks for constructing other high-level secure protocols

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and is used to provide perfect forward secrecy in Transport Layer Security’s ephemeral modes.

A key exchange protocol was introduced by Diffie-Hellman during 1976 in their seminal paper [8]. The Diffie-Hellman (or Elliptic Curve Diffie-Hellman) key exchange protocol security relies on the (or Elliptic Curve) discrete logarithmic problem over a finite field [4,6,8]. The Diffie-Hellman (or Elliptic Curve Diffie-Hellmann) key exchange protocol is vulnerable to a quantum computer attack as a result of the Shor’s algorithm [5]. Recent proposals [1,2,7] for key exchange protocol are alternative candidates against quantum attack. In 2013, Xinyu et al. proposed a public key exchange protocol [7] which is no longer dependent on number-theoretic cryptographic hard assumption problems. Their proposal is based on the post-quantum NTRU-lattice based cryptosystem [3]. The NTRU lattice based encryption is a fast feasibly secured scheme introduced in 1996 by Hoffstein et al. [3] and has been commercially standardized. The security of the NTRU cryptosystem [3] relies on the “shortest vector problem” and the “closest vector problem”. Xinyu et al. proposed an NTRU - key exchange protocol [7] by adapting the NTRU lattice based encryption scheme [3].

A man-in-the-middle attack is a cyber attack where an eavesdropper intercepts a communication between two participated parties and injects his own message. This attack can succeed only when the eavesdropper, Eve can impersonate each endpoint to their satisfaction as expected from the legitimate other end. This paper describes how Xinyu et al.’s NTRU-Key exchange protocol is not safe against man -in-the-middle attack; Eve is able to recover both parties’ private keys.

2. Preliminaries

2.1 Notations and Mathematical Background

Throughout this paper, we denote $\mathbb{Z}$ the integer ring and $\mathbb{Z}_q$ is the ring $\mathbb{Z}/q\mathbb{Z}$. A truncated polynomial ring $R_{N,q} = \mathbb{Z}_q[x]/(x^N - 1)$ consists of polynomials with degree less than $N$ and co-efficients in $\mathbb{Z}_q$. An element $f \in R_{N,q}$ can be written as a polynomial,

$$f = \sum_{i=0}^{N-1} f_i x^i = [f_0, f_1, \ldots, f_{N-1}].$$

Two polynomials $f$ and $g \in R_{N,q}$ are multiplied by the ordinary convolution,

$$(f * g)_k = \sum_{i+j = k \mod N} (f_i, g_j), k = 0, 1, \ldots, N - 1,$$
commutative and associative. The convolution product is presented by * to distinguish it from the multiplication in $\mathbb{Z}_q$. We define a center $l_2$–norm of an element $f \in R_q$ by $\| f \|_2 = \left( \sum_{i=0}^{N-1} f_i - \bar{f} \right)^{1/2}$, where $\bar{f} = \frac{1}{N} \sum_{i=0}^{N-1} f_i$.

3. Xinyu et al.’s NTRU-Key exchange protocol

Xinyu et al.’s NTRU - Key exchange protocol [7] public parameters $(N, p, q)$ are chosen such that $N$ is a prime, $p$ and $q$ are co-prime, $\gcd(p, q) = 1$, and $q$ is larger than $p$. The selection process of parameters same as the NTRU-Encryption scheme [3]. For further details, the reader is referred to [7]. We now present briefly Xinyu et al.’s NTRU - Key exchange protocol in the following algorithm 1.

Algorithm 1 Xinyu et al.’s NTRU-Key Exchange Protocol

Alice                  Bob

Step 1:

$$f_A \leftarrow \mathcal{L}_f, \; g_A \leftarrow \mathcal{L}_g$$

$$h_A = f_A^{-1} * g_A \pmod{q}$$

Step 2:

$$h_B \leftarrow \mathcal{L}_f, \; g_B \leftarrow \mathcal{L}_g, \; r_B \leftarrow \mathcal{L}_r$$

$$f_B \leftarrow \mathcal{L}_f, \; g_B \leftarrow \mathcal{L}_g$$

Step3:

$$h_B e_B = pr_B * h_A + f_B \pmod{q}$$

$$r_A \leftarrow \mathcal{L}_r$$

$$e_B = pr_B * h_A + f_B \pmod{q}$$

Step 4:

$$i_A = f_A * e_B \pmod{q}$$

$$i_B = f_B * e_A \pmod{q}$$

$$K_A = i_A \pmod{p} = f_A * f_B \pmod{p}$$

$$K_B = i_B \pmod{p} = f_B * f_A \pmod{p}$$

4. Man-in-the middle attack on Xinyu et al.’s NTRU-Key exchange Protocol

In this Section, we show weakness of Xinyu et al.’s NTRU-Key exchange protocol against man-in-the middle attacks. To show this Xinyu
et. al.’s protocol is insecure against a man-in-the middle attack, let us suppose an adversary, named Eve listens communication between both parties, Alice and Bob who believe they are communicating with each other. A man-in-the middle attack is described as follows:

**Step 1:** An adversary, Eve intercepts both the public keys $h_A$ and $h_B$ sent by Alice and Bob, respectively and computes two of her own public keys, $h' = f'^{-1} * g'(\text{mod} \ q)$ and $h'' = f''^{-1} * g''(\text{mod} \ q)$ such that there exist the inverse of $f'$ and $f''$ in $R_p$ and $R_q$. Then, she sends $h'$ to Alice and $h''$ to Bob.

**Step 2:** After receiving $h'$, Alice picks $r_A \leftarrow \mathcal{L}$ and computes $e' = pr_A * h' + f_A(\text{mod} \ q)$ and sends $e'$ to Bob, but Eve intercepts it. Similarly, after receiving $h''$, Bob picks $r_B \leftarrow \mathcal{L}$ and computes $e'' = pr_B * h'' + f_B(\text{mod} \ q)$ and sends $e_B$ to Alice, but Eve intercepts it.

**Step 3:** After intercepting $e'$ and $e''$ sent by Alice and Bob, respectively, Eve computes $w_A = e' * f'(\text{mod} \ q)$, $K'_A = w_A * f'^{-1}(\text{mod} \ p) = f_A(\text{mod} \ p)$ and similarly, $w_B = e'' * f''(\text{mod} \ q)$, $K'_B = w_B * f''^{-1}(\text{mod} \ p) = f_B(\text{mod} \ p)$ to recover Alice’s private key $f_A$ and Bob’s private key $f_B$, respectively.

5. Conclusion

In this paper, we have described how Xinyu et al.’s NTRU-KE protocol is insecure against a man-in-the middle attack. The process of this attack has been explained how an attacker can recover private keys of both parties. Note that Xinyu et al.’s NTRU-Key exchange protocol is trivially insecure against a man-in the middle attack for unauthenticated public keys used in this protocol.

References

[1] C. Peikert, “Lattice cryptography for the Internet”, PQ Crypto 2014, Lecture Notes in Computer Science, Vol.8772, 2014, pp:197-219.

[2] J. Ding, X. Xie and X. Lin, “A simple provably secure key exchange scheme based on the learning with errors problem”, https://eprint.iacr.org/2012/688.pdf.

[3] J. Hoffstein, J. Pipher and J. Silverman, “NTRU: A ring-based public key cryptosystem”, in: Algorithmic number theory, Springer 1998, pp:267-288.
[4] N. Koblitz, “Elliptic curve cryptosystems”, Mathematics of Computation, Vol.48, 1987, pp:203-209.

[5] P.W.Shor, “Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer”, SIAM J.Compu., Vol.26, 1997, pp:1484-1509.

[6] V. Miller, “Uses of elliptic curves in cryptography”, Advances in Cryptology-CRYPTO ’ 85, Lecture Notes in Computer Science, Vol.218,1986, pp:417-426.

[7] X.Lei and X.Liao, “NTRU-KE : A lattice-based public key exchange protocol”, https://eprint.iacr.org/2013/718.pdf.

[8] W.Diffie and M.Hellman, “New directions in cryptography”, IEEE Transactions on Information Theory, Vol.22(6), 1976, pp: 644-654.

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