Dynamic Power Balancing Algorithm for Single-Phase Energy Storage Systems in LV Distribution Network with Unbalanced PV Systems Distribution

Watcharakorn Pinthurat, Student Member, IEEE, Branislav Hredzak, Senior Member, IEEE,

Abstract—Unbalanced power, due to high penetration of single-phase PV rooftops into a four-wire multi-grounded LV distribution system, can result in significant rise in the neutral current and neutral voltage. This preprint proposes a distributed clustering algorithm for dynamic power balancing, using single-phase battery storage systems distributed in the LV distribution system, in order to reduce the neutral current and neutral voltage rise. The distributed clustering algorithm aggregates households connected to the same phase into clusters. Within each cluster, another distributed clustering algorithm is applied to calculate the total grid power exchanged but the corresponding phase. Then, the dynamic power balancing control is applied to balance the powers at the bus, based on battery storage systems’ charge/discharge constraints, power minimization and willingness of the households to participate in the power balancing control.

Index Terms—Distributed clustering algorithm, multi-agents, single-phase energy storage system, unbalanced power, power balancing, photovoltaic source, LV distribution network.

I. INTRODUCTION

HIGH penetration of single-phase PV sources into a low voltage (LV) distribution system has been increasing, leading to unbalanced power among phases due to unbalanced allocation of the PV sources [1]. The unbalanced powers among the three phases can result in large neutral current and high neutral-to-ground voltage (NGV) [2]. Large NGV can have negative effects on sensitive loads such as electronic devices and computing equipment because of the common-mode noise effect. NGV of computing equipment should be maintained at less than $0.5\,\text{V}$ as specified by manufacturers in [3]. Also, the high NGV can adversely affect human beings as well as farm animals as reported in [4] and [5] respectively. Therefore, the neutral current and the NGV rise should be limited within predefined values to ensure safety and reliability in the LV distribution system.

Several traditional methods to mitigate the neutral current and NGV rise by resizing neutral conductor, improving grounding and installing a passive harmonic filter were proposed in [6], [7] and [8]. However, owing to high variation of single-phase PV units distributed in LV distribution system, power balancing control between loads and PV sources is difficult, and hence the traditional methods cannot properly solve the issues caused by the unbalanced powers. This is because the traditional methods are considered to be static [9].

There were various mitigation strategies proposed in literature to overcome the before mentioned issues. In [9], community energy storage (ES) was employed to mitigate NGV rise in four-wire multi-grounded LV distribution system. Although the neutral current and NGV rise were regulated within acceptable limits, this control strategy required DC common bus along the feeders to provide required active power to households during power balancing operation. Moreover, centralized communication links were used for households to communicate with the central energy storage. The main drawback of the centralized control is a single point of failure [10]. In [11], the same authors addressed the neutral current and NGV rise by using single-phase distributed energy storage devices...
and distributed communication network. However, only one household in each phase was considered in a given bus. In reality, there are many households connected to the same phase of the bus, as illustrated in Fig. [1]. Therefore, different control approaches for phase power balancing in three-phase four-wire LV distribution system are required. In this preprint, a clustering based approach is proposed. Clustering divides data into groups based on similar properties [12]. Several centralized clustering algorithms were proposed in literature, such as k-means, hierarchical clustering, self-organization map and expectation maximization clustering algorithm in [13], [14], [15] and [16] respectively. Distributed algorithms were also studied for distributed data environment [17]. However, a centralized network was still required for the distributed algorithms. This is because data storage from the distributed algorithms needs to communicate with a main site for clustering operation [18]. Besides, both centralized and distributed based clustering algorithms rely on data collected from the previous time instant, not on real-time states. Hence, the above-mentioned algorithms may be inadequate for clustering households using the distributed communication network in LV distribution power system.

Inspired by the above discussion, this preprint presents a distributed clustering algorithm for dynamic power balancing using distributed single-phase energy storage systems in order to mitigate the neutral current and neutral-to-ground voltage rise caused by unbalanced loads and PV sources in LV distribution system. The distributed clustering algorithm runs in real-time using dynamic states of multi-agent systems. The multi-agent based systems communicate via a sparse distributed communication network. The main contributions of this preprint are:

1) Distributed clustering algorithm to group households connected to the same phase into clusters is proposed. Each phase cluster has information about the real-time grid power exchanged by the other two phase clusters.
2) Dynamic power balancing control strategy, based on ESs’ charge/discharge constraints, power minimization and willingness of the households to participate in the power balancing control, is combined with the distributed clustering algorithm to alleviate the neutral current and neutral voltage rise in the network.

The rest of the preprint is organized as follows. Section II presents the active power flow in three-phase system. Section III describes the distributed clustering algorithms. Application of the proposed algorithm for power balancing control is discussed in Section IV. Finally, Section V concludes the preprint.

II. POWER FLOW IN THREE-PHASE SYSTEM

A. Neutral Current and Neutral Voltage Rise

Neutral current is produced by unbalanced loads and PV units in a three-phase four-wire multi-grounded LV distribution system. A typical LV feeder including several households is shown in Fig. [1]. There are a local load, an ES system and a PV unit in each household. The total neutral current $I_N$, produced by the imbalanced power at any given bus, is defined as,

$$I_N = -(I_{net}^a + I_{net}^b + I_{net}^c)$$

$$I_{net}^φ = \frac{(P_{lφ} + jQ_{lφ})^* - (P_{pvφ} + jQ_{pvφ})^*}{(V_{φre} + jV_{φim})^* - (V_{φnetre} + jV_{φnetim})^*}$$

where $*$ represents the complex conjugate; $P_{lφ}$ and $P_{pvφ}$ are the active powers of the local load and PV output power at the φ-th phase; $Q_{lφ}$ and $Q_{pvφ}$ are the reactive powers of the local load and PV unit at the φ-th phase; $V_{φre}$ and $V_{φim}$ are the real and imaginary parts of the phase voltage; $V_{φnetre}$ and $V_{φnetim}$ are the real and imaginary parts of the neutral voltage.

To investigate the neutral current and its effects, the current unbalance factor (CUF) is widely employed [19], calculated as,

$$CUF = \frac{\sqrt{I_{ns}^2 + I_{zs}^2}}{|I_{ps}|} \times 100\%,$$

$$\begin{bmatrix} I_{ns} \\ I_{ps} \\ I_{zs} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}, a = e^{j(\frac{2\pi}{3})},$$

where $I_{ps}$, $I_{ns}$ and $I_{zs}$ denote the positive, negative and zero sequences of current respectively.

B. Grid Power Exchange in Three-Phase Power System

In a balanced three-phase system, for Y-connection and positive phase sequence, phase voltages are expressed as,

$$v_{an}(t) = V_m \sin(\omega t)$$

$$v_{bn}(t) = V_m \sin(\omega t - 120°)$$

$$v_{cn}(t) = V_m \sin(\omega t + 120°),$$

where $v_{an}$, $v_{bn}$ and $v_{cn}$ are the line-to-neutral voltages of phases $a$, $b$ and $c$ at any bus respectively, with a 120° phase shift from the reference, and $V_m$ is the voltage magnitude. Note that phase $a$ is chosen to be the reference (0°).

It is assumed that the power to be exchanged with the grid ($P_{gφ}$) at each phase and the $i$-th bus is equal to the reference power ($P_{refi}$). The amount of phase power to be charged or discharged by the ES systems for the dynamic power balancing control can be calculated as,

$$P_{bφ} = P_{refi} - P_{φ} = P_{refi} - (P_{lφ} - P_{pvφ}), \phi \in \{a, b, c\},$$

where $P_b$ is the charged or discharged power by the distributed ES devices (+ve indicates the discharging mode while -ve indicates the charging mode), $P_l$ is the active load power, and $P_{pv}$ is the PV output power.

After the power mode $P_{bφ}$ of the ES system is obtained from (4), the charging/discharging current of the ES device at the $φ$-phases and the $i$-th bus can be defined as,

$$I_{bφ} = \frac{P_{bφ}}{V_{bφ}} = \frac{P_{bφ}}{f(SOC_{bφ} , C_{bφ} , Θ_{bφ})}, \phi \in \{a, b, c\},$$

where $I_{bφ}$ is the charging/discharging current of the ES system, which will be used as a reference signal for the power.
balancing control, \( V_{i}^{\text{SoC}} \) is the output voltage of battery, depending on the battery state of charge (SoC), \( C_{i}^{\text{SoC}} \) is the battery capacity, and \( \Theta_{i}^{\text{SoC}} \) is a set of battery modelling parameters adopted from [20].

III. DYNAMIC DISTRIBUTED CLUSTERING ALGORITHM

In this section, the distributed clustering algorithm for multi-agent based systems is introduced. It is defined that the algorithm will cluster \( N \) agent into \( M \) group based on pre-selected feature states. Within a cluster, the estimations of average states can also be accessed by each agent via the network. The algorithms was originally proposed in [21].

A. Distributed Communication Graph

A sparse graph \( \mathcal{G}(\mathcal{V}, \mathcal{E}) \) represents distributed communication links among neighbours, where \( \mathcal{V} = \{1, \ldots, N\} \) and \( \mathcal{E} \) denote the nodes (agents) and edges respectively [22]. The node, \( \mathcal{E} \), has elements \( (i, j) \), in which \( (i, j) \in \mathcal{E} \) if node \( i \) can communicate with node \( j \) via a communication link. The neighbours of the node \( i \) are denoted as \( \mathcal{N}_i \). Node \( j \) is said to be a neighbour of node \( i \) if \( (i, j) \in \mathcal{E} \). The adjacent matrix of the communication graph is expressed by,

\[
A = [a_{ij}] \in \mathbb{R}^{N \times N}, a_{ij} = \begin{cases} \alpha, & (i, j) \in \mathcal{E} \\ 0, & \text{otherwise} \end{cases},
\]

where \( \alpha \) denotes the coupling gain.

The graph Laplacian matrix is defined as,

\[
L = D - A,
\]

where \( D = \text{diag}(d_i) \), and the in-degree of the graph is represented as \( d_i = \sum_{j=1}^{N} a_{ij} \).

In the following subsections, the distributed clustering algorithm, employing the distributed communication graph, is presented.

B. Real-time Distributed Clustering Algorithm

In this subsection, the distributed clustering algorithm is introduced. The feature states of the \( i \)-th agent are denoted by \( x_i \in \mathbb{R}^{n} \). These feature states are used to cluster \( N \) agents into \( M \) clusters in real-time. The average estimations of the states \( x_i \) in cluster \( j \) are defined as \( \bar{x}_{i}^{\text{est}\_j} \), in which \( i = 1, \ldots, N \) and \( j = 1, \ldots, M \). In all clusters, the latest average states \( \bar{x}_{i}^{\text{est}\_j} \) are dynamically accessed by each \( i \)-th agent via the distributed communication network.

Next, the algorithm is described. The initial average estimations of \( \bar{x}_{i}^{\text{est}\_j} \) are selected arbitrarily. Then, the current state \( x_i \) is compared to all current estimations of the \( \bar{x}_{i}^{\text{est}\_j} \), where \( j = 1, \ldots, M \) to decide to which cluster the \( i \)-th agent belongs to. The \( i \)-th agent belongs to the \( k \)-th cluster if its estimated state, \( \bar{x}_{i}^{\text{est}\_k} \), has the smallest distance from its current state \( x_i \), as determined by \((10)\). After all clusters are obtained, for each \( i \)-th agent, average state estimation of the \( k \)-th cluster is compared with its current state \( x_i \) to decide to which cluster it belongs to and the estimations of the \( k \)-th cluster agents are sent to the communication network, as defined by \((9)\). On the other hand, the state estimates of the neighbours, which are not members of the \( k \)-th cluster, \( j \in \{1, \ldots, N\}, j \neq k \), are passed through, as depicted in \((9)\).

\[
\bar{x}_{i}^{\text{est}\_j} = \bar{x}_{i} + \sum_{l \in \mathcal{N}_i} a_{il}(\bar{x}_{l}^{\text{est}\_j} - \bar{x}_{l}^{\text{est}\_k}),
\]

where \( |\mathcal{N}_i| \) is the number of neighbours of the \( i \)-th agent. As depicted in Fig. 2 the following two results can always be accessed by the \( i \)-th agent:

1) \textit{clstr\_rslt\_a}: information that belongs to the \( k \)-th cluster;
2) \textit{clstr\_rslt\_b}: the average estimations of the feature states \( \bar{x}_{i}^{\text{est}\_j} \) in the \( j \)-th cluster, \( j = 1, \ldots, M \).

The dynamic clustering algorithm is summarized as \textbf{Algorithm 1} and Fig. 2 illustrates the implementation of the proposed algorithm.

C. Clustering Utilization

Within the clusters, the auxiliary states \( z_i \in \mathbb{R}^{m} \) can be used for analysis. Therefore, the estimations of the auxiliary state, \( z_i \) are defined as \( \bar{z}_{i}^{\text{est}\_j} \), where \( i = 1, \ldots, N \) and \( j = 1, \ldots, M \). Similar to the estimations of the average state, \( \bar{x}_{i}^{\text{est}\_j} \) from \textbf{Algorithm 1}, the \( \bar{z}_{i}^{\text{est}\_k} \) can be obtained using the result of \textit{clstr\_rslt\_a}. Then, the same procedure as for the estimations of the feature state is followed.\((21)\).

\[
\bar{z}_{i}^{\text{est}\_j} = \bar{z}_{i} + \sum_{l \in \mathcal{N}_i} a_{il}(\bar{z}_{l}^{\text{est}\_j} - \bar{z}_{l}^{\text{est}\_k}),
\]

\[
\bar{z}_{i}^{\text{est}\_k} = \frac{1}{|\mathcal{N}_i|} \sum_{l \in \mathcal{N}_i} \bar{z}_{l}^{\text{est}\_k} + \sum_{l \in \mathcal{N}_i} a_{il}(\bar{z}_{l}^{\text{est}\_k} - \bar{z}_{l}^{\text{est}\_j}), j \neq k.
\]
\[ x_i \in \left\{ k\text{-th cluster} \mid x_i^{cstr_k} = \arg \min_{(k \in 1, ..., M)} \left( \left\| x_i - x_i^{cstr_k} \right\| \right), j = 1, ..., M \right\}. \] (10)

**Algorithm 1** Online Distributed Clustering

1. design \( \mathcal{G}(\mathcal{V}, \mathcal{E}) \) and \( M \)
2. initialize \( \bar{x}_i^{cstr_j}, j = 1, ..., M \)
3. for each \( i \)-th agent at time \( t \) do
   4. measure \( x_i \)
   5. receive \( \bar{x}_{i}^{cstr_j}, l \in N_i, j = 1, ..., M \) from neighbours
   6. \( k \leftarrow \) label of the cluster with the smallest distance between \( x_i \) and \( \bar{x}_i^{cstr_j}, j = 1, ..., M \), where \( k \in \{1, ..., M\} \)
   7. if \( j = k \) then
      8. Eq. (8)
   9. else
      10. Eq. (9)
   11. end if
12. send \( \bar{x}_i^{cstr_j}, j = 1, ..., M \) to neighbours
13. end for
14. Each local agent \( i \) implements the same algorithm.

The result of \( clstr_{rsht} \_c \) is accessed by the \( i \)-th agent node via the distribution network as illustrated in Fig. 3.

**Algorithm 2** is obtained by including the average estimations of auxiliary states into **Algorithm 1**

**Algorithm 2** Utilization of Auxiliary States within Cluster

1. design \( \mathcal{G}(\mathcal{V}, \mathcal{E}) \) and \( M \)
2. initialize \( \bar{x}_i^{cstr_j}, j = 1, ..., M \)
3. for each \( i \)-th agent at time \( t \) do
   4. measure \( x_i \) and \( z_i \)
   5. receive \( \bar{x}_{i}^{cstr_j}, l \in N_i, j = 1, ..., M \) from neighbours
   6. receive \( z_{i}^{cstr_j}, l \in N_i, j = 1, ..., M \) from neighbours
   7. \( k \leftarrow \) label of the cluster with the smallest distance between \( x_i \) and \( \bar{x}_i^{cstr_j}, j = 1, ..., M \), where \( k \in \{1, ..., M\} \)
   8. if \( j = k \) then
      9. Eq. (8) and (11)
   10. else
      11. Eq. (9) and (12)
   12. end if
13. send \( \bar{x}_i^{cstr_j}, j = 1, ..., M \) to neighbours
14. send \( z_i^{cstr_j}, j = 1, ..., M \) to neighbours
15. end for

### IV. Application Example

In this section, an application of the distributed clustering algorithm to dynamic power balancing for mitigating neutral current and neutral voltage rise caused by imbalance of PV sources and load demands is introduced.

ES devices, powers, for balancing operation in (4), should be minimized to reduce required ES capacity. In the case that the power exchange with the grid in all phases has the same sign (either consumed or delivered powers), the reference power \( P_{g}^{ref} \) at a given bus is obtained as,

\[ P_{g}^{ref} = \min(|P_{a}^{ia}|, |P_{b}^{ib}|, |P_{c}^{ic}|). \] (13)

However, if the power exchange with the grid in all phases and at a given bus has different signs (both power consumption and delivery), (14) is required to ensure the sum of power injected and consumed by ES systems in a given bus is minimized.

\[
\begin{align*}
\text{minimize:} & \sum_{\phi \in \{a,b,c\}} |P_{b}^{\phi}|, \text{subject to:} & \\
& \begin{bmatrix} 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ \end{bmatrix} & \begin{bmatrix} P_{a}^{ia} \\ P_{b}^{ib} \\ P_{c}^{ic} \\ D_{c}^{ic} \\ \end{bmatrix} + \begin{bmatrix} P_{a}^{ia} - P_{a}^{ib} \\ P_{b}^{ib} - P_{b}^{ic} \\ P_{c}^{ic} - P_{c}^{ia} \\ \end{bmatrix},
\end{align*}
\] (14)

\[
(P_{a}^{ia} \times P_{b}^{ib}) \geq 0, (P_{b}^{ib} \times P_{c}^{ic}) \geq 0, (P_{a}^{ia} \times P_{c}^{ic}) \geq 0. \] (15)

The equality constraint in (15) ensures equal power exchange with the grid for all three phases while the correct decision made by ES systems (charging/discharging mode) is ensured by the inequality constraints in (16).

#### A. Methodology

Methodology for the distributed clustering algorithm with dynamic power balancing control can be described as follows:

1) In a given bus, the active powers among the three phases should be equally balanced, a sum of the phase currents is equal to zero \( (I_a + I_b + I_c = 0) \). It is assumed that there are nine households connected to a bus, and three households to a phase, as shown in Fig. 1.
2) **Algorithm 1** is required to group the households at the same bus into three clusters (phases a, b and c) by using the phase shifts in (3) as the feature states. Virtual communication links are formed as shown in Fig. 4. A cluster of Agents 1-3 represents the households belonging to phase a, while the other two household clusters belong to phases b and c.

3) **Algorithm 2** is adopted for utilization of the feature states within the cluster. In this step, the grid power exchanges \( P_g^{\phi} \) are calculated using (4) and are subsequently sent to the other two clusters. (Note that each cluster has to be able to communicate with the neighboring clusters.)

4) Next, two possible decisions can be made by the power balancing control method. If the grid power exchanges \( P_g^{\phi} \) have the same sign (either injected power or consumed power from the grid), ES devices’ powers \( I_{b,c}^{\phi} \) for balancing operation are calculated using (13). However, if the grid power exchanges \( P_g^{\phi} \) have opposite signs (both delivered and consumed powers), (14) along with (15) and (16) are used to calculate the ES’s powers \( P_{b,c}^{\phi} \).

5) After ESs’ powers \( P_{b,c}^{\phi} \) are obtained, charge/discharge currents of the ES devices are calculated using (5) to obtain the reference signals for the power control loop. In this step, \( SoC_{max} \) and \( SoC_{min} \) of the ES devices are taken into consideration to ensure that all ES devices operate within the limits.

6) In a realistic situation, not all households may be willing to participate in the power balancing control even though they have sufficient power. In this case, the average state estimations in the clustering algorithm will not include the unwilling households in the power control operation.

7) Furthermore, SoC condition may be taken into consideration. Batteries having a certain SoC, eg close to 50%, are more suitable to participate in the power balancing control. The households having too high or too low SoCs may not be allowed to participate into power control even though they are willing to.

### B. Grid Power Exchange Scenarios

There are three possible scenarios of the grid power exchanges in the four-wire multi-grounded LV distribution system illustrated in Fig. 5:

1) **All three phases inject powers to the grid:** In Fig. 5(a), \( P_g \) is greater than zero for all phases (inject powers to the grid). The algorithm finds the phase which injects the minimum power to the grid using (5) (phase c in this example). Then, the ES devices distributed in the phases a and b are charged to absorb the surplus powers and make the powers injected to the grid equal to that of the phase c. Hence, the active powers for all phases at a given bus are balanced.

2) **All three phases consume powers from the grid:** \( P_g \) is less than zero for all phases (powers consumed by local loads), as depicted in Fig. 5(b). The algorithm finds the phase with the minimum power (absolute value) consumed from the grid using (5) (phase c in this example). Then, the ES devices distributed in the phases a and b are discharged to make the powers consumed from the grid equal to that of the phase c. Hence, the active powers for all phases at a given bus are balanced.

3) **Phases inject/consume powers to/from the grid:** In this case, the powers are both injected to the grid and consumed by loads from the grid simultaneously, as shown in Fig. 5(c). Power balancing is achieved by both charging and discharging or only discharging the ES devices as shown in Figs. 5(d) and 5(e) respectively. In these cases, (14) along with (15) and (16) are required to find the total minimum power consumed or delivered by the ES devices to balance operation at a given bus.

### C. Control System Implementation

Fig. 6 depicts the complete control system of the proposed control strategy for the \( i \)-th ES system.

1) Green block shows the distributed clustering algorithm and the auxiliary states estimations. In the clustering algorithm block, the phase shift in (3) is the pre-selected
feature state $x_i$. In this block, a virtual ES system is formed at each phase at a given bus. Within the cluster, the obtained auxiliary average states $x^g_i$, $x^b_i$ ($= x^g_i - x^b_i$) are sent to the neighbours and to the power balancing control.

2) Red block represents the dynamic power balancing control. In this block, the grid power exchanges from the other two phases are received from the neighbours. Hence, the battery current in $[5]$ for the power balancing can be obtained.

3) Purple block indicates that the current from the dynamic power balancing control is the reference signal for the power control block at the primary level control. The converter controller generates the PWM signal for the converter to achieve the power balancing control.

V. CONCLUSION

This preprint presented a distributed clustering algorithm with dynamic power balancing control using single-phase distributed battery energy storage systems to alleviate the neutral current and neutral voltage rise produced by unbalanced PV allocations in four-wire multi-grounded LV distribution network. The distributed clustering algorithm was used to group the households at the same phase into the same cluster. Within the cluster, another distributed clustering algorithm was applied to calculate the total grid power exchanged by each phase. Then, the dynamic power balancing control was adopted to balance the powers among the phases at a bus, considering charge/discharge constraints, power minimization and willingness of the households to participate in the power balancing control.

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