Do we need photons in parametric down conversion?

Trevor W. Marshall

Department of Mathematics, University of Manchester, Manchester M13 9PL, U. K.

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Abstract

The phenomenon of parametric down conversion from the vacuum may be understood as a process in classical electrodynamics, in which a nonlinear crystal couples the modes of the pumping field with those of the zeropoint, or “vacuum”, field. This is an entirely local theory of the phenomenon, in contrast with the presently accepted nonlocal theory. The new theory predicts a hitherto unsuspected phenomenon — parametric up conversion from the vacuum.

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The widely accepted description of parametric down conversion (PDC) by a pumped nonlinear crystal (NLC) is based on a hamiltonian (for simplicity we confine attention to the scalar version)

\[ H = \sum \hbar \omega_k a_k^\dagger a_k + \sum g_{kk'} a_k^\dagger a_{k'}^\dagger e^{i\omega_0 t}. \]  (1)

The interaction part of \( H \) is interpreted to mean that (I prefer to say “was constructed so that”) an incoming laser photon of frequency \( \omega_0 \) down converts into a correlated pair of photons with frequencies \( \omega \) and \( \omega_0 - \omega \). Such a process is depicted in Fig.1. Naturally, since we know that \( E = \hbar \omega \), that means energy is conserved in the PDC process. The correlation in the properties of the two outgoing photons, according to this description, can only be described as bizarre[2, 3]. Indeed, attempts, by supporters of the theory, to describe effects such as “entanglement” to a nonspecialist public, have used the words “mind boggling”[4] and, in another context, “absurd”[5], but these authors’ implication is that the theory must nevertheless be correct.

The mind boggling feature of the standard theory of PDC, and also of atomic transitions, is its nonlocality. This feature is already clear in Fig.1, which implies that photons, or plane waves filling the whole of space, are created instantaneously by the action of \( H \). But we do not have to believe Fig.1 represents what really happens. The alternative to photons is almost as old as photons themselves, namely the zeropoint electromagnetic field of Max Planck[6, 7].

I have reviewed elsewhere[8] how nonlocal photon descriptions may be more or less systematically replaced by local descriptions which incorporate Planck’s field. This systematic replacement includes, indeed begins with, the celebrated experiments on Bell-inequality tests in atomic cascades[9]. Most specialists, possibly quite rightly, are not impressed, because, although the treatment of the field is unambiguously maxwellian, we have had to improvise, in a rather crude manner, the description of the atom-field interaction. Nobody has yet succeeded in doing, for the Dirac field, what Planck did for the Maxwell field.

Now when it comes to PDC we do not need to know any details about the atom-field interaction; only the relation between the current and the field inside the crystal is relevant at optical and near ultraviolet frequencies. So a purely maxwellian theory of the type I have outlined[8, 10], and which I call the Field Theory, can engage with the theory generated by eq.(1) and Fig.1, which I call the Photon Theory, on equal terms. But the Field Theory is manifestly local and causal; the incoming fields generate a cur-
rent in the crystal, and the outgoing fields are the retarded fields generated by those currents.

The form of maxwellian theory I have considered starts from the current-field relation

\[ J(r,t) = \int_{-\infty}^{\infty} f(\omega) d\omega \int_{-\infty}^{\infty} E(r,t') e^{i\omega t - i\omega t'} dt' \]

\[ + g \cos[\omega_0 t - \omega_0 \mu(\omega_0) z] E(r,t), \]  

(2)

where \( f(\omega) \) is analytic in the lower half plane, so that the integration on \( t' \) may be taken from minus infinity to \( t \) only. The function \( f(\omega) \) is related to the refractive index \( \mu(\omega) \) in the standard way, and it is this function which determines the directions of the down converted waves. Eq.(2), like eq.(1), is a linearized form of the interaction, that is it neglects effects due to depletion of the laser intensity.

We have shown, in a series of articles\(^{[11, 12, 13, 14]}\), that the Photon Theory, when cast in the Wigner representation, gives a coupling between the field modes. The PDC predictions of that theory may then all be obtained by the application of second-order perturbation theory based on \( H \), as given by eq.(1). The Hilbert-space representation is generally preferred, because in that case only first-order perturbation theory is required, but I think this algorithmic simplification obscures what is really happening inside the crystal. I say this because the field-coupling obtained from eq.(2) is very nearly the same as we obtained in the above series of articles, and a calculation of one important set of data, namely the fringe visibility in the Zou-Wang-Mandel experiment\(^{[4]}\), using second-order perturbation theory with this coupling, gives very nearly the same results as the Photon Theory.

The point about first- and second-order contributions is made clear by reference to Fig.2, which may be considered a picture of classical PDC\(^{[4]}\). In order to find the intensities of the outgoing waves to second order, we need the signal amplitude to first order, and the idler amplitude to second order.

Figure 2: Classical PDC. When a wave of frequency \( \omega \) is incident, at a certain angle \( \theta(\omega) \), on a nonlinear crystal pumped at frequency \( \omega_0 \), a signal of frequency \( \omega_0 - \omega \) is emitted in a certain conjugate direction. The modified input wave is called the idler.

There is, of course, a symmetry about Fig.1 which is lacking in Fig.2. This is because we have, in Fig.2, no input corresponding to the signal mode. But the point about our Planck-based theory is that all modes of the field are actually present, with an intensity corresponding to half a photon. If one input is classical, for example a second laser, it is appropriate to leave out its conjugate mode as we have done. But if we wish to consider PDC from the vacuum, which is what Fig.1 purports to describe, we have to take both relevant inputs into account, as in Fig.3. The zero-point inputs, denoted by interrupted lines, do not activate photodetectors, because the threshold of these devices is set precisely at the level of the zeropoint intensity, as discussed in Ref.[4]. However, the two idlers have intensities above that of their corresponding inputs. Also there is no coherence between a signal and an idler of the same frequency, so their intensities are additive in both channels. Hence there are photoelectron counts in both of the outgoing channels of Fig.3.

The small differences between the predictions of eqs.(1) and (2), even for all PDC processes considered hitherto, let alone just the one or two I have already tested, does not appear to be a convincing reason to change our allegiance from (1) to (2); the day-to-day practice of science is such that we do not need a reason for staying with (1), even though the latter is now admitted\(^{[4, 5]}\) to be absurd. Students of history will tell us we need a crucial experiment…

Right then. Here it is.

An incident wave of frequency \( \omega \), as well as being down converted by the pump to give a PDC signal of frequency \( \omega_0 - \omega \), may also be up converted to give a PUC signal of frequency \( \omega_0 + \omega \). We depict this phenomenon, which is well known in classical nonlinear optics, in Fig.4. Note that the angle of incidence, \( \theta_0(\omega) \), at which PUC occurs is quite different from the PDC angle, which in Fig.2 was denoted simply \( \theta(\omega) \), but which we should now call \( \theta_{\text{d}}(\omega) \).

Now, following the same argument which led us from Fig.2 to Fig.3, we predict the phenomenon of PUC from the vacuum, which we depict in Fig.5. When we come to calculate the intensity of the PUC rainbow\(^{[7]}\), there is an important difference from the...
PDC situation, because we find that the idler intensities are now less than the input zeropoint intensities. The signal intensities in both channels almost, but not quite, cancel this shortfall, so that the PUC intensities, above threshold, are only about 3 per cent of the corresponding PDC quantities, which may explain why nobody has yet observed them. Furthermore, the intensity of the \(\omega_0 + \omega\) output is actually less than the zeropoint input, and will not therefore be detected at all.

But the intensity of the other output is above zeropoint and my prediction, therefore, is that, as well as the main PDC rainbow \(\theta_d(\omega)\), there is a satellite rainbow, whose intensity is about 3 percent of the main rainbow, at \(\theta_u(\omega)\). Although the precise position of the satellite rainbow depends on the details of the refractive index as a function of frequency, an approximate calculation indicates[17] that the component of frequency \(\omega_0/2\) will have a value for \(\theta_u\) about 2.5 times the corresponding \(\theta_d\).

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