The hypothesis of a real Cabibbo Kobayashi Maskawa matrix

Preliminary

P. Checchia\textsuperscript{1}, E. Piotto\textsuperscript{2}, F. Simonetto\textsuperscript{3}

\textsuperscript{1}I.N.F.N., Padova (Italy)
\textsuperscript{2}University of Milano, Milano (Italy)
\textsuperscript{3}University of Padova, Padova (Italy)
1 Introduction

In the frame of the Standard Model the mass eigenstates of the quark fields are not eigenstates of the weak interactions: the two set of bases are connected by a unitary transformation which is represented by a three times three unitary matrix. This fact accounts for flavour changing charged currents and flavour changing neutral currents. The mechanism of flavour mixing was originally proposed to account for the different amplitudes of the decays of the muon and of the down and strange quarks, all mediated by weak charged currents [1]. It was extended to three quark generations in 1973 [2] (the beauty quark, down type member of the third generation was later discovered in 1977) as a possible description of the CP violation in $K^0$ decays within the frame of the Standard Model without introducing new (so called Super Weak) interactions. Once removed all arbitrary phases, the three by three unitary matrix must contain four independent parameters, three rotation angles in the quark field space and a phase, which introduces an imaginary part in the Hamiltonian. Therefore the amplitudes for a process and for its CP conjugated (obtained by hermitian conjugation) may differ.

Several representations of the the Cabibbo-Kobayashi-Maskawa [1, 2] mixing matrix

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

(1)

were proposed in the past. The Wolfenstein parametrisation [3] is adopted in this paper as it naturally describes the measured hierarchy among the parameters. The matrix elements are expressed in terms of the four parameters $\lambda$, $A$, $\rho$ and $\eta$

$$V_{CKM} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4).$$

(2)

A simple extension valid to $O(\lambda^6)$ is described e.g. in [4]. The parameter $\eta$ is the complex phase accounting for CP violation of the weak interactions. The $\lambda$ and $A$ parameters are known with a good accuracy ($\sim 1\%$ and $\sim 4\%$, respectively), while many contributions to extract $\rho$ and $\eta$ from the available measurements exist in the literature [1, 3, 4]. To this purpose, the measurements of the CP violation parameter in neutral Kaon decay $|\epsilon_k|$, of the difference between the mass eigenstates in the $B_d^0 - \bar{B}_d^0$ system $\Delta m_d$ and of the ratio $|V_{cb}|$ and the lower limit on the difference between the mass eigenstates in the $B_s^0 - \bar{B}_s^0$ system $\Delta m_s$ can be used. On the other hand, since the only direct experimental evidence for CP violation is given by the fact that $|\epsilon_k| \neq 0$ and the effect could be explained in term of models proposed in alternative to the Standard Model (see, for instance, [4, 8] and references therein), it is suggested to remove the constraint coming from the neutral Kaon system and to investigate the results on parameter $\eta$ or to test the hypothesis of a real $V_{CKM}$ matrix. In [3] two different procedures have been exploited. In the first one the two parameters $\rho$ and $\eta$ has been fitted to the experimental values of all available constraints described therein, apart that coming from measurement of neutral kaon mixing. In the second one the same fit has been computed but forcing $\eta$ to zero. The results of the two procedures in [3] are opposite. In this letter the second procedure has been followed but with a different statistical approach, the Best Linear Unbiased Estimator [5], and the conclusion is opposite to the second procedure in [3] but in agreement...
with the first one. In this letter, the inclusion of different data sets is also discussed and the corresponding results are presented.

2 Measurements and constraints on $V_{CKM}$ parameters

The $\lambda$ parameter is the sine of the Cabibbo angle [10]:
\[
\lambda = |V_{us}| = \sin \theta_c = 0.2196 \pm 0.0023. (3)
\]

The $A$ parameter depends on the matrix element $|V_{cb}| = 0.0395 \pm 0.0017$ (obtained from semileptonic decays of B hadrons [10]) and on $\lambda$:
\[
A = \frac{|V_{cb}|}{\lambda^2} = 0.819 \pm 0.035. (4)
\]

In order to constrain the parameters $\rho$ and $\eta$ without considering CP violation in the neutral Kaon system, three experimental input are used:

2.1 $B_d^0$ oscillations.

The mass difference $\Delta m_d$ between the mass eigenstates in the $B_d^0 - \bar{B}_d^0$ system has been measured with high precision [10, 11, 12]. In the Standard Model it can be related to the CKM parameters in the following way:
\[
\left[(1 - \rho)^2 + \eta^2\right] = \frac{\Delta m_d}{G_F^2 m_t^2 m_{B_d^0} \left(f_{B_d}\sqrt{B_{B_d}}\right)^2 \eta_B F(z) A^2 \lambda^6} (5)
\]

where $m_t$ is the top pole mass scaled according to [15] and $z = m_t^2/m_W^2$. The function $F(z)$ is given by:
\[
F(z) = \frac{1}{4} + \frac{9}{4(1 - z)} - \frac{3}{2(1 - z)^2} - \frac{3z^2 \ln z}{2(1 - z)^3}. (6)
\]

The values of all the parameters are given in table II.

In the $\rho - \eta$ plane, the measurement of $\Delta m_d$ corresponds to a circumference centred in (1,0). By constraining $\eta$ to zero, an evaluation of $\rho$ can be obtained. Unfortunately the term $f_{B_d}\sqrt{B_{B_d}}$, given by lattice QCD calculations, is known with a 20% order uncertainty [13, 14] and therefore it gives the largest contribution to the error on the $\rho$ determination.

2.2 $B_s^0$ oscillations.

The mass difference $\Delta m_s$ between the mass eigenstates in the $B_s^0 - \bar{B}_s^0$ system is expected to be much larger than $\Delta m_d$ and in the Standard Model is related to the CKM parameters:
\[
\left[(1 - \rho)^2 + \eta^2\right] = \frac{\Delta m_d}{\Delta m_s} \frac{1}{\lambda^2} \frac{m_{B_s^0} f_{B_s^0}}{m_{B_d^0}} \xi^2 (7)
\]
Since the ratio
\[ \xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}} \] (8)
is computed by lattice QCD with a better precision than the single terms, a measurement of \( \Delta m_s / \Delta m_d \) could provide a much stronger constraint on the \( \rho - \eta \) plane. However, given the very high frequency in the \( B_0^s - \bar{B}_0^s \) system oscillation, only a lower limit on \( \Delta m_s \) is available, as shown in table 1, which corresponds to a circular bound in the two parameter space or, if the assumption \( \eta = 0 \) is made, to a lower limit for \( \rho \).

2.3 |\( V_{ub} \)| measurements from semileptonic b decay.

Charmless semileptonic b decays have been used to measure |\( V_{ub} \)| or the ratio |\( V_{ub} \)| / |\( V_{cb} \)|. The CLEO collaboration determined that parameter both by measuring the rate of leptons produced in B semileptonic decays beyond the charm end-point [17] and from direct reconstruction of charmless B semileptonic decay [18]. The two results are consistent but both methods are limited by theoretical uncertainties. In [10] they are not combined and a value |\( V_{ub} \)| / |\( V_{cb} \)| = 0.08 ± 0.02 obtained from the former result is given. Recently CLEO gave a new result and an average with their previous results [19]. At LEP, ALEPH [20], L3 [21] and DELPHI [22] have measured the inclusive charmless semileptonic transitions \( b \rightarrow u l \nu \). The average |\( V_{ub} \)| value from LEP measurements is given in [23] and the combination obtained using the latest CLEO result is given in table 1 in terms of |\( V_{ub} \)| / |\( V_{cb} \)|. The ratio of the two CKM matrix elements is related to the \( \rho \) and \( \eta \) parameters by:
\[ \frac{|V_{ub}|}{|V_{cb}|} = \lambda \sqrt{\rho^2 + \eta^2} \] (9)
and hence, in the \( \rho - \eta \) plane, the measurement of |\( V_{ub} \)| / |\( V_{cb} \)| corresponds to a circumference centred at the origin. If \( \eta \) is assumed to be zero, it would be proportional to the \( \rho \) absolute value.
3 Data compatibility with a real CKM matrix hypothesis

The assumption of a real CKM matrix implies that all the constraints described in the previous section are reduced to values of (or limits on) $\rho$. The compatibility of the obtained values can then be used to estimate the goodness of the assumption itself. In [3] and in the explicit reference to it in [8], it is written that the hypothesis of a real CKM matrix can fit the data. However it is unclear which was the statistical approach followed in the second procedure in [3] to come to that statement [4] and a completely different conclusion can be obtained with the same procedure and a standard statistical method. In addition the claim in the second procedure of [3], although using a different input-data-set, contradicts what is reported in [7]. Assuming $\eta = 0$, modifying accordingly eq. (5), (7) and (9) and using exactly the same input parameters as [3] (table 1 second column), the values

$$\rho^{\Delta m_d} = 0.01^{+0.18}_{-0.26}$$

and

$$\rho^{V_{ub}} = \pm (0.42 \pm 0.07)$$

are obtained from eq. (5) and (9), respectively. The limit on $\Delta m_s$ has been obtained by means of the amplitude method [24] which allows to know the exclusion Confidence Level for any value of $\Delta m_s$. Therefore, by a convolution with the dominant uncertainty from the ratio $\xi$ in eq. (7) it is possible to obtain:

$$\rho^{\Delta m_s} > -0.05$$

at the 95% Confidence Level.

In a naive approach on which the errors on $\rho^{\Delta m_d}$ and $\rho^{V_{ub}}$ are assumed to be uncorrelated it is evident that the two values are fairly incompatible. The negative $\rho^{V_{ub}}$ solution is clearly excluded by the $\rho^{\Delta m_s}$ limit and therefore it can be discarded. In order to include all the correlations due to common terms contributing to the errors, namely $|V_{cb}|$ and $\lambda$, a two dimensional error matrix $M$ has been written and the Best Linear Unbiased Estimator [9] has been used:

$$\rho^{\text{BLUE}} = \sum_{i=1}^{2} \sum_{j=1}^{2} \rho_i (M^{-1})_{ij}$$

with the variance

$$\sigma^2_{\rho} = \frac{1}{\sum_{i=1}^{2} \sum_{j=1}^{2} (M^{-1})_{ij}}.$$

The error matrix $M$ includes correlated and uncorrelated contributions:

$$M_{ij} = \delta_{ij} \sigma_i^{\text{uncorr}} \sigma_j^{\text{uncorr}} + \sum_{\alpha=1}^{m} \Delta_{\alpha i} \Delta_{\alpha j}$$

4In the cited paper, a first procedure where both $\rho$ and $\eta$ are fitted excludes the hypothesis $\eta = 0$ at the 99% C.L. A second fit with $\eta$ forced to zero gives $\chi^2 = 6.7$ and the author concludes that this is compatible with a real CKM hypothesis but it is not specified why the mentioned $\chi^2$ value corresponds to a reasonable Confidence Level for such hypothesis.
where the indexes \(i\) and \(j\) run over the two \(\rho\) measurements and \(\Delta \alpha_i\) is the change (with sign) on measurement \(i\) when the common systematic parameter \(\alpha\) is moved by its error. The \(\chi^2\) is obtained by:

\[
\chi^2 = \sum_{i=1}^{2} \sum_{j=1}^{2} \left( [\rho_i - \rho^{BLUE}] (M^{-1})_{ij} [\rho_j - \rho^{BLUE}] \right).
\]

(16)

With the table \(\text{I}\) (second column) parameters and taking the positive error in eq. (10), \(\rho^{BLUE} = 0.36 \pm 0.07\) and \(\chi^2 = 4.3\) (1 degree of freedom) are obtained. The values \(\Delta \alpha_i\) and the more relevant uncorrelated contributions are given in table \(\text{II}\) and table \(\text{III}\) respectively. The correlation between the two measurements is small given the dominance of the uncertainty on \(f_{Bs} \sqrt{B_{Bs}}\) in \(\rho^{\Delta m_d}\) as it can be deduced from the small off-diagonal term in the error matrix \(\mathbf{M}\) (table \(\text{IV}\)). The corresponding \(\chi^2\) probability is 3.9\% and this is clearly in contradiction with what stated after the second procedure of \([6]\).

| Parameter (\(\alpha\)) | \(\Delta \alpha \rho \Delta m_s\) | \(\Delta \alpha \rho V_{ub}\) |
|------------------------|-----------------------------|-----------------------------|
| \(V_{cb}\)            | 4.3                         | -1.8                        |
| \(\lambda\)           | 1.1                         | -0.4                        |

Table 2: Variation \(\times 10^{-2}\) on \(\rho\) when the parameter \(\alpha\) is moved by its positive error.

| Parameter | \(i = \rho^{\Delta m_d}\) | \(i = \rho^{V_{ub}}\) |
|-----------|---------------------------|------------------------|
| \(f_{Bs} \sqrt{B_{Bs}}\) | +17.1 - 26.1              |                        |
| \(m_t\)  | ±3.1                      |                        |
| \(\Delta m_d\) | ±1.7                     |                        |
| \(\eta_B\) | ±1.0                      |                        |
| \(|V_{ub}|\) |                        | ±7.1                    |

Table 3: Uncorrelated contributions \(\times 10^{-2}\) to the \(\rho\) errors \(\sigma_i\) computed from the parameters listed in the second column of table \(\text{I}\).

| Parameter | \(j = \rho^{\Delta m_d}\) | \(j = \rho^{V_{ub}}\) |
|-----------|---------------------------|------------------------|
| \(i = \rho^{\Delta m_d}\) | 3.3 \times 10^{-2}        | -0.8 \times 10^{-3}    |
| \(i = \rho^{V_{ub}}\) | -0.8 \times 10^{-3}       | 5.4 \times 10^{-3}     |

Table 4: Elements of the error matrix \(\mathbf{M}\) \(M_{ij}\) computed from the parameters listed in the second column of table \(\text{I}\).

The effect of the limit on \(\Delta m_s\) is negligible even though the information contained in the amplitude is taken into account as suggested in \([4]\): since the value of \(\Delta m_s\) that one would obtain by inserting \(\rho = \rho^{BLUE}\) and \(\eta = 0\) in eq. (7) is \(\Delta m_s = 31\) ps\(^{-1}\), the present experimental sensitivity \((13.8\) ps\(^{-1}\)) does not give any sizeable information on that \(\rho\) region.

In order to take into account terms of the order up to \(O(\lambda^5)\) \([25]\), the substitution \(\rho \rightarrow \bar{\rho} = \rho(1 - \lambda^2/2)\) must be done in eq. (5) with \(\eta = 0\).
More recent values of \( f_{B_d}\sqrt{B_{B_d}} \) can be taken from [14] (option a: \( f_{B_d}\sqrt{B_{B_d}} = 0.215^{+0.040}_{-0.030} \) GeV) or from [10] (option b: \( f_{B_d}\sqrt{B_{B_d}} = 0.210^{+0.039}_{-0.032} \) GeV) where the latter value has been obtained by considering the measured value of \( f_{D_s} \) and the theoretical evaluation of the ratio \( f_{B_d}/f_{D_s} \). In the following both cases will be considered. Including the last results on \( V_{ub} \) [14, 23] (see table I), the incompatibility of the two \( \bar{\rho} \) measurements is still present:

\[
\bar{\rho}^{\text{BLUE}} = 0.36 \pm 0.05 \text{ and } \chi^2 = 3.9 \text{ (1 degree of freedom)} \quad (18)
\]

corresponding to a \( \chi^2 \) probability of 4.9% (option a) and

\[
\bar{\rho}^{\text{BLUE}} = 0.35 \pm 0.05 \text{ and } \chi^2 = 4.3 \text{ (1 degree of freedom)} \quad (19)
\]

corresponding to a \( \chi^2 \) probability of 3.7% (option b).

Since the dominant error in \( \rho^{\Delta m_d} \) is due to the lattice QCD computation, the hypothesis of a flat error with the same R.M.S. on \( f_{B_d}\sqrt{B_{B_d}} \) has been studied with a simple simulation. Several experimental results for \( \rho^{\Delta m_d} \) and \( \rho^{V_{ub}} \) have been generated with central values equal to \( \bar{\rho}^{\text{BLUE}} \). For \( \rho^{\Delta m_d} \) this is achieved by shifting the value of \( f_{B_d}\sqrt{B_{B_d}} \) and allowing it to vary within a flat distribution with the R.M.S. corresponding to the quoted error. All the other parameters of eq. (5) are allowed to vary with a gaussian distribution corresponding to their error. In each experiment the combined value and the \( \chi^2 \) are computed according to eq. (13) and (16), respectively. Looking at the \( \chi^2 \) distribution, it is possible to determine the fraction of simulated experiments with a \( \chi^2 \) higher than the value found in the evaluation with real data. This procedure has been repeated for the data sets of table I and for none of them the \( \chi^2 \) has been found to be higher than the experimental values of eq. (17), (18) and (19) in more than 5% of the cases.

In [1] it is mentioned that contributions from new physics can modify the value \( \rho^{\Delta m_d} \) and the limit \( \rho^{\Delta m_s} \). In particular, being \( \delta_{d,s} \) the new physics contribution to \( \Delta m_{d,s} \), the changes on \( \rho^{\Delta m_{d,s}} \) depend on the fractional contributions \( F_{d,s} = \delta_{d,s}/\Delta m_{d,s} \) and with the data of the third column of table I \( \Delta \rho^{\Delta m_d} \sim 0.5F_d \) and \( \Delta \rho^{\Delta m_s} \sim 0.5(F_d - F_s) \) are obtained. It is straightforward that in presence of a very large term \( \Delta \rho \) there cannot be any exclusion of the \( \eta = 0 \) hypothesis. It must be noticed, however, that those terms were not foreseen in the second procedure of [3] which has then to be compared with the present result from eq. (17). An evaluation of the allowed range of \( F_d \) from the data of table I 3rd column option b), shows that small positive values of \( F_d \) are excluded (\( F_d > 0.04 \) at the 95% Confidence Level) while all negative values are excluded except for the case of large positive \( F_s \) (\( O(30\%) \)) which would allow for negative values of \( \rho \). This evaluation agrees with [4].

### 4 A method to determine \( \bar{\rho} \) and \( \bar{\eta} \)

To obtain \( \bar{\rho} \) and \( \bar{\eta} \) without using the constraint on \( |\epsilon_k| \) the eq. (5), (7) and (9) have to be used. It must be noticed that, in absence of a measurement of \( \Delta m_{s} \), the number of constraints is such that a solution for the system of eq. (5) and eq. (9) is often found or, in other words, a 0(C) fit with \( \bar{\rho} \) and \( \bar{\eta} \) as free parameters could converge to a pair of values such that the total \( \chi^2 \) function minimum is zero. The addition of terms taking into account the existing uncertainty in the value of the parameters entering into the two
expressions does not modify the number of degrees of freedom. With the present data (table 3rd column option b) such a solution exists and it is:

$$\bar{\rho} = 0.13^{+0.13}_{-0.23} \text{ and } \bar{\eta} = 0.38^{+0.07}_{-0.09}. \quad (20)$$

In the most recent search for $B_s^0$ oscillation [12] with the amplitude method an evidence for such a signal was not found but in the region $13 \text{ ps}^{-1} < \Delta m_s < 17 \text{ ps}^{-1}$ the combined amplitude was greater than zero by more than 1.645 standard deviations. As explained in [24], the amplitude $A$ and its error $\sigma_A$ at any value of $\Delta m_s$ can be related to the log-likelihood referenced to its value for an infinite $\Delta m_s$ (infinite oscillation frequency) by the expression $\Delta L_{\infty}(\Delta m_s) = [1/2 - A(\Delta m_s)]/\sigma_A^2$. From eq. (7) is possible to express $\Delta m_s$ as function of $\bar{\rho}, \bar{\eta}, \xi$ etc. and then the experimental amplitude and its error corresponding to a set of these parameter can be determined. Therefore a global log-likelihood function can be written as:

$$L = \Delta L_{\infty}(\bar{\rho}, \bar{\eta}, \xi) + L^{\Delta m_d,V_{ub}}(\bar{\rho}, \bar{\eta}, f_{B_d} \sqrt{B_{B_d}}, \ldots)$$

where $L^{\Delta m_d,V_{ub}}$ is the log-likelihood obtained from eq. (5) and (9) as function of the input parameters of those equations and their errors. Clearly $L^{\Delta m_d,V_{ub}}$ alone has a minimum in correspondence to the values of eq. (20) while the minimisation of $L$ gives:

$$\bar{\rho} = 0.14^{+0.05}_{-0.06} \text{ and } \bar{\eta} = 0.37 \pm 0.05. \quad (21)$$

The addition of the term $\Delta L_{\infty}$ has the effect of excluding the $\bar{\rho}, \bar{\eta}$ space on the region corresponding to the excluded $\Delta m_s$ values (i.e. negative $\bar{\rho}$) but also of reducing considerably the errors of the two parameters on the opposite direction. The latter fact is related to the presence of a minimum on $\Delta L_{\infty}(\Delta m_s)$ for $\Delta m_s = 14.75 \text{ ps}^{-1}$ (see [12]) which is about 2.8 units below the asymptotic value for very high frequencies [26]. However it has to be emphasised that, in absence of a clear signal for the $B_s^0$ oscillation, the presence of ghost minima on $\Delta L_{\infty}(\Delta m_s)$ cannot be excluded. Moreover, the minimum is slightly above the experimental sensitivity ($14.3 \text{ ps}^{-1}$). As a consequence, the results of eq. (21) must be taken as an indication of a method to include the $\Delta m_s$ information (and of the relevant effects of its inclusion) rather than as a robust evaluation of $\bar{\rho}$ and $\bar{\eta}$ with the present data.

## 5 Conclusions

The hypothesis of a real CKM matrix is tested on the basis of the present published and preliminary data and lattice QCD calculations. The Best Linear Unbiased Estimator has been used for the statistical approach. With all the input data used, included those suggested in [6], that hypothesis is excluded at more than 95% Confidence Level. This result agrees with the result indicated in [7] with an older input data set and with a first procedure reported in [6] but contradicts statements in the same paper from a second procedure. A method to include the $\Delta m_s$ information is proposed showing that a strong reduction on the error on $\bar{\rho}$ and $\bar{\eta}$ is achievable.

## 6 Acknowledgements

We wish to thank R. Barbieri, M. Loreti, G. Martinelli, M. Mazzucato, F.Parodi, A. Stocchi and L. Ventura for comments and useful discussions.
References

[1] N. Cabibbo, *Phys. Rev. Lett.* **10** (1963) 531.

[2] M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49** (1973) 652.

[3] L. Wolfenstein, *Phys. Rev. Lett.* **51** (1983) 1945.

[4] P. Paganini et al., *Phys. Scripta* **58** (1998) 556.

[5] F. Parodi et al., hep-ph/9802289.

[6] S. Mele, *Phys. Rev. D* **59** (1999) 113011.

[7] R. Barbieri et al., *Phys. Lett. B* **425** (1998) 119.

[8] H. Georgi and S. Glashow, HUTP-98/A048, hep-ph/9807399.

[9] See for example L.Lyons at al. *Nucl. Instr. and Methods* **A 270** (1988) 110 and references therein; details of the method are also described in COMBOS manual available at http://www.cern.ch/LEPBOSC/combos.

[10] Particle Data Group, *Eur. Phys. J. C* **3** (1998) 1.

[11] The LEP B Oscillation working group, LEPBOSC 98/3 contribution to the Vancouver 1998 conference.

[12] The LEP B Oscillation working group, LEPBOSC 99/1.

[13] J.M. Flynn and C.T. Sachraida, hep-lat/9710057, Proceedings of: 4th International Workshop on Progress in Heavy Quark Physics Rostock, Germany.

[14] T. Draper, hep-lat/9810063, 14th International Symposium on Lattice Field Theory: Lattice '98 Boulder, CO, UK.

[15] A. J. Buras et al., *Nucl. Phys. B* **347** (1990) 491.

[16] F. Parodi et al., LAL 99-03, DELPHI 99-27 CONF 226.

[17] CLEO collab., J.Bartelt et al., *Phys. Rev. Lett.* **71** (1993) 4111.

[18] CLEO collab, J.P. Alexander et al., *Phys. Rev. Lett.* **77** (1996) 5000.

[19] CLEO collab, B.H. Beherens et al., CLNS 99/1611, hep-ex/9905056.

[20] ALEPH collab., R. Barate et al., *Euro. Phys. J.C* **6** (1999) 555.

[21] L3 collab., M. Acciarri et al., *Phys. Lett. B* **436** (1998) 174.

[22] DELPHI collab., M. Battaglia et al. DELPHI 98-97 CONF 165.

[23] The LEP $V_{ub}$ working group, LEPVUB 99/1.

[24] H.G. Moser and A. Roussarie, *Nucl. Instr. and Methods* **A 384** (1997) 491.
[25] A. J. Buras et al., *Phys. Rev.* D 50 (1994) 3433.

[26] The amplitude values as from [12] are available in: http://www.cern.ch/LEPBOSC/combined_results/may_1999/dms_W.dat