A new approach to overcome the imbalance in three-phase systems using the new proposed fractional-polynomial functions

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Abstract. Non-linear or unbalanced loads, etc., are the typical causes of asymmetry in the three-phase systems. The typical difficulty that exists in many methods of symmetrisation is the calculation of the adjusted values (regulable parameter). In this paper, we propose a new method to overcome all of the above problems, which includes: introduction of the new dependence of the basic electrical quantities (voltages and currents) on the controllable values of the static compensation devices; the proposed new method of overcoming the asymmetric operation regime in the three-phase systems, in which we aim to reduce the number of compensating devices.

1. Introduction
Unbalanced currents generate voltages and currents of the inverse and zero sequences, which are very undesirable in the operation of three-phase systems. To reduce the influence of imbalance, we can take several actions with varying degrees of technical complexity including rearrangement or redistribution of single-phase loads in such a way that the system becomes more balanced, changing operating parameters or using reactive power compensation devices, etc.

The typical difficulty in all of the above methods is the calculation of the adjusted values (controllable parameter) in reducing asymmetry. Its cause is the limitation of describing the relationship between the parameters of the steady-state mode and the adjusted values. And, the solution to the problem is to provide a link between the parameters of the steady state mode and the controllable parameters of the compensator. This relationship is presented by the fractional-polynomials functions, which describe the variations of voltages and currents according to controllable parameters. This proposal is based on some mathematical rules as well as the laws of circuit theory [1, 2, 3, 4, 5, 6, 7, 8].

In order to optimize calculation, intelligent search algorithms are applied to improve the efficiency and speed of the method.

2. New proposed fractional-polynomial functions
To clearly explain our proposition of fractional-polynomial functions (FPF) that can be applied to any electrical system, we agreed to the following clause: the part of the circuit that is observed from a bus to the side of the distributed system equivalent to the voltage source in series with a complex resistor. This clause allows us to prove the accuracy of the proposed FPF based on a simple circuit and infer that it is also accurate for any complex electrical systems. This clause based on the equivalent circuit transformation theorems of Thevenin and Norton [1, 6, 7].
Assume that we have a three-phase circuit consisting of \((n+1)\) nodes and \(m\) \((n < m)\) branches, that is described by the matrix of link nodes:

\[
A = \begin{bmatrix}
    1 & 2 & 3 & 4 \\
    a_{11} & a_{12} & \cdots & a_{1m} \\
    a_{21} & a_{22} & & a_{2m} \\
    \vdots & \vdots & \ddots & \vdots \\
    a_{n1} & a_{n2} & \cdots & a_{nm}
\end{bmatrix}
\]

where \(a_{ij} = 1; i=1+n; j=1+m\); if the positive direction of the current in the \(j\)-th branch goes out from the \(i\)-th node; \(a_{ij} = -1\) – enters; \(a_{ij} = 0\) if the \(j\)-th branch is not connected to the \(i\)-th node.

The vector of the complex conductance of the branches:

\[
\mathbf{Y} = \text{diag}(Y_1,Y_2,\ldots,Y_m).
\]

Vectors of current and electromotive force sources:

\[
\mathbf{J} = (J_1,J_2,\ldots,J_m)^	ext{T}.
\]

\[
\mathbf{E} = (E_1,E_2,\ldots,E_m)^	ext{T}.
\]

The nodal voltage equations are written in matrix form as follows: [1]

\[
AYA'U_0 = -A(J + YE)
\]

where \(U_0 = (U_1,U_2,\ldots,U_n)\) – vector of the nodal voltages.

If let \(Y = AYA'\) – a matrix of the aggregate conductance, its size \((n \times n)\); \(J = -A(J + YE)\) vector equivalent current sources, its size \((n \times 1)\) then (1) becomes \(YU_0 = J\).

The nodal voltages can be calculated using the Cramer formula:

\[
U_i = \frac{\det Y_i}{\det Y}
\]

where \(Y_i\) – the matrix obtained by replacing the \(i\)-th column of the matrix \(Y\) with the column vector of \(J\).

Assume that the compensators are constrained at two phases (parallel/serial) and their values are denoted \(x_1\) (FACTS1) and \(x_2\) (FACTS2) (see Figure 1), the matrix \(Y\) will be:

\[
A = \begin{bmatrix}
    1 & \cdots & i & \cdots & j & \cdots & n \\
    Y_{1,1} & \cdots & Y_{1,i} & \cdots & Y_{1,j} & \cdots & Y_{1,n} \\
    \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
    Y_{i,1} & \cdots & Y_{i,i} & \cdots & Y_{i,j} & \cdots & Y_{i,n} \\
    \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
    Y_{j,1} & \cdots & Y_{j,i} & \cdots & Y_{j,j} & \cdots & Y_{j,n} \\
    \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
    Y_{n,1} & \cdots & Y_{n,i} & \cdots & Y_{n,j} & \cdots & Y_{n,n}
\end{bmatrix}
\]

**Figure 1.** The matrix \(Y\).
and vector $\mathbf{J} = \{J_1, \ldots, J_j, \ldots, J_n\}$.

where $i, j = 1+n$; $\mathbf{Y}_{i,j} = \mathbf{Y}_{j,i}$ [1].

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The matrix determinants $\mathbf{Y}$ and $\mathbf{Y}_i$ are defined as follows:

$$\det \mathbf{Y} = a_0 + a_1x_1 + a_2x_2 + a_3x_1x_2 = a_0 \left( 1 + \frac{a_1}{a_0} x_1 + \frac{a_2}{a_0} x_2 + \frac{a_3}{a_0} x_1x_2 \right); \quad \det \mathbf{Y}_i = b_{0,i} + b_{1,i}x_1 + b_{2,i}x_2 + b_{3,i}x_1x_2$$

In general case, nodal voltages will be

$$U_i = \frac{\det \mathbf{Y}_i}{\det \mathbf{Y}} = \frac{b_{0,i} + b_{1,i}x_1 + b_{2,i}x_2 + b_{3,i}x_1x_2}{a_0 \left( 1 + \frac{a_1}{a_0} x_1 + \frac{a_2}{a_0} x_2 + \frac{a_3}{a_0} x_1x_2 \right)}$$

Let $a_p = a_p / a_0; p = 1+3$ and $c_{q,i} = b_{q,i} / a_0; q = 0+3$, we have

$$U_i = \frac{\det \mathbf{Y}_i}{\det \mathbf{Y}} = \frac{c_{0,i} + c_{1,i}x_1 + c_{2,i}x_2 + c_{3,i}x_1x_2}{1 + a_1x_1 + a_2x_2 + a_3x_1x_2}$$

(2)

where, coefficients $c_0$...$c_3$ and $a_1$...$a_3$ are complex coefficient; $x_1$ and $x_2$ are real numbers; coefficients $a_1$...$a_3$ have the same value for all buses; $i = 1+n$ – index of buses.

Assume that the regulable parameters $x_1$ and $x_2$ are measured in Ohms. By placing $x_1' = 1/x_1$, $x_2' = 1/x_2$ and performing a few simple transformations, the function can be obtained in the same form (2) in which the controllable parameters have the unit of conductance (Siemens). Of course, in this case, its coefficients will be different from (2). Thus, the form of the FPF does not depend on the choice of the regulable parameters such as resistance or conductance, meaning it can be arbitrarily chosen.

If the $i$-th branch connects two nodes $j$-th and $k$-th, the current will flow from $k$ to $j$, in the general case, as follows: (*)

$$I_i = \frac{d_{0,i} + d_{1,i}x_1 + d_{2,i}x_2 + d_{3,i}x_1x_2}{1 + a_1x_1 + a_2x_2 + a_3x_1x_2}$$

(3)

Coefficients $a_1$...$a_3$, in this case, have the same value as the coefficients of the voltages, $i = 1+m$ – index of branches.

Equations (2) and (3) are two proposed FPF, which are the dependencies of voltages and currents on controllable parameters of the compensators. Corresponding to each pair of values $(x_1, x_2)$, we get the values of the voltages and currents by the direct measurement of the state of the system. Putting these values into functions (2) and (3) we get the system of equations whose solution contains the coefficients of the FPF [1, 2].

3. Determine the accuracy of the proposed function

In this example, we determined the currents in the branches and the voltages at the nodes in two ways. The first one (theory), based on the Kirchhoff’s laws. Thanks to the software Matlab, we got the current $I_{k,j}^A(x_1, x_2)$ of phase $A$ from node “$j$” to node “$k$” and the voltage $U_{j,k}^A(x_1, x_2)$ of phase $A$ at node “$h$”. In the second (proposed functions), we performed as described in section 2.
The error of proposed functions was calculated by the formula

\[ \varepsilon (x_1, x_2) = 100\% \left| \frac{g_1(x_1, x_2) - g_2(x_1, x_2)}{g_1(x_1, x_2)} \right| \]

here \( g_m(x_1, x_2) = \left\{ \hat{I}^{(j,k)}_{n,A}(x_1, x_2) \right\} \) or \( t^{(j,k)}_{m,A}(x_1, x_2) \); \( j, k = 0, n; j \neq k; m = 1, 2 \) – currents (voltages) that were determined by the above two ways.

Suppose that the characteristic of consumption load (diagram of load) is constant. This hypothesis does not conform to reality, but in practical cases when the slow variable speed of the characteristics of the load, the deviation of the actual characteristic may be considered negligible. Such a hypothesis makes it easier to verify the accuracy of the FPF.

In Figure 2 presented the errors \( \varepsilon_I(x_1, x_2) \) of current and \( \varepsilon_U(x_1, x_2) \) of voltage of the proposed FPF.

![Figure 2. The error in of the proposed FPF.](image)

Because all complex three-phase electrical systems can be converted into a simple system as mentioned in section 2, so we can affirm that its accuracy is also true for complex systems. In short, based on the errors of fractional-polynomial functions which describe the quantity of electricity in the system, has been discussed above, we can absolutely use these polynomials to optimize any unbalanced power system.

4. Applies to symmetrization of a three-phase unbalanced system

**Solution:** In 1918, Fortescue C.L. [9, 10] who explained the principal and gave experimental verification of its correctness. In a three-phase system, one set of phasors has the same phase sequence as the system under study (positive sequence), the second set has the inverse phase sequence (negative sequence), and in the third set the phasors A, B, and C are in phase with each other (zero sequence). Sequence components are converted from electrical quantities in three-phase systems is described briefly as follows:

\[
\begin{bmatrix}
\hat{I}^{(j,k)}_{1}(x_1, x_2) \\
\hat{I}^{(j,k)}_{2}(x_1, x_2) \\
\hat{I}^{(j,k)}_{0}(x_1, x_2)
\end{bmatrix}
= \begin{bmatrix}
a & 1 & 1 \\
a^2 & a & 1 \\
a & a^2 & 1
\end{bmatrix}^{-1}
\begin{bmatrix}
\hat{I}^{(j,k)}_{A}(x_1, x_2) \\
\hat{I}^{(j,k)}_{B}(x_1, x_2) \\
\hat{I}^{(j,k)}_{C}(x_1, x_2)
\end{bmatrix}.
\]

(4)

where \( j, k = 1, N; j \neq k \); \( N \) – the number of the bus in the system and \( n = A, B, C \) – labels of phase and \( a = e^{\frac{2\pi}{3}j} \).
When there are imbalances in the system appear the inverse and zero sequence components. The vectors of each sequence components are linearly associated with the vector of the three-phase currents, we got:

\[
j_n^{(jk)}(x_1, x_2) = \frac{a_n^{(jk)} + b_n^{(jk)} x_1 + c_n^{(jk)} x_2 + d_n^{(jk)} x_1 x_2}{1 + a_1 x_1 + a_2 x_2 + a_3 x_1 x_2}.
\]

(5)

Where, \(a_n^{(jk)}, b_n^{(jk)}, c_n^{(jk)}, d_n^{(jk)}\) are the complex constants that depend on the number \(j\) and \(k\), and the constants \(a_1, a_2\) and \(a_3\) are invariably for all voltages and currents; \(m = 1, 2, 0\) – positive, negative and zero sequence components.

When the system is symmetric, negative and zero sequence components will approach minimum (equal to zero) and positive component maximum. We have

\[
\frac{[f_1(x_1, x_2)]}{x_1-x_2} \rightarrow \text{max}, \quad \frac{[f_2(x_1, x_2)]}{x_1-x_2} \rightarrow \text{min}, \quad \frac{[f_0(x_1, x_2)]}{x_1-x_2} \rightarrow \text{min}.
\]

or

\[
\frac{[f_1^{(jk)}(x_1, x_2)]}{x_1-x_2} \rightarrow \text{max}, \quad \frac{[f_0^{(jk)}(x_1, x_2)]}{x_1-x_2} \rightarrow \text{min}, \quad \frac{[f_0^{(jk)}(x_1, x_2)]}{x_1-x_2} \rightarrow \text{min}.
\]

(6)

Thanks to the giving of polynomials (2), (3), the problem of minimization of the unbalanced mode in three-phase systems become a problem of optimization (6). In this case, is the multi-objective optimization that consists of more than one conflicting objective which cannot be solved with a single solution with a single run [11]:

Results: In this paper, we aim to reduce the number of compensating devices in overcoming the asymmetric operation regime in the three-phase systems. This method can be applied effectively to any three-phase system, especially for systems in which the active powers at each phase of the load are not too different.

In this section, numerical results are carried out on a 5-bus system in 24 hours and executed on Matlab before and after compensation. The optimization results were presented to illustrate the performance of the proposed method.

We have proposed the use of compensators on two phases at one bus, as presented in figure 3 (FACTS 1 and 2; FACTS 3 and 4, etc.). This proposition generates a small problem that is optimization of the placement (e.g., in phases A and B, B and C, or A and C) [12, 13].

![Figure 3. Electrical system.](image-url)
In Figure 4 presented voltages, currents, apparent powers and sequence components of currents before compensations and in Figure 5 – after compensations at a bus (Load 5). Due to the limitations of this paper we didn’t present all the results of the optimization.

![Figure 4. Before compensation.](image)

![Figure 5. After compensation.](image)

Note that in the result of the multi-objective optimization problem we got the Pareto front [11], and the result shown in Figure 5 represents a point on it. Depending on the type of load and how it works, we can make appropriate choices. In addition, in this simulation result, we were considered the continuous variation of the load.

| Table 1. The average values of the ratio $I_0/I_1$ before and after compensation |
|---------------------------------|--------|--------|--------|--------|--------|
|                                 | L.1    | L.2    | L.3    | L.4    | L.5    |
| Before compensation             | 0.20   | 0.14   | 0.20   | 0.28   | 0.42   |
| After compensation              |        |        |        |        |        |
| Without location optimization   | 0.12   | 0.13   | 0.20   | 0.20   | 0.16   |
| With location optimization      | 0.07   | 0.06   | 0.05   | 0.07   | 0.13   |

| Table 2. Compensator locations |
|---------------------------------|
| FACTS Phase A       | FACTS Phase B       | FACTS Phase C       | Phase A       | Phase B       | Phase C       | Phase B       | Phase C       | Phase C       |
| Load 1              | Load 2              | Load 3              | Load 4              | Load 5              |
As we can see in Table 1, the average values of the ratio $I_0/I_1$ (the zero-current imbalance factor $\text{CIF}_0$) of the system before and after compensation has improved significantly. The zero-sequence current is about 10% of the positive current. The effectiveness of the proposed method was proved, though only two single-phase compensation devices/three-phase was used at a bus. Of course, included in it is the optimal calculation of the fitting positions of the compensation devices (Table 2).

5. Conclusions

The dependences of the currents (voltages) on the values of the compensation devices given above, is the mathematical description of the relationship between the static compensating devices for the three-phase system. They allow for simplified and accelerated computation and can be applied to any three-phase system in the reactive power compensation in which static compensators are used. In this article, we have proposed the use of single-phase compensator on two phases at a bus. However, the function can also be extended to cases which it uses a compensator on one (or three) phase(s). This expansion has also been verified and confirmed to be accurate.

The proposed method may be applied for the solution of other problems including balancing of the most sensitive regarding electric energy quality part of the power system, minimization of active power losses, stabilization of three-phase voltages, enhancement of asynchronous machine performance stability and reduction of errors occurring in power consumption measuring circuits.

In contrast with traditional balancer sets, the use of shunts with controllable reactivity possesses a series of considerable advantages:

- Low-cost value as compared with balance-unbalance transformer;
- Possibility to be connected to existing electric plant;
- Low operating expenses;
- Possibility to continue power supply to consumers while the plant is out of service for repair/maintenance.

The scope of application of this method covers three-phase consumers with insignificant phase unbalance. The proposed approach fits well into the existing Smart grid conceptual design that declares the increased role of a consumer during the process of electric energy generation and supply. A common usage of similar devices for industrial and domestic loads shall help to transform large energy units into multi-agent systems with a multitude of active consumers.

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