Vortices in self-bound dipolar droplets

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Quantized vortices have been observed in a variety of superfluid systems, from 4He to Bose-Einstein condensates and ultracold Fermi gases along the BEC-BCS crossover. In this article we study the stability of singly quantized vortex lines in dilute dipolar self-bound droplets. We first discuss the energetic stability region of dipolar vortex excitations within a variational ansatz in the generalized nonlocal Gross-Pitaevskii functional that includes quantum fluctuation corrections. We find a wide region where an absolute minimum corresponding to a vortex state exists. We then show via large scale dynamical simulations that such vortices are subject to fragmentation into two droplets with equal particle number when the splitting energy, defined as the energy difference of the droplet with vortex and two non-overlapping ground state droplets, is positive. When the splitting energy is negative droplets hosting a vortex do not fragment but develop Kelvin waves, leading eventually to a bending of the vortex line. We conclude with some experimental considerations for the observation of such states and suggest possible extensions of this work.

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Quantized vortices are a direct manifestation of the genuine quantum behavior of superfluid systems. A prime example is superfluid Helium which has been widely studied over the past decades. Ultracold quantum gases offer the possibility to investigate vortex properties in a complementary regime in terms of particle numbers, interaction strength and range [21], with either bosonic [4, 37] or fermionic [4, 38] atoms. Dipolar condensates may also display vortex excitations with peculiar properties as a consequence of the long range and anisotropy of the interactions [5, 6]. For a three dimensional condensate with a vortex line and in the presence of a periodic potential, the spectrum of transverse modes may display a roton-like minimum, which destabilizes the straight vortex and leads to a transition from vortex into helical or snake-like configurations [9, 10]. Theoretical models of superfluid states of ultracold gases at zero temperature are usually based on well established mean-field approximations which accurately describe experiments [11], ranging from analytic treatments and variational approaches to full numerical simulations. Corrections beyond the mean-field picture have been measured for strongly interacting Bose gases [12] and for ultracold fermions along the BCS-BEC crossover [13] and compared with ab initio Quantum Monte Carlo calculations. The recent observation of ultradilute self-bound droplets both in dipolar condensates [14–20], as well as in two-component Bose mixtures [21], together with a combined theoretical effort, established the importance of the fundamental role of quantum fluctuations in ultracold atomic systems [22–35]. Yet, no work has investigated the presence and stability of vortex states in self-bound droplets in ultradilute liquids. Helium droplets hosting several quantized vortices have been recently observed [36] and studied in detail theoretically [37–40] both in pure samples and in presence of impurities. The scales are, nevertheless, completely different. Helium droplets can be easily taken as a homogeneous, infinite superfluid background since vortices are much smaller compared to the system size (given the large interaction strengths). Droplets in quantum ferrofluids are very anisotropic and vortex cores are expected to have a size comparable to that of the whole droplet. Here we address the issue of stability and dynamics of singly quantized vortex lines in dipolar droplets for a wide range of the dipolar interaction strength and particle numbers. We carry out large scale fully three-dimensional simulations, which allow for an efficient determination of energies and shapes of droplets as well as their dynamics. We find a strong anisotropy of such droplets, very elongated along the polarization axis. For small particle numbers, droplets are dynamically unstable towards splitting in two droplets where angular momentum is redistributed into surface collective excitations [11]. For larger sizes splitting is energetically prohibited and vortex lines display bending for long times [10, 32–45]. We conclude with a discussion of a possible experimental implementation and observation of our findings with current experimental setups.

Methods. The dynamics of an untrapped dipolar Bose-Einstein condensate is described by a generalized nonlocal Gross-Pitaevskii equation

$$i\hbar \dot{\psi} = \left(-\frac{\hbar^2 \nabla^2}{2m} + \mathcal{K}_{\text{int}}(r) + g_{\text{LHY}}|\psi|^2\right)\psi, \quad (1)$$

where $\psi(r,t)$ is the BEC wavefunction, $\mathcal{K}_{\text{int}}(r) = g_c |\psi(r)|^2 + \int dr' V_{dd}(r-r') |\psi(r')|^2$ describes the contact and dipolar mean-field interaction of the condensate. Here $g_c = 4\pi a_s \hbar^2 / m$ is the contact interaction strength with $a_s$ being the s-wave scattering length, and $V_{dd}(r) = C_{dd} \frac{1 - \cos \frac{3 \pi r}{3 a_s}}{r^3}$ is the dipolar potential. $C_{dd} \equiv \frac{\hbar^2}{2m} \int dr r^2 |\psi|^2$.
The dipolar coupling constant, $a_{dd}$, is the ratio of dipolar interaction to the wave interaction strengths defining stability of a uniform condensate in the Bogoliubov approach when $\varepsilon_{dd} < 1$ [32, 33]. Quantum fluctuation corrections to the meanfield energy for a uniform dipolar condensate are introduced in Eq. (1) by a Lee-Huang-Yang-type (LHY) term, with coefficient $g_{\text{LHY}} = \frac{128\sqrt{\pi} \hbar^2 a_{dd}^{5/2}}{3m \sigma_x^2} (1 + \frac{3}{2} \varepsilon_{dd}^2)$ [27, 46]. Enegertic stability of dipolar droplets in trapping potentials have been studied in a number of works. A stability diagram for droplets in free space was proposed in [24] via a Gaussian ansatz and checked against numerical simulations. The variational approach well describes collective properties of the condensate, such as the energy, the shape close to the instability region, and excitation spectra [28]. Recent ab initio Quantum Monte Carlo calculations also showed a good agreement with results based on Eq. (1) [32, 33].

**Energetic stability diagram.** We begin our study with the static properties of vortex lines introducing a variational wavefunction

$$\psi_V (r) = \left( \frac{2\ell + 3}{\pi \sigma_x^2 \sigma_y^2} \right)^{1/2} \rho^\ell e^{i \ell \theta} e^{-2 \left( \frac{r^2}{2 \sigma_x^2} + \frac{y^2}{2 \sigma_y^2} \right)},$$

where $N$ is the particle number and $\rho$ the radial coordinate. The choice $\ell = 0$ corresponds to a state with no vortex, whereas for $\ell > 0$ the state has $\ell$ quanta of circulation. In this work we specialize to the case of $\ell = 1$, and leave the investigation of multicharged vortices to a separate study. In Eq. (2) the width of the condensate $\sigma_x$ and $\sigma_y$ are variational parameters to be determined via a minimization of the full energy functional associated with Eq. (1). Therefore, we compute the rescaled energy

$$\frac{E_V}{E_0} = \frac{1}{N \sigma_x^2} \left( 1 + \frac{4}{y^2} \right) + \frac{2\sigma_y^2}{3 \sigma_x^2 N} (\varepsilon_{dd}^{-1} - g(y))$$

$$+ \frac{28192 \sqrt{\pi}}{625 \sigma_x^2 \sigma_y^2 N^2} a_{dd}^4.$$ 

where $E_0 = \frac{\hbar^2}{m a_{dd}^2}$, $y = \sigma_y / \sigma_x$, $g(x) = f(x) + 3x f'(x)/8 + x^2 f''(x)/8$ and $f(x) = \frac{1 + 2x^2}{1-x^2} - 3x^2 \tanh^2 \frac{1-x^2}{2}$. The resulting minimization of Eq. (3) is shown in Fig. 1a as a function of $\varepsilon_{dd}^{-1}$ and particle number $N$. For comparison we show the result of energy minimization for $\ell = 0$ (dashed line) [24]. The shaded region below the full line is the stability region of a droplet with $\ell = 1$, which is shrinked compared to the $\ell = 0$ case. Above the solid line there is no minimum and the minimum energy state is a uniform solution with vanishing density [17].

**Shape of dipolar droplets with a vortex line.** We proceed by characterizing the shape of the droplet in the presence of a vortex for different particle numbers and $\varepsilon_{dd}$. In Fig. 1b we compute the vortex core size in units of $a_{dd}$, whereas in Fig. 1c we plot the horizontal and vertical width $w_r$ and $w_z$. All quantities are computed by taking the average length at which density varies from 90% to 10% of its maximum. Variational calculations are checked against full numerical simulations of Eq. (1) in imaginary-time exploiting the azimuthal symmetry of vortex states, where we assume $\psi_V(r) = e^{i\ell \theta} \Psi_V(\rho, z)$. The problem reduces then to an effective two-dimensional problem for the function $\Psi (\rho, z)$. To efficiently compute the kinetic term, we employ a discrete Hankel transform.
in the radial direction and the usual fast Fourier transform in the longitudinal direction [38]. In Figs[1]b–c lines terminate at the spinodal point of Fig[1]a, for the corresponding value of $N$. We notice that for a wide range of interactions and particle numbers the vortex core is of the same order of the radial width $w_r$, and both are always much smaller than the longitudinal width $w_z$. All lengths decrease slightly by increasing $\varepsilon_{dd}$ for a fixed $N$.

**Real-time dynamics.** A crucial issue, relevant for the experiments, is the stability of such vortex states in self-bound droplets. To address this point we perform fully three-dimensional simulations in real time to take into account possible instabilities which can break azimuthal symmetry (see below). The input states at $t = 0$ are taken from numerical simulations in imaginary time, slightly perturbed with numerical noise. In Fig[2] we illustrate the prototypical real-time dynamics of a droplet with a vortex line at $t = 0$ for two cases of condensates of $N = 10^4$ and $N = 10^5$ particles with $\varepsilon_{dd}^{-1} = 0.2$ and 0.1, respectively. Time is measured in units of $t_0 = m a_{dd}^2 / \hbar$, which equals 0.12 $\mu$s for $^{164}$Dy and 0.03 $\mu$s for $^{168}$Er. The blue shaded region displays the magnitude of the pseudovorticity vector $\omega = \nabla Re(\psi) \times \nabla Im(\psi)$ which is tangent to the vortex line along its length [50]. The two dynamics display very different features. For $N = 10^4$ the system develops a splitting instability and divides in two droplets with $N_f \approx N/2$ at $t \approx 1200 t_0$, where $N_f$ is the particle number of each droplet after splitting. For longer times the two fragments move apart with opposite momenta and display no residual vorticity. All initial angular momentum gets transferred into collective surface excitations of the droplets. For the larger system, splitting instability starts to develop at the same time $t = 1200 t_0$, however the droplet does not fragment. At later times the condensate restores a droplet-like configuration with the development of Kelvin waves along the vortex line, eventually leading to vortex bending for $t = 11400 t_0$ and surface excitations.

To explain the dynamical behaviors of the two cases we compute the splitting energy $\Delta E_s^{(N)}$ defined as the energy difference between the initial state of the droplet for $N$ atoms and $\ell = 1$ and two droplets with $N/2$ atoms and $\ell = 0$ (in units of $E_0$)

$$ \Delta E_s^{(N)} = E_{vortex}^{(N)} - 2E_{droplet}^{(N/2)}. $$

If $\Delta E_s > 0$ splitting is energetically possible and the residual energy $\Delta E_s$ can be transferred to kinetic energy of the fragments as well as to their collective excitations, absorbing the vortex angular momentum. Otherwise, if $\Delta E_s < 0$ the droplet cannot split. In Fig[3] we plot the splitting energy as a function of $\varepsilon_{dd}^{-1}$ for $N = 10^4, 10^5$. Variational calculations (lines) are in good agreement with full numerical calculations (dots). We observe that $N = 10^4$ is always unstable against splitting. Instead, for $N = 10^5$ there exists a critical value $\varepsilon_{dd}^{-1} \approx 0.25$ below which the droplet with vortex is stable against splitting [49]. We verified the criterion based on the calculation of the splitting energy $\Delta E_s^{(N)}$ over a large atom number range and for several interaction strengths with time-dependent simulations. In Fig[4]a we divide the energy diagram in two regions. For weaker dipolar interactions...
and smaller number of particles in between the curves $E_V = 0$ and $\Delta E_s^{(N)} = 0$ the vortex line is unstable toward splitting. For stronger dipolar interactions and particle numbers, below the $\Delta E_s^{(N)} = 0$ curve the droplets do not fragment but the vortex line develops Kelvin waves which lead to its bending. As they increase in amplitude, the Kelvin waves distort the vortex line, which shrinks and eventually disappears, leaving a wobbling, though stable, droplet.

![Graph](image)

**FIG. 3. Splitting energy of a droplet with vortex.** Energy difference between a droplet with a vortex ($\ell = 1$) with $N$ atoms and the sum of the energies of two droplets with $N/2$ atoms without the vortex ($\ell = 0$) as a function of $\varepsilon_{dd}^{-1}$. We notice that for $N = 10^4$ the system with a vortex is always unstable to splitting, whereas for $N = 10^5$ there exists a critical value $\varepsilon_{dd}^{-1} \approx 0.25$ below which the droplet with vortex is stable against splitting. Solid lines are variational results, dots are numerical results from Eq. (1).

**Experimental considerations.** Vortices in ultracold atomic systems are created either by phase imprinting or via an effective rotating potential generated by an applied laser beam. In any case, vortex imprint must be done in-trap, following the droplet preparation and before its release. The optical spoon technique is more invasive and requires longer equilibration times and it becomes less likely to work experimentally. Phase imprinting, on its turn, can be done with high spatial resolution as well as by mean of very short optical pulses, allowing an almost instantaneous imprint of a vortex. Following the simulations presented above, the typical times for the observation of vortex dynamics for $^{164}$Dy range from $100 \mu s$ to just below 2 ms, which is short, but still within experimental resolution. For example, for the parameters as in Fig [2], splitting instability sets on for times $t \approx 10^3 t_0$, which correspond to $t \approx 10^{-4}$ s for $^{164}$Dy. Nevertheless, we have also verified instances in which the onset of the splitting process took slightly longer time. This was the case of $\varepsilon_{dd}^{-1} = 0.6$ and $N = 10^5$, where $t \approx 45000 t_0$, corresponding to $t \approx 6$ ms. Detection, in any case, must be done in-situ, preferably with a levitating external magnetic field gradient to allow for longer observation times [19]. Extraction of dynamical properties, e.g. momentum, shape, atom-number, and/or absence of droplet movement in the cases covered in Fig [2] can be done at longer evolution times, on the order of several milliseconds, when detection is expected to be easier.

**Conclusions.** In this article we studied the stability of quantum vortex lines in dilute self-bound droplets of dipolar atoms. We first discussed the energetic stability region of such vortex excitations via a variational ansatz in the generalized nonlocal Gross-Pitaevskii functional that includes a LHY-type contribution. The region corresponding to an absolute minimum $E_V < 0$ is largely unstable to fragmentation into two droplets when also the condition $\Delta E_s^{(N)} > 0$ is met. When this is not the case we found that Kelvin waves establish along the vortex line, which eventually bends in the central region of the droplet. We confirmed our findings by detailed fully three-dimensional numerical simulations of vortex states created by phase imprinting. The situation where Kelvin waves start developing has also been predicted to appear in a similar context of three-dimensional dipolar BECs [9]. Droplets with vortices may thus serve as promising testbeds to the study of twisted vortex lines in real-life experiments. An extension of this work would include the investigation of the excitation spectrum of these vortex lines, similarly to what has been recently done in vortex free droplets [29], and with vortex states in trapped geometries [51]. These instabilities offer new opportunities for devising stabilization methods, such as temporal or spatial modulation of the scattering length as done for non-dipolar BECs [52, 53] or pinning potentials [54, 55]. Also, the appearance of vortex arrays [56, 57] as well as the effect of impurities and turbulence phenomena may be relevant to current experiments [58, 61]. Finally, we point out the analogy between vortices in droplets and multicharged vortices in harmonically trapped BECs [62, 63]. The latter are known to be dynamically unstable against splitting into singly-charged vortices and their splitting dynamics has been experimentally studied [64].

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