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Published in:
Physical Review A

Publication date:
2012

Document Version
Publisher's PDF, also known as Version of record

Link back to DTU Orbit

Citation (APA):
Harris, G. I., Andersen, U. L., Knittel, J., & Bowen, W. P. (2012). Feedback-enhanced sensitivity in optomechanics: Surpassing the parametric instability barrier. Physical Review A, 85(6), 061802.
Feedback-enhanced sensitivity in optomechanics: Surpassing the parametric instability barrier

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(Received 11 September 2011; published 21 June 2012)

The intracavity power, and hence sensitivity, of optomechanical sensors is commonly limited by parametric instability. Here we characterize the degradation of sensitivity induced by parametric instability in a micron-scale cavity optomechanical system. Feedback via optomechanical transduction and electrical gradient force actuation is applied to suppress the parametric instability. As a result a 5.4-fold increase in mechanical motion transduction sensitivity is achieved to a final value of $1.9 \times 10^{-18}$ mHz$^{-1/2}$.

DOI: 10.1103/PhysRevA.85.061802 PACS number(s): 42.50.Pq, 07.10.Cm

Optical techniques are capable of ultraprecise measurements of parameters such as phase, position, and refractive index. The sensitivity is typically limited by optical shot noise, which can be reduced by increasing optical power. Using coherent states of light the ultimate sensitivity is fundamentally set by the standard quantum limit (SQL) [1,2]. However, well before the SQL is reached, radiation pressure may become sufficiently strong to severely alter the dynamics of the intrinsic mechanical motion of the sensor. This regime, called parametric instability, is characterized by violent mechanical oscillations and was first theoretically investigated by Braginsky [3] in the context of large-scale interferometers for gravitational wave detection, followed by experimental observation in electrical readouts of resonant bar systems [4] and later in optical microcavities [5]. The physical process, described graphically in Fig. 1, is a result of radiation pressure from asymmetric Stokes and anti-Stokes sidebands generated from the mechanical motion of the cavity. If this process, known as dynamical backaction heating [6], amplifies the motion at a rate faster than the mechanical decay rate then parametric instability occurs. Due to a combination of large mechanical oscillations and necessary saturation of amplification, the noise floor of the optomechanical sensor increases, rendering it ineffective at transducing small signals. Consequently, parametric instability is predicted to be a problem in the context of ultraprecise optical sensors such as gravity wave interferometers [3].

Parallel to the development of gravitational wave detectors, there has been a recent push towards real-time readout and control of mesoscopic mechanical oscillators in the quantum regime [7–9], facilitating new quantum information technologies [10], and experimental tests of quantum nonlinear mechanics [11–13] and even potentially quantum gravity [14]. However, parametric instability limits the strength of both entangled and squeezed states which may be produced via the optomechanical interaction [15], and—even when employing strategies such as backaction evasion (BAE)—the ability to transduce the mechanical motion at the quantum level [16].

Radiation-pressure-mediated optomechanical interactions are also of increasing importance to photonic circuits and sensors, where many applications are facilitated by miniaturized integrated architectures [17,18]. Miniaturization is often accompanied by high mechanical compliance. This has the adverse consequence of increasing exposure to parametric instability but provides the possibility of introducing new functionality via mechanical elements such as optomechanical switches [19] and memories [20], and ultraprecise gyroscopes, magnetometers [21], and mass and force sensors [22]. Even hybrid optomechanical circuits have been proposed, where fully integrated phononic and photonic circuits are interfaced via the optomechanical interaction [23], with phononic elements used for memories, filters, and processing elements and photonic elements used for communication. Parametric instability can severely adversely affect the performance of such integrated optomechanical devices.

The growing role of complex optomechanical systems in both fundamental science and applications means that techniques capable of individually addressing and suppressing instabilities in optomechanical elements are of crucial importance. In this Rapid Communication, we propose and experimentally demonstrate such a technique based on optoelectromechanical feedback control, characterizing the parametric-instability-induced degradation in, and feedback-induced revival of, mechanical transduction sensitivity in a cavity optomechanical transducer. Our cavity optoelectromechanical system (COEMS), seen in Fig. 1, consists of a silica microtoroid integrating high $Q$ mechanical and optical modes with strong electrical actuation [24]. Parametric instability of a 14-MHz mechanical mode is found to occur at optical powers of 60 $\mu$W, resulting in a drastic loss of broadband sensitivity for higher power levels and a maximum optomechanical sensitivity still a factor of 300 higher than the SQL. Stabilization of the parametric instability is achieved via a viscous damping force applied to the unstable mechanical mode using electric feedback. Narrowband filtering ensures that the feedback force is applied only at frequencies in close proximity to the unstable mechanical mode using electric feedback. Narrowband filtering ensures that the feedback force is applied only at frequencies in close proximity to the unstable mechanical mode using electric feedback. Narrowband filtering ensures that the feedback force is applied only at frequencies in close proximity to the unstable mechanical mode using electric feedback.

The dynamical interaction between light and mechanical motion including radiation pressure $F_{\text{rad}}$, feedback $F_{\text{fb}}$, and thermal forces $F_{\text{T}}$ can be described through the equations of motion [6]

$$m\ddot{x} + \Gamma_0 \dot{x} + \omega_\text{m}^2 x = F_{\text{rad}} + F_{\text{T}} + F_{\text{fb}},$$

(1)

$$\dot{a} = -[\gamma - i(\Delta_0 + gx)]a + \sqrt{2}\gamma_\text{ad}a_m.$$  

(2)
The first equation describes the motion of the mechanical oscillator where $m$, $\Gamma_0$, and $\omega_0$ are its effective mass, damping rate, and resonance frequency, respectively; $F_{\text{rad}} = \hbar g|a(t)|^2$, and $F_T = \sqrt{\Gamma_0/\hbar} m \xi(t)$, where $\xi(t)$ is a unit white-noise Wiener process. The second equation describes the intracavity optical field where $\Delta_0$ is the optical detuning, $|a|^2$ is the intracavity photon number, and $|a_{\text{in}}|^2$ is the input photon flux, coupled into the cavity at rate $\gamma_{\text{in}}$. The total optical decay rate is $\gamma = \gamma_{\text{in}} + \gamma_0$, where $\gamma_0$ is the intrinsic decay rate. The equations are coupled via the optomechanical coupling parameter, $g$, which gives rise to both static and dynamic effects such as radiation pressure bistability [25], the optical spring effect [26], and dynamical backaction cooling and amplification [27–30]. Due to the nonlinear nature of the equations of motion, linearization is required to reach an analytic solution where a separation of each variable into its mean value and fluctuations is performed; $a = \bar{a} + \delta a$ and $x = \bar{x} + \delta x$. Taking the linearized equations into the frequency domain yields

$$\delta a(\omega) = \frac{\sqrt{2} Y_{\text{in}} \delta a_{\text{in}} + i g \bar{a} \delta x(\omega)}{\gamma - i (\Delta - \omega)},$$

$$\chi_0^{-1} \delta x(\omega) = h g [\bar{a} \delta a^\dagger(-\omega) + \bar{a}^\dagger \delta a(\omega)] + F_T(\omega) + F_{\text{rad}}(\omega),$$

where $\chi_0 = m^{-1}[\omega_m^2 - \omega^2 + i \Gamma_0 \omega]^{-1}$ is the mechanical susceptibility and $\Delta = \Delta_0 + g \bar{x}$ is the static detuning of the cavity in the presence of radiation pressure. As seen in Eq. (3) the mechanical fluctuations are imprinted onto the field $\delta a$, which, in turn, is outcoupled with $\omega_{\text{out}} = \omega_{\text{in}} - \sqrt{2} Y_{\text{in}} a$ and detected on a photodiode, giving a photocurrent $i = \bar{a}_{\text{out}} a_{\text{out}}$. After some work the resulting photocurrent fluctuation, $\delta i(\omega) = \bar{a}_{\text{out}}^\dagger \delta a_{\text{out}} + \bar{a}_{\text{out}} \delta a_{\text{out}}$, is found to be

$$\delta i = \delta x \left( \frac{2 i g [\bar{a} \delta a^\dagger] \Delta [\omega - i 2 \gamma_0]}{\gamma^2 + \Delta^2 - \omega^2 + i 2 \gamma_0 \omega} \right) + \delta i_{\text{a}}.$$
Variable Phase/Amplitude

**FIG. 2.** (Color online) (a) Experimental schematic. Blue (light grey) indicates the optical components allowing transduction and parametric instability; green (dark gray) indicates the electrical components used in the feedback stabilization. FPC, fiber polarization controller; BPF, bandpass filter. (b) Observed mechanical spectra below parametric instability threshold.

60 μm and 6 μm respectively with a 26-μm undercut. The toroid-taper separation was controlled by a piezo stage to allow critical coupling into the optical cavity. The laser was thermo-optically locked [33,34] to the full-width half maximum (FWHM) of the optical mode, which had an intrinsic quality factor of \( Q \approx 10^7 \). This optical detuning allowed simultaneous radiation-pressure-induced mechanical amplification and transduction of the mechanical motion. The absolute mechanical displacement amplitude was calibrated via the optical response to a known reference phase modulation [35]. The mechanical motion, which modulates the optical resonance frequency, was detected via fluctuations in the transmitted power incident on an InGaAs photodiode. Fourier analysis of the photocurrent reveals a mechanical power spectra with peaks corresponding to mechanical resonances. A typical spectra containing many mechanical modes can be seen in Fig. 2(b) at optical powers below the threshold for parametric instability.

**FIG. 3.** (Color online) Mechanical power spectra show the mechanical spectra (b) with and (a) without feedback stabilization of the regenerative 14-MHz mode.

instability, generating harmonics on the transduction signal at 28, 42, and 56 MHz due to the nonlinear process involved in saturation. This is evident by the emergence of a dark narrow band, among broadband noise, at 28 MHz. The noise spectra at a fixed power of 136 μW is shown in Fig. 4(a) (dark line) and reveals the narrowed harmonic at 28 MHz, and beat frequencies from mixing with the groups of mechanical modes, labeled \( G_1 \) and \( G_2 \) in Fig. 2(b). The three peaks in the region labeled \( U + G_2 \) arise from sum frequency generation between the group \( G_2 \) and the unstable mode \( U \), while the peaks in the regions \( 2U \pm G_1 \) arise from sum and difference frequency generation, respectively, between group \( G_1 \) and the second harmonic of the unstable mode \( 2U \). In addition to these harmonics and beats, low-frequency laser phase noise, seen in the inset to Fig. 4(a), is also mixed up via sum and difference frequency generation with the second harmonic, causing broadband noise. This extra noise is clearly illustrated in Fig. 4(a) (dark trace), where mixed-up laser noise completely obscures the motion of the measured mode \( M \) (gray trace). The signal to noise ratio (SNR) of the measured mode is shown as a function of power in Fig. 4(b) (black squares), with severe degradation apparent once the threshold is reached.

Feedback to suppress the parametric instability was implemented by electrically filtering and amplifying the photocurrent and applying it directly to a sharp electrode placed close
phase and gain were controlled inside the feedback loop by an effect of feedback on the measured mode amplification of the feedback signal while eliminating the unstable mechanical mode at 14 MHz, allowing maximum the second harmonic. Inset: Single-sided noise spectrum about threshold, with (gray) and without (black) feedback stabilization of $\mu_{136}$ (circles) and without feedback (squares).

The feedback forces could be applied by electrodes integrated onto each optomechanical element, while the feedback signals could be obtained either directly from the optical output of the device with spectral filtering to select the appropriate unstable modes, or if there are degenerate unstable modes, from weak optical tapoffs integrated throughout the device. This research was funded by the Australian Research Council Centre of Excellence CE110001013 and Discovery Project DP0987146. Device fabrication was undertaken within the Queensland node of the Australian Nanofabrication Facility. We gratefully acknowledge the Danish Council for Independent Research (Sapere Aude program) and George Brawley for measurements of laser frequency noise.

**FIG. 4.** (Color online) (a) Mechanical spectra observed with 136 $\mu$W of input power, well above the parametric instability threshold, with (gray) and without (black) feedback stabilization of the unstable 14-MHz mode. Inset: Single-sided noise spectrum about the second harmonic 2U (solid line) showing the measured scaling of laser frequency noise (dashed line) and the suppression of noise from thermal locking (shaded region). (b) Signal-to-noise ratio of the 28.6-MHz mechanical mode versus optical power with feedback (circles) and without feedback (squares).

This work represents an important step for stabilization in mesoscopic quantum optomechanical systems, particularly when involving BAE or optomechanical entanglement. Extending into applications, this technique could be used for stabilizing mechanical instabilities in miniaturized photonic-phononic circuits and mechanical sensors.

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