Non-commutative Dp-Brane in General Background Fields

Maryam Zoghi and Davoud Kamani

Faculty of Physics, Amirkabir University of Technology (Tehran Polytechnic)
P.O.Box: 15875-4413, Tehran, Iran
e-mails: zoghi, kamani@aut.ac.ir

Abstract

We investigate non-commutativity of open strings, attached to a Dp-brane, in the presence of the linear dilaton, tachyon, U(1) gauge field as well as constant anti-symmetric B-field backgrounds. Non-commutativity parameter, open string metric and some special cases will be studied. Mode-dependent non-commutativity, inspired by the tachyon field, will be discussed in detail.

PACS: 11.25.-w; 11.30.Cp

Keywords: D-brane; non-commutativity; dilaton; tachyon.
1 Introduction

D-branes, as inevitable objects in string theory [1], have been studied from various points of view. On the other hand we have non-commutative geometry which has been used as a framework for many subjects in physics and chiefly in high energy physics. Advent of non-commutative string theory in the $B$-field background and appearance of non-trivial commutators in spatial directions of D-brane [2, 3, 4, 5], revived investigation of non-commutativity in the string theory with different backgrounds fields like dilaton field [6, 7], tachyon field [8, 9], as well as moving D-branes.

Unstable D-branes in background fields are mainly studied in world-volume field theory and boundary state descriptions [10, 11, 12]. In this paper we consider a $D_p$-brane of the bosonic string theory in the presence of a $U(1)$ gauge field, linear dilaton, tachyon and Kalb-Ramond $B$-field, simultaneously. This is a system with generalized set-up. For this system we obtain the boundary conditions and propagator of an open string, attached to the $D_p$-brane. This propagator enables us to extract a set of open string metrics and non-commutativity parameters. We observe that these variables depend on the open string modes. On these variables some scaling limits and approximations will be investigated. Finally, we evaluate our achievements for the D1-brane and D2-brane as special cases.

2 Extended boundary conditions of open string

We consider a $D_p$-brane in a general background, which contains the anti-symmetric field $B_{\mu \nu}$, the dilaton field $\Phi$, the open string tachyon field $T(x)$ and a $U(1)$ gauge field $A_\alpha$ which lives on the $D_p$-brane world-volume. We shall use the indices $\alpha \in \{0, 1, \cdots, p\}$ and $i \in \{p+1, \cdots, d-1\}$ for directions along the world-volume of the $D_p$-brane and perpendicular to it, respectively. Therefore, the string sigma model action in these background fields has the following feature

$$S = \frac{1}{4\pi \alpha'} \int_\Sigma d^2 \sigma (\sqrt{-h} \epsilon^{ab} g_{\mu \nu} \partial_a X^\mu \partial_b X^\nu - 2\pi i \alpha' \epsilon^{ab} B_{\mu \nu} \partial_a X^\mu \partial_b X^\nu + \alpha' \sqrt{-h} R^{(2)} ) + \frac{1}{2\pi \alpha'} \int_\partial \Sigma d\tau \{-2\pi i \alpha' A_\alpha \partial_\tau X^\alpha + T(X)\},$$

(1)

where $\Sigma$ indicates the string worldsheet which has the boundary $\partial \Sigma$ and the metric $h_{ab}$ with $h = -\det h_{ab}$. The scalar curvature $R^{(2)}$ is constructed from the metric $h_{ab}$. The space-time metric also is $g_{\mu \nu}$. Note that the indexes of worldsheet and spacetime take their values from the sets $a, b \in \{\sigma, \tau\}$ and $\mu, \nu \in \{0, 1, \cdots, d-1\}$, respectively.
The variety of the background fields enables us to restrict them to obtain a solvable model. For this we take the background fields $g_{\mu\nu}$ and $B_{\mu\nu}$ to be constant with $g_{\alpha i} = 0$ and only $B_{\alpha\beta} \neq 0$. In addition, for the $U(1)$ gauge field we elect the gauge $A_\alpha = -\frac{1}{2} F_{\alpha\beta} X^\beta$, where the field strength $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$ is constant. Moreover, we suppose that the dilaton field has a linear form along the brane world-volume, i.e, $\Phi = a_\alpha X^\alpha$ where the parameters $\{a_\alpha | \alpha = 0, 1, \cdots, p\}$ are constant. In the conventional case the dilaton is an arbitrary function, which causes appearance of third order derivative in the action, and hence is put away [13]. In the action (1) the scalar curvature $R^{(2)}$ contains second order derivative. This implies that by considering a linear dilaton only the usual second order derivative is obtained.

Remember that the exponential of the dilaton field gives the strength of the string coupling. So the linear dilaton background describes a world in which strings are weakly coupled for large negative $x$ and strongly coupled for large positive $x$. Thus, one could worry about the reliability of the formalism in such set-up. However, adding a tachyon background term of the form $T_0 \exp(u \cdot X)$ suppresses the contribution of the strongly coupled region, and this keeps things under control [14]. The conventional inhomogeneous tachyon profile is a linear form which appears as a squared term in the boundary of the string action [15]. This motivates us to consider tachyon profile as $T(X) = \frac{1}{2} U_{\alpha\beta} X^\alpha X^\beta$ with the constant symmetric matrix $U_{\alpha\beta}$, which originates from the expansion of exponential form $T_0 \exp(u_\alpha X^\alpha)$ for small parameters $\{u_\alpha | \alpha = 0, 1, \cdots, p\}$. This form of the tachyon field accompanied by the linear dilaton gives a Gaussian theory, and hence is exactly solvable [16].

Reparametrization invariance of the bulk part of the action (1) enables us to choose the conformal gauge for the worldsheet metric, i.e, $h_{ab}(\sigma, \tau) = e^{\rho(\sigma, \tau)} \eta_{ab}$. Note that due to the presence of the dilaton, the action does not have the Weyl symmetry. Thus, $\rho(\sigma, \tau)$ is a nonzero worldsheet field.

Vanishing the variation of the action leads to the equations of motion for the worldsheet fields $X^\mu$ and $\rho$ as in the following

\[
\begin{align*}
\partial^2 X_\mu + \frac{1}{2} a_\mu \partial^2 \rho &= 0, \\
a_\alpha \partial^2 X^\alpha &= 0,
\end{align*}
\]

(2) (3)

where $\partial^2 = \eta^{ab} \partial_a \partial_b$. Now we consider a non-critical string theory, i.e. $a^2 = a_\alpha a^\alpha \propto d-26 \neq 0$. Then the Eqs. (2) and (3) split into $\partial^2 X^\mu = \partial^2 \rho = 0$. In addition, vanishing of this variation defines the boundary conditions of the string. For example, for the open string end at $\sigma = 0$ we receive the boundary equations

\[
(g_{\alpha\beta} \partial_\alpha X^\beta + 2\pi i \alpha' F_{\alpha\beta} \partial_\alpha X^\beta + U_{\alpha\beta} X^\beta)_{\sigma=0} = 0,
\]
Here, \( F_{\alpha\beta} = B_{\alpha\beta} - F_{\alpha\beta} \) is total field strength.

Removing \( \partial_\sigma X^0 \) from the first boundary condition, via the third one, leads us to define the variable \( \tilde{g}_{\alpha\bar{\beta}} \) as

\[
\tilde{g}_{\alpha\bar{\beta}} \equiv g_{\alpha\bar{\beta}} - g_{\alpha0} \frac{a_{\bar{\beta}}}{a_0},
\]

where \( \bar{\beta} \in \{1, 2, \ldots, p\} \). To obtain desirable boundary conditions we demand \( \tilde{g}_{0\bar{\beta}} \) to be zero which gives \( a_{\bar{\beta}} = \frac{g_{\alpha0}}{g_{\alpha\bar{\beta}}} g_{0\bar{\beta}} \). In addition, we impose the extra conditions \( \mathcal{F}_{\alpha0} = U_0\beta = 0 \).

Therefore, the boundary condition of open string along the brane directions takes the form

\[
(\tilde{g}_{\alpha\bar{\beta}} \partial_\sigma X^\beta + 2\pi i \alpha' \mathcal{F}_{\alpha\bar{\beta}} \partial_\tau X^\beta + U_{\alpha\beta} X^\beta)_{\sigma=0} = 0.
\]

The symmetric matrix \( \tilde{g}_{\alpha\bar{\beta}} = g_{\alpha\bar{\beta}} - \frac{g_{\alpha0}}{g_{\alpha\bar{\beta}}} a_\alpha a_{\bar{\beta}} \) effectively possesses the treatment of a metric in the brane volume. By having the coordinates \( \{X^\alpha\} \), we are able to specify \( X^0 \) via its equation \( \partial^2 X^0 = 0 \) and the boundary condition: \( \partial_\sigma X^0|_{\sigma=0} = -\frac{a_\alpha}{a_0} \partial_\sigma X^\alpha|_{\sigma=0} \).

## 3 Non-commutativity variables

The open string propagator \( G_{\alpha\bar{\beta}} \) can be calculated via the equation

\[
\partial \partial \bar{G}^{\alpha\bar{\beta}}(z, z') = -2\pi \delta^{(2)}(z - z'),
\]

and the boundary condition

\[
[(\tilde{g} + 2\pi \alpha' \mathcal{F}) \partial \bar{G} - (\tilde{g} - 2\pi \alpha' \mathcal{F}) \partial \bar{G} - iU \bar{G}]^{\alpha\bar{\beta}}|_{\sigma=0} = 0,
\]

where the complex variable is \( z = \tau + i\sigma \). The solution of these equations is given by

\[
\bar{G}^{\alpha\bar{\beta}}(z, z') = -\alpha' \tilde{g}^{\alpha\bar{\beta}} \ln |z - z'| + \frac{\alpha'}{2} \sum_{n=1}^{\infty} \left( \frac{\tilde{g} - 2\pi \alpha' \mathcal{F} - \frac{i}{2n} U}{\tilde{g} + 2\pi \alpha' \mathcal{F} + \frac{i}{2n} U} \right)^{\alpha\bar{\beta}} \frac{(zz')^n + (\bar{z}\bar{z}')^n}{n} + \frac{\alpha'}{2} \sum_{n=1}^{\infty} \left( \frac{\tilde{g} - 2\pi \alpha' \mathcal{F} - \frac{i}{2n} U}{\tilde{g} + 2\pi \alpha' \mathcal{F} + \frac{i}{2n} U} \right)^{\bar{\alpha}\beta} \frac{(z\bar{z}')^n - (\bar{z}z')^n}{in}.
\]

According to the prototype initiated in the Ref. [3], the above propagator defines an open string metric and a non-commutativity parameter, for each string mode, as in the following

\[
G_{\alpha\bar{\beta}} = \left( \frac{1}{\tilde{g} + 2\pi \alpha' \mathcal{F} + \frac{i}{2n} U} \right)^{\alpha\bar{\beta}}
\]

\[
= \left( \frac{1}{\tilde{g} + 2\pi \alpha' \mathcal{F} + \frac{i}{2n} U} \right) \frac{1}{\tilde{g} - 2\pi \alpha' \mathcal{F} - \frac{i}{2n} U} \frac{1}{\tilde{g} + 2\pi \alpha' \mathcal{F} - \frac{i}{2n} U} \right)^{\alpha\bar{\beta}}.
\]
where $n \in \{1, 2, 3, \cdots\}$. Note that these variables depend on the positive string mode numbers. Analog results can be seen for noncommutative $D$-branes in the pp-wave background [17]. The peculiar result that we obtained is a consequence of the tachyon field in the volume of the brane. In the absence of the tachyon all modes of the open string probe the same value for each of these variables. However, the tachyon splits this degeneracy. By adjusting the parameters $\bar{g}_{\bar{\alpha}\bar{\beta}}, F_{\bar{\alpha}\bar{\beta}}$ and $U_{\bar{\alpha}\bar{\beta}}$ one can receive expedient values of the non-commutativity variables.

Now we introduce the averaged values of the non-commutativity variables

$$\bar{G}_{\bar{\alpha}\bar{\beta}} = \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} G_{n}^{\bar{\alpha}\bar{\beta}},$$

$$\bar{\theta}_{\bar{\alpha}\bar{\beta}} = \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} \theta_{n}^{\bar{\alpha}\bar{\beta}}. \quad (12)$$

By using the Eqs. (10) and (11), these variables are acquired as

$$\bar{G}_{\bar{\alpha}\bar{\beta}} = \left( \frac{1}{\bar{g}} + \frac{2\pi \alpha'}{g} \right) \frac{1}{\bar{g} - 2\pi \alpha' F} \bar{G}_{\bar{\alpha}\bar{\beta}}^{\bar{\alpha}\bar{\beta}},$$

$$\bar{\theta}_{\bar{\alpha}\bar{\beta}} = -(2\pi \alpha')^{2} \left( \frac{1}{\bar{g}} - \frac{\alpha'}{2n} \right) F \left( \frac{1}{\bar{g} - 2\pi \alpha' F} \right) \bar{\theta}_{\bar{\alpha}\bar{\beta}}^{\bar{\alpha}\bar{\beta}}. \quad (13)$$

These are independent of the tachyon matrix. Therefore, they are counterparts of the well-known standard non-commutativity variables, as expected. In fact, due to the dilaton parameters $\{a_{\bar{\alpha}} | \bar{\alpha} = 1, \cdots, p\}$ in the matrix $\bar{g}_{\bar{\alpha}\bar{\beta}}$, these are generalized version of the ordinary case.

### 3.1 Non-commutativity after tachyon condensation

Ground state of the bosonic open string has negative mass squared and hence is tachyonic. In fact, the tachyon field describes the dynamics of an unstable $Dp$-brane. That is, after tachyon condensation (i.e. when at least one of the nonzero elements of the matrix $U_{\alpha\beta}$ goes to infinity) the brane becomes unstable. Therefore, a $Dp$-brane in the presence of a tachyon field decays to the lower dimensional branes or decays to the closed string vacuum.

According to the Eqs. (10) and (11), since each mode feels its own non-commutativity, tachyon condensation depends on the mode number. Let tachyon condensation to take
place in the $x^p$-direction of the brane. For the finite mode numbers this implies the limit
\[ \frac{1}{n} U_{pp} \rightarrow \infty. \]
Thus, the last columns and the lowest rows of the matrices $G_n$ and $\theta_n$ vanish.

The nonzero $(p-1) \times (p-1)$ matrices inside the matrices $G_n$ and $\theta_n$ elaborate the open string metric and non-commutativity parameter of a D$(p-1)$-brane. In other words, the D$p$-brane loses its extension along the direction $x^p$, as expected. The features of the new non-commutativity variables are similar to the previous case, i.e. the Eqs. (10) and (11), with $\bar{\alpha}, \bar{\beta} \in \{1, 2, \cdots, p-1\}$. For very large mode numbers if the quantity $\frac{1}{n} U_{pp}$ tends to infinity again we have the above discussion. If the infinite values of $U_{pp}$ and $n$ lead to a finite value for $\frac{1}{n} U_{pp}$, then tachyon condensation does not occurs.

We observe that the averaged values of the non-commutativity variables are independent of the tachyon, and hence they are not affected by condensation of the tachyon. In the Sec. 5 the tachyon condensation again will be illustrated.

4 Scaling limits

4.1 The zero slope limit

Now we impose the zero slope limit ($\alpha' \rightarrow 0$) on the open string variables. This is useful for studying the low energy behavior of the open string. Since open strings are sensitive to $G$ and $\theta$, we should take the limit in the manner that these variables to be fixed rather than the closed string parameters [3]. Therefore, we apply the limits $\alpha' \sim \epsilon^{\frac{1}{2}}, g_{\bar{\alpha} \bar{\beta}} \sim \epsilon \rightarrow 0$ and $U_{\bar{\alpha} \bar{\beta}} \sim \epsilon^{\frac{1}{2}}$, while $\mathcal{F}$ is fixed. The Eq. (5) gives the scaling $\tilde{g}_{\bar{\alpha} \bar{\beta}} \sim \epsilon \rightarrow 0$. In these limits we receive the following quantities

\[ G_{n}^{\bar{\alpha} \bar{\beta}} = -\frac{1}{(2\pi \alpha')^2} (\mathcal{F}^{-1} (\tilde{g} + \frac{\alpha'}{2n} U) \mathcal{F}^{-1})^{\bar{\alpha} \bar{\beta}}, \]
\[ \theta_{n}^{\bar{\alpha} \bar{\beta}} = (\mathcal{F}^{-1})^{\bar{\alpha} \bar{\beta}}, \]
\[ \bar{G}^{\bar{\alpha} \bar{\beta}} = -\frac{1}{(2\pi \alpha')^2} (\mathcal{F}^{-1} \tilde{g} \mathcal{F}^{-1})^{\bar{\alpha} \bar{\beta}}, \]
\[ \bar{\theta}^{\bar{\alpha} \bar{\beta}} = (\mathcal{F}^{-1})^{\bar{\alpha} \bar{\beta}}. \]

Since $\tilde{g}_{\bar{\alpha} \bar{\beta}}$, $\alpha' U$ and $\alpha'^2$ have the same limiting behavior the above variables are finite.

4.2 Small $\mathcal{F}$ and $U$ limit

A small tachyon and total gauge field limit can be obtained by using the scaling $g_{\bar{\alpha} \bar{\beta}} \sim \epsilon$, $U_{\bar{\alpha} \bar{\beta}} \sim \epsilon$ and $\mathcal{F}_{\bar{\alpha} \bar{\beta}} \sim \epsilon^{\frac{1}{2}}$. In this limit the corresponding non-commutativity variables reduce
to

\[
G_{\alpha\beta}^{\hat{n}} = -\frac{1}{(2\pi\alpha')^2} \left( \mathcal{F}^{-1} \left( \bar{g} + \frac{\alpha'}{2n} \mathcal{F} \right) \right)^{\hat{\alpha}\hat{\beta}},
\]

\[
\theta_{\alpha\beta}^{\hat{n}} = (\mathcal{F}^{-1})^{\hat{\alpha}\hat{\beta}},
\]

(16)

\[
G^{\hat{\alpha}\hat{\beta}} = -\frac{1}{(2\pi\alpha')^2} \left( \mathcal{F}^{-1} \left( \bar{g} \mathcal{F}^{-1} \right) \right)^{\hat{\alpha}\hat{\beta}},
\]

\[
\bar{\theta}^{\hat{\alpha}\hat{\beta}} = (\mathcal{F}^{-1})^{\hat{\alpha}\hat{\beta}}.
\]

(17)

Though the features of these variables and their counterparts in the zero slope limit are the same but they have different values. For example, the non-commutativity parameters in the Eqs. (14) and (15) are finite while in the Eqs. (16) and (17) they are very large. However, the closed string metrics in both cases are finite.

4.3 Large $\mathcal{F}$ and $U$ limit

Now we examine large values for the matrices $\mathcal{F}$ and $U$,

\[
\alpha' \sim \epsilon^\frac{1}{4}, \quad \mathcal{F}_{\hat{\alpha}\hat{\beta}} \sim \epsilon^{-\frac{1}{4}}, \quad U_{\hat{\alpha}\hat{\beta}} \sim \epsilon^{-\frac{1}{2}}, \quad g_{\alpha\beta} \sim \epsilon \to 0.
\]

(18)

This scaling leads to the following equations

\[
G_{\alpha\beta} = \frac{2n}{\alpha'} \left( \frac{1}{U + 4\pi n \mathcal{F}} \right) \left( \frac{1}{U - 4\pi n \mathcal{F}} \right) \bar{g} \mathcal{F},
\]

\[
\theta_{\alpha\beta} = -(4\pi n)^2 \left( \frac{1}{U + 4\pi n \mathcal{F}} \right) \left( \frac{1}{U - 4\pi n \mathcal{F}} \right) \bar{g} \mathcal{F},
\]

(19)

\[
\bar{G}^{\hat{\alpha}\hat{\beta}} = -\frac{1}{(2\pi\alpha')^2} \left( \mathcal{F}^{-1} \left( \bar{g} \mathcal{F}^{-1} \right) \right)^{\hat{\alpha}\hat{\beta}},
\]

\[
\bar{\theta}^{\hat{\alpha}\hat{\beta}} = (\mathcal{F}^{-1})^{\hat{\alpha}\hat{\beta}}.
\]

(20)

From the open string metrics only $G_n$ is finite while $\bar{G}$ behaves like $\epsilon$. The non-commutativity parameters $\theta_n$ and $\bar{\theta}$ also go to zero similar to $\epsilon^\frac{1}{4}$.

4.4 Small tachyon approximation

An interesting approximation is given by the small tachyon matrix elements. In this case the non-commutativity variables are truncated to the following versions

\[
\lim_{U \to 0} G_{\alpha\beta}^{\hat{n}} = \bar{G}^{\hat{\alpha}\hat{\beta}} + \frac{\alpha'}{2n} \left( \frac{1}{\bar{g} + 2\pi \alpha' \mathcal{F}} \right) U \left( \frac{1}{\bar{g} - 2\pi \alpha' \mathcal{F}} \right) \mathcal{F}
\]

\[
- U \left( \frac{1}{\bar{g} + 2\pi \alpha' \mathcal{F}} \bar{G} - \bar{G} \left( \frac{1}{\bar{g} - 2\pi \alpha' \mathcal{F}} U \right) \right)^{\hat{\alpha}\hat{\beta}} + \mathcal{O}(U^2)^{\hat{\alpha}\hat{\beta}},
\]

(21)
\lim_{U \to 0} \theta^{\bar{\alpha}\bar{\beta}}_n = \theta^{\bar{\alpha}\bar{\beta}} - \frac{\alpha'}{2n} \left( \frac{1}{\bar{g} - 2\pi \alpha' F} U + \frac{1}{\bar{g} + 2\pi \alpha' F} \bar{\theta} \right)^{\bar{\alpha}\bar{\beta}} + \mathcal{O}(U^2)^{\bar{\alpha}\bar{\beta}}. \quad (22)

We observe that the first terms of these limits are the averaged non-commutativity variables, as expected. The first order corrections also slow down by the factor \(1/n\).

5 Examples

To have more physical intuitive of the above general set-up, we describe the non-commutativity variables for two simple examples. These are D1-brane and D2-brane.

5.1 D1-Brane

Consider a D1-brane lying in the \(X^1\)-direction. The general forms of the closed string metric and the matrix \(U\) can be written as

\[
g_{\alpha\beta} = \begin{pmatrix} -g_0 & g_1 \\ g_1 & g_2 \end{pmatrix}, \quad U_{\alpha\beta} = \begin{pmatrix} 0 & 0 \\ 0 & u_2 \end{pmatrix}. \quad (23)
\]

Note that in our formulation there are \(F_{\bar{\alpha}0} = U_{0\bar{\beta}} = 0\). According to \(\tilde{g}_{11} = g_2 + \frac{g_1^2}{g_0}\), the open string metrics reduce to

\[
G_{(n)11} = g_2 + \frac{g_1^2}{g_0} + \frac{\alpha'}{2n} u_2, \quad \bar{G}_{11} = g_2 + \frac{g_1^2}{g_0}. \quad (24)
\]

Since there is only one spatial direction the non-commutativity parameters vanish.

5.2 D2-Brane

For a D2-brane in the \(X^1X^2\)-plane there are

\[
g_{\alpha\beta} = \begin{pmatrix} -g_0 & g_1 & g_3 \\ g_1 & g_2 & g_4 \\ g_3 & g_4 & g_5 \end{pmatrix}, \quad F_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & b \\ 0 & -b & 0 \end{pmatrix}, \quad U_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & u_2 & u_4 \\ 0 & u_4 & u_5 \end{pmatrix}. \quad (25)
\]

The closed string metric defines the matrix \(\tilde{g}\) as in the following

\[
\tilde{g}_{\alpha\bar{\beta}} = \begin{pmatrix} g_2 + \frac{g_1^2}{g_0} & g_4 + \frac{g_1 g_3}{g_0} \\ g_4 + \frac{g_1 g_3}{g_0} & g_5 + \frac{g_2^2}{g_0} \end{pmatrix}, \quad \bar{\alpha}, \bar{\beta} \in \{1, 2\}. \quad (26)
\]
Therefore, the non-commutativity variables take the forms

\[
G_{\bar{n}n}^{\bar{\alpha}\bar{\beta}} = \begin{pmatrix}
\frac{g_{01}^2 + g_{5} + \alpha' u_5}{P_n} & \frac{2n (2ng_1 g_3 + 2ng_0 g_4 + \alpha' u_4)}{Q_n} \\
\frac{2n (2ng_1 g_3 + 2ng_0 g_4 + \alpha' u_4)}{Q_n} & \frac{g_{01}^2 + g_{2} + \frac{\alpha'}{2n} u_2}{P_n}
\end{pmatrix},
\]

\[
\theta_{\bar{n}n}^{\bar{\alpha}\bar{\beta}} = \begin{pmatrix}
0 & -\frac{8\pi \alpha' \beta g_0}{Q_n} \\
\frac{8\pi \alpha' \beta g_0}{Q_n} & 0
\end{pmatrix},
\]

(27)

where the variables \(P_n\) and \(Q_n\) have the following definitions

\[
P_n = \left( \frac{g_{1} g_{3}}{g_{0}} + g_{4} - 2\pi \alpha' b + \frac{\alpha'}{2n} u_4 \right) \left( \frac{g_{1} g_{3}}{g_{0}} + g_{4} + 2\pi \alpha' b + \frac{\alpha'}{2n} u_4 \right)
+ \left( \frac{g_{2}^2}{g_{0}} + g_{2} + \frac{\alpha'}{2n} u_2 \right) \left( \frac{g_{3}^2}{g_{0}} + g_{5} + \frac{\alpha'}{2n} u_5 \right),
\]

\[
Q_n = 4n^2 g_0 g_4^2 + 2n \alpha' g_5^2 u_2 + 2n \alpha' g_0 g_5 u_2
- 4n \alpha' g_0 g_4 u_4 + 16\pi^2 n^2 \alpha' b^2 g_0 - \alpha' g_0 u_4^2
+ \alpha' g_0 u_2 u_5 - 4ng_1 g_3 (2ng_4 + \alpha' u_4) + 2ng_1^2 (2ng_5 + \alpha' u_5)
+ 2ng_2 (2ng_5^2 + 2ng_0 g_5 + \alpha' g_0 u_5).
\]

(28)

Regarding to the tachyon condensation consider the limit \((U_{22} = u_5) \rightarrow \infty\). The equations (27) implies that the non-commutativity matrix vanishes and the only nonzero element of the open string metric reduces to \(G_{11}^{11} = 1/(g_2 + \frac{g_{1}^2}{g_{0}} + \frac{\alpha'}{2n} u_2)\). This is exactly the inverse of the first equation in (24). That is, the D2-brane has deformed to a D1-brane, as expected.

## 6 Conclusions and summary

The non-commutativity of open strings, which are attached to a Dp-brane in the presence of the massless fields: Kalb-Ramond, a \(U(1)\) gauge field, dilaton and a tachyon field which is massive, was investigated. Presence of the linear dilaton field effectively deforms the closed string metric, and hence the non-commutativity variables. Appearance of various parameters due to the background fields, i.e. \(\{B_{\bar{\alpha}\bar{\beta}}, F_{\bar{\alpha}\bar{\beta}}, a_\alpha, U_{\bar{\alpha}\bar{\beta}}\}\) enables us to adjust these variables to desirable values.

The non-commutativity variables are influenced by all background fields, but the tachyon field has more control on them. Precisely, the tachyon field decomposes a non-commutativity which has been originated by the Kalb-Ramond and the \(U(1)\) gauge fields into infinite number of non-commutativities. That is, each open string mode feels its own non-commutativity. The average values of the non-commutativity variables are independent of the tachyon. Accurately they are equal to the non-commutativity of infinite massive states of open string, or
equivalently they are the ordinary non-commutativity variables, which appear in the systems without tachyon field. As expected, tachyon condensation deform the non-commutativity variables of a $Dp$-brane to that of a $D(p-1)$-brane. However, this reduction for light string modes always takes place, while for very large mode numbers its occurrence depends on the orders of the infinities of “$n$” and “$U_{pp}$”.

Finally, various scaling limits were studied. The tachyon matrix enabled us to introduce new appealing scaling limits.

Note that for a moving $Dp$-brane in our set-up, with velocity perpendicular to the brane directions, the non-commutativity structure is the same as we studied. By contrast, a moving $Dp$-brane along itself generates a new non-commutativity structure which can be investigated.

References

[1] J. Polchinski, Phys. Rev. Lett. 75, 4724, 9510017 (1995)

[2] M. R. Douglas, C. M. Hull, JHEP 9802, 008 (1998)

[3] N. Seiberg, E. Witten, JHEP 9, 32 (1999)

[4] C.S. Chu, P.M. Ho, Nucl. Phys. B550, 151 (1999)

[5] C. Chu, Non-commutative geometry from strings (2005), arXive:hep-th/0502167

[6] S. Bhattacharyya, A. Kumar, S. Mahapatra, Mod. Phys. Lett. A16, 2263-2272(2001)

[7] B. Nikolic, B. Sazdovic, Phys. Chem. and Tech. 4, No. 2, 405-413(2006)

[8] E. Witten, Phys. Rev. D46, 5467 (1992); Phys. Rev. D47, 3405 (1993)

[9] K. Dasgupta, S. Mukhi and G. Rajesh, JHEP 6, 022 (2000)

[10] A. Sen, JHEP 0204:048 (2002)

[11] S. J. Rey, S. Sugimoto, Phys. Rev. D67, 086008 (2003)

[12] S. J. Rey, S. Sugimoto, Phys. Rev. D68, 026003 (2003)

[13] C.G. Callan, C. Lovelace, C.R. Nappi, S.A. Yost, Nucl. Phys. B293, 83 (1987); Nucl. Phys. B308, 221 (1988)
[14] K. Becker, M. Becker, J. Schwarz, String theory and M-theory: a modern introduction, (Cambridge University press, 2007)

[15] D. Kutasov, M. Marino, G. Moore, Remarks on Tachyon Condensation in Superstring Field Theory (2000) arXive:hep-th/0010108; T. Lee, Journal of the Korean Physical Society, Vol. 39, pp. S536-S545(2001)

[16] C.G. Callan and I.R. Klebanov, Nucl. Phys. B465, 473, 12 (1996); E.T. Akhmedov, M. Laidlaw, G.W. Semenoff, JETP Lett. 77, pp.1-6 (2003); Pismazh. Eksp. Teor. Fiz. 77, pp. 3-8 (2003); Z. Rezaei, D. Kamani, J. Exp. Theor. Phys. 140 (2011), arXiv:hep-th/1106.2097

[17] C. S. Chu, P. M. Ho, Nucl. Phys. B636, pp. 141-158 (2002)