The hidden strangeness mechanism in $D_s^+ \to \omega \pi^+$ and $D_s^+ \to \rho^0 \pi^+$ decays

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We study possible contributions to the $D_s^+ \to \omega \pi^+$ and $D_s^+ \to \rho^0 \pi^+$ decay amplitudes. The $D_s^+ \to \omega \pi^+$ decay amplitude vanishes when naive factorization is used, while the $D_s^+ \to \rho^0 \pi^+$ decay amplitude arises due to the annihilation contribution. We find that amplitudes for both decays might be a result of the internal $K, K^*$ exchange. The $D_s^+ \to \omega \pi^+$ amplitude might obtain additional contributions from $D_s^+ \to \rho^0 \eta (\eta')$ re-scattering. The low experimental bound on the $D_s^+ \to \rho^0 \pi^+$ rate can be understood as a result of combination of the $(\pi(1300))$ pole dominated annihilation contribution and the $K, K^*$ internal exchanges. The calculated branching fractions for $D_s^+ \to \omega \pi^+$ and $D_s^+ \to \rho^0 \pi^+$ are in agreement with the current experimental results.

The weak nonleptonic decays of charm mesons were usually approached within the factorization ansatz. A decade ago it was realized that one has to include the effects of final state interactions (FSI), with the simplest approach being to treat the FSI by assuming the dominance of nearby resonances. This leads to rather good overall agreement with the experimental data; however, there are a few cases where none of the existing approaches work. Two such examples are the channels (quoting the PDG experimental values)

$$BR(D_s^+ \to \omega \pi^+) = (2.8 \pm 1.1) \times 10^{-3},$$
$$BR(D_s^+ \to \rho^0 \pi^+) < 7 \times 10^{-4}.$$ (1)

The current theoretical approaches usually predict that the $D_s^+ \to \rho^0 \pi^+$ branching fraction is equal or even larger than the branching fraction for the $D_s^+ \to \omega \pi^+$ decay in contradiction with the present data.

On the other hand, the observation of the $D_s^+ \to \omega \pi^+$ decay has been motivated as a clean signature of the annihilation decay of $D_s^+$ [8]. The sizes of annihilation contributions are very important for phenomenological studies, but are also very hard to obtain from theoretical considerations (see e.g., [9]). Understanding the origin of the $D_s^+ \to \omega \pi^+$ transition is thus of great theoretical interest.

Let us first discuss the two modes [1] using factorization approximation for the weak vertex. In this approximation the $D_s^+ \to \omega \pi^+$ amplitude is zero due to G-parity conservation, which gives a vanishing $\langle \omega \pi^+ \mid (ud)_{V-A} \mid 0 \rangle$ matrix element [10]. The $D_s^+ \to \rho^0 \pi^+$ decay amplitude, on the other hand, already in the factorization limit receives a contribution through the annihilation graph, Fig. 1,

$$M(D_s^+ \to \rho^0 \pi^+) = \frac{G_F}{\sqrt{2}} V_{us} V_{ud}^* a_1 \times \langle \rho^0 \pi^+ \mid (\bar{u}d)_{V-A} \mid 0 \rangle \langle (\bar{s}c)_{V-A} \mid D_s^+ \rangle,$$ (2)

leading to simple $\pi$ pole dominance in the $\langle \rho^0 \pi^+ \mid (\bar{u}d)_{V-A} \mid 0 \rangle$ matrix element. The analysis of [12] indicates that $\pi(1300)$ states dominate this annihilation graph, while the contribution of the lowest lying $\pi$ is negligible. In [10] we have estimated the size of the annihilation contribution coming from the $\pi(1300)$ intermediate state. We found $f_{\pi(1300)} < 4$ MeV [10]. In the factorization approximation for the weak vertex we then get

$$BR(D_s^+ \to \rho^0 \pi^+)_{\pi(1300)} < 7 \times 10^{-4},$$ (3)

where we have used $f_{D_s} = 230$ MeV, to-
Figure 1. Annihilation diagram of $D_s \to \rho \pi$ decay.

together with the conservative assumptions of $BR(\pi(1300) \to \rho \pi) \sim 100\%$ and $\Gamma(\pi(1300))$ equal to its upper experimental bound of 600 MeV. The interference with other annihilation contributions from intermediate $\pi$ and $\pi(1800)$ states can somewhat change the above estimate (using PCAC, the contribution from $\pi$ was found in [12] to be negligible, while the contribution of $\pi(1800)$ is difficult to estimate due to the lack of experimental data). In addition, also the FSI contributions (to be considered shortly) fall in exactly the same range [10]. Therefore, unless there are large cancellations, the value of $BR(D_s^+ \to \rho^0 \pi^+)$ is expected to be near to its present experimental upper bound [11].

In the case of the $\omega\pi^+$ final state there is no such resonance annihilation contribution and one has to explain a relatively large experimental value for $BR(D_s \to \omega\pi^+)$ [11] in a different way.

An important observation is that there are multi-body intermediate states that do have the correct values of $I^G$ and $J^P$, for instance the two-body $K^{(*)} \bar{K}^{(*)}$ states. As we will show in the rest of the talk, it is possible to explain the experimental value for $BR(D_s \to \omega\pi^+)$ by considering the contributions due to the rescattering of these intermediate states.

In estimating the contributions from hidden strangeness intermediate states (that can arise from spectator quark diagrams), we use the following assumptions

- We consider only contributions coming from two body intermediate states with $s,s$ quantum numbers (lowest lying pseudoscalar and vector states). Note that the re-scattering through intermediate $K, K^*$ states is possible for both $\rho^0 \pi^+$ as well as $\omega\pi^+$ final state, while the re-scattering with intermediate $\eta$ or $\eta'$ is possible only in the case of $\omega\pi^+$ final state due to isospin and $G$ parity conservation.

- For the weak transition $D_s^+ \to (K^{(*)}\bar{K}^{(*)})^+$ in the $D_s^+ \to (K^{(*)}\bar{K}^{(*)})^+ \to \rho^0 \pi^+$ and $D_s^+ \to (K^{(*)}\bar{K}^{(*)})^+ \to \omega\pi^+$ decay chains as well as for the weak transition $D_s^+ \to \eta(\eta')\rho^+$ in the $D_s^+ \to \eta(\eta')\rho^+ \to \omega\pi^+$ decay chain we will use the factorization approximation. The weak Lagrangian is therefore

$$\mathcal{L}_{\text{weak}} = \frac{G_F}{\sqrt{2}} V_{cs} \bar{V}_{ud} \times$$

$$
\times (a_1(\bar{u}d)_H(\bar{s}c)_H + a_2(\bar{s}d)_H(\bar{u}c)_H),
$$

(4)

with $(\bar{u}d)_H, \ldots$ the hadronized V-A weak current, $V_{ij}$ the CKM matrix elements and $a_{1,2}$ the effective (phenomenological) Wilson coefficients taken to be $a_1 = 1.26$ and $a_2 = -0.52$ [123].

- Finally, the strong interactions are taken into account through the following effective Lagrangian [14][15][16]:

$$\mathcal{L}_{\text{strong}} = \frac{i g_{\rho\pi\pi}}{\sqrt{2}} Tr(\rho^\mu [\Pi, \partial_\mu \Pi])$$

$$-4 C_{V\Pi} \frac{f}{f} \epsilon^{\mu\nu\alpha\beta} Tr(\partial_\mu \rho_\nu \partial_\alpha \rho_\beta \Pi),$$

(5)

where $\Pi$ and $\rho^\mu$ are $3 \times 3$ matrices containing pseudoscalar and vector meson operators respectively and $f$ is a pseudoscalar decay constant. We used numerical values $C_{V\Pi} = 0.33$, and $g_{\rho\pi\pi} = 5.9$ [11][15][16].

In addition we have checked that the use of factorization for the $D_s^+ \to K^+ \bar{K}^{0}$, $D_s^+ \to K^+ K^{*0}$ and $D_s^+ \to \bar{K}^0 K^{*+}$ decays gives reasonable estimates of the measured rates (note that we do not need $D_s^+ \to \bar{K}^0 K^+$ in further considerations) [10]. In these results the annihilation contributions have been neglected since they are an order of magnitude smaller.
The situation in the case of $\eta, \eta'$ intermediate states is not so favorable. To treat the $\eta, \eta'$ mixing we use the approach of Ref. [10] with the value of the mixing angle transforming between $\eta, \eta'$ and $\eta_q \sim (u\bar{u} + d\bar{d})/\sqrt{2}$, $\eta_s \sim s\bar{s}$ states taken to be $\phi = 40^\circ$. The factorization approach then gives a reasonable description of $D_s^+ \to \rho^0 \eta$ decay, while it does not reproduce satisfactorily the experimental result for $D_s^+ \to \rho^+ \eta'$. This is a known problem as the $D_s^+ \to \rho^+ \eta'$ rate is very difficult to reproduce in any of the present approaches [23,24]. This inevitably introduces some further uncertainty into our approach, yet the resulting uncertainty is not expected to affect significantly our main conclusions.

For the weak current matrix elements between $D_s$ and vector or pseudoscalar final states we use a common form factor decomposition [7,10] with the form factors $F_+(q^2), V(q^2), A_{1,2}(q^2)$ and $A_0(q^2)$. For the $q^2$ dependence of the form factors we use results of [18], based on a quark model calculation combined with a fit to lattice and experimental data. Ref. [18] provides a simple fit to their numerical results with the form factors $F_+(q^2), V(q^2)$ and $A_0(q^2)$ described by double pole $q^2$ dependence

$$f(q^2) = \frac{f(0)}{(1 - q^2/M^2)(1 - \sigma q^2/M^2)},$$

while single pole parameterization

$$f(q^2) = \frac{f(0)}{(1 - \sigma q^2/M^2)},$$

can be used for $A_{1,2}(q^2)$, as the contributing resonances have masses farther away from the physical region (note that this parameterization applies also to $F_0$ form factor, which however does not contribute in the processes we discuss in this paper). The values of $f(0)$ and $\sigma$ are listed in Table 1, and are taken from [18]. We use $M = 1.97$ GeV in the expression for $A_0$, and $M = 2.11$ GeV for all the other form factors [18]. Incidentally, the parameterizations of the form factors [6] and [7] make all the loop diagrams in Figs. 2 and 3 finite.

For the decay constants, defined through $\langle 0|\bar{q}\gamma^{\mu}\gamma_5 q|P(p)\rangle = if_{P}p^{\mu}$ and $\langle 0|\bar{q}\gamma^{\mu}q|V(p)\rangle = f_{V}p^{\mu}$

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
 & $F_+$ & $V$ & $A_0$ & $A_1$ & $A_2$ & $F_{\eta',+}$ \\
\hline
$f(0)$ & 0.72 & 1.04 & 0.67 & 0.57 & 0.42 & 0.78 \\
$\sigma$ & 0.2 & 0.24 & 0.2 & 0.29 & 0.58 & 0.23 \\
\hline
\end{tabular}
\caption{Table 1
The intermediate $\eta, \eta', \rho^+$ contributions in the $D_s^+ \to \omega \pi^+$ decay.}
\end{table}
In Appendix of [10]. The numerical values for spectrally explicit expressions can be found obtained on the lattice [13] and for the rest Fig. 3, which as stated above, does not work factorization approximation for the diagram of Fig. 2 and Fig. 3 respectively. The amplitudes for the \( D_\pi^+ \rightarrow \omega \pi^+ \) decays can be written as:

\[
A(D_s^+ \rightarrow \omega \pi^+) = \frac{g_F}{\sqrt{2}} k_2 \left( \sum_i A_i^{(\omega)} + B \right),
\]

\[
A(D_s^+ \rightarrow \rho^0 \pi^+) = \frac{g_F}{\sqrt{2}} k_2 \sum_i A_i^{(\rho)},
\]

with \( \epsilon \) the helicity zero polarization vector of the \( \omega \) or \( \rho \) vector mesons, while \( k_2 \) is the pion momentum. The reduced amplitudes \( A_i^{(\rho),(\omega)} \) and \( B \) correspond to the diagrams in Figs. 2 and 3 respectively. The explicit expressions can be found in Appendix of [10]. The numerical values for \( A_i^{(\rho),(\omega)} \) and \( B \) are given in Table 2 Combining the above results we arrive at the prediction

\[
BR(D_s^+ \rightarrow \omega \pi^+) = 3.0 \times 10^{-3}.
\] (10)

Note that in this calculation we have used the factorization approximation for the diagram of Fig. 3 which as stated above, does not work well for \( D_s^+ \rightarrow \rho^+ \eta, \eta' \) transition. Including hidden strangeness FSI to the \( D_s^+ \rightarrow \rho^+ \eta' \) decay mode gives an order of magnitude smaller contribution. On the other hand one can use the experimental input to rescale the corresponding amplitudes. This results in the prediction

\[
BR(D_s^+ \rightarrow \omega \pi^+) = 4.4 \times 10^{-3}.
\]

We point out that the loop contributions are finite due to double pole parametrization of the form factors. If a single pole parametrization is used, one has to regularize the amplitudes. We found that the numerical results do not change significantly in this case when the cut-off scale is above but close enough to the \( D_s \) meson mass. We can draw the conclusion that the experimental result for \( BR(D_s^+ \rightarrow \omega \pi^+) \) can be understood as a result of the combined effect of a spectator transition and FSI. Therefore, it makes the attempt to understand the \( D_s^+ \rightarrow \omega \pi^+ \) amplitude as a result of annihilation contributions unsuccessful.

In the case of the \( D_s^+ \rightarrow \rho^0 \pi^+ \) transition, the FSI contributions alone result in

\[
BR(D_s^+ \rightarrow \rho^0 \pi^+)_\text{FSI} = 0.7 \times 10^{-3}. \tag{11}
\]

This is almost exactly the same as our estimate of the upper bound on the annihilation contribution [9]. Both contributions are equal or very close to the present 90% CL upper bound. If there is no destructive interference between these two contributions and the contributions of FSI through higher resonances that we did not take into account, one hopes that the branching fraction for this decay will be determined in the near future. Our prediction is in agreement with the results of other theoretical studies which give the rate for \( D_s^+ \rightarrow \rho^0 \pi^+ \) to be equal [8] or even larger than the rate for \( D_s^+ \rightarrow \omega \pi^+ \) decay [6].

However, one should consider possible cancellation that might occur. Adding the FSI contribution and the maximal annihilation contributions [8] with alternating signs gives a fairly large interval

\[
BR(D_s^+ \rightarrow \rho^0 \pi^+) = (0.05 - 3.5) \times 10^{-3}. \tag{12}
\]

We note that the experimental uncertainties reflected in the input parameters can change the values for \( BR(D_s^+ \rightarrow \rho^0 \pi^+) \) and \( BR(D_s^+ \rightarrow \omega \pi^+) \) by about 20%.

| \( D_s^+ \rightarrow \omega \pi^+ \) | \( A_{iD} \) | \( A_{iA} \) |
|-----------------|--------|--------|
| \( A_1 \)       | -0.7   | -0.7   |
| \( A_2 \)       | 0.7    | 0.7    |
| \( A_3 \)       | -1.1   | 3.3    |
| \( A_4 \)       | -1.4   | 1.5    |
| \( A_5 \)       | 11.3   | -4.0   |
| \( A_6 \)       | 12.5   | -19.7  |
| \( B_\eta \)    | 1.3    | -7.2   |
| \( B_{\eta'} \) | 3.6    | -3.7   |

Table 2
The dispersive \( A_{iD} \) and absorptive \( A_{iA} \) parts of the amplitudes (in units of \( 10^{-3} \) GeV) for the \( D_s^+ \rightarrow \omega \pi^+ \) decay corresponding to the diagrams on Fig. 2 (\( A_i \)) and Fig. 3 (\( B_{\eta,\eta'} \)). The amplitudes for the \( D_s^+ \rightarrow \rho^0 \pi^+ \) decay (neglecting the mass difference between \( m_\rho \) and \( m_\omega \)) are obtained by inverting the sign of \( A_{iD}, A_{iA} \) for even \( i \), while \( B_{\eta,\eta'} = 0 \).
Finally, we mention that the kind of FSI contributions we were considering in this paper is not the leading contribution in the $D_s^+ \rightarrow \phi \pi^+$ transition, which can proceed through spectator quark transition directly. Use of the factorization approximation for the weak vertex leads to a prediction $BR(D_s^+ \rightarrow \phi \pi^+) = 4.0\%$, which is already in excellent agreement with the experimental result of $3.6 \pm 0.9\%$. We found that inclusion of FSI reduces the theoretical prediction from 4% to $\sim 3.6\%$. The size of the shift also indicates that FSI of the type described in the present paper are in the case of $D_s^+ \rightarrow \phi \pi^+$ transition a second order effect. Note as well, that the size of the FSI correction is in agreement with the predictions for $BR(D_s^+ \rightarrow \rho^0 \pi^+)$ and $BR(D_s^+ \rightarrow \omega \pi^+)$, which are of order of magnitude smaller than $BR(D_s^+ \rightarrow \phi \pi^+)$. We summarize that the hidden strangeness final state interactions are very important in understanding the $D_s^+ \rightarrow \omega \pi^+$ and $D_s^+ \rightarrow \rho^0 \pi^+$ decay mechanism. The $D_s^+ \rightarrow \omega \pi^+$ amplitude can be explained fully by this mechanism. In the case of the $D_s^+ \rightarrow \rho^0 \pi^+$ decay rate we obtain a fairly large range due to possible cancellation between FSI and single pole contributions.

The measurement of the $D_s^+ \rightarrow \rho^0 \pi^+$ decay rate will considerably improve our understanding of the $D_s^+ \rightarrow \rho^0 \pi^+$ decay mechanism. The hidden strangeness FSI might fully explain the observed decay rate for $BR(D_s^+ \rightarrow \omega \pi^+)$. Finally, this kind of FSI gives only subdominant contributions in the case of $D_s \rightarrow \phi \pi, KK^*$ decays, which are well described by the factorization approximation.

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