Small-\(x\) asymptotics of structure function \(g_2\).

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Abstract

Nonsinglet structure function \(g_2(x, Q^2)\) for deep inelastic scattering of a lepton on a constituent quark is calculated in the double logarithmic approximation at \(x \ll 1\). Small-\(x\) asymptotics of \(g_2\) is shown to have the same singular behaviour as asymptotics of the nonsinglet structure function \(g_1\).

1 Introduction

Study of small-\(x\) behaviour of structure functions \(g_1\) and \(g_2\) of a constituent quark is essential for understanding of polarization effects in deep inelastic lepton-hadron scattering (DIS). Earlier \(g_2\) has been studied theoretically at \(x \sim 1\) (see e.g. [1, 2, 3, 4, 5, 6, 7] and refs therein).

In framework of the conventional parton model one has to consider the whole process of DIS as a superposition of two independent phases: hadron fragmentation in constituent partons and further deep inelastic scattering of the lepton on one of these partons.

The first phase is controlled by non-perturbative (non-PT) QCD effects and could be described today only in some model approach (see e.g. [8, 9] and refs therein). The second phase can be described in rigor terms of the perturbative (PT) QCD. The cross section of the whole process then could be obtained as convolution of the corresponding two probabilities.

At asymptotically high energies \(s \to \infty\) behaviour of the cross section is governed by the second phase, i.e. by the perturbative QCD, while the first phase, related to hadron-scale distances, just modifies pre-asymptotical factor and invokes strongly suppressed higher twist corrections.

According to dispersion relations the DIS structure functions can be presented as imaginary parts of the corresponding amplitudes for forward compton scattering of the virtual photon on incoming hadron.

For unpolarized leptons and hadrons the cross section depends only on crossing-symmetrical part of the compton amplitude. The antisymmetrical part of the amplitude corresponds to spin-dependent cross sections.

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At asymptotically high energies the symmetrical part of the amplitude strongly dominates over the antisymmetrical one. As a matter of fact the leading log contributions come from Feynman diagrams mainly constructed from gluon ladders. But for symmetrical case these ladders are of BFKL-pomeron type \[10\] whereas for antisymmetrical case they are simply of double log (DL) type similar to the case of a non-singlet fermion ladder. This difference leads to an extra power factor of \(1/x\) for symmetrical amplitude.

The structure functions of unpolarized hadrons were extensively studied both theoretically and experimentally by many authors. The spin-dependent structure functions, though being more clear in technique for theoretical analysis remain nonetheless less studied.

The spin-dependent part of the hadronic tensor, \(W_{\mu\nu}^{[A]}\), of the deep inelastic virtual photon – quark scattering is defined by

\[
W_{\mu\nu}^{[A]} = i\epsilon_{\mu\nu\alpha\beta} \frac{m q_{\alpha}}{p q} \left( S_{\beta} g_1 + \left( S_{\beta} - p_{\beta} \frac{S q}{p q} \right) g_2 \right) \tag{1}
\]

where \(m\) and \(S\) are the mass and spin of the quark. The functions \(g_1\) and \(g_2\) depend on \(x = Q^2/2 pq\) and \(Q^2\) \((Q^2 = -q^2 > 0)\). At \(x \ll 1\) the Eq.(1) can be written in terms of longitudinal and transversal projections of spin, relative to the plane of the 4-vectors \(p\) and \(q\), in a following way:

\[
W_{\mu\nu}^{[A]} \simeq i\epsilon_{\mu\nu\alpha\beta} \frac{m q_{\alpha}}{p q} \left( g_1 S^\parallel + \left( g_1 + g_2 \right) S^\perp_{\beta} \right) \tag{2}
\]

At very small-\(x\), when \(\ln(1/x) \gg \ln(Q^2/\mu^2)\), \(\mu\) – a mass scale which is order of few hundred MeV, all contributions \(\sim (\alpha_s(\ln(x)^2))^k\) are important and must be taken into account. The structure functions obtained from DGLAP evolution equations \[11, 12, 13\] did not account for these terms as far as these equations were derived originally for \(x \sim 1\), though analysis of the experimental data for the polarised DIS within HERA energy range is perfectly compatible at present \[14\] with extrapolating of DGLAP into the small \(x\) region.

The double logarithmic asymptotics of \(g_1\) for a constituent quark (a quark with virtuality \(\sim \mu^2\)) was calculated earlier \[15, 16\] in the double-logarithmic approximation (DLA) for both flavour non-singlet and singlet cases. Unlike \(g_1\) the perturbation series for \(g_2\) begins from higher orders in QCD coupling \(\alpha_s\). In particular, it was shown \[7\] that there were no DL contributions to \(g_2\) at small \(x\) in the \(\sim \alpha_s\) order. We show below that in DLA the dominant contribution to \(g_2\), which corresponds to ladder graphs, can be obtained through differentiation in QCD coupling the analytical expression of \(g_1\).

This paper is organized as follows.

In Section 2 the pure quark ladder digarams are considered and the analytical relation between their contributions to \(g_1\) and \(g_2\) is derived.

In Section 3 the effects of non-ladder contributions to \(g_1\) and \(g_2\) are discussed. Estimations for the small-\(x\) asymptotics of \(g_1\) and \(g_2\) are presented.
2 Quark ladder contributions

The quark ladder diagram with $n$ gluon rungs (see Fig.1),
with a gluon propagator taken in the Feynman gauge

$$G_{\lambda\nu}(k) = \frac{g_{\lambda\nu}}{k^2 + i\epsilon},$$

gives the following contribution to the general structure function

$$W^{(n)}_{\mu\nu} = \frac{1}{2\pi} \text{Disc} \int \ldots \int \prod_{i=1}^{n} \left[ \frac{d^4 k_i}{(2\pi)^4 i} \frac{g_s^2 C_F - g_{\lambda\nu}}{(k_i^2)^2 - (k_i - k_{i-1})^2 - i\epsilon} \right] \frac{e^2 T^{(n)}_{\mu\nu}}{(q + k)^2 - i\epsilon}$$  \hspace{1cm} (3)

$$T^{(n)}_{\mu\nu} = \frac{1}{2} \text{Tr} \left\{ (\hat{p} + m)(1 - \gamma_5 \hat{S}) \gamma_\lambda \hat{k}_1 \ldots \gamma_\lambda \hat{k}_n \gamma_\nu (\hat{q} + \hat{k}_n) \gamma_\mu \hat{k}_n \gamma_\lambda \ldots \hat{k}_1 \gamma_\lambda \right\}$$  \hspace{1cm} (4)

Here $g_s^2 = 4\pi\alpha_s$ is the QCD coupling constant, $C_F = (N_c^2 - 1)/2N_c$ - the color factor and “–” signs are arranged to make explicit the positive imaginary parts of propagators. We have omitted the quark mass $m$ in all internal quark propagators as only linear in $m$ terms are essential and because of the well-known relations

$$\gamma_\lambda \gamma_\mu \gamma^\lambda = -2\gamma_\mu, \quad \gamma_\lambda \gamma_\mu \gamma_\nu \gamma^\lambda = 4g_{\mu\nu}$$

and the restriction on a spin of an incoming on-shell quark, $(pS) = 0$, only the incoming quark $m$-term works.

To make the DL contribution explicit in Eq.(3) let us express all momentums in terms of the light-cone or Sudakov variables:

$$p = p' + \frac{m^2}{s} q', \quad q = -xp' + q', \quad s = 2pq', \quad p'^2 = q'^2 = 0, \quad (5)$$

\hspace{1cm}
\[ k_i = \beta_i p' + \alpha_i q' + k_i^\perp, \quad k_i^2 = \beta_i \alpha_i s - k_i^2_{\perp\perp}, \quad d^d k_i = d\beta_i d\alpha_i s \frac{1}{2} d^2 k_i_{\perp\perp}. \]

Let us remind here that \( x = Q^2/2pq', \quad Q^2 = -q^2 > 0. \)

We shall neglect the power suppressed terms \( \mathcal{O}(m^2/s), \quad \mathcal{O}(q^2/s) \) and thus ignore the difference between \( p, q \) and \( p', q' \) respectively in what follows. It is also convenient to differ the Lorentz-covariant vector \( k_i^\perp \) and its two dimensional projection \( k_i^\perp_{\perp\perp} \): \( (k_i^\perp)^2 = -k_i^2_{\perp\perp}. \)

Taking integrations over all \( \alpha_i \) as residues in the poles of gluon propagators,

\[ s\alpha_i = \frac{-(\vec{k}_i - \vec{k}_{i-1})^2}{\beta_i - \beta_{i-1}} + s\alpha_{i-1}, \quad (6) \]

we see that DL contribution may come only from the integration over the region

\[ 1 \gg \beta_1 \gg \beta_2 \ldots \gg \beta_n \geq x, \quad \frac{k_i^2_{\perp\perp}}{\beta_i} \ll \frac{k_i^2_{\perp\perp}}{\beta_2} \ldots \ll \frac{k_n^2_{\perp\perp}}{\beta_n}, \quad (7) \]

inside which the whole ladder factorizes in the product of independent rungs with quark virtualities

\[ k_i^2 = -\vec{k}_i^2_{\perp\perp} - \frac{\beta_i}{\beta_{i-1}} (\vec{k}_{i-1} - \vec{k}_{i-1})^2 \approx -\vec{k}_i^2_{\perp\perp}. \quad (8) \]

Thus Eq.(3) turns into

\[ W^{(n)}_{\mu\nu} = \prod_{i=1}^{\infty} \left[ \frac{g^2 C_F}{16\pi^2} \int x^{\beta_i-1} \frac{d\beta_i}{\beta_i} \int_0^{2\pi} \frac{d\phi_i}{2\pi} \int_{\beta_{i-1}}^{\beta_i} \frac{d^2 k_i}{[k_i^2]^2} \right] \beta_n e^{2} 2s \delta(\beta_n - x - \vec{k}_n^2_{\perp\perp}) T^{(n)}_{\mu\nu}, \quad (9) \]

with

\[ \kappa_i = \mu^2 + \frac{\beta_i}{\beta_{i-1}} \vec{k}_{i-1}^2_{\perp\perp} \]

where the infrared cutoff \( \mu^2 \) was added to the lower boundary of each transverse momentum to account for color bleaching which turns on at confinement distances. The upper boundary for \( \vec{k}_{i\perp\perp} \) stems from the requirement \( (k_i + q)^2 \geq 0. \)

To obtain the logarithmic integration over each \( k_i^2_{\perp\perp} \) we evidently have to see that \( T^{(n)}_{\mu\nu} \) defined by the Eq.(4) combined with the product of all denominators is proportional to the product of all \( k_{i\perp\perp}^{-2} \) multiplied by some dimensionless tensor which is independent of all \( \beta_i \) and \( k_i^2_{\perp\perp} \).

In the region Eq.(6) one can neglect the extra term \( \vec{k}_n \) which appears together with \( q \) in the middle of the trace Eq.(4). Then invoking a very useful and constructive formula

\[ \gamma_\nu q_\gamma = i\varepsilon_{\mu\nu\lambda\sigma} q^\lambda \gamma_5 \gamma_\sigma + q_\nu \gamma_\mu + q_\mu \gamma_\nu - g_{\mu\nu} q \]

immediately splits the tensor \( T^{(n)}_{\mu\nu} \) into sum of symmetrical and antisymmetrical parts:

\[ T^{(n)}_{\mu\nu} = T^{(n)}_{\{\mu\nu\}} + T^{(n)}_{[\mu\nu]}, \quad (11) \]

\[ T^{(n)}_{\{\mu\nu\}} = (q_\nu g_{\mu\sigma} + q_\mu g_{\nu\sigma} - q_\sigma g_{\mu\nu}) \frac{1}{2} \text{Tr} \left\{ (\hat{p} + m)(1 - \gamma_5 \hat{S}) \Gamma^\sigma \right\}, \]

\[ T^{(n)}_{[\mu\nu]} = i\varepsilon_{\mu\nu\lambda\sigma} q^\lambda \frac{1}{2} \text{Tr} \left\{ (\hat{p} + m)(1 - \gamma_5 \hat{S}) \gamma_5 \Gamma^\sigma \right\}. \]
where
\[ \Gamma^\sigma = \left( \prod_{i=1}^{n} (-g^\lambda_i \lambda'_i) \right) \gamma_{\lambda_i} \hat{k}_i \ldots \gamma_{\lambda_n} \hat{k}_n \gamma^\sigma \hat{k}_n \gamma_{\lambda_n} \ldots \hat{k}_1 \gamma_{\lambda_1} \] (12)
and the anticommutation property of \( \gamma_5 \) was used to take it out of \( \Gamma^\sigma \) for the antisymmetrical part.

The key point for understanding the structure of the matrix \( \Gamma^\sigma \) is that
\[ \hat{k} \gamma^\sigma \hat{k} = -k^2 f^\sigma\sigma'(k) \gamma_{\sigma'}, \quad f^\sigma\sigma'(k) = g^\sigma\sigma' - 2 \frac{k^\sigma k^{\sigma'}}{k^2}. \] (13)

This means that \( \Gamma^\sigma \) is linear in \( \gamma^\sigma \) and can be expressed in terms of convolution of \( n \) similar \( f \)-tensors:
\[ \Gamma^\sigma = \left( \prod_{i=1}^{n} [-2k_i^2] \right) E^\sigma_{\tau} \gamma^\tau, \quad E_{\sigma\tau} = \left( \prod_{i=1}^{n} \otimes f(k_i) \right)_{\sigma\tau}. \] (14)

According to Eq.(8) the dimensional factor in Eq.(14) makes all integrations over \( k_i^2 \) in Eq.(9) the log integrals only if the integral over all azimuthal angles
\[ E_{(n)}^{(n)} = \int_0^{2\pi} \frac{d\phi_1}{2\pi} \left[ \frac{k_{1\perp}^2}{-k_1^2} \right] \ldots \int_0^{2\pi} \frac{d\phi_n}{2\pi} \left[ \frac{k_{n\perp}^2}{-k_n^2} \right] E_{(n)}^{(n)} \] (15)
does not implicitly depend on \( k_{i\perp}^2 \) or \( \beta_i \). We shall prove this later. Substituting Eq.(13) and Eq.(11) into Eq.(3) we finally obtain the following compact formulae which describe all structure functions corresponding to the considered ladder diagram:
\[ W^{(n)}_{\{\mu\nu\}} = -g_{\mu\nu} M^{(n)} \left( E^{(n)} \right)_{\sigma\rho} \frac{q^\rho p^\sigma}{(pq)} \] (16)
\[ W^{(n)}_{[\mu\nu]} = i\varepsilon_{\mu\nu\lambda\rho} \frac{mq^\lambda}{(pq)} M^{(n)} \left( E^{(n)} \right)^{\sigma} \rho S^\rho \] (17)
where
\[ M^{(n)} = \left( \frac{\alpha_s C_F}{2\pi} \right)^n \prod_{i=1}^{n} \left[ \int_{x}^{\beta_i} d\beta_i \int_{\kappa_i}^{\beta_i} d\kappa_i \frac{k_{i\perp}^2}{k_i^2} \right] \beta_n \frac{1}{2} \frac{1}{\delta(\beta_n - x - k_{n\perp}^2 s)} \] (18)
can be considered as a DL integral operator.

Due to the trivial relation
\[ \int_0^{2\pi} \frac{d\phi}{2\pi} k_{\sigma\rho} = (k^\perp)^2 g_{\sigma\rho} \] (19)
the integral
\[ \int_0^{2\pi} \frac{d\phi}{2\pi} f_{\sigma\sigma'}(k^\perp) = g_{\sigma\sigma'} \] (20)
in the DL region acts like a projector on the longitudinal subspace. If one might neglect the longitudinal components of the momenta \( k_i, k_i^\parallel = \beta_i p + \alpha_i q \), then we would immediately obtain the obvious results:
\[ E^{(n)}_{\sigma\rho} p^\rho = p_\sigma, \quad E^{(n)}_{\sigma\rho} S^\rho_\parallel = S^\parallel_\sigma, \quad E^{(n)}_{\sigma\rho} S^\rho_\perp = 0. \] (21)
In reality the account of the longitudinal momenta components does not influence the first two results of Eq.\((21)\) but changes the last prediction dramatically. To see this let us present \(E\) of Eq.\((14)\) as follows:

\[
E^{(n)}_{\sigma \rho} = g_{\sigma \rho} - \frac{n}{2} \sum_{i=1}^{n} \frac{(k_i^\perp)_\sigma (k_i^\perp)_\rho}{k_i^2} + \sum_{i>j}^{n} \frac{2(k_i^\perp)^2(k_i^\perp k_j)(k_j^\perp)_\rho}{k_i^2 k_j^2} + \ldots \tag{22}
\]

We have made it clear here that only \((k_i^\perp)_\sigma\) component can work in Eq.\((16)\), as the \(q\)-component gives zero or power suppressed contribution and the \(p\)-component leads beyond the DL approximation because of the small factor \(\beta_i\).

The first two terms of Eq.\((22)\) obviously contribute to Eq.\((16)\) as

\[
E^{(n)}_{\sigma \rho} = g_{\sigma \rho} - ng_{\sigma \rho}^\perp \tag{23}
\]

One might expect to get some contribution from non-diagonal terms \((k_i^\perp)_\sigma \alpha_i q_\rho\) as all \(\alpha_i\) depend on azimuthal angles according to Eq.\((6)\). But

\[
\int_0^{2\pi} \frac{d\phi_i}{2\pi} (k_i^\perp)_{\alpha_i} q_\rho = -(k_i^\perp-1)_{\alpha_i} \frac{(k_i^\perp)^2}{\beta_{i-1} s} q_\rho \tag{24}
\]

and subsequent integration over \(\phi_{i-1}\) turns this contribution to zero. To be correct at such circumstances one must do not forget to include the small azimuthal dependence of \(k_i^2\) (see Eq.\((8)\)) in denominators in Eq.\((13)\) into consideration:

\[
\int_0^{2\pi} \frac{d\phi_i}{2\pi} \frac{(k_i^\perp)_{\sigma}}{k_{i-1}^2} = \frac{\beta_{i-1} (k_{i-2}^\perp)_{\sigma}}{\beta_{i-2} (k_{i-1}^\perp)^2} \tag{25}
\]

Performing the chain of subsequent integrations over azimuth angles we come eventually to the last integral over \(\phi_1\) which turns to zero.

The third term of Eq.\((22)\) despite its large magnitude

\[-2(k_i k_j) = -2(k_i^\perp k_j^\perp) - \beta_j \alpha_i s \approx -\frac{\beta_j}{\beta_{i-1}} (k_i^\perp - k_{i-1}^\perp)^2 \gg -(k_i^\perp)^2.\]

does not contribute in Eq.\((16)\) at all. As a matter of fact, according to Eqs.\((15, 23)\)

\[
\int_0^{2\pi} \frac{d\phi_i}{2\pi} (k_i^\perp)_{\sigma} 2(k_i k_j)(k_j^\perp)_\rho = (k_i^\perp)^2 \left( (k_j^\perp)_{\sigma} - \frac{\beta_j}{\beta_{i-1}} (k_{i-1}^\perp)_{\sigma} \right) \tag{26}
\]

and subsequent integrations over \(\phi_{i-1}, \phi_{i-2}, \ldots \phi_{j+1}\) transform the result of Eq.\((26)\) into

\[ (k_j^\perp)_{\sigma} - \frac{\beta_j}{\beta_{i-1}} \frac{\beta_{i-1}}{\beta_{i-2}} \ldots \frac{\beta_j}{\beta_{j+1}} (k_j^\perp)_{\sigma} = 0.\]

As soon as large tensor magnitude was integrated down to zero the problem of dangerous small corrections to it arises again. We omit the cumbersome analysis of such corrections here and just present the result: the third term of Eq.\((22)\) indeed contribute nothing in DL approximation.
With this result the next term which is not shown in Eq.(22) and can be expressed as the convolution of already considered terms gives also zero contribution. Therefore Eq.(23) can be considered as the total expression for substitution into Eq.(16):

\[
W^{(n)}_{(\mu\nu)} = -g_{\mu\nu}^{\perp} M^{(n)}
\]

\[
W^{(n)}_{[\mu\nu]} = i\varepsilon_{\mu\nu\alpha\sigma}(pq) \left\{ M^{(n)} S^{\sigma} - \frac{\partial}{\partial \ln \alpha_s} M^{(n)} S^{\perp} \right\}
\]

(27)

where we have used the simple relation

\[
\frac{\partial \alpha_s^n}{\partial \ln \alpha_s} = n\alpha_s^n
\]

to express the transverse spin contribution in terms of the amplitude \( M^{(n)} \) of Eq.(18).

Summing up over all ladders thus leads to a very simple relation between the contributions to DIS functions \( g_1 \) and \( g_2 \):

\[
g_2 = -\frac{\partial g_1}{\partial \ln \alpha_s}
\]

(28)

3 Discussion

Eq.(28) is valid only for ladder Feynman graphs. Though these diagrams determine the small-\( x \) asymptotics of the DGLAP evolution equations, this is not true for \( g_1 \) and \( g_2 \) in the DL region [7].

To understand the effect of nonladder graphs on Eq.(28), let us add a nonladder gluon to a ladder graph (see Fig.(2)). Nonladder gluon momentum \( k \) must be soft enough not to destroy the DL pattern of a ladder graph. Unlike a skeleton ladder gluon a nonladder gluon invokes an additional DL integration only if no any \( k_2^{\perp} \) factor appears in the integrand’s numerator. Such factor would cancel the softest virtuality propagator (of those invoked by a nonladder gluon) in the denominator. Thus adding a nonladder gluon does not change the relation between contributions of the \( S^{\perp} \) and \( S^{\parallel} \)-structures, i.e. the Eq.(23).

Therefore we could use the Eq.(28) if one would separate out the contribution of ladder skeleton gluons to \( g_1 \) and differentiate it in \( \alpha_s \). Fortunately one can use the expression for \( g_1 \) obtained in the work [15] and tag the ladder and nonladder gluon contributions through definition of separate QCD couplings \( \alpha_L \) and \( \alpha_{NL} \) respectively, \( \alpha_L = \alpha_{NL} = \alpha_s \):

\[
g_1 = e^2 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} \left( \frac{1}{x} \right)^{\omega} \frac{\omega}{\omega - f_0} \left( \frac{Q^2}{\mu^2} \right) f_0
\]

(29)

\[
f_0 = \frac{1}{2} \left[ \omega - \sqrt{\omega^2 - \frac{2\alpha_L C_F}{\pi} + \frac{4\alpha_{NL}^2 C_F N_c}{\omega \pi^2}} \right]
\]

\[
V = \frac{d}{d\omega} \ln \int_0^\infty dt t^{-(1+p)} \exp \left( -\frac{t^2}{2} - \frac{t\omega}{\sqrt{\frac{\alpha_{NL} N_c}{2\pi}}} \right).
\]

where \( V \) defines the nonladder contribution, \( p = -1/2N_c^2 \).
Now Eq.(28) can be written in the form

\[ g_2 = - \frac{\partial g_1}{\partial \ln \alpha_L} \bigg|_{\alpha_L = \alpha_{NL} = \alpha_s}. \]  

(30)

Eqs.(29,30) show the same asymptotical behaviour of \( g_1 \) and \( g_2 \) at \( x \to 0 \):

\[ g_2 \sim g_1 \sim \left( \frac{1}{x} \right)^a \left( \frac{Q^2}{\mu^2} \right)^{\frac{2}{a}}, \quad a \simeq \sqrt{\frac{2\alpha_s C_F}{\pi}} \left( 1 + \frac{1}{2N_c^2} \right). \]  

(31)

4 Conclusion

We have considered only nonsinglet contribution to the structure function \( g_2 \). Obviously, one should expect the singlet contribution, i.e. insertion of gluon ladders, to be dominating over the nonsinglet one – similar to the case of the structure function \( g_1 \) [16]. Our preliminary consideration has shown that one could repeat the analysis presented above for the case of gluon ladder contributions, the crucial points of the analysis as well as general features of the result remain the same. The only difference stems from different contributions of a gluon rung to the numerator’s tensor structure (17) for each case: for a quark ladder - Eq.(13) and for a gluon ladder -

\[ h^{\sigma\sigma'}(k) = g^{\sigma\sigma'} - \frac{k^\sigma k^{\sigma'}}{k^2}. \]

Thus one would expect the following generalization of the expression (30) for the total structure function \( g_2 \):

\[ g_2 = - \frac{\partial g_1}{\partial \ln \alpha_L} \bigg|_{\alpha_L = \alpha_{NL} = \alpha_s} - \frac{1}{2} \frac{\partial g_1}{\partial \ln \bar{\alpha}_L} \bigg|_{\bar{\alpha}_L = \alpha_{NL} = \alpha_s}. \]

where we discern the QCD coupling \( \bar{\alpha}_L \) corresponding to gluon radiation in a gluon cell from the QCD coupling \( \alpha_L \) corresponding to gluon radiation in a quark cell of a ladder graph.

A detailed comprehensive consideration which is necessary to prove this relation will be published elsewhere.

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