Jet-quenching of the rotating heavy meson in a $\mathcal{N}=$4 SYM plasma in presence of a constant electric field

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Abstract

In this paper, we consider a rotating heavy quark-antiquark ($q\bar{q}$) pair in a $\mathcal{N}=$4 SYM thermal plasma. We assume that $q\bar{q}$ center of mass moves at the speed $v$ and furthermore they rotate around the center of mass. We use the AdS/CFT correspondence and consider the effect of external electromagnetic field on the motion of the rotating meson. Then we calculate the jet-quenching parameter corresponding to the rotating meson in the constant electric field.

Keywords: AdS/CFT correspondence; Super Yang Mills theory; Black hole; String theory.

1 Introduction

As we know the Maldacena conjecture [1-3] provides useful mathematical tools for studying complicated problems of QCD at strong coupling. In this way one of the important subjects is the motion of charged particles through the strongly coupled thermal medium. Already the subject of a quark in the thermal plasma at weak coupling has been well studied in literature [4-10]. But, QCD at the strong coupling will be a hard problem, however Maldacena conjecture make it easy. According to the Maldacena conjecture there is the relation between type IIB string theory in $AdS_5 \times S^5$ space and $\mathcal{N}=$4 super Yang-Mills (SYM) gauge theory on the 4-dimensional boundary of $AdS_5$ space. So, this duality will be a candidate for the studying strongly coupled plasma. In that case, instead of a heavy quark in the

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gauge theory, one may consider dual picture of the heavy quark which is an open string in AdS space. Also the dual picture of temperature in the gauge theory is a black hole (black brane) in AdS$_5$ space. One of the important fields in the QCD, which is also interesting to experiment in the LHC and RHIC [11], is consideration of the moving heavy quark through the $\mathcal{N}$=4 Super Yang-Mills thermal plasma [12-18]. Recently, the same calculations are done for $\mathcal{N}$=2 supergravity thermal plasma [19]. This subject is important because the solutions of supergravity theory with $\mathcal{N}$=4 and $\mathcal{N}$=8 supersymmetry may be reduced to the solutions of the $\mathcal{N}$=2 supergravity. In Refs. [19] we found that the problem of drag force in the $\mathcal{N}$=2 supergravity thermal plasma at zero non-extremality parameter and finite chemical potential [20] is corresponding to the $\mathcal{N}$=4 SYM plasma for heavy quark. Another interesting problem is to consider a $q\bar{q}$ pair which may be interpreted as a meson. As we know the problem of celebrated Regge behavior of the hadron spectra has been discussed in literature [21]. Also the meson spectrum obtained so far and reasonably describing experiment can be seen in the Ref. [22], where it is found that the angular momentum plays an important role to obtain the meson spectrum. So, this give us good motivation to consider the rotating meson.

Already, the energy of the moving $q\bar{q}$ pair through the $\mathcal{N}$=4 SYM plasma is studied in both rest frames of thermal plasma and $q\bar{q}$ pair, which relate to each other by a Lorentz transformation [23, 24, 25]. Authors in [25] found that the heavy meson in the plasma feels no drag force. Considered system in that paper was an ideal case. Actually, the $q\bar{q}$ pair may have more degrees of freedom such as the rotational motion around the center of mass and the oscillation along the connection axis. In the Refs. [26, 27] the description of quark-antiquark system instead single quark well explained. Also the problem of the spinning open string (meson) in description of the non-critical string/gauge duality [28] considered. In that paper the relationship between the energy and angular momentum of spinning open string for the Regge trajectory of mesons in a QCD-like theory is studied [29, 30]. It is important to note that, in $\mathcal{N}$=4 SYM there is no dynamical quark hence no dynamical mesons, therefore the rotating mesons are non-dynamical in $\mathcal{N}$=4 SYM.

Already a rotational motion for the $q\bar{q}$ pair considered, then the momentum densities for such a system calculated [31]. In that case their method was different with Refs. [28] and [32]. In the Ref. [32] authors considered a rotating quark and calculated drag force on a test quark moving through the plasma. In order to obtain the drag force one needs to calculate the components of the energy-momentum density $\Pi^X$ and $\Pi^Y$. We assumed that $q\bar{q}$ pair moves at a constant speed $v$ along $X$ direction, and also rotates around the center of mass. In [31] authors obtained the effect of the rotational motion in the energy-momentum components of the heavy $q\bar{q}$ pair with the non-relativistic velocity. Therefore, they are determined the motion of the heavy $q\bar{q}$ pair more exactly. As we know, in the case of the single quark, the energy of quark goes from D-brane into the black hole. In the case of quark and antiquark the currents of energy and momentum coming from two points on D-brane. These components cancel each other on the top of the string and therefore effectively the heavy $q\bar{q}$ pair feels no drag force, so the string can save its shape in the quark-gluon plasma. In this paper we would like to consider the effect of constant electromagnetic field on the motion of rotating heavy meson through the $\mathcal{N}$=4 SYM plasma. Also we are going to use results of
the jet-quenching parameter, which is one of the interesting properties of the strongly coupled plasma [33-38]. The jet-quenching parameter supplies a measurement of the dispersion of the plasma. This quantity commonly obtained by using perturbation theory, but in here by using the AdS/CFT correspondence we find jet-quenching parameter in non-perturbative quantum field theory.

This paper organized as the following. In the section 2 we review some important results of the Ref. [31] for completeness. In the section 3 we add an external constant electromagnetic field and obtain the motion of the rotating string. In the section 4 we calculate the jet-quenching parameter in this system and finally in the section 5 we summarized our results and give some suggestions for future works.

2 Rotating String Equation of Motion

The dual picture of a meson in the CFT is a string in the AdS space which both endpoints of it live on the D-brane. This configuration illustrates the strong interaction between two quarks due to a tube of gluon field and describes the confinement mechanism in QCD. In the classical level, these states can be regarded as the rotation of the system. In another word we will dealing with the spinning open string. The spinning string is interesting because it is dual picture of the rotating meson. In the Fig.1 we show configuration of the rotating string in the AdS space.

![Figure 1: A rotating \( \cap \) - shape string dual to a \( q\bar{q} \) pair which can be interpreted as a meson. The points \( A \) and \( B \) represent quark and antiquark with separating length \( l \). The radial coordinate \( r \) runs from \( r_h \) (black hole horizon radius) to \( r = r_{\text{min}} \) (\( r = \infty \)) on the D-brane. The \( r_c \) is the critical radius, obtained for single quark solution. The \( r_{\text{min}} \) is turning point of]
string and one can obtain \( r_{\text{min}} \geq r_c \). The parameter \( \theta \) is assumed to be the angle with \( Y \) axis and the string center of mass moves along \( X \) axis at velocity \( v \).

From Maldacena dictionary, we know that, adding temperature to the system is equal to the existence of a black hole in the center of \( AdS \) space. For the dual picture of \( N=4 \) SYM plasma there is the \( AdS_5 \) black hole solution which is given by \([25]\),

\[
\begin{align*}
\frac{ds^2}{\sqrt{H}} &= \frac{1}{\sqrt{H}}(-hdt^2 + d\vec{x}^2) + \frac{\sqrt{H}}{h}dr^2, \\
\frac{h}{h} &= 1 - \frac{r_h^4}{r^4}, \\
H &= \frac{R^4}{r^4},
\end{align*}
\]

(1)

where \( R \) and \( r_h \) are curvature radius of \( AdS \) space and radius of black hole horizon respectively, also the motion direction described by \( \vec{x} : (X, Y, Z) \). So we choose motion axis as \( X \) and \( Y \) (X-Y plan, \( Z = 0 \)).

We know the dynamics of the open string is described by the Nambu-Goto action,

\[
S = T_0 \int dt dr \mathcal{L},
\]

(2)

where we used static gauge (\( \sigma = r \) and \( \tau = t \)). Therefore the lagrangian density of system is given by,

\[
\mathcal{L} = -\sqrt{-g} = -\left[ 1 + \frac{h}{H}(X'^2 + Y'^2) - \frac{1}{H}(\dot{X}^2 + \dot{Y}^2) - \frac{1}{H}(\dot{X}^2Y'^2 + \dot{Y}^2X'^2 - 2\dot{X}\dot{X}'\dot{Y}\dot{Y}') \right]^\frac{1}{2},
\]

(3)

where prime and dot denote derivative with respect to \( r \) and \( t \) respectively, also \( g \equiv det g_{ab} \), where \( g_{ab} \) is the metric on the world sheet of the string. It is most general lagrangian for the string which its endpoints lie on \( D \)-brane. In that case one can obtain the following expressions for the energy and momentum density components,

\[
\begin{pmatrix}
\Pi_X^0 & \Pi_X^1 \\
\Pi_Y^0 & \Pi_Y^1 \\
\Pi_r^0 & \Pi_r^1 \\
\Pi_t^0 & \Pi_t^1
\end{pmatrix} = -\frac{T_0}{H\sqrt{-g}} \begin{pmatrix}
\dot{Y}Y'X' - \dot{X}X'Y' - \frac{h}{H}\dot{X}
\dot{X}XX' - \dot{Y}YY' - \frac{h}{H}\dot{Y}
\dot{X}\dot{Y}X' + \dot{Y}\dot{X}Y' + hX' + hY'
\frac{H}{h}(\dot{X}X' + \dot{Y}Y')
\frac{H}{h}(\dot{X}X' + \dot{Y}Y')
\frac{H}{h}(X'^2 + Y'^2 - h)
\frac{H}{h}(X'^2 + Y'^2 + h)
\frac{H}{h}(X'X' + Y'Y')
\frac{H}{h}(X'X' + Y'Y')
\end{pmatrix}.
\]

(4)
In order to obtain the total energy, total momentum and angular momentum of the string we use the following relations respectively,

\[
E = - \int_{r_{\text{min}}}^{r_0} dr \Pi^0_t,
\]

\[
P_X = \int_{r_{\text{min}}}^{r_0} dr \Pi^0_X,
\]

\[
P_Y = \int_{r_{\text{min}}}^{r_0} dr \Pi^0_Y,
\]

\[
J = \int_{r_{\text{min}}}^{r_0} dr (X \Pi^0_Y - Y \Pi^0_X).
\]

(5)

It is possible to study Regge trajectory by calculation of \( \frac{E^2}{J} \). In that case we should determine the motion of string, it means that we should find explicit expressions for \( x(r) \) and \( y(r) \).

Now, we are in the place of applying rotational motion to the string, so we consider the following ansatz as solution of the equation of motion,

\[
X(r, t) = vt + x \sin \omega t, \\
Y(r, t) = y \cos \omega t,
\]

(6)

with constants linear and rotational motion which are denoted by \( v \) and \( \omega \) respectively. We choose \( \theta(t) = \omega t \) as an angle with \( Y \) axis (see Fig. 1).

Here, we consider the special case of small velocities to find momentum densities. So, we consider a moving heavy \( q\bar{q} \) with non-relativistic speed \( v \), which rotate by angle \( \theta = \omega t \) around the center of mass, therefore we have \( \omega \ll 1 \). It is corresponding to motion of the heavy meson with large spin. Indeed in the very large angular momentum limit a semiclassical approximation is reliable. In this case, as mentioned above, the angular velocity of the string is very small and the separation value of quark and antiquark is very large. In this limits one can obtain,

\[
\sqrt{-g} \approx \left[ 1 - \frac{v^2}{h} + \frac{h}{H} x'^2 \sin^2 \omega t + \frac{h - v^2}{H} y'^2 \cos^2 \omega t \right]^{\frac{1}{2}},
\]

(7)

and from equation (4) one finds the following expression of momentum currents,

\[
\begin{pmatrix}
\Pi^1_X \\
\Pi^1_Y
\end{pmatrix} = -\frac{T_0}{H \sqrt{-g}} \begin{pmatrix}
x' \sin \omega t \\
y' \cos \omega t
\end{pmatrix},
\]

(8)

where \( \sqrt{-g} \) is given by the equation (7). By using equations (7) and (8) one can obtain the following expressions,

\[
x' \sin \omega t = \frac{H(h - v^2)}{h} \Pi^1_X \sqrt{\frac{1}{(h - v^2)(hT_0^2 - H\Pi^1_X^2) - hH\Pi^1_Y^2}},
\]

\[
y' \cos \omega t = H \Pi^1_Y \sqrt{\frac{1}{(h - v^2)(hT_0^2 - H\Pi^1_X^2) - hH\Pi^1_Y^2}}.
\]

(9)
Then one can fix momentum densities by using the condition $\frac{y'}{x'} = \cot \omega t$ at $r = r_{\text{min}}$ [31]. The reality condition for single quark solution yields to the velocity-dependent critical radius [12, 13, 16, 19],

$$r_c = \frac{r_h}{(1-v^2)^{\frac{1}{4}}}. \quad (10)$$

By using the reality condition in equation (9) for the quark-antiquark system one can find special radius, $r_{\text{min}}$, where functions $x$ and $y$ are not imaginary. Then, by using square root quantity in (9) one can obtain,

$$r_{\text{min}} = \left[r_h^4 + \frac{b}{2T_0^2(1-v^2)} \left(1 - \sqrt{1 - \frac{4T_0^2(1-v^2)v^2r_h^4R^4\Pi^1_X^2}{b^2}}\right)\right]^{\frac{1}{4}} \quad (11),$$

where we define,

$$b \equiv R^4(1-v^2)\Pi^1_X + R^4\Pi^1_Y + T_0^2v^2r_h^4. \quad (12)$$

Indeed, the $r_{\text{min}}$ is the turning point of the string. It is easy to check that $r_{\text{min}} \geq r_c$. If we consider $\Pi^1_X = 0 \ (l = 0)$ the special case of $r_{\text{min}} = r_c$ will be satisfied. We note here $r_{\text{min}} = r_c$ correspond to the single quark solution [25]. Also one can see that if $v = 0$ then $r_{\text{min}} = r_h$ which is expected. The turning point $r_{\text{min}}$ is an important parameter to determine the jet quenching parameter, so we will use relation (11) in the section 4.

In the next section we add an electromagnetic field to the background and will obtain motion of the rotating meson.

### 3 Effect of the constant electromagnetic field

In the previous section we considered the rotating meson through the thermal plasma without any external field. Now, we introduce a constant electromagnetic field in the background. The constant electromagnetic field assumed along the $X$ and $Y$ directions. The constant electric field and the constant magnetic field denoted by $B_{01}$ and $B_{12}$ respectively. This procedure for the single quark in the $N=4$ SYM theory originally studied in [39]. In this configuration the lagrangian density (3) changes to,

$$-g = 1 + \frac{\hbar}{H}(X'^2 + Y'^2) - \frac{1}{\hbar}(\dot{X}^2 + \dot{Y}^2) - \frac{1}{H}(\dot{X}^2Y'^2 + \dot{Y}^2X'^2 - 2\dot{X}\dot{X}'\dot{Y}\dot{Y}')$$

$$- \left(B_{01}X' + B_{12}(\dot{X}Y' - \dot{Y}X')\right)^2. \quad (13)$$

Therefore the equations of motion read as,

$$\frac{\partial}{\partial r} \left[ \frac{1}{\sqrt{-g}} \left( \frac{X'^2 - \dot{X}\dot{X}' - \hbar X'}{H} + (B_{01} - B_{12}) \left(B_{01}X' + B_{12}(\dot{X}Y' - \dot{Y}X')\right) \right) \right]$$

$$+ \frac{1}{\sqrt{-g}} \frac{\partial}{\partial t} \left[ \frac{\dot{X}}{\hbar} + \frac{\dot{Y}Y'^2 - X'Y\dot{Y}}{H} + B_{12}Y' \left(B_{01}X' + B_{12}(\ddot{X}Y' - \ddot{Y}X')\right) \right] = 0, \quad (14)$$
Also the energy and momentum currents in the small velocities approximation obtained as

\[
\frac{\partial}{\partial t} \left[ \frac{1}{\sqrt{-g}} \left( Y' \dot{X'}^2 - \dot{X} \dot{Y}' - h Y' \right) \right] + \frac{1}{\sqrt{-g}} \frac{\partial}{\partial t} \left[ \frac{Y'}{h} + \frac{\dot{X}'^2 - X'' \dot{X}}{H} - B_1 X' \left( B_0 X' + B_12 (\dot{X} Y' - \dot{Y} X') \right) \right] = 0, \tag{15}
\]

for \(X\) and \(Y\) respectively. In this case the string motion described by the following anstaz,

\[
X(r, t) = v_x t + x \sin \omega t, \quad Y(r, t) = v_y t + y \cos \omega t, \quad \tag{16}
\]

Therefore the equation (7) extends to the following relation,

\[
-g \approx 1 - \frac{v_x^2 + v_y^2}{h} + \frac{h}{H} (x'' \sin^2 \omega t + y'' \cos^2 \omega t) + \frac{1}{H} (x' v_y \sin \omega t - y' v_x \cos \omega t)^2 - [B_0 x' \sin \omega t + B_12 (y' v_x \cos \omega t - x' v_y \sin \omega t)]^2 \tag{17}
\]

Also the energy and momentum currents in the small velocities approximation obtained as the following,

\[
\Pi_X^0 = \frac{v_x}{h} + \frac{1}{h} (v_x y' \cos \omega t - v_y x' y' \sin \omega t \cos \omega t) + B_12 y' \cos \omega t (B_0 x' \sin \omega t + B_12 (v_x y' \cos \omega t - v_y x' \sin \omega t))
\]

\[
\Pi_Y^0 = \frac{v_y}{h} + \frac{1}{h} (v_x x' \sin \omega t - v_y x' y' \sin \omega t \cos \omega t) + B_12 x' \sin \omega t (B_0 x' \sin \omega t + B_12 (v_y x' \cos \omega t - v_x x' \sin \omega t))
\]

\[
\Pi_X^1 = -\frac{h}{H} x' \sin \omega t + \frac{1}{h} (x'' v_y \sin \omega t - v_x x' y' \cos \omega t) + (B_0 - B_12 v_y) (B_0 x' \sin \omega t + B_12 (v_x y' \cos \omega t - v_y x' \sin \omega t))
\]

\[
\Pi_Y^1 = -\frac{h}{H} y' \cos \omega t + \frac{1}{h} (y'' v_x \cos \omega t - v_x x' y' \sin \omega t) + B_12 v_x (B_0 x' \sin \omega t + B_12 (v_x y' \cos \omega t - v_y x' \sin \omega t)), \tag{18}
\]

up to the factor \(\frac{T_0}{\sqrt{-g}}\). Now one may consider two special cases as the following. In the simplest case we can choose \(v_x = \nu\) and \(v_y = 0\) which is corresponding to the existence of only electric field, i.e. \(B_12 = 0\). Then one may choose the case of only magnetic field which implies \(B_0 = 0\).

In this paper we consider the case of \(B_12 = 0\), then for small velocities one can obtain,

\[
x' \sin \omega t = \frac{(h - v^2)}{\sqrt{h (h - H B_{01}^2)}} \frac{H \Pi_X^1}{\sqrt{(h - v^2) [(h - H B_{01}^2) T_0^2 - H \Pi_X^1]^2 - (h - H B_{01}^2) H \Pi_Y^2}},
\]

\[
y' \cos \omega t = \frac{\sqrt{h - H B_{01}^2}}{h} \frac{H \Pi_Y^1}{\sqrt{(h - v^2) [(h - H B_{01}^2) T_0^2 - H \Pi_X^1]^2 - (h - H B_{01}^2) H \Pi_Y^2}}, \tag{19}
\]
which are extension of the equation (9) to the case of the existing a constant electric field in the background. In that case, similar to the Ref. [31], one can find the following expression for the momentum densities,

\[
\Pi^1_x = \frac{(h(r_{\text{min}}) - H(r_{\text{min}})B_{01}^2)\tan^2 \omega t}{\sqrt{H(r_{\text{min}})(h(r_{\text{min}}) - v^2) + \frac{h(r_{\text{min}}) - H(r_{\text{min}})B_{01}^2}{H(r_{\text{min}})}\tan^4 \omega t}},
\]

\[
\Pi^1_y = \frac{h(r_{\text{min}}) - v^2}{\sqrt{H(r_{\text{min}}) + \frac{h(r_{\text{min}}) - H(r_{\text{min}})B_{01}^2}{H(r_{\text{min}})(h(r_{\text{min}}) - v^2)}\tan^4 \omega t}}.
\]  

(20)

Now by using reality condition one can obtain,

\[
r_{\text{min}} = \left[r_h^4 + \frac{d}{2T_0^2(1 - v^2)} \left(1 - \sqrt{1 - \frac{4T_0^2(1 - v^2)R^4c}{d^2}}\right)\right]^\frac{1}{4},
\]

(21)

where,

\[
d \equiv R^4(1 - v^2)(\Pi^1_X^2 + T_0^2B_{01}^2) + R^4\Pi^1_Y^2 + T_0^2v^2r_h^4,
\]

\[
c \equiv v^2r_h^4(\Pi^1_X^2 + T_0^2B_{01}^2) + R^4B_{01}^2\Pi^1_Y^2.
\]

(22)

We can see that the effect of the constant electric field is increasing of the radius \(r_{\text{min}}\). Again in the case of the \(B_{01} = 0\) we recover the relation (11) and in the case of \(B_{01} = 0\) and \(v = 0\) we obtain \(r_{\text{min}} = r_h\) which is expected. In the next section we will use relations (11) and (21) to obtain the jet-quenching parameter.

### 4 Calculation of the Jet-Quenching Parameter

In ultra-relativistic heavy-ion collisions at LHC or RHIC, interactions between the high-momentum Parton and the quark-gluon plasma are expected to lead to jet energy loss, which is called jet quenching. It is known that the non-perturbative definition of the jet-quenching parameter may be obtained in terms of light-like Wilson loop [33]. In that case we use results of [33-38] to obtain the jet-quenching parameter in our system. At the first we calculate the jet-quenching parameter without any external field. Then we add a constant electric field to the system and obtain the jet-quenching parameter. In order to find the jet-quenching parameter, first by introducing the new coordinates \(x^\pm = \frac{1}{\sqrt{2}}(t \pm x_1)\), we rewrite the line element (1) in the light-cone coordinates,

\[
ds^2 = \frac{1 - h}{2\sqrt{H}}[(dx^+)^2 + (dx^-)^2] - \frac{1 + h}{\sqrt{H}}dx^+dx^- + \frac{1}{\sqrt{H}}[(dx^2)^2 + (dx^3)^2] + \frac{\sqrt{H}}{h}dr^2.
\]

(23)

We choose word-sheet coordinates as \(r(x^-, \bar{x})\) and use static gauge \(x^- = \tau\) with length \(L^-\) and \(\bar{x} = \sigma\) with length \(L\). Because of condition \(L^- \gg L\) we can assume that the coordinates
$r$ is independent of $\tau$ and one can consider $r(\sigma)$ as world-sheet coordinates, so there is boundary condition as $r(\pm \frac{1}{2}) = \infty$. Other coordinates assuming to be constant. Now, definition of the jet-quenching parameter is given by [33],

$$\hat{q} \equiv 8\sqrt{2} \frac{S - S_0}{L - L^2},$$

(24)

where $S$ obtained from action (2) by using above definitions, so one can obtain,

$$S = \sqrt{2}T_0L^- \int_{r_{\text{min}}}^{\infty} \frac{dr}{r'} \sqrt{(1 - h)(\frac{1}{H} + \frac{r'^2}{h})},$$

(25)

where $r_{\text{min}}$ is given by the equation (11). In order to remove $r'$ in the above expression we use energy conservation law $\mathcal{H} = \mathcal{L} - \frac{\partial}{\partial \sigma} r' = \text{Const.} \equiv E$, which yield us to the following relation,

$$r'^2 = \frac{h}{H}(\frac{r_h^4}{2E^2R^4} - 1),$$

(26)

which implies that $E^2 \leq \frac{r_h^4}{2R^4}$. We are interested in low energy limit where $E \ll 1$. This limit is corresponding to $L \to 0$ limit which is agree with $L \ll L^-$. The action $S_0$ in the equation (24) interpreted as self-energy of the quark and antiquark which is given by [38],

$$S_0 = 2T_0L^- \int_{r_{\text{min}}}^{\infty} dr \sqrt{-g_{rr}},$$

(27)

where $r_{\text{min}}$ is given by the relation (11). Inserting (26) into the action (25) and expanding for infinitesimal $E$ yields to the following relation,

$$S - S_0 = \sqrt{2}T_0L^- E^2R^4 \int_{r_{\text{min}}}^{\infty} \frac{dr}{r'^2 - r_h^4},$$

(28)

On the other hand one can integrate (26) and obtain the following relation between $L$ and constant $E$,

$$\frac{L}{2} = \sqrt{2ER^4} \int_{r_{\text{min}}}^{\infty} \frac{dr}{r'^2 - r_h^4} = (\sqrt{2ER^4} \frac{r_{\text{min}}r_h^2}{r_h^4})_{2F1}[\frac{1}{4}, 1; 2; \frac{r_h^4}{r_{\text{min}}^4}].$$

(29)

Inserting equations (28) and (29) in the equation (24) leads us to the expression for the jet-quenching parameter,

$$\hat{q} = \frac{2T_0}{R^4} \frac{r_{\text{min}}r_h^2}{2F1[\frac{1}{4}, 1; 2; \frac{r_h^4}{r_{\text{min}}^4}]}.$$ 

(30)

It is easy to compare our result with the previous work. In the case of $\nu = \omega = 0$ one can obtain $r_{\text{min}} = r_h$ and therefore hypergeometric function reduced to the gamma function as $2F1[\frac{1}{4}, 1; 2; \frac{r_h^4}{r_{\text{min}}^4}]_{r_{\text{min}}=r_h} = \sqrt{\pi}\Gamma(\frac{5}{4})/\Gamma(\frac{3}{4})$, then by using relations $r_h = \pi R^2T$, $R^2 = \alpha'/\sqrt{\lambda}$ and $T_0 = \frac{1}{2\pi\alpha'}$ we recover the famous relation of the jet-quenching parameter in $\mathcal{N}=4$ SYM
theory in the large $N_c$ and large-$\lambda$ limits, $\hat{q} = \frac{a^2}{a} \sqrt{AT^3}$, where $a \approx 1.311$. However in the case of rotational motion the value of the jet-quenching parameter increases. Now, we consider a two form $B_0 dt \wedge dx$. In that case one can obtain,

$$S - S_0 = \sqrt{2T_0}L^- E^2 R^4 I,$$  \hspace{1cm} (31)

where

$$I = \int_{r_{\min}}^{\infty} \frac{dr}{\sqrt{(r^4 - r_h^4)(r^4 - B_0^2 r^2)}};$$  \hspace{1cm} (32)

and the radius $r_{\min}$ is given by the equation (21). Also one can obtain,

$$r'^2 = \frac{H}{H} \left( \frac{r^4 - B_0^2 r^2}{2E^2 R^4} - 1 \right).$$  \hspace{1cm} (33)

Therefore the jet-quenching parameter for the rotating heavy meson through the $\mathcal{N}=4$ SYM thermal plasma in a constant electric field obtained as the following relation,

$$\hat{q} = \frac{2T_0}{R^4} I^{-1}. $$  \hspace{1cm} (34)

In order to obtain the explicit expression of the jet-quenching parameter, including the electric field, we assume that the electric field is infinitesimal parameter. For the infinitesimal electric field one can obtain,

$$\hat{q} = \frac{\pi T^2}{\alpha'} 2F_1\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}; \frac{r_{\min}^4}{r_h^4}\right] \left( 1 - \frac{r_{\min}^4}{2\pi^2 \alpha' \sqrt{AT^3}} 2F_1\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}; \frac{r_{\min}^4}{r_h^4}\right] \int_{r_{\min}}^{\infty} \frac{r^2 dr}{\sqrt{r^4 - r_h^4}} \right).$$  \hspace{1cm} (35)

It means that the effect of the constant electric field is decreasing of the jet-quenching parameter. This is agree with the fact that electric field decreases the drag force [19]. Therefore we calculated the jet-quenching parameter in rotating meson system under effect of the constant electric field. It is easy to check that the above results are coincide with the previous work, without electric field, if we cancel the electric field, ie. $B_{01} = 0$.

5 Conclusion

In this paper we generalized the problem of the rotation meson in the $\mathcal{N}=4$ SYM thermal plasma to the case of the existing constant electromagnetic field. We considered the constant electromagnetic field in the background and obtained the lagrangian density for the small velocities. Then we calculated the effect of the constant electric field on the motion of the rotating meson and obtained the momentum densities. We have shown that in the presence of a constant electric field the distance of the turning point from the D-brane is smaller than the case of without the electric field. Then, without presence of any external field we obtained the jet-quenching parameter for the rotating mesons in terms of the hypergeometric function and have shown that our results for the $\omega = 0$ are coincide with the
case of non-rotating meson. We found that the rotating mesons have larger jet-quenching parameter than non-rotating mesons. Finally we calculated the jet-quenching parameter under effect of a constant electric field. We found that the electric field decreases the value of the jet-quenching parameter.

Here, there are some interesting problem for future works. For example one can obtain jet quenching parameter [33-38] for rotating $q\bar{q}$ pair or shear viscosity [40] in other backgrounds such as $\mathcal{N}=2$ supergravity [19]. Also it is interesting to consider the effect of higher derivative terms as in the previous cases [41-49]. As a recent work [50] one may consider more quarks, such as four quarks in the baryon through $\mathcal{N}=4$ SYM thermal plasma. At the end, it may be interesting to consider fluctuations of the quark-antiquark pair and obtain the exact solution of such a system.

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