Controlling of nonlinear relaxation of quantized magnons in nano-devices

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Relaxation of linear magnetization dynamics is well described by the viscous Gilbert damping processes. However, for strong excitations, nonlinear damping processes such as the decay via magnon-magnon interactions emerge and trigger additional relaxation channels. Here, we use space- and time-resolved micro-focused Brillouin light scattering spectroscopy and micromagnetic simulations to investigate the nonlinear relaxation of strongly driven spin-waves in yttrium iron garnet nano-conduits. We show that the nonlinear magnon relaxation in this highly quantized system possesses intermodal features, i.e. magnons scatter to higher-order quantized modes through a cascade of scattering events. We demonstrate that this type of scattering saturates on a time scale of approximately 4 ns, and mediates the decay rate of the resonantly driven magnons. We further show how to control such intermodal dissipation processes by quantization of the magnon band in single-mode devices, where this phenomenon approaches to its fundamental limit.

Relaxation of magnons, the quanta of spin waves (SWs) due to magnetic damping, is a complicated process and involves different (non)linear contributions. Relaxation mechanisms which can be described by the phenomenological Gilbert damping drive the magnetization towards its equilibrium state by e.g. dissipating the energy to the lattice. It is one of the key elements of performance in many practical devices and fundamental phenomena [1–10].

Dissipation of the energy can be more intricate for strongly driven excitations, where nonlinear relaxation mechanisms via magnon-magnon interactions open up additional dissipation channels [11–17]. Unlike the Gilbert damping, these types of intrinsic dissipation processes redistribute the magnon energy within the magnon spectrum [18–27].

The classical works of Suhl predicted that large amplitude uniform magnetization oscillations leads to the onset of instability processes, allowing the nonlinear relaxation of strongly driven magnons by a decay into secondary magnon modes [25]. In particular, the second-order Suhl instability process can be: (i) a disadvantage since it come along with detrimental influence on the magnon transport and decay characteristics, potentially dominating the competing linear damping [17,22,28], or, (ii) an advantage by providing additional degrees of freedom of magnon transport for device architectures and quantum computing concepts [23,29,30]. Therefore, a thorough understanding of the prerequisites and dynamics of these phenomena is in general very important from different points of view.

Recent development of ultra-low damping nanoscale systems based on YIG, the most promising hosts for SWs, provides access to quasi-1D systems with highly quantized magnon spectra [31,32]. By imposing limitations on the available relaxation channels, the strong quantization of the magnon
band in these systems can lead to different nonlinear dynamics compared to continuous films and quasi-2D systems with dense spectra [33-34]. Furthermore, recent experimental and theoretical studies of SW dynamics and magnon condensates in nanoscopic systems [32, 35,36] enforce us to better understand nonlinear SW dynamics and magnon thermalization processes in nano-scaled 1D systems. However, little investigations have been carried out in this direction yet.

Here, we use space- and time-resolved micro-focused Brillouin light scattering (µBLS) to uncover the mechanism of nonlinear relaxation of strongly driven magnons via the second-order Suhl instability in YIG nano-conduits. We demonstrate how magnons nonlinearly relax to other quantized modes via four-magnon scattering processes, and such nonlinear processes can be controlled using quantization of the magnon band.

To demonstrate the effect of quantization on the nonlinear dynamics, we use two exemplary magnonic nano-conduits structured from a Liquid Phase Epitaxial (LPE) YIG film grown on top of a Gadolinium Gallium Garnet (GGG) substrate [37]. The multi-mode nano-conduit with a lateral width of \( w = 400 \) nm (Fig. 1a) and a thickness of \( d = 85 \) nm was fabricated using a hard mask and ion beam milling process [31]. A comparative single-mode conduit with a smaller width of \( w = 100 \) nm and \( d = 44 \) nm was fabricated using a similar method (Fig. 1b). SWs in both devices are excited by a microwave antenna which is placed on top of the nano-conduits by electron beam lithography and a lift-off process [31]. Applying a microwave rf current to the antenna generates a dynamic Oersted field which in return excites SWs resonantly, see supplemental materials SM [40]. The detection of the generated SWs has been carried out using space- and time-resolved µBLS [38]. An incident laser light with an effective spot size of 300 nm (focused by a \( \times 100 \) microscope objective with a numerical aperture NA=0.85) is used to probe the SWs through the GGG substrate under the antenna. The inelastically scattered light was analyzed using a tandem Fabry-Perot interferometer to obtain the frequency and intensity of the magnons.

A static external field (\( \mu_0 H_e = 60 \) mT) saturates the nano-conduits along their length. Here, the wave vector of the SWs is parallel to the magnetization vector, \( \mathbf{k} \parallel \mathbf{M} \), where waveguide (WG) modes appears [32]. The width of the multi-mode waveguide is large enough to ensure dipolar pinning of the spins at the edges, while spins at the edges of the single-mode conduit are fully unpinned [32]. Moreover, due to the interplay between the contributions of the dipolar and exchange energy to the SW dispersion, a strong frequency shift among different modes takes place and the different WG modes are well quantized on the frequency axis. The dispersion relation of the fundamental mode and the first two WG modes are shown in Fig 1c-d, in which the dashed lines are analytical results based on method discusses in Ref [32], and the color plot is obtained by micromagnetic simulations using the MuMax 3.0 pack-
age [39, 40]. The fundamental mode and higher order WG modes are labeled as \( n = 0 \) and \( n = 1, 2 \) respectively. Please note that the spectrum is much more dilute in the 100 nm wide conduit due to the higher contribution of the exchange energy to the magnon band, which leads to a strong quantization and the absence of degenerate states among modes (single-mode system).

We first set the rf frequency to \( f = 3.85 \) GHz where dipolar SWs having small wave vector of \( k_x = 1.5 \) rad/\( \mu \)m are excited in the multi-mode device [40]. To characterize the linear response of the conduit, we set the rf power to \( P = 10 \) dBm and measure the intensity of the generated magnons as displayed in Fig 1e (black circles). Up to \( P = 18 \) dBm, only the resonantly driven SW mode is observed (red and green triangles). A further increase in the rf power up to \( P = 20 \) dBm (blue curve) leads to the appearance of two additional peaks in the SW frequency spectrum named as \( f^- \) and \( f^+ \) in Fig. 1e. Thus, this power is high enough to exceed the instability threshold and leads to a pronounced nonlinear excitation of the unstable modes. This evidences a four-magnon scattering process from the driven mode to the unstable modes, whose energy and momentum conservation laws reads [18,20,22,28],

\[
f^1 + f^2 = f^3 + f^4, \quad k^1 + k^2 = k^3 + k^4
\]  

(1)

where two magnons with the frequencies \( f^1 \) & \( f^2 \) and momenta \( k^1 \) & \( k^2 \) scatter to two magnons with the frequencies \( f^3 \) & \( f^4 \) and momenta \( k^3 \) & \( k^4 \). Note that the lateral component of the \( k \) vector is symmetric, and the out of plane component is zero due to the small thickness. In our experiments, two magnons with a frequency of \( f = 3.85 \) GHz scatter to two magnons with the frequencies \( f^- = 3.25 \) GHz and \( f^+ = 4.405 \) GHz. We note that this process is not a special peculiarity of the chosen spectral position, see SM [40].

We now investigate the same nonlinear process in the comparative single-mode waveguide. We set the \( f = 3.71 \) GHz and measure the intensity of the driven mode as shown in Fig. 1f. Clearly, even in the presence of high powers like \( P = 20 \) dBm, side peaks cannot be observed, evidencing the absence of a similar nonlinear dissipation processes. Here, only the \( \mu \)BLS intensity drops at high powers which is caused by the nonlinear frequency shift of the dispersion relation and possible impacts of the higher temperature [31, 41]. In principle, the absence of side peaks demonstrates that such scattering processes can be efficiently suppressed in narrower conduits where the magnon band structure is highly quantized and therefore, the fundamental conservation laws required for the scattering processes cannot be fulfilled, leading to the absence of dissipation channels.

FIG. 2. Spin-wave amplitude in the multi-mode conduit as a function of microwave excitation power when \( f = 3.85 \) GHz. The unstable modes created by the second order Suhl instability are denoted as \( f^- \) and \( f^+ \). The back and gray arrows indicate the onset of instability and the rise of the unstable modes, respectively.

Let us investigate the observed nonlinear dynamics in the multi-mode conduit. In principle, a nonlinear scattering instability is characterized by a clear threshold of the initial magnon intensity which is required for its onset [18,22,24,42]. Neglecting SW radiation losses, the threshold magnon intensity is defined by the effective relaxation frequency of the unstable mode divided by the four-magnon coupling strength [16,22]. To investigate the threshold behavior in the multi-mode conduit in which the scattering is observed, we sweep the rf power for a fixed frequency \( f = 3.85 \) GHz as shown in Fig. 2. Once the instability threshold is reached at \( P = 18 \) dBm (indicated by the black arrow), the growth rate of the magnon intensity via microwave power drops. Increasing the power to \( P = 19 \) dBm let the intensity of the unstable modes labeled as \( f^- \) and \( f^+ \) to increase abruptly with power (indicated by the gray arrow).
From this power ($P = 19$ dBm), the intensity growth rate of the directly excited mode with respect to the power decreases, evidencing that the energy transfers to the unstable modes. A closer look on Fig. 2 shows that the instability threshold opens the question what happens when the instability threshold is approached at $P = 18$ dBm and the μBLS intensity of the initially excited mode drops, while the amplitudes of the unstable modes are still at the thermal level, implying the absence of magnon scattering to these modes.

We perform micromagnetic simulations at room temperature and using realistic distribution of the dynamical Oersted field of the antenna to uncover the wave vector of the scattered magnons and address the discussed question. Figure 3a shows the frequency spectrum of the simulated multi-mode conduit ($f = 3.85$ GHz) in which different amplitude of the rf currents are used to drive the system. For a small rf current equal to $i_{rf} = 4$ mA, only the resonantly excited SWs can be observed in the frequency spectrum (black curve). Increasing the rf current to a higher value of $i_{rf} = 8$ mA increases the amplitude and the linewidth of the resonant SWs [28]. A further increase of the rf current to $i_{rf} = 13$ mA leads to the onset of the sideband peaks in the frequency spectrum (blue curves), similar to the experiments. Indeed, increasing the rf current leads to higher dynamical Oersted fields and a higher intensity of the resonantly injected magnons. Once above a critical value, the second order instability sets in and the unstable modes populates. The frequency of the unstable modes are $f^+ = 3.46$ GHz and $f^- = 4.24$ GHz declaring a qualitative agreement with the experiments. The small difference of the obtained results to the experiments is possibly caused by the laser heating which can shift the local dispersion and, the impact of the structuring process on the lateral edges of the conduit that can change its effective width [31].

To elaborate the momentum conservation laws according to the Eq. 1, in Fig. 3b-d we plot the magnon band structures of the system corresponding to the presented frequency spectra in Fig 3a. Figure 3b shows the linear excitation of the driven mode at $k_x = 1.5$ rad/μm. Increasing the rf current to $i_{rf} = 8$ mA as shown in Fig. 3c permits the onset of the first-level four magnon scattering process in which the frequency of the magnons is conserved. Such a process cannot be observed at the measured frequency spectrum of the conduit.

![Figure 3](image_url)

**FIG. 3.** Results of the micromagnetic simulations in the multi-mode conduit. (a) spin-wave frequency spectra when the microwave current varies. (b-d) Magnon band structures (linear scale) of the driven system correspond to the black, red and blue curves in (a), respectively. The scaling of b-d is independent from each other.

Here, two incoming magnons from the resonantly driven mode with opposite momenta scatter to two outgoing magnons at the same frequency, but with different momenta. The scattered magnons populate the fundamental mode ($n = 0$) at a higher wave number of $k_x = 30$ rad/μm, and two spectral position at the first WG mode ($n = 1$). These frequency-conserving scattering processes which are similar to plane films [25] are indicated by the pink arrows in Fig. 3c, and can also be observed in the single-mode conduit, see SM [40]. A further increase of the rf current to $i_{rf} = 13$ mA (Fig. 3d), leads to the onset of the second-level four magnon scattering process. Under this condition, the frequency-conserving four magnon scattering process increases the density of the scattered magnons at the first WG mode ($n = 1$) far above the threshold for the next level of four-magnon scattering process. This allows that two magnons with the frequency of $f = 3.85$ GHz and momentum of $k_x = 10.7$ rad/μm at the first WG mode ($n = 1$), scatter to two outgoing magnons with the frequen-
cies of $f^- = 3.46 \text{ GHz}$ and $f^+ = 4.24 \text{ GHz}$ at the fundamental mode ($n = 0$) and the second WG mode ($n = 2$), respectively. This type of frequency-nonconserving scattering is represented by the red arrows. The scattered magnons feature $k_x^+ = 14.3 \text{ rad/\mu m}$ and $k_x^- = 7.1 \text{ rad/\mu m}$, assuring momentum conservation laws given by $2k_x = k_x^+ + k_x^-$. The simulations well explain the observed peculiarity in the threshold curve of the experiments as were discussed in the context of Fig. 2. Indeed, the magnons scattered to higher wave numbers via the frequency-conserving scattering process (Fig. 3c) cannot be detected experimentally due to the maximum detectable momentum using μBLS spectroscopy, which is approximately $k_x \sim 21 \text{ rad/\mu m}$ in our experiments [38]. Moreover, the frequency-conserving scattering has a lower threshold compared to the frequency-nonconserving scattering processes. Therefore, to observe the nonlinear scattering to the unstable modes at different frequencies, a slightly higher power than the frequency-conserving instability threshold is needed.

To further prove the impact of the nonlinear relaxation on the total relaxation of the system [16], we perform time-resolved μBLS measurements in the conduit. The measured intensity of the driven and unstable modes at the beginning and the end of a 1µs long microwave rf pulse ($f = 3.85 \text{ GHz}$ and $P = 24 \text{ dBm}$) at the measurement position are shown in Fig. 4a-b. Figure 4a illustrates that the resonantly driven SW mode (blue curve) must reach approximately ~75% of its maximum intensity (which happens after $t \sim 4$ ns) in order to let the second-level four-magnon scattering take place, evidenced by the rise of the unstable modes (yellow and red curves). This is indicated by the black arrow in Fig. 4a. Note that the growth rate of the driven mode drops by the rise of the unstable modes until both saturate, evidencing the conservation of the energy in the nonlinear redistribution process.

The decay of the magnons at the end of pulse is presented in Fig. 4b. In particular, the decay of the unstable modes only begins once the intensity of the driven SW mode is approximately ~75% of its maximum intensity (indicated by the black arrow).

More interestingly, the decay of the magnons at the resonantly driven frequency to the thermal level includes two steps manifesting the high nonlinearity of the dynamics. First, it decays with an exponential decay time of $t_{1,d} = 19$ ns, which is accompanied by the decay of the unstable modes at $f^+$ and $f^-$. Afterwards, it decays with a longer exponential decay time of $t_{2,d} = 24$ ns suggesting a transition from a nonlinear relaxation to a linear relaxation with a lower decay rate. In other words, the first decay includes an energy flow to the unstable modes which acts as an additional dissipation channel for the driven magnons. After the unstable modes decayed to the thermal level, this additional dissipation channel is switched off, which leads to a slower decay time of the driven SWs.

In summary, we explored the nonlinear relaxation of strongly driven magnons in YIG nanoscopic conduits. It was shown that the intermodal dissipation process is strongly suppressed in systems with a strongly quantized magnon band (single-mode systems), suggesting the fundamental limitation of this process in nano-devices. This can open a new avenue for coherent nonlinear nano-magnonics. Our study can be used for several device architectures, namely, frequency mixers [43], squeezed...
states [44], signal and data processing units [29, 45-48], and quantum computing concepts [23], and further open doors to engineered dissipation of magnons in nano-devices.

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