Constraining Modified Chaplygin Gas Parameters

D. Panigrahi¹, ²*, B. C. Paul³**, and S. Chatterjee², ⁴***

¹Sree Chaitanya College, Habra 743268, India
²Relativity and Cosmology Research Centre, Jadavpur University, Kolkata—700032, India
³Department of Physics, University of North Bengal, Dist.—Darjeeling 734013, India
⁴IGNOU Convergence Centre, New Alipore College, Kolkata—700053, India

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Abstract—We study the evolution of a FRW model fueled by a modified Chaplygin gas (MCG) with the equation of state \( p = A \rho - B/\rho^\alpha \). An attempt is made to constrain the free parameters of MCG model through the well-known contour plot technique using observational data. The permissible range of values of two free parameters is determined to study the viability of cosmological models against the observational results. Aside from allowing the desirable feature of a flip of the sign of the deceleration parameter, we also find that a transition from deceleration to acceleration occurs at relatively low value of the redshift in accordance with the observational prediction that the acceleration is a recent phenomenon. Stability of the model against density perturbation is studied in some detail, and it is found that the effective acoustic speed may become imaginary depending on the initial data, signalling that perturbations associated with instability set in, resulting in structure formation. As one considers more negative values of \( A \), the flip in sign is delayed making the density parameter change fast. Again, it is found from the contour plot that compatibility with observational data is admitted with a value of \( A \) which is very near zero or is a small negative number.

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1. INTRODUCTION

Following the high-redshift supernovae data in the last decade [1], we know that when interpreted in the framework of the standard FRW type of universe (homogeneous and isotropic), we are left with the only alternative that the universe is now passing through an accelerated phase of expansion with baryonic matter contributing only four per cent of the total energy budget. Observational results coming from CMBR studies [2] also point to this conclusion engaging a large community of cosmologists ([3] and references therein) to embark on a quest to explain the cause of the acceleration. The teething problem now confronting researchers is the identification of the mechanism that triggered the late inflation. Researchers are mainly divided into two groups, either suggesting a modification of the original general relativity theory or invoking a mysterious fluid in the form of an evolving cosmological constant or a quintessential type of scalar field. But as debated at length in the literature, both alternatives face serious problems mainly at the theoretical and conceptual level. Typically, the late acceleration has been attributed to a mysterious dark energy (DE) which contributes about three fourth of the cosmic substratum. Another unknown matter component of the universe is dark matter (DM), the missing mass necessary to hold together the galaxy clusters and also needed to explain the current large-scale structure of the universe.

In the literature, a good number of DE models are proposed, but little is precisely known about it. Nowadays, the DE problem remains one of the major unsolved problems of theoretical physics [4]. On the way of searching for possible solutions of this problem various models are explored during the last few decades, referring to new exotic forms of matter, e.g., quintessence [5], phantom [6], holographic models [7], string theory landscape [8], Born-Infeld quantum condensate [9], the modified gravity approaches [10], inhomogeneous spacetime [11], etc (readers interested in more detail for a comprehensive overview of existing theoretical models may refer to [12]).

Even though the cosmological constant allows for the cosmic acceleration at late times, the observational bounds on \( \Lambda \) are incompatible with theoretical predictions of a gravitational vacuum state. Grosso modo, the \( \Lambda \) model does well in fitting most of the
observational data, but the density parameter corresponding to $\Lambda$ and matter are of the same order of magnitude, surprisingly close to each other, even though $\Lambda$ is constant during the entire evolutionary history of the universe. These two thorny shortcomings, namely, the fine tuning and the coincidence problems, disturb the otherwise appealing picture of a cosmological constant and dramatically plague the so-called $\Lambda$CDM model. To circumvent this difficulty and host others people invoke a variable $\Lambda$ model, but that too is beset with serious field theoretic problems [13]. Another possibility to obtain an accelerated expansion is provided by theories with large extra dimensions known as braneworlds [14], but one has to assume the existence of extra spatial dimensions in these models.

Among different theories recently put forward in the literature, the single component fluid known as the Chaplygin gas (CG) with an equation of state (EoS) \( p = -B/\rho \) [15], where \( \rho \) and \( p \) are the energy density and pressure, respectively, and \( B \) is a constant, has attracted large interest in cosmology. Although the model is very successful in explaining the SNe Ia data, it shows that the CG does not pass the tests connected with structure formation and observed strong oscillations of matter power spectrum [16]. One can circumvent this situation in the generalized Chaplygin gas (GCG) proposed with \( p = -B/\rho^\alpha \) with \( \alpha \) constrained in the range \( 0 < \alpha < 1 \). Since the inferences from the $\Lambda$CDM and GCG models are almost similar, at least from the observational fallouts, this model is further modified either considering \( B \) as a function of the redshift \( z \) as \( B = B(z) \) [17] or assuming an EoS as

\[
p = A\rho - B/\rho^\alpha
\]

(modified Chaplygin gas, MCG) [18], where \( A \) is a new constant parameter of the theory. Thus MCG may be looked upon as a mixture of a barotropic perfect fluid and GCG. Although it suffers from serious shortcomings, e.g., it violates the time-honored principle of energy conditions, its theoretical conclusions are found to be in broad agreement with the observational results coming from gravitational lensing or recent CMBR and SNe data in varied cosmic probes. This is generally achieved through a careful maneuvering of the value of the newly introduced arbitrary constant. The viability of the scenario has been tested by a number of cosmological tests, including SNe Ia data [19], lensing statistics [20], age-redshift tests [21], CMB measurements [22], measurements of X-ray luminosity of galaxy clusters [23], statefinder parameters [24]. Another interesting feature of the MCG is that it can show the radiation era in the early universe. At late times MCG behaves as a cosmological constant and can be fitted to $\Lambda$CDM.

Furthermore, DM and DE are considered as different manifestations of the same component and describe the dark sector as some kind of fluid whose physical properties depend on the scale: it behaves as DM at high densities and transforms into DE at lower ones. Most of these United Dark Matter models invoke the generalized Chaplygin gas (GCG), a perfect fluid characterized by a negative pressure which is inversely proportional to the energy density.

Using the MCG in a FRW universe, a number of cosmological models have been discussed. In most of the models the values of the free parameters are picked up by hand in order to suit the best fit values with the observational results without much physical consideration. It is important to study cosmological models in observational contexts which naturally impose constraints on the values of the free parameters. In view of the above considerations, in this work we determine the constraints on the free parameters of the model by drawing the usual contour plot diagrams. However, two works in this field are worth mentioning. In an earlier one, Jianbo Lu et al. [25] have apparently come to the conclusion via usual contour plot diagram that the recently observed data give credence to both the MCG and other fashionable models, so that it is very difficult to choose one over the other. This is in line with earlier results in this field [26]. Secondly, in a very recent communication Wang et al. [27] have also studied in depth the constraints on \( \alpha \) in the GCG formalism in two distinct cases: one pertaining to a barotropic equation of state and the other to DM with some specific properties. In what follows we discuss and compare, in brieﬂ, our own findings in this field with the work of Wang et al. at the end of our paper in Section 5. Our work is primarily focused on two domains. First, to study the dynamics of a FRW cosmological scenario with MCG as matter ﬁeld as also to obtain constraints on the three free parameters with the help of contour diagram, to achieve the best ﬁt with current observational findings. The paper is organized as follows: in Section 2 we build up the evolution equations, while the nature of evolution and analysis of instability of our model are dealt with in some detail in Section 3. In Section 4 we discuss the observational data, namely, Stern data, measurements of baryon acoustic oscillations (BAO) peak parameters and CMB shift data to draw a contour diagram for the permissible range of values of the pairs of parameters \((\alpha, A), (\omega_{\text{cdm}}, \dot{A}) \) and \((\Omega_{\text{cdm}}, A)\). Finally, in Section 5 we give a brief discussion.

2. FIELD EQUATIONS

We consider a flat spherically symmetric homogeneous spacetime with the line element

\[
ds^2 = dt^2 - a^2(t)(dr^2 + r^2d\Omega^2),
\]
where the scale factor \(a(t)\) depends on time only. Taking a comoving coordinate system such that \(u^0 = 1, u^i = 0\) \((i = 1, 2, 3)\) and \(g^{\mu\nu}u_\mu u_\nu = 1\), where \(u_\mu\) is the 4-velocity, the energy-momentum tensor of a perfect fluid distribution in these coordinates is given by

\[
T^\mu_\nu = (\rho + p)\delta^\mu_0 \delta^\nu_0 - p\delta^\mu_\nu,
\]

where \(\rho(t)\) is the matter density and \(p(t)\) the isotropic pressure. The Einstein equations for the above metric are

\[
3H^2 = \rho,
\]

\[
2H + 3H^2 = -p,
\]

where \(H = \dot{a}/a\) is the Hubble parameter and we use the units where \(8\pi G = 1\). The conservation equations for matter fields are

\[
\nabla_\nu T^{\mu\nu} = 0,
\]

which in turn yields

\[
\dot{\rho}_{\text{total}} + 3H(\rho_{\text{total}} + p_{\text{total}}) = 0.
\]

We now consider a mixture of fluids where baryons are one of the components, and conservation of baryons leads to the continuity equation that can be separated into two equations:

\[
\dot{\rho}_b + 3H(\rho_b + p_b) = 0,
\]

\[
\dot{\rho} + 3H(\rho + p) = 0,
\]

where \(\rho_{\text{total}} = \rho + \rho_b\) and the subscript \(b\) denotes the baryons. The component of other part is a mixture of dark matter and dark energy. In our model we consider the Modified Chaplygin gas (MCG) as a candidate for DE, and consequently the mathematical expressions are \(p = p_{\text{de}}\) and \(\rho = \rho_{\text{de}} + \rho_{\text{dm}}\), which obeys the EoS given by equation (1). In the above, the subscripts “dm” and “de” denote DM and DE, respectively. Let \(p_{\text{de}} = \omega_{\text{de}}\rho_{\text{de}},\) where the DE EoS parameter is \(\omega_{\text{de}}\). Using Eq. (1) in the conservation equation given (9), one obtains the energy density

\[
\rho = \left[\frac{B}{1 + A} + c(1 + z)^{3(1+\alpha)(1+A)}\right]^{1/(1+\alpha)},
\]

where \(c\) is an integration constant. From Eq. (10) it is evident that \(A \neq -1\) is required for a finite \(\rho\).

3. COSMOLOGICAL DYNAMICS

The DE and DM energy densities are now determined using the set of dynamic equations given in Section 2, such that

\[
\rho_{\text{de}} = \frac{\rho}{\omega_{\text{de}}} \left[\frac{B}{1+A} + c(1 + z)^{3(1+\alpha)(1+A)}\right]^{\alpha/(1+\alpha)},
\]

\[
\rho_{\text{dm}} = \rho - \rho_{\text{de}} \left[\frac{B}{1+A} + c(1 + z)^{3(1+\alpha)(1+A)}\right]^{\alpha/(1+\alpha)} + B.
\]

Now the constants \(B\) and \(c\) can be expressed as

\[
B = A(1 - \Omega_{ob}) - \omega_{\text{ode}}\Omega_{\text{ode}}(1 - \Omega_{ob})^\alpha \rho_{\text{oc}}^{(1+\alpha)},
\]

\[
c = (1 - \Omega_{ob} + \omega_{\text{ode}}\Omega_{\text{ode}})(1 - \Omega_{ob})^\alpha \rho_{\text{oc}}^{(1+\alpha)},
\]

where \(\rho_{\text{oc}}\) represents the critical density and label \(o\) denotes quantities that estimate the present values.

In the case of GCG, \(B\) is a free parameter, but in the MCG case the parameter \(B\) is related to \(A\), the present value of the density parameter, \(\alpha\) and the EoS parameter via Eq. (13), so that if we constrain \(A\) from the observational inputs, another constant, \(B\), is automatically fixed. Further, from Eq. (10) we find that \(B\) is important when \(z\) is very small and the universe is DE dominated. So knowing DE contribution, \(B\) can be fixed, which, however, is not considered in the present paper.

The energy density of the universe can be expressed in terms of the redshift \(z\) as follows:

\[
\rho(z) = \frac{(1 - \Omega_{ob})^\alpha}{(1 + A)^{\alpha/(1+\alpha)}} \left[A(1 - \Omega_{ob}) - \omega_{\text{ode}}\Omega_{\text{ode}}\right].
\]
that the universe's denser for density parameters agrees with that of Fabris et al. [28]. Using the slightly less than zero. The prediction of our model are $\Omega_{\text{odm}} = 0.22; w_{\text{ode}} = -1.05$. Other permitted values of $\Omega_{\text{odm}}$ and $\Omega_{\text{ob}}$ are $(-0.1072, \pm 0.05)$, $(0.002, 0.100)$, $(0.0446, -0.0583)$, respectively. However, from Fig. 3 it is clear that the range of $A$ is further restricted to $(-0.0649$ to $-0.0319)$ at 95.4% confidence level. This implies that the DM parameter dictates that $A$ should be slightly less than zero. The prediction of our model agrees with that of Fabris et al. [28]. Using the WMAP predicted values [2], namely, the current density parameters $\Omega_{\text{ode}} = 0.734 \pm 0.029$, $\Omega_{\text{odm}} = 0.222 \pm 0.026$ and $\Omega_{\text{ob}} = 0.0449 \pm 0.0028$, we explore the density variation in the universe $\rho(z)$ with redshift $z$. It is plotted in Fig. 4 for different $A$ and $\alpha$ values. As expected, the density of the universe increases with increasing $z$ values, and at larger $z$ observations probe the early universe. It is also noted that as one picks up more and more negative values of $A$ with positive $\alpha$, it leads to greater energy density in the early universe for a fixed $z$ value. The permitted range of values of $A$ and $\alpha$ will be discussed in Section 4; the present energy density of the universe is independent of $A$ and $\alpha$ at $z = 0$ and attains a value $\rho_{z=0} = (1 - \Omega_{\text{ob}})$, which also follows from Eq. (15). The study reveals the dynamical property of the universe which shows how fast the universe is accelerating for different values of $A$ and $\alpha$.

Using Eqs. (1), (10), (13) and (14), we obtain the equation of state parameter for the MCG model

$$
\omega(z) = \frac{p}{\rho} = -A(1 - \Omega_{\text{ob}}) - \omega_{\text{ode}} \Omega_{\text{ode}} - A(1 - \Omega_{\text{ob}} + \omega_{\text{ode}} \Omega_{\text{ode}})(1 + z)^{3(1 + \alpha)(1 + A)}
$$

where we use $a(t) = 1/(1 + z)$.

We consider four values of $A$ and $\alpha$ permitted by the contours drawn in Figs. 1 and 2; the density variation with $z$ is plotted, and it is found that the universe is denser for $A = -0.1072$ and $\alpha = 1.376$. Other permitted values of $A$ and $\alpha$ are $(-0.05, 0.50); (0.002, 0.100); (0.0446, -0.0583)$, respectively.

**Fig. 2.** The contour for $\omega_{\text{ode}}$ and $A$.

**Fig. 3.** The contour for $\Omega_{\text{odm}}$ and $A$.

**Fig. 4.** The densities as functions of the redshift, with $\omega_{\text{ode}} = -1.05$, $\Omega_{\text{ode}} = 0.734$, $\Omega_{\text{odm}} = 0.222$ and $\Omega_{\text{ob}} = 0.044$. 

GRAVITATION AND COSMOLOGY Vol. 21 No. 1 2015
independent of $\omega$ that an accelerating universe is permitted. A $\omega$ negative for negative values of $\omega$ ever, we note that $\omega$ in the whole range of values of $\omega$ a barotropic equation of state parameter $\omega(z) = 0$ from the past to the future at $A$, $\alpha = 0$, but was positive in the early epoch for $A$. $\alpha$ and $\alpha$ negative in the whole range of values of $\omega$. It is evident that $\omega$ in Fig. 5. It is evident that $\omega(z)$ is mostly flat at high redshifts ($z > 2.0$) and is very steep at low $z$. We note that $\omega$ is always negative if one extrapolates from the past to the future at $A = -0.1072$ and $\alpha = 1.376$, $A = -0.05$ and $\alpha = 0.50$ and $A = 0.002$ and $\alpha = 0.100$. However, we note that $\omega(z)$ is negative at the present epoch but was positive in the early epoch for $A = 0.0446$ and $\alpha = -0.0583$. Thus a transition to an accelerating universe is permitted.

In Fig. 1 we plot variations of $\omega$ with $\alpha$. It evidently admits $\omega = 0, \alpha = 0.10$. At $z = 0, \omega(z)$ becomes negative for negative values of $\omega_{\text{ode}}$, admitting an accelerating universe. The evolution of $\omega(z)$ is shown in Fig. 5. It is evident that $\omega(z)$ is mostly flat at high redshifts ($z > 2.0$) and is very steep at low $z$. We note that $\omega(z)$ is always negative if one extrapolates from the past to the future at $A = -0.1072$ and $\alpha = 1.376$, $A = -0.05$ and $\alpha = 0.50$ and $A = 0.002$ and $\alpha = 0.100$, but at $A = 0.0446$ and $\alpha = -0.0583$ it is positive in the past (shown with $z \approx 3.80$). In this case we get $\omega(z) = 0$ at $z \approx 3.80$, corresponding to a dust dominated universe in the recent past. Again it is found that at $z = 0, \omega(z)$ attains $-0.80$ which is independent of $A$ and $\alpha$. It also follows from Eq. (16) that $\omega(z)_{z=0} = \omega_{\text{ode}}(1 - \Omega_{\text{ob}})/(1 - \Omega_{\text{ob}})$.

In the context of the theory of linear perturbations the squared speed of sound $v_s^2$ plays a very important role in determining the stability or instability of a given perturbed state. The positive sign indicates a periodic propagating mode of density perturbation and points at stability of a given mode. The negative sign (an imaginary value of the speed) shows an exponentially growing mode of density perturbations indicating an instability of a given mode. Generally the evolution of sound speed in the linear perturbation regime is independent of the dynamics of the background cosmology:

$$v_s^2 = \frac{dp}{d\rho} = \frac{1}{3H dH} \left[ H^2 \left( q - \frac{1}{2} \right) \right].$$

Thus the sign of $q$ determines the sign of $v_s^2$ via a transition epoch from the CDM dominated phase to DE dominated phase. From the above description one can easily find that the growth of perturbation depends on the choice of DE model of background dynamics. For the Chaplygin gas (CG) model we have

$$v_s^2 = -B/\rho^2 = -\omega(z) \quad (19)$$

If we further take the CG as a quintessence DE model with $-1 < \omega(z) < 0$, the squared speed is always positive. So the stability of the CG model against density perturbation at any cosmic scale factor is guaranteed.

In the case of GCG model the situation is almost similar. We see that

$$v_s^2 = -\alpha \omega(z) \quad (20)$$

Once again in the quintessence range the GCG model is unstable against density perturbation if $\alpha < 0$ and stable if $\alpha > 0$. From the observational data we know that $\alpha$ is always positive, and this ensures the model stability. For the MCG model under consideration the situation is, however, more involved. We get for the quintessence zone

$$v_s^2 = A(1 + \alpha) - \alpha \omega(z). \quad (21)$$

Thus for $A \geq 0$ the system is always stable. But for negative $A$ the stability depends on the relative magnitudes of $\omega(z), A$ and $\alpha$.

Now, Eqs. (16) and (21) give for the case under consideration

$$v_s^2 = A(1 + \alpha) + \alpha \left[ \frac{A(1 - \Omega_{\text{ob}}) - \omega_{\text{ode}}(1 - \Omega_{\text{ob}}) + (1 - \Omega_{\text{ob}} + \omega_{\text{ode}})(1 + z)^{3(1+\alpha)(1+A)}}{A(1 - \Omega_{\text{ob}}) - \omega_{\text{ode}}(1 + z)^{3(1+\alpha)(1+A)}} \right]. \quad (22)$$
In Fig. 6 we plot the variation of the squared speed of sound $v_s^2$ with redshift $z$. It is observed that $v_s$ is positive-definite for $A = -0.1072$ and $\alpha = 1.376$, and $A = -0.05$ and $\alpha = 0.50$, (i.e., negative values of $A$) for low values of $z$, which, however, becomes $v_s^2 < 0$ for a high-redshift region. This leads to instability of the MCG fluids as noted by Mei Deng [29]. It is argued that for $\alpha > -1$, $(v_s^2)_z \rightarrow \infty \approx 0$. However, we note that the instability sets in our case with $\alpha \geq -0.0583$ leading to $v_s^2 = 0$ for finite $z$ accommodating the results obtained by Mei Deng.

We note that the squared speed of sound does not exceed that of light for $A = -0.1072$ and $\alpha = 1.376$ at $z = 0$. However, in the future, i.e., $(1 + z < 1)$, it is possible, and $v_s$ exceeds the speed of light for the above value of $A$ and $\alpha$. This results in a perturbation of spacetime, and a perturbative analysis of the whole system shows that it favors structure formation [30]. For $A = 0.0446$ and $\alpha = -0.0583$, the MCG fluids are unstable for lower $z$ value. However, at high redshifts, i.e., in the early universe $v_s^2 > 0$ is permitted. It is very interesting to note that for $A = 0.002$ and $\alpha = 0.100$, $v_s^2$ is always positive. At high redshifts it was lower than in the present epoch. Again in the future $v_s^2$ will be higher but does not exceed the velocity of light. At $A = 0$, $v_s^2 = -\alpha \omega(z)$ and $\omega(0) < 0$ in this case, which gives a positive value of $v_s$ (since $\alpha$ is slightly positive at $A = 0$). From Eq. (22) it follows that the present value of $v_s^2$ is

$$v_s^2(z) = A(1 + \alpha) - \frac{\omega(1 + \alpha)}{1 - \Omega_{ob}},$$

obtained by putting $z = 0$. The squared speed of sound $v_s^2$ depends on both $A$ and $\alpha$ at $z = 0$, unlike the cases of $\rho$ and $\omega(z)$.

It is evident from Fig. 6 that at high redshifts $(1 + z < 1.7)$ $v_s^2 < 0$ for $A = -0.05$, $\alpha = 0.50$ and $A = -0.1072$, $\alpha = 1.376$. This leads to an instability of the MCG fluid high $z$, i.e., in the early universe, which helps in structure formation. But the model is stable at the present epoch ($z = 0$), and also $0 < v_s^2 < c_s^2$.

Now, we consider a spatially flat FRW universe with a background fluid described by an exotic fluid component, namely, MCG and baryons. From the dynamical field equations we obtain the dimensionless Hubble parameter

$$E^2(z) \equiv \frac{H^2}{H_0^2} = \frac{(1 - \Omega_{ob})}{(1 + A)^{\frac{1}{1+r}} + \omega_{ode}(1 + z)^{3(1+\alpha)(1+A)} + \Omega_{ode}(1 + \omega_{ode})(1 + z)^{3(1+\alpha)(1+A)}$$

$$+ \Omega_{ob}(1 + z)^3. \quad (23)$$

Variation of the normalized Hubble parameter with redshift is shown in Fig. 7. It is observed that the Hubble parameter changes more sharply with $z$ for the set $(A = -0.1072, \alpha = 1.376)$ as compared to other sets, where $A$ is less negative in line with the fact that more negative $A$ values imply a more accelerating model. It attains $E^2(z) = 1$ at the present epoch $z = 0$.

Using the above equations, the densities of the energy components $\Omega_{dm}, \Omega_{de}$ and $\Omega_{b}$ are obtained:
\[\Omega_{de} = \frac{(1 - \Omega_{ob})}{\omega_{de} E^2(z)} \left[ \frac{A (1 - \Omega_{ob}) - \omega_{ode} \Omega_{ode}}{1 + A} \right]^{1/\alpha} + (1 - \Omega_{ob} + \omega_{ode} \Omega_{ode}) (1 + z)^{3(1+\alpha)} (1 + A) \right]

\[\Omega_{dm} = \frac{(1 - \Omega_{ob})}{\omega_{de} E^2(z)} \left[ \frac{1}{1 + A} (\omega_{de} - A) \left\{ (1 - \Omega_{ob}) - \omega_{ode} \Omega_{ode} \right\} + (1 - \Omega_{ob} + \omega_{ode} \Omega_{ode}) (1 + z)^{3(1+\alpha)} (1 + A) \right]

\[\Omega_b = \Omega_{ob} (1 + z)^3 / E^2(z). \tag{26}\]

Variations of density parameters \(\Omega_{de}, \Omega_{dm}\), and \(\Omega_b\) with redshift \(z\) are shown in Fig. 8. At \(z = 0\), the density parameters attain the same value for different sets of the values of \(A\) and \(\alpha\). For example, for the set \((A = -0.1072, \alpha = 1.376)\) the density parameters change with \(z\) at a faster rate than for the other sets of the values of \(A\) and \(\alpha\) taken here. Fig. 8 shows that the density parameters change faster for more negative values of \(A\), admitting an accelerating universe. Fig. 8 also shows that \(\Omega_{de}\) dominance leads to acceleration. The gradually increasing DE density is the cause of a transition from deceleration to acceleration.

To understand this transition we study the evolution of the deceleration parameter \(q\).

The deceleration parameter is

\[q \equiv -1 - \frac{\dot{H}}{H^2} = -1 + \frac{3(1 + z)^3}{2E^2(z)} \left[ 1 + A \right]^{1/\alpha} (1 - \Omega_{ob})^{1/\alpha} \times (1 + z)^{3A (1 - \Omega_{ob} + \omega_{ode} \Omega_{ode})} \times \left\{ (1 - \Omega_{ob}) - \omega_{ode} \Omega_{ode} \right\}^{-1/\alpha} + \Omega_{ob} \right]\tag{27}
In Fig. 9, the redshift variation of the deceleration parameter is plotted. It is found that the deceleration parameter at $z = 0$, i.e., $q_0 = -0.656$ is independent of the values of $A$ and $\alpha$. This value is consistent with observations [29] and follows from Eq. (27) leading to $q_{z=0} = \frac{1}{3}(1 + 3\omega_{\text{ode}}\Omega_{\text{ode}})$. A deceleration flip occurs at different values of $z$ (say $z_f$) for different sets of $\alpha$ and $A$. For $A = -0.1072$ and $\alpha = 1.376$, the flip occurs at a more recent past than for other sets of $A$ and $\alpha$ values, in this case $z_f \approx 0.415$. As is also evident from Fig. 9, $z_f \approx 0.60$ for $A = -0.05$ and $\alpha = 0.50$, $z_f \approx 0.786$ for $A = 0.002$ and $\alpha = 0.100$, and $z_f \approx 0.86$ for $A = 0.0446$ and $\alpha = -0.0583$. It is further observed that for more negative values of $A$ a flip occurs at later times. However, in all cases an accelerating universe emerges at low redshifts ($z < 0.8$). We note that at lower redshift $q$ changes slower with $z$ as compared with high $z$ ($z > 2.0$). But at low $z$, i.e., in the recent past $q$ changes at a faster rate accommodating a phantomlike dynamics. It is also observed in Fig. 9 that as $A$ becomes more negative, the rate of decrease of $q$ is faster with $z$, accommodating an accelerated universe.

4. OBSERVATIONAL CONSTRAINTS ON EoS PARAMETERS

Using Eqs. (23)–(25), we draw contours for the pairs of parameters $(\alpha, A)$, $(\omega_{\text{ode}}, A)$ and $(\Omega_{\text{odm}}, A)$ in Figs. 1–3, respectively, to study the viability of cosmological models with observational predictions.

We note the following:

(i) For a given observational values of $\omega_{\text{ode}}$ and $\Omega_{\text{odm}}$ we draw an $(\alpha, A)$ contour in Fig. 1. The range of permitted values lies between $(-0.1072, -0.0984)$ for $A$, $(-0.2241, 1.374)$ for $\alpha$ at 90% confidence level. Similar ranges are $(-0.1166, 0.1417)$ for $A$ and $(-0.307, 1.585)$ for $\alpha$ at 95.4% confidence level.

(ii) For a given $\alpha$, it is possible to draw a contour between $\omega_{\text{ode}}$ and $A$. Fig. 2 is drawn with $\alpha = 0.05$, $\Omega_{\text{odm}} = 0.22$. The range of permitted values of $A$ lies between $(-0.08172, 0.0068)$, and that for $\omega_{\text{ode}}$ between $(-0.9609, -1.15)$ at 68.3% confidence level. Similar ranges are $(-0.0978, 0.0325)$ for $A$ and $(-0.909, -1.18)$ for $\omega$ at 90% confidence level, and $(-0.1072, 0.0464)$ and $(-0.8771, -1.194)$, respectively, at 95.4% confidence level.

(iii) For a given $\omega_{\text{ode}}$, it is possible to draw a contour between $A$ and $\Omega_{\text{odm}}$. Fig. 3 shows the contours for $\alpha = 0.05$ and $\omega_{\text{ode}} = -1.05$ using H-Z data [31], BAO [32], and CMB shift data [33]. The range of permitted values of $A$, lies between $(-0.0583, -0.0378)$, and that for $\Omega_{\text{odm}}$, between (0.1952, 0.2567) at 68.3% confidence level. Similar ranges at 90% confidence level are $(-0.0616, -0.0339)$ for $A$ and (0.1846, 0.2699) for $\Omega_{\text{odm}}$, and at 95.4% confidence level the admissible values of $A$ range between $(-0.0469, -0.0319)$ and those of $\Omega_{\text{odm}}$ between (0.1780, 0.2788).

5. CONCLUDING REMARKS

In this paper we have studied the dynamics of a flat FRW cosmology filled with a modified Chaplygin gas and tried to obtain observational constraints on the free parameters of the MCG model employing the contour plot technique. We have also briefly discussed the observational data, namely the Stern data, measurement of BAO peak parameters and CMB shift data to draw contours of permissible ranges of values of parameter pairs $(\alpha, A)$ and $(\omega_{\text{ode}}, A)$ in Figs. 1, 2 to study the viability of cosmological models against the observational results. We have taken some of the permissible values of $A$ and $\alpha$ as $(-0.1072, 10376)$; $(-0.05, 0.50)$; $(0.002, 0.100)$ & $(0.0446, -0.0583)$, respectively. However, from Fig. 3 we determine the permissible range of $A$ which lies in the range $-0.0649$ to $-0.0319$) at 95.4% confidence level. Thus the permitted value of $A$ for a viable cosmological model is very near zero from the negative side. It may be of some interest to mention that the MCG model has an important application for the case $A = 1/3$, pointing at a radiation dominated era. However, our analysis shows that the permissible range of values of $A$ do not include $A = 1/3$, which is in our opinion quite feasible because here we focussed our attention on the late era only. As pointed out in the introduction, it may not be quite out of place to draw some correspondence with a fairly similar work of Wang et al. [27] where they studied the constraints on
a decomposed GCG model described by dark matter interacting with inhomogeneous vacuum energy. The first case ends with a very stringent constraint, where $\alpha$ tends to a vanishingly low value at 95% confidence level. In the second case, however, the constraint is less restrictive, such that $-0.15 < \alpha < 0.25$. It may be interesting to point out that in our case with the MCG model the range of $\alpha$ is $-0.1166 < \alpha < 0.1417$ at 95.4% confidence level. This weaker constraint matches favorably with the second case of their work. From the analysis presented here we summarize our findings as follows:

(i) Taking the allowed values of $A$, we have studied the $z$ dependence of the density $\rho$. As expected, we found from Fig. 4 that $\rho$ decreases with $z$. We have also calculated $\rho$ at $z = 0$ and found that $(1 - \Omega_{0b})$ is independent of $A$ and $\alpha$, whereas interestingly $d\rho/dz$ is a function of both $A$ and $\alpha$. We also note that the universe is most dense for the set $(A = -0.1072, \alpha = 1.376)$, i.e., a more negative value of $A$ leads to a greater energy density in the early universe.

(ii) The EoS parameter for $B = 0$ is $w(z) = A = \omega_{0b}(1 - \Omega_{0b}(1 - \Omega_{0b}), which is negative if $\omega_{0b}$ is negative. From Fig. 5 it is clear that for $A = 0.0446$, $\alpha = 0.0583$ a transition from ordinary matter to DE dominated universe is permitted since $w(z)$ passes on from positive to negative values with changing $z$. For other sets of values of $A$ and $\alpha$ such a transition is not permitted since $w(z)$ is always negative for any $z$

From Fig. 5 it is also evident that $w(z)$ is mostly flat at high $z$ values $z > 2$ and is very steep at low $z$. We note that at $z = 0$, $w(z)$ attains $-0.8$, which is independent of $A$ and $\alpha$, close to an observational value.

(iii) We have studied in some detail the stability of our model in Section 3. The effective speed of sound $v_s^2 < 0$ at finite $z$ obtained here is similar to that obtained by Mei [29], where $v_s^2 < 0$ at $z \rightarrow \infty$. This is possibly due to the fact that Mei a priori chose a relation between $A$ and $\alpha (A = \alpha/(1 + \alpha))$ unlike us. We further note from Eq. (21) that if $A \geq 0$, the system is always stable, but for $A < 0$ it depends on the relative magnitudes of $A$, $\omega(z)$ and $\alpha$. Moreover, it is argued in the literature that the advent of $v_s^2 < 0$ signals a stage where structure formation is likely to start.

(iv) It is evident from Fig. 7 that the normalized Hubble parameter changes more sharply with $z$ for the set $(A = -0.1072, \alpha = 1.376)$ than for other sets, where $A$ is less negative in line with the fact that more negative $A$ values imply a more accelerating model.

(v) From Fig. 8 it is evident that for the set $(A = -0.1072, \alpha = 1.376)$ the density parameters change with $z$ more rapidly than for other sets of the values of $A$ and $\alpha$ considered here, i.e., the density parameters change faster for more negative values of $A$ admitting an accelerated universe. The gradually increasing DE density is the reason for a transition from decelerated to accelerated expansion.

(vi) From Fig. 9 it is observed that a flip in the sign of $q$ occurs at different values of $z_f$ for different sets of $A$ and $\alpha$. It is also observed that for more negative values of $A$ the flip occurs at later times, and the $q$ decrease rate is faster. However, in all cases the flip occurs at low redshifts value ($z < 0.8$) pointing at the fact that acceleration is a late phenomenon in accordance with observations.

Finally, we note that in our model with more negative values of $A$ one obtains (a) a universe which is more dense, (b) the density parameter changes faster with $z$, and (c) the flip occurs at a later time. Again from Fig. 3 it is observed that $A$ is negative and nearly equal to zero, which is in agreement with the work of Fabris et al. [28]. We have not considered those values of $A$ because they do not change the nature of evolution abruptly for different parameters.

A final remark may be in order. While the original Chaplygin gas (CG) and its different variants are invoked to explain the late-time acceleration of the universe, the CG has recently found an important application concerning the early inflation [34]. In a recent communication del Campo analyzed issues related to early inflation taking the GCG as a source and tried to constrain the value of the constant parameter $\alpha$ with the help of the recently released Planck data [35]. In this study, for the best fit the value of $\alpha$ comes out to be $\alpha = 0.2578$. While, unlike del Campo, we are dealing here with MCG and, moreover, the formalism adopted by del Campo is quite different from ours, a cursory look at Fig. 1 shows that $\alpha = 0.25$ for $A = 0$, the type of CG taken by del Campo. But we argue that it is simply a coincidence and need not be taken too far because, while del Campo dealt with the early era, we here discuss the late era instead. The analysis carried out by del Campo may be extended to our model, which will be discussed elsewhere.

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