An alternative heavy Higgs mass limit

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Abstract

After commenting on the present value of the Higgs particle mass from radiative corrections, we explore the phenomenological implications of an alternative, non-perturbative renormalization of the scalar sector where the mass of the Higgs particle does not represent a measure of observable interactions at the Higgs mass scale. In this approach the Higgs particle could be very heavy, even heavier than 1 TeV, and remain nevertheless a relatively narrow resonance.

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1. Introduction

The very good agreement of the Standard Model predictions with the available experimental data (with the possible exception of neutrino masses) strongly constrains possible frameworks for new physics. At the same time, one essential ingredient of the theory, namely the Higgs boson, is still missing and, at the present, there is an open discussion to understand the marginal consistency between the result from its direct search, \( m_h > 114.4 \) GeV (at the 95% C.L.) [1] and the phenomenological indication from radiative corrections \( m_h = 87^{+36}_{-27} \) GeV (with the corresponding 95% C.L. upper bound \( m_h < 160 \) GeV [2]), see for instance the Introduction of Ref.[3].

In general, there might be many different scenarios for \( m_h \). At the extremes of the mass range one can consider two basically different options. The (minimal) supersymmetry, where the mass of the lightest Higgs scalar is \( m_h = O(M_W) \), and the alternative point of view where the Standard Model, with a large \( m_h = O(1) \) TeV, represents the natural extension beyond the massive vector boson theory. This latter point of view, where a heavy Higgs particle would mark a second threshold for weak interactions, was emphasized by Veltman [4]. In his view, a large \( m_h \) represents a natural cutoff of the non-renormalizable Glashow model [5] just like the W mass is the natural cutoff for the non-renormalizable 4-fermion V-A theory.

Such a heavy Higgs particle seems in contradiction with the precision tests and thus should be ruled out. In our opinion, however, this conclusion might be too restrictive for the following reasons:

i) the phenomenological analysis is based on experimental quantities where the Higgs particle mass enters only through the radiative correction to Veltman’s \( \rho^- \) parameter [6]. Although for \( m_h = O(1) \) TeV the known quadratic two-loop contribution remains considerably smaller than the logarithmic one-loop term, the authors of Ref.[7], from the expected magnitude of the higher-order terms, were estimating that perturbation theory might break down when \( m_h \) is in this region (“It is very unclear what happens if the Higgs mass is in this range”). For this reason, strictly speaking, finite-order calculations in the Higgs sector might become inconsistent for \( m_h = O(1) \) TeV and large values of \( m_h \) be still phenomenologically viable.

ii) one cannot exclude that, if the physical spectrum is richer than that expected in the Standard Model, see e.g. Refs.[8][9], even for a large \( m_h \), the resulting effective \( S \) and \( T \) parameters might mimic those perturbatively computed for small values of \( m_h \).

iii) the quality of the global fit, when including all electroweak data, \( \chi^2/d.o.f. = 29.7/15 \) with a C.L. 1.3% [9], indicates that some subsets of data are marginally consistent with each
other. We emphasize that some of them point to a large $m_h$. In this sense, the present reading of the data is not completely free of ambiguities.

iv) the measured hadronic cross sections at LEP2 show an average excess of about $+1.5\%$ with respect to the SM prediction [10]. Due to the known positive $(m_h, \alpha_s)$ correlation in the radiative corrections, see e.g. [11], extrapolating this effect back on the Z peak would tend to increase considerably the fitted value of $m_h$ from the electroweak data.

On the basis of the previous arguments, it might be worth to re-consider from scratch the case of a heavy Higgs particle $\mathcal{O}(1)$ TeV and the possible phenomenology expected at LHC. Radiative corrections represent then a separate issue. In the end, if the Higgs particle will turn out to be heavy and if consistency with radiative corrections will still require the introduction of new physics, experimental evidences for the needed new physical sector might be found within the same LHC energy range.

Here the basic point we want to emphasize is that a large $m_h$ does not necessarily imply the presence of strong interactions at the Higgs mass scale. In fact, one can consider an alternative, non-perturbative renormalization of the scalar sector where this conclusion is not true. In this approach, the Higgs particle could be very heavy, even heavier than 1 TeV, and remain nevertheless a relatively narrow resonance.

In the following, we shall first review in Sect.2 the standard picture associated with a heavy Higgs particle. Then, in Sect.3, we shall illustrate an alternative description of spontaneous symmetry breaking in $\lambda\Phi^4$ theories where the mass parameter $m_h$ does not represent by itself a measure of any observable interaction. Further, in Sect.4, we shall explore the possible implications of this picture for the Standard Model. Finally, in Sect.5, we shall present a summary and our conclusions.

2. The standard heavy Higgs mass limit

To start with, let us preliminarily observe that the existence of a non-perturbative regime for large $m_h$ is usually deduced in two very different ways. On the one hand, without considering the self-interactions in the Higgs sector and by simply using leading-order calculations in the unitary gauge, one finds [12] the violation of tree-level unitarity in the scattering amplitudes for longitudinal polarized vector bosons for values of $m_h$ such that

$$\frac{3m_h^2}{16\pi v_R^2} \sim 1$$

(1)

$v_R$ being the physical vacuum expectation value of the scalar field $v_R \sim \frac{1}{\sqrt{G_F} \sqrt{2}} \sim 246$ GeV. On the other hand, by explicitly considering the Higgs sector, the previous strong interactions
emerge as a remnant of the scalar dynamics at zero gauge coupling. In fact, the Higgs field is predicted to be self-interacting with a tree-level, contact strength

$$\lambda^{(1)} = \frac{3m_h^2}{v_R^2}$$

(2)

so that Eq. (1), equivalent to $m_h \sim 1$ TeV, can also be expressed as $\frac{\lambda^{(1)}}{m_h} \sim 1$. The link between the two descriptions, in the high-energy limit $M_w \ll \sqrt{s}$, represents the content of the so-called "Equivalence Theorem" [13]. In this context, the value $m_h \sim 1$ TeV does not represent an upper limit but, rather, indicates a mass region where one should expect substantial non-perturbative effects.

Beyond the tree-approximation, in the usual approach, the self-coupling $\lambda^{(1)}$ in Eq. (2) no longer represents the contact interaction but is now interpreted as a sort of boundary condition for the running coupling at a scale $\mu \sim m_h$. To explore the implications of this identification, let us denote in general by $\lambda_s$ the contact interaction at some large energy scale $\Lambda_s$ that, in principle, in a quantum field theory one might also decide to send to infinity. Let us also denote by $\lambda^{(2)}$ the running coupling at the scale $\mu = m_h \ll \Lambda_s$.

By assuming the perturbative $\beta$-function of $\lambda \Phi^4$ theory

$$\beta_{\text{pert}}(\lambda) = \frac{3\lambda^2}{16\pi^2} + O(\lambda^3)$$

(3)

one gets the relation

$$\ln \frac{m_h}{\Lambda_s} = \int_{\lambda_s}^{\lambda^{(2)}} \frac{dx}{\beta_{\text{pert}}(x)}$$

(4)

Therefore, when $\Lambda_s/m_h \to \infty$ one ends up with an infinitesimal low-energy coupling

$$\lambda^{(2)} \sim \frac{16\pi^2}{3\ln \frac{\Lambda_s}{m_h}} = \epsilon$$

(5)

regardless of how large $\lambda_s$ might be.

The identification $\lambda^{(1)} \sim \lambda^{(2)}$ leads to values of $m_h$ that have to decrease by increasing the magnitude of $\Lambda_s$. In particular, if the maximal Higgs mass and the lowest degree of locality of the theory are connected through the order of magnitude relation $\ln \frac{(\Lambda_s)_{\min}}{(m_h)_{\max}} \sim 1$, one obtains

$$\frac{16\pi^2}{3} \sim \frac{3(m_h^2)_{\max}}{v_R^2}$$

(6)

and the order of magnitude estimate $(m_h)_{\max} \sim 1$ TeV [14]. At the same time, if very heavy, say 800-1000 GeV, one should detect some strongly interacting sector at LHC by measuring
some peak cross section that is proportional to $m_h$. In this mass range, the Higgs particle is expected to be a very broad resonance with a decay width into longitudinal W’s

$$\Gamma(h \to W_L W_L) \sim \frac{3m_h^3}{32\pi v_R^2}$$

that is comparable to its mass.

On the other hand, by still accepting a vanishingly small low-energy interaction, as in Eq. (5) when $\Lambda_s \to \infty$, there might be alternative frameworks where the standard assumption $\lambda^{(1)} \sim \lambda^{(2)}$ is not true. This happens in the approach of Refs. [15, 16] where the two couplings in Eq. (2) and Eq. (5) have a qualitatively different meaning. Namely, the coupling in Eq. (2) emerges as a collective self-interaction of the scalar condensate that enters the effective potential. The coupling in Eq. (5), on the other hand, is appropriate to describe the individual interactions among the elementary excitations of the vacuum, i.e. the singlet Higgs field $h(x)$ and the Goldstone boson fields $\chi_i(x)$. Thus, the continuum limit $\Lambda_s \to \infty$ would have a finite $m_h/v_R$ ratio but $\lambda^{(2)} \to 0$ with trivially free Higgs and Goldstone boson fields.

3. An alternative picture of symmetry breaking

Before addressing the Standard Model, we shall first remind the basic results of Refs. [15, 16]. These were obtained by exploring the structure of the effective potential in that particular class of approximations (gaussian and postgaussian approximations both for the discrete-symmetry and continuous-symmetry O(N) theory [17]) where the scalar self-interaction effects can be reabsorbed into the parameters of an effective quadratic hamiltonian.

The key observation is that, within the generally accepted ”triviality” of the theory in 3+1 dimensions, and by retaining an infinitesimal 2-body coupling of the type in Eq. (5), one can nevertheless obtain a non-trivial effective potential $V_{\text{eff}}$. In fact, when $\Lambda_s \to \infty$, “triviality” dictates the vanishing of all observable, i.e. non-zero momentum, scattering processes. Thus it does not forbid a non-trivial zero-momentum dynamics, that enters the effective potential, provided that its effects can be re-absorbed into the first two moments of a gaussian structure of Green’s functions. After reviewing the original argument, we shall present the very same physical conclusions from a different point of view that, perhaps, may sound more familiar to most readers.

To illustrate the basic point, let us consider the familiar one-loop potential, as originally computed by Coleman and Weinberg [18]

$$V_{\text{eff}}^{1-\text{loop}}(\varphi) = \frac{\lambda \varphi^4}{4!} + \frac{\lambda^2 \varphi^4}{256\pi^2} \left( \ln \frac{\lambda \varphi^2}{2\Lambda_s^2} - \frac{1}{2} \right)$$

(8)
Such basic calculation predicts a weakly first order phase transition. In fact, at one loop, the
version of the theory corresponding to the mass renormalization condition $V''_{\text{eff}}(\varphi = 0) = 0$
gives non-trivial absolute minima for values $\varphi = \pm v$ such that

$$m_h^2 = \frac{\lambda v^2}{2} = \Lambda_s^2 \exp -\frac{32\pi^2}{3\lambda}$$

This yields the equivalent form of the effective potential

$$V^{1-\text{loop}}_{\text{eff}}(\varphi) = \frac{\lambda^2 \varphi^4}{256\pi^2} \left( \ln \frac{\varphi^2}{v^2} - \frac{1}{2} \right)$$

and of the vacuum energy

$$\mathcal{E} = V^{1-\text{loop}}_{\text{eff}}(v) = -\frac{m_h^4}{128\pi^2}$$

Notice that Eq.(9) gives the relation $\lambda \sim \epsilon$ for the coupling constant as in Eq.(5) so that the
quadratic shape at the minima $\varphi = \pm v$

$$\frac{d^2V^{1-\text{loop}}_{\text{eff}}}{d\varphi^2} = \frac{\lambda^2 v^2}{32\pi^2} \sim \epsilon m_h^2$$

is infinitesimal in units of $m_h$.

As discussed by Coleman and Weinberg, one may object to such a straightforward mini-
mization procedure. In fact, the “renormalization-group-improved” result

$$V^{LL}_{\text{eff}}(\varphi) \sim \frac{\lambda(\mu^2 = \lambda\varphi^2)}{4!} \varphi^4$$

obtained by resumming leading-logarithmic terms to all orders, gives no non-trivial minima
and confirms the expectations based on a second-order phase transition, as at the classical
level. The conventional view is that the latter improved calculation is trustworthy while the
former is not, the argument being that the minimum occurs where the one-loop correction
is as large as the classical tree-level term. However, there is an equally strong reason to
distrust the RG-improved result [16]: it amounts to re-summimg a geometric series of leading
logarithmic terms that is actually a divergent series. The moral is that one cannot trust
perturbation theory, improved or not, and one needs an alternative approximation scheme.

The gaussian approximation to the effective potential, being of variational nature, gives
one possible clue [19, 15]. At the same time, by accepting the “triviality” of $\Phi^4$ theories in 3+1
space-time dimensions, it should be reliable when taking the continuum limit $\Lambda_s \to \infty$ where
all higher order Green’s functions should be expressible in terms of the first two moments
of a gaussian distribution. Now, the gaussian effective potential is in complete agreement
with the basic one-loop result (8) thus confirming the indications of a weakly first-order
phase transition. Moreover, the same conclusion persists in the more elaborate postgaussian approximations both for the discrete-symmetry and continuous-symmetry O(N) theory \[17\].

The point is that, differently from the leading-log resummation, the alternative infinite set of gaussian and post-gaussian corrections simply redefines the coupling entering the tree-level potential and the mass entering the zero-point energy, thus preserving the basic one-loop structure in Eq.\[16\]. In the gaussian approximation the relevant relations can be given in the reasonably compact form as

\[
V_{\text{eff}}^G(\varphi) = \frac{\hat{\lambda}\varphi^4}{4} + \frac{\Omega^4(\varphi)}{64\pi^2} \left( \ln \frac{\Omega^2(\varphi)}{\Lambda_s^2} - \frac{1}{2} \right)
\]

with

\[
\hat{\lambda} = \frac{\lambda}{1 + \frac{\lambda}{16\pi^2} \ln \frac{\Lambda_s}{\Omega(\varphi)}}
\]

the variational mass \(\Omega(\varphi)\) being determined self-consistently as \(\Omega^2(\varphi) = \hat{\lambda}\varphi^2/2\) with the minimum-value relation \(m_h = \Omega(v)\).

In this sense, the one-loop potential admits a non-perturbative interpretation being the prototype of all “triviality-compatible” approximations to \(V_{\text{eff}}\). The basic results that are found in this class of approximations are therefore the same as at one loop, namely

i) the physical mass of the shifted Higgs field is given by

\[
m_h^2 \sim \epsilon v^2
\]

ii) the vacuum energy density scales as

\[
\mathcal{E} \sim -m_h^4
\]

In the previous relations \(\varphi\) indicates the bare vacuum expectation value of the scalar field, taken as a variational parameter, and \(v\) its stationarity value. Notice that, by introducing the critical temperature \(T_c\) at which symmetry can be restored (by the order of magnitude relation \(|\mathcal{E}| \sim T_c^4\)), one finds that \(T_c\) is finite in units of \(m_h\).

Now, adopting \(m_h\) as the unit mass scale, while the vacuum energy density itself, \(|\mathcal{E}| \sim m_h^4\), is finite, the slope of the effective potential has necessarily to be infinitesimal

\[
V'_{\text{eff}}(\varphi = v) \sim \frac{m_h^4}{v^2} \sim \epsilon m_h^2
\]

because, starting from \(\varphi^2 = 0\), the finite vacuum energy \(\mathcal{E}\) will be attained for the infinitely large value \(\varphi^2 = v^2 \sim m_h^2/\epsilon\). Thus, if one wants to introduce a definition of the vacuum field,
say $\varphi_R = v_R$, to match the quadratic shape of the effective potential to the physical mass, i.e.

$$V''_{\text{eff}}(\varphi_R = v_R) = m^2_h$$  \hspace{1cm} (19)

one needs a potentially divergent re-scaling for the bare vacuum field

$$\varphi^2 = Z_\phi \varphi^2_R$$  \hspace{1cm} (20)

with

$$Z_\phi = \mathcal{O}(1/\epsilon)$$  \hspace{1cm} (21)

since

$$V''_{\text{eff}}(\varphi_R = v_R) = Z_\phi V''_{\text{eff}}(\varphi = v)$$  \hspace{1cm} (22)

The matching condition Eq. (19), that merely expresses the consistency requirement that the renormalized zero-momentum two-point function $V''_{\text{eff}}(v_R)$ gives the $p_{\mu} \to 0$ limit of the inverse connected propagator $(p^2 + m^2_h)$, is implicit when expanding in the broken phase the full scalar field $\Phi(x)$ into the sum of a vacuum part $v_R$ and of a free-field-like quantum fluctuation $h(x)$ with a given physical mass $m_h$. The generalization to the continuous-symmetry case, which is relevant for the Standard Model, is straightforward and consists in expressing the isodoublet scalar field as

$$\Phi(x) = e^{i\theta_a(x)}\sigma_a \frac{1}{\sqrt{2}} \begin{pmatrix} 0, & v_R + h(x) \end{pmatrix}$$  \hspace{1cm} (23)

by defining the Goldstone boson fields $\chi_a(x)$ through the relation $\theta_a(x) = \chi_a(x)/v_R$.

For completeness, we want to mention that the standard perturbative calculations of $V_{\text{eff}}$, while still maintaining $m^2_h \sim \epsilon v^2$, provide however the different scaling law for the vacuum energy density $\mathcal{E} \sim -\epsilon v^4$. Thus, in this other calculation scheme, one finds $V''_{\text{eff}}(v) \sim m^2_h$ and $v \sim v_R$. Since the vacuum energy density $\mathcal{E} \sim -\frac{m^4_h}{\epsilon}$ diverges in units of $m^4_h$ when $\epsilon \to 0$, the critical temperature $T_c \sim \frac{m_h}{\epsilon^{1/4}}$ now diverges in units of $m_h$.

Returning to the picture of Refs. [15, 16], we observe that the introduction of the potentially divergent re-scaling $Z_\phi$ does not violate any rigorous result since the scalar condensate is not a quantum field that enters the asymptotic representation for the S-matrix. Due to $Z_\phi$, $m_h$ and $v_R$ scale uniformly in the continuum limit where $\epsilon \to 0$ and therefore the coupling $\lambda^{(1)}$ emerges as the natural collective interaction of the scalar condensate with itself

$$\epsilon Z_\phi \sim \frac{m^2_h}{v^2} v^2 \sim \lambda^{(1)}$$  \hspace{1cm} (24)

On the other hand, due to “triviality”, the shifted quantum fields have necessarily to undergo a trivial re-scaling $Z_h = 1 + \mathcal{O}(\epsilon)$. For this reason, at low-energy, they remain weakly
interacting entities with typical strength $\lambda^{(2)} \sim \epsilon$, as in Eq. (5). The difference $Z_\phi \neq Z_h$, or $\lambda^{(1)} \neq \lambda^{(2)}$, reflects the basic phenomenon of vacuum condensation \[20\] in a “trivial” theory and would be sensitive to the presence of a very large cutoff $\Lambda_s$.

From this point of view, the situation is similar to the phenomenon of superconductivity in non-relativistic solid state physics. There the transition to the new, non-perturbative phase represents an essential instability that occurs for any infinitesimal two-body attraction between the two electrons forming a Cooper pair. At the same time, however, the energy density of the superconducting phase and all global quantities of the system (energy gap, critical temperature,...) depend on a much larger collective coupling obtained after re-scaling the tiny 2-body strength by the large number of states near the Fermi surface.

Notice also that $Z_\phi$ enters the extraction of the physical $v_R$ from the bare $v$ measured in lattice simulations. In this way, one will not run in contradiction with the existing data where a single wave function renormalization constant $Z = Z_h \sim 1$ has always been assumed both for the condensate and the fluctuation field \[21, 22\]. We emphasize that even the authors of the critical response paper of Ref.\[23\] must admit that “the unconventional picture of symmetry breaking cannot be ruled out by present numerical simulations”.

For a complete description of the $v_R - m_h$ interdependence, we summarize below the main quantitative results. These can be conveniently expressed in terms of the parameter pair $(\zeta, v_R^2)$ that conveniently replaces the bare mass and bare coupling. The parameter $\zeta$, with $0 < \zeta \leq 2$, is defined by the relation \[16\]

$$V''_{\text{eff}}(\varphi_R = v_R) = m_h^2 = 8\pi^2\zeta v_R^2$$

with the upper bound $\zeta = 2$ coming from vacuum stability since one finds

$$\mathcal{E} = -\frac{\pi^2}{2} \zeta(2 - \zeta)v_R^4$$

Also, in terms of $\zeta$ the condensate re-scaling factor is

$$Z_\phi = 3\zeta \ln \frac{\Lambda_s}{m_h}$$

Finally, one obtains

$$V'''_{\text{eff}}(\varphi_R = v_R) = \frac{3m_h^2}{v_R^2} \left( 1 + \frac{m_h^2}{3\pi^2 v_R^2} \right)$$

As shown in Ref.\[24\], in the heavy-Higgs mass region $\zeta \sim 1$, corresponding to $m_h = \mathcal{O}(1)$ TeV, the inclusion of vector bosons introduces only small corrections $\mathcal{O}(M_w^2/m_h^2)$ with respect to the relations \[25\]–\[28\] of the pure scalar case.
As anticipated, we conclude this part by mentioning a more conventional way to understand the crucial relation $\lambda^{(1)} \neq \lambda^{(2)}$. With an alternative, non-perturbative stability analysis of the theory, we have found that the quantity $\lambda^{(1)} \sim m_h^2/v_R^2$ is a collective self-coupling of the condensate and does not describe the low-energy interactions of the Higgs field and of the Goldstone bosons for which the relevant coupling is rather $\lambda^{(2)} = \mathcal{O}(\epsilon)$. It is possible to characterize this alternative approach without mentioning the effective potential and by only considering the possible choices of boundary conditions in Eq. (1). In fact, one could say that standard perturbation theory, assuming $\lambda^{(1)} \sim \lambda^{(2)}$, is limited to parameter pairs $(m_h, \Lambda_s)$ for which $m_h^2/v_R^2$ and $\ln(\Lambda_s/m_h)$ cannot be, at the same time, very large. Now, suppose that one would try to relax this condition and describe the complementary situation where $m_h^2/v_R^2$ and $\ln(\Lambda_s/m_h)$ can be, at the same time, very large. In this case, when starting from values $\lambda^{(1)} \sim 1$, it becomes natural to associate the range of values $\lambda(\mu) \sim \lambda^{(1)} \gg 1$ with an ultraviolet scale $\mu \gg m_h$ rather than with region $\mu \sim m_h$ where $\lambda(\mu) \sim \lambda^{(2)} = \mathcal{O}(\epsilon)$. In this alternative reading, abandoning $\lambda^{(1)}$ and using $\lambda^{(2)}$, to describe the low-energy interactions of the Higgs field and of the Goldstone bosons, is nothing but the standard renormalization-group evolution from $\lambda^{(1)}$ down to $\lambda^{(2)}$.

Thus, our picture is relevant to that corner, where both $m_h^2/v_R^2$ and $\ln(\Lambda_s/m_h)$ might be much larger than unity, that does not exist in the usual approach [25]. In the next section, we shall explore the phenomenology expected in this other version of the theory where $m_h = \mathcal{O}(1)$ TeV and $\Lambda_s$ can be as large as the highest energy boundaries that are usually considered, namely the grand-unification scale $\Lambda_{gauge} \sim 10^{15}$ GeV or even the Planck scale.

4. A new heavy Higgs phenomenology

To explore the phenomenological implications of the alternative picture of symmetry breaking described in Sect.3, we shall first analyze the physical meaning of the Equivalence Theorem, as explained in Ref. [26]. According to these authors, in any renormalizable $R_\xi$ gauge, the limit $g = g_{gauge} \rightarrow 0$ is smooth. This leads to predict that, in the full Standard Model where $g \neq 0$, there can be no physical interaction (say a peak cross section, a decay width,..) that grows proportionally to $m_h$ unless the same coupling is already present in the theory at $g = 0$. With these premises, the scattering amplitudes for longitudinal vector boson scattering in the $g \neq 0$ theory can be obtained from the corresponding amplitudes for the scattering of the Goldstone boson fields $\chi_i$ in the $g = 0$ theory. Namely, in the high-energy limit $M_w \ll \sqrt{s}$, one can establish the following relation between the two T-matrix elements (the S-matrix is
\[ S = 1 + iT \]

\[ T(W_L W_L \rightarrow W_L W_L) = C^4 T_{g=0}(\chi \chi \rightarrow \chi \chi) \quad (29) \]

where \[ C = 1 + \mathcal{O}(g^2) \quad (30) \]

and, from now on, we omit for simplicity the index \( i \) both in the vector boson and Goldstone boson fields.

Since the Equivalence Theorem is valid to lowest order in \( g \) but to all orders in the scalar self-coupling one expects it to remain valid even if the scalar sector were treated non perturbatively.

To illustrate explicitly the basic point, let us consider for simplicity, as in Ref.\[27\], the case of a pure SU(2) symmetry, i.e. setting \( \sin \theta_w = 0 \), with an invariant Lagrangian given by

\[
L_{\text{inv}} = -\frac{1}{4} G_{\mu \nu}^a G_{\mu \nu}^a - \frac{1}{2} M_w^2 W^2 - \frac{1}{2} (\partial_\mu h)^2 - \frac{1}{2} \partial_\mu \chi^a D_\mu \chi^a + \frac{1}{2} g W_\mu^a (h \partial_\mu \chi^a - \chi^a \partial_\mu h)
\]

\[-M_w \chi^a \partial_\mu W_\mu^a - \frac{1}{2} g M_w W^2 h - \frac{1}{8} g^2 W^2 (h^2 + \chi^a \chi^a) - U_{\text{Higgs}} + \ldots \]

and a fourth-order Higgs potential

\[
U_{\text{Higgs}} = \frac{1}{2} m_h^2 h^2 + \epsilon_1 r g M_w h (\chi^a \chi^a + h^2) + \frac{1}{8} \epsilon_2 r g^2 (\chi^a \chi^a + h^2)^2 \quad (31)
\]

with

\[
r = \frac{m_h^2}{4 M_w^2} \quad (32)
\]

This structure is valid to all orders in the scalar self-interactions but to lowest order in \( g^2 \).

The dots indicate higher-order potential and derivative terms in the low-energy expansion of the full effective action for the scalar fields, evaluated in gaussian and post-gaussian approximations and renormalized as in Sect.3 by imposing the condition \( V''_{\text{eff}}(v_R) = m_h^2 \). The two phenomenological parameters \( \epsilon_1 \) and \( \epsilon_2 \) should be set to unity in ordinary perturbation theory. In general, substantial differences from unity would signal new physics in the scalar sector or, as in our case, indicate an alternative description of symmetry breaking where \( \epsilon_1^2 = \epsilon_2 = 1/Z_\phi \sim \epsilon \).

Concerning the high-energy scattering of longitudinal W's, we observe that, at the tree-level, it can be computed directly as in Ref.\[12\]. In this case, one starts from a tree-level amplitude which is formally \( \mathcal{O}(g^2) \). However, after contracting with the longitudinal polarization vectors \( \sim \frac{k}{M_w} \) on the external legs, one ends up, for energies \( \sqrt{s} \to \infty \), with the same
contact coupling $\lambda^{(1)}$ as for the scalar case. Thus longitudinal W’s interact precisely with a tree-level, contact strength

$$\lambda^{(1)} \sim 3g^2 \frac{m_h^2}{4M_w^2}$$

Here the factor $g^2$ comes from the vertices. The factor $m_h^2$ emerges, in the infinite-energy limit, after combining with the other contributions the graphs with the Higgs propagator. Finally the $1/M_w^2$ derives from contracting with the longitudinal polarization vectors. The identification with $\lambda^{(1)}$ only assumes the Standard Model relation $M_w^2 = g^2 v^2 R^4$ (valid up to few percent corrections such as those associated with the renormalization of the fine structure constant from zero energy to the W-mass scale). By taking into account $m_h^2/s$ terms (but neglecting $M_w^2/s$ and $M_w^2/m_h^2$) the tree-level amplitudes can be conveniently expressed as

$$a_0(\chi\chi \rightarrow \chi\chi) \sim a_0(W_LW_L \rightarrow W_LW_L) = -\frac{\lambda^{(1)}}{48\pi} f(s/m_h^2)$$

with

$$f(x) = 2 - \frac{1}{1-x} - \frac{1}{x} \ln(1+x)$$

Let us now consider the effect of loop corrections in the low-energy region $\sqrt{s} \lesssim m_h$. In this case, with a Higgs potential as in Eq.(31), one finds a T-matrix element at $g = 0$

$$T_{g=0}(\chi\chi \rightarrow \chi\chi) = O(\epsilon)$$

As discussed at the end of Sect.3, this result is equivalent to use the $\beta$–function in Eq.(3) to re-sum higher order effects in $\chi\chi$ scattering. Thus one is driven to deduce a similar result for longitudinal W-scattering, namely

$$T(W_LW_L \rightarrow W_LW_L) = (1 + O(g^2)) T_{g=0}(\chi\chi \rightarrow \chi\chi) = O(\epsilon)$$

The power of the Equivalence Theorem is that, knowing the $\chi\chi$ amplitude, one does not need to re-sum the analogous troublesome longitudinal W-scattering graphs to all orders. For $\Lambda_s \rightarrow \infty$, and whatever the tree-level high-energy coupling, they will always interact with a coupling $O(\epsilon)$ at a scale $\sqrt{s} \lesssim m_h$. To the leading-logarithmic accuracy, this means to replace in Eq.(31) the tree-level coupling $\lambda^{(1)}$ with $\lambda^{(2)}$ thus obtaining

$$a(W_LW_L \rightarrow W_LW_L) \sim a(\chi\chi \rightarrow \chi\chi) \sim -\frac{\pi}{9\ln \frac{\Lambda_s}{m_h}} f(s/m_h^2)$$

Just to have an idea, for $m_h \sim 2$ TeV the coefficient of $f(s/m_h^2)$ in Eq.(34) is about 1.3. On the other hand, for $\Lambda_s \sim 10^{15}$ GeV, the coefficient in Eq.(38) is about 0.013. In this way,
both the intermediate energy range $M_w \ll \sqrt{s} \ll m_h$, where $f \sim -s/m_h^2$, and the resonance region $s \sim m_h^2$ correspond to a weak-coupling regime. Finally, in the region $m_h \ll \sqrt{s} \ll \Lambda_s$, where $f \sim 2$, one should replace $m_h$ with $\sqrt{s}$ within the logarithm in Eq. (38).

Analogously, for the Higgs decay width one can write

$$\Gamma(h \rightarrow W_L W_L) = (1 + \mathcal{O}(g^2)) \Gamma_g=0(h \rightarrow \chi \chi)$$

(39)

By expressing the decay width into Goldstone bosons as

$$\Gamma_g=0(h \rightarrow \chi \chi) \sim \frac{3\epsilon^2 m_h^2}{32\pi v_R^2} m_h$$

(40)

one finds for $\epsilon^2 \sim 1$ the conventional result and for $\epsilon^2 \sim \epsilon$ our alternative description. Thus, in this latter approach, where the ratio

$$\gamma = \frac{\Gamma_g=0(h \rightarrow \chi \chi)}{m_h} = \mathcal{O}(\epsilon)$$

(41)

the Higgs particle could be very heavy and still remain a relatively narrow resonance. As a numerical estimate, by using eq.(27) with $\Lambda_s \sim 10^{15}$ GeV and $m_h \sim 2$ TeV (so that $\zeta \sim 1$), one obtains $\epsilon^2 = \frac{1}{\Lambda_s} \sim 10^{-2}$ and

$$\gamma = \frac{\Gamma_g=0(h \rightarrow \chi \chi)}{m_h} \sim 0.024$$

(42)

Notice that both $\lambda^{(2)}$ and $\gamma$ vanish when $\epsilon \rightarrow 0$. Therefore, the only energy region where one can get a non trivial $\chi \chi$ scattering is within the zero-measure set $\frac{\Delta \sqrt{s}}{m_h} \sim \gamma$ around the Higgs resonance where the amplitude is $\sim \frac{\Delta^{(2)}}{\gamma}$.

In conclusion, by renormalizing the pure scalar sector as discussed in Sect.3, one is driven to deduce that in the $g \neq 0$ theory the Higgs decay width can only acquire new contributions proportional to $g^2 m_h \ll m_h$. For a large $m_h$, a decay width proportional to $\frac{m_h^3}{8\pi v_R}$, as well as the existence of peak cross sections proportional to $\frac{m_h^2}{8\pi v_R}$, would imply the existence of large observable interactions at $s \sim m_h^2$ (for $g = 0$) while, according to “triviality”, for $\Lambda_s \rightarrow \infty$ there can be none.

We have also found that, at the leading logarithmic accuracy, by assuming the perturbative $\beta-$function of $\lambda \Phi^4$ theory to describe the evolution of the coupling in $\chi \chi$ scattering, the Equivalence Theorem requires the replacement $\lambda^{(1)} \rightarrow \lambda^{(2)}$ as the correct strength to describe longitudinal $W$‘s scattering at scales $\sqrt{s} \lesssim m_h$.

As anticipated, this picture is just the opposite of the usual perturbative perspective based on the identification $\lambda^{(1)} \sim \lambda^{(2)}$. There, for a large $m_h = \mathcal{O}(1)$ TeV, the lowest degree
of locality of the theory is set by the Higgs sector that at a scale $\Lambda_s \sim m_h \ll \Lambda_{\text{gauge}}$ has to be replaced by new degrees of freedom. Here, independently of $m_h$, there is no problem in taking the $\Lambda_s \rightarrow \infty$ limit. Thus one might be driven to reverse the hierarchical relation as if, in this alternative approach, $\Lambda_s$ might be even larger than $\Lambda_{\text{gauge}}$ (say $\Lambda_s \sim 10^{19}$ GeV).

### 5. Summary and conclusion

Summarizing: in this paper, motivated by the stability analysis of the theory in a class of approximations to the effective potential that are consistent with the basic triviality property, we have explored the phenomenological implications of relaxing the standard equivalence between Eq. (2) and Eq. (5). Namely, the coupling $\lambda^{(1)} = \frac{3m_h^2}{v_R^2}$ in Eq. (2) emerges as a collective self-interaction of the scalar condensate and can remain finite in the continuum limit. The coupling $\lambda^{(2)} = O(\epsilon)$ in Eq. (5), on the other hand, is appropriate to describe the individual low-energy interactions of the elementary excitations of the vacuum, i.e. the singlet Higgs field $h(x)$ and the Goldstone bosons $\chi_i(x)$, and has to vanish when $\Lambda_s \rightarrow \infty$. Therefore a heavy Higgs might not necessarily imply observable strong interactions at the Higgs mass scale.

Similar considerations apply to longitudinal W’s. In fact, from the standard tree-level calculations, and by using the Equivalence Theorem, one obtains a contact coupling $\lambda_s = \lambda^{(1)}$ for $\sqrt{s} \rightarrow \infty$. Analogously, at the level of loop corrections, by assuming a $\beta-$function as in Eq. (3) to compute the downward evolution of the scalar self-coupling, the same Equivalence Theorem implies that at low energies $\sqrt{s} \lesssim m_h$ longitudinal W’s will interact with an observable coupling $\lambda^{(2)} = O(\epsilon)$, regardless of how large their contact coupling $\lambda_s$ may be.

This type of conclusion applies to all sectors of the theory where the ratio $m_h^2/v_R^2$ is assumed to represent a direct measure of observable, low-energy interactions, and thus, for instance, to the structure of the radiative corrections to the $\rho-$parameter beyond one-loop level. A general analysis of the problem requires to separate out preliminarily the contributions where $m_h$ enters as a genuine mass parameter from those where it is traded for a coupling constant entering the scalar 3- and 4-point functions $\Gamma_3$ and $\Gamma_4$ that describe the self-interactions of the Higgs and Goldstone boson fields. By adopting the same notations as in Sect.4, one can express the low-energy boundary conditions in the general form $\Gamma_3 = \frac{3\epsilon_1 m_h^2}{v_R}$ and $\Gamma_4 = \frac{3\epsilon_2 m_h^2}{v_R}$ where $m_h$ and $v_R$ are defined through the effective potential as $V''_{\text{eff}}(\varphi_R = v_R) = m_h^2$.

Now, in the standard perturbative approach, where there is no distinction between the collective self-interactions of the condensate and those of the fluctuating fields, one is forced
to fix $\epsilon_1 = \epsilon_2 = 1$, and thus there will be corrections proportional to $(m_h^2/v_R^2)^n$ at the $(n+1)$ loop level. In our alternative scheme, on the other hand, the $\epsilon_i$ parameters vanish when $\Lambda_s \to \infty$ and thus the leading contribution to $\Delta \rho$ from the Higgs sector would simply consist of the logarithmic one-loop term.

As previously observed, however, our description quantitatively differs from the usual one only for large $m_h$ where, in the presence of a very large cutoff $\Lambda_s$, we predict a relatively narrow Higgs decay width. If $m_h$ is small, the existence or not of observable effects that are proportional to powers of $m_h^2/v_R^2$ becomes irrelevant. For this reason, if in the end the present indication $m_h = \mathcal{O}(100)$ GeV will be confirmed, we cannot see any definite signal to distinguish the two descriptions of symmetry breaking. Our alternative point of view may still represent a logical possibility but there would be no more compelling theoretical reason that can be traced back to the effective nature of the standard perturbative approach for large $m_h$.

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