1. Introduction

Topological defects, and in particular cosmic strings, are natural consequences of phase transitions in the early universe [1, 2, 3]. If detected they could provide windows into physics at very early times and very high energy, giving important information both for particle physics and cosmology. For most cosmological studies the abelian Higgs model is used as a prototypical cosmic string theory. However, in realistic particle physics theories the situation is more complicated and the resulting cosmic strings can have a rich microstructure. Additional features can be acquired at the string core at each subsequent symmetry breaking. This additional microstructure can, in some cases, be used to constrain the underlying particle physics theory to ensure consistency with standard cosmology. For example, if fermions couple to the Higgs field which gives rise to the string then these fermions could become zero modes (zero energy solutions of the Dirac equation) in the string core. The existence of fermion zero modes in the string core can have dramatic consequences for the properties of cosmic strings. For example, the zero modes can be excited and will move up or down the string, depending on whether they are left- or right-movers. This will result in the string carrying a current [4]. An initially weak current on a string loop will be amplified as the loop contracts and could become sufficiently strong and prevent the string loop from decaying. In this case a stable loop or vorton [5] is produced. The density of vortons is tightly constrained by cosmology. For example, if vortons are sufficiently stable so that they survive until the present time then we require that the universe is not vorton dominated. However, if vortons only survive a few minutes then they can still have cosmological implications. We then require that the universe be radiation
dominated at nucleosynthesis. These requirements have been used in [6, 7] to constrain such models.

Vortons are classically stable [8]. If vortons do decay, then they probably do so by quantum mechanical tunnelling [9, 10]. This would result in them being very long lived. In the case of fermion zero modes it has been shown that they are indeed very stable, particularly in the chiral limit [11]. However, in the case of fermion superconductivity, the existence of fermion zero modes at high energy does not guarantee that such modes survive subsequent phase transitions. It is thus necessary to trace the microphysics of the cosmic string from formation through all subsequent phase transitions in the history of the universe.

For example, many popular particle physics theories above the electroweak scale are based on supersymmetry. Such theories can also admit cosmic string solutions [12]. Since supersymmetry is a natural symmetry between bosons and fermions, the fermion partner of the Higgs field forming the cosmic string is a zero mode. Thus, the particle content and interactions dictated by supersymmetry naturally give rise to current-carrying strings. Gauge symmetry breaking can arise either by introduction of a super-potential or by means of a Fayet-Iliopoulos term [14]. In both cases fermion zero modes arise.

However, supersymmetry is not observed in nature and must therefore be broken. General soft supersymmetry breaking terms are introduced and their effect on the fermion zero modes considered [13]. For most soft breaking terms, the zero modes are destroyed. Hence, any vortons formed would dissipate. However, in the case of gauge symmetry breaking via a Fayet-Iliopoulos term [14], the zero modes, and hence vortons, survive supersymmetry breaking. Hence, supersymmetric theories which break a $U(1)$ symmetry in this way would result in cosmologically stable vortons and could therefore be ruled out.

In these lectures I review the necessary properties of cosmic strings and will introduce current-carrying strings. The formation and properties of cosmic vortons is discussed and I derive constraints on the underlying particle physics theories. Cosmic strings in supersymmetric theories are analysed, and it is shown how they become current-carrying. The vorton constraints on such theories is considered. I will discuss how the observed microwave background radiation can be used to constrain particle physics theories which have some form of long lived, but not absolutely stable defect. Finally I address the problem of cosmic strings arising in superstring theory. Superstrings introduce an extra ingredient, the dilaton field. Since dilatons are long-lived their production result in constraints on the underlying theory. These are discussed in an effective field theory.
2. Cosmic Strings and Current-Carrying Strings

In this section I am going to summarise briefly the salient features of cosmic strings and current-carrying strings. The former have been discussed in previous lectures [15, 16] but I will include a summary for completeness. Further detail can be found in the above lectures or in the excellent reviews [1, 2, 3].

Topological defects can be formed when the universe undergoes a phase transition. In particular, if the vacuum manifold is topologically non-trivial then defects result. If the first homotopy group is not trivial then cosmic strings are formed. The simplest model for cosmic strings is the abelian Higgs model with lagrangian

\[ L = D_\mu \bar{\phi} D^\mu \phi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - V(\phi), \]  

(1)

where

\[ V(\phi) = \lambda (\bar{\phi}\phi - \eta^2). \]

(2)

By solving the classical equations of motion one finds the string solution,

\[ \phi = \eta f(r)e^{i\theta}, \]

(3)

where \( f(r) \) is zero for small \( r \) and tends to 1 for \( r \sim \eta \sqrt{\lambda}^{-1} \). This defines the core of the string. Note that in the string core the \( U(1) \) symmetry is restored, but is broken outside the core. For topological reasons cosmic strings are either infinite or closed loops. Immediately after the phase transition the string network interacts strongly with the background plasma. This interaction results in the correlation length, \( \xi(t) \), growing until it catches up with the horizon, after which the correlation length scales with the horizon. Initially the correlation length is microphysical, being \( \xi(t_x) \approx \eta^{-1} \), where \( t_x \) is the time of the phase transition producing the cosmic string network. By calculating the frictional forces, which are dominated by Aharonov–Bohm scattering, one finds that \( \xi(t) \) grows like \( t^{5/4} \) at early times. This is the friction dominated regime. Eventually the correlation length catches up with the horizon at time \( t_* = (G\mu)^{-1}t_x \), where \( \mu \) is the string tension and is approximately \( \eta^2 \). After \( t_* \) the correlation length scales with the horizon; this is denoted the scaling regime. The scaling solution is maintained by the interactions of cosmic strings. That is, when two infinite strings meet they intercommute. Similarly, when an infinite string self-intersects a closed loop and infinite string result. Whilst infinite strings are stable, loops oscillate, lose energy via gravitational radiation and eventually decay. Thus the string network does not dominate the energy density of the universe.

However, in realistic models of particle physics the Higgs field also couples to fermions. Indeed, in the standard model the fermions acquire their
masses by such a Yukawa coupling. This gives rise to a richer structure of cosmic strings. For example, consider the simple model

\[ \mathcal{L} = \bar{\psi} i \gamma_\mu D^\mu \psi + \lambda_f \bar{\psi} L \phi \psi_R + h.c., \]  

(4)

where \( \lambda_f \) is the Yukawa coupling. Thus, when \( \phi \) goes to zero in the core of the string, the fermions become massless; they are zero mode solutions. They can be excited and move up or down the string depending on their charges and whether they are left- or right-movers. This results in the string carrying a current, though not necessarily an electromagnetic current. Nevertheless, this is not the most general case. When the string has bosonic zero modes, new fermion zero modes have been found [17] which acquire mass from Yukawa couplings to scalar fields that are non-zero in the core.

For current-carrying strings the above evolution of the cosmic string network can be altered since string loops can be stabilised by the angular momentum of the current-carriers. This results in stable loops or vortons and there is the possibility of the string network dominating the energy density of the universe. In this case constraints can be put on the underlying particle physics theory. We consider this in the next section. In the case of electromagnetically charged currents the frictional effects are also different from that described above. This is outside the scope of these lectures; further details are in [18].

3. Cosmic Vortons and Particle Physics Constraints

When a string becomes current-carrying the properties of the network are modified. In this section we consider the constraints on the underlying theory when stable loops, or vortons, result. When the string acquires a current as a consequence of fermion zero modes (as in theories when the fermions become massive from the string-forming Higgs field) the zero modes will be present in the string core at formation. If we call the temperature of the phase transition forming the strings \( T_x \), we can estimate the vorton density. The more general case to consider would be when the string becomes current-carrying at a subsequent phase transition, but this is beyond the scope of these lectures; we refer the reader to [6, 7]. The physics of cosmic vortons has also been discussed in [19, 20].

The string loop is characterised by two currents, the topologically conserved phase current and the dynamically conserved particle number current. Thus the string carries two conserved quantum numbers; \( N \) is the topologically conserved integral of the phase current and \( Z \) is the particle number. A non conducting Kibble type string loop must ultimately decay by radiative and frictional drag processes until it disappears completely. However, a conducting string loop may be saved from disappearance by
reaching a state in which the energy attains a minimum for given non-zero values of $N$ and $Z$, i.e., the loop is stabilised by the angular momentum of the current-carriers. Here we are going to consider two cases: the chiral and non-chiral case. In the former there are either left- or right-movers in the string core, but not both.

It should be emphasised that the existence of such vorton states does not require that the carrier field be gauge coupled. If there is indeed a non-zero charge coupling then the loop will have a corresponding total electric charge, $Q$, such that the particle number is $Z = Q/e$. However, the important point is that, even in the uncoupled case where $Q$ vanishes, the particle number $Z$ is perfectly well defined. Indeed, in the strictly chiral case the current is not electromagnetically coupled.

The physical properties of a vorton state are determined by the quantum numbers, $N$ and $Z$. However, these are not arbitrary. For example, to avoid decaying completely like a non-conducting loop, a conducting loop must have a non-zero value for at least one of the numbers $N$ and $Z$. In fact, one would expect that both these numbers be reasonably large compared with unity. There is a further restriction on the values of their ratio $Z/N$ in order to avoid spontaneous particle emission as a result of current saturation. In general we would expect that $|Z| \approx N$; in the chiral case there is only one independent quantum number and $|Z| = N$. The energy of the vorton is

$$E_v \simeq \ell_v m_x^2 ,$$

where $m_x$ is related to the string tension.

In order to evaluate this quantity all that remains is to work out $\ell_v$. Assuming that vortons are approximately circular, with radius given by $R_v = \ell_v/2\pi$ and angular momentum quantum number $J$ given by $J = N Z$ [21] one obtains

$$\ell_v \simeq (2\pi)^{1/2}|NZ|^{1/2}m_x^{-1} .$$

Thus we obtain an estimate of the vorton mass energy as

$$E_v \simeq (2\pi)^{1/2}|NZ|^{1/2}m_x \approx N m_x ,$$

where we are assuming the classical description of the string dynamics. This is valid only if the length $\ell_v$ is large compared with the relevant quantum wavelengths. This will only be satisfied if the product of the quantum numbers $N$ and $Z$ is sufficiently large. A loop that does not satisfy this requirement will never stabilise as a vorton.

We can now calculate the vorton abundance. Assuming that the string becomes current carrying at a scale $T_\chi$ by fermion zero modes then one expects that thermal fluctuations will give rise to a non-zero value for the
current. Hence, a random walk process will result in a spectrum of finite values for the corresponding string loop quantum numbers $N$ and $Z$. Therefore, loops for which these numbers satisfy the minimum length condition will become vortons. Such loops will ultimately be able to survive as vortons if the induced current, and consequently $N$ and $Z$, are sufficiently large, such that

$$|NZ|^{1/2} \gg 1.$$  

Any loop that fails to satisfy this condition is doomed to lose all its energy and disappear.

The total number density of small loops with length and radial extension of the order of $L_{\text{min}}$, the minimum length for vortons, will be not much less than the number density of all closed loops and hence

$$n \approx \nu L_{\text{min}}^{-3},$$

where $\nu$ is a time-dependent parameter. The typical length scale of string loops at the transition temperature, $L_{\text{min}}(T_x)$, is considerably greater than the relevant thermal correlation length, $T_x^{-1}$, that characterises the local current fluctuations. It is because of this that string loop evolution is modified after current carrier condensation. Indeed, since $L_{\text{min}}(T_x) \gg T_x^{-1}$ and loops present at the time of the condensation satisfy $L \geq L_{\text{min}}(T_x)$, then reasonably large values of the quantum numbers $|Z|$ and $N$ build up. If $\lambda$ is the wavelength of the fluctuation of the carrier field then

$$|Z| \approx N \approx \left(\frac{L}{\lambda}\right)^{1/i},$$

where $i = 1$ in the strictly chiral case and $i = 2$ in the more general case, and $\lambda \approx T_x^{-1}$. In the above the difference between the chiral and non-chiral cases arises since there is a random walk effect in the latter case. Thus, one obtains

$$|Z| \approx N \approx (L_{\text{min}}(T_x) T_x)^{1/i} \gg 1.$$ (11)

For current condensation during the friction-dominated regime this requirement is always satisfied.

Therefore, the vorton mass density is

$$\rho_v \approx N m_{3\gamma} n_v.$$ (12)

In the friction-dominated regime the string is interacting with the surrounding plasma. We can estimate $L_{\text{min}}$ in this regime as the typical length scale below which the microstructure is smoothed [6, 7]. This then gives the quantum number, $N$

$$N \approx \left(\frac{m_p}{\beta T_x}\right)^{1/2i},$$ (13)
where $\beta$ is a drag coefficient for the friction-dominated era that is of order unity. We then obtain the number density of mature vortons

$$n_v \approx \nu_* \left( \frac{\beta T_x}{m_\nu} \right)^{3/2} T^3. \tag{14}$$

This gives the resulting mass density of the relic vorton population to be

$$\rho_v \approx \nu_* \left( \frac{\beta T_x}{m_\nu} \right)^{3/2-1/2i} T_x T^3, \tag{15}$$

where we have assumed that $m_x \sim T_x$.

3.1. THE NUCLEOSYNTHESIS CONSTRAINT.

One of the most robust predictions of the standard cosmological model is the abundances of the light elements that were fabricated during primordial nucleosynthesis at a temperature $T_N \approx 10^{-4}$ GeV. In order to preserve this well-established picture, it is necessary that the energy density in vortons at that time, $\rho_v(T_N)$, should have been small compared with the background energy density in radiation, $\rho_N \approx g^* T^4_N$, where $g^*$ is the effective number of degrees of freedom. Assuming that carrier condensation occurs during the friction damping regime and that $g^*$ has dropped to a value of order unity by the time of nucleosynthesis, this gives

$$\nu_* g_s^{-1} \left( \frac{\beta T_x}{m_\nu} \right)^{3/2-1/2i} T_x \ll T_N, \tag{16}$$

where $g_s^*$ is the effective number of degrees of freedom at the time of current condensation and is approximately $10^2$ in the early universe. If we make the rather conservative assumption that vortons only survive for a few minutes, which is all that is needed to reach the nucleosynthesis epoch, we obtain a fairly weak restriction on the theory.

$$T_x \leq 10^8 \text{ GeV} \tag{17}$$

in the strictly chiral case and

$$T_x \leq 10^9 \text{ GeV} \tag{18}$$

in the more general case, where we have assumed that the net efficiency factor $\nu_*$ and drag factor $\beta$ are of order unity. These are the conditions that must be satisfied by the formation temperature of cosmic strings that become superconducting immediately, subject to the rather conservative assumption that the resulting vortons last for at least a few minutes. We note
that both these conditions rule out the formation of such strings during any conceivable GUT transition, but are consistent with their formation at temperatures close to that of the electroweak symmetry breaking transition.

3.2. THE DARK MATTER CONSTRAINT.

Let us now consider the rather stronger constraints that can be obtained if at least a substantial fraction of the vortons are sufficiently stable to last until the present epoch. Indeed one might expect the chiral vortons to be sufficiently stable [10, 11]. It is generally accepted that the virial equilibrium of galaxies and particularly of clusters of galaxies requires the existence of a cosmological distribution of “dark” matter. This matter must have a density considerably in excess of the baryonic matter density, \( \rho_b \approx 10^{-31} \text{ gm/cm}^3 \). On the other hand, on the same basis, it is also generally accepted that to be consistent with the formation of structures such as galaxies it is necessary that the total amount of this “dark” matter should not greatly exceed the critical closure density, namely

\[
\rho_c \approx 10^{-29} \text{ gm cm}^{-3} .
\]  

As a function of temperature, the critical density scales like the entropy density so that it is given by

\[
\rho_c(T) \approx g^* m_c T^3 ,
\]  

where \( m_c \) is a constant mass factor. For comparison with the density of vortons that were formed at a scale \( T_x \) we can estimate this to be

\[
g^* m_c \approx 10^{-28} m_p \approx 1 \text{ eV} .
\]  

The general dark matter constraint is

\[
\Omega_v \equiv \frac{\rho_v}{\rho_c} \leq 1 .
\]  

Inserting our earlier estimate for the vorton density and noting that \( g^*_v \) is essentially unity in the present epoch, we can derive the dark matter constraint. This gives

\[
T_x \leq 10^5 \text{ GeV}
\]  

in the strictly chiral case and

\[
T_x \leq 10^7 \text{ GeV}
\]  

in the non-chiral case, where we have again assumed that the efficiency factor and drag coefficient are of order unity. This result is based on the
assumptions that the vortons in question are stable enough to survive until
the present day. Thus, this constraint is naturally more severe than its
analogue in the previous section. However, we expect it to be realistic for
the chiral case since such vortons are classically and quantum mechanically
very stable. It is to be remarked that vortons produced in a phase transition
occurring at or near the limit that has just been derived would give a
significant contribution to the elusive dark matter in the universe. However,
if they were produced at the electroweak scale, then they would constitute
such a small dark matter fraction, \( \Omega_v \approx 10^{-9} \), that they would be very
difficult to detect.

These constraints are very general for long-lived vortons. They raise
the question of what class of theories give rise to zero modes on cosmic
strings. One such example is that of supersymmetric theories. Indeed a
class of supersymmetric theories where the gauge symmetry is broken by
a so-called D-term, gives rise to chiral cosmic strings, and hence the most
stringent constraints. We consider this in the next section.

4. A Supersymmetric Model

We consider supersymmetric versions of the spontaneously broken gauged
\( U(1) \) abelian Higgs model [12]. These models are related to or are simple
extensions of those found in reference [22]. For abelian theories the gauge
symmetry can be broken either by adding a potential or by a so-called
D-term [14]. The former case also generalises to non-abelian theories [13].
Here we consider both cases.

In component form we can write the lagrangian as

\[
\mathcal{L} = \mathcal{L}_B + \mathcal{L}_F + \mathcal{L}_Y - U,
\]

with

\[
\mathcal{L}_B = (D_\mu^* \phi_i)(D^{\mu \phi}_i) - \frac{1}{4} F^{\mu \nu} F_{\mu \nu},
\]

\[
\mathcal{L}_F = -i \bar{\psi}_i \sigma^\mu D_\mu \psi_i - i \lambda_i \sigma^\mu \partial_\mu \bar{\lambda}_i,
\]

\[
\mathcal{L}_Y = \frac{ig}{\sqrt{2}} q_i \bar{\phi}_i \psi_i \lambda - \left( \frac{1}{2} b_{ij} + c_{ijk} \phi_k \right) \bar{\psi}_i \psi_j + \text{(c.c.)},
\]

\[
U = |F_i|^2 + \frac{1}{2} D^2,
\]

where \( D_\mu^i = \partial_\mu + \frac{1}{2} ig q_i A_\mu \) and \( F^{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \). Here \( \phi_i \) are complex
calar fields of \( U(1) \) charge \( q_i \), \( A_\mu^i \) is the gauge field and \( g \) is the gauge
coupling. These correspond to the familiar boson fields of the abelian Higgs
model. The fermions, \( \psi_i, \bar{\psi}_i, \lambda_i \) and \( \bar{\lambda}_i \), are Weyl spinors and the complex
bosonic fields, $F_i$, and real bosonic field, $D$, are auxiliary fields which can be eliminated by their equations of motion. These are,

$$ F_i^* + a_i + b_{ij} \phi_j + c_{ijk} \phi_j \phi_k = 0 \ , \quad (30) $$

$$ D + \kappa + \frac{g}{2} q_i \overline{\phi}_i \phi_i = 0 \ . \quad (31) $$

Substituting gives the most general potential to be

$$ U = |a_i + b_{ij} \phi_j + c_{ijk} \phi_j \phi_k|^2 + \frac{1}{2} \left( \kappa + \frac{g}{2} q_i \overline{\phi}_i \phi_i \right)^2 \ . \quad (32) $$

Now consider spontaneous symmetry breaking in these theories. Considering the gauge invariance of the underlying superpotential which gave rise to $U$ implies that $a_i \neq 0$ only if $q_i = 0$, $b_{ij} \neq 0$ only if $q_i + q_j = 0$ and $c_{ijk} \neq 0$ only if $q_i + q_j + q_k = 0$. The situation is a little more complicated than in non-SUSY theories, since anomaly cancellation in SUSY theories implies the existence of more than one chiral superfield (and hence Higgs field). In order to break the gauge symmetry, one may either induce SSB through an appropriate choice of superpotential or, in the case of the $U(1)$ gauge group, one may rely on a non-zero Fayet-Iliopoulos, or D term [14].

We shall refer to the theory with superpotential SSB (and, for simplicity, zero Fayet-Iliopoulos term) as theory F and the theory with SSB due to a non-zero Fayet-Iliopoulos term as theory D. Since the implementation of SSB in theory F can be repeated for more general gauge groups, we expect that this theory will be more representative of general defect-forming theories than theory D for which the mechanism of SSB is specific to the $U(1)$ gauge group.

4.1. THEORY F: VANISHING FAYET-ILIOPOULOS TERM

The simplest model with vanishing Fayet-Iliopoulos term ($\kappa = 0$) and spontaneously broken gauge symmetry contains three chiral superfields. It is not possible to construct such a model with fewer superfields which does not either leave the gauge symmetry unbroken or possess a gauge anomaly. The fields are two charged fields $\Phi_{\pm}$, with respective $U(1)$ charges $q_\pm = \pm 1$, and a neutral field, $\Phi_0$. The potential $U$ is minimised when $F_i = 0$ and $D = 0$. This occurs when $\phi_0 = 0$, $\phi_+ \phi_- = \eta^2$ and $|\phi_+|^2 = |\phi_-|^2$. Thus we may write $\phi_\pm = \eta e^{\pm i \alpha}$, where $\alpha$ is some function. We shall now seek the Nielsen-Olesen[23] solution corresponding to an infinite straight cosmic string. We proceed in the same manner as for non-supersymmetric theories. Consider only the bosonic fields (i.e. set the fermions to zero) and in cylindrical polar coordinates $(r, \varphi, z)$ write
so that the z-axis is the axis of symmetry of the defect. The profile functions, \(f(r)\) and \(a(r)\), obey

\[
\begin{align*}
    f'' + \frac{f'}{r} - n^2 \frac{(1-a)^2}{r^2} &= \mu^2 \eta^2 (f^2 - 1) f, \\
    a'' - \frac{a'}{r} &= -g^2 \eta^2 (1-a) f^2,
\end{align*}
\]

with boundary conditions

\[
\begin{align*}
    f(0) &= a(0) = 0, \\
    \lim_{r \to \infty} f(r) &= \lim_{r \to \infty} a(r) = 1.
\end{align*}
\]

Note here, in passing, an interesting aspect of topological defects in SUSY theories. The ground state of the theory is supersymmetric, but spontaneously breaks the gauge symmetry, while in the core of the defect the gauge symmetry is restored but, since \(|F_i|^2 \neq 0\) in the core, SUSY is spontaneously broken there.

We have constructed a cosmic string solution in the bosonic sector of the theory. Now consider the fermionic sector. In our case this is

\[
\mathcal{L}_Y = i \frac{g}{\sqrt{2}} \left( \bar{\psi}^+ \psi^+ - \bar{\psi}^- \psi^- \right) \lambda - \mu \left( \phi_0 \psi^+ \psi^- + \phi_+ \psi_0 \psi^- + \phi_- \psi_0 \psi^+ \right) + c.c.
\]

As with a non-supersymmetric theory, non-trivial zero energy fermion solutions can exist in the string core. Consider the fermionic ansatz

\[
\psi_i = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \psi_i(r, \varphi),
\]

\[
\lambda = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \lambda(r, \varphi).
\]

If we can find solutions for the \(\psi_i(r, \varphi)\) and \(\lambda(r, \varphi)\) then, following Witten [4], we know that solutions of the form
\[ \Psi_i = \psi_i(r, \varphi) e^{\chi(z+t)}, \quad \Lambda = \lambda(r, \varphi) e^{\chi(z+t)}, \]  
with \( \chi \) some function, represent left-moving superconducting currents flowing along the string at the speed of light. Thus, the problem of finding the zero modes is reduced to solving for the \( \psi_i(r, \varphi) \) and \( \lambda(r, \varphi) \).

The fermion equations of motion derived from (25) are four coupled equations given by

\[ e^{-i\varphi} \left( \partial_r - \frac{i}{r} \partial_\varphi \right) \bar{\lambda} - \frac{g}{\sqrt{2}} \eta f \left( e^{i\varphi} \psi_- - e^{-i\varphi} \psi_+ \right) = 0, \tag{46} \]
\[ e^{-i\varphi} \left( \partial_r - \frac{i}{r} \partial_\varphi \right) \bar{\psi}_0 + i\mu \eta f \left( e^{i\varphi} \psi_- + e^{-i\varphi} \psi_+ \right) = 0, \tag{47} \]
\[ e^{-i\varphi} \left( \partial_r - \frac{i}{r} \partial_\varphi \pm n\frac{a}{r} \right) \bar{\psi}_\pm + \eta f e^{\pm i\varphi} \left( i\mu \psi_0 \pm \frac{g}{\sqrt{2}} \lambda \right) = 0. \tag{48} \]

The corresponding equations for the lower fermion components can be obtained from those for the upper components by complex conjugation, and putting \( n \to -n \). The superconducting current corresponding to this solution (like (45), but with \( \chi(t-z) \)) is right-moving.

We may enumerate the zero modes using an index theorem [25]. This gives \( 2n \) independent zero modes, where \( n \) is the winding number of the string. However, in supersymmetric theories we can calculate them explicitly using SUSY transformations. This relates the fermionic components of the superfields to the bosonic ones and we may use this to obtain the fermionic solutions in terms of the background string fields [12].

\[ \lambda_\alpha \to \frac{2n\alpha'}{\eta r} i(\sigma^z)_\alpha \xi_\beta, \]  
\[ (\psi_\pm)_\alpha \to \sqrt{2} \left( i f' \sigma^r \pm \frac{n}{r} (1-a) f \sigma^\varphi \right)_{\alpha\bar{\alpha}} \bar{\xi}_{\bar{\alpha}} \eta e^{\pm i\varphi}, \]  
\[ (\psi_0)_\alpha \to \sqrt{2} \mu \eta^2 (1-f^2) \xi_\alpha, \]  
where we have defined

\[ \sigma^\varphi = \begin{pmatrix} 0 & -ie^{-i\varphi} \\ ie^{i\varphi} & 0 \end{pmatrix}, \tag{52} \]
\[ \sigma^r = \begin{pmatrix} 0 & e^{-i\varphi} \\ e^{i\varphi} & 0 \end{pmatrix}. \tag{53} \]

Let us choose \( \xi_\alpha \) so that only one component is nonzero. Taking \( \xi_2 = 0 \) and \( \xi_1 = -i\delta/(\sqrt{2}\eta) \), where \( \delta \) is a complex constant, the fermions become
\[ \lambda_1 = \delta \frac{n \sqrt{2} a'}{g \eta r}, \quad (54) \]

\[ (\psi_+)_1 = \delta^* \left[ f' + \frac{n}{r}(1-a)f \right] e^{i(n-1)\varphi}, \quad (55) \]

\[ (\psi_0)_1 = -i \delta \mu \eta (1-f^2), \quad (56) \]

\[ (\psi_-)_1 = \delta^* \left[ f' - \frac{n}{r}(1-a)f \right] e^{-i(n+1)\varphi}. \quad (57) \]

It is these fermion solutions which are responsible for the string superconductivity. Similar expressions can be found when \( \xi_1 = 0 \). It is clear from these results that the string is not invariant under supersymmetry, and therefore breaks it. However, since \( f'(r), a'(r), 1 - a(r) \) and \( 1 - f^2(r) \) are all approximately zero outside the string core, the SUSY breaking and the zero modes are confined to the string. We note that this method gives us two zero mode solutions. Thus, for a winding number 1 we obtain the full spectrum, whereas for strings of higher winding number only a partial spectrum is obtained.

The results presented here can be extended to non-abelian gauge theories. This is done in [13]. The results are very similar to those presented here, so we leave the interested reader to consult the original paper.

4.2. THEORY D: NONVANISHING FAYET-ILIOPOULOS TERM

Now consider theory D in which there is just one primary charged chiral superfield involved in the symmetry breaking and a non-zero Fayet-Iliopoulos term. In order to avoid gauge anomalies, the model must contain other charged superfields. These are coupled to the primary superfield through terms in the superpotential such that the expectation values of the secondary chiral superfields are dynamically zero. The secondary superfields have no effect on SSB and are invariant under SUSY transformations. Therefore, for the rest of this section we shall concentrate on the primary chiral superfield which mediates the gauge symmetry breaking.

Choosing \( \kappa = -\frac{1}{2} g \eta^2 \), the theory is spontaneously broken and there exists a string solution obtained from the ansatz

\[ \phi = \eta e^{i\varphi} f(r), \quad (58) \]

\[ A_\mu = -\frac{2}{g} \frac{n}{r} a(r) \delta^\mu, \quad (59) \]

\[ D = \frac{1}{2} g \eta^2 (1 - f^2), \quad (60) \]

\[ F = 0. \quad (61) \]
The profile functions \( f(r) \) and \( a(r) \) then obey the first order equations

\[
f' = n \frac{(1 - a)}{r} f, \quad (62)
\]

\[
\frac{a'}{r} = \frac{1}{4} g^2 \eta^2 (1 - f^2). \quad (63)
\]

Now consider the fermionic sector of the theory and perform a SUSY transformation. This gives

\[
\lambda_\alpha \rightarrow \frac{1}{2} g \eta^2 (1 - f^2) i (I + \sigma^z)_{\alpha \beta} \xi_\beta, \quad (64)
\]

\[
\psi_\alpha \rightarrow \sqrt{2} \eta \frac{n}{r} (1 - a) f (i \sigma^r - \sigma^\varphi)_{\alpha \dot{\alpha}} \xi_{\dot{\alpha}} e^{i \varphi}. \quad (65)
\]

If \( \xi_1 = 0 \) both these expressions are zero. The same is true of all higher order terms, and so the string is invariant under the corresponding transformation. For other \( \xi \), taking \( \xi_1 = -i \delta / \eta \) gives

\[
\lambda_1 = \delta g \eta (1 - f^2), \quad (66)
\]

\[
\psi_1 = 2 \sqrt{2} \delta \eta \frac{n}{r} (1 - a) f e^{i (n-1) \varphi}. \quad (67)
\]

Thus supersymmetry is only half broken inside the string. Vortices with supersymmetry half broken in the string core also arise in the theories considered in [24]. This is in contrast to theory F which fully breaks supersymmetry in the string core. The theories also differ in that theory D’s zero modes will only travel in one direction, while the zero modes of theory F (which has twice as many) travel in both directions. Thus the D theory has chiral zero modes and is subject to the constraints derived in the previous section for chiral vortons. In both theories the zero modes and SUSY breaking are confined to the string core.

Thus, a necessary feature of cosmic strings in SUSY theories is that supersymmetry is broken in the string core and the resulting strings have fermion zero modes. As a consequence, cosmic strings arising in SUSY theories are automatically current-carrying. As discussed in the previous section, the presence of vortons puts severe constraints on the underlying theory since the density of vortons could overclose the universe if they are stable enough to survive to the present time. If they only live for a few minutes then the vorton density could affect nucleosynthesis.

4.3. SOFT SUSY BREAKING

Supersymmetry is not observed in nature. Hence, it must be broken. Supersymmetry breaking is achieved by adding soft SUSY breaking terms which do not induce quadratic divergences.
In a general model, one may obtain soft SUSY breaking terms by the following prescription.

1. Add arbitrary mass terms for all scalar particles to the scalar potential.
2. Add all trilinear scalar terms in the superpotential, plus their hermitian conjugates, to the scalar potential with arbitrary coupling.
3. Add mass terms for the gauginos to the Lagrangian density.

Since the techniques we have used are strictly valid only when SUSY is exact, it is necessary to investigate the effect of these soft terms on the fermionic zero modes we have identified.

As we have already commented, the existence of the zero modes can be seen as a consequence of an index theorem [25]. The index is insensitive to the size and exact form of the Yukawa couplings, as long as they are regular for small $r$ and tend to a constant at large $r$. In fact, the existence of zero modes relies only on the existence of the appropriate Yukawa couplings and that they have the correct $\phi$-dependence. Thus there can only be a change in the number of zero modes if the soft breaking terms induce specific new Yukawa couplings in the theory and it is this that we must check for. Further, it was conjectured in [25] that the destruction of a zero mode occurs only when the relevant fermion mixes with another massless fermion.

We have examined each of our theories with respect to this criterion and list the results below.

4.3.1. Theory-F
The trilinear and mass terms that arise from soft SUSY breaking are

$$m_0^2|\phi_0|^2 + m_-^2|\phi_-|^2 + m_+^2|\phi_+|^2 + \mu M \phi_0 \phi_+ \phi_-.$$  

(68)

The derivative of the scalar potential with respect to $\phi_0^*$ becomes

$$\phi_0 (\mu^2 |\phi_+|^2 + \mu^2 |\phi_-|^2 + m_0^2) + \mu M (\phi_+ \phi_-)^*.$$  

(69)

This will be zero at a minimum, and so $\phi_0 \neq 0$ only if $M \neq 0$.

New Higgs mass terms will alter the values of $\phi_+$ and $\phi_-$ slightly, but will not produce any new Yukawa terms. Thus these soft SUSY-breaking terms have no effect on the existence of the zero modes.

However, the presence of the trilinear term gives $\phi_0$ a non-zero expectation value, which gives a Yukawa term coupling the $\psi_+$ and $\psi_-$ fields. This destroys all the zero modes in the theory since the left- and right-moving zero modes mix. For completeness note that a gaugino mass term also mixes the left and right zero modes, aiding in their destruction. Thus in this case there is a possibility of the fermion zero modes being destroyed by supersymmetry breaking and evading the vorton bounds of the previous section.
4.3.2. Theory-D

The $U(1)$ theory with gauge symmetry broken via a Fayet-Iliopoulos term and no superpotential is simpler to analyse. New Higgs mass terms have no effect, as in the above case, and there are no trilinear terms. Further, although the gaugino mass terms affect the form of the zero mode solutions, they do not affect their existence, and so, in theory-$D$, the zero modes remain even after SUSY breaking. For this class of theories, the strings remain current-carrying, have a vorton problem, and are therefore in conflict with cosmology unless subsequent phase transitions destroy the zero modes.

5. Microwave Background Constraints

In previous lectures [16] we have seen the cosmic string predictions for the cosmic microwave background. However, the microwave background can also be used to constrain theories in another respect. Cosmological models with non-thermal photon production occurring after a redshift of about $10^6$ are strongly constrained by the precision measurements [26] of the black-body spectrum of the Cosmic Microwave Background (CMB). Energy release (either into photons or electrons and other electromagnetically interacting particles with rather general spectra) for redshift in the range between $10^5$ and $3 \times 10^6$ would lead to a Bose–Einstein distortion of the thermal spectra characterised by a chemical potential [27]. Energy release at later times produces a distortion in the Comptonised spectrum. The bottom line is that the fraction of photons produced after the above redshift must be smaller than $7 \times 10^{-5}$. This can be used to constrain any theory with decaying topological defects. For example, theories which give rise to embedded defects [28] or theories resulting in vortons which are not stable to the present time can be constrained in this way [29].

Some types of embedded defects can be stabilised by plasma effects in the early universe when some of the fields are electromagnetically charged via interaction with the surrounding photons [30]. Since the photons fall out of thermal equilibrium at the time $t_{ls}$ of last scattering, the stabilization forces will disappear at this time and the embedded defects will decay, emitting a certain fraction of their energy as non-thermal photons. They can thus be constrained by the FIRAS results.

To illustrate the mechanism that renders some embedded defects stable in the early Universe, we shall consider a model [31] in which the order parameter consists of four real scalar fields with a standard symmetry breaking potential symmetric in the four fields. We will assume that two of the fields are electrically charged and the other two neutral, as is the case in the standard electroweak theory and in the Sigma model description of low energy QCD in the limit of vanishing pion mass.
In an electromagnetic plasma, the interactions with the photon bath will lead to an effective potential for the order parameter which breaks the symmetry between the charged and the neutral fields, while preserving the symmetry within the neutral scalar field sector. The potential is lifted more in the direction of the charged scalar fields. Thus, the vacuum manifold becomes $S^1$, giving rise to cosmic string solutions of the full field equations which look like the standard cosmic string configuration of the neutral fields with the charged scalar fields set to zero.

The thermal averaging implicit in the above analysis breaks down after the time of last scattering, and thus it is expected that the confining potential will disappear and the embedded strings will decay, emitting a fraction $\beta$ (which is expected to be smaller but of the order of 1) of its energy as photons. Since the topological defects are out-of-equilibrium objects, the photons produced by their decay will lead to spectral distortions of the thermal CMB.

Let us now look at this stabilization mechanism in more detail. The Higgs sector of our model is described by the following Lagrangian

$$\mathcal{L} = \frac{1}{2}(D_\mu \chi^* D^\mu \chi + \partial_\mu \phi^* \partial^\mu \phi) - V,$$

with the symmetry breaking potential

$$V = \frac{\lambda}{4} (\chi^* \chi + \phi^* \phi - \eta^2)^2,$$

and with gauge-covariant derivative

$$D_\mu \chi = \partial_\mu \chi + ie A_\mu \chi,$$

where $e$ is the fundamental charge and $A_\mu$ is the gauge field of electromagnetism. The field $\phi$ denotes the neutral Higgs doublet, which gives rise to the string solution and the field $\chi$ is the charged Higgs doublet. Using the language of the low-energy Sigma model of QCD we can write

$$\chi = \pi_1 + i\pi_2, \quad \phi = \pi_3 + i\pi_0.$$

The finite temperature corrections to the potential of the theory given by (70) were worked out in detail in [31]. Keeping only the contributions to the finite temperature effective potential $V_T$ quadratic in the temperature $T$ coming from the photon bath we obtain

$$V_T - V = \frac{T^2}{2} e^2 \left( \frac{1}{6} A_\mu A^\mu + \pi \chi^* \chi \right)$$

$$+ T^2 \frac{\lambda}{4} (\chi^* \chi + \phi^* \phi - \frac{2}{3} \eta^2).$$
The second term in the first line is responsible for the breaking of the degeneracy in the potential between the charged and neutral scalar fields. Equation (74) describes a potential which is lifted by different amounts in the neutral and charged scalar field directions compared to the zero temperature potential. For temperatures below the critical temperature \( T_c = \sqrt{2} \eta \), the space of lowest energy states forms a manifold \( S^1 \) consisting of configurations with \( \chi = 0 \) and \( \phi^* \phi = \eta_r^2 \), where \( \eta_r \) is given by

\[
\eta_r^2 = \eta^2 - \frac{T^2}{2}.
\]  

(75)

Thus, there is a static string solution consisting of a \( U(1) \) cosmic string in the \( \phi \) variables with \( \chi = 0 \), the \emph{embedded string}.

The embedded string is stable in the temperature range immediately below the critical temperature for which the curvature of the potential in the \( \chi \) direction is positive at \( \chi = 0 \). This is the case for

\[
\sqrt{2} \left(1 + \frac{2\pi e^2}{\lambda}\right)^{-1/2} < \frac{T}{\eta} < \sqrt{2}.
\]  

(76)

When the temperature drops below the lower limit given in (76), the string does not disappear, but rather undergoes a core phase transition \([32, 33]\) in which the charged field \( \chi \) acquires a non-vanishing value \( \chi_T \) in the string core which lowers the potential energy density in the core. What results is an asymmetric vortex defect which, as realized in \([31]\), will generically be superconducting, the superconductivity being induced by the phase gradient of \( \chi \) in the string core. This gave rise to the so-called drum vortons, whose properties are outside the scope of these lectures. The value of \( |\chi_T| \) is given by

\[
\chi^* \chi = \eta^2 - \frac{T^2}{2} \left(1 + \frac{2\pi e^2}{\lambda}\right).
\]  

(77)

Since the asymmetric vortices are superconducting, string loops will generically form vortons \([5]\). The strings acquire their current at the time \( T_Q \) of the core phase transition, i.e. (see (76))

\[
T_Q = \sqrt{2} \left(1 + \frac{2\pi e^2}{\lambda}\right)^{-1/2} \eta.
\]  

(78)

Since this temperature is only slightly lower than the temperature at which the strings initially form, the string network will still be in the friction-dominated phase when the current condensation occurs.

In the following, we will use the knowledge of the energy contained in embedded vortices, in particular those of asymmetric core nature, to determine the cosmological constraints.
5.1. CONSTRAINTS

Cosmic vortices (and other defects) that decay into (in part) photons at some temperature $T_d$ corresponding to a redshift of less than $10^6$ will produce spectral distortions of the CMB and will thus be strongly constrained by the COBE/FIRAS data [26]. The constraint on the non-thermal fractional energy density production in photons is

$$\frac{\delta \rho_\gamma}{\rho_\gamma} \lesssim 7 \times 10^{-5}. \quad (79)$$

This constraint arises from the limits to the Compton $y$ parameter and chemical potential, which measures the spectral distortion. Any defects which decay after a redshift of $10^6$ will be subject to these FIRAS constraints. Thus, the following considerations will apply to stabilized embedded defects and to decaying topological defects (such a decay might be induced by a late time phase transition).

The specific bounds on defect models will depend on the type of defect, on the density of defects and on the specifics of the decay. We will focus on cosmic strings, both of topological and of embedded type. First, we consider strings which are in their scaling regime at the time of decay. This applies, for example, to embedded vortices formed at early times and in which superconductivity is absent or too weak to produce vortons. If the strings are formed later than some critical time, the string network will not yet be scaling and the dynamics will still be friction-dominated. Since the density of strings is higher in this phase relative to a string scaling configuration, the bounds in this case are different. This is the second case we treat. Finally, we analyze the case of a gas of vortons produced by the superconducting string loops.

5.1.1. Scaling Cosmic Strings

First let us examine the case of cosmic strings in the scaling regime. The density of strings at time $t$ is

$$\rho_s = \nu \frac{\eta^2}{T^2}, \quad (80)$$

where the constant $\nu$ determines the number of long string segments per Hubble volume, and whose value is $\nu \sim 10$ (see [1] and references therein).

Let us first consider strings decaying at the time $t_{ls}$ of last scattering. From (80) it follows that the density of strings at last scattering is

$$\rho_s = \nu \frac{32\pi^3}{45} \frac{z_{eq} \eta^2}{z_{ls} m_P T_{ls}^4}, \quad (81)$$
where $z_{\text{eq}}$ is the redshift at the time of equal matter and radiation. If a fraction $\beta$ of the energy of the strings goes directly or indirectly into photons, then the COBE/FIRAS constraint (79) becomes

$$
\frac{\beta \rho_s}{\rho} = \beta \nu \frac{32\pi}{3} \left( \frac{\eta}{m_p} \right)^2 \frac{z_{\text{eq}}}{z_{ls}} \lesssim 7 \times 10^{-5}.
$$

(82)

Assuming that $\beta$ is of order 1 then this results in the constraint

$$
\eta \lesssim \nu^{-1/2}10^{16}\text{GeV}.
$$

(83)

Thus, for values of $\nu$ in the range indicated by present cosmic string simulations, the COBE/FIRAS constraint severely constrains decaying cosmic string models with a symmetry breaking scale given by the scale of Grand Unification, though all cosmic string models with this scale of symmetry breaking are already constrained [16].

Consider now strings decaying at a redshift $z_d$ larger than $z_{\text{eq}}$ (but smaller than $10^6$). In this case, the COBE/FIRAS constraint yields a result analogous to above, but modified by a factor of $(z_{eq}/z_{ls})^{1/2}$, resulting in a weaker bound by a factor of about 3 than the bound given in (83). Conversely, if the strings decay after $t_{ls}$, the bound is stronger by a factor of $(z_d/z_{ls})^{1/2}$.

5.1.2. Friction Dominated Strings

If however the string network is friction dominated at the time of decay then the density of strings is much greater than that in the scaling regime. However, the friction domination condition will be satisfied at the time when the strings decay (assumed to be $t_{ls}$) whenever $T_\star$ is less than $T_{ls}$, i.e. less than $10^{-13}$ GeV. This gives

$$
\frac{\eta}{m_p} \lesssim 10^{-16},
$$

(84)

i.e. $\eta \lesssim 10^3$ GeV. In this case, though, strings that are in the friction epoch are not constrained by the COBE/FIRAS data, unless they lead to vortons.

5.1.3. Vortons

In section 3 we saw that cosmic string theories giving rise to vortons are subject to stringent constraints if they are absolutely stable and to slightly less stringent constraints if the vortons survive until the time of nucleosynthesis. Here we consider the additional constraint arising from the limit (79) provided by the COBE/FIRAS data [26] in cases for which vortons decay between a redshift of $10^6$ and today, thus giving rise to decay products that would produce observable distortions of the black body spectrum.
If the string is formed at temperature $T_X$ then the vorton density at temperature $T$ is given by \[ \rho_v = \tilde{\nu} \left(\frac{T}{m_p}\right)^{5/4} T_X T^3, \] where $\tilde{\nu}$ is a constant of the (rough) order of 1. This result holds both in the radiation and matter dominated phases.

As before, we denote the temperature at which the vortons decay by $T_d$. If the vortons emit a fraction $\beta$ of their energy as photons, then the fractional photon energy density input from vorton decay is
\[ \frac{\Delta \rho_\gamma(T_d)}{\rho_\gamma(T_d)} = \kappa \left(\frac{T_X}{m_p}\right)^{5/4} \frac{T_X}{T_d}, \]
in which the constant $\kappa$ is given by
\[ \kappa = \frac{30}{\pi^2 g_*(T_d)} \beta \tilde{\nu}. \]

The COBE/FIRAS constraint (79) thus becomes
\[ \kappa \left(\frac{T_X}{m_p}\right)^{5/4} \frac{T_X}{T_d} < 7 \times 10^{-5}. \] (88)

Assuming that the decay occurs at the time of last scattering and using the estimate $\kappa \sim 1$, the general constraint (88) leads to
\[ T_X \lesssim 10^5 \text{GeV}. \] (89)

We have considered cosmological constraints on models with decaying topological defects which result from demanding that the photons produced in the decay do not lead to spectral distortions of the CMB in excess of the observational limits from the COBE/FIRAS experiment. The strongest limits arise for theories leading to decaying vortons. Any theory giving rise to vortons resulting from a string forming phase transition above $10^5$ GeV, and which decayed from a redshift of $10^6$ to today, would produce a too large spectral distortion as measured by the Compton $y$ parameter, and thus be ruled out. The above constraint applies both to vortons resulting from topological strings or those resulting from embedded strings which have become stabilised by plasma processes. These constraints are in fact stronger than constraints on vorton models requiring compatibility with nucleosynthesis and with the dark matter abundance limits [6] and are comparable to those obtained in the chiral case [7].
Since many types of embedded strings are stabilized by interactions with the electromagnetic plasma, undergo core phase transitions and become superconducting, thus yielding vortons [31] which decay at the time of last scattering, our constraints are very important for theories with embedded defects.

Our analysis also gives rise to constraints on theories with topological cosmic strings with GUT scale symmetry breaking scale, provided they decay in the relevant redshift interval.

6. Dilatonic Cosmic Strings

Superstring theory predicts the existence of light, gauge-neutral fields (the dilaton and moduli) with gravitational strength couplings to ordinary matter. In addition it has an axionic symmetry. In the effective lagrangian a combination of the dilaton and axion symmetry is broken to give rise to cosmic string solutions. These cosmic strings have novel solutions in that there are two energy scales and oscillating string loops copiously emit dilatons in addition to gravitons. Since the cosmic strings arise from a supersymmetric theory, there will also be zero modes in the string core.

Due to the weak couplings with ordinary matter the dilaton is rather long-lived, posing problems for cosmology. It was realised [34] that dilatons are copiously produced by oscillating cosmic string loops, resulting in constraints on the dilaton mass and scale of symmetry breaking. In this section we briefly summarise their results. Since [34] only considered the dilaton field and did not take the axionic symmetry into account we will also extend their results to this case.

6.1. DILATON EMISSION FROM COSMIC STRING LOOPS

In [34], see also [35], the dilaton emission from cosmic string loops was calculated in an analogous way to graviton emission (see [1]). That is the energy loss from an oscillating, periodic source was estimated. The scaling distribution for the string loops was taken. For the case considered by [34] the dominant contribution arose from loops smaller than the critical length,

\[ L_c = \frac{4\pi}{m_s}, \tag{90} \]

where \( m_s \) is the mass of the dilaton. Here they found that the total number of dilatons produced by a decaying loop is

\[ N = \Gamma \alpha^2 G^2 \mu^3 t^2, \tag{91} \]

where \( \Gamma \) is a numerical coefficient which depends on the loop trajectory, but is expected to be of order \( 10^2 \), \( \alpha \) is a dimension-less quantity which
measures the strength of the coupling of the dilaton field to the cosmic strings, and is expected to be of order 1, and $t_f$ is the time the loop decays. The total number of dilatons produced by the string network is

$$Y(t_f) = \frac{n_s N_s}{s(t_f)},$$

(92)

where $n_s$ is the number density of string loops, calculated using the scaling distribution and $s(t_f)$ is the entropy density of the universe at the time of loop decay. This gives,

$$Y(t_f) = \frac{\Gamma \alpha^2 (G\mu)^2 (m_s t_f)^{1/2}}{g^{*1/4}}.$$  

(93)

The dilaton constraints are sensitive to its lifetime, which are determined by its mass and couplings, and is

$$\tau_s \approx \frac{4m_s^2}{N_F m_3^3},$$

(94)

where $N_F$ is the number of gauge bosons with masses less than $m_s$ and factors of order unity have been ignored. The scale of symmetry breaking and dilaton mass can then be bound by assuming that dilaton production does not affect standard nucleosynthesis. It was found that, for

$$m_s = 10^3 \text{ GeV},$$

(95)

$$\eta \leq 10^{11} \text{ GeV},$$

(96)

or for

$$\eta = 10^{16} \text{ GeV},$$

(97)

then

$$m_s \geq 10^5 \text{ GeV}.$$  

(98)

Thus the favoured values of a GUT scale symmetry breaking, $\eta = 10^{16} \text{ GeV}$, and dilaton mass given by the scale of supersymmetry breaking, $m_s = 10^3 \text{ GeV}$, are incompatible with observation.

6.2. A DILATON MODEL WITH AXIONIC SYMMETRY

In the above calculation a simple model was used where the complexities of the superstring effective lagrangian were ignored. I have addressed this in work in progress with Pierre Binetruy and Stephen Davis. Here I present our preliminary results. The superstring effective action results in a model with a pseudo-anomalous $U(1)$ local symmetry and dilaton field. The real
part of the dilaton field defines the gauge coupling, \(1/g^2 = s_R\). The bosonic part is discussed in [36]. In the supersymmetric case the gauge symmetry is broken by a combination of \(F\) and \(D\) terms, resulting in two distinct mass scales. The heavier scale, \(m_D\) is related to the compactification scale in string theory, so should be close to the Planck scale, whilst \(m_F\) is the dilaton scale and should be related to the supersymmetry breaking scale, thought to be around \(10^{3}\) GeV.

We look for cosmic string solutions of the form

\[
\phi = \eta f(r)e^{i\varphi},
\]

\[
A_\theta = n\frac{v(r)}{r},
\]

\[
s = \frac{\delta_{GS}}{\eta^2 \gamma(r)^2} + 2i n \delta_{GS} \varphi,
\]

\[
\phi_Z = 0,
\]

where \(\delta_{GS}\) is the Green-Schwarz parameter in string theory (see [36] for further details) and \(\phi_Z\) is a scalar field of charge \(q_Z\). As \(r\) approaches infinity the functions \(f\), \(\gamma\) and \(v\) all tend to 1, and we allow spatial variations of the dilaton.

These profile functions are then inserted into the equations of motion, which were solved numerically. As expected, two distinct length scales were found, resulting in the string having an inner core of radius \(r_D \sim m_D^{-1}\) \((\rho_D \sim 1)\) in which \(v < 1\) and \(f \neq \gamma\), so \(D \neq 0\). Around that region there is an outer core in which \(v \approx 1\) and \(f \approx \gamma\) but \(f, \gamma < 1\). This region is of radius \(r_F \sim m_F^{-1}\), which can be far greater than \(r_D\) even for moderate values of the parameters.

An approximate analytic solution to the field equations was also found. Inside the inner core \((\rho < 1)\) of the string the kinetic terms dominate the field equations. Imposing the boundary conditions \(\gamma = f = v = 0\) at \(\rho = 0\) and \(f \approx \gamma\), \(v \approx 1\) at \(\rho = 1\) gives the approximate solution for \(\rho < 1\)

\[
f \approx A \rho^{|n|},
\]

\[
\gamma \approx \frac{A}{\sqrt{1 - 2|n|\eta^2 A^2 \log \rho}},
\]

\[
v \approx \frac{\rho^2}{1 - 2|n|\eta^2 A^2 \log \rho}.
\]

Note that there is some analogy to the situation in the \(A\) phase of \(He^3\) where there is a hard inner core and a soft outer core [37]; a similar situation arises in some supersymmetric theories with flat directions [38].
If $m_D \gg m_F$ then an approximate solution for the outer core is also needed. Here the auxiliary field $D \approx 0$ so $f \approx \gamma$, and we obtain

$$f \approx \gamma \approx A + \frac{1 - A}{\log(m_D/m_F)} \log \rho,$$

for $1 < \rho < m_D/m_F$ and $v = 1$. For $\rho > \rho_F$, $f = \gamma = v = 1$. The constant $A$ is determined by taking $\gamma'$ to be continuous at $\rho = 1$. Note that if $m_D \sim m_F$, there is no outer core. Then $|\phi|$ and $s_R$ take their vacuum expectation values for $\rho > 1$, so $v = \gamma = f = 1$ and thus $A = 1$. The energy per unit length is found to be $\mu \sim |n| \eta^2$, with logarithmic dependence on $m_D/m_F$.

Since the theory is supersymmetric there is also a fermionic lagrangian. From this one can investigate the fermion zero mode solutions. We find that there is a rich spectrum in this model, found by analogous methods to those used in the previous sections. In particular these were found by supersymmetry transformations on the boson fields and also by use of the index theorem [25]. We find (for $q_Z < 0$) there are $|n|(1 - q_Z)$ zero modes, all of which are left (right) movers if $n > 0$ ($n < 0$). Thus these strings are again current-carrying strings and since the fermions are either left or right-movers, the current is chiral. However, the existence of the two distinct mass scales, $m_D$ and $m_F$, means that we cannot automatically assume that the vorton constraints derived previously will apply.

### 6.2.1. Constraints

In our case a stable string network is only formed at $T \sim m_F$, which can be far less than the energy scale of the strings, $\eta$, which is determined by $m_D$. This is an unusual feature of our model. If

$$m_F \leq \frac{m_D^2}{m_\nu},$$

the friction domination regime will be absent. In this case the string network may well be formed in the scaling regime.

The unusual form of the axion strings in this model does provide an alternative solution to the vorton problem. Typically the radius of a stable loop will be of order $10^2 m_D^{-1}$. If $m_F \leq 10^{-3} m_D$ the outer cores of each side of a potential vorton will overlap. It is then energetically favourable for the proto-vorton loop to decay, releasing the parent particles as radiation. This will avoid the vorton bounds, but different bounds arising from constraints on dilaton production will apply instead.

We can recalculate the vorton bounds taking $m_F = 10^{-2} m_D$. We find that there were no constraints on the model arising from the nucleosynthesis bound. If the vortons were stable enough to live to today, we found
\[ m_F \leq \left[ m_F m_c \left( \frac{\Gamma_1^2 m_F}{\sqrt{g_s m_p}} \right)^{-1/3} \right]^{1/2}, \tag{108} \]

\( m_c \sim 10^{-9} \text{ GeV is the closure mass). Thus for both } \Gamma, g_s \sim 10^2 \text{ and } m_D = 10^2 m_F \text{ we obtain the following bound} \]

\[ m_F \leq 10^6 \text{ GeV}. \tag{109} \]

This is less tight than the constraint we found in section 3. If we took \( m_F \sim m_D \) then there would only have one scale and we recover the results of section 3. Since \( m_F \) is essentially the mass of the dilaton, the usual choice of \( 10^3 \text{ GeV} \) would evade the vorton constraints.

We should now check our string loops for dilaton emission. Here the calculation is similar, but more complicated than that presented above. This is because the two scales result in the string loops being much larger than those considered in [34]. Using the favoured value of \( 10^3 \text{ GeV} \) for the dilaton mass we find

\[ \eta \leq 10^{14} \text{ GeV}. \tag{110} \]

Whilst this is still much less than the GUT scale it is higher than previous results, opening up the dilaton window in superstring models. We also consider direct dilaton emission from loop decay, but this gave a similar bound.

The final constraint on the theory we need to consider comes from structure formation. The string network will give rise to density perturbations in the microwave background. Experimentally these are detected at the level of about \( 10^{-6} \) [16]. This constrains \( g^2 \delta_{GS} \leq 10^{-6}, \) which is rather less than the usual string theory values.

7. Discussion

In these lectures we have considered the constraints on particle physics models arising from cosmic defects. We have seen that the presence of a current radically changes the cosmology of a cosmic string network, resulting in the formation of stable loops or vortons. By calculating the vorton density we were able to constrain the underlying theory in two ways. If vortons survive for only a few minutes, we demanded that the universe be radiation dominated at nucleosynthesis. If the vortons survive until the present time then we require that they do not overclose the universe. This puts stringent constraints on the theory, in particular in the chiral case.

We showed that cosmic strings arising in supersymmetric theories were current-carrying via fermion zero modes. In some cases the zero modes
survived supersymmetry breaking, so the vorton constraints applied. Chiral zero modes were obtained when the gauge symmetry was broken with a \( D \) term; in this case the zero modes survived supersymmetry breaking.

Cosmological constraints on models with decaying defects were obtained by demanding that the photons produced in the decay do not lead to spectral distortions of the CMB in excess of the observational limits from the COBE/FIRAS experiment. The strongest limits arise for theories leading to decaying vortons. Any theory giving rise to vortons resulting from a string forming phase transition above \( 10^5 \) GeV, and which decayed from a redshift of \( 10^6 \) to today, would produce a too large spectral distortion and thus be ruled out. This constraint applies both to vortons resulting from topological strings or those resulting from embedded strings which have become stabilised by plasma processes. This constraint is comparable to that found for chiral vortons and stronger than the other cases.

Finally we showed that cosmic strings arising in superstring theory were subject to constraints from dilaton production. We showed that in such models there are two distinct scales resulting in cosmic strings with an inner and outer core. This raises the possibility that such theories could evade vortons constraints, though they would still be subject to constraints arising from the overproduction of dilatons.

**Acknowledgments**

These lectures were presented at the Summer School on *Patterns of Symmetry Breaking*, supported by NATO and the European Science Foundation Programme *Cosmology in the Laboratory*. I would like to thank the organisers for the opportunity to deliver these lectures, and creating a friendly and stimulating environment. I would also like to thank my collaborators for the fun we have had over the years developing the ideas presented here, and my colleagues who read preliminary versions of these lectures. This work is supported in part by PPARC and the ESF.

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