Wellbore stability analysis during CO2 injection considering elastoplastic and continuous damage effects.

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Abstract. A double porosity finite element model (FEM) is used to analyse the stability of a wellbore during carbon dioxide (CO2) injection in a carbon capture and storage (CCS) scheme. Elastoplastic deformation and continuous damage effects were considered. Attention was given to the pore and fissure pressures of the rock surrounding the wellbore as these can create stresses which can lead to potential fracturing. The size of the wellbore was also discussed as well as the studied depth and the different injection pressures of CO2.

Keywords: Wellbore stability analysis, CO2 injection, FEM, Damage evolution

Introduction
The increase of the concentration of CO2 in the atmosphere has created serious environmental problems, NASA, and the Intergovernmental Panel on Climate Change (IPCC) have forecasted a temperature rise of 5.56℃ in the next century [1]. Carbon capture and storage is a very promising technology that can safely and permanently store CO2 in geological reservoirs. Direct injection of CO2 from a ship into deep formations is challenging and the pressure and temperature of injection need to be regulated as they can induce stresses on the surrounding rock which can lead to fracturing at the wellbore or even leakage before arriving its destination, the reservoir. CO2 is stored on a ship at around -50℃ and 0.7 MPa so that it remains a dense liquid. Prior its injection it is heated up to almost 4.5℃ and injected at pressures higher than 5 MPa so that it remains in the liquid phase for the whole process [2]. Consequently, appropriate constitutive models are needed to access the rock nearby the wellbore to ensure safe injection and storage of CO2 underground.

Field studies and experimental work to determine the thermo-hydraulic-mechanical effects on rock are difficult due to the occurrence of natural fractures, topological discontinuities, and difficulty in scaling to full size [3,4,5]. Numerical approaches are essential to reproduce different scenarios for wellbores and reservoirs at great depths [6,7]. Early flow-deformation models considered only single porous materials with the first one being Biot’s 1941 [8]. Afterwards, double porosity models appeared as rock formations do not only contain porous in the matrix but also fractures that separate the subdomain [4]. An accurate double porous constitutive model was proposed in [9, 10]. For the numerical modelling of rock stability during CO2 injection, [1] suggested that finite element modelling procedures seem to be the most reliable for simulating the coupling effects on double porous rock.

The main objective of this paper is to present different loading scenarios on different wellbores during CO2 injection, as well as the influence of the wellbore size to the stress distribution and porous/fracture pressures. Thermal effects due to CO2 injection were
neglected and the constitutive model and numerical model were validated against a one dimensional and double porosity consolidation in [4].

1. Constitutive modelling
The geoformation as a dual porosity system can be separated into the porous domain with porosity \( \phi_1 \) and fissure network with porosity \( \phi_2 \) [9]. The primary variables of the constitutive models are the fluid pressures in porous \( (p_1) \), fractures pressures \( (p_2) \) and the displacement vector \( (\mathbf{u}) \) of the matrix. The bulk and shear modulus of the double porous media change according to the pressure and damage of the domain. For this reason, a damage variable \( (D) \) was introduced and the damage evolution law of [10] was adopted and can be found at Appendix A. Additionally, the permeability evolution was considered for the two flow regions \( (k_1) \) and \( (k_2) \) respectively.

1.1. Damage evolution
Based on the power law for brittle materials, for ductile fracturing damage evolution \( (\dot{D}) \) can be expressed as follows by using [11, 12]:

\[
\dot{D} = \begin{cases} 
\frac{1}{m}Y^{1/m}, & Y > Y_D \\
0, & Y \leq Y_D 
\end{cases}
\]

(1)

where \( m > 1 \) is the material constant and can be determined by experimental results. \( H_Y \) and \( Y_D \) are not constant and are the damage evolution rate parameter and the threshold value of damage energy release rate. More details can be found in the Appendix A.

1.2. Deformation model
To simulate the deformation effects for the case of injecting CO\(_2\) inside a wellbore, a representative volume of fracture porous media of the wellbore wall was studied. The element was fully saturated, with a compressive fluid to represent the condition of the rock under the seabed. Ignoring the thermal effects of the injected CO\(_2\) and any internal acting effects, the governing equation for the deformation model was created based on [4] as follows:

\[
\text{div}\left( \frac{1}{2} \text{Sep}(D) (\nabla \mathbf{u}_d + \mathbf{u}_d \nabla) + \beta_1 p_1 \delta + \beta_2 p_2 \cdot \delta \right) + \mathbf{F} = 0
\]

(2)

where \( \text{Sep}(D) \) is the elastoplastic damage stiffness with damage evolution consideration and all the plasticity parameters are presented in the Appendix B. The contribution of pore and fracture fluid pressure to matrix deformation is illustrated by the effective stress parameters \( \beta_1 \) and \( \beta_2 \), which can be found in Appendix D. In Equation (2), \( \mathbf{F} \) is the body force per unit volume.

1.3. Flow model
Pore and fissure pressure will increase at the vicinity of the wellbore due to CO\(_2\) injection and the flow model needs to exist to describe the flow of the water from the wellbore wall to a distance many metres away, where only hydrostatic pressure exists. Combining Darcy’s law and the mass balance equation of the fluid and neglecting any viscous effects, the governing equation for fluid flow through a fully saturated deformable fractured porous media can be expressed, based on [10]:

Fluid flow in porous media:
\[ \text{div} \left( \frac{k_1}{\text{visc}} \left( \nabla p_1 + \rho_l g \right) \right) = -\beta_1 \text{div}\mu + \beta_{11} p_1 - \beta_{12} p_2 + \omega(p_1 - p_2) \] (3)

Fluid flow in fractured network:

\[ \text{div} \left( \frac{k_2}{\text{visc}} \left( \nabla p_2 + \rho_f g \right) \right) = -\beta_2 \text{div}\mu + \beta_{22} p_2 - \beta_{21} p_1 + \omega(p_2 - p_1) \] (4)

where \( \omega \) is the leakage term that is introduced in Appendix E to describe the fluid exchange between the two phases, \( g \) is the vector of gravitational acceleration, \( \text{visc} \) is the fluid viscosity and \( \rho_f \) is the density of fluid. All effective parameters \( \beta_{11}, \beta_{12}, \beta_{22}, \beta_{21} \) are presented in Appendix D and the permeability evolution in Appendix C.

2. Model setup and numerical implementation in FEM

Applications of the theoretical constitutive model are presented in this study. A vertical wellbore was considered based on the example of [15]. All the material parameters were taken from [4] and are presented in Table 1. The full numerical implementation can be found in [4, 10].

Four-noded square elements were used for the analysis. Each node has four degrees of freedom which represent the deformation, stress, pressure of the fluid in porous (\( p_1 \)), and pressure of the fluid in the fracture network (\( p_2 \)), respectively. As the wellbore is circular and symmetric, it is assumed that the pressure of the liquid CO\(_2\) at the same height will be the same everywhere inside the well. Consequently, as the loadings show symmetry, the problem is idealized as axisymmetric. A unit thickness (\( h = 1 \text{m} \)) of the wellbore was simulated with the outer radius being set to 15 m. On the vertical axis, the degrees of freedom were constrained, and the pressure of the CO\(_2\) was applied at the first two nodes as a boundary condition while radial stress and pore pressure were applied at the outer 15 m radius at the two last nodes. The total injection time was 80 sec following [4,15]. Two different values for the stiffness of the material (\( E=20 \text{ and } 100 \text{ GPa} \)) were examined for the case of damage evolution to represent one soft and one hard material. The mesh and boundary conditions are presented in Fig.1.

![Fig. 1. Finite element mesh for axisymmetric problem based on [4] (not to scale)](image)

Different coordinates were used during the numerical simulation according to the size of the wellbore to examine the influence of the radius to the results. These coordinates are presented in the Appendix F.
Table 1. Material Parameters

| Material parameters | [4,15,16] |
|---------------------|-----------|
| Elastic modulus [E(GPa)] | 20, 100  |
| Poisson’s ratio (ν) | 0.25  |
| Storativity of Porous block (β₁₁) [MPa⁻¹] | 7.23 × 10⁻⁹ |
| Storativity of fissure domain (β₂₂) [MPa⁻¹] | 1.8 × 10⁻⁹ |
| Effective stress parameter (β₁) | 0.99  |
| Effective stress parameter (β₂) | 0.01  |
| Fluid viscosity (visc) [MPa] | 10⁻⁹ |
| Leakage parameter (ω) [m²(KNs)⁻¹] | 5.44 × 10⁻⁵ |
| Porosity of the porous block (φ₁) | 0.04985 |
| Porosity of the fissure network (φ₂) | 0.001243 |
| Permeability of porous block (k₁) [m²] | 3.55 × 10⁻¹⁷ |
| Permeability of fissure network (k₂) [m²] | 7.2 × 10⁻¹⁴ |

The numerical simulation was validated against a wellbore stability post-drilling scenario using the results of [4]. In turn, [4] is validated against the results of [15] and shows good agreement with Terzaghi’s equation which was expanded to double porosity consolidation. Terzaghi’s equation is a classical benchmark problem used to verify mechanical responses of fully saturated domains. Fig. 2 shows the porous and fissure pressures produced by the models at 1000m depth. As can be seen from the figure, there is excellent agreement between the models.

![Fig. 2. Validation of borehole stability without CO₂ injection using the results of [4]](image)

3. Numerical Simulations

The purpose of this axisymmetric numerical model designed upwards was to investigate the influence of the injected CO₂ at the pore and fissure pressure near and far away from the wellbore wall, as well as to examine the effective created stresses at the rock nearby by changing every time the injection pressure, the depth and radius of the wellbore.

Assumptions made for the numerical simulation are the constant injection rate of CO₂ with a specific studied pressure, representing in this way a certain condition that the well is not affected by flux fluctuation, the initial damage of the material which was set to D = 0.015.
based on [4], and the absence of temperature thermal effects due to the difference between the injection temperature of the CO₂ and the existing in-situ temperature. Various cases were examined in this study.

The first application was to compare the same wellbore at three different depths during injection. CO₂ pressure was assumed to be 10 MPa and according to the depth difference, in-situ stresses and hydrostatic pressures were considered (Fig.3). Then, a certain depth of 1000 m was selected to discuss the influence of the injection pressure to the created effective stress nearby (Fig.4). As damage can be influenced from the confining pressure and from material’s stiffness, the damage evolution at two different depths 10m and 1000m was analysed (Fig.5 and Fig.6) for different injection pressures and different rocks. Finally, the influence of the wellbore radius to the pore/fissure pressure and to the effective stress distribution for a selected depth of 1000 m was examined (Fig.7).

3.1. Different depth results and injection pressures
For the scenario of an injection pressure at a certain depth of 1000 m at a specific time that the pressure at the wall had 10 MPa value, Fig.3 indicates that a small radial distance near the wellbore at the wellhead height is the most vulnerable to pore and fissure pressure. The pore pressure is increasing at the vicinity of the wellbore to almost 10 MPa and then immediately drops below 1 MPa at less than 0.2 m distance. This increased pore and fissure pressure can create fracturing in the material. For the case of 500m and 1000m depth, the gap is less between the existing pore pressure and the induced one.

It can also be seen that for the three occasions the fissure pressure is retaining some of its water pressure for longer distance because the permeability of the fracture is higher, which results in bigger concentration of water in this phase.

![Fig.3. Pore and fissure pressure comparison illustrated with different depths with the same CO₂ pressure (E=100GPa, r=0.1m, CO₂ injected pressure 10 MPa)](image)

![Fig.4. Effective radial and hoop stresses for different injection pressures (E=100GPa, r=0.1m)](image)

Apart from the pore and fissure pressure, effective radial and hoop stresses were examined for the specific depth of 1000 m and different times of injection were studied as different inputs at the pressure of the applied CO₂. It can be seen from Fig.4 that for higher injection pressures, the radial effective stress is suddenly increasing at the wellbore and then gradually until reaching the specific effective stress of the formation due to the material properties and
depth stresses. On the other hand, the existing hoop effective stress due to borehole construction is being reduced for larger CO₂ pressures as equilibrium between the injection and in-situ stresses is taking place.

The damage evolution due to CO₂ injection was also analysed. Fig.5 and Fig.6 present damage evolution according to the distance from the well for two different in strength materials until reaching the initial damage value of the formation. For the hard formation (E=100GPa), as can be seen at Fig.5, the maximum damage value created at the wellhead was almost two times bigger the initial one, while there was not any huge damage impact at high depths. For the softer rock with Young’s modulus of 20GPa, a huge increase in damage was spotted especially at the wellhead with a value of 0.3, which is 20 times higher than the initial value (Fig.6). There, the formation is not confined and consequently any pressure applied can have huge impact.

For both materials it can be outlined that at 1000 m depth a 5 MPa injection had larger impact on the damage than a 15 MPa injection. This is explained as the initial damage of the material in post-drilling scenario has already a high value due to existing hydrostatic and confining pressure nearby the wellbore wall at the specific depth. Then any applied pressure due to CO₂ injection is coming in equilibrium with the existing stresses and are reducing the damage effects.

![Fig.5. Damage evolution for different injection pressures and wellbore depths (E=100GPa)](image1)

![Fig.6. Damage evolution for different injection pressures and wellbore depths (E=20GPa)](image2)

3.2. Different radius

Fig.7 presents the effect of changing the radius of the wellbore to the effective stress and pore and fissure pressure nearby the wellbore. The studied depth was again the 1000m and the hard rock formation was considered.

It was identified from Fig.7 that the effective stress was barely influenced by the size of the radius of the bore hole. This can be explained as the injected CO₂ was considered to have a constant flux and pressure was not changing with increasing the depth. Additionally, it was noticed that for bigger radius the pore and fissure pressure dropped at their equilibrium value at a lower vertical distance from the well. Both fissure and porous network were achieving the same pressure at a smaller distance from the well.
Fig. 7. Effective radial and hoop stress/ Pore and Fissure pressure for different radius (E=100GPa, 1000m depth, 10MPa CO$_2$ injection)

4. Conclusions

Numerical simulations based on [10] are presented for different depth scenarios. Injection pressures of CO$_2$ and radius sizes were varied to determine their influence on the effective stress, pore and fissure pressure and damage nearby the wellbore. The height and the radius of the wellbore can influence the nearby pore and fissure pressure significantly, which can lead to potential fracturing and leakage of CO$_2$ inside the rock. The injection pressure affects the effective stress at the vicinity of the wellbore wall, which in combination with the created and existing in-situ stresses, can induce damage. The stiffness of the material is also critical as harder material has less damage impact than softer rock, especially at the wellhead. More simulations are needed including thermal stresses for different types of rocks and different depths of the wellbore. Thermal stresses can increase or decrease the effective stress and pore and fissure pressures nearby due to expansion or shrinkage of the rock matrix.

Experimental work is taking place at Newcastle university on the thermal effects on the surrounding rock. The existing constitutive model will be updated to include thermal stresses and the data generated will be used to validate the model. A robust thermo-hydraulic-mechanical model for double porosity medium influenced by CO$_2$ injection will be presented in the future.

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Appendix A

Damage evolution

\[ H_Y = h_y |n| \ln \left| \ln \left( \frac{M_{\text{crit}} + \eta}{2M_{\text{crit}}} \right) \right| \]  

(A.1)
\[ Y_D = Y_{D0} + \kappa \Delta \ln (p') \exp \left( \frac{1}{h_p} \right) \] (A.2)

Where \( M_{\text{crit}} \) is the slope of the critical state line based on [10] and \( \kappa \) is the slope of the unloading-reloading line (URL) in the \( v - \ln p' \) domain and characterises the increase in the rate of damage evolution. \( Y_{D0} \) is the initial value of damage evolution and \( \eta = \frac{q}{p'} \) is the ratio of the deviatoric stress to the applied pressure. \( h_p \) is a material constant and \( h_p \) is the plastic hardening modulus.

\[ Y = (1 - D) \frac{1000}{K(D)} \sqrt{d\sigma} \] (A.3)

\[ w = \frac{p'}{p'_{\text{conf}} + 1} \] (A.4)

Where \( d\sigma \) is the mean deviatoric stress. The \( p'_{\text{a}} \) is the maximum hydrostatic compressive pressure or the historic consolidated pressure and \( p'_{\text{conf}} \) the confining pressure at the certain depth.

### Appendix B

**Elastoplastic design with damage consideration**

The elastoplastic stiffness of the double porous material is mainly influenced by the stress and damage and can be expressed according to [13] as follows:

\[ \text{Sep}(D) = \left\{ [S^e(D)] - \frac{1}{H} \left[ S^c(D) \right] \left( \frac{\partial g}{\partial \sigma} \right) \left( \frac{\partial f}{\partial \sigma} \right)^T \right\} \] (B.1)

Where \( S^e(D) \) is the elastic stiffness of the material and is determined according to the elastic-damaged influenced bulk modulus \( K(D) \) and to the elastic-damage influenced shear modulus \( G^d(D) \) as below:

\[ S^e(D) = \begin{bmatrix} K(D) & 0 \\ 0 & 3G^d(D) \end{bmatrix} \] (B.2)

\( f \) is the plastic yield surface, \( g \) is the plastic potential and \( H \) is the plastic modulus. More details about the bounding surface plasticity can be found in [14].

The hardening modulus \( H \) is expressed based on [5] as follows:

\[ H = h_p + \left( \frac{\partial f}{\partial \sigma} \right)^T \cdot [S^e(D)] \cdot \left( \frac{\partial g}{\partial \sigma} \right) \] (B.3)
The damage elastic bulk modulus can be expressed based on [10] as follows:

$$K(D) = (1 - D)^2 \frac{\nu p^{\text{new}}}{\kappa}$$  \hspace{1cm} (B.4)

Where $p^{\text{new}}$ is a new different hydrostatic pressure, which will be lower compared to the hydrostatic pressure in the undamaged material, $\nu = 1 + e$ is the specific volume of the material with $e$ being the void ratio.

### Plasticity analysis parameters

| Parameter | Value |
|-----------|-------|
| Slope of Isotropic Compression line ($\lambda$) | 0.015 |
| Slope of critical state line ($M_{\text{crit}}$) | 1.7 |
| Bounding parameter (N) | 1.9 |
| Bounding parameter (R) | 2.45 |
| Slope of Unloading-Reloading line ($\kappa$) | 0.031 |
| Hardening parameter ($k_m$) | 10 |
| Hardening parameter ($k_d$) | 1 |

### Appendix C

#### Permeability evolution

The permeability evolution was expressed according to [13] as follows:

$$k = k_0 \left( \frac{\phi}{\phi_0} \right)^\Psi \left( \frac{1}{\tau} \right)^\zeta$$  \hspace{1cm} (C.1)

Where $k, k_0$ are the current and reference permeabilities, $\phi$ is the current porosity and $\phi_0$ is the reference, $\Psi$ and $\zeta$ are the parameters of the permeability and permeability resistance and $\tau$ the tortuosity.

### Appendix D

#### Governing Equations

For the three governing equations (deformation and two flow models) the effective stress parameters are:
\[ \beta_1 = \frac{C_p}{C_f} - \frac{C_s}{C_f} \quad (D.1) \]

\[ \beta_2 = 1 - \frac{C_p}{C_f} \quad (D.2) \]

\[ \beta_{11} = \varphi_1 C_{fl} + \left( \beta_1 - \varphi_1 \right) C_s + \beta_{12} \quad (D.3) \]

\[ \beta_{22} = \varphi_2 C_{fl} + \left( \beta_2 - \varphi_2 \right) C_s + \beta_{21} \quad (D.4) \]

\[ \beta_{12} = \beta_{21} = \left( \beta_1 \beta_2 - \varphi_1 \varphi_2 \frac{\beta_1 + \beta_2}{\varphi_1 + \varphi_2} \right) C_f \quad (D.5) \]

\[ \varepsilon_v = -\delta^T \varepsilon = -\text{div}\mathbf{u} \quad (D.6) \]

\[ \nabla_{\text{sym}} \mathbf{u} = \frac{1}{2} \left( \nabla \mathbf{u} + \mathbf{u} \nabla \right) = \dot{\varepsilon}_{\text{dam}}^p = \dot{\varepsilon} - \frac{2D}{1 - D} \varepsilon^e \quad (D.7) \]

Where \( \rho_f \) is the density of the fluid, \( C_{fl} \) is the fluid compressibility and \( \dot{\varepsilon}_{\text{dam}}^p \) is the damage plastic strain expressed in dependence of total strain rate and elastic strain.

\( c_p, c_f \) and \( c_s \) represent the drained tangent elastic compressibilities of the porous medium, the fracture porous domain and the solid skeleton.

**Appendix E**

**Leakage parameter (\( \omega \))**

The leakage parameter (\( \omega \)) was expressed based on [17] as follows:

\[ \omega = \frac{\hat{S} k_1}{\text{visc}} \quad (E.1) \]

\[ \hat{S} = \frac{4\bar{\omega} (\bar{\omega} + 2)}{l^2} \quad (E.2) \]

\[ l = \frac{3d_1 d_2 d_3}{(d_1 d_2 + d_2 d_3 + d_1 d_3)} \quad , \bar{\omega} = 3 \quad (E.3) \]

In which \( d_1, d_2, d_3 \) are the average fissure spacing, \( \bar{\omega} \) is the number of fissures and \( \text{visc} \) is the viscosity of the material which is given from literature.

**Appendix F**

**Mesh coordinates for numerical simulation**
Coordinates used for the mesh of the studied area near the wellbore.

- For radius $r=0.1$ the coordinates of the horizontal grid points are:
  $(0.1, 0.1025, 0.105, 0.1075, 0.110, 0.115, 0.120, 0.125, 0.130, 0.135, 0.140, 0.145, 0.150, 0.155, 0.160, 0.17, 0.180, 0.190, 0.20, 0.22, 0.25, 0.30, 0.50, 0.7, 1, 1.5, 2, 2.5, 3, 4, 5.5, 8, 10, 15)$.

- For radius $r=0.15$ the coordinates of the horizontal grid points are:
  $(0.15, 0.1525, 0.155, 0.1575, 0.16, 0.1615, 0.162, 0.1625, 0.163, 0.1635, 0.164, 0.1645, 0.1650, 0.1660, 0.167, 0.1690, 0.17, 0.18, 0.19, 0.20, 0.22, 0.250, 0.3, 0.5, 0.7, 1, 1.5, 2, 3, 4, 5.5, 8, 10, 15)$

- For radius $r=0.2$ the coordinates of the horizontal grid points are:
  $(0.2, 0.2025, 0.205, 0.2075, 0.210, 0.215, 0.220, 0.225, 0.230, 0.235, 0.240, 0.245, 0.250, 0.255, 0.260, 0.27, 0.280, 0.290, 0.30, 0.32, 0.35, 0.40, 0.50, 0.7, 1, 1.5, 2, 2.5, 3, 4, 5.5, 8, 10, 15)$

- For radius $r=0.3$ the coordinates of the horizontal grid points are:
  $(0.3, 0.3025, 0.305, 0.3075, 0.310, 0.315, 0.320, 0.325, 0.330, 0.335, 0.340, 0.345, 0.350, 0.355, 0.360, 0.37, 0.380, 0.390, 0.40, 0.42, 0.45, 0.50, 0.60, 0.7, 1, 1.5, 2, 2.5, 3, 4, 5.5, 8, 10, 15)$