A Measure for the Adiabaticity of Quantum Transitions

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We construct a measure for the adiabaticity of quantum transitions in an arbitrary basis. Small deviations from nearly completely adiabatic quantum dynamics can be easily quantified through integrals of non-adiabatic couplings between instantaneous eigenstates, as is usually done. In contrast, the situation is more involved in generic quantum dynamics that may be only partially adiabatic and when transitions between non-eigenstates are of interest. Targeting the latter case, we demonstrate with several examples, how the measure is able to distinguish transitions between basis states that occur due to the adiabatic change of a populated energy eigenstate from those that occur due to beating between several such eigenstates.

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I. INTRODUCTION

The adiabatic evolution of a selected quantum state, modified via slow temporal changes of a Hamiltonian is of widespread utility for quantum state engineering and unravelling complex quantum dynamics. Examples are adiabatic quantum computation protocols [1–4], quantum optimisation [5] and nuclear motion in molecules [6]. Adiabaticity is also frequently central to state preparation schemes in cold atomic physics such as stimulated Raman adiabatic passage (STIRAP) [7] and other adiabatic quantum state transfer schemes [8–10].

Due to the central role of adiabaticity in all of the above, it may be of interest to quantify how adiabatic a certain dynamical process is. If only net total adiabaticity is of interest, this is relatively straightforward using the non-adiabatic coupling terms in Schrödinger’s equation for the instantaneous eigenbasis. These therefore have been used extensively to constrain conditions under which evolution due to a certain Hamiltonian should remain adiabatic. Nonetheless such conditions remain non-trivial to identify in general [11–13].

Many of the protocols above begin with the system in a specific initial quantum state and are designed such that it reaches a specific target state, following an eigenstate of the Hamiltonian. In contrast, there are scenarios in which the initial state is less clear or random and in which we care to what extent quantum transitions in a non-eigenbasis are due to adiabatic evolution or would have also occurred without manipulation of the Hamiltonian. One such scenario is the quantum transport of an electronic excitation in a molecular aggregate [14] through molecular motion [15–20]. There, the instantaneous eigenbasis diagonalizes long-range dipole-dipole interactions between molecules in the aggregate and thus is made of states describing a delocalized excitation, while the basis most useful to study quantum transport is made of states in which the electronic excitations is localized on a certain molecule.

FIG. 1: Quantum transitions due to beating (c) versus adiabatic changes (d). In (c) we show the population in three states $|A\rangle$, $|B\rangle$ and $|C\rangle$ for the level scheme shown in (a) with constant $\Delta E = 3$ and $J_0 = 2$. Population periodically reaches $|C\rangle$ due to beating between eigenstates. (d) For a suitable time-dependent variation of coupling strengths $J_k(t)$ as sketched in (b) with constants $J_{10} = J_{20} = 8$ and same $\Delta E$ as in (a), the population reaching $|C\rangle$ can be significantly increased through adiabatic following of eigenstates. Discriminating contributions as in (b) from those as in (d) is the central objective of this article.

Here we propose a measure to quantify whether transitions in an arbitrary basis are due to adiabatic following of time-evolving eigenstates or rather due to beating between different eigenstates in a superposition. Our motivation is to clarify, whether the enhanced energy transport efficiencies in molecular aggregates due to molecular motion reported in [15] can be attributed to adiabaticity. However, we expect that the measure is of much broader utility.

We illustrate the challenge to be addressed in Fig. 1.
with an abstract three-level system in quantum state

$$|\Psi(t)\rangle = \sum_{n \in \{A,B,C\}} c_n(t)|n\rangle$$ \hspace{1cm} (1)

and with Hamiltonian

$$\hat{H}(t) = \sum_n E_n|n\rangle\langle n| + J_1(t)[|A\rangle\langle B| + \text{c.c.}] + J_2(t)[|B\rangle\langle C| + \text{c.c.}].$$ \hspace{1cm} (2)

When the couplings are constant, $J_k(t) = J_0$, a system initialized in state $|A\rangle$ will typically eventually reach state $|C\rangle$ with some probability, since $|A\rangle$ is not an eigenstate and the initially populated eigenstates will in general contain a $|C\rangle$ contribution. In that case, transport to $|C\rangle$ thus arises through beating of eigenstates. In contrast, consider time dependent couplings $J_1(t)$ and $J_2(t)$ of the form [4],

$$J_1(t) = J_{10}\sin^2\left(\frac{\pi t}{2t_{\text{max}}}\right),$$ \hspace{1cm} (3)

$$J_2(t) = J_{20}\cos^2\left(\frac{\pi t}{2t_{\text{max}}}\right),$$ \hspace{1cm} (4)

where $J_{10}$ and $J_{20}$ are the maximal coupling strength and $t_{\text{max}}$ defines the time-scale for changes of this coupling. As we show in Fig. 1, this can result in a final state $|C\rangle$ with unit fidelity, without ever populating more than one eigenstate. Any generic time-evolution will contain both these features, as we shall see later. Obtaining a relative measure for the importance of the latter, adiabatic, type of transition is our objective here.

The article is organised as follows: In section II we formulate our proposal, first reviewing the general framework of quantum dynamics in a time-independent basis or the adiabatic basis and then proceeding to quantify adiabatic state changes based on this framework.

II. ADIABATIC TRANSITION MEASURE

We now formulate our proposal, first reviewing the general framework of quantum dynamics in a time-independent basis or the adiabatic basis and then proceeding to quantify adiabatic state changes based on this framework.

A. Quantum dynamics

Consider a generic time-dependent Hamiltonian $\hat{H}(t)$ with a discrete spectrum and let $\{|n\rangle\}$ be an arbitrary but time-independent basis of its Hilbertspace, which we refer to as diabatic basis. Meanwhile $|\varphi_k(t)\rangle$ are a solution of the instantaneous eigenproblem

$$\hat{H}(t)|\varphi_k(t)\rangle = U_k(t)|\varphi_k(t)\rangle,$$ \hspace{1cm} (5)

for energy $U_k(t)$, which also form a basis, referred to as adiabatic basis. The total time-evolving state $|\Psi(t)\rangle$ can be expressed in either basis as $|\Psi(t)\rangle = \sum_n c_n(t)|n\rangle$ or $|\Psi(t)\rangle = \sum_k \tilde{c}_k(t)|\varphi_k(t)\rangle$, which defines two different sets of expansion coefficients, related by $\tilde{c}_k(t) = \sum_n U_{kn}(t)c_n(t) = \sum_n \langle \varphi_k(t)|n\rangle c_n(t)$. Projecting Schrödinger’s equation into either basis we reach

$$i\hbar \frac{\partial}{\partial t} c_n(t) = \sum_m H_{nm}(t)c_m(t)$$ \hspace{1cm} (6)

or

$$i\hbar \frac{\partial}{\partial t} \tilde{c}_k(t) = U_k(t)\tilde{c}_k(t) - i\hbar \sum_m \kappa_{km}\tilde{c}_m(t).$$ \hspace{1cm} (7)

It can be seen from (7), that as long as the non-adiabatic couplings $\kappa_{km} = \langle \varphi_k(t)|\frac{\partial}{\partial t}|\varphi_m(t)\rangle$ remain small, the system evolves adiabatically with all populations of eigenstates fixed to their initial value $|\tilde{c}_k(t)|^2 = |\tilde{c}_k(0)|^2$. Thus deviations of these populations from their initial value provide a measure of non-adiabaticity while the size of non-adiabatic coupling terms provides a measure of instantaneous non-adiabaticity.

The situation becomes more subtle if one is instead interested in the root cause of population changes in the basis $\{|n\rangle\}$, for which there are two possibilities: (i) The system may be in a superposition of eigenstates, such as $|\Psi(0)\rangle = (|\varphi_1(0)\rangle + |\varphi_2(0)\rangle)/\sqrt{2}$. Then, even for a time-independent Hamiltonian, the population in a given basis state $p_n = |\langle n|\Psi(t)\rangle|^2$ will experience beating

$$p_n = \frac{1}{2}\left(|U_{1n}|^2 + |U_{2n}|^2 + \text{Re}[U_{2n}U_{1n}^*\cos \left(\frac{(U_2 - U_1)t}{\hbar}\right)] + \text{Im}[U_{2n}U_{1n}^*\sin \left(\frac{(U_2 - U_1)t}{\hbar}\right)]\right).$$ \hspace{1cm} (8)

(ii) The system may be in a unique eigenstate, such as $|\Psi(0)\rangle = |\varphi_1(0)\rangle$, but the Hamiltonian is time-dependent, such that the population $|\langle n|\Psi(t)\rangle|^2 \approx |\langle n|\varphi_0(t)\rangle|^2$ varies due to the resultant change of that eigenstate. In a generic quantum dynamics scenario, both contributions mix and are non-straightforward to disentangle.

B. Extracting adiabatic contributions

We shall refer with $d_n^{(k)}(t) = \langle n|\varphi_k(t)\rangle$ to the component amplitude in basis state $|n\rangle$ for system eigenstate $|\varphi_k(t)\rangle$. Using this shorthand, we can write the diabatic amplitude as

$$c_n(t) = \sum_k \tilde{c}_k(t)d_n^{(k)}(t),$$ \hspace{1cm} (9)

and hence the population in state $|n\rangle$ as

$$p_n(t) = |c_n(t)|^2 = \sum_{k,k'} \tilde{c}_k^{*}(t)\tilde{c}_k(t)d_n^{(k)}(t)d_n^{(k')}^{*}(t).$$ \hspace{1cm} (10)
Its rate of change using the chain rule is
\[
p_n(t) = \sum_{k,k'} \left[ (\dot{c}^*_k(t) \, c_k(t) + \dot{c}^*_k(t) \, \check{c}_k(t)) \, d_n^{(k)}(t) \, d_n^{(k')*}(t) \right. \\
+ \left. \dot{c}^*_k(t) \, \check{c}_k(t) \left( d_n^{(k)}(t) \, d_n^{(k')*}(t) + d_n^{(k)}(t) \, \check{d}_n^{(k')*}(t) \right) \right]
\]
(11)

The last line already clearly contains contributions to \(p_n(t)\) from temporal changes of \(|c_k(t)\rangle\) and thus will be related to adiabatic state following. Let us inspect the first line more closely for the case of a time-independent Hamiltonian. In that case we simply have \(\check{c}_k(t) = \check{c}_k(0) \exp \left[-iU_k t/\hbar \right]\) and hence
\[
\dot{c}^*_k(t) \, \check{c}_k(t) + \dot{c}^*_k(t) \, \check{c}_k(t) \nonumber \\
= i \frac{E_k - E_{k'}}{\hbar} d_n^{*}(0) \, \dot{c}_k(0) e^{-i \left( \frac{E_k - E_{k'}}{\hbar} \right) t}. 
\]
(12)

This expression is non-zero even for a time-independent Hamiltonian, simply quantifying the temporal changes of \(p_n(t)\) due to beating between different eigenstates as in [8]. Importantly, the resultant time-dependence for this term affects the phase of (12) only, not the modulus.

To exploit this, let us write the coefficient \(\check{c}_k(t)\) in polar representation \(\check{c}_k(t) = \tilde{a}_k(t) e^{i\tilde{b}_k(t)}\) with \(\tilde{a}_k, \tilde{b}_k \in \mathbb{R}, \tilde{a}_k > 0\). Then \(\dot{\tilde{c}}_k(t) = \tilde{a}_k(t) e^{i\tilde{b}_k(t)} + \tilde{a}_k(t) i \tilde{b}_k(t) e^{i\tilde{b}_k(t)}\). We insert this expansion into (11), remove the phase evolution \(\tilde{b}_k\) and introduce a new notation for the remaining expression:
\[
t_n(t) = \sum_{k,k'} \left[ \tilde{a}_k(t) e^{-i\tilde{b}_k(t)} \, \check{c}_k(t) \\
+ \tilde{a}_k(t) e^{i\tilde{b}_k(t)} \left( d_n^{(k)}(t) \, d_n^{(k')*}(t) + d_n^{(k)}(t) \, \check{d}_n^{(k')*}(t) \right) \right]
\]
(13)

The resultant real variable \(t_n(t)\) is now a measure for the rate of change of the population in state \(n\) due to temporal changes in the eigen-spectrum of the Hamiltonian. By construction it does not contain any contribution from beating between several occupied eigenstates.

C. Assembling the measure

We now further proceed to reach a single number to quantify adiabatic transitions between basis states using the \(t_n(t)\). Several variants will be possible and the best choice may depend on the type of quantum dynamics for which one intends to characterise adiabaticity. We shall explore the following two options. **Variant 1:**
\[
T_1(t) = \frac{1}{2} \sum_n \int_0^t dt' |t_n(t')|.
\]
(14)

The expression shall give unity if a system makes a transition from one state \(|a\rangle\) into a second state \(|b\rangle\) entirely due to adiabatic following of a single eigenstate. It does treat transitions between all basis states \(|n\rangle\) on equal footing and provides a time-averaged result for the entire duration \(t\) of interest.

However owing to the modulus, (14) also treats transitions into some state equivalent to transitions out of that state, which can be problem in some cases as demonstrated shortly. That problem is remedied by **Variant 2:**
\[
T_2(t) = \int_0^t dt' X(t'),
\]
(15)
evaluated for a target state \(|X\rangle\) only.

We shall explore characteristic features of measures (14) and (15) in the next section for a diverse selection of examples, in the context of Ref. [13].

III. ADIABATIC EXCITATION TRANSPORT IN MOLECULAR AGGREGATES

A. Model

We first briefly review the specific scenario of Ref. [15] as a setting where a measure of adiabaticity is desirable for physical analysis, and difficult to obtain due to dynamics being at best partially adiabatic and the quantum state usually involving a superpositions of eigenstates.

To model a molecular aggregate, we consider \(N\) monomers of some molecular dye with mass \(M\), arranged in a one dimensional (1D) chain along the \(X\) direction, as sketched in Fig. 2(a). The positions of the molecules are given by \(X = (X_1, X_2, \ldots, X_N)\) i.e., the \(n\)th monomer is located at a definite, classical position \(X_n\) and treated as a point particle. Adjacent monomers are assumed to bind to each other with a Morse type potential
\[
V_{mn}(X) = D_e \left[ e^{-2\alpha(|X_{mn}| - X_0)} - 2 e^{-\alpha(|X_{mn}| - X_0)} \right],
\]
(16)
where \(D_e\) is the depth of the well, \(|X_{mn}| = |X_n - X_m|\) the separation of monomers \(n\) and \(m\) with \(X_0\) its equilibrium value and \(\alpha\) controls the width of the binding potential, shown in Fig. 2(b).

Additionally each monomer may be in an electronically excited state \(|e\rangle\) or ground-state \(|g\rangle\). Among the resultant many-body states, we restrict ourselves to the so-called single-exciton manifold, where just monomer number \(n\) is excited, this state is denoted by \(|n\rangle\). The excited state can then migrate to any other monomer via long-range dipole-dipole interactions. Altogether we thus have a classical Hamiltonian for molecular motion
\[
H_{\text{class}} = \sum_{n=1}^{N} \frac{1}{2} M X_n^2 + \sum_{n<m} V_{mn}(X),
\]
(17)
and a quantum mechanical one for excitation transport through dipole-dipole interactions,

\[
\hat{H}(X) = \sum_{n=1}^{N} E_n \langle n \rangle \langle n \rangle + \sum_{n,m=1 \atop n \neq m} \frac{\mu^2}{|X_{mn}|^2} \langle n \rangle \langle m \rangle,
\]

where \(E_n\) is the electronic transition energy of the \(n\)th monomer and \(\mu\) is the transition dipole moment. We find the system dynamics in a quantum-classical approach, where the motion of the molecules is treated classically using Newton’s equations,

\[
M \frac{\partial^2}{\partial t^2} X_n = -\frac{\partial}{\partial X_n} U_s(X) - \sum_m \frac{\partial}{\partial X_n} V_{mn}(X).
\]

Here \(U_s(X)\) are the potential energy surfaces defined by using the adiabatic basis \(|\varphi_s(X(t))\rangle\) as in \([9]\), i.e. solving \(H(X)|\varphi_s(X(t))\rangle = U_s(X(t))|\varphi_s(X(t))\rangle\). The dynamics of excitation transport is obtained by write the electronic aggregate state as \(|\Psi(t)\rangle = \sum_n c_n(t)\rangle\) and using Schrödinger equation,

\[
i \frac{\partial}{\partial t} c_n = \sum_{m=1}^{N} H_{mn}[X_{mn}(t)] c_m.
\]

Here \(H_{mn}[X_{mn}(t)]\) is the matrix element \(\langle n | \hat{H} | m \rangle\) for the electronic coupling in Eq. \((18)\). We have used a similar model in Ref. \([15]\) to show that thermal motion of molecules can enhance the transport of excitation in the presence of disorder, compared to the case where molecules are immobile. That research was motivated by earlier results proposing excitation transport due to adiabatic quantum state following in an ultra-cold atomic system \([21, 22]\). However, in the more complex molecular setting, clearly tagging a contribution of adiabaticity to quantum transport is more challenging and shall be explored in the following. For these simulations and the following ones we have taken \(\mu = 1.12\) a.u., and \(M = 902330\) a.u. roughly matching carbonyl-bridged triaryl-amine (CBT) dyes \([23]\).

### B. Simple Test cases

We first investigate a few simple scenarios, intended to demonstrate that the measures in section II C give useful results in clear cut cases, shown in Fig. 3 for a trimer aggregate.

![FIG. 2:](image)

(a) Energy level schematic for a one dimensional chain of \(N\) molecules, with electronic ground state \(|g\rangle\), excited state \(|e\rangle\), dipole-dipole interaction \(V_{dd}(X)\) and \(E_n\) the site energy of the \(n\)th molecule. (b) Inter-molecular Morse potential for \(\alpha = 0.3\) Å\(^{-1}\) (blue dashed) and \(\alpha = 0.9\) Å\(^{-1}\) (red dot-dashed) and the strength of dipole-dipole interactions \(V_{dd}^{(dd)}\) (red solid line).

![FIG. 3:](image)

FIG. 3: Exciton transport in a trimer aggregate. The first row (1a - 4a) shows the trajectories of individual molecules (white lines) with the excitation probabilities of each molecule shown by the color shading. The second row (1b-4b) shows the excitation probability for each molecule \(p_n = |c_n|^2\) (adiabatic populations) and the third row (1c-1d) of each exciton state \(\bar{p}_n = |\bar{c}_n|^2\) (adiabatic populations). The inset in panel (4c) shows a zoom onto one of these populations. Finally the fourth row (1d-4d) shows the proposed adiabaticity measures \([14]\) (solid blue) and \([15]\) for target site #3 (red dashed). The columns differ by initial state and parameters as discussed in the text.

The first column (1a-1d) shows an immobile case with \(M \to \infty\). At \(t = 0\) the excitation is initialized on site #1. The state \(|1\rangle\) is not an eigenstate, and population quickly reaches site #3 in the resultant beating. Since in
this case eigenstates do not evolve, our adiabacity mea-
sures defined by Eq. (14) and Eq. (15) yield zero per
construction as can be seen in (1d).

In the second column (2a-2d), monomers are mobile
and the excitation initially shared between site one and
two such that the initial electronic state is given by,

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|n = 1\rangle + |n = 2\rangle).$$  \hspace{1cm} (21)

Our parameters are adjusted such that the excitation
reaches the output site solely due to adiabatic quantum
state following, as in [21][22]. This can be inferred from
all population remaining constantly in the initially oc-
cupied eigenstate (2c). At the moment where the excita-
tion has reached the output site with probability $p = 1/2$ at
about $t = 0.5$ ps, also the measures $T_1 = T_2 = 1/2$,
indicating that transport has been entirely adiabatic.

For the third column (3a-3d), we give the second
molecule a significant initial velocity, such that the quan-
tum dynamics no longer is adiabatic. Hence we see in
(3c) that the population in the initially occupied exciton
state has dropped to 0.5 by the time $t = 0.01$ ps. The
adiabaticity measures shown in (3d) are accordingly de-
creased by a factor of about 1/2, compared to the ideal
adiabatic transport in the second column.

For the examples discussed so far, the two measures $T_1$
and $T_2$ by and large agree. However, this is not the case
in the last column (4a-4d) in Fig. 3. It shows transport
where the molecules are allowed to oscillate around their
equilibrium separation after been given random initial
offsets and velocity from a thermal distribution at room
temperature. The initial electronic state at $t = 0$ is as-
sumed to be first site (#1)

$$|\psi(0)\rangle = |n = 1\rangle.$$  \hspace{1cm} (22)

Adiabatic populations for this case show no significant
net change over longer times, but exhibit fast small am-
plitude oscillations, as seen in (4c) and its inset. Any
change in site populations due to motion must be periodic
due to the periodicity of molecular trajectories shown in
(4a). In this more involved case, our measure $T_1$ shows a
slow steady increase, since both, population increase and
decrease on the target site are cumulatively contributing.
This problem is removed for measure $T_2$, as can be seen
in (4d), which is thus here more effective in identifying
long-term useful adiabatic contributions to transport.

C. Thermal motion of molecules

While the examples in section III B were designed to
show the basic functionalities of the measures intro-
duced in section II C for simple cases, we now proceed
to benchmark (14) and (15) in a more complex setting:
energy transport in thermally agitated molecular aggrega-
tes.

For this, Fig. 4 shows the dynamics of excitation trans-
port for molecular aggregates at temperature $T = 300$
K and with increasing energy disorder from (1a) to
(4a). Energy disorder arises due to the coupling of the
monomers with the environment causing slightly differ-
ent transition energy shifts as sketched in Fig. 2. We
assume the energy disorder is Gaussian distributed with
a standard deviation $\sigma_E$,

$$p_E(E_n - E_0) = \frac{1}{\sqrt{2\pi}\sigma_E}e^{-(E_n - E_0)^2/(2\sigma_E^2)},$$  \hspace{1cm} (23)

where $E_n$ is the site energy of $n$’th molecule defined in
Eq. (18) and $E_0$ is the unperturbed electronic transition
energy of each molecule.

FIG. 4: Exciton dynamics similar to Fig. 3 but for a larger
system and with thermally induced motion. The first row
(1a - 4a) shows the trajectories of individual molecules (white
lines) with the excitation probabilities of each molecule shown
by the color shading. The second row (1b-4b) shows the excita-
tion probability on the output site $p_5 = |c_5|^2$ (solid blue)
and its cumulative maximum (red dashed). (1c-1d) Population
of each adiabatic state $p_n = |c_n|^2$. (1d-4d) Adiabaticity
measures (14) (solid blue) and (15) for target site #5 (red
dashed). The parameters varied were for the first column
($\sigma_E, \alpha = (150 \text{ cm}^{-1}, 0.5 \text{ Å}^{-1})$, for the second column ($\sigma_E, \alpha = (300 \text{ cm}^{-1}, 0.5 \text{ Å}^{-1})$, for the third column($\sigma_E, \alpha = (450 \text{ cm}^{-1}, 0.5 \text{ Å}^{-1})$ and for the fourth column ($\sigma_E, \alpha = (550 \text{ cm}^{-1}, 0.3 \text{ Å}^{-1})$).

As in section III B the initial state of excitation is given
by Eq. (22). For the first column (1a-1d) in Fig. 2, the
disorder in energy is relatively small compared to the
electronic dipole-dipole coupling. Due to the weak dis-
order, the excitation can reach the output-site with high
amplitude at early times, before motion had a chance to
impact dynamics. In Ref. [15] we quantify transport ef-
ciciency through the maximum of population on the out-
put site (here 5) over the time of interest, shown as a
red-dashed line in row (b). Probing the adiabaticity mea-
sures at the times where this maximum increases, gives
a correctly constantly low contribution from adiabaticity
only from measure $T_2$, not from $T_1$. The reason is as
discussed for column 4 in Fig. 3.
For slightly increased disorder, in column two, the adiabatic populations show some sharp changes indicating the onset of non-adiabaticity. However this implies that the eigenstate are actually significantly changing in time, and the population that is adiabatically retained will contribute to adiabatic transport. This is now heralded by a significantly larger measure $T_2$ in Fig. 4 (2d) indicating that a fraction of transport to the site #5 is adiabatic.

For the example in column 3, significant adiabatic transport can now be inferred directly from panels (3a), (3c), since exciton populations remain fairly adiabatic while almost the complete site population is transferred from #1 to #3. This leads to stepwise increases in measure $T_1$, however not impacting $T_2$ since the latter is based on site #5 which was not involved.

Finally the fourth column shows a clear cut case where excitation is transported from site 1 directly to 5, since energy disorder has rendered all intervening sites off resonant, while the adiabatic population remains constant. Consequently this shows up as nearly identical step-like increase in both measures.

We have seen that both measures give adequate results for certain regions in parameter-space, however care has to be taken with $T_1(t)$ in Eq. (14) for cases where this adds up fast in- and out- transfer of population among basis states, that does however not yield a significant net transition when averaged over longer times. This is alleviated by measure $T_2(t)$ in Eq. (15) at the expense of being sensitive only to transitions into one specific state.

### IV. CONCLUSIONS

We have constructed a measure that is able to quantify the extent to which adiabatic following of the eigenstates of a quantum system are the root cause of quantum transitions in another basis than the eigenbasis. The basic functionality of the measure was first demonstrated with a few simple examples where adiabaticity is either not at all related to transitions or completely responsible for them. We then explored its behaviour in more complex settings, the main feature of all of which was that transitions due to adiabatic transport following to beating between eigenstates happen concurrently. These examples demonstrate that the measures proposed can at least give a relative indication of the importance of adiabaticity. This can then be useful to assess, whether adiabaticity is significantly impacting some quantum dynamics of interest in a desirable way, in which case known results regarding adiabaticity can be applied in order to enhance its effect.

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