EXISTENCE AND NONEXISTENCE OF ENTIRE POSITIVE RADIAL SOLUTIONS FOR A CLASS OF SCHRÖDINGER ELLIPTIC SYSTEMS INVOLVING A NONLINEAR OPERATOR

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ABSTRACT. In this paper, we study the positive solutions of the Schrödinger elliptic system
\[
\begin{align*}
\text{div}(G(|\nabla y|^{p-2})\nabla y) &= b_1(|x|)\psi(y) + h_1(|x|)\phi(z), \quad x \in \mathbb{R}^n \quad (n \geq 3), \\
\text{div}(G(|\nabla z|^{p-2})\nabla z) &= b_2(|x|)\psi(z) + h_2(|x|)\phi(y), \quad x \in \mathbb{R}^n,
\end{align*}
\]
where $G$ is a nonlinear operator. By using the monotone iterative technique and Arzela-Ascoli theorem, we prove that the system has the positive entire bounded radial solutions. Then, we establish the results for the existence and nonexistence of the positive entire blow-up radial solutions for the nonlinear Schrödinger elliptic system involving a nonlinear operator. Finally, three examples are given to illustrate our results.

1. Introduction and preliminary. The purpose of this paper is to show the existence and nonexistence of entire positive bounded and blow-up radial solutions of the following nonlinear Schrödinger elliptic system involving a nonlinear operator
\[
\begin{align*}
\text{div}(G(|\nabla y|^{p-2})\nabla y) &= b_1(|x|)\psi(y) + h_1(|x|)\phi(z), \quad x \in \mathbb{R}^n \quad (n \geq 3), \\
\text{div}(G(|\nabla z|^{p-2})\nabla z) &= b_2(|x|)\psi(z) + h_2(|x|)\phi(y), \quad x \in \mathbb{R}^n \quad (n \geq 3),
\end{align*}
\]
where $G$ is a nonlinear operator on $\Theta = \{G \in C^2([0, +\infty), (0, +\infty)) \mid \text{there exists a constant } p > 2 \text{ such that for all } 0 < t < 1, G(t) \leq t^{p-2}G(t)\}$.

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In (1), it is assumed that \( b_i, h_i, (i = 1, 2) \), \( \psi \) and \( \varphi \) satisfy the following conditions.

(S\(_1\)) \( : b_i, h_i \in C([0, +\infty), (0, +\infty)), (i = 1, 2); \)

(S\(_2\)) \( : \psi, \varphi \in C([0, +\infty), (0, +\infty)) \) are nondecreasing;

(S\(_3\)) \( : \) There exists a constant \( 0 < \varepsilon < p - 1 \) such that

\[
\psi(kt) \geq k^\varepsilon \psi(t) \quad \text{and} \quad \varphi(kt) \geq k^\varepsilon \varphi(t), \quad k \in (0, 1].
\]

Denote

\[
B(r) = \int_0^r R^{-1} \left( \frac{1}{t^{n-1}} \int_0^t s^{n-1}(b_1(s) + h_1(s)) ds \right) dt, \quad r \geq 0,
\]

\[
H(r) = \int_0^r R^{-1} \left( \frac{1}{t^{n-1}} \int_0^t s^{n-1}(b_2(s) + h_2(s)) ds \right) dt, \quad r \geq 0,
\]

\[
B(\infty) := \lim_{r \to \infty} B(r), \quad H(\infty) := \lim_{r \to \infty} H(r),
\]

and

\[
A(r) = \int_a^r \frac{dt}{(\psi(t) + \varphi(t) + 1)^{\frac{1}{p-1}}}, \quad r \geq a > 0, \quad A(\infty) := \lim_{r \to \infty} A(r).
\]

It is not difficult to know that \( A'(r) = \frac{1}{(\psi(r) + \varphi(r) + 1)^{\frac{1}{p-1}}} > 0, \forall r > a, \) besides, \( A \) has an increasing inverse function \( A^{-1} \) on \([0, \infty)\).

In the sequel, we assume that

(A\(_1\)) \( : A(\infty) = \infty; \)

(A\(_2\)) \( : A(\infty) < \infty; \)

(A\(_3\)) \( : B(\infty) = \infty, H(\infty) = \infty; \)

(A\(_4\)) \( : B(\infty) < \infty, H(\infty) < \infty. \)

First let us review the following model involving the classical Laplacian operator

\[
\triangle z = b(|x|)\psi(z) + h(|x|)\varphi(z), \quad x \in \mathbb{R}^n.
\]

Problem (2) grows out of various areas of applied mathematics and physics \([3, 8, 9, 10, 13, 14]\). Some of the basic results are listed as follows. In 2019, Covei showed the existence of positive bounded and blow-up radially symmetric solutions to the problem (2) and described a real-world model in which such problems may arise. For more detailed information, please refer to Section 3 in \([3]\). When \( \psi(z) = z^\alpha, \ \varphi(z) = z^\beta \) and \( 0 < \alpha \leq \beta \), Lair \([9]\) studied the existence and nonexistence of entire positive blow-up radial solutions to the problem (2). When \( \psi(z) \) satisfies (S\(_2\)), \( b(|x|) = 1 \) and \( h(|x|) = 0 \), Keller \([8]\) and Osserman \([14]\) first gave a necessary and sufficient condition

\[
\int_1^\infty \frac{1}{\sqrt{2\Psi(x)}} dx = \infty, \quad \Psi(x) = \int_0^x \psi(t) dt,
\]

for the existence of entire positive blow-up radial solutions to the problem (2).

Subsequently, considering the following semi-linear elliptic system

\[
\begin{align*}
\triangle y &= b(|x|)\psi(z), \quad x \in \mathbb{R}^n (n \geq 3), \\
\triangle z &= h(|x|)\varphi(y), \quad x \in \mathbb{R}^n.
\end{align*}
\]

Li, Zhang and Zhang \([13]\) investigated the existence of entire positive bounded and blow-up radial solutions to the system (3).
Moreover, when \( \psi(z) = z^\alpha, \varphi(y) = y^\beta \) and \( 0 < \alpha, \beta \leq 1 \), the system (3) is transformed into the following system

\[
\begin{align*}
\Delta y &= b(|x|)z^\alpha, \quad x \in \mathbb{R}^n (n \geq 3), \\
\Delta z &= h(|x|)y^\beta, \quad x \in \mathbb{R}^n.
\end{align*}
\] (4)

Lair [10] has considered the necessary and sufficient conditions for the existence of the nonnegative entire large radial solution of the system (4).

There are the further results in [1, 2, 4, 5, 6, 7, 11, 12, 17, 18, 19, 26, 27, 32] and the references therein.

In 2018, Zhang, Wu and Cui [28] proved the existence and nonexistence of the entire blow-up radial solutions for the following nonlinear Schrödinger elliptic equation by employing the iterative technique

\[
\text{div}(G(|\nabla z|)\nabla z) = b(|x|)\psi(z), \quad x \in \mathbb{R}^n,
\] (5)

where \( n \geq 2 \).

Inspired by the above excellent works, in this paper, by applying the monotone iterative method, we investigate the existence and nonexistence of entire positive bounded and blow-up radial solutions of the nonlinear Schrödinger elliptic system (1) involving a nonlinear operator under some appropriate conditions on \( b_i, h_i (i = 1, 2), \psi, \varphi \). The monotone iterative method, as an effective tool, plays a crucial role in the study of nonlinear problem, see [15, 16, 20, 21, 22, 23, 24, 25, 28, 29, 30, 31, 33, 34] and the references therein.

For convenience, we use the following definition:

**Definition 1.1.** Let \((y, z) \in C^2[0, \infty) \times C^2[0, \infty)\) be a solution of the system

\[
\begin{align*}
\text{div}(G(|\nabla y|^{p-2})\nabla y) &= \varphi(x, y, z), \\
\text{div}(G(|\nabla z|^{p-2})\nabla z) &= \psi(x, y, z).
\end{align*}
\] (6)

A solution \((y, z)\) of the system (6) is called an entire blow-up solution(or explosive solution, or large solution), if it is a classical solution of (6) on \( \mathbb{R}^n \) and \( y(x) \to \infty \) and \( z(x) \to \infty \) as \(|x| \to \infty\).

For the existence of the positive entire bounded radial solutions to (1), we have the following results.

**Theorem 1.2.** Assume \((S_1), (S_2), (A_1)\) and \((A_4)\) hold. Then the Schrödinger system (1) has infinitely many positive entire bounded radial solutions \((y, z) \in C^2[0, \infty) \times C^2[0, \infty)\).

**Theorem 1.3.** Assume \((S_1), (S_2), (A_2)\) and \((A_4)\) hold and there exists a constant \( \gamma > \frac{n}{2} \) such that

\[
B(\infty) + H(\infty) < A(\infty) - A(2\gamma).
\]

Then the Schrödinger system (1) has infinitely many positive entire radial solutions \((y, z) \in C^2[0, \infty) \times C^2[0, \infty)\) satisfying

\[
\gamma + \left( \min \left\{ \frac{1}{3}, \psi(\gamma), \varphi(\gamma) \right\} \right)^{\frac{1}{p-1}} B(r) \leq y(r) \leq A^{-1}(A(2\gamma) + B(r) + H(r))
\]

and

\[
\gamma + \left( \min \left\{ \frac{1}{3}, \psi(\gamma), \varphi(\gamma) \right\} \right)^{\frac{1}{p-1}} H(r) \leq z(r) \leq A^{-1}(A(2\gamma) + B(r) + H(r)).
\]
Regarding the existence and nonexistence of the positive blow-up radial solutions to (1), we obtain the following three theorems.

**Theorem 1.4.** Assume (S1), (S2), (A1) and (A3) hold. Then the Schrödinger system (1) has infinitely many positive entire blow-up radial solutions $(y, z) \in C^2[0, \infty) \times C^2[0, \infty)$.

**Theorem 1.5.** Assume (S1), (S2), (S3) and (A3) hold. Then the Schrödinger system (1) has infinitely many positive entire blow-up radial solutions $(y, z) \in C^2[0, \infty) \times C^2[0, \infty)$.

**Remark 1.** In Theorem 1.5, the condition (A3) is only sufficient for the Schrödinger system (1) to have infinitely many positive entire blow-up radial solutions. But for a single equation, it is a sufficient and necessary condition (see Theorem 3.1 in [28]).

**Theorem 1.6.** Assume (S1), (S2), (S3) and (A4) hold. Then the Schrödinger system (1) has no positive entire blow-up radial solution.

**Remark 2.** In Theorem 1.6, for the condition (S2), $\psi$ and $\varphi$ may not be required to have the monotonicity.

**Remark 3.** Our results improve and extend the works [Li, Zhang and Zhang, A remark on the existence of entire positive solutions for a class of semilinear elliptic system, J. Math. Anal. Appl., 365 (2010), 338-341.] and [Zhang, Wu and Cui, Existence and nonexistence of blow-up solutions for a Schrödinger equation involving a nonlinear operator. Appl. Math. Lett., 82 (2018), 85-91.].

2. The positive bounded radial solutions. In this section, we prove Theorem 1.2 and Theorem 1.3. First, we give the following lemmas, which play important roles in the proof of our results.

**Lemma 2.1.** [28] If $G \in \Theta$, let $R(t) = tG(t^{p-2})$, then

1. $R(t)$ has a nonnegative increasing inverse mapping $R^{-1}(t)$;
2. when $0 < l < 1$, one has
   $$R^{-1}(lt) \geq l^{\frac{1}{p-1}} R^{-1}(t);$$
3. when $l \geq 1$, one has
   $$R^{-1}(lt) \leq l^{\frac{1}{p-1}} R^{-1}(t).$$

With the aid of a standard deduction, we draw the following conclusion.

**Lemma 2.2.** The Schrödinger elliptic system (1) has a radial solution $(y, z) \in C^2[0, \infty) \times C^2[0, \infty)$ if and only if it solves the following ordinary differential system

$$\begin{cases}
G \left( |y'|^{p-2} y' \right) + \frac{n-1}{r} G \left( |y'|^{p-2} \right) y' = b_1(r) \psi(y) + b_1(r) \varphi(z), & r > 0, \\
G \left( |z'|^{p-2} z' \right) + \frac{n-1}{r} G \left( |z'|^{p-2} \right) z' = b_2(r) \psi(z) + b_2(r) \varphi(y), & r > 0, \\
y'(r) \geq 0, & r \in [0, \infty), \quad y(0) = z(0) = \gamma > 0.
\end{cases} \quad (7)$$

**Proof.** Denote $r = |x|$, where $x = (x_1, ..., x_n) \in \mathbb{R}^n$. Since $r = \sqrt{x_1^2 + \cdots + x_n^2}$, we have $[7]
\frac{\partial y}{\partial x_i} = \frac{dy}{dr} \frac{\partial r}{\partial x_i} = \frac{dy}{dr} \frac{x_i}{\sqrt{x_1^2 + \cdots + x_n^2}} = \frac{dy}{dr} \frac{x_i}{r}, \quad i = 1, \ldots, n,$
Thus the Schrödinger elliptic system (1) is equivalent to the ordinary differential equation:

$$\nabla y = \left( \frac{\partial y}{\partial x_1}, \ldots, \frac{\partial y}{\partial x_n} \right) = \left( \frac{dy}{dr} \frac{x_1}{r}, \ldots, \frac{dy}{dr} \frac{x_n}{r} \right) = \frac{1}{r} \frac{dy}{dr} x,$$

and

$$\Delta y = \text{div}(\nabla y) = \text{div}\left( \frac{1}{r} \frac{dy}{dr} x \right) = \frac{d^2 y}{dr^2} + \frac{n-1}{r} \frac{dy}{dr} = y'' + \frac{n-1}{r} y'.$$

Further, we can get that

$$\nabla y \cdot \nabla |y'| = \left( \frac{dy}{dr} \frac{x_1}{r}, \ldots, \frac{dy}{dr} \frac{x_n}{r} \right) \left( \frac{d|y'|}{dr} \frac{x_1}{r}, \ldots, \frac{d|y'|}{dr} \frac{x_n}{r} \right)$$

$$= \frac{dy}{dr} \frac{d|y'|}{dr} \left( \frac{\partial}{\partial x_1}, \ldots, \frac{\partial}{\partial x_n} \right) \left( \frac{\partial}{\partial x_1}, \ldots, \frac{\partial}{\partial x_n} \right)$$

$$= |y'| y'.$$

and

$$\text{div}(G(|\nabla y|^{p-2}) \nabla y)$$

$$= G(|\nabla y|^{p-2}) \Delta y + \nabla y \cdot \nabla G(|\nabla y|^{p-2})$$

$$= G(|\nabla y|^{p-2}) (y'' + \frac{n-1}{r} y') + \nabla y \cdot \left[ (G(|\nabla y|^{p-2}))' (p-2) |\nabla y|^{p-3} \nabla |\nabla y| \right]$$

$$= G(|\nabla y|^{p-2}) (y'' + \frac{n-1}{r} y') + (G(|\nabla y|^{p-2}))' (p-2) |\nabla y|^{p-3} \nabla |\nabla y|$$

$$= G(|\nabla y|^{p-2}) (y'' + \frac{n-1}{r} y') + \frac{n-1}{r} G(|\nabla y|^{p-2}) y'$$

$$= \left( G(|\nabla y|^{p-2}) y' \right) + \frac{n-1}{r} G(|\nabla y|^{p-2}) y'.$$

Similarly,

$$\text{div}(G(|\nabla z|^{p-2}) \nabla z) = \left( G(|\nabla z|^{p-2}) z' \right) + \frac{n-1}{r} G(|\nabla z|^{p-2}) z'.$$

Thus the Schrödinger elliptic system (1) is equivalent to the ordinary differential system (7).

2.1. Proof of theorem 1.2. From Lemma 2.1, we know that the system (7) is equivalent to the following system:

$$\begin{cases} (R(y'))' + \frac{n-1}{r} R(y') = b_1(r)\psi(y(r)) + h_1(r)\varphi(z(r)), \quad r > 0, \\
(R(z'))' + \frac{n-1}{r} R(z') = b_2(r)\psi(z(r)) + h_2(r)\varphi(y(r)), \quad r > 0. \end{cases} \quad (8)$$

It is well known that the solutions of the above system (8) [13] are the solutions of the following integral system:

$$\begin{cases} y(r) = y(0) + \int_0^r R^{-1}\left( \frac{1}{l^{n-1}} \int_0^t s^{n-1}(b_1(s)\psi(y(s)) + h_1(s)\varphi(z(s)))ds \right) dt, \\
z(r) = z(0) + \int_0^r R^{-1}\left( \frac{1}{l^{n-1}} \int_0^t s^{n-1}(b_2(s)\psi(z(s)) + h_2(s)\varphi(y(s)))ds \right) dt. \end{cases}$$
Choose the initial values $y(0) = z(0) = \gamma > 0$. Define $\{y_m\}_{m \geq 1}$ and $\{z_m\}_{m \geq 1}$ on $[0, \infty)$ by

$$
y_m(r) = y_0(r) = \gamma, \\
y_m(r) = \gamma + \int_0^r R^{-1} \left( \frac{1}{r_{n-1}} \int_0^t s^{n-1} \left( b_1(s) \psi(y_{m-1}(s)) + h_1(s) \phi(z_{m-1}(s)) \right) ds \right) dt,
$$

$$
z_m(r) = \gamma + \int_0^r R^{-1} \left( \frac{1}{r_{n-1}} \int_0^t s^{n-1} \left( b_2(s) \psi(z_{m-1}(s)) + h_2(s) \phi(y_{m-1}(s)) \right) ds \right) dt.
$$

We can conclude that for all $r \geq 0$ and $m \in \mathbb{N}, y_m(r) \geq \gamma, z_m(r) \geq \gamma$ and $z_0 \leq z_1$. By Lemma 2.1 and (S2), for all $r \geq 0$, we yield $y_1(r) \leq y_2(r)$, then $z_1(r) \leq z_2(r)$ holds. Continuing this process, we get that $\{y_m\}$ and $\{z_m\}$ are the increasing sequences.

It is inescapably clear that $y'_m(t) \geq 0$ and $z'_m(t) \geq 0$. Furthermore, by applying Lemma 2.1 and monotonicity of $\psi, \phi, \{y_m\}$ and $\{z_m\}$, we find that for each $r \geq 0$,

$$
y_m'(r) = R^{-1} \left( \frac{1}{r_{n-1}} \int_0^r s^{n-1} \left( b_1(s) \psi(y_{m-1}(s)) + h_1(s) \phi(z_{m-1}(s)) \right) ds \right)
\leq R^{-1} \left( \frac{1}{r_{n-1}} \int_0^r s^{n-1} \left( b_1(s) \psi(y_m(s)) + h_1(s) \phi(z_m(s)) \right) ds \right)
\leq R^{-1} \left( \psi(y_m(r)) + \phi(z_m(r)) \cdot \frac{1}{r_{n-1}} \int_0^r s^{n-1} \left( b_1(s) + h_1(s) \right) ds \right)
\leq \left( \psi(y_m(r)) + \phi(z_m(r)) + 1 \right)^{\frac{1}{r_{n-1}}} R^{-1} \left( \frac{1}{r_{n-1}} \int_0^r s^{n-1} \left( b_1(s) + h_1(s) \right) ds \right)
\leq \left( \psi(y_m(r)) + \phi(z_m(r)) + 1 \right)^{\frac{1}{r_{n-1}}} B'(r)
\leq \left[ \psi(z_m(r) + y_m(r)) + \phi(z_m(r) + y_m(r)) + 1 \right]^{\frac{1}{r_{n-1}}} B'(r),
$$

and

$$
z_m'(r) = R^{-1} \left( \frac{1}{r_{n-1}} \int_0^r s^{n-1} \left( b_2(s) \psi(z_{m-1}(s)) + h_2(s) \phi(y_{m-1}(s)) \right) ds \right)
\leq \left[ \psi(z_m(r) + y_m(r)) + \phi(z_m(r) + y_m(r)) + 1 \right]^{\frac{1}{r_{n-1}}} H'(r).
$$

Then

$$
\int_0^r \frac{y_m'(t) + z_m'(t)}{(\psi(y_m(t) + z_m(t)) + \phi(y_m(t) + z_m(t)) + 1)^{\frac{1}{r_{n-1}}}} dt \leq H(r) + B(r).
$$

Consequently,

$$
A(y_m(r) + z_m(r)) - A(2\gamma) \leq B(r) + H(r), \quad \forall r \geq 0.
$$
In the light of the monotonicity of $A^{-1}$, we have
\[ y_m(r) + z_m(r) \leq A^{-1}(A(2\gamma) + B(r) + H(r)), \quad \forall r \geq 0. \tag{13} \]
Since $A(\infty) = \infty$, we can obtain that
\[ A^{-1}(\infty) = \infty. \tag{14} \]
Because $A^{-1}$ is increasing on $[a, \infty)$ and (14), we can see that for arbitrary $c_0 > 0$,
\[ y_m(r) + z_m(r) \leq A^{-1}(A(2\gamma) + B(r) + H(r)) \]
\[ \leq A^{-1}(A(2\gamma) + B(c_0) + H(c_0)) \]
\[ < \infty, \quad \forall r \in [0, c_0]. \tag{15} \]

It follows that $\{y_m\}$ and $\{z_m\}$ are two bounded sequences on $[0, c_0]$ for arbitrary $c_0 > 0$. So do $\{y_m'\}$ and $\{z_m'\}$ from (10) and (11). With the aid of Arzela-Ascoli theorem, we can get the subsequences of $\{y_m\}$ and $\{z_m\}$ converge uniformly to $y$ and $z$ on $[0, c_0]$, respectively. On the other hand, it follows from the arbitrariness of $c_0 > 0$ that $(y, z)$ is a positive radial solution of the system (1). By the arbitrariness of the initial value $\gamma \in (0, \infty)$, we can know that the system (1) has infinitely many positive radial solutions. Moreover, noticing that $B(\infty) < \infty, H(\infty) < \infty$ and (13), we conclude
\[ y(r) + z(r) \leq A^{-1}(A(2\gamma) + B(r) + H(r)) \]
\[ \leq A^{-1}(A(2\gamma) + B(\infty) + H(\infty)) \]
\[ < \infty, \quad \forall r \geq 0. \]

This implies $(y, z)$ is bounded. The proof is completed.

2.2. Proof of theorem 1.3. When $(S_1)$, $(S_2)$ and $(A_2)$ hold, by the same arguments used in Theorem 1.2, we can obtain (13). It follows from $A(\infty) < \infty$ and the monotonicity of $A^{-1}$ that $A^{-1}(\infty) = \infty$. The further part of the proof is the same as the one of Theorem 1.2. So we know that the Schrödinger system (1) has infinitely many positive radial solutions. Noticing that $A(\infty) < \infty, B(\infty) < \infty, H(\infty) < \infty$ and there exists $\gamma > \frac{\alpha}{2}$ such that $B(\infty) + H(\infty) < A(\infty) - A(2\gamma)$, we can know by (12) that
\[ A(y_m(r) + z_m(r)) \leq A(2\gamma) + B(r) + H(r) < A(\infty) < \infty. \]

In the light of the monotonicity of $A^{-1}$, we get
\[ y_m(r) + z_m(r) \leq A^{-1}(A(2\gamma) + B(r) + H(r)) < \infty, \quad \forall r \geq 0. \tag{16} \]

By taking the limit in (16), we find that
\[ y(r) + z(r) \leq A^{-1}(A(2\gamma) + B(r) + H(r)) < \infty, \quad \forall r \geq 0. \]
It follows from Lemma 2.1 and the monotonicity of $\psi$ and $\varphi$ that

$$y(r) = y(0) + \int_0^r \mathcal{R}^{-1} \left( \frac{1}{t^{n-1}} \int_0^t s^{n-1} \left( b_1(s)\psi(y(s)) + h_1(s)\varphi(z(s)) \right) ds \right) dt$$

$$\geq \gamma + \int_0^r \mathcal{R}^{-1} \left( \frac{1}{t^{n-1}} \int_0^t s^{n-1} \left( b_1(s)\psi(y(s)) + h_1(s)\varphi(z(s)) \right) ds \right) dt$$

$$\geq \gamma + \int_0^r \mathcal{R}^{-1} \left( \min \{ \psi(y(s)), \varphi(z(s)) \} \right) \frac{1}{t^{n-1}} \int_0^t s^{n-1} \left( b_1(s) + h_1(s) \right) ds dt$$

$$\geq \gamma + \left( \min \{ \frac{1}{3}, \psi(y(s)), \varphi(z(s)) \} \right) \frac{1}{t^{n-1}} B(r)$$

and

$$z(r) = z(0) + \int_0^r \mathcal{R}^{-1} \left( \frac{1}{t^{n-1}} \int_0^t s^{n-1} \left( b_2(s)\psi(y(s)) + h_2(s)\varphi(z(s)) \right) ds \right) dt$$

$$\geq \gamma + \left( \min \{ \frac{1}{3}, \psi(y(s)), \varphi(z(s)) \} \right) \frac{1}{t^{n-1}} H(r).$$

The proof is completed.

3. The positive blow-up radial solutions. In this section, we prove Theorem 1.4, Theorem 1.5 and Theorem 1.6.

3.1. Proof of theorem 1.4. When $(S_1), (S_2)$ and $(A_1)$ hold, it has been proved from Theorem 1.2 that the Schrödinger system (1) has infinitely many positive radial solutions. Next, we need to prove that $\lim_{r \to \infty} y(r) = \infty$ and $\lim_{r \to \infty} z(r) = \infty$. It follows from Lemma 2.1 and the monotonicity of $\psi$ and $\varphi$ that

$$y(r) = y(0) + \int_0^r \mathcal{R}^{-1} \left( \frac{1}{t^{n-1}} \int_0^t s^{n-1} \left( b_1(s)\psi(y(s)) + h_1(s)\varphi(z(s)) \right) ds \right) dt$$

$$\geq \gamma + \left( \min \{ \frac{1}{3}, \psi(y(s)), \varphi(z(s)) \} \right) \frac{1}{t^{n-1}} B(r)$$

and

$$z(r) = z(0) + \int_0^r \mathcal{R}^{-1} \left( \frac{1}{t^{n-1}} \int_0^t s^{n-1} \left( b_2(s)\psi(y(s)) + h_2(s)\varphi(z(s)) \right) ds \right) dt$$

$$\geq \gamma + \left( \min \{ \frac{1}{3}, \psi(y(s)), \varphi(z(s)) \} \right) \frac{1}{t^{n-1}} H(r).$$

Because of $B(\infty) = \infty$ and $H(\infty) = \infty$, we can obtain that $\lim_{r \to \infty} y(r) = \infty$ and $\lim_{r \to \infty} z(r) = \infty$ from the above two inequalities. Thus the Schrödinger system (1) has infinitely many positive entire blow-up radial solutions $(y, z)$. The proof is completed.
3.2. **Proof of theorem 1.5.**

When \((S_1), (S_2)\) and \((S_3)\) hold, we firstly show that (1) admits a positive radial solution. For this goal, we consider the equivalent integral system of the system (1)

\[
\begin{align*}
\begin{cases}
g(r) = y(0) + \int_0^r R^{-1} \left( \frac{1}{tn-1} \int_0^t s^{n-1} \left( b_1(s) \psi(y(s)) + h_1(s) \varphi(z(s)) \right) ds \right) dt, \\
z(r) = z(0) + \int_0^r R^{-1} \left( \frac{1}{tn-1} \int_0^t s^{n-1} \left( b_2(s) \psi(z(s)) + h_2(s) \varphi(y(s)) \right) ds \right) dt.
\end{cases}
\end{align*}
\]

(17)

In the following, we shall generate two positive increasing sequences \(\{y_m\}_{m \geq 1}\) and \(\{z_m\}_{m \geq 1}\), which are bounded above on \([0, L]\) for fixed \(L > 0\). For convenience, let \(\{y_m\}_{m \geq 1}\) and \(\{z_m\}_{m \geq 1}\) be as defined in Theorem 1.2. It follows from the monotonic increasing property of \(\psi\) and \(\varphi\) that the sequences \(\{y_m\}\) and \(\{z_m\}\) are increasing. Next, what we need to prove is that the sequences \(\{z_m(r)\}_{m \geq 0}\) and \(\{y_m(r)\}_{m \geq 1}\) are bounded on \([0, L]\) for fixed \(L > 0\).

It is easy to see that \(y'_m(t) \geq 0\) and \(z'_m(t) \geq 0\) from Theorem 1.2. For fixed \(L > 0\), by \((S_2)\) and Lemma 2.1, one has

\[
y_m(L) = y(0) + \int_0^L R^{-1} \left( \frac{1}{tn-1} \int_0^t s^{n-1} \left( b_1(s) \psi(y_m-1(s)) + h_1(s) \varphi(z_m-1(s)) \right) ds \right) dt,
\]

\[
\leq y(0) + \int_0^L R^{-1} \left( \frac{1}{tn-1} \int_0^t s^{n-1} \left( b_1(s) \psi(y_m(s)) + h_1(s) \varphi(z_m(s)) \right) ds \right) dt,
\]

\[
\leq y(0) + \int_0^L R^{-1} \left( \psi(y_m(t)) + \varphi(z_m(t)) \right) \frac{1}{tn-1} \int_0^t s^{n-1} \left( b_1(s) + h_1(s) \right) ds \right) dt,
\]

\[
\leq y(0) + \left( (y_m(L) + 1) + (z_m(L) + 1) \right) \frac{1}{tn-1} \int_0^t s^{n-1} \left( b_1(s) + h_1(s) \right) ds \right) dt.
\]

(18)

Similarly,

\[
z_m(L) \leq y(0) + \left( (y_m(L) + 1) + (z_m(L) + 1) \right) \frac{1}{tn-1} \int_0^t s^{n-1} \left( \psi(1) + \varphi(1) + 1 \right) \frac{1}{tn-1} \int_0^t H(L).
\]

(19)

Hence,

\[
z_m(L) + y_m(L) \leq 2y(0) + \left( (y_m(L) + 1) + (z_m(L) + 1) \right) \frac{1}{tn-1} \int_0^t s^{n-1} \left( \psi(1) + \varphi(1) + 1 \right) \frac{1}{tn-1} \int_0^t \left( B(L) + H(L) \right).
\]

(20)

Denote

\[
M(L) := \lim_{m \to \infty} (z_m(L)) \text{ and } N(L) := \lim_{m \to \infty} (y_m(L)).
\]
It is obvious that $M(L)$ and $N(L)$ are finite. If not, by (20), we can discover that

$$1 \leq \frac{2\gamma}{z_m(L) + y_m(L)} + \frac{\left((y_m(L) + 1)^\gamma + (z_m(L) + 1)^\gamma\right)^{\frac{1}{\gamma}}}{z_m(L) + y_m(L)}$$

$$\times \left(\psi(1) + \varphi(1) + 1\right)^{\frac{1}{\gamma}} \left\{H(L) + B(L)\right\} \to 0,$$

as $m \to \infty$, which is a contradiction. Thus $M(L)$ and $N(L)$ are finite. It follows from the fact that $z_m(r), y_m(r), M$ and $N$ are increasing on $(0, \infty)$. Thus for all $r \in [0, L]$ and $m \geq 1$, we can know that

$$\gamma \leq z_m(r) \leq z_m(L) \leq M(L)$$

and

$$\gamma \leq y_m(r) \leq y_m(L) \leq N(L),$$

which state clearly that the sequences $\{z_m(r)\}_{m \geq 0}$ and $\{y_m(r)\}_{m \geq 1}$ are bounded on $[0, L]$. Let

$$z(r) := \lim_{m \to \infty} z_m(r) \text{ and } y(r) := \lim_{m \to \infty} y_m(r), \ \forall r \geq 0.$$ 

Now, taking the limit in (8), we can conclude that $(y, z)$ is a positive solution of (17). Finally, it remains to prove that $y(r)$ and $z(r)$ are blow-up. In fact, it follows from (17) that

$$y(r) = y(0) + \int_0^r \mathcal{R}^{-1} \left(\frac{1}{n-1} \int_0^t s^{n-1} \left(b_1(s)\psi(y(s)) + h_1(s)\varphi(z(s))\right) ds\right) dt$$

$$\geq \gamma + \int_0^r \mathcal{R}^{-1} \left(\frac{1}{n-1} \int_0^t s^{n-1} \left(b_1(s)\psi(\gamma) + h_1(s)\varphi(\gamma)\right) ds\right) dt$$

$$\geq \gamma + \left(\min \left\{\frac{1}{3}, \psi(\gamma), \varphi(\gamma)\right\}\right)^{\frac{1}{\gamma}} B(r).$$

Similarly,

$$z(r) \geq \gamma + \left(\min \left\{\frac{1}{3}, \psi(\gamma), \varphi(\gamma)\right\}\right)^{\frac{1}{\gamma}} H(r).$$

Since $B(\infty) = \infty$ and $H(\infty) = \infty$, the right-hand side of the above two inequalities tends to $+\infty$ as $r \to +\infty$, which shows that $(y, z)$ is a blow-up solution of the system (1). According to the arbitrariness of the initial value $\gamma \in (0, \infty)$, we can know that the Schrödinger system (1) has infinitely many positive entire blow-up radial solutions. The proof is completed.

3.3. Proof of theorem 1.6. Assume that the system (1) exists a positive entire blow-up radial solution $(y, z)$. Since $B(\infty) < \infty$ and $H(\infty) < \infty$, there exists $r_0 > 0$ large enough such that

$$\int_{r_0}^\infty \mathcal{R}^{-1} \left(\frac{1}{n-1} \int_0^t s^{n-1} (b_1(s) + h_1(s)) ds\right) dt < \frac{1}{2 \left(\psi(1) + \varphi(1) + 1\right)^{\frac{1}{\gamma}}} \quad (21)$$

and

$$\int_{r_0}^\infty \mathcal{R}^{-1} \left(\frac{1}{n-1} \int_0^t s^{n-1} (b_2(s) + h_2(s)) ds\right) dt < \frac{1}{2 \left(\psi(1) + \varphi(1) + 1\right)^{\frac{1}{\gamma}}} \quad (22)$$
Thus, by Lemma 2.1 and (S2), for \( r > r_0 > 0 \), we have

\[
y(r) = y(r_0) + \int_{r_0}^{r} \mathcal{R}^{-1} \left( \frac{1}{r^{n-1}} \int_0^t s^{n-1} \left( b_1(s) \psi(y(s)) + h_1(s) \varphi(z(s)) \right) ds \right) dt
\]

\[
\leq y(r_0) + \int_{r_0}^{r} \mathcal{R}^{-1} \left( \left( \psi(y(t)) + \varphi(z(t)) \right) \frac{1}{r^{n-1}} \int_0^t s^{n-1} \left( b_1(s) + h_1(s) \right) ds \right) dt
\]

\[
\leq y(r_0) + \int_{r_0}^{r} \mathcal{R}^{-1} \left( \left( (y(t) + 1)^\varepsilon + (z(t) + 1)^\varepsilon \right) \frac{1}{r^{n-1}} \int_0^t s^{n-1} \left( b_1(s) + h_1(s) \right) ds \right) dt
\]

\[
\leq y(r_0) + \left( (y(r) + 1)^\varepsilon + (z(r) + 1)^\varepsilon \right) \frac{1}{r^{n-1}} \int_0^t s^{n-1} \left( b_1(s) + h_1(s) \right) ds dt
\]

Similarly,

\[
z(r) \leq z(r_0) + \left( (y(r) + 1)^\varepsilon + (z(r) + 1)^\varepsilon \right) \frac{1}{r^{n-1}} \int_0^t s^{n-1} \left( b_2(s) + h_2(s) \right) ds dt
\]

Thus,

\[
y(r) + z(r) \leq y(r_0) + z(r_0) + \left( (y(r) + 1)^\varepsilon + (z(r) + 1)^\varepsilon \right) \frac{1}{r^{n-1}} \int_0^t s^{n-1} \left( b_1(s) + h_1(s) \right) ds dt
\]

\[
+ \int_{r_0}^{r} \mathcal{R}^{-1} \left( \frac{1}{r^{n-1}} \int_0^t s^{n-1} \left( b_2(s) + h_2(s) \right) ds \right) dt
\]

\[
\leq y(r_0) + z(r_0) + \left( (y(r) + 1)^\varepsilon + (z(r) + 1)^\varepsilon \right) \frac{1}{r^{n-1}}.
\]

Since \( 0 < \frac{\varepsilon}{p-1} < 1 \), we can see that \( y(r) \) and \( z(r) \) are bounded, which is obviously contradictory to the assumption. Hence the system (1) has no positive entire blow-up radial solution. This completes the proof.

4. Examples.

Example 1. Consider the following nonlinear Schrödinger elliptic system

\[
\begin{align*}
\text{div}(\mathcal{G}(|\nabla y|^2)\nabla y) &= \frac{1}{1+x^4} \ln(y+1) + \frac{2}{1+x^4} \arctan z, \quad x \in \mathbb{R}^6, \\
\text{div}(\mathcal{G}(|\nabla z|^2)\nabla z) &= \frac{|x|^2}{1+x^4} \ln(z+1) + \frac{2}{1+x^4} \arctan y, \quad x \in \mathbb{R}^6.
\end{align*}
\] (23)

Note \( \mathcal{G}(s) = s^4, p = 4 \), then \( \mathcal{G} \in \Theta \). Here \( b_1(s) = \frac{1}{1+x^4}, b_2(s) = \frac{2}{1+x^4}, h_1(s) = \frac{2}{1+x^4}, h_2(s) = \frac{s^2}{1+x^4} \). \( \psi(t) = \ln(t+1) \) and \( \varphi(t) = \arctan t \), then we know that the condition (S1) is obviously satisfied and \( \psi(t) \) and \( \varphi(t) \) are nondecreasing which
satisfies \((S_2)\). After a simple calculation, on the one hand, one has

\[
B(\infty) = \int_0^\infty R^{-1} \left( \frac{1}{t^{n-1}} \int_0^t s^{n-1} (b_1(s) + h_1(s)) ds \right) dt
\]

\[
= \int_0^\infty \left( \frac{1}{t^{15}} \int_0^t s^{15} \left( \frac{1}{2 + s^{13}} + \frac{2}{1 + s^{14}} \right) ds \right)^{\frac{1}{2}} dt
\]

\[
< \int_0^\infty \left( \frac{1}{t^{15}} \int_0^t s^{15} \left( \frac{1}{s^{13}} + \frac{2}{s^{14}} \right) ds \right)^{\frac{1}{2}} dt
\]

\[
= \int_0^\infty \left( \frac{1}{3} \right)^{\frac{1}{2}} dt
\]

\[
\leq \int_0^\infty \left( \frac{4}{3} \right)^{\frac{1}{2}} dt
\]

\[
= \sqrt{\frac{4}{3}} \int_0^\infty \frac{1}{\sqrt{t}} dt = \infty,
\]

and

\[
H(\infty) = \int_0^\infty R^{-1} \left( \frac{1}{t^{n-1}} \int_0^t s^{n-1} (b_2(s) + h_2(s)) ds \right) dt
\]

\[
= \int_0^\infty \left( \frac{1}{t^{15}} \int_0^t s^{15} \left( \frac{s^2}{1 + s^{15}} + \frac{s^2}{3 + s^{14}} \right) ds \right)^{\frac{1}{2}} dt
\]

\[
< \int_0^\infty \left( \frac{1}{t^{15}} \int_0^t s^{15} \left( \frac{s^2}{s^{15}} + \frac{s^2}{s^{14}} \right) ds \right)^{\frac{1}{2}} dt
\]

\[
= \int_0^\infty \left( \frac{1}{3} \right)^{\frac{1}{2}} \left( \frac{1}{4} \right)^{\frac{1}{2}} dt
\]

\[
\leq \int_0^\infty \left( \frac{7}{12} \right)^{\frac{1}{2}} dt
\]

\[
= \sqrt{\frac{7}{12}} \int_0^\infty \frac{1}{\sqrt{t}} dt = \infty,
\]

which mean \((A_4)\) is satisfied. On the other hand,

\[
A(\infty) = \int_a^\infty \frac{dt}{(\psi(t) + \varphi(t) + 1)^{\frac{1}{2}}}
\]

\[
= \int_a^\infty \frac{dt}{(\ln(t + 1) + \arctan t + 1)^{\frac{1}{2}}} \geq \int_a^\infty \frac{dt}{\sqrt{2(t + 1)}} = \infty,
\]

which means \((A_1)\) is satisfied. By Theorem 1.2, the Schrödinger system \((23)\) has infinitely many positive bounded radial solutions \((y, z) \in C^2[0, \infty) \times C^2[0, \infty)\).

**Example 2.** Consider the following nonlinear Schrödinger elliptic system

\[
\begin{align*}
\text{div}(\mathcal{G}(|\nabla y|)\nabla y) &= \frac{|x|^5}{1 + |x|^2} y^{16} + \frac{|x|^6}{16} (x^2 + 1)^{16}, \quad x \in \mathbb{R}^5, \\
\text{div}(\mathcal{G}(|\nabla z|)\nabla z) &= \frac{4|x|^3}{|x|^2} z^{16} + \frac{x^8}{(y^2 + 3)^{16}}, \quad x \in \mathbb{R}^5.
\end{align*}
\] (24)
Here $b_1(s) = \frac{s^5}{13}$, $b_2(s) = \frac{4e^s}{s}$, $h_1(s) = \frac{s^8}{10}$, $h_2(s) = e^s$, $\psi(t) = \frac{t^{16}}{(2t^3+1)^2}$ and $\varphi(t) = \frac{t^p}{(t^3+1)^2}$. Note $G(s) = s^3$, $p = 3$, then $G \in \Theta$. After a simple calculation, one has

$$A(\infty) = \int_0^\infty \frac{dt}{(\psi(t) + \varphi(t) + 1)^{n-1}}$$

$$= \int_0^\infty \frac{dt}{(\frac{t^{18}}{(2t^3+1)^2} + \frac{t^9}{(t^3+3)^2} + 1)^{\frac{n}{2}}} \geq \int_0^\infty \frac{dt}{\sqrt{2t+1}} = \infty,$$

which means $(A_1)$ is satisfied, and

$$B(\infty) = \int_0^\infty R^{-1} \left( \frac{1}{t^{n-1}} \int_0^t s^{n-1}(b_1(s) + h_1(s))ds \right) dt$$

$$= \int_0^\infty \left( \frac{1}{t^4} \int_0^t s^4 \left( \frac{s^5}{13} + \frac{s^8}{16} \right) ds \right)^{\frac{1}{2}} dt$$

$$= \int_0^\infty \left( \frac{t^6}{130} + \frac{t^9}{208} \right)^{\frac{1}{2}} dt$$

$$\geq \sqrt{\frac{1}{104}} \int_0^\infty t^\frac{3}{2} dt = \infty$$

and

$$H(\infty) = \int_0^\infty R^{-1} \left( \frac{1}{t^{n-1}} \int_0^t s^{n-1}(b_2(s) + h_2(s))ds \right) dt$$

$$= \int_0^\infty \left( \frac{1}{t^4} \int_0^t s^4 (\frac{4e^s}{s} + e^s) ds \right)^{\frac{1}{2}} dt$$

$$= \int_0^\infty \left( \frac{1}{t^4} \int_0^t (s^4 + 4s^3) e^s ds \right)^{\frac{1}{2}} dt$$

$$= \int_0^\infty e^\frac{3}{2} dt = \infty,$$

which means $(A_3)$ is satisfied. The condition $(S_1)$ is obviously satisfied. It is easy to see that $\varphi(t)$ and $\psi(t)$ are nondecreasing, which meets $(S_2)$. By Theorem 1.4, the Schrödinger system (24) has infinitely many positive blow-up radial solutions $(y,z) \in C^2[0, \infty) \times C^2[0, \infty)$.

**Example 3.** Consider the following nonlinear Schrödinger elliptic system

$$\begin{cases}
\text{div}(G(|\nabla y|)|\nabla y|) = 3|x|^5 y^\frac{4}{5} + 5|x|^8 z^\frac{17}{18}, & x \in \mathbb{R}^9, \\
\text{div}(G(|\nabla z|)|\nabla z|) = 6|x|^2 z^\frac{4}{5} + 8e|x|^2 y^\frac{17}{18}, & x \in \mathbb{R}^9.
\end{cases}$$

(25)
Here $b_1(s) = 3s^5$, $b_2(s) = 6s^2$, $h_1(s) = 5s^8$, $h_2(s) = 8e^s$, $\psi(t) = t^{\frac{7}{8}}$ and $\varphi(t) = t^{\frac{11}{16}}$. Note $G(s) = s^6$, $p = 6$, then $G \in \Theta$. After a simple calculation, one has
\[
B(\infty) = \int_0^\infty R^{-1} \left( \frac{1}{t^{n-1}} \int_0^t s^{n-1}(b_1(s) + h_1(s))ds \right) dt \\
= \int_0^\infty \left( \frac{1}{t^{\frac{18}{17}}} \int_0^t s^8(3s^5 + 5s^8)ds \right)^{\frac{1}{17}} dt \\
= \int_0^\infty \left( \frac{3}{14} t^6 + \frac{5}{17} t^9 \right)^{\frac{1}{17}} dt \\
\geq \int_0^\infty \left( \frac{5}{17} t^9 \right)^{\frac{1}{17}} dt \\
= \frac{25}{17} \int_0^\infty t^{\frac{9}{17}} dt = \infty
\]
and
\[
H(\infty) = \int_0^\infty R^{-1} \left( \frac{1}{t^{n-1}} \int_0^t s^{n-1}(b_2(s) + h_2(s))ds \right) dt \\
= \int_0^\infty \left( \frac{1}{t^{\frac{18}{17}}} \int_0^t s^8(6s^2 + 8e^s)ds \right)^{\frac{1}{17}} dt \\
\geq \int_0^\infty \left( \frac{1}{t^{\frac{18}{17}}} \int_0^t s^8(6s^2 + 8s^2)ds \right)^{\frac{1}{17}} dt \\
= \int_0^\infty \left( \frac{14}{11} t^3 \right)^{\frac{1}{17}} dt \\
= \frac{25}{11} \int_0^\infty t^{\frac{3}{17}} dt = \infty,
\]
which mean $(A_2)$ is satisfied. The condition $(S_1)$ is obviously satisfied. It is easy to see that $\varphi(t)$ and $\psi(t)$ are nondecreasing, which meets $(S_2)$. Obviously, for $\varepsilon = \frac{1}{7}$,
\[
\varphi(kt) = (kt)^{\frac{11}{16}} = k^{\frac{11}{16}} t^{\frac{11}{16}} \geq k^{\frac{11}{16}} k^{\frac{11}{16}} = k^\frac{11}{4} \varphi(t), \quad \forall \ t \in [0, \infty), \ k \in (0, 1]
\]
and
\[
\psi(kt) = (kt)^{\frac{7}{8}} = k^\frac{7}{8} t^{\frac{7}{8}} \geq k^\frac{7}{8} k^\frac{7}{8} = k^{\frac{7}{4}} \psi(t), \quad \forall \ t \in [0, \infty), \ k \in (0, 1],
\]
so, $(S_3)$ is satisfied. By Theorem 1.5, the Schrödinger system (25) has infinitely many positive entire blow-up radial solutions $(y, z) \in C^2[0, \infty) \times C^2[0, \infty)$.

5. Conclusions. In this article, we have proved the existence and nonexistence of entire positive bounded and blow-up radial solutions of the nonlinear Schrödinger elliptic system (1) involving a nonlinear operator by using the monotone iterative method. In addition, $G$ is an important nonlinear operator, which makes (1) contain many equations and systems as special cases. For instance, Covei [3] only studied the existence of positive bounded and blow-up radially symmetric solutions for the single equation (2); Li et al. [13] only proved the existence of entire positive bounded and blow-up radial solutions for the semi-linear elliptic system (3); Zhang et al. [28] only proved the existence and nonexistence of the entire blow-up radial solutions for the nonlinear Schrödinger elliptic equation (5). Our results enrich the existing
results[3, 9, 10, 13, 28], which is a meaningful contribution to the topic of nonlinear Schrödinger elliptic system.

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