Gaia kinematics reveal a complex lopsided and twisted Galactic disc warp

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ABSTRACT

Context. Currently there are few warp kinematic models able to characterise both structure and kinematics, since they require high accuracy at large distances. These methods are necessary to shed some light to the lopsidedness of the warp and the twisting of the line-of-nodes of the stellar component, already seen in other components, such as gas and dust, and in external galaxies.

Aims. We want to use the vertical information coming from the Gaia Data Release 2 astrometric data up to $G = 20$ mag to characterise the structure of the Galactic warp, the related vertical motions and the dependency of the Galactic warp on the age.

Methods. We analyse the spatial and kinematic data of two different populations up to galactocentric distances of $R = 16$ kpc, namely a young bright sample mainly formed by OB stars and an older one of Red Giant Branch stars. We use two different but complementary methods ($nGC3$ PCM and LonKin) based on the Gaia observables to study the global shape of the Galactic warp, together with two dimensional projections of the positions and proper motions in the Galactic plane.

Results. This work confirms a high degree of complexity of the vertical structure and motion of the Galactic disc. We definitively confirm the age dependency of the Galactic warp, both in positions and kinematics, being the altitude of the Galactic warp of the order of 0.2 kpc for the OB sample and of 1.0 kpc for the Red Giant Branch at $R = 14$ kpc. Both the LonKin and $nGC3$ PCM methods find that the starting radius of the warp is $12 < R < 13$ kpc for the OB sample and $10 < R < 11$ kpc for the RGB. From the RGB sample, we find from $R > 10$ kpc the line-of-nodes twists away from the Sun-anticentre line towards galactic azimuths $\theta \sim 180 - 200^\circ$ increasingly with radius. Also, the RGB sample reveals a slightly lopsided stellar warp with $|Z(down)| = |Z(up)| \sim 250$ pc. We find the line of maximum of proper motions in latitude is systematically offset from the line-of-nodes estimated from the spatial data, which our warp models predict as a kinematic signature of lopsidedness. We also show a prominent wave-like pattern of a bending mode dominant from the Solar ring to large heliocentric distances different in the OB and RGB samples. Both positions and kinematics also reveal substructures in phase space that might not be related to the large scale Galactic warp nor to the bending mode.

Conclusions. Gaia Data Release 2 data opens the possibility of studying the vertical distribution of the Galactic plane in terms of both positions and velocities of two tracers with different stellar population ages and up to $R = 16$ kpc. It confirms that both the structure and kinematics of the Galactic warp depend on the age. Additionally, it reveals a high degree of complexity that triggers the need for complex kinematic models, flexible enough to combine both wave-like patterns and an S-shaped lopsided warp.

Key words. Astrometry – Proper motions – Galaxy: kinematics and dynamics – Galaxy: structure

1. Introduction

Warped discs still represent a theoretical challenge nowadays. From surveys of edge-on galaxies, it is clear that about 50 – 70% of spiral disc galaxies present stellar warped discs (e.g. Sanchez-Saavedra et al. 1990), suggesting that they are long-lived, or repeatedly generated. Our Galaxy also presents a warped disc, first detected with 21-cm observations of the HI gas (Burke 1957; Westerhout 1957; Oort et al. 1958; Levine et al. 2006; among others), and later in dust and stars using the Two Micron All Sky Survey infrared data (Freudenreich et al. 1994; Drimmel & Spergel 2001; López-Corredoira et al. 2002; Rey et al. 2009; Amores et al. 2017; among others). Most of these studies dedicated their efforts to determine the morphology of the warped disc, that is, the galactocentric radius at which the disc starts bending, the phase angle of the line-of-nodes, its maximum amplitude and its possible dependence upon the tracer.

Very few studies have focused on analysing the warp kinematically, in the sense of finding the effect of a warped
disc on the kinematics of the stars. The first kinematic analyses were conducted using \textit{Hipparcos} proper motions. \textcite{Dehnen1998} selected a set of kinematically unbiased main sequence and giants stars and found evidence of the stellar warp when plotting the vertical velocity as a function of the tangential velocity. \textcite{Drimmel2000} plotted the vertical velocity of the stars as a function of the galactocentric radius for a sample of OB stars, concluding that the kinematics of the stars towards the outer disc was inconsistent with the expectations from a long-lived non-precessing warp. Some years later, studies by \textcite{Seabroke2007} as well as \textcite{Bobylev2010, 2013} concluded the (then) available proper motion surveys do not allow complete studies of the Galactic warp. Using PPMXL proper motions and selecting disc Red Clump stars using 2MASS photometry, \textcite{Lopez-Corredoira2014} found kinematic evidence of the stellar warp, by plotting the vertical velocity as a function of the galactocentric azimuth, concluding that their results cannot be reproduced by a population in statistical equilibrium. The quality of the \textit{Gaia} Data Release 1 and the Tycho-Gaia Astrometric Solution (TGAS, \textcite{Michalik2015, Brown2016}) proper motions allowed \textcite{Schonrich2018} to review the study started by \textcite{Dehnen1998}. Following a similar strategy, these authors find evidence of a kinematic signature of the Galactic stellar warp in the TGAS data in a cone towards the centre and anti-centre directions and estimate the onset of the warp at a guiding radius inside the Solar circle, \( R_g \leq 7 \, \text{kpc} \), in agreement with the previous work by \textcite{Drimmel2001}. \textcite{Carrillo2018} and \textcite{Schonrich2018} also point out that the complexity of the data cannot be explained by a simple warp pattern, but also wave-like patterns, bending and breathing modes are reflected in the kinematic structure. The advent of the \textit{Gaia} Data Release 2 (hereafter, GaiaDR2), and the availability of proper motions for 1.3 billion sources, has allowed to expand the study to larger volumes and fainter sources. \textcite{GaiaCollaboration2018} has already shown the complexity of the vertical motion and \textcite{Poggio2018} review the data selection of OB-type and Giants stars performed in \textcite{GaiaCollaboration2018} and show the kinematic evidence of the Galactic warp in vertical velocity using two-dimensional maps.

The main challenge in our work is going one step forward. We want to use the spatial and kinematic data, together with models, to constrain the morphology of the Galactic warp. We assume this challenge by defining three Galactic warp models and using two characterisation methods developed by the authors specifically for this purpose. The complexity of the data leads to consider not only simple symmetric tilted rings models, but more complex asymmetric models, such as lopsided tilted rings or lopsided S-shaped warp models. We also use two methods to infer the structural parameters of the warp given the observables (positions and proper motions). The first is the LonKin method \textcite{Abedi2015}, which provides information on the level of asymmetry of the warp and shape, when applied to different warp models. The second is \( nG3 \), a method from the Great Circle Cell Counts (GC3) family \textcite{Johnston1996, Mateu2011}, which searches for over-densities of stars along great circle cells. The latter has already been used in simulations by \textcite{Abedi2014} to assess the \textit{Gaia} capabilities to derive the tilt angle, or amplitude of the warp, and the twist angle of the line-of-nodes. In this work, we apply both methods to two different populations from GaiaDR2, namely OB-stars and Red Giant Branch stars (similarly but not equally selected as in \textcite{Poggio2018}). We define a strategy that allows us to select stars in both samples up to magnitude \( G = 20 \, \text{mag} \) in order to have enough statistics up to \( R \leq 16 \, \text{kpc} \) and to disentangle the kinematic signature of the warp. Two tracer populations are necessary to study the dependency of morphology and kinematics on the age of the tracer, as recently suggested by \textcite{Amores2017}. We study the characteristics of the Galactic warp in terms of spatial and kinematic data of the two tracer populations, with intrinsically different ages, in order to give a first step towards understanding the origin of the Galactic warp. Additionally, the application of the methods to samples of test particles, evolved with the different models specified above, allows us to give an interpretation of the results in terms of shape, onset radius and tilt angle.

The paper is organised as follows. In Sect. 2 we give a brief description of the characterisation methods (Sect. 2.1) used and we discuss the need to use an adequate distance estimator and we give the final choice (Sect. 2.2). In Sect. 3 we describe the selection of the working samples from GaiaDR2 data and give an initial characterisation. In Sect. 4 we study the spatial density distribution of both samples (Sect. 4.1) and we give the first spatial characteristics of the Galactic warp (Sect. 4.2). We then continue by studying the kinematic signature of the warp in Sect. 5. We first focus on the kinematics in 2D projection maps (Sect. 5.1) and in Sects. 5.2 and 5.3 we apply the LonKin and \( nG3 \) methods to the two populations extracted from the GaiaDR2 data, respectively. In Sect. 6 we combine the spatial and kinematic distributions and we compare it with the models. Finally, in Sect. 7 we present our conclusions. This work is complemented with three appendices: in appendix A we provide a full description of the warp models used; in Appendix B we detail the position and velocity transformations applied to a flat disc in order to warp it according to each of the three models; in Appendix C we discuss the information the two methods provide when we apply them to synthetic data consisting of three different sets of particles, simulated with increasing complexity and reality.

2. Methods and data treatment

In order to analyse the warp signature in the disc kinematics, we use methods previously developed by the authors \textcite{Abedi2014}, namely the LonKin method and the \( nG3 \) methodology (hereafter, GC3) method and its variations. Both methods are designed to provide structural and kinematic information of the warp from the use of positions, distances and proper motions. First, in Sect. 2.1 the methods are described and specify the requirements needed to apply them. Second, in Sect. 2.2 we use mock catalogues to study and define the appropriate choice for the distance estimator used throughout this paper.

2.1. Methods for warp detection and characterisation

The LonKin method looks for the signature of a possible warped disc in a plot of proper motion in galactic latitude, \( \mu_b \), as a function of the heliocentric galactic longitude \( l \). The method as shown in this paper was developed in \textcite{Abedi2015}, and its main advantage is that it uses directly
the Gaia observables. The method is a variation of those used in Drimmel et al. (2000) and López-Corredoira et al. (2014), which plot the vertical velocity, $W$, as a function of the galactocentric radius and of the galactocentric azimuth, respectively. Therefore, the main advantage of LonKin with respect to these is that it works in the space of the observables, using proper motions instead of the vertical velocity, $W$. In the vertical axis we plot the proper motion in latitude corrected by the reflection of the solar motion, that is, with respect the Local Standard of Rest, $\mu_{b,\text{LSR}}$. In the horizontal axis, we plot the heliocentric galactic longitude, $l$, in segments of 20°. For each segment, we compute the median of the $\mu_{b,\text{LSR}}$ values and the lower (15.85-percentile) and upper (84.15-percentile) 1σ uncertainties on the estimation of the median as defined in the appendix A of Gaia Collaboration et al. (2018). Since the Galactic warp is a galactocentric feature, we require to split the sample into cylindrical galactocentric radial bins, for which we need to assume a Galactic solar radius and propagate the errors to the galactocentric frame to obtain the cylindrical galactocentric radius $R_{\text{gal}}$. If the galactic disc is flat, the median $\mu_{b,\text{LSR}}$ should be constant and equal to zero, but if it is warped, a particular variation will be introduced as a function of $l$. If the disc is symmetrically warped and the line-of-nodes is aligned with the Sun-Galactic Centre line, the LonKin method predicts a maximum in $\mu_{b,\text{LSR}}$ in the anti-centre direction. If, on the other hand, the disc is asymmetric, this is, it is Lopsided, and the line-of-nodes, still on the Sun-Galactic Centre line, the maximum in $\mu_{b,\text{LSR}}$ is no longer in the anti-centre direction, but shifted towards longitudes coinciding the maximum warp amplitude (see detailed discussion in appendix C).

The family of Great Circle Cell Counts (hereafter, GC3) methods (Johnston et al. 1996, Mateu et al. 2011) comprises different ways of searching for overdensities in great circle cells in the sky. In the most general version (mGC3) including full kinematic information –introduced in Mateu et al. (2011) – the method sweeps over the sky counting how many stars have position and velocities lying in a great circle within a given tolerance, each great circle being defined uniquely by its normal vector or pole. The all-sky sweep over all possible great circle cells results in a Pole Count Map (hereafter, PCM), a plot of the number of stars associated to each possible pole, and thus great circle cell, in the celestial sphere. To apply this method for the particular case of characterising the warp, PCMs are made for different galactocentric radial bins and the star counts are made in great circles defined in a Galactocentric reference frame. If the Galactic disc is flat, the peak in stellar density should be located in the North Galactic Pole of the PCM. If the disc is not flat, the peak of over-density should move in the PCM providing information on the tilt angle of the warp, as well as the azimuth (twist) of the line-of-nodes as a function of radii (see Abedi et al. 2014 for detailed examples). If the warp is Lopsided, the signature in the PCM is not a single peak, but it has a shape that depends on the warp model (see detailed discussion in appendix C). In this work in particular, we will use the mGC3 method of the family, which uses 3D position information and proper motions, without requiring line-of-sight velocities (Abedi et al. 2014).

2.2. Selection of the distance estimator

The Lonkin and nGC3 methods both start off by binning the sample in galactocentric distance. This is the most error prone step in both methods (see Abedi et al. 2014), so, in order to reach as far as possible in the disc with the smallest possible distance bias it is crucial to have a well-behaved distance estimator. Thus, our analysis here is focused in estimating the galactocentric distance bias for stars binned in consecutive galactocentric rings, for different distance indicators under DR2 parallax error prescriptions (Lindegren et al. 2018). This will also allow us to estimate the maximum galactocentric radius we can reach without introducing a significant bias.

We use the warped disc test particle simulation for RC stars up to $G = 20$ mag from Abedi et al. (2014) and explore as distance indicators: the distance computed as the reciprocal of the parallax (excluding stars with negative parallaxes), the Bayesian indicator with an exponentially decreasing space density (EDSD) prior proposed by Bailer-Jones (2015) and Astraatmadja & Bailer-Jones (2016), for values of the scale length parameter $L = 1.35$, 2.0 and 2.5 kpc; and the EDSD prior with a variable scale length $L(d, b)$ dependent upon the line of sight (Bailer-Jones et al. 2018). We use the posterior mode as a point estimator for the distance from here on, as in BJ18.

![Fig. 1. Median (top) and mode (bottom) bias of the true $r_{\text{gal}}$ distances of stars selected in bins of inferred $r_{\text{gal}}$, for Gaia DR2 errors. Results for the reciprocal of the parallax are shown with the (yellow) solid+circle line; the prior scale lengths used with the Bayesian estimators are indicated with different colours as shown in the figure legends.](image-url)
Figure 1 shows two summary statistics (median and mode) for the bias in the inferred galactocentric radius \( r_{\text{gal}} \), i.e. for stars in a given inferred \( r_{\text{gal}} \) bin, how deviated is the median or mode of their true \( r_{\text{gal}} \) distributions. In this plot we show the results obtained using stars with observed parallax error smaller than 50%, i.e. \( f_{\text{obs}} = |\alpha_{\text{obs}}/\omega_{\text{obs}}| < 0.5 \). We have checked that this \( f_{\text{obs}} \) threshold allows us to exclude stars with arbitrarily large fractional parallax errors (both positive and negative), while keeping the galactocentric distance bias reasonably low. We are aware that a change in the selected \( f_{\text{obs}} \) threshold would affect the distribution of true distances of the stars selected in the sample and, therefore, the optimal prior scale length.

The figure shows the behaviour of the distributions is fairly stable up to \( r_{\text{gal}} \sim 12 - 13 \) kpc, beyond which there is an abrupt change. For all estimators, the (median and mode) bias first increases up to that distance and then plummets to large negative values systematically. For any given estimator, it also shows the mode of the true distance distribution in a given bin is less biased and more stable than the median, at all distances.

For the median \( r_{\text{gal}} \), the best performance up to the threshold radius (\( \sim 13 \) kpc for RC stars) is obtained with the reciprocal of the parallax and the Bayesian EDSD prior with \( L = 2.5 \) kpc. At larger distances the \( L = 2 \) kpc estimator performs slightly better than \( L = 2.5 \) kpc and much better than the reciprocal of the parallax. In general, the BJ18 \( L(l,b) \) model results resemble the most those for the shortest scale length \( L = 1.35 \) kpc, for both the mode and median. This is consistent with the fact that values of \( L \) shorter than 1.35 kpc are the most common in the BJ18 model (see their Fig. 1).

For the mode \( r_{\text{gal}} \), the best performance is consistently obtained with the \( L = 2 \) kpc estimator, which is effectively unbiased up to \( \sim 14.5 \) kpc and showing the smallest bias (\( \sim 0.5 \) kpc) at 15.5 kpc. From \( r_{\text{gal}} > 13 \) kpc, the short scale estimators \( L = 1.35 \) kpc and \( L(BJ18) \) show a larger bias than for \( L = 2 \) kpc (\( \sim 0.3 \) kpc), half the corresponding bias expected for the median (\( \sim 0.6 \) kpc). Therefore, for the 0.5 kpc bins used in our analysis, the results for these two short scale length priors should not differ too much from those of the \( L = 2 \) kpc estimator when using the mode.

The threshold radius of \( \sim 12 - 13 \) kpc that marks the change in behaviour for the bias depends upon the selected tracer, as this choice will set a particular dependence of the parallax errors with distance, via the intrinsic magnitude, the colour and the distance modulus. Compared to the RC stars used in the tests presented here, we expect the threshold radius to be slightly larger for OB stars, around 16 kpc, since these stars are intrinsically brighter on average, based on similar tests conducted in [Abedi et al. 2014] using the reciprocal of the parallax as an estimator.

As discussed in [Luri et al. 2018], the best estimator depends upon the particular choice of the sample and its specific parallax error distribution. Our results show that there is no single estimator, of those considered here, that simultaneously outperforms all the others at all distance ranges and with all statistics. Overall, we find that for GaiaDR2 errors and \( f_{\text{obs}} < 0.5 \), the mode as the estimator with a prior scale length \( L = 2 \) kpc shows the best performance.

### 3. Selection of the two working samples in GaiaDR2

We are interested in selecting from the GaiaDR2 catalogue two samples characterised by being intrinsically bright, in order to reach the outermost parts of the disc, and with different age ranges, in order to assess whether the structural and kinematic properties of the warp depend on the age of the population. We select in the HR diagram of a young population formed by mainly upper main sequence stars or OB-type stars (hereafter, OB sample) and of an older population composed of all the stars in the red giant branch (hereafter, RGB sample). From GaiaDR2 we select stars up to magnitude \( G = 20 \), with an available parallax measurement and with \( f_{\text{obs}} < 0.5 \) (as discussed in the previous section), i.e. an absolute value of the relative error in parallax less than 50% (i.e., we keep stars with negative parallaxes). This first cut reduces the GaiaDR2 sample to 383,510,799 sources.

For these stars, we compute the distance using a Bayesian estimator with an exponentially decreasing space density prior with scale-length \( L = 2 \) kpc, based on our analysis of the distance bias for different estimators (see Sect. 2.2). Once we have derived the distances, and in order to reduce the computational cost of the process, we make a second cut which consists in removing cool main sequence stars from the sample. To do that, we select stars with \( M_G < 1.95*(G_{\text{BP}} - G_{\text{RP}})+2 \) following the extinction line, where \( M_G \) is the absolute G magnitude of the star uncorrected for extinction; \( M_G \) is given by \( M_G = G - 5 \log 10(d) + 5 \); and \( (G_{\text{BP}} - G_{\text{RP}}) \) is the observed colour. This second cut reduces the sample to 86,814,618 sources. Since we want to characterise the Galactic warp, we only keep stars with galactocentric spherical distance \( r_{\text{gal}} > 7 \) kpc and galactocentric cylindrical distance \( r_{\text{gal}} < 16 \) kpc

This second cut reduces the sample to 45,349,864 stars. For these stars, we compute the V-band absorption using the Drimmel extinction model (Drimmel et al. 2003) with re-scaling factors. Using \( A_V \) and \( (G_{\text{BP}} - G_{\text{RP}}) \), we compute \( A_G \) and the reddening \( E(G_{\text{BP}} - G_{\text{RP}}) \) according to a polynomial fit (J.M. Carrasco, private communication). We refer to this sample as the "All-Stars" sample. Finally, the two tracers are selected as in Gaia Collaboration et al. (2018):

1. Young population, OB sample:

\[
M_G < 2, \quad (G_{\text{BP}} - G_{\text{RP}}) < 0.
\]

(1)

2. RGB sample:

\[
M_G < 3.9, \quad (G_{\text{BP}} - G_{\text{RP}}) > 0.95
\]

where \( M_G = M_G' - A_G \) and \( (G_{\text{BP}} - G_{\text{RP}}) = (G_{\text{BP}} - G_{\text{RP}}) - E(G_{\text{BP}} - G_{\text{RP}}) \). The final young population (OB sample) consists of 1,860,651 stars and the RGB sample has 18,008,025

1 We use the GaiaDR2 catalogue within the Big Data infrastructure known as the Gaia Data Analytics Framework (GDAF) cluster in Universitat de Barcelona (Tapiador et al. 2017), primarily used for a scientific purpose in [Mor et al. 2018]. It runs on Apache Spark (https://spark.apache.org/), which is an engine coming from business science suited to deal with large surveys.

2 The LonKin methods work in galactocentric cylindrical bins, while the PCM works in galactocentric spherical bins.
stars. This RGB sample is a mixture of stars with ages in the range 3 – 7 Gyr (e.g. Robin et al. 2012).

Figure 2 shows the resulting distributions in G magnitude and heliocentric distance, $d$, for both working samples. We emphasise here that both samples reach up to magnitude 20 and they extend well beyond the solar neighbourhood, having $1.1 \times 10^6$ OB type stars and $13.6 \times 10^6$ RGB stars up to $d = 5$ kpc.

4. The spatial distribution of the OB and RGB samples

In this section, we analyse the spatial distribution of both working samples in order to detect the signature of the warp. We apply a coordinate transformation to the galactocentric cartesian frame and we study the structural characteristics of both samples. Throughout this work, we use a right-handed galactocentric reference frame. Its X-axis points along the Sun-Galactic centre line, away from the Sun. The Y-axis is orthogonal to the X-axis and on the Galactic plane in the direction of Galactic rotation. The Z-axis points towards the North Galactic pole. We also use cylindrical coordinates $(R, \theta, Z)$ with $R$ the Galactocentric distance and the azimuthal angle $\theta$ measured on the Galactic plane starting from the positive X-axis and towards the positive Y-axis. To perform these coordinate transforma-

4.1. Peculiar features in the stellar density distribution

In Fig. 3 we plot the stellar density map of the two samples in the $(X, Y)$ cartesian galactocentric projections. In general, both populations show the radial features characteristic of the high extinction lines (under-densities), which point to the presence of dust clouds in the direction of the line-of-sight, and of the error in distance of a given object (shown as elongated over-densities, also known as the finger-of-god effect). The effect of dust clouds is visible in both populations (OB and RGB samples), being the most prominent the ones we find in the directions: $270^\circ < l < 250^\circ$, $130^\circ < l < 150^\circ$, $180^\circ < l < 190^\circ$ and $250^\circ < l < 270^\circ$, well in agreement with the Drimmel extinction map (Drimmel et al. 2003).

Focusing on the stellar density map of the OB sample (see left panel of Fig. 3), the over-densities we observe clearly trace some known star forming regions. Zooming in at heliocentric distances closer than one kpc, we could see the Orion, Vela and the Cygnus star forming regions, well described in Zari et al. (2018). At heliocentric distances between $1 < d < 3$ kpc, we see the Cassiopeia region ($120^\circ < l < 130^\circ$), the Outer Spur ($230^\circ < l < 250^\circ$) and the Carina star forming region ($280^\circ < l < 300^\circ$). In the latter, it is where we have a larger concentration of OB stars. This over-density is well traced by the early type stars, including HII regions (e.g. Molina-Lera et al. 2016) or...
the classical Cepheids (Skowron et al. 2018), and it is often associated to the Carina-Sagittarius spiral arm of gas and dust (e.g. Martos et al. 2004). We observe that this Carina region over-density contributes to the galactocentric rings $8 < R < 10$ kpc in the study of the Galactic warp.

In the right panel of Fig. 3 we plot the stellar density map corresponding to the RGB sample. Although it apparently looks more homogeneous than the map for the OB stars, it presents a clear asymmetry in stellar density with respect to the centre–anticentre line. We show in detail this feature in Fig. 4, where we separate the RGB stars that are located above (left panel) and below (right panel) the Galactic plane. Although some of the under-densities visible in both panels have the characteristic radial trend of high extinction lines, this significant over-density region at galactocentric azimuths $135^\circ < \theta < 180^\circ$ is clearly visible in both hemispheres, and not expected in an axisymmetric Galactic potential. This over-density will require further detailed analysis in the future. Combining the information in Figs. 3 and 4 we conclude that, first, the stellar density maps of the young OB and old RGB samples clearly show opposite over-densities with respect to the centre–anticentre line at galactocentric radial bins between $8 < R < 12$ kpc.

4.2. Spatial signature of the Galactic warp

We now focus on the stellar signature of the warp detected at larger heliocentric distances by plotting the median $Z$ for each of the samples (see Fig. 5). Both samples show a clear positive median in $Z$ at galactic azimuths $90^\circ < \theta < 135^\circ$ and a negative median at $225^\circ < \theta < 270^\circ$, well in agreement with the expected behaviour of the Galactic warp (e.g. Drimmel et al. 2000; Levine et al. 2006). For intermediate azimuths, $135^\circ < \theta < 225^\circ$, where the signature of the line-of-nodes lies, the data shows a high degree of complexity, with significant differences between the two populations. From now on, we associate a stripe with null median $Z$ towards the anticentre direction as the possible location of the line-of-nodes of the Galactic warp. The RGB sample with more than 18 million stars, shows a null median $Z$ stripe inclined with respect to the Sun–anticentre direction towards $\theta \sim 180 - 200^\circ$. The fact that the stripe is curved and with galactic azimuth growing with galactocentric radius suggests that the line-of-nodes is twisted. We emphasise that these results do not agree with previous works. As an example, (Momany et al. 2006) classically placed the line-of-nodes at $\theta < 180^\circ$. Regarding the young population, our OB sample presents a clumpy distribution of median $Z$, without a clear trace of the line-of-nodes. Some recent studies using classical Cepheids, place the line-of-nodes of the warp also at $\theta < 180^\circ$ (Skowron et al. 2018). The Cepheids population is very young, so we would expect the OB sample to be comparable to the (Skowron et al. 2018) results. However, as mentioned above, the clumpy structure of the median $Z$ we find for this young population, does not allow us to support or rule out their conclusion.

Finally, in Fig. 6 we plot the density contours in the $(Y,Z)$ projection. In this projection, the X-axis is perpendicular to the plot and we expect to see the up/down side of the warp towards the Y-positive/Y-negative axis, respectively. We show the median of the three samples, namely All-Stars (black), OB (blue) and RGB (red) samples. As expected, the median of the density in the $(Y,Z)$ projection for the three samples is not flat, but it describes a warped profile. More important, the GaiaDR2 data definitively confirms that the amplitude of the warp is increasing with the age of the population.

The altitude of the warp at $R = 14$ kpc for the OB and RGB samples is given in the top rows of Table 1 where we also provide values given in other works found in the literature, for comparison. Our results show that the altitude of the Galactic warp at $R = 14$ kpc is of the order of 0.2 kpc and 1.0 kpc for the OB and RGB sam-

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**Fig. 4.** Two dimensional projections of the RGB sample. Left panel: $Z > 0$. Right panel $Z < 0$. The size of the bins are 0.16 kpc $\times$ 0.32 kpc for the RGB sample. The colour mapping scales logarithmically with number counts $N$. We mark the Sun’s position with a star and the gray solid lines show the galactocentric azimuths: $135^\circ$, $180^\circ$, and $225^\circ$. The black dotted lines show circles at different galactocentric radii, $R = 8$, 10, 12, 14 and 16 kpc.

**Fig. 5.** Two dimensional projections of the median $Z$ for the two working samples, namely OB sample (left column) and RGB sample (right column). The size of the bins are 0.32 kpc $\times$ 0.64 kpc for the OB sample and 0.16 kpc $\times$ 0.32 kpc for the RGB sample. We only plot the bins with at least 15 sources. We mark the Sun position with a star, the gray solid lines show the galactocentric azimuths: $135^\circ$, $180^\circ$, and $225^\circ$, and the black dotted lines show circles at different galactocentric radii, $R = 8$, 10, 12, 14 and 16 kpc. The color scale is different in each sample.
Table 1. Values of the warp up ($Z_{\text{up}}$) and down ($Z_{\text{down}}$) altitude with respect to the Galactic plane at galactocentric radius $R = 14$ kpc expressed in kpc. If the authors report an average value or find no difference between the up and down side, we specify it with the symbol ~ in the $Z_{\text{down}}$ column. For the Amores et al. (2017) we indicate in parenthesis the mean age of the population.

| Source | $Z_{\text{up}}$ | $Z_{\text{down}}$ |
|--------|----------------|-----------------|
| OB sample (this work) | 0.23 | -0.19 |
| RGB sample (this work) | 0.97 | -1.22 |
| Drimmel & Spergel (2001) | 1.34 | ~ |
| López-Corredoira et al. (2002) | 1.23 | ~ |
| Marshall et al. (2006) (dust) | 0.74 | -0.68 |
| Reyle et al. (2009) | 0.50 | ~ |
| Amores et al. (2014) (0.5Gyr) | 3.3 | -0.4 |
| Amores et al. (2014) (2.5Gyr) | 2.1 | -1.2 |
| Amores et al. (2017) (4.0Gyr) | 2.0 | -1.7 |
| Amores et al. (2017) (6.0Gyr) | 3.9 | -2.7 |

5.1. The kinematics in the (X,Y) plane

In Fig. 6 we plot the median of each of the components of the proper motion corrected for the Sun’s peculiar motion projected into the (X,Y) plane, galactic longitude (top panels) and latitude (bottom panels), and we analyse any substructures seen in this projection. Regarding the proper motion in longitude, $\mu_{\text{LSR}}$, as a first approximation we observe the symmetric trend expected for a flat rotation curve. The differences between the OB and RGB samples, observed as slightly larger proper motions in the range $l < 90^\circ$ and $l > 270^\circ$, are associated with order effects such as the asymmetric drift and the orbits of the stars of the tracer populations. Possible trends induced by the Galactic warp are blurred due to the above mentioned effects, so the kinematic characterisation of the warp is better constrained using the proper motion in the galactic latitude, as seen in the bottom panels of Fig. 6.

In the median $\mu_{\text{LSR}}$ distribution, we clearly see some radial features in the kinematic map, such as those at $l \sim 55, 135, 200, 220^\circ$. We attribute them to high extinction lines together with the fact that the density profile of the populations is not homogeneous above and below the disc. We also see these effects in Foggio et al. (2018), with similar but less extended samples.

5.2. The kinematic signature of the Galactic warp

In this section, we analyse the kinematic distribution of both working samples using three strategies. We start by projecting the median proper motion distribution into the Galactic plane (Sect. 5.1) and then by applying the two methods described in Sect. 2.1, namely the LonKin and the nGC3 PCM methods (Sect. 2.2), which extract and quantify the overall signal of the warp using both position and kinematic information. We show simplified theoretical expectations for three different warp models as a guide on how to interpret these results.

3 The warp models used are described in detail in appendix A.
5.2. The warp as seen by the LonKin method

Here we show the LonKin result for the OB and RGB, together with a toy theoretical prediction. Figure 8 shows the results for the OB sample (blue lines) and the RGB sample (red lines) in galactocentric radial bins of increasing distance from top to bottom panels. Only the longitude bins for which there are at least 300 stars are shown. Figure 9 shows the model predictions for a random realisation of three warp models, described in detail in appendix C.

As already hinted in the previous section, GaiaDR2 data reveals clear differences between the two populations. As for the OB stars, the inner annuli show an almost flat trend with a median proper motion of about \(-0.2\) mas yr\(^{-1}\), while for the RGB, the median is slightly positive. In the middle annuli, the proper motion increases, as expected in a simple warp model, showing the OB stars a quite planar trend in the outer disc, from \(l=90^\circ\) to \(l=270^\circ\), and a sharp decrease towards the inner disc, whereas the RGB stars clearly show that the maximum in the proper motion is not achieved in the anti-centre direction, while it shows two local maxima about \(l=140^\circ\) and \(l=220^\circ\), the one in the second quadrant being slightly higher. As we move towards the outer annuli (bottom panel), the OB stars distribution becomes clearly asymmetric (see top left panel of Fig. 8), and the LonKin only provides information in the third and fourth quadrant, showing a maximum in the proper motion around \(l=220^\circ\). The proper motion for the RGB sample shows a clearly different trend from that of the OB stars. The trend that we could initially glimpse in the middle annuli, is clear here, with the maxima moving slightly towards the inner disc. Also, the median proper motion in the anti-centre direction is almost zero for the radial bin \(14 < R < 15\) kpc, and becomes negative in the outermost ring. We can also distinguish a significant local minimum at \(l=120^\circ\), this corresponds to the "blob" already detected in the two dimensional maps shown above (see Fig. 7). This minimum in proper motion in latitude could be related to a negative median \(Z\) at the same region (see right panel of Fig. 5) due to high extinction and the fact that the distribution of RGB stars is not homogeneous above and below the disk. It might also be caused by some type of substructure present in the Southern hemisphere.

For comparison, in Fig. 9 we show the trend in proper motion predicted by three different, yet simple, warp models, whose free parameters (warp amplitude and onset radius) are fixed using a recent fit by age populations of the Besançon Galaxy model to 2MASS data (Amorès et al. 2017). We have used the parameters corresponding to \(<\text{ age }>=2.5\) Gyr to show each model’s expected trend, even though we have found these to be over-estimated (Sect. 4). The amplitude of the curves in Fig. 8 supports our findings from Sect. 4 that the warp has a low amplitude. The fact that the proper motion for the RGB data has a double-peak tendency and that it becomes negative towards the anti-centre at large galactocentric distances (\(14 < R < 15\) kpc) indicates that an S Lopsided model would be a right choice as a starting model (see further discussion in Sect. 4).

The LonKin method also allows us to estimate the initial radius of the warp. We find that the initial radius for the OB sample is \(12 < R < 13\) kpc, while for the RGB sample is \(10 < R < 11\) kpc. We again confirm an age dependence of the warp characteristics, as already shown in Sect. 4.

![Fig. 7. Two dimensional maps of the proper motion in galactic coordinates of the two working samples, namely OB sample (left column) and RGB sample (right column). Top panels: Median proper motion in the longitude direction corrected for the Sun’s peculiar motion, \(\mu_{l,LSR}\). Bottom panels: Median proper motion in the latitude direction corrected for the Sun’s peculiar motion, \(\mu_{b,LSR}\). The black dotted lines show circles at different galactocentric radii, \(R=8, 10, 12, 14\) and \(16\) kpc. The size of the bins are \(0.32 \text{kpc} \times 0.64 \text{kpc}\) for the OB sample and \(0.16 \text{kpc} \times 0.32 \text{kpc}\) for the RGB sample. We only plot the bins with at least 15 sources. The grey solid lines show azimuths \(135^\circ\) and \(225^\circ\), while the black horizontal line show the centre–anti-centre direction. The position of the Sun is shown with a black small star.](image-url)
In Fig. 10 we check whether the results of applying the LonKin method shown in Fig. 8 depend on the choice of the distance estimator. We show the results of applying the LonKin method to the radial annuli $13 < R < 14$ kpc obtained for two different prior scale-lengths, $L=2$ kpc (solid lines) and $L=1.35$ kpc (dotted lines) in red for the RGB sample and in blue for the OB sample. The peak that traces the warp (cross) reaches co-latitudes close to $4^\circ$ for the RGBs compared to $\sim 2.5^\circ$ for the OBs, at $r_{gal} \sim 14$ kpc. These tilt values estimated from the PCMs are compatible with those shown by the Y-Z density plot in Fig. 6 at the same distance. The behaviour of the tilt angle as a function of $r_{gal}$ is summarised in Fig. 12 where the larger amplitude of the tilt angle for the RGB compared to the OB is evident, at all radii. The figure also compares these results with those obtained with a shorter prior scale length ($L=1.35$ kpc); for the two tracers both sets are in excellent agreement, showing our results are robust against the choice of the distance estimator.

5.3. nGC3 signature of the warp

Here we show the nGC3 result for the OB and RGB, together with a toy theoretical prediction. The nGC3 PCMs for the OB and RGB samples are shown in the left and right panels of Fig. 11, respectively, for $r_{gal}$ bins of 0.5 kpc width. Qualitatively the two sets of PCMs for both tracers have a similar behaviour: there is a peak at the north Galactic pole, i.e. the centre of the PCM, that remains fixed at all $r_{gal}$ and another peak in the lower-left quadrant of the PCM, whose co-latitude increases as $r_{gal}$ does. This latter peak is the clear signature of a warped disc whose amplitude increases with radius. Comparing the PCMs for the two samples, we see that the amplitude of the warp for the RGB stars is larger than for the OB stars at a given radius, as already shown by the LonKin method (Sect. 5.2) and the spatial distribution (Sect. 4). For example, the centroid of the peak that traces the warp (cross) reaches co-latitudes close to $4^\circ$ for the RGBs compared to $\sim 2.5^\circ$ for the OBs, at $r_{gal} \sim 14$ kpc. These tilt values estimated from the PCMs are compatible with those shown by the Y-Z density plot in Fig. 6 at the same distance. The behaviour of the tilt angle as a function of $r_{gal}$ is summarised in Fig. 12 where the larger amplitude of the tilt angle for the RGB compared to the OB is evident, at all radii. The figure also compares these results with those obtained with a shorter prior scale length ($L=1.35$ kpc); for the two tracers both sets are in excellent agreement, showing our results are robust against the choice of the distance estimator.

Figure 13 shows the predictions given by the three simple warp models applied to the same realisation of particles as in Fig. 9 at the galactocentric ring $r_{gal} = 13 - 14$ kpc. The pole counts signal in and around the north pole that appears at all radii is incompatible with the tilted-ring-like models like the SS or the SineLop model. Only the SLop model provides a good fit to the data. The theoretical prediction provided by the nGC3 PCM applied to the three warp models is given in detail in appendix C.3 and Figs. C.4, C.5 and C.6.
model predicts a similar signal, as our tests with random realisation of particles. Also, the inclined shaped of the peak that traces the warp (centroid indicated with a cross) seems most alike the PCM signature shown for the test particle PCMs in the SLop model. We do stress that the theoretical predictions given by the simple warp models do not fully reproduce the features in the observed PCMs, though the S Lop-sided model is the one that qualitatively better matches the data, as already the LonKin method points to.

We emphasise that for the SLop model the mean tilt angle derived from the PCM is not trivially related to the up/down tilt angles of the model due to the overlap of the two signatures (see discussion in appendix C). Deriving values for these parameters would require detailed modelling. Looking at the change of slope of the tilt angle as a function of $r_{gal}$ in Fig. 10, we estimate the starting radius of the warp to be around $12 - 12.5$ kpc for the OB sample and around $10.5 - 11$ kpc for the RGB sample, in agreement with the reported values using the LonKin method.

6. The richness and complexity of the vertical motion

From Sects. 4 and 5, we clearly confirm the presence of the Galactic warp in GaiaDR2 data. The projections of the positions to the plane perpendicular to the Sun-anticentre line (Fig. 6) of the OB and RGB samples reveal that the amplitude of the warp is different according to the age of the population, being the warp in the RGB sample more prominent than in the OB sample.

The simple theoretical predictions we have shown are intended to shed some light and help interpreting the results in terms of different warp models. The two dimensional projection into the Galactic plane clearly shows the the line-of-nodes in the RGB sample is not shifted towards galactocentric azimuth $\theta > 180^\circ$ and twisted in this direction from $R > 12$ kpc. The results of applying both the LonKin and $nGC3$ PCM methods, together with the kinematic two dimensional projections, suggest that we can indeed rule out that the Galactic warp is symmetric. Both the SineLop and SLop models predict that when the warp is lopsided, the maximum proper motion does not coincide with the line-of-nodes, as we have clearly observed with the LonKin method and in the kinematic 2D maps, with both tracers. Poggio et al. (2018), using similar tracers but with smaller radial and magnitude coverage, note that the maximum vertical velocity they observe in their maps is not along the Sun–anticentre line and say this might be due to the Sun not being in the line-of-nodes. Here, we trace that kinematic signature further out in the disc and give an alternative interpretation based on our model predictions: that the offset between the line-of-nodes and the line of maximum vertical proper motion is due to the lopsidedness of the disc’s warp.

Furthermore, the presence of two relative maxima with different amplitude and a negative dip in proper motions in the LonKin plots for the RGB sample favour the SLop model as a more suitable description of the warp. The fact that the LonKin method favours an SLop shape is corroborated by the $nGC3$ PCM method. The over-density in the PCMs is not point-like, as would be expected from a symmetric warp; instead, it is elongated and rotated with respect to the meridians which, coupled, suggest lopsidedness. Additionally, at all radii there is significant signal in the PCMs at the north Galactic pole, a feature characteristic of the SLop model alone, not observed in either of the tilted-rings symmetric (SS) or SineLop models. The signature observed in the GaiaDR2 data PCMs is very similar to what we see in the test particle simulations with an SLop model (see right panel of Fig. C.6). As stated above, the simple tilted-ring-models used here do not reproduce every feature the kinematic data reveal, but they suggest that the SLop model would be an appropriate starting point to model the Galactic warp. The SLop model is designed such that the disc is flat not just along the line-of-nodes, like in the SS and SineLop models, but also in a region around it, out to the warp starting radius roughly (see Fig. A.1 for a schematic representation of the three warp models).

Figures 7, 8, and 11 clearly show the high complexity of the Galactic disc structure and it is clear that, in addition to it being detected in density in Fig. 6, the kinematics are in agreement with the presence of a Galactic warp. The large amount of the data and its high accuracy reveal that the kinematic maps (Fig. 8) are not dominated by a single phenomenon, as we might expect from a Galaxy that has not reached yet equilibrium (Antoja et al. 2018). The two dimensional projection of the median proper motion in latitude corrected from the Sun’s peculiar motion, $\mu_b, LSR$, reveals complex and non-uniform trends. The radial heliocentric features are clearly related to extinction effects. But the data also highlights wave-like trends that remind those of bending and breathing modes. The bending mode, as defined recently by Widrow et al. (2014), is the mean of the vertical component of the velocity measured at two parallel layers above and below the Galactic plane. In this sense, the projection of median proper motion in latitude can be viewed as a measure of the bending mode at (large) heliocentric distances where the line-of-sight velocity does not dominate the motion. Thus, from the analysis of the proper motion in latitude we can glimpse the effect of the Galactic warp (via the LonKin and $nGC3$ PCM methods) and the bending modes (via the two dimensional projection) at large distances. The features with median negative proper motion alternating with median positive values in the two dimensional map clearly have a wave-like pattern reminiscent of a bending motion (as previously found by Schönrich & Dehnen 2018).

Throughout all this analysis we have detected two significant features in the structure and kinematic characterisation of the RGB sample: first, an over-density of RGB stars above the Galactic plane and, possibly, an under-density of RGB stars below the Galactic plane, towards galactocentric azimuths $135^\circ < \theta < 180^\circ$. Second, in a similar but more constrained azimuthal range and in the outer disc, $R > 12$ kpc, the so-called “blob”, a minimum in proper motion in latitude, which is not predicted by the theoretical models and it is not consistent with a wave-like pattern either. We wonder whether these two feature are related. This sudden drop in proper motion is located near the region where there is also a lack of stars in the northern hemisphere at the same longitude range (see right panel of Fig. 5). There are not enough stars in the OB sample in that region to draw this conclusion. A similar gap in this quadrant is also seen in the open cluster distribution in the disc reported by Cantat-Gaudin et al. (2018, private communication), while Bisht et al. (2016) find a bend of the Galactic disc towards the southern latitude using young star
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clusters with an heliocentric distance larger than 2 kpc in the longitude range $l = 130^\circ - 180^\circ$. In this same longitude range and below the galactic plane, there has been long debated the origin of the TriAnd overdensity [Majewski et al. 2004; Deason et al. 2014] which recently using the chemical composition has been related to the Galactic disc, extending well beyond 20 kpc from the Galactic centre. This is a much more distant feature than the "blob", but our lack of data in the range from 15 to 20 kpc prevents us from concluding whether there is a connection or between the two. From all these works, it seems that this region shows different features. So we suggest that the "blob" we detect as an overdensity below the Galactic plane with negative median proper motion, is a physical feature in the Galactic disc.

7. Conclusions

We have used two different tracers from GaiaDR2 data, namely a young bright population, the OB sample (1.8 Million sources), and the Red Giant Branch, the RGB sample (18 Million sources), in order to study their kinematics and relate them to the Galactic warp structure. To achieve this
goal, we have used different and complementary analyses that provide structural (Sect. 4) and kinematic (Sect. 5) information of the warp. Our results using spatial only data are the following:

1. The RGB sample presents a clear asymmetry in stellar density with respect to the centre–anticentre line at galactocentric azimuths $135^\circ < \theta < 180^\circ$ clearly visible in both hemispheres, and not expected in an axisymmetric Galactic potential.

2. The RGB sample shows a null median-Z stripe, the line-of-nodes, rotated with respect to the Sun–anticentre direction towards $\theta \sim 180^\circ - 200^\circ$ and twisted from $R > 10$ kpc. This suggests that the line-of-nodes of the Galactic warp is twisted.

3. The GaiaDR2 data definitively confirms that the altitude of the warp increases with the age of the population. The median of the Galactic warp at $R = 14$ kpc is of the order of $0.2$ kpc and $1$ kpc for the OB and RGB samples, respectively.

4. The RGB sample reveals a slightly lopsided Galactic warp in the sense $|Z(\text{down})| - |Z(\text{up})| > 250$ pc. No solid conclusion can be established from the OB sample, for which lopsidedness seems to be much smaller.

Combining spatial and kinematic information, we conclude:

1. The median vertical proper motion, $\mu_b, \text{LSR}$, values towards the antecentre show a clear vertical modulation. Whereas the OB sample has a transition from negative to positive values, the reverse is observed in the RGB sample. This is a clear first signature of the age dependency of the bending vertical motion.

2. The offset between the line-of-nodes ($\theta \sim 180^\circ - 200^\circ$) and the line of maximum vertical proper motion, detected in a significant stripe towards galactic azimuths $\theta \sim 160^\circ - 170^\circ$, can be naturally associated by our models to the lopsidedness of the disc’s warp.

3. The fact that the vertical proper motion for the RGB data has a double-peak tendency in the LonKin and that it becomes negative towards the anti-centre at large galactocentric distances ($14 < R < 15$ kpc) together with the signature found in the PCM, are again clear indications of lopsidedness and suggests that an S Lopsided model would be a right choice as a starting model.

4. Both the LonKin and the $nGC3$ PCM methods also allow us to estimate the initial radius of the warp. We find that the initial radius for the OB sample is $12 < R < 13$ kpc, while for the RGB sample is $10 < R < 11$ kpc, again with a clear age dependency.

5. The data also highlights wave-like trends that remind those of bending and breathing modes.

The kinematic maps of both samples reveal a complex disc, not in equilibrium, in agreement with recent works (e.g. Antoja et al. 2018), that reflect multiple phenomena. New models with a higher degree of complexity and methods, flexible enough to be able to capture the complexity of the disc kinematics, are required.

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Appendix A: Warp models

We describe the three warp models we use in this study to compare with data: one symmetric as in Abedi et al. (2014), the "Simple Symmetric warp," and two lopsided models first used in this work: the "Sine Lopsided" and the "S Lopsided" models. In Fig. A.1 (first to third row), we show a schematic representation in 3D (left column) and in projection along the line-of-nodes (right column) of the three warp models and an example of the twisted S Lopsided model (fourth row), whose mathematical forms are described in detail in the following subsections.

Appendix A.1: The Simple Symmetric warp

The Simple Symmetric warp (SS) is already described in detail in Abedi et al. (2014). It consists of splitting the Galactic disc into galactocentric rings and tilting each one by an angle $\psi$ as a function of the galactocentric radius. The expression for the tilt angle is:

$$\psi_{SS}(R; R_1, R_2, \alpha, \psi_2) = \psi_2 f(R; R_1, R_2, \alpha)$$  \hspace{1cm} (A.1)

where the function $f(R)$ denotes the dependence of the tilt angle with radius and we express it as a power law:

$$f(R; R_1, R_2, \alpha) = \begin{cases} 0, & R \leq R_1 \\ \left(\frac{R-R_1}{R_2-R_1}\right)^\alpha, & R_1 < R < R_2 \\ 1, & R \geq R_2 \end{cases}$$  \hspace{1cm} (A.2)

where $R_1$ and $R_2$ are the initial and final radius of the warp, and the exponent $\alpha$ fixes the shape of the power law. We notice that once the warp parameters $R_1$, $R_2$, and $\alpha$ have been fixed, the tilt angle only depends on the galactocentric radius $R$, and has a constant maximum amplitude at $R \geq R_2$ of $\psi_2$. This defines a symmetric warp with respect to the line-of-nodes, i.e. the maximum amplitude of the warp is the same at corresponding points on both sides of the line-of-nodes. According to observational evidence, the up (down) side of the Galactic warp is towards the northern (southern) hemisphere (e.g. Oort et al. 1958; López-Corredoira et al. 2002; Levine et al. 2006).

Appendix A.2: The Sine Lopsided Model

The Sine Lopsided model (SL) is a variation of the Simple Symmetric model. It introduces a dependence on the galactocentric azimuth, $\theta$, while keeping the same radial dependence as the Simple Symmetric warp model. Therefore, the maximum amplitude of the warp changes on either side of the line-of-nodes. The tilt angle is given by:

$$\psi_{SL}(R ; \theta; R_1, R_2, \alpha, \psi_{up}, \psi_{down}) = f(R; R_1, R_2, \alpha) \left[ A + B \sin(\theta) \right]$$  \hspace{1cm} (A.3)

where $A = (\psi_{up} + \psi_{down})/2$, $B = (\psi_{up} - \psi_{down})/2$ and $\psi_{up}$ and $\psi_{down}$ are the maximum amplitudes of the warp at the up and down warps, respectively, at the azimuth perpendicular to the line-of-nodes. Note that, using this expression, rings are not planar any more, i.e. a warped ring when seen edge-on is not a flat line, but it is slightly bent at the line-of-nodes, as illustrated by the edge-on view shown in Fig. A.1 (second row, right). This curvature is caused by the sine dependence on the azimuth, that allows us to introduce an arbitrary up/down maximum amplitude asymmetry in a smooth (continuous and differentiable) manner across the line of nodes, which is not attainable by just setting different tilt angles at either side of the line of nodes in the tilted ring model of the previous section. Hence, the expression of Eq. (A.3) provides a lopsided disc (see second row panels of Fig. A.1). In the Galactic disc’s warp, the amplitude in the up side is larger than in the down side (Levine et al. 2006; Marshall et al. 2006).

Appendix A.3: The S Lopsided Model

The S lopsided model (SLop) is designed to have an S-shape when seen edge-on, that is, it is flat at the line-of-nodes. It is a variation of the Sine Lopsided model with a different dependence on the galactocentric azimuth. We first define an angle function given by:

$$g(\theta) = \begin{cases} \psi_{up}, & 0 \leq \theta < 180^\circ \\ \psi_{down}, & 180^\circ \leq \theta < 360^\circ \end{cases}$$  \hspace{1cm} (A.4)

where, as previously, $\psi_{up}$ and $\psi_{down}$ are two constant angles which are the maximum amplitudes of the warp at the up and down sides, respectively.

The tilt angle for this model is given by:

$$\psi_{SLop}(R ; \theta; R_1, R_2, \alpha, \psi_{up}, \psi_{down}) = f(R; R_1, R_2, \alpha) g(\theta) \sin^2(\theta)$$  \hspace{1cm} (A.5)

Appendix A.4: Twisted versions

In all warp models, the line-of-nodes can either be a straight line aligned with the Sun – Galactic centre line or it can be twisted an angle $\phi$:

$$\phi(R; R_1, R_2, \gamma, \phi_{max}) = \begin{cases} 0, & R \leq R_1 \\ \phi_{max} \left(\frac{R-R_1}{R_2-R_1}\right)^\gamma, & R_1 < R < R_2 \\ \phi_{max}, & R \geq R_2 \end{cases}$$  \hspace{1cm} (A.6)

where it increases with radius from the x-axis (see last row of Fig. A.1).

Appendix B: Warp transformations

The purpose of this section is to develop the equations that provide the spatial and velocity coordinates for particles in a warped disc. We apply a warp transformation to the spatial coordinates of particles in a flat disc, and then find the corresponding transformation for the velocities. As we will see, velocities require a different transformation.

Appendix B.1: The spatial transformation

As mentioned in the previous section, to warp a flat disc we use a rotation by an angle $\psi$ with respect to the positive $x$-axis (see Fig. B.1):

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\psi) - \sin(\psi) & \cos(\psi) \\ 0 & \sin(\psi) & \cos(\psi) \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \cos(\psi) - z \sin(\psi) \\ z \cos(\psi) + y \sin(\psi) \end{pmatrix}$$
Appendix B.2: The velocity transformation

One may think that velocities are transformed using the same transformation used for positions, but this is not the case. Velocities are spatial displacements per time, and so they are transformed as:

\[ v'_i = \frac{dx'_i}{dt} = \frac{\partial x'_i}{\partial x_j} \frac{dx_j}{dt} = \Lambda'_{i,j} v_j \]  

(B.2)

where summation over repeated indices is implied.

In order to compute the \( \Lambda' \) matrix, first we obtain the derivatives of the tilt angle in Cartesian coordinates. We note that the dependence of the tilt angle is given by:

\[ \psi = \psi(R, \theta), \quad \text{with} \quad R = (x^2 + y^2)^{1/2}, \quad \theta = \arctan{y/x} \]  

(B.3)

The components of the Jacobian of the transformation between polar and Cartesian coordinates are:

\[ \frac{\partial R}{\partial x} = \frac{x}{R}, \quad \frac{\partial R}{\partial y} = \frac{y}{R}, \quad \frac{\partial \theta}{\partial x} = -\frac{y}{R^2}, \quad \frac{\partial \theta}{\partial y} = \frac{x}{R} \]  

(B.4)

The partial cartesian derivatives of the tilt angle are then:

\[
\begin{align*}
\frac{\partial \psi}{\partial x} &= \frac{\partial \psi}{\partial R} \frac{\partial R}{\partial x} + \frac{\partial \psi}{\partial \theta} \frac{\partial \theta}{\partial x} = \frac{x}{R} \frac{\partial \psi}{\partial R} - \frac{y}{R^2} \frac{\partial \psi}{\partial \theta}, \\
\frac{\partial \psi}{\partial y} &= \frac{\partial \psi}{\partial R} \frac{\partial R}{\partial y} + \frac{\partial \psi}{\partial \theta} \frac{\partial \theta}{\partial y} = \frac{y}{R} \frac{\partial \psi}{\partial R} + \frac{x}{R^2} \frac{\partial \psi}{\partial \theta}
\end{align*}
\]

(B.5)

With these, we can now proceed to get the individual components of the \( \Lambda' \) matrix using equation (B.1):

\[ \Lambda'_{1,1} = (\partial x'/\partial x) = 1, \]

\[ \Lambda'_{1,2} = (\partial x'/\partial y) = 0, \]

\[ \Lambda'_{1,3} = (\partial x'/\partial z) = 0 \]

\[ \Lambda'_{2,1} = \frac{\partial}{\partial x} [y \cos(\psi) - z \sin(\psi)] = -\frac{\partial \psi}{\partial x} [z \cos(\psi) + y \sin(\psi)] \]

\[ \Lambda'_{2,2} = \frac{\partial}{\partial y} [y \cos(\psi) - z \sin(\psi)] = \cos(\psi) - \frac{\partial \psi}{\partial y} [z \cos(\psi) + y \sin(\psi)] \]

\[ \Lambda'_{2,3} = \frac{\partial}{\partial z} [y \cos(\psi) - z \sin(\psi)] = -\sin(\psi) \]

\[ \Lambda'_{3,1} = \frac{\partial}{\partial x} [z \cos(\psi) + y \sin(\psi)] = \frac{\partial \psi}{\partial x} [y \cos(\psi) - z \sin(\psi)] \]

\[ \Lambda'_{3,2} = \frac{\partial}{\partial y} [z \cos(\psi) + y \sin(\psi)] = \sin(\psi) + \frac{\partial \psi}{\partial y} [y \cos(\psi) - z \sin(\psi)] \]

\[ \Lambda'_{3,3} = \frac{\partial}{\partial z} [z \cos(\psi) + y \sin(\psi)] = \cos(\psi) \]

(B.6)

If we now define the following functions: \( P \equiv z \cos(\psi) + y \sin(\psi), \quad Q \equiv y \cos(\psi) - z \sin(\psi), \) the \( \Lambda' \) matrix can be written in a very compact way as:

\[ \Lambda' = \begin{pmatrix} 1 & 0 & 0 \\ -P(\partial \psi/\partial x) & \cos(\psi) - P(\partial \psi/\partial y) & -\sin(\psi) \\ Q(\partial \psi/\partial x) & \sin(\psi) + Q(\partial \psi/\partial y) & \cos(\psi) \end{pmatrix} \]  

(B.7)
where the partial derivatives of $\psi$ are given by equations (B.5) and (B.6).

Note that in the case of a fixed tilt angle (inclined plane), or if we consider pure circular rotation along flat, tilted rings, the partial derivatives are null and the $A'$ matrix coincides with the spatial transformation matrix of equation (B.1). But in the general case, the $\psi$-derivatives make the velocity transformation different from the spatial one.

The transformed (but unscaled) velocity components are then given by:

\[
v'_x = v_x \quad \text{(B.8)}
\]

\[
v'_y = -v_x P(\partial \psi / \partial x) + v_y \{ \cos(\psi) - P(\partial \psi / \partial y) \} - v_z \sin(\psi) \quad \text{(B.9)}
\]

\[
v'_z = v_z Q(\partial \psi / \partial x) + v_y \{ \sin(\psi) + Q(\partial \psi / \partial y) \} + v_x \cos(\psi) \quad \text{(B.10)}
\]

Now, since the velocity transformation redirects the velocity vectors, but should not change their magnitudes, we need to scale the velocity components making sure that the $v_x$ remains constant. In other words, we need to satisfy:

\[
v'_x^2 + v'_y^2 + v'_z^2 = (v_x')^2 + (v_y')^2 + (v_z')^2 \quad \text{and} \quad v_x = v'_x \quad \text{(B.11)}
\]

This implies:

\[
v'_y^2 + v'_z^2 = (v_y')^2 + (v_z')^2 \quad \text{(B.12)}
\]

Which means that the scaling for the transformed velocity components is:

\[
v'_x = v_x, \quad v'_y = v_y \times v_{\text{scale}}, \quad v'_z = v_z \times v_{\text{scale}} \quad \text{(B.13)}
\]

where the scaling factor is:

\[
v_{\text{scale}} = \sqrt{\frac{v'_x^2 + v'_z^2}{(v_y')^2 + (v_z')^2}} \quad \text{(B.14)}
\]

It only remains to specify $(\partial \psi / \partial R)$ and $(\partial \psi / \partial \theta)$ for each of the three warp models used.

For the Simple Symmetric model, the tilt angle derivatives are simply:

\[
\frac{\partial \psi_{\text{SS}}}{\partial R} = \psi_{\text{max}} \frac{df}{dR}, \quad \frac{\partial \psi_{\text{SS}}}{\partial \theta} = 0 \quad \text{(B.15)}
\]

where $f(R)$ is as in Eq. (A.2)

For the Sine Lopsided model, the derivatives of $\psi_{\text{SL}}$ are:

\[
\frac{\partial \psi_{\text{SL}}}{\partial R} = \frac{df}{dR}(A + B \sin(\theta)), \quad \frac{\partial \psi_{\text{SL}}}{\partial \theta} = f(R) B \cos(\theta) \quad \text{(B.16)}
\]

Finally, for the S Lopsided model, the derivatives are:

\[
\frac{\partial \psi_{\text{SLop}}}{\partial R} = \frac{df}{dR} g(\theta) \sin^2(\theta), \quad \frac{\partial \psi_{\text{SLop}}}{\partial \theta} = f(R) g(\theta) \sin(2\theta) \quad \text{(B.17)}
\]

where $g(\theta)$ is as in Eq. (A.4)

In the transformation outlined here, the twist is not included. This for reasons of brevity and clarity. To include a twist, an additional rotation around the positive $z$-axis by an angle $\phi$ should be added to the rotation matrix (Eq. (B.1)) according to equation (Eq. (A.6)). The resulting combined rotation matrix should be used to compute the new version of the $A'$ matrix.

**Appendix C: Prediction of the kinematic signature of the warp as seen by the Lonkin and nGC3 methods**

In this section, we discuss the expected theoretical signatures of the three warp models described in Sect. A for the LonKin and the nGC3 PCM methods.

To predict and understand the kinematic signatures of the different warp models we use three different test scenarios of increasing complexity: a circular orbit, a random realization of warp models and a test particle simulation.

We fix the free parameters of the warp models to the values derived from Amôres et al. (2017) for a population of Red Clump stars (hereafter, RC): $R_1 = 10.1$ kpc, $R_2 = 14$ kpc, $\alpha = 1.1$ and $\psi_2 = 7:5$, $\psi_{\text{up}} = 7:5$ and $\psi_{\text{down}} = 4:25$. In the case of the twisted line-of-nodes, we fix $\phi_{\text{max}} = 20^\circ$ and $\gamma = 1.1$. In the remainder of this section we will assume these numerical values for the warp model parameters.

**Appendix C.1: The three test scenarios**

The first scenario is the simplest case of a planar circular orbit warped following each of the three warp models. We take a circular orbit at $R_{\text{gal}} = R_2 = 14$ kpc, with circular velocity of $V_{\text{c}}(R_2) = 215$ km s$^{-1}$, assuming the rotation curve of Allen & Santillan (1991). The equations we use to warp positions and velocities are detailed in appendix B.

The second scenario is a random realisation, obtained by taking an initial set of particles in a planar disc and applying positions and velocities transformations to each of the particles. As the initial set, we take the set of disc Red Clump (hereafter, RC) initial conditions as in Abedi et al. (2014); Romero-Gómez et al. (2015). They are a subset of the red giant branch stars, well defined in the HR diagram. They are generated according to a disc population of RC stars with the corresponding velocity dispersion and with the density in statistical equilibrium with the Miyamoto-Nagai disc potential (see appendix A of Romero-Gómez et al. 2015). We note that by construction, the realisation has non-zero velocity dispersion.

The third scenario considered is the test particle simulation, for which we take the same initial conditions for a disc RC population as we did for the realisation. The number of particles in the simulation is chosen to have a realistic surface number density of RC stars in the Solar neighbourhood (Czekaj et al. 2014; Romero-Gómez et al. 2015). The integration strategy is the same as in Abedi et al. (2014), where we integrated the initial conditions to obtain a test particle simulation in statistical equilibrium with the imposed potential. This strategy consists of integrating the particles in a flat disc potential for two periods (of an orbit at $R_{\text{gal}} = 14$ kpc), then integrating for another five periods while the disc potential is changed towards the chosen warp model, and in the third step, integrating two further periods to allow particles to settle into the final warped disc potential. We want to stress that the success of this strategy to obtain a set of particles in statistical equilib-
Appendix C.2: Theoretical signature in the LonKin method

First, we consider the simplest case of a warped circular orbit. In Fig. C.1 we show how a warped circular orbit at $R_{\text{gal}} = 14$ kpc looks when we apply the LonKin method using the three warp models described in Sect. A, with and without twist of the line-of-nodes. As expected from Abedi et al. (2015), the proper motion in latitude, $\mu_b , \text{LSR}$, has a maximum in the anti-centre direction when the orbit is tilted using the Simple Symmetric model. As soon as the symmetry is broken, i.e. in the lopsided models, the maximum is no longer in the anti-center and the proper motion in latitude has a different behaviour depending on the model used to warp the orbit. The S Lopsided model simply displaces the maximum towards the longitude of the warp’s maximum amplitude. When the orbit is warped using the S Lopsided model, the proper motion in latitude becomes zero at the anti-centre direction. Note that in this model, the disc is flat at the line-of-nodes. The main characteristic of the S Lopsided model is the presence of two relative maxima at $l \sim 90^\circ$ and $l \sim 270^\circ$.

In Fig. C.2, we apply the LonKin method to a random realisation of particles inside the cylindrical galactocentric radial bin $13 - 14$ kpc. As expected, the curve corresponding to the Simple Symmetric warp is essentially flat in the anti-centre direction and symmetric with respect to the line-of-nodes, while the signature of the Sine Lopsided warp displaces the maximum towards the longitudes of maximum warp amplitude and the S Lopsided warp shows two clear maxima at $l \sim 90^\circ$ and $l \sim 270^\circ$. We note that warping the ensemble of particles using this last model, the proper motion in latitude is no longer zero at the line-of-nodes. This fact is due to the velocity dispersion of the generated particles.

Finally, in Fig. C.3 we show the result of applying the LonKin method to the test particle simulations of disc RC stars. We divide Fig. C.3 in three panels according to the galactocentric radial bin of the particles. From top to bottom: $9 < R < 10$ and $10 < R < 11$ kpc; $11 < R < 12$ kpc.
Fig. C.3. LonKin method applied to RC test particle simulations. Left panel: using the Sine Lopsided model. Right panel: using the S Lopsided model. From top to bottom, different radial galactocentric bins, specified in the legend, expanding from the inner to the outer disc. Horizontal dashed line shows the zero-axis while the vertical dashed line shows the anti-centre direction at $l = 180^\circ$. As in Fig. C.2, the error bars show the lower and upper 1σ uncertainty of the median.

and $12 < R < 13$ kpc; and $13 < R < 14$, $14 < R < 15$ and $15 < R < 16$ kpc. The warp models used are the Sine Lopsided (left) and the S Lopsided (right). We stress here that only the test particle simulation using the Sine Lopsided model is in statistical equilibrium, while the result of the test particle simulation using the S Lopsided model will not follow the trend predicted for such model (compare Figs. C.1 and C.2) because it is not in statistical equilibrium. As for the Sine Lopsided test particle simulation, we note that the median proper motion in latitude increases towards the outer disc and the outermost curve shows the expected shape for this model (compare Figs. C.1 and C.2), namely the maximum of the curve is shifted towards the maximum amplitude of the warp. Thus, if the disc of the Milky Way is warped following a Sine Lopsided model, we should expect a signature in the LonKin method qualitatively similar to that in the left panel of Fig. C.3. As expected, the results regarding the test particle simulation not being in statistical equilibrium using the S Lopsided model have lost the predicted signature for this model in the previous scenarios. The proper motion in latitude does not show the two relative maxima. Since the simulation is not in equilibrium, it is expected that the median values of the proper motion in latitude per longitude bins resemble those for a symmetric warp: we simply recover the increase of the proper motion with respect to the galactocentric radius (Eqs. A.1 and A.2), but it clearly does not show a lopsided behaviour.

Fig. C.4. $nGC3$ PCM signatures for a circular orbit at $r_{gal} = 14$ kpc, for each of the three warp models: Simple Symmetric warp (black), the Sine Lopsided warp (red) and the S Lopsided warp (blue). The dotted circles correspond to parallels at co-latitudes of 2°, 4°, 6°, 8° and 10°, with the outermost one (solid black line) corresponding to 10°.

Appendix C.3: Theoretical signature in the $nGC3$ method

The $nGC3$ PCM signature for the first test scenario, a circular orbit, is shown in Fig. C.4 for each of the three
warp models. The Simple Symmetric model produces a single peak (black dot) at a co-latitude corresponding to the warp’s tilt. Lopsided models produce double-symmetric contours, since each side of the warp has a different amplitude $\psi_{\text{up}/\text{down}}$. For the Sine Lopsided model the PCM signature (red line) is a double or bow-tie contour with extremes at co-latitudes corresponding to the up/down warp amplitudes $\psi_{\text{up}/\text{down}}$. For the S Lopsided model, there is also a double contour (blue line) but, in this case, each peak extends from the north pole to the co-latitude corresponding to the maximum tilt angle of the up/down warp amplitudes. This is clearly illustrated by how the circles of the S Lopsided signature (blue) touch the extremes of the Sine Lopsided signature bow-tie (red). Notice also that the signature goes back to the PCM center in between the two loops. This is because the warp disappears next to the line-of-nodes for this model.

The PCM signature for the second test scenario, a random realisation of particles with $r_{\text{gal}}$ in the range 13–14 kpc, is shown in Fig. C.5. The figure shows that the signatures for the different warp models have similar overall features as in Fig. C.4 with some differences for the S Lopsided model. The SS model produces a single well-defined peak; the SineLop model produces a bow-tie signature caused by two overlapping peaks, barely resolved in this plot; and the SLop model produces two overlapping peaks, in this case with a triangular shape extended in azimuth. For the first two models, the SS and SineLop, the main PCM peaks stay along the $\phi = 270^\circ$ meridian, which corresponds to the pole’s azimuth of the (straight) line of nodes assumed in this example. On the contrary, for the SLop model, even though the line of nodes is straight, there is signal at azimuths approximately $\pm30^\circ$ away from the $\phi = 270^\circ$ azimuth of the line-of-nodes. This means that, for the SLop model, the azimuth of PCM regions were there is signal does not give the twist angle straight away; instead, the twist angle is given by the azimuth of the meridian along which the signature shows reflection symmetry.

The PCM signature for the last test scenario, the test particle simulations, is shown in Fig. C.6. In this figure we show the change in PCM signature going, from top to bottom, from the error-free test particles to the mock catalogue described in Sect. C.1 with GaiaDR2 simulated errors and selection function. For the SineLop model, the left column of the figure shows the bow-tie signature expected from the previous tests, which is only mildly distorted in the last row due to the effect of the observational errors. For the SLop model the signature differs from that of the ensemble of particles, even in the error-free set (top row). As discussed in Sect. 2.1, this model’s test particle set is in an impulsive regime as the SLop warped potential does not allow the particles to reach statistical equilibrium. Therefore, the PCMs shown here are not to be taken as characteristic of the SLop model itself, but as a guide to what the pole count signature might look like for a population of stars that is not in statistical equilibrium with the SLop model.