Quantum Theory of the Smectic Metal State in Stripe Phases

V. J. Emery\textsuperscript{a}, E. Fradkin\textsuperscript{b}, S. A. Kivelson\textsuperscript{c,d}, and T. C. Lubensky\textsuperscript{e}

Brookhaven National Laboratory,\textsuperscript{a} Upton NY 11973-5000, Dept. of Physics, University of Illinois at Urbana-Champaign\textsuperscript{b}
Urbana, IL 61801-3080, Dept. of Physics, U. C. L. A.,\textsuperscript{c}; Los Angeles, CA 90095, Dept. of Physics, Stanford university,\textsuperscript{d}
Stanford CA 94305, and Dept. of Physics, University of Pennsylvania,\textsuperscript{e} Philadelphia PA 19104.

(November 4, 2018)

We present a theory of the electron smectic fixed point of the stripe phases of doped layered Mott insulators. We show that in the presence of a spin gap three phases generally arise: (a) a smectic superconductor, (b) an insulating stripe crystal and (c) a smectic metal. The latter phase is a stable two-dimensional anisotropic non-Fermi liquid. In the absence of a spin gap there is also a more conventional Fermi-liquid-like phase. The smectic superconductor and smectic metal phases (or glassy versions thereof) may have already been seen in Nd-doped LSCO.

In the past few years very strong experimental evidence has been found for static or dynamic charge inhomogeneity in several strongly correlated electronic systems, in particular in high-temperature superconductors\textsuperscript{[1]}, manganites\textsuperscript{[2]}, and quantum Hall systems.\textsuperscript{[3]}

In d-dimensions, the charge degrees of freedom of a doped Mott insulator are confined to an array of self-organized (d - 1)-dimensional structures. In d = 2 these structures are linear and are known as stripes. Stripe phases may be insulating or conducting. We have recently proposed that quite generally the quantum mechanical ground states, and the thermodynamic phases which emerge from them, can on the basis of broken symmetries, be characterized as electronic liquid crystal states.\textsuperscript{[4]}

Specifically, a conducting stripe ordered phase is an electronic smectic state,\textsuperscript{[5]} while a state with only orientational stripe order (such as is presumably observed in quantum Hall systems\textsuperscript{[3]}) is an electronic nematic state.\textsuperscript{[6]}

Here, we use a perturbative renormalization group analysis which is asymptotically exact in the limit of weak inter-stripe coupling, to reexamine the stability of the electronic phases of a stripe ordered system in d = 2 and T \to 0. The results are summarized in Figs. 1 and 2.\textsuperscript{[7]}

In addition to an insulating stripe crystal phase, a variant of a Wigner crystal, we prove that there exist stable smectic phases: 1) An anisotropic smectic metal (non Fermi-liquid) state, which is a new phase of matter. 2) A stripe ordered smectic superconductor. We consider the cases of both spin-gap and spin-1/2 electrons.

One-dimensional correlated electron systems are Luttinger liquids,\textsuperscript{[7]} which are quintessential non-Fermi liquids, and are scale invariant, so that their correlation functions exhibit power law behavior, typically with anomalous exponents. The problem of the stability of arrays of Luttinger liquids\textsuperscript{[8]} has recently been reexamined following a proposal by Anderson\textsuperscript{[9]} that the fermionic excitations of a Luttinger liquid are confined\textsuperscript{[10]} and consequently that inter-chain transport is incoherent. However perturbative studies of the effects of inter-chain couplings at the decoupled Luttinger liquid fixed point have invariably concluded that such systems always order at low temperatures, or cross over to a higher-dimensional Fermi liquid state,\textit{i.e.} that the Luttinger behavior is restricted to a high-energy crossover regime.\textsuperscript{[11]}

In particular, in the important case in which the interactions within a chain are repulsive, the most divergent susceptibility within a single chain, especially when there is a spin gap, is associated with 2k\textsubscript{F} or 4k\textsubscript{F} charge-density wave fluctuations,\textit{i.e.} the decoupled Luttinger fixed point is typically unstable to two-dimensional crystallization.\textsuperscript{[12]} There is however a loophole in this argument. The decoupled Luttinger fixed point is not the most general scale-invariant theory compatible with the symmetries of an electron smectic. In particular, the long-wavelength density-density and/or current-current interactions between neighboring Luttinger liquids are exactly marginal operators, and should be included in the fixed point Hamiltonian (Eq. 5), which we call the generalized smectic non-Fermi liquid fixed point. Our principal results follow from a straightforward analysis of the perturbative stability of this fixed point. To the best of our knowledge, the model presented here is the first explicit example of a system with stable non-Fermi liquid behavior (albeit very anisotropic) in more than one dimension and which exhibits “confinement of coherence”.\textsuperscript{[13]}

Sliding phases, which are classical analogs of the smectic metal state\textsuperscript{[4]} in 3D stacks of coupled 2D planes with XY, crystalline, or smectic order, have, however, been investigated\textsuperscript{[14,15]}

The low energy Luttinger liquid behavior of an isolated system of spinless interacting fermions is described by the fixed-point Hamiltonian of a bosonic phase field, \(\phi(x, \tau)\), whose dynamics is governed by the Lagrangian density (in imaginary time \(\tau\))

\[
\mathcal{L} = \frac{w}{2} \left[ \frac{1}{v} \left( \frac{\partial \phi}{\partial \tau} \right)^2 + v \left( \frac{\partial \phi}{\partial x} \right)^2 \right] \tag{1}
\]

where \(w\) (the inverse of the conventional Luttinger parameter \(K\)) and the velocity of the excitations \(v\) are non-universal functions of the coupling constants and depend on microscopic details. For repulsive interactions we expect \(w \geq 1\) and, for weak interactions, \(w\) and \(v\) are determined by the backward and forward scattering amplitudes \(g_2\) and \(g_4\). Physical observables such as the long wavelength components of the charge density fluctuations...
\[ j_0 \] and the charge current \[ j_1 \], are given by the bosonization formula \[ j_{\mu} = \frac{1}{\pi} e_{\mu
u} \partial^\nu \phi \] where \( e_{\mu
u} \) is the Levi-Civita tensor. If both spin and charge are dynamical degrees of freedom, there are two Luttinger parameters (\( K_s, K_c \)), and two velocities \((v_c, v_s)\).

The one-dimensional correlated electron fluids in the stripe phases of high-temperature superconductors are coupled to an active environment, and so are expected to have gapped spin excitations \[ 13 \]. As such they are best described as Luttinger liquids in the Luther-Emery regime \[ 14 \] whose low-energy physics is described by a single Luttinger liquid for charge. The same is true of the stripe states of the 2DHE in magnetic fields, which are (in almost all cases of interest) spin polarized. \[ 5 \]

Now consider a system with \( N \) stripes, each labeled by an integer \( a = 1, \ldots, N \). We will consider first the phase in which there is a spin gap. Here, the spin fluctuations are effectively frozen out at low energies. Nevertheless each stripe \( a \) has two degrees of freedom \[ 4 \]: a transverse displacement field which describes the local dynamics of the configuration of each stripe, and the phase field \( \phi_a \) for the charge fluctuations on each stripe. The action of the generalized Luttinger liquid which describes the smectic charged fluid of the stripe state is obtained by integrating out the local shape fluctuations associated with the displacement fields. These fluctuations give rise to a finite renormalization of the Luttinger parameter and velocity of each stripe. More importantly, the shape fluctuations, combined with the long-wavelength inter-stripe Coulomb interactions, induce inter-stripe density-density and current-current interactions, leading to an imaginary time Lagrangian density of the form

\[ L_{\text{smectic}} = \frac{1}{2} \sum_{a,a',\mu} j^a_{\mu}(x) \tilde{W}_{\mu}(a-a') j^a'_{\mu}(x). \]  \[ (2) \]

These operators are marginal, i.e. have scaling dimension 2, and preserve the smectic symmetry \( \phi_a \rightarrow \phi_a + \alpha_a \) (where \( \alpha_a \) is constant on each stripe) of the decoupled Luttinger fluids. Whenever this symmetry is exact, the charge-density-wave order parameters of the individual stripes do not lock with each other, and the charge density profiles on each stripe can slide relative to each other without an energy cost. In other words, there is no rigidity to shear deformations of the charge configuration on nearby stripes. This is the smectic metal phase. \[ 13 \]

The fixed point action for a generic smectic metal phase thus has the form (in Fourier space)

\[ S = \sum_Q \frac{1}{2} \left\{ |W_0(Q)|^2 \omega^2 + |W_1(Q)|^2 k^2 \right\} |\phi(Q)|^2 \]

\[ = \sum_Q \frac{1}{2} \left\{ \frac{\omega^2}{W_1(Q)} + \frac{k^2}{W_0(Q)} \right\} |\theta(Q)|^2 \]  \[ (3) \]

where \( Q = (\omega, k, k_\perp) \), and \( \theta \) is the field dual to \( \phi \). Here \( k \) is the momentum along the stripe and \( k_\perp \) perpendicular to the stripes. The kernels \( W_0(Q) \) and \( W_1(Q) \) are analytic functions of \( Q \) whose form depends on microscopic details, e.g. at weak coupling they are functions of the inter-stripe Fourier transforms of the forward and backward scattering amplitudes \( q_2(k_\perp) \) and \( q_4(k_\perp) \), respectively. Thus, we can characterize the smectic fixed point by an effective (inverse) Luttinger function \( w(k_\perp) = \frac{W_0(k_\perp)}{W_1(k_\perp)} \) and an effective velocity function \( v(k_\perp) = \frac{W_1(k_\perp)}{W_0(k_\perp)} \).

In the presence of a spin gap, single electron tunneling is irrelevant \[ 13 \], and the only potentially relevant interactions involving pairs of stripes \( a, a' \) are singlet pair (Josephson) tunneling, and the coupling between the CDW order parameters. These interactions have the form \( H_{\text{int}} = \sum_n (H^n_{\text{SC}} + H^n_{\text{CDW}}) \) for \( a' - a = n \), where

\[ H^n_{\text{SC}} = \left( \frac{\Lambda}{2\pi} \right)^2 \sum_a J_n \cos[\sqrt{2\pi}(\theta_n - \theta_{a+n})] \]

\[ H^n_{\text{CDW}} = \left( \frac{\Lambda}{2\pi} \right)^2 \sum_a V_n \cos[\sqrt{2\pi}(\phi_n - \phi_{a+n})]. \]  \[ (4) \]

Here \( J_n \) are the inter-stripe Josephson couplings, \( V_n \) are the \( 2k_F \) component of the inter-stripe density-density (CDW) interactions, and \( \Lambda \) is an ultra-violet cutoff, \( \Lambda \sim 1/a \) where \( a \) is a lattice constant. A straightforward calculation, yields the scaling dimensions \( \Delta_{1,n} \equiv \Delta_{\text{SC},n} \) and \( \Delta_{-1,n} \equiv \Delta_{\text{CDW},n} \) of \( H^n_{\text{SC}} \) and \( H^n_{\text{CDW}} \):

\[ \Delta_{\pm 1,n} = \int_{-\pi}^\pi \frac{dk_\perp}{2\pi} [\kappa(k_\perp)]^{\pm 1} (1 - \cos nk_\perp), \]  \[ (5) \]

where \( \kappa(k_\perp) \equiv w(0, 0, k_\perp) \). Since \( \kappa(k_\perp) \) is a periodic function of \( k_\perp \) with period \( 2\pi \), \( \kappa(k_\perp) \) has a convergent Fourier expansion of the form \( \kappa(k_\perp) = \sum_n \kappa_n \cos nk_\perp \). We will parametrize the fixed point theory by the coefficients \( \kappa_n \), which are smooth non-universal functions. In what follows we shall discuss the behavior of the simplified model with \( \kappa(k_\perp) = \kappa_0 + \kappa_1 \cos k_\perp \). Here, \( \kappa_0 \) can be thought of as the intra-stripe inverse Luttinger parameter, and \( \kappa_1 \) is a measure of the nearest neighbor inter-stripe coupling. For stability we require \( \kappa_0 > \kappa_1 \). Since it is unphysical to consider longer range interactions in \( H_{\text{int}} \) than are present in the fixed point Hamiltonian, we treat only perturbations with \( n = 1 \), whose dimensions are \( \Delta_{\text{SC},1} \equiv \Delta_{\text{SC}} = \kappa_0 - \frac{\Lambda}{2\pi} \), and \( \Delta_{\text{CDW},1} \equiv \Delta_{\text{CDW}} = 2(\kappa_0 - \kappa_1 + \sqrt{\kappa_0^2 - \kappa_1^2}) \). For a more general function \( \kappa(k_\perp) \), operators with larger \( n \) must also be considered, but the results are qualitatively unchanged. \[ 12, 13 \]

In Figure 1 we present the phase diagram of this model. The dark \( AB \) curve is the set of points where \( \Delta_{\text{CDW}} = \Delta_{\text{SC}} \), and it is a line of first order transitions. To the right of this line the inter-stripe CDW coupling is the most relevant perturbation, indicating an instability of the system to the formation of a 2D stripe crystal. \[ 13 \]

To the left, Josephson tunneling (which still preserves the smectic symmetry) is the most relevant, so this phase is
a 2D smectic superconductor. (Here we have neglected the possibility of coexistence since a first order transition seems more likely). Note that there is a region of \( \kappa_0 \geq 1 \), and large enough \( \kappa_1 \), where the global order is superconducting although, in the absence of interstripe interactions (which roughly corresponds to \( \kappa_1 = 0 \)), the SC fluctuations are subdominant. There is also a (strong coupling) regime above the curve \( CB \) where both Josephson tunneling and the CDW coupling are irrelevant at low energies. Thus, in this regime the smectic metal state is stable. This phase is a 2D smectic non-Fermi liquid in which there is coherent transport only along the stripes.

\[ \rho_{xx} = \frac{\hbar}{e^2 n_s v} \left| V_{\text{back}} \right|^2 \frac{T}{v \Lambda} \Delta_{\text{CDW}} f_{xx}(X^2, Y^2) + \ldots, \]  

where \( f_{xx}(X, Y) \) is a scaling function and \( f_{xx}(0, 0) \sim 1 \). Here, \( n_s \) is the density of stripes, and \( \Delta_{\text{CDW}} \equiv \Delta_{\text{CDW,\infty}} \) is the dimension of the CDW order parameter.

Whether the inter-stripe Josephson coupling, \( J \), is irrelevant or relevant, so long as the temperature is not too low, the component of the conductivity tensor transverse to the stripe direction can be obtained from a perturbative evaluation of the Kubo formula to lowest order in powers of the leading coupling \( J \). Combining this result with a simple scaling analysis we find (to zeroth order in \( V_{\text{back}} \))

\[ \sigma_{yy} = \frac{e^2}{\hbar} n_s b^2 \Lambda \left( \frac{J}{v} \right)^2 \left( \frac{T}{\Lambda v} \right)^{2\Delta_{\text{SC}} - 3} f_{yy}(X^2, Y^2), \]  

where \( b \) is the spacing between stripes, \( f_{yy} \) is a scaling function and \( f_{yy}(0, 0) \sim 1 \). An interesting aspect of this expression is that, in the perturbative (high-temperature) regime, the temperature derivative of \( \sigma_{yy} \) changes from positive to negative at a critical value of \( \Delta_{\text{SC}} = 3/2 \), whereas the actual superconductor to (CDW) insulator transition occurs somewhere in the range \( 1 < \Delta_{\text{SC}} < 2 \), depending on the value of \( n_s/\kappa_3 \).

For a system with Galilean invariance along the stripes

\[ \rho_{xy} = \frac{n_s \hbar e}{B} + \ldots \]  

The physics governing \( n_s \) is rather subtle - neglecting irrelevant couplings, the fixed point Hamiltonian is actually particle-hole symmetric, which implies \( \rho_{xy} = 0 \). Thus \( n_s \) is determined by the leading irrelevant couplings which break particle-hole symmetry, terms of the form \( (\partial_x \phi)^3 \) and \( (\partial_t \phi)^2 \partial_x \phi \). Generically, \( 1/n_s \) approaches a non-zero constant value at low temperatures. However, in special cases \( \phi \), the quarter-filled Hubbard chain in the infinite U limit where there is an effective “particle-hole symmetry” at low energy, \( \rho_{xy} \) will vanish as a power of \( T \).

Let us now discuss what happens if both charge and spin excitations are gapless on the stripes. We now have two Luttinger fluids on each stripe for charge and spin respectively, represented by the fields \( \phi_c \) and \( \phi_s \). \( SU(2) \) spin invariance requires \( K_s = 1 \) whereas \( K_c = K \) as in the spin gap case. Here we will discuss a system in which there is only a coupling of the charge densities between neighboring stripes and no exchange coupling. Since both spin and charge are gapless, electron tunneling has to be considered in addition to CDW coupling and Josephson tunneling. The dimensions of
the most relevant CDW and Josephson interactions in the gapless spin case are \( \Delta_{\text{CDW}} = 1 + \Delta_{\text{(Gap)}} \), and \( \Delta_{\text{SC}} = 1 + \Delta_{\text{(Gap)}} \), where \( \Delta_{\text{CDW}} \) and \( \Delta_{\text{SC}} \) are their dimensions in the spin gap case, Eq. (5). The dimension of the nearest-neighbor single electron tunneling operator is \( \Delta_e = 1 + \Delta_{\text{(Gap)}} \). It is also easy to check that the dimensions of the 2\( k_F \) charge density wave (CDW) and spin density wave (SDW) operators satisfy \( \Delta_{\text{CDW}} = \Delta_{\text{SDW}} \). Similarly, the triplet and singlet superconductor couplings have the same dimension. We can now derive the phase diagram for the spin gapless case, shown in Figure 2. There is a large region of the phase diagram in which the electron tunneling operator is relevant, shown in Figure 2 as the region below the curve \( ABC \) (defined by the marginality condition \( \Delta_{e,1} = 2 \)). In this regime the system initially flows towards a 2D Fermi liquid fixed point, which will itself exhibit a BCS instability in the presence of residual attractive interactions (\( \kappa_0 \) < 1). For stronger inter-stripe couplings the system crystallizes, and there are also strong coupling smectic metal (non-Fermi liquid), and superconducting phases.

![Phase diagram for a system without a spin gap.](image)

**FIG. 2.** Phase diagram for a system without a spin gap.

The non-Fermi liquid smectic metal phase is a remarkable state of matter. Because inter-stripe tunneling of any type is irrelevant, the transport across the stripes is incoherent, whereas transport is coherent (and large) inside each stripe. Recently, evidence of the existence of a “metallic” stripe ordered state, which we identify as such a smectic, has been observed [13] in La\(_{1.4-x}\)Nd\(_{0.6}\)Sr\(_x\)CuO\(_4\); Glassy stripe order has been confirmed by neutron and X-ray scattering studies; the in-plane transport remains metallic (with at most a logarithmic increase) down to low temperatures while the inter-plane resistivity (which is perpendicular to the stripes) appears to diverge as \( T \to 0 \). On the same system photoemission experiments [14] have found strong evidence for one-dimensional electronic structure. Strikingly, Noda et al. [22] have found that for \( x \leq 1/8 \), \( \rho_{xy} \) vanishes (roughly linearly) as \( T \to 0 \), while for \( x > 1/8 \), although \( \rho_{xy} \) still decreases strongly at low temperatures, it appears to approach a finite value. This behavior was taken by Noda et al to indicate a crossover from one to two dimensional metallic conduction at \( x = 1/8 \). We propose, instead, that the system is a smectic for a range of \( x \), and that the crossover indicates that the stripes are nearly quarter filled, and have an approximate particle-hole symmetry for \( x < 1/8 \), while particle-hole symmetry is broken for \( x > 1/8 \). Finally, the present results suggest the existence of a smectic metal state of the 2DEG in large magnetic fields, a result conjectured previously by us [16] and by Fertig [21], although microscopic calculations still yield conflicting conclusions [23].

We thank S. Bacci, D. Barci, H. Esaki, M. P. A. Fisher, and Z. X. Shen for useful discussions. EF and SAK are grateful to S. C. Zhang and the Dept. of Physics of Stanford University, for their hospitality. This work was supported in part by the NSF, grants DMR98-08685 (SAK), DMR98-17941 (EF), DMR97-30405 (TCL), and by DMS, USDOE contract DE-AC02-76CH00016 (VJE).

[1] For a recent perspective on stripe phases, see V. J. Emery et al, Proc. Natl. Acad. Sci. USA, 96, 8814 (1999).
[2] C. Chen and S. Cheong, Phys. Rev. Lett. 76, 4042 (1996).
[3] M. P. Lilly, et al, Phys. Rev. Lett. 82, 394 (1999).
[4] S. A. Kivelson et al, Nature 393, 550 (1998).
[5] E. Fradkin and S. Kivelson, Phys. Rev. B59, 8069 (1999).
[6] E. Fradkin et al., Phys. Rev. Lett. 84, 1982 (2000).
[7] See V. Emery, in Highly Conducting One-Dimensional Solids, J. Devreese, et. al., (Plenum, New York, 1979); M. Stone, Bosonization (World Scientific, Singapore, 1994).
[8] H.J. Schulz, J. Phys. C 16, 6769 (1983)
[9] P. W. Anderson, Phys. Rev. Lett.67, 3844 (1991)
[10] S. Chakravarty and P. W. Anderson, Phys. Rev. Lett.72, 3850 (1994).
[11] L. Golubović and M. Golubović, Phys. Rev. Lett. 80, 4341 (1998). Erratum ibid. 81, 5704 (1998). C. S. O’Hern and T. C. Lubensky, ibid. 80, 4345 (1998).
[12] C. S. O’Hern et. al., Phys. Rev. Lett. 83, 2745 (1999).
[13] V. J. Emery et al, Phys. Rev. B56, 6120 (1997).
[14] A. Luther and V. J. Emery, Phys. Rev. Lett. 33, 589 (1974).
[15] For the model considered \( \Delta_{SC,2} \) is the most relevant operator. An example of a more general model is \( \kappa(k_\perp) = |\kappa_0 + \kappa_1 \cos(k_\perp)|^2 \), for which all perturbations are irrelevant for large \( \kappa_0 \) and small [\( \kappa_0 - \kappa_1 \)].
[16] A. Luther and I. Peschel, Phys. Rev. Lett. 32, 922 (1974); W. Gütze and P. Wölfle, Phys. Rev. B6, 1226 (1972); T. Giamarchi and H. Schulz Phys. Rev. B 37, 325 (1988).
[17] These results also follow from the Kubo formula.
[18] J. M. Tranquada, Nature 375, 561 (1995).
[19] X. J. Zhou et al, Science 286, 269 (1999).
[20] T. Noda al, Science 286, 265 (1999).
[21] H. Fertig, Phys. Rev. Lett. 82, 3693 (1999)
[22] A. H. MacDonald and M. P. A. Fisher, Phys. Rev. B61, 5724 (2000); H. Yi et al., cond-mat/0003139.