A Simple and Efficient Binary Byzantine Consensus Algorithm using Cryptography and Partial Synchrony

Tyler Crain*1
1tcrainwork@gmail.com

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Abstract

This paper describes a simple and efficient Binary Byzantine faulty tolerant consensus algorithm using a weak round coordinator and the partial synchrony assumption to ensure liveness. In the algorithm, non-faulty nodes perform an initial broadcast followed by a executing a series of rounds consisting of a single message broadcast until termination. Each message is accompanied by a cryptographic proof of its validity. In odd rounds the binary value 1 can be decided, in even round 0. Up to one third of the nodes can be faulty and termination is ensured within a number of round of a constant factor of the number of faults. Experiments show termination can be reached in less than 200 milliseconds with 300 Amazon EC2 instances spread across 5 continents even with partial initial disagreement.

1 Introduction and related work.

Binary byzantine consensus concerns the problem of getting a set of distinct processes distributed across a network to agree on a single binary value 0 or 1 where processes can fail in arbitrary ways. It is well known that this problem is impossible in an asynchronous network with at least one faulty process [19]. To get around this, algorithms can employ randomization [1, 2, 4, 5, 9, 20, 23, 27, 31, 33, 35], or rely on an additional synchrony assumption. This work assumes partial synchrony [15, 16] which ensures that after some point in time there exists an (unknown) upper bound on message delay and difference in speed between processes. Furthermore, the algorithm assumes that less than one third of the processes are faulty and ensures termination in $O(t)$ messages steps, both of which are well known lower bounds [24, 18].

While there are many algorithms that solve this problem with the same bounds and assumptions [15, 16], this algorithm focuses on simplicity and efficiency. Namely, it starts with each process broadcasting an initial proposal, then executing a series of rounds that consist of broadcasting a single message then waiting to receive a threshold of valid messages from other processes. In the good case agreement happens in the first round. To ensure termination with the benefit of the partial synchrony assumption, the algorithm uses a weak round coordinator [11]. This coordinator is used to help agreement by suggesting a value to decide when both 0 and 1 are valid. While classic round coordinators [7, 16] will rely on the coordinator for termination in all cases, the weak

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round coordinator is not needed for termination in the expected case. Note that this is a practical distinction as the bounds for termination are not effected.

The design of the algorithm is primarily based on two previous algorithms; [8] and [11]. Similarly to [8], a set of cryptographic signatures are included with each message proving its validity. Differently [8] uses randomization for termination and each process broadcasts several messages per round. Similarly to [11], the algorithm can decide 1 in odd rounds or 0 in even rounds, uses a weak round coordinator, and assumes partial synchrony for termination. Differently, [11] has the advantage of not requiring cryptographic signatures, but requires processes to broadcast several messages per round. Note that the design of [11] is based off of the randomized algorithm of [27].

While the binary consensus problem only allows process to agree on a single binary value, there exist many reductions to multi-value consensus [28, 29, 36, 37] allowing processes to agree on arbitrary values. Furthermore many algorithms [3, 10] exists that solve multi-value consensus directly with the same assumptions. Additionally algorithms exists that make many different assumptions about the model such as synchrony [17], different fault models [25, 26, 32] solve different definitions of consensus [30], and so on.

2 A Byzantine Computation Model.

This section describes the assumed computation model.

Asynchronous processes. The system is made up of a set Π of \( n \) asynchronous sequential processes, namely \( Π = \{ p_1, \ldots, p_n \} \); \( i \) is called the “index” of \( p_i \). “Asynchronous” means that each process proceeds at its own speed, which can vary with time and remains unknown to the other processes. “Sequential” means that a process executes one step at a time. This does not prevent it from executing several threads with an appropriate multiplexing. Both notations \( i \in Y \) and \( p_i \in Y \) are used to say that \( p_i \) belongs to the set \( Y \).

Communication network. The processes communicate by exchanging messages through an asynchronous reliable point-to-point network. “Asynchronous” means that there is no bound on message transfer delays, but these delays are finite. “Reliable” means that the network does not lose, duplicate, modify, or create messages. “Point-to-point” means that any pair of processes is connected by a bidirectional channel. A process \( p_i \) sends a message to a process \( p_j \) by invoking the primitive “send \ TAG(m) \ to \ p_j \”, where \( TAG \) is the type of the message and \( m \) its content. To simplify the presentation, it is assumed that a process can send messages to itself. A process \( p_i \) receives a message by executing the primitive “receive()”. The macro-operation broadcast \ TAG(m) \ is used as a shortcut for “for each \( p_i \in Π \) do send \ TAG(m) \ to \ p_j \ end for”.

Signatures. Asymmetric cryptography allow processes to sign messages. Each process \( p_i \) has a public key known by everyone and a private key known only by \( p_i \). All messages are signed using the private key can be validated by any process with the corresponding public key, allowing the process to identify the signer of the message. Signatures are assumed to be unforgeable. A process will ignore any message that is malformed or contains an invalid signature.

Failure model. Up to \( t \) processes can exhibit a Byzantine behavior [32]. A Byzantine process is a process that behaves arbitrarily: it can crash, fail to send or receive messages, send arbitrary messages, start in an arbitrary state, perform arbitrary state transitions, etc. Moreover, Byzantine
processes can collude to “pollute” the computation (e.g., by sending messages with the same content, while they should send messages with distinct content if they were non-faulty). A process that exhibits a Byzantine behavior is called faulty. Otherwise, it is non-faulty. Let us notice that, as unforgeable signatures are used no Byzantine process can impersonate another process. Byzantine processes can control the network by modifying the order in which messages are received, but they cannot postpone forever message receptions.

**Additional synchrony assumption.** It is well-known that there is no consensus algorithm ensuring both safety and liveness properties in fully asynchronous message-passing systems in which even a single process may crash [19]. As the crash failure model is less severe than the Byzantine failure model, the consensus impossibility remains true if processes may commit Byzantine failures. To circumvent such an impossibility, and ensure the consensus termination property, we enrich the model with additional synchrony assumptions as follows: After some finite time, there exists an (unknown) upper bound on message transfer delays. Furthermore there is an (unknown) upper bound on the difference in speeds between non-faulty processes. This partial synchrony assumption is denoted $\diamond Synch$ [15, 16].

### 3 Binary Byzantine Consensus.

#### 3.1 The Binary Consensus Problem.

In this problem processes input a value to the algorithm, called their *proposal*, run an algorithm consisting of several rounds, and eventually output a value called their *decision*. Let $\mathcal{V}$ be the set of values that can be proposed. While $\mathcal{V}$ can contain any number ($\geq 2$) of values in multi-valued consensus, it contains only two values in binary consensus, e.g., $\mathcal{V} = \{0, 1\}$. Assuming that each non-faulty process proposes a value, the binary Byzantine consensus (BBC) problem is for each of them to decide on a value in such a way that the following properties are satisfied:

- **BBC-Termination.** Every non-faulty process eventually decides on a value.
- **BBC-Agreement.** No two non-faulty processes decide on different values.
- **BBC-Validity.** If all non-faulty processes propose the same value, no other value can be decided.

#### Notations.

- The acronym $BAM\mathcal{P}_{n,t}[\emptyset]$ is used to denote the basic Byzantine Asynchronous Message-Passing computation model; $\emptyset$ means that there is no additional assumption.
- The basic computation model strengthened with the additional constraint $t < n/3$ is denoted $BAM\mathcal{P}_{n,t}[t < n/3]$.
- The computation model strengthened with the partial synchrony constraint $\diamond Synch$ is denoted $BAM\mathcal{P}_{n,t}[t < n/3, \diamond Synch]$.
- A signature of process $i$ is $\theta_i$.
- A message $m$ signed by process $i$ is $(m, \theta_i)$.
3.2 A Safe and Live Consensus Algorithm in $BAMP_{n,t}[t < n/3, \Diamond Synch]$.

**Message types.** The following message types are used by the consensus.

- $AUX[r](v)$. An $AUX$ message contains a round number $r$ and a binary value $v$.
- $\langle (AUX[r](v), \theta_i), proofs \rangle$. A tuple containing an $AUX$ message signed by process $i$ and a set $proofs$ containing signed $AUX$ messages from a previous round that are used to prove $v$ is a valid binary proposal for round $r$.

**Valid Notation.** For a given round $r \geq 1$ a binary value $b$ is valid if $b$ has been proposed by a non-faulty process and $\neg b$ has not been decided in any round before $r$. An $\langle (AUX[r](v), \theta_i), proofs \rangle$ is valid if binary value $v$ is valid in round $r$. The following section describes a function that is used to compute the validity of a message given $r$, $v$, and $proofs$ as input.

**Variables.** The following variables are used throughout all rounds of the consensus.

- $r_i$. Current round number of process $i$.
- $aux\_values_i$. Set of valid signed $AUX$ messages received by process $i$ throughout all rounds of the consensus.
- $timers_i$. A map from a round to a timer at process $i$. There are two timers per round so the timers for round $r$ are entries $r \times 2$ and $r \times 2 + 1$ in the map. Timer variables can be started then expire after a predetermined amount of time (see Section 3.2.1 for how times are chosen). Starting a timer that has already started or expired is a no-op.

**Algorithm description.** Figures 1 and 2 describe the pseudo-code for the algorithm. The $bin\_propose$ operation of Figure 1 contains the main loop of the algorithm. The $perform\_broadcast$ procedure describes the code used to prepare and broadcast valid signed $AUX$ messages for each round. The lines 22-26 handle the reception of signed $AUX$ messages. The $is\_valid$ predicate of Figure 2 describes the procedure used to check if a binary value is valid for a given round and a set of signed $AUX$ messages.

To start the consensus, each process $p_i$ calls $bin\_propose$ with its initial binary proposal $v_i$ (Figure 1). Line 01 initializes local variables, then on Line 02 the process sends an a signed $AUX$ message with round 0, binary value $v_i$, and an empty set for $proofs$ as any round 0 message is considered to be valid. The process then repeats the while loop of Lines 03-12 for each round.

A round starts by incrementing the processes round counter on Line 04. If the process $p_i$ is the round coordinator (i.e. $i = (r_i \mod n)$) it then invokes $perform\_broadcast$ (line 05) to generate and broadcast a valid signed $AUX$ message. Otherwise the process waits for a timeout (line 06) before invoking $perform\_broadcast$ (line 07). If the timer is large enough, non-faulty processes will receive the coordinator’s $AUX$ message before broadcasting their own $AUX$ message using the same binary value if valid.

Non-faulty processes then wait until $(n - t)$ valid $AUX$ messages have been received for the round, before waiting on a second timer to expire (lines 08-09). If the timer is large enough it will ensure all non-faulty processes receive valid signed $AUX$ messages from all non-faulty processes for that round. If $n - t$ of the signed $AUX$ messages support the same binary value $b_i = r_i \mod 2$ then the process decides $b_i$. The process then continues on to the next round. Given that $t < n/3$, if a non-faulty process receives $n - t$ signed $AUX$ messages in round $r_i$ supporting $b_i$, then any set of
operation bin_propose($v_i$) is
(01) $r_i$ ← 0; $aux\_values_i$ ← Ø;
(02) broadcast ($\langle AUX[r_i](v_i), \theta_i, 0 \rangle$); // Broadcast the initial proposal
(03) while (true) do
(04) $r_i$ ← $r_i + 1$;
(05) if ($i = (r_i \mod n)$) then perform_broadcast() end if; // Coordinator broadcasts before the timer
(06) Start timers,$r_i$,$2$ if not yet done; wait until timers,$r_i$,$2$ has expired;
(07) if ($i \neq (r_i \mod n)$) then perform_broadcast() end if; // Non-coordinators broadcast after the timer
(08) wait until $(n-t)$ valid $AUX[r_i]()$ messages have been received from $(n-t)$ different processes;
(09) Start timers,$r_i$,$2 + 1$ if not yet done; wait until timers,$r_i$,$2 + 1$ has expired;
(10) $b_i$ ← $r_i$ mod 2;
(11) if $(n-t)$ valid $AUX[r_i](b_i)$ messages have been received from $(n-t)$ different processes
(12) then decide($b_i$) if not yet done end if
(13) end while.

procedure perform_broadcast() is
(14) $values_i$ ← compute $values_i$ as the set of binary values that satisfy the is_valid predicate for $aux\_values_i$ and round $r_i$;
(15) $bv_i$ ← ($r_i + 1$) mod 2;
(16) if received ($\langle AUX[r_i](p_{val}), \theta_i, (r_i \mod n) \rangle$) from process $p_{val}$, then
(17) then $est_i$ ← $p_{val}$ // Support the coordinator's value for the next round
(18) else if ($bv_i \in values_i$) then $est_i$ ← $bv_i$ // Otherwise prefer the modulo of the next/previous round
(19) else $est_i$ ← ¬$bv_i$ end if;
(20) $proofs_i$ ← compute $proofs_i$ as a set of signed $AUX$ messages from $aux\_values_i$ that satisfy the is_valid predicate for binary value $est_i$ and round $r_i$;
(21) broadcast ($\langle AUX[r_i](est_i), \theta_i, proofs_i \rangle$).

when ($\langle AUX[r_i](est_j), \theta_j, proofs_j \rangle$) is received
(22) if (is_valid($r_j, est_j, proofs_j$)) then
(23) $proofs$ ← $proofs_j$ \ {any messages in $proofs$ not needed to satisfy the is_valid predicate}.
(24) $aux\_values_i$ ← $aux\_values_i \cup \{\langle AUX[r_j](est_j), \theta_j \rangle \} \cup proofs_j$;
(25) end if;
(26) $r_i$ ← the largest round for which $aux\_values_i$ contains $t + 1$ messages from different processes;
(27) For all $v_i \in N$ such that $v_i \leq r_i \times 2$, set timers,$[v_i]$ to expired if not yet done. // Catch-up mechanism

Figure 1: A safe algorithm for the binary Byzantine consensus in $BAMP_{n,t}[t < n/3]$.

$n - t$ signed $AUX$ from round $r_i$ will contain at least one $AUX$ message supporting $b_i$. With this, the is_valid predicate will ensure that ¬$b_i$ is not valid in any round after $r_i$ at all non-faulty processes. As a result, in all following rounds, non-faulty processes will only broadcast $AUX$ messages supporting $b_i$ and decide in round $r_i$ or a following round.

Lines 14-21 describe the perform_broadcast procedure used to create and broadcast valid signed $AUX$ messages for each round. The procedure starts by using the is_valid predicate to compute the set of valid binary values for round $r_i$ given the set $aux\_values_i$ of valid $AUX$ messages received so far (line 14). Lines 16-19 are used to decide which of these values to broadcast. First, if a valid $AUX$ message from the round coordinator was used, then that value is broadcast(lines 16-17). Second, $(r_i + 1)$ mod 2 is chosen if it is valid (line 18), otherwise the only remaining valid value is chosen (line 19). By preferring the binary value of the coordinator, termination is ensured when a non-faulty process is chosen as the coordinator and if large enough timeouts are eventually used, as all non-faulty processes broadcast the same binary value. Note that the protocol would remain correct if lines 18-19 were changed so that the process simply chooses a random valid value to broadcast, but they are included as they encourage non-faulty processes to support the same value and reach a decision even without the presence of the coordinator.

Lines 22-27 describe what happens when a signed $AUX$ message and its proofs are received. If
the \textit{is\_valid} predicate indicates that this message is valid then the signed \textsl{AUX} message and its proofs are added to the \textit{aux\_values} set (lines 22-24). Line 24 ensures that no invalid messages are added to \textit{aux\_values}. Next \(\rho_i\) is computed as the largest round for which the process has received at least \(t + 1\) valid signed \textsl{AUX} messages from different processes (line 25). The process then sets all timers up to round \(\rho_i\) as expired. The threshold of \(t + 1\) ensures that at least one non-faulty processes has reached round \(\rho_i\), then setting the timeouts up to this round as expired helps this process catch up to the faster ones.

\textbf{Is\_valid predicate description.} Figure 2 describes the \textit{is\_valid} predicate that is called by Algorithm 1 to check if a binary value is valid. It takes as input a round \(r\), a binary value \textit{est} and a set of signed \textsl{AUX} messages \textit{proofs}. As previously mentioned, the predicate should return \texttt{true} if \textit{proofs} ensures that (i) \textit{est} was proposed by a non-faulty process and (ii) \texttt{¬est} has not been decided by any non-faulty process in any round before \(r\). Otherwise \texttt{false} should be returned.

For round 0 the predicate immediately returns \texttt{true} as any initial proposal is valid (line 01). For round 1, as no value can be decided in round 0, the predicate returns \texttt{true} if \textit{proofs} contains at least \(t + 1\) round 0 messages with binary value \textit{est} (line 03), i.e. if (i) is satisfied.

For all other rounds the value \(b \leftarrow (r - 1) \text{ mod } 2\) is computed. Note that \(b\) is the binary value that could be decided in round \(r - 1\). If \((b = \textit{est})\) then \texttt{true} is returned in the following cases.

1. \(r = 2\) and \textit{proofs} contains at least \(t + 1\) signed \textsl{AUX}[0](\textit{est}) messages. If \(r = 2\) and \(b = \textit{est}\) then \textit{est} = 1 and by 11-12 of Figure 1 only 1 could have been decided in rounds \(r < 2\). Thus, if \textit{proofs} contains at least \(t + 1\) signed \textsl{AUX}[0](\textit{est}) then both (i) and (ii) are satisfied and \texttt{true} is returned (line 07).

2. \(r > 2\) and \textit{proofs} contains at least \(n - t\) signed \textsl{AUX}[\(r - 2\)](\textit{est}) messages. By lines 11-12 of Figure 1, in round \(r - 2\) only \texttt{¬b} could have been decided. Furthermore, if \(n - t\) signed \textsl{AUX}[\(r - 2\)](\textit{b}) are contained in \textit{proofs} then all sets of \(n - t\) signed \textsl{AUX} messages from round \(r - 2\) must contain at least one \textsl{AUX}[\(r - 2\)](\textit{b}) message, thus \texttt{¬b} could not have been decided
by a non-faulty process in round \( r - 2 \). An induction argument can then be used to show that \( \neg b \) could not have been decided in any round before \( r - 2 \) so true is returned (line 08).

Otherwise if \( (b = \neg \text{est}) \) then true is returned only if \( n - t \) signed \( \text{AUX}[r - 1](\text{est}) \) messages are received (line 11). This is enough to ensure \( \neg \text{est} \) was not decided in round \( r - 1 \) or before using the same argument as case 2 above. If none of these cases are met the false is returned.

3.2.1 Timers

The timers used on lines 06 and 09 of Figure 1 are used to ensure processes eventually execute rounds synchronously given \( \Diamond \text{Synch} \). The duration of the timers must increase by (at least) a constant amount in every constant number of rounds to ensure that they are eventually large enough to encompass the bound given by \( \Diamond \text{Synch} \) and that slow non-faulty processes can catch up to the faster ones. The faster the timeout increases, the fewer rounds will be needed to reach synchrony, but the longer process might have to wait unnecessarily. Finding this ideal trade-off would require a detailed inspection of the specific case where the algorithm is expected to run. Fortunately, as this algorithm uses a weak coordinator, in most cases it does not need a coordinator or timeouts to terminate (in fact the experiments in Section 4.1 never actually needed to use a coordinator). Given this it is suggested to set the timeouts to 0 and disable lines 16-17 of the perform broadcast procedure that are used to support the coordinator for some constant number of rounds. If the algorithm does not terminate quickly in this case then it is suggested to set the timeout at the expected average network delay of the system, and increase it by a small constant each following round.

3.3 Proofs.

This section shows that the algorithm presented in Figure 1 solves the Binary consensus problem in \( \text{BAMP}_{n,t}[t < n/3, \Diamond \text{Synch}] \) through a series of lemmas.

Lemma 1. For a given round \( r \) there can be at most one binary value \( b \) for which there exists at least \( n - t \) signed \( \text{AUX}[r](b) \) messages from different processes.

Proof. This follows from the fact that there are at most \( t < n/3 \) faulty processes and that non-faulty processes sign and broadcast at most one \( \text{aux} \) message per round.

Lemma 2. In round \( r > 0 \) non-faulty processes will only sign and broadcast \( \text{AUX} \) messages containing binary values proposed by non-faulty processes.

Proof. By line 13 of Figure 1 a non-faulty process will only broadcast values that satisfy the is_valid predicate. By lines 03, 07, 08, 11 of the is_valid predicate, in any round \( r > 0 \) a binary value will only satisfy the predicate if the process has received at least \( t + 1 \) signed \( \text{AUX} \) messages from different processes. Given that there are at most \( t \) faults and by induction, non-faulty processes will only broadcast values proposed by non-faulty processes.

Lemma 3. All non-faulty processes decide the same value.

Proof. Assume a non-faulty process decides in round \( r_x \). By line 11 the process must have received \( n - t \) signed \( \text{AUX}[r_x][(r_x \mod 2)] \) messages from different processes and decided \( v_x = (r_x \mod 2) \). Also by line 11 for this or a different non-faulty process to decide \( \neg v_x \), the process must receive \( n - t \) signed \( \text{AUX}[r_y](\neg v_x) \) messages from different processes in some round \( r_y \). Furthermore by lines 10 and 11, in round a round \( r \) only \( (r \mod 2) \) can be decided, thus \( r_y \neq r_x \).
First assume $r_y > r_x$. By Lemma 1 no process will receive $n - t$ signed $\text{AUX}[r_x](¬v_x)$ messages from different processes and by line 14 a non-faulty process will only sign and broadcast a value that satisfies the $\text{is\_valid}$ predicate. Given that there are less than $n - t$ signed $\text{AUX}[r_x](¬v_x)$ messages from different processes, $¬v_x$ will not be valid in either round $r_x + 1$ or $r_x + 2$ (lines 08, 11 of the $\text{is\_valid}$ predicate), thus messages supporting $¬v_x$ will not be added to $\text{aux\_values}_i$ on line 24 of Figure 1 for those rounds, and will not be broadcast by non-faulty processes (by lines 14, 20, 21). Given no non-faulty process broadcasts $¬v_x$ in rounds $r_x + 1$ or $r_x + 2$, the $\text{is\_valid}$ predicate will ensure $¬v_x$ will remain invalid in later rounds and will not be broadcast by non-faulty processes in round after $r_x + 2$ (note that the case on line 02 of the $\text{is\_valid}$ predicate does not apply as $r_y > 0$, and neither does the case on line 07 because if $r_y = 2$ then $¬v_x \neq 1$). Thus no non-faulty process will decide $¬v_x$ in a round after $r_x$.

Next assume $r_y < r_x$. If a process receives $n - t$ signed $\text{AUX}[r_y](¬v_x)$ from different processes and decides $¬v_x$ in round $r_y$ then using the same argument as above, no non-faulty process will receive $n - t$ signed $\text{AUX}[r_x](v_x)$ in any following round and will not decide $v_x$. Thus by contradiction no process will decide $v_x$ in a round prior to $r_x$.

**Lemma 4.** For any round $r > 0$ all non-faulty processes will (eventually) receive enough valid messages to satisfy the $\text{is\_valid}$ predicate of Figure 2 for the round.

**Proof.** By line 01 of the $\text{is\_valid}$ predicate all signed round 0 $\text{AUX}$ messages are valid and by line 02 of Figure 1 all non-faulty processes sign and broadcast a round 0 $\text{AUX}$ message. All non-faulty processes will then receive at least $n - t$ signed round 0 $\text{AUX}$ messages from different processes. Given that $t < n/3$, of these $n - t$ messages at least $t + 1$ messages supporting a single binary value will be received, satisfying line 03 of the $\text{is\_valid}$ predicate for round 1. All non-faulty processes will then sign and broadcast a valid $\text{AUX}$ message for round 1 and advance to round 2.

In round 2 non-faulty processes will receive at least $n - t$ signed valid round 1 $\text{AUX}$ messages from different processes. If $n - t$ of these messages are of the form $\text{AUX}[1](0)$, then by line 11 of Figure 2 the $\text{is\_valid}$ predicate is satisfied for round 2. Otherwise, at least one of the valid signed $\text{AUX}$ messages must be of the form $\text{AUX}[1](1)$. By line 22 of Figure 1 this message must contain proofs generated by the $\text{is\_valid}$ predicate supporting binary value 1 for round 1. This can only happen on line 03 of Figure 2 by including $t + 1$ messages of the form $\text{AUX}[0](1)$. Notice then that by line 07 these proofs also satisfy the $\text{is\_valid}$ predicate for round 2. Thus, all non-faulty processes will then sign and broadcast a valid $\text{AUX}$ message for round 2 and advance to round 3.

Now assume by induction all non-faulty processes have received enough valid messages to satisfy the $\text{is\_valid}$ predicate for a round $r - 1$. All processes will then sign and broadcast a valid $\text{AUX}$ message on line 21 of Figure 1 and advance to round $r$. Following this all non-faulty processes will receive at least $n - t$ valid signed $\text{AUX}$ messages from round $r - 1$. If $n - t$ of these messages are of the form $\text{AUX}[r - 1](¬(r \mod 2))$ then the $\text{is\_valid}$ predicate for round $r$ is satisfied by line 11 of Figure 2.

Otherwise, at least one of the valid signed $\text{AUX}$ messages must be of the form $\text{AUX}[r - 1](¬((r \mod 2)))$. By line 22 of Figure 1 this message must contain proofs generated by the $\text{is\_valid}$ predicate supporting binary value $¬((r \mod 2))$ for round $r - 1$. For this, on line 05 of Figure 2 we have $b = ((r - 2) \mod 2)$, or equivalently $b = (r \mod 2)$ and $\text{est} = ¬((r \mod 2))$, i.e. $\text{est} \neq b$. Therefore by line 11 the proofs for message $\text{AUX}[r - 1](¬((r \mod 2)))$ must be $n - t$ messages of the form $\text{AUX}[r - 2](¬((r \mod 2)))$. Now consider the $\text{is\_valid}$ predicate with input round $r$ and $\text{est} = ¬(r \mod 2)$, or equivalently $\text{est} = ((r - 1) \mod 2)$, in this case the predicate is satisfied by $n - t$ messages of the form $\text{AUX}[r - 2](¬((r \mod 2)))$ (line 08), which is exactly the set of proofs that were included in the $\text{AUX}[r - 1](¬(r \mod 2))$ message, completing the induction proof. 

\(\square\)
Lemma 5. Let $r$ be the smallest round in which a non-faulty process decides. All non-faulty processes will decide in either round $r$ or $r+2$.

Proof. Given line 11 of Figure 1, a non-faulty process decides $v = (r) \mod 2$ in round $r$ after receiving $n - t$ signed $\text{aux}[r](v)$ from different processes. By Lemma 1 no process will receive $n - t$ signed $\text{aux}[r](\neg v)$ messages from different processes, thus by Lemma 3, $\neg v$ will not satisfy the is_valid predicate in round any round after $r$. From this and by Lemma 4, in all rounds after $r$ the is_valid predicate on line 14 will return $\{v\}$ and all non-faulty processes will broadcast $\text{aux}$ messages supporting $v$. Thus by line 08 of Figure 1 a non-faulty process will wait until it receives $n - t$ signed $\text{aux}[r+2](v)$ messages from different processes, and decide on line 12.

Lemma 6. If all non-faulty processes execute synchronous rounds, then termination is ensured within $O(t)$ rounds.

Proof. All non-faulty processes executing synchronous rounds will receive all messages from all $n-t$ non-faulty in the round before preceding to the next round. Now given two consecutive synchronous rounds $r$ and $r+1$, where coordinators $p_{(r, \mod n)}$ and $p_{(r+1, \mod n)}$ are non-faulty processes, all non-faulty processes will broadcast $\text{aux}$ messages with the same binary value in rounds $r+1$ and $r+2$ (i.e. the value broadcast by the coordinators by line 17 of Figure 1) and decide in either round $r$ or $r+1$. Given that there are at most $t$ faulty processes and $t < n/3$, two consecutive rounds that have non-faulty coordinators will be reached after at most $O(t)$ rounds.

Lemma 7. Given the $\Diamond \text{Synch}$ assumption, all non-faulty processes eventually execute synchronous rounds.

Proof. As described in Section 3.2.1 the timer for any round $r$ is larger than the timer for round $r-1$ by at least some constant $\alpha$. By the timers on lines 06, 09 of Figure 1 and that the threshold for skipping a timer (line 26) is $t+1$, no non-faulty process will reach round $r$ faster than the sum of all the timeouts (as given by the fastest non-faulty process bounded by $\Diamond \text{Synch}$) of the previous rounds. Thus to reach round $r$, a non-faulty process must have waited at least $\sum_{j=0}^{r-1} 2 \times j \times \alpha$ units of time for timers to expire (i.e a polynomial number of time units).

Now given that a non-faulty process will only start the timer on line 09 for round $r$ once it has received $n - t$ messages and that $t < n/3$, the process must have received messages from at least $t+1$ non-faulty processes for round $r$ when it starts the timer. Thus all non-faulty processes will receive $t+1$ messages from round $r$ within a constant bound $c$ given by $\Diamond \text{Synch}$ and by line 27 will skip all timeouts until round $r$ and reach the round in a bound given by $c \times r$ (i.e. in a linear amount of time units).

As follows, the fastest non-faulty process reaches round $r$ in a polynomial amount of time bounded by $\Diamond \text{Synch}$, and the slowest non-faulty process reaches round $r$ in a linear amount of time bounded by $\Diamond \text{Synch}$. Given that a polynomial function grows faster than a linear one, eventually the slowest non-faulty processes will reach a round $r$ early enough so that all non-faulty processes receive messages from all other non-faulty processes (given $\Diamond \text{Synch}$) before any non-faulty process progresses to round $r+1$. Furthermore once this threshold is reached it hold for all following rounds (given $\Diamond \text{Synch}$).

Theorem 1. The algorithm presented in Figure 1 solves the Binary consensus problem in $\mathcal{BAMP}_{n,t}[t < n/3, \Diamond \text{Synch}]$.

Proof. First recall the definition of Binary Byzantine Consensus.

- BBC-Termination. Every non-faulty process eventually decides on a value.
• BBC-Agreement. No two non-faulty processes decide on different values.
• BBC-Validity. If all non-faulty processes propose the same value, no other value can be decided.

Lemma 5 ensures that if a non-faulty process decides then all non-faulty processes decide, while Lemmas 6 and 7 ensure all non-faulty processes decide in the presence $\Diamond \text{Synch}$, thus ensuring BBC-Termination. BBC-Agreement and BBC-Validity are ensured by Lemmas 2 and 3 respectively.

4 Implementation and experiments.

Stopping and garbage collection. The algorithm shown in Figure 1 continues to execute rounds forever. To avoid this, if a non-faulty process decides in round $r$ it can simply broadcast a "proof" of decision, containing the $n - t$ messages that allowed it to decide and stop immediately. Furthermore, the broadcast of this message may be delayed until the process receives a valid message from another process from round $r + 1$, ensuring that if all processes decide in round $r$ then no extra messages will be sent. Note that, in implementation, a process can not be immediately garbage collected as it needs to ensure that its messages are reliably delivered (reliable channels are often implemented through the use of re-transmissions when needed). Fortunately, in a system that is executing multiple consensus instances, garbage collection of earlier instances can be easily coordinated in later instances (this is not described here as it depends on the requirements of the specific system).

Timeouts and coordinators. As described in Section 3.2.1, the algorithm does not always need to use timeouts or a coordinator to terminate. Disabling timeouts and not using the round coordinator until round 10 was found to be a good trade-off, allowing the algorithm to terminate quickly, while still ensuring progress.

Cryptographic signatures and validity proofs. Like timeouts and coordinators, including proofs of validity with messages is necessary for the correctness of the algorithm, but are not often needed in the expected case. In fact in the presence of reliable channels the validity proofs are needed only in the case of Byzantine faults. Given this, for efficiency an implementation may choose not to include proofs with messages by default and instead have processes request proofs from the sender of the message if the recipient cannot validate the message itself. Notice that this does not effect the correctness of the algorithm, but only increases the needed synchrony window to include enough time for a non-faulty process to request and receive missing proofs from other non-faulty processes.

Threshold signatures. Another way to implement validity proofs efficiently is through the use of threshold signatures [12, 13, 14, 34]. When using threshold signatures, a set of signatures of the same message from different processes can be combined into a single shared signature. In the case of this algorithm $n - t$ threshold signatures can be used, allowing the validity proof to be reduced to a single value in most cases. To enable threshold signatures, a distributed key generation protocol is usually required to be executed before the first consensus iteration in order to compute the shared keys.
4.1 Experiments

The algorithm has been implemented using the Go [21] programming language. Reliable channels are implemented by using message re-transmission. All received messages are stored to disk in an append only log allowing processes to recover after a failure. Signatures are implemented using the ECDSA implementation included in the Go standard library [22]. Proofs of validity are transmitted on request of the recipient as described previously.

The experiments were run on Amazon EC2 using from 75 to 300 c5.large instances (4 GiB of memory, 2 vCPUs, EBS backed storage). The instances were spread evenly across EC2’s 15 regions in Asia, Australia, Europe, North America, and South America.

In the experiments each node chooses a random initial binary proposal using a threshold given by the experiment, then run consensus 10 times. Results are then calculated as the average of the those runs. The thresholds are chosen as 25, 50, and 75 percent, where for example 25 percent would mean approximately 25 percent of nodes choose 1 and their initial proposal with the remaining nodes choosing 0. All nodes are non-faulty.

Figure 3 shows the results of the experiment. Figure 3(a) shows the average latency of executing a single consensus instance, Figure 3(b) shows the average termination round the consensus instances, Figure 3(c) shows the average number of messages sent for a single consensus instance for all nodes.

Given that the consensus can decide 1 on round 1 and 0 on round 2, with 75 percent 1 proposals termination happens on the first round, and with 25 percent 1 proposals termination happens on the second round. With 50 percent 1 proposals and given the randomization of the experiment, the termination round varies in each case, but stays below 2 on average. A maximum termination round of 8 was observed. The latency is related directly to the termination round, with 195 milliseconds being the minimum latency and 293 milliseconds being the maximum latency. Increasing the number of nodes from 75 to 300 had minimal impact on latency. The number of messages sent is quadratic to the number of nodes multiplied by the number of rounds.

![Figure 3: Experiment results varying the number of instances and the number of initial 1 proposals.](image)

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