Fifth International Conference on
PERSPECTIVES IN HADRONIC PHYSICS
Particle-Nucleus and Nucleus-Nucleus Scattering at Relativistic Energies

22 - 26 May 2006

Color Superconductivity in Ultra-Dense Quark Matter

Mark ALFORD
Washington University
Department of Physics
Campus Box 1105
Saint Louis, MO 63130-4899
U.S.A.

These are preliminary lecture notes, intended only for distribution to participants
Color superconductivity in ultra-dense quark matter

Mark Alford

Washington University
Saint Louis, USA

M. Alford, nucl-th/0512005, hep-ph/0102047

T. Schäfer, hep-ph/0304281. K. Rajagopal, F. Wilczek, hep-ph/0011333
Outline

I High density QCD
   Color-superconducting quark matter, Color-flavor locking (CFL)

II Quark matter in compact stars
   weak interactions, neutrality, $M_s$;

III Signatures of color superconductivity in compact stars
   Transport properties, mass-radius relationship

IV Looking to the future
I. High density QCD

Conjectured QCD phase diagram

heavy ion collisions: chiral critical point and first-order line
compact stars: color superconducting quark matter core
Cooper pairing of quarks at high density

At sufficiently high density and low temperature, there is a Fermi sea of almost free quarks.

\[ \mu = E_F \]

But quarks have attractive QCD interactions.

\[ F = E - \mu N \]

Any attractive quark-quark interaction causes pairing instability of the Fermi surface. This is the Bardeen-Cooper-Schrieffer (BCS) mechanism of superconductivity.
High-density QCD calculations

Guess a color-flavor-spin pairing pattern $P$
minimize free energy wrt $\Delta$: gap equation for $\Delta$.

$$= \langle q^\alpha_{ia} q^\beta_{jb} \rangle_{1PI} = P^\alpha\beta_{ij\ ab} \Delta$$

1. **Weak-coupling** methods. First-principles calculations
direct from QCD Lagrangian, valid in the asymptotic
regime, currently $\mu \gtrsim 10^6$ MeV.

2. **Nambu–Jona-Lasinio models**, ie quarks with four-
fermion coupling based on instanton vertex, single
gluon exchange, etc. This is a semi-quantitative guide
to physics in the compact star regime $\mu \sim 400$ MeV,
not a systematic approximation to QCD.

NJL gives $\Delta \sim 10-100$ MeV at $\mu \sim 400$ MeV.
Both methods agree on the favored pairing pattern.
Color superconductivity in three flavor quark matter: Color-flavor locking (CFL)

Equal number of colors and flavors allows a special pairing pattern
(Alford, Rajagopal, Wilczek, hep-ph/9804403)

\[
\langle q_i^\alpha q_j^\beta \rangle \sim (\kappa+1) \delta_i^\alpha \delta_j^\beta + (\kappa-1) \delta_j^\alpha \delta_i^\beta = \epsilon^{\alpha\beta N} \epsilon_{ijN} + \kappa(\ldots)
\]

color $\alpha, \beta$  
flavor $i, j$  

This is invariant under equal and opposite rotations of color and (vector) flavor

\[ SU(3)_{\text{color}} \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{C+L+R} \times \mathbb{Z}_2 \]
\[ \supset U(1)_Q \]

- Breaks chiral symmetry, but not by a $\langle \bar{q}q \rangle$ condensate.
- There need be no phase transition between the low and high density phases: (“quark-hadron continuity”)
- Unbroken “rotated” electromagnetism, $\tilde{Q}$, photon-gluon mixture.
Color-flavor-locked ("CFL") quark pairing

\[
\begin{array}{cccccccccc}
\tilde{Q} & 0 & 0 & 0 & -1 & +1 & -1 & +1 & 0 & 0 \\
\hline
u & d & s & d & u & s & u & s & d \\
\hline
u & & \Delta & \Delta & & & & & \\
d & & \Delta & \Delta & & & & & \\
s & & \Delta & \Delta & & & & & \\
d & & & & & -\Delta & & & \\
u & & & & & & -\Delta & & \\
s & & & & & & & -\Delta & & \\
u & & & & & & & & -\Delta & \\
s & & & & & & & & & -\Delta \\
d & & & & & & & & & -\Delta \\
\end{array}
\]
II. Quark matter in compact stars

Where in the universe is color-superconducting quark matter most likely to exist? In compact stars.

A quick history of a compact star.

A star of mass $M \gtrsim 10M_\odot$ burns Hydrogen by fusion, ending up with an Iron core. Core grows to Chandrasekhar mass, collapses $\Rightarrow$ supernova. Remnant is a compact star:

\[
\begin{array}{cccc}
\text{mass} & \text{radius} & \text{density} & \text{initial temp} \\
\sim 1.4M_\odot & \mathcal{O}(10 \text{ km}) & \gtrsim \rho_{\text{nuclear}} & \sim 30 \text{ MeV} \\
\end{array}
\]

The star cools by neutrino emission for the first million years.
The real world: $M_s$ and neutrality

In the real world there are three complications that disfavor the CFL phase at realistic compact-star densities ($\mu \sim 400$ MeV).

1. **Strange quark mass** is not infinite nor zero, but intermediate. It depends on density, and ranges between about 500 MeV in the vacuum and about 100 MeV at high density.

2. **Neutrality requirement.** Bulk quark matter must be neutral with respect to all gauge charges: color and electromagnetism.

3. **Weak interaction equilibration.** In a compact star there is time for weak interactions to proceed: neutrinos escape and flavor is not conserved.

So quark matter in a compact star might be CFL, or something else: kaon-condensed CFL, 2SC, 1SC, crystalline (LOFF), diquark BEC ...
Effect of $M_s$ and electrical neutrality

Fermi momenta of flavors tend to split apart. If pairing is strong enough ($\Delta > M_s^2/(2\mu)$) it can hold them together.

So do we get unpaired $\rightarrow$ 2SC $\rightarrow$ CFL as density increases? Not that simple.

No electrons!
Rüster et. al. use an NJL model with coupled chiral and color-superconducting condensates. The “$g$” phases are gapless, and are unstable: we don’t know what replaces them.
Gapless phases

Quasiparticle dispersion relations for the $d_s$ sector:

When $M_s^2/\mu$ reaches $2\Delta_{d_s}$, unpairing begins: a “blocking” or “breached pairing” region opens up, bordered by gapless modes.
Problem: magnetic instability of gapless phases

In the gCFL phase, the $r\bar{g}$ gluons have an imaginary Meissner mass (Huang and Shovkovy, hep-ph/0408268; Casalbuoni, Gatto, Mannarelli, Nardulli, Ruggieri, hep-ph/0410401; K. Fukushima hep-ph/0506080).

This is a generic consequence of gapless quasiquark dispersion relations:

\begin{align*}
\Delta = \delta\mu \\
M^2_{\text{Meissner}}
\end{align*}

2 charged quark species, chem pots $\bar{\mu} \pm \delta\mu$, form Cooper pairs with gap parameter $\Delta$.

The Meissner mass goes imaginary when the gap in the dispersion relation $\Delta - \delta\mu$ reaches zero.

(Alford and Wang, hep-ph/0501078)
What replaces the gapless phases?

Effective Hamiltonian for diquark condensate $\phi$ is

$$\mathcal{L}_{\text{eff}} = \kappa D_i \phi^* D_i \phi + \cdots = \kappa \partial_i \phi^* \partial_i \phi + \kappa |\phi|^2 A_i A_i + \cdots$$

$M_{\text{Meissner}}^2 < 0 \Rightarrow \kappa < 0$: instability towards spatial variation $\phi(\vec{x})$.

Suggestions:

1. Crystalline “LOFF” phase: Cooper pairs have non-zero momentum.
   
   (Alford, Bowers, Rajagopal, hep-ph/0008208, Casalbuoni, Nardulli hep-ph/0305069)

2. $p$-wave $K^0$ meson condensate
   
   (Schäfer hep-ph/0508190, Kryjevski hep-ph/0508180)

3. Mixed phases, secondary pairing, gluon condensate, ...
IV. Signatures of color superconductivity in compact stars

Gaps in spectra affect Transport properties.
Pairing energy affects Equation of state.

Transport properties, mean free paths, conductivities, viscosities, etc.

1. Cooling by neutrino emission, neutrino pulse at birth
   (Page, Prakash, Lattimer, Steiner, hep-ph/0005094; Carter and Reddy, hep-ph/0005228
   Reddy, Sadzikowski, Tachibana, nucl-th/0306015)

2. Gravitational waves: r-mode instability
   (Madsen, astro-ph/9912418)

3. Glitches and crystalline (“LOFF”) pairing
   (Alford, Bowers, Rajagopal, hep-ph/0008208)
Constraints from \textit{r}-modes (Madsen, astro-ph/9912418)

Rotation frequencies above curves are unstable: viscosity is too low to hold back the \textit{r}-modes.

\underline{No pairing}

\underline{CFL pairing}

Solid and dotted lines: quark matter with $m_s = 200, 100$ MeV; Dashed line: nuclear matter. box: LMXBs; crosses: fastest pulsars.

Pure CFL quark matter stars are ruled out.
Cooling of a neutron star with quark matter core

With 2-flavor color superconductivity, and additional weak pairing of the blue quarks. Can accommodate data with masses ranging from $1.1 \, M_\odot$ to $1.7 \, M_\odot$. 

(Grigorian, Blaschke, Voskresensky, astro-ph/0411619)
Equation of state (mass-radius relationship)

pressure \[ p = (1 - c) \mu^4 - (\cdot) M_s^2 \mu^2 + (\cdot) \Delta^2 \mu^2 - B \]

Bag constant \( B \) and QCD correction parameter \( c \) have a strong effect on EoS and hence \( M \) vs \( R \). But \( \Delta \) is like a renormalization of \( M_s \) or even \( B \).
It is hard to find values of $M$ and $R$ that would rule out quark matter in compact stars.
What would rule out quark matter?
Difficult. Most regions of $M-R$ space that cannot be reached by any quark+nuclear matter hybrid also cannot be reached by nuclear matter alone. $M \gtrsim 2.1 \, M_\odot$ seems hard to achieve with quark matter.

What would indicate the presence of quark matter?
Very Difficult. Regions of $M-R$ space that cannot be reached by any nuclear matter EoS also cannot be reached by hybrid NM-QM EoS.

What would indicate the presence of color superconducting quark matter?
Impossibly difficult. Even if we found an $M(R)$ that was characteristic of quark matter, we would need an independent determination of the bag constant and $M_s$ to claim that it was color-superconducting.
V. Looking to the future

• Neutron-star phenomenology of color superconducting quark matter:
  – Structure: nuclear-quark interface
  – Crystalline phase and glitches
  – Vortices but no flux tubes
  – Effects of gaps in quark spectrum
    * conductivity and emissivity (neutrino cooling)
    * shear and bulk viscosity ($r$-mode spin-down)

• Particle theoretic questions:
  – Response of CFL to $M_s$: gapless CFL, kaon condensation, ...?
  – Magnetic instability of gapless phases
  – Better weak-coupling calculations, include vertex corrections
  – Go beyond mean-field, include fluctuations.