Mining terrain curvatures approximation using the polynomials and a subsidence trough profile fragmentation

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Abstract. In an article a method of average course obtaining of measured mining terrain curvatures has been presented. Terrain curvatures values are very important for building objects located on mining areas and which have a considerable length or/and a great cubature. Curvatures observed graph has an irregular course and that’s why there is a need of their average course determination in order to obtain the most probable value. Measured curvatures approximation by use of the polynomial various orders has been done. Additionally, a whole profile of subsidence trough into the parts has been divided. Subsidence trough fragmentation allowed for a better fit of the curvatures average values to the observed values. A right polynomial order and a trough segments proper number taking into account value of variability coefficient of curvatures random dispersion have been chosen. Geodesic surveys, based on which a graph of measured curvatures has been obtained, in the Upper Silesian Coal Basin (Poland) have been done. There was conducted a mining exploitation of hard coal deposits by use of the longwall system with a roof rocks cave-in.

1. Introduction

Terrain curvatures caused by underground mining exploitation of hard coal seams have a great influence on maintenance and damages in objects located on terrain surface. Objects, which have a considerable length e.g. pipelines, highways, trackways or a big horizontal projection e.g. warehouses, production halls, shopping centers, swimming pools, stadiums, are especially susceptible to action of curvatures.

Curvatures observed graph has an irregular course what results mainly from the cracks of subsurface layers of ground [1]. Moreover, sometimes curvatures achieve very high extreme values and it isn’t known, whether is it caused by surveys inaccuracies, measuring errors, geometry of measuring network, random dispersion of observations results, or are these the real values.

Therefore it exists a need for designation of most probable curvatures graph. It can be done by approximation of average courses of curvatures measured values. Smoothed splines functions [2-10] and polynomials [11] to this have been used so far. Previous research shown that approximation conducted along a whole profile of subsidence trough is insufficient. Therefore there is a need of division of trough profile into sections and carrying out approximation in each segment separately. It should help better fit an average graph of curvatures to their measured values and answer the
question about the extreme measured values (are they real or a result of measured values random dispersion?). That’s why in the article an instance of average courses approximation of curvatures observed values with division of curvatures graphs into the parts has been presented. Right number of sections on the basis of minimization of value of variability coefficient of random dispersion of measured curvatures and polynomial order has been chosen. Then an approximation of average graphs of terrain surface curvatures induced by underground mining exploitation of hard coal deposits and observed by geodetic measurements has been conducted. Approximation for each exploitation stage, after the end of a coal excavation from each of four longwalls has been done.

2. Hard coal extraction and geodetic surveys

2.1. Exploitation of the 354 hard coal seam
Exploitation of the 354 hard coal seam in the southern part of Poland (the Upper Silesian Coal Basin), by some hard coal mine has been conducted.

Operation in the years 1967 – 1971 has been placed. It by use of the longwall system with a roof rocks cave-in has been carried out. Four longwalls (the 1, 2, 3 and 4 longwall) were exploited at a height of 1.7 m and a depth of 270 – 350 m. There were conducted in sequence of numbering, from the east to the west. Exploitation in the 1 and 2 longwalls was carried out from the south to the north. Extraction of hard coal from the 3 and 4 longwalls was conducted from the north to the south.

Inclination of a hard coal seam was rather small and amounted 5° in the southern direction. An overlay thickness was amounted 220 m and it was mainly created by the layers of sands, gravels, Quaternary clays, silts and Miocene sands.

Basic information about an exploitation in the 354 coal seam in the Table 1 have been presented.

Table 1. Exploitation parameters in the 354 hard coal seam

| Parameter               | the 354 coal seam          |
|-------------------------|-----------------------------|
| Longwalls numbers       | 1, 2, 3, 4, 4a              |
| Height [m]              | 1.7                         |
| Depth [m]               | 270 ÷ 350                  |
| Time [years]            | 1967 ÷ 1971                |
| Inclination [°]         | 5.0                         |
| Exploitation direction  | from east to west           |

Location, shape and exploitation direction of longwalls in the 354 hard coal seam at the Figure 1 have been shown.

Figure 1. Longwalls in the 354 hard coal seam [12 – 14]
2.2. Geodetic measurements

Geodetic surveys of terrain surface on the 51 ground points have been conducted. These points in shape of measurement line have been stabilized. Observational line No. 1A perpendicularly to the longwalls runs has been situated (Figure 1). Distances between subsequent measuring points amounted from 20 m to 30 m. Observations in monthly or bimonthly cycles have been carried out. There four measuring cycles (the 6th, 9th, 14th and 18th cycle) have been chosen. They demonstrated vertical profiles of static subsidence troughs which after the end of exploitation in particular longwalls have been created.

Measurements of distances between subsequent ground points and their heights by use of the standard methods and a standard surveying instruments have been done. There a precise leveller to measurements of points heights (a geometric levelling from the middle) and a digital range-finder to measurements of sections lengths have been used.

Basic information about geodetic observations conducted on the measuring line No. 1A in the Table 2 have been presented.

| Parameter             | Value                      |
|-----------------------|----------------------------|
| Measuring line        | No. 1A                     |
| Points number         | 51                         |
| Sections lengths [m]  | 20 ÷ 30                    |
| Cycles number         | 23                         |
| Chosen cycles         | 6th, 9th, 14th, 18th       |
| Measurements frequency [months] | 1 ÷ 2                  |

3. Curvatures observed graphs

Observed graphs of terrain surface curvatures on the basis of terrain surface subsidence noted after the end of each exploitation stage (after one, two, three and four longwalls) have been obtained.

Subsidence of each measuring point as a heights difference of this point before and after termination of each exploitation stage has been calculated [15].

Curvature of two neighboring sections of measuring line from the subsidence differences of three neighboring points divided by an average length of two segments has been calculated [15].

Curvatures graphs observed along a measuring line after an extraction end in the 1 longwall (Figure 2a); the 1 and 2 longwalls (Figure 2b); the 1, 2 and 3 longwalls (Figure 2c); the 1, 2, 3 and 4 longwalls (Figure 2d) at the Figure 2 have been presented.

![Curvature graphs](image1.png)

| a) after the 1 longwall | b) after the 1 and 2 longwalls |
4. Curvatures average graphs

4.1. Choice of segments number and polynomial order
To determine a proper order of polynomial and a number of sections on which a vertical profile of subsidence trough should be divided, there is a need to define an unambiguous criterion.

Minimization of value of variability coefficient of random dispersion of observed curvatures ($M_C$) as this criterion has been assumed. A variability coefficient of random dispersion of observed curvatures is calculated as a standard deviation between the observed and average values of curvatures divided by the absolute value of an extreme, average curvature [16]:

| Exploitation stage          | Minimal observed curvature $c_{\text{obs}}$ [-10$^6$/m] | Maximal observed curvature $c_{\text{obs}}$ [-10$^6$/m] |
|-----------------------------|----------------------------------------------------------|---------------------------------------------------------|
| 1. After the 1 longwall     | -195                                                     | +300                                                    |
| 2. After the 1 and 2 longwalls | -200                                                   | +238                                                    |
| 3. After the 1, 2 and 3 longwalls | -200                                                   | +270                                                    |
| 4. After the 1, 2, 3 and 4 longwalls | -192                                                   | +255                                                    |
where:

\[ M_c = \frac{\sigma_{c_{\text{avg}}}}{C_{\text{extr}}^*} \]  

(1)

where:

- \( M_c \) – a variability coefficient of random dispersion of observed curvatures,
- \( \sigma_{c_{\text{avg}}} \) – a standard deviation between observed and average curvatures,
- \( C_{\text{extr}}^* \) – an extreme value of average curvature.

Due to a large random dispersion of observed curvatures, a value of variability coefficient of random dispersion to the value of 33.5 % [17] has been limited.

At the Figure 3 the graphs presenting choice of fragments number of subsidence trough and order of polynomial depending on a value of variability coefficient of random dispersion of observed curvatures have been shown.

A right polynomial order \((n)\) and a fragments number of subsidence trough profile \((k)\) due to minimization of variability coefficient of random dispersion of observed curvatures \((M_c)\), for each stage of exploitation in the 354 hard coal seam, in the Table 4 have been compared.
Table 4. Sections number \((k)\), polynomial order \((n)\) and variability coefficient of random dispersion of observed curvatures \((M_C)\) after the end of exploitation subsequent stages

| Exploitation stage                        | Sections number \(k\) | Polynomial order \(N\) | Variability coefficient of random dispersion \(M_C\) [%] |
|-------------------------------------------|------------------------|-------------------------|--------------------------------------------------------|
| 1. After the 1 longwall                   | 20                     | 3                       | 6.36                                                   |
| 2. After the 1 and 2 longwalls            | 20                     | 3                       | 9.66                                                   |
| 3. After the 1, 2 and 3 longwalls         | 18                     | 3                       | 10.00                                                  |
| 4. After the 1, 2, 3 and 4 longwalls      | 18                     | 10                      | 23.61                                                  |

From the Figure 3 and data presented in the Table 4 results that an optimal number of segments is relatively large and similar for each exploitation stage (for the first two stages \(k = 20\) and for the last two stages \(k = 18\)). For the first three extraction stages an order of polynomial is low and equals \(n = 3\). For the last operation stage a polynomial order is big and equal to 10. Value of a variability coefficient of random dispersion of observed curvatures \((M_C)\) increases together with increasing of exploitation range. A large growth of value of the \(M_C\) coefficient (more than twice) is observed between third \((M_C = 10.0\) %\) and fourth \((M_C = 23.6\) %\) exploitation stage.

4.2. Approximation of curvatures average graphs

Approximation of average graphs of observed curvatures by the use of polynomials and fragmentation of vertical profile of subsidence trough has been done. Sections numbers and polynomials orders determined for all exploitation stages have been used.

Average courses of curvatures measured after the end of an exploitation in subsequent longwalls localized in the 354 hard coal seam at the Figure 4 have been shown.

![Curvature Graphs](image)

**Figure 4.** Average graphs of curvatures observed after the termination of subsequent exploitation stages in the 354 hard coal seam
The extreme values of average curvatures approximated along the measuring line No. 1, after the end of each exploitation stage in the 354 hard coal seam and a mapping coefficient of extreme values of observed curvatures \( S_{\text{extr}} [\%] \) in the Table 5 have been shown.

**Table 5.** Extreme values of average curvatures approximated after the end of exploitation subsequent stages

| Exploitation stage | Minimal average curvature \( L_{\text{min}}^{\text{ave}} \) \( [10^{-6} \text{ m}^{-1}] \) | \( S_{\text{min}} \) [%] | Maximal average curvature \( L_{\text{max}}^{\text{ave}} \) \( [10^{-6} \text{ m}^{-1}] \) | \( S_{\text{max}} \) [%] |
|--------------------|---------------------------------|---------------------|---------------------------------|---------------------|
| 1. After the 1 longwall | -191.84 | 98.38 | +287.40 | 95.80 |
| 2. After the 1 and 2 longwalls | -199.47 | 99.74 | +235.79 | 99.07 |
| 3. After the 1, 2 and 3 longwalls | -196.48 | 98.24 | +267.08 | 98.92 |
| 4. After the 1, 2, 3 and 4 longwalls | -190.02 | 98.97 | +253.16 | 99.28 |

Taking into account the values presented in the Table 5, it can be said that an approximation of average courses of observed curvatures by the use of the polynomials and a fragmentation of vertical profile of subsidence trough correctly has been carried out. Extreme values of average curvatures are mapping well the extreme values of measured curvatures and the \( S_{\text{extr}} \) mapping coefficient reaches a value greater than 95 %. Generally its value amounts around 99 %.

**5. Conclusions**

Analysis of made approximation of average graphs of terrain surface curvatures caused by subsequent stages of underground exploitation of the 354 hard coal seam and carried out by the use of the polynomials with division of a graph into the parts, allowed for the following statements:

- this kind of a mean – squared approximation better fits average courses of curvatures to their measured courses;
- the extreme values of average curvatures reflect the extreme values of observed curvatures very well;
- the extreme values of measured curvatures are real and aren't a result of random dispersion;
- a value of variability coefficient of random dispersion of observed curvatures increases together with enlargement of an exploitation range;
- an optimal order of polynomial grows with increasing the number of longwalls.

**References**

[1] Kowalski A and Jędrzejec E 2015 Influence of Subsidence Fluctuation on the Determination of Mining Area Curvatures, *Arch. Min. Sci.* **60**(2) 487-505

[2] Orwat J and Mielimąka R 2017 Approximation of Average Course of Measured Curvatures of Mining Area with Reference to Their Forecast Values by Bialek’s Formulas, *AIP Conf. Proc.* **1863** 130003-1 – 130003-4

[3] Mielimąka R and Orwat J 2017 Approximation of Average Course of Measured Curvatures of Mining Area with Reference to Their Forecast Values by Knothe’s Formulas, *AIP Conf. Proc.* **1863** 130005-1 – 130005-4

[4] Orwat J 2017 Approximation of Average Course of Measured Subsidences of Mining Area by Smooth Splines, *AIP Conf. Proc.* **1863** 130004-1 – 130004-4

[5] Orwat J and Mielimąka R 2017 Average Course Approximation of Measured Subsidence and Inclinations of Mining Area by Smooth Splines, *Journal of Sustainable Mining* **16**(1) 8-13

[6] Orwat J and Mielimąka R 2018 Smoothing Parameter as Shape Parameter of Function Approximating the Average Course of Terrain Surface Subsidence, *AIP Conf. Proc.* **1978** 390005-1 – 390005-4

[7] Orwat J 2018 Appraisal of Application Possibilities of Smoothed Splines to Designation
of the Average Values of Terrain Curvatures Measured after the Termination of Hard Coal
Exploitation Conducted at Medium Depth, *IOP Conf. Ser.: Mater. Sci. Eng.* 294 012030

[8] Orwat J 2018 Possibility of Using the Smoothed Spline Functions in Approximation of Average
Course of Terrain Inclinations Caused by Underground Mining Exploitation Conducted at
Medium Depth, *IOP Conf. Ser.: Mater. Sci. Eng.* 294 012029

[9] Orwat J 2019 Relation Between the Theoretical and Average Observed Curvatures of Mining
Terrain, *IOP Conf. Ser.: Mater. Sci. Eng.* 477 012043

[10] Orwat J 2019 Linear Regression Equation of Mining Terrain Curvatures Caused by Hard Coal
Excavation from a Few Seams and Their Approximated Values by the Use of Smoothed
Spline, *IOP Conf. Ser.: Mater. Sci. Eng.* 477 012042

[11] Orwat J and Franz S 2019 Average Graph of Land Surface Curvatures Induced Mining
Operation Determined by Use of Fourth Order Polynomial, 17th International Conference of
Numerical Analysis and Applied Mathematics, September 23 – 28, 2019, Rhodes, Greece

[12] Orwat J and Mielimąka R 2017 Approximation of Average Course of Measured Curvatures of
Mining Area with Reference to their Forecast Values by Bialek’s Formulas, *AIP Conf. Proc.* 1863 130003

[13] Orwat J 2017 Approximation of Average Course of Measured Subsidences of Mining Area by
Smooth Splines, *AIP Conf. Proc.* 1863 130004

[14] Mielimąka R and Orwat J 2017 Approximation of Average Course of Measured Curvatures of
Mining Area with Reference to their Forecast Values by Knothe’s Formulas, *AIP Conf. Proc.* 1863 130005

[15] Orwat J 2018 Determination of Equation Describing the Measured and Average Curvatures
Graphs Observed as a Result of Multideposit Exploitation at the Great Depth, *Informatics, Geoinformatics and Remote Sensing – Geoinformatics, Geodesy and Mine Surveying* 18(2.2) 685-691

[16] Orwat J 201 Depth of the Mining Exploitation and Its Progress in the Time, and a Random
Dispersion of Observed Terrain Subsidence and Their Derivatives, *IOP Conf. Ser.: Earth and Environmental Science* 261 012037

[17] Kowalski A 2007 *Nieustalone Górnicze Deformacje Powierzchni w Aspekcie Dokładności Prognoz*, Wydawnictwo Głównego Instytutu Górnictwa, Katowice (polish)