QSO Lifetimes

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Abstract

The QSO lifetime \( t_Q \) is one of the most fundamental quantities for understanding black hole and QSO evolution, yet it remains uncertain by several orders of magnitude. If \( t_Q \) is long, then only a small fraction of galaxies went through a luminous QSO phase. In contrast, a short lifetime would require most galaxies today to have undergone a QSO phase in their youth. The current best estimates or constraints on \( t_Q \) from black hole demographics and the radiative properties of QSOs vary from at least \( 10^6 \) to \( 10^8 \) years. This broad range still allows both possibilities: that QSOs were either a rare or a common stage of galaxy evolution. These constraints also do not rule out the possibility that QSO activity is episodic, with individual active periods much shorter than the total active lifetime.

In the next few years a variety of additional observational constraints on the lifetimes of QSOs will become available, including clustering measurements and the proximity effect. These new constraints can potentially determine \( t_Q \) to within a factor of 3 and therefore answer one of the most fundamental questions in black hole evolution: Do they shine as they grow? This precision will also test the viability of our current model for accretion physics, specifically the radiative efficiency and need for super-Eddington luminosities to explain the black hole population.

1.1 Introduction

Shortly after the discovery of luminous QSOs, several authors suggested that they could be powered by accretion onto supermassive black holes (e.g., Salpeter 1964; Zel’dovich & Novikov 1964; Lynden-Bell 1969). The strong evidence that dormant, supermassive black holes exist at the centers of all galaxies with a spheroid component (e.g., Richstone et al. 1998) supports this hypothesis and suggests these galaxies go through an optically luminous accretion phase. More recent observations suggest that the masses of supermassive black holes are correlated with the sizes of their host galaxy spheroids, which implies some connection between the growth of the subparsec-scale supermassive black holes and the kiloparsec-scale spheroids. The critical parameter for understanding how quickly these present-day black holes grew to their present size is the lifetime of the QSO phase \( t_Q \). The QSO lifetime determines the net growth of supermassive black holes during an optically luminous phase. It may also have implications for the growth rate of galactic spheroids.

For the purpose of the present review, the lifetime of a QSO is defined to be the total
amount of time that accretion onto a supermassive black hole is sufficiently luminous to be classified as a QSO — that is, the net time the luminosity in some wavelength range $L > L_{Q\text{,min}}$, where $L_{Q\text{,min}}$ is the minimum luminosity of a QSO. This is an observational definition, chosen because QSOs are categorized on the basis of luminosity in existing surveys. A more physically motivated definition of a QSO would define the lifetime as the time a black hole is accreting above some minimum mass accretion rate, or emitting radiation above some minimum fraction of the Eddington luminosity (and this definition would also require a minimum black hole mass), as these two parameters are more directly relevant to the details of the accretion physics. The measured QSO luminosity instead depends on the product of one of these quantities with the black hole mass.

A valuable fiducial time scale for supermassive black hole growth is the Salpeter (1964) or $e$-folding time scale:

$$t_S = \frac{M}{\dot{M}} = 4.5 \times 10^7 \left(\frac{\epsilon}{0.1}\right) \left(\frac{L}{L_{\text{Edd}}}\right) \text{yr},$$

(1.1)

where $\epsilon = L/Mc^2$ is the radiative efficiency for a QSO radiating at a fraction $L/L_{\text{Edd}}$ of the Eddington luminosity. Commonly accepted values of these two key parameters for luminous QSOs are $\epsilon = 0.1$ and $L/L_{\text{Edd}} = 1$. The importance of an optically luminous or QSO mode of black hole growth relative to significantly less efficient, or less luminous modes is not well established, primarily due to uncertainties in these two parameters and the uncertainty in the QSO lifetime. If the QSO lifetime is long, then QSOs were exceptionally rare objects and only a small number of the present-day supermassive black holes went through a QSO phase. In this scenario, essentially all of the mass in these black holes can be accounted for by luminous accretion. The remaining supermassive black holes in other galaxies must instead have accreted their mass through less radiatively efficient or less luminous modes. If instead the QSO lifetime is short, most present-day black holes went through an optically luminous accretion phase, yet this phase made a relatively small contribution to the present masses of the black holes.

One long-standing upper limit on the lifetime of QSOs is the lifetime of the entire QSO population. The evolution of the QSO space density is observed to rise and fall on approximately a $10^9$ year time scale (see, e.g., Osmer 2003). More refined demographic arguments, described in detail in the next section, find values on order $10^6 - 10^7$ years. Lower limits to the QSO lifetime are less direct, such as that provided by the proximity effect in the Ly$\alpha$ forest, and they argue for a lower limit of $t_Q > 10^4$ years. Unlike demographic arguments, which constrain the net time a black hole accretes matter as a luminous QSO, the lower limits are based on radiative measures that do not take into account the possibility that a given black hole may go through multiple luminous accretion phases. These radiative methods, described in §1.3, constrain the episodic lifetime of QSOs and place a lower bound on the net lifetime.

Observations of QSOs and the less-luminous Seyfert and LINER galaxies demonstrate that AGNs have a range of different accretion mechanisms and these mechanisms depend on AGN luminosity. In the present review I will only discuss the luminous QSOs, defined to be objects with absolute $B$ magnitude $M_B > -23$ mag. These objects may form a relatively homogeneous subclass of AGNs that accrete at an approximately fixed fraction of the Eddington rate with constant radiative efficiency. This luminosity limit corresponds to $L \approx L_{\text{Edd}}$ for a $10^7 M_\odot$ black hole.
1.2 The Net Lifetime

The main methods used to estimate the net lifetime of QSOs are demographic estimates, which typically take one of two forms: integral or counting. Integral estimates are based on the integrated properties of QSOs over the age of the Universe, while counting arguments compare the numbers of objects at different epochs. The classic integral constraint on $t_{Q}$ is that the amount of matter accreted onto QSOs during their lifetime $t_{Q}$, as represented by the luminosity density due to accretion, should be less than or equal to the space density of remnant black holes in the local Universe (Sołtan 1982). This type of estimate is most sensitive to the assumed value of the radiative efficiency $\epsilon$. One example of a counting argument is to ask what value of $t_{Q}$ is require if all bright galaxies go through a QSO phase in their youth (e.g., Rees 1984). Counting arguments are not sensitive to $\epsilon$, but are affected by the assumed value of $L/L_{\text{Edd}}$ and galaxy (black hole) mergers.

The predicted upper bound for $\epsilon$ is 0.3, attained in models of thin disk accretion onto a maximally rotating Kerr black hole (Thorne 1974). The value of $\epsilon$ is more commonly set to 0.1, based on expectations for thin-disk accretion onto a non-rotating black hole. Determination of $L/L_{\text{Edd}}$ is more tractable observationally, at least once the mass of the central black hole is known. The most common assumption is that $L/L_{\text{Edd}} \sim 1$ for luminous QSOs. If they are substantially sub-Eddington, then this implies the presence of a population of massive ($> 10^{10} M_{\odot}$) black holes not seen in the Universe today. In contrast, substantial super-Eddington luminosities (greater than a factor of a few) have historically been considered unlikely due to the limitations of radiation pressure that define the Eddington limit. However, recent work has suggested that the Eddington luminosity could be exceeded by as much as a factor of 100 in some objects due to small-scale inhomogeneities in thin accretion disks (Begelman 2002).

The demographic constraint set by the evolution of the entire QSO population, $t_{Q} \leq 10^{9}$ years, is essentially the only demographic constraint on $t_{Q}$ that does not depend on some assumptions about the parameters $\epsilon$ and $L/L_{\text{Edd}}$ and is neither a counting nor an integral constraint. If the conventional values for these parameters are assumed and $t_{Q} = 10^{9}$ years, then the black holes in QSOs grow by $\sim 25$ e-folds during this QSO phase. This rapid, prolonged growth would produce a population of extremely massive black holes ($> 10^{10} M_{\odot}$) that have not been observed in the local Universe (Cavaliere & Padovani 1988, 1989), assuming the initial black hole seeds are stellar remnants with $M > 1 M_{\odot}$.

A simple counting argument was considered by Richstone et al. (1998), who compared the space density of QSOs at $z \approx 3$ to supermassive black holes in the local Universe. As the space density of QSOs at their peak is approximately $10^{-3}$ times that of present-day supermassive black holes, the implied lifetime is $t_{Q} \approx 10^{6}$ years. This lifetime is much shorter than the Salpeter time scale and therefore implies that only a small percentage of the present-day mass in black holes was accreted during the QSO epoch.

1.2.1 QSO Evolution and the Local Black Hole Population

Many more detailed demographic models have been put forth, particularly in the last decade (Small & Blandford 1992; Haehnelt & Rees 1993; Haehnelt, Natarajan, & Rees 1998; Salucci et al. 1999; Yu & Tremaine 2002). These models are mostly based on counting arguments that simultaneously address the present-day black hole mass function and the space density evolution of QSOs with a recipe for the luminosity evolution and growth of
supermassive black holes in the QSO phase. Haiman & Loeb (1998) and Haehnelt et al. (1998) find that they can match the present-day black hole mass function and the QSO luminosity function at $z = 3$ with $t_Q$ between $10^6$ and $10^8$ years, depending on the relationship between the mass of the central black hole and the host halo. They describe how a lifetime as short as $10^6$ years requires that much of the present-day black hole mass is not accreted as an optically luminous QSO.

Yu & Tremaine (2002; see also Yu 2003) develop a similar model that is a variation on Soltan’s (1982) integral approach using new data for the present-day black hole mass function, as well as the luminosity function and space density evolution of QSOs. They calculate the present-day black hole mass function with the $M_\bullet - \sigma$ relation (Ferrarese & Merrit 2000; Gebhardt et al. 2000) and velocity dispersion measurements for a large number of elliptical galaxies and massive spheroids from the SDSS (Bernardi et al. 2003) and take the QSO luminosity function from the 2dF (Boyle et al. 2000) and the Large Bright QSO Survey (Hewett, Foltz, & Chaffee 1995). Their estimate of the current mass density in black holes is $\rho_{\text{QSO}}(z = 0) = (2.5 \pm 0.4) \times 10^5 h_{0.65}^2 M_\odot\text{Mpc}^{-3}$. This can be compared to the integrated mass accretion onto black holes due to luminous QSOs, which they calculate to be

$$
\rho_{\text{QSO},\text{acc}}(z = 0) = 2.1 \times 10^5 \frac{C_B}{11.8} \frac{(1 - \epsilon)}{\epsilon} M_\odot\text{Mpc}^{-3},
$$

where $C_B$ is the bolometric correction in the $B$ band scaled by the value of 11.8 calculated by Elvis et al. (1994). The cumulative accretion history they derive is shown in Figure 1.1. Note that half of the total mass density accreted by QSOs occurred by $z \approx 1.9$.

The mean lifetime of QSOs above some black hole mass is then approximately equal to the integrated luminosity of accreted matter by these black holes, divided by their space density:

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Fig. 1.2. Mean QSO lifetime from Yu & Tremaine (2002; their Fig. 5). The two lines represent the calculated lifetime for two values for the radiative efficiency $\epsilon$. The solid parts of the lines show the range of local black hole masses estimated from observations of early-type galaxies. The dotted lines represent regions where the local black hole mass function and QSO luminosity function were extrapolated. The two horizontal, dashed lines are the Salpeter time scale for $\epsilon = 0.3$ and 0.1.

This method is therefore essentially an integral method, but by considering the lifetime above a range of black hole masses Yu & Tremaine (2002) capture some of the elements of a counting argument as well. Figure 1.2 shows the mean lifetime derived by Yu & Tremaine (2002) as a function of black hole mass for two values of the radiative efficiency. The solid lines mark the range of black hole mass constrained by the velocity dispersions of early-type galaxies. The dotted lines are based on extrapolations of the local black hole mass function and the luminosity function of QSOs. This calculation assumes that QSOs are radiating at the Eddington luminosity and that the spectral energy distributions of the entire QSO population are well represented by the Elvis et al. (1994) sample. For $\epsilon = 0.1 - 0.3$ and $10^8 < M_\bullet < 10^9 M_\odot$, Yu & Tremaine calculate that the mean QSO lifetime lies in the range $t_Q = 3 - 13 \times 10^7$ years.

For the standard radiative efficiency of $\epsilon = 0.1$ and a bolometric correction $C_B = 11.8$, optically selected QSOs can account for $\sim 75\%$ of the current matter density in black holes. The X-ray and infrared backgrounds offer a potentially better constraint on the total mass accretion onto black holes, as these wavelengths are less susceptible to obscured or highly beamed AGNs. Barger et al. (2001) combined these multiple backgrounds and estimated $\rho_{\bullet, \text{acc}} \approx 9 \times 10^5 (0.1/\epsilon) M_\odot \text{Mpc}^{-3}$, higher than Yu & Tremaine’s estimate due to optically selected QSOs alone. The discrepancy may be due to a genuine population of obscured AGN, but then comparison of this higher accreted mass density with that estimated for local black holes implies $\epsilon \geq 0.3$, in conflict with the maximum efficiency estimated for accretion onto a Kerr black hole. Other potential causes of this discrepancy include (1) uncertainties...
in the local black hole mass density, which has been extrapolated to $M_\bullet < 10^8 M_\odot$ and $M_\bullet > 10^9 M_\odot$, (2) a different population responsible for these backgrounds, (3) the possibility of a free-floating supermassive black hole population that has been ejected from galaxies, and (4) the possibility that a significant amount of black hole mass has been lost as gravitational radiation in mergers.

1.2.2 Coevolution of QSOs, Black Holes, and Galaxies

An alternative approach to modeling the coevolution of QSOs, black holes, and galaxies is to build detailed models with some adjustable parameters and ask what the lifetime is in that context. This is the approach adopted by Kauffmann & Haehnelt (2000), who model the formation of QSOs during major mergers of gas-rich galaxies. Any merger between galaxies with a mass ratio greater than 0.3 results in some fraction $M_{\text{acc}}$ of gas accreted onto the black hole over a time scale $t_Q$. The galaxy mergers and their cold gas content are then obtained from their semi-analytic model of galaxy evolution, combined with the merging history of dark matter halos from extended Press-Schechter (1974) theory.

Kauffmann & Haehnelt let the peak luminosity of each QSO scale as the mass of gas accreted onto the central black hole, which in turn scales as the mass of available cold gas in the merger and the lifetime of the accretion phase. The radiative efficiency is taken to be constant and chosen so that the most luminous QSOs do not exceed the Eddington luminosity. The luminosities of QSOs after the merger do not remain constant, but decline exponentially from their peak on a time scale $t_Q$. While their high-redshift QSOs emit at close to the Eddington luminosity, the QSOs at low redshift have $L/L_{\text{Edd}} = 0.01 - 0.1$. In addition to a QSO lifetime that is constant with redshift, these authors also parametrize the lifetime to scale as the dynamical time: $t_Q(z) = t_Q(0)(1+z)^{-1.5}$, where $t_Q(0)$ is the lifetime at $z = 0$. The lifetime that best fits the evolution of the QSO population, the merger history of galaxies, and the evolution of their cold gas fraction is $t_Q(0) = 3 \times 10^7$ years (for a given merger event).

1.3 Limits on Episodic Activity

1.3.1 QSOs and Starbursts

The presence of many QSOs in merging systems and the detection of luminous hard X-ray emission from the cores of some merging galaxies strongly suggest that at least some QSOs are triggered by interactions of giant, gas-rich galaxies. The merging process removes angular momentum from a significant fraction of the two galaxies’ ISM, leading it to flow inward toward the circumnuclear region and central black hole. This angular momentum loss is a natural mechanism to ignite QSO activity, as well as significant amounts of circumnuclear star formation.

In extreme circumstances Norman & Scoville (1988) proposed that mass loss from a sufficiently large star cluster formed in such a merger may provide a continuous fuel supply for a long $t_Q$. These authors specifically explore the implications of a star cluster of mass $4 \times 10^9 M_\odot$ within the central 10 pc of the merger remnant. In $\sim 10^8$ years, mass loss from this cluster can power accretion rates of up to $10 M_\odot$ yr$^{-1}$, corresponding to QSO luminosities $L > 10^{12} L_\odot$ due to the black hole alone, and much higher than the stellar luminosity of the
central star cluster. This model therefore predicts an episodic QSO lifetime of $\sim 10^8$ years after a major merger between two massive, gas-rich galaxies.

A number of examples of so-called transition QSOs, which show evidence for recent and substantial star formation, were studied in detail by Canalizo & Stockton (2001). They derived ages for the post-starburst population as high as 300 Myr in some objects, while Brotherton et al. (1999) derived an age of 400 Myr for the post-starburst population in UN J1025-0040. If the QSO and starburst were triggered at the same time, the implied lifetimes for the QSOs are greater than a few $\times 10^8$ years.

Kawakatu, Umemura, & Mori (2003) recently considered a similar coevolution model for black holes and galaxies. They model the formation and growth of the central black hole via radiation drag on the host galaxy ISM. This process gradually increases the mass of the central black hole for $\sim 10^8$ years, during which time the galaxy may appear as an ultraluminous infrared galaxy. Once the central region is no longer obscured, the galaxy appears as an optically identifiable QSO. This evolutionary scenario corresponds to a great deal of supermassive black hole growth, which these authors propose would lead to larger velocities in the broad-line region.

### 1.3.2 Size of Narrow-Line Regions

The size of the narrow-line region (NLR) around AGNs potentially provides a straightforward, geometric estimate of their lifetime. Bennert et al. (2002) recently studied a sample of Seyferts and QSOs and found that the size of the NLR increases approximately as the square root of the $[O \text{ III}]$ luminosity and the square root of an estimate of the $H\beta$ luminosity. For a constant ionization parameter, electron density, and covering factor, the size of the ionized region should scale as the square root of the luminosity. The NLR is therefore ionization bounded and the size of the ionized region sets a lower limit to the lifetime of the ionizing source. As the most luminous QSOs in their sample have NLRs approximately 10 kpc in radius, the episodic lifetime of the most luminous QSOs must be greater than $3 \times 10^4$ yr.

### 1.3.3 Lengths of Jets

Jets are another macroscopic product of accretion onto a supermassive black hole. Their expansion time therefore sets a lower limit to the lifetime of the accretion. The main uncertainty in the use of jets to constrain the lifetime is the expansion speed, although the lengths of the jets themselves are also somewhat uncertain due to the unknown inclination angle.

The expansion speed of jets as a population can be estimated by the mean ratio of the length of the jet to the counterjet (Longair & Riley 1979), although this requires that we assume that all jets have the same expansion speed. For an expansion speed $v = \beta c$ and inclination to the line of sight $\theta$, the observed length ratio of the jet to counterjet (following Scheuer 1995) is

$$Q = (1 + \beta \cos \theta)/(1 - \beta \cos \theta). \quad (1.4)$$

If we assume that the inclination angle of the population is uniformly distributed between zero and $\theta_{\text{max}}$ degrees, the mean of $\log Q$ in the limit of small $\beta$ is
\[ \langle \ln Q \rangle = \beta (1 + \cos \theta_{\text{max}})[1 + \frac{1}{6} \beta^2 (1 + \cos^2 \theta_{\text{max}})\ldots] \]
because these QSOs exist in a neutral, predominantly lower-density IGM. Above \( z \approx 6 \), the Gunn-Peterson trough in the IGM will erase all flux blueward of \( \text{Ly} \alpha \) in the absence of ionization of the surrounding medium by the QSO, i.e. an H II region. The size of this region is set by the lifetime of the QSO as the sole ionizing source, unlike the case for the proximity effect, where the lifetime is only constrained to be the recombination time because only high density regions remain to be (re)ionized.

Haiman & Cen (2002; see also Haiman & Loeb 2001) applied this concept to the \( z = 6.28 \) QSO SDSS 1030+0524. They used the expected density distribution of the IGM with a hydrodynamic simulation (Cen & McDonald 2002) and found that a quasar lifetime of \( 2 \times 10^7 \) years and an H II region \( \sim 4.5 \) Mpc (proper) radius produced a good match to the observed flux in SDSS 1030+524 blueward of \( \text{Ly} \alpha \).

The masses of the black holes powering QSOs such as this one and others with \( z > 6 \) provide some additional information on the early evolution of QSOs. For example, one of the key conclusions of Haiman & Cen (2002) was that the \( z = 6.28 \) QSO was not significantly lensed or beamed. Therefore if the QSO is shining at or below the Eddington rate, the mass of the central black hole is on order \( 2 \times 10^9 \) \( M_\odot \). The other \( z > 6 \) QSOs discovered by SDSS have comparable luminosities, and therefore are presumably similarly massive. To form such a massive black hole at this early epoch requires that it has been accreting at the Eddington rate for nearly the lifetime of the Universe, less than \( 10^9 \) years at \( z = 6 \) assuming \( \epsilon = 0.1, L \approx L_{\text{Edd}}, \) and a \( 100 M_\odot \) seed. Depending on the mass of the seed black hole population, either the QSO lifetime must be quite long or super-Eddington accretion must occur in the early Universe.

### 1.4.2 Transverse Proximity Effect

The proximity effect can be used to set a stronger constraint on the episodic lifetime with a direct measure of its extent in the plane of the sky. This measurement could be made through observations of multiple QSOs that lie in close proximity on the sky. The proximity effect due to a lower-redshift QSO in the spectrum of higher-redshift QSO would provide a lower limit on the lifetime of the lower-redshift QSO equal to the time required to ionize the intervening neutral medium, where the ionization front expands at a speed \( v \approx c \) in the primarily low-density IGM. The angular diameter distance between the two QSOs is then a lower-limit to the lifetime. Such a measurement was first discussed by Bajtlik et al. (1988), although in practice the statistical significance of the proximity effect in neutral hydrogen of any given QSO at \( z < 6 \) is sufficiently weak that this measurement would require combining data from large numbers of QSOs to obtain a mean lower limit to the episodic lifetime.

The proximity effect in neutral hydrogen is difficult to measure because its mean opacity of the intergalactic medium is so low (at least below \( z \approx 6 \)) and the recombination times for the remaining neutral, high-density clouds are quite short. Helium offers a better chance of success as the mean opacity of neutral helium is quite high above \( z \approx 2.9 \). Individual helium ionizing sources, i.e. QSOs, above this redshift will therefore produce distinct opacity gaps. Unfortunately, relatively few He II opacity gaps are known as the He II Ly\( \alpha \) \( \lambda 304 \) \( \text{Å} \) line is still in the space UV regime at \( z \approx 3 \). Current instrumentation therefore requires them to be observed in the spectra of very bright QSOs. These systems are also easily obscured by the much more common Lyman-limit systems.

There are currently five known helium absorption systems in the spectra of four QSOs.
These systems offer potentially excellent lower limits on the episodic lifetime if the origins of these systems are identified with QSOs a significant distance from the He II system. Jakobsen et al. (2003) identified such a candidate QSO for the He II opacity gap at \( z \approx 3.056 \) in the spectrum of the \( z = 3.256 \) QSO Q0302-003. They identified a \( z = 3.050 \pm 0.003 \) QSO at the approximate redshift of the helium feature with an angular separation of 6.5′ from the line of sight to Q0302-003. If the new QSO is responsible for the helium opacity gap, then the light travel time sets a lower limit of \( t_Q > 10^7 \) yr.

The widths of all of the known He II opacity gaps are \( \Delta z \simeq 0.01 - 0.02 \), which corresponds to a size of 2 – 5 Mpc (Jakobsen et al. 2003). These sizes set a strong lower limit of \( \sim 10^6 \) years for the lifetime of the ionizing source. An alternative explanation of these opacity gaps is that they are due to low-density regions in the IGM. Heap et al. (2000) studied the opacity gap in Q0302-003 and argue against this interpretation, as their simulations cannot reproduce the distribution and amplitude of the gap with a low-density region.

### 1.4.3 Clustering

The clustering of QSOs can provide a strong constraint on their net lifetime, under the assumption that QSOs reside in the most massive dark matter halos. If QSOs are long-lived, they need only reside in the most massive, and therefore most strongly clustered, halos in order to match the observed space density of luminous QSOs. In contrast, if QSOs are short-lived phenomena, then a much larger population of host halos is required to match the observed QSO space density at the epoch of observation. As this large population of hosts will be dominated by less massive halos, due to the shape of the halo mass function, the QSO population will not be as strongly clustered. These simple arguments therefore predict that the larger the observed correlation length, the longer the lifetime of QSOs (Haehnelt et al. 1998).

Martini & Weinberg (2001) calculated the expected relation between QSO clustering and lifetime in detail in anticipation of forthcoming measurements from the 2dF and SDSS collaborations (see also Haiman & Hui 2001). The main assumptions of this model are that the luminosity of a QSO is a monotonic function of the mass of its host dark matter halo and that all sufficiently massive halos go through a QSO phase. For some absolute magnitude-limited sample of QSOs at redshift \( z \), the probability that a given halo currently hosts a QSO is simply \( t_Q/t_H \), where \( t_H \) is the lifetime of the host halo. The QSO therefore does not necessarily turn on when the halo forms, which differs slightly from the model developed by Haehnelt et al. (1998).

Calculation of the clustering corresponding to some lifetime \( t_Q \) requires knowledge of the minimum halo mass, which can be found by setting the integral of the mass function of dark matter halos, multiplied by the fraction that host QSOs, equal to the observed space density \( \Phi(z) \):

\[
\Phi(z) = \int_{M_{\text{min}}}^{\infty} dM \frac{t_Q}{t_H(M, z)} n(M, z). \tag{1.8}
\]

The halo lifetime \( t_H \) depends on both the halo mass and redshift and was calculated by solving for the median time a halo of mass \( M \) at redshift \( z \) will survive until it is incorporated into a new halo of mass \( 2M \). The mass function of dark matter halos \( n(M, z) \) is calculated with the Press-Schechter formalism:

\[ \]
The predicted clustering of QSOs depends directly on the minimum halo mass. The bias factor for halos of a given mass was calculated by Mo & White (1996) as

$$b(M,z) = 1 + \frac{1}{\delta_c} \left[ \frac{\delta_c^2(z)}{\sigma^2(M)} - 1 \right].$$  \hspace{1cm} (1.10)

The effective bias factor for the host halo population is then the integral of the bias factor for all masses greater than $M_{\text{min}}$ and weighted by the halo number density and lifetime:

$$b_{\text{eff}}(M_{\text{min}},z) = \left( \int_{M_{\text{min}}}^{\infty} dM \frac{b(M,z) n(M,z)}{t_H(M,z)} \right) \left( \int_{M_{\text{min}}}^{\infty} dM \frac{n(M,z)}{t_H(M,z)} \right)^{-1}. \hspace{1cm} (1.11)$$

The clustering amplitude of QSOs can be parametrized as the radius of a top-hat sphere in which the rms fluctuations of QSO number counts $\sigma_Q$ is unity:

$$\sigma_Q(r_1,z) = b_{\text{eff}}(M_{\text{min}},z) \sigma(r_1) D(z) = 1,$$  \hspace{1cm} (1.12)

where $\sigma(r_1)$ is the rms linear mass fluctuation in spheres of radius $r_1$ at $z = 0$ and $D(z)$ is the linear growth factor. The radius $r_1$ is similar to the correlation length $r_0$, at which the correlation function $\xi(r)$ is unity, but can be determined more robustly because it is an integrated quantity and does not require fitting $\xi(r)$.

For some specified cosmology and comoving QSO space density, $r_1$ depends directly on $t_Q$. Figure 1.3 shows the minimum mass, effective bias factor, and $r_1$ as a function of $t_Q$ for a range of cosmologies at $z = 2, 3,$ and $4$. The bottom panels, which show the relationship between $r_1$ and $t_Q$, demonstrate that the clustering length increases as a function of QSO lifetime and that the clustering length can be used to obtain a good estimate of the lifetime if the cosmological parameters are relatively well known. The difference in the relationship between $r_1$ and $t_Q$ as a function of redshift mostly reflects the evolution in the QSO space density (taken from Warren, Hewett, & Osmer 1994). QSOs are rarer at $z = 4$ than at $z = 2$ or $3$ and therefore the observed space density can be matched with a larger $M_{\text{min}}$ at the same $t_Q$. At all of these redshifts the most massive halos correspond to the masses of individual galaxy halos. At lower redshifts, the most massive halos could contain multiple galaxies and the assumption of one QSO per halo may break down. The relation between clustering and lifetime is much shallower for the $\Omega_M = 1$ models because of their smaller mass fluctuations. For these models the values of $M_{\text{min}}$ also lie out on the steep, high-mass tail of the halo mass function, where a smaller change in $M_{\text{min}}$ is required to compensate for the same change in $t_Q$.

The relationship between lifetime and clustering is sensitive to several model details, which can be used to either determine the precision of the estimated lifetime or observationally test and further refine the relation. One uncertainty lies in the definition of the halo lifetime, which was taken to be the median time before the halo was incorporated into a new halo of twice the original mass. If the definition were increased to a factor of five increase in mass (a strong upper limit), then the QSO lifetimes corresponding to the same observed $r_1$ would increase by a factor of $2 - 4$. Another potential complication is that the correlation between halo mass and QSO luminosity is not perfect. Scatter in this relation will lead to the inclusion of a larger number of less massive halos in an absolute-magnitude limited sample.
of QSOs. Because this scatter always serves to increase the number of lower-mass halos, it decreases the clustering signal from the ideal case of a strict relationship and effectively makes the clustering predictions in Figure 1.3 into lower limits for $t_Q$ for a measured $r_1$. The presence of scatter in this relation could be determined and measured observationally. In the absence of scatter, there should be a direct relationship between the clustering length and QSO luminosity at fixed redshift. The presence of scatter will flatten the predicted relation between clustering and luminosity and could be used to estimate and correct for the amount of scatter in the determination of $t_Q$.

This model also assumes that the luminosity evolution of QSOs is a step function, that is they are either on at some constant luminosity, or off. If QSOs are instead triggered by a mechanism that drives a great deal of fuel toward the center, such as a galaxy merger, this fuel supply will probably gradually diminishes over time and cause the fading of the QSO. An alternative parameterization of QSO luminosity evolution, employed by Haehnelt et al. (1998) is an exponential decay: $L(t) \propto \exp(-t/t_Q)$. Martini & Weinberg (2001) found that if this luminosity evolution is used instead of the “on-off” model, and $t_Q$ redefined to represent
Fig. 1.4. Clustering length vs. $t_Q$ for the $\Lambda$CDM model for two luminosity evolution models and three values of the space density (from Martini & Weinberg 2001; their Fig. 8).

The $e$-folding time of the QSO, there is very little change in the relation between $t_Q$ and $r_1$ (see Fig. 1.4). This figure also demonstrates the scalable nature of the model to QSO surveys with different absolute magnitude limits. A decrease in the absolute magnitude limit is equivalent to increasing the space density of QSOs and therefore the number of lower-mass halos. To first approximation, increasing the space density of the sample by a factor of 10 corresponds to a factor of 10 longer lifetime at fixed $r_1$. These results scale in a similar manner if QSOs are beamed. If QSOs are only visible through some beaming angle $f_B$, then the space density needs to be corrected by a factor $f_B^{-1}$ before the QSO lifetime can be determined from the clustering.

The 2dF and SDSS surveys will provide the best measurements of QSO clustering to date. Preliminary results from these surveys (Croom et al. 2001; Fan 2003) indicate that $r_0 \approx 6h^{-1}\text{Mpc}$, or $r_1 \approx 9h^{-1}\text{Mpc}$ for $\xi(r) = (r/r_0)^{-1.8}$. This corresponds to a lifetime of $t_Q \approx 10^6$ years. A much larger correlation length $r_0 = 17.5 \pm 7.5h^{-1}\text{Mpc}$ (Stephens et al. 1997) was measured in the Palomar Transit Grism Survey (Schneider, Schmidt, & Gunn 1994). This high correlation length may be a statistical anomaly as it is only based on three QSO pairs, but could also be a consequence of the very high luminosity threshold of the survey.
Table 1.1. *Constraints on the QSO Lifetime*

| Method                  | Net/Episodic | Lifetime [yr] |
|-------------------------|--------------|---------------|
| **Current:**            |              |               |
| Evolution               | Net          | $< 10^9$      |
| Local Black Holes       | Net          | $10^7 - 10^8$ |
| Merger Scenario         | Net          | few $\times 10^7$ |
| Mergers & Starbursts    | Episodic     | few $\times 10^6$ |
| Proximity Effect        | Episodic     | $> 10^4$      |
| Radio Jets              | Episodic     | few $\times 10^6$ |
| **Future:**             |              |               |
| QSOs at $z > 6$         | Episodic     | few $\times 10^7$? |
| Transverse Proximity Effect | Episodic   | $10^7$ ??     |
| Clustering              | Net          | $10^6 - 10^7$ ?? |

### 1.5 Summary

Current estimates of the QSO lifetime lie in the range $t_Q = 10^6 - 10^8$ years, still uncertain by several orders of magnitude. Estimates of the net lifetime are primarily demographic and rely on assumptions about the radiative efficiency of accretion $\epsilon$ and the luminosity of QSOs relative to the Eddington rate $L/L_{Edd}$. The new models developed in the last few years (Kauffmann & Haehnelt 2000; Yu & Tremaine 2002) have started to relax the common assumptions about the values of $\epsilon$ and $L/L_{Edd}$, but a complex relationship between $\epsilon$, $L/L_{Edd}$, and $M_\bullet$ has by no means been ruled out.

While the physics of the accretion process and the potential for significant obscured black hole growth may appear daunting obstacles, there are reasons for optimism. Demographic estimates of the net lifetime will be substantially improved as more supermassive black hole masses are measured and the validity of the $M_\bullet - \sigma$ relation better established, including its application to active galaxies. Measurement of the black hole mass in more QSOs via reverberation mapping (e.g., Barth 2003), or estimates using the $M_\bullet - \sigma$ method will lead to direct estimates of $L/L_{Edd}$ for many QSOs. These steps will help to eliminate some of the main uncertainties in the value or range of $L/L_{Edd}$ and the local mass density of supermassive black holes. Finally, measurement of a significant number of redshifts for the sources that comprise the hard X-ray and sub-mm background will greatly improve the integral constraint provided by the net luminosity from accretion. Many of these new observations could be realized in the next several years and reduce the need for sweeping assumptions about $\epsilon$ or $L/L_{Edd}$.

The current best estimates or limits for the episodic lifetime are consistent with a wider range of $t_Q$ than the net lifetime, but constraints from the proximity effect in the Ly$\alpha$ forest do point to $t_Q > 10^4$ years. Observations of QSOs when the opacities of neutral helium and hydrogen was much higher suggest much longer episodic lifetimes, although to date are still only based on observations of two QSOs. Constraints such as these on the episodic lifetime provide a valuable complement to the primarily demographic estimates of the net
lifetime, as they do not depend on assumptions of accretion physics, but instead on much more straightforward radiation physics.

The next few years offer the hope of tremendous progress in measurement of the QSO lifetime via a variety of techniques. The discovery of additional QSOs associated with He II absorption systems (and hopefully more of these systems), as well as additional QSOs at high redshift \( z > 6 \), could provide quite strong lower limits to the episodic lifetime, while clustering measurements hold great promise to constrain the net lifetime. As the clustering method does not depend on assumptions about accretion physics, it will provide a valuable complement to the present demographic approaches.

This potential progress suggests that \( t_Q \) could be determined to within a factor of 3 in the next three years, which is sufficient precision to address some fundamental questions on the growth of black holes and the physics of the accretion process. One important advance will be to determine if the QSO lifetime determined via clustering is in agreement with that estimated via black hole demographics. As these models employ an independent set of assumptions, these measurements could provide strong evidence that \( \epsilon \approx 0.1 \) and \( L/L_{\text{Edd}} \approx 1 \) are valid choices.

Measurement of \( t_Q \) will also determine if a luminous QSO phase was the dominant growth mechanism for present-day supermassive black holes. If \( t_Q > 4 \times 10^7 \) yr, then a substantial fraction of the black hole mass in QSO hosts was accreted via this optically luminous mechanism. However, such a long lifetime would also imply that QSOs were quite rare phenomena and only a small fraction of present-day supermassive black holes were created as QSOs. The remainder may have been identifiable as AGN, or were heavily obscured, but would not have been classified as optically luminous QSOs.

If instead \( t_Q \) is shorter than the Salpeter time scale, then hosting a short-lived QSO phase was a relatively common occurrence. However, only a small fraction of the present-day black hole mass was accreted during this time and the remainder must have occurred via a less luminous, or less radiatively efficient, mode of accretion, either before or after the QSO epoch. This could then imply that advection-dominated accretion (e.g., Narayan, Mahadevan, & Quataert 1998) or inflow-outflow solutions (Blandford & Begelman 1999) played a significant role in black hole growth.

I would like to thank Zoltán Haiman and David Weinberg for helpful comments on this manuscript. I am also grateful to Luis Ho for inviting and encouraging me to prepare this review.

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