Materials Research Express

PAPER

Dual index properties of photonic crystal and its application in subwavelength focusing

Bingming Liang, Jing Ji, Dawei Tang, Yan Huang and Xiao Huang

Shanghai Key Laboratory of Modern Optical System, School of Optical-Electrical and Computer Engineering, University of Shanghai for Science and Technology, Shanghai 200093, People’s Republic of China

E-mail: liangbming@ sina.com

Keywords: dual index, photonic crystal, sub wavelength

Abstract

Based on the two-dimensional dielectric wedge-shaped photonic crystal, we studied the existence of two kinds of equivalent refractive indexes (negative refractive index is $-1$, positive refractive index is 1.897) in the photonic crystal by means of fast Fourier transform (FFT), as well as its influence on the optical propagation characteristics was discussed. By adjusting the wedge angle of the photonic crystal to eliminate the influence of positive refraction beam on the imaging, the sub wavelength focusing with half-width less than half of a wavelength can be realized. The numerical results show that the Full Width Half Maximum (FWHM) of the image point can reach a minimum of $0.41 \lambda$ when the wedge angle of the photonic crystal is $73^\circ$.

1. Introduction

Photonic crystal has attracted many attentions from the academics due to its special properties such as autocollimation effect [1], slow light effect [2, 3], anomalous Doppler effect [4, 5] and negative refraction effect [6, 7]. Also it has broad application prospects in a series of light control fields such as imaging, super lens, optical switch, stealth, and frequency-selective surface. As early as 2006, Schurig et al designed a cylindrical invisibility cloak with special electromagnetic parameter distribution by using the theory of transform optics [8]. Liu et al used the curved surface structure of Ag/Al$_2$O$_3$ multilayer hyperbolic material to make a super lens, which broke through the traditional diffraction limit of imaging, and realized the sub wavelength resolution imaging with a wavelength of 365 nm [9]. Lyubov demonstrated that photonic crystal allows lensless focusing by complex wavefront shaping [10]. Moreover, the sub wavelength imaging can be realized by designing the structure of the photonic crystal based on its negative refraction characteristics [11]. Xia et al verified the negative refraction and sub wavelength imaging ability of two-dimensional ring photonic crystal with hexagonal lattice [12]. Ma achieved sub wavelength imaging with a minimum FWHM of $0.44 \lambda$ by using a silicon based photonic crystal [13].

For two-dimensional photonic crystals, in addition to the negative refraction effect, birefringent optical phenomena also exist besides the negative refraction effect [14, 15]. In 2005, Xian et al designed a two-dimensional photonic crystal structure of metal dielectric triangular lattice, and successfully realized a plate-shaped and wedge-shaped polarization double beam separator [16]. The use of a birefringent graded photonic crystal (GPhC) is proposed for the realization of an efficient polarization beam splitter by Cassan [17]. The two-dimensional square photonic crystal designed by Derby et al realizes the dual focus imaging of the photonic crystal plate, and proposes a new super lens with high resolution and more images [18]. Walid Aroua et al designed a mode conversion optical isolator using two-dimensional cellular lattice photonic crystal structure and birefringence phenomenon, which realized the function of optical isolation [19, 20]. Jiang et al studied the existence of both normal and inverse doppler effects in one two-dimensional wedge-type photonic crystals, proposed the double Doppler effect, and verified this phenomenon [21]. Kim proposed an ultracompact high-efficiency polarizing beam splitter based on a hybrid photonic crystal and a conventional waveguide structure [22]. The polarizing beam splitter separates TM- and TE-polarized modes into orthogonal output waveguides to
achieve light splitting. We design a beam splitter based on a double refractive indexes photonic crystal. By changing the incident angle of the beam, the ratio of the normalized energy intensity of the negative refraction output light and the normalized energy intensity of the positive refraction output light (the splitting ratio) is from 1 to infinite, which is simple and efficient to achieve the beam splitting function.

Different from the researches on the imaging of single index photonic crystal [23, 24], this paper probes into the imaging characteristics of dual index photonic crystal materials. We have realized the sub wavelength focusing by using the silicon dielectric cylindrical photonic crystal with two refractive indexes. Zhang et al made use of the super–convergence of the immersion lens in single refractive index photonic crystal, which makes the far-field super-resolution focal area beyond the diffraction limit of traditional optics [25]. In this paper, in the double equivalent refractive indexes photonic crystal structure (where the negative refractive index is $-1$), by changing the wedge angle of the photonic crystal structure, it breaks the traditional imaging diffraction limit, improves the imaging resolution and realizes sub wavelength focusing. Because the dielectric cylinder photonic crystal is easy to process and has a deep processing depth, many researchers used the dielectric column photonic crystal as the experimental scheme [21, 26, 27]. The design scheme in this paper provides a certain reference value for the realization of super-resolution imaging of dielectric cylindrical photonic crystal.

2. Design and simulation

This research uses air as background, as shown in the upper right corner of figure 1 (a), a photonic crystal with a two-dimensional hexagonal lattice structure of silicon dielectric pillars, in which the radius of the dielectric pillar is $r = 0.413 \times a$, the relative dielectric column diameter $b = 2r/a$, the dielectric constant is $\varepsilon = 11.69$, the refractive index is $n = 3.42$, and the lattice constant is $a = 615$ nm. Rsoft is used to simulate the propagation process of light sources at different incidence angles with normalized frequency $f = 0.3968$ in TE-polarized band by wedge-shaped two-dimensional photonic crystals with wedge angle of $30^\circ$, as shown in figure 1. It is clearly shown in the figure that under the same polarization state, both the positive and negative refraction

![Figure 1](image)
phenomena will occur when the incident beams pass through the photonic crystal at different angles. Snell’s theorem is satisfied when incident at different angles: \( n_1 \sin \theta_1 = n_2 \sin \theta_2 \). We specify that the incident angle on the left side of the normal is positive and the incident angle on the right side of the normal is negative. Compared with the three pictures in figures 1(a)–(c), it can be found that with the incidence angle changing from 0° to −10°, the positive refraction beam on the exit surface increases, while the negative refraction beam weakens. The main reason for this phenomenon is that when the equivalent refractive index of the material is positive or negative, the reflectivity of the incident light passing through the two interfaces at different angles is also different. As shown in figure 1(d), the propagation characteristics of the incident light at different angles are shown. When the incident angle is from —14° to 18°, the normalized energy intensity of the positive refraction light beam is from 0.7 to 0, and it is a continuous attenuation. From 8° to 18°, the ratio of the normalized energy intensity of the negative refraction beam to the normalized energy intensity of the positive refraction beam is from 1 to infinity. This characteristic can be used to design beam splitter.

In order to explore the mechanism of the phenomenon, the amplitude data in the vertical direction of optical transmission as shown in figure 2(a) was processed, added up and averaged by the finite-difference time-domain (FDTD) method. The periodic distribution of optical field along the one-dimensional average amplitude distribution in the direction of optical transmission that is obtained is shown in figure 2(b).

In figure 2(b), the periodicity of the electric field distribution in the photonic crystal region is relatively complex. Therefore, the FFT is applied to the one-dimensional electric field distribution, and the corresponding spectrum is shown in figure 2(c). In this paper, the FFT method is used to deal with the electric field transmitted in the photonic crystal. According to the formula

\[
    n_i = \frac{\lambda_i}{\lambda_0} = \frac{f_i}{f_0}
\]

The \( f_i \) is the spatial frequency of light in vacuum, and the \( f_i \) is the frequency of each electric field component in the photonic crystal. The refractive index of the photonic crystal of this structure is calculated as \( n_1 = -1.0541 \), and \( n_2 = 1.897 \). In other words, when the normalized frequency \( f = 0.3968 \) of the incident light source, the photonic crystal of this structure has two refractive indexes at the same time.

Due to the influence of wavelength shift and processing width, in order to get a negative refractive index closer to −1, the following research has been done. Firstly, the influence of different relative dielectric column diameters \( b \) on refractive index is considered. In this paper, the parallel light with \( \lambda = 1.55 \mu m \) is incident on the photonic crystal plate, and the relative dielectric column diameter \( b \) is adjusted within the range of 0.808 \( \mu m \) and 0.840 \( \mu m \). The change of the refractive indexes is shown in figure 3(a). Figure 3(a) shows that without changing the incident light wave length, the positive refractive index of the photonic crystal increases with the increase of the relative dielectric column diameter \( b \), while the negative refractive index decreases with the increase of the relative dielectric column diameter \( b \). When \( b = 0.826 \mu m, n_1 = -1.0541 \), the negative refractive index closest to −1. On this basis, this paper also discusses the effect of wavelength on the refractive indexes. If you adjust the value of the wavelength \( \lambda = 1.55 \mu m \), the correspondingly change of the refractive indexes is shown in figure 3(b). The data shows that when the relative dielectric column diameter \( b \) is kept constant at \( b = 0.826 \mu m \), and only the wavelength is changed as a single parameter, the positive refractive
index of the photonic crystal decreases with the increase of wavelength, while the negative refractive index also decreases with the increase of wavelength. Moreover, when $\lambda = 1.55 \, \mu m$, the negative refractive index of the photonic crystal is closest to $-1$. To sum up, when $\lambda = 1.55 \, \mu m$, $b = 0.826 \, \mu m$, the negative refractive index of the photonic crystal is closest to $-1$. According to Lou, photonic crystals with negative refraction effect can achieve sub wavelength imaging. And the closer to $-1$ the equivalent refractive index is, the better the sub wavelength imaging effect is [28]. Therefore, to obtain better sub wavelength imaging effect, the experiments of photonic crystal are carried out under this parameter.

According to the wedge angle in the structure in figure 1, the imaging of point light source is simulated in the optical path simulation software of Rsoft. FDTD is used to simulate the electric field distribution of photonic crystal imaging as shown in figure 4(a). The simulation results show that when the wedge angle of the photonic crystal is $30^\circ$, there exist no obvious image points, and much stray light show up. Through theoretical analysis, it is concluded that the main reason why the photonic crystal under this structure cannot achieve imaging is that the positive refraction beam and the negative refraction beam exist at the same time, and therefore interference happens. In order to eliminate the impact of positively refracted light beams on imaging and achieve sub wavelength imaging, the following research was done. As shown in figure 4(b), $\alpha$ can be any wedge angle of the photonic crystal structure. If the incident light enters the photonic crystal at the angle of $\beta_1$ at any angle, the first positive refraction angle passing through the lower surface of the photonic crystal is $\beta_2$, and this refracted light is used as the second incident light. The incident angle is $\beta_3$. To ensure the positive refracted beam of this incident

![Figure 3](image1.png)  
**Figure 3.** (a) The change of refractive indexes when the relative dielectric column diameter $b$ is between $0.808 \, \mu m$ and $0.84 \, \mu m$; (b) when $b = 0.826 \, \mu m$, the change of refractive indexes when the wavelength $\lambda$ is between $1.53 \, \mu m$ and $1.57 \, \mu m$.

![Figure 4](image2.png)  
**Figure 4.** (a) FDTD was used to simulate the electric field distribution of the image of the point light source through a wedge photonic crystal with a wedge angle of $30^\circ$; (b) schematic diagram of eliminating refracted beams.
beam cannot be ejected from the outgoing surface, the total reflection principle should be adopted. The total reflection angle that happens to produce total reflection is called $\beta_3$. When the total reflection condition is satisfied, all angles of positively refracted beam can be eliminated.

In the case of the negative equivalent refractive index of the photonic crystal being close to $-1$, the positive equivalent refractive index is $n_2 = 1.897$. According to the total reflection formula,

$$n_0 \sin 90^\circ = n_2 \sin \beta_3$$

the critical angle of total reflection angle is $\beta_3 \approx 31.8^\circ$.

It can be seen from figure 1(d) that when the incident angle is negative, the normalized energy intensity of the outgoing light of the positive refraction light beam is far greater than that of the negative refraction light beam, while when the incident angle is positive, it is the opposite. In order to achieve sub wavelength focusing, it is necessary to eliminate the positive refraction beam on the exit surface. When the beam is incident from the right side ($\beta_1 = 90^\circ$), the incident beam is least prone to total reflection. In the following, the critical angle of total reflection on the emitting surface will be calculated by taking ($\beta_1 = 90^\circ$) as an example, and then the minimum wedge angle of the photonic crystal structure will be obtained. At the same time, with the beam entering the photonic crystal from the air, the positive refraction beam at the incident plane refracts according to Snell’s theorem

$$n_0 \sin 90^\circ = n_2 \sin \beta_2$$

In order to make the total reflection of the positive refraction beam on the exit surface, according to the total reflection theorem

$$n_2 \sin \beta_3 = n_0 \sin 90^\circ$$

Combined with formula (3) and (4), it can be achieved that

$$\beta_2 = \beta_3$$

According to the angle relationship analysis in figure 5, it is concluded that

$$\alpha = \beta_2 + \beta_3 = 2\beta_3$$

Therefore, it can be concluded that the minimum angle of $\alpha$ is $2\beta_3 \approx 63.6^\circ$. When $\alpha \geq 63.6^\circ$, the positive refraction beam with $\beta_1 = 90^\circ$ is eliminated. At the same time, the positive refraction beam of the incident plane at any angle is eliminated because of total reflection.

In the design of photonic crystal structure, the influence of wedge angle on sub wavelength imaging is studied according to $\alpha \geq 63.6^\circ$. In the actual simulation, as shown in figure 5, when $\alpha = 71^\circ$, the incident angle is $-10^\circ$, only the light with the equivalent refractive index of $-1$ is emitted. In addition, the simulation of the incident angle from $90^\circ$ to $-90^\circ$ has been carried out, and the positive refraction light beams have been eliminated. Therefore, when the wedge angle of the photonic crystal is $71^\circ$, only the light with an equivalent negative refractive index of $-1$ participates in the imaging.

Figure 5. When $\alpha = 71^\circ$, the propagation diagram of the light with an incident angle of $-10^\circ$ passing through the photonic crystal along the direction of $\Gamma M$. 

---

**Mater. Res. Express** 8 (2021) 015902 B Liang et al
When the coordinate of the top angle on the surface of the photonic crystal structure is \((-17.28 \mu m, -12.87 \mu m)\), and the coordinate of the light source is \((-14.7 \mu m, -15.5 \mu m)\), seven sets of simulation experiments were carried out, in which the beam value was \(71^\circ, 72^\circ, 73^\circ, 74^\circ, 75^\circ, 76^\circ\) and \(77^\circ\), and the FWHM was shown in figure 6(a). The data show that with the increase of angle, the half-width decreases at first and then increases. When \(\alpha = 73^\circ\), the minimum FWHM is 0.41 \(\lambda\). The theoretical analysis proved that the main reason for the increase of half-width of image point after reaching the minimum is that the increase of angle leads to the more reflection of negative refraction beams at the exit surface, thus reducing the number of beams participating in negative refraction imaging. When \(\alpha = 73^\circ\), FDTD was used to simulate the electric field distribution of photonic crystal imaging, as shown in figure 6(b) and the FWHM of the corresponding image point, as shown in figure 6(c).

3. Conclusion

In conclusion, this paper, based on the silicon dielectric cylindrical photonic crystal, finds that the photonic crystal has both positive and negative refractive indexes through the fast Fourier transform (FFT). And the sub wavelength focusing properties of a photonic crystal with a negative refraction equivalent index of about \(-1\) are studied. In order to eliminate the influence of positive refraction, a wedge angle of \(\alpha = 73^\circ\) was cut out, and a sub wavelength focusing with a minimum FWHM of 0.41 \(\lambda\) has been obtained under this condition. This paper provides both a new idea for the sub wavelength imaging of the silicon dielectric cylindrical photonic crystal with two refractive indexes, and a reference for the application research of the multiple refractive indexes photonic crystal device.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

Funding

National Natural Science Foundation of China (61177043); Science and Technology Commission of Shanghai Municipality (17590750300).

Disclosures

The authors declare no conflicts of interest.

ORCID iDs

Bingming Liang @ https://orcid.org/0000-0003-2833-5278
References

[1] Rakich P T et al 2006 Achieving centimetre-scale supercollimation in a large-area two-dimensional photonic crystal Nature Mater 5 93–6
[2] Zhang G et al 2004 Phase-coupling-induced ultraslow light propagation in solids at room temperature Phys. Rev. Lett. 93 133903
[3] Yoshimi H, Yamaguchi T, Ota Y, Arakawa Y and Iwamoto S 2020 slow light waveguides in topological valley photonic crystals Opt. Lett. 45 36–40
[4] Reed E J, Solačić M and Joannopoulos J D 2003 Reversed Doppler effect in photonic crystals Phys. Rev. Lett. 91 133901
[5] Seddon N and Bearpark T 2003 Observation of the inverse Doppler effect Science 302 1537–40
[6] Kosaka H et al 1998 Superprism phenomena in photonic crystals Phys. Rev. B 58 10096
[7] Kosaka H et al 1999 Superprism phenomena in photonic crystals: toward microscale lightwave circuits J. Lightwave Technol. 17 2032–8
[8] Schurig D et al 2006 Metamaterial electromagnetic cloak at microwave frequencies Science 314 977–80
[9] Liu Z et al 2007 Far-field optical hyperlens magnifying sub-diffraction-limited objects Science 315 1686
[10] Lyubov V A et al 2016 High-resolution wavefront shaping with a photonic crystal fiber for multimode fiber imaging Opt. Lett. 41 497–500
[11] Li Z Y and Lin J L 2003 Evaluation of lensing in photonic crystal slabs exhibiting negative refraction Phys. Rev. B 68 245110
[12] Xia F et al 2013 Negative refraction and subwavelength imaging in a hexagonal two-dimensional annular photonic crystal J. Appl. Phys. 113 013109
[13] Ma H L, Liang B M, Zhuang S L, Chen J B and Niu J K 2017 Study on focusing of subwavelength imaging of a point source based on two-dimensional photonic crystals Opt. Lett. 42 4012–5
[14] Michael F et al 2000 Giant birefringent optics in multilayer polymer mirrors Science 287 2451–3
[15] Luo Y et al 2004 Wide-angle beam splitting by use of positive-negative refraction in photonic crystals Opt. Lett. 29 2920–2
[16] Ao X and He S 2005 Polarization beam splitters based on a two-dimensional photonic crystal of negative refraction Opt. Lett. 30 2152–4
[17] Cassan E et al 2013 Polarization beam splitting using a birefringent graded photonic crystal Opt. Lett. 38 459–61
[18] Aroua W et al 2014 Mode converter optical isolator based on dual negative refraction photonic crystal IEEE. J. Quantum Elect. 50 633–8
[19] Derbali J and AbdeMalek F 2015 Dual negative refraction in a two-dimension square photonic crystal Opt. Commun. 350 213–6
[20] Cao H et al 2015 Broadband optical isolator based on helical metamaterials J. Opt. Soc. Am. A 32 778–81
[21] Jiang Q et al 2018 Dual Doppler Effect in Wedge-Type Photonic Crystals Sci. Rep. 8 6527
[22] Kim S et al 2003 Dual doppler effect in wedge-type photonic crystals Opt. Lett. 28 2384
[23] Lei Y et al 2019 Subwavelength focusing by combining negative-refractive photonic crystal and silicon lens Opt. Mater. Express 9 3962–7
[24] Ma H et al 2016 Subwavelength imaging of a point source based on two-dimensional photonic crystals Opt. Lett. 41 3833–5
[25] Zhang Q, Li X and Gu M 2013 Super-resolution focal area induced by super-convergence in a photonic crystal immersion lens J. Opt. 15 075102
[26] Gilarlue M M and HadiBadri S 2019 Photonic crystal waveguide crossing based on transformation optics Opt. Commun. 450 308–15
[27] HadiBadri S and Gilarlue M M 2020 Coupling Si3N4 waveguide to SOI waveguide using transformation optics Opt. Commun. 460 125089
[28] Luo C, Johnson S G, Joannopoulos J D and Pendry J B 2003 Subwavelength imaging in photonic crystals Phys. Rev. B 68 045115