QCD Sum Rules: Isospin Symmetry Breakings in Pion-Nucleon Couplings

W-Y. P. Hwang,\textsuperscript{1,2} Ze-sen Yang\textsuperscript{3}, Y.S. Zhong,\textsuperscript{3} Z. N. Zhou,\textsuperscript{3} and Shi-lin Zhu\textsuperscript{3}

\textsuperscript{1}Department of Physics, National Taiwan University, Taipei, Taiwan 10764
\textsuperscript{2}Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
\textsuperscript{3}Department of Physics, Peking University, Beijing 100871, China

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Abstract

We use the method of QCD sum rules in the presence of an external pion field to investigate isospin symmetry breakings in pion-nucleon couplings. Possible manifestations of such isospin symmetry breaking are examined in the context of low-energy nucleon-nucleon ($NN$) scattering. We discuss numerical results in relation to both the existing data and other theoretical predictions.

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I. Introduction

Isospin symmetry and its possible violations constitute an integral part of nuclear physics [1], not only in the sense that good symmetry offers an effective classification of nuclear or hadronic structure but also in the proximity of the isospin symmetry concept to other fundamental ideas such as chiral symmetry. In general, isospin symmetry breaking effects observed in nuclear physics may arise from different sources such as, in addition to the well known electromagnetic interactions, strong interactions but augmented with hadron mass differences (such as the $n - p$ or $\pi^\pm - \pi^0$ mass difference) or with different meson-nucleon couplings (such as non-universal $\pi NN$ couplings) or with isospin mixings (such as $\rho - \omega$ or $\pi - \eta$ mixing). Determination of isospin symmetry breaking effects requires information on quantities which may not be directly accessible from experiments, such as the meson-nucleon couplings or the isospin mixing parameters. Accordingly, reliable theoretical predictions become indispensable for advancing our understanding of the problem.

Unfortunately, it appears that a quantitative treatment of isospin symmetry breakings may involve the capability of treating strong interactions at a quantitative level or, in other words, the ability to handle the complications due to quantum chromodynamics (QCD) in a reliable manner. To some extent, the method of QCD sum rules, as proposed originally by Shifman, Vainshtein, and Zakharov [2] and adopted, or extended, by many others [3, 5, 6], incorporates the nonperturbative QCD effects through various condensates (associated with the nontrivial QCD vacuum), thereby offering some hope of being able to treat strong interactions in a quantitative manner.

In this work, we wish to focus on the determination of isospin symmetry breakings in pion-nucleon couplings, with some emphasis on possible manifestations of such symmetry breaking in nucleon-nucleon scattering. As the present effort is based primarily on the method of QCD sum rules, it differs much from the quark-model calculations [4, 8]; as we shall see, this work is considerably more extensive than a recent work [9] also making use of the QCD sum rule method. Specifically, we wish to adopt a method in which the pion
field \( \pi \) is treated as the external field. The idea of treating the pion as an external field was first suggested in \[3\] and has not been used in any extensive manner until recently \[10, 11\].

II. Isospin Symmetry Breakings in Pion-Nucleon Couplings

In the method of QCD sum rules in the presence of an external field, we attempt to evaluate, both at the quark and hadron levels, the correlation function specified by

\[
\Pi(p) \equiv i \int d^4 x e^{i p \cdot x} \langle 0 | T(\eta_p(x)\bar{\eta}_p(0)|0 \rangle |_\pi, \tag{1}
\]

where we have

\[
\eta_p(x) = \epsilon^{abc} \{ u^{aT}(x) C \gamma_\mu u^b(x) \} \gamma^5 \gamma^\mu d^c(x)
\]

\[
\bar{\eta}_p(y) = \epsilon^{abc} \{ \bar{u}^b(y) \gamma_\nu C \bar{u}^{aT}(y) \} \bar{d}^c(y) \gamma^\nu \gamma^5. \tag{2}
\]

By evaluating the appropriate correlation function in the presence of the external pion field, we may determine the pion-nucleon couplings \( g_{\pi^0 pp}, g_{\pi^0 nn}, \) and \( g_{\pi^+ pn} \). An alternative method is to evaluate correlation functions with \( T \) product of currents sandwiched between the vacuum and the one-pion state, as first suggested by \[12\] and recently adopted by some authors \[13\]. It appears that the induced condensates involved in the external field method are now well understood and it becomes relatively straightforward to perform calculations to higher dimensions (as required for reliable predictions). This is what we choose to use in this paper and we use it to obtain isospin symmetry breakings in \( \pi NN \) couplings.

The above correlation function allows us to determine the \( \pi^0 pp \) coupling. At the quark level, we have

\[
\langle 0 | T\eta_p(x)\bar{\eta}_p(0)|0 \rangle |_{\pi^0} = 2i \epsilon^{abc} \epsilon^{a'b'c'} \text{Tr} \{ S_{a'}^b(x) \gamma_\mu [S_{a''}^{a'}(x)]^T C \gamma_\mu \} \gamma_5 \gamma_\mu S_{d'}^{c'}(x) \gamma_\nu \gamma_5. \tag{3}
\]
Here the quark propagator is given by

$$S(x) = S^{(0)}(x) + S^{(\pi)}(x),$$

with

$$iS^{(0)ab}(x) = \frac{i\delta^{ab}}{2\pi^2 x^2} \hat{x} + \frac{i}{32\pi^2 x^2} \frac{\lambda^a}{2} g_c G^a_{\mu\nu}(\sigma^{\mu\nu} \hat{x} + \hat{x} \sigma^{\mu\nu}) - \frac{i\delta^{ab}}{12} \langle \bar{q} q \rangle$$

$$+ \frac{\delta^{ab}}{192} \langle g_c \bar{q} \sigma \cdot G q \rangle - \frac{m_q \delta^{ab}}{4\pi^2 x^2} + \frac{m_q \delta^{ab}}{24} G^a_{\mu\nu} \sigma^{\mu\nu} \ln(-x^2)$$

$$- \frac{\delta^{ab}}{2^9 \times 3\pi^2} m_q x^2 \ln(-x^2) + \frac{i\delta^{ab} m_q}{48} \langle \bar{q} q \rangle \hat{x} - \frac{1}{2^7 \times 3\pi^2} i m_q \langle g_c \bar{q} \sigma \cdot G q \rangle \delta^{ab} x^2,$$

and

$$iS^{(\pi)ab}(x) = -\frac{i\delta^{ab}}{4\pi^2 x^2} g_q \vec{\tau} \cdot \vec{\tau} \gamma_5 + \frac{i\delta^{ab}}{24} g_q \vec{\tau} \cdot \vec{\tau} \gamma_5 \chi \langle \bar{q} q \rangle$$

$$- \frac{i\delta^{ab}}{384} m_0^5 g_q \vec{\tau} \cdot \vec{\tau} \gamma_5 \langle \bar{q} q \rangle x^2 + \frac{1}{2^6 \pi^2} g_q \vec{\tau} \cdot \vec{\tau} \gamma_5 \frac{\lambda^a}{2} G^a_{\mu\nu} \sigma^{\mu\nu} \ln(-x^2)$$

$$- \frac{i\delta^{ab}}{2^9 \times 3\pi^2} g_q \vec{\tau} \cdot \vec{\tau} \gamma_5 (g_2^a G^2) x^2 \ln(-x^2) - \frac{\delta^{ab}}{48} g_q \vec{\tau} \cdot \vec{\tau} \gamma_5 \langle \bar{q} q \rangle \hat{x},$$

where we have introduced $\hat{x} \equiv x^\mu \gamma^\mu$, $\langle \bar{\psi}(0) i \gamma^5 \tau^j \psi(0) \rangle \equiv g_q \chi \gamma^j < \bar{q} q >$, and $< \bar{\psi}(0) i \gamma^5 \tau^j g_c \sigma \cdot G \psi(0) > \equiv g_q m_0^5 \tau^j < \bar{q} q >$ with $\psi$ the isospin doublet consisting of $u$ and $d$ quarks. The various terms in the quark propagators $iS^{(0)ab}(x)$ and $iS^{(\pi)ab}(x)$ may be represented pictorially by the diagrams shown in Figs. 1 and 2, respectively. We wish to stress that we have in fact performed our calculations in momentum space especially when ambiguities arise and, in addition, special attention has been directed to adoption of the $\gamma_5$ in $d$-dimensions in relation to dimensional regularization.

To incorporate isospin symmetry violations, we note that $< \bar{q} q >$ appearing in the quark propagator needs to be interpreted properly; in particular, it is to be understood as $< \bar{u} u >$ in the $(uu)$ channel, or as $< \bar{d} d >$ in the $(dd)$ channel, or as $(< \bar{u} u > + < \bar{d} d >)/2$ for the $(d \to u)$ or $(u \to d)$ propagation. We also note that Fig. 2(f) has been drawn in a way to indicate that the choice of $q$ in $< \bar{q} q >$ should follow the flavor of the quark connecting to $x = 0$ (on the right hand side of the diagram) — a result following usage of the equation of motion. No potential ambiguity should arise for all the other diagrams shown in Figs. 1 and 2. The assumption has also been made that, once we have made the suitable interpretation of $< \bar{q} q >$, $\chi$ and $m_0^5$ are considered to be universal. In other
words, we refrain from introducing, at the quark level, further unknown sources for isospin symmetry violations; our assumption regarding the universal nature of the susceptibilities $\chi$ and $m_0^{\pi}$ is further supported by the derivation of induced condensates as illustrated in [1]. This is a crucial point since the terms in $\chi < \bar{u}u >$ and $\chi \bar{d}d >$ turn out to be one of the most important contributions which distinguish the coupling $g_{\pi^0 pp}$ from $g_{\pi^0 nn}$.

We note that, for the determination of $\pi^- pn$ or $\pi^+ np$ coupling, we find

\begin{equation}
\langle 0 | T \eta_p(x) \bar{\eta}_n(0) | 0 \rangle_{\pi^-} = -4i e^{abc} e^{a'b'c'} \gamma_\mu \gamma_5 S^{aa'}_d(x) \gamma_\nu C[S^{b'b'}_d(x)]^T C \gamma_\mu S^{cc'}_u(x) \gamma_5 \gamma_\nu. \tag{7}
\end{equation}

We note the absence of $i S^{(0)}(x)$ in the present case, due to a change in the electric charge. We also note that the structure (which does not involve the trace of some product) seems quite different from the $\pi^0 pp$ case, but we will see that isospin symmetry is indeed there should we assume $m_u = m_d$ and $< \bar{u}u > = < \bar{d}d >$. Finally, to obtain QCD sum rules up to dimension eight (as compared to the leading perturbative term), we need to enumerate diagrams which are proportional to 1, $\chi \langle \bar{q}q \rangle$, $\langle g_2^2 G^2 \rangle$, $m_0^{\pi} \langle \bar{q}q \rangle$, $\langle \bar{q}q \rangle^2$, $\langle \bar{q}q \rangle \langle g_\pi \bar{q} \sigma \cdot Gq \rangle$, $m_q \langle \bar{q}q \rangle$, and $m_q \langle \bar{q}g \bar{c} \cdot Gq \rangle$. Here it is clearly of importance to be able to distinguish between $< \bar{d} \hat{O} d >$ and $< \bar{u} \hat{O} u >$ in the quark propagators used to evaluate the correlation functions.

On the other hand, we may parametrize the correlation functions at the hadronic level in the standard manner.

\begin{equation}
\int d^4 x \langle 0 | T \eta_p(x) \bar{\eta}_n(0) | 0 \rangle_{\pi^-} = \lambda_n \lambda_p \frac{i}{p - m_p} \left( -\gamma_\mu \gamma_5 \sqrt{2} g_{\pi^- np} \right) \frac{i}{\bar{p} - m_n} + \cdots, \tag{8}
\end{equation}

\begin{equation}
\int d^4 x \langle 0 | T \eta_p(x) \bar{\eta}_p(0) | 0 \rangle_{\pi^0} = \lambda_p^2 \frac{i}{p - m_p} + \lambda_p^2 \frac{i}{\bar{p} - m_p} \left( -\gamma_\mu g_{\pi^0 pp} \right) \frac{i}{\bar{p} - m_p} + \cdots. \tag{9}
\end{equation}

We consider the various diagrams shown in Fig. 3 and obtain the quark-level expression (referred to as “l.h.s.”) for the $\pi^0 pp$ coupling (directly from the momentum-space
with \( a_q \equiv -(2\pi)^2 \langle \bar{q}q \rangle \), \( \langle \bar{q}q \cdot G \rangle \equiv -m_0^2 \langle \bar{q}q \rangle \) and \( b \equiv g_\pi^2 G^2 \). Accordingly, we obtain, upon Borel transform, the following QCD sum rule for the \( \pi^0 pp \) coupling:

\[
M^6 - \frac{3}{4}a_d M^4 - \frac{11}{24}b M^2 + \frac{4}{3}a_u^2 + \frac{2}{3}a_u^2 \frac{1}{M^2} + m_0^2 \ln \frac{\mu^2}{\Lambda^2} \left[ (a_u m_d + a_d m_u) - \frac{1}{4} m_u a_u \right] + m_0^2 \left[ \frac{5}{12} m_u a_u - \frac{1}{8} m_d a_d + \frac{115}{48} m_d a_u + \frac{115}{48} m_u a_d \right] = \frac{g_{\pi^0 pp}^2}{g_{\pi^0 dd}^2} \tilde{\lambda}_\rho^2 e^{\frac{m_0^2}{\mu^2}} + \text{excited states + continuum},
\]

where \( M \) is the Borel mass and \( \tilde{\lambda}_\rho^2 = (2\pi)^4 \lambda_\rho^2 \). A similar expression can be obtained for the \( \pi^0 nn \) coupling through the replacement \( u \leftrightarrow d \). It is of some interest to note that the contribution from Fig. 3(d-1) is canceled by that from Fig. 3(d-2), so that the induced susceptibility \( m_0^\pi \) does not enter in the determination of strong \( \pi NN \) coupling, an aspect already observed in [10].

For the \( \pi^+ np \) coupling, we have, with \( \bar{a} \equiv (a_u + a_d) \),

\[
M^6 - \frac{3}{4} \bar{a} M^4 - \frac{11}{24}b M^2 + \frac{4}{3}a_U a_d + \frac{1}{3} (a_d - a_u) a_d - (m_u - m_d) a_u M^2 + m_0^2 \left\{ -\frac{1}{2} a_u m_u - \frac{5}{24} a_d m_d + \frac{29}{12} a_u m_d + \frac{19}{8} a_d m_u \right\} + m_0^2 \ln \frac{\mu^2}{\Lambda^2} \left[ a_u m_d + a_d m_u - \frac{1}{4} m_u (a_d + a_u) \right] + m_0^2 \frac{3}{4} a_u a_d \frac{1}{M^2} = \frac{g_{\pi^+ np}}{g_{\pi^+ dd}} (2\pi)^4 \lambda_\rho e^{\frac{m_0^2}{\mu^2}} + \text{excited states + continuum},
\]

As a standard practice, we may improve the applicability of these QCD sum rules by (1) taking into account the anomalous dimension of each term (operator) and (2) invoking the continuum approximation in which contributions from excited states and the continuum are approximated by what we may obtain at the quark level above a certain threshold \( W^2 \). Results from such improvements can be duplicated easily from Yang et al. [14]. In
this way, we obtain, for the $\pi^0 pp$ coupling \[14, 10\],

$$\begin{align*}
M^6 L^{-4/9} E_2 - \frac{2}{3} a_d M^4 L^{2/9} E_1 &+ \frac{11}{24} b M^2 E_0 + \frac{4}{3} u^2 L^{4/9} \\
+ m_0^2 \bar{a} M^2 L^{-2/27} + m_0^2 \ln \frac{M^2}{\mu^2} L^{-26/27} &\left\{ \frac{2}{5} (a_u m_d + a_d m_u) - \frac{1}{4} m_u a_u \right\} \\
+ m_0^2 L^{-26/27} &\left\{ \frac{5}{12} m_u a_u - \frac{1}{8} m_d a_d + \frac{67}{48} m_d a_u + \frac{115}{48} m_u a_d \right\} \\
= \frac{g_{\pi^0 pp}}{g_{\pi^0 dd}} \lambda^2 \rho e^{-\frac{a^2}{M^2}},
\end{align*}$$

and, for the $\pi^+ np$ coupling,

$$\begin{align*}
M^6 L^{-4/9} E_2 - \frac{\alpha}{\mu^2} M^4 L^{2/9} - \frac{11}{24} b M^2 E_0 + \left\{ \frac{4}{3} a_u a_d + \frac{1}{4} (a_d - a_u) a_d \right\} L^{4/9} \\
- (m_u - m_d) a_u M^2 L^{-4/9} E_0 + m_0^2 L^{-26/27} &\left\{ - \frac{1}{2} a_u m_d - \frac{5}{24} a_d m_d + \frac{29}{12} a_u m_u + \frac{19}{8} a_d m_u \right\} \\
+ m_0^2 \ln \frac{M^2}{\mu^2} L^{-26/27} &\left\{ a_u m_d + a_d m_u - \frac{1}{4} m_u (a_d + a_u) \right\} + \frac{m_0^2}{3} a_u a_d L^{-2/27} \frac{1}{M^2} \\
= \frac{g_{\pi^+ pp}}{g_{\pi^+ dd}} (2\pi)^4 \lambda_n \lambda_p e^{-\frac{a^2}{M^2}}.
\end{align*}$$

Here $E_0 = 1 - e^{-x}$, $E_1 = 1 - (1 + x) e^{-x}$, and $E_2 = 1 - (1 + x + \frac{x^2}{2}) e^{-x}$ with $x = W^2/M^2$, and $L = 0.621 ln(10 M)$.

There are several sources for isospin symmetry breakings, viz.:

At the hadronic level (r.h.s.), we have $m_n \neq m_p$, $\lambda_n \neq \lambda_p$, and $W^2_n \neq W^2_p$ (with $W^2$ the threshold parameter in the continuum approximation for treating the excited states and the continuum). Apart from the well-known neutron-proton mass difference, we use the parameters previously adopted by Yang et al. \[14\].

At the quark level (l.h.s.), the isospin symmetry breakings caused by $< \bar{d} d > \neq < \bar{u} u >$ and $m_d \neq m_u$ have been made explicit in the formulae given above. Such effects have also been considered by other authors \[3, 14\].

In addition, there is an effect due to the non-universal pion-quark couplings. Conceptually, we may start with a universal pion-quark coupling, such as in the effective chiral quark theory, but the vertex renormalizations due to electromagnetic interactions, as shown in Fig. 4 and also pointed out by Cao and Hwang \[7\], modify $\pi^0 uu$, $\pi^0 dd$, and $\pi^+ ud$ in a different manner. Here we have adopted the dimensional regularization scheme to work out such differences and conclude that the non-universal pion-quark cou-
plings so obtained could be a reasonably important source for isospin symmetry breaking. Specifically, we obtain the following expressions for the pion-quark couplings.

\[
\begin{align*}
g_{\pi^+ du} & \rightarrow g_{\pi^+ du} \{1 + \frac{\alpha}{4\pi} \left(-\frac{43}{18} + \frac{1}{6} \gamma_E\right)\}, \\
g_{\pi^0 uu} & \rightarrow g_{\pi^0 uu} \{1 + \frac{\alpha}{4\pi} \left(\frac{52}{9} - \frac{4}{3} \gamma_E\right)\}, \\
g_{\pi^0 dd} & \rightarrow g_{\pi^0 dd} \{1 + \frac{\alpha}{4\pi} \left(\frac{13}{9} - \frac{1}{3} \gamma_E\right)\}. \\
\end{align*}
\]

(15)

Numerically, these expressions give rise to $2.18 \times 10^{-3}$ to the charge symmetry breaking term $g_{\pi^0 uu} - g_{\pi^0 dd}$ and $4.2 \times 10^{-3}$ for the charge dependent combination $g_{\pi^0 uu} - g_{\pi^+ du}$.

To perform numerical analyses, we may introduce the following parameters:

\[
\begin{align*}
\lambda & \equiv \frac{\lambda_p + \lambda_n}{2}, & \sigma & \equiv \frac{\lambda_n - \lambda_p}{\lambda}; \\
\bar{m} & \equiv \frac{m_p + m_n}{2}, & \epsilon & \equiv \frac{m_n - m_p}{\bar{m}}. \\
\end{align*}
\]

\[
\begin{align*}
\frac{g_{\pi^0 pp}}{g_{\pi^0 dd}} & \equiv K \\
\frac{g_{\pi^0 nn}}{g_{\pi^0 uu}} & \equiv K(1 + \rho_1) \\
\frac{g_{\pi^- pn}}{g_{\pi^- ud}} & \equiv K(1 + \rho_2) \\
\frac{g_{\pi^+ np}}{g_{\pi^+ du}} & \equiv K(1 + \rho_3) \\
\end{align*}
\]

Moving the pion-quark coupling to the phenomenological side, we may write the r.h.s. as follows:

\[
\begin{align*}
\pi^0 pp & \rightarrow K\lambda^2 e^{-\frac{m^2}{M^2}}(1 - \sigma + \epsilon \frac{m^2}{M^2}), \\
\pi^0 nn & \rightarrow K\lambda^2 e^{-\frac{m^2}{M^2}}(1 + \sigma - \epsilon \frac{m^2}{M^2} + \rho_1), \\
\pi^- pn & \rightarrow K\lambda^2 e^{-\frac{m^2}{M^2}}(1 + \rho_2), \\
\pi^+ np & \rightarrow K\lambda^2 e^{-\frac{m^2}{M^2}}(1 + \rho_3). \\
\end{align*}
\]

The various parameters which we adopt are $\sigma = -10^{-3}$, $\epsilon = 1.3 \times 10^{-3}$, $\gamma = -6.57 \times 10^{-3}$, $a_q = 0.546 $GeV$^3$, $m_u = 5.1 MeV$, $m_d = 8.9 MeV$, $b = 0.474 GeV^4$, $m_0^2 = 0.8 GeV^2$, $\mu = 0.5 GeV$, and $\chi a = -3.14 GeV^2$.
the QCD sum rules for the various $\pi NN$ couplings is $0.8\text{GeV}^2 \leq M_B^2 \leq 1.5\text{GeV}^2$, a standard choice for analyzing the various QCD sum rules associated with the nucleon.

In this way, we find

$$\frac{g_{\pi}^{\sigma_{nn}} - g_{\pi}^{\sigma_{pp}}}{g_{\pi}^{\sigma_{NN}}/g_{\pi}^{\sigma_{QQ}}} |_{\text{average}} = 0.58\%$$  \hspace{1cm} (16)

$$\frac{g_{\pi}^{\sigma_{+np}} - g_{\pi}^{\sigma_{pp}}}{g_{\pi}^{\sigma_{NN}}/g_{\pi}^{\sigma_{QQ}}} |_{\text{average}} = 0.35\%$$  \hspace{1cm} (17)

Combining with the difference in pion-quark couplings (from vertex renormalizations), we therefore conclude that $g_{\pi}^{\sigma_{nn}}$ is numerically bigger than $g_{\pi}^{\sigma_{pp}}$ by $0.80\%$, while $g_{\pi}^{\sigma_{+np}}$ is numerically greater than $g_{\pi}^{\sigma_{pp}}$ by $0.15\%$.

Numerically, we may arbitrarily set $m_d = m_u$, $<\bar{d}d> = <\bar{u}u>$, and $\lambda_n = \lambda_p$, except keeping physical values for $m_n$ and $m_p$. In this way, we find that $|g_{\pi nn}|$ is bigger than $|g_{\pi pp}|$ by $0.19\%$. Analogously, we find that $\lambda_n \neq \lambda_p$ (as found by Yang et al. \cite{14}) has a contribution of $0.20\%$. $m_d \neq m_u$ and $<\bar{d}d> \neq <\bar{u}u>$ combine to give another contribution of $0.20\%$. Therefore, the overall contribution of $80\%$ in fact comes from several sources of comparable magnitudes (and of the same sign in the present case).

It is of some interest to note that our prediction on $|g_{\pi}^{\sigma_{nn}}| - |g_{\pi}^{\sigma_{pp}}|$ (a positive value) differs in sign from the quark-model result \cite{7}, and also from a three-point correlation function approach \cite{9}. It is also of interest to note that the difference in pion-quark couplings represents a source of numerical importance on the charge symmetry breaking effect.
III. Low-Energy Nucleon-Nucleon Scattering

The one-pion-exchange potential (OPEP) in nucleon-nucleon interactions can be obtained in the standard manner [7]. For example, we have, in the $p-p$ case,

$$V_{pp}^\pi(r) = \frac{g_{\pi^0 pp}^2}{4\pi} \frac{\vec{\tau}_1 \cdot \vec{\nabla}_r \vec{\tau}_2 \cdot \vec{\nabla}_r}{4m_p^2} \frac{1}{r} (e^{-m_{\pi^0} r} - e^{-\Lambda r}),$$  

(18)

where $\Lambda$ is the cutoff at the short range which contributes negligibly to the isospin symmetry breakings [7]. The OPEP in the $n-n$ case may be obtained by simple substitution $p \to n$. On the other hand, the $n-p$ OPE potential is given by

$$V_{np}^\pi(r) = \frac{g_{\pi^0 nn} g_{\pi^0 pp}}{4\pi} \tau_3^1 \tau_3^2 \frac{\vec{\tau}_1 \cdot \vec{\nabla}_r \vec{\tau}_2 \cdot \vec{\nabla}_r}{4m_n m_p} \frac{1}{r} (e^{-m_{\pi^0} r} - e^{-\Lambda r})$$

$$+ \frac{g_{\pi^0 np}^2}{4\pi} \left\{ \left( \tau_1^1 \tau_2^2 - \tau_3^1 \tau_3^2 \right) \vec{\sigma}_1 \cdot \vec{\sigma}_2 \cdot \vec{\nabla}_r \right\} \frac{1}{r} (e^{-m_{\pi^0} r} - e^{-\Lambda r}).$$

(19)

Here we have used $\delta \equiv (m_n - m_p)/(m_n + m_p)$ and $M_N = \frac{1}{2}(m_n + m_p)$. The $^1S_0$ phase shift $\delta_0$, as used to describe low-energy nucleon-nucleon scatterings, can be parametrized in the standard manner:

$$k \cot \delta_0 = -\frac{1}{a} + \frac{1}{2} k^2 r_0 + \cdots,$$  

(20)

with $a$ the scattering length and $r_0$ the effective range. Experimentally, we have [16]

$$\delta a_{CSB} \equiv |a_{pp}| - |a_{nn}| = -(1.5 \pm 0.5) \text{fm};$$

$$\delta a_{CD} \equiv |a_{np}| - |a_{nn}| = (5.0 \pm 0.3) \text{fm}.$$  

(21)

As already noted in [7], the differences in OPEP’s, such as $V_{pp}(r) - V_{nn}(r)$, may be treated as a perturbation, making it easy to compute $\delta a_{CSB}$ or other low-energy isospin symmetry breaking observables. In this way, our prediction on the difference between $g_{\pi^0 nn}$ and $g_{\pi^0 pp}$ gives rise to about $-1.0 \text{fm}$ on $\delta a_{CSB}$, consistent with the observation both in sign and roughly in magnitude.

As already known [1, 7] for some time, the bulk of the charge-dependent effect as revealed by a (relatively) large value of $\delta a_{CD}$ can be understood quantitatively as caused
by the pion mass difference $m_{\pi^+} \neq m_{\pi^0}$ while the differences as found in pion-nucleon couplings give rise to relatively small contribution to $\delta a_{\text{CD}}$. As a result, we may safely conclude that the observed $\delta a_{\text{CSB}}$ and $\delta a_{\text{CD}}$ as summarized recently in [16] can be understood, to a large extent, as manifestations of isospin asymmetries associated with the one-pion exchange potential.

**IV. Discussion and Summary**

The observed differences [16] in the $^1S_0$ NN scatterings are by far the best known isospin symmetry breakings in which pion-nucleon couplings play a central role. There might be some isospin symmetry breakings in the $^1S_0$ effective ranges or in the $^3P_J$ scattering lengths, but until now such effects have not been extracted experimentally.

The last term in $V_{\text{np}}^\pi(r)$ [in Eq. (17)], as induced by $m_\rho \neq m_\pi$, gives rise to isospin mixings such as $^3P_1-^1P_1$ or $^3D_2-^1D_2$ mixing, but only in a rather small way. On the other hand, charge-symmetry breakings observed in medium-energy elastic $n-p$ scattering (at TRIUMF and IUCF) are dictated primarily [8] by the charge symmetry breaking force as caused by the $\rho-\omega$ mixing or by $g_{\omega pp} \neq g_{\omega nn}$. However, the short-range nature in this case may require a careful investigation of the quark-interchange mechanisms as addressed in [17], as it is difficult to reconcile the picture of exchanging, between two nucleons, a meson ($\rho$ or $\omega$) with a range of about 0.25 fermi, considerably smaller than the extent of a nucleon (with a radius of at least 0.5 fermi, even assuming a little bag picture for the nucleon).

In any event, the isospin symmetry breakings associated with the one-pion exchange potential (which is clearly on a very sound conceptual ground) can best be accessed through careful measurements of low-energy scattering parameters such as $\delta a_{\text{CSB}}$, $\delta a_{\text{CD}}$, etc. Such efforts should be further encouraged by the currently popular wisdom that the
low-energy sector of the $\pi N$ are $NN$ systems can be described by the long-wave length limit of QCD via Goldstone theorem, yielding the so-called chiral perturbation theory, so that the low-energy scattering parameters seems to be more “fundamental” than those needed in, e.g., the description of medium-energy nucleon-nucleon scatterings.

In summary, we have adopted the method of QCD sum rules in the presence of an external pion field to investigate isospin symmetry breakings in pion-nucleon couplings. Numerically, we find that $|g_{\pi nn}|$ is bigger than $|g_{\pi pp}|$ by 0.80% (and bigger than $|g_{\pi^+ np}|$ by 0.65%). This result accounts for the observed charge-symmetry breaking effect $\delta a_{\text{CSB}}$ both in sign and (roughly) in magnitude. Contrary to many previous calculations in the literature, our result offers, for the first time, the possibility to understand both $\delta a_{\text{CSB}}$ and $\delta a_{\text{CD}}$ in a quantitative manner (both in sign and in magnitude).

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Figure Captions

Fig. 1. Pictorial representation of the various terms in the quark propagator $S^{(0)ab}(x)$.

Fig. 2. Pictorial representation of the various terms in the quark propagator $S^{(\pi)ab}(x)$.

Fig. 3. The diagrams used to determine the QCD sum rule for $g_{\pi^{0}pp}$.

Fig. 4. The vertex renormalizations (including the self-energy contributions) leading to the non-universal pion-quark couplings, an important source for isospin symmetry breakings.
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