Hydrogel core flexible matrix composite (H-FMC) actuators: theory and preliminary modelling

M P M Dicker¹, P M Weaver¹, J M Rossiter² and I P Bond¹

¹ Advanced Composites Centre for Innovation and Science, University of Bristol, Queen’s Building, Bristol BS8 1TR, UK
² Department of Engineering Mathematics, University of Bristol, Merchant Venturers Building, Bristol, BS8 1UB, UK

E-mail: michael.dicker@bristol.ac.uk

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Abstract

The underlying theory of a new actuator concept based on hydrogel core flexible matrix composites (H-FMC) is presented. The key principle that underlines the H-FMC actuator operation is that the three-dimensional swelling of a hydrogel is partially constrained in order to improve the amount of useful work done. The partial constraint is applied to the hydrogel by a flexible matrix composite (FMC) that minimizes the hydrogel’s volume expansion while swelling. This constraint serves to maximize the fixed charge density and resulting osmotic pressure, the driving force behind actuation. In addition, for certain FMC fibre orientations the Poisson’s ratio of the anisotropic FMC laminate converts previously unused hydrogel swelling in the radial and circumferential directions into useful axial strains. The potential benefit of the H-FMC concept to hydrogel actuator performance is shown through comparison of force–stroke curves and evaluation of improvements in useful actuation work. The model used to achieve this couples chemical and electrical components, represented with the Nernst–Plank and Poisson equations, as well as a linear elastic mechanical material model, encompassing limited geometric nonlinearities. It is found that improvements in useful actuation work in the order of 1500% over bare hydrogel performance are achieved by the H-FMC concept. A parametric study is also undertaken to determine the effect of various FMC design parameters on actuator free strain and blocking stress. A comparison to other actuator concepts is also included.

Keywords: chemomechanical, artificial muscle, force–stroke curves

1. Introduction

The theory and modelling of hydrogel core flexible matrix composite (H-FMC) actuators is presented. The components and function of the H-FMC actuator concept are displayed in figure 1. The device consists of a hydrogel (a polymer gel that reversibly swells when exposed to differing stimuli in solution) contained within a flexible matrix composite tube (a cylindrical fibre reinforced composite laminate with an elastomeric matrix).

This novel actuator design was motivated by the desire to develop new functional structures that combine sensing, signal transmission, control and actuation through chemical stimulus and processes alone [1]. Such a structure would be a solid-state intelligent structure, generating autonomous motion in response to its surrounding stimuli without reliance on any electrical process for either power or control.

A concept for such a structure is shown in figures 2 and 3. This structure is a simplified biomimetic analogue of the leaf of the Cornish Mallow (Lavatera cretica L.), one of the most fascinating and highly studied sun-tracking plants. It was investigated extensively by Schwartz and Koller et al [2–7] as well as Fisher et al [8–10] in the 1980s. The Cornish Mallow performs diapheliotropic leaf movements, where the plane of the lamina (leaf) is maintained perpendicular to the incident sunlight throughout the day. This is achieved through
Figure 1. The two states and components of a contraction H-FMC actuator. The actuator has had a cut-out removed from its centre to reveal the internal hydroxyethylmethacrylate acid (HEMA) hydrogel. (a) Actuator in its un-actuated elongated state, here the internal hydrogel is exposed to a solution with pH 2 through the porous end connections. (b) In this state the osmotic pressure generated by the hydrogel is zero, as the polymer chains are not ionized as in (c). (d) The osmotic pressure is now greater than zero, as the hydrogel is exposed to a solution of pH 10, ionizing acidic groups on the hydrogel polymer as in (e). (f) This internal pressure is coupled through the anisotropic FMC (with $\theta < \pm 55^\circ$) which it acts upon to produce a contracting actuation force and stroke (arrows). Detail (b) also outlines the coordinate system used in the FMC model; here $x$ is the actuator longitudinal axis, while $\phi$ is the circumferential direction. 1 and 2 coordinates refer to the fibre and matrix dominated direction of a specific lamina that contributes to the overall FMC laminate.

Figure 2. (a) Un-actuated structure with quarter section removed to reveal internal structure. (b) Full structure in actuated state showing orientation towards the light source.
bending of a pulvinus, an osmosis driven motor organ located at the connection between the leaf and stem (3–4 mm in length). Light perception primarily takes place distant from the pulvinus in the major leaf veins. The work of Koller and Fisher revealed that the sun-tracking movement results from an extraordinarily complex—yet extremely elegant—process of signal perception, generation, filtering and control. Sensing is achieved through anisotropically arranged photoreceptors [2, 6]. Control is imparted by a rearrangement of the vascular network that carries the chemical signal to the pulvinus, integrating the signal from the various leaf veins, whilst providing a means for elimination of signal conflict [10].

This paper focuses on the design, theory and preliminary modelling of the H-FMC actuators for the biomimetic analogue of the sun-tracking Cornish Mallow leaf. Subsequently we determine the potential benefit of the H-FMC concept to hydrogel actuator performance, examine the effect of flexible matrix composite (FMC) parameters on the actuator free

![Figure 3. Cross-section of light tracking structure displaying operating principle.](image)
strain and blocking stress, and provide comparison to other actuator concepts.

1.1. Hydrogels

Hydrogels are three-dimensional polymeric networks with the ability to swell to many times their dry weight by absorption of water [14]. The polymer chains that form the gel contain either acidic or basic groups which can be ionized by changes in environmental factors, including pH, solvent quality, electric field or temperature [15]. The H-FMC device explored in this work uses a pH sensitive hydrogel. It is this process of ionization which switches on and off the swelling of the hydrogel, a process ultimately driven by osmotic pressure set up between ionic concentrations of solution inside the hydrogel (attracted/repelled by the ionized groups), and the solution outside the hydrogel. Swelling continues until an equilibrium is reached between the ionic pressure and the solution outside the hydrogel. Hydrogels have been widely investigated for use in drug delivery, and as sensors, and actuators [17]. Table 1 presents the typical properties of a variety of actuators for comparison. Although hydrogels have excellent static actuation properties — similar to those of natural muscle — their actuation speed is limited by the rate of diffusion of ionic and/or molecular species, making them impractically slow for many applications (although the development of micro and nano-dimensional particles and fibres has helped in this regard [18–24]). In addition, they exhibit limited electronic controllability [25].

As a result, hydrogels are typically employed in applications on the micron-scale, and have had limited success when applied to larger devices [25]. The initial application envisaged for the H-FMC actuator is one of very slow motion (~6 h for full stroke), and employs chemical control, thus it is envisaged that these known limitations will not be a hindrance.

1.2. FMC

FMC actuators, inspired by the fibrillar network present in plant cell walls, were first developed by Philen et al [26]. By tailoring the orientation of the fibres, the actuator can be made to either elongate (fibres oriented at greater than ±55° from the longitudinal axis) or contract (less than ±55°) upon internal pressurization. In the H-FMC actuator concept, the internal pressurization results from the swelling of the hydrogel that is contained within. Contracting FMC actuators can exhibit a large mechanical advantage, whereby the actuation force generated is greater than the force generated by the internal pressure acting on the end area of the actuator alone (the internal pressure multiplied by the area of the actuators cross-section).

To our knowledge the proposed H-FMC is the first time a hydrogel’s swelling has been used as the pressure source within an FMC actuator. The benefit to hydrogel actuator performance provided by this is not only the mechanical advantage obtained, but also the partial volumetric constraint of the hydrogel by the FMC which also yields performance benefits. The benefit of partial constraint had previously been demonstrated in experiment by Etches and Bond among others, who found that the constraint of a hydrogel in one direction could more than double the resulting swelling achieved in the unconstrained direction [13]. This paper examines this phenomenon more closely, finding it relates to the density of fixed ionisable groups on the hydrogel polymer. The theory and modelling of this is presented, and for the first time, the actuation benefit is quantified through comparison of force–stroke curves, and changes in useful work output for constrained and unconstrained hydrogel actuators.

As a result of the potential benefit offered by the H-FMC concept to hydrogel actuation performance it is believed that the concept will find application in a wide variety of fields, with particular potential for use as soft actuators in robotic devices. A similar actuator concept employing hydrogels contained in McKibben actuator tubes has been developed by Tondu et al for robotic applications [27–30]. McKibben actuators are similar to FMC based actuators and consist of an internal elastomeric tube constrained externally by an unbonded fibre weave or braid [26]. Santulli et al also developed an actuator where a hydrogel was contained within a braided fibre structure similar to a McKibben actuator, but with a permeable membrane in lieu of a elastomeric tube [31]. This was intended for application as a vibration neutralizer, but response times proved restrictive.

Although the motivations for the partial constraint of the hydrogel in the work of Tondu et al and Santulli et al appears to be different than our own, both McKibben and FMC tubes could provide this function. However, in this work FMC tubes have been chosen rather than McKibben tubes as a result of the wider array of actuation configurations/properties available through the full spectrum of fibre angles and layup configurations. These can be accurately controlled through the manufacturing process of filament winding used to create them [32].

This paper first presents the materials used in this investigation, followed by the theory and modelling of the H-FMC concept. This includes the hydrogel swelling and FMC constraint. The model is then validated against well known hydrogel swelling data. Results are then presented, with quantification of the benefits of the actuator concept conducted through examination of force–stroke curves and calculation of improvement in useful actuation work output. A parametric study is then undertaken to examine key design

### Table 1. Typical properties of different actuator types [25].

| Actuator  | Energy den. (kJ m⁻³) | Stroke (%) | Pressure (MPa) | Reaction time (ms) |
|-----------|-----------------------|------------|----------------|-------------------|
| Hydrogels | 3.5                   | 90         | 4              | 300–10⁶            |
| Solenoids | 0.25                  | 50         | 0.1            | 5                 |
| SMA       | 100                   | 8          | 900            | 300              |
| Muscle    | 5.9                   | 70         | 1.18           | 0.03              |

- Actuator size in the micrometre and sub-millimetre range to actuator size in the centimetre range.
- Depending on the applied heating power.
inputs that affect the actuator free strain and blocking stress. Finally, the paper compares the H-FMC performance to that of other actuators. The paper concludes with a discussion of study limitations and recommendations for future work.

2. Materials

2.1. Hydrogel

Although the model presented in this paper are universally applicable to all pH activated hydrogels, this work specifically focuses on homogeneous copolymers of hydroxyethylmethacrylate acid (HEMA) and acrylic acid with 1% of a diacrylate crosslinker. The swelling of HEMA hydrogels has been extensively studied, with the work of De et al [16] regularly cited as validation for hydrogel swelling models, including the large body of work by Li et al on the development and application of their multi-effect-coupling pH-stimulus (MECpH) model [33–44]. The HEMA hydrogel is assigned a Young’s Modulus that varies linearly from 0.25 MPa at pH 5.5 to 0.23 MPa at pH 7.5, and a Poisson’s ratio of 0.43 [16].

2.2. FMC

For the sake of model simplicity we represent the FMC as a linear elastic material, although the behaviour of the elastomeric matrix employed is nonlinear. We choose a nominal linear elastic material, although the behaviour of the elastomeric matrix is nonlinear. We choose a Young’s modulus $E$ of 8 MPa with Poisson’s ratio $\nu$, of 0.45. A pitch based carbon fibre with modulus 379 GPa and Poisson’s ratio 0.3 accompanies this.

3. Method

3.1. Hydrogel

The steady state MECpH models of both Li et al [33, 42] and Wallmersperger et al [45, 46] are used to define the hydrogel performance.

Hydrogel swelling occurs when weakly acid or basic groups in the network are ionised as a result of changes in the surrounding environment. For the HEMA hydrogel, weakly acidic carboxyl groups are ionized by an increase in pH,

$$\text{RCOOH} \rightleftharpoons \text{RCOO}^- + \text{H}^+. \quad (1)$$

At which point ionic interactions cause the gel to swell. The dissociation constant for ionization $K_a$ is required to model this swelling, it is a measure of the relationship between the fixed charge groups in the gel and the diffusive hydrogen ions,

$$K_a = \frac{[\text{RCOO}^-][\text{H}^+]}{[\text{RCOOH}]], \quad (2)$$

noting that for the HEMA hydrogel used in this work $K_a = 10^{-2}$ mol m$^{-3}$ [16]. The density of total ionizable fixed charged groups, $c'_{m0}$, is given by the ratio of the number of moles of ionizable groups $n$ and the volume of the solid polymer $V'$,

$$c'_{m0} = \frac{n}{V'}, \quad (3)$$

where $c'_{m0} = 1800$ mol m$^{-3}$ for the HEMA hydrogel [16]. For determination of the ionic concentrations in the hydrogel, this must be converted to a single unit of fluid volume inside the hydrogel, $c_{m0}$, using

$$c_{m0} = \frac{n}{V'f} = \frac{c'_{m0}V'}{V'f} = \frac{c'_{m0}H}{H}, \quad (4)$$

where $H$ is the hydration of the hydrogel, the volumetric ratio of the current fluid inside the hydrogel, $V'$, to the solid polymer according to

$$H = \frac{V'f}{V'}. \quad (5)$$

Noting that $V'f$ is a function of hydrogel deformation, calculated as the difference between the variable total hydrogel volume, and the constant solid polymer volume. The reaction isotherm is then described by the fixed charge density, $c_h$, and the hydrogen ion concentration, $c_{H^+}$:

$$K_a = \frac{c_hc_{H^+}}{c_{m0} - c_h}. \quad (6)$$

Finally, the fixed charge density can be written as:

$$c_h = \frac{K_ac_{m0} c_{H^+}}{K_a + c_{H^+}}. \quad (7)$$

where this expression is used for determining the concentration of all species $c_k$, resulting from electrical potential $\psi$. Subscript $k$ refers to each $k$th ionic species of $N$ present in the solution. The method involves coupling the Nernst–Plank equation (equation (8)) with the Poisson equation (equation (10)),

$$V \left( D_k \psi c_k + \mu_k z_k c_k V \psi \right) = 0, \quad (8)$$

where $\mu_k$ and $z_k$ represent the ionic mobility and valency of the $k$th ionic species in solution. $D_k$ is the effective diffusion constant of the $k$th ion ($D_{H^+} = 9.311 \times 10^{-9}$, $D_{Na^+} = 1.334 \times 10^{-9}$, $D_{Cl^-} = 2.032 \times 10^{-9}$ [47]) they are related to the ionic mobility by the Einstein relationship,

$$\mu_k = \frac{D_k F}{RT} \quad (9)$$

where $R$, $F$ and $T$ are the universal gas constant (8.314 J mol$^{-1}$ K$^{-1}$), Faraday constant (9.64853399 x 10$^4$ C mol$^{-1}$) and temperature (K). Static charge is then determined from the Poisson equation,

$$\nabla^2 \psi = - \frac{F}{\varepsilon \varepsilon_0} \left( \sum_{k=1}^{N} z_k c_k + z_h c_h \right) \quad (10)$$

where the subscript $h$ refers to the fixed charge on the hydrogel. $\varepsilon_0$ and $\varepsilon_i$ are the vacuum permeability and dielectric constant of the solvent (80 for water at 20 °C).
The distribution of ionic species concentrations \(c_k\) is then determined by solving the \(N\) Nernst–Plank equations (equation (8)) with the Poisson equation (equation (10)), such that the condition of electro-neutrality is met in both the hydrogel and surrounding solution. This is done using the COMSOL multiphysics FE package [48], with solution boundary conditions for \(c_k\) equal to the solution initial concentrations and \(w = 0\).

The distribution of \(c_k\) is the key input in determining the osmotic pressure, the driving force behind the hydrogel swelling, given by,

\[
P_{os} = RT \sum_{k=1}^{N} (c_{k,h} - c_{k,s}),
\]

where the subscripts \(h\) and \(s\) refer to the hydrogel and solution domain respectively. \(P_{os}\) is applied as a surface boundary pressure acting on the hydrogel. In the validation case, a cylindrical hydrogel is axially clamped, such that the linear elastic radial strain, \(\varepsilon_r\), is given by [49]:

\[
\varepsilon_r = \frac{P_{os}(1 - \nu_h)}{E_h}.
\]

3.2. FMC

The first step in calculating the contribution to actuation from the FMC tube is to calculate the engineering constants for its component lamina [50]. In the following equations the subscript \(f\) refers to fibre elements, whilst subscript \(m\) refers to matrix components. The 1 and 2 subscripts refer to fibre dominated and matrix dominated lamina coordinates as per detail (b) of figure 1. The volume fraction, \(V\), is such that \(V_f + V_m = 1\) and:

\[
E_1 = E_f V_f + E_m V_m,
\]

\[
E_2 = \frac{E_f E_m}{E_f V_f + E_m V_f},
\]

\[
\nu_{12} = \nu_f V_f + \nu_m V_m,
\]

\[
\nu_{21} = \frac{E_2}{E_1},
\]

\[
G_{12} = \frac{G_f G_m}{G_f V_f + G_m V_f}.
\]

The shear modulus \(G\) for each isotropic component is calculated from classical elasticity, as

\[
G = \frac{E}{2(1 + \nu)}.
\]

These terms are the material inputs used in the classical lamination theory that determines the mechanical properties of the lamina through determination of \(Q_{ij}\), the components of the lamina stiffness matrix [50]:

\[
Q_{11} = \frac{E_1}{1 - \nu_{12} \nu_{21}},
\]

\[
Q_{12} = \frac{\nu_{12} E_2}{1 - \nu_{12} \nu_{21}} = Q_{21},
\]

\[
Q_{22} = \frac{E_2}{1 - \nu_{12} \nu_{21}}.
\]

\[
Q_{66} = G_{12}.
\]

The coordinates for the stress strain relationship of the individual lamina are then transformed from lamina specific (1–2) coordinated, to laminate specific (\(x\–\phi\)) coordinates (detail figure 1(b)). For a balanced symmetric FMC laminate [50]:

\[
\begin{align*}
\tilde{Q}_{11} &= Q_{11} \cos^2 \theta + Q_{22} \sin^2 \theta \\
&\quad + 2(Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta, \\
\tilde{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66}) \sin^2 \theta \cos^2 \theta \\
&\quad + Q_{12} (\cos^2 \theta + \sin^2 \theta), \\
\tilde{Q}_{22} &= Q_{11} \sin^2 \theta + Q_{22} \cos^2 \theta \\
&\quad + 2(Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta,
\end{align*}
\]

where \(\tilde{Q}_{ij}\) are the components of the transformed lamina stiffness matrix. We now consider the behaviour of a laminate composed of multiple lamina. This is done by considering a laminated section of the FMC subjected to forces per unit length \(N_i, N_\phi\), where \(N = \sigma t\) (\(t\) being the laminate thickness).

Such consideration results in the formation of a laminate extension stiffness matrix \(A\):

\[
\begin{bmatrix}
N_x \\
N_\phi
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} \\
A_{12} & A_{22}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_\phi
\end{bmatrix},
\]

where for distance from neutral axis \(z\) of each lamina \(L\) of \(M\):

\[
A_{ij} = \int_{-L/2}^{L/2} (\tilde{Q}_{ij})_k (z_L - z_{L-1}) dz.
\]

From the theory of a thin walled cylindrical pressure vessel with radius \(r\), we obtain:

\[
N_x = \sigma_x t = \frac{P_{os} r}{2},
\]

\[
N_\phi = \sigma_\phi t = \frac{P_{os} r}{2}.
\]

Using these relationships (equations (27)–(29)), strains are calculated as:

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_\phi
\end{bmatrix} = [A]^{-1}
\begin{bmatrix}
\frac{P_{os} r}{2} \\
\frac{P_{os} r}{2}
\end{bmatrix}.
\]

Although equation (30) provides a good first approximation for the actuation of the H-FMC, it does not include the stiffness contribution of the hydrogel and restricts the analysis to thin walled H-FMC actuators (internal radius divided by wall thickness must be greater than 10 [49]).
The stiffness of the hydrogel is included by considering that the proportion of total stress in the structure (force $F_s$ divided by the sum of both the hydrogel and FMC cross-sectional area $\Sigma A$) that is acting in either the FMC or the hydrogel is proportional to the ratio of that component’s stiffness, $E_{s,FMC}$, to the equivalent stiffness of the entire structure, $E_s$.

\[ \sigma_{s,FMC} = \frac{F_s}{\Sigma A} \cdot \frac{E_{s,FMC}}{E_s} = \frac{N_s}{t}, \quad (31) \]

\[ N_s = \frac{P_{os} r^2}{(r + t)^2} \times \frac{E_{s,FMC} (r + t)^2 t}{rE_h + (r + t)^2 rE_{s,FMC}}, \quad (32) \]

where

\[ E_{s,FMC} = \frac{1}{t} \left[ A_{11} - \frac{A_{12}^2}{A_{22}} \right] \quad (33) \]

Similarly,

\[ \sigma_{\phi,FMC} = \frac{F_\phi}{\Sigma A} \cdot \frac{E_{\phi,FMC}}{E_\phi} = \frac{N_{\phi}}{t}, \quad (34) \]

\[ F_\phi = l \int_0^l P_{os} dr = lP_{os} r, \quad (35) \]

where $l$ is the length of the hydrogel, and

\[ E_{\phi,FMC} = \frac{1}{t} \left[ A_{22} - \frac{A_{12}^2}{A_{11}} \right] \quad (36) \]

\[ N_{\phi} = \frac{P_{os} r}{(r + t)^2} \times \frac{E_{\phi,FMC} (r + t)^2 t}{rE_h + tE_{s,FMC}}. \quad (37) \]

Equations (28) and (29) can now be replaced with equations (32) and (37), capturing the contribution from the hydrogel stiffness, and allowing the analysis to be extended beyond thin walled H-FMC actuators.

The actuator stroke or axial displacement $s$ is given by:

\[ s = \varepsilon_s l, \quad (38) \]

the blocking force for the H-FMC actuator is,

\[ F_b = n \varepsilon_s E_s \pi (r + t)^2, \quad (39) \]

where $n$ is a fraction of full stroke between 1 (zero stroke) and 0 (full stroke). The useful work output from an actuator is then given by,

\[ W = \int_0^l F ds, \quad (40) \]

where the work $W$ is equal to the force $F$ multiplied the displacement (stroke) $s$.

### 3.4. Geometric nonlinearity

Although the results presented in this work are primarily linear, the effects of changing fibre orientation in the FMC between different actuation states is incorporated by time stepping the application of $P_{os}$ in the model, and updating the fibre angle $\theta$ at each step such that the fibre angle in the analysis $\theta_{new}$ is a function of the FMC strains, and the reference (initial or un-actuated) fibre angle $\theta_{initial}$:

\[ \theta_{new} = \tan^{-1} \left[ \left( \frac{\varepsilon_{\phi} + 1}{\varepsilon_s + 1} \right) \tan \theta_{initial} \right], \quad (41) \]

Equation (41) has the effect of shifting the strain value towards values found at fibre orientation closer to $\pm 55^\circ$, as observed in figure 6.

### 3.5. Coupling

This involves the N Nernst–Plank equations (equation (8)) with the Poisson equation (equation (10)) being solved in COMSOL [48], for varied fixed charge density $c_h$ for a variety of solution ionic concentrations (initial $c_{Na^+}$ and $c_{Cl^-}$). The results are used to determine $P_{os}$ from equation (11), with the $P_{os}$ vs. $c_h$ relationship shown in figure 4.

The computational flow chart for the H-FMC actuation is shown in figure 5. For each iteration step in the model, $c_h$, which is a function of gel hydration $H$ and solution pH, is calculated with a new osmotic pressure being interpreted from figure 4. The strains resulting from this applied pressure are then calculated, which in turn lead to a new hydration, fixed charge density, and thus pressure. When calculating free strain the computation loops until the percentage error $\delta$ is below a predefined limit:

\[ \delta = 100 \times \sqrt{\frac{\varepsilon_{s,\text{new}}^2 - \varepsilon_{s,\text{old}}^2}{\varepsilon_{s,\text{old}}^2}}, \quad (42) \]

When calculating force stroke curves where axial strain is fixed, $\varepsilon_s$ is replaced with $\varepsilon_{\phi}$, the circumferential strain.
4. Results and discussion

Results for H-FMC axial strain and the osmotic pressure as a function of fibre wrap angle are shown in figure 6. Details of the H-FMC can be found in the figure caption. The figure shows results for both the geometrically linear model, and the nonlinear model described in section 3.4. The linear model is compared against an ABAQUS finite element (FE) model (linear FE). This FE model applies the osmotic pressure determined in the analytical regime as a surface pressure acting on the boundary of the hydrogel and FMC. A tie constraint is also applied at this interface. All material parameters are identical to that used in the analytical model, with the hydrogel’s Young’s modulus set to its high pH value of 0.23 MPa. The only difference between the analytical and FE models is in the end clamping conditions applied to the H-FMC in order to model a rigid fitting connecting the actuator to the fluid delivery system. In the FE analysis this is modelled as a 4 mm thick cylindrical end cap of ABS plastic (Young’s modulus 2.3 GPa). The end cap is not included in the analytical model developed in this work. It can be seen in figure 6(a) that the analytical model is an excellent match for fibre angles close to 0° and 90°, but there is some minor variation in regions when this end clamping condition becomes significant (greater difference between maximum actuated FMC diameter, and fixed end diameter).

4.1. Validation

Validation of the wider model is by means of comparison with the experimental work of De et al [16]. Results for the swelling of the cylindrical hydrogel restricted from axial displacement as a function of pH are shown in figure 7. Although reasonable results are observed for the final swelling of the sample, the model poorly captures the behaviour in the intermediary region near the hydrogels pK_a (log of the acid dissociation constant, and equal to pH 5). The problem is very complex in this transition region, where changes in the chemical composition inside the hydrogel domain are occurring from the ionisation of its carboxylic acid groups. This is accompanied by a change in the hydrogel’s stiffness, further complicating the simulation. Errors and uncertainty

Figure 5. Computational flow chart for the H-FMC actuation.

Figure 6. Results for axial strain (a) and osmotic pressure (b) as a function of fibre wrap angle $\theta$ for $E_m = 8$ MPa, $E_f = 379$ GPa, $V_f = 0.6$, $r = 6$ mm, $l = 100$ mm and $t = 1$ mm. Nonlinear results refer to the varying fibre angle model outlined in section 3.4. Linear FE results from ABAQUS.
regarding such aspects as variation in the pH from the fixed solution domain to the hydrogel domain, and uncertainty regarding exact hydrogel acid dissociation constant and Young’s modulus likely all contribute to the poor prediction of behaviour in this region.

4.2. Actuation advantage of H-FMC concept

The main aim of the preliminary modelling presented in this work was to establish if the actuation performance of a hydrogel could indeed be improved by its partial constraint with a FMC. This analysis is achieved by comparing force–stroke curves for a variety of H-FMC actuators, and the force–stroke curve for a hydrogel with no FMC constraint. Force–stroke curves are a common way of characterising the performance of actuators, which until the recent work of Illeperuma et al had not been applied to hydrogels [51].

Force–stroke curves show how hydrogel and FMC actuators have a maximum stroke (extension or axial strain), at which point they are no longer capable of applying a force, while they produce a maximum force, a so called blocking force, when the stroke is completely restrained.

The force–stroke curves for a 100 mm long, 6 mm radius H-FMC with a variety of fibre wrap angles is shown in figure 8(a), and compared against a hydrogel of the same dimensions, but without an FMC constraint. It can be seen that although the FMC reduces the maximum stroke obtained from the hydrogel, it compensates for this with an increase in blocking force. An interesting result observed here is that the H-FMC with \( \theta = \pm 80^\circ \) has a higher force profile than that with \( \theta = \pm 40^\circ \). This contradicts the expected result for a FMC, where contractile actuators (\( \theta < \pm 55^\circ \)) generate higher forces than extension actuators (\( \theta > \pm 55^\circ \)). For the H-FMC concept, the variable actuating pressure complicates this result. In this case, the \( \theta = \pm 80^\circ \) H-FMC actuates with less hydrogel volume expansion than the \( \theta = \pm 40^\circ \) H-FMC, resulting in it operating at higher pressures, and overriding the benefit of the contractile FMC fibre orientation.

By integrating the the force–stroke curves as per equation (40) we arrive at a value for the full amount of useful work per cycle. This is shown in figure 8(b) and (c) for different fibre wrap angles, and for a range of cylinder diameters and lengths. For (b) and (c) and (d) the % increase in useful work over a bare hydrogel is shown as a function of fibre wrap angle. All these charts use the nonlinear model described in section 3.4.
work capable for both a given H-FMC arrangement, and the bare hydrogel. Figures 8(b) and (c) show the variation in the increase in useful work output provided by the H-FMC concept as a function of fibre wrap angle, and a given actuator radius and length. The potential benefit is found to be in the order of 1500% for fibre wrap angles $\theta = \pm 20^\circ$ to $30^\circ$. This at first seems rather high, but when considering the work the bare hydrogel does in the radial direction which is now

Figure 9. Investigation of parameters effecting H-FMC actuator performance. The parameter study is performed with $E_m = 8$ MPa and $l = 100$ mm. All other values are the medium parameters from study, for example, for the varying radius plots (a) and (b), $t = 1$ mm, $E_f = 579$ GPa and $V_f = 0.6$. 
utilized by the H-FMC, and the large reduction in osmotic pressure this unconstrained radial swelling results in, then this becomes quite reasonable. The benefit of the H-FMC concept diminishes with reducing radius and length, but is found to remain significant for all values examined.

4.3. Parametric study

The second aim of this paper was to examine the effect of varying FMC design parameters, and to compare the resulting trends for a H-FMC actuator, where the actuating pressure is a function of hydrogel hydration (H-FMC volume), and a FMC, where the actuating pressure is constant. The FMC radius, wall thickness, fibre modulus, and fibre volume fraction were varied and the effect on free axial strain (stroke), and blocking stress observed. These results are presented for comparison in figure 9.

In these plots the dashed lines represent the constant pressure FMC case, where the operating pressure is the maximum osmotic pressure observed in the variable pressure case (found for fibre wrap angle $\theta = \pm 55^\circ$). It is observed that the parameter dependent trends in the constant pressure FMC case hold for the variable pressure H-FMC case. However, the benefit of varying a particular parameter is often diminished in the H-FMC case; for example, reducing the wall thickness from 1.5 mm to 0.8 mm with wrap angle $90^\circ$ results in a decrease in axial strain of 95% for the constant pressure case, but only 70% for the variable pressure case.

One of the most interesting results of the parametric study is the contribution of fibre modulus; it has a near negligible effect on strain, but a significant effect on maximum blocking stress when the fibre wrap angle is less than $20^\circ$.

4.4. Actuator comparison

Figure 10 presents a graphical means of actuator comparison based on performance indices of stroke work per unit volume and mass, as suggested by Huber et al [52]. This involves the introduction of the dimensionless stroke work coefficient $C_s$, defined as the ratio of maximum work done in a single stroke to the product of the blocking stress $\sigma$ and free strain $\varepsilon$, and must lie between 0 and 1 for all actuators. For the H-FMC and bare hydrogel actuators this is approximately 0.5. The improvement to the bare actuator performance offered by the H-FMC concept is evident in these charts, with the H-FMC delivering comparable performance to shape memory alloys, pneumatic actuators, and select electro-active polymers in regards to the selected performance indices.

5. Study limitations and recommendations for future work

On the modelling side future work aims to address the major limitations of the current study. This will involve the inclusion of material nonlinearities, and improvement of the inclusion of geometric nonlinearities. It is expected this will enable accurate prediction of the performance of structures incorporating H-FMC actuators. Improving the modelling of the chemical–electrical hydrogel swelling is also recommended in order to better predict swelling close to the hydrogel pH$_a$, whilst if possible moving away from the
computationally expensive and mesh sensitive nonlinear PDE system solved with FE. This could involve the investigation of the use of the Donnan ratio to determine the ion concentrations in the hydrogel and solution domains, or experimental techniques for determining the $P_{os-c_h}$ relationship. Investigation of H-FMC kinetics is also required as part of a refined model. Future modelling may also include H-FMC design optimization, involving the examination of a wider array of FMC laminates.

Preliminary trials have been undertaken to manufacture H-FMC actuators, with significant refinement required in this area. It is expected that this work will conclude with physical static and dynamic testing of H-FMC actuators in order to validate future refined models. Ultimately, the H-FMC concept is to be integrated with a chemical sensing and control system as outlined in the introduction to this paper.

6. Conclusion

This paper has presented the theory and preliminary modelling of the novel H-FMC actuator concept designed for application in a new class of intelligent structure. The key principle that underlines the H-FMC concept is summarized as follows: the three-dimensional swelling of a hydrogel is partially constrained in order to improve the amount of useful work done. The partial constraint provided to the hydrogel by the H-FMC concept reduces hydrogel hydration during swelling by minimizing the volumetric expansion, this serves to maximize the fixed charge density and resulting osmotic pressure, the driving force behind actuation. In addition, the Poisson’s ratio of the anisotropic FMC laminate converts previously unused hydrogel swelling in the radial and circumferential directions into useful axial strains. The model used to achieve this couples chemical and electrical components, represented with the Nernst–Plank and Poisson equations, as well as a linear elastic mechanical material model, encompassing limited geometric nonlinearities. Although this model includes a number of simplifications, it is deemed to be sufficiently accurate for the comparative results presented in this paper. It has been shown that improvements in useful actuation work from a H-FMC with fibre wrap angles $\theta = \pm 20^\circ$ to $30^\circ$ are in the order of 1500% compared to a bare hydrogel. It is also found that the parameters affecting H-FMC strain and blocking force follow the same trends as FMC actuators that operate at constant pressure, but that the magnitude of these trends is reduced. Finally, the H-FMC actuator is shown to compare well to other actuator concepts when considering both stroke work per unit volume and mass. It is hoped that the potential of the H-FMC concept demonstrated in this work will see it develop into a refined actuator design, suitable for a wide variety of applications.

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