What is the present-day status of The Copernican Principle?

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Summary

I point out that according to the Copernican principle our universe is not unique. The way to make sense out of this statement is for us to construct a gravitational instanton that will tunnel out of our vacuum into another, to form a universe other than our Hubble bubble.

Introduction

The Copernican Principle plays the role of a guiding principle in all fundamental theories of physics and in our lives. Arthur Kôstler’s chapter on Copernicus in Sleepwalkers is entitled “The Timid Cannon” owing to the fact that he felt the revolutionary idea he espoused was not acceptable to the public, let alone the authorities that be.

Quite apart from its original formulation that the earth is not the center of the solar system, the Copernican principle has reappeared under different guises throughout the history of science. Its main message that we are not privileged has led to principles of denial that must be respected in formulating fundamental theories of physics. What we came to call principles of denial are in fact basic truths which we do not commonly teach our children as we bring them up. Perhaps the most striking one among them is in thermodynamics where, roughly put, we start with the fact that “there is no free lunch.”

Newton’s formulation of the laws of mechanics as well as Maxwell-Yang-Mills’s field theories of electrodynamics and Einstein’s theory of gravity are based on principles of gauge-invariance and general covariance which are principles of denial that we have inherited from Copernicus. To every principle of denial there corresponds a group of invariance and the field equations of the physical theory must be formulated to transform covariantly under this group.
Our universe, itself, is not unique

The Copernican principles of denial have been so successful in constructing fundamental theories of physics that we have come to regard its original formulation as a historical artifact that has little relevance to the physics of today. Yet, as I shall presently point out, it is still relevant in the context of its original subject, namely cosmology. Soon after Einstein published the general theory of relativity there appeared two of its exact solutions due to Schwarzschild and Friedman that surprised him by their simplicity. Both of these solutions are crucially important for problems of cosmology even though the relevance of the former was recognized only relatively recently. Nowadays a great deal of effort is spent on discussing whether or not ours is a Friedman universe that has closed, open, or flat space sections. Basically this is a question that is only of experimental interest.

I would like to point out that for cosmology the important observation is that our universe, itself, is not unique which surely is the present-day formulation of the Copernican principle.

At first this appears to be an empty statement since we cannot be in communication with any observers in other universes. However, we can make sense of it by designing an experiment, albeit a gedanken one for the present, that will enable us to create an instanton to tunnel out of our vacuum into another and subsequently form a new universe.

Tunnelling between two different vacua is an idea that first appeared in Yang-Mills theories. The “particles” mediating this transition are localized in time, rather than the familiar ones localized in space. Thus they are bound states of the theory with Euclidean signature and are called instantons. There are many parallels between gauge theories and general relativity, so it is tempting to consider gravitational instantons which are solutions of the vacuum Einstein field equations with Euclidean signature. However, we cannot do so naively.

It was shown by Utiyama that Riemannian geometry of space-time can be understood as a gauge theory for the Lorentz group along the lines Yang and Mills had put forth. However, from the perspective of gauge theory the field equations of Maxwell and Yang-Mills are unlike those of Einstein owing to a coincidence between the dimension of the Lorentz group and the space of 2-forms on space-time. This leads to the possibility of constructing a new Lagrangian, Hilbert’s Lagrangian for gravity which is radically different from the Lagrangian for gauge theories. It is an important point to keep in
mind when we are considering gravitational instantons as opposed to Yang-Mills instantons. In the Yang-Mills case the transition between different vacua is constructed using analytic continuation. In contrast, for the case of gravity Calabi has shown that Ricci-flat Riemannian spaces are governed by the elliptic complex Monge-Ampère equation. It is remarkable that for Euclidean signature there is a single equation that replaces all of the Einstein field equations. Furthermore Chern, Levine and Nirenberg had shown that the parabolic complex Monge-Ampère equation replaces Laplace’s equation in the theory of functions of many complex variables. The upshot of this is that solutions of the complex Monge-Ampère equation are notoriously not amendable to analytic continuation. Thus, except in a few very special cases, starting with exact solutions of Euclidean signature the passage to Lorentzian signature remains as a fundamental difficulty. We must radically refine our ideas about tunnelling between different vacua in the case of gravity.

The crucial gravitational instanton is the $K3$ surface that Kummer introduced over a century and half ago. Even though we know a great deal of its properties from the powerful index theorem of Atiyah and Singer, we were no closer to knowing its explicit form than Kummer’s characterization of it as a quartic in complex projective space of 3 dimensions. This situation can be compared to a hypothetical historical scenario for black holes. If we had not known the Kerr solution, we would still be able to prove theorems about black holes. But without knowledge of the exact geometry of a black hole, it would be very difficult to know its precise nature.

The principal difficulty in the construction of the metric on $K3$ lies in the fact that it admits no continuous symmetries. In view of the importance of $K3$ as the most important gravitational instanton which will radically Copernicize our ideas about cosmology, my friends Andrei Malykh, Misha Sheftel’ and I embarked on a long project to study non-invariant solutions of the complex Monge-Ampère equation. Recently this has borne fruit [1]-[3] and we were able to construct anti-self-dual metrics, which are therefore Ricci-flat, without any Killing vectors. The method of constructing the solution, the use of partner symmetries, is a novel one in mathematical physics. At about the same time Maciej Dunajski and Lionel Mason [4], also close friends from Penrose’s group, had introduced ideas which in spirit are very similar to our method.

At the moment it is not clear whether, or not some parts of our solutions characterize the metric on $K3$. However, we now have a novel method to construct non-invariant solutions of non-linear partial differential equations.
The issue of the relevance of these solutions to $K3$ which will play such an important role in our present-day idea of the Copernican principle is still under active investigation.

References

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