Peeking though the Colored Looking Glass: A Perspective on New Directions

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Abstract. I discuss the Color Glass Condensate as a media. I argue that Pomerons, Odderon
s and Reggeons are the small fluctuation excitations of this media. I argue that understanding
the effects of Pomeron loops leads to the idea that this media has a duality symmetry. I discuss
the implications of the Color Glass Condensate for the initial conditions at RHIC.

1. Introduction
In the last several years, an understanding of the wee partons which control the interaction of
high energy particles has emerged.[1] These partons are largely gluons, and they control the
properties of hadrons when the longitudinal momentum of the gluon scaled by that of the high
energy hadron,

\[ x = \frac{p^+_{\text{gluon}}}{P^+_{\text{hadron}}} \]  

is \( x \leq 10^{-2} \). In this region, the number density of gluons is so large that the typical separation
between gluons is much less than the size of the hadron. The total number of gluons is large.
This is the limit in which this gluonic matter can be treated as bulk matter.

This high energy density gluonic matter is called the Color Glass Condensate[2, 3, 4, 5, 6, 7, 8, 9, 10]. The name arises because: The gluons are colored. The matter is ultimately made from the very high energy, large \( x \), constituents of the hadron, and in making the low \( x \) matter, the time scale of evolution, which is Lorentz dilated relative
to natural time scales, is transferred to the low \( x \) gluons. Therefore the low \( x \) gluons evolve on
time scales enormous relative to their natural time scale. This is the property of a glass. Finally
the gluons condense inside the hadron because of a negative potential energy produced by the
high \( x \) gluons. This is stabilized by repulsive interactions among the gluons, which occurs when
\( 1/\alpha_S \) gluons fill each phase space cell, so that

\[ \alpha \frac{dN}{dy d^2 p_T d\eta} \sim 1 \]  

When phase space densities are large, the quantum occupation number per state is large, and
the system is a condensate, with properties similar to traditional Bose-Einstein condensates.
This means the condensate is highly coherent, and there are large classical chromodynamic
fields present. Also, the coupling constant is weak, since the typical separation between gluons is small.

This Color Glass Condensate is a media. In this talk, I will argue that the small amplitude excitations of this media are the Pomerons, Odderons and Reggeons of S-matrix theory.\cite{11, 12} These excitations can form quantum virtual fluctuations, which in the language of Feynman diagrams appear as loops. I will argue that understanding these loops leads to a theory with a duality symmetry.\cite{13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23}

In the last section, I discuss the implications of the CGC for the formation of the Quark Gluon Plasma.\cite{24, 25, 26, 27, 28, 29} There has been much excitement recently concerning the rapid thermalization of the QGP which has been observed at RHIC. This has led to the idea that in the early stages of the collision, the interactions are quite strong. One possibility is that this is due to intrinsic strong self coupling of the system. The other possibility, suggested by the CGC is that this is due to a weakly coupled system which interacts coherently.

Coherent interactions are well known to amplify the strength of intrinsically weak interactions. An example of this build up of interaction strength is gravity. The strongest force we experience daily is that of gravity, which arises from the coherent interaction with all of the constituents of the earth. This is in spite of the fact that gravity is intrinsically the weakest force between elementary particles.

I would like to begin this talk with an advertisement: The LHC heavy ion program will be largely about the Color Glass Condensate. The values of x and transverse momenta at LHC are ideally suited for the study of the CGC, and the order one effects which appear in collisions will be largely determined by CGC effects. Hints of the CGC have been seen in HERA and in RHIC experiments, but a real program can be developed at LHC which can test specific predictions. Alice staring at a colored looking glass is shown in Fig. 1.

2. Reggeons, Pomerons and Odderons
Reggeons are mathematical objects which were originally proposed as an explanation for Regge poles which appear in S-matrix theory. (In Fig. 2, you see Prof. Regge with Mikhail Gorbachev

Figure 1. Alice and the looking glass.
when he won the Pomranchuk Prize awarded at the Institute for Theoretical and Experimental Physics in Moscow.) Regge poles arise when one does an angular momentum decomposition of scattering amplitudes. The series representation for these amplitudes are then continued by dispersion theory into an integral in the complex angular momentum plane. It is found that a few simple poles in the complex angular momentum plane control most of the properties of these analytically continued integrals.

Figure 2. Prof. Regge winning the Pomeranchuk prize. Michail Gorbachev is in the background.

The most famous of these poles is that associated with the Pomeron, named after Dr. Pomeranchuk, shown in Fig. 3. This pole controls the high energy limit of scattering amplitudes. It was proven in the early days of Regge theory that the Pomeron leads to universal high energy behaviour for cross sections of strongly interacting particles. It was also argued, most notably by Gribov, that the Pomerons will interact among themselves, and behave like the elementary excitations of an effective field theory.[30, 31, 32, 33] In effect, Gribov showed that Pomerons were the quasi-particles of some as yet to be defined media, since their self interactions has all of these characteristics.

The odderon is the Pomeron’s younger brother, see Fig. 4.[34] The Pomeron is the Reggeon responsible for the high energy limit of the real part of the S matrix. The Odderon controls the imaginary part. The Pomeron corresponds to charge conjugation even exchange of quantum numbers in high energy collisions, and the Odderon to C odd. Recall that two gluons make a C even color singlet state and the three gluons correspond to C odd. Therefore, insofar as the Pomeron and the Odderon can be thought of as composed of gluons, the Pomeron is the two gluon configuration and the odderon is the three gluon.

The Odderon has never been seen experimentally, whereas there is very good evidence for the Pomeron. This odd state of affairs is similar to living in a world where we knew of the existence of mesons (2 quark states) but had never seen baryons (2 quark states).

3. The Pomeron: A Modern Perspective
In spite of many years of experimental and theoretical work on strong interactions physics, some of the simplest and most common phenomena are only recently becoming understood. For example, the total cross section as a function of energy is shown in Fig. 5 The total cross section is a slowly varying function of energy, and it is widely believed that it saturates the unitarity limit,

\[ \sigma \sim \ln^2(E/E_0) \]  
(3.3)
where $E$ is the center of mass energy. The pomeron was originally introduced to explain constant total cross sections, and the data which showed a rising cross section which originated in the Serpukhov experiments, was a big surprise.

Later, Gribov and colleagues suggested that there was a perturbation theory associated with Pomeron dynamics, and that the bare or lowest order Pomeron actually would predict a power law growth of the cross section with energy. This means that at high energies, the interactions of Pomerons have to become strong, since unitarity requires that these interactions must decrease the rise of the cross section to $\ln^2(E/E)$. This led to the invention of Gribov’s Reggeon calculus.
In modern language, we can think about the bare Pomeron as being composed of gluons.\[11\] The rise with energy of the bare Pomeron occurs because the density of gluons is rising at small $x$ like a power $1/x^\delta$. The phenomena of saturation of the gluon densities must occur because of the generation of a high density Pomeron (in old language) or gluon (in modern language) state. In this way of thinking about the problem, it is natural to imagine that the Pomeron is a dilute quasi-particle excitation of the Color Glass Condensate. This might be the case at for example low energies, or in some limited range of relatively large transverse momentum at high energy.

Using these simple ideas, we can understand how the cross section reaches its unitarity bound.\[35, 36\] Assume we have a distribution of gluons inside a hadron’s transverse space,

$$\frac{dN}{dyd^2r_T} = Q_{sat}^2(y)e^{-2m_{*}r_T}$$  \hspace{1cm} (3.4)

The exponentially falling behaviour in transverse space follows from general conditions for an isosinglet distribution of gluons. The factor of $Q_{sat}^2(y)$ is the total number of gluons per unit rapidity, and grows like $e^{\delta y}$. In scattering, a probe of a hadron will interact after it passes through the hadron and on the average some fixed number of gluons,

$$e^{\delta y}e^{-2m_{*}r_T} \sim \text{constant}$$  \hspace{1cm} (3.5)

The cross section is therefore

$$\sigma \sim r_T^2 \sim y^2 \sim \ln^2(E/\Lambda_{QCD})$$  \hspace{1cm} (3.6)

and saturates the Froissart bound. Saturation of the Froissart bound arises due to a tradeoff of the exponential growth of the number of gluons as a function of rapidity compared to the exponentially falling transverse profile.

4. The Pomeron and the Color Glass Condensate

The Color Glass Condensate description of gluonic matter is given by a renormalization group. The object of computation is the function which controls the fluctuations in the number distribution of gluons $F[\rho]$. If we have a longitudinal momentum cutoff $\Lambda^+$, the theory is
described by Yang-Mills fields, sources of glue ρ, the Yang-Mills action and F[ρ]. The density matrix is
\[ \int_{\Lambda^+} [dA][d\rho] e^{iS[A,\rho]}e^{-F[\rho]} \] (4.7)

If we define
\[ Z[\rho] = e^{-F[\rho]} \] (4.8)
and
\[ \eta = \ln(1/x) \] (4.9)
then Z satisfies the renormalization group equation
\[ \frac{d}{d\eta} Z = -H(\rho, d/d\rho)Z \] (4.10)
This is the equation for Euclidean time evolution of a Hamiltonian system. In this equation, η plays the role of a Euclidean time. For weak sources and small derivatives, this Hamiltonian is
\[ H = \kappa_\alpha \int [dx][dy][dz] \frac{(x-y)^2}{(x-z)^2(y-z)^2} \]
\[ \frac{d}{d\rho(x)} (\rho(x) - \rho(z)) \frac{d}{d\rho(y)} \]
(4.11)
where κ is a numerical constant.

Correlation functions involving the Pomeron are the simplest operators one can construct
\[ \rho(x) \cdot \rho(y) \] (4.12)

5. The Pomeron and Saturation
The operators which describe the Pomeron can be generalized to an operator which describes the pomeron in a highly dense and coherent media. It is the so called dipole operator[37, 38]
\[ O(x,y) = 1 - \frac{1}{N_c} < tr(U(x)U^\dagger(y)) > \] (5.13)
where
\[ U(x) = P exp \left( i \int dx - \frac{1}{\sqrt{2}} \rho(x) \right) \] (5.14)
where \( x^\pm = (t \pm z)/\sqrt{2} \). These dipole operators exist only in the transverse plane. They describe a pomeron and arise from considering a dipole pair of quark-antiqaurk in the color glass field. They solve the Balitsky-JIMWLK equation[39, 7, 8, 40, 9]
\[ \frac{d}{d\eta} O(x,y) = \int [dz] \frac{(x-y)^2}{(x-z)^2(y-z)^2} \]
\[ (O(x,z) + O(z,y) - O(x,y) - O(x,z)O(z,y)) \] (5.15)
The real part of O is the Pomeron amplitude. This equation is the Balitsky-Kovchegov equation in mean field approximation and has been the subject of much study as a paradigm for saturation.

The Balitsky-Kovchegov equation give exponential growth in y for the Pomeron amplitude at transverse momentum scales larger than that of the saturation momentum, \( Q_{sat}(y) \). There is power law growth in y at smaller transverse momenta. The saturation momenta never saturates and grows exponentially in y.
6. The Odderon
The Odderon is the C-odd partner of the pomeron. It is made of three gluons, and in weak field is written as

\[ d_{abc} \rho^a(x) \rho^b(y) \rho^c(z) \]  \hspace{2cm} (6.16)

In strong field for coupling to mesons, it can be taken to be the imaginary part of the dipole operator \( O \). Recall that the Pomeron is the real part of this operator. For coupling to baryons when the fields are strong in SU(3) gauge theory, it is given by the imaginary part of

\[ \epsilon_{\alpha\beta\gamma} \epsilon^{\alpha'\beta'\gamma'} U_{\alpha\alpha'} U_{\beta\beta'} U_{\gamma\gamma'} \]  \hspace{2cm} (6.17)

Note that the real part of the baryon amplitude is the topological operator for baryon number. At high energies, the amplitude for the Odderon can be shown to behave as

\[ |A_{\text{Odderon}}|^2 \sim \text{constant} \]  \hspace{2cm} (6.18)

The Pomeron is the simplest excitation of the Color Glass Condensate. The Odderon is a closely related, and is the second simplest quasi-particle excitation. The Odderon has never been experimentally probed.

7. Reggeons
There are of course many more quasi-particle excitations of the Color Glass Condensate. Reggeons are objects made of \( N \) gluons: [41, 42, 43]

\[ \rho_1 \cdots \rho_N \]  \hspace{2cm} (7.19)

Computing the effects of Reggeons involves solving a \( N \) body problem in QCD. This problem may be simpler in the limit of a large number of colors \( N_c \). In large \( N_c \), the Reggeons form a chain with only nearest neighbor interactions. If one further assumes that this amplitude vanishes whenever any two coordinates of the chain are equal, then this JIMWLK equation which describes this evolution is conformally separable. It has been conjectured by Lipatov-Faddeev-Korchemsky that this system may be exactly solvable and is a conformal field theory. It is not known what happens when the conformal separability ansatz is relaxed, nor how to go beyond large \( N_c \), nor how to deal with reggeons made of quarks.

8. Ploops
If one has quasi-particle excitations, these excitations may interact among themselves. They may have quantum interactions and generate loops. Loops made of Pomerons, we will call ploops.[44, 45, 46, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23]

The JIMWLK equation has no ploop. It describes tree graphs and has the structure that a j-gluon amplitude describing a pomeron evolves as

\[ \frac{d}{dy} A_j = \sum_{j=k}^\infty c_k A_k \]  \hspace{2cm} (8.20)

The evolution always involves more or equal number gluons than was in the original amplitude. This structure will always result when one considers strong fields, since in this limit the amplitude with larger numbers of gluons in them are of increasing importance.

To generate ploops, one needs to have terms with \( k \leq j \) in the sum. These terms are clearly only important in the weak field limit.
We see that the introduction of ploops, introduces the idea of duality. For strong field evolution, the theory involves non-linearities in $\rho$ since the multi-field amplitudes above are generated by inserting more powers of $\rho$ in the JIMWLK Hamiltonian. On the other hand, the weak field limit is generated by powers of $d/d\rho$. A full theory with ploops must include effects of both non-linearities in $\rho$ and non-linearities in $d/d\rho$. There should be some duality relationship between $\rho$ and $d/d\rho$. (After this lecture was prepared, there have appeared many conjectures that ploop theory is self-dual.)

9. The Strongly Interacting QGP and the CGC
There is now compelling evidence that a Quark Gluon Plasma has been produced at RHIC.[24, 25] This has been discussed in several talks at this meeting, and I shall not repeat this discussion here. Suffice it to say, that one the most surprising pieces of information arising from the RHIC experiments is that the matter produced interacts with itself very strongly very early in the collision. This has led many of us to call the matter produced at RHIC as the sQGP: The strongly interacting Quark Gluon Plasma.

The Color Glass Condensate provides a theory of how the Quark Gluon Plasma originates in heavy ion collisions, as shown in Fig. 6. The coherent gluon fields of the Color Glass Condensate's of the two nuclei become time dependent when the nuclei pass through one another. As time passes, these time dependent fields become diluted by expansion and evolve into gluon radiation fields.

![Figure 6](image.png)

Figure 6. The evolution of matter produced in heavy ion collisions.

The details of this process are interesting. One has highly coherent fields below some momentum scale. These fields have a strength $1/g$ so that a particle interacting with them always has strength $g \times 1/g \sim 1$. When the typical momentum scale associated with these soft
fields becomes less than or of the order of the typical momentum scale of all the degrees of freedom, then this strong interaction is of decreasing importance.

The bottom line is, however, that the CGC predicts very strong interactions at early time. The issue is how these fields thermalize. Recently, it has been proposed that these fields evolve into a strongly interacting plasma. This plasma is probably not correctly thought of as thermalized, yet longitudinal plasma instabilities induce a isotropic system. Therefore the magneto-hydrodynamic equation will predict behaviour very similar to that of a system in thermal equilibrium.[27, 28, 29]

This description contrasts with that proposed by Shuryak and collaborators where the system is thermalized by a complicated configuration of resonant states near \( T_c \).[26] The test of these pictures is simple: At LHC where the initial energy per particle is significantly higher, Shuryak’s scenario would have less early thermalization than is the case due to plasma instabilities.

10. Perspective for the LHC
The LHC is a machine which will test our ideas of the Color Glass Condensate. In the central region of such collisions, typical \( x \sim 10^{-4} \) One can probe much smaller \( x \) values and high \( p_T \) by measuring produced particles in the forward region. There will be truly excellent test of the ideas of universality and evolution equations.

This is good for the CGC, but will make life a little more complicated for the studies of the QGP. Because we expect that the gluon distribution functions in nuclei will be much modified relative to \( A \) times that in a proton, they will have to be measured directly before QGP tests become unambiguous. This requires good \( pA \) or \( dA \) and \( pp \) measurements, or alternatively, good \( \gamma - jet \) measurement with realistic extrapolation to \( Q^2 \) values of order a GeV. With this information in hand, one has potentially very interesting information about jet quenching due to the QGP or jet modification due to the CGC.

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