A dynamical model of radiating stars and their thermal behavior

Grishma Verma\textsuperscript{a}, Rajesh Kumar\textsuperscript{b}\textsuperscript{,}\textsuperscript{c}, \textsuperscript{d}

Department of Mathematics and Statistics, Deen Dayal Upadhyaya Gorakhpur University, Gorakhpur, India

Received: 19 March 2022 / Accepted: 6 May 2022 / Published online: 6 June 2022

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Abstract In this paper, we have studied a new class dynamical model of spherically symmetric radiating collapsing stars. The solution of field equations has been discussed by using the boundary condition and obtained all the physical parameters in terms of mass $M$ and surface radius $R_0$ which will be significant in astrophysical applications. We have also studied the thermal behavior of the stars and discussed the horizon formation and its temperature. We have shown that horizon temperature increases with time under certain conditions.

1 Introduction

General relativity theory (GTR) has an important role in the studies of gravitational collapse of massive stars and their remnants like neutron stars or black-holes [1–4]. A star that loses its equilibrium, collapse under its own gravity and ultimately culminates in the state of space-time singularity. The state of space-time singularity may be naked (light rays emanating from it can reach any far-away observer) or, a black hole (implying that the light rays from it may be trapped within a surface called its event horizon). The studies of mathematical models of gravitationally collapsing systems are expected to play a key role in the understanding of the nature of collapsing stellar system and its singularities [5,6]. Therefore, a comprehensive description of gravitational collapse and the modeling of the structure of compact objects under a variety of circumstances remain among the most interesting problems that general relativity theory has to deal with [7–12]. This fact elucidates the attraction that these problems exert on the community of the relativists. The collapsing gravitating system possesses different stages of evolutions involving physical quantities that determine the end state of relativistic fluid configuration [7,13–18].

The space-time in the exterior region of the radiating stars is known to be described by Vaidya metric [9,10]. The formulation of the junction conditions smoothly joining the interior space-time metric of the collapsing matter across its boundary with the appropriate form of Vaidya metric, as first proposed by Santos [11] has given an impetuous for studies in this direction. The thermodynamical nature of black holes in general relativity was first noted by Hawking and his collaborators in their works that established what are known now as the laws of black hole thermodynamics [19,20]. These thermodynamics laws study the area and surface gravity of the event horizon of a stationary black hole in general relativity to the entropy and temperature of a thermodynamic system at equilibrium. A lot of effort has been devoted in the studies of the connection between gravity and thermodynamics that the analogy can be extended to the theory, making it valid not only on the horizon-surface, and therefore showing possibly how gravity could be considered as a thermodynamic theory [21–25].

In the present work, we have considered spherically symmetric relativistic star, whose collapse is accompanied with dissipation in the form of radial heat flux. The geometry of the interior space-time of the collapsing system is chosen to be conformal to that of FLRW metric so that the Weyl tensor is known to vanish (conformally flat). Conformally flat space-times, in the context of radiating star, were first studied by Som and Santos [26]. Later, the most general class of conformally flat solutions for a shear-free radiating star were obtained and examined by various researchers [5,6,27–29]. Herrera et al. [29] have critically examined models of shear-free collapsing fluids accompanied with dissipation of heat on the space-time background subject to the constraint that the associated Weyl tensor should vanish. This paper is organised as follows-after introduction, in Sect. 2, we have provided the field equations for the spherically symmetric conformally flat metric and energy momentum tensor for the interior space time is that of perfect fluid with heat flux. In Sect. 3, the junction conditions (boundary condi-
tions) required for the smooth matching of the interior and exterior space times across a time-like hypersurface are presented and luminosity and red shift expression are obtained. The transport equations for the dissipative fluxes within the framework of non causal thermodynamics are also considered here. The dynamical model of radiating collapsing star is discussed by using boundary condition for \( k = 0 \) and \( k \neq 0 \) in Sect. 4. Last section contains the discussion and concluding remarks.

2 Interior space-time and field equation

Consider a class of spherically symmetric conformally flat space time which describes the interior of collapsing fluids undergoing dissipation in the form of heat flow,

\[
ds_0^2 = \frac{1}{R^2(t, r)} \left[ -dt^2 + \frac{dr^2}{(1 - kr^2)} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right]
\]

(1)

The interior of the collapsing fluid considered to be here perfect fluid with heat flux, is represented by the energy momentum tensor,

\[
T_{ij} = (p + \rho)v_i v_j + p g_{ij} + q_i v_j + v_i q_j
\]

(2)

where \( p \) is isotropic pressure and \( \rho \) is the energy density, \( v_i \) and \( q_i \) are four velocity of fluid and radial heat flux vector respectively, satisfying

\[
v^i = R_0 \delta^i_0, \quad q^i = q \delta^i_1
\]

\[
v^i v_i = -1, \quad v^i q_i = 0
\]

(3)

where \( q \) is function of \( (t, r) \) represents heat flux of radiation.

The shearing tensor \( \sigma_{ij} \) is define as,

\[
\sigma_{ij} = v_{(i;j)} + \frac{1}{2} \left( a_i v^j + a_j v^i \right) - \frac{1}{3} \Theta \left( g_{ij} + v_i v_j \right)
\]

(4)

with the acceleration vector \( (a_i) \) and expansion scalar \( (\theta) \)

\[
a_i = v_{(i;j)} v^j \quad \Theta = v^i_i
\]

(5)

For line-element (1), one obtained the four acceleration vector

\[
a_i = -R'R_{R} \delta^i_1 \quad (i = 0, 1, 2, 3)
\]

(6)

and the expansion scalar

\[
\Theta = -3 \dot{R}
\]

(7)

For gravitational collapsing configuration,

\[
\Theta < 0 \iff \dot{R} > 0
\]

(8)

The Einstein field equations \( G_{ij} = \alpha T_{ij} \) (where \( \alpha = \frac{8\pi G}{c^4} \)) for (1) and (2) yield the following equations,

\[
\alpha p = 3kR^2 + 3\dot{R}^2 - 3(1 - kr^2)R^2
\]

\[
+2(1 - kR^2)RR'' + \frac{2}{r}(2 - 3kr^2)RR'
\]

\[
\alpha p = -kR^2 + 2\dot{R} - 3\dot{R}^2
\]

\[
+3(1 - kr^2)R^2 - \frac{4}{r}(1 - kr^2)RR'
\]

\[
\alpha p = -kR^2 + 2\dot{R} - 3\dot{R}^2 + 3(1 - kr^2)R^2 - 2(1 - kr^2)RR'' - \frac{2}{r}(1 - 2kr^2)RR'
\]

\[
\alpha q = -2(1 - kr^2)\dot{R}^2\dot{R}'
\]

(9)

(10)

(11)

(12)

where dot (‘) and dash (‘’) represent the derivative with respect to \( t \) and \( r \) respectively.

The mass function which describes the mass of collapsing star at \( (t, r) \) is given by [30, 31]

\[
m(t, r) = \frac{1}{2R} \left[ kr^2 + \frac{R'}{R} R^2 - r^2 R^2 \left( 1 - kr^2 \right) \right]
\]

\[
+2r R' \left( 1 - kr^2 \right)
\]

(13)

3 The junction condition and the thermodynamics

The spherically symmetric collapsing fluid which undergoing dissipation in the form of heat flow is bounded by a timelike spherical surface \( \sum \) and it described by the intrinsic metric

\[
ds_\sum^2 = g_{ij}dx^i dx^j = -dt^2 + \frac{r^2}{R_0^2(\tau)} \left( d\theta^2 + \sin^2 \theta d\phi^2 \right)
\]

(14)

where \( \xi = (\tau, \theta, \phi) \) is the intrinsic co-ordinate and \( R_0(\tau) \) intrinsic geometrical radius. The exterior space-time to a radiating star is represented by Vaidya metric which is given by [11, 32, 33]

\[
ds_{\mathbb{R}}^2 = -\left( 1 - \frac{2M(v)}{\Omega} \right)dv^2 - 2dv d\Omega + \Omega^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right)
\]

(15)

with co-ordinate \( x^t = (v, \Omega, \theta, \phi) \) and \( M(v) \) is the mass of star is function of retarded time \( v \) i.e. Newtonian mass of gravitating star as measured by distant observer. The timelike spherical surface \( \sum \) separates the interior space time \( ds_0^2 \) to exterior space time \( ds_{\mathbb{R}}^2 \). The metric (15) is the unique
solution of the spherically symmetric field for the radiating star and the Einstein’s tensors for this,

\[ G_{ij} = -\frac{2}{\Omega^2} \frac{dM}{dv} \delta_i^j \delta_j^i, \quad i, j = 0, 1, 2, 3 \]  

(16)

The energy momentum tensor for radiation (exterior to star)

\[ T_{ij}^+ = \epsilon w_i w_j \]  

(17)

where \(\epsilon\) is energy density of radiation and \(w_i = (1, 0, 0, 0)\) the four velocity vector. Thus from (16) and (17) the field equations produced,

\[ \alpha \epsilon \Sigma = -\frac{2}{\Omega^2} \frac{dM}{dv} \left( \frac{dv}{d\tau} \right)^2 \]  

(18)

which is energy density of radiation measured by an observer on \(\Sigma\). Since the star is radiating (\(\epsilon > 0\)) to the exterior space time, therefore it must have \(\frac{dM}{dv} \leq 0\) that is, mass of the star diminishes during radiation [7,11,32,33].

The junction condition require the continuity of the metric (1) and (15) at surface \(\Sigma\),

\[ (ds^2)\Sigma = ds_+^2 = (ds^2)_\Sigma \]  

(19)

known as continuity of first fundamental form and

\[ [K^+_{ij}]= [K_{ij}]\Sigma \]  

(20)

known as continuity of second fundamental form, where

\[ K^+_{ij} = -\eta^+_{ij} \frac{\partial x^k_m}{\partial \xi^i} + \eta^+_{ij} \Gamma^k_{mn} \frac{\partial x^m}{\partial \xi^i} \frac{\partial x^n}{\partial \xi^j} \]

and \(\eta^+_{ij}\) represent the extrinsic curvature and the normal vector to the surface \(\Sigma\) in \(x^+\) coordinate respectively. Using the junction condition (19) with the metric (1), (14) and (15), one obtain

\[ \frac{dt}{d\tau} \Sigma = R(r_\Sigma, \tau) \]  

(21)

\[ R(r_\Sigma, \tau) \equiv R_0(\tau) \]  

(22)

\[ \Omega \equiv \frac{\Sigma}{R_0(\tau)} \]  

(23)

\[ \left( \frac{dv}{d\tau} \right)^2 \Sigma \equiv \left( 2\frac{d\Omega}{dv} + 1 - \frac{2M(v)}{\Omega} \right)^{-1} \]  

(24)

The non-zero components of extrinsic curvature for the line-element (1) and (15) are,

\[ K_{\tau \tau} \Sigma \equiv \sqrt{1 - kR^2} R'(r, \tau) \]  

(25)

\[ K_{\theta \theta} \Sigma \equiv \left( \frac{r}{\sqrt{1 - kR^2}} - \frac{r^2}{R^2 \sqrt{1 - kR^2}} \frac{R'}{R} \right) \]

\[ = K_{\phi \phi} \text{cosec}^2 \theta \]  

(26)

and

\[ K_{\tau \tau}^+ \Sigma \equiv \frac{d^2v}{d\tau^2} \left( \frac{dv}{d\tau} \right)^{-1} - \frac{M dv}{\Omega^2 d\tau} \]  

\[ K_{\theta \theta}^+ \Sigma \equiv \Omega \frac{d\Omega}{dv} + \Omega \frac{dv}{d\tau} \left( 1 - \frac{2M}{\Omega} \right) = K_{\phi \phi} \text{cosec}^2 \theta \]  

(27)

(28)

Now using the Eqs. (26) and (28), the condition (20) gives

\[ M(\nu) = \left[ \frac{1}{\nu} \frac{R'}{R} \left( kr^2 + R^2 \right) \right] \sum = \frac{r}{\nu} \left( kr^2 \right) \]  

(29)

by taking use of Eqs. (21)–(24) with Eqs. (25) and (27), the condition (20) gives

\[ p \equiv \left( \frac{q}{\nu} \right) \]  

(30)

The energy density of radiation \(\epsilon\) measured by an observed on surface \(\Sigma\)

\[ \alpha \epsilon \Sigma \equiv -\frac{2}{\Omega^2} \frac{dM}{dv} \left( \frac{dv}{d\tau} \right)^2 \]  

(31)

The luminosity observed on the surface \(\Sigma\)

\[ L_\Sigma = -\frac{dM}{dv} \left( \frac{dv}{d\tau} \right) \]  

(32)

and boundary redshift

\[ 1 + Z_\Sigma = \left( \frac{dv}{d\tau} \right) \sum \]  

(33)

The luminosity observed for distant observer at infinity

\[ L_\infty = -\left( \frac{dm}{dv} \right) \sum \equiv -\left[ \frac{dm}{dt} \frac{dt}{d\tau} \left( \frac{dv}{d\tau} \right)^{-1} \right] \sum \]  

(34)

3.1 Thermodynamics

The evolution of a system such as a massive star undergoing gravitational collapse, the temperature profile and luminosity are necessary parameters for discussion [7,24,25]. We consider the non casual thermodynamics to obtain the temperature of collapsing star. The first order Eckart formalism was used to study the temperature where heat flux is governed by the Fourier’s law,

\[ q^i = -\kappa h^{ij} \left( \frac{\partial T}{\partial x^j} + \mathbf{T}a_j \right) \]  

(35)
From Eqs. (10) and (11), one obtained

\[ k = k_T \text{ observer} \]  \[ a = \delta \]  \[ \delta = \frac{\pi^2 k_1^4}{15h^3} \]

where for photon the constant \( \delta \) is given by

where \( k_1 \) and \( h \) denoting respectively Boltzmann’s and Plank’s constant.

4 Dynamics of the collapsing star

From Eqs. (10) and (11), one obtained

\[ \frac{R''}{R} - \frac{R'}{rR} \left( 1 - kr^2 \right) = 0 \]  \[ R(t, t) = \lambda_1(t)r^2 + \mu_1(t) \]  \[ \lambda_1(t) = \frac{1}{2} R_0 \left( 1 \pm \sqrt{\left( 1 - \frac{2M}{r_0} \right) + \frac{R_0^4}{r_0^2} \left( \frac{R_0^2}{r_0^2} \right)^2} \right) \]  \[ \mu_2(t) = \frac{1}{2} R_0 \left( 1 \mp \sqrt{\left( 1 - \frac{2M}{r_0} \right) + \frac{R_0^4}{r_0^2} \left( \frac{R_0^2}{r_0^2} \right)^2} \right) \]

Thus Eq. (41) now becomes

\[ R(t, r) = \frac{1}{2} R_0 \left( 1 + \frac{r^2}{r_0^2} \right) + \left( 1 - \frac{r^2}{r_0^2} \right) \sqrt{1 - \frac{2M}{r_0} + \frac{R_0^4}{r_0^2} \left( \frac{R_0^2}{r_0^2} \right)^2} \]

for collapsing configuration \( r < r_0^2 \). Here \( R_0 \) is the intrinsic geometrical radius of star, \( R_0^* = \frac{dR_0}{dt} \) and \( M \) is the mass (Newtonion) of the star at surface \( r_0^2 \).

Taking use of Eq. (45) into Eqs. (9)–(12), one obtain

\[ \alpha_p = \frac{3}{2} \left[ \frac{1}{2} \left( 1 - \frac{r^2}{r_0^2} \right) \left( \frac{R_0^*}{R_0} + \frac{R_0^4}{2R_0^2} \right) + \frac{R_0^4}{2R_0^2} \left( 1 + \frac{r^2}{r_0^2} \right) \right]^2 \]

\[ \alpha_p = \frac{R_0^*}{2r_0^2} \left( 1 - \frac{r^2}{r_0^2} \right) \times \left[ \frac{R_0^*}{R_0} C - \frac{R_0^2}{R_0} - C \right] \]

\[ \lambda_2(t) + \mu_2(t) \]

where \( \lambda_1, \lambda_2, \mu_1 \) and \( \mu_2 \) are arbitrary function of 't'. Further we shall consider the cases for \( k = 0 \) and \( k \neq 0 \) separately.

4.1 Case I: \( k = 0 \)

The collapsing fluid configuration of this class is described by Eq. (41). To determine the orbitay constants \( \lambda_1, \mu_1 \), we use the boundary conditions given by Eqs. (22)–(29).

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The orbitrary function interaction with matter through the diffusive approximation

Using Eqs. (10), (13), (21), (24) and (45) into Eqs. (33) and (34) one obtained

\[ 1 + Z = \left( \frac{2M R_0^*}{R_t} - \frac{r_{\Sigma} R_0^*}{R_0^2} \right)^{-1} \]  

and

\[ L_\infty = \frac{a}{2} \left[ q R_0^2 \left( \frac{2M R_0^*}{R_t} - \frac{r_{\Sigma} R_0^*}{R_0^2} \right)^2 \right] \]

From Eqs. (56) and (57) one can observe that the horizon formation \((L_\infty \to 0, q \neq 0)\) occurs when \((r_{\Sigma} - 2M R_0) \to 0\)

In view of Eqs. (12) and (36), we have

\[ \alpha T^b R^2 \left( T' - T \frac{R}{R'} \right) = 2 \frac{\dot{R}'}{\alpha} \]

by integrating above, gives

\[ T^{b+1} = 2 \frac{(b + 1) R^{b+1}}{aa} R_t \int \frac{\dot{R}'}{R^{b+1}} dr + A_1 R^{b+1} \]  

where \(A_1(t)\) is arbitrary constant.

In Eq. (59), choosing \(b = 3\) which represents radiation interaction with matter through the diffusive approximation [7]. The arbitrary function \(A_1(t)\) is determined by using boundary conditions (38) and Eq. (57),

\[ A_1(t) = \frac{a q_{\Sigma}}{2 \pi \delta R_0^2} \left[ \frac{2M R_0^*}{R_t} - \frac{r_{\Sigma} R_0^*}{R_0^2} \right]^2 \]

Therefore, the temperature inside the star becomes

\[ T^4 = \frac{8}{aa} \int \frac{\dot{R}'}{R^4} dr + \frac{\alpha q_{\Sigma} R_0^4}{2 \pi \delta R_0^2} \left[ \frac{2M R_0^*}{R_t} - \frac{r_{\Sigma} R_0^*}{R_0^2} \right]^2 \]

One can observe from Eq. (61) that \(T_H\) (the temperature of star at the horizon formation \(r = r_H\)) is,

\[ T_H^4 = \frac{8}{aa} R_H^4 \int_{r_H}^{0} \frac{\dot{R}'}{R^4} dr \]

\[ \frac{\dot{T}_H}{T_H} = \frac{\dot{R}_H}{R_H} + \frac{1}{4} \left[ \frac{\partial}{\partial t} \log \left( \int_{r_H}^{0} \frac{\dot{R}'}{R^4} dr \right) \right] \]

where

\[ R_H = R(t, r_H) = R_0 \left[ 1 + \frac{r_H^2}{4M^2 R_0^2} \left( 1 - \frac{R_H^2}{4M^2 R_0^2} \right) \right] \]

From Eq. (45), the mass of collapsing star at horizon surface \(r = r_H\),

\[ M_H = \frac{1}{2} \frac{R_H}{r_H} \left[ \frac{r_H^2 R_H^2}{R_H^2} - \frac{r_H^2 R_H^2}{R_H^2} \right] + 2 R_H \frac{R_H^2}{R_H} \]

where \(R_H\) and \(M_H\) are geometrical radius and mass of collapsing star at horizon \(r = r_H\).

It can be seen from Eqs. (62) and (63) that the temperature \((T_H)\) of star is increases when \(\frac{\dot{r}}{r} \log \left( \int_{r_H}^{0} \frac{\dot{R}'}{R^4} dr \right) > 0\).

4.2 Case II: \(k \neq 0\)

In this case, we discussed the behaviour of collapsing fluids with equation (42), for \(k \neq 0\).

Using boundary condition (22)–(29) to obtain arbitrary constant \(\lambda_2\) and \(\mu_2\),

\[ \lambda_2(t) = \frac{R_0}{r^2_{\Sigma}} \left[ \sqrt{1 - kr_\Sigma^2} \pm C \right] \]

\[ \mu_2(t) = \frac{R_0}{r^2_{\Sigma}} \left[ 1 \pm C \sqrt{1 - kr_\Sigma^2} \right] \]

and the we have

\[ R(t, r) = \frac{R_0}{r^2_{\Sigma}} \left[ 1 - \sqrt{1 - kr_\Sigma^2} \right] \left( 1 - kr_\Sigma^2 \right) \]

\[ + \left( \sqrt{1 - kr_\Sigma^2} - 1 \right) C \]

using Eq. (68) into Eqs. (9)–(12), one obtained

\[ a \rho = 3C^2 \left( \frac{k R_0^2}{r^2_{\Sigma}} + \frac{R_0^2}{k R_0^2 r^2_{\Sigma}} \right) \]

\[ +3C_2 \frac{A^*}{k R_0^2 r^2_{\Sigma}} \left( \frac{1 - kr_\Sigma^2}{\sqrt{1 - kr^2}} \right) \]

\[ + \left( \frac{4k \frac{R_0^2}{R^2_{\Sigma}} C^2}{\sqrt{1 - kr^2}} \right) \]

\[ -3r^2 R_0^2 \left( \frac{1 - kr_\Sigma^2}{\sqrt{1 - kr^2}} \right) \]
\[ \alpha p = C_4 \left( \frac{R_0^2}{k r^*_\Sigma} + \frac{2 R_0^{*2}}{k^2 R_0 r^*_\Sigma} - 5 \frac{A_0^{*2}}{k^2 R_0^4 r^*_\Sigma} \right) \]
\[ + C_4 \left( \sqrt{1 - k r^2} - \sqrt{1 - k r^*_\Sigma} \right) \]
\[ \times \left( - \frac{R_0^2 C_2}{k^2 r^*_\Sigma C} + \frac{R_0^2 C_3}{k^2 r^*_\Sigma C} - \frac{R_0^2 C^2}{2 k^2 r^*_\Sigma C^2} \right) \]
\[ - \frac{4 R_0^2}{k r^*_\Sigma} \left( \sqrt{1 - k r^2} - \sqrt{1 - k r^*_\Sigma} \right) \]
\[ - \frac{3 R_0^2}{4 k^2 r^*_\Sigma} C^2 \left( \sqrt{1 - k r^2} - \sqrt{1 - k r^*_\Sigma} \right)^2 \]
\[ + \frac{3}{k^4} R_0^2 \left( \sqrt{1 - k r^2} - \sqrt{1 - k r^*_\Sigma} \right)^2 \]
\[ \alpha q = -2 r \sqrt{1 - k r^2} \frac{R_0^2 C_4}{r^*_\Sigma} \]
\[ \times \left[ \frac{R_0}{R_0 r^*_\Sigma} \left( \sqrt{1 - k r^2} - \sqrt{1 - k r^*_\Sigma} \right) - \frac{R_0 C_2}{2 r^*_\Sigma C} \right] \]
\[ \text{where } C, C_1, C_2, C_3 \text{ are given by (51)–(55) and} \]
\[ C_4 = 1 - \sqrt{(1 - kr^2_\Sigma)(1 - kr^2)} + \left( \sqrt{1 - kr^2} - \sqrt{1 - kr^2_\Sigma} \right) C \]
\[ 1 + Z_\Sigma = \left( \sqrt{1 - kr^2_\Sigma} - \frac{r_\Sigma \sqrt{1 - kr^2_\Sigma}}{2 M R_0} \right)^{-1} \]
\[ \text{and the luminosity} \]
\[ L_\infty = \frac{\alpha}{2} \left[ \frac{q r^*_\Sigma \sqrt{1 - kr^2_\Sigma}}{R_0^3} \left\{ 1 - \frac{r_\Sigma}{2 M R_0} \right\}^2 \right] \]
\[ \text{where } A_2(t) \text{ is arbitrary constant. The arbitrary constant} \]
\[ A_2(t) \text{ is determined by using Eq. (38)} \]
\[ A_2(t) = \frac{\alpha q \sum \sqrt{1 - kr^2} - r_\Sigma}{2 \pi \delta R_0^5} \left( 1 - \frac{r_\Sigma}{2 M R_0} \right)^2 \]
\[ \text{Therefore,} \]
\[ T^4 = \frac{8 R^4}{a^3} \int \frac{(1 - kr^2) R'}{R^4} dr + A_2 R^4 \]
\[ \text{Thus the temperature of star at the horizon surface } r = r_H \text{ becomes} \]
\[ T_H = \frac{8 R^4}{a^3} \int_{r_H}^0 \frac{(1 - kr^2) \dot{R}'}{R^4} dr \]
\[ \text{and} \]
\[ \frac{H^4}{T_H} = \frac{\dot{H}}{R_H} + \frac{1}{4} \frac{\partial}{\partial t} \log \left( \int_{r_H}^0 (1 - kr^2) \frac{\dot{R}'}{R^4} dr \right) \]
\[ \text{where} \]
\[ R_H = \frac{1}{4 M^2 R_0} \left[ 1 - \sqrt{(1 - 4 k M^2 R_0^2)(1 - kr^2_H)} \right. \]
\[ + \left. \left( \sqrt{1 - kr^2_\Sigma} - \sqrt{1 - 4 k M^2 R_0^2} \right) \frac{2 M R_0^*}{R_0} \right] \]
\[ M_H = \frac{1}{2} R_H \left[ kr^*_H + r_H \frac{R_H^2}{R_H^4} - R_H^2 \frac{R_H^2}{R_H^4} \left( 1 - kr^2_H \right) \right. \]
\[ + 2 r_H \frac{R_H^2}{R_H^4} \left( 1 - kr^2_H \right) \]
\[ \text{where } R_H \text{ and } M_H \text{ are geometrical radius and mass of collapsing star at horizon surface } r = r_H. \]
\[ \text{It can be seen from Eq. (80) } \frac{T_H}{T_H} > 0 \text{ when } \frac{\partial}{\partial t} \log \left( \int_{r_H}^0 (1 - kr^2) \frac{\dot{R}'}{R^4} dr \right) > 0 \]

5 Discussion and concluding remarks

In this paper, we have described the dynamical solution of relativistic conformally flat spherically symmetric collapsing radiating star and discussed its thermal behaviour. We consider interior metric as a class of conformally flat spherically symmetric consists of dissipative perfect fluid distribution and the exterior region of star is described by the Vaidya space time metric. By employing the Darmois-Israel junction conditions for the smooth matching of interior(\( \Sigma^\mathrm{in} \)) and exterior (\( \Sigma^\mathrm{out} \)) spacetime at the boundary \( \Sigma \), we have obtained the boundary conditions (29) and (30) over hypersurface \( \Sigma \). We
have presented new class of solution of spherical collapsing fluid distribution using boundary conditions in two separate cases $k = 0$ and $k \neq 0$. All the physical quantities namely $p$, $\rho$, and $q$ are obtained in terms of $M$(Newtonian mass) and intrinsic surface radius $R_0$. The redshift ($z_{\infty}$) and luminosity ($L_\infty$) of radiating star to distant observer are also calculated in the terms of $M$ and $R_0$. We can see from Eq. (57) and (74) that $L_\infty \rightarrow 0$ when $r_{\infty} - 2M R_0 \rightarrow 0$ (with $q \neq 0$) which is the horizon formation condition. This shows that the stars still emitting radiation after formation of horizon [35]. Further we discussed the noncausal thermodynamics with the heat transport equation which predicts temperatures of the star given by Eqs. (61) and (78) for $k = 0$ and $k \neq 0$ respectively. Also we showed that the temperature of radiating star is increases under certain condition. The mass of the star at horizon surface are also obtained in both cases. Since black-hole are highly thermal objects in the universe and their thermodynamics properties are well understood. The present study determined the event horizon and its temperature in non-static space time. Since in this work the solutions are described in terms of the mass and radius of the stars, therefore the presented model will be significant for the studies of the astrophysical phenomena such as blackhole and high energy compact objects.

**Author contributions** GV did all the calculation parts of article and the draft of the manuscript was written by the RK. Both the authors read and approved the final manuscript.

**Funding** There is no fund available for the publication of this research article.

**Data availability** This manuscript has no associated data or the data will not be deposited. [Authors’ comment: Since in the present work all the studies/solutions are based on mathematically. There is no required any data here.]

**Declarations**

**Conflict of interest** The authors have no relevant financial or non-financial interests to disclose.

**Ethical statements** The submitted work is original and has not been published elsewhere in any form or language (partially or in full).

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