Charming penguin contributions to charmless $B$ decays into two pseudoscalar mesons

C. Isola$^a$, M. Ladisa$^b$, G. Nardulli$^c$, T. N. Pham$^a$, P. Santorelli$^d$

$^a$Centre de Physique Théorique, Centre National de la Recherche Scientifique, UMR 7644, École Polytechnique, 91128 Palaiseau Cedex, France

$^b$Physics Department, Technion-Israel Institute of Technology, Haifa 32000, Israel

$^c$Dipartimento di Fisica dell’Università di Bari, Italy
Istituto Nazionale di Fisica Nucleare, Sezione di Bari, Italy

$^d$Dipartimento di Scienze Fisiche, Università di Napoli ”Federico II”, Italy
Istituto Nazionale di Fisica Nucleare, Sezione di Napoli, Italy

Abstract

We present estimates of the charming penguin contribution to $B \to K\pi, \pi\pi, K\eta, K\eta'$ decays due to intermediate charmed meson states. We find that this contribution is indeed significant for $B \to K\pi$ decays, and its inclusion, together with the tree and penguin terms, produces large branching ratios in agreement with data, though the analysis is affected by large theoretical uncertainties. On the other hand, for $B \to \pi\pi, K\eta, K\eta'$ decays, the effect of the charming penguin contribution is more modest. We also compute CP asymmetries for $B \to K\pi, \pi\pi$ decays and we obtain rather large results.

PACS: 13.25.Hw
1 Introduction

In a recent paper [1], hereafter referred to as I, we gave an estimate of the so-called charming penguin [2], [3] contributions to the decays $B \to K \pi$. This is a long-distance part of the decay amplitude whose imaginary part results from the decay chains

\[ B \to D(D_s) \to K \pi , \]
\[ B \to D^*(D^*_s) \to K \pi , \]  

(1)

while the real part can be computed by a tree diagram of the effective chiral lagrangian for heavy mesons [4]-[9]. In the present paper, we shall call this amplitude $A_{ChP}$. The relevance of these contributions for $B \to K \pi$ decays was first pointed out in [10]; though suppressed in the factorization approximation, these terms are enhanced by the Cabibbo-Kobayashi-Maskawa (CKM) matrix factor $V_{cb}V_{cs}^*$ in comparison to the short distance terms, i.e., contributions arising from the Tree and Penguin terms in the factorization approximation, whose amplitude we call here $A_{T+P}$. In I we have shown that, even taking into account the uncertainties inherent to this calculation, the contribution of the charming penguins contributions to the decay channels $B^+ \to K^0 \pi^+$ and $B^0 \to K^+ \pi^-$ is indeed significant and can explain the difference between the data and the result obtained by $A_{T+P}$. In the present paper, we wish to extend the analysis to cover other $B$ decay channels with a $K \pi$ pair in the final state, as well as other charmless $B$ decays into two pseudoscalar mesons, i.e.,

\[ B \to \pi \pi , \]
\[ B \to K \eta , \]
\[ B \to K \eta' . \]  

(2)

(3)

(4)

For the processes (3) and (4) we add to $A_{T+P}$ the the charming penguin contributions with $D^{(*)}, D^{(*)}_s$ intermediate charmed meson states; for the $\pi \pi$ final state, the charming penguin contribution is obtained by $D^{(*)}, D^{(*)}_s$ intermediate states. All the relevant formulae are presented in I and can be applied here with some obvious changes. For example the substitution $K \to \pi$ for the channel (2) or the substitution of the pion physical constants with the analogous observables of $\eta$ and $\eta'$ for the channels (3) and (4). A few points, however, deserve a more detailed discussion; let us examine them in the next section.

2 Discussion on the method and its uncertainties

The procedure for obtaining the real part is based on the use of an effective field theory satisfying chiral symmetry as well as heavy flavor symmetries. The main point in this
procedure is the following approximation (we take $B^+ \rightarrow K^0\pi^+$ as the representative channel for the $B \rightarrow PP$ decays):

$$A_{ChP} = \frac{G_F}{\sqrt{2}} a_2 V^*_{cb} V_{cs} < K^0\pi^+ | : J_\mu(0) : | B^+ > \approx \frac{G_F}{\sqrt{2}} a_2 V^*_{cb} V_{cs} \int \frac{d\vec{n}}{4\pi} < K^0\pi^+ | T \{ J_\mu(x_0) : | B^+ >$$

(5)

with $J_\mu = \bar{b} \gamma_\mu (1 - \gamma_5) c$ and $\hat{J}_\mu = \bar{c} \gamma_\nu (1 - \gamma_5) s$; moreover $a_2 = \left( c_2 + c_1/3 \right) \approx 1.03$ where $C_1$ and $C_2$ are Wilson coefficients, and

$$x^\lambda_0 = (0, \vec{n}/\mu) \ ,$$

(6)

where $|\vec{n}| = 1$ and $\mu$ is a scale representing the onset of the scaling behaviour. In (4) we have not considered color octet operators which would have given no contribution as we consider only the color-singlet physical intermediate states. This approximation is based on the light-cone expansion [11], [12], which in the present case reads:

$$< K^0\pi^+ | T \{ J_\mu(x_0) : | B^+ > \approx < K^0\pi^+ | : J_\mu(0) : | B^+ > + O(x_0^2) ,$$

(7)

where the $O(x_0^2)$ terms are negligible for $\mu = 1/|x_0|$ sufficiently large ($\mu \sim m_b$); clearly the integral over $\vec{n}$ corresponds to an average over the directions of $\vec{x}_0$. Moreover, the non-trivial scale dependence of the r.h.s. of eq. (7) is matched by the Wilson coefficient, to give scale-independent physical observables. Also the l.h.s. contains a scale (the cut-off). Ideally this scale dependence should be cancelled by the short distance coefficient as well; in practice, however, the cancellation is not complete as we make a truncation in the long distance physics and we include only the low lying charmed intermediate states. In order to compute eq. (5), we write

$$< K^0\pi^+ | T \{ J_\mu(x_0) : | B^+ > = \int \frac{d^4q}{(2\pi)^4} e^{-iq \cdot x_0} T(q) \ ,$$

(8)

where

$$T(q) = \int d^4x e^{iq \cdot x} < K^0\pi^+ | T \{ J_\mu(x) : | B^+ > .$$

(9)

Let us now show that, after averaging over $\vec{n}$, one obtains a cutoff over the high frequencies in eq. (8). As a matter of fact, one has

$$A_{ChP} = \frac{G_F}{\sqrt{2}} a_2 V^*_{cb} V_{cs} \int \frac{d^4q}{(2\pi)^4} T(q) \int \frac{d\vec{n}}{4\pi} e^{-iq \cdot x_0} = \frac{G_F}{\sqrt{2}} a_2 V^*_{cb} V_{cs} \int \frac{d^4q}{(2\pi)^4} T(q) \theta(\mu, |q|) \ .$$

(10)

where the cutoff function is

$$\theta(\mu, |q|) = \frac{\sin |\vec{q}|/\mu}{|\vec{q}|/\mu} .$$

(11)
For $\mu \to \infty$, $\theta(\mu, |q|) \to 1$; for finite values of $\mu$, this function cuts off from the $q$–integral in eq. (10) the region $-q^2 \geq \mu^2$. Instead of the smooth oscillating function (11), we used in $\mathcal{I}$ the step function
\begin{equation}
\theta(q^2 + \mu^2),
\end{equation}
which allows a considerable reduction of computing time. We also stress that one can extract from the integration in the momentum $q$ the heavy mass contribution according to the formula
\begin{equation}
q = p_B - p_{D(\ast)} \equiv (m_B - m_{D(\ast)})v - \ell
\end{equation}
(see Eq.(30) of $\mathcal{I}$). Here $v^\mu$ is the heavy meson velocity and $\ell$ is a residual momentum. By this the cutoff function on the $\ell$–integration becomes
\begin{equation}
\theta(\ell^2 + \mu_\ell^2),
\end{equation}
and the value of the cut-off $\mu_\ell$ found in $\mathcal{I}$ is $0.5 - 0.7$ GeV. The whole procedure we have described so far has been used several times in the past in the application of the light cone expansion ideas to the nonleptonic weak decays, starting from the pioneering work of K. Wilson (see Section 7 of [11] and the subsequent work of several authors [13]). We repeat it here as it may not be familiar to some readers and also to stress that the correct value of the cut-off $\mu$ (corresponding to $\mu_\ell$) is not the $W$ mass, but a scale of the order of $m_b$ or, better, $m_b - m_c$. It is a consequence of the precocity of the scaling behaviour, a well-known example of which is provided by deep inelastic scattering*. The introduction of non-locality through the cutoff $\mu$ of the order of $m_b$ (or $\mu_\ell$ of the order of 0.5-0.7 GeV) has a clear physical meaning. It corresponds to a separation between short distance physics (whose physical features are embodied in the Wilson coefficients) and long-distance physics, which is dominated by hadronic states and resonances. In considering the long distance part, we have included the low-lying states that could contribute, i.e., $D^{(\ast)}D^{(\ast)}$. This procedure can be avoided for operators having factorizable contributions; in the case of the charming penguin contribution, however, such contributions do not exist, while the non factorizable contribution are Cabibbo enhanced. This is the reason for taking them into account explicitly.

The second point to be stressed is that while in the present paper, as well as in $\mathcal{I}$, we are using the chiral lagrangian effective theory for heavy mesons [4]-[6], the light pseudoscalar mesons in the final state have large momenta. Therefore, the effective lagrangian must be corrected to take into account the hard meson momenta. The procedure we adopted in $\mathcal{I}$ was based on the introduction of form factors, similar to the approach followed in [3], and [9]. We were able to estimate, by using the constituent quark model, the form factor correcting the $B^{\ast}B\pi$ and $D^{\ast}D\pi$ vertices. In addition, one should also consider the form

*On the basis of the previous remarks, the criticism of $\mathcal{I}$ contained in [14] appears to be unjustified.
factor correcting the diagram in fig. 1, which represents the main contribution to the real part of $A_{ChP}$ (in this diagram is depicted in fig. 2a). The weak vertices $D^{(*)} \rightarrow K\pi$ correspond to direct, non-resonant couplings and arise from the weak effective current:

$$L_{\mu a} = \frac{i\alpha}{2} \text{Tr} \gamma_\mu (1 - \gamma_5) H_b \xi^a_b,$$  \hspace{1cm} (15)

which is the effective realization of the quark current $\bar{q}_a \gamma^\mu (1 - \gamma_5) Q$. $\alpha$ is related to the heavy meson leptonic decay constant by the formula $\alpha = f_D \sqrt{m_D}$, valid in the infinite quark mass limit. Moreover,

$$H_a = \frac{1 + \gamma^\mu}{2} (P^*_a \gamma^\mu - P_a \gamma_5)$$  \hspace{1cm} (16)

and

$$\xi = e^{i \frac{\Delta}{f}},$$  \hspace{1cm} (17)

In these formulae, $v$ is the heavy meson velocity; $P_a, P^*_a$ are the annihilation operators of heavy pseudoscalar and vector mesons made up by a heavy quark and a light antiquark of flavour $a \ (a = 1, 2, 3$ for $u, d, s); \ M$ is the usual $3 \times 3$ matrix comprising the octet of pseudo–Goldstone bosons; and $f \approx f_\pi \approx 132 MeV$ is the pseudo–Goldstone bosons decay constant. Equation (15) generates not only weak couplings of $D, D^*$ to hadronic final states with two pseudo–Goldstone bosons, but also the amplitudes with one light pseudoscalar boson in the final state; in particular, it produces the Callan-Treiman relation relating the form factor $F_0^{D\pi}(q^2)$ at $q^2 = m_D^2$ with $f_D$, i.e.,

$$F_0^{D\pi}(m_D^2) \simeq \frac{f_D}{f_\pi}.$$  \hspace{1cm} (18)

At the scale we are interested in, this vertex should be corrected by a form factor that we call $F_a(q^2)$ ($q$ is the four momentum carried by the current),

$$\frac{f_D}{f_\pi} \rightarrow \frac{f_D}{f_\pi} F_a(q^2).$$  \hspace{1cm} (19)
We do not have sufficient information on the behaviour of $F_a(q^2)$, therefore, we leave it as a (constant) parameter and we write

$$F_a = 1.0 \pm 0.5.$$  \hfill (20)

It must be stressed, however, that, in the evaluation of the scaling behaviour (with $1/m_b$) of the charming penguin contributions, the role of this form factor is indeed relevant. Assuming $\frac{1}{m_b}$, as in [16], that $F_{B\pi}^0(m_B^2)$ scales as $\left(\frac{\Lambda}{m_b}\right)^{\frac{3}{2}}$, one gets a scaling law $\frac{1}{m_b}$ for the factorized contribution (in our language $A_{T+P}$). However, in these hypotheses, the form factor $F_{B\pi}^0(q^2)$ should display a pole behaviour, to match with the $\left(\frac{\Lambda}{m_b}\right)^{\frac{1}{2}}$ behaviour predicted by the Callan-Treiman relation at $q^2 \simeq m_B^2$. One can now assume a similar behaviour for the form factor $F_a$, which is reasonable, as the two amplitudes $P \rightarrow M$ and $P \rightarrow MM$ ($P$ heavy, $M$ light mesons) derive from the same effective current $L^\mu$. This implies that the contribution of the charming penguin contribution diagrams should be suppressed by some power $O(1/m_b)$ in the $m_b \rightarrow \infty$ limit in comparison with the factorizable ones. As we are not able to define better the form factor $F_a$, our evaluation should be understood as an order of magnitude estimate.

3 Results

Given these remarks, we are now ready to present our results. The Tree and Penguin contribution to the decay processes $B \rightarrow K\pi$, $B \rightarrow \pi\pi$, and $B \rightarrow K\eta$ ($K\eta'$) is obtained by the usual procedure of factorization using the non leptonic hamiltonian as given e.g. in [17]. As for the charming penguin contribution terms, the explicit formulae can be found in I and need not be reported here (in I they are denoted as $A_{LD}$). The numerical results we obtain for the amplitudes are reported in Table 1. We note that the phase of $A_{T+P}$ is due only to the weak interactions, while the phase in $A_{ChP}$ is purely strong. We use the following set of parameters (with the notations of [17]): For the Wilson coefficients [18]: $c_2 = 1.105$, $c_1 = -0.228$, $c_3 = 0.013$, $c_4 = -0.029$, $c_5 = 0.009$, $c_6 = -0.033$, $c_7/\alpha = 0.005$, $c_8/\alpha = 0.060$, $c_9/\alpha = -1.283$, and $c_{10}/\alpha = 0.266$. Moreover, we use $F_{0}^{B_{M'}}(m_M^2) \approx F_{0}^{B_{M'}}(0) = 0.25$ ($M, M' = K, \pi^\pm$).

The amplitudes are evaluated using, for the CKM matrix elements, the results of the analysis [21]: $A = 0.82$, $\rho = 0.23$, and $\eta = 0.32$. For the $K\eta^{(')}$ final state, we use $SU(3)$ symmetry and the method of [17] with $f_0 = f_8 = f_{\pi} = 132$ MeV and $\theta_0 = \theta_8 = -22^\circ$. We also notice

1A model calculation of this form factor is in [12].

2This assumption is based on the dominance of the hard contribution in the QCD evaluation of the form factor; the actual scaling law may be affected by the behaviour in the soft region, e.g. at the end points.

3We employ the QCD Sum Rule result of [19]; a slightly higher value is in [20].
Table 1: Theoretical values for $A_{T+P}$ (Tree+Penguin amplitude) and $A_{ChP}$ (Charming Penguin amplitude).

| Process          | $A_{T+P} \times 10^8$ GeV | $A_{ChP} \times 10^8$ GeV |
|------------------|---------------------------|---------------------------|
| $B^+ \rightarrow K^0\pi^+$ | +1.69                     | +2.06 + 2.36 i            |
| $B^+ \rightarrow K^+\pi^0$ | +1.21 - 0.498 i           | +1.45 + 1.67 i            |
| $B^0 \rightarrow K^+\pi^-$ | +1.32 - 0.634 i           | +2.06 + 2.36 i            |
| $B^0 \rightarrow K^0\pi^0$ | -0.921 - 0.0497 i         | -1.45 - 1.67 i            |
| $B^+ \rightarrow \pi^+\pi^0$ | -1.35 - 1.79 i            | 0                         |
| $B^0 \rightarrow \pi^+\pi^-$ | -1.85 - 2.16 i            | -0.576 - 0.648 i          |
| $B^0 \rightarrow \pi^0\pi^0$ | +0.0516 + 0.379 i         | -0.576 - 0.648 i          |
| $B^+ \rightarrow K^+\eta$   | -0.0491 - 0.415 i         | +0.0830 + 0.0896 i        |
| $B^+ \rightarrow K^+\eta'$  | +1.40 - 0.261 i           | +2.53 + 2.83 i            |
| $B^0 \rightarrow K^0\eta$   | + 0.172 - 0.0418 i        | +0.0830 + 0.0896 i        |
| $B^0 \rightarrow K^0\eta'$  | +1.54 - 0.0269 i          | +2.53 + 2.83 i            |

that our phase convention is such that the amplitude $\mathcal{A}(B^+ \rightarrow K^0\pi^+)$ differs by a sign from the result of [22]; for $B \rightarrow \pi^0\pi^0$, the statistical factor 1/2 in the branching ratio takes into account the identity of the final mesons. From the results in Table 1, we can compute the

Table 2: Theoretical values for the CP averaged Branching Ratios (BR) compared with experimental data. Data are averages [23] from among CLEO [24], BaBar [25], Belle [26] except for the upper limit that comes from [24].

| Process          | BR $\times 10^6$ (T+P) | BR $\times 10^6$ (T+P+ChP) | BR $\times 10^6$ (Exp.) |
|------------------|-------------------------|---------------------------|-------------------------|
| $B^\pm \rightarrow K^0\pi^\pm$ | $\sim 2.7$       | 18.4 $\pm$ 10.8          | 17.2 $\pm$ 2.5          |
| $B^\pm \rightarrow K^+\pi^0$   | $\sim 1.6$        | 9.5 $\pm$ 5.5            | 12.1 $\pm$ 1.7          |
| $B \rightarrow K^+\pi^+$       | $\sim 1.9$        | 15.3 $\pm$ 9.9           | 17.2 $\pm$ 1.5          |
| $B^0 \rightarrow K^0\pi^0$    | $\sim 0.75$       | 7.4 $\pm$ 4.8            | 10.3 $\pm$ 2.5          |
| $B^\pm \rightarrow \pi^+\pi^0$ | $\sim 4.8$       | $\sim 4.8$               | 5.6 $\pm$ 1.5           |
| $B^0 \rightarrow \pi^+\pi^-$  | $\sim 7.2$        | 9.7 $\pm$ 2.3            | 4.4 $\pm$ 0.9           |
| $B^0 \rightarrow \pi^0\pi^0$  | $\sim 0.06$       | 0.37 $\pm$ 0.35          | $< 5.7$                 |

Branching Ratios (BR) and the CP asymmetries for the $K\pi$ and $\pi\pi$ final states. The CP averaged Branching Ratios are reported in Table 2. In the first numerical column, we report the results obtained by including only the Tree and Penguin contributions, i.e., $A_{T+P}$; in the second column, we give the results obtained by the full amplitude $A_{T+P} + A_{ChP}$; in the final column, we give the available data from the CLEO, Belle and BaBar experiments.
The errors on the branching ratios are obtained varying independently the cut-off $\mu_\ell$ in the range $0.5 \div 0.7$ GeV, $F_a$ in the range $0.5 \div 1.5$, and $F(\vec{p}_\pi) = 0.065 \pm 0.035$, and summing the errors in quadrature. We have not added the errors related with the Tree and Penguin contribution, arising from the CKM matrix elements and from the hadronic parameters. A comparison between and the first and the second column shows the importance of $A_{\text{ChP}}$ for the $K\pi$ final state, while for the $\pi\pi$ final states the charming penguin contribution is either absent ($\pi^\pm \pi^0$) or less important ($\pi^+ \pi^-$). As a matter of fact, as already observed in $\mathcal{I}$, using $SU(3)$ symmetry one obtains for this channel

$$A_{\text{ChP}}(B^0 \to \pi^+ \pi^-) = \frac{V_{cd}}{V_{cs}} \frac{f_K}{f_\pi} A_{\text{ChP}}(B^0 \to K^+ \pi^-),$$

i.e., a CKM suppression in comparison with the $K\pi$ final state. We note a general good agreement with the data; the only significant difference is for the $\pi^+ \pi^-$ final state, which in our opinion should be explained by a more refined analysis of the errors in the inputs of the Tree and Penguin contributions (we repeat that, for the sake of simplicity, we have not introduced these errors in our discussion). In any event, Table 2 show that the uncertainties arising from the charming penguin contribution term are rather large.

The absorptive part of $A_{\text{ChP}}$, which is less sensitive to theoretical uncertainties than the real part, provides a strong argument for a large inelastic final state interaction phase and for an appreciable CP violation even in the absence of the $K\pi$ and $\pi\pi$ elastic rescattering phase shift. To be more quantitative, from the results in Table 1 we compute $CP$ violating asymmetries for the various channels:

$$A^{+ -}_{\pi\pi} = \frac{BR(\overline{B^0} \to \pi^+ \pi^-) - BR(B^0 \to \pi^+ \pi^-)}{BR(\overline{B^0} \to \pi^+ \pi^-) + BR(B^0 \to \pi^+ \pi^-)},$$

$$A^{- 0} = \frac{BR(B^- \to K^- \pi^0) - BR(B^+ \to K^+ \pi^0)}{BR(B^- \to K^- \pi^0) + BR(B^+ \to K^+ \pi^0)},$$

$$A^{0 -} = \frac{BR(B^- \to K^0 \pi^-) - BR(B^+ \to K^0 \pi^+)}{BR(B^- \to K^0 \pi^-) + BR(B^+ \to K^0 \pi^+)},$$

$$A^{- +} = \frac{BR(\overline{B^0} \to K^- \pi^+) - BR(B^0 \to K^+ \pi^-)}{BR(\overline{B^0} \to K^- \pi^+) + BR(B^0 \to K^+ \pi^-)},$$

$$A^{0 0} = \frac{BR(\overline{B^0} \to K^0 \pi^0) - BR(B^0 \to K^0 \pi^0)}{BR(\overline{B^0} \to K^0 \pi^0) + BR(B^0 \to K^0 \pi^0)}. \quad (22)$$

We obtain the following results:

$$A^{+ -}_{\pi\pi} = \frac{BR(\overline{B^0} \to \pi^+ \pi^-) - BR(B^0 \to \pi^+ \pi^-)}{BR(\overline{B^0} \to \pi^+ \pi^-) + BR(B^0 \to \pi^+ \pi^-)} = -0.24 \pm 0.24, \quad (23)$$

while the asymmetries for the $K\pi$ final state are reported in Fig. 2 as a function of the angle $\gamma = \text{arg}(V_{ub}^*)$. We have not reported the asymmetry $A^{0 -}$ that vanishes in our approach.
The regions reported in these graphs correspond to a variation of the three most relevant parameters affecting our numerical results, i.e., the cutoff $\mu_{\ell} \in [0.5, 0.7] \text{ GeV}$ and the form factors $F_a$ in eq. (20) and $F(|\vec{p}_\pi|) \in [0.03, 0.10]$. We see that the variations are rather large, but still compatible with the CLEO [27], BaBar [28], and Belle data [29], which are as follows [23]:

$$\mathcal{A}^{-0} = -0.096 \pm 0.119, \quad \mathcal{A}^{-+} = -0.048 \pm 0.068, \quad \mathcal{A}^{0-} = -0.047 \pm 0.139.$$ (24)

The CP asymmetries we obtain are large, about 20% or more, as shown in Fig. 2. In particular, we find large CP asymmetries for $B^0 \to \pi^+\pi^-$ decays, see Eq. (23). The measurement of the weak angle $\alpha$ from $B^0 \to \pi^+\pi^-$ decays could still be possible once an accurate determination of the long-distance absorptive part from $B \to K\pi$ decays was obtained. Our results for the asymmetries are (in absolute value) compatible with Refs. [3], [30], which also obtain large CP asymmetries for $B \to K\pi, \pi\pi$ decays in phenomenological analyses of the charming penguin contributions. This is in contrast with the QCD-improved factorization model, which predicts small CP asymmetries for $B \to K\pi$ and $B \to \pi\pi$ decays [16, 23, 31].

Let us also briefly comment on the $K\eta$ and $K\eta'$ final states. Our results for these channels are reported in Table 3. For the $K\eta'$ final states, one can clearly see that the charming penguin contribution significantly enhances the results and may be important for producing a large branching ratio; however, it is also clear that some relevant further contribution

Figure 2: First line, CP asymmetries $\mathcal{A}^{-0}$ (left) and $\mathcal{A}^{-+}$ (right); second line $\mathcal{A}^{00}$. The asymmetries are plotted versus the angle $\gamma$. 
is still missing since by no reasonable choice of the parameters can the charming penguin contribution alone solve the puzzle posed by experimentally very large decay fractions. We refer the reader to the existing literature [32] on this subject.

Table 3: Theoretical values of Branching Ratios (BR) for $B \rightarrow K\eta^{(l)}$ compared with experimental data from (a) CLEO [24]; (b) average between Belle [26] and CLEO [24].

| Process          | BR $\times 10^6$ (T+P) | BR$\times 10^6$ (Ch+T+P) | BR$\times 10^6$ (Exp.) |
|------------------|--------------------------|---------------------------|------------------------|
| $B^+ \rightarrow K^+\eta$ | $\sim 0.162$             | $0.099 \pm 0.029$         | --                     |
| $B^+ \rightarrow K^+\eta'$ | $\sim 1.83$             | $20 \pm 10$               | $80.0 \pm 12.2$ (a)   |
| $B^0 \rightarrow K^0\eta$ | $\sim 0.027$             | $0.058 \pm 0.016$         | --                     |
| $B^0 \rightarrow K^0\eta'$ | $\sim 2.00$             | $21 \pm 10$               | $80 \pm 15$ (b)       |

4 Conclusions

In conclusion, we have extended our model of the charming penguin contributions in $B \rightarrow K\pi$ decays to all the significant decays with two pseudoscalar mesons in the final state. Although the calculation presents a number of theoretical uncertainties, it clearly shows that the effect of the charming penguin contribution terms is overwhelming for all the $B \rightarrow K\pi$ decay modes while its role is less significant in the other channels. This dominance is not parametric, i.e., it does not contradict the dominance of the factorized amplitude in the $m_b \rightarrow \infty$ limit discussed by several authors in the last two years [14], [16], [33], [34], [35]. It arises from the CKM enhancement of the non-factorized decay chains (1) and their related real parts. The size of these charming penguin contribution terms can be estimated by an effective field approach, though a complete calculation is beyond the presently available theoretical methods. Therefore, one cannot escape the conclusion that, in spite of the proven theorems, the elusive non-leptonic B-decays still maintain their secrecy.

Acknowledgements

M.L. acknowledges partial support from the Israel-USA Binational Foundation and from the Israel Science Foundation.

References
[1] C. Isola, M. Ladisa, G. Nardulli, T. N. Pham and P. Santorelli, Phys. Rev. D 64 (2001) 014029 [arXiv:hep-ph/0101118].

[2] M. Ciuchini, E. Franco, G. Martinelli and L. Silvestrini, Nucl. Phys. B 501 (1997) 271 [arXiv:hep-ph/9703353].

[3] M. Ciuchini, E. Franco, G. Martinelli, M. Pierini and L. Silvestrini, Phys. Lett. B 515 (2001) 33 [arXiv:hep-ph/0104120].

[4] M. B. Wise, Phys. Rev. D 45 (1992) 2188.

[5] C. L. Lee, M. Lu and M. B. Wise, Phys. Rev. D 46 (1992) 5040.

[6] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio and G. Nardulli, Phys. Rept. 281 (1997) 145 [arXiv:hep-ph/9605342].

[7] A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio and G. Nardulli, Phys. Lett. B 320 (1994) 170 [arXiv:hep-ph/9310320].

[8] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio and G. Nardulli, Phys. Lett. B 299 (1993) 139 [arXiv:hep-ph/9211248].

[9] A. Deandrea, N. Di Bartolomeo, R. Gatto and G. Nardulli, Phys. Lett. B 318 (1993) 549 [arXiv:hep-ph/9308210].

[10] P. Colangelo, G. Nardulli, N. Paver and Riazuddin, Z. Phys. C 45 (1990) 575.

[11] K. G. Wilson, Phys. Rev. 179 (1969) 1499.

[12] R. A. Brandt and G. Preparata, Nucl. Phys. B 27 (1971) 541.

[13] S. Nussinov and G. Preparata, Phys. Rev. 175 (1968) 2180; G. Nardulli, G. Preparata and D. Rotondi, Phys. Rev. D 27 (1983) 557; T. N. Pham and D. G. Sutherland, Z. Phys. C 41 (1988) 327; N. Bilic and B. Guberina, Phys. Lett. B 136 (1984) 440.

[14] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B 606 (2001) 245 [arXiv:hep-ph/0104110].

[15] M. Ladisa, G. Nardulli, T. N. Pham and P. Santorelli, Phys. Lett. B 471 (1999) 81 [arXiv:hep-ph/9909492].

[16] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Phys. Rev. Lett. 83 (1999) 1914 [arXiv:hep-ph/9905312]; Nucl. Phys. B 591 (2000) 313 [arXiv:hep-ph/0006124].

[17] A. Ali, G. Kramer and C. D. Lu, Phys. Rev. D 58 (1998) 094009 [arXiv:hep-ph/9804363].
[18] A. J. Buras, arXiv:hep-ph/9806471.

[19] P. Colangelo and P. Santorelli, Phys. Lett. B 327 (1994) 123 [arXiv:hep-ph/9312258]; P. Colangelo, F. De Fazio, P. Santorelli and E. Scrimieri, Phys. Rev. D 53 (1996) 3672 [Erratum-ibid. D 57 (1996) 3186] arXiv:hep-ph/9510403.

[20] P. Ball and R. Zwicky, JHEP 0110 (2001) 019 arXiv:hep-ph/0110115.

[21] M. Ciuchini et al., JHEP 0107 (2001) 013 arXiv:hep-ph/0012308.

[22] M. Neubert, Phys. Lett. B 424 (1998) 152 arXiv:hep-ph/9712224.

[23] M. Neubert, talk given at the workshop QCD@Work, Martina Franca, Italy, arXiv:hep-ph/0110093.

[24] S. J. Richichi et al. [CLEO Collaboration], Phys. Rev. Lett. 85 (2000) 520 arXiv:hep-ex/9912059; D. M. Asner et al. [CLEO Collaboration], arXiv:hep-ex/0010340; D. Cronin-Hennessy et al. [CLEO Collaboration], arXiv:hep-ex/0010140.

[25] M. Convery [BABAR Collaboration], talk presented at the 15th Les Rencontres de Physique de la Vallee d’Aoste, La Thuile, Italy, Mar 2001, arXiv:hep-ex/0106043.

[26] T. Iijima [BELLE Collaboration], (talk presented at the 4th International Workshop on $B$ physics and CP violation, Ise-Shima, Japan, February, 2001), arXiv:hep-ex/0105003.

[27] S. Chen et al. [CLEO Collaboration], Phys. Rev. Lett. 85 (2000) 525 arXiv:hep-ex/0001009.

[28] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 87 (2001) 151802 arXiv:hep-ex/0105061; arXiv:hep-ex/0107074.

[29] K. Abe et al. [BELLE Collaboration], Phys. Rev. D 64 (2001) 071101 arXiv:hep-ex/0106093.

[30] M. Ciuchini, E. Franco, G. Martinelli, M. Pierini and L. Silvestrini, arXiv:hep-ph/0110022.

[31] T. Muta, A. Sugamoto, M. Z. Yang and Y. D. Yang, Phys. Rev. D 62 (2000) 094020 arXiv:hep-ph/0006022; D. s. Du, C. S. Kim and Y. d. Yang, Phys. Lett. B 426 (1998) 133 arXiv:hep-ph/9711428.

[32] H. Y. Cheng and B. Tseng, Phys. Lett. B 415 (1997) 263 arXiv:hep-ph/9707310; I. Halperin and A. Zhitnitsky, Phys. Rev. D 56 (1997) 7247 arXiv:hep-ph/9704412;
A. S. Dighe, M. Gronau and J. L. Rosner, Phys. Rev. Lett. 79 (1997) 4333 [arXiv:hep-ph/9707521]; A. Ali, J. Chay, C. Greub and P. Ko, Phys. Lett. B 424 (1998) 161 [arXiv:hep-ph/9712372]; M. Z. Yang and Y. D. Yang, Nucl. Phys. B 609 (2001) 469 [arXiv:hep-ph/0012208].

[33] D. S. Du, D. S. Yang and G. H. Zhu, Phys. Lett. B 488 (2000) 46 [arXiv:hep-ph/0005006]; Phys. Rev. D 64 (2001) 014036 [arXiv:hep-ph/0103211].

[34] Y. Y. Keum, H. N. Li and A. I. Sanda, Phys. Lett. B 504 (2001) 6 [arXiv:hep-ph/0004004]; Phys. Rev. D 63 (2001) 054008 [arXiv:hep-ph/0004173].

[35] W. N. Cottingham, H. Mehrban and I. B. Whittingham, [arXiv:hep-ph/0102012].