Research Article
What Can We Learn from (Pseudo)Rapidity Distribution in High Energy Collisions?

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Based on the (pseudo)rapidity distribution of final-state particles produced in proton-proton (pp) collisions at high energy, the probability distributions of momenta, longitudinal momenta, transverse momenta (transverse masses), energies, velocities, longitudinal velocities, transverse velocities, and emission angles of the considered particles are obtained in the framework of a multisource thermal model. The number density distributions of particles in coordinate and momentum spaces and related transverse planes, the particle dispersion plots in longitudinal and transverse coordinate spaces, and the particle dispersion plots in transverse momentum plane at the stage of freeze out in high energy pp collisions are also obtained.

1. Introduction

Due to the complexity of high energy collisions, it is impossible to measure all quantities and characteristics. Instead, some quantities and characteristics can be measured by experiment [1], while others can be obtained from the theoretical explanations based on the experimental results.

In experiments, high energy proton-proton (pp), proton-nucleus (pA), deuteron-nucleus (dA), and nucleus-nucleus (AA) collisions have been performed at available accelerators such as the Super Proton Synchrotron (SPS) [2–4] and colliders such as the Relativistic Heavy Ion Collider (RHIC) [5–8] and the Large Hadron Collider (LHC) [9–12]. Some quantities and characteristics such as the pseudorapidity (or rapidity) distributions, transverse momentum distributions, azimuthal correlations, flow effects, and some other interesting information have been obtained [13].

A lot of models were introduced in the past several decades. Different phenomenological mechanisms including initial interactions, intermediate processes, and final-state statistical laws have been proposed [14–19]. In a workshop held a few years ago at the CERN Theory Institute [20], many models reported their predictions for the heavy ion program at the LHC based on the explanations of the experimental results at the RHIC. Recently, some of the models are tested by pp, pA, and AA collisions at the LHC. Meanwhile, these models are further tested in pp, pA, dA, and AA collisions at the RHIC and SPS.

In the past years, we suggested a multisource thermal model [21–23] to describe the (pseudo)rapidity distributions, multiplicity distributions, transverse momentum distributions, azimuthal distributions, flow effects, and so forth. Some interesting quantities such as the temperature, speed of sound, and number of sources are obtained. It is noticed that some quantities and characteristics are related to others. Usually, based on a few distributions, other distributions can be obtained due to some modelling assumptions.

Very recently, the NA61/SHINE Collaboration reported the negatively charged pion productions in inelastic pp collisions at 20–158 GeV/c at the SPS [1]. The rapidity spectrums are parameterized by the sum of two Gaussian functions symmetrically displaced with respect to midrapidity. It is interesting for us to give a further test for the multisource thermal model by using the rapidity spectrums in pp collisions at SPS momenta (energies). According to the rapidity spectrums, we hope some other distributions can be obtained by the model.
In this paper, in the framework of the multsource thermal model [21–23], we analyze the rapidity spectrums in $pp$ collisions measured by the NA61/SHINE Collaboration at the SPS [1]. Then, a series of other distributions are obtained due to the description of rapidity spectrums.

2. The Model and Calculation Method

In the multsource thermal model [21–23], we assume that many emission sources are formed in high energy $pp$ collisions. In rapidity space in the laboratory or center-of-mass reference frame, these sources distribute at different rapidity $y$ and form a target cylinder in rapidity interval $[y_{T_{min}}, y_{T_{max}}]$ and a projectile cylinder in rapidity interval $[y_{P_{min}}, y_{P_{max}}]$, respectively. Because of the symmetry in $pp$ collision, we have $[y_{T_{min}} = -y_{P_{max}}]$ and $[y_{T_{max}} = -y_{P_{min}}]$.

In the rest frame of a given source, we can use different formalizations to describe the production of particles. For example, we can use the relativistic ideal gas model to give
a description for the source. Then, we have the momentum $p'$ distribution of particles to be

$$ f_{p'}(p') = \frac{1}{n d_{p'}} = C p'^2 \exp \left( -\frac{\sqrt{p'^2 + m_0^2}}{T} \right), \quad (1) $$

where $C = (1/m_0^2 T)(1/K_2(m_0/T))$ is the normalization constant, $K_2(m_0/T)$ is the modified Bessel function of order 2, $m_0$ is the rest mass of the considered particle, $n$ is the particle number, and $T$ is the temperature parameter of the source.

The particles are assumed to emit isotropically in the rest frame of the source. Then, the distributions of emission angle $\theta'$ and the azimuth $\phi'$ can be given by $f_{\theta'}(\theta') = (1/2) \sin \theta'$ and $f_{\phi'}(\phi') = 1/(2\pi)$, respectively.

Let $p'_x$, $p'_T$, $p'_y$, $m'_T$, $E'$, $y'$, and $\eta'$ denote, respectively, the longitudinal momentum, transverse momentum, $x$-component of momentum, $y$-component of momentum, transverse mass, energy, rapidity, and pseudorapidity of a considered particle in the rest frame of the considered source. In the Monte Carlo calculation, let $r_{1,2,3}$ denote, respectively,
Figure 3: The same as Figure 2, but showing the distributions of (a) $E$, (b) $\beta$, (c) $\beta_z$, and (d) $\beta_T$. The circles in Figure 3(c) represent the contribution of the target cylinder to $\beta_z$ distribution at 158 GeV/c.

For other quantities, we have

$$\theta' = \arctan\left[2\sqrt{r_2}(1 - r_2)/(1 - 2r_2)\right]$$ if the right side of the equation is greater than 0; otherwise $\theta' = \arctan[2\sqrt{r_2}(1 - r_2)/(1 - 2r_2)] + \pi$. Further, we have some quantities such as $\phi' = 2\pi r_3$, $p'_z = p' \cos \theta'$, $p'_t = p' \sin \theta'$, $p'_x = p'_t \cos \phi'$, $p'_y = p'_t \sin \phi'$, $m'_t = \sqrt{p'^2 + m'^2}$, $E' = \sqrt{p'^2 + m'^2} = \sqrt{p'^2 + p'^2 + m'^2} = \sqrt{p'^2 + m'^2}$, and $y' = (1/2) \ln[(E' + p'_z)/(E' - p'_z)]$.

The pseudorapidity $\eta \equiv -\ln \tan(\theta/2)$. Then, the probability distributions of the
The space density distributions of considered quantities and their correlations can be obtained. The space density distributions of \( p \) and \( p_T \) are \( \rho(p) = [1/(4\pi p^2)](dn/dp) \) and \( \rho(p_T) = [1/(2\pi p_T n)](dn/dp_T) \), respectively. Besides, some exchangeable quantities in the above expressions are \( p_z = p'_z \cosh y'_z + E' \sinh y'_z \) and \( E = p'_z \sinh y'_z + E' \cosh y'_z \). The values with errors of the three parameters are shown.

Further, the space density distributions of \( r \) and \( R_T \) are \( \rho(r) = [1/(4\pi r^2 n)](dn/dr) \) and \( \rho(R_T) = [1/(2\pi R_T n)](dn/dR_T) \), respectively.

### 3. Comparison and Extraction

The NA61/SHINE Collaboration has measured the rapidity distributions and other features of negatively charged pions produced in 20–158 GeV/c \( pp \) collisions at the SPS [1]. It is shown that the rapidity spectrums are parameterized by the sum of two Gaussian functions symmetrically displaced with respect to midrapidity:

\[
\frac{dn}{dy} = \frac{n}{2\sigma_0 \sqrt{2\pi}} \left[ \exp \left( -\frac{(y - y_0)^2}{2\sigma_0^2} \right) + \exp \left( -\frac{(y + y_0)^2}{2\sigma_0^2} \right) \right],
\]

where \( n, \sigma_0, \) and \( y_0 \) are the normalization constant, distribution width for one Gaussian, and peak position parameter, respectively. The values with errors of the three parameters at different momenta are obtained by the NA61/SHINE Collaboration [1].

We can compare directly our modelling results with the experimental distribution function of the NA61/SHINE Collaboration [1]. In Figure 1, the rapidity and pseudorapidity distributions of negatively charged pions produced in (a) 158, (b) 80, (c) 40, and (d) 20 GeV/c \( pp \) collisions are displayed. The two-dotted curves represent the rapidity distribution range of the NA61/SHINE experiment [1]. The solid curves which fall into the experimental distribution range are our modelling results obtained by using the multisource thermal model. The modelling results in fact give almost whole superpositions to the two-Gaussian functions which are not presented in the panels. Correspondingly, the modelling results on the pseudorapidity distributions are given by the dashed curves. In the calculation, we have used the Monte Carlo technique and \( \chi^2 \)-testing method. The values

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**Table 1:** Values of parameters corresponding to the solid curves in Figure 1.

| Figure | \( T \) (GeV) | \( \gamma_{p_{\text{max}}} \) | \( \gamma_{p_{\text{min}}} \) |
|--------|---------------|----------------|----------------|
| Figure 1(a) | \( 0.150 \pm 0.012 \) | \( 2.00 \pm 0.02 \) | \( -0.21 \pm 0.01 \) |
| Figure 1(b) | \( 0.145 \pm 0.012 \) | \( 1.71 \pm 0.02 \) | \( -0.21 \pm 0.01 \) |
| Figure 1(c) | \( 0.141 \pm 0.011 \) | \( 1.43 \pm 0.01 \) | \( -0.21 \pm 0.01 \) |
| Figure 1(d) | \( 0.138 \pm 0.011 \) | \( 1.20 \pm 0.01 \) | \( -0.21 \pm 0.01 \) |
of parameters $T$, $y_{p_{\text{max}}} (= -y_{T_{\text{max}}})$, and $y_{p_{\text{min}}} (= -y_{T_{\text{min}}})$ for Figures 1(a), 1(b), 1(c), and 1(d) are listed in Table 1. One can see that the model describes the experimental rapidity distributions of negatively charged pions produced in $pp$ collisions at the SPS. The pseudorapidity distribution has a lower peak and wider range than those of the rapidity distribution. The temperature increases and the rapidity shift $y_{p_{\text{max}}}$ increases with increase of the incident momentum, and the rapidity shift $y_{p_{\text{min}}}$ does not change with the momentum.

By using the above parameter values, we can obtain some interesting features for other quantities. Figure 2 presents the distributions of (a) $p$, (b) $p_z$, (c) $p_T$, and (d) $m_T - m_0$ of negatively charged pions produced in $pp$ collisions at the SPS. The solid, dotted, dashed, and dotted-dashed curves correspond to the results at 158, 80, 40, and 20 GeV/c, respectively. The circles in Figure 2(b) represent the contribution of the target cylinder to $p_z$ distribution at 158 GeV/c, while the contribution of the projectile cylinder is symmetrical. For the $p$, $p_T$, and $m_T - m_0$ distributions, both the contributions of the target and projectile cylinders are the same and equal to a half of the distributions. Similarly, Figure 3 presents the distributions of (a) $E$, (b) $\beta$, (c) $\beta_z$, and (d) $\beta_T$ of the
negatively charged pions. The circles in Figure 3(c) represent the contribution of the target cylinder to $\beta_z$ distribution at 158 GeV/c, while the contribution of the projectile cylinder is symmetrical. For the $E$, $\beta$, and $\beta_T$ distributions, both the contributions of the target and projectile cylinders are the same and equal to a half of the distributions.

The space angular distributions of negatively charged pions produced in $pp$ collisions at the SPS are displayed in Figure 4(a). The solid, dotted, dashed, and dotted-dashed curves represent the results at 158, 80, 40, and 20 GeV/c, respectively. The open circles are the contribution of the target cylinder at 158 GeV/c, while the contribution of the projectile cylinder is symmetrical. For comparison, the result of an isotropic emission is given by the closed circles.

To compare the pseudorapidity distributions at different incident momenta, the dashed curves in Figures 1(a)–1(d) are plotted together in Figure 4(b) by the solid, dotted, dashed, and dotted-dashed curves, respectively. The contribution of the target cylinder at 158 GeV/c is shown by the open circles, while the contribution of the projectile cylinder is symmetrical. For comparison, the result of an isotropic emission is given by the closed circles. One can see that the final-state particles are far from the isotropic emission.
The space density distributions of (a) \( r \), (b) \( r_T \), (c) \( p \), and (d) \( p_T \) are presented in Figure 5, where the value of \( t_0 \) is taken to be 2 fm/c for convenience. The solid, dotted, dashed, and dotted-dashed curves correspond to the results at 158, 80, 40, and 20 GeV/c, respectively. One can see that the maximum \( \rho(r) \) appears at the maximum \( r \) and the minimum \( \rho(r) \) appears in the region close to 0. The maximum values of \( \rho(r_T) \), \( \rho(p) \), and \( \rho(p_T) \) appear at 0 and the minimum values appear at the maximum coordinate or momentum (transverse momentum) values.

The dispersion plots in \( x(y) - z \), \( y - x \), and \( p_x(p_y) - p_z \) planes for negatively charged pions produced in (a) 158, (b) 80, (c), 40, and (d) 20 GeV/c \( pp \) collisions are presented in Figures 6, 7, and 8, respectively. The circles and squares correspond, respectively, to the contributions of the target and projectile cylinders in 500 events. We can see that the density in large \( |z| \) and small \( |x(y)| \) region is greater than those in other regions. The density in small \( |p_x(p_y)| \) and \( |p_z| \) region is greater than those in other regions. The contribution points of the two cylinders are mixed in small longitudinal momentum region. The particle number densities in coordinate space and momentum space increase obviously with increase of the incident momentum.

### 4. Conclusions

From the above discussions, we obtain the following conclusions.
(a) The multislence thermal model is used to describe the rapidity distributions of negatively charged pions produced in pp collisions at SPS energies (momenta). A target cylinder and a projectile cylinder are assumed to form in the collisions. For each source, a relativistic ideal gas model is used. The calculated results are in agreement with the rapidity distributions measured by the NA61/SHINE Collaboration [1]. In our calculation, the main parameters are the temperature, the maximum rapidity shift, and the minimum rapidity shift. The temperature and the maximum rapidity shift increase with increase of the incident momentum, and the minimum rapidity shift does not change with the momentum.

(b) According to the parameter values obtained from the rapidity distributions, the other distributions are obtained in the framework of the multislence thermal model. The other distributions include the probability distributions of momenta, longitudinal momenta, transverse momenta (transverse masses), energies, velocities, longitudinal velocities, transverse velocities, space angles, and pseudorapidities, as well as the number density distributions of particles in coordinate and momentum spaces. The fi nal-state particles are far from the isotropic emission. The maximum $\rho(r)$ appears at the maximum $r$ and the minimum $\rho(r)$ appears in the region close to 0. The maximum values of $\rho(r_T), \rho(p)$,
and $\rho(p_T)$ appear at 0 and the minimum values appear at the maximum coordinate or momentum (transverse momentum) values.

(c) According to the parameter values obtained from the rapidity distributions, the particle dispersion plots in $x(y)-z$, $y-x$, and $p_y(p_z)-p_x$ planes are also obtained in the framework of the model. The density in large $|z|$ and small $|x(y)|$ region is greater than those in other regions, and the density in small $|p_y(p_z)|$ and $|p_x|$ region is greater than those in other regions. The contributions of the target cylinder and the projectile cylinder are mixed in small longitudinal momentum region.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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