THE INTRINSIC SHAPES OF MOLECULAR CLOUD FRAGMENTS OVER A RANGE OF LENGTH SCALES

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ABSTRACT

We decipher intrinsic three-dimensional shape distributions of molecular clouds, cloud cores, Bok globules, and condensations using recently compiled catalogs of observed axis ratios for these objects mapped in carbon monoxide, in ammonia, through optical selection, or in continuum dust emission. We apply statistical techniques to compare assumed intrinsic axis ratio distributions with observed projected axis ratio distributions. Intrinsically triaxial shapes produce projected distributions that agree with observations. Molecular clouds mapped in $^{12}$CO are intrinsically triaxial but more nearly prolate than oblate, while the smaller cloud cores, Bok globules, and condensations are also intrinsically triaxial but more nearly oblate than prolate.

Subject headings: ISM: clouds — ISM: globules — ISM: structure — stars: formation

On-line material: color figures

1. INTRODUCTION

Numerous catalogs have now compiled properties of hundreds (or even thousands) of molecular clouds or cloud fragments with a large range of sizes, allowing meaningful statistical analysis of properties of molecular clouds, cloud cores, and smaller condensations. With such data sets, the distribution of the apparent projected core axis ratio $p$ can be used to constrain the intrinsic three-dimensional shapes. In a previous paper (Jones, Basu, & Dubinski 2001, hereafter Paper I), we investigated the shapes of molecular cloud cores mapped in NH$_3$ (Jijina, Myers, & Adams 1999) and cores mapped through optical selection (Lee & Myers 1999) via both statistical and analytical methods. We found that strictly axisymmetric prolate or oblate cores could not reproduce the observed projected axis ratios and that molecular cloud cores were triaxial.

In this paper, we extend our previous work and conduct a statistical analysis of seven recent data sets that include a wide range of sizes of objects, from molecular clouds with effective radius as large as 45 pc (Heyer, Carpenter, & Snell 1995) in mm maps of smaller protostellar condensations (Motte, André, & Neri 1998; Motte et al. 2001) in $^{12}$CO. The information obtained about the intrinsic shapes of these objects can yield insight into the physical processes that govern their evolution and subsequent star formation. We examine the $^{12}$CO catalog of molecular regions in the outer Galaxy compiled by Heyer et al. (2001) in §3.1, cores mapped in NH$_3$ and C$^{18}$O in §3.2 (Onishi et al. 1996; Tachihara, Mizuno, & Fukui 2000), catalogs of Bok globules (Clemens & Barvainis 1988; Bourke et al. 1995) in §3.3, and millimeter and submillimeter continuum maps of smaller protostellar condensations (Motte, André, & Neri 1998; Motte et al. 2001) in §3.4. A discussion and summary are given in §§4 and 5.

2. METHODS

In general, a triaxial ellipsoid can be described by the equation

$$x^2 + \frac{y^2}{\xi^2} + \frac{z^2}{\xi^2} = a^2,$$  

where $a$ is a constant and $1 \geq \xi \geq \zeta$. The geometrical analysis of Stark (1977) and Binney (1985) shows that such a body, when viewed in projection, has elliptical contours. Following Binney (1985), the projection of a triaxial body when viewed from an observing angle $(\theta, \phi)$ (where the angles are defined on an imaginary viewing sphere and have their usual meaning in a spherical coordinate system) is found using the quantities

$$A = \cos^2 \theta \left( \sin^2 \phi + \frac{\cos^2 \phi}{\zeta^2} \right) + \frac{\sin^2 \theta}{\xi^2},$$

$$B = \cos \theta \sin 2\phi \left( 1 - \frac{1}{\zeta^2} \right) \frac{1}{\xi^2},$$

$$C = \left( \sin^2 \phi + \cos^2 \phi \right) \frac{1}{\xi^2}.$$  

The apparent axis ratio in projection then equals

$$p = \left( \frac{A + C - D}{A + C + D} \right)^{1/2},$$

where $D = [(A - C)^2 + B^2]^{1/2}$. Using these equations, one can construct probability distributions for the projected axis ratio, assuming a large number of randomly distributed viewing angles.

We assign a Gaussian distribution of values for each axis ratio $\zeta$ and $\xi$, with a mean in the range [0, 1] and standard deviation $\sigma$ typically equal to 0.1, consistent with our use of 10 bins to sample the data. We did test a range of $\sigma$ from 0.05 to 0.2 and found that our conclusions do not change significantly within this range (see discussion in §4). The drawback to using $\sigma \geq 0.2$ is that a relatively large fraction of the Gaussian distribution can fall outside the allowed range $[0, 1]$. For example, a Gaussian distribution centered at 0.8 with a $\sigma$ of 0.2 has $\approx 16\%$ of the data greater than 1 (see Paper I for an extended discussion of this issue). For similar reasons, we limit the $\sigma = 0.1$ analysis to the range of axis ratios $[0.1, 0.9]$.

In order to find the best-fit intrinsic distribution of triaxial bodies, distributions of axis ratios with peak values $\zeta_0$ and $\xi_0$ (for a given $\sigma$) are input into a Monte Carlo program. We
typically employ at least $10^4$ viewing angles to calculate the projected distribution for each individual pair of axis ratios and at least $10^4$ sets of axis ratios for each Gaussian distribution. This is more than sufficient for comparison with data sets sampled in 10 bins. The program produces the expected observed distribution that results from the assumed intrinsic distributions. We compare this output with the observed data sets via their $\chi^2$ values calculated by comparing the area of each bin to the area under the expected distribution at the location of each bin. Distributions of triaxial shapes for which the mean values $0_0 = 0_0$ emphasize prolate objects, while those distributions with $0_0$ near 1 emphasize oblate objects. Furthermore, distributions with $0_0 > \frac{1}{3}(1 + 0_0)$ can be classified as containing more nearly oblate than prolate objects. For greater detail, consult Paper I.

3. RESULTS

3.1. Molecular Clouds

Heyer et al. (2001) have cataloged the properties of clouds and clumps using $^{12}$CO data from a survey of the outer Galaxy. The catalog consists of 10,156 objects, which include small isolated clouds and clumps within larger clouds. Observations from the outer Galaxy have the advantage that there is no distance ambiguity and the clouds are more widely spaced along the line of sight, which eliminates problematic blending of emission, allowing cloud properties to be determined more accurately. The FWHM beam size for this catalog is $45''$, and there are an impressive 10,134 objects with major and minor axes tabulated. Heyer et al. (2001) point out that the largest clouds, with effective radius $r_c$ of about 10 pc or larger, are self-gravitating, whereas the smaller clouds (comprising the vast majority of observed objects) are not self-gravitating. This boundary also approximates the usual distinction between molecular clouds and giant molecular clouds (GMCs; see Blitz 1991; Williams, Blitz, & McKee 2001).

Given the evidence for a physical distinction based on size, we performed our analysis on the entire data set as well as a subset with $r_c > 10$ pc, which corresponds to a mass range of approximately $10^4-10^5 M_\odot$, typical of GMCs. Furthermore, to avoid any pathological cases where an elliptical fit to the cloud shape can be a very poor approximation (M. H. Heyer 2001, private communication), we restricted our sample to those clouds that span at least 10 spatial pixels in the observations (an even more restrictive threshold of 20 pixels also yields essentially the same final result). This criterion reduces the total set to 5685 and the subset of GMCs to 85. Our separate analyses can reveal whether there is any significant shape difference in the two populations.

Figure 1 shows the results of the $\chi^2$ calculations for the triaxial fitting of the GMC subset. The data set is best fitted by distributions with axis ratios $(0_0, 0_0) = (0.2, 0.2)$ when $\sigma = 0.1$. In order to determine whether values of $(0_0, 0_0)$ closer to zero would improve the fit, we repeated the analysis with a value of $\sigma = 0.05$. (As the mean value of the Gaussian gets closer to the endpoints of the allowed range [0, 1], a larger fraction of the Gaussian distribution falls outside this range for a given $\sigma$. There is no ideal way to correct for this problem, as explained in § 2 of this paper and in § 4 of Paper I.) Even with $\sigma = 0.05$, however, the best-fit mean axis ratios $(0_0, 0_0)$ are not closer to zero, and they agree with our result for $\sigma = 0.1$ within the estimated error. See § 4.1 for a discussion of the errors, which we estimate to be a maximum of ±0.1 in the mean value of each axis ratio.

Figure 2 shows the result of the $\chi^2$ calculation for the complete set of axis ratios based on our selection criteria in the Heyer et al. (2001) catalog. The best fit based on the $\chi^2$ values is $(0_0, 0_0) = (0.3, 0.3)$.

For both the total set and the subset based on large effective radius, the best-fit triaxial distributions require $0_0 = 0_0$, as shown very clearly in Figures 1 and 2. This means that the distributions emphasize prolate objects. Since $0_0$ and $0_0$ are the means of distributions, however, most individual
objects cannot be considered strictly prolate and are, in fact, triaxial. For the clouds with large $r_e$, the distributions that best fit the observations have thinner objects (smaller $\xi$ and $\zeta$) than those that best fit the entire set. The difference is quite small, however, and equal to our maximum estimated error of \pm 0.1.

Figure 3 compares the best-fit distribution of $p$ to the complete binned data set of Heyer et al. (2001). It also reveals that the histogram of projected shapes $p$ of molecular clouds has some unique features. We recall that our previous analysis (Paper I) of dense cores showed that an observed broad peak in the distribution at $p \geq 0.5$ and the presence of a significant number of objects near $p = 1$ favored triaxial but more nearly oblate intrinsic shapes. Indeed, this pattern is reinforced in our subsequent study of other cores, Bok globules, and protostellar condensations (see §§ 3.2–3.4). The shapes of molecular clouds, however, are distinct in that they have a very narrow peak and at a low value, $p \approx 0.3$. The narrow peak favors near-prolate objects, although a pure prolate cloud with $\xi = \zeta = 0.3$ yields a poor fit to the data, as shown in Figure 3. A pure prolate cloud would have too narrow a peak in the observed shape distribution as well as a higher probability than observed of a near-circular projection (see discussion and figures in § 2 of Paper I). In fact, Figure 3 shows that even a Gaussian distribution of triaxial objects implies a higher probability of detecting near-circular objects than observed.

The cutoff in high values of $p$ may be the result of an actual cutoff in the distribution of intrinsic axis ratios ($\xi_0$, $\zeta_0$) above some value or could be the result of some selection effect. One selection effect in the Heyer et al. (2001) sample is the fact that an object must span at least 5 pixels in the map to be classified as a cloud (additionally, we impose the higher threshold of 10 pixels for our shape analysis). We see no evidence in the sample, however, for a trend toward greater circularity as clouds have smaller projected size. Another selection effect is that the edges of the objects in the sample likely correspond to the CO photodissociation boundary and not that of the H$_2$ gas (Heyer et al. 2001). However, it is again not clear that this in any way biases against near-circular objects.

3.2. Dense Cores

Onishi et al. (1996) and Tachihara et al. (2000) have observed dense cloud cores in Taurus and Ophiuchus, respectively, in C$^{18}$O with telescopes at Nagoya University. Since the same telescopes and technique were used to obtain these ratios, and both groups quote the axial ratios to a tenth of a parsec, we combined these two surveys in order to obtain a reasonable sized set for statistical analysis. The final set of data from these two regions includes 80 cores. The best fit determined by calculating the $\chi^2$ values is ($\xi_0$, $\zeta_0$) = (0.4, 0.9) (see Fig. 4). The best fit is compared with the binned data set in Figure 5. We performed this analysis with and without the sources for which there were embedded IR sources and obtained the same result.

Although our best fit underestimates the observed axis ratios in the bin near $p = 1$, it also overestimates the previous bin. The significant bin-to-bin variation is likely the result of the relatively small size of this sample. However, the best fit agrees, within the estimated errors, with the results presented in Paper I using much larger samples of NH$_3$ data (Jijina et al. 1999) and optical data (Lee & Myers 1999) for dense cores. Jijina et al. (1999) cataloged core properties for 264 objects from NH$_3$ observations, and Lee & Myers (1999) cataloged properties for 406 dense cores from contour maps of optical extinction. The Jijina et al. data set is best fitted by triaxial distributions with mean axis ratios ($\xi_0$, $\zeta_0$) = (0.5, 0.9), and the Lee & Myers data set is best fitted by mean values ($\xi_0$, $\zeta_0$) = (0.3, 0.9).

3.3. Bok Globules

Bourke et al. (1995) cataloged physical characteristics of 169 isolated small molecular clouds (Bok globules) in NH$_3$ from the southern sky. These observations were compiled to complement a similar study of 248 optically selected Bok globules in the northern hemisphere by Clemens & Barvainis (1988). We note that Clemens & Barvainis (1988) also found that there was no correlation between the orientation of the clouds relative to the Galactic plane and their projected optical shapes. Ryden (1996) looked at the shapes of

![Fig. 3.—Comparison of the observed axis ratios from Heyer et al. (2001) with the best fit, assuming triaxial clouds (solid line) and assuming a pure prolate cloud (dot-dashed line). The actual number of observations in the bins is shown on the right-hand side vertical axis.](image)

![Fig. 4.—Two-dimensional plot of inverse $\chi^2$ values for triaxial core shape models applied to the combined surveys of Onishi et al. (1996) and Tachihara et al. (2000). [See the electronic edition of the Journal for a color version of this figure.]](image)
the globules from these two data sets based on the assumption of axisymmetry. She found that the Bok globules were consistent with oblate objects having an intrinsic mean axis ratio $q = 0.3$ or prolate objects having $q = 0.5$. Ryden (1996) realized that these sets suffered from rounding errors. This affected the smallest major and minor axes, yielding an artificially large number of axis ratios near 1. For example, Clemens & Barvainis (1988) measured the Palomar Observatory Sky Survey plates to compile their catalog. They rounded the major and minor axes to the nearest millimeter, which corresponds to an angular size of 1.12. The set is composed of a relatively large number of small globules (42 out of 248) that have a major axis less than 2 mm. When these smallest values are rounded, the result is a large number of axis ratios erroneously equal to 1 or close to 1. The top panel of Figure 6 shows very clearly that there is proportionally a large number of objects in the bin with the largest axis ratios. Obviously, this problem does not affect the larger globules as significantly. Ryden (1996) compensated for this problem by adding to each of the original major and minor axes a random error term $\Delta$. She did this hundreds of times and obtained a new distribution of major and minor axes by taking an average from all her trials. See Ryden (1996) for additional details. We followed her procedure to correct for these rounding errors and obtained a new distribution of axis ratios for both data sets. Since the round-off error is likely selected uniformly from within a fixed range, we used a random number generator to select uniformly from a prescribed range of estimated error. For the Clemens & Barvainis (1988) and the Bourke et al. (1995) data sets, we used a range of error $-0.56 < \Delta < +0.56$ and $-0.25 < \Delta < +0.25$, respectively. Figure 6 shows both the original binned data of axis ratios and the corrected distribution of axis ratios for the Clemens & Barvainis (1988) data. Notice that the large number of axis ratios near 1 in the original set has been dramatically reduced. We investigated these two sets and found that intrinsic triaxial shapes produced distributions that agreed with the observations. Figures 7 and 8 show the results of the $\chi^2$ calculations with the triaxial fitting for the Clemens & Barvainis (1988) and the Bourke et al. (1995) corrected data sets, respectively. The Clemens & Barvainis (1988) data set is best fitted by distributions with mean axis ratios $(\xi_0, \eta_0) = (0.4, 0.9)$, and the Bourke et al. (1995) data set is also best fitted by mean values $(\xi_0, \eta_0) = (0.4, 0.9)$. The best fit for the Clemens & Barvainis (1988) data is also shown in Figure 6.

3.4. Condensations Mapped in Millimeter-Submillimeter Wavelengths

Motte et al. (1998) observed the $\rho$ Ophiuchi main cloud at 1.3 mm with the IRAM 30 m telescope and were able to detect 58 individual compact dusty objects with a fragmentation scale size of approximately 6000 AU. After we removed clumps that were thought to be composite in nature, there were 35 clumps for which projected major and minor axes were resolved. Although this is a small sample for statistical analysis, we still present the results of the $\chi^2$
calculations from the triaxial fitting in Figure 9. The Motte et al. (1998) data set is best fitted by distributions with mean axis ratios \( (\xi_0, \zeta_0) = (0.4, 0.9) \).

Motte et al. (2001) surveyed the protoclusters NGC 2068 and NGC 2071 in Orion B at 850 and 450 \( \mu \)m with the Submillimeter Common-User Bolometer Array (SCUBA) on the James Clerk Maxwell Telescope (JCMT) and were able to detect small objects (size \(~5000\) AU), which they call condensations. There were 64 condensations for which the projected major and minor axes were obtained. This is a remarkable data set, which measures objects on size scales approaching that of our solar system. The results of the \( \chi^2 \) calculations with the triaxial fitting are presented in Figure 10 for \( \sigma = 0.1 \). The Motte et al. (2001) data set is best fitted by distributions with mean axis ratios \( (\xi_0, \zeta_0) = (0.4, 0.9) \). Since the binned observed data of axis ratios do not have a strong peak, but instead look rather flat, we repeated our analysis for \( \sigma = 0.2 \). For \( \sigma = 0.2 \), the data set is best fitted by distributions with mean axis ratios \( (\xi_0, \zeta_0) = (0.5, 0.9) \). The distribution with the mean value of 0.9 and \( \sigma = 0.2 \), however, has 31% of the data values outside the allowed range. After the data are truncated, the remaining distribution will not have a final mean at 0.9 and \( \sigma = 0.2 \), and the remaining values inside the allowed range must also be normalized. As a result, the normalized distribution is actually peaked more than one would intuitively expect. Consequently, the \( \chi^2 \) values for \( \sigma = 0.2 \) are no better than the \( \chi^2 \) values for \( \sigma = 0.1 \). The best fit for \( \sigma = 0.1 \) is compared with the observed data in Figure 11.

Interestingly, Figure 11 shows that there are a number of objects with an apparent axis ratio near or exactly equal to 1. A close examination of the original Motte et al. (2001) data reveals that the objects that fall into this bin are well above the resolution limit, and that this large number is not a selection effect. Since this data set has a number of axis ratios near \( p = 1 \), it is interesting to speculate whether or not intrinsic oblate objects (rather than near-oblate triaxial objects) may agree with the observations. We use two additional tests to investigate this hypothesis. First, we fit the histogram of observed shapes with a curve and carry out an analytic inversion to see if it is consistent with a distribution of intrinsic oblate shapes. The resulting intrinsic shape distribution \( \phi(q) \) does go negative near \( q = 1 \), just like in the inversions performed on the dense core shape data of Ji and Meyers (1999) and Lee & Myers (1999) in Paper I. This is caused by the continuing presence of a decline in the observed axis ratio distribution \( \phi(p) \) toward \( p = 1 \). To investigate further, we also experiment with an intrinsic axis distribution that is Gaussian in \( \xi \), with \( \xi_0 = 0.4 \) and \( \sigma = 0.1 \), but has a fixed value \( \zeta = 1 \) for all clouds; i.e., the clouds are all oblate with various degrees of flattening. In this case, our Monte Carlo program yields a probability distribution function, which is plotted in Figure 11. The inverse \( \chi^2 \) values for both the pure oblate objects and the best-fit triaxial objects are \( \approx 0.2 \). The Motte et al. (2001) data, unlike the dense core data presented or reviewed in \( \S \) 3.2, are close enough to being flat near \( p = 1 \) and have a small enough sample of objects that we cannot entirely exclude the pure
oblate hypothesis on the basis of the $\chi^2$ test. If just one of
the six objects (interestingly, one object has an embedded
Class 0 protostar) is removed from the bin closest to $p = 1$,
the near-oblate triaxial objects provide a much better fit.
Further observations and a larger sample will be necessary
before the intrinsic shapes of these very dense objects can be
determined with certainty. Nevertheless, we can conclude
that molecular cloud condensations are either oblate or
near-oblate in shape.

4. DISCUSSION

4.1. Error Bounds

In order to test the reliability of our results, we recalculate
the $\chi^2$ values after randomly removing 20% of the data and
repeat this procedure 10 times for each set. For every set
investigated in this paper, we obtain the same best-fit mean
axis ratios ($\xi_0$, $\zeta_0$) for all the trials, with the exception
of two trials with the Motte et al. (1998) data. This data set has
only 35 noncomposite objects for which the minor and
major axes are published. When 20% of the data are
randomly removed, only 28 objects remain. Nevertheless, we
obtain ($\xi_0$, $\zeta_0$) = (0.4, 0.9) for eight trials, ($\xi_0$, $\zeta_0$) =
(0.5, 0.9) for one trial, and ($\xi_0$, $\zeta_0$) = (0.4, 0.8) for
the remaining trial.

Additionally, adjusting the width of the Gaussian distribu-
tions in $\xi$ and $\zeta$ within the range [0.05, 0.2] yields at most
a change of $\pm 0.1$ in the best-fit values $\xi_0$ and $\zeta_0$. The largest
variation is seen in the smallest data sets. The effect on the
Motte et al. (2001) data is described in § 3.4. Using $\sigma = 0.05$
allows us to test distributions with peaks near the bounda-
ries 0 and 1, while a relatively large $\sigma = 0.2$ allows us to see
if a better fit exists for the broadest observed distributions in
projected axis ratio $p$.

Altogether, our testing allows us to state an approximate
maximum error in our best-fit mean axis ratios of $\pm 0.1$.
Interestingly, absolutely all of the best fits for the data sets
of objects with size scale $\sim 0.1$ pc or smaller agree with one
another within this range of error. This strengthens the case
that these objects are triaxial but preferentially flattened in
one direction and close to oblateness. A small degree of tri-
axiality seems necessary to explain the observed decline in
$\phi(p)$ near $p = 1$, and the pure oblate hypothesis seems
excluded for dense cores (Paper I and § 3.2). Because of the
smaller number of statistics and relatively large number of
objects with $p$ near 1, the dense condensations are most
likely to be compatible with pure oblateness. More extensive
observations of this class of objects are necessary to settle
the issue.

Finally, there is the possibility that our results are biased
by the fact that the spectral line or dust continuum data are
probing emission from regions of varying opacity and/or
temperature, so that the projected shape may not corre-
respond exactly to the physical shape of the gas distribution.
When data from other wavelengths become available, it is
possible that our conclusions may change. This applies par-
ticularly to the molecular cloud data (§ 3.1), for which we
have used a single catalog. However, since we obtained very
nearly the same result using a variety of tracers for the
smaller cloud cores, Bok globules, and condensations, the
results in these cases seem robust.

4.2. Physical Implications

This investigation shows that intrinsic triaxial objects
produce distributions that reasonably match observations
of projected axis ratios for molecular clouds, molecular
cloud cores, Bok globules, and protostellar condensations.
The results clearly fall into two categories: (1) on scales $\gtrsim 1$
pc, mapped in $^{12}$CO, molecular clouds, including GMCs,
have triaxial shapes that are more closely prolate than
oblate; and (2) dense cores, Bok globules, and condensa-
tions, mapped in a variety of tracers, on scales from a few
times 0.1 pc down to 0.01 pc, have triaxial shapes that are
more closely oblate than prolate. The results about the latter
objects reinforce our earlier finding (Paper I) from two other
catalogs of dense core shapes. See Table 1 for a summary
of the best-fit axis ratios, ($\xi_0$, $\zeta_0$), for each of the data sets
we investigated. The results from Paper I are included for
comparison.

The robust tendency for cores, Bok globules, and smaller
condensations to have triaxial fits with $\xi_0 = 0.3$–0.5 and
$\zeta_0 = 0.9$ implies that they are all preferentially flattened in
one direction. This could be the result of flattening along the
direction of a mean magnetic field or of significant rota-
tional support in the smallest objects. The magnetic field
explanation for cores is attractive, since it implies that the
observed near-alignment of core minor axes and magnetic
field direction in Taurus (see Onishi et al. 1996) may be indi-
cative of a more universal phenomenon. We also note that
early submillimeter polarimetry of a few dense cores (Ward-
Thompson et al. 2000) reveals a tendency toward alignment
but also a noticeable angular offset. This is interpreted as
evidence for triaxiality of the cores (Basu 2000), which may
still be preferentially flattened along the direction of the
magnetic field.

While triaxiality is consistent with a nonequilibrium state,
evolving as a result of external turbulence or internal grav-
ity, the near-oblate shape also means that the objects may
not be particularly far from equilibrium and that oblate
equilibrium models may act as a reasonable approximation
to these objects. This can explain why the internal structure
of some Bok globules and prestellar cores can be closely or
approximately fitted by spherical equilibrium Bonnor-Ebert

Fig. 11.—Comparison of the observed axis ratios from Motte et al.
(2001) with the best fit, assuming triaxial cores for $\sigma = 0.1$ (solid line) and
assuming pure oblate cores (dashed line). The actual number of observa-
tions in the bins is shown by the right-hand side vertical axis.
or near-equilibrium oblate magnetic models (Alves, Lada, & Lada 2001; Bacmann et al. 2000; Ciolek & Basu 2000; Zucconi, Walmsley, & Galli 2001). It is also consistent with the observed near-virial equilibrium of most cores (Myers & Goodman 1988).

We also note that the Bok globules, which are by definition isolated sites of star formation, have shapes that are not significantly different from those of molecular cloud cores and condensations embedded within larger clouds. This suggests that the environment in which the cores and condensations are embedded plays a relatively insignificant role in their dynamics; i.e., the external pressure from the parental cloud does not seem to be important at this stage.

The smallest objects in our study, the condensations mapped in millimeter and submillimeter continuum emission, may be the precursors to individual stars, since the mass spectrum appears to match the initial mass function compiled by Salpeter (1955) over a certain mass range (Motte et al. 1998). The estimated triaxial but near-oblate shape of these objects is an important link in understanding the collapse process that leads to star formation.

For the larger molecular cloud scale, we have utilized an exhaustive catalog of the shapes of clouds in the outer Galaxy (Heyer et al. 2001). Although our study of molecular cloud shapes is based on this single available sample of projected axis ratios, and the result should be confirmed when other shape data become available, the sheer size of this catalog is a strong point. The histogram of observed axis ratios (Fig. 3) is very distinct from any of the other samples. It has a very sharp peak and a severe lack of objects with \( p \gtrsim 0.5 \).

While there may be some unknown selection effect that biases against the observation of near-circular objects, we note that the orientations of the projected shapes in the plane of the sky do appear to be truly random. Furthermore, we note that the earlier \(^{13}\)CO catalog of only 23 clouds in Ophiuchus by Nozawa et al. (1991) that was utilized by Ryden (1996) has the same qualitative feature of a narrow peak near \( p = 0.3 \) and a steep decline toward \( p = 1 \).

Heyer et al. (2001) note that the vast majority of their clouds (all but the largest clouds, which we loosely label GMCs) are not self-gravitating. They are either transient features or held together by external pressure. If these clouds are indeed brought together by large-scale turbulence in the interstellar medium (or even confined for some time by an anisotropic ram pressure), we might expect that they have an elongated, filamentary shape (see, e.g., Nagai, Inutsuka, & Miyama 1998; Balsara, Ward-Thompson, & Crutcher 2001; review by Shu et al. 1999). Since even the largest clouds seem to have these shapes, we surmise that all clouds may be brought together by external forcing (due to shock waves or turbulent motions for example), with only the largest clouds or densest regions within smaller clouds able to become self-gravitating. This ties in with the general picture of a rapid formation of molecular clouds due to external triggers (see, e.g., Hartmann, Ballesteros-Paredes, & Bergin 2001; Pringle, Allen, & Lubow 2001).

## 5. SUMMARY

Generally speaking, the decline in the observed probability distribution function \( \phi(p) \) toward \( p = 1 \) favors triaxial rather than axisymmetric intrinsic shapes. In addition, objects observed on scales a few times 0.1 pc and smaller have a broad peak, and a significant number of objects were observed with \( p \approx 1 \). This favors near-oblate triaxial objects, as shown in detail in Paper I. In contrast, molecular clouds observed on scales \( \gtrsim 1 \) pc have an observed \( \phi(p) \) with a much sharper peak and a precipitous decline toward \( p = 1 \). This favors near-prolate triaxial objects. Reviews of the expected distributions \( \phi(p) \) for various shapes can be found in Binney & Merrifield (1998) and Paper I. A summary of our best fits to the various data sets is given in Table 1.

Our new results strengthen the finding that one of the best-fit intrinsic axis ratios is always quite a bit larger than the other axis ratio for cores, condensations, and Bok globules, which means that these objects are preferentially flattened in one direction and close to oblateness. They may then not be particularly far removed from equilibrium or from oblate models often used to fit them. On the other hand, the much larger, lower density clouds have best-fit distributions such that many objects are close to prolateness, consistent with the formation of these objects from large-scale external forcing.

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