Deuteron Stripping on Nuclei at Intermediate Energies

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Abstract

A general analytical expression for the double differential cross section of inclusive deuteron stripping reaction on nuclei at intermediate energies of incident particles was obtained in the diffraction approximation. Nucleon-nucleus phases were calculated in the framework of Glauber formalism and making use of the double-folding potential. The exact wave function of deuteron with correct asymptotics at short and long distances between nucleons was used. The calculated angular dependencies of cross sections are in good agreement with corresponding experimental data.

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1. Introduction

The binding energy of deuteron is low. Therefore, when the latter collides with nuclei, inelastic processes are the most probable ones: the deuteron breakup in the nuclear Coulomb field (mainly at low deuteron energies) and the deuteron stripping, when one of deuteron’s nucleons is absorbed by the target, whereas the other is released as a reaction product. In the intermediate energy interval, the stripping reaction is mainly a result of direct interaction (the capture of deuteron’s nucleon by the nucleus), and the differential cross section of reaction is characterized by a sharp peak at particle emission angles $\Theta \ll 1$. The analysis of the angular and energy distributions of cross sections in the deuteron stripping reaction allows additional information on the residual nucleus structure and reaction mechanisms to be obtained, being one of the most important sources of spectroscopic data in nuclear physics.

For the first time, the theory of deuteron stripping at intermediate energies was proposed by R. Serber [1] for transparent and opaque target nuclei, making no allowance for the diffuseness of their surface. Later, the formalism of inclusive deuteron stripping reaction on nuclei was developed by Akhiezer and Sitenko in
work [2] on the basis of diffraction nuclear model [3, 4], and its various aspects were afterwards analyzed and improved by other authors (see [5, 6] and references therein).

The general formula for the inclusive deuteron stripping cross section [2] is inconvenient for the analysis and direct numerical calculations, because it contains a five-fold integral. Therefore, it is usually modified for practical purposes by introducing additional conditions and restrictions (e.g., the nucleus is opaque and non-diffuse; the deuteron radius is much smaller than the target one; and so on). However, this integral can be transformed into a general analytical expression if Gaussian-like functions are used as integrands. Gaussoid functions can be used here as basis ones for the expansion of both the deuteron wave function (the variational problem) and the profile functions of arbitrary forms. Notice that a similar trick is widely applied in the variational approach to describe bound states [7], to parametrize the charge densities in the ground state of nuclei [8, 9], and in scattering problems [10], which makes it possible to calculate the corresponding scattering phases and form factors analytically.

2. Formalism

Light and medium nuclei were selected as targets, because in this case and in the case of intermediate energies, the Coulomb interaction can be neglected. The spins of the deuteron’s nucleons and the target were also not taken into account.

The general formula for the differential cross section of deuteron stripping is derived as follows [2]. Let a proton be a particle captured by the target nucleus at stripping. The wave function of the neutron released in this reaction will be presented as a plane wave: \( \psi(r_1) = \exp(i k_1 r_1) \), where \( k_1 \) is the neutron momentum, and \( r_1 \) its radius vector. The wave functions of the proton absorbed by the nucleus are coefficients of the integral expansion of deuteron wave function near the nucleus in series of functions \( \psi(r_1) \). In other words, the probability amplitude that the neutron has the momentum \( k_1 \) and the proton is at the point \( r_2 \) equals

\[
 a(k_1, r_2) = \int d^3r_1 \exp(-ik_1 r_1) S_1 S_2 \varphi_0(r) \psi_0(r_d),
\]

where \( S_i = 1 - \omega_i \) are the neutron (\( i = 1 \)) and proton (\( i = 2 \)) diffraction multipliers, \( \omega_i \) are the nucleon-nucleus profile functions, and \( \varphi_0(r) \) and \( \psi_0(r_d) \) the wave functions of deuteron and its center-of-mass motion, respectively. Let the deuteron
move in the positive direction of \( z \)-axis. Then, the proton concentration in the \( xy \)-plane is determined by the squared absolute value of amplitude (1),

\[
|a(k_1, s_2)|^2 = \left| S_2 \int d^{(3)}r_1 \exp(-i k_1 r_1) S_1 \varphi_0(r) \right|^2,
\]

where \( s_2 \) is the impact parameter vector of proton.

Now, integrating the difference between formula (2) taken at \( S_2 = 1 \) and \( S_2 \neq 1 \), i.e. when the target does not absorb and absorb protons, respectively, over the whole impact plane, we obtain the sought expression for the double-differential (with respect to the neutron emission angle and energy) cross section,

\[
d\sigma_1 = B(k_1) \frac{dk_1}{(2\pi)^3},
\]

\[
B(k_1) = \int d^{(2)}s_2 (1 - |S_2|^2) \left| \int d^{(3)}r_1 \exp(-i k_1 r_1) S_1 \varphi_0(r) \right|^2.
\]

In order to find the angular (energy) distribution of the neutrons arising in the deuteron stripping reaction, expression (3) has to be integrated over the longitudinal (transverse) components of vector \( k_1 \).

As \( \varphi_0(r) \) in (4), we use the deuteron wave function (the S-wave) obtained in the framework of variational method in the Gaussoid basis for the triplet nucleon-nucleon potential from work [11],

\[
V(r) = 3720.0 \exp[-(r/0.488)^2] - 528.59 \exp[-(r/0.976)^2],
\]

namely,

\[
\varphi_0(r) = \sum_{j=1}^{N} c_j \exp(-d_j |r_1 - r_2|^2), \quad N = 10.
\]

This function has correct asymptotics at short and long distances between nucleons. Besides, it reproduces the experimental values of deuteron binding energy and deuteron root-mean-square radius [12] with a high accuracy.

The nucleon-nucleus profile functions in (4), which are considered in the framework of Glauber model [13],

\[
\omega_i(s_i) = 1 - \exp[-\phi_i(s_i)],
\]

where \( \phi_i(s_i) \) is the eikonal phase, can be constructed as follows. Let the distribution of nucleon density in the impact parameter plane look like

\[
\rho_i(s_i) = \rho_i(0) \exp(-s_i^2/\alpha_N^2),
\]
where \( a_N^2 = r_0^2/\ln 2 \) and \( r_0^2 = 0.65 \text{ fm}^2 \) \cite{14}. Expanding the density distribution (experimental \cite{9} or model) in series of Gaussoid basis functions,

\[
\rho_T(s) = \sum_{j=1}^{K} \rho_{Tj} \exp(-s^2/a_{Tj}^2), \quad a_{Tj}^2 = R_{\text{rms}}^2/j,
\]

where \( R_{\text{rms}} \) is the root-mean-square radius of target nucleus, the formula for the eikonal phase from work \cite{15} can be generalized:

\[
\phi_i(s_i) = \sum_{j=1}^{K} \phi_{ij}(0) \exp\left(-\frac{s_i^2}{a_{Tj}^2 + a_N^2 + r_0^2}\right), \quad \phi_{ij}(0) = N_W \frac{\pi^2 \bar{\sigma}_{NN} \rho_i(0) a_N^2 \rho_T a_{Tj}^2}{a_{Tj}^2 + a_N^2 + r_0^2},
\]

where \( N_W \) is the normalizing coefficient for the imaginary part of double-folding potential, and \( \bar{\sigma}_{NN} \) the isotopically averaged cross section of nucleon-nucleon interaction. Substituting (10) into (7) and expanding \( \omega_i \) in series once more, we obtain

\[
\omega_i(s_i) = \sum_{j=1}^{K} \alpha_{ij} \exp\left(-\frac{s_i^2}{\beta_{ij}}\right), \quad \beta_{ij} = R_{\text{rms}}^2/j.
\]

Now, substituting functions (6) and (11) into (4) and integrating the result, we obtain the expression

\[
B(k_1) = B(\kappa_1, k_{1z}) = \sum_{i=1}^{N} \sum_{j=1}^{N} c_i c_j Y(\lambda^{-1}, \kappa_1, k_{1z}), \quad \lambda = (d_i + d_j)/2,
\]

where

\[
Y(\lambda^{-1}, \kappa_1, k_{1z}) = y^{(1)}(\lambda, \kappa_1, k_{1z}) - y^{(2)}(\lambda, \kappa_1, k_{1z}),
\]

\[
y^{(1)}(\lambda, \kappa_1, k_{1z}) = 4t(\lambda, k_{1z})(y_{11}(\lambda, \kappa_1) + y_{12}(\lambda, \kappa_1) + y_{13}(\lambda, \kappa_1)),
\]

\[
y^{(2)}(\lambda, \kappa_1, k_{1z}) = t(\lambda, k_{1z})(y_{21}(\lambda, \kappa_1) + y_{22}(\lambda, \kappa_1) + y_{23}(\lambda, \kappa_1)),
\]

\[
t(\lambda, k_{1z}) = \pi^4 \lambda^3 \exp\left(-\frac{\lambda k_{1z}^2}{2}\right),
\]

\[
y_{11}(\lambda, \kappa_1) = \exp\left(-\frac{\lambda \kappa_1^2}{2}\right) \sum_{i=1}^{K} \alpha_{2i} \beta_{2i},
\]

\[
y_{12}(\lambda, \kappa_1) = -2 \sum_{i=1}^{K} \sum_{j=1}^{K} \alpha_{1i} \beta_{1i} \alpha_{2j} \beta_{2j} \exp\left(-\frac{\lambda + 2 \beta_{1i} + 2 \beta_{j} \lambda \kappa_1^2}{\lambda + \beta_{1i} + \beta_{2j}}\right),
\]

\[
y_{13}(\lambda, \kappa_1) = \sum_{i=1}^{K} \sum_{j=1}^{K} \sum_{l=1}^{K} \alpha_{1i} \beta_{1i} \alpha_{1j} \beta_{1j} \alpha_{2l} \beta_{2l} \exp\left(-\frac{\beta_{1ij} \lambda \kappa_1^2}{\lambda + \beta_{1ij} + 2 \beta_{2l}}\right),
\]

\[
y_{21}(\lambda, \kappa_1) = \exp\left(-\frac{\lambda \kappa_1^2}{2}\right) \sum_{i=1}^{K} \alpha_{2i} \beta_{2i}.
\]
\[ y_{22}(\lambda, \kappa_1) = -4 \sum_{i=1}^{K} \sum_{j=1}^{K} \sum_{l=1}^{K} \alpha_{1i} \beta_{1i} \alpha_{2j} \beta_{1j} \beta_{2jl} \exp \left( -\frac{\lambda + 2\beta_{1i} + \beta_{2jl}}{2\lambda + 2\beta_{1i} + \beta_{2jl}} \frac{\lambda \kappa_1^2}{2} \right), \quad (21) \]

\[ y_{23}(\lambda, \kappa_1) = \sum_{i=1}^{K} \sum_{j=1}^{K} \sum_{l=1}^{K} \sum_{n=1}^{K} a_{1i} \beta_{1i} a_{1j} \beta_{1j} a_{2l} \beta_{2l} \beta_{2ln} \exp \left( -\frac{\beta_{1ij}}{\lambda + \beta_{1ij}} \frac{\lambda \kappa_1^2}{2} \right), \quad (22) \]

\[ \beta_{ijl} = 2\beta_{ij} \beta_{il} / (\beta_{ij} + \beta_{il}), \quad (i=1, 2; \ j, l=1, K), \quad (23) \]

\( k_1 = \{\kappa_1, (k/k)k_{1z}\} \), and \( k \) is the vector of incident deuteron momentum, with \( \kappa_1 k = 0 \). The components \( \kappa_1 \) and \( k_{1z} \) of vector \( k_1 \) are related to the neutron energy \( T_1 \) and emission angle \( \Theta_1 \) in the laboratory reference frame by the formulas [5]

\[ \kappa_1 = (k/2 + k_{1z}) \tan \Theta_1, \quad (24) \]
\[ k_{1z} = \sqrt{m/T}(T_1 - T/2), \quad (25) \]

where \( m \) is the nucleon mass, and \( T \) the initial deuteron energy.

Expressing the components of \( dk_1 \) in (3) in the cylindrical coordinates and using (24), we obtain the angular distribution of neutrons,

\[ \frac{d\sigma_1}{d\Omega_1} = \frac{1}{(2\pi)^3 \cos^3 \Theta_1} \int_{-\infty}^{\infty} (k/2 + k_{1z})^2 B(\kappa_1, k_{1z}) dk_{1z}. \quad (26) \]

In order to calculate cross section (26) in the center-of-mass frame, the formulas of relativistic kinematics from work [16] were used.

### 3. Results of calculations

In figure, the neutron angular distributions calculated for the reaction \(^2\text{H}(d, n)^3\text{He} \) at intermediate energies of incident particles are shown as an example. The solid curves demonstrate the results of cross section calculations with exact deuteron wave function (6); the same function was applied while constructing the target density distribution (9) with \( K = 10 \). The dashed curves were calculated making use of the model function

\[ \varphi_0(r) = (2\xi/\pi)^{3/4} \exp(-\xi|\mathbf{r}_1 - \mathbf{r}_2|^2). \quad (27) \]

Here, the parameter \( \xi = 0.049 \text{ fm}^{-2} \) was so chosen that (27) would reproduce the experimental root-mean-square radius of deuteron [12]. The dash-dotted curves reproduce the results of cross section calculations made in work [17] in the framework of the virtual pion exchange model. No fitting parameters were used when calculating cross sections (26), except for the normalization factor \( N_W \) for the imaginary
Figure 1: Angular distributions of neutrons in the reaction $^2\text{H}(d,n)^3\text{He}$ at $T = 787$ (1), 858 (2), and 1242 MeV (3). See other explanations in the text. Experimental data were taken from work [17].

part of double-folding potential in (10). The relevant $N_W$-values were equal to 0.68 (at $T = 787$ MeV), 0.49 (858 MeV), and 0.15 (1242 MeV).

The behavior of calculated curves brings us, first of all, to a conclusion that it is highly important that the wave function of incident particle with correct asymptotics should be used in similar calculations. Model function (27) has a good asymptotic at short internucleon distances, but the corresponding cross sections decrease more rapidly than experimental values as the nucleon emission angle $\Theta$ increases (dashed curves). From a comparison between the cross sections calculated with exact wave function (6) and the experimental data, it follows that the behavior of deuteron nucleon density in the tail section of distribution is crucial for the satisfactory description of experiments (solid curves). Whence a conclusion can be drawn that the deuteron stripping is a surface reaction [18].

4. Conclusions

The majority of experimental and theoretical works devoted to the researches of deuteron stripping reactions on nuclei were published in 1960s-1970s. Interest
revived recently to this reaction (see [6] and references therein) is associated with intensive studies of unstable nuclei. In this connection, the \((d, N)\) processes may turn out a unique tool for extracting spectroscopic information. The main result of this work is the exact analytical expression for the corresponding cross section obtained by transforming integrands in the general formula. Such an approach can also be used in other similar problems if the relevant integrands can be expanded in series of Gaussoid basis functions.

Concerning the result of this work, the universal character of its possible application should be emphasized. The matter is that, in its most general definition [19], the inclusive stripping reaction means that one of the incident particle fragments becomes removed from the particle and participates in an unobserved interaction subprocess with the target. The subprocess can be arbitrary: from inelastic scattering to nuclear fusion (really, general expression (4) contains all information on the input channel and only partial on the output one). Preliminary calculations show that double-differential cross section (3) with \(B(k_1)\) calculated by formulas (12)–(23) successfully describes experimental data for the \((d, pn)\) process, in which the spectrum of output protons is registered [20]. In our opinion, the formulas obtained in this work will also allow one to analyze experimental data on the stripping, pickup, and breakup reactions for light and heavy ions (provided that the projectile wave function and the corresponding cluster-nucleus potential are known).

The stripping problem considered above can also be generalized to the case when the spin-orbit interaction is taken into account. The difference from this work is reduced to the appearance of the corresponding operator in the expression for profile function. Then, using the density matrix formalism and carrying out required expansions in the Gaussoid basis, it is possible to derive an analytical formula for the polarization of particles arising in the stripping reaction.

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