Development and validation of a parameter estimation methodology for two different lithium-ion batteries to optimize their performance and life cycle

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Abstract. This paper presents four procedures developed to analyse the dependence of the discharge curve of a lithium battery on discharge current and working temperature. For this work, two models of lithium batteries have been used, whose discharge curves have different shapes. The first one, the shape of the curve describes an almost horizontal line in most of it (type 1), while in the other one, the shape of the curve describes a negative slope in most of it (type 2). Two of the developed procedures are used for discharge curves with different currents but constant temperature and the other two developed procedures are used for discharge curves with different temperatures but constant current. The information for the development of the simulations is obtained from the datasheet of the analysed lithium batteries.

1. Introduction

The development of a good battery model demands it to be able to reproduce both its dynamic characteristics, as well as the loss of capacity throughout its useful life. This has two very important impacts: the first one allows an adequate sizing of the energy storage system to be carried out, according to the application that will be given to this technology, implying that the initial economic investment is reduced; the second important point is the possibility of obtaining the optimal profile of operation of the battery, maximizing its lifetime, which makes the investment return more profitable.

The wide variety of battery models found in the literature can be grouped into three main categories, depending on the modelling approach [1]: electrochemical models, mathematical models and equivalent circuit models.

Electrochemical models can provide full information about the dynamics of the battery, achieving a high accuracy, on the other hand, they consist of a set of partial differential equations, which demand a high computational effort for performing long time studies. Moreover, the parametrization of those equations is not straightforward, since they require specific data that are usually not provided by battery manufacturers.

Mathematical models are commonly associated with stochastic models developed with fuzzy logic or artificial neural networks techniques. Hence, it is not needed to understand the relationship between the battery dynamics and its influencing factors. Nevertheless, developing those models requires a huge amount of data from a specific battery to train and make them non generalizable.
Equivalent circuit models are developed to imitate the dynamics of a battery with a set of capacitors, resistances, and inductances. The main advantage is their simplicity, since they use ordinary differential equations, reducing considerably the computational effort. Moreover, they are easily implemented and generalizable for different batteries among the lithium ion family, which makes them notably suitable for industry application studies.

The present work is related to equivalent circuit models and uses information provided in the datasheet of lithium batteries manufacturers to analyse the way in which the parameters of the discharge curve vary, depending on the current and temperature. This may lead to future long-term performance predictions for lithium batteries.

The work starts from an already known mathematical model for a lithium battery, furthermore this work focuses on finding a suitable method for extracting the parameters that belong to the aforementioned mathematical model and another method for analysing the dependence of these parameters on current and temperature. This will allow an adequate simulation of the electrical performance of a lithium battery under different operating conditions to those specified in its datasheet, that is, the electrical performance of the battery can be modeled under a desired operating regime.

In the data sheets, two types of discharge curves have been identified, therefore, a different analysis has been carried out for each case, which results in different calculation procedures. Future research should focus on finding a unique method of analysis. A correct estimation of the discharge curve parameters will generate a modeling of the electrical behavior of the battery close to its actual operation. Therefore, it is important to develop a good parameter extraction method. A miscalculation in the discharge curve parameters can lead to the estimation of certain lithium battery working conditions that are not really adequate, which could lead to the reduction of the battery life time.

Chapter 2 presents the equations used to model lithium batteries. Chapter 3 addresses the methodology to obtain the model parameters from the battery manufacturer data. Chapter 4 analyses the dependence of the model parameters on current and temperature. Finally, chapter 5 validates the developed methodology.

2. Mathematical model
For the present work, the datasheets of two lithium batteries were used, in which two different types concerning the shape of the discharge curve have been identified: a) ANR26650 M1-A [2] from A123SYSTEMS, where it is observed that the discharge curve describes an almost horizontal line in most of its shape (type 1, figure 1a) b) CGR18650AF [3] from PANASONIC, where the discharge curve describes a negative slope in most of its shape (type 2, figure 1b).

The difference between the shape of the discharge curves (type 1 and type 2) requires slightly different processes or data treatment, with the aim of making a realistic mathematical description of the actual case and, consequently, making it possible to analyse the dependence of the curve parameters on current and temperature. The need to develop differentiated processes is observed in [4], where the developed method performs an inexact calculation of the shape of the type 2 curves.

2.1. Voltage – Current performance
The electrical characteristic of a lithium battery is modeled using equation (1) [5-7], whose equivalent circuit is shown in figure 2. This model has been chosen because its parameters not need experimental tests, but rather this information can be extracted from the datasheet of the battery to be tested, which is provided by the manufacturers.
Figure 1. Discharge curves (a) type 1 and (b) type 2 for lithium batteries.

\[ V = E_0 - i \cdot R - K \cdot \left( \frac{Q}{Q - it} \right) \cdot (it + i^*) + A \cdot \exp(-B \cdot it) \]  

(1)

Where

- \( V \): battery voltage (V)
- \( E_0 \): battery constant voltage (V)
- \( K \): polarization constant (V/Ah) or polarization resistance (Ω)
- \( Q \): battery capacity (Ah)
- \( it \): instantaneous battery charge (Ah)
- \( A \): exponential zone amplitude (V)
- \( B \): exponential zone time constant inverse (Ah\(^{-1}\))
- \( R \): internal resistance (Ω)
- \( i \): battery discharge current (A)
- \( i^* \): filtered discharge current (A)

Figure 2. Equivalent circuit for discharge battery model.
2.2. Thermal model

Thermal modeling is based on the dependence of the parameters in equation (1) with respect to temperature. For this, equations (2) to (5) [8-10] are used:

\[ E_o(T) = E_o|_{T=ref} + \frac{E_o}{\Delta T}(T - T_{ref}) \]  
\[ (2) \]

\[ R(T) = R|_{T=ref} \cdot \exp \left[ \alpha \left( \frac{1}{T} - \frac{1}{T_{ref}} \right) \right] \]  
\[ (3) \]

\[ K(T) = K|_{T=ref} \cdot \exp \left[ \beta \left( \frac{1}{T} - \frac{1}{T_{ref}} \right) \right] \]  
\[ (4) \]

\[ Q(T) = Q|_{T=ref} + \frac{\partial Q}{\partial T}(T - T_{ref}) \]  
\[ (5) \]

Where:
- \( T \): battery temperature (K)
- \( T_{ref} \): reference battery temperature (K)
- \( \alpha \): Arrhenius constant for the polarization resistance (K)
- \( \beta \): Arrhenius constant for the internal resistance (K)

2.3. Empirical equations

During data processing, it has been observed that certain parameters follow a linear trend with respect to the variation of current and/or temperature. Thus, some empirical equations have been established in order to improve the analysis of the recorded information.

For both types of curves, a linear dependence of the parameter \( Q \) on discharge current has been observed. Similarly, in the case of curve type 1, a linear dependence of parameters \( A \) and \( B \), has been observed, as well as an exponential dependence of parameter \( K \), on discharge current. The next set of equations were used:

\[ Q(i) = Q|_{i=ref} + \frac{\partial Q}{\partial i}(i - i_{ref}) \]  
\[ (6) \]

\[ A(i) = A|_{i=ref} + \frac{\partial A}{\partial i}(i - i_{ref}) \]  
\[ (7) \]

\[ B(i) = B|_{i=ref} + \frac{\partial B}{\partial i}(i - i_{ref}) \]  
\[ (8) \]

\[ K(i) = K|_{i=ref} \cdot \exp \left[ \beta_i(i - i_{ref}) \right] \]  
\[ (9) \]

Where,
- \( i_{ref} \): reference discharge current (A)

Likewise, for both types of curves, a linear dependence of parameter \( B \) on temperature has been observed, where the following equation is used:

\[ B(T) = B|_{T=ref} + \frac{\partial B}{\partial T}(T - T_{ref}) \]  
\[ (10) \]
It has also been observed that the battery capacity in the cut-off voltage \( Q_f \), presents a linear dependence on discharge current, as well as temperature. The following equations were used:

\[
Q_f(i) = Q_f|_{i_{\text{ref}}} + \frac{\partial Q_f}{\partial i}(i - i_{\text{ref}}) \tag{11}
\]

\[
Q_f(T) = Q_f|_{T_{\text{ref}}} + \frac{\partial Q_f}{\partial T}(T - T_{\text{ref}}) \tag{12}
\]

Finally, a new parameter \( V_o \) has been defined.

\[
V_o = E_o - i \cdot R \tag{13}
\]

3. Parameter extraction

Once the model is defined by the battery performance equations, the next step is to obtain the parameters of the system from battery datasheet. The most demanding operation for the batteries is the discharge, therefore the model will be based on the corresponding curves.

The analysis of the electrical performance of lithium batteries begins with the extraction of the parameters in equation (1). This is performed for each discharge curve.

For both types of curves, the value of the terms \( \frac{\partial E_o}{\partial T} \) and \( \alpha \) are found from three discharge curves measured at different temperatures. The following system of equations can be set.

\[
V_{\text{full},1} = E_o |_{T_{\text{ref}}} - i \cdot R_{\text{ref}} + A 
\]

\[
V_{\text{full},2} = E_o |_{T_{\text{ref}}} + \frac{\partial E}{\partial T}(T_2 - T_{\text{ref}}) - i \cdot R |_{T_{\text{ref}}} \cdot \exp \left[ \alpha \left( \frac{1}{T} - \frac{1}{T_{\text{ref}}} \right) \right] + A 
\]

\[
V_{\text{full},3} = E_o |_{T_{\text{ref}}} + \frac{\partial E}{\partial T}(T_3 - T_{\text{ref}}) - i \cdot R |_{T_{\text{ref}}} \cdot \exp \left[ \beta \left( \frac{1}{T} - \frac{1}{T_{\text{ref}}} \right) \right] + A 
\]

Where

\( V_{\text{full},1} \) : voltage at the fully charged state of the curve number 1 (V)
\( V_{\text{full},2} \) : voltage at the fully charged state of the curve number 2 (V)
\( V_{\text{full},3} \) : voltage at the fully charged state of the curve number 3 (V)

3.1. Discharge curve type 1

For this type of curve, the developed procedure is based on the method presented in [1] and [4], for which the same system of equations of the aforementioned method is used, together with equation 13. Furthermore, unlike the base method, the developed procedure does not consider the parameter \( Q \) constant, so one more equation is added from the cut-off voltage, so that the value of \( Q \) can be found using a recursive method.

\[
V_{\text{full}} = V_o + A 
\]

\[
V_{\text{exp}} = V_o - K \cdot \left( \frac{Q}{Q - Q_{\text{exp}}} \right) \cdot (Q_{\text{exp}} + i) + A \cdot \exp (-3) \tag{15b}
\]

\[
V_{\text{nom}} = V_o - K \cdot \left( \frac{Q}{Q - Q_{\text{nom}}} \right) \cdot (Q_{\text{nom}} + i) + A \cdot \exp \left( -\frac{3}{Q_{\text{exp}}} \cdot Q_{\text{nom}} \right) \tag{15c}
\]
\[
V_f = V_o - K \cdot \left( \frac{Q}{Q - Q_f} \right) \cdot (Q_f + i) + A \cdot \exp \left( -\frac{3}{Q_{\text{exp}}} \cdot Q_f \right)
\]

(15d)

Where

- \(V_{\text{full}}\): voltage at the fully charged state (V)
- \(V_{\text{exp}}\): voltage at the exponential section (V)
- \(V_{\text{nom}}\): voltage at the nominal section (V)
- \(V_f\): cut-off voltage (V)
- \(Q_{\text{exp}}\): charge at the exponential section (Ah)
- \(Q_{\text{nom}}\): charge at the nominal section (Ah)
- \(Q_f\): charge at the cut-off voltage (Ah)

3.1.1. Curves with different discharge currents and constant temperature. In this case, the value of the parameters \(E_0\) and \(R\), are found by performing a linear regression of \(V_0\) with respect to the discharge current \((i)\), where \(R\) is the slope of the line and \(E_0\) the intersection with the y axis.

3.1.2. Curves with different temperatures and constant current. In this case, the value of parameter \(A\) is held constant, therefore, although it works with the system of equations 15, equation 15b is excluded and only \(Q_{\text{exp}}\) is used to calculate the value of parameter \(B\) \((= 3/Q_{\text{exp}})\).

3.2. Discharge curve type 2

Due to the shape of this type of curve, it has been considered that parameter \(B\) is not constant, therefore, a system of five equations is proposed that are solved by the substitution method, where parameters \(B\) and \(Q\) are found by recursive methods.

\[
V_{\text{full}} = V_o + A
\]

(16a)

\[
V_{\text{ze}} = V_o - K \cdot \left( \frac{Q}{Q - Q_{\text{ze}}} \right) \cdot (Q_{\text{ze}} + i) + A \cdot \exp (-B \cdot Q_{\text{ze}})
\]

(16b)

\[
V_{\text{exp}} = V_o - K \cdot \left( \frac{Q}{Q - Q_{\text{exp}}} \right) \cdot (Q_{\text{exp}} + i) + A \cdot \exp (-B \cdot Q_{\text{exp}})
\]

(16c)

\[
V_{\text{nom}} = V_o - K \cdot \left( \frac{Q}{Q - Q_{\text{nom}}} \right) \cdot (Q_{\text{nom}} + i) + A \cdot \exp (-B \cdot Q_{\text{nom}})
\]

(16d)

\[
V_f = V_o - K \cdot \left( \frac{Q}{Q - Q_f} \right) \cdot (Q_f + i) + A \cdot \exp (-B \cdot Q_f)
\]

(16e)

Where

- \(V_{\text{ze}}\): voltage inside the exponential zone (V) \(V_{\text{exp}} < V_{\text{ze}} < V_{\text{full}}\)
- \(Q_{\text{ze}}\): charge inside the exponential zone (Ah) \(0 < Q_{\text{ze}} < Q_{\text{exp}}\)

3.2.1. Curves with different discharge currents and constant temperature. Through the first calculations carried out to find the value of the parameters of the discharge equation, it was observed that these values hardly changed, so it was decided to work considering that all the parameters, except for \(Q\), maintain a value constant. In addition, the value of the parameters \(E_0\) and \(R\), are found by taking two curves (or more) and solving the following system of equations:
\[ V_{\text{full}1} - A = E_0 - i_1 \cdot R \]  
(17a)

\[ V_{\text{full}2} - A = E_0 - i_2 \cdot R \]  
(17b)

Where

\( i_1 \) : discharge current of the curve number 1 (A)
\( i_2 \) : discharge current of the curve number 2 (A)

3.2.2. Curves with different temperatures and constant current. In this case, the value of parameter \( A \) is hold constant, therefore, although working with the system of equations 16, equation 16b is excluded.

4. Dependence on current and temperature

Along this chapter the dependence of the model on the variables current and temperature are analysed.

In order to graph the discharge curve, it is must to know how the parameters of equation 1 vary with respect to the discharge current and the operating temperature. On the other hand, despite not being a parameter, the dependence of \( Q_f \) on current and temperature has been analysed (equations 11 and 12), this is because in most cases it has been obtained a correlation coefficient between \( Q_f \) and the discharge current and between \( Q_f \) and the operating temperature, greater than in the case of \( Q \), therefore, for the calculation of this last parameter, first the value of the other parameters and of \( Q_f \) for a given current and temperature conditions are calculated, then equation 1 is realigned in such a way that it allows to calculate \( Q \). The above is possible because the cut-off voltage is constant.

The results of the aforementioned analyses are shown below.

4.1. Discharge curve type 1

For this analysis, seven discharge curves have been considered, four of them at the same operating temperature (25 °C) but with different discharge currents (1 A, 10 A, 30 A and 40 A) and the other three at the same discharge current (2.3 A) but with different operating temperatures (-20 °C, 0 °C and 25 °C).

4.1.1. Curves with different discharge currents and constant temperature. Table 1 shows the values obtained for each parameter belonging to a determined discharge curve analysed, without considering those that remain constant. In this case, a linear dependence of \( V_0 \) (figure 3a), \( \ln K \) (figure 3b), \( A \) (figure 3c), \( B \) (figure 3d), \( Q \) (figure 3e) and \( Q_f \) (figure 3f) on discharge current has been observed.

| \( I \) (A) | \( V_0 \) (V) | \( K \) (V/Ah o \( \Omega \)) | \( A \) (V) | \( B \) (Ah\(^{-1}\)) | \( Q \) (Ah) | \( Q_f \) (Ah) |
|---|---|---|---|---|---|---|
| 1 | 3.312 | 0.007637 | 0.252 | 27.912 | 2.250 | 2.208 |
| 10 | 3.171 | 0.002828 | 0.306 | 19.384 | 2.270 | 2.203 |
| 30 | 2.811 | 0.000337 | 0.495 | 9.968 | 2.201 | 2.171 |
| 40 | 2.624 | 0.000106 | 0.583 | 6.875 | 2.172 | 2.156 |

4.1.2. Curves with different temperatures and constant current. Table 2 shows the values obtained for each parameter belonging to a determined discharge curve analysed, without considering those that remain constant. Likewise, a linear dependence of \( \ln K \) (figure 4a), \( B \) (figure 4b), \( Q \) (figure 4c) and \( Q_f \) (figure 4d) on operating temperature has been observed.
Table 2. Parameters for discharge curve type 1 at different temperatures.

| T (°C) | K (V/Ah o Ω) | B (Ah⁻¹) | Q (Ah) | Q_f (Ah) |
|--------|---------------|-----------|--------|----------|
| -10    | 0.050120      | 12.349    | 2.599  | 2.059    |
| 0      | 0.012230      | 15.217    | 2.312  | 2.206    |
| 25     | 0.005381      | 21.522    | 2.341  | 2.300    |

Figure 3. Dependence of (a) $V_0$, (b) $\ln K$, (c) $A$, (d) $B$, (e) $Q$ y (f) $Q_f$ on discharge current for the curve type 1.
Figure 4. Dependence of (a) $\ln K$, (b) $B$, (c) $Q$ and (d) $Q_f$ on temperature for the curve type 1.

4.2. Discharge curve type 2
For this analysis, seven discharge curves have been considered, three of them at the same operating temperature (25 °C) but with different discharge currents (0.39 A, 1.95 A and 3.9 A) and the other four at the same discharge current (1.95 A) but with different operating temperatures (-10 °C, 0 °C, 25 °C and 45 °C).

4.2.1. Curves with different discharge currents and constant temperature. As explained in section 3.2.1, table 3 shows the $Q$ and $Q_f$ values obtained for each discharge curve analysed. Likewise, a linear dependence of $Q$ (figure 5a) and $Q_f$ (figure 5b) on discharge current has been observed.

Table 3. Parameters for discharge curve type 2 at different currents.

| $I$ (A) | $Q$ (Ah) | $Q_f$ (Ah) |
|--------|----------|------------|
| 0.39   | 2.106    | 2.076      |
| 1.95   | 2.036    | 1.981      |
| 3.90   | 2.038    | 1.930      |

4.2.2. Curves with different temperatures and constant current. Table 4 shows the values obtained for each parameter belonging to a determined discharge curve analysed, without considering those that remain constant. Comparing the corresponding graphs, it is observed that the curve type 1 shows a greater correlation coefficient between the parameter $B$ and the temperature than in the curve type 2. In this case, a linear dependence of $\ln K$ (figure 6a), $B$ (figure 6b), $Q$ (figure 6c) and $Q_f$ (figure 6d) on discharge current has been observed.
Table 4. Parameters for discharge curve type 2 at different temperatures.

| T (°C) | K (V/Ah o Ω) | B (Ah⁻¹) | Q (Ah) | Qᵥ (Ah) |
|-------|---------------|-----------|--------|---------|
| 10    | 0.01566       | 0.905     | 1.809  | 1.669   |
| 0     | 0.00972       | 1.078     | 1.864  | 1.782   |
| 25    | 0.00362       | 1.243     | 2.040  | 2.007   |
| 45    | 0.00363       | 1.126     | 2.136  | 2.102   |

Figure 5. Dependence of (a) Q and (b) Qᵥ on discharge current for the curve type 2.

Figure 6. Dependence of (a) LnK, (b) B, (c) Q and (d) Qᵥ on temperature for the curve type 2.
5. **Validation of the methodology to obtain the model parameters**

The validation of the proposed method consists of two parts: i) from the discharge curves measured by the manufacturer and which are provided in the battery datasheet, the model parameters are obtained, then these parameters are replaced in the equation 1 in order to calculate the points that belong to the measured curve through the mathematical model used in this work, the level of coincidence between the measured curve and the calculated curve is an indicator of the quality of the parameter extraction process, ii) through the linear regression curves the parameters corresponding to each measured curve are calculated, depending on the discharge current and/or the operating temperature, with this information and using equation 1, the points that belong to the measured curves are calculated by the mathematical model being used, the level of coincidence between the measured curves with their respective calculated curves is an indicator of the level of coincidence that will exist between a calculated curve and a measured curve that has not been used for these calculations, that is, it will be an indicator of the quality of the mathematical modeling of the discharge curve, under operating conditions different from those provided in the datasheet.

The process of graphing a discharge curve for desired operating conditions, from a reference curve (provided by the manufacturer) with the same or different operating conditions as above, is what will be referred to hereinafter as translation of the discharge curve, that is, transfer the shape of the discharge curve under initial operating conditions to final conditions.

5.1. **Discharge curve type 1**

The comparison between calculated curve and measured curve for the parameter extraction process and for the translation process of the discharge curve is shown in the figure 7a and 7b, respectively.

![Figure 7. Model and datasheet results for (a) parameters extraction and (b) curve translation of the discharge curve type 1.](image)
If figures 3e and 4c are compared with figures 3f and 4d, respectively, it is observed that there is a higher correlation coefficient in the linear regression of $Q_f$ than in the linear regression of $Q$, therefore, in these cases, instead of calculating $Q$ using its linear regression curve, it was done by first finding $Q_f$ through its own linear regression curve and replacing that value in equation 1, with the other parameters previously calculated and knowing that $V_f$ is constant, $Q$ can be solved from equation 1.

5.2. Discharge curve type 2

The comparison between calculated curve and measured curve for the parameter extraction process and for the translation process of the discharge curve is shown in the figure 8a and 8b, respectively.

![Figure 8](image)

**Figure 8.** Model and datasheet results for (a) parameters extraction and (b) curve translation of the discharge curve type 2.

Similar to the previous case, if figure 5a and figure 5b are compared, it is observed that there is a higher correlation coefficient in $Q_f$ than in $Q$, then this parameter is calculated as in the previous case.

6. Results

Obtaining the discharge curve parameters through the linear regression curve generates an error compared to the values calculated using the parameter extraction procedure. The following figure shows the relative errors to the moment of calculating the parameters for the curve type 1 (figure 9a) and for curve type 2 (figure 9b); in these graphs it can be seen that the parameters that generate the most error in the calculation of the points that belong to the discharge curve are parameter $K$ and parameter $B$, which define the shape of the discharge curve in the nominal zone and in the exponential zone, respectively. To a lesser extent, parameter $A$, which defines the amplitude of the exponential zone, generates a relative error of less than 5% for low currents. Finally, the parameter $V_o$, which helps to define the initial point of discharge and the parameter $Q$ together with $Q_f$, which define the end point of
the discharge curve, are those that generate the least error in the calculation of the points of the discharge curve.

Regarding the coincidence between the calculated discharge curve and the measured discharge curve, it is observed that for the curve type 1, the coincidence is less as the discharge current increases; similarly, for curve type 2, the coincidence is less as the operating temperature is reduced.

Considering the mathematical relationships found in the bibliography and those that have been empirically deduced, it can be said that: i) for both types of discharge curves, the parameters $E_0$ and $R$ depend only on the operating temperature; ii) in the case of the discharge curve type 1, the parameters $K$, $B$ and $Q$ (and $Q_f$) depend on both the discharge current and the operating temperature, while parameter $A$ only depends on the discharge current; iii) in the case of discharge curve type 2, the parameter $Q$ (and $Q_f$) depends on the discharge current and the operating temperature, while the parameters $K$, $A$ and $B$ depend on both the discharge current and the operating temperature.

**Figure 9.** Parameters error for discharge curve (a) type 1 and (b) type 2.
7. Conclusions
This paper presents a procedure to model the electrical performance of a lithium battery from the information available in the datasheet of the battery to be tested.

Two different methods have been developed since two forms of discharge curves were found. Thus future research should focus on finding a single method.

Although the calculation of some parameters has a large relative error, when are compared to their respective values found by the parameters extraction procedure, the final result (that is the simulated curve) has an acceptable similarity with respect to the measured curve, which indicates that not all parameters have the same weight within the equation representing the discharge curve of a lithium battery. Future research should focus on studying the weight of each parameter.

It has been observed that the value of the parameters and the similarity between the measured curve and the calculated curve, depend on the points of the measured curve that are chosen to perform the calculations. A poor choice of those points could generate large errors. Future research may find a set of important or representative points of the discharge curve characterizing its shape. The analyses developed in this work has focused on the study of these points.

The coincidence between the calculated discharge curve and the measured discharge curve has been observed to improve current and temperature values.

The parameters $E_0$ and $R$ were not affected by the discharge current. They only change their value depending on the operating temperature, similarly to the other parameters.

Through the developed methods, it will be possible to estimate the most convenient performance of the lithium battery.

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