On the optical light curves of afterglows from jetted gamma-ray burst ejecta: effects of parameters

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ABSTRACT

Owing to some refinements in the dynamics, we can follow the overall evolution of a realistic jet numerically until its bulk velocity is as small as $\beta c \sim 10^{-3}c$. We find no obvious break in the optical light curve during the relativistic phase itself. However, an obvious break does exist at the transition from the relativistic phase to the non-relativistic phase, which typically occurs at time $t \sim 10^{5} \pm 10^{6.5}$ s (i.e. 10–30 d). The break is affected by many parameters, such as the electron energy fraction $\xi_e$, the magnetic energy fraction $\xi_B^2$, the initial half-opening angle $\theta_0$ and the medium number density $n$. Increasing any of them to a large enough value will make the break disappear. Although the break itself is parameter-dependent, afterglows from jetted GRB remnants are uniformly characterized by a quick decay during the non-relativistic phase, with power-law timing index $\alpha \gg 2.1$. This is quite different from that of isotropic fireballs, and may be of fundamental importance for determining the degree of beaming in $\gamma$-ray bursts observationally.

Key words: radiation mechanisms: non-thermal – stars: neutron – ISM: jets and outflows – gamma-rays: bursts.

1 INTRODUCTION

The discovery of afterglows from gamma-ray bursts (GRBs) has opened up a new era in the field. Up till the end of 1999 August, X-ray, optical and radio afterglows were observed from about 16, 11 and 5 GRBs respectively (Costa et al. 1997; Bloom et al. 1998; Groot et al. 1998; Kulkarni et al. 1998, 1999; Harrison et al. 1999; Stanek et al. 1999; Fruchter et al. 1999; Galama et al. 1999). The so-called fireball model (Goodman 1986; Paczynski 1986; Mészáros & Rees 1992; Rees & Mészáros 1992, 1994; Katz 1994; Sari, Narayan & Piran 1996) is strongly favoured, which is found to be successful in explaining the major features of GRB afterglows (Mészáros & Rees 1997; Vietri 1997; Tavani 1997; Waxman 1997a; Wijers, Rees & Mészáros 1997; Sari 1997a; Sari, Piran & Narayan 1998; Huang, Dai & Lu 1998a; Huang et al. 1998b; Huang, Dai & Lu 1999a,b; Dai & Lu 1999a,b,c; Mészáros, Rees & Wijers 1998; Wijers & Galama 1999). However, we are still far from resolving the puzzle of GRBs, because their ‘inner engines’ are well hidden from direct afterglow observations.

Most GRBs localized by BeppoSAX have indicated isotropic energies of $10^{51} \pm 10^{52}$ erg, well within the energy output from solar-mass compact stellar objects. However, GRB 971214, 980703, 990123 and 990510 have implied isotropic gamma-ray releases of $3.0 \times 10^{53}$ erg (0.17 $M_{\odot}$c$^2$, Kulkarni et al. 1998), $1.0 \times 10^{53}$ erg (0.06 $M_{\odot}$c$^2$, Bloom et al. 1998), $3.4 \times 10^{54}$ erg (1.9 $M_{\odot}$c$^2$, Kulkarni et al. 1999; Andersen et al. 1999) and $2.9 \times 10^{53}$ erg (0.16 $M_{\odot}$c$^2$, Harrison et al. 1999), respectively. Moreover, if really located at a redshift of $z \geq 5$, as suggested by Reichart et al. (1999), GRB 980329 would imply an isotropic gamma-ray energy of $5 \times 10^{54}$ erg (2.79 $M_{\odot}$c$^2$). Such enormous energetics has forced some theorists to deduce that GRB radiation must be highly collimated in these cases, with half-opening angle $\theta \ll 0.2$, so that the intrinsic gamma-ray energy could be reduced by a factor of $10^{2} \sim 10^{3}$, and could still come from compact stellar objects (Pugliese, Falcke & Biermann 1999). Obviously, whether GRBs are beamed or not is of fundamental importance to our understanding of their nature. For theorists, the degree of beaming can place severe constraints on GRB models.

How can we tell a jet from an isotropic fireball? Direct clues may come from afterglow light curves. As argued by Panaiteescu & Mészáros (1999) and Kulkarni et al. (1999), when the Lorentz factor of the ejecta drops to $\gamma < 1/\theta$ the edge of the jet becomes visible. Thus the light curve will steepen by $t^{-3/4}$, where $t$ is the observed time. This is called the edge effect (Mészáros & Rees 1999). Another effect is the lateral expansion effect. Rhoads (1997, 1999a,b) has shown that the lateral expansion (at sound speed) of a relativistic jet ($\gamma \geq 2$) will cause the blast wave to decelerate more quickly, leading to a sharp break in the afterglow light curve. The breaking point is again determined by $\gamma \sim 1/\theta$. The power-law decay indices of afterglows from GRB 980326 and

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980519 are anomalously large, \( \alpha \sim 2.0 \) (Groot et al. 1998; Owens et al. 1998; Halpern et al. 1999), and optical light curves of GRB 990123 and 990510 even show obvious steepening at \( t \geq 1-2\text{d} \) (Kulkarni et al. 1999; Harrison et al. 1999; Castro-Tirado et al. 1999). Recently GRB 970228 was also reported to have a large index of \( \alpha \sim 1.73 \) (Galama et al. 2000). These phenomena have been widely regarded as evidence for relativistic jets (Sari, Piran & Halpern 1999).

However, numerical studies by some other authors (Panaitescu & Mészáros 1998; Moderski, Sikora & Bulik 2000) have shown that, because of the increased swept-up matter and the time delay of the large-angle emission, the sideways expansion of the jet does not lead to an obvious dimming of the afterglow. Thus there are two opposite conclusions about the jet effect: the analytical solution predicts a sharp break, while the numerical calculation shows no such sharp breaks. The condition is quite confusing. We need to clarify this question urgently.

In a recent paper (Huang et al. 2000), we developed a refined model to describe the evolution of jetted GRB remnants. Owing to some crucial refinements in the dynamics, we can follow the overall evolution of a realistic jet until its expanding velocity is as small as \( \sim 10^{-5}\text{c} \). Many new results were obtained in that paper, for example those given below.

(i) We found no obvious break in the optical light curve during the relativistic phase itself, i.e. the time determined by \( \gamma - 1/\theta \) is not a breaking point. In some cases, however, obvious breaks do appear at the relativistic–Newtonian transition point.

(ii) Generally speaking, the Newtonian phase of jet evolution is characterized by a sharp decay of optical afterglows, with the power-law timing index \( \alpha \geq 1.8-2.1 \). The most interesting finding may be that whether the relativistic–Newtonian break appears or not depends on \( \xi_c \), the parameter characterizing the energy equipartition between electrons and protons. This has given strong hints on the solution to the confusing problem mentioned just above: whether an obvious break appears or not may depend on parameters.

In this paper, we go further to investigate the impact other parameters will have on the optical light curves, based on the model developed by Huang et al. (2000). The organization of the paper is as follows. For completeness, the model is briefly described in Section 2. In Section 3 we investigate various parameter effects and present our detailed numerical results, mainly in the form of optical light curves. We find that the light curve break is severely affected by many other parameters. Section 4 is our final conclusion, and Section 5 is a brief discussion.

2 MODEL

We use the model developed by Huang et al. (2000). This model has the following advantages.

(i) It is applicable to both radiative and adiabatic blast waves, and appropriate for both ultrarelativistic and non-relativistic stages. The model even allows the radiative efficiency \( \epsilon \) to evolve with time, so that it can trace the evolution of a realistic GRB remnant, which is believed to evolve from the highly radiative regime to the adiabatic one (Dai, Huang & Lu 1999).

(ii) It takes the lateral expansion of the jet into account. The lateral speed is given by a reasonable expression.

(iii) It also takes many other effects into account, for example the cooling of electrons and the equal arrival time surfaces. The model is very convenient for numerical studies. Here, for completeness, we describe the model briefly. For details please see Huang et al. (2000).

2.1 Dynamics

Let \( R \) be the radial coordinate in the burster frame; \( t \) the observer’s time; \( \gamma_0 \) and \( M_j \) the initial Lorentz factor and ejecta mass and \( \theta \) the half-opening angle of the ejecta. The burst energy is \( E_b = \gamma_0 M_j c^2 \).

The evolution of the radius \( (R) \), swept-up mass \( (m) \), half-opening angle \( (\theta) \) and Lorentz factor \( (\gamma) \) is described by (Huang et al. 2000)

\[ \frac{dR}{dt} = \beta c \gamma (\gamma + \sqrt{\gamma^2 - 1}), \]

\[ \frac{dm}{dR} = 2\pi R^2 (1 - \cos \theta) n_m p, \]

\[ \frac{d\theta}{dR} = \frac{1}{R} \frac{dR}{dt}, \]

\[ \frac{d\gamma}{dm} = -\frac{\gamma^2 - 1}{M_j + \epsilon m + 2(1 - \epsilon)\gamma^2}, \]

where \( \beta = \sqrt{\gamma^2 - 1}/\gamma \), \( n \) is the number density of the surrounding interstellar medium (ISM), \( m_p \) is the mass of a proton, \( \alpha \) is the comoving lateral radius of the ejecta (Rhoads 1999a; Moderski et al. 2000), \( c_s \) is the comoving sound speed and \( \epsilon \) is the radiative efficiency.

A reasonable expression for \( c_s \) is

\[ c_s^2 = \frac{\gamma (\gamma - 1) (\gamma - 1)}{1 + \gamma (\gamma - 1)} c^2, \]

where \( \gamma \approx (4\gamma + 1)/(3\gamma) \) is the adiabatic index. In the ultra-relativistic limit \( (\gamma \geq 1, \gamma = 4/3) \), equation (5) gives \( c_s^2 = c^2 \), and in the non-relativistic limit \( (\gamma \sim 1, \gamma = 5/3) \), we simply obtain \( c_s^2 = 5B^2 c^2/9 \).

As usual, assume that the magnetic energy density in the comoving frame is a fraction \( c^2\) of the total thermal energy density (Dai et al. 1999), \( B^2/8\pi = \xi_c^2 c^2 \), and that the shock-accelerated electrons carry a fraction \( \xi_c \) of the proton energy. The minimum Lorentz factor of the random motion of electrons in the comoving frame is \( \gamma_{\text{emin}} = \xi_c (\gamma - 1) m_p (p - 2)/(m_n (p - 1)) + 1 \), where \( p \) is the index characterizing the power-law energy distribution of electrons, and \( m_c \) is the electron mass. Then the radiative efficiency of the ejecta is (Dai et al. 1999)

\[ \epsilon = \xi_c \frac{t_{\text{syn}}^{-2} - 1}{t_{\text{ex}}^{-2} - 1}, \]

where \( t_{\text{ex}} \) is the comoving frame expansion time, \( t_{\text{ex}} = R/\gamma c \), and \( t_{\text{syn}} = 6\pi m_n c/(\sigma_T B^2 \gamma_{\text{emin}}) \), with \( \sigma_T \) the Thompson cross-section. In this paper, we call the jet, the radiative efficiency of which evolves according to equation (6), a ‘realistic’ one (Dai et al. 1999).

2.2 Synchrotron radiation

In the comoving frame, synchrotron radiation power at frequency \( \nu' \) from electrons is given by (Rybicki & Lightman 1979)

\[ P'(\nu') = \frac{3\sqrt{2} \gamma_c^5 B^2}{m_e c^3 \nu_{\gamma_{\text{max}}} F(\nu'/\nu_c)} d\gamma_c, \]

where \( e \) is electron charge, \( \gamma_{\text{c,max}} = 10^4(B'/1 \text{ G})^{-1/2} \) is the
maximum Lorentz factor of electrons, $dN_e/d\gamma_e$ is the electron distribution function, $\nu_c = 3\gamma_c^2 eB/(4\pi m_e c)$, and

$$F(x) = x^{-1} \int_0^\infty K_{5/3}(k) dk,$$

(8)

with $K_{5/3}(k)$ being the Bessel function.

In the absence of radiation loss, the distribution of the shock-accelerated electrons behind the blast wave is usually assumed to be a power-law function of electron energy, $dN_e/d\gamma_e \propto \gamma_e^{-p}$. However, radiation loss may play an important role in the process. Sari et al. (1998) have derived an equation for the critical electron Lorentz factor, $\gamma_c$, above which synchrotron radiation is significant: $\gamma_c = 6\pi m_e c/(\sigma_T B^2)$. In our model, the actual electron distribution is given according to the following cases (Dai et al. 1999; Huang et al. 2000):

(i) for $\gamma_c \leq \gamma_{c,\text{min}}$,

$$\frac{dN_e}{d\gamma_e} \propto \gamma_e^{-(p+1)}, \quad (\gamma_{\text{min}} \leq \gamma_e \leq \gamma_{\text{max}});$$

(9)

(ii) for $\gamma_{\text{min}} < \gamma_e \leq \gamma_{\text{max}}$,

$$\frac{dN_e}{d\gamma_e} \propto \begin{cases} \gamma_e^{-p}, & (\gamma_{\text{min}} \leq \gamma_e \leq \gamma_c), \\ \gamma_e^{-(p+1)}, & (\gamma_c < \gamma_e \leq \gamma_{\text{max}}); \end{cases}$$

(10)

(iii) for $\gamma_e > \gamma_{\text{max}}$,

$$\frac{dN_e}{d\gamma_e} \propto \gamma_e^{-p}, \quad (\gamma_{\text{min}} \leq \gamma_e \leq \gamma_{\text{max}}).$$

(11)

Let $D_L$ be the luminosity distance and $\Theta$ the angle between the velocity of emitting material and the line of sight, and define $\mu = \cos \Theta$. Then the observed flux density at frequency $\nu$ from this emitting point is

$$S_\nu = \frac{1}{\gamma^4(1-\beta \mu)^2} \frac{1}{4\pi D_L^2} P [\gamma(1-\mu \beta) \nu].$$

(12)

In order to calculate the total observed flux density, we should integrate over the equal arrival time surface (Waxman 1997b; Sari 1997b; Panaitescu & Mészáros 1998) determined by

$$t = \int \frac{1-\beta \mu}{\beta c} dR = \text{const},$$

(13)

within the jet boundaries (Moderski et al. 2000; Panaitescu & Mészáros 1999).

3 NUMERICAL RESULTS

In this section, we follow the evolution of jetted GRB remnants numerically to see what effects those parameters (such as $\xi_e$, $\xi_\Theta$, $\theta_0$, $\theta_{\text{obs}}$, $n$, $p$, ..., etc.) will have on the optical light curves. For convenience, let us define the following initial values or parameters as a set of ‘standard’ parameters: initial energy per solid angle $E_0/\Omega_0 = 10^{54}$ erg/4$\pi$, $\gamma_0 = 300$ (i.e. initial ejecta mass per solid angle $M_0/\Omega_0 = 0.0019 M_\odot/4\pi$), $n = 1$ cm$^{-3}$, $\xi_\Theta = 0.01$, $p = 2.5$, $D_L = 1.0 \times 10^6$ kpc, $\xi_e = 0.1$, $\theta_0 = 0.2$, $\theta_{\text{obs}} = 0$, where the observing angle $\theta_{\text{obs}}$ is defined as the angle between the line of sight and the jet axis. For simplicity, we first assume that the expansion is completely adiabatic all the time (i.e. $\epsilon = 0$; we call it an ‘ideal’ jet, distinguishing it from the ‘realistic’ jet defined in Section 2.1).

Fig. 1 shows the evolution of the Lorentz factor for some exemplary jets. In the ‘standard’ case, the ultrarelativistic phase lasts only for $\sim 10^5$ s, and the non-relativistic phase begins at about $t \sim 10^6$ s. In short, the ejecta will cease to be highly relativistic at time $t \sim 10^5$ s. This again gives strong support to our previous argument that we should be careful in discussing the fireball evolution under the simple assumption of the ultrarelativistic limit (Huang et al. 1998a,b, 1999a,b). In the case of a dense ISM ($n = 10^6$ cm$^{-3}$), the dash-dotted line, the expansion will become non-relativistic as early as $t \sim 10^{5.5}$ s. Fig. 2 is the evolution of the jet opening angle $\theta$. During the ultrarelativistic phase, $\theta$ increases only slightly. At the Newtonian stage, however, the increase of $\theta$ is very quick.

The effect of $\xi_\Theta$ on the optical (R-band) light curves is illustrated in Fig. 3, from which we can clearly see the following.

(i) In no case could we observe the theoretically predicted light curve steepening (with the break point determined by $\gamma \sim 1/\theta$) during the relativistic stage itself; this is consistent with the result of Moderski et al. (2000). Note that in this figure the relativistic phase is restricted by $t \lesssim 10^5$ s. Huang et al. (2000) have given a reasonable explanation for this phenomenon: the edge and the lateral expansion effects begin to take effect when $\gamma \sim 1/\theta$ (in our calculation this occurs at $t \sim 10^{5.5}$ s; however the blast wave is already in its mildly relativistic phase at that moment and it will become non-relativistic soon after that (i.e. when $t \gtrsim 10^5$ s, see Fig. 1). It is therefore not surprising that we could not see any
obvious breaks during the relativistic phase, as they just do not have time to emerge. Another possible reason is that at these stages, because $\gamma$ is no longer much larger than 1, we should be careful in using the analytic approximations.

(ii) When $\xi_e$ is small, an obvious break does appear in the light curve, but it is clearly the result of the transition from the relativistic phase to the non-relativistic phase. The simulation by Moderski et al. (2000) does not show such breaks, because their model is not appropriate for non-relativistic expansion (Huang et al. 2000).

(iii) When $\xi_e$ is large, the break disappears. This is not difficult to understand (Huang et al. 2000). According to the analysis in the ultrarelativistic limit, the time that the light curve peaks scales as (Wijers & Galama 1999; Böttcher & Dermer 2000; Chevalier & Li 2000)

$$t_m \propto \left(\frac{p - 2}{p - 1}\right)^{4/3} \xi_e^{1/3} (\xi_B^2)^{1/3}. \tag{14}$$

From Fig. 3 we can also see that with the increase of $\xi_e$, $t_m$ becomes larger and larger, consistent with equation (14). In the case of $\xi_e = 1.0$, $t_m$ is as large as $\sim 10^5$ s. Note that the expansion has already ceased to be ultrarelativistic at that moment. Then we cannot see the initial power-law decay (i.e., with $\alpha \sim 1.1$) in the relativistic phase, and so it is not strange that the break arising from the relativistic–Newtonian transition does not appear (Huang et al. 2000).

(iv) In all cases, the light curves during the non-relativistic phase are characterized by quick decays, with $\alpha \gg 2.1$. This is quite different from isotropic fireballs, the light curves of which steepen only slightly after entering the non-relativistic phase (Wijers et al. 1997; Huang et al. 1998a). We thus suggest that the most obvious characteristic of jet effects is a sharp decay of afterglows at late stages (with $\alpha \gg 2$). This will be further proved by the figures that follow.

Fig. 4 illustrates the effect of $\xi_B$ on the optical light curves. Again no break appears during the relativistic phase itself. Interestingly but not surprisingly, we see that $\xi_B$ has an effect similar to $\xi_e$: for small values of $\xi_B$, there are obvious breaks at the transition from the relativistic stage to the non-relativistic stage (i.e. at $t \sim 10^5$ s), but for large $\xi_B$ values the break disappears and we could only observe a single steep line with $\alpha \sim 2.1$. The reason is also similar to that for $\xi_e$: With the increase of $\xi_B$, $t_m$ becomes larger and larger, as can be clearly seen from equation (14). When $t_m$ is large enough (i.e. enters the mildly relativistic zone), the initial power-law decaying segment (with $\alpha \sim 1.1$) in the relativistic phase will be hidden completely. Then we can only see the quick decay in the Newtonian phase. The figure also shows that with the decrease of $\xi_B$, the flux density decreases substantially.

Fig. 5 illustrates the effect of $\theta_0$ on the light curves. For small $\theta_0$ values, the breaks arising from the relativistic–Newtonian transition are very obvious, consistent with a recent analytic treatment by Wei & Lu (2000). For large $\theta_0$ values, the breaks are not striking. The dotted line is drawn with $\theta_0 = 1.3$, it in fact can be regarded as an isotropic fireball. The most notable difference between the solid line and the dotted line is their slope disparity in the non-relativistic phase. This is an important difference between a jet and an isotropic fireball, and may be useful in determining the degree of beaming.

The effect of $\theta_{0b}$ on the light curves is shown in Fig. 6. The thick solid line is drawn with $\theta_{0b} = 0$. The light curve with $\theta_{0b} = \theta_0$ deviates from the thick solid line only slightly. For $\theta_{0b} > \theta_0$, however, the observed flux decreases severely. For $\theta_{0b} = 0.3$, the observed peak flux density is dimmer than that of $\theta_{0b} = 0$ by 5 mag. Fortunately, however, their late time behaviour is very similar. It is therefore still possible for us to resort to systematic deep optical surveys, which are expected to find many faint afterglows not associated with any known GRBs if GRBs are highly collimated.

Fig. 7 illustrates the effect of ISM number density $n$. According
to analysis under the ultrarelativistic assumption, $t_m$ is not related to $n$ \( (t_m \propto n^0) \), and the peak flux density scales as $S_{\text{R}} \propto n^{1/2}$ (Wijers & Galama 1999; Böttcher & Dermer 2000). Fig. 7 shows these patterns qualitatively. Fig. 7 also shows that $n$ affects the light curve steepening: with the increase of $n$, the time that the ejecta enters the Newtonian stage comes earlier, so that the light curve break becomes less and less striking. The effect of dense media on afterglows from isotropic fireballs has been discussed by Dai & Lu (1999, 2000).

Fig. 8 illustrates the effect of $p$ on the light curves. From equation (14) we know that $t_m$ is related to $p$. Our numerical results reflect the relation correctly. Generally speaking, although $p$ affects the slope of the light curve notably, it has minor effect on the light curve steepening. This again is very different from that of an isotropic fireball, the timing index $\alpha$ of which varies from $(3p - 3)/4$ in the relativistic phase to $(15p - 21)/10$ in the Newtonian phase (Wijers et al. 1997; Dai & Lu 1999, 2000), i.e. the increment $\Delta \alpha$ strongly depends on $p$: $\Delta \alpha = (27 - 15p)/20$.

Light curves in Fig. 9 are drawn under different assumptions. The solid line corresponds to a ‘standard’, ‘ideal’ jet (adiabatic), the dashed line corresponds to a ‘realistic’ jet (the radiative efficiency of which evolves according to equation 6), and the dash–dotted line corresponds to a jet without lateral expansion (i.e. $c_s = 0$). It is generally believed that the lateral expansion effect tends to make the light curve steepening more obvious. However, Fig. 9 has just shown the opposite tendency: the steepening of the dash–dotted line is obviously more striking than that of the solid line. In fact the numerical simulation by Moderski et al. (2000) also shows this tendency. From this we conclude that it is the jet edge effect that leads to additional light-curve steepening (with respect to an isotropic fireball) at the relativistic–Newtonian transition point. The lateral expansion effect tends to reduce the steepening.

Fig. 10 shows the evolution of the observed spectra for the ‘standard’ jet. At early times, the spectrum can be divided into three segments. However, at time $t \sim 10^6$ s, a fourth segment emerges. It may come from the edge effect in the Newtonian
phase, and it is just this segment that leads to the light-curve steepening.

4 CONCLUSION

We have studied the evolution of jetted GRB remnants numerically, following the convenient model developed by Huang et al. (2000). In typical cases, the remnant enters the mildly relativistic phase at time $t \sim 10^5$–$10^6$ s (i.e. 1–10 d), and it will become non-relativistic at time $t \approx 10^6$ s (37 d). In the analytic approximation, it is usually assumed that the expansion is highly relativistic ($\gamma \approx 1$) all the time. Now we should note that these approximations are appropriate only for early afterglows (Huang et al. 1998a,b, 1999a,b, 2000). Owing to lateral expansion, the half-opening angle $\theta$ will increase with time. However, the increment of $\theta$ is very small at the ultrarelativistic stage. After entering the Newtonian stage, the increase of $\theta$ is very quick.

It has been predicted that, because of the edge effect and the lateral expansion effect, the light curve of afterglows from jetted GRB remnants should show obvious steepening during the relativistic phase, with the break point determined by $\gamma \sim 1/\theta$. However, our numerical results show clearly that no steepening could be observed during the relativistic stage itself (also see Huang et al. 2000). In many cases, however, a striking break does appear in the light curve at later stages, which is obviously caused by the relativistic–Newtonian transition. Typically the power-law timing index $\alpha$ varies from $\sim 1.2$ to $\sim 2.1$.

The light curve break resulting from the relativistic–Newtonian transition is affected by many parameters, such as $\epsilon_e$, $\epsilon_B$, $\theta_0$ and also $n$. Increasing any of them to a large enough value will make the break disappear. We have also demonstrated that it is the jet edge effect (Kulkarni et al. 1999; Mészáros & Rees 1999) that leads to the light-curve break. The lateral expansion effect just tends to reduce the steepening (see Fig. 9); this may be completely unexpected to most researchers.

Although whether the break appears or not depends on parameters, afterglows from jetted GRB remnants are uniformly characterized by a quick decay during the non-relativistic phase, with $\alpha \approx 2.1$. This is quite different from isotropic fireballs, the light curves of which steepen only slightly after entering the non-relativistic phase. It can be regarded as a fundamental characteristic of jets and may be used to judge the degree of beaming.

The effect of $p$ on the light-curve break is also very interesting. For an isotropic fireball, if $p$ is as small as 2.1, the light curve will steepen only slightly after entering the non-relativistic phase. For a jet, however, if other parameters are properly assumed, we still can observe a steep break. In practical observations, if $p$ can be determined to be near 2 from spectral information, and if we observed a steep break in the light curve, then the possibility is large that we were observing afterglows from a jet.

5 DISCUSSION

We have shown that the afterglow from a jetted GRB remnant is characterized by a steep light curve (with $\alpha \approx 2$) at late stages. However, beaming is not the only factor that leads to such a steep light curve. If the GRB progenitor is a massive star, a stellar wind environment will be created, where the density scales as $n \propto R^{-2}$. Then the afterglow light curve will also be quite steep (Chevalier & Li 2000; Halpern et al. 1999; Frail et al. 2000; Dai & Lu 1998a). This makes the problem even more complicated.

In practical afterglow observations, we may encounter four kinds of optical light curves: (1) a single flat line, with $\alpha \sim 1.1$; (2) a single steep line, with $\alpha \approx 2$; (3) a slightly broken curve, with $\alpha$ evolves from $\sim 1.1$ to $\sim 1.5$; (4) a steeply broken curve, with $\alpha$ evolves from $\sim 1.1$ to $\sim 2.1$. Let us discuss their meanings one by one.

(1) A single flat line, with $\alpha \sim 1.1$. In this case, we can safely say that the ejecta is not highly beamed. GRB 970508, 971214, 980329 and 980703 may be good examples.

(2) A single steep line, with $\alpha \approx 2$. This may be the result of either a highly collimated jet or an isotropic fireball in a wind environment. We cannot distinguish them solely by optical light-curve features. Frail et al. (2000) suggested that they can be distinguished by radio afterglows. GRB 980326, 980519 and probably GRB 970228 belong to this case.

(3) A slightly broken curve, with $\alpha$ evolving from $\sim 1.1$ to $\sim 1.5$. In this case the ejecta should be isotropic, and the break is likely caused by the relativistic–Newtonian transition of the remnant.

(4) A steeply broken curve, with $\alpha$ evolving from $\sim 1.1$ to $\sim 2.1$. In this case, the ejecta is probably highly collimated. The break is caused by the relativistic–Newtonian transition of the jet.

Showing obvious breaks in the optical light curves, GRB 990123 and 990510 are the most hopeful candidates for beaming. However their late-time timing indices are still not accurately determined; we cannot definitely say that they belong to case (3) or (4). To determine the late time index accurately, we need a well-defined optical light curve with $t \geq 30$ d. This is a difficult task.

From the above discussions, we see that until now we have observed at least three kinds of optical light curves. They belong to case (1), case (2) and case (3) or (4) respectively. This means that there are at least two kinds of GRB afterglows: one corresponds to an isotropic fireball in a uniform medium (i.e. case 1), the other corresponds to an isotropic fireball in a wind environment or a jet. These kinds of information can provide important clues for our understanding of GRBs. For example, at least we know that $\gamma$-ray emission in some GRBs is isotropic, so the energy crisis is really a problem: GRB 971214 and 980703 indicate isotropic energies of $\sim 0.17 M_{\odot} c^2$ and $\sim 0.06 M_{\odot} c^2$ respectively.

Systematic deep optical surveys may provide another way for determining the degree of beaming. If GRBs are highly collimated, these surveys may reveal many faint decaying optical sources. They are afterglows from jetted GRBs, the $\gamma$-ray emission of which deviates from the line of sight slightly.

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