Abstract: The results of numerical simulation of detection ranges by personnel of ZU-23 anti-aircraft twin barreled autocannon in different conditions of tactical employment and values of statistical probability of aerial target detection for average prepared personnel are presented in this article. Conditional probabilities of target destruction at one and n shots are obtained. Guidance to the target along the tracks of shells and firing by a platoon armed with ZU-23 is also considered. Values of slope distances to the outedge of engagement area under various conditions of tactical employment are found.

Introduction. As an indicator of the firing efficiency of ZU-23 paired anti-aircraft mount, we use a conditional probability of target destruction at n shots $R_n$ [1-3]. Range of target detection and identification depends on background situation, area and target colour, meteorological visibility range (MVR), cut-off angles of fire position and presence of optical interferences. The range of target lock-on to the collimator of the anti-aircraft sight and the values of outedge of engagement area also depend on the same factors.

Analysis of publications [1-4] has shown that massive clouds with fast changes of dark and light areas underestimate the range of targets detection with the naked
eye. The firing limit is related to the direction to the sun not less than \(\pm 15^\circ\) [5, 6]. Although, the sun illumination from the opposite firing direction increases the range of targets detection \(D_d\) and its identification. The target colour can be concealed against the background of the sky. Thus, the target of silver-aluminium colour against the background of the cloudless blue sky reduces \(R_d\) to 50% relatively to the white-gray background [5, 6]. Complex meteorological conditions as rain, fog and application of aerosol or smokescreens used by the enemy are taken into account by the MVR indicator reduction. For the target Su-7B type at MVR is only 4 km \(D_d\) with the naked eye is not more than 3.5 km. The common objective for ZU-23 is Mig-17 aircraft. When MVR is not less than 10 km, the common objective for \(D_d\sim 6.8\) km, and statistical probability of detection \(P_d\) is 0.5 [5, 6].

The quality of personnel combat performance of ZU-23 is estimated from "excellent" to "satisfactory" and determines the values of systematic components of shell firing errors in the image plane of fire. The cut-off angles of fire position do not allow the personnel of AAG (anti-aircraft gun) to fire targets in a timely manner. Pulsed and continuous optical interference, power, can blind the personnel for a while, preventing firing. Signal missiles (shells, mines), routes of anti-aircraft guns, also complicate the process of aerial target detection (firing) [3-6].

The objective of this article is numerical simulation of values of ranges and probabilities of different aerial targets detection, as well as values of conditional probabilities of their destruction and, at the same time, implemented slope distances of the outedge of anti-aircraft mount engagement area.

**Main material.** Determination of ranges and probabilities of various aerial targets detection. Firing from ZU-23 at common objective and unmanned aerial vehicles (UAVs) is being considered. The "Forpost" and "Tahion" were chosen as UAV types. The first relates to the large-size UAV, while the second one relates to the small-size UAV.

To estimate the anti-aircraft mount firing efficiency, we use the approximate highest \(S_{max}\) and the least \(S_{min}\) target area of the image plane of fire. It was assumed for "Forpost" \(S_{max F} = 6.7\ m^2, S_{min F} = 0.58\ m^2\) and for "Tahion" – \(S_{max T} = 0.32\ m^2, S_{min T} = 0.04\ m^2\). The least \(S_{mini}\) is expected when firing at low altitudes (elevation angle \(\varepsilon \leq 5^\circ\)) and the highest \(S_{max i} – \varepsilon > 45^\circ\). The values of UAV flight speed are averaged to 40 m/s. For common objective it is \(S_{max co} = 30.2\ m^2\) and \(S_{min co} = 9.1\ m^2\).

The target detection ranges \(D_{co}(S, \xi_c, \xi_m, \xi_p, \xi_s, K_{co})\) at probability of correct detection \(P_d\) 0.5, depending on various factors we find from the approximate expression [7]:

\[
D_{co}(S, \xi_c, \xi_m, \xi_p, \xi_s, K_{co}) = R_d \sqrt{\frac{S}{S_{min co}}} \xi_c \xi_m \xi_p \xi_s K_{co},
\]

where \(D_{co}\) – CO target detection range (~ 8.5 km [5-7]); \(S, S_{min co} = 9.1\ m^2\) – target image area, which is being fired and the smallest area of CO in the image plane of fire, respectively; \(\xi_c, \xi_m\) – factors that consider the target colour (\(\xi_c\) varies between
0.5 and 1) and MVR ($\tilde{\xi}_m = 0.48...1$); $\tilde{\xi}_p$ – figure of merit of AAG personnel combat performance (“excellent” – 0.9; “good” – 0.8; “satisfactorily” – 0.7; “master” – 1.1); $\tilde{\xi}_s$ – considering factor of solar illumination of a target (in the range from 1.3 to 1.5) [5-7]; $K_{co}$ – considering factor of cut-off angles of the AAG fire position (varies within the range from 0 to 1, during simulation 1 was accepted).

Average ranges of targets detection by the naked eye and its lock-on to collimator of AAG target control, at numerical simulation, were accepted for AAG average prepared personnel by 8.5 km [5, 6].

The results of calculations for the expression (1) are given in Fig. 1.

The first line $D_{co1}(S)$ (indicated by a continuous line) reflects a change in target detection range with probability of 0.5 when $s$ varies within $10^{-2} m^2$ to $10 m^2$ at AAG average prepared personnel ($\tilde{\xi}_p = 0.8$). The second line (dots $D_{co2}(\tilde{\xi}_c)$) and the third line (dashes $D_{co3}(\tilde{\xi}_m)$) give values of detection range of large-dimensional UAV “Forpost” type $S_{minF}$ at influence of target color and MVR, respectively. The significant dependence $D_{co1}$ on target parameters $S, \tilde{\xi}_c, \tilde{\xi}_m$ can be noticed.

The fourth line (dots and dashes $D_{co4}(\tilde{\xi}_p)$) is calculated with account of AAG personnel combat performance. The fifth line (··○··○··○··), $D_{co5}(\tilde{\xi}_s)$ is received when the target is highlighted by the sun. Taking into account the flight speed of different targets, the AAG personnel cannot always fire at the outedge of engagement area. At the same time, firing remains on a pursuit course.

According to the studies, if there is only an alert (no target designation), the probability of CO passing at low altitudes is approximately 0.4, and at altitudes of 2.5…3 km is 0.8, that is the statistical probability of CO detection is within the range of only from 0.6 to 0.2, respectively [5].

On the basis of polygon tests [5, 6] the values of statistical probabilities of different targets detection by ZU-23 personnel, depending on the range to them, are found by approximation [7]:

$$D_{co}(S, \tilde{\xi}_c, \tilde{\xi}_m, \tilde{\xi}_p, \tilde{\xi}_s, K_{co}) = \exp \left( \frac{0.91D}{D_{co}(S, \tilde{\xi}_c, \tilde{\xi}_m, \tilde{\xi}_p, \tilde{\xi}_s, K_{co})} \right)^4 \right)$$

(2)

where $D$ – target range, which varies from 100 m to $10^4$ m.

The results of calculation by expression (2) are given in Fig. 2. The first curve $P_{co1}(D)$ is indicated by continuous line, displays values of statistical probability of CO detection when $S_{minco} = 9.1 m^2$, thus, $P_{co1}(6.8 \cdot 10^3) \approx 0.5$ was obtained. The second curve $P_{co2}(D)$ (dots) and the third curve $P_{co3}(D)$ (dashes) give values of probabilities of detection of the UAV “Forpost” type, when $S_{min F}$, and the third curve under condition of unfavourable UAV colour $P_{co2}(1721) \approx 0.5$ and $P_{co3}(680) \approx 0.5$. The fourth curve $P_{co4}(D)$ (dots and dashes) and the fifth curve $P_{co5}(D)$ (··○··○··○··) accordingly, indicate values of probabilities of detection of the UAV “Forpost” type at MVR $\tilde{\xi}_m = 0.48$ and solar illumination $\tilde{\xi}_s = 1.5$. A significant reduction in the range detection at which the detection probability is 0.5 was obtained. Thus,
$P_{co4}(413) \approx 0.5$ and $P_{co2}(619) \approx 0.5$. It is necessary to note the possibility of trained personnel to detect targets at probability of 0.2...0.3.

**Fig. 1. Target detection range**

$D_{co}$ from variable $S, \xi_c, \xi_m, \xi_p, \xi_s$

**Fig. 2. Statistical probabilities of different targets detection of ZU-23 personnel $P_{coi}$ depending on the range to them $D$**

Calculation of projectiles deviations in the image plane of fire at non-compliance of firing conditions based on external ballistics of a small-caliber anti-aircraft projectile. The matrix of the solution of the system of differential equations by the Runge-Kutta method on the interval $t_1 - t_2$ at $n$ fixed steps of the solution in the MathCAD shell is used [1, 8]. The initial flight time of the projectile is $t_1 = 0$ s, and the final flight time is $t_2 = 5$ s. The number of steps in the interval $t_1 - t_2$, when determining the mean square deviation (MSD) by range and altitude is equal to 100.

Numerical simulation for one of the test points at targeting angle of the AAG $1.5^\circ$ by $D = 2$km gave the MSD by range of $\sigma_d \approx 28m$ and by altitude $\sigma_a \approx 3m$. Although in fire tables [4]. The MSD by range is $\sigma_d \approx 44.4m$ and by altitude $\sigma_a \approx 2.4m$, which indicates slightly improved values relatively to the table ones. Henceforward we find approximate errors of firing from 23 mm AA gun ZU-23 while firing against aerial targets, if the range of fire is ~ 2 km and more.

Analysis of the magnitude of firing errors associated with the initial projectile velocity, air density, projectile weight and wind shown that projectile velocity and air density had the greatest miss impact. On the basis of the conducted numerical simulation, the systematic components $r_i$ of ZU-23 fire errors (fragmentation – high-explosive – incendiary – tracer shells) along the axes $y, z$ in the image plane were assumed to be equal to $r_y \approx r_z \approx 20...60m$.

Finding of values of conditional probabilities of small targets and CO destruction while firing from ZU-23. In the simulation, the number of targets engagement $n$ varies to 100. Systematic components were accepted as $r_y \approx r_z \approx 20...60m$ and corresponding to them MSD errors — $\sigma_y \approx \sigma_z \approx 25...50m$ [8]. According to the research data, these values $r_i, \sigma_i$ take place at the range of fire not less than $1.2 \cdot 10^3$ m.
Calculation of probabilities of the target engagement $P_1(S, \sigma_y, \sigma_z, r_y, r_z)$ is determined by approximate expression [3, 8, 9]:

$$P_1(S, \sigma_y, \sigma_z, r_y, r_z) = \frac{S}{2\pi \sigma_y \sigma_z} \exp\left(-\frac{1}{2} \left[\frac{r_y}{\sigma_y} + \frac{r_z}{\sigma_z}\right]^2\right). \tag{3}$$

Probability of target engagement $G(\omega)$ at target hitting $\omega$ projectiles is equal to $G(\omega) = 1/\omega$. During firing the battlefield fighter (BFF) $\omega$ is within the range from 2 to 6 and for medium bomber is not less than 15 shells [8, 10]. The average values for the BFF $\omega \approx 4$, and for the UAV $\omega \approx 1$.

Determination of values of conditional probabilities of targets engagement $R_1(S, \sigma_y, \sigma_z, r_y, r_z, \omega)$ and $n$ shots $R_n(S, \sigma_y, \sigma_z, r_y, r_z, \omega)$:

$$R_1(S, \sigma_y, \sigma_z, r_y, r_z, \omega) = P_1(S, \sigma_y, \sigma_z, r_y, r_z)G(\omega);$$

$$R_n(S, \sigma_y, \sigma_z, r_y, r_z, \omega) = 1 - \left[1 - R_1(S, \sigma_y, \sigma_z, r_y, r_z, \omega)\right]^n.$$  \tag{4}

We have $n = 100$ (six continuous bursts from AAG) for BFF $R_{100}(9.1; 37; 37; 40; 40; 4) \approx 8.2 \cdot 10^{-3}$ and for UAV $R_{100}(0.58; 37; 37; 40; 40; 1) \approx 2.1 \cdot 10^{-3}$. If ZU-23 personnel directs the projectiles to the target along the tracks, the systematic components of firing errors decrease to the values of $r_y \approx r_z \approx 20m$. In view of the latter, for BFF is $R_{100}(9.1; 37; 37; 20; 20; 1) \approx 0.02$ and for UAV $\approx R_{100}(0.58; 37; 37; 20; 20; 1) \approx 5 \cdot 10^{-3}$.

The results of calculations ($n = 100$) are given in Fig. 3.

Continuous line $R_{n1}(S)$ is obtained for average firing conditions on BFF ($\omega \approx 4$). The second line $R_{n2}(S)$ (dots) shows values of conditional probabilities at firing on UAV ($\omega \approx 1$). The same, but when direct projectiles along the tracks gave the third line (dashes) and the fourth line (dots and dashes). We note an increase in firing efficiency during the guidance of the AAG along the projectile tracks.

We find conditional probabilities of targets destruction by a platoon armed with four 23-mm AAGs $R_{n6i}$ from the expression [3, 8]:

$$R_{n6i}(S, \sigma_y, \sigma_z, r_y, r_z, \omega) = 1 - \left[1 - R_n(S, \sigma_y, \sigma_z, r_y, r_z, \omega)\right]^4.$$  \tag{5}

Analysis of values $R_{n6i}$ showed the expediency of BFF firing and on the UAV "Forpost" type at angles of targets elevation not less than 45°. The result is $R_{n6i}(S_{max\mu}) \approx 0.23$ and $R_{n6i}(S_{max\phi}) \approx 0.21$. At angles of elevation not more than 5° – $R_{n6i}(S_{min\mu}) \approx 0.076$ and $R_{n6i}(S_{min\phi}) \approx 0.02$. Firing at the UAV "Tahion" type gives low efficiency of application $R_{n6i}(S_{maxT}) \approx 0.011$ and $R_{n6i}(S_{minT}) \approx 0.0014$. Though the experience of battle firings from ZU-23 on small UAVs indicates the possibility of their defeat.

*Slope distance of the outedge of engagement area of ZU-23.* If there are no cut-off angles of AAG fire position ($K_{co} = 1$) and the target heading parameter is close
to zero, we find the slope distance of the outedge of engagement area
\( r_d(S, \xi_c, \xi_m, \xi_p, \xi_s, V) \) from the equation:
\[
r_d(S, \xi_c, \xi_m, \xi_p, \xi_s, V) = \frac{D_{co}(S, \xi_c, \xi_m, \xi_p, \xi_s, 1) - V(\tau_d + \tau_i)}{1 + 1.6 \cdot 10^{-3} V},
\]
where \( V \) – airspeed (it was chosen 250m/s for BFF, 40 m/s for UAV); \( \tau_d, \tau_i \) – time of determination and input of initial data, respectively (~ 3 s) and time of designation of fire type (~ 2 c) with the regard for the response time to signals and commands: 
\( r_d(S, \xi_c, \xi_m, \xi_p, \xi_s, V) \leq 2.5 \text{ km} \).

The calculation results are shown in Fig. 4.

\[\text{Fig. 3. Conditional probability of target destruction } R_{ni}(S) \text{ depending on the area of target projection } S \text{ in the image firing plane of the target}\]

\[\text{Fig. 4. Slope distances to the outedge of the engagement area } r_{di} \text{ depending from } S, \xi_c, \xi_m, \xi_p, \xi_s, V \text{ target}\]

The first curve \( r_{d1}(S) \) indicated by a continuous line is obtained at change of projection area of target \( S \) in the image firing plane. When \( S > 4.5 \text{m}^2 \) the imposed limit of \( r_{d1} \) 2.5 km is activated. The influence of the UAV colour of "Forpost" type is shown by the second curve \( r_{d2}(\xi_c) \) (points). If the color is not favorable (\( \xi_c = 0.5 \)) and MSD is different, we have the third curve \( r_{d3}(\xi_m) \) (dashes). The fourth curve \( r_{d4}(\xi_p) \) (dots and dashes) represents the influence of AAG personnel fighting performance quality. Due to solar illumination the range increases – the fifth curve \( r_{d5}(\xi_s) \) (·············). The sixth curve \( r_{d6}(V) \) (·············) indicates the values of engagement zone depending on the UAV speed. The significant impact on the range \( r_{di} \) of targets detection \( D_{coi} \) is noted (6).

It was received \( s_{min,co} \) \( r_{d1}(9.1) \approx 2.5 \text{ km} \), and for the UAV "Forpost" type \( r_{d1}(6.7) \approx 2.5 \text{ km} \) and \( r_{d1}(0.58) \approx 1.4 \text{ km} \).

For the UAV "Tahion" type \( r_{d1}(0.32) \approx 1.0 \text{ km} \) and only \( r_{d1}(0.04) \approx 0.24 \text{ km} \). Negative influence of target colour, MVR, personnel reaction time and target speed reduce values of slope distances to the outedge of the ZU-23 engagement area.
Conclusions. Thus, the results of numerical simulation of detection ranges and probabilities of ZU-23 personnel detection under different conditions of combat performance are proposed. Solutions of equations of external ballistics by Runge-Kutta method were used to determine systematic components of firing errors. Analysis of values of conditional probabilities of different targets destruction using one AAG and by platoon armed with ZU-23 was performed. Values of slope distances to the outedge of the engagement area under different conditions of combat performance are determined. Parameters reducing values of conditional probabilities of targets engagement were taken into account. Proposals have been formed to increase the efficiency of ZU-23 firing.

References:

[1] Herasimov, S., Belevshchuk, Y., Ryapolov, I., Tymochko, O., Pavlenko, M., Dmitriev, O., Zhyvytskyi, M. and Goncharenko, N. (2018). Characteristics of radiolocation scattering of the SU-25T attack aircraft model at different wavelength ranges. Information and controlling systems. Eastern-European Journal of Enterprise Technologies, 69(96), 22–29. https://doi.org/10.15587/1729-4061.2018.152740.

[2] Kriukov, O., Melnikov, R., Bilenko, O., Zozulia, A., Herasimov, S., Borysenko, M., Pavlii, V., Khmelevskiy, S., Abramov, D. and Sivak, V. (2019). Modeling of the process of the shot based on the numerical solution of the equations of internal ballistics. Applied physics. Eastern-European Journal of Enterprise Technologies, 15(97), 40–46. https://doi.org/10.15587/1729-4061.2019.155357.

[3] Manual on the Study of Firing Rules on Anti-Aircraft Artillery Complexes of the Air Defence Troops of the GF. Part 5. Battery (platoon, quad) 23 mm paired mount ZU-23. Moscow, Voenizdat, 1967, 146 p.

[4] Zhuravlev, O., Kolomiytsev, O. and Herasimov, S. (2017). Method for determining coefficient power error of front resistance missile by means station outwardly trajectory measurements. Scientific Works of Kharkiv National Air Force University, 3(52), 72–76.

[5] Firing Rules Manual. Part 7. AA missile system “Kitoboi”, “Strela-10M2”, “Strela-10”. Moscow, Voenizdat, 1989, 248 p.

[6] Firing Rules Manual. Part 8. Man-portable SAM “Igla” (“Igla-1”). Moscow, Voenizdat, 1989, 119 p.

[7] Kolomiytsev, O., Kurdyashov, V. and Shevchenko, A. (2009). Effective fire control and firing from a man portable surface-to-air missile system in the stipulated conditions of its application. Weapon system and military equipment, 2(18), 14–16.

[8] Piskunov, S., Oboronov, M. and Konovalov, O. (2009). Model of the projectile movement when firing surface-to-air missile systems. Weapon system and military equipment, 2(18), 28–32.

[9] Bractslavska, A., Herasimov, S., Zubrytskyi, H., Tymochko, A. and Timochko, A. (2017). Theoretical basic concepts for formation of the criteria for measurement signals synthesis optimality for control of complex radio engineering systems technical status. Information Processing Systems, 5(151), 151–157. https://doi.org/10.30748/sol.2017.151.20.

[10] Borysenko, M., Herasymov, S., Kostenko, O. and Makarchuk, D. (2018). Development of optimum navigation information processing algorithm. Science and Technology of the Air Force of Ukraine, 3(32), 38–44. https://doi.org/10.30748/nhttps.2018.32.06.