Demonstration of the frequency-drift-induced self-comparison measurement error in optical lattice clocks

Xiaotong Lu¹,², Mojuan Yin¹, Ting Li¹,², Yebing Wang¹, and Hong Chang¹,²*¹

¹Key Laboratory of Time and Frequency Primary Standards, National Time Service Center, Chinese Academy of Sciences, Xi’an 710600, People’s Republic of China
²School of Astronomy and Space Science, University of Chinese Academy of Sciences, Beijing 100049, People’s Republic of China
*E-mail: changhong@ntsc.ac.cn

Received April 26, 2020; revised May 17, 2020; accepted June 2, 2020; published online June 17, 2020

The frequency-drift-induced self-comparison measurement error was experimentally demonstrated by measuring the frequency difference between two interleaved clock loops with the same systemic parameters in the ⁸⁷Sr optical lattice clock at the National Time Service Center of China. Combining the experimental and simulated results, this error was precisely determined by the total clock laser frequency drift during the time interval between two adjacent operations of interleaved clock loops. © 2020 The Japan Society of Applied Physics

Benefiting from the development of ultra-stable lasers based on ultra-stable reference cavities,¹⁻⁵ the optical lattice clocks based on neutral atoms have demonstrated unprecedentedly high stability and accuracy.⁶⁻⁸ This excellent performance not only can help to improve the preciseness of the International System of Units (SI),⁹ but also makes optical lattice clocks ideal for detecting physical phenomena which cause minute frequency alterations in the clock transition.¹⁰⁻¹³

In terms of the accuracy and the corresponding uncertainty evaluation of optical lattice clocks, most of the systemic frequency shifts are measured by the method of self-comparison.¹⁴⁻¹⁷ With self-comparison, a clock is separated into two independent clock loops (noted L¹ and L² respectively) which interleaved operate in the time domain and are locked to the same stretched states.¹⁴ The frequency shift can be inferred by comparing the frequency difference of the two interleaved clock loops which usually possess different parameters that need to be measured. The measurement errors will occur when net drifts happen between the two interleaved clock loops. Most of these drifts have been widely studied and carefully evaluated,¹⁸⁻²⁰ while the measurement error caused by the clock laser frequency drift (CLFD) has not been demonstrated, of which the reason is that after the drift cancellation,²¹ the residual drift will always be ignored in the fraction uncertainty of 10⁻¹⁸ in optical lattice clocks. However, with the accuracy of optical lattice clocks improving to 10⁻¹⁹, the CLFD may cause considerable self-comparison measurement error and so the exact relationship between the CLFD and self-comparison measurement error need to be precisely determined.

In this study, the frequency difference, between two interleaved clock loops that possessed the same systemic parameters, was experimentally measured based on the ⁸⁷Sr optical lattice clock at the National Timing Service Center (NTSC). The frequency-drift-induced self-comparison measurement error was demonstrated by this frequency difference and was found to be equal to the total frequency drift of the clock laser during the interval between two adjacent operations of L¹ (or L²). The process of self-comparison was well simulated and the simulations reasonably agreed with the corresponding experimental conclusions.

The schematic of the experimental apparatus used for forming lattice and measuring the clock transition detection. LAL: lattice laser, CLL: clock laser, CL: convex lens; CM: concave mirror, with a radius of 250 mm, PMT: photomultiplier tube. (b) Spin-polarized spectrum for the transition.
One of the

is the reduced Planck constant, \( \hbar \). The typical frequency differences \( f' \) between \( L_1 \) and \( L_2 \). The solid line indicated the self-comparison measurement error \( \Delta_{v-c} \) that was the average of \( \delta_f \). The measured \( \Delta_{v-c} \) at different \( \tau_{loop} \times \alpha_{lin} \). The solid line indicated the linear fitting and the band was the 1σ confidence interval. The fit was carried by the weighted least square method where the weight of each data point was inferred from only the y error bars, considering the \( x \) error bars were almost same and much less than the corresponding y error bars. (d) The overlapping Allan deviation according to all experimental data of \( \delta_f \). The solid line was the white frequency noise fitting with a fixed slope of \(-0.5\), corresponding to the interleaved stability of \(2.43 \times 10^{-15} \). Error bars indicated 1σ confidence intervals.

Fig. 2. (Color online) (a) The typical feedback signals obtained by self-comparison. \( f_{01} \) and \( f_{02} \): the feedback signals of clock loops \( L_1 \) and \( L_2 \), respectively. (b) The typical frequency differences \( f' \) between \( L_1 \) and \( L_2 \). The solid line indicated the self-comparison measurement error \( \Delta_{v-c} \) that was the average of \( \delta_f \). (c) The measured \( \Delta_{v-c} \) at different \( \tau_{loop} \times \alpha_{lin} \). The solid line indicated the linear fitting and the band was the 1σ confidence interval. The fit was carried by the weighted least square method where the weight of each data point was inferred from only the y error bars, considering the \( x \) error bars were almost same and much less than the corresponding y error bars. (d) The overlapping Allan deviation according to all experimental data of \( \delta_f \). The solid line was the white frequency noise fitting with a fixed slope of \(-0.5\), corresponding to the interleaved stability of \(2.43 \times 10^{-15} \). Error bars indicated 1σ confidence intervals.

12000 and the wavelength of the lattice laser was \( \lambda_{L} = 813.42 \) nm at the so-called “magic wavelength”\(^{23} \). The beam waist of the lattice laser was 100 \( \mu \)m and the one-way power was 280 mW, corresponding to a potential depth of 87 \( E_R \). Therein, \( E_R = (\hbar k_L)^2 / 2m \) is the recoil energy, \( \hbar \) is the reduced Planck constant, \( m \) represents the mass of \( ^{87}\text{Sr} \), and \( k_L = 2\pi / \lambda_{L} \) is the wavevector of the lattice laser. The lifetime of the atoms trapped in lattice was more than 7 \( s \). With loading atoms into the lattice, the process of “energy filtering” was used to decreased the atomic temperature, \(^{24} \) where the depth of the lattice was linearly reduced to about 50 \( E_R \) within 10 ms, kept this depth 20 ms and linearly increased to 87 \( E_R \) within 10 ms. After that, the longitudinal and radial temperatures of atoms trapped in the lattice were 1.7 \( \mu \)K and 2.6 \( \mu \)K, respectively. Following, a polarizing laser, resonating with the \( |S_{00}, F = 9/2 \rangle \rightarrow |P_{0}, F = 9/2 \rangle \) transition at \( \lambda = 689 \) nm, was used for driving all atoms to one of the \( |S_{00}, m_{R} = \pm 9/2 \rangle \) stretched states. \(^{25} \)

The wavelength of the clock laser, corresponding to the \( |S_{00} \rangle \rightarrow |P_{0} \rangle \) clock transition at \( \lambda_{L} = 698 \) nm, was stabilized to a resonator made of ULE optical cavity with a fineness of about 200000. This cavity was housed in a passive heat shield in a vacuum system and was actively temperature stabilized at the zero-crossing temperature where the CLFD will be almost linear. \(^{26} \) Both the clock laser system and the ULE cavity setups were mounted on a passive vibration isolation platform. After the clock laser being well stabilized to the ULE cavity, the noises of the output laser were dominated by frequency drift \( \delta_{freq} \) and thermal noise \( \delta_{thermal} \) of the cavity. \(^{5} \) The linewidth of the laser was 1 Hz measured by beating with another similar laser system. \(^{27} \) We used a photomultiplier tube to detect the atomic fluorescence signal and the normalization excitation was determined by the “electron shelving” technique. \(^{28} \) After driving the atoms to stretched states and applying a bias magnetic field about 400 mG in the direction of gravity, the spin-polarized spectrum was obtained by scanning the clock laser frequency by an acousto-optical modulator and detecting the normalization excitation at each frequency detuning. Figure 1(b) represented the typical spin-polarized spectrum with a full width at half maximum of 2.1 Hz as the clock laser pulse duration was 450 ms.

In this experiment, the same systemic parameters of the two interleaved clock loops were same. Therefore, the frequency difference between \( L_1 \) and \( L_2 \) will be zero if there were no any drifts. Otherwise, it indicated the self-comparison measurement error (denoted by \( \Delta_{v-c} \)). When our clock routinely operated with self-comparison, one clock feedback cycle included four clock cycles (each clock cycle last \( T_c = 0.6 \) s) where the first two clock cycles operated \( L_1 \), and the next two clock cycles operated \( L_2 \). In this way, the clock laser frequency was alternately locked to the center
frequency of the $|S_{1p} m_F = +9/2\rangle \rightarrow |P_{1p} m_F = +9/2\rangle$ transition.

Every feedback cycle can obtain two feedback signals, corresponding to $L_1 (f_{01})$ and $L_2 (f_{02})$, respectively, which represented the frequency corrections of the clock laser frequency for making it resonant with clock transition all the time. The typical feedback signals measured by self-comparison was shown in Fig. 2(a), where we can obtain the linear frequency drift rate (denoted as $\alpha_{lin}$ with a unit of $\text{Hz s}^{-1}$) by fitting the slope of $f_{01}$ (or $f_{02}$). Throughout the measurements, the maximum value of the quadratic frequency drift rate was $6 \times 10^{-7} \text{Hz s}^{-2}$, indicating that the linear approximation of the CLFD was reasonable. Figure 2(b) indicated the typical frequency differences $\delta f$ between $L_1$ and $L_2$. The average of $\delta f$ was found to be 0.154 (13) Hz, where the nonzero average was inferred to be caused by CLFD after confirming that other drifts were enough small in our system. To test this prediction, we measured the $\Delta_{v-c}$ at different value of $\tau_{loop} \times \alpha_{lin}$, where the $\tau_{loop} = 2T_c$ indicated the interval between two adjacent operations of $L_1$ (or $L_2$). By linearly fitting the data, the relationship of $\Delta_{v-c}$ and $\tau_{loop} \times \alpha_{lin}$ was expressed by

$$\Delta_{v-c} = 1.06(9) \tau_{loop} \times \alpha_{lin} - 0.01(2),$$

where, the errors mainly come from the systemic uncertainty and the model errors as we ignored other drifts and the nonlinear CLFD. According to the Eq. (1), the long-term interleaved stability, calculated by all measured $\delta f$ of

Fig. 2(c), will be degrade because the drift rates were different between measurements, as shown in Fig. 2(d). Therein, in order to make full use of experimental data sets, the overlapping Allan deviation was used to estimate the interleaved stability. The overlapping Allan deviation can be given by

$$\sigma(\tau) = \frac{1}{\sqrt{2m^2(M - 2m + 1)}} \sum_{j=1}^{M-2m+1} \sum_{j=1}^{M-1} \left( \sum_{j=1}^{M-1} y_{i+m} - y_i \right)^2,$$

where, $\tau$ is the averaging time and $y_i$ is the $i$th of the $M$ fractional frequency values averaged over the measurement interval $\tau = m\tau_0$ where $\tau_0$ is the basic measurement interval and $m$ is the averaging factor (in this experiment, $\tau_0 = T_c$ and $m = 5$).

Simulation can study the pure relationship between $\tau_{loop} \times \alpha_{lin}$ and $\Delta_{v-c}$, which can contribute us to obtaining correct experimental conclusions when used properly. For this reason, a simulation program based on Labview was designed. In this simulation, the $f_{\text{drift}}$ accumulated with cycles and then was added to the clock laser frequency before clock detections. The $f_{\text{thermal}}$, which was the frequency white noise, was generated by a random number generator (RNG). The output range of the RNG indicated the noise magnitude of the clock laser, which had considered the influence of the Dick effect because the Dick effect, performing as white frequency noise, can be simply regarded as increasing the fluctuation of the clock laser frequency. The output range of the RNG was
determined by comparing the experimental interleaved stability with the simulated interleaved stability and was finally determined as 1.7 Hz that kept unchanged in the following simulation. We tested the validity of this simulation from two aspects. On the one hand, we simultaneously varied gain G of servo systems from 0.2 to 2.5 and calculated the overlapping Allan deviation by the simulated δf as shown in Fig. 3(a). The short-term interleaved stability worsened with G increasing, but the long-term stability had little variation. This result was consistent with Ref. 21, preliminarily showing that this simulation was reasonable. On the other hand, we extracted the CLFD rate of each measurement, and then operated the program with the same drift rate as experiments. The overlapping Allan deviation calculated by simulated data of δf greatly agreed with the experimental measurement as shown in Fig. 3(b), indicating that this simulation was proper for our system.

Figure 3(c) shows the simulated Δf,c at different values of τloop × αlin. The fitting line can be expressed as,

$$\Delta_{\text{lin}} = 1.002(3) \times 10^{-1} \times \alpha_{\text{lin}} - 0.0007(9)$$

which agreed well with Eq. (1), showing that the experimental result was reasonable. According to Eq. (1), we removed Δf,c from δf of each measurement and again calculated the interleaved stability as shown in Fig. 3(d). The squares were the same as in Fig. 2(c), the circles indicated the interleaved stability after removing Δf,c from δf, the triangulations were the simulations with αlin = 0. After removing Δf,c from δf, the measured long-term interleaved stability was evidently better than the result in Fig. 2(c) and reasonably agreed with the simulation with free of frequency drift, indicating that the frequency-drift-induced self-comparison measurement error can be cancelled by directly removing Δf,c from δf.

In summary, we experimentally demonstrated the frequency-drift-induced self-comparison measurement error and measured it at different linear frequency drift rate of the clock laser. We eventually experimentally determined Δf,c = 1.06(9)τloop × αlin − 0.01(2), which was well consistent with the simulations. Meanwhile, it meant that in terms of the 87Sr optical lattice clocks, even the αlin = 200 μHz s⁻¹, the fraction self-comparison measurement error caused by the CLFD will be 1 × 10⁻¹⁸ (with typical τloop = 2 s). This work also gives us a new way to cancel this measurement error by directly removing Δf,c from δf where we only need to measure the linear drift rate that usually can be well known by the feedback signals. The nonlinear frequency drift can also be repressed by cutting the data of δf into sections, and removing Δf,c of each section, respectively.

Acknowledgments This research is supported by the National Natural Science Foundation of China (61775220), the National Key R&D Program of China (Grant No. 2016YFF0202001), the Key Research Project of Frontier Science of the Chinese Academy of Sciences (Grants No. QYZDB-SSW-JSC004) and the Strategic Priority Research Program of the Chinese Academy of Sciences (Grant No. XDB21001001).

1) S. A. Webster, M. Oxborrow, S. Puglia, J. Millo, and P. Gill, Phys. Rev. A 77, 033847 (2008).
2) T. Legero, T. Kessler, and U. Sterr, J. Opt. Soc. Am. B 27, 914 (2010).
3) T. Kessler, C. Hagemann, C. Grebing, T. Legero, U. Sterr, F. Riehle, M. J. Martin, L. Chen, and J. Ye, Nat. Photon. 6, 687 (2012).
4) D. G. Matei et al., Phys. Rev. Lett. 118, 263202 (2017).
5) J. M. Robinson, E. Olker, W. R. Milner, W. Zhang, T. Legero, D. G. Matei, F. Riehle, U. Sterr, and J. Ye, Optica 6, 240 (2019).
6) M. Takamoto, F. L. Hong, R. Higashi, and H. Katori, Nature 435, 321 (2005).
7) W. F. McGrew et al., Nature 564, 87 (2018).
8) E. Olker et al., Nat. Photon. 13, 174 (2019).
9) C. Grebing, A. A. Masoudi, S. Dörscher, S. Häfner, V. Gerginov, S. Weyers, B. Lippardi, F. Riehle, U. Sterr, and C. Lisdat, Optica. 3, 563 (2016).
10) C. C. Hsu, D. F. Hume, T. Rosenband, and D. J. Wineland, Science 329, 1630 (2010).
11) A. Derevianko and M. Pospelov, Nat. Phys. 10, 933 (2014).
12) S. Kolkowitz, I. Pikovski, N. Langellier, M. D. Lukin, R. L. Walsworth, and J. Ye, Phys. Rev. D 94, 124043 (2016).
13) T. Takano, M. Takamoto, I. Ushijima, N. Ohmoe, T. Akatsuka, A. Yamaguchi, Y. Kurosaki, H. Munekane, B. Miyahara, and H. Katori, Nat. Photon. 10, 662 (2016).
14) T. L. Nicholson, M. J. Martin, J. R. Williams, B. J. Bloom, M. Bishof, M. D. Swallows, S. L. Campbell, and J. Ye, Phys. Rev. Lett. 109, 230801 (2012).
15) A. A. Masoudi, S. Dörscher, S. Häfner, U. Sterr, and C. Lisdat, Phys. Rev. A 92, 063814 (2015).
16) Q. Wang, Y. G. Lin, F. Meng, Y. Li, B. K. Lin, E. J. Zang, T. C. Li, and Z. J. Fang, Chin. Phys. Lett. 33, 103201 (2016).
17) I. R. Hill, R. Hobson, W. Bowden, E. M. Bridge, S. Donnellan, E. A. Curtis, and P. Gill, J. Phys. Conf. Ser. 723, 012019 (2016).
18) S. Falke et al., Metrologia 48, 399 (2011).
19) T. Takano, R. Mizushima, and H. Katori, Appl. Phys. Express 10, 072801 (2017).
20) T. L. Nicholson et al., Nat. Commun. 6, 6896 (2015).
21) E. Peik, T. Schneider, and C. Tamm, J. Phys. B Atomic Mol. Opt. Phys. 39, 145 (2006).
22) Y. B. Wang, M. J. Yin, J. Ren, Q. F. Xu, B. Q. Lu, J. X. Han, Y. Guo, and H. Chang, Chin. Phys. B 27, 023701 (2018).
23) V. D. Osvianiukov, V. G. Pal’chikov, A. V. Taichenachev, V. I. Yudin, H. Katori, and M. Takamoto, Phys. Rev. A 75, 020501(R) (2007).
24) S. Falke et al., New J. Phys. 16, 073023 (2014).
25) X. T. Lu, M. J. Yin, T. Li, B. Y. Wang, and H. Chang, Appl. Sci. 10, 1140 (2020).
26) C. Wang, Z. H. Ji, T. Gong, D. Q. Su, Y. T. Zhao, L. T. Xiao, and S. T. Jia, J. Phys.: D: Appl. Phys. 52, 455104 (2019).
27) Y. B. Wang, X. T. Lu, B. Q. Lu, D. H. Kong, and H. Chang, Appl. Sci. 9, 2194 (2019).
28) T. Koto, M. Yasuda, K. Hosaka, H. Inaba, Y. Nakajima, and F. L. Hong, Appl. Phys. Express 2, 072501 (2009).
29) W. J. Riley, Handbook of Frequency Stability Analysis (William Riley, Boulder, CO, 2008) NIST Special Publications, 1065, p. 15.
30) G. Santarelli, C. Audoin, A. Makdissi, P. Laurent, G. J. Dick, and A. Clairon, IEEE Trans. Ultrason. Ferroelectric. Freq. Control 45, 887 (1998).