Shortcuts to Quantum Approximate Optimization Algorithm

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The Quantum Approximate Optimization Algorithm (QAOA) is a quantum-classical hybrid algorithm intending to find the ground state of a target Hamiltonian. Theoretically, QAOA can obtain the approximate solution if the quantum circuit is deep enough. Actually, the performance of QAOA decreases practically if the quantum circuit is deep since near-term devices are not noise-free and the errors caused by noise accumulate as the quantum circuit increases. In order to reduce the depth of quantum circuits, we propose a new ansatz dubbed as ‘Shortcuts to QAOA’ (S-QAOA), S-QAOA provides shortcuts of the quantum annealing by including more two-body interactions and releasing the parameter freedom. To be specific, besides the existed ZZ interaction in the QAOA ansatz, other two-body interactions are introduced in the S-QAOA ansatz such that the approximate solutions could be obtained by a less depth of quantum circuit. Considering the MaxCut problem and Sherrington-Kirkpatrick (SK) Model, numerically computation shows the YY interaction has the best performance. The reason for this might arise from the counter-adiabatic effect generated by YY interaction. On top of this, we release the freedom of parameters of two-body interactions, which a priori do not necessarily have to be fully identical, and numerical results show that it is worth paying the extra cost of having more parameter freedom since one has a greater improvement on success rate.

I. INTRODUCTION

In the noisy intermediate-scale quantum (NISQ) era1, the number of reliable quantum operations is limited by the quantum errors which contains quantum decoherence, rotation error, and so on. Thus people are interested in the quantum-classical hybrid algorithm whose quantum circuit depth is decreased with the help of classical optimizers, like the Quantum Approximate Optimization Algorithm (QAOA)2 which is expected to get an approximate solution for combinatorial optimization problems. In this hybrid algorithm, the quantum state is prepared by a quantum computer and the parameters in the quantum circuit are optimized by a classical optimizer to find a evolution path which needs less quantum circuit depth. Some researches show that QAOA is better than quantum adiabatic algorithms (QAA) and is hopeful to get a quantum advantage using the near-term device 3,4. However, the performance of QAOA is limited by the noise of the near-term devices 5, and Google’s experiment for QAOA with their Sycamore superconducting qubit quantum processor shows that errors overwhelm the theoretical performance increase at larger layers6. Therefore executing QAOA on the present near-term quantum computer is a challenging task. It is important to reduce the circuit depth of QAOA to make it achievable for the near-term device.

QAOA can be regarded as a digitized and variational version of QAA. QAA starts with a simple-to-prepared ground state of an initial Hamiltonian, and the adiabatic theorem guarantee that the ground state of the final Hamiltonian can be obtained if the time-dependent Hamiltonian varies slowly. The adiabatic condition requires that the run time of QAA scales as $T \sim O(1/\Delta_{\text{min}}^2)$, $\Delta_{\text{min}}$ is the minimal spectral gap2 during the evolution of the quantum system, thus many problems are hard to optimize using QAA because of the overlong annealing time. Besides, the digitization of QAA may lead to a deep quantum circuit to minimize the trotter error8,9. QAOA will overcome these problems by optimizing parameters (including evolution path and duration for every digitized step) using a classical optimizer. The optimized parameters of QAOA is related to a fast evolution path and QAOA will break through the limits of the adiabatic condition8.

Shortcuts to adiabaticity (STA)10 is a class of methods to accelerate the quantum adiabatic process, counterdiabatic (CD) driving11–13 is a technique of STA to reduce the finite-speed diabatic effect by adding counter terms to the time-dependent Hamiltonian. The CD driving Hamiltonian has a better performance and reduces the evolution time compared to adiabatic evolution14. Recently, there is a research indicates QAOA is at least counteradiabatic and has a better performance than finite time adiabatic evolution14. And there is also an effort to add a counteradiabatic term to the ansatz of QAOA to reduce the quantum circuit depth15.

Our work focus on the MaxCut problems and Sherrington-Kirkpatrick (SK) Model, both of them are NP-hard problems and the QAOA is expected to have a quantum acceleration on these problems. In this work, we investigate the counteradiabatic effect of QAOA, and the simulation result implies that adding a two-gate term associate with the YY interaction to the quantum circuit will accelerate the optimization of QAOA. Besides, we release the QAOA parameter freedom of the two-body in-
teractions (including ZZ interaction and YY interaction) to reduce the quantum circuit depth, e.g. each two-body interaction has its independent parameter. The idea of extending the parameter degree is mentioned by Farhi in 2017\cite{Farhi2017} and they use random initial parameters of each ZZ interaction. In our case, firstly we optimize the QAOA parameters to get a local optimal value, and then use the optimal parameters of QAOA as the initial parameters of each two-body term to do a further local optimization. The proposed algorithm uses the philosophy of STA, so we call it "Shortcuts to QAOA" (S-QAOA), and the simulated result shows that S-QAOA can get a good result with a more shallow quantum circuit compared with QAOA.

II. PROBLEMS AND QUANTUM ALGORITHMS

In this research, we focus on the MaxCut problem and SK model. The MaxCut problem is defined on a graph \(G(V,E)\), \(V = \{1,2,\cdots,n\}\) is the set of vertices, and \(E = \{(i,j),w_{ij}\}\) is the set of edges where \((i,j)\) is a pair of connected vertices and \(w_{ij}\) is the weight of the edge \((i,j)\). We study two classes of graphs: the first class is unweighted 3-regular graphs (u3R) whose weights \(w_{ij}\) are a constant for all edges, e.g. \(w_{ij} = 1, \forall (i,j) \in E\); the second class is weighted 3-regular graphs (w3R) whose weights \(w_{ij}\) are uniform random numbers in range \([0,1]\). The target Hamiltonian of MaxCut problem is:

\[
H_{C} = -\sum_{(i,j)\in E} w_{ij} \frac{I - Z_i Z_j}{2}.
\]

The SK model is defined on the complete graph and the coefficient \(w_{ij}\) is randomly chosen from the set \([-1,1]\). The target Hamiltonian of the SK model is:

\[
H_{C} = \sum_{i<j} w_{ij} Z_i Z_j.
\]

A. Quantum Adiabatic Algorithms

QAA is able to find the ground state of the target Hamiltonian if the annealing process is slow enough, and its feasibility is guaranteed by the adiabatic theorem. In QAA, a simple-to-prepared quantum state is chosen as the initial state, which usually is the uniform superposition over computational basis states: \(\psi(t=0) = |+\rangle^\otimes n\), \(n\) is the number of qubits of the quantum system. \(\psi(t=0)\) is the ground state of Hamiltonian \(H_B = -\sum_{i=1}^{n} X_i\). The time-dependent Hamiltonian of QAA is:

\[
H_{QAA}(t) = (1-\lambda(t))H_B + \lambda(t)H_C,
\]

\[0 \leq t \leq T, \lambda(0) = 0, \lambda(T) = 1.
\]

\(H_{C}\) is the target Hamiltonian. The ground state of \(H_C\) can be obtained in the end \(t = T\) if the Hamiltonian \(H_{QAA}\) varying adiabatically during the evolution:

\[
|\psi(T)\rangle = e^{-i\int_0^T dt H_{QAA}(t)} |\psi(0)\rangle
\]

STA is able to reduce the finite-speed diabatic excitations by adding the CD driving terms\cite{Hauke2016}:

\[
H_{STA} = (1 - \lambda(t))H_B + \lambda(t)H_C + \lambda H_{CD}^{\alpha},
\]

\[
H_{CD}^{\alpha}(\alpha) = i \sum_{k=1}^{l} \alpha_k(t) [H_j, [H, \partial_\alpha H]]
\]

The coefficient \(\alpha = (\alpha_1,\cdots,\alpha_l)\) can be determined variationally. The first order of CD driving terms is the commutator of the initial Hamiltonian \(H_B\) and the target Hamiltonian \(H_C\):

\[
H_{CD}^{\alpha} = i \alpha_1 [H_B, H_C],
\]

and it can be proven that the coefficient \(\alpha_1\) is negative\cite{Liu2018}. In \(l = +\infty\) limits, the CD driving terms are able to compensate the diabatic excitations exactly. However, higher-order CD driving terms will produce long-range interactions between qubits which is hard to implement using the near-term devices, it is necessary to consider the first few orders of \(H_{CD}^{\alpha}\) to avoid the nonlocal operations.

B. Quantum Approximate Optimization Algorithm

The ansatz of QAOA consists of a series of digitized evolution and the parameters of QAOA are optimized by a classical optimizer\cite{Montanaro2017}:

\[
|\psi(\beta, \gamma)\rangle = e^{-i\beta_1 H_B} e^{-i\gamma_1 H_C} \cdots e^{-i\beta_p H_B} e^{-i\gamma_p H_C} |\psi(0)\rangle.
\]

The purpose of the classical optimization is to find the optimal parameters to minimize the expectation of the target Hamiltonian:

\[
E(\beta, \gamma) = \langle \psi(\beta, \gamma) | H_C | \psi(\beta, \gamma) \rangle,
\]

\[
\beta = (\beta_1, \cdots, \beta_p), \gamma = (\gamma_1, \cdots, \gamma_p).
\]

The expectation \(E\) will be convergent during the optimization, and \(E\) will converge to the ground state energy of the target Hamiltonian if the layer \(p\) of QAOA is large enough. Recently, there is a research shows that QAOA is counterdiabatic and is better than QAA\cite{Honda2018}. Using the second order Baker-Campbell-Hausdorff (BCH) expansion, we can show that the effective evolution Hamiltonian of QAOA is:

\[
e^{-i\beta_k H_B} e^{-i\gamma_k H_C} \sim e^{-iH_{eff}}, 1 < k < p,
\]

\[
H_{eff} = \gamma_k H_C + \beta_k H_B - \frac{i\beta_k \gamma_k}{2} [H_B, H_C]
\]

Compare the result of Eq\[3\] and Eq\[5\], we can find that the effective Hamiltonian \(H_{eff}\) of QAOA has a first-order CD driving term, and the coefficient of this term is also negative. So the evolution of QAOA will include some counterdiabatic effects to compensate the diabatic excitation.
C. Shortcuts to QAOA

Inspired by the technique of STA, we consider the counterdiabatic effect of QAOA. In cases of MaxCut problem and SK model, an extra two-body interaction is added to the QAOA ansatz followed by each ZZ interaction to accelerate the quantum evolution, the possible type of the extra two-body interaction is $M_{ij} = \frac{P_i Q_j + Q_i P_j}{2}$, $PQ \in \{YZ, YY, XX, XZ, XY\}$. The compact implementation of the $M_{ij}$ and ZZ interactions does not need extra SWAP gates and makes this technique feasible for the near-term devices. Besides, after the optimization of QAOA parameters, the parameter freedom of two-body interactions are released and a further optimization is performed. More parameters will increase the presentation capability of the quantum circuit and can get a better result compared with QAOA in the same quantum layer. The procedure of S-QAOA is as follows:

1. Optimize the QAOA parameters using INTERP strategy\[3\], get the optimal parameters $(\beta, \gamma)_p$ for layer $p$.

2. Release the parameter freedom of ZZ interaction and add an extra two-body interaction $M_{ij} = \frac{P_i Q_j + Q_i P_j}{2}$, $PQ \in \{YZ, YY, XX, XZ, XY\}$, followed by each ZZ interaction. Operation of each layer $k \in [1, p]$ of S-QAOA is:

$$
\begin{align*}
  e^{i \delta_k \sum_i X_i} & \rightarrow e^{i \beta_k \sum_i X_i - i \gamma_k \sum_{ij} Z_i Z_j} \\
  e^{i \delta_k \sum_i X_i} & \rightarrow e^{i \beta_k \sum_i X_i} \prod_{ij} \{e^{-i \gamma_k' w_{ij} Z_i Z_j} e^{-i \alpha_k \gamma_k' w_{ij} M_{ij}} \}. \\
\end{align*}
$$

(10)

The initial parameters are: $\beta_k = \tilde{\beta}_k, \gamma_k = \tilde{\gamma}_k, \alpha_k = 0$. (This is the ansatz of S-QAOA for SK model, and it is the same for MaxCut problem except for a coefficient $\frac{1}{2}$. For simplicity, we will only show the ansatz for SK model in the following.)

3. Using the finite-difference method to calculate the gradient of parameters $(\{\gamma_{ij}^{(k)}\}, \beta_k, \alpha_k)$, and append the parameters whose absolute value of gradient is bigger than a threshold $\delta_1$ to a set $A$:

$$
\begin{align*}
  g(\theta) & = \frac{E(\theta + \epsilon) - E(\theta)}{\epsilon}, \\
  \forall \theta & \in (\{\gamma_{ij}^{(k)}\}, \beta_k, \alpha_k), k = 1, \ldots, p, \\
\end{align*}
$$

(11)

$\epsilon$ is a small constant. If $|g(\theta)| > \delta_1$, add $\theta$ to set $A$.

4. Optimize the parameters in set $A$ until convergent, if the decrease of energy is smaller than a threshold $\delta_2$ after the optimization, exit; else, update the optimized parameters and return to step3.

III. SIMULATION RESULT

We study the u3R and w3R MaxCut problem on 14-vertex graphs, and SK model on 6-vertex graphs. In each case, the results are averaged on 20 random graphs. There are three ansatz are studied: QAOA; only releasing the parameter freedom of ZZ interaction(This ansatz only contains ZZ interaction, and for simplicity, we call it ‘ZZ technique’ below); adding an extra two-body interaction and releasing the freedom of parameters (S-QAOA). The simulation result implies that the third ansatz has the best performance in all cases, and the proper extra two-gate term is able to accelerate the evolution significantly. A comprehensive study of the optimal type of extra two-gate term in S-QAOA can be found in Fig[6] which shows the performance and the comparison of all the possible extra two-gate types: $M_{ij} = \frac{P_i Q_j + Q_i P_j}{2}$, $PQ \in \{YZ, YY, XX, XZ, XY\}$. In general, YY interaction has the best performance and it is included in S-QAOA ansatz, thus the operation of each layer $k \in [1, p]$ of S-QAOA is:

$$
\begin{align*}
  e^{i \delta_k \sum_i X_i} \prod_{ij} \{e^{-i \gamma_k w_{ij} Z_i Z_j} e^{-i \alpha_k \gamma_k' w_{ij} M_{ij}} \}. \\
\end{align*}
$$

(12)

The results of u3R MaxCut problem are shown in Fig[7] It is obvious that only releasing the freedom of parameters of the existed ZZ interaction will produce a better result in the same layer compared with QAOA. On the top of this, adding the YY interaction will improve the performance further, especially in the low layer where the performance of QAOA is not so good. When the quantum layer $p$ is increased, the difference between the three ansatz is smaller, this is due to that the evolution time is large enough for QAOA if the quantum layer is large and there is little space for improvement. Fig[2] demonstrate the simulation results of w3R graphs, and the superiority of S-QAOA is more obvious in this case, the average fidelity is improved significantly even at $p = 10$. The MaxCut problem on w3R graphs is more difficult than u3R graphs, since the energy gap of the MaxCut Hamiltonian on w3R graph is smaller and it needs a longer evolution time to satisfy the adiabatic condition. The result shows the potential of S-QAOA to solve the problems which are difficult for QAA and QAOA.

SK model is defined on the complete graph and it is challenging to implement SK model on a NISQ device which has a limited qubit connectivity. Because of the all-to-all interaction of SK model, all the nodes can be entangled together at $p = 1$, and there are sufficient parameters to optimize if the parameters of ZZ interaction are independent. So that a pretty good result can be obtained at $p = 1$ if only releasing the parameter freedom (Fig[3]). And if a YY interaction is added to the ansatz meanwhile, the fidelity is improved obviously and reaches about 80% at $p = 1$. S-QAOA introduces only one extra parameter compared with the ZZ technique, and the performance of S-QAOA is significantly better than the...
latter at $p = 1$. The significant improvement produced by $YY$ interaction confirms its effects on countering the diabatic excitations and accelerating the process of quantum annealing. The difference between S-QAOA and ZZ technique becomes smaller and smaller when the quantum layer $p$ is increased, this is due to that the parameter freedom of ZZ technique is enough for the optimization at large layer $p$, and there is not much space for S-QAOA to improve.

S-QAOA has a better performance in all cases we study, e.g. for a specific quantum layer, $R_p = \frac{poss_1}{poss_0} > 1$, $poss_1(poss_0)$ is the possibility of the optimal solution got by S-QAOA(QAOA). S-QAOA does a further optimization and has more parameters compared with QAOA, thus the function evaluate number of S-QAOA is more than that of QAOA, e.g. $R_f = \frac{feval_1}{feval_0} > 1$, $feval_1(feval_0)$ is the function evaluate number of S-QAOA(QAOA). It is necessary to consider whether it is cost-effective to do these extra optimizations. We show the ratio of $R_f$ and $R_p$: $R_{fp} = \frac{R_f}{R_p}$ in Fig. 4, and $R_{fp} < 1$ represents that it is deserved to do the extra optimization of S-QAOA to produce a higher improvement of fidelity. It is clearly that the ratio $R_{fp} < 1$ if $p \leq 4$ for almost cases. If $p$ increases, QAOA can produce a quite high fidelity for SK model and u3R MaxCut problem, and there is little space to get an improvement for S-QAOA. So the ratio $R_{fp}$ of SK model and u3R MaxCut problem approach to 1 or even larger than 1 for large $p$. For w3R MaxCut problem, the fidelity of QAOA is far away from 1, so S-QAOA can improve the fidelity effectively with some further optimization. In all, S-QAOA is an effective way to improve
FIG. 3. Comparison of the performance of QAOA, ZZ technique and S-QAOA for 6-vertex SK model. ZZ technique is able to significantly decrease the fractional error and improve the fidelity at low layers, and the result can be improved further by using S-QAOA with few additional parameters.

IV. DISCUSSION

The quantum circuit of QAOA consists of the alternating implementation of $e^{-i\gamma H_C}$ and $e^{-i\beta H_B}$, the nest commutator of $H_B$ and $H_C$ is able to span the entire Lie algebra associated with the Hilbert space of the n-qubit system, and QAOA is able to approximate any element of the full unitary group $U(2^n)$ if a sufficient quantum circuit depth is applied\cite{18, 19}. Based on the existed generator $H_B$ and $H_C$, S-QAOA provide an extra generator associated with the two-body interaction to accelerate the process of approximating the desired unitary operation. The numerical result shows that the generator associate with the YY interaction has the best performance and the quantum circuit depth is reduced significantly by including it in S-QAOA ansatz (Fig.6). The advantage of the YY interaction might be explained by the connection of it and the CD driving terms (Eq.5).

For MaxCut problem and SK model, the first order of Eq.5 is:

$$H^1_{CD} = i\alpha_1[H_B, H_C]$$

$$= -2\alpha_1 \sum_{\langle i,j \rangle} w_{ij} (Z_i Y_j + Y_i Z_j).$$

There is a little improvement when we add the above YZ term to the quantum circuit Eq.10(Fig.6). The first order of YZ term is possibly due to that the effective Hamiltonian of QAOA contains the first order of CD driving terms (Eq.9). In order to introduce more counter terms to compensate the diabatic excitations, we consider the second order of CD driving terms(Eq.10):

$$H \sim \beta H_B + \gamma H_C,$$

$$H^2_{CD} = i\alpha_2[H_B, H_C][H_B, H_C]$$

$$= -\alpha_2 \left\{ (c_1 \beta^2 + c_2 \gamma^2) \sum_{\langle i,j \rangle} w_{ij} (Y_i Z_j + Z_i Y_j) \right. + c_3 \beta \gamma \sum_{i} \sum_{j \in \{n_i\}} w_{ij} w_{ik} (X_i Z_j Y_k + X_i Y_j Z_k)$$

$$+ c_4 \gamma^2 \sum_{i} \sum_{jkm \in \{n_i\}} w_{ij} w_{ik} w_{im} Y_j Z_k Z_m \}.$$ \hspace{1cm} (15)

There are some positive coefficients $c_1, c_2, c_3, c_4$, and $\{n_i\}$ represent the neighbors of vertex $i$, eg. $\langle i,j \rangle \in \cdots$
The first two terms on the right side of equation Eq.15 can be generated if a series of YY interaction $H_Y = \sum_{ij} w_{ij} Y_i Y_j$ is added to ansatz:

$$e^{-i\alpha H_Y} e^{-i\beta H_B} e^{-i\gamma H_C} \sim e^{-iH_{eff}},$$

$$H_{eff} = \alpha H_Y + \beta H_B + \gamma H_C$$

$$- \frac{i\beta\gamma}{2} [H_B, H_C] - \frac{i\alpha\beta}{2} [H_Y, H_B] - \frac{i\alpha\gamma}{2} [H_Y, H_C]$$

(16)

$$= \alpha H_Y + \beta H_B + \gamma H_C + H_1 + H_2 + H_3.$$  

$H_1$ has the same form as the first order of CD driving terms (Eq.6). $H_2$ and $H_3$ will generate terms same as the first two terms on the right side of equation Eq.15.

$$H_2 = -\frac{i\alpha\beta}{2} [H_Y, H_B] = \alpha \beta \sum_{ij} w_{ij} (Y_i Z_j + Z_i Y_j).$$

(17)

$$H_3 = -\frac{i\alpha\gamma}{2} [H_Y, H_C]$$

$$= \alpha \gamma \sum_i \sum_{j \in \{n_i\}} w_{ij} w_{ik} (X_i Y_j Z_k + X_i Z_j Y_k).$$

(18)

$H_2$ contains the same interaction as the first term on the right side of equation Eq.15 and $H_3$ contains the same interaction as the second term. Besides, the sign of the coefficient of $H_2$ and $H_3$ are same, it is the same for the corresponding terms of Eq.15. So it can partially compensate the excited state by adding the YY interaction to S-QAOA ansatz.

Besides, the parameter freedom of ZZ interaction is released to further reduce the quantum circuit depth, and since the strength of YY and ZZ interactions should be positively related, the coefficient $\gamma_{ij}$ is added to each YY interaction:

$$e^{-i\alpha H_Y} e^{-i\beta H_B} e^{-i\gamma H_C} \rightarrow$$

$$e^{-i\alpha \sum_{(ij)} w_{ij} \gamma_{0i} Y_i Y_j} e^{-i\beta \sum_{i=1}^n X_i} e^{-i \sum_{(ij)} w_{ij} \gamma_{ij} Z_i Z_j}. $$

(19)

The coefficient $\alpha$ plays the same role as in STA, and is also determined variationally.

To reduce the number of swap gates, the order of the operations in the ansatz is adjusted and the operations of the YY and ZZ interactions are combined together:

$$e^{-i\beta \sum_i X_i} \prod_{(ij)} e^{-iw_{ij} \gamma_{ij} (\alpha Y_i Y_j + Z_i Z_j)}. $$

(20)

The compact impletation of two-body interactions in Eq.20 cut the number of swap gates in half compared to the ansatz in Eq.19. Fig.5 shows the comparison of the performance of ansatz in Eq.19 and Eq.20 by considering the MaxCut problem on w3R graphs. The result implies that the performance of those two ansatz are basically the same, so we choose the ansatz in Eq.20 to reduce the quantum circuit depth.

V. SUMMARY AND OUTLOOK

In this work, S-QAOA ansatz is proposed to reduce the required quantum circuit depth. The main innovation of S-QAOA is: firstly, the extra two-body interaction is considered in the S-QAOA to compensate the diabatic effect and accelerate the process of quantum annealing; secondly, the parameter freedom of two-body interactions is released to enhance the capacity of the quantum circuit. We study the performance of S-QAOA and QAOA on MaxCut problem and SK model, the simulation result implies that S-QAOA has better performance at lower quantum layers compared with QAOA. So S-QAOA is a good candidate to solve the combinatorial problems using NISQ.

Releasing the parameter freedom needs extra cost of optimization, the numerical simulation shows it is cost-effective because of the greater improvement on fidelity. In S-QAOA, further optimization is performed on the parameters which have large gradient, and there need more works to explore the way of how to release the parameters properly, eg. how to pick out the most important parameters to do a further optimization. The most influential parameters should be different for different problems, it is important to develop an efficient way to pick them out. Besides, it deserves further exploration to explain the reason of the effectiveness of the YY interaction more clearly. We will study more cases in further works to test and improve our idea in next step.

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Appendix A: Additional Figures

Except for the existed ZZ interaction in QAOA ansatz, S-QAOA contains other two-body interactions to accelerate the process of quantum annealing. Operation of each layer $k \in [1, p]$ of S-QAOA is:

$$\prod_{k=1}^{p} \{ e^{i \beta_k} \sum_{i} \sum_{\{w_{ij}\} \in X} \{ e^{-i \gamma_k^{w_{ij}} w_{ij} Z_i Z_j} e^{-i \alpha_k^{w_{ij}} w_{ij} M_{ij}} \} \}, \quad (A1)$$

$M_{ij}$ is a two-gate term and there are five possible types of it: $M_{ij} = \frac{P_i Q_j + Q_i P_j}{2}, PQ \in \{ YZ, YY, XX, XZ, XY \}$. To choose the best one in these two-gate terms, we do a comprehensive simulation for MaxCut problem and SK model, the result can be found in Fig. 6. The simulation result implies that the $M_{ij} = Y_i Y_j$ has the best performance in all cases. A further study in Fig. 7 is to explore the necessity of adding more two-body interactions in the ansatz, and it is obvious that the ansatz adding more interactions can not get better performance than the ansatz just adding YY interaction. So it is enough to just add the YY interaction in S-QAOA ansatz.
FIG. 6. Comparison of the performance of all the possible additional two-body interactions: \{YZ, YY, XX, XZ, XY\}, the results are averaged on 10 random graphs for every cases. YY interaction performs best for all the three cases we study. The performance of XZ, XY and ZZ type are almost same and it demonstrate that the XZ and XY interactions have little effect to the quantum annealing process.
FIG. 7. Comparison of the performance of only adding the YY interaction to the ansatz and adding two or three additional interactions which are combination of \{YZ, YY, XX\} interactions to the ansatz. Eg. 'YZ YY', 'YZ XX', 'YY XX' represent there are two additional interactions followed by each ZZ interaction; 'YZ YY XX' represents the YZ, YY, XX interactions followed by ZZ interaction one by one. The results are averaged on 10 random graphs and the performance of the ansatz including two or three additional interactions are almost same as the ansatz only adding the YY interaction. So if we just consider two-gate terms, it is enough to add the YY interaction to accelerate the evolution.