Imperfect Dark Matter

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Abstract. We consider cosmology of the recently introduced mimetic matter with higher derivatives (HD). Without HD this system describes irrotational dust — Dark Matter (DM) as we see it on cosmologically large scales. DM particles correspond to the shift-charges — Noether charges of the shifts in the field space. Higher derivative corrections usually describe a deviation from the thermodynamical equilibrium in the relativistic hydrodynamics. Thus we show that mimetic matter with HD corresponds to an imperfect DM which: i) renormalises the Newton’s constant in the Friedmann equations, ii) has zero pressure when there is no extra matter in the universe, iii) survives the inflationary expansion which puts the system on a dynamical attractor with a vanishing shift-charge, iv) perfectly tracks any external matter on this attractor, v) can become the main (and possibly the only) source of DM, provided the shift-symmetry in the HD terms is broken during some small time interval in the radiation domination epoch.

In the second part of the paper we present a hydrodynamical description of general anisotropic and inhomogeneous configurations of the system. This imperfect mimetic fluid has an energy flow in the field’s rest frame. We find that in the Eckart and in the Landau-Lifshitz frames the mimetic fluid possesses nonvanishing vorticity appearing already at the first order in the HD. Thus, the structure formation and gravitational collapse should proceed in a rather different fashion from the simple irrotational DM models.

Keywords: modified gravity, dark matter theory, physics of the early universe

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1 Introduction

The origin of Dark Matter (DM) is one of the oldest and biggest puzzles in cosmology and particle physics. Surprisingly, even basic macroscopic nature of DM is not yet understood — indeed, we do not know whether DM is a gas of some particles\(^1\) beyond the Standard Model (SM), or a gas of primordial black holes see e.g. \[1, 12\], or some other macroscopic objects, Q-Balls \[13\], Topological defects etc, see e.g. \[14–19\], or some fluid e.g. \[20\] or Bose-Einstein condensates — some classical scalar fields e.g. \[21–23\] or even some effective solid \[24\]. In the latter approach, where one assumes high occupation numbers of some new fields, we can also incorporate the relativistic version of MOND \[25–27\] — TeVeS \[28\] and numerous other modifications of general relativity (GR), e.g. \[29\]. The simplest modification of GR can be achieved by promoting it to a scalar-tensor theory.

GR enjoys a very powerful symmetry — diffeomorphism invariance. One of the manifestations of its power is that one can parametrize the metric \(g_{\mu \nu}\) by a scalar field \(\varphi\) and an auxiliary metric \(\ell_{\mu \nu}\) in a general disformal way \[30\]

\[
g_{\mu \nu} = C (\varphi, X) \ell_{\mu \nu} + D (\varphi, X) \varphi_{,\mu} \varphi_{,\nu},
\]

where \(X = \frac{1}{2} \ell^{\mu \nu} \varphi_{,\mu} \varphi_{,\nu}\) and \(C (\varphi, X)\) and \(D (\varphi, X)\) are free functions,\(^2\) and obtain the Einstein equations (for \(g_{\mu \nu}\)) by variation of the action with respect to \(\varphi\) and \(\ell_{\mu \nu}\) instead of \(g_{\mu \nu}\), see \[36\].

\(^1\)This gas can be composed of the so-called Weakly Interacting Massive Particles (WIMPs) \[1\], sterile right-handed neutrinos \[2–4\] and even different, and some times numerous copies of the SM, see e.g. \[5, 6\] just to mention few options. Whether the axion DM \[7–9\] represents a condensate or not is still debated, for the most recent discussion see \[10\].

\(^2\)There is a well known physical disformal transformation — the effective/acoustic metric for the propagation of small perturbations in k-essence \[31–33\] or irrotational hydrodynamics is a particular disformal transformation of the gravitational metric \(g_{\mu \nu}\), see \[34, 35\].
The only exception from this rule corresponds to a singular parameterisation when
\[
D (\varphi, X) = f (\varphi) - \frac{C (\varphi, X)}{2X} .
\] (1.2)

When the transformation is singular, there are new degrees of freedom and new physics
modifying GR. Mimetic Dark Matter [37] is one of the theories of this type and makes use
of the transformation (1.1) with \( C = 2X \) and \( D = 0 \), so that
\[
g_{\mu \nu} = \left( \ell^\alpha_{\beta} \varphi, \alpha \varphi, \beta \right) \ell_{\mu \nu} .
\] (1.3)

It is important that in this case the system is Weyl invariant with respect to the transforma-
tions of the auxiliary metric \( \ell_{\mu \nu} \). Soon it was realised that Mimetic Dark Matter is equivalent
to the fluid description of irrotational dust [38, 39] with the mimetic field \( \varphi \) playing the role
of the velocity potential. Models of this type also appear in the IR limit of the projectable
version of Hofava-Lifshitz gravity [40–42] and correspond to a scalar version of the so-called
Einstein Aether [43]. Surprisingly these models can also emerge in the non-commutative
geometry [44]. In [45] this class of systems was further extended by i) adding a potential
\( V (\varphi) \) which allows to obtain an arbitrary equation of state for this dust-like matter with
zero sound speed, as it was done earlier in [46]; ii) by introducing higher derivatives (HD)
which provide a nonvanishing sound speed. The latter modification allowed one to study
inflationary models with the creation of quantum cosmological perturbations. Moreover, this
finite sound speed can suppress the structure on small scales [47] and have other interesting
phenomenological consequences.

This paper is organised as follows: first, in section 2 we present the main equations
and reformulate the model from [45] in terms of two scalar fields and also we extend this
setup by introducing shift-symmetry breaking in HD terms. In the next section 3 we consider
background cosmology. In particular, we present constraints on the models parameters. Our
main point in this section is a new mechanism which allows mimetic fluid to survive inflation
and later becomes the main source of DM. This mechanism is based on the shift-symmetry
breaking in the HD terms operating during some cosmologically short period of time within
the radiation-domination époque or possibly as early as within reheating. Then in section 4
we consider fluid-like description for the system in the case of shift-symmetry. The most
important point there is that the mimetic fluid does have vorticity and moreover is not a
perfect fluid. Indeed the four-velocity of particles (the Eckart frame) does not coincide with
the four-velocity of energy (Landau-Lifshitz frame). This is very important for studies of
DM on nonlinear scales. Then in section 5 we discuss our results and mention open question
which provide directions for future research.

2 Main setup

The matter part of the action
\[
S [g, \lambda, \varphi] = \int d^4x \sqrt{-g} \left( \lambda \left( g^{\mu \nu} \partial_\mu \varphi \partial_\nu \varphi - 1 \right) + \frac{1}{2} \gamma (\varphi) (\Box \varphi)^2 \right) ,
\] (2.1)

where \( \lambda \) is a Lagrange multiplier field, \( \varphi \) is the mimetic field, \( \gamma (\varphi) \) is a function of the field
and \( \Box = g^{\mu \nu} \nabla_\mu \nabla_\nu \) with \( \nabla_\mu (\cdot) = (\cdot)_\mu \) being the covariant derivative. This action with a
constant \( \gamma \) was introduced in [45] and it generalises [37, 46], see also [48]. Without higher
derivatives this action describes irrotational dust.\footnote{For the Lagrangian description of dust with vorticity see, \cite{49,50}.} We will assume the standard minimal coupling to gravity. Also we will assume that matter is not directly coupled to the mimetic field $\phi$. Throughout the paper we will either use the units where $8\pi G_N = 1$ or write $G_N$ explicitly where it is useful for discussion. We also use the signature convention $(+, -, -, -)$. Here we have omitted the boundary terms which are needed because of the higher derivative structure of the theory.

The Lagrange multiplier $\lambda$ enforces the constraint
\begin{equation}
 g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi = 1. \tag{2.2}
\end{equation}
Similarly to \cite{47} and \cite{48} one could also add a term $\nabla_\mu \nabla_\nu \varphi \nabla^\mu \nabla^\nu \varphi$, to the Lagrangian. However, this term only introduces a direct non-minimal coupling to gravity
\begin{equation}
 \int d^4x \sqrt{-g} \left( \nabla_\mu \nabla_\nu \varphi \right) \left( \nabla^\mu \nabla^\nu \varphi \right) = \int d^4x \sqrt{-g} \left( \Box \varphi \right)^2 - R^{\mu\nu} \nabla_\mu \varphi \nabla_\nu \varphi, \tag{2.3}
\end{equation}
where we have again omitted the boundary terms.\footnote{This condition can be easily violated, if one applies the mimetic ansatz (1.3) in $f(R)$ theories \cite{51}.} We will not consider this term in this paper. One can use an equivalent action to describe the dynamics of the system without use of the higher derivatives
\begin{equation}
 S'[g, \lambda, \varphi, \theta] = \int d^4x \sqrt{-g} \left[ \lambda (\varphi^\mu \varphi_{,\mu} - 1) - \gamma (\varphi) \left( \varphi_{,\mu} \theta^{,\mu} + \frac{1}{2} \theta^2 \right) - \gamma' (\varphi) \theta \varphi_{,\mu} \varphi_{,\mu} \right], \tag{2.4}
\end{equation}
where $\theta$ is an auxiliary field. The equation of motion for this field gives $\theta = \Box \varphi$. In addition, there is an equation of motion for $\varphi$ which can be conveniently written in form of the current (non) conservation
\begin{equation}
 \nabla_\mu J^\mu = \frac{1}{2} \gamma' (\varphi) \theta^2, \tag{2.5}
\end{equation}
where the the prime denotes the derivative $\partial / \partial \varphi$ and the current is given by
\begin{equation}
 J_\mu = \left( 2\lambda - \gamma' (\varphi) \theta \right) \partial_\mu \varphi - \gamma \partial_\mu \theta. \tag{2.6}
\end{equation}
When the theory is shift-invariant: symmetric with respect to $\varphi \to \varphi + c$ the current is conserved and is the Noether current corresponding to the shift symmetry.

The energy-momentum tensor (EMT) is
\begin{equation}
 T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} = 2 \left( \lambda - \gamma' (\varphi) \theta \right) \varphi_{,\mu} \varphi_{,\nu} \tag{2.7}
 + \gamma (\varphi) \left[ g_{\mu\nu} \left( \varphi_{,\alpha} \theta^{,\alpha} + \frac{1}{2} \theta^2 + \frac{\gamma' (\varphi)}{\gamma (\varphi)} \theta \right) - \varphi_{,\mu} \varphi_{,\nu} - \varphi_{,\nu} \varphi_{,\mu} \right],
\end{equation}
where we have assumed that the constraint (2.2) is satisfied together with the equation of motion
\begin{equation}
 \theta = \Box \varphi = \frac{1}{\sqrt{-g}} \partial_\mu \left( \sqrt{-g} g^{\mu\nu} \partial_\nu \varphi \right). \tag{2.8}
\end{equation}
Cosmology

Let us consider a spatially-flat Friedmann universe. From the constraint \((2.2)\) it follows that \(\varphi = t + c\). Hence the auxiliary field \((2.8)\) reads

\[
\theta = 3H ,
\]

where \(H\) is the Hubble parameter. From the EMT \((2.7)\) we obtain the energy density

\[
\varepsilon = T^0_0 = 2\lambda - \dot{\gamma}\theta - \gamma \left( \dot{\theta} - \frac{1}{2} \theta^2 \right) ,
\]

and the pressure

\[
p = -\frac{1}{3} T^i_i = -\dot{\gamma}\theta - \gamma \left( \dot{\theta} + \frac{1}{2} \theta^2 \right) .
\]

Thus the enthalpy density is

\[
h = \varepsilon + p = 2\lambda - \dot{\gamma}\theta - 2\gamma \dot{\theta} = 2\lambda - 3\dot{\gamma}H - 6\gamma\dot{H} .
\]

The second term in the last equality is similar to the so-called inhomogeneous equation of state considered in \([53, 54]\). While the charge density of the current \((2.6)\) is given by

\[
n = J^0 = 2\lambda - \dot{\gamma}\theta - \gamma \dot{\theta} .
\]

For the \(\varphi\) equation of motion \((2.5)\) or current (non)-conservation we can write

\[
\frac{d}{dt} \left( n a^3 \right) = \frac{9}{2} \dot{a}^3 \dot{\gamma} H^2 .
\]

For the Hubble parameter we have the Friedmann equation

\[
H^2 = \frac{1}{3} (\varepsilon + \rho_{\text{ext}}) ,
\]

\[
\dot{H} = -\frac{1}{2} (\varepsilon + \rho_{\text{ext}} + p + P_{\text{ext}}) ,
\]

where \(\rho_{\text{ext}}\), \(P_{\text{ext}}\) is the external energy density and pressure of e.g. radiation etc.

Let us for a while neglect the explicit dependence of \(\gamma (\varphi)\) and consider the case with the shift-symmetry. We will come back to the breaking of the shift symmetry in the subsection 3.3. In that case it was shown in \([45]\) that the density perturbations inquire a sound speed

\[
c^2_s = \frac{\gamma}{2 - 3\gamma} .
\]

Using the Friedmann equations \((3.7), (3.8)\) in the shift-symmetric case we obtain that

\[
\varepsilon = 2\lambda + \frac{3}{2} \dot{\gamma} (2\varepsilon + 2\rho_{\text{ext}} + p + P_{\text{ext}}) ,
\]

and the pressure is

\[
p = \gamma \frac{3}{2} (p + P_{\text{ext}}) ,
\]

so that

\[
p = \frac{3\gamma}{2 - 3\gamma} P_{\text{ext}} = 3c^2_s P_{\text{ext}} ,
\]
in particular if the external pressure is vanishing the pressure of the mimetic fluid is vanishing as well. In particular it implies that when the mimetic fluid is the only matter in the universe, it will always behave like dust or DM from the point of view of the background evolution.\footnote{Note that the presence of dust-like cosmological solutions was showed in \cite{48}, here we prove that there are no other solutions and universe filled with the shift-symmetric mimetic fluid always undergoes the cosmological expansion corresponding to the matter domination époque.}

### 3.1 Attractor and ideal tracking

In the shift-symmetric case it is convenient to exclude $\lambda$ and write the energy density as

$$\varepsilon = \frac{2}{2-3\gamma} n + \frac{3\gamma}{2-3\gamma} \rho_{\text{ext}}. \quad (3.13)$$

In cosmology the shift-charge density will be redshifted as

$$n \propto a^{-3}. \quad (3.14)$$

Thus the whole cosmological effect on the background level is an addition of the DM like component $2n/(2-3\gamma)$ and second part which ideally tracks the external matter. The latter tracking is just a rescaling of the Newton’s constant

$$G_{\text{eff}} = G_{N} \left(1 + \frac{3\gamma}{2-3\gamma}\right) = G_{N} \left(1 + 3c_{S}^{2}\right). \quad (3.15)$$

Thus instead of $G_{N}$ the cosmological expansion will depend on $G_{\text{eff}}$.

However, on the level of perturbations, during the radiation-dominated époque the mimetic fluid adds a component with the sounds speed $c_{S}^{2} = \gamma/(2-3\gamma)$ and with equation of state of radiation $w = 1/3$. Hence at the beginning of the radiation dominated era. Indeed, if the shift-symmetry was not broken during inflation and after that, the charge density will be redshifted as $n \propto a^{-3}$, producing at the end the field configuration without any charge, $n = 0$, with a tremendous exponential precision. In that case from (3.5) we obtain

$$2\lambda_{*} = \gamma \dot{\theta}_{*} = 3\gamma \dot{H}, \quad (3.16)$$

while from (3.13) we see that surprisingly the energy density is not completely redshifted and is proportional to the external energy density (of radiation, additional DM etc):

$$\varepsilon_{*} = \frac{3\gamma}{2-3\gamma} \rho_{\text{ext}} = 3c_{S}^{2}\rho_{\text{ext}}. \quad (3.17)$$

Thus the shift-symmetric mimetic fluid cannot be the only source of DM.

Clearly $\gamma \sim 10^{-10}$ as it is chosen in \cite{47} to suppress the power spectrum on sufficiently small wavelengths would provide a completely unobservable part of DM, if the mimetic fluid is always shift-symmetric.

For the pressure we still have

$$p_{*} = \frac{3\gamma}{2-3\gamma} P_{\text{ext}} = 3c_{S}^{2} P_{\text{ext}}, \quad (3.18)$$

hence the neutral mimetic fluid has exactly the same equation of state as the external matter\footnote{This the opposite situation from the anti-tracking happening in \cite{55}.}
Thus the only background manifestation of the mimetic fluid is the rescaling of the Newton’s constant \( \gamma = 1/3 \) with \( c_S^2 = 1/3 \), while \( \gamma = 1/2 \) corresponds to the speed of light and the speed of sound of a canonical scalar field. Such large \( \gamma \) would not be allowed, because of the bounds on the \( \delta G_N \) and because the system in that case would not cluster like the ordinary CDM.

### 3.2 Bounds on the sound speed and abundance

During the big bang nucleosynthesis (BBN) the bounds on the \( \delta G_N \) imply that:

\[
c_s^2 \bigg|_{BBN} \lesssim 0.02 ,
\]

if we follow the results from [56].\(^8\) Thus from [56] it follows that shift-symmetric mimetic fluid can only provide up to 6% of the DM and build the same 6% of the radiation.

We should stress that we do not know yet what will be the effective Newton’s constant in the Solar System in the presence of the mimetic fluid. This would require knowledge of the profile of mimetic fluid around solar system. This task goes beyond the scope of the current paper. Thus this bound and the bounds below on \( \delta G_N \) should be taken with caution.

If \( \gamma \) changes during the matter/radiation equality, but before that and some time after that the mimetic fluid is practically uncharged \( n \simeq 0 \), one can use data from the high resolution CMB and find that [58]

\[
3 (c_s^2 \big|_{\text{matter}} - c_s^2 \big|_{\text{radiation}}) \lesssim 0.105 \pm 0.049 ,
\]

or if one includes baryonic acoustic oscillations:

\[
3 (c_s^2 \big|_{\text{matter}} - c_s^2 \big|_{\text{radiation}}) \lesssim 0.066 \pm 0.039 .
\]

Moreover, the scalar/irrotational DM (IDM) can be interesting for seeding the primordial black holes (BH), [59]. In this context it was shown that to accelerate the formation of the primordial black holes one has to impose \( c_S^2 \lesssim \Phi \simeq 10^{-5} \). In that case essentially all primordial overdensities of IDM collapse to black holes. Thus the abundance of IDM with respect to CDM was showed to be \( \varepsilon_{\text{IDM}} / \rho_{\text{CDM}} \lesssim 10^{-7} \), because of the ratio between the mass of the Sagittarius A* and the mass of the Milky Way. In particular this bound on abundance would be applicable for \( \varepsilon \simeq 10^{-10} \) taken in [47], provided mimetic fluid were irrotational and perfect. This would place a by far more stringent bound

\[
\gamma \simeq c_S^2 \lesssim 3 \times 10^{-8} .
\]

Note that in this work we are discussing a model which can survive the inflationary red-shifting, whereas energy density for the IDM models studied in [59] would completely disappear during inflation.

Further we should mention that, if \( c_S^2 \geq 10^{-5} \) during the matter domination era the formation of the structure and black holes is not that efficient and the bound \( \varepsilon_{\text{IDM}} / \rho_{\text{CDM}} \lesssim 10^{-7} \) is not applicable.

On the other hand there are constraints on the sound speed of DM [60] where it was claimed that the bound is \( c_S^2 < 10^{-5} \). We expect that the nonlinear collapse proceeds radically

\[^8\]An earlier work [57] provides a weaker bound \( c_S^2 \lesssim 0.1 \).
differently in the mimetic imperfect fluids. Indeed, as we show in the next section 4, contrary to the models from [59], our imperfect DM has an intrinsic anisotropy in form of the energy flow $q_\mu$ (4.14) thus the system is not a perfect fluid. Moreover, there is an intrinsic vorticity. Thus the bound (3.23) is not directly applicable.

3.3 Transition

Now let us consider a short period of time during the radiation domination époque when the shift-symmetry is broken and the shift-charge can be generated. Thus the theory is shift-symmetric, before and after this period of time which is finished at $t_{cr}$. Suppose, for simplicity, that the charge is generated during $\Delta t \ll H^{-1}$ so that we can neglect the cosmological evolution during the charge generation. During the creation of the charge we can assume that the mimetic fluid is completely subdominant in comparison with the radiation. Then equation for the charge generation (3.6) gives us

$$na^3 = \frac{9}{2} \int_{(t_{cr} - \Delta t)}^{t} dt' a^3 \Delta \gamma H^2 \simeq \frac{3}{2} \int_{(t_{cr} - \Delta t)}^{t} dt' a^3 \rho_{rad} \simeq \frac{3}{2} a^3 \rho_{rad} (t_{cr}) \Delta \gamma ,$$

(3.24)

where in the last approximate equality we have used the sudden change $\Delta t \ll H^{-1}$ approximation. Hence, the created charge density is

$$n (t_{cr}) \simeq \frac{3}{2} \rho_{rad} (t_{cr}) \Delta \gamma .$$

(3.25)

During the latter times and, in particular, during the matter domination $\gamma = \gamma_{early} + \Delta \gamma \ll 1$. For example the following $\gamma (\varphi)$ would work

$$\gamma (\varphi) = \gamma_{early} + \frac{1}{2} \Delta \gamma \left( \tanh \left( \frac{\varphi - \varphi_{cr} + \Delta \varphi}{\Delta \varphi} \right) + 1 \right).$$

(3.26)

After the charge is created and the shift-symmetry is restored, the energy density in the mimetic fluid redshifts as DM. In particular,

$$n (t) = n (t_{cr}) \left( \frac{a_{cr}}{a (t)} \right)^3.$$

(3.27)

If the transition generates the same amount of energy density in the mimetic fluid as it would be in the standard CDM picture, then $\varepsilon = \rho_{eq} (a/a_{eq})^{-3}$ where $\rho_{eq}$ is the energy density in radiation at the moment of matter-radiation equality and $a_{eq}$ is the scale factor at this moment. Using (3.13) we obtain that

$$\Delta \gamma = \frac{2}{3} \left( \frac{a_{cr}}{a_{eq}} \right) \simeq \frac{2}{3} \frac{z_{eq}}{z_{cr}},$$

(3.28)

where $a_{cr} = a (t_{cr})$, $z_{eq} \sim 10^4$ is the redshift of the matter-radiation equality, and $z_{cr}$ corresponds to the redshift at the moment of charge creation. Thus the earlier happens the creation of charge the smaller is the change of $\gamma$ and of the the sound speed needed. Note that from the bounds on the change of the Newton’s constant (3.22) one obtains that the charge creation should happen at the redshift $z \gtrsim 10^6$ from now which corresponds to the temperature $T \gtrsim 10^{2} \text{eV}$. In particular, for the choice of [47] $\Delta \gamma \lesssim 10^{-10}$ which was interesting for the small scales phenomenology\footnote{Note that in normal units this means $\Delta \gamma \lesssim (10^{-5} M_{Pl})^2 \simeq (10^{13} \text{GeV})^2$.} (to suppress perturbations with wavelengths below
100 kpc), \(z \gtrsim 10^{14}\) or temperature \(T \gtrsim 10\) GeV. Thus there the charge creation happens before the quark-gluon transition and can easily occur during the electroweak phase transition. In general the temperature at the transition is related to \(\Delta \gamma\) as

\[
T_{cr} \simeq \frac{T_{eq}}{\Delta \gamma} \simeq \frac{eV}{\Delta \gamma}.
\]

Thus to move the temperature to the region of physics currently not probed by accelerators, \(T_{cr} \gtrsim 100\) TeV, we have to assume that \(\Delta \gamma \lesssim 10^{-14}\). In general, it would be rather interesting to connect this charge-creation moment with a phase transition in the early universe or with reheating after inflation. The sounds speed corresponding to the efficient clustering is \(c^2_s \lesssim 10^{-5}\) [59, 60] thus \(\Delta \gamma \lesssim \gamma \lesssim 10^{-5}\), so that \(T_{cr} \gtrsim 0.1\) MeV which is between the primordial nucleosynthesis and electron-positron annihilation.

Here it is important to note that a potential term \(V(\varphi)\) would not help to generate the charge without \(\gamma(\varphi)\). Indeed, if \(n(t_i) = 0\), then

\[
n(t) = \int_{t_i}^{t} dt' \dot{V} \left( \frac{a(t')}{a(t)} \right)^3 < \Delta V,
\]

where in the last inequality we have used the fact that the universe is expanding. Therefore, the increase of the cosmological constant would be larger than the produced charge density making it impossible for the mimetic fluid to become the main source of DM. Of course one could use changes of the cosmological constant (during say phase transitions provided \(\Delta V > 0\)) to generate some of the DM density. But clearly it is not enough to explain the hole DM abundance.

On the other hand, one can invert the argument — exactly during this short phase the breaking of the shift-symmetry can generate a change in the potential, \(\Delta V\), i.e. the observed small cosmological constant. This creation of \(\Lambda\) is accompanied with a production of the negligible amount of the shift-charge.

This mechanism of DM production, potentially can introduce some amount of isocurvature cosmological perturbations. In a forthcoming publication we will show that generically these isocurvature perturbations are negligible.

4 Fluid picture

In this chapter we consider unbroken shift-symmetry: \(\varphi \to \varphi + c\), so that \(\gamma = \text{const}\). We will follow the reviews [61, 62] and the discussion of a similar system — the imperfect fluid from Kinetic Gravity Braiding [55].

The shift-symmetry Noether current can be decomposed using the local rest frame (LRF) given by a natural choice \(u_\mu = \partial_\mu \varphi\) as

\[
J_\mu = n u_\mu - \gamma \lambda_\mu \nabla \lambda \theta,
\]

where the shift-charge density is

\[
n = 2 \lambda - \gamma \dot{\theta},
\]

with the notation \(u_\mu \nabla \mu = D/d\tau = (\dot{\cdot})\),

\[
\theta = \nabla_\mu u^\mu,
\]
is the expansion (note that it is consistent with (2.8)) and
\[ g_{\mu\nu} u^\mu u^\nu, \] (4.4)
is the projector to the hypersurface orthogonal to \( u^\mu \).

Clearly our fluid is irrotational — the twist tensor is vanishing so that for the extrinsic curvature we get
\[ K_{\mu\nu} = \nabla^\lambda u_{\nu} = \nabla^\lambda \phi = \frac{1}{2} \nabla^\mu \left( \nabla^\lambda \phi \nabla^\nu \phi \right) = 0. \] (4.5)

Moreover, because of the constraint (2.2) the four velocity \( u^\mu \) is tangential to the time-like geodesics.

Therefore we will call the \( \partial_\mu \phi \) frame — the natural or the geodesic frame. However, from (4.1) it follows that the shift-charges do not move along the geodesics. If \( \gamma \) is a constant parameter the current (4.1) is conserved
\[ \nabla_\mu J^\mu = 0, \] (4.7)
we obtain
\[ \gamma \Box \theta - 2\dot{\lambda} - 2\lambda \theta = 0, \] (4.8)
which is nothing else as the equation of motion. This equation we can rewrite as
\[ \gamma \left( \ddot{\theta} - \nabla^2 \theta \right) - 2\dot{\lambda} - 2\lambda \theta = 0, \] (4.9)
where
\[ \nabla^2 = - \nabla^\lambda \nabla_\lambda \left( \nabla^\mu \nabla_\mu \right) = - \nabla^\mu \nabla_\mu, \] (4.10)
denotes the spatial Laplacian. Decomposing the EMT (2.7) in the same way we obtain
\[ T_{\mu\nu} = \varepsilon u_{\mu} u_{\nu} - p \perp_{\mu\nu} + q_{\mu} u_{\nu} + q_{\nu} u_{\mu}, \] (4.11)
where the energy density is
\[ \varepsilon = T_{\mu\nu} u^\mu u^\nu = 2\lambda - \gamma \left( \ddot{\theta} - \frac{1}{2} \dot{\theta}^2 \right), \] (4.12)
the pressure
\[ p = -\frac{1}{3} T^{\mu\nu} \perp_{\mu\nu} = -\gamma \left( \dot{\theta} + \frac{1}{2} \dot{\theta}^2 \right), \] (4.13)
and the energy flux
\[ q_{\mu} = \perp_{\mu\lambda} T_{\sigma}^{\lambda} u^\sigma = -\gamma \perp_{\mu} \nabla_\lambda \theta = \perp_{\mu} J_\lambda. \] (4.14)

Thus in the LRF the energy is transported along the spatial gradients of the expansion. However, there is no heat flux because \( q_{\mu} = \perp_{\mu\nu} J^\nu \). Similarly to the imperfect fluid from\( \text{Kinetic Gravity Braiding} \) [55] we can use this fact to claim that the Landau-Lifshitz frame moving with energy coincides up to \( O(\gamma) \) with the Eckart frame moving with the shift-charges, see [63], eq. (2), p. 312. One can also see that the anisotropic stress is vanishing in this frame
\[ \Pi_{\mu\nu} = \left( \perp_{\mu\alpha} \perp_{\nu\beta} - \frac{1}{3} \perp_{\mu\nu} \perp_{\alpha\beta} \right) T^{\alpha\beta} = 0. \] (4.15)
Further one can observe that an analog of the Euler relation holds:

$$\varepsilon + p = n - \gamma \dot{\theta}, \quad (4.16)$$

where the last terms on the r.h.s. appears purely because of the imperfect character of the fluid. Note that this expression corresponds to the unit chemical potential, because of the normalisation of our field. From the expressions (4.12), (4.13) and (4.16) it follows that for nearly neutral $n \simeq 0$ system with a slow expansion a $\dot{\theta} \ll \theta^2$ the equation of state is approximately that of vacuum $p \simeq -\varepsilon$. This happens even, if the mimetic fluid is not a dominant source of curvature.

Further we will need the Raychaudhuri equation for the timelike geodesics

$$\dot{\theta} = -\frac{1}{3} \theta^2 - \sigma_{\mu\nu} \sigma^{\mu\nu} - R_{\mu\nu} u^\mu u^\nu, \quad (4.17)$$

where $\sigma_{\mu\nu}$ is the shear tensor

$$\sigma_{\mu\nu} = \frac{1}{2} \left( \nabla^\lambda u_\nu + \nabla^\lambda u_\mu \right) - \frac{1}{3} \eta_{\mu\nu} \theta, \quad (4.18)$$

$\sigma^2 = \sigma_{\mu\nu} \sigma^{\mu\nu}$. Thus we see that in the presence of the external matter the energy density and pressure of the mimetic fluid depend on the pressure and energy density of this external matter through the Einstein equations in the form

$$R_{\mu\nu} = T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu}. \quad (4.19)$$

These equations give us

$$R_{\mu\nu} u^\mu u^\nu = \frac{\varepsilon + 3p}{2} + \frac{\rho_{\text{ext}} + 3P_{\text{ext}}}{2} = \lambda - 2\gamma \dot{\theta} - \frac{\gamma}{2} \theta^2 + \frac{\rho_{\text{ext}} + 3P_{\text{ext}}}{2}, \quad (4.20)$$

where for the external matter $\rho_{\text{ext}}$ and $P_{\text{ext}}$ are defined in the LRF associated with $u^\mu$. Thus using the Raychaudhuri equation (4.17) we can express the time derivative of the expansion

$$\dot{\theta} \left(1 - 2\gamma\right) = -\lambda - \frac{1}{3} \left(1 - \frac{3\gamma}{2}\right) \theta^2 - \sigma^2 - \frac{\rho_{\text{ext}} + 3P_{\text{ext}}}{2}. \quad (4.21)$$

Now we can write the corresponding formulas for the energy density (4.12):

$$\varepsilon = \left(\frac{2 - 3\gamma}{1 - 2\gamma}\right) \lambda + \frac{\gamma}{6} \left(5 - 9\gamma\right) \theta^2 + \frac{\gamma}{1 - 2\gamma} \left(\sigma^2 + \rho_{\text{ext}} + 3P_{\text{ext}}\right), \quad (4.22)$$

and pressure (4.13):

$$p = \frac{\gamma}{1 - 2\gamma} \lambda - \frac{\gamma}{6} \left(1 - 3\gamma\right) \theta^2 + \frac{\gamma}{1 - 2\gamma} \left(\sigma^2 + \rho_{\text{ext}} + 3P_{\text{ext}}\right). \quad (4.23)$$

It is important to note that the local energy density and pressure of DM explicitly depend the external matter e.g. on baryonic energy density. This could potentially explain dependance of the observable halo profiles on the local baryonic physics, (especially if we consider $\gamma (\varphi)$), see the corresponding discussion on this issue in [23]. We can also differentiate
the Raychaudhuri equation to obtain express $\dot{\lambda}$. For our time-like geodesics the Raychaudhuri equation for the shear tensor after some calculations takes the form

$$\dot{\sigma}_{\mu\nu} = -\frac{1}{3} \perp_{\mu\nu} \dot{\theta} - K_{\mu}^{\alpha} K_{\nu}^{\alpha} + \frac{1}{2} u^\alpha u^\beta \perp^\lambda_{\mu} (R_{\beta\gamma\lambda\alpha} + R_{\beta\lambda\gamma\alpha}),$$

(4.24)

where we have used the notation (4.5). Differentiating the Raychaudhuri for the expansion we obtain

$$-\dot{\lambda} = (1 - 2\gamma) \ddot{\theta} + \left(\frac{2}{3} - \gamma\right) \theta \dot{\theta} + 2\dot{\sigma}_{\mu\nu} \sigma^{\mu\nu} + \dot{\rho}_{\text{ext}} + 3\dot{P}_{\text{ext}},$$

(4.25)

which we can plug into the equation of motion (4.9) to obtain for the highest derivatives of this equation

$$\ddot{\theta} (2 - 3\gamma) - \gamma \nabla^2 \theta + 2u^\alpha u^\beta \sigma^{\mu\nu} C_{\alpha\mu\nu\beta} + \ldots = 0,$$

(4.26)

where the ellipsis stands for the terms with the lower number of derivatives and $C_{\alpha\mu\nu\beta}$ is the Weyl tensor and $\nabla^2$ is the spatial Laplacian (4.10). From this equation one can see that the speed of propagation of the expansion — sound speed is

$$c^2_S = \frac{\gamma}{2 - 3\gamma},$$

(4.27)

exactly as it was calculated for the cosmological case in [45], provided that the shear and the Weyl tensors are vanishing. The coupling between the shear and the Weyl tensor hints to the possible change of the speed of propagation for the gravitons on the backgrounds with non-vanishing shear. However, caution is needed, as the Einstein equations do not contain higher derivatives of the expansion, therefore similarly to [64] the system is triangular and there is no change in the speed of propagation for the gravitons. Moreover, because of the anisotropy given by $q_{\mu}$ one can expect that on general backgrounds with shear and Weyl tensor the phonons of the mimetic fluid propagate with different speeds in different directions even in the LRF. This issues definitely require further investigation using the original dynamical variable $\delta \varphi$. Especially this anisotropy in the sound speed can be important for studies of the nonlinear collapse.

### 4.1 Other frames

As we have shown above the EMT of the mimetic fluid does not take the form of a perfect fluid. Instead, it has a form of a fluid with an energy flow. The natural frame $u_{\mu} = \partial_{\mu} \varphi$ is particular, because i) in this frame there is no anisotropic stress, ii) there is no vorticity, iii) this frame moves along timelike geodesics, iv) the relation of the energy density, pressure etc to the field $\varphi$ and its derivatives is polynomial. Thus in this geodesic frame all equations look less nonlinear than in other frames. It is important to stress that for any non-ideal fluid (EMT) one can use the geodesic frame considered above. However, neither shift-charges nor energy move along this velocity field. One should expect that the velocity fields corresponding to the motion of charges and energy are more important in a physical setup e.g. for a consideration of structure formation or nonlinear gravitational collapse.

#### 4.1.1 Eckart frame

Another useful frame is the so-called Eckart frame. This is the frame moving together with the charges (provided the current (4.1) is timelike and future-directed). In this frame we have

$$U_{\mu} = \frac{J_{\mu}}{n_E} = \frac{nu_{\mu} + q_{\mu}}{n_E} = \frac{2\lambda \partial_{\mu} \varphi - \gamma \partial_{\mu} \theta}{n_E} \simeq \partial_{\mu} \varphi - \frac{\gamma}{2\lambda} \perp^\lambda_{\mu} \partial_{\lambda} \theta,$$

(4.28)
where $n_E$ is the proper density of shift-charges

$$n_E = \sqrt{J^\alpha J_\alpha} = \sqrt{n^2 - q^2} = \sqrt{4\lambda^2 - 4\lambda \gamma \dot{\theta} + \gamma^2 (\dot{\theta})^2} \simeq 2\lambda \left( 1 - \frac{\gamma}{2\lambda} \right),$$

(4.29)

where

$$q^2 = -q^\mu q_\mu.$$  

(4.30)

Further it is convenient to introduce a unit spacelike vector in the direction of the energy transfer

$$\hat{q}_\mu = q_\mu / q.$$  

(4.31)

In this frame the EMT is

$$T_{\mu\nu} = (\mathcal{E}_E + P_E) U_\mu U_\nu - P_E g_{\mu\nu} + \Pi^E_{\mu\nu} + Q_\mu U_\nu + Q_\nu U_\mu,$$

(4.32)

where the notation is the same as in the formulas (4.12), (4.13), (4.14), (4.4). The four-velocity of charges $U^\mu$ is obtained (4.28) from $u^\mu$ by the Lorentz transformation

$$U_\mu = \frac{m u_\mu + q \hat{q}_\mu}{n_E} = \frac{u_\mu + v \hat{q}_\mu}{\sqrt{1 - v^2}},$$

(4.33)

where the relative velocity is given by

$$v = \frac{q}{n},$$

(4.34)

so that as expected

$$n = \frac{n_E}{\sqrt{1 - v^2}}.$$  

(4.35)

Thus the transition to the Eckart frame is the Lorentz boost $(u_\mu, \hat{q}_\mu) \rightarrow (U_\mu, \hat{Q}_\mu)$ with

$$u_\mu = \frac{U_\mu - v \hat{Q}_\mu}{\sqrt{1 - v^2}}, \quad \text{and} \quad \hat{q}_\mu = \frac{\hat{Q}_\mu - v U_\mu}{\sqrt{1 - v^2}},$$

(4.36)

and

$$\hat{Q}_\mu = \frac{v u_\mu + \hat{q}_\mu}{\sqrt{1 - v^2}}.$$  

(4.37)

By plugging in the expressions (4.36) into the EMT (4.11) we obtain EMT from (4.32) with

$$\Pi^E_{\mu\nu} = \beta \left( \hat{Q}_\mu \hat{Q}_\nu + \frac{1}{3} (g_{\mu\nu} - U_\mu U_\nu) \right),$$

(4.38)

$$Q_\mu = - \left( q + \frac{\beta}{v} \right) \hat{Q}_\mu,$$

(4.39)

$$\mathcal{E}_E = \varepsilon + \beta,$$

(4.40)

$$P_E = p + \frac{1}{3} \beta,$$

(4.41)

where we denoted

$$\beta = v^2 \left( \frac{\varepsilon + p - 2n}{1 - v^2} \right) = -2\lambda \left( \frac{v^2}{1 - v^2} \right).$$

(4.42)
In the leading order in $\gamma$ we obtain:

$$\beta \simeq -2\lambda v^2 = -\frac{2\lambda q^2}{n^2} \simeq \frac{\gamma^2 \perp^{\mu\nu} \partial_\mu \theta \partial_\nu \theta}{2\lambda},$$

(4.43)

so that,$^{10}$

$$Q_\mu \simeq \frac{\gamma \dot{\theta}}{n} q_\mu \simeq -\gamma^2 \dot{\theta} \nabla_\mu \theta,$$

(4.44)

and

$$\Pi_{\mu\nu}^E \simeq \beta \left( q_\mu \dot{q}_\nu + \frac{1}{3} \perp_{\mu\nu} \right).$$

(4.45)

It is important that energy transfer and anisotropic stress are both $O(\gamma^2)$ and quadratic in $\theta$. Now we can compare the anisotropic stress with the usual shear viscosity. In the leading order

$$\sigma_{\mu\nu} \simeq \sigma_{\mu\nu} = \phi_{\mu,\nu} - \frac{1}{3} \theta \perp_{\mu\nu},$$

(4.46)

so that

$$\Pi_{\mu\nu}^E \simeq -\frac{\beta}{\theta} (\sigma_{\mu\nu} - \pi_{\mu\nu}),$$

(4.47)

with an additional part

$$\pi_{\mu\nu} = \theta \dot{q}_\mu q_\nu + \phi_{\mu,\nu}.$$  

(4.48)

Both tensors $\sigma_{\mu\nu}$ and $\pi_{\mu\nu}$ are symmetric, traceless and purely spatial. Because these tensors are not positive-definite generically they cannot be simultaneously diagonalized. However, if $\|\pi_{\mu\nu}\| \ll \|\sigma_{\mu\nu}\|$ the anisotropic stress mimics effects of shear viscosity with the shear viscosity coefficient

$$\eta \simeq \frac{\beta}{\theta} = \frac{\gamma^2}{2\lambda} \left( \perp^{\mu\nu} \partial_\mu \theta \partial_\nu \theta \right).$$

(4.49)

In cosmology we can estimate these terms as

$$Q_\mu \sim -\gamma^2 \frac{\dot{H} \nabla_\mu \xi}{H^2},$$

(4.50)

where $\xi$ is the curvature perturbation. Hence the magnitude of the energy transfer on physical scale $\ell$ is suppressed as

$$Q_\ell \sim c_S^2 \frac{\dot{H} \xi}{H} (\ell H)^{-1}.$$  

(4.51)

The anisotropic stress only comes at quadratic order in perturbations. In particular,

$$\eta \sim -\frac{\gamma^2}{H} \delta_{ik} \left( \frac{\partial_i \xi}{aH} \right) \left( \frac{\partial_k \xi}{aH} \right),$$

(4.52)

and on physical scale $\ell$ its magnitude is strongly suppressed

$$\eta_\ell \sim -\frac{c_S^4}{H^2} \xi_\ell^2 (\ell H)^{-2}.$$  

(4.53)

$^{10}$This implies that $\beta < 0$ for most natural physical cases when the leading energy density is positive $\lambda > 0$. 

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4.1.2 Landau-Lifshitz frame

Let us find the so-called Landau-Lifshitz (LL) frame $V^\mu$ in which there is no energy flow — so that $V^\mu$ is a timelike, unit and future directed eigenvector of the EMT $-T^\mu_\nu V^\nu = \mathcal{E} V^\nu$ where $\mathcal{E}$ is the energy density in this frame. The results one can find e.g. in [65] where the same structure of the EMT was considered. One obtains for the eigenvalue

$$\mathcal{E} = \frac{\varepsilon - p}{2} + \sqrt{\left(\frac{\varepsilon + p}{2}\right)^2 - q^2},$$

and the corresponding 4-velocity is

$$V_\mu \propto u_\mu + v_\mu = \phi_\mu \left(1 + \frac{\gamma v}{q} \dot{\theta} - \frac{\gamma v}{q} \theta_\mu, \right),$$

and the relative velocity between the LL frame and the natural geodesic frame is

$$v_{LL} = \frac{\varepsilon + p}{2q} - \sqrt{\left(\frac{\varepsilon + p}{2q}\right)^2 - 1}.\quad (4.56)$$

In this frame the EMT is

$$T_{\mu\nu} = \mathcal{E} V_\mu V_\nu - P_{LL} \perp_{LL}^{\mu\nu} + \Pi_{LL}^{\mu\nu},$$

where the anisotropic stress $\Pi_{LL}^{\mu\nu} = \mathcal{O}(\gamma^2)$. Note that the difference in the relative velocities of the Eckart and Landau-Lifshitz frames is $v - v_{LL} = \mathcal{O}(\gamma^2)$.

4.2 Gradient expansion

The mimetic fluid shares some similarities with the imperfect fluid from *Kinetic Gravity Braiding* [55]. In particular, there is a mixing with the gravity and the energy flux forcing the EMT to deviate from the perfect fluid form. Similarly [55] it is useful to look at the (spatial) gradient expansion around the equilibrium also for the mimetic fluid. For one can make a boost to the new LRF — the Eckart frame (4.28) moving together with the charges:

$$U_\mu \simeq \frac{J_\mu}{n} \simeq u_\mu + \frac{q_\mu}{n},$$

where we have suppressed terms $\mathcal{O}(\gamma^2)$ as we will further constantly do further in this section. It is important to stress that the Eckart frame does have vorticity, because $\partial_\mu \phi$ and $n^{-1} \gamma \nabla_\lambda \nabla_\lambda \theta$ are not proportional to a common gradient, see (4.3). In this frame the EMT takes the form of the perfect fluid

$$T_{\mu\nu} \simeq (\varepsilon + p) U_\mu U_\nu - pg_{\mu\nu} + \mathcal{O}(\gamma^2),$$

where $p$ and $\varepsilon$ are still defined as in (4.12) and (4.13). From equations (4.20) and (4.17) we obtain that

$$\dot{\theta} \simeq \frac{1}{3} \theta^2 - \sigma_{\mu\nu} \sigma^{\mu\nu} - \lambda - \frac{\rho_{ext} + 3P_{ext}}{2},$$

$$p \simeq \gamma \lambda = \frac{\gamma}{2} \varepsilon,$$
thus up to $O(\gamma^2)$ terms $c_S^2 = \gamma/2$. Here we neglected terms with expansion $\theta$, because they are higher order in the gradient expansion $O(\partial^2)$. Also we neglected the external matter contribution, because this is a purely gravitational effect. For the charge density we get

$$n \simeq (2 + \gamma) \lambda,$$

and for the energy density we get

$$\varepsilon \simeq 2 \lambda.$$

Using the formalism from [66] we can write the chemical potential $\mu$ as

$$U_\mu = \mu^{-1} \left( \partial_\mu \phi - n^{-1} \gamma \nabla_\lambda \theta \right) \simeq \mu^{-1} \left( \partial_\mu \phi - \frac{\gamma}{2 \lambda} \partial_\mu \theta \right),$$

so that $V^\mu V_\mu = 1$ and

$$\mu \simeq 1 + \frac{\gamma}{2 \lambda} \dot{\theta} \simeq 1 - \frac{\gamma}{2} = \frac{\partial \varepsilon}{\partial n},$$

where in the last equality we have neglected higher order derivatives and the external contribution $\rho_{\text{ext}} + 3P_{\text{ext}}$. Thus we have demonstrated that mimetic fluid with HD in the gradient expansion corresponds to the perfect fluid with equation of state $\gamma/2$. In this approximation, any deviation from the perfect fluid appears only on the level $O(\gamma^2)$ or through Planck-scale suppressed operators.

### 4.3 Vorticity

As we have seen both Landau-Lifshitz and Eckart frames have the form

$$U_\mu = \mu^{-1} \left( \partial_\mu \varphi + \rho \partial_\mu \theta \right),$$

were $\mu$ and $\rho$ are some scalar functions which together with $\theta$ and $\varphi$ build relativistic Clebsch potentials, see e.g. [66]. Thus the vorticity vector $\Omega^\mu (V)$ for any timelike vector field $V^\mu$ is defined as

$$\Omega^\mu (V) = \frac{1}{2} \varepsilon^{\alpha \beta \gamma \mu} \nabla_\alpha V_\beta \gamma = \frac{1}{2} \varepsilon^{\alpha \beta \gamma \mu} \nabla_\alpha V_\beta \gamma,$$

where $\nabla_\mu = g_{\mu \nu} - V_\mu V_\nu$. For the velocity field (4.66) we have

$$\Omega^\mu (U) = \frac{1}{2 \mu^2} \varepsilon^{\alpha \beta \gamma \mu} \rho \partial_\alpha \varphi \gamma = \frac{1}{2 \mu^2} \varepsilon^{\alpha \beta \gamma \mu} \nabla_\alpha \varphi \nabla_\beta \theta u_\gamma,$$

where $\nabla_\alpha = \nabla_\alpha \nabla_\lambda$ and $\nabla_\mu = g_{\mu \nu} - u_\mu u_\nu$, where we have used the geodesic frame.

In particular, in the Eckart frame we have $\rho = -\gamma/2\lambda$ and $\mu = n_E/2\lambda$ so that

$$\Omega^\mu (U) = \frac{\gamma}{n_E} \varepsilon^{\alpha \beta \gamma \mu} \nabla_\alpha \nabla_\beta \theta u_\gamma \simeq \frac{\gamma}{4 \lambda^2} \varepsilon^{\alpha \beta \gamma \mu} \nabla_\alpha \nabla_\beta \theta u_\gamma,$$

where in the last equality we omitted terms of the order $O(\gamma^2)$. Generically the vorticity does not vanish, because $\lambda$ and $\theta$ correspond to different initial data and can be chosen independently. Indeed, without HD, $\lambda (x)$ corresponds to the initial energy density profile, while $\theta (x)$ is fixed by the initial velocities and the metric. As we have already mentioned the Eckart frame and Landau-Lifshitz frame coincide up to factors $O(\gamma)$. Thus the vorticity is also there in the Landau-Lifshitz frame.
5 Conclusions and open questions

On the level of cosmological background we found that mimetic fluid with higher derivatives (HD), $\gamma (\Box \phi)^2$, renormalises the Newton’s constant in the Friedmann equations by $\delta G_N / G_N \sim \gamma$. We have discussed different bounds on this renormalisation coming from observations, see subsection 3.2. These bounds are not particularly severe. Further we have proved that mimetic fluid has zero pressure when there is no extra matter in the universe. Surprisingly our imperfect DM survives the inflationary expansion and builds a small part of the dominating energy density e.g. radiation after that. However, inflation completely redshifts the shift-charge so that after that the shift-symmetric system is on the dynamical attractor solution perfectly tracking any external matter. Thus, if there is no external matter, the mimetic fluid on attractor with zero shift-charge does not have any energy density and cannot be DM. If one insists on the mimetic origin of the constraint $\phi,\mu \phi^\mu = 1$ and on shift-symmetry, then the system cannot be DM. We have showed that a short period of shift-symmetry breaking in the HD term (through $\gamma (\phi)$) during the radiation domination epoch removes this problem. During this cosmologically short period of time one can create sufficient amount of shift-charges to enter the usual matter domination epoch at the usual redshift. The charge creation should happen in that case at temperatures $T_{\text{cr}} \gtrsim 0.1 \text{MeV}$ for the current DM sound speed $c_S^2 \lesssim 10^{-5}$ and at $T_{\text{cr}} \gtrsim 10 \text{GeV}$ for the $c_S^2 \simeq 10^{-10}$ chosen in [47] to suppress perturbations with wavelengths below 100 kpc. It is important to note that this model essentially has three parameters — the sound speed in the early universe (or $\gamma_{\text{early}}$), the sound speed in the late universe (or $\gamma_{\text{late}}$) and the duration of the shift-symmetry breaking $\Delta t$ which is cosmologically short and would be rather difficult to observe.

We have also demonstrated that the shift-symmetry breaking in form of potential $V (\phi)$ would not help to generate enough shift-charge. On the other hand one can use the phase of shift-symmetry breaking to generate the cosmological constant $\Lambda$. It would be very interesting to see whether this DM picture can be elegantly combined with inflation or with Dark Energy similarly to [46].

We have shown that the HD operator $\gamma (\phi) (\Box \phi)^2$ modifies the simplest irrotational DM to an imperfect fluid with a finite sound speed $c_S^2$ of the order $O (\gamma)$, finite vorticity of the order $O (\gamma)$ in the frames moving either with the shift-charges or with the energy. In the natural rest frame of the field $u_\mu = \partial_\mu \phi$ the fluid has the energy flow $O (\gamma)$. Generically there is no reference frame where the fluid would look locally isotropic. Thus one can expect that around a general configuration the sound speed depends on direction in all reference frames. This can change the whole structure formation picture. On the other hand we presented an argument that for all backgrounds with vanishing shear and Weyl tensor the speed of sound is the same as in cosmology. If the shear is not vanishing there is an intriguing possibility that the speed of propagation of gravity waves changes around such configurations and becomes anisotropic, see (4.26).

Moreover, the appearance of the vorticity, energy flow and anisotropic stress should make the collapse by far less efficient as it is in the case for the irrotational DM [59, 67]. Thus it seems that this scalar field theory of DM is not bound to be a small part of the total DM budget. There is still an open issue with caustics which are quite often formed in the hydrodynamical and in particular scalar models of DM, see e.g. [42, 68–70]. Indeed, it is not clear whether this imperfect DM forms caustics, other nonlinear singularities or supports strong shock waves. Clearly mimetic DM without higher derivatives will form caustics more efficient than standard rotational DM. The HD (or dissipative) terms have potential to avoid
the caustics in some cases [69]. Moreover, on galactic scales DM is virialized and this implies many caustics on the fluid level. Thus it is not clear how to model such strong overdensities in a fluid-like picture and not to form caustics. A proper interpretation of a caustic in the field-theoretical setup is needed. It may happen that the only way to continue the solution through the multivalued region is the UV physics or quantisation of the system. Another related problem is to find the strong coupling scale for the quantised sound waves around a general configuration. Further it is important to understand the Hamiltonian formulation of this theory. The first step without HD was done in [71]. It is known that HD can substantially change the structure of the theory also in the presence of constraints, see e.g. [72]. These issues lie beyond the scope of this paper but definitely requires a further investigation.

Depending on the value of the sound speed, the linear perturbation theory of the mimetic imperfect DM can have rather interesting phenomenological consequences related to the finiteness of the sound speed and the corresponding suppression of the power spectrum on the scales shorter than the sonic horizon, see [47].

We think that the imperfect DM provides a perfect playground for a phenomenologically rich and sophisticated modeling of the universe where we live in.

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