An advance report on particle invariance in particle physics

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Abstract

Since particle such as molecule, atom and nucleus are composite particle, it is important to recognize that physics must be invariant for both the composite particle and its constituent particles, this requirement is called particle invariance. But difficulties arise immediately because for fermion we use the Dirac equation, for boson we use the Klein-Gordon equation. Therefore, the particle invariance demands there is a general wave equation for describing particle motion regardless particle class. In this paper, three advances in this subject are reported: (1) momentum-wavefunction relation is a general relation shared by both fermion and boson, meets the requirement of the particle invariance. As a test, the momentum-wavefunction relation was directly applied to hydrogen atom, and get the correct fine structure and spin effect for the electron. (2) the Dirac equation and Klein-Gordon equation can be derived out from the momentum-wavefunction relation when we abandon some higher order terms. (3) according to the momentum-wavefunction relation a path integral method was developed, differing from Feynman’s path integral, it simplifies quantum computation.

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1 Introduction

Consider a particle of rest mass $m$ and charge $q$ moving in an inertial frame of reference with relativistic 4-vector velocity $u_\mu$, it satisfies

$$u_\mu u_\mu = -c^2$$

(1)

where there is not distinction between covariant and contravariant components in the Cartesian coordinate system. Let $A_\mu$ denote vector potential of electromagnetic field, substituting the momentum-wavefunction relation

$$mu_\mu = \frac{1}{\psi}(-i\hbar \partial_\mu - qA_\mu)\psi$$

(2)

into Eq.(1), we obtain a new quantum wave equation with single component wavefunction

$$[(-i\hbar \partial_\mu - qA_\mu)\psi][(-i\hbar \partial_\mu - qA_\mu)\psi] = -m^2 c^2 \psi^2$$

(3)

where we regard the momentum as momentum itself but not momentum operator, please note that Eq.(3) is not the Klein-Gordon wave equation. In the recent years, H Y Cui has studied this equation for many years, it was found that by solving Eq. (3) for hydrogen atom, the fine structure of hydrogen energy can be calculated correctly, while its wavefunction has only single component in contrast with Dirac’s wavefunction, the spin effect of electron is also revealed by Eq. (3) when the hydrogen atom is in a magnetic field [2] [3] [4] [5]. It was also found that the Dirac wave equation and Klein-Cordon wave equation can be derived out from Eq. (3) when we abandon some higher order terms or nonlinear terms [5]. These results are easily understood regarding that Eq. (1) is a general relation shared by both fermion and boson, or any other particle.

In the present paper, we mainly report that a path integral method based on the momentum-wavefunction relation was developed, it differs from Feynman’s path integral; as a test, the spin effect of electron are calculated by using the path integral method, giving out correct results. It was shown that the path integral method is a rapid quantum computation method which satisfies the particle invariance.

2 Path integral method

Consider Eq.(2), its path integral form is given by

$$\psi = e^{i\int (p_\mu + qA_\mu)dx_\mu}$$

(4)

where $p_\mu = mu_\mu$ is the momentum of the particle. In the Cartesian coordinate system $(x_1, x_2, x_3, x_4 = i ct)$, from the Eq.(4), the momentum components satisfy

$$p_1^2 + p_2^2 + p_3^2 + p_4^2 = -m^2 c^2$$

(5)

In the next sections, we test the path integral for some physical systems, to note that it differs from Feynman’s path integral.
3 Fine structure

In the following, we use Gaussian units, and use \( m_e \) to denote the rest mass of electron, thus

\[
\psi = e^{\frac{i}{\hbar} \int (p_\mu + q A_\mu/c) dx_\mu} \quad (6)
\]

In a spherical polar coordinate system \((r, \theta, \varphi, ic t)\), the nucleus of hydrogen atom provides a symmetric potential \( V(r) = e/r \) for the electron \((q = -e)\), the displacement elements and vector potential are given by

\[
\begin{align*}
dx_r &= dr \\ dx_\theta &= rd\theta \\ dx_\varphi &= r \sin \theta d\varphi \\ A_r &= A_\theta = A_\varphi = 0 \\ A_4 &= i V = ic/r
\end{align*}
\]

Then, the wavefunction is given by

\[
\psi = e^{\frac{i}{\hbar} \int p_r dx_r} e^{\frac{i}{\hbar} \int p_\theta dx_\theta} e^{\frac{i}{\hbar} \int p_\varphi dx_\varphi} e^{\frac{i}{\hbar} \int (p_4 + q A_4/c) dx_4}
\quad (12)
\]

For separating the variables so that \( \psi = R(r) X(\theta) \phi(\varphi) e^{-iEt/\hbar} \) for energy eigenstate, we expect

\[
\phi(\varphi) = e^{\int p_\varphi dx_\varphi} \quad (13)
\]

\[
X(\theta) = e^{\int p_\theta dx_\theta} \quad (14)
\]

\[
R(r) = e^{\int p_r dx_r} \quad (15)
\]

\[
e^{-iEt/\hbar} = e^{\int_0^t (p_4 + q A_4/c) dx_4} \quad (16)
\]

The angular momentum magnitude and its \( z\)-axis component magnitude are denoted by \( J \) and \( J_z \) respectively, we have

\[
p_\varphi \sin \theta = J_z \quad (const.) \quad (17)
\]

\[
(\sqrt{p_\theta^2 + p_\varphi^2}) r = J \quad (const.) \quad (18)
\]

and

\[
p_4 = \frac{-E - ic q A_4/c}{ic} = \frac{-E - e^2/r}{ic} \quad (19)
\]

\[
p_r = \pm \sqrt{-m_e^2 c^2 - p_\theta^2 - p_\varphi^2 - p_4^2} \quad (20)
\]

\[
= \pm \sqrt{-m_e^2 c^2 - \frac{J^2}{r^2} + \frac{1}{c^2}(E + \frac{e^2}{r})^2} \quad (21)
\]

thus we have

\[
\phi(\varphi) = e^{\frac{i}{\hbar} \int p_\varphi dx_\varphi} = C_1 e^{\frac{\pm i}{\hbar} J_z \varphi} \quad (22)
\]

\[
X(\theta) = e^{\frac{i}{\hbar} \int p_\theta dx_\theta} = C_2 e^{\frac{\pm i}{\hbar} \int_0^\theta \sqrt{J^2 - \frac{J_\theta^2}{\sin^2 \theta}} d\theta} \quad (23)
\]

\[
R(r) = C_3 e^{\frac{\pm i}{\hbar} \int_0^\infty \sqrt{-m_e^2 c^2 - \frac{J^2}{r^2} + \frac{1}{c^2}(E + \frac{e^2}{r})^2} dr} \quad (24)
\]

where \( C_1, C_2 \) and \( C_3 \) are integral constants. Since \( \phi(\varphi) \) and \( X(\theta) \) must be periodic functions, and the radical wavefunction \( R(r) \) forms a "standing wave" in the range from \( r = 0 \) to \( r = \infty \), these requirements demand

\[
\frac{1}{\hbar} \int_0^{2\pi} J_z d\varphi = 2\pi m \quad (25)
\]

\[
\frac{1}{\hbar} \int_0^{2\pi} \sqrt{J^2 - \frac{J_\theta^2}{\sin^2 \theta}} d\theta = 2\pi k \quad (26)
\]

\[
\frac{1}{\hbar} \int_0^\infty \sqrt{-m_e^2 c^2 - \frac{J^2}{r^2} + \frac{1}{c^2}(E + \frac{e^2}{r})^2} dr = \pi s \quad (27)
\]

These definite integrals have been evaluated in the author's previous paper \cite{2} by using the residue theorem and contour integrations in complex space, because the last two integrands are multiple-valued functions when over their turning points, the results are given by

\[
J_z = m \hbar \quad (28)
\]

\[
\frac{1}{\hbar} \int_0^{2\pi} \sqrt{J^2 - \frac{J_\theta^2}{\sin^2 \theta}} d\theta = 2\pi \frac{J}{\hbar} - |m| \quad (29)
\]

\[
J = (k + |m|) \hbar = j \hbar \quad (30)
\]

\[
\frac{1}{\hbar} \int_0^\infty \sqrt{-m_e^2 c^4 - \frac{J^2}{r^2} + \frac{1}{c^2}(E + \frac{e^2}{r})^2} dr = \pi E \alpha \sqrt{m_e^2 c^4 - E^2} - \pi \sqrt{j^2 - \alpha^2} \quad (31)
\]

\[
\frac{\pi E \alpha}{\sqrt{m_e^2 c^4 - E^2}} - \pi \sqrt{j^2 - \alpha^2} = \pi s \quad (32)
\]

where \( \alpha = e^2/hc \) is known as the fine structure constant.

Form the last Eq.(30), we obtain the energy levels given by

\[
E = m_e c^2 \left[ 1 + \frac{\alpha^2}{(\sqrt{j^2 - \alpha^2 + s})^2} \right]^{-\frac{1}{2}} \quad (33)
\]

where \( j = k + |m| \), because \( j \neq 0 \) in Eq.(30), we find \( j = 1, 2, 3, ... \)

The result, Eq.(33), is completely the same as that in the calculation of the Dirac wave equation\cite{7} for the hydrogen atom, it is just the fine structure of hydrogen energy.

4 Electronic spin

If we put the hydrogen atom into an external uniform magnetic field \( B \) which is along the \( z \) axis with the vector
potential \((A_x, A_y, A_z) = (0, 0, \frac{1}{2}r \sin \theta B)\), i.e. \(\mathbf{B} = B\mathbf{e}_z\), where \(\mathbf{e}_z\) is the unit vector along \(z\) axis. According to Eq. (1), the energy eigenstate of the hydrogen atom is described by

\[
\psi = R(r)X(\theta)\phi(\varphi)e^{-iEt/\hbar} \quad (34)
\]

\[
\phi(\varphi) = e^{\pm i \int (p_\varphi + q A_\varphi/c)dx_\varphi} \quad (35)
\]

\[
X(\theta) = e^{\pm i \int p_\theta dx_\theta} \quad (36)
\]

\[
R(r) = e^{\pm i \int p_r dx_r} \quad (37)
\]

\[
e^{-iEt/\hbar} = e^{\pm i \int (p_\varphi + q A_\varphi/c)dx_\varphi} \quad (38)
\]

The magnitude of the angular momentum is denoted by \(J\) and its component along \(z\)-axis by \(J_z\), then

\[
p_\varphi r \sin \theta = J_z \quad (const.) \quad (39)
\]

\[
(B^2) = \frac{m^2c_\theta^2}{r} + \frac{1}{c^2}(E + \frac{e^2}{r})^2 \quad (40)
\]

we also have the same expressions as

\[
p_\varphi = \frac{-E - icqA_\varphi/c}{ic} = \frac{-E - e^2/r}{ic} \quad (41)
\]

\[
p_r = \pm \sqrt{m^2c_\theta^2 - p_\theta^2 - p_\phi^2} \quad (42)
\]

\[
= \pm \sqrt{m^2c_\theta^2 - \frac{J^2}{r^2} + \frac{1}{c^2}(E + \frac{e^2}{r})^2} \quad (43)
\]

but we have

\[
\phi(\varphi) = e^{\pm i \int (p_\varphi + q A_\varphi/c)dx_\varphi} \quad (44)
\]

\[
X(\theta) = e^{\pm i \int p_\theta dx_\theta} = C_2 e^{i \int \int \int \int \theta d\theta} = C_2 e^{i \int \int \int \int \theta d\theta} \quad (45)
\]

\[
R(r) = C_3 e^{i \int \int \int \int \theta d\theta} \quad (46)
\]

where \(C_1, C_2\) and \(C_3\) are integral constants, we have neglected \(O(B^2)\) term. Since \(\phi(\varphi)\) and \(X(\theta)\) must be periodic functions, and the radical wavefunction \(R(r)\) forms a "standing wave" in the range from \(r = 0\) to \(r = \infty\), these requirements demand

\[
\frac{1}{\hbar} \int_0^{2\pi} (J_z - \frac{e}{2} \sin^2 \theta B) d\varphi = 2\pi m \quad (47)
\]

\[
(m = 0, \pm 1, \pm 2, \ldots) \quad (48)
\]

\[
\frac{1}{\hbar} \int_0^{2\pi} \sqrt{J^2 - \frac{m^2h^2}{c^2} - \frac{mhe^2B}{c} dr} = 2\pi k \quad (49)
\]

These definite integrals can be evaluated in the same way as in the author’s previous paper [2], given by

\[
J_z - \frac{e}{2} r^2 \sin^2 \theta B = m \quad (50)
\]

\[
\frac{1}{\hbar} \int_0^{2\pi} \sqrt{J^2 - \frac{m^2h^2}{c^2} - \frac{mhe^2B}{c} d\varphi} = 2\pi (1 - \frac{m^2h^2}{c^2}) \quad (51)
\]

we get

\[
J^2 - \frac{mhe^2B}{c} = (k + |m|)^2 \quad (52)
\]

we have

\[
\frac{1}{\hbar} \int_0^{2\pi} \sqrt{J^2 - \frac{m^2h^2}{c^2} - \frac{mhe^2B}{c} d\varphi} = \pi \sqrt{J^2 - \alpha^2} \quad (53)
\]

\[
\frac{\pi E_\alpha}{\sqrt{m^2c^4 + mhe^2B - E^2}} = \pi \sqrt{J^2 - \alpha^2} = \pi \quad (54)
\]

we obtain the energy levels of hydrogen atom in the magnetic field given by

\[
E = \sqrt{m^2c^4 + me^2B} \left[1 + \frac{\alpha^2}{(\sqrt{J^2 - \alpha^2} + s)^2}\right]^{-\frac{1}{2}} \quad (55)
\]

In the usual spectroscopic notation of quantum mechanics, four quantum numbers: \(n, l, m_l\) and \(m_s\) are used to specify the state of an electron in an atom. After the comparison, we get the relations between the usual notation and our notation.

\[
n = j + s, \quad s = 0, 1, \ldots; j = 1, 2, \ldots \quad (57)
\]

\[
l = j - 1, \quad (58)
\]

\[
max(m_l) = max(m_l) - 1 \quad (59)
\]

We find that \(j\) takes over 1, 2, \ldots, \(n\); for a fixed \(j\) (or \(l\), \(m\) takes over \(-(l + 1), -l, \ldots, 0, l, l + 1\). In the present work, spin quantum number is absent.
According to Eq. (59), for a fixed \((n,l)\), equivalent to \((n,l+1)\), the energy level of hydrogen atom will split into 2\(l+3\) energy levels in the magnetic field, given by

\[
E = (m_{e}c^2 + \frac{m_{e}eB}{2m_{e}c}) \left[ 1 + \frac{\alpha^2}{(\sqrt{j^2 - \alpha^2} + s)^2} \right]^{-\frac{1}{2}} + O(B^2)
\]

(60)

Considering \(m = -(l+1), -l, ..., 0, ..., l, l+1\), this effect is equivalent to the usual Zeeman splitting in the usual quantum mechanics given by

\[
E = E_{nl} + \frac{(m \pm 1)eB}{2m_{e}c}
\]

(61)

But our work works on it without spin concept, the so-called spin effect has been revealed by Eq. (60) without spin concept, this result indicates that electronic spin is a kind of orbital motion. In the present calculation, the so-called spin has been merged with the orbital motion of the electron.

Spin concept objects to particle invariance, bear in mind that simplicity is always a merit for the physics.

5. Discussion

1. influence of nucleus motion

Since the nucleus and the electron in hydrogen atom consists an atom system with a rest center of mass, the nucleus is moving, therefore, the nucleus should provide a modified Coulomb’ force which contains both electric field and magnetic field for the electron. In the author’s previous paper [8], the modified Coulomb’s 4-force \(f\) is expressed in the Minkowsky space as (ref. to Fig. 1 in the paper)

\[
f = \frac{kqq'}{c^2r^3} [(u \cdot X)u' - (u \cdot u')X]
\]

(62)

If we consider the nucleus influence on the electron, then the calculation of hydrogen atom can be improved further.

2. gyrated state

To note that \(J_z\) and \(J\) are the magnitudes of the angular momentum vector and its z-axis component, since they are constants, the direction of the angular momentum vector is definitely uncertain, thus, there are two kinds of state: (1) when \(J_z = J\), the direction of \(J\) is in the z-axis, the state of electron motion is in a circular or elliptical orbit; (2) when \(J_z \neq J\), the state should be gyrated state, i.e. \(J\) vector keeps its magnitude unchanged but varies its direction, this state can only appear in non Coulomb’s field by the nucleus or external field.

3. rapid quantum computation

Since the path integral method for quantum mechanics needs not to evaluate quantum wave equation (2 order or nonlinear ones), it definitely is a rapid quantum computation method. This path integral method provides a great prospects for computer computation in some research fields such as \(X\alpha\), ab-initio, LMTO, DV, etc.

The path integral method developed in the present paper differs essentially from Feynman’s path integral.

4. the particle invariance in particle physics

Since particles such as molecules, atoms and nuclei are composite particles, it is important to recognize that physics must be invariant for both the composite particles and their constituent particles, this requirement is called particle invariance [6].

To note that the Eq. (14) is suitable for any kind of particle, therefore, it satisfies the particle invariance in particle physics.

5. the explanation of the wavefunction

The wavefunction \(\psi\) we employed in the calculation for hydrogen atom differs from the wave function in the usual quantum mechanics, because it was found that the wavefunction \(\psi\) keeps \(|\psi| = 1\) everywhere in the hydrogen atom. But this kind of wavefunction can interference with each other, for the detail discussion see the papers [11, 12].

6. others

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6. Conclusion

Using equation

\[
(mu_{\mu} + qA_{\mu})\psi = -i\hbar \partial_{\mu}\psi
\]

and its integral solution

\[
\psi = e^{\frac{i}{\hbar} \int (p_{\mu} + qA_{\mu})dx_{\mu}}
\]

a path integral method for calculating quantum state of a particle was developed. The approach has a great advantage: it is a rapid computation method, because it needs only to evaluate integration for quantum problem, instead of solving quantum wave equation as in usual quantum mechanics.

In this paper, three advances in the particle invariance are reported: (1) momentum-wavefunction relation is a general relation shared by both fermion and boson, meets the requirement of the particle invariance. As an example, the momentum-wavefunction relation was directly applied to hydrogen atom, and get the correct fine structure and spin effect for the electron. (2) the Dirac equation and Klein-Gordon equation can be
derived out from the momentum-wavefunction relation when we abandon some higher order terms. (3) according to the momentum-wavefunction relation a path integral method was developed, differing from Feynman’s path integral, it simplifies quantum computation.

The present calculation is characterized by using the usual momentum-wavefunction relation directly, it provides an insight into the foundations of quantum mechanics. The particle invariance is a basic principle for particle physics.

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