Multiple Residual Dense Networks for Reconfigurable Intelligent Surfaces Cascaded Channel Estimation

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Abstract—Reconfigurable intelligent surface (RIS) constitutes an essential and promising paradigm that relies programmable wireless environment and provides capability for space-intensive communications, due to the use of low-cost massive reflecting elements over the entire surfaces of man-made structures. However, accurate channel estimation is a fundamental technical prerequisite to achieve the huge performance gains from RIS. By leveraging the low rank structure of RIS channels, three practical residual neural networks, named convolutional blind denoising network, convolutional denoising generative adversarial networks and multiple residual dense network, are proposed to obtain accurate channel state information, which can reflect the impact of different methods on the estimation performance. Simulation results reveal the evolution directions of these three methods and reveal their superior performance compared with existing benchmark schemes.

Index Terms—Channel estimation, deep learning, multiple residual dense network, reconfigurable intelligent surface.

I. INTRODUCTION

To greatly enhance ultra-high data rate and ubiquitous coverage requirements of the sixth-generation (6G) wireless networks, as one of the promising and innovative techniques, reconfigurable intelligent surface (RIS) aided massive multiple-input multiple-output (MIMO) is envisioned to significantly reduce link blocking probability and system energy consumption to improve link quality with sophisticated beamforming [1]–[3]. RIS aided MIMO has been explored with near-passive array to obtain green and sustainable communications between the user equipment (UE) and the base station (BS), by appropriately and dynamically adjusting the magnitude and phase response, wireless signals can be coherently combined and steered to desired directions [4], [5]. Each RIS reflective element can individually control the amplitude response and phase shift of the incident electromagnetic waves at the nanosecond level to achieve energy concentration. Through reflection, refraction, absorption, and transmission, the reshaped electromagnetic waves will form new paths. Based on the passive and low-cost characteristics of RIS reflective elements, the RIS system requires very low energy consumption to improve the electromagnetic environment and increase propagation environment coverage.

However, benefits from a systematic performance improvement, the RIS system relies on the perfect channel state information (CSI) assumption. Unfortunately, the above work assumes a perfect CSI but not consider the difficulty of obtaining it. First of all, it is quite difficult to estimate the RIS-UE and RIS-BS channels separately, unless the RIS can be equipped with radio frequency (RF) chains. Secondly, the cascaded channel of the RIS between the BS and the UE can be very high-dimensional due to the massive number of reflecting elements. Currently, assuming that RIS elements are connected to RF chains, the channel estimation can be performed with acceptable performance through compressed sensing (CS) based methods. However, due to extremely low deployment, hardware and communication costs, purely passive RIS reflecting elements are undoubtedly more attractive.

By assuming active reflection patterns to achieve a smaller active array size to reduce hardware complexity, a conventional least squares method (LS) is proposed. In addition, by using the low-rank characteristics of the MIMO channel, the training overhead can be reduced through sparse matrix decomposition. Considering the sparse representation of cascaded channels, the CS method is proposed in [6]. Furthermore, as the number of antennas of the UE and BS are equipped with more antennas, the channel estimation complexity increases sharply. Using the angular-domain channel sparsity, a CS-based channel estimation scheme is proposed in [7]. However, the difference in structure sparsity between different channels will cause performance loss. Moreover, deep learning (DL) were proposed to predict the optimal RIS phase shift matrices [8], but it is still significant to get accurate CSI.

In the field of image denoising, the previous convolutional neural network (CNN) structure can construct a pair of training images by adding synthetic noise to the noise-free images [9]. Considering the similarity between image noise reduction and channel estimation, a deep residual learning approach was
As shown in Fig. 1, we consider one RIS, one controller, and one RIS in horizontal and vertical orientations. Defining $h$ for the mmWave RIS-aided MIMO system, we assume that the planar RIS is equipped with $N_u$ antennas and $N_v$ user equipments (UEs) equipped with $N_u$ antennas for the mmWave RIS-aided MIMO system, we assume that the planar RIS is equipped with $N = N_u N_v$ passive reflecting elements, where $N_u$ and $N_v$ denote the number of unit elements for RIS in horizontal and vertical orientations. Defining $h_{r,u,k} \in \mathbb{C}^{N \times N_u}$, $h_{r,b} \in \mathbb{C}^{N \times N_u}$ as the channels from the $k$th UE to the RIS and the BS to the RIS, $h_{u,k,b} \in \mathbb{C}^{N_u \times N}$ as the direct channel between the $k$th UE and the BS, respectively. $\mathbb{C}^{M \times N}$ represents an $M \times N$ complex-valued matrix. Then we can express the received signal as

$$y = \sum_{k=1}^{K} \left( h_{r,b}^T \Phi_k h_{r,u,k} \Phi_k^T h_{u,k,b} + h_{u,k,b} \Phi_k^T \right) + n,$$  

where $n \sim \mathcal{CN}(0, \sigma_n^2 I)$ denotes the noise vector at the BS and $\sigma_n^2$ is the noise power at each antenna, $\Phi_k = [\phi_{k,1}, \phi_{k,2}, \ldots, \phi_{k,N_u}] \in \mathbb{C}^{r \times N_u}$ denotes pilot matrix for the $k$th UE, $\phi_{n,k} \in \mathbb{C}^{r \times 1}$ denotes the orthogonal pilot sequence sent by the $n$th antenna of $k$th UE ($\phi_{k,j}^H \Phi_k = 0$, if $k_1 \neq k_2$ or $i \neq j$; $\phi_{k,j}^H \Phi_k = 1$, if $k_1 = k_2$ and $i = j$, $\forall k_1, k_2 \in \{1, 2, \ldots, K\}$). For transmitting the pilots, all antennas of each UE adopt different pilot sequence. In particular, a pilot would only be allocated to one UE, resulting in a orthogonal pilot matrix. Considering a simple model where one or more users in each slot have different optimal RIS phase shift matrix. Therefore, the RIS phase shift matrix $\Psi_k$ represents the phase shift introduced by the RIS to the impinging signal from the transmitter in the $k$th time slot. In addition, $\Psi_k = \text{diag}\{\psi_k\} \in \mathbb{C}^{N \times N}$, with $\psi_k \in \mathbb{C}^{N \times 1}$ representing the effective RIS phase shifts of the RIS reflecting elements and its $n$th element is $\psi_{k,n} = \tau_n e^{j\phi_n}, \forall n \in \{1, 2, \ldots, N\}$. Without loss of generality, we can assume $\psi_k = 1$.

By exploiting $\Phi_k^T \Phi_k = I$ and for simplifying the designing and analysing of the channel estimation algorithms in this work, we assume that there is no direct link between UE and BS due to blockages or negligible receive power, then, the processed received signal of the $k$th UE at the BS is given by

$$y_k = y^T \Phi_k^* = h_{r,b}^T \Psi_k h_{r,u} + n \Phi_k^*.$$  

Since practical mmWave channels usually have limited number of scatters, a LoS is expected in RIS systems. The mmWave channel of the $k$th UE to the RIS and BS to the RIS are, respectively, given as

$$h_{r,u} = \sum_{l=1}^{L_r} z_l \alpha_{R,t} \left( \phi_{R,t,l} \phi_{R,t,l}^e \right) \alpha_{H} \left( \phi_{H,t,l} \phi_{H,t,l}^e \right), \quad (3)$$

$$h_{r,b} = \sum_{l=1}^{L_r} z_l \alpha_{R,b} \left( \phi_{R,b,l} \phi_{R,b,l}^e \right) \alpha_{H} \left( \phi_{H,b,l} \phi_{H,b,l}^e \right), \quad (4)$$

where $L \ll \min(N_{act}, N_u)$ denotes the number of multipaths, $z_l \in \mathbb{C}$ denotes the distance-dependent pathloss of the $l$th path, $\phi_{R,t,l} \phi_{R,t,l}^e$ denotes the elevation (azimuth) angle-of-arrival of the $l$th path for both $h_{R,t}$ and $h_{R,b}$. The steering vectors depend on the array geometry. For the typical channel $h_{T,k,b}$ and $h_{T,k,b}$, variables $\alpha_{R,t} \phi_{R,t,l} \phi_{R,t,l}^e \in \mathbb{C}^{N_u \times 1}$ and $\alpha_{T} \phi_{T,l} \phi_{T,l}^e \in \mathbb{C}^{N_v \times 1}$ are given by

$$\alpha_{R,t} \left( \phi_{R,t,l} \phi_{R,t,l}^e \right) = \left[ 1, e^{2\pi k d \sin \phi_{R,t,l} \sin \phi_{R,t,l}^e / \lambda}, \ldots, e^{2\pi d (N_u - 1) \sin \phi_{R,t,l} \sin \phi_{R,t,l}^e / \lambda} \right] \otimes \left[ 1, e^{2\pi k d \cos \phi_{R,t,l} / \lambda}, \ldots, (5) \right]$$

$$\alpha_{T} \left( \phi_{T,l} \phi_{T,l}^e \right) = \left[ 1, e^{2\pi d \sin \phi_{T,l} \sin \phi_{T,l}^e / \lambda}, \ldots, e^{2\pi d (N_v - 1) \sin \phi_{T,l} \sin \phi_{T,l}^e / \lambda} \right] \otimes \left[ 1, e^{2\pi \cos \phi_{T,l} \phi_{T,l}^e / \lambda}, \ldots, (6) \right]$$

where $\lambda$ denotes the wavelength, $d$ denotes the antenna spacing, and $\otimes$ is the Kronecker product. $\phi_{T,l}^e$ and $\phi_{R,t,l}^e$ denote the elevation (azimuth) angle-of-departure of the $l$th path for both $h_{T,k,b}$ and $h_{T,k,b}$. $\alpha_{R,t} \phi_{R,t,l} \phi_{R,t,l}^e$, $\alpha_{T} \phi_{T,l} \phi_{T,l}^e$, and $\alpha_{R,t} \phi_{R,t,l}$ denote the steering vectors at the sender side and the receive side, respectively.
III. PROPOSED CHANNEL ESTIMATION METHODS

In this section, we introduce CBDNet, GAN-CBD and MRDN for the cascaded channel estimation of RIS systems. Leveraging the sparsity of cascaded mmWave channel, we naturally introduce CBDNet-based method into cascaded channel estimation in line with previous works. And we use the GAN structure to improve the network structure. Specifically, the proposed method MRDN combines the application of residual dense network (RDN) structure and the convolutional block attention module (CBAM) [13], which serves as a building block and can obtain accurate CSI for the cascaded sparse channel. Compared with existing baseline schemes, MRDN can reduce the complexity of RIS hardware. In the following, we will show the CBDNet, GAN-CBD and MRDN structure channel estimator. In addition, x and z represent the input and output of the universal layers and networks, respectively, in this correspondence.

A. CBDNet-based Channel Estimator

\( D_{N_e} \) and \( D_{N_D} \) denote the noise level estimation sub-network and the non-blind denosing subnetwork, respectively. \( \Theta_E \) and \( \Theta_D \) are the network parameters of \( D_{N_e} \) and \( D_{N_D} \), respectively.

1) Basic Structure: Assuming that \( \ast \) denotes \( \text{Conv} \) function, as \( x \) and \( z \) are the input and output of the \( k \)th \( \text{Conv} \) layer, the mathematical deduction for convolutional layer is

\[
\mathbf{z} = W_k \ast \mathbf{x} + b_k,
\]

where the weight and bias matrices \( W_k \) and \( b_k \) are the \( k \)th \( \text{Conv} \) parameters. \( \mathbf{z} = c(x), \Theta_{k,c} = \{W_{k,c}, b_{k,c}\} \) for “\( \text{Conv} \)” layers, \( \mathbf{z} = s(x), \Theta_{k,s} = \{W_{k,s}, b_{k,s}\} \) for “\( \text{SoftMax} \)” layers. Assuming that \( \max \) denotes “\( \text{ReLU} \)” layer function, the mathematical deduction for “\( \text{ReLU} \)” layer is

\[
\mathbf{z} = \max(0, \mathbf{x}),
\]

count as \( \mathbf{z} = r(x) \) for “\( \text{ReLU} \)” layers.

2) Noise Level Estimation Subnetwork:

- Input Layer: As the real and imaginary parts of the received signal matrix \( \mathbf{y} \in \mathbb{C}^{N_b \times N_u} \) are independent at the BS, we first combine them into a super matrix \( \mathbf{Y} \in \mathbb{R}^{N_b \times 2N_u} \) as the input of \( D_{N_e} \). \( \mathbb{R}^{M \times N} \) represents an \( M \times N \) real-valued matrix.
- Convolutional sensing: The \( D_{N_e} \) consists of \( B \) \( \text{Conv} \) layers and \( K \) \( \text{SoftMax} \) layers. The recurrence relation of main body for \( D_{N_e} \) is

\[
\sigma = \mathcal{F}_E(\mathbf{Y}, \Theta_E) = c \circ \cdots \circ c \circ \cdots \circ s(\mathbf{Y}) = \left( (c)^{B_1} \circ (s)^K \right)(\mathbf{Y}),
\]

where the operator \( \circ \) denotes a function composition, \( \sigma \) denotes the noise level for the space-invariant AWGN, \( \mathbf{M} \in \mathbb{R}^{N_b \times 2N_u} \) is a uniform map where all elements are

\[
\sigma, \Theta_E = \{\Theta_{1,c}, \ldots, \Theta_{B_1,c}, \Theta_{1,s}, \ldots, \Theta_{K,s}\}.
\]

The \( \mathcal{F}_E : \mathbb{R}^{N_b \times 2N_u} \rightarrow \mathbb{R}^{1 \times 1} \) is the mapping function for \( D_{N_e} \).

3) Non-Blind Denosing Subnetwork:

- Input Layer: The \( D_{N_D} \) takes both \( \mathbf{Y} \) and \( \mathbf{M} \) as input to obtain the estimated channel \( \hat{\mathbf{H}} \).
- Residual Blocks: The \( D_{N_D} \) consists of \( B \) residual blocks \( c \circ b \circ r \), then, the recurrence relation of main body for \( D_{N_D} \) is

\[
\mathbf{H}_m = \mathcal{F}_D(\mathbf{Y}, \mathbf{M}, \Theta_D) = c \circ b \circ r \cdots c \circ b \circ r(\mathbf{Y}, \mathbf{M}) = (c \circ b \circ r)^B(\mathbf{Y}, \mathbf{M}).
\]

The middle output \( \mathbf{H}_m = \mathcal{F}_D(\mathbf{Y}, \mathbf{M}, \Theta_D) \), where \( \mathcal{F}_D : \mathbb{R}^{N_b \times 2N_u} \rightarrow \mathbb{R}^{N_b \times 2N_u} \) is the mapping function for stacking residual blocks.
- Output Layer: By reversing the combining, the middle output of \( D_{N_D} \) \( \mathbf{H}_m \in \mathbb{R}^{N_b \times 2N_u} \) produces the estimated channel matrix \( \hat{\mathbf{H}} \in \mathbb{C}^{N_b \times N_u} \).
- Loss Function: In asymmetric learning, the noise level is estimated to improve the loss function, to quantify the effectiveness of \( D_{N_D} \) criterion. The loss function is denoted as

\[
\mathcal{L}_{rec} = \frac{1}{\sigma} ||\hat{\mathbf{H}} - \mathbf{H}||^2
\]

Given the estimated noise level \( \sigma(\mathbf{Y}) \) and the truth \( \sigma(\mathbf{Y}_1) \), more penalty is incorporated into their MSE when \( \sigma(\mathbf{Y}) < \sigma(\mathbf{Y}_1) \).

B. GAN-based Channel Estimation

Motivated by the development of generative adversarial networks (GAN) structure technique, based on the previous CBDNet as our own generator subnetwork, we develop our own GAN-CBD for denoise modeling. The GAN paradigm generates samplers through training and fitting as CBDNet works, and the results of GAN-CBD network compare with and label results, making the discriminator \( D \) work well. Training \( D \) can distinguish the training examples from the samples generated by \( G \), and \( G \) undergoes the judgment of \( D \) to reduce the possibility of samples being misclassified.

1) Generator Network: In addition, in order to verify the effectiveness of GAN structure, we use CBDNet as the generator network. The \( G_{N_D} \) consists of \( B \) residual blocks. We have

\[
\hat{\mathbf{H}} = G_{\theta_d}(\mathbf{Y}, \mathbf{M}, \Theta_{\theta_d}) = c \circ b \circ r \cdots c \circ b \circ r(\mathbf{Y}, \mathbf{M}) = (c \circ b \circ r)^B(\mathbf{Y}, \mathbf{M}),
\]

where \( \theta_d : \mathbb{R}^{N_b \times 2N_u} \rightarrow \mathbb{C}^{N_b \times N_u} \) is the mapping function for the generator network. \( \mathbf{M} \in \mathbb{R}^{N_b \times 2N_u} \) is a uniform map from \( D_{N_e} \), \( \sigma = \mathcal{G}_c(\mathbf{Y}, \Theta_{\mathcal{G}_c}) \).

2) Discriminator Network: In the original formulation, the training procedure defines a continuous minimax game as

\[
\arg \min_D \arg \max_G \mathbb{E}[\log D(x)] + \mathbb{E}[\log(1 - D(G(n)))]
\]

where \( D \) is a function that maps \( \mathbb{R}^{N_b \times 2N_u} \) to the unit interval, and \( G \) is a function that maps a noise vector \( n \in \mathbb{R}^{N_b \times 2N_u} \) drawn from a simple distribution \( p(n) \), to the ambient space of the training data \( \mathbb{R}^{N_b \times 2N_u} \).

C. MRDN-based Channel Estimation

We define this feature concatenation part of RDN and CBAM in Fig. 2 and use it as a building module of MRDN.
Estimated Channel Matrix

for MRDN is

and output of the CBAM. The recurrence relation of CBAM

eter

where the weight and bias matrices consist the CBAM param-

2) Residual Dense Network Structure: RDN performs well

in addressing denoising image problems. Motivated by many

recent image restoration networks including RDN, we include

the global residual connection such that the network can focus

on learning the difference between the noisy and ground-truth

channel matrix. The main body of RDN have B layers. The

recurrence relation of main body for RDN is

F = g(F_{n-1}(Y), \cdots, F_1(Y), Y), \forall n \in \{2, \ldots, B\}. \quad (16)

3) Convolutional Block Attention Module:

z_{-1} = W_{-1,a} * x + b_{-1,a},

z_0 = \max(0, z_{-1}),

z_1 = W_{1,a} * z_0 + b_{1,a},

where the weight and bias matrices consist the CBAM param-

eter \Theta_a = \{W_{-1,a}, W_{1,a}, b_{-1,a}, b_{1,a}\}. x and z_1 are the input and output of the CBAM. The recurrence relation of CBAM

for MRDN is A(x) = c \circ r \circ c(x).

4) Input Layer: As the real and imaginary parts of the

received signal matrix y_k \in \mathbb{C}^{N_x \times N_u} are independent at the

RIS, we first combine them into a super matrix Y \in \mathbb{R}^{N_x \times 2N_u}. In this case, the channel matrix can be treated as a 2D image and the super matrix Y is the input of MRDN.

5) Multiple Residual Dense Network Structure: We take

advantages of novel ideas in RDN and RCAN as follows.

- RDN itself is an image restoration network, but we use it with modifications as a component of our network and construct a cascaded structure of N_R RDNs as our image denoising network.
- The recurrence relation of main body for RDN is

\[ M(x) = F_{n,N_R} \circ F_{n,N_R-1} \circ \cdots \circ F_{n,1}(x), \quad (20) \]

\[ F(x) = M \circ A(x) = F^{N_R}_n \circ A(x), \quad (21) \]

where the operator \circ denotes a function composition and

F^{N_R}_n denotes the N_R-fold product of F_n. The middle output \hat{H}_m = F(Y), where F : \mathbb{R}^{N_x \times 2N_u} \mapsto \mathbb{R}^{N_x \times 2N_u} is the mapping function for MRDN.
6) Computational Complexity Analysis: The computational complexity of the training phase in CBDNet is given by

\[ O(N^2K^2s(tL_dD_1^2 + L_eE_1^2)), \]  

where \( s \) donates the size of mini-batch, \( t \) donates the number of iterations, \( K^2 \) donates the size of kernels, \( L_d \) and \( L_e \) denote the number of “Conv” for DNN\(_D\) and DNN\(_E\), \( D_1 \) and \( E_1 \) denote the number of features for the \( l \)th layer of DNN\(_D\) and DNN\(_E\), respectively. The computational complexity of the training phase in GAN-CBD and MRDN are given by

\[ O(N^2K^2s(tL_gD_g,t^2 + L_gE_g,t^2 + L_aE_a,t^2)), \]  

and

\[ O(N^2K^2sL_m^2D_m^2). \]

IV. SIMULATION RESULT

We consider the RIS-aided mmWave massive MIMO system with 20 UEs, where \( N_b = 64 \), \( N_u = 32 \), \( N = 4096 \), \( L = 3 \) and \( d = \lambda/2 \). In terms of hardware, we use Intel Core i7-9700K @3.60GHz, 32 GB RAM and NVIDIA GeForce RTX 2080Ti to implement the above three models through PyTorch library. From the perspective of normalized mean square error (NMSE) performance, this section illustrates the pros and cons of the three proposed channel estimators in terms of structure. All simulation results are derived in PyCharm Community Edition (Python 3.8 environment). The training rate is set as 0.0001 for MRDN and 0.001 for CBDNet and GAN-CBD and the mini-batch size is 20 for all three methods. The training, validation, and testing sets include 16,000, 6,000, and 8,000 samples, respectively. The training, validation and testing sets for the three methods use the same data set samples. The number of RDN for MRDN, residual blocks for both CBDNet and GAN-CBD are 6 and 12, respectively. The MRDN has 80 features, CBDNet and GAN-CBD have 96 features. Note that NMSE is defined as

\[ \text{NMSE} = E \left( \frac{\| \hat{\mathbf{H}} - \mathbf{H} \|^2}{\| \mathbf{H} \|^2} \right). \]  

Figure 6 compares the NMSE performance of the three methods with different model capacities. We can find that the MRDN can achieve best NMSE performance and fastest convergence. Because the GAN-CBD brings the advantage of judging the network, it shows better performance than the CBDNet. The computational complexity of training and offline operation can be hugely reduced. Also, the robustness of the channel estimator to different scenarios is enhanced. The average running time of MRDN (in seconds) is 0.0075, while the CBDNet and the GAN-CBD are 0.0094 and 0.0098 respectively, the computational complexity of training and offline operations for the MRDN can reduced compared with the CBDNet and the GAN-CBD. However, for almost the same computational complexity, the GAN-CBD can achieve better NMSE performance and fast convergence compared with the CBDNet. But compared with the MRDN, the improvement of network structure is not significant.

Figure 5 compares the NMSE performance for different number of features and RDN. With more RDNs for the global residual dense connection, and more comprehensive perceptual fields, the MRDN with 80 features and 6 dense connections for RDN performs better. Consequently, the main challenge in accurately describing noise is the lack of observational dimensions and modeling capabilities of neural networks, such as features and layers.

V. CONCLUSION

We proposed the CBDNet, GAN-CBD and MRDN based cascaded channel estimators for RIS-aided mmWave massive MIMO communication systems. Utilizing the sparsity of the cascaded RIS channels and classic image processing techniques, we regard the channel matrix as a two-dimensional image. The proposed residual dense network structure can increase the flexibility of the overall network to obtain better generalization and fitting capabilities, while the advantages brought by the GAN structure are not significant. Compared with the previous generation method, based on the above advantages, the MRDN-based deep learning network is designed to estimate the cascaded RIS channels. The simulation results show that the performance of the proposed the MRDN estimator increases with the increase of the scale of the network structure under the same order of complexity as the CBDNet and the GAN-CBD.
APPENDIX A
FORWARD AND BACKWARD PROPAGATION

Assume that the weight matrix $W_i \in \mathbb{R}^{n_i+1 \times n_i}$ and the bias vector $b_i \in \mathbb{R}^{n_i+1}$ are the parameters at a “Conv” layer $c_i$. In the multilayer perceptron (MLP), we can explicitly write $c_i(x; W_i, b_i) = r_i(W_i \cdot x + b_i)$, where $r_i$ is an elementwise function and the definition for Conv is $r_i(x) = \max(0, x)$, as the first derivative is $r_i'(x) = H(x)$. For any $x_i \in \mathbb{R}^{n_i}$ and vectors $U_i \in \mathbb{R}^{n_i+1 \times n_i}$ in inner product space,

$$\nabla_{W_i} c_i(x_i) \cdot U_i = D\Psi_i(z_i) \cdot U_i \cdot x_i, \quad (26)$$

$$\nabla_{b_i} c_i(x_i) = D\Psi_i(z_i), \quad (27)$$

where $z_i = W_i \cdot x_i + b_i$, $\Psi(v) = \sum_{k=1}^{n_i} z_i(k) e_k$, and

$$Df(x; \theta) = \frac{d}{dt} f(x + tv; \theta) \bigg|_{t=0}. \quad (28)$$

The loss function gradients in MLP is

$$J(x, z; \theta) = \frac{1}{2}\|z - F(x; \theta)\|^2 = \frac{1}{2}\|z - F(x; \theta) - z - F(x; \theta)\|^2.$$ \quad (29)

Let $(x, z) \in E_3 \times E_{L+1}$ be a network input-output pair,

$$\nabla_{W_i} J(x, z; \theta) = \left[ \Psi_i'(z_i) \odot (D^* \omega_{i+1}(x_{i+1}) \cdot e) \right] x_i^T, \quad (30)$$

$$\nabla_{b_i} J(x, z; \theta) = \left[ \Psi_i'(z_i) \odot (D^* \omega_{i+1}(x_{i+1}) \cdot e) \right], \quad (31)$$

where $x_i = \alpha_{i-1}(x)$, and the prediction error is $e = F(x; \theta) - z$,

$$F(x; \theta) = (c_L \cdot \cdots \cdot c_1)(x), \quad (32)$$

for all $i \in [L]$ and $\theta \in \{W_i, b_i\}$,

$$\nabla_{\theta_i} J(x, z; \theta) = \nabla^*_{\theta_i} c_i(x_i) \cdot D^* \omega_{i+1}(x_{i+1}) \cdot e. \quad (33)$$

The generic layer of a CNN as a parameter-dependent map that takes as input an $m_1$-channeled tensor, where each channel is a matrix of size $m_1 \times \ell_1$, and outputs an $m_2$-channeled tensor, where each channel is a matrix of size $m_2 \times \ell_2$. The parameters $W \in \mathbb{R}^{n_i \times n_i} \otimes \mathbb{R}^{m_2}$, the input $x \in \mathbb{R}^{n_1 \times \ell_1} \otimes \mathbb{R}^{m_1}$, $\{e_j\}_{j=1}^{m_1}$ denotes an orthonormal basis for $\mathbb{R}^{m_1}$, and $\{e_j\}_{j=1}^{m_2}$ denotes an orthonormal basis for $\mathbb{R}^{m_2}$, the $x$ and $W$ is:

$$x = \sum_{j=1}^{m_1} x_j \otimes e_j, \quad W = \sum_{j=1}^{m_2} W_j \otimes \hat{e}_j. \quad (34)$$

And the convolution operator $C$ can be written as

$$C(W, x) = \sum_{j=1}^{m_2} c_j(W, x) \otimes \hat{e}_j, \quad (35)$$

where $c_j$ is a bilinear operator that defines the mechanics of the convolution:

$$c_j(W, x) = \sum_{k=1}^{n_1} E_k \cdot \sum_{l=1}^{n_i} \langle W_j, K_{\gamma(k, l, \Delta)}(x) \rangle \cdot \hat{E}_{k, l}, \quad (36)$$

where $K$ is the cropping operator that defines the action of convolution, $\gamma(k, l, \Delta) = (1 + (k - 1)\Delta, 1 + (l - 1)\Delta)$, $\Delta$ defines the stride of the Conv. The generic layer $c_i$ is

$$c_i(x_i) = \Psi_i(C_i(W_i, x_i)), \quad (37)$$

For the “Conv” layer,

$$\nabla_{W_i} c_i(x_i) = (\Psi_i, x_i) \cdot D\Psi_i(C_i(W_i, x_i)). \quad (38)$$

$$D^* c_i(x_i) = (W_i, c_i) \cdot D\Psi_i(C_i(W_i, x_i)) \cdot D^* \Psi_i(C_i(W_i, x_i)). \quad (39)$$

where $(e_1, B) \cdot B = (e_1, e_2)$, and $(B, e_2) \cdot e_1 = B(e_1, e_2)$. The learning rate $\eta \in \mathbb{R}_+$, the gradient descent step algorithm update the parameter for backpropagation is

$$\nabla_{W_i} J(x, z; \theta) \leftarrow (C_i(x_i) \cdot D\Psi_i(C_i(W_i, x_i))) e_i, \quad (40)$$

the parameters can be updated by $W_i \leftarrow W_i - \eta \nabla_{W_i} J(x, z; \theta)$. Due to the application of the derivative chain rule and error backpropagation, the high-dimensional neural network demonstrates excellent results.

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