Numerical simulation of the exposed area of high-altitude electromagnetic pulse on the ground and its influencing factors

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Abstract. In traditional High-Altitude Electromagnetic Pulse (HEMP) research, the line-of-sight area on the earth’s surface of the burst is approximately regarded as the exposed area of HEMP on the ground. However, the line-of-sight area is only affected by the height of burst, and cannot reflect the influence of yield and the distribution of peak HEMP fields on the ground. In this paper, the ground distance between ground zero and the southernmost point of peak HEMP field contour is defined as south radius for explosions in the northern hemisphere. The numerical simulation results of south radius of different peak HEMP field contours are given, and the impact of burst height and yield on south radius is analysed. It is concluded that when the yield is fixed, the south radius increases first and then decreases as the burst height increases. When the burst height is fixed, the south radius increases as the yield increases, but the rate of increase keeps decreasing.

1. Introduction

HEMP is one of the important damaging effects of high-altitude nuclear explosions, which has the characteristics of large amplitude, short rise time, short duration, and wide exposed area. HEMP can couple into electronic systems through the gaps and holes on the cable, antenna, and metal shield, causing interference or damage to electronic systems [1]. Therefore, it attracts the attention of researchers.

The magnetic dipole model of HEMP was proposed by Longmire [2], Kazas and Latter [3-4], which explains the production mechanism of HEMP and has been widely accepted. Based on this theory, a series of studies have been carried out, which generally focus on peak HEMP field, rise time, half-height width, and the variations of these characteristic parameters with explosion conditions, such as the burst height, yield, and latitude [5-7]. While these studies pay little attention to the exposed area of HEMP. However, the exposed area of HEMP is an important factor that needs to be considered in the evaluation of the damage capability of HEMP and the evaluation of electromagnetic protection capability of critical infrastructure.

In traditional research, researchers approximately regard the line-of-sight area on the earth’s surface of the burst as the exposed area of HEMP on the ground. However, the line-of-sight area can
only reflect the influence of yield, but cannot reflect the influence of yield and the distribution of peak HEMP fields on the ground. The peak HEMP fields observed in the line-of-sight area from a high-altitude explosion varies significantly. The peak field at the edge of the line-of-sight area may be close to zero, and its damage capability to electronic systems is weak. Therefore, the exposed area bounded by a specified peak HEMP field contour can more effectively reflect the damage capability of HEMP. For explosions in the northern hemisphere, the ground distance between ground zero and the southernmost point of peak HEMP field contour is defined as south radius, which is selected as a characteristic parameter of the exposed area. Refs. [8] and [9] give the relationship between the exposed area and the burst height, but they only give the results in specific explosion scenes without systematic calculation and analysis.

In this paper, the numerical simulation results of south radius of different peak HEMP field contours are given, and the impact of burst height and yield on south radius is analyzed. The results can provide references for the HEMP protection design of electronic systems.

2. Physical model and method

Figure 1 shows the production process of HEMP. The γ-rays released by a high-altitude nuclear explosion interact with atmosphere atoms at the altitude of 20~40 km, generating Compton electrons. The recoil electrons deflect in the geomagnetic field and constitute Compton current, stimulating electromagnetic pulses. At the same time, the high-energy recoil electrons ionize the air and produce low-energy secondary electrons. The secondary electrons drift in the induced electric field, which limits electromagnetic pulses.

![Figure 1. Production process of HEMP.](image)

For γ-rays emitted from the burst, the number density at \( r \) (vector from the burst point to source point) is,

\[
N_\gamma (r) = N_0 \frac{\exp(-\int_0^r \frac{dr'}{\lambda(r')})}{4\pi r^2}
\]

where \( N_0 \) is the total number of the prompt γ-rays released from the burst and \( \lambda \) is the mean free path of γ-rays. The production rate of Compton electrons is,

\[
\frac{dN_c}{dt} (r,t) = \frac{N_\gamma (r)}{\lambda(r)} f(t)
\]

where \( N_c \) is the number density of the Compton electrons and \( f(t) \) is the time history of γ-rays. The formula of Compton current is [4],
\[ J(r, \tau) \approx -e \int_0^{R/v_0} v(\tau') \times \frac{dN_0}{dt}(r, \tau - \tau' + \frac{x(\tau')}{c}) \, d\tau' \]  

(3)

where \( v(\tau') \), \( x(\tau') \), \( R \), and \( v_0 \) are the velocity vector, the distance traveling along the LOS, the range, and the initial speed of the Compton electron, respectively. \( e \) is the electron charge and \( c \) is the speed of light.

The number density of conduction electrons is identified as \( N_e \), satisfying the equations of equilibrium model,

\[
\frac{dN_e}{dt} = S - k_1 N_e - k_2 N_e N_+ - k_3 N_e N_{-} 
\]  

(4)

\[
\frac{dN_+}{dt} = S - k_2 N_e N_+ - k_1 N_+ N_{-} 
\]  

(5)

\[
\frac{dN_{-}}{dt} = k_1 N_+ - k_3 N_e N_{-} 
\]  

(6)

where \( S \) is the production rate of conduction electrons, \( N_+ \) and \( N_{-} \) are the number densities of positive ions and negative ions. \( k_1, k_2, \) and \( k_3 \) are the rate constants for the attachment process, dissociative recombination, and mutual neutralization, respectively.

The air conductivity is

\[
\sigma = e(N_e \mu_+ + N_+ \mu_e + N_{-} \mu_+) 
\]  

(7)

where \( \mu_e, \mu_+, \) and \( \mu_{-} \) are the mobilities of electrons, positive ions, and negative ions, respectively. The conductivity model and parameters used in equations (4)-(7) are taken from [10].

The total current density is the sum of the Compton current and conduction current density,

\[ J = J_c + \sigma E \]  

(8)

The differential form of Maxwell’s equations are,

\[
\nabla \times E = -\frac{\partial B}{\partial t} 
\]  

(9)

\[
\nabla \times B = \varepsilon \mu \frac{\partial E}{\partial t} + \mu J 
\]  

(10)

where \( B \) is the magnetic induction, \( E \) is the electric field, \( J \) is the total current density, \( \varepsilon \) and \( \mu \) are the permittivity and permeability of free space.

Introducing the transformations to the retarded time \( \tau = t - r/c \), we have

\[
\nabla \times E - \frac{1}{c} e_r \times \frac{\partial E}{\partial \tau} = -\frac{\partial B}{\partial \tau} 
\]  

(11)

\[
\nabla \times B - \frac{1}{c} e_r \times \frac{\partial B}{\partial \tau} = \varepsilon \mu \frac{\partial E}{\partial \tau} + \mu J 
\]  

(12)

where \( t \) is the real time and \( e_r \) is a radial unit vector from the burst center.

Based on a high-frequency approximation [4], Maxwell’s equations reduce to one-dimensional (along the LOS) differential equations,

\[
\frac{1}{r} \frac{\partial}{\partial r} (rE_r) = \frac{\mu c}{2} J_\phi 
\]  

(13)

\[
\frac{1}{r} \frac{\partial}{\partial r} (rE_\phi) = -\frac{\mu c}{2} J_r 
\]  

(14)

\[
\frac{1}{r} \frac{\partial}{\partial r} (rB_\phi) = -\frac{1}{2\varepsilon c^2} J_r 
\]  

(15)

\[
\frac{1}{r} \frac{\partial}{\partial r} (rB_r) = \frac{1}{2\varepsilon c^2} J_\phi 
\]  

(16)
3. Results and discussion

We choose a gamma source with energy of 1.66 MeV and a double-exponential time history \( f(t) \) as

\[
f(t) = \frac{\alpha \beta}{\beta - \alpha} \left( e^{-\alpha t} - e^{-\beta t} \right)
\]

\( \alpha = 4 \times 10^6 \text{s}^{-1}, \beta = 4.76 \times 10^8 \text{s}^{-1} \). The geomagnetic field \( B_0 \) is set as 0.5 Gauss in the north direction and the dip angle of \( B_0 \) is 60°.

3.1. Peak HEMP field contours on the ground

Figure 2 shows the definition of tangent radius identified as \( R_t \) and \( R_s \) as a function of the height of burst (HOB) identified as \( H \) [11],

\[
R_s \approx 110 \sqrt{H} \text{(km) for } H \leq 500 \text{ km}
\]

![Figure 2. Definition of tangent radius and tangent radius as a function of the height of burst. [11]](image)

In the simulation, the above-mentioned model is used to calculate HEMP. The peak HEMP field distribution on the ground within the tangent radius is shown in Figure 3 at the burst height of 75 km and 200 km and the yield of 100 kt and 1000 kt. For nuclear explosions in the northern hemisphere, the maximum peak electric field occurs south of ground zero, and the minimum peak electric field occurs north of ground zero, which is consistent with the IEC standard [11] and verifies the correctness of the simulation program. Figure 3 also shows the peak electric field contours of 20 kV/m, 15 kV/m, 10 kV/m and 5 kV/m, respectively. In order to quantitatively analyze, south radius \( R_s(E_{th}) \), the ground distance between ground zero and the southernmost point of peak HEMP field contour of the specific peak electric field value \( E_{th} \), is used as a characteristic parameter of the size of the exposed area. According to the geometric relationship, the maximum value of \( R_s \) is \( R_t \). Thus the relation \( R_s \leq R_t \) always holds.

When the burst height is 75 km and the yield is 1000 kt, the minimum peak electric field within the line-of-sight area is still greater than 5 kV/m. As a result, there is no peak electric field contour of 5 kV/m in Figure 3(b). When the burst height is 200 km and the yield is 100 kt, the maximum peak electric field within the line-of-sight area is still less than 15 kV/m. As a consequence, there is no peak electric field contour of 15 kV/m and 20 kV/m in Figure 3(c). According to Figure 3(b) and Figure 3(d), when the yield is 1000 kt, the maximum peak electric field on the ground at the burst height of 75 km is greater than that at the burst height of 200 km. While the south radius \( R_s(E_{th}=5 \text{ kV/m}) \) of a 200 km burst is greater than that of a 75 km burst.
Figure 3. Peak electric field contours of 20 kV/m (red dash dot line), 15 kV/m (red dot line), 10 kV/m (red dash line) and 5 kV/m (red solid line).

3.2. Effect of burst height

For a fixed yield $Y$ and a fixed threshold of peak electric field $E_{\text{th}}$, when the burst height $H$ varies between 60 km and 400 km, the variation of the south radius $R_s(E_{\text{th}})$ with the burst height is shown in Figure 4. In general, south radius $R_s$ increases first and then decreases as burst height increases. With the increase of burst height, there are two competing factors affecting the south radius. One is the increase of the line-of-sight area and the other is the decrease of the peak HEMP field on the ground. When the burst height is low, with the increase of the burst height, the increase of the line-of-sight area is the dominant factor. At this time, the south radius $R_s$ increases with the increase of the burst height. When the burst height is high, with the increase of the burst height, the decrease of the peak HEMP field on the ground is the dominant factor. At this time, the south radius $R_s$ decreases with the increase of the burst height.

As shown in Figure 4, the variation of $R_s(E_{\text{th}})$ with the burst height is also related to the yield $Y$ and the threshold of peak electric field $E_{\text{th}}$. In the case of a small yield explosion, with the increase of the burst height, the peak HEMP field on the ground decreases significantly and the south radius $R_s(E_{\text{th}})$ tends to decrease. While in the case of a large yield explosion, the south radius $R_s(E_{\text{th}})$ increases first and then decreases as the burst height increases. When the yield is large enough, the peak electric field at the edge of line-of-sight area is still greater than the threshold $E_{\text{th}}$, and the south radius $R_s(E_{\text{th}})$ is equal to the tangent radius $R_t$. At this time, the south radius increases with the increase of the burst height. For a fixed yield, the optimal burst height, which maximizes the south radius, increases with
the decrease of $E_{th}$. For a nuclear explosion, the south radius increases with the decrease of $E_{th}$ until it is equal to the tangent radius.

![Figure 4](image-url)

**Figure 4.** South radius varying with burst height for different yields.

### 3.3. Effect of yield

For a fixed burst height $H$ and a fixed threshold of peak electric field $E_{th}$, when the yield $Y$ varies between 50 kt and 1000 kt, the variation of the south radius $R_s(E_{th})$ with the yield is shown in Figure 5. The south radius $R_s$ increases with the increase of the yield, for the reason that the peak HEMP field on the ground increases as the yield increases. While the increase rate of south radius keeps decreasing with the increase of the yield, until $R_s(E_{th})$ is equal to $R_t$. The lower the burst height, the more obvious the decreasing trend of increase rate. This is because the HEMP field has a saturation effect as the yield increases, and the lower the burst height, the greater the HEMP field strength for the same yield and the more significant the saturation effect. For a fixed burst height, the optimal yield, the minimum yield which maximizes the south radius, decreases with the decrease of $E_{th}$. 
Figure 5. South radius varying with yield for different burst heights.

4. Conclusion
In this paper, the variations of the exposed area of HEMP on the ground with burst height and yield at different thresholds of peak electric field are systematically presented. In order to quantitatively analyze, the ground distance between ground zero and the southernmost point of peak HEMP field contour is defined as south radius for explosions in the northern hemisphere. The results of numerical simulation indicate that the south radius tends to increase first and then decrease as burst height increases for a fixed yield. When the yield is large enough, the peak electric field at the edge of the line-of-sight area is still greater than the threshold, and the south radius is equal to the tangent radius. At this time, the south radius increases with the increase of the burst height. For a fixed height of burst, the south radius increases with the increase of the yield. While the increase rate of south radius keeps decreasing with the increase of the yield, until the south radius is equal to the tangent radius. The lower the burst height, the more obvious the decreasing trend of increase rate for the reason that HEMP field has a saturation effect as the yield increases. For a fixed yield, the optimal burst height, which maximizes the south radius, increases with the decrease of the threshold of peak field. While for a fixed burst height, the optimal yield, the minimum yield which maximizes the south radius, decreases with the decrease of the threshold of peak field. For a nuclear explosion, the south radius increases with the decrease of the threshold of peak field until it is equal to the tangent radius. The results can be useful for the HEMP protection of electronic systems.
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