Mathematical Model of Rotary Machined Helical Surfaces

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Abstract. The rotary cutting method of materials has a number of advantages over the existing traditional cutting methods, e.g. temperature decrease in the cutting zone, also noncumulative blade wear. Due to its high durability, the rotary tool allows processing hardened and difficult-to-machine materials, high-temperature alloys, as well as composite and laminated materials. However, this machining method is usually not applied for machining various shaped surfaces, which is mainly due to the lack of mathematical calculation of the resulting profiles, and the absence of a wide variety of methods for rotary tools installation. The article discusses the mathematical foundation of the resulting profile when processing helical surfaces when processing the flanks of rotary tools.

1 Introduction

Rotary machining has found a wide application at the beginning of the 20th century [1,2,3]. The research on rotary machining took place mainly in the second half of the 20th century, with the concept of rotary cutting (RC) [4,5,6], which described the machinability of simple-shaped surfaces, primarily cylindrical, consequently flat in difficult-to-machine materials [7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. The greatest contribution to the development of RC was made in the work [17], which described the machinability of shaped surfaces located on a flat plane by means of RC. The further development of RC proceeded in accordance with the advanced machining methods, e.g. the usage of NC machine tools [18, 19, 20, 21, 22]. However, no research on the obtaining of helical surfaces by means of RC has been conducted thus far.

Two basic RC techniques have been known by now, having a cup-shaped cutting element located either end up (the first technique) or laterally (the second technique) to the machined surface [23]. In this case, the second technique provides a greater number of different positions than the first one. Besides, there are a number of advantages of the second technique over the first one, viz.: no restricted size of bearing assemblies related to the cutting element diameter; a more rigid and reliable support for the cutter; the roughness of 2.5 to 0.63 Ra obtained by high feed speed cutting, due to the larger radius of curvature of the active cutting edge [23]. As for the disadvantages of the technique, the main one is the difficulty of the design of a rigid structure allowing the adjustment of the cutting element towards the workpiece.

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Mathematical simulation of the machined helical profile by means of RC allows establishing the relationship between the rotary tool installation and the resulting real workpiece profile. The existing mathematical research on rotary cutters is mainly aimed at the simulation of both cut layers and the active cutting edges of rotary tools [24, 25]. However, no information on current profound research on the rotary tool machining shaped profiles has been provided thus far. Therefore, recently, mathematical simulation of helical surfaces obtained by tools with renewable cutting edges has become urgent.

2 Mathematical model of helical surfaces machined by rotary tools

The profile of the machined helical surface does not match the profile of the machining tool. Such discrepancy is explainable by the fact that any helical surface is obtained by the envelope of a number of successive positions of the moving machining tool. Therefore, there must be established the relation of the tool-fixed coordinate system with the workpiece-fixed coordinate system, by means of successive transitions from one coordinate system to another.

Let us consider a mathematical model of the machined helical profile when installing the cutting element of a rotary tool at two angles (Fig. 1). In this model, let us introduce a workpiece-fixed coordinate system X1Y1Z1 (see Fig. 1, a). The origin of this coordinate system O1 is located on the workpiece axis. The X1 axis is directed along the workpiece axis, whereas the Y1 axis is directed along the centre distance line of the workpiece and the cutting cup. In the initial position, the cutting element axis is perpendicular to the workpiece axis. Subsequently, let us introduce a tool-fixed coordinate system X2Y2Z2. The origin of this coordinate system O1 is located on the cutting element, in the plane where the end of the largest radius is located. The Z2 axis is directed along the axis of the cutting element rotation, and the Y2 axis is directed along the Y1 axis. To form the angle of the cutting element rotation, let us rotate it, together with the X2Y2Z2 coordinate system, around the Y2 axis through an angle θ. After rotating, the cutting element, together with its coordinate system, takes the position X2'Y2'Z2' (see Fig. 1, b). Consequently, to set the angle of inclination of the cutting element in relation to the workpiece axis, let us rotate the cutting element through an angle ß. The rotation through the angle ß is performed by rotating the cutting element together with the X2'Y2'Z2' coordinate system around the X2' axis. Let us fix the X2''Y2''Z2'' coordinate system with this position.

To establish the relation between the tool-fixed coordinate system and the workpiece-fixed coordinate system, let us sequentially perform the transitions between these coordinate systems. Initially, let us establish the cutter-fixed coordinate system:

\[
\begin{align*}
X'_1 &= X_2 \\
Z'_2 &= Z'_2 \cos \beta + Y'_2 \sin \beta \\
Y'_2 &= Y'_2 \cos \beta - Z'_2 \sin \beta \\
X'_2 &= X'_2 \cos \theta - Z'_2 \sin \theta \\
Z' = Z'_2 \cos \theta + X'_2 \sin \theta \\
Y'_2 &= Y'_2
\end{align*}
\]  

(1)
While forming a helical profile, the cutting element and the workpiece, together with their coordinate systems, make a relative helical motion. When the workpiece-fixed coordinate system $X_1 Y_1 Z_1$ is rotated around the $X_1$ axis through an angle $\varphi$ (see Fig. 1, e), the $X_2 Y_2 Z_2$ tool-fixed coordinate system will move by the value $P \cdot \varphi$ (where $P$ is the helical parameter, and $\varphi$ is the angle of rotation taken in radians). The helical parameter itself is calculated from the well-known formula ($H$ is the pitch of the helical surface, mm per rotations).

Considering that the tool-fixed coordinate system is located at a distance $a$ from the workpiece-fixed coordinate system, we have the following:

$$
\begin{align*}
X_i &= X_2 - P \varphi \\
Z_i &= Z_2 \cos \varphi - (Y_2 - a) \sin \varphi \\
Y_i &= (Y_2 - a) \cos \varphi + Z_2 \sin \varphi
\end{align*}
$$

(3)

Considering the above-given equations (1) and (2), we obtain the final relation between the coordinates of the cutting element and the workpiece:

$$
\begin{align*}
X_i &= X'_2 \cos \theta - (Z'_2 \cos \beta + Y'_2 \sin \beta) \sin \theta - P \varphi \\
Z_i &= ((Z'_2 \cos \beta + Y'_2 \sin \beta) \cos \theta + X'_2 \sin \theta) \cos \varphi - (Y'_2 \cos \beta - Z'_2 \sin \beta - a) \sin \varphi \\
Y_i &= (Y'_2 \cos \beta - Z'_2 \sin \beta - a) \cos \varphi + ((Z'_2 \cos \beta + Y'_2 \sin \beta) \cos \theta + X'_2 \sin \theta) \sin \varphi
\end{align*}
$$

(4)
Let us give a calculation for a specific point of the cutting edge of the rotary tool. When rotating through an angle $\beta$ (Fig. 2), the cutting element will move to the distance $b = r \cdot \sin \beta$, while the cutting tip will rise above the centre distance line by $h = b \cdot \sin \alpha$, or $h = r \cdot \sin \beta \cdot \sin(90 - \theta)$.

![Design scheme of the coordinates of the cutter tip point.](image)

Fig. 2. Design scheme of the coordinates of the cutter tip point.

The coordinates of the calculated point along the $Z_1$, $X_1$, and $Y_1$ axes in the workpiece-fixed coordinate system are as follows:

\[
Z_i = h = r \cdot \sin \beta \cdot \sin(90 - \theta) ;
\]
\[
X_i = -b \cdot \cos \alpha = -r \cdot \sin \beta \cdot \cos(90 - \theta)
\]
\[
Y_i = -(R \cdot \cos \gamma - t) = -(R \cdot \cos(\arcsin h / R) - t)
\]

Before calculating the coordinates, let us calculate the parameter $a$ for this case:

\[
a = R + r - l - t ,
\]

where $l$ is the displacement of the cutting element providing its contact with the workpiece, mm.

When rotated through an angle $\beta$, the cutting element shifts from the workpiece at the distance $l = c + k$. Based on the design diagram in Fig. 2, the following values are obtained: $k = r - r \cdot \cos \beta$, $c = R - R \cdot \cos \gamma$, $\gamma = \arcsin(h / R)$, and consequently the distance $l = R - R \cdot \cos(\arcsin(h / R)) + r - r \cdot \cos \beta$ is obtained. Finally, the following result is obtained:

\[
a = R \cos \left( \arcsin \left( \frac{r \cdot \sin \beta \cdot \sin(90 - \theta)}{R} \right) \right) + r \cos \beta - t
\]

(5)
The obtained profile coordinates and the centre distance allows further calculation of parametrized workpiece profiles.

4 Conclusions

The calculated mathematical model for the formation of a helical surface with the lateral surface of the cutting element of a rotary tool allows obtaining the coordinates of the helical profile points on the workpiece, with the consideration of the workpiece parameters, the tool parameters and tool-and-workpiece installation. The calculations of the machined helical profiles allows geometric assessment of the mathematical model. The installation parameters of the rotary cutter influence differently on the resulting profile. The angle of rotation of the cutting element axis relative to the feed vector has a significant influence on the curvature and the width of the helical groove, whereas the radius of the groove depends on the angle of inclination of the cutting element relative to the workpiece axis.

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