Dynamical Coupling of Pygmy and Giant Resonances in Relativistic Coulomb Excitation

N. Brady, 1,2 T. Aumann, 2,3 C.A. Bertulani, 1,4 and J. Thomas 1,2
1Department of Physics and Astronomy, Texas A & M University-Commerce, Commerce, Texas 75429, USA
2Institut für Kernphysik, Technische Universität Darmstadt, Schlossgartenstr. 9, 64289 Darmstadt, Germany
3GSI Helmholtzzentrum für Schwerionenforschung, Planckstr. 1, 64291 Darmstadt, Germany
4Department of Physics and Astronomy, Texas A & M University, College Station, Texas 77843, USA
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We study the Coulomb excitation of pygmy dipole resonances (PDR) in heavy ion reactions at 100 MeV/nucleon and above. The reactions $^{68}$Ni+$^{197}$Au and $^{68}$Ni+$^{208}$Pb are taken as a practical examples. Our goal is to address the question of the influence of giant resonances on the PDR as the dynamics of the collision evolves. We show that the coupling to the giant resonances affects considerably the excitation probabilities of the PDR, a result that indicates the need of an improved theoretical treatment of the reaction dynamics at these bombarding energies.

The existence of collective vibrations in neutron-rich nuclei at low energies was suggested by Kubono, Nomura, and collaborators in a 1987 proposal for the Japanese Hadron project which eventually became the J-PARC facility [1]. This proposal was later given theoretical support by Ikeda [2] and collaborators. It took nearly two decades for experimental evidences of the existence of a collective low energy response to be found in neutron-rich nuclei, far from the valley of stability. It is worth mentioning that direct breakup of light and loosely-bound projectiles, such as $^{11}$Be and $^{11}$Li were initially thought to be indicative of a collective nuclear response but it was shown to be a direct Coulomb dissociation of the weakly-bound valence nucleons [3]. The fragmentation of the nuclear response in the low-lying energy is now called Pygmy Dipole Resonances (PDR) [4]. The energy spectrum is typically obtained with the experimental probe of choice, i.e., relativistic Coulomb excitation of projectiles produced and accelerated in radioactive beam facilities (for related reviews, see Refs. [5, 6]). In such a process, the identification of pygmy resonances is done via their decay modes, usually via gamma or neutron emission, and the energy spectrum is obtained by invariant mass reconstruction from the energy of the fragments [4]. PDRs are typically interpreted as due to the oscillation of the excess neutrons against a more tightly bound core.

Theories for giant resonances date back to when a simple hydrodynamical interpretation of protons oscillating against the neutrons was used [7,8]. Later on microscopic calculations were developed based on the linear response theory [9]. Nowadays, an effort is being undertaken to describe nuclear collective motion with more elaborated models such as the time-dependent superfluid local density approximation [10][11]. Similarly, theoretical studies of the pygmy resonances have been developed based on the improvements of the hydrodynamical model [12][13], and with microscopic theories such as the random phase approximation (RPA) and its variants [15][16]. When re-
The study of the PDR. This has been observed in the past in the context of the excitation of double giant dipole resonances (DGDR) [24–26]. The observation of the DGDR in experiments is a consequence of higher-order effects in relativistic Coulomb excitation and arises because the large excitation probabilities of giant resonances in heavy ion collisions at small impact parameters. The dynamical coupling between the usual giant resonances and the DGDR is very strong, as shown, e.g., in Ref. [27]. In the present work we make an assessment of this effect on the excitation of the PDR using the relativistic coupled channels (RCC) equations introduced in Ref. [28]. As we do not want to dwell with the microscopic properties of the pygmy and giant resonances, we will assume that they are described by Lorentzian functions centered around the GDR and GQR resonances.

The S-matrix, \( S_\alpha(z, b) \), for Coulomb excitation is obtained from the RCC equations [28]

\[
iv \frac{\partial S_\alpha(z, b)}{\partial z} = \sum_{\alpha'} \langle \alpha | O_{EL} | \alpha' \rangle S_{\alpha'}(z, b) e^{-i(E_{\alpha'} - E_\alpha)z/\hbar},
\]

where \( v \) is the projectile velocity and \( O_{EL} \) is the electromagnetic operator for electric dipole (E1) and quadrupole (E2) transitions connecting states \( \alpha \) and \( \alpha' \) satisfying the selection rules of their intrinsic angular momenta and parities. The ground state is denoted by \( |0\rangle = |E_0 J_0 M_0\rangle \) and the excited states by \( |\alpha\rangle = |E_\alpha J_\alpha M_\alpha\rangle \), where \( |EJM\rangle \) labels intrinsic energy and angular momentum quantum numbers. In the long-wavelength approximation the electromagnetic operators are given by [28]

\[
O_{E1m} = \sqrt{2\pi} \frac{3}{(b^2 + \gamma z^2)^{3/2}} \left\{ \begin{array}{ll}
+ b & \text{if} \ m = \pm 1 \\
\mp 2 \gamma z & \text{if} \ m = 0
\end{array} \right.
\]

(3)

where \( \xi \) is the intrinsic coordinate of the excited nucleus and \( Ze \) is the charge of the nucleus giving rise to the electromagnetic field (in our case, the target). For E2 transitions the electromagnetic operator is [28]

\[
O_{E2\mu} = \sqrt{2\pi} \frac{3\pi \xi^2 Y_2(\xi)}{10} \frac{\gamma Z e^2}{(b^2 + \gamma z^2)^{5/2}} \left\{ \begin{array}{ll}
\frac{b^2}{2} & \text{if} \ m = \pm 2 \\
\pm 2 \gamma z & \text{if} \ m = \pm 1 \\
\sqrt{2/3} \left(2\gamma z^2 - b^2\right) & \text{if} \ m = 0
\end{array} \right.
\]

(4)

The coupled equations \([3]\) are solved by using \( S_\alpha(z \rightarrow -\infty) = \delta_{\alpha 0} \). For high energies and very forward angles, the cross sections for the \( |0\rangle \rightarrow |\alpha\rangle \) transition is given by

\[
\sigma_\alpha = 2\pi \int db \exp[-2\chi(b)] |S_\alpha(z \rightarrow \infty, b)|^2,
\]

where \( b \) is the impact parameter in the collision, and \( \chi(b) \) is the eikonal absorption phase given by

\[
\chi(b) = \frac{\sigma_{NN}}{4\pi} \int dq q \rho_1(q) \rho_2(q) J_0(qb),
\]

(6)

with \( \sigma_{NN} \) the nucleon-nucleon cross section with medium corrections [29–31] and \( \rho_1(q) \) is the Fourier transform of

![FIG. 1. Strength function for the E1 RPA response in $^{68}$Ni calculated with formalism described in Ref. [22]. The calculation is performed for several Skyrme interactions, shown in the figure inset. The arrow shows the location of the pygmy resonance.](image)

![FIG. 2. Coulomb excitation cross section as a function of the excitation energy of 600 MeV/nucleon $^{68}$Ni projectiles incident on $^{197}$Au targets. The filled circles represent the calculations using first-order perturbation theory, while the filled squares are the results of coupled-channel calculations.](image)
the ground state densities of the nuclei obtained from either fitting to electron scattering experiments [31] for \(^{197}\)Au and Hartree-Fock-Bogoliubov calculations for \(^{68}\)Ni with the SLy4 interaction. It is worth mentioning that the use of the eikonal absorption phase to cut down the Coulomb excitation mechanism at small impact parameters has been introduced in Ref. [32] for the first time to calculate cross sections relevant to GDR and DGDR excitations.

We consider the excitation of \(^{68}\)Ni on \(^{197}\)Au and \(^{208}\)Pb targets at 600 and 513 MeV/nucleon, respectively. These reactions have been experimentally investigated in Refs. [33] [34]. In the first experiment a pygmy dipole resonance in \(^{68}\)Ni was identified at \(E_{PDR} \approx 11\) MeV with a width of \(\Gamma_{PDR} \approx 1\) MeV, exhausting about 5% of the Thomas-Reiche-Kuhn (TRK) energy-weighted sum rule. The identification was done by identifying the excitation and decay via gamma emission. In the second experiment the PDR centroid energy was found to be at 9.55 MeV, with a 2.8% fraction of the TRK sum rule, a width of 0.5 MeV, and the PDR identification was done by measuring the neutron decay channel of the PDR. In this work our objective is to study the effects of the coupling between the several giant resonances with the PDR, and therefore we will only calculate the excitation function \(d\sigma/dE\) without concern for the decay channels. We use \(E_{PDR} = 11\) MeV, consistent with Refs. [33] [34], but a full width at half maximum of 2 MeV, which is more in line with theoretical calculations [13] [20] than with the experimental data [33] [34]. The larger PDR width also allows us to better determine the higher-order effects on the modification of the tails of the PDR.

For the (isovector) \(^1\) giant dipole resonance (GDR) we assume \(E_{GDR} = 17.2\) MeV and \(\Gamma_{GDR} = 4.5\) MeV and for the (isoscalar) \(^2\) giant quadrupole resonance (GQR) we take \(E_{GQR} = 15.2\) MeV and \(\Gamma_{GQR} = 4.5\) MeV. The total number of channels are \(35 \times 3 + 35 \times 5 + 35 \times 5 + 1 = 456\) including all magnetic substates and the ground state, assume to be a \(^0\)\(^+\) state. The calculations are CPU intensive but can be reduced for the practical purposes because the major dynamical effect arises from the coupling of the PDR with the GQR via the dominant \(E1\) interaction at relativistic bombarding energies. The number of channels can also be reduced by a factor of 2 by means of a coarser binning of the PDR and GQR states with a loss of accuracy at the level of 10%. With the number of channels mentioned earlier our calculations converge to within 1%.

In Figure 2 we show the first-order Coulomb excitation cross sections of the PDR and GQR separately, as a function of the excitation energy and for 600 MeV/nucleon \(^{68}\)Ni projectiles incident on \(^{197}\)Au targets. The filled circles represent the calculations using first-order perturbation theory, while the filled squares are the results of coupled-channel calculations.

FIG. 3. Coulomb excitation cross section as a function of the excitation energy of 600 MeV/nucleon \(^{68}\)Ni projectiles incident on \(^{197}\)Au targets. The filled circles represent the calculations using first-order perturbation theory, while the filled squares are the results of coupled-channel calculations.

tained by means of the relation

\[
\frac{d\sigma}{dE} = \sum_{\pi L} \frac{N_{\pi L}(E)}{E} \sigma_{\pi L}(E),
\]

where \(\pi L\) denotes the multipolarity, \(N_{\pi L}\) are the virtual photon numbers, and \(\sigma_{\pi L}(E)\) are the cross sections for real photons with multipolarity \(\pi L\). The sum runs over the relevant multipoles, here \(E1\) and \(E2\) stand for \(^1\) and \(^2\) excitations, respectively. It is quite evident from the figure that the coupling between these states has a visible impact on the energy dependence of the cross sections. We notice that according to the Brink-Axel hypothesis, a giant resonance can be excited on top of any other state in a nucleus [35] [36]. Therefore, the couplings are in fact a manifestation of \((PDR \otimes GQR)_{1-}\), \((PDR \otimes PDR)_{2+}\), \((PDR \otimes GDR)_{2+}\) and \((GDR \otimes GQR)_{1-}\) states which are of our interest, as they build up components of the PDR, GDR and GQR. The importance of our findings lies on the reliability of the experimental extraction of the PDR strength relative to that of the GDR.

The dynamical calculations show that not only the strength, but also the width of the PDR is modified ap-
precisely due to the coupling to the GQR. In Figure 2, the modification of the population of PDR and GQR states are shown separately. Because the $1^-$ states in the GDR region are very weakly affected by the coupling to the other states, we left them out of the figure as we want to concentrate on the PDR excitation spectrum. The GDR excitation dominates in the high energy region by a factor of $2 - 3$ times that of the $2^+$ states. The main modifications in the excitation spectrum come from the couplings PDR $\leftrightarrow$ GQR $\leftrightarrow$ PDR by E2 fields, while the couplings PDR $\leftrightarrow$ GDR $\leftrightarrow$ PDR by E2 fields contribute very little to the $1^-$ states in the PDR energy region. We also see in Figure 2 that the tails of the PDR, and to a minor extent those of the GQR, are appreciably modified. A small shift of the peaks also occurs, although barely visible for the PDR, it is evident for the GQR.

In Figure 3 we singled out the energy region of the pygmy resonance and we plot the results of our calculations for two different bombarding energies: 100 MeV/nucleon and 2 GeV/nucleon, with the same notation as in Figure 2. The coupling effects change dramatically. At the lower energy the influence of the giant resonances is to increase appreciably the response in the energy region of the PDR, while at the higher energy the effect of coupling is much smaller and the tendency is to slightly decrease the PDR excitation cross section. This result is expected because at energies around 100 MeV/nucleon the E2 field is dominant, with an appreciable increase of the excitation of the GQR and a consequently strong feedback to the PDR via subsequent E1 transitions.

In Figure 4 we show the Coulomb excitation cross sections of the PDR as a function of the bombarding energy of $^{68}$Ni projectiles incident on $^{197}$Au targets. The filled circles represent the calculations using first-order perturbation theory, while the filled squares are the results of coupled-channel calculations.

![FIG. 4. Coulomb excitation cross sections of the PDR as a function of the bombarding energy of $^{68}$Ni projectiles incident on $^{197}$Au targets. The filled circles represent the calculations using first-order perturbation theory, while the filled squares are the results of coupled-channel calculations.](image)
derstanding of neutron-star properties. The low-energy response is particularly important for the polarizability due to the inverse weighting with energy. This opens really exciting possibilities for the studies of the pygmy resonance in nuclei and use it as a tool for applications in nuclear astrophysics.

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