Min–Max Latency Optimization for IRS-Aided Cell-Free Mobile Edge Computing Systems

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Abstract—Mobile edge computing (MEC) is expected to provide low-latency computation service for wireless devices (WDs). However, when WDs are located at cell edge or communication links between base stations (BSs) and WDs are blocked, the offloading latency will be large. To address this issue, we propose an intelligent reflecting surface (IRS)-assisted cell-free MEC system consisting of multiple BSs and IRSs for improving the transmission environment. Consequently, we formulate a min–max latency optimization problem by jointly designing multiuser detection (MUD) matrices, IRSs' reflecting beamforming vectors, WDs' offloading data size and edge computing resource, subject to constraints on edge computing capability and IRSs phase shifts. To solve it, an alternating optimization algorithm based on the block coordinate descent (BCD) technique is proposed, in which the original nonconvex problem is decoupled into two subproblems for alternately optimizing computing and communication parameters. In particular, we optimize the MUD matrix based on the second-order cone programming (SOCP) technique, and then develop two efficient algorithms to optimize IRSs' reflecting vectors based on the semi-definite relaxation (SDR) and successive convex approximation (SCA) techniques, respectively. Numerical results show that employing IRSs in cell-free MEC systems outperforms conventional MEC systems, resulting in up to about 60% latency reduction can be attained. Moreover, numerical results confirm that our proposed algorithms enjoy a fast convergence, which is beneficial for practical implementation.

Index Terms—Cell-free network, intelligent reflecting surface (IRS), min–max latency, mobile edge computing (MEC).

I. INTRODUCTION

RECENTLY, the rapid development of the Internet of Things (IoT) and artificial intelligence (AI) techniques have enabled various new applications (such as natural language processing, face/fingerprint recognition, autonomous driving, 3-D media, etc.) based on real-time communication. Generally, these applications require low latency and need more computing resources. However, this is challenging for IoT devices due to their limited computing capabilities [1], [2], [3]. To address this challenge, mobile edge computing (MEC) was proposed, where the computing tasks of IoT devices can be offloaded to the edge servers that are usually equipped with huge computing resources [4], [5], [6], [7]. However, the transmission latency needs to be considered in a MEC system, which is determined by the offloading links. When wireless devices (WDs) are located at the cell edge or communication links between base stations (BSs) and WDs are blocked, the offloading links will be poor, which leads to large transmission latency. Therefore, it is crucial to study how to improve the wireless communication environment for further exploiting the potential of MEC systems.

To provide better offloading links, the ultradense network (UDN) architecture with a lot of small BSs can be applied in order to shorten the distances between BSs and WDs or provide direct links between them [8]. However, as the number of BSs increases, intercell interference (ICI) becomes the bottleneck [9], [10], [11], [12]. Thus, the user-centric-based cell-free network structure is employed, where BSs simultaneously serve all WDs to avoid the multicell interference [13]. On the other hand, the energy consumption (EC) and hardware cost are still high for the cell-free network due to the deployment of massive BSs. Recently, an intelligent reflecting surface (IRS) composed of a large number of low-power passive reflecting elements was developed, which can focus signal energy in the desired spatial direction by adjusting phase shifts, thus enlarging the wireless coverage [14], [15]. Therefore, to reduce the system cost, some BSs can be replaced by IRSs. In this article, we will investigate the latency optimization problem in the IRS-aided cell-free MEC systems.
A. Related Works

Recently, there have been many works focusing on MEC. Initially, the binary offloading model was considered, and each computation task is treated as a whole. Specifically, Zhang et al. [16] investigated to minimize the EC of a single user by dynamically configuring the clock frequency of the local central processing unit (CPU) for local computing and varying the offloading rate for cloud computing according to the stochastic channel condition. Then, Wang et al. [17] extended the single-user MEC system to the multiuser one, and aimed to minimize the weighted sum of all users’ EC. For the binary offloading model, the offloading design in multiuser MEC is usually a typical mixed-integer nonlinear programming problem that is usually nonconvex. To deal with it, Wang et al. [17] proposed a low-complexity two-stage algorithm by iteratively optimizing the offloading decision and resource allocation. Chen et al. [18], [19] and Wang et al. [20] formulated the offloading decision optimization problem as a multiuser noncooperative computing offloading game, and then an effective algorithm based on game theory was proposed. Dinh et al. [21] and Kuang et al. [22] considered a partial offloading model, in which a task can be further divided into multiple modules, which can be performed locally or remotely. The weighted sum of EC and computing latency was minimized by jointly optimizing offloading decision and CPU frequency in [21], or offloading decision and transmit power as in [22]. Additionally, several researchers also investigated the edge computing problem under multi-tier computing systems. For example, Ning et al. [23], Yang et al. [24], and Gao et al. [25] integrated cloud computing with MEC and proposed a three-tier system model (“terminal-edge-cloud”). Particularly, Ning et al. [23] considered both single-user and multiuser scenarios, and for the single-user scenario, a branch and bound algorithm was proposed. For the multiuser scenario, an iterative heuristic MEC resource allocation algorithm was developed for making the offloading decision dynamically. In [24], the optimal offloading node selection problem was formulated as a Markov decision process, and then an iterative optimization algorithm was proposed. Gao et al. [25] considered a three-tier system consisting of multiple WDs, multiple edge nodes, and a central cloud, and then proposed two offloading algorithms by iteratively optimizing the WD-edge matching strategy and resource allocation.

Additionally, there are several works investigating edge computing problem in IRS-aided MEC systems. Haber et al. [26] proposed an IRS-aided single-user MEC system, where the computing task can be offloaded to multiple MEC nodes for guaranteeing high reliability. Later, Sun et al. [27] considered both the flat-fading channel and frequency-selective channel model under the IRS-aided multiuser MEC scenario, and proposed an alternating optimization algorithm to minimize users’ EC. Bai et al. [28] and Li et al. [29] investigated the EC problem, where, compared with the conventional MEC system, about 80% EC reduction was achieved in the proposed IRS-aided wireless powered MEC system. Besides, to avoid the interference among WDs, Mao et al. [30] and Chu et al. [31] applied the time division multiple access protocol in the computing offloading stage. Similarly, Mao et al. [30] also considered the IRS-aided wireless powered MEC system, and they jointly optimized downlink/uplink IRS reflecting beamforming vector, transmit power, time allocation for energy transmission of downlink and computing offloading of uplink, and local CPU frequency. Chu et al. [31] jointly optimized offloading time, CPU frequency, transmit power, and IRS phase shift vector in the IRS-aided MEC system. Note that both [30] and [31] aim to maximize the total computation data bits of all users. To minimize the total latency, Zhou et al. [32] proposed a new flexible time-sharing nonorthogonal multiple-access (NOMA) scheme, in which the offloading data can be divided into two parts, and then the optimal solution was obtained for both the cases of finite and infinite edge computation capacities.

Based on the above brief literature survey, we note that, although there have been several works considering the latency problem in IRS-aided MEC system, WDs’ fairness was not investigated. Furthermore, for future IRS-aided cell-free MEC systems, how to jointly optimize multiple BSs’ received matrix, multiple IRSs’ reflecting beamforming, WDs’ offloading data size, and edge computing resource based on WDs’ fairness is challenging.

B. Contributions

In this article, considering WDs’ fairness during the computation offloading, we investigate the min–max WD’s latency in IRS-aided cell-free MEC systems, and the main contributions are summarized as follows.

1) We consider an IRSs-aided cell-free MEC network consisting of multiple BSs and IRSs for improving the transmission performance. To guarantee the WDs’ fairness, we formulate a min–max WDs’ latency optimization problem by jointly designing computing and communication resource. Furthermore, the limited edge computing resource is considered. To the best of the authors’ knowledge, this is the first time to investigate such model. For the formulated min–max latency optimization problem, we first introduce auxiliary variable $t$, then the block coordinate descent (BCD) technique is adopted to decouple the reformulated problem into two subproblems.

2) The first subproblem is to jointly optimize the offloading data size and edge computing resource-based fixed multiuser detection (MUD) matrix and reflecting beamforming vector. To solve it, we first analyze the relation between offloading data size and edge computing resource, and obtain a semi-closed solution of the optimal offloading data size. Then, the first subproblem is transformed into a convex optimization problem with the help of the bisection search method over $t$.

3) The second subproblem is to jointly optimize the MUD matrix and reflecting beamforming vector based on obtaining offloading data size and edge computing resource from the first subproblem. We solve for the MUD matrix based on the second-order cone programming (SOCP) technique, and solve for the
reflecting beamforming vector based on the semi-definite relaxation (SDR) and successive convex approximation (SCA) techniques.

4) Extensive simulation results verified the benefits of deploying IRSs in MEC systems under different parameter settings. For example, compared with the conventional MEC system, the latency can be effectively reduced from 160 to 100 ms for distances $L = 60$ and 100 m. Moreover, our simulation results confirmed that a) multiple BSs’ collaboration is better; b) to minimize the latency, adding appropriate computing capability at the edge server is cost-effective; and c) our proposed algorithms enjoy a fast convergence, which validates its engineering viability.

The remainder of this article is organized as follows. Section II introduces the system model and the formulated problem. In Section III, we develop an alternating algorithm for solving the formulated min-max latency optimization problem. Section IV investigates the latency optimization problem in an IRS-aided cell-free MEC system with single WD. Section V provides the performance evaluation. Finally, the conclusions are given in Section VI. For clarity, we list the mathematical operations adopted throughout this article in Table I.

| Parameter  | Definition                                                                 |
|------------|---------------------------------------------------------------------------|
| $\ast\ast$ | Conjugate transpose                                                        |
| $E \{ \cdot \}$ | Expectation                                                                 |
| $\\{ \}$ | Transpose                                                                   |
| $\\{ \}$ | Trace                                                                      |
| $\text{arg} \{ \}$ | The argument of a complex number                                            |
| $\mathbb{C}^m \times n$ | The space of $m \times n$ complex-valued matrices                           |
| $\text{det} (\mathbf{A})$ | Determinant of $\mathbf{A}$                                               |
| $\text{diag} \{ \}$ | Diagonalization operator                                                   |
| $|$ | Absolute value of a scalar                                                 |
| $\| \cdot \|_2$ | Euclidean norm / 2-norm                                                   |
| $\mathbb{V} \succeq 0$ | Positive semi-definite matrix                                              |
| Re | Real part                                                                   |
| Im | Imaginary part                                                             |
| $\lfloor \cdot \rfloor$ | The floor operations                                                       |
| $\lceil \cdot \rceil$ | The ceiling operations                                                     |
| $\mathbb{V}_{m,n}$ | The element of $\mathbb{V}$ in the $m$-th row, $n$-th row.               |
| $\ell_k$ | The total input data size                                                  |
| $c_k$ | The offloading volume                                                      |
| $f_k^l$ | The number of CPU cycles required to process one input bit                |
| $f_k^e$ | The local computing capability of the $k$-th WD                          |
| $D_k^l$ | The edge computing resource allocated to the $k$-th WD                    |
| $D_k^e$ | The time required for carrying out the local computation                  |

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. Communication Model

As shown in Fig. 1, we consider an IRS-aided cell-free MEC system, where multiple distributed BSs cooperatively serve WDs with the help of IRSs. All IRSs and BSs are connected to a CPU by high-throughput optical cables. The considered system consists of $K$ single-antenna WDs, $I$ IRSs, and $B$ BSs. The number of elements at the $i$-th IRS and number of antennas at the $b$-th BS are denoted by $N_i$ and $M_b$, respectively. For simplicity, we set $N_i = N$ and $M_b = M$ for any $i$ and $b$, and let $N \in \{1, \ldots, N\}$, $B \in \{1, \ldots, B\}$, $I \in \{1, \ldots, I\}$, and $K \in \{1, \ldots, K\}$ denote the index sets of IRS elements, BSs, IRSs, and WDs, respectively. We consider a block-fading channel model, where the wireless channels remain constant during the current time block but change over different time blocks, and assume that the channel state information (CSI) of all involved channels are available by using existing advanced channel estimation methods [33], [34].

As shown in Fig. 2, the direct channel vector from the $k$th WD to the $b$th BS is denoted by $\mathbf{h}_{d,b,k} \in \mathbb{C}^M \times 1$, and the reflect channel vectors from the $k$th WD to the $i$th IRS and the $i$th IRS to the $b$th BS are denoted by $\mathbf{h}_{r,i,k} \in \mathbb{C}^N \times 1$ and $\mathbf{G}_{b,i} \in \mathbb{C}^{M \times N}$, respectively. The phase shift coefficient vector of the $i$th IRS is denoted by $\theta_i = [\theta_{i,1}, \theta_{i,2}, \ldots, \theta_{i,N}]^T$, where $\theta_{i,n} \in [0, 2\pi)$ for all $i \in I$ and $n \in N$. Then, the diagonal reflecting matrix of the $i$th IRS is given by

$$\Theta_i \triangleq \text{diag}(\beta_{i,1} e^{j\theta_{i,1}}, \ldots, \beta_{i,N} e^{j\theta_{i,N}}) \quad \forall i \in I$$

where $\beta_{i,n} \in [0, 1]$ stands for the reflection amplitudes of the IRS elements and each of them is fixed to one for maximizing the reflected signal power. It is worth noting that double/multihop reflections between IRSs also occur in the multi-IRS communication environment. However, due to the severe multiplicative path loss in multihop reflection, we only
consider the one-way reflection as in [35], [36], and [37]. Thus, the combined effective channel from the $k$th WD to the $b$th BS can be defined as

$$h_{b,k} = h_{d,b,k} + \sum_{i=1}^{I} G_{b,i} \Theta h_{r,i,k}. \quad (2)$$

Let $P_t$ and $s = [s_1, s_2, \ldots, s_K]^T$ denote the offloading power and signal of $K$ WDs, respectively. Here, we assume that all WDs transmit the same power for simplicity as in [38]. Let $w_{b,k} \in \mathbb{C}^{M \times 1}$ denote the MUD vector of the $b$th BS for the $k$th WD. Hence, the detected signal at BSs for the $k$th WD can be formulated as

$$y_k = \sum_{b=1}^{B} w_{b,k}^H \left[ \sqrt{P_t} \sum_{j=1}^{K} \left( h_{d,j} + \sum_{i=1}^{I} G_{i} \Theta h_{r,i,j} \right) s_j + n_b \right] \quad (3)$$

where $n_b \in \mathbb{C}^{M \times 1}$ denotes the received noise vector of the $b$th BS, $w_k \in \mathbb{C}^{MB \times 1}$ is the $k$th column of the MUD matrix $W \in \mathbb{C}^{MB \times K}$, where $W = [w_1^T, \ldots, w_K^T]^T$. (a) holds by defining $w_k = [w_1^T, \ldots, w_K^T]^T$, $h_{d,k} = [h_{d,1,k}^T, \ldots, h_{d,K,k}^T]^T$, $G_i = [G_{i,1}^T, \ldots, G_{i,K}^T]^T$, and $\Theta = \text{diag}(\Theta_1, \ldots, \Theta_I)$, and (c) holds according to $h_{k} = h_{d,k} + G \Theta h_{r,k}$. Then, the received SINR for the $k$th WD is

$$\gamma_k(w_k, \theta) = \frac{P_t |w_k^H (h_{d,k} + G \Theta h_{r,k})|^2}{P_t \sum_{j=1}^{K} |w_k^H (h_{d,j} + G \Theta h_{r,j})|^2 + \sigma^2} \quad (4)$$

and the corresponding achievable rate is given by

$$R_k(w_k, \theta) = \eta \log_2(1 + \gamma_k(w_k, \theta)) \quad (5)$$

where $\eta$ is the system bandwidth.

### B. Computing Model

In this article, the partial offloading scheme is considered, and the computing models for both local and edge processing are presented, respectively.

1) **Local Computing**: Let $f_0$, $L_k$, $\ell_k$, and $c_k$ denote the local CPU-cycle frequency (cycles/$s$), total input data size to be processed, offloading data size, and the number of CPU cycles required to compute a single bit for the $k$th WD, respectively. Thus, the latency imposed by local computing is formulated as $D_{l}(\ell_k) = (L_k - \ell_k)c_k/f_0$.

2) **Edge Computing**: The latency for edge computing usually includes three parts: a) offloading latency for transmitting computing data to BSs; b) processing latency for executing offloaded data at MEC server; and c) return latency for transmitting computing results to WDs. Let $f_0^e$ and $f_k^e$ denote the total computing resource of MEC server and the computing resource allocated to the $k$th WD, respectively, satisfying $\sum_{k=1}^{K} f_k^e \leq f_0^e$. Here, we ignore the return latency since the returning results are usually of small size [17], [39]. Therefore, the total latency for edge processing can be written as

$$D_{k}(w_k, \theta, \ell_k, f_k^e) = \ell_k R_k(w_k, \theta) + \ell_k c_k/f_k^e.$$ 

On this basis, the overall latency for the $k$th WD can be formulated as

$$D_k(w_k, \theta, \ell_k, f_k^e) = \max \left\{ D_l(\ell_k), D_{k}(w_k, \theta, \ell_k, f_k^e) \right\} = \max \left\{ \ell_k R_k(w_k, \theta) + \ell_k c_k/f_k^e \right\}.$$ 

### C. Problem Formulation

Considering WDs’ fairness, we minimize the maximum WD’s latency by jointly optimizing offloading data size $\ell = [\ell_1, \ell_2, \ldots, \ell_K]^T$, edge computing resource $f = [f_0^e, f_1^e, \ldots, f_K^e]^T$, MUD matrix $W$, and reflecting beamforming vector $\theta$, which can be formulated as

$$\mathcal{P}0 : \min_{\ell, f, W, \theta} \max_{k \in K} D_k(w_k, \theta, \ell_k, f_k^e) \quad \text{s.t.} \quad 0 \leq \ell_k \leq 2\pi, \forall \theta \in \mathcal{D}, \forall n \in \mathcal{N} \quad (7a)$$

$$\ell_k \in \{0, 1, \ldots, L_k\}, \forall k \in K \quad (7b)$$

$$\sum_{k=1}^{K} f_k^e \leq f_0^e \quad (7c)$$

$$f_k^e \geq 0, \forall k \in K \quad (7d)$$

$$\|w_k\|^2 \leq 1, \forall k \in K \quad (7e)$$

where (7a) is IRS reflection coefficients constrains, (7b) implies that the $k$th WD’s offloaded data size should be an integer between zero and the total input data $L_k$. Equation (14c) denotes the computing resource allocated to all WDs should not exceed the total edge computing resource. Equation (7e) represents unit-norm detection vector constrains for the $k$th WD. It is clear that $\mathcal{P}0$ is difficult to be directly solved. Next, we first introduce an auxiliary variable $t$ to transform $\mathcal{P}0$ into the following $\mathcal{P}1$, given as:

$$\mathcal{P}1 : \min_{\ell, f, W, \theta} t \quad \text{s.t.} \quad D_k(w_k, \theta, \ell_k, f_k^e) \leq t, \forall k \in K \quad (8a)$$

$$\ell_k \in \{0, 1, \ldots, L_k\}, \forall k \in K \quad (7a), \quad (7b), \quad (14c), \quad (7d), \quad (7e). \quad (8b)$$

**Remark 1**: Although the objective function (OF) of $\mathcal{P}1$ and constraints in (8b) are linear, it is still challenging to directly solve $\mathcal{P}1$ due to the following three aspects: 1) the segmented form of (8a); 2) MUD matrix $W$ and reflecting beamforming vector $\theta$ are coupled together; and 3) Equation (8a) is nonconvex with respect to $\theta$. In general, there is no standard method to find a globally optimal solution of such a nonconvex optimization problem. To proceed, we develop an iterative framework to obtain a locally optimal solution. Specifically, the segmented form of (8a) is reformulated as a linear form.
relying on the BCD technique. Then, fixing the computing setting, we optimize the MUD matrix and reflecting beamforming vector alternately. Finally, we develop two efficient algorithms based on SDR and SCA techniques to obtain a locally optimal solution of $\theta$, respectively.

III. Problem Solutions

In this section, we first divide $P1$ into two independent sub-problems relying on the BCD technique. Specifically, given $W$ and $\theta$, the offloading data size $\ell$ and edge computing resource $f^*$ are optimized. Then, based on the obtained $\ell$ and $f^*$, the MUD matrix $W$ and reflecting beamforming vector $\theta$ are optimized. The above procedure is repeated until convergence.

A. Jointly Optimizing $\ell$ and $f^*$ for Given $W$ and $\theta$

For given $W$ and $\theta$, $P1$ can be reformulated as follows:

$$P2 : \min_{\ell, f^*} t$$

s.t.

$$D_k(\ell_k, f^*_k) \leq t \quad \forall k \in K \quad (9a)$$

$$\ell_k \in \{0, 1, \ldots, L_k\} \quad \forall k \in K \quad (9b)$$

$$\sum_{k=1}^{K} f^*_k \leq f^\text{total} \quad (9c)$$

$$f^*_k \geq 0 \quad \forall k \in K. \quad (9d)$$

Based on the following Proposition 1, we optimize the offloading data size $\ell$.

**Proposition 1:** For given $f^*$, the optimal offloading data size is given by

$$\hat{\ell}^*_k = \arg\min_{\ell_k \in [\lfloor \hat{\ell}_k \rfloor, \lceil \hat{\ell}_k \rceil]} D_k(\ell_k) \quad (10)$$

where $\lfloor \cdot \rfloor$ and $\lceil \cdot \rceil$ denote the floor and ceiling operations, respectively. Thus, the value of $\hat{\ell}^*_k$ is selected for ensuring that $D_k'(\hat{\ell}^*_k) = D_k'(\hat{\ell}_k)$, i.e.,

$$\hat{\ell}^*_k = \frac{L_k c_k R_k f^*_k}{f^*_k + c_k R_k (f^*_k + f^*_k)}. \quad (11)$$

**Proof:** See Appendix A.

After obtaining the relation between offloading data size and edge computing resource, we substitute (11) into (9a), then $P2$ can be written as

$$P2.1 : \min_{f^*} t$$

s.t.

$$\left( L_k c_k^2 R_k + L_k c_k f^*_k \right) \leq t \quad \forall k \in K \quad (12a)$$

$$\sum_{k=1}^{K} f^*_k \leq f^\text{total} \quad (12b)$$

$$f^*_k \geq 0 \quad \forall k \in K. \quad (12c)$$

Note that $P2.1$ is a nonconvex problem. To proceed it, we first reformulate (12a) as $(L_k c_k^2 R_k + L_k c_k f^*_k) - t(f^*_k + c_k R_k (f^*_k + f^*_k)) \leq 0$. Then, with the help of a bisection search over $t$, $P2.1$ can be equivalently transformed into the following feasibility problem, given as:

$$P2.2 : \text{Find } f^*$$

s.t.

$$L_k c_k^2 R_k + L_k c_k f^*_k \leq t \quad \forall k \in K \quad (13a)$$

$$\sum_{k=1}^{K} f^*_k \leq f^\text{total} \quad (13b)$$

$$f^*_k \geq 0 \quad \forall k \in K. \quad (13c)$$

where $t^{(l)}$ is the value of $t$ at the $l$th iteration. For a given latency target $t^{(l)}$, the above optimization problem is convex, and thus the global optimal solution of $P2.2$ can be found via existing convex optimization techniques, e.g., CVX solver.

Note that if $P2.2$ is feasible, the given latency $t$ can be achieved. Assuming that the optimal solution of $P2.1$ is $\hat{\ell}$, we can infer that, for any given $t$, if $P2.2$ is feasible, we have $t \leq \hat{\ell}$, while if $P2.2$ is infeasible, we have $t \geq \hat{\ell}$. Therefore, with the aid of a bisection search over $t$, $P2.1$ can be equivalently solved by checking the feasibility of $P2.2$ for a given $t \geq 0$.

In summary, the procedure for solving $P2$ is presented as Algorithm 1.

**Algorithm 1** Joint Optimization Scheme for Solving $P2$

**Input:** $h_k$, $B$, $p_i$, $\sigma^2$, $L_k$, $c_k$, $K$, $f^\text{total}$, $\epsilon_1$, $W$, and $\theta$

**Output:** Optimal $\ell$ and $f^*$

1. **Initialization**
   Initialize $l_1 = 0$, calculate $R_k$ according to (5), $\forall k \in K$

2. **Joint optimization of $t$ and $f^*$**
   repeat:
   - Calculate $t^{(l)}$ using the bisection search method
   - Solving $P2.2$ to obtain optimal $f^{*(l)}$
   - Update $l_l \leftarrow l_l + 1$
   until Convergence

3. **Calculate $\{\ell_k\}$ using (11), $\forall k \in K$**

B. Jointly Optimizing $W$ and $\theta$ for Given $\ell$ and $f^*$

Given a set of offloading data size $\ell$ and edge computing resource allocation $f^*$, $P1$ can be simplified as

$$P3 : \min_{W, \theta, t} t$$

s.t.

$$D_k(w_k, \theta) \leq t \quad \forall k \in K \quad (14a)$$

$$0 \leq \theta_{in} < 2\pi \quad \forall i \in I \quad \forall n \in N \quad (14b)$$

$$\|w_k\|^2 \leq 1 \quad \forall k \in K. \quad (14c)$$

The segmented form of $D_k(w_k, \theta)$ makes $P3$ difficult to be solved directly. Bearing in mind that the optimal solution of $P0$ results in $D_k = D_k^L = D_k^f$ as proved in Proposition 1. Hence, upon replacing $D_k$ by $D_k^L$, $P3$ can be rewritten as

$$P3.1 : \min_{W, \theta, t} t$$

s.t.

$$\frac{\ell_k}{R_k(w_k, \theta)} + \frac{\ell_k c_k}{f^*_k} \leq t \quad \forall k \in K. \quad (15a)$$

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0 \leq \theta_{n,i} < 2\pi \quad \forall i \in \mathcal{I} \quad \forall n \in \mathcal{N} \tag{15b}

\|w_k\|^2 \leq 1 \quad \forall k \in \mathcal{K}. \tag{15c}

Remark 2: Solving $\mathcal{P}3.1$ involves a joint optimization of MUD matrix $W$, reflecting beamforming vector $\theta$ and latency $t$, which is challenging. Next, we develop an alternate iterative optimization algorithm to solve $\mathcal{P}3.1$.

1) Optimization of MUD Matrix $W$: For given reflecting beamforming vector $\theta$, $\mathcal{P}3.1$ can be rewritten as

\[ \mathcal{P}3.2 : \min_{w_k} t \]

s.t. \[ \frac{\ell_k}{R_k(w_k, \theta)} \geq t \quad \forall k \in \mathcal{K} \]

\[ \|w_k\|^2 \leq 1 \quad \forall k \in \mathcal{K}. \]

Since $\mathcal{P}3.2$ is still nonconvex, we introduce the following feasibility-check problem by fixing $t$ like $\mathcal{P}2.2$:

\[ \mathcal{P}3.3 : \text{Find } \{w_k\} \]

s.t. \ (16a), (16b). \tag{17a}

Assuming that the optimal solution of problem $\mathcal{P}3.2$ is $t^*$, similar to the analysis in Section III-A, for any given $t \geq t^*$, $t$ is a feasible solution to $\mathcal{P}3.3$. Whereas, if $t \leq t^*$, $\mathcal{P}3.3$ is infeasible. Thus, with the aid of a bisection search over $t \geq 0$, $\mathcal{P}3.2$ can be solved equivalently by checking the feasibility of $\mathcal{P}3.3$.

Next, we investigate how to solve $\mathcal{P}3.3$. The inequality in (16a) can be rewritten as

\[ \left( 1 + \frac{1}{2^{(t-t_{k,c})}} - 1 \right) P_t |w_k^H (h_{d,k} + G \Theta h_{r,k})|^2 \geq P_t \sum_{j=1}^{K} |w_{k,j}^H (h_{d,j} + G \Theta h_{r,j})|^2 + \sigma^2 \quad \forall k \in \mathcal{K} \tag{18} \]

where $t_{k,c} = \ell_k c_k / f_k^c$. According to (18), it is clear that for any feasible solution $\{w_k\}$, $\mathcal{P}3.3$ is still feasible after any phase rotation. Without loss of optimality, we select a set of $\{w_k\}$ so that $w_k^H (h_{d,k} + G \Theta h_{r,k})$ is nonnegative value for any WDs $k \in \mathcal{K}$. Hence, we have the following constraints:

\[ w_k^H (h_{d,k} + G \Theta h_{r,k}) \geq 0 \quad \forall k \in \mathcal{K}. \tag{19} \]

Let $\text{Re}(x)$ and $\text{Im}(x)$ denote the real and imaginary parts of $x$. The real part and imaginary part of $w_k^H (h_{d,k} + G \Theta h_{r,k})$ are nonnegative and zero, respectively, i.e., $\text{Re}(w_k^H (h_{d,k} + G \Theta h_{r,k})) \geq 0$ and $\text{Im}(w_k^H (h_{d,k} + G \Theta h_{r,k})) = 0$. Then, we define a matrix $A$, in which the $(j,k)$th element is $\sqrt{P_j} w_{k,j}^H (h_{d,j} + G \Theta h_{r,j})$. Let $\| \cdot \|$ denote the 2-norm of a vector, and thus (18) can be reformulated as

\[ \sqrt{1 + \frac{1}{2^{(t-t_{k,c})}} - 1} w_k^H (h_{d,k} + G \Theta h_{r,k}) \geq \frac{A^H e_k}{\sigma_k} \quad \forall k \in \mathcal{K}. \tag{20} \]

where $e_k \in \mathbb{C}^{K \times 1}$ represents a vector, in which the $k$th element is one and other elements are zero. Hence, for a given $t$ at the $l$th iteration, $\mathcal{P}3.3$ can be equivalently expressed as

\[ \mathcal{P}3.4 : \text{Find } \{w_k\} \]

s.t. \ (16b), (19). \tag{21a}

It is not difficult to observe that $\mathcal{P}3.4$ is an SOCP problem and can be solved using the existing standard convex optimization techniques (i.e., CVX solver) \cite{40}.

2) Optimization of Reflecting Beamforming Vector $\theta$: Different from solving $W$, the SOCP technique cannot be used to find the optimal $\theta$ since $\Theta$ is common, resulting in

\[ w_k^H (h_{d,k} + G \Theta h_{r,k}) \]

cannot be ensured to be real numbers for all $k \in \mathcal{K}$.

To proceed, we first define $v = [\beta_1 e^{j\theta_1}, \ldots, \beta_N e^{j\theta_N}]^T$, $a_{k,j} = \sqrt{P_j} \text{diag}(h_{r,j}^H)G^H w_k$ and $d_{k,j} = \sqrt{P_j} w_k^H h_{d,j} \forall k, j \in \mathcal{K}$. Consequently, we have

\[ P_t |w_k^H (h_{d,j} + G \Theta h_{r,j})|^2 = v^H C_{k,j} v + 2 \text{Re} \{v^H u_{k,j}\} + |d_{k,j}|^2 \tag{22} \]

where $C_{k,j} = a_{k,j} a_{k,j}^H$ and $u_{k,j} = \sqrt{P_j} w_k^H h_{d,j} \text{diag}(h_{r,j}^H) G^H w_k$. After obtaining $W$, $\mathcal{P}3.1$ can be reformulated as

\[ \mathcal{P}3.5 : \min_{v,l} t \]

s.t. \[ \sum_{j=1, j \neq k}^{K} (v^H C_{k,j} v + 2 \text{Re} \{v^H u_{k,j}\} + |d_{k,j}|^2) + \sigma^2 \geq 2^{(t-t_{k,c})} - 1 \quad \forall k \in \mathcal{K} \tag{23a} \]

\[ |v_n|^2 = 1, \quad n = 1, 2, \ldots, N. \tag{23b} \]

$\mathcal{P}3.5$ is still difficult to be solved directly due to nonconvex constraints (23a) and (23b). Next, we develop two effective schemes based on SDR and SCA techniques to solve it.

Solving $\mathcal{P}3.5$ Based on SDR: To solve $\mathcal{P}3.5$, we first reformulate it as

\[ \mathcal{P}3.6 : \min_{v,l} t \]

s.t. \[ \sum_{j=1, j \neq k}^{K} (v^H R_{k,j} v + |d_{k,j}|^2) + \sigma^2 \geq \alpha_k(t) \quad \forall k \in \mathcal{K} \tag{24a} \]

\[ |v_n| = 1, \quad n = 1, 2, \ldots, N + 1 \tag{24b} \]

where $\alpha_k(t) = 2^{(t-t_{k,c})} - 1$, $R_{k,j}$ and $v$ are defined as

\[ R_{k,j} = \begin{bmatrix} C_{k,j} & u_{k,j}^H \\ u_{k,j} & 0 \end{bmatrix} \tag{25} \]

respectively. Define $V = vv^H$, which needs to satisfy $V \succeq 0$ and rank($V$) = 1. Accordingly, $\mathcal{P}3.6$ can be re-expressed as
Algorithm 2 Joint Optimization of $t$, $W$, and $\theta$ With SDR Technique

**Input:** $l_2 = 0$, $\theta^{(0)}$ and accuracy threshold $\epsilon_2 > 0$.

1: repeat
2: Under given $(t^{(l_2)}, \theta^{(l_2)})$, solving $P3.4$ to obtain $t^*$ and $W^*$, and set $t^{(l_2)} = t^*$, $W^{(l_2)} = W^*$
3: Under given $(t^{(l_2)}, W^{(l_2)})$, solving $P3.8$ to obtain $t^*$ and $\theta^*$, and set $t^{(l_2+1)} = t^*$, $\theta^{(l_2+1)} = \theta^*$
4: Update $l_2 \leftarrow l_2 + 1$
5: until Convergence

$P3.7$ : $\min_{t, V} t$
\[ \text{s.t.} \quad \frac{\text{Tr}(R_k^V)}{\sum_{j=1}^K (\text{Tr}(R_{kj}^V) + |d_{kj}|^2) + \sigma_k^2} \geq \alpha_k(t) \quad \forall k \in K \] (26a)
\[ V_{n,n} = 1, \quad n = 1, \ldots, IN + 1 \] (26b)
\[ V \succeq 0 \] (26c)
\[ \text{rank}(V) = 1 \] (26d)

where $\text{Tr}(.)$ represents the trace operation. $P3.7$ is nonconvex due to the rank-one constraint. To proceed it, we relax this rank-one constraint and solve $P3.7$ by solving the following feasibility problem based on the above analysis, with a bisection search over $t$, which is formulated as:

$P3.8$ : Find : $V$
\[ \text{s.t.} \quad \text{Tr}(R_k^V) + |d_{kj}|^2 \geq \alpha_k(t^l) \left( \sum_{j \neq k} (\text{Tr}(R_{kj}^V) + |d_{kj}|^2) + \sigma_k^2 \right) \quad \forall k \in K \] (27a)
\[ V_{n,n} = 1, \quad n = 1, \ldots, IN + 1 \] (27b)
\[ V \succeq 0 \] (27c)

where $t^l$ is the value of $t$ at the $l_2$th iteration. It is obvious that $P3.8$ is a classical SDR problem, and the optimal solution can be found by CVX solver. However, the SDR may not be tight for $P3.7$. In this case, we can use the Gaussian randomization technique [41] to obtain a feasible solution to $P3.7$.

In summary, we solve $P3.1$ by alternately solving $P3.4$ and $P3.8$, and the detailed procedure is presented in Algorithm 2. For each iteration, $P3.4$ is first solved based on obtained $\theta$ at the previous iteration, and then $P3.8$ is solved based on obtained $W$ in step 3. We start from solving $P3.4$ instead of solving $P3.8$ because $P3.4$ is always feasible under any given $\theta$, but the reverse may not be true. Furthermore, the rigorous proof of convergence is provided in Proposition 2.

**Proposition 2:** The OF of $P3.1$ is monotonically nonincreasing based on the proposed Algorithm 2.

Proof: See Appendix B.

Remark 3: When the solution of $P3.8$ is not rank-one, we need to re-construct a rank-one solution by leveraging the Gaussian randomization procedure. Therefore, the performance of the reconstructed solution depends critically on the generated Gaussian randomization number. For example, due to the uncertainty in randomizations, we may find a highly suboptimal solution to $P3.5$, which will lead to a compromised performance. In addition, to find a better solution, we need to generate a large number of Gaussian randomizations. Besides, solving the SDR problem is time-consuming, especially when the matrix dimension is large. Therefore, in order to reduce the computational complexity and guarantee performance, we need to further develop an efficient algorithm.

**Solving $P3.5$ Based on SCA:** To overcome the drawbacks of the SDR technique, we propose an efficient scheme to update the reflecting beamforming vector $v$ based on the SCA technique. Instead of solving $P3.5$ directly, we try to find a feasible solution $v$ to reduce the maximum edge computing latency. Particularly, for a specific iteration $l \geq 1$, we first define $v^{(l-1)}$ as the value of $v$ obtained at the previous iteration. Then, for given $(\{w_k^l\}, v^{(l-1)})$, the achieved maximum WD's edge computing latency is denoted by $t^{(l)} = \max_{k \in K} D_k^e(\{w_k^l\}, v^{(l-1)})$.

First, according to (23a), we introduce an auxiliary function $F_k$, which is defined as:

$$F_k(v, \{w_k^l\}, t) = \alpha_k(t) \left[ \sum_{j \neq k} (v^H C_{kj} v + 2 \text{Re} \{v^H u_{kj}\} + |d_{kj}|^2) + \sigma_k^2 \right]$$

$$- (v^H C_{kj} v + 2 \text{Re} \{v^H u_{kj}\} + |d_{kj}|^2) \quad \forall k \in K.$$ (28)

If $(v, \{w_k^l\}, t)$ is a set of feasible solutions to $P3.5$, we have $F_k \leq 0 \quad \forall k \in K$. After solving $P3.2$ at each iteration, the maximum $F_k$ at WDs is equal to 0, i.e., $\max_{k \in K} F_k(v^{(l-1)}, \{w_k^l\}, t^{(l)}) = 0$. Consequently, the reflecting beamforming vector can be updated by equivalently solving the following optimization problem:

$P3.9$ : $\min_{v, \{w_k^l\}} \max_{k \in K} F_k(v, \{w_k^l\}, t^{(l)})$

s.t. (23b).

When $\max_{k \in K} F_k(v^{(l)}, \{w_k^l\}, t^{(l)}) < 0$, it can be easily proved that $\max_{k \in K} D_k^e(v^{(l)}, \{w_k^l\}) < \max_{k \in K} D_k^e(v^{(l-1)}, \{w_k^l\})$, i.e., the maximum edge computing latency is reduced. Thus, $P3.5$ can be equivalently solved by solving $P3.9$. Next, we investigate how to solve $P3.9$.

Since the OF of $P3.9$ is nonconvex, it is difficult to solve $P3.9$ directly. Inspired by the SCA technique, we first obtain a convex upper bound on $F_k$ by approximating the second convex term based on its first-order Taylor expansion. Therefore, under given $t^{(l)}$, $(w_k^l)$, and local point $v^{(l-1)}$, the lower bounded of $F_k$ is:

$$F_k(v, \{w_k^l\}, t^{(l)}) \leq \alpha_k(t^{(l)}) \left[ \sum_{j \neq k} (v^H C_{kj} v + 2 \text{Re} \{v^H u_{kj}\} + |d_{kj}|^2) + \sigma_k^2 \right]$$

$$- (v^H C_{kj} v + 2 \text{Re} \{v^H u_{kj}\} + |d_{kj}|^2)$$

$$\quad - 2(\text{Re} \{v^H u_{kj}\} + |d_{kj}|^2) \quad (v - v^{(l-1)})^H (v - v^{(l-1)})$$

$$\triangle F_k^{\text{up}}(v, \{w_k^l\}, t^{(l)}, v^{(l-1)}).$$ (30)
Algorithm 3 Joint Optimization of $t$, $W$, and $\theta$ With SCA Technique

**Input:** Initialize the phase shift vector $v(0)$, set $l_3 = 1$ and accuracy threshold $\epsilon_3 > 0$.

1. **repeat:**
   2. Under given $(t(l_3 - 1), v(l_3 - 1))$, solve $P3.4$ to obtain $t^*$ and $W^*$, and set $t(l_3) = t^*$, $W(l_3) = W^*$.
   3. Under given $(t(l_3), W(l_3))$, solve $P3.10$ to update $v$, and set $v^{l_3} = v^*$.
   4. Update $l_3 \leftarrow l_3 + 1$
   5. **until** Convergence.

Next, we replace $F_k(v; \{w_k^{(l)}\}, t^{(l)})$ by $F_k^{up}(v; \{w_k^{(l)}\}, t^{(l)}, v^{(l-1)})$ and introduce another auxiliary variable $z$, $P3.9$ can be approximated as

$$P3.10 : \min_{v, z} \quad z$$

$$\text{s.t.} \quad F_k^{up}(v; \{w_k^{(l)}\}, t^{(l)}, v^{(l-1)}) \leq z \quad \forall k \in K$$

(31b)

So far, we have transformed $P3.9$ into a convex form, which can be solved by existing convex optimization solvers such as CVX [40]. Suppose that $v^*$ is the optimal solution to $P3.10$. By substituting $(v^*, \{w_k^{(l)}\}, t^{(l)})$ into $P3.9$, it is observed that the feasible solution of $P3.10$ is still feasible to $P3.9$, and thus solving $P3.10$ can achieve a smaller value than solving $P3.9$. The details of the SCA-based algorithm are presented in Algorithm 3.

C. Overall Algorithm to Solve $P1$

In Algorithm 4, we provide the details of the BCD-based algorithm for solving $P1$. It is worth noting that the auxiliary variable $t$, which is also the OF of $P1$, is constantly updated with the implementation of steps 2 and 3. Let $g(\ell(t^{(l-1)}-1), \mathbf{f}(t^{(l-1)}), \mathbf{w}(t^{(l-1)}), \theta(t^{(l-1)}))$ and $g(\ell(t^{(l)}), \mathbf{f}(t^{(l)}), \mathbf{w}(t^{(l)}), \theta(t^{(l)}))$ denote the value of $t$ after implementing step 3 at the $(l-1)$th iteration, and the value of $t$ after implementing step 2 at the $l$th iteration, respectively. It follows that $g(\ell(t^{(l)}), \mathbf{f}(t^{(l)}), \mathbf{w}(t^{(l)}), \theta(t^{(l)})) \leq g(\ell(t^{(l)}-1), \mathbf{f}(t^{(l)}), \mathbf{w}(t^{(l)}-1), \theta(t^{(l)}))$, this is due to that in step 2, $t = g(\ell(t^{(l)}), \mathbf{f}(t^{(l)}), \mathbf{w}(t^{(l)}-1), \theta(t^{(l)}-1))$ can be regarded as the overall maximum WD’s latency, while in step 3, $t = g(\ell(t^{(l)}), \mathbf{f}(t^{(l)}), \mathbf{w}(t^{(l)}), \theta(t^{(l)}))$ refers to the maximum WD’s latency for edge computing. However, the convergence of Algorithm 4 can be guaranteed due to the WD’s overall latency and WD’s edge processing latency are both reduced at different iterations. Hence, we have $g(\ell(t^{(l)}), \mathbf{f}(t^{(l)}), \mathbf{w}(t^{(l)}), \theta(t^{(l)})) \leq g(\ell(t^{(l)}-1), \mathbf{f}(t^{(l)}), \mathbf{w}(t^{(l)}), \theta(t^{(l)}))$.

The termination condition is given by $\epsilon_4(t_{l=4}^{(l)}-\epsilon_4 \leq \frac{L_{t=4}^{(l)}}{L_{total}^{(l)}}$.

To further describe the proposed algorithm, a flowchart is provided in Fig. 3. As the flowchart shows, we first introduce auxiliary variable $t$ to transform problem (7) into problem (8), which is difficult to be directly solved. To proceed, we decouple problem (8) into two subproblems relying on the BCD technique. Specifically, in the first subproblem, we optimize the offloading data size and edge computing resource with fixed communication setting, while in the second subproblem, we optimize the MUD vector based on SOCP technology and the reflecting beamforming vector based on BCD and SCA technology with fixed computing setting. Note that auxiliary variable $t$ is constantly updated at each step using the bisection search method.

IV. SINGLE WD SCENARIO

To further investigate the beneficial role of IRS in the cell-free MEC system, we consider a single WD scenario in this section. Compared with the multi-WD scenario, the single-user scenario is more tractable. This is because 1) the min–max latency optimization problem $P0$ becomes a simple latency minimization problem, which is more tractable; 2) there is no involved computing resource allocation; and 3) there is no multi-WD interference. Thus, the overall latency is reformulated as

$$D(w, \theta, \ell) = \max\left\{\frac{(L - \ell)c}{f^I}, \frac{\ell}{R(w, \theta)} + \ell c / f_{total}\right\}.$$  (32)

On this basis, $P0$ is simplified to

$$P4 : \min_{w, \theta, \ell} \quad D(w, \theta, \ell)$$

s.t. $0 \leq \theta_n < 2\pi, \quad n = 1, 2, \ldots, N$  (33a)

$\ell \in \{0, 1, 2, \ldots, L\}$  (33b)

$\|w\|^2 \leq 1.$  (33c)
Algorithm 4 Joint Optimization of $\ell$, $f'$, $W$, and $\theta$

Input: $h_k$, $B$, $p_i$, $\sigma^2$, $L_k$, $c_k$, $K$, $f_{total}^e$, $e_4$, and maximum number of iterations $l_4^{\max}$.
Output: Optimal $\ell$, $f'$, $W$ and $\theta$

1. Initialization
   Initialize $l_4 = 1$, $e_4^{(0)} \geq 0$
   Initialize $W^{(0)}$ satisfying (7e) $\theta^{(0)}$ satisfying (7a), and calculate $R_k$ according to (5), $\forall k \in K$.

2. Joint optimization of $t$, $\ell$, and $f'$, given $R^{(l_4-1)}$
   Calculate $f^{(l_4)}$ and $\theta^{(l_4)}$ and update $t$ relying on Algorithm 1

3. Joint optimization of $t$, $W$ and $\theta$, given $\ell^{(l_4)}$ and $f^{(l_4)}$
   Calculate $W^{(l_4)}$ and $\theta^{(l_4)}$ and update $t$ relying on the Algorithm 2 or Algorithm 3

4. Convergence checking
   $e_4^{(l_4)} = \frac{\text{obj}(\ell^{(l_4)}, f^{(l_4)}, W^{(l_4)}, \theta^{(l_4)}) - \text{obj}(\ell^{(l_4-1)}, f^{(l_4-1)}, W^{(l_4-1)}, \theta^{(l_4-1)})}{\text{obj}(\ell^{(l_4)}, f^{(l_4)}, W^{(l_4)}, \theta^{(l_4)})}$
   if $e_4^{(l_4)} > e_4$ & $l_4 < l_4^{\max}$ hold then
     Calculate $R^{(l_4)}$ using (5)
     $l_4 \leftarrow l_4 + 1$
     Go to Step 2
   else
     Output $\ell$, $f'$, $W$ and $\theta$
   end if

Like Proposition 1, fixing $w$ and $\theta$, $D(w, \theta, \ell)$ reaches its minimum threshold when $\ell$ is chosen to ensure that local computing latency is equivalent to edge computing latency, i.e., $[L - \ell]c/f' = [(\ell)/(R(w, \theta))] + [(\ell c)/f_{total}^e]$. Thus, the optimal offloading data size is given by

$$\hat{e} = \frac{LcR_{total}^e}{f_{total}^e + cR_{total}^e + f_4^e}. \quad (34)$$

Based on obtaining $\ell$, $\mathcal{P}4$ can be equivalently transformed into

$$\mathcal{P}4.1: \min_{w, \theta} \frac{\ell}{R(w, \theta)} + \frac{\ell c}{f_{total}^e}$$

$$\text{s.t. } 0 \leq \theta_n < 2\pi, \quad n = 1, 2, \ldots, \text{IN} \quad (35a)$$

$$\|w\|^2 \leq 1. \quad (35b)$$

Based on (4) and (5), $\mathcal{P}4.1$ can be simplified to

$$\mathcal{P}4.2: \max_{w, \theta} \frac{P_t}{\sigma^2} \left| w^H (h_d + G\theta_h)^2 \right|$$

$$\text{s.t. } 0 \leq \theta_n < 2\pi, \quad n = 1, 2, \ldots, \text{IN} \quad (36a)$$

$$\|w\|^2 \leq 1. \quad (36b)$$

Next, we jointly optimize $w$ and $\theta$ by using the BCD technique. Specifically, fixing $\theta$, the maximum ratio combining (MRC) technique can be applied to find optimal $w$, which is formulated as

$$w = \sqrt{P_t/(h_d + G\theta_h)}/\sigma. \quad (37)$$

Then, after obtaining $w$, we optimize the reflecting beamforming vector $\theta$. The SINR is upper bounded by

$$\frac{P_t}{\sigma^2} \left| w^H (h_d + G\theta_h)^2 \right| \leq \frac{P_t}{\sigma^2} \left| w^H h_d^2 \right| + \frac{P_t}{\sigma^2} \left| w^H G\theta_h^e \right|^2 \quad (38)$$

The equality in (38) is achieved when $\theta$ satisfies $\arg\{w^H h_d\} = \arg\{w^H G\theta_h\}$. Thus, the optimal $\theta$ is given by

$$\theta = \arg\{w^H h_d\} - \arg\{\text{diag}\{w^H G\} h_r\}. \quad (39)$$

V. Simulation Results

In this section, we evaluate the performance of the proposed algorithms by simulation. Similar to [42], we consider a 3-D system model, which is presented in Fig. 4. Five BSs simultaneously serve two WDs, and WDs can choose to offload a portion of the computing data to MEC nodes for remote execution with the help of two IRSs. It is worth noting that the IRSs are usually installed on the transmitter and receiver side to fully exploit the benefit of IRS. The detailed location settings for BSs and IRSs are given by [8] and provided in the “Location model” block of Table II.

The distance-dependent path loss model is given by [43]

$$L(d) = C_0 \left(\frac{d}{d_0}\right)^{-\kappa} \quad (40)$$

where $C_0 = -30$ dB refers to the path loss at a reference distance $d_0 = 1$ m, $d$ stands for the individual channel distance, $\kappa$ denotes the path loss exponent. The specific setting of “Communication model” is given by [38] and presented.
Then, for the small-scale fading, we consider a Rician fading channel model for all involved channels. Thus, the channel model \( \mathbf{H} \) is given by

\[
\mathbf{H} = \sqrt{\frac{\beta_{\text{UB}}}{1 + \beta_{\text{UB}}}} \mathbf{H}^{\text{LoS}} + \sqrt{\frac{1}{1 + \beta_{\text{UB}}}} \mathbf{H}^{\text{NLoS}} \tag{41}
\]

where \( \beta_{\text{UB}} \) refers to the Rician factor, \( \mathbf{H}^{\text{LoS}} \) and \( \mathbf{H}^{\text{NLoS}} \) denote the LoS deterministic components and the non-LOS Rayleigh fading components, respectively. \( \mathbf{H} \) is equivalent to a Rayleigh fading channel when \( \beta_{\text{UB}} \to 0 \) and a LoS channel when \( \beta_{\text{UB}} \to \infty \). Note that for the WD-BS channel model, we need to multiply the square root of the large-scale fading coefficient by the elements of the small scale fading coefficient. Similarly, the WD-IRS and IRS-BS channels can also be generated by following the above procedure with \( \beta_{\text{UI}} \) and \( \beta_{\text{IB}} \) denoting the Rician factors of them. Furthermore, we set \( \beta_{\text{IB}} \to \infty \), \( \beta_{\text{UI}} \to \infty \), and \( \beta_{\text{UB}} = 0 \) as \[43\]. The default settings of these parameters are specified in the communications model block of Table II. Besides, the computing settings follow \[38\], and are specified in the “Computing model” block of Table II.

In the following, we provide simulation results to evaluate the maximum WD’s latency achieved by our proposed BCD-based algorithm in various simulation environments, and compare them with the following baseline algorithms.

1) **Without IRS**: The reflection matrix \( \Theta \) is set to a zero matrix, while the other optimization variables are optimized using Algorithm 4.

2) **Without Direct Link**: Assume that all direct links between WDs and BSs are completely blocked by some moving or static object, i.e., \( \mathbf{H} = 0 \), then, we optimize all variables using Algorithm 4.

3) **Random Phase Shift**: The IRS phase shift is set as a uniformly distributed random value in the range of \([0, 2\pi]\), while other optimization variables are optimized as Algorithm 4.

Figs. 5–9 show the latency under different parameter settings.

### A. Impact of WDs’ Locations

Fig. 5 shows the latency achieved under different algorithms/schemes versus WDs’ locations. It is observed that, compared to baseline algorithms, our proposed two algorithms can achieve a lower latency, especially when WDs are close to IRSs. Besides, for all considered schemes with IRS, there are two obvious valleys at \( L = 60 \) and \( L = 100 \) m. This is because IRSs can receive stronger signals transmitted from WDs when WDs approach either of two IRSs. Additionally, we observe that the latency increases when WDs are far away from BSs under the “Without IRS” scheme. Compared to without IRS scheme, the reduced latency by the “Random phase shift” scheme is very limited. Furthermore, it is observed that “Without direct link” scheme achieves the highest latency, and the latency gap is larger than that of the proposed schemes when WDs approach one of two IRSs. Therefore, we conclude that deploying IRSs with optimized phase shift design can extend the signal coverage, improve signal transmission environment, and reduce latency.
with the increase of $f_{\text{total}}^e$, while the latency reduces slowly when $f_{\text{total}}^e$ reaches a certain value, i.e., $30 \times 10^9$ cycle/s. This is because when $f_{\text{total}}^e$ is small, the edge processing latency plays a dominant role, while when $f_{\text{total}}^e$ becomes large, the offloading latency dominates. This indicates that, to minimize latency, adding appropriate computing capability at edge server is cost-effective.

### C. Convergence

To show the convergency of the proposed algorithms, we present the latency against the number of iterations $I_o$ in Fig. 7. It is observed that all the considered algorithms enjoy a fast convergence, which verifies its practical implementation.

### D. Impact of WDs’ Transmit Power

Fig. 8 shows the latency versus WDs’ transmit power $P_t$. It can be readily observed that, for all schemes, the increase of transmit power $P_t$ leads to a decrease in latency. Moreover, the reduced latency by employing IRSs is significant when WDs’ transmit power is moderate. To elaborate, when WDs’ transmit power is low (i.e., $-20$ dBm), the signals reflected by IRSs are so weak that the contribution of employing IRSs is limited. In addition, the reduced latency by employing IRSs is also negligible when the transmit power is high (i.e., $20$ dBm), and this is because the WD-BS link is dominant for high-transmit power. Additionally, it can be observed that as the transmit power increases, the latency gap between without direct link scheme and without IRS scheme becomes large. Here, without direct link scheme means that BSs only receive signal transmitted from the WD-IRS-BS links, while without IRS means that BSs only receive signals transmitted from the WD-BS links [43]. This implies that, compared to without direct link scheme, the without IRS scheme can achieve a lower latency. In this case, the BSs will tend to design sophisticated MUD matrix to receive more power from the WD-BS links, which weakens the role of IRSs.

### E. Impact of Intercell Interference

It is worth pointing out that the ICI also affects computing offloading in realistic scenarios. Fig. 9 shows the latency versus ICI-to-noise power ratio, where the BS is assumed to know the received interference power but not the specific signal transmitted from other cells. It is observed that, when the ICI-to-noise power ratio is small, the latency remains unchanged. Whereas, when the ICI-to-noise power ratio reaches a certain threshold, (i.e., $30$ dB), the latency remains unchanged. This is because that, increasing the ICI-to-noise power ratio, WDS will choose to offload less data for edge computing, and when the ICI-to-noise power ratio reaches a certain threshold, all data computing will be performed locally. Therefore, to fully exploit the potential of IRSs and improve the network capacity, the ICI need to be eliminated. Thus, this also demonstrates the necessity of BSs’ collaboration.
VI. CONCLUSION

In this article, we aimed to minimize the maximum WD’s latency by jointly optimizing offloading data size $\ell$, edge computing resource $P$, reflecting beamforming vector $\theta$, and MUD matrix $W$, subject to the edge computing capability constraint and IRS phase shift constraint. To address this nonconvex problem, we developed a BCD-based algorithm, in which the optimization variables related to the computing and communication setting were solved via an alternating iterative manner. Extensive simulation results verified the necessity of BSs’ collaboration and the benefits of deploying IRSs in MEC systems under different parameter settings. Meanwhile, we can conclude that employing IRSs with optimized phase shift design can extend the signal converge and reduce latency. Furthermore, to minimize the latency, adding appropriate computing capability at the edge server is cost-effective. Compared with the conventional MEC system, the latency can be effectively reduced from 160 to 100 ms for distances $L = 60$ and 100 m. Moreover, our simulation results confirmed that our proposed algorithms enjoy a fast convergence, which validates its engineering viability.

APPENDIX A

PROOF OF PROPOSITION 1

For notational convenience, let $\hat{\ell}_k \in [0, L_k]$ denote the relaxation of integer value $\ell_k \in \{0, 1, \ldots, L_k\}$ [44]. In addition, fixing $P$, we denote the overall latency of the $k$th WD by $D_k(\hat{\ell}_k) \triangleq \max\{D_k(\hat{\ell}_k), D_k(\hat{\ell}_k)\}$. Based on (6), $D_k(\hat{\ell}_k)$ can be re-expressed as

$$
D_k(\hat{\ell}_k) = \begin{cases}
(\ell_k - \hat{\ell}_k) c_k \ell_k, & 0 \leq \hat{\ell}_k \leq \frac{L_k c_k R_k f_k}{f_k f_k + c_k R_k (f_k^2 + f_k^2)}, \\
\frac{L_k c_k R_k f_k}{f_k f_k + c_k R_k (f_k^2 + f_k^2)}, & \frac{L_k c_k R_k f_k}{f_k f_k + c_k R_k (f_k^2 + f_k^2)} < \hat{\ell}_k \leq L_k.
\end{cases}
$$

(42)

It is obvious that, when $\hat{\ell}_k$ increases from 0 to $\left\{L_k c_k R_k f_k \right\} / \left\{f_k f_k + c_k R_k (f_k^2 + f_k^2) \right\}$, latency $D_k(\hat{\ell}_k)$ reduces, whereas, when $\hat{\ell}_k$ increases from $\left\{L_k c_k R_k f_k \right\} / \left\{f_k f_k + c_k R_k (f_k^2 + f_k^2) \right\}$ to $L_k$, latency $D_k(\hat{\ell}_k)$ increases. Thus, when we set $\hat{\ell}_k = \hat{\ell}_k^* = \left\{L_k c_k R_k f_k \right\} / \left\{f_k f_k + c_k R_k (f_k^2 + f_k^2) \right\}$, the minimum latency of the $k$th WD is achieved. Note that the value of offloading data size has to be a nonnegative integer, and thus, we can obtain the optimal $\ell_k$ after carrying out the operation

$$
\ell_k^* = \arg\min_{\ell_k} \ell_k \left\{D_k(\hat{\ell}_k^*), D_k(\hat{\ell}_k^*) \right\}.
$$

APPENDIX B

PROOF OF PROPOSITION 2

We define the value of OF in $P3.1$ based on a set of solutions $(W, \theta)$ as $t = g(W, \theta) = \max_{\ell_k \in [0, L_k]} D_k(\hat{\ell}_k, \theta)$. At the $(l)^{th}$ iteration, if $P3.8$ is feasible, the solution $(W(l), \theta(l+1))$ is also a feasible solution to problem $P3.4$. Denote $(W(l), \theta(l))$ and $(W(l+1), \theta(l+1))$ as the optimal solutions to $P3.4$ at the $(l)^{th}$ and $(l+1)^{th}$ iteration, respectively. Since $W(l+1)$ is the optimal solution of $P3.4$, then, we have $g(W(l+1), \theta(l+1)) \leq g(W(l), \theta(l+1))$. As a result, we have the following inequality

$$
g(W(l+1), \theta(l+1)) \leq g(W(l), \theta(l+1)) \leq g(W(l), \theta(l))
$$

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