Power system solution based on root matching method in FPGA environments

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Abstract—Digital electromagnetic transient simulation is an important means to analyze the operation, planning and control of power systems. Numerical integration algorithm is the core of electromagnetic transient simulation analysis. By analyzing the existing electromagnetic transient simulation methods of power systems, it is difficult to solve the numerical oscillation and decoupling problems, and the root matching method is proposed to solve such problems. From the perspectives of basic component modeling methods and basic simulation algorithms, the disadvantages of mainstream algorithms in solving electromagnetic oscillations are analyzed. Combining the calculation and simulation features in FPGA, the three-phase two-level inverter model of the power system built is verified.

1. Introduction

Real-time simulation of the power system needs to reproduce the output (voltage/current) waveforms with the required accuracy, and these waveforms represent the behavior of the actual power system being modeled. In order to achieve this goal, the digital real-time simulator needs to solve a model equation of a time step at the same time in the actual clock[1]. Therefore, it produces output at discrete time intervals, where the system state is calculated using a fixed time step at a specific discrete time. The system is represented by the component models available in the software tool library by using a graphical interface, and simulation is performed on a hardware platform using parallel computing. Since RTDS Technologies Inc. used DSP to demonstrate the first commercial real-time digital simulator (RTDS) in 1991, more and more real-time simulators such as RT-LAB, dSPACE, starsim, etc. appeared on the market[2]. The real-time simulator needs to solve the mesh model with a time step of about 50μs or less to faithfully reproduce the transient. Computing power can be defined as the product of the number of nodes/buses in the simulated power grid and the number of time steps taken per second. It requires more computing power and the ability to simulate in a small time step. However, obtaining these hardware emulation platforms often requires high costs[3]. And these commercial simulation platforms often use FPGAs to build their own company's products. Compared with traditional application specific integrated circuits, FPGA has the advantages of fast hardware speed and flexible software. In the past ten years, the application field of FPGA has been expanding continuously[4]. With the rapid development of the current power electronic technology and the introduction of a large number of power electronic devices in the power grid, many calculation and control problems in the power system have become increasingly complex[5]. At present, the modeling and analysis methods used in the more mature power system simulation platform for power system simulation can be roughly divided into: nodal analysis method, state-space techniques and state space nodal method. SSN)[7]. These three
analysis methods use different numerical calculation methods to solve the circuit. Each algorithm has
its own advantages and disadvantages. Different algorithms show significant differences in response to
model switching events. It is very necessary to choose a suitable algorithm. And the modeling is subject
to three constraints of space scale, time scale, and component complexity. The core part of power system
simulation is the calculation part with matrix calculation as the core. This paper considers the different
modeling analysis methods and matrix multiplication architectures used in power system solving, and
is used in FPGA hardware implementation. The structure of the paper is as follows. Section 2 briefly
summarizes the basic simulation algorithm and component modeling analysis method; section 3
introduces the difference between the root matching method and the above method and its advantages
in component decoupling; section 4 introduces the application of this method in matrix operation of the
electrical systems; section 5 introduces the design and implementation on FPGA.

2. Power system basic simulation algorithm and component modeling methods
For different types of applications, electromagnetic transient simulation can be divided into offline
simulation tools and real-time simulators[8]. Offline simulation tools include various common software
packages, such as EMTP-RV, ATP, EMTDC, MicroTran, etc. In the analysis and calculation of the
transient stability of the power system, the computer simulation of the dynamic process of the power
grid including the components is very valuable, and the basic method is the numerical integration
method[9]. At this stage, the solution of the electrical system is essentially to use different methods for
the solution of the system in the time domain. The main analysis methods are divided into three types:
1. Nodal analysis method, 2. State variable method, 3. State space node method. Since it is possible to
observe the amount of electricity that cannot be measured in a live power system due to technical
requirements, the simulation results are helpful to analyze the impact of the transient (abnormal) state
of the power system operation and also provide valuable data[10]. If you want to use a computer to realize
system simulation, it is necessary to convert a continuous model to a discrete model. There are many
different ways of expressing differentiation and integration, and transformations are not unique.
Therefore, the choice of numerical method directly determines the nature of the discrete model. The
numerical value of the discrete model is uncertain, which is specifically manifested as undamped
oscillation, even if the corresponding continuous model is stable. Arithmetic summaries that affect the
accuracy of numerical calculations may also lead to instability of the discrete model. In the power grid,
the basic differential equation of the branch with linear components (R, L, C) is in the form:

$$\frac{dy(t)}{dt} + \lambda y(t) = bw(t)$$  

(1)

Among them, w(t) is voltage, y(t) is current; λ, b are network parameters respectively.

2.1. Nodal analysis
The use of nodal analysis methods such as RTDS, offline simulation software PSCAD/EMTDC and so
on. This type of software is called EMTP[11]. The root of this type of software is that the transient process
of the circuit is solved by trapezoidal integration.

$$y(k) = y(k - 1) + \frac{Th(w(k) + w(k - 1))}{2}$$  

(2)

The core idea of the implicit ladder method is to replace all component models in the network with
constant equivalent resistances and equivalent current sources that reflect historical records. Then
establish the nodal equation according to this equivalent circuit, form the admittance matrix, and perform
the inversion operation to find the equation solution. The implicit trapezoidal integration method has the
advantages of small error, good numerical stability, local truncation error of o(h(3)), and large step size
simulation. However, when there is a circuit breaker operating in the electrical system, numerical
oscillations may occur.
2.2. State variable analysis

State variable (or state space) analysis uses a set of first-order differential equations to characterize the power system, which is then solved by numerical integration. The solution of the state equation is based on the number of state variables and the number of independent energy storage elements. The identification of state variables can be obtained by establishing a path correlation matrix[11,12]. The ladder column of the branch using Gaussian elimination calculation indicates the state variables, and then the state variables are specified in a special order (such as current source, voltage source, inductance, capacitance, resistance). But there are also certain problems. The system must re-operate every time the system topology changes. Therefore, this method is not suitable when the circuit contains similar switching components frequently breaking. One possible way to reduce the amount of calculation is to divide the power system into two parts: the steady part and the frequent topology change part. Then use the voltage source and the current source to link the two parts of the interface. At present, the simulation software using state variable analysis method includes Simulink, PLECS, etc. There are two main methods for the formation of the main state equations: transformation method and graphic method.

Based on what type of branch is connected to the node, it can be roughly divided into three types: (1) node α is connected to at least one capacitive branch; (2) node β is connected to at least one resistive branch, without capacitance; (3) γ The node is only connected to the inductor. You can use \( A_{RN}, A_{LN}, A_{CN} \) to represent the branch nodes of R, L, and C respectively. The basic branch equation can be derived:

**Resistance branch**
\[
I_r = R_r^{-1}(A_{R\alpha}V_a + A_{R\beta}V_b)
\]

(3)

**Inductive branch**
\[
E_L p(l_i * I_i) - R_i * I_i + A_{l\alpha}V_a + A_{l\beta}V_b + A_{l\gamma}V_c = 0
\]

(4)

**Capacitor branch**
\[
A_{n\alpha}I_c + A_{n\beta}I_r + A_{n\gamma}I_l + A_{ns}I_s = 0
\]

(5)

For calculations within one step, if the integral calculation converges, subsequent calculations need to calculate the dependent variable after calculating the state variable, and then calculate the derivative of the state variable from the state equation.

2.3. State space method

For power systems with short transmission lines, complex networks, and numerous power equipment, it is difficult to ensure that modeling and analysis are completed under the premise of a certain step size and accuracy by using traditional power system model decoupling methods, and the circuits are processed in parallel to achieve real-time The purpose of transformation. The basic principle of the state space method is to divide the circuit model into several sub-networks of arbitrary size, and use the state space method to model and calculate each sub-network, and solve the equivalent circuit corresponding to the network. By dividing the state space group, reducing the number of equivalent electrical nodes in the system, reducing the size of the sub-network, and reducing the number of switches connected to the Zhouwei network in each small grid network, high-level algorithms can be used inside the sub-network. The node method is used between each sub-network to establish an instant solution of the node network equation, and within the same simulation step, the equivalent circuit calculated by each network is combined to obtain the global result of the entire circuit. At the same time, avoid the
space complexity caused by decoupling and the corresponding artificial delay, and improve the simulation accuracy.

According to the known circuit, the differential equation of the circuit can be obtained. The exact solution obtained after the state equation is transformed into discretization is:

\[ x_{k+1} = e^{A\Delta t}x_k + \int_t^{t+\Delta t} e^{A(t-\tau)} Bu(\tau)d\tau \]  

(6)

2.4. Component modeling method

In a power grid with lumped parameters, the basic differential equation describing the dynamic relationship between the physical quantities observed in the branch with linear elements (R, L, C) takes the following form. Resistor is the simplest circuit element. It is shaped like a resistor connected between nodes k and m. It is described by the following equation based on Ohm’s law:

\[ i_{km}(t) = \frac{1}{R} [v_k(t) - v_m(t)] \]  

(7)

Inductance:

\[ v_L = v_k - v_m = \frac{di_{km}}{dt} \]  

(8)

Obtained by using the trapezoidal method:

\[ i_{km}(t) = \frac{1}{R_{eff}} [v_k(t) - v_m(t)] \]  

Similarly, the capacitor can also use the trapezoidal integral to obtain the equivalent current history item:

\[ I_{\text{history}}(t - \Delta t) = -i_{km}(t - \Delta t) - \frac{2C}{\Delta t} [v_k(t - \Delta t) - v_m(t - \Delta t)] \]  

(10)

Any form of implicit integral formula can be brought into the differential equation to form different differential equations and at the same time form the corresponding Norton equivalent circuit. Numerical integration substitution technology can be used to solve electromagnetic transient problems as a universal method. Its main characteristics are simplicity, comprehensiveness of application and high efficiency of calculation. Simple circuit components such as resistors, inductors, capacitors, and various transmission lines can be simplified to simply solve the Norton equivalent circuit with nodal equations. By choosing an appropriate integration step size, numerical integration substitution can be applied to all transient phenomena and any modeling system. However, in some cases, the truncation error of the trapezoidal method based on the truncated Taylor series will cause numerical oscillations. When simulating a power system that includes switches, the network will contain a large number of step responses, causing numerical oscillations.

3. The advantages of root matching method component decoupling

In recent years, as the development of electromagnetic transient programs has gradually deepened, it is inevitable that several major companies have invested a lot of time and energy to maintain their simplicity and efficiency. One of the major characteristics of the power system is that the power devices frequently complete the switching action under the action of the control system, which is also one of the difficulties that needs to be overcome in the development of electromagnetic transient simulation programs. The fundamental cause of numerical oscillations is that when some form of change occurs in the network, the non-state variables have abrupt changes. Methods to eliminate numerical oscillations:

1. Correctly obtain the non-state variables after the mutation; 2. Choose an integral format to avoid non-state variables. The former method is difficult to achieve for large-scale power system calculations. We can only find a way to avoid non-state variables and use integration methods with higher accuracy
instead. The integral method of truncated Taylor series has numerical oscillation problems during simulation, and the alternative method of numerical integration is to use numerical integration.

The purpose of root matching method in eliminating numerical oscillations is to accurately transfer the zeros and poles of the s-plane to the z-plane. Ensure that the continuous time of the simulation is lacking. For the RL series branch, the numerical integration substitution method and the trapezoidal rule can be used to obtain the following branch difference equation:

\[
i_k = \frac{1 - \Delta t R/(2L)}{1 + \Delta t R/(2L)} i_{k-1} + \frac{\Delta t}{2L} (v_k + v_{k-1})
\]

(11)

From the above formula, we can find that the first coefficient term is a first-order approximation of \(e^x\), where \(x = \Delta t R/L\). The second coefficient term is the first-order approximation of \((1-e^x)/2\), and we can see that the elimination of truncation error can be in the form of exponential expression. Then (9) can be expressed as:

\[
i_k = e^{-\Delta t R/L} i_{k-1} + \left(1 - e^{-\Delta t R/L}\right) v_k/R
\]

(12)

Although the difference equation can be obtained through numerical integration, and then the corresponding exponential form can be obtained from the difference equation, it is difficult to identify the original data form from the truncated formula. The root matching method completes the conversion of the continuous time system to the discrete time system mapping, and equivalently converts the s domain to the z domain. The root matching method changes the mapping process, thereby changing the resulting error.

Digital simulation needs to be carried out in the z-domain, or expressed in the form of a transfer function, or based on an equivalent difference equation. When the transfer function is used for research, the bilateral recursive forms are:

\[
\begin{align*}
(a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_m z^{-m})U(z) & = (1 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_m z^{-m})Y(z) \\

\end{align*}
\]

(13)

The corresponding difference equation is:

\[
\begin{align*}
y(k\Delta t) & = (a_0 u + a_1 u_{-1} + a_2 u_{-2} + \cdots + a_m u_{-m}) \\
& - (b_1 y_{-1} + b_2 y_{-2} + \cdots + b_m y_{-m})
\end{align*}
\]

(14)

The poles remain stable in the s-domain on the left half plane, and the corresponding z-domain poles fall inside the unit circle. The purpose of digital simulation for frequency domain conversion is for the correct conversion of zeros and poles. In order to ensure the continuity of the simulation, the final value of the poles and zeros must match the actual system. Therefore, as long as the system is stable, regardless of the step size change, the difference equation will be stable, so \(z^{-1} = e^{-\Delta t}\) transformation equation.

4. Acknowledgments

In power system simulation, electromagnetic transient simulation is one of the main system simulation applications, and it is also a key component of power system safety analysis and operation. The core algorithm of this application is the solution of large-scale linear equations \(Ax = b\). In the third section, through the root matching method, we get a matrix expression that avoids oscillations. Field Programmable Gate Array has the advantages of large on-chip buffer, customizable transmission bandwidth, flexible scheduling, etc.\cite{13}, which can maximize the use of computing power of the computing system. The design of FPGA mainly involves several processes of design input, functional simulation, synthesis optimization, placement and routing, and online debugging. When it comes to matrix operations, the first problem to be solved is to perform matrix inversion operations on FPGA. Convert \(Ax = b\) into the form of \(x = A^{-1}b\), and then solve the value of \(x\) through a multiplier\cite{14}.

Matrix decomposition is to decompose a complex or unidentified matrix into several matrix products with simple forms or obvious features. The decomposed matrix can still effectively reflect some of the
numerical features before the tapping matrix, such as the rank and singularity of the determinant. Value, characteristic value, etc. Most of the process of matrix inversion is implemented according to different matrix decomposition methods. A single matrix triangulation decomposition mainly includes QR decomposition, LU decomposition and Cholesky decomposition. Design three IP cores respectively corresponding to the inverse of the matrix L, the lower triangle multiplying the diagonal matrix IP core and the upper triangle multiplying the lower triangle IP core.

In order to avoid the square root operation, we update the inversion algorithm of the matrix to:

\[
A^{-1} = (LDL^H)^{-1} = (L^H)^{-1}D^{-1}L^{-1} = B^H DB
\]  

(15)

This conversion brings certain advantages. The main performance is that the operation is easy to implement by FPGA, and it can take advantage of the parallel operation of FPGA to only perform multiplication and addition and reciprocal operations, and the results generated in the continuous storage and reading required during the operation can be obtained by Realization of RAM resources in FPGA.

5. Example verification and conclusion

Our design uses a three-phase inverter system simulation to verify. The circuit is divided into three parts: a 3-phase inverter part, the power electronic switch converts the input direct current under the action of the PWM control signal; a filter part; a 3-phase load output part

![Fig.2 Three-phase inverter](image)

For the FPGA board, we choose the ARTIX-7 XC7A35T FGG484ABX2017 (black gold series) board. After comparing the same model simulation we built in Matlab, we can see that except for the inevitable glitch, the rest are basically similar and meet the simulation requirements.

![Fig.3 FPGA on-chip simulation](image)  ![Fig.4 matlab simulation](image)

This section introduces the simulation test verification using the root matching method in the FPGA environment, and initially verifies the feasibility, correctness and effectiveness of the root matching method through the steady-state simulation test of the three-phase inverter circuit.

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