Is Hubble’s Expansion due to Dark Energy

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Abstract

The universe is expanding is known (through Galaxy observations) since 1929 through Hubble’s discovery ($V = HD$). Recently in 1999, it is found (through Supernovae observations) that the universe is not simply expanding but is accelerating too. We, however, hardly know only 4% of the universe. The Wilkinson Microwave Anisotropy Probe (WMAP) satellite observational data suggest 73% content of the universe in the form of dark-energy, 23% in the form of non-baryonic dark-matter and the rest 4% in the form of the usual baryonic matter. The acceleration of the universe is ascribed to this dark-energy with bizarre properties (repulsive-gravity). The question is that whether Hubble’s expansion is just due to the shock of big-bang & inflation or it is due to the repulsive-gravity of dark-energy? Now, it is believed to be due to dark-energy, say, by re-introducing the once-discarded cosmological-constant $\Lambda$. In the present paper, it is shown that ‘the formula for acceleration due to dark-energy’ is (almost) exactly of same-form as ‘the acceleration formula from the Hubble’s law’. Hence, it is concluded that: yes, ‘indeed it is the dark-energy responsible for the Hubbles expansion too, in-addition to the current on-going acceleration of the universe’.

Key words: Cosmology, Dark Energy, Hubble’s expansion
PACS: 98.80.-k, 95.36.+x, 04.20.-q

1 Introduction

The scenario before the discovery, in early 20th century (in 1929), of Hubble’s expansion was almost stagnant & static in astrophysics & cosmology. Though revolutionary changes occurred when Ptolemy geo-centric model was replaced by Copernicus helio-centric model, and advancement made then-after with the work of many scientists (such as Galileo, Kepler, Newton & Einstein) too are notable; but as a whole the research situation was progressing slowly, it was almost...
stagnant. Even the universe was considered, for sure, to be static. Einstein theory, through Friedman's equation, was predicting the expanding (or contracting) universe; but Einstein himself, being over-confident as per the then prevailing belief that the universe is static, introduced a fuzz-factor (cosmological constant \( \Lambda \)) into his equation to make the universe static. Einstein, later, after hearing the Hubble's expansion, regretted the introduction of \( \Lambda \) and said it to be his 'greatest blunder' of life \([1, 2]\). Though Einstein abandoned \( \Lambda \) but some other scientist kept it alive, however, for one reason or the other. The expansion implies that in the past the universe was smaller. If we rewind the conceptual-film of expansion, we reach to a point (singularity) wherein the so-called big-bang occurred. If we run the film forward from the big-bang, after about 1 lac years we pass through an era of de-coupling (of matter & radiation), the relic of which is known as cosmic-microwave-background (CMB) radiation and it is considered as the firm evidence of big-bang theory \([1-3]\). Soon after the big-bang, within extremely-short-time there was a super-rapid exponential-expansion, called inflation \([1-3]\). Hubble's expansion is quite different, however. Much later at the end of 20th century (in 1999), with the Supernovae observations, scientists came to conclusion that universe is not only expanding but accelerating too \([4-9]\). These observations of type Ia supernovae (SNe Ia) suggest that the accelerated expansion of the universe is possibly powered by a smooth energy component with negative pressure dubbed as dark energy. Therefore, the cosmological constant \( \Lambda \) re-appeared once again, as vacuum-energy as a candidate for dark-energy causing the acceleration. Note that the Hubble's expansion was reported in 1929, 70 years earlier than the recognition of dark-energy in 1999 responsible for the current-ongoing acceleration of the expanding-universe. Is the dark-energy also responsible for Hubble’s expansion, in addition to the acceleration of the universe? Moreover, what is the content and constituent(s) of the universe and that what is that which expands in the expanding-universe?

It is rather unbelievable but true, that in general, we even don’t know that ‘we don’t know with what (constituents) the universe is made of’. It is surprising that we hardly know only 4% of the universe. Rest of the universe is said to be made up of 73% of dark-energy and 23% of dark-matter. Though science (including astrophysics & cosmology) has progressed a lot; but even the scientists don’t know exactly what are the dark-energy & dark-matter, albeit a few theories have been suggested in literature. Dark-energy \([4-9]\) is responsible for acceleration of the expanding universe; whereas dark-matter \([10-14]\) is said to be necessary as extra-mass of bizarre-properties to explain the anomalous rotational-velocity of galaxy.
2 About the Content and Constituent(s) of the Universe

How much matter and energy are there in the universe? It is now well established that universe-expansion began with a big-bang \([1-3]\). The ultimate fate of the universe (Fig.1) depends \([1-3]\) on the universe’s matter & energy density \((\rho)\) as compared to a certain value called critical-density \((\rho_c)\). If \(\rho > \rho_c\), the universe is said to be ‘closed’; and its expansion will slow down (decelerate) and start contracting leading finally to a big-crunch (meaning hot-death of the universe). If \(\rho < \rho_c\), the universe is said to be ‘open’; and will expand forever even much faster (leading the universe to cold-death). If \(\rho = \rho_c\), the universe is said to be ‘flat’; and will continue to expand but not that-fast to lead to cold-death soon. The density-ratio (Omega) \(\Omega = \rho/\rho_c\), determines (the nature & fate of the universe) that whether the universe is closed \((\Omega > 1)\), open \((\Omega < 1)\) or flat \((\Omega = 1)\). It has been estimated that the universe would have collapsed (to hot-death) much sooner than the present-age of the universe if \(\Omega > 1\); and it would have cooled down (to cold-death) much earlier than the present-age.
of the universe if $\Omega < 1$. The present-age (14 billion years) constraint of the
universe, compel the scientists to believe that $\Omega = 1$, i.e., the universe must be
flat [1-3].

Once agreed-upon that the universe density $\rho = \rho_c$, the next question arises
that ‘what is the universe made of’? Estimation of visible-type matter like
galaxies, stars, planets etc. hardly leads only to about 2% of $\rho_c$; and when other
all such things like inter-galactic gases, black-hole, white-dwarf, neutron-stars
etc. are also included, the estimate hardly reaches a mere 4% of $\rho_c$. What is then
96% of the remaining-part? It seems invisible and unknown, hence thought as
dark constituent(s). Scientists have, presently, estimated that the major-chunk
of the universe is repulsive-gravity type dark-energy (about 73%) causing the
universe’s accelerated expansion [4-9], and the rest is non-baryonic invisible but
gravitating dark-matter (about 23%) causing anomalous high rotational-speed
of galaxies [10-14].

It is intriguing that most abundant stuff (dark energy) is least understood.
The major candidate for the dark-energy is considered to be the cosmologi-
cal constant $\Lambda$ (vacuum-energy), though other theories have been proposed,
for dark-energy having wide-range origin(s) ranging from tiny-nucleus [15] to
mighty brane-world [16].

3 Acceleration due to dark energy

Initially it was thought that the universe would be decelerating due to gravity
inside, but now it is well established from several clues [4-9] such as Super-
novae observations that the universe is actually accelerating due to repulsive
gravity of dark-energy. The deceleration-parameter $q$ is defined as follows in
Eq. (1). (Note that even though universe is actually accelerating, but the old
name deceleration-parameter retained; but $q$ comes out to be actually negative,
implying that $\ddot{S}$ is positive i.e., universe is accelerating). Note that though
Scale-factor $S$ (ratio of the co-moving distance at previous-time at $Z > 0$ to
the co-moving distance at present-time $Z = 0$) is dimensionless whereas Size-
of-universe (or co-moving-distance) has dimension of length; but sometimes all
these are denoted by the same symbol $S$, mainly in view that the scale-factor is
proportional to the size-of-universe (co-moving-distance) and incidentally both
the words begin with the letter S. Sometimes, as in references [15, 17], scale-
factor and universe-size are denoted by symbol ‘a’ (but we can not use such
symbol here because in this paper ‘a’ is used for acceleration). So it is better for
clarity, if all these are denoted by different symbols as follows; scale-factor as $S$,
co-moving distance between two points as $D$, universe size as $D_{\text{max}}$, all being
function of time due to expansion of universe. Thus scale factor $S(t) = D(t)/D_0$
or simply $S = D/D_0$ where $D$ is the co-moving distance in the past (at $Z > 0$)
and $D_0$ is the co-moving distance at present (at $Z = 0$). Hence it is obvious
that $S$ is proportional to $D$. Universe-size is $D_{\text{max}}$ i.e., the distance of the visi-
ble universe-horizon, such that as per Hubble’s law $V_{\text{max}} = c = H D_{\text{max}}$. (Note that even if universe-tip may be moving with speed higher than light-speed, as it was during inflation, the observable ‘visible’ horizon will be limited by the equation $c = H D_{\text{max}}$).

The deceleration parameter $q$ is conventionally defined by

$$q = -\frac{S \ddot{S}}{S^2}. \quad (1)$$

Putting the experimental (See the Ref. [18]) value of $q = 0.67$, the expression for the acceleration ($a_d$) of the universe (or galaxy as the case may be) due to dark-energy is given by,

$$a_d = \ddot{S} = 0.67 \left( \frac{S^2}{S} \right). \quad (2)$$

But in Eq. (2) what should we use for $S$? It seems for galaxy observations, more appropriately $S$ should be (being proportional) the co-moving-distance $D$, say, between the observed galaxy (say, Andromeda galaxy) from the earth (situated in the Milky-way galaxy). Hence, the galactic acceleration due to dark-energy can be obtained as follows by replacing $S$ by $D$ in the previous equation,

$$a_d = \ddot{D} = 0.67 \left( \frac{D^2}{D} \right). \quad (3)$$

## 4 Acceleration due to Hubble’s expansion

Note that from Hubble’s law of the expansion, velocity is proportional to distance i.e., $V = H D$, $H$ being Hubble’s constant. Hubbles law $V = H D$ is rewritten as $\dot{D} = H D$. This also gives the acceleration

$$\ddot{D} = H \dot{D} = \left( \frac{\dot{D}}{D} \right) \dot{D} = \frac{\dot{D}^2}{D}, \quad (4)$$

which is (almost) exactly of the same-form (equivalent) as the Eq. (3). Thus, this also reinforces the understanding that the co-moving distance $D$ is proportional to scale-factor or vice-verse. This also indicates an important possibility that the Hubbles expansion is due to dark-energy. Therefore, galactic acceleration due to Hubble’s expansion is,

$$a_h = \ddot{D} = \left( \frac{\dot{D}^2}{D} \right). \quad (5)$$

## 5 Is $a_d$ and $a_h$ are same or similar?

The two equations, one for the galactic acceleration $a_d$ (Eq. 3) due to dark-energy and the other for the galactic acceleration (Eq. 5) due to Hubbles law, are
of (exactly) the same-form, possibly it could be equal too, but the accelerations
differ by a little factor of 0.67. The small factor 0.67 may be ignored, albeit
it may due to some reason (coincidence). It, however, seems that the Hubble’s
expansion as well as its acceleration is due to the dark-energy.

6 Is Hubble’s expansion due to aftermath-effect
of Big-bang or due to Dark-energy?

The big question is that whether Hubble’s expansion is just due to the shock of
big-bang & inflation or it is due to the repulsive-gravity of dark-energy? Earlier
it was thought that the Hubble’s expansion is the aftermath effect since big-bang
& inflation, but now it is believed to be (accelerating) due to dark-energy. The
similarity in Eq. (3) & Eq. (5) is not a mere coincidence; it indicates that
Hubble’s expansion is indeed due to dark-energy. The expansion occurred much
earlier; the acceleration, however, is comparatively a recent phenomenon much
later (at Z = 0.5) than the galaxy-formation era (at Z = 3), as described in
reference [15] in accordance with the astrophysical constraints [19]. The common
understanding of the universe-expansion is through re-introducing the once-
discarded cosmological-constant Λ; though there are other theories/candidates
[20] for dark-energy such as scalar-field, nuclear-energy, brane-world. What is
that which is expanding? It is the fabric of universe or the scale-factor S (=
D/D0) or the co-moving distance D or its maximum-value Dmax (the universe)
expanding, as mentioned in the section-3.

7 The 2/3 Coincidences

There are several coincidences of appearance of a factor 2/3 (= 0.67 or nearly70%)
in the physics & cosmology (which may have some reasons, a few of which are
explained in reference [19]). These 2/3 coincidences are listed as follows. There
seems to be some reasons for it (as just indicated), or these coincidences of 2/3
are purely by chance (coincidence)? The regime of the 2/3 coincidences appear
to be too wide, ranging from tiny strings & quark to mighty universe & cosmos.

• For dark-energy (quintessence) the equation of state parameter \( w = -\frac{2}{3} \).

• Dark-energy is about 2/3 (about 70%) of the universe-content.

• Deceleration parameter (Experimental [18]) \( q = -0.67 \) i.e., \( q = -\frac{2}{3} \).

• As per the Eqs. (3 & 5) mentioned in this paper \( a_d = 0.67 a_h \) or \( a_d = \frac{2}{3} a_h \).

• Hubble’s age \( t_H = 1/H \) and present age \( t_0 \) (for matter universe) are
related as \( t_0 = (2/3)t_H \). Most likely this could be to the answer for the
difference of 0.67 (= 2/3) in the equations (3 & 5).
- Our Solar-system is situated at about \( \frac{2}{3} \) radial-distance from the centre of the milky-way galaxy.

- Planetary-orbit \( R \) and its time-period \( T \) are linked with \( \frac{2}{3} \) power Kepler’s law as \( R \propto T^{2/3} \).

- About \( \frac{2}{3} \) of the earth-surface is covered by water (sea).

- Up-quark has a fractional charge of \( +\frac{2}{3} \).

- In 10-dimensional super string theory, the ratio of the usual 4-dimensional space-time to the 6 curled-up dimensions is \( \frac{4}{6} = \frac{2}{3} \).

8 Conclusions

Hubble’s expansion is a slow expansion of universe and from galactic-observation Hubble’s famous formula is \( V = H D \) or \( \dot{D} = H D \). This also leads to a galactic-acceleration \( a_h = \ddot{D} = (\dot{D}^2/D) \). This incidentally is of the same-form as obtained by the formula of deceleration (acceleration) parameter; the acceleration caused by dark-energy is as \( a_d = \ddot{D} = 0.67(D^2/D) \). These two equation are of exactly same-form, except by a factor of \( 2/3 \), the coincidence of this yet to be fully explained. Moreover, in view of the striking-similarity in the equations (3 & 5), it is concluded that: yes, ‘it is the dark-energy responsible for Hubble’s expansion too, in-addition to the current on-going acceleration of the universe’.

Acknowledgements

The authors thank Sushant Gupta for his comments & view. The authors also thank UPTU, Lucknow, GLA University, Mathura and IUCAA, Pune for direct & indirect support & facility.

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