ON LIMITING THE THICKNESS OF THE SOLAR TACHOCLINE

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Received 2010 December 20; accepted 2011 February 25; published 2011 April 28

ABSTRACT

We present axisymmetric simulations of the coupled convective and radiative regions in the Sun in order to investigate the angular momentum evolution of the radiative interior. Both hydrodynamic and magnetohydrodynamic models were run. We find an initial rapid adjustment in which the differential rotation of the convection zone viscously spreads into the radiative interior, thus forming a “tachocline.” In polar regions, the subsequent spread of the tachocline is halted by a counterrotating meridional circulation cell which develops in the tachocline. Near the equator such a counterrotating cell is more intermittent and the tachocline penetration depth continues to increase, albeit more slowly than previously predicted. In the magnetic models, we impose a dipolar field initially confined to the radiative interior. The behavior of the magnetic models is very similar to their non-magnetic counterparts. Despite being connected to the convection zone, very little angular momentum is transferred between the convective and radiative regions. Therefore, while it appears that a magnetic field is not necessary to stop the tachocline spread, it also does not promote such a spread if connected to the convection zone.

Key words: hydrodynamics – magnetohydrodynamics (MHD) – Sun: rotation

Online-only material: color figures

1. INTRODUCTION

Helioseismic observations (Brown et al. 1989; Goode et al. 1991) have revealed a surprising picture of the solar rotation profile. These observations show that the differential rotation observed at the surface, with the equator spinning faster than the poles, persists to the base of the convection zone with very little radial variation. On the other hand, the radiative interior is revealed to be rotating uniformly. The transition between these very different rotation profiles is now commonly known as the tachocline, with a still undetermined extent, although Elliott & Gough (1999) estimate a width of 0.019 $R_{\odot}$.

The observed rotation profile is puzzling for many reasons. First, it was previously thought that the rotation profile of the solar convection zone would obey the Taylor–Proudman (TP) constraint, in which rotation is constant on cylinders. Indeed, it has proven quite difficult to break this constraint and recover differential rotation constant on cones, similar to the observed profile. Rempel (2005) showed that the TP constraint could be overcome with a latitudinal entropy gradient imposed in a subadiabatic tachocline. This thermal wind would break the TP constraint and lead to more realistic differential rotation in the convection zone. Follow-up numerical experiments by Miesch et al. (2006) showed that indeed, when a latitudinal temperature gradient of order 10 K is imposed as a boundary condition at the base of their model convection zone, solar-like differential rotation is recovered.

The studies of the convection zone differential rotation discussed above suggest the importance of the underlying tachocline and adjacent stable region in understanding the differential rotation of the convection zone. This leads to the second (and third) puzzling aspect of the solar rotation profile. Despite the constant forcing by the overlying differential rotation of the convection zone the tachocline remains thin and the radiative interior is maintained in uniform rotation. Although a thin tachocline does not guarantee a uniformly rotating radiative interior, the two questions have often been treated as one. In the defining work on the tachocline, Spiegel & Zahn (1992) attribute the thinness of the tachocline to anisotropic turbulence. They argue that, similar to Earth’s atmosphere, the solar tachocline, lying mostly in the stably stratified interior, opposes vertical turbulence. They show that the width of the tachocline is then determined by the degree of anisotropy in the turbulence. Simulations carried out by Elliott (1997) confirmed this trend.

An alternative solution was offered by Gough & McIntyre (1998). They argue that there are many examples in Earth’s atmosphere in which two-dimensional (2D) turbulence of the sort envisioned by Spiegel & Zahn (1992) conserve potential vorticity and thus drive a system away from uniform rotation. These authors take the helioseismic results as an indication that there must exist a large-scale magnetic field in the radiative interior. Such a field would easily establish uniform rotation of the radiative interior via Ferraro’s isorotation law (Ferraro 1937) and a thin tachocline could be formed as something of a magnetic boundary layer.

The magnetic solution to the rotation profile of the radiative interior was not unique to Gough & McIntyre (1998); it was investigated by other authors (Charbonneau & MacGregor 1993; Rudiger & Kitchatinov 1997; MacGregor & Charbonneau 1999). In particular, MacGregor & Charbonneau (1999) elucidated a problem with this theory that is now known as the “confinement” problem. That is, magnetic field naturally diffuses and any remnant field in the radiative interior would diffuse into the convection zone. Once the field is in contact with the differentially rotating convection zone, a toroidal field and an associated Lorentz force act to spread the differential rotation of the convection zone into the radiative interior. The contribution of Gough & McIntyre (1998) was to suggest that this diffusion could be controlled by meridional circulation driven in the convection zone and burrowing into the tachocline, effectively holding down the magnetic field. Subsequent studies on the ability of a meridional flow to confine the tachocline have produced mixed results: Garaud (2002) and Kitchatinov & Rüdiger (2006) initially showed that such confinement by meridional circulation was possible. Time-dependent, three-dimensional (3D)
simulations by Brun & Zahn (2006) indicated that it was not. The differences in these works amounted to how deep the meridional circulation, driven in the convection zone, could penetrate into the radiative interior, and with what strength. Kitchatinov & Rüdiger (2006) estimated that a flow speed of $10^3$ cm s$^{-1}$ penetrating $10^8$ cm would be sufficient to confine the field. The strength of meridional circulation and its ability to penetrate the stable tachocline depends, in turn, on how the boundary condition with the convection zone is treated. In all of these studies only the radiative interior was modeled, with the convection zone treated purely as a boundary condition which imposed the differential rotation. Garaud & Brummell (2008) showed that the depth of penetration of meridional circulation into the radiative interior and its strength depended sensitively on the boundary conditions assumed. In reality, the convection zone imposes a complex boundary condition with (at least) large-scale inflows and outflows, penetration of plumes, and pumping of magnetic field. However, coupling the convection zone and radiative interior is a difficult undertaking because of the vast discrepancy in the governing timescales for the convective and radiative zones in the Sun. Here we report on axisymmetric, time-dependent simulations which solve self-consistently both the convection zone and the underlying radiative region.

2. NUMERICAL MODEL

2.1. Equations

The numerical method we employ here is identical to that described in Rogers (2011), here we give a brief review and refer the reader there for more details. We solve the full set of axisymmetric MHD equations in the anelastic approximation:

$$\nabla \cdot (\overline{\rho} \mathbf{u}) = 0. \tag{1}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{2}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla P - C \overline{g} \overline{r} + 2(\mathbf{u} \times \Omega) + \frac{1}{\rho} \left( \mathbf{j} \times \mathbf{B} \right) + \overline{v} \left( \nabla^2 \mathbf{u} + \frac{1}{3} \nabla (\nabla \cdot \mathbf{u}) \right) \tag{3}$$

$$\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T = -u_r \left( \frac{d T}{d r} - (\gamma - 1) T h_r \right) + (\gamma - 1) T h_r u_r + \gamma \kappa \left[ \nabla^2 T + (h_p + h_v) \frac{\partial T}{\partial r} \right] \tag{4}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}. \tag{5}$$

Equations (1) and (2) ensure the mass and magnetic flux are conserved. In Equation (3), the momentum equation, $C$ denotes the co-density (Braginsky & Roberts 1995; Rogers & Glatzmaier 2005), $g$ the gravity, $\Omega$ the rotation rate, and $\overline{v}$ and $\overline{\kappa}$ the viscous and thermal diffusivity, respectively. The inverse density and thermal diffusivity scale heights are denoted by $h_p$ and $h_v$, $\gamma$ is the adiabatic index, and all other variables take their usual meanings. In all of the above equations, reference state variables are denoted by an overbar. Equation (4) represents the energy equation written as a temperature equation, where $u_r$ represents the radial velocity. The first term on the right-hand side of Equation (4) represents the difference between the reference state temperature gradient and the adiabatic temperature gradient and allows us to represent strongly subadiabatic regions in addition to convection zones. Equation (5) is the magnetic induction equation, in which $\eta$ represents the magnetic diffusivity. The diffusivities have the following forms:

$$\overline{\kappa} = \kappa_m \frac{16 \sigma T^3}{3 \rho k c_p^2}$$

$$\overline{\nu} = \frac{v_m}{\rho}$$

$$\eta = \text{const},$$

where $\kappa_m$ is $10^6$, $v_m$ is $5 \times 10^{11}$, and $\eta$ is $5 \times 10^{12}$, giving units of cm$^2$ s$^{-1}$. Values at the base of the convection zone are shown in Table 1. It is worth mentioning here that because our thermal diffusivity, $\kappa$, is $10^6$ larger than solar the flux through the simulation is $10^6$ times solar. Using mixing length theory this would imply velocities which are $(10^6)^{1/3} \approx 100$ times solar.

These equations are solved in axisymmetric spherical coordinates using a spherical harmonic decomposition in latitude and a finite difference scheme in the radial direction. The maximum spherical harmonic degree, $l_{\text{max}}$, is 340, so that the grid resolution is 512 zones in latitude. The radial resolution is 1500 zones, with 600 zones dedicated to the radiative interior and 900 zones covering the convection zone. We solve the above equations from $0.15 R_\odot$ to $0.95 R_\odot$. The radiative region extends from $0.15 R_\odot$ to $0.71 R_\odot$ and the convection zone extends from $0.71 R_\odot$ to $0.9 R_\odot$, for numerical reasons we impose an additional stably stratified region from $0.9 R_\odot$ to $0.95 R_\odot$. The reference state thermodynamic variables are taken from a polynomial fit to the standard solar model from Christensen-Dalsgaard et al. (1996). We should note that because of the convection and the gravity waves in the deep radiative interior our time step is severely limited and therefore, we are unable to run these simulations very long. Our average time step is approximately 10 s and all of these simulations have run approximately a few hundred thousand processor hours.

2.2. Model Setup

We recognize that without fully 3D simulations we would be unable to recover the observed solar differential rotation profile.
in the convection zone. However, we are mainly interested in studying the response of the underlying radiative interior to this differential rotation in the presence of physical phenomena such as convective overshoot and magnetic pumping. Therefore, as in Rogers (2011) we artificially force the differential rotation of the convection zone to be that inferred from helioseismology (Thompson et al. 1996) and described by

$$\Omega_c = A + B \cos^2 \theta + C \cos^4 \theta$$  \hspace{1cm} (7)

where $A$ is 456 nHz, $B$ is $-72$ nHz, and $C$ is $-42$ nHz. Unlike Rogers (2011), we force the differential rotation only in the convection zone, with a step function to the uniform rotation relative to the constant as where the last term, $F$, represents the forcing function, defined as

$$F = \begin{cases} \frac{1}{7} \left( \Omega'(r, \theta) - \frac{u(r, \theta)}{r \sin \theta} \right) & \text{for } r > 0.71 R_\odot \\ 0 & \text{for } r < 0.71 R_\odot \end{cases}$$  \hspace{1cm} (9)

where $\Omega'(r, \theta)$ represents the observed differential rotation relative to the constant $\Omega$ and $\tau$ is a forcing timescale. In all simulations, $\Omega = 441$ nHz and $\tau = 10^3$ s.$^2$

We ran three models: one purely hydrodynamic model and two magnetic models. In the magnetic models, we start with a hydrodynamic model until convection is sufficiently established at which point we imposed a dipolar field with the form

$$B_1 = B_0 r^2 \left( 1 - \frac{r}{0.71 R_\odot} \right)^2$$  \hspace{1cm} (10)

so that initially all field lines close just beneath the convection zone. Figure 1 shows a schematic of the initial setup and model, reproduced from Rogers 2011. Although we ran models with two magnetic field strengths, $B_0 = 40G$ and $B_0 = 4000G$, all of the results presented will be only for the strong field case because the weak field case is virtually identical to the non-magnetic case.

In all models, the boundary conditions are stress free and impermeable at the top and bottom boundaries. The temperature boundary conditions are constant temperature perturbations on top and bottom and the magnetic field is matched to an internal potential field at the bottom of the domain and an external potential field at the top of the domain. The current density, $J_1$, is zero on the top and bottom boundary.

1 While we have run models both with an imposed tachocline and those without, we feel that imposing a tachocline does not accurately represent the situation we are trying to investigate. This is because if a tachocline is forced, there is no stress presented to the radiative interior.

2 We have run models with $\tau = 10^3$ and $10^4$ for a limited time (not as long as the full simulations presented here, but much longer than the forcing time) and see very little difference.

Figure 1. Model schematic (reproduced from Rogers 2011). Radiation zone occupies the inner 75% of the simulated domain, with convection occupying the outer 25%. In the magnetic models, a dipolar field is imposed in the radiative interior (field lines shown). Gravity-Alfvén waves are generated in the radiative interior by impinging plumes.

3. RESULTS

3.1. Meridional Circulation

The fundamental assumption of the Gough & McIntyre (1998) model is that meridional circulation driven by the differential rotation in the convection zone is able to penetrate (at least minimally) the radiative interior and confine the magnetic field. One of the persistent issues in testing this model has been regarding how deep this meridional circulation penetrates the radiative interior, if at all. Gilman & Miesch (2004) argued that the depth of penetration of the meridional circulation driven within the convection zone was limited to an Ekman depth and therefore, would penetrate no further than the overshoot layer. Garaud & Brummell (2008) showed that the circulation driven beneath the convection zone depends strongly on the boundary conditions imposed at the top of the radiative layer and that radial forcing at the interface could produce small but sustained flows deep within the interior.

The simulations presented here couple the convective and radiative regions. Moreover, the radiative interior is sufficiently “stiff,” using the Brunt–Väisälä frequency from the standard solar model. Therefore, these simulations are able to address the penetration of meridional circulation in the most self-consistent manner to date. Furthermore, because we artificially impose the differential rotation in the convection zone we expect a faithful representation of the meridional circulation driven in the convection zone from the differential rotation.

In Figure 2 we show the latitudinal velocity, both in time average and time snapshots. In this figure, red represents flow toward the south pole, while blue represents flow toward the north pole. One can immediately see the poleward flow at the surface, with equatorward flow near the base of the convection zone (the overlaid white line in panels (a) and (b) shows the base of the convection zone). In both the magnetic and non-magnetic cases,
Figure 2. Time-averaged latitudinal flow in (a) non-magnetic and (b) magnetic models. Red represents flow toward the south pole, while blue represents flow toward the north pole. The white overlaid line in (a) and (b) represents the base of the convection zone. Flow amplitudes are approximately $9 \times 10^4$ cm s$^{-1}$ at the top of the convection zone and $5 \times 10^4$ cm s$^{-1}$ near the base of the convection zone. One can also see hints of internal gravity waves, particularly in the non-magnetic case and in the time snapshots (these can be seen more clearly in Figure 3).

While Figure 2 clearly shows the general circulation setup in the convection zone, it is hard to discern the amplitudes of the meridional flows within the radiative interior. In Figure 3, we show these flows more clearly. In that figure, we show the absolute magnitude of both the radial and latitudinal velocities as a function of radius below the convection zone for three different latitudes. Very little latitudinal dependence is seen. However, one can clearly see a rapid decrease in the depth of the flow velocities, which are down nearly to five orders of magnitude by $0.60 R_\odot$. Figures 3(a) and (c) show this amplitude for non-magnetic cases while Figures 3(b) and (d) show it for the magnetic cases. One can see a slightly faster decay in the magnetic cases as well as shorter vertical wavelengths of the mixed gravity-Alfvén waves, which is expected given the dispersion relation for these mixed waves (MacGregor & Rogers 2011).

According to the analytic expression derived by Garaud & Acevedo-Arreguin (2009), the amplitude of the radial

\[ 3 \text{ The amplitudes of the meridional circulation here are larger than those observed in the solar convection zone because of an increased thermal diffusivity, as discussed in Section 2.1. While this is not ideal, our overestimate of the flow speeds likely leads to enhanced penetration of flows. Weaker flows would lead to less penetration.} \]

Figure 3. Absolute magnitude of the time-averaged radial and latitudinal velocities in non-magnetic (a and c) and magnetic (b and d) models in cm s$^{-1}$. Different line types represent different latitudes: $70^\circ$ (solid line), $40^\circ$ (dotted line), and $10^\circ$ (dashed lines). We clearly see exponential decay of the amplitudes as well as the signs of gravity waves. Amplitudes in magnetic models show slightly faster decay and waves have shorter vertical wavelengths.
velocity should fall off exponentially with a length-scale determined by the parameter $\sigma = \sqrt{PrN/\Omega}$, where $N$ is the Brunt–Vaisala frequency. In these simulations the Prandtl number and Brunt–Vaisala frequency, $N$, vary substantially over the domain, so a direct comparison is difficult, but using $N \approx 100 \mu Hz$ and a Prandtl number of 0.05 (the values just beneath the convection zone), the analytically predicted length-scale is $\approx 9 \times 10^8$ cm. Investigation of the data plotted in Figure 3 gives a length-scale of $\approx 4 \times 10^8$ cm, in fair agreement with the analytic prediction (given the simplified comparison).

By forcing the differential rotation in the convection zone, we are artificially breaking the Taylor–Proudman constraint there. As predicted by Rempel (2005) and tested by Miesch et al. (2006) in 3D simulations, breaking the Taylor–Proudman constraint could be achieved by a latitudinal temperature gradient within the tachocline (or as a bottom boundary condition in the case of the simulations). Here we can ask the question in reverse; given the imposed differential rotation, is a latitudinal temperature gradient recovered? In Figure 4 we show the temperature fluctuation as a function of latitude at three radii within the tachocline (the tachocline will be defined in the next section; see figure caption for actual depths beneath the convection zone). Here we see poles which are much warmer than the equator at the top of the tachocline (solid line) and slightly lower in the tachocline (dotted line). However, we also see a fair amount of variation at lower latitudes. In addition, near the middle of the tachocline (dashed line) this temperature variation in latitude switches sign (particularly at high latitudes), so that the poles are slightly cooler than the equator.

Investigation of Figure 2 indicates that the middle tachocline (where the latitudinal temperature gradient changes sign) is also associated with a meridional circulation cell which rotates counter to the main meridional circulation driven in the convection zone. This counterrotating cell (CRC) is robust in the polar regions. Near the equator the cell is more time dependent, but longer time averages show the CRC becoming more prevalent. We should note that such a CRC has been seen in 3D simulations by Miesch et al. (2010); however, in that work they attribute this cell to their artificial impermeable boundary at the base of their convection zone. We confirm here the presence of such a cell even under realistic boundary conditions. This CRC has important implications for the spread of the tachocline as we will discuss in Section 3.4.

3.2. Tachocline Penetration Depth

In the simulations discussed here we do not impose a tachocline, but instead want to investigate the spread of differential rotation from the convection zone into the radiative interior. Thus, we look at the formation of the tachocline and its evolution in time. Figure 5 shows both time-averaged and time snapshots of the azimuthal velocity. One can clearly see that the differential rotation quickly penetrates into the radiative interior. However, between Figures 5(e) and (f), there is very little change in that penetration, although Figure 5(f) represents a substantial increase in time. Given that the observationally determined tachocline coincides with the region of strongest radial shear in the azimuthal velocity, one way to measure the tachocline depth is by looking at this shear. Looking at $|\partial(v_\phi/r)/\partial r|$ as a function of radius and time we can then measure the tachocline penetration depth as the depth beneath the convection zone at which this is half its peak value. This is shown in Figure 6, which shows the radial gradient of angular velocity as a function of radius at both high (left-hand panels) and low (right-hand panels) latitudes at various times. The dashed lines represent the secondary peak and the solid lines represent half of that peak. If the tachocline penetration depth is measured as the distance between the base of the convection
Figure 6. Absolute value of the radial gradient of the angular velocity as a function of radius beneath the convection zone for the non-magnetic model (solid line) and the magnetic model (dotted line), in units of cm s$^{-1}$. The low latitude values are averaged from 60° to 120° and the high latitude values are averaged from 0° to 30° and 150° to 180°. The half-maximum value is defined as half of the amplitude at the secondary peak (denoted by the vertical dashed line) and marked with the vertical solid line. Defining the tachocline depth as the distance between the base of the convection zone and this position, the tachocline penetration depth is measured as 0.021 $R_\odot$ for high latitudes and 0.022 $R_\odot$ for low latitudes in the non-magnetic model.

The tachocline thickness at a given latitude is by the depth beneath the convection zone at which the angular velocity perturbation (initially zero) is $X$% of its forced value at that latitude within the convection zone. We plot this value for $X = 50\%$ in Figure 7, and note that the general behavior in time at different values of $X$ is the same even if the amplitude is not.

Figure 7 shows the tachocline thickness, measured this way, as a function of time, averaged over polar regions (0°–30°) and equatorial regions (60°–90°) for the non-magnetic case (solid line) and the strong field case (dashed line). Also shown in that figure is the time derivative of the tachocline penetration depth as a function of time (panels (b) and (d), and zoomed in over the last half of the simulation in the insets). The first obvious feature of these results is that the tachocline thickness increases rapidly, and subsequent evolution is slow. In fact, in the polar regions the tachocline penetration depth has stopped increasing, as can be seen in Figure 7(a), as well as Figure 7(b), where the time derivative is sometimes negative. In the equatorial regions the tachocline penetration depth is still increasing, though at a very slow rate, approximately 1–5 cm s$^{-1}$ (as seen in the insets of panels (b) and (d) of Figure 7), which would take longer than the solar age to spread through the entire solar radiative interior. Measuring the tachocline depth

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4 This provides an upper limit for this way to measure the tachocline depth. If we had measured from the secondary peak to its half max, the depth would be much smaller.
Figure 7. Tachocline thickness as a function of time for the non-magnetic model (solid line) and the strong magnetic case (dashed line) in cm. This thickness is measured as described in the text and is averaged over the polar regions (0°−30°, panel (a)) and the equatorial regions (60°−90°, panel (c)). The time derivative of the tachocline penetration depth is shown in panels (b) and (d) in cm s$^{-1}$. Polar regions show that the tachocline thickness has stopped increasing. Lower latitudes still show a slow increase. The insets in panels (b) and (d) show the time derivative of the tachocline penetration depth over just the last half of the simulation.

this way, we recover tachocline depths of $\approx 8 \times 10^9$−$10^{10}$ cm or 0.12−0.16 $R_{\odot}$.

The tachocline widths measured this way are larger and inconsistent with helioseismic results. However, given the diffusivities used in these simulations a thicker tachocline is not surprising. Furthermore, the most important feature is that the tachocline thickness has virtually stopped spreading (particularly at high latitudes). This is seen in both methods for determining tachocline thickness (Figures 6 and 7). At high latitudes, the tachocline thickness at the end of the simulation is 97% that at half the simulation time for the non-magnetic case and 98% for the magnetic case. At low latitudes the tachocline thickness is 2% higher at the end of the simulation compared to that at half the simulation time for both magnetic and non-magnetic cases.

The viscous spread of the tachocline should give a tachocline thickness $\Delta \approx R(t/t_\nu)^{1/2}$. After $4 \times 10^7$ s (the time of initial rapid tachocline spread in these simulations) and using the viscous diffusivity at the top of the tachocline, the viscous depth is approximately 0.16 $R_{\odot}$ (10$^{10}$ cm), using the viscous diffusivity at the bottom of the tachocline this depth is approximately 0.08 $R_{\odot}$ (5.5 $\times 10^9$ cm), in excellent agreement with the tachocline thicknesses seen here. However, if we were to assume viscous spread over the entire simulation time, we would get tachocline penetration approximately equal to the depth of the radiative interior.

If, on the other hand, the subsequent evolution of the tachocline (after the rapid initial spread) obeyed the circulation-enhanced, "hyper-diffusive" spreading described in Spiegel & Zahn (1992) in which $\Delta \propto t^{1/4}$, then the tachocline penetration depth would be approximately 1.7 times larger at the end of the simulation than it is at one-third of the simulation time. This is also not seen, clearly indicating that something is preventing both the viscous and the thermal hyper-diffusion spread of the tachocline, even in the non-magnetic case.\footnote{The initial rapid adiabatic response predicted by Spiegel & Zahn (1992) occurs on a timescale of approximately $2 \times 10^6$ s, shorter than what we see here. Furthermore, the depth over which it extends is predicted to be approximately $4 \times 10^7$ cm, again shorter than what we see here.}

In the following sections, we refer to the "tachocline" in these models as defined by the radial gradient of the azimuthal velocity or 0.02 $R_{\odot}$. Although we recognize that there are other ways to define the tachocline depth (as studied above) we find that the relevant force balance which stops the tachocline spread occurs in this narrow depth. We now turn to the question of what is stopping the spread of the tachocline.

\footnote{In these simulations the Prandtl number $\nu/\kappa$ in the tachocline is approximately 0.05, but varies with depth. This is not low enough to ensure that the spread due to thermal hyper-diffusion is larger than that due to viscous diffusion, as in the Sun.}
3.3. The Spiegel and Zahn Model

The defining work on the tachocline by Spiegel & Zahn (1992) proposed that the spread of the tachocline could be stopped by an anisotropic turbulence, akin to that seen in Earth’s atmosphere. This is argued based on the fact that the bulk of the tachocline lies within the radiative interior, where the Brunt–Vaisala frequency is large and likely inhibits substantial radial motion. Since our non-magnetic model also shows a halt of the tachocline penetration depth this seems a likely candidate. We can test this proposal by investigating the stresses within the tachocline. The Reynolds stress can be broken into mean and fluctuating components, such that

\[ \nabla \cdot (\bar{u}_{\phi} u_{\phi}) = \nabla \cdot (\bar{u}_{\phi}' u_{\phi}) + \nabla \cdot (\bar{u}' u_{\phi}), \]  
(11)

where \( \bar{u}_{\phi} \) represents a time average and \( u_{\phi}' \) represents the fluctuation, so that \( u_{\phi} = \bar{u}_{\phi} + u_{\phi}' \).

The first term on the right-hand side of Equation (11), the divergence of the Reynolds stress, can be split into horizontal and radial terms and we can ask if there is any anisotropy:

\[ \nabla \cdot (\bar{u}_{\phi}' u_{\phi}) = \frac{1}{r^2} \frac{\partial}{\partial r} (\bar{u}_{\phi}' u_{\phi} r) + \frac{\dot{u}_{\phi}' u_{\phi}}{r} + \frac{u_{\phi}' u_{\phi} \cot \theta}{r}. \]  
(12)

The anisotropy envisioned by Spiegel & Zahn (1992) would then require that the ratio of the radial to latitudinal terms on the right-hand side of Equation (12) be small. We show this ratio in Figure 8 as a function of latitude at various depths within the tachocline. In both models and at all radii evaluated in the tachocline there is no detectable anisotropy of the kind envisioned in Spiegel & Zahn (1992). This could be due to our lack of resolution, so we cannot rule this model out completely, but we can say it is not responsible for the halt of the tachocline spread seen here. In fact, in much of the tachocline these Reynolds stresses are small compared to other terms in Equation (8) (see the next section).

3.4. Force Balance

The above discussion then implies that the Spiegel & Zahn (1992) model is not at work in these simulations. Furthermore, given that the tachocline spread is halted in the non-magnetic cases, we also see that a magnetic field is not necessary in stopping the tachocline spread.\(^7\) This leads to the important question of what is responsible for limiting the tachocline penetration depth. For an answer, we can look directly at the force balance within the tachocline to determine which forces are responsible for stopping this spread. The relevant equation is the azimuthal component of the momentum equation given by

\[ \frac{\partial}{\partial t} \left( \frac{u_{\phi}}{r \sin \theta} \right) = -\frac{1}{r \sin \theta} (\nabla \cdot \bar{u}_{\phi} + \nabla \cdot \bar{u}_{\phi}') + \left[ \frac{1}{\rho \mu r \sin \theta} \right] (2 \bar{u} \times \Omega)_{\phi} + \frac{v}{r \sin \theta} \nabla^2 u_{\phi} + \frac{\nu u_{\phi}}{r^3 \sin^3 \theta}, \]  
(13)

where \( F \) has been omitted because there is no forcing in this region and, as in Equation (11), the total azimuthal velocity, \( u_{\phi} \), is made up of the mean (time-averaged, \( \bar{u}_{\phi} \)) plus the fluctuating component, \( u_{\phi}' \).

In Figure 9 we show the various force terms as a function of latitude, at two different radii within the tachocline, with the top four panels representing the non-magnetic case and the bottom four panels representing the strong magnetic case.\(^8\) The left-hand panels represent high latitudes, while the right-hand panels represent low latitudes. In the following discussion we will concentrate on the non-magnetic case, but note here that the magnetic case shows the same behavior. At high latitudes, at the top of the tachocline (top left panel) the total force (denoted by the solid line) is very small. The force there is dominated by the Coriolis force (dashed line, defined as the sum of the first and fourth terms on the right-hand side of Equation (13)) and the divergence of the Reynolds stress (dotted line, defined as the second term on the right-hand side of Equation (13)). The divergence of the Reynolds stress in this region is largely due to convective overshoot and is therefore, unsurprisingly large. The sign of the Coriolis force at the top of the tachocline is negative indicating its tendency to slow the poles. However, this Coriolis term is offset entirely by the Reynolds stress and diffusive terms, leading to very little net angular momentum transport there.

At the bottom of the tachocline, again at high latitudes, the Coriolis term which is still dominant has changed sign and leads to an acceleration of the azimuthal flow at high latitudes.

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\(^7\) Although see magnetic section for further discussion.

\(^8\) The magnetic stresses are hardly seen because they are substantially smaller than the other stresses in the tachocline; see Section 3.5.
change of sign of the Coriolis force is because, at the bottom of
the tachocline, the Coriolis force is acting on the counterrotating
meridional circulation cell discussed in Section 3.1, which is
poleward. Therefore, this CRC acts to change the sign of the
dominant force in the tachocline, effectively turning off the
spread of the tachocline in radius.

At lower latitudes, the situation is more complicated. There is
substantially more time and latitudinal variation. This is because
a persistent CRC has not developed at lower latitudes. We
believe that over longer time averages the low-latitude CRC
will become more persistent and halt the spread there as well.
This expectation is supported by Figure 7, which shows the
spread of the tachocline nearing zero, even at low latitudes.

3.5. Magnetic Models

As stated above we ran three models: two with a magnetic
field and one without. Although we ran two magnetic cases, as
stated previously we only present results from our strong field
case, for ease of presentation, but also because the results in
the weak field case are virtually identical to the non-magnetic
case. The results for the tachocline penetration depth shown in

Figures 6 and 7 indicate that the Gough & McIntyre (1998)
model cannot be solely responsible for halting the spread of the
tachocline. However, it is also expected that an internal poloidal
field once “connected” to the differentially rotating convection
zone would cause the differential rotation to be spread into the
radiative interior on approximately an Alfvén crossing time. In
Figure 10 we show the evolution of the poloidal (and toroidal)
field, where we can clearly see that the poloidal field connects
to the convection zone. Yet, despite this connection we see no
additional spread of the tachocline in the magnetic case.

For our strong field case, with an initial internal field strength
of 4 kG, assuming no diffusion, nor any turbulent diffusion
due to small-scale motion, the Alfvén velocity varies from a
few cm s\(^{-1}\) at the base of the convection zone to a few hun-
dred cm s\(^{-1}\) in the deep radiative interior. After the initial rapid
spread of the tachocline as seen in Figure 7, that is, at \(\approx 0.6 \, R_\odot\),
the Alfvén velocity (\(v_A\)) is approximately 100 cm s\(^{-1}\). Therefore,
in the subsequent \(10^8 \, s\) after the initial spread of differential
rotation, one would (very simply) expect an additional spread of
\(v_A \times t \approx 10^{10} \, cm\) if that spread were being mediated by a magnetic
field. However, we see no spread of this magnitude. Indeed, we
see very little effect of the magnetic field at all.
Figure 10. Evolution of the toroidal (top panels) and poloidal (bottom panels) fields. The dark line represents the base of the convection zone. Toroidal field evolution: initially, there is no toroidal field (a). Differential rotation acts on the poloidal field to produce a toroidal field which initially changes sign at mid-latitudes (b). This configuration then becomes unstable and leaves behind a single dominant sign in each hemisphere (c). The time-averaged toroidal field is shown in (d). Poloidal field evolution; after a short time the poloidal field diffuses into the convection zone (f), aided by convective overshoot and dredge-up, thus connecting the radiative interior to the convection zone.

(A color version of this figure is available in the online journal.)

There are (at least) three reasons for this (that this author can think of). The first, and most obvious, is that these simulations have not run long enough to see the additional effects of the magnetic field. The estimate above is just that, a very crude estimate. Because we wanted our magnetic Prandtl number to be less than one, our magnetic diffusivity is rather large. Compounding this problem, we have no dynamo and therefore, the field strength and Alfvén velocity are decreasing (after an initial rise; see Figure 11). Estimating the decay of the magnetic field strength using the imposed diffusivity, the amplitude of the magnetic field (and hence, Alfvén velocity) would have dropped by approximately 30% and hence, the spread might have only amounted to $7 \times 10^9$ cm, but this would still be measurable. The picture is more complicated though, because the evolution of the magnetic field is not governed solely by diffusion. As can be seen in Figure 11, initially the magnetic energy increases due to differential rotation and convection and only later does exponential decay set in. Depending on the field component, the field decay requires nearly half of the simulation time to decay back to its original amplitude. Therefore, estimating how far the differential rotation should have spread if mediated by the magnetic field is not straightforward, given the complications of a time and space dependent field. In light of the uncertainty in estimating the timescale of tachocline spread mediated by magnetic field, it is wise to at least investigate the trend of the magnetic stresses.

3.5.1. The Lorentz Force in the Tachocline

The scenario in which a magnetic field aids the spread of differential rotation into the radiative interior depends on a few factors. First, the field must be sufficiently “connected” to the convection zone. Second, the Lorentz force must be larger than other forces in the region (at least on long time averages). And finally, the configuration of $B_\phi$, $B_\theta$, and $B_r$ must be specific.

As we have seen in Figure 10, the initial poloidal field indeed becomes “connected” to the convection zone and is not confined by the meridional circulation. Given this clear connection one would expect the differential rotation of the convection zone to be rapidly imprinted into the radiative interior. However, a magnetically aided tachocline spread depends on the Lorentz force being dominant or comparable to the other forces in the azimuthal momentum equation (at least on long time averages). This depends not only on the strength of the magnetic field in this region but also on the correlation of these fluctuating fields. In Figure 9, we showed the various forces contributing to the maintenance of the differential rotation. In that figure the
Figure 11. Magnetic energy as a function of time for the three different field components; radial field (solid line), latitudinal field (dotted line), and toroidal field (dashed line). The total integrated energy for the entire domain is shown in (a), the convection zone (b), the tachocline (c), and the radiative interior (d). One can clearly see an initial increase in all components of the magnetic field energy in the convection zone and tachocline followed by exponential decay. The toroidal field is amplified substantially and shows oscillatory behavior where the toroidal energy grows, decays, and repeats. In the radiative interior, the field predominantly decays, except the toroidal field.

Lorentz forces are seen as zero because they are substantially smaller than the other forces shown. In Figure 11, we show the volume integrated magnetic energy in the various regions of the simulation domain (total: (a), convection zone: (b), tachocline: (c), and radiation zone: (d)). We see there that the magnetic energy is initially amplified in the convection zone and tachocline by the action of differential rotation and convection. In particular, the energy in the toroidal field (initially zero) is amplified to the extent that it becomes the dominant field component (dashed line). Simply estimating the Lorentz force given these amplitudes, one would expect it to be comparable to the divergence of the Reynolds stress in the tachocline (where the velocities are approximately 10–10³ cm s⁻¹), but as shown in Figure 9, the Lorentz forces are quite small, indicating that the field fluctuations are correlated in such a way that they do not lead to a substantial Lorentz force. This can possibly be explained by considering that wherever $B_\phi$ is large it is so because differential rotation and/or convection have acted on $B_r$ or $B_\theta$ to generate it, so that wherever $B_\phi$ is large $B_r$ and $B_\theta$ are not. Therefore, while the field clearly “connects” to the tachocline and convection zone (see Figure 10), the Lorentz force in the tachocline is small and therefore, little angular momentum is transported by magnetic terms. This is the second possible reason the magnetic field has no effect on the propagation of differential rotation into the radiative interior.

### 3.5.2. Sense of the Lorentz Force

Finally, the spread of differential rotation by an internal magnetic field depends on the configuration of the magnetic field generated at the convective-radiative interface. Simply looking at the azimuthal component of the induction equation (neglecting diffusion),

$$\frac{\partial B_\phi}{\partial t} = r B_r \frac{\partial}{\partial r} \left( \frac{u_\phi}{r} \right) + \frac{\sin \theta B_\theta}{r} \frac{\partial}{\partial \theta} \left( \frac{u_\phi}{\sin \theta} \right) - r u_r \frac{\partial}{\partial r} \left( r B_\phi \right) - \frac{\sin \theta u_\theta}{r} \frac{\partial}{\partial \theta} \left( \frac{B_\phi}{\sin \theta} \right), \quad (14)$$

one can see that in the tachocline, where the radial gradient of differential rotation is initially dominant in inducing toroidal field (first term in Equation (14)), a toroidal field which switches sign at mid-latitudes is induced (Figure 10(b)). Considering the azimuthal component of the Lorentz force,

$$[(\nabla \times \mathbf{B}) \times \mathbf{B}]_\phi = \frac{B_\phi}{r \sin \theta} \frac{\partial (B_\phi \sin \theta)}{\partial \theta} - \frac{B_r}{r} \frac{\partial (r B_\phi)}{\partial r}, \quad (15)$$

a toroidal field that switches sign at mid-latitudes, coupled with a latitudinal field which is positive, generates a Lorentz
force which promotes decelerating angular velocity at high latitudes and accelerating angular velocity at low latitudes, thereby promoting the spread of the differential rotation of the convection zone into the radiative interior.9

In these simulations, we indeed find that the radial gradient of differential rotation dominates the initial induction of the toroidal field, leading to a toroidal field that switches sign at mid-latitude (Figure 10(b)). Later in the simulation, however, the latitudinal component of differential rotation becomes dominant. This leads to a toroidal field configuration that has one sign toroidal field in the northern hemisphere (positive) and one sign in the southern hemisphere (negative); see Figure 10(c). The amplitude of the toroidal field peaks at around mid-latitudes (see Figure 10(d)), so that the latitudinal gradient of toroidal field changes sign at mid-latitudes. Furthermore, as the field diffuses out of the radiative interior and connects with the convection zone, the sign of the latitudinal magnetic field changes at high latitudes. Whereas initially the latitudinal field was positive everywhere in the tachocline, later in the simulation the latitudinal field is negative at high latitudes and positive at low latitudes. Combining the toroidal field configuration seen in Figure 10(d), with positive latitudinal field at low latitudes and negative latitudinal field at high latitudes, one can then simply see that this field configuration gives acceleration at high latitudes and deceleration at low latitudes, as seen in Figure 12. This field configuration would then act to stop tachocline spread (if Lorentz forces were dominant over long timescales).

Of course, it is not clear that such a field configuration would develop in the Sun. But this highly simplified simulation indicates that the magnetic field configuration and associated Lorentz force in a tachocline that is bounded on top by convection and fed from below by a space and time-dependent magnetic field is quite complicated and possibly not well described without considering instabilities and poloidal field evolution.

3.6. Weaknesses

Clearly, this model has several weaknesses. The first is that it is essentially 2D and there is some possibility that 2D convection conspires to limit tachocline spread. It is impossible to test this until 3D simulations, using similar parameters as those described here are conducted. Such simulations are likely not too far off.

The second major limitation of these simulations is that the diffusivities are too large. This prevents us from directly testing the case in which the thermal hyper-diffusion described by Spiegel & Zahn (1992) dominates the viscous diffusion. It also prevents us from getting the force balance in the tachocline exactly right. For instance, our meridional circulation appears to be only about 45% too large, leading to Reynolds stresses and Coriolis forces in the tachocline which are larger than expected. However, this should be offset by the larger than solar viscous diffusivity. In the end, the viscous forces are not dominant in the tachocline in these simulations and that is the first order goal.

Finally, because our time step is severely limited by gravity waves and convection we are not able to run this simulation very long. We have only run these simulations approximately 20%
of a viscous diffusion time or 40% of a magnetic diffusion time. Although only a small time, we still would have expected more tachocline spread. Furthermore, over this short time we can still investigate the trend of the stresses in the tachocline region. The following discussion then proceeds with these weaknesses in mind.

4. DISCUSSION

These results represent the first attempt at coupling the convective and radiative interior of the Sun in order to understand the angular momentum evolution of the radiative interior. Although troubled by computational limitations (as all simulations are), there are still some very interesting and important results to be gleaned from this model. First, we find that a purely hydrodynamical model of the solar interior could prevent the spread of the tachocline by setting up a CRC that straddles the base of the convection zone/top of the tachocline. The Coriolis force acting on this CRC acts to accelerate the flow at high latitude, thus shutting off the spread of the tachocline.

In the magnetic models the meridional circulation appears unable to prevent the field from connecting to the convection zone; both by diffusion and dredge-up by convective plumes. However, despite being connected to the convection zone very little angular momentum is transported between the two regions by magnetic stresses. This is because the magnetic stresses in the tachocline are small. Despite a rather large toroidal field growing in the tachocline due to differential rotation, the radial and latitudinal field there are quite small and the Maxwell stress resulting from the correlations of these field components are similarly small. Therefore, while it does not appear necessary for a magnetic field to prevent the spread of the tachocline, we also find that such a internal poloidal field does not promote differential rotation if it connects to the convection zone.

We are grateful to G. Glatzmaier, K. B. MacGregor, M. Rempel, and P. Garaud for helpful discussions. The manuscript was substantially improved due to very helpful and insightful suggestions by an anonymous referee. Support for this research was provided by a NASA grant NNG06GD44G. T. Rogers is supported by an NSF ATM Faculty Position in Solar physics under award number 0457631. Computing resources were provided by NAS at NASA Ames, aastex-help@aas.org.

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