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Coupled Effects between Solid Particles and Gas Velocities on Erosion of Elbows in Natural Gas Pipelines

Nan Lin¹, Hui-Qing Lan¹*, Yu-Gong Xu¹, Yue Cui¹, Gary Barber²

¹School of Mechanical, Electronic and Control Engineering, Beijing Jiaotong University, Beijing, China
²Department of Mechanical Engineering, Oakland University, MI, U.S.A

Abstract

The damage of materials caused by erosion of solid particles occurs on a large scale in different fields, including machinery, aviation, and pipeline transportation, resulting in considerable economic losses in the world. Therefore, the erosion of elbows has been extensively researched, with emphasis on single factors such as properties of solid particles, gas velocities, incidence angles, and temperatures. Erosion is generally affected by two or more coupled factors, however very few studies have been conducted on this subject. Thus, this study investigates the coupled effect between solid particles and gas velocities on the erosion of elbows in natural gas pipelines. A simulation was performed by combining the theoretical model of erosion and the Lagrange discrete phase model. The simulation result was then compared with the results of the working conditions and test platform based on a similar principle. Moreover, the properties of solid particles and the speed of natural gas in pipelines were coupled to analyze the change of erosion rate at different velocities and properties of solid particles (i.e., sizes, shapes, quantities, and incidence angles). Results showed that the erosion rate increases with the increase in size and quantity of solid particles. The small shape coefficient of solid particles causes severe erosion, and their incidence angle influences the rate and position of erosion of the elbows. A complicated coupling effect between solid particles and gas velocities on the erosion of elbows in natural gas pipelines occurs.

Keywords: Coupled effect; Solid particles; Velocity; Erosion; Elbow

* Corresponding author. Tel.: +86 010 51684699; address: No.3 Shangyuancun, Haidian District, Beijing, China, 100044.
E-mail address: hqlan@bjtu.edu.cn

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1. Introduction

The production of sand in gas pipelines is one of the major causes of erosion\(^1\). Subsequently, these solid particles damage the pipelines\(^2\). The particle erosion that occurs on elbows is a challenging problem because of the complexity of the flow fields\(^3\). Many scholars have explored the particle erosion of metals. Brenton et al. studied the distribution of sand particles in horizontal and vertical multi-phase flows in pipelines by conducting numerous tests and then discussed the effects of this distribution on sand erosion\(^4\). Kesana et al. investigated the different positions of straight pipes and elbows by employing angle-head and flat-head ER probes and the ultrasonic measurement method to determine the effects of particle size and fluid viscosity on erosion\(^5\)\(^6\). Zhang et al. considered the location of maximum erosive wear damage in elbows and U-shaped bends under different directions of gravity\(^7\). However, these studies mainly focused on the properties of the materials and the effects of elbows and pipelines on erosion. Research on the coupled effect between solid particles and gas velocities on the erosion of elbows is lacking. In the present study, a coupled model between solid particles and gas velocity was established based on a similarity test, and then erosion of elbows was simulated based on the model. Moreover, the effects of different coupling factors on the erosion of elbows under working conditions in the Daqing Oilfield were analyzed to provide guidance on the safe operation and protection of pipelines.

2. Theoretical model

2.1. Mechanical balance model

Turbulence model

The turbulent viscosity \(\mu_t\) can be expressed as a function of \(k\) and \(\varepsilon\):\(^8\)

\[
\mu_t = \rho C_{\mu} \frac{k^2}{\varepsilon}
\]

where \(\rho\) is the density of fluid, \(C_{\mu}\) is an empirical constant with a usual value of 0.09, \(k\) is the turbulent kinetic energy, and \(\varepsilon\) is the dissipation rate of turbulent kinetic energy.

The transport equation of the standard \(k-\varepsilon\) model is given by\(^9\)\(^10\):

\[
\frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho u_i k)}{\partial x_i} = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + G_k + G_b - \rho \varepsilon - Y_H + S_k
\]

\[
\frac{\partial (\rho \varepsilon)}{\partial t} + \frac{\partial (\rho \varepsilon u_i)}{\partial x_i} = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_{\varepsilon}} \right) \frac{\partial \varepsilon}{\partial x_j} \right] + C_{1\varepsilon} \frac{\varepsilon}{k} (G_k + C_3 \varepsilon G_b) - C_{2\varepsilon} \rho \frac{\varepsilon^2}{k} + S_{\varepsilon}
\]

where,

\[
G_k = \mu_t \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j}
\]

\[
G_b = \beta g_i \frac{\mu_t}{\rho \varepsilon} \frac{\partial T}{\partial x_i}
\]

where,

\[
\beta = - \frac{1}{\rho} \frac{\partial \rho}{\partial T}
\]

\[
Y_H = 2 \rho \varepsilon M_i^2
\]

where,

\[
M_i = \sqrt{k/\alpha^2}
\]
\[ a = \sqrt{\gamma R T} \]  

where \( G_k \) is the generated item of turbulent kinetic energy \( k \), which is caused by the average velocity gradient and can be calculated using Equation (4). \( G_b \) is the generated item of turbulent kinetic energy \( k \), which is caused by the buoyancy and can be calculated by Equation (5), where \( Pr \) and \( \beta \) are the turbulent Prandtl number and coefficient of thermal expansion, respectively. \( Y_M \) is the effect of expansion on total dissipation rate in compressible turbulence and can be calculated using Equation (7), where \( M_t \) and \( a \) are the turbulent Mach number and velocity of voice, respectively. \( C_{1\varepsilon} \), \( C_{2\varepsilon} \), and \( C_{3\varepsilon} \) are empirical constants with typical values of 1.44, 1.92, and 0.09, respectively. \( \sigma_k \) and \( \sigma_\varepsilon \) are the Prandtl numbers of turbulent kinetic energy and dissipation rate of turbulent kinetic energy with the usual values of 1.0 and 1.3, respectively. \( S_k \) and \( S_\varepsilon \) are the source items defined by the user.

**Lagrange discrete model**

The Lagrange discrete model was adopted to calculate the discrete phases\(^{11-12}\)

\[
\frac{du_p}{dt} = F_D(u - u_p) + \frac{g_x(p_p - \rho)}{\rho_p} + F_x 
\]  

where \( F_D \) is the drag force of particles, which can be calculated by Equation (11), \( u \) is the natural gas velocity, \( u_p \) is the particles' velocity, \( \rho \) is the density of the natural gas, \( \rho_p \) is the density of the particles, and \( F_x \) is an additional acceleration term (force/unit particle mass).

\[
F_D = \frac{18\mu C_D Re}{\rho_p d_p^2} 
\]  

where \( \mu \) is the molecular viscosity of the natural gas, \( d_p \) is the particle diameter, and \( Re \) is the relative Reynolds number, which is defined as

\[
Re = \frac{\rho d_p |u_p - u|}{\mu} 
\]

**Momentum exchange coupling model**

The momentum exchange model was used to couple the particle and gas phases\(^{13}\).

\[
F = \sum_i \left( \frac{18\mu C_D Re}{\rho_p d_p^2} \left(u_p - u\right) + F_{other}\right) \cdot m_p \Delta t 
\]

where \( m_p \) is the mass flow rate of the particles.

**Discrete particle hard sphere model**

The collision between each particle was described by using the hard sphere model, which is based on binary quasi-instantaneous collisions. The collision process of particles conformed to the theorem of momentum\(^{14-15}\).

When two particles, namely, \( i \) and \( j \), slide during the collision process,

\[
v_i = v_i^0 - (n + f \hat{t}) \left(n \cdot G^{(0)}\right) \left(1 + e\right) \frac{m_j}{m_i + m_j} 
\]

\[
v_j = v_j^0 - (n + f \hat{t}) \left(n \cdot G^{(0)}\right) \left(1 + e\right) \frac{m_i}{m_i + m_j} 
\]
When the two particles stop sliding during the collision process,

\[
\omega_i = \omega_i^0 + \left( \frac{5}{2r_i} \right) ( \mathbf{n} \cdot \mathbf{G}^{(0)} ) (\mathbf{n} \times \mathbf{t})(1+e) \frac{m_j}{m_i + m_j} \\
\omega_j = \omega_j^0 + \left( \frac{5}{2r_j} \right) ( \mathbf{n} \cdot \mathbf{G}^{(0)} ) (\mathbf{n} \times \mathbf{t})(1+e) \frac{m_i}{m_i + m_j}
\]

(16) (17)

where \( \mathbf{n} \) is the unit vector that points from the center of particle \( i \) to the contact point, \( f \) is the friction coefficient, \( t \) is the unit tangential vector at the contact point, \( e \) is the restitution coefficient, \( \mathbf{v} \) is the particle velocity, \( \omega \) is the angular velocity, \( \mathbf{G}^{(0)} \) is the relative velocity between the two contact points of the particles before collision, \( m \) is the particle mass, \( \mathbf{G}^{(0)}_{ct} \) is the tangential component of the relative velocity before collision, and \( r \) is the particle radius.

**Probabilistic model**

The collision frequency between particle \( i \) (the radius is \( r_i \)) and particle \( j \) can be calculated using the model proposed by Oesterle et al.\cite{16}

\[
p = \sqrt{2} \pi r_i r_j | \mathbf{G}_{ij} | N \frac{V}{2}
\]

(22)

where \( \sqrt{2} \) is the correction coefficient for extending from one dimension to three dimensions, \( V \) is the volume, \( \mathbf{G}_{ij} \) is the relative velocity, and \( N \) is the quantity of particles in a unit. The model assumed that the relative particle velocity (about the partial average particle velocity) followed the distribution in which the velocity (and rotation velocity) of sample particles is valued as the average velocity (and rotation velocity) in the grid, while the collision points are calculated by implementing a random method.

**2.2. Erosion model**

The erosion rates of the walls caused by the particles can be calculated as follows\cite{17}:

\[
E = \sum_{p=1}^{N_{\text{particles}}} \frac{\dot{m}_p C(d_p) f(\alpha) v^{h(v)}}{A_{\text{face}}}
\]

(23)

where \( E \) is the erosion rate, \( C(d_p) \) is a function of particle diameter, \( f(\alpha) \) is a function of impact angle \( \alpha \),
is a function of relative velocity, and $A_{face}$ is the area of the cell face at the wall. $C\left(d_p\right), f\left(\alpha\right)$, and $b\left(v\right)$ are parameters that reflect the effect of particle and wall materials on erosion. The Tulsa angle dependent model is described as follows:

$$C\left(d_p\right) = 1559B^{-0.59}F_s$$

(24)

$$v^{1.73} = v^{b\left(v\right)}$$

(25)

$$E = 1559B^{-0.59}F_s v^{1.73} f\left(\alpha\right)$$

(26)

where $B$ is the Brinell hardness and $F_s$ is a particle shape coefficient.

3. Model verification

The relation between the relative erosion volume of different metals and the angle of attack was studied by Finnie. Some of his results are shown in Fig. 1. It can be seen that the erosion of metals was highest when the value of attack angle was approximately 20°. Meanwhile, the curved wall of elbows as the particles bounced on its surface contributed to a complicated process. In analyzing the erosion of elbows, the particles were assumed to impact the elbow from one side in a straight line (without deflection angle) and then distributed equably. The red circle in Fig. 2 indicates the theoretical maximum erosion position of the elbows based on the relationship between the erosion of materials and the attack angle of particles. The figure demonstrates that the maximum erosion occurred on the outboard side of the elbows and toward the outlet near 50°. These findings conform to the results presented by Gabriel et al. 21.

![Graph](image_url)

Fig. 1 Relationship between erosion of materials and attack angles of particles(16)
To verify the reliability of the model established by FLUENT in this study, a similarity test was designed by adopting the similarity principle. The model of the test was established using the parameters of the similarity test, which are shown in Table 1.

Table 1 Parameters of the similarity test

| Name                | Value                      |
|---------------------|----------------------------|
| Elbow Type          | DN25 (galvanized pipe)     |
| Elbow Size          | \( \Phi 31 \times 1.5 \text{mm} \) |
| Operating Pressure  | 0.26 MPa                   |
| Operating Velocity  | 4.68 m/s                   |
| Test Temperature    | 16 \( ^\circ \text{C} \)   |

The image of the damaged elbow in the test and the simulation result are compared in Fig. 3. The maximum erosion positions are similar and concur well to the theoretical maximum erosion position shown in Fig. 2. The erosion rates of simulations and tests under different particle diameters and velocities are compared in Fig. 4. The figure shows that the findings are in good agreement. Thus, studying the coupled effect of solid particles and gas velocities on the erosion of elbows by conducting the above method is reasonable.
4. Application of the model under working conditions

The model was applied to the elbows under working conditions in the Daqing Oilfield, and the results were compared in an onsite detection report. The parameters of the elbows in the Daqing gathering gas station are shown in Table 2. The model was evaluated using the parameters presented in Table 2.

Table 2 Parameters of elbows in Daqing gathering gas station

| Name                | Value             |
|---------------------|-------------------|
| Elbow Type          | DN65 (20G)        |
| Elbow Size          | Φ76×9mm           |
| Operating Pressure  | 15.5MPa           |
| Operating Velocity  | 10m/s             |
| Test Temperature    | 25℃               |

According to the onsite detection report of Daqing gathering pipelines, the maximum wall thinning of the elbows was approximately 1.2 mm (the calculated erosion rate was about 7.468×10⁻⁸ kg/m²-s). The maximum erosion rate of the elbows calculated by using the coupling model presented in this study was 6.912×10⁻⁸ kg/m²-s, which was relatively close to the detection value.

5. Analysis of the coupled effects of solid particles and gas velocities on erosion

The erosion on the elbows of the gathering gas pipes under different properties of solid particles and gas velocities was analyzed by adopting the validated model in this paper.

5.1. Coupled effects of particle sizes and gas velocities on erosion

The coupled effects of different particle sizes and velocities on the erosion rates of the elbows are illustrated in Fig. 5. The kinetic energy of the solid particles increases as the velocity increases, and the erosion on the elbows caused by the particles becomes more severe. The erosion caused by particles also becomes greater as the particle size increases. However, large particles are more likely to collide with each other. Hence, the erosion rate changes slowly as the size of the particles increase between 0.4 and 0.6 mm. Under the coupled effects of different particle
sizes and gas velocities, the influence of different sizes of solid particles on the maximum erosion rate is more
obvious for high gas velocity, and the influence of changing gas velocities on the maximum erosion rate is more
obvious when the particles are larger in size. With gas velocities, there are some differences on effects of different
particle sizes on erosion rate.\textsuperscript{25}

5.2. \textit{Coupled effects of particle shapes and gas velocities on erosion}

The coupled effects of different shapes of particles (defined by the shape coefficient) and gas velocities on the
erosion rate were thoroughly investigated. When the particles have a small shape coefficient, they exhibit more
edges and corners than particles with a large shape coefficient. As the value of the particle shape coefficient
becomes closer to 1, the shape of the particle becomes spherical.\textsuperscript{26} The coupled effects of the shape of different
particles and velocities on the erosion rates of elbows are exhibited in Fig. 6. Erosion is more severe as the shape
coefficient of the particles decreases, and the erosion rate increases sharply as the shape coefficient changes from
0.9 to 0.7. Under the coupled effects of different particle shape coefficients and flow velocities, the effect of flow
velocity on the erosion of elbows is more obvious for small shape coefficients than for large shape coefficients. The
influence of the shapes on the erosion of elbows is also more obvious for high gas velocities.
5.3. Coupled effects of particle quantity and gas velocities on erosion

The coupled effects of quantity of different particles and velocities on the erosion rates of elbows are shown in Fig. 7. The frequency of particles bouncing on the wall increases as the quantity of the particles increases, and then the erosion on the elbows becomes more severe. However, with more particles, they will likely collide with each other and the erosion rate of the elbows slowly changes as the flow rate exceeds 0.012 kg/s. Under the coupled effects of different particle quantity and gas velocities, the erosion of the elbows becomes more obvious when the flow rate is below 0.012 kg/s. The influence of gas velocity on the erosion of the elbows is more obvious for high flow rates.

![Fig. 7 Maximum erosion rates of the elbows for different particle quantities and gas velocities](image)

5.4. Coupled effects of particle incidence angles and gas velocities on erosion

The position of maximum erosion on the elbows changes as the incidence angle changes. The erosion of the elbows under different incidence angles is shown in Fig. 8. Erosion occurs more on the outboard wall of the elbows as the incidence direction approaches the negative angle. Otherwise, the particles bounce on the surface of the inboard wall of the elbows as the incidence direction approaches a positive angle, resulting in erosion occurring on the inboard wall near the entrance of the elbows.

![Fig. 8 Erosion of the elbows for different incidence angles](image)

The coupled effects of different incidence angles and velocities on the erosion rates of the elbows are presented in Fig. 9. Under the coupled effects of incidence angles and gas velocities, the erosion rate slightly changes as the
incidence angle changes in low flow velocities. Meanwhile, the erosion rate changes sharply as the incidence angle changes along with an increase in gas velocity. As presented in Fig. 1, the erosion caused by particles on metals is greater at an attack angle of approximately 20º. The erosion rate decreases as the incidence angle approaches the positive angle (inboard wall of the elbows). Maximum erosion occurs on the elbow at the incidence angle of approximately –15º, and the erosion rate slightly decreases as the incidence angle approaches the negative angle.

![Fig. 9 Maximum erosion rates of the elbows under different incidence angles and gas velocities](image)

6. Conclusions

In this paper, a coupled model of the properties of particles and the gas velocities of elbows is proposed to analyze erosion in gathering gas pipes. The maximum erosion rate and the eroded area on the elbows can be predicted by adopting a coupled model. The simulation results concurred well with the results in the onsite detection report of gathering gas pipelines in Daqing Oilfield.

Based on the analysis of the coupled effects between particles and gas velocities on the erosion of elbows in gathering gas pipes, the following conclusions are drawn:

1. Changing gas velocities significantly affect erosion rate under the coupled effects of the properties of particles and gas velocities, and the erosion caused by the particles increases sharply with increase in velocity.

2. The erosion rate of elbows increases as the size or quantity of particles increases. However, the erosion rate slowly changes or becomes constant in sufficiently large-sized particles. The erosion of the elbows increases as the shape coefficient of the particles decreases (with the particles exhibiting more edges and corners). The incidence angles of the particles also affect erosion rate and the position of maximum erosion on the elbows.

3. The coupled effects between solid particles and gas velocities slightly affected the erosion of elbows at a low flow velocity. The erosion rate changes sharply as the particle properties change at high flow velocity. When the properties of the particles (size, quantity or shape coefficient) have high or large values, the erosion of the elbows (under coupled effects between solid particles and gas velocities) changes slightly as the flow velocity changes. When the properties have low values, the change in erosion rate is more obvious. The relationship between coupled effects such as the properties of particles and flow velocity on the erosion of elbows is complex. These variables may influence each other, resulting in changes in the erosion rates. Further research should be conducted to fully understand coupled effects of different factors on erosion.
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