Research Article

Decentralized Adaptive Control for Quasi-Consensus in Heterogeneous Nonlinear Multiagent Systems

Jiaju An,1 Wei Yang,1 Xiaohui Xu,2 Tianxiang Chen,1 Bin Du,1 Yi Tang,1 and Quan Xu1

1School of Mechanical Engineering, Xihua University, Chengdu 610039, China
2School of Automobile and Transportation, Xihua University, Chengdu 610039, China

Correspondence should be addressed to Quan Xu; quanxnjd@sina.com

Received 19 May 2021; Accepted 2 July 2021; Published 14 July 2021

Academic Editor: Guoguang Wen

Copyright © 2021 Jiaju An et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This paper proposes some novel decentralized adaptive control protocols to settle the quasi-consensus problem of multiagent systems with heterogeneous nonlinear dynamics. Based on local communication with the leader and between the followers, some innovative control protocols are put forward to adapt the control gains and coupling weights simultaneously and to steer the consensus errors to some bounded areas. In particular, two new inequalities are proposed to establish the Lyapunov-based adaptive controller design approach for quasi-consensus. Some quasi-consensus criteria are derived by utilizing the designed controllers, in which the error bound can be modulated on the basis of the adaptive controller parameters. Numerical tests are conducted to show the feasibility of the theoretical derivation. Our findings highlight quasi-consensus in heterogeneous multiagent systems without adding some additional complex nonlinear control terms to cancel the dynamical differences between agents.

1. Introduction

Collaborative control of multiagent systems (MASs) has become a research hotspot of distributed artificial intelligence because of its wide application in intelligent energy, multirobot formation, intelligent transportation, multi-unmanned system collaboration, and other engineering systems [1–5].

Among them, consensus or synchronization is a key common scientific issue of cooperative control of MASs, which has aroused great concern of multidisciplinary scholars. In brief, a core issue is to design appropriate control protocols to reach an agreement between agents. So far, many significant results have been acquired, regarding the consensus patterns, models, control algorithms, etc (please see [6–14] and some other results).

However, the aforesaid works [1–14] on the consensus of MASs mainly concentrate upon the case in which an agent has the same dynamics as all the neighbors or the leader. In some real scenes, the mismatched parameters or dynamical differences among agents may be almost inevitable, which will thus result in heterogeneous (or nonidentical) multiagent systems (HMASs) [15–17]. It is remarkably that, due to heterogeneity, it is even impossible for HMASs to reach the complete consensus just by state feedback control when the coupling weights are constant. Up to now, there are few thorough research studies on complete consensus in HMASs because one has to add some additional complex nonlinear control terms or design compensators to cancel the dynamical differences [18–26]. Another alternative technique to deal with the heterogeneity is to transform HMASs to homogeneous ones [27, 28]. Unfortunately, all of the aforementioned approaches are complex and nonintuitive, which are thus not suitable for engineering applications. Nevertheless, in many practical HMASs, the consensus error may be bounded, even small enough, which is, namely, the so-called quasi-consensus (QC) [29].

Instead, an immediate and natural question, then, is how to design a simple controller to reach QC in HMASs. Similar to it, work to date has considered the quasi-synchronization (QS) in heterogeneous complex dynamical networks (HCDN) [30–34]. For example, in [33, 34], some QS criteria
for fractional HCDN are derived via state feedback control and impulsive control, respectively. By contrast, there are few research studies about QC of HMASs [29, 35–38]. The definition of QC for MASs was first proposed in [38], and then, the definition of QC was further broadened in [29]. After that, the QC of HMASs has been further studied. The QC problem of nonlinear HMASs is studied via sampled-data control in [35]. In [36], sufficient conditions for the QC in switched HMASs are given, considering cooperation and competition interactions simultaneously. Ye and Shao [37] attempted to prove that QC in HMASs under DOS attacks can be realized by impulsive control. It should be noted, however, that, for large scale HCDN or HMASs, the computational complexity and conservativeness of QS or QC conditions in [29–38] impede their applications. In particular, it is difficult, even impossible, to check the LMI-based conditions without adaptive schemes for large-scale HCDN or HMASs. Obviously, it is an interesting and open problem to improve the QC conditions for HMASs by exploring some new control algorithms.

Motivated by the applications of decentralized adaptive control for synchronization in integer-order and fractional-order complex dynamical networks [39–43], this paper aims to design some decentralized adaptive protocols to reach QC in HMASs. Different from some previous studies [37, 44, 45], in our controller, the coupling and feedback values between agents change adaptively. The main contributions in this paper are as follows. First, two new inequalities are proposed to establish the Lyapunov-based adaptive controller design approach for QC in HMASs. Second, some innovative control protocols are introduced to accommodate the control gains and coupling weights adaptively, to steer the consensus errors to some bounded areas. Third, some QC criteria under the designed controllers are derived by the Lyapunov function method and the new inequalities.

2. Preliminaries and Problem Formulation

2.1. Graph Theory. To carry out later research, we present some important concepts about graph theory in this section.

\[ G = (V, E, \mathcal{A}) \] is defined as a weighted undirected graph with the network topology of \( N \) agents, where \( V = \{0, 1, \ldots, N\} \) and \( E = \{(i,j)\} \subseteq V \times V \) are the separate sets of nodes and undirected edges. \( \mathcal{A} = (a_{ij})_{N \times N} \) is the weighted adjacency matrix of which the elements are nonnegative. The \( i \)-th agent and the leader are modeled as the node \( i \in V, V = \{0, 1, \ldots, N\} \) and node 0, respectively. As a rule, the undirected edge \( (i, j) \in E \) in the weighted undirected graph \( G \) denoted by an ordered pair \((V, E, \mathcal{A})\) represents that agent \( i \) and agent \( j \) become a pair of neighbors which can get their information from each other. There is an undirected spanning tree in an undirected graph if one agent exits an undirected path to every other distinct node.

There are two matrices that are considered as the network topology, i.e., the weighted adjacency matrix \( \mathcal{A} = (a_{ij})_{N \times N} \) with \( a_{ij} = a_{ji} > 0 \) if \( e_{ij} \in E \), else \( a_{ij} = 0 \) if \( e_{ij} \notin E \), and the Laplacian matrix \( L = (L_{ij})_{N \times N} \) which is defined as \( L_{ii} = \sum_{j=1,j \neq i}^{N} a_{ij} = \text{deg}(i) \) and \( L_{ij} = -a_{ij}, i \neq j \), noted that all of them are asymmetric for the undirected graph.

**Lemma 1** (see [46]). The Laplacian matrix \( L \) is constructed from the undirected network. There are several properties in the following.

1. **Eigenvalues of \( L \) satisfy** \( 0 = \lambda_1 (L) < \lambda_2 (L) \leq \cdots \leq \lambda_N (L) \) and the smallest positive eigenvalue \( \lambda_1 (L) = \min_{x^T L x \neq 0} \{x^T L x / x^T x\} \) if and only if the network is connected.

2. For any vector \( \eta = (\eta_1, \eta_2, \ldots, \eta_N)^T \in \mathbb{R}^N \), the equation satisfies
   \[
   \eta^T L \eta = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} G_{ij} (\eta_i - \eta_j)^2.
   \] (1)

2.2. Problem Formation. In this paper, we consider that there are \( N \) follower multiagent systems, which can be described by

\[
\dot{\omega}_i (t) = A_i \omega_i (t) + B_i f (\omega_i (t), t) + u_i (t),
\] \[ \text{for any vector } \omega, \eta, \eta_1, \ldots, \eta_N \in \mathbb{R}^N \text{, the vector function } \eta = (\eta_1, \eta_2, \ldots, \eta_N)^T \in \mathbb{R}^N \text{, the equation satisfies}
\]

\[
\eta^T L \eta = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} G_{ij} (\eta_i - \eta_j)^2.
\] (1)

where \( \omega_i (t) \in \mathbb{R}^n \) can be regarded as the position vector of the agent \( i, f: \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^n \) represents a continuous nonlinear vector function. \( A_i, B_i \in \mathbb{R}^{n \times n} \) represent the system matrices of the agent \( i \), respectively. \( u_i (t) \in \mathbb{R}^n \) denotes the control protocol to be designed.

The leader agent can be described as follows:

\[
\dot{\omega}_0 (t) = A_0 \omega_0 (t) + B f (\omega_0 (t), t),
\] \[ \text{where } \omega_0 (t) \in \mathbb{R}^n \text{ represents the leader’s state vector and } A, B \in \mathbb{R}^{n \times n} \text{ represent the leader’s system matrices, respectively.}
\]

For the above heterogeneous leader-follower multiagent system, we give the following assumptions.

**Assumption 1.** Suppose there is a normal quantity \( l \) so that the vector function \( f \) for any vector \( \lambda, v \in \mathbb{R}^n \) satisfies

\[
\| f (\lambda, v) - f (\nu, v) \| \leq l (\| \lambda - \nu \|).
\] (4)

**Lemma 2** (see [47]). For any vector \( x, y \in \mathbb{R}^n \), the following holds:

\[
x^T y \leq \frac{1}{2} x^T x + \frac{1}{2} y^T y.
\] (5)

**Lemma 3.** Any two continuous functions satisfy

\[
\dot{v} (t) + w (t) \leq -\gamma v (t),
\] \[ \text{where } \gamma > 0. \text{ Then, there is a } t_0 \geq 0 \text{ so that the following holds:}
\]

\[
v (t) \leq (v (0) + w (0)) e^{-\gamma t} - w (t) + ye^{-\gamma t} * w (t), \quad t \geq t_0,
\] \[ \text{where } * \text{ is the convolution.}
\] (7)
Proof. Since \( v(t) + \dot{w}(t) \leq -\gamma v(t) \), the existence of \( z(t) \geq 0 \) makes the following equation:

\[
\dot{v}(t) + \dot{w}(t) + z(t) = -\gamma v(t).
\]

We calculate the Laplace transform of (8) to obtain

\[
sv(s) + sw(s) - (v(0) + w(0)) + Z(s) = -\gamma V(s).
\]

Then, we can obtain

\[
V(s) = (v(0) + w(0)) - \frac{s}{s + \gamma} W(s) - \frac{s}{s + \gamma} Z(s).
\]

The inverse Laplace transform of (10) can be obtained as

\[
v(t) = (v(0) + w(0))e^{-\gamma t} - w(t) + ye^{-\gamma t} \cdot w(t) + \frac{\varepsilon}{\gamma}, \quad t \geq 0.
\]

Through (10), we can obtain

\[
v(t) \leq (v(0) + w(0))e^{-\gamma t} - w(t) + ye^{-\gamma t} \cdot w(t) + \frac{\varepsilon}{\gamma}.
\]

Lemma 4. Any two continuous functions satisfy

\[
\dot{v}(t) + \dot{w}(t) \leq -\gamma v(t) + \varepsilon,
\]

where \( \gamma > 0 \) and \( \varepsilon > 0 \). Then, there is a \( t \geq 0 \) so that the following formula holds:

\[
v(t) \leq (v(0) + w(0))e^{-\gamma t} - w(t) + ye^{-\gamma t} \cdot w(t) + \frac{\varepsilon}{\gamma}, \quad t \geq 0.
\]

Proof. Since \( \dot{v}(t) + \dot{w}(t) \leq -\gamma v(t) + \varepsilon \), then \( v(t) \leq (v(0) + w(0))e^{-\gamma t} - w(t) + ye^{-\gamma t} \cdot w(t) + \frac{\varepsilon}{\gamma} \).

By Lemma 3, we can obtain

\[
v(t) \leq (v(0) + w(0))e^{-\gamma t} - w(t) + ye^{-\gamma t} \cdot w(t) + \frac{\varepsilon}{\gamma}.
\]

We can further obtain

\[
v(t) \leq (v(0) + w(0))e^{-\gamma t} - w(t) + ye^{-\gamma t} \cdot w(t) + \frac{\varepsilon}{\gamma}.
\]

Lemma 5 (see [47]). If \( A, B, C, \) and \( D \) represent four different matrices, respectively, and the matrix products \( AC \) and \( BD \) makes sense, the Kronecker product \( \otimes \) satisfies

\[
(1) A \otimes (B + C) = A \otimes B + A \otimes C,
(2) (A \otimes B)(C \otimes D) = (AC) \otimes (BD).
\]

Definition 1 (see [29]). The leader-follower HMASs are decided to reach QC if

\[
\lim_{t \to \infty} \| \omega_i(t) - \omega_0(t) \| \leq \xi, \quad i = 1, 2, \ldots, N,
\]

where \( \xi \) is a nonnegative constant.

Assumption 2. The network topology between agents is undirected and connected, and each agent can acquire the status information of the agent that has a connection relationship with it and the leader agent at any time.

2.3. Our Controller. To obtain QC between HMASs (2) and (3), we design the control input for all follower agents as

\[
u_i(t) = -c \sum_{j=1}^{N} L_{ij}(t)(\omega_j(t) - r_j(t)(\omega_i(t) - \omega_0(t))),
\]

where \( c \) is a positive constant.

The adaptive law for the control gains is described as

\[
\dot{r}_i(t) = \mu(\omega_i(t) - \omega_i(t))^T (\omega_i(t) - \omega_0(t)),
\]

where \( \mu \) is a positive constant to be selected.

The adaptive law for the coupling weights is described as

\[
\dot{L}_{ij}(t) = -\alpha_{ij}(\omega_i(t) - \omega_j(t))^T(\omega_i(t) - \omega_j(t)),
\]

where \( \alpha_{ij} = \alpha_{ji} \) are the positive constants to be selected.

Remark 1. It should be noted that, in controller (19), adaptive laws (20) and (21), the coupling weights \( L_{ij}(t) \) and the control gains \( r_i(t) \) are adjusted adaptively based on local communication with the leader and between the followers. Combining adaptation of the coupling weights and control gains, adaptive law (20) ensures the QC of the follower agents, while adaptive law (21) drives the follower agents to the leader agent.

Let the QC error vector be \( e_i(t) = \omega_i(t) - \omega_0(t) \). Then, the error model with the controller iscom

\[
\dot{e}_i(t) = A_ie_i(t) + B_i\bar{f}(e_i(t), t) + h_i(\omega_0(t), t)
\]

\[
= -c \sum_{j=1}^{N} L_{ij}(t)e_j(t) - r_i(t)e_i(t),
\]

where \( \bar{f}(e_i(t), t) = f(\omega_i(t), t) - f(\omega_0(t), t) \) and

\[
h_i(\omega_0(t), t) = (A_i - A)\omega_0(t) + (B_i - B)f(\omega_0(t), t).
\]

\( h_i(\omega_0(t), t) \) represents the difference between different agents. It can be obtained by Assumption 1:

\[
\| h_i(\omega_0(t), t) \| = \| (A_i - A)\omega_0(t) + (B_i - B)f(\omega_0(t), t) \|
\]

\[
\leq \| A_i - A \| \| \omega_0(t) \| + \| B_i - B \| \| f(\omega_0(t), t) \|
\]

\[
\leq \| A_i - A \| \| \omega_0(t) \| + \| B_i - B \| \| f_{\text{max}} \| \omega_0(t). \]

Since \( \omega_0(t) \) is bounded, it can be obtained that

\( h_i(\omega_0(t), t) \) is bounded.

3. Main Results

In this section, we present some theorems for achieving QC in HMASs.
3.1. Adaptive Control Protocol

**Theorem 1.** It is assumed that \( f(\omega_i(t), t) \) satisfies Assumption 1 and the follower system (2) satisfies Assumption 2. Under the action of controller (19) and adaptive laws (20) and (21), HMASs (2) and (3) can achieve QC.

The Lyapunov function we constructed is

\[
V_1(t) = \frac{1}{2} \sum_{i=1}^{N} e_i^T(t)e_i(t) + \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \frac{c}{4\omega_{ij}} (L_{ij}(t) + k_{ij})^2 + \frac{1}{\mu} \sum_{i=1}^{N} (r_i(t) - d_i^*)^2,
\]

where \( k_{ij} = k_{ji} \) \((i \neq j)\) is a nonnegative constant if and only if \( L_{ij}(t) = 0 \) is \( k_{ij} = 0 \). \( d_i^* \) is the normal constant waiting for the value.

We take the derivative of \( V_1(t) \) along (22) together with controller (19) and adaptive laws (20) and (21), and we can obtain

\[
\dot{V}_1(t) = \sum_{i=1}^{N} e_i^T(t)e_i(t) + \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \frac{c}{4\omega_{ij}} (L_{ij}(t) + k_{ij})^2 + \sum_{i=1}^{N} \omega_i(t) \dot{\omega}_i(t) - \sum_{i=1}^{N} \dot{r}_i(t)e_i(t)
\]

\[
= \sum_{i=1}^{N} e_i^T(t)A_i e_i(t) + \sum_{i=1}^{N} e_i^T(t)B_i \tilde{f}(e_i(t), t) + \sum_{i=1}^{N} e_i^T(t)h_i(\omega_0(t), t) - c \sum_{i=1}^{N} L_{ij}(t)e_j(t) - r_i(t)e_i(t)
\]

\[
- \frac{c}{2} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} (L_{ij}(t) + k_{ij})(\omega_i(t) - \omega_j(t))^T(\omega_i(t) - \omega_j(t))
\]

Through Assumption 1 and Lemma 2, one has

\[
\sum_{i=1}^{N} e_i^T(t)B_i \tilde{f}(e_i(t), t) = \sum_{i=1}^{N} e_i^T(t)(f(\omega_i(t), t) - f(\omega_0(t), t))
\]

\[
\leq \frac{1}{2} \sum_{i=1}^{N} e_i^T(t)B_iB_i^T e_i(t) + \frac{1}{2} \sum_{i=1}^{N} \|f(\omega_i(t), t) - f(\omega_0(t), t)\|^2
\]

\[
\leq \frac{1}{2} \sum_{i=1}^{N} e_i^T(t)(B_iB_i^T + \tilde{L}_i) e_i(t),
\]

\[
\sum_{i=1}^{N} e_i^T(t)h_i(\omega_0(t), t) \leq \frac{1}{2} \sum_{i=1}^{N} e_i^T(t)e_i(t) + \frac{1}{2} \sum_{i=1}^{N} \|h_i(\omega_0(t), t)\|^2.
\]
From (25)–(27), we can obtain

\[
\dot{V}_i (t) \leq \sum_{j=1}^{N} \frac{N}{2} \dot{c}_i^T (t) A_i c_i (t) + \frac{1}{2} \sum_{j=1}^{N} \dot{c}_i^T (t) c_i (t) + \frac{1}{2} \sum_{j=1}^{N} \| h_{i} (\omega_{0} (t), t) \|^2 \\
- \sum_{j=1}^{N} \dot{c}_i (t) \sum_{j=1}^{N} L_{ij} (t) c_j (t) + \frac{1}{2} \sum_{j=1}^{N} \dot{c}_i (t) (B_i B_i^T + \bar{I} I_{n}) c_i (t) - \sum_{j=1}^{N} d_i^T \dot{c}_j (t) c_j (t) \\
- \sum_{j=1}^{N} \sum_{j \neq i}^{N} (L_{ij} (t) + k_{ij}) \left( \omega_i (t) - \omega_j (t) \right)^T \left( \omega_i (t) - \omega_j (t) \right) .
\]

Combining with (28) and (29), we have

\[
\sum_{j=1}^{N} \frac{N}{2} \dot{c}_i (t) L_{ij} (t) c_j (t) + \frac{1}{2} \sum_{j=1}^{N} \dot{c}_i (t) c_j (t) - \sum_{j=1}^{N} \sum_{j \neq i}^{N} \tau_{ij} \dot{c}_i (t) c_j (t)
\]

\[
= -c \sum_{i=1}^{N} \sum_{j=1}^{N} L_{ij} (t) \dot{c}_i (t) c_j (t) + c \sum_{i=1}^{N} \sum_{j=1}^{N} \tau_{ij} \dot{c}_i (t) c_j (t).
\]

Then, there is a unitary matrix \( U = (u_1, \ldots, u_N) \) so that

\( U^T \Omega U = \Delta, \ m (t) = (U^T \otimes I_n) e (t) \),

\[
\dot{V}_i (t) \leq e^T (t) \left[ A + \frac{1}{2} \left( BB^T + I_N \otimes (\bar{I}^2 + 1) I_{n} - 2 (D^* \otimes I_{n}) \right) - \frac{1}{2} \| h_{i} (\omega_{0} (t), t) \|^2 \right] \cdot e (t) + \frac{1}{2} \| h_{i} (\omega_{0} (t), t) \|^2 \\
- c \left( \frac{1}{2} \left( BB^T + I_N \otimes (\bar{I}^2 + 1) I_{n} - 2 (D^* \otimes I_{n}) \right) \right) \cdot e (t) + \frac{1}{2} \| h_{i} (\omega_{0} (t), t) \|^2 \\
= e^T (t) \left[ A + \frac{1}{2} \left( BB^T + I_N \otimes (\bar{I}^2 + 1) I_{n} - 2 (D^* \otimes I_{n}) \right) \right] \cdot e (t) - \frac{1}{2} \| h_{i} (\omega_{0} (t), t) \|^2 \\
+ \frac{1}{2} \| h_{i} (\omega_{0} (t), t) \|^2 .
\]
In view of this, the adaptive pinning control neither realistic nor economical to control all follower agents and (21) for all follower agents are designed. However, it is seen in the previous section, an adaptive controller (19) and an adaptive laws (20) and the coupling weights are adapted. 

Since \( I_n \) is positive definite, by Lemma 1, we can get the following:

\[
\dot{V}_1(t) \leq e^T(t) \left[ A + \frac{1}{2} (BB^T + I_N \otimes (I^2 + 1)I_n - 2(D^* \otimes I_n)) \right] e(t) + \frac{1}{2} \| h(\omega_0(t), t) \|^2 - c \lambda_2(\Omega) m^T(t)(I_n \otimes I_n)m(t) 
\]

\[
= e^T(t) \left[ A + \frac{1}{2} (BB^T + I_N \otimes (I^2 + 1)I_n - 2(D^* \otimes I_n)) \right] e(t) + \frac{1}{2} \| h(\omega_0(t), t) \|^2 - c \lambda_2(\Omega)e^T(t)(P \otimes I_n)(I_N \otimes I_n)(P^T \otimes I_n) e(t) 
\]

\[
= e^T(t) \left[ A + \frac{1}{2} (BB^T + I_N \otimes (I^2 + 1)I_n - 2(D^* \otimes I_n)) - c \lambda_2(\Omega)(I_N \otimes I_n) \right] e(t) + \frac{1}{2} \| h(\omega_0(t), t) \|^2.
\]

We can choose large enough \( \kappa_{ij} \) and \( d_i^* \) so that \( k_{max} \) is positive definite, by Lemma 1, we can get the following:

\[
\lambda_{max}(A + \frac{1}{2} (BB^T + I_N \otimes (I^2 + 1)I_n - 2(D^* \otimes I_n)) - c \lambda_2(\Omega)(I_N \otimes I_n)) + \theta \leq 0, 
\]

where \( \theta > 0 \) is a positive constant.

\[
\dot{V}_1(t) \leq -\theta e^T(t)e(t) + \frac{1}{2} \| h(\omega_0(t), t) \|^2, 
\]

where \( e(t) = (e^T_1, e^T_2, \ldots, e^T_N)^T \).

By Lemma 4, it yields

\[
e^T(t)e(t) \leq \left( \sum_{i=1}^{N} e^2_i(0)e_i(0) + \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \frac{c}{2\omega_{ij}} (L_{ij}(0) - k_{ij})^2 \right) + \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \frac{c}{2\omega_{ij}} (L_{ij}(0) - k_{ij})^2 + \frac{\| h(\omega_0(t), t) \|^2}{2\theta}.
\]

The error will eventually converge to finite region \( \Xi = \{ e(t) \in R^n \| e(t) \| \leq \sqrt{\| h(\omega_0(t), t) \|^2/2\theta} \} \) as \( t \to +\infty \). This completes the proof.

### 3.2. Adaptive Pinning Control Protocol

In the previous section, an adaptive controller (19) and an adaptive laws (20) and (21) for all follower agents are designed. However, it is neither realistic nor economical to control all follower agents in engineering. In view of this, the adaptive pinning control schemes are considered, where a fraction of the control gains and the coupling weights are adapted.

Suppose that \( \bar{\Xi} \) is a subset of \( \Xi \) and HMASs (2) and (3) are connected by the undirected edge. \( \bar{\Xi} \) is connected. The pinning adaptive protocol is written as follows:

\[
u_i(t) = -c \sum_{j=1}^{N} L_{ij}(t)\omega_j(t) - \theta_i r_i(t)(\omega_i(t) - \omega_0(t)),
\]

where \( \theta_i = \begin{cases} 1, & \text{for } i = 1, 2, \ldots, N_1, \\ 0, & \text{for } i = N_1 + 1, \ldots, N \end{cases} \)
The adaptive law for $r_i(t)$ is the same as (20). The adaptive law for $L_{ij}(t)$ is described as

$$\dot{L}_{ij} = -\alpha_{ij}(\omega_i(t) - \omega_j(t))^T(\omega_i(t) - \omega_j(t)),$$

where $(i, j) \in \bar{E}$ and $L_{ij}(0) = L_{ji}(0) > 0$. \hfill (38)

**Theorem 2.** Suppose that Assumption 1 is valid and the follower system (2) satisfies Assumption 2. The HMASs (2) and (3) can achieve QC under the pinning adaptive protocol combined with (20), (21), and (37).

**Proof.** Take into account the Lyapunov function candidate:

$$V_2(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)e_i(t) + \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \frac{c}{2\alpha_{ij}}(L_{ij}(t) + k_{ij})\dot{L}_{ij}(t) + \sum_{i=1}^{N} \frac{1}{2\mu}(r_i(t) - d_i^*)^2,$$

where $c_{ij} = c_{ji}(i \neq j)$ is a nonnegative quantity if and only if $L_{ij}(t) = 0$ and $k_{ij} = 0$. $d_i^*$ is the normal constant waiting for the value if $i = 1, 2, \ldots, N$, $N_1, N_2 \geq 1$, or $d_i^* = 0$. \hfill (39)

The derivative of $V_2(t)$ along (22), controller (37), and the decentralized adaptive pinning laws (20) and (21) gives

$$\dot{V}_2(t) = \sum_{i=1}^{N} e_i^T(t)\dot{e}_i(t) + \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \frac{c}{2\alpha_{ij}}(L_{ij}(t) + k_{ij})\dot{L}_{ij}(t) + \sum_{i=1}^{N} \frac{1}{2\mu}(r_i(t) - d_i^*)\dot{d}_i(t)$$

$$\leq \sum_{i=1}^{N} e_i^T(t)A_e e_i(t) + \frac{1}{2} \sum_{i=1}^{N} e_i^T(t)(BB^T + (I^2 + 1)I_n - 2d_i^* \otimes I_n)e_i(t)$$

$$+ \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \tau_{ij} e_i^T(t)e_j(t)$$

$$\leq e^T(t)\left[A + \frac{1}{2}(BB^T + I_N \otimes (I^2 + 1)I_n - 2(\bar{D}^* \otimes I_n)) - \alpha_3 (1) (I_n \otimes I_n)\right]e(t) + \frac{1}{2}\|h(\omega_0(t), t)\|^2,$$

where $\bar{D}^* = \text{diag}(d_1^*, d_2^*, \ldots, d_N^*, 0, \ldots, 0)$.

The proof can be completed by using the similar analysis method in Theorem 1. \hfill (40)

**Theorem 3.** Suppose that Assumptions 1 and 2 hold. The HMASs (2) and (3) can achieve QC under the pinning adaptive protocol combined with (19), (20), and (38).

**Proof.** Construct the Lyapunov functional as

$$V_3(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)e_i(t) + \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \frac{c}{4\alpha_{ij}}(L_{ij}(t) + \bar{k}_{ij})^2 + \sum_{i=1}^{N} \frac{1}{2\mu}(r_i(t) - d_i^*)^2,$$

where $\bar{k}_{ij} = \bar{k}_{ji} > 0$, $(i, j) \in \bar{E}$, and $\bar{k}_{ij} = 0(i \neq j)$, else. \hfill (41)

Let $\bar{K} = (\bar{k}_{ij})_{N \times N}$, $\bar{k}_{ii} = -\sum_{i=1}^{N} \bar{k}_{ij}$; then,

$$G_{ij} = \begin{cases} L_{ij}(0), & (i, j) \in E - \bar{E}, \\ -\sum_{j=1, j \neq i}^{N} L_{ij}(0), & i = j, \\ 0, & \text{other.} \end{cases}$$

$$\dot{L}_{ij} = -\alpha_{ij}(\omega_i(t) - \omega_j(t))^T(\omega_i(t) - \omega_j(t)),$$

where $(i, j) \in \bar{E}$ and $L_{ij}(0) = L_{ji}(0) > 0$. \hfill (38)
After the derivative of $V_3(t)$ along (22), controller (19), and the decentralized adaptive pinning laws (20) and (38), the following holds:

\[
\dot{V}_3(t) = \sum_{i=1}^{N} e_i^T(t)A e_i(t) + \sum_{i=1}^{N} e_i^T(t)L_{ij}(t) + \sum_{i=1}^{N} \left( \frac{c}{4a_{ij}} \right) (L_{ij}(t) + \bar{\kappa}_{ij})L_{ij}(t) + \frac{1}{\mu} \sum_{i=1}^{N} \left( r_i(t) - d_i^* \right) \dot{d}_i(t)
\]

\[
= \sum_{i=1}^{N} e_i^T(t)A e_i(t) + \sum_{i=1}^{N} e_i^T(t)B_i \bar{f}_i(e_i(t), t) + \sum_{i=1}^{N} e_i^T(t)h_i(\omega_0(t), t)
\]

\[
- \sum_{i=1}^{N} e_i^T(t)c \sum_{i=1}^{N} L_{ij}(t)e_j(t) - \sum_{i=1}^{N} d_i^* e_i^T(t) e_i(t)
\]

\[
- \frac{c}{2} \sum_{i=1}^{N} \sum_{(i,j) \in \mathcal{E}} (L_{ij}(t) + \bar{\kappa}_{ij})(\omega_i(t) - \omega_j(t))^T(\omega_i(t) - \omega_j(t))
\]

\[
\leq \sum_{i=1}^{N} e_i^T(t)A e_i(t) + \frac{1}{2} \sum_{i=1}^{N} e_i^T(t)c \sum_{i=1}^{N} L_{ij}(t)e_j(t) + \frac{1}{2} \sum_{i=1}^{N} \|h_i(\omega_0(t), t)\|^2
\]

\[
+ \frac{1}{2} \sum_{i=1}^{N} e_i^T(t)(B_iB_i^T + I_N) e_i(t) - c \sum_{i=1}^{N} \sum_{j=1}^{N} G_{ij}(t)e_i^T(t)e_j(t)
\]

\[
- c \sum_{i=1}^{N} \sum_{j=1}^{N} \bar{\tau}_{ij} e_i^T(t)e_j(t) - \sum_{i=1}^{N} d_i^* e_i^T(t)e_i(t)
\]

\[
= e^T(t) \left[ A + c(G \otimes I_N) + \frac{1}{2} (I_N \otimes (BB^T + (l_2^2 + 1)I_N)) \right]
\]

\[
(D^* \otimes I_N) - c\lambda_2(\Omega^*) (I_N \otimes I_N) \|e(t)\|^2 + \frac{1}{2} \|h(\omega_0(t), t)\|^2,
\]

where $G = (G_{ij})_{N \times N}$, $\Omega^* = (\bar{\tau}_{ij})_{N \times N}$, $\bar{\tau}_{ij} = -\bar{\kappa}_{ij}, i \neq j$, and

\[
\bar{\tau}_{ii} = -\sum_{j=1}^{N} \bar{\tau}_{ij}.
\]

The following proof is similar to the previous derivation in Theorem 1; thus, we will omit this part here.

**Theorem 4.** Suppose that Assumptions 1 and 2 hold. The HMASs (2) and (3) can achieve QC under the pinning adaptive protocol combined with (20), (37), and (38).

**Proof.** The Lyapunov functional is considered as follows:

\[
V_4(t) = \frac{1}{2} \sum_{i=1}^{N} e_i^T(t) e_i(t) + \sum_{i=1}^{N} \sum_{(i,j) \in \mathcal{E}} \left( \frac{c}{4a_{ij}} \right) (L_{ij}(t) + \bar{\kappa}_{ij})^2 + \sum_{i=1}^{N} \frac{1}{2\mu} (r_i(t) - d_i^*)^2.
\]
Remark 2. In Theorems 2–4, some sufficient conditions are
given to realize QC of HMASs (2) and (3) by using the
adaptive pinning control schemes. Actually, if the follower
agents (3) are connected, one can randomly choose a small
fraction of coupling weights and/or the control gains to
adapt. In particular, it is possible to obtain the QC by
pinning one follower agent.

4. Numerical Examples

In this section, we will confirm the theory proposed in the
paper by using digital simulation experiments.

We use five following agents and one leader agent, and
the initial communication Laplacian matrix for follower
agents is

\[
L = \begin{pmatrix}
1.5 & -0.3 & -0.3 & -0.4 & -0.5 \\
-0.3 & 0.9 & -0.6 & 0 & 0 \\
-0.3 & -0.6 & 0.9 & 0 & 0 \\
-0.4 & 0 & 0 & 0.7 & -0.3 \\
-0.5 & 0 & 0 & -0.3 & 0.8 \\
\end{pmatrix}
\]  

where \( G = (G_{ij})_{N \times N} \), \( \Omega^* = (\tau_{ij})_{N \times N} \), \( \tau_{ij} = -\tau_{ji}, i \neq j \),
and \( D^* = \text{diag}(d_{1}^*, d_{2}^*, \ldots, d_{N}^*, 0, 0, \ldots, 0) \).

The rest of the proof is similar to Theorem 1. To save
space, it is thus omitted here.

The leader’s system matrices can be described as

\[
A = \begin{pmatrix}
-2.5 & 10 & 0 \\
1 & -1 & 1 \\
0 & -18 & 0 \\
\end{pmatrix}
\] \quad and \quad \( B = \begin{pmatrix}
35/6 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{pmatrix} \),

and the nonlinear function of the entire system can be assumed to be

\[
f(t, x) = \begin{pmatrix}
0.5(|x_1(t) + 1| - |x_1(t) - 1|) \\
0 \\
0 \\
\end{pmatrix}.
\]

The following system matrices are, respectively, assumed to be

\[
A_i = \begin{pmatrix}
-2.5 + 0.1 \ast i & 10 + 0.2 \ast i & 0 \\
1 + 0.2 \ast i & -1 + 0.2 \ast i & 1 + 0.2 \ast i \\
0 & -18 + 0.2 \ast i & 0 \\
\end{pmatrix}
\] \quad and \quad \( B_i = \begin{pmatrix}
35/6 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{pmatrix} \quad (i = 1, 2, \ldots, 5). \)

We arbitrarily choose

the initial value as \( x_0(0) = (3.3, 0.66, 0.1)^T \) and
\( x_i(0) = (11.5 + 1.8 \ast i, 4.2 + 1.1 \ast i, 4.9 + 1.5 \ast i)^T \quad (i = 1, 2, \ldots, 5). \) Figure 1 shows the change of \( \|e_i(t)\|_2 \quad (i = 1, 2, \ldots, 5) \) for HMASs without the controller.

4.1. Example 1. In this case, the parameters of controller (19) and
adaptive laws (20) and (21) are chosen according to
Theorem 1. All agent system parameters are in accordance
with \( A_i \) and \( B_i \). We choose \( c = 2.6, \mu = 0.05, \alpha_{12} = \alpha_{13} = 2.7, \alpha_{13} = \alpha_{23} = 2.4, \alpha_{14} = \alpha_{41} = 2.1, \alpha_{15} = \alpha_{21} = 1.8, \alpha_{23} = \alpha_{32} = 2.1, \) and \( \alpha_{35} = \alpha_{54} = 2.2. \) We arbitrarily choose
the initial value as \( r(0) = (2.3, 2.1, 1.8, 1.8, 2.1)^T, \)
\( x_0(0) = (3.3, 0.66, 0.1)^T, \) and \( x_i(0) = (11.5 + 1.8 \ast i, 4.2 + 1.1 \ast i, 4.9 + 1.5 \ast i)^T \quad (i = 1, 2, \ldots, 5). \) Figure 2 shows

the change in the state of the system under the increment controller. It visibly shows that, under the action of the adaptive controller and adaptive law, the error of the leader and follower finally converges to a finite region.

4.2. Example 2. In this example, we only use the coupling between followers and leaders 1 and 2 to achieve QC of the entire system. All agent system parameters are in accordance with $A_1$ and $B_1$. We choose $c = 2.6, \mu = 0.01, \alpha_{12} = \alpha_{31} = 8.1, \alpha_{13} = \alpha_{31} = 7.2, \alpha_{14} = \alpha_{41} = 6.3, \alpha_{15} = \alpha_{51} = 5.4, \alpha_{23} = \alpha_{32} = 6.3, \alpha_{36} = 7.2$. We arbitrarily choose the 1 and 2 agents, and their initial values are selected as $r(0) = (2.3, 2.1)^T$. The initial value of each agent is selected as $x_0(0) = (3.3, 0.66, 0.1)^T$ and $x_i(0) = (-6 - 1.6 * i, 5 + 1.1 * i, 9.4 + 2 * i)^T (i = 1, 2, \ldots, 5)$. Figure 3 shows $e_i(t)_2 (i = 1, 2, \ldots, 5)$ eventually tends to a finite region.

4.3. Example 3. In this example, we use a control strategy that pinning the coupling between followers to QC of HMASs (2) and (3). All agent system parameters are in accordance with $A_1$ and $B_1$. We choose $c = 1.2$. The parameters in the adaptive law are $\mu = 0.01, \alpha_{14} = \alpha_{41} = 7.2, \alpha_{15} = \alpha_{51} = 6.3, \alpha_{23} = \alpha_{32} = 5.4$. We arbitrarily choose the initial value as $r(0) = (2.1, 2.1, 2.1, 2.1)^T, x_0(0) = (3.3, 0.66, 0.1)^T$ and $x_i(0) = (-5.3 - 0.5 * i, 3 + 1.6 * i, 2.7 + 1.4 * i)^T (i = 1, 2, \ldots, 5)$. From Figure 4, we can distinctly see that, under the control strategy designed by Theorem 3, $e_i(t)_2 (i = 1, 2, \ldots, 5)$ of HMASs (2) and (3) are concentrated in a limited area.

4.4. Example 4. In this case, by controlling the coupling between part of the follower agent and the leader and the coupling between the follower agents, we realize the QC of the whole system. All agent system parameters are in accordance with $A_1$ and $B_1$. We choose $c = 3$. The parameters in the adaptive law are $\mu = 0.01, \alpha_{14} = \alpha_{41} = 4.9, \alpha_{15} = \alpha_{51} = 4.2, \alpha_{23} = \alpha_{32} = 4.9$. We arbitrarily choose the 1, 2, and 3 agents, and their initial values are selected as $r(0) = (1.8, 2.1, 1.8)^T$. The initial value of each agent is selected as $x_0(0) = (3.3, 0.66, 0.1)^T$ and $x_1(0) = x_2(0) = x_3(0) = x_4(0) = x_5(0) = (12.3, 5, 5.1)^T$. 

**Figure 1:** The changes of $\|e_i(t)\|_2 (i = 1, 2, \ldots, 5)$ for the system without the controller.

**Figure 2:** The changes of $\|e_i(t)\|_2 (i = 1, 2, \ldots, 5)$ for the system under the control strategy designed by Theorem 1.
Figure 3: The changes of $\|e_i(t)\|_2$ ($i = 1, 2, \ldots, 5$) for the system under the control strategy designed by Theorem 2.

Figure 4: The changes of $\|e_i(t)\|_2$ ($i = 1, 2, \ldots, 5$) for the system under the control strategy designed by Theorem 3.
Figure 5 also verifies that, under the control strategy designed by Theorem 4, the entire system gradually converges to a limited area.

5. Conclusions

The decentralized adaptive control for QC of HMASs has been studied. The combined adaptation of the coupling weights and control gains allows to drive HMASs (2) and (3) to some bounded areas. In addition, some pinning schemes have been proposed to adjust a fraction of the coupling weights and control gains. To deal with the heterogeneity, two new lemmas are proposed to derive the QC criteria. It has been shown that the QC can be obtained without requiring any global. In future works, we will extend the results to more general HMASs, such as fractional-order HMASs, HMASs with time delay, and cooperative-competitive interaction. At the same time, we will attempt to optimize control protocol and extend it to some other systems, i.e., fractional-order systems [48], memristive neural networks [49, 50], and complex-valued neural networks [51, 52].

Data Availability

The data used to support the findings of the study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work was supported jointly by the “Chunhui Plan” Cooperative Research for Ministry of Education, under Grant 191657, the Foundation of Science and Technology Department of Sichuan Province, under Grant 2020ZHCG0076, the Key Scientific Research Fund Project of Xihua University, under Grant Z17124, the Graduate Innovation Fund of Xihua University, under Grant YCJJ2019035, the Open Research Subject of Artificial Intelligence Key Laboratory of Sichuan Province, under Grant 2017YJY03, the Key Research and Development Project of Sichuan Province, under Grant 2021YFG0071, and the Major Scientific and Technological Innovation Project of Chengdu, Sichuan Province, under Grant 2019-YF08-00003-GX.

References

[1] M. Xu, K. An, L. H. Vu, Z. Ye, J. Feng, and E. Chen, “Optimizing multi-agent based urban traffic signal control system,” Journal of Intelligent Transportation Systems, vol. 23, no. 4, pp. 357–369, 2019.

[2] S.-X. Tang, J. Qi, and J. Zhang, “Formation tracking control for multi-agent systems: a wave-equation based approach,” International Journal of Control, Automation and Systems, vol. 15, no. 6, pp. 2704–2713, 2017.

[3] S. M. Muyeen, A. Ghosh, S. M. Islam, and M. S. Baptista, “Multi-agent systems in ICT enabled smart grid: a status update on technology framework and applications,” IEEE Access, vol. 7, pp. 97959–97973, 2019.

[4] P. P. Li, S. Yang, and S. C. Wang, “A decentralized multi-agent control approach for robust robot plan execution,” International Journal of Advanced Robotic Systems, vol. 15, no. 2, pp. 1–14, 2018.

[5] J. Qin and Q. Ma, “Recent advances in consensus of multi-agent systems: a brief survey,” IEEE Transactions on Industrial Electronics, vol. 64, no. 6, pp. 4972–4983, 2016.

[6] Z.-H. Zhu, Z.-H. Guan, B. Hu, D.-X. Zhang, X.-M. Cheng, and T. Li, “Semi-global bipartite consensus tracking of singular multi-agent systems with input saturation,” Neurocomputing, vol. 432, pp. 183–193, 2021.

[7] J. Ni, P. Shi, Y. Zhao, and Z. Wu, “Fixed-time output consensus tracking for high-order multi-agent systems with directed network topology and packet dropout,” IEEE/CAA Journal of Automatica Sinica, vol. 8, no. 4, pp. 817–836, 2021.
Discrete Dynamics in Nature and Society

[8] X. Guo, J. Liang, and J. Lu, “Scaled consensus problem for multi-agent systems with semi-Markov switching topologies: a view from the probability,” Journal of the Franklin Institute, vol. 358, no. 6, pp. 3150–3166, 2021.

[9] C. Du, X. Liu, W. Ren, P. Lu, and H. Liu, “Finite-time consensus for linear multiagent systems via event-triggered strategy without continuous communication,” IEEE Transactions on Control of Network Systems, vol. 7, no. 1, pp. 19–29, 2020.

[10] Y. Su and J. Huang, “Two consensus problems for discrete-time multi-agent systems with switching network topology,” Automatica, vol. 48, no. 9, pp. 1988–1997, 2012.

[11] T. Wang, M. Hu, and Y. Zhao, “Consensus of linear multiagent systems with stochastic noises and binary-valued communications,” International Journal of Robust and Nonlinear Control, vol. 30, no. 13, pp. 4863–4879, 2020.

[12] Q. Shen, P. Shi, J. Zhu, S. Wang, and Y. Shi, “Neural networks-based distributed adaptive control of nonlinear multivehicle systems,” IEEE Transactions on Neural Networks and Learning Systems, vol. 31, no. 3, pp. 1010–1021, 2020.

[13] H. Su, Y. Liu, and Z. Zeng, “Second-order consensus for multiagent systems via intermittent sampled position data control,” IEEE Transactions on Cybernetics, vol. 50, no. 7, pp. 2063–2072, 2020.

[14] M. Ali, R. Agalya, Z. Orman, and S. Arik, “Leader-following consensus of non-linear multi-agent systems with interval time-varying delay via impulsive control,” Neural Processing Letters, vol. 53, no. 1, pp. 69–83, 2020.

[15] Z. Wei, J. Huang, and P. Wei, “Weak synchronization of chaotic neural networks with parameter mismatch via periodically intermittent control,” Applied Mathematical Modelling, vol. 35, no. 2, pp. 612–620, 2011.

[16] H. Kim, H. Shim, and J. H. Seo, “Output consensus of heterogeneous uncertain linear multi-agent systems,” IEEE Transactions on Automatic Control, vol. 56, no. 1, pp. 200–206, 2010.

[17] G. Wen, Y. Zhang, Z. Peng, Y. Yu, and A. Rahmani, “Observer-based output consensus of leader-following fractional-order heterogeneous non-linear multi-agent systems,” International Journal of Control, vol. 93, no. 10, pp. 2516–2524, 2020.

[18] S. Luo, X. Xu, L. Liu, and G. Feng, “Output consensus of heterogeneous linear multi-agent systems with communication, input and output time-delays,” Journal of the Franklin Institute, vol. 357, no. 17, pp. 12825–12839, 2020.

[19] Y.-Y. Qian, L. Liu, and G. Feng, “Output consensus of heterogeneous linear multi-agent systems with adaptive event-triggered control,” IEEE Transactions on Automatic Control, vol. 64, no. 6, pp. 2606–2613, 2019.

[20] J. Han, H. Zhang, H. Jiang et al., “H∞ consensus for linear heterogeneous multi-agent systems with state and output feedback control,” Neurocomputing, vol. 18, no. 10, pp. 2468–2481, 2020.

[21] W. Hu, L. Liu, and G. Feng, “Output consensus of heterogeneous linear multi-agent systems by distributed event-triggered/self-triggered strategy,” IEEE Transactions on Cybernetics, vol. 47, no. 8, pp. 1914–1924, 2017.

[22] Q. Ma and G. Miao, “Output consensus for heterogeneous multi-agent systems with linear dynamics,” Applied Mathematics and Computation, vol. 271, pp. 548–555, 2015.

[23] Y. Cai, H. Zhang, Z. Gao, and S. Sun, “The distributed output consensus control of linear heterogeneous multi-agent systems based on event-triggered transmission mechanism under directed topology,” Journal of the Franklin Institute, vol. 357, no. 6, pp. 3267–3298, 2020.

[24] Z. Meng, Z. Lin, and W. Ren, “Robust cooperative tracking for multiple non-identical second-order nonlinear systems,” Automatica, vol. 49, no. 8, pp. 2363–2372, 2013.

[25] P. Gong and W. Lan, “Adaptive robust tracking control for uncertain nonlinear fractional-order multi-agent systems with directed topologies,” Automatica, vol. 92, pp. 92–99, 2018.

[26] J. Sun and Z. Geng, “Adaptive consensus tracking for linear multi-agent systems with heterogeneous unknown nonlinear dynamics,” International Journal of Robust and Nonlinear Control, vol. 26, no. 1, pp. 154–173, 2016.

[27] C.-J. Li and G.-P. Liu, “Consensus for heterogeneous networked multi-agent systems with switching topology and time-varying delays,” Journal of the Franklin Institute, vol. 355, no. 10, pp. 4198–4217, 2018.

[28] C. D. Cruz-Ancona, R. Martinez-Guerra, and C. A. Perez-Pinacho, “A Leader-following consensus problem of multi-agent systems in heterogeneous networks,” Automatica, vol. 115, pp. 108–899, 2020.

[29] Z. Wang and J. Cao, “Quasi-consensus of second-order leader-following multi-agent systems,” IET Control Theory & Applications, vol. 6, no. 4, pp. 545–551, 2012.

[30] L. Wang, W. Qian, and Q.-G. Wang, “Bounded synchronization of a time-varying dynamical network with nonidentical nodes,” International Journal of Systems Science, vol. 46, no. 7, pp. 1234–1245, 2013.

[31] H. Yang, Z. Wang, M. Xiao, G.-P. Jiang, and C. Huang, “Quasi-synchronization of heterogeneous dynamical networks with sampled-data and input saturation,” Neurocomputing, vol. 339, pp. 130–138, 2019.

[32] L. Pan and J. Cao, “Stochastic quasi-synchronization for delayed dynamical networks via intermittent control,” Communications in Nonlinear Science and Numerical Simulation, vol. 17, no. 3, pp. 1332–1343, 2012.

[33] F. Wang, Z. Zheng, and Y. Yang, “Quasi-synchronization of heterogeneous fractional-order dynamical networks with time-varying delay via distributed impulsive control,” Chaos, Solitons and Fractals, vol. 142, pp. 110–465, 2021.

[34] W. Fei and Y. Yang, “Quasi-synchronization for fractional-order delayed dynamical networks with heterogeneous nodes,” Applied Mathematics and Computation, vol. 339, pp. 1–14, 2018.

[35] Z. Wang, J. Fan, G. P. Jiang et al., “Consensus in nonlinear multi-agent systems with nonidentical nodes and sampled-data control,” Science China-Information Sciences, vol. 61, no. 12, pp. 122–203, 2018.

[36] W. Zhang, D. W. C. Ho, and Y. Liu, “Quasi-consensus of heterogeneous-switched nonlinear multiagent systems,” IEEE Transactions on Cybernetics, vol. 50, no. 7, pp. 3136–3146, 2020.

[37] D. Ye and Y. Shao, “Quasi-synchronization of heterogeneous nonlinear multi-agent systems subject to DOS attacks with impulsive effects,” Neurocomputing, vol. 366, pp. 131–139, 2019.

[38] W. Yu, G. Chen, and M. Cao, “Delay-induced consensus and quasi-consensus in multi-agent dynamical systems,” IEEE Transactions on Circuits & Systems I Regular Papers, vol. 60, no. 10, pp. 2679–2687, 2010.

[39] Q. Xu, S. Zhuang, S. Liu, and J. Xiao, “Decentralized adaptive coupling synchronization of fractional-order complex-variable dynamical networks,” Neurocomputing, vol. 186, pp. 119–126, 2016.
[40] Q. Xu, X. Xu, S. Zhuang, J. Xiao, C. Song, and C. Che, “New complex projective synchronization strategies for drive-response networks with fractional complex-variable dynamics,” *Applied Mathematics and Computation*, vol. 338, pp. 552–566, 2018.

[41] J.-L. Wang, H.-N. Wu, T. Huang, S.-Y. Ren, and J. Wu, “Passivity analysis of coupled reaction-diffusion neural networks with Dirichlet boundary conditions,” *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 47, no. 8, pp. 2148–2159, 2017.

[42] J.-L. Wang, H.-N. Wu, T. Huang, S.-Y. Ren, and J. Wu, “Passivity and output synchronization of complex dynamical networks with fixed and adaptive coupling strength,” *IEEE Transactions on Neural Networks and Learning Systems*, vol. 29, no. 2, pp. 364–376, 2018.

[43] J.-L. Wang, H.-N. Wu, T. Huang, S.-Y. Ren, and J. Wu, “Passivity of directed and undirected complex dynamical networks with adaptive coupling weights,” *IEEE Transactions on Neural Networks and Learning Systems*, vol. 28, no. 8, pp. 1827–1839, 2017.

[44] Q. Shen, P. Shi, J. Zhu et al., “Neural networks-based distributed adaptive control of nonlinear multiagent systems,” *IEEE Transactions on Neural Networks and Learning Systems*, vol. 31, no. 3, pp. 1010–1021, 2019.

[45] Y. Fan, T. Xiao, and Z. Li, “Distributed fuzzy adaptive control for heterogeneous nonlinear multiagent systems with similar composite structure,” *Complexity*, vol. 2020, Article ID 4081904, 10 pages, 2020.

[46] W. Yu, P. DeLellis, G. Chen, M. Di Bernardo, and J. Kurths, “Distributed adaptive control of synchronization in complex networks,” *IEEE Transactions on Automatic Control*, vol. 57, no. 8, pp. 2153–2158, 2012.

[47] D. D’Angeli and A. Donno, “Shuffling matrices, kronecker product and discrete fourier transform,” *Discrete Applied Mathematics*, vol. 233, pp. 1–18, 2017.

[48] Q. Xu, S. Zhuang, X. Xu, C. Che, and Y. Xia, “Stabilization of a class of fractional-order nonautonomous systems using quadratic Lyapunov functions,” *Advances in Difference Equations*, vol. 2018, pp. 1–15, 2018.

[49] Y. Huang, S. Lin, and E. Yang, “Event-triggered passivity of multi-weighted coupled delayed reaction-diffusion memristive neural networks with fixed and switching topologies,” *Communications in Nonlinear Science and Numerical Simulation*, vol. 89, pp. 1–28, 2020.

[50] Y.-L. Huang, S.-H. Qiu, and S.-Y. Ren, “Finite-time synchronisation and passivity of coupled memristive neural networks,” *International Journal of Control*, vol. 93, no. 12, pp. 2824–2837, 2020.

[51] X. Xu, Q. Xu, J. Yang, H. Xue, and Y. Xu, “Further research on exponential stability for quaternion-valued neural networks with mixed delays,” *Neurocomputing*, vol. 400, pp. 186–205, 2020.

[52] X. Xu, J. Yang, Q. Xu, Y. Xu, and S. Sun, “Exponential stability for delayed complex-valued neural networks with reaction-diffusion terms,” *Advances in Difference Equations*, vol. 2021, no. 1, pp. 1–27, 2021.