Elastic Turbulence in Channel Flows at Low Reynolds number

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Abstract

We experimentally demonstrate the existence of elastic turbulence in straight channel flow at low Reynolds numbers. Velocimetry measurements show non-periodic fluctuations in the wake of curved cylinders as well as in a parallel shear flow region. The flow in these two locations of the channel is excited over a broad range of frequencies and wavelengths, consistent with the main features of elastic turbulence. However, the decay of the initial elastic turbulence around the cylinders is followed by a growth downstream in the straight region. The emergence of distinct flow characteristics both in time and space suggests a new type of elastic turbulence, markedly different from that near the curved cylinders. We propose a self-sustaining mechanism to explain the sustained fluctuations in the parallel shear region.

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Viscoelastic fluids such as polymeric and surfactant solutions do not flow like water. One of the most striking differences is the presence of flow instabilities in these fluids even in the absence of inertia, i.e. low Reynolds number (Re) \[1-8\]. At high flow rates, flows of viscoelastic fluids exhibit a completely new type of chaotic behavior – elastic turbulence – that has no analogues in Newtonian liquids \[9-12\]. The precise mechanism of purely elastic instabilities is still not well understood, even though it is fundamental to our knowledge of how biological fluids (e.g. blood, vesicles, mucus) flow \[13-16\], chemical and polymer industries where flow instabilities have been plaguing processing for years \[17, 18\], and micro- and nano-fluidics where purely elastic instabilities were proposed as a way of effective mixing at small length scales \[11, 19-21\].

The origin of these flow instabilities is believed to be the polymeric elastic stress that is both anisotropic and strain-dependent \[22\]. These stresses are often observed in systems where the mean flow has sufficient curvature, such as the flow between rotating disk \[10, 23-24\], between concentric cylinders \[1, 2, 4, 12\], curved channels \[11\], and around obstacles \[5, 25\]. In these systems, high velocity gradients and curved streamlines can stretch the polymer molecules, inducing elastic stress and flow instabilities \[22\]. In fact, it has been argued that curvature is a necessary condition for infinitesimal perturbations to be amplified by the normal stress imbalances in viscoelastic flows \[26-28\], and much of recent work on elastic turbulence has been devoted to geometries with curvature \[9, 25\].

Recently it has been shown that flows of viscoelastic fluids in parallel geometries (pipes and channel flows) can be nonlinearly unstable to finite perturbations \[8, 29\]. Theoretical investigations using non-linear perturbation analysis \[29-32\] predict a subcritical bifurcation from stable base states while non-modal stability analysis predicts transient growth of perturbation before decay \[33, 34\]. Recent experiments have shown the existence of such subcritical transition (and hysteretic behavior) for the flow of a viscoelastic fluid in a straight channel geometry in the absence of inertia (i.e. low Re) \[8\]. This subcritical transition in viscoelastic channel flows was shown to be akin to the transition from laminar to turbulent flows of simple Newtonian fluids (e.g. water) in pipes, except that the governing parameter is the Weissenberg number (Wi), defined as the product of the fluid relaxation time \(\lambda\) and the flow shear-rate \(\dot{\gamma}\). However, the main features of the resulting unstable flow have yet to be well characterized and as a result the flow of viscoelastic fluids in channel flows remain poorly understood.
In this manuscript, we investigate the flow of a polymeric fluid in a straight micro-channel at low Re using particle tracking methods. The flow is excited using a linear array of cylinders and is monitored (i) immediately after the array of cylinders and (ii) far downstream. We find that both the flow next to the cylinder and far downstream show features of elastic turbulence including velocity fluctuations excited on a broad range temporal frequencies and spatial length scales. There are, however, significant differences between the flow in the parallel shear region and near the cylinders including the flow structure (c.f. Fig. 1b,c), velocity time series statistics, and temporal and spatial spectra decay. A simple mechanism is proposed for the sustained fluctuations observed in the parallel shear flow region based on a simple self-sustaining mechanism of energy flow between the flow fluctuation and polymer elastic energy.

The flow of a dilute polymeric solution is investigated using a straight micro-channel
with a squared cross-sectional area \((W = D = 100 \, \mu m)\). The microchannel is made of polydimethylsiloxane (PDMS) using standard soft-lithography methods. The length of the microchannel is much larger than its width \(L/W = 330\), and it is partitioned into two regions. The first region is comprised of a linear array of cylinders that extends for \(30W\). A total of 15 cylinders \((n = 15)\) are used in the linear array; an schematic is shown in Fig. 1(a). Each cylinder has a diameter of \(d\) of \(0.5W\) and are evenly spaced by a distance \(\ell = 2W\); the last cylinder is at position \(x = 0\). A long parallel shear flow region \(300W\) in length follows this initial linear array of cylinders. More details on the channel design can be found elsewhere \[8\].

The polymeric solution is prepared by adding 300 ppm of polyacrylamide (PAA, \(18 \times 10^6\) MW) in a viscous Newtonian solvent (90% by weight glycerol aqueous solution). This solution possesses a nearly constant viscosity of approximately \(\eta = 200 \, mPa \cdot s\); for more information on the rheological properties of the fluid, please see \[8\]. A Newtonian solution, 90% by weight glycerol in water, is also used for comparison. The Reynolds number (Re), is kept below 0.01, where \(Re = \rho U H/\eta\), \(U\) is the mean centerline velocity, \(H\) is the channel half width, and \(\rho\) is the fluid density. The strength of the elastic stresses compared to viscous stresses is characterized by the Weissenberg number \[3, 35\], here defined as \(Wi = N_1/2\dot{\gamma}\eta\), where \(\dot{\gamma} = U/H\) is the shear rate and \(N_1\) is the first normal stress difference. For the experiments presented here, the Weissenberg number is kept constant at approximately 10 and the number of cylinders at 15. We expect the flow of the polymeric solutions to be unstable under such conditions \[8\].

The flow is characterized using particle tracking methods. Fluorescent particles (0.6 \(\mu m\) in diameter) are dispersed in the fluids and imaged using an epifluorescent microscope and a high speed CMOS camera (up to \(10^4\) fps). Spatially-resolved velocity fields are obtained by tracking particles in a rectangular window \((width=0.9W, length=1.2W\), centered at \(y = 0\)) with a grid resolution of \(\sim 1 \, \mu m\). The resultant time resolution is \(\Delta t = 25 \, ms\). However, we can increase the time resolution and duration of the velocimetry measurements by decreasing the window size \((width=0.1W, length=0.7W)\). This time-resolved measurement produces velocity time series with high resolution \((\Delta t = 1 \, ms)\) and relatively long sampling duration \((up to 300 \, s)\).

We begin our flow analysis by measuring the flow streamwise velocity \(u\) in the wake of the last cylinder \((x = 2W)\) as well as in the parallel shear region \((x = 200W)\) using the
spatially-resolved measurement (i.e. large window size). The streamwise velocity fluctuation \( u' \) is obtained by subtracting the ensemble average \( \langle u \rangle \) from the measured signal, \( u' = u - \langle u \rangle \).

Figure 1(b) shows the space time plot of \( u' \) in the cylinder wake region \( (x = 2W \text{ in Fig. 1a}) \) of the channel, where the wall-normal \( y \) is the spatial coordinate. Note that the channel centerline is at \( y = 0 \). The data show relatively large velocity fluctuation in the cylinder wake, with the amplitude reaching approximately 2 mm/s or 28% of the overall channel centerline mean speed (~ 7 mm/s). Along the \( y \) direction, we find that high intensity fluctuations are concentrated in the form of “spots”, which are manifestations of streamwise streaks of high and low local velocity fluctuations. These streaks have a wide range of temporal durations and spatial sizes, as large as the cylinder diameter (~ 50 \( \mu \text{m} \)) and as small as the velocity grid spacing (~ 1 \( \mu \text{m} \)). Far downstream \( (200W, \text{Fig. 1c}) \), however, the flow is significantly different from that in the cylinder wake. We find that velocity fluctuations at 200\( W \) exist in the form of aperiodic “bursts” of various durations and appear to be spatially smoother in the wall-normal direction. We note that no appreciable fluctuations are found in the Newtonian case under similar conditions.

![FIG. 2. (color online). Time series and the associated probability distribution of centerline velocity fluctuations \( u'_c \) for \( n = 15 \) and \( Wi= 10 \) (a) Velocity records measured in the cylinder wake \( (x = 2W) \). An interval of 60 s is shown out of the total duration of 300 s (b) Velocity records measured far downstream in the parallel shear flow region \( (200W) \) (c) Probability distribution of the associated time series, normalized by the maximum of the probability density. Each curve includes \( 1.3 \times 10^6 \) samples.](image)
To quantify the temporal dynamics of the flow, we measure the centerline velocity fluctuations $u'_c(t)$ for both Newtonian and polymeric solutions in the wake of the cylinder (Fig. 2a) and in the parallel shear region (Fig. 2b) using the small interrogation window. The data show significant velocity fluctuations for the viscoelastic fluid; the standard deviation (i.e. fluctuations) reaches approximately 10% of the centerline mean, in both regions of the flow. No significant fluctuations are found in the Newtonian fluid case, shown in gray, under same conditions (i.e. flow rates). At both locations, the velocity fluctuations of the polymeric solution show an irregular pattern without an apparent periodicity, and the amplitudes of the centerline velocity variations are quite similar. There are, however, differences between the flow in the wake of the cylinder ($2W$) and in the parallel shear region ($200W$). For example, the data show that far downstream (Fig. 2b), the velocity fluctuations in the high frequency range are weaker compared to those in the cylinder wake (Fig. 2a), as will be discussed below. In addition, the mean of $u'_c(t)$ at the cylinder wake is negatively biased towards the low velocity values. Physically, this means the flow at $2W$ is characterized by frequent jumps to high velocities amidst dwelling at lower velocities, while far downstream ($200W$) the flow seems to fluctuate around the mean evenly.

The contrast between the flow in these two locations can be quantified by computing the normalized probability distribution of $u'_c(t)$ (Fig. 2c). In the cylinder wake, we find that the mode of the distribution has a negative bias towards lower velocity, consistent with the data in Fig. 2(a). We also find a pronounced tail towards high velocities, which indicates that the distribution is positively skewed. In the parallel shear region (Fig. 2c), by contrast, we find a symmetric distribution that is well represented by a Gaussian fit (solid line). Consequently, the skewness of the distribution is 0.41 at $2W$, compared to the much lower 0.06 at $200W$. We believe that near the cylinder ($2W$), the observed aperiodic jumps in $u'_c(t)$ are associated with the sudden release of elastic energy by polymer molecules into the flow (analogous to the intermittently injection of elastic energy in [24, 36]). Far downstream ($200W$), on the other hand, the even likelihood of velocity above and below the mean value indicates a unbiased energy transfer back and forth between the polymer and the flow. This idea is further developed below by monitoring the fluctuations of the spatial velocity gradients, the random components of the flow that drive the stretching of polymers [37].

Next, we analyze the velocity fluctuations by computing the frequency power spectra. Figure 3(a) shows the power spectra of the centerline velocity for $n = 15$ and $Wi = 10$, both
FIG. 3. (color online). Development of the frequency power spectra and total power of the centerline velocity along the channel. (a) frequency spectra, at positions immediately around the curved cylinder to far down the parallel shear region, Wi = 10, n = 15. (b) total spectral power contained in the velocity fluctuation, summed from the dominant range 0.01 to 100Hz. Inset is a zoom-in of the Wi = 10 case.

Polymeric and Newtonian solutions. The data show that the viscoelastic fluid flow is excited at a broad range of frequencies $f$ at all measured channel locations (from $2W$ to $200W$). This feature is one of the main hallmarks of elastic turbulence, which is most often observed in curved geometries [10].

Figure 3(a) also shows a gradual decay of the frequency power spectrum, following $f^{-1.7}$ in the wake of the cylinder ($2W$). We note that this value ($-1.7$) is significantly higher than the value of $-3.4$ reported in a recent 2D simulation of an Oldroyd-B fluid flowing in a channel with periodic array of cylinders [25]. Moreover, the power law exponent $-1.7$ is relatively high compared to experiments of viscoelastic flows in closed systems with curved streamlines; for example, a $-3.3$ exponent was reported in a serpentine flow [11]. However, Groisman and Steinberg [9] reported a high power decay exponent, between $-1.1$ and $-2.2$, in experiments in a Taylor-Couette geometry, where rotation period is close the polymer relaxation time. We note that in our flow geometry, the time scale associate with the flow round the linear array of cylinders ranges from $U/n\ell \sim 1$ Hz to $U/d \sim 100$ Hz and represents the frequency by which the mean flow is perturbed by the periodic cylinder array. This range overlaps with the polymer relaxation time scales (1-10 Hz) over a frequency decade. We believe that the abnormally gradual decay of the power spectra in the immediate wake
of the cylinders is likely the result of the overlap of these two time scales.

As the flow moves downstream from the array of cylinders into the parallel shear flow region, we observe clear developments in the frequency spectra. We find that, in a few channel widths after the last cylinder, the energy decreases in the high frequency range (10-100 Hz), which corresponds to the periodic perturbation introduced by the cylinders. At $x = 20W$, the decrease in high frequency fluctuations intensifies across two frequency decades. On the other hand, the power in the low frequency range (0.01-0.1 Hz) of the spectrum increases. The combined result is that, after $20W$, velocity fluctuations are increasingly dominated by low frequency variations and the power law decay becomes steeper, following $f^{-2.7}$.

Next, we compute the total spectral power by summing over the dominant frequency range (0.01-100 Hz). This is equivalent to the standard deviation of the time series if all valid frequencies are used. The evolution of the total energy down the channel is shown in Fig. 3(b) for $Wi=4$ and $Wi=10$. For $Wi=4$ case, where the flow is not energetic enough to trigger velocity fluctuations downstream in the parallel shear flow, we find a sharp decay of total power by two orders of magnitude. The $Wi=10$ case sees a initial decay in total power within the first $20W$. However, after $x = 20W$, the trend reverses and follows a steady increase downstream into the parallel shear flow region (Fig. 3b inset), despite the dissipative environment ($Re \sim 0.01$). Such persistence of fluctuation energy suggests a self-sustaining mechanism that has yet to be fully elucidated.

We now turn our attention to the spatial features of the viscoelastic flow. Figure 4(a) shows the spatial spectra of $u'$ along the wall-normal direction $y$. This direction is chosen to reflect the flow structure across the channel width. We find that the flow is activated at a wide range of spatial frequencies $k$ and that spatial variations in $u'$ are much stronger near the array of cylinders than in the parallel shear region; see also Fig. 1(b,c). The spatial spectrum of the viscoelastic flow near the cylinder follows a $k^{-3}$ decay. As the fluid travels downstream into the parallel shear flow, the spatial fluctuations weaken and the spectrum follows $k^{-2}$; the data also shows that these spatial fluctuations are almost uniform across the channel (see Fig. 1c).

So far we have shown that the flow of a polymeric fluid in a parallel shear geometry can sustain relatively large velocity fluctuations in both space and time even at low $Re$. A possible mechanism for these sustained fluctuations may be due to an energy transfer
FIG. 4. (color online). Spatial characteristics of the turbulence evolution along the channel for \( \text{Wi}=10, n=15 \). (a) Spatial power spectra of the velocity fluctuation fields at various channel positions. (b) The rms variation \( \sigma \) of shear \( \partial u/\partial y \) and elongational \( \partial u/\partial x \) components of the velocity gradient, normalized by that of the Newtonian case \( \sigma_N \). The inset shows the elongation component profile across channel width \( y \). Three locations along the channel are used: immediately in the cylinder wake \((2W)\), at the end of the cylinder flow decay \((20W)\), and far downstream in the parallel shear flow region \((150W)\).

between the elastic energy from polymer stretching and the kinetic energy in the flow. To test this hypothesis, we measure the root mean square (rms) variation of the shearing \( (\partial u/\partial y) \) and elongational \( (\partial u/\partial x) \) components of the velocity gradient; these components (quantities) are known to mediate polymer stretching in random flows \([37–40]\).

Figure 4(b) shows the rms variation \( \sigma \) of \( \partial u/\partial y \) and \( \partial u/\partial x \) of the viscoelastic flow normalized by the Newtonian value \( \sigma_N \). Near the linear array the cylinders, we find that the \( \partial u/\partial y \) component dominates \( \partial u/\partial x \) and both components decay as the polymeric solution flows downstream. These trends persist up to approximately \( 20W \). Further into the parallel shear region \((x \gtrsim 20W)\), however, both components of \( \sigma/\sigma_N \) start to increase. Concurrently, we observe that the fluctuations in the elongation component become comparable to fluctuations in the shearing component. This non-monotonic trend is also captured by plotting the quantity \( \sigma/\sigma_N \) for \( \partial u/\partial x \) across the channel width for three different channel locations (Fig. 4b, inset). The data suggest that polymer molecules will be increasingly stretched by flow gradient in the streamwise direction above \( 20W \) (even though there is an initial
decrease from 2W to 20W). This increase in the rms of streamwise velocity gradient leads to polymer stretching, and sustains the velocity fluctuations far downstream in the parallel flow region. The subcritical nature of the flow transition observed in a recent experiment [8] can thus be viewed in a new way under this proposed mechanism. It is likely that a finite level of polymer stretching around the cylinders needs to be reached before it can sustain the velocity fluctuations downstream in the parallel shear region.

In summary, we investigated the temporal and spatial dynamics of the flow of elastic fluid in parallel shear flow. The growth in the velocity fluctuation and polymer stretching in the parallel shear flow, following the initial decay near the cylinder, suggests the emergence of a new type of flow. Particle velocimetry reveals temporal and spatial characteristics that contrast in many ways with the counterpart near the curved cylinders. Yet it remains to be seen whether such fluctuations of velocity and polymer stretching will ultimately decay far enough downstream, analogous to the finite life-time of turbulent spots and puffs in Newtonian pipe flow [11].

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