Thermocapillary structures in a heated liquid film

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Abstract. The system of the equations which in long-wave approach describes a non-stationary 3D non-isothermal film flow in the presence of thermocapillarity is derived. The used model, unlike lubrication theory, is applicable not only for small, but also for a moderate Reynolds number in a wide range of hydrodynamic and thermal parameters of a flow and does not assume a priori set a temperature profile in a film. Based on the derived equations the numerical simulation of stationary 3D thermocapillary rivulets in a liquid film flowing over a "semi-infinite" heater is executed.

1. Introduction
The study of non-isothermal liquid film flow is an important problem of hydrodynamics and heat transfer not only from a theoretical point of view, but also from practical standpoint, since the falling films are used in many industrial devices. In contrast to the isothermal falling liquid films, the heated film flows are poorly understood both in theoretically and experimentally. Dependence of the surface tension on temperature (thermocapillary Marangoni effect) leads to occurrence of the stress shear which influences velocity of a fluid. Thus, the hydrodynamics and a heat transfer appear the interconnected processes, it in essence complicates the theoretical analysis of a problem. Experimental investigations [1-4] of thin films falling down locally heated plates reveal the rivulet structures, horizontal platen and lateral waves. Thermocapillary instabilities in a heated liquid films induce a film thinning process causing the film rupture in regions of high surface temperature. This paper presents the results of numerical simulation of stationary 3D thermocapillary rivulets in a heated liquid film flowing over a "semi-infinite" heater.

2. Theoretical model
Let us consider a vertical laminar liquid film flowing over a "semi-infinite" heater and use Cartesian coordinate system with Ox axis directed downward, Oy axis directed normal to the plate, and Ox axis directed in spanwise. Let us assume the following basic simplifications, acceptable for a wide range of practically important flow conditions. 1) Temperature of the heater's surface \( T_w = \text{const} \), the liquid surface is in contact with stagnant gas with temperature \( T_g = \text{const} \). 2) The perturbation in the film is considered to be long-wave. 3) Density \( \rho \), dynamic viscosity \( \mu \), liquid conductivity \( \lambda \) are assumed constant, but surface tension depends on temperature linearly \( \sigma = \sigma_0 - \gamma(T - T_0) \). Based on the IBL model [5], taking into account thermocapillarity, we derive a system of equations which describes the unsteady 3D film flow.
\[
\frac{\partial q}{\partial t} + \frac{\partial J_1}{\partial x} + \frac{\partial J_{1,2}}{\partial z} = \frac{3}{Re_m} \left( h \left( \sin \theta - \cos \theta \frac{\partial h}{\partial x} \right) - \frac{Ma \partial T_i}{2} - \frac{q}{h^2} \right) + Weh \left( \frac{\partial^3 h}{\partial x^3} + \frac{\partial^3 h}{\partial x \partial z^2} \right),
\]
\[
\frac{\partial m}{\partial t} + \frac{\partial J_2}{\partial z} + \frac{\partial J_{1,2}}{\partial z} = \frac{3}{Re_m} \left( h \cos \theta \frac{\partial h}{\partial x} + \frac{Ma \partial T_i}{2} + \frac{m}{h^2} \right) + Weh \left( \frac{\partial^3 h}{\partial z^3} + \frac{\partial^3 h}{\partial z \partial x^2} \right),
\]
\[
\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} + \frac{\partial m}{\partial z} = 0,
\]
\[
\frac{\partial T}{\partial t} + h \frac{\partial T}{\partial x} + w \frac{\partial T}{\partial z} + V \frac{\partial T}{\partial \eta} = \frac{1}{h^2 Re_m Pr} \frac{\partial^2 T}{\partial \eta^2},
\]
\[
\left( \frac{\partial T}{\partial \eta} + BihT \right)_{\eta=1} = 0, \quad T|_{\eta=0} = 1.
\]

Here \( h(t,x,z) \) is the film thickness, \( T(t,x,\eta,z) \) is the liquid temperature, \( T_i \) is the interface temperature, \( q(x,z,t) = \int u dy \) and \( m(x,z,t) = \int w dy \) are the liquid flow rates along the \( Ox \) and \( Oz \) axes, \( u = \frac{3q}{h} \left( \eta - \frac{\eta^2}{2} \right) - \frac{Ma}{4} (3\eta^2 - 2\eta) \frac{\partial T_i}{\partial z} \) and \( w = \frac{3m}{h} \left( \eta - \frac{\eta^2}{2} \right) - \frac{Ma}{4} (3\eta^2 - 2\eta) \frac{\partial T_i}{\partial z} \) are velocity components along \( Ox \) and \( Oz \) axes, \( \eta = y/h \),

\[
J_{1,2} = \frac{6mq}{5h} h \frac{\partial T_i}{\partial z} + \frac{h^3 Ma^2}{120} \frac{\partial T_i}{\partial z} + \frac{h^3 Ma^2}{120} \frac{\partial T_i}{\partial z}
\]

\[
J_i = \frac{6q^2}{5h} Ma \frac{\partial T_i}{\partial z} + \frac{h^3 Ma^2}{120} \frac{\partial T_i}{\partial z} + \frac{h^3 Ma^2}{120} \frac{\partial T_i}{\partial z}
\]

\[
V = \left( \frac{\partial q}{\partial x} + \frac{\partial m}{\partial z} \right) \left( \eta - \frac{3\eta^2}{2} + \frac{\eta^3}{2} \right) - \frac{Ma}{4} \left( \eta^2 - \eta \right) \frac{\partial}{\partial z} \left( \frac{h^3}{\partial x} \frac{\partial T_i}{\partial z} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{\partial T_i} \frac{\partial T_i}{\partial z} \right), \quad \Delta h = \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2}.
\]

Here, unperturbed film thickness \( h_m \) is used as the distance scale. We also introduced scales for velocity \( u_m = gh_m^3/3\nu \), time \( t_m = h_m/\nu \), flow rate \( q_m = u_m h_m \), and temperature \( T_m = T_m - T_g \). Relation (3) gives boundary conditions for the equation (2).

Non-isothermal film flow is determined by the following dimensionless criteria: Reynolds number \( Re_m = gh_m^3/3\nu^2 \); Weber number \( We = (3 Fi/Re_m)^{1/3} \); Kapitza number \( Fi = \sigma^3/\rho^2 g \nu \); Marangoni number \( Ma = \gamma T_m/\mu u_m \); Bio number \( Bi = \sigma h_m^3/\lambda \); and Prandtl number \( Pr \). Since \( h_m \sim Re_m^{1/3} \), \( u_m \sim Re_m^{2/3} \), than \( Bi \sim Bi \) \( Re_m^{1/3} \) and \( Ma \sim Ma \) \( Re_m^{2/3} \), where the dimensionless complexes of \( Bi \) and \( Ma \) are determined only by the liquid properties and heating conditions \( Bi = \sigma (3\nu^2/\gamma)^{1/3} \), \( Ma = \gamma (3/g \nu^2)^{1/3} (T_m - T_g)/\mu \).

3. Simulation results
Thermocapillary rivulets in heated vertical film are obtained by solving of the equations (1) and (2) with applying the finite difference method. The numerical algorithm is the same used in \([5, 6]\) for 2D film flow. The formation of rivulets was simulated through small perturbation periodic along \( Oz \) axes, which was imposed on unperturbed two-dimensional flow by two ways: 1) by periodic roughness of edge of the heater, i.e. \( x_{heater} = x_0 + A \cos(2\pi \cdot z/L) \); 2) by periodic variation of the flow rate in entry
of a calculation area, i.e. \( q = q_0(1 + A \cos(2\pi \cdot z / L_z)) \). The equations (1), (2) were solved by the relaxation method; unperturbed 2D heated film flow on "semi-infinite" heater was used as the initial condition. The calculation was ended upon establishing 3D rivulet structure, not changing in time. Periodic rivulet structure was calculated at various values of the parameters \( Re_m, Bi, Ma \) and the period \( L_z \). The calculations were performed for a film section with a width equal to a single period, i.e., the computational domain represented a rectangle \( 0 \leq x \leq X \), \(-L_z / 2 \leq z \leq L_z / 2 \). On the \( x \)-axis where the rivulet crest is arranged, the symmetry condition \( \partial / \partial z = 0 \) was provided, while the lateral boundaries of the computational domain (at \( z = \pm L_z / 2 \)) were provided by periodicity condition along \( z \): \( m = 0, \partial q / \partial z = 0, \partial h / \partial z = 0, \partial T / \partial z = 0 \). Unperturbed thickness, flow rate, and temperature were set on the inlet (at \( x = 0 \)).

In Fig. 1 the rivulet structure is visible, that branch off from the horizontal platen available in the edge of a heater (two periods along \( z \) are shown). In a gap between rivulets a film thickness is much less than unperturbed value \( h_m \). At so high value \( Ma^* = 120 \) the structure immediately behind the platen is already developed, and amplitude of rivulet almost does not change downstream.

![Figure 1](image1.png)

**Figure 1.** Rivulet structure at \( Re_m = 1, Ma^* = 120, L_c = 1 \text{ cm}, Bi^* = 0.2 \) (periodic roughness of edge of the heater is set). The front view (left) and rear view (right)

In Fig. 2 the rivulet structure is shown at same value \( Ma^* \), as on Fig. 1, but at essential large value Rem. From figure it follows, that growth of Rem leads to essential increasing of the distance of the development of perturbation, and amplitude of rivulet increase downstream. This effect is quite clear, as parameter Rem characterizes inertial properties of a film.

![Figure 2](image2.png)

**Figure 2.** Rivulet structure at \( Re_m = 5, Ma^* = 120, L_c = 1.2 \text{ cm}, Bi^* = 0.2 \) (periodic variation of the flow rate in entry of a calculation area is set). The front view (left) and rear view (right)
Development of rivulet structures happens only if period $L_z$ belongs to range of instability with respect to spanwise perturbations. In Fig. 3 the case, when flow parameters do not belong the range of instability (for the spanwise perturbations) is shown. Provided that the spanwise perturbation, set on an edge of a heater, does not develop therefore the rivulets are damped downstream.

**Figure 3.** Rivulet structure at $Re_m = 51, Ma^* = 80, L_z = .26$ cm, $Bi^* = 0.2$ (periodic roughness of edge of the heater is set). The front view (left) and rear view (right).

### 4. Conclusions

Based on the developed model, we carried out a numerical simulation of thermocapillary 3D rivulets on the surface of a liquid film flowing over a heater with a constant temperature. The numerical simulation of 3D rivulet structure which develops downstream due to the small transversal perturbation is carried out. At preset values of Marangoni number and Reynolds number the development of rivulet structures happens only in the event that space period $L_z$ belongs to some range. If the parameters do not belong to this range, then the rivulet structure damps downwards on a stream.

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