Heavy Hitters over Interval Queries

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Abstract

Heavy hitters and frequency measurements are fundamental in many networking applications such as load balancing, QoS, and network security. This paper considers a generalized sliding window model that supports frequency and heavy hitters queries over an interval given at query time. This enables drill-down queries, in which the behavior of the network can be examined in finer and finer granularities. For this model, we asymptotically improve the space bounds of existing work, reduce the update and query time to a constant, and provide deterministic solutions. When evaluated over real Internet packet traces, our fastest algorithm processes packets 90–250 times faster, serves queries at least 730 times quicker and consumes at least 40% less space than the known method.

1 Introduction

Heavy hitters are network flows that consume a significant percentage of the total network bandwidth. Identifying such flows is essential for many applications. For example, a load balancer could use a sophisticated scheduling algorithm for the heavy hitters, while resorting to a random allocation for all other flows. Since by definition the number of heavy hitters is small, such a balancing scheme is much more computationally efficient than trying to optimize the allocation of all flows. Similarly, bandwidth enforcement is only important for the heavy hitters, a large number of network security attacks are associated with heavy hitters, etc. Further, since the recent characteristics of the various flows are typically more relevant than old historical behavior, it is important to track the heavy hitters over a sliding window.

Indeed, the problem of finding heavy hitters over sliding windows was extensively studied [19, 6, 3, 21]. Yet, sometimes the window of interest is not known apriori. Further, the ability to perform drill-down queries, in which the behavior of the network can be examined in finer and finer granularities may also be beneficial, especially for network security applications. For example, this enables detecting when precisely a particular anomaly has started and who was involved in it.

In this work, we explore a generalized sliding window model that supports interval queries for the heavy hitters. That is, in our model, a sub-interval of a maximal window is passed as a parameter for each heavy hitters query and the goal is to reply in a corresponding manner as depicted in Figure 1. Naturally, one could maintain an instance of a known sliding window algorithm for each possible interval within the maximal sliding window. However, that is both computationally and space inefficient. Hence, the challenge is to come up with efficient solutions.

This model was previously explored in [27], which based their solution on exponential histograms [15]. However, their solution is both memory wasteful and computationally inefficient. Further, they only provide probabilistic guarantees.

Contributions

Our work focuses on providing efficient solutions to the problems of answering frequency and heavy hitters queries over an ad-hoc interval given at query time. Specifically, our initial contribution is a formal definition of these generalized estimation problems, nicknamed (W,\(\epsilon\))-IntervalFrequency and (W,\(\epsilon\))-IntervalHeavyHitters. A high-level overview of our model is illustrated in Figure 1.

In order to systematically explore the problem, we first present a naive strawman algorithm (RAW), in which multiple instances of a state-of-the-art fixed window algorithm are invoked. In such an approach, an interval query is satisfied by querying the instances that are closest to the beginning and end of the interval, and then subtracting their results. This algorithm is wasteful in terms of memory and its update time is slow, but it serves as a baseline for comparing our more sophisticated solutions.
Older Elements

Stream ● ● ● a b b c a d d a d a c e f b e b b b f

Cannot be queried

HH Query(0.3,8,18)

Figure 1: We process packets and support queries for the flow’s frequency and heavy hitters within an interval specified at query time. While previous work can answer queries for a fixed window, our approach allows us to consider any interval that is contained in the last W packets. In this example, assuming that we allow an additive error of 1, the answer to the query must contain items that appear at least 0.3 * (18 – 8) = 3 times in the interval (a & d) and may or may not report those that appeared twice (c).

| Algorithm | Space | Update Time | Query Time | Comments |
|-----------|-------|-------------|------------|----------|
| RAW       | $O(c^{-2} \log(W[d]))$ | $O(c^{-1})$ | $O(1)$ | Uses prior art as a black box. |
| ACC$_k$   | $O(c^{-1} \log(W[d]) + k e^{-1} \log e^{-1})$ | $O(k + e^{-2}/W)$ | $O(k)$ | Constant time operations for $k = O(1) \land e = \Omega(W^{-1/2})$. |
| HIT       | $O(c^{-1}(\log(W[d]) + \log^2 e^{-1}))$ | $O(1 + \log e^{-1} - 1/W)$ | $O(\log e^{-1})$ | Optimal space when $\log e^{-1} = O(\log(W[d]))$, $O(1)$ time updates when $e = \Omega\left(\frac{\log W}{W}\right)$. |

Table 1: Comparison of the algorithms proposed in the paper. ACC$_k$ can be instantiated for any $k \in \mathbb{N}$.

While developing our advanced algorithms, we discovered that both of them intrinsically solve a common problem that we nicknamed $n$-Interval. Hence, our next contribution is in identifying and formally defining $n$-Interval and then showing a simple reduction from $n$-Interval to the $(W, \epsilon)$-IntervalFrequency and $(W, \epsilon)$-IntervalHeavyHitters problems. This made our algorithms shorter, simpler, and easier to prove, analyze and implement.

Our advanced algorithms, nicknamed HIT and ACC$_k$ (in fact, $\{\text{ACC}_k\}_{k \geq 1}$ is a family of algorithms), process packets in constant time – asymptotically faster than RAW. HIT is asymptotically memory optimal while serving queries in logarithmic time. Conversely, ACC$_k$ answers queries in constant time and incurs a sub-quadratic space overhead. We present formal correctness proofs as well as space and runtime analysis. We summarize our solutions’ asymptotic performance in Table 1.

Our next contribution is a performance evaluation study of our various algorithms along with (i) ECM-Sketch [27], the previously suggested solution for interval queries and (ii) the state-of-the-art fixed window algorithm (WCSS) [6], which serves as a best case reference point since it solves a simpler problem. Our evaluation is based on real world traces from Internet backbone routers, from a university datacenter, and from a university’s border router. Overall, we demonstrate that our methods (HIT and ACC$_k$) process packets 75–2000 times faster and consume at least 20 times less space than the naive approach (RAW) while requiring a similar amount of memory as the state-of-the-art fixed size window algorithm (WCSS). Compared to the previously known solution to this problem [27], all our advanced algorithms are both faster and more space efficient. In particular, our fastest algorithm ACC$_1$ processes packets 90–250 times faster than ECM-Sketch, serves queries at least 730 times quicker and consumes at least 40% less space.

Last, we extend our results to time based intervals, hierarchical heavy hitters, and for detecting traffic volume heavy-hitters, i.e., when counting each flow’s total traffic rather than packet count. We also discuss applying our algorithms in a distributed settings, in which measurements are recorded independently by multiple sites (e.g., multiple routers), and the goal is to obtain a global network analysis.

2 Related Work

The problem of identifying the frequent items in a data stream, known as heavy hitters, dates back to the 80’s [25]. There, Misra and Gries (MG) proposed a space optimal algorithm for computing an $N\epsilon$ additive approximation for the frequency of elements in an $N$-sized stream. Their algorithm had a run time of $O(\log e^{-1})$, which was improved to a constant [17, 20]. Later, the Space Saving (SS) algorithm was proposed [24] and shown to be empirically superior to prior art (see also [12, 29]). Surprisingly, Agarwal et al. recently showed that MG and SS are isomorphic [1], in the sense that from a $k$-counters MG data structure one can compute the estimate that a $k + 1$ SS algorithm would produce.

For streams with item deletions, linear sketches such as Count Sketch [11] and Count Min Sketch [13] were proposed. For weighted streams, where we try to estimate flow volumes, the logarithmic time of MG was recently improved to a constant [7]. The problem of hierarchical heavy hitters, which has important security and anomaly
3 Preliminaries

Given a universe $U$, a stream $S = x_1, x_2, \ldots \in U^*$ is a sequence of universe elements. We denote by $W \in \mathbb{N}$ the maximal queryable window size; that is, we consider algorithms that answer queries for an interval contained with the last $W$ elements window. Given an element $x \in U$ and an integer $0 \leq w \leq W$, the $w$-frequency, denoted $f_w^x$, is the number of times $x$ appears within the last $w$ elements of $S$. For integers $i \leq j \leq W$, we further denote by $f_x^{i,j} \triangleq f_x^j - f_x^i$ the frequency of $x$ between the $i^{th}$ and $j^{th}$ most recent elements of $S$. Finally, for integers $i \leq j \leq W$ and a real number $\theta \in [0, 1]$, we denote by $HH^i_{\theta,j} \triangleq \{ x \in U : f_x^{i,j} \geq \theta \cdot (j-i) \}$ the set of heavy hitters items that appeared at least a $\theta$ fraction of the queried interval.

We seek algorithms that support the following operations:

- **ADD($x$):** given an element $x \in U$, append $x$ to $S$.

- **IntervalFrequencyQuery($x, i, j$):** given an element $x \in U$ and indices $i \leq j \leq W$, return an estimate $\hat{f}_x^{i,j}$ of $f_x^{i,j}$.

- **IntervalHeavyHittersQuery($\theta, i, j$):** given indices $i \leq j \leq W$, return an estimate $\hat{HH}^i_{\theta,j}$ that approximates $HH^i_{\theta,j}$.

We now define the guarantees we require from the solutions. **Problem definitions:**

- **($W, \epsilon$)-IntervalFrequency:**
  \[
  \text{IntervalFrequencyQuery}(x, i, j) \text{ satisfies } f_x^{i,j} \leq \hat{f}_x^{i,j} \leq f_x^{i,j} + W \epsilon.
  \]

- **($W, \epsilon$)-IntervalHeavyHitters:**
  \[
  \text{IntervalHeavyHittersQuery}(\theta, i, j) \text{ satisfies } \hat{HH}^i_{\theta,j} \subseteq \hat{HH}^i_{\theta,j} \subseteq \{ x \in U : f_x^{i,j} \geq \theta \cdot (j-i) - W \epsilon \}.
  \]
Table 2: List of Symbols

That is, the estimated set must contain all elements that appear at least a $\theta$ fraction of the interval and should not have any members whose frequency is lower than $\theta \cdot (j - i) - W \epsilon$.

For simplicity of presentation we assume that $W \epsilon / 12$ and $\epsilon^{-1}$ are integers. For ease of reference, Table 2 includes a summary of basic notations used in this work.

4 Reducing Heavy Hitters to Frequency Estimation

Our goal is to solve the $(W, \epsilon)$-IntervalFrequency and $(W, \epsilon)$-IntervalHeavyHitters problems. Yet, we observe that the two problems are related, which allows us to focus on the frequency estimation problem.

**Observation 1.** Any algorithm $A$ that solves $(W, \epsilon)$-Interval-Frequency can answer an INTERVALHEAVYHITTERSQUERY by returning

$$\hat{HH}^{i,j}_\theta \triangleq \{ x \in U : \hat{f}^{i,j}_x \geq \theta \cdot (j - i) \}.$$

Specifically, all algorithms presented in this paper can compute the set $\hat{HH}^{i,j}_\theta$ in time $O(\epsilon^{-1})$ and without iterating over all universe elements. Thus, we henceforth consider only $(W, \epsilon)$-IntervalFrequency and note that all proposed algorithms can efficiently compute the $\hat{HH}^{i,j}_\theta$ suggested above.

5 Strawman Algorithm

Here, we present the simple Redundant Approximate Windows (RAW) algorithm that uses several instances of a black box algorithm $A(w, \epsilon)$ for solving the frequency estimation problem over a fixed $W$-sized window. That is, we assume that $A(w, \epsilon)$ supports the ADD$(x)$ operation and upon QUERY$(x)$ produces an estimation $\hat{f}_x$ that satisfies:

$$f_x \leq \hat{f}_x \leq f_x + w \epsilon.$$

We note that the WCSS algorithm [6] solves this problem using $O(\epsilon^{-1})$ counters and in $O(1)$ time for updates and queries. Both its runtime and space are optimal.

Specifically, we maintain $4 \epsilon^{-1}$ separate solutions $A_1, \ldots, A_{4 \epsilon^{-1}}$, where each $A_\ell$ is an $A(\ell \cdot W \epsilon / 4, \epsilon / 4)$ instance. We perform the ADD$(x)$ operation simply by calling $A_\ell$.ADD$(x)$ for every instance $\ell$. When asked to return an answer for INTERVALFREQUENCYQUERY$(x,i,j)$, we output

$$\hat{f}^{i,j}_x \triangleq A_{\lfloor j / (W \epsilon / 4) \rfloor}.\text{QUERY}(x) - A_{\lfloor i / (W \epsilon / 4) \rfloor}.\text{QUERY}(x) + W \epsilon / 4. \quad (1)$$

Next, we analyze the properties of RAW.
Theorem 2. Let $\mathcal{A}$ be a black box algorithm as above that uses $S(w, \epsilon)$ space and runs at $U(w, \epsilon)$ time for updates and $Q(w, \epsilon)$ time for queries. Then RAW requires $O(\epsilon^{-1}S(w, \epsilon))$ space, performs updates in $O(\epsilon^{-1}U(w, \epsilon))$ time, and answers queries in $O(Q(w, \epsilon))$ time. Further, RAW solves the $(W, \epsilon)$-IntervalFrequency problem.

Proof. The run times above follows immediately from the fact that RAW utilizes $O(\epsilon^{-1})$ instances of $\mathcal{A}(\cdot, \epsilon/4)$, updates each of them when processing elements, and queries only two instances per interval query. Next, we will prove the correctness of RAW.

Notice that we can express the interval frequency as:

$$f_{i,j} = f_x^j - f_x^i = f_x^j/(W\epsilon/4) \cdot W\epsilon/4 - f_x^i/(W\epsilon/4) \cdot W\epsilon/4 - f_x^j/(W\epsilon/4) \cdot W\epsilon/4 - f_x^i/(W\epsilon/4) \cdot W\epsilon/4.$$  \hspace{1cm} (2)

Next, we note that

$$\forall n, d \in \mathbb{N} : 0 \leq |n - d [n/d]|, |d [n/d] - n| \leq d,$$

and since $\forall a \geq b : 0 \leq f_x^a,b \leq b - a$, we have

$$0 \leq f_x^i/(W\epsilon/4) \cdot W\epsilon/4, f_x^j/(W\epsilon/4) \cdot W\epsilon/4 \leq W\epsilon/4.$$

Plugging this into (2), we get

$$\begin{cases} f_x^{i,j} \geq f_x^j/[i/(W\epsilon/4)] \cdot W\epsilon/4 - f_x^i/[i/(W\epsilon/4)] \cdot W\epsilon/4 \\ f_x^{i,j} \leq f_x^j/[i/(W\epsilon/4)] \cdot W\epsilon/4 - f_x^i/[i/(W\epsilon/4)] \cdot W\epsilon/4 + W\epsilon/2. \end{cases}$$  \hspace{1cm} (3)

Now our estimation in (1) relies on the estimations produced by $A_{[i/(W\epsilon/4)]}, A_{[j/(W\epsilon/4)]}$. By the correctness of $\mathcal{A}$, we are guaranteed that

$$\begin{cases} A_{[i/(W\epsilon/4)]}.\text{QUERY}(x) \geq f_x^i/(W\epsilon/4) \cdot W\epsilon/4 \\ A_{[j/(W\epsilon/4)]}.\text{QUERY}(x) \leq f_x^j/(W\epsilon/4) \cdot W\epsilon/4 + W\epsilon/4 \\ A_{[i/(W\epsilon/4)]}.\text{QUERY}(x) \geq f_x^i/(W\epsilon/4) \cdot W\epsilon/4 \\ A_{[j/(W\epsilon/4)]}.\text{QUERY}(x) \leq f_x^j/(W\epsilon/4) \cdot W\epsilon/4 + W\epsilon/4. \end{cases}$$  \hspace{1cm} (4)

Combining (4) with (1) we establish

$$f_x^{i,j} \geq |f_x^j/[i/(W\epsilon/4)] \cdot W\epsilon/4 - f_x^i/[i/(W\epsilon/4)] \cdot W\epsilon/4 + W\epsilon/4|.$$  \hspace{1cm} (5)

Similarly,

$$f_x^{i,j} \leq |f_x^j/[i/(W\epsilon/4)] \cdot W\epsilon/4 + W\epsilon/4 - f_x^i/[i/(W\epsilon/4)] \cdot W\epsilon/4 + W\epsilon/2.|$$  \hspace{1cm} (6)

Finally, we substitute (5) and (6) in (2) to obtain the desired

$$f_x^{i,j} \leq f_x^{i,j} \leq f_x^{i,j} + W\epsilon.$$  \hspace{1cm} $\square$

While RAW does not assume anything about $\mathcal{A}$, WCSS was shown to be asymptotically optimal both in terms of runtime and memory \cite{6}. Thus, obtaining an improved fixed-window algorithm can only allow constant factor reductions in time and space. Also, while $\mathcal{A}$’s error is proportional to the window size (i.e., the error in the estimation of $A_x$ is at most $\ell \cdot W\epsilon/4$, which may be smaller than the $W\epsilon$ we used in the analysis), optimizing the error for each individual instance does not reduce the space by more than 50%. In the next section, we propose novel techniques to asymptotically reduce both space and update time. Taking into account that every counter consists of an $O(\log |U|)$ bits identifier and an $O(\log W)$ bits value, we conclude the following:

Corollary 1. Using WCSS as the black box algorithm $\mathcal{A}$, RAW requires $O(\epsilon^{-2}(\log W + \log |U|))$ bits, performs updates in $O(\epsilon^{-1})$ time and answers queries in constant time.
Figure 2: The block stream setting. Here, after the EndBlock, $x$ appears in two blocks out of the last 9 and thus $g_x^n = 2$.

6 Block Interval Frequency

In section 7, we propose algorithms that improve over RAW in both memory and update time. The two algorithms present a space-time tradeoff while achieving asymptotic reductions over RAW. While developing these algorithms, we noticed that both intrinsically solve the same auxiliary problem in which the arriving elements are inserted into $O(W\epsilon)$-sized “blocks” and we are required to compute exact interval frequencies within the blocks. Hence, below we formally define this auxiliary problem, nicknamed $n$-Interval, show a reduction to the $(W, \epsilon)$-IntervalFrequency problem and formally analyze the reduction’s cost. Doing so simplifies the presentation and analysis of the algorithms in Section 7. It also has the benefit that any improved reduction between these problems would improve both algorithms.

6.1 The Block Interval Frequency Problem

Here, instead of frequency, we consider items’ block frequency. Namely, for some $x \in U$, we define its window block frequency $g^n_x$ as the number of blocks $x$ appears in within the last $n$ blocks. For integers $i \leq j \leq n$, we define $g_{i,j}^x = g_j^x - g_i^x$. Block algorithms are required to support three operations:

- **ADD($x$):** given an element $x \in U$, add it to the stream.
- **ENDBLOCK():** a new empty block is inserted into the window, and the oldest one leaves.
- **IntervalQuery($x$, $i$, $j$):** given an element $x \in U$ and indices $i \leq j \leq n$, compute $g_{i,j}^x$ (without error).

An example of the above setting is given in Figure 2. We say that an algorithm solves the $n$-Interval problem if given an IntervalQuery($x$, $i$, $j$) it is able to compute the exact answer for any $i \leq j \leq n$ and $x \in U$. For analyzing the memory of algorithms for this problem, we denote by $N$ the sum of cardinalities of the blocks within the $n$-sized window.

6.2 A Reduction to $(W, \epsilon)$-IntervalFrequency

We now show a reduction from the $n$-Interval problem to $(W, \epsilon)$-IntervalFrequency. Specifically, we assume that $A$ is an algorithm that solves that $n$-Interval problem for $n = 6\epsilon^{-1}$; such $A$ could be HIT or ACC$k$, which we introduce in sections 7.1 and 7.2, or any other algorithm for $n$-Interval.

The setup of the reduction is illustrated in Figure 3. Our reduction relies on the observation that by applying such $A$ on a data structure maintained by the Space Saving algorithm [24], we can compute interval queries and not only fixed window size frequency estimations. We break the stream into $W$ sized frames, which are further divided into blocks of size $O(W\epsilon)$. We employ a Space Saving [24, 6] instance to track element frequencies within each frame. Whenever a counter reaches an integer multiple of the block size, we add its associated flow’s identifier to the most recent block of $A$. When a frame ends, we flush the Space Saving instance and reset all of its counters. We note that an implementation that supports constant time flush operations was suggested in [6]. Also, the max sum of block’s cardinalities within a window (overlapping up to 2 frames) is $N = 12/\epsilon$. Finally, we reduce each IntervalFrequencyQuery to an IntervalQuery by computing the indices of the blocks in which the intervals starts and ends. The variables of the reduction algorithm are described in Table 3 and its pseudocode appears in Algorithm 1.
Figure 3: The stream is logically divided into intervals of size $W$ called frames and each frame is logically partitioned into $k$ equal-sized blocks. The window of interest is also of size $W$, and overlaps with at most 2 frames and $k + 1$ blocks.

Table 3: Variables used by the Algorithm

| Variable | Description |
|----------|-------------|
| $fo$ | the offset within the current frame. |
| $A$ | an algorithm that solves $(W\epsilon/6)$-Interval. |
| $SS$ | a Space Saving instance with $\lceil 6\epsilon - 1 \rceil$ counters. |
| $s$ | the size of blocks (fixed at $s = W\epsilon/6$). |

Algorithm 1 From Blocks to Approximate Frequencies

Initialization: $fo \leftarrow 0, s = W\epsilon/6$, initialize $A, SS(\epsilon/6)$.

1: function $Add(x)$
2: $fo \leftarrow (fo + 1) \mod W$
3: $SS.Add(x)$
4: if $SS.Query(x) \mod s = 0$ then
5: $A.Add(x)$
6: if $fo \mod s = 0$ then
7: $A.EndBlock()$
8: if $fo = 0$ then
9: $SS.Flush()$
10: function $IntervalFrequencyQuery(x, i, j)$
11: return $s \cdot (A.IntervalQuery(x, \lceil i/s \rceil, \lfloor j/s \rfloor) + 2)$

6.3 Theoretical Analysis

Given a query $IntervalFrequencyQuery(x, i, j)$, we are required to estimate $f_{x}^{i,j} = f_{x}^{j} - f_{x}^{i}$. Our estimator is $\hat{f}_{x}^{i,j} = A.IntervalQuery(x, \lceil i/(W\epsilon/6) \rceil, \lfloor j/(W\epsilon/6) \rfloor) + W\epsilon/3$. Intuitively, we query $A$ for the block frequency of $x$ in the minimal sequence of blocks that contain interval $i,j$. Every time $x$’s counter reaches an integer multiple of the block size, the condition in Line 4 is satisfied and the block frequency of $x$, as tracked by $A$, increases by 1. Thus, multiplying the block frequency by $s = W\epsilon/6$ allows us to approximate $x$’s frequency in the original stream.

There are several sources of estimation error: First, we do not have a counter for each element but rather a Space Saving instance in which counters are shared. Next, unless the counter of an item reaches an integer multiple of $s$, we do not add it to the block stream. Additionally, the queried interval might not be aligned with the blocks. Finally, when a frame ends, we flush the counters and thus lose the frequency counts of elements that are not recorded in the block stream. Despite these, we prove the correctness of our algorithm.

Theorem 3. Let $A$ be an algorithm for the $6\epsilon^{-1}$-Interval problem. Then Algorithm 1 solves $(W, \epsilon)$-IntervalFrequency.

Proof. We begin by noticing that once an element’s counter reaches $s = W\epsilon/6$, it will stay associated with the element till the end of the frame. This follows directly from the Space Saving algorithm, which only disassociates elements whose counter is minimal among all counters. Recall that the number of elements in a frame is $W$ and that the Space Saving instance is allocated with $\lceil 6\epsilon^{-1} \rceil$ counters. Since the sum of counters always equals the number of elements processed, any counter that reaches $s$ will never be minimal. Thus, once an element was added to a block (Line 5), its block frequency within the frame is increased by one for every $s$ subsequent arrivals. This means that an item might be added to a block while appearing just once in the stream, but this gives an overestimation of
at most $s - 1$. As the queried intervals can overlap with two frames, this can happen at most twice, which imposes an overestimation error of no more than $2s$.

Our next error source is the fact that the queried interval may begin and end anywhere within a block. By considering the blocks that contain $i$ and $j$, regardless of their offset, we incur another overestimation error of at most $2s$.

We have two sources of underestimation error, where items frequency is lower than $s$ times its block frequency. The first is the count we lose when flushing the Space Saving instance. Since we record every multiple of $s$ in the block stream, a frequency of at most $s - 1$ is lost due to the flush. Second, in the current frame, the residual frequency of an item (i.e., the appearances that have not been recorded in the block stream) may be at most $s - 1$. We make up for these by adding $2s$ to the estimation (Line 11). As we have covered all error sources, the total error is smaller than $6s \leq W\epsilon$.

**Reducing the Error** Above, we used a block size of $s = W\epsilon/6$, which can be reduced to $W\epsilon/5$ as follows: One of the error sources in Theorem 3 is the fact that the queried interval $i, j$ may begin and end in the middle of a block and we always consider the entire blocks that contain $i$ and $j$. This can be optimized by considering $i$’s and $j$’s offsets within the relevant blocks, and including the block’s frequency only if the offset crosses half the size of the block. This incurs an overestimation error of at most $s$ instead of $2s$, allowing blocks of size $W\epsilon/5$ and reduces the number of blocks to $n = 5/\epsilon$.

## 7 Improved Algorithms

### 7.1 Approximate Cumulative Count (ACC)

We present a family of algorithms for solving the above problem. Approximate Cumulative Count (ACC) of level $k$, denoted $ACC_k$, aims to compute the interval frequencies while accessing at most $k$ hash tables for updates and $2k + 1$ for queries. To reduce clutter, we assume in this section that $n^{1/k} \in \mathbb{N}$; this assumption can be omitted with the necessary adjustments while incurring a $1 + o(1)$ multiplicative space overhead. This family presents a space-time trade-off — the larger $k$ is, $ACC_k$ takes less space but is also slower.

The ACC algorithms break the block stream into consecutive frames of size $n$ (the maximal window size). That is, blocks $B_1, B_2, \ldots, B_n$ are in the first frame, $B_{n+1}, B_{n+2}, \ldots, B_{2n}$ in the second frame and so on. Notice that each actual $n$-sized window intersects with at most two frames. Within each frame, ACC uses a hierarchical structure of tables that allow it to compute element’s block frequency in constant time.

$ACC_1$ and $ACC_2$ are illustrated in Figure 4 and are explained below. The simplest and fastest algorithm, $ACC_1$, computes for each block a frequency table that tracks how many times each item has arrived from the beginning of the frame. For example, the table for block $5n + 7$ (for $n > 7$) will contain an entry for each item that is a member of at least one of $B_{5n+1}, B_{5n+2}, \ldots, B_{5n+7}$. The key is the item identifier and the value is its block frequency from the frame’s start. This way, we can compute any block interval frequency by querying at most 3 tables. Within a frame, we can compute any interval by subtracting the queried item’s block frequency at the beginning of the interval from its block frequency at the end; if the interval spans across two frames, we need a single query for reaching the beginning of the frame.

$ACC_2$ saves space at the expense of additional table accesses. Tables now have “levels”, such that each table is either in level-0 or level-1. The core idea is that $ACC_1$ is somewhat wasteful as it may create $O(n)$ table entries for each element, as it will be tracked in all tables within the frame after its arrival. Instead, we “break” each frame into $\sqrt{n}$ sized segments. At the end of each segment, we keep a single level-1 table that counts element frequencies from the beginning of the frame. Since we can use these tables just as in $ACC_1$, we are left with computing the queried element frequency within a segment. This is achieved with a level-0 table we maintain for each block.

Alas, unlike the level-1 tables, the lower level tables only keep the block frequency counts from the beginning of the segment the block belongs to. Thus, each appearance of an element (within a specific block) can appear on all $\sqrt{n}$ level-1 tables, but on at most $\sqrt{n}$ level-0 tables. Thus, compared with $ACC_1$, we reduced the overall number of table entries from $O(N \cdot n)$ to $O(N \cdot \sqrt{n})$. For interval $i, j$, let $block_i$ and $block_j$ be the block numbers of $i$ and $j$ respectively. To answer any interval frequency query of element $x$, we consider two cases: If $block_i$ and $block_j$ are in the same frame, we simply access $block_i$’s and $block_j$’s tables to get $x$ frequency from the beginning of the frame till $block_i$, $block_j$ and subtract the results. If $block_i$ and $block_j$ are in different frames, we should consider $x$’s frequency in the $j$’s blocks within the current frame by accessing $block_j$’s tables, plus its frequency within the last $i$ blocks of the previous frame. To do so, we need to know $x$’s frequency from the beginning of the previous frame. But the corresponding tables also count blocks that left the window, yet are part of the previous frame. We solve
ACC Algorithm representations

\begin{figure}
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
$A^{C1}$ & $x_2$ & $y_2$ & $x_3$ & $x_4$ & $x_5$ & $x_5$ & $x_5$ & $x_5$ \\
\hline
$d_1$ & $d_1$ & $y_1$ & $y_2$ & $y_2$ & $y_2$ & $y_2$ & $y_2$ & $y_2$ \\
\hline
$\sqrt{n}$ level-0 tables & $\sqrt{n}$ level-1 tables & between each level-1 table & & & & & & \\
\hline
$A^{C2}$ & $x_2$ & $y_1$ & $x_2$ & $x_2$ & $x_5$ & $x_5$ & $x_5$ & $x_5$ \\
\hline
$d_1$ & $b_1$ & $d_1$ & $d_1$ & $d_1$ & $d_4$ & $d_4$ & $d_4$ & $d_4$ \\
\hline
level-0 tables & level-1 tables & & & & & & & \\
\hline
\end{tabular}
\caption{ACC Algorithm representations}
\end{figure}

Table 4: Variables used by $ACC_k$ algorithm.

| BlockSize | the number of segments from a level that consist of a next-level segment. |
| Tables$[\ell,idx]$ | used for tracking block frequencies. Each table is identified with a level $\ell$ and the index of the last block in its segments. |
| incTables$[\ell]$ | tables for incomplete segments. |
| ghostTables$[\ell]$ | tables for leaving segments. |
| offset | The offset within the current frame. |

it by maintaining $\text{ghostTables}[\ell]$ of leaving segments: for $1 \leq \ell \leq k$, $\text{ghostTables}[\ell]$ contains the table of last leaving block that has a table at level $\ell$. Hence, we can subtract the corresponding $\text{ghostTables}$ entries as well.

Next, we generalize this to arbitrary $k$ values. In $ACC_k$, we have $k$ levels of tables and segments. We consider each block to be in its own level-0 segment and maintain a level-0 table for it. Inductively, each level-$\ell$ segment (for $1 \leq \ell \leq k$) consists of $n^{1/k}$ level-$(\ell - 1)$ segments. That is, each level-1 segment contains $n^{1/k}$ blocks, level-2 segments each consists of $n^{1/k}$ level-1 segments for a total of $n^{2/k}$ blocks, etc. As each element may now appear in at most $n^{1/k}$ tables of each level, we get that the overall number of table entries is $O(N \cdot k \cdot n^{1/k})$. In order to avoid long computations at the end of each segment, we maintain $k$ additional “incomplete” tables that contain the cumulative counts for segments that already started but not all of their blocks have ended yet. A pseudo-code of the $ACC_k$ algorithm appears in Algorithm 2.

7.1.1 Analysis

The following theorem bounds the memory consumption of the ACC algorithms.

\textbf{Theorem 4.} Denote the sum of cardinalities of the last $n$ blocks by $N$. Algorithm 2 requires $O(N \cdot n^{1/k} \cdot k \cdot (\log n + \log |U|))$ space.

\textbf{Theorem 5.} Algorithm 2 solves the $n$-Interval problem.

\textit{Proof Sketch.} We need to prove that upon an $\text{INTERVALQUERY}(x,i,j)$ query, $ACC_k$ is able to compute the exact answer for any $i \leq j \leq n$ and $x \in U$. Notice that in handling such queries in Algorithm 2 we split the computation
in two: If $i = 0$, this means that the end of the interval is also the last block, and thus we only need to return the frequency w.r.t. the last block in the window, as calculated by $\text{WinQuery}(x, j)$. Otherwise, we subtract the frequency that is calculated $\text{WinQuery}(x, i)$ from the result of $\text{WinQuery}(x, j)$. Hence, we need to show that the frequency calculated by $\text{WinQuery}(x, w)$ is correct.

As is evident from the code in Lines 1–13, $incTables$ store the frequency of items within the current block while $Tables$ store the frequencies of items completed blocks from the beginning of their frame. Consider the case where the entire range is within the current frame. Hence, in principle, the frequency of an item in the last $w$ blocks can be calculated as its frequency in the current block ($incTables$) plus its frequency in the preceding blocks, as is done in Line 15, and stored in $cFreq$. Notice that to reduce query time, we access the highest level containing this information. However, since $Tables$ store the frequency from the beginning of the frame, we need to subtract from $cFreq$ obtained in Line 15 the frequency of this item in prior blocks, which is done in Line 17 (here again, accessing the highest level tables that include this data).

The second case is when the range crosses into the previous frame. In this case, we need to add to $cFreq$ the frequency of the blocks that are included in the previous frame. Once again, we need to find the table holding the frequency in the last relevant block of that frame, and subtract from it the frequency in the preceding tables. Yet, as some of these tables might be beyond an entire window limit, their information might be stored in $ghostTables$ rather than $Tables$. This is handled in Lines 20–21.

### 7.2 Hierarchical Interval Tree (HIT)

Hierarchical Interval Tree, denoted HIT, tracks flow frequencies using a hierarchical tree structure, where each node in the tree stores the partial frequency of its sub-tree. Specifically, the levels of the tree are defined as follows: level_0 includes frequency tables, one for each block of the stream, that track how many times each item arrived within the corresponding block. At level_1, tables of block_i track how many times each item has arrived between block_{i-2^l+1} and block_i, where $0 < l \leq traling_zeros(i)$. That is, these tables contain partial queries results for each item and track item's multiplicity from the previous same level block. Hence, each level contains tables for
half the blocks of the previous level, and thus each \( block_i \) has tables in \( \text{trailing_zeros}(i) \) levels; we assume that the number of trailing zeros can be computed efficiently with the \( \text{ctz} \) machine instruction in modern CPUs. An illustration of the algorithm appears in Figure 5.

For example, consider blocks 9, 10, 11 and 12. During block 9, items \( x \) and \( d \) arrive; \( x \) also arrives in blocks 10 and 11, while there are no items arrivals in block 12. So the tables of block 12 will be as follow:

- Level 0 table is empty because there is no items arrival within block 12.
- Level 1 table tracks items arrival between block 11 and 12; its content will be item \( x \) with count 1.
- Level 2 table counts the item arrival between block 9 and 12, so it will contain item \( x \) 3 times (blocks 9, 10 and 11) and \( d \) once (in block 9). Note that each table at level \( \ell + 1 \) merges two level \( \ell \) frequency tables.

We can compute any interval frequency by using the hierarchical tree tables. While this can be done using linear scan, the higher levels of the tree are designed to allow efficient time computation by using the stored partial queries.

Notice that some of the partial queries results stored in the higher levels may be invalid. For example, in case a new block is added, the oldest one departs the window, so the content of tables that refer to the departing block become invalid. We solve this problem by choosing the levels to use such that we only consider valid tables. Let \( block_i \) and \( block_j \) be the block numbers of the first interval index and the last one. Here, we scan backward from \( block_j \) to \( block_i \), greedily using the highest possible level at each point. This minimizes the number of needed steps.

If \( block_j > block_i \), all tables along the way are valid. In this case, we only need \( \log_2(block_j - block_i + 1) \) value lookups. Otherwise, we choose level0 tables between blocks 1 and \( block_j \), so we need \( \log_2(block_j + 1) \) value lookups, and then another \( \log_2(n - block_i + 1) \) lookups for querying the remaining interval. Overall, our computation takes at most \( 2 \log(n) \) steps.

The pseudo code of the algorithms appears in Algorithm 8 and a list containing the variables description appears in Table 5. We use incremental tables for incomplete blocks. In an \( \text{ADD}(x) \) operation, we increment element \( x \)'s entries.

7.2.1 Analysis

We now analyze the HIT algorithm. We start by proving the correctness of HIT.

**Theorem 6.** Algorithm 8 solves the n-Interval problem.
**Algorithm 3 HIT**

Initialization: \( \text{offset} \leftarrow 0 \), initialize \( \text{Tables}, \text{incTable} \).

1: \textbf{function} \( \text{Add}(x) \)
2: \( \text{incTable}(x) += 1 \) \quad \triangleright \text{Update the incomplete block’s tables}

3: \textbf{function} \( \text{EndBlock}() \)
4: \( \text{offset} \leftarrow (\text{offset} + 1) \mod n \)
5: \( \text{Tables}[0, \text{idx}] \leftarrow \text{incTable} \)
6: \text{empty \ incTable} \quad \triangleright \text{Delete all entries.}

7: \textbf{for} \( \ell \in 1, \ldots, \text{ctz}(\text{offset}) \) \textbf{do}
8: \( \text{Tables}[\ell, \text{idx}] \leftarrow \text{incTable} \)

9: \textbf{function} \( \text{IntervalQuery}(x, i, j) \)
10: \( \text{last} = (\text{offset} - j) \mod n \) \quad \triangleright \text{The most recent block’s index}
11: \( \text{first} = (\text{offset} - i) \mod n \) \quad \triangleright \text{The oldest queried block’s}
12: \( b \leftarrow \text{first} \)
13: \( \text{count} \leftarrow 0 \)
14: \( d \leftarrow 1 + (\text{first} - \text{last} \mod n) \)
15: \textbf{while} \( d > 0 \) \textbf{do}
16: \( \text{level} \leftarrow \min(\text{ctz}(b), \lfloor \log d \rfloor) \)
17: \( \text{count} \leftarrow \text{count} + \text{Tables}[\text{level}, b](x) \)
18: \( d \leftarrow d - 2^\text{level} \)
19: \( b \leftarrow b - 2^\text{level} \)
20: \textbf{if} \( b = 0 \) \textbf{then}
21: \( b \leftarrow k \)
22: \textbf{return} \( \text{count} \)

---

| BlockSize | Number of blocks from a level that consist of a next-level block. |
|-----------|---------------------------------------------------------------------|
| Tables[\ell, \text{idx}] | used for tracking block frequencies. Each table is identified with a level \( \ell \) and the index of the last block in its block. |
| incTable | A table for the most recent, incomplete, block. |
| offset | The offset within the current frame. |

Table 5: Variables used by HIT algorithm.

**Proof.** We need to prove that upon an \( \text{IntervalQuery}(x, i, j) \) query, HIT is able to compute the exact answer for any \( i \leq j \leq n \) and \( x \in U \) without error. We first introduce some notations. \( x \) denotes the queried element; \( f_x[i] \) indicates the frequency of item \( x \) during the \( i \)th block, so that the newest block’s index is 1. According to Line 15, we iterate over blocks from last to first, query \( \text{level} \) table, which contains partial query of \( 2^\text{level} \) blocks, thus we can advance by \( 2^\text{level} \), Lines 18 and 19. The output of the algorithm when queried for \( x \) in interval \( i \leq j \leq n \) is:

\[
\text{IntervalQuery}(x, i, j) = \sum_{i=1}^{\text{first}} f_x[i] - \sum_{i=1}^{\text{last}} f_x[i] = \sum_{i=1}^{(\text{offset} - j) \mod n} f_x[i] - \sum_{i=1}^{(\text{offset} - i) \mod n} f_x[i]. \tag{7}
\]

According to the definition of \( g_k^x \) in section 6.1 and Line 4, which indicates that \( \text{offset} \) is actually the number of blocks within current frame, (7) is equal to: \( g_k^x = g_x^k \triangleq g_x^k \).

**Theorem 7.** Denote the sum of cardinalities of the last \( n \) blocks by \( N \). Algorithm 3 requires \( O(N \log n \log (n|U|)) \) space.

**Proof.** As described above, each element’s appearance may reflect in \( O(\log n) \) tables. Every table entry consumes \( O(\log |U|) \) for the key and \( O(\log n) \) for the value, and thus the overall space is \( O(N \log n \log (n|U|)) \).

**7.3 Optimizations**

This section includes optimizations that can be applied to the \( \text{ACC}_K \) and/or HIT algorithms.
Short IDs  Element IDs are often quite long (alternatively, \( \mathcal{U} \) is large), e.g., a 5-tuple identifier per flow may take over 100 bits and Internet URLs can be even longer. Hence, when the size of item IDs are large, we can reduce their required space as follows: For each frame, we maintain an \( O(N) \) sized array of items identifiers that were added to some block during the frame. Every time a new (distinct) item arrives, we add it to the array. To find the index of each ID in the array, we maintain an additional table that maps IDs to their array indices. Clearly, the combined space requirement of the array and map table is \( O(N \cdot (\log \mathcal{U} + \log n \log N)) \). Finally, we replace the keys in the algorithms’ tables (at all levels) such that instead of storing identifiers we use the array indices as keys. Given a query, we first find the array index using the new table and then follow the same procedure as before, but with the index as key. This optimization can be applied to both \( ACC_k \) and \( HIT \). Thus, we always store at most \( O(N) \) IDs. This reduces HIT’s space to \( O(N (\log |\mathcal{U}| + \log n \log N)) \) and that of \( ACC_k \) to \( O(N (\log |\mathcal{U}| + n^{\frac{k}{k}} \log N)) \).

Deamortization  Algorithm \[ \text{Algorithm 1} \] shows a reduction from the \( n \)-Interval problem to \( \text{INTERVALFREQUENCYQUERY} \). For any \( A \) algorithm that solves \( n \)-Interval, we notice that the operation \( A.\text{Add}(x) \) cannot be called more than once a block for the same element \( x \). We can then \textit{deamortize} the \( A.\text{Add}(x) \) operation for reducing the worst case update time. Namely, we can spread the time required for \( \text{Add}(x) \) over an entire block. This means that if queried for \( x \), we may miss 1 from its block frequency. Nevertheless, this only adds an error of \( s \) and can be compensated for by slightly reducing the block size.

8 Evaluation  We developed a C++ prototype of all algorithms described in this work: \( HIT \), \( RAW \), and instantiations of the \( ACC_k \) protocols for \( k = 1, 2, 4, 8 \). Here, the \( HIT \) and \( ACC_k \) algorithms are implemented using the \( WCSS \) algorithm \[ \text{Algorithm 2} \] as a building block. Besides, we also implemented ECM-Sketch \[ \text{Algorithm 3} \] (a.k.a. \( ECM \)) in C++ for comparison because their code is in Java. \( ECM \) is configured for error probability \( \delta = 0.01\% \). We also use the \( WCSS \) al-
Figure 7: Query operation runtime comparison as a function of $\epsilon$ and maximum window size ($W$).

Algorithm [6] as a general baseline since it is the state of the art for the simpler problem of a fixed sliding window. For each algorithm, we evaluate the performance of executing $\text{ADD}(x)$ and $\text{INTERVALFREQUENCYQUERY}(x,i,j)$ operations as well as its memory requirements.

The evaluation is performed on an Intel(R) 3.20GHz Xeon(R) CPU E5-2667 v4 running Linux with kernel 4.4.0-71. Each data point in all runtime measurements is shown as a 95% confidence interval of 10 runs.

8.1 Datasets

Our evaluation includes the following datasets:

1. A CAIDA 2016 backbone Internet trace [18], denoted Backbone. It consists of 97M packets, which are a mix of UDP/TCP/ICMP packets collected from the ‘equinix-chicago’ high-speed monitor, a major backbone router.

2. University campus data centers traces, denoted Datacenter [9]. These data centers are located in the western/mid-western U.S. and are hosted on the premises of the organizations to serve local users. The Datacenter dataset consists of 17M packets.

3. A trace from the border router of the Computer Science Department, University of California Los Angeles, denoted Edge consisting of 16M packets [28].

8.2 Update Speed Comparison

We start by exploring the trade-off of $\epsilon$ parameter with a fixed maximal window of size $W = 2^{20}$. Then, we explain the trade-off of window size parameter with a fixed $\epsilon = 2^{-8}$. Figure 6 compares the update speed for each dataset.
8.2.1 Effect of $\epsilon$ on Runtime

Throughout, as $\epsilon$ decreases, more tables must be updated on every overflowed element. Thus, update operations become slower when $\epsilon$ decreases. As depicted, HIT update performance is close to ACC$_1$ and ACC$_2$. As $k$ increases, there are more tables to update on every overflowed element, so the performance decreases. This difference becomes especially noticeable with small $\epsilon$ values.

Recall that every update operation in RAW means $4\epsilon^{-1}$ ADD operations, one for every $4\epsilon^{-1}$ instances of the $A(\cdot, \epsilon/4)$ algorithm, which is WCSS in our implementation. So, as $\epsilon$ decreases, update operations take more time. Figure 8 shows RAW’s update performance for the Backbone dataset. We managed to run this only for $\epsilon \geq 2^{-8}$ due to space limitation on the server. Since the update speed of RAW is orders of magnitudes slower than the other algorithms, we have placed it in a separate figure. This echoes Table 1, which presents the analytical performance summary of the different algorithms. Among our algorithms depicted in Figure 6, the slowest one is ACC$_8$. As can be seen in the inner graphs, even ACC$_8$ processes packets 57-210 times faster than ECM.

8.2.2 Effect of Window Size on Runtime

Figure 6 shows the effect of window size when $\epsilon$ is fixed to $2^{-8}$. All algorithms perform better when the window size is larger as this means fewer blocks and table accesses. The ACC$_k$ algorithms get slower as $k$ increases as they need to update more tables. Again, we compared the most inefficient algorithm ACC$_8$ with ECM in the inner graphs; ACC$_8$ processes packets 50-218 times faster than ECM for the given $\epsilon$ values.

8.3 Query Speed Comparison

For query speed comparison, we choose random intervals, each of size 1% of the total window’s size. We begin the evaluation by exploring the impact of the $\epsilon$ parameter with a fixed window of size $2^{20}$. Then, we explain the trade-off of the window size parameter with fixed $\epsilon = 2^{-8}$. We compare the performance of the improved algorithms with the existing work, ECM, and with WCSS, which can only answer queries with fixed window size.

8.3.1 Effect of $\epsilon$ on Runtime

As shown in Figure 7, RAW is the fastest as each interval query is translated to two WCSS queries. We managed to run RAW only up to $\epsilon = 2^{-8}$ due to its memory consumption limitation (see section 8.4).

HIT computes any block interval frequency by using the hierarchical tree tables, greedily choosing the highest possible level each time. For decreasing $\epsilon$ values, the blocks numbers are increasing, so the queried interval crosses more blocks and accesses more tables. Consequently, we get slower interval query operations.

For the ACC$_k$ algorithms, for increasing $k$ values we get fewer queries per seconds as we read more tables on average. For example, ACC$_1$ computes any block frequency by querying at most 3 tables, while ACC$_2$ does the same by accessing no more than 5 tables as explained in Section 7.1. Query operations runtime depends also on the interval itself; there are “good” intervals in which the corresponding blocks have table at level $k - 1$, so one table access for each is sufficient. Therefore, we choose random intervals for every query. As $\epsilon$ value decreases, block sizes become smaller and the number of tables grow. In this case, not all the tables fit in memory and we experience paging that causes lower query performance. Recall that while WCSS is the fastest, it solves the much simpler problem of a fixed window size and only serves as a best case reference point. ECM answers queries in
a very inefficient way compared to our algorithms. We only run ECM up to $\epsilon = 2^{-6}$ due to time limitation. As expected, its performance decreases for decreasing $\epsilon$ values.

### 8.3.2 Effect of Window Size on Runtime

As mentioned before, we evaluate queries by choosing random intervals of size 1% of window’s size, when $\epsilon$ is fixed to $2^{-8}$. Figure 7 shows that all algorithms’ query performance is not very sensitive to the window size. This is because the number of tables accessed depends on the ratio between the interval and window sizes. We run ECM only up to $2^{14}$ due to time limitation. The performances of our algorithms are orders of magnitudes better than ECM also in this case.

### 8.4 Memory Consumption Comparison

Figure 8 shows the space consumed by our algorithms and also ECM sketch for a given $\epsilon$ value. As seen, the smaller $\epsilon$ gets, all algorithms consume more space. We can see that ECM is more compact than RAW but consumes more space than the others. As mentioned before, RAW maintains $4\epsilon^{-1}$ separate WCSS instances so its space consumption is the largest. For the $ACC_k$ algorithms, as $k$ increases, the overall number of tables entries for overflowed elements decreases resulting is better space consumption. Yet, ECM consumes more space than $ACC_1$ which has the highest memory consumption among the $ACC_k$ algorithms family. As shown, the memory consumption of $ACC_k$ is close to HIT. Yet, HIT is the most efficient among the algorithms that solve $n$-Interval because its data structure is the most compact. Recall that while WCSS is the most compact algorithm in term of space, it solves the much simpler problem of a fixed window size and only serves as a best case reference point.

### 9 Extensions and Applications

#### 9.1 Time Based Intervals

In this section, we describe how to extend our algorithms for supporting time intervals. The idea is that sometimes what matters is the flow frequencies in a time interval rather than during a packet-count interval. For example, if we want to allow a user to make 100 queries/sec to an API, we need to measure the number of times this user has accessed the system during the last second. In such a setting, we consider a time-interval query as $\langle x, i, j, h \rangle$: given an element $x \in \mathcal{U}$ and indices $j \leq i \leq W$, return an estimate $\overline{h_{i,j}}$ of $h_{i,j}$.

Finally, if an algorithm’s error is at most an $\epsilon$ fraction of the overall possible traffic in $T$ time, we say that it solves the $(T, R, \epsilon)$-TimeIntervalFrequency problem. That is, its estimation needs to satisfy

\[
\forall j \leq i \leq T : h_{i,j}^x \leq \overline{h_{i,j}}^x \leq h_{i,j}^x + T \cdot R \cdot \epsilon.
\]

Our construction has two parts: We maintain a $(T \cdot R, \epsilon/2)$-IntervalFrequency solution in addition to a data structure that translates time intervals into packet intervals. For this, we use Ben Basat’s Sliding Ranker (SR) algorithm [1] that can compute a sliding window sum over an integer stream, where the size of the window is given at query time. SR has parameters $(R, \mathfrak{M}, \Delta)$: it processes a stream in $\{0, 1, \ldots, R\}$ such that upon a query for some $i \leq \mathfrak{M}$, it computes a $\Delta$-additive approximation for the sum of the last $i$ elements. Every timestamp, we feed the number of packets that arrived into an SR with parameters $(R, T, T \cdot R \epsilon/2)$. Given a time-interval query $x, i, j$, we use the SR for computing the number of packets sent since time $i$ and from time $j$. We then use these estimations to query the IntervalFrequency instance for the estimated packet-interval. Since the SR and IntervalFrequency each has an error of $T \cdot R \epsilon/2$, we satisfy the error guarantee. The memory consumption of SR for $\Delta = \Theta(\mathfrak{M})$ is just $O(R\mathfrak{M}\Delta + \log \mathfrak{M}) = O(\epsilon^{-1} + \log \mathfrak{M})$ bits.
9.2 Hierarchical Heavy Hitters

Next, we describe how our algorithms can be used for answering interval HHH queries (see [26] for formal definitions). In [26], Mitzenmacher et al. proposed combining their approach (which originally utilized Space Saving [24]) with sliding window algorithms such as [19, 6] to solve HHH on sliding windows. However, such an approach yields a fixed window size algorithm. By replacing the underlying black box algorithm by our interval query solutions, we get an algorithm that solves HHH on interval queries.

9.3 The Distributed Model

We now consider applying our algorithms in distributed settings. Here, multiple streams are received at various distributed sites $S_1, \ldots, S_r$ ($r > 1$) and each site maintains its own instance of the chosen algorithm, e.g., $ACC_k$, $HIT$, etc. Obtaining a global view of the network status requires merging data structures from all individual sites. The common way of serving such queries is to have all individual sites transmit a copy of their data structures to a central controller $C$, which merges them into a global data structure. This can be done either periodically assuming synchronized clocks between the sites, or in a coordinated manner initiated periodically by $C$. Queries are forwarded to the controller that computes the reply based on its merged data-structure. This model is communication efficient when queries are frequent, since queries are served directly by the controller and the rate in which the distributed sites need to communicate with the controller can be lower than the query rate.

In contrast, when queries are not as frequent, the above solution is inefficient, since the sites needlessly update the controller. To that end, by applying the time based intervals adaptation, our solution enable the reverse model. That is, given a range query, it is directly propagated to each of the $r$ distributed sites. Each site then returns its locally computed portion and all replies are then merged into a global one. The reason why time based intervals are needed is that individual streams might arrive at different rates to the various sites. Hence, it is meaningless to merge the results of queries on a packet based window or range. For this reason, the above approach cannot be applied to packet based sketch algorithms.

Last, as mentioned before, our algorithms provide an $\epsilon$ error guarantee. Hence, when each of the sites runs its independent instance, the overall error guarantee of the distributed model becomes $r\epsilon$. Another way of looking at this is that since the space requirement is inversely proportional to the error guarantee, the space requirement for a given error grows with $r$. Since usually $r$ is a small constant, for most systems this is acceptable.

9.4 Supporting Traffic Volume Heavy-Hitters

It is often desired to find the heavy hitters in terms of traffic volume. That is, consider a stream in which its packet has a size and we wish to find the flows that account for most of the bandwidth in a given interval. Formally, we consider a weighted stream $S = (x_1, w_1), (x_2, w_2), \ldots \in (U \times \{1, 2, \ldots, M\})^*$ and define a flow’s volume as the sum of sizes for packets that belong to it.

Intuitively, to address this problem we can add the weight of the packet in Line 3 of Algorithm 1 and change the condition of Line 4 to consider whether the current estimation exceeds a new multiple of $s \cdot M$. The Space Saving algorithm [24] can find weighted heavy hitters over a stream with $O(\log \epsilon^{-1})$ update time [10]. Recent breakthroughs [7, 5, 2] improve this runtime to a constant. Thus, we can solve the interval volume estimation and (weighted) heavy hitters problems with the same asymptotic complexity as the unweighted variants and with an error of at most $WM\epsilon$. This is a generalization of the result of [25] that finds weighted heavy hitters over fixed size windows.

10 Discussion

In this paper, we studied the problems of flow frequency estimation and heavy hitters identification over intervals that are passed at query time. Such capabilities can be useful when one wishes to maintain the above statistics over multiple sliding windows and for performing drill down queries, e.g., for root cause analysis of network anomalies.

We presented formal definitions of these generalized problems and explored three alternative solutions: a naive approach (RAW) and two more sophisticated solutions called $HIT$ and $ACC_k$. Both $HIT$ and $ACC_k$ process updates in $O(1)$, but differ in their space vs. query time tradeoff: $HIT$ is asymptotically memory optimal but answers queries in logarithmic time whereas $ACC_k$ processes queries in $O(1)$ but consumes more space. In fact, $HIT$’s space requirement is similar to the memory requirement of the state of the art algorithm that can only cope with fixed size windows. Hence, $HIT$ is adequate when space is tight while $ACC_k$ is suitable for real time query processing. Both our advanced algorithms are faster and more space efficient than ECM-Sketch [27], the previously
known solution for interval queries. This is true both asymptotically and in measurements over real-world traces, in which we demonstrated orders of magnitude runtime improvements as well as at least 40% memory reductions for equivalent estimation errors.

Our approach can be applied to additional related problems. For example, as discussed in Section 9 in the Hierarchical Heavy Hitters (HHH) problem [26], which is useful in detecting distributed denial of service attacks (DDoS), one can replace the Space Saving algorithm employed by [26] with HIT or ACC_k to detect HHH over query intervals!

**Code Availability:** All code used in this work will be made available in open source soon.
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