Thermodynamic potential of the Periodic Anderson Model with the X-boson method: Chain Approximation

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Abstract

The Periodic Anderson Model (PAM) in the $U \rightarrow \infty$ limit has been studied in a previous work employing the cumulant expansion with the hybridization as perturbation (M. S. Figueira, M. E. Foglio and G. G. Martinez, Phys. Rev. B 50, 17933 (1994)). When the total number of electrons $N_t$ is calculated as a function of the chemical potential $\mu$ in the “Chain Approximation” (CHA), there are three values of the chemical potential $\mu$ for each $N_t$ in a small interval of $N_t$ at low $T$ (M. S Figueira, M. E Foglio, Physica A 208 (1994)). We have recently introduced the “X-boson” method, inspired in the slave boson technique of Coleman, that solves the problem of non conservation of probability (completeness) in the CHA as well as removing the spurious phase transitions that appear with the slave boson method in the mean field approximation. In the present paper we show that the X-boson method solves also the problem of the multiple roots of $N_t(\mu)$ that appear in the CHA.

Key words: A. Periodic Anderson Model, B. Cumulant Expansion, C. Slave Boson, D. X-Boson.

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1 Introduction

In the Periodic Anderson Model (PAM) there are four local states at each site \( j \) of the lattice (identified by a single index in this work): the vacuum state \( |j,0\rangle \), the two states \( |j,\sigma\rangle \) of one electron with spin component \( \sigma = \pm 1 \) and the state \( |j,2\rangle \) with two local electrons [1]. In the limit of infinite Coulomb repulsion \( U \to \infty \), the state \( |j,2\rangle \) is empty, and we shall use the Hubbard operators [2] to project it out from the space of local states at site \( j \). The Hamiltonian of the system is then [3]:

\[
H = \sum_{k,\sigma} E_{k,\sigma} c_{k,\sigma}^\dagger c_{k,\sigma} + \sum_{j,\sigma} \varepsilon_{j,\sigma} X_{j,\sigma\sigma} + \sum_{j,\sigma,k} \left( V_{j,\sigma,k} X_{j,0\sigma}^\dagger c_{k,\sigma} + V_{j,\sigma,k}^* c_{k,\sigma}^\dagger X_{j,0\sigma} \right),
\]

where the first term is the Hamiltonian of the conduction electrons (\( c \)-electrons), the second term describes independent localized electrons (\( f \)-electrons), and the last term is the hybridization Hamiltonian giving the interaction between the \( c \)-electrons and the \( f \)-electrons.

The \( X \) Hubbard operators do not satisfy the usual commutation relations so that diagrammatic methods based on Wick’s theorem are not applicable, and one has to use product rules instead:

\[
X_{j,ab} X_{j,cd} = \delta_{b,c} X_{j,ad}.
\]

The identity decomposition in the reduced space of local states at site \( j \) is then

\[
X_{j,00} + X_{j,\sigma\sigma} + X_{j,\sigma\bar{\sigma}} = I_j,
\]

where \( \bar{\sigma} = -\sigma \), and the three \( X_{j,aa} \) are the projectors into \( |j,a\rangle \). Because of the translational invariance, the occupation numbers \( n_{j,a} = \langle X_{j,aa} \rangle \) satisfy \( n_{j,a} = n_a \) (independent of \( j \)), and from Eq. (3) we obtain the “completeness” relation

\[
n_0 + n_\sigma + n_{\bar{\sigma}} = 1.
\]

The occupation numbers can be calculated from appropriate Green’s functions (GF), and it has been found that Eq. (4) is not usually satisfied when the \( n_a \) are calculated with approximate cumulant Green’s functions (GF) [4].
An approximation with this behavior is the “Chain Approximation” (CHA), which was first obtained by Hewson [5,6]. This approximation has interesting properties: it is $\Phi-$derivable [7,8], and it is also the most general cumulant expansion with only second order cumulants. We have employed several procedures to restore completeness to the CHA: renormalization of the one-electron Green’s functions (GF) or adding diagrams to the GF in the CHA. The second method lead us to a conjecture on a systematic way of achieving completeness by adding a set of diagrams to an arbitrary family [8]. An alternative to these techniques was inspired in the mean field treatment of the slave boson technique [9,10], in which the correlated problem is transformed into an uncorrelated one with one condition that forces to zero the occupation of $|j,2\rangle$ states. This condition turns out to be just our Eq. (4), and following Coleman we minimized the free energy of the system calculated with the CHA but forcing the validity of Eq. (3), that implies the completeness. This is the essence of the X-boson method [11,12] and, differently from the slave boson treatment, the correlation is kept at the final stage because it is intrinsic to the CHA [13]. An important consequence of this fact is that the spurious phase transition that appears in the slave boson treatment for several regions of the system parameters (for intermediate temperatures or when $\mu >> E_f$) [14], disappears completely from our treatment. The results of the X-boson method are fairly close to those obtained by the slave boson treatment in the region of its validity, while the X-boson method gives results that are close to those of the CHA when the slave boson is not valid any more (at high temperatures and when $\mu >> \varepsilon_f$).

A rather inconvenient aspect of the CHA is that its results show a region of instability [15], apparent because the dependence of $N_t$ with $\mu$ shows multiple values of $\mu$ for a given $N_t$ within a small interval of values of $N_t$, and this leads to negative compressibility. The main result of the present work is to show that this difficulty of the CHA is removed by the X-boson treatment, and $N_t(\mu)$ becomes then a monotonous function.

The paper is organized as follows: the X-boson approach is presented in section 2, the GF in the CHA are given in section 3, and in section 4 we describe the calculation of the free energy. Our results and conclusions are presented in section 5.

2 X-boson Cumulant Method

In Coleman’s “slave boson” method [9,10], the Hubbard X operators are written as a product of ordinary bosons and fermions: $X_{j,oo} \rightarrow b_j^+ b_j$, $X_{j,oo} \rightarrow b_j^+ f_{j,\sigma}$, $X_{j,oo} \rightarrow f_{j,\sigma}^+ b_j$, and a condition, that is equivalent to Eq. (4), is imposed to avoid states with two electrons at each site $j$. In the spirit of the
mean field approximation $b_i^+ \to < b_i^+ > = r$ and the method of Lagrangian multipliers is employed to minimize the free energy subject to that condition. The problem is then reduced to an uncorrelated Anderson lattice with renormalized hybridization $V \to rV$ and $f$ level $\varepsilon_f \to \varepsilon_f + \lambda$, and the conservation of probability in the space of local states is automatically satisfied because they are described by Fermi operators.

The approximate GF obtained by the cumulant expansion [3,16] do not usually conserve probability (i.e., they do not satisfy Eq. (4)), and the procedure we adopt to recover this property in the X-boson method is to introduce

$$R \equiv < X_{j,0} > = < b_j^+ b_j >,$$

as variational parameter, and to modify the approximate GF so that it minimizes an adequate thermodynamic potential while being forced to satisfy Eq. (4). To this purpose we add to Eq. (1) the product of each Eq. (4) into a Lagrange multiplier $\Lambda_j$, and employ this new Hamiltonian to generate the functional that shall be minimized by employing Lagrange’s method. To simplify the calculations we use a constant hybridization $V$, as well as site independent local energies $\tilde{\varepsilon}_{f,j,\sigma} = \varepsilon_{f,\sigma}$ and Lagrange parameters $\Lambda_j = \Lambda$. We then obtain a new Hamiltonian with the same form of Eq. (1):

$$H = \sum_{\vec{k},\sigma} E_{\vec{k},\sigma} c_{\vec{k},\sigma}^+ c_{\vec{k},\sigma} +$$

$$\sum_{j,\sigma} \tilde{\varepsilon}_{f,\sigma} X_{j,\sigma\sigma} + N_s \Lambda (R - 1) +$$

$$V \sum_{j,\vec{k},\sigma} \left( X_{j,0\sigma}^+ c_{\vec{k},\sigma}^+ + c_{\vec{k},\sigma}^+ X_{j,0\sigma} \right),$$

but with renormalized localized energies

$$\tilde{\varepsilon}_{f,\sigma} = \varepsilon_{f,\sigma} + \Lambda.$$

The parameter

$$R = 1 - \sum_{\sigma} < X_{\sigma\sigma} >$$

is now varied independently to minimize the thermodynamic potential, choosing $\Lambda$ so that Eq. (4) be satisfied. While at this stage the electrons in the slave boson Hamiltonian have lost all the correlations, the Eq. (6) is still in the projected space and it is not necessary to force the correlations with an extra condition. On the other hand we do not have an exact solution for this new problem, and we then consider the most simple approximation obtained within the cumulant formalism, the Chain approximation (CHA) [5,6]. The need of minimizing a thermodynamic potential arises because the completeness relation is not automatically satisfied for approximate cumulant solutions,
and although the two procedures are formally very similar, they have a rather different meaning.

The Grand Canonical Ensemble of electrons is employed in the present treatment, and instead of Eq. (1) we have to use

$$\mathcal{H} = H - \mu \left\{ \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k},\sigma} c_{\mathbf{k},\sigma}^\dagger c_{\mathbf{k},\sigma} + \sum_{j\alpha} \nu_a X_{j,\alpha\alpha} \right\}, \quad (9)$$

where $\nu_a = 0, 1$ is the number of electrons in state $| a \rangle$. It is then convenient to define

$$\varepsilon_{j,a} = E_{f,j,a} - \mu \nu_a \quad (10)$$

and

$$\varepsilon_{k\sigma} = E_{k\sigma} - \mu, \quad (11)$$

because $E_{f,j,a}$ and $E_{k\sigma}$ appear only in that form in all the calculations. The exact and unperturbed averages of the operator $A$ are denoted in what follows by $< A >_H$ and $< A >$ respectively.

### 3 The Chain Approximation Green’s Functions

The GF in the Chain approximation (CHA) are given by [8,12]:

$$G_{\sigma}^{f\mathbf{k}}(z_n) = \frac{-D_{\sigma}(z_n - \varepsilon_{\mathbf{k}\sigma})}{\left( z_n - \varepsilon_{f,\sigma} \right) \left( z_n - \varepsilon_{\mathbf{k}\sigma} \right) - \left| V_{\sigma}(\mathbf{k}) \right|^2 D_{\sigma}}, \quad (12)$$

$$G_{\sigma}^{cc}(z_n) = \frac{-\left( z_n - \varepsilon_{f,\sigma} \right)}{\left( z_n - \varepsilon_{f,\sigma} \right) \left( z_n - \varepsilon_{\mathbf{k}\sigma} \right) - \left| V_{\sigma}(\mathbf{k}) \right|^2 D_{\sigma}}, \quad (13)$$

$$G_{\sigma}^{fc}(z_n) = \frac{-D_{\sigma} V_{\sigma}(\mathbf{k})}{\left( z_n - \varepsilon_{f,\sigma} \right) \left( z_n - \varepsilon_{\mathbf{k}\sigma} \right) - \left| V_{\sigma}(\mathbf{k}) \right|^2 D_{\sigma}}, \quad (14)$$

where

$$D_{\sigma'} = < X_{oo} > + < X_{\sigma'\sigma} > = R + n_{f,\sigma'}. \quad (15)$$

and $\sigma' = \sigma, \bar{\sigma}$. The lattice slave-boson GF are recovered if we put $D_{\sigma} = 1$ and $V_{\mathbf{k}} \rightarrow r V_{\mathbf{k}} = \nabla_{\mathbf{k}}$ in Eqs.(12,13,14). The usual GF for the lattice
in the CHA are obtained if we use the bare \( D_\sigma \), while \( D_\sigma \) must be calculated self-consistently in the X-boson approach.

In an earlier paper [15] we considered the atomic limit of the PAM, i.e. with a conduction band of zero width, to study both the free energy and \( N_t \) as a function of \( \mu \), but here we shall consider the PAM with a wide band.

4 Calculation of the Lattice Helmholtz Free Energy

When the total number of electrons \( N_t \), the temperature \( T \) and the volume \( V_s \) are kept constant the equilibrium state corresponds to a minimum of the Helmholtz free energy, but the same state of equilibrium is obtained by minimizing the thermodynamic potential \( \Omega = -k_B T \ln(Q) \), (where \( Q \) is the Grand Partition Function) at constant \( T \), \( V_s \), and chemical potential \( \mu \) (this result is easily obtained by employing standard thermodynamic techniques). We shall then employ \( \Omega \) as the thermodynamic potential that is minimized in the X-boson method with Eq. (4) as constraint. The Helmholtz free energy \( F \) is then given by

\[
F = N_t \mu + \Omega, \quad (16)
\]

and our first step would be the calculation of \( \Omega \).

A convenient way to obtain \( \Omega \) is to employ the method of \( \xi \) parameter integration [15,17,18]. This method introduces a \( \xi \) dependent Hamiltonian \( H(\xi) = H_o + \xi H_1 \) through a coupling constant \( \xi \) (with \( 0 \leq \xi \leq 1 \)), where \( H_1 \) is the hybridization in our case. One obtains [12]

\[
\Omega = \Omega_o + \int_0^1 d\xi \langle H_1(\xi) \rangle, \quad (17)
\]

where \( \langle A \rangle_{\xi} \) is the ensemble average of an operator \( A \) for a system with Hamiltonian \( H(\xi) \) and the given values of \( \mu \), \( T \), and \( V_s \), while \( \Omega_o \) is the thermodynamic potential of the system with \( \xi = 0 \). This value of \( \xi \) corresponds to a system without hybridization, and one obtains (in the absence of magnetic field \( \varepsilon_{k\sigma} = \varepsilon_k \) and \( \tilde{\varepsilon}_{f\sigma} = \tilde{\varepsilon}_f \))

\[
\Omega_o = -\frac{2}{\beta} \sum_k \ln \left[ 1 + \exp(-\beta \varepsilon_k) \right] - \frac{N_s}{\beta} \ln \left[ 1 + 2 \exp(-\beta \tilde{\varepsilon}_f) \right] + N_s \Lambda(R - 1), \quad (18)
\]
and to calculate $\Omega$ in Eq.(17) we use

$$\langle H_1 \rangle_\xi = 2 \text{Re} \left[ \sum_{k,\sigma} V_{j,k,\sigma}^* \langle c_{k,\sigma}^\dagger X_{0,\sigma} \rangle_\xi \right].$$

(19)

Employing standard Green’s functions techniques [12] we find

$$\langle H_1 \rangle_\xi = \frac{1}{\pi} \int_{-\infty}^{\infty} d\omega \ n_F(\omega) \sum_{k,\sigma} \text{Im} \frac{\xi |V_\sigma(k)|^2 D_\sigma}{(\omega^+ - \varepsilon_{f,\sigma})(\omega^+ - \varepsilon_{k,\sigma}) - \xi^2 |V_\sigma(k)|^2 D_\sigma},$$

(20)

where $n_F(x) = 1/[1 + \exp(\beta x)]$ is the Fermi-Dirac distribution and $\omega^+ = \omega + i0$. This equation has an interesting scaling property: it is equal to the corresponding expression of the uncorrelated system for the scaled parameter $V_{j,k,\sigma} = \sqrt{D_0} V_{j,k,\sigma}$ (it is enough to remember that by replacing $D_0 = 1$ in the GF of the CHA one obtains the corresponding GF of the uncorrelated system). Rather than performing the $\xi$ and $\omega$ integrations, we shall use the value of the $\Omega^u$ for the uncorrelated system with $V_{k,\sigma} = 0$. The uncorrelated Hamiltonian for the lattice problem is then

$$H^u = \sum_{k,\sigma} \varepsilon_{k,\sigma} c_{k,\sigma}^\dagger c_{k,\sigma} + \sum_{j,\sigma} \tilde{\varepsilon}_f f_{j,\sigma}^\dagger f_{j,\sigma} + N_s \Lambda (R - 1)$$

$$+ \sum_{j,k,\sigma} \left( V_{j,k,\sigma} f_{j,\sigma}^\dagger c_{k,\sigma} + V_{j,k,\sigma}^* c_{k,\sigma}^\dagger f_{j,\sigma} \right),$$

(22)

and this Hamiltonian can be easily diagonalized. The corresponding $H^u$ can be written

$$H^u = \sum_{i,\sigma} \omega_{i,\sigma} \alpha_{i,\sigma}^\dagger \alpha_{i,\sigma} + \Lambda (R - 1),$$

(23)

where $\alpha_{i,\sigma}^\dagger (\alpha_{i,\sigma})$ are the creation (destruction) operators of the composite particles of energies $\omega_{i,\sigma}$ (there are $N_s + 1$ states for each spin $\sigma$). The calculation of $\Omega^u$ is then straightforward [12]:

$$\Omega = \overline{\Omega}_0 + \frac{-1}{\beta} \sum_{i,\sigma} \ln \left[ 1 + \exp(-\beta \omega_{i,\sigma}) \right] + \Lambda (R - 1),$$

(24)
where
\[
\Omega_0 \equiv \Omega_o - \Omega_u = -\frac{N_s}{\beta} \ln \left[ \frac{1 + 2 \exp(-\beta \tilde{\varepsilon}_f)}{(1 + \exp(-\beta \tilde{\varepsilon}_f))^2} \right],
\] (25)
and the eigenvalues \(\omega_{i,\sigma}\) of the \(H^\sigma\) are just given by the poles of the GF in the CHA (Eq.(12)). In the present case the \(\omega_{i,\sigma}\) can be calculated analytically, because the Hamiltonian for each spin component \(\sigma\) is reduced into \(N_s\) matrices \(2 \times 2\), and one finds
\[
\omega_{i,\sigma} = \omega_{\pm,\sigma}(k) = \frac{1}{2} (\varepsilon_{k,\sigma} + \tilde{\varepsilon}_f) \pm \frac{1}{2} \sqrt{(\varepsilon_{k,\sigma} - \tilde{\varepsilon}_f)^2 + 4 |V_\sigma(k)|^2 D_\sigma}. \] (26)

The parameter \(\Lambda\) is obtained minimizing \(\Omega\) with respect to \(R\) [12]. To simplify the calculations we shall consider a conduction band with a constant density of states, width \(W = 2D\), an hybridization constant \(V_\sigma(k) = V\), and \(\varepsilon_{k,\sigma} = \varepsilon_k\); we then obtain
\[
\Lambda = \frac{V^2}{D} \int_{-D}^{D} d\varepsilon_k \frac{n_F(\omega_+(k)) - n_F(\omega_-(k))}{\sqrt{(\varepsilon_k - \tilde{\varepsilon}_f)^2 + 4V^2 D_\sigma}}. \] (27)

All the correlation effects in Eq. (24) are included in \(\Omega_o\) (cf. Eq. (16)), and the hybridization redistributes the quasi-particle energies in the same way that an uncorrelated system with \(V_{j,k,\sigma} = \sqrt{D_{0\sigma}} V_{j,k,\sigma}\) would do; one then expects Fermi liquid behavior of the quasi-particles [12].

5 Results and Conclusions

Employing the CHA to calculate the GF of the PAM one discovers an undesirable feature in several regions of the parameter space and for sufficiently low \(T\) [15]. When the total number of particles in the system \(N_t\) is plotted against the chemical potential \(\mu\) one finds that there are three possible values of \(\mu\) for each \(N_t\) in an interval of \(N_t\), originating an instability in a small interval of \(\mu\), characterized by a negative derivative of \(N_t\) with respect of \(\mu\). The problem was first observed in a wide conduction band, but further analysis was done for a band of zero width [15], because one could then obtain an exact solution of the problem and compare with the approximate result. The value of the free energy \(F\) was then calculated for the three states to analyze their relative stability. Here we shall consider the problem for a wide conduction band, and
compare the results of the X-boson with those obtained with the CHA and with the slave-boson. The main result of the present work is that the X-boson does not present the multiple solutions of the CHA.

In Figure 1 we plot \( N_t(\mu) \) for CHA, slave boson and X-boson for the same parameters used in [15]. The multiple solutions of the CHA are apparent, as well as the thermodynamically unstable region (part b-c of the Fig. 1, where \( N_t \) decreases when \( \mu \) increases), i.e. the same behavior that was observed for the atomic limit of the PAM [15]. The problem of thermodynamic unstable solutions does not appear in the slave boson and X-boson, but the slave boson breaks down at higher \( \mu \) (\( \mu \gg \tilde{\varepsilon}_f \), magnetic regime) while the X-boson presents a continuous evolution in all regimes of the model.

\[ \mu \]

\[ \varepsilon_f = -0.15; \ W = \pi; \ V = 0.1; \ T = 0.001. \]

The slave boson breaks down when \( \mu \approx -0.12 \).

In Figure 2 we present the Helmholtz free energy \( F \) vs. \( N_t \) in the slave boson, X-boson, and CHA approximations. The CHA present three values of free energy \( F \) for the same \( N_t \) that have three values of \( \mu \), while this problem is absent in the X-boson and slave boson methods.

In Figure 3 we plot the grand thermodynamic potential \( \Omega \) vs. \( N_t \) for the slave boson and X-boson, at two values of \( T \). In this plot the grand thermodynamic potential \( \Omega \) for the slave boson is always lower or equal than for the X-boson, and its values increase with temperature in the two methods; its minimum values are in the Kondo region at the lowest \( T \). The values of \( \Omega \) are different at the larger value of \( T \) for all \( N_t \).

In Figure 4 we show that \( \Omega(N_t) \) has multiple values for the CHA at \( T = 0.001 \) and \( T = 0.003 \) when \( N_t \sim 1.5 \), while that behavior is absent at higher \( T \). Note that there is no minimum of \( \Omega \) in the CHA, and that this quantity changes

Fig. 1. Total Number of particles \( N_t \) vs. \( \mu \), in the three approaches with the following parameters: \( \varepsilon_f = -0.15; \ W = \pi; \ V = 0.1; \ T = 0.001. \) The slave boson breaks down when \( \mu \approx -0.12. \)
Fig. 2. Helmholtz free energy $F$ vs. $N_t$, for the X-boson, Slave boson and CHA approaches with the same parameters of Figure 1.

Fig. 3. Grand thermodynamic potential $\Omega$ vs. $N_t$, for the slave boson and X-boson approaches with the same parameters of Figure 1, but for two values of the temperature: $T = 0.1$ and $T = 0.001$. with $N_t$ between maximum and minimum values that are independent of $T$.

In Figure 5 we plot $N_t$ vs. $\mu$ for the CHA at different values of $T$, and the multiple values of $\mu$ for a given $N_t$ disappear at the larger value of $T$.

In Figure 6 we plot $\rho_f(\mu)$ (the density of states $f$ on the Fermi level $\mu$) vs. $N_t$ for the X-boson and slave boson at $T = 0.001$, the lower value of temperature that we consider in Figure 3. From this figure is clear that the Kondo effect is present when $N_t \approx 1.6$ where $\rho_f(\mu)$ has a maximum in the two approaches.
5.0.1 Conclusions

Some years ago we studied the thermodynamic properties of the CHA [15], an approximation that was obtained by employing a diagrammatic cumulant expansion for the PAM. We then found that for a region of the system parameters at very low temperatures, there are three states with different chemical potential $\mu$ for the same total number of electrons $N_t$. We observed that one of these states is thermodynamically unstable (part b-c of the Fig. 1) because $N_t$ decreases when $\mu$ increases. The X-boson is essentially a generalized CHA approximation which satisfies completeness through a minimization of the thermodynamic potential as a function of the average occupation of the empty state while keeping Eq. (3) as a constraint. As it is shown in Figure (1)
Fig. 6. Density of states $f$ on the Fermi level $\mu$, $\rho_f(\mu)$ vs. $N_f(\mu)$, for the X-boson and Slave boson approaches with the same parameters of Figure 1.

This approach does not present thermodynamically unstable states or multiple solutions for a given $N_t$. We conclude that the X-boson combines the simplicity and usefulness of the CHA and slave boson methods, without their more obvious defects: the presence of spurious phase transitions of the slave boson and the absence of Kondo resonance, of multiple solutions and instabilities, as well as the failure to satisfy completeness of the CHA.

References

[1] A. C. Hewson, The Kondo Problem to Heavy Fermions - Cambridge Studies in Magnetism - Cambridge University Press, (1993).
[2] J. Hubbard, Proc. R. Soc. London, Ser. A 285 (1965) 542; A 296 (1966) 82.
[3] M. S. Figueira, M. E. Foglio and G. G. Martinez, Phys. Rev. B 50 (1994) 17933.
[4] M. E. Foglio, M. S. Figueira, International Journal of Modern Physics B 12 (1998) 837.
[5] A. C. Hewson J. Phys. C: Solid State Phys. 10 (1977) 4973.
[6] E. V. Anda, J. Phys. C: Solid State Phys. 14 (1981) L1037.
[7] G. Baym and L. P. Kadanoff Phys. Rev. 124 (1961) 287.
[8] M. S. Figueira and M. E. Foglio J. Phys.: Condens. Matter 8 (1996) 5017.
[9] P. Coleman, Phys. Rev. B 29 (1984) 3035.
[10] D. M Newns, N. Read Advances in Physics 36 (1987) 799.
[11] R. Franco, M. S. Figueira and M. E. Foglio, J. Magn. Magn. Mat 226-230 (2001) 194.

[12] R. Franco, M. S. Figueira and M. E. Foglio, unpublished. The submitted paper can be found in preprint (2001) [cond-mat/0109037].

[13] M. E. Foglio, Phys. Rev. B 43 (1991) 3192.

[14] P. Coleman, J. Magn. Magn. Mat 47&48 (1985) 323.

[15] M. S. Figueira, M. E Foglio, Physica A 208 (1994) 279.

[16] M. E. Foglio, M. S. Figueira, J. Phys. A: Math. Gen. 30 (1997) 7879.

[17] A. A. Abrikosov, L. P. Gorkov and I. E. Dzyaloshinski, Quantum Field Theoretical Methods in Statistical Physics (Pergamon, Oxford), (1965).

[18] S. Doniach and E. H. Sondheimer, Green’s Functions for Solid State Physicists (Benjamin, New York), (1974).