INSTANTON-INDUCED PARTICLE PRODUCTION
IN DEEP INELASTIC SCATTERING

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ABSTRACT

I discuss the instanton-induced contributions to the coefficient functions in front of parton densities. They correspond to the spherically symmetric particle production at the level of $10^{-2} - 10^{-5}$ of the total cross section of deep inelastic scattering from the parton.

1. Introduction

The deep inelastic lepton-hadron scattering at large momentum transfers $Q^2$ and not too small values of the Bjorken scaling variable $x = Q^2/2pq$ is studied in much detail and presents a classical example for the application of perturbative QCD. The factorization theorems allow one to separate the $Q^2$ dependence of the structure functions in coefficient functions $C_i(x, Q^2/\mu^2, \alpha_s(\mu^2))$ in front of parton (quark and gluon) distributions of leading twist $P_i(x, \mu^2, \alpha_s(\mu^2))$

$$F_2(x, Q^2) = \sum_i C_i(x, Q^2/\mu^2, \alpha_s(\mu^2)) \otimes P_i(x, \mu^2, \alpha_s(\mu^2)),$$

where the summation goes over all species of partons, and $\mu$ is the scale separating "hard" and "soft" contributions to the cross section. At $\mu^2 = Q^2$ the coefficient functions can be calculated perturbatively and are expanded in power series in the strong coupling

$$C(x, 1, \alpha_s(Q^2)) = C_0(x) + \frac{\alpha_s(Q^2)}{\pi} C_1(x) + \left(\frac{\alpha_s(Q^2)}{\pi}\right)^2 C_2(x) + \ldots$$

whereas their evolution with $\mu^2$ is given by famous Dokshitzer-Gribov-Lipatov-Altarelli-Parisi equations. Going over to a low normalization point $\mu^2 \sim 1GeV$, one obtains the structure functions expressed in terms of the parton distributions in the nucleon at this reference scale. The parton distributions absorb all the information about the dynamics of large distances and are fundamental quantities extracted from the experiment. Provided the parton distributions are known, all

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the dependence of the structure functions on the momentum transfer is calculable and is contained in the coefficient functions $C_i$. Corrections to this simple picture come within perturbation theory from the parton distributions of higher twists and are suppressed by powers of the large momentum $Q^2$.

The picture described above presents a part of the common wisdom about hard processes in the QCD, and in a more or less detailed presentation can be found in any textbook. Less widely known is the fact that from the theoretical point of view this picture is not complete. An indication that some contributions may be missing comes from the asymptotic nature of the perturbative series in $\mathcal{O}(g^2)$. This series is non-Borel-summable, which means that any attempt to attribute a quantitative meaning to the sum of the series in $\mathcal{O}(g^2)$ would produce an exponentially small imaginary part $i \exp\{-\text{const} \cdot \pi/\alpha_s(Q^2)\}$, which is to be cancelled by the imaginary part coming from nonperturbative contributions. Thus, separation between perturbative and nonperturbative pieces in the cross section as the ones which contribute to the coefficient function and the parton distribution, respectively, cannot be rigorous. A modern discussion of the asymptotical properties of the perturbation series in QCD can be found in [1, 2].

In addition to imaginary exponential corrections which must cancel identically against the corresponding nonperturbative contributions, the coefficient functions may acquire also real exponential corrections, which potentially produce observable effects. In this talk I shall report on recent results by V.Braun and myself [3, 4], indicating that these corrections are indeed present. We have found that the deep inelastic cross section indeed possesses exponential contributions of the form $F(x) \exp(-4\pi S(x)/\alpha_s(Q^2))$, where $S(x)$, $F(x)$ are certain functions of Bjorken $x$, which we are able to calculate in a certain kinematical domain. Since the experimental data are becoming more and more precise, it is of acute interest to find a boundary for a possible accuracy of the perturbative approach, which is set by nonperturbative effects. Our study has been fuelled by recent findings of an enhancement of instanton-induced effects at high energies in a related problem of the violation of baryon number in the electroweak theory [5, 6]. In the case of QCD the instanton-induced effects could turn out to be significant at high energies, despite the fact that they correspond formally to contributions of a very high fractional twist $\exp(-4\pi S(x)/\alpha_s(Q^2)) \sim (A_{QCD}^2/Q^2)^{bS(x)}$.

Note that, in principle, the instanton contributions in QCD are infrared-unstable - in a typical situation integrations over the instanton size are strongly IR-divergent. However, this problem does not affect calculation of instanton contributions to the coefficient functions which are IR-protected, as we shall see below (the detailed discussion can be found in [4]).

2. Instanton contribution to the structure function of a gluon

Let us start from the instanton-induced contribution to the coefficient function in front of gluon density. Following Zakharov [7], we calculate the cross section of the $\gamma^* g$ scattering by means of the optical theorem. The trick is to evaluate the
The instanton-induced contribution to the structure function of a gluon (b) and of a quark (c,d). Wavy lines are (nonperturbative) gluons. Solid lines are quark zero modes in the case that they are ending at the instanton (antiinstanton), and quark propagators at the $\bar{\Pi}$ background otherwise.

contribution to the functional integral coming from the vicinity of the instanton-antiinstanton configuration in Euclidian space, and calculate the cross section by the analytical continuation to Minkowski space and by taking the imaginary part. The true small parameter in our calculation is the value of the coupling constant at the scale $Q^2$, which ensures that the effective instanton size is sufficiently small. We prefer to start, however, from the well-studied situation where not only the instanton sizes are small, but also the $\bar{\Pi}$ separation is much larger than these sizes. As we shall see below, this corresponds to $1 - x \ll 1$ (but of course $1 - x \gg \alpha_s$ still).

After that we shall move to smaller $x$ (which correspond to strongly interacting $I$ and $\bar{\Pi}$) trying to be accurate to collect to the semiclassical accuracy all the dependence on $\rho^2/R^2$ in the exponent. To this end, we shall have in mind the valley method \[11\], in which all the dependence on the $\bar{\Pi}$ separation is absorbed in the action $S(\xi)$ on the $\bar{\Pi}$ configuration. However, we do not take into account corrections of order $\rho^2/R^2$ in the preexponent, and to this accuracy need the first nontrivial term only in the cluster expansion of the quark propagator at the $\bar{\Pi}$ background \[8\]:

$$\langle x|\nabla^{-2}_I \nabla_{\bar{\Pi}}|0\rangle = \int dz \langle x|\nabla^{-2}_1 \nabla_1|z\rangle \sigma_\xi \frac{\partial}{\partial z} \langle z|\nabla_2 \nabla^{-2}_2|0\rangle.$$ 

The leading contribution to the gluon matrix element of the $T$-product of the electromagnetic currents is given by the following expression

$$\int dz e^{iqz} \langle A^a(p), \lambda|T\{j_\mu(z)j_\nu(0)\}|A^a(p), \lambda\rangle^{\bar{\Pi}} =$$

$$= \sum_q e_q^2 \int dU \int \frac{d\rho_1}{\rho_1^2} d(\rho_1) \int \frac{d\rho_2}{\rho_2^2} d(\rho_2) \int dT \int dz e^{iqz}$$

$$\times \frac{1}{8} \lim_{\rho^2 \to 0} p^4 e_\lambda e_\beta^2 T \{ A^I_\alpha(p) A^I_\beta(-p) \} e^{-\frac{\rho^2}{\rho^2} S_{\bar{\Pi}}}$$

$$\times (a^I a)^{n-1} \left\{ a^I_\lambda \phi_0(0) \bar{\sigma}_\nu(0) |\nabla^{-2}_2 \nabla_2 \bar{\nabla}_1 \nabla^{-2}_1|z\rangle \bar{\sigma}_\mu \phi_0(z) \right\}$$

$$+ a^I_\lambda \bar{\phi}_0(z) \sigma_\nu \langle z|\nabla^{-2}_1 \nabla_1 \bar{\nabla}_2 \nabla^{-2}_2|0\rangle \sigma_\mu \bar{\phi}_0(0) + (\mu \leftrightarrow \nu, z \leftrightarrow 0) + \ldots \right\}$$

which corresponds to the diagram shown in Fig.1a. The full expression contains many more terms \[4\] which are not shown because we have found that all of them.
are of order $O(\alpha_s(Q^2))$ compared to the expression in Eq. 3. The subscript ‘1’ refers to the antiinstanton with the size $\rho_1$ and the position of the center $x_1 = R + T$, and the subscript ‘2’ refers to the instanton with the size $\rho_2$ and the center at $x_2 = T$. We use conventional notations $\nabla = \nabla_\mu \sigma_\mu$ and $\bar{\nabla} = \nabla_\bar{\mu} \sigma_{\bar{\mu}}$, etc., where $\sigma_\mu^{\alpha \dot{\alpha}} = (-i\sigma_1^1, 1)$, $\sigma_{\bar{\mu}}^{\dot{\alpha} \alpha} = (i\sigma_1^1, 1)$, and $\sigma$ are the standard Pauli matrices. Also, we write the quark zero modes in terms of the two-component Weil spinors $\psi_0 = \left(\kappa_0 \phi_0\right)$, $\psi_0^\dagger = \left(\bar{\phi}_0 \bar{\kappa}_0\right)$, and $a$ and $a^\dagger$ denote the overlap integrals $a = -\int dx \left(\bar{\kappa} \partial \kappa\right)$, $a^\dagger = -\int dx \left(\bar{\phi} \partial \phi\right)$. Here $S_{II}$ is the action of the instanton-antiinstanton configuration and $\xi = R^2 + \rho_2^2 - \rho_1^2$ is the conformal parameter [10] (the normalization is such as $S(\xi) = 1$ for an infinitely separated instanton and antiinstanton). Writing the action as a function of $\xi$ ensures that the interaction between instantons is small in two different limits: for a widely separated $II$ pair, and for a small instanton put inside a big (anti)instanton, which are related to each other by the conformal transformation. In the limit of large $\xi$ the expansion of $S(\xi)$ for the dominating maximum attractive $II$ orientation reads

$$S(\xi) = \left(1 - \frac{6}{\xi^2} + O(\ln(\xi)/\xi^4)\right)$$

(4)

where the $1/\xi^2$ term corresponds to a slightly corrected dipole-dipole interaction. Thus, the action $S(\xi)$ decreases with the distance between instantons, so that the instanton and the instanton effectively attract each other. This attraction results in the exponential increase of the cross section — the effect found by Ringwald [7]. Further terms in the expansion of the action can be obtained by the so-called valley method [11], and a typical solution (conformal valley) [10] gives a monotonous function of the conformal parameter, which turns to zero at $R \rightarrow 0$. In the traditional language, the valley approach corresponds to the summation of all so called ”soft-soft” corrections arising from the particle interaction in the final state. Main problem is in the evaluation of ”hard-hard” corrections [12], which come from particle interaction in the initial state. These corrections are likely to decrease the cross section, and in physical terms must take into account an (exponentially small) overlap between the initial state, which involves a few hard quanta, with the semiclassical final state [13]. Thus, the instanton-antiinstanton action is substituted by an effective ”holy grail” function, which determines the leading exponential factor for the semiclassical production at high energies, and which received a lot of attention in recent years. Unitarity arguments [14, 15] suggest that the decrease of the action will stop at values of order $S(\xi) \approx 0.5$. In a recent preprint [16] Diakonov and Petrov argue that $S(\xi)$ indeed decreases up to the value $1/2$ at a certain energy of order of the sphaleron mass, and then starts to increase, so that the semiclassical production cross section is resonance-like. The question seems to us to be not settled finally. In this study, we have taken the value $S = 1/2$ as a reasonable guess for the residual suppression, and assumed that the behavior of the ”true” function $S(\xi)$ for $S(\xi) > 1/2$ is close to that given by the conformal valley [10]. The latter assumption is supported by numerical studies, e.g. in [16].

We shall see below that the leading contribution in the strong coupling comes
from the following regions of integration:

\[ \frac{z^2}{(z - R - T)^2 + \rho_1^2} \sim \frac{1}{Q^2 \alpha_s} \]
\[ (z - R - T)^2 + \rho_1^2 \sim T^2 + \rho_2^2 \sim z^2/\alpha_s \]

and additionally \( \rho^2/R^2 \sim 1 - x \) when \( x \) is close to 1. Note that these regions of integration correspond to imaginary part of the \( \Pi \) contribution so effectively the \( z_i^2 \) are negative.

Since \( z^2 \) is small we can use the lightcone expansion (see e.g. [19]) for the quark propagator in the \( \Pi \) background. Using the explicit expressions for the propagators from [20] we find:

\[ \tilde{\kappa}_0(x) \sigma_\mu(x \nabla_1 \nabla_2 \nabla_3 \nabla_4 \sigma_\nu \kappa_0(0)) = \]

\[ = -\frac{1}{2\pi^4} \int_0^1 d\gamma \frac{(\rho_1 \rho_2)^{3/2}}{[(z - R - T)^2 + \rho_1^2][T^2 + \rho_2^2]} \frac{1}{\sqrt{R^2}} \text{Tr} \left\{ \frac{\bar{\sigma}_\nu \bar{z} \sigma_\mu}{z^4} [(z - R - T)
+ \rho_2^2 (\gamma z - R - T)] \right\} + \ldots \]

Omitted terms have turned out to be of order \( O(\alpha_s) \).

Let us at first consider the toy example of the Eq.3 without extra integration over \( \gamma \) which brings only technical complexities:

\[ \int d^4z \frac{e^{iqz}}{z^2} \int dT \int d\rho_1^2 (\rho_1^2)^{\mu_1} \int d\rho_2^2 (\rho_2^2)^{\mu_2} \int dRe^{iR - \frac{4\pi}{\alpha_s}(1 - \frac{2}{3})} \frac{\Gamma(m_1)}{[(z - R - T)^2 + \rho_1^2][T^2 + \rho_2^2]^{m_2} \Gamma(m_2) \Gamma(n)} \]

This integral diverges at \( \rho \to \infty \). However, the divergent part corresponding to instanton with size \( \rho \sim \Lambda_{QCD} \) contributes only to parton densities and not to the coefficients in front of them. Moreover, these divergent parts possess no imaginary part so we shall imply them subtracted in what follows. (Strictly speaking we need \( \mu - m \) subtractions of the type \( (z - R - T)^2 + \rho^2) - \rho^{-4} + 2(x - R - T)^2 \rho^{-6} + \ldots \) After performing the integrations one obtains

\[ \pi^5 \int dz \frac{\Gamma(n)\Gamma(-l)}{(z^2)^{n-l}} \int_0^1 dx \frac{u^m + u^2 \xi^{l-1}}{2m_1 + m_2 - 2n + 2l + 2(1 + x)n - l - \mu - \mu_2} \frac{6\pi u}{\alpha_s(1 + u)^2} \frac{\rho_1^2}{\rho_2^2} e^{i4\pi \frac{1}{\alpha_s}(1 - \frac{4\pi}{\alpha_s})} \]

where we use the notation \( l = \mu_1 + \mu_2 - m_1 - m_2 + 4 \) and \( \bar{u} = 1 - u \). After continuation to Minkowski space the imaginary part of this integral take the form:

\[ \pi^8 B(n, l) \frac{\Gamma(n)\Gamma(-l)}{\Gamma(n-l)} \frac{\rho_1^2}{\rho_2^2} e^{i4\pi \frac{1}{\alpha_s}(1 - \frac{4\pi}{\alpha_s})} \]

where \( \bar{x} = 1 - x \), \( B(n, l) = \frac{\Gamma(n)\Gamma(-l)}{\Gamma(n-l)} \) and \( \xi \) is now \( 2(1 + x)/\bar{x} \). It is easy to see now that the characteristic distances correspond to Eq.7.

One may also account for for the \( \rho \) dependence in the argument of \( \alpha_s \). Careful analysis [4] shows that \( \frac{1}{\alpha_s} \) should be changed to \( \frac{4\pi}{\alpha_s} + 2b \) (where \( b = \frac{11}{2} N_c - \frac{3}{2} n_f \)) and the argument of \( \alpha_s \) obeys the equation

\[ \rho_s = \frac{4\pi}{\alpha_s(\rho_s)} \frac{12(\xi_s - 2)}{Q^2} \]

(10)
Figure 2: The non-perturbative scale in deep inelastic scattering (instanton size \( \rho^{−1} \)), corresponding to the solution of equation Eq.10 as a function of \( Q \) and for \( S(\xi) \sim 0.5 − 0.6 \) (\( \xi \sim 3 − 4 \)).

A numerical solution of this equation for the particular expression of the action \( S(\xi) \) corresponding to the conformal instanton-antiinstanton valley is shown in Fig.2. Note that the difference between the hard scale \( Q^2 \) and the effective scale for non-perturbative effects \( \rho^{−2} \) is numerically very large. This is a new situation compared to calculations of instanton-induced contributions to two-point correlation functions, see e.g. [8, 17, 18], where the size of the instanton is of order of the large virtuality. The effect is that the instanton-induced contributions to deep inelastic scattering may turn out to be non-negligible at the values \( Q^2 \sim 1000 GeV^2 \), which are conventionally considered as a safe domain for perturbative QCD.

Now, in order to find the \( \gamma^g \) amplitude Eq.3 we should take the real answer Eq.7 instead of our toy example. After a considerable algebra (for details see [4]) we obtain the following answer for the \( \bar{II} \) contribution to the structure function of a real gluon:

\[
F_1^{(G)}(x,Q^2) = \sum_q \epsilon_q^2 \frac{1}{9\pi^2} \frac{d^2 \pi^{9/2}}{bS(\xi_*)[bS(\xi_*) - 1]} \left( \frac{16}{\xi_*^3} \right)^{n_f-3} \\
\times \left( \frac{2\pi}{\alpha_s(\rho_*)} \right)^{19/2} \exp \left[ - \left( \frac{4\pi}{\alpha_s(\rho_*)} + 2b \right) S(\xi_*) \right].
\]

(11)

To our accuracy, we find that the instanton- induced contributions obey the Callan-Gross relation \( F_2(x,Q^2) = 2xF_1(x,Q^2) \).

The expression in Eq.11 presents our main result. It gives the exponential correction to the coefficient function in front of the gluon distribution of the leading twist in Eq.2. The exponential factor is exact to the accuracy of Eq.4. The preexponential factor is calculated to leading accuracy in the strong coupling and up to corrections of order \( O(1 - x) \). The corresponding contribution to the structure function of the nucleon is obtained in a usual way, making a convolution of (11) with a distribution of gluons in the proton at the scale \( \rho^2_\pi \).
The instanton-antiinstanton contribution to the structure function of a quark contains a similar contribution shown in Fig.1b. The answer reads

\[
F_1^{(q)}(x, Q^2) = \left[ \sum_{q' \neq q} \epsilon_q^2 + \frac{1}{2} \epsilon_q^2 \right] \frac{128}{81 \pi^3} \frac{d^2 \pi^{9/2}}{bS(\xi_\ast)|bS(\xi_\ast) - 1|} \left( \frac{16}{\xi_\ast^2} \right)^{n_f - 3} \times \left( \frac{2\pi}{\alpha_s(\rho_\ast)} \right)^{15/2} \exp \left[ - \left( \frac{4\pi}{\alpha_s(\rho_\ast)} + 2b \right) S(\xi_\ast) \right]
\]

(12)

However, in this case additional contributions exist of the type shown in Fig.1c. They are finite (the integral over instanton size is cut off at \( \rho^2 \sim x^2 / \alpha_s \)), but the relevant instanton-antiinstanton separation \( R \) is small, of order \( \rho \). This probably means that the structure of nonperturbative contributions to quark distributions is more complicated. This question is under study. The answer given in Eq.12 presents the contribution of the particular saddle point in Eq.10.

3. Value of cross section and structure of final state for instanton-induced particle production

The instanton-induced contribution to the structure function of a gluon in Eq.11 is shown as a function of Bjorken \( x \) for different values of \( Q \sim 10 - 100 \text{GeV} \) in Fig.3. The contribution of the box graph is shown by dots for comparison. The low boundary for possible values of \( Q \) is determined by the condition that the effective instanton size is not too large. At \( Q = 10 \text{GeV} \) we find \( \rho_\ast \sim 1 \text{GeV}^{-1} \), cf. Fig.2. This value is sufficiently small, so that instantons are not distorted too strongly by large-scale vacuum fluctuations. Another limitation is that the valley approach to the calculation of the "holy grail" function \( S(\xi) \) is likely to be justified at \( S(\xi) \geq 1/2 \), which translates to the condition that \( x > 0.3 - 0.35 \). Numerical results are strongly sensitive to the particular value of the QCD scale parameter. We use the two-loop expression for the coupling with three active flavors, and the value \( \Lambda_{\overline{\text{MS}}}^{(3)} = 365 \text{MeV} \) which corresponds to the coupling at the scale of \( \tau \)-lepton mass \( \alpha_s(m_\tau) = 0.33 \) [21]. Since the dependence on the coupling is exponential, the 20% increase of \( \alpha_s(\rho_\ast) \) induces the increase of the cross section by almost an order of magnitude! Together with uncertainties in the function \( S(\xi) \) and in the preexponential factor, this indicates that the particular curves given in Fig.3 should not be taken too seriously, and rather give a target for further theoretical (and experimental?) studies to shoot at.

To summarize, we have found that instantons produce a well-defined and calculable contribution to the cross section of deep inelastic scattering for sufficiently large values of \( x \) and large \( Q^2 \sim 100 - 1000 \text{GeV}^2 \), which turns out, however, to be rather small — of order \( 10^{-2} - 10^{-5} \) compared to the perturbative cross section. This means that the accuracy of standard perturbative analysis is sufficiently high, and that there is not much hope to observe the instanton-induced contributions to the total deep inelastic cross section experimentally. However, instantons are likely to produce events with a very specific structure of the final state, and such peculiarities may be subject to experimental search. The dominating Feynman diagrams in our
Figure 3: Nonperturbative contribution to the structure function $F_1(x,Q^2)$ of a real gluon (11) as a function of $x$ for different values of $Q$ (solid curves). The leading perturbative contribution is shown for comparison by dots. The dashed curves show lines with the constant effective value of the action on the $\bar{\Pi}$ configuration.

calculation correspond to $2\pi/\alpha(\rho_*) \sim 15$ gluons and $2n_f - 1 = 5$ quarks in the final state with the low energy of order $\rho_*^{-1} \sim 1\text{GeV}$. They are produced in the spherically symmetric way in the c.m. frame of the partons colliding through the instanton. (The transverse momentum of the quark coming to the instanton is $k_\perp^2 \sim Q^2\alpha_s \sim \text{few Gev}$ and so is the transverse momentum of the current quark jet). It is not likely that quarks and gluons emitted from the instanton can be resolved as minijets (they have $k_\perp \sim Q\alpha_s \sim \text{1 Gev}$), and we rather expect a spherically symmetric production of final state hadrons in this frame. The effect is likely to be resonance-like, that is present in a narrow interval of values of Bjorken $x$ of order 0.25–0.35 (in the parton-parton collision). In any case, finding of an instanton-induced particle production at high energies is a challenging problem, and further theoretical efforts are needed to put it as a practical proposal to experimentalists.

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Instanton Contribution to DIS

\[ \log_{10}[F_{1}(x, Q^2)] \]

- \[ S = 0.42 \]
- \[ S = 0.52 \]
- \[ S = 0.61 \]
- \[ S = 0.7 \]
- \[ Q = 10 \]
- \[ Q = 15 \]
- \[ Q = 22 \]
- \[ Q = 32 \]
- \[ Q = 46 \]
- \[ Q = 68 \]
- \[ Q = 100 \text{ GeV} \]