Verifying Termination of General Logic Programs with Concrete Queries

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Abstract

We introduce a method of verifying termination of logic programs with respect to concrete queries (instead of abstract query patterns). A necessary and sufficient condition is established and an algorithm for automatic verification is developed. In contrast to existing query pattern-based approaches, our method has the following features: (1) It applies to all general logic programs with non-floundering queries. (2) It is very easy to automate because it does not need to search for a level mapping or a model, nor does it need to compute an interargument relation based on additional mode or type information. (3) It bridges termination analysis with loop checking, the two problems that have been studied separately in the past despite their close technical relation with each other.

Keywords: Logic programming, termination analysis, loop checking, automatic verification.

1 Introduction

For a program in any computer language, in addition to having to be logically correct, it should be terminating. Due to the recursive nature of logic programming, however, a logic program may more likely be non-terminating than a procedural program. Termination of logic programs then becomes one of the most important topics in logic programming research. Because the problem is extremely hard (undecidable in general), it has been considered as a never-ending story; see [12, 15] for a comprehensive survey.

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The goal of termination analysis is to establish a characterization of termination of a logic program and design algorithms for automatic verification. A lot of methods for termination analysis have been proposed in the last decade (e.g., see [1, 2, 6, 13, 15, 17, 19, 22, 30, 31, 43, 45]). A majority of these existing methods are the norm- or level mapping-based approaches, which consist of inferring mode/type information, inferring norms/level mappings, inferring models/interargument relations, and verifying some well-founded conditions (constraints). For example, Ullman and Van Gelder [43] and Plümer [30, 31] focused on establishing a decrease in term size of some recursive calls based on interargument relations; Apt, Bezem and Pedreschi [1, 2], and Bossi, Cocco and Fabris [6] provided characterizations of Prolog left-termination based on level mappings/norms and models; Verschaetse [46], Decorte, De Schreye and Fabris [16], and Martin, King and Soper [28] exploited inferring norms/level mappings from mode and type information; De Schreye and Verschaetse [13], Brodsky and Sagiv [7], and Lindenstrauss and Sagiv [23] discussed automatic inference of interargument/size relations; De Schreye, Verschaetse and Bruynooghe [14], and Mesnard [29] addressed automatic verification of the well-founded constraints. Very recently, Decorte, De Schreye and Vandecasteele [15] presented an elegant unified termination analysis that integrates all the above components to produce a set of constraints that, when solved, yields a termination proof.

It is easy to see that the above methods have among others the following features.

1. They are compile-time approaches in the sense that they make termination analysis only relying on some static information about the structure (of the source code) of a logic program, such as modes/types, norms (i.e. term sizes of atoms of clauses)/level mappings, models/interargument relations, and the like.

2. They are suitable for termination analysis with respect to (abstract) query patterns [14]. A query pattern defines a class of concrete queries, such as ground queries, bounded queries, well-moded queries, etc.

Our observation shows that some dynamic information about the structure of a concrete infinite SLDNF-derivation, such as repetition of selected subgoals and clauses and recursive increase in term size, plays a crucial role in characterizing the termination. Such dynamic features are hard to capture unless we evaluate some related concrete queries. This suggests that methods of extracting and utilizing dynamic features for termination analysis should be exploited.

Another observation comes from real programming practices. Consider the following situation: Given a logic program \( P \) and a query pattern \( Q \), applying a termination analysis yields a conclusion that \( P \) is not terminating w.r.t. \( Q \). In most cases, this means that there are a handful of concrete queries of the pattern \( Q \) evaluating which

\[1\text{The difference between an abstract query pattern and a concrete query is similar to that between a class and an object in object-oriented programming languages.}\]
may lead to infinite SLDNF-derivations. In order to improve the program, users (programmers) most often want to figure out how the non-termination happens by posing a few typical concrete queries and evaluating them step by step while determining which derivations would most likely extend to infinite ones. Such a debugging process is both quite time consuming and tricky. Doing it automatically is of great significance. Obviously, the above mentioned termination analysis techniques cannot help with such job. This suggests that methods of termination analysis for concrete queries should be developed.

The above two observations motivated the research of this paper. In this paper, we introduce an effective method for termination analysis w.r.t. concrete queries. The basic idea is as follows: First, since non-termination is caused by an infinite (generalized) SLDNF-derivation, we directly make use of some essential structural characteristics of an infinite derivation (such as variants, expanded variants, etc.) to characterize the termination. Then, given a logic program and a set of concrete queries, we evaluate these queries while dynamically collecting and applying certain structural features to predict (based on the characterization) if we are on the track to an infinite derivation. Such a process of query evaluation is guaranteed to terminate by a necessary condition of an infinite derivation. Finally, we provide the user with either an answer \textit{Yes}, meaning that the logic program is terminating w.r.t. the given set of queries, or a finite (generalized) SLDNF-derivation that would most likely lead to an infinite derivation. In the latter case, the user can improve the program following the guidance of the informative derivation.

Although the termination problem is undecidable in general, our method works effectively for a vast majority of general logic programs with non-floundering queries. In fact, the methodology used in this paper is partly borrowed from loop checking — another research topic in logic programming, which focuses on detecting and eliminating infinite loops in SLD-trees (e.g., see [3, 8, 26, 27, 35, 36, 37, 44, 48]). Therefore, our work bridges termination analysis with loop checking, the two problems which have been studied separately in the past despite their close technical relation with each other [12].

The plan of the paper is as follows. In Section 2, we introduce a notion of a generalized SLDNF-tree, which is the basis of our method. Roughly speaking, a generalized SLDNF-tree is a set of SLDNF-trees augmented with an ancestor-descendant relation on their subgoals. In Section 3, we prove a necessary and sufficient condition for an infinite generalized SLDNF-derivation. In Section 4.1, we formally define the notion of termination, which is slightly different from that of De Schreye and Decorte [12]. In Section 4.2, we develop an algorithm for automatically verifying termination of a general logic program with concrete queries and prove its properties. We will use some representative logic programs to illustrate the effectiveness of the algorithm. In Section 5, we mention some related work on termination analysis and on loop checking. We end in Section 6 with some concluding remarks and further work.
1.1 Preliminary

We present our notation and review some standard terminology of logic programs as described in [25].

Variables begin with a capital letter, and predicate, function and constant symbols with a lower case letter. Let $A$ be an atom/function. The size of $A$, denoted $|A|$, is the count of function symbols, variables and constants in $A$. We use $\text{rel}(A)$ to refer to the predicate/function symbol of $A$, and use $A[i]$ to refer to the $i$-th argument of $A$, $A[i][j]$ to refer to the $j$-th argument of the $i$-th argument, and so on. Let $S$ be a set or a list. We use $|S|$ to denote the number of elements in $S$.

**Definition 1.1** Let $A$ be an atom with the list $[X_1, ..., X_m]$ of distinct variables. By variable renaming on $A$ we mean to substitute the variables of $A$ with another list $[Y_1, ..., Y_m]$ of distinct variables. Two atoms $A$ and $B$ are said to be variants if after variable renaming (on $A$ or $B$) they become the same.

For instance, let $A = p(a, X, Y, X)$ and $B = p(a, Z, Y, Z)$. By substituting $[X, Y]$ for $[Z, Y]$, $B$ becomes the same as $A$, so $A$ and $B$ are variants. However, $A$ and $C = p(a, Z, Y, W)$ are not variants because there is no variable substitution that makes them the same. Note that any atom $A$ is a variant of itself.

**Definition 1.2** A (general) logic program is a finite set of clauses of the form

$$A \leftarrow L_1, ..., L_n$$

where $A$ is an atom and $L_i$s are literals. $A$ is called the head and $L_1, ..., L_n$ is called the body of the clause. If a general logic program has no clause with negative literals in its body, it is called a positive program.

**Definition 1.3** A goal is a headless clause $\leftarrow L_1, ..., L_n$ where each literal $L_i$ is called a subgoal. $L_1, ..., L_n$ is called a (concrete) query. When $n = 0$, the “$\leftarrow$” symbol is omitted.

The initial goal, $G_0 = \leftarrow L_1, ..., L_n$, is called a top goal. Without loss of generality, we shall assume throughout the paper that a top goal consists only of one atom (i.e. $n = 1$ and $L_1$ is a positive literal).

**Definition 1.4** A control strategy consists of two rules: one rule for selecting one goal from among a set of goals, and one rule for selecting one subgoal from the selected goal.

The second rule in a control strategy is usually called a selection or computation rule in the literature. Throughout the paper we use a fixed depth-first, left-most control strategy (as in Prolog). So the selected subgoal in each goal is the left-most subgoal.

Trees are commonly used to represent the search space of a top-down proof procedure. For convenience, a node in such a tree is represented by $N_i : G_i$ where $N_i$ is the name of the node and $G_i$ is a goal labeling the node. Assume no two nodes have the same name. Therefore, we can refer to nodes by their names.
2 Generalized SLDNF-Trees

Non-termination of general logic programs results from infinite derivations. In order to characterize infinite derivations more precisely, in this section we extend the standard SLDNF-trees [25] to include some new features.

To characterize an infinite derivation we need first to define the ancestor-descendant relation on its selected subgoals. Informally, \( A \) is an ancestor subgoal of \( B \) if the proof of \( A \) needs (or in other words goes via) the proof of \( B \). For example, let \( M : \leftarrow A, A_1, ..., A_m \) be a node in an SLDNF-tree, and \( N : \leftarrow B_1, ..., B_n, A_1, ..., A_m \) be a child node of \( M \) that is generated by resolving \( M \) on the subgoal \( A \) with a clause \( A \leftarrow B_1, ..., B_n \). Then \( A \) at \( M \) is an ancestor subgoal of all \( B_i \)s at \( N \). However, such relationship does not exist between \( A \) at \( M \) and any \( A_j \) at \( N \). It is easily seen that all \( B_i \)s at \( N \) inherit the ancestor subgoals of \( A \) at \( M \).

The ancestor-descendant relation can be explicitly expressed using an ancestor list introduced in [36], which is a set of pairs \((\text{Node}, \text{Atom})\) where \( \text{Node} \) is the name of a node and \( \text{Atom} \) is the selected subgoal at \( \text{Node} \). The ancestor list of a subgoal \( L_j \) is \( AL_{L_j} = \{(N_1, A_1), ..., (N_k, A_k)\} \), showing that \( A_1 \) at node \( N_1 \), ..., and \( A_k \) at node \( N_k \) are all ancestor subgoals of \( L_j \). For instance, in the above example, if the ancestor list of the subgoal \( A \) at node \( M \) is \( AL_A \), then the ancestor list of each \( B_i \) at node \( N \) is \( AL_{B_i} = \{(M, A)\} \cup AL_A \).

Augmenting SLDNF-trees with ancestor lists leads to the following definition of SLDNF*-trees.

Definition 2.1 (SLDNF*-trees) Let \( P \) be a general logic program, \( G_0 = \leftarrow A_0 \) a top goal, and \( R \) a depth-first, left-most control strategy. The SLDNF*-tree \( T_{G_0} \) for \( P \cup \{G_0\} \) via \( R \) is defined as follows.

1. The root node is \( N_0 : G_0 \) with the ancestor list \( AL_{A_0} = \{\} \) for \( A_0 \).

2. Let \( N_i : \leftarrow L_1, ..., L_m \) be a node in the tree selected by \( R \). If \( m = 0 \) then \( N_i \) is a success leaf, marked by \( \Box \). Otherwise, we distinguish between the following two cases.

   (a) If \( L_1 \) is a positive literal, then for each clause \( B \leftarrow B_1, ..., B_n \) such that \( L_1 \) and \( B \) are unifiable, \( N_i \) has a child node

   \[
   N_s : \leftarrow (B_1, ..., B_n, L_2, ..., L_m)\theta
   \]

   where \( \theta \) is an mgu (i.e. most general unifier) of \( L_1 \) and \( B \), the ancestor list for each \( B_k\theta \) is \( AL_{B_k\theta} = \{(N_i, L_1)\} \cup AL_{L_1} \), and the ancestor list for each \( L_k\theta \) is \( AL_{L_k\theta} = AL_{L_k} \). If there exists no clause whose head can unify with \( L_1 \) then \( N_i \) has a single child node — a failure leaf, marked by \( \Box_f \).

   (b) If \( L_1 = \neg A \) is a ground negative literal, then build a partial SLDNF*-tree \( T_{\neg A} \) for \( P \cup \{\neg A\} \) via \( R \) where \( A \) inherits the ancestor list of \( L_1 \), until the first success leaf is generated. If \( T_{\neg A} \) has a success leaf then \( N_i \) has
a single child node — a failure leaf, \( \Box_f \). Otherwise, if all branches of \( T_{\neg A} \) end with a failure leaf then \( N_i \) has a single child node

\[
N_s : \leftarrow L_2, ..., L_m
\]

where all \( L_k \) inherit the ancestor lists of \( L_k \) at node \( N_i \).

Note that in this paper we do not discuss floundering — a situation where a non-ground negative subgoal is selected by \( R \) (see [9, 18, 24, 32] for discussion on such topic). In contrast to SLDNF-trees, an SLDNF*-tree has the following two new features.

1. An ancestor list \( AL_{L_j} \) is attached to each subgoal \( L_j \). In particular, subgoals of a subsidiary SLTNF*-tree \( T_{\neg A} \) built for solving a subgoal \( L_1 = \neg A \) inherit the ancestor list of \( L_1 \) (see item 2b). This is especially useful in identifying infinite derivations across SLTNF*-trees (see Example 2.1). Note that a negative subgoal will never be an ancestor subgoal.

2. To handle a ground negative subgoal \( L_1 = \neg A \), only a partial subsidiary SLTNF*-tree \( T_{\neg A} \) is generated by stopping at the first success leaf (see item 2b). The reason for this is that it is totally unnecessary to exhaust the remaining branches of \( T_{\neg A} \) because they would have no new answer for \( A \). This can not only improve the efficiency of query evaluation, but also avoid some possible infinite derivations (see Example 2.2). In fact, Prolog achieves this by using cuts to skip the remaining branches of \( T_{\neg A} \) (e.g. see SICStus Prolog [21]).

For convenience, we use dotted edges “\( \cdots \)" to connect parent and child SLDNF*-trees, so that infinite derivations across SLDNF*-trees can be clearly identified. Moreover, we refer to \( T_{G_0} \), the top SLDNF*-tree, along with all its descendant SLDNF*-trees as a generalized SLDNF-tree for \( P \cup \{G_0\} \), denoted \( GT_{G_0} \). Therefore, a path of a generalized SLDNF-tree may come across several SLDNF*-trees through dotted edges. Any such a path starting at the root node \( N_0 : G_0 \) is called a generalized SLDNF-derivation. A generalized SLDNF-derivation is successful (resp. failed) if it ends at a success leaf (resp. at a failure leaf).

Thus, there may occur two types of edges in a generalized SLDNF-tree, “\( \xrightarrow{C} \)” and “\( \cdots \)". For convenience, we use “\( \Rightarrow \)” to refer to either of them. We also use \( N_i : G_i \xrightarrow{C_1} ... \xrightarrow{C_m} N_k : G_k \) to represent a segment of a generalized SLDNF-derivation, which generates \( N_k : G_k \) from \( N_i : G_i \) by applying the set of clauses \( \{C_1, ...C_m\} \). Moreover, for any node \( N_i : G_i \) we use \( L_1^i \) to refer to the selected (i.e. left-most) subgoal in \( G_i \).

**Example 2.1** Let \( P_1 \) be a general logic program and \( G_0 \) a top goal, given by

\[
P_1 : \quad p(X) \leftarrow \neg p(f(X)).
\]

\[
G_0 : \quad \leftarrow p(a).
\]
The generalized SLDNF-tree $\text{GT}_{\neg p(a)}$ for $P_1 \cup \{G_0\}$ is shown in Figure 1, where $\infty$ represents an infinite extension. We see that $\text{GT}_{\neg p(a)}$ consists of one infinite generalized SLDNF-derivation.

Example 2.2 Consider the following general logic program and top goal.

\[
P_2: \quad p \leftarrow \neg q. \quad q. \quad q \leftarrow q. \quad G_0: \quad \leftarrow p.
\]

The generalized SLDNF-tree $\text{GT}_{\neg p}$ for $P_2 \cup \{G_0\}$ is depicted in Figure 2 (a). For the purpose of comparison, the SLDNF-trees for $P_2 \cup \{\leftarrow p\}$ are shown in Figure 2 (b). Note that Figure 2 (a) is finite, whereas Figure 2 (b) is not.

We now formally define the ancestor-descendant relation.

**Definition 2.2** Let $N_i : G_i$ and $N_k : G_k$ be two nodes in a generalized SLDNF-derivation, and $A$ and $B$ be the selected subgoals in $G_i$ and $G_k$, respectively. We say that $A$ is an ancestor subgoal of $B$, denoted $A \prec_{\text{ANC}} B$, if $A$ is in the ancestor list $\text{AL}_B$ of $B$. When $A$ is an ancestor subgoal of $B$, we refer to $B$ as a descendant subgoal of $A$, $N_i$ as an ancestor node of $N_k$, and $N_k$ as a descendant node of $N_i$. 

![Figure 1: A generalized SLDNF-tree $\text{GT}_{\neg p(a)}$.](image1)

![Figure 2: A generalized SLDNF-tree $\text{GT}_{\neg p}$ (a) and its two corresponding SLDNF-trees (b).](image2)
3 Characterizing an Infinite Generalized SLDNF-Derivation

In this section we establish a necessary and sufficient condition for an infinite generalized SLDNF-derivation.

In [37], a concept of expanded variants is introduced, which captures some key structural characteristics of certain subgoals in an infinite SLD-derivation. We observe that it applies to general logic programs as well. That is, infinite generalized SLDNF-derivations can be characterized based on expanded variants.

**Definition 3.1** Let \( A \) and \( A' \) be two atoms or functions. \( A' \) is said to be an expanded variant of \( A \), denoted \( A' \supseteq_{EV} A \), if after variable renaming on \( A' \) it becomes \( B \) that is the same as \( A \) except that there may be some terms at certain positions in \( A \) each \( A[i]...[k] \) of which grows in \( B \) into a function \( B[i]...[k] = f(\ldots, A[i]...[k], \ldots) \). Such terms like \( A[i]...[k] \) in \( A \) are then called growing terms w.r.t. \( A' \).

As an illustration, let \( A = p(X, h(X)) \) and \( A' = p(Y, h(h(Y))) \). By renaming \( Y \) with \( X \), \( A' \) becomes \( B = p(X, h(h(X))) \), which is the same as \( A \) except that \( B[2][1] = h(A[2][1]) \). Therefore, \( A' \) is an expanded variant of \( A \) with a growing term \( A[2][1] \). Here are a few more examples: \( p(X, Y) \supseteq_{EV} p(Z, W) \), \( p(f(a)) \supseteq_{EV} p(a) \), \( p(g(a), f(h(X))) \supseteq_{EV} p(a, f(h(Y))) \), and \( p([X_1, X_2, X_3]) \supseteq_{EV} p([X_1, X_4]) \) (note that \([X_1, X_2, X_3] = [X_1][X_2, X_3]]\)).

It is immediate from Definition 3.1 that variants are expanded variants with the same size.

**Theorem 3.1** Let \( D \) be an infinite generalized SLDNF-derivation without infinitely large subgoals. Then there are infinitely many goals \( G_{g_1}, G_{g_2}, \ldots \) in \( D \) such that for any \( j \geq 1 \), \( L^i_{g_j} \triangleleft_{\ANC} L^i_{g_{j+1}} \) and \( L^i_{g_j} \) and \( L^i_{g_{j+1}} \) are variants.

**Proof.** Let \( D \) be of the form

\[
N_0 : G_0 \Rightarrow N_1 : G_1 \Rightarrow \ldots \Rightarrow N_i : G_i \Rightarrow N_{i+1} : G_{i+1} \Rightarrow \ldots
\]

For each derivation step \( N_i : G_i \xrightarrow{C} N_{i+1} : G_{i+1} \), where \( L^i_1 \) is a positive subgoal and \( C = A \leftarrow B_1, \ldots, B_n \) such that \( A\theta = L^i_1\theta \) under an mgu \( \theta \), we do the following:

1. If \( n = 0 \), which means \( L^i_1 \) is proved at this step, mark node \( N_i \) with \( \# \).

2. Otherwise, the proof of \( L^i_1 \) needs the proof of \( B_j\theta \) (\( j = 1, \ldots, n \)). If all descendant nodes of \( N_i \) in \( D \) have been marked with \( \# \), which means that all \( B_j\theta \) have been proved at some steps in \( D \), mark node \( N_i \) with \( \# \).

Note that the root node \( N_0 \) will never be marked by \( \# \), for otherwise \( G_0 \) would have been proved and \( D \) should have ended at a success leaf. After the above marking process, let \( D \) become
where all nodes except $N_0, N_{i_1}, N_{i_2}, \ldots, N_{i_k}, \ldots$ are marked with #. Since we use the depth-first, left-most control strategy, for any $j \geq 0$ the proof of $L_{i_j}^1$ needs the proof of $L_{i_{j+1}}^1$ (let $i_0 = 0$), for otherwise $N_{i_j}$ would have been marked with #. That is, $L_{i_j}^1$ is an ancestor subgoal of $L_{i_{j+1}}^1$. Moreover, $D$ must contain an infinite number of such nodes because if $N_{i_k} : G_{i_k}$ was the last one, which means that all nodes after $N_{i_k}$ were marked with #, then $L_{i_k}^1$ would be proved, so that $N_{i_k}$ should be marked with #, a contradiction.

The above proof shows that $D$ has an infinite number of selected subgoals $L_{i_1}^1, L_{i_2}^1, \ldots$ such that $L_{i_j}^1 \prec_{ANC} L_{i_{j+1}}^1 \ (j \geq 1)$. Since all subgoals in $D$ are bounded in size, and any general logic program has only a finite number of clauses and predicate, function and constant symbols, there must be an infinite number of subgoals $L_{g_1}^1, L_{g_2}^1, \ldots$ among the $L_{i_j}^1$s that are variants. This concludes the proof.

**Theorem 3.2** Let $D$ be an infinite generalized SLDNF-derivation with infinitely large subgoals. Then there are infinitely many goals $G_{g_1}, G_{g_2}, \ldots$ in $D$ such that for any $j \geq 1$, $L_{g_j}^1 \prec_{ANC} L_{g_{j+1}}^1$ and $L_{g_{j+1}}^1 \not\equiv_{EV} L_{g_j}^1$ with $|L_{g_{j+1}}^1| > |L_{g_j}^1|$.  

The following lemma is required to prove this theorem.

**Lemma 3.3** Let $S = \{C_1, \ldots, C_n\}$ be a finite set of clauses. Let $D$ be an infinite generalized SLDNF-derivation of the form

$$N_0 : G_0 \Rightarrow \ldots N_{i_1} : G_{i_1} \Rightarrow \ldots \Rightarrow N_{i_2} : G_{i_2} \Rightarrow \ldots \Rightarrow N_{i_k} : G_{i_k} \Rightarrow \ldots$$

where for any $j \geq 1$, $\{C_{j_1}, \ldots, C_{j_{n_j}}\} = S$, $L_{i_j}^1 \prec_{ANC} L_{i_{j+1}}^1$, and $L_{i_j}^1$ is a variant of $L_{i_{j+1}}^1$ except for a few terms (at least one) at certain positions in $L_{i_{j+1}}^1$ that increase in size w.r.t. $L_{i_j}^1$. Then there are an infinite sequence of subgoals $L_{g_1}^1, L_{g_2}^1, \ldots$ among the $L_{i_j}^1$s such that for any $k \geq 1$, $L_{g_{k+1}}^1 \not\equiv_{EV} L_{g_k}^1$ with $|L_{g_{k+1}}^1| > |L_{g_k}^1|$.  

**Proof.** Since $L_{i_{j+1}}^1$ being an expanded variant of $L_{i_j}^1$ with $|L_{i_{j+1}}^1| > |L_{i_j}^1|$ is determined by those arguments of $L_{i_j}^1$ whose size increases w.r.t. $L_{i_{j+1}}^1$, for simplicity of presentation we ignore the remaining arguments of $L_{i_j}^1$ that are variants of the corresponding ones in $L_{i_{j+1}}^1$. Since the $L_{i_j}^1$s are generated by repeatedly applying the same set $S$ of clauses, the increase in their term size must be made in a fixed, regular way (assume that our programs contain no built-in’s such as $\text{assert}(.)$ and $\text{retract}(.)$). In order to facilitate the analysis of such regular increase in term size, with no loss in generality, let each $L_{i_j}^1$ be of the form $\rho([T_{i_j}^1, \ldots, T_{m_j}^1])$, which contains a single argument that is a list, such that the list of $L_{i_{j+1}}^1$ is derived from the list $L = [T_{1_j}^1, \ldots, T_{m_j}^1]$ of $L_{i_j}^1$ via a DELETE-ADD-SHUFFLE process which consists of deleting the first $0 \leq n^- \leq m_j$ elements from $L$, then adding $n^+ \geq n^-$ elements to the front of the remaining part of $L$, and finally shuffling the list of elements obtained (in a fixed way).2

Let $\vec{X}$ represent a sequence of elements, such as $X_1, X_2, \ldots, X_m$. We distinguish the following two cases based on the ADD operation.

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2See Example 4.4 for an illustration.
1. The \( n^+ \) added elements are independent of the list \( L \) of elements of \( L_{i_1}^1 \). Since the same set \( S \) of clauses is applied, the set \( E_1 \) of elements added to \( L_{i_2}^1 \) from \( L_{i_1}^1 \) must be the same as or a variant of the set added to \( L_{i_3}^1 \) from \( L_{i_2}^1 \) that must be the same as or a variant of the set added to \( L_{i_4}^1 \) from \( L_{i_3}^1 \), and so on. Let \( E_2 \subseteq \{ T_1^1, ..., T_{m_1}^1 \} \) be such that each \( T \in E_2 \) occurs in the derivation \( D \) infinite times. That is, no \( T \in E_2 \) will be removed by the \DELETE{} operation. (But all \( T \in \{ T_1^1, ..., T_{m_1}^1 \} - E_2 \) will be deleted at some derivation steps in \( D \) by the \DELETE{} operation.) Then there must be an infinite sequence \( L_{j_1}^1, L_{j_2}^1, ..., \) among the \( L_{i_j}^1 \)'s such that for any \( k \geq 1 \), all elements in \( L_{j_k}^1 \) come from \( E_1 \cup E_2 \) (or its variant). Since \( E_1 \cup E_2 \) contains only a finite number of elements, no matter what shuffling approach is used, there must be an infinite sequence \( L_{g_1}^1, L_{g_2}^1, ..., \) among the \( L_{j_k}^1 \)'s such that for any \( k \geq 1 \), let \( L_{g_k}^1 = p([S_1, ..., S_n]) \), then after variable renaming \( L_{g_{k+1}}^1 \) will become like \( p([\bar{S}_1, ..., \bar{S}_n]) \) such that each \( S_l \) is in \( \bar{S}_l \). Obviously, these \( S_l \)'s in \( L_{g_k}^1 \) are growing terms w.r.t. \( L_{g_{k+1}}^1 \). That is, \( L_{g_{k+1}}^1 \equiv_{EV} L_{g_k}^1 \) with \( |L_{g_{k+1}}^1| > |L_{g_k}^1| \).

2. Some of the \( n^+ \) added elements depend on the list of \( L_{i_j}^1 \). For simplicity of presentation and without loss of generality assume that from \( L_{i_1}^1 \) to \( L_{i_2}^1 \) only one added element, say of the form \( f(A_1^1, g(A_2^1)) \), depends on the list \( [T_1^1, ..., T_{m_1}^1] \) of \( L_{i_1}^1 \). Let \( E_1 \) be the set of elements added to \( L_{i_2}^1 \) that are independent of the list \( [T_1^1, ..., T_{m_1}^1] \). That is, the set of added elements from \( L_{i_1}^1 \) to \( L_{i_2}^1 \) is \( E_1 \cup \{ f(A_1^1, g(A_2^1)) \} \). Let \( E = E_1 \cup \{ T_1^1, ..., T_{m_1}^1 \} \). Then \( E \) can be considered to be the domain of the two arguments \( A_1^1, A_2^1 \) in \( f(A_1^1, g(A_2^1)) \), i.e. \( A_1^1, A_2^1 \in E \). Since all the \( L_{i_j}^1 \)'s in the derivation \( D \) are generated recursively by applying the same set \( S \) of clauses, for any \( j \geq 1 \) from \( L_{i_j}^1 \) to \( L_{i_{j+1}}^1 \) the set of added elements should be \( E_1 \cup \{ f(A_1^1, g(A_2^1)) \} \) where \( A_1^1, A_2^1 \in E \cup \{ f(A_1^1, g(A_2^1)) \} \). It is easy to see that any element of \( L_{i_j}^1 \) with an infinitely large size must be of the form \( f(A_1^\infty, g(A_2^\infty)) \) where \( A_1^\infty \) or \( A_2^\infty \) or both are of the form \( f(A_1^\infty, g(A_2^\infty)) \).

We now consider the following two cases.

(a) No \( L_{i_j}^1 \) in \( D \) contains elements with an infinitely large size. Let \( N \) be the largest size of an element \( f(\_ , g(\_ )) \) in the \( L_{i_j}^1 \)'s and let \( E_2 = \{ f(A_1^k, g(A_2^k)) \} \) of size \( k \geq 1 \) such that \( |f(A_1^k, g(A_2^k))| \leq N \). Obviously \( E_2 \) is finite. So the elements of any \( L_{i_j}^1 \) in \( D \) come from \( E \cup E_2 \). Since \( E \cup E_2 \) is finite, the number of the combinations of elements of \( E \cup E_2 \) is finite. Since the \( L_{i_j}^1 \)'s in \( D \) grow towards an infinitely large size, some combinations must repeat in \( D \) infinite times. This suggests that no matter what shuffling approach is used, there must be an infinite sequence \( L_{g_1}^1, L_{g_2}^1, ..., \) among the \( L_{i_j}^1 \)'s such that for any \( k \geq 1 \), let \( L_{g_k}^1 = p([S_1, ..., S_n]) \), then after variable renaming \( L_{g_{k+1}}^1 \) will become like \( p([\bar{S}_1, ..., \bar{S}_n]) \) such that each \( S_l \) is in \( \bar{S}_l \). These \( S_l \)'s in \( L_{g_k}^1 \) are growing terms w.r.t. \( L_{g_{k+1}}^1 \), thus \( L_{g_{k+1}}^1 \equiv_{EV} L_{g_k}^1 \) with \( |L_{g_{k+1}}^1| > |L_{g_k}^1| \).

(b) As \( j \to \infty \), some elements in \( L_{i_j}^1 \) grow towards an infinitely large size. Let \( T^{j+1} \) be an element in the list of \( L_{i_{j+1}}^1 \) that (as \( j \to \infty \)) grows towards an
element with an infinitely large size. Then there must be an element $T^j$ in the list of $L^1_{ij}$ that grows towards an element with an infinitely large size via $T^{j+1}$ (otherwise, $T^{j+1}$ would not grow towards an infinitely large element since we apply the same set $S$ of clauses from $L^1_{ij+1}$ to $L^1_{ij+2}$ as from $L^1_{ij}$ to $L^1_{ij+1}$). This means that $T^{j+1}$ is the same as (or a variant of) $T^j$ or $T^{j+1} = f(A_i^j, g(A_2^j))$ such that $T^j$ (or its variant) is in $A_1^j$ or $A_2^j$. Obviously $T^{j+1}$ is an expanded variant of $T^j$. Generalizing such argument, for each infinitely large element $T^j_\infty$ in the list of $L^1_{\infty}$ we have an infinite sequence

$$T^1, T^2, ..., T^j, T^{j+1}, ..., T^\infty$$

with each $T^j$ in the list of $L^1_{ij}$ such that $T^1$ grows towards $T^\infty$ via $T^2$ that grows towards $T^\infty$ via $T^3$, and so on. Obviously, for any $k > j \geq 1$ $T^k$ is an expanded variant of $T^j$. So in this case each $T^j$ is called a growing element w.r.t. $T^\infty$. Note that if there are more than one element in some $L^1_{ij}$ that grow towards $T^\infty$ via $T^{j+1}$, such as in the case $T^{j+1} = f(T^j_1, g(T^2_1))$ with $T^j_1, T^2_1$ in $L^1_{ij}$, only one $T^j$ of them is selected as the growing element w.r.t. $T^\infty$ based on the following criterion: $T^j$ must be an expanded variant of $T^{j-1}$. If still more than one element meet such criterion, select an arbitrary one.

We now partition the list $[T^j_1, ..., T^j_m]$ of each $L^1_{ij}$ into two parts: the sublist $GE_{ij}$ of growing elements and the sublist $NGE_{ij}$ of non-growing elements. That is, $T \in [T^j_1, ..., T^j_m]$ is in $GE_{ij}$ if it is a growing element w.r.t. some $T^\infty$. Clearly, for each $k > j \geq 1 |GE_{ik}| \geq |GE_{ij}|$. Since for any $V_1, ..., V_m$ in $GE_{ij}$ there are $m$ elements $V'_1, ..., V'_m$ in $GE_{ik}$ such that each $V'_i$ is an expanded variant of $V_i$, there must be an infinite sequence $L^1_{j_1}, L^1_{j_2}, ...$ among the $L^1_{is}$ such that for any $k > j \geq 1$ $GE_{jk}$ is an expanded variant of $GE_{jk}$. That is, let $GE_{j_k} = [V_1, ..., V_m]$, then after variable renaming $GE_{j_k}$ becomes $[\tilde{V}_1, ..., \tilde{V}_m]$ such that each $\tilde{V}_i$ contains an element that is an expanded variant of $V_i$.

Now consider the elements of $L^1_{j_1}$. Since all infinitely large elements have been covered by the growing elements, the size of any non-growing element is bounded by some constant, say $N$. Let $E_2 = \{f(A^k_1, g(A^k_2)) | k \geq 1 \text{ such that } |f(A^k_1, g(A^k_2))| \leq N\}$. So all non-growing elements of any $L^1_{j_1}$ come from $E \cup E_2$. Since $E \cup E_2$ is finite and the sequence $L^1_{j_1}, L^1_{j_2}, ...$ is infinite, some combinations of elements of $E \cup E_2$ must occur in infinitely many $L^1_{j_1}$s. This implies that no matter what shuffling approach is used, there must be an infinite sequence $L^1_{g_1}, L^1_{g_2}, ...$ among the $L^1_{j_1}$s such that for any $k \geq 1$, let $L^1_{g_k} = p([S_1, ..., S_n])$, then after variable renaming $L^1_{g_{k+1}}$ will become like $p([\tilde{S}_1, ..., \tilde{S}_n])$ such that each $\tilde{S}_i$ contains an element that is an expanded variant of $S_i$. That is, $L^1_{g_{k+1}} \equiv_{EV} L^1_{g_k}$ with $|L^1_{g_{k+1}}| > |L^1_{g_k}|$.

**Proof of Theorem 3.2.** By the proof of Theorem 3.1, $D$ contains an infinite number of selected subgoals $L^1_1, L^1_2, ...$ such that $L^1_j \prec_{ANC} L^1_{j+1}$ ($j \geq 1$). Since any
logic program has only a finite number of clauses, there must be a set of clauses in the program that are invoked an infinite number of times in \( D \). Let \( S = \{ C_1, ..., C_n \} \) be the set of all different clauses that are used an infinite number of times in \( D \). Then \( D \) can be depicted as

\[
N_0 : G_0 \Rightarrow ... N_{i_1} : G_{i_1} \xrightarrow{C_{i_1}} ... \xrightarrow{C_{i_n}} N_{i_2} : G_{i_2} \Rightarrow ... \xrightarrow{C_{2n_2}} N_{i_3} : G_{i_3} \xrightarrow{C_{3_1}} ...
\]

where for any \( j \geq 1 \), \( L^1_{i_j} \prec_{\text{ANC}} L^1_{i_{j+1}} \) and \( \{ C_{j_1}, ..., C_{j_n} \} = S \). Since any logic program has only a finite number of predicate, function and constant symbols and \( D \) contains subgoals with infinitely large size, there must be an infinite sequence \( L^1_{i_1}, L^1_{i_2}, ... \) among the \( L^1_{i_j} \)'s such that for any \( l \geq 1 \), \( L^1_{i_l} \) is a variant of \( L^1_{i_{l+1}} \) except for a few terms in \( L^1_{i_{l+1}} \) that increase in size. Hence by Lemma 3.3, there is an infinite sequence \( L^1_{g_1}, L^1_{g_2}, ... \) among the \( L^1_{i_l} \)'s such that for any \( k \geq 1 \), \( L^1_{g_{k+1}} \equiv \text{EV} L^1_{g_k} \) with \( |L^1_{g_{k+1}}| > |L^1_{g_k}| \).

**Theorem 3.4** \( D \) is an infinite generalized SLDF-derivation if and only if it is of the form

\[
N_0 : G_0 \Rightarrow ... N_{g_1} : G_{g_1} \xrightarrow{C_{1}} ... \xrightarrow{C_{1_n}} N_{g_2} : G_{g_2} \xrightarrow{C_{2}} ... \xrightarrow{C_{2n_2}} N_{g_3} : G_{g_3} \xrightarrow{C_{3}} ...
\]

such that

1. For any \( j \geq 1 \), \( L^1_{g_j} \prec_{\text{ANC}} L^1_{g_{j+1}} \) and \( L^1_{g_{j+1}} \equiv \text{EV} L^1_{g_j} \).
2. For any \( j \geq 1 \), \( |L^1_{g_j}| = |L^1_{g_{j+1}}| \), or for any \( j \geq 1 \), \( |L^1_{g_j}| < |L^1_{g_{j+1}}| \).
3. For any \( j \geq 1 \), the set of clauses used to derive \( L^1_{g_{j+1}} \) from \( L^1_{g_j} \) is the same as that of deriving \( L^1_{g_{j+2}} \) from \( L^1_{g_{j+1}} \), i.e. \( \{ C_{j_1}, ..., C_{j_n} \} = \{ C_{(j+1)_1}, ..., C_{(j+1)n+1} \} \).

**Proof.** (\( \Leftarrow \)) Straightforward.

(\( \Rightarrow \)) By Theorems 3.1 and 3.2, \( D \) is of the form

\[
N_0 : G_0 \Rightarrow ... N_{i_1} : G_{i_1} \Rightarrow ... \Rightarrow N_{i_2} : G_{i_2} \Rightarrow ...
\]

where for any \( j \geq 1 \), \( L^1_{i_j} \prec_{\text{ANC}} L^1_{i_{j+1}} \) and \( L^1_{i_{j+1}} \equiv \text{EV} L^1_{i_j} \). In particular, when all subgoals in \( D \) are bounded in size, by Theorem 3.1 for any \( j \geq 1 \), \( |L^1_{i_j}| = |L^1_{i_{j+1}}| \). Otherwise, by Theorem 3.2 for any \( j \geq 1 \), \( |L^1_{i_j}| < |L^1_{i_{j+1}}| \).

Since any logic program has only a finite number of clauses, there must be a set \( S = \{ C_1, ..., C_n \} \) of clauses in the program that are invoked an infinite number of times in \( D \). This means that there exists an infinite sequence of subgoals \( L^1_{g_1}, L^1_{g_2}, ... \) among the \( L^1_{i_j} \)'s such that for any \( j \geq 1 \), \( L^1_{g_{j+1}} \) is derived from \( L^1_{g_j} \) by applying the set \( S \) of clauses. That is, \( D \) is of the form

\[
N_0 : G_0 \Rightarrow ... N_{g_1} : G_{g_1} \xrightarrow{C_{1}} ... \xrightarrow{C_{1_n}} N_{g_2} : G_{g_2} \xrightarrow{C_{2}} ... \xrightarrow{C_{2n_2}} N_{g_3} : G_{g_3} \xrightarrow{C_{3}} ...
\]
such that the three conditions of this theorem hold.

Theorem 3.4 is the principal result of this paper. It captures two crucial characteristics of an infinite generalized SLDNF-derivation: repetition of selected subgoals and clauses, and recursive increase in term size. Repetition leads to variants, whereas recursive increase introduces growing terms. It is the characterization of these key (dynamic) features that allows us to design a mechanism for automatically testing termination of general logic programs.

4 Testing Termination of General Logic Programs

4.1 Definition of Termination

In [12], a generic definition of termination of logic programs is presented.

**Definition 4.1** Let $P$ be a general logic program, $S_Q$ a set of queries and $S_R$ a set of selection rules. $P$ is terminating w.r.t. $S_Q$ and $S_R$ if for each query $Q_i$ in $S_Q$ and for each selection rule $R_j$ in $S_R$, all SLDNF-trees for $P \cup \{\leftarrow Q_i\}$ via $R_j$ are finite.

Observe that the above definition considers finite SLDNF-trees for termination. That is, if $P$ is terminating w.r.t. $Q_i$ then all (complete) SLDNF-trees for $P \cup \{\leftarrow Q_i\}$ must be finite. This does not seem to apply to Prolog where there exist cases in which, although $P$ is terminating w.r.t. $Q_i$ and $R_j$, some (complete) SLDNF-trees for $P \cup \{\leftarrow Q_i\}$ are infinite. Example 2.2 gives such an illustration, where Prolog terminates with a positive answer.

In view of the above observation, we present the following definition based on a generalized SLDNF-tree.

**Definition 4.2** Let $P$ be a general logic program, $S_Q$ a finite set of queries and $R$ a fixed depth-first, left-most control strategy. $P$ is terminating w.r.t. $S_Q$ and $R$ if for each query $Q_i$ in $S_Q$, the generalized SLDNF-tree for $P \cup \{\leftarrow Q_i\}$ via $R$ is finite.

The above definition implies that $P$ is terminating w.r.t. $S_Q$ and $R$ if and only if there is no infinite generalized SLDNF-derivation in any generalized SLDNF-tree $GT_{-Q_i}$. So the following result is immediate from Theorem 3.4.

**Theorem 4.1** $P$ is terminating w.r.t. $S_Q$ and $R$ if and only if for each query $Q_i$ in $S_Q$ there is no infinite generalized SLDNF-derivation in $GT_{-Q_i}$ of the form

$$N_0 : G_0 \Rightarrow \ldots N_{g_1} : G_{g_1} \overset{C_{11}}{\rightarrow} \ldots \overset{C_{1n_1}}{\leftarrow} N_{g_2} : G_{g_2} \overset{C_{21}}{\rightarrow} \ldots \overset{C_{2n_2}}{\leftarrow} N_{g_3} : G_{g_3} \overset{C_{31}}{\rightarrow} \ldots$$

that meets the three conditions of Theorem 3.4.
4.2 An Algorithm for Automatically Testing Termination

Theorem 4.1 provides a necessary and sufficient condition for termination of a general logic program. Obviously, such a condition cannot be directly used for automatic verification because it requires generating an infinite generalized SLDNF-derivation to see if the three conditions of Theorem 3.4 are satisfied.

As we mentioned before, the three conditions of Theorem 3.4 capture two most important structural features of an infinite generalized SLDNF-derivation. Therefore, we may well use these conditions to predict possible infinite generalized SLDNF-derivations based on some finite generalized SLDNF-derivations. Although the predictions may not always be guaranteed to be correct (since the termination problem is undecidable in general), it can be correct in a vast majority of cases. That is, if the three conditions of Theorem 3.4 are satisfied by some finite generalized SLDNF-derivation, the underlying general logic program is most likely non-terminating. This leads to the following definition.

Definition 4.3 Let $P$ be a general logic program, $S_Q$ a finite set of queries and $R$ a depth-first, left-most control strategy. Let $d > 1$ be a depth bound. $P$ is said to be most-likely non-terminating w.r.t. $S_Q$ and $R$ if for some query $Q_i$ in $S_Q$, there is a generalized SLDNF-derivation of the form

\[ N_0 : \leftarrow Q_i \Rightarrow ... N_{g_1} : G_{g_1} \overset{C_{1_1}}{\longrightarrow} ... \overset{C_{1_{n_1}}}{\longrightarrow} N_{g_{2}} : G_{g_{2}} \overset{C_{2_1}}{\longrightarrow} ... \overset{C_{2_{n_2}}}{\longrightarrow} \ldots \overset{C_{d_1}}{\longrightarrow} ... \overset{C_{d_{n_d}}}{\longrightarrow} N_{g_{d+1}} : G_{g_{d+1}} \]

such that

1. For any $j \leq d$, $L_{g_j}^1 \prec_{ANC} L_{g_{j+1}}^1$ and $L_{g_{j+1}}^1 \supseteq_{EV} L_{g_j}^1$.
2. For any $j \leq d$ if $|L_{g_j}^1| = |L_{g_{j+1}}^1|$, or for any $j \leq d$ if $|L_{g_j}^1| < |L_{g_{j+1}}^1|$.
3. For any $j \leq d$, the set of clauses used to derive $L_{g_{j+1}}^1$ from $L_{g_j}^1$ is the same as that of deriving $L_{g_j}^1$ from $L_{g_{j+2}}^1$, i.e. $\{C_{j_1}, ..., C_{j_{n_j}}\} = \{C_{(j+1)_{1}}, ..., C_{(j+1)_{n_{j+1}}}\}$.

Theorem 4.2 Let $P$, $S_Q$ and $R$ be as defined in Definition 4.3.

1. If $P$ is not terminating w.r.t. $S_Q$ and $R$ then it is most-likely non-terminating w.r.t. $S_Q$ and $R$.
2. If $P$ is not most-likely non-terminating w.r.t. $S_Q$ and $R$ then it is terminating w.r.t. $S_Q$ and $R$.

Proof: 1. If $P$ is not terminating w.r.t. $S_Q$ and $R$, by Definition 4.2 for some query $Q_i \in S_Q$ there exists an infinite generalized SLDNF-derivation in $G_{T\rightarrow Q_i}$. The result is then immediate from Theorem 3.4.
2. If $P$ is not most-likely non-terminating w.r.t. $S_Q$ and $R$ and, on the contrary, it is not terminating w.r.t. $S_Q$ and $R$, then by the first part of this theorem we reach a contradiction.

It is easily seen that the converse of the above theorem does not hold. The following algorithm is to determine most-likely non-termination.

**Algorithm 4.1** Testing termination of a general logic program.

- **Input:** A general logic program $P$, a finite set of queries $S_Q = \{Q_1, ..., Q_m\}$, and a depth-first, left-most control strategy $R$.
- **Output:** Yes or a generalized SLDNF-derivation $D$.
- **Method:** Apply the following procedure.

  \[
  \text{procedure Test}(P, S_Q, R) \\
  \text{begin} \\
  1. \text{For each query } Q_i \in S_Q, \text{ construct the full generalized SLDNF-tree } \text{GT}_{\rightarrow Q_i} \text{ for } P \cup \{\leftarrow Q_i\} \text{ via } R \text{ unless a generalized SLDNF-derivation } D \text{ is encountered that meets the three conditions of Definition 4.3, in which case return } D \text{ and stop the procedure;} \\
  2. \text{Return Yes} \\
  \text{end}
  \]

**Theorem 4.3** Algorithm 4.1 terminates. It returns Yes if and only if $P$ is not most-likely non-terminating w.r.t. $S_Q$ and $R$.

**Proof:** If for each query $Q_i \in S_Q$ the generalized SLDNF-tree $\text{GT}_{\rightarrow Q_i}$ for $P \cup \{\leftarrow Q_i\}$ is finite, line 1 of Algorithm 4.1 will be completed in finite time, so that Algorithm 4.1 will terminate in finite time. Otherwise, let all generalized SLDNF-trees $\text{GT}_{\rightarrow Q_i}$ with $i < m$ be finite and $\text{GT}_{\rightarrow Q_{i+1}}$ be infinite. Let $D$ be the first infinite derivation in $\text{GT}_{\rightarrow Q_{i+1}}$. By Theorem 3.4, $D$ must be of the form

\[
N_0 : G_0 \Rightarrow ... N_{g_1} : G_{g_1} \xrightarrow{c_{11}} ... \xrightarrow{c_{1n_1}} N_{g_2} : G_{g_2} \xrightarrow{c_{21}} ... \xrightarrow{c_{2n_2}} N_{g_3} : G_{g_3} \xrightarrow{c_{31}} ...
\]

such that the three conditions of Theorem 3.4 hold. Obviously, such an infinite derivation will be detected at the node $N_{g_{d+1}} : G_{g_{d+1}}$, thus Algorithm 4.1 will stop here.

When Algorithm 4.1 ends with an answer Yes, all generalized SLDNF-trees for all queries in $S_Q$ must have been generated without encountering any derivation $D$ that meets the three conditions of Definition 4.3. This shows that $P$ is not most-likely non-terminating w.r.t. $S_Q$ and $R$. Conversely, if $P$ is not most-likely non-terminating w.r.t. $S_Q$ and $R$, Algorithm 4.1 will not stop at line 1. It will proceed to line 2 with an answer Yes returned.
Theorem 4.4 The following hold:

1. If Algorithm 4.1 returns Yes then \( P \) is terminating w.r.t. \( S_Q \) and \( R \).

2. If \( P \) is not terminating w.r.t. \( S_Q \) and \( R \) then Algorithm 4.1 will return a generalized SLDNF-derivation \( D \) that meets the conditions of Definition 4.3.

Proof: 1. By Theorem 4.3 Algorithm 4.1 returning Yes implies \( P \) is not most-likely non-terminating w.r.t. \( S_Q \) and \( R \). The result then follows from Theorem 4.2.

2. By Theorem 4.2, when \( P \) is not terminating w.r.t. \( S_Q \) and \( R \), it is most-likely non-terminating w.r.t. \( S_Q \) and \( R \). So there exist generalized SLDNF-derivations in some generalized SLDNF-trees \( GT_{-Q} \), that meet the three conditions of Definition 4.3. Obviously, the fist such derivation \( D \) will be captured at line 1 of Algorithm 4.1, which leads to an output \( D \).

4.3 Examples

We use the following very representative examples to illustrate the effectiveness of our method. In the sequel, we choose the smallest depth bound \( d = 2 \).

Example 4.1 Applying Algorithm 4.1 to the logic program \( P_1 \) of Example 2.1 with a query \( Q_1 = p(a) \) will return a generalized SLDNF-derivation \( D \), which is the path from \( N_0 \) to \( N_4 \) in Figure 1. \( D \) is informative enough to suggest that \( P_1 \) is not terminating w.r.t. \( Q_1 \).

Example 4.2 Applying Algorithm 4.1 to the logic program \( P_2 \) of Example 2.2 with a query \( Q_1 = p \) will return an answer Yes. That is, \( P_2 \) is terminating w.r.t. \( Q_1 \). However, for the query \( Q_2 = q \) applying Algorithm 4.1 will return the following generalized SLDNF-derivation

\[
N_0 \leftarrow q \xrightarrow{C_{q2}} N_1 \leftarrow q \xrightarrow{C_{q2}} N_2 \leftarrow q
\]

which strongly suggests that \( P_2 \) is not terminating w.r.t. \( Q_2 \).

Example 4.3 Consider the following widely used program:

\[
P_3 : \quad \text{append}([], X, X).
\]

\[
append([X|Y], U, [X|Z]) \leftarrow append(Y, U, Z).
\]

Assume the following types of queries (borrowed from [12]):

\[3\text{It represents a large class of well-known logic programs such as member, subset, merge, quick-sort, reverse, permutation, and so on.}\]
Example 4.4  The following program illustrates how a list of terms grows recursively through a DELETE-ADD-SHUFFLE process.

\[
P_4: \quad p([X_1, X_2|Y]) \leftarrow q([X_1, X_2|Y], Z), reverse(Z, [], Z_1), p(Z_1). \quad C_{p_1}
\]

\[
q([X_1, X_2|Y], [X_3, f(X_1, X_2, X_2|Y)]).
\]

\[
reverse(Z, [], Z_1) \leftarrow Z_1 \text{ is the reversed list of } Z. \quad C_{rev}
\]

Given a subgoal \( p([X_1, X_2|Y]) \), applying \( C_{p_1}, C_{q_1}, C_{rev} \) successively will
Note that the addition of $X_3$ is independent of the original list $[X_1, X_2[Y]]$, but $f(X_1, X_2)$ is generated based on the list. This means that given a query of the form $p([T_1, ..., T_m])$, a new variable $X$ and a function $f(A_1, A_2)$ will be added each cycle \{C_{p_1}, C_{q_1}, C_{rev}\} is applied, where the domain of $A_1$ and $A_2$ is the closure of the function $f(\_, \_)$ over $[X, T_1, ..., T_m]$ (up to variable renaming). As an illustration, consider an arbitrary query $Q_1 = p([a, b])$. Applying Algorithm 4.1 to $Q_1$ will return a generalized SLDNF-derivation as shown in Figure 4, where for the selected subgoals $L_{12}, L_6, L_0$ at nodes $N_{12}, N_6$ and $N_0$, we have $L_0^{\prec anc} L_6^{\prec anc} L_{12}^{\prec anc} L_1^{\prec EV} L_6^{\prec EV} L_0^{\prec EV}$, and $|L_{12}| > |L_6^{\prec EV}| > |L_0^{\prec EV}|$. We see the following terms added due to the repeated applications of $C_{p_1}, C_{q_1}, C_{rev}$:

\[
\begin{align*}
\text{From } N_0 \text{ to } N_3 & \quad X_1, f(a, b), \\
N_3 \text{ to } N_6 & \quad X_2, f(b, f(a, b)), \\
N_6 \text{ to } N_9 & \quad X_3, f(X_1, f(a, b)), \\
N_9 \text{ to } N_{12} & \quad X_4, f(X_2, f(b, f(a, b))).
\end{align*}
\]

 Apparently, the generalized SLDNF-derivation of Figure 4 can be infinitely extended. Thus $P_4$ is non-terminating w.r.t. $Q_1$ (and all queries of the form $p([T_1, ..., T_m])$).

\[
\begin{align*}
* & N_0: \quad p([a, b]) \\
& \quad C_{p_1} \\
& N_1: \quad q([a, b], Z), reverse(Z, \[], Z_1), p(Z_1) \\
& \quad C_{q_1} \\
& N_2: \quad reverse([X_1, f(a, b), \[], \[], Z_1), p(Z_1) \\
& \quad C_{rev} \\
& N_3: \quad p([b, f(a, b), X_1]) \\
& \quad C_{p_1} \\
& N_4: \quad q([b, f(a, b), X_1], Z_2), reverse(Z_2, \[], Z_3), p(Z_3) \\
& \quad C_{q_1} \\
& N_5: \quad reverse([X_2, f(b, f(a, b)), f(a, b), X_1, \[], \[], Z_3), p(Z_3) \\
& \quad C_{rev} \\
* & N_6: \quad p([X_1, f(a, b), f(b, f(a, b)), X_2]) \\
& \quad C_{p_1}, C_{q_1}, C_{rev} \\
& N_7: \quad p([X_2, f(b, f(a, b)), f(a, b), f(X_1, f(a, b)), X_3]) \\
& \quad C_{p_1}, C_{q_1}, C_{rev} \\
* & N_{12}: \quad p([X_3, f(X_1, f(a, b)), f(a, b), f(b, f(a, b)), f(X_2, f(b, f(a, b))), X_4]) \\
& \quad C_{p_1}, C_{q_1}, C_{rev}
\end{align*}
\]

Figure 4: A generalized SLDNF-derivation for $P_4 \cup \{\leftarrow Q_1\}$
Example 4.5 ([1]) Consider the following well-known game program:

\[ P_5 : \begin{align*}
\text{win}(X) & \leftarrow \text{move}(X,Y), \neg \text{win}(Y). \\
\text{move}(a,b) & \leftarrow \text{for} \ (a,b) \in \mathcal{G} \text{ where } \mathcal{G} \text{ is an acyclic finite graph.}
\end{align*} \]

Assume the following two types of queries:

\[ Q_1 = \text{win}(a), \quad Q_2 = \text{win}(X). \]

Since \( \mathcal{G} \) is an acyclic finite graph, no expanded variants occur in any generalized SLDNF-derivations. Therefore, Algorithm 4.1 will terminate for both \( Q_1 \) and \( Q_2 \) with an answer Yes. That is, \( P_5 \) is terminating w.r.t. \( \{Q_1, Q_2\} \).

Example 4.6 ([1]) The following general logic program is used to compute the transitive closure of a graph.

\[ P_6 : \begin{align*}
\text{r}(X,Y,E,V) & \leftarrow \text{member}([X,Y], E). \\
\text{r}(X,Z,E,V) & \leftarrow \text{member}([X,Y], E), \neg \text{member}(Y,V), \text{r}(Y,Z,E,[Y|V]). \\
\text{member}(X,[X|T]). \\
\text{member}(X,[Y|T]) & \leftarrow \text{member}(X,T).
\end{align*} \]

Queries over this program are of the form \( \text{r}(X,Y,e,[X]) \) where \( X, Y \) are nodes and \( e \) is a graph specified by a finite list of its edges denoted by \([\text{Node}, \text{Node}]\). Such a query is supposed to succeed when \([X,Y]\) is in the transitive closure of \( e \). The last argument of \( \text{r}(X,Y,e,[X]) \) acts as an accumulator in which a list of nodes is maintained which should not be reused when looking for a path connecting \( X \) with \( Y \) in \( e \) (to keep the search path acyclic). As an example, let \( e = \{[[a,b],[b,c],[c,a]]\} \). We consider the following three types of queries:

\[ Q_1 = \text{r}(a,c,e,[a]), \quad Q_2 = \text{r}(a,Y,e,[a]), \quad Q_3 = \text{r}(X,Y,e,[X]). \]

The generalized SLDNF-trees \( GT_{\_Q_1}, GT_{\_Q_2}, \) and \( GT_{\_Q_3} \) are depicted in Figures 5, 6 and 7, respectively. Since there is no expanded variant in any generalized SLDNF-derivations, Algorithm 4.1 will return Yes when executing \( \text{Test}(P_6, \{Q_1, Q_2, Q_3\}, R) \). That is, \( P_6 \) is terminating w.r.t. these three types of queries.

It is interesting to observe that for each of the above logic programs, \( P_1 - P_6 \), it is terminating if and only if applying Algorithm 4.1 to it yields an answer Yes. In fact, this is true for all representative logic programs we have currently collected in the literature. However, due to the undecidability of the termination problem, it is unavoidable that there exist cases in which Algorithm 4.1 returns a generalized SLDNF-derivation \( D \), but \( P \) is a terminating logic program. We have created such a rarely used program.
\[ N_0: \leftarrow r(a, c, e, [a]) \]
\[
\text{\hspace{1cm}} \rightarrow C_{r_1} \text{\hspace{1cm}} \rightarrow C_{r_2}
\]

\[ N_1: \leftarrow \text{member}(a, Y, e), \neg \text{member}(Y, [a]), r(Y, c, e, [Y, a]) \]
\[
\text{\hspace{1cm}} Y = b \text{\hspace{1cm}} \rightarrow C_{r_1} \text{\hspace{1cm}} \rightarrow C_{r_2}
\]

\[ N_2: \leftarrow \neg \text{member}(b, [a]), r(b, c, e, [b, a]) \]

\[ N_3: \leftarrow r(b, c, e, [b, a]) \]
\[
\text{\hspace{1cm}} \rightarrow C_{r_1} \text{\hspace{1cm}} \rightarrow C_{r_2}
\]

\[ N_4: \leftarrow r(c, e, [c, b, a]) \]
\[
\text{\hspace{1cm}} \rightarrow C_{r_1} \text{\hspace{1cm}} \rightarrow C_{r_2}
\]

\[ N_5: \leftarrow \text{member}(c, Y_2, e), \neg \text{member}(Y_2, [c, b, a]), r(Y_2, c, e, [Y_2, c, b, a]) \]
\[
\text{\hspace{1cm}} Y_2 = a \text{\hspace{1cm}} \rightarrow C_{r_1} \text{\hspace{1cm}} \rightarrow C_{r_2}
\]

\[ N_6: \leftarrow \neg \text{member}(a, [c, b, a]), r(a, c, e, [a, c, b, a]) \]

\[ \boxed{f} \]

Figure 5: \( GT_{_Q_1} \) for \( P_6 \cup \left\{ \leftarrow Q_1 \right\} \)

\[ N_0: \leftarrow r(a, Y, e, [a]) \]
\[
\text{\hspace{1cm}} Y = b \text{\hspace{1cm}} \rightarrow C_{r_1} \text{\hspace{1cm}} \rightarrow C_{r_2}
\]

\[ \boxed{t} \]

\[ N_1: \leftarrow \text{member}(a, Y_1, e), \neg \text{member}(Y_1, [a]), r(Y_1, Y, e, [Y_1, a]) \]
\[
\text{\hspace{1cm}} Y_1 = b \text{\hspace{1cm}} \rightarrow C_{r_1} \text{\hspace{1cm}} \rightarrow C_{r_2}
\]

\[ N_2: \leftarrow \neg \text{member}(b, [a]), r(b, Y, e, [b, a]) \]

\[ N_3: \leftarrow r(b, Y, e, [b, a]) \]
\[
\text{\hspace{1cm}} Y = c \text{\hspace{1cm}} \rightarrow C_{r_1} \text{\hspace{1cm}} \rightarrow C_{r_2}
\]

\[ \boxed{t} \]

\[ N_4: \leftarrow r(c, Y, e, [c, b, a]) \]
\[
\text{\hspace{1cm}} Y = a \text{\hspace{1cm}} \rightarrow C_{r_1} \text{\hspace{1cm}} \rightarrow C_{r_2}
\]

\[ \boxed{t} \]

\[ N_5: \leftarrow \text{member}(c, Y_3, e), \neg \text{member}(Y_3, [c, b, a]), r(Y_3, Y, e, [Y_3, c, b, a]) \]
\[
\text{\hspace{1cm}} Y_3 = a \text{\hspace{1cm}} \rightarrow C_{r_1} \text{\hspace{1cm}} \rightarrow C_{r_2}
\]

\[ N_6: \leftarrow \neg \text{member}(a, [c, b, a]), r(a, Y, e, [a, c, b, a]) \]

\[ \boxed{f} \]

Figure 6: \( GT_{_Q_2} \) for \( P_6 \cup \left\{ \leftarrow Q_2 \right\} \)
Example 4.7 Consider the following logic program and top goal, where the function \( size(Z) \) returns the number of elements in the list \( Z \) (e.g. \( size([a, b]) = 2 \)).

\[
P_7 : \quad p([X|Y], N) \leftarrow size([X|Y]) < N, p([X, X|Y], N).
\]

The generalized SLDNF-tree \( GT_{G_0} \) for \( P_7 \cup \{G_0\} \) is shown in Figure 8. It is easy to see that for any \( i \geq 0 \), the subgoal \( L^1_{2^*i} \) at \( N_{2^*i} \) is an ancestor subgoal of the subgoal \( L^1_{2^*(i+1)} \) at \( N_{2^*(i+1)} \), while \( L^1_{2^*(i+1)} \) is an expanded variant of \( L^1_{2^*i} \) with \( |L^1_{2^*(i+1)}| > |L^1_{2^*i}| \). \( P_7 \) is terminating w.r.t. the query \( p([a], 100) \). However, applying Algorithm 4.1 (with \( d = 2 \)) will return a generalized SLDNF-derivation \( D \) that is the segment between \( N_0 \) and \( N_4 \) in Figure 8. Apparently, in order for Algorithm 4.1 to return \( \text{Yes} \) the depth bound \( d \) should not be less than 100.
5 Related Work

Our work is related to both termination analysis and loop checking.

5.1 Work on Termination Analysis

Concerning termination analysis, we refer the reader to the survey of Decorte, De Schreye and Vandecasteele [12, 15] for a comprehensive bibliography.

There are two essential differences between existing termination analysis techniques and ours. The first difference is that theirs are static approaches, whereas ours is a dynamic one. Static approaches only make use of the syntactic structure of the source code of a logic program to establish some well-founded conditions/constraints that, when satisfied, yield a termination proof. Since non-termination is caused by an infinite generalized SLDNF-derivation, which contains some essential dynamic characteristics (such as expanded variants and the repeated application of some clauses) that are hard to capture in a static way, static approaches appear to be less precise than a dynamic one. For example, it is difficult to apply a static approach to prove the termination of program $P_2$ in Example 2.2 with respect to a query pattern $p$. Moreover, although some static approaches (e.g., see [13, 30, 43, 45]) are automatable, searching for an appropriate level mapping or computing some interargument relations could be very complex. For our approach, the major work is to identify expanded variants, which is easy to complete.

The second difference is that existing methods are suitable for termination analysis with respect to query patterns, whereas ours is for termination analysis with respect to concrete queries. The advantage of using query patterns is that if a logic program $P$ is shown to be terminating with respect to a query pattern $Q$, it is terminating with respect to all instances of $Q$ that could be an infinite set of concrete queries. However, if $P$ is shown to be not terminating with respect to $Q$, which usually means that $P$ is terminating with respect to some instances of $Q$ but is not with respect to the others, we cannot apply existing termination analysis methods to make such a further distinction. In contrast, our method can make termination analysis for each single concrete query posed by the user and provide explanations about how non-termination happens. This turns out to be very useful in real programming practices. Observe that in developing a software in any computer languages we always apply some typical cases (i.e. concrete parameters as inputs) to test for the correctness or termination of the underlying programs, with an assumption that if the software works well with these typical cases, it would work well with all cases of practical interests.

From the above discussion, it is easy to see that our method plays a complementary role with respect to existing termination analysis approaches (i.e. static versus dynamic and query patterns versus concrete queries).
5.2 Work on Loop Checking

Loop checking is a run-time approach towards termination. It locates nodes at which SLD-derivations step into a loop and prunes them from SLD-trees. Informally, an SLD-derivation

\[ N_0 : G_0 \Rightarrow N_1 : G_1 \Rightarrow ... \Rightarrow N_i : G_i \Rightarrow ... \Rightarrow N_k : G_k \Rightarrow ... \]
is said to step into a loop at a node \( N_k : G_k \) if there is a node \( N_i : G_i (0 \leq i < k) \) in the derivation such that \( G_i \) and \( G_k \) are sufficiently similar. Many mechanisms related to loop checking have been presented in the literature (e.g. see [3, 8, 11, 20, 27, 26, 35, 36, 37, 40, 41, 44, 48]). We mention here a few representative ones.

Bol, Apt and Klop [3] introduced the Equality check and the Subsumption check. These loop checks can detect loops of the form

\[ N_0 : G_0 \Rightarrow N_1 : G_1 \Rightarrow ... \Rightarrow N_i : G_i \Rightarrow ... \Rightarrow N_k : G_k \]

where either \( G_k \) is a variant or an instance of \( G_i \) (for the Equality check), i.e. \( G_k = G_i \theta \) under a substitution \( \theta \), or \( G_i \) is included in \( G_k \) under a substitution \( \theta \) (for the Subsumption check), i.e. \( G_k \supseteq G_i \theta \). However, they cannot handle infinite SLD-derivations of the form

\[ N_0 : p(X) \Rightarrow N_1 : p(f(X)) \Rightarrow ... \Rightarrow N_i : p(f(...f(X)...)) \Rightarrow ... \]

Sahlin [34, 35] introduced the OS-check (see also [4]). It determines infinite loops based on two parameters: a depth bound \( d \) and a size function \( size \). Informally, OS-check says that an SLD-derivation may go into an infinite loop if it generates an oversized subgoal. A subgoal \( A \) is said to be oversized if it has \( d \) ancestor subgoals in the SLD-derivation that have the same predicate symbol as \( A \) and whose size is smaller than or equal to \( A \).

Bruynooghe, De Schreye and Martens [8, 26, 27] presented a framework for partial deduction with finite unfolding that, when applied to loop checking, is very similar to OS-check. That is, it mainly relies on term sizes of (selected) subgoals and a depth bound. See [4, 26] for a detailed comparison of these works.

OS-check (similarly the method of Bruynooghe, De Schreye and Martens) is complete in the sense that it cuts all infinite loops. However, because it merely takes the number of repeated predicate symbols and the size of subgoals as its decision parameters, without referring to the informative internal structure of the subgoals, the underlying decision is fairly unreliable; i.e. many non-loop derivations may be pruned unless the depth bound \( d \) is set sufficiently large.

Using expanded variants, in [37] we proposed a series of loop checks, called VAF-checks (for Variant Atoms loop checks for logic programs with Functions). These loop checks are complete and much more reliable than OS-check. However, they cannot deal with infinite recursions through negation like that in Figure 1.

The work of the current paper can partly be viewed as an extension of [37] from identifying infinite SLD-derivations to identifying infinite generalized SLDNF-derivations. It is worth noting that termination analysis is merely concerned with the
characterization and identification of infinite derivations, but loop checking is also concerned about how to prune infinite derivations. The latter work heavily relies on the semantics of a logic program, especially when an infinite recursion through negation occurs. Bol [4] discussed loop checking for locally stratified logic programs under the perfect model semantics [33].

6 Conclusions

We have presented a method of verifying termination of general logic programs with respect to concrete queries. A necessary and sufficient condition is established and an algorithm for automatic testing is developed. Unlike existing termination analysis approaches, our method does not need to search for a model or a level mapping, nor does it need to compute an interargument relation based on additional mode or type information. Instead, it detects infinite derivations by directly evaluating the set of queries of interest. As a result, some key dynamic features of a logic program can be extracted and employed to predict its termination. Such idea partly comes from loop checking. Therefore, the work of this paper bridges termination analysis with loop checking, the two problems which have been studied separately in the past despite their close technical relation with each other.

It is worth mentioning that the practical purpose of termination analysis is to assist users to write terminating programs. Our method exactly serves for this purpose. When Algorithm 4.1 outputs Yes, the logic program is terminating; otherwise it provides users with a generalized SLDNF-derivation of the form as shown in Figures 3 or 4. Such a derivation may most likely lead to an infinite derivation, thus users can improve their programs following the informative guidance. (In this sense, our method is quite like a spelling mechanism used in a word processing system, which always indicates most likely incorrect spellings.)

Due to the undecidability of the termination problem, there exist cases in which a logic program is terminating but Algorithm 4.1 would not say Yes unless the depth bound $d$ is set sufficiently large (see Example 4.7). Although $d = 2$ works well for a vast majority of logic programs (see Examples 4.1 - 4.6), how to choose the depth bound in a general case then presents an interesting open problem.

Tabled logic programming is receiving increasing attention in the community of logic programming (e.g. see [5, 10, 38, 39, 42, 48, 49]). Verbaeten, De Schreye and K. Sagonas [47] recently exploited termination proofs for positive logic programs with tabling. For future research, we are going to extend the work of the current paper to deal with general logic programs with tabling.

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