Chiral approach to weak radiative hyperon decays and the $\Xi^0 \to \Lambda \gamma$ asymmetry

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March 27, 2022

Abstract

We reanalyse the recent version of the chiral model of weak radiative hyperon decays, proposed by Borasoy and Holstein. It is shown that predictions of the analysed model are significantly changed when one accepts the usual classification of $\Lambda(1405)$ as an $SU(3)$-singlet. In particular, for the $\Xi^0 \to \Lambda \gamma$ decay large negative asymmetry is obtained. This is contrasted with the Hara’s-theorem-violating approaches where this asymmetry is large and positive.

PACS numbers: 14.20.Jn;13.30.-a;11.30.Rd;11.30.Ly
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1 Introduction

In 1964 Hara proved a theorem [1], according to which the parity-violating amplitude of the $\Sigma^+ \to p\gamma$ decay should vanish in the limit of exact $SU(3)$ symmetry. For weak breaking of $SU(3)$, one then expects a small asymmetry in this decay. The experimental evidence accumulated over the years proves, however, that the asymmetry in question is large and negative [2, 3], $\alpha(\Sigma^+ \to p\gamma) = -0.76 \pm 0.08$. Understanding this experimental result and related data on other weak radiative hyperon decays (WRHD’s) constitutes an important issue for low-energy physics of weak interactions.

WRHD’s were studied in many approaches (for a review see ref.[3]). Generally, in most models in which Hara’s theorem is satisfied, a small value of the $\alpha(\Sigma^+ \to p\gamma)$ asymmetry is predicted (ref.[4] is an important exception here). This was in particular the case of the chiral approach in which it was found [2] that $|\alpha(\Sigma^+ \to p\gamma)| < 0.2$. Recently, Borasoy and Holstein (BH) attempted a new description of WRHD’s within the chiral framework [6]. In the BH approach, pole model contributions from low-lying excited $J^P = 1/2^+$ intermediate states were studied, in addition to the usually considered contributions from the ground-state baryons and the $1/2^-$ baryon resonances. Model parameters were determined from a fit to nonleptonic hyperon decays and used as an input for the description of WRHD’s. It was found that in the parity-conserving amplitudes, the contribution of the $1/2^+$ resonances is substantial (especially for the $\Sigma^+ \to p\gamma$ decay). Furthermore, a large negative $\Sigma^+ \to p\gamma$ asymmetry (around $-0.50$) was obtained. Although the detailed BH predictions do not fit the WRHD data well, the observation that inclusion of the $1/2^+$ resonances permits a significant contribution to the parity-conserving $\Sigma^+ \to p\gamma$ amplitude is interesting. With the inclusion of the $1/2^+$ resonances, the parity-conserving $\Sigma^+ \to p\gamma$ amplitude does not vanish in the $SU(3)$ limit. Such vanishing, occurring when only ground-state baryons are considered as intermediate states, constituted a problem for the authors of ref.[4]. In their paper, the parity-conserving $\Sigma^+ \to p\gamma$ amplitude depended on the difference
(dissappearing in the $SU(3)$-limit) of the anomalous parts of the $\Sigma^+$ and $p$ magnetic moments. Predictions published in [4] were obtained using experimental values of the relevant magnetic moments. On the other hand, if quark model formulas for $\mu_{\Sigma^+}, \mu_p$ are employed, these predictions lead to a positive sign for the $\alpha(\Sigma^+ \rightarrow p\gamma)$ asymmetry [4]. Thus, it is certainly interesting that a contribution to parity-conserving amplitudes, which does not vanish in the $SU(3)$ limit and leads to a definitely negative asymmetry, was identified in ref.[6].

Although in the BH paper the $\Sigma^+ \rightarrow p\gamma$ asymmetry is fairly large (this was also the case in ref.[4]), other BH predictions do not seem to be in good agreement with experiment. One such prediction is the branching ratio of the decay $\Sigma^+ \rightarrow p\gamma$, which is larger than data by an order of magnitude. The huge size of this branching ratio stems directly from the large value of the relevant parity-conserving amplitude. (In ref.[4] this branching ratio, although somewhat smaller than the experimental one, is still of reasonable size.) This discrepancy suggests that the size of contributions from excited $1/2^+$ states is overestimated in ref.[4]. Another problem for ref.[4] is that, in those places where the contribution from the $1/2^+$ excited states is already small, there is no agreement between the predictions of ref.[4] and the original model of ref.[4]. It seems natural that such agreement should exist because both Gavela et al. [4] as well as Borasoy and Holstein [6] intended to include all important contributions from the ground-state and excited $1/2^-$ baryons in their calculations. Thus, there must be an additional difference between the two approaches. In the present paper we analyze where this difference comes from and show its main reason. It turns out that in fact (and contrary to the statements contained therein) ref.[6] omits contributions from the low-lying $J^P = 1/2^-$ excited $SU(3)$-singlet baryon $\Lambda(1405)$. When this singlet contribution is taken into account, one reproduces more or less closely the predictions of ref.[4]. In particular, the chiral BH approach with the contribution of intermediate singlet baryon included leads to a significantly negative asymmetry in the $\Xi^0 \rightarrow \Lambda\gamma$ decay.
2 Intermediate states in parity-violating amplitudes

Prescriptions of the chiral approach of ref. [6] reduce to the familiar formulas of the pole model. Thus, what is analysed in ref. [6] is a pole model consistent with chiral symmetry conditions. It is this pole model that is ultimately compared with experiment. Clearly, predictions of any pole model depend in an essential way on the intermediate states taken into account. As the intermediate states ($B^*$ in Fig. 1) in the parity-violating amplitudes, the authors of ref. [4] consider all those $J^P = 1/2^-$ states from the $(70, 1^-)$ multiplet which have appropriate flavour quantum numbers. That is, they consider all relevant states from the two $SU(3)$ octets $^28$ and $^48$ (of quark spins $S = 1/2$ and $S = 3/2$) and a singlet $^21$ (of quark spin $S = 1/2$). (Contributions from the decuplet $^210$ vanish.) In the BH paper, on the other side, only states from the lowest-lying octet of the excited $J^P = 1/2^-$ baryons ($^28$ in theoretical models) are taken into account. While one may perhaps expect that contributions from the higher-lying octet $^48$ are not very important, neglecting the singlet is not justified as explained below.

In ref. [6] the $\Lambda(1405)$ baryon is treated as a member of the lowest-lying octet of excited $J^P = 1/2^-$ baryons. However, $\Lambda(1405)$ is usually classified as a singlet [7], while the octet $\Lambda$ is identified with $\Lambda(1670)$. A corresponding $\Sigma$ state is found at 1750 MeV. The PDG flavour assignment of the lowest-lying $J^P = 1/2^-$ states [8] is corroborated by theoretical calculations and decay analyses [8]. Isgur and Karl [9] predict a dominantly singlet state just below 1500 MeV, and a dominantly octet $\Lambda$ state at around 1650 MeV. Large spin-orbit splitting between the $\Lambda(1405)$ and the $J^P = 3/2^-$ state $\Lambda(1520)$ (also a singlet) is not reproduced in their model, though. This discrepancy between the predictions of quark model and experiment, as well as proximity of $\Lambda(1405)$ mass to the sum of $N$ and $\bar{K}$ masses are sometimes regarded as an indication that $\Lambda(1405)$ is an unstable $N\bar{K}$ bound state akin to the deuteron, and not a quark-model $SU(3)$-singlet resonance. However, as stressed by Dalitz in ref. [7].
in order to accommodate the quark-model singlet $J^P = 1/2^-$ state, another Λ baryon should then be present in the vicinity of Λ(1520). No such state has been observed experimentally. Furthermore, it turns out that inclusion of hadron-loop effects (i.e. of the coupling to the $NK$ channel in particular) splits the two $J^P = 1/2^-$ and $J^P = 3/2^-$ quark-model SU(3)-singlet Λ states in the correct way with the $J^P = 1/2^-$ state shifted down in mass. All other hadron-loop-induced shifts and mixings in the whole $(70, 1^-)$ multiplet (as well as those of ground-state baryons) are also in good agreement with the data. In particular, in the model of ref.[10] the downward shift of Λ($J^P = 1/2^-$) is substantial (though somewhat small). All this shows clearly that Λ(1405) should indeed be considered an approximate SU(3)-singlet, and not an octet. Thus, when taking into account the lowest-lying $J^P = 1/2^-$ states, in addition to the $N(1535)$, the $Σ(1750)$, and the Ξ(?), we have to include two Λ states: the dominantly singlet Λ(1405) and the dominantly octet Λ(1670).

3 Relative size of singlet and octet contributions

Clearly, SU(3) symmetry cannot predict the relative size and sign of the contributions from the singlet and octet excited Λ’s. To get this crucial information, one has to employ a broader symmetry that would put all the considered $J^P = 1/2^-$ states into a single multiplet. This is usually achieved through the use of $SU(6) \times O(3)$ symmetry of the quark model. Such an approach was employed in ref.[4]. In order to see how important the contributions neglected in the BH paper are, we must therefore study ref.[4] in more detail.

With the help of Tables 1 and 2 of ref.[4], the contributions from various intermediate states (i.e. from each of the two octets and from the singlet) may be easily reconstructed. These contributions are gathered in Table 1 here. Normalization of entries in Table 1 is such that the totals for each decay are equal to the numbers given in Table 7.2 of ref.[3] multiplied by a common
factor $2 + K$. This reflects full agreement between the $SU(6)_W \times VMD$ approach of [13, 14] (which in turn is based on ref. [15]) and the approach of ref. [4] (apart from the question of the relative sign of contributions from diagrams (b1) and (b2) in Fig. 1). The parameter $K$ is of order 1. In ref. [4] it is calculated within the framework of the harmonic oscillator constituent quark model as $K = R^2 \omega^2 = \omega/m = 500 \text{MeV}/400 \text{MeV} = 1.25$ with $R$ being baryon radius, $\omega$ - excitation frequency and $m$ - constituent quark mass.

Table 1

Weights of amplitudes corresponding to diagrams (b1) and (b2) in ref. [4].

| process     | $^{2s+1}{\bf R}_{SU(3)}$ | diagram (b1)          | diagram (b2)          |
|-------------|---------------------------|------------------------|------------------------|
| $\Sigma^+ \to p\gamma$ | $^2\bf 8$                | $-\frac{1}{3\sqrt{2}}(2 + K)$ | $-\frac{1}{3\sqrt{2}}(2 + K)$ |
|             | $^4\bf 8$                | 0                      | 0                      |
|             | total                    | $-\frac{1}{3\sqrt{2}}(2 + K)$ | $-\frac{1}{3\sqrt{2}}(2 + K)$ |
| $\Lambda \to n\gamma$ | $^2\bf 8$                | $\frac{1}{6\sqrt{3}}(2 + \frac{K}{3})$ | $\frac{1}{3\sqrt{3}}(2 + \frac{K}{3})$ |
|             | $^4\bf 8$                | $\frac{1}{9\sqrt{3}}K$ | $\frac{2}{9\sqrt{3}}K$ |
|             | $^2\bf 1$                | 0                      | 0                      |
|             | total                    | $\frac{1}{6\sqrt{3}}(2 + K)$ | $\frac{1}{2\sqrt{3}}(2 + K)$ |
| $\Xi^0 \to \Lambda\gamma$ | $^2\bf 8$                | $-\frac{1}{6\sqrt{3}}(2 + \frac{K}{3})$ | $-\frac{1}{3\sqrt{3}}(2 + \frac{K}{3})$ |
|             | $^4\bf 8$                | $-\frac{1}{9\sqrt{3}}K$ | $-\frac{2}{9\sqrt{3}}K$ |
|             | $^2\bf 1$                | $\frac{1}{6\sqrt{3}}(2 + K)$ | 0                      |
|             | total                    | 0                      | $-\frac{1}{3\sqrt{3}}(2 + K)$ |
| $\Xi^0 \to \Sigma^0\gamma$ | $^2\bf 8$                | $\frac{1}{6}(2 + \frac{K}{3})$ | 0                      |
|             | $^4\bf 8$                | $\frac{1}{9}K$ | 0                      |
|             | $^2\bf 1$                | $\frac{1}{6}(2 + K)$ | 0                      |
|             | total                    | $\frac{1}{3}(2 + K)$ | 0                      |
The weights given in Table 1 have to be multiplied by appropriate pole factors. In an idealized $SU(3)$-symmetric case, when all states of a given $SU(6) \times O(3)$ multiplet have the same mass, these factors are equal for $(b1)$ and $(b2)$ diagrams. One can then see from Table 1 that Hara’s theorem is satisfied when the parity-violating WRHD amplitudes are proportional to the differences of weights appropriate for diagrams $(b1)$ and $(b2)$. This proportionality to weight differences is indeed obtained when the relevant calculations are performed in a Hara’s-theorem-satisfying framework, as it was done in ref.[4].

Omission in ref.[6] of the contribution from the singlet intermediate state may affect asymmetries of the neutral hyperons only. Thus, only the estimates of the parity-violating amplitudes of $\Lambda \rightarrow n\gamma$, $\Xi^0 \rightarrow \Lambda\gamma$, and $\Xi^0 \rightarrow \Sigma^0\gamma$ performed in ref.[6] should be done anew. On the basis of Eqs.(18) of ref.[6] we may form a table of octet contributions arising there from diagrams $(b1)$ and $(b2)$.

**Table 2**

Weights of amplitudes corresponding to intermediate octet states in diagrams $(b1)$ and $(b2)$ in ref.[6]. Simplified formulas obtained for $\omega_f = -\omega_d$ and corresponding directly to Table 1 are given for each decay in bottom rows.

| process | parameters | diagram (b1) | diagram (b2) |
|---------|------------|--------------|--------------|
| $\Lambda \rightarrow n\gamma$ | $\omega_d$, $\omega_f$ | $\frac{1}{6\sqrt{3}}(\omega_d + 3\omega_f)$ | $\frac{1}{6\sqrt{3}}\cdot 2\omega_f$ |
| | $\omega_f = -\omega_d$ | | $\frac{1}{6\sqrt{3}}(\omega_d - 3\omega_f)$ |
| $\Xi^0 \rightarrow \Lambda\gamma$ | $\omega_d$, $\omega_f$ | $-\frac{1}{6\sqrt{3}}(\omega_d + 3\omega_f)$ | $\frac{1}{6\sqrt{3}}(\omega_d - 3\omega_f)$ |
| | $\omega_f = -\omega_d$ | $-\frac{1}{6\sqrt{3}}\cdot 2\omega_f$ | $-\frac{1}{3\sqrt{3}}\cdot 2\omega_f$ |
| $\Xi^0 \rightarrow \Sigma^0\gamma$ | $\omega_d$, $\omega_f$ | $-\frac{1}{6}(\omega_d - \omega_f)$ | $\frac{1}{6}(\omega_d + \omega_f)$ |
| | $\omega_f = -\omega_d$ | $\frac{1}{6} \cdot 2\omega_f$ | 0 |

We rewrite Eqs.(18) of ref.[6] in the form of Table 2, in notation analogous to that used in Table 1. In Table 2 the pole factors are omitted and the
normalization is adjusted so as to simplify comparison of Tables 1 and 2. (In ref.\cite{3,4,6}, conventions of relative phases between the $\Lambda \to n\gamma$, $\Xi^0 \to \Lambda\gamma$, and $\Xi^0 \to \Sigma^0\gamma$ amplitudes are identical.)

The fit used in ref.\cite{6} is characterized by $\omega_f \approx 2370 \cdot 10^{-7}$ MeV, $\omega_d \approx -1780 \cdot 10^{-7}$ MeV i.e. by $\omega_f/\omega_d \approx -1.3$. Proximity of the latter ratio to $-1$ is necessary if a reasonable description of the experimentally very small $\Xi^- \to \Sigma^-\gamma$ branching ratio is to be achieved. Indeed, in ref.\cite{3} the parity-violating amplitude of the $\Xi^- \to \Sigma^-\gamma$ is smaller than the parity-violating amplitudes for the decays of neutral hyperons by a factor of the order of $(\omega_f+\omega_d)/(m_{\Xi^\pm}-m_{\Sigma^\pm})$. The value of $\omega_f/\omega_d$ is equal to $-1$ if only $W$-exchange processes contribute: the decay $\Xi^- \to \Sigma^-\gamma$, being wholly due to a single quark transition, cannot then occur. This was the assumption made in ref.\cite{4}. Consequently, in order to compare ref.\cite{6} with the original paper \cite{1}, we have to set $\omega_f = -\omega_d$ as it is done in respective rows in Table 2. One can see that the pattern of the $^2S$ and $^4S$ contributions to the parity-violating amplitudes of neutral hyperon decays in ref.\cite{4} (Table 1) is identical to that in the BH paper for $\omega_d = -\omega_f$ with the correspondence $2 + \frac{K}{3} \leftrightarrow 2\omega_f$ for $^2S$ (or $\frac{2}{3}K \leftrightarrow 2\omega_f$ for $^4S$). In the fit in refs.\cite{3,14}, apart from the contribution of $W$-exchange processes determined (without any free parameters) from nonleptonic hyperon decays, the contribution from single quark processes responsible for $\omega_d \neq -\omega_f$ was taken into account (and described by fit parameter) as well.

Since in ref.\cite{1} $K \approx 1.25$, it follows from Table 1 that the weights of contributions from $^4S$ resonances are smaller by a factor of around 3 than those due to $^2S$. In addition, the $^4S$ resonances are heavier than the $^2S$ resonances and the corresponding pole factors should be smaller. Thus, in the first approximation one might neglect the contribution of $^4S$ states, as it was done in ref.\cite{1}. However, in ref.\cite{1} the weights of singlet contributions are (for $K$ of the order of 1) of the same size as (or somewhat larger than) those of the lowest-lying octet (Table 1). Since the singlet $\Lambda(1405)$ has the lowest mass of all the excited $1/2^-$ resonances, its contribution is important and has to be taken into account.
4 Estimates of parity-violating amplitudes and of asymmetries

Below we estimate the sign and size of the Λ(1405) contribution to the parity-violating Ξ⁰ → Λγ amplitude, relative to the sign and size of the octet contribution calculated in ref. [3]. In ref. [3] the contributions from the Λ(1670) and Ξ⁰ (?) combine to give the total parity-violating amplitude (in units of 10⁻⁷ GeV⁻¹ used in ref. [3]) as a sum of contributions from diagrams (b₁) and (b₂):

\[ B_{ΛΞ⁰} = 8\sqrt{2} \, e_r d \left( \frac{1}{m_Ξ - m_{R₁}} \frac{1}{6\sqrt{3}}(ω_d + 3ω_f) + \frac{1}{m_Λ - m_{R₂}} \frac{1}{6\sqrt{3}}(ω_d - 3ω_f) \right) \]

\[ = -0.58 + 0.50 = -0.08 \] (1)

where \( e_r d \approx 0.022 \) GeV⁻¹ and resonance masses \( m_{R₁} \approx m_{R₂} \approx 1540 \) MeV were used. The first (second) term above comes from excited \( J^P = 1/2^- \) octet Λ (Ξ⁰) respectively. In reality, both of these resonances are heavier than the value of 1540 MeV employed in [3], and one may expect their contributions to \( B_{ΛΞ⁰} \) to be somewhat smaller. The near cancellation of the two contributions in Eq.(1) occurs also if more realistic masses of the resonances are used. Thus, for \( m_{R₁} = 1670 \) MeV and \( m_{R₂} = 1830 \) MeV one gets \( B_{ΛΞ⁰} = -0.36 + 0.30 = -0.06 \).

In Table 1 the contribution from the singlet is similar in absolute size to that from the octet Λ, but of opposite sign. Thus, when the lowest-lying singlet and octet states are both taken into account, we expect that the parity-violating \( B_{ΛΞ⁰} \) amplitude should be around

\[ B_{ΛΞ⁰} \approx -0.58 + 0.50 + 0.4 \approx +0.3 \] (2)

with the third term in the sum in Eq.(2) resulting from a very rough (assuming identical pole factors) estimate of the singlet contribution: from Table 1 it should be close to +0.58 if comparison with the \( ^28 \) Λ weight (column (b₁)) is employed or around 0.50/2 = 0.25 if comparison with the \( ^28 \) Ξ⁰ weight (column (b₂)) is made. In Eq.(2) the average of these two estimates is used. Similar estimates are obtained if one first uses \( ω_f \approx -ω_d \approx 2075 \cdot 10^{-7} \) MeV (the
average of values used in \[3\]) and then estimates the singlet contribution on
the basis of Table 1. Using \(m_{\Lambda_1} = 1520\ MeV\) one then obtains for the singlet contribution
\[
\Delta B^{\Lambda_{\Xi^0}}(\Lambda_1) \approx \frac{1}{m_{\Lambda_1} - m_{\Xi}} \cdot er_d \frac{8 \sqrt{2}}{3 \sqrt{3}} \omega_f \approx +0.5
\]
instead of the value of +0.4 in Eq.(2).

I think that putting the quark-model mass for \(m_{\Lambda_1}\) (equal to \(m(\Lambda(3/2^-)) \approx 1520\ MeV\)) instead of the real mass of \(\Lambda(1405)\), is more appropriate here. Explanation of the small \(\Lambda(1405)\) mass is presumably connected with coupling to \(\bar{K}N\) and related channels. If we were to use the value of 1405 MeV for the mass of \(\Lambda_1\), we would also have to include the effects of hadron-level corrections to the size of weak and electromagnetic hadronic transition amplitudes used in the model. At present there is no model which could estimate such corrections in a reliable way.

One may also give an experiment-based argument that in WRHD’s the contribution from \(\Lambda(1405)\) should follow symmetry predictions given by the weights of Table 1, with all pole factors of approximately the same size. Namely, one may look at data on hyperon nonleptonic decays and, assuming the dominance of \((70, 1^-)\) contributions, try to learn from the data about the properties of the contribution from the singlet \(\Lambda\). It turns out (compare Table 2 in ref.\[16\]) that the excited singlet \(\Lambda\) contributes to \(\Sigma^+_\pm\) and \(\Sigma^-\) parity-violating transitions, but not at all to those of \(\Lambda\) or \(\Xi\) decays. For \(\Sigma\) decays, the weights of contributions from the singlet \((^21)\) and the lighter octet \((^28)\) were determined in ref.\[16\] to be of roughly the same size (up to a factor of 1.5 for \(\Sigma^-\), and about −0.5 for \(\Sigma^+_\pm\)). As a result, if the size of the singlet contribution relative to that of the octet were modified too much by a completely different pole factor, we should not be able to describe the \(\Sigma\) decay amplitudes with the same parameters that may be extracted from the s-wave amplitudes of \(\Lambda\) and \(\Xi\).

Indeed, using only the \(\Lambda\) and \(\Xi\) s-wave amplitudes, one can extract \(f - d \approx -2.83\), \(f + d \approx -0.91\) (in units of \(10^{-7}\)). As can be checked in Table 2 of ref.\[16\], the dominant contribution in these decays comes from the heavier \((^48)\)
intermediate states (the ratio of weights from $^4S$ and $^2S$ is 8 : 1). Assuming (for justification see below) that the contributions from the $^2S$ and $^4S$ intermediate states are given by symmetry considerations (i.e. disregarding possible difference of the size of the relevant pole factors) and denoting the contributions from both octets and the singlet by $O$ and $S$ respectively, we then have for the $\Sigma$ decays (from [16]):

$$
\Sigma^+_+ = +0.13 = \; O - S = 0
$$

$$
\Sigma^+_0 = -3.27 = 3O/\sqrt{2} = \sqrt{3/2}(f - d)
$$

$$
\Sigma^- = +4.27 = -2O - S = -\sqrt{3}(f - d)
$$

(4)

where the column of numbers represents the data, and the rightmost entries give standard expressions for the amplitudes in terms of $f$ and $d$. The choice $S = O$ corresponds then to the symmetry situation in which the contribution from the $\Lambda(1405)$ is evaluated with the pole factor identical to that used for octet contributions. Using $f - d$ determined from $\Lambda$ and $\Xi$ decays one then predicts that $\Sigma^+_0 = -3.47$, and $\Sigma^- = +4.90$, which is in fair agreement with the data. (An overall fit to $\Lambda$, $\Xi$ and $\Sigma$ decays gives slightly different values for $f$ and $d$ and describes the data a little better. Also, one has to remember that the $\Delta I = 3/2$ contributions are of the order of a few percent, thus defining what is the acceptable accuracy.)

Let us note now that the $\Sigma^+_0$ amplitude depends on intermediate octets only. In fact, it follows from Table 2 of ref. [16] that the contribution to $\Sigma^+_0$ comes entirely from the $^2S$ intermediate states. The approximate equality $-3.47 \approx -3.27$ confirms therefore that the relative size of contributions from $^2S$ and $^4S$ is properly given by symmetry considerations (as assumed above), without taking into account the difference in the size of pole factors. The agreement $+4.90 \approx +4.27$ for the $\Sigma^-_0$ amplitude indicates the same for the contribution from the singlet. Had we used the singlet contribution enhanced by (say) 30% (or 80%) due to a larger pole factor, we would have obtained $\Sigma^- = +5.39$ (or +6.21) respectively, in much worse agreement with the data. At the same time we would have obtained $\Sigma^+_0 = +0.49$ (or +1.30). Clearly, in nonleptonic hyperon
decays the singlet contribution follows the symmetry prescription. I think that this is a good hint that in WRHD’s the relative size of the contribution from the singlet $\Lambda$, relative to that from the octet $\Lambda$, should follow the pattern of weights in Table 1.

With $\Delta B^{\Xi^0}(\Lambda_1)$ as in Eq.(3), one obtains

$$B^{\Lambda \Xi^0} = -0.36 + 0.30 + 0.5 = +0.44$$

(5)
in good agreement with Eq.(2).

Using the value of the parity-conserving amplitude $A^{\Lambda \Xi^0} = -0.34$ from the BH paper, one obtains the asymmetry:

$$\alpha(\Xi^0 \to \Lambda \gamma) \approx -1.$$  \hspace{1cm} (6)

(in ref. \[6\] this asymmetry was calculated to be +0.46), and the decay rate (in units of GeV):

$$\Gamma(\Xi^0 \to \Lambda \gamma) \approx (5 \text{ to } 6) \cdot 10^{-18}$$

(7)

(2.5 $\cdot$ 10$^{-18}$ in the BH paper \[6\] and in the experiment). Let us note that in ref.\[6\] the contribution of the excited $1/2^+$ states to the $\Xi^0 \to \Lambda \gamma$ parity-conserving amplitude was found to be negligible. In reality, this contribution is probably even smaller: the size of the $\Sigma^+ \to p \gamma$ branching ratio seems to indicate that the overall size of the contribution from excited $1/2^+$ states has been overestimated in BH. Consequently, the parity-conserving $\Xi^0 \to \Lambda \gamma$ amplitude is well estimated by the ground-state contribution.

The present analysis of the BH approach, with the $SU(3)$-singlet baryon contribution taken into account, shows that in Hara’s-theorem-satisfying chiral framework the $\Xi^0 \to \Lambda \gamma$ asymmetry is large and negative, in complete agreement with previous studies \[3, 4\]. Note also that if the weights ($b_1$) and ($b_2$) are added (as predicted by calculations in Hara’s-theorem-violating approaches \[4\]), one gets

$$B^{\Lambda \Xi^0} = -0.58 - 0.50 + 0.40 \approx -0.7$$

(8)

(alternatively: $-0.36 - 0.30 + 0.5 = -0.16$) leading to a large and positive $\Xi^0 \to \Lambda \gamma$ asymmetry (around +0.8).
For the $\Lambda \rightarrow n\gamma$ decay, Table 1 shows that in Hara’s-theorem-satisfying approach of BH one obtains $B^{n\Lambda} \approx 0.30 - 0.35 - 0.25 \approx -0.3$ (the first two figures in this sum come from ref. [6], the third one, i.e. $-0.25 \approx -0.3 \approx -0.35/2$ is a rough estimate of the singlet contribution, obtained in a way analogous to that for $\Xi^0 \rightarrow \Lambda\gamma$). Since in ref. [6] the parity-conserving amplitude $A^{n\Lambda}$ is positive and equal to $+0.52$ (with contribution from excited $1/2^+$ states amounting to 25% only), one concludes that the asymmetry should be large and negative. Thus, when the singlet is taken into account, the near-zero negative asymmetry of BH remains negative but becomes much larger. On the other hand, if amplitudes corresponding to diagrams (b1) and (b2) are added (as in Hara’s-theorem-violating cases), one gets $B^{n\Lambda} \approx 0.30 + 0.35 + 0.25 \approx 0.9$ and a large positive asymmetry should be observed.

For the $\Xi^0 \rightarrow \Sigma^0\gamma$ decay, Table 1 shows that with singlet contribution included, the parity-violating amplitude will be of approximately twice the value given in ref. [6], i.e. $B^{\Sigma^0\Xi^0} \approx +1.4$. Within the BH approach, this leads to a very small positive asymmetry (around $+0.07$). Should the contribution of the excited $1/2^+$ states be smaller than in ref. [6], the parity-conserving amplitude and the resulting asymmetry would become negative, irrespectively of whether the excited $J^P = 1/2^-$ singlet is or is not taken into account. This prediction of negative $\Xi^0 \rightarrow \Sigma^0\gamma$ asymmetry agrees nicely with the recent experiment, according to which $\alpha(\Xi^0 \rightarrow \Sigma^0\gamma) = -0.65 \pm 0.13$ [17, 18], and is another indication that the contribution of the excited $1/2^+$ resonances is overestimated in BH. Because for $\Xi^0 \rightarrow \Sigma^0\gamma$ there is almost no contribution from diagram (b2) (in the original BH paper this contribution is negligible, while in ref. [4] it is zero), the total $\Xi^0 \rightarrow \Sigma^0\gamma$ amplitude for Hara’s-theorem-satisfying case is almost the same as for Hara’s-theorem-violating case. Consequently, measurement of the asymmetry of the $\Xi^0 \rightarrow \Sigma^0\gamma$ decay alone does not provide useful information on the question of the violation of Hara’s theorem.
5 Summary

In this paper we have analysed an extended version of the chiral model of WRHD’s discussed recently by Borasoy and Holstein [6]. In this version the contribution from the intermediate singlet baryon has been properly taken into account. The analysis of the signs of the $\Xi^0 \to \Lambda \gamma$, $\Xi^0 \to \Sigma^0 \gamma$, and $\Lambda \to n \gamma$ asymmetries is then in complete accord with the discussion given previously in [3]. Of course, one has to remember that predictions of ref. [6] are based on a fit to nonleptonic hyperon decays obtained without taking into account the usual SU(3)-singlet classification of $\Lambda(1405)$. Consequently, the original fit should in principle be redone with the singlet included, and only then WRHD’s should be considered. It might seem that the discussion of the present paper would be meaningful only provided such an improved fit had been done first. Fortunately, this is not the case. Experimental smallness of the $\Xi^- \to \Sigma^- \gamma$ branching ratio proves that $\omega_f \approx -\omega_d$, irrespectively of model details. Our analysis is based on this assumption (which is also approximately satisfied in BH), and on the relative smallness of contributions from excited $1/2^+$ states in $\Xi^0 \to \Lambda \gamma$ and $\Lambda \to n \gamma$ decays.

In summary, in the chiral approach of Borasoy and Holstein, asymmetries of the $\Xi^0 \to \Lambda \gamma$ and $\Lambda \to n \gamma$ decays are both large and negative when the $SU(3)$ singlet assignment of $\Lambda(1405)$ is taken into account. This should be contrasted with Hara’s-theorem-violating approaches in which these asymmetries are large and positive.

6 Acknowledgments

I would like to thank B. Holstein for a discussion concerning the issue of (un?)assailability of fundamental assumptions needed to prove Hara’s theorem.
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Fig. 1. Baryon-pole diagrams for WRHD’s