TWO-PERIOD PRICING AND ORDERING DECISIONS OF
PERISHABLE PRODUCTS WITH A LEARNING PERIOD FOR
DEMAND DISRUPTION

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ABSTRACT. In this paper, we develop a two-period inventory model of perishable products with considering the random demand disruption. Faced with the random demand disruption, the firm has two order opportunities: the initial order at the beginning of selling season (i.e., Period 1) is intended to learn the real information of the disrupted demand. When the information of disruption is realized, the firm places the second order, and also decides how many unsold units should be carried into the rest of selling season (i.e., Period 2). The firm may offer two products of different perceived quality in Period 2, and therefore it must trade-off between the quantity of carry-over units and the quantity of young units when the carry-over units cannibalize the sales of young units. Meanwhile, there is both price competition and substitutability between young and old units. We find that the quantity of young units ordered in Period 2 decreases with the quality of units ordered in Period 1, while the pricing of young units is independent of the quality level of old units. However, both the surplus inventory level and the pricing of old units monotonically increase with their quality. We also investigate the influence of two demand disruption scenarios on the optimal order quantity and the optimal pricing when considering different quality situations. We find that in the continuous random disruption scenario, the information value of disruption to the firm is only related to the disruption mean, while in the discrete random disruption scenario, it is related to both unit purchase cost of young units and the disruption levels.

1. Introduction. In most inventory management decisions, it is assumed that the product value does not deteriorate over time. However, for some types of inventories, such as baked goods, milk, fresh fruits, meat products, vegetables, pharmaceutical and healthcare products, the impact of perishability cannot be ignored, and these stocks may be partially or completely unsuitable for consumption as time goes on. The main characteristics of perishable products are that their value deteriorates significantly over time. According to the website statistics, China Food and Grocery

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retail market to reach US$3.63 trillion by 2018. In 2019, the total market value of grocery retailing came to approximately 193.6 billion British pounds (GBP) in the United Kingdom (www.giiresearch.com/report). Therefore, they have become an increasingly important source of income for the Grocery industry. In the actual business operation, the demand of consumers for perishable products is highly uncertain, which can lead to the situation of supply shortage or oversupply. It is observed that in 2014 over 38 million tons of wasted food were discarded in the United States, while according to the report from U.S. Department of Agriculture, approximately 13% of American households had difficulty in providing enough food for all their members in 2016 due to a lack of resources. On the basis of relevant research, there were billions of dollars’ worth of food expiration and waste per month ([39]). Therefore, it is essential to provide an effective pricing and inventory decisions for firms selling perishable products.

From the view of perishable product management, there usually exists the substitution among the products of different quality. In order to ensure the continuousness of providing product service, the firms need to replenish their inventory before the units are sold out. In our paper, we assume the firm has two order opportunities in the whole selling season, and the perceived quality of perishable products will decline if the new batch of products goes on sale in the same time. For example, the supermarkets usually cut its price of pre-ordered fresh products when the products in new order arrive. Besides the initial order quantity at the beginning of selling season, the firm also has a reorder opportunity to refill the inventory. When the second order arrives, the products ordered at the beginning of the selling season will age and become old, while the perceived quality of old units will not reach the value of zero. We therefore assume that the old units also have their sales market, competing with the young units in pricing. The decline in perceived quality does ensure that customers assess the old units less highly than the young ones. In actual situation, when the perceived quality of old units is lower than that of young units, those old units tend to be marked down. Therefore, firms must differentiate between the old and the new in terms of the retail price.

With considering the coexistence of products with different quality, dynamic pricing plays an important role in the firm’s revenue management since the price is one of the most effective variables that decision makers can control to stimulate market demand ([25]). In our paper, we consider the differential pricing for the products of different perceived quality. Differential pricing, as a way of dynamic pricing, is mainly based on the diversity of market demand and the difference of consumers’ recognition of product value at different times, so as to set the product or service at different price levels. There are many examples of such products with differential pricing between the old and the young, and our study applies to the following two categories of perishable products. In the first category, the degraded functions of the product will deteriorate over time, such as milk and agricultural products. We find that supermarkets like Walmart display products of different ages, and the prices of products that are about to expire are lower than those of fresh products. The second includes products with non-degraded functions, but the customers’ perceived utility of the products will decline over time. For example, fashion products such as fashion clothing at a discount at the end of a season. In this work, we focus on the pricing and inventory decisions of perishable products, and explore how the different perceived quality levels affect decisions and profit.
Additionally, our paper also investigates how the demand disruption affects the optimal decisions and the total profit. Market demand is often disrupted by events such as supply disruption, raw material markup, or policy changes. In the information age, supply chain management often faces the risk of demand disruption. Demand disruption, which occurs when the market demand is suddenly hit by unexpected events such as supply disruption, raw material markup or policy changes, is a critical issue in business and management, and has been for a long time. Recent real-world examples of demand disruption include a significant drop in demand for poultry meats as a result of the sudden outbreak of bird flu (H7N9) in China in 2013. The direct economic consequences of demand disruption, listed by the world economic forum (2012) as one of the greatest risks faced by the global economy, include loss of efficiency, competitive disadvantage and loss of revenue ([47]). Nowadays, demand disruption can also be caused by the technological innovation and process changes that are taking place in the digital economy based on big data, which is leading to greater and more frequent demand disruption. Meanwhile, the randomness of demand disruption may bring a greater challenge to actual supply chain operation ([31]). To overcome this challenge, sophisticated optimization strategies and planning measures are required.

These disruptions have led to adjustments in original production and inventory plans. For example, on the one hand, the outbreak of avian flu can lead to a sudden decreased demand for poultry, and on the other hand, it will also result in the sudden increased demand for respirators and disinfectors. Meanwhile, in agricultural-related industries, planting decisions are often made without the ability to accurately predict weather conditions ([22]). In this paper, for the scenario of the demand disruption, we define the operation without disruption as the normal operation and the operation with disruption as the irregular operation. In the irregular operation, the sudden change of market demand or other factors will cause certain extra deviation costs for the decision-makers ([43]). After introducing deviation costs, it would be very valuable for us to analyze the robustness of original order plan.

However, the information on demand disruption is often uncertain. To obtain the real information on demand disruption, firms usually adopts the way of two-period sales. Faced with the uncertain information on disruption, the firm tends to have a tentative selling period to learn the possible real information by deciding the initial order quantity and retail price. Thus, this selling period is regarded as the learning period in our paper. When the uncertain information of demand disruption is realized, the firm determines the second order quantity of young units. The units leftover from the first period can be carried into the second period but, the quality of leftover units declines and become a partial substitute for new units. From the market analysis of a single type of product, the customer’s demands in different selling periods are usually highly correlated. Research shows that for most of the brands to enter the market, the uncertainty of market scale can be determined by a 6-week pre-sale of the preliminary market test ([2]). With the realization of market potential by the first selling period, the firm needs to decide how many units ordered at the beginning of selling season should be carried into the rest of selling season and how many young units should be reordered. For the irregular operations, two distribution functions on the disrupted demand are considered, i.e., continuous or a two-point discrete random distribution function. We find that the value of demand disruption to the firm is only related to the disruption mean for the case of the
continuous random disruption amount, while it is related to both unit purchase cost of young units and the disruption levels for the case of the discrete random disruption amount.

2. Literature review. Our study is mainly related to four research streams: the perishable inventory management, dynamic pricing, information update, and disruption management. Here, we provide a review of prominent research for each stream and position our study at the point of their intersection.

2.1. Perishable inventory management. Perishable inventory management always focuses on multi-indicator performance to improve overall system performance, rather than using approximate methods to optimize the total cost and profit of individual departments (see, [24, 44, 56]). Therefore, it is essential to establish an effective management system that can optimize the cost. In the existing literature, perishable products usually have one or more periods of shelf life. On the one hand, the surplus stock left over the previous period will be fully recycled ([36]), eliminated ([11]) or salvage at a lower profit ([38]). Therefore, there may be no price differentiation among products. On the other hand, the surplus stock left over from the previous period can be carried into the next period ([30, 33]). For this case, the competition between the products of different perceived quality should be considered, and it is necessary to differentiate the new and old units in pricing to meet the needs of price-sensitive demand ([49]). Facing with the randomness of demand, firms dealing with the perishable can use the following approaches managing their inventory, e.g., safety stocks for overcoming possible out-of-stock situations ([7]), reinforcement learning on age information ([4]), or implementing price discrimination ([3]). To control effectively waste risks of ordering a large number of perishable products, Mallidis et al. [14] propose a novel inventory scheme that determines the number of donations based on the relationship between the net inventory quantity and the donation threshold. Furthermore, increasing displayed stock level that is ending-stock level in a supermarket can induce consumers to purchase more due to its visibility, variety, and seasonality ([26, 27, 36]).

Unlike the above literature, our paper considers a special two-period inventory model, which is characterized as follows: the information of demand is uncertain at the beginning of selling season (i.e., Period 1) and there is the price competition between the products of different quality in the rest of selling season (i.e., Period 2). The surplus inventory left over from Period 1 can be carried into Period 2 with mark-down price. For our model, we pay attention to not only the optimal initial order quantity of Period 1 (or the learning period), but also the effect of perceived quality of old products on the price and order decisions.

2.2. Dynamic pricing. In order to maintain the continuity of services, products of different ages will always appear on the shelf at the same time. Dynamic pricing is a common practice for selling perishable products with different age groups. For a review of the literature on dynamic pricing, we refer the readers to [45]. For inventory management of perishable products, the impact of product freshness on customer purchase decisions needs to be considered when considering product pricing. Pricing policies that ignore declining freshness (quality) can also hurt the firms dealing in perishable products ([41]). Therefore, it is also necessary to maximize utilization rate of different-age products while ensuring the sustainability of the inventory. To obtain a high profit, the price policies, such as price reductions
and discounts, can be used by the manager to improve the competitiveness of old products ([26, 40]). In practice, differentiated pricing for products of different ages (quality) can effectively stimulate consumer purchase behavior ([54]). In addition, inventory strategies and pricing policies usually interact in perishable inventory management. For example, Li, Lim, and Rodrigues [52] explore the joint decisions of dynamic pricing and the inventory for perishable products with two-period shelf life. Further, Li and Teng [36] consider an infinite time horizon dynamic pricing and inventory management for a perishable product, and study how reference prices affects firm’s optimal pricing and inventory strategies. Meanwhile, the joint effects of dynamic pricing and freshness are also investigated with considering menu cost ([16]), partial myopia /forward-looking behavior ([28]), variable non-instantaneous deterioration ([32]), strategic consumers ([12]), or psychic stock effect ([8]).

Based on the existing literature, we assume that different-age products are priced separately according to their inventory time, thus forming price competition and demand competition between old and new units. In this paper, dynamic pricing of perishable product is adopted to stimulate demand and thus improve the firm’s profit, which is rarely considered in the studies above.

2.3. Information update. Since the 1950s, information updates have been extensively studied in supply chain management ([13]). The existing literature related to information updates can be mainly divided into two categories: demand information update and supply information update ([6]). Uncertainty from supply side or demand side seriously undermines supply chain performance ([20, 29]). If the information on the sale or supply is updated in time, the performance of firm or supply chain can be significantly improved ([15]). Our research is closely related with demand information update. Usually, demand information update refers to retailers using information to modify previous demand or forecast information to the next sales period, so as to make more accurate demand forecasts. The existing literature on demand information update mainly uses (i) Bayesian update methods ([34]) or (ii) predictive evolution methods in time series ([50]) to explore the value of continuous updates. For perishable products with short-term demand patterns and long-term service targets, the retailer determines its long-term orders in the first stage and revises the order according to a short-term forecast update at the second stage ([1]). Furthermore, when trading, retailers can also use historical demand information to update estimates of unknown parameters in order to make an optimal plan ([18]). In addition, information updates are also used in the booming order delivery market. Based on customer feedback, the online ordering platform can update the average food quality and waiting time in time to make an optimal strategy ([55]).

In our paper, we assume that the whole selling season can be divided into two periods, which are similar to the assumption used in [17, 53]. However, unlike their researches, we assume that the uncertain information on the market scale potential can be obtained from the sale of the first period, and then the period is regarded as a learning period. With the information update, the firm places a reorder plan and provides the differential retail prices for different-quality units in the second period. We will focus on the effects of information update obtained from the learning period on the order and pricing decisions.

2.4. Disruption management. The literature on disruption management can be roughly divided into (i) supply disruption, (ii) demand disruption, and (iii) both
supply and demand disruptions, while our paper mainly focuses on demand disruption. Generally speaking, changes in original production or order plan caused by demand disruptions may result in considerable deviation costs to the rectification system ([48]). Under the circumstance of the demand disruption, the disruption amount can greatly affect decisions and profits of supply chain members ([21]).

In terms of asymmetric demand information on the supply side or the retail side, Chen et al. [23] establish a game model of two competing supply chains, and discuss the design of information disclosure mechanism and its impact on the system. For the decisions of supply chain management, the optimal production volume has the characteristics of robustness due to the introduction of deviation costs ([37]). By analyzing the combination impacts of demand and production cost disruption on optimal pricing and production decisions, it has been found that it is always beneficial for central decision makers to understand the exact information of the disruption ([5]). From the perspective of decision-making robustness, the decision change in the supplier alliance decision mode is smaller than that in the decentralized decision mode when demand disruption occurs frequently ([46]). For the literature on the supply disruption, one can refer to [23] and [15]. In addition, there also exists a lot of literature on the disruptions from both supply side and demand side (e.g., [10, 51]).

The existing disruption models mainly focus on the supply chain inventory management, and have following characteristics: (i) they are developed in the static single-period environment; (ii) they do not consider the changes of product’s perceived quality as the sale process goes. Therefore, different from the current studies, our paper studies a two-period dynamic pricing and inventory decisions of perishable products with a learning period for the uncertain demand disruption. In addition, we also investigate how the perceived quality of products affects the firm’s optimal decisions and profit.

2.5. The distinctiveness of our research. Compared with the existing literature, our model is innovative in the following aspects. First, our model considers two-period perishable inventory management, in which Period 1 is characterized by uncertainty of information, while Period 2 is characterized by pricing competition between old and young units. Second, unlike the existing literature on the inventory management, we explore the joint dynamic pricing policy and inventory strategy under the uncertain disrupted demand, which can be realized after a learning period. Third, we employ the bi-level programming approach to obtain the optimal solutions for different optimization problems. From the pricing perspective, the higher the perceived quality of old inventory left over from Period 1, the lower the order quantity of young units; meanwhile, the retail price of new units is independent of the perceived quality of old units. From the ordering perspective, we find that (i) if the perceived quality is sufficiently low, the firm will sell out the units before placing the reorder. (ii) The perceived quality is sufficiently high, the firm will only carry the old inventory to Period 2 without reordering. (iii) The perceived quality is medium, the firm will not only carry old inventory, but also reorder young units for the sale of Period 2. Finally, we explore the effects of demand disruption on the inventory decisions and profit, and offer several new insights to the related literature ([30]). For example, there are robust regions for the firm’s order plan when considering the demand disruption. We also find that in the case of continuous random disruption amount, the information value of demand disruption to the firm is only
related to the disruption levels, while in the case of discrete random disruption, it is related to both the unit purchase cost of young units and the disruption levels.

3. Problem description. We develop a two-period dynamic pricing and inventory model for perishable products with the random demand disruption. At the beginning of the selling season, the firm needs to determine the initial order quantity and retail price before knowing the complete characterization of disrupted demand. When mastering the real demand disruption, the firm also has a second opportunity to refill the inventory to satisfy the demand. Here, we give some assumptions on both product features and demand market as follows. First, we assume that the perceived quality of the previously ordered products decreases when the newly ordered products are available for sale at the same time. In fact, it is very quite common practice for supermarkets and large retail stores to mark down older products after restocking. Therefore, the perishable products have the following characteristics: (i) the perceived quality of products ordered in the same sales period is undifferentiated and sold at the same retail price. (ii) The products ordered in different sales periods have different perceived quality, and the perceived quality of the products not sold in the previous period is lower than that of products ordered in the following period. To gain intuition, the perishable product in our paper can be classified into two types: 'young' if it is sold in the same period it was ordered or 'old' if it is sold in the period after it was ordered. With the arrival of new units, the perceived quality of old units declines, thus offering lower valuations for customers. When old and young units coexist in the same period, the price competition between young and old units will be considered due to their difference of perceived quality from customers.

Second, we assume that the disrupted demand market has the following characteristics: (i) Before learning the characteristics of the demand disruption, the firm tends to make tentative sales at the beginning of the selling season, exploring the disrupted demand market by deciding the initial order quantity or retail price of perishable product. Thus, the tentative sale stage can be regarded as the learning period (i.e., Period 1 in this paper). Namely, the firm will place a new order when the inventory on hand drops down to the reorder point. The surplus inventory at the end of the learning period will be carried into the rest of the selling season (i.e., Period 2). (ii) With the tentative sale, the firm can master the real information on demand disruption, and then decides the reorder quantity for Period 2 based on the leftover units (old inventory) from Period 1. (iii) There may be retail price competition between the old and young units in Period 2. To simplify the analysis, we assume that the lead-time of reorder plan is zero, which coincides with the perishable inventory literature. From the perspective of decision-making, the firm needs to determine the initial order quantity in Period 1, the quantity of old units to be carried into Period 2, the reorder quantity at the beginning of Period 2, and their retail prices. Our inventory plan and pricing strategy should ensure that the inventory of the young and old units reaches the optimum in Period 2, thus obtaining the optimum two-period profit.

When considering the retail price competition in Period 2, we employ the similar assumptions used by [30], [35] and [42]. That is, to distinguish old units from the units of the new order in term of retail price, we introduce the customer’s perceived quality $q$, and the retail prices of young and old units in Period 2 are presented as
follows, respectively.

\[ P^o_2 = R - x_2 - qE \quad \text{and} \quad P^n_2 = q(R - x_2 - E), \]

(1)

where \( x_2 \) represents the reorder quantity of young units and \( E \) represents the ending-stock level at the end of the learning period (or the old inventory level). The value of \( E \) also implies that the quantity of old units carried from Period 1 to Period 2. \( q \) represents the perceived quality of old units when young units go on sale. It can be seen that the perceived quality of old units has an important impact on the pricing of both old and young units. From Eq. (1), we find that the higher the perceived quality of old units, the higher the retail price of old units will be. On the contrary, with the increases of the perceived quality of old units, the retail price of young units will decrease accordingly. That is, the higher the perceived quality of old units, the lower the retail price of the young unit and the stronger substitution of the old unit will be. Therefore, the retail price of young units will decrease accordingly with the perceived quality of old units.

Since there are only young units in Period 1, the unit retail price of products is therefore given as \( P_1 = R - x_1 \), where \( x_1 \) represents the quantity of the young units ordered at the beginning of Period 1.

For convenience, the notations and functions used in paper are presented as Table 1.

### Table 1. Notations used in the stochastic model

| Parameters | Description |
|------------|-------------|
| \( R \)    | Market potential in Period 1, \( R > c \) |
| \( \Delta R \) | Random disrupted amount in Period 1, i.e., continuous or discrete random variable |
| \( \bar{\Delta}R \) | Mean of random disrupted amount \( \Delta R \) |
| \( \bar{R} \) | Determined market potential in Period 2, \( \bar{R} > c \) |
| \( h \) | Unit cost to carry old products |
| \( q \) | Perceived quality of old units when the young units go on sale, \( 0 \leq q \leq 1 \) |
| \( c \) | Unit cost to purchase young products |
| \( c_u \) | Unit penalty cost for a unit increased quantity, \( c_u \geq 0 \) |
| \( c_s \) | Unit penalty cost for a unit decreased quantity, \( c_s \geq 0 \) |
| \( P_1 \) | Retail price of young products in Period 1 |
| \( P^n_2 \) | Retail price of young products in Period 2 |
| \( P^o_2 \) | Retail price of old products in Period 2 |

| Decision Variables | Description |
|-------------------|-------------|
| \( x_i \) | Order quantity of young products in Period \( i \), \( i = 1, 2 \) |
| \( E \) | Reorder point, or ending-stock level at the end of Period 1, where \( E \geq 0 \) |

| Functions | Description |
|-----------|-------------|
| \( \Pi_2(x_2, E) \) | Profit of Period 2 given the surplus inventory \( E \) and the reorder quantity \( x_2 \) |
| \( \Pi_{12}(x_1) \) | Expected profit of both periods given the initial order quantity \( x_1 \) |

For the whole selling season, the firm is assumed to have two order opportunities to replenish its inventory. The decisions on the initial order quantity and the retail price at the beginning of selling season can be used to learn the uncertain information of disrupted demand. After the tentative sales, the firm needs to place
the second order and retail price decisions. We will examine the optimal two-period order and dynamic pricing decisions of perishable products, in which Period 1 is characterized by the uncertainty in the disrupted demand and Period 2 is characterized by the potential pricing competition between the young units reordered at the beginning of Period 2 and the surplus old units left over from Period 1. Meanwhile, we will also explore the optimal order or pricing decisions for both periods and the optimal reorder point under a regular operation and two irregular operations.

To be specific, our paper focuses on the following issues.

(i) In Period 1, we explore how the possibility of carrying unsold inventory to Period 2 affects the firm’s order decision in Period 1 when it only orders young units.

(ii) We examine how the perceived quality of surplus units at the end of Period 1 affects the firm’s order and pricing decisions, which can be captured by the optimization problem of Period 2. The firm makes the following decisions on both new and old units: (a) how many leftover units should be carried to Period 2; (b) how many new units should be reordered for Period 2; (c) how to price them.

We assume initial order quantity is $x_1$ and the reorder quantity in the second order opportunity is $x_2$. Therefore, the available quantity at the beginning of Period 2 will increase to $x_2 + E$. Please see Fig. 1 for illustration.

Based on the above sequence of events, we start from the analysis of Period 2 and solve it by backward induction due to the characteristics of bi-level programming approach.

4. Inventory decisions in Period 2. According to the price elasticity of demand of the learning period, the firm can master the real information on the demand scale by the tentative sale. Here, we use $R_i$ to represent the realized demand scale after disruption. When the inventory in Period 1 falls to the reorder point $E$, the second order for young units will be placed by the firm. Thus, both the surplus units left over from Period 1 and the young units are sold in Period 2 simultaneously. In this period, the firm needs to determine how many old inventory left over from
Period 1 should be carried into Period (reorder point \(E\)) and the reorder quantity \(x_2\). Therefore, the firm’s profit function in Period 2 is as follows:

\[
\Pi_2(x_2^*, E^*) = \max_{x_2 \geq 0, E \geq 0} \left( P^n_2 - c \right) x_2 + \left( P^o_2 - h \right) E,
\]

where \(P^n_2 = R - x_2 - qE\) and \(P^o_2 = q(R - x_2 - E)\).

The first item in Eq. (2) is the profit from selling young units with the retail price \(P^n_2\), and the second item is the profit from selling old units with the retail price \(P^o_2\).

**Lemma 1.** The objective function \(\Pi_2(x_2^*, E^*)\) is a jointly concave function with respect to \(x_2\) and \(E\).

**Proof.** Proofs of all lemmas, propositions and corollaries are given in Appendix.

Lemma 1 indicates that there is a unique optimal point \((x_2^*, E^*)\) to maximize the profit of Period 2. Namely, the firm determines the optimal quantity of old inventory \(E^*\) that is carried into Period 2 and the reorder quantity of young units \(x_2^*\). Then, according to Lemma 1, we can obtain the optimal solution for Eq. (2) with the Kuhn-Tucker conditions, and the optimal decisions are presented in the following proposition.

**Proposition 1.** The optimal decisions and the corresponding profit of Period 2 are presented in Table 2.

| Cases | \(x_2^*\) | \(E^*\) | \(P^n_2^*\) | \(P^o_2^*\) | \(\Pi_2(x_2^*, E^*)\) |
|-------|--------|--------|--------|--------|----------------|
| \(q \leq \frac{h}{c}\) | \(\frac{R-c}{2}\) | 0 | \(\frac{P^n_2}{2}\) | \(NA\) | \(\frac{(R-c)^2}{4q}\) |
| \(\frac{h}{c} < q < \frac{R-c+h}{R}\) | \(\frac{(1-q)R-c+h}{2(1-q)}\) | \(\frac{2qh-q^2-h^2}{4}\) | \(\frac{P^n_2}{2}\) | \(\frac{P^o_2}{2}\) | \(\frac{R^2-2Rc}{4q} + \frac{2qh-q^2-h^2}{4}\) |
| \(q \geq \frac{R-c+h}{R}\) | 0 | \(NA\) | \(\frac{R-h}{2q}\) | \(\frac{P^n_2}{2}\) | \(\frac{(qR-h)^2}{4q}\) |

From Proposition 1, we can derive the following insights:

(i) When \(q \leq \frac{h}{c}\) and \(q \geq \frac{(R-c+h)}{R}\), the firm only sells the products with the same quality level. That is, only young units are sold when \(q \leq \frac{h}{c}\), and only old units are sold when \(q \geq \frac{(R-c+h)}{R}\);

(ii) When \(\frac{h}{c} < q < \frac{(R-c+h)}{R}\), the firm sells the products with two different quality levels at the same time, and there is retail price competition between young and old products.

From the perspective of customer’s purchasing behavior, the lower the perceived quality level of old inventory, the lower the purchase value for the buyer will be. Therefore, the products with the low perceived quality level will not be carried into Period 2. For this case, the units ordered at the beginning of Period 1 will be sold out and only the young units will be sold in Period 2. When the perceived quality level of old units left over from Period 1 is sufficiently high, the customer still has the strong purchasing willing for the old units due to their favorable price. Therefore, the firm will not reorder young units at the end of Period 1 and only sell old units left over from Period 1. The benefit to the firm is greater than selling a young unit. Thus, in this case, the firm has a higher incentive to carry and prepare a higher level initial inventory rather than to mid-cycle reorder plan. When the perceived
quality level of leftover old units is medium, the firm needs to reorder some young units based on the surplus old inventory left over from Period 1. In this case, we can find that there is internal pricing competition between the old and the young.

In addition, From Proposition 1, we can also obtain the following results.

**Corollary 1.** Considering the different quality levels, we find that

(i) The optimal pricing of young units is always independent of the old inventory’s perceived quality level;

(ii) The optimal pricing of old units is always increasing with their perceived quality level.

Part (i) of Corollary 1 illustrates that the optimal retail price for young units is always remained as a fixed constant as \((\overline{R} + c)/2\) even if there exists the pricing competition from old units. In fact, the customer’s value perception for young units does not depend on the perceived quality level of old units. As Part (ii) states, the optimal retail price for old units always has a fixed increasing function with their quality level, i.e., \((q\overline{R} + h)/2q\), regardless of the retail competition from young units. Therefore, the perceived quality of old units on its pricing validity is clear and definite. Since the consumer’s perceived value of old units is positively related to their quality level. That is, the higher the quality level, the higher the perceived value will be. Therefore, consumers are willing to pay a higher price for a higher quality level.

**Corollary 2.** Considering the price competition of young and old units, i.e., \(h/c < q < (\overline{R} - c + h)/\overline{R}\), the reorder quantity of young units in Period 2 decreases with the perceived quality of old units.

It is natural that the higher the perceived quality level of old units, the higher the substitutability between the old and young units will be. That is, when the perceived quality level of the surplus units left over from Period 1 is sufficiently high, the firm will cut down the reorder quantity in Period 2.

**Corollary 3.** The optimal ending-stock level \((E^*)\) is continuous and monotonously increasing with the quality level.

Corollary 3 implies some interesting insights on the relationship between the product quality and the time length of the learning period. Since the time length of tentative selling period is determined by the value of \(E^*\), we can find that the higher perceived quality level of the surplus products, the shorter the time length of the learning period will be. Therefore, Corollary 3 implies that the disrupted demand scale can be anticipated to be clear and definite in short time when the products have the higher quality.

5. **Inventory decisions of Period 1.** Now, we move back to Period 1. To benchmark model for the disruption case, we first present a regular inventory model that the market demand is not disrupted, which is presented in Subsection 5.1. Second, for the case of demand disruption, we consider the following cases according to the characteristics of demand disruption. Namely, the amount of demand disruption may have a continuous random distribution function (Subsection 5.2) or have a two-point discrete random distribution function (Subsection 5.3).
5.1. Benchmark model: Regular inventory decisions. When there is no demand disruption in Period 1, the firm’s expected profit of both periods is written as follows:

\[
\Pi_{12}(x_1^*) = \max_{x_1 > E} P_1(x_1 - E^*) - c x_1 + \rho \Pi_2^*(x_2^*, E^*),
\]

where \( P_1 = R - x_1 \).

The first item of Eq. (3) represents the revenue from the sales in Period 1, the second item represents the replenishment cost, and the third item represents the discounted profit of Period 2. We can derive the following results by solving Eq. (3).

**Proposition 2.** Considering the different product quality levels, the optimal decisions and profits of Period 1 based on the perceived quality of old inventory are given in Table 3.

**Table 3.** Optimal decisions and profits of Period 1 based on the perceived quality of old inventory.

| \( q \) | \( E^* \) | \( x_1^* \) | \( P_1^* \) | \( \Pi_{12}(x_1^*) \) |
|-------|-------|-------|-------|-------|
| \( q \leq \frac{h}{c} \) | \( 0 \) | \( \frac{R-c}{2} \) | \( \frac{R+c}{2} \) | \( \frac{(1+\rho)(R-c)^2}{4} \) |
| \( \frac{h}{c} < q < \frac{R-c+E^*}{R} \) | \( \frac{cq-h}{2q(1-q)} \) | \( \frac{R-c+E^*}{2} \) | \( \frac{R+c-E^*}{2} \) | \( \frac{(R-c-E^*)^2}{4} - c E^* + \rho \Pi_2^*(E^*) \) |
| \( q \geq \frac{R-c+E^*}{R} \) | \( \frac{QR-h}{2q} \) | \( \frac{R-c+E^*}{2} \) | \( \frac{R+c-E^*}{2} \) | \( \frac{(R-c-E^*)^2}{4} - c E^* + \rho \Pi_2^*(E^*) \) |

From Proposition 2, we find that when the reorder point equals zero (i.e., there is no leftover inventory at the end of Period 1 will be carried into Period 2), the optimal pricing and inventory decisions in Period 2 are the same as those in Period 1. Therefore, the whole selling season can be divided into two independent and irrelevant selling periods. When the reorder point satisfies \( E^* > 0 \), the optimal pricing and inventory decisions of young units in Period 1 are closely related to the reorder point decision. Therefore, it is natural that the existence of the reorder decision will affect the optimal decision of Period 1. According to Proposition 2, the optimal decisions and profit based on the different perceived quality levels can be written in the same forms as \( x_1^* = (R - c + E^*)/2 \), \( P_1^* = (R + c - E^*)/2 \), and \( \Pi_{12}(x_1^*) = (R - c - E^*)^2/4 - c E^* + \rho \Pi_2^*(x_2^*, E^*) \). For the validity of our model, we assume that the market scale satisfies \( R > 2c - h \) to ensure a non-negative order quantity (i.e., \( x_1^* > 0 \)). Otherwise, there is no need to satisfy the market demand, since it is not profitable to produce.

5.2. Model I: Continuous random disruption amount (\( \Delta R \)). In the actual operation, the demand is often disrupted unexpectedly by the haphazard event, weather reasons, the impact of new technology, etc. With the unpredicted changes, the firm needs to adjust the original strategies correspondingly, such as the price and the inventory. In the following, we will study the influence of uncertain market disruption on the pricing and inventory decisions and the profit of the firm. When the demand disruption occurs, we assume that the market scale \( R \) is changed into \( R = R + \Delta R \), where \( \Delta R > 0(\Delta R < 0) \) implies that the market scale increases (decreases). The changed demand can result in the change of the retail price (thus the change of order quantity) of the firm. In addition, the information on the disruption amount is more likely to be uncertain to the firm in practice. We will find
the optimal solution of the disruption model with the disruption amount satisfying a general probability distribution.

Similar to the existing studies ([43, 48]), we assume that $c_u$ represents a unit penalty cost for a unit increased quantity, while $c_s$ represents a unit penalty cost for a unit decreased quantity. Then the firm’s optimal expected profit, denoted as $E[\tilde{\Pi}_{12}^*]$, can be obtained from the following optimization problem.

$$
\begin{align*}
E[\tilde{\Pi}_{12}^*] &= \max_{\tilde{x}_1} E[\tilde{\Pi}_{12}^*(\tilde{x}_1)], \\
&= E[(R + \Delta R - \tilde{x}_1)(\tilde{x}_1 - E^*) - c\tilde{x}_1 - c_u(\tilde{x}_1 - x_1^*)\
&\quad - c_s(x_1^* - \tilde{x}_1)^* + \rho E[x_1^* - \tilde{x}_1](x_2^*, E^*)].
\end{align*}
$$

(4)

Let $(\tilde{x}_1, \tilde{P}_1)$ be the optimal solution for Eq. (4), and the optimal order quantity $(\tilde{x}_1)$ has the following property.

**Lemma 2.** The optimal initial order quantity in Period 1 satisfies

$$
\tilde{x}_1 \begin{cases} 
  \geq x_1^*, & \text{if } \Delta R \geq 0 \\
  \leq x_1^*, & \text{if } \Delta R \leq 0
\end{cases}
$$

Lemma 2 presents that the firm’s ordering plan closely depends on the disruption mean of market scale. When the disruption mean is greater than zero, the firm will increase the initial order quantity for young units, and vice versa. According to Lemma 2, we can get the optimal results.

**Proposition 3.** The optimal order quantity and pricing for the young product in Period 1 are as follows:

$$
\tilde{x}_1^* = \begin{cases} 
  x_1^* + (\Delta R - c_u)/2, & \text{if } \Delta R \geq c_u \\
  x_1^*, & \text{if } -c_s < \Delta R < c_u \\
  x_1^* + (\Delta R + c_s)/2, & \text{if } 2c - R - h \leq \Delta R \leq -c_s
\end{cases}
$$

and

$$
\tilde{P}_1^* = \begin{cases} 
  P_1^* + (\Delta R + c_u)/2, & \text{if } \Delta R \geq c_u \\
  P_1^* + \Delta R, & \text{if } -c_s < \Delta R < c_u \\
  P_1^* + (\Delta R - c_s)/2, & \text{if } 2c - R - h \leq \Delta R \leq -c_s
\end{cases}
$$

Demand disruption can change the optimal sales quantity and optimal retail price strategies according to the different demand disruption levels. From Proposition 3, we have the following insights.

(i) When the demand increases greatly, i.e., $\Delta R \geq c_u$, the firm should increase the initial order quantity by $(\Delta R - c_u)/2$. When the demand is reduced greatly, i.e., $2c - R - h \leq \Delta R \leq -c_s$, the firm should reduce the initial order quantity by $(\Delta R + c_s)/2$ accordingly.

(ii) When the demand disruption is small, i.e., $-c_s < \Delta R < c_u$, the firm will not change the initial ordering plan. Similar to the literature on disruption management, we refer to the corresponding range of the changed demand as a robust region, which shows robustness in the optimal output for the firm’s production plan. The reason for robustness lies in the basic characteristics of deviation penalties. When the scale of the disruption amount of demand is beyond the region, the optimal output should be adjusted.

(iii) The firm needs to adjust the retail price according to the disruption amount of the demand. For example, with the changed demand, the firm should adjust
the unit retail price, and increase the unit retail price when the demand increases ($\Delta R \geq 0$), or lower the unit retail price when the demand decreases ($\Delta R \leq 0$).

According to Proposition 3, the expected profit of both periods will be written as follows.

$$
\mathbb{E}[\Pi_{12}] = \begin{cases}
\frac{(R + \Delta R - c - E^*)^2 - c^2}{2} / 4 - cE^* + \rho \Pi_2^*(x^*_2, E^*), & \text{if } \Delta R \geq c_u \\
\frac{(R + \Delta R - c - E^*)^2 - \Delta R^2}{4} / 4 - cE^* + \rho \Pi_2^*(x^*_2, E^*), & \text{if } -c_u < \Delta R < c_u \\
\frac{(R + \Delta R - c - E^*)^2 - c^2}{4} / 4 - cE^* + \rho \Pi_2^*(x^*_2, E^*), & \text{if } 2c - R - h \leq \Delta R \leq -c_u.
\end{cases}
$$

(5)

According to Eq. (5), we can derive the following result.

**Proposition 4.** The expected profit for both periods increases monotonically with the disruption mean $\Delta R$.

From Proposition 4, we can find that as the demand increases, the increase of the firm’s expected profit mainly comes from two aspects: (i) the increase of the sale quantity, (ii) the increase of the unit revenue with the increase of the retail price. For example, for the cases of $\Delta R \geq c_u$ and $\Delta R \leq -c_u$, the firm will benefit from the increase of demand disruption amount with the increase of both order quantity and retail price. However, when the demand disruption amount is small, i.e., $-c_u < \Delta R < c_u$, the firm just only raises the retail price of products, but the initial order quantity is unchanged. Therefore, as the increase of demand, the firm will benefit from the markup of products without expanding sale for the case $-c_u < \Delta R < c_u$. Naturally, $\Delta R \geq 0 (\Delta R \leq 0)$ implies that the market scale increases (decreases), which will result in higher (less) revenue for the firm, i.e., $\mathbb{E}[\Pi_{12}] \geq \Pi_{12}^* (\mathbb{E}[\Pi_{12}]^* \leq \Pi_{12}^*)$.

Here, we define the value of demand disruption as $\Delta^G \Pi = \mathbb{E}[\Pi_{12}^*] - \Pi_{12}^*$, which can be presented as follows.

$$
\Delta^G \Pi = \mathbb{E}[\Pi_{12}^*] - \Pi_{12}^* = \begin{cases}
\frac{(2\Delta R(R - c - E^*) + \Delta R^2 - c^2)}{4}, & \text{if } \Delta R \geq c_u \\
\frac{\Delta R(R - c - E^*)}{2}, & \text{if } -c_u < \Delta R < c_u \\
\frac{(2\Delta R(R - c - E^*) + \Delta R^2 - c^2)}{4}, & \text{if } 2c - R - h \leq \Delta R \leq -c_u.
\end{cases}
$$

(6)

From Eq. (6), we can derive the following results.

**Proposition 5.** (i) When $\Delta R \geq 0$, $\Delta^G \Pi \geq 0$ and $\Delta^G \Pi$ decreases monotonically with $q$ and $c$, respectively; (ii) When $\Delta R \leq 0$, $\Delta^G \Pi \leq 0$ and $\Delta^G \Pi$ increases monotonically with $q$ and $c$, respectively.

Part (i) of Proposition 5 indicates that when the expected market scale increases, the value of demand disruption is positive. From Corollary 3, we know that the reorder point increases with the perceived quality. Then, as the increase of the perceived quality, and the effect of disruption information on the profit becomes less, which will result in a less value of demand disruption. On the other hand, with the increase of unit purchase cost, firms will appropriately reduce the order quantity of young units in order to reduce the total cost. However, the reorder point is non-decreasing with the unit purchase cost, therefore, the quantity of young products sold in the learning period will be reduced, and then the value of demand disruption will become less. Part (ii) of Proposition 5 indicates that when the expected market scale decreases, the profit under demand disruption will always be lower than the under complete information. But as the perceived quality and unit purchase
cost of the product increase, the difference between them will gradually narrow. Therefore, combining Parts (i) and (ii), we can find that \(|\Delta^G|\) decreases monotonically with the perceived quality \((q)\) and the unit purchase cost \((c)\), respectively.

5.3. Model II: Discrete random disruption amount \((\Delta R)\). Here, we proceed with the second irregular operation: A two-point discrete random disruption amount. This uncertainty on the demand disruption can also be represented in terms of the firm’s uncertainty over which probability distribution characterizes two disrupted demand types, high or low disruption. We assume that the changed amount of demand scale, \(\Delta R\), is a random variable representing demand uncertainty and is given as follows.

\[
\Delta R = \begin{cases} 
\Delta R_H, & \text{with probability } \beta_H; \\
\Delta R_L, & \text{with probability } \beta_L,
\end{cases}
\]  

where \(\Delta R_H\) and \(\Delta R_L\) correspond respectively to the high and low disruption amounts of market scale with \(\Delta R_H \geq \Delta R_L\) and \(\beta_H + \beta_L = 1\). \(\beta_H\) can be interpreted as the fraction of high-disruption demand in the market and \(\beta_H \in [0, 1]\). The assumption of only two disruption types is a simplification of reality, but it is sufficient to capture the main feature of market uncertainty in our problem. In addition, we denote the disruption type as \(D\) throughout the paper, where \(D = H, L\). The two-type disrupted demand distribution is often employed in much of the supply chain literature ([9]).

Similar to Subsection 5.2, the optimal expected profit of the firm, denoted as \(E[\Pi^B_{12}]\), can be obtained from the following optimization problem.

\[
\begin{cases} 
\max_{x_{1D} > E^*} E[\Pi^B_{12}(x_{1D})], \\
\text{where } E[\Pi^B_{12}(x_{1D})] = \sum_{D=H,L} \beta_D[(R + \Delta R_D - x_{1D})(x_{1D} - E^*) - c x_{1D}]
\end{cases}
\]

Let \((\bar{x}_{1D}, \bar{P}_{1D})\) be the optimal solution for Eq. (8), the optimal order quantity under the two-point probability distribution of demand disruption has the following property.

**Lemma 3.** The optimal initial order quantity in Period 1 satisfies

\[
\bar{x}_{1D} \begin{cases} 
\geq x^*_1, & \text{if } \Delta R_D \geq 0 \\
\leq x^*_1, & \text{if } \Delta R_D \leq 0
\end{cases}
, \quad \text{where } D = H \text{ or } L.
\]

For each type of demand disruption, Lemma 3 also illustrates that when the market scale disruption is greater than zero, the market demand will increase, and then the firm will increase the initial order quantity, and vice versa. According to Lemma 3, we can get the optimal solution of Eq. (8).

**Proposition 6.** The optimal decision for young units in Period 1 is as follows:

(i) If \(\Delta R_D \geq c_a\), we have \(\bar{x}_{1D} = x^*_1 + (\Delta R_D - c_a)/2\), where \(D = H, L\);

(ii) If \(\frac{\Delta R_H \geq c_a}{\Delta R_L < c_a} \quad \text{and } \frac{\Delta R_L \geq c_a}{\Delta R_H < c_a}\), we have \(\bar{x}_{1H} = x^*_1 + (\Delta R_H - c_a)/2\) and \(\bar{x}_{1L} = x^*_1 + (\Delta R_L - c_a)/2\); and \(\bar{P}_{1H} = P^*_1 + (\Delta R_H + c_a)/2\) and \(\bar{P}_{1L} = P^*_1 + (\Delta R_L + c_a)/2\);

(iii) If \(\Delta R_H \geq c_a\), \(\Delta R_L \leq c_a\), we have \(\bar{x}_{1H} = x^*_1 + (\Delta R_H - c_a)/2\) and \(\bar{x}_{1L} = x^*_1 + (\Delta R_L - c_a)/2\);

(iv) If \(-c_a < \Delta R_D < c_a\), we have \(\bar{x}_{1D} = x^*_1\), \(\bar{P}_{1D} = P^*_1 + \Delta R_D\), where \(D = H, L\).
(v) If \( -c_s < \Delta R_H < c_u \) \( \Delta R_L \leq -c_s \), we have \[ \{ \tilde{x}_1^* = x_1^* \}
[ \{ \tilde{P}_{1H} = P_{1H} + \Delta R_H \} \text{ and } \{ \tilde{x}_1^* = x_1^* + (\Delta R_D + c_s)/2 \} \], \{ \tilde{P}_{1L} = P_{1L} + (\Delta R_D - c_s)/2 \} ;
\]

(vi) If \( \Delta R_D \leq -c_s \), we have \[ \{ \tilde{x}_1^* = x_1^* + (\Delta R_D + c_s)/2 \} \]
[ \{ \tilde{P}_{1D} = P_{1D} + (\Delta R_D - c_s)/2 \} , where \( D = H, L \).]

Proposition 6 illustrates how the firm adjusts the initial order and pricing plans according to both the disruption type and the disruption levels. To better understand relationship between the optimal decisions and the corresponding disruption regions, we plot the following Fig.2, which describes the optimal solution region for each disruption case presented in Proposition 6.

![Figure 2](image-url.png)

**Figure 2.** The optimal solution regions on the disrupted demand, where \( M \) is sufficiently large number

Proposition 6 has the following insights for the disruption amount satisfying the two-point probability distribution:

1. When the demand increases greatly, i.e., \( \Delta R_D \geq c_u \), the firm should increase not only the extra order quantity by \((\Delta R_D - c_u)/2\), but also the retail price by \((\Delta R_D + c_u)/2\), where \( D = H, L \); \( D = H \) for Zones (i), (ii) and (iii), and \( D = L \) for Zone (i).

2. When the demand decreases greatly, i.e., \( \Delta R_D \leq -c_s \), the firm should lower not only the order quantity by \(|(\Delta R_D + c_s)/2|\), but also the retail price by \(|(\Delta R_D - c_s)/2|\), where \( D = H, L \); \( D = L \) for Zones (iii), (v) and (vi), and \( D = H \) for Zone (vi).

3. The change of the demand can result in the change of the firm’s retail price. However, when the demand change is small \((-c_s < \Delta R_D < c_u)\), i.e., robust region, the firm’s optimal order plan will be unchanged. To be specific, the optimal order quantity will be unchanged for the case of high disruption type when \(-c_s < \Delta R_H < c_u \) (Zones (iv) and (v)), i.e., the firm’s initial order plan has robustness for the high-disruption demand. When \(-c_s < \Delta R_L < c_u \) (Zones (ii) and (iv)), the firm’s initial order quantity will be unchanged for the low-disruption demand. In addition, the initial order plan has robustness for the both disruption types in Zone (iv).

4. According to the disruption amount of demand, the firm should raise the retail price when the demand increases \((\Delta R_D \geq 0)\) and lower the retail price when the demand decreases \((\Delta R_D < 0)\).
Corollary 4. The optimal expected profit, $E[\Pi_i^{B*}]$, increases with probability $\beta_H$ or decreases with probability $\beta_L$.

The higher the probability of high disruption implies the higher demand scale. Therefore, as the increase of the probability of high disruption, the firm can obtain more expected profit by changing the initial order plan or retail price. The optimal decision of the firm in Period 1 is only related to the disruption type and independent of the probability of the type of demand disruption. However the expected profit of both periods is related to the type of disruption and its probability of occurrence.

Similarly, with the discrete random disruption amount ($\Delta R$), we define the value of demand disruption as $\Delta B^*\Pi = E[\Pi_i^{B*}] - \Pi_i^{H_L}$. Similar to Proposition 5, we have the following results.

Proposition 7. The relationship between profit under complete information and expected profit under incomplete information is as follows:

(i) If $\Delta R_D \geq c_u$, we always have $\Delta B^*\Pi \geq 0$, where $D = H, L$;

(ii) If $\Delta R_H \geq c_u$ and $\Delta R_L \leq c_u$, we have $\Delta B^*\Pi \geq 0$, where $D = H, L$;

(iii) If $\Delta R_H \geq c_u$ and $\Delta R_L \leq c_u$, we have $\Delta B^*\Pi \geq 0$, where $D = H, L$;

(iv) If $-c_s \leq \Delta R_D < c_u$, we have $\Delta B^*\Pi \geq 0$, where $D = H, L$;

(v) If $-c_s \leq \Delta R_D < c_u$, we have $\Delta B^*\Pi \geq 0$, where $D = H, L$;

(vi) If $\Delta R_D \leq c_u$, we have $\Delta B^*\Pi \geq 0$, where $D = H, L$.

From Part (i) of Proposition 7, we find that the total profit with demand disruption is always larger than the total profit without demand disruption, regardless of the unit purchase cost ($c$). As for Part (iv), $\Delta R \geq 0(\Delta R \leq 0)$ implies that the market scale increases (decreases), which will result in higher (less) profit for the firm, i.e., $\Delta B^*\Pi \geq 0(\Delta B^*\Pi \leq 0)$. This is similar to the continuous random disruption model. However, from Parts (ii), (iii), (v) and (vi), we can find that value with discrete random disruption amount ($\Delta B^*\Pi$) will be always positive if the disruption mean of demand is higher than the certain disruption threshold $D_i$, where $i = 1, 2, 3, 4$. That is, it will be better for the firm to master the information on demand disruption if the disruption mean is less than $D_i$. However, when the disruption mean is less than the disruption threshold $D_i$, the value of demand disruption may be negative if the unit purchase cost is higher than a certain cost threshold $c_i$ and $i = 1, 2, 3, 4$. That is, the less unit purchase cost can induce the firm to master disruption information even when the disruption amount mean is sufficiently low. In addition, for the discrete random disruption amount ($\Delta R$), both
cost thresholds $c_i$ and disruption thresholds $D_i$ depend on the disruption levels of two disruption types.

The following Corollary 5 gives some comparisons between these two disruption models in a special case with $\beta = 0$ or $\beta = 1$, i.e., only the low (high) disruption case occurs.

**Corollary 5.** (i) When $\Delta R = \Delta R_L$ with $\beta = 0$ (or $\Delta R = \Delta R_H$ with $\beta = 1$), we have $E[\tilde{\Pi}_{12}^G] = E[\tilde{\Pi}_{12}^B]$;
(ii) When $\Delta R > \Delta R_L$ with $\beta = 0$ (or $\Delta R < \Delta R_H$ with $\beta = 1$), we have $E[\tilde{\Pi}_{12}^G] > E[\tilde{\Pi}_{12}^B]$ (or $E[\tilde{\Pi}_{12}^G] < E[\tilde{\Pi}_{12}^B]$).

From Corollary 5, it can be found that two random disruption models can result in the same expected profit when $\Delta R = \Delta R_L$ with $\beta = 0$ (or $\Delta R = \Delta R_H$ with $\beta = 1$). Meanwhile, when $\Delta R > \Delta R_L$ and $\beta = 0$, the firm obtains more expected profit in the continuous disruption case than in the discrete disruption case. Similarly, the firm obtains less expected profit in the continuous disruption case when $\Delta R < \Delta R_H$ and $\beta = 1$. However, for a more general analysis discussion, it is very difficult to compare the expected profits under these two disruption cases due to complexity. In the following, we will further compare and analyze the continuous disruption model and the discrete disruption model through numerical experiments.

6. **Numerical analysis.** In this section, we will further investigate the effects of different disruption types ($\Delta R_H$ and $\Delta R_L$) on the optimal decisions of the order quantity and pricing in Period 1 and the expected profit of both periods through numerical experiments. According to the requirements of parameters in the model, we assume the default values of parameters as follows: $R = 65$, $c = 20$, $h = 5$, $c_u = 6$, $c_s = 4$, $\beta = 0.4$, $\rho = 0.9$. Let $\delta = \Delta R_H - \Delta R_L$, and $\delta = 2$.

The above data are simulating the behaviors of the industry practice, and consistent with the assumptions made in our paper. Our numerical examples are intended to provide more insights and suggestions for the corresponding industrial operation. We can find that the main insights and suggestions obtained from the numerical examples will not be changed if the default values of the above parameters are changed. This simulation method of handling the numerical data is an economical way and used in many researches on the optimization of supply chain management.

First, we present numerical examples to explore the relationship between the order and pricing decisions under different disruption types.

From Fig.3 and Fig.4, when the demand disruption amount is sufficiently large, we find that the increase of low-disruption demand will obviously result in the increase of order quantity in each information type, while it will also promote the firm to raise the unit retail prices. When the disruption amount is sufficiently small, the original order plan will be independent of the disruption amount. However, the retail price of unit product is still going up in this case because the market scale increases. Since the optimal decisions of the firm will be affected by the change of demand to a certain degree, the retail price in the robust scale is the steepest. That is, the firm takes a vigorous price leverage to prevent from deviating the planned order quantity. When the demand disruption is sufficiently large, both the optimal retail price and the optimal order quantity are strictly increasing with the disrupted market scale even if there are the deviation costs. Additionally, the retail price under the high-disruption type is always higher than that under the low-disruption type due to the increased market scale.
Fig. 5 and Fig. 6 illustrate how both low-disruption amount ($\Delta R_L$) and high-low disruption difference ($\delta$) affect the optimal order quantities under the high-disruption type ($x_{1H}$) and under the low-disruption type ($x_{1L}$), respectively. With different low-disruption amounts ($\Delta R_L$), we find that the order quantity $x_{1H}$ mainly shows three cases of change trend with the increase of the high-low disruption difference ($\delta$). Here, we consider the following three representative cases. For example, when $\Delta R_L = -10$, the change of order quantity is divided into three intervals and the robustness appears in the middle interval. When $\Delta R_L = 0$, the change of order quantity is divided into two intervals, and the robustness appears in the first interval. However, when $\Delta R_L = 10$, the order quantity of young units is without robustness and presents monotonously increasing with the same growth rate. Similarly, we find that under different high-low disruption difference ($\delta$), the order quantity also presents three cases with the increase of low-disruption amount. The reason is that the change of order quantity mainly depends on the change range.
of disruption amount. From Fig.6, when the low-disruption amount is fixed as a constant, the order quantity under the low disruption will not change even if the high-low disruption difference ($\delta$) changes. In addition, we also find that with the increase of low disruption amount, the change trend of order quantity under the low-disruption type is similar to that under the high-disruption type, which mainly depends on the size of the low-disruption amount.

From Fig.7 and Fig.8, we find that the retail price of young units under high-disruption type also has three kinds of change trends, which depend on the low disruption amount ($\Delta R_L$) as well as the high-low disruption difference ($\delta$). The same disruption intervals in Fig.5 and Fig.6 are also considered for the study on the retail prices for the high and low disruption types. It indicates that the firm will adjust the two decision variables simultaneously according to the different market disruption levels. From Fig.8, we find that the retail price under the low disruption
Figure 7. Optimal pricing, $\tilde{P}_{1H}$ versus disruption amounts

Figure 8. Optimal pricing, $\tilde{P}_{1L}$ versus disruption amounts

type depends on the low disruption amount, and is independent of the high disruption type. Unlike the order decision, the retail price decision is always changing with the amount of disruption. That is, as long as the market scale changes, the firm will definitely adjust the retail price of the product. Comparing Fig.5 with Fig.7, we find that the three intervals of adjusting the retail price decision and the three intervals of adjusting the order decision coincide exactly according to the different disruption amount. When both disruption amounts are sufficiently large, the change rates of order quantities under both low and high disruption types are same as the change rates of retail prices. However, the retail prices will become steeper in the robust scale (i.e., both disruption amounts are sufficiently small). That is, with the unchanged order plan, the firm should respond to the market disruption by adjusting the retail price dramatically.

Obviously, from Fig.9, we find that the profit will increase with the low disruption amount ($\Delta R_L$), since the increase of demand disruption amount result in the
increase of the market scale. Similarly, when the low disruption amount remains unchanged, the total market scale increases with the high-low disruption difference ($\delta$). The increase of either $\Delta R_L$ or $\delta$ is essentially an increase in market scale. Therefore, when the market scale increases, the firm can satisfy the market demand by increasing simultaneously both the order quantity and the retail price of the young units or increasing only the retail price in the robust scale, thus making the firm more profitable. In fact, we can find that with the increasing market scale, the curve of the expected profit will become steeper. That is, when the market demand is relatively large, both the price leverage and order quantity leverage play a more active and effective role in the expected profit of both periods.

Finally, we give the numerical comparison of expected profits under two random disruption models, as shown in Figs.10 and 11 below.

As shown in Figs.10 and 11, we consider the three cases in the continuous random model, i.e., $\overline{\Delta R} = -5$, $\overline{\Delta R} = 0$, and $\overline{\Delta R} = +5$ (see Fig.10), and three cases in the
discrete random model, i.e., $\beta = 0$, $\beta = 0.5$ and $\beta = 1$ (see Fig. 11). We can find that the average value of high-disruption amount and low-disruption amount in the discrete random model equals to zero when $\beta = 0.5$. However, the expected profit under the continuous random disruption with the mean $\Delta R = 0$ is still different with that under the discrete random disruption with $\beta = 0.5$. In fact, in this case, the expected profit of the discrete model is higher slightly than that of the continuous model, but overall it is almost the same. Besides, we find that the expected profit of the continuous model increases with the expected value of the disruption amount (Fig. 10), and the expected profit of the discrete model increases with the probability of high disruption (Fig. 11). This can be considered as an increase in the scale of demand leading to an increase in expected profit. In addition, from Figs. 10 and 11, when the perceived quality of the old units is small, the expected profits under two models are independent of the perceived quality. The reason is that the reorder point is zero in the case of low quality, that is, no old product is carried to Period 2. Therefore, the expected profit is also independent of the perceived quality. When the perceived quality is medium, the expected profit for the two periods decreases monotonically with respect to quality. In Period 2, the firm sells new and old units at the same time. That is, the higher the perceived quality of old units, that is, the higher the substitutability will be. Therefore, the firm will reduce the reorder quantity of new units, which will reduce the expected profit. However, when the perceived quality is high, the firm only sells old units in Period 2. The quantity and selling price of old units monotonously increase with the perceived quality. Therefore the expected profit in both periods is also monotonous increase.

7. Conclusion. In this paper, we study the order quantity and dynamic pricing decisions of perishable products, and the perceived quality of products deteriorates when the new ordered units go on sale. We extend the traditional newsvendor model by considering two order opportunities in the whole selling season, i.e., an initial order plan in a learning period for uncertain information, and the reorder plan when the information is realized. For the initial order plan, the firm makes pricing and quantity decisions under uncertainty over the disrupted market scale. For the reorder plan, the firm needs to decide how many of the unsold units to
carry over to the next period. Meanwhile, the firm also decides how many new units should be reordered for the sale of the next period. Then, the firm may offer the products of different quality in the second period. Therefore, when the carry-over unit cannibalizes the sales of new units, the firm must balance the quantity of carry-over units provided to the market with the quantity of young units.

We find that firms will adjust their inventory decisions according to the perceived quality of products as follows. (i) When the perceived quality is low, the firm will sell all the products in the first period, and will not carry the surplus stock to the second period; (ii) when the perceived quality is medium, the firm will not only carry the surplus stock to the second period, but also reorder some young units. (iii) When the perceived quality is high, the firm only carries the surplus stock to the second period, and does not reorder. We find that when there is price competition between the old and young units, the quality of old units only affects the reorder quantity of young units, and does not affect the pricing of young units. The quality of old units has an impact on their own inventory and pricing, which is consistent with our basic model. Furthermore, comparing the regular operation and two kinds of disruption operations, we explore the effects of demand disruption on the optimal decisions such as the order quantity and pricing. We find that when the disruption amount of market scale is high, the firm responds to market disruption by adjusting the pricing and order quantity of young units simultaneously; when the disruption amount is small, there is the robustness for the original order plan. By comparing irregular cases with the regular case, we find that with the increase of perceived quality or unit purchase cost, the length of the learning period becomes shorter, and then the effect of disruption information on profit becomes smaller. We find that the information value of demand disruption to the firm is only related to the disruption mean in the continuous random disruption scenario, while it is related to both unit purchase cost of the young units and the disruption levels in the discrete random disruption scenario.

It’s meaningful and interesting to creatively combine perceived quality of perishable products with the demand disruption management, and there are several directions that our research could continue. For example, we can apply this combination to a simple one-retailer-one-supplier supply chain to study the impact of perceived quality and demand disruption on decisions and profits of channel members, and design information disclosure mechanisms when the retailer has the private information on the disrupted amount of market demand (see, [19]). Second, we can also consider that the unit purchase cost of the learning period is lower than that of the sales period, so as to study the impact of different purchase costs on the ordering decisions of the firm. Finally, we will also consider the advertising costs of introducing new products and the exploration of cost-sharing contracts for supply chain members.

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Appendix.

Proof of Lemma 1. For the function $\Pi_2(x_2, E) = (R - x_2 - qE - c)x_2 + (q(R - x_2 - E) - h)$, we can derive that the second-order partial derivatives satisfy $\partial^2 \Pi_2(x_2, E)/\partial x_2^2 = -2 < 0$, $\partial^2 \Pi_2(x_2, E)/\partial E^2 = -2q < 0$, and $\partial^2 \Pi_2(x_2, E)/\partial x_2 \partial E = -2q < 0$. Thus the Hessian matrix $H = \begin{pmatrix} -2 & -2q \\ -2q & -2q \end{pmatrix}$ is negative definite. $\Pi_2(x_2, E)$ is a joint concave function with respect to $x_2$ and $E$, respectively.

Proof of Proposition 1. Using K-T conditions to solve
\[
\max \quad \Pi_2(x_2, E) = (P_2^o - c)x_2 + (P_2^o - h)E,
\]
s.t. $x_2 \geq 0, E \geq 0$.
and the Lagrangian function is $L(x_2, E, \lambda, \mu) = (R - x_2 - qE - c)x_2 + (q(R - x_2 - E) - h)E + \lambda x_2 + \mu E$, where $\lambda$ and $\mu$ is the optimal Lagrangian multiplier. Therefore, we have

Case 1. When $E > 0$ and $x_2 > 0$ (i.e., $\lambda = 0$ and $\mu > 0$). According to the first-order condition, we have $x_2 = (R - c + h)/(1 - q)$, $E = (cq - h)/2(1 - q)$. Due to $\mu > 0$, we have $q \leq h/c$. Thus $P_2^o = (R - c)/2$, $\Pi_2 = (R - c)^2/4$.

Case 2. When $E > 0$ and $x_2 > 0$ (i.e., $\lambda = 0$, $\mu > 0$). According to the first-order condition, we have $x_2 = (1 - q)(R - c + h)/2(1 - q)$, $E = (cq - h)/(2q(1 - q))$. Due to $x_2 > 0$ and $E > 0$, we have $h/c < q < (R - c + h)/R$. Therefore, for this case, the optimal decisions and profit are $P_2^o = (R - c)/2$, $\Pi_2 = (qR + h)/2$ and
\[
\Pi_2 = ((R^2 - 2Rc)(q^2 - q) - qc^2 + 2(ch - h^2)/4(q^2 - q).
\]

Case 3. When $E > 0$ and $x_2 = 0$ (i.e., $\lambda > 0$, $\mu = 0$). Similarly, we can derive that $E = (qR - h)/2q$, $\lambda = qR - (R - c + h)$. From the condition $\lambda \geq 0$, we have $q \geq (R - c + h)/R$. Thus $P_2^o = (qR + h)/2$ and $\Pi_2 = (qR - h)^2/4q$.

Proof of Corollary 3. First, we will show the continuity of $E^\ast$. According to Proof of Proposition 1, the optimal stocks of the old products in the three cases are $E^\ast_{q \leq h/c} = 0$, $E^\ast_{h/c < q < (R - c + h)/R} = (cq - h)/2q(1 - q)$ and $E^\ast_{q \geq (R - c + h)/R} = (qR - h)/2q(q^2 - q)$, respectively.

For the convenience, we will record the optimal ordering points of these three cases as $E^\ast_i (i = 1, 2, 3)$. When $q = h/c$, $E^\ast_1 = E^\ast_2 = 0$; and when $q = (R - c + h)/R$, $E^\ast_2 = E^\ast_3 = (R - c + h)/R$. Therefore $E^\ast$ is continuous with respect to $q$.

The following proves the monotonicity of $E^\ast$.

1. $E^\ast_1 = 0$ is a constant (can be regarded as monotonically increasing).
2. $E^\ast_2 = (cq - h)/2q(1 - q)$, we have $dE^\ast_2/dq = 2q(cq - h) + (1 - h)/4q^2(1 - q)^2$, and then $dE^\ast_2/dq > 0$.
3. $E^\ast_3 = (qR - h)/2q$, obviously, $dE^\ast_3/dq = h/2q^2 > 0$.

It can be seen that $E^\ast$ is monotonically increasing with respect to $q$.

In summary the optimal inventory of old products $E^\ast$ is continuous and monotonically increasing with $q$.

Proof of Proposition 2. Case 1. When $q \leq h/c$, we have $\Pi_{12}(x_1) = (P_1 - c)x_1 + \rho \Pi_2(0) = (R - x_1 - c)x_1 + \rho (R - c)^2/4$. Then let $d\Pi_{12}/dx_1 = R - 2x_1 - c = 0$, we have $x_1 = (R - c)/2$, $P_1 = (R + c)/2$ and $\Pi_{12}(x_1^\ast) = (1 + \rho)(R - c)^2/4$. Since condition $x_1 - E^\ast_1 > 0$ needs to be satisfied, we assume that $R - c > 0$.

Case 2. When $h/c < q < (R - c + h)/R$, we have $\Pi_{12}(x_1) = (P_1 - c)(x_1 - E^\ast_2) - cE^\ast_2 + \rho \Pi_2(x_2^\ast, E_2^\ast)$, where $P_1 = R - x_1$. From the first-order condition, i.e.,
\[
\frac{d\Pi_{12}}{dx_1} = R - 2x_1 - c + E_2^* = 0, \text{ we have } x_1 = \frac{(R - c + E_2^*)}{2}, \text{ and } P_1 = \frac{(R + c - E_2^*)}{2}.
\]

Therefore, we have \(\Pi_{12}(x_1^*) = \frac{(R - c - E_2^*)^2}{4} - cE_2^* + \rho \Pi_2^*(x_2^*, E_2^*)\). Since it is necessary to satisfy the condition \(x_1 - E_2^* > 0\), we have \(R - c > E_2^*\). It is known that \(E_2^*\) is monotonically increasing with respect to \(q\), and the maximal value of \(E_2^*\) is \(R(R - c)/2(R - c + h)\), therefore, the condition \(R - c > c - 2h\) should be satisfied to ensure \(x_1 - E_2^* > 0\).

Case 3. When \(q \geq (R - c + h)/R\), we have \(\Pi_{12}(x_1) = (P_1 - c)(x_1 - E_2^*) - cE_3^* + \rho \Pi_2(x_2^*, E_2^*)\), where \(P_1 = R - x_1\). Then according to \(d\Pi_{12}/dx_1 = R - 2x_1 - c + E_3^* = 0\), we can obtain \(x_1 = \frac{(R - c + E_3^*)}{2}\) and \(P_1 = \frac{(R + c - E_3^*)}{2}\). Thus there is \(\Pi_{12}(x_1^*) = \frac{(R - c - E_3^*)^2}{4} - cE_3^* + \rho \Pi_2(x_2^*, E_3^*)\). Since it is necessary to satisfy the condition \(x_1 - E_3^* > 0\), that is, \(R - c > E_3^*\). Similar to Case 2, the condition \(R - c > c - h\) should be satisfied to ensure \(x_1 - E_3^* > 0\).

To combine the above three cases, the condition \(R - c > c - h\) should be satisfied to ensure a non-negative order quantity of the first period, i.e., \(x_1 - E^* > 0\).

**Proof of Lemma 2.** We will show it by contradiction. Assume \(\tilde{x}_1^* < x_1^*\) when \(\Delta R > 0\), and we have

\[
\begin{align*}
\tilde{\Pi}_{12}(\tilde{x}_1^*) &= (P_1^* - c)(\tilde{x}_1^* - E^*) - cE^* - c_0(\tilde{x}_1^* - \tilde{x}_1^*) + \rho \Pi_2^*(x_2^*, E^*) \\
&= (R + \Delta R - \tilde{x}_1^*) - (\tilde{x}_1^* - E^*) - cE^* - c_0(\tilde{x}_1^* - \tilde{x}_1^*) + \rho \Pi_2^*(x_2^*, E^*) \\
&= (R - \tilde{x}_1^*) - (\tilde{x}_1^* - E^*) - cE^* + \Delta R(\tilde{x}_1^* - E^*) - c_0(\tilde{x}_1^* - \tilde{x}_1^*) + \rho \Pi_2^*(x_2^*, E^*) \\
&< (R - \tilde{x}_1^*) - (\tilde{x}_1^* - E^*) - cE^* + \Delta R(\tilde{x}_1^* - E^*) - c_0(\tilde{x}_1^* - \tilde{x}_1^*) + \rho \Pi_2^*(x_2^*, E^*) = \tilde{\Pi}_{12}(x_1^*).
\end{align*}
\]

The inequality holds because the function \((R - \tilde{x}_1^*) - (\tilde{x}_1^* - E^*)\) is concave and increasing with \(\tilde{x}_1^*\) when \(\tilde{x}_1^* < x_1^*\), and \(\Delta R(\tilde{x}_1^* - E^*) < \Delta R(x_1^* - E^*)\). Therefore, the optimal ordering quantity \(\tilde{x}_1^*\) is not the optimal solution of \(\tilde{\Pi}_{12}(\tilde{x}_1^*)\), which is contradict with our assumption. Therefore, when \(\Delta R \geq 0\), we have \(\tilde{x}_1^* \geq x_1^*\).

Similarly, we can show that when \(\Delta R \leq 0\), we have \(\tilde{x}_1^* \leq x_1^*\) by contradiction.

**Proof of Proposition 3.** The function \(E[\Pi_{12}^G(\tilde{x}_1)] = (R + \Delta R - \tilde{x}_1)(\tilde{x}_1 - E^*) - c\tilde{x}_1 - c_0(\tilde{x}_1 - x_1^*) + c_0(\tilde{x}_1 - \tilde{x}_1) + \rho \Pi_2^*(x_2^*, E^*)\) can be transformed into the following two cases equivalently:

\[
\begin{align*}
\max E[\Pi_{12}^G(\tilde{x}_1)] &= (R + \Delta R - \tilde{x}_1)(\tilde{x}_1 - E^*) - c\tilde{x}_1 - c_0(\tilde{x}_1 - x_1^*) + \rho \Pi_2^*(x_2^*, E^*) \\
\text{s.t.} \quad &\tilde{x}_1 - x_1^* \geq 0 \\
\end{align*}
\text{and}
\begin{align*}
\max E[\Pi_{12}^G(\tilde{x}_1)] &= (R + \Delta R - \tilde{x}_1)(\tilde{x}_1 - E^*) - c\tilde{x}_1 - c_0(\tilde{x}_1 - \tilde{x}_1) + \rho \Pi_2^*(x_2^*, E^*) \\
\text{s.t.} \quad &\tilde{x}_1 - \tilde{x}_1 \geq 0
\end{align*}
\]

(A1) \hspace{1cm} \text{and} \hspace{1cm} (A2)

The Kuhn-Tucker of Eq. (A1) is

\[
\begin{align*}
\frac{\partial E[\Pi_{12}^G(\tilde{x}_1)]}{\partial \tilde{x}_1} + \lambda \frac{\partial (\tilde{x}_1 - x_1^*)}{\partial \tilde{x}_1} &= 0 \\
\lambda (\tilde{x}_1 - x_1^*) &= 0 \\
\lambda &\geq 0 \\
\tilde{x}_1 - x_1^* &\geq 0
\end{align*}
\]

Solving the above equation, we obtain the following cases.

When \(\Delta R \geq c_0\), it means that the Lagrangian multiplier \(\lambda = 0\). By differentiating the profit function, we can derive that \(\tilde{x}_1^* = \frac{(R + \Delta R - c + E^* - c_0)}{2}\). Therefore, the optimal order quantity and optimal pricing of young products can be recorded as \(\tilde{x}_1^* = x_1^* + (\Delta R - c_0)/2\) and \(\tilde{P}_1^* = P_1^* + (\Delta R + c_0)/2\), respectively.
When $\overline{\Delta R} < c_u$, it means that the Lagrangian multiplier $\lambda > 0$, indicates $\hat{x}_1 - x_1^* = 0$. From the first order condition of the profit, we can derive that $\hat{x}_1 = x_1^*$ and $\hat{P}_1 = P_1^* + \overline{\Delta R}$.

Similarly, using the K-T conditional method to solve Eq. (A2), we can get the following results:

When $\Delta R \leq -c_s$, we can get $\hat{x}_1^* = (R + \Delta R - c + E^* + c_s)/2$. Therefore, the optimal order quantity and optimal pricing can be recorded as $\hat{x}_1 = x_1^* + (\overline{\Delta R} + c_s)/2$ and $\hat{P}_1 = P_1^* + (\Delta R - c_s)/2$, respectively.

When $\Delta R > -c_s$, it indicates $\hat{x}_1 - x_1^* > 0$. We can get the optimal solutions $\hat{x}_1 = x_1^*$ and $\hat{P}_1 = P_1^* + \overline{\Delta R}$.

Combining two cases, we can obtain the conclusion of Proposition 3.

Proof of Proposition 4. According to Proposition 3, the firm’s profit is expressed as follows:

$$E[\Pi_{12}^G] = \begin{cases} 
\frac{(R + \overline{\Delta R} - c - E^*)^2 - c_u^2}{4} - cE^* + \rho \Pi_2(x_2^*, E^*), & \text{if } \overline{\Delta R} \geq c_u \\
\frac{(R + \overline{\Delta R} - c - E^*)^2 - \overline{\Delta R}^2}{4} - cE^* + \rho \Pi_2(x_2^*, E^*), & \text{if } -c_s < \overline{\Delta R} < c_u \\
2R - c - E^*, & \text{if } 2c - R - h \leq \overline{\Delta R} \leq -c_s
\end{cases}$$

Thus,

$$\frac{\partial E[\Pi_{12}^G]}{\partial \overline{\Delta R}} = \begin{cases} 
2(R + \overline{\Delta R} - c - E^*) > 0, & \text{if } \overline{\Delta R} \geq c_u \\
2(R - c - E^*) > 0, & \text{if } -c_s < \overline{\Delta R} < c_u \\
2(R + \overline{\Delta R} - c - E^*) > 0, & \text{if } 2c - R - h \leq \overline{\Delta R} \leq -c_s
\end{cases}$$

Therefore, the total optimal profit for both periods is monotonically increasing with respect to the disruption amount mean $\overline{\Delta R}$.

Proof of Proposition 5. From Proposition 2 and 4, we can obtain the value of demand disruption is written as follows:

$$\Delta^{G^*}\Pi = E[\Pi_{12}^{G^*}] - \Pi_{12}$$

$$= \begin{cases} 
\frac{(2\overline{\Delta R}(R - c - E^*) + \overline{\Delta R}^2 - c_u^2)}{4}, & \text{if } \overline{\Delta R} \geq c_u \\
\overline{\Delta R}(R - c - E^*), & \text{if } -c_s < \overline{\Delta R} < c_u \\
\frac{(2\overline{\Delta R}(R - c - E^*) + \overline{\Delta R}^2 - c_s^2)}{4}, & \text{if } 2c - R - h \leq \overline{\Delta R} \leq -c_s
\end{cases}$$

Obviously, we can get the following results:

$$\frac{\partial (\Delta^{G^*}\Pi)}{\partial q} = -\frac{\overline{\Delta R}}{2} \frac{dE^*}{dq} \quad \text{and} \quad \frac{\partial (\Delta^{G^*}\Pi)}{\partial c} = -\frac{\overline{\Delta R}}{2} \left(1 + \frac{dE^*}{dc}\right).$$

Therefore, when $\overline{\Delta R} \leq 0$, $\Delta^{G^*}\Pi \leq 0$ and $\Delta^{G^*}\Pi$ monotonically increases with respect to the perceived quality ($q$) and unit purchase cost ($c$), respectively. While, when $\overline{\Delta R} \geq 0$, $\Delta^{G^*}\Pi \geq 0$ and $\Delta^{G^*}\Pi$ decrease monotonically with the perceived quality ($q$) and unit purchase cost ($c$), respectively.

Proof of Proposition 6.

$$\max E[\Pi_{12}^R(\hat{x}_{1D})] = \sum_{D=H,L} \beta_D ((R + \Delta R_D - \hat{x}_{1D})(\hat{x}_{1D} - E^*) - c\hat{x}_{1D})$$

$$-c_u(\hat{x}_{1D} - x_1^*)^+ - c_s(x_1^* - \hat{x}_{1D})^+ + \rho \Pi_2(x_2^*, E^*).$$
can be transformed into the following four cases.

\[
\begin{align*}
\text{max} \ E[\Pi^B_{12}(\tilde{x}_{1D})] &= \beta [(R + \Delta R_H - \tilde{x}_{1H})(\tilde{x}_{1H} - E^*) - c\tilde{x}_{1H} - c_u(\tilde{x}_{1H} - x_1^*) + \rho\Pi^*_L(x_1^*, E^*)] \\
&+ (1 - \beta) [(R + \Delta R_L - \tilde{x}_{1L})(\tilde{x}_{1L} - E^*) - c\tilde{x}_{1L} - c_u(\tilde{x}_{1L} - x_1^*) + \rho\Pi^*_L(x_1^*, E^*)] \\
\text{s.t.} & \quad \tilde{x}_{1H} - x_1^* \geq 0, \tilde{x}_{1L} - x_1^* \geq 0
\end{align*}
\]  
\tag{A3}

\[
\begin{align*}
\text{max} \ E[\Pi^B_{12}(\tilde{x}_{1D})] &= \beta [(R + \Delta R_H - \tilde{x}_{1H})(\tilde{x}_{1H} - E^*) - c\tilde{x}_{1H} - c_u(\tilde{x}_{1H} - x_1^*) + \rho\Pi^*_L(x_1^*, E^*)] \\
&+ (1 - \beta) [(R + \Delta R_L - \tilde{x}_{1L})(\tilde{x}_{1L} - E^*) - c\tilde{x}_{1L} - c_u(\tilde{x}_{1L} - x_1^*) + \rho\Pi^*_L(x_1^*, E^*)] \\
\text{s.t.} & \quad \tilde{x}_{1H} - x_1^* \geq 0, \tilde{x}_{1L} - x_1^* \geq 0
\end{align*}
\]  
\tag{A4}

\[
\begin{align*}
\text{max} \ E[\Pi^B_{12}(\tilde{x}_{1D})] &= \beta [(R + \Delta R_H - \tilde{x}_{1H})(\tilde{x}_{1H} - E^*) - c\tilde{x}_{1H} - c_u(\tilde{x}_{1H} - x_1^*) + \rho\Pi^*_L(x_1^*, E^*)] \\
&+ (1 - \beta) [(R + \Delta R_L - \tilde{x}_{1L})(\tilde{x}_{1L} - E^*) - c\tilde{x}_{1L} - c_u(\tilde{x}_{1L} - x_1^*) + \rho\Pi^*_L(x_1^*, E^*)] \\
\text{s.t.} & \quad \tilde{x}_{1H} - x_1^* \geq 0, \tilde{x}_{1L} - x_1^* \geq 0
\end{align*}
\]  
\tag{A5}

\[
\begin{align*}
\text{max} \ E[\Pi^B_{12}(\tilde{x}_{1D})] &= \beta [(R + \Delta R_H - \tilde{x}_{1H})(\tilde{x}_{1H} - E^*) - c\tilde{x}_{1H} - c_u(\tilde{x}_{1H} - x_1^*) + \rho\Pi^*_L(x_1^*, E^*)] \\
&+ (1 - \beta) [(R + \Delta R_L - \tilde{x}_{1L})(\tilde{x}_{1L} - E^*) - c\tilde{x}_{1L} - c_u(\tilde{x}_{1L} - x_1^*) + \rho\Pi^*_L(x_1^*, E^*)] \\
\text{s.t.} & \quad \tilde{x}_{1H} - x_1^* \geq 0, \tilde{x}_{1L} - x_1^* \geq 0
\end{align*}
\]  
\tag{A6}

First, note that Eq. (A6) means that \(\tilde{x}_{1L} > x_1^* > \tilde{x}_{1H}\). From Lemma 3, we can draw the conclusion that \(\Delta R_H < \Delta R_L\), which contrary to assumptions. Therefore, it should be removed.

Next, the Kuhn-Tucker condition of Eq. (A3) is

\[
\begin{align*}
\frac{\partial E[\Pi^B_{12}(\tilde{x}_{1H}\tilde{x}_{1L})]}{\partial \tilde{x}_{1H}} + \lambda \frac{\partial (\tilde{x}_{1H} - x_1^*)}{\partial \tilde{x}_{1H}} + \mu \frac{\partial (\tilde{x}_{1L} - x_1^*)}{\partial \tilde{x}_{1L}} &= 0 \\
\frac{\partial E[\Pi^B_{12}(\tilde{x}_{1H}\tilde{x}_{1L})]}{\partial \tilde{x}_{1L}} + \lambda \frac{\partial (\tilde{x}_{1H} - x_1^*)}{\partial \tilde{x}_{1L}} + \mu \frac{\partial (\tilde{x}_{1L} - x_1^*)}{\partial \tilde{x}_{1L}} &= 0 \\
\lambda (\tilde{x}_{1H} - x_1^*) = 0, \mu (\tilde{x}_{1L} - x_1^*) = 0 \\
\lambda \geq 0, \mu \geq 0, \tilde{x}_{1H} - x_1^* \geq 0, \tilde{x}_{1L} - x_1^* \geq 0
\end{align*}

Solving the above equation, we obtain the following.

When \(\Delta R_D \geq c_u\), it means that the Lagrangian multipliers \(\lambda = 0\) and \(\mu > 0\). From the first order condition of the expected profit, we can derive that \(\tilde{x}_{1H} = x_1^* + (\Delta R_H - c_u)/2\) and \(\tilde{x}_{1L} = x_1^* + (\Delta R_L - c_u)/2\). Therefore, the optimal order quantity and optimal pricing of young products in Period 1 can be recorded as \(\tilde{x}_{1D} = \tilde{x}_1 = (\Delta R_D - c_u)/2\) and \(P^*_D = P^*_1 + (\Delta R_D + c_u)/2\), respectively.

When \(\Delta R_H \geq c_u\) and \(\Delta R_L < c_u\), it means that the Lagrangian multiplier \(\lambda = 0\) and \(\mu > 0\). From the first order condition of the expected profit, we can derive the optimal decisions \(\tilde{x}_{1H} = x_1^* + (\Delta R_H - c_u)/2\), \(\tilde{P}^*_1 = P^*_1 + (\Delta R_H + c_u)/2\), \(\tilde{x}_{1L} = x_1^*\) and \(\tilde{P}^*_L = P^*_1 + \Delta R_D\).

When \(\Delta R_H < c_u\) and \(\Delta R_L < c_u\), it means that the Lagrangian multiplier \(\lambda > 0\) and \(\mu > 0\). We can get the optimal solutions \(\tilde{x}_{1H} = \tilde{x}_{1L} = x_1^*\). Therefore, the optimal order quantity and optimal pricing of young products are \(\tilde{x}_{1D} = x_1^*\) and \(\tilde{P}^*_D = P^*_1 + \Delta R_D\), respectively.

When \(\Delta R_H < c_u\) and \(\Delta R_L \geq c_u\), it implies that \(\lambda > 0\) and \(\mu = 0\). At this time, \(\Delta R_L > \Delta R_H\), which is contradicts with the assumption of \(\Delta R_H > \Delta R_L\). Therefore, this case should be removed.

Proofs of Eq. (A4) and Eq. (A5) are similar to that of Eq. (A3), and here are omitted. Combining two cases, we can obtain the conclusion of Proposition 6. \(\square\)

**Proof of Corollary 4.** By the optimal decision of Proposition 6, we can calculate the expected profit for both periods as follows:
(i) If $\Delta R_H \geq c_u$ and $\Delta R_L \geq c_u$, we have
\[
\mathbb{E} \left[ \tilde{\Pi}_{12}^{B^*} \right] = \beta \frac{(R + \Delta R_H - c - E^*)^2 - c_u^2}{4} + (1 - \beta) \frac{(R + \Delta R_L - c - E^*)^2 - c_u^2}{4} - cE^* + \rho \tilde{\Pi}_1^* (x^*_2, E^*),
\]
and then
\[
\frac{\partial \mathbb{E} \left[ \tilde{\Pi}_{12}^{B^*} \right]}{\partial \beta} = \frac{(\Delta R_H - \Delta R_L) (2(R - c - E^*) + (\Delta R_H + \Delta R_L))}{4} > 0;
\]
(ii) If $\Delta R_H \geq c_u$ and $-c_s < \Delta R_L < c_u$, we have
\[
\mathbb{E} \left[ \tilde{\Pi}_{12}^{B^*} \right] = \beta \frac{(R + \Delta R_H - c - E^*)^2 - c_u^2}{4} + (1 - \beta) \frac{(R + \Delta R_L - c - E^*)^2 - \Delta R_L^2}{4} - cE^* + \rho \tilde{\Pi}_1^* (x^*_2, E^*),
\]
and then we have
\[
\frac{\partial \mathbb{E} \left[ \tilde{\Pi}_{12}^{B^*} \right]}{\partial \beta} = \frac{(\Delta R_H^2 - c_u^2) + 2(R - c - E^*) (\Delta R_H - \Delta R_L)}{4} > 0
\]
(iii) If $\Delta R_H \geq c_u$ and $\Delta R_L \leq -c_s$, we have
\[
\mathbb{E} \left[ \tilde{\Pi}_{12}^{B^*} \right] = \beta \frac{(R + \Delta R_H - c - E^*)^2 - c_s^2}{4} + (1 - \beta) \frac{(R + \Delta R_L - c - E^*)^2 - c_s^2}{4} - cE^* + \rho \tilde{\Pi}_1^* (x^*_2, E^*),
\]
and then we have
\[
\frac{\partial \mathbb{E} \left[ \tilde{\Pi}_{12}^{B^*} \right]}{\partial \beta} = \frac{(R + \Delta R_H - c - E^*)^2 - c_s^2}{4} - \frac{(R + \Delta R_L - c - E^*)^2 - c_s^2}{4} \begin{cases} \Delta R_L \leq -c_s > 0; \\ \Delta R_L \geq -c_s \end{cases}
\]
(iv) If $-c_s \leq \Delta R_H \leq c_u$ and $-c_s \leq \Delta R_L \leq c_u$, we have
\[
\mathbb{E} \left[ \tilde{\Pi}_{12}^{B^*} \right] = \beta \frac{(R + \Delta R_H - c - E^*)^2 - \Delta R_H^2}{4} + (1 - \beta) \frac{(R + \Delta R_L - c - E^*)^2 - \Delta R_L^2}{4} - cE^* + \rho \tilde{\Pi}_1^* (x^*_2, E^*),
\]
and then we have
\[
\frac{\partial \mathbb{E} \left[ \tilde{\Pi}_{12}^{B^*} \right]}{\partial \beta} = \frac{(R - c - E^*) (\Delta R_H - \Delta R_L)}{2} > 0;
\]
(v) If $-c_s < \Delta R_H < c_u$ and $\Delta R_L \leq -c_s$, we have
\[
\mathbb{E} \left[ \tilde{\Pi}_{12}^{B^*} \right] = \beta \frac{(R + 2\Delta R_H - c - E^*) (R - c - E^*)}{4} + (1 - \beta) \frac{(R + \Delta R_L - c - E^*)^2 - c_s^2}{4} - cE^* + \rho \tilde{\Pi}_1^* (x^*_2, E^*),
\]
and then
\[
\frac{\partial \mathbb{E} \left[ \tilde{\Pi}_{12}^{B^*} \right]}{\partial \beta} = \frac{(R + 2\Delta R_H - c - E^*) (R - c - E^*)}{4} - \frac{(R + \Delta R_L - c - E^*)^2 - c_s^2}{4} \begin{cases} \Delta R_L \leq -c_s > 0; \\ -c_s < \Delta R_L \leq c_u \end{cases}
\]
(vi) If $\Delta R_H \leq -c_s$ and $\Delta R_L \leq -c_s$, we have

$$E[N_{12}^B] = \beta ((R + \Delta R_H - c - E^*)^2 - c^2) + (1 - \beta) (\Delta R^2 + (c - E^*)^2 - c^2)$$

and then we have

$$-e^* + \beta R^2 (x_1^*, E^*) ,$$

Therefore, according to the above analysis, we can get the main conclusion of Corollary 4.

Proof of Proposition 7. (i) If $\Delta R_H \geq c_u$ and $\Delta R_L \geq c_u$, we have

$$\Delta^* = E[\Pi_{12}^B] - \Pi_{12}^* \geq [\beta \Delta R^2 + (1 - \beta) \Delta R^2 + 2\Delta R - (c - E^*)^2] - c^2] / 4 \geq 0 ;$$

(ii) If $\Delta R_H \geq c_u$ and $-c_s < \Delta R_L < c_u$, we have $\Delta^* \geq [\beta \Delta R^2 + (1 - \beta) \Delta R^2 + 2\Delta R - (c - E^*)^2] / 4$. When $\Delta R \geq 0$, $\Delta^* \geq 0$. However, when $\Delta R < 0$, $\Delta^* \geq 0$. Let $\Delta^* = 0$; then, we can obtain $\Delta R = -\beta (\Delta R^2 + c^2) / 2(R - E^*)$ and record it to be $D_1$. If $\Delta R \geq 0$, $\Delta^* \geq 0$; and if $\Delta R \leq 0$, $\Delta^* \geq 0$; when $\Delta R \leq D_2$, $\Delta^* \geq 0$; if $\Delta R \leq D_3$, $\Delta^* \geq 0$; and if $\Delta R \leq D_4$, $\Delta^* \geq 0$. Therefore, we can get the results:

$$\Delta^* < 0 ; \Delta^* < 0 ; \Delta^* \geq 0 ; \Delta^* \geq 0 ;$$

where $D_2 = -\varphi / 2(R - E^*)$, $c_2 = R - E^* + \varphi / 2\Delta R$ and $\varphi = \beta (\Delta R^2 + c^2) + (1 - \beta) (\Delta R^2 - c^2)$. (iv) If $-c_s < \Delta R \leq c_u$, we have $\Delta^* = \Delta R(R - c - E^*) / 2$. Obviously, we have

$$\Delta^* \geq 0 ; \Delta^* \geq 0 ; \Delta^* \geq 0 ; \Delta^* \geq 0 .$$

(v) If $-c_s < \Delta R_H < c_u$ and $\Delta R_L \leq -c_s$, we have $\Delta^* = [2\Delta R(R - c - E^*) + (1 - \beta)(\Delta R^2 - c^2)] / 4$. Similar to part (ii), we can get the following results: when $\Delta R \geq D_3$, $\Delta^* \geq 0$; and if $\Delta R \leq D_4$, $\Delta^* \geq 0$. Therefore, we can obtain $c_1 = R - E^* + \beta (\Delta R^2 + c^2) / 2\Delta R$. Therefore, we can get the main conclusion of Corollary 4.
Therefore, we can get the result: when $\Delta R \geq D_4$, $\Delta B^* \Pi \geq 0$; when $\Delta R \leq D_4$,
\[
\begin{align*}
\Delta B^* \Pi & \leq 0 \quad \text{if } 0 \leq c \leq c_4 \\
\Delta B^* \Pi & \geq 0 \quad \text{if } c_4 \leq c \leq R - E^*.
\end{align*}
\]

To summarize the above analysis, we can obtain the main results of Proposition 7.

Proof of Corollary 5: Part (i) is obvious, and here is omitted. Part (ii) can be obtained from Proposition 4.

REFERENCES

[1] Z. Azadi, S. D. Eksioglu, B. Eksioglu and G. Palak, Stochastic optimization models for joint pricing and inventory replenishment of perishable products, *Computers & Industrial Engineering*, **127** (2019), 625–642.

[2] Y. Aviv, The effect of collaborative forecasting on supply chain performance, *Management Science*, **47** (2001), 1331–1440.

[3] İ. S. Bakal, Z. P. Bayındır and D. E. Emir, Value of disruption information in an EOQ environment, *European J. Oper. Res.*, **263** (2017), 446–460.

[4] M. A. Begen, H. Pun and X. Yan, Supply and demand uncertainty reduction efforts and cost comparison, *International Journal of Production Economics*, **180** (2016), 125–134.

[5] A. Bensousan, Q. Feng, S. Luo and S.P. Sethi, Evaluating long-term service performance under short-term forecast updates, *International Journal of Production Research*, (2003), 1–14.

[6] E. Cao, C. Wan and M. Lai, Coordination of a supply chain with one manufacturer and multiple competing retailers under simultaneous demand and cost disruptions, *International Journal of Production Economics*, **141** (2013), 425–433.

[7] E. P. Chew, C. Lee and R. Liu, Joint inventory allocation and pricing decisions for perishable products, *International Journal of Production Economics*, **120** (2009), 139–150.

[8] J. Chen, M. Dong, Y. Rong and L. Yang, Dynamic pricing for deteriorating products with menu cost, *Omega*, **75** (2018), 13–26.

[9] K. B. Chen and P. Zhang, Disruption management for a dominant retailer with constant demand-stimulating service cost, *Computers & Industrial Engineering*, **61** (2011), 936–946.

[10] K. B. Chen and T. J. Xiao, Production planning and backup sourcing strategy of a buyer-dominant supply chain with random yield and demand, *International Journal of Systems Science*, **46** (2015), 2799–2817.

[11] K. B. Chen, R. Xu and H. Fang, Information disclosure model under supply chain competition with asymmetric demand disruption, *Asia-Pacific Journal of Operational Research*, **33** (2016), 1650043, 35pp.

[12] Z. X. Chen, Optimization of production inventory with pricing and promotion effort for a single-vendor multi-buyer system of perishable products, *International Journal of Production Economics*, **203** (2018), 333–349.

[13] P. Chintapalli, Simultaneous pricing and inventory management of deteriorating perishable products, *Annals of Operations Research*, **229** (2015), 287–301.

[14] J. Danusantoso and S. A. Moses, Disruption management in a two-period three-tier electronics supply chain, *Cogent Business & Management*, **3** (2016), 1137138.

[15] P. S. Desai, O. Koenigsberg and D. Purohit, Research note-the role of production lead time and demand uncertainty in marketing durable goods, *Management Science*, **53** (2007), 150–158.

[16] L. Duong, L. Wood and W. Wang, A review and reflection on inventory management of perishable products in a single-echelon model, *International Journal of Operational Research*, **31** (2018), 313–329.

[17] C. Y. Dye, Optimal joint dynamic pricing, advertising and inventory control model for perishable items with psychic stock effect, *European Journal of Operational Research*, **283** (2020), 576–587.

[18] A. Ehrenberg and G. Goodhardt, New brands: Near-instant loyalty, *Journal of Targeting, Measurement & Analysis for Marketing*, **16** (2001), 607–617.
L. Feng, Y. L. Chan and L. E. Cárdenas-Barrón, Pricing and lot-sizing policies for perishable products, *International Transactions in Operational Research*, 25 (2018), 2031–2051.

L. Feng, Y. L. Chan and L. E. Cárdenas-Barrón, Pricing and lot-sizing polices for perishable goods when the demand depends on selling price, displayed stocks, and expiration date, *International Journal of Production Economics*, 185 (2017), 11–20.

M. E. Ferguson and O. Koenigsberg, How should a firm manage deteriorating inventory?, *Production and Operations Management*, 16 (2007), 306–321.

Y. He and S. Wang, Analysis of production-inventory system for deteriorating items with demand disruption, *International Journal of Production Research*, 50 (2012), 4580–4592.

Z. He, G. Han, T. C. E. Cheng, B. Fan and J. Dong, Evolutionary food quality and location strategies for restaurants in competitive online-to-offline food ordering and delivery markets: An agent-based approach, *International Journal of Production Economics*, 215 (2019), 61–72.

A. Herbon, Potential additional profits of selling a perishable product due to implementing price discrimination versus implementation costs, *International Transactions in Operational Research*, 26 (2019), 1402–1421.

S. Huang, C. Yang and X. Zhang, Pricing and production decisions in dual-channel supply chains with demand disruptions, *Computers & Industrial Engineering*, 62 (2012), 70–83.

X. Ji, J. Sun and Z. Wang, Turn bad into good: Using transshipment-before-buyback for disruptions of stochastic demand, *International Journal of Production Economics*, 185 (2017), 150–161.

A. Kara and I. Dogan, Reinforcement learning approaches for specifying ordering policies of perishable inventory systems, *Expert Systems with Applications*, 91 (2018), 150–158.

M. Lashgari, A. A. Taleizadeh and S. S. Sana, An inventory control problem for deteriorating items with back-ordering and financial considerations under two levels of trade credit linked to order quantity, *Journal of Industrial & Management Optimization*, 12 (2016), 1091–1119.

C. Y. Lee and R. Yang, Supply chain contracting with competing suppliers under asymmetric information, *IIE Transactions*, 45 (2013), 25–52.

B. Li, C. Yang and S. Huang, Study on supply chain disruption management under service level dependent demand, *Journal of Networks*, 9 (2014), 1432.

R. Li and J. T. Teng, Pricing and lot-sizing decisions for perishable goods when demand depends on selling price, reference price, product freshness, and displayed stocks, *European Journal of Operational Research*, 270 (2018), 1099–1108.

S. K. Li, J. X. Zhang and W. S. Tang, Joint dynamic pricing and inventory control policy for a stochastic inventory system with perishable products, *International Journal of Production Research*, 53 (2015), 2937–3950.

T. Li and H. Zhang, Information sharing in a supply chain with a make-to-stock manufacturer, *Omega*, 50 (2015), 115–125.

Y. Li, A. Lim and B. Rodrigues, Note-Pricing and inventory control for a perishable product, *Manufacturing & Service Operations Management*, 11 (2009), 538–542.

W. Liu, Y. Liu, D. Zhu, Y. Wang and Z. Liang, The influences of demand disruption on logistics service supply chain coordination: A comparison of three coordination modes, *International Journal of Production Economics*, 179 (2016), 59–76.

I. Mallidis, D. Vlachos, V. Yakavenka and Z. Eleni, Development of a single period inventory planning model for perishable product redistribution, *Annals of Operations Research*, (2018), 1–17.

S. Minner and S. Transchel, Order variability in perishable product supply chains, *European Journal of Operational Research*, 260 (2017), 93–107.

S. Minner and S. Transchel, Periodic review inventory-control for perishable products under service-level constraints, *OR spectrum*, 32 (2010), 979–996.

C. Muriana, An EOQ model for perishable products with fixed shelf life under stochastic demand conditions, *European Journal of Operational Research*, 255 (2016), 388–396.

X. Qi, J. F. Bard and G. Yu, Supply chain coordination with demand disruptions, *Omega*, 32 (2004), 301–312.
TWO-PERIOD PRICING AND ORDERING DECISIONS

[43] P. E. Rossi and G. M. Allenby, Bayesian statistics and marketing, Marketing Science, 49 (2003), 230–230.

[44] M. R. G. Samani and S. M. Hosseini-Motlagh, An enhanced procedure for managing blood supply chain under disruptions and uncertainties, Annals of Operations Research, 283 (2019), 1413–1462.

[45] H. Scarf, Bayes solutions of the statistical inventory problem, Annals of Mathematical Statistics, 30 (1959), 490–508.

[46] B. Shen, T. M. Choi and S. Minner, A review on supply chain contracting with information considerations: Information updating and information asymmetry, International Journal of Production Research, (2018), 1–39.

[47] N. Tashakkor, S. H. Mirmohammadi and M. Iranpoor, Joint optimization of dynamic pricing and replenishment cycle considering variable non-instantaneous deterioration and stock-dependent demand, Computers & Industrial Engineering, 123 (2018), 232–241.

[48] T. S. Vaughan, A model of the perishable inventory system with reference to consumer-realized product expiration, Journal of the Operational Research Society, 45 (1994), 519–528.

[49] T. J. Xiao and X. T. Qi, Price competition, cost and demand disruptions and coordination of a supply chain with one manufacturer and two competing retailers, Omega, 36 (2008), 741–753.

[50] X. Xu and X. Cai, Price and delivery-time competition of perishable products: Existence and uniqueness of Nash equilibrium, Journal of Industrial & Management Optimization, 4 (2008), 843–859.

[51] M. Xue and G. Zhu, Partial myopia vs. forward-looking behaviors in a dynamic pricing and replenishment model for perishable items, Journal of Industrial & Management Optimization, (2019).

[52] G. Yi, X. Chen and C. Tan, Optimal pricing of perishable products with replenishment policy in the presence of strategic consumers, Journal of Industrial & Management Optimization, 15 (2019), 1579–1597.

[53] J. Zhang, J. Zhang and G. Hua, Multi-period inventory games with information update, International Journal of Production Economics, 174 (2016), 119–127.

[54] Y. Zhao, T. M. Choi, T. C. E. Cheng and S. Wang, Supply option contracts with spot market and demand information updating, European Journal of Operational Research, 266 (2018), 1062–1071.

[55] J. Zhou, R. Zhao and B. Wang, Behavior-based price discrimination in a dual-channel supply chain with retailer’s information disclosure, Electronic Commerce Research and Applications, 39 (2020), 100916.

[56] J. Zhou, R. Zhao and W. Wang, Pricing decision of a manufacturer in a dual-channel supply chain with asymmetric information, European Journal of Operational Research, 278 (2019), 809–820.

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