The width of the $\xi(2230)$ meson

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Abstract

A lower bound of 135 MeV for the width of the $\xi$ meson is obtained from analyzing the $pp$ and $\bar{p}p$ interactions by use of Regge theory. The $pp$ data exclude a narrow $\xi$ as the latter would lead to $\sigma_{tot}^{pp}$ far exceeding the measured ones. The broad width explains why the $\xi$ may not be seen in $\bar{p}p$ experiments.

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One of the important predictions of the quantum chromodynamic (QCD) theory is the existence of bound states of the gluons (the glueballs), which arises from the self-coupling of the gluons. Calculations \[1\] - \[3\] have predicted that the lightest \(2^{++}\) glueball has a mass about 2.2\(\pm\)0.3 GeV. It is also generally believed that a pure glueball should decay flavor symmetric or flavor blind. The \(\xi\) meson has been the center of attention in recent years because its reported mass and flavor-symmetric decay characteristics fit what one would expect from a glueball. This meson was first observed by the MARK III collaboration \[4\] in the decay of \(J/\psi\) to \(K_S K_S\) and \(K^+ K^-\) channels and was found to have quantum numbers \(J^P = (even)^+\), masses \(M = 2230^{+6}_{-6} \pm 15\) and \(2232^{+7}_{-7} \pm 7\) MeV, and widths \(\Gamma = 18^{+23}_{-15} \pm 10\) and \(26^{+26}_{-16} \pm 7\) MeV, respectively. The \(\xi\) was also seen by the WA91 collaboration \[5\], but with low statistics. Although the \(\xi\) was not seen in a number of other experiments, in 1996 the BES collaboration \[6\] reported the observation of \(\xi\) in the radiative decay of the \(J/\psi\) to \(p p, K^+ K^-, K_S^0 K_S^0\), and \(\pi^+ \pi^-\) channels. The mass and the width of \(\xi\) obtained in the BES-experiment agreed with those given in ref. \[4\] and the data were found to be compatible with spin 2. Subsequently, the BES collaboration reported \[7\] \[8\] the observation of \(\xi \rightarrow \pi^0 \pi^0, \eta \eta, \eta' \eta', \eta' \eta'\) and concluded \[8\] that the decays of \(\xi\) to \(K^+ K^-, \pi^+ \pi^-\), and \(\pi^0 \pi^0\) are flavor symmetric. The BES results raised, therefore, the expectation that the \(\xi(2230)\) meson may be the lightest tensor glueball predicted by the QCD theory. As a result, there have been many experimental efforts aiming at verifying the BES results. One interesting experiment consisted in looking for a resonance structure in the \(p\bar{p} \rightarrow \eta \eta, \pi^0 \pi^0\) reactions. However, a systematic study in the mass region of 2222.7 to 2239.7 MeV led to negative results \[9\], casting serious doubts on the very existence of the \(\xi\). The challenge of understanding these conflicting experimental results has motivated this study. I will show how the high-energy (> 50 GeV) \(p p\) cross-section data can be used to set a limit for the width of the \(\xi\).

For notational convenience, let me denote the \(s\)-channel \(p p \rightarrow p p\) and the \(t\)-channel \(p \bar{p} \rightarrow p \bar{p}\) scatterings, respectively as \(12 \rightarrow 34\) and \(13 \rightarrow 24\). The total cross section of the \(t\)-channel \(p \bar{p} \rightarrow \xi \rightarrow p \bar{p}\) scattering is given by
\[
\sigma_{\text{tot}}^{\bar{p}p}(\bar{s}) = \frac{1}{8q^2} \sum_{\lambda'\lambda} G_{\lambda'\lambda}(t) \left| Tm[A^{(t)}_{\lambda'\lambda}(\bar{s}, \bar{t})] \right|_{\bar{t}=0}^2
\]

where I have used the variables \( \bar{s} \) and \( \bar{t} \) to represent, respectively, the total c.m. energy and the momentum transfer in the \( t \)-channel. In eq.(1) \( q^2 = \bar{s}/4 - m^2 \) is the square of the c.m. three-momentum, with \( m \) being the mass of the proton or antiproton. Furthermore, \( \lambda' = \lambda_{\bar{s}} - \lambda_4 \) and \( \lambda = \lambda_1 - \lambda_{\bar{s}} \), with \( \lambda_i \) denoting the helicity of the particle \( i \). In the helicity basis, the \( t \)-channel Feynman amplitude due to the exchange of \( \xi \) can be written as

\[
A^{(t)}_{\lambda'\lambda}(\bar{s}, \bar{t}) = \frac{-4\pi(2J + 1)c_{\lambda'J}c_{\lambda\lambda}d_{\lambda\lambda}(\theta_t)}{\sqrt{\bar{s} - M^2 + iM\Gamma}},
\]

where \( J, M, \) and \( \Gamma \) are the spin, mass, and the width of the \( \xi \). The \( c_{\lambda'J} = 2mC_{1}'G_{\lambda'J}\xi(\bar{s}) \) and \( c_{\lambda\lambda} = 2mC_{1}G_{\lambda\lambda}\xi(\bar{s}) \) denote the coupling strength between the \( \xi \) and the \( pp \) system, with \( C_{1} = 1/\sqrt{2} \) being the isospin coefficient for \( \bar{p}p \) coupling to the isospin zero \( \xi \) meson. The \( H_{\lambda\xi;J} \) and \( H_{\lambda;\lambda,\lambda}^J \) are the form factors for the \( \xi p\bar{p} \) coupling vertex in the helicity basis with \( G_{\lambda'} \) and \( G_{\lambda} \) being the corresponding coupling constants. There are 16 helicity amplitudes but only 5 are independent. For forward \( \bar{p}p \) scattering \( (\theta_t = 0) \),

\[
\sigma_{\text{tot}}^{\bar{p}p}(\bar{s}) = \frac{1}{8q^2} Tm[A_0^{(t)}(\bar{s}, \bar{t} = 0) ].
\]

The helicity-basis form factors are related to the canonical-basis form factors \( F_{LS} \) by a unitary transformation \([10]\). For the \( \bar{p}p \) system coupling to a \( J^P = 2^+ \) state, the parity conservation leads to \( L(= J \pm 1) = 1 \) or 3, and \( S = 1 \) only. Since only one value of \( S \) is allowed, \( F_{LS} \) will henceforth be denoted as \( F_L \). The relations between the form factors in the above two bases are \( G_0 H_{\uparrow\downarrow} = G_0 H_{\downarrow\uparrow} = 2/5g_1F_1 - \sqrt{3}/5g_3F_3 \) and \( G_1 H_{\uparrow\downarrow} = G_{-1} H_{\downarrow\uparrow} = \sqrt{3}/5g_1F_1 + 2/5g_3F_3 \). Here, \( \uparrow \) and \( \downarrow \) represent, respectively, \( \lambda = +1/2 \) and \(-1/2 \). The \( F_L \) has the general form \( F_L = q^L f(t) \). Here, the factor \( q^L \) ensures the correct threshold behavior and \( f(t) \) is an analytical function in both the \( s \)- and \( t \)-channels \([11]\). The form factor can be normalized in such a way that \( F_L(M^2)=1 \). Hence, at \( \bar{s} = M^2 \) and \( J = 2 \), \( A_0^{(t)} = i 40m^2\pi(\sqrt{2}/5g_1 - \sqrt{3}/5g_3)^2/M\Gamma \).

The coupling constants \( g_1 \) and \( g_3 \) can be determined from the \( pp \) total and elastic differential cross sections by use of Regge theory \([12,13]\). We recall that the Regge theory
has been very successful in understanding high-energy hadronic scatterings. In recent years, Regge-theory based models have also been used to predict diffractive production of vector mesons [14]. According to Regge theory, hadron-hadron scattering at very high energies can be described by the pole contribution of the Pomeron trajectory alone. This Pomeron dominance holds in the energy domain where \( \sigma_{ph}(s) = \sigma_{\overline{p}h}(s) \). Here \( h \) and \( \overline{h} \) denote, respectively, a hadron and its antiparticle. Experiments have shown that this cross-section equality occurs [15] at \( \sqrt{s} > 40 \text{ GeV} \). The Pomeron trajectory has the linear form \( \alpha(t) = 1.08 + \alpha' t \) at low \( t \)'s. Here, \( t \) is the four-momentum transfer in the \( s \)-channel or the square of the mass of the exchanged particle. Both the \( \alpha \) and \( \alpha' \) are complex functions of \( t \), i.e., \( \alpha = \alpha_R + i\alpha_I \), and \( \alpha' = \alpha'_R + i\alpha'_I \). For real \( t \) lesser than the \( t \)-channel threshold, \( \alpha'_I = 0 \) and, therefore, \( \alpha(t) = \alpha_R(t) = 1.08 + \alpha'_R t \). The Pomeron trajectory indicates that the mass of an exchanged particle having spin \( J = \alpha_R(M^2) \) is \( M = [(J - 1.08)/\alpha'_R]^{1/2} \). Model-independent analyses gave \( \alpha'_R = 0.20 \pm 0.02 \) [13] [16] [17]. Hence, a particle of spin 2 will have a mass \( M = 2.05 - 2.26 \text{ GeV} \), which overlaps with the mass range of the \( \xi \). The Regge-pole amplitude \( A^{(t)}_{\lambda'\lambda} \) due to the exchange of the Pomeron trajectory is given by [11]

\[
A^{(t)}_{\lambda'\lambda;\lambda_1\lambda_2}(t, s) = C_I \frac{-4\pi^2 (2\alpha + 1) \beta_{\lambda'\lambda}(t) (-1)^{\alpha + \lambda} \frac{1}{2}[1 + (-1)^\alpha]}{\sin \pi(\alpha + \lambda')} d_{\lambda'\lambda}(z_t),
\]

where \( \alpha \equiv \alpha(t) \) and \( z_t \equiv \cos \theta_t \). The variables \( t \) and \( s \) are the momentum transfer and energy in the \( s \)-channel. It is easy to verify that \( t = \overline{s} \) and \( s = \overline{t} \). The residue function \( \beta_{\lambda'\lambda} \) is given by

\[
\beta_{\lambda'\lambda}(t) = 4\alpha'_R m^2 G_{\lambda'\lambda}(t) H_{\lambda'\lambda;\lambda_1\lambda_2}(t) G_{\lambda;\lambda_1\lambda_2}(t). \]

Detailed expressions for the \( pp \) total cross section and \( pp \) elastic differential cross sections in terms of \( A^{(t)}_{\lambda'\lambda} \) can be found in ref. [11]. While the exponent \( L \) in the threshold-behavior factor \( q^L \) of the form factor was fixed with the integer values \( J \pm 1 \) in [11], in the present work I have used the relation \( L = \alpha_R(t) \pm 1 \) to continue \( L \) to noninteger values. The \( g_1 \) and \( g_3 \) obtained from fitting the \( pp \) total cross sections and the diffractive peak of the \( pp \) elastic differential cross sections [15] [16] at \( \sqrt{s} = 53 \) and 62 GeV are: \( g_1 = 1.55 \pm 0.06 \) and
$g_3 = 0.24 \pm 0.02$, which are not too different from what was obtained with fixed integer $L$’s [18]. Using these values in eq. (3), I have calculated the $\sigma_{\pi \pi}^{pp}(s = M^2)$ as a function of $\Gamma$. These calculated cross sections are situated inside the zone between the two solid curves in fig.1. As one can see, small total widths lead to calculated cross sections that exceed the experimental $\sigma_{\pi \pi}^{pp}$ (the dashed line). In order not to violate this experimental constraint, I deduce from figure 1 a lower bound of 135 MeV for the width of the $\xi$, which is much greater than the 22 MeV reported in the literature [3] [4].

It is worth noting that the above broad width is in line with the finding of an earlier experiment by Alde et al. [19] who observed a broad enhancement ($\Gamma \sim 140$ MeV) with spin 2 at $M = 2220$ MeV in the reaction $\pi^- p \rightarrow nX, X \rightarrow \eta \eta'$. The broad width given by the present analysis is also consistent with the non-observation of a resonance structure in the 2.23 GeV region in the $pp \rightarrow \pi^0 \pi^0, \eta \eta$ experiments of Seth et al. [6].

Using the same notation as before, the $\bar{p}p \rightarrow hh$ cross sections can be written as

$$
\left( \frac{d\sigma(\bar{s}, t)}{dt} \right)_{a'a} = \frac{1}{256\pi q^4} \sum_{\lambda'\lambda} \frac{-4\pi(2J + 1)c'_{\lambda', J}c_{\lambda, J}d'_{\lambda, \lambda}(\theta_t)}{\bar{s} - M^2 + iM\Gamma} \right|^2
$$

where the indices $a$ and $a'$ denote, respectively, the initial($\bar{pp}$) and the final ($hh$) channels, with $c'_{\lambda', J}$ being the coupling strength of the final state $a'$ to the $\xi$. Clearly, a resonance structure will not show up in the energy dependence of the cross section if $c'_{\lambda', J} = 0$, i.e., if the resonance $\xi$ does not exist. However, I will show below that even if $\xi$ exists, a resonance structure may still not be seen when its width is broad.

Let me introduce the angle-integrated cross section

$$
\Sigma(\bar{s}) = \int_{\Omega} \left( \frac{d\sigma}{dt} \right)_{a'a} d\Omega
$$

and the variation of the cross sections

$$
V(\sqrt{\bar{s}_\delta}) \equiv \frac{\Sigma(M^2)}{\Sigma(\bar{s})}
$$

with $\Omega$ being the solid angle over which the experimental differential cross sections were summed and $\sqrt{\bar{s}_\delta} \equiv M - \delta$. If $2\delta$ represents the mass interval covered by the measurement,
then the greater the value of $V(\sqrt{s_\delta})$, the more visible will be the resonance structure in this mass region. As the numerator in eq.(6) does not vary rapidly with $\sqrt{s}$, we have $V(\sqrt{s_\delta}) \simeq [(\sqrt{s_\delta} - M^2)^2 + M^2\Gamma^2]/M^2\Gamma^2$. For $M = 2.23$ GeV, $\sqrt{s_\delta} = 2.223$ GeV, and $\Gamma \geq 135$ MeV, we have $V(2.223) \leq 1.01$. In other words, the energy dependence of the cross sections is practically flat in the mass region $2230 \pm 7$ MeV. This was exactly the finding of ref. [9]. In fact, even if $\Gamma$ were 50 MeV, one would still only have $V(2.223) = 1.08$, i.e. only an 8% variation. Only when the width is 22 MeV, can a 40% variation be seen. Hence, the $\bar{p}p$ experiment of ref. [9] can only confirm the existence of a narrow resonance. However, a non-observation of the resonance structure is not sufficient grounds for rejecting the existence of $\xi$. Had $\xi$ been an isolated resonance, it would have been possible to check the resonance by choosing a sufficiently large $\delta$. However, there are two isoscalar tensor resonances, the $f_2(2150)$ and $f_2(2300)$, situated only 80 MeV away and each of them has a width of about 150 to 160 MeV. These nearby resonances prevents us from probing the $\xi$ across a large mass interval.

In conclusion, the high-energy $pp$ data and the experimental $\bar{p}p$ total cross section at c.m. energy $\sqrt{s_{\bar{p}p}} = 2.23$ GeV imply that the total width of the $\xi$ is at least 135 MeV. This broad width is compatible with the non-observation of the $\xi$ in the $\bar{p}p$ experiment of ref. [9]. As the mass of the $\xi$ is situated in the region where the lightest $2^{++}$ tensor glueball is expected, it is of great importance to ascertain the existence of the $\xi$. We note that $q\bar{q}$ states can also have the quantum number $2^{++}$ and, therefore, can mix with the $2^{++}$ gluonium. In this latter case, an experimentally observed $2^{++}$ state can have a broad width. The experimental results of ref. [9] and the present analysis all suggest that the $\xi$ meson, if it exists, should have a broad width. It was reported at the Hadron2001 Conference in Protvino, Russia, that the BES collaboration had repeated the measurement of $J/\psi$ radiative decays with much improved statistics. It will be of great interest to learn the new result.
FIG. 1. Total $\bar{p}p$ cross sections at c.m. energy $\sqrt{s_{\bar{p}p}} = 2.23$ GeV. Theoretical cross sections as a function of the width $\Gamma$ are represented by the zone between the two solid curves. The dashed line indicates the experimental $\bar{p}p$ total cross section [15].
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