Distribution of pairing functions in superconducting spin-valve switching modes

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Abstract. We investigated the critical temperature \( T_c \) of SF1F2 trilayers (S is a singlet superconductor, F1 and F2 are ferromagnetic metals), where the long-range triplet superconducting pairing is generated at canted magnetizations of the F layers. We examined the spin-singlet and spin-triplet pairing distributions and their amplitudes as a function of the layers thicknesses under different values of the angle \( \alpha \) in the SF1F2 structure to clarify which one of the pairing distributions and how may affect the superconducting \( T_c \).

1. Introduction
We investigated the critical temperature \( T_c \) of SF1F2 trilayers (S is a singlet superconductor, F1 and F2 are ferromagnetic metals), where the long-range triplet superconducting pairing is generated at noncollinear magnetizations of the F layers [1]. An asymptotically exact numerical method [2] is employed to calculate \( T_c \) as a function of the trilayer parameters, such as mutual orientation of magnetizations, interfaces transparencies, and the layers thicknesses. Earlier we demonstrated that \( T_c \) in the semi-infinite SF1F2 structures can be a non-monotonic function of the angle \( \alpha \) between magnetizations of the two F layers [3], contrary to the monotonic \( T_c(\alpha) \) behavior calculated for the F1SF2 superconducting spin-valve design [4]. The existence of the anomalous dependence of the spin-triplet correlations on the angle \( \alpha \) in FFS structures in limit of thin F films was shown recently [5]. We examined the spin-singlet and spin-triplet pairing distributions and amplitudes as a function of the layers thicknesses at different values of the angle \( \alpha \) in the SF1F2 structure to clarify which one of the pairing distributions and how may impact on the superconducting \( T_c \).

2. The model and numerical method
At first we found a nonmonotonic dependence of \( T_c \) in a SF1F2 trilayer as a function of the angle \( \alpha \) between the exchange fields of the two F layers (figure 1).
The S layer is of the thickness $d_S (-d_S < x < 0)$, the middle F1 layer is of the thickness $d_{F1}$ ($0 < x < d_{F1}$), the outer F2 layer is of the thickness $d_{F2}$ ($d_{F1} < x < d_{F1}+d_{F2}$), the x axis is normal to the plane of the layers. The exchange field in the middle F1 layer is along the $z$ direction, $h = (0, 0, h)$, while the exchange field in the outer F2 layer is in the $yz$ plane: $h = (0, h \sin \alpha, h \cos \alpha)$. The angle $\alpha$ varies between 0 (parallel configuration, P) and $\pi$ (antiparallel configuration, AP).

Figure 1. SF1F2 trilayer. The S/F1 interface corresponds to $x = 0$. The thick arrows in the F layers denote direction of the exchange fields $h$ lying in the $(y, z)$ plane. The angle between the in-plane exchange fields is $\alpha$.

We consider SF1F2 structure in the dirty limit, which is described by the Usadel equations. Near $T_c$, the Usadel equations are linearized and contain only the anomalous Green function $\hat{F}$ [1]:

$$\frac{D}{2} \nabla^2 \hat{f} - |\omega| \hat{f} - \frac{i \text{sgn} \omega}{2} \{ \hat{\xi}_i (\hbar \hat{\sigma}), \hat{f} \} + \Delta \hat{f} \hbar = 0. \quad (1)$$

Here, $D$ is the diffusion constant, $\hat{F}$ is a $4 \times 4$ matrix, $\omega = \pi T_c (2n + 1)$ where the integer $n$ is the Matsubara frequency, $\hat{\xi}_i$ and $\hat{\xi}_s$ are the Pauli matrices in the Nambu-Gor’kov and spin spaces, respectively. $\hat{\xi}_i \hat{\xi}_s$ is direct product of these matrices. The order parameter $\Delta$ is real-valued in the superconducting layer, while in the ferromagnetic layers it is zero. In general, the diffusion constant $D$ acquires a proper subscript, S or F, when equation (1) is applied to the superconducting or ferromagnetic layers, respectively.

The Green function has the following components:

$$\hat{F} = \hat{\tau}_i \left( f_0 \hat{\sigma}_0 + f_1 \hat{\sigma}_1 + f_2 \hat{\sigma}_2 \right). \quad (2)$$

Here, $f_0$ is the singlet component, $f_1$ is the triplet with zero projection on the $z$ axis, and $f_2$ is the triplet with $\pm 1$ projections on $z$ (the latter is present only at $\alpha \neq 0, \pi$).

There are the following symmetries:

$$f_0 (-\omega) = f_0 (\omega), \quad f_0 \text{ is purely real}$$
$$f_1 (-\omega) = -f_1 (\omega), \quad f_1 \text{ is purely imaginary} \quad (3)$$
$$f_2 (-\omega) = -f_2 (\omega), \quad f_2 \text{ is purely imaginary},$$

which makes it sufficient to consider only positive Matsubara frequencies, $\omega > 0$.

Problem of calculating $T_c$ can be reduced to an effective set of equations for the singlet component in the S layer: the set includes the self-consistency equation and the Usadel equation,

$$\Delta \ln \frac{T_c}{T_c^0} = 2\pi T_c \sum_{\omega > 0} \left( \frac{\Delta}{\omega} - f_0 \right), \quad (4)$$
$$\frac{D}{2} \frac{d^2 f_0}{dx^2} - \alpha f_0 + \Delta = 0, \quad (5)$$

with the boundary conditions:

$$\frac{df_0}{dx} = 0 \bigg|_{x = d_S}, \quad -\xi_S f_0 \bigg|_{x = d_S} = \xi f_0 \bigg|_{x = 0}. \quad (6)$$

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Here, $T_{cS}$ and $x_S = \sqrt{D_S/2pT_{cS}}$ are the superconducting transition temperature and coherence length for an isolated S layer, respectively. This is exactly the problem for which the multimode method (as well as the method of fundamental solution) was developed in [2] and then applied to F1SF2 [4] and semi-infinite SF1F2 [3] spin valves. We only need to determine the explicit expression for $W$ in equation (6), solving the boundary problem for the SF1F2 structure.

The Usadel equation (1) generates the following characteristic wave vectors:

$$k_ω = \sqrt{\frac{2ω}{D}}, \quad k_κ = \sqrt{\frac{h}{D}}, \quad \vec{k}_κ = \sqrt{k_ω^2 + 2iγ_κ^2}. \quad (7)$$

In the S layer the solution of equation (1) is ($A$ and $B$ are purely imaginary):

$$\begin{align*}
&\begin{pmatrix}
    f_0 \\
    f_s \\
    f_2
\end{pmatrix}
= C_1 \begin{pmatrix}
    0 & \cosh(k_ω x) + C_2 & 1 \\
    \cosh(\vec{k}_κ x) + C_3 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
    1 \\
    1 \\
    0
\end{pmatrix}
\begin{pmatrix}
    0 \\
    0 \\
    1
\end{pmatrix}
\begin{pmatrix}
    0 \\
    1 \\
    1
\end{pmatrix}
\begin{pmatrix}
    0 \\
    0 \\
    1
\end{pmatrix}
\begin{pmatrix}
    \cosh(\vec{k}_κ x) \\
    \sinh(\vec{k}_κ x) + S_2 \\
    0
\end{pmatrix}
\begin{pmatrix}
    \cosh(\vec{k}_κ x) \\
    \sinh(\vec{k}_κ x) + S_3 \\
    0
\end{pmatrix}.
\end{align*} \quad (8)$$

Note that here we keep $f_0(x)$ as is, because we cannot solve the equation for this component since it contains $\Delta(x)$.

In the middle F1 layer the solution of equation (1) has the following form ($C_1$ and $S_1$ are purely imaginary, $C_3 = -C_2^*$, $S_3 = -S_2^*$):

$$\begin{align*}
&\begin{pmatrix}
    f_0 \\
    f_s \\
    f_2
\end{pmatrix}
= C_1 \begin{pmatrix}
    0 & \cosh(k_ω x) + C_2 & 1 \\
    \cosh(\vec{k}_κ x) + C_3 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
    1 \\
    1 \\
    0
\end{pmatrix}
\begin{pmatrix}
    0 \\
    0 \\
    1
\end{pmatrix}
\begin{pmatrix}
    0 \\
    1 \\
    1
\end{pmatrix}
\begin{pmatrix}
    0 \\
    0 \\
    1
\end{pmatrix}
\begin{pmatrix}
    \cosh(\vec{k}_κ x) \\
    \sinh(\vec{k}_κ x) + S_1 \\
    0
\end{pmatrix}
\begin{pmatrix}
    \cosh(\vec{k}_κ x) \\
    \sinh(\vec{k}_κ x) + S_2 \\
    0
\end{pmatrix}.
\end{align*} \quad (9)$$

In the outer F2 layer ($E_1$ is purely imaginary, $E_3 = -E_2^*$):

$$\begin{align*}
&\begin{pmatrix}
    f_0 \\
    f_s \\
    f_2
\end{pmatrix}
= E_1 \begin{pmatrix}
    0 & -\sin \alpha & 1 \\
    \cos \alpha & \cosh(k_ω x) + E_2 & \cosh(\vec{k}_κ x) + E_3
\end{pmatrix}
\begin{pmatrix}
    1 & \cos \alpha & 1 \\
    \sin \alpha & \cosh(\vec{k}_κ x) + E_3
\end{pmatrix}
\begin{pmatrix}
    0 \\
    0 \\
    1
\end{pmatrix}
\begin{pmatrix}
    0 \\
    0 \\
    1
\end{pmatrix}
\begin{pmatrix}
    -\sin \alpha \\
    \cos \alpha \\
    1
\end{pmatrix}
\begin{pmatrix}
    \cosh(\vec{k}_κ x) \\
    \sinh(\vec{k}_κ x) + H_2 \\
    0
\end{pmatrix}
\begin{pmatrix}
    -\sin \alpha \\
    \cos \alpha \\
    1
\end{pmatrix}
\begin{pmatrix}
    \cosh(\vec{k}_κ x) \\
    \sinh(\vec{k}_κ x) + H_3 \\
    0
\end{pmatrix}.
\end{align*} \quad (10)$$

The boundary conditions at the free interface F2 of the structure take the form:

$$\begin{align*}
&\frac{df}{dx} = 0 \quad \text{at} \quad x = d_1 + d_2. \quad (11)
\end{align*}$$

The boundary conditions at the SF1 and F1F2 interfaces have the form [6]:

$$\begin{align*}
&\begin{pmatrix}
    f_0 \\
    f_s \\
    f_2
\end{pmatrix}
= \begin{pmatrix}
    f_0^{\text{left}} & \xi f_s^{\text{left}} & \xi f_2^{\text{left}} \\
    \gamma f_s^{\text{left}} & \gamma f_2^{\text{left}} & \gamma f_2^{\text{left}}
\end{pmatrix}
\begin{pmatrix}
    f_0^{\text{right}} \\
    f_s^{\text{right}} \\
    f_2^{\text{right}}
\end{pmatrix}.
\end{align*} \quad (12)$$

where $g_*$ and $g$ are the spin-independent suppression parameters:

$$\begin{align*}
&\gamma_{BSF2} = R_{BSF2} A_B / \rho_S, \quad \gamma_{SF} = \rho_{F2} / \rho_S, \\
&\gamma_{BF1F2} = R_{BF1F2} A_B / \rho_{F1F2}, \quad \gamma_{F1F2} = \rho_{F2} / \rho_{F1F2}.
\end{align*} \quad (13)$$

$R_{BSF1S}, R_{BF1F2}$ and $A_B$ are the resistance and the area of the SF1 and F1F2 interfaces, $r_S$, $r_{F1}$ and $r_{F2}$ are the resistivities of the S, F1 and F2 layers, respectively.

We choose the simplest formulation: all the interfaces are transparent ($g_{*} = 0$), the diffusion constants and the conductivities are the same ($g = 1$), the absolute values of the exchange fields in
the two F layers coincide, \( f_0 \) is a constant within the S layer. Our strategy now is to obtain the effective boundary conditions (6) for \( f_0(x) \) by eliminating all other components in the three layers.

The boundary conditions (11) and (12) give 14 equations. We are mainly interested in one of them, determining the derivative of the singlet component on the S side of the SF1 interface (\( x = 0 \)):

\[
\frac{df_0}{dx} = \tilde{k}_S S_2 - \tilde{k}_S S_1. \tag{14}
\]

The remaining 13 boundary conditions form a system of 13 linear equations for 13 coefficients in equations (8)-(10). The solution of this system is nonzero due to \( f_0(x) \) in equation (8). Then, we substitute the \( S_2 \) coefficient into equation (14) and thus explicitly find \( W \) in equation (6). All the information about the two F-layers is contained in the single real-valued function \( W(\alpha) \).

3. Results and discussions

The results of numerical calculations of \( T_c \) as a function of the angle \( \alpha \), and pairing distributions under different values of the layers thicknesses in SF1F2 trilayer are given in figures 2 - 4.

Figure 2 (a) demonstrates the direct spin-valve effect (\( T_{c,AP}(\alpha = 180^\circ) > T_{c,P}(\alpha = 0^\circ) \)). The basic physical reason of the difference \( \Delta T_c = T_{c,AP} - T_{c,P} \) is partial compensation of the pair-breaking ferromagnetic exchange field, when the magnetizations of the F1 and F2 layers are aligned antiparallel. As far as the F-layers are thin compared with the coherent lengths, the compensation is pretty good providing large \( \Delta T_c \). The both triplet pairing components \( f_2 \) and \( f_3 \) tend to have a maximum mostly at the outer surface of the F2 layer.

![Figure 2](image-url)
Figure 3(a) demonstrates the triplet spin-valve effect ($T_c(\alpha = 180^\circ) < T_c(\alpha = 0^\circ)$). In this mode, the oscillating behavior of the singlet superconducting pairing in the F layers is observed. The minimal critical temperature $T_c$ corresponds to the long-range triplet pairing $f_2$ changing sign in the outer F2 layer, and double crossing zero by the singlet pairing component $f_0$ in the F layers (figure 3(c)), which does not present at the maximum temperatures (figures 3(b) and 3(d)). The zero spin-projection triplet component $f_3$ is located close to the S/F1 interface.

Figure 3. Critical temperature $T_c$ vs. misalignment angle $\alpha$ (a). Spin-singlet and spin-triplet pairing distributions at $\alpha = 0^\circ$ (b), $90^\circ$ (c) and $180^\circ$ (d) for $n = 2$; $d_{F1}/\xi_{F1} = 0.73$, $d_{F2}/\xi_{F2} = 2$, and table 1.

Figure 4(a) demonstrates the inverse spin-valve effect ($T_c(\alpha = 0^\circ) > T_c(\alpha = 180^\circ)$). In this mode the long-range triplet pairing component $f_2$ has a maximum at the outer surface of the F2 layers, while the zero spin-projection triplet component $f_3$ is always located close to the S/F1 interface. The minimum critical temperature $T_c$ corresponds to the peculiar behavior – double crossing of zero by the singlet superconducting component $f_0$ in the F layers (figure 4(d)) which is not the case at $\alpha = 0^\circ$ (figure 4(b)).

| Parameter | value |
|-----------|-------|
| $d_s/\xi_s$ | 2.75  |
| $\gamma$ | 1     |
| $\gamma_B$ | 0     |
| $\xi_{F}/\xi_{S}$ | 1     |
Figure 4. Critical temperature $T_c$ vs. misalignment angle $\alpha$ (a). Spin-singlet and spin-triplet pairing distributions at $\alpha = 0^\circ$ (b), $90^\circ$ (c) and $180^\circ$ (d) for $n = 2$; $d_{F1}/\xi_{F1} = 1.1$, $d_{F2}/\xi_{F2} = 0.3$, and table 1.

4. Conclusion
We have considered a finite-thickness SF1F2 spin valve. We visualized distributions of spin-singlet and spin-triplet superconducting pairing components and attributed their peculiarities to different spin-valve switching modes of the SF1F2 trilayer heterostructure. This may be important to plan experimental detecting of the triplet pairings and the inverse proximity effect in SF hybrids by, for example, polarized neutron reflection [7, 8].

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