Tensor effects in shell evolution using non-relativistic and relativistic mean field

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Abstract. Tensor effects in shell evolution are studied within the mean-field approach. Particular attention is paid to the analysis of the proton magic gap \( Z = 8 \). Hartree-Fock calculations with Skyrme and Gogny interactions are performed where the tensor term has a zero and finite range, respectively. Results obtained with and without the tensor component are compared between them and with the experimental data. To complete this analysis, the tensor effect is also investigated within the relativistic Hartree-Fock model.

It turns out that the tensor effect can be easily identified in the evolution of the proton magic gap \( Z = 8 \). We suggest to explicitly include the data associated to this gap evolution as new constraints in the fitting procedures when the tensor contribution is taken into account at the mean-field level.

1. Introduction
The tensor component is a non-central contribution of the nucleon-nucleon interaction which has been extensively discussed in the last years. It is reasonable to expect that some nuclear observables are strongly sensitive to the tensor force owing to the rich spin-isospin structure of finite nuclei. For instance, the shell evolution and the magicity modification far from stability are often interpreted in terms of tensor effects [1]. In the framework of the shell model, the tensor contribution in shell evolution has been explored by Otsuka and collaborators [2-4]. Within the mean-field models, the study of the nuclear structure at the ground-state level is usually done either by using effective nuclear interactions (non-relativistic framework), like the zero-range Skyrme [5-7] or the finite range Gogny [8,9] forces, or by employing covariant effective Lagrangians (relativistic framework). In general, in all these models, the tensor component has been systematically neglected and considered only in very recent investigations [10-20].

In this work, we discuss how the tensor component contributes to the shell evolution at the mean-field level, using non relativistic Skyrme- and Gogny-Hartree-Fock (HF) models as well as relativistic HF (RHF) models. In particular, we are interested in the behavior of the proton
gap at the magic number $Z = 8$. We are mainly interested in the effects due to the neutron-proton interaction related to the tensor contribution: i.e., the effects on proton levels due to the filling of neutron orbits. The theoretical gaps obtained with and without the tensor contribution are compared with the experimental ones to evidence the cases where the tensor effects are unambiguously important in determining the shell evolution.

The article is organized as follows. In Sec. 2 some formal aspects related to the tensor component are briefly recalled for Gogny (2.1), Skyrme (2.2) and RHF (2.3). Results are illustrated in Sec. 3. Conclusions are drawn in Sec. 4.

2. Models
We work in spherical symmetry and consider only even nuclei in all the adopted models.

2.1. Tensor force in the Gogny case
We use the GT2 parametrization [10] that has been introduced by adding to the standard Gogny interaction a term depending on the tensor force only in the isospin-isospin channel. Thus, the new channel that is included in the effective interaction GT2 is:

$$ v_T = F_T \tau_1 \cdot \tau_2 \frac{\left( \frac{\sigma_1 \cdot r_{12}}{(r_{12})^2} \right) \left( \frac{\sigma_2 \cdot r_{12}}{(r_{12})^2} \right) - \sigma_1 \cdot \sigma_2}{f_G(r)}, $$

where $F_T = 50.79506$ MeV and $f_G(r)$ is a Gaussian function, with a range of $1.2$ fm. The parameter $F_T$ has been fixed assuming that the volume integral reproduces that of the AV8'. For simplicity, the range of $f_G(r)$ has been chosen equal to the longest range of the central part.

2.2. Tensor force in the Skyrme case
The tensor interaction was already written as a zero-range force in the original articles of Skyrme [5,6]. It modifies the spin-orbit potential by

$$ \Delta W_{so}^{(q)} = \alpha J_q + \beta J_{q'}, $$

where $q$ ($q'$) indicates protons (neutrons) or neutrons (protons) and $J_q$ is the spin-orbit density:

$$ J_q(r) = \frac{1}{4\pi r^3} \sum_i (2j_i + 1) \left[ j_i (j_i + 1) - l_i (l_i + 1) - \frac{3}{4} \right] R_i^2(r). $$

In the above expression, $R_i$ are the radial wave functions. The parameters $\alpha$ and $\beta$ are expressed as $\alpha = \alpha_C + \alpha_T$ and $\beta = \beta_C + \beta_T$, where $\alpha_C$ and $\beta_C$ are related to the central exchange part of the interaction. They can be written in terms of the usual parameters of the Skyrme interaction: $\alpha_T$ and $\beta_T$ are related to the tensor part. If $\beta > 0$, when neutrons (protons) fill a $j_\pi = l + 1/2$ orbital, the proton (neutron) spin-orbit splitting is reduced because the neutron (proton) spin-orbit density constructed with the $j_\pi$ orbital is positive. This mechanism reproduces the picture of Otsuka and collaborators [3] that implies an attractive (repulsive) neutron-proton interaction between $j_\pi$ and $j'_\pi$ ($j'_\pi$). The mechanism would be the opposite if $\beta < 0$. In this work, we use as an illustration the values of $\alpha_T$ and $\beta_T$ fitted in Ref. [11] on top of the SLy5 parametrization which already contains the central exchange part, i.e., $(\alpha_T, \beta_T) = (-170, 100)$ MeV fm$^5$. With the parametrization SLy5, $\alpha_C$ and $\beta_C$ are equal to 80.2 and -48.87 MeV fm$^5$, respectively (this means that $\beta = \beta_C + \beta_T > 0$ in this case).
2.3. Tensor in RHF

In the relativistic framework, the tensor correlations are deduced from the pion-nucleon and ρnucleon tensor interactions. The part of the Lagrangian containing the pion and ρ-meson fields is written as [20,21]

\[
\mathcal{L}_{\pi+\rho} = + \frac{1}{2} \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} - \frac{1}{2} m_\pi^2 \vec{\pi} \cdot \vec{\pi} - \frac{f_\pi}{m_\pi} \bar{\psi}_\gamma \gamma^\mu \partial_\mu \vec{\pi} \cdot \vec{\tau} \psi + \frac{1}{4} \vec{R}^{\mu\nu} \cdot \vec{R}_{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho} \cdot \vec{\rho} - g_\rho \bar{\psi} \gamma^\mu \vec{\rho} \cdot \vec{\tau} \psi + \frac{f_\rho}{2M} \bar{\psi} \sigma_{\mu\nu} \partial^\mu \vec{\rho}^\nu \cdot \vec{\tau} \psi,
\]

where \( \vec{R}^{\mu\nu} = \partial^\mu \vec{\rho}^\nu - \partial^\nu \vec{\rho}^\mu \). In the parametrization PKA1 [20], the coupling strengths \( f_\pi, g_\rho, \) and \( f_\rho \) are exponentially density-dependent.

In principle, we should perform similar calculations as in the Skyrme and Gogny cases where we switch off the tensor terms without changing the rest of the force. However, using the relativistic Lagrangian, it is difficult to isolate the tensor contributions from the central ones. Setting \( f_\pi = f_\rho = 0 \) would lead to huge changes also in the central part of the mean field and in most cases the mean-field calculation would not even converge. To compare with results without tensor, we thus perform RMF calculations with DDME2 [22] where the pion and ρ-tensor interactions are not included. The results obtained with these two parametrizations do not coincide for the spin saturated nuclei because the two relativistic models PKA1 and DDME2 differ not only by the tensor couplings.

3. Results

The theoretical gaps are calculated as differences of HF single-particle energies. For the experimental data, we have used an approximation (the same procedure is also adopted in Ref. [1]). For the proton gap at \( Z_{\text{magic}} \), we have calculated the single-particle energies of the last occupied (below) and the first unoccupied (above) orbit, \( \epsilon_b \) and \( \epsilon_a \), as:

\[
\begin{align*}
\epsilon_b(Z_{\text{magic}}, N) &= -(E(Z_{\text{magic}}, N) - E(Z_{\text{magic}} - 1, N)) \\
\epsilon_a(Z_{\text{magic}}, N) &= -(E(Z_{\text{magic}} + 1, N) - E(Z_{\text{magic}}, N)) \\
&= -S_p(Z_{\text{magic}} + 1, N),
\end{align*}
\]

where \( E \) are the binding energies and \( S_p \) the proton separation energies. The energy of the gap is then evaluated as \( \epsilon_a - \epsilon_b \). All the experimental values are taken from Ref. [23]. The separation energies are supposed to be very similar to the single-particle energies if one assumes that the core \( Z_{\text{magic}} \) remains almost unchanged when one nucleon is added to or removed from the magic core. This assumption is of course more adapted for treating spherical nuclei rather than deformed ones where the presence of strong correlations may deteriorate the quality of this schematic picture. We stress that we consider here the experimental values just as indications to provide qualitative (and not precise) experimental trends to compare with the theoretical results.

We calculate the difference of the HF single-particle energies between the first empty level (proton 1d\( _{5/2} \)) and the last occupied one (proton 1p\( _{1/2} \)). It is worth noting that \( d_{5/2} \) is the \( j_\pi = l + 1/2 \) orbit of the \( d \) spin-orbit pair while \( p_{1/2} \) is the \( j_\pi = l - 1/2 \) orbit of the \( p \) spin-orbit partners. The two orbitals will be thus affected in the opposite way by the tensor force. Going from \( ^{16}\text{O} \) to \( ^{22}\text{O} \), the neutron \( d_{5/2} \) orbital \( (j_\pi') \) is filled. Owing to this, one expects that the energy of the proton \( p_{1/2} \) state is lowered whereas that of the proton \( d_{5/2} \) one is pushed up by
the tensor force. The values of the gaps obtained for $^{16}$O and $^{22}$O are shown in the three panels of Fig. 1 for the Gogny (a), Skyrme (b) and relativistic (c) cases, respectively. Results obtained with the tensor contribution (full lines) are compared with the corresponding results obtained without tensor (dotted lines) and with the experimental values (dashed lines). In this figure (and in the following ones) the lines are plotted only to guide the eye and see more clearly the trend from one nucleus to the other.

Figure 1. Proton gap $Z = 8$ with Gogny (a), Skyrme (b) and relativistic calculations (c). In GT2$_{nT}$ the tensor part of GT2 has been suppressed. In SLy5$_{wT}$ the tensor part has been added on top of the SLy5 parametrization.

Figure 2. (a) Proton gap $Z = 8$ with GT2 for different choices of the parameter $F_T$. The current choice is indicated as $F_T$ in the figure. The other choices have been drawn relative to it. (b) Proton gap $Z = 8$ with Skyrme SLy5 for different choices of the parameter $\beta_T$. The current choice is indicated as $\beta_T$ in the figure. The other choices have been expressed in terms of it. The parameter $\alpha_T$ is kept equal to -170 MeV fm$^5$ in all cases.

Going from $^{16}$O to $^{22}$O, the tensor produces, as expected, an increase of the gap energy of 4 MeV approximately for the GT2 interaction and 2 MeV for the Skyrme and the PKA1-RHF cases (one observes that the gap increases more strongly with PKA1 than with DDME2 going from $^{16}$O to $^{22}$O). In the relativistic case, it is important to notice that the two calculations with and without tensor are not based on the same Lagrangian. This explains why the results obtained with DDME2 (no tensor) and PKA1 (with tensor) do not coincide for the spin saturated nucleus $^{16}$O. The effect of the tensor force is very clear in all the calculations: the slope is always increased by tensor going from $^{16}$O to $^{22}$O and the gap $Z = 8$ is more strongly enhanced with respect to the results without tensor. This is coherent with what expected according to the mechanism described in Ref. [3]. However, the experimental gap decreases when going from $^{16}$O to $^{22}$O: the magicity at $Z = 8$ is reduced when one goes towards the neutron drip line. This means that the tensor contribution acts in the opposite way with respect to what would be necessary to reproduce the experimental trend.

To analyze this more in detail, we consider as an illustration the case of Skyrme: it has been underlined that the predicted trend depends on the parameters, namely on the sign of the parameter $\beta$ which is positive in the present calculations ($\beta = \beta_T + \beta_C = 51.13$ MeV fm$^5$). Since oxygen isotopes are spin saturated for protons, the tensor effect is practically due only to the filling of neutron orbitals and is only sensitive to the parameter $\beta$. A value of $\beta < 0$ would trivially lead to a decreasing of the gap from $^{16}$O to $^{22}$O: we plot in Fig. 2 (b) the gap $Z = 8$ for
the nuclei $^{16}$O and $^{22}$O for different values of $\beta$ while $\alpha$ is kept fixed at the value $\alpha_T + \alpha_C = -89.8$ MeV fm$^5$. For $\beta_T = 0$, -50 and -100 MeV fm$^5$, $\beta$ is negative and the slope for the gap can be modified. For Gogny, the tensor parameter (and its sign) is fixed on a realistic case. However, if $F_T$ in Eq. (1) is treated as a free parameter one can obtain similar results as in the Skyrme case by changing its sign (see Fig. 2(a)).

In the relativistic case the scenario is more complicated and the tensor contribution cannot be easily isolated. $^{16}$O is a $Z = N$ nucleus. To extract the experimental single-particle energies, we have used an approximate procedure where masses of nuclei are employed. This is of course an approximation which is valid in nuclei where correlations are quite weak, as it has been already emphasized. Magic nuclei are thus expected to be typically good systems where this approximation can be used. However, the case of $N = Z$ nuclei deserves a special attention. In these nuclei, the last orbitals for protons and neutrons are quite similar and this feature strongly favors correlations associated to neutron-proton pairing. The problem of extracting single-particle energies from the experimental masses in this case is thus very crucial. The so-called Wigner energy is used in many cases to actually correct these values, even if a consensus on this specific point and on the procedures to use is still missing. Several procedures have been adopted (see, e.g., Ref.[24]). Since the problem is still open, we have decided not to include this correction in the previous figures but we evaluate here the importance of this eventual correction on the gap for $^{16}$O. By using as an illustration one of the prescriptions introduced in Ref. [25], we have estimated a correction of -3.7 MeV for $^{16}$O. Fig. 3 shows the same quantities as Fig. 1 but the experimental data corrected by the Wigner energy are also plotted. This figure gives an idea of how much delicate is actually this point. The experimental slope is strongly changed if the correction is applied. Before making any fit on experimental values in these region, one should thus finally clarify this point on the way of extracting the experimental energies.

![Figure 3](image-url)

**Figure 3.** The same as Fig. 1, but including the Wigner correction for the experimental data (red dashed line).

4. Conclusions

We have analyzed the effects of the tensor force in the shell evolution at the magic number $Z = 8$ within non-relativistic and relativistic mean-field approaches. A more systematic analysis extended also at the cases $Z = 20$ and 28 and $N = 8$, 20 and 28 is done in Ref. [26].

The experimental values have been estimated by using the same approximated procedure as that of Ref. [1]. The issue of the Wigner correction has been discussed. We have neglected pairing correlations in our analysis and performed only Hartree-Fock calculations for all nuclei,
both closed and open-shell. The main reason for this is that the GT2 parametrization which we adopt in the Gogny case has been fitted at the Hartree-Fock level. It is also important mentioning that our mean field predictions for the gaps are expected to be modified by beyond-mean-field effects which we are not taking into account, e.g., the particle-vibration coupling.

The considered nuclei are spin saturated for protons and the only tensor contribution comes from the filling of neutrons (neutron-proton interaction). The tensor effect is easily identified in the evolution of the $Z = 8$ magic gap. In the two non relativistic cases, we have observed that it is possible to modify the theoretical trends by changing the signs of the parameter $\beta$ in the Skyrme case and $F_T$ in the Gogny case. This means that, in principle, a fitting procedure including the experimental gap evolution at the magic number 8 could generate some sets of parameters able to reproduce these experimental trends owing to the tensor contribution.

To conclude, since the evolution of the gaps is an important feature characterizing exotic nuclei we strongly suggest that the observables related to this feature should be included in the fit procedures when the tensor terms have to be constrained. It is also important to properly choose the regions where to perform these fits and we recommend $Z = 8$ as a suitable region where the role played by the tensor force can be identified and is not mixed with other mean-field effects.

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