Theoretical Model for Prediction of Durable Life of RC Square Piles under Marine Environment

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Abstract. This paper presents a model for predicting the durable life of reinforced concrete (RC) square piles under marine environment. Firstly, according to Fick's first law and mass conservation law, the diffusion equation of chloride ion in RC square pile is established, and found the analytical solution of the diffusion equation; secondly, the critical rust depth of the concrete cover is generated according to Faraday's law. Finally, the purpose of predicting the durable life of RC square piles is achieved.

1. Introduction

Corrosion of steel bars caused by chloride ions is one of the main reasons for the deterioration of RC square piles, which has seriously affected the durability life of RC square piles. The steel bars in the concrete square piles are in a highly alkaline medium before the chloride ion is eroded, and a passivation film is formed on the surface. The passivation film effectively suppresses the corrosion of the steel bars. However, when the square pile is in the chloride ion environment, the chloride ions will pass through the concrete square pile protective layer and reach the steel surface through diffusion, capillary adsorption, infiltration and electrochemical migration, once the chloride ion concentration reaches the critical chloride ion concentration. And if there is enough moisture and oxygen, the passivation film on the surface of the steel will be damaged, and the steel will be rusted (Chen and Mahadevan, 2008 [1]). Steel corrosion will produce a large amount of rust products, which are about 2 to 6 times the volume of the original reinforcement (Lu et al., 2011a [2]). The volume expansion of the rust product will cause the expansion stress inside the concrete cover. When the expansion stress exceeds the tensile strength of the concrete cover, it will cause the crack generation and expansion of the concrete cover (El Maaddawy and Soudki, 2007 [3]). At the same time, the corrosion of steel bars caused by chloride ions will also cause the effective section of the steel bars to decrease, and the bond between the steel bars and the concrete cover will degenerate, eventually leading to the loss of structural properties of the RC square piles (Martín-Pérez et al., 2000 [4]).

In this paper, the corrosion process of chloride ion on RC square pile is divided into two stages: corrosion initiation and crack initiation. Firstly, according to Fick's first law and mass conservation law, the diffusion equation of chloride ion in RC square pile is established. According to the initial and boundary conditions of the diffusion equation, the analytical solution of the diffusion equation is derived, and the prediction of durable life for corrosion initiation is realized; secondly, based on the
thick-walled cylinder theory, the critical rust depth of the steel bar when the rust expansion crack is generated in the concrete cover is derived, and the critical rust depth of the concrete cover is generated according to Faraday's law. The theoretical model of durable life prediction of chloride ion-contaminated RC square piles is established.

2. Durable life analysis of RC square piles

2.1. Corrosion initiation

The traditional chloride ion diffusion model was first proposed by Collepardi, which assumes that the concrete is a homogeneous, isotropic material, and that the materials in the concrete do not react with chloride ions. The transport of chloride ions in concrete can be described by Fick's law, and the expression of the diffusion equation is

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2}$$

where \( t \) is the chloride ion diffusion time; \( C \) is the chloride ion concentration; \( D \) is the chloride ion diffusion coefficient; \( x \) is the depth of the chloride ion diffusion, that is, the distance from the boundary.

Under the assumption that the premier chloride concentration in the RC square pile is 0 and the chloride ion concentration on surface is constant. It may be obtained that the initial condition is \( t=0, x>0, C=C_0 \); the boundary conditions are \( x=0, t>0, \) and \( C=C_s \). According to the initial and boundary conditions, the analytic solution can be obtained as

$$C = C_0 + (C_s - C_0) \left[ 1 - \text{erf} \left( \frac{x}{2\sqrt{Dt}} \right) \right]$$

where \( C_o \) is the initial chloride concentration; \( C_S \) is the chloride ion concentration on surface; \( \text{erf} \) is the error function.

According to the traditional chloride ion diffusion model, it is assumed that there is no chloride ion initially in the RC square pile, and there is also a chloride ion diffusion coefficient in a constant state. Combined with Fick's second law, the diffusion equation in RC square piles is expressed as

$$\frac{\partial C}{\partial t} = D \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right)$$

where \( D \) is the chloride ion diffusion coefficient, \( m^2/s; t \) is the chloride ion diffusion time, \( s; C \) is the current concentration of chloride ion mass, \( kg/m^3 \). The initial conditions and boundary conditions corresponding to equation (3) are

$$\begin{cases}
C(P, \Omega, t = 0) = C_0 \\
C(P, \Gamma, t \neq 0) = C_s
\end{cases}$$

where \( C_r \) is the chloride ion concentration on the surface (\( kg/m^3 \)); \( \Omega \) is the transport space of chloride inside the concrete; \( P \) is any point in the RC square pile; the coordinates are \( P(x, y) \); \( \Gamma \) is the boundary of concrete members with at least one of \( x, y \) is zero. Assuming that the diffusion coefficient and the chloride ion concentration on the surface are both constant, the solution of Eqs. (4) is

$$C = C_0 + (C_s - C_0) \left[ 1 - \text{erf} \left( \frac{x}{2\sqrt{Dt}} \right) \text{erf} \left( \frac{y}{2\sqrt{Dt}} \right) \right]$$
The error function in Eq. (5) can be expressed as

\[ \text{erf}(u) = \frac{2}{\sqrt{\pi}} \int_{0}^{u} e^{-t^2} dt \] (6)

As the surface chloride concentration of the rebar achieves the critical chloride ion concentration \(C_{th}\), the corrosion initiation period \(t_i\) can be determined by verifying the condition \(C = C_{th}\).

2.2. Crack initiation

2.2.1 Square pile cover cracking

This paper assumes that the thickness of the concrete cover is \(c\), the initial diameter of the rebar is \(d_0\), and there is a porous area at the boundary surface between the rebar and the concrete called thick-walled concrete cylinder model. The porous zone is evenly distributed and its thickness is \(\delta_0\). The steel corrosion product will produce radial expansion stress \(p_r\) on the concrete cover at the interface. As the circumferential tensile stress in the concrete cover at the boundary surface reaches the tension strength of the concrete cover, it will generate cracks. Under the action of the radial expansion stress \(p_r\), the radial displacement at the boundary surface between the steel bar and the corrosion product is \(\delta_c\), which can be expressed as

\[ \delta_c = \frac{r_0}{E_{cef}} \left[ \frac{(r_0 + c)^2 + r_0^2}{(r_0 + c)^2 - r_0^2} + \nu_c \right] p_r \] (7)

where \(\nu_c\) is the Poisson's ratio of the concrete cover, and \(r_0 = d_0/2 + \delta_0\). Considering the creep effect of the concrete cover, the effective elastic modulus \(E_{cef}\) is (Bhargava et al. 2005 [5])

\[ E_{cef} = \frac{E_c}{1 + \varphi_c} \] (8)

where \(\varphi_c\) is creep coefficient of concrete cover. Defining \(\eta = 2r_0^2 / c(2r_0 + c)\), then Eq.(7) can be rewritten as

\[ \delta_c = \frac{r_0}{E_{cef}} \left[ \eta + 1 + \nu_c \right] p_r \] (9)

With the corrosion continues, the increase in volume can be expressed as (El Maaddawy and Soudki 2007 [3])

\[ \frac{M_r}{\rho_r} - \frac{M_{loss}}{\rho_s} = \pi \left[ d_0 (\delta_0 + \delta_c) + (\delta_0 + \delta_c)^2 \right] \approx \pi d_0 (\delta_0 + \delta_c) \] (10)

where \(\rho_r\) and \(\rho_s\) are the density of corrosion and steel, separately; and \(M_r\) and \(M_{loss}\) are the mass of rust and steel corrosion, respectively. Defining \(\alpha_s\) as the volume expansion ratio and based on Eq. (9) and Eq. (10), the radial expansion stress \(p_r\) can be given by

\[ p_r = \frac{E_{cef}}{(\eta + 1 + \nu_c)} \left[ \frac{M_{loss}}{\rho_s} \left( \alpha_s - 1 \right) - \frac{\delta_0}{r_0} \right] \] (11)
According to the research, the thickness of the concrete cover ($c$) is related to the tension strength of the concrete cover ($f_{ct}$). The equation can be expressed as (Papakonstantinou and Shinozuka 2013[6])

$$p_{cr} = \frac{2cf_{ct}}{d_0}$$

(12)

where $p_{cr}$ is critical expansion stress. Based on Faraday's law, the mass of steel erosion can be calculated from the following equation

$$\frac{dM_{loss}}{dt} = \frac{I_{corr}A}{zF}$$

(13)

where $I_{corr}$ is the corrosion current (A); $A$ is the atomic weight of iron ions (g/mol); $z$ is the valence, generally assuming $z=2.5$; $F$ is the Faraday constant (C/mol); $t$ is the erosion time of the steel (s).

Equation (13) can be simplified as

$$\frac{dM_{loss}}{dt} = 2.315 \times 10^{-4} I_{corr}$$

(14)

Assuming that $d_0$ (mm) is initial diameter of the steel and the rebar per unit length $L=1$ m. The corrosion current $I_{corr}$ can be expressed as

$$I_{corr} = \pi d_0 i_{corr} L = 10^{-3} \pi d_0 i_{corr}$$

(15)

where $i_{corr}$ is the erosion current density ($\mu$A/cm$^2$). Substituting the Eq. (15) into the Eq. (14), the mass of steel corrosion can be given by

$$M_{loss} = 2.315 \times 10^{-9} \pi d_0 i_{corr} t$$

(16)

When $p_r$ equals $p_{cr}$, the concrete cover crack occurs. According to Eq. (16), the durable life of the crack initiation can be obtained as

$$t_i = 4.32 \times 10^8 \frac{r_0 \rho_s}{(\alpha_v - 1) i_{corr}} \left[ \frac{2c(\eta + 1 + \nu_c)}{d_0} \frac{f_{ct}}{E_{cef}} + \delta_0 \right]$$

(17)

3. **Illustrative example**

Assume that the section size of RC square pile is 500 mm×500 mm, the thickness of concrete cover is $c=50$ mm, the diameter of steel bar is $d_0=20$ mm, the surface chloride ion concentration of square pile is $C_s=4.0$ kg/m$^3$, and the diffusion coefficient of chloride ion is $D=0.6 \times 10^{-12}$ m$^2$/s, the critical chloride ion concentration is $C_{th}=1.4$ kg/m$^3$. According to Eq. (5), the durable life of RC square pile for corrosion initiation is $t_i=39.21$ years.

3.1 **Analysis of dominant factors for corrosion initiation**

From the chloride ion diffusion model, it can be known that the durable life of RC square piles in the marine environment for corrosion initiation is mainly affected by factors such as surface chloride ion concentration, chloride ion diffusion coefficient, concrete cover thickness and critical chloride ion concentration.

The relationship between the durable life of RC square piles for corrosion initiation and the chloride ion diffusion coefficient is shown in Fig. 1. With the increase of the chloride ion diffusion coefficient, the durable life for corrosion initiation decreases sharply. This is because the diffusion...
Coefficient of chloride ions increases, which accelerates the diffusion of chloride ions in the RC square piles, thereby reducing the durable life for corrosion initiation. Therefore, the diffusion coefficient of chloride ions in the RC square pile can be reduced by increasing the density of the concrete cover of the square pile, thereby achieving the purpose of prolonging the durable life for corrosion initiation.

The relationship between the durable life of RC square piles for corrosion initiation and the surface chloride ion concentration in the RC square pile is depicted in Fig. 2. As the surface chloride ion concentration increases, the durable life of RC square piles for corrosion initiation decreases sharply. This is because the increase of the concentration of chloride ions on the surface promotes the concentration gradient of chloride ions inside and outside the RC square pile, which accelerates the diffusion of chloride ions in the square piles, thus accelerating the end of the durable life of RC square piles for corrosion initiation. Therefore, the surface of the pile can be coated with a waterproof material such as epoxy resin to reduce the chloride ion concentration on the surface of the RC square pile, thereby prolonging the durable life of RC square piles for corrosion initiation.

The relationship between the durable life of RC square piles for corrosion initiation and the critical chloride ion concentration in the RC square pile is expressed in Fig. 3. The critical chloride ion concentration has a significant effect on the durable life of RC square piles for corrosion initiation. As the critical chloride ion concentration increases, the durable life for corrosion initiation increases significantly. This is because the increase of the critical chloride ion concentration prolongs the time that the chloride ion concentration on the surface of the steel bar reaches the critical chloride ion concentration, thereby prolonging the durable life for corrosion initiation.

The relationship between the durable life of RC square piles for corrosion initiation and the thickness of the concrete cover is shown in Fig. 4. As the thickness of the concrete cover increases, the durable life for corrosion initiation increases significantly. This is because the increase in the thickness of the concrete cover prolongs the time that chloride ions reach the surface of the steel bar, thereby prolonging the durable life of RC square piles for corrosion initiation.
3.2 Analysis of dominant factors for crack initiation

Assuming that the tensile strength of the square pile concrete cover is $f_{ct}=1.71$ MPa, the concrete elastic modulus is $E_c=32.5$ GPa, the initial diameter of the steel bar is $d_0=20$ mm, the thickness of the concrete cover is $c=50$ mm, and the concrete Poisson’s ratio is $\nu_c=0.18$. The volume expansion ratio of the rust product is $\alpha_v=3.0$, the thickness of the pore region is $\delta_0=12.5$ μm, the creep coefficient of concrete is $\phi_c=2.0$, the mass density of the steel bar is $\rho_s=7.85$ g/cm$^3$, and the corrosion current density is $i_{corr}=1.2$ μA/cm$^2$. According to Eq. (17), the durable life of RC square pile for crack initiation is $t_c=1.0$ years. It can be known from the above theoretical model that the durable life of RC square pile in the marine environment for crack initiation is mainly influenced by factors such as volume expansion ratio, concrete cover thickness, concrete cover tensile strength, steel corrosion rate, and concrete elastic modulus.

The relationship between the durable life for crack initiation and the volume expansion ratio of the rust product is shown in Fig. 5. As the volume expansion ratio increases, the durable life for crack initiation gradually decreases. This is because the volume expansion ratio of the rust product increases, and the critical rust amount required for the critical rust expansion stress of the concrete cover is reduced, thereby reducing the durable life for crack initiation. Therefore, the oxidation level of the steel bar can be reduced by reducing the supply of oxygen and moisture during the corrosion process of the steel bar, thereby achieving the purpose of extending the durable life for crack initiation period.

The relationship between the durable life for crack initiation and the thickness of the concrete cover is presented in Fig. 6. It can be seen from the figure that as the thickness of the concrete cover increases, the durable life for crack initiation enhances significantly. This is because the increase of the thickness of the concrete cover causes the increase of critical rust expansion stress of the concrete cover, which in turn leads to an enhancement in the critical rust amount required for the critical rust expansion stress of the concrete cover. Therefore, increasing the concrete cover thickness is a very effective method to prolong the durable of the RC square pile. The increase of the concrete cover can not only extend the durable life for corrosion initiation of the steel bar, but also is very effective for preventing the rust expansion crack of the RC square pile.

The relationship between the durable life for crack initiation and the tensile strength of the concrete is shown in Fig. 7. As the tensile strength of the concrete increases, the durable life for crack initiation improves gradually because the increase of the tensile strength causes an enhancement in the radial expansion stress, thereby reducing the durable life for crack initiation period.

The relationship between the durable life for crack initiation and the corrosion rate of the steel bar is depicted in Fig. 8. As the corrosion rate of the steel increases, the durable life for crack initiation is significantly reduced, because the increase of rebar corrosion rate of reinforcement corrosion leads to
the decrease of the time for the amount of steel corrosion to reach the critical amount of corrosion, which causes the reduction of the durable life for crack initiation.

The relationship between the durable life for crack initiation and the concrete elastic modulus is shown in Fig. 9. As the elastic modulus of the concrete improves, the durable life for crack initiation gradually decreases. This is because the increase in the concrete elastic modulus causes an enhancement in the critical expansion stress, thereby prolonging the durable life for crack initiation period.

4. Conclusions
In this paper, the corrosion process of chloride ion in RC square piles is divided into two periods: corrosion initiation and crack initiation. Firstly, according to Fick's first law and mass conservation law, the diffusion equation of chloride ion in RC square pile is established. According to the initial and boundary conditions of the diffusion equation, the analytical solution of the diffusion equation is obtained. Furthermore, the prediction of the durable life for corrosion initiation of steel bars is realized; secondly, based on the theory of thick-walled cylinders, the critical corrosion depth of the steel bars in the rust expansion cracks of the concrete cover is derived, and according to Faraday's law, the critical corrosion of the concrete cover when the rust expansion crack is generated is established. The relationship between the depth and the corrosion rate of the steel bar has realized the prediction of the durable life for crack initiation, and established the theoretical model of the life prediction of the chloride ion-contaminated RC square pile. According to the analysis results, the following conclusions can be obtained:

(I) The increase of surface chloride ion concentration and chloride ion diffusion coefficient accelerates the diffusion of chloride ions in the RC square pile, thus reducing the durable life for
corrosion initiation. The increase of the concrete cover thickness can not only prolong the durable life for corrosion initiation, but also increase the critical rust expansion stress of the rust expansion crack of the concrete cover, which leads to the enhance of the critical rust required for the critical rust expansion stress of the concrete cover, thereby prolonging the durable life for crack initiation. The increase of the critical chloride concentration extends the time that the chloride ion concentration on the surface of the steel bar reaches the critical chloride ion concentration, thereby prolonging the durable life for corrosion initiation.

(II) The increase of the corrosion rate of the steel reduces the time that the amount of steel corrosion reaches the critical amount of corrosion, thereby reducing the durable life for crack initiation. The increase of the tensile strength of the concrete causes an improvement in the radial expansion stress, thereby decreasing the durable life for crack initiation. The enhancement of the volume expansion ratio of the rust product reduces the critical rust amount required for the critical rust expansion stress of the concrete cover, thereby reducing the durable life for crack initiation. The increase of the elastic modulus of the concrete causes an increase in the critical expansion stress, thereby prolonging the durable life for crack initiation period.

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