The effect of the trapping of dust grains on the sheath structure and the charging in a plasma

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Abstract

A model of the trapping of dust grains is shown in a plasma for the first time. The multiple sheath potential, the space charge density and the multiple electric field associated with our model are simulated. Our result explains the confinement of the dust grains observed in the experiment. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Due to the development of plasma engineering, the dust grains attract the attention [1–3]. It is well known that the dust grains occur in the devices of the plasma process and near the insulator materials of fusion reactors [4,5]. Recently, the experiments in relation to trapped dust grains are performed [6–8]. The investigation of trapped dust grains is important in making a plan for the removal of the contamination of dusts in plasma devices. However, not many simulational works have treated the model of the trapped dust grains. In order to simulate the behavior of trapped dust grains and the sheath structures, we obtain the trapped dust density by integration. For the purpose of the simulation, we treat the plasma with the components of electrons, positively charged potassium ions and negatively charged kaolinite Al₂Si₂O₅(OH)₄ as dust grains. Since the behavior of the plasma varies due to the materials of the dust grains, we study the dependency of the plasma on the physical parameters.

In Section 2, we show the modeling of the plasma including trapped dust grains. In Section 3, we illustrate the sheath potential and the electric field by changing the parameters such as the secondary electrons, the ion density and temperature. For numerical calculation, we consider the electrons, K⁺ ions and negatively charged kaolinite ions as dusts. We also investigate the charging process of dust grains. In Section 4, we compare our simulation with the observation results, and summarize this investigation.

2. The modeling of the trapped dust grains

We assume that the orbits of the dust grains are finite and close. We obtain the density by integrating the distribution function with the velocity components of the radial and tangential directions

\[ f_d = n_d \left( \frac{m_d}{2\pi k_B T_d} \right)^{3/2} \exp \left[ - \left( \frac{m_d}{2k_B T_d} \right)(v_r^2 + v_\theta^2) - \frac{Ze\Phi}{k_BT_d} \right], \]

where \( k_B \), \( T_d \), \( n_d \), \( v_r \), \( v_\theta \), \( m_d \), \( e \) and \( Z \) are the Boltzmann constant, dust temperature, background dust density, radial velocity, tangential velocity, dust grain mass, electronic charge and charge number, respectively. We consider the electrons of the Boltzmann distribution, flowing ions, secondary electrons and trapped dust grains in the plasma. In the sheath, since the potential is negative, Poisson’s equation is

\[ -\frac{d^2 \Phi}{dx^2} = \frac{e}{\varepsilon_0}(n_e - n_i - n_s + Z n_d), \]

where \( \Phi \) and \( \varepsilon_0 \) are the electrostatic potential, one dimensional direction and permittivity in vacuum, respectively, and \( n_e \), \( n_i \), \( n_s \) and \( n_d \) are the densities of electrons, positive ions, secondary electrons and dust grains, respectively. The right hand side of this equation expresses the space charge density \( \rho \). We simulate the sheath voltage, the space charge density and the electric field by solving this equation. The
densities $n_e$, $n_i$, $n_a$, and $n_d$ are

$$n_e = n_{e0} \exp\left(-\frac{e\Phi}{k_B T_e}\right), \quad (2)$$

$$n_i = \frac{n_{i0}}{\sqrt{1 + \frac{2e\Phi}{k_B T_e (v_i^2/m_i)(v_i^2/2 - T_i/T_e)}}}, \quad (3)$$

$$n_a = 14.8 \frac{\delta_m}{E_m} n_{e0} \exp\left(-\frac{e\Phi}{k_B T_e} + aZ\right) \frac{2e\Phi_w}{k_B T_e \sqrt{1 - \Phi/\Phi_w}}, \quad (4)$$

$$n_d = n_{d0} \left\{ \exp\left(-\frac{Ze\Phi}{k_B T_{dif}}\right) \left(1 - \text{erf}\left(\frac{Ze\Phi}{k_B T_{dif}}\right)\right) \right. \\
+ \frac{2}{\sqrt{\pi} T_{dif}/T_{dif}} \left(\frac{Ze\Phi}{k_B T_{dif}} - \left(\frac{-\Phi + \Phi_0}{Z}\right)Ze\right) \right. \left. \right. \\
+ \frac{1}{1 + \sqrt{T_{dif}/T_{dif}}} \exp\left[\frac{-Ze(\Phi - \Phi_0)}{k_B T_{dif}}\right] \right. \\
\times \text{erf}\left[\left(1 + \frac{1}{\sqrt{T_{dif}/T_{dif}}}\right)\left(\frac{-Ze(\Phi - \Phi_0)}{k_B T_{dif}}\right)\right] \right\}. \quad (5)$$

Here we define the parameter that $\Phi = e\Phi/k_B T_e$, $\xi = x/\lambda_{D_e}$, $M = v_i/v_A$, $\mu_i = m_i/m_e$, $a = e^2/4\pi\varepsilon_0 r_T T_e$, $\phi_0 = e\Phi_0/k_B T_e$, where the Debye length $\lambda_{D_e}$, $v_i$, $v_A$, $T_{dif}$ and $T_{dif}$ denote the Debye length, ion velocity, ion acoustic velocity, free and trapped dust temperatures, respectively. We use these parameters in Section 3.

3. Simulation of the sheath structure

For the purpose of the study of the interaction between the plasma and the wall, we perform the simulation of the sheath structure. For numerical calculation, we limit the components of the plasma to electrons, K$^+$ ions and negatively charged kaolinite ions as dusts in this section. Fig. 1 shows the voltage with $\tau_i = T_i/T_e = 0.02$, 0.1 and 0.5, where the parameters $\delta_i = n_{d0}/n_{a0} = 25$, $\tau_i = T_{dif}/T_{dif} = 10$, $\tau_i = T_{dif}/T_e = 1$, $a = 1.44 \times 10^{-3}$, $Z = 100$, $\phi_0 = 0$, the wall potential $\phi_w = -10$, the secondary electron coefficient $\delta_m = 1.3$, the maximum energy of secondary electrons $E_m = 600$, the Mach number $M = 1.5$ and $\mu_i = m_i/m_e = 71787.6$ for potassium ions. We assume that ions, dust grains and electrode are potassium, kaolinite and tantalum. The sheath voltage decreases in the vicinity of $x/\lambda_D = 0.24$ in Fig. 1. It forms double sheath and does not depend on the ion temperature.

We show the magnified figure in the region $x/\lambda_D = 0.49$–0.52 of Fig. 1 in Fig. 2. It is observed that the triple sheath structure is formed. The voltages for $\delta_i = 3, 25$ and 200 are shown in Fig. 2, where $\tau_i = 0.1$, $\tau_i = 10$, $\tau_i = 1$, $a = 1.44 \times 10^{-3}$, $Z = 100$, $\phi_0 = 0$, $\phi_w = -10$, $\delta_m = 1.3$, $E_m = 600$, $M = 1.5$ and $\mu_i = 71787.6$. The single sheath voltage is formed when $\delta_i = 3$. The drop in the voltage is shown in Fig. 2. Fig. 3 shows the electric field corresponding to Fig. 2. The single, triple and double electric fields are observed for $\delta_i = 3, 25$ and 200, respectively. It turns out that the electric field depends on the ion density.

Next, we show the sheath structure including the secondary electrons. Figs. 4 and 5 show the examples of the voltage and electric field, where $\tau_i = 0.1$, $\delta_i = 3$, $\delta_i = 0.1$, $\tau_i = 0$, $\tau_i = 1$, $a = 1.44 \times 10^{-3}$, $Z = 100$, $\phi_0 = 0$, $\phi_w = -10$, $\delta_m = 1.3$, $E_m = 600$, $M = 1.5$, and $\mu_i = 71787.6$.

4. The charging of the dust grains

As the compositions of the current flowing to the dust grains, we consider the electrons, secondary electrons and positive ions. In the bulk plasma, the conservation law of the charge and current is

$$\frac{dQ}{dt} = J_e + J_s + J_i. \quad (6)$$

![Fig. 1: Potential profiles of the sheath for the solid and dotted ($\tau_i = 0.02$), solid($\tau_i = 0.1$) and dotted ($\tau_i = 0.5$) lines, respectively.](image-url)
We obtain \( t_0 = \frac{k_B T_e}{4 \pi e^2 n_0 \nu_{te}} \) by solving \( \frac{dQ}{dt} = 0 \). The quantities, \( J_e, J_s, \) and \( J_i \) are the currents of electrons, secondary electrons and positive ions, respectively. They are \( J_e = -J_0 \nu_{te} \exp(\alpha Z_d), \ J_s = J_0 (14.8 \delta_m/E_m) J_c, \ J_i = J_0 \sqrt{\tau_i/\mu_i} (1 - \alpha Z_d/\tau_i) n_i, \) where \( J_0 = 4 \pi e^2 e n_0 \nu_{te} \) and \( \nu_{te} \) is the thermal velocity. Substituting \( J_e, J_s, J_i \) into Eq. (6), we reduce Eq. (6) to

\[
\frac{dQ}{dt^*} = -\left(1 - 14.8 \frac{\delta_m}{E_m}\right) \exp(\phi + \alpha Z_d) \\
+ \delta_i \sqrt{\frac{\tau_i}{\mu_i}} \left(1 - \frac{\alpha Z_d}{\tau_i}\right) \frac{\delta_i}{\sqrt{1 - \frac{2 \phi}{\mu_i (M^2 - \tau_i)}}}.
\]

(7)

Here we define \( t^* = t/t_0 \), where \( t_0 \) is the time taken to attain the equilibrium charge.

We show the example of the charging of the dust grains in Fig. 6. These curves are the case of \( \tau_i = 0.02, 0.1 \) and 0.5

\[
\frac{dQ}{dt^*} = -\left(1 - 14.8 \frac{\delta_m}{E_m}\right) \exp(\phi + \alpha Z_d) \\
+ \delta_i \sqrt{\frac{\tau_i}{\mu_i}} \left(1 - \frac{\alpha Z_d}{\tau_i}\right) \frac{\delta_i}{\sqrt{1 - \frac{2 \phi}{\mu_i (M^2 - \tau_i)}}}.
\]

(7)

where \( T_e = 1 \text{ eV}, \mu_i = 71787.6, \delta_i = 25, a = 1.44 \times 10^{-3}, \delta_m = 1.3 \) and \( E_m = 600 \), respectively.

5. Discussion

In this investigation, we simulate the behavior of trapped dust grains when we assume the kaolinite as the dust grain. We consider that the confinement of the dust grains occurs at the peak of the curve of the electric field. We find the following facts:

- When the ion temperature increases, there is no change in the sheath structure.
- When the temperature of trapped dust grains increases, the trapped region of dust grains becomes wider.
- When the ion density changes, the trapping of dust grains changes more complicatedly.
- When the charge number increases, the sheath structure changes from triple to quadrupole according to the ion density.


fig. 6. The charge number of the dust grains depends on the time, where the curves denote the solid and dotted (τₖ = 0.02), solid (τₖ = 0.1) and dotted (τₖ = 0.5) lines, respectively.

fig. 7. Comparison between the cases of our result and the experimental one.

- When the secondary electrons are included, the confinement of dust grains becomes complicated.
- When the ion temperature increases and the ion density decreases, the charge number increases.

In Fig. 7, we make a sketch of the cloud from a picture of the experimental result [7]. In the laboratory plasma, it is reported that the thickness of the sheath in Ref. [8] lies in the range from 1 to 3λₑ, i.e., 1.0–7.7 mm, where \( n_{e_0} = 6 \times 10^{14} \text{ m}^{-3} \) and \( T_e = 3 \text{ eV} \). As is seen in Figs. 1–5, we can show that the dust grains trapped in the region 1.0–5.9 mm, from the electrode or the wall, where the Debye length \( \lambda_\text{de} = 1.5 \text{ mm} \), \( T_e = 4 \text{ eV} \) and \( n_{e_0} = 10^{14} \text{ m}^{-3} \). Comparing our result with that obtained in the devices, we understand that our result is within the experimental data. Since our simulation explains the confinement of the dust grains, this investigation offers the important information as the basic data on the removal of the dusts in engineering devices and in fusion reactors.

**Appendix A**

Fig. A1 shows the velocity distribution function of trapped dust grains. In order to obtain the dust grain density having rapped dusts, we consider the distribution functions of the velocities.

The functions \( f_1, f_2, \) and \( f_3 \) are defined as

\[
f_1 = \frac{m_d}{2\pi k_B T_{\text{df}}} \exp \left( -\frac{m_d v_{\text{df}}^2}{2k_B T_{\text{df}}} + \frac{Ze\Phi}{k_B T_{\text{df}}} \right) \times \left( -\infty < v_d < -\sqrt{-\frac{2Ze\Phi}{k_B T_{\text{df}}}} \right),
\]

\[
f_2 = \frac{m_d}{2\pi k_B T_{\text{df}}} \times \left( -\sqrt{-\frac{2Ze\Phi}{k_B T_{\text{df}}}} < v_d < -\sqrt{-\frac{2Ze(\Phi - \Phi_0)}{k_B T_{\text{df}}}} \right),
\]

\[
f_3 = \frac{m_d}{2\pi k_B T_{\text{df}}} \exp \left( -\frac{m_d (v_d + \Delta v)^2}{2k_B T_{\text{df}}} + \frac{Ze(\Phi - \Phi_0)}{k_B T_{\text{df}}} \right) \times \left( -\sqrt{-\frac{2Ze(\Phi - \Phi_0)}{k_B T_{\text{df}}}} < v_d < 0 \right)
\]

where \( v_d, \Phi, \Phi_0, Z, T_d, T_{\text{df}}, \) and \( \Delta v \) are the velocity of dust grains, voltage in the sheath, voltage of the sheath edge, charge number, temperatures of free and trapped dust grains and the increase of the velocity of trapped dust grains. Integrating the velocity distribution functions

\[
n_d = 2n_{e_0} \left[ \int_{-\infty}^{-\sqrt{-2Ze\Phi/(k_B T_{\text{df}})}} f_1 \, dv_d + \int_{-\sqrt{-2Ze\Phi/(k_B T_{\text{df}})}}^{-\sqrt{-2Ze(\Phi - \Phi_0)/(k_B T_{\text{df}})}} f_2 \, dv_d + \int_{-\sqrt{-2Ze(\Phi - \Phi_0)/(k_B T_{\text{df}})}}^{0} f_3 \, dv_d \right],
\]

Fig. A1. A model of the distribution function of the velocity on the trapped dust grains.
we derive the density of dust grains as

\[
n_d = n_{d0} \left[ \exp\left( -\frac{Ze\Phi}{k_BT_{df}} \right) \left( 1 - \text{erf}\left( \frac{-Ze\Phi}{k_BT_{df}} \right) \right) \right.
\]

\[+ \frac{2}{\pi T_{df}} \left( \sqrt{\frac{k_BT_{df}}{k_BT_{dt}}} - \sqrt{\frac{(-\Phi + \Phi_0)Ze}{k_BT_{dt}}} \right)
\]

\[+ \frac{1}{1 + \sqrt{T_{df}/T_{dt}}} \exp\left( -\frac{Ze(\Phi - \Phi_0)}{k_BT_{dt}} \right) \left( 1 + \frac{1}{\sqrt{T_{df}/T_{dt}}} \right) \right] \right] \] (A.1)

Eq. (A1) is equal to Eq. (5).

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