Classical and quantum: some mutual clarifications

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ABSTRACT This paper presents two unconventional links between quantum and classical physics. The first link appears in the study of quantum cryptography. In the presence of a spy, the quantum correlations shared by Alice and Bob are imperfect. One can either process the quantum information, recover perfect correlations and finally measure the quantum systems; or, one can perform the measurements first and then process the classical information. These two procedures tolerate exactly the same error rate for a wide class of attacks by the spy. The second link is drawn between the quantum notions of "no-cloning theorem" and "weak-measurements with post-selection", and simple experiments using classical polarized light and ordinary telecom devices.

1 Introduction

The boundary between classical and quantum physics is a fascinating region, that in my opinion, in spite of several important explorations, has not delivered its deepest treasures. I will try to motivate this optimistic view on the future of research in physics
by presenting some remarkable links between "quantum" and "classical" physics.

We have often read in old textbooks or popular books that quantum physics is the physics of the "infinitely small", while the "everyday world" is governed by classical physics. This might be considered, and probably is, a very naive view. Bohr maintained that the distinction between the classical measurement device and the quantum measured system is arbitrary but is necessary for our understanding. The current view of the physicists working in the field, is that everything is quantum, the classicality emerging through interactions (the "everyday world" appears then to be classical because of the huge amount of interacting particles involved). This last view, the emergence of classical behavior simply because of interaction, is nowadays unchallenged by observation: no phenomenon can be produced as an evidence of its falseness. Thus, it is a satisfactory description for any practical purpose, although one may question its validity as a Weltanschauung.

The links between "classical" and "quantum" that am I going to present here are of a different nature: they do not seem to arise simply from many interacting quantum objects that together exhibit classical behavior. The first link (Section 2) deals with quantum cryptography\(^\text{1}\). The second link (Section 3) shows how typical "quantum" notions (namely, the no-cloning theorem and the idea of weak measurements with post-selection) manifest themselves in phenomena that can be described using an entirely classical theory of light, and that can be revealed using the devices of ordinary telecommunication networks.

2 Classical bounds in quantum cryptography

Quantum cryptography is nowadays the most developed application of quantum information theory \[\Pi\]. A more exact name for quantum cryptography would be quantum key distribution

\(^1\text{This was the topic of my talk during the Workshop Multiscale Methods in Quantum Mechanics, Rome, 16-20 December, 2002.}\)
(QKD): the goal of the quantum processing is to establish a secret key between two distant partners, Alice and Bob, avoiding the attacks of a possible eavesdropper Eve. Once a common secret key is established, Alice and Bob will encode the message using classical secret-key protocols, known to be unbreakable even if the message is sent on a public authenticated channel.

![Diagram](image)

**FIGURE 1.** The scheme of the QKD implementation with entangled states. Alice prepares a maximally entangled state $|\Phi\rangle$. She measures one particle (here, a two-level system) and forwards the other one to Bob. The spy Eve accesses the quantum channel and tries to obtain some information by interacting with the flying particle.

We describe (fig. 1) an implementation of QKD that uses a source of entangled states, and for clarity we speak of two-dimensional quantum systems (qubits). Alice has a source that produce a pair of qubits in the maximally entangled state

$$|\Phi\rangle_{AB} = \frac{1}{\sqrt{2}} (|+z\rangle \otimes |+z\rangle + |-z\rangle \otimes |-z\rangle) = \frac{1}{\sqrt{2}} (|+x\rangle \otimes |+x\rangle - |-x\rangle \otimes |-x\rangle). \quad (1)$$

She keeps one qubit and forwards the other one to Bob. In the absence of Eve: (i) if Alice and Bob measure the same observable, either $\sigma_z$ or $\sigma_x$, they obtain the same result, the same random bit; (ii) if one of the partners measures $\sigma_z$ and the other $\sigma_x$, they obtain completely uncorrelated random bits. This protocol is repeated a large number of times. At the end, the items in which Alice and Bob have performed different measurements are discarded later by public communication on the classical
channel, leaving Alice and Bob with a list of perfectly correlated random bit: the secret key. This is what happens in the absence of the spy.

Eve can in principle do whatever she wants on the quantum channel. The security of QKD comes from the fact that, since any measurement or interaction perturbs the state, Eve’s intervention cannot pass unnoticed: Alice and Bob know that someone is spying. Two situations are then possible. (I) Eve has got a ”small” amount of information; in this case, Alice and Bob can process their data in order to obtain a shorter but completely secret key. Such classical protocols are the object of important studies in classical information theory. (II) Eve has got ”too much” information; then Alice and Bob discard the whole key. This may seem a failure, but it is not: it simply means that the spy has no other alternative than cutting the channel and forbid any communication; and this achieves the goal of cryptography, because no encrypted message is ever sent that the spy could decode.

It is then important to quantify the words ”small” and ”too much” in the previous discussion: what is the amount of Eve’s information that Alice and Bob can tolerate, that is, at what critical value are they obliged to discard the whole key? Here is where remarkable links appear between classical and quantum information.

$$\Psi(A,B,E) \rightarrow \Phi(A,B)\psi(E)$$

$$\downarrow$$

$$P(A,B,E) \rightarrow P'(A,B)P(E)$$

FIGURE 2. Possible ways for the extraction of a secret key. One starts from a global quantum state $\Psi(A,B,E)$ of Alice, Bob and Eve, and wants to end up with a classical secret key $P'(A,B)P(E)$ with $P'(A = B) = 1$. Grey arrows (horizontal): distillation, quantum or classical; black arrows (vertical): measurement of the quantum system, leading to a classical probability distribution.
We refer to fig. 2. Because of Eve’s intervention, before any measurement the quantum system is in a three-party entangled state of Alice-Bob-Eve, $\Psi_{ABE}$. Alice and Bob on their own share the mixed state $\rho_{AB}$, obtained from $\Psi_{ABE}$ by partial trace on Eve’s system. Two procedures are then possible:

(a) The one that we described above: all the partners make a measurement, ending in a classical probability distribution $P(A, B, E)$. Then, Alice and Bob apply classical protocols (advantage distillation) in order to extract a shorter secret key, that is, a shorter list of bits distributed according to a new distribution $P'(A, B)P'(E)$ in which Eve is uncorrelated and $P'(A = B) = 1$.

(b) If the state $\rho_{AB}$ is entangled, Alice and Bob can delay any measurement and process many copies of $\rho_{AB}$, to obtain a smaller number of copies of $|\Phi\rangle_{AB}$ — and in this case, automatically Eve is uncorrelated. This procedure is known as entanglement distillation, and is one of the fundamental processes of quantum information. Once Alice and Bob have $|\Phi\rangle_{AB}$, the measurement provide them immediately with the secret key.

Having understood this, we can state the main results that have been obtained:

- Classical advantage distillation of $P(A, B, E)$ is possible for bits if and only if quantum entanglement distillation is possible for the state $\rho_{AB}$ (which is equivalent of asking that $\rho_{AB}$ is entangled in the case of qubits). This was demonstrated by Gisin and Wolf when Eve uses the so-called optimal individual attack [2], and has been recently extended to all individual attacks [3].

- The same holds for dits (d-valued random variables) and qudits (d-dimensional quantum systems), under Eve’s individual attack that is supposed to be the optimal one [4, 5]. The demonstration is more involved because not all entangled states of two qudits are distillable.

- If $\rho_{AB}$ is entangled enough to violate a Bell inequality, then a secret key can be extracted from $P(A, B, E)$ in
an efficient way, that is, using only one-way communication. This was first proven in Ref. [6]; for the state-of-the-question, see [4]. A similar result holds for a multi-partite scheme of key distribution known as ”quantum secret sharing” [7].

Mainly because Eve’s optimal attack is not generally known, there are still several open questions. The most important ones are reviewed in the last section of Ref. [4].

This concludes my first ”unconventional” link between the classical and the quantum worlds: at the level of information processing, specifically of the extraction of a secret key from an initially noisy distribution/state, the critical parameters are exactly the same, irrespective whether the purification of the correlations is performed at the quantum or at the classical level. Moreover, a typically quantum feature such as the violation of Bell’s inequalities is related to the efficiency of the classical key-extraction procedure.

3 Quantum physicists meet telecom engineers

This section is devoted to another kind of unconventional link between the classical and the quantum world. I prefer let the examples speak first and draw my conclusions later.

3.1 No-cloning theorem

The first example concerns the no-cloning theorem, a well-known primitive concept of quantum information [8]. In its basic form, it states that no evolution (or more generally, no trace-preserving completely positive map) can bring $|\psi\rangle \otimes |0\rangle$ onto $|\psi\rangle \otimes |\psi\rangle$ for an unknown state $|\psi\rangle$.

This no-go theorem has motivated the search for an optimal quantum cloner: given that perfect cloning is impossible, what is the best one can do? Optimal cloners have indeed been found and widely studied; all the meaningful references can be found in any basic text on quantum information, e.g. [9]. In the course
of these investigations, a sharp link has been found between optimal cloning and the well-known phenomenon of amplification of light: stimulated emission of light in a given mode (perfect amplification, or cloning) cannot be done without spontaneous emission (random amplification). Suppose that $N$ photons enter an amplifier, and at the output one selects the cases in which exactly $M > N$ photons are found: it turns out that this process realizes the optimal quantum cloning from $N$ to $M$ copies. The fidelity of the amplification is the ratio between the mean number of photons found in the initial mode (i.e. the mean number of correct copies) and the total number of copies, $M$ here. The optimal fidelity is found to be

$$F_{opt}^{N\rightarrow M} = \frac{MN + M + N}{M(N + 2)}.$$  \hspace{1cm} (2)

We realized an experimental demonstration of optimal cloning using the principle just described to clone the polarization of light [11]. Polarized light of intensity $\mu_{in}$ is sent into a conventional fiber amplifier (exactly as those that are used in telecommunications); at the output, we have an intensity $\mu_{out}$; we separate the input polarization mode from its orthogonal, and measure the fidelity. The theoretical prediction for this experiment is

$$F_{\mu_{in}\rightarrow \mu_{out}} = \frac{Q\mu_{out}\mu_{in} + \mu_{out} + \mu_{in}}{Q\mu_{out}\mu_{in} + 2\mu_{out}}$$ \hspace{1cm} (3)

where $Q \in [0, 1]$ is a parameter related to the quality of the amplification process. The experimental results are in excellent agreement (fig. 3).

It is striking to notice that for $Q = 1$, formula (3) is exactly the same as (2). Its meaning is however rather different. In our experiment, everything is classical: the laser light is in a coherent state, therefore it can be described by a classical field; the amplifier is “classical” in the sense that it transforms coherent states into coherent states. The quantities $\mu_{in}$ and $\mu_{out}$ that appear in eq. (3) are not photon numbers as the $N$ and $M$ of eq.
FIGURE 3. Inset: $\mu_{\text{out}}$ as a function of $\mu_{\text{in}}$; the linear fit shows that we are far from the saturation of the amplifier. Main figure: fidelity as a function of $\mu_{\text{in}}$. Solid line: $Q = 0.8$, best fit with eq. (3). Dotted lines: upper: $Q = 1$ (optimal cloning); lower: $Q = 0$ (no cloning). From Ref. [11].

\[ \mu_{\text{out}} [\text{photons/mode}] \]

\[ \mu_{\text{in}} [\text{photons/mode}] \]

3.2 Weak measurements with post-selection

The second example is related to the meaning and physics of the measurement process, a widely debated topic of the foundations of quantum mechanics. In this context, Aharonov, Vaidman and others introduced the notion of weak measurement with post-selection [12], sometimes called the ”two-state formalism” of quantum mechanics. The authors’ intention in studying this formalism is strongly motivated by interpretational issues; that is why most physicists tend to look at these concepts as artificial
ones, introduced on purpose, and that do not add anything to physics itself. To date, apart from some experiments that were designed on purpose, only some complex tunnelling phenomena had received some clarification through this formalism.

We have found however that this formalism does apply to something that exists and is extremely widespread: once again, the optical telecommunication network. Telecom engineers are performing weak measurements with post-selection in basically all that they do! A modern optical network is composed of different devices connected through optical fibers. With respect to polarization, two main physical effects are present. The first one is polarization-mode dispersion (PMD): due to birefringency, different polarization modes propagate with different velocities; in particular, the fastest and the slowest polarization modes are orthogonal. PMD is the most important polarization effect in the fibers. The second effect is polarization-dependent loss (PDL), that is, different polarization modes are differently attenuated. PDL is negligible in fibers, but is important in devices like amplifiers, wavelength-division multiplexing couplers, isolators, circulators etc.

The first piece of the connection we want to point out is the following: a PMD element performs a measurement of polarization on light pulses (Fig. 4). In fact, PMD leads to a separation $\delta \tau$ of two orthogonal polarization modes in time. If $\delta \tau$ is larger

\begin{figure}[h]
\centering
\includegraphics[width=1\textwidth]{figure4.png}
\caption{When a polarized pulse passing through a PMD fiber, the polarization mode $H$ (parallel to the birefringency axis in the Poincaré sphere) and its orthogonal $V$ are separated in time. A measurement of the time-of-arrival (TOA) is a measurement, strong or weak, of the polarization.}
\end{figure}
than the pulse width \( t_c \), the measurement of the time of arrival is equivalent to the measurement of polarization — PMD acts then as a “temporal polarizing beam-splitter”. However, in the usual telecom regime \( \delta \tau \) is much smaller than \( t_c \). In this case, the time of arrival does not achieve a complete discrimination between two orthogonal polarization modes anymore; but still, some information about the polarization of the input pulse is encoded in the modified temporal shape of the output pulse. We are in a regime of *weak measurement* of the polarization. The formulae introduced by Aharonov and co-workers are recovered by measuring the mean time of arrival, that is, the “center of mass” of the output pulse.

The second piece of the connection defines the role of PDL: *a PDL element performs a post-selection of some polarization modes*. Far from being an artificial ingredient, post-selection of some modes is the most natural situation in the presence of losses: one does always post-select those photons that have not been lost! This would be trivial physics if the losses were independent of any degree of freedom, just like random scattering; but in the case of PDL, the amount of losses depends on the meaningful degree of freedom, polarization. An infinite PDL would correspond to the post-selection of a precise polarization mode (a pure state, in the quantum language); a finite PDL corresponds to post-selecting different modes with different probabilities (a mixed quantum state).

In summary: by tuning the PMD, we can move from weak to strong measurements of polarization; the PDL performs the post-selection of a pure or of a mixed state of polarization. Any telecom network, devices connected by fibers, is performing ”weak measurements with post-selection”. Just as in the example of quantum cloning discussed above, all this can be (and is actually) described by the classical theory of light.

### 3.3 The fundamental role of entanglement

We have shown that two results thought to be ”typically quantum”, namely the no-cloning theorem and the theory of weak
measurements, can be demonstrated with classical light and standard telecom devices. The key for a deep understanding is the conceptual distinction between two superposition principles: the classical one, which is dynamical (fields superpose because Maxwell's equations are linear); and the quantum one, which is kinematical: states are superposed. These two superposition principles, at the level of interpretation, have a completely different meaning. However, it may difficult to tell which is acting in a real situation.

I would like to extend this observation to stress the fundamental role of entanglement. In the traditional textbooks of quantum mechanics, entanglement has been considered a kind of a side-issue, and in any case a derived notion: if a composed quantum system is described by a tensor product of Hilbert spaces, and if the superposition principle has to hold in this total space, then non-factorizable states must appear. In other words, traditionally one starts with the quantum physics of the single system, states the superposition principle in this context, and derives the existence of entanglement a posteriori. While this may be an unavoidable approach for a didactic course, I don't think that the view so conveyed is really the whole story. Students meeting the Stern-Gerlach experiment in their "quantum physics" course fail to realize that they have studied its analog with light polarization some months before, in their lectures on "classical electrodynamics". But what does it mean? Is a spin $\frac{1}{2}$ classical? Or is polarization a quantum intruder in the classical theory of light?

The solution comes by noticing that the Stern-Gerlach experiment is not the only experimental result involving the spin! The spins of the electrons explain the Mendeleev table via the Pauli exclusion principle, a principle that has no classical analogue; different cross-sections have been observed in scattering experiments, according to whether the full spin was in a symmetric or in the anti-symmetric state; spins couple coherently to one another in nuclear magnetic resonance, or to the polarization of photons in atomic physics... The list may rapidly become very large. But if we give a second glance to this list, we notice
that it contains only phenomena in which two or more quantum systems are involved. And if we finally notice that "coherent interaction" means "entanglement", we have the solution: we know that a single spin $\frac{1}{2}$ is a quantum object because we observe the consequences of its entanglement with other spins or other degrees of freedom. The same can be said for polarization.

The difference between the "classical superposition principle of waves" and the "quantum superposition principle of states" lies in the fact that only the second gives rise to entanglement. If we'd have only the quantum physics of the single particle (the Stern-Gerlach experiment, Young's double-slit...), the most economic solution would be to adopt once for all the de Broglie-Bohm view of a real particle guided by a hidden wave — and we'd lose all the fascinating view of the world that is inspired by quantum physics.

4 Conclusion

The main message I wanted to convey is that "classical" and "quantum" physics — or information — are tightly connected. Specifically, I have discussed how in the analysis of the security of quantum cryptography, we discover numbers that come from the analysis of the security of classical cryptography (Section 2); and how experiment with classical light and standard telecom devices can provide demonstrations of the no-cloning theorem and of the theory of weak measurements with post-selection (Section 3).

In this text, I reported on results obtained at the University of Geneva under the direction of prof. Nicolas Gisin, together with Antonio Acín, Nicolas Brunner, Daniel Collins, Sylvain Fasel, Grégoire Ribordy and Hugo Zbinden. I also benefited from several discussions with François Reuse and Antoine Suarez.

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