Do weak values show the past of a quantum particle?

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We investigate the use of weak values to determine the presence of an individual quantum particle. Specifically, we show issues with both Vaidman’s weak trace approach, and other approaches where a nonzero weak value of the spatial projection operator is implicitly assumed to indicate the presence of an individual quantum particle.

I. INTRODUCTION

According to weak value approaches to particle presence, non-zero weak values show where a quantum particle has been, even when this would normally be impossible due to the pre- and post-selected states not being eigenstates of spatial projection operators [1–2]. These weak values are obtained via weak measurement and pre- and post-selection [3], of sub-ensembles from ensembles of particles. However, in these approaches, particle presence is attributed to individual particles.

A version of this is the ‘weak trace’ approach, according to which particles leave traces along the paths they traverse, due to interaction with the environment, but such a trace is left even when this interaction with the environment is vanishing. Such a non-zero weak value is taken to show that an individual particle travelled along that path [4], in this vanishing limit of interaction (a weak trace).

This approach claims there is a weak trace in a region when any operator formed by the product of the spatial projection operator for that region with an operator for a property of the particle (e.g. spin) has a non-zero weak value. This can happen even if the weak value of the spatial projection operator itself is zero. Weak traces are supposed to give us more information about the particle than von Neumann measurement of the spatial projection operator would allow. The weak trace approach even allows discontinuous particle trajectories, hinting at a mechanism by which seemingly disconnected events can affect one another, such as in counterfactual communication [5]. Therefore, this approach has been controversial [6–15].

Another approach connecting weak values and particle presence, due to Aharonov et al, requires that the spatial projection operator itself for a location has a non-zero weak value, for us to say the particle was present at that location. In the quantum Cheshire cat [16], these approaches come apart since the weak value of the spatial projection operator on a path can be zero, but the weak value of the spin operator times the spatial projection operator non-zero — in these cases, the weak trace approach asserts the particle was on the path with a zero weak value for the spatial projection operator, contrary to what Aharonov et al claim.

In this paper, we show how one obtains the weak value of an operator. We then give a case where the weak value of an operator containing the spatial projection operator is unexpectedly non-zero (and so, by weak value approaches, a particle has left an unexpected trace in its environment). After this, we look specifically at Vaidman’s weak trace approach, and show it is inconsistent. Peleg and Vaidman argue that the weak trace approach is justified because there is some non-vanishing local interaction between the quantum particle and its environment along whatever path it travels [17]. This implies the weak value of the spatial projection operator along a path should always be non-zero whenever a particle travels along that path, and so the particle was not present if the weak value of the spatial projection operator is zero. However, there are cases where the weak trace approach asserts a particle was present on a path, despite the spatial projection operator for that path having weak value zero (e.g. in the quantum Cheshire cat set-up).

We then point out four major problems more generally with weak value approaches.

First, even weak measurements disturb a system, so any approach relying on such a perturbation to determine the location of a quantum particle will only describe this disturbed system, not a hypothetical undisturbed state. We highlight this using the case of a balanced interferometer tuned to have destructive interference (i.e. no light exiting at its dark port) where such a perturbation changes the nature of the system. The unperturbed state is the vacuum, while the perturbed state has light present. While the measurement effect can be made arbitrarily small, this is not the same as removing it entirely. Further, we show Peleg’s and Vaidman’s attempt to respond to this line of argument by saying there are no completely unperturbed systems, as weak interactions are ubiquitous in nature [17], contradicts their association of presence with non-zero weak values of some operation multiplied by the spatial projection operator, when the weak value of the spatial projection operator is zero.

Secondly, even assuming no disturbance, there is no reason to associate the non-zero weak value of an operator containing the spatial projection operator with the
classical idea of ‘particle presence’. Indeed, in some situations, just taking the weak value gives features inconsistent with classical ideas associated with a particle being present (e.g., giving discontinuous particle trajectories, or particles not being in coarse-grainings of their location).

Thirdly, weak values are only measurable over ensembles, and so to infer properties of individual particles from values of them is problematic at best.

Finally, weak values approaches to the path of a particle are not useful—they do not give us any new physics beyond standard quantum theory [18], to explain the causes of counterintuitive quantum effects (or even the paradoxes the approach itself creates). They simply assume a connection between particle presence and weak values without contributing anything testable.

II. WEAK VALUES

We first go over the derivation of a weak value of an operator, and how it is argued this relates to the trace left by a particle on its environment.

Aharonov et al defined the weak value \( O_w \) of an operator \( \hat{O} \) [3], where

\[
O_w = \langle \hat{O} \rangle_w = \frac{\langle \psi_f | \hat{O} | \psi_i \rangle}{\langle \psi_f | \psi_i \rangle} \tag{1}
\]

As this weak value increases as \( \langle \psi_f | \psi_i \rangle \) goes to zero, weak value protocols are used in metrology to amplify signals from delicate results so they can be observed experimentally [19][21]. This is at the expense of postselection reducing success probability. Weak values however also lead to a range of paradoxes [22].

To derive Eq. (1) we first couple our initial system \( |\psi_i\rangle \) to our initial pointer state, \( |\phi\rangle \), by weakly measuring them with the probe Hamiltonian \( \hat{H} = \lambda \hat{O} \otimes \hat{P}_d/T \) for small coupling constant \( \lambda \) and state-probe interaction time \( T \). This produces the state

\[
|\psi_w\rangle = e^{-\frac{i\lambda T}{\hbar}} |\psi_i\rangle \otimes |\phi\rangle = e^{-\frac{i\lambda}{\hbar} \hat{O} \otimes \hat{P}_d} |\psi_i\rangle \otimes |\phi\rangle \tag{2}
\]

where \( \hat{P}_d \) is the momentum of that pointer. We then strongly measure this weak-measured state \( |\psi_w\rangle \) with the operator

\[
\hat{P}_1 = |\psi_f\rangle \langle \psi_f | \otimes \hat{I}_d \tag{3}
\]

Assuming \( \hat{P}_d \) has Gaussian distribution around 0 with low variance (so \( \hat{X}_d \), the position of the pointer, has Gaussian distribution with high variance), we can say

\[
e^{-\frac{i\lambda}{\hbar} \hat{O} \otimes \hat{P}_d} = \sum_{k=0}^{\infty} \left(-\frac{i\lambda}{\hbar} \hat{O} \otimes \hat{P}_d\right)^k / k! = |1 - \frac{i\lambda}{\hbar} \hat{O} \otimes \hat{P}_d + O(\lambda^2) \approx |1 - \frac{i\lambda}{\hbar} \hat{O} \otimes \hat{P}_d| \tag{4}
\]

so this strong measurement gives the result

\[
|\psi_f\rangle \langle \psi_f | e^{-\frac{i\lambda}{\hbar} \hat{O} \otimes \hat{P}_d} |\psi_i\rangle \otimes |\phi\rangle \\
\approx |\psi_f\rangle \langle \psi_f | (1 - \frac{i\lambda}{\hbar} \hat{O} \otimes \hat{P}_d) |\psi_i\rangle \otimes |\phi\rangle \\
= |\psi_f\rangle \otimes \langle \psi_f | \psi_i \rangle \left(1 - \frac{i\lambda}{\hbar} O_w \hat{P}_d\right) |\phi\rangle \\
\approx |\psi_f\rangle \otimes \langle \psi_f | \psi_i \rangle e^{-\frac{i\lambda}{\hbar} O_w \hat{P}_d} |\phi\rangle \tag{5}
\]

This means the position of our initial pointer state \( |\phi\rangle \), acting as our “readout needle”, shifts, with measurement of the pointer position giving a read-out value \((\langle x \rangle - a)\). For an initial Gaussian pointer state, this shift causes the average position to move from \(\langle x \rangle\) to \(\langle (x) - \pi \rangle\) due to the application of the effective operator \(1 - i\lambda O_w \hat{P}_d/\hbar\). \(a\) is distributed over a wide range of values for many repeats, but the distribution of “\(a\)’s will be a Gaussian centred on (the real part of) \(i\lambda O_w\). Peculiarly, this weak value \(O_w\) can be very far from any of the eigenvalues of \(\hat{O}\), or even imaginary [3][23][24]. This is odd, given this weak value appears in exactly the place in the equation that an eigenvalue of \(\hat{O}\) would for a Von Neumann measurement (where the variance of \(\hat{X}_d\) on the measured state is vanishingly small). This led to the belief that the weak value \(O_w\) represented some fundamental value of the operator \(\hat{O}\) between measurements.

Due to their interrelatedness, we can graphically show where the spatial projection operator has a non-zero weak value through the Two-State Vector Formalism (TSVF). Aharonov et al developed the TSVF [25], which considers both the backwards and forwards-evolving quantum states, rather than just the forward as in standard quantum mechanics. (A similar intuition led Watanabe to develop the Double Inferential-state Vector Formalism [26].) For some operator \(\hat{O}\), the forwards travelling initial state \(|\psi_i\rangle\), and the backwards travelling final state \(\langle \psi_f |\), the TSVF gives out a conditional probability amplitude \(\langle \psi_f | \hat{O} | \psi_i \rangle\), where \(\langle \psi_f | | \psi_i \rangle\) is referred to as the Two-State Vector. This conditional probability amplitude is of the same form as the numerator of the weak value, and so we can use the TSVF to graphically plot where the weak value of an operator containing the spatial projection operator is nonzero. This involves plotting the forward-evolving state (possible paths the particle could have travelled via from its original position) and the backwards-evolving state (possible locations the particle could have come from to reach its final position). Weak value approaches assert a quantum particle is present wherever these states visibly overlap (as shown...
in Fig. 1.

The nested-interferometer model, which we show in Fig. 1, was pointed out as a case where the Two-State Vector Formalism contradicted “common-sense” continuous-path approaches [4]. The inner interferometer is balanced such that a photon entering from arm D exits to detector D3. Consequently, the outer interferometer is unbalanced, so there is an equal probability of a photon introduced from the source ending in D1 or D2. When the photon ends at D2, common sense would tell you it must have travelled via path A. This is as, had it travelled via D into the inner interferometer, it could not have exited onto path E, or reached D2. However, supporters of the weak trace approach claim, while the photon never travelled paths D or E, it travelled along paths B and C as well as along path A.

III. VAIĐMAN’S WEAK TRACE APPROACH

Consider a case where the weak value of either the spatial projection operator, or an operator formed by multiplying the spatial projection operator with the operator for some property of a particle, along a certain path was non-zero for a given $|\psi_i\rangle$ and $\langle\psi_f|$. Vaidman argues we should interpret this as meaning a particle was on that path and left a weak trace along it, on two grounds [4]. Firstly, if it were a strong measurement, an operator containing the spatial projection operator having non-zero eigenvalues would determine if a quantum particle was present. Secondly, if a particle was present in a region, he argues, it should have non-zero interaction with the local environment, due to decoherence, and so leave a “trace” along its local path. While this interaction would necessarily be weak, as we can see by the environmental coupling not collapsing position superpositions in the way a projective measurement would, supporters of the weak trace approach claim it would still be in principle detectable (i.e. only the first order in the expansion of the weak coupling constant), and so would be equivalent to the weak value of an operator containing the spatial projection operator along that path (which is also only to first order in $\lambda$).

However, this approach is inconsistent because there are cases where it asserts a particle was present on a path, despite the spatial projection operator for that path having weak value zero (e.g. Aharonov’s quantum Cheshire cat). This contradicts Peleg’s and Vaidman’s claim that the weak trace is justified because there is some non-vanishing local interaction, and so a quantum particle interacts with the environment along whatever path it travels [17]. This claim implies the weak value of the spatial projection operator along a path should always be non-zero whenever a particle travels along that path, and so the particle was not present if the weak value of the spatial projection operator is zero. This contradicts Vaidman’s claims in [4] that a particle is present along a path even when the weak value of the spatial projection operator along that path is zero, so long as the weak value of some other operator multiplied by the spatial projection operator is nonzero (e.g. the weak value of the spin along the right-hand path in the quantum Cheshire cat set-up).

IV. ISSUES WITH THE WEAKNESS ASSUMPTION

The process of weak measurement and strong postselection given above leads to a small shift in the position $X_d$ of our pointer system $|\phi(x)\rangle$, with an average value

$$\bar{\sigma} = \lambda \text{Re}(O_x) << \Delta X_d$$  \hspace{1cm} (6)

Performing the measurement on a pre-and-postselected ensemble of $N$ particles allows measurement of the shift to precision $\Delta X_d/\sqrt{N}$. Therefore, supporters of the weak values approach claim, as long as $N > (\Delta X_d)^2O(\lambda^{-2})$, the presence of the particle is revealed. This is still however a coupling — so long as $\lambda \neq 0$, there is measurement. This is as it must be, because Busch’s Theorem says we cannot gain information about the state of the system without disturbing it [27].

This is at odds with the “weakness assumption” about weak measurement—that if we measure weakly enough, we can effectively see how the system behaves when it is not measured. As argued above, weak interaction is still interaction and so disturbs the system (albeit negligibly so for some purposes). As long as there is a non-zero coupling, as in all weak value experiments, the ‘negligibly’ small disturbance is still a disturbance, and there are differences in observable effects between small coupling and no coupling (see [28] for an example of such a difference).

Popular interpretations of weak values claim that they exist in the absence of measurement and pertur-
bation [18, 29, 30]. However, this is clearly not the case [28, 31, 38]. To defend against this point Peleg and Vaidman say, “in a hypothetical world with vanishing interaction of the photon with the environment, Vaidman’s definition is not applicable, but in the real world there is always some non-vanishing local interaction. Unquestionably, there is an unavoidable interaction of the photon with the mirrors and beam splitters of the interferometer” [17].

This statement contradicts the approach taken practically in obtaining spatial weak values. If the purpose of weak value approaches is to identify the non-vanishing interaction a quantum particle has with its environment, to trace its path, then a definition of this trace which neglects some of these non-vanishing local interactions is only telling us part of the story. Proponents of weak value approaches only consider the first-order trace, as per the approximation given in Eq. 4 due to the supposedly vanishing nature of the terms of second-order or higher in λ. Despite this, Peleg and Vaidman specifically say that these environmental terms provide non-vanishing local interaction [17]. A typical photon interaction with a mirror for instance transfers of order $10^{-33}$ of its energy per reflection (for a 1500nm-wavelength photon and a 1 gram mirror), a suitably weak interaction that is mostly safely ignored. Hence even the higher order terms are many orders of magnitude larger than the strongest perturbation from the environment in optics experiments, and ignoring higher-order terms, while invoking the weak trace naturally left by the quantum object on its surroundings, is misleading.

It could be argued that this is more an objection to the experimental methods used to detect spatial weak values than the application of the concept itself—however, it is from consideration of these experimental methods that supporters of weak value approaches claim we should neglect the higher-order terms (due to their comparative undetectability)—so therefore, if we are instead looking more theoretically at the non-vanishing local interactions, we should associate presence with all terms, rather than just the first order terms in λ.

V. DO WEAK VALUES REVEAL PARTICLE PRESENCE?

The classical conception of a particle presence — being present at a certain place at a certain time — can be characterised as follows:

i). Every particle is located in space at all times.

ii). Particles cannot be on more than one path simultaneously.

iii). Particle trajectories are continuous (or at least as continuous as space is) so particles cannot get from one place to another without passing through the space in between.

iv). Particles interact with other objects/fields local to their location.

v). If a particle is on a path at a given time, and that path is within some region, then the particle is also located in that region at that time.

vi). If a particle’s property is at a location, the particle must be at that location too.

In standard quantum mechanics, when a particle is in an eigenstate of a position operator, then it is attributed a position. However, quantum particles can be in superpositions of position eigenstates. Whether they then have positions at all, or even have two positions at once, is contentious and interpretation-dependent. To require that there is always some location at every time where conditions (i) and (ii) are satisfied is to advocate a hidden variable approach to quantum mechanics, where the hidden variable is the particle’s location. However, despite quantum tunnelling and other phenomena, condition (iii) still applies for quantum particles in superpositions of position evolving according to the Schrödinger equation, since the evolution of the superposition is simply the superposition of the classical evolutions of the classical components of the superposition, each of which has a continuous trajectory. (iv-vi) are also compatible with standard quantum mechanics.

Particles that are not in eigenstates of position can nonetheless have nonzero weak values for the spatial projection operator at a given location (or some composite operator formed of this multiplied by some other operator). Perhaps a condition for a particle being present at a location in a quantum context is that it has a non-zero weak value for an operator containing the spatial projection operator at that location. Whenever a particle satisfies condition (ii), it also satisfies this condition, as its forwards and backwards-travelling states always overlap. However, this can only be a necessary condition for particle presence in the sense of (i) to (vi) above, rather than a sufficient condition, as there are cases where:

1). A particle has a non-zero weak value of an operator containing the spatial projection operator at a location, but has no continuous path to/from this location.

2). A particle has a non-zero weak value of an operator containing the spatial projection operator at a location, but we cannot say for certain that the particle would interact with other objects/fields in that location, as any such effect would cause a breakdown in interference and so change the system.

3). A particle has a non-zero weak value of an operator containing the spatial projection operator at a location, but a weak value of zero for an operator containing the spatial projection operator at a coarse-graining of that location (e.g. having a non-zero weak value of an operator containing the spatial projection operator on path $B$, and a non-zero
weak value of an operator containing the spatial projection operator on path $C$, but a weak value of zero for an operator containing the spatial projection operator for the space composed of paths $B$ and $C$).

4). A particle has a weak value of zero for the spatial projection operator along a certain path, yet has a non-zero weak value for some other operator (e.g. the spin operator in a given direction) multiplied by the spatial projection operator along that path (e.g. in the quantum Cheshire cat set-up).

Point (2) could be challenged by Peleg’s and Vaidman’s claim that quantum objects leave a ubiquitous weak trace wherever they travel, which would have an effect on the environment there. However, Dziewior et al. show there are locations where “no effect on local external systems can be observed, even when the forward-evolving wavefunction did not vanish”, and further claim that certain key cases where an effect on local systems (“coupling to the external degrees of freedom”) can be thought of “as being due to misalignment of the interferometer.” This reinforces the idea that there is a difference between a non-zero weak trace and presence in the setup as claimed, rather than a misaligned version (with a different actual state to that claimed through the ‘weakness’ assumption).

Using the two-state vector formalism and weak value tools, we can quantitatively analyse the nested interferometer set-up to show Point (3). We first define the forwards-travelling initial vector and backwards-travelling final vector by the paths via which they evolve:

$$
|\psi_i\rangle = \frac{\sqrt{2}|A\rangle + |B\rangle + |C\rangle}{2}
$$

$$
\langle \psi_f | = \frac{\sqrt{2}(|A\rangle + |B\rangle - |C\rangle)}{2}
$$

Using these, and defining the spatial projection operator for arm $A$ as $\hat{P}_A = |A\rangle\langle A|$ (similarly for $B$ and $C$), we get the weak values Vaidman does in [4]:

$$
\langle \hat{P}_A \rangle_w = \frac{\langle \psi_f | |A\rangle\langle A| |\psi_i\rangle}{\langle \psi_f | \psi_i \rangle} = 1
$$

$$
\langle \hat{P}_B \rangle_w = \frac{\langle \psi_f | |B\rangle\langle B| |\psi_i\rangle}{\langle \psi_f | \psi_i \rangle} = \frac{1}{2}
$$

$$
\langle \hat{P}_C \rangle_w = \frac{\langle \psi_f | |C\rangle\langle C| |\psi_i\rangle}{\langle \psi_f | \psi_i \rangle} = -\frac{1}{2}
$$

If we want to see however if the particle was present in the inner interferometer as a whole (as in either arm $B$ or $C$), we define $\hat{P}_{BC} = |B\rangle\langle B| + |C\rangle\langle C|$, as we are allowed to do, given projectors in standard quantum mechanics are linear. We find

$$
\langle \hat{P}_{BC} \rangle_w = \frac{\langle \psi_f | (|B\rangle\langle B| + |C\rangle\langle C|) |\psi_i\rangle}{\langle \psi_f | \psi_i \rangle}
$$

$$
= \frac{1}{2} - \frac{1}{2} = 0
$$

If we assumed a non-zero weak trace implies particle presence, this would mean the photon was never in the inner interferometer (made up of arms $B$ or $C$) overall. (Vaidman explicitly says that weak values obey the sum rule, and so allows us to say $\langle \hat{P}_{BC} \rangle_w$ must equal $\langle \hat{P}_B \rangle_w + \langle \hat{P}_C \rangle_w$, [18, 40].) This seems incoherent, given, taken separately, it claims the particle was in arm $B$, and was in arm $C$. (Aharonov et al consider a similar scenario in their three-box experiment, and also discuss this idea of negative weak value in the equivalent of arm $C$ cancelling the positive weak value in the equivalent of arm $B$ [29].) This illustrates the importance of the sign of the weak values, which the weak trace approach, and the graphical form of the TSVF (as in Fig. [1]), neglect.

For weak value experiments aiming to distinguish which-path information, we can link this to the Visibility-Distinguishability Inequality [11, 22].

$$
\mathcal{D}^2 + \mathcal{V}^2 \leq 1
$$

where $\mathcal{D}$ is the distinguishability of which path the light travelled, and $\mathcal{V}$ is the fringe-visibility at the output of the interferometer. Therefore, doing anything which would increase the distinguishability between the two paths (e.g. placing different tags on $B$ and $C$) will affect the interference pattern at the $BCE$ beamsplitter. Given perfect interference is required to ensure all light that enters the inner interferometer from $D$ exits into $D3$, anything causing distinguishability between paths $B$ and $C$ will allow light to leak through onto $E$, and cause a trace to show in $D2$. Therefore, it will never be showing what would happen in an unperturbed system.

Given Eq. 10, $\langle \hat{P}_{BC} \rangle_w$ is a far better measure of whether light reaching $D2$ was ever in the undisturbed inner interferometer. This is as measuring $\langle \hat{P}_{BC} \rangle_w$ does not cause distinguishability between paths $B$ and $C$ in the inner interferometer, and so does not affect the interference pattern required to output all inner interferometer light into $D3$. $\langle \hat{P}_{BC} \rangle_w$ being 0 provides support for a ‘common-sense’ path (i.e. light only travelling via $A$ to reach $D2$) in an unperturbed system.

An argument against looking at the weak value of the projector over multiple paths (e.g. $\hat{P}_{BC}$) specific to Vaidman’s weak trace approach is that the weak trace is only defined as the effect of the particle on the environment local to it—and combining the two weak values for these two paths is not a local operation, so does not indicate anything about the weak trace. However, this still goes against classical ideas of presence, given it shows that the presence in each of the two arms must have opposite sign, so they can cancel when added, and so implies that being
realist about weak values implying presence means being realist in some way about ‘negative’ presence. This negative presence is yet another way the ‘presence’ asserted by weak values approaches differs from classical presence.

Point (4) seems to support Vaidman’s weak trace approach to particle presence, whereby a non-zero weak value of any operator corresponding to a property of the particle, multiplied by the spatial projection operator along a path, implies the particle was present along that path. However, as we saw in Section III, taking this to imply presence even when the weak value of the spatial projection operator alone is zero, contradicts Peleg and Vaidman’s claim that the weak trace comes from the necessarily nonzero interaction between a particle and its environment, given if this were the case, we would expect a particle to necessarily have a nonzero weak value for the spatial projection operator for any path along which it travels. This is a dilemma for weak value approaches—they either contradict the rationale for weak values indicating particle presence, or imply that a particle’s properties are not necessarily in the same location as that particle.

From these four points, we see that the “nonzero weak value for an operator containing the spatial projection operator” condition can be satisfied while the conditions for a particle being present at a location are not satisfied. Therefore, it is not a sufficient condition for particle presence in the standard sense of the term.

VI. WEAK VALUES ARE ONLY DEFINED OVER ENSEMBLES

It is important to note that advocates of the weak value approach to particle presence use it to attribute presence to individual particles. This is despite the experimental results of measurements of weak values necessarily being produced by ensembles, because weak values are obtained by post-selection. It is non-obvious that we can go from facts about ensembles to facts about individual elements of that ensemble. As an example, we see that one can have a weak value of the velocity of electrons in a given set-up that is greater than the speed of light in a vacuum [43–45]. However, this does not necessarily mean than any of the individual electrons in the ensemble used to obtain this weak value travelled at superluminal speeds—otherwise this would be a violation of special relativity. Instead, similar to how group velocities of wavepackets can be superluminal despite the phase velocity (or velocity of the components) being below c, this superluminal weak value for speed is a fact only about the ensemble, rather than any of its constituents. Similarly, there is no reason to infer facts about the presence of individual quantum particles from weak values of operators containing the spatial projection operator.

FIG. 2. Two-State Vector Formalism analysis of Salih et al’s polarisation-based single-outer-cycle protocol for counterfactual communication. Bob communicates with Alice by turning off/on his switchable mirrors to determine whether the photon goes to D1 or D0 respectively. We specify the polarisation, given it determines direction of travel through the polarising beamsplitters (PBSs). The forwards- and backwards-travelling states do not overlap anywhere on the inner interferometer chain when there is a detection at D0, meaning, by weak value approaches, the particle detected at D0 was never at Bob. This means Bob’s ability to communicate with Alice is not explained by weak value approaches any more than it is by standard quantum mechanics.

VII. USEFULNESS OF WEAK VALUE APPROACHES TO PARTICLE PRESENCE

Supporters of weak value approaches argue they provide more (interpretational) information about the underlying state of the system than standard quantum mechanics. They claim that an issue with Wheeler’s “common-sense” approach to particle trajectories [46], is that it is entirely operational: not telling us anything about the underlying mechanisms at work, just the final result [4]. Similarly, Vaidman claims “the von Neumann description of the particle alone is not sufficient to explain the weak trace” [4]—implying weak value approaches provide something more than the standard von Neumann approach does. We however turn this criticism
back on these supporters—weak value approaches do not tell us anything about the underlying system either, beyond standard quantum mechanics.

A key motivation for weak value approaches was to explain how the results of interference are affected by changes to disconnected regions. They were intended to show that phenomena such as Wheeler’s Delayed Choice, or Salih et al’s Counterfactual Communication protocol and related effects aren’t as “spooky” as they appear. Salih et al however have shown that in their communication protocol, the weak value of the spatial projection operator (or any compound operator including the spatial projection operator) at Bob is always zero. These both show that weak value approaches do not explain this phenomenon.

While suggesting a time-symmetry to quantum processes through the TSVF, weak value approaches do not imply any new physics beyond standard quantum theory, to explain the causes of counterintuitive quantum effects when there is a nonzero weak value of the spatial projection operator (or any operator containing the spatial projection operator) at Bob is zero. These approaches simply assume the particle was present wherever the weak value of an operator containing the spatial projection operator is nonzero. Hence they add a claim of particle presence but contribute nothing testable to our ontology. This label confuses matters by oversimplifying a complex concept: what it means for a specific particle to be sequentially “present” at a two specific places in quantum field theory. It also causes a number of paradoxes by itself, such as the discontinuous photon trajectories in the nested interferometer set-up above.

VIII. CONCLUSION

Not only do weak value approaches to particle presence give incoherent results (claiming that a particle can be in B, or in C, but not in B or C), but they also rest on the faulty assumption that weak coupling is equivalent to no coupling. Regardless of disturbance, a non-zero weak value of an operator containing the spatial projection operator is insufficient for particle presence.

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