Fracture Considerations for the Thermal Shock Resistance of Porous Ceramic Materials

Yunxia Zhang1*, Baolin Wang2, Yanliang Shang3*

1Langfang Normal University, Langfang 065000, P.R. China
2Graduate School at Shenzhen, Harbin Institute of Technology, Harbin 150001, P.R. China
3Department of Civil Engineering, Shijiazhuang Tiedao University, Shijiazhuang, 050043, China

Email: yunxiazhang2003@163.com, sty1980926@163.com.

Abstract: The influence factors of thermal shock resistance of porous ceramic materials are analysed from the point of stress criterion and energy criterion. Results show that material properties, relative density and initial crack have important effects to thermal shock fracture of porous ceramic materials. In energy criterion, the total elastic energy is given. The driving force for crack propagation is provided by the elastic energy stored at fracture. When the elastic energy stored in porous ceramic materials is larger than the fracture energy, the crack will propagate. The relationship between crack length and initial crack length is given. In addition, the reason why the thermal shock resistance of porous ceramic materials is larger than dense ceramics is explained.

1. Introduction

Porous ceramic materials consist of an interconnected network of solid struts or walls which form the edges and faces of cells. There is a growing demand of porous ceramic materials for high temperature structural applications mainly due to their excellent properties, such as corrosion resistance, high-temperature stability and moderate mechanical properties. Porous ceramic materials can be used as electrodes, catalysts and light-weight structural components [1]. Porous ceramic materials with a pore size in the range of smaller and a special shape offer great potential for the design structures with improved properties. For structure components, the existence of pores can be useful in increasing the strain tolerance, and reducing the weight of the structure. Moreover, the low elastic modulus together with a high strength leads to superior thermal shock resistance.

Thermal shock resistance is an important method to measure thermal shock fracture of porous ceramic materials. Generally, there are many parameters to express thermal shock resistance, such as thermal cycles [2], thermal shock resistance parameters (including $R$, $R'$, $R''$, $R'''$) [3], the maximum temperature difference [4] etc. However, there is few reports investigating influence factors about thermal shock resistance of porous ceramic materials. That’s the main goal that I want to reach to discuss them from the point of stress criterion and energy criterion. In this paper, factors affecting the thermal shock resistance (such as medium parameters, relative density, initial crack length, surface energy etc.) are discussed. The relationship between crack length and initial crack length is given. Moreover, the probable reasons about why porous ceramic materials have higher thermal shock resistance are given. The results can be used to guide design porous materials to satisfy some thermal protection applications.
2. Distribution of temperature and stress

Shown in Fig. 1 is a finite-thickness porous ceramic material within the Cartesian coordinate system. At the beginning, the porous ceramic material is at a uniform temperature $T_0$. Its lower boundary ($z=0$) is thermally insulated and upper boundary ($z=2H$) is exposed to thermal environment with the temperature $T_\infty$. For such a problem, the boundary conditions are:

$$\frac{\partial T}{\partial z} = 0, \text{ at } z=0 $$

$$-\kappa \frac{\partial T}{\partial z} = h(T - T_\infty), \text{ at } z=2H $$

The problem under consideration is the isotropic materials. The strain components are given by the following forms:

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{v}{E} & -\frac{v}{E} \\ -\frac{v}{E} & \frac{1}{E} & -\frac{v}{E} \\ -\frac{v}{E} & -\frac{v}{E} & \frac{1}{E} \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \end{bmatrix} + \begin{bmatrix} \alpha \\ \alpha \\ \alpha \end{bmatrix} (T - T_0) $$

where $v$, $E$ and $\alpha$ are, respectively, Poisson’s ratio, the elastic modular and thermal expansion coefficient of the porous ceramic materials.

The configuration in Fig. 1 is analyzed under the constraint that the porous ceramic material is free to expand with disappearing axial force

$$\int_0^{2H} \sigma_{xx} \, dz = 0, \int_0^{2H} \sigma_{yy} \, dz = 0 $$

and disappearing normal stress in thickness direction $\sigma_{zz}=0$. The strain is independent of all coordinates, and depends only on time under the geometry and boundary conditions. The porous ceramic material expands uniformly but does not produce warping deformations.

It follows from Eq. 3 and Eq. 4 that the transient thermal stress $\sigma_{xx}(z, t)$ associated with the temperature field $T(z, t)$ is

$$\sigma_{xx}(z, t) = \frac{E\alpha}{2(1-v)H} \int_0^{2H} (T - T_0) \, dz - \frac{E\alpha(T - T_0)}{1-v} $$

Here the similar relation is present in thermal stress $\sigma_{yy}(z, t)$ and it can be obtained from Eq. 4 and boundary conditions. In general, it is assumed that the stress component initiating fracture is $\sigma_{xx}(z, t)$ and attention will be restricted to it only.

The mathematical formulation of the heat conduction in the $z$ direction shown in Fig. 1 is given by

$$\kappa \frac{\partial^2 T(z, t)}{\partial z^2} = \frac{\partial T(z, t)}{\partial t} $$

Fig. 1. A finite-thickness porous ceramic material: geometry and thermal boundary conditions.
where $\kappa$ is the thermal diffusivity, which can be dealt with constant since porosity has no effect on it if the pores contain gases only [5].

Separation variable method is applied to solve Eq. 6 with heat transfer boundary conditions, Eq. 1 and Eq. 2, giving

$$
T(z,t) - T_{\infty} = \frac{-k\sigma_{m}^2}{4H^2} \sum_{m=1}^{\infty} \exp(\frac{-k\sigma_{m}^2}{4H^2}) \cos(\frac{\gamma_{m} z}{H}) \sin(\frac{\gamma_{m} t}{H}) \frac{2(\gamma_{m}^2 + B_i^2)}{\gamma_{m}((\gamma_{m}^2 + B_i^2) + Bi)}
$$

where $\gamma_{m}$ are the roots of $\gamma_{m}\tanh\gamma_{m} = Bi$ and the non-dimensional heat transfer coefficient $Bi = 2hH/k$ is the Biot number for the isotropic porous ceramic materials. In the limit of perfect thermal insulation $Bi = 0$, $\gamma_{m} = m\pi$ and $T = T_{\infty}$. At the other limit of perfect heat transfer, $Bi = \infty$, $\gamma_{m} = (m+1/2)\pi$.

Thermal stress $\sigma_{xx}(z, t)$ is got directly from Eq. 5 and Eq. 7 as

$$
\sigma_{xx} = \frac{E\alpha(T_{0} - T_{\infty})}{1 - \nu} \sum_{m=1}^{\infty} \exp(\frac{-k\sigma_{m}^2}{4H^2}) \sin(\frac{\gamma_{m} t}{H}) \frac{2(\gamma_{m}^2 + B_i^2)}{\gamma_{m}((\gamma_{m}^2 + B_i^2) + Bi)} \left( \sin(\frac{\gamma_{m} z}{H}) - \cos(\frac{\gamma_{m} z}{H}) \right)
$$

In the limit $Bi = 0$, the foam is everywhere stress free. On the other hand, if $Bi = \infty$, the thermal stress $\sigma_{xx}(z, t)$ can be easily got by setting $\gamma_{m} = (m+1/2)\pi$ in Eq. 8.

For porous ceramic materials, porosity has no effect on coefficient of thermal expansion and Poisson’s ratio [6,7] which have the same values as the corresponding dense materials. Supposing the dominant deformation mechanism is the bending of the pore strut under tensile and compressive loading, Zhang and Ashby [8] gave the equation: $E = 0.5E_{s}(\rho/\rho_{s})^{3}$, where $\rho_{s}$ is the density of the ceramic materials, $\rho$ is the density of the porous ceramic material and $E_{s}$ is the elastic modulus of ceramic materials. Therefore, the thermal stress is

$$
\frac{\sigma_{xx}}{\sigma_0} = 0.5 \left( \frac{L}{\rho_{s}} \right)^{3} \sum_{m=1}^{\infty} \exp(\frac{-k\sigma_{m}^2}{4H^2}) \sin(\frac{\gamma_{m} t}{H}) \frac{2(\gamma_{m}^2 + B_i^2)}{\gamma_{m}((\gamma_{m}^2 + B_i^2) + Bi)} \left( \sin(\frac{\gamma_{m} z}{H}) - \cos(\frac{\gamma_{m} z}{H}) \right)
$$

where $\sigma_0 = E_{s}\alpha(T_{0} - T_{\infty})/(1 - \nu)$. This provides the closed form expression for the thermal stress in finite porous ceramic materials. According to stress criterion, if the thermal stress which the porous ceramic material affords is larger than stress strength, the crack in the porous ceramic material will propagate. Eq. 3 is the thermal stress equation which indicates that the thermal stress at fracture is proportional to the relative density and the material parameters. Because the relative density is generally less than 0.3, the thermal stress that porous ceramic materials afford is small. Therefore, the porous ceramic material has higher thermal shock resistance comparing dense material.

3. Energy criterion

Crack propagation under sudden thermal load conditions generally occurs without exterior force. The driving force leading to crack growth is derived only from the thermal stress field within the porous ceramic materials. Suppose the porous ceramic material contains an initial crack only, any other form of defects are absent. When the total energy of the porous ceramic material attains minimum, the crack propagates. The total energy per unit volume is the sum of elastic energy plus energy of the crack propagation. For the present model (simple geometry and thermal environment) subjected to cold shock, using the expression for the effective bulk modulus of materials containing an edge crack[9], the total energy can be expressed:

$$
W = \frac{2(\alpha_{c} \Delta T)^2 E_{s}}{4(1 - \nu_{c})} \left( \frac{\rho_{s}}{\rho} \right)^{3} \left[ 1 + \frac{16(1 - \nu_{c})a^{3}}{9(1 - 2\nu_{c})} \right]^{1/2} \pi a^{2} \gamma
$$

where $\gamma$ is the surface energy of ceramics and $a$ is the length of crack.

Based on the general theory of Griffith, cracks are unstable between those limits for which:

$$
\frac{dW}{da} = 0
$$

Substitution of Eq. 10 into Eq. 11, result of the critical temperature difference becomes:
\[ \Delta T_i = \left( \frac{\pi \gamma(1-2v_s)}{E_0 \alpha^2_s \left( \frac{\rho}{\rho_i} \right)^3 (1+v_s)} \right)^{1/2} \left[ 1 + \frac{16(1-v_s^2)a^3}{9(1-2v_s)} \right] \frac{1}{\sqrt{a}} \]  
(12)

For short crack, the last term in Eq. 12 can be neglected. In this condition, Eq. 12 becomes:

\[ \Delta T_i = \left( \frac{\pi \gamma(1-2v_s)}{E_0 \alpha^2_s \left( \frac{\rho}{\rho_i} \right)^3 (1+v_s)} \right)^{1/2} \frac{1}{\sqrt{a}} \]  
(13)

Consider two conditions when the porous ceramic materials are in the initial condition that the crack length is \( a_0 \) and in the final condition that the crack length is \( a \). The energy difference between the initial condition and the final condition are the driving force to make the crack propagate. The expression is:

\[ 2 \pi \gamma a^2 (a^2 - a_0^2) = 3(\alpha_2 \Delta T)^2 E_0 \left( \frac{\rho}{\rho_i} \right)^2 \left[ 1 + \frac{16(1-v_s^2)a^3}{9(1-2v_s)} \right] - \frac{3(\alpha_2 \Delta T)^2 E_0 \left( \frac{\rho}{\rho_i} \right)^2 \left[ 1 + \frac{16(1-v_s^2)a^3}{9(1-2v_s)} \right]}{8(1-v_s^2)} \]  
(14)

Substitution of Eq. 13 into Eq.14 when the final length of crack is larger than initial length, the final length of crack can be written:

\[ a = \left( \frac{3(1-2v_s)}{8(1-v_s^2)}a_0 \right)^{1/2} \]  
(15)

That means the final crack length is related with Poisson’s ratio and initial crack length for the porous ceramic materials containing pre-existing crack. To improve the thermal shock resistance, the porous ceramic materials containing moderate initial crack length should be selected.

4. Discussion

General aspect of thermal stress fracture of porous ceramic materials can be discussed in terms of the present theory. As discussed in the above, the thermal shock resistance is related with material parameters, relative density and initial length of crack. Fig. 2 is the relationship of crack length of porous ceramic materials with the initial crack length, it can be seen that the final crack length \( a \) decreases with the increase of initial length of crack. In this figure, the Poisson’s ratio 0.3 is selected. In fact, the figure is similar when other value of Poisson’s ratio is selected. That means initial crack length has important effect on crack propagating.

Another factor which may influence the crack propagation through the body is the existence of elastic discontinuities in the form of pores. Elastic discontinuities will disperse or scatter the kinetic energy which lead to crack propagation. Energy is thereby removed from the crack propagation, causing it to be arrested rapidly. That’s maybe the reason that porous ceramic material has higher thermal shock resistance than dense ceramic materials.

![Fig. 2 Relationship between the crack length after thermal shock and initial crack length](image)
5. Conclusion

In this paper, key factors controlling the thermal shock resistance of porous ceramic materials are considered from the points of stress criterion and energy criterion. Results show that material properties, microscopic structure such as relative density and pre-existing crack have larger effects on the thermal shock resistance of porous ceramic materials. The possible reasons that porous ceramic materials have higher thermal shock resistance are given. Because the porous ceramic materials have special structure, the kinetic energy of crack propagating can be dispersed, which makes the crack propagating arrested quickly. In addition, the numerical result that the relationship between crack growth length and the initial crack size is given. This paper can provide theoretical basis and technical support for high temperature strength design of thermal protection system.

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