DONOR STARS IN BLACK HOLE X-RAY BINARIES

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Received 1998 July 21; accepted 1999 March 30

ABSTRACT

We study theoretically the formation of black hole (BH) X-ray binaries. Consistency of the models with the observed relative numbers of systems with low-mass ($\lesssim 2 M_\odot$) and intermediate-mass ($\sim 2 M_\odot - M_{\odot}$) donors leads to severe constraints on the evolutionary parameters of the progenitors. In particular, we find that (1) BH progenitor masses cannot exceed about 2$M_{\odot}$; (2) high values of the common envelope efficiency parameter ($\alpha_{CE} > 1$) are required, implying that energy sources other than orbital contraction must be invoked to eject the envelope; and (3) the mass-loss fraction in helium star winds is limited to $\lesssim 50\%$. Outside of this limited parameter space for progenitors we find that either BH X-ray binary formation cannot occur at all or donors do not have the full range of observed masses. We discuss the implications of these results for the structure of massive hydrogen-rich stars, the evolution of helium stars, and BH formation. We also consider the possible importance of asymmetric kicks.

Subject headings: binaries: close — black hole physics — stars: evolution — stars: mass loss — stars: Wolf-Rayet — X-rays: stars

1. INTRODUCTION

Radial velocity measurements of the nondegenerate donors in X-ray binaries provide limits on the masses of the accreting objects. When combined with additional information (e.g., spectra of the donor, orbital light curves) actual measurements of the masses are obtained (for a recent review see Charles 1998). In this way, the masses of 11 compact objects in X-ray binaries have been found to exceed the maximum mass possible for a neutron star (the most recent one by Orosz et al. 1998; for earlier determinations see the review by Wijers 1996). These binaries are thought to be black hole (BH) X-ray binaries.

In addition to these 11 systems, there are X-ray sources for which radial velocity measurements have not yet been obtained but whose X-ray spectral and variability properties show similarities to the dynamical BH X-ray binaries (e.g., Chen, Shrader, & Livio 1997; Stella et al. 1994). These sources are regarded as possible BH candidates. We note, however, that similar spectral and variability characteristics have also been observed in some neutron star binaries (van der Klis 1994a, 1994b). Indeed, some of the systems once considered to be BH candidates have later turned out to contain neutron stars, while others turned out to contain black holes (e.g., 4U 1543$-$47). Therefore, the presence of a black hole in these binaries is somewhat uncertain.

The recent increase of the number of known binaries with accreting black holes in our Galaxy allows us to study them as a separate population of X-ray binaries. In particular, the subset of eight dynamically identified BH systems with Roche lobe filling, low-mass companions appears to form a uniform class of sources, typically discovered as X-ray transients.

One issue of interest concerns the total number of BH binaries in our Galaxy and their birth frequency. Empirical estimates have been hampered by the transient nature of the systems and the lack of good constraints on their recurrence times from present data. Recently, Romani (1998) considered in detail the sensitivity and sky coverage of the main X-ray surveys. He estimated that there are $\sim 1700$ low-mass BH binaries in our Galaxy with a typical discovery rate of $\sim 2$ yr$^{-1}$. On the theoretical side, the birthrate of low-mass BH binaries can be calculated using population synthesis methods. Based on such calculations it has been pointed out that the formation of low-mass BH binaries can be considerably favored relative to neutron star binary formation if there is limited mass loss during the collapse and if kick magnitudes are small (Romani 1992, 1996). It has also been suggested (Portegies Zwart, Verbunt, & Ergma 1997) that the formation of low-mass BH binaries at rates comparable to the (notably rather uncertain) empirical estimates ($\sim 10^{-7}$ yr$^{-1}$) may require high values of the efficiency for common envelope (CE) ejection. However, different choices for the mass ratio distribution in primordial binaries and for the average kick magnitude can affect the predicted birthrates by 1 or 2 orders of magnitude (see Fryer, Burrows, & Benz 1998; Kalogera & Webbink 1998, hereafter KW98).

Another point of interest concerns the types and masses of donors found in the observed BH binaries. Just like neutron star binaries, BH binaries can be separated into two classes: one with black holes accreting from the wind of a massive star (three out of 11 systems) and another with low-mass Roche lobe filling companions. For accreting neutron stars this division has been understood in the context of unstable mass transfer, which occurs approximately when the companion is more massive than the accretor (van den Heuvel 1975; Kalogera & Webbink 1996). The development of this instability limits the masses of Roche lobe filling donors to $\lesssim 1$--2 $M_\odot$. Analogous considerations in the case of low-mass BH binaries would naively lead us to expect donors as massive as the black holes ($\sim 10 M_\odot$). However, it has been noticed that intermediate-mass donors ($\gtrsim 2 M_\odot$) to black holes appear to be rare (Wijers 1996). Of the eight Roche lobe overflow systems only two, J1655$-$40 and 4U 1543$-$47, are thought to have donor masses of $\sim 2.3$ and $\sim 2$--$3$ $M_\odot$, respectively; the other six have masses well below 1 $M_\odot$ (e.g., Charles 1998). Of the systems thought to harbor a black hole based on their spectral and variability properties, there are six more sources for which we have some information about their visual magnitudes, $m_v$. Measurements or just lower limits on $m_v$ combined with estimated distances and
extinction strongly indicate that the donors have masses \( \lesssim 1 \, M_\odot \) (Ritter & Kolb 1998; Chen et al. 1997; Stella et al. 1994).

In this paper we address the issue of the apparent paucity of intermediate-mass donors in low-mass BH binaries by considering a larger set of constraints, beyond the stability of mass transfer, imposed both on BH binaries with donors filling their Roche lobe on the main sequence (MS) and on their progenitors. In the present study we assume that no asymmetric kicks are imparted to black holes at the time of their formation. This is consistent with our limited understanding of the physical mechanism for supernova kicks and for BH formation. Under this assumption, we find that formation of BH binaries is actually not possible for a wide range of evolutionary parameters. When possible, it is often the case that systems are formed either with only low-mass or only intermediate-mass donors. This "dichotomy" has its origin mainly in the fact that magnetic braking operates only for low-mass stars. Furthermore, we find that in cases where formation of both types of donors is possible, consistency with observations requires that the efficiency for CE ejection be relatively high (in agreement with the results of Portegies Zwart et al. 1997, although based on different considerations). We also show that the progenitors of black holes can be at most about twice as massive as the black hole being formed. Finally, we are able to place very strict limits on the wind mass loss from relatively massive helium stars. For the formation of BH binaries with MS companions to be at all possible, the amount of mass lost from helium stars must be smaller than about 50% of their initial mass.

In the next section we discuss the BH formation path considered here and describe the evolutionary constraints that are imposed on the systems and their progenitors. In § 3 we calculate the corresponding limits set on the binary parameters. In § 4 we present our results on the masses of donors in low-mass BH binaries that are allowed to form when all the constraints are taken into account. We also investigate their dependence on various evolutionary parameters, varied in the entire permitted range. In § 5 we compare the lifetimes and predicted birthrates for the two groups of systems (with low- and intermediate-mass donors). We find that the results are consistent with the observations if certain evolutionary parameters are restricted within relatively narrow ranges. Finally, in § 6 we discuss the implications of our results for our understanding of the formation of BH X-ray binaries, the possible selection effects acting on the observed sample, the viability of other evolutionary paths, and the relevance of kicks possibly imparted to black holes at birth.

2. FORMATION OF BLACK HOLE BINARIES POWERED BY ROCHE LOBE OVERFLOW

2.1. Formation Channel

Over the years, one evolutionary sequence has developed as being the standard formation path for low-mass X-ray binaries with neutron stars (Sutantyo 1975; van den Heuvel 1983). It involves the evolution of binaries with extreme mass ratios (\( \leq 0.1 \)) through a CE phase and the subsequent collapse of a helium star (the post-CE primary) into a neutron star. The X-ray phase is initiated when the low-mass companion to the compact object fills its Roche lobe, either because of its own radial expansion on the giant branch or because of orbital angular momentum losses due to gravitational radiation and magnetic braking. Naturally, this evolutionary channel can be extended to account for the formation of low-mass BH binaries starting from primordial binaries with presumably more massive primaries so that the helium star collapses into a black hole instead of a neutron star. This evolutionary sequence has already been considered for the formation of low-mass BH binaries in earlier studies examining the birthrate of such systems or the masses of the BH progenitors (Romani 1996; Portegies Zwart et al. 1997; Ergma & van den Heuvel 1998).

Another channel has also been proposed that involves the evolution of a hierarchical triple system (Eggleton & Verbunt 1986), which consists of an inner massive binary and a third, distant low-mass companion. This channel has so far not been studied in any quantitative way. However, in recent years, multiple stellar systems have been discovered at a significant rate (for a recent catalog see Tokovinin 1997), and detailed modeling of triple-star evolution could prove quite interesting. We note, however, that a crucial phase in this sequence is the formation of a Thorne-Żytkow object (a massive envelope accreting on a compact object) and its subsequent evolution. The stability and fate of such a configuration is still an issue of debate (Biehle 1991; Cannon et al. 1992; Fryer, Benz, & Herant 1996).

One issue common to both formation paths is that progenitor systems with extreme mass ratios are required, since the donor star in the low-mass BH binary is much more massive than the progenitor (massive single-star or inner binary system) of the black hole. Among the observed binaries in our Galaxy most have stellar components of comparable mass, a smaller fraction have mass ratios (secondary to primary mass) as low as \( \approx 0.2-0.3 \), and no systems have been observed with mass ratios \( \lesssim 0.1 \), primarily because such systems are inaccessible to current instrumentation. It has been shown, however, that this kind of distribution is only a result of strong selection effects favoring systems with stars of comparable brightness and hence mass (Hogeveen 1991). Nevertheless, even correcting for the selection effects cannot provide us with any information about the existence or the true relative frequency of systems with extreme mass ratios (\( \lesssim 0.1 \)).

If systems with extreme mass ratios do not exist in nature, then there is no alternative channel at present for the formation of low-mass BH binaries. This is in contrast to low-mass X-ray binaries (LMXBs) with neutron stars, which, in the absence of very low mass ratio binaries, could still all be formed by accretion-induced collapse (AIC) of accreting white dwarfs. The possibility of neutron star formation via AIC is still highly controversial. However, AIC of neutron stars cannot possibly account for the observed BH binaries since the measured BH masses exceed the sum of those of neutron stars and their low-mass companions (necessary for stable, long-term, and—in the most favorable case—conservative mass transfer). One way to avoid the requirement of extreme mass ratios would be for the black holes to acquire their low-mass companions after their formation, but this is extremely unlikely in the low-density environment of the Galactic disk; ejection of systems from globular clusters also appears unlikely given the small vertical scale height inferred for the population (White & van Paradijs 1996; however, see Mardling 1996). Therefore, it appears that parent multiple systems consisting of stars massive enough to produce \( \sim 10 \, M_\odot \) black holes and of compan-
ions less massive than the black hole are required for the formation of the observed low-mass BH binaries.

In what follows, we study the formation of BH X-ray binaries with Roche lobe–filling donors on the MS via the channel that involves CE evolution and the collapse of a helium star, where the existence of binaries with low mass ratios is assumed, although their relative birth frequency is not strictly specified. In § 6 we also consider the fate of those primordial binaries which are wide enough to avoid evolution through a CE phase.

2.2. Evolutionary Constraints

As the evolution of a binary system can follow a wide variety of paths, it is reasonable to expect that evolution through a specific channel requires that binaries satisfy a given set of constraints. The complete set of constraints relevant to the He star collapse channel for neutron star binaries has been discussed in detail by KW98. We also list them here in favor of completeness in the case of BH binaries (see also Portegies Zwart et al. 1998).

Before the formation of the black hole the following conditions must be present:

1. The binary orbit must be small enough for the massive star to fill its Roche lobe and the system to enter a CE phase. This sets an upper limit on the initial orbital separation of the binary.
2. The helium-rich primary and its companion must fit within their Roche lobes at the end of the CE phase for the phase to actually be terminated. This constraint set lower limits on the orbital separation of the binary after the CE phase. In addition, the helium star must fit inside the Roche lobe even after it experiences wind mass loss as it evolves toward collapse, so that it avoids complete merging with its companion or losing its envelope (in most cases the carbon core will not be massive enough to collapse to a black hole). For the formation of black holes of several solar masses relatively massive helium stars are required (the lower limit on their mass being set by the mass of the black hole). Calculations of helium star evolution (Habets 1985; Woosley, Langer, & Weaver 1995) indicate that such massive helium stars do not become giants at the end of their evolution. Therefore, the requirement that they fit in their Roche lobes turns out not to be a significant constraint.

In the last few years, the evidence for kicks being imparted to neutron stars at birth has greatly increased (Kaspi et al. 1996; Hansen & Phinney 1997; Lorimer, Bailes, & Manchester 1997; Fryer & Kalogera 1997; Fryer et al. 1998). Depending on the physical origin of the kicks and on the way black holes are formed, kicks may or may not be imparted to black holes. Based on the spatial distribution and kinematic properties of BH binaries, White & van Paradijs (1996) conclude that no asymmetric kicks are imparted to black holes. In any case, even if kicks are associated with BH formation, they are probably smaller in magnitude than the neutron star kicks and possibly inversely proportional to their mass (for a given amount momentum transfer). In what follows, we assume that kicks are not imparted to black holes, although we discuss their possible effect in § 6. The constraints imposed on the progenitors of BH binaries after the formation of the black hole are then as follows:

3. The system must remain bound after the collapse. For a symmetric collapse, an upper limit is set on the amount of mass lost during BH formation and hence on the mass of the collapsing helium star.
4. For the formation of BH binaries with Roche lobe filling donors on the MS, the companion to the black hole must fill its Roche lobe before it evolves away from the MS. For donors less massive than \( \sim 1 M_\odot \), there is an additional time constraint that the postcollapse binary be tight enough for Roche lobe overflow to occur within the age of the Galactic disk (assumed here to be \( 10^{10} \) yr). These constraints set an upper limit on the postcollapse orbital separation.
5. Mass transfer must proceed stably—the donor must remain in hydrostatic and thermal equilibrium—and at sub-Eddington rates. This sets an upper limit on the mass of the donor when it is on the zero-age MS and an upper limit on the orbital separation when the mass of the donor is comparable to that of the black hole and the star has evolved away from the zero-age MS.

3. LIMITS ON THE PROGENITORS

The constraints discussed in § 2.2 impose specific limits on the progenitor characteristics at different stages of their evolution. Based on the effect of each evolutionary phase on binary parameters we translate all of the constraints to a given stage in the formation path, which we choose here to be the one of circularized postcollapse orbits.

For a specific value of the mass, \( M_{\text{BH}} \), of the black hole, we calculate all the limits imposed on the circularized post-collapse orbital separations, \( A \), and the donor masses, \( M_d \). The exact position of the limits on the \( A - M_d \) space depends on the BH mass and on three additional quantities related to the precollapse evolution of the progenitors:

1. The CE efficiency, \( \xi_{\text{CE}} \).—Since the details of CE evolution are still not well understood (for a review see Iben & Livio 1993), we adopt the usual formulation for the orbital contraction in the CE phase as suggested by Webbink (1984). Accordingly, we assume that the CE becomes unbound as the low-mass companion spirals inward and orbital energy is dissipated in the envelope with some assumed efficiency \( \xi_{\text{CE}} \) defined by

\[
\xi_{\text{CE}} = \frac{GM_{\text{He}} M_d}{2A_f} - \frac{GM_p M_d}{2A_f} = \frac{GM_p M_d}{\lambda R_p} = \frac{GM_p M_d}{\lambda r_{Lp} A_f}.
\]

Here \( M_p \) is the mass of the primary at the onset of the CE phase (which can be different from the initial mass, \( M_1 \), of the primary because of wind mass loss); \( M_e \) is the mass of the envelope, and \( M_{\text{He}} \) is the mass of the helium core of the massive primary (which becomes the post-CE primary); \( A_f \) and \( A_i \) are the orbital separations at the onset and at the end, respectively, of the CE phase; \( \lambda \) is a numerical factor of order unity that depends on the degree of the central concentration of the primary and for evolved stars is typically assumed to be equal to 0.5; and \( R_p \) is the radius of the primary at the onset of the CE phase that is also the radius of the Roche lobe of the primary and hence equal to \( r_{Lp} A_f \) (\( r_{Lp} \) being the dimensionless radius of the primary’s Roche lobe in units of the initial orbital separation; we adopt the approximation of \( r_{Lp} \) as function of the mass ratio as given by Eggleton 1983).
Although intuitively one would expect that the value of $\alpha_{CE}$ is restricted to between zero and unity, it has been realized that additional sources of energy, other than orbital, may contribute to the ejection of the envelope; these include the thermal and ionization energy of the envelope (e.g., Han, Podsiadlowski, & Eggleton 1995) or possibly nuclear energy generated in the region close to the inspiraling companion (Taam 1994). In the formulation of equation (1) such additional energy sources imply values of $\alpha_{CE}$ higher than unity.

Scrutiny of equation (1) indicates that, for a given decrease in orbital separation, the absolute normalization of $\alpha_{CE}$ is not well defined as it depends on the central concentration parameter $\lambda$. The value of $\lambda$ is quite uncertain, as the usual assumption of $\lambda = 0.5$ is based on models for solar-mass giant stars and an extrapolation is necessary for the high stellar masses of interest here. In a similar way (see eq. [1]), the definition of $\alpha_{CE}$ depends on how accurately known the radii of massive stars typically are after they have left the MS ($R_\odot = r_{1, A}$). In calculating the upper limit on $A$ for CE evolution, the maximum stellar radius, $R_{\text{max}}$, for which Roche lobe overflow is possible enters. For the massive primaries of interest here this maximum radius is reached typically at core helium ignition and sensitively depends on the details of the stellar evolution models (treatment of convection, mass-loss rates, etc.). For a given decrease in orbital separation, deviations from the assumed values of $\lambda$ (0.5) and $R_{\text{max}}$ ($R_{\text{Sch}}$ from the Schaller et al. 1992 models) affect the normalization of $\alpha_{CE}$, $\alpha_{CE}(R_{\text{max}}/R_{\text{Sch}})_{\lambda/0.5}$, and therefore any conclusions regarding contributions from energy sources other than that of the orbit.

2. The ratio of the helium star mass at the time of its collapse, $M_{\text{He,f}}$, to that at the end of the CE phase, $M_{\text{He}}$.—Bare helium stars are known to lose mass in often strong Wolf-Rayet winds (e.g., Barlow, Smith, & Willis 1981). Although observational determinations of Wolf-Rayet mass-loss rates have been improved in recent years, the dependence of these rates on fundamental stellar parameters (e.g., mass, radius, luminosity, etc.) still remains very uncertain. Langer (1989) proposed a mass-loss law such that the mass-loss rate depends only on the stellar mass (power-law dependence with an index of 2.5). The results of stellar evolution calculations adopting this law (Woosley et al. 1995) suggest that, for a wide range of initial helium star masses (up to 20 $M_\odot$), the final masses are concentrated in a narrow range between 3 and 4 $M_\odot$. Such final masses are certainly too small when compared with the BH masses (5–10 $M_\odot$) in the observed X-ray binaries. Given the uncertainties in determining the mass-loss rates and their dependence on stellar parameters, we choose to treat the final helium star mass as a free parameter. The limits set on the binary parameters based on the constraints discussed in § 2.2 depend only on the ratio $M_{\text{He,f}}/M_{\text{He}}$, which can in principle have any value between zero and unity.

3. Mass ejection during the formation of the BH. —Black hole formation during the collapse of massive stars is still not well understood. From core collapse simulations (Fryer 1999; for a recent review, see Burrows 1998) it has become clear that the nature (NS or BH) and the mass of the remnant, the amount of fall-back, and the amount of mass ejection depend sensitively on the details of the collapse mechanism, the energy involved, and the structure of the collapsing star. Therefore, given a BH mass, we are not yet in a position to identify the mass of the progenitor (there may not even be a unique answer). Mass ejection can be significant both in cases where BH formation occurs promptly and when it is preceded by the formation of a neutron star, which then collapses to a black hole because of heavy fallback (e.g., Woosley & Weaver 1995; Fryer 1999). Given our current limited understanding of these processes, we allow for the mass of the collapsing helium star $M_{\text{He,f}}$ to exceed the mass of the black hole.

An upper limit on $M_{\text{He,f}}$ is set by the constraint that the postcollapse system be bound. For a symmetric collapse this implies

$$M_{\text{He,f}} - M_{\text{BH}} \leq \frac{M_{\text{He,f}} + M_\bullet}{2}.$$  

In what follows we let the ratio $M_{\text{He,f}}/M_{\text{BH}}$ be a free parameter constrained in the range

$$1 \leq \frac{M_{\text{He,f}}}{M_{\text{BH}}} \leq 2 + \frac{M_\bullet}{M_{\text{BH}}} \lesssim 3,$$  

where the first inequality follows from the minimal requirement that the mass of the collapsing helium star is at least as massive as the black hole and the last inequality follows from the condition that the donor mass, $M_\bullet$, has to be smaller than the BH mass (approximately) for the mass transfer in the X-ray phase to remain stable and at sub-Eddington rates. Note that whether helium stars with final masses in the range defined by equation (3) will actually collapse to black holes is quite uncertain. It is possible that there is a more restrictive lower limit on $M_{\text{He,f}}$ or that there are additional physical parameters (e.g., magnetic field, rotation, etc.) that determine the nature of the remnant (Ergma & van den Heuvel 1998; Fryer 1999).

For a given BH mass, the above three parameters uniquely specify the efficiency of energy deposition in the CE, the mass of the collapsing helium star, $M_{\text{He,f}}$, and the initial (post-CE) helium star mass, $M_{\text{He}}$. In the present study, we derive our results for a wide enough range of normalized $\alpha_{CE}$ values (as shown below in § 3) and for the complete range of values allowed a priori for the last two ratios, $M_{\text{He,f}}/M_{\text{He}}$ and $M_{\text{He,f}}/M_{\text{BH}}$.

To calculate the exact limits imposed by the first two constraints (§ 2.2) we also need information regarding the masses and radii of hydrogen- and helium-rich, mass-losing stars. For those we adopt the fitting relations given byKW98 (see eqs. [A1], [A4], and [A10] inKW98), extended to the mass ranges of interest here:

$$\log M_{1,i} = 0.58 \log M_1 + 0.62, \quad \log M_1 \geq 1.551,$$  

$$\log R_{1,i} = -2.43 \log M_1 + 7.31, \quad \log M_1 \geq 1.78,$$  

$$\log M_{\text{He}} = 1.68 \log M_{1,i} - 1.482, \quad \log M_1 \geq 1.551,$$  

where $M_1$ is the initial mass of the primary, $M_{1,i}$ and $R_{1,i}$ are the mass and radius, respectively, of the primary at core helium ignition, and $M_{\text{He}}$ is the initial (post-CE) mass of the helium star. Note that we include the effect of wind mass loss (on timescales longer than the orbital period) from both the massive BH progenitors and the helium stars, under the assumption that the specific angular momentum of the wind is equal to that of the mass-losing star (Jeans mode of mass loss). In this case, the ratio of the final to the initial orbital
Fig. 1.—Limits on the donor masses, $M_d$, and postcollapse circularized orbital separations, $A$, of BH X-ray binaries with a $7 M_\odot$ black hole. Thick solid line: Upper limit on $A$ so that low-mass ($M_d \lesssim 1.5 M_\odot$) donors will fill their Roche lobes within $10^{10}$ yr because of magnetic braking and so that more massive donors fill their Roche lobes before reaching the TMS and drive at most Eddington mass transfer rates. Thick dotted line: Upper limit on $A$ so that the progenitors experience a CE phase. Thin solid line: Lower limit on $A$ so that the companion, $M_d$, lies within its Roche lobe at the end of the CE phase. Limits are shown for three different choices of $a_{\text{CE}}$ (for $\lambda = 0.5$), $M_{\text{He}, f}/M_{\text{He}}$, and $M_{\text{He}, f}/M_{\text{BH}}$: (a) 1.0, 1.0, 1.0 (only intermediate-mass donors, $M_d > 2 M_\odot$); (b) 2.0, 1.0, 1.0 (both low- and intermediate-mass donors); (c) 2.0, 0.6, 1.3 (only low-mass, $M_d < 2 M_\odot$, donors), respectively.

i.e., the orbit expands. For the massive hydrogen-rich primaries, however, this expansion is overtaken by the stellar expansion in radius as they leave the MS; the upper limit on $A$ for CE evolution is then determined by the maximum radius of the mass-losing star typically reached just before core-He ignition (Schaller et al. 1992).

The first of the two precollapse constraints of § 2.2 for CE evolution to occur requires that

$$A_{\text{pre}} < R_1/r_{L_1}. \quad (8)$$

Using equation (1) the upper limit on $A_{\text{pre}}$ can be obtained. The phases of wind mass loss from the helium star, as well as of mass loss during BH formation and subsequent circularization, also affect the orbit:

$$A_{\text{precoll}} = \frac{M_{\text{He}, f} + M_d}{M_{\text{He}, f} + M_d}, \quad (9)$$

$$A_{\text{circ}} = (1 + e) = \frac{M_{\text{He}, f} + M_d}{M_{\text{BH}} + M_d}, \quad (10)$$

respectively, under the assumptions of Jeans mass-loss mode and symmetric collapse; $e$ is the postcollapse eccentricity. Using equations (9) and (10) we obtain the upper limit (for CE evolution) on the separation of the binary at the postcollapse, circularized stage. The effect of circularization is calculated assuming conservation of orbital angular momentum. We have also evaluated the circularization timescale for stars with radiative envelopes and, as the stars approach Roche lobe overflow, these timescales become short ($< 10^7$ yr) compared with the timescale of stellar expansion or orbital contraction; for stars with convective envelopes this timescale is even shorter (Zahn 1977, 1992).

The second constraint of § 2.2 sets a lower limit to the post-CE binary separation,

$$A_{\text{post-CE}} > R_d/r_{L_d}, \quad (11)$$

where $r_{L_d}$ is the radius of the Roche lobe of the secondary $M_d$ in units of the orbital separation and $R_d$ is the (zero-age MS) radius of the secondary (eq. [A1] in Kalogera & Webbink 1996). We then translate the above lower limit at the postcollapse, circularized stage with the use of equations (9) and (10).

The limits imposed by the last two constraints of § 2.2 are independent of the above three parameters and depend only on the masses of the black hole and its companion. The upper limit set on the orbital separation (for given $M_d$ by the age of the Galactic disk and the terminal main sequence (TMS) also depends on the strength of angular momentum losses mainly due to magnetic braking (gravitational radiation is included as well). Here, we use the parameterization of magnetic braking as derived by Rappaport, Verbunt, & Joss (1983) and modified by a mass-dependent efficiency estimated by KW98 to fit the rotational data of main-sequence stars more massive than the Sun (eqs. [A14] and [9] in KW98). The magnetic braking strength implied by this specific law has also been found to...
be consistent with the fraction of short-period low-mass X-ray binaries with neutron stars and the transient fraction among them (Kalogera, Kolb, & King 1998).

Finally, to calculate the limit imposed on the binary parameters by the constraint that mass transfer is stable and proceeds at sub-Eddington rates, we extend the results of Kalogera & Webbink (1996) to more massive accretors and hence more massive donors, making sure that this extension is consistent with the detailed calculations of Hjellming (1989). Note that if we were to adopt the less strict requirement of only hydrodynamic equilibrium for the donor and relax the constraints for thermal equilibrium or sub-Eddington mass-transfer rates, our conclusions are further strengthened, as the upper limit on the donor mass increases and the formation of BH binaries with intermediate-mass donors is more favored.

We are now in the position to study all the limits relevant to BH binary formation. In what follows we examine the case of X-ray binaries forming with 7 \( M_\odot \) black holes, as an example, motivated by the results of Bailyn et al. (1998); they conclude that the sample of dynamically measured BH masses in binaries with Roche lobe main-sequence donors is consistent with a relatively narrow range of values around 7 \( M_\odot \). The sensitivity of our results on the assumed BH mass is discussed in § 6.

The limits from all the constraints for \( \alpha_{CE} = 1.0, M_{He, f}/M_{He} = 1.0 \) (no wind mass loss from the helium star), and \( M_{He, f}/M_{BH} = 1.0 \) (no mass ejection during the collapse) are shown in Figure 1a. The upper and lower limits on orbital separation are shown as thick and thin lines, respectively. For this specific case, it is clear that only intermediate-mass (\( > 2 M_\odot \)) donors are allowed to form in an X-ray binary with a 7 \( M_\odot \) black hole. The strongly restrictive character of these limits as shown in Figure 1a indicates that the detailed study of the constraints may provide us with some answers to the question of the physical origin of the binary parameters seen among the observed low-mass BH binaries.

We now investigate the behavior of the limiting curves as the three parameters, \( \alpha_{CE}, M_{He, f}/M_{He}, \) and \( M_{He, f}/M_{BH} \) are varied within their full ranges of allowed values. Changes in the value of \( \alpha_{CE} \) only affect the upper limit on \( \lambda \) (Fig. 1, thick dotted line); ensuring that the progenitor experiences a CE phase, while changes in the two ratios, \( M_{He, f}/M_{He} \) and \( M_{He, f}/M_{BH} \) also affect the lower limit (Fig. 1, thin solid line; ensuring that the donor fits within its Roche lobe at the end of CE evolution). The rest of the limits are independent of changes in these three parameters. From Figures 1a–1c, we see that as these three parameters are varied, the allowed region in the \( \lambda-M_\odot \) space varies as well, so that stars of different masses and evolutionary stages can become donors to 7 \( M_\odot \) accreting black holes. Indeed, if we increase \( \alpha_{CE} \) to 2 (Fig. 1b), we see that both low- (\( < 2 M_\odot \)) and intermediate-mass donors are now allowed. If in addition we let \( M_{He, f}/M_{He} = 0.6 \) and \( M_{He, f}/M_{BH} = 1.3 \) (Fig. 1c), the lower limit (\( \text{thin solid line} \)) on \( \lambda \) moves upward and now excludes donors of intermediate mass. This is because the wind mass loss from the helium star leads to orbital expansion and the mass ejection during collapse leads to an increase in the postcollapse eccentricity.

The three cases shown in Figure 1 are merely indicative of how the evolutionary and structural constraints that are imposed on BH binary progenitors can affect the properties of the X-ray binaries that are formed. In the following section we examine in more detail the constraints imposed on the masses of the donors in BH X-ray binaries.

4. MASSES OF DONOR STARS

The characteristics of BH binaries are determined by the five main constraints discussed in § 2.2. Depending on the degree of orbital contraction in the CE phase, the extent of wind mass loss from helium stars, and the amount of mass ejection at during BH formation, three different outcomes with regard to the donor masses are possible: BH binaries can be formed with (1) only low-mass donors; (2) only intermediate-mass donors; (3) both low- and intermediate-mass donors. Here and throughout this paper the separation between low- and intermediate-mass donors is chosen to be 2 \( M_\odot \).

For a specific BH mass and for a range of values of the CE efficiency \( \alpha_{CE} \), we can study the types of BH donors in the plane of the other two parameters: \( M_{He, f}/M_{He} \) and \( M_{He, f}/M_{BH} \), examining the complete range of their possible, a priori, values, 0–1 and 1–3, respectively. The results for a 7 \( M_\odot \) black hole are shown in Figure 2 for six values of \( \alpha_{CE} \). The three different types of main-sequence donors are indicated by different shadings (light gray: low mass; dark gray: intermediate mass; black: both). Regions of the parameter space that are not shaded correspond to conditions for which the formation of X-ray binaries with the assumed BH mass is never possible.

Given the high masses of the original primaries, the contraction of the orbits, for low values of \( \alpha_{CE} \) (\( \lesssim 0.5 \)), is so strong that the donor stars cannot fit within their Roche lobes at the end of the CE phase. As a consequence, the binaries merge and no BH X-ray binaries are formed. This would of course be in contrast to the observations and therefore such low values of \( \alpha_{CE} \) can be excluded. As the degree of CE orbital contraction decreases (\( \alpha_{CE} \) increases), the more massive of the donors are able to eject the massive envelopes and avoid merging with the cores of their companions. Therefore, for \( 0.5 \lesssim \alpha_{CE} \lesssim 1.0 \), BH X-ray binaries with only intermediate-mass donors are being formed. The exclusion of all low-mass donors would still be in disagreement with the observed donor masses, forcing us to examine the limits for even higher values of \( \alpha_{CE} \). The formation of BH binaries with donor masses lower than 2 \( M_\odot \) becomes possible only when \( \alpha_{CE} \) exceeds unity. For \( \alpha_{CE} \gtrsim 1.8–2.0 \), the limit for the occurrence of CE evolution is pushed far enough up in the \( \lambda-M_\odot \) plane (Fig. 1) that it does not at all interfere with the limit for the donor stars to be accommodated within their Roche lobes at the end of the CE phase. As a result, the masses of donors allowed for the BH binaries become independent of \( \alpha_{CE} \). Consistency with the observed sample at a minimum level, i.e., BH binaries with low-mass donors are allowed to form, would then require that the normalized \( \alpha_{CE} \) exceed unity.

It is also interesting to note the behavior of the donor types as the amount of mass loss associated with helium star winds and BH formation varies. Both of these parameters primarily affect the way the lower limit, related to the donor fitting within its Roche lobe at the end of the CE phase, is translated through the subsequent evolutionary stages of wind mass loss, symmetric BH formation, and circularization of the orbits as the onset of mass transfer is approached. The larger the mass loss is, the more the orbits expand, either adiabatically from the helium star wind or nonadiabatically following the collapse to a black hole.
FIG. 2.—Limits on the parameter space of the final (precollapse) helium star mass, $M_{\text{He},f}$, and the ratio, $M_{\text{He},f}/M_{\text{He}}$, that describe the extent of mass loss from helium stars, for six values of the $\alpha_{\text{CE}} = 0.6, 1.0, 1.4, 1.6, 2.0, 3.0 (\lambda = 0.5)$. Conditions in the unshaded areas do not allow the formation of BH binaries with MS Roche lobe filling donors; conditions in the light gray, dark gray, and black areas allow the formation of systems with only low-mass, only intermediate-mass, and both types of donors, respectively. The BH mass has been assumed to be equal to 7 $M_\odot$. It is evident that relatively high values of $\alpha_{\text{CE}}$ are favored, while mass loss both in a helium star wind and at BH formation are constrained to within a factor of about 2.

Overall, we can identify the following systematic behavior (in the range $\alpha_{\text{CE}} \gtrsim 1.8$–2.0, where the precise value of $\alpha_{\text{CE}}$ becomes irrelevant). If mass loss is absent or small, then both low- and intermediate-mass donors can be feeding 7 $M_\odot$ black holes in X-ray binaries. For moderate mass loss, low-mass donors only are allowed, as binaries with $M_d \gtrsim 2 M_\odot$ expand so much that mass transfer could not be initiated while the donors are on the MS. For low-mass donors, magnetic braking is expected to be efficient and the associated orbital contraction brings them at Roche lobe overflow. Very strong helium star winds that decrease the helium star mass by more than a factor of about 3 ($M_{\text{He},f}/M_{\text{He}} \lesssim 0.35$), prevent BH binary formation altogether, because the expansion during the wind phase leads to orbits too wide for BH binaries with MS donors of any mass to be formed. Mass loss following BH formation also leads to orbital expansion, but it is further constrained to be at most comparable to the BH mass ($M_{\text{He},f}/M_{\text{BH}} \lesssim 2$) for the binary to be bound after the collapse. Therefore, mass loss in both phases has to be sufficiently small for BH binaries like the observed ones to form.

It is interesting to examine in some detail the effect of the evolution of the helium star on two of the limiting curves (first two of the constraints discussed in § 2.2). Mass loss from the helium star wind and during BH formation combined with circularization of the postcollapse orbit results in the orbital expansion described by equations (9) and (10). Both precollapse constraints are actually set at the beginning and the end of the CE phase; therefore, by combining these two equations we find that the translation of the corresponding limits through the helium star wind and collapse phases becomes independent of the mass, $M_{\text{He},f}$, of the collapsing helium star:

$$\frac{A_{\text{circ}}}{A_{\text{post-CE}}} = \frac{M_{\text{He}} + M_d}{M_{\text{BH}} + M_d}. \quad (12)$$

Therefore, for specific BH and donor masses, the limits on the orbital separations of bound systems for CE evolution (upper) and for the donor fitting within its Roche lobe are expected to be independent of $M_{\text{He},f}$. Indeed Figure 2 shows that the character (low- or intermediate-mass) of donors in BH binaries changes along straight lines on the parameter plane of $M_{\text{He},f}/M_{\text{He}}$ and $M_{\text{He},f}/M_{\text{BH}}$. From the definition of these two ratios, it becomes clear that the change of donor types occurs along lines of constant helium star mass, $M_{\text{He}}$, at the end of the CE phase as expected.

We should note, however, that, although the limits in the $A-M_d$ parameter space are independent of $M_{\text{He},f}$, we cannot completely eliminate from the problem the final helium star mass, because knowledge of it is necessary to decide whether the system will remain bound after the collapse or not. Then, given that the pair remains bound, the orbital separation of the binary is determined independently of the mass of the collapsing helium star. The fact that we still need to know the value of $M_{\text{He},f}$ is what actually enables us to set quantitative limits both on the extent of wind mass losses from massive helium stars and on the masses of BH progenitors.

We can use this simplifying independency of the limits on $M_{\text{He},f}$ and further examine how the results change in response to varying only the initial helium star mass $M_{\text{He}}$ (and correspondingly that of the massive initial BH progenitor), and the normalized CE efficiency, $\alpha_{\text{CE}}$. As
already discussed (§ 3), for a given decrease in the orbital separation, the normalization of $\alpha_{CE}$ is somewhat arbitrary as it is sensitive to the estimate of the binding energy of the envelope, which depends on the uncertain values of the central concentration parameter, $\lambda$, and the maximum stellar radius, $R_{\text{max}}$. Taking into account this lack of absolute normalization, we identify the ranges of initial helium star masses, $M_{\text{He}}$, and of the product $\alpha_{CE}(R_{\text{max}}/R_{\text{Sch}}) (\lambda/0.5)$, for which X-ray binaries with $7 M_{\odot}$ BHs and low-mass, intermediate-mass, or a mixture of these donors are expected to be formed (Fig. 3). As was also evident from Figure 2, the formation of any X-ray binaries is possible only if $\alpha_{CE}$ (as normalized above) is higher than $\sim 0.5$. Binaries with exclusively low-mass donors are formed when $\alpha_{CE} \gtrsim 1$ and the progenitor helium stars are more massive than the black hole by a factor of $2$ ($M_{\text{He}} \gtrsim 15 M_{\odot}$ and $M_{i} \gtrsim 45$). Formation of X-ray binaries with both low- and intermediate-mass donors is required to explain the observations, and this requirement constrains $\alpha_{CE}$ to be relatively high and the initial helium stars to be at most twice as massive as the black hole (initial primaries in the range $25\rightarrow 45 M_{\odot}$).

5. Relative Numbers of Low- and Intermediate-Mass Donors

As discussed in § 1, the majority of the observed BH X-ray binaries harbor low-mass ($\sim 1 M_{\odot}$) donor stars, and only two of the 14 BH candidates (including the ones with only spectral and variability evidence for the presence of a black hole) have masses between 2 and $3 M_{\odot}$. For the results of the present study to be consistent with the current observational sample, the three parameters that enter the problem (assuming a specific BH mass) should be constrained within the ranges of values for which BH X-ray binaries with both low- and intermediate-mass donors can be formed (Figs. 2 and 3 black areas).

In this section we examine the predicted relative numbers of systems with low- and intermediate-mass donors. The ratio of their numbers depends both on the lifetimes of the X-ray phases and on the birthrates of systems in each group.

5.1. X-Ray Lifetimes

Angular momentum losses due to magnetic braking are thought to dominate the evolution of BH X-ray binaries with low-mass donors. For donor masses exceeding $\sim 1 M_{\odot}$ magnetic braking is thought to become unimportant as the stars lose their convective envelopes. Convective envelopes are necessary for the generation of magnetic fields by dynamo processes and effective magnetic braking. Here, we have adopted the magnetic braking law proposed by Rappaport et al. (1983), along with a mass-dependent magnetic braking efficiency, $b(M_d)$, derived to match the rotational velocity data of more massive MS stars (of spectral type F0 and F5; for more details see KW98). We define the magnetic braking timescale as $\tau_{\text{MB}} = J/orb$ where $J/orb$ and $J$ are the orbital angular momentum and its derivative, respectively (see eq. [A14] in KW98). For systems with MS donors less massive than $\sim 1.2 M_{\odot}$, we calculate this timescale to be in the range $(2 \times 10^9)\rightarrow 10^9$ yr.

In BH binaries with intermediate-mass donors the mass transfer is driven by the nuclear evolution of the donor. Using the results of Schaller et al. (1992) and adopting a fitting formula (eqs. [A9] and [A10] in Kalogera & Webbink 1996) for the evolution of stellar radius with time on the MS, we find that, for $2\rightarrow 7 M_{\odot}$ stars, the MS lifetimes and the corresponding timescales of radial expansion lie in the range $10^8\rightarrow 10^9$ yr. Therefore, it appears that the lifetimes of the BH binaries with low- or intermediate-mass donors are comparable within a factor of $\sim 2$. As a result, the relative numbers of low- and intermediate-mass BH binaries depend mainly on the relative birthrates for the two groups of systems.

5.2. Birthrates

So far we have identified the regions in the parameter space of circularized postcollapse orbital separation, $A$, and donor mass, $M_d$, of both low- and intermediate-mass BH binaries, for the full range of values of the relevant parameters describing the evolution of their progenitors. The probability density with which these regions are populated depends directly on the distribution function that describes primordial binaries with extreme mass ratios and the effects on it of the various evolutionary stages leading to the formation of BH X-ray binaries.

In order to calculate the predicted birthrates for the two types of BH binaries, we need to synthesize a primordial binary population and evolve it through the formation channel of interest to us here. Since we have assumed that the helium star collapse is symmetric, the population synthesis calculation is extremely simplified and can be performed fully analytically using Jacobian transformations of the distribution function (this method has been described in detail by KW98), for the general case where the collapse is not necessarily symmetric.

We assume that the primordial binary population is characterized by three parameters: the mass of the primary, $M_1$, the mass ratio, $q = M_d/M_1$, and the orbital separation, $A$, (orbits are assumed to be circular). We adopt a field star
initial mass function as derived by Scalo (1986) and a distribution over orbital separations that is constant in log $A_i$ (Abt 1983), and we parameterize the mass ratio distribution as a power law, proportional to $q^{-a}$ (for more details regarding these assumptions, see KW98). Since we are ultimately interested in ratios of predicted birthrates, all normalization constants become irrelevant. The distribution function of the primordial binary population is¹

$$F(\log M_1, q, \log A_i) \propto M_1^{-1.7} q^{-a_1},$$

or

$$F(\log M_1, \log M_d, \log A_i) \propto M_1^{7.2} q^{1-a_1}.$$  

We transform the above distribution function through all the evolutionary phases and calculate it at the stage of circulared postcollapse orbits. The stages we have to consider are the following:

1. Wind mass loss from the massive primary as it evolves in isolation until the CE phase.—The mass of the primary decreases (down to $M_{1,L,i}$ (eq. [4]) for the CE upper limit on $A$, and the orbit expands according to the Jeans mode of mass loss (eq. [7]).

2. Orbital contraction and loss of the primary’s envelope in the CE phase.—The mass of the primary decreases to $M_{He}$ (eq. [6]), and the post-CE orbital size depends on the CE efficiency and the stellar masses involved (eq. [1]).

3. Wind mass loss from the helium star primary.—The final helium star mass depends on the value of the assumed ratio $M_{He,f}/M_{He}$, and the orbit again expands according to the Jeans mass-loss mode (eq. [7]).

4. Symmetric BH formation and circularization.—The primary mass decreases to the BH mass and the orbit expands (eq. [10]).

In each of the above evolutionary stages, the ratio of the final to the initial orbital separation depends only on the stellar masses involved and the CE parameters and not on any of the orbital separations themselves. This results in all the derivations of the form $\partial \log A_i/\partial \log A_i$ being equal to unity, and therefore the distribution function in log $A$ remains unaffected by these transformations. On the other hand, the changes in primary mass (the donor mass remains constant throughout the evolution) depend only on constants that appear in the fitting relations, while for the transformations from $M_{He}$ to $M_{He,f}$, the relevant derivative is equal to unity. Given the simplicity of these steps, the distribution of binaries over log $M_{He}$, log $M_d$, and log $A$, remains the same (modulo normalization constants) as in equation (8):

$$F(\log M_{He}, \log M_d, \log A) \propto M_{He}^{7.2} M_d^{-a_1}.$$  

To calculate the ratio $BR_1/BR_2$ of the predicted birthrates for BH binaries with intermediate-mass donors over that of systems with low-mass donors, we integrate the above distribution function over the ranges of orbital separations and donor masses that are relevant to each of the two groups:

$$BR_1 \propto \int_{M_d}^{M_d^{max}} \left( \log \frac{M_d^{max}}{M_d^{min}} \right) M_d^{-a_2} d \log M_d.$$  

For specific values of the BH mass, CE efficiency, and initial helium star mass (or initial primary mass), $M_d^{min}$ and $M_d^{max}$ are determined by the limits discussed in § 3, and they are functions of $M_d$ only. The logarithm of their ratio that appears in equation (16) is the result of the integration of equation (15) over log $A$ at a given $M_d$. For the systems with low- and intermediate-mass donors the values of $M_d^{max}$ and $M_d^{min}$, respectively, are equal to 2 $M_\odot$. Finally, the dependence on $M_P$ that appears in equation (15) becomes irrelevant when we take the ratio of the two birthrates at a given $M_{He}$, hence initial primary mass, $M_1$.

It is evident that two unknowns enter the calculation of the predicted birthrates: the distributions of initial orbital separations and mass ratios. Given the narrow ranges of orbital separations of interest (see Fig. 1) the exact form of their distribution does not affect the results in a significant way. However, this is not true for the mass ratio distribution. The primordial binaries that are relevant for the formation of low-mass BH binaries have mass ratios typically smaller than 0.1, a range which is observationally inaccessible. This situation is identical to that of LMXBs with neutron stars, which has been discussed in detail by KW98. In short, the essentially arbitrary choice of the power-law index $a_q$ in the mass ratio distribution significantly affects the absolute normalization of the predicted birthrates (but essentially not the distribution of the newborn LMXBs in orbital parameters). In what follows, we adopt the results of Hogeveen (1991) and extend them to mass ratios smaller than ~0.3. His results show that the observed mass-ratio distribution is biased toward high values, while the corrected distribution can be described by a steep power law with index $a_q = 2.7$.

We use equation (16) and the limits on orbital separations as calculated in § 3 and evaluate the birthrate ratio $BR_1/BR_2$ for $a_q = 2.7$ and a 7 $M_\odot$ black hole. We plot the results against the initial (post-CE) helium star mass for three values of the CE efficiency in Figure 4. For smaller and higher values of $a_q$, the behavior of the ratio is very similar to what is shown for $a_q = 1.6$ and 3, respectively. It is clear that in the parameter space in which both types of BH X-ray binaries are allowed to form (Figs. 2 and 3, black areas), there are only small regions where systems with low-mass donors dominate the population (i.e., the ratio $BR_1/BR_2$ is smaller than unity). At first, this result may appear counterintuitive, given the choice of $a_q = 2.7$, which strongly favors very small initial mass ratios and hence systems with low-mass companions. However, there is a counteracting effect: the width of the range of orbital separations allowed to be populated by the two types of systems. Scrutiny of Figure 1 indicates that this range is considerably wider for intermediate-mass donors than for low-mass donors. The origin of this difference lies in the steep slopes of the radius mass relations of both evolved massive stars and of the low-mass zero-age MS stars. Consistency with the observations appears to be possible only if the CE efficiency as normalized in equation (1) is relatively high (higher than about 2). Given the strong peak of the assumed mass ratio distribution at low values, the results shown in Figure 4 should be regarded as lower limits. We

¹ For simplicity, we use the same symbol, $F$, for the distribution functions at all evolutionary stages. The individual stages can be identified based on the notation used for the masses and orbital separations.
have calculated the same birthrate ratios for $q = 0$ (flat mass ratio distribution, which is often used in the literature) and found that the ratios are increased, i.e., intermediate-mass donors are favored even more strongly. This is expected, given the decrease of $q$, and contrasts even more with the properties of the observed sample.

From the observed sample without any corrections and for the roughly comparable lifetime estimates for the two groups (§5.1), a value of $\approx 2/14 \sim 10^{-1}$ is implied for $BR_1/BR_2$. Given the uncertainties and possible selection effects acting on the population of BH binaries, we choose to be relatively conservative and regard model results as being consistent with observations when the ratio of birthrates, $BR_1/BR_2$, is lower than unity (since the lifetimes of the two groups have been found to be roughly comparable). For normalized $\alpha_{\text{CE}}$-values in the range 1.5–2.5, this can be obtained only for narrow ranges of initial helium star masses, either very close to the BH mass ($7M_\odot$) or between $\sim 11$ and $14M_\odot$ (Fig. 4). For smaller values ($\alpha_{\text{CE}} \lesssim 1.5$), $BR_1/BR_2$ always exceeds unity, while for higher values ($\alpha_{\text{CE}} \gtrsim 2.5$), the full range ($\sim 7–15M_\odot$) of $M_{\text{hel}}$-values (initial primary masses in the range $\sim 25–45$) could be consistent with the observed sample. We note that for a flat mass ratio distribution ($q = 0$), masses are restricted in the narrow range $11–14M_\odot$, even for high $\alpha_{\text{CE}}$-values ($\alpha_{\text{CE}} > 2$).

6. SUMMARY AND DISCUSSION

We find that the models for BH formation are consistent with the properties of the observed sample if (1) wind mass loss from helium stars is limited so that they lose less than half of their initial mass, (2) helium stars that form black holes are at most about a factor of 2 more massive than the black holes, and (3) CE efficiencies are relatively high and, depending on the exact radii of massive stars and their density profiles, significant contributions from energy sources other than the orbit may be required. Our results show weak dependence on the assumed BH mass. Although the results presented in this paper are all for $M_{\text{BH}} = 7M_\odot$, we have also examined models for $M_{\text{BH}} = 5M_\odot$ and found that only the shape of the boundaries in the parameter space between the different donors is somewhat changed. However, both the qualitative conclusions and quantitative constraints on $\alpha_{\text{CE}}$, helium star wind mass loss, and BH progenitors remain essentially unaltered.

Based on the approximate formulation of the CE phase (eq. [1]) we stress that the CE efficiency, $\alpha_{\text{CE}}$, lacks absolute normalization and only relative limits can be set on it. With this scaling in mind, it is worth noting that an investigation of the birthrate of low-mass BH binaries by Portegies Zwart et al. (1997) also pointed to a need for relatively high values of $\alpha_{\text{CE}}$. On the other hand, a study of the properties (orbital periods and transient character) of LMXBs with neutron stars (Kalogera et al. 1998) favors relatively low values of $\alpha_{\text{CE}}$ ($\sim 0.5$ with the formulation used here). This difference could possibly be attributed to the different range of primary masses that are relevant in the formation of the two types of LMXBs ($M_1 < 25 M_\odot$ for neutron stars) and therefore to the differences in the structure and energetics of their massive envelopes.

For primordial binaries with extreme mass ratios like those needed for the formation of BH binaries, a CE phase is inevitable once the primary fills its Roche lobe. Based on our results, formation of a black hole in the range of masses observed is then possible only if the helium star is able to retain a significant amount of its mass—at least half of it (see also Ergma & van den Heuvel 1998). This provides a new constraint on the mass-loss process in helium stars; it suggests that the parameterization of the mass-loss rate adopted so far (Langer 1989), which leads to a very narrow range of final masses at $\sim 4M_\odot$ (Woosley et al. 1995), may not apply to stars with relatively high initial masses.

Formation of low-mass BH binaries through helium star collapse is expected once the primordial binary experiences a CE phase. There is, however, the possibility that the initial binary is so wide that the primary never fills its Roche lobe during its entire lifetime and collapses directly to a black hole. If the primary is more massive than $\sim 40M_\odot$, then it is possible that the star loses its hydrogen-rich envelope in a wind (Schaller et al. 1992) and the helium core is once again revealed. In this case, the problems discussed above are again relevant and we would have to conclude that helium star winds cannot be as strong as thought previously. If the star is able to retain part of its hydrogen envelope, then its final mass is much larger than the observed BH masses; subsequent mass loss at the time of the collapse should lead to disruption of the binary, if the collapse had been symmetric. Even if the final mass is low enough that the mass loss at BH formation is not disruptive (does not exceed half of the precollapse total mass), using the radius-mass relation for such massive stars (Schaller et al. 1992), we find that the precollapse orbital separation is so wide ($\sim 1000R_\odot$) that the postcollapse system will never reach a Roche lobe overflow phase and appear as an X-ray binary (Romani 1992; Portegies Zwart et al. 1997).
average kick magnitude must be at least comparable to or higher than the typical relative orbital velocity in the pre-collapse system (Kalogera 1996), i.e., higher than ~100 km s^{-1}. Given the mass difference between black holes and neutron stars, these kick magnitudes would require a momentum transfer several times larger in the case of black holes. Although this cannot be excluded (since the physical origin of kicks is not known), it imposes significant quantitative constraints on the process. In the case of CE evolution, the helium star binaries are close, with orbital separations ~10–20 R_\odot and orbital velocities in excess of ~300 km s^{-1}, up to ~900 km s^{-1}. As shown above, we cannot exclude the possibility that kicks could affect our conclusions, but then the kick mechanism should be such that the amount of momentum transfer is higher for black holes than for neutron stars, by a factor comparable to their mass ratio (~10).

So far we have implicitly assumed that the observed sample of low-mass BH binaries is representative of the total underlying Galactic population, at least from the point of view of their donor masses. However, the apparent paucity of systems with intermediate-mass donors could be merely the result of selection effects. We note that accurate dynamical measurements of the compact object masses are possible only for soft X-ray transients, because detailed observations of the low-mass secondaries can be undertaken only in quiescence (e.g., Charles 1998). Therefore, if BH binaries with companion masses in excess of 2 M_\odot were persistent sources, then a significant population of these systems could exist in the Galaxy and remain undetected. However, if this were the case, to the extent that the emission processes were similar, then there should be some evidence of such a population among the X-ray binaries that are thought to be BH candidates based only on their spectral and rapid variability characteristics. As already discussed, all the systems (the sample includes transient and persistent sources) for which there are at least lower limits on their visual magnitudes appear to harbor low-mass companions (Ritter & Kolb 1998; Chen et al. 1997; Stella et al. 1994). Furthermore, one can estimate the mass transfer rates expected for intermediate-mass BH binaries, using the radius expansion rates from nuclear evolution on the MS. For a substantial range of masses (up to ~4 M_\odot), these are found to be low enough for the disk instability to develop, and therefore such systems should appear as soft X-ray transients just like their low-mass counterparts (U. Kolb 1998, private communication). The apparent paucity of intermediate-mass donors appears therefore to be real.

In the present study, we have not addressed the issue of formation of BH binaries with Roche lobe–filling donors on the giant branch (V404 Cyg is the only example in the observed sample). Our results on the limits imposed on orbital separations (Fig. 1) indicate that the upper limit (for CE evolution) is so strong that it may be hard for binaries ever to form with wide enough orbits. Such orbits would be required for the systems to evolve to longer periods with evolved donors as described by Poylyser & Savonije (1988). Another way to form such systems could possibly be through secular evolution of intermediate-mass BH binaries. As magnetic braking is not expected to be efficient for these higher mass stars, orbital evolution is driven by nuclear expansion on the MS. Predictions of the fate of such systems in terms of their binary parameters require detailed modeling of the mass transfer phase; it may be possible that systems with low-mass donors on the giant branch form through secular evolution and not from initially wide post-collapse binaries.

It is a pleasure to thank R. Narayan, D. Psaltis, and F. Rasio for many stimulating discussions and useful comments in the course of this study. I would also like to thank S. Woosley for discussions on the issue of mass loss from helium stars and R. Webbink and J. McClintock for discussions on selection effects on the BH X-ray binary population. Furthermore, I would like to thank U. Kolb, A. King, A. Esin, K. Menou, and E. Quataert for discussions. I am grateful to D. Psaltis and F. Rasio for a critical reading of an earlier version of the manuscript. I would also like to thank an anonymous referee for important comments that improved the clarity of the text. Finally, I am thankful for the warm hospitality of the Aspen Center of Physics, the Astronomy Group of the University of Leicester, and the “Anton Pannekoek” Astronomical Institute at the University of Amsterdam. Support by the Smithsonian Astrophysical Observatory through a Harvard-Smithsonian Center for Astrophysics Postdoctoral Fellowship is also acknowledged.

REFERENCES

Abt, H. A. 1983, ARA&A, 21, 343
Bailyn, C. D., Jain, R. K., Coppi, P., & Orosz, J. A. 1998, ApJ, 499, 367
Barlow, M. J., Smith, L. J., & Willis, A. J. 1991, MNRAS, 196, 191
Biehle, G. T. 1991, ApJ, 380, 167
Burrows, A. 1998, in Proc. 9th Workshop on Nuclear Astrophysics, Ring-Biehle, G. T. 1991, ApJ, 380, 167
Hjellming, M. S. 1989, Ph.D. thesis, Univ. Illinois
Hansen, C. M. S., & Phinney, S. L. 1997, MNRAS, 291, 569
Han, Z., Podsiadlowski, P., & Eggleton, P. P. 1995, MNRAS, 270, 121
Hansen, C. M. S., & Phinney, S. L. 1997, MNRAS, 291, 569
Hjellming, M. S. 1989, Ph.D. thesis, Univ. Illinois
Hogan, S. J. 1991, Ph.D. thesis, Univ. Amsterdam
Iben, J., &Livio, M. 1993, PASP, 105, 1373
Kalogera, V. 1996, ApJ, 471, 352
Kalogera, V., Kolb, U., & King, A. R. 1998, ApJ, 504, 967
Kalogera, V., & Webbink, R. F. 1996, ApJ, 458, 301
Kaspi, V. M., Bailes, M., Manchester, R. N., Stappers, B. W., & Bell, J. F. 1996, Nature, 381, 884
Langer, N. 1989, A&A, 220, 135
Lorimer, D. R., Bailes, M., & Harrison, P. A. 1997, MNRAS, 289, 592
Mardling, R. A., 1996, in IAU Symp. 174, Dynamical Evolution of Star Clusters: Confrontation of Theory and Observations, ed. P. Hut & J. Makino (Dordrecht: Reidel), 273
Orosz, J. A., et al. 1998, ApJ, 499, 375
Portegies Zwart, S. F., Verbunt, F., & Ergma, G. 1997, A&A, 321, 207
Pylyser, E., & Savonije, G. J. 1998, A&A, 191, 57
Rappaport, S. A., Verbunt, F., & Joss, P. C. 1983, ApJ, 275, 713
Ritter, H., & Kolb, U. 1998, A&AS, 129, 83
Romani, R. W. 1992, ApJ, 399, 621
Romani, R. W. 1996, in IAU Symp. 165, Compact Stars in Binaries, ed. J. van Paradijs, E. J. van den Heuvel, & E. Kuulkers (Dordrecht: Reidel), 93
—. 1998, A&A, 333, 583
Scalo, J. M. 1986, Fundam. Cosmic Phys., 11, 1
Schaller, G., Schaerer, D., Meynet, G., & Maeder, A. 1992, A&AS, 269, 331

No. 2, 1999 DONOR STARS IN BLACK HOLE X-RAY BINARIES 733

Higbeven, D. R., Bailes, M., & Harrison, P. A. 1997, MNRAS, 289, 592
Kalogera, V. 1996, ApJ, 471, 352
Kalogera, V., Kolb, U., & King, A. R. 1998, ApJ, 504, 967
Kalogera, V., & Webbink, R. F. 1996, ApJ, 458, 301
Kaspi, V. M., Bailes, M., Manchester, R. N., Stappers, B. W., & Bell, J. F. 1996, Nature, 381, 884
Langer, N. 1989, A&A, 220, 135
Lorimer, D. R., Bailes, M., & Harrison, P. A. 1997, MNRAS, 289, 592
Mardling, R. A. 1996, in IAU Symp. 174, Dynamical Evolution of Star Clusters: Confrontation of Theory and Observations, ed. P. Hut & J. Makino (Dordrecht: Reidel), 273
Orosz, J. A., et al. 1998, ApJ, 499, 375
Portegies Zwart, S. F., Verbunt, F., & Ergma, G. 1997, A&A, 321, 207
Pylyser, E., & Savonije, G. J. 1998, A&A, 191, 57
Rappaport, S. A., Verbunt, F., & Joss, P. C. 1983, ApJ, 275, 713
Ritter, H., & Kolb, U. 1998, A&AS, 129, 83
Romani, R. W. 1992, ApJ, 399, 621
Romani, R. W. 1996, in IAU Symp. 165, Compact Stars in Binaries, ed. J. van Paradijs, E. J. van den Heuvel, & E. Kuulkers (Dordrecht: Reidel), 93
—. 1998, A&A, 333, 583
Scalo, J. M. 1986, Fundam. Cosmic Phys., 11, 1
Schaller, G., Schaerer, D., Meynet, G., & Maeder, A. 1992, A&AS, 269, 331
Stella, L., Israel, G. L., Mereghetti, S., & Ricci, D. 1994, in Proc. 7th Marcel Grossmann Meeting: Recent Developments in Theoretical and Experimental General Relativity, Gravitation, and Relativistic Field Theory, ed. R. Ruffini & M. Keiser (Singapore: World Scientific)

Sutantyo, W. 1975, A&A, 41, 47

Taam, R. E. 1994, in ASP Conf. Ser., 56, Interacting Binary Stars, ed. A. W. Shafter (San Francisco: ASP), 208

Tokovinin, A. A. 1997, A&AS, 124, 751

van den Heuvel, E. P. J. 1975, ApJ, 198, L109

———. 1983, in Accretion-driven Stellar X-Ray Sources, ed. W. H. G. Lewin & E. P. J. van den Heuvel (Cambridge: Cambridge Univ. Press), 303

van der Klis, M. 1994a, A&A, 283, 469

———. 1994b, ApJS, 92, 511

Webbink, R. F. 1984, ApJ, 277, 355

White, N. E., & van Paradijs, J. 1996, ApJ, 473, 25

Wijers, R. A. M. J. 1996, in Evolutionary Processes in Binary Stars, ed. R. A. M. J. Wijers, M. B. Davies, & C. A. Tout (Dordrecht: Kluwer), 327

Woosley, S. E., Langer, N., & Weaver, T. A. 1995, ApJ, 448, 315

Woosley, S. E., & Weaver, T. A. 1995, ApJS, 101, 181

Zahn, J.-P. 1977, A&A, 57, 383

———. 1992, in Binaries as Tracers of Stellar Formation, ed. A. Duquennoy & M. Mayor (Cambridge: Cambridge Univ. Press), 253