Routing and Spectrum Allocation in Spectrum-sliced Elastic Optical Path Networks

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Abstract—The Spectrum-sliced Elastic Optical Path (SLICE) networks adopt the Orthogonal Frequency Division Multiplexing (OFDM) technology to offer flexible and elastic bandwidth allocation. The OFDM technology enables both sub-wavelength and super-wavelength traffic accommodation by allocating appropriate number of sub-carriers while providing high signal quality and mitigating various impairments. To accommodate traffic demands in SLICE networks, one fundamental problem is to establish spectrum paths by allocating sub-carriers along the corresponding routes, which is referred to as the routing and spectrum allocation (RSA) problem. The optimal RSA problem is NP-Hard and different from the traditional routing and wavelength assignment (RWA) problem in WDM networks. In this work, we model the RSA problem using a set of proposed Integer Linear Programming (ILP) formulations to achieve different optimization objectives. We also present new approaches to analyze the lower/upper bounds for the sub-carrier number in a SLICE network. Two efficient heuristic algorithms are also studied in our simulation under different optimization goals.

I. INTRODUCTION

Traditional WDM networks adopt a fixed-size frequency allocation per wavelength, where a wavelength is the smallest granularity to accommodate the traffic demands in all-optical domain. To facilitate the signal filtering, any two adjacent wavelengths have to be separated by guard-band frequencies [1]. This fixed-size frequency allocation, however, can lead to the underutilization of the spectrum resources [2], [3]. For example, sub-wavelength traffic may have to be overprovisioned due to the coarse granularity of the wavelength. When a traffic demand requires multiple wavelengths (i.e., super-wavelength traffic), guard-band frequencies (or spectrum gap) between those wavelengths can lead to underutilization of the available spectrum resources. The inflexibility and coarse granularity of WDM frequency allocation, recently, motivates the emerging of the spectrum-sliced elastic optical path (SLICE) networking technologies [2], [4], [5].

The elastic and flexible bandwidth allocation is enabled in the SLICE network with the aid of the Orthogonal Frequency Division Multiplexing (OFDM) technology [2]. With in-born advantages of robustness against the physical impairments (e.g., chromatic dispersion (CD)), OFDM can efficiently accommodate both the sub-wavelength and super-wavelength traffic. In specific, OFDM distributes the data on a bunch of sub-carriers. Each sub-carrier has a much lower data rate than a single wavelength in WDM networks, which ensures sub-wavelength traffic demands accommodated efficiently in SLICE networks. Moreover, due to the orthogonality, adjacent sub-carriers can partially overlap with each other without a spectrum gap. When a super-wavelength traffic demand requires multiple sub-carriers, consecutive sub-carriers can be allocated and overlapped in the spectrum domain without using the spectral gap or guard-band frequencies.

Similar to the routing and wavelength assignment (RWA) problem in WDM networks, the SLICE network has to deploy the routing and spectrum allocation process to accommodate the traffic demands [2]. In our previous work [3], we studied the static RSA problem in SLICE networks with the goal of minimizing the maximum sub-carrier number over any fiber. In [3], the sub-carrier number analysis was restricted to the ring network. In this work, we study the optimal RSA problem by further incorporating the goal of minimizing the total sub-carriers, and propose new approaches for the analysis of sub-carrier number in a general mesh network. Particularly, we first present Integer Linear Programming (ILP) formulations for optimal RSA with two optimizations goals. We then present new methods to analyze the bounds of the required sub-carriers in the SLICE network. Those methods generalize our previous analysis to the mesh network and improve the bound for the case with predetermined routing knowledge. Furthermore, our simulations investigate two efficient heuristic algorithms, namely balanced load spectrum allocation (BLSA) and shortest path with maximum spectrum reuse (SPSR) under two different optimization goals.

The rest of the paper is organized as follows. In Section II, we formally define the RSA problem. In Section III, we model the optimal RSA problem using the ILP formulations. In Section IV, we present the sub-carrier bounds analysis in the SLICE network. In Section V, we present the numerical analysis of the proposed schemes. We review the related work in Section VI and conclude the work in Section VII.

II. ROUTING AND SPECTRUM ALLOCATION IN SLICE NETWORKS

In SLICE networks, a spectrum path (SP) is an all-optical trail established between the source and sink nodes by using

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one or multiple consecutive sub-carriers. The fundamental issue in the SLICE network is to route and set up spectrum paths to accommodate the traffic demands, which is called as the routing and spectrum allocation (RSA) problem [2]. Although both RSA and RWA have to consider the allocation of spectrum resources and the spectrum continuity for any spectrum path (or lightpath), the RSA problem is different from and more challenging than the traditional RWA problem [3]. First, for a spectrum path with multiple sub-carriers, the allocated sub-carriers have to be consecutive in spectrum domain to be effectively modulated at the OFDM transponders [5], which is referred to as the sub-carrier consecutiveness constraint [3]. Second, although multiple sub-carriers within one spectrum path can be partially overlapped, various spectrum paths have to be separated in the spectrum domain by guard frequencies when two spectrum paths share at least one common fiber. These guard frequencies are used to facilitate the physical frequency filtering and are referred to as guard-carriers. The guard-carrier can be in the order of one or multiple sub-carrier(s) (say GC sub-carriers) [3], [5].

In the frequency domain, one sub-carrier normally corresponds to several GHz, and the capacity of one sub-carrier is in the order of Gbps. Assuming that each fiber consists of $W$ sub-carriers (with index $1, 2, ..., W$), for a given traffic demand, the request can be translated into a number of sub-carriers (with consecutive index), and accommodated through the establishment of a spectrum path. To form spectrum paths, the SLICE network needs to deploy bandwidth-variable (BV) transponders at the network edge and bandwidth-variable WXC s in the network core, which can be built based on the continuous BV wavelength-selective switch (WSS) [4]. In the following, we formally define the routing and spectrum allocation (RSA) problem in the case with off-line or static traffic.

Definition: Static Routing and Spectrum Allocation problem - given a network $G(V, E, S)$, where $V$ is the set of nodes, $E$ is the set of directional fibers between nodes in $V$, and $S$ is the set of sub-carriers on each fiber. For a predefined set of requests $\{t_i\}$, where $t_i$ is the request size (in terms of the number of sub-carriers) of the $i$-th traffic demand, is it possible to determine the path for each request and establish each spectrum path in the set using consecutive sub-carriers, while satisfying the guard-carrier constraint?

As shown in the definition, RSA contains both the routing decision and the sub-carrier allocation to create spectrum paths. When the routing is known or predetermined, the RSA problem turns out to be the static spectrum allocation (SRA) problem, which was shown to be NP-Complete [3]. Therefore the optimal RSA problem which jointly optimizes the routing and spectrum allocation is NP-Hard.

III. INTEGER LINEAR PROGRAMMING (ILP) MODEL FOR THE OPTIMAL RSA

In this section, we present a set of mathematical formulations to model the optimal RSA problem using the ILP techniques.

### A. Notations and Variables

- **$W$**: The maximum sub-carrier index in a fiber;
- **$I_n$**: The set of nodes connected to Node $n$ by incoming fibers to $n$;
- **$O_n$**: The set of nodes connected to Node $n$ by outgoing fibers from $n$;
- **$\Xi_{n,m}$**: Traffic demands matrix; the element $T_{n,m}$ represents traffic demands or the required sub-carrier number between Node $n$ and $m$;
- **$V_w^s$**: 1, if there is a spectrum path using sub-carrier $w$ to satisfy the traffic demand between node-pair $(s, d)$ going from Node $i$ to $o$ and 0 otherwise;
- **$MS$**: The maximum index of the sub-carriers allocated among all the fibers in the network;
- **$MI_{i,o}$**: The maximum index of the sub-carriers over the fiber from Node $i$ to $o$.

### B. Objectives of the RSA problem

One objective considered in this study is to minimize the maximum sub-carrier index among all the fibers, which is shown in Eq. (1) $^1$. Another objective is to minimize the total allocated sub-carriers over all the fibers as shown in Eq. (2).

### C. Constraints

**Traffic Demand Constraint**: Equations (5-6) specify that the traffic demands for node-pair $(s, d)$ should be exactly added at Node $s$ and dropped at Node $d$. Equation (7) makes sure that no traffic is added and dropped at the same node.

**Sub-carrier Capacity Constraint**: Equation (8) guarantees that one sub-carrier can only be used for satisfying one spectrum path.

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$^1$This maximum index determines how many sub-carriers per fiber should be deployed in a green-field network design, hence implying the cost, and power consumption of the switching equipments.
Spectrum Continuity Constraint: The spectrum continuity constraint specifies that the spectrum path should use the same spectrum(s) or sub-carrier(s) along its routing path, which is shown in Eq. (9).

\[ \sum_{w,o \neq d \in I_o} V_{w,o,s,d}^w = \sum_{w,o \neq d \in O_o} V_{w,o,p,s,d}^w \quad \forall s, d, w. \]  

Guard-Carrier Constraint: If \( V_{w,o,s,d}^w = 1 \) for some \( w \) on fiber \( i \rightarrow o \), then Guard-Carrier Constraint requires that all the sub-carriers within \([w - GC, w + GC]\) cannot be used for any other spectrum paths. To model above if-then relationship using ILP, we introduce a large number \( B \) shown in Eq. (10). Clearly, if \( V_{w,o,s,d}^w \) equals 1, then it exactly represents the above if-then relationship; if \( V_{w,o,s,d}^w = 0 \), this constraint is virtually omitted from the ILP model since the left side of Eq. (10) is small enough (\( B \) dominates) to make it a tautology. The same technique is used in Eq. (11) and Eq. (12).

Sub-carrier Consecutiveness Constraint: Equivalently, we transform the Sub-carrier Consecutiveness Constraint as: if \( V_{w,o,s,d}^w = 1 \) and \( V_{w+1,o,s,d}^w = 0 \), all the sub-carriers with index higher than \( w + 1 \) will not be used for the spectrum path of node-pair \((s, d)\) on Fiber \( i \rightarrow o \). The equation to represent this constraint is shown in Eq. (11). And Eq. (12) makes sure that the size of consecutive sub-carriers is \( T_{s,d} \) if \( V_{w,o,s,d}^w \) is 1.

\[ (V_{w,o,s,d}^w - V_{w+1,o,s,d}^w - 1) \geq \sum_{(s,d) \notin [w + 1, W]} V_{w,o,s,d}^w \quad \forall w, i, o. \]  

\[ (V_{w,o,s,d}^w - V_{w+1,o,s,d}^w - 1) \geq B + \sum_{(s,d) \notin [w + 2, W]} V_{w,o,s,d}^w \quad \forall w, i, o. \]  

The above ILP model may not be tractable to resolve the RSA problem in a large SLICE network. Two heuristic algorithms for routing and spectrum allocation were proposed in [3]. One is the shortest path with maximum reuse (SPSR) algorithm, which combines the shortest path routing and the Maximum Reuse Spectrum Allocation (MRSA) algorithm. And the other is the balanced load spectrum allocation (BLSA) algorithm, which combines the balanced load routing and the MRSA algorithm. In this work, we apply above two different optimization goals, and compare the performance of the ILP model, SPSR, BLSA, and the following bounds analysis.

IV. LOWER/UPPER BOUNDS ANALYSIS

In this section, we analyze the lower/upper bounds for the maximum sub-carrier index (i.e., \( MS \)) in a SLICE network with a general topology. We assume that for a network with \( N \) nodes, there are 2 unidirectional fibers per link and uniform traffic demands \( X \) between each node-pair.

A. \( MS \) in mesh networks without predetermined routing

Lower Bound: For the case where the routing is not predetermined, we use the cut-set (CS) technique [6], [7] to analyze the lower bound of \( MS \). A cut \( U \) separates the network with \( N \) nodes into 2 disjoint induced sub-graphs (with \( S \) and \( N - S \) nodes, respectively). The cut carries all the traffic demands \((2 \ast S \ast (N - S) \ast X)\) between 2 sub-graphs. Adding \( 2 \ast S \ast (N - S) \ast G \) sub-carriers as the guard-carriers, the sub-carrier number required on one fiber is the ratio between the whole traffic demands over the cut \( U \) and the number of fibers in the cut \((2 \ast |U|)\). Moreover, each \( SP \) requires the guard-carrier except for the \( SP \) with the largest sub-carrier index. Accordingly, we obtain the lower bound as in Theorem 1.

Theorem 1: For a mesh network with \( N \) nodes, \( MS \geq \frac{(\max_{u \in U} \left| S \times (N - S) \right|)}{|U|} \ast (X + GC) - GC \).

The cut-set method is simple and useful when the routing path is unknown. However, the number of various cuts is significant and can be up to \( \sum_{n=1}^{\left| E \right| - 1} \binom{\left| E \right|}{n} \) in a mesh network. Hence, we propose an alternative method, namely even-load (EL) method to approximate the lower bound. The EL method assumes that the network load is evenly distributed over all the \( 2 \ast |E| \) fiber links within the network. We can obtain the average load per fiber \( L_{avg} = \left[ \frac{(X + GC) \ast N \ast (N - 1) \ast H_{avg}}{2 \ast |E|} - GC \right] \), where \( H_{avg} \) is the average shortest path length over all the node-pairs and \( N \ast (N - 1) \ast H_{avg} \) is the total path length. Then we have Theorem 2.

Theorem 2: For a mesh network with \( N \) nodes, \( MS \geq L_{avg} = \left[ \frac{(X + GC) \ast N \ast (N - 1) \ast H_{avg}}{2 \ast |E|} - GC \right] \).

Upper Bound: To obtain the upper bound, we assume that the network utilizes the shortest path routing since which potentially minimize the path length and hence the overlapping. We construct an interference graph (IG) by viewing each spectrum path as a vertex, where vertexes are adjacent if the corresponding spectrum paths overlap. In the IG, the maximum node degree \( M_o \) corresponds to the maximum overlapping \( M_o \) of spectrum paths along the shortest paths in the original SLICE network. In specific, if there are \( M_o \) spectrum paths overlapping with spectrum path \( i \), then \( M_o = \max_{i \neq i'} M_{i,i'} \). We have the upper bound as shown in Theorem 3.

Theorem 3: In a mesh network with \( N \) nodes, if the spectrum paths under the shortest path routing has the maximum overlapping degree \( M_{o} \), \( MS \leq (M_{o} + 1) \ast (X + GC) - GC \).

Proof: In the IG, the maximum degree is \( M_o \). According to Brook’s Theorem [8], for a graph with maximum degree \( \Delta \), a greedy coloring requires \( \Delta + 1 \) different colors. In the IG, we thus only requires \( \Delta + 1 = M_o + 1 \) set of sub-carriers. Since each spectrum path requires at most \( X + GC \) consecutive sub-carriers, the required sub-carrier number on one fiber is bounded by \( MS \leq (M_o + 1) \ast (X + GC) - GC \) after excluding the guard-carrier for the spectrum path that owns the largest sub-carrier index.

B. \( MS \) in mesh networks with predetermined routing

Lower Bound: In the case that the routing path is predetermined (i.e., an instance of the SRA problem), we can
obtain the lower bound by estimating the load on the most congested fiber. We estimate that the load on a given fiber $i$ as $L_i = \sum_{k \in P_i} t_k + GC \times (I - 1)$, where $I$ is the total number of spectrum paths using the fiber and $p_k$ is the index of the $k$-th spectrum path. Consequently, we can obtain the minimum number of sub-carriers on a fiber as shown in Theorem 4. Note that Theorem 4 can be applied to the case with non-uniform traffic.

**Theorem 4:** If the routing is predetermined, then $MS \geq LD = \max_{i \in \mathcal{F}} L_i$.

**Upper Bound:** In the case that the paths are predetermined, we can obtain the upper bound using the same technique for deriving Theorem 3. The only difference is that the routing path is known and may not be the shortest path. We hence have the upper bound shown in Theorem 5, whose proof is omitted due to the similarity to the proof of Theorem 3.

**Theorem 5:** If the routing is predetermined and $M_o$ is the maximum overlapping among all the spectrum paths, then $MS \leq (M_o + 1) \times (X + GC) - GC$.

Figure 1 shows 4 spectrum paths along the path $A-B-C-D$, $B-C-D-A$, $C-D-A-B$, and $D-A-B-C$, respectively. We apply Theorem 4 and 5 on the network shown in Fig. 1 while assuming $GC = 1$. Since the estimated load on each fiber is $3 + 2 = 5$, the lower bound of $MS$ is 5 according to Theorem 4. We note that, however, this lower bound cannot be achieved in this case (where each spectrum path overlaps with the others) due to the spectrum continuity constraint. Instead, at least 7 sub-carriers have to be deployed in each fiber to accommodate all the spectrum paths. On the other hand, since $M_o = 3$, Theorem 5 can yield 7 as the upper bound.

**V. PERFORMANCE EVALUATION AND ANALYSIS**

In this section, we present the simulation results of the proposed bound analysis, ILP model (using the ILOG CPLEX), and heuristic algorithms.

Table I shows the results when applying the bound analysis on the 14-node NSF network with $GC = 1$. The uniform traffic demand $X$ between each node-pair is 1 or 2. The lower bound (LB) and upper bound (UB) in the first two columns are obtained using Theorem 2 and 3, the LB/UB for BLSA and SPSR are obtained using Theorem 4 and 5. From Table I, we can see that the BLSA and SPSR can achieve the lower bound in both cases.

**TABLE I**

**BOUND ANALYSIS ON THE 14-NOSE NSF NETWORK**

| $X$ | LB | UB | BLSA | LB/UB for BLSA | SPSR | LB/UB for SPSR |
|-----|----|----|------|---------------|------|---------------|
| 1   | 18 | 63 | 27   | [27,84]       | 29   | [29,63]       |
| 2   | 51 | 95 | 41   | [41,128]      | 44   | [44,95]       |

We simulate the ILP model and heuristic algorithms (SPSR and BLSA) on a random 6-node network where the traffic demands are randomly generated within $[0, 3]$ sub-carriers for each node-pair. The $MS$ in the objective of Eq. (1) and the total sub-carrier number in the objective of Eq. (2) for 3 representative traffic demands where the summation of sub-carrier requests are 10, 20, 30, respectively, are shown in Table II. For the $MS$, BLSA has a slightly better performance than SPSR. This is because SPSR adopts the shortest-path routing scheme while BLSA can balance the traffic load in the network. For the total sub-carrier number required in the network as shown in the last row of Table II, however, SPSR outperforms BLSA. This is because shortest path routing potentially minimizes the total hops that spectrum paths span.

**TABLE II**

**RESULTS FOR THE 6-NOSE NETWORK**

| $X$ | LB | UB | BLSA | LB/UB for BLSA | SPSR | LB/UB for SPSR |
|-----|----|----|------|---------------|------|---------------|
| 1   | 10 | 20 | 30   | [10,20]       | 30   | [30,30]       |
| 2   | 30 | 50 | 60   | [30,50]       | 60   | [60,60]       |

The results for various combinations of uniform traffic demands and guard-carrier size are shown in Table III. Clearly, larger $GC$ for the same $X$ indicates more overhead for the guard-carrier, hence requiring both more total sub-carriers and $MS$. Interestingly, we observe that the cases with $(X = 1, GC = 3)$, $(X = 2, GC = 2)$, and $(X = 3, GC = 1)$ require almost the same $MS$. This is because the $(X + GC)$ value is the same for the three cases and the small difference among the above cases is due to the difference in the guard-carrier size for the spectrum path with the largest sub-carrier index. The total sub-carrier number for above 3 cases, however, is not close since the difference at each fiber is accumulated when counting the total sub-carrier number. For the total sub-carrier number, the results show that SPSR outperforms BLSA due to its shortest path routing.

**TABLE III**

**RESULTS FOR THE 14-NOSE NETWORK**

| $X$ | LB | UB | BLSA | LB/UB for BLSA | SPSR | LB/UB for SPSR |
|-----|----|----|------|---------------|------|---------------|
| 1   | 10 | 20 | 30   | [10,20]       | 30   | [30,30]       |
| 2   | 30 | 50 | 60   | [30,50]       | 60   | [60,60]       |

We also collect the results for non-uniform traffic by randomly generating the traffic within $[0, r]$, where $r$ is the maximum traffic demands. Figure 2 shows the lower bound (LB) (using Theorem 4) for $MS$ under the balanced load routing and shortest path routing as well as the $MS$ from BLSA and SPSR. As shown in Fig. 2, the LB of BLSA is smaller than that of SPSR due to the load-balanced routing. However, the gap of $MS$ between BLSA and its LB is larger than that of SPSR. This is because the shortest path routing can potentially reduce the overall path lengths and path overlapping, while balanced load routing may introduce longer routing paths and overlapping as a tradeoff of the load balancing. When comparing the total sub-carrier number used over the whole network in Fig. 3, we observe that BLSA consumes more sub-carriers than SPSR, which implies that the shortest path routing facilitates the goal of minimizing total sub-carrier number. In general, we may conclude SPSR outperforms BLSA when minimizing the total sub-carrier number, while BLSA outperforms SPSR when minimizing the maximum sub-carrier index (i.e., $MS$).
VI. RELATED WORK

In the literature, the study in [2] raised the challenges for the future optical networks and explored the possibility and feasibility of adopting SLICE networks as a mid-term solution for next-decade network. The enabling technologies of SLICE networks were firstly elaborated in [4]. For example, the node architecture based on bandwidth-variable wavelength-selective switch (WSS) was shown in [4], which can support the spectrum path switching in SLICE networks. The authors of [3], [10] further studied some unique features of SLICE networks. In [5], the filtering characteristics of SLICE network were studied, and the spectrum efficiency of SLICE networks was shown better than that of WDM networks by a large margin. The authors of [9] investigated a unique feature of bandwidth-squeezing restoration in SLICE networks, where the bandwidth of the failed spectrum path can be squeezed to achieve the minimal connectivity.

The concept of routing and spectrum allocation was introduced in [2] for the first time, and detailed algorithms for RSA problem were firstly given in [3], [10]. Recently, the study in [4] introduced the concept of distance-adaptive spectrum resource allocation in SLICE networks, where the modulation level and filter width for a given spectrum path can be adaptively chosen based on the path length. The extended optical reach in SLICE networks supported by the OFDM technology as well as the flexibility in modulation level, enables the virtualization of the spectrum resource in the optical domain, which was discussed in [11]. The efforts on the standardization of the Frequency Slot in SLICE networks was also discussed in [4], which indicates that one can realize the routing and spectrum allocation based on the Frequency Slot instead of sub-carriers.

VII. CONCLUSION

In this work, we have studied the routing and spectrum allocation (RSA) problem in the SLICE network. We have presented the Integer Linear Programming (ILP) models with different goals and analyzed the lower/upper bounds of the maximum sub-carrier number. Our simulations have also compared the performance of two heuristic algorithms under different goals. In the future, we plan to extend the RSA study to the case with on-line or dynamic traffic. Topics such as how to effectively incorporate the features of the SLICE network into the RSA process also deserve further investigation.

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