A Modularized Design for Output Synchronization of LTI Dynamical Networks with Communication Delays

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Abstract: In this paper, we address output synchronization for a network of heterogeneous agents with linear time invariant single-input-single-output dynamics in the presence of communication delays. To this end, we present a three-stage modularized design procedure: (i) a parallel feedforward compensator is designed to convert the dynamics to a minimum-phase system with relative degree one, (ii) the resulting system is then transformed into a feedback equivalent to a passive system, and (iii) the agents are interconnected based on a passivity-based output synchronization law. The benefit of the present control scheme is that not only implementation but also the design stage of the controller is distributed, namely the agents do not need to know the entire network structure. The present solution is then shown to achieve output synchronization. We finally demonstrate the present controller through simulation.

Key Words: output synchronization, communication delay, parallel feedforward compensator, passivity.

1. Introduction

Output synchronization is a distributed control problem to synchronize outputs for a network of dynamical components. During the last few decades, a number of publications have been devoted to the problem, motivated by a wide range of engineering applications such as robot motion coordination [1], formation control [2], distributed estimation for sensor networks [3], distributed optimization [4],[5], power grids [6], coordination of automatic vehicles [7], and scientific interest in biological/social networks. A comprehensive survey of synchronization is presented in [8].

In this paper, we treat output synchronization for a network of linear time invariant (LTI) dynamical systems in the presence of communication delays. The problem has been studied, e.g., in [9]–[13] and references therein. They present conditions for output synchronization to be met by the aggregate network system, e.g., in the form of linear matrix inequalities (LMIs) [9]–[11]. However, these conditions would not always be available for both analysis and design due to imperfect knowledge on the overall network and computational costs to solve the large-scale LMIs. Münz et al. [14] present a condition checkable for each component, but the intended system is restricted to double integrators interconnected by an undirected and connected graph.

Meanwhile, [1],[15] show that a relaxed definition for output synchronization is ensured by arbitrarily strong inter-agent mutual feedbacks if the components have passive dynamics. Similar results are also reported in [16],[17]. However, practical system dynamics do not always meet passivity. In this regard, [1],[18],[19] present a delay output synchronization law applicable to the minimum phase dynamics with relative degree one, wherein the dynamics is first converted to a feedback equivalent to passive system based on [20] and then the resulting systems are interconnected via the passivity-based synchronization law in [1],[15]. Whereas, one of the authors’ antecessor [21] presents a necessary and sufficient condition for converging general single-input-single-output (SISO) LTI systems into minimum phase systems with relative degree one via a so-called parallel feedforward compensator (PFC), and the technique is applied to output synchronization. However, communication delays are not investigated in [21].

In this paper, we present a modularized design procedure to achieve output synchronization for a network of heterogeneous SISO LTI components with communication delays by combining [21] and [1],[18],[19]. Namely, we take a three-stage design procedure: a PFC is designed to convert the dynamics to a minimum-phase system with relative degree one, the resulting system is then transformed into a passive system, and the agents are interconnected based on a passivity-based output synchronization law in [1],[15]. Since the PFC design and transformation to the feedback equivalent to a passive system are closed in the component level and [1],[15] inherently ensure stable interactions regardless of the coupling strength, the present solution does not require any conditions across the overall network. In other words, the present controller is distributed not only in implementation but also in the design stage. The direct combination of [21] and [1],[18],[19] immediately proves delay synchronization of the outputs transformed by a PFC but we still have two technical questions: (i) is the original definition of output synchronization rather than the relaxed version in [1],[15] guaranteed? and (ii) do the original outputs rather than the ones transformed by the PFC synchronize? To address these concerns, we rigorously prove that the original outputs synchronize in the strict sense. Finally, the present design scheme is demonstrated through simulation.

The contents of this paper are in part presented in the confer-
2. Preliminary

The control design presented in this paper relies on a so-called parallel feedforward compensator (PFC), and we thus first introduce the concept and a related technical result.

Consider an SISO LTI system $\mathcal{P}$ with the state space representation

$$
\mathcal{P} : \begin{cases}
\dot{x}^p = A^p x^p + B^p u, & x^p \in \mathbb{R}^n, u \in \mathbb{R}, \\
y^p = C^p x^p, & y^p \in \mathbb{R},
\end{cases}
$$

where $x$ is the state, $u$ is the input, and $y$ is the output of the system. In addition to this, $(A^p, B^p, C^p)$ is assumed to be minimal. The structure of PFC is then illustrated in Fig. 1, where a compensator $\mathcal{F}$ is connected in parallel to the original system $\mathcal{P}$. In particular, the PFC is said to be stable if $\mathcal{F}$ is stable.

Let us now introduce the following assumption on the system $\mathcal{P}$.

**Assumption 1** Let us define the transfer function of $\mathcal{P}$ as $G^p(s) = N^p(s)/D^p(s)$, where $N^p(s)$ and $D^p(s)$ are coprime polynomials. Then, the signs of $N^p(s_i)$, $s_i \in \{s \in \mathbb{R} : D^p(s) = 0 \} \geq 0$ are the same.

Then, the following result is shown to be true in one of the authors’ antecessor [21].

**Lemma 1** There exists a stable PFC

$$
\mathcal{F} : \begin{cases}
\dot{x}^f = A^f x^f + B^f u, & x^f \in \mathbb{R}^n, u \in \mathbb{R}, \\
y^f = C^f x^f, & y^f \in \mathbb{R},
\end{cases}
$$

with minimal $(A^f, B^f, C^f)$, which makes

$$
\mathcal{P} + \mathcal{F} : \begin{cases}
\dot{x} = A x + O x + B u, & x \in \mathbb{R}^n, u \in \mathbb{R}, \\
y = C x, & y \in \mathbb{R},
\end{cases}
$$

be of minimum phase and have relative degree one if and only if Assumption 1 holds.

In this paper, we also use the following concept.

**Definition 1** Consider a system with the state space representation

$$
\dot{x} = \phi(x, u), \quad y = \varphi(x, u),
$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^p$ is the input, and $y(t) \in \mathbb{R}^p$ is the output. The system is then said to be passive if there exists a positive semidefinite function $S : \mathbb{R}^n \to \mathbb{R}$, called a storage function, such that

$$
S(x(t)) - S(x(0)) \leq \int_0^t y^T r(\tau) u(\tau) d\tau
$$

holds for all inputs $u(t)$, all initial states $x(0) \in \mathbb{R}^n$, and all $t \geq 0$.

If the storage function $S$ is differentiable, (1) is equivalent to the inequality

$$
\dot{S} \leq y(t)^T u(t).
$$

3. Problem Formulation

Consider a group of $N$ dynamical systems called agents, where the set of agents’ IDs is denoted by $\mathcal{V} = \{1, \ldots, N\}$. We suppose that agent $i$ is modeled by the following SISO LTI system $\mathcal{P}_i$:

$$
\mathcal{P}_i : \begin{cases}
\dot{x}^i = A^i x^i + B^i u, & x^i \in \mathbb{R}^n, u \in \mathbb{R}, \\
y^i = C^i x^i, & y^i \in \mathbb{R},
\end{cases}
$$

where $x^i$ is the state, $u$ is the input and $y^i$ is the output of agent $i$. Throughout this paper, we assume that (2) satisfies the following assumption and Assumption 1 as well.

**Assumption 2** $(A^i, B^i, C^i)$ is minimal.

Suppose that agents can communicate their own information with some of the other agents. The communication structure is modeled by a weighted digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, W)$, $E \subseteq \mathcal{V} \times \mathcal{V}$, $W \in \mathbb{R}^{N \times N}$, where $(i, j)$ entry of $W$, denoted by $w_{ij}$, is positive if $(i, j) \in E$ and $w_{ij} = 0$ otherwise. We suppose that agent $j \in \mathcal{V}$ can receive information of agent $i \in \mathcal{V}$ if $(i, j) \in \mathcal{E}$, and each weight $w_{ij}$ represents the reliability of information from agent $j$ to agent $i$. We also denote, by $L$, the weighted graph Laplacian matrix defined as

$$
L := \begin{cases}
\sum_{j \in N_i} w_{ij} & \text{if } j = i, \\
-w_{ij} & \text{if } j \in N_i, \\
0 & \text{otherwise},
\end{cases}
$$

where $N_i$ is the set of neighbors for agent $i \in \mathcal{V}$ defined by $N_i := \{ j \in \mathcal{V} : (j, i) \in \mathcal{E} \}$.

In this paper, we assume the following property on $\mathcal{G}$.

**Assumption 3** The graph $\mathcal{G}$ is fixed and strongly connected.

Under this assumption, there exists a vector $\eta$ with positive elements such that

$$
\eta^T L = 0
$$

holds [1].

In this paper, we suppose that the inter-agent communication suffers from delays, which is assumed to be constant but heterogeneous. The time delay for the communication from agent $i$ to $j$ is denoted by $T_{ij}$. The objective of this paper is to achieve output synchronization, defined below, for a network of agents with the heterogeneous dynamics (2) even in the presence of time delays:

$$
\lim_{t \to \infty} (y^i_t - y^j_t) = 0, \quad \forall (i, j) \in \mathcal{V} \times \mathcal{V}.
$$

Fig. 1 Overview of PFC.
Remark The output synchronization for LTI dynamical agents with delays is investigated, e.g., [9]–[11],[13]. However, they require careful selections of the gains in the inter-agent mutual feedback so as to not lose stability under the knowledge on the number of agents, the network connection, and the agent dynamics. Meanwhile, it is shown in [1] and the references therein that if the dynamics ensures passivity, output synchronization is achieved by arbitrarily strong mutual feedbacks. The contribution of this paper is to show that output synchronization is achieved without careful mutual feedback gain selections for a class of possibly non-passive LTI dynamical systems by combining the PFC [21] and the passivity-based synchronization scheme [1],[18],[19].

4. Controller Design

In this section, we present a distributed controller to achieve the output synchronization (4). The design procedure is summarized as below: We first design a PFC for each agent so that the system with the PFC is of minimum phase with relative degree one. We then transform the system into controllable canonical form for the sake of brevity, and transform the system into a passive system. The resulting systems are then interconnected with each other in the same way as the passivity-based output synchronization scheme in Section 8.3.2 of [1].

Let us first design a PFC for the system (2)

\[
\begin{align*}
T_i: \quad \dot{x}_i &= A_i^p x_i^p + B_i^p u_i, \\
&= C_i^p x_i^p, \\
\text{so that the resulting system} \quad &P_i + T_i: \\
\dot{y}_i &= C_i x_i
\end{align*}
\] (5)

is of minimum phase and has relative degree one, where

\[A_i^p := \begin{bmatrix} A_i^p & O \\ O & A_i^p \end{bmatrix}, \quad B_i^p := \begin{bmatrix} B_i^p \\ B_i^p \end{bmatrix}, \quad C_i := \begin{bmatrix} C_i^p & C_i^p \end{bmatrix}.\]

The existence of such a PFC is guaranteed by Lemma 1.

Let us next transform (6) into the controllable canonical form for the sake of simplicity. As \(P_i + T_i\) is of minimum phase and has relative degree one, its transfer function \(G_i(s)\) is formulated in the form of

\[G_i(s) = \frac{b_{i,0}}{s^{n_i} + a_{i,n_i-1}s^{n_i-1} + \cdots + a_{i,0}}.\]

where \(n_i = n_i^0 + n_i^1\) and \(b_{ij} > 0 \quad \forall i = 0,1,\ldots,n_i - 1\). Then, choosing an appropriate \(M_i\) and defining \(\tilde{x}_i\) as

\[
\begin{align*}
\dot{x}_i &= M_i \tilde{x}_i, \\
\dot{y}_i &= C_i M_i \tilde{x}_i
\end{align*}
\] (7)

where

\[
\tilde{A}_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & 1 \\ -a_{i,0} & -a_{i,1} & \cdots & \cdots & -a_{i,n_i-1} \end{bmatrix}
\]

\[
\tilde{B}_i = \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix}^T,
\]

\[
\tilde{C}_i = [b_{i,0} \ b_{i,1} \cdots \ b_{i,n_i-1}].
\]

As an LTI system with minimum phase and relative degree one is known to be feedback equivalent to a passive system [1], we next design the following state feedback controller for the system (6) to render the system passive:

\[
u_i = (\tilde{C}_i \tilde{B}_i)^{-1} (v_i - \tilde{C}_i \tilde{A}_i \tilde{x}_i),\]

(8)

where \(v_i\) is a new input. Note that \(\tilde{C}_i \tilde{B}_i\) is not zero because the system has relative degree one. Then, we have the following lemma.

Lemma 2 The system (7) with minimum phase and relative degree one and the state feedback controller (8) is passive from \(v_i\) to \(y_i\).

Proof. Substituting (8) into (7), we obtain the following equations:

\[
\begin{align*}
\dot{x}_i &= \tilde{A}_i \tilde{x}_i + B_i \tilde{C}_i \tilde{A}_i \tilde{x}_i + \tilde{B}_i \tilde{C}_i \tilde{B}_i v_i,
\dot{y}_i &= \tilde{C}_i \tilde{A}_i \tilde{x}_i + \tilde{C}_i \tilde{B}_i v_i.
\end{align*}
\]

(9)

As \(\dot{y}_i\) is \(v_i\) holds, agent \(i\) is passive from \(v_i\) to \(y_i\). \qed

Next, we introduce a new state variable \(z_i\) as below:

\[
z_i = \tilde{x}_i - \tilde{B}_i (\tilde{C}_i \tilde{B}_i)^{-1} v_i
\]

(10)

Transforming the states from \(\tilde{x}_i\) to \(z_i\), the system (9) is reformulated as

\[
\begin{align*}
\dot{z}_i &= A_i z_i + B_i y_i,
\dot{y}_i &= v_i
\end{align*}
\]

(11)

where

\[
A_i = \tilde{A}_i - \tilde{B}_i (\tilde{C}_i \tilde{B}_i)^{-1} \tilde{C}_i \tilde{A}_i
\]

\[
= \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & 1 \\ -b_{i,0} & -b_{i,1} & \cdots & \cdots & -b_{i,n_i-2} \end{bmatrix}
\]

\[
B_i = \tilde{A}_i - \tilde{B}_i (\tilde{C}_i \tilde{B}_i)^{-1} \tilde{C}_i \tilde{A}_i \tilde{B}_i (\tilde{C}_i \tilde{B}_i)^{-1} = A_i B_i (\tilde{C}_i \tilde{B}_i)^{-1}
\]

\[
= 1 \begin{bmatrix} 0 & \cdots & 0 & 1 & -b_{i,n_i-1} \end{bmatrix}^T.
\]

The bottom right \((n_i - 1)\)-by-\((n_i - 1)\) block in \(A_i\) is now denoted by \(\tilde{A}_i\). Then, by considering the characteristic polynomial of \(\tilde{A}_i\), it is immediate to see that the eigenvalue of the matrix \(\tilde{A}\) coincide with the zeros of the transfer function \(G_i(s)\). Consequently, \(\tilde{A}\) is Hurwitz because \(G_i(s)\) is the transfer function of a minimum phase system.

Now, define a positive semi-definite matrix \(P_i\) by
where $R_i \neq 0$ is an arbitrary matrix determined by a designer. Then, we get the following equation:

$$\dot{\overline{A}}_i^T P_i + P_i \overline{A}_i = -R_i^T R_i.$$ 

We also define positive semi-definite matrices $P_i$ and $Q_i$ as:

$$P_i = \begin{bmatrix} 0 & O \\ O & \overline{P}_i \end{bmatrix}, \quad Q_i = \begin{bmatrix} 0 & O \\ O & R_i^T R_i \end{bmatrix}.$$ 

By calculation, we then have the following equation:

$$\dot{A}_i^T P_i + P_i A_i = -Q_i.$$ (12)

Based on the passivity-based synchronization scheme in Section 8.3.2 of [1], we design $v_i$ using the above matrix $P_i$ and real number constants $\epsilon_i$, $\eta_i$ as follows:

$$v_i(t) = \sum_{j \in N_i} w_{ij} \epsilon_i \left( \epsilon_j y_j(t - T_j) - e_j y_j(t) \right) - \dot{B}_i^T P_i z_i(t).$$ (13)

For notational simplicity, we denote signals of time $t$, like $y_i(t)$, as $y_i$ dropping the time $t$ unless mentioned otherwise. The overall agent dynamics is illustrated in Fig. 2.

5. Convergence Analysis

In this section, we prove that the controller designed in the previous section achieves output synchronization. To this end, we first prove a lemma by following a procedure similar to Section 8.3.2 in [1].

**Lemma 3** Consider the system (11) with (13). Then, $\lim_{t \to \infty} \left( \epsilon_i y_i(t - T_j) - e_i y_i \right) = 0 \quad \forall (i,j) \in \mathcal{V} \times \mathcal{V}$ and $\lim_{t \to \infty} \epsilon_i^T Q z_i = 0 \quad \forall i \in \mathcal{V}$ hold under Assumption 3.

**Proof.** Define the energy functional $U_i$ as

$$U_i = \sum_{i=1}^{N} \eta_i \left( \epsilon_i^T P_i z_i + \| y_i \|^2 \right) + \sum_{i=1}^{N} \eta_i \sum_{j \in N_i} w_{ij} \int_{t-T_j}^{t} \| \epsilon_j y_j(\tau) \|^2 d\tau,$$

where $\eta_i$ is the $i$-th element of $\eta$ satisfying (3). Then, using (11), we obtain

$$\dot{U}_i = \sum_{i=1}^{N} \eta_i \left[ \epsilon_i^T P_i z_i + 2 \epsilon_i y_i \right] - \sum_{i=1}^{N} \epsilon_i^T \epsilon_i^T P_i z_i = \sum_{i=1}^{N} \eta_i \left[ \epsilon_i^T (P_i + P_i A_i) z_i + 2 \epsilon_i y_i \right] - \sum_{i=1}^{N} \epsilon_i^T \epsilon_i^T P_i z_i = \sum_{i=1}^{N} \eta_i \left[ \epsilon_i^T (P_i + P_i A_i) z_i + 2 \epsilon_i y_i \right] - \sum_{i=1}^{N} \epsilon_i^T \epsilon_i^T P_i z_i.$$ (14)

Substituting (13) into (14) and using (12) yield:

$$\dot{U}_i = \sum_{i=1}^{N} \eta_i \left[ \epsilon_i^T (P_i + P_i A_i) z_i + 2 \epsilon_i y_i \right] - \sum_{i=1}^{N} \epsilon_i^T \epsilon_i^T P_i z_i = \sum_{i=1}^{N} \eta_i \left[ \epsilon_i^T (P_i + P_i A_i) z_i + 2 \epsilon_i y_i \right] - \sum_{i=1}^{N} \epsilon_i^T \epsilon_i^T P_i z_i.$$ (15)

By calculation, (16) is further rewritten as

$$\dot{U}_i = \sum_{i=1}^{N} \eta_i \| \epsilon_i y_i \|^2 - \sum_{i=1}^{N} \eta_i \| \epsilon_i y_i \|^2 - \sum_{i=1}^{N} \eta_i \| \epsilon_i y_i \|^2 - \sum_{i=1}^{N} \eta_i \| \epsilon_i y_i \|^2 = \sum_{i=1}^{N} \eta_i \| \epsilon_i y_i \|^2 - \sum_{i=1}^{N} \eta_i \| \epsilon_i y_i \|^2 - \sum_{i=1}^{N} \eta_i \| \epsilon_i y_i \|^2.$$ (17)

Now, we have the following equation from (3):

$$\sum_{i=1}^{N} \eta_i \sum_{j \in N_i} w_{ij} \| \epsilon_j y_j \|^2 - \sum_{i=1}^{N} \eta_i \| \epsilon_i y_i \|^2 = \sum_{i=1}^{N} \eta_i \sum_{j \in N_i} w_{ij} \| \epsilon_j y_j \|^2 - \sum_{i=1}^{N} \eta_i \| \epsilon_i y_i \|^2.$$ (18)

Substituting (18) into (17) yields

$$\dot{U}_i = - \sum_{i=1}^{N} \eta_i \left[ \epsilon_i^T Q z_i + \sum_{j \in N_i} w_{ij} \| \epsilon_j y_j(t - T_j) - \epsilon_j y_j \|^2 \right] \leq 0.$$ (19)

The last inequality holds because $Q_i$ is positive semi-definite. Thus, the finite limit $\lim_{t \to \infty} U_i$ exists. Then, from the definition of the energy function, $\epsilon_i^T P_i z_i$ and $y_i$ are bounded. It is also

Fig. 2 Block diagram of entire system.
confirmed from (11) and (13) that ˙yi is bounded. Let us now represent each element of zi as zi := [z1i, · · · , zni,]. Then, as yi is bounded and Ai is Hurwitz, the dynamics of [z1i, · · · , zni,] in (11) is regarded as a stable LTI system with the bounded input yi, and hence [z1i, · · · , zni,] is bounded. On the other hand, the time derivative of zT i Qzi is calculated as follows:

$$
\frac{d}{dt} z_i^T Q z_i = z_i^T (A_i^T Q_i + Q_i A_i) z_i + B_i^T Q_i z_i + y_i^T B_i y_i.
$$

By calculation, we can confirm that the right-hand side of (19) is independent of the element zi,1, which is the only element in zi not proved to be bounded, and hence zT i Qzi is bounded. In summary, we can conclude that ˙Ui is uniformly continuous. Then, invoking Barbalat’s Lemma, we can conclude

$$
\lim_{t \to \infty} z_i^T Q z_i = 0 \forall i.
$$

Under Assumption 3, (20) is equivalent to

$$
\lim_{t \to \infty} \left[ \begin{array}{c} \varepsilon y_i (t - T_i) - \varepsilon y_i \end{array} \right] = 0 \forall (i, j) \in E.
$$

Next, we use such notations to prove the next theorem that

$$
\lim_{t \to \infty} \left[ \begin{array}{c} \varepsilon y_i (t - T_i) - \varepsilon y_i \end{array} \right] = 0 \forall (i, j) \in V \times V.
$$

This completes the proof. □

Let us focus on the bottom row of the first equation of (9), which is reformulated as

$$
\tilde{x}_{i,1} = -\frac{b_{j,0}}{b_{j,0-1}} \tilde{x}_i + \beta_i (\tilde{x}_{i,2}, \ldots, \tilde{x}_{i,n-1}, v_i),
$$

where βi is a linear function and hence is vanishing from (22) and (24). As b_{j,0-2} and b_{j,0-1} are positive, (25) is an exponentially stable system with a vanishing input, and hence ˙xi,1 → 0 holds from Theorem 4.4 of [1]. Then, let us investigate the relationship between ˙xi and ˙yi. The outputs of (2) and (5) can be rewritten as this form:

$$
\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} = \begin{bmatrix} C_i & O \\ O & C_i \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} C_i & O \\ O & C_i \end{bmatrix} M \tilde{x}_i.
$$

According to the above discussion, all the elements of ˙xi except ˙xi,1 vanish. As a result, (26) means

$$
\lim_{t \to \infty} \dot{y}_i = \frac{\delta_i}{b_{j,0}} y_i = \varepsilon y_i.
$$

Finally, we can confirm that ˙yi converges to ˙yi. Consequently, ˙yi will also synchronize according to Lemma 3.

In addition to this, ˙xi converges to a constant value according to (24), and hence the objective (4) is achieved. This completes the proof. □

In the same way as above, we can also prove the following corollary.

**Corollary 1** Consider the system in Lemma 3. Then, ˙yi → 0 ∀ i ∈ V is achieved under Assumption 3 if R_i is nonsingular.

The result is less interesting in the context of output synchronization since the final output is always zero regardless of the network structures and initial states. However, it would be informative in the context of stabilization of network systems with delayed couplings.

6. Simulation

Consider a group of 20 agents. the dynamics of agent i is described as
Theorem 1. \(\lambda\) is an eigenvalue of \(A_i\), then the output synchronize, which demonstrates validity of the controller.

Having relative degree two, each agent is not passive. Suppose that the communication structure is described as the graph shown in Fig. 3. Note that the graph satisfies Assumption 3. We then have proved synchronization of not only the outputs corrected by PFC but also the original system outputs despite arbitrary and unknown constant inter-agent communication delays. Finally, we have demonstrated the above results through simulation.

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7. Conclusion

In this paper, we have investigated output synchronization for a network with heterogeneous SISO LTI dynamics in the presence of communication delays. To this end, we have presented a three-stage modularized design procedure, namely conversion to minimum phase systems with relative degree one using PFC, transformation into a feedback equivalent to passive system, and interconnection of agents via a passivity-based synchronization law. We then have proved synchronization of not only the outputs corrected by PFC but also the original system outputs despite arbitrary and unknown constant inter-agent communication delays. Finally, we have demonstrated the above results through simulation.

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