NUMERICAL MODEL FOR DAM BREAK OVER A MOVABLE BED USING FINITE VOLUME METHOD

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ABSTRACT: In this paper, we study dam break phenomenon over a movable bed through a mathematical model. The model dynamically links the hydrodynamic and sediment transport. To solve the model numerically, we use finite volume method on a staggered grid that is simpler than the generalized collocated finite volume method. This is because the Riemann invariants problem in collocated grid approach can be particularly complex and costly in simulating dam break floods with sediment transport. The quality of the dam break flood simulations with our numerical scheme is verified by comparing the results against analytical solutions, laboratory tests and some experimental data available in the literature on fixed and mobile bed conditions. The numerical results reproduce the experimental evidence quite well, proving that the model is capable of predicting the temporal evolution of the free-surface and the bed. Our numerical scheme, which is not only simple to implement but also both accurate and computationally efficient, is proposed as an appropriate tool for simulating dam break problem over a movable bed.

Keywords: Dam break, Movable bed, Finite volume method, Sediment transport.

1. INTRODUCTION

Dam break is an event where a massive volume of water is suddenly released due to the dam failure. Dam failure can be caused by several factors such as flood, earthquake and other factors. The event may leads to a catastrophic disaster to the downstream of the dam. The dam break induced flow may leads to a severe destruction along its flow path. In general, the generated flow shares a common characteristic with tsunami waves. The water depth and velocity are high. This may also leads to the surface erosion along the flow path.

There have been many studies on dam break flow. Yakti et al. [1] has shown the damaging effect of a dam break to its downstream area. They have analysed the overland flow, generated by the Waia Ela Dam failure in 2013. The event nearly destroyed the whole Negeri Lima Village, Ambon, Indonesia. Satellite images and aerial photo also showed severe erosion due to the flow. However, their study did not cover the erosion profile.

The dam break generated flow is generally simulated using the Shallow Water Equations. Finite difference, finite element, or finite volume scheme are used to solve the set of equations. Finite difference scheme is widely used due to its simplicity. Yet, the scheme is less flexible than the others. Thus, its application to a real case scenario may be limited. In addition, the lower order scheme may suffer from instability due to shock waves. Hence, it requires extra term to achieve stability [2]. Finite element provides a better flexibility in terms of grid. Therefore, it may cover a wide range of application. Nevertheless, it is temperamental in terms of stability in handling shock waves [3, 4]. The finite volume has both grid flexibility and simplicity. In general, it also has a faster computation time than the others. Therefore, in this research, we propose finite volume method to solve the mathematical model. Peng [5] has developed a model to simulate an experimental dam break case. His experiment also includes buildings, situated downstream of the dam. Thus, the effect of the dam break generated flow to these buildings were also observed. The shock waves were captured using a Total Variation Diminishing (TVD) scheme. TVD is commonly used to capture shock [6, 7]. There are other ways to capture shock in finite volume scheme without the need to use a higher order method. Shock capturing in finite volume scheme may also utilize slope limiter function [8, 9, 10]. Pudjaprasetya and Magdalena [11] employed a momentum conservative scheme to handle shock and discontinuity. Moreover, I. Magdalena et. al [12] have investigated radial dam break problem using finite volume method on a staggered grid.

There are limited studies on the dam break induced erosion. Spinewine and Zech [13] have studied the phenomenon by conducting a laboratory experiment in a flume. The bed of the flume was covered with sediment grain. The flume was separated by a gate. The upstream of the gate was filled with water. The dam break was simulated by a sudden opening of this gate. They investigated the velocity profile and sediment grain concentration as the flow propagates. Spinewine and Capart [14] updated the experiment by using a longer flume and...
enhancing the gate opening. They also provided a method to estimate the sediment movement.

In this study, a model is developed to simulate a dam break flow and the generated erosion. The model is based on the shallow water equations, solved using a finite volume method with a momentum conservative scheme. It is coupled with a sediment transport equation, thus enhancing its capability for simulating erosion. The model is applied to an experimental dam break case with movable bed.

2. GOVERNING EQUATIONS

In this section, the governing equations of transport sediment due to the movement of a fluid in contact with a sediment layer will be discussed. The equation is based on coupled shallow water-Exner equations where Shallow Water Equations simulate the flow of water and Exner equation for movement of sediment layer. The coupled Shallow Water-Exner equations in one-dimensional case read as

\[
\begin{align*}
\partial_t h + \partial_x (hu) &= 0, \\
\partial_t (hu) + \partial_x \left( hu^2 + \frac{1}{2} g h^2 \right) + g h \partial_x (z - H) + S_f &= 0, \\
\partial_t z + \zeta \partial_x Q_s &= 0,
\end{align*}
\]

where \( h(x,t) \) is total water depth, \( u(x,t) \) is horizontal velocity, \( z(x,t) \) is sediment elevation, \( g \) is gravitational acceleration, \( S_f \) is friction term and sediment, \( Q_s \) is sediment discharge, and \( H \) is depth of bedrock layer to reference level. The coefficient \( \zeta \) is computed by \( \zeta = (1 - \sigma)^{-1} \), where \( \sigma \) is bed sediment porosity. In natural systems, the good value of \((1 - \sigma)\) range for 0.45 to 0.75, so we let \( \sigma = 0.3 \).

To model the friction term \( S_f \) and sediment discharge \( Q_s \), we use some of most used empirical formulas. In most of cases, the flow is turbulent. Therefore, we use Manning’s friction law read as

\[
S_f = \frac{k^2 u |u|}{h^3}
\]

where \( k \) is Manning’s roughness coefficient. For the bedload \( Q_s \), we will use Grass Formula. This formula is proposed by Grass for solid transport discharge read as

\[
Q_s = A_g |u|^{m-1} u,
\]

where \( 0 \leq A_g \leq 1 \) and \( 1 \leq m \leq 4 \) are based on experimental data. Letting \( A_g \) be close to 1 shows there is strong interaction between fluid and sediment. We currently use value \( m = 3 \).

![Fig. 1. The domain of this work.](image1)

![Fig. 2. Illustration of finite volume method on staggered grid.](image2)

3. NUMERICAL APPROXIMATION

In this section, we solve Eqs. (1-3) numerically using finite volume method on a staggered grid. The time interval \([0, T]\) is divided into \( N_t \) time steps of length \( \Delta t \) and for all \( n \in \{0, ..., N_t\}, \ t^n = n \Delta t \). Consider a spatial domain \([0, L]\) with a staggered grid partition \( x_0 = 0, x_1, x_2, ..., x_{N_x}, \ x_{N_x+1} = L \). Following the staggered momentum conservative scheme for the Shallow Water Equations as seen in [11]. By modifying, we can implement an analogous scheme for governing equations (1), (2), and (3). The values of \( h \) and \( z \) are computed at every full grid points \( x_j = j \Delta x, \) with \( j = 1, 2, ..., N_x \). Whereas velocity \( u \) is computed at every staggered grid points \( x_{j+\frac{1}{2}} = \left(j + \frac{1}{2}\right) \Delta x \) with \( j = 0, 1, 2, ..., N_x \). This space of discretization is shown in Fig. 2.
In Eq. (6), the water depth \( h_{j+1/2} \) and \( h_{j-1/2} \) are unknown, so we approximate their values using the first order upwind method

\[
\frac{\Delta t}{\Delta x} \left( h_{j+1/2}^{n+1} - h_{j-1/2}^{n+1} \right) + g \frac{(z - H)^{n+1}_{j+1} - (z - H)^{n+1}_{j-1}}{\Delta x} + \frac{S_f}{\Delta x} u_j^{n+1/2} = 0.
\]

Using the same method as used in momentum conservation, the discrete form of Eq. (8) is

\[
\frac{u_{j+1/2}^{n+1} - u_{j-1/2}^{n+1}}{\Delta t} + g \frac{h_{j+1}^{n+1} - h_{j-1}^{n+1}}{\Delta x} + \frac{S_f}{\Delta x} u_{j+1/2}^{n+1/2} = 0.
\]

It is important to notice that in order to prevent the presence of non-entropic shock, we are using semi-implicit approximation \( g \frac{h_{j+1}^{n+1} - h_{j-1}^{n+1}}{\Delta x} \) instead of explicit approximation \( g \frac{h_{j+1}^{n+1} - h_{j-1}^{n+1}}{\Delta x} \) in the momentum conservation [15].

The friction term \( S_f \) is discretized implicitly to reduce the restriction for the stability condition

\[
S_f^{n+1/2} = k u^{n+1} \left( \frac{u_{j+1/2}}{h_{j+1/2}^{n+1}} \right)^{4/3}.
\]

Advection term \( uu_x \) related to momentum variable \( q = hu \) is given by

\[
uu_x = \frac{1}{h} \left( \frac{\partial (qu)}{\partial x} - u \frac{\partial q}{\partial x} \right).
\]

Discretization for advection terms Eq. (11) read as

\[
(uu_x)_{j+1/2} = \frac{1}{h_{j+1/2}^{1/2}} \left( \frac{\tilde{q}_{j+1} u_{j+1} - \tilde{q}_j u_j}{\Delta x} - u_{j+1/2} \frac{\tilde{q}_{j+1} - \tilde{q}_j}{\Delta x} \right)
\]

whereas

\[
\tilde{h}_{j+1/2}^{1/2} = \frac{1}{2} \left( h_j + h_{j+1} \right),
\]

\[
\tilde{q}_j = \frac{1}{2} (q_{j+1} + q_{j-1}), q_{j+1} = \tilde{h}_{j+1/2} u_{j+1/2},
\]

with the following upwind approximation of \( u_j \)

\[
\tilde{u}_j = \begin{cases} u_{j+1/2} & \text{if } \tilde{q}_j \geq 0, \\ u_{j-1/2} & \text{if } \tilde{q}_j < 0. \end{cases}
\]

Using the same method, the discretization of Exner Equation (3) is then

\[
\frac{z_{j+1}^{n+1} - z_j^n}{\Delta t} + \zeta - \frac{\Delta x}{\Delta t} = 0.
\]

Discretization for transport sediment \( Qs \) read as

\[
Q_{s,j+1/2}^{n+1} = A_g \left( |u|_{j+1/2}^{n+1} \right)^2 u_{j+1/2}^{n+1}.
\]

Then, we approximate Shallow Water Equations and Exner Equation sequentially. First, we calculate \( h_{j+1}^{n+1} \) using Eq. (6), then we calculate \( u_{j+1}^{n+1} \) using Eq. (9), and finally we calculate \( z_{j+1}^{n+1} \) using Eq. (14). Notice that we are using implicit evaluation in Eq. (14) in order to prevent additional stability condition. Since the sediment transport does not influence the fluid flow, a sufficient stability condition for our scheme is same as stability condition for one-dimensional Shallow Water Equations given by

\[
\frac{\Delta t}{\Delta x} \sqrt{gH} \leq 1.
\]

Note that we only consider first-order scheme in space and time for all equations. An advantage of this is that it seems to maintain stability properties, while giving sufficiently accurate results, as shown in Section 4.

4. NUMERICAL SIMULATION

In evaluating the accuracy of our proposed numerical scheme, we will conduct some of numerical simulations. First, we will evaluate our scheme with cases from literature. Second, we will compare our scheme to experimental data of dam break simulations.

4.1 Comparison with analytical solution.

To show the performance of our scheme, we consider a case where an analytical solution is
available in [16]. In this case, we use $A_g = 0.005$, $\Delta x = 0.07$, and $\Delta t = 0.01$. The initial conditions are given by

$$(hu)_{ini}(x) = 1,$$  

$h_{ini}(x) = \frac{(hu)_{ini}(x)}{u_{ini}(x)},$  

$u_{ini}(x) = \frac{\alpha x + \beta}{A_g}^{\frac{1}{3}},$  

$z_{ini}(x) = 1 - \frac{u^3_{ini}(x) + 2g(hu)^2_{ini}(x)}{2gu_{ini}(x)},$

where the coefficient $\alpha$ and $\beta$ are given by $\alpha = \beta = 0.005$. In this simulation, we assume that there is no friction, or $S_f = 0$.

From Fig. (3), we can see that our numerical scheme has successfully approximated the analytical solution for water level, sediment elevation and wave velocity. Using Root Mean Square Error (RMSE) method, we have found that the error between the analytical and numerical solutions for explicit scheme and semi-implicit scheme. Consider domain $[0,200]$ and the initial conditions which are given by

$h_{ini}(x) = \begin{cases} 
1 & \text{if } x \leq 50, \\
0 & \text{otherwise.}
\end{cases}$  

$u_{ini}(x) = 0, \quad z_{ini}(x) = 0.$

In this simulation, we assume that there is no transport sediment, or $A_g = 0$. Using $\Delta x = 1$ and $\Delta t = 0.5$, the simulation gives us Fig. (4) and (5).

Fig. 4. Comparison between analytical and semi-implicit numerical solutions of water level in dam break case at final time $T = 50$ s.

As seen in Fig. (5), the analytical solution seems to be well approximated by our semi-implicit scheme compared to the explicit scheme. Using RMSE, the error between analytical and numerical $h + z$ for semi-implicit scheme is 0.00918. This is much smaller than the error for the explicit scheme which is 0.04803. This proves that our semi-implicit scheme works much better than the explicit scheme to simulate the dam break model.

4.2 Dam break test.

In this section, we will conduct numerical simulation of dam break. The first case is devoted to wet dam break and the second one is for dry dam
break. First, consider domain $[0, 10]$ with a dam located in the middle of the domain. The initial condition is given by

$$h_{\text{init}}(x) = \begin{cases} 2 & \text{if } x \leq 5 \\ 0.125 & \text{otherwise} \end{cases}, \quad u_{\text{init}}(x) = 0, \quad z_{\text{init}}(x) = 0,$$

and assume there is no friction term. Using $A_g = 0.005$ and $\Delta x = 0.025$, we will get results as shown in Fig. (6).

In Fig. (6.b), we can see that as the dam breaks, the water that propagates to the right side brings along the sediment under it. It makes sense and agree with the real phenomenon of dam break cases. Notice that in Fig. (6.c), at the right side, the sediment is elevated and becomes about $0.08$ m over the flat bed $z = 0$. That is actually $4.267\%$ of the water level difference which is $1.875$ m. The sediment elevation could be higher if we consider the energy of the water waves caused by the breaking process of the dam.

Next, consider domain $[0, 10]$ with a dam located in the middle of the domain. Initial conditions are given by

$$h_{\text{init}}(x) = \begin{cases} 1 & \text{if } x \leq 5 \\ 0 & \text{otherwise} \end{cases}, \quad u_{\text{init}}(x) = 0, \quad z_{\text{init}}(x) = 0,$$

and assume there is no friction term. This case is dry dam break because there is no water at right side of the dam. Using $A_g = 0.005$ and $\Delta x = 0.01$, we will get result as shown in Fig. (7).

In Fig. (7), we consider the dam break case in the dry condition. From the figure, the sediment transported to the right side is about $0.05$ m. Compared to the water level difference, which is $1$ m, the sediment elevation is exactly $5\%$ of it. This is bigger than the one produced in the wet dam break condition. This might be caused by the fact that in the wet condition, there are water at the right side which will restrain the flow of the water waves as well as restrain the sediment transport. On the other hand, in dry condition, there are no water to restrain the flow, consequently, the sediment that is transported is higher.

Furthermore, we will show a comparison between numerical solution and experimental data. Some laboratory experiments of dam break over a granular bed have been reported. We focus on the experiment performed in Louvain-la-Nueve, Universite of Catholique de Louvain [14].

Consider the domain of experiment is $[-3, 3]$ where the dam is located in the middle of the domain. The dam separates two initial water heights $h_{\text{init}}(x) = 0.35$ at the left side and $h_{\text{init}}(x) = 0$ at the other side. Initially, the sediment level is $z_{\text{init}}(x) = 0$, while the fluid velocity is $u_{\text{init}} = 0$. Since we use Grass Formula, we assume that $A_g =$
In this section, we use $\Delta x = 0.025$ and coefficient Manning $k = 0.001$.

The experiment was carried out in the dam-break flume at Universite of Catholique de Louvain. As in Fig. (8), the flume has a horizontal bottom, a total length $L = 6 \text{ m}$, an adjustable width set at $25 \text{ cm}$ and a sidewall height of $70 \text{ cm}$. More details about the flume and experiment are given in [13].

Results at different times are presented in Fig. (10). They show a good comparison between numerical solutions and experimental data in aspect of water level and sediment profiles. We prove this statement by calculate the error between numerical and experimental results in both water level and sediment elevation for both $t = 0.6$ and $t = 1$. Using RMSE, the error for $h + z$ is 0.02289 at $t = 0.2$ and 0.02187 at $t = 1$. For $z$, the numerical-experimental error is 0.02118 at $t = 0.2$ and 0.02509 at $t = 1$.

4.2 Comparison with experimental data.

In this section, we will compare our numerical results to the data collected in a non-dambreak experiment which taken place at Tohoku University [8]. The set-up of the experiment can be seen in Fig. (11).

The experiment was conducted by generating a constant water flow to the flume to keep $h$ constant. At the right side, we have an absorbing boundary. In the area behind the structure, sediment with 0.2 m deep is placed, to see how the flow and structure affect the sediment transport.

For simulation, we begin with illustrating the evolution of the incoming flow, before and after its interaction with the structure, using our numerical scheme. The evolution of the water flow for different observation time can be seen in Fig. (12).

Fig. (12.a) shows the water profile when it just started to propagate into the domain with the initial height. After a while, the flow starts to interact with the structure, causing a condition similar to wave run-up condition, showed in Fig. (12.b). In Fig. (12.c), because of the flow that is constantly generated, there will be an overtopping condition occur over the structure.

In order to be able to compare our numerical results to the experimental data, we need to adjust the initial condition. Because there is a limitation regarding the incoming flux that is used in the
实验，我们假设水槽在实验开始时已经充满了水。因此，该比较的初始条件如图（13）所示。

在该模拟中，我们关注的是域$[2.05, 2.5]$，其中的沉积物剖面发生变化。然后，我们的模拟结果与实验数据进行比较，如图（14）所示。

图（14）展示了使用我们的数值方案模拟的沉积物剖面与基于实验数据的模拟的比较。

图像12。流入的水流量在（a）$t = 4.5$ s，（b）$t = 8.6$ s，（c）$t = 13.5$ s。

在这种情况下，我们关注的是观察域$[2.05, 2.5]$中的沉积物剖面是否发生变化。然后，我们比较了我们的模拟结果与实验数据，如图（14）所示。

图像13。模拟的初始条件。

图像14。沉积物剖面的比较。

5. 结论

一个数学模型已开发用于模拟崩溃问题，具有可移动床条件。该模型基于浅水方程，结合沉积物传输方程。这些方程组是通过使用有限体积方法在交错网格上求解的，采用半-隐式动量守恒方案。

该模型通过分析性崩溃的案例进行了验证。此外，我们的半-隐式方案也与显式方案进行了比较。半-隐式方案的结果比显式方案更接近于分析性解。开发的模型被应用于模拟假设的崩溃问题，包括可移动床。在两种情况下，模型表现良好。该模型进一步应用于验证一个带有移动床的实验崩溃问题。我们的结果与实验数据有很好的比较。在未来，该模型可以发展成2-D模型，以考虑沉积物传输的体积。此外，它也可以与实际模拟结果进行比较，以验证模型的性能。
phenomenon of sediment transport in dam break cases.

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