Methodology based on cross modal strain energy method to estimate the damage severity in a metallic structure

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Abstract. The studies related to the Structural Health Monitoring have been important to evaluate the operation condition of structures, and let reduce the maintenance costs. The modal strain energy method has been used for this goal. This method requires to compare the information of mechanical vibrations acquired from the structure vs the same structure in initial conditions, called structure in health condition. This work is focused on developing a methodology that differs from the traditional modal strain energy. The methodology allows us to identify and evaluate the damage severity using a severity indicator. The methodology has been implemented in an offshore structure. The results have been satisfactory for the damage quantification for single and multiple damages.

1. Introduction
The metallic structures may reduce its capabilities due to hard work conditions. So, it has been important to develop techniques that allow detecting incipient damages in this kind of structures. Many researchers have worked on this topic and they have implemented different methods to get this goal [1–3]. We can check in the bibliography that techniques based on mechanical vibrations are very interesting to develop this area [4, 5]. The premise of these techniques is: changes in the structure dynamic response means changes in its physical properties [6].

The common methods to detect early damages in metallic structures based on modal analysis involve two general stages: first, identifying the natural frequencies and associated modal shapes. To achieve this step the modal testing method, by theoretical and experimental way, is used; and the second step, developing an algorithm to identify, locate, and evaluate the damage. It supposes different levels of the algorithm complexity, depending on the information that it will provide [4].

Some vibration-based damage identification techniques (VBDIT) have been developed. The mode shape curvature method is an example. In this case, the modal shape curvature for damage and undamaged structure are calculated. The damage indicator is obtained by the difference between these values. Several works have been reported using and improving this method [7, 8].

Change in flexibility matrix is another method to detect damage in structures. It uses the flexibility matrix to estimate changes in static behavior of the structure. Its sensitivity and change are then used to detect structural damage location and damage extent [9]. Other papers based on this method were reported by Zare [10], Weng [11] and Stutz [12].

There are still several issues to improve in this area. One of them is related to the accuracy of the damage severity evaluation. Thus, this work is focused on developing a method called cross...
modal strain energy decomposition (CMSED), bases presented by Hu et al [13]. In this case the CMSED method is applied to detect damage in an offshore structure. The experimental and numerical information is collected. An algorithm is developed to identify failures in a structure and evaluate its damage severity.

2. Modal strain energy method

The modal strain energy is calculated for both damage and undamaged structure by the Equation (1) and Equation (2), where \( n \) is the \( n \)th element, \( i \) the \( i \)th mode \( \Phi \), \( d \) is related to the damaged structure and \( K_n \) is the undamaged element stiffness matrix.

\[
MSE_{ni} = (\Phi_i, K_n \Phi_i) \quad (1)
\]

\[
MSE_{ni}^d = (\Phi_i, K_n \Phi_{di}) \quad (2)
\]

Equation (3) represents the energy change due to the damage presence, called modal strain energy criterium (MSECR). It will be greater for the damage element than others.

\[
MSECR_n^i = \frac{|MSE_{ni}^d - MSE_{ni}|}{MSE_{ni}} \quad (3)
\]

If several modes extracted are taking into account, the average energy change is calculated for all of them by Equation (4). The MSECR indicator is normalized with respect to the largest indicator \( MSECR_{max} \).

\[
MSECR_n^i = \frac{1}{N_i} \sum_{i=1}^{N_i} \frac{MSECR_n^i}{MSECR_{max}} \quad (4)
\]

3. Cross modal strain energy method

The CMSE has into account information from the reference structure, related to the healthy structure by modal strain energy method (MSE), and from the unhealthy structure. This approach is different from other methods based on MSE. The baseline structure model is obtained by Equation (5), and the corresponding equation for the damaged structure is obtained from Equation (6).

\[
K \Phi_i = \lambda_i M \Phi_i \quad (5)
\]

\[
K^* \Phi^*_i = \lambda^*_i M^* \Phi^*_i \quad (6)
\]

Where \( K \) is the stiffness matrix, \( M \) is the mass matrix, \( \Phi \) is the modes matrix, \( \lambda \) is the natural frequencies, \( i \) and \( j \) indicate the \( i \)th and \( j \)th eigenvalue and eigenvector, and the * denote information from damaged structure.

Operating Equation (5) and Equation (6), and proposing that the stiffness matrix of the damaged structure can be calculated by Equation (7), Equation (8) is obtained.

\[
K^* = K + \sum_{n=1}^{N_d} \lambda_n K_{ln} \quad (7)
\]

\[
\sum_{n=1}^{N_d} \alpha_n(\Phi_i)^T K_{ln} \Phi^*_j = \left( \frac{\lambda^*_i}{\lambda_i} - 1 \right) (\Phi_i)^* K \Phi^*_j \quad (8)
\]

In the above equations, \( n \) is a counter of damaged elements, \( N_d \) is the number of damaged elements, \( \alpha \) is the lost stiffness factor by damage and \( l \) is the element with damage.

From Equation (8), the structural cross modal strain energy is defined by Equation (9).
\[ C_{ij} = (\Phi_i)^* K \Phi_j^* \]  
\[ (9) \]

It can be written for each element by Equation (10).

\[ C_{n,ij} = (\Phi_i)^* K_{in} \Phi_j^* \]  
\[ (10) \]

Equation (8) can be written as Equation (11).

\[ \sum_{n=1}^{N_d} \alpha_n C_{n,ij} = \left( \frac{\lambda_j^*}{\lambda_i} - 1 \right) C_{ij} \]  
\[ (11) \]

Replacing \( ij \) by \( m \) and reducing the expression, the Equation (11) can be written as Equation (12) and Equation (13).

\[ \sum_{n=1}^{N_d} \alpha_n C_{n,m} = b_m \]  
\[ (12) \]

where

\[ b_m = \left( \frac{\lambda_j^*}{\lambda_i} - 1 \right) C_{ij} \]  
\[ (13) \]

Considering that the modes from the reference structure and the damaged structure are available, \( N_i \) and \( N_j \) modes respectively, the Equation (12) can be written in a matrix form as Equation (14).

\[ C \alpha = b \]  
\[ (14) \]

\( \alpha \) can be estimated by the Equation (15).

\[ \hat{\alpha} = (C^T C)^{-1} C^T b \]  
\[ (15) \]

CMSE requires the damage locations, however, sometimes these are not known. Thus, it is proposed to perform a residual analysis for each failure scenario of the true damage locations. So, \( b \) can be estimated by Equation (16).

\[ \hat{b} = C \hat{\alpha} \]  
\[ (16) \]

The normalized residue for each CMSE equation is calculated as Equation (17).

\[ e_m = \frac{\hat{b}_m - b_m}{b_m} \]  
\[ (17) \]

Calculating \( e \) for all of \( e_m \), the norm of \( e \) can be used as a severity indicator, Equation (18).

\[ \| e \| = \sqrt{(e_1)^2 + \cdots + (e_{N_q})^2} \]  
\[ (18) \]

4. Numerical and experimental studies
The physical model used to apply the methodology proposed is an offshore structure, see Figure 1. The 3D offshore structure was made with 48 elements distributed in 4 floors. The mechanical properties of the constituent material are: modulus of elasticity \( E = 2e11 \) N/m, density \( \rho = 7900 \) kg/m\(^3\) and Poisson coefficient \( \nu = 0.3 \). The length of the structure’s elements is: 21 cm, 13.12 cm, 14 cm and 8.8 cm. The diameter for all elements is 6.35 mm.

The extracted natural frequencies, by modal testing, related to the two first modes were 102.81 Hz and 144.59 Hz.
The numerical model was validated by comparing the theoretical and experimental results by modal analysis. For the numerical model, the damages were induced by reduction to the modulus of elasticity $E^*$ on the $n$th damaged element ($n$). Hence, Equation (19) lets to calculate the new $E$ by lost factor $\alpha$.

$$E_n^* = (1 - \alpha_n)E$$ (19)

![Figure 1. Offshore structure.](image)

5. Results
Five damage simulations were induced in different elements, see Figure 2. The CMSE method, of section 3, was applied for each case. The induced damages and natural frequencies are presented in Table 1.

![Figure 2. Damages induced in the offshore structure.](image)
Table 1. Induced damages in the offshore structure

| Damage case | Damaged member | Damage severity | Modal Frequencies (Hz) |
|-------------|----------------|-----------------|-----------------------|
| Undamage    | none           | none            | 102.81 144.60         |
| A           | 20 30          | 102.75 144.58   |
| B           | 17 20          | 102.81 144.45   |
| C           | 25 10          | 102.63 144.56   |
| D           | 21-34 25       | 102.50 143.85   |
| E           | 20-30 25       | 102.73 144.56   |

The first three cases are focused on single damages. The results are presented in Figure 3. Multiple damages were induced in the structure. Figure 4 shows the detection results for two different cases.

Figure 3. One element with damage for three different cases.
6. Conclusions
This study is focused on the implementation of Cross Modal Strain Energy Method, by an algorithm, to determine the damage severity on a metallic structure. An offshore structure was used to apply the methodology and validate the results. The CMSE method requires to know the damage location and the MSE method can solve this problem. The numerical model was implemented to develop this project, and simulations were conducted for failure location. The model was validated by comparing the numerical and experimental results. Results indicate that CMSE is an effective method to identify single and multiple damages in a structure.

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