Decoherence of Homogeneous and Isotropic Geometries in the Presence of Massive Vector Fields

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Abstract
Decoherence of Friedmann-Robertson-Walker (FRW) geometries due to massive vector fields with SO(3) global symmetry is discussed in the context of Quantum Cosmology.

The Universe on the large scale behaves classically to a high degree of accuracy. On the other hand, all matter in the Universe is ultimately described by quantum fields, and one expects the gravitational field will eventually be described in a similar way. Progress in this direction has lead in recent decades to an extensive research about the quantization of the whole Universe, assuming it can be described by a single wave function \( \Psi \). A crucial issue in quantum cosmology is to understand how the classical behaviour can be recovered in order to make predictions.

The conditions that must be satisfied in order that a system may be regarded as classical are the following:

1. Firstly, its the evolution should be driven by classical laws, which means that from the wave function \( \Psi \) a strong correlation between the configuration space coordinates and the conjugate momenta should exist.
2. The second condition involves the notion of decoherence, meaning that any interference between different quantum states should vanish.

The decoherence process may emerge in the following way. One considers that the original system is part of a more complex world and interacts with another subsystem, usually designated as environment and which is formed by unobserved (or irrelevant) degrees of freedom. Then, under some circumstances, quantum interference effects on the original system can be supressed by the interaction with the environment. Recent works shows that the two above mentioned criteria for the retrieval of classical behaviour, i.e., correlation and decoherence, must be considered as

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interdependent aspects of the quantum to classical transition and should be analysed altogether. To be more precise, if one adopts the conditional probability interpretation, according to which only peaks on the wave function, or on a probability distribution function derived from $\Psi$, are tantamount to predictions in a quantum cosmological scenario \([1, 3]\), then a more careful analysis indicates \([6]\) that the wave function or probability distribution does not have a peak around a single trajectory and hence, does not exhibit the expected classical correlations unless some kind of coarse-graining is introduced. Thus, a system coupled to an environment induces it to decohere.

Most of the literature about the retrieval of classical behaviour in quantum cosmology considers minisuperspace models in the presence of scalar fields \([3]-[10]\) and as environment inhomogeneous modes either of the gravitational or of the matter fields\([4, 12]\).

In this work we propose an alternative model which possesses a non-Abelian global symmetry, and where the scalar field is replaced by a massive vector field. The main reason to consider this model is to achieve an increasingly realistic description of the early Universe incorporating particle-physics-motivated quantum fields, such as vector fields which acquire their masses through spontaneous symmetry breaking mechanisms (cf.ref.\([13]\)). Furthermore, the presence of a mass term is of crucial importance in the quantum cosmological context since for the massless case the conformal invariance of the Yang-Mills action gives origin to a Wheeler-DeWitt equation that allows for a decoupling of the gravitational and gauge degrees of freedom \([14]\) (similarly to the case of a free massless conformally invariant scalar field \([1]\)). Hence, an interaction process could not in that situation induce the minisuperspace variables to decohere. The presence of a mass term breaks the conformal invariance, thus allowing interaction terms between the gravitational and gauge degrees of freedom and providing a scale to which the decoherence process can be studied.

The action of a model with a non-Abelian global symmetry with a massive vector field is given by

$$S = \int_{M^4} d^4x \sqrt{-\det g} \left\{ \frac{M_{Pl}^2}{16\pi} R + \frac{1}{8\varepsilon^2} \text{Tr}(F_{\mu\nu}F^{\mu\nu}) + \frac{1}{2} m^2 \text{Tr}(A_{\mu}A^{\mu}) \right\} - \frac{M_{Pl}^2}{8\pi} \int_{\partial M^4} d^3x \sqrt{-\det h} K,$$

(1)

where $g$ is the Lorentzian metric, with signature $(-, +, +, +)$ in the four dimensional manifold $M^4$, $h$ is the induced spatial metric on the spatially hypersurfaces, $R$ and $K$ are the scalar curvature corresponding to $g$ and the trace of the extrinsic curvature $K_{\mu}^\nu$ of $M^4$, respectively. The constant $\varepsilon$ denotes a coupling constant, $M_{Pl}$ the Planck mass, $m$ the mass of the vector field $A_{\mu}$ and $F_{\mu\nu}$ is the usual field strength tensor.

We are interested in spatially closed FRW cosmologies associated with the action (1) for the case where the internal global symmetry is $\hat{G} = SO(3)$. The most general form of a metric that is spatially homogenous and isotropic in a $M^4 = IR \times S^3$ topology is given by the FRW ansatz

$$g = \sigma^2(-N^2(t)dt^2 + a^2(t)\sum_{a=1}^{3}\omega^a \otimes \omega^a),$$

(2)

where $N(t)$ and $a(t)$ are respectively the lapse function and the scale factor, $\sigma^2 = \frac{2}{3\pi M_{Pl}}$ and $\omega^a$ are left-invariant 1-forms in $SU(2) \cong S^3$ satisfying $d\omega^a = -\epsilon_{abc}\omega^b \wedge \omega^c$.

For the vector field we make the ansatz

$$A = \frac{1}{2} \left[ 1 + \sqrt{\frac{2\alpha}{3\pi}} \bar{\chi}_0(t) \right] T_{bc} \epsilon_{bac} \omega^a,$$

(3)

where $T_{bc}$ are the generators of $\hat{G}$, $\bar{\chi}_0$ is an arbitrary function of time and $\alpha \equiv \frac{\varepsilon^2}{4\pi}$. The ansatz (3) follows the method presented in ref.\([13]\), and corresponds to a vector field which is homogeneous.
and isotropic (i.e., $SO(4)$-invariant, where $G = SO(4)$ is the group of isometries of $S^3$) up to a compensating internal global transformation. This means that the energy-momentum tensor obtained from the action (1) and using the ansatz (3) is of the form of a perfect fluid.

When the transition from quantum to classical states is studied in a system with scalar fields, one usually considers a perturbed Friedmann model and proceeds to an expansion of the scalar fields in terms of spherical harmonics [2],[4]-[10]. In our model, however, we have a vector field with only spatial components and therefore it should be expanded in terms of spin-1 spinor hyperspherical harmonics (cf. ref. [13] and references therein). In order to get some insight we shall instead discuss here an approach that consists in taking the vector field in the following form

$$A = \frac{1}{2} \left[ 1 + \frac{2\alpha}{3\pi} \left\{ \chi_0(t) + \Sigma_{n,l,m} f_{nml}(t) Q_{lm}^n(\chi, \theta, \varphi) \right\} \right] \mathcal{T}_{bc} \epsilon_{bac} \omega^e,$$  

(4)

where $Q_{lm}^n(\chi, \theta, \varphi)$ are spherical harmonics on $S^3$ and $n = 2, 3, \ldots, \infty$; $l = 0, 1, \ldots, n - 1$; $m = -l, \ldots, l$. The scale factor $a(t)$, the function $\chi_0(t)$ and the infinite modes $f_{nml}$ form the “superspace”. Reduction to a minisuperspace is performed truncating the higher modes ($n \geq 2$) which will play the role of the environment. The assumption we are making here is that this simplification contains the main features of the full model. A more complete account of this problem will be discussed elsewhere [14].

Substituting (2) and (4) into (1) and remembering that the integration over spatial coordinates gives the volume $2\pi^2$ for $S^3$ ($a = 1$), one obtains an effective action:

$$S_{\text{eff}} = \frac{1}{2} \int d\eta \left\{ -\left( \frac{da}{d\eta} \right)^2 + a^2 + \left( \frac{d\chi_0}{d\eta} \right)^2 - \frac{2\alpha}{3\pi} \left[ \lambda^0_0 - \frac{3\pi}{2\alpha} \right]^2 - \frac{16\mu^2 a^2}{3} \left( \sqrt{\frac{3\pi}{2\alpha} + \chi_0^2} \right)^2 + \frac{1}{2\pi^2} \Sigma \left( \frac{df_n}{d\eta} \right)^2 - \frac{2}{3\pi^2} \Sigma f_n^2 \left( n^2 - \frac{5}{3} \right) - \frac{2\alpha^2}{\pi^3} \Sigma f_n^2 - \frac{4\alpha}{3\pi^2} \chi_0 a_{nm} f_n f_m f_p - \frac{\alpha}{3\pi^3} b_{nm} f_n f_m f_p f_q \right\},$$  

(5)

where the conformal time $d\eta = \frac{dt}{a(t)}$ has been used and

$$a_{nm} \equiv \int_{S^3} d\Omega Q^n Q^m Q^p, \quad b_{nm} \equiv \int_{S^3} d\Omega Q^n Q^m Q^p Q^q,$$  

(6)

where $d\Omega = \sqrt{\det \Omega} d\chi d\theta d\varphi$, $\Omega$ being the metric on $S^3$, $\mu^2 \equiv \frac{m^2}{M_{P_1}}$, and the subscripts on the $f$’s and $Q$’s collectively denote the quantum numbers ($nlm$).

Let us now turn to the derivation of the Wheeler-DeWitt equation. The canonical momenta conjugate to $a(t)$, $\chi_0$ and $f_n$ are given by

$$\pi_a = \frac{\partial L_{\text{eff}}}{\partial (da/d\eta)} = -\frac{da}{d\eta}, \quad \pi_{\chi_0} = \frac{\partial L_{\text{eff}}}{\partial (d\chi_0/d\eta)} = \frac{d\chi_0}{d\eta}, \quad \pi_{f_n} = \frac{\partial L_{\text{eff}}}{\partial (df_n/d\eta)} = \frac{1}{2\pi^2} \frac{df_n}{d\eta}.$$  

(7)

The Hamiltonian constraint, $\mathcal{H} = 0$, can be written as
Wheeler-DeWitt equation in this case is given by

\[
\frac{1}{2} \left\{ -\pi_a^2 - a^2 + \pi_{\chi_0}^2 + \frac{2\alpha}{3\pi} \left[ \chi_0^2 - \frac{3\pi}{2\alpha} \right]^2 - \frac{16\mu^2 a^2}{3\pi} \left( \sqrt{\frac{3\pi}{2\alpha} + \chi_0^2} \right)^2 + 2\pi^2 \Sigma_n \pi_n^2 + \frac{2}{3\pi^2} \Sigma_n f_n^2 \left( n^2 - \frac{5}{3} \right) + \frac{2\mu^2 a^2}{\pi} \Sigma_n f_n^2 + \frac{2\alpha}{\pi^3} \chi_0^2 \Sigma_n f_n^2 + \frac{4\alpha}{3\pi^2} \chi_0 a_{nmp} f_n f_m f_p + \frac{\alpha}{3\pi^3} b_{nmpq} f_n f_m f_p f_q \right\} = 0.
\]

(8)

Quantization proceeds by promoting the canonical conjugate momenta in (7) into operators, in the following way

\[
\pi_a = -i \frac{\partial}{\partial a}, \quad \pi_{\chi_0} = -i \frac{\partial}{\partial \chi_0}, \quad \pi_{f_n} = -i \frac{\partial}{\partial f_n}, \quad \pi_a^2 = -a^{-p} \frac{\partial}{\partial a} \left( a^p \frac{\partial}{\partial a} \right),
\]

(9)

where the last substitution reflects the operator ordering ambiguity being \( p \) a real constant. The Wheeler-DeWitt equation is then found to be

\[
\frac{1}{2} \left\{ a^{-p} \frac{\partial}{\partial a} \left( a^p \frac{\partial}{\partial a} \right) - \frac{\partial^2}{\partial \chi_0^2} - 2\pi^2 \Sigma_n \frac{\partial^2}{\partial f_n^2} + \mathcal{V}_0(a, \chi_0) + \mathcal{V}(a, \chi, f_n) \right\} \Psi[a, \chi_0, f_n] = 0.
\]

(10)

where

\[
\mathcal{V}_0 = -a^2 + \frac{2\alpha}{3\pi} \left[ \chi_0^2 - \frac{3\pi}{2\alpha} \right]^2 + \frac{16\mu^2 a^2}{3\pi} \left( \sqrt{\frac{3\pi}{2\alpha} + \chi_0^2} \right)^2
\]

(11)

\[
\mathcal{V} = \frac{2\alpha}{\pi^3} \chi_0^2 \Sigma_n f_n^2 + \frac{2}{3\pi^2} \Sigma_n f_n^2 \left( n^2 - \frac{5}{3} + 3\pi^2 a^2 \right) + \frac{4\alpha}{3\pi^2} \chi_0 a_{nmp} f_n f_m f_p + \frac{\alpha}{3\pi^3} b_{nmpq} f_n f_m f_p f_q
\]

(12)

It is interesting to compare this result with the case of a FRW minisuperspace model with a massive scalar field with a self-interaction term \( \lambda \Phi^4 \) and a conformal coupling term \( \frac{1}{12} R \Phi^2 \). The Wheeler-DeWitt equation in this case is given by

\[
\left\{ \frac{1}{2} a^{-p} \frac{\partial}{\partial a} \left( a^p \frac{\partial}{\partial a} \right) - \frac{1}{2} \frac{\partial^2}{\partial \Phi_0^2} + \frac{1}{2} \Sigma_n \frac{\partial^2}{\partial f_n^2} + \mathcal{V}_0 + \mathcal{V} \right\} \Psi[a, \Phi_0, f_n] = 0.
\]

(13)

where

\[
\mathcal{V}_0 = -\frac{1}{2} a^2 + \frac{1}{2} m^2 a^2 \Phi_0^2 + \frac{\lambda}{4!} \Phi_0^4
\]

(14)

\[
\mathcal{V} = \frac{6\lambda}{4!} \Sigma_k \Phi_0^2 f_k^2 + \frac{1}{2} \Sigma_k (k^2 + m^2 a^2) f_k^2 + \frac{\lambda}{4!} a_{kln} f_k f_l f_n + \frac{\lambda}{4!} b_{klmn} f_k f_l f_m f_n.
\]

(15)

One clearly sees that both models are fairly similar. In particular, if one drops the \( a_{kln} \) and \( b_{klmn} \) terms, retaining in (12) and (15) only the terms quadratic in the \( f_n \)'s, one gets a non-linear interaction between the lowest mode \( (n = 1) \) with the the environment, i.e, the higher modes. Phenomena such as dissipation, fluctuation, decoherence, back-reaction and particle creation are particularly interesting in this last minisuperspace model, suggesting they may be also worth studying in our model (cf.refs.17). It is important to stress that up to ref.7 the generality of the models that were
considered containing only free scalar fields (in some cases with a conformal coupling term) on a
FRW background presented no interaction between the lowest and the higher modes.

However, there are some important differences between our model and the one in ref. [7]. In
the first place, the effective potential of self-interaction for $\chi_0$ in (11) is of the anharmonic type
[14]. Moreover, the effective action (5) describes the dynamics of the early Universe via variational
principles in a setting where earlier than the inflationary period the Universe is dominated by gravity
and non-Abelian vector fields, being therefore more realistic. In this context, our approach represents
a step forward towards the understanding of the transition from quantum to classical regime in the
very early universe.

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