Quantum glass phases in the disordered Bose-Hubbard model

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The phase diagram of the Bose-Hubbard model in the presence of off-diagonal disorder is determined using Quantum Monte Carlo simulations. A sequence of quantum glass phases intervene at the interface between the Mott insulating and the Superfluid phases of the clean system. In addition to the standard Bose glass phase, the coexistence of gapless and gapped regions close to the Mott insulating phase leads to a novel Mott glass regime which is incompressible yet gapless. Numerical evidence for the properties of these phases is given in terms of global (compressibility, superfluid stiffness) and local (compressibility, momentum distribution) observables.

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\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{(Color online) Phase diagram of the 1-D BHM with spatially correlated off-diagonal disorder: $t_{i,i+1} = t/3$ and $t$ with equal probability. For comparison, the phase boundaries of the clean system ($t_{i,i+1} = t$) are shown with dashed lines. The phases are characterized by the stiffness $\rho_s$ and global compressibility $\kappa$: superfluid (SF) ($\rho_s > 0$, $\kappa > 0$), Bose glass (BG) ($\rho_s = 0$, $\kappa > 0$), Mott insulator (MI) ($\rho_s = 0$, $\kappa = 0$). The Mott glass (MG) and MI phases have identical global properties and differ only in local properties.}
\end{figure}

The competition between disorder, interactions and commensurability in quantum many-body systems is known to produce novel quantum glassy phases, characterized by a gapless spectrum and by the absence of a global order parameter\textsuperscript{1}. He adsorbed on porous media\textsuperscript{2}, granular superconductors\textsuperscript{3}, disordered magnets\textsuperscript{4} are but a few manifestations of localization effects due to random potentials in interacting bosons. In recent years, ultracold atomic gases in magneto-optical traps have opened a new frontier in the study of strongly correlated systems, as unprecedented control over experimental parameters in these systems makes them ideally suited for studying many-body phenomena. Disorder can be generated in optical lattices by exposure to speckle lasers\textsuperscript{5,6}, incommensurate small-particle disorder\textsuperscript{7,8,9}, and by other means\textsuperscript{10}. The interplay between disorder and interactions in trapped Bose-Einstein condensates has recently been explored experimentally in $^87\text{Rb}$, both in the continuum\textsuperscript{11,12} and in an optical lattice\textsuperscript{9,13}.

Theoretically, the effects of random potentials on interacting bosons in periodic lattices have been studied using analytic\textsuperscript{14,15,16,17} and numerical techniques\textsuperscript{18,19}. It is now well established that even infinitesimally small potential disorder can destroy the direct superfluid (SF) to Mott insulator (MI) transition in one dimension by introducing an intervening insulating, but compressible, Bose glass (BG) phase\textsuperscript{18,19}. Surprisingly, while the effects of potential disorder have been widely investigated, other kinds of disorder, e.g. hopping or interaction strengths, have remained largely unexplored until recently, when it was demonstrated that these can lead to qualitatively quite different phenomena\textsuperscript{19,20,21}. In the quantum rotor model with off-diagonal disorder, the SF to MI transition takes place via an intermediate Mott glass (MG) phase\textsuperscript{21,22} – a unique incompressible, yet gapless, glassy regime that was first reported in disordered fermions with extended range interactions\textsuperscript{23}. It is thus of great interest to investigate whether such a phase also appears in the Bose-Hubbard model (BHM), and if particle-hole symmetry is essential for the stabilization of such a MG.

We use large-scale QMC simulations to study the effects of off-diagonal disorder in the Bose-Hubbard model on a one-dimensional (1D) lattice. We find that, in contrast to diagonal disorder, the Mott lobes do not shrink in the presence of off-diagonal disorder. Instead, there is an extended BG phase separating the MI lobes from the SF regime, and an additional MG regime emerges.

The Bose-Hubbard model is given by the Hamiltonian

$$H = \sum_{i=1}^{L} \left[ -t_{i,i+1}(b_{i+1}^\dagger b_i + h.c.) + \frac{U}{2} n_i(n_i - 1) - \mu n_i \right],$$

where $b_i^\dagger$ ($b_i$) creates (annihilates) a boson at site $i$, $t_{i,i+1}$
is the hopping matrix element between sites $i$ and $i+1$, $U$ is the on-site interaction strength, and $\mu$ is the chemical potential. Off-diagonal disorder is introduced in the form of a bimodal distribution of the hopping matrix elements $-t_{i,i+1}=t$ or $t/3$ with equal probability. The Stochastic Series Expansion (SSE) method is used to simulate the model in chains of length $L=64-256$. For each set of parameters, 200-1000 realizations of disorder are sampled. To characterize the emerging phases, we compute the superfluid stiffness and the global compressibility. In simulations employing updates that sample different winding number sectors, the stiffness is conveniently obtained from the fluctuations in the winding numbers of the world lines as $\rho_s=\langle(W^2)/2n\rangle_{av}$, where $[\ldots]_{av}$ denotes averaging over multiple realizations of disorder. The global compressibility, $\kappa$, is the energy cost of adding a particle to the system. It is defined by $\kappa=\beta(n^2)-\langle n\rangle^2_{av}$, where $n$ is the density of particles.

At sufficiently large $U$, local variations of the hopping amplitudes create a non-trivial landscape of high-mobility domains ("lakes") that coexist with gapped, localized regions. In order to tune the size of these lakes, we introduce spatially correlated disorder. The Fourier filtering method is used to generate a sequence of random numbers, $\eta(i)$, with long-range algebraic correlation,

$$C(i-j) \equiv \langle \eta(i)\eta(j) \rangle \sim |i-j|^{-\alpha}. \quad (2)$$

Without any loss of generality, we choose $\alpha = 0.3$. This is then mapped onto a bimodal distribution of hopping integrals, $t_{i,i+1}$, between sites $i$ and $i+1$ of a one-dimensional lattice. While the qualitative features are same for correlated and random disorder, spatially correlated disorder favors extended domains in the disorder realizations. This, in turn, allows for a more reliable identification of the phase boundaries of the unique MG phase which differs from the MI only in terms of short-range properties. Its proper characterization thus depends crucially on the sampling of finite-ranged domains. With uncorrelated disorder, typical domain sizes are smaller, which renders the MG phase more difficult to detect numerically.

The results of the simulations are summarized in Fig. I rendering a rich phase diagram. In the absence of disorder, the ground state of the BHM is in either the SF or the MI phase. The SF-MI transitions (dashed lines) are marked by the simultaneous vanishing of the compressibility, $\kappa$, and stiffness, $\rho_s$, leading to the well-known sequence of MI lobes, with integer fillings, as $\mu/U$ is varied. In the presence of diagonal disorder, the two transitions -- $\rho_s=0$ and $\kappa=0$ -- decouple and a compressible ($\kappa > 0$), insulating ($\rho_s = 0$) BG phase appears. With off-diagonal disorder, an additional glassy phase -- the MG -- is realized. The MG has global properties identical to the MI ($\rho_s = 0$, $\kappa = 0$ and integer filling), but differs in local properties. In contrast to diagonal disorder, the Mott lobes do not shrink with respect to the clean case. There are no glassy phases in the atomic limit ($t/U \rightarrow 0$). At finite $t/U$, the direct SF-MI transitions of the clean limit are replaced by SF-BG-MG-MI sequences. Our results are consistent with all the transitions being continuous.

Details of the simulations are illustrated in Fig. 2. The global stiffness and compressibility are shown as a function of the chemical potential $\mu/U$ at constant $t/U=0.16$ (cut along the dashed line in Fig. 1). For comparison, data of the clean system are also shown. In the absence of disorder, stiffness and compressibility vanish simultaneously at the SF-MI boundary. In the presence of disorder, the $\rho_s = 0$ and the $\kappa = 0$ transitions decouple, resulting in the glassy phases BG ($\rho_s = 0$, $\kappa > 0$) and MG ($\rho_s = 0$, $\kappa = 0$). The MG regimes I, II and III differ from the MI in terms of local quantities. Note that there is a finite MG phase with $n = 2$ ($1.28 < \mu/U < 1.34$) while there is no corresponding MI phase.

![FIG. 2: (Color online) Stiffness ($\rho_s$) and compressibility ($\kappa$) as functions of the chemical potential ($\mu$) at constant $t/U = 0.16$ (cut along the dashed line in Fig. 1). For comparison, data of the clean system are also shown. In the absence of disorder, stiffness and compressibility vanish simultaneously at the SF-MI boundary. In the presence of disorder, the $\rho_s = 0$ and the $\kappa = 0$ transitions decouple, resulting in the glassy phases BG ($\rho_s = 0$, $\kappa > 0$) and MG ($\rho_s = 0$, $\kappa = 0$). The MG regimes I, II and III differ from the MI in terms of local quantities. Note that there is a finite MG phase with $n = 2$ ($1.28 < \mu/U < 1.34$) while there is no corresponding MI phase.](image-url)
lobes of the clean system with \( t_{i,i+1} = t \) (in particular, there is no \( n = 2 \) MI phase for this value of \( U/t \)), the ground state consists of a mixture of SF \((t_{i,i+1} = t)\) and MI \((t_{i,i+1} = t/3)\) domains as in the BG phase, but differs from the BG by being globally incompressible.

Having explored the global properties, we focus on the local compressibility and the momentum profile of the ground state in each phase. The local compressibility at site \( i \) is defined as the local number fluctuation, \( \kappa_i = \beta(\langle n_i^2 \rangle - \langle n_i \rangle^2)_{av} \), where \( n_i \) is the particle density at site \( i \).

The momentum distribution in the different phases averaged over 800 realizations of disorder. The double-peaked distribution in the BG and MG phases indicate the presence of co-existing SF and MI domains. (b) Momentum profiles of the corresponding ground states. The BG and MG phases have similar momentum profiles, reflecting the similarity in their local structure.

\[ n(q) = \frac{1}{N} \sum_{i,m} e^{-i(qr_i-r_m)} \langle b_i^\dagger b_m \rangle. \]  

\[ \chi(q) = \left[ \int_0^\beta d\tau \sum_{j,k} e^{-i\tau(qr_j-r_k)} \langle n_j(\tau)n_k(0) \rangle_{av} \right]. \]

A finite value of \( S(q) \) as \( q \to 0 \) implies the phase is compressible, i.e., the energy gap towards adding or removing a particle vanishes in the thermodynamic limit. Conversely, a vanishing \( S(q) \) as \( q \to 0 \) is a signature of an incompressible phase with a gapped charge excitations. Additionally, the ratio \( 2\chi(q)/S(q) \) as \( q \to 0 \) gives an upper bound for the charge excitation gap. Fig. 4 shows the structure factor and the susceptibility as a function of momentum for representative points in each of the different phases. Consistent with global compressibility measurements, \( S(q) \) vanishes as \( q \to 0 \) in the MI and MG phases, whereas it remains finite for SF and BG ground states. The upper bound of the charge excitation gap, \( \Delta = 2\chi(q)/S(q) \) as \( q \to 0 \), on the other hand, sheds light on the nature of the MG phase. Although \( \Delta_{MG} \) is finite, it is less than the effective chemical potential difference \( \delta\mu_{MG}^{I-II} \) between type I \((t = 1)\) and type II \((t = 1/3)\) domains. While there is a gap towards overall addition or removal of bosons, there exists a gapless mode involving the transfer of particle across the boundaries of the two types of domains which is driven by the effective potential energy difference between adjacent sites at the domain boundaries. This is in contrast to the MI phase where the effective potential difference \( \delta\mu_{MI}^{I-II} \) between the two types of domains is less than \( \Delta_{MI} \). This confirms the existence of a gapless mode in the MG phase.

In conclusion, using large-scale quantum Monte Carlo
correlations in the disorder realization. It is also evident glass can be tuned by varying the range of the spatial off-diagonal disorder, and that the extent of the Mott insulating phase are basically unaffected by properties. It is remarkable that the phase boundaries leads to a novel Mott glass phase which is incompressible and gapped regions close to the Mott insulating phase. In particular, the coexistence of gapless the Mott insulating and the superfluid regimes of the tum glass phases that intervene at the interface between (random hopping), yields a non-trivial sequence of quan-
existence of a gapless mode.

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