Mathematical model for assessing lateral stability of articulated tracked vehicles

A P Chajkin¹,², R Yu Dobretsov¹, V A Sokolova², I A Teterina³, A V Kamenchukov⁴, E A Tikhonov² and V I Bazykin⁶

¹ Peter the Great St. Petersburg Polytechnic University, st. Polytechnicheskaya, 29, St. Petersburg, 195251, Russia
² Saint Petersburg State Forest Technical University named after S.M. Kirov, Institutskiy per., 5, St. Petersburg, 194021, Russia
³ Siberian State Automobile and Highway University (SibADI), 5, Mira ave., Omsk, 644080, Russia
⁴ Pacific National University, 136 Tihookeanskaya St., Khabarovsk, 680035, Russia
⁵ Petrozavodsk State University, Lenin Ave., 33, Republic of Karelia, Petrozavodsk, 185000, Russia
⁶ Federal State Budgetary Scientific Institution "Federal Scientific Agroengineering Center VIM", branch in Saint Petersburg, Filtrovskoje shosse, 3 p.o. Tiarlevo, Saint Petersburg, 196625, Russia

²E-mail: kondraga@mail.ru

Abstract. The article considers the problem of increasing transport productivity, operational reliability and safety during cargo transportation by using articulated tracked vehicles and road trains with active trailers. The influence of the introduction of an electromechanical drive, the modernization of the propulsion unit and the steering control system on the lateral stability of an articulated tracked vehicle is analyzed. A mathematical model is described for calculating the lateral stability of the chassis of articulated tracked vehicles used in the regions of the Far North, Arctic and Antarctic. The model is based on developments carried out for the chassis of an articulated wheeled vehicle. The model allows calculating to determine the key geometric and kinematic parameters of the rotation, taking into account the action of external forces. The use of holonomic constraints in determining the critical speed of movement is determined by the physical picture of the beginning of overturning, which corresponds to the achievement of the critical folding angle of the sections. This approach makes it possible not to use empirical coefficients when assessing the instantaneous position of the center of gravity of the system, the center of rotation, the radius of rotation of the center of mass, and the critical speed of the chassis.

The moment of the beginning of the rollover is determined by the disappearance of the normal reaction under the link caterpillar. The onset of lateral sliding is determined by the lateral force exceeding the lateral adhesion limit.

1. Introduction
One of the key problems in the regions of the Far North and the Arctic is the difficulties with transport accessibility. The development and development of the region is impossible without solving this problem, moreover, it is highly desirable that the solution be economically viable.
It is known that one of the ways to increase the economic efficiency of cargo transportation is the use of multi-link transport vehicles. If the condition of the track bed is satisfactory, it is sufficient to use wheeled road trains with one or more passive trailers. To increase efficiency, improve the traction and dynamic properties of the chassis and reduce the environmental load in the articles [1,2], it is proposed to use the principle of an active trailed link with an electromechanical drive. The principles of constructing such electromechanical systems are considered, for example, in works [3,4,5].

It seems economically feasible to develop the direction of pairing a tracked tractor with an active tracked trailer by analogy with proposals for wheeled vehicles [1,2,6,7]. In especially difficult road conditions, as well as on soils with a weak surface layer, it is necessary to use two-link tracked vehicles with active links. In Russia, two-link conveyors of the DT series are serially produced [8]. The machines of this series have exceptionally high traction characteristics and unique mobility, and are well suited for off-road operation typical for the regions of the Far North, for work on snow and ice. Prospects for a comprehensive improvement in the operational properties of such machines are associated with the use of an electromechanical drive of the rear section (see, for example, article [9]) and modernization of the steering control system through the use of differential power distribution mechanisms (see articles [10,11]). A characteristic feature of the region, in which two-link tracked conveyors are mainly operated, is the vulnerability of soils when exposed to a contact propeller. The problem of interaction of a caterpillar mover with the ground is considered in detail, for example, in works [12,13,14].

The issue of reducing the environmental load on the supporting surface leads to a number of proposals, the implementation of which leads to an increase in the position of the center of gravity of the machine, complication and weighting of the running system, but, as expected, will reduce the destructive effect on the soil, which is very important for the unique soils of the Far North. Such concepts include the use of a pneumatic caterpillar mover [15], modernization of link tracks [16,17], modernization of the steering control system [11,18].

In turn, an increase in the position of the center of mass negatively affects stability. It is necessary at the stage of design or modernization of the chassis, as well as during the operational assessment of the transport accessibility of objects on the ground for this type of equipment in real conditions of use, to assess the possibility of overturning the chassis.

This problem makes the development of a mathematical model of lateral stability of an articulated tracked vehicle urgent. The model should take into account the key geometric and power characteristics of the chassis, be simple enough to use and undemanding to computing power.

The purpose of the study is to create a methodology for assessing the lateral stability of an articulated two-link tracked vehicle.

The objects of research are articulated tracked vehicles, using the example of DT series transporters.

Research methods - methods of theoretical mechanics, theory of transport tracked and wheeled vehicles, tractor theory, system analysis, mathematical analysis.

2. Statement of the problem
Working with literary sources and analyzing the experience of designing the chassis of transport and traction machines [19,20,21,22,23] allow us to formulate the following tasks.

• Based on the analysis of the industry experience, propose an approach to the construction of a design model and the selection of basic assumptions.
• Propose a mathematical apparatus and methodology (sequence of actions) of calculation.
• To test the model on the example of assessing the lateral stability of serially produced two-link tracked conveyors.

3. Results of theoretical research
In figure 1 shows a design diagram that allows evaluating the lateral stability of an articulated tracked vehicle. The scheme developed for a wheeled tractor is taken as a basis [20]. When simplifying the design according to the principles formulated in [2], it is possible to abandon the principle of control of
the machine rotation by turning the sections in the plan and go to the coordinated steering control by changing the traction forces or angular speeds of the sides. This can be done at the head link due to the steering mechanisms available in the transmission of a caterpillar tractor, on the trailed link - due to the use of a drive with a traction electric motor (TED) and a controlled power distribution mechanism [7,10]. With this approach, the design of the machine is simplified and facilitated by eliminating the hydraulically driven section swing mechanism and a complex coupling device that transfers torque to the transmission of the second section. It becomes possible to use commercially available tractors as part of such a machine. It should be noted that in the latter case, it seems problematic to obtain a design that could seriously compete with the DT series machines. However, such a design could be actively used in solving transport problems.

When assessing lateral stability against overturning, the loss of stability is determined by the disappearance of the normal reaction under the link caterpillar. The onset of lateral sliding is determined by the lateral force exceeding the lateral adhesion limit.

When constructing the model, we use the basic assumptions: turning occurs on a horizontal surface, skidding and slipping of the tracks are not taken into account, the movement is uniform, the links between the normal reaction, longitudinal and lateral reactions are linear [23].

Let us introduce a coordinate system \( XOY \) with the origin at the intersection of the axis of longitudinal symmetry of the rear section and a line normal to this axis and passing through the center of rotation \( O_n \). When turning the sections by an angle, the center of gravity of the machine will shift to a point \( G \).

Centrifugal force acts on the machine \( F = mV^2/R_G \). Here \( m \) is the mass of the car; \( V \) - movement speed; \( R_G \) - radius of rotation of the center of mass. This force tends to overturn the car relative to the

![Figure 1. Design diagram of the turning of a two-link tracked vehicle.](image)
line $A*C*$. Let us introduce the designation for the folding angle of the sections $\psi$. Then, for the scheme under consideration, the critical overturning speed will be determined by the dependence:

$$V_{sp} = \sqrt{\frac{g l_G R_G}{h_G \cos \psi}}$$  \hspace{1cm} (1)

In the last expression, $g$ is the acceleration of gravity; $h_G$ - the height of the center of mass of the system; $l_G$ - the shoulder of the overturning force (see figure 1).

The coordinates of the center of gravity of the machine are determined by the expressions (2):

$$x_G = \frac{G_1 \left( l_1 - a_{A1} \right) \cos \Theta + l_2}{G_1 + G_0 + G_2} ;$$

$$y_G = \frac{G_1 \left( l_1 - a_{A1} \right) \sin \Theta}{G_1 + G_0 + G_2} ;$$

$$z_G = \frac{G_1 z_{A1} + G_0 z_{A0} + G_2 z_{A2}}{G_1 + G_0 + G_2}$$  \hspace{1cm} (2)

In these dependencies: $G_1$, $G_2$, $G_0$ - the weights of the front and rear sections of the machine and the coupling device (connecting link); $a_{A1}$, $a_{A2}$, $a_{A0}$ - coordinates of the center of gravity of the machine and the center of gravity of the connecting link along the axes of the sections; $z_{A1}$, $z_{A2}$, $z_{A0}$ - are the heights of the centers of gravity relative to the supporting surface.

In the general case, the values of $a_{A1}$ and $a_{A2}$ (the longitudinal displacement of the turning poles of the tracks) depend on the transverse force (such a dependence is essential and is considered, for example, in the source [23]) and the calculation of the critical speed should be carried out by the iterative method. As a first approximation, it is assumed that $a_{A1} \rightarrow 0$ and $a_{A2} \rightarrow 0$.

The pivot center $O_1$ of the machine lies at the intersection of straight lines $AB$ and $CD$:

$$\begin{cases} a_1 x + b_1 y + c_1 = 0 \\ a_2 x + b_2 y + c_2 = 0 \end{cases}$$  \hspace{1cm} (3)

In this system (3):

$$a_1 = -2 b \cos \Theta ; \ b_1 = 2 b \sin \Theta ; \ c_1 = 2 b (l_1 + l_2 \cos \Theta) ; \ a_2 = -2 b ; \ b_2 = c_2 = 0.$$

The coordinates of the center of rotation are determined by the expression (4):

$$x_{op} = 0 ; \ y_{op} = \left( l_1 + l_2 \cos \Theta \right) / \sin \Theta$$  \hspace{1cm} (4)

The radius of movement of the center of gravity of the machine is determined by the expression (5):

$$R_G = \sqrt{\left( x_G - x_{on} \right)^2 + \left( y_G - y_{on} \right)^2}$$  \hspace{1cm} (5)

To determine the quantity $l_G$, we use the equation of the straight line $A*C*$: $a_3 x + b_3 y + c_3 = 0$, where

$$a_3 = (y_{C*} - y_{A*}) ; \ b_3 = (x_{A*} - x_{C*}) ; \ c_3 = (y_{A*} x_{C*} - x_{A*} y_{C*})$$

Then:
\[
y_{C^*} = y_C = l_2 + l_1 \cos \Theta + b \sin \Theta; \quad x_{C^*} = L_2/2; \quad y_{A^*} = b \cos \Theta - (l_1 - L_4/2) \sin \Theta;
\]
\[
x_{A^*} = l_2 + (l_1 - L_4/2) \cos \Theta + b \sin \Theta \quad \text{and} \quad l_G = \frac{|a_4 x_G + b_3 y_G + c_3|}{\sqrt{a_3^2 + b_3^2}} \quad (6)
\]

Similarly, to determine the value \( \psi \): \( a_4 x + b_4 y + c_4 = 0 \), where

\[
a_4 = (y_G - y_{OP}); \quad b_4 = (x_{OP} - x_G); \quad c_4 = (y_{OP} x_G - x_{OP} y_G), \quad \text{and} \quad \psi = 90^\circ - \arctg \frac{a_4 b_4 - a_3 b_3}{a_3 a_4 + b_3 b_4} \quad (7)
\]

Obtained for the maximum folding angle of the sections \( \Theta = 38^\circ \) (data for DT-10P [1]) the calculated minimum turning radius \( R_{G,\min} = 13.89 \) m is very close to that stated in [8] (13 m), which confirms the adequacy of geometric constructions and associated calculations.

The value \( R_{G,\min} = 13.89 \) m corresponds to the critical overturning speed of 11.6 m/s. Thus, in the real range of travel speeds and turning radii, overturning does not limit the possibility of operating the machine in question under given conditions, in contrast to the case of a wheeled tractor [20]. This situation is common for military transport and combat tracked vehicles with a low center of gravity, however, for vehicles using a tracked tractor and an active trailer with a high center of gravity at full load, a preliminary assessment is necessary at the design stage.

The anti-skid resistance limits the machine's ability to turn. In figure 2 shows the calculation results when turning on rolled snow).

4. Practical implications and perspectives
These are: expansion of the fleet of equipment used in cargo transportation; increasing the transport performance of individual machines; reducing the destructive impact on the ecosystem; increasing the durability and operational reliability of transport and transport-technological machines in the Arctic and subarctic zones.

Separately, it is necessary to highlight the possibility of increasing traffic safety due to the provided opportunity to assess the threat of the chassis overturning when operating under specified conditions.

Further interest is the development of the presented approach for the case of a multi-link transport vehicle and the transition to taking into account the dynamic aspects of the rotation of the tracked chassis.

5. Conclusions
Increased transport productivity in the logging industry can be achieved through the use of tracked tractors with an active linkage.

One of the technical solutions for activating the trailer link is the use of a traction motor with a controlled power distribution mechanism.

An assessment of the lateral stability of such machines can be carried out on the basis of a model developed for wheeled vehicles, taking into account the necessary corrections for the specifics of the tracked propulsion unit.
Figure 2. Calculation results stability of the machine against skidding $V_{CR1}(R_G)$ and $V_{CR2}(R_G)$.

References

[1] Vasiliev A et al. 2018 On the way to driverless road-train: Digital technologies in modeling of movement, calculation and design of a road-train with hybrid propulsion unit IV International Scientific Conference “The Convergence of Digital and Physical Worlds: Technological, Economic and Social Challenges” (CC-TESC 2018) 1-9

[2] Vasiliev A et al. 2020 Methodology for assessing the energy consumption of a road train with an active trailer International Forum KAZAN DIGITAL WEEK - 2020: collection of materials (Kazan: GBU "NCBZD") 1 234-42

[3] Porshnev G P et al. 2020 The Estimation of Main Parameters of the Power Plant and Electromechanical Powertrain for the Wheeled Vehicle International Review of Mechanical Engineering (IREME) 14(2) 139-45

[4] Porshnev G P et al. 2020 Transmission with electromechanical transmission for tractors and road-building machines Izvestiya MGTU "MAMI" 2(44) 33-41

[5] Dobretsov R Yu and Semenov A G A method of increasing the traction class of a tractor or road-building machine on its chassis and a device for its implementation 2 741 850 Rus. Federation: MPK51 B60K 17/12 (2006.01); B60K 17/08 (2006.01); B60K 6/20 (2007.10); F16H 47/07 (2006/01) / applicant and patentee Peter the Great St. Petersburg Polytechnic University. - No 2020115242; declared 04/30/2020; publ. 01/29/2021 4 19

[6] Bukashkin A Yu et al. 2017 Split Transmission of Tractor with Automatic Gearbox Procedia Engineering 206 1728-34

[7] Galyshnev Yu V, Porshnev G P et al. 2018 Transmission of the Perspective Wheel Tractor with Automatic Gearbox: Management of the Power Distribution Mechanism International Review of Mechanical Engineering (IREME) 12(9) 790-6

[8] 1988 Two-link conveyors DT-10 and DT-10P: technical description and operating instructions (M.: "Military publishing house") 160

[9] Dobretsov R Y, Voinash S A et al. 2021 Comprehensive Modernization of the Chassis of a Two-
LinkTracked Transporter *IOP Conf. Series: Materials Science and Engineering* **1079** 072002
[10] Dobretsov R Yu, Voinash S A, Sokolova V A, Krivonogova A S, Pushkov Yu L and Andronov A V 2020 Power distribution mechanism for the transmission of forest tracked and wheeled vehicles *Journal of Physics: Conference Series* **1679** 042046
[11] Dobretsov R Yu, Grigoriev I V and Gazizov A M 2017 Ways to improve the controllability of forest and transport tracked vehicles *Bulletin of the Bashkir State Agrarian University* **3(43)** 97-106
[12] Dobretsov R Yu 2009 Features of the caterpillar drive in the area of small specific traction forces *Tractors and Agricultural Machines* **6** 25-31
[13] Dobretsov R Yu 2009 Ways to reduce the environmental hazard of interaction of tracked propellers with soil *Ecology and Industry of Russia* **5** 24-7
[14] Galyashev Yu V and Dobretsov R Yu 2013 Efficiency of using the support surface of a caterpillar propeller when transferring normal loads *Scientific and technical statements of SPbSPU. Ser.: Science and Education* **3** 272-8
[15] Dobretsov R Y, Voinash S A et al. 2021 Pneumatic Caterpillar Mover for a Light Transport Vehicle *IOP Conf. Series: Earth and Environmental Science* **666** 042003
[16] Dobretsov R Y and Uvakina D V 2020 The Mechatronic Device Impulse Control in Vehicle Powertrains *Lecture Notes in Mechanical Engineering* 63-73
[17] Dobretsov R Yu and Semenov A G 2011 New designs of caterpillars for snow-swamp and agricultural machinery *Tractors and agricultural machines* **5** 10-4
[18] Lozin A V, Medvedev M S et al. 2019 Hyperbolic steering for tracked vehicles *Lecture Notes in Mechanical Engineering* 2367-74
[19] Skotnikov V A, Maschenksy A A, Solonsky A S and Skotnikov V A 1986 *Fundamentals of theory and calculation of a tractor and an automobile* (Moscow: Agropromizdat) 383
[20] Shuvalov E A, Boykov A V, Dobryakov B A and Pantyukhin M G ed. Boykova A V 1980 *Theory and calculation of the Kirovets tractor* (Leningrad: Mechanical Engineering) 208
[21] ASTU; comp.: Bachin O I and Zhigalov A M 2003 *Forest machines. Kinematic schemes of transmissions of vehicles of the forestry complex: methodological instructions for course and diploma design* (Arkhangelsk: AGTU) 65
[22] J Y Wong-3rd ed. 2001 *Theory of ground vehicles* 528
[23] Shelomov V B 2013 *The theory of motion of multipurpose tracked and wheeled vehicles. Traction calculation of curvilinear motion: a textbook for universities in the specialty "Automotive and tractor construction"* (St. Petersburg: Polytechnic Publishing House University) 90