Local Superfield Lagrangian BRST Quantization

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Abstract

A $\theta$-local formulation of superfield Lagrangian quantization in non-Abelian hypergauges is proposed on the basis of an extension of general reducible gauge theories to special superfield models with a Grassmann parameter $\theta$. We solve the problem of describing the quantum action and the gauge algebra of an $L$-stage-reducible superfield model in terms of a BRST charge for a formal dynamical system with first-class constraints of $(L + 1)$-stage reducibility. Starting from $\theta$-local functions of the quantum and gauge-fixing actions, with an essential use of Darboux coordinates on the antisymplectic manifold, we construct superfield generating functionals of Green’s functions, including the effective action. We present two superfield forms of BRST transformations, considered as $\theta$-shifts along vector fields defined by Hamiltonian-like systems constructed in terms of the quantum and gauge-fixing actions and an arbitrary $\theta$-local boson function, as well as in terms of corresponding fermion functionals, through Poisson brackets with opposite Grassmann parities. The gauge independence of the S-matrix is proved. The Ward identities are derived. Connection is established with the BV method, the multilevel Batalin–Tyutin formalism, as well as with the superfield quantization scheme of Lavrov, Moshin, and Reshetnyak, extended to the case of general coordinates.

1 Introduction

The construction of superfield counterparts of the Lagrangian \cite{1} and Hamiltonian \cite{2,3} quantization schemes for gauge theories on the basis of BRST symmetry \cite{4} has been covered in a number of papers \cite{5,6,7}. These works are based on nontrivial (represented by the operator $D = \partial_\theta + \theta \partial_t$, \([D, D]) = 2\partial_t$) and trivial relations between the even $t$ and odd $\theta$ components of supertime\textsuperscript{1).} In \cite{5,6,7}, the geometric interpretation \cite{10} of BRST transformations is realized by special translations in superspace, which originally provided a basis for the superspace description \cite{11} of quantum theories of Yang–Mills type.

It should be noted that superfield quantization is closely related to generalized Poisson sigma-models, used in \cite{12} to realize the concept of star-product within the deformation quantization of Poisson manifolds \cite{13}, described from a superfield geometric viewpoint in \cite{14} and developed algorithmically by Batalin and Marnelius in \cite{15}. The geometry of $D = 2$ supersymmetric sigma-models \cite{16} with an arbitrary, $N \geq 1$, number of Grassmann coordinates was adapted to the classical and quantum description of $D = 1$ sigma-models by Hull, and, independently, the introduction of $N = 2$ nilpotent parameters was applied to the construction of the partition function for a classical mechanics by Gozzi et al \cite{17}. Quantization with a single fermion supercharge, $Q(t, \theta)$, containing the BRST charge and the unitarizing Hamiltonian \cite{15}, was recently extended to $N = 2$ (non-spacetime) supersymmetries \cite{15}, and then, in \cite{19}, to the case of an arbitrary number of supercharges, $Q^k(t, \theta^1, \ldots, \theta^N)$, $k = 1, \ldots, N$, depending on Grassmann variables $\theta^k$. The superfield modification \cite{20} of the procedure \cite{5} reveals a close interplay between the quantum action of the Batalin–Vilkovisky (BV) method \cite{11} and the BRST charge of the Batalin–Fradkin–Vilkovisky (BFV) method \cite{2}. Finally, note that the superfield approach is used in the description of second-class constrained systems as gauge models \cite{21} as well as in the second quantization of gauge theories \cite{22}.

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\textsuperscript{1)} One of the first attempts to apply the concept of odd time to Lagrangian quantization was made in \cite{5}.
The Lagrangian superfield partition function of [5] is derived from the Hamiltonian partition function through functional integration over so-called Pfaffian ghosts and momenta. On the other hand, the quantization rules [6, 7] present a superfield modification of the BV formalism by including non-Abelian hypergauge [23]. The corresponding hypergauge functions enter a gauge-fixing action which obeys (following the ideas of [24]) the same generating equation that holds for the quantum action [6, 7], except that the first-order operator $V$ in this equation is replaced by the first-order operator $U$ (these operators are crucial ingredients of [6, 7] from the viewpoint of a superspace interpretation of BRST transformations).

The formalism [6, 7] provides a comparatively detailed analysis of superfield quantization (BRST invariance, S-matrix gauge-independence). This analysis is based on solutions to the generating equations; however, a detailed correspondence between these solutions and a gauge model is not established. To achieve a better understanding of the quantum properties based on solutions of the superfield generating equations, it is natural to equip the formalism [6, 7] with an explicit superfield description of gauge algebra structure functions that determine a given model. So far, this problem has remained unsolved. For instance, the definition of a classical action of superfields, $A^i(\theta) = A^i + \lambda^i \theta$, on a superspace with coordinates $(x^\mu, \theta)$, $\mu = 0, \ldots, D - 1$, as an integral of a nontrivial odd density, $\mathcal{L}(A(x, \theta), \partial_\mu A(x, \theta), \ldots; x, \theta) \equiv \mathcal{L}(x, \theta)$, is a problem for every given model. Here, by trivial densities $\mathcal{L}(x, \theta)$ we understand those of the form

$$\int d^Dx d\theta \mathcal{L}(x, \theta) = \int d\theta \theta S_0(A(\theta)) = S_0(A),$$

where $S_0(A)$ is a usual classical action.

A peculiar feature of the vacuum functional $Z$ and generating functional of Green’s functions $Z[\Phi^*]$ in the formalism [6, 7] is the dependence of the gauge fermion $\Psi[\Phi]$ and quantum action $S[\Phi, \Phi^*]$ on the components $\lambda^A$ of superfields $\Phi^A(\theta)$ in the multiplet $(\Phi^A, \Phi^A_\lambda)(\theta) = (\phi^A + \lambda^A \theta, \phi^A - \theta J_A)$, where $(\phi^A, \phi^A_\lambda, \lambda^A, J_A)$ are the complete set of variables of the BV method. Another peculiarity of [6, 7] is that, due to the manifest structure of $\Phi^A(\theta)$ and $Z[\Phi^*]$, an effective action $\Gamma$ with the standard Ward identity $(\Gamma, \Gamma) = 0$ in terms of a superantibracket [6] can be introduced by a Legendre transformation of $\ln Z[\Phi^*]$ with respect to $P_1(\theta) \Phi^*_A(\theta)$,$^2$ \begin{equation}
\Gamma[P_0(\Phi, \Phi^*)] = \frac{\hbar}{i} \ln Z[\Phi^*] + \partial_\theta \{ [P_1(\theta) \Phi^*_A(\theta)] \Phi^A(\theta) \}, \quad \Phi^A(\theta) = -\frac{\hbar}{i} \delta \ln Z[\Phi^*] \frac{\delta}{\delta (P_1(\theta) \Phi^*_A(\theta))}. \end{equation}

Although non-contradictory, such an introduction of $\Gamma$ violates the superfield content of the variables.$^3$

In this paper, we propose a local formalism of superfield Lagrangian quantization in which the quantities of an initial classical theory are realized in the framework of a $\theta$-local superfield model (LSM). The idea of LSM is to represent the objects of a gauge theory (classical action, generators of gauge transformations, etc.) in terms of $\theta$-local functions, trivially related to the spacetime coordinates, in the sense that (as compared to the formalism [5]) the derivatives with respect to the even $t$ and odd $\theta$ component of superfield are taken independently. Using an analogy with classical mechanics (or classical field theory), we reproduce the dynamics and gauge invariance (in particular, BRST transformations) of the initial theory (the one with $\theta = 0$) in terms of $\theta$-local equations, called Lagrangian and Hamiltonian systems (LS, HS) with a dynamical odd time $\theta$, which implies that this coordinate enters an LS or HS not as a parameter but as part of a differential operator $\partial_\theta$ that describes the $\theta$-evolution of a system.

On the basis of the suggested formalism, we circumvent the peculiarities of the functionals $Z$ and $Z[\Phi^*]$ in [6, 7] as well as solve the following problems:

1. We develop a dual description of an arbitrary reducible LSM of Ref. [24] in the case of irreducible gauge theories (with bosonic classical fields and parameters of gauge transformations), in terms of a BRST charge related to a (formal) dynamical system with first-class constraints of a higher stage of reducibility. By dual description, we understand such a treatment of a gauge model that interrelates the Lagrangian and Hamiltonian formulations (the latter understood in the sense of formal dynamical systems).

2. An HS constructed from $\theta$-local quantities, i.e., a quantum action, a gauge-fixing action, and an arbitrary bosonic function, encodes (through a $\theta$-local antibracket) both BRST and so-called anticanonical-like transformations, in terms of a universal set of equations underlying the gauge-independence of the

$^2$The objects $P_1(\theta)$ and $\delta/\delta (P_1(\theta) \Phi^*_A(\theta))$ are, respectively, an element of the system of projectors $\{ P_n(\theta) = \delta_{n0}(1 - \theta_0) + \delta_{n1}\theta_0, n = 0, 1 \}$, acting on the supermanifold with coordinates $(\Phi^A, \Phi^A_\lambda)$, and a superfield variational derivative with respect to $P_1(\theta) \Phi^*_A(\theta)$.

$^3$By violating the superfield content, we understand the fact that the derivative of $Z[\Phi^*]$, which defines the effective action through a Legendre transformation, is taken with respect to only one superfield component, namely, the $\theta$-component of $\Phi^*_A(\theta)$, so that the resulting effective action depends only on $\phi^A$ and $\phi^A_\lambda$, which can be formally expressed as $P_0(\theta) (\Phi^A, \Phi^A_\lambda)(\theta) = (\phi^A, \phi^A_\lambda)$. 

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S-matrix. This set of equations is generated, in terms of an even superfield Poisson bracket, by a linear combination of fermionic functionals corresponding to the mentioned $\theta$-local quantities, e.g., the quantum and gauge-fixing actions as well as the bosonic function.

3. For the first time in the framework of superfield approach, we introduce a superfield effective action (also in the case of non-Abelian hypergauges).

4. We extend the superfield quantization of Refs. [3, 4] to the case of general coordinates on the manifold of super(anti)fields and establish a relation with the proposed local quantization.

The paper is organized as follows. In Section 2, a Lagrangian formulation of an LSM is proposed as an extension of a usual model of classical fields $A^i$, $i = 1, \ldots, n = n_+ + n_-$, to a $\theta$-local theory, defined on the odd tangent bundle $T_{\text{odd}}M_{\text{CL}} \equiv \Pi T^*M_{\text{CL}} = \{A^i, \partial_0 A^i\}$, $I = 1, \ldots, N = N_+ + N_-$, $(n_+, n_-) \leq (N_+, N_-)$. The superfields $(A^i, \partial_0 A^i)$ are defined in a superspace $\mathcal{M} = \bar{\mathcal{M}} \times \bar{\mathcal{P}}$ parameterized by $(z^M, \theta)$, where the spacetime coordinates $z^M \in I$ include Lorentz vectors and spinors of the superspace $\mathcal{M}$. We investigate the superfield equations of motion, introduce the notions of reducible general and special superfield gauge theories and apply Noether’s first theorem to $\theta$-translations. Section 3 is devoted to the Hamiltonian formulation of an LSM on the odd cotangent bundle $T^*_{\text{odd}}M_{\text{CL}} \equiv \Pi^* T^*M_{\text{CL}} = \{A^i, A^i_\theta\}$. Here, we establish a connection to the Lagrangian formalism and investigate the existence of a Noether integral, related to $\theta$-translations, that leads to the validity of a $\theta$-local master equation. The quantization rules are given in Section 4. In particular, we construct the dual description of an LSM and define a generating functional of Green’s functions, $Z(\theta)$, and an effective action, $\Gamma(\theta)$, using an invariant description of super(anti)fields on a general antisymplectic manifold. An essential feature in introducing $Z(\theta)$ and $\Gamma(\theta)$ is a choice of Darboux coordinates $(\varphi, \varphi^\ast)(\theta)$ compatible with the properties of the quantum action. In Section 5, on the basis of a component form of the local superfield quantization, we establish its connection with the first-level formalism [24], with the BV method, and with an extension of the superfield scheme [0, 7]. In the Conclusion, we discuss the results of the present work.

In addition to DeWitt’s condensed notation [26], we partially use the conventions of Refs. [6, 7]. We distinguish between two types of superfield derivatives: the right (left) variational derivative $\delta_{\bar{\varphi}} F / \delta \Phi$ of a functional $F$, and the right (left) derivative $\partial_{\theta} F(\theta) / \partial \Phi(\theta)$ of a function $F(\theta)$ for a fixed $\theta$. Derivatives with respect to super(anti)fields and their components are understood as right (left), for instance, $\delta / \delta \Phi(\theta)$ or $\partial / \partial \lambda(\theta)$, while the corresponding left (right) derivatives are labelled by the subscript $l(r)$. For right-hand derivatives with respect to $A^i(\theta)$, with a fixed $\theta$, we use the notation $F_{,i}(\theta) = \partial F(\theta) / \partial A^i(\theta)$. The $\delta(\theta)$-function and integration over $\theta$ are given, respectively, by $\delta(\theta) = \theta$ and left-hand differentiation over $\theta$. We refer to a function $F(\theta)$, regarded as an element of the superalgebra $C^\infty(T_{\text{odd}}M_{\text{CL}})$, as a $C^\infty(T_{\text{odd}}M_{\text{CL}})$-function. The rank of an even $\theta$-local supermatrix $K(\theta)$ with $\mathbb{Z}_2$-grading $\varepsilon$ is characterized by a pair of numbers $\bar{m} = (m_+, m_-)$, where $m_+$ ($m_-$) is the rank of the Bose–Bose (Fermi–Fermi) block of the $\theta$-independent part of the supermatrix $K(\theta)$: rank$[K(\theta)] = \text{rank}[K(0)]$. With respect to the same Grassmann parity $\varepsilon$, we understand the dimension of a smooth supersurface, also characterized by a pair of numbers, in the sense of the definition [27] of a supermanifold, so that the above pair coincides with the corresponding numbers of the Bose and Fermi components of $z(0)$, being the $\theta$-independent parts of local coordinates $z^i(0)$ parameterizing this supersurface$^5$. On these pairs, we consider the operations of component addition, $\bar{m} + \bar{n} = (m_++n_+, m_-+n_-)$, and comparison,

$$m = \bar{n} \Leftrightarrow m_+ = n_+, \quad m > \bar{n} \Leftrightarrow (m_+ > n_+, m_- \geq n_-) \text{ or } (m_+ \geq n_-, m_+ > n_-).$$

## 2 Odd-time Lagrangian Formulation

The basic objects of the Lagrangian formulation of an LSM are a Lagrangian action $S_L$: $T_{\text{odd}}M_{\text{CL}} \times \{\theta\} \to \Lambda_1(\theta; \mathbb{R})$, being a $C^\infty(T_{\text{odd}}M_{\text{CL}})$-function taking values in a real Grassmann algebra $\Lambda_1(\theta; \mathbb{R})$, and (independently) a functional $Z[A]$, whose $\theta$-density is defined with accuracy up to an arbitrary function $f((A, \partial_0 A)(\theta), \varepsilon) \in \ker(\partial_\theta)$, $\varepsilon(f) = 0$,

$$Z[A] = \partial_0 S_L(\theta), \quad \varepsilon(Z) = \varepsilon(\theta) = (1, 0, 1), \quad \varepsilon(S_L) = 0.$$  

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$^5$All denotes the operation that changes the coordinates of a tangent fiber bundle $TM_{\text{CL}}$ over a configuration $A^i$ into the coordinates of the opposite Grassmann parity [23], and $N_+, N_-$ are the numbers of bosonic and fermionic fields, among which there may exist superfields corresponding to the ghosts of the minimal sector in the BV quantization scheme (in terms of the condensed notation [20] used in this paper).

$^6$IIn the infinite-dimensional case (which we preferably use in this paper) the concept of dimension has to be clarified. Thus, for the vector bundle $M_{\text{CL}} \to \mathcal{M}$, we formally understand that dim $M_{\text{CL}}$ is the dimension of a fiber $F^p_{\text{MC}}$ over an arbitrary point $p \in \mathcal{M}$. 

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The values $\varepsilon' = (\varepsilon_P, \varepsilon_J, \varepsilon)$, $\varepsilon = \varepsilon_P + \varepsilon_J$, of $Z_2$-grading, with the auxiliary components $\varepsilon_J, \varepsilon_P$ related to the respective coordinates $(z^M, \theta)$ of a superspace $\mathcal{M}$, are defined on superfields $A^I(\theta)$ by the relation $\bar{\varepsilon}(A^I) = ((\varepsilon_P) I, (\varepsilon_J) J, \varepsilon)$. Note that $\mathcal{M}$ may be realized as the quotient of a symmetry supergroup $J = J \times P$, $P = \exp(i \mu_0 p)$, for the functional $Z[\mathcal{A}]$, where $\mu$ and $p_0$ are, respectively, a nilpotent parameter and a generator of $\theta$-shifts, whereas $J$ is chosen as the spacetime SUSY group. The quantities $\varepsilon_J, \varepsilon_P$ are the respective Grassmann parities of the coordinates of representation spaces of the supergroups $J, P$. The introduced objects allow one to achieve a correct quantization of the spin-statistic relation into operator quantization.

Among the objects $S_L(\theta)$ and $Z[\mathcal{A}]$, invariant under the action of a $J$-superfield representation $T$ restricted to $J$, $T|_J$, it is only $S_L(\theta)$ that transforms nontrivially (in view of the $J$-scalar nature of $Z[\mathcal{A}]$) with respect to the total representation $T$ under $A^I(\theta) \to \bar{A}^I(\theta) = (T|_J A)^I(\theta - \mu)$,

$$\delta S_L(\theta) = S_L(A^I(\theta), \partial_0 A^I(\theta), \theta) - S_L(\theta) = -\mu \left[ \frac{\partial}{\partial \theta} + P_0(\theta)(\partial_0 U)(\theta) \right] S_L(\theta). \quad (3)$$

Here, we have introduced the nilpotent operator $(\partial_0 U)(\theta) = \partial_0 A^I(\theta) \partial_0 / \partial A^I(\theta) = [\partial_0, U(\theta)]_-, U(\theta) = P_1 A^I(\theta) \partial_0 / \partial A^I(\theta)$.

Assuming the existence of a critical superfield configuration for $Z[\mathcal{A}]$, one presents the dynamics of an LSM in terms of superfield Euler–Lagrange equations:

$$\frac{\delta Z[\mathcal{A}]}{\delta A^I(\theta)} = \left[ \frac{\partial}{\partial A^I(\theta)} \frac{\partial^2 S_L(\theta)}{\partial (\partial_0 A^I(\theta))} \right] S_L(\theta) = L^I_0(\theta) S_L(\theta) = 0, \quad (4)$$

equivalent, in view of $\partial^2_0 A^I(\theta) = 0$, to an LS characterized by $2N$ formally second-order differential equations in $\theta$,

$$\frac{\partial^2 A^I(\theta)}{\partial (\partial_0 A^I(\theta))^2} \equiv \partial^2_0 A^I(\theta)(S_L')_{IJ}(\theta) = 0,$n$$
$$\Theta_I(\theta) \equiv \frac{\partial_0 S_L(\theta)}{\partial A^I(\theta)} - (1)^{\varepsilon_I} \frac{\partial}{\partial (\partial_0 A^I(\theta))} \left( \frac{\partial}{\partial \theta} \frac{\partial S_L(\theta)}{\partial (\partial_0 A^I(\theta))} + (\partial_0 U)(\theta) \frac{\partial S_L(\theta)}{\partial (\partial_0 A^I(\theta))} \right) = 0. \quad (5)$$

The Lagrangian constraints $\Theta_I(\theta) = \Theta_I(A(\theta), \partial_0 A(\theta), \theta)$ restrict the setting of the Cauchy problem for the LS and may be functionally dependent, as first-order equations in $\theta$.

Provided that there exists (at least locally) a supersurface $\Sigma \subset \mathcal{M}_{CL}$, such that

$$\Theta_I(\theta)|_\Sigma = 0, \quad \text{dim} \Sigma = \overline{M}, \quad \text{rank} \| L^I_0(\theta) [L^I_0(\theta) S_L(\theta) (1)^{\varepsilon_I}] \|_\Sigma = \overline{N} - \overline{M}, \quad (6)$$

there exist $M = M_+ + M_-$ independent identities:

$$\int d\theta \frac{\delta Z[\mathcal{A}]}{\delta A^I(\theta)} \hat{R}^I_{A_0}(\theta; \theta_0) = 0, \quad \hat{R}^I_{A_0}(\theta; \theta_0) = \sum_{k \geq 0} \left( (\partial_0)^k \delta(\theta - \theta_0) \right) \hat{R}^I_{A_0}(A(\theta), \partial_0 A(\theta), \theta). \quad (7)$$

The generators $\hat{R}^I_{A_0}(\theta; \theta_0)$ of general gauge transformations,

$$\delta g A^I(\theta) = \int d\theta_0 \hat{R}^I_{A_0}(\theta; \theta_0) \xi^A_0(\theta_0), \quad \xi^A_0(\xi^A_0) = \varepsilon_0, \quad A_0 = 1, \ldots, \quad M_0 = M_0_+ + M_0_-, \quad (\theta, \xi^A_0) \in \Sigma$$

that leave $Z[\mathcal{A}]$ invariant, are functionally dependent under the assumption of locality and $\bar{J}$-covariance, provided that

$$\text{rank} \left\| \sum_{k \geq 0} \hat{R}^I_{A_0}(\theta) (\partial_0)^k \right\|_\Sigma = \overline{M} < \overline{M}_0.$$
As a result, the relations of dependence for eigenvectors that define a general $L_g$-stage reducible LSM are given by

$$\int d\theta' \hat{Z}^{A_{s-2}}_{A_{s-1}}(\theta_{s-2}; \theta') \hat{Z}^{A_{s-1}}_{A_s}(\theta'; \theta_s) = \int d\theta' \Theta_j(\theta') \mathcal{L}^{A_{s-2}J}_{A_{s-1}}((A, \partial_0 A)(\theta_{s-2}), \theta_{s-2}, \theta'; \theta_s),$$

for $s = 1, \ldots, L_g$, $A_s = 1, \ldots, M_s = M_{s+} + M_{s-}$, $\mathcal{M} = \mathcal{M}_{s-1}$. For $L_g = 0$, the LSM is an irreducible general gauge theory.

In case an LSM admits the form $S_{\bar{L}}(\theta) = T(\partial_0 A(\theta)) - S(A(\theta), \theta)$, the functions $\Theta_j(\theta)$ are given in the extended configuration space $\mathcal{M}_{CL} \times \{\theta\}$ by the relations

$$\Theta_j(\theta) = -\delta \mathcal{L}_{ij} \delta A_i(\theta, \theta)(-1)^{\varepsilon_j} = 0,$$

being the extremals of the functional $S_0(A) = S(A(0), 0)$, corresponding to $\theta = 0$. Condition $\mathfrak{b}$ and identities $\mathfrak{a}$ take the usual form (in case $\theta = 0$)

$$\text{rank} \|S_{j,i}(A(\theta), \theta)\|_{\Sigma} = \mathcal{M} - \mathcal{M}, \quad S_{j,i}(A(\theta), \theta) \mathcal{R}_{A_0} \epsilon^I_{A_0} (A(\theta), \theta) = 0,$$

with linearly-dependent ($\mathcal{M}_0 > \mathcal{M}$) generators of special gauge transformations,

$$\delta \mathcal{L}^I(\theta) = \mathcal{R}_{A_0} \epsilon^I_{A_0} (A(\theta), \theta) \mathcal{L}^I_{A_0}(\theta),$$

that leave invariant only $S(\theta)$, in contrast to $T(\theta)$. The dependence of generators $\mathcal{R}_{A_0} \epsilon^I_{A_0} (\theta)$, as well as of their zero-eigenvalue eigenvectors $\hat{Z}^{A_0}_{A_1}(A(\theta), \theta)$, and so on, can also be expressed by special relations of reducibility for $s = 1, \ldots, L_g$, namely,

$$Z_{A_{s-2}}^{A_{s-1}}(A(\theta), \theta) Z_{A_{s-1}}^{A_{s}}(A(\theta), \theta) = S_{j,i}(A(\theta), \theta) \mathcal{L}_{A_{s-2}}^{A_{s-1}J}(A(\theta), \theta), \quad \varepsilon(Z_{A_{s-1}}^{A_{s}}) = \varepsilon_{s} - \varepsilon_{A_{s-1}},$$

$$Z_{A_0}^{A_{s-1}}(\theta) \equiv \mathcal{R}_{A_0} \epsilon^{A_{s-1}}(\theta), \quad \mathcal{L}_{A_1}^{A_{s-1}}(\theta) = K^{A_{s-1}}_{A_1}(\theta) = (-1)^{\varepsilon_{j+1}} K^{A_{s-1}A_1}(\theta).$$

For $\mathcal{M}_{L_g} = \sum_{k=0}^{L_g} (-1)^k \mathcal{M}_{L_g-k-1} = \text{rank} \|Z_{A_{s}}^{A_{s-1}}\|_{\Sigma}$, relations $\mathfrak{c}$–$\mathfrak{e}$ determine a special gauge theory of $L_g$-stage reducibility. The gauge algebra of such a theory is $\theta$-locally embedded into the gauge algebra of a general gauge theory with the functional $Z[A] = \partial_0(T(\theta) - S(\theta))$, which implies the relation between the eigenvectors

$$\hat{Z}^{A_{s-1}}_{A_s}(A(\theta_{s-1}), \theta_{s-1}; \theta_s) = -\delta(\theta_{s-1} - \theta_0) Z_{A_{s-1}}^{A_{s-1}}(A(\theta_{s-1}), \theta_{s-1})$$

and the fact that the structure functions of the gauge algebra of a special gauge theory may depend on $\partial_0 A^I(\theta)$ only parametrically. Note that an extended (as compared to $\{A_0(\theta)\}$, $a = 0, 1$) system of projectors onto $C^\infty(T_{odd}\mathcal{M}_{CL}) \times \{\theta\}$, $\{p_0(\theta), \theta \partial \theta, \partial U(\theta)\}$, selects from $\mathfrak{f}$ two kinds of gauge algebra: one with structure equations and functions $S(A(\theta))$, $Z_{A_{s-1}}^{A_{s-1}}(A(\theta))$ not depending on $\theta$ in an explicit form; another with the standard relations for the gauge algebra of a reducible model with quantities $S_0(A)$, $Z_{A_{s-1}}^{A_{s-1}}(A)$, in case $\theta = 0$, $(\varepsilon_{p})_{\bar{L}} = (\varepsilon_{p})_{A_0} = 0$, $s = 1, \ldots, L_g$, and under the assumption of completeness of the reduced generators $\mathcal{R}_{A_0} \epsilon^I_{A_0} (\theta)$ and eigenvectors $Z_{A_{s-1}}^{A_{s-1}}(A(\theta))$; see Subsection 4.1.

An extension of a usual field theory to a $\theta$-local LSM permits one to apply Noether’s first theorem $\mathfrak{h}$ to the invariance of the density $d\theta S_{\bar{L}}(\theta)$ with respect to global $\theta$-translations as symmetry transformations of the superfields $A^I(\theta)$ and coordinates $(z^M, \theta)$, $(A^I, z^M, \theta) \rightarrow (A^I, z^M, \theta + \mu)$. By direct verification, one establishes that the function

$$S_E((A, \partial_0 A)(\theta), \theta) \equiv \frac{\partial S_{\bar{L}}(\theta)}{\partial (\partial_0 A^I(\theta))} \partial_0 A^I(\theta) - S_{\bar{L}}(\theta)$$

(13)
is an LS integral of motion, i.e., a conserved quantity under the \( \theta \)-evolution, in case there holds the equation

\[
\frac{\partial}{\partial \theta} S_L(\theta) + 2(\partial_\theta U)(\theta) S_L(\theta) \bigg|_{\theta=0} = 0. \tag{14}
\]

In contrast to its analogue in a \( t \)-local field theory, the energy \( E(t) \), the function \( S_E(\theta) \) is an LS integral also in the case of an explicit dependence on \( \theta \). This fact takes place in case \( S_L(\theta) \) admits the structure

\[
S_L((A, \partial_\theta A)(\theta), \theta) = S_0^L((A, \partial_\theta A)(\theta)) - 2\theta(\partial_\theta U)(\theta) S_0^L(\theta), \quad \varepsilon(S_0^L) = 0. \tag{15}
\]

### 3 Odd-time Hamiltonian Formulation

Independently, an LSM can be formulated in terms of a Hamiltonian action, being a \( C^\infty(T^*\mathcal{M}_{CL}) \)-function, \( S_H : T^*\mathcal{M}_{CL} \times \{ \theta \} \to \Gamma_1(\theta; \mathbb{R}) \), depending on superantifields \( \Lambda^*_\theta = (A^*_\theta - \theta J_1) \), included in the local coordinates of \( T^*\mathcal{M}_{CL} : \Gamma^R_{CL}(\theta) = (A^\prime, \Lambda^\prime)(\theta), \varepsilon(A^\prime) = \varepsilon(A^\prime) + (1, 0, 1) \). The equivalence of the Lagrangian and Hamiltonian formulations is implied by the nondegeneracy of the supermatrix \( \begin{bmatrix} (S^0_L)_{1,1}(\theta) \end{bmatrix} \) given by \( \mathcal{M} \), in the framework of a Legendre transformation of \( S_L(\theta) \) with respect to \( \partial^0_\theta A^\prime(\theta) \),

\[
S_H(\Gamma_{CL}(\theta), \theta) = A^\prime_1(\theta) \partial^0_\theta A^\prime(\theta) - S_L(\theta), \quad A^\prime_1(\theta) = \frac{\partial S_L(\theta)}{\partial (\partial^0_\theta A^\prime(\theta))}, \tag{16}
\]

where \( S_H(\Gamma_{CL}(\theta), \theta) \) coincides with \( S_E(\theta) \) in terms of the \( T^*\mathcal{M}_{CL} \)-coordinates.

The dynamics of an LSM is given by a generalized Hamiltonian system of \( 3N \) first-order equations in \( \theta \), equivalent to the LS equations in \( \mathcal{M} \), and expressed through a \( \theta \)-local antibracket \( \langle \cdot, \cdot \rangle_\theta \), namely,

\[
\begin{align*}
\partial^0_\theta \Gamma^R_{CL}(\theta) &= (\Gamma^R_{CL}(\theta), S_H(\theta))_\theta, \quad \Theta^H_1(\Gamma_{CL}(\theta), \theta) = \Theta_1(A(\theta), \partial_\theta A(\Gamma_{CL}(\theta), \theta), \theta) = 0, \\
(\mathcal{F}_1(\theta), \mathcal{F}_2(\theta))_\theta &= \frac{\partial \mathcal{F}_1(\theta)}{\partial A^\prime(\theta)} \frac{\partial \mathcal{F}_2(\theta)}{\partial A^\prime(\theta)} - \frac{\partial \mathcal{F}_1(\theta)}{\partial A^\prime_1(\theta)} \frac{\partial \mathcal{F}_2(\theta)}{\partial A^\prime_1(\theta)} - \mathcal{F}_1(\theta) \mathcal{F}_2(\theta) \quad \mathcal{F}_1(\theta) \in C^\infty(I\Pi^*\mathcal{M}_{CL}), t = 1, 2
\end{align*} \tag{17}
\]

with Hamiltonian constraints \( \Theta^H_1(\Gamma_{CL}(\theta), \theta) = -\partial^0_\theta A^\prime(\theta) - S_{H, I}^{1}\theta) (\theta)^{-1} \).

Formula \( \mathcal{I}3 \) establishes the equivalence of an HS with a generalized HS, and hence with an LS in the corresponding formal [formal, in view of the degeneracy conditions \( \mathcal{I}6 \)] setting \( (\theta = 0, k = \text{CL}) \) of the Cauchy problem for integral curves \( A^\prime, \Gamma^R_k(\theta) \),

\[
\left(A^\prime, \partial^0_\theta A^\prime \right)(0) = (\mathcal{A}^\prime, \partial^0_\theta \mathcal{A}^\prime), \quad \Gamma^R_k(0) = (\mathcal{A}^\prime, \mathcal{A}^\prime), \quad \mathcal{A}^\prime = P_0 \frac{\partial S_L(\theta)}{\partial (\partial^0_\theta A^\prime(\theta))} (\mathcal{A}^\prime, \partial^0_\theta \mathcal{A}^\prime), \tag{19}
\]

where we have ignored the continuous part of \( I \). The equivalence between an HS and a generalized HS is valid due to the coincidence (mutual inclusion) of the corresponding sets of solutions. Indeed, the solutions of a generalized HS are included into those of an HS by construction, while the reverse is valid due to \( \mathcal{I}8 \).

The HS is defined through a variational problem for a functional identical to \( Z[\mathcal{A}] \),

\[
Z[\Gamma_k] = \int d\theta \left[ \frac{1}{2} \Gamma^P_k(\theta) \omega_{PQ}(\theta) \partial^0_\theta \Gamma^Q_k(\theta) - S_H(\Gamma_k(\theta), \theta) \right],
\omega^P_k(\theta) = \left( \Gamma^P_k(\theta), \Gamma^Q_k(\theta) \right)_\theta, \quad \omega^P(\theta) = \omega^P(Q(\theta)) = \delta^P Q. \tag{20}
\]

Definitions \( \mathcal{I}9 - \mathcal{I}11 \) remain the same for special gauge theories, while definitions \( \mathcal{I}7, \mathcal{I}8 \), in the case of general gauge theories of \( L_0 \)-stage reducibility, are transformed by the rule

\[
\hat{Z}_{s, s-1}^A(\Gamma_k(\theta_s-1), \theta_{s-1} ; \theta_s) = \hat{Z}_{s, s-1}^A(A(\theta_s-1), \partial_\theta A, A(\Gamma_k(\theta_s-1), \theta_{s-1}), \theta_s ; \theta_s) , \quad s = 0, \ldots, L_0. \tag{21}
\]

Eqs. \( \mathcal{I}1 \), transformations \( \mathcal{I}6 \) and their consequence \( \partial^0_\theta (S_L + S_H)(\theta) = 0 \) imply the invariance of \( S_L(\theta) \) under \( \theta \)-shifts along arbitrary solutions \( \hat{\Gamma}^P_k(\theta) \), or, equivalently, along an \( (\varepsilon, \varepsilon) \)-odd vector field \( Q(\theta) = \text{ad}S_H(\theta) \equiv (S_H(\theta), \cdot)_\theta \). Hence,

\[
\delta_\mu S_H(\theta)|_{\hat{\Gamma}^P_k(\theta)} = \mu \left[ \frac{\partial}{\partial \theta} \left( S_H(\theta) \right) - \left( S_H(\theta), S_H(\theta) \right)_\theta \right] = 0, \quad \delta_\mu S_H(\theta) = \mu \partial_\theta S_H(\theta). \tag{22}
\]
holds true, provided that $S_H(\theta)$ can be presented, according to (14), in the form

$$S_H(\Gamma_k(\theta), \theta) = S^0_H(\Gamma_k(\theta)) + \theta \left(S_H^0(\Gamma_k(\theta)), S^0_H(\Gamma_k(\theta))\right)_\theta,$$

(23)

where $(\partial_\theta U)(\theta)S_L(\theta) = 1/2 (S_H(\theta), S_H(\theta))_\theta$, and $S^0_H(\Gamma_k(\theta))$ is the Legendre transform of $S^0_L(\theta)$, defined by (15).

If $S_H(\theta)$ or $S_L(\theta)$ does not depend on $\theta$ explicitly, then eq. (22) or (14) implies the fulfilment of the equation $(S_H(\theta), S_H(\theta))_\theta = 0$, or $(\partial_\theta U)(\theta)S_L(\theta)|_{\lambda(\theta)} = 0$, which has no counterpart in a $t$-local field theory, and imposes the known condition (11) that $S_H(\theta)$ or $S_L(\theta)$ be proper, although for an LSM at the classical level. In this case, a $\theta$-superfield integrability of the HS in (17) is guaranteed by the standard properties of the antibracket, including the Jacobi identity:

$$\left(\partial_\theta^2 \Gamma^P_k(\theta)\right) = \frac{1}{2} \left(\Gamma^P_k(\theta), (S_H(\Gamma_k(\theta)), S_H(\Gamma_k(\theta)))_\theta\right) = 0.$$

(24)

This fact ensures the validity on $C^\infty(T^*_\text{odd} M_{\text{CL}} \times \{\theta\})$ of the $\theta$-translation formula

$$\delta_\mu \mathcal{F}(\theta)|_{\Gamma_k(\theta)} = \mu \left[\frac{\partial}{\partial \theta} - \text{ad}S_H(\theta)\right] \mathcal{F}(\theta) = \mu \delta^i(\theta) \mathcal{F}(\theta),$$

(25)

as well as the nilpotency of a BRST-like generator of $\theta$-shifts along $Q(\theta)$, $s^i(\theta)$.

Depending on the realization of additional properties of a gauge theory (see Section 4), we shall henceforth assume the validity of the equation

$$\Delta^k(\theta) S_H(\theta) = 0, \; \Delta^k(\theta) \equiv \frac{1}{2} (-1)^k (\Gamma^Q_k(\theta), \omega_{QP}^k(\theta)) \left(\Gamma^P_k(\theta), \left(\Gamma^Q_k(\theta), \cdot \right)_\theta\right) .$$

(26)

Eq. (26) is equivalent to a vanishing divergence of the vector field $Q(\theta)$, namely,

$$\text{div} \left(\partial^P_k \Gamma_k(\theta)|_{\Gamma_k(\theta)}\right) = -\frac{\partial}{\partial \Gamma^P_k(\theta)} \left(\partial^P_k \Gamma^P_k(\theta)|_{\Gamma_k(\theta)}\right) = 2 \Delta^k(\theta) S_H(\theta) = 0.$$

(27)

This condition is trivial for the symplectic counterpart of formula (24). The validity of the Hamiltonian master equation $(S_H(\theta), S_H(\theta))_\theta = 0$ in case $\frac{\partial}{\partial \theta} S_H(\theta) = 0$ justifies the interpretation of the equivalent equation in (14), for $\frac{\partial}{\partial \theta} S_L(\theta) = 0$, $(\partial_\theta U)(\theta)S_L(\theta)|_{\mathcal{L}_\theta^P(S_L)} = 0$, as a Lagrangian master equation.

### 4 Local Superfield Quantization

In order to set up the rules of local superfield quantization for a gauge model, we should first extract such a model from a general LSM. Then we should consider a procedure of constructing a quantum action for the restricted LSM, and, finally, investigate the possibility (inherent in the $\theta$-local approach) of a dual description of the LSM in terms of the quantities of the BFV formalism.

#### 4.1 Superfield Quantum Action in Initial Coordinates

In this subsection, we transform the reducibility relations of a restricted special LSM into a sequence of new gauge transformations for the ghost superfields of the minimal sector. Together with the gauge transformations of the classical superfields $A^i(\theta)$, extracted from $A^i(\theta)$, the new gauge transformations are translated into a Hamiltonian system related to the initial restricted HS. A requirement of superfield integrability for the resulting HS produces a deformation of the $\theta$-local Hamiltonian in powers of the ghosts and superantifields of the minimal sector, and leads to a quantum action, and, independently, to a gauge-fixing action (Subsection 4.3), subject to different $\theta$-local master equations.

Given the standard distribution of ghost number (11) for $\Gamma^P_k(\theta)$, $\text{gh}(A^i_j) = -1 - \text{gh}(A^i_j) = -1$, the choice $\text{gh}(\theta, \partial_\theta) = (-1, 1)$ implying the absence of ghosts among $A^i_j$, and, in particular, the relations $(\varepsilon P)_{ij} = 0$, the quantization rules consist, firstly, in restricting an LSM (in both Lagrangian and Hamiltonian formulations) by the equations

$$\left(\text{gh}, \frac{\partial}{\partial \theta}\right) S_{H(\text{L})}(\theta) = (0, 0).$$

(28)

\[6\] The notion of $\theta$-superfield integrability is introduced by analogy with the treatment of Ref. [18].
Given the existence of a potential term in $S_{H(L)}(\theta)$, $S(A(\theta),0) = S(A(\theta))$, and the absence in $S_{H(L)}(\theta)$ of a dimensional constants with a nonzero ghost number, solutions of eqs. select from an LSM a usual gauge model with a classical action $S_0(A)$ in which the fields $A'$ are extended to $A'(\theta)$. Then the generalized HS in (17) is transformed into a $\theta$-integrable system defined in $H_{*}A_{cl} = \{\Gamma_{cl}^{p}(\theta)\} = \{(A',A')'(\theta)\}$, with $\Theta_{H}(A(\theta)) = \Theta_{L}(A(\theta))$,

$$\frac{\partial}{\partial x} \Gamma_{cl}^{p}(\theta) = (\Gamma_{cl}^{p}(\theta), S_{0}(A(\theta)))_{\theta}, \quad \Theta_{H}(A(\theta)) = -(-1)^{\gamma} S_{0;\nu}(A(\theta)). \quad (29)$$

The restricted special gauge transformations $\delta A'(\theta) = \mathcal{R}_{\epsilon}(\theta \otimes A(\theta))$ of $\xi_{\alpha_{0}}(\theta)$, $\bar{\xi}(\xi_{\alpha_{0}}(\theta)) = \varepsilon_{\alpha_{0}}$, with the condition $(\varepsilon_{p})_{\alpha_{0}} = 0$, are embedded by the substitution $\xi_{\alpha_{0}}(\theta) = d\xi_{\alpha_{0}}(\theta) = C_{\alpha_{0}}(\theta) d\theta$, $\alpha_{0} = 1, ..., m_{0} = m_{0-} + m_{0+}$, into a Hamiltonian system with 2$n$ equations for unknown $\Gamma_{cl}^{p}(\theta)$, with the Hamiltonian $S_{cl}^{1}(\Gamma_{cl}, C_{0}) = (A_{L}', \mathcal{R}_{\alpha_{0}}(A_{L})C_{\alpha_{0}}(\theta))$. A union of this system with the HS in (29), extended to $2(n+m_{0})$ equations, has the form

$$\frac{\partial}{\partial x} \Gamma_{cl}^{p}_{[0]}(\theta) = (\Gamma_{cl}^{p}_{[0]}(\theta), S_{0;[1]}(\theta))_{\theta}, \quad \Gamma_{cl}^{p}_{[0]}(\theta), \quad \Gamma_{cl}^{p}_{[0]}(\theta) \equiv (\Gamma_{cl}^{p}, \Gamma_{cl}^{p}), \quad \Gamma_{0}^{p} \equiv (C_{\alpha_{0}}, C_{\alpha_{0}}). \quad (30)$$

By virtue of (11), the function $S_{1}^{n}(\theta)$ is invariant, modulo $S_{0;\nu}(\theta)$, under special gauge transformations of ghost superfields $C_{\alpha_{0}}(\theta)$, with arbitrary functions $\xi_{\alpha_{0}}(\theta)$, $(\varepsilon_{p})_{\alpha_{0}} = 0$, on the superspace $\mathcal{M}$:

$$\delta C_{\alpha_{0}}(\theta) = Z_{\alpha_{0}}(A(\theta))\xi_{\alpha_{0}}, \quad (\varepsilon, gh)\xi_{\alpha_{0}}(\theta) = (\varepsilon_{\alpha_{0}} + (1,0,1), 1). \quad (31)$$

Making the substitution $\xi_{\alpha_{0}}(\theta) = d\xi_{\alpha_{0}}(\theta) = C_{\alpha_{0}}(\theta) d\theta$, $\alpha_{0} = 1, ..., m_{1}$, and an enlargement of $m_{0}$ first-order equations in $\theta$, with respect to the unknowns $C_{\alpha_{0}}(\theta)$ in transformations (31), to an HS of $2m_{0}$ equations with the Hamiltonian $S_{1}^{1}(A_{L}, C_{0}, C_{1}) = (C_{\alpha_{0}}, Z_{\alpha_{0}}(A_{L})C_{\alpha_{0}}(\theta))$, we obtain a system of the form (30), written for $\frac{\partial}{\partial \Gamma_{cl}^{p_{0}}(\theta)}$. The enlargement of the union of the latter HS with eqs. (30) is formally identical to the system (30) under the replacement

$$(\Gamma_{cl}^{p_{0}}, S_{0;[1]}^{1}) \rightarrow (\Gamma_{cl}^{p_{1}}, S_{1;[1]}^{1}): \quad \Gamma_{cl}^{p_{1}} = (\Gamma_{cl}^{p_{0}}, \Gamma_{cl}^{p_{1}}), \quad \Gamma_{cl}^{p_{1}} = (C_{\alpha_{0}}, C_{\alpha_{0}}), \quad S_{1;[1]} = S_{0;[1]}^{1} + S_{1;[1]}^{1}.$$
is a solution of the master equation with accuracy up to $O(C^{\alpha_r})$, modulo $S_{0;i}(\theta)$. The integrability of the HS in \([33]\) is guaranteed by a double deformation of $S_{\text{min}}^\Psi(\theta)$: first in powers of $\Phi_k^a(\theta)$ and then in powers of $C^{\alpha_r}(\theta)$, in the framework of the existence theorem \([29]\) for the classical master equation in the minimal sector:

\[
(S_{H;k}(\Gamma_k(\theta)), S_{H;k}(\Gamma_k(\theta)))_\theta = 0, \quad \left(\varepsilon, \text{gh}, \frac{\partial}{\partial \theta}\right) S_{H;k}(\Gamma_k(\theta)) = \left(\tilde{o}_k, 0, 0\right), \quad k = \text{min}.
\] (35)

The proposed superfield algorithm for constructing the function $S_{H,\text{min}}(\theta)$ may be considered as a superfield version of the Koszul–Tate complex resolution \([30]\).

Let us consider an extension of $S_{H,\text{min}}(\theta)$ to $S_{H;k}(\theta) = S_{H,\text{min}}(\theta) + \sum_{s=0}^{L_s} \sum_{s'=0}^{s'} (C^{\alpha}_{s',k} B^{\alpha}_{s',k})(\theta)$, being a proper solution \([1]\) in $\Pi^T, \hbar$ constant with the pyramids of ghosts and Nakanishi–Lautrup superfields, and with a deformation in the Planck constant $\hbar$. Then $S_{H;k}(\theta)$ determines the quantum action $S^\Psi_H(\Gamma(\theta), \hbar)$, e.g., in case of an Abelian hypergauged defined as an anticanonical phase transformation:

\[
\Gamma_k(\theta) \rightarrow \Gamma_k(\theta) = \left(\Phi^A_k(\theta), \Phi^*_{k}(\theta) - \frac{\partial \Psi(\Phi(\theta))}{\partial \Phi^A_k(\theta)}\right), \quad S^\Psi_H(\Gamma(\theta), \hbar) = e^{\text{ad}_{\Phi}} S_{H;k}(\Gamma_k(\theta), \hbar).
\] (36)

The functions $(S^\Psi_H, S_{H;k})(\theta, \hbar)$ obey eqs. \([20], \[55]\) in case the $\hbar$-deformation of $S_{H,\text{min}}(\theta)$ is a solution of these equations. It is known that this choice of equations ensures the integrability of a non-equivalent HS constructed from $S^\Psi_H$, $S_{H;k}$, as well as the anticanonical preserving the volume element $dV_k(\theta) = \prod_{p} d\Gamma_{kp}(\theta)$ nature of this change of variables, corresponding to a $\theta$-shift by a constant parameter $\alpha$ along the corresponding HS solutions. In its turn, the quantum master equation

\[
\Delta^k(\theta) \exp \left[\frac{i}{\hbar} E(\theta, \hbar)\right] = 0, \quad E \in \{S_H^\Psi, S_{H;k}\}
\] (37)

determines a non-integrable HS, with the respective anticanonical change of variables preserving $dV_k(\theta) = \exp \left[(i/\hbar) E(\theta, \hbar)\right] dV_k(\theta)$. It is the latter nonintegrable HS with the Hamiltonian $S^\Psi_H(\theta, \hbar)$ that is crucial, for $\theta = 0$, in the BV formalism. This HS defines in $\Pi^T, \theta$-local, but not nilpotent, generator of BRST transformations, $\hat{s}'(\Psi)(\theta)$, which is associated with its $\theta$-nonintegrable consequence:

\[
\frac{\partial}{\partial \theta} \left(\Phi^A_k, \Phi^*_{k}(\theta)\right) = \left(\left(\partial \Psi(\Phi(\theta), S^\Psi_H(\theta, \hbar))_\theta, 0\right), \hat{s}'(\Psi)(\theta) = \frac{\partial}{\partial \theta} + \frac{\partial S^\Psi_H(\theta, \hbar)}{\partial \Phi^A_k(\theta)} \frac{\partial}{\partial \Phi^*_{k}(\theta)}.
\] (38)

### 4.2 Duality between the BV and BFV Superfield Quantities

An embedding of a restricted LSM gauge algebra, described by $S_{H,\text{min}}(\theta)$ and eq. \([55]\), into the gauge algebra of a general gauge theory in the Lagrangian formalism, see eqs. \([7, 12]\), can be effectively realized by means of dual functional counterparts, with the opposite $(\varepsilon, \varepsilon)$-parity, of the action and antibracket, by analogy with the approach of Refs. \([11, 20]\). To this end, let us consider the functional

\[
Z_k[\Gamma_k] = -\partial_k S_{H;k}(\theta), \quad \left(\varepsilon, \text{gh}\right) Z_k = \left((1, 0, 1, 1)\right)
\]
on the supermanifold $\Pi^T(\Pi^T, \theta)$ with natural $(\varepsilon, \varepsilon)$-even, symplectic, and $(\varepsilon, \varepsilon)$-odd Poisson structures. These structures define an $(\varepsilon, \varepsilon)$-even functional $\{\cdot, \cdot\}$ with canonical pairs $\{(\Phi^A_k, \Phi^*_{k}(\theta), (\partial \Phi^A_k, \Phi^*_{k}(\theta), (\cdot, \cdot)_{\theta})\}$ Poisson brackets. The latter act on the superalgebra $C^\infty(\Pi^T(\Pi^T, \theta))$ and provide a lifting of the antibracket $(\cdot, \cdot)_{\theta}$ defined on $\Pi^T$. For arbitrary functionals $F_1, F_2 = \partial_k F_1 ((\Gamma_k, \partial_k \Gamma_k)(\theta, \theta), \theta = 1, 2$, one has the following correspondence between the Poisson brackets of opposite Grassmann grading:

\[
\{F_1, F_2\} = \int d\theta \left[\frac{\delta F_1}{\delta \Phi^A_k(\theta)} \frac{\delta F_2}{\delta \Phi^*_{k}(\theta)} - \frac{\delta_r F_1}{\delta \Phi^A_k(\theta)} \frac{\delta_{\theta} F_2}{\delta \Phi^*_{k}(\theta)}\right] = \int d\theta (F_1(\theta), F_2(\theta))^{(\Gamma_k, \partial_k \Gamma_k)}_{\theta} = \left[\left(\mathcal{L}_{\Phi_k} F_1, \mathcal{L}^* \Phi^A_k F_2 - \mathcal{L}^* \Phi^A_k F_1 \mathcal{L}^\dagger_{\Phi_k} F_2\right)(\theta), \right.
\] (39)
where the Euler–Lagrange superfield derivative, e.g., with respect to \( \Phi_{A_k}(\theta) \), for a fixed \( \theta \), has the form
\[
\mathcal{L}^{A_k} = \frac{\partial}{\partial \Phi_{A_k}(\theta)} \frac{\partial}{\partial (\partial \Phi_{A_k}(\theta))} \cdot \delta \Phi_{A_k}(\theta) \cdot (\partial \Phi_{A_k}(\theta)).
\]

By construction, the functional \( Z_k \) is nilpotent:

\[
\{ Z_k, Z_k \} = \int d\theta (S_{H,k}(\theta), S_{H,k}(\theta))_\theta = 0, \quad k = \text{min},
\]

and, due to the absence of the additional time coordinate, is formally related to the BRST charge of a dynamical system with first-class constraints \([2]\). Indeed, after identifying the fields \((\Gamma_k, \partial \Gamma_k)(0)\) with the phase-space coordinates of the minimal sector, canonical with respect to the \((\varepsilon, \varepsilon)\)-even brackets in the framework of the BFV method \([2]\) for first-class constrained systems of \((L + 1)\)-stage reducibility,

\[
(q^i, p_i) = (A^i, \partial \Phi A^i)(0), \quad (C^{A_s}, \mathcal{P}_{A_s}) = \left( (\partial \delta C^{\alpha_{s-1}}, C^{\alpha_s}), (C^{\alpha_{s-1}}, \partial \Phi C^{\alpha_s}) \right)(0),
\]

\[
A_s = (\alpha_{s-1}, \alpha_s), \quad s = 0, \ldots, L, \quad \left( C^{A_{L+1}}, \mathcal{P}_{A_{L+1}} \right) = (\partial \delta C^{\alpha_L}, C^{\alpha_L})(0),
\]

the functional \( Z_k \) takes the form

\[
Z_k[\Gamma_k] = T_{A_0}(q, p) C^{A_0} + \sum_{s=1}^{L+1} \mathcal{P}_{A_{s-1}} Z_{A_s}^{A_{s-1}}(q) C^{A_s} + O(C^2).
\]

With allowance for the gauge algebra structure functions of the original \( L \)-stage-reducible restricted LSM described by the enhanced eqs. \([11]\), the constraints \( T_{A_0}(q, p) \) and the set of \((L + 1)\)-stage-reducible eigenvectors \( Z_{A_s}^{A_{s-1}}(q) \) are defined by the relations (the symbol \( T \) below stands for transposition)

\[
T_{A_0}(q, p) = \left( S_{0,q}(q), -p_i \mathcal{R}_{0,q}(q) \right), \quad Z_{A_s}^{A_{s-1}}(q) = \text{diag} \left( Z_{\alpha_{s-2}}, Z_{\alpha_{s-1}} \right)(q),
\]

\[
s = 1, \ldots, L, \quad \left( Z_{A_{L+1}}^{A_{L}} \right) = (Z_{\alpha_{L-1}}, 0) (q),
\]

\[
Z_{A_s-1}^{A_{s-1}} = T_{B_0} L_{A_{s-2}}^{-\alpha_0} T_{B_0}, \quad s = 1, \ldots, L + 1, \quad Z_{A_0} = T_{A_0}, \quad L_{A_s}^{-\alpha_0} = 0,
\]

\[
L_{A_s}^{-\alpha_0} = \text{diag} \left( L_{\alpha_{s-2}}, L_{\alpha_{s-1}} \right), \quad L_{\alpha_{s}} = L_{\alpha_{s+1}} = 0, \quad L_{\alpha_{s-1}}(q, p) = (-1)^{s+1} p_i K_{\alpha_i}^{A_s}(q, p).
\]

Relations \([40] - [44]\) generalize, to the case of arbitrary reducible theories, the results of Ref. \([20]\) concerning a dual description (for \( \varepsilon = \varepsilon_{\alpha_0} = L = 0 \)) of the quantum action and classical master equation in terms of a nilpotent BRST charge.

By the rule \([11]\), the variables \( (C_{s', \alpha}, B_{s', \alpha}, F_{s', \alpha})(\theta) \) are identical to the respective ghost momenta \( \mathcal{P}_{s', A_s} \), Lagrangian multipliers \( \lambda_{s', A_s} \), and their conjugate momenta \( \pi_{s', \alpha} \) in \([2]\). Then a comparison of the superfields \( C_{s'}^\alpha(\theta) \), \( s' = 0, \ldots, s \), selected from the non-minimal configuration space of an \( L \)-stage-reducible LSM, with the coordinates \( C_{s'}^\alpha \) selected from the non-minimal phase space of the corresponding \((L + 1)\)-stage-reducible dynamical system \([2]\) demonstrates the only possible embedding of \( \Pi(\Pi^* M_{\text{ext}}) \) into the phase space of the BFV method. Indeed, for the coordinates \( C_0^{A_{L+1}}, \text{gh}(C_0^{A_{L+1}}) = -L - 2 \), there exists no pre-image among \( (C_{s'}^\alpha, \partial \Phi C_{s'}^\alpha)(0) \), because the ghost number spectrum for the latter variables is bounded from below:

\[
\text{min gh}(C_{s'}^\alpha, \partial \Phi C_{s'}^\alpha) = \text{gh}(C_0^{A_L}) = -L - 1.
\]

As a consequence, the nilpotent functional \( Z_k[\Gamma_k] = -\partial \delta S_{H,k}(\theta), k = \text{ext}, \) is embedded into the total BRST charge constructed by the prescription of Ref. \([2]\).

It should be noted that the systems constructed with respect to the Hamiltonians \( S_H^{\beta_k}(\Gamma(\theta), \hbar) \) and \( S_{H,k}(\theta), k = \text{min, ext}, \) are equivalently described by dual fermion functionals \( Z_k[\Gamma_k] \) and \( Z^\Psi[\Gamma] = -\partial \delta S_H^{\beta_k}(\Gamma(\theta), \hbar) \), in terms of even Poisson brackets, for instance,

\[
\partial^\Psi \Gamma^p(\theta) = (\Gamma^p(\theta), S_{H,k}^{\Psi}(\Gamma(\theta), \hbar)) \theta = -\{ \Gamma^p(\theta), Z^\Psi[\Gamma] \}.
\]

Thereby, BRST transformations in the Lagrangian formalism with Abelian hypergauge can be encoded by a formal BRST charge, \( Z^\Psi[\Gamma] \), related to \( Z_k[\Gamma_k], k = \text{ext}, \) by a phase canonical transformation with the \((\varepsilon, \varepsilon)\)-even phase \( F^\Psi[\Phi] = \partial \delta \Psi(\Phi(\theta)) \),

\[
Z^\Psi[\Gamma] = e^{\Phi^\Psi F^\Psi} Z_k[\Gamma_k], \quad \text{ad} F^\Psi \equiv \{ F^\Psi, \cdot \}.
\]
On the assumption that an additional gauge invariance does not appear in deriving the restricted LSM from the initial general gauge theory, i.e., \( \overline{m}_s \leq \overline{m}_s \), and, therefore, \( L \leq L_g \), cf. footnote 7, the problem of including the restricted LSM gauge algebra into the initial gauge algebra, defined by \( (2), (7), (8), = 1 \) and

On the assumption that an additional gauge invariance does not appear in deriving the restricted LSM path integral, for a fixed \( \theta \)

\[ \text{coordinates } \Gamma \]

\[ \text{with the standard ghost number only in the sector of } (\Phi^{A_{\text{MIN}}}, \Phi^{*}_{A_{\text{MIN}}}) \text{, for } gh(\Phi^{I}, C^{A_{\ast}}) = (0, 1 + s) \text{, and having the spectrum } \]

\[ gh_g (\Phi^{I}, C^{A_{\ast}}) = (0, 1 + s), \quad gh_g (\Phi^{A_{\text{MIN}}}) = -1 - gh_g (\Phi^{A_{\text{MIN}}}), \quad gh_g (\theta, \partial_{\theta}) = (0, 0). \]

Conditions (23), applied to \( S_{L;k}(\theta) \) in case \( (\varepsilon, \varepsilon) = (\varepsilon, \varepsilon) \), extract from \( \hat{Z}_k \) the functional \( Z_k \) in (24), so that the \( (\varepsilon, \varepsilon) \)-even \( \theta \)-density \( S_{L;k}(\theta) \) lifts the function \( S_{L;k}(\theta) \in C^\infty (\Pi^{*} \mathcal{M}_{\text{MIN}}) \text{ to the superalgebra } C^\infty (\Pi^{*} \mathcal{M}_{\text{MIN}}) \times \theta \). In general, \( S_{L;k}(\theta) \) does not obey the generalized master equation (46) with the antibracket (53) acting on \( C^\infty (\Pi^{*} \mathcal{M}_{\text{MIN}}) \times \theta \),

\[ (S_{L;k}(\theta), S_{L;k}(\theta))_{\theta}^{(\varepsilon, \varepsilon, \varepsilon)} = \hat{f} (((\Gamma_{k}, \partial_{\theta} \Gamma_{k})(\theta)), \hat{f}(\theta) \in \ker \{ \partial_{\theta} \}, k = \text{MIN}. \]}

\[ (\Delta^{\mathcal{N}}(\theta))_{\theta}^{\lambda, \lambda} = \frac{1}{2} (\varepsilon)^{N} (\varepsilon, \varepsilon) \rho - 1 \omega^{\mathcal{N}}(\theta) (\Gamma^{\rho}(\theta), \rho (\Gamma^{\rho}(\theta), \cdot )_{\rho}^{\mathcal{N}}). \]}

The definition of a generating functional of Green’s functions \( Z((\partial \varphi^{*}, \varphi^{*}, \partial \varphi, \varphi, \mathcal{J}))(\theta) \equiv Z(\theta) \) as a path integral, for a fixed \( \theta \), is possible, within perturbation theory, by introducing on \( \mathcal{N} \) the Darboux coordinates, \( \Gamma^{\rho}(\theta) = (\varphi^{a}, \varphi^{a}) \), in a vicinity of solutions of the equations \( \partial W(\theta) / \partial \Gamma^{\rho}(\theta) = 0 \), so that \( \rho = 1 \) and \( \omega^{\mathcal{N}}(\theta) = \text{antidiag}(-\delta^{\alpha}_{\beta}, \delta^{\alpha}_{\beta}) \). The function

\[ Z(\theta) = \int d\mu(\tilde{\Gamma}(\theta)) d\mathcal{A}(\theta) \exp \{ i / h \} \left[ W(\tilde{\Gamma}(\theta), h) + X ((\tilde{\varphi}^{*} \varphi^{*} - \varphi^{*} \Lambda^{*} \varphi^{*})(\theta), h) \big|_{\lambda^{*} = 0} \right] \]}

depends on an extended set of sources,

\[ (\partial \varphi^{*}, \partial \varphi^{a}_{\theta}, \partial \varphi^{a}_{\theta}, \mathcal{I}^{a}) = (J_{a}, \lambda^{b}, J_{0a} + I_{1a} \theta), \]

\[ (\varepsilon, gh) \partial \varphi^{a}_{\theta} = (\varepsilon, gh) \mathcal{I}^{a} + ((1, 0, 1), 1) = (\varepsilon, \lambda^{a}) \]

\[ \text{to the superfields } (\varphi^{*, \varphi^{a}_{\theta}, \Lambda^{a}})(\theta), \text{ where } \Lambda^{a}(\theta) = \lambda^{a} \]}

\[ G_{a}(\Gamma(\theta)), a = 1, ... , k = n + \sum_{r=0}^{L} (2r + 3) m_{r}, k_{+} + k_{+}, \]

\[ \text{rank } \| \partial G_{a}(\theta) / \partial \Gamma^{\rho}(\theta) \|_{\partial \theta^{r} = -1, l_{+} + l_{-} = k}. \]
The functions \( G_a(\Gamma(\theta)) \), \((\varepsilon, \text{gh}) G_a = (\varepsilon, \text{gh}) I_a \), determine a boundary condition for the gauge-fixing action, 
\[ X(\theta) = X((\Gamma, \Lambda, \Lambda^*)(\theta), \hbar) \]
\[ \partial_r X(\theta)/\partial \Lambda^a(\theta)|_{\Lambda^r=0} = G_a(\theta), \]
defined on the direct sum \( \mathcal{N}_\text{tot} = \mathcal{N} \oplus \Pi \mathcal{T}^* \mathcal{K} \) of the manifolds \( \mathcal{N} \) and \( \Pi \mathcal{T}^* \mathcal{K} = \{(\Lambda^a, \Lambda^a_0(\theta))\} \). Hypergauges in involution, \( (G_a(\theta), G_b(\theta)) \mathcal{N}^N = G_c(\theta) U_{ab}^c(\Gamma(\theta)) \), obey different types of unimodularity relations \[ \text{depending on a set of equations for which} \]
\[ (\cdot, \cdot)_0 = (\cdot, \cdot) \mathcal{N} + (\cdot, \cdot) \mathcal{K} \]
and the operator \( \Delta(\theta) = (\Delta^N + \Delta^C)(\theta) \), trivially lifted from \( \mathcal{N} \) to \( \mathcal{N}_\text{tot} \).

1. \( (E(\theta), E(\theta))_\theta = 0, \Delta(\theta)E(\theta) = 0 \); 2) \( \Delta(\theta) \exp \left[ \frac{i}{\hbar} E(\theta) \right] = 0, \ E \in \{W, X\} \).

The functions \( G_a(\theta) \), assumed to be solvable with respect to \( \varphi_0^a(\theta) \), determine a Lagrangian surface, \( \mathcal{Q} = \{(\varphi, \varphi^*, \Lambda(\theta)) \} \subset \mathcal{N}_\text{tot} \), on which the restriction \( \mathcal{X}(\theta)|_{\mathcal{Q}} \) is non-degenerate. Given this, integration over \( (\varphi^*, \Lambda(\theta)) \) in eq. \[ \text{determines a function, for} \]
\[ \partial_\theta \varphi^a = I_a = 0, \] whose restriction to the Lagrangian surface \( \{\varphi(\theta)\} \subset \mathcal{N} \) is also non-degenerate.

In the properties of \((W, X)(\theta)\), one can introduce an effective action \( \Gamma(\theta) \equiv \Gamma(\varphi, \varphi^*, \partial_\theta \varphi, I(\theta)) \) defined, in the usual manner, by means of a Legendre transformation of \( \ln Z(\theta) \) with respect to \( \partial_\theta \varphi^a(\theta) \),
\[ \Gamma(\theta) = \frac{\hbar}{i} \ln Z(\theta) + ((\partial_\theta \varphi^a) \varphi^a)(\theta), \varphi^a(\theta) = \frac{-\hbar}{i} \partial_\theta \ln Z(\theta) \].

The analysis of the properties of \((Z, \Gamma)(\theta)\) is based on the following \( \theta \)-nonintegrable Hamiltonian-like system, which contains an arbitrary \((\varepsilon_p, \varepsilon)\)-even \( C^\infty(\mathcal{N}_\text{tot}) \)-function, \( R(\theta) = R \left( (\tilde{\Gamma}, \Lambda, \Lambda^*)(\theta), \hbar \right) \), with a vanishing ghost number:
\[ \begin{array}{l}
\partial_\theta \tilde{\Gamma}^p(\theta) = -i \hbar T^{-1}(\theta) \left( \tilde{\Gamma}^p(\theta), T(\theta) R(\theta) \right) \bigg|_{\Lambda^r=0} , \\
\partial_\theta \Lambda^a(\theta) = -2i \hbar T^{-1}(\theta) \left( \Lambda^a(\theta), T(\theta) R(\theta) \right) \bigg|_{\Lambda^r=0} , \\
\partial_\theta (\varphi_0^a, \Lambda^a)(\theta) = 0 ,
\end{array} \]
where the function \( T \left( (\tilde{\Gamma}, \Lambda, \Lambda^*)(\theta), \hbar \right) \equiv T(\theta) \) has the form \( T(\theta) = \exp [(i/\hbar) (W - X)(\theta)] \). Let us enumerate the properties of \((Z, \Gamma)(\theta)\).

1. The integrand in \[ \text{is invariant, for} \]
\[ \partial_\theta \varphi^* = \partial_\theta \varphi = I = 0, \] with respect to the \textit{superfield BRST transformations}
\[ \tilde{\Gamma}_\text{tot}(\theta) = (\tilde{\Gamma}, \Lambda, \Lambda^*)(\theta) \rightarrow \left( \tilde{\Gamma}_\text{tot} + \delta_\mu \tilde{\Gamma}_\text{tot} \right)(\theta), \delta_\mu \tilde{\Gamma}_\text{tot}(\theta) = \left( \partial_\theta \tilde{\Gamma}_\text{tot} \right)|_{\tilde{\Gamma}_\text{tot} \mu}, \]
having the form of a \( \theta \)-shift by a constant parameter \( \mu \) along an arbitrary solution \( \tilde{\Gamma}_\text{tot}(\theta) \) of the system \[ \text{or, equivalently, along a vector field determined by the r.h.s. of} \]
\[ \text{for} \ R(\theta) = 1. \] Here, the arguments of \((W, X)(\theta)\) are the same as in definition \[ \text{The above statement can be verified with the help of the identities}
\[ \partial_r X(\theta)/\partial F(\theta)|_{\Lambda^r=0} = \partial_r (X(\theta)|_{\Lambda^r=0}/\partial F(\theta), F \in \{\Gamma^p, \Lambda^a\}). \]
Notice that the system \[ \text{for} \ R(\theta) = \text{const}, \] admits the integral \((W + X)(\theta)\) in case \( W \) and \( X \) obey the first system in \[ \text{2. The vacuum function} \ Z_X(\theta) \equiv Z(0, \varphi^*, 0, 0)(\theta) \text{is gauge-independent, namely, it does not change when} \]
\[ \text{is replaced by an} \ (X + \Delta X)(\theta) \text{subject to the same system in} \]
\[ \text{that holds for} \ X(\theta) \text{and conforming to nondegeneracy on the surface} \mathcal{Q}. \] Indeed, this hypothesis implies that the variation \( \Delta X(\theta) \) obeys a system of linearized equations with a nilpotent operator \( Q_j(X) \), \( j = 1, 2 \),
\[ Q_j(X) \Delta X(\theta) = 0, \delta_j \Delta(\theta) \Delta X(\theta) = 0 ; Q_j(X) = \text{ad} X(\theta) - \delta_j (i/\hbar \Delta(\theta)), \]
where \( j \) is identical to the number that labels that system in eqs. \[ \text{for which} \ X(\theta) \text{is a solution. Using the fact that solutions} \ X(\theta) \text{of every system in} \]
\[ \text{are proper, one can prove, by analogy with the theorems of Ref.} \]
\[ \text{that the cohomologies of the operator} \ Q_j(X) \text{on the functions} \ f(\Gamma_\text{tot}(\theta)) \in C^\infty(\mathcal{N}_\text{tot}) \text{vanishing for} \]
\[ \Gamma_\text{tot}(\theta) = 0 \text{are trivial. Hence, the general solution of eq.} \]
\[ \text{has the form}
\[ \Delta X(\theta) = Q_j(X) \Delta Y(\theta), \left( \varepsilon, \text{gh}, \frac{\partial}{\partial \theta} \right) \Delta Y(\theta) = ((1, 0, 1), -1, 0), \]
\[ \Delta Y(\theta)|_{\Gamma_\text{tot}=0} = 0. \]
with a certain $\Delta Y(\theta)$. Now, making in $Z_{X+aX}(\theta)$ a change of variables induced by a $\theta$-shift by a constant $\mu$, related to the system \[33\], and choosing

$$2R(\theta)\mu = \Delta Y(\theta),$$

we find that $Z_{X+aX}(\theta) = Z_X(\theta)$, and conclude that the S-matrix is gauge-independent\footnote{Properties 1, 2 of $Z_X(\theta)|_{\Lambda^*=0}$ are valid for arbitrary $\rho(\theta)$, $\Gamma^\nu(\theta)$ on the manifold $N$.} in view of the equivalence theorem \[32\].

The above proof shows, due to \[34\], that the system \[33\] encodes the BRST transformations for $R(\theta) = \text{const}$, as well as continuous anticanonical-like transformations in an infinitesimal form, with the scalar fermionic generating function $R(\theta)\mu$, where $R(\theta)$ is arbitrary and $\mu$ is constant.

Equivalently, following the ideas of Subsection 4.2, the above characteristics of the generating functional of Green's functions can be derived from a Hamiltonian-like system presented in terms of an even superfield Poisson bracket in general coordinates (see footnote 8),

$$\left\{ \partial_\theta \tilde{\Gamma}^\nu(\theta), Z^W[\tilde{\Gamma}] - (Z^X + i\hbar Z^R)[\tilde{\Gamma}_{\text{tot}}] \right\}_{\Lambda^*=0} = 0,$$

$$\left\{ \partial_\theta \Lambda^a(\theta), Z^W[\tilde{\Gamma}] - (Z^X + i\hbar Z^R)[\tilde{\Gamma}_{\text{tot}}] \right\}_{\Lambda^*=0} = 0,$$

with a linear combination of fermionic functionals related to the above actions and a bosonic function by the rule

$$Z^E[\tilde{\Gamma}_{\text{tot}}] = -\partial_\theta E(\Gamma_{\text{tot}}(\theta), \hbar), \quad E \in \{W, X, R\}.$$  

(57)

If the actions $(W, X)(\theta)$ obey the first system in \[34\], then the functionals $Z^W, Z^X$, formally playing the role of the usual and gauge-fixing BRST charges, are nilpotent with respect to the even Poisson bracket \{ , \} $\Pi^\mu(\cdot, \cdot)\Pi^\nu(\cdot, \cdot)$, and choosing, for instance, the first bracket in the sum is defined on arbitrary functionals on $\Pi^\mu \times \{\theta\}$, via a $\theta$-local extension of the odd bracket $(\cdot, \cdot)^\Pi^\mu$ in \[35\], as follows:

$$\{F_1, F_2\}^\Pi^\mu = \int d\theta \frac{\delta_{\theta} F_1}{\delta \Gamma^\nu(\theta)} \omega^\mu(\Gamma(\theta)) \frac{\delta_{\theta} F_2}{\delta \Gamma^\nu(\theta)} = \partial_\theta \{\Gamma(\theta), F_2(\theta)\}^\Pi^\mu, \quad \{F_1(\theta), F_2(\theta)\}^\Pi^\mu = (\ell^\mu_{\theta} F_1) \omega^\mu(\Gamma(\theta)) \ell^\mu_{\theta} F_2(\theta), \quad F_1(\Gamma) = \partial_\theta F_1(\Gamma, \partial_\theta \Gamma(\theta), \theta),$$

(58)

where $\ell_{\theta}(\cdot)$ is the left-hand Euler–Lagrange superfield derivative\footnote{The antibracket $(\cdot, \cdot)^\Pi^\mu$, identical for $N = \Pi^\mu M_k$, with $(\cdot, \cdot)^\nu(\Gamma_k, \partial_\theta \Gamma_k), k = \text{ext}$, in \[36\] lifts the operator $\Delta^N$ in \[37\] to the nilpotent operator $\Delta^\Pi^\mu$ acting in $\subset^\infty$ (Poisson brackets $\Pi^\mu \times \{\theta\}$), defined exactly as $\Delta^N(\theta)$, although in terms of the antibracket \[35\].} with respect to $\Gamma^\nu(\theta)$. Therefore, as in the case of the IS in \[38\], we arrive at an interpretation of BRST transformations, for a gauge theory with non-Abelian hypergauges in Lagrangian formalism, in terms of the formal “BRST charges” $Z^W, Z^X$, as well as in terms of the functional $Z^R$ and the even Poisson bracket\footnote{The construction of the latter bracket is different from that of \[36\], where an odd superfield Poisson bracket was derived from a $(t, \theta)$-local even bracket; however, it is similar to the construction of Ref. \[39\]; see eqs. (27).} 10.

The system \[33\] encodes the BRST transformations, for $Z^R = 0$, as well as the BRST and continuous anticanonical-like transformations with the bosonic generating functional $Z^R\mu$, for an arbitrary $Z^R$ and a constant $\mu$.

3. The functions $(Z, \Gamma(\theta))$ obey the Ward identities

$$\left\{ \left[ \partial_\theta \phi^*_{\text{a}}(\theta) - \left( \frac{\partial W}{\partial \phi_{\text{a}}(\theta)} \right) \left( i\hbar \frac{\partial}{\partial \phi_{\text{a}}(\theta)}, i\hbar \frac{\partial}{\partial \phi^*_{\text{a}}(\theta)} \right) \right] \partial_{\theta} \phi^*_{\text{a}}(\theta) + \frac{i}{\hbar} \Lambda_{\text{a}}(\theta) \partial_{\theta} \phi^*_{\text{a}}(\theta) \right\}_{\Lambda^*=0} = 0,$$

(60)

$$\Lambda_{\text{a}}(\theta) \partial_{\theta} \phi^*_{\text{a}}(\theta) \left. X \left( i\hbar \frac{\partial}{\partial \phi_{\text{a}}(\theta)}, i\hbar \frac{\partial}{\partial \phi^*_{\text{a}}(\theta)} - \phi^*, \frac{\hbar}{i} \frac{\partial}{\partial \phi_{\text{a}}(\theta)} \right) \right|_{\Lambda^*=0} = 0,$$

(61)

with the notation $\Gamma''_{\text{ab}}(\theta) = \frac{\partial}{\partial \phi_{\text{a}}(\theta)} \partial_{\theta} \phi^*_{\text{b}}(\theta)$, $\Gamma''_{\text{ab}}(\theta)(\Gamma''^{-1})_{\text{bc}}(\theta) = \delta_{\text{a}}^\text{b}$. In the symmetric form of these identities, we have extended the standard set of sources $\partial_{\theta} \phi^*_{\text{a}}(\theta)$ used in the definition of the generating functional of Green's functions in Abelian hypergauges.
The technique used in deriving the above identities is analogous to the corresponding procedure of Refs. [33, 34], applied, in the framework of the BV [1] and Batalin–Lavrov–Tyutin [31] methods, to the problem of gauge dependence in theories with composite fields. Thus, identities (60) and (61) follow from the corresponding system in (51) for (W, X)(θ). For instance, making the functional averaging of the second system in (51) for X(θ),

\[ \int d\Lambda(\theta)d\mu (\tilde{\Gamma}(\theta)) \exp \left[ \frac{i}{\hbar} (W - (\partial_\theta \varphi_\alpha^* \varphi_\alpha - \varphi_\alpha^* \partial_\theta \varphi_\alpha + \mathcal{I}_a \Lambda^a)(\theta) \right] \times \left\{ \Delta(\theta) \exp \left[ \frac{i}{\hbar} X ((\varphi, \varphi^* - \varphi^*, \Lambda, \Lambda^*))(\theta) \right) \right\} \right|_{\Lambda^* = 0} = 0, \]  

and integrating by parts in (52), with allowance for \((\partial/\partial \varphi^* + \partial/\partial \varphi^*) X(\theta) = 0\), we obtain identity (60). Identities (60) and (61) take the standard form in case \(\partial_\theta \varphi^a = \mathcal{I}_a(\theta) = \theta = 0\), which becomes more involved due to the quantities \((\partial_\theta W(\theta)/\partial \varphi^a(\theta))\), in the case of non-Abelian hypergauges.

In the special case of Abelian hypergauges, \(G_A ((\Phi, \Phi^*)(\theta)) = \Phi^a(\theta) - \partial \Psi(\Phi(\theta))/\partial \Phi^A(\theta) = 0\), corresponding to the change of variables \(50\), for \((\varphi, \varphi^*, W) = (\Phi, \Phi^*, S_{\text{H,ext}})\), \(\partial_\theta \Phi^A = \mathcal{I}_A = 0\) (locally, \(\mathcal{N} = \Pi^* \mathcal{M}_\text{ext}\)), the object \(Z(\partial_\theta \Phi^*, \Phi^*)(\theta)\) takes the form

\[ Z(\partial_\theta \Phi^*, \Phi^*)(\theta) = \int d\Phi(\theta) \exp \left\{ \frac{i}{\hbar} \left[ S^\Phi_H(\Gamma(\theta), \hbar) - ((\partial_\theta \Phi^A)(\Phi^A))(\theta) \right] \right\}. \]  

A \(\theta\)-local BRST transformation for \(Z(\partial_\theta \Phi^*, \Phi^*)(\theta)\) is given, for an HS defined on \(\Pi^* \mathcal{M}_\text{ext}\), with the Hamiltonian \(S^\Phi_H(\theta, \hbar)\) and a solution \(\tilde{\Gamma}(\theta)\), by the change of variables

\[ \Gamma^p(\theta) \rightarrow \Gamma^{(1)p}(\theta) = \exp \left[ \mu s^{(1)}(\Phi(\theta)) \right] \Gamma^p(\theta), \quad s^{(1)}(\Phi(\theta)) = \frac{\partial}{\partial \theta} - \text{ad} S^\Phi_H(\theta, \hbar). \]  

Transformation (64) with a constant \(\mu\) is anticanonical, with \(\text{Ber} \| \frac{\partial \Gamma^{(1)p}(\theta)}{\partial \theta} \| = \text{Ber} \| \frac{\partial \Phi^{(1)}(\theta)}{\partial \Phi^A} \| = 1\), provided that \(S^\Phi_H(\theta, \hbar)\) is subject to the first system in (51).

The obvious permutation rule of the functional integral, \(\varepsilon(\partial \Phi(\theta)) = 0\),

\[ \partial_\theta \int d\Phi(\theta) F((\Phi, \Phi^*)(\theta), \theta) = \int d\Phi(\theta) \left[ \frac{\partial}{\partial \theta} + (\partial_\theta V)(\theta) \right] F(\theta), \quad (\partial_\theta V)(\theta) = \partial_\theta \Phi^A(\theta) \frac{\partial}{\partial \Phi^A(\theta)}, \]  

yields, for \(i\hbar \partial_\theta^* \ln Z(\theta) = (\partial_\theta \Phi^A \partial_\theta^* \Phi^A(\theta) - \partial_\theta^* \Gamma(\theta)\), the following relations:

\[ \partial_\theta Z(\theta)|_{\Gamma(\theta)} = (\partial_\theta V)(\theta) Z(\theta) = 0, \quad \partial_\theta^* \Gamma(\theta)|_{\Gamma(\theta)} = (\Gamma(\Gamma(\theta)), \Gamma(\Gamma(\theta)))_\theta = 0. \]  

When deriving eqs. (65), we have taken into account the fact that the functional averaging of the HS with respect to \(Z(\theta)\) and \(\Gamma(\theta)\) has the form

\[ (\partial_\theta \Gamma^p)|_Z = \left( \frac{\hbar}{i} Z^{-1} \frac{\partial Z(\theta)}{\partial \Phi^A(\theta)}, -\partial_\theta \Phi^A(\theta) \right), \quad (\partial_\theta^* \Gamma^p) = ((\Gamma^p(\theta)), \Gamma((\Gamma(\theta))))_\theta = \partial_\theta^* (\Gamma^p), \]  

without the sign of average in (65) for \(\tilde{\Gamma}^p(\theta)\) and \(\Gamma^p(\theta)\). Expressions (65) relate the explicit form of the Ward identities in a theory with Abelian hypergauges to the invariance of the generating functional of Green’s functions with respect to the superfield BRST transformations.

## 5 Connection between Lagrangian Quantizations

The problem of establishing a correspondence between an LSM and a usual gauge theory can be solved on the basis of a component form of the local quantization in the following two ways: one is applicable to an arbitrary LSM, another applies to theories of Yang–Mills type. This makes it possible to establish a relation of the local superfield scheme with the known formulations of Lagrangian quantization [11, 20], as well as with an extension (proposed below) of the superfield method [6, 17] to the case of general coordinates.
5.1 Component Formulation and its Relation to Batalin–Vilkovisky, Batalin–Tyutin and Superfield Methods

The objects of $\theta$-local quantization in the Lagrangian and Hamiltonian formulations are related to the conventional description of a gauge theory by means of a component representation of the variables $\Gamma^{\mu}_{n}$, $\Gamma^{\mu}_{k}, \Lambda^{a}, I_{a}, \Gamma^{L,k}_{\theta}$, $k = \text{tot}$, under the restriction $\theta = 0$, for instance, $(M, N_{k}, \Lambda_{a}, I_{a}) \rightarrow (M, N_{k}, 0)$. Extracting a standard field model from a classical description of a general gauge theory can be effected, in addition to $\theta = 0$, by various kinds of eliminating the functions $\partial_{\theta} A^{I}(\theta)$, $A^{I}_{\theta}(\theta)$, as well as the superfields $A^{I}(\theta)$ that contain objects with an incorrect spin-statistics relation, $\varepsilon_{p}(A^{I}) \neq 0$. A possible way of such elimination is provided by the conditions $\varepsilon_{g}(A^{I}) = -1$ and $\varepsilon_{g}(A^{I}) = 0$, $(\varepsilon_{P})_{r} = 0$, and $(\varepsilon_{g}, \delta_{\theta}(\theta)) S_{\lambda}(\theta, \theta) = (0, 0)$, mentioned in Subsection 4.1. Another possibility is related to superfield BRST transformations for theories of Yang–Mills type [11 35 39], in which a Lagrangian classical action $S_{\text{LYM}}(\theta) = S_{\text{L}}(A, D_{U} A, \hat{A}, D_{U} \hat{A}) (\theta)$ is defined in terms of generalized Yang–Mills superfields, $A^{B}_{u}(z)$, $A^{u}_{B} = (A^{u}, C^{u})$, $u = 1, ..., r$, and matter superfields, $(z) = (\Psi^{g}, \overline{\Psi}^{g}, \varphi^{g}, \varphi^{g}_{+}) (z)$, where $\Psi^{g}$, $\overline{\Psi}^{g}$, $g$, $\varphi^{g}$, $\varphi^{g}_{+}$, $g, h = 1, ..., k_{2}$, are spinless ones. The superfields $A^{u}_{B}(z)$ and $(z)$ are defined on the superspace $M = \mathbb{R}^{1,3} \times \hat{P} = \{ z^{B} = (x^{\mu}, \theta) \}$ and take values, respectively, in the adjoint and vector representation spaces of an $r$-parametric Lie group. The action $S_{\text{LYM}}(\theta)$ can be written as

$$S_{\text{LYM}}(\theta) = \int d^{4} x \left[ \frac{1}{4} G_{B^{C}u}G^{C}^{Bu}(-1)^{e_{B}} - i\overline{\theta}^{\delta} g \nabla B_{\delta} \Psi^{g} - \nabla B_{\delta}^{h} A^{g}_{+} \nabla B_{\delta}^{h} \theta^{f} + M(\hat{A}) \right] (z),$$

with an $(z)$-local gauge-invariant polynomial $M(\hat{A})$, containing no derivatives over $z^{B}$. In expression [67], we have introduced the superfield strength $G_{B^{u}} = i[D_{B}, D_{u}]_{\theta} = \partial_{B} A^{C}_{u}(-1)^{e_{B} e_{C}} c_{A B}^{C} f_{u w v} A^{w}_{B} A^{v}_{C}$, $\partial_{B} = (\partial_{\mu}, \partial_{\theta})$ and the following covariant derivatives, expressed through the matrix elements of the Hermitian generators $\Gamma^{\mu} = \text{diag} \left( T^{A}, \overline{T}^{A}, \tau^{A}, \tau^{A} \right)$ of the corresponding Lie algebra:

$$(D_{B}^{u}, \nabla B_{\delta}^{e}, \nabla B_{\delta}^{e}, \nabla B_{\delta}^{e}) = \partial_{B} \left( \delta^{u}, \delta^{e}, \delta^{e}, \delta^{e} \right) + (f_{u v w}, -i(T^{w})^{e}_{e}, -i(w)^{e}_{f}, -i(w)^{e}_{g}) A^{w}_{B},$$

where the coupling constant is absorbed into the totally antisymmetric structure coefficients $f_{u v w}$. We have also used a generalization of Dirac’s matrices, $\gamma^{B} = (\gamma^{\mu}, \gamma^{g} \theta)$, $\gamma^{B} = (\gamma^{g} \theta)^{+} = \xi_{B}^{4}$, with a Grassmann scalar $\xi$, $(\varepsilon, \varepsilon^{g}) = ((1, 0, 1), -1)$. The $\varepsilon$-grading and ghost number are nonvanishing for the superfields $(\Psi, \overline{\Psi}, C^{u})$, namely, $(\varepsilon(\Psi, \overline{\Psi})), (0, 1, 1), (\varepsilon^{g}) = (1, 0, 1), (\varepsilon^{g}) = (0, 1, 0)$, $g = 1$. The functional $Z[A, \hat{A}] = \partial_{\theta} S_{\text{LYM}}(\theta)$ is invariant under the infinitesimal general gauge transformations

$$\delta_{\theta} A^{I}(\theta) = A^{I} A^{B^{u}} ; (z) = - \int d^{4} z_{0} \left( B^{u} v^{c}_{B} (-1)^{e_{B}} ; iD^{u} \hat{A}(z) (-1)^{c}(\hat{A}) \right) \delta(z(z_{0}) \xi^{u}(z_{0}),$$

with arbitrary bosonic $(\varepsilon_{A}, = 0)$ functions $\xi^{u}(z_{0})$ on $M$, and with functionally-independent generators $R^{u}_{A} (\theta, \theta) \equiv R^{u}_{A} (\theta)$. The condensed indices $I, A_{0}$ of the theory in question, $(I, A_{0}) = ((B, u, \delta, h) ; (h, x)) ; (v, x))$, conform to the relations, $\overrightarrow{N} \geq \overrightarrow{N}, \overrightarrow{M} = \overrightarrow{M}, (\overrightarrow{m}, \overrightarrow{M}) = (\overrightarrow{m}, \overrightarrow{M})$, provided that

$$(4r + 2k_{2}, 3k_{1}) ; (0, 0, 0),$$

which holds for a reduced theory with the action $S_{YM}(\theta) = S_{\text{LYM}}(\hat{A}, 0, 0, 0, 0, 0, 0, 0)$, in view of special horizontality conditions for the strength $G_{B^{u}}$ and certain subsidiary conditions for the matter superfields $A^{I}(\theta)$ in [11 35],

$$G_{B^{u}}(z) = G_{B^{u}}(z) ; \left( \nabla_{\delta}^{u} \Psi^{g}, \nabla_{\delta}^{u} \overline{\Psi}^{g}, \nabla_{\delta}^{u} \varphi^{g}, \nabla_{\delta}^{u} \varphi^{g}_{+} \right) (z) = (0, 0, 0),$$

To extract a standard component model defined on $M_{cl}(\theta) = 0$ from a Hamiltonian LSM, it is sufficient to eliminate, for $\theta = 0$, the antifields $A^{I}_{\theta}(\theta)$ of a theory of Yang–Mills type, by analogy with the prescription [70], i.e., by taking into account the relation between $A^{I}_{\theta}(\theta)$ and $\partial_{\theta} A^{I}(\theta)$: see Section 3 and the final remarks (item 1) of the Conclusion.

For the restricted LSM used in the Feynman rules of Section 4, the reduction to the model of the multilevel formalism of Ref. [23] is realized by the conditions

$$\theta = 0, \partial_{\theta} \varphi^{a}_{\alpha} = \partial_{\theta} \varphi^{a}_{\alpha} = \varphi^{a}_{\alpha} = I_{a} = 0.$$
In this case, the identification \((\rho, \omega^{pq})(\Gamma_0) = (M, E^{pq})(\Gamma_0)\) implies the coincidence of \((\cdot, \cdot)_{\theta}|_{\theta=0}\) and \(\Delta(0)\) with their counterparts of \(23\). Then the first-level functional integral \(Z^{(1)}\) and its symmetry transformations \(23\)

\[
Z^{(1)} = \int d\lambda_0 d\Gamma_0 M(\Gamma_0) \exp \left\{ \frac{i}{\hbar} (W(\Gamma_0) + G_a(\Gamma_0)\lambda_0^a) \right\},
\]

\[
\delta \Gamma_0^p = (\Gamma_0^p, -W + G_a \lambda_0^a)\mu,
\]

\[
\delta \lambda_0^a = (-U_{cb}^a \lambda_0^b(-1)^{\epsilon_c} + 2i\hbar V^a_\gamma \lambda_0^a + 2(i\hbar)^2 \tilde{G}_a^a)\mu,
\]

coincide (\(\lambda_0^a\) being replaced by the notation \(\pi^a\) of \(23\)), respectively, with \(Z_X(0)|_{\omega^a_0=0}\) and the BRST transformations \(\delta_\theta \Gamma_{\text{anot}}\) (having the opposite signs) generated by the system \(23\) for \(R(\theta) = 1\). This coincidence is guaranteed by the choice of \(X(\theta)\) in the form

\[
X(\theta) = \left\{ G_a(\Gamma)\Lambda^a - \Lambda_a^* \left[ \frac{1}{2} U_{cb}^a(\Gamma)\Lambda^b\Lambda^c(-1)^{\epsilon_c} - i\hbar V^a_\gamma (\Gamma)\Lambda^b - (i\hbar)^2 \tilde{G}_a^a(\Gamma) \right] \right\}(\theta) + o(\Lambda^a),
\]

(72)

where \((V^a_\gamma, \tilde{G}_a^a)(\theta)\), together with \((U_{cb}^a, G_a)(\theta)\), define the unimodularity relations \(23\). The relation of the \(\theta\)-local quantization to the generating functional of Green’s function \(Z[J, \phi^*] \) of the BV method \(11\) is obvious from the identification \(Z(\partial_\theta \Phi^*, \Phi^*) (0) = Z[J, \phi^*]\) in \(23\), where the action \(S^0_\theta (\Gamma_0, \hbar)\) of \(23\) obeys eq. \(23\).

Another aspect of the restriction \(\theta = 0\) is that an arbitrary function \(F(\theta) = F((\Gamma, \partial_\theta \Gamma(\theta), \theta) \in C^\infty (\Pi T N \times \{\theta\})\) is represented by a functional \(F[\Gamma]\) of the superfield methods \(6,7\) (in case \(\Gamma^p = (\Phi^A, \Phi^*_A)\), see the Introduction)

\[
F[\Gamma] = \int d\theta F(\theta) = F(\Gamma(0), \partial_\theta \Gamma, 0) = F(\Gamma_0, \Gamma_1).
\]

(73)

In the first place, formula \(23\) implies the independence of \(F[\Gamma]\) from \(\partial_\theta \Gamma p(\theta) = \Gamma_0^p\), in case \(F(\theta) = F(\Gamma(\theta), \theta)\). Secondly, formula \(23\) is fundamental in establishing a relation between the \(\theta\)-local antibracket \((\cdot, \cdot)_{\theta}\) and operator \(\Delta^N(\theta)\), acting on \(C^\infty (N \times \{\theta\})\), with a generalization to arbitrary \((\Gamma, \omega^{pq}, \rho(\theta))\) of the flat functional operations \((\cdot, \cdot), \Delta\) of Refs. \(6,7\), identical to their counterparts of the BV method in case \(\Gamma^p = (\Phi^A, \Phi^*_A), \omega^{pq}(\Gamma(\theta)) = \text{antidiag} (-\delta^A_B, \delta^B_A), \rho(\theta) = 1\), and in case of a different odd Poisson bivector, \(\tilde{\omega}^{pq}(\Gamma(\theta), \theta') = (1 + \theta' \partial_\theta)\omega^{pq}(\theta)\).

The correspondence follows from

\[
(F(\theta), G(\theta)) = \left\{ \frac{\partial G(\Gamma_0)}{\partial \Gamma_0^p} \omega^{pq}(\Gamma_0) \frac{\partial F(\Gamma_0)}{\partial \Gamma_0^p} \right\} = (F[\Gamma], G[\Gamma])^N,
\]

\[
(F[\Gamma], G[\Gamma])^N = \partial_\theta \left[ \frac{\partial F[\Gamma]}{\partial \Gamma^p(\theta)} \frac{\partial G[\Gamma]}{\partial \Gamma^p(\theta')} \right] (-1)^{\epsilon(\Gamma^p) + 1},
\]

\[
\Delta^N F(\theta) |_{\theta=0} = \Delta^N (0) F(\Gamma_0) = \Delta^N F[\Gamma],
\]

\[
\Delta^N = \frac{1}{2} (-1)^{\epsilon(\Gamma^p)} \partial_\theta \partial_{\theta'} \left[ \rho^{-1}[1] \tilde{\omega}_{pq}(\theta', \theta) \left[ \Gamma^p(\theta), \rho[\Gamma(\theta')] \right] ^N \right],
\]

(74)

(75)

where \((\rho[\Gamma], \tilde{\omega}_{pq}(\theta', \theta)) = (\rho(\Gamma_0), \theta' \partial_\theta \omega_{pq}(\theta))\) and

\[
\int d\theta' \omega^{pq}(\theta', \theta') \omega_{pq}(\theta', \theta) = \theta \delta^{pq}.
\]

When establishing the correspondence with the operations \((\cdot, \cdot)\) and \(\Delta\) of \(6,7\) in \(74, 75\), we have used a relation between the superfield and component derivatives:

\[
\delta_i / \delta \Gamma^p(\theta) = (-1)^{\epsilon(\Gamma^p)} (\theta \delta_i / \delta \Gamma_0^p - \delta_i / \delta \Gamma_1^p), \Gamma_1^p = (\lambda^A, -(-1)^{\epsilon A} J_A).
\]

In general coordinates, the action of the sum and difference \(\partial_\theta (V \pm U)^N(0)\) for \(N = \Pi T^* M_{\text{ext}}\big|_{\theta=0}\) reduced to

\[
\partial_\theta (V \pm U(0)) = \partial_\theta \Phi^*_A(\theta) \partial / \partial \Phi^*_A(0) \pm \partial_\theta \Phi^A(\theta) / \partial \Phi^*_A(0),
\]

is identical to the action of the generalized sum and difference of their counterparts \(V, U\) in \(9\):

\[
\partial_\theta (V - (-1)^t U)^N(0, \theta) = (S^t(\theta), F(\theta))^N |_{\theta=0}
\]

\[
= (V - (-1)^t U)^N F[\Gamma] = (S^t[\Gamma], F[\Gamma])^N, \ t = 1, 2,
\]

\[
S^t(\theta) = (\partial_\theta \Gamma^p) \omega_{pq} \Gamma(\theta) \Gamma^q(\theta), \ S^t[\Gamma] = \partial_\theta \left[ \Gamma^p(\theta) \partial_{\theta'} \partial_\theta \left[ \omega_{pq}(\theta, \theta') \Gamma^q(\theta') \right] \right] = S^t(0),
\]

(76)
where the functions $\omega_{pq}^{t}(\theta), \tilde{\omega}_{pq}^{t}(\theta, \theta')$, identical with $\omega_{pq}(\theta)$ and $\tilde{\omega}_{pq}(\theta, \theta')$ for $t = 1$, are defined by

$$\tilde{\omega}_{pq}^{t}(\theta, \theta') = \theta \theta' \omega_{pq}^{t}(\theta, \theta') = -(-1)^{1+\varepsilon(\nu^t)(\nu^t)} \omega_{pq}^{t}(\theta', \theta), \quad \omega_{pq}^{t}(\theta) = (-1)^{\varepsilon(\nu^t)(\nu^t)+t} \omega_{pq}^{t}(\theta).$$

The $\varepsilon$-bosonic quantities $S^{t}(\theta)$ and $S^{t}[\Gamma]$ with a vanishing ghost number play the role of the symmetric $\text{Sp}(2)$-tensor $S_{ab}^{(a, b = 1, 2)}$ and anti-Hamiltonian $S_{\text{th}}^{(a, b = 1, 2)}$ of Ref. [37], which define (in terms of extended antibrackets) the first-order operators of the modified triplex algebra. In this case, the additional functions $\omega_{pq}^{2}(\theta), \tilde{\omega}_{pq}^{2}(\theta, \theta')$ may be considered as quantities that define another non-antisymplectic (non-Riemannian) nondegenerate structure on $N$. The $\theta$-local functional operators $\{\Delta^{N}, \partial_{\theta}V^{N}, \partial_{\theta}U^{N}\}(\theta)$ anticommute for a fixed $\theta$,

$$[E_{i}^{N}(\theta), E_{j}^{N}(\theta)]_{+} = 0, \quad [E_{i}^{N}(\theta), E_{j}^{N}(\theta)]_{-} = (E_{1}, E_{2}, E_{3}) = (\Delta, \partial_{\theta}V, \partial_{\theta}U),$$

provided that $S^{t}(\theta)$ or $S^{t}[\Gamma]$ is subject to

$$\Delta^{N}(\theta)S^{t}(\theta) = 0, \quad (S^{t}(\theta), S^{t}(\theta))_{+}^{N} = 0, \quad t, u, v = 1, 2.$$  

Relations (78), which hold, due to eqs. (74)–(77), also for functional objects (those without a $\theta$-dependence), follow from the well-known properties of the antibracket (bilinearity, graded antisymmetry, Leibniz rule, Jacobi identity), and from the rule of antibracket differentiation by the operator $\Delta^{N}(\theta)$. The system (77) determines the geometry of $N$ by restricting the choice of both quantities $\omega_{pq}^{t}(\theta), \tilde{\omega}_{pq}^{t}(\theta, \theta')$. Notice that a solution of eqs. (78) always exists, for instance, $\omega_{pq}^{t}(\theta) = \text{antidiag}(\delta_{B}^{t}, (-1)^{t} \delta_{B}^{A})$.

### 5.2 Superfield Functional Quantization in General Coordinates

Let us consider a generalization of the vacuum functional of the superfield method [30], namely,

$$Z^{N}_{N} = \int d\mu[\Gamma]q^{N}[\Gamma] \exp \left\{ \frac{i}{\hbar} (W' + X' + \varkappa_{2}S^{2}) \right\}.$$

where $\varkappa_{2}$ is an arbitrary real number; $W'$, $X'$ are the quantum and gauge-fixing actions, defined on $N$ and subject to the equations

$$\frac{1}{2}(W', W')^{N} + \nabla W' = i\hbar \Delta^{N}W', \quad \frac{1}{2}(X', X')^{N} + \mathcal{U}X' = i\hbar \Delta^{N}X',$$

while the integration measure and the weight functional $q^{N}[\Gamma]$ have the form

$$d\mu[\Gamma] = \rho[\Gamma]d\Gamma, \quad d\Gamma = d\Gamma_{0}d\Gamma_{1}, \quad q^{N}[\Gamma] = \delta(G_{0}^{N}(\Gamma(\theta))), \quad a_{1} = 1, \ldots, \dim_{+}N.$$  

In [30], we have introduced a two-parameter set $\mathcal{U}(\varkappa_{1}, \varkappa_{2}), \mathcal{V}(\varkappa_{1}, \varkappa_{2})$ of anti-commuting operators,

$$\mathcal{U} = \frac{1}{2}(-1)^{\varkappa_{1}}\varkappa_{2}(S^{t}[\Gamma], \cdot)^{N}, \quad \mathcal{V} = \frac{1}{2}(\mathcal{S}^{t}[\Gamma], \cdot)^{N},$$

satisfying, together with $\Delta^{N}$, the algebra (84), for arbitrary real numbers $\varkappa_{1}$, whose choice admissible for the existence of the functional integral fixes the form of $Z^{N}_{N}$. This choice also fixes equations (81), the admissible boundary conditions for $W'$, $X'$, and the form of the additional hypergauge conditions, $G_{0}^{N}(\Gamma(\theta)) = 0$, which are required to retain the explicit superfield form of the vacuum functional. The independent functions $G_{0}^{N}(\Gamma(\theta))$ are equivalent to the set of functions $\mathcal{V} \mathcal{G}_{p}^{N}(\theta)$: $G_{0}^{N}(\theta_{1}) = \partial_{\theta} [Y_{a_{1}}(\Gamma(\theta_{1}), \theta) \mathcal{V} \mathcal{G}_{p}^{N}(\theta)]$ with certain $Y_{a_{1}}(\theta_{1}, \theta)$ such that

$$\text{rank} \left[ P_{0}(\theta) \frac{\delta E_{i}(\theta_{1})}{\delta \Gamma^{q}(\theta)} \right]_{\delta W' = \delta X' = \delta \mathcal{G} = \mathcal{G}_{p}^{N}} = (L_{1}^{Y}, \dim_{+}N - L_{1}^{Y}), \quad (E_{1}, E_{2}) = (G_{a_{1}}^{N}, \mathcal{G}_{p}^{N}),$$

for some integers $L_{1}^{Y}, L_{2}^{Y}, 0 \leq L_{1}^{Y}, L_{2}^{Y} \leq \dim_{+}N$.

The basic properties of the functional $Z^{N}_{N}$ are analogous to properties 1, 2 of $Z(\theta)$ in [30], which are encoded by a Hamiltonian-like system with an arbitrary functional $R[\Gamma], (\varepsilon, gh) = (0, 0),$

$$\partial_{\theta}^{t} \mathcal{G}_{p}^{N}(\theta) = \frac{\hbar}{i} T^{-1}[\Gamma] \mathcal{G}_{p}(\theta), \mathcal{T}[\Gamma] R^{N}, \quad \mathcal{T}[\Gamma] = \exp \left[ \frac{i}{\hbar} (W' - X' + \varkappa_{1}S^{1}) \right].$$

(84)
For instance, the superfield BRST transformations $\delta_\mu \Gamma^p(\theta) = \partial_\mu^G \Gamma^p(\theta) \mu$ for $Z_N^X$, are derived from $\mathcal{S}$, with $R = 1$, and from the additional equations

$$(G^N_{a1}(\Gamma(\theta)), W' = X' + \kappa_1 S^1)^N = 0 \iff \delta_\mu G^N_{a1}(\Gamma(\theta)) = 0,$$  

(85)

which ensure the BRST invariance of $q^N$. In order to be valid for any gauge theory with an admissible action, eqs. $\mathcal{S}$ impose strong restrictions on all the quantities $\gamma_{a1}(\theta, \theta')$, $\tilde{\omega}^\mu_{pq}(\theta, \theta')$, and consequently on the geometry of $N$. For example, the constant functions $\gamma_{a1}(\theta, \theta')$, $\tilde{\omega}^\mu_{pq}(\theta, \theta')$ belong to solutions of eqs. $\mathcal{S}$. We, however, do not restrict the consideration to this special case, assuming that eqs. $\mathcal{S}$ are fulfilled for any $W'$, $X'$.

Some remarks are in order concerning the status of the functional $q^N[\Gamma]$. Here, we do not consider the possibility of presenting the functions $G^N_{a1}(\theta)$ by an integral over new additional superfields $\Lambda^a(\theta)$, following in part the prescription of Ref. [23] that introduces so-called “unimodularity

involution relations” for $G^N_{a1}(\theta)$ and modifies the BRST transformations for the extended set of variables $(\Gamma^p, \Lambda^a)(\theta)$.

Choosing

$$(\kappa_1, \Gamma^p, \rho, \tilde{\omega}^\mu_{pq}(\theta, \theta'), Y_{a1p}(\theta, 1), \theta) = (1, (\Phi^A, \Phi^*_A), 1, \theta \theta' \text{antidiag}(\delta^A_\theta, (-1)^\delta^A_\theta, \delta(\theta_1 - \theta) \delta A_p),$$

(86)

we obtain

$$(V, U, S^2, q^N) = (V, U, \partial_\theta (\Phi^*_A \Phi^A)(\theta), \delta(J_\Lambda)), $$

(87)

where $(V, U) = (-1)^{\rho_2} \theta_\theta \partial_\theta (-\Phi^*_A(\theta) \theta_\theta \delta / \delta \Phi^*_A(\theta), \Phi^*(\theta) \theta_\theta \delta_1 / \delta \Phi^A(\theta))$, according to $\mathcal{G}$, and hence $Z^N$, as well as equations $\mathcal{S}$, and BRST transformations, implied by $\mathcal{S}$ for $R = 1$, coincide, respectively, with the vacuum functional $Z$,

$$Z = \int d\Phi d\Phi^* \delta (\partial_\theta \Phi^\ast(\theta)) \exp \left\{ \frac{i}{\hbar} (W[\Phi, \Phi^*] + X[\Phi, \Phi^*] + \partial_\theta (\Phi_A^* \Phi^A)) \right\},$$

with the equations $1/2(W, W) + VW = i\hbar \Delta W, 1/2(X, X) - UX = i\hbar \Delta X, W = W', X = X', \text{and with the BRST symmetry transformations }$ $\mathcal{G}$ for $Z$ (having the opposite signs in the r.h.s.)

$$\delta \Phi^A(\theta) = \mu U \Phi^A(\theta) + (\Phi^A(\theta), X + W) \mu, \delta \Phi^*_A(\theta) = \mu V \Phi^*_A(\theta) + (\Phi^*_A(\theta), X - W) \mu.$$

In particular, choosing $X$ in terms of the gauge fermion $\Psi[\Phi] = \Psi(\phi, \lambda), X[\Phi, \Phi^*] = U \Psi[\Phi]$, first realized in $\mathcal{G}$, we obtain the generating functional of Green’s functions $Z[\Phi^*]$ used in Section 1 in order to determine the superfield effective action in Abelian hypergauges.

A complete correspondence between $Z^N_X$ and the functional $Z_X(0)|_{\varphi^*_a = 0}$ in $\mathcal{S}$ can be established as follows: First, the functional $\kappa_2 S^2$ is represented as $(1/2)(1 + (-1)^\delta)\kappa_1 S^1$, so that the redefined actions

$$W'' = W' + \frac{1}{2} \kappa S_1 S^1, X'' = X' + \frac{1}{2} (-1)^\delta \kappa_1 S^1$$

(88)

obey eqs. $\mathcal{S}$ without the operators $V$ and $U$. Second, the actions $W(\theta)$ in $\mathcal{S}$ and $W''[\Gamma]$, as well as the quantities $X(\theta)|_{\lambda^a = 0}$ in $\mathcal{S}$ and $X''[\Gamma]$, are related by formula $\mathcal{R}$. Third, the solvability of the hypergauges $G_a[\Gamma]$ with respect to the fields $\varphi^*_a(\theta)$, on condition that $\Lambda^a(\theta) = \partial^a_\theta \varphi^*_a(\theta)$, implies, together with the previous restriction, a linear dependence of $X''[\Gamma]$ on $\Lambda^a(\theta)$ and its independence from $\partial_\theta \varphi^*_a(\theta)$.

Next, one should take into account the structure of the generating equation for $X''[\Gamma]$, as well as the second system in $\mathcal{S}$ with $\mathcal{R}$ for $X(\theta)$, and the fact that the corresponding systems $\mathcal{S}$, $\mathcal{S}$, encoding the BRST transformations, coincide with each other. The latter requires the commutativity of $G_a[\Gamma]$ and the triviality of the unimodularity relations, i.e., $\Delta^N G_a = V^0_\theta = \tilde{G}^a = 0$. Finally, the measure $d\mu[\Gamma]q^N$ in $\mathcal{S}$ is made identical to $d\mu(\Gamma(\theta)) d\Lambda(\theta)|_{\theta = 0}$ in $\mathcal{S}$ by the choice $q^N = \delta(\partial_\theta \varphi^*_a(\theta))$. This choice can be realized by $(\kappa_1, \tilde{\omega}^\mu_{pq}(\theta, \theta'), a_1, Y_{a1p}(\theta, 1), \theta) = (1, \theta \theta' \text{antidiag}(\delta^A_\theta, (-1)^\delta^A_\theta, \delta(\theta_1 - \theta) \delta A_p), a, \delta(\theta_1 - \theta) \delta_{a1p})$.

6 Conclusion

Let us summarize the main results of the present work:

We have proposed a $\theta$-local description of an arbitrary reducible superfield theory as a natural extension of a usual gauge theory, defined on a configuration space $\mathcal{M}_G[\theta = 0]$ of classical fields $A^i$, to a superfield model defined on extended cotangent, $T^*_{odd} \mathcal{M}_G \times \{\theta\}$, and tangent, $T_{odd} \mathcal{M}_G \times \{\theta\}$, odd bundles, in respective Hamiltonian and Lagrangian formulations. It is shown that the conservation under a $\theta$-evolution
(defined by a Hamiltonian or Lagrangian system providing a superfield extension of the usual extremals) of a Hamiltonian action \( S_H ((\mathcal{A}, \mathcal{A}^*) (\theta), \theta) \), or, equivalently, of an odd counterpart of energy, \( S_E ((\mathcal{A}, \theta)_* A (\theta), \theta) \), is equivalent, in view of Noether’s first theorem, to the validity of a Hamiltonian or Lagrangian master equation, respectively.

Using non-Abelian hypergauges, we have constructed a \( \theta \)-local superfield formulation of Lagrangian quantization for a reducible gauge model, extracted from a general superfield model by the conditions of a manifest \( \theta \)-independence of the classical action and the vanishing of ghost number and auxiliary Grassmann parity (related to \( \theta \)) for the action and \( \mathcal{A}^l (\theta) \). In particular, we have proposed a new superfield algorithm for constructing a first approximation to the quantum action in powers of ghosts of the minimal sector, on the basis of interpreting the reducibility relations as special gauge transformations of ghosts for an HS with the Hamiltonian chosen as the quantum action. To investigate the properties of BRST invariance and gauge-independence in a superfield form, for the introduced generating functionals of Green’s functions (including the effective action), we have used \textit{two equivalent} Hamiltonian-like systems. The first system is defined by a \( \theta \)-local antibracket, in terms of a quantum action, a gauge-fixing action, and an arbitrary \( \theta \)-local boson function, while the second (dual) system is defined by an even Poisson bracket, in terms of fermion functionals corresponding to the above functions. The two systems allow one to describe the BRST transformations and the continuous (anti)canonical-like transformations in a manner analogous to the relation between these transformations in the superfield Hamiltonian formalism \( [23] \). We emphasize that, as a basis for the local quantization, we have intensely used the first-level formalism of \( [23] \), whose main ingredient is the vacuum functional (however, without recourse to the gauge-fixing action in a manifest form).

We have considered the problem of a \textit{dual description} for an \( L \)-stage reducible gauge theory in terms of a BRST charge for a formal dynamical system with first-class constraints of \((L + 1)\)-stage reducibility. It is shown that this problem is a particular case of embedding a reducible special gauge theory into a general gauge theory of the same stage of reducibility.

We have proposed an extension of functional superfield quantization \( [6, 17] \) to the case of general antisymplectic manifold without connection. It is shown that the condition of anti-commutativity for all operators as well as the requirement of a correct transformation of the path integral measure impose strong restrictions on the geometry of the manifold as well as on \textit{additional hypergauge conditions} that determine the measure.

We have established the coincidence of the first-level functional integral \( Z^{(1)} \) in \( [23] \) with the local vacuum function of the proposed quantization scheme, in case \( \theta = 0 \) and \( \varphi^* (\theta) = 0 \), \( Z_X (0)|_{\varphi^*_X = 0} \). A correspondence is found between \( Z_X (0)|_{\varphi^*_X = 0} \) and the vacuum functional \( Z^N_X \) of the proposed extension of the superfield quantization \( [6, 17] \). It is shown that the above functionals coincide only in Abelian hypergauges, with a trivial choice of the additional hypergauge conditions.

From the obtained results there follow the generating functional of Green’s functions and the effective action of the first-level formalism \( [23] \). It is observed that in case the quantum action \( W^* [\Gamma] \) depends on the superfields \( \partial_0 \Gamma^p (\theta) \), or the gauge-fixing action \( X^l [\Gamma] \) depends on the same superfields more than linearly, the functional \( Z^N_X \) differs from \( Z_X (0)|_{\varphi^*_X = 0} \) exactly as the functional \( Z \) in \( [17] \) differs from \( Z^{(1)} \) in \( [23] \).

In connection with the discussed points, the following open problems seem to be of interest:

1. One could obtain a Hamiltonian formulation of an LSM from a Lagrangian formulation in the case of a degenerate Hessian supermatrix \( (S''_{ij})_{L,I} (\theta) \) in \( [17] \), and consider its relation to the standard component description of a model. In this case, Dirac’s algorithm in terms of a \( \theta \)-local antibracket, under the conservation of primary constraints in the course of \( \theta \)-evolution along a vector field defined by an HS with a primary Hamiltonian in terms of antifields, would determine all antisymplectic constraints for the classical superfields \( \Gamma^p_{CL} (\theta) \). Among these constraints, there may exist a subsystem of second-class ones, in the case of the degeneracy of the supermatrix \( \left[ \mathcal{L}^l_j (\theta_1) \mathcal{L}^l_i (\theta_1) S_L (\theta_1) ( -1)^{\gamma^l} \right] \) in \( [17] \). It is interesting to apply the BFV method to construct, in terms of a \( \theta \)-local Dirac’s antibracket, \( (\cdot, \cdot)_BD \), a triplet of \( \theta \)-local quantities \( \hat{S}_H (\theta), \hat{\Omega} (\theta), \hat{\Psi} (\theta) \): \((\varepsilon, \varepsilon)_\text{even functions} \hat{S}_H (\theta), \hat{\Omega} (\theta), \hat{\Psi} (\theta)\), commuting with respect to \((\cdot, \cdot)_BD \) [by analogy with the Hamilton function and the BFV–BRST charge in a \( t \)-local field theory] and an \((\varepsilon, \varepsilon)_\text{odd function} \hat{\Psi} (\theta) \), which encodes the dynamics of an LSM and its first-class constraint algebra, as well as fixes the “gauge” arbitrariness in a space larger than \( T^*_{\text{odd}} \mathcal{M}_{CL} \times \{ \theta \} \). In this connection, it seems interesting to consider the question of how the construction of \( \hat{S}_H (\theta), \hat{\Omega} (\theta) \) and of the “unitarizing Hamiltonian” \( \hat{S}_H (\theta) = \hat{S}_H (\theta) + (\hat{\Omega} (\theta), \hat{\Psi} (\theta))_BD \) is related to the quantum action of the BV method.

2. From the solution of the dual problem of Subsection 4.2, found within the classical description,
there arise two natural questions: “How does the operator description of a formal dynamical system with a nilpotent BRST charge and a quantum counterpart of the even Poisson bracket correspond to the Lagrangian quantization of a gauge model?” and “Which ingredient of the Lagrangian formulation should correspond to the formal supercommutator and the Hilbert space of states?” The mentioned problems seem to be related to the correspondence \[39\] between Poisson brackets and their operator counterparts of the opposite parity, as well as to the possibility \[40\] of constructing a Lagrangian quantization procedure for more general gauge theories that are determined, like higher-spin gauge fields \[41\], by non-Lagrangian equations of motion,

\[
T_i(A) \neq \delta S(A)/\delta A^i \quad \text{for any \( \varepsilon \)-bosonic} \quad S(A) \in C^\infty(M_{cl})|_{\theta=0}.
\]

3. Notice that one of the possibilities of describing theories with non-Abelian hypergauges within the superfield method \[42\] consists in enlarging the component spectrum of superfields \((\Phi^A, \Phi^*_A)(\theta)\) by a Grassmann parameter \(\tilde{\theta}\) unrelated to an additional antiBRST symmetry. In this case, the inclusion of \((\Phi^A, \Phi^*_A)(\theta)\) and the fields \(\lambda^*_A\), anticanonically conjugate to \(\lambda^A\), into the general superfields \((\Phi^A, \overline{\Phi}_A)(\theta, \tilde{\theta})\)

is provided by the relations

\[
(\Phi^A, \partial_\theta \overline{\Phi}_A)(\theta, 0) = (\Phi^A, \Phi^*_A)(\theta), \quad \overline{\Phi}_A(0, 0) = \lambda^*_A.
\]

Finally, note that the procedure of \(N = 1\) local quantization has been recently developed in \[42\] as applied to the case of reducible general hypergauges when independent hypergauge conditions cannot be determined in a covariant manner on an antisymplectic manifold.

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