Influence of acceleration on multi-body entangled quantum states

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We study the influence of acceleration on the twin-Fock state which is a class of specific multi-body entangled quantum state and was already realized experimentally with high precision and sensitivity. We show that the multi-body quantum entanglement can be increased with the acceleration, consistent with the “anti-Unruh effect” in reference to the counterintuitive cooling previously pointed out for an accelerated detector coupled to the vacuum. In particular, this kind of entanglement increase can lead to the improvement of the phase sensitivity, which provides a way to test the anti-Unruh effect in the future experiments.

I. INTRODUCTION

In 1976, Unruh discovered that an observer with uniform acceleration would feel a thermal bath of particles in the Minkowski vacuum of a free quantum field [1], which implicates that the particle content of a quantum field is observer dependent [2]. This effect was put forward soon after that Hawking discovered that black hole could emit thermal radiation [3] and could help to clarify some conceptual issues [4] raised by black hole evaporation due to the equivalence. So the understanding of the Unruh effect is also significant for Hawking radiation and the related problems (i.e. information loss problems). In the past years, the Unruh effect was digested and extended to many different situations (see the review [2] and references therein), but the observation of Unruh effect has not been realized up to now, because of the pretty low Unruh temperature \( T = h\alpha/(2\pi c k_B) \) where \( \alpha \) is the proper acceleration of the observer, \( h \) is the reduced Planck constant, \( c \) is the speed of the light, and \( k_B \) is the Boltzman constant. The acceleration must be about \( 10^{20} m/s^2 \) in order to realize a photon bath at \( 1K \).

Although it was claimed that the Unruh effect is a direct result of quantum field theory and does not require any experimental confirmation if the quantum field theory is correct [5], there exists still some problems needed to be clarified through experiments or observations, i.e. whether the particles felt by the accelerated observers are real [6], whether the effect is applicable to the extended systems [7], and even some theoretical calculation implies that the possible inversion from Bose to Fermi statistics for many-particle states observed by an accelerated observer [8]. Thus, the experimental quest for the evidence of the Unruh effect is necessary for the final confirmation. As well-known, the most observational proposals are related to a model called the Unruh-DeWitt detector [9]. Based on the model, it is found that a quantum system consisting of such a detector uniformly accelerating in Minkowski vacuum sees a thermal field and thus cause decoherence due to the coupling with the thermal field. The first attempt is to observe such effect by the deexcitation of the electron in storage rings by the thermal Unruh radiation [10]. Then some other possible detections related to proton decays [11, 12], accelerated charges [13, 14], neutrino oscillations [15] and the recent theoretical [16] and observational [17] methods using Larmor radiation were proposed. In particular, an interesting observation for Unruh radiation using quantum simulation in Bose-Einstein condensates was reported, which is significant for the future research of the dynamics of quantum many-body systems in a curved spacetime [18].

On the other hand, the recent found anti-Unruh effect [19] states that a particle detector in uniform acceleration coupled to the vacuum can cool down with increasing acceleration under certain conditions, which is opposite to the celebrated Unruh effect. Since the experiments are always made in the range of finite length and time, it must distinguish the two situations of Unruh and anti-Unruh effects carefully. An interesting way for this is to see the change of quantum entanglement by acceleration. According to the previous results [20–28], the quantum entanglement would be degraded by the Unruh effect, which helps to establish the general conclusion that entanglement is also observer dependent. In particular, a recent calculation showed that the anti-Unruh effect can lead to the increase for the quantum entanglement [29], which might be significant for the task of quantum information in large spatial or temporal scale. In this paper, we will consider the influence of acceleration on the spin squeezed states [30, 31] and the corresponding experimental feasibility through the change of entanglement. Spin squeezed states have attracted much attention due to their use in the measurement of the correlation or entanglement among particles and in the improvement of measurement precision in quantum metrology. We will focus on twin-Fock (TF) states [32] which can be seen as a kind of limit for spin squeezed states and had been realized in a recent experiment with more than \( 10^4 \) atoms [33].

This paper is organized as follows. First, in section II we review the theory about two-level Unruh-DeWitt (UDW) detector in Minkowski spacetime, and the change of entanglement between two atoms for the Unruh and anti-Unruh effect. This is followed in section III by the discussions on the influence of acceleration on entanglement for TF states, where the spin squeezing parameter
is used to measure the change of entanglement. Then, when the atoms in the TF state are accelerated, how the phase sensitivity is changed under the background of the Ramsey interferometer is investigated in section IV. Finally, we give a conclusion in section V. In this paper, we use units with $c = \hbar = k_B = 1$, except the part of analyzing the experimental feasibility in section IV.

II. THE UNRUH-DEWITT MODEL

We start with the model of UDW detector in order to investigate the interaction between accelerated atoms and vacuum. The detector, usually considered as a point-like two-level quantum system or atom (as required in this paper), consists of two quantum states, i.e. the ground $|g\rangle$ and excited $|e\rangle$ states, which are separated by an energy gap $\Omega$ while experiencing accelerated motion in a vacuum field. But for the accelerated atom, the vacuum appears thermal due to the Unruh effect, which will influence the state of the atom. This could be described according to the following interaction Hamiltonian in a $(1 + 1)$-dimension model,

$$H_I = \lambda \chi (\tau/\sigma) \mu (\tau) \phi (x (\tau)),$$

where $\phi$ is a scalar field related to the vacuum in Minkowski spacetime and interacts with the accelerated atom, $\lambda$ is the coupling strength, $\tau$ is the atom’s proper time along its trajectory $x (\tau)$, $\mu (\tau)$ is the atom’s monopole momentum, and $\chi (\tau/\sigma)$ is a switching function that is used to control the interaction time scale $\sigma$. This can be easily generalized to more complex situations such as a quantum oscillator [34], as confirmed with KMS conditions for thermal equilibrium [35]. For an atom accelerating in a vacuum cavity, the evolution of the total quantum state is determined perturbatively by the unitary operator up to first order given by,

$$U = I - i \int d\tau H_I (\tau) + O (\lambda^2).$$

The atom is accelerated along the trajectory

$$t (\tau) = a^{-1} \sinh (at),$$
$$x (\tau) = a^{-1} (\cosh (at) - 1),$$

with the proper acceleration $a$. Note that the extra term $a^{-1}$ in the expression of $x (\tau)$ is related to the initial condition and only for convenience of the calculation below without changing the influence of the acceleration. Thus, within the first-order approximation and in the interaction picture, the evolution of the atom could be described by [19],

$$U |g \rangle |0\rangle = D_0 (|g\rangle |0\rangle - i\eta_0 |e\rangle |1\rangle),$$
$$U |e\rangle |0\rangle = D_1 (|e\rangle |0\rangle + i\eta_1 |g\rangle |1\rangle),$$

where $k$ is the mode of the $(1 + 1)$-dimension scalar field with (bosonic) annihilation (creation) operator $a_k$ (or $a_k^\dagger$), $a_k |0\rangle = 0$ and $a_k^\dagger |0\rangle = |1\rangle$, and $D_{0,1}$ is the state normalization factor. It is noted that the created state $|1_k\rangle$ is dependent on the wave vector $k$, so the coupling in Eq. (4) has to be understood by writing $\eta_0 |1\rangle = \lambda \int d\tau I_{+,k} |1_k\rangle$ and $\eta_1 |1\rangle = \lambda \int d\tau I_{-,k} |1_k\rangle$ where $I_{\pm,k}$ is given as

$$I_{\pm,k} = \frac{1}{\sqrt{4\pi\omega}} \int_{-\infty}^\infty \chi (\tau/\sigma) \exp[\pm i\Omega \tau + i\omega t (\tau) - ikx (\tau)] d\tau.$$

The notations $\eta_0 |1\rangle$ and $\eta_1 |1\rangle$ is inseparable, but in this paper we consider the $|1\rangle$ is the same for the two cases under the spirit of single mode approximation [36, 37], and $\eta_0$ and $\eta_1$ are related to the excitation and deexcitation probability of the atom, i.e. $|\eta_0|^2 = \sum_k |\langle 1_k, e | U (1) |0, g\rangle|^2$ and $|\eta_1|^2 = \sum_k |\langle 1_k, g | U (1) |0, e\rangle|^2$ where $U (1) = -i \int d\tau H_I (\tau)$.

It is worth pointing out that the change of the quantum state, i.e. the transition probability, is dependent on the concrete parameters like the interaction time scale $\sigma$ and the energy gap $\Omega$ [19]. In particular, under some conditions, for example when the interaction timescale is far away from the timescale associated to the reciprocal of the detector’s energy gap, the probability decreases as the acceleration or the Unruh temperature increases, which makes the atom “feel” cooler instead of warm up expected by the Unruh effect. This effect was called as anti-Unruh effect. Although the initial discussion for the anti-Unruh effect is made in Ref. [19] for accelerated detectors coupled to a massless scalar field either in a periodic cavity or under a hard-IR momentum cutoff for the continuum, it has been shown to represent a general stationary mechanism that can exist under a stationary state satisfying Kubo-Martin-Schwinger (KMS) condition [38, 40] and is independent of any kind of boundary conditions [19, 33]. Thus, like the Unruh effect, the anti-Unruh effect constitutes another new phenomenon for the accelerated observers. Although the physically essential reasons remain to be explored for their difference, some important elements, like the interaction time, the detector’s energy gap, the mass of the quantum field, etc, had been pointed out to distinguish them operationally. Here we consider massive field with e.g. $\omega = \sqrt{k^2 + m^2}$ as in Ref. [32] so that the anti-Unruh effect discussed will not be constrained by the finite interaction time and its validity can be extended to situations where the detector is switched on adiabatically over an infinite long time. Without loss of generality, $m = 1$ is used for all numerical calculations.

With that, the change of bipartite entanglement for two atoms was investigated before [29], in which the initial state is assumed to take the form

$$|\Psi_i\rangle = (\alpha |g\rangle_A |e\rangle_B + \beta |e\rangle_A |g\rangle_B) |0\rangle_A |0\rangle_B,$$

with the complex coefficients satisfying $|\alpha|^2 + |\beta|^2 = 1$. Here we consider the vacuum as a product state and thus the interaction between either one of two atoms and
the scalar field is independent of each other. This could help us to understand the influence of acceleration on the quantum state of the atoms without the disturbance of the complicated vacuum (i.e. it is regarded as an entangled state) [24]. This means that the subscripts A and B in the vacuum state \(|\tilde{0}\rangle \equiv |0\rangle_A |0\rangle_B\) represents the locations related to the atoms A and B. For the case we consider, each atom is independently [41] accelerating in the vacuum and has the same coupling with the scalar field in its respective (spatial) place by the same process presented in Eq. (4). When the two atoms are accelerated simultaneously, the state becomes

\[
|\Psi_f\rangle = D_0 D_1(\alpha |g\rangle_A |e\rangle_B + \beta |e\rangle_A |g\rangle_B) |0\rangle_A |0\rangle_B
- i (\alpha \eta |g\rangle_A |e\rangle_B + \beta \eta |e\rangle_A |g\rangle_B) |0\rangle_A |1\rangle_B
- i (\beta \eta |g\rangle_A |e\rangle_B + \alpha \eta |e\rangle_A |g\rangle_B) |1\rangle_A |0\rangle_B
+ (\alpha \eta \eta |e\rangle_A |e\rangle_B + \beta \eta \eta |g\rangle_A |e\rangle_B) |1\rangle_A |1\rangle_B,
\]

where \(|\tilde{1}\rangle \equiv |0\rangle_A |1\rangle_B \equiv |1\rangle_A |0\rangle_B\) represents the single-mode state from a global perspective but \(|1\rangle_A |0\rangle_B\) might be different from \(|1\rangle_A |0\rangle_B\) locally when the two atoms are separated far apart, and \(|\tilde{2}\rangle \equiv |1\rangle_A |1\rangle_B\) represents the two-mode state from a global perspective. It is necessary to keep the last term in Eq. (7) in order to make the evolution in Eq. (8) intact formally, since the so-called single-mode approximation in this paper is made for the interaction between a single atom and the scalar field. When the two atoms locates nearly at the same place, the forms \(|0\rangle_A |1\rangle_B\) and \(|1\rangle_A |0\rangle_B\) for the vacuum can be regarded as the same and the two related terms in Eq. (7) can be combined into one, i.e. \(i(\alpha + \beta)(\eta |g\rangle_A |g\rangle_B + \eta |e\rangle_A |e\rangle_B) |0\rangle |1\rangle\). We consider the two atoms (or many atoms considered in the next section) staying nearly in the same place in the whole process of acceleration, but the loss of atoms due to acceleration is not considered in this paper.

The change of entanglement can be quantified by concurrence [42] which is a widely used entanglement measure for bipartite mixed state. Concurrence is defined by

\[
C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\},
\]

where \(\lambda_1, \lambda_2, \lambda_3, \lambda_4\) are the eigenvalues of the Hermitian matrix \(\sqrt{\rho} \sigma_\rho \sqrt{\rho}\) with \(\rho = (\sigma_y \otimes \sigma_y) \rho^* (\sigma_y \otimes \sigma_y)\) the spin-flipped state of \(\rho\), \(\sigma_y\) being the y-component Pauli matrix, and the eigenvalues listed in decreasing order. For the case of two atoms being accelerated, the change of entanglement can be calculated using concurrence for the reduced density matrix \(\rho_{AB}\) by tracing out the scalar field from the final accelerated quantum state [4], which is shown in the upper plot of Fig.1 with the initial state \(|\Psi_i\rangle\) are taken by \(\alpha = \beta = \frac{1}{\sqrt{2}}\). The solid red line represents the case of the anti-Unruh effect, as discussed in the paragraph after Eq. (4). More detailed discussion and other cases for different initial states refers to Ref. [24]. Since the experiment is always made within certain timescale and using the certain energy gap, the appearance of anti-Unruh effect is possible when the experiment is implemented. Therefore, the experimental test must consider this point, for which the increase of entanglement would also be the result of acceleration.

III. TWIN-FOCK STATE

The previous section presents the UDW model and the change of entanglement between two atoms in this model. Since it is not easy to implement the corresponding experiment to observe the effect for two atoms, we now attempt to apply it to the case of multi-body quantum states for multiple atoms being simultaneously accelerated. Before that, it is noticed that the bipar-
tite quantum state for the maximal entangled atoms, \(|\psi\rangle = \frac{1}{\sqrt{2}} (|e\rangle_A |e\rangle_B + |e\rangle_A |g\rangle_B)\), can be regarded as the simplest TF state. TF state is one kind of Dicke states \([43]\). For a collection of \(N\) identical (pseudo-) spin-1/2 particles, Dicke states can be expressed in Fock space as \(|\frac{N}{2} + m\rangle\) with \(\frac{N}{2} + m\) particles in spin-up and \((\frac{N}{2} - m)\) particles in spin-down modes for \(m = -\frac{N}{2}, -\frac{N}{2} + 1, \ldots, \frac{N}{2}\). In particular, \(m = 0\) represents just the TF state where the number of the particles is the same for each one of the two spin states. On the other hand, Dicke states can be described by the common eigenstate \(|j, m\rangle\) of the collective spin operators \(J^2\) and \(J_z\), with respective eigenvalues \(j(j + 1)\) and \(m\). For the system consisted of \(N\) two-level atoms we will consider, the state \(|j = (\frac{N}{2}, m)\rangle\) indicates that \((j + m)\) atoms are at the excited state \(|e\rangle\), \((j - m)\) atoms are at the ground state \(|g\rangle\), and it is the TF state when \(m = 0\). \(J^2 = \frac{1}{2}(n_e - n_g)\) represents the difference of the number of atoms between excited \((n_e)\) and ground \((n_g)\) states, and \(J^2 = \frac{N}{2}(\frac{N}{2} + 1)\) is related to the total number of atoms. With this description, the state of two atoms can be written as \(|\psi\rangle = |1, 0\rangle\), with which the average of the difference of the number of atoms, \(\langle J_0 \rangle = 0\).

When the two atoms are accelerated, according to the UDW model discussed in last section, the state \(|\psi\rangle\) will become \(\rho_f = Tr_{\phi} (|\Psi_f\rangle \langle \Psi_f|)\) where \(\alpha = \beta = \sqrt{2}\) are taken for the state \(|\Psi_f\rangle\), and \(Tr_{\phi}\) indicates the calculation of tracing out the part of the scalar field. Thus, the difference of the number of atoms between excited and ground states is obtained as

\[
\langle J_0 \rangle = Tr (\rho_f J_0) = D_0^2 D_1^2 \left( |\eta_0|^2 - |\eta_1|^2 \right),
\]

where \(Tr\) represents the trace of a matrix. The result means that the atom’s number at the excited state is not equal to that at the ground state, different from the requirement of TF state, unless the probability of transition from the ground state to the excited state equates the probability for the inverse transition.

Now we extend this to the case of \(N\) atoms with the initial TF state \(|j, 0\rangle\). When all atoms are accelerated simultaneously, the TF state becomes

\[
\rho_t = B^t_0 |j, 0\rangle \langle j, 0| + \sum_{m=-N/2}^{N/2} \sum_{m'=-N/2}^{N/2} B_m B^*_{m'} |j, m\rangle \langle j, m'|,
\]

up to the normalization factor which is included in our numerical calculation. \(B^t_0 = \sum_{k=0}^{N/2} C_k^{N/2} D_0 D_1 \sum_{m=-N/2}^{N/2} C_{N/2}^{k+m} D_0 D_1 \sum_{m}^{N/2} (|\eta_0|^2)^k \langle \theta (m) (-i \eta_0)^m + \theta (-m) (i \eta_0)^{|m|}| \rangle\), in which the function \(\theta (x) = 1\) when \(x > 0\) and \(\theta (x) = 0\) otherwise, the star * in \(B^*_{m'}\) represents the complex conjugate, and \(C^n_r = \frac{n!}{r!(n-r)!}\) denotes the combinatorial factor of choosing \(r\) out of \(n\). When all atoms are accelerated, according to Eq. (11), the ground state would change into the form \(D_0 (|g\rangle |0\rangle - i \eta_0 |\epsilon\rangle |1\rangle)\), and at the same time, the excited state \(|\epsilon\rangle\) would change into the form \(D_1 (|\epsilon\rangle |0\rangle + i \eta_0 |g\rangle |1\rangle)\). Then, expanding these terms, recombining them and tracing the vacuum state out give the expression (10) of the final state after acceleration. The parameter \(B^t_0\) represents the probability of remaining the original form of the TF state, which includes those cases that if \(l \{0 \leq l \leq \frac{N}{2}\}\) atoms are changed from the ground states to the excited states, there must be other \(l\) atoms which are changed from the excited states to the ground states simultaneously. Similarly, the parameter \(B_m\) can be worked out by choosing the terms that in every term either there are \(m\) more excited states than ground states (that is the case for \(m > 0\)) or there are \(m\) more ground states than excited states (that is the case for \(m < 0\)). No crossed terms like \(|j, 0\rangle \langle j, m|\), because we consider the vacuum state including the same number of photons as the same state no matter which atoms emitted these photons. It is not difficult to confirm this for the cases \(N = 2\) and \(N = 4\).

In order to quantify the change of entanglement in the process of accelerating atoms that is initially in the TF state, we choose the spin squeezing parameter \([31]\),

\[
\xi_E^2 = \frac{\min_{\eta} \left[ (N - 1) \langle \Delta J^x \rangle^2 + \langle J^z \rangle^2 \right]}{\langle J^2 \rangle - N/2},
\]

where \(\min_{\eta}\) denotes the minimum value of \(\eta\) with respect to \(\eta\) and \(\langle \Delta J^x \rangle^2\), \(\langle J^z \rangle^2\) and \(\langle J^2 \rangle\) are the variance of \(J^x\), \(J^z\) and \(J^2\) respectively. The inequality

\[
(N - 1) \langle \Delta J^x \rangle^2 + \langle J^z \rangle^2 \geq \langle J^2 \rangle - N/2
\]

holds for any separable states, and the violation of this inequality indicates entanglement. More related works refer to Ref. \([10]\). If \(\xi_E^2 < 1\), the state is spin squeezed and entangled. In particular, the smaller the value of \(\xi_E^2\), the more the entanglement will be, which can be seen by comparing the upper and lower panels of Fig.1. Since the mean-spin direction of Dicke states can be set along the z direction, we take z-direction as the direction of \(\overline{\eta}\) and the expression for the spin squeezing parameter is written as

\[
\xi_E^2 = \frac{(N - 1) \langle \Delta J_z \rangle^2 + \langle J^2 \rangle}{\langle J^2 \rangle - N/2}.
\]

For TF states, \(\xi_E^2 = 0\) due to \(\langle J^z \rangle = \langle J_z \rangle = 0\), which means that the initial TF state is the most spin-squeezed and entangled state under the measure of \(\xi_E^2\).

After acceleration, the TF state becomes \(\rho_t\) described in Eq. (11). With this, we can calculate

\[
\langle J_z \rangle = Tr (\rho_t J_z) = \sum_{m=-N/2}^{N/2} m |B_m|^2,
\]
and

$$\langle J_z^2 \rangle = Tr (\rho J_z^2) = \sum_{m=-N/2}^{N/2} m^2 |B_m|^2. \quad (15)$$

Thus, according to $$(\Delta J_z)^2 = \langle J_z^2 \rangle - \langle J_z \rangle^2$$, ones can calculate $\xi_E^2$ by substituting these results (13) and (14) into Eq. (13), which is presented in Fig.2 for different energy gaps (see also the lower panel of Fig.1 for $N = 2$). As seen, the anti-Unruh effect is represented with the solid red line, and it shows that the entanglement increases with the acceleration, as expected. It is noted that the entanglement at $a = 0$ for the accelerated state (10) is less than that for the initial maximal entangled state due to the presence of switching function. This had been pointed out before [19, 34] and its corresponding behavior in entanglement was also presented clearly [29]. Moreover, we calculate the change of $\xi_E^2$ with regard to the total number $N$ of atoms, which is presented in Fig.3. It shows that entanglement with regard to the initial entanglement or the change of entanglement increases when the number $N$ increases for a given acceleration, no matter what energy gap is taken. This makes the observation easier experimentally for the influence of acceleration on the quantum state with a larger number. Although the trend of the change is the same both for the Unruh and anti-Unruh effects, it appears that the change of entanglement with $N$ atoms from the Unruh effect is more violent than that from the anti-Unruh effect. This can be understood by noticing that the atoms “feel” hotter in the case that the Unruh effect works than that the anti-Unruh effect works, as seen for $a = 10$ from Fig.2.

IV. PHASE SENSITIVITY

Since the change of spin squeezing or entanglement influences the phase sensitivity of the measurement, in this section we will study the influence of acceleration on the phase sensitivity and compare it with the present experiment. In order to do this, we first give the general expressions, and then compare the results for the TF state and its corresponding accelerated state (10).

Consider the Ramsey interferometer [47, 48] with the initial input state $\rho_i$ and the output state $\rho_o = U \rho_i U^\dagger$ where $U = \exp (-i \theta J_\theta)$ is the unitary operator for the evolution and $\theta$ is the phase shift. According to the error propagation formula [51], the phase sensitivity $\Delta \theta$ can be calculated as

$$\langle \Delta \theta \rangle^2 = \frac{(\Delta J_z)_o^2}{d (J_z^2)_o/d \theta^2}, \quad (16)$$

where the subscript $o$ denotes that the average is taken under the output state. Using $U J_i U^\dagger = J_z \cos \theta - J_x \sin \theta$, it is easy to calculate $\langle J_z^2 \rangle_o = \langle J_z^2 \rangle_i \cos^2 \theta + \langle J_z^2 \rangle_i \sin^2 \theta$ where the subscript $i$ denotes the average is taken under the input state. It is seen that the phase shift can be deduced by measuring $\langle J_z^2 \rangle_o$ in the experiment. The corresponding fluctuation of $J_z^2$ is $$(\Delta J_z^2)_o = \langle J_z^2 \rangle_i - \langle J_z \rangle^2_o = \langle J_z^2 \rangle_i \cos^2 \theta + \langle J_z^2 \rangle_i \sin^2 \theta + V_{zz} \sin^2 \theta \cos^2 \theta,$$

where $V_{zz} = \langle (J_z J_z + J_z J_x)^2 \rangle_i + \langle J_x J_z + J_z J_x \rangle_i^2 - 2 \langle J_z \rangle_i \langle J_x \rangle_i$. Thus, the phase sensitivity becomes

$$\langle \Delta \theta \rangle^2 = \frac{(\Delta J_z^2)_i \cos^2 \theta + \langle J_z^2 \rangle_i \tan^2 \theta + V_{zz}}{4 \langle (J_z^2)_i - \langle J_z \rangle_i \rangle^2}.$$  

(17)
When the phase shift satisfies $\tan^2 \theta_p = \frac{(\Delta J_z^2)}{2(\Delta J_z^2)}$, the optimal phase sensitivity is obtained as

$$ (\Delta \theta)^2 p = \frac{2(\Delta J_z^2)}{4(\langle J_z^2 \rangle - \langle J_z^2 \rangle)^2}, \quad (18) $$

which is our main formula for investigating the change of phase sensitivity due to the influence of acceleration on the spin squeezing or entanglement of TF states.

For Dicke states $|j, m\rangle$, the optimal phase sensitivity occurs at $\theta = 0$ due to $\langle \Delta J_z^2 \rangle = 0$. It is calculated easily that $\langle J_z^2 \rangle = \frac{1}{2} [j(j+1) - m^2]$, $\langle J_z^2 \rangle = m^2$, $V_{zz} = \frac{1}{2} (4m^2 + 1) [j(j+1) - m^2] - 2m^2$. According to Eq. (18), the optimal phase sensitivity for Dicke states is obtained as

$$ (\Delta \theta)^2 PD = \frac{(4m^2 + 1) [j(j+1) - m^2] - 4m^2}{2[j(j+1) - 3m^2]^2}. \quad (19) $$

When $m = j$, the result is $\frac{1}{2j}$ which is the standard quantum limit and can be reached by the spin coherent state $|\psi\rangle$. When $m = 0$, we have

$$ (\Delta \theta)^2 PD = \frac{1}{2j(j+1)}, \quad (20) $$

which gives the phase sensitivity with $\sqrt{\frac{2}{N(N+2)}}$ approaching the Heisenberg limit $\frac{B}{2}$. For the accelerated state in Eq. (10), a direct but tedious calculation with the approximation, $m, m' << j$ and $|B_m|^2 << B_0^2$ gives

$$ \langle J_z^2 \rangle = \sum_{m=-N/2}^{N/2} m^2 |B_m|^2, $$

$$ \Delta J_z^2 = \sqrt{\sum_{m=-N/2}^{N/2} m^4 |B_m|^2 - \left( \sum_{m=-N/2}^{N/2} m^2 |B_m|^2 \right)^2}, $$

$$ \langle J_z^2 \rangle \simeq \frac{1}{2} j(j+1) B_0^2, $$

$$ \Delta J_z^2 \simeq \frac{B_0^2}{2\sqrt{2}j} (j+1), $$

$$ V_{zz} \simeq \frac{1}{2} j(j+1) \left[ 1 + \sum_{m=-N/2}^{N/2} |B_m|^2 (4m^2 + 1) \right]. \quad (21) $$

Put these results into the Eq. (18) and the phase sensitivity is obtained as

$$ (\Delta \theta)^2 PA \simeq \frac{1}{2j(j+1)} + \frac{\sqrt{2}B_0 \Delta J_z^2}{2j(j+1)} + \sum_{m=-N/2}^{N/2} \left\{ \frac{|B_m|^2 (4m^2 + 1)}{2j(j+1)} \right\}. \quad (22) $$

FIG. 4: (Color online) The phase sensitivity as a function of the acceleration $a$. We make the total atom number $N = 100$, and the other parameters are the same as in Fig. 1.

FIG. 5: (Color online) The phase sensitivity as a function of the acceleration $a$. We take the parameters according to the experiment made in Ref. 33 with $\lambda = 1$, $\sigma = 30$, $\Omega = 2\pi$, and $N = 10000$. The terms related to the summation are ignored in the denominator and we have confirmed this approximation numerically. Fig.4 presents the behavior of the phase sensitivity with regard to the acceleration. It is seen that when the acceleration increases, the anti-Unruh effect can lead to the improvement of the phase sensitivity. This is expected, since entanglement will increase (this corresponds to the decrease of the squeezing parameter) when the quantum state is accelerated in the case that the anti-Unruh effect works.

Now we estimate the feasibility to test the effect through the corresponding experiments. In the recent experiment that generates the TF state $^{33}$, the temperature is decreased to the level of $10^{-9}$ K that is required to form Bose-Einstein condensates. Thus, in order to test the Unruh or anti-Unruh effect, the acceleration has to reach the level of $10^{10}$ m/s$^2$ at least, which is smaller than other experimental proposals $^{11,14,16}$ to test such ef-
fect, in which the acceleration is more than $10^{17}$ m/s$^2$. Together with the experimentally allowable parameters, $\Omega \sim 2\pi$ Hz, $N \sim 10000$, it is gotten that

$$\left( \Delta \theta \right)_A^2 \sim 10^{-6}, \quad (23)$$

which is consistent with the present sensitivity in the experiment. This doesn’t mean that the Unruh or anti-Unruh effect can be tested in the experiment instantly, because the required acceleration is still too large for the practical implementation. However, such suggestion is promising by reducing the acceleration through decreasing the experimental temperature or increasing the number of atoms under the present sensitivity. It is also feasible to reduce the acceleration by improving the sensitivity of measurement by some means other than changing the temperature or the number of atoms. In particular, for the anti-Unruh effect, it is noticed that the influence of acceleration on the TF state could be extracted, even though the temperature generated by the acceleration is lower than the temperature of background, due to the fact that the thermal effect from the background cannot lead to the increase of entanglement similar to the analysis made in Ref. [35]. This is attractive for the future experiment with higher sensitivity. Fig. 5 presents the possibility to realize the case of anti-Unruh effect with the accelerating TF state.

V. CONCLUSION

In this paper, we revisit the influence of acceleration on quantum entanglement and the possible test for this effect through accelerating one class of experimentally feasible multi-body entangled quantum states. We have calculated the change of the TF state due to acceleration and studied the change of entanglement among atoms using the spin squeezing parameter as the measurement of entanglement. It is shown that entanglement among atoms not only decreases but also increases with the acceleration for the certain range of the parameters. We have also compared the measurement of entanglement for two atoms using concurrence and spin squeezing parameter respectively and found that the same conclusion are obtained. In order to investigate the feasibility of testing the effect from acceleration, we have calculated the phase sensitivity of measurement using the distorted TF state due to acceleration. It is interesting to note that the case for anti-Unruh effect can appear for such accelerated states, which is favorable for the possible future experiment since this effect is distinctive and different from that coupled to a thermal environment directly by inertial observers [35].

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