Gauge boson masses dominantly generated by Higgs-triplet contributions?

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Abstract

We discuss a model in which the Standard Model (SM) Higgs sector has been extended by additional real and complex triplets. The $\rho \approx 1$ constraint is satisfied by restricting the potential to have an enlarged $SU(2)_L \otimes SU(2)_R$ global symmetry. This is fine tuning, which leads to a decreased predictability in next-to-leading order. In this model, however, the triplet vacuum expectation values may give the dominating contribution to the gauge boson masses. Using a renormalization group argument we constrain this region of the parameter space. Another interesting feature of this model is that one of the neutral scalars doesn’t couple to the fermion sector at tree level and therefore could have a relatively large branching ratio to $2\gamma$’s. It is coupled, however, to the Z-boson and therefore it could be produced at LEP via the standard $e^+e^- \rightarrow Z\phi$ mechanism with rates comparable to the ones of the Standard Model.
The Standard Model (SM) of electroweak and strong interactions successfully describes all elementary particle physics phenomena. Recently, it has been successfully confronted with the precision measurements at LEP [1]. The Higgs sector, and so the mechanism of the electroweak symmetry breaking, however, is weakly constrained by the data. The most important constraint is given by the measured value of the rho parameter:

\[
\rho = \frac{M_W^2}{M_Z^2 \cos^2 (\theta_W)} \quad (1)
\]

\[
\rho_{\text{exp}} = 1.003 \pm 0.004 \quad (2)
\]

Models involving only \(SU(2)_L\) doublets and singlets, satisfy in a natural way the \(\rho \approx 1\) condition. This is due to the fact that in the SM the \(SU(2)_L \otimes U(1)_Y\) gauge symmetry forces the scalar potential to exhibit a \(SU(2)_L \otimes SU(2)_R\) global symmetry. Breaking the gauge symmetry the Higgs potential still exhibits an \(SU(2)\) custodial symmetry which ensures \(\rho = 1\) at tree level. Because of this, \(SU(2)_L \otimes SU(2)_R\) violating radiative corrections to the Higgs potential are finite and small. This also means that the rho parameter is close to one naturally, without fine tuning parameters.

If we want to allow also for Higgs triplets or even higher multiplets, the value of the measured rho parameter becomes a strong constraint. One obvious possibility is that we demand the triplet vacuum expectation values (VEV’s) to be small. This requires fine tuning and a effective decoupling of the triplets from gauge bosons and fermions. Alternatively, we could require that the scalar potential, even with these higher multiplets, will remain invariant with respect to the enlarged \(SU(2)_L \otimes SU(2)_R\) global symmetry [3, 4]. For triplets, however, the \(SU(2)_L \otimes SU(2)_R\) symmetry of the Higgs potential will not be obtained automatically as a result of the gauge symmetry. It can only be achieved by fine tuning the parameters of the Higgs potential. Therefore, the rho-parameter in such models will suffer from a fine tuning problem and will depend ”directly” on the fine-tuned parameters of the scalar potential. In the SM model the rho parameter is protected from this problem by the custodial symmetry.

With fine tuned \(SU(2)_L \otimes SU(2)_R\) symmetry, however, the higher dimensional multiplets may generate a major part of the W and Z masses. Furthermore, in these models the doublet Yukawa couplings could be strongly enhanced with respect to the SM, indicating an interesting phenomenology. In this note we shall discuss the phenomenological viability of this scenario.

The most straightforward extension of the SM Higgs sector, realizing the above situation, has been presented by Georgy and Machacek [3] who added a complex and a real triplet to the SM doublet. With this content of the scalar sector it is indeed possible to restrict the potential to a \(SU(2)_L \otimes SU(2)_R\) symmetric version, as has been investigated in some detail by Chanowitz and Golden [4]. More recent papers [5, 6, 7] have taken a further look at the renormalization problems [7] and the phenomenology of this model.

From the tree level point of view the Higgs sector presents itself as follows [8]

\[
\Phi = \begin{pmatrix} \phi^o & \phi^+ \\ \phi^- & \phi^* \end{pmatrix}, \quad X = \begin{pmatrix} \chi^o & \xi^+ & \chi^{++} \\ \xi^- & \chi^o & \xi^+ \\ \chi^{--} & \xi^- & \chi^o \end{pmatrix} \quad (2)
\]

denoting a \((1, 1/2)\) and a \((1, 1)\) multiplet of \(SU(2)_L \otimes SU(2)_R\), with hypercharge assignments of \(Y = 1, 0\) and \(2\) for the doublet, the real and the complex triplet respectively and with the following phase conventions: \(\phi^- = -\phi^+\), \(\chi^- = -\chi^+\), \(\xi^- = -\xi^+\), \(\chi^{--} = \chi^{++}\) and \(\xi^o = \xi^o\). They transform under \(SU(2)_L \otimes SU(2)_R\) global transformations as \(\Phi \rightarrow U_L \Phi U_R^\dagger\) (\(\Phi \leftrightarrow X\)), with \(U_L, U_R\) being the transformation matrices in the appropriate representation. The restricted Higgs potential takes the form [4]

\[
V(\Phi, X) = \lambda_1 (Tr[\Phi^\dagger \Phi] - a^2)^2 + \lambda_2 (Tr[X^\dagger X] - b^2)^2 + \lambda_3 (Tr[\Phi^\dagger \Phi] - a^2 + 2 Tr[X^\dagger X] - 3b^2)^2 + \lambda_4 (Tr[\Phi^\dagger \Phi] Tr[X^\dagger X] - 2 Tr[\Phi^\dagger \Phi^T X^\dagger X^T]\) + \lambda_5 (3 Tr(X^\dagger X X^\dagger X) - 2 Tr[X^\dagger X X^\dagger X]) \quad (3)
\]

\[1\]Fit of experimental data with \(M_{\text{top}} = 100\ GeV.\]

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where the sum is taken over $i$ and $j$. The vacuum states $|\Phi\rangle_o=\frac{a}{\sqrt{2}}|1\rangle$ and $|X\rangle_o=b|1\rangle$ are defined by $V(|\Phi\rangle_o,|X\rangle_o)=0$ together with the positivity conditions

$$\begin{align*}
\lambda_1 + \lambda_2 + 2\lambda_3 & \geq 0 \\
\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 & \geq 0 \\
\lambda_4 & \geq 0 \\
\lambda_5 & \geq 0 .
\end{align*}$$

(4)

The two VEV’s $a$ and $b$ are related through the $W$-mass

$$M_W^2 = M_Z^2 \cos^2 \theta_w = \frac{g^2}{4} (a^2 + 8b^2) = \frac{g^2}{4} v^2$$

(5)

with $g$ being the $SU(2)_L$ coupling constant and $v (\sim 250 \text{ GeV})$ the VEV of the SM Higgs. This allows the definition of a mixing angle $\theta_H$ denoted by its cosine $c_H$, sine $s_H$ (and tangent $t_H \equiv \tan(\theta_H)$)

$$c_H = \frac{a}{v}, \quad s_H = \frac{2\sqrt{2}b}{v} .$$

(6)

Within the Higgs sector the gauged $SU(2)_L \otimes U(1)_Y$ can be regarded as a subgroup of $SU(2)_L \otimes SU(2)_R$ with $T^3_R$ playing the role of the hypercharge. A diagonal subgroup of $SU(2)_L \otimes SU(2)_R$ survives the spontaneous symmetry breakdown. The physical Higgs bosons will form multiplets, which are degenerate in mass, under this custodial $SU(2)_C$ global symmetry. Expressed in terms of the fields defined in eq(2) they are

$$\begin{align*}
H_{5}^{++} &= \chi^{++} \\
H_{5}^+ &= \frac{1}{\sqrt{2}}(\chi^+ - \xi^-) \\
H_3^o &= \frac{1}{\sqrt{6}}(\chi^o + \chi^{o*} - 2\xi^o) \\
H_1^o &= \frac{(\chi^o + \chi^{o*} + \xi^o)}{\sqrt{3}}
\end{align*}$$

$$\begin{align*}
H_{3}^+ &= c_H \frac{\chi^+ + \xi^+}{\sqrt{2}} - s_H \phi^+ \\
H_3^o &= c_H \frac{\chi^o - \chi^{o*}}{\sqrt{2}} + s_H \frac{\phi^o - \phi^{o*}}{\sqrt{2}} \\
H_1^o &= \frac{\phi^o + \phi^{o*}}{\sqrt{2}}
\end{align*}$$

(7)

with $H_5$, $H_3$, $H_1$ and $H_1^\prime$ denoting a $SU(2)_C$ fiveplet, triplet and two singlets respectively. The Goldstone triplet is orthogonal to the $H_3$-plet

$$\begin{align*}
G_3^+ &= s_H \frac{\chi^+ + \xi^+}{\sqrt{2}} + c_H \phi^+ \\
G_3^o &= s_H \frac{\chi^o - \chi^{o*}}{\sqrt{2}} - c_H \frac{\phi^o - \phi^{o*}}{\sqrt{2}} .
\end{align*}$$

(8)

In terms of those fields one sees that the quantity $c_H$ defined in eq(3) indeed denotes the cosine of a $SU(2)_L$ multiplet rotation to $SU(2)_C$ triplets. The masses are\footnote{Note that the positivity of the squared masses is guaranteed by eq(1).} \footnote{Note that by means of this definition $s_S$ takes values between $-\frac{1}{\sqrt{2}}$ and $+\frac{1}{\sqrt{2}}$.}

$$\begin{align*}
M_{H_5}^2 &= 3v^2(c_H^2\lambda_4 + s_H^2\lambda_5) \\
M_{H_3}^2 &= v^2\lambda_4 \\
M_{H_1, H_1^\prime}^2 &= 8v^2 \left( \frac{c_H^2(\lambda_1 + \lambda_3)}{3c_H s_H \lambda_3} \sqrt{\frac{3}{8}c_H s_H \lambda_3} \frac{3}{8}s_H^2(\lambda_2 + \lambda_3) \right) .
\end{align*}$$

(9)

It’s convenient to rotate the two $SU(2)_C$ singlets to their mass eigenstates $H_1^o$ and $H_1^\prime$. The rotation angle shall be denoted by its cosine $c_S$ and sine $s_S$. $H_1^o (H_1^\prime)$ defines the mass eigenstates with the larger $SU(2)_C$ doublet $(H_1^o)$, (triplet $(H_1^\prime)$) portion.
Here we consider only Dirac type Yukawa couplings. Of course, only those fields of eq(3) which have a nonzero doublet admixture will couple (at tree level) to the fermion sector. Due to $\alpha \lesssim v$ the Yukawa couplings are enhanced compared with the SM. They are (omitting an overall factor $ig$):

$$H_i^f f \tilde{f} : - \frac{M_f}{2M_W} \frac{1}{\epsilon_H}$$
$$H_3^f f \tilde{f} : \pm \frac{M_f}{2M_W} t_H \gamma_5$$
$$H_3^V \bar{v} e : \frac{M_3}{2M_W} t_H \frac{(1 + \gamma_5)}{\sqrt{2}}$$
$$H_5^\pm \bar{p} n : \frac{\mu}{2M_W} \left[ M_n \frac{(1 + \gamma_5)}{\sqrt{2}} - M_p \frac{(1 - \gamma_5)}{\sqrt{2}} \right] ,$$

where the $\pm$ in the second term refers to up, down type fermions, $p$ and $n$ denote up and down type quarks and, $\nu$ and $e$ stand for neutrino type and electron type leptons respectively. $V_{\mu n}$ is the CKM mixing matrix. In the following let $V$ denote either a $W$ or a $Z$ boson. The VVH type vertices of this model are (omitting an overall factor $ig g^\mu
u$):

$$W^+ W^- H_5^0 : \sqrt{2} M_W s_H, \quad Z Z H_5^0 : \frac{2}{\sqrt{3}} M_W c_H s_H$$
$$W^+ W^- H_3^0 : - \frac{M_W}{\sqrt{3}} s_H, \quad Z Z H_3^0 : \frac{2}{\sqrt{3}} M_W c_H s_H$$
$$W^+ W^- H_1^0 : \frac{2}{\sqrt{3}} M_W s_H, \quad Z Z H_1^0 : M_W c_H$$
$$W^+ W^- H_5^+ : - \frac{M_W}{\sqrt{3}} s_H .$$

Again the $H_i^f H_3^0$ mixing has been taken to be zero. The absence of $H_3^0 V V$ couplings is due to $H_3^0$ being CP-odd.

The seven parameters of the restricted potential (eq(3)) can be reexpressed in terms of the $W$-mass, the 4 Higgs masses, the singlet mixing ($c_S$) and $t_H = (\frac{\lambda}{\lambda_{min}})$.

Let’s first have a brief look at the limit $t_H \gg 1$ ($b \approx \sim v$). In this limit the mixing angle of the two singlets approaches zero: $c_S^2 = 1 - \frac{4}{9} \left( \frac{\lambda}{\lambda_{min}} \right)^2 t_H^2 + O(t_H^4)$. The $SU(2)_L$ doublet VEV generates the gauge boson masses and the Goldstone bosons are the same as in the SM (eq(8)). The field $H_3^0$ has SM $H f f$ and $H V V$ and suppressed $H H V$ couplings, whereas all other Higgs bosons have suppressed couplings to the fermion sector (eq(3)) and to the $H V V$ sector. The whole situation therefore resembles very much to the SM with $H_3^0$ playing the role of the SM Higgs boson. The main differences come from the non vanishing $H H V$ type couplings (which can serve to bound the masses of the other (not $H_3^0$) Higgs fields from below, and the fact that the other singlet $(H_3^0)$ is likely to be very light: $M_{H_3^0}^2 \approx 3 \frac{t_H}{\lambda_{min}} \frac{(\lambda_{min} + \lambda_2 + \lambda_3)}{(\lambda_{min} + \lambda_2 + \lambda_3)}$. The opposite limit $t_H \ll 1$ ($a \approx b$) is far more interesting, because here the $SU(2)_L$ triplet fields $(X)$ are the source of the SSB, generating the $W$ and $Z$ masses. Again there is almost no singlet mixing: $c_S^2 = 1 - \frac{4}{9} \left( \frac{\lambda}{\lambda_{min}} \right)^2 t_H^{-2} + O(t_H^{-4})$. Here we have the remarkable situation that $SU(2)_C$ multiplets either couple to the fermion sector $(H_1, H_3)$, those couplings being strongly enhanced with respect to the SM, or to the $H V V$ sector $(H_1, H_3)$, with couplings being roughly of the same order of magnitude as the SM ones, but not both. Again there is a field which is likely to be lighter than the others: $M_{H_3^0}^2 \approx 4 \frac{t_H}{\lambda_{min}} \frac{(\lambda_{min} + \lambda_2 + \lambda_3)}{(\lambda_{min} + \lambda_2 + \lambda_3)}$.

For medium values of $t_H \sim O(1)$ there can be significant $H_i^f H_3^0$, mixing allowing the mass eigenstates $H_3^0$ and $H_3^0$ to have SM like couplings in both the fermion and the $H V V$ sector. As in the

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4 Majorana type Yukawa couplings induced by the $SU(2)_L$ triplets can be introduced but are unlikely to be phenomenologically important unless $t_H \ll 1$.

5 For simplicity we have assumed no singlet mixing ($c_S = 1$) in the following expressions.

6 For a list of the $H V V$ type couplings see e.g.
previous case $H_3$ couples to the fermion sector while $H_5$ has $HVV$ couplings, but not the other way round.

The restricted potential (eq(3)) emerges after the additional parameters of the general $SU(2)_L \otimes U(1)_Y$ invariant potential have been fine tuned to be zero on a certain mass scale (e.g. on the $M_{H_3}$ shell). However, due to the running of those parameters this fine-tuned custodial symmetry won’t be exact anymore at another mass scale (e.g. the triplet mass shell $M_{H_3}$). It is therefore reasonable to understand the term fine-tuned custodial symmetry as an approximate symmetry, which is valid only at tree level. Fine tuning in our case means that the model looses its predictive power at the next-to-leading order level.

At this point one might be tempted to supersymmetrize the model. In fact, supersymmetry provides a solution to those fine tuning problems since it ensures that the parameters of the superpotential, from which the scalar potential derives, do not receive radiative corrections. This is due to cancellations among various diagrams (Non Renormalization Theorems). After supersymmetry has been softly broken those parameters will only receive logarithmic corrections. An extension of the supersymmetric SM containing one (necessarily complex) triplet with zero hypercharge $\xi$ has been recently studied by Espinosa and Quirós. In this model, however, the triplet VEV has to be kept small to fulfill the constraint $\rho \approx 1$. If we want to have large triplet VEV’s, then only one chiral triplet with nonzero hypercharge has to be added as becomes clear from the tree level relation

$$\rho = \frac{\sum_{\Phi} |<\Phi>_o|^2 (T_L(T_L+1)-(Y/2)^2)}{\sum_{\Phi} |<\Phi>_o|^2(Y/2)^2}$$

where the sum goes over all (complex) multiplets, $T_L$ denotes the largest eigenvalue of the $SU(2)_L$ generator $T_3$ in the appropriate representation and $Y$ is the hypercharge. Adding e.g. a $\chi : (3,-2)$ to $\xi : (3,0)$, $H : (2,1)$ and $H : (2,-1)$ would require $|<\xi>_o| = \frac{\sqrt{2}}{\sqrt{2}}$ for $\rho = 1$. But since $\chi$ misses its "counterpart" $(3,2)$ the Higgsinos remain massless and give rise to a chiral anomaly. One then has to introduce a third chiral triplet $\bar{\chi} : (3,2)$. The conditions for $\rho = 1$ are now $|<\chi>_o|^2 + |<\bar{\chi}>_o|^2 = 2|<\xi>_o|^2$, resembling, in a special case, to the non supersymmetric model: $<\chi>_o = <\bar{\chi}>_o = <\xi>_o = b$. In this model the scalar Higgs sector has 23 physical degrees of freedom corresponding to 2 doubly charged, 5 singly charged and 9 neutral scalars. The most general gauge invariant superpotential has 8 parameters (corresponding to the singlet combinations: $HH, \chi \bar{\chi}, \xi \xi, H\bar{H}, H\chi \bar{H}, \bar{H} \chi H, \xi \xi \xi, \chi \xi \bar{\chi}$). The soft breaking terms add another 13 parameters, which are, however, not independent if one assumes that soft breaking is produced by spontaneous breakdown of local supersymmetry. In this case the 13 parameters could, in principle, be computed by renormalizing the 4 independent parameters at the scale of supergravity breaking down to a low scale. In the end this leaves us with 12 parameters and 18 masses (including W and Z) indicating the existence of mass relations. Still there is quite a number of independent parameters. It is therefore clear that, although in the supersymmetrized model the parameters of the scalar potential can be restricted such that the theory has a vacuum which satisfies $\rho = 1$ at tree level, with stabilized fine tuning, such a model has difficulties with predictability due to the larger multiplet content and large number of parameters.

Resuming the discussion of the non supersymmetric model we see, that the Higgs sector, as presented before, undergoes minor changes in terms of an approximate custodial symmetry. Note that the quantity $t_H$ is only defined in the tree level approximation, since the VEV’s of $\chi$ and $\xi$ are not identical in general. Also there will now be mixings between $SU(2)_C$ representations of different dimensionalities, leading to a breaking of the mass degeneracy of the $SU(2)_C$ multiplets. This means that $\rho \neq 1$ in general, with $(\rho - 1)$ being small, of the order of 1-loop corrections, but not computable, since in this order additional new parameters of the theory also contribute. Similarly, new parameters contribute also to mixing effects breaking the custodial $SU(2)_C$.

\footnote{For the explicit form of this potential see e.g. \[7\].}

\footnote{Note that one cannot simply impose that those terms don’t exist, because they are automatically introduced by means of counterterms after the renormalization procedure.}

\footnote{Note that custodial $SU(2)_C$ symmetry is sufficient but not necessary for $\rho = 1$. One can, however, think of this model as an effective theory of a more fundamental structure from which this tuned property might emerge on some dynamical reason.}
In fact, at 1-loop level, all mixings, that respect charge and CP conservation, among Higgs and gauge boson fields are allowed now\footnote{H^3_{\nu}, consisting of the same imaginary field parts as G^u_{\nu}, has the same CP assignment (-1), whereas the other neutral scalars H^1_{\nu}, H^2_{\nu}, and H^3_{\nu} have CP (+1). It can be shown directly that the various 1-loop corrections to e.g. $H^3_{\nu} H^3_{\nu}$ mixing cancel each other \footnote{1}}. The fields $H^3_{\nu}$ and $H^3_{\nu}$ for instance couple now to the fermion sector not only by means of triangle diagrams, but also by $H^3_{\nu} H^3_{\nu}$ and $H^3_{\nu} H^3_{\nu}$ mixing terms. Decays of those fields to fermion pairs are therefore likely to be dominant below the $W$-threshold. Unfortunately the corresponding branching ratios cannot be computed due to the loss of next-to-leading order predictability.

When trying to constrain the Higgs sector of this model it seems reasonable first to concentrate on $t_H$, since it distinguishes between uninteresting ($t_H \ll 1$) and interesting ($t_H \gg 1$) regions of the parameter space. Strict upper bounds on $t_H$ can only be got with the help of unitarity arguments, because one can always render the enhanced Yukawa couplings phenomenologically negligible by demanding that the corresponding Higgs masses are big enough. Tree level partial wave unitarity constraints from $H_i^i t t^\dagger$ and $H_i^i H_i^i$ scattering combined with the lower bound on $M_{H_1} (\geq 5 \text{ GeV})$ coming from the $\Upsilon$ decay gives roughly 30 as an upper bound on $t_H$\footnote{1}. Unitivity limits coming from longitudinal $W_L W_L$ and $W_L Z_L$ scattering give upper bounds on $H_1^o$ and $H_5^o$ masses ($t_H \gg 1$) that are similar to the SM bounds, roughly $M_{H_1} M_{H_5} \leq 1 \text{ TeV}$\footnote{1}. Some ad hoc arguments show what the general unitarity limit on $t_H$ might look like: Assuming for instance that the $\lambda_i$ parameters are about similar in magnitude would yield $M_{H_1} M_{H_5} \leq 1 \text{ TeV}$. Combining this with constraints coming from $B\bar{B}$ mixing\footnote{1} (see below) results in limits like $t_H \leq \sim 10$.

The strongest bound on $t_H$ derives from a renormalization group argument, based on two simple assumptions: "perturbative unification" and "desert". First one wants perturbation theory to be valid up to a high scale (e.g. an unification scale $M_U$), furthermore one excludes new physics in between $M_W$ and $M_U$, so that the renormalization group equations (RGE) evolve all the way up to $M_U$ in an effective $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ theory. The point is that the RGE of the doublet Yukawa couplings have an infrared fixed point \footnote{2} and that those couplings inevitably blow up at some high scale if their value is close to or higher than this fixed point (at the $W$-scale). This upper bound on the Yukawa couplings gives upper bounds on the sum of the squared masses of quarks or leptons\footnote{1}. For the case of three families this results in an upper bound on the top mass\footnote{1} as

$$M_{t_{\text{top}}} \leq 245 \text{ GeV} \; .$$

which translates in our case to $M_{t_{\text{top}}} \leq 245 \cdot c_H (\text{GeV})$ or

$$t_H \leq \sqrt{\left(\frac{245 \text{ GeV}}{M_{t_{\text{top}}}}\right)^2 - 1} .$$

With given lower bounds on the top mass one gets the following upper bounds on $t_H$

$$t_H \leq 2.5 \quad \text{for } M_{t_{\text{top}}} > 92 \text{ GeV} ,$$

$$t_H \leq 4.4 \quad \text{for } M_{t_{\text{top}}} > 54 \text{ GeV} .$$

The first bound\footnote{1} is valid under the assumption that the decay $t \to H^+ b$ and other non-SM decays do not occur. It therefore requires that $M_{H_5}$ and $M_{H_3}$ both are bigger than $92 \text{ GeV}$. $M_{H_5}$ or $M_{H_3}$ below $92 \text{ GeV}$ asks for a decay mode independent lower bound on the top mass. Such a bound can be derived from the $W$ width ($M_{t_{\text{top}}} > 54 \text{ GeV}$)\footnote{1}. A low mass charged Higgs boson, therefore, strongly weakens the unitarity bound on $t_H$. The result given by eq.(13) is rather significant: it tells us that if we demand to have a Higgs sector with Higgs triplets such that the triplet contribution to the $W$-mass is as large as possible, the assumptions of "perturbative unification" and "desert" then restrict the triplet contribution to less than $78\%$ or $63\%$, respectively.

The decay $Z^0 \to H^+ H^-$ provides a $t_H$ independent lower bound on the five- and three-plet masses $M_{H_5}$ and $M_{H_3}$\footnote{4} as

$$M_{H_5}, M_{H_3} \geq 41.7 \text{ GeV} .$$


One might believe that, in addition to this high energy bound, there could also be a low energy bound coming from the mixing of the $B$ and $B$ mesons, since the field $H_3^+$ can give a dominant contribution, for high $t_H$, to the flavor changing neutral currents responsible for this mixing. This bound depends strongly on $t_H$ (i.e. the Yukawa couplings) and on $M_{top}$, in the sense that a low $M_{top}$ weakens the constraint as does a low $t_H$. It turns out that the unitarity bound on $t_H$ is too strong for the $BB$ bound to take effect. The 3-plet mass $M_{H_3}$ is not constrained by $BB$ mixing. Results for certain two Higgs doublet models [14, 17] can be directly adapted to our case. Fortunately there is another low energy bound, that does constrain the triplet mass. It is coming from the bottom decay $b \to s\gamma$, which is mediated by penguin diagrams involving the top, $H_3^+$ and $W$ fields. Again this constraint is weakened for low $M_{top}$ and low $t_H$. The results obtained in two-Higgs-doublet models [18] apply to our case directly. The combined constraints on $M_{H_3}$ are shown in figures (1).

The fiveplet mass $M_{H_5}$ is, in addition to eq.(16), indirectly constrained by the lower bounds on $M_{H_3}$. Combining eqs(1,2) one gets a lower bound on $M_{H_3}$ as

$$ M_{H_3} \geq \sqrt{\frac{3}{1 + t_H^2}} M_{H_3}. $$

Constraints on $M_{H_5}$ are shown in figure (2).

"$Z^o \to Z^* H_1^o$, $H_1^o \to$ hadrons" bremsstrahlung can be used to bound the $H_1^o$ mass from below. Results for the SM Higgs boson [14] translate to our case with help of the ratio

$$ B = \frac{\Gamma(Z^o \to Z^* H_1^o)}{\Gamma(Z^o \to Z^* H_{SM}^o)} = \frac{c_{S}^2 c_{H}^2}{c_{S}^2 s_{H}^2} + \frac{8}{3} s_{S}^2 s_{H}^2 + 2 \sqrt{\frac{8}{3}} s_{S} s_{H} c_{H} s_{H}. $$

(18)

Those bounds are displayed in figure (3). The $Y$ decay yields the additional low energy bound [2]

$$ M_{H_1^o} \geq \sim 5 \text{ GeV} $$

(19)

as long as the Yukawa couplings of this field are enhanced with respect to the SM, or in other words as long as the condition $c_{S}^2 c_{H}^2 / c_{H}^2 \geq 1$ is met. For the case of zero mixing this is always the case. This bound is also valid for the field $H_1^o$ if $s_{H}^2 / c_{H}^2 \geq 1$.

In the limit of $t_H \ll 1$ and $c_{S} \approx 1$ (no $H_1^o H_1^o$ mixing) the field $H_1^o$ is likely to be lighter than the other scalar Higgs fields, but it can not be constrained due to its strongly suppressed HVV and Hff type vertices. In this model, therefore, it is possible to have a very light Higgs boson, that escapes detection.

Let’s now concentrate for a moment on the properties of $H_3^o$. Assuming that $M_{H_3^o} \leq 92$ GeV weakens, as mentioned before, the unitarity bound on $t_H$ (4.4). An $H_3^o$ below the $Z$ threshold, therefore, could be produced by bremsstrahlung ($Z^o \to Z^* H_3^o$) at LEP with rates comparable to the SM. The relative production rates are equal or enhanced, compared to the SM, for $t_H$ being in the interval [1.8, 4.4] with a maximal value of 1.27 for $t_H = 4.4$. Such a Higgs could also be produced through WW fusion at hadron colliders, but there the production rate is suppressed by a factor of three or more compared to the case of SM Higgs production.

The decay channels of $H_3^o$ within the Higgs sector are $H_3^o \to H_2^o H_1^o$, and $H_3^o \to H_2^o H_3^o$ or $H_3^o H_4^o$. A "close to the unitarity limit" ($t_H > 1.8$) $H_3^o$ below the $W$, $H_1^o$, and $H_3$ threshold could therefore only decay through $H_3^o H_1^o$, $H_3^o$, $V^* V^*$, $H_3^o H_3^o$, 1-loop and SU(2)$_C$ violating mixing diagrams, since it has no tree level couplings to the fermion sector (eq(13)). Decays to $f \bar{f}$ pairs are mediated by triangle diagrams involving gauge bosons and members of the $H_3$-plet and by the $H_2^o H_2^o$ mixing term, which is taken to be of the same order of magnitude as other 1-loop diagrams. Such a $H_3^o$ could therefore have a relatively large (~10%) branching ratio to $2\gamma$'s given by 1-loop diagrams, with all the charged particles, that couple to $H_3^o$, running around the loop. Again it should be emphasized that, due to the fine tuning in this model, the corresponding branching ratio cannot be computed.

Let’s now focus on the Higgs triplet models where the $\rho \approx 1$ constraint is satisfied by imposing a custodial SU(2)$_C$ symmetry on the potential, the gauge boson masses still can be dominated.

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11Again we consider only the "no singlet $H_1^o H_1^o$ mixing" sublimit.
12Compare with the 2\gamma branching ratios of the SM Higgs boson (see e.g. ref. [14]).
by triplet contributions. Unfortunately (or fortunately?) this idea can only be implemented invoking fine tuning of some of the parameters of the general $SU(2)_L \otimes U(1)_Y$ invariant potential, which leads to a loss of the next-to-leading order predictability. The assumptions of ”perturbative unification” and ”desert” then lead to a renormalization group argument which requires that at least $\approx 20\%$ of the $W - mass$ must come from doublet contributions. Close to this limit a neutral scalar with a mass below the $W$-threshold could have a large $B(H^0 \rightarrow 2\gamma)$ branching ratio, and could be produced with rates comparable to the SM at LEP through $Z^0 \rightarrow Z^0*H^0$ bremsstrahlung. In the other limit, with the gauge boson masses produced by the $SU(2)_L$ doublet field, the model resembles very much to the SM. Although here one of the neutral scalars could be very light and still escape detection.

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Figure captions

Figures 1
Combined bounds on $M_{H^5}$, for $M_{H^5}$ being smaller (A) (resp. higher (B)) than $92 \text{ GeV}$. "$\tan(\theta_H) \equiv t_H$" denotes the tangent of the $SU(2)_L$ doublet triplet mixing angle as defined in eq(6). In both diagrams the high energy bound of eq(16) is represented by the dashed line, the solid line refers to the unitarity bound of eq(15) and the dash-dotted line denotes the constraint coming from $b \to s\gamma$ decay [18]. In both figures the shaded region is excluded. The dotted line in picture (B) and the upper dotted line in picture (A) represent the $B\bar{B}$ mixing bound for "nominal" values of the hadronic matrix elements, that enter the computation of $B\bar{B}$ mixing, as used in [16]. For comparison the $B\bar{B}$ bound for "pessimistic" values of the parameters (which would correspond to an actual bound) has been plotted in figure (A) (lower dotted line). It obviously doesn’t cover any new parameter space.

Figure 2
Combined bounds on $M_{H^5}$. Again the horizontal dashed line denotes the high energy bound of eq(16), whereas the vertical solid line marks the unitarity limit on $\tan(\theta_H)(\equiv t_H)$. The dotted line represents the bound coming from eq(17) and eq(16). The $b \to s\gamma$ constraint as plotted in fig(1) yields together with eq(17) the dash-dotted line. The shaded region is excluded. Again the strict bound from $B\bar{B}$ mixing ("pessimistic" values) covers no new parameter space. For $\tan(\theta_H) = 0$ the limit reads $M_{H^5} > 72 \text{ GeV}$.

Figure 3
Bremsstrahlung bound on $M_{H^5}$ for the case of zero singlet mixing ($s_S = 0$ (dashed)) and maximal singlet mixing ($s_S = \frac{1}{\sqrt{2}}$ (dotted), $s_S = -\frac{1}{\sqrt{2}}$ (dot-dashed)) and for $\tan(\theta_H) < 2.5$, which corresponds to the stronger bound on $\tan(\theta_H)$ as given in eq(18). This experimental bound represents the combined results of the four LEP experiments for the process $Z \to Z^* H^0$ with $H^0$ decaying to hadrons [14]. In each case the region below the corresponding line is excluded.