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Robustness of the Starobinsky inflationary model

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Abstract. The first inflationary model conceived was the one proposed by Starobinsky which includes an additional term quadratic in the Ricci-scalar $R$ in the Einstein-Hilbert action. The model is now considered a target for several future cosmic microwave background experiments given its compatibility with current observational data. In this paper, we analyze the robustness of the Starobinsky inflation by inserting it into a generalized scenario characterised by an additional parameter $\beta$. In the Einstein frame, the generalized model recovers the original model for $\beta = 0$ whereas $\forall \beta \neq 0$ represents an extended class of models that admits a wider range of solutions. We investigate limits on $\beta$ from current cosmic microwave background and baryonic acoustic oscillation data and find that only a small deviation from the original scenario is allowed, $\beta = -0.08 \pm 0.12$ (68% C.L.), which is fully compatible with zero and confirms the robustness of the Starobinsky inflationary model in light of current observations.

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1 Introduction

The inflationary framework yields a viable explanation for some problems of the Big Bang cosmology, as well as for the process of growth of the primordial cosmological perturbations which produced the observed large-scale structures and temperature fluctuations in the Cosmic Microwave Background (CMB). The simplest models of inflation involve a single scalar field $\phi$ slowly rolling down its potential $V(\phi)$, which generates primordial scalar perturbations with a nearly scale-invariant power spectrum [1, 2] (see also [3] for a recent review). The recent CMB observations [4, 5] have not only confirmed this framework but also allowed to test the observational viability of a number of inflationary models (see e.g. [6]).

Although the majority of models of inflation involve scalar fields, the very first model proposed was driven by quantum corrections to the Einstein-Hilbert Lagrangian [7] (usually called Starobinsky or $R^2$ inflation), i.e.,

$$S = \frac{M_{Pl}^2}{2} \int d^4x \sqrt{-g} \left( R + \frac{R^2}{\mu^2} \right),$$

which includes a quadratic term of Ricci scalar, $R^2$, that dominates the Lagrangian density during the primordial universe – in the above expression, $M_{Pl}$ is the Planck Mass and $\mu$ is a given mass scale. The equivalence between Einstein and Jordan frames through a conformal transformation of the metric allows to deal with an inflaton potential of type

$$V(\phi) = V_0 \left[ 1 - \exp \left( -\frac{\sqrt{2}}{3} \frac{\phi}{M_{Pl}} \right) \right]^2,$$

where $V_0$ is the amplitude of the potential. The above expression is the equivalent of the $R^2$ contribution to the Lagrangian density and describes a class of potentials that obeys the slow-roll approximation, necessary for inflation to happen and produce the inhomogeneity pattern observed in the CMB data.

From the theoretical side, recent investigations have shown that inflationary potentials of several unrelated inflationary models coincide, leading to identical predictions for the slow-roll parameters $n_s$ and $r$, which well fit observational data [5]. The original Starobinsky model, for instance, is a particular case whose potential emerges in i) the Higgs model with a non-minimal coupling to gravity, $\xi \phi^2 R - \frac{1}{4} (\phi^2 - v^2)^2$, for $\xi < 0$, in the limit $1 + \xi v^2$ [8], ii) as a simple conformally invariant theory with spontaneous symmetry breaking, in the
context of superconformal theory and supergravity [9, 10], and iii) in the large field regime of a Superconformal D-Term Inflation [11].

From the observational viewpoint, analyses of different classes of inflationary models using current CMB data have shown that the Starobinsky model provides an excellent fit to the data [6], being now considered as a “target” model for some planned CMB experiments (see e.g. [12–14]). The model predicts a spectral index \( n_s \simeq 0.96 \) with a small spectral running and also a small amount of gravitational waves. Given its compatibility with current observational data, extensions of the Starobinsky model have been investigated. For instance, a simple extension including an extra scalar field was studied in [15]. Furthermore, attempts in the context of higher derivative theories of the type \( R^2p \) and other extensions of the Starobinsky \( R^2 \) model were also considered in [16–18]. The analysis performed in [16] considered the Einstein frame in searching for deviations from the benchmark value of the tensor amplitude for the case with \( p \simeq 1 \), which recovers the Starobinsky model. It was found that the original Starobinsky model provides an excellent fit to the CMB data, despite the fact that uncertainties on \( n_s \) may modify the expected value of \( r \).

Our goal in this paper is to investigate the robustness of the Starobinsky scenario in light of current observational data. In principle, to check the robustness or validity of a theory or model, it is important to insert it into a more general framework. Here, we introduce the following potential (hereafter \( \beta \)-Starobinsky model)

\[
V(\phi) = V_0 \left[ 1 - \left( 1 - \beta \sqrt{\frac{2}{3M_{Pl}}} \phi \right)^{\frac{1}{\beta}} \right]^2 ,
\]

which fully recovers the Starobinsky potential (1.2) for \( \beta = 0 \) whereas \( \forall \beta \neq 0 \) represents a generalized model that admits a wider range of solutions. Note also that constraints on the parameter \( \beta \) quantify directly the allowed deviations from the original model and, therefore, its robustness with respect to increase in the number of degrees of freedom and also to the observational data. In the potential (1.3), we use the well-known relation \( \lim_{\beta \to 0} (1 + \beta x)^{1/\beta} = e^x \). Inflationary models driven by generalized exponential potentials have been investigated in [19–21].

We organize this paper as follows. In Sec. II, we discuss the main features of the potential (1.2) through a slow-roll analysis and compare its theoretical predictions with the latest results of the Planck Collaboration. Sec. III presents the method employed to calculate the theoretical predictions for the amplitude of fluctuations of the CMB temperature and the statistical analyses performed using the current CMB data. A discussion of the main results of our analysis is shown in Sec. IV. We present our conclusions in section V.

### 2 \( \beta \)-Starobinsky inflation

In this section, we discuss some theoretical predictions of the \( \beta \)-Starobinsky potential given by Eq. (1.3). As mentioned above, the free parameter \( \beta \) quantifies how much it deviates from the current best-fit inflationary model (\( \beta = 0 \)). Figure 1 shows the behaviour of the potential (1.3) for some arbitrary values of \( \beta \), and note that for both intervals \( \beta < -4 \) and \( \beta > 0.3 \) the potential behaviour differs significantly from the Starobinsky model. As expected, in the limit \( \beta \to 0 \) the potential (1.2) is fully recovered. Furthermore, for \( \beta < 1.2 \) one finds a large-field behaviour when \( 0 < \phi < 10 \) whereas for values of \( \beta > 1.2 \) we do not retrieve the large-field behaviour.
As is well known, one can characterize the slow-roll inflationary regime by parameters that depend on the field potential and its derivatives w.r.t the scalar field $\phi$ (denoted by the prime in the equations below). The slow-roll parameters for the model under consideration can be written as

$$
\epsilon(\phi) = M_{Pl}^2 \left[ \frac{V'(\phi)}{V(\phi)} \right]^2 = \frac{4}{3} \chi^{\frac{2}{3}}-2 \left(1 - \chi^{\frac{1}{3}} \right)^{-2},
$$

(2.1a)

$$
\eta(\phi) = M_{Pl}^2 \frac{V''(\phi)}{V(\phi)} = \frac{4}{3} \chi^{\frac{1}{3}}-2 \left[ \beta - 1 - (\beta - 2) \chi^{\frac{1}{3}} \right] \left(1 - \chi^{\frac{4}{3}} \right)^{2},
$$

(2.1b)

where $\chi = 1 - \beta \sqrt{\frac{2}{3} M_{Pl}}$.

Inflation happens while $\epsilon, \eta \ll 1$ and the condition $\epsilon(\phi) = 1$ defines the value of the field $\phi$ when inflation ends, $\phi_{end}$. Since (2.1a) does not allow a direct inversion one needs to solve it numerically. We interpolated the points of $\beta$ and $\phi$ that satisfies the constraint $\epsilon(\phi) = 1$ with two polynomial fits of 12th order, which are solutions of (2.1a): one is valid for $\phi_{end} > 0$ and the other for $\phi_{end} < 0$, and we call them solutions 1 and 2, respectively. Note that the Starobinsky model must be recovered when $\beta \rightarrow 0$, which happens only for the solution 1 (with $\phi_{end} \sim 0.94$). Hence, we discard the solution 2 as a viable extension of the Starobinsky model and, throughout this paper, we focus only on the investigation of the solution 1.

The potential amplitude, $V_0$, is obtained considering the primordial power spectrum of curvature perturbations, calculated when the CMB mode exits from horizon at the scale $\phi_*$,

$$
P_R = \frac{1}{24 \pi^2} \frac{V(\phi)}{\epsilon} \bigg|_{k=k_*}.
$$

(2.2)

The value of $P_R(k_*)$ is determined by Planck normalization, i.e., $2.0933 \times 10^{-9}$ for the pivot choice $k_* = 0.05 \text{Mpc}^{-1} [5]$. Combining Eqs. (1.3) and (2.2), and inverting for $V_0$, we obtain

$$
V_0 = \frac{32 \pi^2 P_R \chi_*^{2/\beta - 2}}{(1 - \chi_*^{1/\beta})^4},
$$

(2.3)
where $\chi_\star \equiv 1 - \beta \sqrt{\frac{2}{3} M_{Pl}}$.

In order to find the value of $\phi_\star$ we consider the number of e-folds,

$$N = \int_{\phi_{\text{end}}}^{\phi} \frac{d\phi}{\sqrt{2 \epsilon}} = \int_{\phi_{\text{end}}}^{\phi} \frac{3}{8} \left[ \left( 1 - \sqrt{\frac{2}{3} \beta \frac{\phi}{M_{Pl}}} \right)^{1-1/\beta} - 1 + \beta \sqrt{\frac{2}{3} M_{Pl}} \right] d\phi$$

$$= \sqrt{\frac{3}{8} M_{Pl}} \left[ - \left( \frac{\phi}{M_{Pl}} \right) + \beta \frac{2}{3} \left( \frac{\phi}{M_{Pl}} \right)^2 - \sqrt{\frac{3}{2 (2 \beta - 1)}} \left( 1 - \beta \sqrt{\frac{2}{3} M_{Pl}} \right)^{2-1/\beta} \right]_{\phi_{\text{end}}},$$

with $\phi = \phi(N)$. Again, this expression cannot be inverted and then we solve it numerically for $\phi_\star(N)$. In the case in which the pivot scale crosses the Hubble horizon during inflation, we find the values of $\phi_\star$ and $\beta$ for which $N_\star = 55$ is valid and interpolate with a polynomial fit for $\phi_\star$. Correspondingly, the value of the field in the beginning of inflation $\phi_{\text{ini}}$ is obtained when considering $N_{\text{ini}} = 70$ in Eq. (2.4). Similarly to $\phi_{\text{end}}$, the polynomial fits for $\phi_\star$ and $\phi_{\text{ini}}$ are of 12th order and retrieve the Starobinsky model in the limit $\beta \to 0$. In addition, the slow-roll conditions are fully met for values of $-4 < \beta < 0.6$.

Finally, the spectral index, $n_s$, and the tensor-to-scalar ratio, $r$, are written as

$$n_s = 1 + \frac{8}{3} \left( \frac{\chi^2}{1 - \chi^2} \right)^2 \left[ \beta \left( 1 - \chi^2 \right) - 1 - \chi^{1/\beta} \right],$$

$$r = \frac{64}{3} \left( \frac{\chi^2}{1 - \chi^2} \right)^2 \left( 1 - \chi^{1/\beta} \right)^{-2},$$

and it will continue to operate across the text also.

$$r = 8(n_s - 1) \chi^{1/\beta} \left[ (\beta - 1) - \beta \chi^{1/\beta} \right].$$

The $n_s - r$ plane is shown in figure 2. We display different values of $\beta$ satisfying the Eq. (2.4) and consider two different numbers of e-folds, i.e., $N = 50$ and $N = 60$. The contours correspond to 68% and 95% confidence levels (C.L.) obtained from the most recent Planck CMB data [5]. Notice that the values of $n_s$ and $r$ increase as the value of $\beta$ decreases. These results are not very restrictive because all the values predicted are within the 95% region. The constrained values of $\beta$, $-4 < \beta < 0.2$, are consistent both with Planck results at $2\sigma$ and with the slow-roll regime discussed earlier. Finally, it is also worth mentioning that even if the theoretical predictions of a given model are in agreement with the $n_s - r$ plane, it does not necessarily mean that it is a good model when compared with other inflationary scenarios [22]. Therefore, in what follows we will analyze the predictions of the power spectrum of temperature fluctuations and compare them with current CMB data.

3 Method and analysis

The theoretical predictions of the $\beta$-Starobinsky model are calculated modifying the latest version of the Code for Anisotropies in the Microwave Background (CAMB) [23], to include the $\beta$ parameter, since in its standard realization it assumes a power-law parameterization for the primordial perturbation spectrum, $P_R = A_s (k/k_*)^{n_s-1}$. In this context, we modify CAMB following the lines of the MODECODE adapted for our primordial potential choice,
in order to calculate the dynamic and perturbations of our model and then construct the primordial power spectrum.

MODECODE calculates the spectrum of CMB temperature fluctuations solving numerically the equations of inflationary dynamics, namely the Friedmann and Klein-Gordon equations, as well as the Fourier components associated with curvature perturbations produced by the fluctuations of the scalar field $\phi$. These components are solution of the Mukhanov-Sasaki equations [1, 2]

$$u''_k + \left( k^2 - \frac{z''}{z} \right) u_k = 0,$$

where $u \equiv -zR$ and $z \equiv a\dot{\phi}/H$. The primordial power spectrum of curvature perturbations $P(k)$ defined as function of the vacuum expected value of $R$ is

$$\langle R^*(k)R(k') \rangle \equiv \frac{2\pi^2}{k^3} \delta^3(k-k')P(k),$$

where $\delta$ is the Dirac delta function and the factor $2\pi^2/k^3$ is chosen to obey the usual Fourier conventions. It then follows that $P_R(k)$ is related with $u_k$ and $z$ via:

$$P_R(k) = \frac{k^3}{2\pi^2} \left| \frac{u_k}{z} \right|^2.$$
Figure 3. The theoretical predictions for the angular power spectra considering different values of $\beta$.

Table 1. Priors on the cosmological parameters considered in the analysis.

| Parameter | Prior Ranges         |
|-----------|----------------------|
| $\Omega_b h^2$ | [0.005 : 0.1]       |
| $\Omega_c h^2$ | 0.001 : 0.99        |
| $\theta$        | [0.5 : 10.0]        |
| $\tau$          | [0.01 : 0.8]        |
| $\beta$         | [-4 : 0.2]          |

evaluates the spectrum of curvature perturbations when the mode crosses the horizon. The theoretical predictions of the $\beta$–Starobinsky potential are shown in figure 3. Note that the effect of the parameter $\beta$ is to slightly modify the amplitude of the temperature power spectrum.

In order to constrain the cosmological parameters associated with the $\beta$-Starobinsky model we perform an analysis using the latest version of COSMOMC code [24], necessary to explore the cosmological parameter space. In addition to the parameter $\beta$ we also vary the usual cosmological variables, namely, the baryon and the cold dark matter density, the ratio between the sound horizon and the angular diameter distance at decoupling, and the optical depth: \{ $\Omega_b h^2$, $\Omega_c h^2$, $\theta$, $\tau$ \}. We consider purely adiabatic initial conditions, fix the sum of neutrino masses to 0.06 eV and the universe curvature to zero, and also vary the nuisance foregrounds parameters [25]. The flat priors on the cosmological parameters used in our analysis are shown in Table 1. Moreover, the interval of values of the parameter $\beta$ is chosen from the considerations made in the previous section, i.e., $-4 < \beta < 0.2$ (see e.g. figure 2).
Table 2. 68% confidence limits and best fit values for the cosmological parameters. The first columnblock show the constraints on the parameters of the Starobinsky and $\beta$-Starobinsky models, using the extended data set, i.e. the joint PLC+BAO+BKP15 data. The table is divided into two sections: the upper section shows the primary parameters, while in the lower part shows the derived ones and lastly the BIC values. The values indicated with (*) are calculated for the pivot choice of $N = 55$.

| Parameter | Starobinsky | $\beta$-Starobinsky |
|-----------|-------------|----------------------|
|           | mean | best fit | mean | best fit |
| Primary   |       |        |       |        |
| $\Omega_b h^2$ | 0.02218 ± 0.00018 | 0.022276 | 0.022198 ± 0.00019 | 0.022227 |
| $\Omega_c h^2$ | 0.1195 ± 0.0009 | 0.11916 | 0.1192 ± 0.0009 | 0.11897 |
| $\theta$ | 1.04092 ± 0.00041 | 1.040603 | 1.04098 ± 0.00041 | 1.040920 |
| $\tau$ | 0.0526 ± 0.0028 | 0.0547 | 0.0542 ± 0.0044 | 0.0523 |
| $\beta$ | – | – | –0.08 ± 0.12 | –0.11 |
| Derived  |       |        |       |        |
| $H_0$ | 67.37 ± 0.40 | 67.46 | 67.50 ± 0.41 | 67.60 |
| $\Omega_m$ | 0.3136 ± 0.005 | 0.3122 | 0.3119 ± 0.006 | 0.3104 |
| $\Omega_\Lambda$ | 0.6864 ± 0.005 | 0.6878 | 0.6881 ± 0.006 | 0.6896 |
| $n_s$ | – | 0.9652* | – | 0.9675* |
| $r_{0.002}$ | – | 0.0035* | 0.0044 ± 0.0018 | 0.0048 |
| $\Delta$BIC | Reference | Positive |

We use the CMB data set from the latest Planck (2018) Collaboration release [5], considering the high multipoles Planck temperature data from the 100-,143-, and 217-GHz half-mission T maps, and the low multipoles data by the joint TT, EE, BB and TE likelihood, where EE and BB are the E- and B-mode CMB polarization power spectrum and TE is the cross-correlation temperature-polarization (hereafter PLC18). We also combine the CMB data with an extended background data sets composed of i) Baryon Acoustic Oscillations (BAO) from the 6dF Galaxy Survey (6dFGS) [26], Sloan Digital Sky Survey (SDSS) DR7 Main Galaxy Sample galaxies [27], BOSSgalaxy samples, LOWZ and CMASS [28] and ii) the tensor amplitude of B-mode polarization from 95, 150, and 220 GHz maps, which are the tightest and least model-dependent constraints on the tensor amplitude coming from the Keck Array and BICEP2 Collaborations [29, 30] analysis of the BICEP2/Keck field, in combination with Planck high-frequency maps to remove solve the Boltzmann equations and to e polarized Galactic dust emission, used to constrain the parameters associated with the tensor spectrum (hereafter BK15).

4 Results

The main results of our analysis are shown in Table 2, where we summarize the constraints on the cosmological parameters of the Starobinsky and $\beta$-Starobinsky models obtained using the Planck 2018 likelihood combined with BAO and Bicep/Keck 2015 data. We also show in figure 4 the confidence intervals at 68% and 95% and the posterior probability distribution for the most interesting behaviours. As we can see in the second column of Table 2, all the primary and the derived cosmological parameters of $\beta$-starobinsky model are consistent within 1$\sigma$ with the standard Starobinsky inflation. We found no evidence for a non-zero $\beta$ parameter, which is allowed to vary within the range $-0.08 \pm 0.12$ (1$\sigma$). These results are
Figure 4. Two-dimensional probability distribution and one-dimensional probability distribution for the \( \beta \)-Starobinsky model (green contours) and the reference Starobinsky model (blue contours), both using the extended dataset (PLC18 + BAO + BK15). The dotted lines indicate the predicted value for the tensor-to-scalar ratio \( r \) for the standard Starobinsky model.

also consistent with previous analyses [16], which have investigated a generalization of the Starobinsky inflation of the type \( f(R) \propto R^{2p} \) and found \( p \simeq 1 \).

As discussed earlier, MODECODE calculates the spectrum of CMB temperature fluctuations from the numerical solutions of inflationary dynamics, instead of a power-law parametrization in terms of the scalar amplitude \( A_s \) and the spectral index \( n_s \). This amounts to saying that the analyses we performed for both Starobinsky and \( \beta \)-Starobinsky models did not obtain direct constraints on those parameters, but we still can derive the spectral index through the Eq. (2.5a) (see the derived \( n_s \) values tagged with 's' in Table 2). The constraints on tensor-to-scalar ratio \( r \) for the \( \beta \)-Starobinsky model displayed in figure 4 and Table 2 show
Figure 5. The best-fit angular power spectrum for the standard and \( \beta \)–Starobinsky models. The data points correspond to the latest release of Planck data [5] and the lower panel show the residuals with respect to the reference model (Starobinsky).

perfect agreement with Starobinsky inflation within 1\( \sigma \) for the theoretical value calculated here, with the upper limit reported in Planck 2018 release \((r < 0.106 \text{ at } 95\% \text{ C.L.})\) and also with the upper limit of \( r > 0.0017 \) at 95\% C.L. found by [16].

Finally, in the last line of Table 2 we also show the Bayesian Information Criterion (BIC), which consider only the point that maximizes the posterior probability distribution to compare the models, taking into account both the number of data points and the number of free parameters of the models under consideration. The BIC value is given by [31]

\[
\text{BIC} = -2 \ln \mathcal{L}(d|\theta) + k \ln N,
\]

where the number of free parameters are \( k = 4 \) and \( k = 5 \), for Starobinsky and \( \beta \)–Starobinsky models, respectively. We can rank the models using the \( \Delta \text{BIC} \equiv \text{BIC}_i - \text{BIC}_{\text{ref}} \) value, which represents the preference of the reference model over model \( i \), with \( \Delta \text{BIC} \leq 2 \), \( 2 < \Delta \text{BIC} \leq 6 \), \( 6 < \Delta \text{BIC} \leq 10 \) and \( \Delta \text{BIC} \geq 10 \) meaning weak, positive, strong and very strong support for the reference model, respectively [32]. We compare our generalized model with the original Starobinsky model and find \( \Delta \text{BIC} = 2.3 \), which means that the Starobinsky model has a positive preference over the extended \( \beta \)–Starobinsky scenario. Therefore, even providing a good description of the data (for a small deviation of the Starobinsky model, \( \Delta \beta = \pm 0.12 \)), the generalized scenario is penalized by the presence of an extra parameter, that is, the data do not justify the extension of the Starobinsky model, preferring the minimum model. This result, therefore, reinforces the robustness of Starobinsky model to describe the primordial inflationary phase.
5 Conclusions

In order to check the robustness or validity of a model, it is important to insert it into a more general framework. In this work, we have proposed and analyzed an extension of the Starobinsky model motivated mainly by its remarkable observational success and by the works of [9, 15, 16], which show that the Starobinsky model can be retrieved by different approaches.

Using the most recent CMB measurements along with BAO and B-mode polarization data, we have found that only small departures from the Starobinsky inflation is allowed within the range of $\beta = -0.08 \pm 0.12$ (68% C.L.), which implies a tensor-to-scalar ratio of $r_{0.002} = 0.0044 \pm 0.0018$ (68% C.L.). Such a result is in a good agreement with the observational data, as shown in figure 5.

As pointed out in [16], the prediction of the Starobinsky model carries the uncertainties on $n_s$, thus $r$ could be due not to a real presence of tensor perturbations in Planck data but rather arising from the correlation between $r$ and $n_s$. Considering a generalisation of the type $R^{2p}$ in the Einstein-Hilbert action (1.1), these authors found limits on $r$ of the order of $r < 0.04$. On the other hand, the limits derived in our analysis show that the predicted value of the tensor-to-scalar ratio by the Starobinsky inflation differs by 0.5$\sigma$ or, equivalently, $\Delta r \sim 0.0009$ from the estimate obtained in the context of our extended scenario. Although such a small difference is not expected to be detectable by some future CMB experiments, such as LiteBIRD satellite [13] or Simons Observatory [14], whose sensitivities are $\Delta r \sim 0.001 - 0.002$ [13, 14], it might be detected by the CMB-S4 [12], which is expected to reach the sensitivity of $\Delta r \sim 0.0006$.

Finally, despite the small deviations from the conventional Starobinsky model allowed by current observations, the BIC analysis indicates positive support for the Starobinsky model over the extended one. Therefore, the generalized potential proposed in this paper has allowed us to investigate the robustness of the Starobinsky inflation, and the statistical analysis performed has confirmed its remarkable success to describe current observational data.

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