Brane Boxes: Bending and Beta Functions

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**Abstract**

We study the type IIB brane box configurations recently introduced by Hanany and Zaffaroni. We show that even at finite string coupling, one can construct smooth configurations of branes with fairly arbitrary gauge and flavor structure. Limiting our attention to the better understood case where NS-branes do not intersect over a four dimensional surface gives some restrictions on the theories, but still permits many examples, both anomalous and non-anomalous. We give several explicit examples of such configurations and discuss what constraints can be imposed on brane-box theories from bending considerations. We also discuss the relation between brane bending and beta-functions for brane-box configurations.

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1 Introduction

Recently, following the ideas of [1], it has become possible to investigate non-perturbative properties of supersymmetric gauge theories by studying brane configurations in string and M theory [2–5]. However, constructing chiral $N = 1$ supersymmetric gauge theories has proven more difficult. Several classes of chiral gauge theories were constructed in Refs. [6–10]. An interesting development appeared in ref. [11]. In this paper it was shown that in the limit of vanishing string coupling it is possible to construct a large class of chiral theories (including models known to exhibit dynamical supersymmetry breaking). One would like to understand the dynamics of these brane constructions at non-zero coupling. In this paper, we take a step in this direction and discuss some of the properties of the brane configurations introduced in [11].

Let us briefly recall the setup of the model. Following [11], we consider field theory on D5-branes with world-volume in dimensions (012346). To obtain a four-dimensional low energy effective theory, we need these D-branes to be finite in two dimensions. Therefore we will let them end on NS5-branes with world volume in (012345) and NS$^\prime$5-branes with world-volume in (012367). In addition to dimensional reduction from 6 to 4 dimensions, the inclusion of NS-branes also reduces the amount of supersymmetry to $N = 1$, and in general gives rise to chiral theories. The basic configuration with one gauge group is shown in Figure 1. If we put $N_c$ D5-branes of finite extent into the box we obtain an $SU(N_c)$ gauge theory, while product group theories arise from configurations with several finite boxes. If we add (semi-infinite) D5-branes to the surrounding boxes this will give rise to matter in the fundamental representation given by strings between “flavor” and “color” D-branes as shown in Figure 1. By convention, incoming arrows correspond to anti-fundamental fields, while outgoing arrows correspond to fundamental fields (the same rules apply to the determination of charges under global symmetries). It is also possible to introduce more general matter content by adding D7-branes and orientifold 7-planes to the configuration.

The original [11] construction was done for zero string coupling. In this limit, one did not have to worry about the intersection of the D5 and NS- or NS$^\prime$-branes; all branes are orthogonal. It is much more interesting if one can understand the bending of the branes at nonzero coupling. This is important because it should somehow reflect the dynamics of the field theories they represent. Here we should distinguish between bending from perturbative and
non-perturbative effects; we will only address the former in this paper. A further motivation for the investigation of brane-bending is that it can potentially provide restrictions on the class of allowed theories. This is important because it appears that arbitrary configurations of the type described above (including those leading to anomalous field theories) are allowed.

In the case of weak coupling, it is possible to consider the bending of the NS- (NS') branes and check if there exist continuous non-intersecting solutions. A first step in this direction was made in [12]. It was required that asymptotically (for large values of \( x_7 \)) the angle under which the NS'-branes bend (and their \( x_4 \) coordinate) should be independent of the \( x_6 \) coordinate on the brane (an analogous conditions on the NS-brane bending did not lead to an independent constraint). The constraint required that:

\[
\begin{align*}
  n_{21} &= n_{11} - n_{12} + N_c \\
  n_{23} &= n_{13} - n_{12} + N_c \\
  n_{31} &= n_{33} + n_{11} - n_{13} \\
  n_{32} &= n_{33} + n_{12} - n_{13}
\end{align*}
\]  

While these conditions are sufficient to guarantee anomaly freedom, they also allowed only theories with \( N_f \geq N_c \). In particular, the constraints imposed by Eq. (1) do not allow theories to flow in parameter space to pure \( N = 1 \) SYM; neither do they allow one to construct SYM theory itself, which is rather unsatisfactory.

In this paper we will show that the conditions of [12] are too restrictive by demonstrating the existence of smooth solutions to the brane bending
problem. The existence of such solutions is to be expected given the new configurations proposed in [13] which avoid this restriction, though it was not explicitly demonstrated there why the conditions of [12] are unnecessary. The only constraint we derive from bending arises from restricting brane intersections. Unfortunately, while our “relaxed” conditions will exclude some anomalous models, in general anomalous theories are still permitted. Therefore, it remains an open question to find consistency conditions on the brane-box configurations (or understand how anomalies can be cancelled due to the inflow from the bulk theory in the spirit of [14]). Presumably this problem is solved by a better understanding of the underlying string degrees of freedom of the theory.

We will also show a connection between the running of the gauge coupling constant and the bending of the branes. We will first argue that in some simple configurations, the bending is logarithmic at large distances from the brane intersections, reflecting 4-dimensional physics. While we do not yet know the detailed connection between the running of the coupling constant and the brane bending in general cases, we will argue that the brane bending correctly reflects the leading term in the beta function.

The paper is organized as follows: In Section 2 we will find solutions for the brane bending in several illustrative cases and show that there exist smooth solutions for the brane in general configurations. In Section 3 we will discuss what restrictions might be imposed on brane-box configurations from the requirement that the NS- (NS’-) branes do not intersect over a 4-dimensional surface. In Section 4 we will discuss how the logarithmic running of the 4-dimensional gauge coupling is reflected in the bending of the NS-(NS’-) branes in some simple configurations. In Section 5 we will argue that for general configurations the beta function is encoded in the bending of the branes. We summarize our results in Section 6.

2 Bending Branes

Consider an NS’-brane in flat space. Its dynamics is governed by a Dirac-Nambu-Goto type action. Its equation of motion derived from the action is

\[
\frac{1}{\sqrt{-\text{det} \gamma}} \partial_a \left( \sqrt{-\text{det} \gamma} \gamma^{ab} \partial_b X \right) = 0,
\]  

(2)
where $\gamma_{ab}$ is the induced metric $\partial_a X^M \partial_b X_M$. The equation reduces to the flat space Laplace equation whenever $\gamma_{ab}$ is constant and proportional to the identity. If some other branes end on the NS-brane they will act as sources for the Laplace equation and modify the right hand side of (2).

Now consider a D5-brane ending on an NS$'$-brane. As a result the NS$'$-brane will buckle in the $x_4$ direction. Because of symmetry, the $x_4$ coordinate of the NS$'$-brane depends only on $x_7$, the distance from the D5-brane end point inside the NS$'$-brane. Consequently, for large values of $x_7$ (far away from the region where the source is strong), the buckling is determined by the one dimensional flat space Laplace equation\footnote{One could find a more precise solution by compactifying type IIB theory on a circle and solving this configuration as M-theory on a torus \cite{16}.}. The solution is given by:

$$x_4 = k N_c |x_7|,$$

where $N_c$ is the number of D5-branes and $k$ is the proportionality constant which vanishes in the limit $g_s \to 0$. (Note that we have chosen integration constants differently from \cite{15} so that the configuration is more symmetric; see Figure 2).

If the D5-branes also terminate on NS-branes, and thus are finite in the $x_6$ direction, the situation changes. In that case the solution of equation (2) is no longer independent of $x_6$, and one therefore has to solve a two dimensional Laplace equation. The solutions for the branes will not surprisingly look very different. For simplicity, we will solve the flat space Laplace equation which is only an approximation to the true equation of motion. As a consistency check, one can plug the solution into the full equation of motion (2) and verify that far away from the source we get back the flat space Laplace equation.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.png}
\caption{D5-branes ending on a NS$'$-brane}
\end{figure}
Generally, this means we trust our solution better far from the intersection. A relevant question to ask is what happens to supersymmetry when the D5-brane is finite and the NS-brane therefore bends in two dimensions as discussed in the previous paragraph. The answer can be found by examining the $\kappa$-symmetry of the 5-brane \textsuperscript{17}. We find that supersymmetry is preserved when the following matrix has zero eigenvalues

\[
1 + \frac{e^{a_1...a_6}}{\sqrt{-\det(\gamma + F)}} \left( \frac{1}{3!} F_{a_1 a_2} F_{a_3 a_4} F_{a_5 a_6} 
+ \frac{1}{2!} F_{a_1 a_2} F_{a_3 a_4} \gamma_{a_5 a_6} + F_{a_1 a_2} \gamma_{a_3...a_6} + \gamma_{a_1...a_6} \right). \tag{4}
\]

Here $\Gamma_{a_1...a_6} = \Gamma_{M_1...M_6} \partial_{a_1} X^{M_1} ... \partial_{a_6} X^{M_6}$, the pullback of the space-time gamma matrices and $F$ contains the field strength of the world volume gauge field. When the 5-brane is flat $F$ is zero and we get the usual equation for a supersymmetric configuration. When the brane is curved we find that we have to turn on some non-trivial $F$ field to preserve supersymmetry in analogy with \textsuperscript{18}.

We now consider explicitly the form of the solution in several relevant examples.

- The first example we consider is the one describing pure N=1 SYM theory. We will solve for the bending of the NS$'$-brane. In our simplified discussion of the NS$'$-brane bending the only role played by the NS-branes is to provide a point in the 6-7 plane on which the D5-branes can terminate. To find the bending of the NS$'$-brane caused by one D5-brane of length 2$L$ ending on it, we integrate the solution of the two-dimensional Laplace equation for a point-like charge along the line $x_7 = 0$, $-L < x_6 < L$ and find the solution:

\[
x_4 = 2x_7 \left( \arctan\left( \frac{L+x_6}{x_7} \right) + \arctan\left( \frac{L-x_6}{x_7} \right) \right) + 
(L + x_6) \log((L + x_6)^2 + x_7^2) + (L - x_6) \log((L - x_6)^2 + x_7^2) \tag{5}
\]

The first conclusion to be drawn from Eq. (5) is that there indeed exists a continuous and smooth solution, and thus this brane configuration is consistent. Note that when viewed far from the intersection (large $x_7$), the solution exhibits logarithmic (not linear) bending. This is to be expected, since we no longer have an infinite line source of charge. When viewed from far away, the finite line of charge appears point-like;
one expects logarithmic bending. This is similar to the bending of the NS-branes in the type IIA configuration [2]. On the other hand, close to where the D5-brane is located the solution looks almost linear. To illustrate this point we plot a picture of a NS'-brane being pulled by a D5-brane in Figure 3.

We next consider the solution for QCD with matter. We can add \( N_L = N_R = N_f \) semi-infinite D5-branes on the left and on the right (see Figure 4). As was discussed in [2] this corresponds to an \( SU(N_c) \) gauge theory with \( N_f \) flavors in the fundamental representation. It is convenient to find the solution for 3 finite boxes of length \( L_L - L, 2L, L_R - L \), and then take a limit \( L_L = L_R \to \infty \). The bending of the NS'-branes is given by

\[
x_4 = 2(N_c - N_f) x_7 \left[ \arctan \left( \frac{L + x_6}{x_7} \right) + \arctan \left( \frac{L - x_6}{x_7} \right) \right] +
(N_c - N_f)(L - x_6) \log((L - x_6)^2 + x_7^2) +
(N_c - N_f)(L + x_6) \log((L + x_6)^2 + x_7^2) +
2\pi N_f |x_7|,
\]

(6)
where we subtracted constant terms. At sufficiently large distances from the finite box, the bending is approximately given by the superposition of the linear and logarithmic terms. Again, the solution we find is smooth.

In the case we are considering, the solution for the NS-brane bending is still logarithmic at large distances. It is identical to the solution in Eq. (5) with a factor of $N_c - N_f$ and an obvious relabelling of directions. When $N_f > N_c$, we find that the NS-branes intersect. This is because the bending of one set of NS-brane does not carry the full information about the breaking of supersymmetry down to $N = 1$. Thus the NS-brane intersection reflects the loss of asymptotic freedom in the “parent” $N = 2$ theory. We conclude that in addition to SQCD with $N_f$ flavors the field theory of this brane configuration contains additional degrees of freedom which have not been identified. We therefore would not describe the $N_f > N_c$ theories in this manner but instead would construct configurations with $N_f > N_c$ by putting some of the D5-branes in different boxes [12].

• The final configuration we consider is a case with no finite boxes, but two horizontal NS'-branes and a single vertical NS-brane. One could similarly consider instead the example with more vertical NS-branes which can be interpreted as a gauge theory, but this simpler case is sufficient to illustrate our point. Assume that the number of D5 branes on the left (right) is $N_L$ ($N_R$). The solution contains infinite terms of the form $x_6(N_L \log L_L - N_R \log L_R)$. We can choose $L_L$ and $L_R$ in such a way that these terms cancel, and take the limit in a correlated fashion. The solution is then

$$x_4 = x_6 \log((L - x_6)^2 + x_7^2) - N_L \log((L + x_6)^2 + x_7^2) + \frac{2(N_R - N_L)x_7 \arctan(\frac{x_6}{x_7})}{x_7} + (N_L + N_R)|x_7|. \quad (7)$$
Figure 5: The angle of NS'-brane bending as a function of $x_6, x_7$

Note that if either $N_L$ or $N_R$ were zero, we could not cancel the divergence. In that case the NS'-brane would become vertical in the 4-6 plane. This problem suggests that it may be necessary for the D5-branes to end on D7-branes so that all D5-branes in the configuration are finite. These are presumably necessary if one is to describe correctly the moduli fields, analogously to D6-branes in the type IIA theory [5].

In [12], it was argued that for $N_L \neq N_R$, there would be a sharp transition as one moves along $x_6$, which would literally tear the branes apart at large $x_7$ distances. As we can see from the solution in Eq. (7), the situation is quite different. We find that the transition region is smooth everywhere and in fact becomes more smooth as one goes to large $x_7$. This is illustrated in Figure 5 where we have plotted the angle at which the brane bends ($\frac{\partial x_4}{\partial x_7}$) as a function of $x_6$ and $x_7$. As can be seen, there is a transition between two different bending angles but it is smooth everywhere and the branes do not break.

From these examples, the general behavior of the solution should be clear. A general solution is a superposition of elementary solutions and in our approximation NS- and NS'-branes bend independently. For instance, if we
Figure 6: A section of NS'-brane at $x_7 = 0$ in a model with anomalous matter content.

Consider many finite boxes in a row (corresponding to product group theories) the solution is given by a linear superposition of many solutions from the first example; again the general tendency should be clear. There is always a smooth solution to the two dimensional Laplace equation which approximates the true answer. However, we have not yet considered restrictions which might be imposed when there is more than one NS- or NS'-brane which is bending, so that there can be intersections which lead to new field theory degrees of freedom. We do this in the next section.

3 Restrictions from Brane Intersections and Their Solutions

In the previous section, we found that there exist smooth solutions for the NS'-(NS-) brane shapes for arbitrary D-brane configurations. Thus it appears that one can relax the conditions imposed in [12]. Can one impose any restrictions on allowed brane configurations by considering their bending? We will require that all brane configurations are such that the NS- (NS') branes do not intersect over a 4-dimensional manifold. If the branes would intersect, there would be additional light degrees of freedom living on the intersection and the content of the effective field theory would be different. Let us illustrate the consequences of this requirement on an anomalous model with $N_L \neq N_R$. Combining the solution for pure SYM theory in Eq. (5) with
the solution for semi-infinite D5-branes in Eq. (7) we plot the shape of one NS' -brane at \( x_7 = 0 \) in Figure 6. Since the other NS'-brane bends in the opposite direction we find that two (classically) parallel NS'-branes intersect over a 4-dimensional surface. Thus the anomalous theory is problematic as a brane configuration. We therefore impose the requirement \( N_L = N_R \). Notice that this condition was imposed by hand from a knowledge of the field theory in [11] but here we derive it from the brane configuration itself. For any theory constructed out of two NS-branes and two NS'-branes the above condition becomes:

\[
\begin{align*}
    n_{31} &= n_{11} + n_{33} - n_{13}, \\
    n_{32} &= n_{33} + n_{12} - n_{13}, \\
    n_{21} &= n_{11} + n_{23} - n_{13}.
\end{align*}
\] (8)

It is easy to further generalize our conditions to an arbitrary array of brane boxes constructed out of \( H \) NS'-branes and \( V \) NS-branes. Requiring that branes do not intersect imposes \( H + V - 1 \) independent conditions on \((H+1)(V+1)\) numbers of D5-branes, which is far less than the \((H-1)(V-1)\) conditions imposed in [12]. Yet we note that these conditions exclude some of the interesting models, including the 3–2 model of dynamical supersymmetry breaking.

It is, however, not quite clear to us whether even these less restrictive conditions imposed above are necessary. For example, instead of introducing matter by having semi-infinite D5-branes one could allow them to end on D7-branes as noted earlier. Then moving the D7-branes sufficiently close to the gauge theory box, one could guarantee that there are no 4-dimensional intersections of NS- (NS')-branes. If this were possible, then the physics would depend on the \( x_6 \) position of the D7-branes and the \( x_4 \) position of the D7' branes. This is in contrast to the role played by the D6-branes in type IIA configurations, where holomorphic quantities do not depend on the \( x_6 \) coordinate of the D6-branes. This might be a reasonable difference, since it is difficult to identify the holomorphic structure of the field theory in the brane box configuration. We should mention however that the D7-branes change the geometry of space, introducing a deficit angle, so our flat space arguments may not be valid in their presence.

While we lack a complete understanding of the conditions which should be imposed on brane configurations due to bending, we nonetheless can go on and consider implications of the above conditions together with anomaly freedom conditions imposed by hand. The total number of conditions would then be \((H-1)(V-1)\) which is the same as the number of conditions in [12].
It turns out that the combination of our constraints and anomaly cancellation is always satisfied by the conditions of [12]. However, in certain cases some of the anomaly freedom conditions are redundant, allowing theories with non-intersecting branes which do not satisfy conditions [12]. We have seen an example of this in supersymmetric QCD. In Figure 7, we give another example of an anomaly free, consistent brane configuration which does not satisfy the condition [12].

Finally, there is an additional subtlety. Suppose we had started with a configuration in which the \(i\)’th and \((i+1)\)’th NS (or NS’) branes remain parallel (even if they bend), so that the distance between them does not depend on the coordinate in the 4-5 (6-7) plane. Then we can add \(\Delta N_{i-1}, \Delta N_i,\) and \(\Delta N_{i+1}\) D5-branes to the corresponding (finite) boxes. As a result, the distance between the NS- (NS’-) branes will change logarithmically at large values of \(x_4, x_5 (x_6, x_7)\). If \(\delta N_{i-1} + \delta N_{i+1} > 2\delta N_i\), the two NS- (NS’-) branes under consideration will intersect. We therefore exclude such configurations.

\[\begin{array}{cccccc}
\text{NS}' & \text{NS}' & & & & \\
4 & 4 & 4 & 3 & 0 \\
5 & 6 & 5 & 5 & 1 \\
4 & 4 & 4 & 3 & 0 \\
\end{array}\]

Figure 7: A non-intersecting anomaly free configuration which does not satisfy conditions of Eq. (1).

4 The Running Coupling Constant

Our discussion of brane bending is also useful for understanding the relation of the running coupling constant to the brane configuration. Due to dimensional reduction, the 4-dimensional coupling constant is related to the area of the D5-brane bounded by the NS-branes. Usually, the running of
the coupling constant is associated with the bending of the NS-branes. However, since the intersection between the D5-brane and the NS- (NS′-) brane is 4-dimensional, naively one would expect that the bending is linear, which would not reflect the running of the coupling constant in 4 dimensions.

As we have seen, in a pure SYM configuration and at sufficiently large distances from the finite D5-branes, the solution (and therefore, the distance between two NS-branes) changes logarithmically:

\[ \Delta x_4 \propto 2N_c \log R, \]  

where \( R = \sqrt{x_6^2 + x_7^2} \). It is tempting to identify this logarithmic bending with the properties of a 4-dimensional theory in analogy with [2]. However, the coefficient of the log does not reproduce the beta function coefficient correctly. In fact the coefficient gives the correct value for \( N = 2 \) SYM. This is not surprising because our configuration has originated as an \( N = 2 \) theory, and has been reduced to \( N = 1 \) due to the presence of NS-branes. This is similar to the situation in the type IIA constructions [3] of \( N = 1 \) SYM. These models were constructed by rotating one of the NS-branes in the \( N = 2 \) theory. The bending of each of the NS-branes is determined only by the number of D4-branes attached to it, and reflects the \( N = 2 \) running coupling constant [4, 19]. One infers the beta function coefficient and the running of the coupling constant of the \( N = 1 \) theory by taking into account the mass of the adjoint chiral multiplet which is not reflected in the bending of the branes. Similarly in our case, the running coupling constant should be given by some function of the distances between both NS and NS′-branes.

Nevertheless, there do exist configurations with a simple relation between brane bending and the running gauge coupling. As has been argued in [20], this is the case in the configurations where one set of NS-branes does not bend. In [20] the one-loop coefficient \( b_0 \) of the beta function for such models (see Figure 8) was determined. The model in Figure 8 corresponds to an \( SU(N) \) gauge theory with \( N_f = N + M + K \) flavors. Let us give an argument for the running coupling constant which is somewhat different from that in [20]. The gauge coupling in the model is given by \( \frac{1}{g^2} = \frac{\Delta x_4 \Delta x_6}{g_s^2} \). The distance between the NS′-branes depends linearly on \( x_7 \):

\[ \Delta x_4(x_7) = \Delta x_4 + (3N - N_f)x_7. \]  

Let us reconnect \( n \) D5-branes in the middle row to form an infinite D5-brane and move them away in \( x_7 \), thus Higgsing the theory to \( N - n \) colors.
Figure 8: A configuration corresponding to an $SU(N)$ gauge group with $N + M + K$ flavors in which the NS-branes do not bend.

and $N_f - n$ flavors. If we require that the asymptotic positions of the NS- and NS'-branes do not change (this would require an infinite amount of energy from a 4-dimensional point of view), then the area bounded by the NS- and NS'-branes (and, therefore the gauge coupling) will change:

$$\Delta A \propto 2nx_7$$

where $x_7$ is the final position of the D5-branes. The coefficient in front of $x_7$ correctly reproduces the change in the one-loop beta-function coefficient $b_0$. We could give analogous arguments for turning on mass terms by reconnecting an arbitrary number of D5-branes in the lower and/or bottom rows. Thus it is tempting to identify the coefficient in front of $x_7$ in Eq. (10) with $b_0$. However, this would imply linear running of the gauge coupling, which seems to reflect that the origin of the theory is a five-dimensional theory. In fact, we can Higgs the theory to take the D-branes off the NS-branes so that the theory really seems to be five-dimensional.

As we have noted before, even some potentially problematic configurations may be acceptable (as far as the bending goes) if all the D5-branes are finite (which requires that flavor D5-branes end on D7 or D7' branes). In the configuration of Figure 8, we therefore introduce D7-branes. Then

\footnote{Note that in the anomalous theory discussed in Section 3 some of the semi-infinite D5-branes can not be reconnected into infinite ones, and the theory can never look truly five-dimensional. This is in agreement with the fact that one does not expect inconsistencies in a five-dimensional configuration.}

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asymptotically, the distance between the NS$'$-branes changes logarithmically with the coefficient appropriate for $N = 1$ theory:

$$\Delta x_4 \propto (2N - M - K) \log R = (3N_c - N_f) \log R.$$ (12)

The arguments we have given above for matching couplings of the high energy and low energy theories are still valid, and thus we see the correct logarithmic running of the coupling constant.

5 The Beta Function

While we cannot extract the running coupling constant from brane bending in an arbitrary configuration, it is possible to constrain the form of the beta function (or more precisely, the one-loop coefficient of the beta function $b_0$) in the brane-box theories. The argument goes as follows. Assume that the beta function is an arbitrary function of $N_c$ and $N_f$ ($b_0 = f(N_c, N_f)$) and that it can be related to the bending of the branes. A precise knowledge of this relation is not necessary for the argument. From the assumption we conclude that any deformation of the brane configuration that does not change the bending of the branes cannot change the beta function. In particular we can add one D5-brane that covers all the boxes. Doing so we see that $N_c \rightarrow N_c + 1$ and that $N_f \rightarrow N_f + 3$. The requirement that the beta function is invariant under this tells us that it is a function of the particular combination $3N_c - N_f$ which is indeed correct for $SU(N_c)$ gauge theories with $N_f$ chiral multiplets in the fundamental representation.

Note that the same type of argument could be applied to the type IIA configurations studied in [2]. There one realizes $N = 2$ supersymmetric gauge theories on the world volume of D4-branes suspended between NS-branes. If one adds a D4-brane that extends from minus infinity to plus infinity in $x^6$, one does not change the bending of the NS-branes but one does indeed change the number of colors by one and the number of flavors by two giving the invariant combination $2N_c - N_f$ which agrees with the $N = 2$ beta function.

So far, using the assumption that the brane bending encodes the beta function, we have established that (the one loop coefficient of) the beta function is an arbitrary function of $3N_c - N_f$: $b_0 = f(3N_c - N_f)$. Assuming that this function is universal, we can determine its exact form by explicitly calculating its value in some simple cases. In Section [3] we were able to directly
relate the change in brane bending of the configuration given in Figure 8 to the change of the value of $b_0$. This tells us that $b_0 = 3N_c - N_f + c$ where $c$ is an undetermined constant. Finally we note that configurations without brane bending corresponds to field theories where $b_0 = 0$ \cite{21}. Such configurations have $N_f = 3N_c$ and thus we find that $c = 0$.

In the presence of orientifolds, the identification of the degrees of freedom is less well understood. However, we believe that the argument given above should still apply in this case. The argument could in fact be used as a consistency check on any proposed counting of degrees of freedom in the presence of an orientifold plane.

6 Conclusions

In this paper, using the fact that the bending of NS-branes in brane-box theories is determined by a two dimensional Laplace equation, we have demonstrated the existence of consistent solutions. A D5-brane ending on the NS-brane looks like a line of charge and a line of charge of finite extent looks like a point charge at long distance and thus gives rise to logarithmic bending rather than linear bending. This means that we can relax some of the consistency conditions on these theories proposed in \cite{12} since they were derived assuming that all branes bend linearly. At long distance the brane bends as a smooth solution of the two dimensional Laplace equation and the branes do not break. What can happen though is that two of the NS-branes intersect at some distance. In that case there can be new massless states that has not been taken into account in the field theory and it does not capture the physics on the branes correctly.

The requirement that branes do not intersect is much less restrictive than the condition given in \cite{12}. In particular, we find configurations of branes that do not intersect but that give rise to field theories that are anomalous. This shows that there is no direct relation between field theory anomalies and the brane bending. Presumably to understand the anomalies better one would have to look at non-conservation of Ramond-Ramond charge in the brane set-up.

We also argued that if all “flavor” D5-branes are finite ending on D7 and/or D7’ branes, and if the D7- (D7’-) branes are sufficiently close, there may be even less restrictions. We have also seen that in some cases, the D7-branes may be required in order to have a truly 4-dimensional theory.
However, we have observed a mysterious dependence on the $x_6$ ($x_4$) coordinate of the D7- (D7$'$-) branes. Also, the D7-branes introduce a deficit angle into the geometry and there might be restrictions on the allowed number of D7-branes. The physics associated with the introduction of D7-branes into the system requires further investigation.

We also showed that in some simple configurations and in the presence of D7 branes, the logarithmic running of the gauge couplings can be simply read off from the brane bending. Although it is not manifest how this relation works in general, we have demonstrated that the bending of the branes does correctly reflect the beta function dependence on $N_c$ and $N_f$. We commented that this should also be true for theories with orientifolds. The argument makes it clear that the bending of the branes does encode the beta function and provides a nice consistency check on the brane-box theories since it relies only on the way flavors are counted in these theories. It would be interesting to identify more precisely how the branes encode this information about the beta function. Also of importance ultimately is to see how the branes incorporate quantum effects. Since these are presumably most important in the vicinity of the intersection of D5 and NS- or NS$'$-branes, one might require a more exact solution to the equations of motion in order to explore non-perturbative gauge dynamics with brane-box theories.

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