\{Q\bar{q}\}\{\bar{Q}'q\} molecular states

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(Dated: September 12, 2009)

Masses for \{Q\bar{q}\}\{\bar{Q}'q\} molecular states are systematically studied in QCD sum rules. The interpolating currents representing the related molecular states are proposed. Technically, contributions of the operators up to dimension six are included in operator product expansion (OPE). Mass spectra for molecular states with \{Q\bar{q}\}\{\bar{Q}'q\} configurations are obtained.

PACS numbers: 11.55.Hx, 12.38.Lg, 12.39.Mk
I. INTRODUCTION

The field of heavy hadron spectroscopy has attracted much attention nowadays. Experimentally, plentiful hadronic resonances have been observed, such as the so-called X, Y, and Z states (for experimental reviews, e.g., see [1, 10]). In theory, different from conventional charmonium states, some of these resonances may not reconcile with the quark model picture, hence it is not easy to find appropriate positions for them in mesonic spectroscopy. Many authors [11, 12, 13, 14, 15, 16] tend to interpret some of the hadrons as possible mesonic molecule candidates for their masses are very close to the meson-meson thresholds. For example, it has been proposed that X(3872) could be a $D^*\bar{D}$ molecular state [12], $Z^+(4430)$ be a $D^*\bar{D}$, $\Lambda_0(3930)$ be a $D^*\bar{D}$, $\Lambda_1(4630)$ be a $D^*\bar{D}$, $\Lambda_2(4930)$ be a $D^*\bar{D}$, $\Lambda_3(5460)$ be a $D^*\bar{D}$ etc.. As a matter of fact, the charmed molecular states were put forward long ago in [17], and it has also predicted that the molecular states involving hidden $c\bar{c}$ pair do exist and have a rich spectroscopy in [18]. What is also very important, the existence of molecular states is not excluded by QCD itself. Indubitably, investigations of molecular states could deepen one’s understanding of quark-gluon interaction. Thus, the quantitative descriptions of molecular states’ properties like masses are quite needed for well comprehending their structures. In a word, it is interesting and significative to study mass spectra for the molecular states.

Motivated by the above reasons, we devote to obtain the spectra for molecular states with $\{Q\bar{q}\}\{\bar{Q}'(\prime)q\}$ configurations in this work. However, it is a great challenge to extract information on the spectrum from first principles. While QCD has been widely accepted as the correct theory describing strong interaction, it is still far from clear how to gain hadron masses from the simple Lagrangian. That’s because QCD is highly nonperturbative in the low energy region where futile to attempt perturbative calculations. Therefore, one has to deal with a genuinely strong field in nonperturbative methods. Before thoroughly grasping the essence of QCD confinement, for the moment, one could make use of QCD sum rule [19] (for reviews see [20, 21, 22, 23] and references therein), which is a comprehensive and reliable working tool for evaluating the nonperturbative effects. The basic idea of this approach is bridging the gap between the perturbative and nonperturbative sectors by employing the language of dispersion relations. Already, there have been several works testing some molecular states from QCD sum rules up to now [24, 25, 26]. At present, we’d like to gain the spectra for $\{Q\bar{q}\}\{\bar{Q}'(\prime)q\}$ molecular states with QCD sum rules, which could serve as an extension of our previous work on $\{Q\bar{s}\}\{\bar{Q}'(\prime)s\}$ states [22].

The paper’s framework is as follows. In Sec. II, QCD sum rules for the molecular states are introduced, and both the phenomenological representation and QCD side are derived, followed by the numerical analysis to extract the hadronic masses and a brief summary in Sec. III.

II. MOLECULAR STATE QCD SUM RULES

The QCD sum rule attempts to link the hadron phenomenology with the interactions of quarks and gluons, the elementary point of which is the choice of interpolating current. Following the standard scheme [10], the $Q\bar{q}$ mesons with $J^P = 0^-, 1^-, 0^+$, and $1^+$ are named $D, D^*, D_0^*$, and $D_1$ for charmed mesons, with $B, B^*, B_s^*$, and $B_1$ for bottom mesons, respectively. Here the configurations for these mesons are represented as $(Q\bar{q})_j$, $(Q\bar{q})^*_{j'}$, $(Q\bar{q})^\prime_{j''}$, and $(Q\bar{q})_{j'''}$. In full theory, the interpolating currents for these mesons can be found in Refs. [27, 28]. Consequently, the interpolating currents for the related molecular states are constructed in following forms, with

\[
\begin{align*}
\tilde{j}_{(Q\bar{q})(Q'\bar{q})} &= \langle \bar{q}_a i\gamma_5 Q_a \rangle \langle \bar{Q}'_b i\gamma_5 q_b \rangle, \\
\tilde{j}_{(Q\bar{q})^*(Q'\bar{q})^*} &= \langle \bar{q}_a \gamma_{\mu} Q_a \rangle \langle \bar{Q}'_b \gamma^\mu q_b \rangle, \\
\tilde{j}_{(Q\bar{q})^\prime_{j'}(Q'\bar{q})_{j''}} &= \langle \bar{q}_a Q_a \rangle \langle \bar{Q}'_b q_b \rangle, \\
\tilde{j}_{(Q\bar{q})_{j'''}(Q'\bar{q})_{j'''}} &= \langle \bar{q}_a \gamma_{\mu} \gamma_5 Q_a \rangle \langle \bar{Q}'_b \gamma^\mu \gamma_5 q_b \rangle,
\end{align*}
\]
for one type of hadrons, and

\[
\begin{align*}
\bar{q} \gamma_\mu Q_a ((\bar{Q}\gamma_5 Q)_{\mu

\begin{align*}
\bar{q} \gamma_\mu \gamma_5 Q_a ((\bar{Q}\gamma_5 Q)_{\mu \gamma_5 q_b}), \\
\bar{q} \gamma_\mu Q_a ((\bar{Q}\gamma_5 Q)_{\mu \gamma_5 q_b}), \\
\bar{q} \gamma_\mu Q_a ((\bar{Q}\gamma_5 Q)_{\mu \gamma_5 q_b}), \\
\bar{q} \gamma_\mu Q_a ((\bar{Q}\gamma_5 Q)_{\mu \gamma_5 q_b}),
\end{align*}
\]

for another type, where \( q \) indicates the light quark \( u \) or \( d \), \( Q \) and \( Q' \) denote heavy quarks (\( Q = Q' \) or \( Q \neq Q' \)), with \( a \) and \( b \) are color indices.

For the former case, the starting point is the two-point correlator

\[
\Pi(q^2) = i \int d^4 x e^{i q \cdot x} \langle 0 | T [j(x) j^+(0)] | 0 \rangle. \tag{1}
\]

The correlator can be phenomenologically expressed as a dispersion integral over a physical spectral function

\[
\Pi(q^2) = \frac{\lambda_H^2}{M_H^2 - q^2} + \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\text{Im} \Pi_{\text{phen}}(s)}{s - q^2} + \text{subtractions}, \tag{2}
\]

where \( M_H \) denotes the mass of the hadronic resonance, and \( \lambda_H \) gives the coupling of the current to the hadron \( \langle 0 | j | H \rangle = \lambda_H \). In the OPE side, the correlator can be written in terms of a dispersion relation as

\[
\Pi(q^2) = \int_{(m_Q + m_{Q'})^2}^{\infty} ds \frac{\rho_{\text{OPE}}(s)}{s - q^2} (m_Q = m_{Q'} \text{ or } m_Q \neq m_{Q'}), \tag{3}
\]

where the spectral density is given by the imaginary part of the correlator

\[
\rho_{\text{OPE}}(s) = \frac{1}{\pi} \text{Im} \Pi_{\text{OPE}}(s). \tag{4}
\]

After equating the two sides, assuming quark-hadron duality, and making a Borel transform, the sum rule can be written as

\[
\lambda_H^2 e^{-M_H^2/M^2} = \int_{(m_Q + m_{Q'})^2}^{s_0} ds \rho_{\text{OPE}}(s) e^{-s/M^2}, \tag{5}
\]

where \( M^2 \) indicates Borel parameter. To eliminate the hadronic coupling constant \( \lambda_H \), one reckons the ratio of derivative of the sum rule and itself, and then yields

\[
M_H^2 = \int_{(m_Q + m_{Q'})^2}^{s_0} ds \rho_{\text{OPE}}(s) e^{-s/M^2} \bigg/ \int_{(m_Q + m_{Q'})^2}^{s_0} ds \rho_{\text{OPE}}(s) e^{-s/M^2}. \tag{6}
\]

For the latter case, one starts from the two-point correlator

\[
\Pi^{\mu \nu}(q^2) = i \int d^4 x e^{i q \cdot x} \langle 0 | T [j^{\mu}(x) j^{\nu+(0)}(0)] | 0 \rangle. \tag{7}
\]

Lorentz covariance implies that the two-point correlation function can be generally parameterized as

\[
\Pi^{\mu \nu}(q^2) = \left( \frac{q^\mu q^\nu}{q^2} - g^{\mu \nu} \right) \Pi^{(1)}(q^2) + \frac{q^\mu q^\nu}{q^2} \Pi^{(0)}(q^2). \tag{8}
\]
The part of the correlator proportional to $g_{\mu\nu}$ will be chosen to extract the mass sum rule here. In phenomenology, $\Pi^{(1)}(q^2)$ can be expressed as a dispersion integral over a physical spectral function

$$\Pi^{(1)}(q^2) = \frac{[\lambda^{(1)}]^2}{M_H^2 - q^2} + \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\text{Im}\Pi^{(1)}_{\text{phen}}(s)}{s - q^2} + \text{subtractions}, \tag{9}$$

In the OPE side, $\Pi^{(1)}(q^2)$ can be written in terms of a dispersion relation as

$$\Pi^{(1)}(q^2) = \int_{(m_Q + m_Q')^2}^{\infty} ds \rho_{\text{OPE}}(s) \left( m_Q = m_{Q'} \text{ or } m_Q \neq m_{Q'} \right), \tag{10}$$

where the spectral density is given by

$$\rho_{\text{OPE}}(s) = \frac{1}{\pi} \text{Im} \Pi^{(1)}(s). \tag{11}$$

After equating the two sides, assuming quark-hadron duality, and making a Borel transform, the sum rule can be written as

$$[\lambda^{(1)}]^2 e^{-M_H^2/M^2} = \int_{(m_Q + m_Q')^2}^{s_0} ds \rho_{\text{OPE}}(s) e^{-s/M^2}. \tag{12}$$

To eliminate the hadronic coupling constant $\lambda^{(1)}$, one reckons the ratio of derivative of the sum rule and itself, and then yields

$$M_H^2 = \int_{(m_Q + m_Q')^2}^{s_0} ds \rho_{\text{OPE}} e^{-s/M^2} / \int_{(m_Q + m_Q')^2}^{s_0} ds \rho_{\text{OPE}} e^{-s/M^2}. \tag{13}$$

Calculating the OPE side, one works at leading order in $\alpha_s$ and considers condensates up to dimension six with the similar techniques in Refs. [29, 30]. To keep the heavy-quark mass finite, one uses the momentum-space expression for the heavy-quark propagator. One calculates the light-quark part of the correlation function in the coordinate space, which is then Fourier-transformed to the momentum space in $D$ dimension. The resulting light-quark part is combined with the heavy-quark part before it is dimensionally regularized at $D = 4$. For the heavy-quark propagator with two and three gluons attached, the momentum-space expressions given in Ref. [27] are used. After some lengthy OPE calculations, the concrete spectral densities can be acquired, which are collected in the Appendix.

III. NUMERICAL ANALYSIS

In this section, the sum rules (9) and (13) will be numerically analyzed. The input values are taken as $m_c = 1.23$ GeV, $m_b = 4.20$ GeV [11] with $\langle \bar{q}q \rangle = -(0.23)^3$ GeV$^3$, $\langle q \sigma \cdot Gq \rangle = m_c^2 \langle \bar{q}q \rangle$, $m_b^2 = 0.8$ GeV$^2$, $\langle y^2G^2 \rangle = 0.88$ GeV$^4$, and $\langle y^3G^3 \rangle = 0.045$ GeV$^6$ [22, 23, 29]. Complying with the standard procedure of sum rule analysis, the threshold $s_0$ and Borel parameter $M^2$ are varied to find the optimal stability window, in which the perturbative contribution should be larger than the condensate contributions while the pole contribution larger than continuum contribution. Thus, the regions of thresholds are taken as those values presented in the related figure captions, with $M^2 = 3.5 \sim 4.5$ GeV$^2$ for $D \bar{D}$, $D^* \bar{D}$, $D^* \bar{D}^*$, $D_0^* \bar{D}_0^*$, $D_1 \bar{D}_1$, $D \bar{D}^*$, $D^* \bar{D}^*$, and $D^* \bar{D}^*$, $M^2 = 7.5 \sim 9.0$ GeV$^2$ for $B \bar{D}$, $B^* \bar{D}$, $B^* \bar{D}^*$, $B_0^* \bar{D}_0^*$, $B_1 \bar{D}_1$, $B \bar{D}_0^*$, $B^* \bar{D}_0^*$, $B^* \bar{D}_1$, $B^* \bar{D}^*$, $D_1^* \bar{D}_0^*$, $D_0^* \bar{D}_0^*$, $D_0^* \bar{D}_1$, $D^* \bar{D}_1$, $D^* \bar{D}^*$, and $M^2 = 9.5 \sim 11.0$ GeV$^2$ for $B \bar{B}$, $B^* \bar{B}$, $B^* \bar{B}^*$, $B^*_0 \bar{B}_0^*$, $B^*_0 \bar{B}_1$, $B^*_1 \bar{B}_0^*$, $B^*_1 \bar{B}$, $B^*_1 \bar{B}^*$, and $B^* \bar{B}_1$, respectively. The corresponding Borel curves are exhibited in Figs. 1-18, and the numerical results are listed in Tables I-II. It is worth noting that the numerical errors reflect the uncertainty due to sum rule windows (variation of the threshold $s_0$ and Borel parameter $M^2$) only; the uncertainty resulted from the variation of the quark masses and QCD parameters is not included. After a comparison, one can find that the numerical results for $D^* \bar{D}$,
TABLE I: The mass spectra of molecular states with same heavy quarks.

| Hadron | configuration | mass (GeV)   | Hadron | configuration | mass (GeV)   |
|--------|---------------|--------------|--------|---------------|--------------|
| $D\bar{D}$ | $(c\bar{q})(\bar{c}q)$ | $3.76 \pm 0.10$ | $BB$ | $(b\bar{q})(\bar{b}q)$ | $10.58 \pm 0.10$ |
| $D^*\bar{D}$ | $(c\bar{q})^*(\bar{c}q)$ | $3.88 \pm 0.10$ | $B^*B$ | $(b\bar{q})^*(\bar{b}q)$ | $10.62 \pm 0.10$ |
| $D^*\bar{D}^*$ | $(c\bar{q})^*(\bar{c}q)^*$ | $3.91 \pm 0.11$ | $B^*\bar{B}^*$ | $(b\bar{q})^*(\bar{b}q)^*$ | $10.67 \pm 0.10$ |
| $D^*_1D^*_0$ | $(c\bar{q})_1^*(\bar{c}q)_0^*$ | $4.56 \pm 0.11$ | $B^*_1\bar{B}^*_0$ | $(b\bar{q})_1^*(\bar{b}q)_0^*$ | $11.28 \pm 0.08$ |
| $D^*_1D^*_0$ | $(c\bar{q})_1^*(\bar{c}q)_0^*$ | $4.62 \pm 0.11$ | $B^*_1\bar{B}^*_0$ | $(b\bar{q})_1^*(\bar{b}q)_0^*$ | $11.32 \pm 0.09$ |
| $D^*_1D^*_0$ | $(c\bar{q})_1^*(\bar{c}q)_0^*$ | $4.66 \pm 0.13$ | $B^*_1\bar{B}^*_1$ | $(b\bar{q})_1^*(\bar{b}q)_1^*$ | $11.33 \pm 0.12$ |
| $D^*_2D^*_1$ | $(c\bar{q})_2^*(\bar{c}q)_1^*$ | $4.21 \pm 0.07$ | $B^*_2\bar{B}^*_1$ | $(b\bar{q})_2^*(\bar{b}q)_1^*$ | $11.03 \pm 0.09$ |
| $D^*_2D^*_1$ | $(c\bar{q})_2^*(\bar{c}q)_1^*$ | $4.34 \pm 0.07$ | $B^*_2\bar{B}^*_1$ | $(b\bar{q})_2^*(\bar{b}q)_1^*$ | $11.04 \pm 0.09$ |
| $D^*_2D^*_1$ | $(c\bar{q})_2^*(\bar{c}q)_1^*$ | $4.26 \pm 0.07$ | $B^*_2\bar{B}^*_1$ | $(b\bar{q})_2^*(\bar{b}q)_1^*$ | $11.02 \pm 0.09$ |
| $D^*_3D^*_1$ | $(c\bar{q})_3^*(\bar{c}q)_1^*$ | $4.44 \pm 0.09$ | $B^*_3\bar{B}^*_1$ | $(b\bar{q})_3^*(\bar{b}q)_1^*$ | $11.03 \pm 0.09$ |

TABLE II: The mass spectra of molecular states with differently heavy quarks.

| Hadron | configuration | mass (GeV)   | Hadron | configuration | mass (GeV)   |
|--------|---------------|--------------|--------|---------------|--------------|
| $B\bar{D}$ | $(b\bar{q})(\bar{c}q)$ | $7.12 \pm 0.09$ | $B^*\bar{D}^*_0$ | $(b\bar{q})^*(\bar{c}q)_0^*$ | $7.67 \pm 0.06$ |
| $B^*\bar{D}$ | $(b\bar{q})^*(\bar{c}q)$ | $7.28 \pm 0.09$ | $B^*\bar{D}^*_1$ | $(b\bar{q})^*(\bar{c}q)_1^*$ | $7.74 \pm 0.07$ |
| $B^*\bar{D}^*$ | $(b\bar{q})^*(\bar{c}q)^*$ | $7.29 \pm 0.10$ | $B^*\bar{D}^*_1$ | $(b\bar{q})^*(\bar{c}q)_1^*$ | $7.21 \pm 0.09$ |
| $B^*_1D^*_0$ | $(c\bar{q})_1^*(\bar{c}q)_0^*$ | $8.04 \pm 0.08$ | $D^*_1\bar{B}^*_0$ | $(c\bar{q})_1^*(\bar{c}q)_0^*$ | $8.04 \pm 0.10$ |
| $B^*_1D^*_0$ | $(c\bar{q})_1^*(\bar{c}q)_0^*$ | $8.06 \pm 0.13$ | $D^*_1\bar{B}^*_0$ | $(c\bar{q})_1^*(\bar{c}q)_0^*$ | $7.70 \pm 0.06$ |
| $B^*_1D^*_1$ | $(c\bar{q})_1^*(\bar{c}q)_1^*$ | $8.07 \pm 0.11$ | $D^*_1\bar{B}^*_1$ | $(c\bar{q})_1^*(\bar{c}q)_1^*$ | $7.74 \pm 0.07$ |
| $B^*_2D^*_0$ | $(c\bar{q})_2^*(\bar{c}q)_0^*$ | $7.68 \pm 0.06$ | $D^*_2\bar{B}^*_0$ | $(c\bar{q})_2^*(\bar{c}q)_0^*$ | $7.76 \pm 0.06$ |
| $B^*_2D^*_1$ | $(c\bar{q})_2^*(\bar{c}q)_1^*$ | $7.77 \pm 0.06$ | $D^*_2\bar{B}^*_1$ | $(c\bar{q})_2^*(\bar{c}q)_1^*$ | $7.76 \pm 0.07$ |

$D^*_1\bar{D}$, and $D^*\bar{D}^*$ are in good agreement with the experimental data for $X(3872)$, $Z^+(4430)$, and $Y(3930)$, respectively, which could support the molecular interpretations for these hadrons.

FIG. 1: The dependence on $M^2$ for the masses of $D\bar{D}$ and $BB$ from sum rule [13]. The continuum thresholds are taken as $\sqrt{s_0} = 4.1 \sim 4.3$ GeV and $\sqrt{s_0} = 10.9 \sim 11.1$ GeV, respectively.
FIG. 2: The dependence on $M^2$ for the masses of $D^* \bar{D}$ and $B^* \bar{B}$ from sum rule (13). The continuum thresholds are taken as $\sqrt{s_0} = 4.3 \sim 4.5$ GeV and $\sqrt{s_0} = 11.0 \sim 11.2$ GeV, respectively.

FIG. 3: The dependence on $M^2$ for the masses of $D^* \bar{D}^*$ and $B^* \bar{B}^*$ from sum rule (6). The continuum thresholds are taken as $\sqrt{s_0} = 4.3 \sim 4.5$ GeV and $\sqrt{s_0} = 11.0 \sim 11.2$ GeV, respectively.

FIG. 4: The dependence on $M^2$ for the masses of $D_1 D_0^*$ and $B_1 B_0^*$ from sum rule (13). The continuum thresholds are taken as $\sqrt{s_0} = 5.0 \sim 5.2$ GeV and $\sqrt{s_0} = 11.6 \sim 11.8$ GeV, respectively.
FIG. 5: The dependence on $M^2$ for the masses of $D_0^*\bar{D}_0^*$ and $B_0^*\bar{B}_0^*$ from sum rule (6). The continuum thresholds are taken as $\sqrt{s_0} = 4.9 \sim 5.1$ GeV and $\sqrt{s_0} = 11.5 \sim 11.7$ GeV, respectively.

FIG. 6: The dependence on $M^2$ for the masses of $D_1\bar{D}_1$ and $B_1\bar{B}_1$ from sum rule (6). The continuum thresholds are taken as $\sqrt{s_0} = 5.3 \sim 5.5$ GeV and $\sqrt{s_0} = 12.0 \sim 12.2$ GeV, respectively.

FIG. 7: The dependence on $M^2$ for the masses of $DD_0^*$ and $BB_0^*$ from sum rule (6). The continuum thresholds are taken as $\sqrt{s_0} = 4.5 \sim 4.7$ GeV and $\sqrt{s_0} = 11.4 \sim 11.6$ GeV, respectively.
FIG. 8: The dependence on $M^2$ for the masses of $D_1\bar{D}$ and $B_1\bar{B}$ from sum rule (13). The continuum thresholds are taken as $\sqrt{s_0} = 4.7 \sim 4.9$ GeV and $\sqrt{s_0} = 11.4 \sim 11.6$ GeV, respectively.

FIG. 9: The dependence on $M^2$ for the masses of $D^*\bar{D}_0$ and $B^*\bar{B}_0$ from sum rule (13). The continuum thresholds are taken as $\sqrt{s_0} = 4.6 \sim 4.8$ GeV and $\sqrt{s_0} = 11.4 \sim 11.6$ GeV, respectively.

FIG. 10: The dependence on $M^2$ for the masses of $D^*D_1$ and $B^*B_1$ from sum rule (13). The continuum thresholds are taken as $\sqrt{s_0} = 4.9 \sim 5.1$ GeV and $\sqrt{s_0} = 11.4 \sim 11.6$ GeV, respectively.
FIG. 11: The dependence on $M^2$ for the masses of $B\bar{D}$ and $B^*\bar{D}^*$ from sum rule (6). The continuum thresholds are taken as $\sqrt{s_0} = 7.5 \sim 7.7$ GeV and $\sqrt{s_0} = 7.7 \sim 7.9$ GeV, respectively.

FIG. 12: The dependence on $M^2$ for the masses of $B_0^*\bar{D}_0^*$ and $B_1\bar{D}_1$ from sum rule (6). The continuum thresholds are taken as $\sqrt{s_0} = 8.3 \sim 8.5$ GeV and $\sqrt{s_0} = 8.6 \sim 8.8$ GeV, respectively.

FIG. 13: The dependence on $M^2$ for the masses of $D^*\bar{B}$ and $B^*\bar{D}$ from sum rule (13). The continuum thresholds are taken as $\sqrt{s_0} = 7.6 \sim 7.8$ GeV and $\sqrt{s_0} = 7.7 \sim 7.9$ GeV, respectively.
FIG. 14: The dependence on $M^2$ for the masses of $D_1 \bar{B}_0^*$ and $B_1 \bar{D}_0^*$ from sum rule (13). The continuum thresholds are taken as $\sqrt{s_0} = 8.4 \sim 8.6$ GeV and $\sqrt{s_0} = 8.4 \sim 8.6$ GeV, respectively.

FIG. 15: The dependence on $M^2$ for the masses of $D^* \bar{B}_0^*$ and $B^* \bar{D}_0^*$ from sum rule (13). The continuum thresholds are taken as $\sqrt{s_0} = 8.1 \sim 8.3$ GeV and $\sqrt{s_0} = 8.0 \sim 8.2$ GeV, respectively.

FIG. 16: The dependence on $M^2$ for the masses of $D^* B_1$ and $B^* D_1$ from sum rule (15). The continuum thresholds are taken as $\sqrt{s_0} = 8.1 \sim 8.3$ GeV and $\sqrt{s_0} = 8.1 \sim 8.3$ GeV, respectively.
In summary, the QCD sum rules have been employed to calculate the masses for \( \{Q\bar{q}\}\{\bar{Q}^{'(1)}q\} \) molecular states, including the contributions of the operators up to dimension six in OPE. We have attained mass spectra for \( \{Q\bar{q}\}\{\bar{Q}^{'(1)}q\} \) molecular states in the end. In molecular pictures, the numerical results for the masses of \( X(3872) \), \( Z^+ (4430) \), and \( Y(3930) \) agree well with their corresponding experimental values, which can support that \( X(3872) \) could be a \( D^* \bar{D} \) molecular state, \( Z^+ (4430) \) be a \( D^* \bar{D}_1 \) molecular state, and \( Y(3930) \) be a \( D^* \bar{D}^{*} \) one. On all accounts, all the numerical results are expecting further experimental identification and it is looking forward to more experimental evidence on molecular states.

**APPENDIX**

The spectral densities are distinguished for two kinds of doubly heavy molecular states, namely, containing the same or differently heavy quarks. It is defined that \( r(m_{Q_1}, m_{Q_2}) = \alpha m_{Q_1}^2 + \beta m_{Q_2}^2 - \alpha \beta s \). Concretely, \( r(m_Q, m_{Q'}) = \alpha m_Q^2 + \beta m_{Q'}^2 - \alpha \beta s \) and \( r(m_Q, m_{Q'}) = \alpha m_Q^2 + \beta m_{Q'}^2 - \alpha \beta s \). First, with

\[
\rho_{\text{OPE}}(s) = \rho_{\text{pert}}(s) + \rho^{(\bar{q}q)}(s) + \rho^{(\bar{q}q)^2}(s) + \rho^{(\bar{q}q \sigma \cdot Gq)}(s) + \rho^{(\sigma \cdot G^2)}(s) + \rho^{(g^3 c^2)}(s),
\]
\[
\rho_{\text{OPE}}(s) = -\{ \rho_{\text{pert}}(s) + \rho^{(qq)}(s) + \rho^{(\bar{q}q)^2}(s) + \rho^{(q\bar{q}\sigma-Gq)}(s) + \rho^{(g^2G^2)}(s) + \rho^{(g^3G^3)}(s) \},
\]

\[
\rho_{\text{pert}}(s) = -3 \frac{m_Q}{2^{11/6} \pi^{11/2}} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\text{min}}}^{\alpha} \frac{d\beta}{\beta^{11/2}} \frac{1}{(1 - \alpha - \beta)(1 + \alpha + \beta)} r(m_Q, m_Q)^4,
\]

\[
\rho^{(qq)}(s) = \frac{3}{2^{7/4} \pi^{11/2}} m_Q \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\text{min}}}^{\alpha} \frac{d\beta}{\beta^{11/2}} (1 - \alpha - \beta)(1 + \alpha + \beta) r(m_Q, m_Q)^2,
\]

\[
\rho^{(\bar{q}q)^2}(s) = -\frac{\langle \bar{q}q \rangle^2}{m_Q^2} \sqrt{1 - 4m_Q^2/s},
\]

\[
\rho^{(g^2G^2)}(s) = \frac{\langle g^2G^2 \rangle}{2^{11/6} \pi^{11/2}} m_Q \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\text{min}}}^{\alpha} \frac{d\beta}{\beta^{11/2}} (1 - \alpha - \beta)(1 + \alpha + \beta) r(m_Q, m_Q),
\]

\[
\rho^{(g^3G^3)}(s) = \frac{\langle g^3G^3 \rangle}{2^{11/6} \pi^{11/2}} m_Q \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\text{min}}}^{\alpha} \frac{d\beta}{\beta^{11/2}} (1 - \alpha - \beta)(1 + \alpha + \beta) r(m_Q, m_Q) + 2m_Q^2 \beta,
\]

for \((Qq)(\bar{Q}q)\),

\[
\rho^{(q\bar{q}\sigma-Gq)}(s) = \frac{3}{2^{11/6} \pi^{11/2}} m_Q \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\text{min}}}^{\alpha} \frac{d\beta}{\beta^{11/2}} \left\{ -r(m_Q, m_Q) + \frac{2}{1 - \alpha} [m_Q^2 - \alpha(1 - \alpha)s] \right\},
\]

\[
\rho^{(g^2G^2)}(s) = -\frac{\langle g^2G^2 \rangle}{2^{11/6} \pi^{11/2}} m_Q \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\text{min}}}^{\alpha} \frac{d\beta}{\beta^{11/2}} (1 - \alpha - \beta)(1 + \alpha + \beta) r(m_Q, m_Q),
\]

\[
\rho^{(g^3G^3)}(s) = -\frac{\langle g^3G^3 \rangle}{2^{11/6} \pi^{11/2}} m_Q \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\text{min}}}^{\alpha} \frac{d\beta}{\beta^{11/2}} (1 - \alpha - \beta)(1 + \alpha + \beta) r(m_Q, m_Q) + 2m_Q^2 \beta,
\]

for \((Qq)^*(\bar{Q}q)\),
\[ \rho_{\text{OPE}}(s) = \rho_{\text{pert}}(s) + \rho^{(\bar{q}q)}(s) + \rho^{(\bar{q}q)^2}(s) + \rho^{(g\bar{q}\sigma \cdot Gq)}(s) + \rho^{(g^2G^2)}(s) + \rho^{(g^3G^3)}(s), \]

\[ \rho_{\text{pert}}(s) = \frac{3}{2^{9}\pi^6} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta^3} (1-\alpha-\beta)r(m_Q, m_Q)^4, \]

\[ \rho^{(\bar{q}q)}(s) = -\frac{3}{2^{9}\pi^4} m_Q \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta} r(m_Q, m_Q)^2, \]

\[ \rho^{(\bar{q}q)^2}(s) = \frac{(\bar{q}q)^2}{2^{4}\pi^2} m_Q^2 \sqrt{1-4m_Q^2/s}, \]

\[ \rho^{(g\bar{q}\sigma \cdot Gq)}(s) = \frac{3}{2^{9}\pi^4} m_Q \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta} (1-\alpha-\beta)r(m_Q, m_Q), \]

\[ \rho^{(g^2G^2)}(s) = \frac{\frac{3}{2^{7}\pi^4} m_Q^2}{2^{10}\pi^6} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta} r(m_Q, m_Q) + 2m_Q^2 \beta, \]

for \((Q\bar{q})^*(\bar{Q}q)^*\),

\[ \rho_{\text{OPE}}(s) = \rho_{\text{pert}}(s) + \rho^{(\bar{q}q)}(s) + \rho^{(\bar{q}q)^2}(s) + \rho^{(g\bar{q}\sigma \cdot Gq)}(s) + \rho^{(g^2G^2)}(s) + \rho^{(g^3G^3)}(s), \]

\[ \rho_{\text{pert}}(s) = \frac{3}{2^{11}\pi^6} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta^3} (1-\alpha-\beta)r(m_Q, m_Q)^4, \]

\[ \rho^{(\bar{q}q)}(s) = \frac{3}{2^{9}\pi^4} m_Q \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta} r(m_Q, m_Q)^2, \]

\[ \rho^{(\bar{q}q)^2}(s) = \frac{(\bar{q}q)^2}{2^{4}\pi^2} m_Q^2 \sqrt{1-4m_Q^2/s}, \]

\[ \rho^{(g\bar{q}\sigma \cdot Gq)}(s) = \frac{3}{2^{9}\pi^4} m_Q \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta} (1-\alpha-\beta)r(m_Q, m_Q), \]

\[ \rho^{(g^2G^2)}(s) = \frac{\frac{3}{2^{7}\pi^4} m_Q^2}{2^{10}\pi^6} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta} (1-\alpha-\beta)r(m_Q, m_Q), \]
\[
\rho^{(g^3G^3)}(s) = \frac{(g^3G^3)}{2^{12}\pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1 - \alpha - \beta)\left[r(mQ, mQ) + 2m_Q^2/\beta\right],
\]
for \((Q\bar{q})_0(\bar{Q}q)_0\),

\[
\rho^{\text{OPE}}(s) = \rho^{\text{pert}}(s) + \rho^{(\bar{q}\bar{q})}(s) + \rho^{(\bar{q}\bar{q})^2}(s) + \rho^{(g\bar{q}\sigma Gq)}(s) + \rho^{(g^2G^2)}(s) + \rho^{(g^3G^3)}(s),
\]

\[
\rho^{\text{pert}}(s) = \frac{3}{2^{9}\pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1 - \alpha - \beta)r(mQ, mQ)^4,
\]

\[
\rho^{(\bar{q}\bar{q})}(s) = \frac{3(\bar{q}\bar{q})}{2^{9}\pi^4} mQ \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta} r(mQ, mQ)^2,
\]

\[
\rho^{(\bar{q}\bar{q})^2}(s) = \frac{(\bar{q}\bar{q})^2}{2^{10}\pi^2} m_Q^2 \sqrt{1 - 4m_Q^2/s},
\]

\[
\rho^{(g\bar{q}\sigma Gq)}(s) = \frac{3(g\bar{q}\sigma \cdot Gq)}{2^8\pi^4} mQ \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha} [m_Q^2 - \alpha(1 - \alpha)s],
\]

\[
\rho^{(g^2G^2)}(s) = \frac{(g^2G^2)}{2^8\pi^6} m_Q^2 \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta} (1 - \alpha - \beta)r(mQ, mQ),
\]

\[
\rho^{(g^3G^3)}(s) = \frac{(g^3G^3)}{2^{10}\pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1 - \alpha - \beta)[r(mQ, mQ) + 2m_Q^2/\beta],
\]
for \((Q\bar{q})_1(\bar{Q}q)_1\),

\[
\rho^{\text{OPE}}(s) = -\{\rho^{\text{pert}}(s) + \rho^{(\bar{q}\bar{q})}(s) + \rho^{(\bar{q}\bar{q})^2}(s) + \rho^{(g\bar{q}\sigma Gq)}(s) + \rho^{(g^2G^2)}(s) + \rho^{(g^3G^3)}(s)\},
\]

\[
\rho^{\text{pert}}(s) = -\frac{3}{2^{12}\pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1 - \alpha - \beta)(1 + \alpha + \beta)r(mQ, mQ)^4,
\]

\[
\rho^{(\bar{q}\bar{q})}(s) = -\frac{3(\bar{q}\bar{q})}{2^{9}\pi^4} mQ \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta} (1 + \alpha + \beta)r(mQ, mQ)^2,
\]

\[
\rho^{(\bar{q}\bar{q})^2}(s) = -\frac{(\bar{q}\bar{q})^2}{2^{10}\pi^2} m_Q^2 \sqrt{1 - 4m_Q^2/s},
\]

\[
\rho^{(g\bar{q}\sigma Gq)}(s) = \frac{3(g\bar{q}\sigma \cdot Gq)}{2^8\pi^4} mQ \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha} \{\int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta} r(mQ, mQ) - \frac{2}{1 - \alpha}[m_Q^2 - \alpha(1 - \alpha)s]\},
\]
\[
\rho^{(g^2G^2)}(s) = -\frac{(g^2G^2)}{2^{11}\pi^6}m_Q^2 \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\text{min}}}^{1-\alpha} d\beta (1-\alpha-\beta)(1+\alpha+\beta)r(m_Q,m_Q),
\]

\[
\rho^{(g^3G^3)}(s) = -\frac{(g^3G^3)}{2^{12}\pi^6}m_Q^2 \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\text{min}}}^{1-\alpha} d\beta (1-\alpha-\beta)(1+\alpha+\beta)[r(m_Q,m_Q) + 2m_Q^2\beta],
\]

for \((Q\bar{q})(Q\bar{q})_0^0\),

\[
\rho^{\text{OPE}}(s) = \rho^{\text{pert}}(s) + \rho^{(\bar{q}q)^2}(s) + \rho^{(\bar{q}q)^2}(s) + \rho^{(g^2\sigma \cdot Gq)}(s) + \rho^{(g^2G^2)}(s) + \rho^{(g^3G^3)}(s),
\]

\[
\rho^{\text{pert}}(s) = \frac{3}{2^{11}\pi^6} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta^3} (1-\alpha-\beta)r(m_Q,m_Q)^4,
\]

\[
\rho^{(\bar{q}q)^2}(s) = -\frac{(q\bar{q})^2}{2^{14}\pi^6}m_Q^2 \sqrt{1-4m_Q^2}/s,
\]

\[
\rho^{(g^2G^2)}(s) = -\frac{(g^2G^2)}{2^{12}\pi^6}m_Q^2 \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\text{min}}}^{1-\alpha} d\beta (1-\alpha-\beta)r(m_Q,m_Q),
\]

\[
\rho^{(g^3G^3)}(s) = -\frac{(g^3G^3)}{2^{12}\pi^6}m_Q^2 \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\text{min}}}^{1-\alpha} d\beta (1-\alpha-\beta)[r(m_Q,m_Q) + 2m_Q^2\beta],
\]

for \((Q\bar{q})(Q\bar{q})_0^0\),

\[
\rho^{\text{OPE}}(s) = -\{\rho^{\text{pert}}(s) + \rho^{(\bar{q}q)^2}(s) + \rho^{(\bar{q}q)^2}(s) + \rho^{(g^2\sigma \cdot Gq)}(s) + \rho^{(g^2G^2)}(s) + \rho^{(g^3G^3)}(s)\},
\]

\[
\rho^{\text{pert}}(s) = -\frac{3}{2^{12}\pi^6} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta^3} (1-\alpha-\beta)(1+\alpha+\beta)r(m_Q,m_Q)^4,
\]

\[
\rho^{(\bar{q}q)}(s) = -\frac{3(q\bar{q})}{2^{14}\pi^6}m_Q \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta^2} (1-\alpha-\beta)r(m_Q,m_Q)^2,
\]

\[
\rho^{(\bar{q}q)^2}(s) = -\frac{(q\bar{q})^2}{2^{14}\pi^6}m_Q^2 \sqrt{1-4m_Q^2}/s,
\]

\[
\rho^{(g^2\sigma \cdot Gq)}(s) = -\frac{3(g^2\sigma \cdot Gq)}{2^{15}\pi^6}m_Q \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta} r(m_Q,m_Q),
\]

\[
\rho^{(g^2G^2)}(s) = -\frac{(g^2G^2)}{2^{11}\pi^6}m_Q^2 \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\text{min}}}^{1-\alpha} d\beta (1-\alpha-\beta)(1+\alpha+\beta)r(m_Q,m_Q),
\]
\[ \rho^{(g^3G^3)}(s) = -\frac{(g^3G^3)}{2^{15}\pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1 - \alpha - \beta)(1 + \alpha + \beta)r(m_Q, m_Q) + 2m_Q^2], \]

for \((Q\bar{q})_1(\bar{Q}q),\)

\[ \rho_{\text{OPE}}(s) = -\{\rho_{\text{pert}}(s) + \rho^{(\bar{q}q)}(s) + \rho^{(\bar{q}q)^2}(s) + \rho^{(q\bar{q}\sigma\cdot Gq)}(s) + \rho^{(g^2G^2)}(s) + \rho^{(g^3G^3)}(s)\}, \]

\[ \rho_{\text{pert}}(s) = -\frac{3}{2^{12}\pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1 - \alpha - \beta)(1 + \alpha + \beta)r(m_Q, m_Q)^4, \]

\[ \rho^{(\bar{q}q)}(s) = \frac{3\langle \bar{q}q \rangle}{2^{7}\pi^4} m_Q \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta}(1 - \alpha - \beta)r(m_Q, m_Q)^2, \]

\[ \rho^{(\bar{q}q)^2}(s) = \frac{\langle \bar{q}q \rangle^2}{2^{4}\pi^2} m_Q^2 \sqrt{1 - 4m_Q^2/s}, \]

\[ \rho^{(g\bar{q}q\cdot Gq)}(s) = \frac{3\langle g\bar{q}q\cdot Gq \rangle}{\pi^4} m_Q \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta} r(m_Q, m_Q), \]

\[ \rho^{(g^2G^2)}(s) = -\frac{(g^2G^2)}{2^{11}\pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta}(1 - \alpha - \beta)(1 + \alpha + \beta)r(m_Q, m_Q), \]

\[ \rho^{(g^3G^3)}(s) = -\frac{(g^3G^3)}{2^{15}\pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta}(1 - \alpha - \beta)[r(m_Q, m_Q) + 2m_Q^2], \]

for \((Q\bar{q})^*(\bar{Q}q)_0\), and

\[ \rho_{\text{OPE}}(s) = \rho_{\text{pert}}(s) + \rho^{(\bar{q}q)}(s) + \rho^{(\bar{q}q)^2}(s) + \rho^{(q\bar{q}\sigma\cdot Gq)}(s) + \rho^{(g^2G^2)}(s) + \rho^{(g^3G^3)}(s), \]

\[ \rho_{\text{pert}}(s) = \frac{3}{2^{9}\pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1 - \alpha - \beta)r(m_Q, m_Q)^4, \]

\[ \rho^{(\bar{q}q)^2}(s) = -\frac{\langle \bar{q}q \rangle^2}{2^{4}\pi^2} m_Q^2 \sqrt{1 - 4m_Q^2/s}, \]

\[ \rho^{(g^2G^2)}(s) = \frac{(g^2G^2)}{2^{7}\pi^4} m_Q \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta} r(m_Q, m_Q), \]

\[ \rho^{(g^3G^3)}(s) = \frac{(g^3G^3)}{2^{11}\pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta}(1 - \alpha - \beta)[r(m_Q, m_Q) + 2m_Q^2]. \]
for \((Q\bar{q})(\bar{Q}q)\). The integration limits are given by 
\[ \alpha_{\text{min}} = (1 - \sqrt{1 - 4m_{Q}^2/s})/2, \quad \alpha_{\text{max}} = (1 + \sqrt{1 - 4m_{Q}^2/s})/2 \]
and \(\beta_{\text{min}} = \alpha m_{Q}^2/(s \alpha - m_{Q}^2)\).

Second, with

\[
\rho_{\text{OPE}}(s) = \rho_{\text{pert}}(s) + \rho^{(\bar{q}q)}(s) + \rho^{(\bar{q}q)^2}(s) + \rho^{(g\bar{q}\bar{q}Gq)}(s) + \rho^{(g^2G^2)}(s) + \rho^{(g^3G^3)}(s),
\]

\[
\rho_{\text{pert}}(s) = \frac{3}{2^{11} \pi^6} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta^3} (1 - \alpha - \beta) r(m_Q, m_{Q'})^4,
\]

\[
\rho^{(\bar{q}q)}(s) = -\frac{3}{2^7 \pi^4} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} d\alpha \int_{\beta_{\text{min}}}^{1-\alpha} d\beta \left( \frac{m_Q^2}{\alpha^2} + \frac{m_Q}{\alpha \beta} \right) r(m_Q, m_{Q'})^2,
\]

\[
\rho^{(\bar{q}q)^2}(s) = \frac{\langle \bar{q}q \rangle^2}{2^4 \pi^2} m_Q m_{Q'} \sqrt{(s - m_Q^2 + m_{Q'}^2)^2 - 4m_Q^2 s/s},
\]

\[
\rho^{(g\bar{q}\bar{q}Gq)}(s) = -\frac{3}{2^8 \pi^4} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\text{min}}}^{1-\alpha} d\beta \left( \frac{m_Q^2}{\alpha^2} + \frac{m_Q}{\alpha \beta} \right) r(m_Q, m_{Q'})^4,
\]

\[
\rho^{(g^2G^2)}(s) = \frac{\langle g^2 G^2 \rangle}{2^1 \pi^6} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} d\alpha \int_{\beta_{\text{min}}}^{1-\alpha} d\beta \left( 1 - \alpha - \beta \right) \left( \frac{1}{\alpha^3} + \frac{1}{\beta^3} \right) r(m_Q, m_{Q'})^2,
\]

\[
\rho^{(g^3G^3)}(s) = \frac{\langle g^3 G^3 \rangle}{2^3 \pi^4} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} d\alpha \int_{\beta_{\text{min}}}^{1-\alpha} d\beta \left( \frac{m_Q^2}{\alpha^2} + \frac{m_Q}{\alpha \beta} \right) r(m_Q, m_{Q'})^2,
\]

for \((Q\bar{q})(\bar{Q}q)\).
\[ \rho(g^2 G^2)(s) = -\frac{(g^2 G^2)}{2^{12} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta (1 - \alpha - \beta)(1 + \alpha + \beta)\left(\frac{m_Q^2}{\alpha^3} + \frac{m_{Q'}^2}{\beta^3}\right)r(m_Q, m_{Q'}), \]

\[ \rho(g^3 G^3)(s) = -\frac{(g^3 G^3)}{2^{14} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta (1 - \alpha - \beta)(1 + \alpha + \beta)[\frac{1}{\alpha^3} + \frac{1}{\beta^3}]r(m_Q, m_{Q'}) + 2\left(\frac{m_{Q'}^2}{\alpha^3} + \frac{m_Q^2}{\beta^3}\right), \]

for \((Qq)^*(\bar{Q}q)\),

\[ \rho_{\text{OPE}}(s) = \rho_{\text{pert}}(s) + \rho(\bar{q}q)(s) + \rho(q\bar{q})^2(s) + \rho(gq\sigma \cdot Gq)(s) + \rho(g^2 G^2)(s) + \rho(g^3 G^3)(s), \]

\[ \rho_{\text{pert}}(s) = \frac{3}{2^{9} \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta (1 - \alpha - \beta)r(m_Q, m_{Q'})^4, \]

\[ \rho(\bar{q}q)(s) = -\frac{3(gq)}{2^{7} \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta (\frac{m_{Q'}}{\alpha^2 \beta} + \frac{m_Q}{\alpha \beta^2})r(m_Q, m_{Q'})^2, \]

\[ \rho(q\bar{q})^2(s) = \frac{(\bar{q}q)^2}{2^{12} \pi^2} m_Q m_{Q'} \sqrt{(s - m_Q^2 + m_{Q'}^2)^2 - 4 m_Q^2 s/s}, \]

\[ \rho(gq\sigma \cdot Gq)(s) = -\frac{3(gq\sigma \cdot Gq)}{2^{9} \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \left(\frac{m_{Q'}}{\alpha} + \frac{m_Q}{1 - \alpha}\right)[am_Q^2 + (1 - \alpha) m_{Q'}^2 - \alpha(1 - \alpha)s], \]

\[ \rho(g^2 G^2)(s) = \frac{(g^2 G^2)}{2^{12} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta (1 - \alpha - \beta)\left(\frac{m_{Q'}^2}{\alpha^3} + \frac{m_Q^2}{\beta^3}\right)r(m_Q, m_{Q'}), \]

\[ \rho(g^3 G^3)(s) = \frac{(g^3 G^3)}{2^{14} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta (1 - \alpha - \beta)[\frac{1}{\alpha^3} + \frac{1}{\beta^3}]r(m_Q, m_{Q'}) + 2\left(\frac{m_{Q'}^2}{\alpha^3} + \frac{m_Q^2}{\beta^3}\right), \]

for \((Qq)^*(\bar{Q}q)^*\),

\[ \rho_{\text{OPE}}(s) = \rho_{\text{pert}}(s) + \rho(\bar{q}q)(s) + \rho(q\bar{q})^2(s) + \rho(gq\sigma \cdot Gq)(s) + \rho(g^2 G^2)(s) + \rho(g^3 G^3)(s), \]

\[ \rho_{\text{pert}}(s) = \frac{3}{2^{11} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta (1 - \alpha - \beta)r(m_Q, m_{Q'})^4, \]

\[ \rho(\bar{q}q)(s) = \frac{3(\bar{q}q)}{2^{7} \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta (\frac{m_{Q'}}{\alpha^2 \beta} + \frac{m_Q}{\alpha \beta^2})r(m_Q, m_{Q'})^2, \]

\[ \rho(q\bar{q})^2(s) = \frac{(\bar{q}q)^2}{2^{12} \pi^2} m_Q m_{Q'} \sqrt{(s - m_Q^2 + m_{Q'}^2)^2 - 4 m_Q^2 s/s}, \]
\[ \rho^{(q\bar{q}\sigma-Gq)}(s) = \frac{3}{2^8 \pi^4} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \left( \frac{m_{Q'}}{\alpha} + \frac{m_{Q}}{1-\alpha} \right) |am_{Q}^2 + (1-\alpha)m_{Q'}^2 - \alpha(1-\alpha)s|, \]

\[ \rho^{(g^2G^2)}(s) = \frac{g^2G^2}{2^{11} \pi^6} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \int_{\beta_{min}}^{1-\alpha} d\beta (1-\alpha - \beta) \left( \frac{m_{Q'}}{\alpha^3} + \frac{m_{Q}}{\beta^3} \right) r(m_Q, m_{Q'}), \]

\[ \rho^{(g^3G^3)}(s) = \frac{g^3G^3}{2^{13} \pi^6} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \int_{\beta_{min}}^{1-\alpha} d\beta (1-\alpha - \beta) \left[ \frac{1}{\alpha^3} + \frac{1}{\beta^3} \right] r(m_Q, m_{Q'}) + 2 \left( \frac{m_{Q'}^2}{\alpha^3} + \frac{m_{Q}^2}{\beta^3} \right), \]

for \((Q\bar{q})_0(Q'\bar{q})_0^*,\)

\[ \rho^{OPE}(s) = \rho^{pert}(s) + \rho^{(q\bar{q})^2}(s) + \rho^{(g\bar{q}q-Gq)}(s) + \rho^{(g^2G^2)}(s) + \rho^{(g^3G^3)}(s), \]

\[ \rho^{pert}(s) = \frac{3}{2^9 \pi^4} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \int_{\beta_{min}}^{1-\alpha} d\beta \left(1-\alpha - \beta \right) r(m_Q, m_{Q'})^4, \]

\[ \rho^{(q\bar{q})}(s) = \frac{3}{2^8 \pi^4} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \int_{\beta_{min}}^{1-\alpha} d\beta \left( \frac{m_{Q'}}{\alpha^2 \beta} + \frac{m_{Q}}{\alpha \beta^2} \right) r(m_Q, m_{Q'})^2, \]

\[ \rho^{(q\bar{q})^2}(s) = \frac{\langle q\bar{q} \rangle^2}{2^2 \pi^2} m_Q m_{Q'} \sqrt{(s-m_Q^2 + m_{Q'}^2)^2 - 4m_Q^2 s/s}, \]

\[ \rho^{(g\bar{q}q-Gq)}(s) = \frac{3}{2^8 \pi^4} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \left( \frac{m_{Q'}}{\alpha} + \frac{m_{Q}}{1-\alpha} \right) |am_{Q}^2 + (1-\alpha)m_{Q'}^2 - \alpha(1-\alpha)s|, \]

\[ \rho^{(g^2G^2)}(s) = \frac{g^2G^2}{2^{11} \pi^6} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \int_{\beta_{min}}^{1-\alpha} d\beta (1-\alpha - \beta) \left( \frac{m_{Q'}}{\alpha^3} + \frac{m_{Q}}{\beta^3} \right) r(m_Q, m_{Q'}), \]

\[ \rho^{(g^3G^3)}(s) = \frac{g^3G^3}{2^{13} \pi^6} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \int_{\beta_{min}}^{1-\alpha} d\beta (1-\alpha - \beta) \left[ \frac{1}{\alpha^3} + \frac{1}{\beta^3} \right] r(m_Q, m_{Q'}) + 2 \left( \frac{m_{Q'}^2}{\alpha^3} + \frac{m_{Q}^2}{\beta^3} \right), \]

for \((Q\bar{q})_1(\bar{Q}'q)_1^*,\)

\[ \rho^{OPE}(s) = -\{ \rho^{pert}(s) + \rho^{(q\bar{q})}(s) + \rho^{(q\bar{q})^2}(s) + \rho^{(g\bar{q}q-Gq)}(s) + \rho^{(g^2G^2)}(s) + \rho^{(g^3G^3)}(s) \}, \]

\[ \rho^{pert}(s) = -\frac{3}{2^9 \pi^4} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \int_{\beta_{min}}^{1-\alpha} d\beta \left(1-\alpha - \beta \right)(1 + \alpha + \beta) r(m_Q, m_{Q'})^4, \]

\[ \rho^{(q\bar{q})}(s) = -\frac{3}{2^8 \pi^4} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha \int_{\beta_{min}}^{1-\alpha} d\beta \frac{m_{Q'}(\alpha + \beta)}{\alpha^2 \beta} + \frac{m_{Q}}{\alpha \beta^2} |r(m_Q, m_{Q'})^2, \]
\[ \rho^{(\bar{q}q)^2}(s) = -\frac{(\bar{q}q)^2}{24\pi^2}m_Qm_{Q'}\sqrt{(s - m_Q^2 + m_{Q'}^2)^2 - 4m_Q^2}m/s, \]

\[ \rho^{(g\bar{q}Gq)}(s) = \frac{3\langle g\bar{q}Gq \rangle}{2^{8/4}2^{16}} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\min}}^{1-\alpha} d\beta (1-\alpha - \beta)(1+\alpha + \beta)\frac{m_Q^2}{\alpha^3} + \frac{m_{Q'}^2}{\beta^3}r(m_Q,m_{Q'}), \]

\[ \rho^{(g^2G^2)}(s) = -\frac{\langle g^2G^2 \rangle}{2^{14}\pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^4} \int_{\beta_{\min}}^{1-\alpha} d\beta (1-\alpha - \beta)(1+\alpha + \beta)\left[\frac{1}{\alpha^3} + \frac{1}{\beta^3}\right]r(m_Q,m_{Q'}), \]

\[ \rho^{(g^3G^3)}(s) = -\frac{\langle g^3G^3 \rangle}{2^{14}\pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^6} \int_{\beta_{\min}}^{1-\alpha} d\beta (1-\alpha - \beta)(1+\alpha + \beta)\left[\frac{1}{\alpha^3} + \frac{1}{\beta^3}\right]r(m_Q,m_{Q'}), \]

for \((\bar{Q}q)(\bar{Q}'q)\) and \((\bar{Q}q)(\bar{Q}'q)\) states.

\[ \rho^{\text{OPE}}(s) = \rho^{\text{pert}}(s) + \rho^{(\bar{q}q)}(s) + \rho^{(\bar{q}q)^2}(s) + \rho^{(g\bar{q}Gq)}(s) + \rho^{(g^2G^2)}(s) + \rho^{(g^3G^3)}(s), \]

\[ \rho^{\text{pert}}(s) = \frac{3}{2^{14}\pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} d\beta (1-\alpha - \beta)(1+\alpha + \beta)\left[\frac{1}{\alpha^3} + \frac{1}{\beta^3}\right]r(m_Q,m_{Q'}). \]
\[
\rho^{(q\bar{q})}(s) = \frac{3(q\bar{q})}{2^{7\pi^4}} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} d\alpha \int_{\beta_{\text{min}}}^{1-\alpha} d\beta [\frac{m_{Q'}(\alpha + \beta)}{\alpha^2 \beta} - \frac{m_Q}{\alpha \beta^2}] (m_Q, m_{Q'})^2,
\]
\[
\rho^{(q\bar{q})^2}(s) = \frac{(q\bar{q})^2}{2^{4\pi^2}} m_Q m_{Q'} \sqrt{(s-m_Q^2 + m_{Q'}^2)^2 - 4m_Q^2 s/s},
\]
\[
\rho^{(gq\sigma \cdot Gq)}(s) = \frac{3(gq\sigma \cdot Gq)}{2^{8\pi^4}} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} d\alpha \times \{-\frac{m_{Q'}}{\alpha} \int_{\beta_{\text{min}}}^{1-\alpha} d\beta [r(m_Q, m_{Q'}) + \frac{m_Q}{1-\alpha}] (am_Q^2 + (1-\alpha)m_{Q'}^2 - \alpha(1-\alpha)s)\},
\]
\[
\rho^{(g^2G^2)}(s) = -\frac{(g^2G^2)}{2^{12\pi^6}} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} d\alpha \int_{\beta_{\text{min}}}^{1-\alpha} d\beta (1-\alpha - \beta)(1 + \alpha + \beta)\left[\frac{1}{\alpha^3} + \frac{1}{\beta^3}\right] r(m_Q, m_{Q'})^4,
\]
\[
\rho^{(g^3G^3)}(s) = -\frac{(g^3G^3)}{2^{14\pi^6}} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} d\alpha \int_{\beta_{\text{min}}}^{1-\alpha} d\beta (1-\alpha - \beta)(1 + \alpha + \beta)\left[\frac{1}{\alpha^3} + \frac{1}{\beta^3}\right] r(m_Q, m_{Q'})^4,
\]
for \((Q\bar{q})\),

\[
\rho^{\text{OPE}}(s) = \rho^{\text{pert}}(s) + \rho^{(q\bar{q})}(s) + \rho^{(q\bar{q})^2}(s) + \rho^{(gq\sigma \cdot Gq)}(s) + \rho^{(g^2G^2)}(s) + \rho^{(g^3G^3)}(s),
\]
\[
\rho^{\text{pert}}(s) = -\frac{3}{2^{12\pi^6}} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} d\alpha \int_{\beta_{\text{min}}}^{1-\alpha} d\beta (1-\alpha - \beta)(1 + \alpha + \beta) r(m_Q, m_{Q'})^4,
\]
\[
\rho^{(q\bar{q})}(s) = \frac{3(q\bar{q})}{2^{4\pi^4}} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} d\alpha \int_{\beta_{\text{min}}}^{1-\alpha} d\beta [-\frac{m_{Q'}(\alpha + \beta)}{\alpha^2 \beta} + \frac{m_Q}{\alpha \beta^2}] r(m_Q, m_{Q'})^2,
\]
\[
\rho^{(q\bar{q})^2}(s) = \frac{(q\bar{q})^2}{2^{4\pi^2}} m_Q m_{Q'} \sqrt{(s-m_Q^2 + m_{Q'}^2)^2 - 4m_Q^2 s/s},
\]
\[
\rho^{(gq\sigma \cdot Gq)}(s) = \frac{3(gq\sigma \cdot Gq)}{2^{8\pi^4}} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} d\alpha \left(\frac{m_{Q'}}{\alpha} \int_{\beta_{\text{min}}}^{1-\alpha} d\beta r(m_Q, m_{Q'}) + \left(-\frac{m_{Q'}}{\alpha} + \frac{m_Q}{1-\alpha}\right)(am_Q^2 + (1-\alpha)m_{Q'}^2 - \alpha(1-\alpha)s)\right),
\]
\[
\rho^{(g^2G^2)}(s) = -\frac{(g^2G^2)}{2^{12\pi^6}} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} d\alpha \int_{\beta_{\text{min}}}^{1-\alpha} d\beta (1-\alpha - \beta)(1 + \alpha + \beta)\left[\frac{1}{\alpha^3} + \frac{1}{\beta^3}\right] r(m_Q, m_{Q'})^4,
\]
\[
\rho^{(g^3G^3)}(s) = -\frac{(g^3G^3)}{2^{14\pi^6}} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} d\alpha \int_{\beta_{\text{min}}}^{1-\alpha} d\beta (1-\alpha - \beta)(1 + \alpha + \beta)\left[\frac{1}{\alpha^3} + \frac{1}{\beta^3}\right] r(m_Q, m_{Q'})^4,
\]
for \((Q\bar{q})\),

\[
\rho^{\text{OPE}}(s) = \rho^{\text{pert}}(s) + \rho^{(q\bar{q})}(s) + \rho^{(q\bar{q})^2}(s) + \rho^{(gq\sigma \cdot Gq)}(s) + \rho^{(g^2G^2)}(s) + \rho^{(g^3G^3)}(s),
\]
\[
\rho_{\text{bert}}(s) = \frac{3}{2^9\pi^6} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta^3} (1 - \alpha - \beta) r(m_Q, m_{Q'})^4,
\]

\[
\rho^{(q\bar{q})}(s) = \frac{3\langle q\bar{q} \rangle}{2^9\pi^4} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta} \left( \frac{m_Q}{\alpha^2} - \frac{m_Q}{\alpha\beta} \right) r(m_Q, m_{Q'})^2,
\]

\[
\rho^{(q\bar{q})^2}(s) = -\frac{\langle q\bar{q}\rangle^2}{2^2\pi^2} m_Q m_{Q'} \sqrt{(s - m_Q^2 + m_{Q'}^2)^2 - 4m^2_{Q'} s / s},
\]

\[
\rho^{(g\bar{g}G\bar{g})}(s) = \frac{3\langle g\bar{g}\sigma \cdot G\bar{g} \rangle}{2^7\pi^4} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta} \left( \frac{m_{Q'}^2}{\alpha^2} + \frac{m_{Q'}^2}{\beta^2} \right) r(m_Q, m_{Q'})^2.
\]

\[
\rho^{(g^3G^3)}(s) = \frac{\langle g^3G^3 \rangle}{2^41\pi^4} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta} \left( \frac{1}{\alpha^2} + \frac{1}{\beta^2} \right) [r(m_Q, m_{Q'}) + 2\left( \frac{m_{Q'}^2}{\alpha^3} + \frac{m_{Q'}^2}{\beta^3} \right)],
\]

for \( (Q\bar{q})^*(Q'q)_1 \). The integration limits are given by \( \alpha_{\text{min}} = [s - m_Q^2 + m_{Q'}^2 - \sqrt{(s - m_Q^2 + m_{Q'}^2)^2 - 4m_{Q'}^2 s}] / (2s) \), \( \alpha_{\text{max}} = [s - m_Q^2 + m_{Q'}^2 + \sqrt{(s - m_Q^2 + m_{Q'}^2)^2 - 4m_{Q'}^2 s}] / (2s) \), and \( \beta_{\text{min}} = \alpha m_{Q'}^2 / (s\alpha - m_Q^2) \).

**ACKNOWLEDGMENTS**

J. R. Zhang is very indebted to Ming Zhong for helpful discussions. This work was supported in part by the National Natural Science Foundation of China under Contract No.10675167.

[1] S. K. Choi et al., (Belle Collaboration), Phys. Rev. Lett. 91, 262001 (2003); V. M. Abazov et al., (D0 Collaboration), Phys. Rev. Lett. 93, 162002 (2004); D. Acosta et al., (CDF Collaboration), Phys. Rev. Lett. 93, 072001 (2004); B. Aubert et al., (BaBar Collaboration), Phys. Rev. D 71, 071103 (2005).

[2] S. K. Choi et al., (Belle Collaboration), Phys. Rev. Lett. 94, 182002 (2005); B. Aubert et al., (BaBar Collaboration), Phys. Rev. Lett. 101, 082001 (2008).

[3] B. Aubert et al., (BaBar Collaboration), Phys. Rev. Lett. 95, 142001 (2005); Q. He et al., (CLEO Collaboration), Phys. Rev. D 74, 091104(R) (2006); C. Z. Yuan et al., (Belle Collaboration), Phys. Rev. Lett. 99, 182004 (2007).

[4] S. Uehara et al., (Belle Collaboration), Phys. Rev. Lett. 96, 082003 (2006).

[5] K. Abe et al., (Belle Collaboration), Phys. Rev. Lett. 98, 082001 (2007).

[6] K. Abe et al., (Belle Collaboration), Phys. Rev. Lett. 100, 142001 (2008).

[7] R. Mizuk et al., (Belle Collaboration), Phys. Rev. D 78, 072004 (2008).

[8] T. Aaltonen et al., (CDF Collaboration), arXiv:0903.2229

[9] E. S. Swanson, Phys. Rep. 429, 243 (2006).

[10] C. Amsler et al., (Particle Data Group), Phys. Lett. B 667, 1 (2008).
[11] X. Liu, X. Q. Zeng, and X. Q. Li, Phys. Rev. D 72, 054023 (2005); Y. J. Zhang, H. C. Chiang, P. N. Shen, and B. S. Zou, Phys. Rev. D 74, 014013 (2006); J. L. Rosner, Phys. Rev. D 76, 114002 (2007); G. J. Ding, Phys. Rev. D 79, 014001 (2009); G. J. Ding, J. F. Liu, and M. L. Yan, Phys. Rev. D 79, 054005 (2009); G. J. Ding, arXiv:0905.1188.

[12] F. E. Close and P. R. Page, Phys. Lett. B 578, 119 (2004); M. B. Voloshin, Phys. Lett. B 579, 316 (2004); C. Y. Wong, Phys. Rev. C 69, 055202 (2004); E. S. Swanson, Phys. Lett. B 588, 189 (2004); N. A. Törnqvist, Phys. Lett. B 590, 209 (2004); E. S. Swanson, Phys. Lett. B 598, 197 (2004); C. E. Thomas and F. E. Close, Phys. Rev. D 78, 034007 (2008); Y. R. Liu, X. Liu, W. Z. Deng, and S. L. Zhu, Euro. Phys. J. C 56, 63 (2008); Y. R. Liu and Z. Y. Zhang, Phys. Rev. C 79, 035206 (2009).

[13] C. Meng and K. T. Chao, arXiv:0708.4222; X. Liu, Y. R. Liu, W. Z. Deng, and S. L. Zhu, Phys. Rev. D 77, 034003 (2008); X. Liu, Y. R. Liu, W. Z. Deng, and S. L. Zhu, Phys. Rev. D 77, 094015 (2008).

[14] X. Liu, Z. G. Luo, Y. R. Liu, and S. L. Zhu, arXiv:0808.0073.

[15] X. Liu and S. L. Zhu, arXiv:0903.2529.

[16] N. Mahajan, arXiv:0903.3107; T. Branz, T. Gutsche, and V. E. Lyubovitskij, arXiv:0903.5424; G. J. Ding, arXiv:0904.1782.

[17] M. B. Voloshin and L. B. Okun, JETP Lett. 23, 333 (1976).

[18] A. D. Rujula, H. Georgi, and S. L. Glashow, Phys. Rev. Lett. 38, 317 (1977).

[19] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B147, 385 (1979); B147, 448 (1979); V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Fortschr. Phys. 32, 585 (1984).

[20] M. A. Shifman, Vacuum Structure and QCD Sum Rules, North-Holland, Amsterdam 1992.

[21] B. L. Ioffe, in “The spin structure of the nucleon”, edited by B. Freis, V. W. Hughes, N. de Groot, World Scientific (1997), arXiv:9511401.

[22] S. Narison, QCD Spectral Sum Rules, World Scientific, Singapore, 1989.

[23] P. Colangelo and A. Khodjamirian, in: M. Shifman (Ed.), At the Frontier of Particle Physics: Handbook of QCD, vol. 3, Boris Ioffe Festschrift, World Scientific, Singapore, 2001, pp. 1495-1576, arXiv:0010175; A. Khodjamirian, talk given at Continuous Advances in QCD 2002/ARKADYFEST, arXiv:0209166.

[24] J. R. Zhang and M. Q. Huang, arXiv:0905.4178; J. R. Zhang and M. Q. Huang, arXiv:0905.4672.

[25] S. H. Lee, M. Nielsen, and U. Wiedner, Phys. Rev. D 78, 014005 (2008); S. H. Lee, K. Morita, and M. Nielsen, Nucl. Phys. A 815, 29 (2009); R. M. Albuquerque and M. Nielsen, Nucl. Phys. A 815, 53 (2009); R. M. Albuquerque, M. E. Bracco, and M. Nielsen, arXiv:0903.5540.

[26] Z. G. Wang, arXiv:0903.5200.

[27] L. J. Reinders, H. R. Rubinstein, and S. Yazaki, Phys. Rep. 127, 1 (1985).

[28] L. J. Reinders, H. R. Rubinstein, and S. Yazaki, Nucl. Phys. B186, 109 (1981).

[29] R. D. Matheus, S. Narison, M. Nielsen, and J. M. Richard, Phys. Rev. D 75, 014005 (2007); S. H. Lee, K. Morita, and M. Nielsen, Phys. Rev. D 78, 076001 (2008); M. E. Bracco, S. H. Lee, M. Nielsen, and R. R. daSilva, Phys. Lett. B 671, 240 (2009).

[30] J. R. Zhang and M. Q. Huang, Phys. Rev. D 77, 094002 (2008); Phys. Rev. D 78, 094007 (2008); Phys. Rev. D 78, 094015 (2008); Phys. Lett. B 674, 28 (2009).