Research on Key Technologies of the Non-Bracket Construction Method for an Annular Cable Supported Grid Structure

Lifan Huang 1,2,++, Yangjie Ruan 1,2,*, Bin Luo 1,2,+++, Mingmin Ding 3 and Hao Gu 4

1 College of Civil Engineering, Southeast University, Nanjing 211189, China; huanglifan@seu.edu.cn (L.H.); seurobin@seu.edu.cn (B.L.)
2 National Prestress Engineering Research Center of China, Southeast University, Nanjing 211189, China
3 College of Civil Engineering, Nanjing Forestry University, Nanjing 211189, China; andyming1989seu@foxmail.com
4 China Resources Land Limited (Wuxi), Wuxi 214000, China; guhaoseu@163.com
* Correspondence: ruanyj@seu.edu.cn; Tel.: +86-180-8207-3566

Abstract: A cable strut structure uses a tension cable net as the main load-bearing system, which can allow full play of the high-strength material characteristics of the cables, greatly reduce the burden on the lower supporting system, and increase the span in an economic and effective manner. The annular cable supported grid structure is a new type of cable strut structure, which uses rigid grids to replace the flexible cables at the top chord of the spoke cable truss to meet the requirements of laying a heavy rigid roof. In this paper, first the structural mechanical characteristics of the annular cable supported grid structure are introduced, showing that the structural characteristics derived from the cable truss are the basis of non-bracket construction, while the presence of the upper grid results in difficulties with structure installation configuration control. Second, considering the characteristics of the cable supported grid structure and the difficulties in construction, the non-bracket construction method for annular cable supported grid structure with the commonly used nonlinear dynamic finite element method (NDFEM) in construction simulations is proposed. Finally, a numerical example is given and analyzed to verify the accuracy and feasibility of this method. The results indicate that the non-bracket construction technology proposed in this paper is suitable for the construction of a cable supported grid structure, and has the advantages of convenient prestress flow control, no need for brackets, simple and economical construction equipment, flexible arrangement of construction period, safe and reliable construction, and high construction accuracy.

Keywords: cable supported grid structure; construction method; form finding; non-bracket construction

1. Introduction

The annular cable supported grid structure is a new type of hybrid cable strut tension structure combining the mechanical characteristics of wheel spoke cable truss [1] and beam string structure [2]. Similar to the suspended dome [3,4], by using a rigid grid to replace the upper cable net of the double-layer wheel spoke cable truss, the annular cable supported grid structure can meet the requirements of laying a rigid roof while maintaining the advantages of simple and clear stress.

Cable supported grid structures have been studied all over the world. In the field of structural design, Feng et al. (2019) initially proposed the idea and structural form, and explained in detail the geometric composition, stress principle, and force transmission mechanism of the hybrid structure. In that paper, they also optimized the existing local analysis form finding method for cable supported grid structures to keep the strut vertical and control the strut length. They also give an example to verify the effectiveness of the form finding method [5]. Feng et al. (2019) conducted a comprehensive elastic-plastic nonlinear analysis of the spoke cable supported grid structure, and studied the static characteristics of the structural system by analyzing the influencing factors, such as the
pre-tension of the cable strut system, the strut height, the height difference of the outer compression ring, the plane shape of the tension ring, the vector height and form of the rigid grid, the curvature of the rigid grid rib beam, and the height difference of the inner ring [6].

For the cable strut tension structure, stiffness relies on the pre-tension applied in the cables; therefore, it is of great significance to study the process of establishing pre-tension in this kind of cable net construction. The research in the field of constructing cable strut tension structures generally focuses on two aspects: construction process simulation and construction method.

The process of simulating the construction of a cable strut tension structure needs to include a simulation of the dynamic process of the cable net from mechanism to structure, which includes large deformation and rigid body motion. Existing finite element software generally cannot achieve accurate results. The dynamic process of a cable strut tension structure from a state with no or low stress to high stress is a continuous process, which means that the whole structure is in static equilibrium at each time point during the forming process. Therefore, the simulation of the dynamic construction process is transformed into the solution of the equilibrium state of the cable strut system at each given time point, which means form finding.

In recent years, the research on form finding method has matured, and mainly includes the force density method [7], dynamic relaxation method [8,9], nonlinear force method [10], and finite element method [11]. Schek et al. (1974) first introduced the concept of force density to deform the node balance equation, turning the complex solution of the nonlinear equation group into an independent solution of the first-order equation group, which largely reduced the complexity of calculation [12]. Vassart and Motro et al. (1999) presented an analytical method after a lot of calculation and analysis based on the force density method. They solved the balance matrix by using the Gaussian elimination method, which solved the problem of finding the rank deficiency of the node balance matrix in the force density method, and took the lead in applying the force density method to the form finding research of the tensegrity structure [13]. Based on the research of Schek and Motro, Zhang and Ohsaki (2006) proposed the self-adaptive density method based on eigenvalue analysis and spectral decomposition of the force density matrix. In an iterative calculation, the force density matrix was automatically adjusted to constantly find the required rank deficiency value, which solved the problem of difficulty in applying the force density method to structures with massive cable and strut elements [14]. Based on the self-adaptive density method, Lee et al. (2010) adjusted the iterative strategy, presented an iterative calculation method that could be applied to a super-stable structure, and successfully carried out form finding analysis of the cable system and tensegrity structures [15,16]. Recently, Trinh et al. (2022) proposed a force density informed neural network method by utilizing a fully connected neural network, which can obtain the feasible pre-stress design of multiple tensegrity models without depending on the amount of data samples and considers the structural symmetric property effectively [17]. Otter (1966) first put forward the concept of the dynamic relaxation method, which is essentially applied to the numerical solution of a series of nonlinear equations [18]. Day and Bunc (1970) first applied the dynamic relaxation method to the analysis of cable structures [19]. Lewis (1984) used the dynamic relaxation method to solve the static response of cable truss and other pre-tension cable structures [20]. Recently, Maca et al. compared nine schemes of dynamic relaxation methods and summarized the most efficient and stable method [21]. Ding et al. (2015) introduced the operation principle of the nonlinear force method into the dynamic relaxation method and proposed the nonlinear dynamic finite element theory [22]. In addition, the research on applying the idea of optimization algorithm to form finding analysis increase gradually, such as the genetic algorithm [23,24], evolutionary algorithm [25] and so on, which could realize the optimization of the form finding results of numerical solutions through intelligent algorithms. At present, it is used in various types of long-span space...
structures, such as reticulated shell structure [26,27], cable dome [28], cable membrane structure [29] and so on.

In terms of construction methods, the traditional tower erection method is still applicable to the cable supported grid structure, which generally includes erecting a supporting tower, hoisting the upper grid structure on the tower, lifting and installing a high-altitude cable net, and ensuring cable tension. Recently, based on a typical cable supported grid structure composed of a rigid saddle-shaped, single-layer grid structure at the top chord, 68 radial cables at the bottom chord, one tension ring, and a vertical brace, Liu et al. (2021) successfully applied the traditional tower erection construction method and simulated the whole construction process using ANSYS finite element software to verify the accuracy and rationality of the method [30]. However, the erection of the supporting tower is bound to have some problems, such as the time it takes during the construction period, the lack of flexibility in arranging the construction process due to site occupation, and large expenditure on temporary construction measures. Therefore, it is of great research value to improve the traditional method of constructing a cable supported grid structure and put forward a method that does not require the set of brackets. In fact, in the research of cable strut tension structures of other systems, some non-bracket methods have already been proposed and applied in practice.

With regard to cable domes, Zong et al. (2010) proposed a cumulative tower lifting cable strut installation technology based on the understanding that the process of constructing a cable dome is essentially a process of assembling components with known original length according to a certain topological relationship. By arranging a lifting tower at the construction site to lift the central tension ring and drive the spine cable to rise together, most components can be installed near the ground as much as possible, so as to reduce the internal force and traction of components in the construction process. Taking a 100 m cable dome as an example, the whole construction process is analyzed using the dynamic relaxation method to verify the feasibility of the method [31]. Based on cumulative tower lifting cable strut installation technology, Luo et al. (2011) ultimately canceled the erection of the central tower and proposed a non-bracket construction method suitable for the cable dome, which uses the compression ring of the structure as the lifting support of the cable strut system to integrally pull and lift it. Taking the cable dome roof of the Wuxi New Area Science and Technology Exchange Center (Wuxi, China) as an example, the whole construction process was simulated using the nonlinear dynamic finite element method, verifying the advantages of this method, such as good tension efficiency and high cable network stability in the whole process [32]. For the cable truss, Luo et al. (2014) proposed an inclined non-bracket tow-lifting construction technology with fixed jacks, which utilizes jacks connected to the anchor node around the cable truss to obliquely pull and lift the assembled cable strut system from low altitude to high altitude. Taking the tension ring truss of Yueqing Stadium as an example, an analysis of the whole process of self-mass initial state, lifting traction and tension, and disturbance stability was carried out. The results show that this method has the advantages of less high-altitude operation, lower cost and shorter construction period for erecting brackets, simple construction equipment, and high construction efficiency [33,34].

Therefore, considering the advantages of less aerial work, no need to erect brackets, simple and economical construction equipment, flexible arrangement of construction period, and a safe and reliable construction process, this paper introduces the idea of non-bracket construction for annular cable supported grid structures, optimized according to the characteristics of semi-rigid hybrid structures. A non-bracket construction method suitable for cable supported grid structures is proposed. Taking the annular cable supported grid structure of the roof of the Education City Stadium in Qatar (Ar Rayyan, Qatar), a simulation of the whole construction process is carried out using the nonlinear dynamic finite element method to verify the feasibility and the key technologies of this method are introduced in detail.
2. Introduction of Annular Cable Supported Grid Structure and Education City Stadium

The annular cable supported grid structure is a new type of large-span space structure that uses rigid grids to replace the flexible cables at the top chord of the cable truss spokes. It includes a peripheral support structure with an upper single-layer grid and lower cable strut system. The upper chord of the structure is a rigid single-layer grid with a large opening that usually does not have strong vertical stiffness and bearing capacity. Therefore, it is essential to establish the pre-tension load in the cable strut system to make the whole structure rigid, provide vertical support for the upper structure, and greatly improve the bearing capacity of the whole structure. Meanwhile, the horizontal component of the radial cable force can be balanced by the upper grid or compression ring, which means that the cable net, upper grid, and compression ring form a self-balancing system. This kind of structure uses a tension cable net as the main load-bearing system, which allows full play of the high-strength material characteristics of cables, greatly reducing the burden on the lower supporting system and increasing the span in an economic and effective manner. Existing annular cable supported grid structures are usually divided into three categories according to the pre-tension cable balancing mode: upper reticulated shell, outer compression ring, and inner compression ring balancing radial cable force, as shown in Figure 1.

![Figure 1. Types of cable supported grid structures: (a) upper reticulated shell, (b) outer compression ring, and (c) inner compression ring balancing radial cable force.](image)

The roof of Education City Stadium is a typical annular cable supported grid structure; the length of the whole structure along the long axis is 225 m, the length along the short axis is 196 m, and the maximum cantilever span is about 55 m. The cable strut system is composed of 56 straight radial cables, spatial curve tension rings, and vertical struts. The elevation is higher at the 45° tension ring than at the long and short axes, and the plane is projected into a ring. A three-dimensional axonometric drawing of the stadium is shown in Figure 2.

The composition of the cable-supported grid structure is clear (Figure 3), which can be divided into the following components: structural columns, outer compression ring (CR), cable net system, upper main steel members, and upper secondary steel members. The cable net system includes radial cables and tension rings, all of which adopt fully enclosed steel strands; the tensile strength grade of the steel wire is 1670 MPa, and all cable heads use hot-cast anchor sockets. The tension rings are composed of eight cables in parallel, and radial cables are divided into five specifications based on stress level. Figure 4 shows the layout of all structural cables, and Table 1 lists all cable parameters in detail.
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Figure 2. Integral 3D axonometric drawing.

Figure 3. Detailed 3D axonometric drawings: (a) upper main steel members (radial main beams, braces, etc.), (b) roof purlins, (c) outer compression ring, (d) cable net system (radial cables + tension rings), (e) structural columns and crossing braces between columns, (f) partial overall structure.

Table 1. Cable materials and specifications.

| Specification       | Cable Section (mm$^2$) | Cable Weight (kg·m$^{-1}$) |
|---------------------|------------------------|---------------------------|
| Tension rings       | 8D110                  | 67,680                    | 544                      |
| Radial cables       | D75                    | 3890                      | 31.3                     |
|                     | D80                    | 4420                      | 35.5                     |
|                     | D90                    | 5600                      | 45                       |
|                     | D115                   | 9280                      | 74.5                     |
|                     | D135                   | 12,920                    | 104                      |
3. Non-Bracket Construction Method

The construction of a cable strut tension structure is a dynamic process that involves gradually integrating basic components into a partial structure and then the complete structure. In this process, the shape, stiffness, boundary, and load conditions are constantly changing, and the transformation from mechanism to structure is realized according to a certain topological relationship. For a spatial structure with such characteristics, the non-bracket construction method can realize a natural integral transition of the cable net from the non-stress state mechanism to the designed stress level state structure and ensure the gradual development of the structure configuration and stiffness in the whole process of construction.

However, some new difficulties will be encountered in the construction of an annular cable supported grid structure when the brackets are canceled. The core problem is that the cable net and grid configuration will be out of control due to a temporary formation failure of the self-balancing system of the structure during the construction process. With the non-bracket construction method proposed in this paper, the accuracy of the cable net configuration and the grid installation position can be controlled by tensioning the tie-downs at the tension ring nodes (i.e., the cable clamps) when the self-balancing system is not fully formed.

Compared with the traditional construction method, the non-bracket construction method has the following advantages:

(1). For the cable supported grid structure with external CR, the upper grids basically do not participate in the balance of pre-tension; the pre-tension should be balanced by the CR. The traditional method of first completing the upper grid and then installing and tensioning the cable net will result in the installed upper grids participating in the balance of pre-tension of cable force during the construction process, which will lead to differences in pre-stress distribution between the final formed state and the designed equilibrium state. The non-bracket construction method divides the entire
structure into a tension substructure and post-installed components (Figure 5). The tensioning substructure is completed first to establish the pre-stress distribution to meet the design expectation without interference from other components. After that, the incomplete structure is maintained at the design equilibrium position by adjusting the cable forces of tie-downs during the subsequent installation of components such as the upper grid. This means that the pre-stress flow is distributed in the designed pre-stress components during the entire construction process, and the non-bracket method can ensure a more accurate pre-stress distribution especially in the cable supported grid structures with CR.

2. Tie-downs can realize bidirectional, real-time and accurate adjustment of the elevation of the control nodes in the construction process and the erection of brackets is unnecessary, which means that it has fewer requirements for the construction site and is conducive to a flexible arrangement of the construction period for the grandstand and other structures. At the same time, due to the relatively simple construction equipment and fewer temporary construction facilities, this method is also economical.

Figure 5. Prestress flow control process.

Figure 6 presents the entire construction process adjusted for the calculation example in this paper, and the construction steps are as follows: (1) first, install the steel columns and compression ring; (2) spread the radial cables and tension rings on the stand and install the cable clamps; (3) pull the radial cables, lift the cable net to the high altitude, and connect the radial cables with the CR; (4) install the tie-downs on the tension ring clamps, connect them with the ground, and pull the tension rings down to the design elevation; (5) hoist the grid structure sections (including braces) onsite, and adjust the cable forces of tie-downs during the subsequent installation of components such as the upper grid. This means that the pre-stress flow is distributed in the designed equilibrium state. The non-bracket construction method divides the entire structure into a tension substructure and post-installed components (Figure 5). The traditional method of first completing the upper grid and then installing the CR. The traditional method can ensure a more accurate pre-stress distribution especially in the cable supported grid structures with CR.

3.1. Analysis Method

In this paper, the calculations mainly rely on the nonlinear dynamic finite element method (NDFEM) to determine the static equilibrium state of the cable strut system. NDFEM modifies the traditional method by introducing the concepts of virtual inertial force and viscous damping force, which are widely used in cable strut tension structure form finding. In this method, by continuously solving the motion equation and updating the configuration of the cable strut system, the dynamic equilibrium will gradually converge to the static equilibrium state. The cable strut system is in the static unbalanced state before the analysis and in the dynamic equilibrium state during the analysis, and reaches the static equilibrium state after convergence, which means the system moves intermittently (discontinuous motion) from the initial unbalanced state to the stable equilibrium state.
Step 1: Install CR, structural columns, CR brackets, and additional cross-bracing between columns, and lay cable net on stand.

Step 2: Pull and lift cable net.

Step 3: Install and tension tie-downs and keep tension ring clamps at design elevation (i.e., vertical deformation is 0).

Step 4: Install cross-bracing between columns.

Step 5: Install upper steel structure and roof, then adjust cable forces of tie-downs to keep tension ring clamps at design elevation (i.e., vertical deformation is 0).

Step 6: Remove tie-downs.

Step 7: Remove CR brackets and additional cross-bracing between columns.

Step 8: Install other roof components, and entire structure is formed.

Figure 6. Schematic diagram of detailed construction steps for roof of Education City Stadium in Qatar.
The overall steps are as follows: (1) establish the initial finite element model; (2) perform nonlinear dynamic finite element analysis. When the total kinetic energy reaches the peak, the finite element model is updated and the dynamic analysis is carried out again until the configuration iteration converges; (3) perform nonlinear static analysis of the finite element model with iterative convergence of configuration to test the static equilibrium state; and (4) extract analysis results.

The main components of NDFEM are nonlinear dynamic equilibrium iteration and configuration updating iteration.

3.1.1. Nonlinear Dynamic Equilibrium Iteration

Dynamic Balance Equation

The dynamic balance equation (Equation (1)) adopts the Rayleigh damping matrix (Equation (2)), in which the natural vibration circle frequency and damping ratio can be set virtually:

\[ [M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = \{F(t)\} \]

\[ [C] = \alpha[M] + \beta[K] \]

\[ \alpha = \frac{2\omega_i\omega_j(\xi_i\omega_j - \xi_j\omega_i)}{\omega_j^2 - \omega_i^2} \]

\[ \beta = \frac{2(\xi_i\omega_j - \xi_j\omega_i)}{\omega_j^2 - \omega_i^2} \]

where \{U\}, \{\dot{U}\}, and \{\ddot{U}\} are deformation, velocity, and acceleration vectors, respectively; \{F(t)\} is the load time history vector; [C] is the Rayleigh damping matrix; [M] is the quality matrix; [K] is the stiffness matrix; \alpha and \beta are the Rayleigh damping coefficients; \omega_i and \omega_j are the \textit{i}th and \textit{j}th natural vibration circle frequencies, respectively; and \xi_i and \xi_j are the damping ratios corresponding to \omega_i and \omega_j, respectively.

If \xi_i = \xi_j = \xi, Equations (3) and (4) can be simplified to Equations (5) and (6):

\[ \alpha = \frac{2\omega_i\omega_j \xi}{\omega_j + \omega_i} \]

\[ \beta = \frac{2\xi}{\omega_j + \omega_i} \]

Time Step

Time step \Delta T_s is one of the key factors determining the convergence speed of NDFEM form finding analysis. The shorter \Delta T_s is, the easier the convergence of dynamic analysis will be, while the sum of total time steps to reach static equilibrium \( \Sigma N_{ts} \) will be large and the analysis will not be efficient enough. In a dynamic analysis, reasonable \Delta T_s should ensure that total kinetic energy reaches the peak in fewer time steps based on the convergence of dynamic analysis. Considering all above, the form finding analysis of NDFEM is divided into three stages: In the initial stage, the cable strut system moves violently, and a short time step can be set for dynamic analysis to facilitate the iterative convergence of dynamic balance equation. In the middle stage, the main deformation direction of the cable strut system is clear and the whole system tends to reach the static equilibrium configuration. At this time, a large time step should be set to quickly approach the static equilibrium state with fewer time steps and configuration updating times. In the third stage, the cable strut system vibrates near the static equilibrium state. At this time, a larger time step should be set to make the configuration iteration converge as soon as possible and the whole cable strut system finally reach the static equilibrium state.

In view of the important influence of the time step on dynamic balance iteration and analysis efficiency, it is proposed to automatically adjust the time step of each dynamic
analysis by using the adjustment coefficient $C_{ts}$ in the analysis process. The adjustment strategy is as follows:

1. The initial time step $\Delta T_{s(1)}$ is used in the first configuration iteration;
2. If the time steps of the $(m - 1)$th dynamic analysis $N_{ts(m-1)} = [N_{ts}]$ and the total kinetic energy still do not decrease, then the time step of the $m$th dynamic analysis is $\Delta T_{s(m)} = \Delta T_{s(m-1)} \times C_{ts}$;
3. If the $(m - 1)$th dynamic analysis does not converge, then $\Delta T_{s(m)} = \Delta T_{s(m-1)}/C_{ts}$.

Determining Total Kinetic Energy Peak and Corresponding Time Point

Total kinetic energy $E_{(k)}$ of the structure in time step $k$ in the dynamic analysis is shown in Equation (7):

$$E_{(k)} = \frac{1}{2} \{\ddot{U}(k)\}^T \{M\} \{\ddot{U}(k)\}$$

where $\{\ddot{U}(k)\}$ is the velocity vector of time step $k$.

The strategy for determining the peak value of total kinetic energy and its time point is as follows:

Let $E_{(0)} = 0$. The dynamic equilibrium iteration converges in time step $k$. If $k < [N_{ts}]$ and $E_{(k)} > E_{(k-1)}$, then the total kinetic energy does not reach the peak and the $(k + 1)$th dynamic analysis should be carried out continuously. If $k \leq [N_{ts}]$ and $E_{(k)} < E_{(k-1)}$, then fit three consecutive total kinetic energy time steps $E_{(k)}$, $E_{(k-1)}$, $E_{(k-2)}$ with a quadratic parabola curve and calculate peak total kinetic energy $E_{(p)}$ through the curve and the corresponding time point $T_{s(p)}$. If $k = [N_{ts}]$ and $E_{(k)} \geq E_{(k-1)}$, then $E_{(p)} = E_{(k)}$ and $T_{s(p)} = T_{s(k)}$. The $k$th time step dynamic balance iteration does not converge. If $k = 1$, the configuration will not be updated and the next dynamic analysis will be carried out after adjusting the time step. If $1 < k \leq [N_{ts}]$, then $E_{(p)} = E_{(k-1)}$ and $T_{s(p)} = T_{s(k-1)}$.

3.1.2. Model Updating Iteration

Model updating includes configuration, internal force, and original length updating. After the node coordinates are updated by the deformation of nonlinear dynamic finite element analysis, the member length in the model also changes, and the updated member is in a stress-free state. However, theoretically stress and strain exist in the member before the model is updated, so the stress and strain should be applied again after the model is updated. Different element types need different methods.

(1). Cable strut element

In cable system structures, the equivalent initial strain or temperature difference is often used to simulate the tension cable or control the original length. For a cable with an original length that needs to be controlled, the equivalent initial strain or temperature difference should be adjusted according to the updated length based on the principle that the original length should remain unchanged before and after updating. For cables that need to control internal forces (such as lifting traction and tension), the original length is updated directly. The equivalent initial strain ($\varepsilon_p$) or equivalent temperature difference ($\Delta T_p$) can be determined according to the following formulae:

$$\varepsilon_p = \frac{S}{S_0} - 1$$

$$\Delta T_p = -\frac{\varepsilon_p}{\alpha} = \frac{(1 - S/S_0)}{\alpha}$$

where $S$ is the unit length in the model; $S_0$ is the original length of the unit; $E$, $A$, and $\alpha$ are elastic modulus, cross-sectional area, and coefficient of temperature expansion, respectively; and $F$ is the internal force of the cable strut.
(2) Beam element

The instate method is used considering that beam element contains axial, bending, and tangential strain.

3.2. Nonlinear Analysis Method

Structural nonlinearity includes state change, material nonlinearity, and geometric nonlinearity. For tension structures, nonlinear analysis must be adopted because the structure has strong geometric nonlinearity. For complex nonlinear problems, a common effective method in nonlinear analysis is to use linear equations to gradually approximate the nonlinear solution. Therefore, the essence of solving a nonlinear problem is to linearize the nonlinear equations and choose linear approximation strategies and algorithms.

The common methods for solving nonlinear algebraic equations include direct iteration, full and modified Newton–Raphson, and load increment methods. The full Newton–Raphson method is used in this paper. This method linearizes the nonlinear equations, and the tangent stiffness matrix is modified every iteration until the convergence condition is reached [35].

For nonlinear problems, the following nonlinear equations are obtained by discretizing the structural elements:

\[
[K(u)]u = F
\]  

(12)

The above formula can be expressed as:

\[
\{\Psi(u)\} = [K(u)]u - F = 0
\]  

(13)

where \( u \) is the displacement vector, the stiffness matrix \( K(u) \) is the n-order square matrix about \( u \), and \( F \) is the load vector.

After the approximate solution \( u^n \) is obtained, in order to further obtain the approximate solution \( u^{n+1} \), \( \{\Psi(u)\} \) can be expanded by the Taylor series at \( u^{n+1} \) and only retain the linear term:

\[
\{\Psi(u)^{n+1}\} = \{\Psi(u)^n\} + \left[\frac{d(\Psi)}{d(u)}\right]_n \cdot \Delta u^n
\]  

(14)

where \( k = \frac{d(\Psi)}{d(u)} \) is the tangent matrix.

Repeat the above iterative solution process until the iterative convergence condition is satisfied:

\[
\|\Delta u\|_\infty = \|u^{n+1} - u^n\|_\infty < \epsilon
\]  

(15)

where \( \epsilon \) is an infinitesimal quantity.

The positive definiteness of \( \frac{d(\Psi)}{d(u)} \) is a necessary condition for the system of equations to have a solution. The analysis process of using the Newton–Raphson method to solve the single degree of freedom system is shown in Figure 7.
After the approximate solution $\nu$ is obtained, in order to further obtain the approximate solution $\nu_1 + \nu$,

$$\Psi(u)$$

can be expanded by the Taylor series at $\nu_1$ and only retain the linear term:

$$\Psi(u) = \Psi + \Delta_{u},$$

where $\Delta_{u}$ is the tangent matrix.

Repeat the above iterative solution process until the iterative convergence condition is satisfied:

$$\Delta_{u} < \epsilon$$

where $\epsilon$ is an infinitesimal quantity.

The positive definiteness of $\Psi(u)$ is a necessary condition for the system of equations to have a solution. The analysis process of using the Newton–Raphson method to solve the single degree of freedom system is shown in Figure 7.

4. Calculation Example

In order to verify the effectiveness and accuracy of the non-bracket construction method proposed in this paper, the method was applied to the annular cable supported grid structure of the roof of Education City Stadium, and a simulation calculation of the whole construction process was carried out.

4.1. Establishing Finite Element Model

In this paper, ANSYS finite element analysis software (Version 2019 R1) was used to establish the finite element model and carry out construction analysis. Figure 8 presents the finite element model of the whole structure. Tables 2 and 3 list the element types and material characteristics of steel and cable.

![Finite element model of whole structure](image1)

**Figure 7.** Nonlinear response of single degree of freedom system [35].

**Figure 8.** Finite element model of whole structure.
Table 2. Element types.

| Component                          | Element Type               |
|------------------------------------|----------------------------|
| Out compression ring               | Link8/Beam188              |
| Structure column                   | Beam188 + Beam44          |
| Cross-braces between structure columns | Link8                    |
| Top chord steel grids              | Beam188 + Beam44          |
| Cables                             | Link10                    |
| Cable clamps                       | Beam188                   |
| Cable end ear plates               | Mass21                    |

Table 3. Statistical table of material characteristics.

| Parameter          | Steel  | Cable |
|--------------------|--------|-------|
| E (GPa)            | 210    | 165   |
| E (GPa)            | 80.8   |       |
| ρ (kg/m$^3$)       | 7850   | 8050  |
| α (1/°C)           | 11.7 × 10$^{-6}$ | 12.0 × 10$^{-6}$ |

4.1.1. Loads and Constraints

(1). For the whole structure, purlins and radial beams are connected by pin shafts, and the rotation constraints need to be released at the nodes. The purlins and supporting purlins adopt chute beams, and the rotational and axial constraints need to be released at the connecting nodes. The short axis bending moment of all reinforced purlins and the torsional bending moment at the other end need to be released. The braces are connected to the radial beam and tension ring clamp by pin shafts, and the rotation constraints are released.

(2). The load condition of the whole structure consists of dead load and pre-tension load. The dead load is the weight of structural components, and the pre-tension load includes equivalent pre-tension in the cable net and outer compression ring, which can be achieved by applying an equivalent temperature difference calculated through cable forces using Equation (16):

\[ \Delta T = \frac{P}{\alpha E A} \]  

where $P$ pre-tension force, $\alpha$ is temperature linear expansion coefficient, $E$ is elastic modulus, $A$ is section area, and $\Delta T$ is equivalent temperature difference.

Considering the symmetry of the structure, Table 4 shows the equivalent temperature difference of 1/4 CR, and Table 5 shows the equivalent temperature difference of 1/4 radial cables.

Table 4. Equivalent temperature difference of 1/4 CR.

| CR Element | PCR (kN) | α    | E    | A (m$^2$) | ΔT (°C) |
|------------|----------|------|------|-----------|---------|
| 1          | -24,001  | 1.20 × 10$^{-5}$ | 2.10 × 10$^{11}$ | 0.1604 | 59.378   |
| 2          | -24,001  | 1.20 × 10$^{-5}$ | 2.10 × 10$^{11}$ | 0.1604 | 59.378   |
| 3          | -24,001  | 1.20 × 10$^{-5}$ | 2.10 × 10$^{11}$ | 0.1604 | 59.378   |
| 4          | -24,025  | 1.20 × 10$^{-5}$ | 2.10 × 10$^{11}$ | 0.1604 | 59.437   |
| 5          | -24,025  | 1.20 × 10$^{-5}$ | 2.10 × 10$^{11}$ | 0.1604 | 59.437   |
| 6          | -24,015  | 1.20 × 10$^{-5}$ | 2.10 × 10$^{11}$ | 0.1866 | 51.071   |
| 7          | -24,015  | 1.20 × 10$^{-5}$ | 2.10 × 10$^{11}$ | 0.1866 | 51.071   |
| 8          | -24,015  | 1.20 × 10$^{-5}$ | 2.10 × 10$^{11}$ | 0.1866 | 51.071   |
| 9          | -23,964  | 1.20 × 10$^{-5}$ | 2.10 × 10$^{11}$ | 0.25   | 38.038   |
| 10         | -23,964  | 1.20 × 10$^{-5}$ | 2.10 × 10$^{11}$ | 0.25   | 38.038   |
Table 4. Cont.

| CR Element | PCR (kN)   | α         | E           | A (m²) | ΔT (°C) |
|------------|------------|-----------|-------------|--------|---------|
| 11         | −24,014    | 1.20 × 10⁻⁵ | 2.10 × 10¹¹ | 0.25   | 38.117  |
| 12         | −24,014    | 1.20 × 10⁻⁵ | 2.10 × 10¹¹ | 0.25   | 38.117  |
| 13         | −23,972    | 1.20 × 10⁻⁵ | 2.10 × 10¹¹ | 0.25   | 38.051  |
| 14         | −23,972    | 1.20 × 10⁻⁵ | 2.10 × 10¹¹ | 0.25   | 38.051  |
| 15         | −24,013    | 1.20 × 10⁻⁵ | 2.10 × 10¹¹ | 0.25   | 38.116  |
| 16         | −24,013    | 1.20 × 10⁻⁵ | 2.10 × 10¹¹ | 0.25   | 38.116  |
| 17         | −23,971    | 1.20 × 10⁻⁵ | 2.10 × 10¹¹ | 0.25   | 38.049  |
| 18         | −23,971    | 1.20 × 10⁻⁵ | 2.10 × 10¹¹ | 0.25   | 38.049  |
| 19         | −24,012    | 1.20 × 10⁻⁵ | 2.10 × 10¹¹ | 0.2128 | 44.777  |
| 20         | −24,012    | 1.20 × 10⁻⁵ | 2.10 × 10¹¹ | 0.2128 | 44.777  |
| 21         | −23,966    | 1.20 × 10⁻⁵ | 2.10 × 10¹¹ | 0.1604 | 59.291  |
| 22         | −23,966    | 1.20 × 10⁻⁵ | 2.10 × 10¹¹ | 0.1604 | 59.291  |
| 23         | −23,966    | 1.20 × 10⁻⁵ | 2.10 × 10¹¹ | 0.1604 | 59.291  |
| 24         | −24,011    | 1.20 × 10⁻⁵ | 2.10 × 10¹¹ | 0.1604 | 59.403  |
| 25         | −24,011    | 1.20 × 10⁻⁵ | 2.10 × 10¹¹ | 0.1604 | 59.403  |
| 26         | −23,964    | 1.20 × 10⁻⁵ | 2.10 × 10¹¹ | 0.1604 | 59.286  |
| 27         | −23,964    | 1.20 × 10⁻⁵ | 2.10 × 10¹¹ | 0.1604 | 59.286  |
| 28         | −23,964    | 1.20 × 10⁻⁵ | 2.10 × 10¹¹ | 0.1604 | 59.286  |
| 29         | −24,011    | 1.20 × 10⁻⁵ | 2.10 × 10¹¹ | 0.1604 | 59.403  |
| 30         | −24,011    | 1.20 × 10⁻⁵ | 2.10 × 10¹¹ | 0.1604 | 59.403  |
| 31         | −23,979    | 1.20 × 10⁻⁵ | 2.10 × 10¹¹ | 0.1604 | 59.323  |
| 32         | −23,979    | 1.20 × 10⁻⁵ | 2.10 × 10¹¹ | 0.1604 | 59.323  |
| 33         | −23,979    | 1.20 × 10⁻⁵ | 2.10 × 10¹¹ | 0.1604 | 59.323  |
| 34         | −23,979    | 1.20 × 10⁻⁵ | 2.10 × 10¹¹ | 0.1604 | 59.323  |

Table 5. Equivalent temperature difference of 1/4 radial cables.

| Radial Cable Axis | Pr/kN | α         | E           | A/mm² | ΔT/°C  |
|-------------------|-------|-----------|-------------|-------|--------|
| 1                 | 1648  | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 3671.7 | −187.02 |
| 2                 | 1476  | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 3200.5 | −192.16 |
| 3                 | 1698  | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 3671.7 | −192.69 |
| 4                 | 3381  | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 7559.5 | −186.35 |
| 5                 | 4715  | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 10,198.4 | −192.64 |
| 6                 | 4481  | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 10,198.4 | −183.08 |
| 7                 | 4668  | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 10,198.4 | −190.72 |
| 8                 | 4466  | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 10,198.4 | −182.46 |
| 9                 | 4494  | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 10,198.4 | −183.61 |
| 10                | 2069  | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 4614.2 | −186.83 |
| 11                | 1659  | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 3671.7 | −188.26 |
| 12                | 1478  | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 3200.5 | −192.42 |
| 13                | 1615  | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 3671.7 | −183.27 |
| 14                | 1418  | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 3200.5 | −184.61 |
| 15                | 1570  | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 3671.7 | −178.16 |

4.1.2. Model Calibration: Oasys GSA Model and ANSYS Model Analysis Comparison

In this section, under the same load (dead load + pre-tension), the accuracy of the ANSYS model is verified by comparing it with the calculation results of Oasys GSA provided by the design institute.

ANSYS Calculation Results

Figure 9a–f shows the structure response under the dead load state in the ANSYS model. For the horizontal displacement of the CR and tension ring, the results are given in the cylindrical coordinate system.
Table 5. Equivalent temperature difference of 1/4 radial cables.

| Radial Cable Axis | Pr/kN E A/mm² | ΔT/°C | E A/mm² | ΔT/°C |
|-------------------|--------------|-------|---------|-------|
| 1                 | 1648         | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 3671.7 | -187.02 |
| 2                 | 1476         | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 3200.5 | -192.16 |
| 3                 | 1698         | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 3671.7 | -192.69 |
| 4                 | 3381         | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 7559.5 | -186.35 |
| 5                 | 4715         | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 10,198.4 | -192.64 |
| 6                 | 4481         | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 10,198.4 | -183.08 |
| 7                 | 4668         | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 10,198.4 | -190.72 |
| 8                 | 4466         | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 10,198.4 | -182.46 |
| 9                 | 4494         | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 10,198.4 | -183.61 |
| 10                | 2069         | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 4614.2 | -186.83 |
| 11                | 1659         | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 3671.7 | -188.26 |
| 12                | 1478         | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 3200.5 | -192.42 |
| 13                | 1615         | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 3671.7 | -183.27 |
| 14                | 1418         | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 3200.5 | -184.61 |
| 15                | 1570         | 1.20 × 10⁻⁵ | 2.00 × 10¹¹ | 3671.7 | -178.16 |

4.1.2. Model Calibration: Oasys GSA Model and ANSYS Model Analysis Comparison

In this section, under the same load (dead load + pre-tension), the accuracy of the ANSYS model is verified by comparing it with the calculation results of Oasys GSA provided by the design institute.

Figure 9. ANSYS calculation results: (a) radial displacement of CR (m); (b) radial displacement of tension ring (m); (c) vertical displacement of tension ring (m); (d) equivalent stress of CR (Pa); (e) radial cable forces (N); (f) tension ring cable forces (N).

Comparison of Results

(1). Comparison of vertical displacement of tension rings
Figure 10 shows comparison curves of the vertical displacement of 56 typical tension ring nodes between the results of ANSYS and Oasys GSA. The vertical displacement of the tension rings calculated by ANSYS is $-19$ to $-22.6$ mm and calculated by Oasys GSA is $-16.5$ to $-23.5$ mm. The difference in vertical displacement between the two calculations is slight (maximum difference is 4.4 mm and minimum difference is 0.1 mm).

(2). Comparison of vertical displacement of cantilever end of radial beam

There are 56 radial beams in the whole roof structure. Figure 11 shows comparison curves of vertical displacement of 56 cantilever ends between ANSYS and Oasys GSA. The vertical displacement calculated by ANSYS is $-14.3$ to $-33.4$ mm and by GSA is $-5$ to $-34.5$ mm. The absolute value of Oasys GSA results is slightly smaller than that of ANSYS, with a maximum displacement difference of 9.4 mm and minimum displacement difference of 0.2 mm. The difference is again slight, and the variation trend is similar.

(3). Comparison of vertical reaction forces at the bottom of structural columns

Figure 12 shows that the vertical reaction forces at the bottom of the column obtained by ANSYS are between 396.5 and 1320 kN, and the results of Oasys GSA are between 410 and 1317.8 kN, so the two results are basically the same.
Figure 12 shows that the vertical reaction forces at the bottom of the column obtained by ANSYS are between 396.5 and 1320 kN, and the results of Oasys GSA are between 410 and 1317.8 kN, so the two results are basically the same.

![Comparison curve of vertical reaction forces of structural columns (kN).](image)

**Figure 12.** Comparison curve of vertical reaction forces of structural columns (kN).

4. Comparison of radial cable forces

It can be seen in Figure 13 that the radial cable forces obtained by ANSYS are between 1407.7 and 4682.3 kN, and the results of Oasys GSA are between 1435 and 4755 kN, so the two results are basically the same.

![Comparison curve of radial cable force (kN).](image)

**Figure 13.** Comparison curve of radial cable force (kN).

5. Comparison of tension ring cable force

The tension ring is divided into 56 segments by 56 radial cables. Figure 14 shows a comparison curve of the tension ring cable forces. The cable forces obtained by ANSYS are between 23,716 and 23,825 kN, and the results of Oasys GSA are between 23,980 and
24,090 kN. The results of the latter are slightly larger than those of the former, while the cable force difference ratios are still relatively small at 1.1% and 1.2%.

![Comparison curves of tension ring cable forces (kN).](image)

Therefore, it can be considered that the calculation results of ANSYS are basically consistent with the design model results (Oasys GSA), and the accuracy of the ANSYS model is verified.

4.2. Analysis Results of the Construction Technology Application

4.2.1. Analysis of Cable Net Traction Lifting

The traction lifting of the cable net is a process of transformation from a stress-free or low-stress state to a high-stress state. In this process, the pre-load established by the cable should be completely taken up by the CR. Due to the combined effect of the large length difference of the initial traction cable and the irregular elliptical CR, the cable forces are uneven and will ultimately cause uneven stress in the CR and excessive deformation. Meanwhile, the distribution of cable forces is also greatly influenced by the traction lifting sequence. Therefore, choosing a reasonable traction lifting sequence seems to be important. This section first adopts a synchronous and equally proportioned integral traction lifting sequence and analyzes the variation in CR deformation, the traction cable forces, and the internal forces of the CR brackets during the traction process to determine the key factors that affect them. Then the cable net traction lifting sequence is improved through the results of synchronous and equally proportioned integral sequence, and the final traction process of the cable net is analyzed according to the adjusted one.

Analysis Model

Through NDFEM, the cable net traction lifting analysis model is built, which is shown in Figure 15. This model displays the initial state of the construction, which is the state where all cables are laid on the stand. The FEA model includes structural columns, CR, cable net, and CR brackets. The load conditions include the weight of the structure, radial cable heads, tension ring heads, and cable clamps, and the material characteristics and boundary conditions can be found in Section 4.1.

Results of Synchronous Equal Proportion Integral Lifting

Synchronous and equal proportion integral lifting means that all traction cables are pulled at the same time according to the same proportion. Specifically, each traction cable will be shortened by 10% of its initial length during each traction step. All corresponding
calculation conditions are listed in Table 6, and the lengths of all traction cables are shown in Table 7 (quarter symmetry of structure).

Table 6. Calculation conditions during synchronous equal proportion integral lifting.

| Calculation Condition | State                                      |
|-----------------------|--------------------------------------------|
| C1                    | Table net laid on stand                    |
| C2                    | Traction cable length reduced by 10%       |
| C3                    | Traction cable length reduced by 20%       |
| C4                    | Traction cable length reduced by 30%       |
| C5                    | Traction cable length reduced by 40%       |
| C6                    | Traction cable length reduced by 50%       |
| C7                    | Traction cable length reduced by 60%       |
| C8                    | Traction cable length reduced by 70%       |
| C9                    | Traction cable length reduced by 80%       |
| C10                   | Traction cable length reduced by 90%       |
| C11                   | Traction lifting completed (100%)          |

Table 7. Length of traction cable under various calculation conditions (m).

| Axis | C1   | C2   | C3   | C4   | C5   | C6   | C7   | C8   | C9   | C10  | C11  |
|------|------|------|------|------|------|------|------|------|------|------|------|
| 1    | 13.85| 12.47| 11.08| 9.70 | 8.31 | 6.93 | 5.54 | 4.16 | 2.77 | 1.39 | 0    |
| 2    | 13.82| 12.44| 11.06| 9.67 | 8.29 | 6.91 | 5.53 | 4.15 | 2.76 | 1.38 | 0    |
| 3    | 14.47| 13.02| 11.58| 10.13| 8.68 | 7.24 | 5.79 | 4.34 | 2.89 | 1.45 | 0    |
| 4    | 14.73| 13.26| 11.78| 10.31| 8.84 | 7.37 | 5.89 | 4.42 | 2.95 | 1.47 | 0    |
| 5    | 15.42| 13.88| 12.34| 10.79| 9.25 | 7.71 | 6.17 | 4.63 | 3.08 | 1.54 | 0    |
| 6    | 15.89| 14.30| 12.71| 11.12| 9.53 | 7.95 | 6.36 | 4.77 | 3.18 | 1.59 | 0    |
| 7    | 16.06| 14.45| 12.85| 11.24| 9.64 | 8.03 | 6.42 | 4.82 | 3.21 | 1.61 | 0    |
| 8    | 16.05| 14.45| 12.84| 11.24| 9.63 | 8.03 | 6.42 | 4.82 | 3.21 | 1.61 | 0    |
| 9    | 15.77| 14.19| 12.62| 11.04| 9.46 | 7.89 | 6.31 | 4.73 | 3.15 | 1.58 | 0    |
| 10   | 15.08| 13.57| 12.06| 10.56| 9.05 | 7.54 | 6.03 | 4.52 | 3.02 | 1.51 | 0    |
| 11   | 14.00| 12.60| 11.20| 9.80 | 8.40 | 7.00 | 5.60 | 4.20 | 2.80 | 1.40 | 0    |
| 12   | 12.93| 11.64| 10.34| 9.05 | 7.76 | 6.47 | 5.17 | 3.88 | 2.59 | 1.29 | 0    |
| 13   | 10.64| 9.58 | 8.51 | 7.45 | 6.38 | 5.32 | 4.26 | 3.19 | 2.13 | 1.06 | 0    |
| 14   | 10.16| 9.14 | 8.13 | 7.11 | 6.10 | 5.08 | 4.06 | 3.05 | 2.03 | 1.02 | 0    |
| 15   | 10.17| 9.15 | 8.14 | 7.12 | 6.10 | 5.09 | 4.07 | 3.05 | 2.03 | 1.02 | 0    |
Typical configuration in the process of cable net traction lifting is shown in Figure 16, and the maximum and minimum vertical deformation of the structure (relative to the design configuration) under each calculation condition are listed in Table 8, reflecting the continuous variation in cable net system elevation in the whole process of traction lifting.

![Typical Uz configuration (m).](image)

Figure 16. Typical Uz configuration (m).

Table 8. Vertical deformation of lifted structure.

| Calculation Condition | Vertical Deformation | Vertical Deformation |
|-----------------------|----------------------|----------------------|
|                       | −Uz Min (m)          | +Uz Max (m)          |
| C1                    | −28.44               | 0                    |
| C2                    | −26.37               | 0                    |
| C3                    | −24.20               | 0                    |
| C4                    | −22.03               | 0                    |
| C5                    | −19.92               | 0                    |
| C6                    | −17.66               | 0                    |
| C7                    | −15.14               | 0                    |
| C8                    | −12.28               | 0                    |
| C9                    | −8.91                | 0                    |
| C10                   | −4.97                | 0.32                 |
| C11                   | −0.002               | 2.047                |

In order to intuitively express the radial deformation of the CR, radial deformations of 56 typical CR nodes through local cylindrical coordinates are extracted to be drawn as a deformation curve, which is shown in Figure 17. (Local cylindrical coordinate system: Taking the arc center of the node itself as the origin, the direction from the origin to the node is the X-axis, the tangential direction is the Y-axis, and the vertical direction is the Z-axis.) Figure 18 shows the maximum equivalent stress variation in CR under various traction steps. From Figures 17 and 18, it is evident that the radial deformation Ux and the maximum equivalent stress of the CR vary greatly from C1 to C2. At C2, the maximum outward expansion deformation reaches 150 mm, which appears at 45° of the CR, and the maximum inward contraction deformation is 161 mm, which appears at the minor axis. From C2 to C11, the 45° CR outward expansion, short axis CR inward contraction deformation, and maximum equivalent stress decrease continuously with continuous traction. (The long axis refers to axis...
1 and 29, the short axis refers to axis 15 and 43, and the 45° refers to axis 8, 22, 35, and 51.)

![Figure 17](image1.png)

**Figure 17.** Radial deformation Ux of CR under local coordinate system at each step.

![Figure 18](image2.png)

**Figure 18.** Maximum equivalent stress curve of CR.

(3). Table 9 and Figure 19 present the traction cable forces under each calculation condition in the whole lifting process, indicating that all traction cable forces increase continuously from C1 to C11 and reach the maximum value when the traction is completed. The maximum traction force is 1006.9 kN, which appears in the traction cable at 45°. During the whole process, the traction cable force at 45° is always larger than the others and significantly larger than those at the long and short axes when the traction lifting process is finally completed.
The above findings indicate that when synchronous and proportional traction lifting is adopted, the deformation of the CR is too large, which exceeds the allowable deformation range of the project, and there is a tendency of local torsion.

From C2 to C10, the internal force of the inner supports decreases and that of the external supports increases. In the process of traction and lifting, the internal force of the external supports near the long and short axes is basically zero, indicating that the CR has a tendency to twist inwards.

A total of 56 CR brackets are set at the connection between each radial cable and the CR, and each bracket is divided into inner and external support. In order to clearly show the changes in internal forces of CR brackets during the whole listing process, four groups of representative brackets are selected, which are shown in Figure 20: axis 1 (long axis), axis 6 and 8 (45°), and axis 15 (short axis). The changes in internal force under traction conditions are also drawn as variation curves, as shown in Figures 21 and 22.

From C2 to C10, the internal force of the inner supports decreases and that of the external supports increases. In the process of traction and lifting, the internal force of the external supports near the long and short axes is basically zero, indicating that the CR has a tendency to twist inwards.

A total of 56 CR brackets are set at the connection between each radial cable and the CR, and each bracket is divided into inner and external support. In order to clearly show the changes in internal forces of CR brackets during the whole listing process, four groups of representative brackets are selected, which are shown in Figure 20: axis 1 (long axis), axis 6 and 8 (45°), and axis 15 (short axis). The changes in internal force under traction conditions are also drawn as variation curves, as shown in Figures 21 and 22.

The above findings indicate that when synchronous and proportional traction lifting is adopted, the deformation of the CR is too large, which exceeds the allowable deformation range of the project, and there is a tendency of local torsion.

Table 9. Traction cable force (kN).

| Axis | C1   | C2   | C3   | C4   | C5   | C6   | C7   | C8   | C9   | C10  | C11  |
|------|------|------|------|------|------|------|------|------|------|------|------|
| 1    | 18.54| 106.76| 108.17| 109.95| 112.31| 115.57| 120.37| 128.09| 142.24| 174.69| 332.54|
| 2    | 21.54| 146.63| 145.39| 144.40| 143.83| 143.96| 145.31| 148.91| 157.37| 180.36| 310.53|
| 3    | 18.64| 87.72 | 90.18 | 93.17 | 96.90 | 101.77| 108.50| 118.74| 136.67| 175.17| 347.06|
| 4    | 47.56| 206.77| 210.15| 214.73| 221.02| 229.83| 242.66| 262.44| 296.13| 369.25| 705.32|
| 5    | 58.66| 198.90| 208.38| 220.35| 235.01| 253.93| 279.49| 316.50| 376.38| 497.38| 1006.90|
| 6    | 66.51| 176.75| 186.91| 199.16| 214.31| 233.67| 259.53| 296.54| 355.85| 473.84| 960.40|
| 7    | 49.51| 191.54| 202.28| 215.16| 231.01| 251.13| 277.86| 315.84| 376.13| 496.19| 999.79|
| 8    | 22.22| 36.23 | 40.35 | 45.43 | 51.97 | 60.52 | 71.97 | 86.46 | 106.44| 145.94| 321.44|
| 9    | 51.29| 166.46| 176.36| 188.39| 203.41| 222.80| 248.99| 286.75| 347.46| 468.45| 965.38|
| 10   | 21.70| 94.78 | 98.55 | 103.15| 108.90| 116.37| 126.54| 141.44| 165.84| 216.27| 440.92|
| 11   | 21.29| 101.84| 104.13| 106.83| 110.11| 114.37| 120.42| 130.16| 147.33| 183.84| 346.69|
| 12   | 22.22| 36.23 | 40.35 | 45.43 | 51.97 | 60.52 | 71.97 | 86.46 | 106.44| 145.94| 321.44|
| 13   | 19.94| 230.72| 225.82| 220.67| 215.16| 209.36| 203.55| 199.96| 202.56| 219.90| 342.30|
| 14   | 18.48| 138.89| 138.13| 137.65| 137.60| 138.25| 140.01| 143.73| 152.05| 175.05| 308.26|
| 15   | 20.16| 109.90| 111.35| 113.21| 115.69| 119.13| 124.14| 131.98| 145.92| 177.82| 336.30|
Figure 20. Schematic diagram of four groups of typical CR brackets.

Figure 21. Variation curve of internal force of typical inner CR brackets.

Figure 22. Variation curve of internal force of typical external CR brackets.

The main reasons for excessive deformation and local torsion of the CR are as follows:
(1). Different lengths of traction cables

The unevenness of the stand results in a large difference in the lengths of the traction cables when the cables are laid on it. The lengths of traction cables at the long and short axes are shorter than those at 45°. Therefore, the tension ring at the long and short axes will leave the grandstand first when traction lifting starts, where the cable force is relatively large, resulting in inward contraction of the CR at the long and short axes.

(2). Quasi-elliptical CR plane

The plane of the CR is quasi-elliptical, and is composed of eight arc segments, four segments at the long and short axes and four segments at 45°. The arc radius at the short and long axes is large, and the curvature is small. At the beginning of lifting, the CR fails to form a circumferential force, resulting in large out-of-plane bending, obvious inward shrinkage of the CR at the long and short axes, and a trend of local torsion. From C2 to C11, as the cable net is lifted continuously, the traction cable force increases gradually. At this time, the CR tends to form a circumferential force, and its deformation starts to decrease continuously.

Therefore, the concerned structure responses are set as adjusting objectives [36], which are listed below:

(1). The radial deformation of the CR must be positive;
(2). All CR supports internal force must be negative.

The adjusted lifting process of the project adopts cumulative section lifting: First, pull radial cables at the 45°, where the CR has a relatively small radius and large curvature, and the tension ring elevation is the lowest. When the typical tension ring nodes elevated at 45° reach the position flush with those at the long axis, the long axis and 45° radial cables are pulled together. Finally, when all tension rings are horizontal, integral traction lifting at the same time can make the CR form an annular force, which is conducive to the configuration control of the CR.

Analysis of Cumulative Section Traction Lifting

According to the adjustment measures obtained from the analysis in the previous section, the cable net is divided into four sections, A, B, C and D, as shown in Figure 23. In this section, the cable net is pulled and lifted in the order of (A), (A, B), (A, B, C), (A, B, C, D).

![Figure 23. Section division of cable net.](image)

The cumulative section traction lifting sequence is shown in Figure 24, in which the red lines represent the traction cable.
The cumulative section traction lifting sequence is shown in Figure 24, in which the red lines represent the traction cable.

Step 1: Cable net is laid on stand.

Step 2: Pulling cable net in section A.

Step 3: Pulling cable net in sections A, B.

Step 4: Pulling cable net in sections A, B, C.

Figure 24. Cont.
Step 5: Integrally lifting cable net.

Step 6: Traction lifting completed.

Figure 24. Schematic diagram of cumulative sectional traction lifting.

According to the adjusted traction lifting sequence, the analysis is divided into 11 calculation conditions, as shown in Table 10. The lengths of traction cables under each calculation condition are shown in Table 11 (quarter symmetry of structure).

Table 10. Calculation conditions during cumulative sectional traction lifting.

| Step | Calculation Condition | State                                           |
|------|-----------------------|-------------------------------------------------|
| 1    | C1                    | Cable net laid on stand                         |
| 2    | C2                    | Traction cable length reduced 10% in section A  |
| 3    | C3                    | Traction cable length reduced 10% in sections A and B |
| 4    | C4                    | Traction cable length reduced 10% in sections A, B, and C |
|      | C5                    | Traction cable length reduced 20% in sections A, B, and C |
|      | C6                    | Traction cable length reduced 25% in sections A, B, and C |
| 5    | C7                    | Entire length of traction cable reduced 20%    |
|      | C8                    | Entire length of traction cable reduced 40%    |
|      | C9                    | Entire length of traction cable reduced 60%    |
|      | C10                   | Entire length of traction cable reduced 80%    |
| 6    | C11                   | Traction lifting completed                      |
Table 11. Length of traction cables under various calculation conditions (m).

| Axis | C1   | C2   | C3   | C4   | C5   | C6   | C7   | C8   | C9   | C10  | C11  |
|------|------|------|------|------|------|------|------|------|------|------|------|
| 1    | 13.85| 13.85| 13.85| 12.47| 9.97 | 7.48 | 5.98 | 4.49 | 2.99 | 1.50 | 0.00 |
| 2    | 13.82| 13.82| 13.82| 12.44| 9.95 | 7.46 | 5.97 | 4.48 | 2.99 | 1.49 | 0.00 |
| 3    | 14.47| 14.47| 14.47| 13.02| 10.42| 7.81 | 6.25 | 4.69 | 3.13 | 1.56 | 0.00 |
| 4    | 14.73| 13.26| 11.93| 10.74| 8.59 | 6.44 | 5.15 | 3.87 | 2.58 | 1.29 | 0.00 |
| 5    | 15.42| 13.88| 12.49| 11.24| 8.99 | 6.74 | 5.40 | 4.05 | 2.70 | 1.35 | 0.00 |
| 6    | 15.89| 14.30| 12.87| 11.58| 9.27 | 6.95 | 5.56 | 4.17 | 2.78 | 1.39 | 0.00 |
| 7    | 16.06| 14.45| 13.01| 11.71| 9.37 | 7.02 | 5.62 | 4.21 | 2.81 | 1.40 | 0.00 |
| 8    | 16.05| 14.45| 13.00| 11.70| 9.36 | 7.02 | 5.62 | 4.21 | 2.81 | 1.40 | 0.00 |
| 9    | 15.77| 14.19| 12.77| 11.50| 9.20 | 6.90 | 5.52 | 4.14 | 2.76 | 1.38 | 0.00 |
| 10   | 15.08| 15.08| 13.57| 12.21| 9.77 | 7.33 | 5.86 | 4.40 | 2.93 | 1.47 | 0.00 |
| 11   | 14.00| 14.00| 12.60| 11.34| 9.07 | 6.80 | 5.44 | 4.08 | 2.72 | 1.36 | 0.00 |
| 12   | 12.93| 12.93| 11.64| 10.47| 8.38 | 6.28 | 5.03 | 3.77 | 2.51 | 1.26 | 0.00 |
| 13   | 10.64| 10.64| 10.64| 10.64| 10.64| 10.64| 8.51 | 6.38 | 4.26 | 2.13 | 0.00 |
| 14   | 10.16| 10.16| 10.16| 10.16| 10.16| 10.16| 8.13 | 6.10 | 4.06 | 2.03 | 0.00 |
| 15   | 10.17| 10.17| 10.17| 10.17| 10.17| 10.17| 8.14 | 6.10 | 4.07 | 2.03 | 0.00 |

- Cumulative section traction lifting analysis results
  Table 12 shows the lowest cable net node elevation during traction lifting, indicating that the cable net is continuously raised.

Table 12. Vertical deformation of cable net.

| Calculation Condition | Vertical Deformation |
|-----------------------|----------------------|
|                       | −Uz Min (m)          | +Uz Max (m)         |
| C1                    | −28.443              | 0                   |
| C2                    | −27.924              | 0                   |
| C3                    | −25.862              | 0                   |
| C4                    | −25.424              | 0                   |
| C5                    | −22.093              | 0                   |
| C6                    | −18.502              | 0                   |
| C7                    | −15.692              | 0                   |
| C8                    | −12.944              | 0                   |
| C9                    | −9.631               | 0                   |
| C10                   | −5.011               | 0.22                |
| C11                   | −0.001               | 2.05                |

The radial deformation Ux curve of CR nodes under each calculation condition in the local cylindrical coordinate system is shown in Figure 25. During the traction lifting process, the CR is in the outward expansion state, and the maximum outward expansion deformation is 99 mm. From C2 to C10, the outward expansion deformation at 45° of the CR gradually decreases, and the outward expansion deformation at the minor axis gradually increases.

Figure 26 shows the maximum equivalent stress curve of the CR at various traction steps. From C1 to C6, the maximum equivalent stress of the CR gradually increases, and from C6 to C10, the maximum equivalent stress decreases because the cable net starts to be lifted as a whole and the CR gradually forms a ring stress. The maximum equivalent stress of the CR is 46 MPa, which occurs when the cable net traction lifting is completed (C11).
The traction cable forces during each traction step are shown in Table 13, and the variation curve of cable forces during traction lifting is shown in Figure 27. In the process of traction lifting, the traction cable forces at 45° are always greater than those at the long and short axes. The maximum traction cable force is 1006.9 kN, which appears in the traction cable at 45° when the traction lifting is completed (C11).

Table 13. Traction cable force in process of traction and lifting.

| Axis | C1  | C2  | C3  | C4  | C5  | C6  | C7  | C8  | C9  | C10 | C11 |
|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1    | 18.54 | 24.309 | 25.053 | 67.994 | 103.62 | 114.49 | 121.04 | 130.39 | 145.88 | 178.73 | 332.54 |
| 2    | 21.54 | 33.757 | 33.489 | 67.057 | 86.278 | 100.3 | 112.93 | 129.45 | 151.84 | 183.53 | 310.53 |
| 3    | 18.64 | 17.778 | 16.456 | 17.777 | 329.98 | 362.96 | 369.3 | 380.11 | 401.95 | 448.89 | 705.32 |
| 4    | 47.56 | 173.84 | 275.35 | 304.78 | 329.98 | 362.96 | 369.3 | 380.11 | 401.95 | 448.89 | 705.32 |
| 5    | 58.66 | 148.7 | 168.78 | 184.83 | 216.55 | 264.51 | 290.89 | 327.46 | 386.46 | 505.93 | 1006.90 |
| 6    | 66.51 | 133.45 | 149.97 | 165.51 | 195.79 | 241.45 | 268.09 | 304.8 | 363.39 | 480.32 | 960.40 |
| 7    | 49.51 | 141.24 | 160.39 | 178.25 | 211.05 | 259.07 | 286.21 | 323.61 | 383.21 | 502.55 | 999.79 |
| 8    | 63.26 | 137.95 | 156.45 | 173.38 | 202.46 | 244.71 | 270.57 | 306.3 | 363.19 | 477.59 | 954.06 |
| 9    | 51.29 | 167.79 | 221.4 | 243.53 | 277.24 | 319.14 | 341.67 | 372.57 | 421.65 | 521.54 | 965.38 |
| 10   | 21.70 | 23.299 | 37.581 | 38.506 | 37.683 | 42.018 | 51.905 | 69.922 | 104.42 | 176.51 | 440.92 |
| 11   | 21.29 | 41.687 | 77.758 | 87.148 | 98.792 | 111.02 | 122.04 | 135.14 | 152.44 | 185.52 | 346.69 |
| 12   | 22.22 | 31.647 | 67.478 | 133.69 | 279.99 | 424.65 | 416.01 | 399.15 | 378.98 | 361.19 | 321.44 |
| 13   | 19.94 | 50.635 | 37.108 | 28.227 | 16.72 | 11.989 | 13.012 | 14.732 | 18.467 | 33.415 | 342.30 |
| 14   | 18.48 | 48.979 | 56.25 | 54.945 | 31.335 | 18.301 | 25.772 | 40.415 | 77.783 | 157.85 | 308.26 |
| 15   | 20.16 | 45.181 | 54.813 | 61.304 | 51.607 | 34.076 | 64.017 | 105.37 | 143.48 | 182.94 | 336.30 |
Figures 27 and 29 show internal force curves for four selected groups of CR brackets. From C1 to C6, due to the partial lifting of the cable net and the eccentric force on the CR, the internal forces of the inner supports at 45° gradually increase and those of the outer supports gradually decrease. From C6 to C11, all cables start to be lifted together. At this time, the eccentric force decreases and the internal forces are gradually distributed evenly. Notably, all supports remain under pressure during the whole traction lifting process, indicating that local torsion of the CR does not occur.

Figure 27. Variation curve of traction force during traction lifting.

Figure 28. Internal force variation curves of four groups of typical inner CR brackets.
Comparisons and Effects

Figure 30 shows an envelope diagram of CR radial deformation before and after adjusting. It indicates that the radial deformation of the CR reduced from between $-161$ mm and $149.6$ mm to between $21.5$ mm and $99$ mm, which means that the CR deformation was greatly improved and there was no internal shrinkage.

The maximum equivalent stress of CR before and after adjusting is plotted in Figure 31, indicating that it was reduced to $46$ MPa.

Figure 32 shows the internal forces of the axis 1 and axis 15 CR external supports under various calculation conditions before and after adjusting. During the traction lifting process, the internal force of the external support near the long and short axes, which was zero before adjusting, turned to negative through adjusting; this means that all CR brackets are in compression and local torsion can be avoided.

Figure 33 shows that the traction cable forces are almost the same before and after adjusting, and there are no higher requirements for the traction cable due to the adjustment.
Figure 32 shows the internal forces of the axis 1 and axis 15 CR external supports under various calculation conditions before and after adjusting. During the traction lifting process, the internal force of the external support near the long and short axes, which was zero before adjusting, turned to negative through adjusting; this means that all CR brackets are in compression and local torsion can be avoided.

![Figure 32](image)

**Figure 32.** Internal forces of axis 1 and axis 15 CR external supports before and after adjusting.

Figure 33 shows that the traction cable forces are almost the same before and after adjusting, and there are no higher requirements for the traction cable due to the adjustment.

![Figure 33](image)

**Figure 33.** Traction cable forces are almost the same before and after adjusting.

### 4.2.2. Analysis of Tie-Down Tension

After the cable net traction lifting is completed, the tie-downs are installed and tensioned in the subsequent construction to ensure the accuracy of the tension ring elevation, which is also the key to non-bracket construction. Due to the different tension sequences of the tie-downs, the cable force distribution in the cable net varies accordingly. Moreover, the change in cable forces in the cable net tends to increase the in-plane bending moment in the CR, and finally increase its deformation. Therefore, in this section synchronous equal proportion tension is first carried out to study the variation in the CR configuration, the cable forces of the tie-downs, and the elevation of the tension ring clamps during the tie-down tension process, so as to determine the key factors affecting those changes. Then, the tension sequence of the tie-downs is improved and analyzed to determine a reasonable sequence.

**Analysis Model**

The tie-down tension analysis model is shown in Figure 34, including structural columns, compression ring, cable net, CR brackets, tie-downs, and other components.
A total of 58 tie-downs connecting cable clamps and their corresponding anchorage are installed, among which the axis 35 and axis 51 tie-downs have a herringbone shape considering the actual construction site requirements. The link 10 element (tension-only element) is adopted for the new tie-downs in the model, and its material characteristics are shown in Table 14.

Table 14. Material characteristics of tie-downs.

| Material Characteristics | Parameter Value |
|--------------------------|-----------------|
| E (GPa)                  | 190             |
| A (m²)                   | 0.00084         |
| P (kg/m³)                | 7850            |
| α (1/°C)                 | 12.0 × 10⁻⁶     |

Results of Synchronous Proportional Tension

Synchronous equal proportion tension is adopted in this section, which means that the tension rings are pulled down to the design elevation at the same time according to the same proportion from the configuration after traction lifting. According to the analysis results in Section 4.2.1, the maximum distance of the tension ring elevation relative to the design elevation is about 2 m after the cable net is lifted. The integral tension of the tie-downs is divided into five steps, and each step reduces the tension ring by about 0.4 m. The corresponding setting of tie-down tension calculation conditions is shown in Table 15. “Tie-down tension 20%” means that the reduction in tie-downs is 20% of the distance between the tension ring elevation and the relative design elevation, and the tie-downs pulled down by 100% means that the tension ring is pulled to the design elevation.

Table 15. Calculation conditions during synchronous proportional tension tension.

| Calculation Condition | State                                           |
|-----------------------|-------------------------------------------------|
| C1                    | Traction lifting completed                       |
| C2                    | Tie-down tension 20%                            |
| C3                    | Tie-down tension 40%                            |
| C4                    | Tie-down tension 60%                            |
| C5                    | Tie-down tension 80%                            |
| C6                    | Tie-down tension 100% (pull down tension ring to design elevation, i.e., pull-down is completed) |

During the tensioning of tie-downs, the vertical elevation of the tension ring gradually decreases until the pull-down is completed. The tension ring elevation of the pull-down
The tension ring elevation of the pull-down completion state is between $-11 \text{ mm}$ and $38.8 \text{ mm}$, basically reaching the designed equilibrium state elevation, as shown in Figure 35.

| Calculation Condition | State                        |
|-----------------------|------------------------------|
| C1                    | Traction lifting completed   |
| C2                    | Tie-down tension 20%         |
| C3                    | Tie-down tension 40%         |
| C4                    | Tie-down tension 60%         |
| C5                    | Tie-down tension 80%         |
| C6                    | Tie-down tension 100% (pull down tension ring to design elevation, i.e., pull-down is completed) |

During the tensioning of tie-downs, the vertical elevation of the tension ring gradually decreases until the pull-down is completed. The tension ring elevation of the pull-down completion state is between $-11 \text{ mm}$ and $38.8 \text{ mm}$, basically reaching the designed equilibrium state elevation, as shown in Figure 35.

**Figure 35.** Vertical deformation $U_z (\text{m})$ of C6.

The radial deformation $U_x$ of 56 CR nodes under each tension calculation condition through local cylindrical coordinate system was drawn as a deformation curve, which is shown in Figure 36. From C1 to C3, the CR radial deformation at the minor axis expands outward to 144 mm. The CR radial deformation at 45$^\circ$ is relatively small and gradually decreases, and at the long axis it remains small and stable. From C4 to C6, CR radial deformation gradually decreases and finally reaches about 0, which means that the plane configuration of the CR reaches the designed one when tie-down tension is completed.

**Figure 36.** Radial deformation $U_x$ of CR under various calculation conditions in local coordinate system.

Figure 37 shows that the maximum equivalent stress of the CR during various tension construction steps gradually increases and finally reaches the maximum value of 144.7 MPa.
Figure 36. Radial deformation $U_x$ of CR under various calculation conditions in local coordinate system.

Figure 37 shows that the maximum equivalent stress of the CR during various tension construction steps gradually increases and finally reaches the maximum value of 144.7 MPa.

Nephograms of tie-down forces under each calculation condition are shown in Figure 38, which indicate that the tie-down force at the short axis is largest, followed by the long axis, and the cable force at the quarter is relatively the smallest. When the pull-down is completed, the cable force of the tie-downs reaches a maximum value of 547.6 kN.

![Figure 38. Tie-down force nephograms (N): (a) C2, (b) C3, (c) C4, (d) C5, (e) C6.](image-url)
When synchronous equal proportion tension is adopted, the maximum CR radial deformation of 144 mm, which occurs at the short axis (axis 15), is still unsatisfactory. After the lifting is completed, the expansion of the short axis of the CR is basically large due to the analysis in Section 4.2.1. At the initial stage of synchronous equal proportional tension of the tie-downs, the changes in cable force in the cable net make the forces acted on the CR more uneven, and its uneven deformation further expands, resulting in more obvious outward expansion at the short axis.

Therefore, the adjusting objective is set as: reduce radial deformation of the CR.

The tie-down tension strategy is adjusted to proportional section tension according to the outward expansion degree of the CR. The initial tension proportion of the tie-downs at the short axis should be larger than that at the long axis and 45° angle, so as to control the outward expansion deformation at the short axis.

Results of Proportional Section Tension

According to the adjusted tension idea obtained from the analysis in the previous section, the tie-downs are divided into three areas: A, B, and C, as shown in Figure 39.

![Figure 39. Division of tie-downs. (1~56 represent the NO. of radial cables corresponding to Axis NO.)](image)

According to the tie-down tension sequence, the tension process analysis is divided into six calculation conditions, as shown in Table 16.

| Calculation Condition | State |
|-----------------------|-------|
| C1                    | Traction lifting completed |
| C2                    | Tie-downs tensioned by 20% in A, 10% in B, and 5% in C |
| C3                    | Tie-downs tensioned 35% in A, 20% in B, and 15% in C |
| C4                    | Tie-downs tensioned 50% in A, 40% in B, and 30% in C |
| C5                    | Tie-downs tensioned 65% in A, 60% in B, and 50% in C |
| C6                    | All tie-downs tensioned 100% (tension ring pulled down to design elevation, i.e., pull-down is completed) |
During the tensioning of tie-downs, the vertical elevation of the tension ring decreases gradually. When pulling down of the cable net is finished, the elevation of the tension ring is between –11 and 38.8 mm, which basically reaches the design elevation (±0). The vertical deformation of the tension ring under each calculation condition is shown in Table 17.

| Calculation Condition | Vertical Displacement of Tension Ring (m) |
|-----------------------|------------------------------------------|
| G1                    | 1.031–2.047                              |
| G2                    | 0.789–1.773                              |
| G3                    | 0.699–1.575                              |
| G4                    | 0.523–1.275                              |
| G5                    | 0.325–0.981                              |
| G6                    | −0.011–0.038                             |

Figure 40 presents deformation curves for 56 typical CR nodes under each calculation condition in the local coordinate system, which shows that the short axis expanding CR deformation reaches 108 mm at C2 and then decreases continuously during the whole process. When the tension of the tie-downs is completed, the CR basically reaches the design configuration.

Figure 41. Maximum equivalent stress curve of CR.

The maximum equivalent CR stress curve under various tension calculation conditions is shown in Figure 41. It can be found that with increased tension force of the tie-downs, the stress of the CR resulting from the radial cable force gradually increases. The maximum equivalent stress value reaches 145.3 MPa when the tensioning of tie-downs is completed. The stress level of CR is acceptable.
Tie-down force nephograms in Figure 42 show the distribution of tie-down forces during the tension process. All tie-down forces increase during the whole tension process. The maximum tension force is 547.6 kN, and the cable force of tie-downs at the long and short axes is significantly greater than at 45°.

Figure 42. Tie-down force nephograms (N): (a) C2, (b) C3, (c) C4, (d) C5, (e) C6.

Comparisons and Effects

The envelope diagram of CR radial deformation before and after adjusting (Figure 43) shows that when the adjusted tie-down tension strategy is adopted, the maximum radial outward expansion deformation of CR can be reduced from 144 to 108 mm.
4.2.3. Analysis of the Whole Construction Process

The previous two parts focused on verifying the two core technologies in the non-bracket construction of an annular cable supported grid structure. In this part we carry out the simulation of the whole construction process of Education City Stadium according to the model and load conditions in Section 4.1 to track the deformation, stress, cable force, and other key technical indices of the structure at each stage of the assembly process, and then compare the results of construction forming state and dead load state.

Figure 43. Envelope diagram of CR radial deformation before and after adjusting.

Figure 44 shows that the adjusted tension strategy can effectively reduce the CR stress during the tension process and has little effect on the stress level when tension is completed. The maximum equivalent stress is 145.3 MPa.

Figure 44. Maximum equivalent stress of CR before and after adjusting.

Four representative tie-downs are selected: axis 1 (long axis), axis 5, axis 8 (45°), and axis 15 (short axis). The changes in cable force under various tension conditions before and after adjusting were drawn as cable force curve, shown in Figure 45. The slight difference in cable forces indicates that additional tooling cables will not be wasted due to different tension strategies of tie-downs.

Figure 45. Four selected groups of tie-down forces before and after adjusting.
4.2.3. Analysis of the Whole Construction Process

The previous two parts focused on verifying the two core technologies in the non-bracket construction of an annular cable supported grid structure. In this part we carry out the simulation of the whole construction process of Education City Stadium according to the model and load conditions in Section 4.1 to track the deformation, stress, cable force, and other key technical indices of the structure at each stage of the assembly process, and then compare the results of construction forming state and dead load state.

Table 18 lists the corresponding calculation conditions set according to the sequence of the whole construction process in Section 3. Especially, in step 5, the counterclockwise hoisting of the roof steel structure will be carried out gradually according to the section and direction in Figure 38.

Table 18. Calculation conditions during construction.

| Step   | Calculation Condition | State                                                                 |
|--------|-----------------------|----------------------------------------------------------------------|
| Step 1 | C1                    | Install CR, structural columns, CR brackets, and additional cross-bracing between columns. |
| Step 2 | C2                    | Pull and lift cable net.                                             |
| Step 3 | C3                    | Install and tension tie-downs to keep tension ring at design elevation. |
| Step 4 | C4                    | Install cross-bracing between columns.                               |
| Step 5 | C5–C40                | Hoist upper grid counterclockwise according to sequence in Figure 46, maintain design tension ring configuration by adjusting tie-down cable forces simultaneously. |
| Step 6 | C41                   | Remove tie-downs.                                                   |
| Step 7 | C42                   | Remove CR brackets and additional cross-bracing between columns.     |
| Step 8 | C43                   | Install remaining roof components, and entire structure is formed.    |

Figure 46. Grid hoisting steps: (a) installation section diagram (blue and red indicate hoisting and closure units, respectively; E1, E2, E3, . . . E56 are codes of 56 units, marked according to specific assembly sequence; 1,15,29,43 represent the main axis); (b) installation sequence diagram.

Analysis Results of the Whole Construction Process

Figure 47 shows selected quarter CR node radial deformation under typical calculation conditions (C1 to C4 and C40 to C43), which indicates the following:
From C41 to C42, the tie-downs are removed, and the CR radial deformation values increase again. Finally, in C43, the radial deformation values return to a level close to 0.

**Figure 47.** CR radial deformation curve (mm).

At the initial stage of structural construction (C1, C2), the internal forces of all cables are at a low level, which has little effect on radial CR deformation, and overall CR outward expansion deformation is large;

From C3 to C40, the pre-tension of the cable net is gradually established, and the tie-downs start to control the tension ring at the design configuration during the construction process. Therefore, the cable forces in the cable net are large and stable, and the radial deformation values of the CR are relatively stable and close to 0, which means that the CR can basically remain in the design state configuration by adjusting the tie-down cable forces while the entire structure is not yet fully formed;

From C41 to C42, the tie-downs are removed, and the CR radial deformation values increase again. Finally, in C43, the radial deformation values return to a level close to 0.

Figures 48 and 49 present the radial and vertical deformation of selected quarter tension ring nodes (Tension ring clamps’ nodes NO. are shown in Figure 50.) under typical calculation conditions (C1 to C4 and C40 to C43), which indicates the following:

**Figure 48.** Radial deformation curve of central nodes of tension ring clamps (mm).
Figure 49. Vertical deformation Uz curve of typical nodes of tension ring (mm).

Figure 50. Tension ring clamp nodes. (1~15 represent the tension ring cable clamps’ nodes NO. corresponding to the axis.)

In the initial stage of structural construction (C1, C2), the traction lifting of the cable net makes the vertical and radial deformation of tension ring large;

From C3 to 40, the deformation values are relatively stable and close to 0 because the tie-downs keep the tension ring at the design configuration during the whole upper grid installation process, ensuring stable and accurate grid installation;

From C41 to 42, the tie-downs are removed, and the deformation values change again. Finally, the deformation (especially vertical deformation) of the tension ring returns to a level close to 0 in C43.

Figure 51 shows the variation in maximum equivalent stress of CR, roof radial truss, structural columns, and CR brackets, which clearly illustrates the three stages of the whole construction process. In the first stage (C1–C2), the maximum equivalent stress of the whole structure is low and increases rapidly, which reflects the cable net construction process. In the middle stage (C3–C40), tie-downs start to control the configuration of the cable net, resulting in stable stress for the whole structure. In the third stage (C41–C43), the maximum equivalent stress of the CR and the structural column further increases with the structure formation, and the maximum equivalent stress in the radial main truss and CR brackets will decrease after the tie-downs are removed. In the whole construction process:
the maximum equivalent stress of the CR is 171 MPa, which occurs in the third stage (C42); the maximum radial main truss stress is 125 MPa, in the third stage (C41); the maximum structural column stress is 59.4 MPa, in the third stage (C43); and the maximum CR bracket stress is 82.5 MPa, in the middle stage.

Figure 51. Maximum stress variation curve of four types of components under various calculation conditions (MPa).

Figure 52 shows the change in radial cable forces under typical calculation conditions. Considering the symmetry of the structure, 1–15 axis 1/4 symmetrical cables are selected, as shown in Figure 53. At first (C1–C2), the pre-tension is preliminarily established inside the cable net while the radial cable forces are still relatively small. Then, in the subsequent tie-down tension and upper grid hoisting conditions (C3–C40), the tie-downs basically control the cable net configuration at the design position, and the radial cable forces are large and stable. Finally (C41–C43), the whole structure is lifted up and the radial cable forces decrease slightly after the tie-downs are removed, and then return to the stable middle stage level when the entire structure is finally formed.

Figure 52. Cable force variation curve of three typical radial cables under various calculation conditions (kN).
Figure 53. Radial cables. (1~15 represent the radial cables’ NO. corresponding to the axis.)

Figure 54 shows that during the whole construction process, the maximum internal force of all CR brackets is 792 kN, which occurs at 45°, and the CR support force at the long and short axes is relatively small.

Figure 54. Variation curve of maximum internal force of each CR bracket (kN).

Comparison between Analysis Results of Construction Process Forming State and Dead Load State

(1). Comparison of vertical deformation of tension ring nodes

Figure 55 compares the vertical deformation of 56 typical nodes of the tension ring in the construction forming state and the dead load state, which shows that the simulation results are slightly larger for the non-bracket construction forming method than the dead load state, with maximum deformation of $-32.25$ mm and minimum deformation of $-22.26$ mm. The maximum difference between the two results is $16.5$ mm, which can be considered negligible.
(2). Comparison of vertical deformation at the inner cantilever end of radial main beam

Figure 56 compares the cantilever end deformation of 56 radial main beams in the construction forming state and the dead load state. The analysis results of the construction process are slightly larger than the dead load state results, with maximum deformation of $-49.35$ mm and minimum deformation of $-8.3$ mm, and the difference between the two results is small.

(3). Comparison of structural column vertical reaction forces and cable forces

Figures 57–59, respectively, compare the structural column bottom reaction forces, radial cable forces, and tension ring forces in the construction forming state and the dead load state. The results show that the vertical reaction forces and radial cable forces are almost consistent in the two states, and the tension ring forces in the construction forming state are slightly lower compared to the dead load state, with a difference of about 0.8%.
The integrity of the cable strut system gives this kind of structure a natural foundation for non-bracket construction. The upper grid still mainly depends on the tensioned cable net. The structural stiffness of the self-equilibrium stress system, composed of the cable strut system, is almost consistent with a cable truss with the upper flexible cables replaced by rigid grids. The structural state is slightly lower compared to the dead load state, with a difference of about 0.8%.

**Figure 57.** Comparison curve of vertical reaction force of structure in two states (kN).

**Figure 58.** Comparison curve of radial cable force in two states (kN).

**Figure 59.** Comparison curve of tension ring force in two states (kN).

This paper introduces a new non-bracket construction method for annular cable supported grid structures. Based on these characteristics, a new non-bracket construction method is proposed. This method utilizes jacks fixed near the radial cable anchor points to reliably and stably lift the cable net, and creatively solves the problem of accurate control of the cable net and grid installations. The situation where components in the self-equilibrium stress system cannot be constructed as a whole results in great difficulty controlling the structure configuration. In the construction process, the CR and cable net configuration, radial cable forces, vertical reaction forces of the structure, and structural stress level are all relatively stable, and basically consistent with the corresponding response of the step when the final roof is formed (construction step 5).

From construction steps 3 to 5, the displacement of the tension ring nodes and CR nodes is basically close to 0, which means that the CR and tension ring can basically maintain the design configuration during the whole process of cumulative hoisting of the upper rigid grid, ensuring that the grid can be assembled accurately according to the design size; and the stress and deformation of the upper flexible cables are significantly reduced. The displacement of the radial cable nodes is significantly reduced in steps 3–5, which means that the radial cables can basically maintain the design configuration during the whole process of cumulative hoisting of the upper rigid grid, ensuring that the grid can be assembled accurately according to the design size; and the stress and deformation of the upper flexible cables are significantly reduced. The situation where components in the self-equilibrium stress system cannot be constructed as a whole results in great difficulty controlling the structure configuration. In the construction process, the CR and cable net configuration, radial cable forces, vertical reaction forces of the structure, and structural stress level are all relatively stable, and basically consistent with the corresponding response of the step when the final roof is formed (construction step 5).

In the early stage of the construction (i.e., construction steps 1–3), the CR and cable net comprise the basic system of the structure. At this time, the pre-tension level in the initial stage is equal, and the radial cables are on both sides of the CR. The pre-tension level is designed to be relatively high, but the actual pre-tension level is low. The CR pre-tension level is designed to be relatively high, but the actual pre-tension level is low. The difference is about 0.8%. From construction steps 3 to 5, the CR and radial cable forces, vertical reaction forces of the structure, and structural stress level are all relatively stable, and basically consistent with the corresponding response of the step when the final roof is formed (construction step 5). The radial cable forces are almost consistent in the two states, and the tension ring forces in the construction forming state are slightly lower compared to the dead load state, with a difference of about 0.8%.
5. Conclusions

This paper introduces a new non-bracket construction method for annular cable supported grid structures. First, the characteristics of the structure are discussed. It is essentially a cable truss with the upper flexible cables replaced by rigid grids. The structural stiffness of the self-equilibrium stress system, composed of cable strut system, compression ring, and upper grid, still mainly depends on the tensioned cable net. The integrity of the cable strut system gives this kind of structure a natural foundation for non-bracket construction. The situation where components in the self-equilibrium stress system cannot be constructed as a whole results in great difficulty controlling the structure configuration in the non-bracket process. Based on these characteristics, a new non-bracket construction method for annular cable supported grid structure is proposed. This method utilizes jacks fixed near the radial cable anchor points to reliably and stably lift the cable net, and creatively solves the problem of accurate control of the cable net and grid installation position through the setting of tie-downs. Finally, based on the actual Education City Stadium in Qatar, the new construction method is applied in practice and the whole construction process is simulated in ANSYS finite element software (Version 2019 R1).

The following conclusions could be drawn from the calculation results:

(1). In the early stage of the construction (i.e., construction steps 1–3), the CR and cable net comprise the basic system of the structure. At this time, the pre-tension level in the cable net is still relatively low and the structural flexibility is large, and the excessive deformation and local torsion of the compression ring require special attention. In the actual construction process, it is necessary to analyze the compression ring deformation, local torsion, and other structural responses to improve the cable net traction lifting and tie-down tensioning strategy. In this paper, the original synchronized equally proportioned strategies are adjusted to cumulative sectional strategies according to the analysis results;

(2). In the construction process, when the tie-downs come into effect (i.e., construction steps 3–5), the displacement of the tension ring nodes and CR nodes is basically close to 0, which means that the CR and tension ring can basically maintain the design configuration during the whole process of cumulative hoisting of the upper rigid grid, ensuring that the grid can be assembled accurately according to the design size;

(3). From construction steps 3 to 5, the CR and cable net configuration, radial cable forces, and structural stress level are all relatively stable, and basically consistent with the corresponding response of the step when the final roof is formed (construction step 43). This indicates that the tie-downs can effectively and accurately simulate the uninstalled components to load on the installed structure before the self-balancing system of the cable supported grid structure is formed, to keep the structure in the designed balance state during the construction process. The stable structural deformation, cable force, and stress levels also indicate that the setting of tie-downs can ensure construction stability and safety when the self-balancing system is not formed;

(4). The radial cable force distribution keeps the same trend during the entire construction process, and radial cable forces from step 3 to step 5 are basically consistent with the corresponding response of the step when the final roof is formed (construction step 43). This indicates that the non-bracket method in this paper could accurately control the prestress flow;

(5). The results of the structure formed after construction (including structural deformation, structural stress, and cable force) are similar to the corresponding results of the dead load state, indicating that the construction method is feasible and has good accuracy.

Therefore, the non-bracket construction method proposed in this paper is suitable for the construction of annular cable supported grid structures and can maintain the accuracy and stability of the structure during the entire construction.
Author Contributions: Conceptualization, L.H.; Data curation, L.H.; Formal analysis, L.H. and H.G.; Writing—original draft, L.H.; Writing—review & editing, Y.R., B.L. and M.D. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the National Natural Science Foundation of China, grant number 11673039; the Key Research and Development Project of Jiangsu Province, grant number BE2020703; and the Natural Science Foundation of Jiangsu Province, grant number BK20190753.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: All data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest: The authors declare no conflict of interest.

References
1. Acic, M.; Vlajic, L.M.; Kostic, D. Determination of prestressing levels for cable trusses as a function of their stability. Gynécologie Obst. Fertil. 2014, 65, 192–193.
2. Masao, S.; Nakakawaji, I. A Study on Structural Characteristic of Beam String Structure: Part 1 Prestressing for Dead Load; Summaries of Technical Papers of Annual Meeting Architectural Institute of Japan; Architectural Institute of Japan: Tokyo, Japan, 1987.
3. Kawaguchi, M.; Masaru, A.; Tatemichi, I. Design, tests and realization of ‘suspen-dome’ system. J. Int. Assoc. Shell Spat. Struct. 1999, 40, 179–192.
4. Kaveh, A.; Rezaei, M. Optimum topology design of geometrically nonlinear suspended domes using ECBO. Struct. Eng. Mech. 2015, 56, 667–694. [CrossRef]
5. Feng, Y.; Xiang, X.; Wang, H. Mechanical behavior and form-finding research on large opening spoke-wheel-type cable supported grid structure. J. Build. Struct. 2019, 40, 12. (In Chinese)
6. Feng, Y.; Xiang, X.; Wang, H. Static performance parameter analysis of large opening spoke-wheel-type cable supported grid structure. J. Build. Struct. 2019, 40, 11. (In Chinese)
7. Jorquera-Lucerga, J.J. Form-Finding of Funicular Geometries in Spatial Arch Bridges through Simplified Force Density Meth-od. Appl. Sci. 2018, 8, 2553. [CrossRef]
8. Rezaiee-Pajand, M.; Sarafraz, S.R.; Rezaeie, H. Efficiency of dynamic relaxation methods in nonlinear analysis of truss and frame structures. Comput. Struct. 2012, 112–113, 295–310. [CrossRef]
9. Wang, Q.; Ye, J.; Wu, H.; Gao, B.; Shepherd, P. A triangular grid generation and optimization framework for the design of free-form grid shells. Comput. Des. 2019, 113, 96–113. [CrossRef]
10. Kooshestani, K. Nonlinear force density method for the form-finding of minimal surface membrane structures. Commun. Nonlinear Sci. Numer. Simul. 2014, 19, 2071–2087. [CrossRef]
11. Pagitz, M.; Tur, J. Finite element based form-finding algorithm for tensegrity structures. Int. J. Solids Struct. 2009, 46, 3235–3240. [CrossRef]
12. Schek, H.J. The force density method for form finding and computation of general networks. Comput. Methods Appl. Mech. Eng. 1974, 3, 115–134. [CrossRef]
13. Motro, V. Multi-parametered Form finding Method: Application to Tensegrity Systems. Int. J. Space Struct. 1999, 14, 147–154.
14. Zhang, J.Y.; Ohsaki, M. Adaptive force density method for form-finding problem of tensegrity structures. Int. J. Solids Struct. 2006, 43, 5586–5673. [CrossRef]
15. Tran, H.C.; Lee, J. Advanced form-finding for cable-strut structures. Int. J. Solids Struct. 2010, 47, 1785–1794. [CrossRef]
16. Tran, H.C.; Lee, J. Advanced form-finding of tensegrity structures. Comput. Struct. 2010, 88, 237–246. [CrossRef]
17. Trinh, D.T.; Lee, S.; Kang, J.; Lee, J. Force density-informed neural network for prestress design of tensegrity structures with multiple self-stress modes. Eur. J. Mech.-A/Solids 2022, 94, 104584. [CrossRef]
18. Otter, J.R.H.; Cassell, A.C.; Hobbis, R.E. Dynamic Relaxation. Proc. Inst. Civ. Eng. 1966, 35, 633–656. [CrossRef]
19. Day, A.S.; Bunce, J.H. Analysis of cable networks by dynamic relaxation. Civ. Eng. Public Work. Rev. 1970, 4, 383–386.
20. Lewis, W.J.; Jones, M.S.; Rushton, K.R. Dynamic relaxation analysis of the non-linear static response of pretensioned cable roofs. Comput. Struct. 1984, 18, 989–997. [CrossRef]
21. Hüttner, M.; Maca, J.; Fajman, P. The efficiency of dynamic relaxation methods in static analysis of cable structures. Adv. Eng. Softw. 2015, 89, 28–35. [CrossRef]
22. Ding, M.M.; Luo, B.; Guo, Z.X.; Pan, J. Integral tow-lifting construction technology of a tensile beam-cable dome. J. Zhejiang Univ. Sci. A Appl. Phys. Eng. 2015, 16, 935–950. [CrossRef]
23. Tomei, V.; Grande, E.; Imbimbo, M. Design optimization of grid shells equipped with pre-tensioned rods. J. Build. Eng. 2022, 52, 104407. [CrossRef]
24. Richardson, J.N.; Adriaenssens, S.; Coelho, R.F.; Bouillard, P. Coupled form-finding and grid optimization approach for single layer grid shells. Eng. Struct. 2013, 52, 230–239. [CrossRef]
25. Kociecki, M.; Adeli, H. Shape optimization of free-form steel space-frame roof structures with complex geometries using evolutionary computing. *Eng. Appl. Artif. Intell.* **2015**, *38*, 168–182. [CrossRef]

26. Teranishi, M.; Ishikawa, K. Grid patterns optimization for single-layer latticed domes. *Adv. Struct. Eng.* **2021**, *24*, 359–369. [CrossRef]

27. Laccone, F.; Malomo, L.; Froli, M.; Cignoni, P.; Pietroni, N. Automatic design of cable-tensioned glass shells. *Comput. Graph. Forum* **2020**, *39*, 260–273. [CrossRef]

28. Quagliaroli, M.; Malerba, P.G.; Albertin, A.; Pollini, N. The role of prestress and its optimization in cable domes design. *Comput. Struct.* **2015**, *161*, 17–30. [CrossRef]

29. Sun, F.; Zhu, D.; Liang, M.; Zhang, D. Study on Form-Finding of Cable-Membrane Structures Based on Particle Swarm Optimization Algorithm. *Math. Probl. Eng.* **2020**, **2020**, 1281982. [CrossRef]

30. Liu, Y.; Zhang, Q.; Luo, X. Construction simulation analysis of spoke-type cable-supported grid structure in a stadium. *Build. Struct.* **2021**, *51*, 6. (In Chinese)

31. Zong, Z.; Gao, M.; Guo, Z. Simulation Analysis of Cable Dome Construction Forming based on Tower Lifting and Accumulative Installation. *Build. Sci.* **2012**, *28*, 5. (In Chinese)

32. Luo, B.; Guo, Z.; Gao, F. Research on non-bracket tow-lifting construction technology and complete process analysis of cable dome. *J. Build. Struct.* **2012**, *33*, 7. (In Chinese)

33. Luo, B.; Guo, Z.; Qiu, R.; Gao, F.; Yang, X. Cable Truss Non-Bracket Construction Method of Inclined Towering and Lifting Cable-Strut Integrality by Fixed Jack.: CN. CN102493664 B[P], 21 December 2011. (In Chinese).

34. Luo, B.; Guo, Z.; Yang, X. Research on non-bracket inclined tow-lifting construction technology with fixed jacks and complete process analysis of cable-truss. *J. Build. Struct.* **2014**, *35*, 7. (In Chinese)

35. Qian, R.; Yang, L. *Tension Structure: Analysis, Design, Construction*; Southeast University Press: Nanjing, China, 2003. (In Chinese)

36. Feng, R.Q.; Ge, J.M. Shape optimization method of free-form cable-braced grid shells based on the translational surfaces technique. *Int. J. Steel Struct.* **2013**, *13*, 435–444. [CrossRef]