The Hermann Weyl Prize 2014

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1. Introduction by Jean-Pierre Gazeau

The Hermann Weyl Prize was created in the year 2000 by the Standing Committee of the International Colloquia on Group Theoretical Methods in Physics (ICGTMP), under the initiative of its chairman H.D. Doebner. The purpose of the Weyl Prize is to provide recognition for young scientists (in principle younger than 35) who have performed original work of significant scientific quality in the area of understanding physics through symmetries.

The Hermann Weyl prize consists of a certificate citing the accomplishments of the recipient, prize money of US$ 500 and an allowance towards attendance of the bi-annual International Colloquium on Group Theoretical Methods in Physics at which the award is presented.

The previous winners of the award are: Edward Frenkel (2002), Nikita A. Nekrasov (2004), Boyko Bakalov (2006), Mohammad M. Sheikh-Jabbari (2008), Giulio Chiribella (2010) and Razvan Gurau (2012).

The Selection Committee of the Weyl Prize 2014 consisted of Edward Frenkel (Chair, UC Berkeley, USA), Gitta Kutyniok (TU Berlin, Germany), Joris Van der Jeugt (Ghent University, Belgium), Katrin Wendland (Freiburg University, Germany) and Pavel Winternitz (University of Montreal, Canada).

The Committee received the nominations of 7 excellent candidates by members of the Standing Committee of ICGTMP. The committee members came to the unanimous conclusion to award the 2014 Hermann Weyl Prize to

Yuji Tachikawa
Department of Physics, University of Tokyo

for outstanding contributions to our understanding of supersymmetric quantum field theories; in particular, to the discovery of the Alday-Gaiotto-Tachikawa correspondence that has led to spectacular advances in both mathematics and quantum physics.

According to Edward Frenkel, Chairman of the Selection Committee, Tachikawa and his collaborators found unexpected connections between models of quantum physics in two and four dimensions. “This work shed new light on many important phenomena in physics, and it also led to some astonishing predictions in geometry that have been rigorously proved by mathematicians” he said. “Without a doubt, this is one of the most important developments in mathematical physics of the past decade.”

The ceremony for the Hermann Weyl Prize took place on July 16, 2014 in the Aula Academica of Ghent University, a historical building dating from 1826. The ceremony was graced by a
saxophone quintet, “Sax Obsession”, playing music from Gershwin and Shostakovich, amongst others. The participants were welcomed by Debora Van Durme, master of ceremonies, who welcomed the awardees, committee members and the invitees. She presented an introduction to some history of the university and the Aula building, and to Adolphe Sax and his instruments. The laudatio for Professor Tachikawa was delivered by Edward Frenkel. The full text of his laudatio is given in the next section. Then followed the actual presentation of the Certificate and the Prize by Jean-Pierre Gazeau and Joris Van der Jeugt. The ceremony was concluded by a reception with drinks and cocktail snacks in the adjacent Peristylium.

In figure 1 the certificate of the 2014 Hermann Weyl Prize is are shown. Figure 2 shows some pictures taken at the ceremony.

![Certificate of the 2014 Hermann Weyl Prize](image)

**Figure 1.** The 2014 Hermann Weyl Prize certificate

2. **Laudatio for Professor Yuji Tachikawa by Edward Frenkel**

It gives me a great pleasure to present, on behalf of the Selection Committee, the 2014 Hermann Weyl Prize to Yuji Tachikawa of University of Tokyo.

Hermann Weyl was a pioneer of applications of symmetries in physics. He once said: “I tried to unite the true and the beautiful. And when I had to choose one or the other, I usually chose beautiful.”

True and beautiful are the attributes that indeed may be used to describe the work of Yuji Tachikawa for which he is receiving the Prize today. The main focus of this work is on supersymmetric gauge theories, or Yang–Mills theories, in four dimensions. These are very important models of quantum field theory, because three out of four known forces of nature (electromagnetic, weak, and strong) are described by gauge theories. The latter are non-supersymmetric, but physicists have been studying in great detail the supersymmetric cousins
of these models in recent decades. Even if supersymmetry is not found in nature, these models are very useful because they are simpler and allow a more detailed analysis. The hope is that they will shed light on various important effects in gauge theories, such as the appearance and behavior of instantons.

The origin of the work of Tachikawa and his collaborators that we honor today is in the observation made by Edward Witten and others in the mid-1990s that for each simply-laced simple Lie algebra \( g \) one can construct a remarkable six-dimensional quantum field theory (known as the “(0,2) superconformal field theory” or “theory X”). While these theories are still quite mysterious, their very existence provides us with a host of useful information. Indeed, we can consider such a theory on a product \( M_d \times M_{6-d} \) manifolds of dimensions \( d \) and \( 6-d \) and consider the limit when the size of \( M_d \) goes to zero. The limiting theory is effectively a theory on \( M_{6-d} \) which we can analyze.

For example, it has been known that if we take as \( M_d \) the elliptic curve \( E_\tau = \mathbb{C}/(\mathbb{Z} + \mathbb{Z}\tau) \), we
obtain the maximally supersymmetric four-dimensional gauge theory with the gauge group $G$ (simply-connected compact Lie group associated to $\mathfrak{g}$) and the complexified coupling constant $\tau$. This has far-reaching consequences, such as the celebrated electromagnetic duality of gauge theories which is explained by the fact that $E_{\tau}$ is isomorphic to $E_{-1/\tau}$ (it may be generalized to non-simply connected and non-simply laced Lie groups). The electromagnetic duality plays a paramount role both in quantum physics (possibility to relate strongly coupled and weakly coupled quantum field theories) and in mathematics (connection to the Langlands Program).

The work of Yuji Tachikawa and his collaborators, Luis Alday and Davide Gaiotto, focused on what happens if we consider instead of $E_{\tau}$ an arbitrary Riemann surface $X$. It turns out that it is also possible to introduce certain defects corresponding to a collection of $n$ marked points $(x_i)$ on $X$. For any such $(X, (x_i))$ and any simply-laced simple Lie algebra $\mathfrak{g}$ we then obtain a four-dimensional supersymmetric quantum field theory and these theories are expected to exhibit a plethora of dualities generalizing the electromagnetic duality.

In the simplest case that $\mathfrak{g} = sl_2$, it has been argued that these four-dimensional theories have several Lagrangian descriptions: one for each decomposition of the marked curve $(X, (x_i))$ into pairs of pants. Note that by shrinking the cycles appearing in such a decomposition, we obtain a degenerate curve. The gauge group of this theory turns out to be $SU(2)$ to the power equal to the number of shrinking cycles, and hence the theory has coupling constants corresponding to the lengths of these shrinking cycles.

This is a very powerful statement because it immediately tells us that there should be a non-perturbatively defined four-dimensional quantum field theory parametrized by the entire moduli space of $n$-pointed curves $(X, (x_i))$, which has many different perturbative descriptions at different “corners” corresponding to the degenerate curves.

And there is more. We can study various quantities in these four-dimensional theories and ask whether they may be interpreted in terms of two-dimensional field theories on the Riemann surface $X$. One such quantity that has recently been studied extensively is the “Nekrasov partition function” $Z_{\text{Nek}}$, which is the generating function of the equivariant volumes of the moduli spaces of instantons of this theory (note that Nikita Nekrasov was the recipient of the 2004 Hermann Weyl Prize). This function depends on various parameters, including two parameters $\epsilon_1, \epsilon_2$ corresponding to the action of two circles on two orthogonal planes in $\mathbb{R}^4$.

Since $Z_{\text{Nek}}$ is produced by compactifying a six-dimensional theory on a punctured curve $(X, (x_i))$, we ask how exactly $Z_{\text{Nek}}$ depends on $(X, (x_i))$. A startling conjecture of Alday–Gaiotto–Tachikawa is that $Z_{\text{Nek}}$ is a conformal block in a two-dimensional conformal field theory on $X$, with operator insertions at the punctured points. This theory has been identified with the Liouville theory with the Virasoro central charge $c = 1 + 6(\epsilon_1 + \epsilon_2)^2$.

Thus, Alday, Gaiotto and Tachikawa discovered an unexpected link between four-dimensional supersymmetric gauge theories and two-dimensional conformal field theories. This has already led to an avalanche of papers by both physicists and mathematicians in which the conjecture has been proved in many cases and new conjectures have been formulated.

For example, for compactifications of the six-dimensional theories of other types one expects the Nekrasov partition functions to reproduce conformal blocks of conformal field theories with $W$-algebra or an affine Kac–Moody algebra symmetry. Also, in the limit $\epsilon_1 \to 0$, the leading term of $\log Z_{\text{Nek}}$ is intimately connected to quantum integrable systems, closely related to the quantization of the Hitchin system.

The work of Tachikawa and his collaborators has illuminated an exciting area of research. This work has brought together symmetries of different kinds in quantum field theory: gauge symmetries and conformal symmetries (such as Virasoro and Kac–Moody). It represents an important new step in our quest to understand the mysteries of the physical world using its symmetries.