The high tunability of the density of states of graphene [1] makes it an ideal probe of quantum transport in different regimes. In particular, the supercurrent that can flow through a non-superconducting (N) material connected to two superconducting electrodes, crucially depends on the length of the N relative to the superconducting coherence length. Using graphene as the N material we have investigated the full range of the superconducting proximity effect, from short to long diffusive junctions. By combining several S/graphene/S samples with different contacts and lengths, and measuring their gate-dependent critical currents ($I_c$) and normal state resistance $R_N$, we compare the product $eR_N I_c$ to the relevant energies, the Thouless energy in long junctions and the superconducting gap of the contacts in short junctions, over three orders of magnitude of Thouless energy. The experimental variations strikingly follow a universal law, close to the predictions of the proximity effect both in the long and short junction regime, as well as in the crossover region, thereby revealing the interplay of the different energy scales. Differences in the numerical coefficients reveal the crucial role played by the interfacial barrier between graphene and the superconducting electrodes, which reduces the supercurrent in both short and long junctions. Surprisingly the reduction of supercurrent is independent of the gate voltage and of the nature of the electrodes. A reduced induced gap and Thouless energy are extracted, revealing the role played by the dwell time in the barrier in the short junction, and an effective increased diffusion time in the long junction.

We compare our results to the theoretical predictions of Usadel equations and numerical simulations which better reproduce experiments with imperfect NS interfaces.

PACS numbers:
Indeed, graphene’s carrier density can be controlled by a gate voltage, leading to the possible continuous spanning, in a single sample of given length $L$ and aspect ratio, of both the Fermi wave-vector and the diffusion constant $D$, and thus the Thouless energy $E_{Th} = \hbar D/L^2$ \cite{11} [14].

We track the critical current $I_c$ of seven diffusive graphene samples, using three different superconducting materials, over a wide range of gate voltage. We find that the product $R_N I_c$ of the critical current by the normal state resistance is proportional to an effective Thouless energy that is a fraction of the Thouless energy, for all long junctions investigated. In the short junction limit, we find that the $R_N I_c$ product is independent of the Thouless energy, and that it is smaller than the electrodes superconducting gap $\Delta$. We find that data of all samples collapse on a single curve, that we compare to the theoretical result of the Usadel equations. In addition, we perform numerical simulations of the proximity effect in the experimentally relevant situation of an interface with a partial transmission. The simulations reproduce qualitatively the behavior suggested by the experiments, underscoring the role played by multiple inner reflections of Andreev pairs that increase the dwell time in the N conductor. The critical current of short diffusive SNS junctions ($\Delta/E_{Th} \to 0$) is predicted to obey \cite{6, 7}

$$eR_N I_c \simeq 1.326\pi \Delta/2 \simeq 2.07\Delta.$$ \hspace{1cm} (1)

Whereas the full superconducting gap $\Delta$ is induced in N in short junctions, in long-junctions ($\Delta/E_{Th} \to \infty$) a much smaller ‘mini’ gap $\Delta_g$ is induced in the N. $\Delta_g$ is proportional to the Thouless energy: $\Delta_g \simeq 3.1 E_{Th}$ \cite{28}. The product $eR_N I_c$ at zero temperature is also proportional to $E_{Th}$ \cite{9}:

$$eR_N I_c(T = 0) = 10.82 E_{Th} = 3.2 \Delta_g.$$ \hspace{1cm} (2)

Expressions (1) and (2) show that it is the smallest of the two energies, $\Delta$ and $E_{Th}$, that limits the critical current in diffusive SNS junctions. The crossover between the short and long junction regimes was also investigated using Usadel equations \cite{9}, and it is found that throughout the full proximity effect range, only the diffusive constant, sample length and superconducting gap determine the critical current, regardless of sample geometry. This universality is unique to the diffusive regime: in ballistic SNS junctions, the critical current is expected to depend on the detailed geometry of the samples \cite{24}. Expression (2) was found to reproduce quite well experiments on long metallic SNS junctions \cite{30} with a good transmission at the SN interface. It was however shown \cite{8, 10, 31} that (1) and (2) are modified by interfacial barriers. The barriers are characterized by an energy scale $\gamma = \hbar/\tau_D$, with $\tau_D$ the typical time associated with the barrier transmission. The barrier can be of various types: tunnel, Schottky or due to disorder at the NS interface, a higher barrier corresponding to a longer dwell time and shorter $\gamma$. When $\gamma$ is smaller than the superconducting gap, $\epsilon R_N I_c$ for short junctions is limited by $\gamma$, and independent of $\Delta$ \cite{10}. The situation is more complex in long junctions with an interfacial barrier such that $\gamma > E_{Th}$, and was less investigated theoretically. The interfacial barrier is often modeled by a simple resistance $R_c \sim h/e^2 M r$ due to M conduction channels of identical transmission $\tau < 1$, and characterized by the ratio $r = R_c/(R_N - 2R_c)$, where $R_N$ is the total normal state resistance, i.e. that of the conductor and barrier resistances in series. In the high $r$ limit, it was found that in short junctions the induced gap $\Delta^*$ (defined as $eR_N I_c/2.07$) is reduced according to $\Delta^* = \Delta/r$. In long junctions \cite{24} is also predicted to be modified with a reduction of the minigap and critical current that essentially depends on $r$ and practically not on $\tau$ \cite{31}.

In the following we present our experimental results on S/graphene/S junctions differing by their superconducting electrodes, length and mobility. Varying the doping changes $r$ substantially, revealing a striking universal behavior, and offering a stringent test of theoretical predictions.

All the samples reported in this paper were prepared by mechanical exfoliation onto oxidized substrates of highly doped Si. Sample parameters are given in table I. The Ti/AI contacts are e-beam evaporated and the Pd/Nb and Pd/ReW contacts are dc-sputtered. The sample length $L$ varies from 300 nm to 1.2 $\mu$m and the ratio $\xi_s/L$, where $\xi_s = \sqrt{\hbar D/\Delta}$ is the superconducting coherence length, varies from 10 to 0.3, so that the full range from short to long junction is accessed for the first time.

The gate-voltage-dependence of the normal state resistance $R_N = 2R_c + R_G$ yields both the contact resistance $2R_c$ and the intrinsic graphene resistance $R_G$. Within a good approximation $R_G$ is found to vary like $1/|V_g - V_D|$ \cite{11} at high gate voltage $V_g$ relative to the Dirac point $V_D$. $R_c$ is found to be independent of $V_g$, and is obtained by the linear extrapolation of $R_N = f(x = 1/|V_g - V_D|)$ close to $x = 0$. We can then determine the conductivity $\sigma = \rho^{-1} = (R_G W/L)^{-1} = (2e^2/h)(k_F l_c)$ and deduce the mean free path $l_c$, the diffusion coefficient $D = 1/2\nu l_c$, and the Thouless energy. The Fermi wave-vector $k_F$ is deduced from a simple capacitance model, valid away from the Dirac point \cite{27}. The elastic mean free path $l_c$ varies with $V_g$ from 50 nm to 160 nm. Our samples are thus always in the diffusive regime. The contact resistance $R_c$, between tens and hundreds of Ohms (see table), corresponds to a rather uniform product of contact resistance by sample width, of the order of 250 $\Omega \pm 5 \mu$m. $R_c$ is thus negligible at low doping, but can be of the order of, or even larger than, the intrinsic resistance of graphene at high doping. This is an ideal parameter range to test the dependence of the proximity effect with. Using the expression for the conduction channels $M = k_F W/\pi$, that yields roughly 80 channels for a micron-wide sample at $V_g - V_D = \pm 30$ V, one can...
then deduce the average transmission $\tau$ of the contacts via $R_c = \left(\frac{h}{4e^2}\right)M^{-1}(1/2 + (1 - \tau)/\tau)$ \cite{35}, and we find $\tau = 0.25 \pm 0.1$.

The differential resistance of the samples was measured at 100 mK via filtered lines, using a standard lock-in technique. Fig. 1 displays the colour-coded differential resistance as a function of the bias current and gate voltage, showing a gate-dependent critical current of the SGS junction Al1. The critical current is strongest at high doping, and depressed at gate voltages close to the Dirac point. Peaks in the differential resistance at $V_n = 2\Delta/ne$ are manifestations of the multiple Andreev reflections (MAR), typical of SNS junctions \cite{35} and enable the determination of $\Delta$.

All samples show qualitatively similar behaviors, with quantitative differences: in the long junction samples (Nb, ReW), the critical current is not just depressed, but is actually destroyed near the Dirac point. We attribute this striking suppression to the charge puddles in the sample near half filling, and the specular Andreev reflections across their boundaries, that randomize the phase of Andreev pairs \cite{24}. In the following we focus on data sufficiently far from the Dirac point (Table I) so that the critical current is higher than 100 nA. This ensures that thermal fluctuations have a negligible influence, since the corresponding Josephson energy $E_J = \Phi_0 I_c/2\pi$ is above 3 K, more than ten times the sample temperature\cite{34}. We show in the following that all samples exhibit a universal behavior.

To follow and compare the critical current of all samples, we plot the experimentally determined $eR_N I_c/\Delta$ as a function of $x = E_{Th}/\Delta$ (Fig. 2), along with the numerical solution $F(x)$ of the Usadel equations for perfect interfaces \cite{9}. We find that all experimental data nearly collapses on a single curve, $eR_N I_c/\Delta = F(x)$, with two asymptotic behaviors that clearly correspond to the long and short junction limits. This universal behavior of all the graphene-based SNS junctions we have investigated is the central result of our paper. In the short junction limit, $\lim_{x \to \infty} F(x) = a$ with $a = 0.55$, and in the long junction limit $\lim_{x \to 0} F(x) = bx$ with $b \simeq 0.39$. This behavior is qualitatively similar to the result of Usadel equations, although the Usadel equation coefficients are different: $a_U \simeq 2.07$ and $b_U = 10.82$ \cite{9}. This comparison with Usadel equations leads us to define effective energies $\Delta^* = (a/a_U)\Delta \simeq 0.3\Delta$ and $E_{Th}^* = (\Delta/\Delta^*)(b/b_U)E_{Th} \simeq 0.14 E_{Th}$ such that $eR_N I_c/\Delta^* \rightarrow a_U$ in the short junction limit and $eR_N I_c/\Delta^* \rightarrow b_U E_{Th}/\Delta$ in the long junction limit. The full dependence, including the crossover between short and long junctions, can be fitted by a generic expression:

$$\frac{eR_N I_c}{\Delta} = F(x) = \frac{abx}{(a^n + b^n x^n)^{1/n}}. \quad (3)$$

FIG. 1: Top: Color-coded plot of $dV/dI$ as a function of gate voltage and dc current. Black corresponds to zero resistance. Bottom left: Differential resistance $dV/dI(I_{DC})$ at three different gate voltages, including at $V_G=0\text{V}$, close to the Dirac point (red curve). The resistance jumps from zero to the normal state resistance at the critical current $I_c$.

FIG. 2: Variations of $R_N I_c/\Delta$ with $x = E_{Th}/\Delta$ for seven diffusive SGS junctions, of different lengths and with different superconducting electrodes. The superconductor used as a contact is indicated in the legend. For each sample, a continuous range of Thouless energy is accessed by varying the gate voltage. Three orders of magnitude of Thouless energy over Delta are accessed. All data practically collapse on a single curve $F(x)$ (red continuous line), see expression (3), whose shape describes both the short and long junction limits, as well as the crossover between the two regimes. This shape is similar to the theoretical curve for a perfect interface, computed from the Usadel equations by Dubos et al.\cite{9} (dashed black curve).
TABLE I: Characteristic parameters of the investigated samples.
The large contact resistances measured for the Al4 and Al5 samples are not intrinsic to the sample but due to silver paste connection problems.
| Sample | Short/intermediate junction (Ti/Al(6nm/70nm)) | long junction (Pd/Nb vs. Pd/ReW(8/70 nm)) |
|---------|----------------------------------|----------------------------------|
| Sample  | L (nm)   | W (µm) | Ic (nm) | ξc (nm) | R0 (Ω) | Vg (V) range | \(Ic(0)\) \(Ic(T)\) |
| A1      | 350      | 4      | 120     | 400     | 500    | -35 / +16.5 | -5 / +15 |
| A2      | 400      | 4      | 140     | 500     | 860    | -35 / +16.5 | -5 / +15 |
| A3      | 350      | 4      | 150     | 430     | 862    | -35 / +16.5 | -5 / +15 |
| A4      | 450      | 4      | 120     | 420     | 853    | -35 / +16.5 | -5 / +15 |
| A5      | 500      | 4      | 170     | 520     | 40     | -35 / +16.5 | -5 / +15 |

Fig. 2 shows that the Usadel results are very well fitted by \(n \approx 1\), and the experiments, with a sharper crossover, by \(n \approx 2\). Fig. 2 also shows that an imperfect interface does not change the main features of the proximity effect: in short junctions \(R_N I_c\) is independent of \(E_{Th}\), and in long junctions \(R_N I_c\) varies linearly with \(E_{Th}\). According to refs. 8, 10, the reduced effective gap \(\Delta^*\) should just be \(\gamma\), the inverse characteristic transmission time through the NS barrier in the limit where \(\Delta \gg \gamma \gg E_{Th}\). It is interesting that \(\Delta^*\) we find is sample independent, for the three samples with Ti/Al contacts for which the crossover between long and short junction is accessed. One would have liked to test samples with different superconducting gaps in this short junction regime to determine whether \(\Delta^* = \gamma\) is independent of \(\Delta\). This was not possible for the Pd/ReW and Pd/Nb contacts, whose very small superconducting coherence length would require sub 30-nm-size junctions to reach the short junction limit.

The physical meaning of the crossover between short and long junctions in the presence of barriers can be heuristically understood writing that \(eR_N I_c = h/\tau_{dw}\), where \(\tau_{dw}\) stands for the typical traversal time of the SNS junction, which is the sum of \(\tau\) the time spent in the barriers, and \(\tau_D\), the diffusion time through the normal junction, yielding \(eR_N I_c = \gamma E_{Th}/(E_{Th} + \gamma)\). This expression reproduces quite well the solution of the Usadel equations 9, with \(\gamma\) instead of \(\Delta\), and corresponds to eq. 2 with \(n = 1\). Its dependence is similar to the experimental curve, although the theoretical crossover is smoother than the experimental curve, that is better described by \(n = 2\).

We now turn to the long junction regime, in which the critical current varies linearly with \(E_{Th}\), but is smaller than the theoretical prediction for perfect interface by a factor \(b_{Th}/b \approx 33 = 3E_{Th}/E_{TTh}\). In 22 we show that this reduced Thouless energy \(E_{Th}\) also determines the temperature dependence of \(I_c\).

Modifications of \(I_c(0)\) and \(I_c(T)\) due to imperfect interfaces were investigated by Hammer et al. 31 using the Usadel equations formalism. They predict that the renormalised critical current and its variations with temperature depend not only on \(E_{Th}/\Delta\) but also on \(r = R_c/(R_N - R_c)\), with a drastic reduction of \(R_N I_c\) at high \(r\). Since \(r\) varies in graphene by a factor 50 (\(r \approx 0.1\) close to \(V_g = V_D\), and \(r \approx 5\) around \(V_g \approx 30 V\)), one would expect \(E_{Th}/E_{TTh}\) to vary with doping, in strong contrast to the universal behavior suggested from our data. The same calculation also predicts a critical current that is only barely reduced so long as the interface resistance is small relative to the normal conductor’s resistance (\(r < 1\)). For \(r = R_c/(R_N - R_c) \approx 0.1\) for instance, as in our experiments at low doping, the prediction would be \(b_{Th}/b \approx 1\) (using our notations). This is in stark contrast with our experimental finding of \(b_{Th}/b \approx 0.03\). Such a high reduction due to a relatively small interface resistance was already reported by Dubos et al. 9 in a metal SNS junction.

A possible interpretation of a strongly reduced effective Thouless energy \(E_{Th} \ll E_{TTh}\) could be the repeated inner reflections of Andreev pairs at the interfacial barriers, leading to an increased typical time spent in the SNS junction from \(\tau_D / N_D\), where \(N\) is the number of reflections at the NS interfaces.

We now present numerical simulations, in which we do find a strongly (ten-fold) reduced critical current, even for \(r \ll 1\), i.e. a graphene sheet whose intrinsic resistance is much higher than the interface resistance. We implement the Bogoliubov-de Gennes Hamiltonian that describes the electron- and hole-like wavefunction components of a hybrid NS ring in a tight-binding 2D Anderson model 37. The graphene sheet is a hexagonal lattice oriented along the armchair direction with \(N_x \times N_y\) sites, and is connected to two superconducting electrodes \((N^S = N_x^S \times N_y^S)\) sites on a square lattice), see inset of Fig. 3. Disorder is described by random on-site energies of variance \(W^2\). The hopping matrix element is restricted to nearest neighbors \(t_{ij} = t\). The SN interface barrier is taken into account via a reduced hopping amplitude between the N and S sites: \(|t_{SN}/t|^2 = \tau\), with
Josephson junctions in the diffusive regime show a remarkable universal behavior, with a crossover between long and short junctions regimes. The full dependence of $R_N I_c$ vs $E_{Th}/\Delta$ can be described by the result of the Usadel theory with perfect interfaces, provided a sample-independent rescaling of the superconducting gap and Thouless energy down to lower energies is performed. We understand this reduction as due to the barriers at the NS interface, whose transmission is estimated to be of the order of 0.25. We find that the predictions of Usadel equations in the long junction limit with opaque interfaces do not agree with the universal behavior we observe. A better agreement is obtained with numerical computation of the Andreev spectrum using a tight binding model of graphene. These results call for a better theoretical understanding of the influence of barriers at the N/S interface on the transmission of Andreev pairs through long SNS junctions.

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