Solitonics effects in supercontinuum generation in highly nonlinear fibers

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Abstract. When ultrashort pulses with large enough power are launched into highly nonlinear fibers, soliton fission gives origin to multiple fundamental solitons of different widths and peak powers. Two soliton related effects become then particularly important in the context of supercontinuum generation: the soliton self-frequency shift and the emission of dispersive radiation in the normal dispersion region. The dispersive characteristics of the optical fiber assume play a key role in these circumstances.

1. Introduction

Supercontinuum generation (SCG) occurs when narrow-band incident pulses undergo extreme nonlinear spectral broadening to yield a broadband spectrally continuous output. It results generally from the synergy between several fundamental nonlinear processes, such as self-phase modulation, cross-phase modulation, stimulated Raman scattering, and four-wave mixing [1]. Soliton related effects also play a significant role whenever light with sufficient power propagates in the anomalous dispersion regime [2].

The first observation of supercontinuum was realized in 1970 by Alfano and Shapiro in bulk borosilicate glass [3]. SCG in fibers occurred for the first time in a 1976 experiment, when 10 ns pulses with more than 1-kW peak power were launched in a 20-m long fiber, producing a 180-nm-wide spectrum [4]. Several types of highly nonlinear fibers, namely microstructured optical fibers (MOFs) with optimized designs, have been developed during the recent years in order to enhance the supercontinuum generation [2,5].

Pumping with ultrashort pulses in the anomalous dispersion region of the fiber, the soliton fission process gives origin to multiple fundamental solitons of different widths and peak powers [6]. Among the host of soliton related effects, there are two which become then particularly important: the soliton self-frequency shift [7] induced by intrapulse Raman scattering (IRS) and the emission of dispersive (or “Cherenkov”) radiation, which comes from the overlap of a soliton spectrum with the normal dispersion range [8]. The peculiar dispersive characteristics of highly nonlinear fibers play a key role in these circumstances.
2. Modelling of the supercontinuum generation

Modelling the supercontinuum generation can be realized considering a generalized nonlinear Schrödinger equation that includes higher-order dispersion effects, as well as IRS. Such equation can be written as [1,2,9]:

\[
\frac{\partial U}{\partial z} - i \sum_{k \geq 2} \frac{\beta_k}{k!} \frac{\partial^k U}{\partial t^k} + \frac{\alpha(\omega)}{2} U = i\gamma \left( 1 + \frac{i}{\omega_0} \frac{\partial}{\partial t} \right) \int_{-\infty}^{t} R(t') \left[ U(z, t-t') \right]^2 dt' \]

where \( U(z, t) \) is the electric field envelope, \( \omega_0 \) is the center frequency, \( \alpha(\omega) \) is the frequency-dependent fiber loss, \( \gamma \) is the nonlinear parameter, and \( \beta_k = \left( \frac{d^k \beta}{d\omega^k} \right) \) (\( k = 0, 1, 2 \ldots \) are the dispersion coefficients at the center frequency, \( \beta(\omega) \) being the mode-propagation constant. The group-velocity dispersion (GVD) is characterized by the parameter \( \beta_2 \). The nonlinear response function \( R(t) \) in Eq. (1) can be written as \( R(t) = (1 - f_R) \delta(t) + f_R h(t) \), where the \( \delta \)-function represents the instantaneous electron response (responsible for the Kerr effect), \( h(t) \) represents the delayed ionic response (responsible for the Raman scattering) and \( f_R \) is the fractional contribution of the delayed Raman response to the nonlinear polarization, in which a value \( f_R = 0.18 \) is often assumed. It is common to approximate \( h(t) \) in the form [9]:

\[
h(t) = \frac{\tau_1^2 + \tau_2^2}{\tau_1 \tau_2} \exp(-t/\tau_2) \sin(t/\tau_1)
\]

where \( \tau_1 = 12.2 \) fs and \( \tau_2 = 32 \) fs.

3. Solitonic effects

Eq. (1) can be used to describe the propagation of femtosecond pulses in optical fibers, in both the normal and anomalous dispersion regimes. The soliton order is given by \( N = \sqrt{L_D / L_{NL}} \), where \( L_D = T_0^2 \beta_2 \) is the dispersion distance and \( L_{NL} = 1/\gamma P_0 \) is the nonlinear length. In the presence of higher-order dispersion, an Nth-order soliton gives origin to \( N \) fundamental solitons whose widths and peak powers are given by \( T_k = T_0 / (2N + 1 - 2k) \) and \( P_k = P_0 (2n + 1 - 2k)^2 / N^2 \), respectively, for \( k = 1, \ldots, \bar{N} \), where \( \bar{N} \) is the integer closest to \( N \) when \( N \) is not an integer. Soliton fission occurs generally after a propagation distance \( L_{fiss} \sim L_D / N \), at which the injected higher-order soliton attains its maximum bandwidth.

The fission phenomenon can be observed in Fig. 1, which shows the temporal and spectral evolution of an optical pulse along a highly nonlinear MOF with hole diameter \( d = 1.4 \) μm and pitch \( \Lambda = 1.6 \) μm. In this case, the dispersion curve has only one zero dispersion wavelength, located at 780 nm. We consider a pump in the anomalous dispersion region at 835 nm, where the nonlinear parameter is \( \gamma = 11 \text{W}^{-1}\text{km}^{-1} \). An input pulse \( U(0, \tau) = \sqrt{P_0} \text{sech}(\tau/T_0) \) is assumed, where \( P_0 = 10 \text{kW} \) and \( T_0 = 28.4 \) fs, which corresponds to an intensity full width at half maximum (FWHM) of 50 fs. For the assumed parameters, we have \( N = 8.5 \) and \( L_{fiss} \sim 2.3 \) cm.
Figure 1 – (a) Spectral and (b) temporal evolution of an optical pulse along a MOF obtained from Eq. (1). The input sech pulse at 835 nm has a peak power of 10 kW and 50 fs FWHM.

Besides higher-order dispersion, another main effect affecting the dynamics of a higher-order soliton fission is the intrapulse Raman scattering. This phenomenon leads to a continuous red-shift of the soliton carrier frequency, an effect known as the soliton self-frequency shift (SSFS) [7]. The rate of frequency shift of a sech pulse propagating as a fundamental soliton is given by [1]

$$\frac{df}{dz} = -\frac{4t_R \left| \beta_2 \right|}{15\pi T_s^4} = -\frac{4t_R \left( P_s \right)^2}{15\pi \left| \beta_2 \right|}$$  \hfill (3)

where \( t_R \equiv f_R \int_{-\infty}^{+\infty} th(t)dt \approx 5 \text{ fs} \) is the Raman parameter, whereas \( T_s \) and \( P_s \) are the soliton width and peak power, respectively. Since the SSFS effect is proportional to \( \left( P_s \right)^2 \), it will be enhanced if short pulses with high peak pulses are propagated in highly nonlinear fibers.

The red-shift of the solitons created by the fission process is clearly seen in Fig. 1. As a consequence of the SSFS, such solitons separate from each other. Since the SSFS is the largest for the shortest soliton, its spectrum shifts the most toward the red side. The change of the soliton’s frequency determines a reduction in the soliton’s speed because of dispersion. This deceleration appears as a bending of the soliton trajectory in the time domain.

Besides the fission phenomenon and the resulting Raman-induced spectral shifts of constituent fundamental solitons, Fig. 1 shows also the appearance of spectral components on the short wavelength side of the pulse spectrum. Such spectral components correspond to nonisolonic radiation (NSR), also called Cherenkov radiation [8], which is emitted by the solitons resulting from the fission process in the presence of higher-order dispersion. NSR is emitted at a frequency \( \omega \) such that its phase velocity matches that of the soliton. This situation occurs when the phase-matching condition \( \beta(\omega) = \beta(\omega_s) + \beta(\omega - \omega_s) + \gamma P_s / 2 \) is satisfied, where \( P_s \) and \( \omega_s \) are the peak power and the frequency, respectively, of the fundamental soliton formed after the fission process. Expanding \( \beta(\omega) \) around \( \omega_s \), this phase-matching condition reduces to
\[ \sum_{n \geq 2} \frac{\Omega^n}{n!} \beta_n (\omega_n) = \frac{\gamma P_2}{2} \] (4)

where \( \Omega = \omega - \omega_s \) is the frequency shift between the dispersive wave and the soliton. Neglecting the fourth- and higher-order dispersions, this frequency shift is given approximately by [8]:

\[ \Omega \approx -\frac{3\beta_2}{\beta_3} + \frac{\gamma P_0 \beta_3}{3\beta_2^2} \] (5)

For solitons propagating in the anomalous-GVD regime we have \( \beta_2 < 0 \). In these circumstances, Eq. (5) shows that \( \Omega > 0 \) when \( \beta_3 > 0 \), in which case the NSR is emitted at wavelengths shorter than that of the soliton, as observed in Fig. 1. On the contrary, if \( \beta_3 < 0 \) the NSR is emitted at a longer wavelength than that of the soliton and it can result in the suppression of the SSFS effect [1].

In the special case that the third-order dispersion vanishes and the soliton is perturbed by the fourth-order alone, Eq. (4) gives the result:

\[ \Omega_\pm \approx \pm \sqrt{-12\beta_2 / \beta_4} \] (6)

where the last term on the right-hand side of Eq. (4) has been neglected. In these circumstances, since solitons propagate in the anomalous-GVD regime (\( \beta_2 < 0 \)) both solutions in Eq. (6) become imaginary and no dispersive waves are produced if \( \beta_4 < 0 \). However, if \( \beta_4 > 0 \), both solutions are real and have opposite signs, indicating that NSR occurs on both sides of the soliton frequency.

4. Conclusions
In this paper we discussed the main solitonic effects contributing to the supercontinuum generation process in highly nonlinear fibers. Such effects become important whenever light propagates in the anomalous dispersion regime. Soliton fission occurs if ultrashort pulses with large enough power are launched into highly nonlinear fibers. Multiple fundamental solitons of different widths and peak powers are generated in this case. Among the host of soliton related effects, there are two which become particularly important: the soliton self-frequency shift and the emission of dispersive radiation in the normal dispersion region. For typical dispersion conditions, solitons dominate the long-wavelength edge of the supercontinuum, while the dispersive radiation determines the short wavelength edge.

5. References
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