Simplified Method for Analysis of Tall Buildings in an Earthquake

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Received September 14, 2019; Revised October 19, 2019; Accepted November 05, 2019

Abstract
As for the earthquake design and assessment of the high rise structures, dynamic analysis is always required. However, dynamic analyses can be computationally expensive and require expert judgement. This research aims to introduce simplified methods to analyse the displacement and inter-storey drift of multi-storey buildings in regions of low to moderate seismicity. The generalised force method (GFM) is an approach that has been proved to be suitable for low rise buildings. GFM was first introduced by Lam, Lumantarna and Wilson to analyse the seismic effects of low-medium rise buildings. It can give accurate results of estimates of the deflection and internal forces. In the analysis of low rise building, GFM is based on the fundamental mode of vibration and the higher mode effects are not taken into consideration. However, in this research project, the existing GFM method is extended to be used for high rise building, which means that the higher vibration mode effects (including nth vibration modes) of the building is considered. In addition, the torsional effects of high rise buildings are also taken into consideration. This research will verify the accuracy of GFM by making comparison to the results from ETABS simulation. Four different tall buildings with varied heights ranging from 100m to 200m are analysed by both GFM and ETABS. And the results show that the displacement and inter-storey drift estimated by GFM are accurate and reliable.

Keywords: earthquake, generalised force method, ETABS, seismic response, high rise buildings, torsional effects

Cite This Article: Yifei Xiao, Huaying Li, Jinzhao Chen, Tuo Zhou, and Elisa Lumantarna, “Simplified Method for Analysis of Tall Buildings in an Earthquake.” American Journal of Civil Engineering and Architecture, vol. 7, no. 5 (2019): 190-201. doi: 10.12691/ajcea-7-5-1.

1. Introduction

Nowadays, more and more multi-storey buildings arise in Australia and over the whole world. Most of them were built of reinforced concrete. Meanwhile, from structural perspective, these tall buildings utilise shear walls or core walls to resist horizontal loads such as wind load and earthquake load. In this research project, a new method for earthquake action analysis (Generalised Force Method) will be introduced and validated.

In past decades, a large number of researches have been done to investigate the seismic response of multi-storey buildings, such as the effect of pile size on seismic response of buildings [1], the relationship between new proposed ground motions intensity measure and structural responses [2], the equivalent static analysis [3,4] and linear modal time history analysis [5,6] on seismic response of building, etc. According to recent research [7,8,9,10], those simplified methods for calculating the seismic response of buildings based on the fundamental mode (the first mode) are suitable for low rise building but could not give accurate and reliable results of displacement, inertial forces and the seismic floor accelerations of high rise building. Therefore, higher mode effects need to be considered when analysing tall buildings.

Generalised Force Method (GFM), an enhanced version of analysis method, is aimed to analyse the performance and seismic response behaviour of buildings in regions of low to moderate seismicity [11]. It can give accurate results of estimates of the lateral displacement and storey shear. This method has been proved to be suitable for low rise buildings [11] and expected to be used for tall buildings. For low rise building, only the first mode is needed to be considered but it can give accurate results of seismic response.

Since GFM is based on the linear behaviour of building, a number of linear elastic structural analysis methods have been studied. The advantages and advantages of these methods are listed in the Table 1.

As can be seen from the Table 1, all these analyses and methods focus on providing a prediction of the seismic response of the building. Some of them are easily conducted, but the results are not accurate, especially when the building is tall and complex. Some are accurate based on the earthquake design standards, but the procedure is complex and needs expert judgement, such as linear dynamic analysis. However, GFM, which considers higher mode effects and response spectrum analysis, is relatively easy to conduct and can give reasonably accurate results of seismic responses of buildings [11].
Table 1. Advantages and limitations of different analysis methods

| Method                        | Advantages                                                                 | limitations                                                                 |
|-------------------------------|-----------------------------------------------------------------------------|-----------------------------------------------------------------------------|
| Equivalent static analysis    | • Seismic loads were modified by various factors [4]                        | • Without considering the higher mode effect [3]                            |
|                               | • Applicable for low rise structure [1,3]                                   | • Without a significant contribution from lateral torsion modes [4]         |
| Response spectrum analysis     | • Multiple modes of vibration are considered [13,14]                        | • Square Root of Sum of Squares (SRSS) is not always applicable [17]        |
|                               | • Give relatively accurate analysis results [15]                           | • Complete Quadratic Combination (CQC) is only applied when the modal frequencies are close to each other [18] |
|                               | • Equivalent static analysis would give more conservative results compared to the response spectrum analysis [16] |                                                                             |
| Uniform hazard spectrum in modal analysis | • Provide a composite of maximum responses at different natural periods for given damping [19] | • The results are still very conservative [19]                           |
| Linear dynamic analysis       | • Applicable for complex buildings [20]                                    | • Calculation is relatively complex [13]                                    |
|                               | • Both time history analysis and modal spectral analysis can be used [14,21]|                                                                             |
|                               | • Higher mode effects are considered, thus results are accurate [13]       |                                                                             |
| Linear Modal Time History Analysis | • Accurate, simple and fast [5,6]                                         | • Higher mode effects are not considered [22]                              |
|                               | • Provide the largest seismic response of the buildings [6]                |                                                                             |
| Linear Direct Integration Time History Analysis | • Have more accurate results compared to the linear modal time history analysis [6] | • Modal superposition is not taken into consideration [22]               |
|                               | • Sets up full equations of motion [22]                                    |                                                                             |
| GFM                           | • Consider higher mode effects [11,23,24,25,26]                            | • Based on the linear elastic stage of the structure, not consider nonlinear stage [11,23,24,25,26] |
|                               | • Apply response spectrum in the analysis [11,23,24,25,26]                 |                                                                             |
|                               | • Easy to conduct and give accurate results for low rise buildings [11,23,24,25,26] |                                                                             |

In this research project, the major objective is to validate the GFM through result comparison between GFM and dynamic modal (computer simulation). To obtain precise results, four buildings with different heights ranging from 100m to 200m will be chosen to ensure the GFM can be extensively utilised. GFM will be utilised to determine the displacement and inter-storey drift of buildings when torsional effects are not taken into consideration, while stiff and flexible edge displacement will be evaluated when torsional effects are considered. Meanwhile, ETABS will be used to perform dynamic analysis and to obtain the seismic performance of these buildings under earthquakes. Results estimation and comparison between the GFM and dynamic analysis through ETABS will determine the feasibility of GFM in seismic analysis and design.

2. Building Selection and Modelling

According to the calculation procedures of GFM, which shown in Eq. (1) of AS 1170.4 [27], the height of building affects the result of earthquake base shear by affecting the fundamental natural period of building.

\[ T_1 = 1.25k_f T_{n,0.75} \]  

In addition, the height of building can have significant impacts on the contribution to the response of higher modes of vibration [28,29]. Generally, for the low rise building, only the first mode needs to be considered. However, as the height of the building increases, the higher modes effects must be taken into consideration. Otherwise, the analysis results of seismic response might be not accurate enough.

In order to explore whether GFM is applicable for high rise buildings, four buildings (Figure 1 to Figure 4) with varied heights ranging from 100m to 200m are analysed. The detailed information of each building is shown in Table 2.

Table 2. Building information

|          | 35-storey building (105m) | 39-storey building (132.6m) | 41-storey building (143.5m) | 46-storey building (193.2m) |
|----------|---------------------------|----------------------------|---------------------------|---------------------------|
| Beam size (mm) | 600×900 & 300×300 | 700×900 & 300×300 | 560×720 & 300×300 | 700×900 & 300×300 |
| Perimeter beam size (mm) | 700×1300 | 700×1300 | 700×1300 | 700×1300 |
| Column size (mm) | 1100×1100 | 1200×1250 & 1000×1200 | diameter 1400 & 1300 | 1200×1500 & 1400×1400 |
| Slab thickness (mm) | 250 | 300 | 250 | 300 |
| Stair thickness (mm) | 100 | 200 | 200 | 200 |
| Core wall thickness (mm) | 400 | 400 | 450 | 250, 400 & 600 |
| Storey height (m) | 3 | 3.4 | 3.5 | 4.2 |
| Material of beams, slabs, stairs & columns | Concrete (f’c=30 Mpa) | Concrete (f’c=30 Mpa) | Concrete (f’c=30 Mpa) | Concrete (f’c=30 Mpa) |
| Material of core walls | Concrete (f’c=40 Mpa) | Concrete (f’c=40 Mpa) | Concrete (f’c=40 Mpa) | Concrete (f’c=40 Mpa) |
Figure 1. Structure plan view of 105m building

Figure 2. Structure plan view of 132.6m building

Figure 3. Structure plan view of 143.5m building
3. Analysis and Results without Torsional Effects

GFM is used to determine the displacement and inter-storey drift of each building. In this paper, the existing GFM method is extended to be able to consider \( n \) vibration modes, but only the first three modes (\( j = 1, 2 \) and 3) effects will be analysed for the purpose of simpler calculation.

Generally, ETABS [30] is the appropriate software which can be used to generate dynamic analysis results based on the simulation. The selection of software is based on what types of buildings and what kind of parameters are going to be analysed.

Building height, structural plan and core layout are the main parameters in this research, which can be simulated well in ETABS. Also, in order to verify whether the results from GFM is accurate or not, the first three modes effects (\( j = 1, 2 \) and 3) will be applied on the ETABS model to get the seismic response.

For each parameter mentioned above, the results from GFM and ETABS (displacement and inter-storey drift) will be assessed and compared. These comparisons will be utilised to validate whether GFM is accurate, as well as assess which parameter has a potential impact on the calculation results of GFM.

3.1 Analysis of GFM

According to the previous work done by Lumantarna [25,26], this method can be described in detail as follows:

Firstly, the seismic response for first mode effect should be identified. According to the section 6.2.1 in Australian standard AS 1170.4: 2007, the earthquake base shear needs to be calculated by using Eq. 2:

\[
V = k_p Z C_h T_1 S_p / \mu W_t
\]  

(2)

Where \( T_1 \) is the fundamental natural period of building which can be determined by using Eq. (1). Then, the calculated base shear can be distributed along the height of building to get the seismic inertia force at each level \( (F_i) \) which can be calculated using Eq. (3):

\[
F_i = \frac{W_i h_i^k}{\sum_{i=1}^{n} (W_i h_i^k)} V
\]  

(3)

Where \( W_i \) is the seismic weight of the building at each floor, \( h_i \) is the height of level \( i \) and \( n \) is the total number of floors in building. \( k \) can be taken as 1 when \( T_1 \leq 0.5 \) sec, 2 when \( T_1 \geq 2.5 \). When \( T_1 \) is between 1 and 2, \( k \) can be calculated by using linear interpolation method.
Based on the value of \( F_i \), next step is to calculate 1st mode displacement value at each floor \( (\delta_{i,1}) \) by using equivalent static analysis method (Figure 5).

After determining the deflection of each floor, the MDOF system of a high rise building can be simplified into a SDOF system by calculating the effective mass \( (m_{eff}) \) and effective displacement \( (\delta_{eff}) \) of the building (Eqs. (4) – (5)).

\[
m_{eff} = \frac{\sum m_i \delta_{i,1}^2}{\sum m_i \delta_{i,1}^2} \quad (4)
\]
\[
\delta_{eff} = \frac{\sum m_i \delta_{i,1}^2}{\sum m_i \delta_{i,1}} \quad (5)
\]

Where \( m_i \) is the mass of each floor.

Then, based on the calculated effective mass \( (m_{eff}) \) and effective displacement \( (\delta_{eff}) \), the effective stiffness \( (k_{eff}) \) can be figured out using Eq. (6).

\[
k_{eff} = \frac{V}{\delta_{eff}} \quad (6)
\]

Following this, the effective natural period \( (T_{eff}) \) and the effective acceleration \( (a_{eff}) \) can then be calculated using Eqs. (7) – (8):

\[
T_{eff} = 2\pi \sqrt{\frac{m_{eff}}{k_{eff}}} \quad (7)
\]
\[
a_{eff} = \frac{V}{m_{eff}} \quad (8)
\]

After calculating the effective acceleration \( (a_{eff}) \) and the effective displacement \( (\delta_{eff}) \), the capacity diagram in terms of acceleration and displacement can be drawn as the format shown in the Figure 6:

The demand diagram of the structure sets up criteria to determine whether the velocity, displacement and acceleration of the building meet the basic requirements [23]. The approach of how to construct demand diagram is also illustrated by Lam & Wilson [23]. Figure 7 shows a demand diagram in terms of acceleration and displacement response spectrum (RSA and RSD). It identifies the displacement and acceleration demand of a building.

Then, in order to determine the performance point which indicates the demand of displacement and acceleration of the buildings, the capacity diagram should be superposed onto the demand diagram to get the displacement demand \( (\delta^*_{i,1}) \) and acceleration demand \( (a^*_{i,1}) \) which is shown in the Figure 8:

\[
\delta^*_{i,1} = \delta_{eff} \frac{\delta_{i,1}}{\delta_{eff}} \quad (9)
\]

Where \( \delta_{i,1} \) is the 1st mode seismic displacement of floor \( i \).

Then, the 1st inter-storey drift \( (\theta_{i,1}) \) can be calculated by using the Eq. (10):

\[
\theta_{i,1} = \frac{\delta_{i,1} - \delta_{i-1,1}}{h_i} \quad (10)
\]

After determining basic parameters for the first mode effect, next step is to add higher modes effects to the seismic response, which includes participation factor \( (\Gamma_j) \) and modal deflection shape factor \( (\phi_{i,j}) \). The values of \( \Gamma_j \times \phi_{i,j} \) are obtained from the previous work [25,26] and converted into the values corresponding to our 4 specific building model. So the displacement response \( (\delta^*_{i,j}) \) and inter-storey drift \( (\theta_{i,j}) \) for \( j^th \) mode effect can be calculated using Eqs. (11) – (12):

\[
\delta^*_{i,j} = \Gamma_j \cdot \phi_{i,j} \cdot RSD(T_j) \quad (11)
\]
\[
\theta_{i,j} = \frac{(\Gamma_j \cdot (\phi_{i,j} - \phi_{i-1,j}) \cdot RSD(T_j))}{h_i} \quad (12)
\]

Where \( i \) represents the level of the building and \( j \) represents the \( j^th \) mode effect of the structure. Therefore, \( \phi_{i,j} \) is the modal deflection shape factor at the \( j^th \) mode. \( T_j \) is the natural period of \( j^th \) mode. Typically, the mean ratio of second mode period \( (T_2) \) and third mode period \( (T_3) \) to effective mode period \( (T_{eff}) \) can be assumed to be 0.25 and 0.1 respectively \( (T_2 = 0.25 T_{eff} \ and \ T_3 = 0.1 \ T_{eff}) \) [11]. And \( RSD(T_j) \) is the spectral displacement at the \( j^th \) mode period \( (T_j) \) which is obtained from the response displacement spectrum vs natural period diagram. \( \delta_{ij} \) is the \( j^th \) modal displacement at level \( i \) of the building.

Finally, all the mode effects are combined together to get a total seismic response of the building. The method is to use square-root-of-the-sum-of-the-square (SRSS) approach and the equations are shown (Eqs. (13) – (14)):
\[ \delta^*_i = \sqrt{\delta_{i,1}^2 + \delta_{i,2}^2 + \delta_{i,3}^2 + \ldots + \delta_{i,f}^2} \]  
(13)

\[ \theta^*_i = \sqrt{\theta_{i,1}^2 + \theta_{i,2}^2 + \theta_{i,3}^2 + \ldots + \theta_{i,f}^2} \]  
(14)

Where \( \delta^*_i \) is the combined displacement of level \( i \). \( \theta^*_i \) is the combined inter-storey drift of level \( i \).

### 3.2 Analysis of ETABS

When torsional effects are not considered, the seismic response of ETABS building model needs to be considered in 2D plane. So the seismic response at X direction is considered for 105m building and 132.6m building (the active degrees of freedom are set in XZ plane). On the contrary, the seismic response at Y direction is considered for 143.5m building and 193.2m building (the active degrees of freedom are set in YZ plane).

Then, the response spectrum is used to conduct the dynamic analysis, and the response spectrum is in accordance with AS1170.4-2007. The site subsoil class D is used in our research. The probability factor \( (k_p) \), hazard factor \( (Z) \), structural performance factor \( (S_p) \) and structural ductility factor \( (i) \) are set to be 1, 0.008, 1 and 1 respectively.

Finally, the Load Case is defined as the Response Spectrum and the dynamic analysis is conducted.

### 3.3. Results Comparison between GFM and ETABS

Based on the Eqs. (4) – (8), the key parameters \( m_{\text{eff}}, \delta_{\text{eff}}, k_{\text{eff}}, T_{\text{eff}}, a_{\text{eff}} \) of each building are summarized in Table 3.

The results of the comparison between GFM and ETABS including combined displacement and combined inter-storey drift are shown in graph format.

| Height (m) | Displacement (mm) | Dynamic analysis |
|-----------|-------------------|------------------|
| 105m building | 26016.25 | 69.83 | 282.57 | 1.91 | 0.76 |
| 132.6m building | 29445.52 | 92.76 | 178.09 | 2.56 | 0.56 |
| 143.5m building | 54597.37 | 166.01 | 173.16 | 3.53 | 0.53 |
| 193.2m building | 86413.67 | 125.02 | 229.14 | 3.86 | 0.33 |

#### 3.3.1. Results of Displacement

As can be seen from Figure 9 - Figure 11, the displacement results from GFM and ETABS modal match very well, especially for the 132.6m and 143.5m buildings. The maximum displacement difference for these four analysed buildings is around 2.5mm, which is acceptable. In addition, it is obvious that the displacement difference is larger in the middle level but getting smaller when reaching the bottom and top level.

#### 3.3.2. Results of Inter-storey Drift

According to the results of inter-storey drift (Figure 13 - Figure 16), it can be found that the results also match reasonably. The maximum difference between GFM and computer dynamic model is approximately 0.02%, which is small and acceptable. And there is no regular pattern of the discrepancies between inter-storey drift and the height of building, some occur at the bottom level (143.5m building) and some appear at the middle level (132.6m building).
3.4. Error analysis

Potential factors contributing to the error:
1. In GFM, the 1st modal displacement ($\delta_{i,1}^*$) is calculated by using the results of static analysis method and ADRS curve. While in dynamic analysis, $\delta_{i,1}^*$ is calculated by using Eq. (15) where $T_1$ is calculated by using Eigenvalue of the model.

$$\delta_{i,1}^* = \Gamma_1 \cdot \phi_{i,1} \cdot RSD(T1) \cdot T_1.$$  \hspace{1cm} (15)

Different equations used in the two methods cause discrepancies of 1st displacement profiles ($\delta_{i,1}^*$), but this error is small enough to be neglected.

2. The approximation of $T_2$ and $T_3$ might result in the discrepancies.

When torsional effects are not taken into consideration, in GFM, higher modal displacement ($\delta_{i,2}$ & $\delta_{i,3}$) are calculated by using the Eqs. (16) – (17):

$$\delta_{i,2} = \Gamma_2 \cdot \phi_{i,2} \cdot RSD(T2) \cdot T_2.$$  \hspace{1cm} (16)

$$\delta_{i,3} = \Gamma_2 \cdot \phi_{i,3} \cdot RSD(T3) \cdot T_3.$$  \hspace{1cm} (17)
Where \( T_2 = 0.25T_{\text{eff}} \) & \( T_3 = 0.1T_{\text{eff}} \).

While in dynamic analysis, the equations used remain the same, but \( T_2 \) and \( T_3 \) are calculated by using Eigenvalue of the model, which leads to the different modal displacement results (\( \delta_{i,2} \) & \( \delta_{i,3} \)) compared with GFM. The comparison of \( T_2 \) and \( T_3 \) is shown in Table 4.

3. Inter-storey drift is based on the calculation of displacement. As mentioned above, displacement results (\( \delta_{*1,1}, \delta_{*2,1} \) & \( \delta_{*3,1} \)) of the first three modes have some discrepancies between GFM and ETABS, thus causing the discrepancies of the inter-storey drift results.

4. Another minor contributing factor is the sum of modal participation mass ratios less than 90%. It is expected to exceed 90% (\( M_{\text{eff,1}} + M_{\text{eff,2}} + M_{\text{eff,3}} > 0.9 \) total mass) to ensure the accuracy of the results. If not, higher mode effects (4th mode and above) need to be considered. In this research, only the first three modes effects are considered. Table 5 shows ETABS results of modal participation mass ratios of each building.

According to the results of ETABS, for 132.6m and 143.5m buildings, which sum of participation mass ratios less than 0.9, 4th mode effect or higher vibration modes may need to be considered to ensure the accuracy of the results.

By considering the displacement ratio \( \Delta/\Delta_o \), the torsional displacement (\( \Delta \)) at stiff edge and flexible edge can be determined, where \( \Delta_o \) is the same as \( \delta_{i,1} \).

### 4. Analysis and Results with Torsional Effects

#### 4.1 Analysis of GFM

The building layout is considered as a uni-axial asymmetry plan when Center of Rigidity (CR) is away from only one axis where Centre of Mass (CM) is located (Figure 17a). Otherwise, the building is considered as bi-axial asymmetry if CR is away from both X and Y axis where CM is located (Figure 17b). According to the structural layouts of 4 selected buildings, the 132.6m building is bi-axial asymmetry while other 3 buildings can be considered as uni-axial asymmetry. When considering torsional effects, the flexible edge which is far from CR of building will have larger displacement compared to the stiff edge which is on the near side of CR.

The displacement calculation is based on the results of 1st mode displacement (\( \delta_{i,1} \)) calculated using Eq. (9).

#### Table 4. Comparison of T2 and T3 in GFM & Dynamic analysis

| Buildings | 105m | 132.6m | 143.5m | 193.2m |
|-----------|------|--------|--------|--------|
| GFM       |      |        |        |        |
| \( T_2 \) | 0.477s | 0.589s | 0.989s | 1.115s |
| \( T_3 \) | 0.205s | 0.235s | 0.395s | 0.446s |
| Dynamic analysis | | | | |
| \( T_2 \) | 0.563s | 0.751s | 1.014s | 1.257s |
| \( T_3 \) | 0.3s | 0.378s | 0.473s | 0.646s |

#### Table 5. ETABS results of modal participation mass ratios of each building

| Buildings | 105m | 132.6m | 143.5m | 193.2m |
|-----------|------|--------|--------|--------|
| Modal 1   | 0.7516 | 0.7291 | 0.6996 | 0.722 |
| Modal 2   | 0.1252 | 0.1215 | 0.1429 | 0.1482 |
| Modal 3   | 0.0426 | 0.0491 | 0.0549 | 0.0417 |
| Sum       | 0.9194 | 0.8997 | 0.8974 | 0.9119 |

#### 4.1.1. Edge Displacement for Uni-axial Asymmetry

GFM to analyse both uni-axial asymmetry buildings and bi-axial asymmetry buildings has been fully introduced [25,26]. The edge displacement can be calculated in three controlled conditions:

1. Acceleration controlled condition:

\[
\frac{\Delta}{\Delta_0} = \sqrt{\sum_{j=1}^{n} \left(1 + \theta_j (\pm B_r) \times \frac{1}{\lambda_j^2} \right)^2} \quad (18)
\]

2. Velocity controlled condition:

\[
\frac{\Delta}{\Delta_0} = \sqrt{\sum_{j=1}^{n} \left(1 + \theta_j (\pm B_r) \times \frac{1}{\lambda_j^2} \right)^2} \quad (19)
\]

3. Displacement controlled condition:

\[
\frac{\Delta}{\Delta_0} = \sqrt{\sum_{j=1}^{n} \left(\frac{1 + \theta_j (\pm B_r)}{1 + \theta_j^2} \right)^2} \quad (20)
\]

Where \( \Delta \) represents the displacement when the lateral load is applied at the CM. \( \Delta_0 \) represents the displacement demand when the lateral load is applied at the CR. \( B_r = B/r \) where \( r \) is mass radius of gyration and \( B \) is the half length of the edge which is perpendicular to the direction of excitation. \( \lambda_j \) is the eigenvalue which can be calculated using Eq. (21) below:

\[
\lambda_j = \frac{1 + \left( b_r^2 + e_r^2 \right)}{2} \pm \sqrt{\left[1 - \left( b_r^2 + e_r^2 \right) \right] + e_r^2} \quad (21)
\]

\( \theta_j \) is the mode shape vector and it can be calculated as:

\[
\theta_j = \frac{\lambda_j^2 - 1}{e_r} \quad (22)
\]

In order to calculate \( \lambda_j \), the values of \( b_r \) and \( e_r \) should be calculated first. \( e_r \) is \( \psi/r \) and \( \psi \) represents eccentricity perpendicular to the direction of motion. \( b_r \) can be calculated using Eq. (23). The details of the calculation of \( e_r \) and \( b_r \) is introduced in section 4.1.3.
4.1.2. Edge Displacement for Bi-axial Asymmetry

When analysing bi-axial asymmetry buildings, eccentricities at both x and y direction need to be considered, which are nominated as \(e_x\) and \(e_y\) respectively. \(e_x\) is the eccentricity perpendicular to the direction of motion and \(e_y\) is the eccentricity in the direction parallel to the motion. Then, \(e_{xr}\) and \(e_{yr}\) can be solved by simply calculating \(e_x/r\) and \(e_y/r\) respectively.

Similar to the calculation for edge displacement of uni-axial asymmetry building, the displacement ratio \(\Delta/\Delta_0\) needs to be calculated in three controlled conditions:

1. Acceleration controlled condition:

\[
\frac{\Delta}{\Delta_0} = \sqrt{\sum_{j=1}^{n} \left[ (1 + \theta_j (\pm B_{rj}) )PF_j \times \frac{1}{\lambda_j} \right]^2}
\]  

2. Velocity controlled condition:

\[
\frac{\Delta}{\Delta_0} = \sqrt{\sum_{j=1}^{n} \left[ (1 + \theta_j (\pm B_{rj}) )PF_j \times \frac{1}{\lambda_j} \right]^2}
\]  

3. Displacement controlled condition:

\[
\frac{\Delta}{\Delta_0} = \sqrt{\sum_{j=1}^{n} \left[ (1 + \theta_j (\pm B_{rj}) )PF_j \times \frac{1}{\lambda_j} \right]^2}
\]

Where \(\lambda_j\) is the eigenvalue which can be solved by using Eq. (27):

\[
\lambda_j = \begin{vmatrix}
a - \lambda_j^2 & 0 & ae_{yr} \\
0 & 1 - \lambda_j^2 & e_{er} \\
ae_{yr} & e_{er} & \left( ae_{yr}^2 + e_{yr}^2 + b_{yr}^2 \right) - \lambda_j^2
\end{vmatrix} = 0
\]

\(PF_j\) is the participation factor and it can be calculated using Eqs. (28) – (29):

\[
PF_j = \frac{1}{x_{r,j}^2 + y_{r,j}^2 + \theta_j^2}
\]

\[
y_{r,j} = 1, x_{r,j} = \frac{e_{yr}}{e_{er}} \left( \frac{a - a\lambda_j^2}{a - \lambda_j^2} \right)
\]

In the equation for calculating \(x_{r,j}\), \(a\) is the ratio of translational stiffness in the x-direction \(k_x\) to translational stiffness in the y-direction \(k_y\).

4.1.3. Determination of Eccentricity \((e)\) and Torsional Stiffness \((b)\)

- First step: find the location of the CR of building.
  The lateral load caused by the base shear is applied to the building to determine the values of \(\delta_{\text{eff}}\) and \(\phi\). When the lateral load is applied at X direction, the CM at X direction \((X_{CM})\) can be obtained. In our paper, the lateral loads of 105m building and 132.6m building are applied at X direction. And the lateral loads of 143.5m building and 193.2m building are applied at Y direction. The \(\delta_{\text{eff}}\) can be calculated using Eq. (30):

\[
\delta_{\text{eff}} = \frac{m_i \delta_i^2}{\sum m_i \delta_i^2}
\]

Then, a second lateral load is applied away from the CM of the building. In our research, the lateral load is applied 0.05L from CM. Again, using Eq. (30) to obtain new values of \(\delta_{\text{eff}}\) and \(\phi\). The summary of \(e\) and \(b\) can be seen in Table 6.

- Second step: Determination of \(e_x\) and \(e_y\).
  As mentioned in section 4.1.1, the value of eccentricity \((e)\) should be determined first before \(e_x\) and \(e_y\) are calculated. Based on the values of \(\phi\) and \(\phi_2\), the location of CR can be got by extrapolation. Then, \(e\) can be expressed as the difference of the location of CM and CR. For a rectangular shape, the value of \(r\) can be calculated using Eq. (31). It should be noticed that the structural plan of 105m and 132.6m buildings are irregular. However, their shapes are much closed to rectangular. So they are treated as rectangular when calculating the value of \(r\). Finally \(e\) can be easily calculated using \(e/r\). The extrapolation results are shown in Figure 18 - Figure 21.

\[
r = \sqrt{\frac{L^2 + W^2}{12}}
\]

Where L and W is the length and width of the building.

After getting the values of \(e\), \(\delta_{\text{eff}}\) and \(\delta_{\text{eff}2}\), the value of \(\Delta_0\) which represents the displacement demand when the lateral load is applied at the CR, can be obtained by extrapolation. As a result, \(\Delta/\Delta_0\) can be calculated. The extrapolation results can be seen in Figure 22 - Figure 25.

Based on the value of \(\Delta/\Delta_0\) and Eq. (23), the value of \(b\) can be calculated.

Figure 18. Obtain \(e\) by extrapolation of 105m building

![Figure 18](image)

Figure 19. Obtain \(e\) by extrapolation of 132.6m building

![Figure 19](image)
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Figure 20. Obtain e by extrapolation of 143.5m building

Figure 21. Obtain e by extrapolation of 193.2m building

Figure 22. Obtain Δo by extrapolation of 105m building

Figure 23. Obtain Δo by extrapolation of 132.6m building

Figure 24. Obtain Δo by extrapolation of 143.5m building

Figure 25. Obtain Δo by extrapolation of 193.2m building

It should also be noted that the 132.6m building is bi-axial asymmetry, which is different from other 3 buildings. So the method in section 4.1.2 should be used to calculate displacement ratio Δ/Δo. Based on the analysis above, the value of e_r, e_y, and a of 132.6m building can be determined as 0.769, 0.521 and 1.451 respectively.

Table 6. Summary of e_r, b_r, location of CM and location of CR

| Building   | X_CM or Y_CM (m) | X_CR or Y_CR (m) | e_r | b_r |
|------------|------------------|------------------|-----|-----|
| 105m building | 7.73             | 8.29             | 0.047 | 1.131 |
| 132.6m building | 9.28             | 0.82             | 0.521 | 7.458 |
| 143.5m building | 19.12            | 16.14            | 0.205 | 0.983 |
| 193.2m building | 19.24            | 18.20            | 0.065 | 1.520 |

4.2 Analysis of ETABS

When torsional effects are considered, the seismic response of ETABS building model needs to be considered in 3D including X, Y and Z directions. The active degrees of freedom are set to be full 3D in ETABS.

4.3. Results Comparison between GFM and ETABS

4.3.1. Results of Stiff Edge Displacement

According to the comparison results in Figure 26 - Figure 29, the stiff edge displacement calculated from GFM matches reasonably with ETABS simulation, especially for the 132.6m building and 193.2m building. However, relatively larger discrepancies could be observed at the upper levels for 105m building and 143.5m building. The largest discrepancies are approximately 6mm and 8mm respectively. Since the maximum lateral displacements at stiff edge for 105m building and 143.5m building are around 70mm and 100mm, these discrepancies are acceptable when applying GFM to analyse tall buildings.

4.3.2. Results of Flexible Edge Displacement

As can be seen from the results of flexible edge displacement (Figure 29), the displacement from GFM also matches reasonably with ETABS simulation. The discrepancy of 143.5m building is relatively larger compared to other buildings, and the maximum value reaches approximately 6mm. In addition, it is also obvious that the discrepancies for both stiff and flexible edge displacement of 143.5m building match worst among the four selected buildings. However, the maximum discrepancies for both stiff and flexible edge displacement are all acceptable for such a tall building.
4.3.3. Error Analysis

As the same reason discussed in section 3.4, the sum of modal participation mass ratios should be greater than 90%. However, only the first three modes are considered in our research and the sum of modal participation mass ratios of 4 buildings are just around 90%. So more vibration modes may need to be considered in the analysis to ensure the accuracy of the results.

5. Conclusion

Although a variety of methods are currently available to analyse buildings in an earthquake, some of them are only used for low rise building and some of them cannot provide accurate results due to the ignorance of the higher modes effects. In this research program, GFM which considers the higher mode effects is proposed to analyse the performance and seismic response behaviour of tall buildings. By performing this task, four different multi-storey buildings with varied heights ranging from 100m to 200m are analysed. In addition, in order to verify the accuracy of GFM, computer simulation is conducted by applying the ETABS software package. The results
show that whether the torsion effect is considered or not, the displacement and inter-storey drift are reliable and accurate enough when applying GFM. And the estimated results from GFM can be used in engineering practice. The main factor contributing to the error is the different results from GFM can be used in engineering practice. And the estimated approach for T2 and T3 should be further improved in future to further verify the accuracy of GFM for tall buildings.

References

[1] Shet, P. A., Kumar, C. M. R., & Ernama, H. The Effect of Size of Piles on the Seismic Response of Multi-Storey Buildings Considering Soil-Structure Interaction. IUP Journal of Structural Engineering. 2019, 12(2), 7-18.

[2] Su, N., Lu, X., Zhou, Y., & Yang, T. Y. Estimating the peak structural response of high-rise structures using spectral value-based intensity measures. The Structural Design of Tall and Special Buildings. 2017, (8).

[3] Roy, R., & Mahato, S. Equivalent lateral force method for buildings with setback: adequacy in elastic range. Earthquakes Struct. 2013, 4(6), 685-710.

[4] Abdel-Karim, Riyad. Seismic Analysis of a Masonry Reinforced Concrete Shear Wall Building with Severe Architectural Irregularity, A Colloquial Discourse. International Journal of Research and Analytical Reviews. 2016, 3. 39-48.

[5] Cuia, A. D., Lombardi, L., Luca, F. D., Risi, R. D., Caprii, S., & Salvatore, W. Linear Time-History Analysis for EC8 design of CBF structures. Procedia Engineering, 2017, 199, 3522-3527.

[6] Stana, B. H. Linear and Nonlinear Analysis of a High-rise Building Excited by Earthquake: Linear og ikke-linear analyse av høybygget bygning (Master's thesis, Institutt for konstruksjonsteknikk). 2014.

[7] Chao SH., Goel SC., & Lee SS. A seismic design lateral force distribution based on inelastic state of structures. Earthq Spectra 2007, 23(3):547-69.

[8] Rodriguez, M. E., Restrepo, J. I., & Carr, A. J. Earthquake-induced floor horizontal accelerations in buildings. Earthquake Engineering Structure Dynamics 2002, 1:693-718.

[9] Medina RA., Sankaranarayanan R., & Kingston KM. Floor response spectra for light components mounted on regular moment-resisting frame structures. Engineering Structure 2006, 28: 1927-40.

[10] Lee HJ., Kuchma D., & Aschheim MA. Strength-based design of flexible diaphragms in low-rise structures subjected to earthquake loading. Eng Struct 2007, 29(7):1277-95.

[11] Lam, N. K., Lunantarna, E., & Wilson, J. L. Simplified elastic design checks for torsionally balanced and unbalanced low-medium rise buildings in lower seismicity regions. Earthquakes And Structures, 2016, 11(5), 741-777.

[12] Finley, D. T., & Cribs, R. A. Equivalent static vs. response spectrum: a comparison of two methods. In Astronomical Structures and Mechanisms Technology (Vol. 5495, pp. 403-411). International Society for Optics and Photonics. 2004.

[13] Gupta, A. K. Response spectrum method in seismic analysis and design of structures. Blackwell Scientific Publ. 1990.

[14] Datta, T. K. Seismic analysis of structures. Singapore; Hoboken, NJ: John Wiley & Sons Asia. 2010.

[15] Costa, J. L. D. Standard methods for seismic analyses. Report BYG-DTU R-064, Technical University of Denmark. 2003.

[16] Arman, Ibrahim & Helou, Samir. Equivalent Lateral Load Method vs. Response Spectrum Analysis Which Way is Forward. Asian Journal of Engineering and Technology (ISSN: 2321 – 2462), 2014, 02. 366-374.

[17] Shepherd, R. Some Limitations of Modal Analysis in Seismic Design. Bulletin of the New Zealand Society for Earthquake Engineering. 1969, 2(3), 284-288.

[18] Wilson, E. L., & Kweh, A., & Bayo, E. P. A replacement for the SRSS method in seismic analysis. Earthquake Engineering & Structural Dynamics. 1981, 9(2), 187-192.

[19] Humar, J., & Mahgoub, M. A. Determination of seismic design forces by equivalent static load method. Canadian Journal of Civil Engineering. 2003, 30(2), 287-307.

[20] Hughes, T. J. The finite element method: linear static and dynamic finite element analysis. Courier Corporation, 2012.

[21] Estekanchi, H. E., Valamanesh, V., & Vafai, A. Application of endurance time method in linear seismic analysis. Engineering Structures, 2007, 29(10), 2551-2562.

[22] Computers and Structures Inc. (2000). CSI Analysis Reference Manual For SAP2000, ETABS, SAFE and CSIBridgeTM. 2000.

[23] Lam, N. K., & Wilson, J. L. Earthquake design of buildings in Australia using velocity and displacement principles. Australian Journal of Structural Engineering, 2006, 6(2): p. 103-118.

[24] Lunantarna, E., Sofi, M., Lam, N. T. K., & Tsang, H. H. Simplified Method for Seismic Assessment of Reinforced Concrete Buildings in Australia. In Simplified Method for Seismic Assessment of Reinforced Concrete Buildings in Australia. Invited presentation in the Seminar on Best Practice in Civil and Structural Engineering Works, 29 November 2016, Universitas Kristen Petra, Surabaya, Indonesia.

[25] Lunantarna, E., Mehdipunah, A., Lam, N., & Wilson, J. Methods of structural analysis of buildings in regions of low to moderate seismicity. The 2017 World Congress on Advances in Structural Engineering and Mechanics (ASEM17), Ilsan (Seoul), Korea, 2017, 28 August – 1 September.

[26] Lunantarna, E., Lam, N.T.K., and Wilson, J.L. Method of analysis for buildings with uni-axial and bi-axial asymmetry in region of lower seismicity. Submitted and approved for Earthq. Struc. journal. 2017.

[27] Standards Australia. AS 1170.4-2007 Structural Design Actions – Part 4 Earthquake Actions. Sydney: Standards Australia. 2007.

[28] Clough, R. W. On the importance of higher modes of vibration in the earthquake response of a tall building. Bulletin of the Seismological Society of America, 1955, 45(4), 289-301.

[29] Tirca, L., & Tremblay, R. Influence of building height and ground motion type on the seismic behaviour of zipper concentrically braced steel frames. In 13th World Conference on Earthquake Engineering. 2004, August.

[30] Computers & Structures Inc. (2013). User’s Guide ETABS 2013: Integrated Building Design Software, Computers & Structures, Inc, Berkeley, California, USA. 2003.