Thermodynamics of Hidden Sector Gaugino Condensation in the Expanding Universe

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Abstract

This work examines the confining-deconfining phase transition in a hidden Yang Mills sector with scale $\Lambda \sim 10^{14}$ GeV appropriate to dilaton stabilization and SUSY-breaking via formation of a gaugino consensate. If the transition is assumed to take place through homogenous nucleation, then under reasonable assumptions it is found that a critical bubble, formed at a temperature which provides enough supercooling, is not large enough to accommodate an adequate number ($\gtrsim 100$) of quanta of the confined phase ('hidden hadrons') to allow a consistent thermodynamic description. Thus, a first order transition in the hidden sector may not be possible in the expanding universe.
1 Introduction

An unbroken hidden supersymmetric gauge sector, supporting a gaugino condensate \( \langle \lambda\lambda \rangle \sim (10^{14} \text{ GeV})^3 \), can through its coupling to the dilaton (and other moduli) serve the related dynamical functions of providing a superpotential for the dilaton and a mass for the gravitino [1]. Because of its possibly central role in the understanding of these phenomena, the dynamics associated with this hidden gauge sector may merit additional study. As a zero temperature field theory, against a Minkowski space-time background, this has been done (without dilaton) in the original paper of Veneziano and Yankielowicz [2] (see also Amati et al [3]). In the effective theory, the ground state emerges as containing a gaugino condensate \( \langle \lambda\lambda \rangle \) with residual \( Z_N \) symmetry. The changes which evolve with the introduction of supergravity and the dilaton are discussed in [4].

In the context of the standard Robertson-Walker cosmology, it is of interest to examine the transition from the hot unconfined phase in the hidden sector to the confined phase [5,6]. Is the transition first or second order? If first order, is the transition completed in a manner consistent with the observed smoothness of the universe? Some possible problems associated with a transition described by the evolution of the condensate as a field theoretic order parameter were discussed in [6]. However, the results depended critically on the Kähler potential of the effective condensate field, and this is certainly not known. Thus, a more phenomenological approach is indicated.

In this work, I present a preliminary study of the transition to the confining phase of the unbroken hidden Yang-Mills sector, under a few well-defined assumptions:

(1) There exists a hot, unconfined phase of the hidden sector.

(2) The transition to confining phase is first order, proceeding through the spontaneous nucleation and expansion of critical bubbles of confined phase.

\(^1\)It is possible that screening, rather than confinement, characterizes the phase transition [3]. The only property to be used in this paper is the change in the degrees of freedom from perturbative non-singlet quanta to gauge-singlet ‘hadrons’ as one passes through the critical temperature.
Neither of these assumptions is necessarily correct. The aim of the present study is to simply examine the consequences of taking them to be true.

The principal conclusion of this study, under the stated assumptions, is the following: if the lightest composite particles in the confined phase (‘hidden hadrons’) are more massive than $\sim 2T_c$ (where $T_c$ is the critical temperature), then the requirement of sufficient supercooling in order to complete the phase transition implies a critical bubble size too small to contain enough (i.e., $> 100$) hidden hadrons to allow a meaningful thermodynamic description. This result is largely traceable to the high value of the transition temperature. The conclusion will be shown to hold as well in the presence of (near) massless vector-like matter fields. Thus a first order transition in a cosmological context may be unlikely in the hidden sector.

2 Review of Homogeneous Nucleation

In this section, I will briefly sketch the derivation of the relevant formulae, initially following Fuller, Mathews, and Alcock [7] in their discussion of the quark-hadron transition, with some modification for ease of application to the present situation.

The formation of the confined phase proceeds through the spontaneous nucleation of critical bubbles of radius $R_c$, at which the free energy difference between confined and deconfined phases (pressures $P_{\text{conf}}(T)$ and $P_{\text{deconf}}(T)$, respectively)

$$\Delta F = -\frac{4\pi}{3}R_c^3 (P_{\text{conf}} - P_{\text{deconf}}) + 4\pi\sigma R_c^2$$ (1)

has a saddle point at

$$R_c(T) = \frac{2\sigma}{P_{\text{conf}}(T) - P_{\text{deconf}}(T)}.$$ (2)

Here $\sigma$ is the surface energy density at the interface between the phases. The probability of nucleation of a single bubble is then

$$p(T) = CT^4 e^{-\Delta F/T}.$$ (3)

The critical temperature $T_c$ is defined through the coexistence condition in the infinite volume limit

$$P_{\text{conf}}(T_c) = P_{\text{deconf}}(T_c).$$ (4)
If the amount of supercooling $\eta \equiv (T_c - T)/T_c$ is small, then one may expand

$$P_{\text{conf}}(T) - P_{\text{deconf}}(T) \simeq L\eta ,$$

(5)

where

$$L = T_c \frac{\partial}{\partial T} (P_{\text{deconf}} - P_{\text{conf}})|_{T_c}$$

(6)

is the latent heat released per unit volume during the phase change. Combining these results, we have

$$p(T) \simeq CT_c^4 e^{-A(T)} ,$$

(7)

where

$$A(T) = \frac{16\pi}{3} \frac{\sigma^3}{L^2 T_c \eta^2}$$

(8)

is the 3-dimensional critical bubble action. The rapid increase of $p(T)$ with increased supercooling is then explicit.

The condition for the complete nucleation of the universe at time $t_f$ is

$$\int_{t_c}^{t_f} dt' p(T'(t')) \frac{4\pi}{3} v_s^3 (t_f - t')^3 = 1 ,$$

(9)

where $v_s \simeq 1/\sqrt{3}$ is the speed of the expanding shock wave of the confined phase into the metastable unconfined phase.

For small supercooling, one may expand about $t' = t_f$:

$$\ln p(T) = \ln p(T_f) + \frac{d\ln p}{dT} \bigg|_{T_f} \frac{dT}{dt} \bigg|_{t_f} (t - t_f)$$

(10)

with

$$\frac{dT}{dt} \bigg|_{t_f} = -T_f H(T_f) \simeq -T_c H(T_c) .$$

(11)

Thus

$$p(t) = p(t_f) e^{-\lambda(t_f - t)}$$

(12)

with $\lambda = 2A(T_f)H(T_c)/\eta_f$. The condition (9) then becomes

$$\frac{8\pi v_s^3}{\lambda^4} p(T_f) = 1 ,$$

(13)
or

$$\frac{8\pi C}{3\sqrt{3}} \left( \frac{T_c}{2} \right)^4 \frac{\alpha}{H(T_c)} = A^6 e^A$$  \hspace{1cm} (14)$$

where

$$\alpha \equiv \left( \frac{16\pi}{3} \frac{\sigma^3}{L^2 T_c} \right)^{\frac{1}{4}},$$  \hspace{1cm} (15)$$

and \( A \equiv A(T_f) \). Note that

$$\eta_f = \frac{\alpha}{\sqrt{A}}$$  \hspace{1cm} (16)$$

and the Hubble constant at \( T_c \) is

$$H(T_c) = \frac{1}{M_{Pl}} \sqrt{\frac{8\pi \rho(T_c)}{3}},$$  \hspace{1cm} (17)$$

with

$$\rho(T_c) = \text{total energy density at } T_c = N_{tot} \frac{\pi^2}{30} T_c^4.$$  \hspace{1cm} (18)$$

Here \( N_{tot} \) is an effective statistical weight for the degrees of freedom at \( T_c \). If the universe happens to be dominated by a vacuum energy \( \rho_0 \) at the time of supercooling, then Eq. (18) is just a redefinition of \( \rho_0 \) in terms of \( N_{tot} \).

Combining Eqs. (15)-(18), the condition (14) becomes

$$A^6 e^A = \left( \frac{0.45 C^{1/4} \alpha}{\sqrt{N_{tot}} \frac{M_{Pl}}{T_c}} \right)^4.$$  \hspace{1cm} (19)$$

3  Example of Pure Non-Supersymmetric SU(3) Yang Mills

As a preliminary for the hidden sector, consider the case of purely gluonic \( SU(3)_c \) (no quarks). Then \( N_{tot} = 16, \ T_c \approx 200 \text{ MeV} \), and a lattice-based estimate of \( \alpha \) is \( 8 \)

$$\alpha \approx 0.0065 \pm 0.0015.$$  \hspace{1cm} (20)$$

With \( C \approx 1 \), Eq. (19) yields

$$A = 124.$$  \hspace{1cm} (21)$$
and Eq. (16) gives $\eta_f = 6 \times 10^{-4}$. From (2), (5), (15), and (16), the radius of a critical bubble is

$$R_c = \frac{2\sigma}{L\eta_f} = \sqrt{\frac{A}{4\pi}} \left( \frac{T_c^3}{\sigma} \right)^{1/2} T_c^{-1} . \tag{22}$$

From the data of [8], the surface energy density is not quite scale invariant with present statistics; nevertheless, it will suffice for the present application to take [9]

$$\sigma \simeq 0.025 \ T_c^3 , \tag{23}$$

which gives

$$R_c = 35 \ T_c^{-1} . \tag{24}$$

I now come to the point of departure of the present work.

With no quarks, the relevant degrees of freedom in the confined phase are glueballs with masses $M_i \gtrsim 1 \text{ GeV} \gg T_c$, and number densities

$$n_i(T_c) \simeq T_c^3 \left( 2S_i + 1 \right) \left( \frac{M_i}{2\pi T_c} \right)^{3/2} e^{-M_i/T_c} . \tag{25}$$

Then the number of glueballs in the critical bubble is

$$N_c = \sum n_i(T_c) \ 4\pi R_c^3 . \tag{26}$$

Phenomenological studies [10] have suggested that a density of states $\tau(M) \propto M^3$ (for each isospin and hypercharge) provides a good fit to the observed hadron spectrum. I adopt this for the glueball spectrum, and normalize to 1 state in the interval $(M_0^2 - \frac{1}{2} \mu^2, M_0^2 + \frac{1}{2} \mu^2)$, where $M_0$ is the mass of the lightest glueball and $\mu^2$ is the inverse of the Regge slope. This gives for the spectral density

$$\tau(M) = (2/M_0^2 \mu^2) \ M^3 , \tag{27}$$

and for the number density

$$n_c = \sum n_i(T_c) = 2(2\pi)^{-3/2} M_0^{-2} \mu^{-2} \int_{M_0}^{\infty} dM \ M^3 \ (M/T_c)^{3/2} e^{-M/T_c} \ T_c^3$$

$$\equiv 2(2\pi)^{-3/2} \ (T_c/\mu)^2 \ f(M_0/T_c) \ T_c^3 . \tag{28}$$
Combining (24), (26), and (28), one finds

\[ N_c = 22,800 \left( \frac{T_c}{\mu} \right)^2 f(M_0/T_c) \]  

(29)

A thermodynamic description, and hence a first order phase transition, will be cosmologically viable only for \( N_c \gg 1 \), say \( N_c > 100 \). Once \( \mu/T_c \) is specified, this will translate via (29) to a bound on \( M_0 \). Since \( \mu^2 \sim 1 \text{ GeV}^2 \), and \( T_c \simeq 250 \text{ MeV} \), one finds

\[ M_0 \leq 8 T_c \simeq 2000 \text{ MeV} \]  

(30)

Before continuing, I will comment briefly on the question of possible temperature dependence of the glueball masses near \( T_c \). A recent theoretical calculation [11] in the context of the dual Ginzburg-Landau theory shows that there is some reduction in mass of the gauge singlet QCD monopole at \( T_c \), although the effect is totally dependent on an assumed (and unknown) temperature dependence of the dual Higgs quartic coupling. There is no basis for supposing that the glueball mass goes to zero (or is even much reduced) at \( T_c \) in a strongly first order phase transition, and I will simply reinterpret (30) as a bound on an effective glueball mass, with the expectation that it does not differ significantly from the zero temperature mass.

I now turn to examine the implication of these ideas when applied to the hidden gauge sector which is relevant to gaugino condensation and SUSY-breaking.

4 The Hidden Sector: Pure SUSY Yang-Mills

There are several significant differences between the hidden sector SUSY-Yang-Mills theory and the non-SUSY SU(3)\( _c \) theory just discussed:

- The theory is supersymmetric: there are Majorana fermions in the adjoint representation, and only rudimentary lattice results are available for such a theory [12].
- The vacuum properties are totally unlike those in ordinary QCD [3].
The gauge group is either much larger than SU(3), or is manifest at a Kac-Moody level \( k \geq 2 \), in order that strong coupling sets in at a scale \( \sim 10^{14} \) GeV.

A question which immediately arises as a consequence of these differences is whether the transition is first order. The simplest approach here is to assume that it is first order as a working hypothesis, and examine the consequences.

Since the interface energy \( \sigma \) and the parameter \( \alpha \) are completely unknown for the hidden sector, the nucleation condition (19) must be rewritten in a suitable manner. I assume that the specific entropy of the hadronic phase is much less than that of the gauge phase, so that I take for the latent heat

\[
L = T_c \left. \frac{dP_{\text{deconf}}}{dT} \right|_{T=T_c} = 4 \frac{\pi^2}{90} N_{\text{hidden}} T_c^4 .
\]

(31)

It is also convenient to set

\[
\dot{\sigma} = \sigma / T_c^3 ,
\]

(32)

so that the condition (13) becomes

\[
\left( \frac{A}{\sigma} \right)^6 e^A = \left( \frac{4.8 C^{1/4}}{N_{\text{hidden}} \sqrt{N_{\text{tot}}}} \frac{M_{\text{Pl}}}{T_c} \right)^4 .
\]

(33)

What is \( T_c \)? For zero cosmological constant, the gravitino mass is given in terms of the effective superpotential \( W \) and Kahler \( K \) by

\[
m_{3/2} = e^{K/2 |W_{\text{eff}}| / M^2} ,
\]

(34)

where \( M = M_{\text{Pl}} / \sqrt{8 \pi} \). In the effective theory, with a simple gauge group and no matter fields, one obtains after integrating out the gaugino condensate [13]

\[
W_{\text{eff}} = -\frac{b}{6e} M_{\text{string}}^3 e^{-3S/2b} ,
\]

(35)

where \( b = \beta(g) / g^3 = 3N / 16\pi^2 \) for SU(\( N \)), Re \( S = 1 / g_{\text{string}}^2 \simeq 2.0 \) at the correct minimum for the dilaton field \( S \). \( M_{\text{string}} \) sets the scale for the logarithmic term in the condensate superpotential. The SU(\( N \)) theory becomes strong (\( g^2 / 4\pi = 1 \)) at a scale

\[
\Lambda = M_{\text{string}} e^{-S/2b} ,
\]

(36)
so that
\[ m_{3/2} = e^{K/2} \frac{b}{6e} \frac{\Lambda^3}{M^2} \cdot (37) \]
For \( m_{3/2} \simeq 10^3 \) GeV, one obtains
\[ \Lambda = e^{-K/6} N^{-1/3} \cdot 1.7 \times 10^{14} \text{ GeV} \cdot (38) \]
As a heuristic example, I will choose as the hidden gauge group SU(6), which is consistent with Eqs. (36) and (38) for \( M_{\text{string}} \simeq 10^{18} \) GeV, \( e^{-K/6} \simeq 1 \). With \( T_c \simeq \Lambda \simeq 10^{14} \) GeV, \( \mathcal{N}_{\text{hidden}} = 2 \left( \frac{15}{8} \right) (6^2 - 1) = 131.25, \mathcal{N}_{\text{tot}} = \mathcal{N}(\text{Standard Model}) + \mathcal{N}_{\text{hidden}} = 213.75 + 131.25 = 345, \) and \( C^{1/4} \simeq 1 \), the constraint Eq. (33) becomes
\[ A + 6 \ln(A/\hat{\sigma}) = 21 \cdot (39) \]
For consistency, we must require that \( A \) not be small, so that for \( A \geq 1 \), one obtains an upper bound
\[ A/\hat{\sigma} \leq 28 \cdot (40) \]
As before, I now proceed to calculate the number of (hidden) hadrons in a critical bubble.

With the same spectrum of glueballs as for the non-SUSY example of the last section (Eq. (28) with a factor of 4 included for the supermultiplet), and Eq. (22) for the bubble radius, one finds
\[ N_c = \left( \sqrt{6}/\pi^2 \right) (T_c/\mu)^2 f(M_0/T_c) (A/\hat{\sigma})^{3/2} \cdot (41) \]
where again \( 1/\mu^2 \) is the Regge slope for the hidden sector glueballs. As previously, the requirement that the cosmological description be thermodynamically viable requires that \( N_c \gg 1 \), which I take to mean \( N_c \gtrsim 100 \). With the use of (40), this devolves to a constraint on \( f(M_0/T_c) \), and hence on \( M_0 \) : for \( \mu/T_c \geq 3 \), I find
\[ M_0 \leq 1.4 T_c \cdot (42) \]
which is of dubious credibility since we expect \( M_0 > \mu \). For \( \mu/T_c \geq 2 \), the bound is
\[ M_0 \leq 2.0 T_c \cdot (43) \]
which is marginally possible. Thus, the conclusion at this point is that a bubble description for the first order transition is barely possibly (for $M_0 \approx \mu \approx 2 T_c$), but seemingly unlikely.

### 5 Hidden Sector with Matter Fields

Suppose that the hidden sector contains $N_f < N$ flavors of vector-like pairs of chiral superfields $Q + \overline{Q}$. If these are massive ($M_Q \gg \Lambda_N$, the SU($N$) confining scale), then they effectively decouple from the dynamics discussed in this paper. If they are massless (or nearly massless), then the discussion in Refs. [3] and [14] is germane: the existence of flat directions $v_{ir} = v_i \delta_{ir} (i = \text{color}, r = \text{flavor})$ in the field space of the $Q + \overline{Q}$ breaks the symmetry to SU($N'$), $N' \equiv N - N_f$, at the scale $v$. Between $v$ and $\Lambda_{N'}$, the effective massless degrees of freedom are the Goldstone bosons of the broken flavor symmetry and the SU($N'$) gauge degrees of freedom. (I assume that it is $\Lambda_{N'}$ which establishes the condensate scale of interest in this work.) If the Goldstones are in thermal equilibrium with the SU($N'$) fields, then they would contribute to the pressure of the critical bubble with high number density, and the problems encountered earlier would be alleviated. I will now show that the Goldstones are not in thermal equilibrium with the SU($N'$) fields, and thus the bubble is transparent to their existence.

Thermal equilibrium requires that the ratio $\Gamma/H$ be $> 1$ during the era of interest, where $\Gamma$ is the reaction rate of the Goldstones in the SU($N'$) plasma. The coupling of a Goldstone to a pair of SU($N'$) gluons is $\frac{g^2}{32\pi^2 v \sqrt{N'}}$, and the cross section in the plasma is easily calculated:

$$\sigma \sim \frac{g^2}{4\pi} \left( \frac{g^2}{32\pi^2 v} \right)^2,$$

independent of temperature and $N'$. The Hubble constant $H \simeq \sqrt{N_{tot}} T^2 / M_{Pl}$, so that

$$\frac{\Gamma}{H} = \frac{\sigma n v_G}{H} \simeq 10^{-8} \frac{N_{N'}}{\sqrt{N_{tot}}} \frac{T M_{Pl}}{v^2},$$

where $n = \text{plasma number density}$, $v_G = \text{Goldstone velocity}$. For the SU(6) example ($N' = 6$), with $T = T_c \simeq 10^{14}, N_{N'} = 131.25, N_{tot} = 345$, one finds

$$\frac{\Gamma}{H} \simeq 0.007 (v/T_c)^{-2} \ll 1$$

(46)
for any $v \geq T_c$. Thus, the Goldstones decouple from the SU($N'$) plasma, and do not contribute to the bubble dynamics.

6 Summary and Conclusions

(a) The field theoretic description of hidden gluino condensates must imply a parallel thermal/cosmological description of the phase transition between the unconfined and confined phases of the unbroken Yang-Mills theory. This work has examined the conditions under which a first order transition in terms of classical bubble nucleation is possible. The result found is that only if the mass of the confined phase glueballs (and superpartners) is very near to $2T_c$, are critical bubbles large enough to contain an adequate number of quanta of the confined phase particles to satisfy the thermodynamic conditions for a first order transition in the expanding universe. This is true whether or not there are Goldstones associated with massless vector-like matter fields. The highly restrictive conditions on glueball mass leads one to question whether a first order transition is possible in the expanding universe.

(b) If a first order transition is not feasible, then a field theoretic description of a second (or higher) order transition may be of interest. This entails some difficulty with the Witten index theorem[15]: at the critical temperature, the order parameter (presumably the gaugino condensate) must change in a continuous manner from zero to a non-zero value. However, the index for the final state is $N$ (for SU($N$)), whereas for the initial state it is (presumably) zero. Resolution of this problem will no doubt involve some non-trivial input to the effective theory.

(c) The critical input to the present analysis is the ratio $M_0/T_c$, where $M_0$ is the mass of the lightest glueball. It would be extremely useful to have some indication, possibly from a lattice study, of this quantity. A continued pursuit of SUSY on the lattice, following the initial effort in Ref. [12] would be very welcome.
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