WHY CANONICAL DISKS CANNOT PRODUCE ADVECTION-DOMINATED FLOWS

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ABSTRACT

Using simple arguments we show that the canonical thin Keplerian accretion disks cannot smoothly match any plain advection-dominated accretion flow (ADAF) model. By “plain” ADAF models we mean the ones with zero cooling. The existence of sonic points in exact solutions is critical and imposes constraints that cannot be surpassed adopting “reasonable” physical conditions at the hypothetical match point. Only the occurrence of new critical physical phenomena may produce a transition. We propose that exact advection models are a class of solutions that do not necessarily involve the standard thin cool disks and suggest a different scenario in which good ADAF solutions could eventually occur.

Subject headings: accretion, accretion disks — black hole physics — hydrodynamics

1. INTRODUCTION

Recently, the need to explain black hole candidate spectra and the low luminosity of some galactic nuclei, supposed to contain black holes, prompted more attention to advective accretion disk models (Narayan & Yi 1995). The basic idea is that the gravitational energy content does not go into local radiation emission as in standard thin and cool Keplerian disks (Shakura-Sunyaev accretion disks, or SSDs) (Shakura & Sunyaev 1973) but goes into heating and into radial motion. Therefore, a new class of disks has been proposed: advection-dominated accretion flow (ADAF) disks. These disk models take into account the advection and pressure terms in the fluid dynamics equations. Indeed, exact solutions of accretion flows (including viscosity, but only for isothermal cases) were found years ago by Chakrabarti (1990). The literature has been enriched by many kinds of such advective disks with different physical properties; see Bisnovatyi-Kogan (1999) for an extensive review. We will examine the very simple ADAF models initially proposed (Narayan & Yi 1995; Narayan, Kato, & Honma 1997; Igumeshev, Abramowicz, & Novikov 1998; Dullemond & Turolla 1998; Lu, Gu, & Yuan 1999), i.e., the ones without any cooling. As stated by the authors, although this model may appear very crude, it contains the basic ingredients: the roles of pressure gradient and of the advection term. Analytical solutions have been obtained only for the self-similar case (Narayan & Yi 1995), but this approximation is not tenable for the case of black hole (BH) accretion, since all standard ADAF solutions rely on the conditions at the sonic point, which is close to the BH.

Other solutions have typically been obtained with numerical integration of the steady state equations. The most used technique is the relaxation method with boundary conditions given by “reasonable” physical approximations. In all works there is the meaningful but a priori idea that the ADAF must necessarily spring out of a canonical disk. Thus, all the numerical procedures compel the solutions to fit the standard Kepler disk. It is worthwhile to add that it is well known that the relaxation methods can relax onto unphysical solutions.

A simple energy consideration gives a cutting argument that the canonical thin Keplerian accretion disks cannot smoothly match any ADAF to SSD disks. The fact that self-similar solutions by Narayan & Yi cannot be automatically connected to cold thin disks has already been indicated by Kato & Nakamura (1998). Here we point out that even the more sophisticated (not self-similar) ADAF solutions cannot have an asymptotic approach to the SSD models. We conclude that, possibly, standard “plain” ADAF models are “numerically forced” solutions and the claimed asymptotic approach to a Keplerian disk is not demonstrated.

2. BASIC EQUATIONS

In the plain ADAF model it is assumed that it does not contain any diffusive term and the cooling processes are irrelevant, as they occur on timescales much longer than the advection fall time. No heat conduction is considered as in Narayan & Yi (1995), Narayan et al. (1997), and Dullemond & Turolla (1998). Then the basic equations are the ones given by classical viscous fluid dynamics. We adopt Newtonian physics, and the BH force on the fluid can be derived from the appropriate Paczyński & Wiita potential (Paczyński & Wiita 1980). This is also a standard procedure in many papers. On the other hand, the classical approach makes more clear the basic physical point without inessential complications due to an exact relativistic treatment. Therefore, an exact disk solution has to obey

1. the mass conservation equation

\[ \dot{M} = r \Sigma v_r = \text{const}; \]

2. the radial momentum equation

\[ v_r \frac{dv_r}{dr} = -\frac{1}{\Sigma} \frac{d\Pi}{dr} - G \frac{M_*}{(r-r_s)^2} + \frac{\lambda^2}{r^3}, \]

where \( \Sigma \) and \( \Pi \) are the vertically integrated density and pressure;

3. the stress definition

\[ \tau_{\theta \phi} = \eta v_r \frac{\partial \Omega}{\partial r}, \]

where \( \eta = \alpha \rho a \) with \( a = [\gamma (P/p)]^{1/2} \) the sound speed;
4. the angular momentum equation
\[ \frac{\rho v_r d\lambda}{r dr} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \tau_{rb} \right) \]
that leads to
\[ \dot{M} \lambda = 4\pi r^2 \tau_{rb} + \dot{J}; \]

5. the vertical equilibrium, which gives the disk thickness equation
\[ Z_{\text{disk}} = \frac{a}{\Omega_\star}; \]

6. the energy equation
\[ \frac{de}{dt} = v_r \frac{de}{dr} = - \frac{P}{\rho} \frac{1}{r^2} \frac{d(r\nu)}{dr} + \frac{\Phi}{\rho}, \]
with \( \epsilon \) the thermal energy per unit mass and \( \Phi \) the dissipation function given by \( \Phi = (\tau_{rb})^2/\eta \); and

7. the equation of state
\[ P = (\gamma - 1) \rho \epsilon. \]

The term \( \Omega_\star \) is the Keplerian angular velocity; all other terms have their standard meaning accepted in the referenced papers.

3. BOUNDARY CONDITIONS

With the help of the energy equation, which will be discussed in the next section, our problem is reduced to solving a set of three algebraic equations (vertical equilibrium, mass, energy) and two differential equations (radial momentum and angular momentum). The existence of sonic points, requiring the continuity of the variables and of the speed derivatives, links these boundary conditions. In particular, once the sonic point is determined, then also the derivative of the radial speed at that point is fixed. The derivation of the analytical formula of \( \partial V/\partial r \) at the sonic point is algebraically complicated but straightforward. The usual assumption \( \partial \lambda/\partial r = 0 \) at the sonic point is reasonable only for a sonic point close to the BH and for low viscosities, and it is not true in general. Because of the different approaches to the numerical solutions (relaxation from fixed inner and outer points, standard Runge-Kutta integration from sonic point), there is no consensus on the exactness of the adopted boundary conditions. Our solutions are computed integrating the equations starting from the sonic point with the appropriate analytical derivative values calculated by the l’Hôpital rule. Artemova et al. (2001), using a similar treatment of the sonic point conditions, have obtained sub-Keplerian flows starting from canonical disks. However, their results do not disprove our conclusions since they refer to disks obtained taking into account radiation transfer and using different viscosity prescriptions. Apart from any further consideration on boundary conditions, we show that some crucial conditions are imposed by the energy equation.

4. THE ENERGY ARGUMENT

An interesting point, apparently not exploited in the ADAF context although appearing in many papers (Hoshi 1984; Honma 1996), is the fact that it is possible to have a constant energy property, reformulating the energy equation in the following way:
\[ \text{div} \left\{ \rho v \left[ \frac{1}{2} v^2 + \Psi(r) + h - v \cdot \mathbf{T} \right] \right\} = 0. \]

The quantity \( h = P/\rho + \epsilon \) is the enthalpy; \( \mathbf{T} \) is the stress tensor.

This expression, after insertion of the stress definition, integration over \( Z \), and use of the mass conservation equation, in general gives the following relationship, even in the differential form of the viscosity tensor:
\[ \frac{1}{2} v^2 = \frac{GM_\star}{(r - r_*)} + \frac{a^2}{r^2} + \frac{\lambda^2}{2r^2} + \frac{\lambda \epsilon}{r^2} = \text{const}/M. \]

The quantity \( \lambda_\star \) is the angular momentum at the inner edge of the disk, usually set equal to the value corresponding to the last stable orbit around the BH.

The energy equation is valid over the whole ADAF solution up to the eventual crossing point with the Keplerian disk. A smooth transition between ADAF and SSD disk requires a continuous smooth limit of the ADAF quantities to the SSD quantities: temperature, radial speed, density, etc.

What is crucial in this formula is that, at the crossing point of the solutions, where the angular momentum is Keplerian, the following relation is obviously valid:
\[ \frac{\lambda_\star}{2r_*} < \frac{\lambda_\star}{r^2} \]
since it is reasonable and common to choose \( \lambda_\star \ll \lambda_r \). Now, if the ADAF solution has to match smoothly a standard Keplerian disk at a radial distance far from the BH \( (r \gg r_*) \), we can substitute \( \lambda_\star \) for \( \lambda_r \); another reasonable and usual assumption is that at the match point the flow be subsonic (Igumenshchek et al. 1998).

Therefore, at the crossing point \( r_* \) we must have
\[ \frac{a^2}{(\gamma - 1) r_*} = K + \frac{GM_\star}{r_*} + \frac{\lambda_\star}{2r_*^2} = K + \frac{3}{2} GM_\star/r_* \]

Now, requiring that the energy constant be the same (or of the same order) as the energy per unit mass of a standard Keplerian disk, i.e., \( K = -\frac{1}{2}(GM_\star/r_*) \), we have
\[ \frac{a^2}{(\gamma - 1) r_*} = \frac{GM_\star}{r_*}. \]

\( GM_\star/r_* \) corresponds to the virial temperature, much larger than the standard temperature of a canonical thin Keplerian disk. We have a strong inconsistency: the ADAF temperature, at the matching point, is always much larger than the canonical Keplerian disk temperature (Frank, King, & Raine 1992).

Indeed, Lu et al. (1999) use our same numerical approach (i.e., starting from the sonic point), and apparently they find cases with a smooth convergence from ADAF to SSD; however, it is worth nothing that their parameter space, corresponding to the Narayan ADAF model, has zero measure. Those configurations lie on a line in their parameter space \( R_\star, \lambda_r \). There-
Fig. 1.—Angular momentum per unit mass vs. radial distance for $\alpha = 0.005, 0.01, 0.02$.

Fig. 2.—Mach number and sound speed vs. radial distance for $\alpha = 0.005, 0.01, 0.02$.

fore, they can be obtained only for a single $\lambda_s$ if the sonic point is given.

5. SOLUTIONS

We performed several integrations using the direct method, starting from the sonic point inward and outward. Figure 1 shows the result of the Runge-Kutta integration of our equations. We plotted the angular momentum per unit mass versus the radial distance for $\alpha = 0.005, 0.01, 0.02$. As the viscosity increases, the angular momentum crosses the Keplerian angular momentum at distances closer to the BH. Figure 2 shows the sound speed and the Mach number for the same $\alpha$ values. The sound speed at the crossing point does not go to extremely low values. The values we obtain are exactly the ones predicted by the energy equation and go to a finite limit at a finite radius located beyond the Keplerian crossing point. The angular momentum of ADAF solutions crosses the Kepler angular momentum value in a non-smooth way. The exact solution continues beyond that point and stops at a larger distance with disk conditions (temperature, density, radial speed) not corresponding to the canonical Keplerian disk. Our correct ADAF solutions are, per se, standing solutions. This kind of solution is a particular case of the more general solutions drawn by Chakrabarti & Titarchuck (1995) and Chakrabarti (1996), who explored a more general case including an ad hoc cooling treatment. We add a further comment on the shock or no shock solutions, since many authors consider the Chakrabarti solutions untenable, as the shock he finds had not been confirmed by their calculations. As has been pointed out in early works (Liang & Thompson 1980), there are many sonic points in the flow solutions. The flow on a BH has no rigid surface, and shock occurs only in solutions passing through the outer sonic point as clearly demonstrated by Chakrabarti (1990) and Chakrabarti & Molteni (1993). All the ADAF solutions we are discussing pass through the inner sonic point, so it is quite obvious that no shock can occur in these branches: the flow is supersonic only after the sonic point, which is close to the BH horizon. Lanzafame, Molteni, & Chakrabarti (1998) tested these solutions with a time-dependent code and found that these solutions are stable, while the standard ADAF solutions have been demonstrated unstable by numerical simulations (Igumenshchev, Chen, & Abramowicz 1996).

6. CONCLUSIONS

We conclude that plain ADAF solutions that do not respect the full set of sonic point conditions, including derivatives at sonic point, are incorrect. However, since ADAF models seem to offer explanations of relevant phenomena, to account for their existence, and in particular for the formation of a hot Comptonizing corona, we propose that a sub-Keplerian thin flow exists by itself, superposed to the standard Keplerian disk. In accretion occurring in active galactic nuclei, these sub-Keplerian flows could easily come from the star cluster, which is known to have a very slow average rotation (Ho 1999). In a low-mass binary system this sub-Kepler flow could come from a wind induced by the high-energy radiation impinging on the normal star surface, or from gas recirculating in the binary potential well (Bisikalo et al. 1998). We are also testing the hypothesis that a physically motivated change in $\alpha$ of the viscosity prescription can lead to differentially rotating flows.

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