Implications evinced by the phase diagram, anisotropy, magnetic penetration depths, isotope effects and conductivities of cuprate superconductors

T Schneider and H Keller
Physik-Institut der Universität Zürich, Winterthurerstrasse 190, CH-8057 Zürich, Switzerland
E-mail: toni.schneider@swissonline.ch

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Abstract. Anisotropy, thermal and quantum fluctuations and their dependence on dopant concentration appear to be present in all cuprate superconductors, interwoven with the microscopic mechanisms responsible for superconductivity. Here we review anisotropy, in-plane and c-axis penetration depths, isotope effect and conductivity measurements to reassess the universal behaviour of cuprates as revealed by the doping dependence of these phenomena and of the transition temperature.

Establishing and understanding the phase diagram of cuprate superconductors in the temperature–dopant concentration plane is one of the major challenges in condensed matter physics. Superconductivity is derived from the insulating and antiferromagnetic parent compounds by partial substitution of ions or by adding or removing oxygen. For instance La$_2$CuO$_4$ can be doped either by alkaline earth ions or oxygen to exhibit superconductivity. The empirical phase diagram of La$_{2-y}$Sr$_y$CuO$_4$ [1]–[9] depicted in figure 1(a) shows that after passing the so-called underdoped limit ($x_u \approx 0.047$), $T_c$ reaches its maximum value $T_c(x_m)$ at $x_m \approx 0.16$. With further increase of $x$, $T_c$ decreases and finally vanishes in the overdoped limit $x_o \approx 0.273$. This phase-transition line is thought to be a generic property of cuprate superconductors [10] and is well described by the empirical relation

$$T_c(x) = \frac{4T_c(x_m)}{(x_o - x_u)^2(x - x_u)(x_o - x)},$$

(1)
Figure 1. (a) Variation of $T_c$ for $\text{La}_2-x\text{Sr}_x\text{CuO}_4$. Experimental data taken from [1]–[9]. The solid line is equation (1) with $T_c(x_m) = 39$ K. (b) $\gamma_T$ versus $x$ for $\text{La}_2-x\text{Sr}_x\text{CuO}_4$. The squares are the experimental data for $\gamma_T$ [1, 2, 4, 6, 7] and the triangles for $\gamma_T=0$ [8, 9]. The solid curve and dashed lines are equation (2) with $\gamma_{T,0} = 2$ and $\gamma_{T=0,0} = 1.63$.

proposed by Presland et al [11]. Approaching the endpoints along the $x$-axis, $\text{La}_2-x\text{Sr}_x\text{CuO}_4$ undergoes at zero temperature doping-tuned quantum phase transitions. As their nature is concerned, resistivity measurements reveal a quantum superconductor-to-insulator (QSI) transition in the underdoped limit [12]–[18] and in the overdoped limit a quantum superconductor-to-normal state (QSN) transition [12].

Another essential experimental fact is the doping dependence of the anisotropy. In tetragonal cuprates, it is defined as the ratio $\gamma = \xi_{ab}/\xi_c$ of the correlation lengths parallel ($\xi_{ab}$) and perpendicular ($\xi_c$) to CuO$_2$ layers ($ab$-planes). In the superconducting state it can also be expressed as the ratio $\gamma = \lambda_c/\lambda_{ab}$ of the London penetration depths due to supercurrents flowing perpendicular ($\lambda_c$) and parallel ($\lambda_{ab}$) to the $ab$-planes. On approaching a non-superconductor to superconductor transition $\xi$ diverges, while in a superconductor to non-superconductor transition $\lambda$ tends to infinity. In both cases, however, $\gamma$ remains finite as long as the system exhibits anisotropic but genuine 3D behaviour. There are two limiting cases: $\gamma = 1$ characterizes isotropic 3D- and $\gamma = \infty$ 2D-critical behaviour. An instructive model where $\gamma$ can be varied continuously is the anisotropic 2D Ising model [19]. When the coupling in the $y$ direction goes to zero, $\gamma = \xi_x/\xi_y$ becomes infinite, the model reduces to the 1D case, and $T_c$ vanishes. In the Ginzburg–Landau description of layered superconductors the anisotropy is related to the interlayer coupling. The weaker this coupling is, the larger $\gamma$ is. The limit $\gamma = \infty$ is attained when the bulk superconductor corresponds to a stack of independent slabs of thickness $d_s$. With respect to experimental work, a considerable amount of data is available on the chemical composition dependence of $\gamma$. At $T_c$ it can be inferred from resistivity ($\gamma = \xi_{ab}/\xi_c = \sqrt{\rho_{ab}/\rho_c}$) and magnetic torque measurements, while in the superconducting state it follows from magnetic torque and penetration depth ($\gamma = \lambda_c/\lambda_{ab}$) data. In figure 1(b) we displayed the doping dependence of $\gamma_T$ evaluated at $T_c$ ($\gamma_{T,T}$) and $T = 0$ ($\gamma_{T=0,T}$). As the dopant concentration is reduced, $\gamma_{T,T}$ and $\gamma_{T=0,T}$ increase systematically, and tend to diverge in the underdoped limit. Thus the temperature range where superconductivity occurs shrinks in the underdoped regime with increasing anisotropy.
This competition between anisotropy and superconductivity raises serious doubts about whether 2D mechanisms and models, corresponding to the limit $\gamma_T = \infty$, can explain the essential observations of superconductivity in the cuprates. From figure 1 it is also seen that $\gamma_T(x)$ is well described by

$$\gamma_T(x) = \frac{\gamma_{T,0}}{x - x_u},$$  \hspace{1cm} (2)$$

where $\gamma_{T,0}$ is the quantum critical amplitude. Having also other cuprate families in mind, it is convenient to express the dopant concentration in terms of $T_c$. From (1) and (2) we obtain the correlation between $T_c$ and $\gamma_T$:

$$\frac{T_c(x)}{T_c(x_m)} = 1 - \left( \frac{\gamma_T(x_m)}{\gamma_T(x)} - 1 \right)^2, \quad \gamma_T(x_m) = \frac{\gamma_{T,0}}{x_m - x_u}. \hspace{1cm} (3)$$

Provided that this empirical correlation is not merely an artefact of La$_{2-x}$Sr$_x$CuO$_4$, it gives a universal perspective on the interplay of anisotropy and superconductivity, among the families of cuprates, characterized by $T_c(x_m)$ and $\gamma_T(x_m)$. For this reason it is essential to explore its generic validity. In practice, however, there are only a few additional compounds, including HgBa$_2$CuO$_{4+\delta}$ [20], for which the dopant concentration can be varied continuously throughout the entire doping range. It is well established, however, that the substitution of magnetic and nonmagnetic impurities, depress $T_c$ of cuprate superconductors very effectively [21, 22]. To compare the doping and substitution driven variations of the anisotropy, we depicted in figure 2 $T_c(x)/T_c(x_m)$ versus $\gamma_T(x_m)/\gamma_T(x)$ for a variety of cuprate families. The collapse of the data on the parabola, which is the empirical relation (3), reveals that this scaling form appears to be universal. Thus, given a family of cuprate superconductors, characterized by $T_c(x_m)$ and $\gamma_T(x_m)$, it gives a universal perspective on the interplay between anisotropy and superconductivity.

Close to 2D-QSI criticality various properties are not independent but related by [13]–[18]

$$T_c = \frac{\Phi_0^2 R_2}{16\pi^3 k_B \lambda_{ab}(0)} \propto d_s/\lambda_{ab}(0) \propto \delta z \propto \delta \nu, \hspace{1cm} (4)$$

independently of the nature of the putative quantum critical point. $k_B$ is the Boltzmann constant, and $\Phi_0$ the elementary flux quantum. $\lambda_{ab}(0)$ is the zero temperature in-plane penetration depth, $z$ is the dynamic critical exponent, $d_s$ the thickness of the sheets, and $\nu$ the correlation length critical exponent of the 2D-QSI transitions. $\delta$ measures the distance from the critical point along the $x$ axis (see figure 1(a)) and $R_2$ is a universal number. To appreciate (4) we note that it is just the quantum analogue of the relation between $T_c$ and $1/\lambda_{ab}(T_c)$ at the Berezinskii–Kosterlitz–Thouless transition in 2D-superconductors [29]. Together with the scaling form (4) the empirical relation (1) implies 2D-QSI and 3D-QSN transitions with $z\nu = 1$, while the empirical behaviour of the anisotropy (equations (2) and (3)) yields $\nu = 1$ at the 2D-QSI criticality. Thus, the empirical correlations point to a 2D-QSI transition with $z = 1$ and $\nu = 1$. These estimates coincide with the theoretical predictions for the transition associated with the onset of superfluidity in a 2D disordered bosonic system with long-range Coulomb interactions where $z = 1$ and $\nu \simeq 1$ [30]–[32]. Here the loss of superfluidity is due to the localization of the pairs, which ultimately drives the transition. This reveals that in cuprates the loss of phase coherence, due to the localization of Cooper pairs, is responsible for the 2D-QSI transition. Since $T_c \propto d_s/\lambda_{ab}(0) \propto n_s^{\square}$, where $n_s^{\square}$ is the aerial superfluid density, is a characteristic 2D property, it also applies to the onset
of superfluidity in $^4$He films adsorbed on disordered substrates, where it is well confirmed [33]. A great deal of experimental work has also been done in cuprates on the so-called Uemura plot, revealing an empirical correlation between $T_c$ and $\frac{ds}{\lambda_{ab}(0)}$ [34]. Approaching 2D-QSI criticality, the data of a given family tends to fall on a straight line, consistent with (4). Differences in the slope reflect the family dependent value of $d_s$, the thickness of the sheets, becoming independent in the 2D limit [14]–[18]. The relevance of $d_s$ was also confirmed in terms of the relationship between the isotope effect on $T_c$ and $\frac{1}{\lambda_{ab}(T)}$ [16, 35]. From the scaling relation (4) it is seen that measurements of the out-of-plane penetration depth of sufficiently underdoped systems allow to estimate the dynamic critical exponent $z$ directly, in terms of $T_c \propto \left(\frac{1}{\lambda_{c}(0)}\right)^{\frac{1}{z(\gamma_T^2+2)}}$, which follows from (4) with $\gamma_T = \frac{\lambda_{c}(0)}{\lambda_{ab}(0)}$. In figure 3 we displayed the data of Hosseini et al [36] for heavily underdoped YBa$_2$Cu$_3$O$_{7-\delta}$ single crystals. The solid line is $1/\lambda_c^2(T) = 2.44 \times 10^{-4} T_c^3$ and uncovers the consistency with the 2D-QSI scaling relation $T_c \propto \left(\frac{1}{\lambda_{c}(0)}\right)^{\frac{1}{z(\gamma_T^2+2)}}$ with $z = 1$.

Furthermore, Hosseini et al [36] found that at low temperature $1/\lambda_{c}^{2}(T) - 1/\lambda_{c}^{2}(0)$ is nearly doping independent. Given this behaviour close to the 2D-QSI transition, scaling predicts that in leading order $1/\lambda_{c}^{2}(T) - 1/\lambda_{c}^{2}(0) \propto T^3$ holds, in good agreement with the experimental data displayed in figure 4. As the in-plane penetration depth is concerned, there is mounting experimental evidence that $1/\lambda_{ab}^{2}(T) - 1/\lambda_{ab}^{2}(0)$ is in the limit $T \to 0$ nearly doping independent as well [15]. In this case, 2D-QSI scaling predicts in leading order $1/\lambda_{ab}^{2}(T) - 1/\lambda_{ab}^{2}(0) \propto T$, in agreement with the experimental data of numerous cuprates [15].

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**Figure 2.** $T_c(x)/T_c(x_m)$ versus $\gamma_T(x_m)/\gamma_T(x)$ for various cuprate families: La$_{2-x}$Sr$_x$CuO$_4$ (●, $T_c(x_m) = 37$ K, $\gamma_T(x_m) = 20$) [1, 2, 4, 6, 7], (○, $T_c(x_m) = 37$ K, $\gamma_T=0(x_m) = 14.9$) [8, 9], HgBa$_2$CuO$_{4+\delta}$ (▲, $T_c(x_m) = 95.6$ K, $\gamma_T(x_m) = 27$) [20], Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ (★, $T_c(x_m) = 84.2$ K, $\gamma_T(x_m) = 133$) [23], YBa$_2$Cu$_3$O$_{7-\delta}$ (◆, $T_c(x_m) = 92.9$ K, $\gamma_T(x_m) = 8$) [24], YBa$_2$(Cu$_{1-y}$Fe$_y$)$_3$O$_{7-\delta}$ (■, $T_c(x_m) = 92.5$ K, $\gamma_T(x_m) = 9$) [25], Y$_{1-x}$Pr$_x$Ba$_2$Cu$_3$O$_{7-\delta}$ (▼, $T_c(x_m) = 85.4$ K, $\gamma_T=0(x_m) = 94.3$) [27] and YBa$_2$(Cu$_{1-y}$Zn$_y$)$_3$O$_{7-\delta}$ (▲, $T_c(x_m) = 92.5$ K, $\gamma_T=0(x_m) = 9$) [28]. The solid and dashed curves are (3), marking the flow from the maximum $T_c$ to QSI and QSN criticality, respectively.
We have seen that the doping tuned flow to the 2D-QSI critical point is associated with a depression of $T_c$ and an enhancement of $\gamma T$. It implies that when the 2D-QSI transition is approached, a no vanishing $T_c$ is inevitably associated with an anisotropic but 3D condensation mechanism, because $\gamma T$ is finite for $T_c > 0$ (see figures 1 and 2). This represents a serious problem for 2D models [37] as candidates to explain superconductivity in the cuprates, and serves as a constraint on future work towards a complete understanding. Note that the vast majority of
Figure 5. Data for the oxygen isotope effect in underdoped La$_{2-x}$Sr$_x$CuO$_4$ ($\bigcirc$: $x = 0.15$ [39], $\blacktriangle$: $x = 0.08, 0.086$ [38], Y$_{1-x}$Pr$_x$Ba$_2$Cu$_3$O$_{7-\delta}$ ($\blacklozenge$: $x = 0, 0.2, 0.3, 0.4$) [39]–[41] and YBa$_2$Cu$_3$O$_{7-\delta}$ ($\blacktriangle$ [39], $\blacksquare$ [42]) in terms of $\Delta\lambda_{ab}(0)/\lambda_{ab}(0)$ versus $-\Delta T_c/T_c$. The solid line indicates the flow to 2D-QSI criticality and provides with (5) an estimate for the oxygen isotope effect on $d_s$, namely $\Delta d_s/d_s = 3.3(4)\%$.

Theoretical models focus on a single Cu-O plane, i.e., on the limit of zero intracell and intercell $c$-axis coupling.

Since (4) is universal, it also implies that the changes $\Delta T_c$, $\Delta d_s$ and $\Delta(1/\lambda_{ab}^2(T = 0))$, induced by pressure or isotope exchange are not independent, but related by

$$\frac{\Delta T_c}{T_c} = \frac{\Delta d_s}{d_s} + \frac{\Delta(1/\lambda_{ab}^2(0))}{(1/\lambda_{ab}^2(0))} = \frac{\Delta d_s}{d_s} - 2\frac{\Delta(\lambda_{ab}(0))}{\lambda_{ab}(0)}. \tag{5}$$

In particular, for the oxygen isotope effect ($^{16}$O versus $^{18}$O) of a physical quantity $X$ the relative isotope shift is defined as $\Delta X/X = (^{18}X - ^{16}X)/^{18}X$. In figure 5 we show the data for the oxygen isotope effect in La$_{2-x}$Sr$_x$CuO$_4$ [38, 39], Y$_{1-x}$Pr$_x$Ba$_2$Cu$_3$O$_{7-\delta}$ [39]–[41] and YBa$_2$Cu$_3$O$_{7-\delta}$ [39, 42], extending from the underdoped to the optimally doped regime, in terms of $\Delta(\lambda_{ab}(0))/\lambda_{ab}(0)$ versus $\Delta T_c/T_c$.

It is evident that there is a correlation between the isotope effect on $T_c$ and $\lambda_{ab}(0)$ which appears to be universal for all cuprate families. Indeed, the solid line indicates the flow to the 2D-QSI transition and provides with (5) an estimate for the oxygen isotope effect on $d_s$, namely $\Delta d_s/d_s = 3.3(4)\%$. Approaching optimum doping, this contribution renders the isotope effect on $T_c$ considerably smaller than that on $\lambda_{ab}(0)$. Indeed, even in nearly optimally doped YBa$_2$Cu$_3$O$_{7-\delta}$, where $\Delta T_c/T_c = -0.26(5)\%$, a substantial isotope effect on the in-plane penetration depth, $\Delta\lambda_{ab}(0)/\lambda_{ab}(0) = -2.8(1.0)\%$, has been established by direct observation, using the novel low-energy muon-spin rotation technique [42]. Note that these findings have been obtained using various experimental techniques on powders, thin films and single crystals.
In this context it is important to recognize that the substantial isotope effect on the in-plane penetration depth uncovers the coupling between local lattice distortions and superfluidity. Indeed, since the lattice parameters remain essentially unaffected [43, 44] by isotope exchange, while the dynamics associated with the mass of the respective ions is modified, the isotope effect on \( ds \), the thickness of the superconducting sheets, the relative shift, \( \Delta d_s/d_s \approx 3.3(4)\% \), apparent in figure 5, implies the coupling to local oxygen distortions which do not modify the lattice parameters but are coupled to the superfluid. Although the majority opinion on the mechanism of superconductivity in the cuprates is that it occurs through a purely electronic mechanism involving spin excitations, and lattice degrees of freedom are supposed to be irrelevant, this shift uncovers clearly the existence and relevance of the coupling between the superfluid and local lattice distortions.

Finally, we turn to the finite temperature critical behaviour. Close to the critical temperature \( T_c \) of the superconductor to normal state transition, in a regime roughly given by the Ginzburg criterion [14], [45]–[47], order parameter fluctuations dominate critical properties. In recent years, the effect of the charge of the superconducting order parameter in this regime in three dimensions has been studied extensively [48]–[55], [57]. While for strong type-I materials, the coupling of the order parameter to transverse gauge field fluctuations is expected to render the transition first order [49], it is well-established that strong type-II materials should exhibit a continuous phase transition, and that sufficiently close to \( T_c \), the charge of the order parameter is relevant [51]–[57]. However, in cuprate superconductors within the fluctuation dominated regime, the region close to \( T_c \), where the system crosses over to the regime of charged fluctuations turns out to be too narrow to access. For instance, optimally doped \( \text{YBa}_2\text{Cu}_3\text{O}_{7-\delta} \), while possessing an extended regime of critical fluctuations, is too strongly type-II to observe charged critical fluctuations [14], [45]–[47], [58]. Indeed, the effective dimensionless charge \( \tilde{e} = \xi/\lambda = 1/\kappa \) is small in strongly type II superconductors (\( \kappa \gg 1 \)). The crossover upon approaching \( T_c \) is thus initially to the critical regime of a weakly charged superfluid where the fluctuations of the order parameter are essentially those of an uncharged superfluid or XY-model [45]. Furthermore, there is the inhomogeneity-induced finite-size effect which renders the asymptotic critical regime unattainable [59]–[61]. However, underdoped cuprates could open a window onto this new regime because \( \kappa_{ab} \) is expected to become rather small. As outlined above, in this regime cuprates undergo a quantum superconductor to insulator transition [14]–[18] and correspond to a 2D disordered bosonic system with long-range Coulomb interactions. Close to this quantum transition \( T_c \), \( \lambda_{ab} \) and \( \xi_{ab} \) scale as \( T_c \propto \lambda_{ab}^{-z} \propto \xi_{ab}^{-2(c+2)/z} \) [14]–[18], while \( \xi_c \rightarrow d_s \). These relations yield with the dynamic critical exponent \( z = 1 \) [14], [18], [30]–[32], \( \kappa_{ab} \propto T_c^{1/2} \) and \( \kappa_c \propto T_c^{-3/2} \). Noting that \( T_c \) decreases by approaching the underdoped limit, the in-plane penetration depth appears to be a potential candidate to observe charged criticality in sufficiently homogeneous and underdoped cuprates, while the \( c \)-axis counterpart is expected to exhibit neutral critical behaviour.

When charged fluctuations dominate the in-plane penetration depth and the correlation length are related by [53]–[57]

\[
\lambda_{ab} = \kappa_{ab} \xi_{ab}, \quad \lambda_{ab} = \lambda_{0ab} |t|^{-\nu}, \quad \nu \approx 2/3, \tag{6}
\]

contrary to the uncharged case, where \( \lambda \propto \sqrt{\xi} \) and with that

\[
\lambda_{ab} = \lambda_{0ab} |t|^{-\nu/2}, \tag{7}
\]
Figure 6. \((T_c \, d \ln \lambda_{ab}/dT)^{-1}\) with \(\lambda_{ab}\) in \(\mu m\) versus \(t\) for YBa\(_2\)Cu\(_3\)O\(_{6.59}\) under hydrostatic pressure of 0.5 kbar (△) and 10.5 kbar (○) taken from [62]. The straight line with slope \(1/\nu \simeq 3/2\) corresponds according to (6) to charged critical behaviour with \(T_c = 57.82\) and 61.1 K.

where \(t = T/T_c - 1\). In a plot \((d \ln \lambda_{ab}/dT)^{-1}\) versus \(T\) charged critical behaviour is then uncovered in terms of a temperature range where the data falls on a line with slope \(1/\nu \simeq 3/2\), while in the neutral (3D-XY) case it collapses on a line with slope \(2/\nu \simeq 3\). Clearly, in an inhomogeneous system the phase transition is rounded and \((d \ln \lambda_{ab}/dT)^{-1}\) does not vanish at \(T_c\). In figure 6 we displayed \((T_c \, d \ln \lambda_{ab}/dT)^{-1}\) versus \(t\) for YBa\(_2\)Cu\(_3\)O\(_{6.59}\) with applied hydrostatic pressures of 0.5 and 10.5 kbar, derived from the measured in-plane penetration depth data of [62]. The solid line with slope \(1/\nu \simeq 3/2\) indicates according to (6) the charged criticality in homogeneous systems with \(T_c = 57.82\) and 61.1 K. Although the charged criticality is attained, there is no sharp transition because \(\lambda_{ab}\) does not diverge at \(T_c\). Inhomogeneities prevent the correlation length \(\xi_{ab} = \lambda_{ab}/\kappa_{ab}\) to grow beyond the spatial extent \(L_{ab}\) of the homogenous domains in the \(ab\)-plane. For a discussion of the associated finite-size effect we refer to [62]. In any case, these measurements clearly reveal that the temperature dependence of the in-plane penetration depth opens a window onto charged criticality at finite temperature in underdoped cuprates.

Extended \(c\)-axis penetration depth measurements on YBa\(_2\)Cu\(_3\)O\(_{6+x}\) single crystals with \(T_c\)'s ranging from 4 to 20 K have been performed by Hosseini et al [36]. In figure 6 we displayed their data for the sample with \(T_c \approx 17.95\) K in terms \(\lambda_{c}^{-2}\) and \((d \ln \lambda_{c}^{-2}/dT)^{-1}\) versus \(T\). The solid curve is \(\lambda_{c}^{-2} = \lambda_{0c}^{-2} (1 - T/T_c)\nu\) with \(T_c = 19.75\) K and \(\lambda_{0c}^{-2} = 2.15 (10^9\) m\(^{-2}\) and the dashed one \((d \ln \lambda_{c}^{-2}/dT)^{-1} = -1/\nu(T_c - T)\) with \(\nu \simeq 2/3\).

They indicate according to (7) the leading uncharged (3D-XY) critical behaviour. Noting that in this underdoped regime, the anisotropy \(\gamma = \lambda_c/\lambda_{ab}\) is rather large (see figure 2), the critical regime, where thermal 3D fluctuations dominate, will be much narrower compared to optimum doping. Here it extends to \(t \approx 77/93.7 - 1 \simeq -0.18\) [58]. Nevertheless, a glance to figure 7 shows that the 3D-XY critical regime is attained for \(18 \lesssim T \lesssim 19\) K, corresponding
Figure 7. $\lambda_c^{-2}$ and $(d \ln \lambda_c^{-2}/dT)^{-1}$ versus $T$ for a YBa$_2$Cu$_3$O$_{6+\delta}$ single crystal derived from the data of Hosseini et al [36]. The solid curve is $\lambda_c^{-2} = \lambda_{0c}^{-2} (1 - T/T_c)^{\nu}$ with $T_c = 19.75$ K and $\lambda_{0c}^{-2} = 2.15$ $(10^9$ m$^{-2}$) and the dashed one $(d \ln \lambda_c^{-2}/dT)^{-1} = -1/\nu(T_c - T)$ with $\nu \simeq 2/3$. They indicate according to (7) the leading uncharged (3D-XY) critical behaviour.

to $0.09 \gtrsim |t| \gtrsim 0.04$. The tail above 19 K is attributable to an inhomogeneity-induced finite-size effect. Indeed, according to the scaling relation $\kappa_c \propto T_c^{-3/2}$, it is unlikely that the crossover to charged criticality is attainable. In contrast and consistent with $\kappa_{ab} \propto T_c^{1/2}$, charged criticality turned out to be accessible in the $ab$-plane penetration depth.

We have seen that the linear relationship between $T_c$ and $1/\lambda_{ab}^2(0)$ in the underdoped regime, referred to as the Uemura relation, is a consequence of the dimensional crossover associated with the flow to the 2D-QSI transition (see (4)). To illustrate this behaviour we displayed in figure 8 $T_c$ versus $1/\lambda_{ab}^2(0)$ for La$_{2-x}$Sr$_x$CuO$_4$. The straight line is (4) in terms of $T_c = 25\lambda_{ab}^2(0)$ and the arrow indicates the flow to the 2D-QSI criticality. However, approaching the optimally doped regime, where $T_c$ reaches its maximum value (see figure 1), the observed $T_c$'s are systematically lower than the 2D-QSI line and this trend continues in the overdoped regime.

To provide an understanding we invoke 3D-XY universality expected to hold along the phase transition line, $T_c(x)$ in La$_{2-x}$Sr$_x$CuO$_4$ (see figure 1), as long as the charge of the pairs can be neglected. In this case the transition temperature $T_c$ and the critical amplitudes of the penetration depths $\lambda_{ab,0}$ and transverse correlation lengths $\xi_{ab,0}$ are related by [14, 18]

$$k_B T_c = \frac{\Phi_0^2}{16\pi^3} \frac{\xi_{ab,0}^r}{\lambda_{ab,0}^2} = \frac{\Phi_0^2}{16\pi^3} \frac{\xi_{c,0}^r}{\lambda_{c,0}^2},$$

where $\lambda_c^2(T) = \lambda_{0c}^2(1 - T/T_c)^{-\nu}$ and $\xi_c^r(T) = \xi_{c,0}^r(1 - T/T_c)^{-\nu}$ with $\nu \simeq 2/3$. For our purpose it is convenient to replace the transverse correlation length by the corresponding correlation lengths above $T_c$ in terms of [14, 18, 64]

$$\frac{\xi_{ab,0}^r}{\xi_{c,0}^r} = f \approx 0.453, \quad \frac{\xi_{ab,0}^r}{\xi_{c,0}^r} = \gamma f.$$
Figure 8. \( T_c \) versus \( 1/\lambda_{ab}^2(0) \) for La\(_{2-x}\)Sr\(_x\)CuO\(_4\). Data taken from Uemura et al [34, 63] and Panagopoulos et al [9]. The straight line is (4) with \( R_2d_s = 6.5 \text{ Å} \) and the arrow indicates the flow to 2D-QSI transition criticality.

where the anisotropy is given by

\[
\gamma^2 = \left( \frac{\lambda_{c0}}{\lambda_{ab0}} \right)^2 = \frac{\xi_{tr}^c}{\xi_{tr}^{ab}} = \left( \frac{\xi_{ab0}}{\xi_{c0}} \right)^2. \tag{10}
\]

Combining equations (8) and (9) we obtain the universal relation

\[
T_c \lambda_{ab}^2(0) = \frac{\Phi_0^2 f}{16\pi^2 k_B} \xi_{c0}. \tag{11}
\]

It holds, as long as cuprates fall into the 3D-XY universality class, irrespective of the doping dependence of \( T_c, \lambda_{ab0}^2 \) and \( \xi_{c0} \). For this reason it provides a sound basis for universal plots. However, there is the serious drawback that reliable experimental estimates for \( T_c \) and the critical amplitudes \( \lambda_{ab0} \) and \( \xi_{c0} \) measured on the same sample are not yet available. Nevertheless, some progress can be made by noting that in the underdoped regime, approaching the 2D-QSI transition the universal scaling forms (4) and (11) should match. This requires

\[
\lambda_{ab}(0) = \Lambda_{ab0}, \quad f\Lambda_{ab}^2 \xi_{c0} \rightarrow R_2d_s, \tag{12}
\]

so that away from 2D-QSI criticality

\[
T_c^2(0) \approx \frac{\Phi_0^2 f}{16\pi^2 k_B} \Lambda_{ab}^2 \xi_{c0}, \tag{13}
\]

holds. Thus, when both \( T_c \) and \( 1/\lambda_{ab}^2(0) \) increase, \( T_c \) values below \( T_c = (\Phi_0^2 R_2/6\pi^2 k_B)d_s/\lambda_{ab0}^2(0) \) (equation (4)) require \( \xi_{c0} \) to fall off from its limiting value \( \xi_{c0} = d_s R_2/(f\Lambda_{ab}^2) \). The doping dependence of \( \Lambda_{ab}^2 \xi_{c0} \) deduced from (13) and the experimental data for \( T_c \) and \( \lambda_{ab0}^2(0) \) is displayed...
in figure 9(a) in terms of $T_c$ versus $\Lambda_{ab}^2 \xi_{c,0}$ and $\Lambda_{ab}^2 \xi_{c,0}$ versus $x$. Approaching the underdoped limit ($x \approx 0.05$), where $T_c$ vanishes (figure 1) and the 2D-QSI transition occurs, $\Lambda_{ab}^2 \xi_{c,0}$ increases nearly linearly with decreasing $x$ to approach a fixed value. Indeed, the data is consistent with

$$\Lambda_{ab}^2 \xi_{c,0} = (16\pi^3 k_B/(\Phi_0^2 f)) T_c \Lambda_{ab}^2(0) \approx 14.34 - 60.47(x - 0.05) \text{Å},$$

yielding the limiting value $\Lambda_{ab}^2 \xi_{c,0}(x = 0.05) \approx 14.34 \text{Å}$ and $f \Lambda_{ab}^2 \xi_{c,0}(x = 0.05) = R_2 d_e \approx 6.5 \text{Å}$ used in figure 8. An essential result is that $\xi_{c,0}$ adopts in the underdoped limit ($x \approx 0.05$) where the 2D-QSI transition occurs and $T_c$ vanishes at the maximum value $\Lambda_{ab}^2 \xi_{c,0} \approx 14.34 \text{Å}$, which is close to the $c$-axis lattice constant $c \approx 13.29 \text{Å}$. Thus, a finite transition temperature requires a reduction of $\xi_{c,0}$, well described over an unexpectedly large doping range by equation (14). Noting that this relation transforms with (2) to

$$\Lambda_{ab}^2 \xi_{c,0} \approx 14.34 - 60.47\gamma_0/\gamma \text{Å},$$

this behaviour is intimately connected to the doping dependence of the anisotropy. Hence, a finite $T_c$ requires unavoidably a finite anisotropy $\gamma$. The lesson is, in agreement with the empirical relation (3), that superconductivity in La$_{2-x}$Sr$_x$CuO$_4$ is an anisotropic but 3D phenomenon which disappears in the 2D limit. From the plot $1/\Lambda_{ab}^2(0)$ versus $T_c/(\Lambda_{ab}^2 \xi_{c,0})$ displayed in figure 9(b), where according to equation (13) universal behaviour is expected to occur, the data is seen to fall rather well on a straight line. This is significant, as moderately underdoped, optimally and overdoped La$_{2-x}$Sr$_x$CuO$_4$ falling according to figure 8 well off the 2D-QSI behaviour $T_c \propto 1/\Lambda_{ab}^2(0)$ now scale nearly onto a single line. Thus the approximate 3D-XY scaling relation (13), together with the empirical doping dependence of the $c$-axis correlation length (equations (14) and (15)) are consistent with the available experimental data for La$_{2-x}$Sr$_x$CuO$_4$ and uncovers the relevance of the anisotropy. However, the linear doping dependence of $\xi_{c}$ is not
expected to hold closer to the overdoped limit ($x \approx 0.27$) where $T_c$ vanishes and a 3D quantum superconductor-to-normal state (3D-QSN) transition is expected to occur (see figure 1).

Since (11) is universal, it should hold for all cuprates falling in the accessible critical regime into the 3D-XY universality class, irrespective of the doping dependence of $T_c$, $\lambda_{ab0}$, $\xi_{ab0}$, $\gamma$ and $\xi_{c0}$. Having seen that charged criticality is accessible in the heavily underdoped regime only, (11) rewritten in the form

$$\frac{\xi_{c0}}{\lambda_{ab0}^2 T_c} = \frac{\xi_{ab0} \gamma}{\lambda_{c0}^2 T_c} = \frac{16\pi^3 k_B}{\Phi_0^2 f},$$

(16)

provides a sound basis for universal plots. However, previously mentioned there is the serious drawback that reliable experimental estimates for the critical amplitudes and the anisotropy at $T_c$ measured on the same sample and for a variety of cuprates are not yet available. Nevertheless, as in the case of $La_{2-x}Sr_xCuO_4$ outlined above, progress can be made by invoking the approximate scaling form (13) and by expressing the correlation lengths in terms of the real part of the frequency-dependent conductivity $\sigma_{dc}(\omega)$ in direction $i$ extrapolated to zero frequency at $T \gtrsim T_c$ [65] in terms of

$$1/\xi_{c0} \simeq \sigma_{dc}^{ab}/s_{ab}, \quad 1/(\gamma \xi_{ab0}) \simeq \sigma_{dc}^{c}/s_{c},$$

(17)

$s_i$ in units $\Omega^{-1}$ incorporates the temperature dependence. With that the universal relation (16) transforms with (13) to

$$\frac{1}{\lambda_{ab}(0) T_c \sigma_{dc}^{ab}} \simeq \frac{1}{\lambda_{ab}^2 (0) T_c \sigma_{dc}^{ab} \Lambda_c^2} \simeq \frac{16\pi^3 k_B}{\Phi_0^2 f \sigma_{ab} \Lambda_{ab}^2},$$

$$\frac{\Lambda_c}{\Lambda_{ab}} = \frac{\gamma T_c}{\gamma T=0},$$

(18)

because $\sigma_{dc}^{ab}/\sigma_{dc}^{c} = \gamma^2$ and with that $s_{ab} = s_{c}$. To check this relation we note that the 2D-QSI scaling form (4) transforms with the sheet conductivity $\sigma_{sheet} = d_s \sigma_{dc}^{ab}$ to

$$\frac{1}{\lambda_{ab}^2 (0) T_c \sigma_{dc}^{ab}} = \frac{16\pi^3 k_B}{\Phi_0^2 R_2 \sigma_{sheet}},$$

$$\sigma_{sheet} = \frac{h}{4e^2} \sigma_0 \simeq \sigma_0 (1.55 \times 10^{-4}) \Omega^{-1},$$

(19)

where $h/4e^2 = 6.45$ kΩ is the quantum of resistance and $\sigma_0$ is a dimensionless constant of order unity [68]. Thus,

$$\frac{1}{\lambda_{ab}^2 (0)} \simeq \frac{10.3 \times 10^{-5}}{R_2 \sigma_0} T_c \sigma_{dc}^{ab},$$

(20)

with $\lambda_{ab}(0)$ in $\mu$m, $T$ in K and $\sigma_{dc}^{ab}$ in $(\Omega^{-1} \text{cm}^{-1})$, and the structure of the $ab$-expression in (18) is recovered. Similarly, approaching 2D-QSI criticality $\lambda_{c}(0)$ and $\sigma_{dc}^{c}$ scale as [66]

$$\lambda_{c}(0) = \Omega_0 (\sigma_{dc}^{c})^{-(2z+2)/4},$$

(21)

where $z = 1$ is the dynamic critical exponent of the 2D-QSI transition. $\Omega_0$ is a non-universal factor of proportionality. Noting that in this limit $\sigma_{dc}^{c}$ and $T_c$ scale as $T_c \propto (\sigma_{dc}^{c})^{z/2}$ the $c$-expression in (18) is readily recovered. Accordingly, there is no contradiction between the scaling forms (18) and (21). In this context it should be kept in mind that the universal relation is valid for the neutral case only. However, we have seen that the effective dimensionless charge $\tilde{e}_{ab} = 1/\kappa_{ab}$
scales as $\tilde{e}_{ab} \propto T^{-1/2}$ and the charged critical regime becomes accessible. This is not the case for the $c$-axis penetration depth because $\tilde{e}_{c} \propto T^{3/2}$. For this reason, as the underdoped limit (2D-QSI transition) is approached there should be a crossover from (18) to the universal relation (19), becoming manifest in different constants on the right-hand side.

On the other hand, approaching the 3D quantum superconductor to normal state (QSN) transition $\lambda_{ab,c}(0)$, $T_c$ and $\sigma_{ab,c}^{dc}$ scale as [18] $1/\lambda_{ab,c}^{2}(0) \propto T^{(1+z)/z}$, $\sigma_{ab,c}^{dc} \propto T^{-(z_{cl}-1)/z}$, where $z$ is the dynamic critical exponent of this quantum transition and $z_{cl}$ the exponent associated with the finite temperature critical dynamics. Indeed, in this limit the correlation length $\xi_{c}$ associated with the finite temperature critical dynamics cannot be eliminated. Since $\xi_{c}$ scales as $\xi_{c} \propto \xi_{ab}^{z_{cl}}$ we obtain $\sigma_{c}^{dc} \propto \xi_{c}/\xi_{ab}^{2} \propto \xi_{c}/(\gamma \xi_{ab}) \propto \xi_{ab}^{z_{cl}-1} \propto T^{-(z_{cl}-1)/z}$. Accordingly,

$$\lambda_{ab}^{2}(0)T_{c}\sigma_{ab}^{dc} \propto \lambda_{c}^{2}(0)T_{c}\sigma_{c}^{dc} \propto T_{c}^{-z_{cl}/z} \tag{22}$$

should hold close to 3D-QSN criticality. Furthermore, in this regime the effective charge becomes negligibly small so that 3D-XY scaling applies. Indeed $\tilde{e}_{ab,c} = 1/\kappa_{ab,c}$ scales as $\tilde{e}_{ab,c} \propto T^{(z-1)/2z}$ because $\lambda_{ab,c} \propto T_{c}^{-z_{cl}+1/2z}$ and $\xi_{ab,c} \propto T_{c}^{-1/z}$. A potential candidate for the 3D-QSN transition is the model proposed by Herbut [69]. It describes the disordered d-wave superconductor to disordered metal transition at weak coupling and is characterized by the dynamic critical exponents $z = 2$. When this holds true, the effective charge scales at $\tilde{e}_{ab,c} \propto T_{c}^{-3/4}$ and 3D-XY scaling is no longer applicable. Furthermore, there is experimental evidence for $z_{cl} = 2$ [47, 70]. Accordingly, the scaling form (18) does not hold in the overdoped regime close to 3D-QSN criticality. In this regime (21) is replaced by [66]

$$\lambda_{c}(0) \propto (\sigma_{c}^{dc})^{(1+z)(2z_{cl}-1)} \tag{23}$$

with a non-universal factor of proportionality. What is particularly remarkable is then that the 3D-XY scaling from (18) predicts that all points of $1/\lambda_{ab}^{2}(0)$ versus $T_{c}\sigma_{ab}^{dc}$ and $1/\lambda_{c}^{2}(0)$ versus $T_{c}\sigma_{c}^{dc}$ should fall on to a single line with the exception of the overdoped regime where the scaling form (22) applies.

In figure 10 we displayed $1/\lambda_{ab}^{2}(0)$ versus $T_{c}\sigma_{ab}^{dc}$ and $1/\lambda_{c}^{2}(0)$ versus $T_{c}\sigma_{c}^{dc}$ as collected by Homes et al [67]. In agreement with the scaling from (18) and the observation that $\xi_{c}/\xi_{ab}$ is of order one (see figure 1) the $ab$-plane and $c$-axis data appear to be well described by the same line, namely $1/\lambda_{ab,c}^{2}(0) \simeq 5.2 \times 10^{-5} T_{c}\sigma_{ab,c}^{dc}$, yielding for $R_{2}\sigma_{0}$ in the 2D-QSI relation (equation (20)) the estimate $R_{2}\sigma_{0} \cong 1.98$. Furthermore, all points of $1/\lambda_{ab}^{2}(0)$ versus $T_{c}\sigma_{ab}^{dc}$ (open symbols) nearly fall onto a single line with a slope of unity. This is significant, as moderately underdoped, optimally and overdoped materials, which fell well off of the 2D-QSI behaviour $T_{c} \propto 1/\lambda_{ab}^{2}(0)$ (see equation (4) and figure 8) now scale nearly onto a single line, in agreement with figure 9(b).

This agreement demonstrates that the approximate scaling relation (18) captures the essentials of the exact 3D-XY scaling form (16) by expressing the critical amplitudes of the penetration depths in terms of their zero-temperature counterparts. This evidence for anisotropic 3D-XY scaling raises again serious doubts that 2D models [37] are potential candidates to explain superconductivity in the cuprates. However, the data do not extend sufficiently close to 2D-QSI and 3D-QSN criticality in order to uncover the aforementioned crossovers from the scaling form (18) to (19) and from (18) to (22).

In contrast, the plot $\lambda_{c}(0)$ versus $\sigma_{c}^{dc}$ displayed in figure 11 provides according to the scaling form (21) information on the flow to 2D-QSI criticality. The straight line is equation (21) with
Figure 10. $1/\lambda_{ab,c}^2(0)$ versus $T_c \sigma_{dc}^{ab,c}$ (open symbols) and $1/\lambda_{ab,c}^2(0)$ versus $T_c \sigma_{dc}^{ab,c}$ (full symbols) for various cuprates as collected by Homes et al [67]. The straight line is $1/\lambda_{ab,c}^2(0) = 5.2 \times 10^{-5} T_c \sigma_{dc}^{ab,c}$. The experimental data is taken from [71]–[76] for YBa$_2$Cu$_3$O$_{7-\delta}$ (Y-123), [73] for Pr/Pb-YBa$_2$Cu$_3$O$_{7-\delta}$(Pr/Pb-123), [74, 77] for YBa$_2$Cu$_3$O$_8$ (Y-124), [74, 77] for Tl$_2$Ba$_2$CuO$_{6+\delta}$, [73, 78] for Bi$_2$Ca$_2$Sr$_2$Cu$_2$O$_{8+\delta}$ and Y/Pb Bi$_2$Ca$_2$Sr$_2$Cu$_2$O$_{8+\delta}$, [79, 80] for Nd$_{1.85}$Ce$_{0.15}$CuO$_4$, [67, 81] for La$_{2-x}$Sr$_x$CuO$_4$ (214), and [67] for HgBa$_2$CuO$_{4+\delta}$.

Figure 11. $1/\lambda_c(0)$ versus $\sigma_{dc}^{c}$ using the data shown in figure 10 and taken from [82, 83] for YBa$_2$Cu$_3$O$_{7-\delta}$ (Y-123), La$_{2-x}$Sr$_x$CuO$_4$ (214), and from [84] for Bi$_2$Ca$_2$Sr$_2$Cu$_2$O$_{8+\delta}$. The straight line corresponds to equation (21) with $\Omega_s = 24$ appropriate for La$_{2-x}$Sr$_x$CuO$_4$ (214).
Ω, = 24 appropriate for La2-x,SrxCuO4 (214) and the arrow indicates the direction of this flow. The data for La2-x,SrxCuO4 cover the range from x = 0.08 to 0.2. Although the data are sparse one anticipates that the deviations from the straight-line behaviour increase systematically as the overdoped region is approached. As noted previously [66], this behaviour is attributable to the initial crossover to 3D-QSN criticality, where the scaling form (23) applies. Clearly, more experimental data extending much closer to the overdoped limit are needed to uncover this crossover. Actually, this also applies to the crossover from the scaling form (18) to (22).

Since equation (18), with \( \frac{\Delta_5^2}{\lambda_{ab}^2} \simeq 1 \) and \( s_{ab} \Delta_5^2 \simeq \text{const} \) turned out to be nearly universal, it also implies that the changes of \( \Delta T_c, \Delta (1/\lambda_{ab,c}^2) \) and \( \Delta \sigma_{ab,c}^{dc} \), induced by pressure or isotope exchange are not independent, but related by

\[
\frac{\Delta (1/\lambda_{ab,c}^2(0))}{(1/\lambda_{ab,c}^2(0))} = \frac{\Delta T_c}{T_c} + \frac{\Delta \sigma_{ab,c}^{dc}}{\Delta \sigma_{ab,c}^{dc}}.
\]

In particular, for the oxygen isotope effect (16O versus 18O) of a physical quantity \( X \) the relative isotope shift is defined as \( \Delta X/X = (18X - 16X)/18X \). Since close to 2D-QSI criticality \( d_s = \sigma_{\text{sheet}}/\sigma_{\text{dc}} \) and with that \( \Delta \sigma_{ab}^{dc}/\sigma_{ab}^{dc} = -\Delta d_s/d_s \) we recover the universal relation (5). Close to optimum doping where in YBa2Cu3O7-\( \delta \) \( \Delta T_c/T_c = -0.26(5)\% \) and \( \Delta (1/\lambda_{ab,c}^2(0))/\Delta (1/\lambda_{ab,c}^2(0)) = -5.6(2.0)\% \) [42] it predicts \(-5.6\% \) effect on \( \Delta \sigma_{ab}^{dc}/\sigma_{ab}^{dc} \) evaluated above and extrapolated to \( T_c \). Hence, the absence of a substantial isotope effect on the transition temperature of cuprates does not rule out phonons as the bosons responsible for the coupling between the charge carriers, as previously suggested [85].

To summarize, we observed remarkable agreement between the experimental data of underdoped cuprates and the flow to the 2D-QSI critical endpoint in the underdoped limit. Although an inhomogeneity-induced finite-size effect makes the asymptotic critical regime unattainable, there is considerable evidence that the 2D-QSI transition falls into the universality class of a 2D disordered bosonic system with long-range Coulomb interactions. The loss of superfluidity is due to the localization of the pairs, which ultimately drives the transition. Important implications include the following. (i) A finite transition temperature and superfluid density in the ground state are unalterably linked to a finite anisotropy. This finding raises serious doubts that 2D models [37] are potential candidates to explain superconductivity in cuprates. (ii) The doping dependence of the large oxygen isotope effects on the zero-temperature in-plane penetration depth confirms the flow to 2D-QSI criticality, poses a fundamental challenge to this understanding. Although the majority opinion on the mechanism of superconductivity in the cuprates is that it occurs via a purely electronic mechanism involving spin excitations, and lattice degrees of freedom are supposed to be irrelevant, the relative isotope shift of the thickness of the superconducting sheets \( \Delta d_s/d_s \approx 3.3(4)\% \) uncovers clearly the existence and relevance of the coupling between the superfluid and local lattice distortions. (iii) Given the striking feature that at low temperature the absolute change in \( 1/\lambda_{ab}^2 \) and \( 1/\lambda_{c}^2 \) has essentially no doping dependence, in contrast to \( 1/\lambda_{ab}^2(0) \) and \( 1/\lambda_{c}^2(0) \), scaling around the 2D-QSI critical endpoint implies a \( T^3 \) power law for \( 1/\lambda_{c}^2(T) \) and \( T \) for \( 1/\lambda_{ab}^2(T) \), consistent with experiment. Such a power-law behaviour also follows from the nodes characteristic of a d-wave energy gap in the one-particle density of states. However, our derivation shows that in underdoped cuprates these power laws are characteristic features of the 2D-QSI transition at the critical endpoint and hold for both s- and d-wave pairing. (iv) Similarly, we have seen that the large changes of \( 1/\lambda_{ab}^2(0), 1/\lambda_{c}^2(0) \) and \( T_c \), consistent with the relation \( T_c \propto \lambda_{ab}^{-2}(T = 0) \propto \lambda_{ab}^{-2/3}(T = 0) \), follow from the flow to the 2D-QSI criticality.
as well. (v) Because the Ginzburg parameters scale as $\kappa_{ab} \propto T_c^{1/2}$ and $\kappa_c \propto T_c^{-3/2}$, this flow was shown to open a door to observe charged criticality in the in-plane penetration depth at finite temperature. An intriguing implication for microscopic models is then the increasing value of the effective charge of the pairs in the $ab$ plane, where $\tilde{e}_{ab} = 1/\kappa_{ab} \propto T_c^{1/2}$, while in the $c$-direction it disappears as $\tilde{e}_c = 1/\kappa_c \propto T_c^{-3/2}$. (vi) We demonstrated here that the empirical relation $\lambda_{ab}^2(0)T_c \sigma_{dc}^c = \lambda_c^2(0)T_c \sigma_{dc}^c \simeq \text{const}$ [67] is a consequence of 3D-XY universality extended to anisotropic systems. Nevertheless, these relations clearly reveal that the absence of a substantial isotope effect on the transition temperature of cuprates does not rule out a coupling between local lattice distortions and superfluidity, as previously suggested.

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