Advection of Accelerated Electrons in Radio/X-ray Knots of AGN Jets

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ABSTRACT

The X-ray emission from the knots of the kilo-parsec scale jet of active galactic nuclei (AGN) suggests the high energy emission process is different from the radio/optical counterpart. Interpretation based on the Inverse Compton scattering of cosmic microwave photons has been ruled out through Fermi gamma-ray observations for low redshift sources. An alternate explanation, synchrotron emission from a different electron population is suggested. We propose a model considering the advected electron distribution from the sites of particle acceleration in AGN knots. This advected electron distribution is significantly different from the accelerated electron distribution and satisfies the requirement of the second electron population. The synchrotron emission from the accelerated and the advected electron distribution can successfully reproduce the observed radio–to–X-ray fluxes of the knots of 3C 273. For the chosen combination of the model parameters, the spectrum due to inverse Compton scattering of cosmic microwave photons falls within the Fermi gamma-ray upper limits.

Key words: galaxies: active – galaxies: individual: 3C 273 – galaxies: jets – X-rays: galaxies

1 INTRODUCTION

The jets from active galactic nuclei (AGN) can extend up to kpc/Mpc scales with bright emission regions embedded called knots (Harris & Krawczynski 2006). The knots of AGN jet have been well resolved in radio and optical wavebands. With the advent of Chandra, with its superior spatial resolution, today we know most of these knots are even bright in X-rays. The mechanism responsible for radio-to-optical emission from the knots is understood to be synchrotron process; where a relativistic electron distribution loses its energy as radiation in the jet magnetic field (Kataoka & Stawarz 2005; Zhang et al. 2010). However, the X-ray emission process is modeled either as synchrotron or inverse compton scattering of soft target photons depending up on the hardness of optical-to-X-ray spectral index ($\alpha_{ox}$) in comparison to radio-to-optical spectral index ($\alpha_{ro}$) (Jester et al. 2006). When $\alpha_{ox} > \alpha_{ro}$, the X-ray emission can be interpreted as synchrotron emission from a broken power-law electron distribution. On the other hand, when $\alpha_{ox} < \alpha_{ro}$ it demands convex (concave upward) electron distribution which is difficult to obtain and hence it is attributed to inverse compton emission or emission from a different/second electron population (Atoyan & Dermer 2004; Harris et al. 2004; Kataoka & Stawarz 2005; Uchiyama et al. 2006).

The X-ray emission from the knots, when attributed to inverse compton process; viable source of the target photons can be the synchrotron photons themselves, commonly referred to as Synchrotron Self Compton (SSC) process. However, this interpretation is disfavored since it demands a magnetic field that deviates largely from the equipartition condition (Harris & Krawczynski 2006; Zhang et al. 2010). Alternatively, Tavecchio et al. (2000) showed if the AGN jets are relativistic even at kpc scales, then the ambient cosmic microwave background (CMB) will be relativistically boosted in the knot frame and can supersede the synchrotron photon energy density. Inverse Compton scattering of this CMB (IC/CMB) can successfully explain the X-ray emission from the knots of PKS 0637-752 while maintaining the equipartition condition. The IC/CMB model is also found to be the preferred emission mechanism to explain the X-ray knots for several other AGN jets (Sambruna et al. 2002; Jorstad & Marscher 2006; Tavecchio et al. 2007; Perlman et al. 2011; Kharb et al. 2012; Stanley et al. 2015).

Modeling the X-ray knots through IC/CMB emission suggests, this spectral component peaks at $\gamma$-ray energies. Hence, a crucial prediction by IC/CMB model is that the X-ray bright kpc scale jets of nearby AGN can be detectable by Fermi space telescope operating at $\gamma$-ray energies (Georganopoulos et al. 2006). On the contrary, $\gamma$-ray observation of 3C 273 by Fermi during 2008 to 2013 resulted only in flux upper limits, even though the IC/CMB model for the knots/jet of the source predicted detectable flux (Meyer & Georganopoulos 2014). A similar result was
also obtained for the source PKS 0637-752, where six years of Fermi observation did not support the IC/CMB origin of the X-ray emission (Meyer et al. 2015). Apart from these sources, the Fermi non-detection of four more sources PKS 1136-135, PKS 1229-021, PKS 1354+195 and PKS 2209+080 with prominent X-ray jets further questioned the validity of IC/CMB emission as a plausible interpretation for the X-ray emission from the jets of low redshift AGN. Nevertheless, IC/CMB emission is still preferred for the sources which are not yet disproved by the Fermi observations (Zargaryan et al. 2017; Meyer et al. 2019).

Failure of IC/CMB model to explain the Fermi γ-ray upper limit favors the presence of second electron population which is responsible for the observed X-ray emission from the knots. If we assume the electrons are accelerated at the knot sites by a shock front, then the particle injection into the upstream and the downstream region will be asymmetric due to the compression of the downstream fluid (Liu et al. 2015). This can develop two independent electron populations and can explain the radio-to-X-ray emission from the knots successfully. Alternatively, electrons can be accelerated to ultra-high energy at the sheared boundary of the jets in addition to the shock acceleration (Tavecchio 2021). Under this scenario, the electrons accelerated at the shock front contribute to radioactive emission; while the X-ray emission is produced by the electrons undergoing shear acceleration.

The drawback of the second electron population interpretation of the X-ray knots is, it demands ultra high energy electrons which are subject to fast cooling (Wang et al. 2020). Hence, these electrons would expend all their energy close to their production site itself and may not explain the extended X-ray emission from the knots. Wang et al. (2020), therefore, suggested a lepto-hadronic origin for the radio-to-X-ray emission for the knots. The radio-to-X-ray emission from the knot can also be explained by models involving emission from hadrons alone (Kundu & Gupta 2014). Advantage of models involving the X-ray emission from hadrons is, they lose their energy much slower than the leptons and therefore can diffuse over long distances.

In this paper, we present an emission scenario for the knots where the advection of electrons from the main acceleration site is considered. This can produce two distinct electron populations with a high energy electron distribution arising from the acceleration site and a low energy one surrounding it. The synchrotron emission from such combined electron distribution is capable to explain the radio-to-X-ray emission from the knots of the jet of 3C 273. In the next section, we deduce the advected electron population and explain the total synchrotron spectrum. In section 3, we apply this model on the knots of 3C 273 and discuss the results. Throughout this work we consider a cosmology where \( H_0 = 71 \text{ km s}^{-1} \text{ Mpc}^{-1} \), \( \Omega_m = 0.27 \) and \( \Omega_\Lambda = 0.73 \).

2 THE ADVECTED ELECTRON POPULATION

We consider a model in which the particle acceleration in the AGN knot is confined within a spherical region of size \( R_0 \). The electron distribution in this region is assumed to be a broken power-law described by

\[
\begin{align*}
    n_0(\gamma)d\gamma &= \begin{cases} 
    K\gamma^{-p}d\gamma & \gamma_{\min} < \gamma < \gamma_b \\
    \gamma_0^{-q}d\gamma & \gamma_b < \gamma < \gamma_{\max}
    \end{cases}
\end{align*}
\]

Such an electron distribution can be an outcome of multiple acceleration sites embedded within \( R_0 \) (Sahayathanath 2008; Pope & Melrose 1994). The accelerated electrons are advected outside \( R_0 \) where they do not undergo further acceleration but lose their energy through radiative processes and adiabatic cooling. We consider the magnetic field at the regions \( R < R_0 \) and \( R > R_0 \) as \( B_0 \) and \( B_{out} \) respectively. The evolution of the electron distribution in the region \( R > R_0 \) can be described by (Kardashev 1962; Atoyan & Aharonian 1997)

\[
\frac{\partial n(\gamma, t)}{\partial t} = \frac{\partial}{\partial \gamma} \left[ P(\gamma) n(\gamma, t) \right] - \frac{n(\gamma, t)}{t_{esc}(R)} + q(\gamma)
\]

Here, \( P(\gamma) \) is total energy loss rate due to radiative processes and adiabatic expansion, \( t_{esc}(R) \) (Ginzburg & Syrovatskii 1964) is electron escape timescale at a distance \( R \) from \( R_0 \) and \( q(\gamma) \) is the rate of electron injection.

For a constant fluid velocity, the distance \( R \) will be linearly dependent on time \( t \) and this let us to express equation (2) in terms of \( R \) as

\[
\frac{\partial n(\gamma, R, R_0)}{\partial R} = \frac{\partial}{\partial \gamma} \left[ P(\gamma) n(\gamma, R, R_0) \right] - \frac{n(\gamma, R, R_0)}{R_{esc}(R_0)} + n_0(\gamma) \delta(R-R_0)
\]

where, \( R_0 \) is the characteristic particle escape radius corresponding to \( t_{esc} \) and

\[
P(\gamma) = \frac{d\gamma}{dR} = \frac{\xi \gamma^2 + \frac{\gamma}{R}}{R}
\]

where, \( \xi \) will be a function of magnetic field and the velocity of the fluid (Rybicki & Lightman 1986; Atoyan & Aharonian 1997). The last term in equation (3) implies the particle injection happens only at \( R_0 \).

The solution of equation (3) can be obtained through the Green’s function for \( R > R_0 \) as (Atoyan & Aharonian 1997)

\[
n(\gamma, R, R_0) = \frac{R_0 \Gamma_0^2}{R \gamma^2} n_0(\Gamma_0) \exp \left[ -\int_{R_0}^{R} \frac{dx}{R_0} \frac{\Gamma_0}{R_{esc}(x)} \right]
\]

where, \( \Gamma_0 \) defines the energy of the electron at injection radius \( R_0 \) which reduces to \( \gamma \) at \( R \) due to radiative and adiabatic losses. From equation (4) we find

\[
\Gamma_0(\gamma, R) = \frac{\gamma \frac{R}{R_0}}{1 - \xi \gamma R \ln \frac{R}{R_0}}
\]

The maximum distance \( R_{max} \) up to where electrons with energy \( \gamma \) will be available and can be obtained by numerically

\[
1 \text{ We express the advection term in terms of } t_{esc} \text{ as follows: for a constant velocity } v, \text{ the advection can be expressed as } -v \frac{\partial n}{\partial R}.
\]

\[
\frac{\partial n}{\partial t} = -\frac{n}{t_{esc}(R)}
\]
for the cases of $R/R_0 = 0.1, 0.2, 0.3, 0.5, 0.8, 1, 1.5, 2, 3, 8, 10, 20, 30, 50, 80$ and 100. The parameters chosen are of knot A. The bold line corresponds to $R/R_0 = 1$

The observed flux on earth after accounting for the relativistic effects of jet and cosmological corrections will be

$$F_{\text{obs}}(\nu_{\text{obs}}) = \frac{\delta_D^3 (1+z)}{d_L^2} J_{\text{syn}} \left( \frac{1+z}{\delta_D} \nu_{\text{obs}} \right)$$

(13)

Here, $d_L$ is the luminosity distance and $\delta_D$ is the Doppler factor given by

$$\delta_D = \frac{1}{\Gamma (1 - \beta \cos \theta)}$$

(14)

with $\Gamma$ being the bulk Lorentz factor, $\beta$ the velocity and $\theta$ the viewing angle of the relativistic jet.

### 3 RESULTS AND DISCUSSION

The model described above, incorporating the advection of electrons from the sites of particle acceleration, is used to reproduce the radio/optical/X-ray fluxes from the knots of 3C 273. The primary accelerated electron distribution is assumed to be a broken power-law and confined within a region $R < R_0$ with magnetic field $B_{\text{in}}$ (equation (1)). The synchrotron emission by this electron distribution is used to model the X-ray flux from the knots of 3C 273. The advected electron distribution from the region $R > R_0$ creates a low energy excess and the synchrotron emission from this electron population can reproduce the observed radio fluxes.

In Fig. 3, we show the observed radio–optical–X-ray spectrum from the knots A, B1, B2, B3, C1, C2, D1 and D2H3 along with the model curves. The dot-dash line is the synchrotron emission from the accelerated electron distribution while the dotted line is from the advected electron population. The total emission is shown as solid line. The parameters used to reproduce the model curves are given in Table 1. Besides these parameters, we have assumed $\alpha = 1$, $\gamma_{\text{min}} = 10$, $\gamma_{\text{max}} = 10 \gamma_{\text{b}}$ and $\theta = 20$ degrees.

The choice of model parameters also decide whether the IC scattering of CMB photons by the total electron distribution will exceed the Fermi upper limits. To verify this we estimate IC/CMB spectral component using approximate analytical solution for a monochromatic external photon field (Sahayanathan et al. 2018). The IC/CMB emissivity at frequency $\nu$ can be expressed as

$$j_{\text{ec}}(\nu) \approx \frac{c \sigma_T U_*}{8 \pi \nu_*} \sqrt{\frac{\Gamma(1+\mu)\nu}{\nu_*}} N_{\text{tot}} \left[ 1 - \frac{\nu}{\nu_*} \right]$$

(15)

Here, $\nu_*$ and $U_*$ are the frequency and the energy density of the external photon field and $\mu$ is the jet viewing angle measured in the proper frame of the AGN. In Fig. 3, the spectral energy distribution due to IC/CMB emission shown as dashed line. For the choice of parameters given in Table 1, the spectrum due to IC/CMB falls below the Fermi upper limits.

In this work, we have assumed the accelerated electron distribution to be a broken power-law and this can be an outcome of multiple acceleration processes (Sahayanathan 2008). For instance, if we consider the case where the particle acceleration happens due to a standing shock buried in a turbulent plasma, then the electrons injected into the shock front are already accelerated under stochastic process. If the confinement time at the turbulent region is longer than the region in the vicinity of the shock front, the electron distribution accelerated through stochastic process will be harder (Rieger et al.
This eventually give rise to a broken power-law distribution with indices governed by the ratio of acceleration to escape timescales in both the regions.

The model considered here demands large electron energies to explain the X-ray emission. Such large electron energies can be achieved under moderate loss or escape time scales. If we assume the escape timescale is governed by gyro radius \( R_g \), then for an electron with Lorentz factor \( 10^8 \), \( R_g \) is \( 10^{-3} \)pc, which is much smaller than the size of inner region \( R_0 \). Similarly, the acceleration timescale for the given gyro-radius is approximately \( R_g/c \) and is of the order of \( 10^5 \) s \( (\text{Inoue} \& \text{Takahara} \, 1996; \text{Kusunose \ et \ al.} \, 2000) \). Whereas, the synchrotron radiative loss time scale for electrons of energy \( \approx 10^4 \) is of orders of \( 10^9 \)s. Hence, the acceleration of electrons to such high energies is viable under the scenario considered here.

It is interesting to study the radial evolution of the advected electron distribution which is governed by the synchrotron and adiabatic cooling (Fig. 1). The integrated distribution over \( R \) in combination with the accelerated electron distribution will then reflect a convex (concave upward) shape which can explain the radio–optical–X-ray spectrum of AGN knots (Fig. 4). The radial evolution of the advected electron distribution also depends up on the choice of \( \alpha \). In the present study, \( \alpha \) is chosen to be unity for simplicity; however, the observed knot spectrum can also be explained with \( \alpha \neq 1 \) and a modified set of parameters.

The synchrotron cooling time scale estimated for the radio, optical and X-ray emitting electron (for the case of \( B \approx 10^{-5} \)) is approximately \( 10^{14} \), \( 10^{15} \), and \( 10^{16} \) s respectively \( (\text{Atoyan} \& \text{Aharonian} \, 1997) \). Equivalent length scale can be estimated as \( 100 \) kpc (radio), \( 1 \) kpc (optical) and \( 10 \) pc (X-ray) for a velocity \( v \approx 0.1c \). This suggests the radio emitting electrons will travel the extended jet before losing its energy as compared to the X-ray emitting electrons. This is consistent with the morphological feature of the source where the radio jet extends over large scales whereas the X-ray emission dies out beyond the knot locations \( (\text{Harris} \& \text{Krawczynski} \, 2006) \).

\textit{Fermi} non-detection of gamma-ray emission from the knots of AGN jets favored the two-electron population hypothesis for the radio-to-X-ray emission. Nevertheless, IC/CMB model for the X-ray emission is still preferred for the sources which are not yet ruled out by \textit{Fermi} observations \( (\text{Ighina \ et \ al.} \, 2021; \text{Worrall \ et \ al.} \, 2020; \text{Kharb \ et \ al.} \, 2012) \). Irrespective of \textit{Fermi} observations, the IC/CMB model for the X-ray emission fail to explain many morphological features of the AGN jet. For example, the electron energies required to produce the X-ray emission through IC/CMB process is similar to the ones emitting radio through synchrotron process. Hence, the radio/X-ray jet morphology should be comparable \( (\text{Harris} \& \text{Krawczynski} \, 2006) \). On the contrary, positional offset has been detected between the radio and X-ray maxima for the knots of many AGN \( (\text{Hardcastle \ et \ al.} \, 2003; \text{Bai} \& \text{Lee} \, 2003) \). Similarly, IC/CMB model demands significant jet speed to explain the X-ray emission which in turn predict one-sided jets due to relativistic debeaming of the counter jet. However, the detection of faint counterjet in Pictor A disfavors the IC/CMB origin of the X-ray emission \( (\text{Hardcastle \ et \ al.} \, 2016) \).

### Table 1.

| Knot | \( R_0 \) (kpc) | \( R_{size} \) (kpc) | \( B_{in} \) \( (10^{-5} G) \) | \( \omega = \frac{B_{in}}{B_{out}} \) | \( \gamma_b \) \( (10^5) \) | \( \nu_{ad} \) \( (10^{-2} c) \) | \( p \) | \( q \) | \( \Gamma \) | \( \zeta \) |
|------|----------------|---------------------|------------------|----------------|----------------|----------------|---|---|---|---|
| A    | 0.12           | 2.9                 | 1.5              | 2.08           | 7.5            | 2.0            | 2.0 | 4.0 | 1.3 | 7.0 |
| B1   | 0.10           | 5.5                 | 1.8              | 3.3            | 6.5            | 1.6            | 2.13| 4.0 | 2.0 | 7.0 |
| B2   | 0.10           | 5.5                 | 2.0              | 3.6            | 6.18           | 1.6            | 2.13| 4.0 | 2.0 | 7.0 |
| B3   | 0.09           | 5.5                 | 1.3              | 2.4            | 4.85           | 1.6            | 2.13| 4.0 | 2.0 | 7.0 |
| C1   | 0.08           | 8.0                 | 1.9              | 3.45           | 3.64           | 9.0            | 2.12| 4.0 | 1.7 | 5.5 |
| C2   | 0.05           | 6.0                 | 1.9              | 3.17           | 4.70           | 5.0            | 2.13| 4.2 | 1.7 | 9.5 |
| D1   | 0.05           | 6.0                 | 1.5              | 3.0            | 3.58           | 4.0            | 2.13| 4.2 | 1.6 | 9.5 |
| D2H3 | 0.05           | 6.5                 | 1.1              | 2.4            | 4.62           | 2.5            | 2.11| 4.0 | 1.3 | 9.5 |

Figure 2. Advected electron population for different \( \zeta \) in the case of knot A. Solid line corresponds to \( \zeta=100 \) and dotted line to \( \zeta=0.1 \). All other plots below solid line corresponds to \( \zeta=50, \ 10, \ 1 \ & \ 0.5 \).

4 SUMMARY

The explanation for the radio-to-X-ray flux from the knots of many AGN jets through synchrotron process suggests the underlying electron distribution to be convex (concave upward). Production of such a distribution demands an acceleration process that can provide excessive power at high electron energies. Alternatively, the synchrotron emissions from...
two spatially separated electron populations are capable to explain the radio–to–X-ray emission from the AGN knots. In this work, we show such distributions can naturally arise if we incorporate the advection of accelerated electrons from the sites of particle acceleration. The synchrotron emission received from the combination of accelerated and advected electron distributions are used to model the radio/optical/X-ray fluxes from the knots of 3C 273. The model parameters are chosen to reproduce the observed fluxes while the predicted IC/CMB emission fall within the Fermi upper limits.
The model assumes the accelerated electron distribution to occupy a spherical region and the advected electrons in a spherical shell around it. If we relax this spherical symmetry, the proposed model is also capable to explain the observed offsets between the radio, optical and X-ray knot positions. Reproduction of the radio, optical and X-ray observations of the knots of 3C 273 using this model suggests the spectral energy distribution of these knots to peak at infrared frequencies. High resolution observation at this frequency in future have the potential to constrain/rule out the model presented in this work.

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DATA AVAILABILITY

The data and the codes used in this work will be shared on the reasonable request to the corresponding author Amal A. Rahman (email:amalar.amal@gmail.com)

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APPENDIX A: PARTICLE DISTRIBUTION FOR THE CASE $\alpha \neq 1$

With definitions of $\Gamma_0$ as in section 2, solution to equation (5) for the case of $\alpha \neq 1$ is given by

$$n(\gamma, R, R_0) = n_0(\Gamma_0) \frac{\Gamma_0^2}{\gamma^2} \exp \left[ \frac{-1}{\zeta(-\alpha + 1)} \left( R^{-\alpha+1} - R_0^{-\alpha+1} \right) \right] \quad (A1)$$

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