de Sitter invariant special relativity and galaxy rotation curves

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Abstract

Owing to the existence of an invariant length at the Planck scale, Einstein special relativity breaks down at that scale. A possible solution to this problem is arguably to replace the Poincaré-invariant Einstein special relativity by a de Sitter invariant special relativity. Such replacement produces concomitant changes in all relativistic theories, including general relativity, which changes to what we have called de Sitter modified general relativity. In this paper, the Newtonian limit of this theory is used to study the circular velocity of stars around the galactic center. It is shown that the de Sitter modified Newtonian force—part of which becomes effective only in the Keplerian region of the galaxy—could possibly explain the flat rotation curve of galaxies without necessity of supposing the existence of dark matter.

1 Introduction

The de Sitter spacetime is usually interpreted as the simplest dynamical solution to the sourceless Einstein equation in the presence of a cosmological constant, standing on an equal footing with all other gravitational solutions, like for example Schwarzschild and Kerr. However, as a non-gravitational spacetime (in the sense that its metric does not depend on Newton’s gravitational constant), the de Sitter solution should instead be interpreted as a fundamental background for the construction of physical theories, standing on an equal footing with the Minkowski solution. Note that, as a quotient spaces [1], both Minkowski and de Sitter are known a priori, independently of Einstein equation. General relativity, for example, can be constructed on any one of them. Of course, in either case gravitation will have the same dynamics, only their local kinematics will be different. If the underlying spacetime is Minkowski, the local kinematics will be ruled by the Poincaré group of ordinary special relativity. If the underlying spacetime is de Sitter, the local kinematics will be ruled by the de Sitter group, which amounts then to replace the Poincaré-invariant special relativity by a de Sitter invariant special relativity [2, 3].

When general relativity is constructed on a de Sitter spacetime, it changes to a new theory we have called de Sitter modified general relativity [5]. This theory implicitly incorporates the idea that any physical system with energy density $\varepsilon_m$ induces a local cosmological term $\Lambda$ in the

*The first ideas about a de Sitter special relativity are due to L. Fantappié, who in 1952 introduced what he called Projective Relativity, a theory that was further developed by G. Arcidiacono. The relevant literature can be traced back from Ref. [4].
background spacetime, which is necessary to comply with the local kinematics of spacetime, governed now by the de Sitter group. It should be noted that this local cosmological term $\Lambda$ is different from the usual notion in the sense that it is no longer required to be constant [7, 8]. For example, outside the region occupied by the physical system, where $\varepsilon_m$ vanishes, the cosmological term $\Lambda$ vanishes as well. In a sense, it can be said to represent an asymptotically flat de Sitter spacetime.

In order to get a better insight into this mechanism, let us consider a Planck photon, that is, a photon whose wavelength is given by the Planck length $l_P$. Such photon induces locally in spacetime the Planck cosmological term

$$\Lambda_P = 3/l_P^2,$$

which has the value $\Lambda_P \simeq 1.2 \times 10^{70}$ m$^{-2}$. This expression can be rewritten in the form

$$\Lambda_P = \frac{4\pi G}{c^4} \varepsilon_P,$$

where

$$\varepsilon_P = \frac{m_P c^4}{(4\pi/3) \ell_P^3}$$

is the Planck energy density, with $m_P$ the Planck mass. Now, the very definition of $\Lambda_P$ can be considered an extremal particular case of a general expression relating the local cosmological term to the corresponding energy density of a physical system. Accordingly, to a physical system of energy density $\varepsilon_m$ will be associated the local cosmological term [9]

$$\Lambda = \frac{4\pi G}{c^4} \varepsilon_m.$$

As an example, let us consider the present-day universe. We know from WMAP that the space section of spacetime is nearly flat today: $k \simeq 0$. As a consequence, the mean energy density of the universe is of the same order of the critical energy density

$$\varepsilon_c = \frac{3H_0^2 c^2}{8\pi G} \simeq 10^{-9} \text{ Kg m}^{-1} \text{ s}^{-2},$$

with $H_0 \simeq 75$ (Km/s)/Mpc the current value of the Hubble parameter. Using this value, the effective cosmological term of the present-day universe is found to be

$$\Lambda \simeq 10^{-52} \text{ m}^{-2},$$

which is of the order of magnitude of the observed value [10–12]. It is important to remark that, since $\Lambda$ is now connected to the local kinematics of spacetime, equation (4) is not a dynamic

† The idea that the presence of matter with an energy density $\varepsilon_m$ could somehow change the underlying spacetime from Minkowski to de Sitter, was first put forward by F. Mansouri in 2002 [6].‡ We note that, according to the de Sitter modified general relativity, the values of the cosmological term $\Lambda$ at the Planck scale and at the large scale of the present-day universe differ roughly by 120 orders of magnitude.
equation, but just an algebraic relation. This means that, even though \( \Lambda \) is allowed to change in space and time, it is not a dynamical variable but just an external parameter, similar to the energy density \( \varepsilon_m \) or the mass density \( \rho_m \). Accordingly, relation (4) could be interpreted as a kind of equation of state.

By considering the de Sitter modified general relativity, we have recently obtained its Newtonian limit, as well as the corresponding Newtonian Friedmann equations. Using these equations, we have shown that this theory is able to give a reasonable account of the dark energy content of the present-day universe [5]. The purpose of the present paper is to use the same Newtonian limit to study galaxy rotation curves.

2 Basics of the de Sitter modified general relativity

For the sake of completeness, we present here a brief description of the de Sitter modified general relativity. For a detailed presentation of this theory, as well as for the computation of its Newtonian limit, we refer the reader to the paper of Ref. [5].

2.1 de Sitter modified Einstein equation

As a local transformation, diffeomorphisms are able to detect the local structure of spacetime. For example, if the underlying spacetime is Minkowski, in which case the local kinematics will be ruled by the Poincaré group of ordinary special relativity, a diffeomorphism is defined by

\[ \delta x^\mu = \delta^\mu_\alpha \epsilon^\alpha(x), \]

where \( \delta^\mu_\alpha \) are the Killing vectors of ordinary translations in Minkowski spacetime.\(^8\) Analogously, if the underlying spacetime is de Sitter, in which case the local kinematics will be ruled by the de Sitter group, a diffeomorphism is defined by

\[ \delta x^\mu = \xi^\mu_\alpha \epsilon^\alpha(x), \]

where \( \xi^\mu_\alpha \) are the Killing vectors of the de Sitter “translations” [13]. Let us then consider the action integral of a general source field

\[ S_m = \frac{1}{c} \int \mathcal{L}_m \sqrt{-g} \, d^4x, \]

with \( \mathcal{L}_m \) the lagrangian density. Invariance of this action under the diffeomorphism (8) yields, through Noether’s theorem, the conservation law [14]

\[ \nabla_\mu \Pi^{\mu \nu} = 0, \]

\(^8\)This is the reason, by the way, why the invariance of any source lagrangian under diffeomorphism in locally Minkowski spacetimes yields the covariant conservation of the energy-momentum tensor \( T^{\rho \mu} = \delta^\rho_\alpha T^{\alpha \mu} \).
where the conserved current has the form

$$\Pi^\rho{}^\mu = \xi_\alpha^\rho T^{\alpha \mu}$$  (11)

with $T^{\alpha \mu}$ the symmetric energy-momentum current.

On the other hand, the Einstein-Hilbert action of general relativity in a locally-de Sitter spacetime resembles that of ordinary special relativity:

$$S_g = \int R \sqrt{-g} \, d^4x.$$  (12)

There is a difference though: the scalar curvature $R$ in this case is obtained from the de Sitter-Cartan curvature tensor $R^{\alpha \beta \mu \nu}$, which is a tensor that represents both the dynamical curvature of general relativity and the kinematic curvature of the underlying de Sitter spacetime. The invariance of the total action integral

$$S = S_g + S_m$$  (13)

under general coordinate transformations, therefore, yields the de Sitter-modified Einstein equation

$$R_{\mu \nu} - \frac{1}{2}g_{\mu \nu} R = \frac{8\pi G}{c^4} \Pi_{\mu \nu}.$$  (14)

This is the equation that replaces ordinary Einstein equation when the Poincaré-invariant Einstein special relativity is replaced by a de Sitter-invariant special relativity. In the contraction limit $l \to \infty$, which corresponds to $\Lambda \to 0$, it reduces to the ordinary Einstein equation

$$R_{\mu \nu} - \frac{1}{2}g_{\mu \nu} R = \frac{8\pi G}{c^4} T_{\mu \nu}. $$  (15)

It is important to note that there are two different transformations involved in the variation of the total action functional (13). Concerning the gravitational action $S_g$, the metric variations $\delta g_{\mu \nu}$ are arbitrary dynamical variations unrelated to any spacetime transformations. Concerning the source action $S_m$, on the other hand, the metric variations $\delta g_{\mu \nu}$ are those coming from general coordinate transformations, as discussed in Section 2.1. This peculiar procedure, it should be noted, holds also in the variational principle of ordinary general relativity.

A crucial point of this approach is that, since the curvature of the underlying de Sitter spacetime is included in Riemann tensor describing the dynamical curvature of gravitation, the cosmological term $\Lambda$ does not appear explicitly in the gravitational field equation. As a consequence, the (contracted form of the second) Bianchi identity,

$$\nabla_\mu \left( R^{\mu \nu} - \frac{1}{2} g^{\mu \nu} R \right) = 0,$$  (16)

does not require $\Lambda$ to be constant. It should be noted that in this theory Lorentz transformations remain a symmetry at all energy scales, including the Planck scale. What is broken down is ordinary translations, which implies that the usual notions of energy and momentum are no longer conserved. What is conserved now is the projection of the energy-momentum tensor along the Killing vectors associated to the de Sitter “translations”, as given by Eq. (11).
2.2 Newtonian limit

In a de Sitter-Cartan geometry, in which the background spacetime is de Sitter instead of Minkowski, the spacetime metric is expanded in the form

$$g_{\mu\nu} = \hat{g}_{\mu\nu} + h_{\mu\nu},$$ (17)

where $\hat{g}_{\mu\nu}$ represents the background de Sitter metric and $h_{\mu\nu}$ is the gravitational metric perturbation. The background connection, which corresponds to the zeroth-order connection, is

$$\hat{\Gamma}^\rho_{\mu\nu} = \frac{1}{2} \hat{g}^{\rho\lambda} \left( \partial_\mu \hat{g}_{\lambda\nu} + \partial_\nu \hat{g}_{\mu\lambda} - \partial_\lambda \hat{g}_{\mu\nu} \right).$$ (18)

The corresponding Riemann tensor $\hat{R}^\alpha_{\beta\mu\nu}$ represents the curvature of the (non-gravitational) de Sitter background. In harmonic coordinates, which is expressed by the condition

$$\hat{\nabla}_\nu h^\rho_{\mu} - \frac{1}{2} \hat{\nabla}^\rho h = 0$$ (19)

with $h = h^\mu_{\mu}$, the first-order Ricci tensor is found to be

$$R^{(1)}_{\mu\nu} = -\frac{1}{2} \hat{\Box} h_{\mu\nu} + h^\sigma_{(\nu} \hat{R}_{\sigma\mu)} - h^\rho_{\sigma} \hat{R}^\sigma_{(\mu\rho\nu)}. $$ (20)

The de Sitter modified Einstein equation is then given by

$$-\frac{1}{2} \hat{\Box} h_{\mu\nu} + h^\sigma_{(\nu} \hat{R}_{\sigma\mu)} - h^\rho_{\sigma} \hat{R}^\sigma_{(\mu\rho\nu)} = \frac{8\pi G}{c^4} \left( \Pi_{\mu\nu} - \frac{1}{2} \hat{g}_{\mu\nu} \Pi \right).$$ (21)

The Newtonian limit is obtained when the gravitational field is weak and the particle velocities are small. In the presence of a cosmological term $\Lambda$, however, it has some subtleties related to the process of group contraction. Notice, to begin with, that the Galilei group is obtained from Poincaré in the contraction limit $c \to \infty$. The Newton-Hooke group, on the other hand, does not follow straightforwardly from the de Sitter group through the same limit. The reason is that, under such limit, the boost transformations are lost. In order to obtain a physically acceptable result, one has to simultaneously consider the limits $c \to \infty$ and $\Lambda \to 0$, but in such a way that

$$\lim c^2 \Lambda = \frac{1}{\tau^2}$$ (22)

with $\tau$ a time parameter. This means that the usual weak field condition of Newtonian gravity must be supplemented by the small $\Lambda$ condition [16]

$$\Lambda r^2 \ll 1,$$ (23)

which is equivalent to $r^2/l^2 \ll 1$. In this limit, and identifying

$$h_{00} = 2\phi/c^2,$$ (24)
with $\phi$ the gravitational scalar potential, the de Sitter modified Einstein equation (21) becomes

$$\hat{\Delta} \phi + 2\phi \hat{R}_{00} = \frac{4\pi G}{c^2} \Pi_{00},$$

(25)

where $\hat{\Delta}$ is the Laplace operator in the background de Sitter metric $\hat{g}^{ij}$, and

$$\Pi_{00} = \xi_0^0 T_{00},$$

(26)

with $\xi_0^0$ the zero-component of the Killing vectors of the de Sitter “translations” and $T_{00} = \rho c^2$. In static coordinates, we can write $\Pi_{00} = \rho \Pi c^2$, where

$$\rho \Pi \simeq \rho \left(1 - \frac{r^2}{2l^2}\right).$$

(27)

Equation (25) can then be show to assume the form

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \phi}{\partial r}\right) - \frac{r^2}{l^2} \frac{\partial^2 \phi}{\partial r^2} - \frac{6}{l^2} \phi = 4\pi G \rho \Pi.$$

(28)

It solution yields the de Sitter modified Newtonian potential

$$\phi(r) = -\frac{GM}{r} - \frac{GMA}{6} r,$$

(29)

where we have used the relation $\Lambda = 3/l^2$. In the limit of a vanishing cosmological term $\Lambda \to 0$, it reduces to the ordinary Newtonian potential.

### 3 Galaxy rotation curves

A generic galaxy rotation curve can be divided into three regions: (i) an inner region (aka bulge) in which the rotation velocity of the stars rises linearly with the distance from the center; (ii) a region where the speed reaches a maximum and then begins to decrease (at the so-called turn over radius); (iii) and a Keplerian region in which the whole mass of the galaxy can be assumed to be given by the bulge mass $M_0$, and located at the central point. For this reason, the Newtonian gravitational force in the Keplerian region resembles that of a point mass force,

$$F = -\frac{GM_0}{r^2}.$$  

(30)

The corresponding rotation velocity of galaxies is found to be [17]

$$v(r) \equiv \sqrt{|F(r)|} = \sqrt{GM_0/r},$$

(31)

from where we see that the velocity falls off as $v \sim r^{-1/2}$, as schematically depicted in curve A of Figure 1. However, instead of such behavior, galaxies show in general a flat rotation curve,
as depicted in curve B of Figure 1. We can then say that either there is some unaccounted
matter in the galaxy—usually called dark matter—or gravity behaves different from the usual
Newtonian limit.

We consider now the de Sitter modified general relativity, whose Newtonian limit has already
been shown to yield the gravitational potential (29), with $\Lambda$ given by Eq. (4). Considering
that the galaxy mass density $\rho_m$ is not constant, $\Lambda$ is not constant either, and the corresponding
gravitational force $F = -d\phi(r)/dr$ assumes the form

$$ F = -\frac{GM}{r^2} + \frac{GM\Lambda(r)}{6} + \frac{GM}{6} r \frac{d\Lambda(r)}{dr}. $$

(32)

The first term on the right-hand side represents the usual attractive Newtonian force. The
background de Sitter spacetime contributes with an additional repulsive force proportional
to $\Lambda(r)$, as well as with a force proportional to the radial derivative of $\Lambda(r)$, which will be
attractive or repulsive depending on the sign of $d\Lambda(r)/dr$. In what follows we are going to use
the gravitational force (32) to study the circular velocity of a star around the galactic center.

Let us begin with the inner region of the galaxy $r \ll r_0$, where the circular velocity of the
stars rises almost linearly with $r$. This means that in this region only the Newtonian force
is in action, and the mass density $\rho_m(r)$ of the galaxy decreases slowly with the radius $r$. In
fact, for a nearly constant mass density $\rho_m(r) \simeq \rho_0$, the inner mass $M(r)$ assumes the form

$$ M(r) = \frac{4}{3} \pi \rho_0 r^3, $$

and the Newtonian star velocity is easily seen to grow linearly with $r$

$$ v(r) = \sqrt{\frac{4}{3} \pi G \rho_0 r}, $$

(33)
in agreement with observations.

At the turn over point $r_0$, on the other hand, using the Milky Way values as the typical
values for the mass density and radius of the bulge [18],

$$ \rho_0 \simeq 10^{-12} \text{ Kg m}^{-3} \quad \text{and} \quad r_0 = 3 \text{ kpc} \simeq 10^{20} \text{ m}, $$

(34)
the corresponding bulge cosmological term is found to be
\[ \Lambda_0 = \frac{4\pi G}{c^2} \rho_0 \simeq 10^{-40} \text{ m}^{-2}. \] (35)

Considering furthermore that around \( r_0 \) the mass density \( \rho_m(r) \) still decreases slowly with the radius \( r \), the same will happen to cosmological term \( \Lambda(r) \), and consequently the last term of the gravitational force (32) can be neglected. In this case it is easy to see that, at the turn over point \( r_0 \), the first two terms of the force (32) are of the same order:
\[ |GM_0/r_0^2| \simeq |GM_0\Lambda_0/6|. \] (36)

The region around the turn over radius \( r_0 \) can accordingly be considered a transition region from the Newtonian to the de Sitter modified Newtonian regimes.

For \( r \gg r_0 \), which defines the Keplerian region, the Newtonian force becomes negligible and the relevant force assumes the form
\[ F = \frac{GM_0}{6} \Lambda(r) + \frac{GM_0}{6} r \frac{d\Lambda(r)}{dr}, \] (37)
where we have assumed that in this region the whole mass of the galaxy can be represented by the bulge mass \( M_0 \). The squared circular velocity of a star at a distance \( r \) from the galactic center is now given by
\[ v^2(r) \equiv r |F(r)| = \frac{GM_0}{6} \left[ \Lambda(r)r + r^2 \frac{d\Lambda(r)}{dr} \right]. \] (38)

Then comes the point: the above expression has a solution in which \( v^2(r) \) is constant. Such solution is obtained when \( \Lambda(r) \) satisfies the first-order differential equation
\[ r^2 \frac{d\Lambda(r)}{dr} + \Lambda(r)r = \beta, \] (39)
with \( \beta \) a constant. It is convenient at this point to introduce the dimensionless coordinate \( r' = r/r_0 \), in terms of which the differential equation (39) assumes the form
\[ r'^2 \frac{d\Lambda(r)}{dr'} + \Lambda(r)r' - \frac{\beta}{r_0} = 0. \] (40)

In terms of the original variables, its solution is
\[ \Lambda(r) = \frac{\beta}{r} \ln \left( \frac{r}{r_0} \right) + \gamma \frac{r_0}{r}, \] (41)
with \( \gamma \) an integration constant. Since at \( r = r_0 \) the cosmological term has the value \( \Lambda(r) = \Lambda_0 \), we can immediately infer that \( \gamma = \Lambda_0 \). Imposing furthermore that \( d\Lambda(r)/dr = 0 \) at \( r = r_0 \), we find that \( \beta = \Lambda_0 r_0 \). The final form of the solution is consequently
\[ \Lambda(r) = \Lambda_0 \left[ \frac{r_0}{r} \ln \left( \frac{r}{r_0} \right) + \frac{r_0}{r} \right]. \] (42)
On account of the relation (4), in terms of the mass density the solution is written as

$$\rho(r) = \rho_0 \left[ \frac{r_0}{r} \ln \left( \frac{r}{r_0} \right) + \frac{r_0}{r} \right].$$ (43)

The combination of this fiducial mass density profile $\rho(r)$ with the de Sitter modified Newtonian force (32) yields a flat rotation curve for the galaxy without necessity of supposing the presence of dark matter. It should be noted that the mass density profile (43) represents a small correction to the power law $\rho(r) \simeq \rho_0 (r_0/r)$, which is within the class of physically acceptable profiles [17].

Furthermore, note also that, from Eqs. (38) and (39), the squared velocity of the flat portion of a galaxy rotation curve is given by

$$v_0^2 \equiv \frac{GM_0}{6} \beta = \frac{GM_0}{6} \Lambda_0 r_0.$$ (44)

Using the average values for the mass and radius of a typical galaxy bulge, given respectively by [18]

$$M_0 = 10^{10} M_\odot \simeq 2 \times 10^{40} \text{ Kg} \quad \text{and} \quad r_0 = 3 \text{ kpc} \simeq 10^{20} \text{ m},$$

as well as $\Lambda_0$ given by Eq. (35), the squared circular velocity of a star around the galactic center is found to be

$$v_0^2 = \frac{GM_0}{6} \Lambda_0 r_0 \simeq 10^{10} \text{ m}^2\text{s}^{-2},$$ (45)

which is of the order of magnitude of the flat portion of a typical galaxy rotation curve. For example, the squared velocity of the Sun around the galactic center is $v^2 \simeq 5 \times 10^{10} \text{ m}^2\text{s}^{-2}$. In addition to explain the flat rotation curve of galaxies without necessity of supposing the presence of dark matter, the de Sitter modified Newtonian force gives the correct order of magnitude for the circular velocities of the stars.

### 4 Final remarks

Due to the existence of an invariant length at the Planck scale, which is not allowed by ordinary special relativity, there is a widespread belief that Lorentz symmetry should break down at that scale. However, this is not necessarily true. In fact, if one replaces the Poincaré-invariant special relativity by a de Sitter invariant special relativity, it is possible to reconcile Lorentz symmetry with the existence of an invariant length at the Planck scale. This replacement will of course produce concomitant changes in all relativistic theories, including general relativity, giving rise to what we have called de Sitter modified general relativity.

In a recent companion paper [5], we have obtained the Newtonian limit of this theory, as well as the corresponding Newtonian Friedmann equations. Using these equations, we have shown that such theory is able to give a reasonable account of the dark energy content of the present-day universe. In the present paper, we have used the same Newtonian limit to
study galaxy rotation curves. The main difference of the de Sitter modified Newtonian limit in relation to the usual one is the existence two new forces, as can be seen from Eq. (32). The first one is repulsive and is proportional to $\Lambda$, whereas the second is proportional the radial derivative of the cosmological term $\Lambda$. Since $d\Lambda/dr < 0$ in the case of galaxies, this new force is attractive. Most importantly, it vanishes in the galactic bulge, becoming active only in the Keplerian region of the galaxy, where $\Lambda$ decays faster. Using the de Sitter modified Newtonian force, we have obtained a kind of fiducial mass density profile, given by Eq. (43), which yields a flat rotation curve for the galaxy without necessity of supposing the existence of dark matter.

It should be remarked that the de Sitter modified general relativity is entirely constructed from first principles: one has just to replace Einstein special relativity, whose kinematics is ruled by the Poincaré group, by a de Sitter invariant special relativity. This is a conceptual advantage in relation to the MOND paradigm [19], which is a theory empirically motivated, and does not follow as the weak field limit from a relativistic theory for gravitation. In addition, not all galaxies show a perfectly flat rotation curve. In spite of this fact, these galaxies can still be studied in our approach: one has simply to replace Eq. (39) by

$$r^2 \frac{d\Lambda(r)}{dr} + \Lambda(r)r = \beta(r),$$

with $\beta(r)$ a function describing the behavior of the galaxy rotation curve in the Keplerian region. The solution to this equation is easily found to be

$$\Lambda(r) = \frac{1}{r} \int \frac{1}{r} \beta(r)dr + \gamma \frac{r_0}{r}.$$  

(47)

Considering that the explicit form of $\beta(r)$ can be inferred from observations, one can then find the explicit form of $\Lambda(r)$, or equivalently, the explicit form of the mass density profile $\rho(r)$ that gives rise to the observed rotation curve.

As an illustration, let us consider the specific case in which, instead of a flat rotation curve, the circular velocity of the stars rises slowly and linearly with the distance $r$. This corresponds to a $\beta(r)$ of the form

$$\beta(r) = a \Lambda_0 r,$$

(48)

where $a$ is a constant that determines how fast the circular velocities rise. In this case, and using the boundary condition $\Lambda(r_0) = \Lambda_0$, solution (47) assumes the form

$$\Lambda(r) = \Lambda_0 \left[ a + (1 - a) \frac{r_0}{r} \right].$$

(49)

which corresponds to the mass density profile

$$\rho(r) = \rho_0 \left[ a + (1 - a) \frac{r_0}{r} \right].$$

(50)

Conversely, given a specific mass density profile, we can proceed backwards to find $\beta(r)$, which determines the corresponding galaxy rotation curve through Eq. (38). This means that the
theory can be applied individually for each galaxy, taking into account their different specifici-
ties. By establishing a correlation between the mass density profile of a galaxy to its rotation
curve, the theory can be experimentally tested for each individual galaxy. It is worth men-
tioning finally that the results presented in this paper are in agreement with recent evidences
favoring a gravitational solution to the missing mass problem [20, 21], as well as with the lack
of experimental signs of particles that could play the role of dark matter [22–25].

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