Quantum mechanics and irreversible time flow
Miloš V. Lokajíček
Institute of Physics, AS CR, 18221 Prague 8, Czech Republic

Abstract
Time flow has been embodied in time-dependent Schrödinger equation representing one of the foundations of quantum mechanics. Pauli’s criticism (1933) has, however, indicated that the assumptions concerning representation Hilbert space have led to some contradictions. Many authors have tried to solve this discrepancy practically without any actual success. The reason may be seen in that two different problems have been mixed: the problem of Pauli (being more general and more important) and non-unitarity of exponential phase operator of linear harmonic oscillator introduced by Dirac, as demonstrated in 1964. The problem will be discussed in a broad historical context and a solution based on extension of representation Hilbert space will be shown.

1. Introduction
Time flow in quantum mechanics represented an important problem practically during the whole past century. It is, therefore, necessary to start with a brief survey concerning the beginning of quantum mechanics. This beginning is linked closely with the name of E. Schrödinger (1925, [1]), who believed firmly in wave nature of matter. He proposed his famous wave equation and was successful when he was able to reproduce all main results obtained earlier with the help of Hamilton equations. It was possible to derive all characteristics of a physical system from a wave function \( \Psi(\tilde{x}, t) \) where \( \tilde{x} \) represented coordinates of all matter objects. A great success has been seen in that the discrete atom energy levels have corresponded to eigenvalues of Hamiltonian [2]. The wave function \( \Psi \) was interpreted as probability distribution by M. Born [3]. N. Bohr [4] attributed then the probabilistic properties directly to individual matter particles; Heisenberg’s [5] uncertainty relations being linked closely with their properties.

A further step was done by J. von Neumann [6] who showed that individual \( \Psi \)-functions at given \( t \) values may be represented by vectors in the Hilbert space spanned on eigenfunctions of the corresponding Hamiltonian (determining the total energy of a given physical system). Some additional assumptions have been introduced, however, into the corresponding mathematical model. And Pauli’s criticism [7] has concerned just them. Pauli has showed that under the mentioned conditions the time evolution (introduction of the time operator) has required for the Hamiltonian to possess continuous spectrum belonging to the whole real interval \((-\infty, +\infty)\), which contradicts the necessity of energy being positive (or at least limited from below).

This problem has been, however, often related to another one concerning the non-unitarity of exponential phase operator

\[
\mathcal{E} = e^{-i\Phi}
\]  

\(^1\)Presented at the conference "Irreversible quantum dynamics", ICTP Trieste, 29.7.-2.8.2002
where the phase $\Phi$ is proportional to time. The exponential phase operator for linear harmonic oscillator was defined by Dirac \cite{8} already in 1927 and it was assumed to be unitary. However, it was shown later by Suskind and Glogower \cite{9} that the operator $\mathcal{E}$ defined in such a way was isometric only; see also the review by Lynch \cite{10}. The non-unitarity has been then assumed to follow from the same source as Pauli’s criticism. It will be shown, however, in the following that they must be handled as two separate problems.

Both the problems will be described to a greater detail in Sec. 2. Pauli’s problem is closely related to two basic assumptions of quantum-mechanical model: time-dependent Schrödinger equation and superposition principle (as will be explained in Sec. 3), while the other problem is more formal and relates rather to some features of quantum field theory. And we should ask how it was possible that the standard quantum-mechanical model with known paradoxes was regarded as the only model of microworld during the whole past century, even if critical arguments have been repeatedly formulated (e.g., Einstein’s criticism \cite{11}); the corresponding story being described in Sec. 4. In Sec. 5 it will be then shown that it has not been only the known mistake of von Neumann, but also two other mistakes that have played yet more important role in common conviction and in commonly accepted conclusions.

The possible solutions of both the problems (concerning the introduction of time operator and avoiding the mistakes described in Sec. 5) will be given in the other half of the paper. In Sec. 6 it will be shown that Pauli’s problem may be solved by extending the Hilbert space according to the theory of Lax and Phillips. The non-unitarity of exponential phase operator may be then solved by doubling further such (already once extended) Hilbert space according to the proposal of Fajn, or of Newton and Bauer, as shown in Sec 7. Some differences in the evolution of states belonging to discrete or continuous spectra of Hamiltonian will be discussed in Sec. 8. Several concluding and summarizing remarks will be given in Sec. 9.

2. Quantum mechanics and time operator

As already mentioned the problem of time operator in quantum mechanics started to be solved by Dirac \cite{8} for the case of linear harmonic oscillator described with the help of Hamiltonian

$$H = \frac{p^2}{2m} + kq^2$$  \hspace{1cm} (2)

when annihilation and creation operators

$$a = p - im\omega q, \quad a^\dagger = p + im\omega q$$  \hspace{1cm} (3)

have fulfilled relations

$$[H, a] = -\omega a, \quad [H, a^\dagger] = \omega a^\dagger; \quad \omega = \sqrt{\frac{k}{m}}.$$  \hspace{1cm} (4)

It has been possible to introduce operators

$$\mathcal{E} = (aa^\dagger + 1)^{1/2}a, \quad \mathcal{E}^\dagger = a^\dagger(aa^\dagger + 1)^{1/2}$$  \hspace{1cm} (5)

fulfilling the relations

$$[H, \mathcal{E}] = -\omega \mathcal{E}, \quad [H, \mathcal{E}^\dagger] = +\omega \mathcal{E}^\dagger$$  \hspace{1cm} (6)
representing exponential phase operator corresponding to Eq. (1).

However, it has been shown later by Susskind and Glogower [9] that the operator $\mathcal{E}$ is not unitary, but only isometric, as it holds $\mathcal{E}^\dagger \mathcal{E} u_{1/2} = 0$ (and not $u_{1/2}$). It means that the unitarity condition is not fulfilled for the state vector corresponding to the minimum-energy (vacuum) state. And we may ask whether the problem may be correlated to the problem of Pauli or not.

As to Pauli’s criticism it represents more important problem. Starting from the standard quantum-mechanical model the time evolution of a physical system is fully determined by the wave function

$$\Psi(\tilde{x}, t) = \int dE a_E(t) u_E(\tilde{x})$$

where $u_E(\tilde{x})$ are eigenfunctions of corresponding Hamiltonian. And according to Pauli the existence of time operator in the corresponding Hilbert space requires for Hamiltonian to have continuous spectrum $E \in (-\infty, +\infty)$.

And one can hardly regard these two discrepancies as consequences of one common problem. Each of them must be removed in a proper way. However, before being possible to propose the solutions of these two problems it is necessary to go back to foundations of quantum mechanics and to the corresponding story in the 20th century.

3. Time-dependent Schrödinger equation and superposition principle

It is possible to say that the standard quantum mechanics is based on two following basic assumptions:

- any state of a physical system and its time evolution is represented by the wave function $\psi(\{x_{k,j}\}, t)$ that is obtained by the solution of time-dependent Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} \psi(\{x_{k,j}\}, t) = H \psi(\{x_{k,j}\}, t)$$

where $H$ is corresponding Hamiltonian

$$H = \sum_{j=1}^{N} \sum_{k=1}^{3} \frac{p_{k,j}^2}{2m_j} + V(\{x_{k,j}\})$$

and $\{x_{k,j}\}$ and $\{p_{k,j}\}$ are coordinates and momentum components of individual objects; $V(\{x_{k,j}\})$ is the sum of potential energies between $N$ individual mass objects; it is put $\{p_{k,j}\} = \frac{\hbar}{i} \frac{\partial}{\partial x_{k,j}}$ in Eq. (8); and $\{x_{k,j}\}$ is coordinates and momentum components of individual objects; $V(\{x_{k,j}\})$ is the sum of potential energies between $N$ individual mass objects; it is put $\{p_{k,j}\} = \frac{\hbar}{i} \frac{\partial}{\partial x_{k,j}}$ in Eq. (8);

- any physical state is represented by a vector in Hilbert space being spanned on one set of Hamiltonian eigenfunctions

$$H \psi_E(\{x_{k,j}\}) = E \psi_E(\{x_{k,j}\})$$

and all states are bound together with the help of superposition principle.

However, these two assumptions exhibit mutual contradiction, if applied to a physical system. It has been argued that any superposition of two solutions of Schrödinger equation...
has been again a solution of the same equation. However, such a statement is entitled and has regular physical meaning only if both these solutions correspond to the same initial conditions. Superposing solutions belonging to different initial conditions one obtains solutions corresponding to fully different initial conditions, which means that significantly different physical states have been combined in a unallowed way.

The general mathematical superposition principle holding for linear differential equations has nothing to do with physical reality, as actual physical states and their evolution is uniquely defined by corresponding initial conditions. These initial conditions characterize individual solutions of Schroedinger equation (8); they correspond to different properties of a physical system, some of them being conserved during the whole evolution.

The physical superposition principle has been deduced from the linearity of Schrödinger differential equation without any actual reason. This drastic assumption was introduced into the physics without any need and any proof; the solutions belonging to diametrically different initial conditions have been arbitrarily superposed. Statements that quantum mechanics (including superposition rules) has been experimentally verified must be regarded as wrong. All hitherto tests have concerned consequences following from Schrödinger equations only.

However, the story of quantum mechanics in the 20th century was paved by other more serious mistakes. One of them (i.e., the mistake of von Neumann) has been usually well known, while the other two have remained practically hidden till now. We shall follow the whole story in the next section.

4. Quantum mechanics in the 20th century

The foundations of quantum mechanics have been mentioned already in Sec. 1. They have related mainly to the names of E. Schrödinger (1925), M. Born (1926), W. Heisenberg (1927), N. Bohr (1927) and J. von Neumann (1932). A deep ontological change has been introduced into matter nature without any actual physical reason. Paradoxical ideas have been accepted by the most physicists even if A. Einstein, who discovered the photon [12] and should be held for one of main founders of quantum theory, criticized strongly physical characteristics following from the given quantum-mechanical mathematical model. Critical comments (“circle” proof) have been published also by G. Herrmann [13], but they have not been taken into account, either.

Einstein [11] denoted the given model as incomplete and required to add some other parameters, the so called hidden variables, necessary for a full description of concerned physical systems. His critical arguments have been, however, refused by N. Bohr [14] and practically by the whole physical community, as well.

Nevertheless, they have been permanently discussed during the whole century. The discussion started to continue when D. Bohm [15] showed in 1952 that a kind of hidden variable was contained already in the time-dependent Schrödinger equation. A decisive progress came then when J. Bell [16] specified exactly and modified non-physical assumption (made use of by von Neumann) and derived his known inequalities. It was believed that the time came when the controversy between Einstein and Bohr might be solved on experimental basis. And really some suitable experiments (measurement of coincidence
transmission of polarized photons through a pair of polarizers) have been proposed and also performed. The final results provided by L. Aspect et al. [17] in 1982 came then to the following conclusions:

- Bell’s inequalities have been violated by experimental data;
- the experimental results have been practically in agreement with quantum-mechanical predictions.

It might seem that the given problem has been solved by these experiments. In the first years it was actually spoken about the victory of quantum mechanics. However, the basic logical controversy has remained unsolved and the discussions have continued until now.

While the mistaking assumption of von Neumann has concerned general mathematical structure (measurement problem in quantum mechanics) all other discussions has concerned concrete points of experimental arrangement: coincidence transmission of equally polarized photons through two polarizers. And one must ask whether all arguments (or assumptions) made use of in interpretation of these coincidence measurements have corresponded to reality.

The two additional arguments, which may be considered misleading in a similar way as that of von Neumann, seem to have played a very decisive role in the common acceptance of given conclusions. One (not reasoned) assumption has been introduced in deriving Bell’s inequalities. And a seemingly convincing (but false) argument was given in the book by Belifante in 1973 [18]. Both these arguments will be analyzed in the next section.

5. Mistaking arguments

It is possible to say that the way to paradoxical interpretation of quantum mechanics has been fundamentally influenced by three mistakes. The consequences of von Neumann’s mistake have lasted practically until now, even if it has been recognized and removed in principle already earlier. It has had a close relation to the disagreement between the physical superposition principle and the time-dependent Schrödinger equation. Not only the wave function but also its first derivatives must be known at one time moment if the evolution of a physical system should be determined.

It means that all properties and the whole evolution cannot be fully defined by one vector of the standard Hilbert space if something more is not added from outside, e.g. superposition coefficients defined as functions of coordinates taken from corresponding solutions of Schroedinger equation. A momentum direction cannot be derived from a mere linear combination of eigenvectors if energy corresponds to momentum squared. All properties may be determined by one vector only if the Hilbert space is correspondingly doubled (see Sec. 6).

However, in the last time the public conviction has been influenced more by two other arguments that must be denoted as misleading. The mistake of von Neumann has been removed by J. Bell only partially. A similar (even if much weaker) assumption has been involved in derivation of Bell’s inequalities, as well. These inequalities were being derived in different ways (see, e.g., [19]), but in all an important (latent) assumption has been
involved.

We will comment now very briefly two kinds of their derivations (shown, e.g., in Ref. [19]), which have been discussed to a necessary detail in Ref. [20]. In the first approach it is possible to pass from Eq. (3.11) to Eq. (3.12) (in Ref. [19]) only if the probabilities (or expectation values) in different pair measurements might be interchanged, which means that the influence of internal structures of both the polarizers (e.g., the influence of the impact of individual localized photons into the atom grid) has been neglected. Bell’s inequalities cannot be derived if this degree of freedom is taken into account; in addition to photon polarization and polarizer axis orientation (i.e., if all three vector characteristics are respected). In the other approach (discussed in [20], as well) practically the same neglection has been involved in the passage from Eq. (3.14) (or (3.15)) to Eq. (3.16) (see Ref. [19]).

It means that the measuring device has been regarded as a black (or at least semi-black) box in a similar way as in the standard quantum mechanics. Actual localization of photon impact into measuring device has been fully omitted. And one must conclude that the violation of Bell inequalities in coincidence polarization experiments is quite irrelevant as to the support for standard quantum mechanics.

The quantum-mechanical interpretation of EPR experiments seems, however, to have been much more supported by the argument of Belifante who stated that the predictions of a hidden-variable theory had to differ significantly from those of quantum-mechanical model (see the corresponding graph on p. 284 in Ref. [18]), which is not true. According to a hidden-variable theory the transmission of a photon (or of two equally polarized photons in coincidence arrangement) through a polarizer pair should equal

\[ m(\alpha) = \int_{-\pi/2}^{\pi/2} p_1(\lambda) p_1(\lambda - \alpha) \, d\lambda \]  

where \( p_1(\lambda) \) is transmission probability through one polarizer; \( \lambda \) - deviation of photon polarization from the axis of the first polarizer; and \( \alpha \) - the angle deviation of the second polarizer. The same formula (11) holds exactly in both the arrangements if the photon polarization does not change in passing through a polarizer.

It is known from the one-side arrangement that it holds

\[ m(\alpha) = (1 - \varepsilon) \cos^2(\alpha) + \varepsilon \]  

where for real (imperfect) polarizers is \( \varepsilon > 0 \) (generalized Malus law); \( \varepsilon \) being very small. Belifante has chosen

\[ p_1(\lambda) = \cos^2(\lambda); \]  

i.e., he has interchanged quite arbitrarily the transmission through one polarizer and through a pair of them. Malus law may be reproduced easily also in hidden-variable alternative if \( p_1(\lambda) \) is chosen in a corresponding way as shown in Fig. 1 (full line).

And one must conclude that any preference for the quantum-mechanical interpretation of coincidence EPR experiments does not follow from experimental data. Nothing
Figure 1: Transmission probability through a polarizer pair leading to Malus law; $p_1(\lambda)$ - full line; $m(\lambda)$ - dashed line; Malus law - individual points.

prevents us from interpreting all available experiments on the basis of a hidden-variable theory. This fact has opened also a new way for answering the question of the time operator and/or of Pauli’s problem.

6. Pauli’s problem and theory of Lax and Phillips

In the standard quantum-mechanical model the Hilbert space has been spanned on a simple vector basis consisting of Hamiltonian eigenfunctions (independently of spectrum kind). In such a case the states corresponding to momenta of different signs are represented in principle by the same vector (Hamiltonian being a function of momentum squares). The sign of wave function derivatives depends on the choice of coordinate system and must be always added from outside; additional information taken from Schroedinger equation. And Pauli’s critique [7] has concerned just this fact. The given deficiency may be removed if the standard Hilbert space is doubled in a way, as it was done by Lax and Phillips [21] already in 1967 (see also [22]) and derived independently by Alda et. al. [23] in solving the problem of unstable particles exhibiting purely exponential decay.

Let us demonstrate at least shortly the given Hilbert structure on the example of a system consisting of two free particles. The corresponding Hilbert space consists then of two subspaces:

$$\mathcal{H} \equiv \{\Delta^-, \Delta^+\}$$

that are mutually related by evolution operator

$$U(t) = e^{-iHt} \ (t \geq 0).$$

It holds, e.g.,

$$\mathcal{H} = \sum_{t} U(t)\Delta^- = \sum_{t} U(-t)\Delta^+$$

7
Individual subspaces $\Delta^-$ and $\Delta^+$ are spanned on Hamiltonian eigenfunction in the usual way. Any $t$-dependent wave function obtained by solving Schrödinger equation may be then represented by a trajectory corresponding to given initial conditions.

In the case of continuous Hamiltonian spectrum (free particles) any point on such a trajectory may be characterized by expectation values of the operator $R = \frac{1}{2}\{p,q\}$, where $q$ and $p$ are coordinates and momentum components of one particle in CMS. The states belonging to $\Delta^-$ are incoming states, and those of $\Delta^+$ - outgoing states (independently of the chosen coordinate system). The evolution goes always in one direction from ”in” to ”out”.

As these two different kinds of states may be experimentally distinguished it is useful to separate ”in” and ”out” states into two mutually orthogonal subspaces. It is then also possible to join an additional orthogonal subspace that might represent corresponding resonances formed in particle collisions (see [23]); i.e.

$$\mathcal{H} = \{\Delta^- \oplus \Theta \oplus \Delta^+\}. \quad (17)$$

It is only necessary to define the action of evolution operator between $\Theta$ and other subspaces in agreement with evolution defined already in individual $\Delta^\pm$.

The evolution goes in one direction, at least from global view; some transitions between internal states of $\Theta$ may be reversible and chaotic. However, global trajectories tend always in one direction; see the scheme in Fig. 2.

$$R = \frac{1}{2}\{p,q\}; \quad \langle i[H,R] \rangle > 0 \quad \Delta^-(\langle R \rangle < 0) \quad \downarrow \quad \Delta^+(\langle R \rangle > 0) \quad \text{"in"} \quad \rightarrow \quad \text{"out"}$$

Fig. 2: Scheme of the Hilbert space (for a two-particle system) extended according to Lax and Phillips; three mutually orthogonal subspaces and direction of time evolution (continuous spectrum).

In the case of discrete Hamiltonian spectrum (e.g., harmonic oscillator) the wave function has similar $t$-dependent form. However, the evolution is periodical as a rule. The Hilbert space will consist now of two non-orthogonal subspaces (or of an infinite series of such pairs if one wants to number individual periods). The evolution may be again characterized by trajectories corresponding to different initial conditions. Different points of these trajectories may be now determined by expectation values of the phase operator from the interval $(0, 2\pi)$ in individual (neighbour) subspace pairs; or in analogy with the continuous case (for one period) by those of $tg(\Phi/2)$ or $cotg(\Phi/2)$ increasing again from $-\infty$ to $+\infty$. 
Such an extended model enables to represent the time evolution in the Hilbert space in a full agreement with actual behavior of physical systems. It gives the full answer to Pauli’s criticism that was misunderstood in the past having been related to non-unitarity of exponential phase operator found in the case of linear harmonic oscillator.

7. Exponential phase operator and its unitarity

We have already mentioned that the exponential phase operator defined by Dirac in the case of linear harmonic oscillator is not unitary. Unitarity breaks at minimum-energy (vacuum) state, which means that the phase operator is not regularly defined in the whole standard Hilbert space. The problem concerns the Hamiltonian possessing discrete spectrum. And the question may be put, whether the problem exists also in the case of continuous spectrum when the vacuum state corresponds practically to zero energy.

As to the discrete spectrum the response was given by Fain [24] already in 1967, who showed that the problem may be solved by doubling the standard Hilbert space. The doubled Hilbert space should consist of two identical mutually orthogonal subspaces:
\[ \mathcal{H} = \mathcal{H}_+ \oplus \mathcal{H}_- , \]
(18)
the orthogonality of them remaining conserved during the whole time evolution:
\[ U(t) \mathcal{H}_+ \in \mathcal{H}_+, \quad U(t) \mathcal{H}_- \in \mathcal{H}_-. \]
(19)
The minimum-energy states in both subspaces have been mutually linked with the help of the annihilation-type operators. The states in the two mutually orthogonal subspaces (with separated time evolution) may be distinguished only by different signs in the relation between the phase and the flowing time: \( \Phi = \pm \omega T. \)

Newton [25] and Bauer [26] have proposed a similar solution independently later. However, they have not recognized the role of phase sign; he regarded the other subspace as a ghost space without any physical meaning.

In fact the given sign may be interpreted probably in a physical sense. It follows from the analysis of three-dimensional oscillator (see [27]) that it may be related to the orientation of the co-ordinate system or to the orientation of the corresponding component of resulting spin (or of angular momentum of a two-particle system). The individual Hilbert subspaces \( \mathcal{H}_+ \) and \( \mathcal{H}_- \) should be, of course, extended according to the proposal of Lax and Phillips (see Sec. 6).

8. Twice extended Hilbert space

Both the problems concerning regular description of time evolution in the framework of Hilbert space could be solved as two different problems by suitable extensions of the standard Hilbert space. The solution could be hardly reached in the past when the problems were combined together.

The twice extended model may provide a regular mathematical basis for the description of irreversible processes in the microscopic physics. All objects are described semiclassically, which enables to take into account also the microscopic characteristics of measuring devices, as needed.
The extension proposed by Lax and Phillips should be regarded as basic and more important. It allows to represent truly corresponding solutions of time-dependent Schrödinger equation for both the kinds of spectrum (continuous and discrete). The physical evolution operator $U(t)$ moves vectors in one direction only; from "in" to "out" (evolution into infinitum) in the continuous case, and alternatively from one subspace to another in the discrete periodical case. Points on individual evolution trajectories may be characterized by expectation values of operators $R$ or $\Phi$ (resp. $tg(\Phi/2)$ in one half-period). The given characteristics for the discrete spectrum has been derived by solving the problem of three-dimensional harmonic oscillator [27] since the simplified picture in the linear case could not provide a sufficient answer.

As to the continuous case it is useful to introduce the orthogonality condition between "in" and "out" subspaces as considered by Lax and Phillips. An additional subspace may be included that may represent corresponding unstable resonances formed in collision processes. The problem will be mentioned yet in the next section (conclusion).

To remove the non-unitarity of exponential phase operator a further doubling of the Hilbert structure in addition to that required by Pauli's problem is necessary. This second doubling differs from the preceding approach in that the two subspaces must be now permanently orthogonal; there is not any linkage of theirs by evolution operator, either. The evolution trajectories do not leave individual orthogonal subspaces. And one should ask whether these two independent evolutions represent identical or different physical processes. One can hardly expect any measurable difference (and distinguishing) in a (two-particle) system corresponding to a continuous spectrum. However, a question has remained open in the case of discrete spectrum as the sign of the phase may relate closely to the space orientation of angular momentum.

9. Conclusion

It has been shown that the paradoxical quantum-mechanical picture accepted by the physical community in the past century was supported by several mistakes in corresponding argumentations. A different picture of microworld following from all experimental data if these mistakes have been removed should be now accepted; being in agreement with Einstein's view. It is possible to represent corresponding evolution trajectories in a Hilbert space if this space is extended (in principle doubled) in comparison to that used standardly in quantum mechanics. There have been, of course, two different problems that have had to be solved separately.

As to the Pauli's problem it could be fully solved by introducing the extension proposed by Lax and Phillips in 1967. It has been then possible to represent physical evolution by trajectories characterized by different initial conditions. However, a kind of superselection rules should be applied to states belonging to trajectories corresponding at least to different values of physical quantities being conserved during the evolution. Physical systems (consisting of stable objects) exhibit the evolution that may be derived from Schrödinger or Hamilton equations. The points on individual evolution trajectories may be then characterized by expectation values of operators $R$ or $\Phi$ being in unique relation
to the time operator. It means that evolution trajectories exhibit similar properties as those in classical phase space.

The other problem concerning the non-unitarity of exponential phase operator should be regarded as less important. It was solved by Fain also in 1967, at least in principle. However, the solution was demonstrated with the help of linear harmonic oscillator where its full physical meaning has remained hidden. It may be understood only when the approach is applied to a three-dimensional system (harmonic oscillator). The question concerns the meaning of the phase sign (relatively to flowing time). It has been shown that it may have a sense for bound systems (with discrete Hamiltonian spectrum) as it may be related to the orientation of angular momentum. However, a final question of actual physical meaning has remained yet open and should be further analyzed.

And it is possible to conclude:

(i) The evolution of all matter world may be described as continuous and irreversible.

(ii) The existence of quantum energy states cannot be regarded as a support for quantum jumps (denoted as damned by Schrödinger); however, they do not represent any support for waves, either.

It is evident that all evolution goes always in an irreversible way, i.e. from its origin to its end. It concerns living as well as non-living objects. Only exactly periodical systems may return to some preceding states, which holds, of course, when these systems are mutually isolated; anyway, there is not any reversible evolution. When they are a part of greater systems the total evolution should be always described as fully irreversible. And the mathematical model based on the given extension is able to involve the description of complex physical systems in agreement with reality.

The given approach seems to be in agreement also with the recent proposal of W. Lamb [28] who has tried to interpret the quantum-mechanical approach in a semiclassical way. The only difference seems to consist in that some serious problems have been removed by narrowing the validity of superposition principle (by further extension of validity of superselection rules). The new interpretation of quantum-mechanical approach is also decisively supported by experimental results gained in measuring light transmission through three polarizers (see [29]).

References

[1] E. Schrödinger: Quantisierung als Eigenwertproblem; Ann. Phys. 79 (1926), 361-76; 489-527; 80 (1926), 437-90; 81 (109-39.
[2] E. Schrödinger: Über das Verhältnis der Heisenberg-Born-Jordanschen Quantenmechanik zu der meinen; Ann. Phys. 79 (1926), 734-56.
[3] M. Born: Zur Quantenmechanik der Stossvorgänge; Zeitschr. f. Physik 37 (1926), 863-7.
[4] N. Bohr: The quantum postulate and the development of atomic theory; Nature 121 (1928), 580-90.
[5] W. Heisenberg: Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik; Zeitschr. Phys. 43 (1927), 172-98.
[6] J. von Neumann: Mathematische Grundlagen der Quantenmechanik; Springer 1932.

[7] W. Pauli: Die allgemeinen Prinzipien der Wellenmechanik; Handbuch der Physik XXIV, Springer Berlin 1933, p. 140.

[8] P.A.M. Dirac: The quantum theory of the emission and absorption of radiation; Proc. Roy. Soc. (London) A 114 (1927), 243-65.

[9] L. Susskind, J. Glogower: Quantum mechanical phase and time operator; Physics 1 (1964), 49-61.

[10] R. Lynch: The quantum phase problem, a critical review; Phys. Rep. 256 (1995), 367-437.

[11] A. Einstein: Can quantum-mechanical description of physical reality be considered complete?; Phys. Rev. 47 (1935), 777-80.

[12] A. Einstein: Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt; Ann. Phys. 17 (1905), 132-48.

[13] Grete Herrmann: Die Naturphilosophischen Grundlagen der Quantenmechanik; Abhandlungen der Fries'schen Schule 6 (1935), 75-152.

[14] N. Bohr: Can quantum-mechanical description of physical reality be considered complete?; Phys. Rev. 48 (1935), 696-702.

[15] D. Bohm: A suggested interpretation of the quantum theory in terms of "hidden variables"; Phys. Rev. 85 (1952), 180-93.

[16] J.S. Bell: On the Einstein Podolsky Rosen paradox; Physics 1 (1964), 195-200.

[17] A. Aspect, P. Grangier, G. Roger: Experimental realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment: A new violation of Bell's inequalities; Phys. Rev. Lett. 49 (1982), 91-4.

[18] F.J. Belinfante: A survey of hidden-variable theories; Pergamon, Oxford 1973, p. 284.

[19] J.F. Clauser, A. Shimony: Bell’s theorem: experimental tests and implications; Rep. Prog. Phys. 41 (1978), 1881-91.

[20] M. Lokajíček: Locality problem, EPR experiments and Bell’s inequalities; http://xxx.lanl.gov/quant-ph/9808005.

[21] P.D. Lax, R.S. Phillips: Scattering theory; Academic Press 1967.

[22] P.D. Lax, R.S. Phillips: Scattering theory for automorphic functions; Princeton 1976.

[23] V. Alda, V. Kundrát, M. Lokajíček: Exponential decay and irreversibility of decay and collision processes; Aplikace matematiky 19 (1974), 307-15.

[24] V. Fajn: Quantum harmonic oscillator in phase representation and the uncertainty relation between the number of quanta and the phase (in Russian); J. Exp. Theor. Phys. 52 (1967), 1544-8.

[25] R.G. Newton: Quantum action-angle variables for harmonic oscillators; Ann. Phys. (N.Y.) 124 (1980), 327-46.

[26] M. Bauer: A time operator in quantum mechanics; Ann. Phys. (NY) 150 (1983), 1-21.

[27] P. Kundrát, M. Lokajíček: Three-dimensional harmonic oscillator and time operator in quantum mechanics; submitted to Phys. Rev. A

[28] W.E. Lamb, Jr.: Super classical quantum mechanics: The best interpretation of nonrelativistic quantum mechanics; Am. J. Phys. 69 (2001), 413-22.

[29] M. Lokajíček: Quantum mechanics and EPR paradox; FZU-D 20020207, Inst. of Phys. AS CR, Prague; http://xxx.lanl.gov/quant-ph/0211012.