D2 Delta Robot Structural Design and Kinematics Analysis

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Abstract. In this paper, a new type of Delta robot with only two degrees of freedom is proposed on the basis of multi-degree-of-freedom delta robot. In order to meet our application requirements, we have carried out structural design and analysis of the robot. Through SolidWorks modeling, combined with 3D printing technology to determine the final robot structure. In order to achieve the precise control of the robot, the kinematics analysis of the robot was carried out. The SimMechanics toolbox of MATLAB is used to establish the mechanism model, and the kinematics mathematical model is used to simulate the robot motion control in Matlab environment. Finally, according to the design mechanism, the working space of the robot is drawn by the graphic method, which lays the foundation for the motion control of the subsequent robot.

1. Introduction

In 1965, Stewart in his article presented a six-degree-of-freedom parallel mechanism, which became the prototype of many Delta robots. In 1979, Mccallino developed the world's first Delta parallel robot on the basis of Stewart's organization, and many scholars have been involved in the study of parallel robots. Compared with the traditional series of robots, Delta parallel robot move fast, high precision, strong carrying capacity, and light structure. After more than 30 years of development, Delta robot has been successfully applied in the food packaging industry, medicine, sorting products and a variety of needs to capture the work of the work environment.

Nowadays, the study of multi-degree-of-freedom Delta robot has been deep and mature, but it is not a lot of research on less freedom. Many work does not require more freedom of the operation, only two degrees of freedom can be completed. Two degrees of freedom Delta robot structure is simpler, cheap, move platform can only move on a plane, it is easy to achieve robot motion control. So the two degrees of freedom delta robot has become one of the hotspots of international robotics research.

The research on Delta robot is mainly focused on structural design and analysis, kinematics analysis, dynamic analysis, and control strategy analysis. In the reference [4], a two-degree-of-freedom Delta robot for the grab product is introduced in detail, from concept design to kinematics modeling, and finally the robot is applied to the device for detecting the quality of the battery. In the reference [5], the kinematic parameters of the mechanism are optimized by analyzing the singularity of the Delta parallel robot and the analysis of the working space, and the dynamic analysis of the mechanism is carried out by the Newton-Euler method. In the literature [6], on the basis of the 3-UPU (universal-prismatic-universal) mechanism, a new two-degree-of-freedom parallel robot for high-speed operation is derived, and the robot is combined with a degree of freedom Organization, forming a three-degree-of-freedom composite robot, widening the application of two-degree-of-freedom Delta robots.
This paper consists of five parts. In the first part, we designed the structure of the D2 Delta Robot. In the second part, the kinematics analysis of the robot is carried out, and the mathematical model is established. In the third part, the robot model is built by Simmechanics toolbox in MATLAB, and the kinematic mathematical model is used to simulate the robot's trajectory control. In the fourth part, according to the limitation of the robot structure, the working space of the end effector is drawn by the graphic method. The last part is the conclusion.

2. Structural design and analysis
D2 Delta Robot is a planar six-bar mechanism as a design basis, as shown in Figure 1. The whole body is distributed in the XY plane, A point, point B as the rotation axis, the lever AB as the fixing bracket, the lever BC and the lever AF as the driving arm, the lever CD and the lever FE as the slave arm, and the lever ED as the end effector.

![Figure 1 D2 Delta Robot structure schematic](image)

During the movement, the active arms AB and BC are rotated around the rotation axis A and the rotation axis B, respectively, to drive the slave arms CD and the FE movement, and finally to control the end effector ED movement. In order to keep the end effector ED from rotating during the movement and always keep parallel to the fixed bracket AB, in the actual mechanical structure design, the active arm BC and the slave arm CD are designed as parallelogram link mechanism. In order to avoid the robot in the course of the movement of interference, from the slave arms FE and CD is designed to crank arm. Through several changes and corrections, the CAD model of the robot is drawn using SolidWorks, as shown in Figure 2-2. Using the 3D printing technology, the CAD structure of the robot is printed out (shown in Figure 2-3), which is adapted to our application requirements by actual detection.

![Figure 2 D2 Delta Robot CAD model](image) ![Figure 3 D2 Delta Robot 3D printing model](image)
3. D2 Delta Robot kinematics analysis
The kinematics analysis of the robot is divided into two parts: forward kinematics analysis and inverse kinematics analysis. Forward kinematics analysis refers to known active arm motion parameters (such as angular displacement, angular velocity, angular acceleration), to solve the end of the end effector motion parameters (such as displacement, speed, acceleration); Inverse kinematics is the case where the motion parameters of the end effector are known to solve the motion parameters of the active arm.

3.1 Forward Kinematics Analysis of D2 Delta Robot
As shown in Figure 1, the X-Y axis is used to establish the coordinate relationship. The whole robot is translated or rotated in the X-Y plane. The midpoint O of the end effector is taken as the object of study, the coordinates of the point are (Tx, Ty). Let the length of AF is L1, the length of FE is L2, the length of DC is L3, the length of BC is L4, the length of AB is L5, the length of ED is L6. The mathematical model of motion is established by the motion relation of the organization:

\[
\begin{cases}
-L_1 \cos \theta_1 + L_2 \cos \theta_2 + L_3 \cos \theta_3 - L_4 \cos \theta_4 - L_5 = 0 \\
-L_1 \sin \theta_1 - L_2 \sin \theta_2 + L_3 \sin \theta_3 + L_4 \sin \theta_4 = 0
\end{cases}
\]  

(1)

\[
\begin{cases}
T_x = -\frac{1}{2} L_5 - L_1 \cos \theta_1 + L_2 \cos \theta_2 + \frac{1}{2} L_6 \\
T_y = -L_1 \sin \theta_1 - L_2 \sin \theta_2
\end{cases}
\]  

(2)

\[
\begin{cases}
T_x = \frac{1}{2} L_5 + L_4 \cos \theta_4 - L_3 \cos \theta_3 - \frac{1}{2} L_6 \\
T_y = -L_4 \sin \theta_4 - L_3 \sin \theta_3
\end{cases}
\]  

(3)

The coordinates (Tx, Ty) of the end effector O are solved by using the rotation angles \( \theta_1 \) and \( \theta_4 \) of the two active arms as the input angle. The relationship between \( \theta_2, \theta_3 \) and \( \theta_1 \) and \( \theta_4 \) can be derived by the calculation of equation (1). The final equation is:

\[
A + B \sin \theta_3 + C \cos \theta_3 + D = 0
\]  

(4)

In equation (4):

\[
A = L_1^2 - L_2^2 + L_3^2 + L_4^2 + L_5^2 + L_6^2
\]  

(5)

\[
B = -2L_1 L_3 \sin \theta_1 + 2L_3 L_4 \sin \theta_4
\]  

(6)

\[
C = -2L_1 L_3 \cos \theta_3 - 2L_3 L_4 \cos \theta_4 + 2L_3 L_6 - 2L_3 L_5
\]  

(7)

\[
D = -2L_1 L_4 \sin \theta_1 \sin \theta_4 + 2L_1 L_4 \cos \theta_1 \cos \theta_4 - 2L_1 L_6 \cos \theta_4 + 2L_4 L_5 \cos \theta_4 - 2L_5 L_6
\]  

(8)

Use the trigonometric function rule, \( \tan \frac{\theta_2}{2} = \delta, \tan \theta_3 = \frac{2\tan(\frac{\theta_3}{2})}{1-\tan^2(\frac{\theta_3}{2})} = \frac{2\delta}{1-\delta^2}, \sin \theta_3 = \frac{2\delta}{1+\delta^2}, \cos \theta_3 = \frac{1-\delta^2}{1+\delta^2}. \) Put \( \sin \theta_3, \cos \theta_3 \) into the equation (4), we can obtain that:

\[
A + B \frac{2\delta}{1+\delta^2} + C \frac{1-\delta^2}{1+\delta^2} + D = 0
\]  

(9)

The equation (9) is simplified as:

\[
\delta^2 (A - C + D) + \delta (2B) + (A + C + D) = 0
\]  

(10)
\[
\delta = \frac{-B \pm \sqrt{B^2 - (A-C+D)(A+C+D)}}{(A-C-D)} \\
\theta_3 = 2 \tan^{-1} \delta \\
\theta_2 = \sin^{-1} \left( \frac{-L_3 \sin \theta_1 + L_3 \sin \theta_3 + L_4 \sin \theta_4}{L_2} \right) 
\]

The equation (13) into the equation (2) to find the end of the actuator coordinates:

\[
\begin{align*}
T_x &= -\frac{1}{2} L_5 - L_1 \cos \theta_1 + L_2 \cos \theta_2 + \frac{1}{2} L_6 \\
T_y &= -L_1 \sin \theta_1 - L_2 \sin \theta_2 \\
\theta_2 &= \sin^{-1} \left( \frac{-L_3 \sin \theta_1 + L_3 \sin \theta_3 + L_4 \sin \theta_4}{L_2} \right)
\end{align*}
\]

The equation (13) into the equation (3) to find the end of the actuator coordinates:

\[
\begin{align*}
T_x &= \frac{1}{2} L_5 + L_4 \cos \theta_4 - L_3 \cos \theta_3 - \frac{1}{2} L_6 \\
T_y &= -L_4 \sin \theta_4 - L_3 \sin \theta_3 \\
\theta_3 &= 2 \tan^{-1} \delta
\end{align*}
\]

3.2 Inverse Kinematics Analysis of D2 Delta Robot

D2 Delta Robot's inverse kinematics analysis is based on the coordinates of the end effector \(o(T_x, T_y)\) for the known input, to solve the rotation angle \(\theta_1\) of the active arm AF and the rotation angle \(\theta_4\) of BC.

By equation (2), we can get the equation:

\[
A_1 + B_1 \cos \theta_1 + C_1 \sin \theta_1 + D_1 = 0
\]

In equation (3.16):

\[
\begin{align*}
A_1 &= T_x^2 + T_y^2 + L_1^2 - L_2^2 + 0.25L_5^2 + 0.25L_6^2 \\
B_1 &= 2T_xL_1 + L_1L_5 - L_1L_6 \\
C_1 &= 2T_yL_1 \\
D_1 &= T_xL_5 - T_yL_6 - 0.5T_5L_6
\end{align*}
\]

Use the trigonometric function rule, \(\tan \frac{\theta_3}{2} = \sigma, \tan \theta_3 = \frac{2\tan(\frac{\theta_3}{2})}{1 - \tan^2(\frac{\theta_3}{2})} = \frac{2\sigma}{1 - \sigma^2}, \sin \theta_1 = \frac{2\sigma}{1+\sigma^2}, \cos \theta_1 = \frac{1 - \sigma^2}{1+\sigma^2}\). Put \(\sin \theta_1, \cos \theta_1\) into the equation (4), we can obtain that:

\[
A_1 + B_1 \frac{2\sigma}{1+\sigma^2} + C_1 \frac{1 - \sigma^2}{1+\sigma^2} + D_1 = 0
\]

The equation (21) is simplified as:

\[
\sigma^2(A_1 - B_1 + D_1) + \delta(2B_1) + (A_1 + C_1 + D_1) = 0
\]
$$\sigma = -B_1 \pm \frac{B_1^2 - (A_1 - C_1 + D_1)(A_1 + C_1 + D_1)}{(A_1 - C_1 + D_1)}$$  \hspace{1cm} (23)

$$\theta_1 = 2 \tan^{-1} \sigma$$  \hspace{1cm} (24)

By equation (3), we can get the equation:

$$A_2 + B_2 \cos \theta_4 + C_2 \sin \theta_4 + D_2 = 0$$  \hspace{1cm} (25)

In equation (25):

$$A_2 = T_x^2 + T_y^2 + L_4^2 - L_5^2 + 0.25L_5^2 + 0.25L_6^2$$  \hspace{1cm} (26)

$$B_2 = -2T_xL_4 + L_4L_5 - L_4L_6$$  \hspace{1cm} (27)

$$C_2 = 2T_yL_4$$  \hspace{1cm} (28)

$$D_2 = -T_xL_5 + T_xL_6 - 0.5T_5L_6$$  \hspace{1cm} (29)

Use the trigonometric function rule $\tan \frac{\theta_4}{2} = \rho$, $\tan \theta_5 = \frac{2\tan(\frac{\theta_4}{2})}{1 - \tan^2(\frac{\theta_4}{2})} = \frac{2\rho}{1 - \rho^2}$, $\sin \theta_4 = \frac{2\rho}{1 + \rho^2}$, $\cos \theta_4 = \frac{1 - \rho^2}{1 + \rho^2}$. Put $\sin \theta_4$, $\cos \theta_4$ into the equation (4), we can obtain that:

$$A_2 + B_2 \frac{2\rho}{1 + \rho^2} + C_2 \frac{1 - \rho^2}{1 + \rho^2} + D_2 = 0$$  \hspace{1cm} (30)

$$\rho^2(A_2 - B_2 + D_2) + \rho(2B_2) + (A_2 + C_2 + D_2) = 0$$  \hspace{1cm} (31)

$$\rho = \frac{-B_2 \pm \sqrt{B_2^2 - (A_2 - C_2 + D_2)(A_2 + C_2 + D_2)}}{(A_2 - C_2 + D_2)}$$  \hspace{1cm} (32)

$$\theta_4 = 2 \tan^{-1} \rho$$  \hspace{1cm} (33)

Through the equation (14), (15), if we know the active arm input angle, we can calculate the end effector coordinates. Through the equation (24), (33), if we know the end effector coordinates, we can calculate the input angle of the active arm.

4. D2 Delta Robot motion simulation experiment

Through simulation experiments, we can verify the accuracy of the D2 Delta Robot kinematics model and prepare for the following studies. Matlab as a good calculation and simulation software, it is widely used in robot technology. Simmechanics in Matlab Simulink Toolbox, which provides modules for rigid body, joints, restraints, sensors and drives. It can model and simulate the rigid body connected by various sports pairs, and realize the analysis of the dynamic performance of the mechanism.

First, the CAD model of D2 Delta Robot is drawn from SolidWorks software, and all components are modularized. The assembled CAD model is imported into MATLAB via the Simmechanics toolbox. MATLAB automatically generates the Simmechanics model of D2 Delta Robot, establishes the connection between each robot arm, as shown in Figure 4.
In order to detect the motion performance of the robot, the driving force and the sensor are added into the SimMechanics model. As shown in Figure 5, A and B are the input ports for the driving force of the active arm. C and D are the output ports for detecting the kinematic parameters of the active arm. E is the sensor for measuring the parameters of the end effector.

**Figure 4** D2 Delta Robot SimMechanics Model

**Figure 5** SimMechanics Model Based on Driving Force and Sensor

On the basis of the SimMechanics model in Figure 5, we add the forward kinematics mathematical model and inverse kinematics mathematical model. Plan the trajectory of an end effector so that the end actuator travels around a fixed center at a certain speed and acceleration. The kinematic parameters of the end effector are transformed into the angular motion parameters of the active arm by the inverse kinematics model. The input ports A and B in Fig. 6 provide power for the whole robot to drive the robot movement. In Figure 6, C Sensor and D Sensor will detect the motion parameters of the active arm, through the forward kinematics model to calculate the end effector motion parameters.

In Simulink, the whole kinematic model structure is shown in Figure 2-8. In the model, AA is the circular trajectory generator with the specified coordinates as the center and 20mm as the radius of motion. BB is the inverse kinematics model of D2 Delta Robot, CC is the SimMechanics model of D2 Delta Robot, DD is D2 Delta Robot forward kinematics model.
The motion parameters calculated by the forward kinematics model of the end effector are compared with the motion parameters of the end effector detected by the Sensor E and the motion parameters of the initial end actuator plan. It is found that the three sets of motion parameters are identical and the correctness of the established kinematics model is verified. All motion parameters are shown below.

**Figure 6** D2 Delta Robot Kinematics Model

![Kinematics simulation - X axis motion trajectory](image)

**Figure 7** Kinematics simulation - X axis motion trajectory
Figure 8 Kinematics simulation - Y axis motion trajectory

Figure 9 Kinematics simulation - X axis speed trajectory

Figure 10 Kinematics simulation - Y axis speed trajectory
5. D2 Delta Robot's workspace

When planning the end effector operating position, in order to avoid exceeding the working range of the robot, you must understand the robot's movement space. Through the analysis of the actual institutions, set the the limit position of the left active arm of the robot is $\frac{\theta}{4}$ (rad) and $-\frac{\theta}{2}$ (rad), the limit position of the right active arm of the robot is $\frac{\theta}{2}$ (rad) and $-\frac{\theta}{4}$ (rad). According to the set of two active arm rotation angle limit, combined with forward kinematics model, we can get the work space of the end effector, as shown in Figure 14.

The entire workspace presents a fan-shaped pattern. When the angular position of the two active arms is the clockwise maximum position (Figure 2-13-a), the end effector will be located at position A (Figure 2-14), the limit coordinates are (-132.6, -90.2); When the angular positions of the two active arms are in the counterclockwise maximum position (Figure 2-13-b), the end effector will be located at position B (Figure 2-14), the limit coordinates are (132.6, -90.2); When the left side of the active arm is located in the clockwise maximum position, the right side of the active arm is located in the counterclockwise maximum position (Figure 2-13-c), the end effector will be located at position C (Figure 2-14), the limit coordinates are (0, -100); When the right side of the active arm is located in the clockwise maximum position, the left side of the active arm is located in the counterclockwise maximum position (Figure 2-13-d), the end effector will be located at position D (Figure 2-14), the limit coordinates are (0, -209);
6. Conclusion

This paper begins with the structure of Delta Robot, according to its performance requirements, design the structure of D2 Delta Robot. In the design of the slave arms, we used the form of the crank arm. Compared with the straight arm in reference [4], reducing the occurrence of institutional stuck.
In order to enable us to achieve precise control of Robot, the kinematics analysis and mathematical modeling of the robot are carried out. Using the SimMechanics toolbox in MATLAB Simulink, The Simmechanics model of D2 Delta Robot was built. Combined with the kinematic mathematical model, to achieve the Robot motion simulation experiment. And the motion parameters of the end effector are compared with the motion parameters of the output terminal to verify the correctness of the established kinematic model. Finally, according to the limit position of the robot, combining the forward kinematics model, we got the workspace of D2 Delta Robot.

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