Modeling the loading factor characteristic of an industrial centrifugal compressor impeller

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Abstract. Test data on compressor stages demonstrate that loading factor versus flow coefficient at an impeller exit is of linear character independent of compressibility. Mathematical models of the Universal Modeling Method use this pattern at their core. In the first versions of the Method, two points determined the loading factor characteristic: loading factor at the design point, and at zero flow rate. To calculate these parameters, the user had to define values of two empirical coefficients. The choice of the coefficient is based on experience. An attempt to obtain approximating formulas for these coefficients did not lead to a result with the required accuracy.

In the modern version of the Method, the loading factor characteristic is defined by the angle of its inclination to the ordinate axis and by the loading factor at zero flow rate. The authors have researched test performances of model stages. Simple and definite equations with four geometric parameters were proposed for loading factor characteristics. The calculation error is 1%.

A trailing edge shape influences the loading factor. Blunt and symmetrically tapered blades have the same loading factor. Non-symmetric tapering changes the loading factor by up to plus/minus 4%. The results of impeller tests with different blade trailing edges and with vane and vaneless diffusers were generalized. The influence of these factors was taken into account as correcting coefficients in the formulas. The alternative model of loading factor characteristic modeling is included in the Universal Modeling Method and demonstrates good results in design practice.

1. Introduction

Research on centrifugal compressors has been carried out at LPI-SPbPU for decades [1],[2]. Dr-Ing. hab., Prof. Yu.B. Galerkin created a method for gas-dynamic design of centrifugal compressors and a mathematical model based on experiments and experience. Dimensions of a compressor flow path and
similarity criteria (isentropic coefficient, Mach and Reynolds numbers) determine its gas dynamic characteristics. The mathematical model was implemented in the form of computer programs called the Universal Modeling Method [3], [4], [5], [6], [7]. The method is constantly being developed and improved at the Research Laboratory "Gas Dynamics of Turbomachines".

A centrifugal compressor does not always operate at design flow rate. When discharge pressure changes in a pipeline, the gas flow rate changes too. The discharge pressure versus gas flow rate characteristic is important for assessing the compressor-pipeline interaction.

The mathematical model of the Method involves calculation and provision of gas-dynamic characteristics in a dimensionless form. Head according to Euler's formula, is $h_T = c_{u2} u_2 - c_{u1} u_1$ (where $c_u$ is the tangential component of absolute flow velocity, $c$ is blade velocity, subscripts 1 and 2 indicate control sections). For centrifugal process compressors $c_{u1}=0$. The dimensionless loading factor is $\psi_T = c_{u2} / u_2$. Figure 1 presents velocity triangles at the outlet of a centrifugal impeller with infinite and finite number of blades ($w$ is relative velocity, $c_r$ is the radial component of absolute velocity, $\beta_{bl}$ is the blade angle).

![Figure 1. Velocity triangles at the outlet of a centrifugal impeller with infinite and finite number of blades](image)

For an impeller with an infinite number of blades, the loading factor:

$$\psi_{\infty} = 1 - \varphi_{2,des} \cot \beta_{bl2},$$

where $\varphi_{2,des}$ is the design flow rate coefficient at the impeller outlet.

For a real impeller with a finite number of blades, the flow angle $\beta_{\Delta}$ differs from the blade angle due to the flow lag. The value of lag in engineering formulas is usually determined by $\Delta \varphi_{\Delta 2}$, Figure 1. For industrial compressors in the middle of the last century, the well-known formula of A. Stodola was used. This formula was introduced into Russian practice by the author of [8]:

$$\psi_1 = 1 - \varphi_{2,des} \cot \beta_{bl2} - \frac{\pi}{z_{imp}} \sin \beta_{bl2},$$

where $z_{imp}$ is the number of blades.
The formula is based on the representation of the relative velocities field in the impeller channel as the sum of the transit flow and the axial vortex. Stodola's formula does not take into account the influence of viscosity, blade dimensions and shape, etc. The accuracy of the formula is acceptable for a limited class of impellers.

In [9], B. Eckert analyzed equilibrium and continuity equations and presented the loading factor formula without taking into account the effect of viscosity:

\[
\psi_{\text{viscic}} = \frac{\psi_{\text{visc}}}{1 + \frac{\pi D_2^2 b_2}{8 z_{\text{imp}} S} \sin \beta_{bl}}
\]

where \( D_2 \) is the impeller diameter; \( b_2 \) is the outlet blade height; \( S = \int b \, r \, dr \).

The book [9] presents the integral \( S \) for "the most common case \( b r = \text{const} \). To take into account the effect of viscosity, the ratio \( h'/h \) is introduced. There \( h \) is the breadth of the channel, \( h' \) is the width of the active part of the flow ("jet"). Then the formula for an impeller with \( b r = \text{const} \) will be:

\[
\psi_x = \frac{\psi_{\text{visc}}}{1 + \frac{h'}{h} \frac{\pi}{2 z_{\text{imp}} (1 - r_i / r_f)} \sin \beta_{bl}}
\]

B. Eckert [9] recommended \( h'/h = 0.7 - 0.8 \) (a smaller value for blades with \( \beta_{bl} < 90^\circ \)). Referring to him, we can write the following empirical formula:

\[
\frac{h'}{h} \pi \sin \beta_2 = 1.5 + 1.1 \frac{\beta_2}{90^\circ}
\]

The formula (4) for impellers with an arbitrary dependence \( b = f(r) \) taking into account the viscosity becomes:

\[
\psi_x = \frac{\psi_{\text{visc}}}{1 + \frac{h'}{h} \frac{\pi D_2^2 b_2}{8 z_{\text{imp}} S} h' \sin \beta_{bl}}
\]

Experiments of the LPI Compressor Engineering Problem Laboratory back in the 1980s showed [10] that for impellers with 2D blades, the most advantageous meridional shape is a linear function \( b = f(r) \). In this case, \( S = \int b \, r \, dr \) and the loading factor (linear function \( b = f(r) \), blade load along \( r \) is constant) is:

\[
\psi_x = \frac{\psi_{\text{visc}}}{1 + \frac{h'}{h} \frac{\pi \sin \beta_{bl}}{2 z_{\text{imp}} (1 - \bar{D}_1)}}
\]

where \( \bar{D}_1 \) is blade inlet relative diameter.

B. Eckert's approach seems to be well grounded. K.I. Strakhovich [11] introduced this approach in Russian practice. However, the samples of velocity diagrams calculated by B. Eckert in [9] show that blade load is far from being constant along \( r \). All others formulas of this kind do not take into account actual blade load distribution too.

The flow lag at the impeller outlet is the result of the pressure field created by the blade load. In the scheme proposed by Yu. Galerkin [12], [13], blade load is replaced by a vortex with circulation \( \Gamma_{bl} = c u z r_i / z_{\text{imp}} \). The vortex is located on the radius \( r_i \). If the load is uniform, the vortex is located in the middle between \( r_i \) and \( r_f \). If the shape of the blade is such that the load is greater at
the beginning of the blade, \( r_p \) is closer to \( r_i \), and vice versa. In accordance with such a scheme, the vortex induces the following velocity component at the impeller outlet:

\[
\Delta c_{u2} = \Gamma_{\theta} \sin \beta_{blav} \frac{2\pi (r_2 - r_p)}{r_2 - r_1}.
\]  

(8)

The radius of the vortex position is set by the empirical coefficient:

\[
K_{pc} = \frac{r_2 - r_p}{r_2 - r_1}.
\]  

(9)

Formula (8) in dimensionless form becomes:

\[
\Delta \bar{c}_{u2} = \frac{\psi_{c}}{2 z_{imp} \bar{L}_{bl} K_{pc}}.
\]  

(10)

where \( \bar{L}_{bl} \) is dimensionless length of a blade.

The load distribution along the radius depends on the shape of the blade mean line and the meridional shape of the blade channel. To take into account the real nature of the flow, an empirical coefficient \( K_{\mu} \) is introduced. The coefficient takes into account the increase in flow lag due to viscosity. The formula for the loading factor at the design flow rate is:

\[
\psi_{T_{des}} = \frac{1 - \psi'_{des}, \cot g \beta_{bl2}}{1 + K_{\mu} \frac{1}{2(1 - \bar{D}) K_{pc,des}} \sin \beta_{bl1} + \beta_{bl2}}
\]  

(11)

where \( \psi'_{des} \) is the flow rate coefficient taking into account the blade blockade factor.

The accuracy of modeling the loading factor characteristics of a compressor affects the efficiency of its gas-dynamic design results, as well as the results of calculating the stage efficiency. This paper is devoted to the choice of the most correct and user-friendly method for modeling the loading factor characteristics of a centrifugal compressor stage.

2. Research objects

The equations presented below for calculating the loading factor characteristic \( \psi_{T} = f (\varphi_{T}) \) are based on gas dynamic characteristics of the 20CE family of model stages [14]. The stages consist of 2D impellers (the blade midline shape does not change along its height), vaneless or vane diffusers and return channels. The impeller blades have the mean line as a circular arc, or had a "profiled" mean line, to provide one or another velocity diagram, load distribution.

The gas dynamic and geometric parameters of the 20CE stages vary as follows: design flow rate coefficient \( \Phi_{des} \) =0.028–0.080, design loading factor \( \psi_{T_{des}} = 0.45–0.65 \), hub relative diameter \( \bar{D}_{hub} = 0.25–0.373 \), diffuser outlet diameter \( \bar{D}_{4} = 1.428–1.615 \), impeller Mach number \( M_{u} = 0.60–0.86 \), impeller diameter Reynolds number \( \text{Re}_{u} = 4.8 \cdot 10^{6} – 6.9 \cdot 10^{6} \). The experiments were carried out on a test rig with an open circuit. The relative uncertainty in determining the gas-dynamic characteristics of the model stages \( \pm 1.0 \% \) [15].

In experiments with model stages, characteristics are measured in the form \( \psi_{T} = f (\Phi) \), where the work coefficient \( \psi_{T} = c_{p} (T_{out} - T_{in}^{*}) / u_{2}^{2} \) (\( c_{p} \) is heat capacity at constant pressure; \( T_{out}^{*} \) is total temperature at the stage outlet; \( T_{in}^{*} \) is total temperature at the stage inlet). To obtain empirical equations \( \psi_{T} = f (\varphi_{T}) \), the flow rate coefficient at the outlet from the impeller is calculated using the Universal Modeling Method. To calculate the loading factor, the proposed by V.F. Ries [8] schematization of the power transmission process can be applied:
\[
\psi_i = \psi_T \left(1 + \beta_{fr} + \beta_{leak}\right),
\]
where \(\beta_{leak}\) is the labyrinth seal leakage coefficient; \(\beta_{fr}\) is the disk friction coefficient. The Universal Modeling Method also performs the calculation of the coefficients of disk friction and leakage, calculation formulas are presented in [12].

### 3. Two ways of presenting the loading factor characteristic

The simulation of the loading factor characteristic is based on the linear dependence of the loading factor versus the flow rate at the outlet of the impeller \(\varphi_2\) [16]. The first versions of the Universal Modeling Method contained formulas for calculating the loading factor characteristic based on the loading factor at the design flow rate and the loading factor at zero flow rate. To determine these two, the user chose two empirical coefficients: a coefficient for a design flow rate \(K_\mu\) (equation (11)), and a coefficient \(X_{\psi_{T0}}\) that determines the loading factor at zero flow rate:

\[
\psi_{T0} = 1 - X_{\psi_{T0}} \frac{\Delta T_{u_{2des}}}{2}.
\]

A loading factor \(\psi_T = f(\varphi'_2)\):

\[
\psi_T = \psi_{T0} - \frac{\psi_{T0} - \psi_{Tdes}}{\varphi'_{2des}} \varphi'_2.
\]

The wide range of recommended empirical coefficients \(K_\mu\) and \(X_{\psi_{T0}}\) values creates certain difficulties. In [17] and [18], the authors proposed to model the loading factor characteristic using the angle of inclination \(\beta_T\) and the coefficient of loading factor at zero flow rate \(\psi_{T0}\), Figure 2.

**Figure 2.** The loading factor characteristics of ideal impellers with an infinite number of blades and the linear loading factor characteristic of the impeller with \(\beta_{bl2} < 90^0\)

The alternative description of the loading factor characteristic is:
\[
\Psi_T = \Psi_{T0} - \varphi_2 \cdot \cot \beta_T.
\]  

(15)

3.1. First way of modeling. Approximation formulas for coefficients \( K_\mu \) and \( X_{\psi T0} \)

Model stages of the 20CE family and multistage compressors tests from [14] were reduced for \( K_\mu \) and \( X_{\psi T0} \) approximation formulas. The authors investigated several variants of formulas with different arguments and for different groups of impellers. The simple formulas demonstrated better results:

\[
X_{\psi T0} = 2.588 - 0.000576 \beta_{b2} + 3.655 \cdot 10^{-6} \beta_{b2}^2; \]

(16)

\[
K_\mu = 1.545 - 3.141 \cdot 10^{-5} \beta_{b2}.
\]  

(17)

Figure 3 demonstrates the quality of approximation.

![Figure 3](image)

**Figure 3.** Empirical coefficients in equations (11, 13) versus blade exit angle. Blue is test, red is approximation.

The authors accept validity of the formulas (16, 17) for preliminary design. Q3-D PC program 3DM-023 [12] calculates head characteristics of final design quite precisely. However, for calculating head characteristics of a compressor of unknown design we need a solution that is more precise.

3.2. Second way of modeling. Approximation formulas for coefficients \( \beta_T \) and \( \psi_{T0} \)

The authors of [17] investigated the influence of the main geometric parameters of impellers (the outlet blade angle, the thickness of the blades, the number of blades, the ratio \( h_2 / h_1 \), the shape of the blade mean line, etc.) on head characteristics. In addition to developing formulas for calculating parameters \( \beta_T \) and \( \psi_{T0} \), it is also necessary to take into account the shape of the impellers trailing edge and the type of diffuser. Both of these parameters affect the shape of the stage head characteristic.

The tests of model stages with different shapes of blade trailing edges and types of diffusers demonstrated different head characteristics [12]. Figure 4 shows studied trailing edges configurations.
Figure 4. Experimentally studied blade trailing edge shape [12]: (a) blunt; (b) symmetrically tapered; (c) tapered to the pressure side; (d) tapered to the suction side

Welded impellers usually have trailing edge (a). Recommended configuration for an impeller with a riveted shroud is (b). Loading factor is the same. Efficiency is a bit better with (b). The configuration (d) ensures highest efficiency at the cost of up to −4% of the loading factor, and with (c) it is the other way around. To take into account the effect of the blade edge type on a loading factor characteristic, correction coefficients $K_{\text{red}}$ and $K_{\text{pr red}}$ are introduced into approximating formulas. Vanes of a diffuser influence flow conditions in an impeller. Loading factor of an impeller coupled with a vane diffuser is higher. To take into account the influence of the vane diffuser, correction coefficients $K_{\text{red}}$ and $K_{\text{pr red}}$ were introduced.

The formula for the inclination angle $\beta_1^0$ of the loading factor characteristic provides accuracy acceptable for design practice:

$$\beta_1^0 = \left( 0.00478 - \beta_{bl}^0 \right) + 17.4802 \left( 1 - \frac{b_1}{b_2} \right)^{0.4} + 18.22 \left( \frac{b_2}{b_1} \right)^{0.8} \cdot K_{\text{red}} \cdot K_{\text{pr red}}. \quad (18)$$

This formula (18) was obtained as a result of processing the results of experiments with blunt and symmetrical trailing edges. For tapered trailing edges, the values of the empirical coefficients $K_{\text{red}}$ were selected. The results showed that for the trailing edge tapered from the pressure side, the coefficient $K_{\text{red}}$ is constant and does not depend on $\psi_{\text{des}}$. For the trailing edge tapered from the suction side, a graph was built (Figure 5), the approximation the data of which made it possible to obtain a linear dependence $K_{\text{red}}$ on $\psi_{\text{des}}$. Thereby:

- for a blunt and symmetrical trailing edges:
  $$K_{\text{red}} = 1; \quad (19)$$
- for the trailing edge tapered to the pressure side:
  $$K_{\text{red}} = 0.97; \quad (20)$$
- for the trailing edge tapered to the suction side:
  $$K_{\text{red}} = -1.2457 \psi_{\text{des}} + 1.6218. \quad (21)$$

Accuracy of approximation is demonstrated in figures below.
Similar researches have been done to account for the influence of the diffuser type. When developing formula (18), the experimental results of model stages with VLD were used. The experimental data were approximated using a linear dependence presented in formula (23). It shows insufficient accuracy of the approximation. However, due to the current lack of experimental results for model stages with different values $\psi_{rdes}$, it is not possible to refine formula (23). Correction coefficients for a diffuser type:

- for a stage with a vaneless diffuser:
  \[ K_{\beta T \text{ dif}} = 1 \; ; \]  
  \[ \text{(22)} \]

- for a stage with a vane diffuser:
  \[ K_{\beta T \text{ dif}} = -0.3413 \cdot \psi_{rdes} + 1.1375 \; . \]  
  \[ \text{(23)} \]

The accuracy of approximation by formula (18) is 98.96%. Comparison of the calculated and experimental data is shown in Figure 7.
**Figure 7.** Comparison of the inclination angle of the loading factor characteristic calculated by formula (18) and the experimental data. Blue is test, red is approximation.

The formula for the loading factor at zero flow rate:

\[
\psi_{T0} = \left(1 - 0.0479 \left(1 - \frac{1}{t}\right)^{0.9} - 0.0025 \left(\frac{b_s}{b_l}\right)^2 + 0.0255 \left(\frac{\beta_{bl2}}{40}\right)^{0.7} - 3.7462b_l^{1.5}\right) \cdot K_{\psi_{T0\text{ed}}} \cdot K_{\psi_{T0\text{dif}}},
\]  

(24)

where

- for a blunt and symmetrical trailing edge of the blade:
  \[K_{\psi_{T0\text{ed}}} = 1;\]
  (25)

- for the trailing edge of the blade tapered to the pressure side:
  \[K_{\psi_{T0\text{ed}}} = 1.04;\]
  (26)

- for the trailing edge of the blade tapered to the suction side:
  \[K_{\psi_{T0\text{ed}}} = 21.349 \cdot \psi_{\text{vdes}} - 20.673 \cdot \psi_{\text{vdes}} + 5.9484;\]
  (27)
Figure 8. Coefficient $K_{\psi_{0\text{ed}}}$ for the trailing edge of the blade tapered to the suction side on the design loading factor. × is test data, line is approximation

- for a stage with a vaneless diffuser:
  $$K_{\psi_{0\text{dif}}} = 1;$$  \hspace{1cm} (28)
- for a stage with a vane diffuser (the approximating equation is shown in Figure 9):
  $$K_{\psi_{0\text{dif}}} = 0.0979 \cdot \psi_{rdes} + 0.9747.$$  \hspace{1cm} (29)

Figure 9. Coefficient $K_{\psi_{0\text{dif}}}$ for stages with vane diffusers. × is test data, line is approximation

Comparison of the calculated and experimental data is shown in Figure 10.
Figure 10. Comparison of the loading factor at zero flow rate calculated by the formula (24) and the experimental data. Blue is test, red is approximation.

The approximation accuracy is 88.62%.

4. Conclusion
Any model of the loading factor characteristic is based on its linear dependence on the flow rate coefficient. The model determining the characteristic by an inclination angle and the loading factor at zero flow rate demonstrates its advantages. The algebraic formulas use blade exit angle and blade relative width for calculating characteristics. The correcting coefficients take into account the shape of blades trailing edge and the diffuser type. The application of this model in the Universal Modeling Method shows that the model accuracy satisfies the preliminary design demand, however, calculations of performance of arbitrary compressors are less reliable. The authors will continue to correct the approximating formulas as the experimental data accumulate on the new test rig ECC-55 [19] that was built for the compressor group of the Laboratory “Technological Processes Modeling and Design of Power Equipment”. The proposed approaches to modeling the loading factor characteristics of centrifugal compressors impellers are based on processing the results of experimental researches of single-row radial impellers without splitter blades for a shroud without cutouts and holes in it. The applicability of the proposed model for other impellers will be tested separately when comparing the calculated and experimental data.

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