Error Analysis of Calibration Length Baseline based on μ-base Distance Meter

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Abstract. This paper introduces in detail methods about calibration length baseline based on μ-base distance meter, and errors in the calculation of oblique distance model; the measurement of meteorological parameters and the correction of height difference are analyzed quantitatively by means of equation derivation. Through the calibration results of μ-base and invar tape, systematic differences between the two measurement methods were found out at the Yellow River Embankment Length Baseline in Zhengzhou. Based on a TS30 total station instrument with stable performance, according to the calculation results of "analytical method" and "comparative method", the accuracy of the two calibration results between μ-base and invar tape is compared and evaluated. In the end, some suggestions to improve the accuracy of using μ-base to calibrate length baseline are given.

1. Introduction
Baseline is a spatial line consisting of two points with a precise distance and a firm mark. All points for observation that constitute the baseline are called baseline points, which are generally piers higher than the ground, and may also be marlstones buried under the ground. Length baseline is generally a general term for multiple baseline points that are associated outdoors. It is mainly used to calibrate the additive and multiplication constant of length measuring instruments such as total station. At present, the standard length of length baseline needs to be measured regularly, there are three calibration methods: optical interferometry by Vaisala interference comparator, direct measurement by 24m invar tape and high precision electro-optical measurement[1]. According to the JJF 1214-2008, the length baseline’s calibrating period is recommended to be three years[4]. Most length baselines in China are calibrated by 24m invar tape, calibration services are expensive and foreign developed countries have not adopted this method in the last century, the method to calibrate baseline length based on μ-base is worth popularizing[5].

2. Calibration length baseline methods for μ-base
Leica produced the μ-base distance meter in 2012. The laser frequency of μ-base is 2.4GHz. Using with the recommended RRR spherical prism, ranging accuracy can be higher than 10 microns within 160m. It is obvious that this can meet the needs of calibration length baseline. In order to reduce the loss of ranging accuracy, we should try to reduce the impact of atmospheric turbulence by choosing a sunny, fog-free morning or evening, and continuously collecting on-site meteorological parameters through two sets of sensors under ventilation conditions, then inputting the data of temperature, air pressure and relative humidity into Pilot software for real-time atmospheric correction.

Next, the general measurement methods of μ-base are briefly introduced[7].
2.1 Interior coplanar method
As shown in Figure 1, for line A-B of 150m to 300m in length, RRR spherical prisms are placed on baseline piers A and B, and we locate μ-base on tripod C, constantly adjusting the height and position of the tripod and μ-base to ensure that the ranging center of μ-base is on the line A-B and three points are coplanar.

\[ S_{A-B} = d_1 + d_2 + 2K \]  

(1)

Before μ-base is used for measurement, its additive constant \( K \) has been tested and corrected by the high-precision 80m large-scale laser comparator of National Institute of Metrology, the measurement error caused by \( K \) value is within 5 microns, so it's negligible.

2.2 Lateral coplanar method
As shown in Figure 2, for line A-B less than 200m in length, in a similar way, ensure that optical axis of the μ-base can pass through the prism center of points A and B, and three points are in the same vertical surface.

\[ S_{A-B} = d_1 - d_2 \]  

(2)

3. Measurement error analysis of μ-base
In order to give full play to the measuring ability of μ-base, the errors of the above two measurement methods are analyzed.

3.1 Errors in the calculation of oblique distance model
When calculating the oblique distance, the distance measuring center of μ-base and the center of two spherical prisms are required to be in a straight line. So there must be errors in this model.

3.1.1 Error of interior coplanar method. Take interior coplanar method as an example, as shown in Figure 3, when setting up μ-base, A, B and C are roughly in a straight line after repeated adjustments. After the measurement of the spherical prism at point A is completed, the μ-base needs be rotated 180 degrees to measure point B. To ensure that the reflected signal intensity displayed by Pilot software is greater than 150, the μ-base also needs to be slightly adjusted, this will result that point B is not on line A-C.
The error calculation equation of oblique distance model is as follows.

$$\Delta S_{d} = d_{1} + d_{2} - \sqrt{(d_{1}^{2} - e^{2}) + e^{2} - (d_{1} - d_{2})}$$ (3)

In equation (3), $e$ is the distance deviation, which is the distance from point B to the extension of line A-C, considering that the diameter of the spherical prism is about 3cm, so the range of values of $e$ is $0 \leq e \leq 3cm$, we take $e$ to be equal to 3cm in the extreme case. The effective measurement range of $\mu$-base is 1.5m-160m and the $\mu$-base is located between two adjacent baseline piers, it is explained that in the past, in order to facilitate the use of 24m invar tape to calibrate the length baseline, the distance between baseline piers in China is designed as an integer multiple of 24m as far as possible, so let $10m \leq d_{1} \leq 160m$ and $10m \leq d_{2} \leq 160m$.

As shown in Figure 4, the calculation results show that the error caused by oblique distance model is not greater than 0.04mm.

3.1.2 Error of lateral coplanar method. Similar to interior coplanar method, the error analysis diagram of lateral coplanar method is shown in figure 5.

![Figure 5](image_url)

Figure 5. Error analysis diagram of lateral coplanar method.

When the $\mu$-base measures point A first, then point B, as shown in (a), the error calculation equation of oblique distance model is as follows.

$$\Delta S_{d} = \sqrt{(d_{1}^{2} - e^{2}) + e^{2} - (d_{1} - d_{2})}$$ (4)

When the $\mu$-base measures point B first, then point A, as shown in (b), so:

$$\Delta S_{d} = \sqrt{(d_{1}^{2} - e^{2} - d_{2}) + e^{2} - (d_{1} - d_{2})}$$ (5)

In equation (4) and (5), we take $e=3cm$, $20m \leq d_{1} \leq 160m$ and $2m \leq d_{2} \leq 140m$. 
As shown in Figure 6, the calculation results show that the errors caused by oblique distance model are less than 0.25mm and 0.02mm respectively. Therefore, when using lateral coplanar method, the distant spherical prism should be measured first to improve the measurement accuracy.

Figure 6. Error calculation results of oblique distance model.

3.2 Errors caused by meteorological parameter measurement

Both at home and abroad, the length baseline is established in the field of natural conditions, the common problem is that atmospheric refraction cannot be completely or effectively eliminated when using high-precision rangefinder, the impact of this greatly limits the accuracy of the instrument itself[8]. The best way to reduce atmospheric refraction error is to install meteorological sensors intensively along the baseline, establish an automatic collection system of environmental parameters, and realize real-time and accurate measurement of temperature, humidity and air pressure, in order to weaken μ-base calibration baseline error, such as Physikalisch Technische Bundesanstalt and National Institute of Metrology[9]. It is conservatively estimated that more than 90 baselines have been built by metrological departments of China, although establishing an automatic collection system of environmental parameters is very effective, the cost is very high and it is difficult to popularize it widely. Most metrological departments adopt the two-point method, that is, the average of the observation station’s and prism station’s meteorological parameters is the meteorological value of the observation line.

For μ-base distance meter, the correction equation of the air refractive index is as follows:

\[
\frac{1}{n} = 1 + \{0.2914269 \times 10^{-6} p + 0.6609 \times 10^{-6} t + 0.01041 \times 10^{-6} t^2 + 0.0036610 \times 10^{-6} R\} + 550.51 \times 10^{-6} \times 10^{7.51/237.3} \times 0.6609 \times 10^{-6} R \times 10^{-6}
\]

In equation (6), \(t\) is temperature, the unit of \(t\) is degrees centigrade, \(p\) is atmospheric pressure, the unit of \(p\) is hPa, \(R\) is relative humidity, the unit of \(R\) is %.

The standard meteorological environment of μ-base is \(t=12\,^{\circ}\mathrm{C}\), \(p=1013.25\,\text{hPa}\) and \(R=60\%\), substitute it into equation (6), and the reference refractive index is \(n_0=1.0002830\).

Calculate the full differential of equation (6) and according to law of error propagation.

\[
m^2 = \left(\frac{\partial n}{\partial t}\right)^2 m_t^2 + \left(\frac{\partial n}{\partial p}\right)^2 m_p^2 + \left(\frac{\partial n}{\partial R}\right)^2 m_R^2
\]

Taking the meteorological data of Zhengzhou in October as an example, substitute \(t=20\,^{\circ}\mathrm{C}\), \(p=1000\,\text{hPa}\) and \(R=40\%\) into equation (7), and get:

\[
m^2 = \pm \sqrt{0.98472 m_t^2 + 0.27182 m_p^2 + 0.00972 m_R^2} \times 10^{-6}
\]

As in equation (8), when temperature changes by 1°C or atmospheric pressure changes by 3.7hPa, it will cause the air refractive index to change by 1×10^{-6}, and the effect of relative humidity on air refractive index is negligible.

Assuming that the reference refractive index is \(n_0\), the distance measured by μ-base is \(D\)[11].
But the actual value of air refractive index is \( n \), the distance is \( D \).

\[
D' = \frac{C\Phi}{4\pi fn_0}
\]

(9)

So under the condition that the actual value of air refractive index is \( n \), the meteorological correction equation of distance is as follows.

\[
D - D' = \frac{C\Phi}{4\pi fn_0} - \frac{C\Phi}{4\pi fn_0} = \frac{C\Phi(n_0 - n)}{4\pi fn_0} = \frac{D(n_0 - n)}{n_0}
\]

(10)

Error of meteorological correction calculation model is shown in equation (12).

\[
\Delta D = |\frac{\partial (D-D')}{\partial (n)}| m_n = \frac{D}{n_0} m_n
\]

(12)

Considering the existence of meteorological representativeness error, suppose the standard deviation of atmospheric pressure and relative humidity measurement is \( m_p=0.5\text{hPa} \) and \( m_R=10\% \) respectively, the maximum error in temperature measurement is \( T \) and the range of values of \( T \) is \( 0\leq T\leq 0.8 \text{℃} \), according to uniform distribution, the standard deviation of temperature is \( m_T=T/1.73 \), the measured distance \( D \) ranges from 2m to 160m. By substituting these dependent variables into equation (12), it can be considered that in a single measurement, the error caused by meteorological parameters is not greater than 0.076mm, then the error between any two baseline piers will be less than 0.15mm.

3.3 Errors in the correction of height difference

At the beginning of design, length baseline is generally built in the flat area and its total gradient is less than 1°. It also indicates that all baseline piers are not equal to the height, so correction of height difference is necessary.

As shown in Figure 7, A and B are two adjacent baseline piers, slant distance is \( S \), height difference is \( h \), the angle is \( \alpha \), we use the Pythagorean theorem to figure out the distance \( L \).

\[
L = \sqrt{S^2 - h^2}
\]

(13)

Calculate the full differential of equation (13) and according to law of error propagation.

\[
m_L = \frac{S^2 - h^2}{S^2 - h^2} m_S^2 + \frac{h^2}{S^2 - h^2} m_h^2 = \frac{(m_S^2 + \sin^2 \alpha m_h^2)}{\cos^2 \alpha}
\]

(14)

The relative accuracy of length baseline should be greater than \( 1\times10^{-6} \), that is to say,

\[
\left(\frac{m_L}{L}\right)^2 = \left(\frac{m_S^2 + \sin^2 \alpha m_h^2}{L^2 \cos^2 \alpha}\right) \leq (1\times10^{-6})^2
\]

(15)

It can be intuitively concluded from equation (15) that the accuracy requirement of height difference measurement is proportional to \( \alpha \). The larger \( \alpha \) is, the higher accuracy requirement of height difference measurement will be. We take \( L=160m \) and \( m_S=0.1mm \), when \( \alpha \) is equal to 0.5 degrees, \( m_h \) is less than 14.3mm, when \( \alpha \) is equal to 1 degree, \( m_h \) is less than 7.2mm.

Therefore, it is necessary to consider what kind of height difference measurement method is adopted to reduce the precision loss in the conversion process. At present, there are four commonly used methods for height difference measurement: leveling surveying, trigonometric leveling,
hydrostatic leveling and GNSS surveying. The observation accuracy and time of these four methods are different, and reasonable methods should be selected according to the slope of the length baseline to meet requirements of accuracy.

- Leveling surveying is a traditional method. It is completely possible for the accuracy of height difference between baseline piers to reach within 1mm by using second-class leveling. However, this method is time-consuming, and the baseline points are generally piers 1 to 1.5 meters above the ground, making it more difficult.

- At present, total stations are more and more accurate, but the method of trigonometric leveling will be affected by atmospheric refraction. Within the range of 1km, a high-precision total station can be set up on each pier to measure the height difference of adjacent piers, the meteorological parameters of the observation station and prism station can be accurately measured and inputted into total station for real-time correction, replacing the method that a total station is set up outside the length baseline to observe elevation of all piers can greatly reduce the impact of atmospheric refraction, however, its precision can’t reach second-class leveling.

- Hydrostatic leveling can effectively reduce the influence of atmospheric refraction, the accuracy can reach submillimeter level and better stability, strong anti-interference ability, we can use the method of eye reading or based on wireless communication and computer connection to achieve automatic data collection. However, its measuring range is limited to a certain extent, the height difference between piers may be too great to allow hydrostatic leveling system to be installed arbitrarily.

- The advantage of GNSS measurement is easy to measure the middle-Long baseline. When the observation time is 1-2h, the height difference accuracy in the vertical direction is low, which can only reach the accuracy of centimeters. Therefore, this method is not used.

4. Comparing experiment
This experiment is at the Yellow River Embankment Length Baseline in Zhengzhou. In June 2018, the first geodetic survey team of ministry of natural resources calibrated the baseline, and the calibration result are given, as shown in table 1, and the certificate number is JX2018-23.

| Sequence Number | Measurement route | Length of calibration | Expanded uncertainty (k=2) |
|-----------------|-------------------|-----------------------|---------------------------|
| 1               | 0-1               | 24+10.38              | 0.15                      |
| 2               | 1-2               | 96+0.62               | 0.29                      |
| 3               | 2-3               | 48-1.35               | 0.20                      |
| 4               | 3-4               | 144+2.02              | 0.35                      |
| 5               | 4-5               | 264+5.28              | 0.47                      |
| 6               | 5-6               | 72+6.42               | 0.25                      |

In October 2018, the baseline was measured again by μ-base, the observation diagram is shown in figure 8, and schematic diagram of field observation is shown in figure 9.

![Observation diagram](image-url)
Figure 9. Schematic diagram of field observation.

The height difference between all baseline piers was measured by trigonometric leveling, the results of consistency comparison between μ-base and invar tape are shown in table 2.

Table 2. Results of consistency comparison between μ-base and invar tape.

| Measurement route | Slope distance of μ-base/m | Height difference /m | Horizontal distance of μ-base/m | Horizontal distance of invar tape /m | System error mm | System error mm/km |
|-------------------|---------------------------|----------------------|-------------------------------|-------------------------------------|----------------|-------------------|
| 0-1               | 24.01045867               | -0.0592              | 24.010453                     | 24.01038                            | 0.079          | 3.1               |
| 1-2               | 96.00034233               | 0.0466               | 96.000354                     | 96.00062                            | -0.278         | -2.8              |
| 2-3               | 47.998048                 | 0.0456               | 47.998070                     | 47.99865                            | -0.602         | -12.1             |
| 3-4               | 144.002379                | 0.0177               | 144.002380                    | 144.00202                           | 0.359          | 2.5               |
| 4-5               | 264.0048567               | -0.0347              | 264.004859                    | 264.00528                           | -0.423         | -1.6              |
| 5-6               | 72.00656733               | 0.0023               | 72.006567                     | 72.00642                            | 0.147          | 2.0               |

According to measurement results of the two methods, although the direct error is not large, it cannot be ignored for the accuracy requirements of baseline calibration. Therefore, a TS30 total station with stable performance is selected for the "six-segment method" measurement. The reliability of the calibration result of multiplying constant is directly proportional to the baseline length, and the total length of the Yellow River Embankment Length Baseline is less than one kilometer, so we only analyze the calculation results of additive constant[12], as shown in table 3.

Table 3. Calculation results of additive constant.

| Analytical method | Comparative method |
|-------------------|--------------------|
| Additive constant | -0.3514 -0.5686 -0.7030 |
| Standard deviation | 0.4230 0.3874 0.5052 |

It can be intuitively seen from table 3 that the standard deviation of 'analytical method' is slightly higher than 'comparative method' of invar tape, this length baseline is relatively stable and the standard value error of the baseline can be negligible. Among the calculation results, the ‘comparison method’ of μ-base has the maximum standard deviation, which indicates that the measurement method needs to be further improved to improve the accuracy of using μ-base to calibrate baseline.

5. Conclusions
Through this measurement experiment, some suggestions are given to improve the accuracy of using μ-base to calibrate baseline.

(a) Some auxiliary piers can be constructed along the baseline to replace the tripod, in order to better make the distance measuring center of μ-base and the center of spherical prisms in a straight line.
(b) Install meteorological sensors intensively along the baseline, establish an automatic collection system of environmental parameters, and realize real-time and accurate measurement of temperature, humidity and air pressure, in order to weaken μ-base calibration baseline error.

(c) Try to use second-class leveling method to accurately measure the height difference among all base piers, and hydrostatic leveling can be used in some conditional length baselines.

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