Strut and tie model optimization for reinforced concrete deep beam using genetic algorithm

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Abstract: This paper presents strut and tie model structural optimization of reinforced concrete deep beam using genetic algorithm. Genetic algorithm is used as the optimization platform as it does not require differentiation of the exact mathematical formulation to get the optimum solution. The force analysis is carried out using two-dimensional linear finite element method with truss element. The struts and ties design are based on ACI 318. One RC deep beam example is presented as an example. During optimization, there are two constraints which consisted of strength of the member alone and combination with deformation limit of the nodes. The stress ratio for both struts and ties are set to not exceed unity while the deformation was limited to 2.0 mm. From the optimization analysis, it can be concluded that genetic algorithm can be used to get the most optimum structural configuration which yield the most economical solution for design purposes. On the other hand, if deformation constraint is added in the optimization parameters, larger deep beam depth is required to satisfy the deformation limits.

Keywords: Structural optimization, strut and tie model, genetic algorithm, reinforced concrete

INTRODUCTION

The use of strut and tie model (STM) to design reinforced concrete element which governed by shear failure due to the presence of disturbed region is now being widely used as a more valid design method than the conventional reinforced concrete design. However, the STM modeling sequence requires a valid structural model that should be manually chosen based on established examples of known geometry and problems. For unexperienced designer, this modeling sequence may pose some difficulties. Limited guidelines on choosing the appropriate truss analogy system are also one of the main concerns.

For design purposes, some researchers provide guidance for the minimum angle of the truss system to provide the optimum design parameters for the STM. Schlaich and Weischede [1] suggested that the angle for STM element should have an angle larger than fifteen degree towards the main principal stress. The ACI 318-05 [2] noted that the angle between members that enters a node should not be less than twenty-five degrees. Rogowsky et. al [3] and Ramirez and Brenn [4] suggested that the angle between members should be between twenty-five to sixty-five degrees. Grob and Thürllmann [5] also suggested that the angle between members should be between 26.6 to 63.4 degrees. Most of the references in the above consider the angle measurement based on many experimental studies.

In 2003, Zhu and coworkers [6] proposed a crack width prediction using compatibility strut and tie model. [6] raises the importance to check the deformability of shear dominated RC members which was caused by localized cracks. Hence, not only the shape of the STM that matters in design but also the deformability of the RC members should be also considered. To obtain the correct topological strut and tie model shapes, many researchers have used evolutionary optimization technique for various RC structures [7-12]. Liang et. al [13] use evolutionary optimization technique for RC deep beam with and without opening, and beam-column joint which was successful to get the best topology shape to be used in the STM analysis. In 2002, Liang et. al [14] extend their work to get the best topology shape of STM for a pier head structure which often used in bridges. However, this method only considers

the elastic behavior of concrete elements which may cracks at lower tensile loads. Hence, the proposed method is only good to get the best topology of the STM.

To combine between optimizing the topology of STM model, the design of strength member, and to cater for the deformation of the structure, the author has develops an in house computer program which incorporates STM analysis and design based on truss finite element analysis and genetic algorithm optimization to get the best known topological shape of the STM model [15]. In this paper, only RC deep beam being investigated using the developed inhouse software. There are two objective function being considered in the GA optimization. The first objective only considers the strength of the member and the second objective includes the nodal vertical deformation checks which should be less than the deformation limit allowed.

RESEARCH SIGNIFICANCE

This paper presents optimization for strut and tie model in reinforced concrete deep beam using genetic algorithm. The GA optimization used in this paper is classified as topological optimization. An inhouse computer program is developed to support this research. There are two kind of objective function being investigated. The first objective function is optimization based on strength of the member alone and the second objective function is optimization based on strength combined with limited deformation. The additional limited deformation in the objective function is to cater for the need to check the deformation limit during the design.

METHODOLOGY

The methodology in this paper is divided into three steps. In the first step, a code-based design of reinforced concrete deep beam using STM is checked. The reinforced concrete deep beam being evaluated is taken from Singh et. al [16]. The RC deep beam have continuous span without interior span. Hence, there are three point of supports for the deep beam and two loading points positioned at the mid-span for each exterior span. Load control is considered in the analysis. The load for each loading points are not equal to
demonstrate the asymmetric loading pattern. The design of the members is based on ACI 318-05 [2].

In the second step, optimization of the RC deep beam is carried out with the objective function to optimize the topology of the truss element by using the member strength alone. In this step, for each populated gene in GA iterations are checked for the stress ratio for its member where it should be lower than unity. The topological optimization is constrained only in the vertical direction to ensure the beam have the same height throughout the span. In addition, the loading points and support positions remain in its location. Hence, it is not necessary to do topological optimization in the horizontal direction.

In the third step, optimization of the RC deep beam is carried out with the objective function to optimize the topology of the truss element by using the member strength combined with limited deformation. The similar justification as in the second step is used except with the additional limited deformation to be checked in the analysis. The purpose to add this limited deformation in the GA optimization is to accommodate the needs to check the deformation of the member during the analysis.

ANALYSIS AND DISCUSSIONS

A. ANALYSIS AND DESIGN OF THE REINFORCED CONCRETE DEEP BEAM USING STRUT AND TIE MODEL

Figure 1 shows the geometry details of the RC deep beam and forces that act on the member. The total span length of the beam is 6.0 m with each exterior span width is 3.0 m. The beam width and height are 500 mm and 2000 mm, respectively. The load applied at the left and right loading plates are 1500 kN and 2000 kN, respectively. The distance between the loading plates measured from each centerline is 3.0 m. Figure 2 shows the internal forces acting on the truss member and reaction at the support.

To determine the ties width, for each member with tensile forces, it is necessary to firstly compute the required bar area. Here, the bar yield strength was set to 414 MPa and the uniaxial concrete compressive strength was set to 24 MPa.
For member AB, the required area can be computed as 
199.2*1000/(0.75*414) = 640 mm². For member AE, the 
required area is 500.4*1000/(0.75*414) = 1607 mm². For 
member ED, the required area is 700.4*1000/(0.75*414) = 
2250 mm². It should be noted that the reduction factor for 
tension is taken as 0.75. A bar with 16 mm diameter (Aₜₜ = 
201 mm²) is used for design. Hence, for member BC, the bar 
reinforcement also increased from four to twelve bars. The 
minimum reinforcement for ties element is 0.04*(fₑ/fₜ)*b*d = 0.04*(24/414)*500*1925 = 2226.5 
mm². Hence, for beam BC, the bar reinforcement also 
increased from four to twelve.

For compressive member, the capacity of compressive 
strut (ϕfₑₕₑ) can be computed as 0.85*0.85*0.75*24 = 11.47 MPa. Hence, the width for 
member AB = 801.1*1000/(11.47*500) = 139.68 mm. For 
member BE = 1120.5*1000/(11.47*500) = 195.37 mm. For 
member CB = 1438.8*1000/(11.47*500) = 250.88 mm. 
From minimum required member width, the design width 
for member AB, BE, CB, and CD, are 150 mm, 200 mm, 
260 mm, 200 mm, respectively.

For the nodal zone, where node A, B, C, and D can be 
considered as CCT node. Therefore, the nodal strength 
based (ϕfₑₕₑ) on ACI 318-05 [2] can be computed as 
0.85*0.85*0.75*0.8*24 = 12.24 MPa. The minimum strut width to be extended to node A can be 
computed based on the width member AD and AB. From
the calculation, the minimum required member width $AD$ and $AB$ are 81.76 mm and 130.89 mm, respectively. The reserved width for strut $AD$ and $AB$ were not less than 150 mm and thus it can be concluded that node A is adequate. The same calculation for node B, C, D, and E which also found to have adequate strength.

Figure 3 shows the RC deep beam design results where the bottom and top longitudinal bars are consisted of twelve D16 bars. The bars are placed within two layers with the distance between the extreme outer concrete fiber to the bar centroid are 75 mm. Hence, the width of ties is 150 mm.

Figure 4 shows the analysis and design results of the designed RC deep beam with the configuration as outlined in the previous section. Table 1 shows the element properties, effective width (as designed), element force, and stress ratio for each member. As shown in Figure 4 and Table 1, the maximum stress ratio was controlled by compression strut BE which is 0.982.

### B. GA OPTIMIZATION OF THE RC DEEP BEAM STM USING THE FIRST OBJECTIVE FUNCTION

In the first objective function, the code-based design STM is optimized by looking only at the member strength alone. Figure 5 shows the final forces and the stress ratio of the RC deep beam optimized using GA with the first objective function. As noted from Table 1, the maximum stress ratio for strut BE is 0.982 which still can be optimized if the stress limit can reach unity. From Figure 5, the stress ratio for member BE is 1.000 which then can be considered as the critical member design. From the GA optimization, the beam height was reduced to 1941 mm. The final internal forces and stress ratio for all members optimized with the first objective function are shown in Table 2.

### C. GA OPTIMIZATION OF THE RC DEEP BEAM STM USING THE SECOND OBJECTIVE FUNCTION

In the second objective function, the code-based design STM is optimized not only considering the member strength but also the deformation limits on node 5. Figure 6 shows the final forces and the stress ratio of the RC deep beam optimized using GA with the second objective function. Figure 6 and Table 3 show the details on the forces and stress ratio of the members. As shown in Figure 6 and Table 3, the previous critical member stress ratio (member BE) from 0.982 to 0.914. This clearly indicates that the previous deformed shape in node 5 was higher than the allowed limit which is 2.0 mm. To satisfy the deformation limit, the optimized beam height increased from 2000 mm to 2271 mm. Table 4 shows the nodal deformation for all the generated STM models. As shown in Table 4, at node 5, for original STM designed using ACI 318-05 [2] code had a deformed value of 2.212 mm in the vertical direction. Hence, when it was optimized using strength alone as the

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**Table 3 STM design results (optimized – second objective)**

| No | ID | Nodal I | ID | Nodal J | Element Properties | Effective Width (mm) | Element Force (kN) | Stress Ratio |
|----|----|---------|----|---------|--------------------|---------------------|-------------------|-------------|
| 1  | 1  | 1       | 2  | 2       | 12#16              | 150                 | -454.350         | 0.613       |
| 2  | 2  | 3       | 12#16 |        |                    | -630.911           | 0.851            |
| 3  | 3  | 3       | 5  | 4       | ACI Bottle Shaped with Steel | 200               | 1093.432        | 0.954       |
| 4  | 4  | 4       | 2  | 5       | ACI Bottle Shaped with Steel | 150               | -150.663        | 0.203       |
| 5  | 4  | 2       | 2  | 5       | ACI Bottle Shaped with Steel | 200               | 1048.550        | 0.914       |
| 6  | 2  | 5       | 5  | 4       | ACI Bottle Shaped with Steel | 260               | 1354.547        | 0.909       |
| 7  | 4  | 1       | 1  | 1       | ACI Bottle Shaped with Steel | 150               | 787.435         | 0.916       |

**Table 4 Nodal deformation for all STM models**

| Nodal ID | Code-based design | Optimized 2nd objective function | Optimized 1st objective function |
|----------|------------------|---------------------------------|---------------------------------|
|          | x                | y                              | x                              | y                              | x                | y                |
| 1        | 0.000            | 0.000                          | 0.000                          | 0.000                          | 0.000            | 0.000            |
| 2        | 0.946            | 0.000                          | 0.795                          | 0.000                          | 0.984            | 0.000            |
| 3        | 2.726            | 0.000                          | 2.273                          | 0.000                          | 2.840            | 0.000            |
| 4        | 0.866            | 1.847                          | 0.749                          | 1.730                          | 0.894            | 1.884            |
| 5        | 1.497            | 2.212                          | 1.231                          | 1.996                          | 1.565            | 2.276            |
objective function, GA will ignore the deformation limits. With a reduced beam height due to the optimization, the vertical deformation in node 5 increases from 2.212 mm to 2.276 mm. However, as the deformation limits was added into the objective function, the vertical deformation in node 5 reduced from 2.212 mm to 1.996 mm which was close but still lower than the allowed limits (2.0 mm).

D. FINAL DESIGN RESULTS BASED ON THE FIRST AND SECOND OBJECTIVE FUNCTION

From the analysis in the previous sections, it can be inferred that there are three design results. The first one is the code-based design results which was shown in Figure 3. The second and the third results are the GA optimized STM results using the 1st and the 2nd objective function. Figure 7 shows the final design of the optimized RC deep beam geometry. In Figure 7, the optimum RC deep beam height with the 1st and the 2nd objective function are 1941 mm and 2271 mm, respectively. It should be noted that here the reinforcing bar design was not changed during optimization.

![Figure 7](image)

In the future work, more complex optimization parameters which includes the number of bars, check on the nodal zone capacity, width of both the struts and ties, and also topology of the truss element should be investigated for more flexibility option in the optimization. This way, the most economical but safe RC member can be designed with sufficiently fast and efficient.

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