NEWTONIAN HYDRODYNAMICS OF THE COALESCENCE OF BLACK HOLES WITH NEUTRON STARS. I. TIDALLY LOCKED BINARIES WITH A STIFF EQUATION OF STATE

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Abstract

We present a detailed Newtonian study of the last stages of binary evolution of a black hole and a neutron star, when the components are separated by a few stellar radii. Our simulations are carried out using a three-dimensional smooth particle hydrodynamics (SPH) code. We calculate the gravitational radiation waveforms as well as the gravitational radiation luminosity in the quadrupole approximation. The neutron star is modeled with a stiff polytropic equation of state, \( P = K \rho^\gamma \), with the adiabatic index \( \Gamma = 3 \). We have performed runs with two different resolutions, using 16,944 and 8121 particles initially. Our equilibrium initial conditions correspond to tidally locked binaries with different initial values of the mass ratio \( q = M/M_{\text{BH}} \) (where \( M \) is the mass of the neutron star and \( M_{\text{BH}} \) is that of the black hole). The dynamical evolution of the system was simulated using an ideal gas equation of state for a time equivalent to several initial orbital periods. We find that for high mass ratios \( q = 1 \) and \( q = 0.8 \), but not for \( q = 0.31 \) there is a critical separation at which the binary becomes unstable due to hydrodynamical effects and decays on a dynamical timescale. The neutron star is not completely tidally disrupted, and its core continues to orbit the black hole. For a mass ratio of unity, an accretion torus forms around the black hole and survives for several dynamical times, but no comparable accretion structure is present for lower mass ratios. For \( q = 0.31 \) we have performed two separate runs, one with gravitational radiation reaction included in our calculations, and one with no gravitational radiation reaction—in both cases we find intermittent mass transfer through Roche lobe overflow. For the stiff polytrope considered here, the binary system always survives the initial mass transfer—the encounter results in a decreased mass ratio and increased separation. In all cases, the binary axis is free of baryons.

Subject headings: binaries: close — equation of state — gamma rays: bursts — hydrodynamics — stars: neutron

1. INTRODUCTION

It is our objective to investigate the outcome of the coalescence of a black hole with a neutron star, to find out to what extent the neutron star is tidally disrupted and, in particular, to determine if an accretion structure forms around the black hole as a result of the encounter. We also wish to explore the role of the mass ratio and the stiffness of the equation of state in the evolution of the system and in the emission of gravitational radiation. For simplicity—and to perform accurate comparisons with the work of Rasio & Shapiro (1994, hereafter RS) on double neutron star binaries—we have chosen to model the neutron star as a polytrope and explore the dependence of the results on the equation of state by varying the stiffness through the adiabatic exponent \( \Gamma \). In this paper we consider stiff polytropes with \( \Gamma = 3 \). For this value of the adiabatic index, the stellar radius decreases with mass \( (M \propto R^{1.5}) \). Elsewhere (Lee & Kluźniak 1999b), we explored a different value of the adiabatic index, yielding a more realistic mass-radius relationship.

We restrict our work to quasi-Newtonian binaries which are initially tidally locked. The construction of a completely self-consistent initial condition is then entirely straightforward. This assumption represents a major simplification in our calculations and can be looked upon as an extreme case of angular momentum distribution in the system. The opposite extreme would be a case where the binary components exhibit no rotation at all as viewed from an external, inertial frame of reference, and it poses a much greater problem regarding the construction of self-consistent initial conditions in equilibrium. Since tidal locking is not expected (Bildsten & Cutler 1992), the full range of initial configurations deserves to be explored.

We report results of simulating the binary interaction of a neutron star with a black hole. The related problem of the coalescence of double neutron star binaries has been the object of numerous previous studies in a Newtonian framework. The gravitational radiation signal was studied by Nakamura & Oohara (1989) and Oohara & Nakamura (1989, 1990), while Davies et al. (1994) considered the effect of different initial spin configurations on the outcome of the merger. In more recent simulations the neutrino emission has been calculated, as well as the gravitational waves from the coalescence (Ruffert et al. 1996; Ruffert & Janka 1997a; Ruffert, Rampp, & Janka 1997b). RS and Lai, Rasio, & Shapiro (1993b, hereafter LRSb) have investigated in great detail the destabilization effect of tidal forces in a Newtonian framework, using an analytical approach based on an energy variational method to examine equilibrium configurations of two polytropes, and a three-dimensional numerical treatment to study the hydrodynamical aspects of the coalescence.

The motivation for our work is presented in § 2. The numerical method we have used to carry out our simulations is described in § 3. Calibration to previous work and
our modeling of the black hole–neutron star system is presented in § 4. In § 5 we present our numerical results, and a discussion of these results follows in § 6.

2. MOTIVATION

Because of the very small radii of neutron stars, binaries containing either two neutron stars or a neutron star and a black hole are detached throughout their evolutionary history, except for the final several orbital periods. The two compact objects in such short-period binaries are inexorably drawn together by the loss of orbital angular momentum to gravitational radiation. Binary systems of this type have been found (PSR 1913 + 16—Hulse & Taylor 1975; PSR 1534 + 12—Wolszczan 1991) and the observed rate of change of the orbital parameters matches the inspiral predicted by general relativity to high accuracy (Taylor & Weisberg 1982; Taylor et al. 1992). An extrapolation of their current evolution indicates that the neutron stars in these binaries will merge in less than the Hubble time. In the final stages of such a coalescence, a powerful burst of gravitational waves is expected, so systems like these are primary candidate sources for detection by instruments such as LIGO and VIRGO, expected to begin operation within a few years (Thorne 1995). Estimates for the event rate can be inferred from the statistics of the known Hulse-Taylor type binaries and from theoretical studies of stellar evolution—a rate of $10^{-6}$ to $10^{-5}$ per galaxy per year is expected (Lattimer & Schramm 1976; Narayan, Piran, & Shemi 1991; Portegies Zwart & Yungelson 1998), which would imply that several coalescences per year should be occurring out to a distance of 1 Gpc.

With current techniques, detection of the gravitational radiation requires an accurate knowledge of the form of the expected signal, probably at least to the third post-Newtonian order beyond the quadrupole formula (Blanchet 1997 and references therein). Computations of the wave templates in general relativity are being carried out by several groups using analytical techniques, under the assumption that the neutron star is in hydrostatic equilibrium and that the only energy and angular momentum losses from the binary are those to gravitational radiation. This latter assumption may be incorrect if significant mass motions are induced in the star (Wilson, Salmonson, & Mathews 1997). Further, it is usually assumed that the final stage of evolution is an inspiral resulting in the disappearance (e.g., coalescence) of the binary, or more generally, that the binary period decreases monotonically, so that the system passes only once through the crucial frequency window of greatest effective sensitivity ($\sim 10^{2}$ Hz) and subsequent mass transfer is irrelevant to the gravitational signal—this is particularly important because the LIGO/VIRGO signal will be integrated over time. This last assumption is just the opposite of what has been suggested by Blinnikov et al. (1984), who discuss steady Roche lobe overflow in the system, resulting in continuous expansion of the binary. Which of these two variants of evolution is true can be checked only with hydrodynamic simulations. Our quasi-Newtonian suggest that the truth may lie in between: for the stiff and initially tidally locked polytrope investigated here we always find an increase of the binary period after the first episode of mass transfer, but no evidence of steady mass transfer or continuing expansion of the orbit.

It has been shown (Lai, Rasio, & Shapiro 1993a), at least in the Newtonian case, that hydrodynamic effects play an important role in the orbital evolution of a double neutron star binary system. At small enough separations (on the order of a few stellar radii), tidal interactions can make a binary dynamically unstable, when the spin angular momentum becomes comparable to the orbital angular momentum. This effect alone can produce orbital decay on a timescale comparable to that of angular momentum loss to gravitational waves.

A three-dimensional hydrodynamical study is therefore essential to understanding the gravitational radiation signal. Eventually, the problem must be solved using a fully general-relativistic formalism. This has not yet been achieved. We hope that some qualitative insights can be obtained from a Newtonian treatment. The gravitational waves will certainly carry a copious amount of information about the emitting source. Among other things, because of hydrodynamical effects and the finite size of the stars, the waves are expected to exhibit strong departure from point-mass behavior, as well as a dependence on the details of the equation of state for matter at supranuclear densities. To gauge this dependence, following RS, we present the waveforms calculated in the quadrupole approximation, even though our simulations are essentially Newtonian.

There are approximate schemes for incorporating gravitational radiation reaction forces in SPH calculations. We use one such scheme to compute the evolution of a binary with mass ratio $q = 0.31$. However, we mostly focus on results obtained with purely Newtonian forces, because this is a uniquely defined problem. We are offering our results as a benchmark against which the hydrodynamic part of future relativistic codes may be tested.

There is yet another reason why these systems are the focus of intense study—they have been suggested as a possible source for the production of gamma-ray bursts (GRBs) (Narayan, Paczyński, & Piran 1992). It is well established that the GRB distribution on the sky is isotropic and it is generally believed that their sources lie at cosmological distances (Fishman & Meegan 1995), with redshifts $z = 0.835, z = 3.4$, and $z = 0.97$ reported for three particular sources (Metzger et al. 1997; Kulvari et al. 1998; Djorgovski et al. 1998). The energy released in gamma rays may exceed $(\Delta \Omega/4\pi) \times 10^{53}$ erg s$^{-1}$ for the most powerful sources (Kulkarni et al. 1998), where $\Delta \Omega$ is the solid angle of emission. The gamma rays must be released in a region not smaller than many astronomical units cubed (otherwise the system would be opaque to pair creation), the observed very short rise times then necessarily imply ultrarelativistic motion of the source. In the relativistic blast wave model (Meszaros & Rees 1992, 1993), the interaction of this outflow with the intermediate medium leads to shock acceleration of electrons and emission of $\gamma$-rays through synchrotron radiation. Such relativistic outflows can be attained only if there is a relatively baryon-free line of sight from the source to the observer, along which matter can be accelerated to speeds close to the speed of light. The spatial distribution of baryonic matter transferred from the neutron star is, therefore, of particular interest in our simulations.

In the double neutron star merger scenario of GRB formation (Paczynski 1986; Goodman 1986; Eichler et al. 1988) the merger initially produces a burst of neutrinos and antineutrinos. Newtonian simulations (Janka & Ruffert 1996) suggest that the energy release into neutrinos may be insufficient to power the observed GRBs. General relativistic calculations carried out in a conformally flat metric
(Wilson, Mathews, & Marronetti 1996) have shown that in some cases each neutron star may collapse into a black hole many orbits prior to merging, and this could produce a GRB (Wilson et al. 1997).

It has been suggested that the coalescence of a black hole with a neutron star may be a more likely source of the energy powering a GRB (Paczynski 1991; Jaroszynski 1996). In this scenario, it was expected that the neutron star would be disrupted by the black hole and a thick accretion torus would form. As in any other case when the central object is enveloped by matter, the release of energy in this process could lead to the formation of an ultrarelativistic blast wave if energy can be channelled along a fairly baryon-free direction (the allowed baryon loading of the blast wave is no more than \( \sim 10^{-4} \, M_\odot \), for the binaries under consideration here). Our simulations suggest that the distribution of matter is favorable in coalescing black hole–neutron star binaries, and that the relevant timescales are of interest at least for one class of GRBs. A more detailed discussion of the implication of our results for GRB models is presented elsewhere (Kluźniak & Lee 1998 and Lee & Kluźniak 1998).

3. NUMERICAL METHOD

For the calculations presented in this paper, we have used the numerical technique known as smooth particle hydrodynamics (SPH). This is essentially a Lagrangian method, where forces are evaluated by interpolation over a grid of points comoving with the fluid, which can be considered as particles. This method was originally developed by Lucy (1977) and Gingold & Monaghan (1977) as an alternative to Eulerian computations on a fixed grid. An excellent review has been given by Monaghan (1992). The principal advantages of SPH are that no assumptions need to be made a priori about the nature of the fluid that will be studied and that no computational effort is wasted in modeling regions over which matter is not present. This is highly desirable when studying complicated three-dimensional astrophysical flows. Another feature of SPH, particularly advantageous in a discussion of the GRB baryon-loading problem, is the banal implementation of vacuum boundary conditions.

SPH has been tested successfully and applied to a variety of problems, such as the shock wave in a tube problem (Monaghan & Gingold 1983), redistribution of angular momentum in a thick accretion torus (Zurek & Benz 1986), static stellar structure (Gingold & Monaghan 1977), astrophysical jets (Coleman & Bicknell 1985), and hydrodynamics of close binary systems (Benz et al. 1990; Rasio & Shapiro 1992; Davies et al. 1994). We have developed our own SPH code (Lee 1998) and successfully tested it in one, two and three dimensions with several of these problems. Agreement with previous results has been excellent in every case. A brief description of our code is given in the Appendix.

4. CALIBRATION AND INITIAL CONDITIONS

4.1. Units

In this paper, we measure distance and mass in units of the radius \( R \) and mass \( M \) of the unperturbed (spherical) polytrope representing the neutron star (13.4 km and 1.4 \( M_\odot \), respectively), except where noted, so that the units of time, density and velocity are

\[
\tilde{t} = 1.146 \times 10^{-4} \, s \left( \frac{R}{13.4 \, \text{km}} \right)^{3/2} \left( \frac{M}{1.4 \, M_\odot} \right)^{-1/2},
\]

\[
\tilde{\rho} = 1.14 \times 10^{18} \, \text{kg m}^{-3} \left( \frac{R}{13.4 \, \text{km}} \right)^{-3} \left( \frac{M}{1.4 \, M_\odot} \right),
\]

\[
\tilde{v} = 0.39c \left( \frac{R}{13.4 \, \text{km}} \right)^{-1/2} \left( \frac{M}{1.4 \, M_\odot} \right)^{1/2}.
\]

4.2. Simulation of a Double Neutron Star Binary

Given that the problem of merging black hole–neutron star binaries is closely related to the study of coalescing double neutron star or white dwarf binaries, we have performed a rigorous calibration of our code to the results presented by RS for two polytropes with a stiff equation of state \( (\Gamma = 3) \) in a tidally locked binary with an initial mass ratio of unity. We used a binary tree structure in the code to compute gravitational interactions, RS used a Fourier transform. While our test simulations had a much lower resolution (\( \sim 2000 \) particles per star vs. 40,000 for RS), qualitative and quantitative agreement of our results with RS, including the amplitudes and frequencies of the gravitational radiation waveforms, was excellent (Lee & Kluźniak 1995; Lee 1998).

We repeated the simulation with \( N = 4224 \) particles. The initial separation for the double neutron star binary was \( r = 2.95 \) as in RS, and the coalescence resulted in a massive central core containing about 82% of the total mass, surrounded by a massive halo and extended spiral arms. At the end of the calculation, the central object is not azimuthally symmetric and thus continues to emit gravitational waves, of amplitude \( h_{\text{final}} \). In Table 1 we present a comparison of some of the more important parameters in the simulations, of RS and our own, of the coalescence of two neutron stars. We show the maximum and final amplitude in the gravitational radiation waveforms and the maximum gravitational wave luminosity. These values are given in geometrized units.

| Table 1 |
| --- |
| **COMPARISON OF RESULTS FOR THE COALESCENCE OF TWO IDENTICAL POLYTROPES WITH A STIFF EQUATION OF STATE (\( \Gamma = 3 \))** |
| **Result** | \( (r_0 R/M)^2 \rho_{\text{max}} \) | \( (r_0 R/M)^2 \rho_{\text{f}} \) | \( (R/M)^2 L_{\text{max}}/L_{0} \) | \( \rho_{\ast} \) | \( M_{\text{halo}}/M_{\ast} \) |
| This work …… | 2.2 | 0.2 | 0.39 | 0.44 | 0.16 |
| RS* …………… | 2.2 | 0.2 | 0.37 | 0.40 | 0.18 |

* The peak and final gravitational radiation amplitudes at a distance \( r_{0} \) from the source (see Appendix).

*\( L_{0} = c^{3} / G = 3.59 \times 10^{50} \) ergs s\(^{-1}\).*

* Rasio & Shapiro 1994.
units, where \( G = c = 1 \), and the peak luminosity \( L_{\text{max}} \) is normalized to \( L_0 = c^5/G = 3.59 \times 10^{59} \) ergs s\(^{-1}\). The last two columns display the central density of the resulting core, and the mass contained in the halo surrounding it as a fraction of the total mass in the system.

4.3. Modeling of the Black Hole

In our simulations, the black hole is modeled by a point mass with a Newtonian potential:

\[
\Phi_{\text{BH}}(r) = -\frac{GM_{\text{BH}}}{r}.
\]

The contribution from the black hole, to the force on particle \( i \) in the star is then simply given by

\[
F_{\text{BH}}^i = -\frac{GM_{\text{BH}}m_i}{|r_i - r_{\text{BH}}|}(r_i - r_{\text{BH}}),
\]

and symmetrically,

\[
F_{\text{BH}}^i = -\frac{Gm_i M_{\text{BH}}}{|r_i - r_{\text{BH}}|^3}(r_{\text{BH}} - r_i),
\]

is the contribution from particle \( i \) to the force on the black hole.

The (Schwarzschild) horizon of the black hole is modeled by placing an absorbing boundary at a distance \( r_{\text{Sch}} = 2GM_{\text{BH}}/c^2 \) from the point mass. At every time step during the dynamical simulations, any particle that crosses this boundary is absorbed by the black hole and removed from the simulation. The mass, position and velocity of the black hole are adjusted so that total mass and linear momentum are conserved. We disregard any spin angular moment that might be gained by the black hole during the process. This does not present any problems, since our calculation is Newtonian throughout, and we make no attempt to model frame dragging.

4.4. Construction of Initial Conditions

Our simulations begin with the construction of a spherical, unperturbed polytrope, which for simplicity we will refer to as the neutron star. To do this, we proceed essentially as Rasio & Shapiro (1992). \( N \) particles with masses \( m_i = \rho_{\text{LE}}/n \) are placed on a cubic lattice, where \( \rho_{\text{LE}} \) is the density calculated from the Lane-Emden solution to the equation of hydrostatic equilibrium with an equation of state \( P = K \rho^n \), and \( n \) is the number density of particles. The smoothing length assigned to each particle is such that the number of overlapping neighbors per particle is \( v = 64 \). This amounts to setting the smoothing length \( h_i \sim l \), where \( l \) is the lattice spacing. There is considerable advantage in having variable particle masses, as this increases the spatial and density resolution near the edge of the star, where the density gradient is largest. The radius of the unperturbed polytrope is 13.4 km, and its mass is 1.4 \( M_\odot \). For the simulations presented in this paper, we have used \( N \sim 8000 \) for every case except two, where \( N \sim 17,000 \).

To obtain tidally locked equilibrium configurations for the binary system, the unperturbed polytrope and the black hole are placed in the corotating Keplerian frame of initial angular velocity \( \Omega = [GM + M_{\text{BH}}]/r^3]^{1/2} \), where \( r \) is the binary separation defined as the distance between the black hole and the center of mass of the neutron star. A damping term linear in the velocity is introduced into the equations of motion for the SPH particles to allow the star to adjust to the presence of the tidal gravitational field. We now have

\[
m_i \dot{v}_i = F_{\text{Io}} + F_{\text{BH}} - m_i \frac{v_i}{t_{\text{damp}}} + m_i \Omega^2 r_i,
\]

\[
M_{\text{BH}} \dot{v}_{\text{BH}} = F_{\text{BH}} + M_{\text{BH}} \Omega^2 r_{\text{BH}},
\]

where \( F_{\text{Io}} \) and \( F_{\text{BH}} \) are the gravitational and hydrodynamical forces on particle \( i \) (of mass \( m_i \)), respectively. The value of \( t_{\text{damp}} \) is chosen so that oscillations are critically damped, reaching an equilibrium as quickly as possible; \( t_{\text{damp}} = (GM/R^3)^{-1/2} \). We neglect Coriolis forces since we are interested in equilibrium configurations with no bulk motion in the corotating frame. The position of the black hole and the center of mass of the star are adjusted at every time step so that the binary separation remains at a desired value. In the same manner, \( \Omega \) is adjusted so that the total (gravitational plus centrifugal) force on the black hole in the corotating frame, \( F_{\text{BH}} + M_{\text{BH}} \Omega^2 r_{\text{BH}} \) is zero, i.e.,

\[
\Omega = \frac{F_{\text{BH}}}{\sqrt{M_{\text{BH}} r_{\text{BH}}^2}},
\]

where \( r_{\text{BH}} \) is the position vector of the black hole. This procedure ensures that the configuration reaches equilibrium in a state of synchronization.

For a given value of the binary separation, we allow the system to relax for a period of 20 time units, keeping the specific entropies of all particles constant, i.e., \( K = \) constant in \( P = K \rho^\gamma \). In all cases, our initial conditions satisfy the virial ratio to better than three parts in \( 10^3 \).

5. RESULTS

5.1. Introduction

In this section we present the results of our simulations for different values of the mass ratio \( q \) in the binary, beginning with high values of \( q \) and proceeding in descending order. We have varied the mass ratio in the binary, \( q = M/M_{\text{BH}} \), by changing the mass of the black hole only. The

![Fig. 1.—Total angular momentum vs. binary separation for a black hole–neutron star binary with mass ratio \( q = 1 \). The solid line is the result for two point masses, the dotted line is computed using the analytical approach of LRSb, treating the neutron star as a compressible triaxial ellipsoid, and the crosses are the result of our SPH relaxation calculations. The Roche limit is at \( r = 2.78 \).](https://example.com/f1.png)
units used are as defined in equations (1), (2), and (3) except where noted. The highest value of the mass ratio is unity. Although the production of a binary system with such a low-mass black hole is rather unlikely, we nevertheless wish to carry out a comparison with the case for two neutron stars, and use this as a starting point in our investigation. For each mass ratio we present the initial configurations that were constructed for tidally locked binaries, followed by a description of the dynamical runs.

To investigate the dynamical evolution of the system for a given initial separation, we remove the damping term from the equations of motion and give every SPH particle and the black hole the azimuthal velocity corresponding to the equilibrium value of $\Omega$ in an inertial frame, with the origin at the center of mass of the system. Each SPH particle is assigned a specific thermal energy $u_i = K\rho^{(\Gamma - 1)/\Gamma}$, and the equation of state is changed to that of an ideal gas where $P = (\Gamma - 1)\rho u$. The specific thermal energy of each SPH particle is then evolved individually throughout the simulation according to the first law of thermodynamics, taking into account the contribution from the viscous terms (see Appendix).

5.2. Mass Ratio $q = 1$
5.2.1. Equilibrium Configurations

As described above (§ 4.4), equilibrium configurations were constructed for different values of the binary separation $r$. We show in Figure 1 a plot of the total angular momentum of the system, $J$, versus binary separation $r$. The solid line is the result for two point masses in Keplerian orbit. The dotted line is the result of approximating the neutron star as a compressible triaxial ellipsoid (LRSb), and

![Graphs showing particle positions projected onto the equatorial plane for initial configurations at various binary separations. One dot is plotted for every SPH particle.](image1)

Fig. 2.—Particle positions projected onto the equatorial plane for initial configurations at various binary separations. One dot is plotted for every SPH particle.
one cross is plotted for each SPH calculation at a fixed separation (with \( N = 8121 \) SPH particles for the neutron star). The presence of a minimum in \( J \) is a direct result of the assumption of tidal locking of the extended star. Essentially, as the separation is decreased, the spin component of angular momentum must grow (just as the orbital component is decreasing) and eventually become comparable to the orbital component. The result is that the total angular momentum, \( J \), reaches a minimum and as the separation decreases further, it will increase. Such a turning point in the curve of total angular momentum as a function of separation marks the onset of an instability, which will lead to orbital decay (Lai et al. 1993a). Note the strong departure from point-mass behavior, and the excellent agreement of the full SPH calculations with the compressible triaxial ellipsoid treatment before the minimum in \( J \) is achieved. Close to the minimum, the ellipsoidal approximation breaks down and a full numerical treatment is necessary. The cross with the smallest value of \( r \) corresponds to Roche lobe overflow \( (r = r_{RL} = 2.78) \). We find the dynamical stability limit to be \( r_{dynam} = 2.78 = r_{RL} \). Our initial determination of the stability limit (Lee & Kluzniak 1995) was higher than this value because the initial conditions did not correspond to proper tidal locking. Figure 2 exhibits particle positions projected onto the equatorial plane for different separations. As the separation is decreased, the effect of the tidal potential becomes more apparent and the neutron star is distorted. The configuration at \( r = 3.0 \) can be approximated by a triaxial ellipsoid, while for those at \( r = 2.8 \) and \( r = 2.78 \) this is increasingly no longer the case.

5.2.2. Dynamical Runs

We have used several of the configurations thus constructed to perform dynamical runs. In Figure 3 we show the binary separation as a function of time during several of these calculations. It is clear that the configurations with initial separations \( r = 3.0 \) and \( r = 2.9 \) are stable (see also Table 2). The separation exhibits oscillations of numerical origin on a period close to the orbital period (respectively, \( P = 22.81 \) and \( P = 21.6 \) in our units, see eq. [1]). However, the configuration with \( r = 2.78 \) and initial period \( P = 20.09 \) is a direct result of the tidal locking of the extended star. Essentially, as the separation is decreased, the spin component of angular momentum must grow (just as the orbital component is decreasing) and eventually become comparable to the orbital component. The result is that the total angular momentum, \( J \), reaches a minimum and as the separation decreases further, it will increase. Such a turning point in the curve of total angular momentum as a function of separation marks the onset of an instability, which will lead to orbital decay (Lai et al. 1993a). Note the strong departure from point-mass behavior, and the excellent agreement of the full SPH calculations with the compressible triaxial ellipsoid treatment before the minimum in \( J \) is achieved. Close to the minimum, the ellipsoidal approximation breaks down and a full numerical treatment is necessary. The cross with the smallest value of \( r \) corresponds to Roche lobe overflow \( (r = r_{RL} = 2.78) \). We find the dynamical stability limit to be \( r_{dynam} = 2.78 = r_{RL} \). Our initial determination of the stability limit (Lee & Kluzniak 1995) was higher than this value because the initial conditions did not correspond to proper tidal locking. Figure 2 exhibits particle positions projected onto the equatorial plane for different separations. As the separation is decreased, the effect of the tidal potential becomes more apparent and the neutron star is distorted. The configuration at \( r = 3.0 \) can be approximated by a triaxial ellipsoid, while for those at \( r = 2.8 \) and \( r = 2.78 \) this is increasingly no longer the case.

![Fig. 3.—Binary separation as a function of time for dynamical calculations starting at three different separations \( (r = 3.0, r = 2.9 \), and \( r = 2.78 \) for the black hole–neutron star binary with mass ratio \( q = 1 \). For \( r = 2.78 \), the solid line is based on a simulation with \( N = 8121 \) particles, and the dashed line on one with \( N = 16944 \) particles.](image)

### Table 2

| \( q_i \) | \( r_{min}^a \) | \( r_{RL} \) | \( r_{i}^b \) | Outcome |
|---|---|---|---|---|
| 1.00 : | 2.82 : | 2.78 : | 2.78 : | Mass transfer |
| 1.00 : | 2.82 : | 2.78 : | 2.90 : | Stable binary |
| 1.00 : | 2.82 : | 2.78 : | 3.00 : | Stable binary |
| 0.80 : | 2.97 : | 2.94 : | 2.93 : | Mass transfer |
| 0.80 : | 2.97 : | 2.94 : | 2.98 : | Stable binary |
| 0.31 : | ... : | 3.76 : | 3.76 : | Mass transfer |
| 0.31 : | ... : | 3.76 : | 3.76 : | Mass transfer |

| *a* \( r_{min} \) is the binary separation for which total angular momentum \( J \) is minimum. |
| *b* \( r_i \) is the initial separation chosen for a dynamical run. |

This run included gravitational radiation reaction as described in § 5.4.

In Figure 4 we show density contours at various times during the simulation, which was run from \( t = 0 \) to \( t = 100 \). The left column shows contours in the orbital plane, and the right column shows contours in the meridional plane containing the black hole and the center of mass of the SPH particles. The black disk with radius \( r_{sch} \) represents the black hole. Mass transfer from the neutron star to the black hole begins almost immediately (within one orbit) through the inner Lagrangian point. The accretion stream gradually becomes thicker and begins to wrap around the black hole. By \( t = 50 \) the star has become considerably elongated and a toroidal structure has begun to form around the black hole. The accretion stream from the neutron star essentially breaks already at \( t < 65 \), mass transfer stops, and a stellar core remains in orbit around the black hole \( (t = 65 \) through \( t = 80 \). Accretion of the remaining matter onto the black hole then gradually tapers off. At the end of the simulation, the core is modeled by approximately 5600 particles, and about 1200 constitute the accretion structure around the black hole.

The result of this coalescence, then, appears to be a stable binary with a greatly altered separation and mass ratio \( (r \sim 4.5, \text{corresponding to} 60 \text{km, and} q_{final} = 0.19) \), consisting of a remnant core with mass \( M_{core} = 0.307 \) (corresponding to \( 0.43 M_{\odot} \)) in orbit around a black hole with \( M_{BH} = 1.607 \) (or \( 2.25 M_{\odot} \)). The mass transfer event is very brief, as can be seen in Figure 5, where we plot the mass of the black hole and the mass accretion rate onto the black hole as a function of time. The peak accretion rate is \( \dot{M}_{BH}/dt = 0.0275 \) (corresponding to \( 0.3 M_{\odot} \text{ ms}^{-1} \)). The black hole is still accreting matter toward the end of our simulation, but the decreased resolution (recall that accretion entails a loss of particles in the simulation) does not allow us to determine the distribution of matter accurately beyond \( t = 100 \) (\( \sim 11 \text{ ms} \)). What can be determined is that the toroidal structure that forms does not extend to form a halo that engulfs the black hole completely. The region...
Figure 4.—Density contours at various times during the dynamical Newtonian evolution of the black hole-neutron star binary with mass ratio $q = 1$ and initial separation $r = 2.78$. (a, b) $t = 0$, (c, d) $t = 40$, (e, f) $t = 55$, (g, h) $t = 60$, (i, j) $t = 65$, (k, l) $t = 80$. The initial orbital period is $P = 20.09$. The logarithmic contours are evenly spaced every 0.25 decades, and bold contours are plotted at log $\rho = -3$, $-2$, $-1$, in the units defined in §4.1. The panels on the left (a, c, e, g, i, k) show density contours in the orbital plane (the rotation is counterclockwise), the right panels show contours in the meridional plane containing the binary axis. The black disk of radius $r_{\text{in}}$ represents the black hole. In this run, $N = 16944$.

directly above the black hole remains devoid of matter down to the limit of our resolution of $7.1 \times 10^{-5}$ (or $10^{-4} M_\odot$) within a cone of opening angle $\sim 5^\circ$ (we have obtained this number by searching for the SPH particle that comes closest to the vertical axis containing the black hole during the course of the simulation and identifying its mass).

The accretion structure around the black hole was originally not reported (Lee & Kluźniak 1995; Kluźniak & Lee 1997) because of the low resolution ($N = 2176$ particles) of our early runs, where the final configuration of the system was that of a stable binary with an unresolved halo of matter surrounding the system. We progressively increased the resolution of our simulations, first using $N = 4224$ particles for the neutron star, then $N = 8121$ and $N = 16944$. A hint of a toroidal accretion structure appeared when $N = 4224$ and was clearly present with $N = 8121$ and $N = 16944$, with minor differences between the two cases. We show in Figure 6 density contours for the case where $N = 8121$ particles model the neutron star initially. A comparison with Figures 4g and 4h shows that the accretion structures are essentially the same (the only significant difference is that the region directly above and below the black hole (along the rotation axis) is devoid of matter to a greater extent for the run with higher resolution). This clearly indicates that we have identified the accretion torus unambiguously. The simulation was stopped at $t = 100$ for two
main reasons: first, the system now consists of a stable binary, and in the absence of gravitational radiation reaction the orbit will not decay on a dynamical timescale; second, the accretion torus cannot be resolved with greater detail due to the loss of SPH particles to the black hole.

The nature of the encounter is reflected in the gravitational radiation waveforms, presented in Figure 7. The amplitude of the emitted waves initially rises from \((r_0 R/M^2)h = 1.55\) to \((r_0 R/M^2)h_{\text{max}} = 1.75\), reflecting the decrease in separation. The subsequent drop in amplitude and decrease in frequency are a result of the short episode of mass transfer, and the final waveforms reflect the fact that the binary has survived the encounter (the new binary orbital period is \(P \approx 39\) in our units, or 4.5 ms). Thus, the presence of a remnant core in orbit around the black hole would be immediately apparent from an observation of the waveforms emitted during such a coalescence. We also show the gravitational wave luminosity in Figure 8. For comparison purposes, the luminosity for the coalescence of two identical neutron stars is plotted as well, from the results of § 4.2. The maximum luminosity and total energy radiated in gravitational waves are \((R/M)^5 T_{\text{NS-NS}} = 0.39\) (or \(3.4 \times 10^{52}\) ergs) and \((R/M)^5 T_{\text{BH-NS}} = 0.15\) (or \(2 \times 10^{52}\) ergs) for the double neutron star and black hole–neutron star case, respectively. Although in both cases there is a clear rise and decay in the luminosity curve, the peak is much broader for the black hole binary (\(t_{\text{BH-NS}} \approx 25\)) than for two neutron stars (\(t_{\text{NS-NS}} \approx 15\)). The slower rise in luminosity and longer timescale of the peak in the black hole–neutron star case are a direct consequence of only one star being deformed and disrupted through tidal interactions.

Finally, we show in Figure 9 the binary separation as a function of time for the simulation we have just presented, together with a plot of the separation for a point-mass
binary with the same mass ratio and initial separation decaying via the emission of gravitational waves in the quadrupole approximation. This last curve is given by (e.g., Shapiro & Teukolsky 1983) $r = r_i(1 - t/t_0)^{1/4}$, where $t_0 = (5r_i^4 c^5)/(256\mu M_t^2 G^3)$ and $r_i$ is the initial separation at $t = 0$, the total mass is $M_t = M_{\text{BH}} + M$ and the reduced mass is $\mu = M_{\text{BH}} M_t$.

Clearly, hydrodynamical effects and the instability they induce drive the coalescence process on a timescale shorter than that of purely gravitational radiation losses, and so we may neglect qualitative effects of radiation reaction on the evolution of the system over the time period which we have modeled. This is not to say that the inclusion of gravitational radiation reaction will not affect the results quantitatively—for example, the total energy which should be lost to gravitational waves, approximately $2 \times 10^{52}$ ergs, is comparable to the final orbital energy of the system, $|E_{\text{orb}}| = GM_{\text{BH}} M_{\text{core}}/r \approx 4 \times 10^{52}$ ergs.

In the calculation described above, we have integrated the thermal energy equation according to the first law of thermodynamics by setting

$$\Delta U = \Delta W + \Delta Q ,$$

where $\Delta Q$ is calculated using the contribution from the artificial viscosity (see eq. [A3] in the Appendix). We will denote this as Case I. To investigate the effect of radiative losses on the outcome of this configuration, we have performed an additional calculation for the extreme case in which all the energy produced by viscosity is lost by the system (e.g., to neutrinos). This will be denoted as Case II and is achieved by simply removing the term associated with viscous heating from equation (4), so that we now have $\Delta U = \Delta W$.

Density contours for Case II (with $N = 16944$ particles) are shown in Figure 10 for $t = 60$ and $t = 80$. An accretion torus, well separated from the neutron star, is clearly visible.
Fig. 5.—Black hole mass (dotted line) and mass accretion rate onto the black hole (solid line) as a function of time for the black hole–neutron star binary with mass ratio $q = 1$ and initial separation $r = 2.78$.

Fig. 6.—Same as Fig. 4 (g), (h), but for a run with $N = 8121$ particles

Fig. 7.—Gravitational radiation waveforms for the coalescence of a black hole–neutron star binary with mass ratio $q = 1$ (solid lines). Two polarizations (left, $h_+$; right, $h_\times$) in geometrized units (such that $G = c = 1$) are shown. The dashed lines show the analytical waveforms computed in the quadrupole approximation for two point masses at the same initial separation.
We realize that it does not correspond to a physical viscosity, but believe it is nevertheless instructive to compare the two cases mentioned above, since this permits exploring the range of possible outcomes for extreme cases of energy loss from the system (Case I is completely adiabatic, while in Case II all the internal energy generated through dissipation is lost from the system). Furthermore, in simulations involving accretion disks in two and three dimensions, it has been shown that there is a definite relationship between the term linear in the velocity differences in the SPH artificial viscosity (see eq. [A2]) and the coefficient of kinematic viscosity in the continuum limit (Meglicki, Wickramasinghe, & Bicknell 1993; Murray 1996; Maddison, Murray, & Monaghan 1996; Murray 1997), which in turn can be related to the standard $\alpha$-prescription for viscosity in accretion disks. Although we are not strictly dealing with an accretion disk in this case, we believe these calculations can provide a useful guide as regards the energy release. We find that for the fluid contained in the accretion structure around the black hole, the $\alpha$ parameter has value on the order of unity.

5.3. Mass Ratio $q = 0.8$

5.3.1. Equilibrium Configurations

For a mass ratio $q = 0.8$, the black hole mass is $M_{BH} = 1.25$ (or $1.75 M_\odot$). Hydrodynamical effects are still important in the construction of equilibrium configurations in this case, as can be seen in Figure 12, where we have plotted the equilibrium angular momentum values for a range of binary separations. Approximating the neutron star as a compressible triaxial ellipsoid gives results which are close to the full numerical calculations before the minimum in angular momentum is reached, but for $r \leq 3.0$ this is no longer the case. We find minimum angular momentum at separation $r = 2.98$, and Roche lobe overflow at separation $r_{KL} = 2.935$.

5.3.2. Dynamical Runs

For this value of the mass ratio, we used $N = 8121$ particles to model the neutron star. As for the case with $q = 1$, several equilibrium configurations were used as initial conditions for dynamical runs (see Table 2). Figure 13 shows the binary separation as a function of time for these calculations. The initial configuration with separation $r = 2.935$ becomes unstable on a dynamical timescale. However, it is apparent that the development of the instability (and hence the orbital decay) proceeds at a slower rate than for a higher mass ratio. The reason for this is that since the black hole is more massive, Roche lobe overflow occurs at a greater separation than previously observed (for $q = 1$), and the tidal effects are less pronounced on the neutron star. Density contours in the orbital plane for the binary with initial separation $r = 2.935$ are shown in Figure 14. Mass transfer proceeds through the formation of an accretion stream from the neutron star to the black hole, lasting from $t \sim 45$ to $t \sim 75$ (i.e., for about 3.5 ms), with a peak accretion rate of 0.017 (0.2 $M_\odot$ ms$^{-1}$). A substantial amount of mass is stripped from the neutron star during the encounter (see Fig. 15), and the final black hole mass is $M_{BH} = 1.65$ (equivalent to 2.3 $M_\odot$). The core of the neutron star, with a final mass $M_{core} = 0.595$ (corresponding to 0.83 $M_\odot$) again

There is once more a baryon-free axis along the rotation axis of the binary, only this time it is clear of matter within a cone of opening angle $\sim 20^\circ$ (down to $4 \times 10^{-5}$, equivalent to $6 \times 10^{-5}$ $M_\odot$—this limit was obtained as for the Case I simulation described above) since the torus is more nearly confined to the orbital plane. This is clearly a consequence of a lack of pressure support due to the removal of thermal energy from the system. In Figure 11 we show for Case II the rate of energy loss, $L_\nu$, due to viscous heating. The peak rate is $(R/M)^2 L_\nu \approx 0.65$, corresponding to $\approx 2 \times 10^{55}$ ergs s$^{-1}$ and the total energy loss from $t = 0$ to $t = 100$ is $\approx 5 \times 10^{52}$ ergs. The mass transfer rate and gravitational radiation waveforms are essentially the same as for Case I.

We note that the artificial viscosity used in SPH was originally devised to prevent particle interpenetration during simulations, and to allow the modeling of shocks.
survives the encounter. At $t = 100$, 4671 particles model the surviving neutron star core. However, the most striking difference between this case and the one presented previously ($q = 1$) is the absence of a toroidal accretion structure around the black hole. All matter stripped from the neutron star is directly accreted by the black hole, and thus in this scenario as well, the rotation axis above the black hole is devoid of matter. As a result of mass transfer (which is clearly over by $t = 100$, when we have stopped the simulation) the binary separation increases to $r \sim 3.6$, and thus the final configuration in this case is again a stable binary.

As before, the hydrodynamical evolution of the binary is reflected in the gravitational radiation waveforms, presented in Figure 16. There is an initial rise in amplitude in the waveforms, followed by a rapid decline and increase in the
period, reflecting the episode of mass transfer and subsequent increase in binary separation. The peak amplitude in this case is \( (r_0 R/M^2)h_{\text{max}} = 1.93 \), and again since the binary survives the encounter, the final amplitude is not zero but \( (r_0 R/M^2)h_{\text{final}} = 1.25 \). The luminosity emitted in gravitational waves is shown in Figure 17. The general shape of the curve resembles very much that for \( q = 1 \), as one can expect after studying the density contour plots for each case (since the toroidal structure that was formed for \( q = 1 \) is essentially azimuthally symmetric, it does not contribute significantly to the emission of gravitational waves). The curve presents again a broad peak, with a duration \( \Delta t \sim 30 \) (3.4 ms). The peak luminosity is \( (R/M)^3 L_{\text{max}} = 0.16 \) (or \( 4.8 \times 10^{54} \) ergs s\(^{-1}\)) and the total energy radiated away in gravitational waves is \( (R^{7/2}/M^{9/2})\Delta E_{\text{GW}} \approx 10 \) (or \( 3.4 \times 10^{52} \) ergs). These values are higher than for the case with \( q = 1 \) because we have normalized our results to the mass of neutron star, and lowering the mass ratio implies the system has a greater total mass. The total energy carried away by gravitational waves is again comparable to the final orbital energy of the system (\( |E_{\text{orb}}| \approx 10^{53} \) ergs), indicating that in a realistic simulation the effect of radiation reaction cannot be ignored.

In Figure 18 we compare (as for \( q = 1 \) in Fig. 9) the orbital evolution of this binary with that of a point-mass binary emitting gravitational waves in the quadrupole approximation. It is apparent that hydrodynamical effects are less important than for \( q = 1 \) in determining the initial orbital evolution but are nevertheless substantial.

5.4. Mass Ratio \( q = 0.31 \)

5.4.1. Newtonian Simulation

Continuing the trend toward lower mass ratios, we chose a value of \( q = 0.31 \), giving \( M_{\text{BH}} = 3.22 \) (corresponding to 4.51 \( M_\odot \)), to study the evolution of a binary system where most of the mass is contained in the black hole. We again used \( N = 8121 \) particles to model the neutron star. A qualitative difference appears in the plot of total angular momentum as a function of binary separation (Fig. 19). At a separation of \( r = 3.76 \) (or 50.5 km), Roche lobe overflow occurs, and this point is reached before the minimum in total angular momentum is attained, contrary to what occurred for \( q = 1 \) and \( q = 0.8 \). Thus, we expect all configurations down to the Roche limit \( r_{\text{RL}} = 3.76 \) to be dynamically stable (see the discussion for \( q = 1 \) in § 5.2.1).

The binary separation for dynamical runs with an initial separation \( r = r_{\text{RL}} = 3.76 \) is shown in Figure 20. The initial orbital period is \( P = 21.92 \) (or 2.51 ms). For the Newtonian simulation there is no orbital decay at all, but rather a slow increase in separation resulting from gradually increasing Roche lobe overflow from the neutron star onto the black hole, which accretes \( \Delta M = 0.0104 \) (or 0.0146 \( M_\odot \)) over \( \sim 4.5 \) orbital periods. At this rate, it would take about 1 s to exhaust the star, which is not disrupted at all during this process (as can be seen in Fig. 21, where we plot density contours in the orbital plane over the course of the simulation, and in Fig. 22, where we plot the gravitational radiation waveform \( h_+ \) for this case). However, we terminated the simulation at \( t = 100 \) (11 ms). The actual mass transfer itself is not visible in the contour plots of Figure 21 because the number of particles being transferred is low (a few hundred), and their mass density is below the lowest density contour plotted. The increase in separation is
consistent with conservative evolution.\textsuperscript{3} We find 
\[
\left(\frac{M_{\text{BH}}}{M_i}\right)^2 = 1.01453 \approx r_f/r_i = 1.0167.
\]
Evidently, the loss of angular momentum down the black hole (whose spin we do not model) is nearly negligible in its effects on the binary evolution in this simulation.

\textsuperscript{3} For conservative mass transfer in a Keplerian point-mass binary, 
\[J = \mu\Omega r^2 = \text{const},\] where \(\mu\) is the reduced mass, and \(\Omega \propto r^{-3/2}\).

This case gives a clear indication of the limitations of the present method of studying the coalescence of a black hole with a neutron star. In the first place, since hydrodynamical effects do not cause a rapid evolution of the orbit, gravitational radiation reaction (which would cause this binary to coalesce in a time \(t_0 = 31.35, \text{ or } 3.59 \text{ ms}\)) cannot be ignored. Second, as is apparent in the density contour plots shown in Figure 21, the black hole has become almost as large as the neutron star itself (the Schwarzschild radius is \(r_{\text{Sch}} = 0.986,\)
Fig. 14.—Continued

Fig. 14.—Black hole mass (dashed line) and mass accretion rate onto the black hole (solid line) as a function of time for the black hole–neutron star binary with mass ratio $q = 0.8$ and initial separation $r = 2.935$.

Fig. 16.—Gravitational radiation waveforms for the coalescing black hole–neutron star binary with mass ratio $q = 0.8$ (solid lines). Two polarizations (left, $h_+; \text{right}, h_\times$) in geometrized units ($G = c = 1$) are shown. The dashed lines show the analytical waveforms computed in the quadrupole approximation for two point masses at the same initial separation.
or 13.2 km, at the start of the simulation), making the radius of the innermost circular stable orbit for a test particle $r_{\text{ms}} = 3r_{\text{sch}} = 2.958$ (or 39.75 km). This means that many of the particles composing the neutron star are at the outset at a distance $r \leq r_{\text{ms}}$ from the black hole. Inclusion of relativistic effects is clearly necessary in this case for the results to be meaningful. Directly below, we describe the results of a run in which we have alleviated the first of these two limitations of a Newtonian study.

5.4.2. Simulation with Gravitational Radiation Reaction

We have performed an additional run, using the same initial conditions as described above (§5.4.1), but including a term in the equations of motion that approximates gravitational radiation reaction on the components of the binary system. Using the quadrupole approximation, the rate of change of energy and that of angular momentum are given by (Landau & Lifshitz 1975)

$$\frac{dE}{dt} = -\frac{32}{5} \frac{G^4(M + M_{\text{BH}})(MM_{\text{BH}})^2}{(cr)^5}, \quad (5)$$

and

$$\frac{dJ}{dt} = -\frac{32}{5c^5} \frac{G^{7/2} M_{\text{BH}}^2 M^2 \sqrt{M_{\text{BH}} + M}}{r^{1/2}}. \quad (6)$$

From these equations one can obtain the acceleration on each member of the binary, assuming they can be represented by point masses (this implementation of a back-reaction force has been used in SPH before; see, e.g., Davies et al. 1994). We apply the same acceleration to all particles within the neutron star.

The binary separation now decays much faster as can be seen in Figure 20, and intense mass accretion from the neutron star onto the black hole begins within one orbital period (see Fig. 23). As a result, the separation increases and, as for all the other cases presented here, the neutron star core is not completely disrupted. The peak mass accretion rate reaches $M_{\text{BH}} = 0.037$, corresponding to 0.45 $M_\odot$ ms$^{-1}$. The orbit after the initial episode of mass transfer is elliptical and is small enough (4.5 $\leq r \leq 5.5$) to allow mass transfer at every successive periastron passage (the simulation was run until $t = 200$ and at this stage 3335 particles modeled the surviving neutron star core) without tidally disrupting the neutron star core (see Figs. 23b–23d and 20). The maximum accretion rates during the successive episodes are lower than the initial (highest) one by approximately 1 order of magnitude.

Since the neutron star is not tidally disrupted, our approximation of the gravitational radiation back-reaction force is reasonable throughout the calculation, in the sense that the neutron star can be represented by a point mass. Strictly speaking, equations (5) and (6) are valid only for circular orbits. However, the eccentricity in the elliptical orbit observed in our simulation is relatively small (in the range 0.1–0.2), and so we believe the errors in $dE/dt$ and $dJ/dt$ are acceptable (20% and 10%, respectively).

The gravitational radiation waveforms (Fig. 22) are markedly different than for the case without radiation reaction, and resemble more the case of a high mass ratio (compare Fig. 16), reflecting the survival of the core after a substantial accretion episode (Table 3). The maximum and...
final amplitude in the waveforms are now \((r_0 R/M^2) h_{\text{max}} = 3.98\), and \((r_0 R/M^2) h_{\text{final}} = 1.54\). The gravitational radiation luminosity shows a first maximum \([(R/M)^2 L_{\text{max}} = 0.71\) (or \(2.13 \times 10^{55} \text{ ergs s}^{-1}\)] during the first episode of mass transfer, and subsequent peaks at each perihelion passage \([(R/M)^2 L_{\text{peak}} = 0.07\) (or \(2.1 \times 10^{54} \text{ ergs s}^{-1}\)]. The total energy radiated away in gravitational waves until the end of the calculation at \(t = 200\) is \((R^{7/2}/M^{9/2}) \Delta E_{\text{GW}} \approx 18\) (or \(6 \times 10^{52}\) ergs).

6. DISCUSSION

Table 2 gives a summary of our dynamical runs, including ones in which no mass transfer was expected (or found) in the Newtonian binary. We present the binary separations \(r_{\text{RL}}\) and \(r_{\text{min}}\), corresponding to the Roche limit and to the equilibrium (tidally locked) configuration with minimum total angular momentum for each mass ratio, as well as the values of the initial separations \(r_i\) that were used to search for the dynamical stability limit (if present) for each case. The lack of an entry for \(r_{\text{min}}\), when \(q = 0.31\), reflects the fact that Roche lobe overflow is initiated in an orbit in which \(J\) has not yet reached a minimum, i.e., all wider orbits are dynamically stable.

In Table 3 we present a summary of the calculated results of the initial mass exchange for the three configurations explored in this paper. The first column indicates the number of SPH particles initially present in the simulation; the second and third columns show the initial and final mass ratios, respectively; the fourth column indicates whether an accretion torus was formed; the fifth, sixth and seventh columns show the maximum and final gravitational radiation amplitude and the maximum gravitational radiation luminosity, respectively, all in geometrized units such that \(G = c = 1\); the eighth column shows the mass of the remnant core.

### Table 3

| \(N^b\) | \(q_i^c\) | \(q_f^d\) | Torus | \(r_0 R M^{-2} h_{\text{max}}\) | \(r_0 R M^{-2} h_f\) | \((R/M)^2 L_{\text{max}}/L_0\) | \(M_{\text{core}}^f\) |
|---|---|---|---|---|---|---|---|
| 16944 | 1.000 | 0.190 | Yes | 1.75 | 0.55 | 0.145 | 0.3070 |
| 8121 | 0.800 | 0.360 | No | 1.93 | 1.25 | 0.160 | 0.5950 |
| 8121 | 0.310 | 0.306\(^e\) | No | 3.59 | 3.48\(^f\) | 0.425 | 0.9896\(^g\) |
| 8121 | 0.310 | 0.106\(^f\) | No | 3.98 | 1.54\(^f\) | 0.710 | 0.4047\(^h\) |

\(^a\) Notation is the same as in Tables 1 and 2.

\(^b\) Initial number of particles in the simulation.

\(^c\) Initial mass ratio.

\(^d\) Final mass ratio, if two distinct cores survive.

\(^e\) Mass of the stellar core at the end of simulation.

\(^f\) At the end of simulation (the binary continues to evolve).

\(^g\) This run included gravitational radiation reaction.
A striking result is that the neutron star is not completely disrupted in any of the simulations presented here—a remnant core of the stiff polytrope always survives in orbit around the black hole of increased mass. The survival of the neutron star core suggests a different outcome than what was initially expected for the coalescence of a black hole with a neutron star (Wheeler 1971) and identifies this process as the only one suspected of producing low-mass neutron stars (Blinnikov et al. 1984). But this result appears to be very sensitive to the equation of state of dense matter—for a softer polytrope the star is completely disrupted and a toroidal structure forms around the black hole (Lee & Kluźniak 1999a, 1999b).

For high mass ratios ($q = 0.8$ and $q = 1$), due to the violent mass transfer episode, the core is transferred to a higher orbit on a timescale of one orbital period. For now, we are unable to model the system on much longer timescales than we have presented here, but there is no doubt, that because of the emission of gravitational waves, mass transfer will eventually resume as the orbit decays and the...
Fig. 22.—Gravitational radiation for a coalescing black hole–neutron star binary with mass ratio \( q = 0.31 \) (solid lines) for the two runs performed (see Fig. 20), compared with the result for a point-mass binary with the same mass ratio and initial separation, computed in the quadrupole approximation (dashed line). The curves corresponding to the run with gravitational radiation reaction are labeled “RR” in both panels—the other (Newtonian) run did not include any effects of gravitational radiation on the binary evolution and was terminated at \( t = 100 \). (Left) Gravitational radiation waveforms, one polarization \( (h_\perp) \) in geometrized units \( (G = c = 1) \) is shown. Note that the ellipticity of the orbit in the RR case is clearly reflected in the gravitational signal. (Right) Gravitational radiation luminosity.

Fig. 23.—Density contours at two moments during the dynamical coalescence of the black hole–neutron star binary with mass ratio \( q = 0.31 \) and initial separation \( r = 3.76 \) (including gravitational radiation reaction). The logarithmic contours are evenly spaced every 0.25 decades, and bold contours are plotted at \( \log \rho = -3, -2, -1 \), in the units defined in eq. (2). The rotation is counterclockwise and the initial orbital period is \( P = 21.92 \). The black disk represents the black hole.
neutron star fills its Roche lobe. Thus, the length of the coalescence process is extended from a few milliseconds to possibly several tens of milliseconds. For \( q = 0.31 \), in addition to the Newtonian simulation, we have also performed a run in which the angular momentum loss to gravitational radiation was taken into account during the simulation (Figs. 20, 21, and 22). Here we can see directly how the effects of mass transfer and of gravitational radiation nearly balance each other for an extended period of time, perhaps for a good fraction of one second. The violent episodes of mass transfer strip the neutron star of so much matter that it may be driven below the minimum mass required for stability. If this happens, then the core explodes (Blinnikov et al. 1984; Sumiyoshi et al. 1998; Colpi, Shapiro, & Teukolsky 1991) in approximately 0.1 s after a slow expansion phase that may last 20 s (Sumiyoshi et al. 1998). The presence of the black hole would complicate the outcome of this event.

Note that only in the simulation with a high mass ratio \((q = 1)\) did an accretion torus appear around the black hole.

\[ F_{\text{int}} = -\sum_j m_j \left( \frac{\nabla_i W_{ij}}{\rho_i} + \Pi_{ij} \right) \nabla_i W_{ij}, \]

where

\[ \Pi_{ij} = \begin{cases} \frac{-\mu_i}{r_{ij}} + \beta \mu_i^2 / \rho_i, & v_{ij} \cdot r_{ij} < 0, \\ 0, & v_{ij} \cdot r_{ij} > 0, \end{cases} \]

\[ \mu_i = \frac{h_i (v_{ij} \cdot r_{ij})}{r_{ij}^2 + \eta^2 h_i^2}. \]

**APPENDIX**

**NUMERICAL CODE**

The equations of motion in SPH are essentially those of an \(N\)-body problem, with each particle representing a fluid element. We split the force on a given particle \(i\) into hydrodynamical and gravitational contributions \((F_{\text{HH}}\) and \(F_{\text{GG}}\), respectively).

It is convenient to perform calculations with a kernel that has compact support. We use the form of Monaghan & Lattanzio (1985):

\[ W(r, h) = \frac{1}{\pi h^3} \begin{cases} 1 - \frac{3}{2} \left( \frac{r}{h} \right)^2 + \frac{3}{4} \left( \frac{r}{h} \right)^3, & 0 \leq r/h < 1, \\ \frac{1}{4} \left( 2 - \frac{r}{h} \right)^3, & 1 \leq r/h < 2, \\ 0, & 2 \leq (r/h). \end{cases} \]

The density at the position of particle \(i\) is then given by \(\rho_i = \sum \rho_{ij} W_{ij}\), where \(W_{ij}\) is the symmetrized kernel for particles \(i\) and \(j\). With \(h_i\) representing the smoothing length for particle \(i\), we have \(W_{ij} = W(|r_i - r_j|, h_i), h_{ij} = (1/2)(h_i + h_j)\).

\(F_{\text{HH}}\) includes the contribution from the pressure gradient and from artificial viscosity. In symmetrized form, it can be written as

\[ F_{\text{HH}} = -\sum_j m_j \left( \frac{\sqrt{P_i P_j}}{\rho_i \rho_j} + \Pi_{ij} \right) \nabla_i W_{ij}, \]
Here \( v_{ij} = v_i - v_j \), \( r_{ij} = r_i - r_j \), \( \alpha \) and \( \beta \) are constants, \( c_i \) is the speed of sound at the position of particle \( i \), and \( \bar{c}_{ij} = (c_i + c_j)/2 \) and \( \bar{p}_{ij} = (\rho_i + \rho_j)/2 \). In the simulations presented in this paper we have used \( \alpha = 1 \), \( \beta = 2 \), and \( \eta^2 = 10^{-2} \).

The value \( u_i \) of the thermal energy per unit mass is evolved according to (see, e.g., Monaghan 1992)

\[
\frac{du_i}{dt} = \frac{1}{2} \sum_j m_j \left( \frac{2}{\rho_i \rho_j} \sqrt{\frac{P_i}{P_j}} \right) + \Pi_{ij} v_{ij} \cdot V_i W_{ij}.
\]

(A3)

The thermal energy equation is integrated by essentially using a two-step procedure (Hernquist & Katz 1989) in order to preserve accuracy. This is necessary because the heating rate depends on the pressure, and via the equation of state, on the thermal energy itself.

Individual smoothing lengths \( h_i \) are adjusted so that every particle has an approximately constant number \( v \) of hydrodynamical neighbors, i.e., those for which \( \bar{c}_{ij} \) is the smoothing length for particle \( i \) at step \( n \), the value at step \( n + 1 \) is found according to (Hernquist & Katz 1989):

\[ h_i^{n+1} = \left( h_i^n / 2 \right) \left[ 1 + (v/v_b)^{1/3} \right] \]

where \( v_b \) is the number of hydrodynamical neighbors at step \( n \). We have found that a value of \( v_b = 64 \) yields adequate sampling of the fluid without requiring excessive computation time.

We have incorporated a binary tree structure (Hernquist & Katz 1989) into our code to calculate gravitational forces efficiently. This procedure makes the number of required operations per time step of \( O(N \log N) \) (instead of \( O(N^2) \) for a direct calculation), thus allowing for simulations with several thousand particles (for a detailed description see Benz et al. 1990). The subroutine corresponding to this approach simultaneously provides \( F_{ij} \) and the hydrodynamical neighbors for every particle that are used to obtain \( F_{ij} \) and \( du_i/ dt \) in equations (A1) and (A3). Our calculations of the gravitational forces are completely Newtonian.

We use a leapfrog algorithm accurate to second order to integrate the equations of motion. The time step is adaptive and is taken to satisfy a combination of the Courant criterion for stability and a further restriction on the maximum change allowed in velocity for any particle during one time step to conserve accuracy. We follow Monaghan (1989) and set the time step to be

\[ \Delta t = \min \left( \Delta t_1, \Delta t_2 \right) \]

where

\[ \Delta t_1 = \min \left( \frac{h_i}{v_b} \right)^{1/2} \]

and

\[ \Delta t_2 = 0.15 \min \left( \frac{h_i}{c_s} + 1.2 x_c + 1.2 f_{\text{max}}(\mu_i) \right) \]

In our Newtonian simulations we calculate gravitational radiation in the quadrupole approximation. The retarded amplitudes for an observer a distance \( r_0 \) away along the axis of the binary are given by

\[
F_{ij} = \frac{G M_i M_j}{r_{ij}^2},
\]

while the luminosity of gravitational radiation is

\[
L_{\text{GW}} = \frac{dE}{dt} = \frac{1}{5} \left( \frac{G}{c^3} \right) \left( \frac{\sigma v}{\Delta t} \right) \left( \frac{\Delta M}{\Delta t} \right),
\]

where

\[
\sigma = \frac{M_i M_j}{r_{ij}} \frac{\Delta M}{\Delta t}.
\]

The summation is over the SPH particles and the superscripts indicate the Cartesian components. We identify the gravitational acceleration on particle \( i \) as \( g_i \) (Finn 1989; Rasio & Shapiro 1992). We have added the terms arising from the presence of the black hole as a point mass contribution.

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