A topological Josephson junction platform for creating, manipulating, and braiding Majorana bound states

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As part of the intense effort towards identifying platforms in which Majorana bound states can be realized and manipulated to perform qubit operations, we propose a topological Josephson junction architecture that achieves these capabilities and which can be experimentally implemented. The platform uses conventional superconducting electrodes deposited on a topological insulator film to form networks of proximity-coupled lateral Josephson junctions. Magnetic fields threading the network of junction barriers create Josephson vortices that host Majorana bound states localized in the junction where the local phase difference is an odd multiple of $\pi$, i.e. attached to the cores of the Josephson vortices. This enables us to manipulate the Majorana states by moving the Josephson vortices, achieving functionality exclusive to these systems in contrast to others, such as those composed of topological superconductor nanowires. We describe protocols for: 1) braiding localized Majorana states by exchange, 2) controlling the separation and hence the coupling of adjacent localized Majorana states to effect non-Abelian rotations via hybridization of the Majorana modes, and 3) reading out changes in the non-local parity correlations induced by such operations. These schemes make use of the application of current pulses and local magnetic field pulses to control the location of vortices, and measurements of the Josephson current-phase relation to reveal the presence of the Majorana bound states. We describe the architecture and schemes in the context of experiments currently underway.

I. INTRODUCTION

Topologically-protected quantum processing is rapidly emerging as a viable route to next generation advances in quantum information, computational science and technology. In contrast to conventional qubits, topological qubits are based on exotic quasiparticle excitations in condensed matter systems that exhibit non-Abelian statistics. Systems having anyons obeying braiding rules are expected to show resilience to environmental interference, making them excellent candidates for fault-tolerant quantum computation. Within the last decade, the proposal and subsequent evidence for the experimental realization of topological superconductivity capable of hosting Majorana bound states (MBS) has created intense attention and activity from the perspective of such topological qubit technology. In these fermionic systems, these localized Andreev bound states are predicted to exist as zero energy states at the Fermi energy. A non-local pair of such MBSs share an electronic state that can either be occupied or empty, making such a pair a parity qubit. Implementing qubit operations requires positioning and manipulating MBSs and exploiting their non-Abelian nature through braiding. While such MBSs cannot alone span universal quantum computation, they are currently the forerunners for achieving topological quantum computation. Critical steps required for realizing topological quantum processing are under development, including experimental verification of the existence of MBSs, creating architectures that offer a platform for qubit operations, and designing complex non-Abelian braiding-based quantum computational protocols.

As with conventional qubits, now realized in a wide range of systems including coupled spins, superconducting transmons, photonic circuits, and cold atom systems, it is imperative that multiple promising approaches be explored to optimize progress toward successful implementation of topological quantum computing schemes. The past years have indeed witnessed a growth of potential candidate systems for hosting topological qubits mainly centered around MBSs. Most attention has been on nanowires having strong spin-orbit coupling and proximitized by contact with a conventional superconductor. By applying a magnetic field oriented along the length of the wire, this system can be tuned into a topological state in which MBSs are predicted to nucleate at the ends of the wire. More recently, chains of ferromagnetic atoms fabricated on a superconducting surface have received prominent attention for their ability to host MBSs, also attached to the ends of the wire. Novel materials that exhibit quantum Hall physics, such as graphene, have revived the initial interest of over a decade ago of exploiting certain fractional states, such as $\nu = 5/2$, for their potential to harbor Majorana bound states, as well as other states having more exotic fractional quasiparticle excitations (e.g. parafermions) that can, in principle, perform universal quantum computation.

As a viable alternative to these systems in which the topological excitations of interest are physically bound to the end or edges of 1D or 2D structures, here we propose a platform consisting of multiply-connected lateral Superconductor-Topological Insulator-Superconductor (S-TI-S) Josephson junctions networks...
for realizing MBSs whose locations can be controllably moved, providing additional functionality for braiding and hybridization that perform non-Abelian operations. In this paper, we delineate the key features required to realize these MBSs and carry out topological quantum processes in this system. Our proposed lateral S-TI-S Josephson junctions offer an attractive platform for MBS manipulations for the following reasons: (1) MBSs in this system are zero-energy Andreev bound states enabled by the spin-momentum locking of topological surface states in the TI and stabilized by the phase of the Josephson coupling. (2) In contrast to other systems such as semiconductor nanowires, nucleation of the MBSs does not require a large magnetic field, enabling phase-sensitive Josephson measurements. (3) Magnetic fields instead play a different role by localizing MBSs at Josephson vortex cores, which allows us to move the MBSs by moving the vortices, easily done in controlled ways by applying currents, voltages, or phase differences. (4) The MBSs can be created in a controlled way in uniform junction regions and are not subject to interface issues, unlike with nanowires in which the MBSs exist at interface between topological and non-topological regions. (5) Junction networks are easily scalable to create complex circuits, and surface codes for performing universal quantum computing in networks of Josephson junctions have already been proposed. As briefly reviewed in a later section, there have already been extensive measurements of the transport and Josephson properties of S-TI-S junctions and many of the features expected to result from MBSs in this system have been observed.

In order to show the suitability of these S-TI-S platforms for the purposes of quantum processing, it is required to explicitly demonstrate certain benchmark tasks. First and foremost, one requires the nucleation of MBSs and their detection of through various local probes (tunnel junctions, quantum dots, single-electron transistors) and interferometry techniques (critical current diffraction patterns, current-phase relation measurements). Here, threading magnetic flux through Josephson junctions and extracting the critical current modulation patterns provides a natural means for realizing both these aspects. Second, one requires the measurement of parity states encoded in pairs of these Majorana modes. The platform proposed here, in addition to allowing for charge sensitive measurements, such as proposed in the nanowire case, can reveal parity transitions through switches in the relative sign of the $4\pi$-periodic component of the Josephson current-phase relation that arises from MBS currents. Finally, non-Abelian rotations in the ground state manifold require the manipulation of at least four Majorana modes. Though this can be strictly shown through actual braiding of Majorana modes through their motion in physical space, a simpler set-up performs this in a non-universal way involves bringing two of the MBSs within each other’s proximity. Within the proposed architecture of networks formed by Josephson junctions, the motion of MBSs bound to Josephson vortices enables both kinds of braiding (exchange and hybridization).

The purpose of this work is to present an appropriate topological Josephson junction architecture that can demonstrate all these tasks crucial for a functional MBS-based quantum processing platform. By bringing together theory and experimental expertise, we design and model principles for nucleating and braiding MBSs in realistic geometries formed of the best candidate materials, informed by experiments being performed in tandem by some of the authors towards realizing the first steps of the design. In what follows, in Section II, we begin with a short summary of what the platform entails. In Section III, we provide a theoretical modeling of the extended topological Josephson junction, focusing on an effective one-dimensional description of the two dispersive Majorana states at the S-TI-S interface. We demonstrate and analyze cases where the application of flux results in multiple zero energy MBSs formed by localizing the dispersive modes along the junction. In Section IV, we calculate the modulation of the critical current as a function of applied flux that is sensitive to the interference of the Josephson supercurrents in the junction, which reveals the effect of the Majorana modes, specifically, node-lifting of odd nodes in comparison with the Fraunhofer diffraction patterns expected for uniform junctions with the usual sinusoidal current-phase relation. In Section V, we present schemes for performing non-Abelian rotations of the MBSs via exchange and hybridization. In Sections VI and VII, we turn to realistic experimental situations, discussing design and fabrication of the platform, and implementation of schemes for braiding, hybridization, and readout. Finally, in Section VIII, we recapitulate our proposal and findings in the context of the broader outlook for topological quantum processing.

II. S-TI-S JOSEPHSON JUNCTIONS AS A PLATFORM FOR MAJORANA BOUND STATE NUCLEATION AND MANIPULATION

Here we provide an overview of the proposed S-TI-S Josephson junction platform for nucleating and manipulating Majorana bound states (MBSs).

Platform architecture: The basic S-TI-S Josephson junction building blocks are made of superconducting islands deposited on top of a topological insulator to form single junctions and trijunctions consisting of three superconducting regions adjacent to each other, as shown in Figure 1. More complex architectures consisting of appropriate networks of junctions can be constructed from these building blocks. For instance, a typical repetitive network pattern could take on a honeycomb structure consisting of a lattice of hexagonal shaped superconducting regions. The architecture would integrate leads, electrodes, single-electron transistors, or microwave cavities depending on the manipulation and read-out schemes.
FIG. 1: Nucleation of Majorana bound states in S-TI-S structures: (a) Lateral S-TI-S Josephson junction in a magnetic field with MBSs at the locations of Josephson vortices, (b) trijunction in zero magnetic field with a single MBS in the center induced by appropriate adjustment of the phases on the electrodes, and (c) trijunction in a magnetic field with multiple MBSs.

Nucleating and identifying Majorana bound states: A simple method for nucleating MBSs involves applying a magnetic flux through a Josephson junction, which induces a gradient in the phase across the junction and a non-uniform supercurrent in the junction. As the magnetic field is increased, Josephson vortices enter the junction symmetrically from each side of the junction at zero applied current, with cores located at the points where the relative phase between the two superconductors is equal to $\pi$ or an odd multiple thereof. These vortices are evenly-spaced in a uniform junction, separated by a flux of one $\Phi_0$ threading the junction barrier, as shown in Figure 2. Localized MBSs are stabilized at these points, effectively bound to the Josephson vortices\cite{25,27}. Such bound states will form the basis of our proposed schemes. While applying a relative phase shift of $\pi$ between the three superconducting regions in the trijunction geometry of Figure 1(b) can also nucleate a MBS at the intersection of its junctions, such states will not be our focus here\cite{29}. As signatures of the flux-induced appearance of MBSs in extended junction geometries, we will show that the critical current diffraction patterns, which track critical current as a function of applied flux, exhibit characteristic features due to low-energy Majorana-mode contributions to the Josephson critical current.

Non-Abelian rotations via braiding and hybridization: A main focus of this work involves performing non-Abelian rotations related to the MBSs. Pairs of MBSs define electronic parity states which can either be occupied or not. The rotations are in this Hilbert space. We propose two schemes for performing rotation based on a theoretical framework developed to describe the localization of Majorana bound states in a magnetic field and their manipulation by local fields and currents. The first approach relies on applying a series of phase pulses in a trijunction S-TI-S device resulting in the exchange of Josephson vortices containing MBSs, resulting in braiding (Figure 3(a)). The second uses magnetic field pulses to control the separation of vortices in a single Josephson junction (Figure 3(b)), resulting in hybridization of MBSs that creates an energy splitting away from zero energy and an associated rotation in the Hilbert space of parity states. This scheme lacks the full topological protection of braiding, but is highly implementable in our geometry.

Parity readout: Either as a means of initializing parity states or doing readouts, determining the parity of electronic states shared by MBS pairs is a crucial ingredient. In the situation here, Josephson junction physics provides a natural way of determining such non-local parity–critical current switching measurements\cite{44}. As in the nanowire situation, other options involve coupling to quantum dot single-electron transistors (SET) or embedding the platform in a transmon geometry. In this work, we briefly survey the possibilities based on the capabilities of the Josephson junction architecture.

In the next section, we describe the theoretical principles behind modeling these S-TI-S junctions, obtaining critical current modulation patterns, and performing non-Abelian rotations.
III. MODELING S-TI-S JUNCTIONS

Here we begin our extensive treatment of the proposed S-TI-S architecture by describing a single extended Josephson junction and the low-energy dispersive Majorana modes that reside on the TI surface in the proximity-induced superconducting region between the superconductor electrodes. We review the manner in which localized MBS nucleate in the presence of applied flux. We then analyze in depth situations having multiple MBSs, calculating their energy spectra, wavefunctions, effect of a non-uniform magnetic flux, and hybridization arising from tunnel coupling of the Majorana modes.

A. Effective model of low-energy junction modes

The basic structure of the junctions studied here is as shown in Figure 3. It consists of a topological insulator slab with two superconducting islands deposited on its upper surface. We assume here that the TI slab is much thicker than coherence length of the proximity-induced superconductivity so that the Josephson supercurrents are confined to the top surface. We restrict ourselves to a well-established effective model that focuses on the low-energy states found along the junction interfaces.

Let us consider a S-TI-S system, similar to the one described in Ref. [27] having a line junction of width $W$ along $y$-axis as shown in Figure 3. The superconducting gap varies as: $\Delta(x) = \Delta e^{i\phi(y)}$ for $x > L/2$ and $\Delta(x) = \Delta e^{-i\phi(y)}$ for $x < -L/2$. A magnetic field pierces through the junction with flux $\Phi_B$. The flux leads to a spatial variation of the superconducting phase difference along the junction to vary as

$$\phi(y) = 2\pi y/l_B,$$

where $l_B = W\Phi_0/\Phi_B$. Here $\Phi_0 = h/2e$ is the flux quantum appropriate for paired superconductivity.

It can be shown [27] when the two SC islands are decoupled, at each S-TI interface there exists a dispersive Majorana mode at zero field, thus yielding a pair of counter-propagating states $\gamma_L$ and $\gamma_R$ in the barrier on the surface. (b) Applied magnetic flux through the junction creates a spatial gradient in the phase difference between the superconductors and, if sufficiently large, generates Josephson vortices that localize the Majorana modes into discrete Majorana bound states pairs at locations where the phase difference is an odd-multiple of $\pi$. The red dots indicate the location of the localized MBSs on the top surface of the junction. The other MBS of the pair is delocalized in this geometry. The situation is modeled by Eq. [2].

$$H = i\hbar v_M(\gamma_L \partial_y \gamma_L - \gamma_R \partial_y \gamma_R) + i\Delta \cos(\phi(y)/2) \gamma_L \gamma_R$$

with $v_M = v[\cos(\mu W/\mu e) + (\Delta/e)\sin(\mu W/\mu e)]/(\mu + \Delta)^2$, where $v$ is the velocity corresponding to the edge state of the TI and $\mu$ is the chemical potential. For a S-TI-S junction of Al-Bi$_2$Se$_3$-Al the estimated values are $v = 10^5 m/s^{-1}$, $\Delta = 150 \mu eV$, $\mu = 10 neV$. The energy gap is an order of magnitude larger for Nb electrodes often used in experiments.

The form of Eq. [2] respects the Dirac equation for a massive particle, where the gap function $\Delta \cos(\phi(y)/2)$ represents a spatially varying mass function. For a linear variation of the flux-dependent $\phi(y)$, the gap function too can be linearized around regions where $\phi(y)$ crosses an odd integer multiple of $\pi$. In this case, there exists a zero-energy eigenstate that shows exponential decay away from the crossing point. This eigenstate has...
the appropriate linear combination of $\gamma_R$ and $\gamma_L$ such that the desired Majorana bound states (MBSs) are real functions. As the magnetic field piercing through the junction is increased, the number of zeros of the gap function increases, thus capturing more number of Majorana modes in the junction. A new Majorana mode appears with the incremental change of the net flux by one quantum, thus confining one Majorana bound state per one Josephson vortex.

A few comments are in order here with regards to several simplifying assumptions made in this model. Here we assume the dimension $L$ to be small. The profile for the phase variation will in general be altered in realistic situations having thicker width. For example, one could consider an extended Josephson junction in which the gap function shows a $\tanh$-like spatial variation yielding a Josephson soliton. Here too, respecting the generic change of sign in the gap function, it can be shown that there exists a Majorana bound state.\(^\text{(29)}\) Another issue is that the physical Hilbert space requires that the MBSs appear in pairs while in our continuum model on the surface, it is possible to obtain a single MBS. In Ref.\(^\text{(27)}\) the full three-dimensional nature of the system is taken into account and it is assumed that the partner of a single MBS is at the bottom surface of the TI. For this to hold, the induced superconducting penetration length within the TI ought to be much greater than its thickness. Here, we consider the opposite limit. We thus expect that for an isolated MBS in the junction, there exists a partner that is delocalized, likely extended along the periphery of the superconducting islands. We now turn to a detailed analysis of the MBS within the context of our model, employing numerical simulations.

### B. Multiple vortices and numerical analysis

Here we analyze the situation in which the applied flux is strong enough to generate multiple vortices and MBSs. In particular, we study the case of four MBSs present along the junction; such a situation is the minimum necessary for quantum information protocols. Through numerical simulation of the model presented in the subsection above, we show the explicit realization of these MBS states, their mid-gap spectral properties, and the manner in which these features can be controlled by altering the local phase profile.

Our numerical technique is straightforward in discretizing the low-energy degrees of freedom given in Eq.\(^\text{(2)}\). These Majorana fermion states are thus confined to a one-dimensional lattice having hard boundary conditions. The Hamiltonian can thus be represented in matrix form, consisting of the kinetic term and the coupling mediated by the spatially varying gap function. The eigenvalues obtained from diagonalizing the matrix thus correspond to discrete low energy states. We note here that for these Dirac-like models, discretization results in a ”fermion doubling” issue\(^\text{(25)}\) we are thus left with taking into account only half the eigenstates as a true representation of the spectrum.

We first consider the instance where the phase variation in Eq.\(^\text{(2)}\) varies linearly and increases from $-\pi N/2$ to $\pi N/2$ as the coordinate along the junction, $y$, spans the junction from $-W/2$ to $W/2$ and $N$ is the number of flux quanta. This situation encompasses four half-flux quanta within the junction, which ought to lead to four MBSs. We explicitly ascertain this MBS distribution and related features by numerically diagonalizing the Hamiltonian in Eq.\(^\text{(2)}\). Figure\(^\text{6}\) shows the numerical results.

For this case of four flux quanta piercing the junction, Figure\(^\text{5}(a)\) shows the variation of the gap function along the junction. Correspondingly, Figure\(^\text{5}(b)\) shows the energy spectrum. Most energy states lie outside a gap region centered around zero energy. As expected, four states however are mid-gap states effectively at zero energy. Our analyses also show that with increasing flux, the formation of new MBSs occurs through select states lying outside the gap entering the gap region and nucleating towards zero energy. Plotting the eigenstates of the corresponding wavefunctions in Figure\(^\text{4}(b)\) indeed shows them to be isolated, evenly spaced, bound states localized along the junction at the zeroes of the gap function. Each of the bound states shows exponential decay in isolation. Moreover, the eigenfunction is completely real, making it of the Majorana form. The MBS wavefunction at a distance $\delta y$ away from its center respects the form

$$\gamma(y) \approx f(x)e^{-|\delta y|/\lambda_M}.$$ \(^\text{(3)}\)

Here, the decay length is characterized by $\lambda_M = \sqrt{\hbar v_M/|B|}$ and $f(x)$ describes the confinement of the MBS in the transverse direction.

The MBSs are effectively isolated when their separation is significantly greater than their decay length. However, when brought closer, a pair of MBSs becomes coupled due to the overlap in their wavefunctions.\(^\text{(29)}\) This coupling between the neighboring MBSs, say $\gamma_a$ and $\gamma_b$ separated by distance $L_{ab}$, leads to an effective Hamiltonian of the tunneling form

$$H_{ab} \approx it_{ab}\gamma_a\gamma_b, \quad t_{ab} \approx e^{-L_{ab}/\lambda_M}.$$ \(^\text{(4)}\)

This is due to the overlap of the wavefunctions of the two MBS in the junction across the distance $L_{ab}$. This coupling results in a tunnel splitting between the degenerate zero energy states associated with the MBS pair.

The proposed S-TI-S architecture here hinges on the ability to move and couple MBS pairs. Our proposed schemes for such manipulation primarily involve changing the local phase variation. As an example, consider the case of two MBSs initially far apart, as shown in Figure\(^\text{6}\). Now, changing the local phase more rapidly between the two MBSs, as shown in Figure\(^\text{6}(a)\), decreases their separation, as shown in Figure\(^\text{6}(b)\). The inset in Figure\(^\text{6}(a)\) shows that the MBSs have come close enough to result in a numerically discernible tunnel splitting.

Such controlled MBS mobility and tunable coupling are essential ingredients in braiding schemes considered...
The application of a magnetic field leads to variation of the phase difference along the Josephson junction and the gap function. The gap function, plotted as a function of distance along the junction, goes to zero when the SC phase difference crosses multiples of $\pi$. Corresponding to every spatial location where the gap function goes to zero, there exists a localized Majorana mode - the corresponding wavefunction profiles are shown here. (b) The spectrum of Andreev bound states (in units of $\hbar v_M$) obtained from the diagonalisation of model Hamiltonian Eq. 2 for the given gap function profile. The mid gap states correspond to the MBS.

FIG. 5: (a) The application of a magnetic field leads to variation of the phase difference along the Josephson junction and the gap function. The gap function, plotted as a function of distance along the junction, goes to zero when the SC phase difference crosses multiples of $\pi$. Corresponding to every spatial location where the gap function goes to zero, there exists a localized Majorana mode - the corresponding wavefunction profiles are shown here. (b) The spectrum of Andreev bound states (in units of $\hbar v_M$) obtained from the diagonalisation of model Hamiltonian Eq. 2 for the given gap function profile. The mid gap states correspond to the MBS.

IV. JOSEPHSON SUPERCURRENTS AND SIGNATURES OF MBS STATES

Having established the description of the S-TI-S Josephson junction, we are now in a position to derive the form of Josephson currents across the junction and the crucial role played by MBS contributions. In semiconductor nanowire systems, the zero-bias conductance peak in transport through the end of the wire is a signature of the presence of MBSs. Here, we propose that the onset of a $4\pi$-periodic component in the Josephson current-phase relation, revealed by characteristic features in the critical current modulation patterns in a magnetic field, play a similar role in providing an indication of the presence of MBSs.
A. Josephson Interferometry

Josephson junctions are an important class of superconducting devices that can be extensively probed using various electrical circuit analysis methods. They are composed of two superconducting islands separated by a barrier made of an insulator, normal metal, or in our specific case, a topological insulator in which conductance through the topological surface states plays a key role. The defining features of these junctions are captured by specific relationships among the gauge-invariant phase difference $\phi$ across the junction, the voltage across the junction $V$ and the supercurrent though the junction $I_s$: (i) the Josephson supercurrent $I_s$ is related to the phase via the current-phase relation. In an ordinary Josephson junction, $I_s = I_c \sin \phi$, where $I_c$, the critical current, is the maximum current that the superconductor can sustain, and (ii) a voltage causes a rate of change in phase given by $d\phi/dt = 2\pi V/\Phi_0$, where $\Phi_0 = h/2e$ is the flux quantum. If a magnetic field is penetrating through the junction, the gauge invariant phase difference across the superconducting islands is given by:

$$\phi = \phi_1 - \phi_2 - 2\pi/\Phi_0 \int A \cdot dA.$$ Here, $A$ represents the vector potential associated with the applied field. The junction can be characterized by a free energy that depends on the SC phase difference $F(\phi) = -I_{C0}(\Phi_0/2\pi) \cos \phi$. The current-phase relation can be simply derived by taking a derivative of the free energy with respect to the phase.

When a uniform magnetic field is applied to a uniform junction with a sinusoidal CPR, the maximum of the supercurrent varies with magnetic field according to a Fraunhofer pattern familiar from single-slit optical diffraction:

$$I_C(\Phi) = I_{C0} \left[ \frac{\sin(\pi \Phi/\Phi_0)}{\pi \Phi/\Phi_0} \right],$$

where $\Phi$ is the total flux passing through the junction. Josephson interferometry, the measurement of the maximum supercurrent of Josephson junctions vs. magnetic field, has played an important role in determining the pairing symmetry of unconventional superconductors such as d-wave and p-wave superconductors. This technique is also sensitive to deviations from a sinusoidal current-phase relation, inducing changes from the conventional Fraunhofer diffraction pattern.

In Josephson junctions that harbor MBSs, such as the S-TI-S junctions under study here, the current-phase relation is altered by the addition of a 4$\pi$ periodic contribution to the CPR. This arises because although conventional Josephson junction processes involve only tunneling of Cooper pairs across the junctions and hence 2e-processes, junctions with MBSs also exhibit single-electron processes that lead to such fractional Josephson phenomena. The CPR thus acquires an additional current contribution $I_{g,I_s} = I_{Mc} \sin(\phi/2)$. Here, based on the model established in Sec. III, we evaluate the manner in which MBS states contribute to the supercurrent and modify the critical current diffraction pattern in a characteristic way.

B. Derivation of MBS contribution to Josephson current

To obtain the MBS contribution to the supercurrent, we first diagonalize the effective Hamiltonian in Eq. 2 as in Sec. III. The ground state energy of the system is given by summing over all the negative energy states in an applied magnetic field $B_z$:

$$E_g(\phi_0, B) = -\sum_{E_j > 0} E_j(\phi_0, B).$$

The phase $\phi_0$ is the reference phase set in the experiment either at the ends or in the middle of the junction. It appears in the phase profile as

$$\phi(y) = 2\pi y/l_B + \phi_0.$$ The total number of MBSs localized within the junction region depends on the net flux piercing through, i.e. $\Phi = B L d$ The super-current is then given by

$$I_{Ms}(\phi_0) = \partial_{\phi_0} E_g(\phi_0, B).$$

The critical current provided by the MBS is obtained by maximizing the supercurrent contribution above, i.e.,

$$I_{Mc}(\phi_0, B) = \max_{\phi_0} I_{Ms}(\phi_0, B).$$

The signature feature of Majorana physics in the supercurrent, as we shall see, is the 4$\pi$ periodicity; this behaviour can be traced back to $\cos(\phi/2)$ variation of $\phi$ in Eq. 2.

While this approach is able to capture the key feature of dispersive Majorana modes and MBS physics, experimental settings could be more complex. One could have a complicated current-phase relationship in the junction due to various factors such as the nature of the non-superconducting region and the link, dispersive modes, and scattering. Alternative derivations of the Josephson current can incorporate features such as inhomogeneities by assuming that the extended junction is a combination of multiple node-junctions in parallel and summing over their contributions from each point along the junction. Our model has yet to reconcile this approach with MBS physics and its non-local features. We still, however, have enormous flexibility to incorporate disorder, unconventional current phase relationships, and non uniform magnetic field distributions.

C. Signatures of Majorana bound states in diffraction patterns

Critical current diffraction patterns measure the maximum supercurrent current of Josephson devices as a function of applied magnetic field and are one of the most powerful methods to characterize their properties. The presence of magnetic flux threading the junction barrier creates a continuous variation of the superconducting
phase difference across the width of the junction. This
results in interference of the supercurrents at different
points along the junction having different phases and
leads to a diffraction pattern. Diffraction patterns that
deviate from the Fraunhofer form \( I_{\text{Co}}(\sin(\pi \Phi/\Phi_0)) \) familiar from single-slit optical interference can reveal valuable
information about the junction structure and the mecha-
isms that carry supercurrent. In this case, we will use
this to probe the MBSs and associated single-electron
tunneling processes they enable.

We employ the procedure outlined above to numeri-
cally evaluate the diffraction in the S-TI-S junction; the
results are shown in Figure 7. As the flux through the
total junction is varied, the number of vortices in the
junction changes as does the corresponding number of
MBSs. In addition to the Majorana contribution to the
supercurrent, there is a regular Cooper pair contribution.
We have included this contribution in the simulation by
adding a \( \sin \phi \) term in the CPR. In Figure 7(a), the inset
on the right shows the MBS contribution and the Cooper
pair contribution to the supercurrent. It can be seen that
the oscillations from the MBS are at double the period of
the Cooper pair as a function of applied flux, reflecting
single electron processes (and associated flux period \( h/e \))
versus Cooper pair processes (and associated flux period
\( h/2e \)).

The main panel in Figure 7(a) shows the critical cur-
rent calculated for the combined Majorana and Cooper
pair processes, which are separately shown in the inset.
Both contributions vanish at the locations of the even-
valued nodes in the conventional Fraunhofer patterns,
but Majorana modes contribute supercurrent at the odd-
valued points, lifting those nodes. This node lifting is
seen more distinctly in the logarithmic plot of Figure
7(b). This feature is a characteristic signature of MBSs
localized within the junction.

Connecting with our prior comments on alternative
approaches, here we derive the Majorana contribution
in terms of energy eigenstates and thus our incorpora-
tion of spatial dependence is only indirect through the
wavefunction profiles shown in the previous section. As
a result, this model predicts a uniform \( \sin\phi/2 \) con-
tribution to the current-phase relation from the Major-
ana bound states. As has been done with conventional
Josephson junctions, more physical modeling would treat
the current-phase relation as a local relationship and ex-
plicitly confine local currents in the vicinity of the Major-
ana bound states confined to Josephson vortices. That
approach has been carried out in interpreting the exper-
imental data on S-TI-S junctions and predicts features in
the critical current diffraction patterns at magnetic
field at which MBSs enter and leave to junctions, in good
agreement with experiment. Such detailed agreement
is important in unequivocally verifying the existence of
MBSs in this system but it is not critical for the discus-
sions of braiding that we address here.

We also note that the results shown in Figure 7 are
merely proof-of-principle. A primary consideration is the
magnitude of single electron tunneling to that of Cooper
pair tunneling. Here, an arbitrary value was chosen. The
actual values depend on junction and interface prop-
ties and could widely vary between different experimental
settings. Nevertheless, as long as the Majorana contribu-
tion is even a small discernible fraction, the node lifting
feature would be measurable. Experiments have indeed
observed such features and allow determination of
those details.

As with nanowire systems, we acknowledge that there
are subtleties in interpreting the zero-bias peaks as a
definitive signature of MBS. In the nanowire case, a
zero-bias conductance peak is the hallmark of MBSs; in
principle, other bound states due to disorder, reso-
nances etc. that contribute to single electron processes could lead to spurious effects. Similarly in our system of S-TI-S junctions such single electron processes could provide a challenge in definitively pinpointing the contribution of MBS to the critical current diffraction patterns. As with the nanowire system, here too, combining signatures of single-electron processes in diffraction patterns with demonstration of braiding and associated parity switches would constitute the definitive evidence for Majorana fermion physics.

V. NON-ABELIAN ROTATION IN THE GROUND STATE MANIFOLD OF MAJORANA MODES

The primary building blocks for non-Abelian rotations and braiding within the Josephson junction architecture are contained in Figure 1. The elements forming the basis of these non-Abelian rotation are the MBSs localized at Josephson vortices. Here we briefly outline the underlying principles, discussing the relevant Hilbert space, operations, and the physical manifestations associated with these rotations.

A. Braiding through physical motion of the MBSs

The simplest instance of braiding through involves four MBSs, say denoted by $\gamma_1, \gamma_2, \gamma_3, \gamma_4$. Any pair of MBSs forms a Dirac (electronic) state that can be occupied or not. As a specific choice, consider $c_A = (\gamma_1 + i\gamma_2)/2$, $c_B = (\gamma_3 + i\gamma_4)/2$. In the absence of coupling between the MBSs, the degenerate ground state manifold is spanned by

$$|N_A, N_B\rangle : |0, 0\rangle, |1, 1\rangle, |1, 0\rangle, |0, 1\rangle$$

where $N_A, N_B$ denote the occupation of the electronic states. For $N$ pairs of MBSs, the ground state is $2^N$ fold degenerate. The occupation of all such parity states decides the net fermionic parity of the ground of the system. Thus unlike conventional superconducting ground state, which is always a superposition of states having an even number of electrons in form of Cooper pairs, a topological superconductor can have states with net fermion parity to be either even or odd.

The simplest braiding operation is an exchange in the positions of the two MBSs. How does this exchange in the position space affect the space of ground state? It can be shown that the exchange of two Majoranas $\gamma_i, \gamma_j$ is represented in the ground state manifold as a unitary rotation in the space $\{ |0, 0\rangle, |1, 1\rangle, |1, 0\rangle, |0, 1\rangle \}$ given by

$$U_{ij} = exp(\pm i\pi \gamma_i \gamma_j / 4).$$

For example, if we start with a state $|0, 0\rangle$, then exchanging $\gamma_2, \gamma_3$ results in

$$U_{23} |0, 0\rangle = (|0, 0\rangle - i |1, 1\rangle) / \sqrt{2}$$

In principle, one can track such rotations by measuring the fermion occupation i.e the fermion parity of the electronic states in the ground-state manifold. The order of consecutive exchanges matter as the unitary operations do not commute: $U_{12}U_{23} \neq U_{23}U_{12}$. Thus the name non-Abelian rotation.

The actual implementation of such rotations in our proposed architecture involves sequences of vortex motion such as shown in the trijunction geometry of Figure 3. In the next section, we provide the exact experimental steps to perform these sequences. We remark here that these sequences are in the spirit of the original proposal by Ivanov13 for performing exchange operations.

B. Effective braiding through tuning MBS coupling

A key feature of the topological qubit formed by the electron parity state is its non-locality. It is shared by two MBS states confined to vortices that can be very far apart. We have seen the manner in which physical exchange results in non-Abelian rotations in the Hilbert space of these parity states. An alternate method for performing non-Abelian rotations without physical exchange involves tuning the coupling between an MBS pair.

As a specific example, consider four MBSs $(\gamma_1, \gamma_2, \gamma_3, \gamma_4)$, this time with their vortex cores aligned along a junction, as in Figure 3(b). The effective low energy Hamiltonian of this system, as discussed in Sec. III, is given by

$$H_{12} = it_{12}\gamma_1 \gamma_2 + it_{23}\gamma_2 \gamma_3 + it_{34}\gamma_3 \gamma_4.$$  (13)

Here, MBSs $\gamma_1, \gamma_2$ are coupled with strength $t_{12}$, MBSs $\gamma_2, \gamma_3$ with strength $t_{23}$, and $\gamma_3, \gamma_4$ with $t_{34}$ as shown in Figure 3. Now let us denote the non-local electronic states by $\Gamma_1 = (\gamma_1 + i\gamma_2)/\sqrt{2}$ and $\Gamma_2 = (\gamma_3 + i\gamma_4)/\sqrt{2}$. The occupation of these modes is given by $N_1 = \Gamma_1 \Gamma_1^\dagger$ and $N_2 = \Gamma_2 \Gamma_2^\dagger$. As with the exchange braiding case, the Hilbert space of the system is given by the occupation of these 2 states $|N_1, N_2\rangle : |0, 0\rangle, |1, 1\rangle, |1, 0\rangle, |0, 1\rangle$. The Hamiltonian in this Hilbert space is then block diagonal. We focus only on even parity block corresponding to $|0, 0\rangle, |1, 1\rangle$; the odd parity block is decoupled and contains analogous physics. In the reduced two-component basis of even parity states, the tunnel coupled Hamiltonian of Eq. 13 takes the form

$$H_{e12} = \begin{pmatrix}
  t_{12} + t_{34} & t_{23} \\
  t_{23} & -(t_{12} + t_{34})
\end{pmatrix}$$

Treating the two states of the Hilbert space $|0, 0\rangle, |1, 1\rangle$ as the “spin-up” and “spin-down” eigenstates of Pauli matrix $\sigma_z$ respectively, we can cast the Hamiltonian in terms of Pauli matrices as:

$$H_{e12} = (t_{12} + t_{34})\sigma_z + t_{23}\sigma_x$$  (15)

Preparing the system in an initial state, say ”spin up” $|0, 0\rangle$ and then changing the $t_{23}$ (“a transverse field”)
would result in the rotation of the state in the spin basis. Effectively, changing the coupling in two Majorana modes $\gamma_2, \gamma_3$ would induce non-local parity correlations. It has been explicitly shown that these rotations are equivalent to braiding operations we discussed in the previous section.\textsuperscript{41}

Specific sequences of such effective braiding would involve preparing the system in a prescribed initial state in the degenerate Hilbert space, bringing a pair of MBSs to break the degeneracy via coupling, and time evolving the initial state in a manner prescribed by Eq. 13. The time scale for varying the coupling is set by the maximum degeneracy splitting; compared to actual braiding, which involves the robust topological operation of exchange, this time scale dependence poses a limitation. Nevertheless, given enough experimental control and knowledge of tuning parameters, qubit operations can be made viable through this procedure.

VI. EXPERIMENTAL IMPLEMENTATION OF THE LATERAL S-TI-S JOSEPHSON JUNCTION ARCHITECTURE

Having laid out the principles of the proposed platform, we now turn to actual implementation. Currently, several experimental groups have begun to actively study S-TI-S junctions for prospective MBS realizations and manipulations, including a subset of authors of this work.\textsuperscript{36–38,52,53} As the setting we know best for its strengths and challenges, here we survey the current status of experiments performed by our group\textsuperscript{40} as directly relevant to the proposed platform. This survey sets the stage for the next section designing an implementation of the schemes proposed above in this setting.

A. Design and fabrication of S-TI-S lateral Josephson junctions

Our devices are lateral Josephson junctions made by depositing superconductor electrodes on the surface of the topological insulator. Our experiments to date have used both exfoliated crystals and MBE-grown films of Bi$_2$Se$_3$ for the topological insulator, and Nb for the superconductor electrodes deposited on the top surface. We have found that this combination produces reliable junctions with good uniformity of the critical current, and requires only ion-milling of the surface to produce good electrical contacts. We use electron-beam lithography and a lift-off process to fabricate junctions with barriers 100nm-300nm long (along the current direction), chosen to give reasonable Josephson coupling. With typical widths of 1\,\mu m-3\,\mu m, the critical currents range from 10nA-10\,\mu A. The topological insulator barrier can be gated with a top-gate electrode separated with an Atomic Layer Deposition (ALD) grown dielectric of Al$_2$O$_3$ or HfO$_2$ in order to tune the carrier density and adjust the location of the Fermi level.

For single junctions, we use the parallel electrode configuration shown in Figure 8. This geometry provides a uniform barrier width and minimizes flux-focusing of the applied magnetic field, allowing us to maintain a uniform field in the junction and reduce trapped magnetic vortices in the electrodes. In this geometry, the effective magnetic width of the junction (junction barrier length plus twice the magnetic penetration depth of the Nb electrodes) is of order 1\,\mu m, so the applied magnetic fields needed to populate the junction with 4 vortices/MBSs is a few mT, a range easily accessible with superconducting Helmholtz coils.

B. Experimental indications of topological surface states and Majorana modes

Measurements of the transport and Josephson properties of S-TI-S junctions reveal a number of signatures consistent with the formation of topological surface states and Majorana bound states.\textsuperscript{40} There is evidence that the Josephson current is primarily carried by surface states on the top surface between the electrodes from the comparison of the sensitivity to top and bottom gating, and from critical current diffraction pattern measurements with the magnetic field applied laterally (parallel to the surface) between the electrodes. It is observed that top-gating induces a sharp drop in the critical current at a particular negative voltage which arises from the depletion of the trivial surface states.\textsuperscript{35,36} This phenomenon can be understood as a transition in the vertical location
of the topological surface state from below the trivial surface states to the top surface with gating, resulting a reduced critical current.

Direct measurements of the Josephson current-phase relation of our devices show a pronounced skewness, indicating higher-order harmonics arising from high-transparency states that should dominate near locations in the junction where the phase difference is near an odd-multiple of \( \pi \) and the Andreev bound state energy gap closes. Most significantly we observe a lifting of the odd-numbered nodes in the critical current, evidence for a \( \sin(\phi/2) \)-component in the junction current-phase relation carried by the Majorana bound states that nucleate at the same locations. This magnitude of the node-lifting is independent of a top-gate voltage, suggesting that the critical current at the expected node location comes from the topological states. We also observe kinks in the diffraction patterns at the magnetic fields at which Josephson vortices enter the junctions, further evidence for these current-carrying Majorana states.

None of these observations is conclusive evidence for Majorana modes, which only demonstrating braiding induced parity transitions with non-Abelian statistical behavior can provide. However, it does give justification to our picture and point-the-way to the braiding schemes we propose that are designed to provide a definitive verification of Majorana states.

### VII. IMPLEMENTATION OF SCHEMES FOR BRAIDING AND HYBRIDIZATION

We have proposed two schemes for braiding MBSs, both based on the ability to manipulate MBSs by moving the Josephson vortices to which they are bound in networks of S-TI-S junctions. This can be done easily and controllably by applying a combination of magnetic fields and phase biases (via currents) to the junctions. For a uniform junction structure, in which the separation of the superconducting electrodes is the same along the width of the junction, in a uniform magnetic field perpendicular to the direction of the supercurrent, the separation of MBSs/vortices is set only by the magnitude of the field, with one quantum of flux threading the junction between each adjacent pair of MBSs. This assumes that supercurrents in the junction do not generate significant magnetic fields, the so-called short junction limit which is satisfied in the proposed devices because of the small magnitude of their supercurrent densities. Figure 2(b) shows the location of vortices/MBSs in the junction as a function of field — with no currents in the junction the vortices enter the junction symmetrically from the edges as the field is increased. The location of the vortex chain within the junction can be shifted by applying a current through the junction that induces a phase drop across the junction. If the critical current is exceeded, a voltage is induced across the junction. This causes the vortices to move across the junction as the phase winds according to the Josephson relation. This provides a way to move the MBSs (at very fast speeds > 1km/s), but it also generates quasiparticles that could cause undesirable parity transitions.

#### A. Non-Abelian rotations by exchange

In a trijunction consisting of three Josephson-coupled superconducting islands on a topological insulator film, as in Figure 9, MBSs nucleate at the locations where the phase difference is an odd multiple of \( \pi \), i.e. at the cores of Josephson vortices. As we have described above, for a uniform structure in a uniform applied magnetic field, the separation of MBSs/Josephson vortices is set only by the applied field, and the vortex pattern is symmetric in the absence of applied currents. However, this pattern can be manipulated by adjusting the relative phases of the islands, changing the location of the chain of vortices in each segment of the trijuction.

One approach is to apply a sequence of Rapid Single Flux Quantum (RSFQ) voltage pulses across the junctions. Such pulses apply an integrated flux of one flux quantum, changing the phase across the junction by \( 2\pi \) and shifting the vortices by one vortex spacing. They can be be generated by Josephson junction circuitry developed for superconducting logic technology.

In Figure 9, we show a sequence of RSFQ pulses that effects an exchange braiding operation of two MBSs. In junctions in which the \( I_c R \)-product is of order 1\( \mu \)V, typical RSFQ pulses are of duration of order 1ns, so braiding operations can be accomplished at GHz frequencies. The disadvantage is that the finite voltage pulses will create heating and generate quasiparticle excitations that can cause quasiparticle poisoning which limits the parity lifetimes, degrading speed and performance.
A better technique can be achieved by shorting two of the junctions with superconducting loops and coupling flux into the loops via applied currents to induce phase differences, as in Figure 10(a). Note that the multiply-connected geometry allows us to access all possible phases across the junction so that we have complete control of the vortex locations without exceeding the critical currents and generating quasiparticle excitations.

In the proposed system, we can use this scheme to braid (exchange) pairs of MBSs, as illustrated in Figure 10. Winding the relative phase of island 1 by $2\pi$ shifts all of vortices in the top and left junctions by one vortex spacing; repeating this for islands 0 and then 2 returns a vortex pattern with two vortices exchanged.

As described in previous sections, in both schemes, the MBSs in each segment of the junction form parity qubits; the exchange operation corresponds to a braiding rotation in the basis of these qubit states, which should result in a change in the parity of the Majorana pair. The principle behind this operation is the same as that proposed for braiding MBSs in semiconducting nanowires in the T-junction geometry\cite{2}.

B. Non-Abelian rotation by hybridization

As also described in previous section, we can alternatively carry out non-Abelian operations in a single S-TI-S Josephson junction, by bringing them close together resulting in hybridization and level splitting. As a physical implementation of such a scheme, an applied field creates a row of Josephson vortices evenly spaced by one flux quantum threading the junction, as in Figure 11. A current pulse applied to a narrow loop of wire crossing the junction generates a localized increase in the magnetic field in a region between the vortices, bringing the vortices closer together. This in turn changes the distance between the MBSs, creating an overlap of the wavefunctions of the MBSs and inducing a parity flip through hybridization. Such controlled dynamic coupling constitutes the realization of an alternate scheme. The related non-Abelian operation depend on the magnitude of the energy level splitting in their time evolution. They thus are not robust unlike the action of exchange.

VIII. IMPLEMENTATION OF PARITY READOUT SCHEMES

A key measurable consequence of MBS based non-Abelian operations is the associated non-local fermion parity transitions. To demonstrate the occurrence of those transitions requires developing fast and high fidelity schemes to read the parity associated with specific MBS pairs. Although that is not the primary subject of the work presented here, we will comment on how multiple viable schemes for achieving this. These include:

1. Measurements of the critical current distribution of S-TI-S Josephson junctions performed by ramping the applied current and recording the value at the onset of a finite voltage across the junction. The supercurrent has two contributions, one from the Cooper pair and one from the MBS. The parity states of MBS pairs results in the splitting of the critical current distribution into a bimodal distribution\cite{58}. This approach is straightforward but is an intrinsically dissipative process, potentially contributing to quasiparticle generation and resulting parity transitions, so-called quasiparticle poisoning.

2. Coupling the junction to a microwave resonator and detecting the shift in the resonant frequency that depends on the energy-splitting from the Majorana fermion parity states. This non-dissipative readout scheme is the basis of the measurements of transmon superconducting qubits\cite{19}. Microwave induced-qubit transitions involve a basis of states composed of multiple Cooper pairs. Here
In conclusion, we have presented an extended topological Josephson junction based platform for nucleation of MBSs and their manipulation towards an eye for quantum processing. Our proposed platform provides an alternative for those based on topological superconducting nanowires. Well-established phase-sensitive measurements, such as those using SQUIDs, form versatile probes of the system. The technique of magnetic field piercing through the junctions, the key to nucleating vortices which harbor MBSs, is standard practice evolved over decades for conventional superconductors. These vortices have added advantage of mobility. By meticulously engineered protocols of applying currents, voltages and local magnetic fields, the motion of MBSs can be realised in controlled ways.

Here, building on these excellent capabilities of extended topological Josephson junctions, we have propose a series of steps to demonstrate the feasibility of these platforms for performing MBS-based exchange braiding and quantum operations. As parallels to proposals in nanowire systems, we have identified two kinds of junction geometries for performing non-Abelian operations in the Hilbert space of parity qubit states associated with four MBSs. First, the trijunctions are a viable alternative to T-junctions for performing the exchange braiding. Second, effective braiding/fusion of MBSs can be achieved in linear junctions threaded by perpendicular magnetic fields. By tuning the local magnetic field, the coupling between neighbouring Majorana modes can be tuned and this coupling can be used to perform an effective non-Abelian rotation. We have also mentioned a few accompanying schemes for initializing parity states and performing readouts, essential for quantum processing. They range from observing characteristic Fraunhofer patterns in the Josephson junction to quantum-dot sensing to transmon-based measurements. While the lofty goal of performing topological quantum computational operations may not be accessible in the near future, a proof-or-principle Josephson junction measurement suggesting the existence of MBSs, we find, is within sight and would go a long way.

In this work we have proposed the S-TI-S junctions as a platform for realisation and manipulation of Majorana bound states. While nanowire systems have enjoyed about a decade's worth of attention, there have been several theoretical and experimental efforts in establishing S-TI-S systems as a viable alternative to realise topological superconductivity. We have shown in this work that S-TI-S junctions offer several advantages stemming from extended Josephson junction physics. Manipulation of the superconductor phase difference profile is a natural component in extended Josephson junction systems. Associated magnetic fields are of much lower magnetic fields compared to current requirements for realizing topological superconducting nanowires. Well-established phase-sensitive measurements, such as those using SQUIDs, form versatile probes of the system. The technique of magnetic field piercing through the junctions, the key to nucleating vortices which harbor MBSs, is standard practice evolved over decades for conventional superconductors. These vortices have added advantage of mobility. By meticulously engineered protocols of applying currents, voltages and local magnetic fields, the motion of MBSs can be realised in controlled ways.
ORIZATION of quantum computational schemes and quantum information processing would undoubtedly require investigating multiple routes. The platform is amenable to scaling beyond the first steps of nucleating and non-Abelian rotations proposed here. These steps would include measurement of non-local Majorana correlations, integrating multiple circuits and qubits, and forming hybrid systems coupling to conventional qubits, given that MBSs alone cannot form a universal quantum computer. As with nanowire systems, it would be crucial to recognize exactly which circumstances enjoy topological protection and which ones are prone to dissipation and decoherence, be it from quasiparticle poisoning, finite temperature effects, or other factors. While these are all longer-term goals, unequivocal evidence of the existence of Majorana fermions as ascertained by their braiding properties seems to be well within reach in the near future, given the characteristic zero-bias conductance peak signatures in nanowires and signatures of $4\pi$–periodicity of the current-phase relation seen in the critical diffraction patterns in extended Josephson junctions. At the fundamental level, with Majorana’s initial postulation of a particle being its own antiparticle dating back to the 1930s, while at the quasiparticle level as opposed to the elementary particle level, a realization of such a quantum entity is a tour de force step in and of itself.

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