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Implications of WMAP observations on Li abundance and stellar evolution models

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ABSTRACT

The WMAP determination of the baryon-to-photon ratio implies, through Big Bang nucleosynthesis, a cosmological Li abundance larger, by a factor of 2 to 3, than the Li abundance plateau observed in the oldest Pop II stars. It is however inescapable that there be a reduction by a factor of at least 1.6 to 2.0 of the surface Li abundance during the evolution of Pop II field stars with \([\text{Fe/H}] \leq -1.5\). That the observed Li be lower than cosmologically produced Li is expected from stellar evolution models. Since at turnoff most of the Li abundance reduction is caused by gravitational settling, the presence of \(^6\text{Li}\) in some turnoff stars is also understood. Given that the WMAP implications for Li cosmological abundance and the Li Spite plateau can be naturally explained by gravitational settling in the presence of weak turbulence, there appears little need for exotic physics as suggested by some authors. Instead, there is a need for a better understanding of turbulent transport in the radiative zones of stars. This requires simulations from first principles. Rather strict upper limits to turbulent transport are determined for the Sun and Pop II stars.

Subject headings: globular clusters: general — globular clusters: individual

1. Astrophysical context

Most low metallicity ([Z/H] < -1.5) halo field stars with \(6300 > T_{\text{eff}} > 5500\) K have nearly the same surface Li abundance (Spite & Spite 1982). This result has frequently been confirmed since and has lead many to suggest that those stars have the cosmological Li abundance.
In stellar evolution models however, one must include all physical processes that follow from first principles, including gravitational settling which leads to a reduction in the surface Li abundance; deeper in, nuclear reactions destroy Li. This combination, together with the similarity of the Li concentration in stars of different $T_{\text{eff}}$ and $[Z/H]$, led Michaud et al. (1984) to conclude that “the measured lithium abundance is not exactly the original abundance. It must have been reduced by a factor of at least 1.5 and more probably 2, from the abundance with which the star formed.” (p. 212).

On the other hand, one may argue that, with the measurements of WMAP, the cosmic microwave background determines the baryon–to–photon–ratio, $\eta$, (Cyburt et al. 2002) so that Big Bang models predict the original Li abundance (Cyburt et al. 2003). This predicted original Li abundance is a factor of 2–3 larger than the Li abundance in stars of the plateau (Spite et al. 1984). More precisely, the WMAP best fit, which includes the use of other data sets (see sec. 3 of Cyburt et al. 2003) leads to

$$N(^{7}\text{Li})/N(\text{H}) = 3.82^{+0.73}_{-0.60} \times 10^{-10}. \quad (1)$$

In this paper we assume the cosmological Li abundance to be given by equation (1) and determine the implications for stellar evolution models of Pop II stars. To this end we first briefly review some pertinent Li observations (Sec. 2) and then (Sec. 3) the surface Li abundance to be expected from a comprehensive series of stellar evolution models that treat particle transport in great detail (Richard et al. 2002b,a; VandenBerg et al. 2002). In section 4, a comparison to observations leads to a discussion of the implications for turbulence modeling, of the link to solar type stars as well a discussion of other points of view. In the conclusion section, by putting all results together, we emphasize that atomic diffusion may be the main cause of the reduction of Li abundance between the original value given by WMAP and that observed in Pop II stars, with turbulent transport acting as a perturbation. The constancy of the Li abundance as a function of $T_{\text{eff}}$ is then less surprising than originally thought. Potential observational tests are also suggested.

### 2. $^6\text{Li}$ and $^7\text{Li}$ Observations

In the halo, Spite & Spite (1982) have obtained very similar Li abundances for nearly all low metallicity dwarf stars with $T_{\text{eff}} \geq 5500$ K. Following those original observations, the existence of the Li abundance plateau in Pop II halo stars has been repeatedly confirmed. In particular Spite et al. (1984), Ryan et al. (1999), Bonifacio et al. (2002), Thorburn (1994) and Bonifacio (2002) have confirmed a plateau at $^6 \text{A(Li)} \simeq 2.1$ or at $^7 \text{A(Li)} \simeq 2.3$ depending on the $T_{\text{eff}}$ scale used. $^6 \text{A(Li)} = 2.32$ has been obtained by Bonifacio et al. (2002) in turnoff stars of NGC 6397, a low metallicity cluster with $[\text{Fe/H}] = -2.01$. While the Li concentration is nearly the same in stars

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$^6 \text{A(Li)} \equiv \log[N(\text{Li})/N(\text{H})] + 12.$
whose Fe concentration differs by more than a factor of 100, from [Fe/H] = −3.7 to −1.5 (Cayrel 1998), a small progressive increase with [Fe/H] has perhaps been detected by Thorburn (1994) and Ryan et al. (1999).

The A(Li) ≃ 2.1 value follows if one uses the lower $T_{\text{eff}}$ scale preferred for instance by Ryan et al. (1999) whereas the A(Li) ≃ 2.3 value follows if one uses the high $T_{\text{eff}}$ scale for dwarf stars (Gratton et al. 1996). Partly because of the discussion in Sec. 3 of VandenBerg et al. (2002) and the better agreement with low metallicity cluster isochrones shown in that paper if the high $T_{\text{eff}}$ scale is used, we tend to prefer that scale. Consistency between field and cluster stars also favors the high $T_{\text{eff}}$ scale. We will however do comparisons of models to observations using both values of $A(\text{Li})$.

The results of 3D–NLTE simulations confirm the 1D–LTE Li abundance determinations in low metallicity Pop II stars (Asplund et al. 2003) though changes to collision rates used in NLTE calculations might slightly lower the measured Li abundance value (Barklem et al. 2003).

The Big Bang produces $^7\text{Li}$ but no $^6\text{Li}$. $^6\text{Li}$ is currently believed to be the product of galactic cosmic rays (GCR) (Reeves 1994). However any observation of $^6\text{Li}$ puts stringent constraints on internal transport processes in stars and observations of $^6\text{Li}$ so shed light on $^7\text{Li}$ observations. $^6\text{Li}$ is more fragile than $^7\text{Li}$ in that it is destroyed at a lower $T$ than $^7\text{Li}$ by nuclear reactions. If $^6\text{Li}$ has survived, very little $^7\text{Li}$ may have been destroyed by nuclear reactions. Its surface abundance is consequently another marker of internal turbulence. Its presence has been confirmed in a few Pop II field stars. In low metallicity stars, Nissen et al. (2000) obtained the ratio $^6\text{Li}/^7\text{Li}= 0.02 \pm 0.01$ in G 271-162. Smith et al. (1998) obtained the ratio $^6\text{Li}/^7\text{Li}= 0.05 \pm 0.03$ in BD +26$^0$3578 and $^7\text{Li}/H = 1.7 \times 10^{-10}$. In HD 84937, Cayrel et al. (1999) obtained the ratio $^6\text{Li}/^7\text{Li}= 0.052 \pm 0.019$ and $^7\text{Li}/H = 1.6 \times 10^{-10}$; they also note that $^6\text{Li}$ appears to be present only in the turnoff (or just past turnoff) halo stars. This suggests that, at least in these three turnoff halo stars, very little surface $^7\text{Li}$ has been destroyed. Their surface $^7\text{Li}$ is then the one from which the star formed or it may have been reduced only by atomic diffusion.

Since $^6\text{Li}$ is made by GCR, its presence in a star implies that some of the original $^7\text{Li}$ in that star also came from GCR and so from another source than the Big Bang. Since $^6\text{Li}$ was measured only in stars with [Fe/H] ≃ −2.4, one estimates$^2$ that all stars with that metallicity had a +0.06 to +0.1 dex contribution to $^7\text{Li}/H$ (see for instance Ryan et al. 1999). Since $^6\text{Li}$ is the product of galactic production, its abundance increases at the same time as that of Fe, though the relation between the two depends on the galactic nucleosynthesis model used. That it should have been observed in turnoff stars only, appears to be consistent with stellar evolution models stating that it is more easily destroyed in cooler stars than in turnoff stars (see below). One then expects a 0.06 to 0.1 dex contribution to $A(\text{Li})$ from GCR in all stars with [Fe/H] ≃ −2.4 and $T_{\text{eff}}$ ≃ 6300 K, so

$^2$Since in those stars where it was measured, $^6\text{Li}$ is about 1/20 of the $^7\text{Li}$ and since GCR produce approximately 1.5 times more $^7\text{Li}$ than $^6\text{Li}$ according to Reeves (1994).
that their $A(^{7}\text{Li})$ from cosmological origin becomes 2.0 dex in the low $T_{\text{eff}}$ scale and 2.2 dex in the high $T_{\text{eff}}$ scale. The high $T_{\text{eff}}$ scale gives a factor of 2.4 $\pm$ 0.4 decrease from the original cosmological value (Eq. 1). Given the various uncertainties, we will consider both a factor of 2.0 and of 3.0 as the reduction factor from the original cosmological $^{7}\text{Li}$ to that observed today.

3. Evolutionary model predictions

3.1. Two series of models

For the first series of models, details of the calculations may be found in Turcotte et al. (1998), Richard et al. (2001) and Richer et al. (1998). All effects of atomic diffusion are taken into account. Only quantities determined from first principles are used except for the mixing length parameter, $\alpha$, which is calibrated using the Sun (Turcotte et al. 1998). These are then the first self consistent models calculated from first principles. This was made possible by the recent availability of large atomic data bases that include the data needed to calculate radiative accelerations, $g_{\text{rad}}$, throughout stellar models (Iglesias & Rogers 1991, 1993, 1995, 1996; Rogers & Iglesias 1992a,b; Seaton 1993). Using this data, the $g_{\text{rad}}$ and Rosseland opacities are continuously calculated during evolution as the relative concentration of species changes (Richer et al. 1998). They play a major role in the particle transport equations. The formalism of Burgers (1969) is used to calculate the transport velocities leading to 56 (28 chemical species and 2 equations per species) non-linear coupled differential equations. In this series, evolutionary models have been calculated for Pop II stars of 0.5 to 1.2 $M_{\odot}$ with $[\text{Fe/H}]_{0}$ from $-4.31$ to $-0.71$ (Richard et al. 2002b,a; VandenBerg et al. 2002) and for solar metallicity clusters by Michaud et al. (2004). These models are called the models with atomic diffusion in what follows. In this paper, we mainly use models with $[\text{Fe}/\text{H}] = -2.31$.

The second series of models is similar to the first with $[\text{Fe}/\text{H}] = -2.31$, except that a turbulent transport coefficient, $D_{T}$, is added. Models for this series are described in Richard et al. (2002b) but additional models were calculated for this paper. Our aim in calculating models with turbulence is to determine the level of turbulence that the $^{7}\text{Li}$ and $^{6}\text{Li}$ observations require. This may later be used to determine the physical mechanism causing this turbulence. These models are called the models with turbulence in what follows; in addition to turbulent diffusion they also include all effects of atomic diffusion.

3.2. Depth of convection zones and turbulence parametrization

For surface Li abundance variations, perhaps the most important property of evolutionary models is how temperature (see Figure 1) and mass (see Figure 1, 2 and 12 of Richard et al. 2002b) vary at the base of the surface convection zone. The temperature, $T_{\text{bcz}}$, is directly related to Li burning while the mass, $M_{\text{bcz}}$, is directly related to the time scale of gravitational settling (see
Michaud 1977). Even in one single model, say the 0.77 $M_\odot$ one, the $M_{\text{bcz}}$ decreases by a factor of 30 during main–sequence evolution while $T_{\text{bcz}}$ decreases by a factor of 3. Most of the gravitational settling should occur around turnoff when $M_{\text{bcz}}$ is smallest. Because of the high sensitivity of Li burning to $T$, any burning of surface Li should occur early in the star’s evolution when $T_{\text{bcz}}$ is largest. As will be shown in §3.4 the reduction of surface Li does not necessarily occur through burning however.

On the lower part of Figure 1, main–sequence stars are on the lower branch and stars past turnoff are on the upper branch of each curve. An important characteristic is the independence of the $T_{\text{bcz}}$ on metallicity during main–sequence evolution. It depends only on $T_{\text{eff}}$. This makes it possible for processes that reduce the Li abundance to be independent of [Fe/H].

Results will be shown for two different parametrizations of turbulent diffusion coefficients. One parametrization allows to determine the turbulence which minimizes the reduction of surface $^7\text{Li}$ abundance throughout the evolution of stars of different masses. The turbulent diffusion coefficient is defined as a function of $T$ in order to allow adjusting it to limit gravitational settling of $^7\text{Li}$ while not burning $^7\text{Li}$. It is then essential to link turbulence to $T$ since the rate of the nuclear reaction $^7\text{Li}(p, \alpha)^4\text{He}$ is highly $T$ sensitive; the Li burning occurs at log $T \simeq 6.4$ (see Lumer et al. 1990 for a detailed discussion). To minimize Li abundance reduction one then tries to adjust the turbulent diffusion coefficient to be smaller than atomic diffusion slightly below that $T$ so that turbulence reduces settling as much as is possible in surface layers without forcing $^7\text{Li}$ to diffuse by turbulence to log $T \simeq 6.4$. The parameters specifying turbulent transport coefficients are indicated in the name assigned to the model. For instance, in the T6.0D400-3 model, the turbulent diffusion coefficient, $D_T$, is 400 times larger than the He atomic diffusion coefficient at log $T_0 = 6.0$ and varying as $\rho^{-3}$ or:

$$D_T = 400D_{\text{He}}(T_0) \left( \frac{\rho}{\rho(T_0)} \right)^{-3}.$$  (2)

To simplify writing, T6.0 will also be used instead of T6.0D400-3 since all models discussed in this paper have the D400-3 parametrization. The $\rho^{-3}$ dependence is suggested (see Proffitt & Michaud 1991a) by observations of the Be solar abundance today showing it is hardly smaller than the original Be abundance (see for instance Bell et al. 2001). This imposes a rapid decrease of the turbulent transport coefficient as $\rho$ or $T$ increases. Examples of those turbulent diffusion coefficients and how they compare to atomic diffusion coefficients are shown in Figure 6 of Richard et al. (2002b). Series of evolutionary models were calculated with T6.0, T6.09, T6.25 and T6.28 in order to investigate the effect of various levels of turbulence and determine the parameters that lead to the observed level of surface Li abundance reduction.

The second parametrization of turbulent diffusion coefficients that is used here was introduced into their solar model by Proffitt & Michaud (1991a). In this parametrization $D_T$ has always the same value immediately below the convection zone. It is, in this model, implicitly assumed that turbulence is generated from motions in the convection zone. Proffitt & Michaud (1991a)
determined as a maximum level of turbulent transport allowed by the observed solar Li abundance:

\[ D_T = 7500 \left( \frac{\rho}{\rho_{bcz}} \right)^{-3} . \]  

This model will be labeled PM7500. When used in the solar model that serves as benchmark for calculations of this paper, it reduces \(^7\text{Li}\) at the solar age by a factor \(1.1 \times 10^{-3}\), whereas replacing 7500 by 4000 (not shown) and 2000 (labeled PM2000) in equation (3) leads respectively to reduction factors of \(1.2 \times 10^{-2}\) and \(6.4 \times 10^{-2}\). This was verified by detailed stellar evolution calculations. The observed solar photospheric Li abundance is approximately \(5 \times 10^{-3}\) what is thought to be the original abundance but a pre–main–sequence reduction by a factor of 3 to 15 is expected (see Table 1 of Piau & Turck-Chièze 2002) leaving a reduction of Li during the main–sequence by a factor of 15 to 70. Solar models were calculated for this paper both with the PMx and Tx parametrizations. In the case of the Sun, one can obtain the desired Li value at the age of the Sun with either parametrization.

The time evolution of the turbulent diffusion coefficients at \(\log \rho = -0.5\) are compared in Figure 2. The density \(\log \rho = -0.5\) was chosen for the comparison because it is, during most of the evolution, between the convection zone and the region where Li burns both in the solar and in the \(0.77 \, M_\odot\) models. Since for all calculations described in this paper \(D_T\) varies as \(\rho^{-3}\), the comparison at one density is valid for all densities.

One notes from Figure 2 that, at a given \(\rho\), the \(D_T\) of the \(0.77 \, M_\odot\) T6.25 model is nearly equal that of the T6.4 solar model and of PM2000 throughout the evolution. In the \(0.70 \, M_\odot\) T6.25 model it is a factor of 2.5 larger. The \(0.77 \, M_\odot\) T6.09 model has a factor of \(10^{-30}\) smaller \(D_T\) than the other model. Using the PM2000 parametrization throughout the evolution of Pop II stars leads however to excessive Li destruction in the lower mass objects. The extent to which this is true will become more evident by considering Li isochrones in the next section. The range of \(D_T\) values compatible with Li surface abundance in both the Sun and Pop II dwarfs is surprisingly small.

3.3. Lithium isochrones

Lithium abundance isochrones for models with atomic diffusion and different series of models with turbulence are shown in Figures 3 and 4. Pre–main–sequence destruction is independent of the level of turbulence since during the pre–main–sequence the whole star is mixed by a convection zone. It is indicated separately in the upper part of the figures and is not included in the lower part. The reduction factor on the pre–main–sequence multiplies the reduction factor on the main–sequence.

On the main–sequence, the \(^7\text{Li}\) abundance drops by at least 0.2 dex in any star whatever the turbulence. The situation is a little complex at turnoff in that, while the \(^7\text{Li}\) abundance is smaller (the reduction factor is larger) in the model with atomic diffusion than in those with turbulence, at \(T_{\text{eff}} = 6000 \, \text{K}\) the \(^7\text{Li}\) abundance is smaller in most of the models with turbulence and decreases
as turbulence is increased. At $T_{\text{eff}} = 6000$ K, the model with T6.0 turbulence has a very slightly larger $^7$Li abundance than the model with atomic diffusion.

Also shown in Figure 3 is the isochrone calculated with turbulence linked to the position of the convection zone given by equation (3), labeled PM7500, and the PM2000 one. They both produce a large variation of Li. This parametrization extends the mixed zone below the bottom of the convection zone by a fixed factor, large enough to minimize settling around 13.5 Gyr for instance in the 0.77 $M_\odot$ star (a factor of 100 or so is needed). This leads to complete destruction of Li in the early evolution of the stars with $M < 0.7 M_\odot$.

There has been virtually no pre–main–sequence destruction of $^7$Li in PopII stars with $T_{\text{eff}} > 5500$ K (Fig. 3). Pre–main–sequence destruction of $^7$Li is important for $T_{\text{eff}} \lesssim 5400$ K. The level of destruction on the pre–main–sequence is relatively uncertain. Proffitt & Michaud (1989) concluded that pre–main–sequence Li destruction was a sensitive function of a number of parameters such as metallicity and He content (see also Piau & Turck-Chièze 2002) and details of the convection model (D’Antona & Montalbán 2003). Piau & Turck-Chièze (2002) obtain a factor of 20 reduction of surface $^7$Li during the solar pre–main–sequence (see their Fig. 10). Pre–main–sequence destruction of $^6$Li is large for $T_{\text{eff}} \gtrsim 6000$ K (see Fig. 4). This limiting $T_{\text{eff}}$ is uncertain but pre–main–sequence nuclear destruction appears unlikely to play a role at turnoff.

The original $^6$Li abundance (Fig. 4) is arbitrary and chosen to reproduce the observed values in the turnoff stars where it was observed. The $^6$Li/$^7$Li ratio is reduced by a factor of about 4 on the pre–main–sequence at 6000 K and by very large factors at smaller $T_{\text{eff}}$ so that one should expect to observe $^6$Li only above 6000 K. $^6$Li is less affected by atomic diffusion than $^7$Li so that the ratio $^6$Li/$^7$Li is larger than initially at turnoff in stars where only atomic diffusion is important for both $^6$Li and $^7$Li. In both the T6.25 and T6.28 models $^6$Li is destroyed by large factors even in turnoff stars.

One important characteristic of the isochrones is that, at a given $T_{\text{eff}}$ before turnoff, the Li abundance does not depend on metallicity. This is related to the property of $M_{\text{bcz}}$, shown in Figure 2 of Richard et al. (2002a), and of $T_{\text{bcz}}$ shown in Figure 1 above. Throughout main–sequence evolution, $M_{\text{bcz}}$ and $T_{\text{bcz}}$ depend only on $T_{\text{eff}}$ independent of metallicity over the metallicity range of interest. Consequently, for the small metallicities considered, settling depends only on $T_{\text{eff}}$ and the distance between the $M_{\text{bcz}}$ and the temperature where Li burns depends only on $T_{\text{eff}}$. This property may be seen, for the models with atomic diffusion, in Figures 9 and 11 of Richard et al. (2002a). After turnoff variations are seen as a function of metallicity however. That the Li abundance reduction at a given $T_{\text{eff}}$ is independent of metallicity before turnoff is then easy to understand. What is more surprising is that there should be a relatively wide plateau as a function of $T_{\text{eff}}$. 
3.3.1. Effect of He abundance variations

At the same time as Big Bang models predict a Li abundance from WMAP observations, they also predict an original He mass fraction of $Y_{\text{init}} = 0.2484$ which is slightly larger than $Y_{\text{init}} = 0.2352$ used in all calculations presented in this paper except in this section. The original He abundance used for most calculations is the same as used in Richard et al. (2002b), Richard et al. (2002a) and VandenBerg et al. (2002). This choice was made partly for consistency with previous calculations and partly because we had a large number of calculations available with that He abundance. Furthermore previous results have shown that such a small He abundance change had little effect on the turnoff luminosity vs age relation (see Fig. 4 of VandenBerg et al. 1996).

One may however wonder if the quantities that depend most sensitively on the value of the $M_{\text{bcz}}$, such as the surface Li isochrones may not be affected by such a change in He initial abundance. We have calculated a series of models with diffusion with $Y_{\text{init}} = 0.2484$ in order to determine the size of the effect. Li abundance isochrones calculated in both the diffusion models calculated with $Y_{\text{init}} = 0.2352$ and those with $Y_{\text{init}} = 0.2484$ are compared in Figure 5. The differences are very small and are mainly due to our use of straight line segments between calculated points. In stellar models of a given mass, the differences are significant but at a given age and $T_{\text{eff}}$, the differences in surface Li abundance are negligible. The main change is in the mass of the model that is at a given evolutionary step. At 13.5 Gyr the $Y_{\text{init}} = 0.2484$ model with maximum $T_{\text{eff}}$ has $M_* = 0.75 M_\odot$, whereas the model with $Y_{\text{init}} = 0.2352$ with maximum $T_{\text{eff}}$ has 0.77 $M_\odot$. The models with the minimum values of Li have respectively masses of 0.76 and 0.78 $M_\odot$. So at a given age, the isochrones are very nearly the same but for different mass models.

In Figure 5, we have continued the isochrones on the subgiant branch where the surface Li abundance is most sensitive to the $M_{\text{bcz}}$. This was not done on Figure 3 in order to avoid confusion with other curves.

We have also verified (not shown) that the $(T_{\text{eff}}, L)$ isochrones were not significantly modified, confirming the analysis of VandenBerg et al. (1996).

3.4. Settling vs destruction of surface Li

To understand what the constancy of the Li abundance as a function of $T_{\text{eff}}$ implies, one needs to understand the transport mechanisms below surface convection zones. Nuclear burning, atomic diffusion and turbulence are involved.
3.4.1. Nuclear burning

From an analysis of nuclear reaction rates and a comparison to detailed calculations, Proffitt & Michaud (1989) obtained that (see their Eq. 1) the relationship between the remaining fraction of the two Li isotopes may be expressed by:

\[
\frac{^{6}\text{Li}}{^{6}\text{Li}_0} = \left( \frac{^{7}\text{Li}}{^{7}\text{Li}_0} \right)^r.
\]  
(4)

The value of \( r \) comes from the ratio of the nuclear reaction rates and varies from \( r = 84 \) to 94 depending on the value of \( T \) where most \(^6\text{Li}\) burns. Proffitt & Michaud (1989) obtained equation (4) in pre–main–sequence stars where there is nearly instantaneous mixing of the whole star by convection. During main–sequence evolution, the situation is more complex.

By comparing the isochrones for \(^7\text{Li}\) (Fig. 3) with those for the \(^6\text{Li}\) to \(^7\text{Li}\) ratio (Fig. 4), it is clear that the abundance of \(^6\text{Li}\) varies much more slowly compared to that of \(^7\text{Li}\) than is given by equation (4) with \( r = 84 \). For instance at turnoff with T6.25 turbulence the \(^6\text{Li}\) to \(^7\text{Li}\) ratio is reduced by about 1.2 dex while \(^7\text{Li}\) is reduced by about 0.4 dex whereas equation (4) would have led to more than 30 dex reduction of \(^6\text{Li}\) for that reduction of \(^7\text{Li}\). This leads us to investigate the various contributions to atomic and turbulent diffusion velocities immediately below surface convection zones. Burning occurs a finite distance from the bottom of the surface convection zone and the surface Li is destroyed only in so far as it is transported to the burning region by turbulence (see Vauclair et al. 1978 for approximate evaluation formulas). The reduction rate of the surface \(^6\text{Li}\) and \(^7\text{Li}\) abundance depends on the transport efficiency from the bottom of the surface convection zone to the layer where \(^6\text{Li}\) and \(^7\text{Li}\) burn. It also depends on how much of the surface abundance reduction is due to atomic diffusion and how much to turbulent transport to the burning region.

3.4.2. Settling vs turbulent transport

While, in our evolutionary models, a complex set of 56 coupled differential equations is used, in order to interpret the results of the evolutionary models, it is convenient to use a diffusion equation developed for ternary mixtures by Aller & Chapman (1960). One must add to that equation the differential radiative accelerations, \( g_{\text{rad}} \), as introduced by Michaud (1970) as well as a turbulent diffusion term, \( D_T \), as introduced by Schatzman (1969):

\[
v_D = (D_{ip} + D_T) \left[ -\frac{\partial \ln c_i}{\partial r} \right] + D_{ip} \left[ \frac{A_im_p}{kT} (g_{\text{rad},i} - g) + \frac{Z_im_p g}{2kT} + kT \frac{\partial \ln T}{\partial r} \right] .
\]  
(5)

Within the first brackets on the right is the purely diffusive term which includes a contribution both from atomic diffusion, \( D_{ip} \), and from turbulent diffusion, \( D_T \). Within the second brackets on

\(^3\)Turbulence is made up of a wide spectrum of macroscopic advective motions, but Schatzman showed they could be modeled by a diffusive term. Anisotropic turbulence is slightly more complex to model (see Vincent et al. 1996).
the right, is an advective term caused by radiative acceleration, gravity, electric field, and thermal
diffusion. In the models with atomic diffusion, $D_T = 0$. In the models with turbulence, the
parametrization used for $D_T$ is defined by equations (2) and (3). The $g_{\text{rad},i}$ dominate all other
terms in turnoff low metallicity stars for Be, B and metals (see Richard et al. 2002a,b).

In Figure 6 are shown separately the diffusive (dashed) and the advective (dot-dashed) com-
ponents\(^4\) of the $^6\text{Li}$ and $^7\text{Li}$ diffusion velocities as well as the total velocity (continuous) at an age
of $13.5\ \text{Gyr}$ in a $0.77\ M_\odot$ model with T6.25 turbulence. Immediately below the convection zone,
the diffusive part of the velocity, $v_c(^7\text{Li})$, is towards the surface for $^7\text{Li}$ while
$v_c(^6\text{Li})$ is towards the interior but equals only $1/4$ of the total downwards velocity for $^6\text{Li}$. The advective part of
the diffusion velocity is towards the interior for both isotopes and causes the reduction of the $^7\text{Li}$
abundance and most of that of $^6\text{Li}$ in the surface convection zone. The diffusive part of the velocity
is mainly driven by turbulence. When it is toward the interior, it is linked to internal nuclear
burning. Immediately below the convection zone, it is however towards the surface for $^7\text{Li}$ and is
reacting to Li settling being larger close to the surface. The diffusive term is then reducing the effect
of gravitational settling, just as it does reduce the effect of the advective term for other metals.
However for Li, it has only a limited effect because nuclear reactions have virtually eliminated Li
deeper in. The corresponding Li profiles are shown in Figure 7. There is only a very small buffer
which turbulence can use to maintain the surface abundance.

The $^7\text{Li}$ profile for the T6.25 model is very nearly horizontal for $\log T < 6.2$ (Fig. 7). The transport
due to the diffusive term is a large fraction of the total transport velocity and changes
sign (at $\log T \sim 5.9$) but the Li abundance appears constant because the $D_T$ used are very large
so that a very small Li abundance gradient is sufficient to cause the purely diffusive term of the
velocity. The burning $^7\text{Li}$ at $\log T \sim 6.4$ forces the downward diffusive term for $\log T \gtrsim 5.9$ but is
unable to affect more superficial regions. The diffusive term is upwards for $\log T < 5.9$.

In the absence of turbulence (continuous line in Fig. 7), the sign change of the slope of the Li
profile is well defined at $\log T \sim 6.3$. The purely diffusive term leads to a downwards velocity for
$\log T > 6.3$ but to an upwards velocity for $\log T < 6.3$ (not shown). The purely diffusive term of
the T6.0 and T6.09 models changes sign also at $\log T \sim 6.3$ but, because $D_T + D_{ip}$ is very large
compared to $D_{ip}$ alone, the Li abundance gradient is very small for $\log T < 6.15$. Note that the
total diffusion velocity (including both diffusive and advective components) is always downwards.

The $^7\text{Li}$ burning causes a sink at $\log T \sim 6.4$. In the model with atomic diffusion, the diffusive
term causes the burning to extend its effect to $\log T \sim 6.3$. The advective terms in equation (5)
cause a reduction of the surface Li by a factor of $-0.7$ dex (see Fig. 7); the surface underabundance
is limited by the upwards diffusion caused the purely diffusive term. If T6.0 turbulence is present,
the Li burning is hardly affected but the upwards diffusive term is increased and cancels the

\(^4\)The diffusive component shown here was calculated in post processing using Eq. (5) and might differ slightly from
the more accurate value used by the evolutionary code. The advective component was available in the code output
and the one shown is the same as used by the code.
downwards advective term for a very small abundance gradient limiting surface underabundance to $\sim 0.22$ dex. With $T6.09$ turbulence, it is limited to $\sim 0.19$ dex again with little additional burning. As turbulence is further increased to $T6.25$, additional burning occurs however.

Atomic diffusion is then largely responsible for the transport of $^6$Li and $^7$Li immediately below the surface convection zone not only in the model with atomic diffusion but even in the $T6.25$ model. The transport by atomic diffusion determines the reduction of $^6$Li/$^7$Li. Even when turbulence reduces the effect of atomic diffusion for metals below 0.1 dex, atomic diffusion is still the dominant transport process for $^7$Li below the surface convection zone. We have also verified (not shown) that, in the absence of atomic diffusion, the surface $^6$Li/$^7$Li abundance reduction is dominated by turbulent transport and the reduction factor does not approach the value of equation (4) with $r \approx 90$. It is only when one assumes instantaneous mixing between the Li burning region and the surface that the reduction factor approaches this value. This is appropriate for instance on the pre–main–sequence (Proffitt & Michaud 1989). Otherwise, the transport mechanism, be it atomic or turbulent diffusion, plays the dominant role in determining the surface reduction factor.

4. Discussion

4.1. Li abundance and upper limits to turbulent transport

In Figure 8 calculated surface abundances are compared to observations of Li in halo stars and in two globular clusters. On the upper part of Figure 8 is shown the calculated surface Li concentration (black open circles) in 50 stars of initial metallicity $[\text{Fe/H}] = -2.31$. These are the result of a Monte Carlo simulation based on interpolations among a dozen complete evolutionary tracks (see Fig. 15 of Richard et al. 2002a for the results of a different draw in which $[\text{Fe/H}]$ was also allowed to vary). No turbulence is assumed outside of convection zones. The age of stars was randomly generated around 13.5 Gyr with a gaussian distribution of 0.3 Gyr standard deviation; 90% of the generated stars are between 13 and 14 Gyr. Only stars with $\log g \geq 3.8$ are included. Observations of Li abundance in metal poor halo stars by Thorburn (1994), in very metal poor stars by Bonifacio et al. (2003) and in the globular clusters M 92 and NGC 6397 (Bonifacio 2002; Bonifacio et al. 2002) are also shown. The Li observations below 5500 K may have been affected by pre-main sequence evolution (see the upper part of Fig. 3) which might explain the lower Li abundances observed in those stars. However it does not appear possible to avoid, with our models with atomic diffusion, a progressive reduction of the predicted Li abundance as $T_{\text{eff}}$ increases above 6000 K. Two different values of Li abundance at a given $T_{\text{eff}}$ are expected above 6000 K since post–turnoff stars have smaller Li abundance than pre–turnoff stars. A few stars with smaller Li abundance than plateau stars have been observed, and may be explained by these results (for instance the star represented by a magenta cross at $T_{\text{eff}} = 6350$ K). However no general reduction of Li abundance is observed above 6000 K. As in Pop I stars, additional turbulence appears to be required at least in some stars (Richer et al. 2000).
Series of models with different assumptions about the strength of turbulence were then calculated. One series uses T6.25 turbulence and the result of a simulation for 50 stars is shown on the lower part of Figure 8. Observations of Li abundance in metal poor halo stars by Spite et al. (1984) and by Ryan et al. (1999) are also shown. For $T_{\text{eff}} \geq 5600$ K, the Spite plateau is very well reproduced. This level of turbulent transport leads to a slight overdestruction of $^7$Li for $T_{\text{eff}} \leq 5600$ K.

The hottest stars simulated with models with turbulence are some 150 K hotter than the hottest ones simulated with models with atomic diffusion (see also VandenBerg et al. 2002). If one uses the hotter $T_{\text{eff}}$ scale, as used for some of the observations reproduced on the upper part of the figure, the hottest stars observed have a $T_{\text{eff}}$ compatible with that of the hottest stars simulated with models with turbulence; however if one uses the lower $T_{\text{eff}}$ scale, as used for some of the observations included in the lower part of the figure, the hottest stars observed have a $T_{\text{eff}}$ compatible with that of the hottest stars simulated with models with atomic diffusion. This assumes an age of 13.5 ±0.3Gyr which is compatible with the one determined by WMAP.

Lithium isochrones of Figures 3 and 4 and the comparison to observations of Figure 8 show that, if halo low metallicity stars have 13.5 ± 0.3 Gyr:

1) The model with atomic diffusion leads to 0.8 dex underabundance at turnoff or 0.5 dex more than at 6000 K. This suggests the presence of some weak turbulence below the convection zone.

2) With the T6.09 model, the underabundance is approximately −0.2 dex in all stars within the $T_{\text{eff}}$ interval of the Spite plateau as discussed in Richard et al. (2002b). Figure 15 of that paper also shows that the high constancy of the Li abundance observations above 5600 K can be reproduced and even exceeded in a self consistent model with turbulence.

3) The level of required turbulence depends on the $T_{\text{eff}}$ scale used to analyze the data. Given the various error bars, T6.09 turbulence is probably not excluded. However if the low $T_{\text{eff}}$ scale used by Ryan et al. (1999) is the right one, T6.25 turbulence would be required in turnoff stars. The absence of observed stars above 6350 K (see Fig. 8) then poses a problem. Turbulence should be limited to lower $T$ in cooler stars than in turnoff stars since otherwise, there is excessive destruction below 5600 K.

4) Turbulence appears linked to $T$ or to density more than to the bottom of the convection zone. All Tx models lead to fairly horizontal isochrones but with progressively larger Li abundance reduction as x increases. Depending on how close to the Li burning temperature the turbulence extends determines the destruction factor. The reason for the Li plateau in a given Tx model is relatively easy to understand. In a given Tx model, the turbulence extends to the same $T$ for all stellar masses and so leads to similar burning flux in all stellar masses. Since the mass above a given $T$ does not vary rapidly as a function of $M_*$, the Li reservoir to be burned is always about the same for all $M_*$. At a given age, the remaining Li is then about the same independent of $M_*$. 

5) Turbulence that extends the convection zone by a fixed factor, such as given by equation
(3) and which may be produced by overshooting, leads to completely wrong $T_{\text{eff}}$ dependence of the Li isochrones. It overdestroys Li in low $T_{\text{eff}}$ stars. It is not possible to extend convection zones by approximately a fixed level of turbulence (Proffitt & Michaud 1991b). The value given by equation (3) had been found by Proffitt & Michaud (1991a) to slightly overdestroy Li in their solar model. It is an upper limit because part of the Li destruction is believed to have been caused by pre–main–sequence burning. In the solar models of this paper, it however overdestroys Li by at least a factor of 10, probably because of a deeper surface convection zone as compared to the model of Proffitt & Michaud (1991a). One has to reduce the turbulence given by equation (3) by a factor of 3 to 4. The model compatible with surface solar Li abundance is that labeled PM2000 in Figure 3.

6) It is of course possible to determine a value of $x$ such that $T_x$ models have the observed Li destruction in the Sun since one has one observation and one adjustable parameter. The level of turbulent transport required to explain the solar value is around $T_{6.40}$ or about $0.2 - 0.3$ dex deeper in $T$ than required to reproduce the Li abundance of the Pop II Li plateau.

It is however surprising that the $T_x$ required for the Sun should be so close to the $T_x$ model that best reproduces Li in Pop II stars. As may be seen in Figure 2, the turbulent diffusion coefficient below convection zones in a 0.77 $M_\odot$ $T_{6.25}$ Pop II star is very nearly the same as in the $T_{6.40}$ Sun throughout evolution.

7) Clearly there remains considerable uncertainty in the value of the turbulent diffusion coefficients. There are many potential sources of such turbulence since, for instance, convective motions inside convection zones must lead to some turbulent motions beyond the frontier. Differential rotation is another potential source. In fact what is surprising is how little turbulent extension of convection zones is allowed by Li abundance observations.

8) Surface $^6\text{Li}$ is not destroyed by nuclear reactions on the main–sequence in models with atomic diffusion for $T_{\text{eff}} \gtrsim 5500$ K. In models with $T_{6.09}$ turbulence, destruction of surface $^6\text{Li}$ occurs only in stars with $T_{\text{eff}} \lesssim 6000$ K. However, in models with $T_{6.25}$ and $T_{6.28}$ turbulence, surface $^6\text{Li}$ is destroyed by more than a factor of 30 even in turnoff star. The presence of $^6\text{Li}$ in some turnoff stars appears to rule out $T_{6.25}$ and $T_{6.28}$ turbulence.

9) In all stars appearing in Figure 4, the ratio of $^6\text{Li}$ to $^7\text{Li}$ destruction is much smaller than $10^{80}$. A large fraction of the Li transport has been caused by atomic diffusion in all turbulence models considered (see §3.4.2). We have also verified that even if one neglects atomic diffusion, one still does not get the very large ratio given by equation (4). Such a ratio applies only if $^6\text{Li}$ destruction occurs in the convection zone. As soon as a transport process is involved, the depletion factor depends more on the properties of the transport process than on ratios of nuclear reaction rates. In most stars, Li destruction occurs in the convection zone only while it is on the pre–main–sequence.
4.2. Link to other work

The Li abundance in Pop II stars has generated a large number of papers and we do not intend to review all of them. It is however appropriate to analyze briefly the link to other points of view.

Salaris & Weiss (2001) argued that the current Spite plateau observations were consistent with evolutionary models including only atomic diffusion if all plateau stars were 13.5 to 14 Gyr old and they also argued that this implied a reduction from the original abundance by a factor of about two, consistent with analyses of the cosmic microwave background. To be consistent with observations as presented in Figure 8, this requires, as discussed by those authors, uncertainties in the $T_{\text{eff}}$ scale conspiring to minimize the $T_{\text{eff}}$ dependence of the Li abundance on halo stars. The small sampling scale may also play a role and they argue that more observations are needed to exclude, at an acceptable certainty level, that surface abundances are determined by stellar evolution models with atomic diffusion without significant contribution from macroscopic mixing processes. We agree that more observations of Li in halo stars are needed to firmly establish the origin of the Li abundance reduction and also that the $T_{\text{eff}}$ scale may well be off by some 100 or, perhaps, 200 K. This may reduce the difference between the WMAP value and the Spite plateau to a factor of $\sim 2$. It seems more difficult to explain the continuity of the Spite plateau to the highest observed $T_{\text{eff}}$ without the presence of some weak turbulence in those stars.

The more frequently discussed point of view has followed Vauclair (1988) analysis of the possible role of turbulent transport in causing Li destruction and/or reducing Li settling. Prescriptions for the initial angular momentum distribution, angular momentum loss, turbulent transport of angular momentum and of Li are required. Given the current level of understanding of turbulent transport processes, different arbitrary parameters are used for angular momentum and particle transport even though efforts are made to link the two. Various prescriptions were studied but, given input uncertainties, obtaining credible models is a formidable task. Pinsonneault et al. (1991, 1992) and Chaboyer & Demarque (1994) obtained destruction factors of up to 10 and linked their assumptions of the effect of internal differential rotation to observations of rotation on the horizontal branch. One has to assume an initial distribution of angular momentum and one then tends to obtain excessive Li abundance dispersion (Vauclair & Charbonnel 1995). Pinsonneault et al. (1999, 2002) strove to satisfy the small dispersion observed on the Spite plateau by varying input parameters and turbulent transport coefficients for Pop I and II stars. They were able to define a set of original angular momentum distribution and transport coefficients calibrations consistent with observed Li dispersion. While they do not explicitly say so, they appear to have neglected atomic diffusion in their calculations. Pinsonneault et al. (1999) concluded that the Li abundance depletion had to be between 0.2 and 0.4 dex. Since little is known about either initial angular momentum, turbulent transport of angular momentum or turbulent transport of Li, one should perhaps not be too surprised that these studies have not been very conclusive.

One difficulty with these models is that they neglect any potential role of internal magnetic fields in transporting angular momentum. In particular models based on diffusive transport of
angular momentum never reproduced the near solid body rotation of the Sun revealed by helio-
sismology (Charbonneau et al. 1998; Couvidat et al. 2003). The flat rotation profile of the Sun is
much more easily reproduced once magnetic fields are introduced (Barnes et al. 1999). As an alter-
native, Talon et al. (2002) have suggested that the solar interior could be slowed down by internal
gravity waves that might lead also to a Li Spite plateau (Talon & Charbonnel 2004). Both the
magnetic and gravity wave models have the advantage of being compatible with solar Be abundance
observations while models which strive to explain momentum transport by turbulent diffusion are
not (Balachandran 2000).

The difficulties with reproducing in detail the observations once transport processes are taken
into account led some to consider standard stellar evolution where it is arbitrarily assumed that
there is no transport process outside of convection zones so that no depletion from the original Li
occurs on the Spite plateau (Chaboyer 1994; Crifo et al. 1998). There is no physical justification
to assume that physical processes described from first principles do not apply to the Spite plateau
stars.

5. Conclusion

We have confirmed, using the latest evolutionary model calculations, the result of Michaud
et al. (1984) that atomic diffusion imposes a reduction by a factor of at least 1.6, and more probably
2.0, of the Li abundance observed in the oldest stars as compared to that produced in the Big Bang.
As briefly described in §4.2, this result is consistent with most other calculations done since, except
when atomic diffusion is arbitrarily neglected in the calculations. This result is also confirmed
observationally by the Li abundance of the Spite plateau being a factor of 2 to 3 smaller than the
primordial value inferred from WMAP as described in §1 and §2. Independent of the degree of
turbulence in the outer regions, VandenBerg et al. (2002) (p. 496) have obtained for M92 an age of
13.5 Gyr which is consistent with the WMAP determination of the age of the universe and which is
2 Gyr smaller than the age determined by Grundahl et al. (2000) who used models in which atomic
diffusion was neglected.

The limitation of the approach comes from the constancy of the Li abundance in low metallicity
Pop II stars above $T_{\text{eff}} \simeq 5600$ K. When special care is taken to reduce as much as possible the
observational errors, as done for instance by Ryan et al. (1999), one obtains a smaller dispersion for
most stars with $T_{\text{eff}} > 6000$K. This is the reason for part of the difference between the observations
on the upper and lower parts of Figure 8. Part of the scattering of the observations of Thorburn
(1994) appears to be observational. It appears that the star to star variation of the Li abundance
is at most 0.1 dex at a given $T_{\text{eff}}$ except for a number of stars that are a factor of 2 or more
below the plateau. This excludes any process that would lead to large abundance variations and
would depend on a property varying from star to star such as original stellar angular momentum,
for instance. It is however consistent with atomic diffusion being the main Li transport process
from the convection zone, as found in §3.4 to be the case even in presence of turbulent transport.
Turbulent transport is then a perturbation to the main transport process from the convection zone which is gravitational settling.

We have learned a number of properties of the evolutionary models of Pop II stars that make this constancy less surprising than it appeared when first noticed. What is needed to reproduce the Spite plateau may be inferred from Figure 7. A small amount of turbulence below the surface convection zone (the T6.0 or perhaps the T6.09 model) is all that is required to produce a sufficiently flat Li abundance plateau. Whether turbulent transport needs to extend deeper in, to the Li burning region, is currently uncertain. This extension would cause the destruction of the Li peak seen at log $T$ = 6.25 in Figure 7. That turbulence does not directly modify gravitational settling from the convection zone. In the absence of turbulence, the value of the Li concentration in that peak is determined by atomic diffusion but is drastically reduced by any turbulence between log $T$ = 6.25 and 6.40. A reduction of this peak is needed if the Spite plateau is more than a factor of 1.6 to 2 below the cosmological Li abundance given by equation (1) so that gravitational settling from the convection zone may proceed without being excessively reduced by a backward turbulent diffusion transport (see Fig. 6). If the high $T_{\text{eff}}$ scale is the right one, the Li reduction factor from the WMAP value can probably be explained without a reduction of this peak.

If, on the other hand, the low $T_{\text{eff}}$ scale is the right one, turbulent transport must approach the T6.25 model. It then plays a role in destroying the Li peak mentioned above. The T6.25 turbulent transport model is surprisingly similar to the turbulent transport which is needed to reproduce the Li abundance reduction in the Sun (see §3.2). The low temperature scale however leads to some difficulties with cluster isochrones (VandenBerg et al. 2002).

It is often assumed that the Pop II stars with a Li abundance a factor of 2 or more below the Spite plateau and a $T_{\text{eff}} > 6000$ K (see for instance Bonifacio et al. 2003 and Ryan et al. 2001) are caused by a different process and should not be included in the search for an explanation of this plateau. If one looks at the upper part of Figure 8 one notes that, in the absence of turbulence, post turnoff stars could have a factor of 2 to 3 lower surface abundance than pre turnoff stars. Such objects appear much less numerous than the number of post turnoff stars expected in this simulation where all stars are assumed to have no turbulence below the convection zone. It is however tempting to make the same suggestion as we made (see §5 of Richard et al. 2002b) to explain the King et al. (1998) observations of Fe abundance variations in some M92 turnoff stars. Just as in Pop I, some 20 % of A stars have lower turbulence than most and develop abundance anomalies by atomic diffusion processes (Richer et al. 2000), similarly a fraction (say 20 %) of Pop II turnoff stars could have lower turbulence and have anomalously small Li. This may be tested by a confirmation of the abundance variations in post turnoff stars of M92. While King et al. (1998) suggest the presence of such anomalies, higher signal to noise observations appear needed to confirm this result especially since such anomalies are not seen in higher metallicity clusters, where however they would not be expected to form by atomic diffusion according to Figure 10 of Richard et al. (2002a).
The surface abundance of $^6$Li in the hotter stars of the plateau offers another test of the role of transport processes. While consistent with the transport being due to atomic diffusion, as described in §3.4.2, the presence of $^6$Li appears to rule out the T6.25 model, that is the most turbulent transport model consistent with WMAP and the Spite plateau (see §4). It is then a very important additional constraint on turbulent transport and it would be important to confirm and or extend the few positive $^6$Li detections mentioned in §2.

Given that the WMAP implications for Li cosmological abundance and the Li Spite plateau can be naturally explained by gravitational settling in presence of weak turbulence, there appears little need for exotic physics as suggested by Ichikawa et al. (2004) and Coc et al. (2004) for instance. Instead, there is a need for a better understanding of turbulent transport in the radiative zones of stars. This requires simulations from first principles such as are being attempted for instance by Talon et al. (2003) and Théado & Vaucclair (2003).

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Fig. 1.— Temperature at the bottom of the surface convection zone as a function of $T_{\text{eff}}$. In the upper part, are shown the paths followed during the evolution of stars of 0.6, 0.7, 0.77 and 0.8 $M_{\odot}$ with [Fe/H] $= -2.31$. Evolution starts on the upper right hand section of each curve. In the lower part of the figure are shown $T_{\text{bcz}}$ isochrones for three metallicities at 13.5 Gyr. The 0.77 $M_{\odot}$ star is at turnoff on the [Fe/H] $= -2.31$ isochrone. At 13.5 Gyr, all main–sequence stars of a given $T_{\text{eff}}$ have the same $T$ at the bottom of their surface convection zone, $T_{\text{bcz}}$, irrespective of their metallicity. This is not the case past turnoff however.
Fig. 2.— Turbulent diffusion coefficient as a function of age at $\log \rho = -0.5$ in 0.7 and 0.77 $M_\odot$ Pop II star and in the solar model. The parameters are defined by Eq. (2) and (3) and the curves identified on the figure. Bottom of convection zones are indicated by nearly vertical lines.
Fig. 3.— Isochrones of $^7$Li at 13.5 Gyr in models where atomic diffusion only is included (continuous line) and where four different levels of Tx turbulence are introduced. Two isochrones for PMx models are also shown. In the upper part of the figure is shown the pre–main–sequence destruction. In the lower part is included the destruction during the main–sequence only. Open circles show the value calculated for the various models in stars of 0.77, 0.75, 0.7, 0.65, 0.6, and 0.55 $M_\odot$ at 13.5 Gyr. No post turnoff star is included here to avoid overcrowding. See Figure 8 where post turnoff stars are included. The horizontal dotted line identifies initial values.
Fig. 4.— Isochrones of the $^6$Li to $^7$Li ratio at 13.5 Gyr in models where atomic diffusion only is included (continuous line) and where four different levels of turbulence are introduced. In the upper part of the figure is shown the pre–main–sequence destruction. In the lower part, is included the destruction during the main–sequence only. See the legend of Fig. 3 for further details.
Fig. 5.— Isochrones of $^7$Li at 13.5 Gyr in models where atomic diffusion only is included and where the original He abundance is varied. The continuous line is the same as in Fig. 3 and was calculated with the same original He mass fraction of 0.2352 whereas the dotted line was calculated for a slightly larger original He mass fraction. The effect is negligible and the difference between the two curves is mainly caused by the use of straight line segments between calculated points.
Fig. 6.— Diffusion velocities of $^6$Li and $^7$Li at 13.5 Gyr in a 0.77 $M_\odot$ model with T6.25 turbulence. The continuous lines are the total velocities, $v_{\text{tot}}$, while the dot-dash line is the advective velocity, $v_{\text{drift}}$, and the dash line the purely diffusive term, $v_c$, containing both atomic diffusion and turbulent diffusion. Negative velocities are towards the interior. The bottom of the convection zone is indicated by a vertical long dashed line. Immediately below the convection zone, the main inward contribution to the total velocity comes from the advective term and is caused by gravitational settling. The dashed line was calculated in postprocessing where only a limited number of significant figures was available leading to the noisy character of those curves.
Fig. 7.— $^7$Li profile in 0.77 $M_\odot$ stars at 13.5 Gyr of [Fe/H] = $-2.31$ with various turbulence models. The smallest surface $^7$Li abundance is in the model with atomic diffusion (continuous line). As turbulence is increased, the surface abundance first increases in the T6.0 and T6.09 models but then decreases as turbulence is further increased in the T6.25 and T6.28 models. On the other hand the maximum in Li concentration seen at log $T \sim 6.25$ in the model with atomic diffusion is progressively reduced as turbulence is increased.
Fig. 8.— Predicted Li abundance in stars without turbulence (upper part) and with the T6.25 turbulence (lower part). For the calculations, the initial value of $A(^{7}\text{Li})$ is 2.58 and open black circles are used. The size of the circles is a function of the radius of the stars in order to indicate roughly their evolutionary stage. There are 50 simulated stars in each panel and 90% of them are between 13 and 14 Gyr. Further description is found in the text. Also shown are observations in metal poor halo stars in the upper part by Thorburn (1994) (blue triangles), Bonifacio et al. (2003) (magenta crosses), Bonifacio et al. (2002) (green squares), Bonifacio (2002) (red triangles) and in the lower part by Spite et al. (1984) (blue filled triangles) and by Ryan et al. (1999) (red filled squares with error bars).