Numerical research of probabilistic characteristics of quality by the piece the component of mix with increase in number of doses given by the batcher

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Abstract. In work it is carried out the mathematical analysis of these pilot studies of the automatic conveyor mixer (non-mixer) and drum dozers for receiving mix of single elements of the components having similar standard overall dimensions but differing in internal properties at effective end use. The relevance and novelty of development are shown on the example of receiving the general party of composite product from small similar parties, at the determined forming of its uniformity, with the set qualitative and quantitative characteristics from single heterogeneous components. The analysis of influence of ratios of components in synthesizable mix on its quality is carried out. The main evaluation criterion was the quantity of doses of components at the same time given by batchers and its influence on final parameters of the received mixes.

1. Introduction
In work [1] were the pilot study of conveyor non-mixing installation for receiving mix of tubular lengthy products at the determined formation of its uniformity is conducted. More than 1000 experiments and the presented results were made allowed to claim that the products received thus possess higher rates of quality, than received in two other ways with which numerical comparison was carried out [2-4].
Use of these specialized products is most caused by accuracy of formation of a particular volume of the mix called further effective selection of use on which amount the result efficient application of a finished product which uses this selection depends. The amount of this selection also is not strictly the fixed, and is tied to the wide product range [5-7].
Therefore there was expedient a finding of dependences between quantity of the given doses of each of the formulation constituents created in a nonmixer from them single doses of mix and the end result – the effective selection of use of mix created from them [1.4]. It will allow to find the solution of the main task of a research: receiving mix of necessary volume high-performance qualities, namely at the necessary ratio of the components making it [8.9].
The decision is caused by finding of number of doses at which the law of delivery of doses of components by the drum feeding devices, the most close to normal will be kept.
Therefore in this work the primal scientific problem, namely assessment of number at which the maximum compliance of the law of delivery with the feeding devices [10, 11] of each of the blending components normal will be provided that most defines indicators of quality of mix and also finding of
its minimum value at which effective selection of use of mix will keep necessary indicators of quality at the determined formation of its uniformity [1, 4, 12] is solved.

2. Problem definition
Let us study quality of the mix made by non-mixer with increase in number of the doses given by separately chosen batcher. Basic concepts of probability theory and carrying out calculations in the environment of Mathcad are the basis for calculations.

Operation of separately chosen batcher is studied. For unit of time \( T \) it gives out on the conveyor \( \xi \) particles, where \( \xi \) – discrete integer random variable with the following (received after statistical processing of results of observations [1]) the final number of distributions (table 1).

| Table 1. Final number of distributions |
|---------------------------------------|
| \( \xi \) | \( \xi_1 \) | \( \ldots \) | \( \xi_n \) |
| \( p \)  | \( p_1 \)  | \( \ldots \) | \( p_n \)  |

Here in the first line values of random variable (in abbreviated form - RV) are written down \( \xi \) as its increase, in the second line – the probabilities corresponding to them. Value \( \xi = k_j (1 < j < n) \) - the required number of particles per time \( T \) from the batcher. It is close to population mean \( m \) RV \( \xi \).

The sum of all probabilities of row is equal to unit, i.e. \( p_1 + p_2 + \ldots + p_n = 1 \). This fact is used below as check of correctness of ranks of distributions of the new random variables received during calculations.

Number of the particles given by the batcher in time \( T \), is called dose. Let \( \eta_1 = n_1 \) – number of the particles given by the batcher in the first period \( 0 \leq t < T \) since the beginning of its work, \( \eta_2 = n_2 \) – number of particles for the second period \( 0 \leq t < 2T \) etc. Then \( \eta_N = n_1 + n_2 + \ldots + n_N \) – are \( N \) component doses in mix. Sizes \( n_1, n_2, \ldots \) are considered as independent random variables.

Let us designate through \( s \) average quadratic deviation of RV \( \xi \). According to probability theory population mean and average quadratic deviation of RV \( \eta_N \) are respectively equal \( N \cdot m \) and \( \sqrt{N} s \). These facts can be also used as intermediate checks when carrying out calculations.

By means of the subroutines-functions enclosed below written in the environment of Mathcad for RV \( \eta_N \) it is possible:

a) to construct the graph \( f = f(x) \) ground of frequencies;

b) to compare this graph to the graph \( f = f(x) \) the normal law of distributions with population mean and average quadratic deviation of RV \( \eta_N \), in relation to conclusions from work [1] taking into account number of doses, of which selection of effective use of mix is formed

\[
f(x) = \frac{1}{\sqrt{2\pi ns}} \exp \left( -\frac{(x-m)^2}{2ns^2} \right);
\]

(1)

c) to track at present value \( \varepsilon \) behind change of probability of event

\[|\eta_N - m| \leq \varepsilon \cdot m\]

with increase in number \( N \).

3. Algorithm of the task solution
At first, we will give the short description of the used subroutines-functions (S-F).
First, in them the functions, which are built in Mathcad, are used:

- \( \text{ceil}(x) \) – is defines the smallest integer, bigger or equal \( x \),
- \( \text{floor}(x) \) – is defines the greatest integer, smaller or equal \( x \),
- \( \text{ORIGIN} \) – is the system variable defining number of the first column and number of the first matrix;
- \( \text{cols}(X) \) – is defines number of columns of matrix \( X \),

- \( \text{int}(x) \) – is defines the smallest integer, bigger or equal \( x \),
- \( \text{max}(x) \) – is defines the greatest integer, smaller or equal \( x \),
- \( \text{min}(x) \) – is defines the smallest integer, bigger or equal \( x \).
rows($X$) – is defines number of lines of matrix $X$,
rsort($X$) – is rearranges in places matrix columns $X$ so that it has been sorted in ascending order of elements the first lines of the specified matrix,

$\text{min}(V)$– is defines the minimum value from all coordinates of vector $V$,
$\text{max}(V)$ – is defines the maximum value from all coordinates of vector $V$,

$linterp(U,V,x)$ – is program of finding of function of one variable $x$ (in our case this function has designation $f(x)$), allowing to construct the diagram of the ground of frequencies of number of distributions $X$. $U, V$ – the first and second columns of the transposed matrix $X$.

Now we will describe purpose of the made S-F.

$SM(X,Y)$ – is sums up two random variables which ranks of distributions are presented in the form of matrixes $X,Y$ with two lines, also finds number of distributions of the sum of these random variables in the disordered look.

$DZ(X)$ – is pre-sorting program of distributions $X$, received by the program $SM(X,Y)$.
$DDBZ(X)$ – is final sorting program of the row received by the program $DZ(X)$.
$Mo(X)$ – is finds population mean of random variable from number of distributions $X$.
$Di(X)$ – is calculates dispersion of random variable from number of distributions $X$.
$SummaP(X)$ – is the test program of check (checks to what the sum of all probabilities from number of distributions is equal $X$.

$F(x,m,s)$ – is calculates value in point $x$ functions (1) at values $m = M, s = S$.

Let us give example of use of the specified S-F.

Let us consider a series of distributions of random variable $\eta_1$ (table 2), found on the ground of frequencies of the first component of mix from work [1]

| $\xi$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|------|---|---|---|---|---|---|---|---|
| $p$  | .06| .012 | .26 | .25 | .17 | .08 | .04 | .02 |

Table 2. Series of distributions of random variable $\eta_1$

The program is gradually outlined below with the necessary explanations:

$Q = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ .06 & .12 & .26 & .25 & .17 & .08 & .04 & .02 \end{bmatrix}$ - number of distributions of RV $\eta_1$  (2)

$ORIGIN: = 1$
$m: = Mo(Q) = 3.85$ – is population mean $\eta_1$
$d: = Di(Q) = 2.4275$ – is dispersion $\eta_1$
$s: = \sqrt{d} = 1.55804$ – is average quadratic deviation

$QT: = Q^T$ – is transposing of matrix $Q$

$n: = cols(Q) = 8$ – is number of columns of matrix $Q$

$X: = QT^1$ – is column of values of number of distributions of RV $\eta_1$

$Y: = QT^2$ – is column of probabilities of number of distributions

$f = ff(X): = linterp(X,Y,x)$ – is function of creation of the ground of frequencies of RV $\eta_1$

$f(x): = Fn(x, m, s)$ – is function (1) for $\eta_1$

$a: = X_1 = 1$ – is the smallest value of RV $\eta_1$

$b: = X_2 = 8$ – is the greatest value $\eta_1$

$A: = \text{min}(Y) = 0.02$ – is the smallest value of probability of row $Q$

$B: = \text{max}(Y) = 0.26$ – is the greatest value of probability of row $Q$

$C: = B \cdot 1.05$ – is the upper bound of values of functions for the diagram in figure 2

$X: = a, 0.1..b$ – is cycle of creation of diagrams
Figure 1. Listings of S-F for components

```plaintext
DZ(X) :=
    j ← 1
    n ← cols(X)
    C(j) ← \begin{pmatrix} X_{1,1} \\ X_{2,1} \end{pmatrix}
    for i ∈ 2..n
        if X_{1,i} = X_{1,j}
            D(X) :=
                \begin{pmatrix} C_{2,j} \\ X_{2,i} \end{pmatrix}
            continue
        if X_{1,i} > X_{1,j}
            j ← j + 1
            C(j) ← \begin{pmatrix} X_{1,1} \\ X_{2,i} \end{pmatrix}

    SM(X, Y) :=
        C ← DZ(X)
        while cols(Y) < cols(X)
            X ← Y
            Y ← DZ(X)
            C ← X

SummaP(X) :=
    s ← 0
    n ← cols(X)
    for i ∈ 1..n
        s ← s + X_{2,i}

Mo(X) :=
    n ← cols(X)
    s ← 0
    for i ∈ 1..n
        s ← s + X_{1,i} X_{2,i}

PN(X, M, e) :=
    r ← e M
    A ← M - r
    B ← M + r
    p ← 0
    n ← cols(X)
    for i ∈ 1..n
        if A ≤ X_{1,i} ≤ B
            p ← p + X_{2,i}
            continue

DDZ(X) :=
    Y ← DZ(X)
    while cols(Y) < cols(X)
        X ← Y
        Y ← DZ(X)
        C ← X

f(X, x) = \frac{1}{\sqrt{2\pi} S} \exp\left[\frac{-(X - M)^2}{2 S^2}\right]

Figure 2. Diagrams of the ground f f(X) frequencies of RV η₁ and functions f(x) normal distribution with parameters m = 3.85, s = 1.55804 sizes η₁
We pass to finding of RV $\eta_2$.

$N_1 = 2$

$Q: = SM(Q, Q)$ – is ranks of distributions of RV $\eta_1$ and $\eta_1$ are summed up disordered series turns out $Q$ distributions of RV $\eta_2$

$Q: = rsort(Q, 1)$ – is pre-sorting the resulting series $Q$

$Q: = DDZ(Q)$ – is final sorting of series $Q$ distributions. Number of distributions of RV $\eta_2$

$Q:= \left( \begin{array}{cccccccccc}
2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16
\end{array} \right)$

$q: = SummaP(Q) = 1$ – check of the sum of probabilities of all values of RV $\eta_2$

$m: = Mo(Q) = 7.7$ – population mean $\eta_2$

$d: = Di(Q) = 4.855$ – dispersion $\eta_2$

$s: = \sqrt{d} = 2.2034$ – average quadratic deviation

$QT: = Q^T$ – transposing of matrix $Q$

$n: = cols(Q) = 15$ – number of columns of matrix $Q$

$X: = QT^{(3)}$ – column of values of number of distributions of RV $\eta_2$

$Y: = QT^{(2)}$ – column of probabilities of number of distributions of RV $\eta_2$

$f = ff(X): = interp(X, Y, x)$ - function of creation of the ground of frequencies of RV $\eta_2$

$f(x): = Fn(x, m, s)$ – function (1) for $\eta_2$

$a: = X_1 = 2$ – the smallest value of RV $\eta_2$

$b: = X_2 = 15$ – the greatest value $\eta_2$

$A: = min(Y) = 0.02$ – the smallest value of probability of row $Q$

$B: = max(Y) = 0.26$ – the greatest value of probability of row $Q$

$C: = B \cdot 1.05$ – the upper bound of values of functions for the diagram in figure 3

$X: = a, 0.1..b$ – cycle of creation of diagrams

Results of work of the provided program are displayed in figure 3.

![Diagram of the ground of frequencies of RV $\eta_2$ and functions of normal distribution with parameters $m = 7.7$, $s = 2.20341$ sizes $\eta_2$](image)

**Figure 3.** Diagrams of the ground of frequencies of RV $\eta_2$ and functions of normal distribution with parameters $m = 7.7$, $s = 2.20341$ sizes $\eta_2$

Addition $\eta_2$ and $\eta_2$ gives $\eta_4$. Addition $\eta_4$ and $\eta_4$ gives $\eta_8$. The calculations like that have been carried out at stay $\eta_2$ have finished in the following results shown in figure 4 and figure 5.
Figure 4. Diagrams of the ground of frequencies of RV $\eta_4$ and functions of normal distribution with parameters $m = 15.4$, $s = 3.11609$ sizes $\eta_4$

Figure 5. Diagrams of the ground of frequencies of RV $\eta_8$ and functions of normal distribution with parameters $m = 30.8$, $s = 4.40681$ sizes $\eta_8$

For the second component of mix [1] with distributions are near

$$Q = \begin{pmatrix} 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 \\ .01 & .03 & .04 & .06 & .1 & .12 & .18 & .16 & .11 & .06 & .02 & .01 \end{pmatrix}$$

the following graphs turn out.
Figure 6. Diagrams of the ground of frequencies of RV $\eta_1$ and functions of normal distribution with parameters $m = 10.21$, $s = 2.46696$ sizes $\eta_1$.

Figure 7. Diagrams of the ground of frequencies of RV $\eta_2$ and functions of normal distribution with parameters $m = 20.42$, $s = 3.48881$ sizes $\eta_2$.

Figure 8. Diagrams of the ground of frequencies of RV $\eta_4$ and functions of normal distribution with parameters $m = 40.84$, $s = 4.93392$ sizes $\eta_4$. 
Figure 9. Diagrams of the ground of frequencies of RV $\eta_8$ and functions of normal distribution with parameters $m = 81.68$, $s = 6.97762$ sizes $\eta_8$.

For the third component of mix [1] with distributions are near

$$Q := \begin{pmatrix} 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 & 21 & 22 \\ 0.01 & 0.03 & 0.02 & 0.06 & 0.1 & 0.14 & 0.15 & 0.14 & 0.1 & 0.06 & 0.03 & 0.02 & 0.01 \end{pmatrix}$$

such graphs turn out.

Figure 10. Diagrams of the ground of frequencies of RV $\eta_1$ and functions of normal distribution with parameters $m = 15.5$, $s = 2.60576$ sizes $\eta_1$. 
4. Discussion of results
From the provided diagrams (Figure 2-12), it is visible that the diagram of the ground of frequencies of RV $\eta_N$ with increase in number $N$ doses of component of mix approaches distribution under the normal law that will be coordinated with the central limit theorem [12] of probability theory. In this case, continuous curves describe experimentally obtained dependences when taking into account the number of doses from which an effective selection of the use mixture is formed, and dashed curves are approximated diagrams of the normal distribution found in the MATHCAD environment. Have led calculations for two other components of mix from work [1] to the same conclusion and for other mixes made by non-mixers.

Further it is necessary to calculate probabilities of formation of indexes of quality of mixes according to the results received earlier and presented on schedules (Figure 2-12) for finding of minimum assessment $N$.

On the reviewed example we will show, how at present value of accuracy $\varepsilon$ of maintenance of component in mix event probability changes
with increase in number $N$ also we will compare this probability $p_1$ with probability $p_2$, which turns out at the normal law of distributions of random variable with population mean and average quadratic deviation of size $\eta_N$.

Let us make it by means of table 3 in which values $p_1$ and $p_2$ are received by means of the subroutine-functions given in the appendix: $PN(X, M, \varepsilon)$ - calculates the probability of event (3) on row $X$ distributions of RV $\eta_N$, determined previously by this row by means of population mean $M$ and size $\varepsilon = \varepsilon$, $p_2(M, S, \varepsilon)$ - calculates the probability of event (3) for normal distribution with population mean $M$ and average quadratic deviation $S$, RV $\eta_N$ with the set accuracy $\varepsilon = \varepsilon$. 

| Number of doses | Probability $p_1$ | Probability $p_2$ |
|----------------|-------------------|-------------------|
| $N$            | 0.3               | 0.25              |
| 1              | 0.68              | 0.51              |
|                | 0.25              | 0.25              |
|                | 0.25              | 0.25              |
|                | 0.25              | 0.25              |
| 2              | 0.5415            | 0.46327           |
|                | 0.37884           | 0.28911           |
|                | 0.19517           | 0.09833           |
| 4              | 0.7364            | 0.6419            |
|                | 0.4939            | 0.3553            |
|                | 0.3553            | 0.1749            |
| 8              | 0.70553           | 0.61769           |
|                | 0.5154            | 0.39985           |
|                | 0.27326           | 0.13871           |
|                | 0.51              | 0.1749            |
|                | 0.25              | 0.09833           |
|                | 0.25              | 0.13871           |
|                | 0.25              | 0.13871           |

From table 3 it is visible that at the low requirement ($\varepsilon = 0.3$, $\varepsilon = 0.25$) to the accuracy of maintenance of component in mix with high probability number $N = 8$ gives the necessary quality of mix.

The table also shows that use of the normal law of distributions allows to receive acceptable assessment of number $N$, as it is made, for example, in work [2].

Receiving by means of computer modelling of ranks of distributions of RV $\eta_N$ on the set number of random variables $\eta_1$ at great values $N$ becomes complicated and even impracticable. It relates to increase in data array in several distributions. If number of distributions of RV $\eta_1$ contains $n$ columns, number of distributions $\eta_N$ contains (with an accuracy of unit) $N \cdot n$ columns.

After told, assessment of the minimum number $N = N_*$ the doses in portion of mix providing with high probability $P$ the set accuracy of maintenance of component in mix $\varepsilon$ on number of RV $\eta_N$, it is possible to carry out by means of the relevant laws of normal distribution.

Let us notice still that number of distributions of RV, even if it is received after the numerous experiments, generally contains small mistakes in probabilities of values of this random variable. Therefore, parameters $m, s$ are defined not precisely, and finding of size $N_*$ it is necessary to carry out in some small ranges

$$m_* - \Delta \leq m \leq m_* + \Delta, s_* - \delta \leq s \leq s_* + \delta.$$  \hspace{1cm} (4)

We consider that in our case for the mix considered components with distributions are near (2)

$$m_* = 3.85, s_* = 1.55804, \Delta = \delta = 0.03.$$  \hspace{1cm} (5)

Calculation of values $N_*$ at parameters $K = k = 4, e = \varepsilon = 0.1, P = 0.9$ and values $M = m, S = s$, meeting the imposed conditions it was carried out by means of the subroutine-function $NPn(M, K, S, e, P)$. At the release of this program, we receive vector column from two numbers. The
first is number $N_*$, the second, corresponding to number $N_*$, is probability $p \geq S$. Results of calculations are reduced in table 4 where only values are given $N_*$. Here, the number is highlighted in bold type $N_*$, received at $S = m_*$, $S = s_*$ and fat italics – the number, most remote from this number $N_* = 86$.

Let us note big deviations of number $N_*$ within three-five units at change of population mean of RV by one hundredth. Change of average quadratic deviation of RV on 0.01 gives change of number $N_*$ on zero - one unit.

**Table 4.** Assessment of the minimum number the doses in portion of mix

| $s$   | 3.82 | 3.83 | 3.84 | 3.85 | 3.86 | 3.87 | 3.88 |
|-------|------|------|------|------|------|------|------|
| 1.528054 | 80   | 73   | 68   | 63   | 59   | 56   | 53   |
| 1.538054 | 81   | 74   | 69   | 64   | 60   | 56   | 53   |
| 1.548054 | 82   | 75   | 70   | 65   | 61   | 57   | 54   |
| 1.558054 | 83   | 76   | 71   | 66   | 61   | 58   | 55   |
| 1.568054 | 84   | 77   | 71   | 66   | 62   | 58   | 55   |
| 1.578054 | 85   | 78   | 72   | 67   | 63   | 59   | 56   |
| 1.588054 | 86   | 79   | 73   | 68   | 64   | 60   | 57   |

Therefore, to tell that: «66 doses in portion of mix with probability not less than 0.9 and size $\varepsilon = 0.1$ provide performance of event (3) » at not precisely defined population mean and dispersion, it is impossible. Within conditions (4)-(5) portion will be made by 86 doses.

5. Conclusions

Thus, computer modelling allows making highly the specifying (adjusting) calculations of definition of the minimum (optimum) dose of mix in which with high probability the set ratio of components will be provided.

The conducted research has shown prospects of use of computer modelling when studying probabilistic characteristics of random variable $\eta_N$, defining keeping of one the allocated component in $N$ mix doses. Display of the received results in the form of diagrams and tables allows accelerating process of finding of the minimum number of doses in portion of mix providing its necessary quality.

6. References

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