NMR Relaxation Rate for One-Dimensional Multicomponent Spin-Orbital Systems

Akira Kawaguchi, Tatsuya Fuji and Norio Kawakami

Department of Applied Physics, Osaka University, Suita, Osaka 565-0871

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The NMR relaxation rate $1/T_1$ is studied for one-dimensional multicomponent spin-orbital systems. By combining the bosonization techniques and the exact solution of the SU(n) model, we evaluate the power-law exponent and the enhancement factor for $1/T_1$ at low temperatures. We discuss how the band splitting affects the relaxation rate, and find that $1/T_1$ may be enhanced around the critical value of the band splitting. The crossover behavior in $1/T_1$ around the critical point is discussed in terms of the low-frequency dynamical spin susceptibility. The effect of the hole-doping is also addressed.

KEYWORDS: NMR, one-dimension, spin-orbital model, bosonization

§1. Introduction

Quantum spin systems with orbital degeneracy have attracted considerable attention recently, since it was recognized that the orbital degrees of freedom give rise to a variety of interesting phenomena in correlated electron systems. These hot topics have stimulated theoretical studies on the orbitally degenerate Hubbard model and the related multicomponent spin-orbital models. As a first step to investigate such orbital effects, the one-dimensional (1D) version of the SU(4) spin-orbital model has been studied numerically and analytically. Such analyses have been also extended to the SU(2)$\times$SU(2) spin-orbital model.

However, these studies have been mainly concerned with the phase diagram and/or the static properties, which naturally motivates us to study dynamical quantities for the above models.

The NMR measurement may be a powerful probe to study low-energy spin dynamics in 1D strongly correlated systems, as demonstrated theoretically and experimentally. In this paper, we study the NMR relaxation rate for 1D multicomponent spin-orbital systems. For this purpose, we first give a multicomponent generalization of the formula for the NMR relaxation rate $1/T_1$ in terms of the bosonization method. We then evaluate the power-law exponent and the enhancement factor for $1/T_1$ by exploiting the integrable SU(n) spin-orbital model. It is discussed how the band splitting affects the relaxation rate. We further study the dynamical spin susceptibility to discuss the crossover behavior in $1/T_1$ around the critical point. The effect of the hole-doping on the relaxation rate is also discussed.

This paper is organized as follows. In §2, we obtain a generalized expression for the NMR relaxation rate, and then discuss the the effect of the band splitting on $1/T_1$ in §3. We present the results for the dynamical spin susceptibility in §4 to discuss how the crossover behavior emerges when the band splitting is changed. Brief summary is given in §5.

§2. Relaxation Rate for Spin-Orbital Systems

2.1 Bosonization approach

We first derive a generalized expression for the NMR relaxation rate $1/T_1$ for 1D multicomponent electron systems. To illustrate the procedure clearly, let us consider the degenerate Hubbard model with the band splitting, which possesses $N$-fold orbitals and spins $\sigma=\uparrow, \downarrow$.

$$H = -t \sum_{i=1}^{L} \sum_{\sigma} \sum_{n' = 1}^{N} (c_{i\sigma n}^\dagger c_{i+1\sigma n'} + h.c.) + \frac{U}{2} \sum_{i=1}^{L} \sum_{\sigma \neq \sigma'} \sum_{n' \neq n''} n_{i\sigma n'} n_{i\sigma' n''} - \Delta \sum_{i} \sum_{\sigma} (n_{i\downarrow}^{(l)} - n_{i\downarrow}^{(u)}),$$

where $c_{i\sigma n}^\dagger$ is the creation operator for an electron with the orbital labeled by $n'=1,2,\cdots,N$. The band splitting $\Delta$, which is assumed to be caused either by the crystal-field effect or by the Zeeman effect, gives rise to the energetically separated bands with the dispersion, $\varepsilon(k) = -2t \cos k a_0 \pm \Delta$ where $a_0$ is the lattice spacing.

We deal with the case that the lower (upper) band is $p$-fold ($q$-fold) degenerate, for which the factor 2 due to the spin sector is included. The number of electrons is denoted as $n_{i\downarrow}^{(l)}$ ($n_{i\downarrow}^{(u)}$) for the lower (upper) band. Note that the system possesses SU(2)$N$ spin-orbital symmetry in the absence of the band splitting. For convenience, we use the number $n=2N$ and the index $m=1,2,\cdots,n(=2N)$, which specifies the spin as well as the orbital degrees of freedom on equal footing.

Passing to the continuum limit, we now introduce the right- and left-going fermion fields.

$$c_{im} \to \sqrt{a_0} \left[ e^{ik_F x} \psi_{+m}(x) + e^{-ik_F x} \psi_{-m}(x) \right],$$

where the Fermi momentum, $k_F$ represents either $k_F^{(l)}$ for
the lower band \((m = 1, \cdots p)\) or \(k_F^{(u)}\) for the upper band \((m = p + 1, \cdots n)\). These fermion fields are bosonized in terms of \(\Phi_m\) and its dual field \(\Theta_m\),

\[
\psi_{\text{ren}} = \frac{\eta_{\text{ren}}}{\sqrt{2\pi a_0}} \exp(-i [r\Phi_m - \Theta_m]),
\]

where the boson fields satisfy the commutation relation, \([\Phi_m, \Theta_n'] = (\pi/2) \delta_{mn} \cdot \text{sgn}(x' - x)\). The anti-commutation relation between fermions with different species is preserved by Majorana-fermion operator, \(\eta_{\text{ren}}\).

Let us now bosonize the Hubbard interaction. We consider a metallic case, for which the charge, spin, and orbital excitations are all massless. In this case, we can neglect both of the backward scattering and the Umklapp scattering around the low-energy fixed point. This simplification reduces the system to an effective multi-component Tomonaga-Luttinger (TL) model in external fields,

\[
H = \sum_m \int \frac{dx}{2\pi} \left\{ v_m [\partial_x \Phi_m(x)]^2 + v_m [\partial_x \Theta_m(x)]^2 \right\} + \frac{a_0 U}{\pi^2} \sum_{m \neq m'} \int \frac{dx}{\partial_x \Phi_m(x)} [\partial_x \Phi_{m'}(x)] ,
\]

where \(v_m = v_l\) for \(m = 1, \cdots p\) and \(v_m = v_u\) for \(m = p + 1, \cdots n\) are the Fermi velocities of elementary excitations.

Note that the Fermi velocities in the lower and upper bands are different, making the diagonalization of the Hamiltonian (4) a little bit complicated. Nevertheless, we can perform the diagonalization via the canonical transformation following the way done for a two-channel TL model [4]. To this end, we introduce the new boson fields, \(\Phi_c\), \(\Phi_\Delta\) and \(\Phi_{\sigma\nu}\) for the charge, orbital and spin sectors, respectively. For the lower band, the transformation is given by

\[
\Phi_m = \left[ \frac{1}{\sqrt{n}} \cos \alpha + \frac{\sqrt{pq}}{q^\sqrt{n}} y \sin \alpha \right] \Phi_c + \left[ -\frac{1}{\sqrt{n}} \sin \alpha + \frac{\sqrt{pq}}{q^\sqrt{n}} \cos \alpha \right] \Phi_\Delta + \sum_{m'=3}^{p+1} \sqrt{(m' - 1)(m' - 2)} \times \left[(m' - m - 1) - (m' - 2) \delta_{m',m+1}\right] \Phi_{\sigma_{m'-2}},
\]

for \(m = 1, 2, \cdots, p\), whereas for the upper band,

\[
\Phi_m = \left[ \frac{1}{\sqrt{n}} \cos \alpha - \frac{\sqrt{pq}}{q^\sqrt{n}} y \sin \alpha \right] \Phi_c + \left[ -\frac{1}{\sqrt{n}} \sin \alpha - \frac{\sqrt{pq}}{q^\sqrt{n}} \cos \alpha \right] \Phi_\Delta + \sum_{m'=p+2}^{n} \sqrt{(m' - p)(m' - p - 1)} \times \left[(m' - m) - (m' - p - 1) \delta_{m',m}\right] \Phi_{\sigma_{m'-2}},
\]

for \(m = p + 1, \cdots, n\). We thus end up with the effective Hamiltonian in the diagonalized form,

\[
H_c = \int \frac{dx}{2\pi} \left\{ \tilde{v}_c \left[\partial_x \Phi_c(x)\right]^2 + \tilde{v}_c \left[\partial_x \Theta_c(x)\right]^2 \right\},
\]

\[
H_\Delta = \int \frac{dx}{2\pi} \left\{ \tilde{v}_\Delta \left[\partial_x \Phi_\Delta(x)\right]^2 + \tilde{v}_\Delta \tilde{K}_\Delta \left[\partial_x \Theta_\Delta(x)\right]^2 \right\},
\]

\[
H_{\sigma\nu} = \int \frac{dx}{2\pi} \left\{ \tilde{v}_{\sigma\nu} \left[\partial_x \Phi_{\sigma\nu}(x)\right]^2 + \tilde{v}_{\sigma\nu} \tilde{K}_{\sigma\nu} \left[\partial_x \Theta_{\sigma\nu}(x)\right]^2 \right\},
\]

where \(\tilde{v}_{\sigma\nu} = \tilde{v}_l\) for \(\nu = 1, \cdots, p - 2\) and \(\tilde{v}_{\sigma\nu} = \tilde{v}_u\) for \(\nu = p - 1, \cdots, n - 2\). The explicit formulae for the parameters introduced in the transformation as well as the TL parameters are listed in the Appendix. Note that \(H_c\) \((H_\Delta)\) describes the Gaussian theory for charge excitations (inter-band excitations), for which the TL parameter \(K_c\) \((\tilde{K}_\Delta)\) characterizes the U(1) critical line. On the other hand, spin excitations in each band are described by \(H_{\sigma\nu}\), which still possess SU\((p)\) \((SU(2))\) symmetry for the upper (lower) band. This fixes the value \(K_{\sigma\nu} = 1\).

2.2 Expression for 1/T1 at low temperatures

We now derive the formula for the NMR relaxation rate, 1/T1. Let us first write down the time-dependent spin-spin correlation function at finite temperatures, \(\langle S^+(x,t)S^-(0,0) \rangle\),

\[
\sum_{\nu} \prod_{\mu} \left[ \sinh \frac{p u_{\nu} (x - \hat{\nu} \mu t)}{\sinh \frac{p u_{\nu} (x - \hat{\nu} \mu t)}{2 \Delta^\mu}} \right]^{2 \Delta^\mu},
\]

with \(\beta = \pi k_B T\), where \(\mu = (c, \Delta, \sigma, \nu)\) classifies massless modes in eq. (7). The phase factor \(e^{i\chi x}\) comes from the large momentum transfer across the Fermi points, where the momentum \(\kappa(= 2k_{F,L}^{(u)} + k_{F,R}^{(u)} + k_{F}^{(u)})\) specifies such excitations. Here \(\Delta^\mu\) are conformal dimensions which are related to the scaling dimension \(x_{\mu}\) as \(x_{\mu} = \Delta^c + \Delta^\mu\). The dynamical spin susceptibility is expressed in terms of the above correlation functions,

\[
\chi_{\perp}(k, \omega) = -i \int dt d\theta(t) \langle [S^+(x,t), S^- (0,0)] \rangle e^{i(\omega t - kx)}.
\]

The transverse spin correlation function, \(\langle S^z S^z \rangle\), can be expressed in a similar manner. By computing eqs. (8) and (9) with the use of eqs. (5), (6) and (7), we obtain a generic formula for 1/T1 at low temperatures,

\[
\frac{1}{T_1} = \lim_{\omega \to 0} \frac{2k_B T}{\hbar \omega} \int \frac{dk}{2\pi} \frac{A^2_k}{\Omega^2_k} \text{Im} \chi(k, \omega)
\]

\[
\sim \sum_{\kappa} B_{\kappa} \Gamma_{\kappa}(\Delta) T^{m_{\kappa} - 1},
\]

where

\[
\Gamma_{\kappa}(\Delta) = \left[ \prod_{\mu} \frac{1}{\bar{v}_l} \right]^{2 \Delta^\mu}.
\]
Here $A_\perp$ is hyperfine form factor and $B_c$ is a constant independent of the temperature. The corresponding critical exponents $\eta_\kappa = 2\sum_\mu \eta_\mu$ are obtained as,

$$
\eta_{2k_F^{(i)}} = 2 - \frac{2}{p} + \frac{2}{n} \left\{ \cos \alpha + \sqrt{\frac{2}{p}} \sin \alpha \right\}^2 \tilde{K}_e \\
+ \frac{2}{n} \left\{ -\sin \alpha + \sqrt{\frac{2}{q}} \cos \alpha \right\}^2 \tilde{K}_\Delta, \quad (12)
$$

$$
\eta_{2k_F^{(s)}} = 2 - \frac{2}{q} + \frac{2}{n} \left\{ \cos \alpha - \sqrt{\frac{2}{q}} \sin \alpha \right\}^2 \tilde{K}_e \\
+ \frac{2}{n} \left\{ -\sin \alpha - \sqrt{\frac{2}{q}} \cos \alpha \right\}^2 \tilde{K}_\Delta, \quad (13)
$$

$$
\eta_{k_F^{(i)} + k_F^{(s)}} = 2 - \frac{1}{p} - \frac{1}{q} \\
+ \frac{1}{2n} \left[ 2 \cos \alpha + \left( \sqrt{\frac{2}{p}} - \sqrt{\frac{2}{q}} \right) y \sin \alpha \right]^2 \tilde{K}_e \\
+ \frac{1}{2n} \left( \sqrt{\frac{2}{p}} + \sqrt{\frac{2}{q}} \right) \left( \sin \alpha \cos \alpha \right)^2 \tilde{K}_e^{-1} \\
+ \frac{1}{2n} \left[ -2 \sin \alpha + \left( \sqrt{\frac{2}{p}} - \sqrt{\frac{2}{q}} \right) \cos \alpha \right]^2 \tilde{K}_\Delta \\
+ \frac{1}{2n} \left( \sqrt{\frac{2}{p}} + \sqrt{\frac{2}{q}} \right) \left( \cos \alpha \right)^2 \tilde{K}_\Delta^{-1}. \quad (14)
$$

This completes the derivation of the formula for $1/T_1$, which generalizes the results of Sachdev to multicomponent cases. It is to be noted here that we have explicitly obtained the enhancement factor $\Gamma_\kappa$ in (10), which has been neglected in ordinary literatures. We shall see that this quantity plays an important role for the NMR relaxation rate in 1D systems, particularly when the system is located around the quantum phase transition point around which the velocities are dramatically renormalized. Some explicit examples are shown in the following section.

We have so far treated the model in the continuum limit within a weak coupling approach, so that it is not straightforward to evaluate $1/T_1$ as a function of the microscopic model parameters. In the following section we evaluate the renormalized parameters $\tilde{v}_\mu$ and $\tilde{K}_\mu$, by exploiting the solvable model to deduce characteristic behaviors in $1/T_1$.

§3. Exact Critical Properties of $1/T_1$

In this section, we exactly evaluate the renormalized parameters $\tilde{v}_\mu$ and $\tilde{K}_\mu$ to discuss critical properties of $1/T_1$ for the SU($n$) spin-orbital chain with band splitting. Also, the effect of hole-doping is discussed.

3.1 Spin-orbital systems

Let us start with the spin-orbital system in a insulating phase. In the limit $U \to \infty$, the spin-orbital sector in the degenerate Hubbard model (1) is reduced to the SU($n$) antiferromagnetic Heisenberg model (each site is occupied by one electron), which is written down in terms of the fermion operators,

$$
H_J = J \sum_{\mu=1}^L \sum_{i=1}^N \sum_{\sigma=\sigma'} \sum_{\mu''} \left( c_{\sigma \mu}^\dagger c_{\sigma' \mu''}^\dagger c_{\sigma' \mu''} c_{\sigma \mu} - n_i n_{i+1} \right), \quad (15)
$$

where only the single occupation of electrons is allowed at each site. The band splitting $\Delta$ due to, e.g. a crystalline field, forms two energetically separated bands, which are composed of the $p$ lower and $q$ upper bands including spin degrees of freedom, $n = 2N = p + q$.

The exact solution of the model (15) was obtained by Sutherland, and its properties have been clarified thus far. Following standard procedures, we can evaluate the velocities of elementary excitations and the critical exponents by applying finite-size scaling techniques to the excitation spectrum.

In the insulating phase, charge excitations are frozen, while spin excitations are still gapless which possess SU($p$) (SU($q$)) symmetry for the lower (upper) band. By applying the finite-size scaling to the excitation spectrum, we obtain the TL parameter as,

$$
\tilde{K}_\Delta = \frac{n}{pq} \xi^{\Delta}_{(p,q)}, \quad \alpha = 0, \quad (16)
$$

which generalizes the results of the SU($d$) case. In the above expression, the dressed charge $\xi^{\Delta}_{(p,q)} = \xi_{\Delta(p,q)}(\lambda_{p}^0)$, which features the critical line of the TL liquid, is given by the integral equation

$$
\xi_{\Delta(p,q)}(\lambda_p) = 1 + \int_{-\lambda_p^0}^{\lambda_p^0} G(\lambda_q - \lambda_p) \xi_{\Delta(p,q)}(\lambda_q) d\lambda_q, \quad (17)
$$

where $G(\lambda)$ is defined as

$$
G(\lambda) = F_{p,p}(\lambda) + F_{q,q}(\lambda) - \frac{1}{2n} \int_{-\infty}^{\infty} x^2 e^{ik\lambda} dk, \quad (18)
$$

$$
F_{\mu,\nu}(\lambda) = \frac{1}{2n} \int_{-\infty}^{\infty} x^{\mu-1} x^{\nu-1} e^{ik\lambda} dk, \quad (19)
$$

with $x = \exp(-|k|/2)$. Here the cut-off parameters $\pm \lambda_p^0$ are determined by minimizing the free energy for a given band splitting $\Delta$.

Having determined the TL parameter $\tilde{K}_\Delta$, it is now straightforward to derive the critical exponents for various correlation functions by exploiting the scaling relations obtained in eqs.(12)-(14). Note that the TL parameter $\tilde{K}_\Delta$ for the charge degree of freedom vanishes in the insulating system. Let us discuss the longitudinal spin susceptibility $\chi_{\perp}$, as an example. For excitations with momentum transfer $2k_F^{(i,u)}$ within the lower and upper bands, we obtain the exponents $\eta_{2k_F^{(i,u)}}$ from (12) and (13),

$$
\eta_{2k_F^{(i)}} = 2 \left[ 1 - \frac{1}{p} \right] + \frac{2}{np} \tilde{K}_\Delta, \quad (20)
$$

$$
\eta_{2k_F^{(u)}} = 2 \left[ 1 - \frac{1}{q} \right] + \frac{2}{nq} \tilde{K}_\Delta. \quad (21)
$$

On the other hand, for the momentum transfer $k_F^{(i)} \pm k_F^{(i)}$
we use the relation (14), which yields

\[
\eta_{k_F^{(i)}+k_F^{(j)}} = \left[2 - \frac{1}{p} - \frac{1}{q}\right] \frac{1}{2pq} K^{-1} + \frac{1}{2n} \left(\frac{q}{p} - \frac{p}{q}\right) K\Delta. \tag{22}
\]

The scaling relations (20), (21) and (22), which are the generalizations of the ordinary TL relations to multicomponent cases, characterize critical properties of the degenerate model with band splitting.

We note here that for the relaxation via excitations around the zero momentum transfer, the critical exponent is fixed to the canonical value, resulting in the specific formula for this relaxation channel,

\[
\frac{1}{T_1} \sim \left(\frac{1}{v}\right)^2 T, \tag{23}
\]

which holds for any value of the band splitting, although the velocity \(v\) may be changed according to the splitting. Note that the quantity \(1/v^2\), which originates from the enhancement factor \(\Gamma_\kappa\) in (10), is proportional to the square of the density of states for elementary excitations. Therefore, this expression is regarded as a 1D analogue of the Korringa relation.

In Fig. 1 we show the critical exponents as a function of the band splitting \(\Delta\). In the absence of the band splitting, \(\Delta = 0\), the TL parameter \(K\Delta\) is given by \(K\Delta = 1\), which results in \(\eta_{2k_F} = 2(1-1/n)\). Hence the power-law temperature dependence becomes less singular as the orbital degeneracy is increased. Although the critical exponents vary continuously when \(\Delta\) is increased up to its critical value \(\Delta_c\), their values do not change so dramatically to induce the qualitative difference in the temperature dependence except for \(n = 2\) (single orbital case). On the other hand, beyond \(\Delta_c\), the upper band becomes massive, being separated from the lower band. Thus for \(\Delta > \Delta_c\), the low-energy physics can be described by the SU\((n/2)\) spin model. Correspondingly, the critical exponent shows a discontinuity at \(\Delta_c\). For example, \(\eta_{2k_F}\) abruptly changes from \(2(1-1/p+1/p^2)\) to \(2(1-1/p)\) for the SU\((n)\) model at \(\Delta = \Delta_c\). As claimed by Yamashita and Ueda,\[ this discontinuity does not imply that the relaxation rate itself possesses such singularity, but there instead occurs the crossover behavior in \(1/T_1\). This point is discussed in more detail in the next section.

We now observe how the enhancement factor \(\Gamma_\kappa\) in (10) behaves as a function of the band splitting. The computed results for \(\Gamma_\kappa\) are shown in Fig. 2. As \(\Delta\) is increased, \(\Gamma_\kappa\) is increased monotonically and is divergently enhanced near \(\Delta = \Delta_c\). This characteristic behavior is caused by the renormalization of the velocities, which gives rise to the edge singularity in density of states. This may be regarded as a 1D analog of the “mass enhancement” well known for the Fermi liquid in three dimension. Note that \(\Gamma_\eta\) as well as \(\Gamma_{2k_F}\) within the lower band still has the finite values even for \(\Delta > \Delta_c\), whereas the others should vanish after divergently increased at \(\Delta_c\). As mentioned before, such a singularity is apparent, and should be replaced by the crossover behavior. We note that the above enhancement factor has been usually neglected to analyze the temperature dependence of \(1/T_1\), however, when the field-dependence of \(1/T_1\) is studied at low temperatures, this should be properly taken into account, in particular around the critical point \(\Delta = \Delta_c\).
3.2 Hole-doped systems

We now turn to a hole-doped case. For a metallic system with finite holes, we consider the multicomponent \( t-J \) model,

\[
H = -t \sum_{i=1}^{L} \sum_{\sigma} \sum_{n=1}^{N} \left( c_{i\sigma n}^\dagger c_{i+1\sigma n'} + h.c. \right) + H_J.
\]

For the supersymmetric case, \( t = J \), the exact solution was obtained \( \mathcal{H} \) which will be used here. Though this model is rather special in its appearance, some essential properties for a doped system can be described well, as is shown below.

In Fig. 3, we show the critical exponents for the SU(\( n \)) symmetric case without the band-splitting \( \Delta = 0 \) (\( n = 2, 4, 6 \)). In this case, the critical exponent \( \eta = \eta_{2k_p} = \eta_{k_p}^{(s)} = \eta_{k_p}^{(i)} + \eta_{k_p}^{(s)} \) is

\[
\eta = 2 - \frac{2}{n} + \frac{2}{n} \tilde{K}_c, \quad \tilde{K}_c = \frac{1}{n} \xi_2^{(n)},
\]

where the dressed charge \( \xi_{h(n)} = \xi_{h(n)}(\lambda_n^0) \) is given by

\[
\xi_{h(n)}(\lambda_n) = 1 + \int_{-\lambda_n^0}^{\lambda_n^0} F_{n,n}(\lambda_n - \lambda_n') \xi_{h(n)}(\lambda_n') d\lambda_n'.
\]

We note that the relation (25) generalizes the TL scaling relation to SU(\( n \)) electron systems. The computed results are shown in Fig.3. When the hole concentration is increased (\( \mu \) is decreased), the critical exponent for the NMR relaxation rate, \( \eta \), is monotonically increased from \( 2(1 - 1/n + 1/n^2) \) up to 2 (band bottom), because \( \tilde{K}_c = 1/n \rightarrow 1 \) in this case. We are now discussing a rather special solvable model, so that the global feature of the critical exponent should depend on the model. However, characteristic properties close to the insulating phase (\( \mu \approx \mu_c \)) is expected to be rather general because in the small hole-doping region the charge sector is scaled to the strong coupling fixed point (hard-core bosons) irrespective of the bare value of \( J \).

Fig. 3. Critical exponents of the NMR relaxation rate for the SU(\( n \)) supersymmetric \( t-J \) model (\( n = 2, 4, 6 \)) as a function of the chemical potential \( \mu \). At \( \mu = \mu_c \), the system becomes the Mott insulator: \( \mu_{c1} \approx 1.39, \mu_{c2} \approx 0.44 \) and \( \mu_{c3} \approx 0.22 \), respectively for \( n = 2, 4, 6 \).

To see such properties for the critical exponents, we now focus on the hole-doped case close to insulator (\( \mu \sim \mu_c \)). In this limiting case, the TL parameters in (12), (13) and (14) are reduced to,

\[
\tilde{K}_c = \frac{n}{pq} \xi_{\Delta(p,q)}, \quad \tilde{K}_c = \frac{1}{n}, \quad \cos \alpha = 1, \quad y \sin \alpha = -\sqrt{\frac{p}{q}} \left( 1 - \frac{n}{p} \xi_{B(p,q)} \right) \left( 1 - \frac{1}{y} \right) \sin \alpha = 0,
\]

where in addition to the dressed charge (16), we have introduced another dressed charge \( \xi_{B(p,q)} = \xi_{B(p,q)}(\lambda_n^0) \) which incorporates the interference between the charge and spin degrees of freedom,

\[
\xi_{B(p,q)}(\lambda_n) = \int_{-\lambda_n^0}^{\lambda_n^0} F_{2,q}(\lambda_n - \lambda_n') \xi_{B(p,q)}(\lambda_n') d\lambda_n'.
\]

In Fig. 4, the critical exponents in a metallic system with the small hole concentration are shown as a function of the band splitting \( \Delta \) (\( n = 2, 4, 6 \)). As soon as holes are doped into the insulator, all the exponents are increased (compare Fig.1 with Fig.4), because the charge sector couples with the spin sector, and thus affects the relaxation process via "gauge interaction" due to Fermi statistics. For the limiting case, \( \Delta \rightarrow \Delta_c \), the critical exponents approach

\[
\eta_{2k_p}^{(i)} = 2 - \frac{4}{n} + \frac{8}{n^2},
\]

\[
\eta_{2k_p}^{(s)} = 2 - \frac{4}{n} + \frac{16}{n^2},
\]

\[
\eta_{k_p}^{(i)+k_p} = 2 - \frac{4}{n} + \frac{1}{2} + \frac{2}{n^2}.
\]

Beyond this critical band-splitting, the NMR relaxation...
rate is given by that for the SU(n/2) t-J model without band-splitting. Thus the critical exponent is discontinuously changed to smaller values at $\Delta = \Delta_c$ for any case of $n$.

We wish to mention that the coefficient $\Gamma_c$ is enhanced for small hole-doping, since the charge velocity $v_c$ in eq. (11) is very small in this region. However, it may not be easy to observe this enhancement due the renormalized charge velocity, because the hole-doping itself increases the value of the exponents, thus having a tendency to hide singular behaviors at low temperatures.

§4. Dynamical Spin Susceptibility

We have shown that the critical exponents as well as the enhancement factor exhibit singularities around the critical point $\Delta_c$. As already mentioned, such singularities may not be observed in NMR measurements, but should be replaced by the crossover behavior. In order to clearly see how the crossover behavior emerges, we discuss the dynamical spin susceptibility $\Im \chi(k, \omega)$ in the low frequency regime.

By exploiting the velocities and the TL parameters determined by the exact solution, we have computed $\Im \chi(k, \omega)$ at finite temperatures by using the expression $\frac{1}{\omega}$. To be specific, we restrict ourselves to the SU(4) spin-orbital model with the band splitting.

In Fig. 5, the spectral function $\Im \chi(k, \omega)/\omega$ is shown for $\Delta = 0$ at finite temperatures. For all the momenta shown in Fig. 5 (a), the broad maximum structure is found in the spectral function, indicating the presence of overdamped excitations. This is because we are dealing with the case of higher temperatures in (a), $h\nu q \leq k_B T$ ($\nu = \pi/2$). On the other hand, if the temperature is decreased with $q$ being fixed, the sharp peak structure is developed around $\omega = h\nu q$ in the spectral function (see (b) for which $h\nu q \gg k_B T$) which indicates the presence of propagating spinons with the above dispersion relation. These behaviors for the propagating mode are analogous to those observed for the SU(2) spin chain. Note that although there are three kinds of massless modes, all the velocities are the same for the SU(4) symmetric case with $\Delta = 0$, so that we can observe only one spinon mode in the spectral function $\Im \chi(k, \omega)/\omega$. The integration over $q$ in the case of (b) determines the low-temperature power-law behavior of $1/T_1$ discussed in the previous section.

We next discuss the case with the finite band splitting close to its critical value $\Delta_c$, where we have encountered the singularities in the NMR relaxation rate. In Fig. 6, we display the spectral function $\Im \chi(k, \omega)/\omega$ by choosing two typical values of the temperature. There are three dominant low-energy excitations with different velocities around the momenta $2k_F^{(u)}$, $k_F^{(l)} + k_F^{(s)}(= \pi/2)$ and $2k_F^{(l)}$. At higher temperatures shown in panels (a), (b) and (c), we can see the broadened maximum structure at low frequencies, which is caused by overdamped spin excitations, as mentioned above. When the temperature is lowered, each excitation starts to show different behaviors in the spectral function. We begin with the spectral function for the $2k_F^{(u)}$ mode in (d), and the $k_F^{(l)} + k_F^{(s)}$ mode in (e). By comparing them with those for the $\Delta = 0$ case, we find that the peak structure in the spectral weight is shifted to the extremely low frequency regime, and its peak-position is monotonically increased with the increase of the momentum $q$, reflecting the fact that the velocities of these excitations become very small around $\Delta_c$.

Recall here that the NMR relaxation rate (10) is determined by the integration of the spectral function over the momentum in the limit $\omega \rightarrow 0$. In the previous section, we have shown that $1/T_1$ is extremely enhanced by the velocity renormalization around $\Delta_c$. However, we should keep in mind the following things. We are now considering the continuum limit of the original lattice model, so that when the system is close to the critical point $\Delta_c$, the energy range where such a treatment may be valid becomes pretty small. For instance, in the above case, the energy range where the dispersion relation $\omega \simeq h\tilde{v}_q q$ holds should be in $-k_F^{(u)} \sim q \sim k_F^{(u)}$ (the cutoff is given by $k_F^{(u)} \simeq 0.15$ for $\Delta = 0.66$). Therefore the enhancement of $1/T_1$ can be seen in the very low frequency regime within the small cutoff parameter. Hence, even if such enhancement is naively expected to be observed at low temperatures, it may be obscured when the temperature is increased beyond the above cutoff energy.

In contrast to the above two modes, the spectral function for the $2k_F^{(l)}$ mode in Fig. 6(f) shows quite different behaviors; it develops two peaks of propagating spinons around $2k_F^{(l)}$, whose dispersions are given by $\omega \simeq h\tilde{v}_q q$ and $\omega \simeq h\tilde{v}_q q$, respectively. When the system is close to
the critical point, the interband excitations with the dispersion $\Delta g$ becomes less important. On the other hand, in such cases, the excitation of $\omega \approx \hbar \tilde{v} \Delta q$ within the lower band becomes more dominant to control the relaxation process. In fact, beyond $\Delta_c$, this excitation completely determines the $1/T_1$ at low temperatures.

From the above analyses of the dynamical spin susceptibility, we can see how the enhancement of $1/T_1$ in the vicinity of the critical field naturally crossovers to the region where $1/T_1$ is dominated by excitations in the lower band, when the energy or the temperature is raised.

§5. Summary

We have studied the NMR relaxation rate $1/T_1$ for 1D multicomponent spin-orbital systems. By generalizing the bosonization approach of Sachdev, we have obtained a generalized formula of $1/T_1$ for multicomponent quantum systems with the band splitting. We have then exactly estimated the relaxation rate by using the SU($n$) integrable model for the insulating as well as metallic phases. It has been found that the power-law temperature dependence of $1/T_1$ becomes less singular as the orbital degeneracy is increased. This is also the case for the finite band splitting, as far as $\Delta$ is smaller than its critical value $\Delta_c$. We have also pointed out that the relaxation rate may be enhanced around $\Delta_c$ due to the dramatic renormalization of the velocities. This type of the enhancement may be common to 1D quantum systems when the spin/orbital gap is formed or collapsed by external fields. In particular, this effect becomes important to analyze the field-dependence of $1/T_1$ at low temperatures. It may be quite interesting to experimentally observe the enhancement in $1/T_1$, or the corresponding crossover behavior at very low temperatures in the NMR experiments.

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Appendix

The parameters for the canonical transformation in (5) and (6) are obtained as,

$$\tan 2\alpha = \frac{\delta v}{v_0(y - \gamma y^{-1})}, \quad v^2 = \frac{K_c^2 + \gamma}{\gamma K_{c\Delta}^2},$$

$$v_0 = \frac{1}{n}(p v_l + q v_u), \quad \delta v = \frac{1}{2}(v_l - v_u),$$

$$\gamma = \frac{q v_l + p v_u}{p v_l + q v_u},$$

where $v_l$ and $v_u$ are the bare velocities, and the TL parameters $K_c$ and $K_{c\Delta}$ are given by

$$K_c = \left(1 + \frac{2(n-1) a_0 U}{\pi v_0} \right)^{-1/2},$$

$$K_{c\Delta} = \left(1 - \frac{2(n-1) a_0 U}{\pi v_0} \right)^{-1/2}.$$

The above set of the parameters in the transformation gives the diagonalized Hamiltonian in the text, for which the renormalized velocities, $\tilde{v}_c(\Delta)$ and the TL parameters, $\tilde{K}_c(\Delta)$ read,

$$\tilde{v}_c^2 = \frac{4pq}{N^2} \delta v^2 + \frac{1}{2} v_0^2 \left[ (K_c^{-2} + \gamma^2 K_{c\Delta}^{-2}) \pm T(\alpha) \right],$$

$$\tilde{K}_c^2 = \frac{\gamma^2 K_{c\Delta}^{-2} + 2\gamma}{[K_c^2(1 + 2\gamma K_{c\Delta}^{-2}) + \gamma^2 K_{c\Delta}^{-2}] \pm T(\alpha)},$$

where $T(\alpha) = (K_c^{-2} - \gamma^2 K_{c\Delta}^{-2})\sqrt{1 + \tan 2\alpha}$.

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