Frequency-up conversion and quantum swap gate in an optical cavity with atomic cloud

Gong-Wei Lin¹, Xu-Bo Zou², Ming-Yong Ye¹, Xiu-Min Lin¹ and Guang-Can Guo²

¹School of Physics and Optoelectronics Technology, Fujian Normal University, Fuzhou 350007, People’s Republic of China and ²Key Laboratory of Quantum Information, Department of Physics, University of Science and Technology of China, Hefei 230026

A scheme is presented for realizing frequency-up conversion and a two-qubit quantum swap gate for intracavity fields. In the scheme, a V-type atomic ensemble prepared in their ground states collectively mediates the interaction between the two cavity modes. Under certain conditions, the cavity-field degree of freedom is decoupled from the atomic degrees of freedom, and the effective coupling strength between the two cavity modes scales up with $\sqrt{N}$ (N is the number of atoms). The numerical simulation shows that the quantum swap gate still has a high fidelity under the influence of the atomic spontaneous emission and the decay of the cavity modes.

I. INTRODUCTION

Cavity quantum electrodynamics (QED) devices hold great promise as basic tools for quantum networks since they provide an interface between computation and communication, i.e., between atoms and photons. In this context, it is a very important task to coherently manipulate and control quantum state of not only the atom but also the intracavity fields. Throughout the last decade, in cavity QED including the microwave and optical regimes many efforts have been devoted to generation of entangled states, nonclassical states, and the implementation of quantum logical gates for cavity fields. Some relevant examples are Schrödinger cat state, Fock state of a single-mode cavity field, the maximally entangled state of two cavity modes, pair coherent states and SU(2) coherent states. In fact, three different schemes were proposed to realize a quantum phase gate of two intracavity modes in which a single atom with Ξ-type, V- type, or Λ-type configuration mediated the interaction between them. Recently, Serra et al. and Zou et al. showed the methods for realizing frequency up- and down-conversions in a two-mode microwave cavity based on dispersive interaction and resonant interaction, respectively.

Most of the schemes mentioned above are based on the interaction of single atoms and the cavity fields. In the past few years, many interesting schemes have been proposed that use atomic ensembles with a large number of identical atoms placed in an optical cavity as the basic system for quantum state engineering and quantum information processing. In particular, several experiments have been reported about continuous variable entanglement using cold atoms and the photon statistics of the light emitted from an atomic ensemble into a single cavity mode. Theoretically, some schemes have also been proposed for realizing frequency down-conversions— two-mode field squeezing using atomic ensemble as medium and carrying out quantum phase-gate operation for two single photons using an N-type atomic ensemble trapped in an optical cavity or an M-type atomic ensemble trapped in a gas cell. The above schemes based on atomic ensembles have some special advantages compared with those based on the control of single atoms: first, laser manipulation of atomic ensembles without separately addressing the individual atoms is normally easier than the coherent control of single particles; second, and more important, atomic ensembles could have some kinds of collectively enhanced coupling to certain optical mode due to the many-atom interference effects.

In this paper, we present a method that realizes frequency up-conversions and a quantum swap gate for two cavity modes with a V-type atomic ensemble in an optical cavity. In the scheme, the cavity-field degree of freedom is decoupled from the atomic degrees of freedom and the effective coupling strength scales up with $\sqrt{N}$ (here N is the number of atoms). The scheme is robust against decoherence processes, such as spontaneous emission, and does not require a strong coupling regime or precise atomic localization. The numerical simulation shows that the swap gate has high fidelity and small error rate under the influence of the atomic spontaneous emission and the decay of the cavity modes.

II. THE FUNDAMENTAL MODEL AND REALIZATION OF FREQUENCY-UP CONVERSION

Our model consists of an ensemble of N identical V-type atoms inside a two-mode optical cavity. The concrete atomic level structure and relevant transitions are shown in Fig. 1. Each atom has a stable ground state $|g\rangle$ and two excited states $|e_1\rangle$ and $|e_2\rangle$. The transition $|g\rangle \leftrightarrow |e_{j1}\rangle$ ($|g\rangle \leftrightarrow |e_{j2}\rangle$) of the jth atom couples to the cavity mode a (b) with coupling strength $g_{ja}$ ($g_{jb}$).
while laser field with Rabi frequency $\Omega_j$ drives the transition $\ket{e_j} \leftrightarrow \ket{g_j}$ ($\Omega_j e^{i\phi_j}$) can be realized by using a two-photon Raman transition via level $\ket{g_j}$, or a fourth level [20]). The Hamiltonian for the whole system in the interaction picture can be written as

$$H_I = H_{\text{cav}} + H_{\text{cla}},$$

with $H_{\text{cav}} = \sum_{j=1}^{N} (g_{ja} \ket{e_j} \bra{g_j} + g_{jb} \ket{g_j} \bra{e_j}) + H.c.$ and $H_{\text{cla}} = \sum_{j=1}^{N} \Omega_j e^{i\phi_j} \ket{e_j} \bra{e_j} + H.c.$

After defining the new atomic basis $\ket{+} = (\ket{e_{j1}} + e^{i\phi_j} \ket{e_{j2}})/\sqrt{2}$, $\ket{-} = (\ket{e_{j1}} - e^{i\phi_j} \ket{e_{j2}})/\sqrt{2}$ [21], one can rewrite $H_{\text{cav}}$ and $H_{\text{cla}}$ as $H_{\text{cav}} = \sum_{j=1}^{N} (g_{ja} \ket{+} \bra{+} + g_{jb} \ket{-} \bra{-})$ and $H_{\text{cla}} = \sum_{j=1}^{N} \Omega_j \ket{+} \bra{+} - \ket{-} \bra{-}$.

The time evolution of this system is decided by Schrödinger’s equation

$$i [d |\Psi(t)\rangle /dt] = H_I |\Psi(t)\rangle .$$

After performing the unitary transformation

$$|\Psi(t)\rangle = e^{-iH_{\text{cla}}t} |\Psi'(t)\rangle ,$$

one can obtain

$$i [d |\Psi'(t)\rangle /dt] = H_{11} |\Psi'(t)\rangle ,$$

where

$$H_{11} = \sum_{j=1}^{N} \left( \frac{g_{ja}}{\sqrt{2}} (e^{i\Omega_j t} \ket{+} \bra{+} + e^{-i\Omega_j t} \ket{-} \bra{-}) + \frac{g_{jb}}{\sqrt{2}} b( e^{i\Omega_j t} \ket{+} \bra{+} - e^{-i\Omega_j t} \ket{-} \bra{-}) \right) + H.c.$$  

(5)

Assuming that all atoms are initially in the ground state $\prod_{j=1}^{N} |g_j\rangle$, and

$$\Omega_j \gg \sqrt{N} |g_{ja}|, \sqrt{N} |g_{jb}| ,$$

(6)

we neglect the effect of rapidly oscillating terms. Using the time-averaging method of Ref. [22, 23], we can further reduce $H_{11}$ to an effective interaction Hamiltonian

$$H_{eff} = -(\xi a^\dagger b + \xi^* b^\dagger a) \prod_{j=1}^{N} |g_j\rangle \langle g_j| ,$$

(7)

where $\xi = \sum_{j=1}^{N} \frac{g_{ja} g_{jb} e^{-i\phi_j}}{\Omega_j}$ is the effective coupling constant. Eq. (7) is the expected frequency up-conversion process, which belongs to a group of nonlinear phenomena of great practical importance and also has very interesting theoretical aspects [24]. Compared with the previous theoretical protocols for realizing frequency up-conversion with an atom in a microwave cavity [8, 10], the effective coupling constant $\xi$ in our scheme scales with $\sqrt{N}$ though the Rabi frequency of laser field $\Omega_j$ should satisfy the conditions in Eq. (6). In principle we can obtain a desired coupling strength as long as enough atoms simultaneously interact with the cavity modes and laser field.

### III. QUANTUM SWAP GATE

Next we will show in detail the implementation of a quantum swap gate based on Eq. (7). Suppose that the zero- and one-photon Fock states of two different polarization (or different frequency) modes of the radiation field inside the cavity represent two logical states of a qubit. When the atoms are initially prepared in their ground states $\prod_{j=1}^{N} |g_j\rangle$ through optical pumping [14] and are driven by the two cavity modes and the appropriate laser field (Fig. 1), the time evolution of four logical states for two qubits, under the effective Hamiltonian in Eq. (7), are given by
where we have assumed $\xi$ is real without loss of generality.

After the effective interaction time $t = \frac{\tau}{\gamma}$, one has

\[
\begin{align*}
|0\rangle|0\rangle &\rightarrow |0\rangle|0\rangle, \quad |0\rangle|1\rangle \rightarrow i|1\rangle|0\rangle, \\
|1\rangle|0\rangle &\rightarrow i|0\rangle|1\rangle, \quad |1\rangle|1\rangle \rightarrow -|1\rangle|1\rangle,
\end{align*}
\]

which represent a swap gate operation, apart from phase factors that can be eliminated by an appropriate setting of the phase of subsequent logic operations [25]. We note that the swap gate is not a required composition of elementary gates from a universal set for quantum computation, which can be decomposed into three quantum phase gates and six Hadamard gates [26, 27]. Our aim for the direct construction of a specific gate—swap gate is to reduce the number of required physical logical gates in practical quantum information processes. We also note that Sangouard et al. [28] and Yavuz [29] have proposed the scheme for direct swap gate for the atoms by adiabatic passage and for the photons by using electromagnetically induced transparency, respectively.

In the following, we analyze the fidelity of the swap gate and the photon loss during the gate operation. We suppose that no photon is detected by the atomic spontaneous emission or by the leakage of a photon through the cavity mirrors. The evolution of the system is governed by the non-Hermitian Hamiltonian

\[
H'_1 = H_I - \frac{i}{2} \sum_{m=1}^{N} \sum_{j=1}^{N} \gamma_m |e_{jm}\rangle\langle e_{jm}| - \frac{i}{2}(\kappa_a a^\dagger a + \kappa_b b^\dagger b),
\]

where $\kappa_a$ and $\kappa_b$ denote the decay rate of the cavity mode fields $a$ and $b$, $\gamma_m$ is spontaneous emission rate of the excited state $|e_m\rangle$. For the sake of convenience, we assume $\kappa_a = \kappa_b = \kappa$, $\gamma_1 = \gamma_2 = \gamma_s$, $|g_{ja}\rangle = |g_j\rangle$, $g_{jb} = g$, and $\Omega_j = \Omega$ in the following numerical simulations [30], so it can be seen $\xi = \frac{N^{1/2}}{\sqrt{2}}$. If the initial state of the atom-cavity system is prepared in $|\Psi(0)\rangle = \prod_{j=1}^{N} |g_j\rangle \otimes (|0\rangle|0\rangle + |0\rangle|1\rangle + |1\rangle|0\rangle + |1\rangle|1\rangle)/\sqrt{2}$, after the effective interaction time $t = \frac{\tau}{\gamma}$ under the non-Hermitian Hamiltonian in Eq. (10), the conditional state can be described by $|\Psi(t_1)\rangle = \prod_{j=1}^{N} |g_j\rangle \otimes (\alpha_{00} |0\rangle|0\rangle + \alpha_{01} |0\rangle|1\rangle + \alpha_{10} |1\rangle|0\rangle + \alpha_{11} |1\rangle|1\rangle) + (\Phi_1) \otimes (\beta_{00} |0\rangle|0\rangle + \beta_{01} |0\rangle|1\rangle + \beta_{10} |1\rangle|0\rangle + \beta_{11} |1\rangle|1\rangle) + (\Phi_2) \otimes (\eta_{00} |0\rangle|0\rangle + \eta_{01} |0\rangle|1\rangle + \eta_{10} |1\rangle|0\rangle + \eta_{11} |1\rangle|1\rangle) + (\Phi_3) \otimes (\zeta_{00} |0\rangle|0\rangle + \zeta_{01} \langle 0|1\rangle + \zeta_{10} |1|0\rangle + \zeta_{11} \langle 1|1\rangle) + (\Phi_4) \otimes (\delta_{00} |0\rangle|0\rangle + \delta_{01} |0\rangle|1\rangle + \delta_{10} |1\rangle|0\rangle + \delta_{11} |1\rangle|1\rangle)$

here $|\Phi_1\rangle = \sum_{n=1}^{N} \sqrt{n} |e_n\rangle \otimes \prod_{j=1,j\neq n}^{N} |g_j\rangle$, $|\Phi_2\rangle = \sum_{n=1}^{N} \sqrt{n} |e_n\rangle \otimes \prod_{j=1,j\neq n}^{N} |g_j\rangle$, $|\Phi_3\rangle = \sum_{n=1,m=1,m\neq n}^{N} \sqrt{n} |e_n\rangle \otimes \prod_{j=1,j\neq n,j\neq m}^{N} |g_j\rangle$, $|\Phi_4\rangle = \sum_{n=1,m=1,m\neq n}^{N} |e_n\rangle \otimes \prod_{j=1,j\neq n,j\neq m}^{N} |g_j\rangle$. So the photon loss due to quantum jump can be calculated by $P_{\text{loss}} = 1 - \langle \Psi(t_1)| \Psi(t_1) \rangle$. Fig. 2 (a) plots the numerical calculation of the photon loss as a function of $g/\kappa$. The fidelity of the swap gate, which depends on the initial state of the system, is an efficient measure of the distance between the quantum logic gates. We define the fidelity ($F$) of the swap gate as $F = |\langle \Psi_{\text{ideal}}| \Psi_{\text{out}} \rangle|^2$, where $|\Psi_{\text{ideal}}\rangle = \prod_{j=1}^{N} |g_j\rangle \otimes (|0\rangle|0\rangle + i|1\rangle|0\rangle + i|0\rangle|1\rangle - |1\rangle|1\rangle)/2$ and $|\Psi_{\text{out}}\rangle = |\Psi(t_1)\rangle / \sqrt{\langle \Psi(t_1)| \Psi(t_1) \rangle}$ since we have assumed that no quantum jump actually occurs during our implementation of the scheme [29]. As shown in Fig. 2 (b), the swap gate has high fidelity even if the strongly coupled condition is not satisfied.

### IV. DISCUSSION AND CONCLUSION

In an optical cavity, the coupling strengths $g_{ja}$, $g_{jb}$, and $\Omega_j$ depend on the position $r$ of atom $j$. However, in presence of a large number of atoms, due to the many-atom interference effects resulting in the atoms collectively enhanced coupling to certain optical mode, it is possible to estimate that even for a high effective coupled parameter $\xi$ and does not require a strong coupling regime or precise atomic localization inside a wavelength [17], which is important for current experimental technique. In our scheme the atoms are prepared in their ground states $\prod_{j=1}^{N} |g_j\rangle$ and no atomic transition is required. Thus our scheme is robust against atomic spontaneous emission.

Next we briefly address the experimental feasibility of the proposed scheme. We consider a low density vapor of $^{85}$Rb in an optical cavity. The required atomic level configuration can be chosen from the hyperfine states of $^{85}$Rb. For instance, the lower level $|g\rangle$ is the $F = 2$ hyperfine state of the $5S_{1/2}$ electronic ground state, while $|e_1\rangle$ and $|e_2\rangle$ are the $F = 3$ hyperfine states of the $5P_{3/2}$ electron excited states, respectively. Using the cavity parameters of Ref. [31], we have the waist of the cavity modes $\omega \sim 35 \mu m$, the waist of the homogeneous
FIG. 2: (Color online) Photon loss and fidelity of the phase gate vs $g/\kappa$ in (a) and (b), respectively. Other common parameters: $\kappa = \gamma_s$, $g_a = g_b = g$, $N = 4 \times 10^4$, $\Omega = 20\sqrt{N}g$.

classical laser beam $d \sim 50\mu m$, and $(g, \kappa, \gamma_s)/2\pi \sim (16, 1.4, 3) kHz$. With the choices of $N = 4 \times 10^4$ and $\Omega = 20\sqrt{N}g$, we obtain an approximate atom number density of $0.8 \times 10^{12}/cm^3$ (small enough to prevent coherence losses due to collisions), and the effective coupling constant $\xi \sim 10g$. Thus the swap gate operation time is $t \sim 1.6ns$, which is much smaller than the photon lifetime $t_c \sim 1/\kappa \sim 0.2\mu s$. Hence, our scheme fits well the status of current experimental technology.

In summary, we have proposed a scheme to realize frequency-up conversion and a swap gate for intracavity fields with atomic ensemble. In our scheme, the atoms collectively enhance coupling strength to the intracavity fields due to the many-atom interference effects. In presence of a large number of atoms, our scheme may relax the requirements of the strong-coupling condition and Lamb-Dicke limit, which is important for current experimental technique. We also discuss the fidelity of the swap gate and the probability of the photon loss under the influence of the atomic spontaneous emission and the decay of the cavity modes. It is shown that the swap gate has a high fidelity and small error rate.

Acknowledgments: This work was funded by National Natural Science Foundation of China (Grant No. 10574022), the Natural Science Foundation of Fujian Province of China (Grant No. 2007J0002 and No. 2006J0230), the Foundation for Universities in Fujian Province (Grant No. 2007F5041), and “Hundreds of Talents” program of the Chinese Academy of Sciences.

[1] J. I. Cirac et al., Phys. Scr., T 76, 223 (1998).
[2] M. Brune et al., Phys. Rev. Lett. 77, 4887 (1996).
[3] S. Brattke, B. T. H. Varcoe, and H. Walther, Phys. Rev. Lett. 86, 3534 (2001); P. Bertet et al., ibid. 88, 143601 (2002).
[4] A. Rauschenbeutel et al., Phys. Rev. A 64, 050301 (R) (2001).
[5] S. B. Zheng, Phys. Rev. A 74, 043803 (2006).
[6] E. Solano et al., Phys. Rev. A 64, 024304 (2001); M. Suhail Zubairy et al., Phys. Rev. A 68, 033820 (2003).
[7] R. García-Maraver et al., Phys. Rev. A 70, 062324 (2004).
[8] J. Shu et al., Phys. Rev. A 75, 044302 (2007).
[9] R. M. Serra et al., Phys. Rev. A 71, 045802 (2005).
[10] X. B. Zou et al., Phys. Rev. A 73, 025802 (2006).
[11] A. Søndberg Sørensen and K. Mølmer, Phys. Rev. A 66, 022314 (2002).
[12] M. D. Lukin et al., Phys. Rev. Lett. 84, 4232 (2000).
[13] A. Dantan et al., Phys. Rev. Lett. 94, 050502 (2005).
[14] L. M. Duan et al., Nature (London) 414, 413 (2001).
[15] V. Josse, A. Dantan, A. Bramati, M. Pinard, and E. Giacobino, Phys. Rev. Lett. 92, 123601 (2004).
[16] M. Heinrich, A. Kuhn, and G. Rempe, Phys. Rev. Lett. 94, 053604 (2005).
[17] R. Guzmán, J. C. Retamal, E. Solano, and N. Zagury, Phys. Rev. Lett. 96, 010502 (2006); A. S. Parks, E. Solano, and J. I. Cirac, ibid. 96, 053602 (2006).
[18] Y. F. Xiao et al., Phys. Rev. A 74, 044303 (2006).
[19] C. Ottaviani et al., Phys. Rev. A 73, 010301 (R) (2006).
[20] J. Metz et al., Phys. Rev. Lett. 97, 040503 (2006); L. M. Duan, Phys. Rev. Lett. 88, 170402 (2002).
[21] Shi-Biao Zheng, Phys. Rev. A 65, 051804 (R) (2002).
[22] M. S. Zubairy, M. Kim, and M. O. Scully, Phys. Rev. A 68, 033820 (2003).
[23] D. F. V. James, Fortschr. Phys. 48, 823 (2000).
[24] For a review see F. Zernike and I. E. Midwinter, Applied Nonlinear Optics (Wiley, New York, 1973); R. G. Byer,
in Nonlinear Optics, edited by P. G. Harper and B. S. Wheerett (Academic, New York, 1977).

[25] A. Barenco, D. Deutsch, A. Ekert, and R. Jozsa, Phys. Rev. Lett. 74, 4083 (1995).

[26] F. Vatan and C. Williams, Phys. Rev. A 69, 032315 (2004).

[27] N. Sangouard et al., Phys. Rev. A 72, 062309 (2005).

[28] D. D. Yavuz, Phys. Rev. A 71, 053816 (2005).

[29] M. B. Plenio and P. L. Knight, Rev. Mod. Phys. 70, 101 (1998).

[30] For simplification of the numerical simulations, we assume all the atoms are in the same situation. In our scheme, this hypotheses is not prerequisite due to the collective effect of the atomic ensemble (see [17]).

[31] M. Hennrich et al., Phys. Rev. Lett. 85, 4872 (2000); P. Maunz et al., ibid. 94, 033002 (2005).