Multi-missile Coordination High Precision Guidance and Control Method for Beam-riding Guidance

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Abstract. Due to the limitation of target indication error and steady-state tracking error, it is difficult for the existing beam-riding guidance weapon system to attack long-range and high-maneuvering targets accurately. In order to solve this problem, this paper proposes a high precision guidance and control method for beam-riding guidance multi missile coordination: through sharing error information between multiple missiles in the same guiding beam, correcting the center line of guided beam and eliminating the steady-state error in the process of missile tracking target, thus the strike capability of the guided missile for long distance and high maneuvering target is enhanced. The six degree of freedom controlled trajectory simulation is used to verify this method. The simulation results show that the guidance and control method is correct and effective. Compared with the existing beam-riding guidance system, it has a higher hit precision and better application prospect.

1. Introduction

In complex battlefield environment, it is always an important direction of ammunition development to be able to hit targets accurately at a long distance[1]. Among many types of guided munition, the guidance technology mainly includes autonomous guidance technology, remote control guidance technology, homing guidance technology and composite guidance technology. The remote control guidance is a guidance technology which sends guidance information to the ammunition by the guidance station and directs the ammunition to the target or the predetermined area[2]. Remote control guidance can be divided into remote command guidance and beam-riding guidance, beam-riding guidance can be divided into radio (radar) beam-riding guidance and optical (laser) beam-riding guidance. Promoted by the development of high-energy continuous-wave solid-state laser technology, ammunition using laser beam-riding guidance mechanism has been equipped with a large number of national troops.

The beam-riding guidance weapon system is widely used because of its strong anti-interference capability, simple structure and low cost[3]. The basic principle of the guided weapon is that the guidance Station emits the guiding beam and directs the center line of the beam to the target. When the missile flies in the guiding beam, it relies on the missile borne equipment to perceive its position in the beam, and then produces the control instruction guided missile to fly along the beam center line, eventually leads the missile to the target[4]. However, it is difficult for the existing beam-riding guidance system to be used to attack long-range and high-maneuvering targets accurately. There are two main factors affecting the system: on the one hand, the guidance station has the target indication error in the process of controlling the guidance beam to track the target; on the other hand, the missile has the steady-state tracking error when tracking the guidance beam center line. In order to solve the
above problems, this paper proposes a multi missile cooperative guidance control method under the beam-riding guidance system. By sharing the error information between the pre launched missile and the following missile, this method can greatly improve the hit precision of the beam-riding guidance system to the long distance and high maneuvering target.

The weapon system currently retrieved to fly multiple missiles in the same guiding beam is the British ‘Starstreak’ air defense missile, with 3 submunitions in the front end of the main projectile. When the two level engine of the projectile is burned out, the 3 submunitions are separated and fly by laser beam-riding guidance to the target[5]. There is no literature on multi-missile coordination in the same guided beam in China, and the ‘Starstreak’ air defense missile in Britain is the most mature product at present.

2. Multi missile coordination high precision guidance and control method

As shown in Figure 1, the guidance accuracy of three missiles flying in the same guidance beam is mainly affected by two aspects:

(1) The guidance station has the target indication error in the process of controlling the guidance beam tracking target. The error includes the angle tracking error in the course of tracking the target in the center line of the target angle tracking device of the guidance station, and the angular error between the center line of the guiding beam central line and the target angle tracking device. The principle of target indication error determines that the linear deviation of missile tracking target will be scaled up with the increase of target distance. The principle of target indication error determines that the linear deviation of missile tracking target will be scaled up with the increase of target distance. For example, when tracking a target with a distance of 10 km, the target indication error of 0.5mil will make the tracking line of the missile deviate to 5 meters, for a target with a distance of 3 km, the tracking error of the missile is only 1.5 meters[6].

(2) The beam-riding guidance system is essentially three point guidance. Its guiding principle determines the steady tracking error in the center line of the missile tracking guided beam when the target is high maneuvering. The size of the error depends on the design of the guidance control system and the motion characteristics of the target. This error will not only increase the miss distance of the missile, but also even cause the missile to fly out of the guidance beam.

Therefore, this paper proposes the multi-missile cooperative high-precision guidance and control method to improve the hit accuracy of the missile under the Beam-riding guidance system includes the following steps:

Step1: the guidance station acquires the target and then launches the guided beam to track the target.  
Step 2: according to a certain time interval, the guided missile will launch 3 missiles in turn.  
Step 3: the follow-up missiles send the tracking error formed in the course of tracking the center line of the guided beam to all the missiles launched in advance.  
Step 4: after the missile receives the tracking error of all the subsequent missiles, the tracking error is used as the open loop error compensation instruction, and the tracking error is compensated in its guidance and control system.  
Step 5: when the first missile that completes the tracking error compensation is close to the target, the position information of the target in its body coordinate system is obtained by the strapdown detector, then the position of the target in the guiding beam is calculated based on the position information.  
According to the position of the target in the guiding beam, the target indication error of the guide beam center line is obtained, and the target indication error is sent to all subsequent missiles.  
Step 6: the subsequent missile corrects the guide beam center line according to the target indication error and tracks the corrected pilot beam center line, until the next missile approaches the target, repeating step 5 to get the new target indication error. According to the new target indication error, the subsequent missile corrects the guide beam center line again, and so on, until all the missiles hit the target.
2.1. Correction method of steady state tracking error

When the target is maneuvering in high dynamic state, the steady-state tracking error exists when the single missile tracks the center line of the guidance beam, which will cause the missile to miss the target or even fly out of the guidance beam and lose control. However, by means of multi-missile cooperative guidance, the tracking error formed in the process of tracking the beam center line of the guided missile launched subsequently is transmitted to the pre launched missile. By introducing the received tracking error into the pre launched missile's control system as an open-loop error compensation instruction, the type of the guidance control system can be greatly improved, and the problem of missile miss target and even fly out of the guidance beam can be effectively solved when the missile tracks the center line of the guidance beam.

2.1.1. Mathematical model. Taking the 3 missile cooperative operation in the longitudinal plane as an example, establishing the mathematical model of steady state tracking error correction method. As shown in Figure 2, a typical longitudinal guidance control loop of a beam-riding guided missile is taken as an example, the system consists of a lead correction network and an autopilot[7]. It is assumed that the desired position of the missile determined by the moving state of the target is represented by \( R(s) = \frac{A}{s^n} \) ( \( n \) is signal order, \( A \) is input signal amplitude, \( s \) is complex variable).

From the open loop transfer function of the system, it is known that the system is a "II" type system, and under the action of \( R(s) \), the steady state tracking error of the guidance and control system is \( e_{ss} = \lim_{s \to 0} (R(s) - Y(s)) = 0 \) only when the motion law of the desired position of the missile satisfies \( n \leq 2 \) [8]. Therefore, when the target is high dynamic maneuver (\( n > 2 \)), there is a steady state tracking error when the single missile tracks the centerline of the beam, which can cause the missile to miss the target or even fly out of the guidance beam. By using the multi missile cooperative guidance method, the steady tracking error of the missile is corrected by the steady tracking error of the following missile. It can improve the type of the control system and effectively solve the problem that the missile is out of the target to fly out of the guidance beam when the missile tracking guide beam center line.

![Figure 2. Structure diagram of typical longitudinal guidance and control system for Missile.](image-url)
During the actual operation, the third missile measures the dynamic tracking error when tracking the center line of the guide beam, and then sends them to the first, second missile by the data chain on the missile. The second missile receives the dynamic tracking error of the third missile and forms an open loop error compensation command to compensate the guidance and control system error. Then the second missile sends the dynamic tracking error of the tracking guide beam center line to the first missile through the data chain on the missile, and the first missile uses the dynamic tracking error of the received second, third missile to form the open loop error compensation instruction to compensate its guidance and control system error, in order to improve the precision of the hit. From Figure 3, we can see that the theoretical tracking error of the third missile is:

$$E_3(s) = \frac{1}{1 + G(s)\frac{1}{s^2}} R(s) \quad (1)$$

After the compensation signal is introduced into the guidance and control system of the second missile, the theoretical tracking error of the second missile is:

$$E_2(s) = \frac{1}{\left(1 + G(s)\frac{1}{s^2}\right)} R(s) \quad (2)$$

After the compensation signal $E_3(s)$ and $E_2(s)$ are introduced into the guidance and control system of the first missile, the theoretical tracking error of the first missile is:

$$E_1(s) = \frac{1}{\left(1 + G(s)\frac{1}{s^2}\right)} R(s) \quad (3)$$

Therefore, when the target motion law satisfies $n < 5$, the steady-state error of the central line of the first, second missile tracking guidance beam is:

$$e_{s,1} = \lim_{s \to 0} sE_1(s) = \lim_{s \to 0} \frac{A}{s^{(n-1)/3} + G(s)\frac{1}{s^{(1-n)/3}}} = 0$$

$$e_{s,2} = \lim_{s \to 0} sE_2(s) = \lim_{s \to 0} \frac{A}{s^{(n-1)/2} + G(s)\frac{1}{s^{(1-n)/2}}} = 0$$

Thus, under the guidance of multi-missile cooperative guidance strategy, the first and second missile can track the center line of the guidance beam with high precision.

![Figure 3. Schematic diagram of multi missile cooperative tracking error compensation structure.](image)

2.1.2. Simulation result. The steady-state tracking error component in the longitudinal plane is taken as an example to verify the correctness of the steady-state tracking error correction method. Steady-
state tracking error correction method is used in the simulation program, and longitudinal deviation comparison charts of three missile during flight are obtained as shown in Figure 4. From this figure, you can see that the error of the missile 3 is the largest because it did not correct the steady-state error. The missile 2 was corrected once, and the error was reduced a lot. The missile 1 was corrected two times, with the smallest error and the highest hit accuracy of the missile, which accords with the theoretical expectation. It is proved that the error correction method of steady state tracking error is correct and effective.

![Figure 4. Contrast diagram of three missile longitudinal deviations.](image)

2.2. Estimation method of indicator error

When the missile can accurately track the center line of the guidance beam, the hit accuracy of the missile mainly depends on the target indication error in the process of tracking the target by the guidance station. In this paper, a cooperative operation method is proposed to eliminate the indication error: by installing strapdown detectors on the head of the pre launched missile, the target indication error is measured by the pre launched missile, and then the error information is sent to the following missile to correct the guidance beam of the subsequent missile launched. So as to improve the hit accuracy of following missiles.

2.2.1. Mathematical model. Taking the 3 missile cooperative operation in the longitudinal plane as an example, establishing the mathematical model of target indication error estimation method. For the convenience of modeling, the \( \Delta \varepsilon \) is first projected into the longitudinal and lateral planes to obtain two target indication error components \( \Delta \varepsilon_y \) and \( \Delta \varepsilon_z \), and then the mathematical models of component \( \Delta \varepsilon_y \) and \( \Delta \varepsilon_z \) are derived respectively.

Taking the component \( \Delta \varepsilon_y \) in the longitudinal plane of the target indication error as an example, deducing the mathematical model. The process of deriving the mathematical model of \( \Delta \varepsilon_z \) is similar to \( \Delta \varepsilon_y \). As shown in Figure 5, it is assumed that the strapdown detector is installed on the missile head, and the detector can measure the distance \( d \) and azimuth \( \lambda \) of the target relative to the original point \( O \) of the coordinate system in the body coordinate system \( O, x_i, y_i \). The missile body coordinate system origin \( O \) coincides with the detector center, the \( O, x_i \) axis coincides with the missile axis and points to the missile in front of the movement is positive, the \( O, y_i \) axis is perpendicular to the \( O, x_i \) axis, and is positive to the right[9]. Suppose that in a short time interval \( \Delta t = t_2 - t_1 \), the relative motion of the target and the missile makes the target moving from the \( T_1 \) point to the \( T_2 \) point relative to the missile position. As shown in Figure 6, the distance and azimuth angle of the \( t_1 \) time target in the body coordinate system \( O, x_i, y_i \) are respectively \( d_1 \) and \( \lambda_1 \), and the distance and azimuth of the \( t_2 \) time target in the body coordinate system \( O, x_i, y_i \) are \( d_2 \) and \( \lambda_2 \) respectively. When \( \Delta t \) is sufficiently small,
it can be considered that the attitude of the missile remains unchanged relative to the guiding beam. At this time, the connection between the $T_1$ point and the $T_2$ point can be used as an estimate of the velocity direction of the missile relative to the center line of the guided beam. When the missile can track the beam center line stably, the velocity direction of the missile is basically the same as that of the beam center line. Thus, the relative attitude angle estimation formula of the missile relative to the guidance centerline can be established:

$$\theta = \arccos \left( \frac{d_1^2 + \overline{T_1T_2}^2 - d_2^2}{2d_1\overline{T_1T_2}} \right) - \lambda_1$$

(5)

$$\overline{T_1T_2} = \sqrt{d_1^2 + d_2^2 - 2d_1d_2 \cos(\lambda_2 - \lambda_1)}$$

(6)

According to the target measurement value of the first missile, the following formula can be used to calculate the position $x_{2,T}$ and $y_{2,T}$ of the target in the guidance beam information field coordinate system $O_2x_2y_2$. As shown in Figure 5, the origin of the $O_2x_2y_2$ coordinate system is located in the guidance station, the $O_2x_2$ axis is coincided with the guiding beam center line, and the $O_2y_2$ axis is perpendicular to the $O_2x_2$ axis. Then:

$$x_{2,T} = R_{M1} + (d \cos \lambda + L) \cos \theta - d \sin \lambda \sin \theta$$

$$y_{2,T} = (d \cos \lambda + L) \sin \theta + d \sin \lambda \cos \theta$$

(7)

Where, $L$ is the distance between known strapdown detectors and missile riding guidance information receivers. $R_{M1}$ is the distance between the first missile and the guidance station, and can be estimated by the missile speed plan. Based on the above estimation formula, the formula of target indication error component in longitudinal plane can be further obtained:

$$\Delta x_\epsilon = \arctan \frac{y_{2,T}}{x_{2,T}}$$

(8)

In the same way, the mathematical model of a can be deduced as follows:

$$x_{2,T} = R_{M1} + (d \cos \lambda + L) \cos \theta - d \sin \lambda \sin \theta$$

$$z_{2,T} = (d \cos \lambda + L) \sin \theta + d \sin \lambda \cos \theta$$

$$\Delta x_\epsilon = \arctan \frac{z_{2,T}}{x_{2,T}}$$

(9)

(10)

During the actual operation, the strapdown detector begins to work when the first missile approaches the target, and uses the measured data to estimate the target indication error and then uses the data chain on the missile to transmit the indication error to the second missile and the third missile. The central line of the first modified guide beam can be obtained by the original guiding beam center line and the indication error. The second missile and the third missile tracking modified guided beam center line can greatly eliminate the effect of the target indication error on the missile hit precision. When the first missile exploded and the second missile approaches the target, the target indication error is recalculated according to the same method. The target indication error calculated by the second missile is passed to the third missile, and the guidance process of the third missile is further corrected.
2.2.2. Simulation result. A low cost strapdown detector installed on a missile is used to measure the parameters needed to estimate the mathematical model of the target indication error. When the first missile is 300 meters away from the target, the strapdown detector begins to work and calculates the target indication error, which is passed to the second, third missile for the first time correction of the guiding beam center line. After the first missile is detonated, when the second missile is 300 meters away from the target, the target indication error is calculated and transferred to the third missile to correct the guide beam center line again. Taking the indication error of 0.5mil in the longitudinal plane as an example, the relationship between the estimated value of the indicator error and the distance between the missile and the target is shown as shown in Figure 7. It can be seen from the graph that the smaller the distance between the missile and the target, the more accurate the target indication error estimate is. This deviation is due to the assumption that when $\Delta t$ is sufficiently small, the relative guided beam of the missile remains unchanged when the mathematical model is derived from the derivation of the indicator error.

Figure 5. Schematic diagram of multi missile synergistic guidance in a longitudinal plane.  
Figure 6. Schematic diagram for estimating the relative attitude angle of a missile.

Figure 7. Relationship between the estimated value of the indicator error and the distance between the missile and the target.
The correctness and validity of the mathematical model of the estimation method of target indication error is verified by Monte Carlo simulation with the example of correcting the error component $\Delta \varepsilon_y$ in the longitudinal plane. Multiple mil, such as (-0.5,-0.4,...,0,...,0.4,0.5), are selected as target indication errors, and the error of the 11 sets of indication error estimates is obtained by simulation as shown in Figure 8. It is divided into 11 groups according to the indication error, each group carries out 200 Monte Carlo simulation, analyzes the data obtained by each group of simulation, and gets the CEP of the falling points of three missiles under the different target indication error as shown in Figure 9.

Figure 8. Comparison diagram of the estimated value of the indication error with the accuracy.

In Figure 8, M1 represents the indicator error estimate calculated by first missile, which is used for the first correction of the guiding beam center line of the second and third missile. M1+M2 represents the sum of the indicator error estimates calculated by the first and second missile, which is used to revise the guiding beam center line of the third missile. It can be seen from the diagram that the error of the estimated value is very small under different indication errors, and after two calculations of the first and second missile, the error of the estimated value is smaller and the corrected guide beam center line is more accurate, indicating that the mathematical model of indication error estimation method is correct.

In Figure 9, the first missile only calculates the indicator error without correcting the guiding beam center line, the second missile receives the first missile calculation indication error correction 1 times, the third missile receives the first and second missile calculation indication error correction 2 times. As can be seen from the diagram, the first missile follows the guided beam center line with error, and the CEP of the falling points becomes larger as the absolute value of the target indication error increases, and basically satisfies the conclusion that the indication error of the 0.5mil has a 5m deviation at 10 km. The second missile follows the guided beam center line which has been modified one time, and the CEP of the falling points is almost unchanged with the variation of the indication error, and the CEP is very small. The third missile follows the center line of the guided beam after two times correction, and the CEP of the falling points is the smallest. It is consistent with the conclusion obtained in Figure 8, which shows that it is correct and effective to use the indication error calculated by the mathematical model of the indicator error estimation method to correct the guiding beam center line.

3. Controlled ballistic simulation of six degree of freedom

3.1. Setting of simulation conditions

In the six degree of freedom simulation program, the steady state tracking error correction model and the target indication error correction model are used simultaneously to verify the correctness and effectiveness of the multi missile coordination high precision guidance and control method. The simulation conditions are set as follows:
The position coordinates of the guidance Station in the inertial coordinate system are (0, 2, 0) m. The muzzle of the missile launcher is 0.5m below the guidance station and 2m on the right. The target flight height is 50m, and the cruising speed is 200m/s. When the distance between the missile and the target is 15000 meters, three missiles are launched in turn, and the initial velocity of the missile is 700m/s. The simulation should ensure that the target attacked by three missiles is in the same motion state. And the standard deviation of the indication error pulling deviation is 0.3mil.

### 3.2. Simulation result

The Monte Carlo method is used to carry out 200 simulation tests on the working conditions of the multi missile high precision guidance and control method before and after the correction, and the CEP data of the three missiles are shown in Table 1. The falling points of the first, second, third missile are shown in Figures 10, 11 and 12 respectively, where (a) is the scatter plot before correction, and (b) is the corrected scatter plot.

#### Table 1. CEP contrast table for three missiles.

|               | Pre modified CEP(m) | Modified CEP(m) |
|---------------|---------------------|-----------------|
| Missile 1     | 3.670               | 3.260           |
| Missile 2     | 3.570               | 0.662           |
| Missile 3     | 3.100               | 0.972           |

As shown in Figures 10, 11 and 12 (a), the falling points of 3 missiles are basically consistent before correction, which is consistent with the situation that error of 0.3mil has a 3m deviation at 10 km. From Table 1, it can be seen that after the correction of the steady-state tracking error of the first missile, its CEP is lower than that before the correction, and from Figure 10 it can be seen that the corrected missile drop distribution is closer to the target and the fall points’ density increases, but the CEP is still up to 3 meters because the indication error is not corrected. As shown in Figure 11(b), the second missile corrected the indication error and steady state tracking error. Compared with the pre correction, the precision of the missile was greatly improved, and the accuracy of the second missile in the 3 revised missiles was also the highest, consistent with the theoretical expectation. The third missile only corrected the indication error, so the CEP was much lower than that before the correction. However, because the steady state tracking error was not corrected, the falling points was concentrated on the side of the target, as shown in Figure 12 (b).

![Figure 10. Monte Carlo Simulation Falling Points Chart of the First Missile before and after Correction.](image-url)
4. Conclusions
(1) When the steady-state tracking error of the missile is corrected, the deviation between the missile and the target decreases, the falling point is closer to the target and the density is higher.
(2) After the target indication error of the missile is corrected, the CEP of the falling point of the missile is obviously reduced, and the hit accuracy of the missile is greatly improved.
(3) Through the simulation verification, the multi missile coordination high precision guidance and control method proposed in this paper is correct and effective. Compared with the current beam-riding guidance system, the multi missile cooperative combat system has higher hit precision and better application prospect.

5. References
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