World Sheet Superstring and
Superstring Field Theory: a new
solution using Ultradistributions of
Exponential Type *

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Abstract

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In this paper we show that Ultradistributions of Exponential Type (UET) are appropriate for the description in a consistent way world sheet superstring and superstring field theories. A new Lagrangian for the closed world sheet superstring is obtained. We also show that the superstring field is a linear superposition of UET of compact support (CUET), and give the notion of anti-superstring. We evaluate the propagator for the string field, and calculate the convolution of two of them.

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1 Introduction

In a series of papers [1, 2, 3, 4, 5] we have shown that Ultradistribution theory of Sebastiao e Silva [6, 7, 8] permits a significant advance in the treatment of quantum field theory. In particular, with the use of the convolution of Ultradistributions we have shown that it is possible to define a general product of distributions (a product in a ring with divisors of zero) that sheds new light on the question of the divergences in Quantum Field Theory. Furthermore, Ultradistributions of Exponential Type are adequate to describe Gamow States and exponentially increasing fields in Quantum Field Theory [9, 10, 11].

Ultradistributions also have the advantage of being representable by means of analytic functions. So that, in general, they are easier to work with and, as we shall see, have interesting properties. One of these properties is that Schwartz’s tempered distributions are canonical and continuously injected into Ultradistributions of Exponential Type and as a consequence the Rigged Hilbert Space with tempered distributions is canonical and continuously included in the Rigged Hilbert Space with Ultradistributions of Exponential Type.

Another interesting property is that the space of UET is reflexive un-
nder the operation of Fourier transform (in a similar way to that tempered distributions of Schwartz)

In two recent papers ([12, 13]) we have shown that Ultradistributions of Exponential type provide an adequate framework for a consistent treatment of string and string field theories. In particular, a general state of the closed string is represented by UET of compact support, and as a consequence the string field is a linear combination of UET of compact support.

In this paper we extend the treatment to world sheet superstrings.

This paper is organized as follows: in sections 2 and 3 we define the Ultradistributions of Exponential Type and their Fourier transform. They are part of a Guelfand’s Triplet (or Rigged Hilbert Space [14]) together with their respective duals and a “middle term” Hilbert space. In sections 4 and 5 we give the main results of ref. [12] for the bosonic string used in the present work. In section 6 we obtain a expression for the Lagrangian of a closed world sheet superstring. We give a solution of the equations of motion and show that the Lagrangian is supersymmetrically invariant In section 7 we give a new representation for the states of the string using CUET. In section 8 we give expressions for the field of the superstring, the superstring field propagator, the creation and annihilation operators of a superstring and
a anti-superstring. In section 9, we give expressions for the non-local action of a free string and a non-local interaction lagrangian for the string field in a way similar to that given in Quantum Field Theory. Also we show how to evaluate the convolution of two superstring field propagators. Finally, section 10 is reserved for a discussion of the principal results.

2 Ultradistributions of Exponential Type

Let $S$ be the Schwartz space of rapidly decreasing test functions. Let $\Lambda_j$ be the region of the complex plane defined as:

$$\Lambda_j = \{ z \in \mathbb{C} : |\Im(z)| < j : j \in \mathbb{N} \} \quad (2.1)$$

According to ref.[6, 8] the space of test functions $\hat{\phi} \in V_j$ is constituted by all entire analytic functions of $S$ for which

$$||\hat{\phi}||_j = \max_{k \leq j} \left\{ \sup_{z \in \Lambda_j} \left[ e^{|\Re(z)|} |\hat{\phi}^{(k)}(z)| \right] \right\} \quad (2.2)$$

is finite.

The space $Z$ is then defined as:

$$Z = \bigcap_{j=0}^{\infty} V_j \quad (2.3)$$
It is a complete countably normed space with the topology generated by the
system of semi-norms \(\{|| \cdot ||_j\}_{j \in \mathbb{N}}\). The dual of \(Z\), denoted by \(B\), is by definition
the space of ultradistributions of exponential type (ref.\cite{6,8}). Let \(S\) be the
space of rapidly decreasing sequences. According to ref.\cite{14} \(S\) is a nuclear
space. We consider now the space of sequences \(P\) generated by the Taylor
development of \(\hat{\phi} \in Z\)

\[
P = \left\{ Q : Q \left( \hat{\phi}(0), \hat{\phi}'(0), \frac{\hat{\phi}''(0)}{2}, \ldots, \frac{\hat{\phi}^{(n)}(0)}{n!}, \ldots \right) : \hat{\phi} \in Z \right\} \tag{2.4}
\]

The norms that define the topology of \(P\) are given by:

\[
||\hat{\phi}||'_p = \sup_n \frac{n^p}{n!} |\hat{\phi}^{(n)}(0)| \tag{2.5}
\]

\(P\) is a subspace of \(S\) and therefore is a nuclear space. As the norms \(|| \cdot ||_j\) and
\(|| \cdot ||'_p\) are equivalent, the correspondence

\[
Z \leftrightarrow P \tag{2.6}
\]

is an isomorphism and therefore \(Z\) is a countably normed nuclear space. We
can define now the set of scalar products

\[
< \hat{\phi}(z), \hat{\psi}(z) >_n = \sum_{q=0}^{n} \int_{-\infty}^{\infty} e^{2n|x|} \overline{\hat{\phi}(q)(z)} \hat{\psi}(q)(z) \, dz = \\
\sum_{q=0}^{n} \int_{-\infty}^{\infty} e^{2n|x|} \overline{\hat{\phi}(q)(x)} \hat{\psi}(q)(x) \, dx \tag{2.7}
\]
This scalar product induces the norm
\[
\|\hat{\phi}\|''_n = \left\langle \hat{\phi}(x), \hat{\phi}(x) \right\rangle_n^{\frac{1}{2}}
\]  \hspace{1cm} (2.8)

The norms \(\| \cdot \|_j\) and \(\| \cdot \|''_n\) are equivalent, and therefore \(Z\) is a countably hilbertian nuclear space. Thus, if we call now \(Z_p\) the completion of \(Z\) by the norm \(p\) given in (2.8), we have:
\[
Z = \bigcap_{p=0}^{\infty} Z_p
\]  \hspace{1cm} (2.9)

where
\[
Z_0 = H
\]  \hspace{1cm} (2.10)

is the Hilbert space of square integrable functions.

As a consequence the “nested space”
\[
U = (Z, H, B)
\]  \hspace{1cm} (2.11)

is a Gelfand’s triplet (or a Rigged Hilbert space=RHS. See ref.[14]).

Any Gelfand’s triplet \(G = (\Phi, H, \Phi')\) has the fundamental property that a linear and symmetric operator on \(\Phi\), admitting an extension to a self-adjoint operator in \(H\), has a complete set of generalized eigen-functions in \(\Phi'\) with real eigenvalues.

\(B\) can also be characterized in the following way ( refs.[6],[8] ): let \(E_\omega\) be the space of all functions \(\hat{F}(z)\) such that:
I- \( \hat{F}(z) \) is analytic for \( \{ z \in \mathbb{C} : |\text{Im}(z)| > p \} \).

II- \( \hat{F}(z)e^{-\rho|\text{Re}(z)|}/z^p \) is bounded continuous in \( \{ z \in \mathbb{C} : |\text{Im}(z)| \geq p \} \), where \( p = 0, 1, 2, \ldots \) depends on \( \hat{F}(z) \).

Let \( N \) be: \( N = \{ \hat{F}(z) \in E_\omega : \hat{F}(z) \text{ is entire analytic} \} \). Then \( B \) is the quotient space:

III- \( B = E_\omega/N \)

Due to these properties it is possible to represent any ultradistribution as ( ref.\cite{6,8}):

\[
\hat{F}(\hat{\phi}) = <\hat{F}(z), \hat{\phi}(z)> = \oint_{\Gamma} \hat{F}(z)\hat{\phi}(z) \, dz \tag{2.12}
\]

where the path \( \Gamma \) runs parallel to the real axis from \(-\infty\) to \(\infty\) for \( \text{Im}(z) > \zeta \), \( \zeta > p \) and back from \(\infty\) to \(-\infty\) for \( \text{Im}(z) < -\zeta \), \(-\zeta < -p \). ( \( \Gamma \) surrounds all the singularities of \( \hat{F}(z) \)).

Formula \( (2.12) \) will be our fundamental representation for a tempered ultradistribution. Sometimes use will be made of “Dirac formula” for exponential ultradistributions ( ref.\cite{6}):

\[
\hat{F}(z) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{\hat{f}(t)}{t-z} \, dt \equiv \frac{\cosh(\lambda z)}{2\pi i} \int_{-\infty}^{\infty} \frac{\hat{f}(t)}{(t-z)\cosh(\lambda t)} \, dt \tag{2.13}
\]

where the “density” \( \hat{f}(t) \) is such that

\[
\oint_{\Gamma} \hat{F}(z)\hat{\phi}(z) \, dz = \int_{-\infty}^{\infty} \hat{f}(t)\hat{\phi}(t) \, dt \tag{2.14}
\]
(2.13) should be used carefully. While \( \hat{f}(z) \) is analytic on \( \Gamma \), the density \( \hat{f}(t) \) is in general singular, so that the r.h.s. of (2.14) should be interpreted in the sense of distribution theory.

Another important property of the analytic representation is the fact that on \( \Gamma \), \( \hat{f}(z) \) is bounded by an exponential and a power of \( z \) ( ref. [6, 8] ):

\[
|\hat{f}(z)| \leq C|z|^p e^{p|\Re(z)|} \tag{2.15}
\]

where \( C \) and \( p \) depend on \( \hat{f} \).

The representation (2.12) implies that the addition of any entire function \( \hat{G}(z) \in \mathbb{N} \) to \( \hat{f}(z) \) does not alter the ultradistribution:

\[
\oint_{\Gamma} (\hat{f}(z) + \hat{G}(z)) \hat{\phi}(z) \, dz = \oint_{\Gamma} \hat{f}(z) \hat{\phi}(z) \, dz + \oint_{\Gamma} \hat{G}(z) \hat{\phi}(z) \, dz
\]

But:

\[
\oint_{\Gamma} \hat{G}(z) \hat{\phi}(z) \, dz = 0
\]

as \( \hat{G}(z) \hat{\phi}(z) \) is entire analytic ( and rapidly decreasing ),

\[
\therefore \oint_{\Gamma} (\hat{f}(z) + \hat{G}(z)) \hat{\phi}(z) \, dz = \oint_{\Gamma} \hat{f}(z) \hat{\phi}(z) \, dz \tag{2.16}
\]

Another very important property of \( B \) is that \( B \) is reflexive under the Fourier transform:

\[
B = \mathcal{F}_c \{ B \} = \mathcal{F} \{ B \} \tag{2.17}
\]
where the complex Fourier transform $F(k)$ of $\hat{F}(z) \in B$ is given by:

$$F(k) = \Theta[J(k)] \int_{\Gamma_+} \hat{F}(z)e^{ikz} \, dz - \Theta[-J(k)] \int_{\Gamma_-} \hat{F}(z)e^{ikz} \, dz =$$

$$\Theta[J(k)] \int_0^\infty \hat{f}(x)e^{ikx} \, dx - \Theta[-J(k)] \int_{-\infty}^0 \hat{f}(x)e^{ikx} \, dx \quad (2.18)$$

Here $\Gamma_+$ is the part of $\Gamma$ with $\Re(z) \geq 0$ and $\Gamma_-$ is the part of $\Gamma$ with $\Re(z) \leq 0$.

Using (2.18) we can interpret Dirac’s formula as:

$$F(k) \equiv \frac{1}{2\pi i} \int_{-\infty}^\infty \frac{f(s)}{s-k} \, ds \equiv \mathcal{F}_c \{ \mathcal{F}^{-1}\{f(s)\} \} \quad (2.19)$$

The treatment for ultradistributions of exponential type defined on $\mathbb{C}^n$ is similar to the case of one variable. Thus

$$\Lambda_j = \{ z = (z_1, z_2, \ldots, z_n) \in \mathbb{C}^n : |J(z_k)| \leq j \quad 1 \leq k \leq n \} \quad (2.20)$$

$$\|\hat{\phi}\|_j = \max_{k \leq j} \left\{ \sup_{z \in \Lambda_j} \left| e^{\sum_{p=1}^n |\Re(z_p)|} \left| D^{(k)}\hat{\phi}(z) \right| \right| \right\} \quad (2.21)$$

where $D^{(k)} = \partial^{(k_1)} \partial^{(k_2)} \ldots \partial^{(k_n)}$, $k = k_1 + k_2 + \ldots + k_n$.

$B^n$ is characterized as follows. Let $E^n_\omega$ be the space of all functions $\hat{F}(z)$ such that:

$I' - \hat{F}(z)$ is analytic for $\{ z \in \mathbb{C}^n : |\Im(z_1)| > p, |\Im(z_2)| > p, \ldots, |\Im(z_n)| > p \}$.

$II' - \hat{F}(z)e^{-\sum_{i=1}^n |\Re(z_i)|}/z^p$ is bounded continuous in $\{ z \in \mathbb{C}^n : |\Im(z_1)| \geq p, |\Im(z_2)| \geq p, \ldots, |\Im(z_n)| \geq p \}$, where $p = 0, 1, 2, \ldots$ depends on $\hat{F}(z)$. 

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Let \( N^n \) be: \( N^n = \{ \hat{F}(z) \in E^n_\omega : \hat{F}(z) \text{ is entire analytic at least in one of the variables } z_j \ 1 \leq j \leq n \} \) Then \( B^n \) is the quotient space:

\[
\text{III'} - B^n = E^n_\omega / N^n
\]

We have now

\[
\hat{F}(\phi) = <\hat{F}(z), \phi(z)> = \oint_{\Gamma} \hat{F}(z) \phi(z) \ dz_1 \ dz_2 \cdots d z_n \quad (2.22)
\]

\( \Gamma = \Gamma_1 \cup \Gamma_2 \cup ... \Gamma_n \) where the path \( \Gamma_j \) runs parallel to the real axis from \(-\infty \) to \( \infty \) for \( \text{Im}(z_j) > \zeta, \zeta > p \) and back from \( \infty \) to \(-\infty \) for \( \text{Im}(z_j) < -\zeta, -\zeta < -p \).

(Again \( \Gamma \) surrounds all the singularities of \( \hat{F}(z) \)). The \( n \)-dimensional Dirac’s formula is

\[
\hat{F}(z) = \frac{1}{(2\pi i)^n} \int_{-\infty}^{\infty} \frac{\hat{f}(t)}{(t_1 - z_1)(t_2 - z_2) \cdots (t_n - z_n)} \ dt_1 \ dt_2 \cdots dt_n \quad (2.23)
\]

where the “density” \( \hat{f}(t) \) is such that

\[
\oint_{\Gamma} \hat{F}(z) \phi(z) \ dz_1 \ dz_2 \cdots d z_n = \int_{-\infty}^{\infty} f(t) \phi(t) \ dt_1 \ dt_2 \cdots dt_n \quad (2.24)
\]

and the modulus of \( \hat{F}(z) \) is bounded by

\[
|\hat{F}(z)| \leq C|z|^p e\left[p \sum_{j=1}^{\infty} |\text{Re}(z_j)|\right] \quad (2.25)
\]

where \( C \) and \( p \) depend on \( \hat{F} \).
3 The Case $N \to \infty$

When the number of variables of the argument of the Ultradistribution of Exponential type tends to infinity we define:

\[
d\mu(x) = \frac{e^{-x^2}}{\sqrt{\pi}} \, dx \quad (3.1)
\]

Let $\hat{\phi}(x_1, x_2, ..., x_n)$ be such that:

\[
\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} |\hat{\phi}(x_1, x_2, ..., x_n)|^2 \, d\mu_1 \, d\mu_2 \cdots d\mu_n < \infty \quad (3.2)
\]

where

\[
d\mu_i = \frac{e^{-x_i^2}}{\sqrt{\pi}} \, dx_i \quad (3.3)
\]

Then by definition $\hat{\phi}(x_1, x_2, ..., x_n) \in L_2(\mathbb{R}^n, \mu)$ and

\[
L_2(\mathbb{R}^\infty, \mu) = \bigcup_{n=1}^{\infty} L_2(\mathbb{R}^n, \mu) \quad (3.4)
\]

Let $\hat{\psi}$ be given by

\[
\hat{\psi}(z_1, z_2, ..., z_n) = \pi^{n/4} \hat{\phi}(z_1, z_2, ..., z_n) e^{\frac{z_1^2 + z_2^2 + \cdots + z_n^2}{2}} \quad (3.5)
\]

where $\hat{\phi} \in Z^n$(the corresponding n-dimensional of Z).

Then by definition $\hat{\psi}(z_1, z_2, ..., z_n) \in G(\mathbb{C}^n),

\[
G(\mathbb{C}^\infty) = \bigcup_{n=1}^{\infty} G(\mathbb{C}^n) \quad (3.6)
\]
The dual $G'(\mathbb{C}^\infty)$ given by
\[
G'(\mathbb{C}^\infty) = \bigcup_{n=1}^{\infty} G'(\mathbb{C}^n)
\] (3.7)
is the space of Ultradistributions of Exponential type.

The analog to (2.11) in the infinite dimensional case is:
\[
W = (G(\mathbb{C}^\infty), L_2(\mathbb{R}^\infty, \mu), G'(\mathbb{C}^\infty))
\] (3.8)

Let us now define:
\[
\mathcal{F} : G(\mathbb{C}^\infty) \to G(\mathbb{C}^\infty)
\] (3.9)
via the Fourier transform:
\[
\mathcal{F} : G(\mathbb{C}^n) \to G(\mathbb{C}^n)
\] (3.10)
given by:
\[
\mathcal{F}[\hat{\psi}](k) = \int_{-\infty}^{\infty} \hat{\psi}(z_1, z_2, ..., z_n) e^{i k \cdot z + \frac{k^2}{2}} d\rho_1 d\rho_2 ... d\rho_n
\] (3.11)
where
\[
d\rho(z) = \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}} dz
\] (3.12)
we conclude that
\[
G'(\mathbb{C}^\infty) = \mathcal{F}(G'(\mathbb{C}^\infty)) = \mathcal{F}[G'(\mathbb{C}^\infty)]
\] (3.13)
where in the one-dimensional case

\[ F_c \{ \hat{\psi} \} (k) = \Theta [\mathcal{I}(k)] \int_{r_c}^{r_+} \hat{\psi}(z) e^{ikz + \frac{k^2}{2}} \, d\rho - \Theta [-\mathcal{I}(k)] \int_{r_-}^{r_c} \hat{\psi}(z) e^{ikz + \frac{k^2}{2}} \, d\rho \]  

(3.14)

4 The Constraints for a Bradyonic Bosonic String

The constraints for a bradyonic bosonic string have been deduced in ref. [12].

As a consequence we can describe the bosonic string by a system composed of a Lagrangian, one constraint and two initial conditions:

\[
\begin{align*}
\mathcal{L} &= |\dot{X}^2 - X'^2| \\
(\dot{X} + X')^2 &= 0 \\
X_\mu(\tau, 0) - X_\mu(\tau, \pi) &= 0
\end{align*}
\]  

(4.1)

or equivalently

\[
\begin{align*}
\mathcal{L} &= |\dot{X}^2 - X'^2| \\
(\dot{X} - X')^2 &= 0 \\
X_\mu(\tau, 0) - X_\mu(\tau, \pi) &= 0
\end{align*}
\]  

(4.2)
5 A representation of the states of the closed bosonic string

The case $n$ finite

From ref. [12] we have

$$a = -z ; \quad a^+ = \frac{d}{dz}$$ \hspace{1cm} (5.1)

Then

$$[a, a^+] = 1$$ \hspace{1cm} (5.2)

Thus we have a representation for creation and annihilation operators of the states of the string. The vacuum state annihilated by $z_\mu$ is the UET $\delta(z_\mu)$, and the orthonormalized states obtained by successive application of $\frac{d}{dz_\mu}$ to $\delta(z_\mu)$ are:

$$F_n(z_\mu) = \frac{\delta^{(n)}(z_\mu)}{\sqrt{n!}}$$ \hspace{1cm} (5.3)

A general state of the string can be written as:

$$\phi(x, \{z\}) = [a_0(x) + a_{i_1}^{i_1}(x) \delta^{i_1}_{i_1} + a_{i_1i_2}^{i_1i_2}(x) \delta^{i_1}_{i_1} \delta^{i_2}_{i_2} + ... + ...

+ a_{i_1i_2...i_n}^{i_1i_2...i_n}(x) \delta^{i_1}_{i_1} \delta^{i_2}_{i_2} ... \delta^{i_n}_{i_n} + ... + ...] \delta(\{z\})$$ \hspace{1cm} (5.4)

where $\{z\}$ denotes $(z_{1\mu}, z_{2\mu}, ..., z_{n\mu}, ..., ...)$, and $\phi$ is a UET of compact support
in the set of variables \{z\}. The functions \( a^{i_1i_2...i_n}_{\mu_1\mu_2...\mu_n}(x) \) are solutions of

\[
\Box a^{i_1i_2...i_n}_{\mu_1\mu_2...\mu_n}(x) = 0
\]  

(5.5)

**The case \( n \to \infty \)**

In this case

\[
a = -z \quad ; \quad a^+ = -2z + \frac{d}{dz}
\]

(5.6)

we have

\[
[a, a^+] = 1
\]  

(5.7)

The vacuum state annihilated by \( a \) is \( \delta(z)e^{z^2} \). The orthonormalized states obtained by successive application of \( a^+ \) are:

\[
\hat{F}_n(z) = 2^{\frac{1}{2}}\pi^{\frac{1}{2}} \frac{\delta^{(n)}(z)e^{z^2}}{\sqrt{n!}}
\]

(5.8)

**6 The World Sheet Supersymmetric String**

We take as starting point the action given by:

\[
S = -\frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{0}^{\pi} |\partial_{\nu}X_{\mu}(\sigma)\partial^{\nu}X^{\mu}(\sigma) - i\overline{\psi}^{\mu}(\sigma)\rho^{\nu}\partial_{\nu}\psi_{\mu}(\sigma)| \, d^2\sigma
\]

(6.1)

where

\[
\rho^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \rho^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \quad \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}
\]

(6.2)
Following a similar treatment to that of ref. [12] we have the constraints

\[
\begin{aligned}
(\dot{X} + X')^2 &= 0 \\
\dot{\psi} + \psi' &= 0 \\
\end{aligned}
\] (6.3)

or

\[
\begin{aligned}
(\dot{X} - X')^2 &= 0 \\
\dot{\psi} - \psi' &= 0 \\
\end{aligned}
\] (6.4)

Thus, to describe the superstring we have:

\[
\begin{aligned}
\mathcal{L} &= |\partial_\nu X_\mu(\sigma)\partial^\nu X^\mu(\sigma) - i\bar{\psi}^\mu(\sigma)\rho^\nu \partial_\nu \psi_\mu(\sigma)| \\
(\dot{X} + X')^2 &= 0 \\
\dot{\psi} + \psi' &= 0 \\
X_\mu(\tau, 0) - X_\mu(\tau, \pi) &= 0 \\
\psi_\mu(\tau, 0) - \psi_\mu(\tau, \pi) &= 0 \\
\end{aligned}
\] (6.5)

or

\[
\begin{aligned}
\mathcal{L} &= |\partial_\nu X_\mu(\sigma)\partial^\nu X^\mu(\sigma) - i\bar{\psi}^\mu(\sigma)\rho^\nu \partial_\nu \psi_\mu(\sigma)| \\
(\dot{X} - X')^2 &= 0 \\
\dot{\psi} - \psi' &= 0 \\
X_\mu(\tau, 0) - X_\mu(\tau, \pi) &= 0 \\
\psi_\mu(\tau, 0) - \psi_\mu(\tau, \pi) &= 0 \\
\end{aligned}
\] (6.6)
If we define:

\[ \mathcal{L}_1 = \partial_\alpha X_\mu(\sigma) \partial^\alpha X^\mu(\sigma) - i \overline{\psi}^\mu(\sigma) \rho^\alpha \partial_\alpha \psi_\mu(\sigma) \]  

(6.7)

the Euler-Lagrange equations for the string are:

\[ \frac{\partial}{\partial \tau} [\text{Sgn}(\mathcal{L}_1) \dot{X}^\mu] - \frac{\partial}{\partial \sigma} [\text{Sgn}(\mathcal{L}_1) X^\prime_\mu] = 0 \]  

(6.8)

\[ \text{Sgn}(\mathcal{L}_1) \rho^\nu \partial_\nu \psi^\mu = 0 \]  

(6.9)

Equation (6.9) implies that \( \text{Sgn}(\mathcal{L}_1) \neq 0 \). The solution to (6.5) is:

\[
\begin{cases}
\psi_1^\mu = \sum_{n=-\infty}^{\infty} c_n^\mu e^{-2in(\tau-\sigma)} \\
\psi_2^\mu = 0 \\
X^\mu = x^\mu + l^2 \rho^\mu \tau + \frac{i l}{2} \sum_{n=-\infty; n \neq 0}^{\infty} a_n^\mu e^{-2in(\tau-\sigma)} \\
p^2 |\Phi> = 0
\end{cases}
\]  

(6.10)

and for (6.6) is:

\[
\begin{cases}
\psi_1^\mu = 0 \\
\psi_2^\mu = \sum_{n=-\infty}^{\infty} c_n^\mu e^{-2in(\tau+\sigma)} \\
X^\mu = x^\mu + l^2 \rho^\mu \tau + \frac{i l}{2} \sum_{n=-\infty; n \neq 0}^{\infty} a_n^\mu e^{-2in(\tau+\sigma)} \\
p^2 |\Phi> = 0
\end{cases}
\]  

(6.11)
where $|\Phi>$ is the physical state of the string.

We will show that (6.1) is a supersymmetric invariant. For this purpose we use the equality for UET:

$$-\frac{1}{2\pi i}(z + y)[\ln(z + y) + \ln(-z - y)] =$$

$$-\frac{1}{2\pi i}z[\ln(z) + \ln(-z)] - \frac{1}{2\pi i}[\ln(z) + \ln(-z)]y +$$

$$2 \sum_{n=-\infty}^{\infty} \delta^{(n)}(z)\frac{y^{2+n}}{(2+n)!}$$  \hspace{1cm} (6.12)

that on the real axis transforms into:

$$|x + y| = |x| + \text{Sgn}(x)\ y + 2 \sum_{n=-\infty}^{\infty} \delta^{(n)}(x)\frac{y^{2+n}}{(2+n)!}$$  \hspace{1cm} (6.13)

As is known supersymmetry transformations are given by (see ref.[15]):

$$\begin{cases}
\delta X^\mu = \overline{\psi} \gamma^\mu \\
\delta \psi^\mu = -i\gamma^\nu \partial_\nu x^\mu \epsilon
\end{cases}$$  \hspace{1cm} (6.14)

To show the invariance we use:

$$\begin{cases}
\text{Sgn}(L_1) \neq 0 \\
\dot{\psi} \pm \psi' = 0
\end{cases}$$  \hspace{1cm} (6.15)

The variation of $L$ is:

$$\delta L = 2\text{Sgn}(L_1)(\dot{X}^\mu \overline{\psi} \gamma^\mu - X^\mu \overline{\psi}' \gamma^\mu)$$  \hspace{1cm} (6.16)
and as a consequence:

$$\delta S = -2 \int_{-\infty}^{\infty} \int_{0}^{\pi} \text{sgn}(L)(\dddot{X}^{\mu} - X^{\mu''})\tau\phi_{\mu} = 0 \quad (6.17)$$

7 A representation of the states of the closed supersymmmetric string

The case n finite

As in ref. [12], for n finite we have:

$$\begin{cases} a = -z & ; \quad a^+ = \frac{d}{dz} \\ c = \frac{d}{d\theta} & ; \quad c^+ = \theta \end{cases} \quad (7.1)$$

$$[a, a^+] = [c, c^+] = 1 \quad (7.2)$$

where $-z$ and $d/dz$ are operators over CUET and $\theta$ is a Grassman variable with scalar product defined by:

$$<f, g> = \int f(\theta_1)e^{\theta_1\theta_2}g(\theta_2) \ d\theta_1 \ d\theta_2 \quad (7.3)$$

In a similar way to that for the bosonic string, a general state of the supersymmetric string can be written as:

$$\Phi(x, \{z\}, \{\theta\}) = [c_0 a_0(x) + c(1, 0) a_{\mu_1}^{i_1}(x) \partial_{i_1}^{\mu_1} + \ldots]$$
\[ c(0, 1) a_{\alpha_1}^{j_1}(x) \theta_{j_1}^{\alpha_1} + \cdots + \]
\[ c(m, n) a_{\mu_1 i_1 \cdots \mu_m j_1 \cdots j_n}^{i_1 \cdots i_m} \delta_{\nu_1 j_1 \cdots j_1}^{\mu_1 \cdots \mu_m} \delta_{\alpha_1 j_1 \cdots j_n}^{\nu_1 \cdots \nu_m} + \]
\[ + \cdots + \cdots \delta([z]) \]  
(7.4)

where \( c(m, n) \) are constants to be determined.

\( \Phi \) satisfy:

\[ \Box \Phi(x, \{z\}, \{\theta\}) = 0 \]  
(7.5)

\[ \Box a_{\mu_1 i_1 \cdots \mu_m \alpha_1 j_1 \cdots j_n}^{i_1 \cdots i_m} (x) = 0 \]  
(7.6)

**The case \( n \rightarrow \infty \)**

In this case:

\[
\begin{cases}
  a = -z & ; & a^+ = -2z + \frac{d}{dz} \\
  c = \frac{d}{d\theta} & ; & c^+ = \theta
\end{cases}
\]  
(7.7)

\[ [a, a^+] = [c, c^+] = 1 \]  
(7.8)

and the expression for the physical state of the string is similar to that for the finite case.
8 The Field of the Supersymmetric String

According to (6.10),(6.11) and section 7 the equation for the string field is given by

\[ \square \Phi(x, \{z\}, \{\theta\}) = 0 \]  

(8.1)

where \(\{z\}\) denotes \((z_{1\mu}, z_{2\mu}, ..., z_{n\mu}, ..., ...)\), and \(\Phi\) is a CUET in the set of variables \(\{z\}\). Any UET of compact support can be written as a development of \(\delta(\{z\})\) and its derivatives. Thus we have:

\[ \Phi(x, \{z\}, \{\theta\}) = [c_0 \mathcal{A}_0(x) + c(1, 0) \mathcal{A}_{1}^{ij}(x) \partial_{i}^{\mu_1} + c(0, 1) \mathcal{A}_{2}^{ij}(x) \theta_{j}^{\alpha_1} + \cdots + c(m, n) \mathcal{A}_{m,n}^{i_1 \cdots i_m j_1 \cdots j_n}(x) \partial_{i_1}^{\mu_1} \cdots \partial_{i_m}^{\mu_m} \theta_{j_1}^{\alpha_1} \cdots \theta_{j_n}^{\alpha_n} + \cdots + \cdots + \cdots \delta(\{z\}) ] \]  

(8.2)

where \(C(m, n)\) are constants to be determined and the quantum fields \(\mathcal{A}_{m,n}^{i_1 \cdots i_m j_1 \cdots j_n}(x)\) are solutions of

\[ \square \mathcal{A}_{m,n}^{i_1 \cdots i_m j_1 \cdots j_n}(x) = 0 \]  

(8.3)

The propagator of the string field can be expressed in terms of the propagators of the component fields:

\[ \Delta(x - x', \{z\}, \{z\}', \{\theta\}, \{\theta\}') = [c_0^2 \Delta(x - x') + \cdots + \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdOTS
\[ \begin{align*}
\frac{d^{m+n}(x-x')}{dt_1 \cdots dt_m dk_1 \cdots dk_m dj_1 \cdots dj_n kl_1 \cdots kl_n}
\mu_1 \cdots \mu_m \alpha_1 \cdots \alpha_n \nu_1 \cdots \nu_m \beta_1 \cdots \beta_n
\left( x - x' \right)

\partial_{t_1}^{\mu_1} \cdots \partial_{t_m}^{\mu_m} \partial_{k_1}^{\nu_1} \cdots \partial_{k_m}^{\nu_m} \theta_{j_1}^{\alpha_1} \cdots \theta_{j_n}^{\alpha_n} \theta_{l_1}^{\beta_1} \cdots \theta_{l_n}^{\beta_n}
+ \cdots \right) \delta([z], [z'])
\end{align*} \]

(8.4)

Writing

\[ \begin{align*}
A_{\mu_1 \cdots \mu_m \alpha_1 \cdots \alpha_n}^{i_1 \cdots j_1 \cdots j_n}(x) &= \int_{-\infty}^{\infty} \left( a_{\mu_1 \cdots \mu_m \alpha_1 \cdots \alpha_n}^{i_1 \cdots j_1 \cdots j_n}(k) e^{-ik \cdot x} + \right.
\left. b^{+i_1 \cdots j_1 \cdots j_n}_{\mu_1 \cdots \mu_m \alpha_1 \cdots \alpha_n}(k) e^{ik \cdot x} \right) d^{-1}k
\end{align*} \]

(8.5)

We define the operators of annihilation and creation of a string as:

\[ \begin{align*}
a(k, \{z\}, \{\theta\}) &= [c_0 a_0(k) + c_1 0 a_0^{i_1}(k) \delta_{i_1}^{\mu_1} + \\
&+ c(0, 1) a_{\alpha_1}^{j_1}(k) \theta_{j_1}^{\alpha_1} + \cdots + \\
c(m, n) a^{i_1 \cdots i_m j_1 \cdots j_n}_{\mu_1 \cdots \mu_m \alpha_1 \cdots \alpha_n}(k) \delta_{i_1}^{\mu_1} \cdots \delta_{i_m}^{\mu_m} \theta_{j_1}^{\alpha_1} \cdots \theta_{j_n}^{\alpha_n} + \\
&+ \cdots + \cdots \delta([z])
\end{align*} \]

(8.6)

\[ \begin{align*}
a^{+}(k, \{z\}, \{\theta\}) &= [c_0 a_0^{+}(k) + c_1 0 a_0^{i_1}(k) \delta_{i_1}^{\mu_1} + \\
&+ c(0, 1) a_{\alpha_1}^{j_1}(k) \theta_{j_1}^{\alpha_1} + \cdots + \\
c(m, n) a^{+i_1 \cdots i_m j_1 \cdots j_n}_{\mu_1 \cdots \mu_m \alpha_1 \cdots \alpha_n}(k) \delta_{i_1}^{\mu_1} \cdots \delta_{i_m}^{\mu_m} \theta_{j_1}^{\alpha_1} \cdots \theta_{j_n}^{\alpha_n} + \\
&+ \cdots + \cdots \delta([z])
\end{align*} \]

(8.7)
where the constants $c(m, n)$ are solution of:

$$c^*(m, n)\theta_{j_n}^{\alpha_n} \cdots \theta_{j_1}^{\alpha_1} a_{\mu_1 \cdots \mu_m, \alpha_1 \cdots \alpha_n}^{i_1 \cdots i_m j_1 \cdots j_n}(k) =$$

$$c(m, n) a_{\mu_1 \cdots \mu_m, \alpha_1 \cdots \alpha_n}^{i_1 \cdots i_m j_1 \cdots j_n}(k) \theta_{j_1}^{\alpha_1} \cdots \theta_{j_n}^{\alpha_n}$$  \hspace{1cm} (8.8)

and define the creation and annihilation operators of the anti-string:

$$b^+(k, \{z\}, \{\theta\}) = [c_0 b_0^+(k) + c(1, 0) b_{\mu_1}^{i_1}(k) \partial_{i_1}^{\mu_1} +$$

$$c(0, 1) b_{\alpha_1}^{j_1}(k) \theta_{j_1}^{\alpha_1} + \cdots +$$

$$c(m, n) b_{\mu_1 \cdots \mu_m, \alpha_1 \cdots \alpha_n}^{i_1 \cdots i_m j_1 \cdots j_n}(k) \partial_{i_1}^{\mu_1} \cdots \partial_{i_m}^{\mu_m} \theta_{j_1}^{\alpha_1} \cdots \theta_{j_n}^{\alpha_n} +$$

$$+ \cdots + \cdots \delta(\{z\})$$  \hspace{1cm} (8.9)

$$b(k, \{z\}, \{\theta\}) = [c_0 b_0(k) + c(1, 0) b_{\mu_1}^{i_1}(k) \partial_{i_1}^{\mu_1} +$$

$$c(0, 1) b_{\alpha_1}^{j_1}(k) \theta_{j_1}^{\alpha_1} + \cdots +$$

$$c(m, n) b_{\mu_1 \cdots \mu_m, \alpha_1 \cdots \alpha_n}^{i_1 \cdots i_m j_1 \cdots j_n}(k) \partial_{i_1}^{\mu_1} \cdots \partial_{i_m}^{\mu_m} \theta_{j_1}^{\alpha_1} \cdots \theta_{j_n}^{\alpha_n} +$$

$$+ \cdots + \cdots \delta(\{z\})$$  \hspace{1cm} (8.10)

As a consequence we have

$$\Phi(x, \{z\}, \{\theta\}) = \int_{-\infty}^{\infty} (a(x, \{z\}, \{\theta\}) e^{-ik_\mu x^\mu} +$$

$$b^+(x, \{z\}, \{\theta\}) e^{ik_\mu x^\mu}) \, d^{y-1}x$$  \hspace{1cm} (8.11)
If we define

\[
\begin{cases}
[ , ]_n = [ , ]; & \text{n even} \\
[ , ]_n = [ , ]; & \text{n odd}
\end{cases}
\]  
(8.12)

with

\[
[a_{i_1 \cdots i_m j_1 \cdots j_n}^{i_1 \cdots i_m \alpha_1 \cdots \alpha_n}(k), a_{\nu_1 \cdots \nu_m \beta_1 \cdots \beta_n}^{i_1 \cdots i_m j_1 \cdots j_n}(k')]_n =
\]

\[
f_{i_1 \cdots i_m j_1 \cdots j_n k_1 \cdots k_m l_1 \cdots l_n}^{i_1 \cdots i_m \alpha_1 \cdots \alpha_n \nu_1 \cdots \nu_m \beta_1 \cdots \beta_n}(k)\delta(k-k')
\]  
(8.13)

Then

\[
\{ a(k, \{z\}, \{\theta\}), a^+(k', \{z'\}, \{\theta'\}) \} = [c_0^2 f_0(k)\delta(k-k') + \cdots +
\]

\[
c^2(m, n) f_{i_1 \cdots i_m j_1 \cdots j_n k_1 \cdots k_m l_1 \cdots l_n}^{i_1 \cdots i_m \alpha_1 \cdots \alpha_n \nu_1 \cdots \nu_m \beta_1 \cdots \beta_n}(k)\delta(k-k')
\]

\[
\partial_{i_1}^{\mu_1} \cdots \partial_{i_m}^{\mu_m} \partial_{\nu_1}^{\nu_1} \cdots \partial_{\nu_m}^{\nu_m} \theta_{\alpha_1}^{\alpha_1} \cdots \theta_{\alpha_n}^{\alpha_n} \theta_{\beta_1}^{\beta_1} \cdots \theta_{\beta_n}^{\beta_n}
\]

\[
+ \cdots \} \delta(\{z\}, \{z'\})
\]  
(8.14)

and for the anti-string

\[
[b_{i_1 \cdots i_m j_1 \cdots j_n}^{i_1 \cdots i_m \alpha_1 \cdots \alpha_n}(k), b^+_{\nu_1 \cdots \nu_m \beta_1 \cdots \beta_n}^{i_1 \cdots i_m j_1 \cdots j_n}(k')]_n =
\]

\[
g_{i_1 \cdots i_m j_1 \cdots j_n k_1 \cdots k_m l_1 \cdots l_n}^{i_1 \cdots i_m \alpha_1 \cdots \alpha_n \nu_1 \cdots \nu_m \beta_1 \cdots \beta_n}(k)\delta(k-k')
\]  
(8.15)

Thus

\[
\{ b(k, \{z\}, \{\theta\}), b^+(k', \{z'\}, \{\theta'\}) \} = c_0^2 g_0(k)\delta(k-k') + \cdots +
\]

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\[ c^2(m, n) \mathcal{d}_{i_1 \cdots i_m j_1 \cdots j_n k_1 \cdots k_m l_1 \cdots l_n}(k) \delta(k - k') \]

\[ \partial_{i_1} \cdots \partial_{i_m} \partial_{k_1}^v \cdots \partial_{k_m}^v \theta_{\alpha_1} \cdots \theta_{\alpha_n} \theta'_{i_1} \cdots \theta'_{i_n} \]

\[ + \cdots ] \delta([z], [z']) \] (8.16)

9 The Action for the Field of the Supersymmetric String

The case \( n \) finite

The action for the free supersymmetric closed string field is:

\[ S_{\text{free}} = \int \int \int \int \partial_{\mu} \Phi(x, [z_1], \{\theta_1\}) e^{[z_1] \cdot [z_2]} e^{[\theta_1] \cdot [\theta_2]} \]

\[ \partial^\mu \Phi^+(x, [z_2], \{\theta_1\}) \ d^\nu x \ [dz_1] [dz_2] [d\theta_1] [d\theta_2] \] (9.1)

A possible interaction is given by:

\[ S_{\text{int}} = \lambda \int \int \int \int \int \Phi^+(x, [z_1], \{\theta_1\}) e^{[z_1] \cdot [z_2]} e^{[\theta_1] \cdot [\theta_2]} \]

\[ \Phi^+(x, [z_2], \{\theta_2\}) e^{[z_2] \cdot [z_3]} e^{[\theta_2] \cdot [\theta_3]} \Phi^+(x, [z_3], \{\theta_3\}) e^{[z_3] \cdot [z_4]} e^{[\theta_3] \cdot [\theta_4]} \]

\[ \Phi^+(x, [z_4], \{\theta_4\}) d^\nu x \ [dz_1] [dz_2] [dz_3] [dz_4] [d\theta_1] [d\theta_2] [d\theta_3] [d\theta_4] \] (9.2)

Both, \( S_{\text{free}} \) and \( S_{\text{int}} \) are non-local as expected.
The case $n \rightarrow \infty$

In this case:

$$S_{\text{free}} = \int \int \int \int e^{(z_1 \cdot z_2)} e^{(\theta_1 \cdot \theta_2)}$$

$$\partial^\mu \Phi^+(x, \{z_1\}, \{\theta_1\}) \ d^\gamma x \ (d\eta_1)(d\eta_2)(d\theta_1)(d\theta_2) \quad (9.3)$$

where

$$d\eta(z) = \frac{e^{-z^2}}{\sqrt{2 \pi}} \ dz \quad (9.4)$$

and

$$S_{\text{int}} = \lambda \int \int \int \int e^{(z_2 \cdot z_3)} e^{(\theta_2 \cdot \theta_3)} \Phi^+(x, \{z_3\}, \{\theta_3\}) e^{(z_3 \cdot z_4)} e^{(\theta_3 \cdot \theta_4)}$$

$$\Phi(x, \{z_4\}, \{\theta_4\}) \ d^\gamma x \ (d\eta_1)(d\eta_2)(d\eta_3)(d\eta_4)(d\theta_1)(d\theta_2)(d\theta_3)(d\theta_4) \quad (9.5)$$

Gauge Conditions

The gauge conditions for the string field are:

$$\int z_1^\mu \cdots z_k^\mu \partial_{\mu_k} \cdots z_m^\mu \partial_{\mu_m} \partial_{\theta_{j_1}^{\alpha_1}} \cdots \partial_{\theta_{j_l}^{\alpha_l}} \ cd \Phi(x, \{z\}, \{\theta\}) \ (dz)(d\theta) = 0 \quad (9.6)$$

$$\partial_{\mu_k} = \partial/\partial x^{\mu_k} \ ; \ \partial_{\theta_{j_1}^{\alpha_1}} = \partial/\partial \theta_{j_1}^{\alpha_1} \ ; \ 1 \leq k \leq m \ ; \ m \geq 1 \ ; \ 1 \leq l \leq n \ ; \ n \geq 1.$$ 

With these gauge conditions the number of the components fields of the
superstring field is finite, and the temporal components of all fields are eliminated.

Another gauge conditions that can be added to (9.6) are

\[
\int \oint \{ \Gamma \} z_{\mu_1}^{\mu_1} \cdot \cdots \cdot z_{\mu_k}^{\mu_k} \cdot z_{\mu_m}^{\mu_m} \cdot \partial_{\theta_{i_1}}^{\alpha_{i_1}} \cdot \cdots \cdot \partial_{\theta_{i_l}}^{\alpha_{i_l}} \cdot \partial_{\theta_{j_1}}^{\alpha_{j_1}} \cdots \partial_{\theta_{j_n}}^{\alpha_{j_n}} \Phi(x, \{z\}, \{\theta\}) \{dz\}\{d\theta\} = 0 \quad (9.7)
\]

1 ≤ k ≤ m; m ≥ 1; 1 ≤ l ≤ n; n ≥ 1.

These additional gauge conditions permit us nullify other component fields according to experimental data. It should be noted that gauge conditions (9.6) and (9.7) does not modify the movement equations of superstring field.

The convolution of two propagators of the string field is:

\[
\hat{\Delta}_{\alpha \beta}(k,\{z_1\},\{\theta_1\},\{z_2\},\{\theta_2\}) \ast \hat{\Delta}_{\alpha \beta}(k',\{z_3\},\{\theta_1'\},\{z_4\},\{\theta_2'\}) \quad (9.8)
\]

where \(\ast\) denotes the convolution of Ultradistributions of Exponential Type on the k variable only. With the use of the result

\[
\frac{1}{\rho} \ast \frac{1}{\rho} = -\pi^2 \ln \rho \quad (9.9)
\]

\((\rho = k_1^2 + k_2^2 + \cdots + k_\nu^2)\) in euclidean space and

\[
\frac{1}{\rho \pm i0} \ast \frac{1}{\rho \pm i0} = \mp i\pi^2 \ln (\rho \pm i0) \quad (9.10)
\]

\((\rho = k_0^2 - k_1^2 - \cdots - k_{\nu-1}^2)\) in minkowskian space,
10 Discussion

We have decided to begin this paper, for the benefit of the reader, with a summary of the main characteristics of Ultradistributions of Exponential Type and their Fourier transform.

We have shown that UET are appropriate to describe in a consistent way string and string field theories. By means of a new Lagrangian for the closed superstring we have obtained the movement equation for the field of the string and solve it with the use of CUET. We show that this string field is a linear superposition of CUET. We give a definition of anti-superstring. We evaluate the propagator for the string field, and calculate the convolution of two of them, taking into account that string field theory is a non-local theory of UET of an infinite number of complex variables. For practical calculations and experimental results we have given expressions that involve only a finite number of variables.

As a final remark we would like to point out that our formulae for convolutions follow from general definitions. They are not regularized expressions
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