Direct phase extraction of self-mixing displacement measurement using Hilbert transform

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Abstract. Signals of a self-mixing interferometer established on a semiconductor laser diode are analyzed. Phase is extracted out for decoding measurement which contained in self-mixing fringes. The semiconductor laser diode works as light source and receiver without modulation. By combining Hilbert transform with phase condition of self-mixing interference, micron-displacement is reconstructed by phase information at week or even moderate feedback level. Theoretical analysis and simulation results are presented before verification of experimental measurement. Practical feedback level is estimated by a data fitting technique with a programmable high-resolution PZT. Consistence of the results promises that direct phase extraction on self-mixing interferometer is available for micron-displacement measurement with a nanometer accuracy.

1. Introduction

There is a continuously rising requirement of displacement monitoring with a high accuracy as well as a wide dynamic range in semiconductor industries or biotechnology. In recent years, self-mixing interference (SMI) has been paid to considerable attention due to compact structure and easy demodulation. Fundamental theoretical three model of SMI had been constructed in Refs [1,2,3,4] which resulted in potential applications of measuring micron displacement or vibration as Refs [5,6,7]. Among piles of lasers employed in SMI, semiconductor laser diode which is equipped with an in-built photo diode gradually becomes an ideal light source and receiver in SMI interferometer without high cost. Consequently, a variety of demodulation methods have emerged as Refs[8,9,10] reported, for instance, triangle frequency modulation is introduced into SMI system by Q.Wu and his team, an accuracy more than 98% is achieved by adjusting a sampling frequency. D.Guo of our team introduces double modulations into SMI for obtaining a more competitive accuracy of \( \lambda/65 \).

However, technologies by injecting current modulation or introducing phase shift are imperfect. The electronics or modulators make SMI interferometer complicated and expensive, the feedback level is limited within very weak level to neglect multi-reflection effect, besides, frequency of modulation frequency will limit the bandwidth of measured displacement, all of these problems need desirable solutions. In this paper, the simplest SMI configuration is analyzed by a novel phase extraction method based on Hilbert transform, feature of displacement is captured by extracting the phase of the interference signal. Semiconductor laser driven by constant current avoids unwanted intensity fluctuation or limitation of bandwidth, which allows a larger range of feedback level comparing with previous SMI interferometer.
Although a lot of algorithm methods have been employed in SMI for demodulation, such as Fourier transform method, beat frequency method and modified fringe counting and so on. But Fourier transform method is based on current modulation, beat frequency method is performed requiring additional reference optical path, accuracy of modified fringe counting is convenient but not high as phase demodulation technology.

The proposed phase extraction without modulation is appropriate for existing SMI system because of easy implementation and immunity to current fluctuation. Phase information of weak, moderate and high feedback level self-mixing signal is analyzed at first, then, measurement principle is demonstrated with numerical simulations before introduction of the experimental setup. In last section comparison between the SMI and Poly Tec 5000 vibration meter is given for drawing the conclusion that direct phase extraction method achieves a nanometer accuracy at weak or moderate feedback level.

2. Measurement Principle

2.1. Modulated phase obtained by Hilbert transform

The basic theory of self-mixing interference has been deeply studied by previous authors, behavior of semiconductor laser is described using a three-mirror model with optical feedback. Implementing Hilbert transform on three-mirror model is an effective method to reflect modulated phase, its process is presented as follows.

Assuming reflectivity of three mirrors are \( r_1, r_2 \) and \( r_3 \), travel time of light in resonant and external cavity are \( \tau_1 \) and \( \tau_2 \), which are expressed:

\[
\tau_1 = \frac{2n_1 l}{c}, \quad \tau_2 = \frac{2L}{c}
\]

where \( l \) denotes length of the resonant cavity, \( L \) denotes length of the external cavity. The whole phase condition of self-mixing interference is written:

\[
\phi_1(t) - \phi_2(t) = C \sin(\phi_1(t) - \arctan(\alpha))
\]

\[
C = (1 - r_1 r_2) r_3 \frac{\tau_1}{\tau_2} \sqrt{1 + \alpha^2 / r_3}
\]

\[
P = P_0 [1 + m \cos(\phi_1(t))]
\]

\[
\alpha = \frac{X}{\rho}
\]

where \( C \) stands for feedback level, \( \alpha \) is line-width enhancement factor depending on change rate of refractive index \( X \) and stimulated emission gain index \( \rho \) of laser, \( m \) is the undulation coefficient determined by reflectivity of the target mirror. \( P_0 \) and \( P \) are optical power without and with feedback, correspondingly, \( \phi_1(t) \) and \( \phi_2(t) \) are time-dependent phase of the external cavity without and with feedback respectively. The optical power is often observed with various levels of fringe inclination due to that SMI interference is operated at different feedback level.

For the bias part and amplitude of SMI signal can be easily and precisely measured by existing tool in software, the optical power expressed in equation (2) usually should be normalized as below before Hilbert transform:

\[
\cos(\phi_1(t)) = (P/P_0 - 1)/m
\]

To extract modulated phase information \( \phi_1(t) \), Hilbert transform is performed on \( \cos(\phi_1(t)) \) firstly, mathematical equation of which is expressed:

\[
H[\cos(\phi_1(t))] = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\cos(\phi_1(t))}{t} \, dt
\]

Hilbert transform of normalized SMI signal is consisted of a real part and an imaginary part. The imaginary part is introduced with an ideal 90° phase shift, which is equal to \( \sin(\phi_1(t)) \), so, \( \phi_1(t) \) is extracted by arc-tangent calculation:
\[ \varphi(t) = \arctan \left( \frac{\text{Im} [H(\cos(\varphi_0(t)))]}{\cos(\varphi_0(t))} \right) \]  

Unwrapping result of equation (6) is a continuous modulated phase information curve.

### 2.2. Correction by phase condition.

As analyzed in front selection, weak feedback level decreases the possibility of mode-hopping dramatically, laser frequency is independent to the optical feedback, the \( \varphi(t) \) is approximate to \( \varphi_0(t) \). But higher feedback level, larger the difference between \( \varphi(t) \) and \( \varphi_0(t) \). It is undeniable that the modulated phase is not linear to measured displacement, hence, a correction is added into its data process.

Combining phase condition of equation (2), the \( \varphi_0(t) \) can be expressed as:

\[ \varphi_0(t) = \varphi(t) + C \sin(\varphi(t) - \arctan(\alpha)) \]  

where \( C \sin(\varphi(t) - \arctan(\alpha)) \) is called as modification factor in next paragraphs, the line-width enhancement factor is induced by below equation:

\[ \alpha = \pm \sqrt{1 - \frac{|r_2 T_r C(1 - r_1^*) r_1^*|}{|r_1^* T_r C(1 - r_1^*) r_1^*|}} \]  

When variation of external cavity length is at micron level, the variation of travel time \( \tau \) is neglected. Reflectivity \( r_1 \) is an intrinsic parameter of semiconductor laser diode, reflectivity of the target mirror is a stable value as well, \( \alpha \) is approximately determined by feedback level \( C \) conclusively. Instantaneous displacement between the laser and the object is linear to initial phase information as below:

\[ \varphi(t) = \frac{4\pi L(t)}{\lambda_0} \]  

where \( \lambda_0 \) is central wavelength of the single mode laser. The relationship is used to obtain displacement curve.

To illustrate the above principle, the number simulation on a computer is shown in below figures, which plot the phase curve, normalized SMI signal, imaginary part of Hilbert transform, and the results of equation (5). Data sampling ratio is set at 250KHz/s, data point number is 2500, the frequency of phase converted from displacement is 100Hz, and amplitude is 20rad, initial phase is zero. Feedback levels in Figure 1 and Figure 2 are 0.5 and 1.5 respectively. The line-width enhancement factor is equal to 4 at both figures.

The Figure 1 and Figure 2 indicate a good point and a weak point, Hilbert transform indeed has a desirable ability in phase extraction, however, arc-tangent calculation is insensitive to changes in direction of displacement.
Figure 1. (a). The sinusoidal phase curve in simulation. (b). Self-mixing signal. (c). Imaginary part of Hilbert transform on picture(b). (d). The modulated phase $\phi(t)$.

Figure 2. Self-mixing signal with feedback level is 1.5. (b). Imaginary part of Hilbert transform on picture (a). (c). The modulated phase $\phi(t)$.
Figure 3. (a). Modification factor of Fig.1. (b). Curve of $\varphi(t)$. (c). Multiplication result of $\varphi(t)$ with sign function. (d). The unwrapped real phase $\varphi(t)$.

Figure 4. (a). Modification factor of Fig.2. (b). Curve of $\varphi(t)$. (c). Multiplication result of $\varphi(t)$ with sign function. (d). The unwrapped real phase $\varphi(t)$.

Subsequently, the equation (6) is illustrated in Figure 3 and Figure 4 with modification factor. A sign function is used to depict the phenomenon of fringe inclination in mathematics. When fringes incline to left,
the sign is negative, when fringes incline to right, the sign is positive. Multiplying $\phi(t)$ with the sign of direction, the true phase can be reconstructed at weak and moderate feedback levels as shown in the picture(c) and (d) of Figure 3 and Figure 4. After unwrapping, actual $\phi(t)$ is modified in a new expression:

$$\phi(t) = \text{sign} \times \phi(t) \pm 2\pi n$$

(9)

where $n$ is an integer, sign = ±1, $\phi(t)$ is a continuous phase curve in time domain.

2.3. Error discussion

To evaluate the accuracy, the simulative error is calculated by computing standard deviation. Peak to peak values of the reconstructed phase curve in Figure 3 are -19.91rad and 20.07rad, peak to peak values of reconstructed phase curve in Figure 4 are -19.93rad and 20.09rad. The amplitude errors are both less than 0.12rad which prove the availability of phase extraction method. Equation (10) is used to calculate the standard deviation between the reconstructed phase and the ideal phase:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{2499} (\phi_{\text{ext}}(i) - \phi_{\text{ideal}}(i))^2}{2499}}$$

(10)

Assuming reconstructed phase is sampled at the instant $t = \frac{i}{N\text{f}}$, sampling ratio is $f$, Hz/s, number of data is $N = 2500$, $\phi_{\text{ext}}(i)$ represents the extracted phase at the i-th point, $\phi_{\text{ideal}}(i)$ represents an ideal phase at the i-th point. Standard deviations of two reconstructed phase curves are 0.043rad, 0.067rad.

The errors of Figure 3 and Figure 4 between reconstructed phase and initial phase curve are plotted in Figure 5.

![Figure 5](image)

**Figure 5.** (a). Phase error obtained by subtraction between reconstructed phase of Figure 3 and initial phase. (b). Phase error curve of Figure 4.

Displacement $L(t)$ is calculated with equation (11):

$$L(t) = \lambda \phi_{\text{ext}}(t)/4\pi$$

(11)

Corresponding standard deviations of displacement are 2.47nm and 3.13nm. It is obviously seen that, proposed method is applicable at weak or moderate feedback level in displacement measurement.

3. Experimentation

3.1. Experimental setup
The experimental setup as illustrated in Figure 4 is mainly divided into 3 parts: the SMI part, the data process part and the comparison part. With the merits of long lifespan, inexpensive price and tiny size, semiconductor laser diode (Thorlabs, L650P007-650, USA) works as light source and receiver in the established experimental system. An aspheric compensating lens is inserted in the optical path to focus emission of semiconductor laser into a point for alignment, 22mA constant current is output from precise supplier (ILX-Lightwave, LDX-3220, USA) for driving semiconductor. In order to control the intensity of reflected light from the target mirror, a various neutral density filter (ND) is used.

![Figure 6. The experimental setup of the semiconductor self-mixing interferometer](image)

The object mirror is fixed on a commercial PZT (P-841, Physik Instrument Co., Germany) whose displacement range is 20um and resolution is 1nm. A 16-bit A/D converter (USB-6361, National Instrument Co., USA) is used to act for data sampling and analog to digital conversion. The digital signal is filtered for removing noises and magnified in pre-process in a personal computer (PC), the SMI signal can be observed simultaneously by connecting to Tektronix oscilloscope as shown Figure 6.

### 3.2. Estimation of parameters in self-mixing model

All parameters appearing in simulation are well known beforehand. But before analyzing experimental data, the feedback level needed to be estimated as accurate as possible, a data fitting method in Ref [11] is adopted to estimate the value of feedback level $C$. With the help of programmable PZT, a group of self-mixing signals are generated in Figure 7 with displacement volts.
Figure 7. The screen shots of digital Tektronix Oscilloscope TDS 2000C, upper curve is monitor volts of PZT, lower curve is self-mixing interference signals, displacement frequency is 100Hz, peak to peak amplitudes of displacement are (a)1.2um, (b)1.6 um, (c)2.1um and (d)2.8um.

Then, simulative self-mixing signal is generated and expressed:

\[ A(C_m, \alpha_m, t) = \cos[\phi(t) - C_m \sin((...) - \arctan(\alpha_m))] \]

where number of data point N is equal to that of observed self-mixing signals, \( i \in (0,1,2,...,N-1) \), omitted part is \( \phi(t) = \phi_i(t) - C_m \sin(\phi_i(t) - \arctan(\alpha_m)) \). \( C_m \) is estimated feedback level, \( \alpha_m \) is estimated line-width enhancement factor, times of iteration of omitted part should exceed 10 to ensure signal veracity. Meanwhile, the direct intensity of experimental signal acquired by A/D card is removed and normalized written as:

\[ B(C, \alpha, t) = \cos(\phi(t)) \]

Set initial \( C_m \) and \( \alpha_m \) at 0.5 and 1, then calculate average data deviation \( D \) as below:

\[ D = \sum_{i=1}^{N} \frac{(A(C_m, \alpha_m, t) - B(C, \alpha, t))^2}{N} \]

Increment of \( C_m \) is 0.01, equation (14) reaches its bottom value when \( C_m \) is most close to experimental feedback level. The estimated feedback level in Figure 6 are 0.87, 0.83 and 0.85. With the best fit \( C_m, \alpha_m \) can be calculated on equation(7) as well.

Figure 8. Process of iteration in data fitting, where SMI signal stands for observed signals.

3.3. Experimental results

One group of displacement measurement has been done to confirm the validity of the proposed method. Displacement of the target mirror was realized by driving the PZT with a frequency of 50 Hz and amplitude of 2000 nm (peak to peak) sinusoidal volts, results of SMI (red color) and Doppler vibration meter (blue color) are recorded and reconstructed error are presented, average error is 8.91nm and maximum error value is 16.7nm.

Setting PZT move in a sinusoidal wave with frequency of 100 Hz and amplitude of 3000 nm (peak to peak), another result in Figure 9 shows an average error of 18.1nm with maximum error of 29.5nm.
Control program of NI DAQ device and the data process is written on Labview2014. Experimental results prove that the reconstructed displacement of the SMI is in good consistency with data from the Ploy Tec-5000 and verify that a displacement measurement accuracy of about 30 nanometers was achieved. In conclusion, the proposed method an improved phase demodulation.

4. Conclusion

The principle of direct phase extraction is presented with number simulations and experimentation. The modulated phase information is corrected by the phase condition of self-mixing and sign function denoting fringe inclination. The real phase is extracted from SMI signal without any reference signals during data process and reconstructed phase curves show an error less than 0.1rad in amplitude of 20rad. The dynamical range of the proposed method is without limitation theoretically, which is convenient and effective to implement in SMI interferometers or sensors.

To check the availability, a compact semiconductor self-mixing interferometer has been built and demonstrated as verification. No additional electronics or modulator exist in the experimental system, which is operated with a constant current at weak or moderated feedback level. The experimental signals mirror the simulative signals with laboratory noise. The parameters used in demodulation are estimated by data fitting between observed signals and simulative signals. Micron displacement measurement and comparison with Doppler vibration meter show an accuracy of less than 30nm has been achieved in measurement, which proves that the direct phase extraction is practical for monitoring micron-displacement position or displacement.

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