PROCEEDINGS

Gravitational memory effects in Brans-Dicke theory

Shaoqi Hou*

1School of Physics and Technology, Wuhan University, Hubei, China

Correspondence
* Email: hou.shaoqi@whu.edu.cn

Present Address
School of Physics and Technology, Wuhan University, Wuhan, Hubei 430072, China

Funding Information
National Natural Science Foundation of China, 11633001 and 11920101003.
Strategic Priority Research Program of the Chinese Academy of Sciences, XDB23000000. China Postdoctoral Science Foundation, 2020M672400.

1 INTRODUCTION

Gravitational memory effects are fascinating phenomena that occur in general relativity (GR) (Braginsky & Grishchuk (1985); Christodoulou (1991); Thorne (1992); Zel'dovich & Polnarev (1974)). There are several types of memories. The displacement memory refers to the permanent change in the relative distance between two test particles after the passage of gravitational waves (GWs). The spin memory manifests itself in the different propagating times it takes for two test particles, orbiting in a circle in opposite directions, to return to their original positions, when GWs pass by (Pasterski, Strominger, & Zhiboedov (2016)). And the center-of-mass (CM) memory is the lasting shift of the CM of an isolated system due to GWs (Nichols (2018)). There are other memories, e.g., velocity memory effect (Zhang, Duval, Gibbons, & Horvathy (2017a, 2018, 2017b)), none of which will be discussed in this work. Both displacement and spin memories might be detected by interferometers and pulsar timing arrays (Boersma, Nichols, & Schmidt (2020); Hübner, Talbot, Lasky, & Thrane (2020); Johnson, Kapadia, Osborne, Hixon, & Kennefick (2019); Lasky, Thrane, Levin, Blackman, & Chen (2016); Madison (2020); McNeill, Thrane, & Lasky (2017); Seto (2009); Wang et al. (2013), but it is difficult to observe the CM memory experimentally (Nichols (2018)).

Memory effects might also exist in modified theories of gravity (Du & Nishizawa (2016); Kilicarslan (2018, 2019); Kilicarslan & Tekin (2019); Lang (2014, 2015)). Brans-Dicke (BD) theory is the simplest modified gravity (Brans & Dicke (1961)), whose action is

\[ S = \frac{1}{16\pi G_0} \int d^4 x \sqrt{-g} \left( \varphi R - \frac{\partial \varphi}{\partial \varphi} \nabla_{\alpha} \varphi \nabla^{\alpha} \varphi \right), \]

(1)

where \( \omega \) is a constant, \( G_0 \) is the bare gravitational constant. The memory effect in this theory would be the topic of this work. It is found out that there are not only the same memories in BD as those in GR, but also a new memory effect due to the BD scalar field, named S memory in Du & Nishizawa (2016). These memories also have something to do with the asymptotic symmetries of an isolated system, the so-called Bondi-Metzner-Sachs (BMS) group, which is a semi-direct product of the supertranslation group and the Lorentz group as in GR. On the one hand, the supertranslation causes the vacuum transition in the tensor sector, and is responsible for the displacement memory, while the Lorentz transformation causes the vacuum transition in the scalar sector, so the S memory takes place. On the other hand, the BMS symmetries imply the existence of supermomenta, angular momenta and Lorentz boost charges at the null infinity \( \mathcal{J} \). When there are GWs, these
quantities vary over time, and their variations equal the corresponding fluxes escaping from the source of gravity. The flux-balance laws provide constraints on the memory effects. For more details, please refer to Hou & Zhu (2020a, 2020b; Tahura, Nichols, Saffer, Stein, & Yagi (2020)).

This work mainly discusses the displacement memories in the tensor and the scalar sectors. Their relations with the BMS symmetries will be deciphered. Finally, we will compute the constraints on various memory effects using the flux-balance laws. We hope this work will set up the basis for using memory effects to probe the nature of gravity (Koyama (2020)). So the “conserved charges” and fluxes are determined in Sec. 2.2.

Section 3 focuses on memory effects to probe the nature of gravity (Koyama (2020)). So this work is organized in the following way. Section 2 discusses the asymptotically flat spacetime in BD. Based on that, the BMS symmetry is defined and computed in Sec. 2.1 and then, the “conserved charges” and fluxes are determined in Sec. 2.2. Section 3 focuses on memory effects. There, the displacement memories are analyzed in Sec. 3.1 and Sec. 3.2 is devoted to the spin and the CM memories. In the end, there is a short summary in Sec. 4. In this work, the abstract index notation is used (Wald (1984)), and the speed of light in vacuum is $c = 1$.

### 2.1 Asymptotically Flat Spacetimes

Memory effects also happen in cosmological background (Bonga & Prabhu (2020); Donnay & Giribet (2019)), but here, we will focus on those happening in an isolated system. At the distances very far away from the source of gravity, the spacetime is nearly Minkowskian, so such kind of spacetime is said to be asymptotically flat. An asymptotically flat spacetime at null infinity in GR can be defined in a coordinate independent manner; see Wald (1984). In BD, one can also propose a similar definition as presented in Hou & Zhu (2020a). But here, we will simply define such a spacetime using the generalized Bondi-Sachs coordinates $(u, r, \theta, \phi)$ ($x^A = \theta, \phi$), in which the metric is

$$ds^2 = g_{uu}du^2 + 2g_{ur}dudr + 2g_{aA}dudx^A + h_{AB}dx^Adx^B. \quad (2)$$

One requires that the metric components satisfy the following boundary conditions,

$$g_{uu} = -1 + \mathcal{O}(r^{-1}), \quad g_{ur} = -1 + \mathcal{O}(r^{-1}), \quad g_{uA} = \mathcal{O}(1), \quad h_{AB} = r^2 \gamma_{AB} + \mathcal{O}(r), \quad (3a)$$

where $\gamma_{AB}$ is the round metric on a unit 2-sphere,

$$\gamma_{AB}dx^Adx^B = d\theta^2 + \sin^2 \theta d\phi^2. \quad (4)$$

The scalar field behaves like $\varphi = \varphi_0 + \mathcal{O}(r^{-1})$ with $\varphi_0$ a constant. One also imposes the determinant condition,

$$\det(h_{AB}) = r^4 \left( \frac{\varphi_0}{\varphi} \right)^2 \sin^2 \theta, \quad (5)$$

so that $r$ becomes the luminosity distance as it approaches infinity.

With these conditions, one can obtain the series solutions to the equations of motion, i.e.,

$$\varphi = \varphi_0 + \frac{\varphi_1}{r} + \frac{\varphi_2}{r^2} + \mathcal{O}\left( \frac{1}{r^3} \right), \quad (6a)$$

$$g_{uu} = -1 + \frac{2m + \varphi_1/\varphi_0}{r} + \mathcal{O}\left( \frac{1}{r^2} \right), \quad (6b)$$

$$g_{ur} = -1 + \frac{\varphi_1}{\varphi_0} + \frac{1}{r^2} \left[ \frac{1}{16} \epsilon^{AB} \epsilon_{A^B} + \frac{2}{8} \left( \frac{\varphi_1}{\varphi_0} \right)^2 + \frac{\varphi_2}{\varphi_0} \right] + \mathcal{O}\left( \frac{1}{r^3} \right), \quad (6c)$$

$$g_{uA} = \frac{\partial_B \epsilon^A}{2} + \frac{2}{3r} \left[ N_A + \frac{1}{4} \epsilon_{AB} \partial_C \epsilon_{BC} - \frac{\varphi_1}{12\varphi_0} \partial_D \epsilon^A \right] + \mathcal{O}\left( \frac{1}{r^2} \right), \quad (6d)$$

$$g_{AB} = r^2 \gamma_{AB} + r \left( \hat{\epsilon}_{AB} - \frac{\varphi_1}{\varphi_0} \right) + \hat{d}_{AB} + \gamma_{AB} \left( \frac{1}{4} \hat{\epsilon}^C \hat{\epsilon}_C + \frac{\varphi_2}{\varphi_0} - \frac{\varphi_1}{\varphi_0} \right) + \mathcal{O}\left( \frac{1}{r} \right). \quad (6e)$$

Here, $\varphi_1, \varphi_2,$ $\hat{\epsilon}_{AB},$ and $\hat{d}_{AB}$ are functions of $u$ and $x^A$ with $\gamma_{AB} \hat{\epsilon}_{AB} = \gamma_{AB} \hat{d}_{AB} = 0,$ and $\mathcal{O}_A$ is the covariant derivative for $\gamma_{AB},$ $m = m(u, x^A)$ and $N_A = N_A(u, x^B)$ are called the Bondi mass and angular momentum aspects, respectively. Their evolution equations are

$$\dot{m} = -\frac{1}{4} \partial_D \partial_B N^{AB} - \frac{N_A N^{AB}}{8} - \frac{2\varphi_3 + 3}{4} \left( \frac{N}{\varphi_0} \right)^2 \quad (7a)$$

$$\dot{N}_A = \partial_A m + \frac{1}{4} \left( \mathcal{D}_B \partial_D \gamma_{BC} - \mathcal{D}_D \mathcal{D}_B \gamma_{BC} \right)$$

$$- \frac{1}{16} \partial_D (N^{BC} \delta^D_A) + \frac{1}{4} \frac{N_B \partial_D \gamma^C}{N^B}$$

$$+ \frac{1}{4} \partial_B (N_A \gamma^C - \gamma^C \gamma^D), \quad (7b)$$

where $N_{AB} = -\partial \epsilon_{AB}/\partial u$ is the news tensor, and $N = \partial \varphi_1/\partial u.$ When there are no GWs, both $N_{AB}$ and $N$ vanish. So they are called the radiative degrees of freedom. In particular, $N_{AB}$ corresponds to the tensor GW, while $N$ to the scalar GW (Hou, Gong, & Liu (2018)).

At the null infinity, although the spacetime is very similar to the flat one, the symmetry group living there is not simply the Poincaré group. It is a much larger group, instead. In the next subsection, this group will be analyzed. After that, the “conserved charges” and fluxes will be obtained.

### 2.1 Bondi-Metzner-Sachs Symmetries

The asymptotic symmetries, or BMS symmetries, are diffeomorphisms that preserve the boundary conditions defined in
where $\mathcal{C}$ represents a cross section on $\mathcal{J}$, and $d^2\Omega = \sin \theta d\theta d\phi$. If $a = 1$, one has the Bondi mass, and if $a$ is a linear combination of $Y_{lm}$ with $l = 1$, one has the spatial momentum. The flux is
\[
F_a[\mathcal{R}] = \frac{\varphi_0}{16\pi G_0} \int \alpha \left[ D_A D_B N^{AB} + \frac{1}{2} N^A_B N^A_B \right.
+ (2\omega + 3) \left( \frac{\varphi_1}{\varphi_0} \right)^2 d^2\Omega,
\]
where $\mathcal{R}$ is a subset of $\mathcal{J}$. The first term in the square brackets represents the so-called soft part, and the remaining are the hard part. If $\mathcal{C}$ and $\mathcal{C}'$ are the past and the future boundaries of $\mathcal{R}$, the flux-balance law is
\[
F_a[\mathcal{R}] = -(P_a[\mathcal{C}'] - P_a[\mathcal{C}]).
\]
This will be useful for constraining the displacement memory in the tensor sector.

For an infinitesimal Lorentz transformation, one writes $Y^A = D_A \mu + e^A B D_B \nu$, then the angular momentum is
\[
J_\nu[\mathcal{C}] = \frac{\varphi_0}{8\pi G_0} \int_{\mathcal{C}} u e^{AB} D_A N_B d^2\Omega,
\]
and the Lorentz boost charge
\[
\mathcal{K}_\mu[\mathcal{C}] = -\frac{\varphi_0}{8\pi G_0} \int_{\mathcal{C}} \mu \left( D_A N_A + 2\mu m \right.
- \frac{\zeta^B A B}{16} - \frac{2\omega + 3 \varphi_0^2}{8} \left) d^2\Omega. \right)
\]
In the above relations, $\mu$ and $\nu$ are linear combinations of $Y_{lm}$ with $l = 1$. The corresponding flux is
\[
F_\nu[\mathcal{R}] = F_\nu[\mathcal{R}] + \frac{\varphi_0}{32\pi G_0} \int_{\mathcal{R}} Y^A \left[ \frac{1}{2} (D_C D_A N_B^B - N^C_B D_A \zeta^C) \right.
+ D_B (N^B_C \zeta_{AC} - \zeta^C B N^B A)\left.
+ \frac{2\omega + 3}{\varphi_0^2} \right) d^2\Omega, \right)
\]
where $\alpha' = \omega / 2$. The flux-balance law in this case is
\[
F_\nu[\mathcal{R}] = -(Q_\nu[\mathcal{C}'] - Q_\nu[\mathcal{C}]),
\]
with $Q_\nu$ either $J_\nu$ or $\mathcal{K}_\mu$.

The flux-balance laws are very useful to constrain various memories, which will be discussed in the next section.

3 MEMORIES

Finally, the memory effect is discussed. Let us first use the obtained metric for an isolated system to compute the geodesic deviation equation, since the ground-based interferometer uses
it to detect GWs. Let $T^a$ be the 4-velocity of the test particles, and $S^a$ be the deviation vector. Then one has (Wald 1984)
\begin{equation}
T^c \nabla_c (T^b \nabla_b S^a) = -R^a_{\ cba} T^c S^b T^d.
\end{equation}
Assume the test particles are placed at a fixed radius $r_0$ and the fixed directions $x^a$, so these particles are called BMS detectors (Strominger & Zhiboedov 2016). In general, these particles will be accelerated, but as long as $r_0$ is very large, they approxi-

mately freely fall. One then has $T^a \approx (\partial_a)\phi^\alpha$ at a far distance $r_0$. Define orthogonal spatial vectors $(e^a)\phi^\alpha = r^{-1}(\partial_a \phi^\alpha)$, and $(e^a)\phi^\alpha = (r \sin \theta)^{-1}(\partial_a \phi^\alpha)$. Then, let $S^a = S^A(e^a)\phi^\alpha$, and the above equation becomes
\begin{equation}
\hat{S}^\lambda \approx -R_{\ ab\lambda} \hat{S}^b.
\end{equation}
where the electric part of the Riemann tensor $R_{abc}^d$ is
\begin{equation}
R_{\ ab\lambda} = \frac{-1}{2r} \left( \Delta \hat{\epsilon}_{\ ab} - \delta_{\ ab} \frac{\Delta \phi_1}{\phi_0} \right) S^\lambda + O\left( \frac{1}{r^2} \right).
\end{equation}
Now, integrating eq. (18) twice results in
\begin{equation}
\Delta S^\lambda_A \approx \frac{1}{2r} \left( \Delta \hat{\epsilon}_{\ ab} - \delta_{\ ab} \frac{\Delta \phi_1}{\phi_0} \right) S^\lambda + O\left( \frac{1}{r^2} \right).
\end{equation}
in which $S^\lambda$ is the initial deviation vector at the retarded time $u_0$ when there were no GWs, i.e., $N_{\ ab} (u_0, x^A) = 0$ and $N_{\ ab} (u_0, x^A) = 0$. A radiating isolated system will eventually set-
down to a state in which no GWs can be emitted, and so $N_{\ ab}$ and $N$ vanish again. But $S^\lambda$ may not return to its original value, that is,
\begin{equation}
\Delta S^\lambda_A \neq 0.
\end{equation}
If so, there exists a permanent change in the relative distances between test particles. This is the displacement memory effect. By Eq. (20), there are two contributions to the total displacement,

memory, one of which is from the tensor part $\Delta \hat{\epsilon}_{\ ab}$, and the other from the scalar part $\Delta \phi_1$. So we will discuss the two contributions separately in Sec. 3.1. Section 3.2 concentrates on the spin and CM memories, mainly the constraints.

For the following discussion, one writes
\begin{equation}
\hat{\epsilon}_{\ ab} = \left( \mathcal{D}_{\ a} \mathcal{D}_{\ b} - \frac{1}{2} \gamma_{\ ab} \mathcal{D}^2 \right) \Phi + \epsilon_{\ A(b} \mathcal{D}_{\ a)} \mathcal{D}^C \gamma^D,
\end{equation}
where $\Phi$ is the electric part and $\gamma$ the magnetic part.

### 3.1 Displacement memories

The displacement memory in the tensor sector is very similar to the one in GR. It is related to the vacuum transition caused by the supertranslation (Strominger & Zhiboedov 2016). Here, a vacuum state in the tensor sector is the spacetime with $\hat{\epsilon}_{\ ab} = \left( \mathcal{D}_{\ a} \mathcal{D}_{\ b} - \frac{1}{2} \gamma_{\ ab} \mathcal{D}^2 \right) \Phi$ for some function $\Phi(x^A)$, so $N_{\ ab} = 0$ (Hou & Zhu 2020b). This kind of state can be transformed by a supertranslation $\alpha$ to a state with $\hat{\epsilon}_{\ ab}' = \left( \mathcal{D}_{\ a} \mathcal{D}_{\ b} - \frac{1}{2} \gamma_{\ ab} \mathcal{D}^2 \right) \Phi' \Phi' = \Phi - 2\alpha$. Therefore, the new state is also a vacuum, but different from the original one. This observation suggests that there are infinitely many vacua in the tensor sector, and the transition between any pair of them is induced by a supertranslation. Usually, one chooses one of the vacua as the physical one, e.g., $\hat{\epsilon}_{\ ab} = 0$. An infinitesimal Lorentz transformation results in $\delta_{\ Y} \hat{\epsilon}_{\ ab} = \mathcal{D}_{\ Y} \hat{\epsilon}_{\ ab} - \frac{\gamma_{\ ab}}{2} \hat{\epsilon}_{\ Y} \Phi$ which is not a vacuum state, but it preserves the physical vacuum, so the $S$-matrix and the soft theorem are still Lorentz covariant.

The displacement memory in the scalar sector, or $S$ memory, is new. Let the vacuum in the scalar sector be described by $\phi_1 = \phi_1(x^A)$ and $N = 0$. Then one finds out a supertranslation does not transform it, i.e., $\delta_{\ Y} \phi_1 = 0$, but a Lorentz generator $Y^A$ changes it according to $\delta_{\ Y} \phi_1 = L_Y \phi_1 + \frac{\gamma}{2} \phi_1$. It is interesting to find out that the new state $\phi_1' = \phi_1 + \delta_{\ Y} \phi_1$ is also a vacuum state. Therefore, like the displacement memory in the tensor sector, the displacement memory in the scalar sector is also the vacuum transition due to the Lorentz transformation.

The flux-balance law Eq. (12) can be used to constrain the displacement memory in the tensor sector. In terms of the variation in $\Phi$, one has
\begin{equation}
\int_a^b \mathcal{D}^2 (\mathcal{D}^2 + 2) \Delta \Phi \mathcal{D}^2 \Omega = \frac{32\pi G_0}{\phi_0} (\mathcal{S}_a + \Delta P_a).
\end{equation}
Here, $\mathcal{S}_a$ is Eq. (11) without the first term in the square brackets, that is, it is the null energy fluxes of the tensor and the scalar GWs. The displacement memory in the scalar sector can be constrained by the evolution equation Eq. (26), i.e.,
\begin{equation}
\Delta \phi_1^2 = \frac{16\Phi_0^2}{2\omega + 3} \left\{ \frac{1}{32} \Delta (\mathcal{D}^2 \hat{\epsilon}_{\ ab}) + \mathcal{D}^{-2} \mathcal{D}^A \Delta \mathcal{N}_A \right. \\
- \int_{u_i}^{u_f} du \left[ m + \frac{1}{2} \mathcal{D}^{-2} \mathcal{D}^A J_A \right],
\end{equation}
where
\begin{equation}
J_A = \frac{1}{2} N_{\ ab} \mathcal{D}^A \mathcal{D}_{\ a} - \frac{2\omega + 3}{\phi_0^2} N \mathcal{D}^A \phi_1,
\end{equation}
and $\mathcal{D}^{-2}$ is the inverse operator of $\mathcal{D}^2$ and is explicitly given in Hou & Zhu (2020b).

### 3.2 Spin and center-of-mass memories

As mentioned in the Introduction, the spin memory can be detected by observing two counter-orbiting particles in a cir-
cle. If there are GWs, they will return to their starting points at different times, given that they were released at the same
time. This effect is related to the leading term in $g_{\ a0}$ component (Pasterski et al. 2016), which does not depend on the scalar
field. So the spin memory exists only in the tensor sector, and is very similar to the one in GR.

As in GR, the spin memory should be constrained by the flux-balance law associated with $Y^A$. However, for this purpose, one allows $Y^A$ to be a local conformal Killing vector field for $\gamma_{\ ab}$. Then $Y^A$ has a finite number of singular points on the
unit 2-sphere, and the flux-balance law obtained in Sec. 2.2 should be modified, for example, keeping the charges Eq. (13) and Eq. (14) while adding to Eq. (15) the following term,

$$\Delta F_\gamma [\mathcal{B}] = \frac{\Phi_0}{32\pi G_0} \int_\mathcal{B} Y^A \mathcal{D}^B (\mathcal{D}_A \mathcal{D}_C \mathcal{C}_B - \mathcal{D}_B \mathcal{D}_C \mathcal{C}_A) \text{d}u \text{d}^2 \Omega$$

$$= \frac{\Phi_0}{64\pi G_0} \int_\mathcal{B} \epsilon_{ABC} Y^A \mathcal{D}^B \mathcal{D}^2 (\mathcal{D}^2 + 2) \text{d}u \text{d}^2 \Omega. \quad (26)$$

Then, the constraint on the spin memory, measured by $\Delta R = \int \text{d}u Y$ (Flanagan & Nichols (2017)), reads,

$$\int_\mathcal{B} \nu \mathcal{D}^2 (\mathcal{D}^2 + 2) \Delta R \text{d}^2 \Omega = - \frac{32\pi G_0}{\Phi_0} (\Delta \mathcal{J}_v + \Phi_v + \mathcal{J}_v). \quad (27)$$

where one defines

$$\mathcal{J}_v = \frac{\Phi_0}{16\pi G_0} \int_\mathcal{B} \nu \mathcal{D}^A \mathcal{D}_A \mathcal{J}_B \text{d}u \text{d}^2 \Omega. \quad (28a)$$

$$\mathcal{J}_v = \frac{\Phi_0}{16\pi G_0} \int_\mathcal{B} \nu \mathcal{D}^A \mathcal{D}_A \mathcal{J}_B \text{d}u \text{d}^2 \Omega. \quad (28b)$$

Note that here, $\nu$ is generally not a linear combination of $l = 1$ spherical harmonics.

Finally, consider the constraint on the CM memory. One may split $\Phi = \Phi_\gamma + \Phi_\alpha$ such that

$$\int_\mathcal{B} \alpha \mathcal{D}^2 (\mathcal{D}^2 + 2) \Delta \Phi_\alpha \text{d}^2 \Omega = \frac{32\pi G_0}{\Phi_0} \mathcal{E}_\alpha, \quad (29a)$$

$$\int_\mathcal{B} \beta \mathcal{D}^2 (\mathcal{D}^2 + 2) \Delta \Phi_\beta \text{d}^2 \Omega = \frac{32\pi G_0}{\Phi_0} \Delta \mathcal{P}_\beta. \quad (29b)$$

Then the CM memory effect is measured by (Nichols (2018; Tabura et al. 2020))

$$\Delta \mathcal{K} = \int_{u_j}^{u_i} \nu \partial_u \Phi_\alpha \text{d}u, \quad (30)$$

which is contained in $F_\mu [\mathcal{B}]$ in Eq. (15), i.e.,

$$F_\mu [\mathcal{B}] = - \frac{\Phi_0}{64\pi G_0} \int_\mathcal{B} \mu \mathcal{D}^2 \mathcal{D}^2 (\mathcal{D}^2 + 2) \Delta \mathcal{K} \text{d}^2 \Omega. \quad (31)$$

So the flux-balance law Eq. (16) can be used to obtain,

$$\int_\mathcal{B} \mu \mathcal{D}^2 \mathcal{D}^2 (\mathcal{D}^2 + 2) \Delta \mathcal{K} \text{d}^2 \Omega = \frac{64\pi G_0}{\Phi_0} (\mathcal{J}_\mu - \Delta \mathcal{K}'_\mu). \quad (32)$$

where one defines

$$\Delta \mathcal{K}'_\mu = - \frac{\Phi_0}{8\pi G_0} \int_\mathcal{B} \nu \Delta (\mathcal{D}^A N_A + 2\mu) \text{d}^2 \Omega \quad (33)$$

Therefore, the CM memory is constrained by Eq. (32), as long as $\mu$ is not simply a linear combination of $l = 1$ spherical harmonics. Since CM memory is related to $\Phi_\alpha$, which is a part of $\hat{\mathcal{C}}_{AB}$, then there does not exist an analogous effect in the scalar sector.

4 CONCLUSION

This work shows that the memory effect of an isolated system in BD shares some similarities with that in GR, and at the same time, has its own unique features. Both theories share the displacement memory effect for the tensor GW, which is induced by the passage of the null energy fluxes through $\mathcal{I}$, because the supertranslations transform the degenerate vacua among each other in the tensor sector. However, in BD, there exists the scalar GW. It not only contributes to the tensor displacement memory effect by providing a new energy flux, but also has its own memory effect. The $S$ memory effect is due to the angular momentum fluxes penetrating $\mathcal{I}$, and degenerate vacua in the scalar sector are transformed into each other due to Lorentz transformations. Using flux-balance laws, one obtains the constraints on the displacement memories, the spin memory and the CM memory. These constraints are very useful to predict how strong memory effects are, and estimate whether they can be detected by the interferometer or pulsar timing arrays. This might provide a new method to test the nature of gravity.

ACKNOWLEDGMENTS

I was grateful to the Organizing Committee of IWARA 2020 for such a great event during the pandemic of COVID-19. This work was supported by the National Natural Science Foundation of China under grant Nos.11633001 and 11920101003, and the Strategic Priority Research Program of the Chinese Academy of Sciences, grant No. XDB23000000. This was also a project funded by China Postdoctoral Science Foundation (No. 2020M672400).

Conflict of interest

The author declares no potential conflict of interests.

REFERENCES

Boersma, O. M., Nichols, D. A., & Schmidt, P. 2020, Phys. Rev. D, 101(8), 083026. doi:
Bonga, B., & Prabh, K. 2020, 9.
Braginsky, V., & Grishchuk, L. 1985, Sov. Phys. JETP, 62, 427-430. 1961, Phys. Rev., 124, 925-935. doi:
Christodoulou, D. 1991, Sep, Phys. Rev. Lett., 67., 1486-1489. Retrieved from https://link.aps.org/doi/10.1103/PhysRevLett.67.1486 
doi:
Donnay, L., & Giribet, G. 2019, Class. Quant. Grav., 36(16), 165005. doi:
Du, S. M., & Nishizawa, A. 2016, Phys. Rev. D, 94(10), 104063. doi:
Flanagan, E. E., & Nichols, D. A. 2017, *Phys. Rev. D*, 95(4), 044002. doi:
Hou, S., Gong, Y., & Liu, Y. 2018, *Eur. Phys. J. C*, 78, 378. doi:
Hou, S., & Zhu, Z.-H. 2009a, 8,
Hou, S., & Zhu, Z.-H. 2009b, 5,
Hübner, M., Talbot, C., Lasky, P. D., & Thrane, E. 2020, January, *Phys. Rev. D*, 101(2), 023011. doi:
Johnson, A. D., Kapadia, S. J., Osborne, A., Hixon, A., & Kennefick, D. 2019, *Phys. Rev. D*, 99(4), 044045. doi:
Kılıçarslan, E. 2018, *Phys. Rev. D*, 98(6), 064048. doi:
Kılıçarslan, E. 2019, *Turk. J. Phys.*, 43(1), 126-134. doi:
Kılıçarslan, E., & Tekin, B. 2019, *Eur. Phys. J. C*, 79(2), 114. doi:
Koyama, K. 2020, *Phys. Rev. D*, 102(2), 021502. doi:
Lang, R. N. 2014, *Phys. Rev. D*, 89(8), 084014. doi:
Lang, R. N. 2015, *Phys. Rev. D*, 91(8), 084027. doi:
Lasky, P. D., Thrane, E., Levin, Y., Blackman, J., & Chen, Y. 2016, *Phys. Rev. Lett.*, 117(6), 061102. doi:
Madison, D. R. 2020, *Phys. Rev. Lett.*, 125(4), 041101. doi:
McNeill, L. O., Thrane, E., & Lasky, P. D. 2017, *Phys. Rev. Lett.*, 118(18), 181103. doi:
Nichols, D. A. 2018, *Phys. Rev. D*, 98(6), 064032. doi:
Pasterski, S., Strominger, A., & Zhiboedov, A. 2016, *JHEP*, 12, 053. doi:
Seto, N. 2009, *Mon. Not. Roy. Astron. Soc.*, 400, L38. doi:
Strominger, A., & Zhiboedov, A. 2016, *JHEP*, 01, 086. doi:
Tahura, S., Nichols, D. A., Saffer, A., Stein, L. C., & Yagi, K. 2020, 7,
Thorne, K. S. 1992, *Phys. Rev. D*, 45(2), 520–524. doi:
Wald, R. M. 1984, General Relativity. Chicago, IL: University of Chicago Press. doi:
Wald, R. M., & Zoupas, A. 2000, *Phys. Rev. D*, 61, 084027. doi:
Zel’dovich, Y. B., & Polnarev, A. G. 1974, *Sov. Astron.*, 18, 17.
Zhang, P.-M., Duval, C., Gibbons, G., & Horvathy, P. 2017a, *Phys. Lett. B*, 772, 743–746. doi:
Zhang, P.-M., Duval, C., Gibbons, G., & Horvathy, P. 2018, *JCAP*, 05, 030. doi:
Zhang, P.-M., Duval, C., Gibbons, G. W., & Horvathy, P. A. 2017b, *Phys. Rev. D*, 96(6), 064013. doi:

**How cite this article:** S. Hou (...), Gravitational memory effects in Brans-Dicke theory, *Q.J.R. Meteorol. Soc.*, ....