LOW ENERGY CONSEQUENCES FROM SUPERSYMMETRIC MODELS WITH LEFT_RIGHT SYMMETRY

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ABSTRACT

We consider several low energy consequences arising from a class of supersymmetric models based on the gauge groups $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and $SU(4)_C \times SU(2)_L \times SU(2)_R$ in which the gauge hierarchy and $\mu$ problems have been resolved. There are important constraints on the MSSM parameters $\tan \beta(\approx m_t/m_b)$, $B$ and $\mu$, and we discuss how they are reconciled with radiative electroweak breaking. We also consider the ensuing sparticle and Higgs spectroscopy, as well as the decays $b \rightarrow s\gamma$ and $\mu \rightarrow e\gamma$. The latter process may be amenable to experimental tests through an order of magnitude increase in sensitivity.
1. Introduction

In a couple of recent papers [1, 2], the minimal supersymmetric standard model (MSSM) arose from the low energy limit of a special class supersymmetric models based on $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [3] and $SU(4)_C \times SU(2)_L \times SU(2)_R$ [4]. By imposing a suitable $R$-symmetry, $U(1)_R$, which contains the unbroken $Z_2$ ‘matter’ parity of MSSM, it was shown that this class of models has some interesting ‘low energy’ consequences. For instance, the magnitude of the supersymmetric $\mu$-term of MSSM gets related to the supersymmetry (SUSY) breaking scale $m_{3/2}$. The $B\mu$ term of MSSM is also generated and found to be of order $m_{3/2}$, while the MSSM parameter $\tan \beta \simeq m_t/m_b$, where $m_t$ and $m_b$ are the top and bottom quark masses respectively. The apparent stability of the proton ($\tau_p > 10^{32} - 10^{33}$ yrs.) is understood to be a consequence of an ‘accidental’ global $U(1)_B$ symmetry. The $SU(4) \times SU(2)_L \times SU(2)_R$ model also suggest the existence of ‘heavy’ charge $\pm e/6$ (colored) and $\pm e/2$ (color singlet) states.

Motivated by these results we propose to investigate additional ‘low energy’ implications of these left-right symmetric models. In particular, we would like to focus on the important issues of radiative electroweak (EW) breaking, sparticle and Higgs spectroscopy, composition of the lightest supersymmetric particle (LSP), and implications of the radiative processes $b \to s\gamma$ and $\mu \to e\gamma$. Since $\tan \beta(\simeq m_t/m_b)$ is large and the parameter $B\mu$ is also constrained, the requirement of radiative EW breaking turns out to be non-trivial. In particular, non-universal soft SUSY breaking terms and some deviation from the minimal Kähler potential must be considered. The requirement that SUSY correction to the bottom (b) quark mass should not be excessive ($\leq 20\%$) imposes additional constraints on the parameters of the model.

The plan of the paper is as follows. In the next section (2), we briefly describe the underlying left-right symmetric models, the mechanism for resolving the $\mu$-problem, and the origin of the $B\mu$ term. We also discuss deviations from Refs. [1, 2] needed to obtain a $B$ term that is consistent with radiative EW breaking. In section (3) the EW symmetry breaking is discussed in detail, while constraints arising from the b-quark mass are taken up in section (4). Section (5) deals with the ensuing SUSY spectrum as well the composition of the LSP. The corresponding Higgs spectroscopy is briefly considered in section (6). Sections (7) and (8) focus on the radiative processes $b \to s\gamma$ and $\mu \to e\gamma$ respectively. Our conclusions are summarized in section (9).
2. The Model

For definiteness, we will take the underlying symmetry group to be \( G = SU(4)_C \times SU(2)_L \times SU(2)_R \) and follow the notation used in Ref. [1]. The breaking of \( G \) at the GUT scale \( (M_{\text{GUT}}) \) to \( SU(3)_C \times SU(2)_L \times U(1)_Y \) is achieved by introducing non-zero vacuum expectation values (VEVs) for the Higgs superfields \( H \) and \( \bar{H} \), which transform under \( G \) as:

\[
H = (4, 1, 2),
\]
\[
\bar{H} = (\bar{4}, 1, 2). \tag{1}
\]

The MSSM Higgs doublets are contained in the representation \( h \) of \( G \), where

\[
h = (1, 2, 2). \tag{2}
\]

A color sextet superfield \( D = (6, 1, 1) \) is also included to ensure that the ‘low energy’ particle sector coincides with that of MSSM. Finally, the quarks and leptons belong to the \((4, 2, 1)_i + (\bar{4}, 1, 2)_i\) representations of \( G \), where \( i = 1, 2, 3 \) denotes the generation index.

The superpotential of the minimal \( G \) model is given by (after imposing a suitable \( U(1)_R \) symmetry) [1, 2],

\[
W = S[\kappa(\bar{H}H - M^2) + \lambda h^2] + \lambda_H DHH + \lambda_R D\bar{H}\bar{H} + \lambda_{33} \bar{F}_3 F_3 h + \lambda_{ij} \bar{F}_i F_j \frac{(\bar{H}H)^n}{M_P^{2n}} + \lambda_{\nu ij} \bar{F}_i \bar{F}_j HH \frac{M^2}{M_P}, \tag{3}
\]

where \( S \) denotes a gauge singlet superfield, the parameters \( \kappa, \lambda \) and \( M \) can be taken to be real and positive, and \( h^2 \) denotes the unique bilinear invariant \( e^{ab} h_a^{(1)} h_b^{(2)} \). Also, \( M_P(\approx 2.4 \times 10^{18} \text{ GeV}) \) denotes the ‘reduced’ Planck mass. The Higgs fields develop VEVs, \(|\langle H \rangle| = |\langle H \rangle| \approx M\), which lead to the symmetry breaking

\[
SU(4)_C \times SU(2)_L \times SU(2)_R \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y. \tag{4}
\]

Note that supersymmetry is unbroken at this stage. The inclusion of soft SUSY breaking terms will induce an expectation value (proportional to \( m_{3/2} \)), namely

\[
\langle S \rangle = -\frac{A_\kappa - A_1}{2\kappa^2}. \tag{5}
\]

Here and throughout, as is customary, the scalar components of the superfields are denoted by the same symbols as the corresponding superfields. \( A_\kappa \) and \( A_1 \) denote the coefficients of the soft trilinear and linear terms that contain \( S \). This means that the \( \lambda \langle S \rangle h^2 \) term in
eq. (3) provides an effective MSSM $\mu$ parameter of the correct order of magnitude. With $A_\kappa = \kappa A m_{3/2}$, $A_1 = \kappa (A - 2) m_{3/2}$ (minimal supergravity),

$$\mu = -\frac{\lambda}{\kappa} m_{3/2}.$$  

(6)

Furthermore, the bilinear term is given by

$$B = 2 m_{3/2}.$$  

(7)

This model implies Yukawa unification for the third family (see eq. (3)), which leads to a large top mass $m_t > 165$ GeV and $\tan \beta \sim m_t/m_b$ [5]. The first and second family Yukawa couplings, as well as mixings, eventually must be generated by non-renormalisable operators and/or the inclusion of additional states. We will not address this important issue here.

Since $\tan \beta \gg 1$, it is necessary to employ non-universal soft SUSY breaking to satisfy the EW breaking conditions [4]. However, it turns out that, even with non-universal soft SUSY breaking, the condition

$$\sin 2\beta = \frac{2 B \mu}{m_{h_1}^2 + m_{h_2}^2 + 2 \mu^2}$$  

(8)

cannot be satisfied with the value of $B$ at $M_{GUT}$ of order $2m_{3/2}$. We can modify the value of $B$ as follows. Consider the relevant superpotential terms, namely

$$\delta W = S[\kappa (\bar{H}H - M^2) + \lambda h^2],$$  

(9)

which leads to the potential

$$\begin{align*}
\delta V &= |\kappa (\bar{H}H - M^2) + \lambda h^2|^2 + |S|^2 (\kappa^2 |\bar{H}|^2 + \kappa^2 |H|^2 + \lambda |h|^2) \\
&\quad + m_{3/2} (|S|^2 + |\bar{H}|^2 + |H|^2 + |h|^2) + (A_\kappa S H H + A_\lambda S h^2 - A_1 S M^2 + h.c).
\end{align*}$$  

(10)

We now depart from the minimal Kähler potential considered in Ref. [2] by assuming non-universality between the trilinear couplings $A_\lambda$ and $A_\kappa$, namely we assume

$$A_\kappa = \kappa A m_{3/2},$$  

(11)

$$A_\lambda = \lambda A' m_{3/2},$$  

(12)

but keep the assumption

$$A_1 = \kappa (A - 2) m_{3/2}.$$  

(13)

Then, we still have $\langle S \rangle = -m_{3/2}/\kappa$, and $\mu = -\frac{\lambda}{\kappa} m_{3/2}$.
The bilinear coupling $B\mu$ is given by

$$B\mu = \lambda F^*_S + A\lambda S.$$  \hfill (14)

Since

$$F^*_S = -\frac{1}{\kappa}(\kappa^2|S|^2 + m^2_{3/2}) - \frac{1}{\kappa}A\lambda S,$$  \hfill (15)

we find that

$$B\mu = -2\frac{\lambda}{\kappa}m^2_{3/2} + \frac{\lambda}{\kappa}(A - A')m^2_{3/2},$$  \hfill (16)

so that

$$B = (2 - (A - A'))m_{3/2}.$$  \hfill (17)

For $A > A'$, we can have $B < m_{3/2}$, which is needed to realize the EW breaking scenario in the large tan $\beta$ case.

3. Electroweak Symmetry breaking

The phenomenological aspects of models with large tan $\beta$ can be quite different from those with small tan $\beta$ values. In particular, radiative EW symmetry breaking is an important issue. This has been discussed under the assumption of universal soft SUSY breaking parameters in Ref. [7, 8]. In the large tan $\beta$ scenario the mass squared parameters for the down (up) sector Higgs $H_1$ ($H_2$) run from the higher energy scale $M_{GUT}$ to the weak scale $M_Z$ in very similar ways if these masses are universal at $M_{GUT}$. This is not conducive for successful EW symmetry breaking, especially in view of the above constraints on $\mu$ and $B$. Requiring non-universality at $M_{GUT}$ such as

$$m^2_{H_1} > m^2_{H_2},$$  \hfill (18)

turns out to be favorable for symmetry breaking with large tan $\beta$. Also the trilinear coupling should be larger than the gaugino masses. Large values of the $A$ parameter are crucial to reduce the value of $B$ during the running from $M_{GUT}$ to $M_Z$. Also, radiative breaking requires non-universality among the gaugino masses. As we will show in the next section, the supersymmetric correction to the bottom quark mass constrains the gluino mass to not be very heavy and therefore implies a constraint on $M_3$. The running of $B$ imposes a constraint on $M_2$, while $M_1$ is essentially unconstrained. However, at the weak scale it turns out that in all cases we have $M_1 < M_2 < M_3$. 
We will assume the following boundary conditions on the soft scalar masses at $M_{\text{GUT}}$:

$$
m^2_{H_1} = m^2_Q = m^2_U = m^2_D = m^2_{3/2},
$$
$$
m^2_{H_2} = m^2_{3/2}(1 - \delta),
$$

(19)

where values of $\delta$ of order unity are preferred by the electroweak symmetry breaking. As we will discuss in the next section, the SUSY corrections to the bottom mass require $\delta$ to be close 0.3. In this case we find that $B$ is sufficiently small at the weak scale, which is very important for successful electroweak breaking with such large value of $\tan \beta$. Figure (1) shows the running of $B$ from $M_{\text{GUT}}$ to $M_Z$.

![Figure 1. Running of the bilinear term $B$ from $M_{\text{GUT}}$ to $M_Z$.](image)

With the choice made in (19) the Higgs masses easily satisfy the constraint

$$
m^2_{H_1} - m^2_{H_2} > M^2_Z.
$$

(20)

Moreover, from the electroweak breaking condition

$$
\mu^2 = \frac{m^2_{H_1} - m^2_{H_2} \tan^2 \beta}{\tan^2 \beta - 1} - M^2_Z/2,
$$

(21)

we can determine the factor $\lambda/2\kappa$ (see eq.(8)).
4. SUSY CORRECTION TO THE BOTTOM QUARK MASS

It is well known [10] that in models with large $\tan \beta$ the bottom quark mass can receive a sizable SUSY correction. The dominant contributions are due to the sbottom-gluino and stop-chargino loops. The tree level value of the bottom mass is $m_b(M_Z) = \lambda_b(M_Z) v \cos \beta \simeq 3.3 \text{ GeV}$, to be compared with the ‘measured’ value [9]

$$m_b(M_Z) = 2.67 \pm 0.50 \text{GeV}.$$  

We therefore would like the SUSY corrections to be negative and $\leq 20\%$. In this section we estimate the SUSY corrections to $m_b$, and we are interested in finding regions of the parameter space which simultaneously allow small SUSY corrections and acceptable electroweak breaking.

The dominant contributions to the bottom quark mass $\delta m_b$ are given by [10]

$$\delta m_b = \mu \tan \beta \left[ \frac{2\alpha_S}{3\pi} \tilde{M}_\beta I(m_{b_1}^2, m_{b_2}^2, M_\tilde{\beta}) + \frac{\lambda_t^2}{16\pi^2} A_t I(m_{t_1}^2, m_{t_2}^2, \mu^2) \right] ,$$  

where $\lambda_t$ is the top Yukawa coupling and

$$I(x,y,z) = -\frac{xy \ln(x/y) + yz \ln(y/z) + zx \ln(z/x)}{(x-y)(y-z)(z-x)}.$$  

The sign of $\delta m_b$ is the same as the sign of $\mu$. Since we require a negative SUSY correction to reduce the tree level value ($\simeq 3.3 \text{ GeV}$) of $m_b$, we must choose $\mu < 0$. The first contribution to $\delta m_b$ in eq.(22) is the dominant one. For SUSY corrections to remain small ($\leq 20\%$), the gluino mass and $\mu$ should not be too large. The experimental lower limit on the gluino mass is about 150 GeV, and so a plausible solution is to have $\mu$ small. In fact this can be achieved for $\delta \leq 0.3$. It is important to mention that for $\delta \leq 0$, the EW breaking conditions are not satisfied. In Figure (2) we see that $\mu \ll m_{\tilde{q}}$ for $\delta \leq 0.3$ ($\mu \simeq 50 \text{ GeV}$ corresponds to $m_{\tilde{q}} \simeq 300 \text{ GeV}$). We also find that the corresponding values of $\delta m_b$ for this region of the parameter space are less than 20\%.

5. SUSY SPECTRUM AND THE LSP

In this section we investigate the SUSY spectrum in this class of large $\tan \beta$ models arising from the parameter space that also lead to successful EW breaking and small SUSY correction to the mass of the bottom quark. As mentioned above, non-universality between the gaugino masses is preferred for EW breaking and other phenomenological aspects.
From the correction to \( m_b \) we have the constraint that the gluino mass should be comparable (more or less) to the experimental limit, and \( \mu \) should be small. This, as we will see, has important implications for phenomenology and cosmology of these models. We observe that with \( M_1 < M_2 < M_3 \) at \( M_{\text{GUT}} \), the value of \( M_1 \) is quite low due to the constraint on \( M_3 \). This implies masses for the lightest neutralino (LSP) which are far below the experimental limit. To avoid this we must consider sufficiently large values for the gaugino masses (and hence \( m_{3/2} \) too). Note that \( M_2 \) is constrained to be small from the running of the bilinear term \( B \), or else we need very large values of the trilinear coupling to reduce the value of \( B \) at \( M_Z \). However, the experimental limit on the lightest chargino can impose a lower bound on the value of \( M_2 \). The gaugino mass \( M_1 \) is essential unconstrained. An interesting and viable region is given by \( M_3 < M_2 < M_1 \) at \( M_{\text{GUT}} \). However, taking account of the different ‘running’, this again leads to \( M_1 < M_2 < M_3 \) at the weak scale \( M_Z \).

In all of the above mentioned regions, and for \( m_{3/2} \) not too large, the LSP is expected to be Higgsino like since \( \mu \) is small. For large values of \( m_{3/2} \), \( \mu \) becomes larger than \( M_1 \), and hence the LSP starts to be more bino like. Figure (3) shows the neutralino composition function \( f_g \) versus the neutralino mass, with

\[
 f_g = |N_{11}|^2 + |N_{12}|^2, 
\]  

(23)
where $N_{ij}$ is the unitary matrix that diagonalize the neutralino mass matrix. It relates the neutralino field $\chi^0_1$ to the original ones, namely

$$\chi^0_1 = N_{11} \tilde{B} + N_{12} \tilde{W}^3 + N_{13} \tilde{H}_1^0 + N_{14} \tilde{H}_2^0. \quad (24)$$

![Figure 3](image-url)  # Figure 3. The value of the neutralino composition function $f_g$ versus the neutralino mass $m_\chi$.

The model is characterized by heavy SUSY scalar masses and ‘light’ gaugino masses. For instance, when the chargino mass is of order the experimental bound, $m_{\chi^+} \simeq 90$ GeV, the lightest scalar mass which corresponds to one of the stop squarks is $\simeq 500$ GeV. Furthermore, the right selectron turns out to be the lightest slepton, with a lower bound $\sim 500$ GeV. The positivity of the eigenvalues of the stau mass squared matrix is an important condition and usually imposes a severe constraint on models with large $\tan \beta$ [11]. The relevant matrix is

$$
\begin{pmatrix}
    m_{\tilde{\tau}_L}^2 + M_Z^2 \cos 2\beta \left( -\frac{1}{2} + \sin^2 \theta_W \right) & v Y_\tau (A_\tau \cos \beta - \mu \sin \beta) \\
    v Y_\tau (A_\tau \cos \beta - \mu \sin \beta) & m_{\tilde{\tau}_R}^2 - M_Z^2 \cos 2\beta \sin^2 \theta_W
\end{pmatrix}, \quad (25)
$$

where $v^2 = \langle H_2 \rangle^2 + \langle H_1 \rangle^2$. In the case of large $\tan \beta$ the tau Yukawa coupling is large and hence the off diagonal elements relative to the diagonal elements cannot be ignored. This could lead to a negative eigenvalue. However, with $m_{\tilde{\tau}_L}^2$ and $m_{\tilde{\tau}_R}^2$ of order $m_{3/2}^2$ at $M_{GUT}$, it turns out that this is not the case and even the lowest eigenvalue of this matrix is larger than the mass squared of the right selectron.
6. Higgs spectrum

In the limit $\lambda = \kappa$, the superpotential $W = \kappa S[(H \bar{H} - M^2) + h^2]$ has an accidental $U(4)$ symmetry under which $(H, h^{(1)}) \in \mathbf{4}$ and $(\bar{H}, \varepsilon h^{(2)}) \in \bar{\mathbf{4}}$, i.e., they transform as the fundamental and antifundamental representation respectively. When $H$ and $\bar{H}$ acquire their VEV, the $U(4)$ symmetry breaks to $U(3)$. Hence, we expect seven ‘goldstone’ superfields, only three of which are true goldstone superfields that are absorbed by the massive gauge superfields. The remaining four superfields correspond to the physical state $h^{(1)}$ and $h^{(2)}$. This accidental symmetry of the superpotential is broken when supersymmetry is broken, so that $h^{(1)}$ and $h^{(2)}$ are ‘psuedogoldstone’ bosons. For $\lambda \neq \kappa$ the $U(4)$ symmetry is explicitly broken in the superpotential and the above arguments must be reconsidered.

The lightest Higgs scalar ($h^0$) has the well-known mass at tree level,

$$m_{h^0}^2 = \frac{1}{2}(m_A^2 + m_Z^2 - \sqrt{(m_A^2 + m_Z^2)^2 - 4m_Z^2 m_A^2 \cos^2 2\beta}),$$

(26)

where

$$m_A^2 = m_{H_1}^2 + m_{H_2}^2 + 2\mu^2.$$  

(27)

For $\tan \beta \simeq \frac{m_t}{m_b}$, eq.(26) gives

$$m_{h^0} \simeq m_Z.$$  

However, one loop corrections [12] can increase this by about (40-60) GeV, while two loop corrections [13] can lower the value by approximately 10 GeV (see also [14]).

Since the value of $\mu$ is quite constrained, we expect that the neutral pseudoscalar Higgs boson $A$, whose mass $m_A$ is given in eq.(27), is not too heavy. Indeed we find that $m_A \simeq 100$ GeV for $m_{3/2} \simeq 500$ GeV. However, $h^0$ turns out to be the lightest supersymmetric Higgs. In Figure 4 we display the correlation between the masses of $h^0$ and $A$.

One could expect that in the region where $m_A$ is of order $\mathcal{O}(100)$ GeV, the charged Higgs boson mass is of the same magnitude,

$$m_{H^\pm}^2 = m_Z^2 + m_A^2,$$

which may lead to a large value for the branching ratio of $b \to s\gamma$. However, the chargino mass in this model is very close to the experimental bound, so that we have a large chargino contribution which gives rise to destructive interference with the SM and charged.
Higgs contributions, as will be explained in the next section. Hence, a relatively light psuedoscalar Higgs is allowed.

7. Constraints from $b \to s\gamma$

In this section we focus on the constraints on the parameter space which arise from the decay $b \to s\gamma$. The CLEO experiment [15] has confirmed that $1 \times 10^{-4} < BR(b \to s\gamma) < 4 \times 10^{-4}$. In supersymmetric models there are three significant contributions to the total amplitude, namely from the $W$-loop, charged Higgs loop and the chargino loop. The inclusive branching ratio for $b \to s\gamma$ is given by

$$R = \frac{BR(b \to s\gamma)}{BR(b \to ce\bar{\nu})}. \quad (28)$$

The computation of $R$ yields [16]

$$R = \frac{|V_{ts}V_{tb}|^2 6\alpha_{em}}{|V_{cb}|^2 \pi} \frac{[\eta^{16/23}A_\gamma + \frac{8}{3}(\eta^{14/23} - \eta^{16/23})A_g + C]^2}{I(x_{cb})[1 - \frac{2}{3}\alpha_S(m_b)f(x_{cb})]} \quad (29)$$

Here, $\eta = \frac{\alpha_S(m_W)}{\alpha_S(m_b)}$, and $C$ represents the leading-order QCD corrections to $b \to s\gamma$ amplitude at the scale $Q = m_b$ [17]. The function $I(x)$ is given by

$$I(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln x,$$

where $x_{cb} = \frac{m_c}{m_b}$, and $f(x)$ is a QCD correction factor, with $f(x_{cb}) = 2.41$. The amplitude $A_\gamma$ is from the photon penguin vertex, the amplitude $A_g$ is from the gluon penguin vertex, and they are given in Ref. [16]. It was shown in MSSM [16], and in models with dilatons.
dominated SUSY breaking \[18\] that with \( \tan \beta \simeq 2 \), the chargino contribution gives rise to a destructive interference with SM contribution and charged Higgs \((H^+)\) contribution, but it is generally smaller than the latter. This leads to a severe constraint on the parameter space of these models. It was also realized that the constraint is less severe if the soft terms are non-universal. In the moduli-dominant SUSY breaking model \[19\] it was shown that the chargino contribution gives rise to substantial destructive interference with SM and \( H^+ \) amplitude, so that the branching ratio of \( b \to s \gamma \) is less than the SM value.

In figure (5) we show that we have a similar situation here since the model is characterized by 'not too large' gaugino masses. Therefore, the chargino contribution which is inversely proportional to its mass square becomes significant. This result is quite interesting since, as pointed out in \[20\], the SM prediction is above the CLEO measurement at the \( 1\sigma \) level. Hence, any new physics beyond the SM should provide destructive interference with the SM amplitude and our model has this feature.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{branching_ratio_b_to_s_gamma}
\caption{The branching ratio of \( b \to s \gamma \) versus the lightest chargino mass. The horizontal lines at \( 1 \times 10^{-4} \) and \( 4 \times 10^{-4} \) correspond to the experimental bounds.}
\end{figure}

It is important to note that for \( m_{\chi^+} \simeq 200 \text{ GeV} \) the gravitino mass \( m_{3/2} \) is of order 1.5 TeV. Furthermore, for this value of \( m_{3/2} \) the signs of the quantity \( M_2^2 - \mu^2 - 2M_W^2 \cos 2\beta \) as well as the chargino contribution are reversed. Thus, destructive interference from the chargino contribution now corresponds to the case \( \mu > 0 \) (instead of \( \mu < 0 \)). However, from \( \delta m_b < 0 \), we obtained the constraint that \( \mu \) should be negative. Therefore, from the
supersymmetric correction to the bottom quark mass and the experimental bound on the branching ratio of the $b \to s\gamma$ we find an upper bound on $m_{3/2}$ of about 1.5 TeV.

8. **Enhancement of $\mu \to e\gamma$ at large $\tan \beta$**

Lepton flavor violation (LFV) is considered a significant prediction of many supersymmetric models and provides a sensitive probe of physics beyond the standard model. In this section we show that with $\tan \beta \simeq m_t/m_b$ the LFV process $\mu \to e\gamma$ may be enhanced in the class of models under discussion and presumably amenable to ongoing and planned experiments.

This result can be understood as follows. After symmetry breaking we can write the superpotential in (3) as

$$W = W_{\text{MSSM}} + \lambda_{i\nu} V_{\nu\nu}^{ij} \tilde{N}_i L_j H_2 + M_{\nu R}^{ij} \tilde{N}_i \tilde{N}_j.$$  

Here, $i, j = 1, ..., 3$ are the generation indices, and the superfields $L$ and $\tilde{N}$ represent the leptons ($\nu_L, e_L$) and $\nu_R^c$ respectively. The lepton sector has a mixing matrix $V_{\nu\nu}^{\text{CKM}}$ analogous to the CKM matrix in the quark sector, which contributes to lepton flavor violation. In particular, the off-diagonal components of the matrices $m_{\tilde{e}}^2, m_{\tilde{l}}^2$ and $A_{l}$ are the sources for LFV.

In our model, the amplitude of the photino contribution is given in terms of the mass insertion $\delta'_{AB}$ defined by $\delta'_{AB} = \frac{\Delta_{AB}}{\tilde{m}^2}$, where $\tilde{m}$ is an average slepton mass and $\Delta^l$ denote the off-diagonal terms in the slepton mass matrices. The mass insertion to accomplish the transition from $\tilde{l}_i$ to $\tilde{l}_j$ is given by [21]

$$\begin{align*}
(\Delta^l_{LL})_{ij} & \simeq \frac{1}{8\pi^2} \lambda_{i\nu} (V_{\nu\nu}^{\text{CKM}})^i V_{\nu\nu}^{\text{CKM}} 3 m_{3/2}^2 + A^2 \ln \left( \frac{M_{\text{GUT}}}{M_{\nu R}} \right), \\
(\Delta^l_{LR})_{ij} & \simeq \frac{1}{8\pi^2} \lambda_{i\nu}^i (V_{\nu\nu}^{\text{CKM}})^i V_{\nu\nu}^{\text{CKM}} \lambda_{ij} v A \ln \left( \frac{M_{\text{GUT}}}{M_{\nu R}} \right), \\
(\Delta^l_{RL})_{ij} & = (\Delta^l_{LR})_{ij}^\dagger, \\
(\Delta^d_{RR})_{ij} & = 0.
\end{align*}$$

Here the neutrino Yukawa coupling constants except for $\lambda_{\nu3}$ are ignored. Since $\Delta^l_{LR}$ is proportional to $\lambda_{i\nu} = m_t / \cos \beta$ this quantity, and hence the branching ratio, is enhanced
for large $\tan \beta$. The branching ratio for the process $\mu \to e\gamma$ is given by \[22\]

$$BR(\mu \to e\gamma) = \frac{\alpha^3}{G_F^2} \frac{12 \pi}{m_e^4} \left[ |M_3(x)(\delta_{21}^l)_{LL} + \frac{m_{\tilde{g}}}{m_l} M_1(x)(\delta_{21}^l)_{LR}|^2 + L \leftrightarrow R \right] BR(\mu \to e\nu\bar{\nu}),$$

(35)

where $x = \frac{m_{\tilde{g}}}{m_l}$, and the functions $M_1(x)$ and $M_3(x)$ are given by

$$M_1(x) = \frac{1 + 4x - 5x^2 + 4x \ln(x) + 2x^2 \ln(x)}{2(1 - x)^4},$$

(36)

$$M_3(x) = \frac{-1 + 9x + 9x^2 - 17x^3 + 18x^2 \ln(x) + 6x^3 \ln(x)}{12(x - 1)^5}.$$

(37)

From eqs.(31-34) the branching ratio depends on the neutrino Yukawa couplings. Several forms for these Yukawa matrices were studied in the supersymmetric $SU(4)_C \times SU(2)_L \times SU(2)_R$ model \[23\]. Here we consider the ansatz given in Ref.\[24\] which is compatible with the solar and the atmospheric neutrino data. It remains to be seen whether this ansatz or some form close to it can be realized in the present scheme which contains $U(1)_R$ symmetry.

In Figure (6) we exhibit the branching ratio $BR(\mu \to e\gamma)$ versus the chargino mass. It is interesting that the predicted values of the branching ratio are very close to the experimental bound, and even for very heavy sleptons only about one order of magnitude below the current limits.

![Figure 6. The branching ratio $BR(\mu \to e\gamma)$ versus the lightest chargino mass](image)
We have studied the low energy consequences of a class of supersymmetric models with left-right symmetry, in particular the $SU(4)_C \times SU(2)_L \times SU(2)_R$ scheme. In these models the gauge hierarchy and $\mu$ problems are first resolved and $\tan \beta$ is constrained to be of order $m_t/m_b$. We have shown that non-universality between $m^2_{H_1}$ and $m^2_{H_2}$ is favorable for successful EW symmetry breaking. On the other hand, the requirement that SUSY corrections to the bottom quark mass should not be exceed 20% gives strong constraints on the allowed parameter space, namely it leads to $\mu < 0$ and not too large, while the gluino mass should be of order the experimental bound.

We have investigated the SUSY spectrum in this class of large $\tan \beta$ models. It turns out that the lightest chargino and neutralino are almost gaugino-like for large ($\sim$ TeV) values of $m_{3/2}$, and they become more Higgsino-like if $m_{3/2}$ is not too large, since in this region $\mu$ is small. Furthermore, we have shown that the lightest Higgs mass is of order 120 GeV, and the neutral pseudoscalar Higgs boson $A$ is not too heavy ($\sim O(100)$ GeV).

We also examined the radiative process $b \to s\gamma$. This process imposes the constraint that the gravitino mass to be less than 1.5 TeV. Finally, we find that the LFV process $\mu \to e\gamma$ is expected to be enhanced due to the large value of $\tan \beta$.

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