Spontaneous Symmetry Breaking from Non-Zero Value of Vacuum Energy Density

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The Higgs mechanism plays an essential role to provide the origin of mass to elementary particles in the Standard Model. However, the origin of Higgs mass itself could not be explained by its mechanism. In this research, we propose a mathematical model of spontaneous symmetry breaking without the addition of the negative mass term to the Higgs potential. The simple requirements of this model before symmetry breaking to happen are that the Higgs boson must be a massless particle in the quartic potential with the non-vanishing vacuum energy density and interact non-minimally with the gravity. If we assume that there is no physics beyond the Planck scale, we show that the spontaneous symmetry breaking of the Higgs boson can be triggered from the existence of vacuum energy density ($\rho$) with the possible values in the regime $10^{24}$ GeV$^4$ to $10^{41}$ GeV$^4$.

I. INTRODUCTION

In the present, the Higgs boson plays an essential role in modern particle physics for the understanding of the origin of mass of the elementary particles [1–4] and this particle could guide the exploration of the new physics beyond the Standard Model by the argument from the naturalness problem [5]. Additionally, in the context of cosmology, this particle could be a candidate to drive the cosmic inflation [6] and could be the portal to the dark matter [7–11]. It is amazing that the Higgs particle could possibly be used to contribute to many areas of modern particle physics.

According to our humble opinion, the origin of the Higgs mechanism is still ambiguous. One of the most questionable features of the Higgs boson is the origin of the Higgs potential. The spontaneous symmetry breaking of the Higgs boson in the potential with the shape of the Mexican hat can address the origin of mass of the particles in the Standard model [12–14]. However, the origin of mass term of the Higgs boson could not be explained by its mechanism. This term is artificially added by hand. Moreover, it artificially requires a mass term with a negative sign. This seems to suggest that there is a missing piece of our understanding about the Standard Model to naturally explain the spontaneous symmetry breaking process of the Higgs boson.

Up to now, several famous scenarios have been proposed to explain the origin of the electroweak phase transition. Some explanations suggest that the unusual sign of the mass term of Higgs boson could be obtained by the quantum correction to Higgs potential seen in the original version of the Wilson effective potential [15]. Another explanation suggests that spontaneous symmetry breaking can happen from the scale-invariant Lagrangian [16–18]. However, with the current experimental technology of collider physics, the search for the real shape of Higgs potential from the Higgs self-coupling sector has not yet come to the conclusion [19–21]. The Higgs self-coupling constants are not yet determined at the end of Run 2 of the Large Hadron Collider [22–29]. Thus, the mechanism behind the electroweak phase transition due to the Higgs boson is still mysterious. As a consequence, we believe that there is still a room for a new idea providing the clue of the origin of the spontaneous symmetry breaking.

In this work, we will present an alternative explanation based on a mathematical point of view that can provide the origin of the Mexican-hat shape of the Higgs potential without the addition of anomalous mass term to Higgs potential, and without the addition of the new degree of freedom of new hypothetical particles interacting to Higgs boson. To pursue our goal, we consider the action of a scalar field ($\phi$) coupled non-minimally to gravity which has been studied in several contexts in cosmology, especially in the inflation model [30–36]

\[
S = \int d^4x \sqrt{-g} \left\{ X - V(\phi) - \frac{M^2 + \xi \phi^2}{2} R \right\}, \tag{1}
\]

where $X = g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi / 2$, $R$ is the scalar curvature, $M$ is an arbitrary mass scale, $\xi$ is the dimensionless coupling constant, and $V(\phi)$ is the arbitrary potential. We shall point out that this theory of non-minimal gravity with the non-zero vacuum energy density ($\rho$) can possibly trigger the spontaneous symmetry breaking to the massless scalar particle in the $\phi^4$ potential

\[
S = \int d^4x \sqrt{-g} \left\{ -\rho + X - \frac{\lambda}{4} \phi^4 - \frac{M^2 + \xi \phi^2}{2} R \right\}. \tag{2}
\]
To simplify the model in this paper, we focus on the value of the mass scale in the order of Planck scale, which is $M \approx M_p = 2.44 \times 10^{18}$ GeV, and the value of $\rho$ and $\xi$ are restricted in the positive region.

This paper is organized as follows. In Sec-II, we shall show that, in the Einstein frame, the massless scalar particle in the action (2) would naturally provides the potential similar to the Mexican hat from the vacuum energy density without the requirement of the artificial addition of the mass term with the anomalous sign. In Sec-III, we try to answer the following questions, if this model is considered to be Higgs boson, what is the validity of this Higgs model specified from the power counting of the effective operators? How big the coupling constants of Higgs self-interaction could be modified by the effect of gravity at the tree level? What is the possible range of energy scale of vacuum energy density $\rho$ for driving the electroweak phase transition? Then, in Sec-IV, we summarize on our spontaneous symmetry breaking model and discuss on some remarkable points.

II. EINSTEIN FRAME ACTION

Let us start with the standard scenario that transforms the theory of scalar field coupling non-minimally to gravity (Jordan frame) to the theory of scalar field coupling minimally to gravity (Einstein frame) by the conformal transformation [37–44]. We introduce the conformal transformation of the metric tensor

$$g_{\mu\nu} = \Omega^{-2} \tilde{g}_{\mu\nu},$$

(3)

where the parameter with tilde notation refers to the quantity in the Einstein frame. The transformations of the inverse metric tensor and its determinant are

$$g^{\mu\nu} = \Omega^2 \tilde{g}^{\mu\nu}, \quad \sqrt{-g} = \Omega^{-4} \sqrt{-\tilde{g}}.$$  

(4)

Under these transformations, the kinetic energy of the scalar field and the Ricci scalar curvature become

$$X = g^{\mu\nu} \partial_{\mu}\phi \partial_{\nu}\phi = \Omega^2 \tilde{g}^{\mu\nu} \partial_{\mu}\phi \partial_{\nu}\phi = \Omega^2 \tilde{X},$$

(5)

$$R = \Omega^2 \tilde{R} + 6 \Omega \tilde{g}^{\mu\nu} \nabla_{\mu} \nabla_{\nu} \Omega - 12 \tilde{g}^{\mu\nu} \tilde{\nabla}_{\mu} \Omega \tilde{\nabla}_{\nu} \Omega.$$  

(6)

Using Eq. (6), the last term in the action (2) can be rewritten as

$$-\frac{M_p^2}{2} \int d^4x \sqrt{-g} \left(1 + \frac{\xi \phi^2}{M_p^2}\right) \tilde{R},$$

$$= -\frac{M_p^2}{2} \int d^4x \sqrt{-\tilde{g}} \tilde{\Omega}^{-2} \left(1 + \frac{\xi \phi^2}{M_p^2}\right) \tilde{\tilde{R}},$$

$$= -\frac{M_p^2}{2} \int d^4x \sqrt{-\tilde{g}} \tilde{\Omega}^{-2} \left(1 + \frac{\xi \phi^2}{M_p^2}\right)$$

$$\times \left(6 \Omega^{-1} \tilde{g}^{\mu\nu} \tilde{\nabla}_{\mu} \tilde{\nabla}_{\nu} \phi - 12 \tilde{g}^{\mu\nu} \tilde{\Omega}^{-2} \tilde{\nabla}_{\mu} \Omega \tilde{\nabla}_{\nu} \Omega\right).$$  

(7)

To decouple the non-minimal coupling term of the scalar field and the scalar curvature in the conformal frame, the conformal factor is

$$\Omega^2 = 1 + \frac{\xi \phi^2}{M_p^2}.$$  

(8)

Substituting Eq. (8) into Eq. (7) and performing integration by parts on the term with the second order derivative $(\tilde{g}^{\mu\nu} \tilde{\nabla}_{\mu} \nabla_{\nu} \Omega)$ term, we obtain

$$-\frac{M_p^2}{2} \int d^4x \sqrt{-\tilde{g}} \left(1 + \frac{\xi \phi^2}{M_p^2}\right) \tilde{R}$$

$$= -\frac{M_p^2}{2} \int d^4x \sqrt{-\tilde{g}} \left(R - \frac{6 \xi \phi^2}{M_p^2 (1 + \frac{\xi \phi^2}{M_p^2})^2} \tilde{X}\right).$$  

(9)

Next, applying the metric transformations (3)-(4) to the vacuum energy density and the scalar field’s kinetic and potential terms, we have

$$\int d^4x \sqrt{-\tilde{g}} (X - V(\phi))$$

$$= \int d^4x \sqrt{-\tilde{g}} \left(1 + \frac{\xi \phi^2}{M_p^2}\right) \tilde{X} - \tilde{V}(\phi),$$  

(10)

where the potential term is defined as

$$V(\phi) = \rho + \frac{\lambda}{4} \phi^4,$$  

(11)

and the transformed potential is

$$\tilde{V}(\phi) = \Omega^{-4} \left(\rho + \frac{\lambda}{4} \phi^4\right) = \frac{\rho + \frac{\lambda}{4} \phi^4}{(1 + \frac{\xi \phi^2}{M_p^2})^2}.$$  

(12)

Upon using Eq. (9) and Eq. (10), the full action in the Jordan frame (2) can be expressed in the Einstein frame as

$$S = \int d^4x \sqrt{-\tilde{g}} \left(\frac{1}{2} F(\phi) \tilde{X} - \tilde{V}(\phi) - \frac{M_p^2}{2} \tilde{R}\right).$$  

(13)

Here the dynamics of the spacetime fluctuation is specified by the metric tensor $g_{\mu\nu}$ and

$$F(\phi) = \frac{M_p^2 \left(M_p^2 + \xi (6 \xi + 1) \phi^2\right)}{(M_p^2 + \xi \phi^2)^2}. $$  

(14)

In the Einstein frame, the shape of potential depends on the value of the vacuum energy ($\rho$) shown in Fig. 1.
In the case of \( \rho = 0 \), the \( Z_2 \) symmetry of scalar field is stable at the lowest minimum of the Einstein frame potential. The vacuum expectation value of field is zero, so the spontaneous symmetry breaking is not required.

Next, in the case of \( \rho \neq 0 \), the vacuum energy density influences the potential \( \tilde{V}(\phi) \) creating the Mexican hat-like shape. This results in the meta-stable state for the scalar particle to live within. The intriguing point is that the scalar field naturally obtains a squared effective mass with the minus sign (as we artificially demanded in the situation of the traditional potential for spontaneous symmetry breaking)

\[
m^2 = \left. \frac{\partial^2 \tilde{V}}{\partial \phi^2} \right|_{\phi = 0} = -\frac{4\xi \rho}{M_p^4} = -\mu^2. \tag{15}\]

This potential with the non-zero vacuum energy density, the vacuum expectation value of the field is non-zero. Its value can be given from the lowest minimum of the potential as

\[
\langle \phi \rangle^2 = -\frac{4\xi \rho}{4\lambda M_p^4} = \frac{\mu^2}{\lambda} = v^2. \tag{16}\]

This then leads to the spontaneous symmetry breaking of the scalar field automatically.

### III. POSSIBILITIES TO BE HIGGS BOSON

In this section, we shall consider the quantum fluctuation from the action (13) in the background value of field at the lowest minimum of the potential \( \tilde{V}(\phi) \), and we are going to address the questions “If this model is interpreted as Higgs boson (in unitary gauge), how much value of Standard Model parameters in the Higgs sector could possibly be modified? What is the validity of this Higgs Model identified by the power counting?, What is the possible value of vacuum energy density to determine the physical scale of the origin of physics behind this spontaneous symmetry breaking?”

Next, we consider the action (13) as the Higgs action in unitarity gauge which the Goldstone mode is neglected in the calculation. We are going to proceed to an investigation on the contribution to the Higgs Lagrangian in the Standard Model. We expand the gravitational field and scalar field in the action (13) around the background value. We consider the metric tensor \( \tilde{g}_{\mu\nu}(x) \) in the flat Minkowski metric \( \eta_{\mu\nu} \), and expand the scalar field \( \phi(x) \) around the background value \( v \) at the minimum of potential \( \tilde{V} \) plus with a quantum fluctuation \( h(x) \)

\[
\tilde{g}_{\mu\nu}(x) = \eta_{\mu\nu}, \tag{17}
\]

\[
\phi(x) = v + h(x). \tag{18}\]

From the action (13), the Lagrangian function of the scalar field \( \phi \) under the field expansion (17) and (18) is written as

\[
\mathcal{L} = \frac{1}{2} F(v + h) \eta^{\mu\nu} \partial_\mu h \partial_\nu h - \tilde{V}(v + h). \tag{19}\]

where functions \( F \) and \( \tilde{V} \) can be rewritten in term of the quantum field \( h \) by Taylor expansion as

\[
F(v + h) = F(v) + \sum_{n=1}^{\infty} \frac{1}{n!} \frac{d^n F}{dh^n} \bigg|_{h=v} h^n, \tag{20}\]

\[
\tilde{V}(v + h) = \tilde{V}(v) + \sum_{n=1}^{\infty} \frac{1}{n!} \frac{d^n \tilde{V}}{dh^n} \bigg|_{h=v} h^n. \tag{21}\]

To proceed with this model as a candidate to Higgs boson, there are two options. The first one is to work directly with the field \( h \) and evaluate the momentum-dependent coupling constant in tree-level amplitude from the non-linearity of kinetic energy to check the validity of our model. As an alternative, one may perform the following field redefinition,

\[
h = \sum_{n=1}^{\infty} c_n h^n, \tag{22}\]

to rearrange the kinetic energy into a renormalized canonical form. The Lagrangian (19) can be rewritten in the canonical way as

\[
\mathcal{L} = \frac{1}{2} \eta^{\mu\nu} \partial_\mu \tilde{h} \partial_\nu \tilde{h} - \tilde{V}(v + h(\tilde{h})), \tag{23}\]

where the field \( \tilde{h} \) will be interpreted as the Higgs field after the symmetry breaking, and the exact expressions of \( c_n \) are derived and given in Appendix A.

The Taylor expansion of potential term around the field \( h(\tilde{h}) \) up to dimension-four operator gives

\[
\tilde{V} \approx \frac{\mu^2 M_p^4}{4\xi (v^2 \xi^2 + M_p^2)} + \mu^2 M_p^6 \left( \frac{25v^4 \xi^2 - 22v^2 \xi M_p^2 + M_p^4}{4v^2 (v^2 \xi^2 + M_p^2)^5} h(\tilde{h})^4 \right)
+ \frac{\mu^2 M_p^6 (25v^4 \xi^2 - 22v^2 \xi M_p^2 + M_p^4)}{4v^2 (v^2 \xi^2 + M_p^2)^5} h(\tilde{h})^4 \tag{24}\]
where we have employed relations (15) and (16). The field redefinition of \( h(\tilde{h}) \) from (22) will be considered up to order \( \tilde{h}^3 \),

\[
h = c_1 \tilde{h} + c_2 \tilde{h}^2 + c_3 \tilde{h}^3 + O(\tilde{h}^4),
\]

(25)
to correct the potential in terms of field \( \tilde{h} \) up to the order \( \tilde{h}^4 \) (see also in Appendix-A). Substituting Eq. (25) into Eq. (24), the Lagrangian (23) in terms of the field \( \tilde{h} \) is written as

\[
\mathcal{L} = -\tilde{\rho} + \frac{1}{2} \partial_j \tilde{h} \partial^j \tilde{h} - \frac{M^2_h}{2} \tilde{h}^2 - \frac{\lambda_3}{3!} \tilde{h}^3 - \frac{\lambda_4}{4!} \tilde{h}^4 + O(\tilde{h}^5)
\]

(26)
where \( M_h \) is a mass of the \( \tilde{h} \) field, \( \lambda_i \) is the modified Higgs self coupling, \( \tilde{\rho} \) is the vacuum energy density after symmetry breaking. All these parameters are defined in terms of Higgs mass, vacuum expectation value, Planck mass, and non-minimal coupling constant as

\[
\tilde{\rho} = \frac{M^2_h M_p^2}{8\xi} \left( 1 + \xi(6\xi + 1) \frac{v^2}{M_p^2} \right),
\]

(27)
\[
M^2_h = 2\mu^2 \left( 1 + \xi \frac{v^2}{M_p^2} \right)^{-1} \left( 1 + \xi(6\xi + 1) \frac{v^2}{M_p^2} \right)^{-1},
\]

(28)
\[
\lambda_3 = d_3 \frac{3 M^2_h}{v^2},
\]

(29)
\[
\lambda_4 = d_4 \frac{3 M^2_h}{v^2},
\]

(30)
where

\[
d_3 = (1 - \sigma_3) \left( 1 + \xi(6\xi + 1) \frac{v^2}{M_p^2} \right)^{-\frac{3}{2}},
\]

(31a)
\[
d_4 = (1 - \sigma_4) \left( 1 + \xi(6\xi + 1) \frac{v^2}{M_p^2} \right)^{-3},
\]

(31b)
\[
\sigma_3 = \frac{3 \xi v^2}{M_p^4} + \frac{2(1 + 6\xi)v^4}{M_p^4},
\]

(31c)
\[
\sigma_4 = \frac{\xi v^2}{M_p^4} + \frac{\xi^2(1 + 6\xi)v^4}{M_p^4} - \frac{4\xi^3(1 + 6\xi)v^6}{M_p^4} - \frac{7\xi^4(1 + 6\xi)^2v^8}{M_p^4}.
\]

(31d)
In the Standard Model, \( d_3 = d_4 = 1 \). On the other hand, in our proposed model predicts the relative deviation from the Standard Model prediction. The parameters \( d_3, d_4 \) depend on the unobservable parameter \( \xi \) shown in Eqs. (31a) and (31b) which comes from the theory of gravity. The direct probe of the value of the Higgs self coupling at future colliders could imply that how strong the dimensionless coupling constant \( \xi \) from the theory of gravity could it be.

### A. Vacuum expectation value

We show the relative deviation of the vacuum expectation value from the Standard Model prediction to our model. The vacuum expectation value \( v \) is traditionally defined by matching the electroweak theory to the four-fermion theory in the low energy limit of muon decay channel. The calculation is shown as follows.

The coupling constant in the four-fermion theory \((G_F \approx 1.166 \times 10^{-5} \text{ GeV}^{-2}) [45]\) could be written in terms of the mass of \( W^\pm \) boson and gauge coupling constant \( g \) as

\[
g^2 \frac{8 M^2_W}{2} = G_F \sqrt{2}
\]

(32)
where \( M_W \) is the mass of \( W^\pm \) boson defined from the coefficient in front of quadratic term after spontaneous symmetry breaking of Higgs boson

\[
g^2 \phi^2 W^\pm W^\mu_\mu = \frac{g^2 v^2_{\text{SM}}}{4} W^\pm W^\mu_\mu + O(h).
\]

(33)
The mass of \( W^\pm \) bosons can be written in terms of Higgs vacuum expectation value as

\[
M_W = \frac{g v_{\text{SM}}}{2}.
\]

(34)
From Eq. (32) and Eq. (34), the vacuum expectation value can be obtained from the \( G_F \) as follow

\[
v_{\text{SM}} = \frac{1}{\sqrt{2} G_F} \approx 246 \text{GeV}.
\]

(35)
In our case, the mass term of \( W^\pm \) boson is needed to be defined in the Einstein frame. The mass term of gauge boson in the Jordan frame has to be transformed by the conformal factor,

\[
g^2 \frac{4}{4\Omega^2} \phi^2 W^\pm W^\mu_\mu \rightarrow g^2 \frac{4}{4\Omega^2} \phi^2 W^\pm W^\mu_\mu,
\]

(36)
from the Jacobian determinant of metric tensor and through appropriate field transformation of gauge field for reorganizing the kinetic energy into canonical form as follow,

\[
\sqrt{-g} \rightarrow \Omega^{-4} \sqrt{-g} \quad W^\mu_\pm \rightarrow \Omega W^\mu_\pm.
\]

(37)
After spontaneous symmetry breaking, we expand the conformal factor of mass term (36) around the field \( h \),

\[
g^2 \frac{4}{4\Omega^2} (v + h)^2 W^\pm W^\mu_\mu = \frac{1}{4} \left( \frac{g^2 v^2}{M_p^2} \right) W^\pm W^\mu_\mu + O(h).
\]

(38)
The gauge boson mass in the Einstein frame could be defined as

\[
M^\pm_W = \frac{g v}{2} \left( 1 + \frac{\xi v^2}{M_p^2} \right)^{-1/2}.
\]

(39)
Substituting $G_F$ from Eq. (35) and $M_W$ from Eq. (39) into Eq. (32), the vacuum expectation value $v$ could be written in terms of $v_{\text{SM}}$ as

$$v^2 = v_{\text{SM}}^2 \left( 1 + \frac{\xi v_{\text{SM}}^2}{M_p^2} \right)^{-1}, \quad (40)$$

As can be seen, our vacuum expectation value varies as a function of dimensionless coupling constant $\xi$. This variation is illustrated as a plot between $v/v_{\text{SM}}$ versus $\xi$ in Fig. 2.

![FIG. 2. The ratio between vacuum expectation value of $h$ field from the model (13) and the vacuum expectation value of Higgs boson from the Standard Model in arbitrary value of $\xi$ where $v_{\text{SM}} = 1/\sqrt{2G_F} \approx 246$ GeV, $M_p = 2.44 \times 10^{18}$ GeV](image)

Comparatively, our vacuum expectation value behaves uniformly similar to the value predicted by the Standard Model in the regime $0 < \xi < 10^{28}$. The different of $v$ and $v_{\text{SM}}$ is less than 0.01 percent in this region. Beyond $10^{28}$, our vacuum expectation value start to falls. This could be one evident to support the compatibility between our model and the Standard Model.

### B. Tree-level unitarity

On the general ground, to check the perturbativity of our model is to use the unitarity of amplitude. According to the constraint from tree-level unitarity, one expects the cubic and quartic self-coupling modification $[46, 47]$ of the Higgs Standard Model from new physics are

$$|\lambda_3/\lambda_3^{\text{SM}}| < 65, \quad |\lambda_4/\lambda_4^{\text{SM}}| < 6.5, \quad (41)$$

where

$$\lambda_3^{\text{SM}} = 3M_h^2/v_{\text{SM}}, \quad \lambda_4^{\text{SM}} = 3M_h^2/v_{\text{SM}}^2, \quad (42)$$

are the cubic and quartic Higgs self-coupling constants from the Standard Model prediction, respectively, and $M_h = 125$ GeV is Higgs mass $[48, 49]$. These theoretical bounds come from the evaluation of the partial wave amplitude $J = 0$ $[50]$ in tree-level containing with $s,t,u$-channel from the cubic self-interaction and the four legged channel from the quartic self-interaction without the graviton exchange. Namely, if the values of $\lambda_3$ and $\lambda_4$ do not satisfy the bound (41), the perturbativity of model breaks down in tree-level amplitude.

Fig. 3 shows the plot of $\lambda_3/\lambda_3^{\text{SM}}$ and $\lambda_4/\lambda_4^{\text{SM}}$ against $\xi$. The value of $\lambda_3$ and $\lambda_4$ calculated from our model obey the restriction (41) for all value of $\xi$. According to Fig. 3, our model strongly agree with the Standard Model when $\xi < 10^{15}$.

![FIG. 3. The ratio of $\lambda_3/\lambda_3^{\text{SM}}$ and $\lambda_4/\lambda_4^{\text{SM}}$ with the various value of $\xi$](image)

When $\xi > 10^{15}$, both coupling constants come close to zero. In other word, the tree-level amplitude of this theory is insensitive to the effect of non-minimal coupling to gravity until $\xi \approx 10^{15}$. As a consequence, no sign of unitarity violation for arbitrary value of $\xi$ exists in the tree-level scattering of $hh \rightarrow hh$.

### C. Cutoff energy

In the framework of the effective field theory, the validity of perturbation theory of $n$-particle scattering amplitude must break down at some energy scale $\Lambda$ called an ultra-violet cutoff (UV cutoff), which is the largest energy that explicitly appears in the propagator or vertices of the calculation. It means that the theory is predictive only with the use of standard perturbative method at $E < \Lambda$, where $E$ is the energy of particle. An easy way to specify the cutoff energy without the calculation of $n$-particle scattering is the power counting method. For operator with the dimension that is larger than four, the cutoff energy is estimated by the inverse coefficient of the operator, which can be found in many literature about the scalar field coupling non-minimally to gravity $[51–53]$, namely

$$\frac{\hat{O}_n(\Phi)}{[\Lambda_n]^{n-4}}, \quad (43)$$

where $\hat{O}_n(\Phi)$ is effective field operator with dimension $n$, and $\Phi$ is a generic field, and $\Lambda_n$ is a cutoff energy of $\hat{O}_n(\Phi)$. In this sense, if the model contains several effective field operators with various cutoff energy $\Lambda_n$,
the UV cutoff of the theory can be defined from the minimum value of $\Lambda_n$ from the operator order $n$: $\Lambda = \min \Lambda_n$.

If Lagrangian (26) is considered to be the model of spontaneous symmetry breaking of Higgs boson (in unitary gauge), the higher dimension operator contributing to Lagrangian (26) starts at a dimension-five operator

$$\mathcal{O}(\hat{h}^5) = \sum_{n=5}^{\infty} \frac{1}{\Lambda_n^n} \hat{h}^n.$$  \hfill (44)

The factor $1/\Lambda_n$ can be explicitly evaluated by the equation

$$(\Lambda_n)^{d-n} = \frac{1}{n!} \frac{d^n \hat{V}}{dh^n} \bigg|_{\hat{h}=0}.$$  \hfill (45)

To derive the cutoff energy of the effective operator with dimension $n$ in the Einstein frame potential $\hat{V}(\hat{h})$, it requires the contribution from field redefinition (22) up to the order $\hat{h}^{n-1}$ to correct the value of cutoff at the order $n$. Then, we plot the cutoff energy of dimension-five to dimension-sixteen operators varying on the dimensionless coupling $\xi$ shown in Fig. 4.

![Graph showing cutoff energy vs. $\xi$]

FIG. 4. The cutoff energy (GeV) from dimension five operator to dimension sixteen operator varying on the value of $\xi$ where $M_h = 125$ GeV, $v_{\text{SM}} = 246$ GeV, $M_p = 2.44 \times 10^{18}$ GeV.

The behavior of the cutoff energy is the following.

- In the region $\xi \lesssim 10^{16}$, we have $\Lambda_n^{(\text{even})} < \Lambda_n^{(\text{even})}$, and, $\Lambda_{n+2}^{(\text{odd})} < \Lambda_{n}^{(\text{odd})}$, and the value of cutoff $\Lambda_n^{(\text{odd})}$ is evidently larger than the value of cutoff $\Lambda_n^{(\text{even})}$.

- In the region $\xi \gtrsim 10^{16}$, we obtain $\Lambda_{n+1} < \Lambda_n$.

In these two different regions, the UV cutoff cannot be specified by the cutoff from the effective operator with the lowest dimension, but it is given by the cutoff of the operator at order infinity

$$\Lambda = \Lambda_{\infty}.$$  \hfill (46)

However, to specify the exact value of the UV cutoff $\Lambda$ from the Eq. (46), it might be impossible through the method of our calculation as it requires us to solving of the system of infinite linear equations in the process of field redefinition.

We therefore choose another route. In this work, the UV cutoff will be estimated parametrically (ignore the non-physical structure coefficient) from the asymptotic value of $\Lambda_n$ for $n \to \infty$. The cutoff can be roughly classified into four major regions shown in Fig. 4. This implies that the UV cutoff takes different value for different interval of $\xi$. The four regions are as follows.

- Region $\xi \lesssim 1$. In this region, the cutoff of this model can be approximated parametrically as a function of

$$\Lambda_n^{(\text{even})} \approx M_p \frac{v_{\text{SM}}}{\sqrt{\xi} M_h},$$  \hfill (47)

$$\Lambda_n^{(\text{odd})} \approx \left( \frac{M_p}{\sqrt{\xi}} \right)^{(n-3)/(n-4)} \left( \frac{v_{\text{SM}}}{M_h^2} \right)^{1/(n-4)}.$$  \hfill (48)

and its asymptotic value as $n \to \infty$ for the UV cutoff is estimated as

$$\Lambda \approx \frac{M_p}{\sqrt{\xi}}.$$  \hfill (49)

It is evident that this region of $\xi$ gives the $\Lambda$ that is bigger than $M_p$. This suggests that the Higgs particle with $E > M_p$ could be possibly allowed in the perturbative calculation. However, this model is an low energy effective theory below $M_p$. Then, we can argue that the Higgs particle with the energy range $E > M_p$ could lead to the breakdown of perturbativity in computing the scattering amplitude. So, this implies that the situation

$$\xi \lesssim 1$$  \hfill (50)

should be excluded.

- Region $1 < \xi \lesssim 10^{16}$. The cutoff scale is proportional to inverse function of $\xi$

$$\Lambda_n^{(\text{even})} \approx M_p \frac{v_{\text{SM}}}{\xi} \left( \frac{v_{\text{SM}}}{M_h} \right)^{2/(n-4)},$$  \hfill (51)

$$\Lambda_n^{(\text{odd})} \approx \left( \frac{M_p}{\xi} \right)^{(n-3)/(n-4)} \left( \frac{v_{\text{SM}}}{M_h^2} \right)^{1/(n-4)}.$$  \hfill (52)

In this region the cutoff tends to the electroweak scale, when $\xi$ increases. For $n \to \infty$, the cutoff for even and odd operator approach to the same asymptotic value. The UV cutoff in this region of $\xi$ is estimated in the form

$$\Lambda \approx \frac{M_p}{\xi}.$$  \hfill (53)

This UV cutoff has the same form as the cutoff scale from Higgs Inflation model [51–53].

- Region $10^{16} \lesssim \xi \lesssim 10^{24}$. The value of the cutoff starts to increase from the electroweak scale at $\xi \approx 10^{16}$ to the Planckian scale with functions

$$\Lambda_n^{(\text{even})} \approx v_{\text{SM}} \left( \frac{v_{\text{SM}}}{M_h} \right)^{2/(n-4)} \left( \frac{v_{\text{SM}}}{M_p} \right)^{2(n-1)/(n-4)},$$  \hfill (54)

$$\Lambda_n^{(\text{odd})} \approx v_{\text{SM}} \left( \frac{v_{\text{SM}}}{M_h} \right)^{2/(n-4)} \left( \frac{v_{\text{SM}}}{M_p} \right)^{(2n-3)/(n-4)}.$$  \hfill (55)
Asymptotic value of the cutoff in this region is directly proportional to $\xi^2$ as

$$\Lambda \simeq v_{\text{SM}} \left( \frac{v_{\text{SM}} \xi}{M_p} \right)^2. \tag{56}$$

When $\xi \to 10^{24}$, the value of the UV cutoff (56) is limited to the energy scale of quantum gravity $M_p$.

- **Region** $\xi \gtrsim 10^{24}$. All orders of the cutoff follow Eqs. (54)-(55) for certain ranges. After that, they are all independent of $\xi$ and the cutoff for even and odd operator behave as

$$\Lambda_n \simeq \left( \frac{M_p}{M_h} \right)^{2/(n-4)} M_p. \tag{57}$$

In this region, the UV cutoff is directly proportional to Planck mass,

$$\Lambda \simeq M_p. \tag{58}$$

The possible lowest value of the UV cutoff in this model is estimated in the electroweak scale. Since the validity of model must be below Planck scale, the value of $\xi$ must be

$$\xi > 1. \tag{59}$$

If this model of symmetry breaking is considered to be Higgs Lagrangian, this condition of $\xi$ naturally predicts the cutoff energy of Higgs sector around

$$v_{\text{SM}} \lesssim \Lambda \lesssim M_p, \tag{60}$$

where $v_{\text{SM}} \approx 246$ GeV in the lowest limit is given from the lowest value of (53)-(56) at $\xi \approx 10^{16}$, and $M_p \approx 2.44 \times 10^{18}$ GeV in the upper limit is given from $\Lambda$ in Eq. (53) at $\xi = 1$, $\Lambda$ in Eq. (56) at $\xi \approx 10^{24}$, and $\Lambda$ in Eq. (58).

**D. The vacuum energy density and mass scale of physics background**

In our proposed model, the vacuum energy density is an artificial parameter that is used to trigger symmetry breaking. If this model is to explain the origin of spontaneous symmetry breaking of Higgs boson, it is important that this model requires the non-zero value of vacuum energy density ($\rho$) possibly generated from unknown source or unknown mechanism below Planck scale to the action in Jordan frame.

In this section, we would like to identify the possible value of $\rho$ to drive the symmetry breaking of the Higgs boson from the constraint in the previous section. By substituting $\mu$ from Eq. (28) into (16), the vacuum energy density can be written in terms of the Higgs mass, vacuum expectation value, Planck mass, and $\xi$ as

$$\rho = \frac{M_p^2 M_h^2}{8 \xi} \left( 1 + \frac{v^2}{M_p^2} \right) \left( 1 + \xi (6 \xi + 1) \frac{v^2}{M_p^2} \right). \tag{61}$$

The plot of $\rho$ with respect to the parameter $\xi$ is shown in Fig. 5.

![Fig. 5. The vacuum energy density $\rho$ in the various value of $\xi$ where $v \approx 246$ GeV, $M_h \approx 125$ GeV, and $M_p \approx 2.44 \times 10^{18}$ GeV.](image)

From our approximation $M \simeq M_p$ in Sec-I, it implies that $\xi < M_p^2/v^2 \approx 10^{32}$. Beyond this value of $\xi$, the result of $\rho$ in Eq. (61) is ineffective and comes to failure. The other condition comes from perturbativity. The cutoff energy implies the value of $\xi$ must be in the regime $\xi > 1$. Combining all conditions on parameter $\xi$, we have $1 \lesssim \xi \lesssim 10^{32}$. Therefore, the possible value of $\rho$ for driving spontaneous symmetry breaking to the Higgs boson is in the range

$$10^{24} \lesssim \rho \lesssim 10^{41} \text{ GeV}^4. \tag{62}$$

This implies that there might possibly be some new physical mechanism generating the vacuum energy density which associate to energy scale $10^6 \lesssim E_{\text{new}} \lesssim 10^{10}$ GeV. The remaining question is that “what is the source of the vacuum energy density that causes spontaneous symmetry breaking and then drive the electroweak phase transition?”.

**IV. SUMMARY AND DISCUSSION**

In summary, the spontaneous symmetry breaking of the massless scalar field can be induced by the non-vanishing vacuum energy density instead of that mass term in the traditional Higgs mechanism. The spontaneous symmetry breaking of scalar field in the Mexican-hat potential is a natural consequence in the Einstein frame. In our scenario, the mass term is an effective mass constructed from the vacuum energy density and comes out naturally which is evident in Eq. (15). If the value of the non-minimally coupling constant $\xi$ is in the range $\xi > 1$, our model of spontaneous symmetry breaking can describe Higgs boson property such as the origin of the mass term without unitarity violation in tree-level scattering and the validity of the model is estimated above the electroweak scale up to Planck scale. In the viewpoint of our symmetry breaking model, this theoretical
bound of $\xi$ also implies that before the electroweak phase transition happens at the early period of the universe, the spontaneous symmetry breaking of Higgs boson has to be triggered by the non-zero vacuum energy density in the regime $10^{24} \lesssim \rho \lesssim 10^{41}$ GeV$^4$.

A. Remark on the Naturalness problem

The quadratic divergence in the radiative correction to Higgs mass is

$$\delta M_h^2 = \frac{c_{\text{SM}} A^2}{16\pi^2}. \quad (63)$$

It plays an essential role in the concept of naturalness, where $c_{\text{SM}}$ contains the summation of the mass of the Standard Model particles. From the standard model contribution, the UV-cutoff must be below 10 TeV [5] to avoid the fine-tuning problem of Higgs bare mass. This leads to the question that why the cutoff energy of SM is very low comparing to the quantum gravity scale. This problem is known as in the name of hierarchy problem.

If our model of spontaneous symmetry breaking is interpreted to be that of the Higgs boson, this model can naturally provide the cutoff energy to the Higgs sector from the framework of the effective field theory. Now, the cutoff energy is modeled using parameter $\xi$ which is a free variable. To make the model free from the fine-tuning problem on the Higgs mass without the addition of hypothetical particle, the coupling constant $\xi$ should satisfy the condition

$$10^{14} \lesssim \xi \lesssim 10^{16}, \quad (64)$$

in order to naturally provide cutoff energy below 10 TeV. If the value of $\xi$ is in this scale, this might possibly be one of an answer of the question that why the Standard Model cutoff is very low comparing to the scale of quantum gravity.

B. Remarkable point on inflaton

From the Higgs chaotic inflation [6], the value of $\xi$ could be specified by the data from WMAP. The appropriate value for $\xi$ is evaluated from the slow roll method at the large field limit with the number of e-folding for the COBE scale $N_{\text{COBE}} \approx 62$ [54]. Namely,

$$\xi \simeq \frac{\sqrt{\lambda N_{\text{COBE}}}}{3} \approx 49000\sqrt{\lambda}. \quad (65)$$

From the Standard Model prediction, $\lambda = M_h^2/2v_{\text{SM}}^2$ where $M_h \approx 125$ GeV and $v_{\text{SM}} \approx 246$ GeV, the value of $\xi$ is numerically 17610. To find appropriate value of $\xi$ for cosmic inflation from our model, we apply the scalar potential of our model from Eq. (11). We obtain the leading order approximation of the slow roll parameters which is

identical to that from Higgs inflation model [6] and the parameter $\xi$ could be written in the form of Eq. (65). However, the prediction of parameter $\lambda$ is different from Standard Model prediction due to gravity effect, by inserting $\mu$ in (16) into (28),

$$\lambda = \frac{M_h^2 (v^2 \xi + M_p^2) (v^2 \xi (6 \xi + 1) + M_p^2)}{2v^4 M_p^4}. \quad (66)$$

Substituting Eq. (65) into Eq. (66), we obtain three possible values of $\xi$ to explain the cosmic inflation

$$\xi^{\pm} \simeq \pm \frac{49000 M_h^2}{\sqrt{2}v^2} + O(M_p)^{-2} \approx \pm 17610, \quad (67)$$

$$\xi^0 \simeq \frac{M_p^4}{3(49000)^2v^2 M_h^2} + O(M_p)^2 \approx 10^{54}. \quad (68)$$

The appropriate value of $\xi$ for inflationary physics is approximately $+17610$ which is not significantly different from the model of Higgs chaotic inflation while the other two solutions do not satisfy the conditions $\xi > 0$ and $M \simeq M_p$ proposed in Sec-I.

With this appropriate value of $\xi$, this model of symmetry breaking requires the value of vacuum energy at $(10^8)^4$ GeV$^4$ to make electroweak phase transition happen and the relevant coupling constants ($\lambda_3$ and $\lambda_4$) are slightly modified from the effect of gravity shown in Fig. 3. The remaining points are the unknown source of the vacuum energy density at $(10^8)^4$ GeV$^4$, together with the new physics that stay in this energy scale affecting the origin of the cosmic inflation and the spontaneous symmetry breaking. This might be one of the interesting areas in physics that the large-scale cosmology and quantum scale phenomenology overlap together.

V. CONCLUSION

In this paper, we have shown that the spontaneous symmetry breaking of massless scalar boson can be stimulated naturally from the vacuum energy density at the early period of the universe. From this point of view, the Mexican-hat potential in the traditional method of spontaneous symmetry breaking is a low energy effective potential from our model. From the simple constraint from the perturbativity bound, this boson could be Higgs boson without the violation on tree-level unitarity in $2h \rightarrow 2h$ scattering amplitude. The electroweak phase transition could be driven by the existence of vacuum energy density around $10^{24}$ (GeV$^4$) to $10^{41}$ (GeV$^4$). The interesting fact is that if the non-minimal coupling constant $\xi$ is close to the order of $10^{16}$, this model naturally provides the cutoff energy below TeV scale. This might be the cure to the concept of naturalness in Higgs radiative correction without the addition of massive hypothetical particle. However, the price to pay is that this Higgs particle could not be the candidate particle to drive cosmic inflation since the observation data from WMAP
requires $\xi \approx 10^4$. However, if we ignore the concept of the naturalness in Higgs radiative correction, this Higgs model could possibly behave like both Higgs boson and inflaton at $\xi \approx 10^4$. This might advocate the concept of $N$-naturalness [55] which does not require the existence of hypothetical particle to fix the fine-tuning in Higgs radiative correction. Finally, another important problem in this theoretical framework is the tremendous remainder of vacuum energy density after symmetry breaking of Higgs boson in Eq. (27). There may be the existence of another mechanism needed to explain the large discrepancy of our prediction and the infinitesimal present value of the vacuum energy density in our universe.

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**Appendix**

This section shows the calculation of the field redefinition to rearrange the kinetic energy of $h$ field in Eq. (19). Firstly, the kinetic energy of $h$ field in Eq. (19) can be written as

$$\frac{1}{2} F(v + h) \partial_\mu h \partial^\mu h = (a_0 + a_1 h + a_2 h^2 + a_3 h^3 + a_4 h^4 + \mathcal{O}(h^5)) \partial_\mu h \partial^\mu h,$$

where the parameter $a_i$ is defined in equation (20) as

$$a_i = \frac{1}{2} \sum_{n=0}^{\infty} \frac{1}{n!} \frac{d^n F}{dh^n} \bigg|_{h=v}. \quad (A.2)$$

Then, we apply field redefinition (25) on (A.1). We obtain

$$\frac{1}{2} F(v + h) \partial_\mu h \partial^\mu h = (b_0 + b_1 \tilde{h} + b_2 \tilde{h}^2 + b_3 \tilde{h}^3 + b_4 \tilde{h}^4 + \mathcal{O}(\tilde{h}^5)) \partial_\mu \tilde{h} \partial^\mu \tilde{h}, \quad (A.3)$$

where $b_i$ is

$$b_0 = a_0 c_1^2,$$

$$b_1 = a_1 c_1^3 + 4 a_0 c_1 c_2,$$

$$b_2 = a_2 c_1^4 + 5 a_1 c_1^2 c_2 + 4 a_0 c_2^2 + 6 a_0 c_1 c_3,$$

$$b_3 = a_3 c_1^5 + 6 a_2 c_1^3 c_2 + 8 a_1 c_1 c_2^2 + 7 a_1 c_1^2 c_3 + 12 a_0 c_2 c_3 + 8 a_0 c_1 c_4,$$

$$b_4 = a_4 c_1^6 + 7 a_3 c_1^4 c_2 + 13 a_2 c_1^2 c_2^2 + 4 a_1 c_2^3 + 8 a_2 c_1 c_3^2 + 22 a_1 c_1 c_2 c_3 + 9 a_0 c_3^2 + 9 a_1 c_1^2 c_4 + 16 a_0 c_2 c_4 + 10 a_0 c_1 c_5. \quad (A.7)$$

Then, we need to solve $c_i$ to obtain the renormalized kinetic energy. We need the conditions

$$b_0 = \frac{1}{2}, \quad b_{i>0} = 0. \quad (A.9)$$

Then solving the coefficients $c_i$ from Eqs. (A.4)-(A.8), therefore, we obtain a set of coefficient $c_i$ for reorganizing the kinetic energy into the renormalized canonical form as

$$c_1 = \frac{1}{\sqrt{2 \alpha_0}}, \quad (A.10)$$

$$c_2 = -\frac{a_1}{8 \alpha_0}, \quad (A.11)$$

$$c_3 = \frac{\sqrt{2} \alpha_1^2 - \sqrt{2} \alpha_0 \alpha_2}{24 \alpha_0^{3/2}}, \quad (A.12)$$

$$c_4 = \frac{-7 \alpha_1^3 + 13 \alpha_0 \alpha_2 \alpha_1 - 6 \alpha_0^2 \alpha_3}{192 \alpha_0^2}, \quad (A.13)$$

$$c_5 = \frac{35 \sqrt{2} \alpha_1^4 - 94 \sqrt{2} \alpha_0 \alpha_2 \alpha_1^2 + 57 \sqrt{2} \alpha_0^2 \alpha_3 \alpha_1}{192 \alpha_0^{13/2}}, \quad (A.14)$$

where $a_i$ is defined in (A.2). Finally, we obtain

$$c_1 = \frac{M_p^2 + \xi \nu^2}{M_p \sqrt{M_p^2 + \xi (6 \xi + 1) \nu^2}}, \quad (A.15)$$

$$c_2 = \frac{\xi \nu (M_p^2 + \xi \nu^2) \left( (1 - 6 \xi) M_p^2 + \xi (6 \xi + 1) \nu^2 \right)}{2 M_p^2 (M_p^2 + \xi (6 \xi + 1) \nu^2)^2}, \quad (A.16)$$

$$c_3 = \frac{\xi (M_p^2 + \xi \nu^2) \left( 3 \xi^2 (6 \xi + 1) \nu^4 M_p^2 \right)}{6 M_p^3 (M_p^2 + \xi (6 \xi + 1) \nu^2)^{7/2}} + \frac{\xi (M_p^2 + \xi \nu^2) \left( 3 \xi (36 \xi^2 + 1) \nu^2 M_p^4 \right)}{6 M_p^3 (M_p^2 + \xi (6 \xi + 1) \nu^2)^{7/2}} + \frac{\xi (M_p^2 + \xi \nu^2) \left( (1 - 6 \xi) M_p^2 \xi + \xi^3 (6 \xi + 1)^2 \nu^6 \right)}{6 M_p^3 (M_p^2 + \xi (6 \xi + 1) \nu^2)^{7/2}}. \quad (A.17)$$
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