Simple Sinflaton-less $\alpha$-attractors

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ABSTRACT: We construct the simplest inflationary $\alpha$-attractor models in supergravity: it has only one scalar, the inflaton. There is no sinflaton since the inflaton belongs to an orthogonal nilpotent superfield where the sinflaton depends on fermion bilinears. When the local supersymmetry is gauge-fixed, these models have only one single real scalar (the inflaton), a graviton and a massive gravitino. The sinflaton, sgoldstino and inflatino are all absent from the physical spectrum in the unitary gauge. The orthogonality condition leads to the simplest Kähler potential for the inflaton, while preserving the Poincaré disk geometry of $\alpha$-attractors. The models are particularly simple in the framework of the $\overline{D3}$ induced geometric inflation.
1 Introduction

Non-linearly realized supersymmetry of the Volkov-Akulov type [1] plays an important role in cosmological model building with de Sitter vacua/dark energy and near de Sitter inflationary evolution of the universe. A closely related to Volkov-Akulov supersymmetry is an approach based on a nilpotent superfield [2–6]. Various more general constrained superfields were studied in [7–9].

A significant interest to these formal developments in non-linearly realized supersymmetry started in applications to cosmology in [10–12] when it became clear that certain difficulties in construction cosmological models with supersymmetric embedding can be solved with the help of constrained superfields. The new supergravity actions were constructed which involve a supergravity multiplet coupled to a nilpotent chiral multiplet $S^2 = 0$. They have natural de Sitter vacua [13, 14]. In string theory the role of the $D3$ brane is analogous to that of the nilpotent chiral multiplet $S^2 = 0$ in supergravity [15].

Many new and interesting properties of supersymmetric theories with constrained multiplets were studied during the last few years, see for example [16–21]. For our work here we will focus on the case when there is one nilpotent chiral superfield $S$

$$S^2 = 0$$

and one orthogonal superfield $\Phi$, which has a nilpotency degree 3,

$$S(\Phi - \bar{\Phi}) = 0, \quad (\Phi - \bar{\Phi})^k = 0, \quad k \geq 3.$$  

The orthogonal multiplet was first introduced in [7] and studied in global supersymmetry in [8] and in local supersymmetry in [16, 18, 19]. It was employed in the cosmological setting.
in [16, 22] where it was recognized that the corresponding cosmological models are very simple, the inflatino and the sinflaton both absent. A string theory origin of constrained multiplets in the context of a non-linearly realized spontaneously broken supersymmetry of the $\overline{D3}$-brane action in type IIB string theory was investigated in [23, 24].

At the time when the cosmological models in [16, 22] were studied, the most advanced form of the $\overline{D3}$ induced geometric inflation [25, 26] was not known. It was therefore necessary in the description of the cosmological models to provide two holomorphic functions in the superpotential, $W = g(\Phi) + f(\Phi)S$ with the potential of the form $V = f^2(\varphi) - 3g^2(\varphi)$. The advantage of the geometric inflation [25, 26] is that the basic information about the model is codified in the Kähler geometry of the nilpotent superfield $S$, in $K_{SS}(\Phi, \bar{\Phi})$ as follows.

$$K_{SS}(\Phi, \bar{\Phi}) = \frac{W_0^2}{3W_0^2 + V(\Phi, \bar{\Phi})} \tag{1.3}$$

and the superpotential can be a constant. These models are significantly simpler than the ones in [16, 22] and lead directly to desired potentials.

However, it was pointed out in [27, 28] that there might be a physics issue with the cosmological models using constrained multiplets. It was at the time when there was a hope that LHC might discover supersymmetric particles and one would expect that the gravitino is light. In such case it was pointed out in [27, 28] that the models with orthogonal nilpotent multiplet in [16, 22] as well the geometric models in [25, 26] might suffer from gravitino overproduction after inflation. At present there is no compelling reason to assume that gravitino is light, since the superpartners of standard model particles have not been discovered so far. Therefore, assuming that gravitino is not light, we may safely study the cosmological models with constrained superfields, as well as the geometric models of inflation with the purpose to establish a simple version of these supersymmetric cosmological models.

Recent cosmological observations [29] suggest that the $\alpha$-attractor models of inflation [26, 30] remain at the sweet spot of the data. Therefore the purpose of this note is to present the simplest possible supergravity version of $\alpha$-attractor models of inflation. For this purpose we will combine the advantages of the orthogonal nilpotent superfield with the geometric model of inflation. In particular, we will define our new supergravity theories with one nilpotent multiplet $S^2 = 0$ and an inflaton orthogonal multiplet with $S(\Phi - \bar{\Phi}) = 0$. We will present for these models the Kähler function $G$, and also the Kähler potential $K$ and the superpotential $W$, in agreement with the definition of these functions

$$G \equiv K + \ln W + \ln \overline{W}, \tag{1.4}$$

and

$$V = e^G (G^\alpha \overline{G}_\alpha G - 3) = e^K (K^\alpha \overline{D}_\alpha W D \overline{W} - 3W \overline{W}). \tag{1.5}$$

## 2 Orthogonal nilpotent superfield in supergravity

We describe the constrained chiral superfields in supergravity following [18], [19], and we also use the notations of these papers.
Consider two unconstrained chiral multiplets, $S$ and $\Phi$ which have the following independent components each: a holomorphic scalar, a left-handed spinor and an auxiliary field:

\[
S: \{ s_r + is_i, P_L \Omega^s, F^s \}, \quad (2.1)
\]
\[
\Phi: \{ \varphi + ib, P_L \Omega^\phi, F^\phi \}. \quad (2.2)
\]

The first superfield $S$ in the context of our cosmological models is often called a stabilizer superfield. Its first component $s = s_r + is_i$ is associated with sgoldstino, the second one, a spinor $\Omega^s$ is a goldstino, and $F^s$ is an auxiliary field of the goldstino multiplet.

The second multiplet $\Phi$ in the context of our cosmological models is an inflaton multiplet. The first component, $\varphi + ib$ has an inflaton $\varphi$ and a sinflaton $b$, the second component is an inflatino, a spinor $\Omega^\phi$, and finally $F^\phi$ is an auxiliary field of the inflaton multiplet.

We impose the conditions that $S$ is a nilpotent chiral superfield defined in eq. (1.1) and that $\Phi$ is an orthogonal real nilpotent superfield defined in eq. (1.2).

Using the supergravity multiplet calculus defined for the constrained superfields in [18] one finds from $S^2 = 0$ that the scalar component of the nilpotent chiral multiplet $S$ is a bilinear combination of the fermions in the same multiplet, under condition that the auxiliary field $F^s$ is not vanishing

\[
s \equiv s_r + is_i = \frac{\overline{\Omega}^s P_L \Omega^s}{2F^s}, \quad F^s \neq 0. \quad (2.3)
\]

Orthogonality condition $SB = 0$, where $B = \frac{1}{2i}(\Phi - \bar{\Phi})$, can be resolved and one finds as shown in [18], [19] that all component fields of the inflaton multiplet $b, \Omega^\phi, F^\phi$ with the exception of the inflaton field $\varphi$ are not independent anymore. Namely, $b, \Omega^\phi, F^\phi$ depend on $\Omega^s$ and on $F^s$ and on the inflaton field $\varphi$. We present explicit solutions for $b, \Omega^\phi, F^\phi$, as functions of $\Omega^s, F^s$ and $\varphi$, in Appendix B.

3 Locally supersymmetric action

To present a complete locally supersymmetric action for a given Kähler function $G(S, \Phi, \bar{S}, \bar{\Phi})$, or a Kähler potential $K(S, \Phi, \bar{S}, \bar{\Phi})$ and a superpotential $W(S, \Phi)$ where the superfields are constrained, one has to use the the rules in [13, 14, 16, 31–35]. For example the kinetic terms require to differentiate the Kähler function $G(S, \Phi, \bar{S}, \bar{\Phi})$ before any constraints on the superfields are imposed, to construct the complete locally supersymmetric action for a given Kähler potential and a superpotential. Once the full action is established, and all kinetic terms and the potential are available, one can impose the constraints $S^2 = 0, SB = 0$ and their consequences which show that the superfields’ components are not independent.

There are two possibilities to proceed in the context of the cosmological applications:

1. One can focus only on the bosonic action of a locally supersymmetric supergravity action and neglect all terms with fermions.

\[
\Omega^s = \Omega^\phi = \psi_\mu = 0. \quad (3.1)
\]
In such case, we can deduce already from eqs. (2.3), (B.1), (B.2), (B.3) that the bosonic action depends only on gravity and on the inflaton field. There is no sgoldstino scalar \( s \) and no sinflaton scalar \( b \) anymore since they are functionals of fermions,

\[
\Omega^s = \Omega^\phi = \psi_\mu = 0 \quad \Rightarrow \quad s = b = 0. \tag{3.2}
\]

2. One can use the local supersymmetry and \textit{gauge fix the action in the unitary gauge} \cite{16, 31} where gravitino is massive and sgoldstino is absent

\[
\Omega^s = 0. \tag{3.3}
\]

It follows\(^1\) that in this gauge that the sgoldstino \( s \), the sinflaton \( b \), the inflatino \( \Omega^\phi \) and the auxiliary field \( F^\phi \) all vanish,

\[
\Omega^s = 0, \quad \Rightarrow \quad s = b = \Omega^\phi = F^\phi = 0 \tag{3.4}
\]

as one can see from equations (2.3), (B.1), (B.2), (B.3). In this gauge therefore

\[
S|_{\text{unitary}} : \{0, 0, F^s\}, \tag{3.5}
\]

\[
\Phi|_{\text{unitary}} : \{\varphi, 0, 0\}, \tag{3.6}
\]

i. e. the nilpotent multiplet \( S \) has only a non-vanishing auxiliary field whereas the inflat on multiplet \( \Phi \) has only a non-vanishing real scalar, an inflat on field.

This is the reason why we find that the cosmological \( \alpha \)-attractor supergravity models with the orthogonal inflat on multiplet are particularly simple. Note, however, that we cannot use the constraints on superfields in the Kähler function \( G(S, \Phi, \bar{S}, \bar{\Phi}) \), or in a Kähler potential \( K(S, \Phi, \bar{S}, \bar{\Phi}) \) or in a superpotential \( W(S, \Phi) \). We can use the constraints in the action, where we encounter also the derivatives of these functions over the superfields, for example in kinetic terms for superfields and in the potential.

### 4 Simple T-models with one nilpotent and one orthogonal multiplet

Below we will mostly skip the boldface notation, we will use only for constraints on superfields. We impose the following constraints on T-model superfields

\[
S^2 = 0, \quad S(Z - \bar{Z}) = 0, \quad \Rightarrow \quad \left(Z - \bar{Z}\right)^k = 0, \quad k \geq 3. \tag{4.1}
\]

The models are defined by a Kähler function \( G \) so that the potential is

\[
G = -\frac{1}{2}G_{Z\bar{Z}}(Z - \bar{Z})^2 + G_{SS}S\bar{S} + S + \bar{S} + \ln W_0^2. \tag{4.2}
\]

\(^1\)When the inflat on is not in an orthogonal multiplet, the gauge-fixing \( \Omega^s = 0 \) does not imply \( \Omega^\phi = 0 \), so that gravitino and inflat on actions are complicated, in general. In case the inflat on is in an orthogonal multiplet the inflat on vanishes in the gauge \( \Omega^s = 0 \) due to eq. (B.2).
Here we take the Kähler metric as
\[ G_{Z\bar{Z}} = \frac{3\alpha}{(1 - ZZ)^2}, \quad G_{SS} = \frac{W_0^2}{|F_S|^2 + f(Z\bar{Z})} \] (4.3)

According to Proposal 1. in Sec. 3, the scalar dependent part of the supergravity action with local supersymmetry depends on one real scalar \( Z = \bar{Z} \) since the sinflaton field \( b = \frac{1}{2i}(Z - \bar{Z}) \) is a combination of fermions, as shown in eq. (B.1).

\[ \mathcal{L}_{sc} = \frac{3\alpha}{(1 - ZZ)^2} \partial Z \partial \bar{Z} - V(Z). \] (4.4)

Thus, there is no need here to stabilize the sinflaton scalar, since \( Z - \bar{Z} \) is a product of fermions and drops from the bosonic part of the action. Here the potential for the geometric field is
\[ V(Z) = \Lambda + f(Z^2), \] (4.5)

where
\[ \Lambda \equiv |F_S|^2 - 3W_0^2 \] (4.6)
is a cosmological constant. The same action in terms of a canonical field \( \varphi \), where \( \frac{1}{2}(Z + \bar{Z}) = \tanh \left( \frac{1}{\sqrt{6\alpha}} \varphi \right) \) is
\[ \mathcal{L}_{sc} = \frac{1}{2} (\partial \varphi)^2 - \left( \Lambda + f \left( \tanh^2 \frac{\varphi}{\sqrt{6\alpha}} \right) \right). \] (4.7)

Now, according to proposal 2 in Sec. 3, we perform a gauge-fixing of the total action. In such case we find that since many fields vanish according to (3.4), the total gauge-fixed action consists of a gravity part, gravitino part and a real scalar part
\[ \mathcal{L}_{\text{total\-fixed}} = \mathcal{L}_{\text{gravity}} + \mathcal{L}_{\text{gravitino}} + \mathcal{L}_{sc}. \] (4.8)

In particular, the fermion spectrum has only a massive gravitino and no other fermions, like inflatino or goldstino, and there is only one real scalar besides the graviton in the bosonic sector.

We can take a simple function in the Kähler metric in (4.3)
\[ f(Z\bar{Z}) = m^2 ZZ \] (4.9)
which results in a simplest T-model potential
\[ V_T = \Lambda + m^2 ZZ = \Lambda + m^2 \tanh^2 \frac{\varphi}{\sqrt{6\alpha}}. \] (4.10)

### 4.1 Kähler potential and superpotential

Instead of a Kähler function \( \mathcal{G} \) as shown in eqs. (4.2),(4.3) we can use a Kähler potential and a superpotential for presenting the models above, in agreement with eq. (1.4),
\[ K = -\frac{1}{2} K_{Z\bar{Z}}(Z - \bar{Z})^2 + K_{S\bar{S}} S\bar{S} + (S + \bar{S}), \] (4.11)
where
\[ K_{Z\bar{Z}} = \frac{3\alpha}{(1 - ZZ)^2}, \quad K_{S\bar{S}} = \frac{W_0^2}{|F_S|^2 + f(Z\bar{Z})}, \] (4.12)
and
\[ W = W_0. \] (4.13)

The total action in the unitary gauge \( \Omega^s = 0 \) is very simple
\[ e^{-L} = \frac{1}{2\kappa^2} \left[ R(\omega(e)) - \bar{\psi}_\mu \gamma^{\mu\nu} D_\nu \psi_\rho + \mathcal{L}_{\text{SG, torsion}} \right] + \frac{W_0}{2} \bar{\psi}_\mu \gamma^{\mu\nu} \psi_\nu - \frac{1}{2} (\partial \varphi)^2 - \left( \Lambda + f \left( \tanh \frac{\varphi}{\sqrt{6\alpha}} \right) \right), \] (4.14)
since
\[ e^K \bigg|_{S_2 = 0, S(Z - \bar{Z}) = 0} = 1, \quad K_Z \bigg|_{S_2 = 0, S(Z - \bar{Z}) = 0} = 0. \] (4.15)

5 Simple E-models with one nilpotent and one orthogonal multiplet

We impose the following constraints on E-model superfields
\[ S_2 = 0, \quad S(T - \bar{T}) = 0, \quad \Rightarrow \quad (T - \bar{T})^k = 0, \quad k \geq 3. \] (5.1)

The E-models are defined by the following Kähler function \( \mathcal{G} \)
\[ \mathcal{G} = -\frac{1}{2} \mathcal{G}_{TT}(T - \bar{T})^2 + \mathcal{G}_{SS}S\bar{S} + S + \bar{S} + \ln W_0^2. \] (5.2)

Here the Kähler metric is
\[ \mathcal{G}_{TT} = \frac{3\alpha}{(T + \bar{T})^2}, \quad \mathcal{G}_{SS} = \frac{W_0^2}{|F_S|^2 + f \left( 1 - \frac{T + \bar{T}}{2} \right)}. \] (5.3)
and the potential for the geometric field is
\[ V = \Lambda + f \left( 1 - \frac{T + \bar{T}}{2} \right). \] (5.4)

The potential for the canonical field \( \varphi \), where \( \frac{1}{2}(T + \bar{T}) = e^{-\sqrt{\frac{2}{3\alpha}} \varphi} \) is
\[ V = \Lambda + f \left( 1 - e^{-\sqrt{\frac{2}{3\alpha}} \varphi} \right). \] (5.5)

In particular we can take a simple function
\[ f \left( 1 - \frac{T + \bar{T}}{2} \right) = m^2 \left( 1 - \frac{T + \bar{T}}{2} \right)^2 \] (5.6)
which results in a simplest E-model potential
\[ V_E = \Lambda + m^2 \left( 1 - \frac{T + \bar{T}}{2} \right)^2 = \Lambda + m^2 \left( 1 - e^{-\sqrt{\frac{2}{3\alpha}} \varphi} \right)^2. \] (5.7)
5.1 Kähler potential and superpotential

Instead of a Kähler function $G$ as shown in eq. (4.2) we can use a Kähler potential and a superpotential for presenting the models above, in agreement with eq. (1.4).

$$K = -\frac{1}{2} K_{TT}(T - \bar{T})^2 + K_{SS} S \bar{S} + S + \bar{S},$$

where

$$K_{TT} = \frac{3\alpha}{(T + \bar{T})^2}, \quad K_{SS} = \frac{W_0^2}{|F_S|^2 + f\left(1 - \frac{T + \bar{T}}{2}\right)},$$

$$W = W_0.$$

The total action in the unitary gauge $\Omega^* = 0$ is very simple,

$$e^{-1} \mathcal{L} = \frac{1}{2\kappa^2} \left[ R(\omega(e)) - \bar{\psi}_\mu \gamma^{\mu\rho} D_\rho \psi_\rho + \mathcal{L}_{SG,torsion} \right]$$

$$+ \frac{W_0}{2} \bar{\psi}_\mu \gamma^{\mu\nu} \psi_\nu - \frac{1}{2} (\partial \varphi)^2 - \left(\Lambda + f\left(1 - e^{-\sqrt{2/3}\varphi}\right)\right),$$

since

$$e^K \bigg|_{S^2 = 0, S(T - \bar{T}) = 0} = 1, \quad K_T \bigg|_{S^2 = 0, S(T - \bar{T}) = 0} = 0.$$

6 Summary and discussion

In this paper, we proposed the simplest class of the inflationary $\alpha$-attractors in supergravity. The orthogonal and the cubic nilpotent conditions (1.1), (1.2) not only reduce the number of physical degrees of freedom to one real scalar, the inflaton, but also simplify the Kähler potential describing hyperbolic geometry (4.12), (5.9). As one can see from eqs. (4.11), (4.12) for T-model and from eqs. (5.8), (5.9) for the E-model, the absence of the sinflaton scalar, $Z - \bar{Z} = 0$ and $T - \bar{T} = 0$, keeps the Kähler metric to be the same as the one which follows from the original more complicated Kähler potentials. These are presented in (A.4) and (A.1), respectively. The $D3$ induced geometric inflation [25, 26] further enables us to build the models realizing inflation, supersymmetry breaking and a cosmological constant in the present universe. We applied this formalism to construct the simplest supersymmetric $\alpha$-attractors in Secs. 4 and 5.

One can check that our inputs for the Kähler function in eqs. (4.2), (4.3) for T-models and in (5.2), (5.3) for E-models, lead to $\alpha$-attractor cosmological models with potentials in eqs. (4.5) for T-models and (5.4) for E-models. For this purpose one can either check the derivation in appendix A, or just run the corresponding Mathematica notebook. Either way, one can easily see that there is a relatively simple action with local supersymmetry, which results in data-compatible cosmological potentials for a single scalar field, inflaton.
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A Validation of new simple inflaton Kähler potentials

Consider the first inflaton shift symmetric Kähler potential for E-models [36], [26]:

$$K = -\frac{3\alpha}{2} \log \left( \frac{(T + \bar{T})^2}{4TT} \right). \quad (A.1)$$

Obviously, our new Kähler potential in (5.2), (5.3) for orthogonal inflaton superfield

$$K = -\frac{3\alpha}{2} \frac{(T - \bar{T})^2}{(T + \bar{T})^2} \quad (A.2)$$

is much simpler than (A.1), for example, there are no log’s. We will show here that (A.1) is actually reduced to (A.2) when the cubic nilpotent condition (5.1), $(T - \bar{T})^k = 0\, , k \geq 3$, is applied.

$$K = -\frac{3\alpha}{2} \log \left( \frac{(T + \bar{T})^2}{4TT} \right)$$

$$= -\frac{3\alpha}{2} \log \left( \frac{(T + \bar{T})^2}{(T + \bar{T})^2 - (T - \bar{T})^2} \right)$$

$$= \frac{3\alpha}{2} \log \left( 1 - \frac{(T - \bar{T})^2}{(T + \bar{T})^2} \right)$$

$$= -\frac{3\alpha}{2} \frac{(T - \bar{T})^2}{(T + \bar{T})^2} \quad (A.3)$$

where we have used $(T - \bar{T})^k = 0, k \geq 3$ in the last step. Thus, we have shown the equivalence between the old Kähler potential (A.1) and the simplified one (A.2) for the orthogonal inflaton supermultiplet.

In the disk variables $Z$ we have the first inflaton shift symmetric Kähler potential [36], [26]

$$K = -\frac{3\alpha}{2} \log \left( \frac{(1 - Z\bar{Z})^2}{(1 - Z^2)(1 - \bar{Z}^2)} \right). \quad (A.4)$$

The $T$- and $Z$-variables are related by the Cayley transformations

$$T = \frac{1 + Z}{1 - \bar{Z}}, \quad Z = \frac{T - 1}{T + 1}. \quad (A.5)$$
It is known from [36, 37] that by making a change of variables
\( T = 1 + \frac{Z}{1 - Z} \) one finds that (A.1) becomes (A.4) and \((T - T)^k = 0, k \geq 3\) condition becomes \((Z - \bar{Z})^k = 0, k \geq 3\). It follows that
\[
K = -\frac{3\alpha}{2} \log \left( \frac{(1 - Z\bar{Z})^2}{(1 - Z^2)(1 - \bar{Z}^2)} \right) \bigg|_{(Z - \bar{Z})^k = 0} = -\frac{3\alpha}{2} \frac{(Z - \bar{Z})^2}{(1 - Z\bar{Z})^2}.
\] (A.6)

**B  Details of Orthogonality Conditions in Local Supersymmetry**

Sinflaton \( b \) depends on goldstino \( \Omega^s \), on goldstino \( s = \frac{\varphi \bar{P}_L \Omega^s}{2F^s} \), on \( F^s \) and on the inflaton \( \varphi \) as follows:
\[
b = \frac{i}{4} \left[ \frac{\Omega^s}{F^s} \gamma^\mu P_L \frac{\Omega^s}{F^s} - \frac{s}{F^s} \left( D_\nu \frac{\Omega^s}{F^s} \right) \gamma^\mu_\nu P_L \frac{\Omega^s}{F^s} \right.
\]
\[
- \frac{s \bar{s}}{2F^s} \left( D_\nu \frac{\bar{\Omega}^s}{F^s} \right) \left( \gamma^{\mu_\nu + \nu_\rho} P_L \left( D_\rho \frac{\Omega^s}{F^s} \right) - \text{c.c.} \right) D_\mu \varphi. \] (B.1)

Inflatino depends on goldstino \( \Omega^s \), on \( F^s \), on the sinflaton \( b \), as shown above, and on the inflaton \( \varphi \) as follows:
\[
P_R \Omega^\phi = \left[ \mathcal{D}(\varphi - ib) \right] P_L \frac{\Omega^s}{F^s}. \] (B.2)

Finally, the inflaton multiplet auxiliary field \( F^\phi \) is a functional of goldstino \( \Omega^s \), goldstino \( s = \frac{\varphi \bar{P}_L \Omega^s}{2F^s} \), of \( F^s \), of the sinflaton \( b \), and on the inflaton \( \varphi \),
\[
F^\phi = \left( D_\nu \frac{\bar{\Omega}^s}{F^s} \right) P_L \gamma^\mu_\nu \frac{\Omega^s}{F^s} D_\mu (\varphi - ib) + \frac{s}{F^s} D^2 (\varphi - ib). \] (B.3)

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