Can Higgs inflation be saved with high-scale supersymmetry?

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Abstract It is shown whether Higgs inflation can be saved as high-scale supersymmetry critically depends on the magnitude of the non-minimal coupling constant $\xi$. For small $\xi \leq 500$, the threshold correction at scale $M_P/\xi$ is constrained with high precision. Its magnitude is in the narrow range of $(-0.03, -0.02)$ and $(-0.05, -0.04)$ for the wino and higgsino–singlino dark matter, respectively. In the large-$\xi$ region with $\xi \geq 10^4$, such high-scale supersymmetry is excluded by a too large threshold correction as required by Higgs inflation.

1 Introduction

The collider facilities such as the Large Hadron Collider (LHC) and astrophysical experiments are two main tools for exploring new physics beyond the standard model (SM). Recently, the Planck Collaboration, designed to detect the cosmic microwave background (CMB) temperature anisotropy and polarization, has reported the latest value of the tensor-to-scalar ratio $r \leq 0.11$ at 95% CL [1,2]. This data excludes a few well-known inflation models such as quadratic inflation, but it still allows some simple examples such as Starobinsky-like inflation [3], $\alpha$-attractor inflation [4], and Higgs inflation [5].

Among these survivors, Higgs inflation is rather special due to two considerations. First, there is only one new parameter $\xi$ in this model in the Lagrangian in this model, which reads, as in the Jordan frame,

$$L_j = \frac{M^2 + \xi h^2}{2} - \frac{1}{2} (\partial h)^2 - \frac{\lambda_H}{4} (h^2 - v^2)^2. \tag{1}$$

Here, $\xi$ measures the non-minimal coupling between the Higgs scalar $h$ and gravity. It was found that the Higgs scalar potential $V_E(h)$ in the Einstein frame rapidly approaches a plateau potential in the large field region $h > M_P/\sqrt{\xi}$, and the model predicts that, for an e-folding number $N = 60$, $n_s \simeq 0.970$, $r \simeq 0.0033$, which is in perfect agreement with the Planck 2015 data [1,2]. In terms of the present cosmological data, $M$ in Eq. (1) approximates the reduced Planck mass $M_p = 2.4 \times 10^{18}$ GeV for our present universe. Moreover, unlike Starobinsky-like inflation or $\alpha$-attractor inflation, it is obvious that Higgs inflation is of special interest from the viewpoint of particle physics.

Unfortunately, there are two arguments against Higgs inflation. The first shows that the SM is not valid above the scale $M_P/\xi$ [6–8]. It implies that the plateau potential $V(h)$ at large field value, $h > M_P/\sqrt{\xi}$, may be significantly modified by the ultra-violet completion so that realistic inflation driven by the Higgs field cannot occur at all. It is proposed in [9] that a scale symmetry in the ultra-violet completion may be the prescription to this problem. The second argument arises because of the discovery of the Higgs mass $m_h = 125.5 \pm 0.5$ GeV [10–12], which implies that the SM Higgs quartic coupling $\lambda_H$ at the weak scale is not large enough, so that $\lambda_H < 0$ above the high-energy scale of $10^9–10^{11}$ GeV. The uncertainty about this critical scale mainly arises from the uncertainty of the top quark mass, as the renormalization group equation (RGE) for $\lambda_H$ is very sensitive to the top Yukawa coupling. Obviously, a positive $\lambda_H$ is required during the inflationary epoch. This problem can be solved through introducing some new fields into the SM, which give rise to either a correct threshold correction to $\lambda_H$ or slowing down the RG evaluation for $\lambda_H$ along the high energy scale; see, e.g., [13,14].

In this letter, we consider saving the Higgs inflation with high-scale supersymmetry (SUSY). High-scale SUSY can provide the observed Higgs mass at the LHC and a stable DM candidate [15]. However, the detection of such models at colliders is unpromising. In this sense the astrophysical probe is an important direction. Fortunately, they can be probed directly or indirectly in the light of the cosmological observations on the early universe [16,17].

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We identify the cut-off scale $M_P/\xi$ as the typical mass scale $\tilde{m}$ in the SUSY mass spectrum. Above the scale $\tilde{m} = M_P/\xi$ supergravity is the natural ultra-violet (UV) completion, and as regards the matter content the same as the next-to-minimal SUSY model (NMSSM); it may maintain the plateau potential [18]. For earlier discussions, see [19–21]. On the other hand, the SUSY dark matter (DM), whose mass is around the weak scale, is the natural choice on the new fields added to the SM below the cut-off scale $\tilde{m}$. In this paper, we consider two possibilities [15], where the wino $\tilde{w}$ and the higgsino–singlino mixing state serves as the DM, respectively.

The paper is organized as follows. In the next section, we discuss the RGE for $\lambda_H$ in two classes of high-scale SUSY below the scale $\tilde{m}$. In Sect. 3, we discuss embedding Higgs inflation into supergravity, where we uncover the constraint on the parameters $\lambda_H$ and $\xi$ at the end of inflation $h_{\text{end}} = M_P/\sqrt{\xi}$, arising from the present cosmological data. In Sect. 4, we discuss the constraint on the SUSY mass spectrum in terms of the constraint on the threshold correction to $\lambda_H$. Finally, we discuss our results in Sect. 5.

2 RGE for $\lambda_H$ below scale $M_P/\xi$

Below the scale $\tilde{m}$, the effective theory is described by the SM together with either wino-like or higgsino–singlino-like DM. In this section we use the 2-loop RGEs for relevant couplings for our analysis. We refer the reader to the appendix in our previous work [15], where the 2-loop RGEs for SM gauge and Yukawa couplings, and Higgs quartic coupling in models of SM+wino and SM+higgsino–singlino are explicitly shown. We ignore other SM Yukawa couplings in the following discussion. For the calculation of beta functions, see the references therein.

Figure 1 shows the RG running for $\lambda_H$ in high-scale SUSY discussed in this letter, which is modified by the SUSY DM (wino or higgsino–singlino mixing state) with the mass of the weak scale being compared with the SM. In this figure, black, blue, and green curves corresponds to the SM, SM+wino, and SM+higgsino–singlino, respectively. The solid lines refer to the central value $m_t = 173.1$ GeV, $m_h = 125.5$ GeV, and $\alpha_s(m_Z) = 0.1184$, while the dotted lines show the uncertainty due to the top quark mass with $1\sigma$ deviation. From Fig. 1 we observe that a more positive $\lambda_H$ is obtained in high-scale SUSY with wino DM for a smaller top quark pole mass $m_t = 172.2$ GeV along the RG running to the scale $M_P/\xi$. This is mainly due to the correction to the SM beta function $\beta_{\lambda_H}$ induced by the wino DM.

Unlike the case with the wino DM, the sign of the correction induced by the higgsino–singlino DM is only positive when the model parameter $g_3$ is small. Otherwise, for large $g_3 \geq 0.4$ the sign of the correction will reverse and $\lambda_H$ will become negative more rapidly along the RG running to scale $M_P/\xi$. This implies that the correction in this case is upper bounded.

Figure 2 shows the sensitivity of the RG running for $\lambda_H$ to the Higgs mass. One finds that for about $\sim 0.5$ GeV deviation to the central value $m_h = 125.5$ GeV the correction in each case is tiny. Among the three models, $\lambda_H$ approaches zero mostly in the case with wino DM, similarly to what Fig. 1 indicates.

In summary, for about $1\sigma$ deviation to the central value of top quark mass, $\lambda_H$ is tuned to be more positive in high-scale SUSY with the low-energy theory below the scale $M_P/\xi$ described by either SM+wino or SM+higgsino–singlino DM being compared with the SM. As we will see in the next section, a more positive $\lambda_H$ at scale $M_P/\xi$ is favored by Higgs inflation. This is so because the threshold correction at the scale $M_P/\xi$ may be unable to tune $\lambda_H$ into a positive parameter above this scale as required by Higgs inflation.
3 Embedding Higgs inflation into supergravity

We proceed to a discussion of the constraint arising from the condition of a plateau potential for the Higgs scalar in the context of supergravity, which is automatically the ultra-violet completion. For this purpose, we focus on the scalar-gravity part of the supergravity Lagrangian, which is defined by the frame function \( \Omega(z_i, \bar{z}_i) \), the Kähler potential \( K(z_i, \bar{z}_i) \) and the superpotential \( W(z_i, \bar{z}_i) \). We use \( z_i \) to label the chiral superfields including the two Higgs doublets \( H_u, d \).

We follow the notation and conventions in [18], where the Kähler potential and frame function are given in units of the Planck mass by

\[
K(z_i, \bar{z}_i) = -3 \log(\Omega),
\]

\[
\Omega(z_i, \bar{z}_i) = 1 - \frac{1}{3} \delta_{ij} z^i z^j + \ldots
\]

(3)

Given the explicit form of these functions, the scalar potential in the Einstein frame is directly derived from the well-known formula\(^1\)

\[
V_E = e^K(D_i W K^{ij} D_j \bar{W} - 3 W \bar{W}) + V_E^{(D)},
\]

(4)

where \( D_i W = \partial_i W + K_i W \), \( V_E^{(D)} \) represents the D-term contribution.

The first attempt to embed Higgs inflation into supergravity was shown in [20]. In the light of [20] there are two important observations. (1) The supergravity version of Eq. (1) requires a holomorphic function \( X = -\frac{1}{2} \chi H_u H_d + h.c \) in order to reproduce the \( \xi \)-term. \( \chi \) is a dimensionless coupling constant. (2) The matter content of MSSM is not viable for the Higgs inflation. By following this line, the authors in [18] proposed that the matter context of NMSSM is a realistic choice, in which the potential in the NMSSM depends on three complex superfields,

\[
z_i = (S, H_u^0, H_d^0) = \left\{ \sin^{\alpha_1 \alpha_2} / \sqrt{2}, h \cos \beta e^{i \alpha_1} / \sqrt{2}, h \sin \beta e^{i \alpha_2} / \sqrt{2} \right\},
\]

(5)

as long as the scalar \( s \) in the singlet superfield \( S \) can be stabilized at \( s = 0 \) and the D-flat condition \( \beta = \pi/4 \) is satisfied. This idea was first considered in [20], but a new \( \xi \)-term should be added to the frame function [18] for stabilizing the \( s \) scalar.

In summary, the frame function and superpotential for the purpose of Higgs inflation are given by, respectively,

\[
\Omega(z_i, \bar{z}_i) = 1 - \frac{1}{3} (|H_u^0|^2 + |H_d^0|^2 + |S|^2)
\]

\[
+ \frac{\xi}{3} |S|^4 - \left( \frac{1}{2} H_u^0 H_d^0 + h.c \right),
\]

\[
W(z_i, \bar{z}_i) = \lambda S H_u^0 H_d^0 + \frac{\rho}{3} S^3.
\]

(6)

Substituting Eq. (6) into Eq. (4) gives rise to the potential

\[
V_E = V_E^{(F)} = \frac{9\lambda^2 h^4}{(3\chi h^2 - 2h^2 + 6)^2},
\]

(7)

for \( s = 0 \) and \( \beta = \pi/4 \). Here we have used \( W |_{s=0} = 0 \). For more details as regards the stabilization of \( s \) and angles in Eq. (5), see [18,21]. In the region \( \chi h^2 \gg 1 \gg h^2 \), Eq. (7) approaches \( (\lambda/\chi)^2 \) (in units of the Planck mass), which verifies our statements above. Moreover, in terms of Eq. (7) one can also verify Eq. (2).

The parameters \( \xi \) and \( \lambda_H \) in Eq. (1) are related to the parameters \( \chi \) and \( \lambda \) in Eq. (6) as

\[
\xi = -\frac{1}{6} + \frac{1}{4} \chi, \quad \lambda_H(\mu_i) = \frac{\lambda^2}{4}.
\]

(8)

Here \( \mu_i \) denotes the RG scale corresponding to inflation. They are constrained by the present cosmological data as follows. Recall that \( V_E^{1/4} = (24\pi^2 M_P^4 \epsilon A_s)^{1/4} \), where \( A_s \) is the amplitude of the power spectrum of the curvature perturbation and \( \epsilon = r/16 \) in the context of single field inflation. For \( A_s^{1/2} \simeq 3.089 \times 10^{-5} \) as reported by Plank Collaboration [22] one obtains

\[
\xi \simeq 58789/\lambda_H(\mu_i).
\]

(9)

4 Constraints on SUSY mass spectrum

Until now, we have obtained the value of \( \lambda_H \) below the RG scale \( \mu = M_P/\xi \) (as shown in Figs. 1 and 2) and above the RG scale \( M_P/\sqrt{\xi} \) (as shown in Eq. (9)) given a choice on \( \xi \). The effective theory at the intermediate scale between scale \( M_P/\xi \) and \( M_P/\sqrt{\xi} \) is described by the SM together with gauginos and squarks. Other SUSY particles such as higgsinos and charged Higgs particles have masses of order \( M_P/\sqrt{\xi} \) [18,21], so they should be integrated out at this intermediate energy scale [25], especially for the discussion of the RGE for \( \lambda_H \).

The threshold correction \( \delta \lambda_H \) at scale \( \tilde{m} = M_P/\xi \) arising from integrating out gauginos and squarks can be determined in terms of the differences between the values of \( \lambda_H \) below and above the scale \( \tilde{m} = M_P/\xi \),

\[
\lambda_H(\tilde{m} - \epsilon) = \lambda_H(\tilde{m} + \epsilon) + \delta \lambda_H(m_{g_1}, m_{g_3}, m_{\tilde{g}_1}),
\]

(10)

where \( 0 < \epsilon << 1 \), and \( m_{g_1}, m_{g_3}, \) and \( m_{\tilde{g}_1} \) refer to the gluino mass, bino mass, and squark mass, respectively. As we will see below, the threshold correction \( \delta \lambda_H(m_{g_1}, m_{g_3}, m_{\tilde{g}_1}) \) can

\[ \end{quote}
be measured at high precision, so it is a new and useful factor to constrain the GUT-scale SUSY mass spectrum.

The value of $\lambda_H(M_p/\xi + \epsilon)$ in Eq. (10) is determined by the RGE for $\delta\beta_{\lambda H}$ in the effective theory at the intermediate scale, with the boundary value at the end of inflation

$$\lambda_H\left(M_p/\sqrt{\xi}\right) = \lambda_H(\mu_I) + \delta\lambda_H(m_{\tilde{u},d}, m_b).$$ (11)

Here, we have ignored the effects due to higgsino-induced operators with mass dimension higher than four, which are at least one order of magnitude smaller than the threshold correction in our case.\(^2\)

The threshold correction $\delta\lambda_H(m_{\tilde{h},a}, m_b)$ in Eq. (11) arises from the heavy higgsinos and singlet $s$, which are given by, respectively [24],

$$\delta\lambda_H |_{\tilde{u}} \simeq -\frac{1}{6} \cos^2(2\beta)\left(\frac{9}{25}g_1^2 + g_2^2\right)\ln\left(\frac{\mu^2}{m^2}\right),$$

$$\delta\lambda_H |_b \simeq 2\lambda_H(\mu_I)\sin^2(2\beta),$$ (12)

where we have assumed that the A-term $A_\lambda$ is smaller than $m_s$. Substituting the stabilized value $\beta = \pi/4$ into Eq. (12) we find that $\lambda_H(M_p/\sqrt{\xi}) \simeq \frac{1}{2}\lambda_H(\mu_I)$. As long as the mass scale $m_\beta$, which is of order $M_p/\sqrt{\xi}$, is far larger than the Hubble parameter $H$ and the reheating temperature after inflation [23], $\beta$ rapidly approaches the stabilized value during and after inflation.

The matter content of the effective theory at the intermediate scale is composed of SM fermions, squarks, gauginos, and electrically neutral scalars in the Higgs sector. The beta function coefficients $b_1$ for the SM gauge coupling in this effective theory are given by, respectively,

$$b_1 = \frac{n}{2}, \quad b_2 = -\frac{3}{2} + \frac{n}{2}, \quad b_3 = -\frac{9}{4} + \frac{n}{2},$$ (13)

where $n = 3$ is the number of SM fermion generations. In terms of Eq. (13) and one-loop RGEs for $\lambda_H$ and SM top Yukawa coupling we show in Fig. 3 the constraint on the magnitude of $\delta\lambda_H(m_{\tilde{g}_1}, m_{\tilde{g}_2}, m_{\tilde{q}_\mu})$ for the range $300 \leq \xi \leq 50000$. It is shown that the value for $\delta\lambda_H(m_{\tilde{g}_1}, m_{\tilde{g}_2}, m_{\tilde{q}_\mu})$ as required by Higgs inflation is very sensitive to the parameter $\xi$.

In the light of Fig. 3 we conclude that for small non-minimal coupling, $\xi \leq 500$, the threshold correction at scale $M_p/\xi$ induced by Higgs inflation is about $-0.03 \sim -0.02$ and $-0.05 \sim -0.04$ in wino and higgsino–singlino DM, respectively. The uncertainties mainly arise from the uncertainty of the top quark pole mass. In the large $\xi$ region with $\xi \geq 1 \times 10^4$, the required threshold correction is $\leq -0.1$, which is too large to exclude such models.

\(^2\) Operators of type $c_{HGG} | H |^2 F_{\mu\nu}F^{\mu\nu}/M^2$ with mass dimension six, as induced by heavy SUSY particles with mass $M$, contribute to the leading corrections to the RGEs for SM EW gauge coupling [26, 27]. As a result, they modify the RGE for $\lambda_H$ indirectly, the significance of which is determined by the modified beta function with SM gauge coupling, $16\pi^2\delta\beta_{\lambda H} \sim c_{HGG}(m_{\tilde{g}_1}^2\lambda_{\tilde{g}_1}/M^2)$. For the three stages we set in this paper, the modification to the RGE for $\lambda_H$ during inflation is maximal, with the heavy SUSY particle identified as $\tilde{H}_{u,d}$. In contrast, the modification after inflation is only mild. In our case, $(m_{\tilde{g}_1}^2)_{\text{eff}} \leq H$ and $M \sim M_p$, which implies that this modification is smaller than the threshold correction $\delta\lambda_H(m_{\tilde{h},a}, m_b)$.
\(\frac{M_\phi}{\xi}\) with vanishing \(A_t\) terms. For large \(\xi \sim 10^4\), the required threshold correction is smaller than \(\simeq -0.1\), which excludes such high-scale SUSY. Finally, we believe our analysis can be applied to Split SUSY, although this model is more complicated in comparison with what have been discussed here.

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