1 Introduction

This lecture is concerned with the structure of hadrons at low energies, where the strong coupling constant is large. Most of the hadronic world discussed here will be made up of the light u, d (and s) quarks since these are the constituents of the low-lying hadrons. The best way to gain information about the strongly interacting particles is the use of well-understood probes, such as the photon or the massive weak gauge bosons. At very low energies, the dynamics of the strong interactions is governed by constraints from chiral symmetry. This leads to the use of effective field theory methods which in the present context is called baryon chiral perturbation theory. In this lecture, I will briefly outline the basic framework of this effective field theory and use nucleon Compton and pion–nucleon scattering to discuss the strengths and limitations of it. The basic degrees of freedom are the pseudoscalar Goldstone bosons chirally coupled to the matter fields like e.g. the nucleons. The very low-energy face of the low-lying baryons is therefore of hadronic nature, essentially point-like Dirac particles surrounded by a cloud of Goldstone bosons. Naturally, I can only cover a small fraction of the many interesting phenomena related to low energy hadron physics. I have chosen to mostly talk about the nucleon since after all it makes up large chunks of the stable matter surrounding us and also is a good intermediary between the nuclear and the high energy physicists present at this workshop. Most of the methods presented here can easily be applied to other problems, and as it will become obvious at many places, we still have a long way to go to understand all the intriguing features of the nucleon in a systematic and controled fashion. Whenever possible, I will avoid to talk about models, with the exception of some circumstances where they can be used to estimate some of the low–energy constants entering the chiral perturbation theory machinery. In fact, I will consider one of these constants and discuss to what extent we can understand its numerical value from the so–called resonance exchange saturation picture. Further aspects of nucleon structure related to photo- and electropionproduction within the framework of CHPT are discussed in V. Bernard’s lecture [1].
2 Chiral Perturbation Theory with Nucleons

The interactions of the strongly interacting particles at low energies are severely constrained by the approximate chiral symmetry of the QCD Lagrangian. This is particularly evident for the pseudoscalar Goldstone bosons which are directly related to the spontaneous symmetry violation. In this section I will be concerned with the inclusion the low-lying spin-1/2 baryons (the nucleons) to the effective field theory. I will consider the two flavor case and mostly work in the isospin limit \( m_u = m_d = \hat{m} \). For a more detailed account, I refer to A. Manohar’s lecture [2]. The inclusion of such matter fields is less straightforward since these particles are not related to the symmetry violation. However, their interactions with the Goldstone bosons is dictated by chiral symmetry. Let us denote by \( \Psi \) the isospinor doublet including the neutron and the proton. It is most convenient to choose a non-linear realization of the chiral symmetry so that

\[
\Psi \rightarrow K \Psi
\]

under \( SU(2)_L \times SU(2)_R \), where \( K \) is a complicated function that does not only depend on the group elements \( g_{L,R} \) of the \( SU(2)_L \times SU(2)_R \) but also on the Goldstone boson fields collected in \( U(x) \), i.e. \( K(x) = K(g_L, g_R, U(x)) \) defines a local transformation. Expanding \( K \) in powers of the Goldstone boson fields, one realizes that a chiral transformation is linked to absorption or emission of pions (which was the theme in the days of "current algebra" techniques). Let us restrict the discussion to processes with one incoming and one outgoing baryon, such as \( \pi N \) scattering, pion photo- and electroproduction or nucleon Compton scattering (otherwise, we would have to add contact \( n \)-fermion terms with \( n \geq 4 \)). In that case, the underlying effective Lagrangian formulated in terms of the asymptotically observed fields takes the form

\[
\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi} \left( i \gamma^\mu D_\mu - m + \frac{1}{2} g_A \gamma^\mu \gamma_5 u_\mu \right) \Psi
\]

with \( m \) the nucleon mass (in the chiral limit), \( u_\mu = i u^\dagger \nabla_\mu U u^\dagger \), \( u = \sqrt{U} \), and \( D_\mu (\nabla_\mu) \) the chiral covariant derivative acting on the nucleons (pions). Finally, \( g_A \) is the axial-vector coupling constant measured in neutron \( \beta \)-decay, \( g_A = 1.26 \). Notice that the lowest order effective Lagrangian contains one derivative and therefore is of dimension one as indicated by the superscript \( '1' \). In contrast to the meson sector, \( \mathcal{L}_{\pi \pi}^{(2,4,...)} \), odd powers of the small momentum \( q \) are allowed (thus, to leading order, no quark mass insertion appears since \( \hat{m} \sim q^2 \)). It is instructive to expand (2.1) in powers of the Goldstone and external fields. From the vectorial term, one gets the minimal photon-baryon coupling, the two-Goldstone seagull (Weinberg term) and many others. Expansion of the axial-vectors leads to the pseudovector meson-baryon coupling, the celebrated Kroll-Rudermann term and much more. Calculating tree diagrams based on (2.1) leads to the current algebra results. This is, however, not sufficient. First, tree diagrams are always real (i.e. unitarity is violated) and second, the Goldstone nature of the pions can lead to large (non-analytic) corrections. Therefore, one has to include loop diagrams making use of the chiral power counting first spelled out by Weinberg [3] for the meson sector. In the presence of baryons, the loop expansion is more complicated. First, since odd powers in \( q \) are allowed, a one-loop calculation of order \( q^3 \) involves contact terms of dimension two and three, i.e. combinations of zero or one quark mass insertions with zero to three derivatives. These terms are
collected in $\mathcal{L}_{\pi N}^{(2,3)}$ and a complete list of them can be found in Krause’s paper [4] (for the case of SU(3)). Second, the finiteness of the nucleon mass in the chiral limit and the fact that its value is comparable to the chiral symmetry breaking scale $\Lambda \sim M_{\rho}$ complicates the low energy structure. This has been discussed in detail by Gasser et al. [5]. Let me just give one illustrative example. The one loop contribution to the nucleon mass not only gives the celebrated non-analytic contribution proportional to $M_{\Sigma}^3 \sim \hat{m}^{3/2}$ but also an infinite shift of $m$ which has to be compensated by a counterterm of dimension zero. It is a general feature that loops produce analytic contributions at orders below what one would naively expect (e.g. below $q^3$ from one loop diagrams). Therefore, in a CHPT calculation involving baryons one has to worry more about higher order contributions than it is the case in the meson sector. There is one way of curing this problem, namely to go into the extreme non-relativistic limit [6] and consider the baryons as very heavy (static) sources. Then, by a clever definition of velocity-dependent fields, one can eliminate the baryon mass term from the lowest order effective Lagrangian and expand all interaction vertices and baryon propagators in increasing powers of $1/m$. To be specific, one writes (I follow here ref.[7])

$$\Psi(x) = \exp[-imv \cdot x](H(x) + h(x)) \quad (2.2)$$

where $H(x)$ and $h(x)$ are velocity–eigenstates (remember that a non–relativistic nucleon has a good four–velocity $v_\mu$) and then eliminates the "small" component $h(x)$. This is similar to a Foldy-Wouthuysen transformation known from QED. The lowest order effective Lagrangian takes the form

$$\mathcal{L}_{\pi N}^{(1)} = \bar{H}(iv^\mu D_\mu + g_A S^\mu u_\mu)H \quad (2.3)$$

with $S_\mu$ the covariant spin–vector (à la Pauli-Lubanski). In this limit one recovers a consistent derivative expansion since the troublesome mass term has been shifted into a string of interaction vertices. A lucid discussion of the chiral counting rules in the presence of heavy baryons can be found in ref.[8]. For example, the one loop contribution of the Goldstone bosons to the baryon self-energy is nothing but the non-analytic $M_{\Sigma}^3 (\phi = \pi, K, \eta)$ terms together with three contact terms from $\mathcal{L}_{MB}^{(2)}$ (in SU(3)). However, one has to be somewhat careful still. The essence of the heavy mass formalism is that one works with old-fashioned time-ordered perturbation theory. So one has to watch out for the appearance of possible small energy denominators (infrared singularities). This problem has been addressed by Weinberg [9] in his discussion about the nature of the nuclear forces. The dangerous diagrams are the ones were cutting one pion line (this only concerns pions which are not in the asymptotic in- or out-states) separates the diagram into two disconnected pieces (one therefore speaks of reducible diagrams). These diagrams should be inserted in a Schrödinger equation or a relativistic generalization thereof with the irreducible ones entering as a potential. So the full CHPT machinery is applied to the irreducible diagrams. This should be kept in mind. For the purposes I am discussing, we do not need to worry about these complications. Being aware of them, it is then straightforward to apply baryon CHPT to many nuclear and particle physics problems [10-14]. I will illustrate this on two particular examples in the next sections. Before doing that, however, I would like to stress that most calculations are only in their infancy. It is believed that for a good quantitative description one has to perform systematic calculations to
order $q^4$, i.e. beyond next-to-leading order, as I will discuss in the context of nucleon Compton scattering. A systematic analysis to this order in the chiral expansion is not yet available. In Manohar’s lecture [2,15], an alternative approach of including the low–lying spin-3/2 decuplet in the effective field theory is discussed (based on phenomenological considerations supplemented with some arguments from the large $N_c$ world). In that fashion, one sums up a certain subset of graphs starting at order $q^4$. A critical discussion of this approach can e.g. be found in ref.[16].

3 Nucleon Compton Scattering

Consider low-energy (real) photons scattering off a proton, $\gamma(p_1)+p(p_2) \rightarrow \gamma(p_2)+p(p_1)$ in the gauge $\epsilon_0 = \epsilon'_0 = 0$ (with $\epsilon_\mu$ denoting the polarization vector of the incoming photon). In the cm-system we have $k_0 = k'_0 = \omega$ and the invariant momentum transfer squared is $t = (k - k')^2 = -2\omega^2(1 - \cos \theta)$. The $T$-matrix takes the form

$$T = e^2 \sum_{i=1}^{6} A_i(\omega, t) O_i$$

in the operator basis of ref.[17]. Under crossing ($\omega \rightarrow -\omega$) the $A_{1,2}$ are even and the $A_{3,4,5,6}$ are odd. Furthermore, below the single pion production threshold, $\omega_{\pi \pi} = M_\pi$, the $A_i$ are real. Clearly, the nucleon structure is encoded in these invariant functions. With them at hand, one can readily calculate the differential cross section and polarisation observables like the parallel asymmetry $A_\parallel$ (polarized photons scatter on polarized protons with the proton spin (anti)parallel to the photon direction) or the perpendicular asymmetry $A_\perp$ (with the proton spin perpendicular to the photon direction) (explicit formulae are given in ref.[14]). In forward direction, the scattering amplitude takes the form

$$\frac{1}{4\pi} T(\omega) = f_1(\omega^2) \; \epsilon'^* \cdot \epsilon + i\omega f_2(\omega^2) \; \sigma \cdot (\epsilon'^* \times \epsilon) .$$

The energy expansion of the spin-independent amplitude $f_1(\omega^2)$ reads

$$f_1(\omega^2) = -\frac{e^2 Z^2}{4\pi m} + (\bar{\alpha} + \bar{\beta}) \omega^2 + O(\omega^4)$$

where the first energy-independent term is nothing but the Thomson amplitude mandated by gauge invariance. Therefore, to leading order, the photon only probes some global properties like the mass or electric charge of the spin-1/2 target. At next-to-leading order, the non-perturbative structure is parametrized by two constants, the so-called electric and magnetic polarizabilities. To lowest order, $q^3$, these are given by a few loop diagrams, i.e. they belong to the rare class of observables free of low–energy constants. The lowest order results [18]

$$\bar{\alpha}_p = \bar{\alpha}_n = 10\bar{\beta}_p = 10\bar{\beta}_n = \frac{5e^2 g_A^2}{384\pi^2 F_F^2} \frac{1}{M_\pi} = 13.6 \cdot 10^{-4} \text{fm}^3$$

already describe the two main features of the data, namely that (a) the neutron and the proton behave essentially as (induced) electric dipoles and that (b) $(\bar{\alpha} + \bar{\beta})_p \simeq$
(\bar{\alpha} + \bar{\beta})_{p,n} (see e.g. the contributions by Nathan and Bergstrom \[19\]). A few remarks concerning the results (2.7) are in order. In the chiral limit of vanishing pion mass, \(\bar{\alpha}_{p,n}\) and \(\bar{\beta}_{p,n}\) diverge as \(1/M_\pi\). This is expected since the two photons probe the long-ranged pion cloud, i.e. there is no more Yukawa suppression as in the case for a finite pion mass. Furthermore, a well-known dispersion sum rule relates \((\bar{\alpha} + \bar{\beta})\) to the total nucleon photoabsorption cross section. The latter is, of course, also well-behaved in the chiral limit which at first sight seems to be at variance with the behaviour of the expansion of the scattering amplitude. But be aware that the general form of (2.6) has been derived under the assumption that there is a well defined low-energy limit. However, as has been pointed out by many \[20\], the strong magnetic (M1) \(N\Delta\gamma\) transition leads to a potentially large \(\Delta\) contribution, \(\bar{\beta}_{p,n} \Delta \approx 10\cdot10^{-4}\) fm\(^3\). From the CHPT point of view, such contributions start at order \(q^4\) since they are \(\sim F_{\mu\nu}F^{\mu\nu}\) (with \(F_{\mu\nu}\) the canonical photon field strength tensor which counts as \(q^2\)).

This problem was addressed in a systematic fashion in refs.\[21\], where all terms of \(\mathcal{O}(q^4)\) were considered (not only some as in previous works). These new terms fall into two categories. The first one consists of one loop diagrams with exactly one insertion from \(\mathcal{L}_{\pi N}^{(2)}\). The corresponding low–energy constants \(c_{1,2,3}\) can be estimated from resonance exchange or determined from data on elastic \(\pi N\) scattering (as discussed in section 4 and 5). The second class are genuine new counter terms from \(\mathcal{L}_{\pi N}^{(4)}\), their coefficients could only be estimated making use of the resonance saturation principle (which works well in the meson sector \[22\]). I will come back to this in section 5. The pertinent results for the electromagnetic polarizabilities take the generic form \[21\]

\[
(\bar{\alpha}, \bar{\beta})_{p,n} = C_1\frac{1}{M_\pi} + C_2 \ln M_\pi + C_3
\]  

(2.8)

where the constant \(C_1\) can be read off from eq.(2.7). The loops of order \(q^4\) contribute to the second and third term whereas the large local \(\Delta\) contribution enters prominently in \(C_3\). Including the theoretical uncertainties in estimating the corresponding low–energy constants and also the possible contributions from loops involving strangeness, one arrives at the following theoretical predictions:

\[
\bar{\alpha}_p = 10.5 \pm 2.0, \quad \bar{\alpha}_n = 13.4 \pm 1.5, \quad \bar{\beta}_p = 3.5 \pm 3.6, \quad \bar{\beta}_n = 7.8 \pm 3.6,
\]  

(2.9)

in units of \(10^{-4}\) fm\(^3\). These agree (with the exception of \(\bar{\beta}_n\)) very well with the data. The two main lessons learned from this improved calculation are: (1) The chiral expansion for electric polarizabilities converges quickly and (2) in the case of \(\bar{\beta}_p\), the coefficient \(C_2\) is large so that the \(\ln M_\pi\) term cancels most of the large and positive \(\Delta\) contribution. This is a novel effect which goes in the right direction and shows once more that one has to include all terms at a given order. However, since there are large cancellations in the predictions for the magnetic polarizabilities, one would like to see the result of a \(q^5\) calculation. On the experimental side, it would be of importance to perform independent measurements of the electric and magnetic polarizabilities to (a) test the dispersion sum rule and (b) to lower the uncertainties in the individual polarizabilities (these are considerably larger than the usually quoted ones if one does not impose the constraint from the sum rule).

The spin-dependent amplitude \(f_2(\omega^2)\) has an expansion analogous to (2.6),

\[
f_2(\omega^2) = f_2(0) + \gamma \omega^2 + \mathcal{O}(\omega^4)
\]  

(2.10)
with the Taylor coefficient \( f_2(0) \) given by celebrated LET due to Low, Gell-Mann and Goldberger [23], \( f_2(0) = -(e^2 \kappa^2)/(8\pi m^2) \), with \( \kappa \) denoting the anomalous magnetic moment of the particle the photon scatters off. In CHPT, \( \kappa \) does not appear in the lowest order effective Lagrangian but is given by loops and counter terms from \( \mathcal{L}_{\pi N}^{(2,3)} \) (this is frequently overlooked). The physics of the so-called "spin–dependent" polarizability \( \gamma \) is discussed in some detail in refs.[7,24]. Here, I just want to point out that the LEGS collaboration at Brookhaven intents to measure this interesting nucleon structure constant [25]. Also, in ref.[26] the interesting observation was made that the multipole predictions for the nucleon spin–polarizability and for the so-called Drell–Hearn–Gerasimov sum rule are incompatible. This again points to wards the importance of independent experimental determinations of these quantities.

4 Topics in Pion–Nucleon Scattering

In this section, I will mostly discuss the chiral corrections to the S–wave \( \pi N \) scattering lengths and give some necessary definitions for the following section. Consider first the S–wave scattering of pions off a nucleon at rest in forward direction,

\[
T_{ba} = T^+(\omega)\delta_{ba} + T^-(\omega)ie^{bac}\tau^c \tag{2.11}
\]

with \( a(b) \) the isospin index of the incoming (outgoing) pion and \( \omega = v \cdot q = q_0 \) denotes the pion cms energy. Under crossing, the functions \( T^{\pm} \) behave as \( T^\pm(\omega) = \pm T^\pm(-\omega) \). At threshold, \( \omega_{\text{thr}} = M_\pi \) (remember that I work to lowest order in the \( 1/m \) expansion), the amplitude is given by its scattering lengths,

\[
a^\pm = \frac{1}{4\pi} \frac{1}{1 + \mu} T^\pm(\omega_{\text{thr}}) \tag{2.12}
\]

These are related to the also often used \( a_{1/2} \) and \( a_{3/2} \) via \( a_{1/2} = a^+ + 2a^- \) and \( a_{3/2} = a^+ - a^- \), respectively. For the later discussion, we also need the so-called axial polarizability. For that, consider \( T^+ \) not longer in forward direction and subtract the nucleon born terms (as indicated by the overbar),

\[
\bar{T}^+(\omega, \vec{q}, \vec{q}') = t_0(\omega) + t_1(\omega)\vec{q}' \cdot \vec{q} + \ldots \tag{2.13}
\]

with the kinematics \( \omega = \nu = v \cdot q = v \cdot q' \) and \( t = (q - q')^2 = 2(M_\pi^2 - \omega^2 + \vec{q}' \cdot \vec{q}) \). The axial polarizability is then defined via

\[
\alpha_A = 2c_0^+ \equiv t_1(0) \tag{2.14}
\]

where for completeness I have also given the relation to the low–energy expansion parameter \( c_0^+ \) commonly used in the \( \pi N \) community.

One of the most splendid successes of current algebra in the sixties was Weinberg’s prediction [27] of the S–wave \( \pi N \) scattering lengths,

\[
a^- = \frac{M_\pi}{8\pi F^2}\approx 8.8 \cdot 10^{-2}/M_\pi, \quad a^+ = 0 \tag{2.15}
\]
in good agreement with the data, \( a^- = 9.2 \pm 0.2 \) and \( a^+ = -0.8 \pm 0.4 \) (in units of \( 10^{-2}/M_\pi \)) [28]. I should stress that in view of the confused situation about low–energy \( \pi N \) scattering, these scattering lengths certainly should be assigned much larger uncertainties [29]. For the sake of the argument, I will however stick to the Karlsruhe–Helsinki values [28]. Of course, one has to worry whether the chiral corrections will spoil this remarkable agreement. In ref.[30], this question was addressed. Besides the canonical one–loop diagrams, one has to include three finite contact terms from \( \mathcal{L}_{\pi N}^{(2)} \), which due to crossing contribute to \( T^+ \),

\[
\mathcal{L}_{\pi N}^{(2)} = c_1 \bar{H} H \mathrm{Tr} (\chi_+) + (c_2 - \frac{g^2}{8m}) \bar{H}(v \cdot u)^2 H + c_3 \bar{H} u \cdot u H
\]

but in fact, only the combination \( C = c_2 + c_3 - 2c_1 + \frac{g^2}{8m} \) enters the result for \( a^+ \). The isospin–odd amplitude \( T^- \) has to be renormalized via a combination of four scale–dependent counter terms, \( b^\prime (\lambda) = b_1^\prime (\lambda) + b_2^\prime (\lambda) + b_3^\prime (\lambda) - 2b_4^\prime (\lambda) \) (here, \( \lambda \) is the scale of dimensional regularization) with the corresponding contact terms from \( \mathcal{L}_{\pi N}^{(3)} \) given in [30] together with \( b_4 \bar{H}[\chi_-, v \cdot u]H \) (which was omitted in that paper, the conclusions and numbers, however, remain unchanged). Due to crossing, \( \mathcal{L}_{\pi N}^{(3)} \) terms contribute to \( a^- \). Defining \( L \equiv M_\pi/8\pi F_\pi^2 \), one arrives at

\[
a^- = L \left[ 1 - \mu - \mu^2 (1 + \frac{g^2}{4}) \right] + \frac{L^2 M_\pi}{\pi} \left( 1 - 2 \ln \frac{M_\pi}{\lambda} \right) - 64\pi L^2 M_\pi F_\pi b^\prime (\lambda) + \frac{3}{4} \frac{g^2}{\lambda} L^2 M_\pi
\]

which shows that the exact knowledge of the low–energy constants is much more important for \( a^+ \) than for \( a^- \) because in the latter case their contribution is suppressed with respect to the leading term by two powers of \( M_\pi \). To get a handle on the numerical values for \( c_{1,2,3} \) and \( b_{1,2,3,4} \), the following procedure was used in ref.[30]. While \( c_1 \) is uniquely fixed from the pion–nucleon \( \sigma \)–term [7], the other low–energy constants were estimated from resonance exchange. In this case, one has contributions from the \( \Delta \), the Roper and also from scalar exchange. The quality of this procedure will be discussed in section 5.

Let me first consider the result for the isospin–odd scattering length. One finds

\[
a^- = (8.76 + 0.40) \cdot 10^{-2}/M_\pi = 9.16 \cdot 10^{-2}/M_\pi
\]

which shows that the chiral corrections of order \( M_\pi^2 \) and \( M_\pi^3 \) are small (approximately 5% of the lowest order result) and move the prediction closer to the empirical value. Furthermore, the dependence of this result on the actual value of \( b^\prime (\lambda) \) is very weak. Matters are different for \( a^+ \). Here, the contact terms play a prominent role and the chiral prediction is very sensitive to the choice of certain resonance parameters, one related to the scalar exchange and the other to the \( \Delta \) contribution (for a more detailed discussion, see ref.[30]). Therefore, in the absence of more stringent bounds on these parameters one can only draw the conclusion that the chiral prediction for \( a^+ \) is within the empirical bounds for reasonable values of the resonance parameters. Also, while for \( a^- \) the convergence in \( \mu = M_\pi/m \) is rapid, it is much slower in the case of \( a^+ \). This indicates that one should perform a \( q^4 \) calculation as it was done in case for the nucleon polarizabilities discussed in section 3.
5 Anatomy of a Low–Energy Constant

To get an idea about the quality of the resonance saturation principle used in the previous sections, I will consider here the low–energy constant $c_3$ defined in eq.(2.16). First, however, let me briefly review the underlying idea of estimating low–energy constants from resonance exchange [22]. As the starting point, consider meson resonances $(M = V,A,S,P)$ chirally coupled to the Goldstone fields collected in $U$ and the matter fields $(N)$ plus baryonic excitations $(N^*)$ and integrate out the meson and nucleon resonances

$$\exp i \int dx \mathcal{L}_{\text{eff}}[U,N] = \int [dM][dN^*] \exp i \int dx \mathcal{\tilde{L}}_{\text{eff}}[U,M,N,N^*]$$

so that one is left with a string of higher dimensional operators contributing to $\mathcal{L}_{\text{eff}}[U,N]$ in a manifestly chirally invariant manner and with coefficients given entirely in terms of resonance masses and coupling constants of the resonance fields to the Goldstone bosons. In the meson sector, i.e. considering neither baryons nor their excitations, this scheme works remarkably well,

$$L_i = \sum_{M=V,A,S,P} L_i^M + L_i(\lambda)$$

where the scale–dependent remainder $L_i(\lambda)$ can be neglected if one choses $\lambda$ to take a value in the resonance region (say between 500 MeV and 1 GeV in the meson sector). But what about the baryon sector? To get an idea, I calculate the axial polarizability defined in eq.(2.14). This amounts to the evaluation of 5 finite one loop diagrams (and their crossed partners) plus the contact term contribution proportional to $c_3$,

$$\alpha_A = -\frac{2c_3}{F^2} - \frac{g_A^2 M}{8\pi F^2} \left(\frac{77}{48} + g_A^2\right) = 2.28 \pm 0.10 \text{ MeV}^{-3}$$

using the central value given in [31] but enlarging the uncertainty by a factor 2.5 (as was suggested to me by M. Sainio [32]). This amounts to

$$c_3 = -5.2 \pm 0.2 \text{ GeV}^{-1}.$$  

In the resonance exchange picture, the dominant contributions to $c_3$ stem from intermediate $\Delta$’s and scalar exchange with a small correction from excitation of the Roper resonance. Varying the corresponding couplings within their allowed values leads to

$$c_3^{\text{res}} = c_3^\Delta + c_3^{N^*} + c_3^S$$

$$= (-2.5 \ldots -3.2) + (-0.1 \ldots -0.2) + (-1.0 \ldots -1.6) \text{ GeV}^{-1}$$

```latex
= -3.6 \ldots -5.0 \text{ GeV}^{-1}
```

at the scale $\lambda = m_\Delta$. Comparison of (2.23) with (2.22) reveals that at least for this particular low–energy constant, the resonance exchange saturation principle seems to work. Clearly, a more systematic analysis has to be performed to draw a final conclusion. However, it is also mandatory to get more high precision low–energy data, at present there are just too few of these to determine all low–energy constants up–to–and–including order $q^3$ (or higher) and compare with predictions from resonance exchange.
6 An amazingly accurate QCD Prediction

The structure of the nucleon as probed by the weak charged currents is encoded in two form factors, the axial and pseudoscalar ones,

\[
< N(p')|A_{\mu}^a|N(p) > = \bar{u}(p') \left[ \gamma_{\mu}G_A(t) + \frac{(p'-p)_{\mu}}{2m}G_P(t) \right] \gamma_5 \frac{\tau^a}{2} u(p)
\]  

(2.24)

with \( t = (p'-p)^2 \) and \( A_{\mu}^a = q\gamma_{\mu}\gamma_5(\tau^a/2)q \) the isovector axial current. The axial form factor \( G_A(t) \) is discussed in V. Bernard’s lecture [1] so I will here concentrate on the induced pseudoscalar form factor \( G_P(t) \) as measured e.g. in muon capture, \( \mu^- + p \rightarrow \nu_\mu + n \), i.e. at \( t = -0.88M_\mu^2 \simeq -0.5M_\pi^2 \). One defines the induced pseudoscalar coupling constant \( g_P \) via

\[
\frac{M_\mu}{2m}G_P(t = -0.88M_\mu^2)
\]  

(2.25)

The best empirical determination of \( g_P = 8.7 \pm 1.6 \) [33] is consistent with the PCAC (lowest order) prediction \( g_P^{PCAC} = 8.9 \). It is therefore believed that a measurement of \( g_P \) can test pion pole dominance but not more. However, one can do better in baryon chiral perturbation theory. For that, one simply uses the chiral Ward identity

\[
\partial^\mu[\bar{q}\gamma_{\mu}\gamma_5\frac{\tau^a}{2}q] = \hat{m}\bar{q}\gamma_5\tau^aq
\]  

(2.26)

and sandwiches it between nucleon states. One arrives at [34]

\[
g_P = \frac{2M_\mu g_{\pi N}F_\pi}{M_\pi^2 + 0.88M_\mu^2} - \frac{1}{3}g_A M_\mu mr_A^2 + O(q^2)
\]  

(2.27)

with \( g_{\pi N} = 13.31 \pm 0.34 \) the strong pion–nucleon coupling constant and \( r_A = 0.65 \pm 0.03 \) fm the nucleon axial radius. The relation (2.27) is known since long [35] but its derivation solely based on the chiral Ward identity of QCD is new. The resulting prediction is [34]

\[
g_P = (8.89 \pm 0.23) - (0.45 \pm 0.04) = 8.44 \pm 0.23
\]  

(2.28)

if one adds the uncertainties in quadrature. In fact, the largest uncertainty stems from the much debated value of the pion–nucleon coupling constant. Consequently, if one could measure \( g_P \) within an accuracy of 2% (as it seems to be feasible within present day technology [36]), one could cleanly test the QCD versus the lowest order (PCAC) prediction. In fact, one could turn the argument around and use such an accurate measurement to pin down the allowed range for the strong pion–nucleon coupling constant. Finally, let me make a remark on the form factor at other values of \( t \). The recently published data on \( G_p(t) \) for \( t = -0.07, -0.139 \) and \( -0.179 \) GeV\(^2\) [37] are not accurate enough to cleanly distinguish between the lowest order and the one–loop QCD prediction.
7 Concluding Remarks

The standard model of the strong and electroweak interactions enjoys a spectacular success, particularly at high energies. At low energies, the symmetries help to formulate an effective field theory (EFT) which can be used to perform precise calculations. The relevant degrees of freedom of this EFT called chiral perturbation theory are the pseudo–Goldstone bosons and other hadrons, but not the quarks and gluons. The pions, kaons and etas play a special role in that they are linked directly to the spontaneous symmetry violation QCD is believed to undergo. The exact mechanism of this phase transition which generates the almost massless degrees of freedom is not yet understood. In the effective Lagrangian, a whole string of terms with increasing energy dimension is present, rendering the theory non–renormalizable. This is, however, of no relevance since the EFT is not supposed to be of use at all scales, but in the case at hand for energies below the typical resonance masses (say 1 GeV). The power of chiral perturbation theory stems from the observation that it is a systematic and simultaneous expansion in small momenta (energies) and quark (pion) masses. It is of utmost importance that to a given order one includes all terms demanded by the symmetry requirements. This means that beyond leading order the so–called low–energy constants enter, which are not given by the symmetries. The finite parts of these constants have to be determined from experiment or can be estimated from some principles like resonance exchange saturation. While in the meson sector the machinery exists and is fully operative, calculations in the baryon sector are hampered by the fact that not sufficiently many accurate low–energy data exist to pin down all appearing low–energy constants. However, with the new CW machines and the renewed interest in low–energy domain, we will eventually leave this transitional stage and will achieve a more satisfactory description of the effective pion–nucleon field theory [38]. The extension to the case of three flavors is also only in its infancy since the small parameter $M_K/4\pi F_\pi \simeq 0.4$ is not that small whereas in the two–flavor sector we deal with $M_\pi/4\pi F_\pi \simeq 0.1$. Furthermore, the closeness of the spin-3/2 decuplet has triggered some speculations that one should include these degrees of freedom in the EFT. Again, a systematic investigation of this approach is not yet available, so that at present one can not draw a final conclusion on it. To summarize, let me emphasis that low–energy hadron physics is as interesting as any other field in physics and that exciting times are ahead of us. Many challenging problems, both theoretical and experimental, remain to be solved.

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