Correlation Equations for Condensing Heat Exchangers Based on an Algorithmic Performance-Data Classification

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Abstract. In this work a new set of correlation equations is developed and introduced to accurately describe the thermal performance of compact heat exchangers with possible condensation. The feasible operating conditions for the thermal system correspond to dry-surface, dropwise condensation, and film condensation. Using a prescribed form for each condition, a global regression analysis for the best-fit correlation to experimental data is carried out with a simulated annealing optimization technique. The experimental data were taken from the literature and algorithmically classified into three groups -related to the possible operating conditions- with a previously-introduced Gaussian-mixture-based methodology. Prior to their use in the analysis, the correct data classification was assessed and confirmed via artificial neural networks. Predictions from the correlations obtained for the different conditions are within the uncertainty of the experiments and substantially more accurate than those commonly used.

1. Introduction
Single-phase and condensing heat exchangers are important in a variety of engineering applications, and their design and control require the accurate prediction of their performance. This task is difficult due to complexities arising from geometry and the occurring phenomena which prevent efficient modeling from a first-principles perspective. The usual direction on this problem is to develop nondimensional correlations based upon experimental data of specific prototypes, and use them later to transfer the information to the thermal design engineer.

Correlation equations effectively compress the experimental information about the heat exchanger behavior into two transfer coefficients from which the heat rate can be obtained. The drawback of the compression process, however, is that often the resulting models provide predictive errors much larger than the uncertainty in the data [1]. Recent investigations have found that assumptions for the heat exchanger analysis [2], the multiplicity of parameter sets in the correlation equation [3], and the specific function assumed for the model [4] are among the sources associated with such errors. Another factor that also plays a significant role in prediction errors from correlations is the data classification process.

Correlation equations for condensing heat exchangers are built from experimental data commonly classified by visual procedures into the corresponding physical conditions; e.g., dry- or wet-surface, etc. However, despite important progress in technology and visualization techniques [5], physical processes related to both the latent heat and the alteration of the flow field by
water film or droplets make it very difficult to distinguish the different forms of condensation by direct observation, hence increasing the uncertainty of the measurements. To address this issue, Pacheco-Vega and Avila [6]–[8] recently proposed the use of a pattern-recognition technique based on cluster analysis to successfully identify and classify condensing heat exchanger data.

The current work aims to develop a new set of correlation equations that accurately describe the thermal performance of compact heat exchangers with possible condensation. To this end, the experimental data used in the analysis are first described and later classified into three groups, roughly resembling the typical conditions of dry-surface, dropwise and film condensation, with a Gaussian-mixture algorithm [6, 7]. Using a prescribed form for each condition, a global regression analysis is then performed using a simulated annealing optimization technique to find, in each case, the best-fit correlation parameters. Finally, the accuracy in predictions from the correlations obtained in this work is assessed, for the different conditions, by comparison to the experiments.

2. Experimental data and classification

2.1. Heat exchanger data

The data sets used in this study were collected and published by McQuiston [9] from experiments on five multiple fin-tube devices of nominal size of 127 mm × 305 mm and different fin spacing, shown schematically in Fig. 1. These data (graphically illustrated in Fig. 2), and the correlations derived from them [10], have become standards in HVAC and refrigeration applications. The tests were done using atmospheric air and cold water as outer- and inner-side fluids for operating conditions that might include possible condensation, either as droplets or as thin film on the fins. These conditions were determined by direct observations and recorded. The data were given as

![Figure 1. Compact heat exchanger.](image1)

![Figure 2. Data represented by cut-planes.](image2)

Colburn $j$-factors for the air-side transfer coefficients, while high Reynolds-number flowrates rendered a negligible thermal resistance for the water side. For the dry surface conditions, the air-side heat transfer coefficients were determined using the log-mean temperature difference. A similar procedure was followed for determining the air-side heat transfer coefficients under
condensing conditions using the enthalpy difference as the driving potential. Details are in [11, 12] and the original papers [9, 13].

The number of datasets collected by McQuiston [9] was 327; which were visually classified as dry surface (91), dropwise condensation (117) and film condensation (119). The reported variables were: inlet water temperature \( T_{\text{in}}^w \), mass flow rate of the in-tube fluid given in terms of Reynolds numbers \( Re_D \), dry-bulb and wet-bulb inlet temperatures of air \( T_{\text{in}}^{a, \text{db}} \) and \( T_{\text{in}}^{a, \text{wb}} \), fin spacing \( \delta \), and the heat rate \( Q \). The heat rate was determined from enthalpy balances between the inlet and the outlet of the external-side of the heat exchanger as \( Q = \dot{m}_a (h_{\text{out}}^a - h_{\text{in}}^a) \). The corresponding ranges are: \( Re_D \in [220, 4266] \), \( T_{\text{in}}^{a, \text{db}} \in [22, 31] \, ^\circ \text{C}, T_{\text{in}}^{a, \text{wb}} \in [11, 28] \, ^\circ \text{C}, T_{\text{w}}^w \in [1, 72] \, ^\circ \text{C}, \delta \in \{1.81, 2.11, 2.54, 3.18, 6.35\} \, \text{mm}, \) and \( Q \in [200, 4900] \) W. Figure 2 provides visual information about the structure of the data \( Q = Q(Re_D, T_{\text{in}}^w, T_{\text{in}}^{a, \text{db}}, T_{\text{in}}^{a, \text{wb}}, \delta) \), by a set of planes passing through the corresponding manifold in a six-dimensional parameter space. The information is arranged in a matrix form, where the top horizontal set of graphs provides information of \( Re_D \) versus \( T_{\text{in}}^{a, \text{db}}, T_{\text{in}}^{a, \text{wb}}, T_{\text{w}}, \delta, \) and \( Q \), whereas the bottom horizontal set gives information about \( Q \) versus the other five variables involved. Although only partial information is obtained this way, it gives an idea of the interrelations among the variables and the number of groups in which the data may be classified. It is observed that, in some cases, the groupings can be easily established, whereas for others this classification is less clear. For instance the set containing \( T_{\text{w}}^w \) as dependent variable shows two potential groups; however, that of \( Q \) illustrates that either one, two, three or five groups are feasible.

2.2. Algorithmic classification: Gaussian-mixtures technique

An alternative approach to visual data-classification is the use of clustering techniques [14, 15]. These are soft computing methodologies able of extracting the main features of a system directly from experiments, and organizing them in groups. Although several schemes have been proposed [16], and few applications to thermal engineering reported [17, 18], the Gaussian-mixtures clustering method is useful in allocating data into groups (which can be found as part of the solution), when relationships among the data points are unknown. This is achieved based on the assumption that the data can be placed into a number of \( K \) clusters (groups), each described by a Gaussian probability density distribution. Once \( K \) is known, both the group structure and the data classification are computed using a maximum likelihood criterion. Good descriptions of the technique are given in [14, 19]. Thus, only a brief account of the method is provided here; additional details about the algorithm are in [20], and the references therein.

Given a set of \( N \) data \( \mathbf{X} = \{ \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N \} \subset \mathbb{R}^d \), the probability distribution of each measurement \( \mathbf{x}_j \) is given by a linear combination of \( K \) mixture components as

\[
p(\mathbf{x}_j|\Theta) = \sum_{k=1}^{K} p(\mathbf{x}_j|\omega_k, \theta_k)p(\omega_k), \quad j = 1, \ldots, N, \tag{1}
\]

where \( p(\omega_k) \) is the probability of group \( \omega_k \) occurring in the sample data, and \( p(\mathbf{x}_j|\omega_k, \theta_k) \) is the conditional probability of \( \mathbf{x}_j \) being a member of group \( \omega_k \), which is given mathematically by

\[
p(\mathbf{x}_j|\omega_k, \theta_k) = \frac{1}{(2\pi)^{d/2}||\Sigma_k||^{1/2}} \exp \left[ -\frac{1}{2}(\mathbf{x}_j - \mu_k)^T \Sigma_k^{-1}(\mathbf{x}_j - \mu_k) \right]. \tag{2}
\]

In Eqs. (1) and (2), \( \Theta = \{(p(\omega_k), \theta_k) : k = 1, \ldots, K \} \) gives the set of parameters of the mixture model (with \( \theta_k = \{\mu_k, \Sigma_k\} \), where \( \mu_k \) is the vector of mean values, and \( \Sigma_k \) the covariance matrix, which define the shape of each component density.

The unknowns in the above equations are the number of groups \( K \), the parameters of the Gaussian distributions \( \theta_k \) and the mixing proportions \( p(\omega_k) \). To estimate the model parameters
Θ, for a prescribed K, and the set of N observations, several methods –based on the maximum likelihood (ML) estimation approach along with the expectation-maximization (EM) iterative algorithm [21]– have been proposed. The result is a maximum likelihood estimate of \( \Theta \), as

\[ \hat{\Theta} = \{ (\hat{p}(w_k), \hat{\mu}_k, \hat{\Sigma}_k) : k = 1, \ldots, K \} = \arg \max \sum_{j=1}^{N} \log \left( \sum_{k=1}^{K} p(x_j | \omega_k, \theta_k) p(\omega_k) \right), \]

(3)

where the datum \( x_j \) can now be assigned to the group \( \omega_k \) according to its maximum posterior probability, i.e., the probability that data point \( x_j \) belongs to the group \( w_k \), given by

\[ \hat{p}(\omega_k)p(x_j | \omega_k, \hat{\theta}_k) > \hat{p}(\omega_l)p(x_j | \omega_l, \hat{\theta}_l), \quad \text{for all} \quad k \neq l; \quad l = 1, \ldots, K. \]

(4)

Since the number of groups needs to be known before the classification can proceed, \( K \) can be estimated based on the minimum description length (MDL) criterion [22], which is given as

\[ \text{MDL}(K, \theta) = -\sum_{j=1}^{N} \log \left( \sum_{k=1}^{K} p(x_j | \omega_k, \theta_k) p(\omega_k) \right) + \frac{1}{2}Kq \log(Nq). \]

(5)

Note that Eq. (5) provides a trade-off between data representation and the complexity of the model. Minimizing \( \text{MDL}(K, \theta) \) with respect to \( K \) establishes the number of clusters (groups) of the simplest Gaussian-mixtures model that best describes the data.

2.3. Classification results and assessment

Application of the Gaussian-mixtures clustering technique [20], to the heat exchanger data of McQuiston [9], has been reported by Pacheco-Vega and Avila [6, 7]. Using the \( N = 327 \) datasets described earlier, convergence of the algorithm to the minimum MDL value is achieved with \( K = 3 \) clusters (Gaussian mixtures), which seems to agree with the possible physical phenomena; i.e., dry-surface, dropwise- and film-condensation, occurring during the operation of the device. The corresponding data classification onto these groups, as computed by the algorithm, is presented quantitatively and qualitatively in Table 1 and Fig. 3. From the table, which shows the agreement (in percentage) between the clustering results and McQuiston [9] visual analysis, it can be observed the two methods agree completely in the dry-surface data, which were all assigned to group I. However, the agreement in the allocation of the data corresponding to humid conditions into groups II and III is less crisp; each of these two groups appear to contain data (in different proportions), that were visually classified as either drop or film condensation.

The figure, on the other hand, illustrates the algorithmic separation of the data on the planes

Table 1. Algorithmic classification of heat exchanger data.

| Condition \ Group | I  | II | III |
|-------------------|----|----|-----|
| Dry surface       | 100% | 0  | 0   |
| Drop condensation | 35.89% | 64.11% |
| Film condensation | 25.21% | 74.79% |

\( Re_D \) vs. \( T_{in}^{w} \) [Fig. 3(a)], and \( Q \) vs. \( Re_D \) [Fig. 3(b)]. From Fig. 3(a) it can be seen that the set of dry-surface data (placed into group I), is well separated from that of the wet-surface, being \( T_{in}^{w} \) the discriminant variable. Even though the algorithm was able to cluster the wet-surface data into two groups (as observed in the table), for the range in the values of \( T_{in}^{w} \) and \( Re_D \), there is no clear separation between the sets of dropwise- and film-condensation data; i.e., there
is a strong overlap in the data between the different types of condensation and both clusters. From Fig. 3(b), which considers only the data for humid conditions, it can be seen that the Gaussian-mixtures method is able to clearly separate the data onto the groups II and III. In this case, it is seen that the discriminant variable is now $Q$, with higher values of it resembling the case of dropwise condensation and, for the same operating conditions (e.g., $Re_D$-values), lower $Q$-values matching the case of film condensation [1, 23].

Assessment of the Gaussian-mixtures classification, was carried out and recently reported by Pacheco-Vega and Avila [6, 8], with a data-classification methodology based on the popular feedforward artificial neural network (ANN) technique. The method has been used to predict the performance of condensing and refrigerating heat exchangers [11, 24], among other applications. Since the heat rate $Q$, is a function of inlet temperatures, flow rates and the geometry, i.e., $Q = Q(Re_D, T_{in}^w, T_{in}a, T_{in}b, \delta)$, an ANN model of the thermal system can be built, with $Q$ being the output variable of the network while the other variables are all inputs to it. The discrimination methodology is grounded on the idea that given two sets of data (one for training and the other for testing), if these sets have common structures the prediction error from a trained network on the testing set will be small; otherwise such error will have a large value. This error is defined as the percentage difference between the target values of the heat rate and the predictions from the ANN model as

$$E_p = \frac{|Q_j - Q_{j,ANN}|}{Q_j} \times 100, \quad j = 1, \cdots, N, \quad (6)$$

where $Q_j$ and $Q_{j,ANN}$ are the target values and the predictions, respectively. Discrimination of data that belong to a specific group –from those that do not– is done by comparing the prediction error $E_p$, to the error obtained by the ANN during the training process $E_t$; i.e., if $E_p \leq E_t$ then testing and training data have common patterns, as opposed to the case $E_p > E_t$, in which testing and training data do not have similar structures, indicating that they do not belong to the same group. Details of the methodology are in [6, 8].

The network chosen for the analysis is a 5-5-3-1 configuration, which contains one input layer, two hidden layers and one output layer; the input and first layers both have five nodes, three
Table 2. ANN classification of heat exchanger data.

| Condition     | Group | I    | II   | III  |
|--------------|-------|------|------|------|
| Dry surface  | 100%  | 0    | 0    |      |
| Drop condensation |     0 | 34.19% | 65.81% | |
| Film condensation |   0 | 21.85% | 78.15% | |

nodes are in the third layer, and the output layer has only one node [11]. Results from the ANN-based discrimination technique are given quantitatively in Table 2. As seen from the table, the neural network classification agrees very well with that obtained by the clustering methodology, being the best case 100% and the worst 85%; i.e., the entire set of data visually recognized as that of dry-surface was also placed into Group I by the ANN, whereas the maximum percentage difference between the two methods for the data classification corresponding to humid conditions (visually grouped as dropwise- and film-condensation), onto groups II and III is only 15% and 4.9%, respectively.

3. Global-regression correlations

3.1. Data analysis and functional forms

The correlations proposed by McQuiston [10] to predict the air-side heat transfer coefficient in terms of the Colburn \textit{j}-factor are

\[
\begin{align*}
    j_s &= 0.0014 + 0.2618 Re_D^{-0.4} A_r^{-0.15} f_s(\delta), \\
    j_t &= 0.0014 + 0.2618 Re_D^{-0.4} A_r^{-0.15} f_t(\delta),
\end{align*}
\]

where \( j_s \) and \( j_t \) are, respectively, the Colburn \textit{j}-factor for the sensible and total heat; \( Re_D \) is the Reynolds number based on tube diameter \( D \), and \( A_r \) is the ratio of the total heat transfer area \( A \), to the surface area of tubes only \( A_{tb} \). These are given as

\[
    j_s = \frac{h}{G c_p} Pr^{2/3}, \quad j_t = \frac{h_t}{G Sc^{2/3}}, \quad Re_D = \frac{GD}{\mu}, \quad A_r = \frac{A}{A_{tb}},
\]

where \( h \) and \( h_t \) are heat and total (enthalpy) transfer coefficients, \( G \) is the mass velocity, \( c_p \) the specific heat, \( \mu \) the dynamic viscosity of air, and \( Pr \) and \( Sc \) the Prandtl and Schmidt numbers. The functions \( f_s(\delta) \) and \( f_t(\delta) \) are defined as

\[
    f_s = \begin{cases} 
        1.0 & \text{dry surface} \\
        (0.90 + 4.3 \times 10^{-5} Re_D^{1.25}) \Delta_r^{-1} & \text{dropwise condensation} \\
        0.84 + 4.0 \times 10^{-5} Re_D^{1.25} & \text{film condensation}
    \end{cases},
\]

\[
    f_t = \begin{cases} 
        1.0 & \text{dry surface} \\
        (0.80 + 4.0 \times 10^{-5} Re_D^{1.25}) \Delta_t^{-1} & \text{dropwise condensation} \\
        (0.95 + 4.0 \times 10^{-5} Re_D^{1.25}) \Delta_t^{-1} & \text{film condensation}
    \end{cases},
\]

where \( Re_\delta = Gr_\delta / \mu \) is the Reynolds number based on fin spacing \( \delta \), and \( \Delta_r = \delta / (\delta - t) \) is the ratio of fin spacing to the spacing between fins, with \( t \) being the fin thickness. From the equations above, in the case of dry-surface conditions \( j_s \) and \( j_t \) are the same. The range of validity of the correlations was also given.

To develop the set of correlation equations from the experimental measurements previously classified by the Gaussian-mixtures clustering technique, let us first consider the functional form of Eqs. (7)–(10); i.e.,

\[
    j = a + b \ Re_D^{-c} A_r^{-d} f(\delta),
\]
with
\[ f(\delta) = (n_1 + n_2 \text{Re}^{n_3}) \Delta^{n_4}, \]
where \( j = j_s \) or \( j = j_t \), and \( f = f_s \) or \( f = f_t \). The goal here is to compute the sets of constants \( C_1 = (a, b, c, d) \) and \( C_2 = (n_1, n_2, n_3, n_4) \) by performing a regression analysis. This is done by first defining the variance of the error between experiments and predictions of the \( j \)-factor as
\[ S_j = \frac{1}{N} \sum_{i=1}^{N} (j^c_i - j^p_i)^2, \]
where \( j^c_i \) and \( j^p_i \), for \( i = 1, \ldots, N \), are the experimental measurements and the values predicted by Eqs. (11)–(12), respectively, and then searching for the values of the unknown constants, such that \( S_j(C_1, C_2) \) is a minimum. It was found that \( S_j \) has multiple solutions [11]; i.e., multiple local minima, the multiplicity stemming from the assumed mathematical form of the correlation which is not only non-linear but also not linearizable. Thus, a global search for the set of constants must be conducted.

On looking into Eqs. (7) and (8), along with the functional form in Eq. (11), it is clear that in the case of dry-surface conditions (where \( f_s = f_t = 1 \)), the two equations not only have the same functional form but also the same values of the constants, and hence \( j_s = j_t \). For humid-surface conditions, on the other hand, the term \( b \text{Re}^{\gamma} A^{\alpha} r^{\beta} \) is modified by \( f(\delta) \). Thus, following the approach of [10], the regression analysis is divided in two steps, the first used to find \( (a, b, c, d) \) whereas the second focused on seeking the values of \( (n_1, n_2, n_3, n_4) \).

The search for the global minimum is carried out by means of the recently-developed optimization technique named simulated annealing (SA). Although several techniques, including the popular genetic algorithm (GA) have become available in the literature [25], SA ensures probabilistically, though not deterministically, that the minimum of a function is found [2].

3.2. Simulated annealing
Simulated annealing (SA) is a stochastic computational technique, derived from statistical mechanics, which discriminates among different local optima to find the best solution of an optimization problem. The technique is inspired by the molecular calculation of the cooling of a physical system in which random agitation is used to avoid entrapment in local extrema. Detailed discussions of the SA algorithm are given in [26, 27], and only a brief description is provided here.

Given a multi-variable objective function \( F(y_1, y_2, \cdots, y_Q) \), in a \( Q \)-dimensional parameter space:

(i) A starting point in the space of unknowns is randomly selected on which a cycle of random moves along each coordinate direction is performed.

(ii) The new point is accepted if it gives a better value of \( F \).

(iii) If the new value of \( F \) is worse than the current best, it is accepted only with a certain probability \( \exp(-\Delta F/T^*\alpha) \).

(iv) The process is repeated with decreasing \( T^* \) and step size until convergence within a certain tolerance is reached.

In the description above, \( \Delta F \) is the change in value of \( F \), analogous to energy in thermodynamics, and \( T^* \) is a dynamic parameter that is analogous to the temperature of a system being cooled [28]. The algorithm described by Corana et al. [29] is followed here.
3.3. Results and discussion

In Section 2.3, the $N = 327$ datasets reported by McQuiston [10] were algorithmically classified into Group I (91), denoted below as D/G-I, Group II (72) – which can be loosely associated to film condensation (thus designated as F/G-II), and Group III (164) – loosely resembling the dropwise condensation phenomenon (hence marked as DW/G-III). Importantly, the results indicated complete correspondence between the data assembled by the Gaussian-mixtures algorithm into group I and the set of data visually organized as dry-surface by [10]. Moreover, for the three data sets of [10], Pacheco-Vega et al. [11] found that multiplicity of solutions exist, for which a global search must be conducted. This type of search was carried out for the dry-surface conditions using three different global optimization algorithms, among them the simulated annealing (SA) [2], with the assumed functional form (11). The correlation obtained is

$$ j = 0.0169 - 6.52 \times 10^{-5} \, Re_{D}^{0.552} \left( \frac{A}{A_{th}} \right)^{0.16} \, f(\delta) \quad (14) $$

with the function $f(\delta) = 1$.

By taking Eq. (14), along with the functional form for $f(\delta)$, given in Eq. (12), the results from minimizing the error in Eq. (13) with the SA algorithm are shown in Table 3. The table presents the correlation constants found from the data for humid conditions, clustered in groups II (F/G-II) and III (DW/G-III), along with the root-mean-square (rms) values of the percentage differences between the predicted and experimental data for both the sensible and total Colburn $j$-factors. The corresponding results for the data of group I; i.e., the dry-surface case (D/G-I), have also been included for completeness. The table illustrates the accuracy of the correlations to predict the data from which they were derived. In all cases, the prediction errors are confined to less than 12.3%, with those of the $j$-factor for dry-surface conditions/Group I (D/G-I) being even smaller (6.2%). Qualitative results of the $j$-factor from the correlations, for the wet-surface conditions classified into Groups II and III, are illustrated in Fig. 4. Figure 4(a) shows a comparison between the experimental and predicted sensible $j$-factor, $j_{s}$, with the data of case F/G-II, whereas Fig. 4(b) depicts that of $j_{s}$ for the data of case DW/G-III. As seen from both figures, the accuracy in predictions from the correlations are very good with the majority of the data being confined to within the ±10% error lines, and only few outliers. Similar dispersion in the predictions was also obtained for the total $j$-factor, $j_{t}$.

It is important to note that, since the data in group I matches those of dry-surface conditions, accuracy of the correlation given by Eq. (14) with $f(\delta) = 1$ can be directly compared to that of McQuiston [10]. The rms error of the proposed correlation ($E = 6.2\%$), which is the best-possible from a given functional form [2, 11], is much smaller to that of [10] for the same data (i.e., $E = 14.6\%$). On the other hand, for the humid-surface conditions, a direct comparison is not possible due to fact that the experimental data were allocated differently by the Gaussian-mixtures algorithm as compared to the visual classification. However, it is expected that the
correlations proposed here be more accurate than those of [10] due to fact that the set of constants in them have been found by a global- instead of a local-regression process.

Figure 4. Experiments vs. predictions of $j_s$ from correlations. Straight line is the perfect prediction.

4. Concluding remarks
In the current work a set of correlation equations capable of describing the thermal performance of compact heat exchangers with possible condensation has been developed on the basis of a novel methodology that incorporates an algorithmic classification along with global regression. The classification has been carried out with a previously-introduced Gaussian-mixture-based clustering technique, and its accuracy assessed by a methodology based on artificial neural networks. The search for the best-possible correlation, based on a prescribed form for each operating condition (dataset in a specific group), was performed using the simulated annealing optimization technique. The results from the application to published condensing heat exchanger data show that the methodology is able to both accurately compute the number of groups and classify the data onto each (as demonstrated via an independent assessment with artificial neural networks), and to find optimal correlation constants which provides the best model of the system behavior. Predictions from the correlations obtained for the different conditions are within the uncertainty of the experiments and substantially more accurate than those commonly used. Although the application of this methodology has been on compact heat exchangers, it may be a useful alternative for analysis of complex physical phenomena occurring in other thermal systems, and results on these will be reported in the future.

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5. References
[1] Incropera F and DeWitt D 2002 Fundamentals of Heat and Mass Transfer (New York, NY: John Wiley & Sons)
[2] Pacheco-Vega A, Sen M and Yang K 2003 Int. J. Heat Mass Transfer 46 1029–1040
[3] Pacheco-Vega A, Sen M, Yang K and McClain R 1998 *Proceedings of the Eleventh International Heat Transfer Conference* vol 6 ed Lee J (New York, NY: Taylor & Francis) pp 137–142

[4] Cai W, Pacheco-Vega A, Sen M and Yang K 2006 *Int. J. Heat Mass Transfer* 49 4352–4359

[5] Olivier J, Liebenberg L, Thome J and Meyer J 2007 *Int. J. Refrigeration.* 30 609–623

[6] Pacheco-Vega A and Avila G 2009 *Proceedings of the ASME 2009 Heat Transfer Summer Conference* (San Francisco, CA) HT2009-88627

[7] Pacheco-Vega A and Avila G 2016 *Int. J. Heat Mass Transfer* Submitted

[8] Pacheco-Vega A and Avila G 2016 *Int. J. Heat Mass Transfer* Submitted

[9] McQuiston F 1978 *ASHRAE Transactions* 84 266–293

[10] McQuiston F 1978 *ASHRAE Transactions* 84 294–309

[11] Pacheco-Vega A, Díaz G, Sen M, Yang K and McClain R 2001 *ASME J. Heat Transfer* 123 348–354

[12] Avila de la Rosa G 2007 *Identification of the Operating Conditions in Compact Heat Exchangers via Cluster Analysis* (in Spanish) MS Thesis Universidad Autonoma de San Luis Potosi San Luis Potosi, Mexico

[13] McQuiston F 1976 *ASHRAE Transactions* 82 87–106

[14] Everitt B, Landau S and Morven L 2001 *Cluster Analysis* (New York, NY: Ed. Arnold)

[15] Abonyi J and Feil B 2007 *Cluster Analysis for Data Mining and System Identification* (Berlin, Germany: Birkhäuser Verlag AG)

[16] Jain A, Murty M and Flynn P 1999 *ACM Computing Surveys* 31 264–323

[17] Avila G and Pacheco-Vega A 2009 *Numer. Heat Transfer, Part A* 56 880–896

[18] Pacheco-Vega A 2011 *Soft Computing in Green and Renewable Energy Systems* vol 269 ed Gopalakrishnan K, Khaitan S and Kalogirou S (Springer-Verlag) pp 1–35

[19] Webb A 2002 *Statistical Pattern Recognition* (New York, USA: John Wiley & Sons, LTD)

[20] Chen S, Bouman C and Lowe M 2004 *IEEE Transactions on Medical Imaging* 23 85–98

[21] Dempster A, Laird N and Rubin D 1977 *J. Royal Statist. Soc. B* 39 1–38

[22] Rissanen J 1983 *Annals of Statistics* 11 417–431

[23] Rose J 2002 *Proc. Inst. Mech. Eng. Part A: J. Power and Energy* 216 115–128

[24] Pacheco-Vega A, Sen M, Yang K and McClain R 2001 *Int. J. Heat Mass Transfer* 44 763–770

[25] Gosselin L, Tye-Gringas M and Mathieu-Potvin F 2009 *Int. J. Heat Mass Transfer* 52 2169–2188

[26] Aarts E and Korst J 1989 *Simulated annealing and Boltzman machines* (Tiptree Essex, Great Britain: Wiley)

[27] S Kirkpatrick CD Gelatt M V 1983 *Science* 220 671–680

[28] Metropolis N, Rosenbluth A, Rosenbluth M, Teller A and Teller E 1953 *J. Chem. Phys.* 21 1087–1092

[29] Corana A, Marchesi M, Martini C and Ridella S 1987 *ACM Trans. Math. Software* 13 262–280