EFFECTS OF TURBULENT TRANSFER ON CRITICAL BEHAVIOR

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Using the field theory renormalization group, we study the critical behavior of two systems subjected to turbulent mixing. The first system, described by the equilibrium model $A$, corresponds to the relaxational dynamics of a nonconserved order parameter. The second system is the strongly nonequilibrium reaction–diffusion system, known as the Gribov process or directed percolation process. The turbulent mixing is modeled by the stochastic Navier–Stokes equation with a random stirring force with the correlator $\propto \delta(t-t')p^{4-d-y}$, where $p$ is the wave number, $d$ is the space dimension, and $y$ is an arbitrary exponent. We show that the systems exhibit various types of critical behavior depending on the relation between $y$ and $d$. In addition to known regimes (original systems without mixing and a passively advected scalar field), we establish the existence of new strongly nonequilibrium universality classes and calculate the corresponding critical dimensions to the first order of the double expansion in $y$ and $\varepsilon = 4 - d$ (one-loop approximation).

Keywords: renormalization group, critical behavior, turbulent transfer

1. Introduction

Various systems of very different physical nature exhibit interesting singular behavior in the vicinity of their critical points. Their correlation functions acquire a self-similar form with universal critical dimensions: they depend only on few global characteristics of the system (such as symmetry or the space dimension). The field theory renormalization group (RG) provides a quantitative description of critical behavior. In the RG approach, possible types of critical regimes (universality classes) are associated with infrared (IR) attractive fixed points of renormalizable field theory models. The more typical equilibrium phase transitions belong to the universality class of the $O_n$-symmetric $\psi^4$ model of an $n$-component scalar order parameter. Universal characteristics of the critical behavior depend only on $n$ and the space dimension $d$ and can be calculated within various systematic perturbation schemes, in particular, in the form of expansions in $\varepsilon = 4 - d$ or $1/n$ (see [1], [2] and the references therein).

Aleksandr Nikolaevich Vasiliev contributed significantly to the development of field theory methods and their application in the theory of critical behavior and the theory of turbulence. His work in this field is summarized in [2]–[4]. The most remarkable specific achievements are probably the calculation of the Fisher exponent $\eta$ in the $O_n$-symmetric $\psi^4$ model to the order $1/n^3$ [5] and the third-order calculation of the anomalous exponents in the Kraichnan model of turbulent advection [6]. Here, we apply the field theory RG and generalized $\varepsilon$-expansion to the problem of the effects of turbulent transfer on various types of...
of critical behavior.

During recent decades, constant interest has been attracted by spreading processes and the corresponding nonequilibrium phase transitions (see, e.g., [7], [8] and the references therein). Spreading processes are encountered in physical, chemical, biological, and ecological systems: autocatalytic reactions, percolation in porous media, epidemic diseases, etc. The transitions between the fluctuating (active) and absorbing (inactive) phases, where all the fluctuations cease entirely, are especially interesting as examples of nonequilibrium critical behavior.

It has long been realized that the behavior of a real critical system is extremely sensitive to external disturbances, gravity, impurities, and turbulent mixing (see the general discussion and references in [9]). Moreover, some disturbances (randomly distributed impurities or turbulent mixing) can produce completely new types of critical behavior with rich and rather exotic properties.

These issues become even more important for nonequilibrium phase transitions because the ideal conditions of a “pure” stationary critical state can hardly be achieved in real chemical or biological systems and the effects of various disturbances can never be completely excluded. In particular, intrinsic turbulence effects cannot be avoided in chemical catalytic reactions or forest fires. We can also speculate that atmospheric turbulence can play an important role in the spread of an infectious disease by flying insects or birds. Effects of different kinds of regular and turbulent flows on critical behavior were studied in [10]–[17].

In several papers [14]–[17], the critical behavior of various systems subject to turbulent mixing was studied using the field theory RG. As a rule, the turbulence was modeled by a time-decorrelated Gaussian velocity field with a pair correlation function of the form $\langle vv \rangle \propto \delta(t - t') p^{-d - \xi}$, where $p$ is the wave number and $0 < \xi < 2$ is a free parameter with the real (“Kolmogorov”) value $\xi = 4/3$. This “Kraichnan rapid-change model” has attracted enormous attention recently because of the insight it offers into the origin of intermittency and anomalous scaling in fully developed turbulence (see [18] and references therein). The RG approach to that problem was reviewed in [19]. In the context of our study, it is especially important that the Kraichnan ensemble allows easily modeling the anisotropy of flow [16] and the compressibility of a fluid [17], which appears much more difficult if the velocity is described by full-scale dynamical equations.

But the Gaussianity and zero correlation time are drastic simplifications of the real situation, and it is desirable to investigate the effects of turbulent mixing caused by more realistic velocity fields. Here, we study the effects of a strongly non-Gaussian velocity field with a finite correlation time governed by a stochastic dynamical equation. More precisely, we use the stochastic Navier–Stokes equation for an incompressible velocity with a random stirring force with the correlator $\propto p^4 - d - y$, where $y$ is an arbitrary exponent with the physical (“Kolmogorov”) value $y = 4$. The RG approach to this model was reviewed in [2], [4].

We consider two representative cases of dynamical critical behavior: the equilibrium critical dynamics of a nonconserved order parameter with a $\psi^4$-type Hamiltonian; a nonequilibrium system near its transition point between the absorbing and fluctuating states. The first model corresponds to critical fluid systems (binary mixtures or liquid crystals), and the second describes the spreading processes in reaction–diffusion systems, belongs to the universality class known as the Gribov process or directed percolation process, and is equivalent (up to the Wick rotation) to the Reggeon field theory [7], [8].

We show that both systems, depending on the relation between $y$ and $d$, exhibit various types of critical behavior associated with different IR attractive fixed points of the RG equations. In addition to known asymptotic regimes (such as equilibrium critical dynamics without mixing or passively advected scalar without self-interaction), we establish the existence of new strongly nonequilibrium types of critical behavior (universality classes) and calculate the corresponding domains of stability in the $yd$ plane and the critical dimensions in the leading order of the double expansion in $y$ and $\varepsilon = 4 - d$, which corresponds to the one-loop approximation of the RG.