The problem of phase breaking in the electronic conduction in mesoscopic systems: a linear-response theory approach

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We study the problem of electronic conduction in mesoscopic systems when the electrons are allowed to interact not only with static impurities, but also with a scatterer (a phase breaker(PB)) that possesses internal degrees of freedom. We first analyze the role of the PB in reducing the coherent interference effects in a one-electron quantum mechanical system. In the many-electron system we can make a number of quite general statements within the framework of linear-response theory and the random-phase approximation. We cannot calculate the conductivity tensor in full generality: we thus resort to a model, in which that tensor can be expressed entirely in a single-electron picture. The resulting zero-temperature conductance can be written in terms of the total transmission coefficient at the Fermi energy, containing an additional trace over the states of the PB.

I. INTRODUCTION

The scattering approach to quantum electronic transport in mesoscopic systems was devised by Landauer¹ and later extended by a number of other authors (see, for instance, as representative articles, Refs. ²,³ and the references contained therein). In an independent-electron picture, it aims at understanding the electric conductance of a sample in terms of its scattering properties. The problem of electronic transport is thus converted into that of solving the quantum-mechanical scattering problem of an electron that impinges on the sample through leads that, ideally, extend to infinity, once the experimental environment the sample is attached to in the laboratory is disconnected. An approach to this problem using the methods of linear-response theory (LRT) has been given, for instance, by the authors of Refs. ²³ (see also other publications referred to there).

In the original conception of the scattering approach to electronic transport, inelastic electron scattering or other phase-breaking mechanisms are not allowed inside the sample. As a result, the phase of the wave function is completely coherent in that region. Yet, in various circumstances the effects of the electron-electron interaction or the interaction with the phonon field may not be negligible. In a further development of the theory, the single-electron picture is maintained and phase-breaking events in a given region are modelled by attaching the system to a “fake wire”, which in turn is connected to a phase-randomizing reservoir, so that there is no phase coherence in the wave function for an electron entering and exiting the reservoir. The chemical potential of the reservoir is chosen so that the net current along the fake wire vanishes. This model provides sensible answers and has been used, for instance, in the study of electric transport through quantum dots.⁴

A number of authors have attempted to generalize the scattering approach to include inelastic scattering explicitly, instead of modelling it as described above. In Ref. ⁵ the problem of quantum transport in the presence of phase-breaking scattering is formulated using an exactly soluble model for the electron-phonon interaction and, using linear-response theory, a generalized conduction formula is found. Ref. ⁶ uses Landauer’s approach and analyzes the effect of a single impurity scatterer, at which both elastic and inelastic processes can occur. The form of the electron-impurity interaction can be quite general; in the weak-scattering limit, in which Born approximation is invoked, the authors arrive at a generalized conduction formula. The possibility of energy exchange with the scatterer makes Pauli blocking effects important and an extension beyond Born approximation seems difficult.

The authors of Ref. ⁷ analyze in great generality the problem of quantum-mechanical phase breaking: they consider the interference between the terms of the electron wave function arising from two different electron paths and study the effect on that interference of an “environment” the electron can interact with. In an experiment in which the electron and not the environment is measured, the coordinate of the latter is integrated upon: as a result, the interference is lost if, in the two interfering partial waves, the states of the environment are orthogonal to each other. It is emphasized that, for this to occur, energy exchange between the electron and the environment need not be invoked. That this loss of interference is irretrievable is a quantum-mechanical effect, common to a number of situations, like the two-slit experiment.

In the present paper we plan to incorporate the mechanism of Ref. ⁷—briefly described in the previous paragraph—to the LRT approach to transport provided by Refs. ²³, in order to study the problem of phase breaking in the electronic conduction in mesoscopic systems. The emphasis on the mechanism of Ref. ⁷ for phase breaking is the main difference between the present and previous work on the problem. Another characteristic of the present paper is that the applications are dis-
discussed in terms of the scattering matrices of the various impurities (static or non-static), a procedure that has been found advantageous in a number of previous publications (see Refs. 11,12 and references contained therein).

Quite generally, we can pose the problem by thinking that the electrons, besides suffering elastic collisions with static scatterers, interact with a number of scatterers, or phase breakers (PB), that possess internal degrees of freedom and can live, say, in m possible quantum-mechanical states altogether. Even in the absence of the electron-electron Coulomb interaction, the problem is now no longer one of a single-electron, but is a full many-body problem: one electron is incident on the PB, this being in some state \( \mu \); after the interaction there is a certain probability to find the PB in state \( \nu \), and this is what the next electron coming in will see. This memory effect, or, equivalently, the electron-electron interaction induced by the PB, gives rise to a situation similar to the one found in Kondo problem, and we are bound to find similar complications.

The paper is organized as follows. In order to become acquainted with the physical phenomena produced by a PB we first study, in the next section, the problem of a single electron scattered by two static impurities and a PB. We show how the interference terms that occur with the static impurities alone are affected by the presence of the PB. The discussion parallels that given in Ref. 10 and is done in terms of the scattering matrices of the various impurities.

In Sec. II we pose the conduction problem (a many-electron problem) of an electronic system with static impurities and a PB [not but subject to a magnetic field, so that we have time-reversal invariance (TRI)], from the standpoint of LRT. We show that we can make a number of quite general statements. However, we reach a point where we are unable to calculate the conductivity tensor in full generality: we thus resort to a simplified, soluble model that we introduce in Sec. III. The conductivity tensor is calculated within that model and is found to be expressible entirely in terms of a single-electron picture, i.e., in terms of single-electron Green’s functions. It is then shown that the resulting zero-temperature dc conductance can be expressed in terms of a total transmission coefficient at the Fermi energy \( \epsilon_F \), but now containing a trace over the m states of the PB, Eq. (1.34). Within the restrictions of the model, the result does not depend on the strength of the electron-PB interaction, as other analysis do. We present (see discussion at the end of section III around equation (3.17)) a speculation as to the validity of our main result beyond the assumptions of the soluble model, in the strict linear-transport regime and above the Kondo temperature (which is taken to be extremely low) associated with the m-level PB.

In Sec. IV we set up a random-matrix description of the electron-PB system, with possible applications to chaotic cavities, in order to calculate the effect of the PB on the average conductance and its fluctuations. The limitations of the model become apparent here.

Our conclusions are discussed in Sec. VI.

II. QUANTUM INTERFERENCE IN A ONE-ELECTRON SCATTERING PROBLEM: THE EFFECT OF A PHASE BREAKER.

We analyze in this section the scattering problem of a single electron interacting with a combination of three scatterers in series: two static ones and a PB in the middle. We show how the interference terms that would occur with the static scatterers alone are affected by the presence of the PB.

For simplicity, the problem is treated as a 1D one. It is described by the Hamiltonian

\[
H = \frac{p^2}{2m} + V_1(x) + \sum_{\mu,\nu} [\mu] V_{\mu\nu}(x) [\nu] + V_3(x). \tag{2.1}
\]

In this equation, \( V_1(x) \) and \( V_3(x) \) are the potentials arising from the two static scatterers, and the third term represents the interaction of the electron with the PB; the m states of the latter, denoted by \( \mu, \nu \), are degenerate in energy. Below, we shall find it convenient to write our states as m-component “spinors”, so that \( H \) acquires a matrix form, where rows and columns are associated with the m states of the PB.

Instead of modelling the potentials for the various scatterers, we have found it advantageous to model their scattering properties through the corresponding scattering matrices. In this language, the three scatterers are described by the S matrices \( S_1, S_2 \) and \( S_3 \), respectively. We write the S matrix \( S_2 \) for the PB as

\[ S_2 = \begin{bmatrix} r_2 & t_2' \\ t_2 & r_2' \end{bmatrix}, \tag{2.2} \]

where the reflection and transmission matrices (for incidence from the left or from the right, respectively) \( r_2, r_2' \), \( t_2, t_2' \) are m-dimensional, with matrix elements \( r_2^{\mu\nu} \), etc. The \( S \) matrices for the elastic scatterers on the left and on the right of the PB are written, respectively, as

\[ S_1 = \begin{bmatrix} r_1 I_m & t_1' I_m' \\ t_1 I_m & r_1' I_m' \end{bmatrix}, \tag{2.3} \]

\[ S_3 = \begin{bmatrix} r_3 I_m & t_3' I_m' \\ t_3 I_m & r_3' I_m' \end{bmatrix}, \tag{2.4} \]

where \( r_1, \ldots, r_3, \ldots \) are just complex numbers (we are in 1D) and \( I_m \) is the m-dimensional unit matrix in the space of the PB states: recall that scatterers 1 and 3 do not change the state of the PB.

The total transmission matrix \( t \) for the chain of three scatterers in series is given by

\[ t = (t_3 I_m) \frac{1}{I_m - r_{12} r_3} t_{12}. \tag{2.5} \]
In this equation, $t_{12}$ is the transmission matrix for the system formed by $S_1$ and $S_2$, given by

$$t_{12} = t_2 \frac{1}{I_m - (r_1^*) r_2} (t_1 I_m). \quad (2.6)$$

Similarly, $r'_{12}$, the reflection matrix for the combination $S_1, S_2$, is given by

$$r'_{12} = r'_2 + t_2 \frac{1}{I_m - (r'_1) r_2} (r'_1 I_m) t'_2. \quad (2.7)$$

To be more specific, we now choose $r_2 = r'_2 = 0$ in the matrix $S_2$ of Eq. (2.3) that defines the PB. Thus the PB will not change the electron momentum; however, we shall see that it reduces, in the electronic current, the interference among the multiply reflected paths occurring between the two elastic scatterers. As a consequence of this choice, the matrices $t_2$ and $t'_2$ are $m$-dimensional unitary matrices

$$t_2 t_2^\dagger = I_m, \quad t'_2 t_2^\dagger = I_m. \quad (2.8)$$

Also, $t_{12}$ reduces to

$$t_{12} = t_2 t_1 \quad (2.9)$$

and $r'_{12}$ to

$$r'_{12} = t_2 r'_1 t'_2 \quad (2.10)$$

We thus have, for $t$

$$t = (t_3 t_1) \frac{1}{I_m - (r'_1 r_3) t_2 t'_2}$$

$$= (t_3 t_1) \frac{1}{I_m - (r'_1 r_3) t_2 t'_2} = (t_3 t_1) t_2 \frac{1}{I_m - a u}, \quad (2.11)$$

where we have defined the complex number

$$a = r'_1 r_3 = |a| \exp(i \rho) \quad (2.12)$$

and the unitary matrix

$$u = t'_2 t_2. \quad (2.13)$$

The total transmission matrix $t$ of Eq. (2.11) is $m$-dimensional. Its element $t^{\mu \nu}$ gives the probability amplitude for the process: \{the electron comes from the left, the PB being in state $\nu$\} $\rightarrow$ \{the electron is transmitted to the right, the PB being shifted to the state $\mu$\}. The corresponding probability is $T^{\mu \nu} = |t^{\mu \nu}|^2$. Now, $T^\nu = \sum_{\mu} T^{\mu \nu}$ is the transmission probability when the PB is initially in state $\nu$ and the PB is not observed; $T^\nu$ can be written as

$$T^\nu = (t^t t)^{\nu \nu} = T_3 T_1 \left[ \frac{1}{I_m - a^\dagger u I_m - au} \right]^{\nu \nu}, \quad (2.14)$$

where Eq. (2.8) was used. Let us emphasize that the superscript $\nu$ specifies the initial pure state of the PB. If, however, the PB is initially in a mixed state (a situation of particular interest for the conductance problem, Sec. [V]), then an additional sum over $\nu$ is needed. Assuming that the PB can be found, with equal probability, in each of its $m$ states, one obtains for the transmission coefficient $T$:

$$T = \frac{1}{m} \sum_{\nu} T^\nu = \frac{1}{m} \text{tr}(t^t t) \quad (2.15)$$

$$= T_1 T_3 \frac{1}{m} \text{tr} \left[ \frac{1}{I_m - a^\dagger u I_m - au} \right]. \quad (2.16)$$

It is instructive to expand $t$ of Eq. (2.11) as the power series

$$t = (t_3 t_1) t_2 \left[ I_m + au + a^2 u^2 + \cdots \right].$$

This series can be easily interpreted in terms of the multiple scattering processes that occur between the two elastic scatterers, influenced by the PB in the middle. The transmitted wave function is a linear combination of all these terms, arising from internal multiple reflections.

We now study a number of particular cases.

A. The Case $m=1$

We set

$$t_2 = t'_2 = 1. \quad (2.18)$$

From Eq. (2.11) we have

$$t = \frac{t_3 t_1}{1 - r'_2 r_3} = \frac{t_3 t_1}{1 - a}. \quad (2.19)$$

In this case, scatterer 2 is not a PB, but a static scatterer: it is thus like having just the two elastic scatterers $1$ and $3$; that extra phase could equally well be of particular interest for the conductance problem, Sec. [V]. Let us emphasize that the PB is not observed; $T^\nu$ can be written as

$$T^\nu = T_3 T_1 \left[ \frac{1 + |a|^2}{1 - a^2} \right]$$

$$= T_3 T_1 \frac{1 + |a|^2 + 2 \text{Re} a}{1 - a^2} = T_\text{coh}. \quad (2.20)$$

This result will be termed the fully coherent response $T_\text{coh}$.
B. The Case \(m=2\)

Now the PB has two (orthogonal) states. We follow the various multiply scattered terms occurring between the two elastic scatterers –as given by Eq. (2.11)– in order to understand more closely what the PB does to them. We consider two examples.

1. Let

\[
t_2 = t_2' = \sigma_z = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},
\]

(2.21)

so that, from Eq. (2.13)

\[
u = t_2' t_2 = I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.
\]

(2.22)

We examine the multiple-scattering series (2.17). In each passage through the PB, the state of the latter is shifted to the orthogonal state. But, after the pair of reflections described by the product \(a = r_1' r_3\), the PB is visited twice and is then back to the original state. In other words, in each passage, the PB exactly undoes what it did in the previous one. This is the significance of \(u = t_2' t_2 = I_2\) in Eq. (2.22).

We thus find for \(t\), Eq. (2.11)

\[
t = (t_3 t_1) t_2 \frac{1}{1 - r_1' r_3},
\]

(2.23)

which leads to Eq. (2.20), exactly as for the case with no PB.

2. Let

\[
u = t_2' t_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \sigma_x.
\]

(2.24)

This could be obtained, for instance, with the choices

\[
t_2' = -i \sigma_y \quad \text{and} \quad t_2 = \sigma_z,
\]

(2.25)

or

\[
t_2 = t_2' = \frac{1}{2} \begin{bmatrix} 1 + i & 1 - i \\ 1 - i & 1 + i \end{bmatrix}.
\]

(2.26)

Now \(u\) shifts the two PB states: i.e., state \(|1\rangle\) is shifted to \(|2\rangle\) and viceversa. This fact has important consequences. The multiple-scattering series (2.17) for \(t\) now gives

\[
t = (t_3 t_1) t_2 \frac{1}{1 - a} \left[ I_2 + a \sigma_x + a^2 I_2 + a^3 \sigma_x + \cdots \right],
\]

(2.27)

which divides naturally into an even-order and an odd-order contribution, which can be summed up to give

\[
t = (t_3 t_1) t_2 \left[ \frac{1}{1 - a^2} I_2 + \frac{a}{1 - a^2} \sigma_x \right].
\]

(2.28)

Suppose that in the incident state the electron comes from the left and the PB is in the pure state

\[
|0\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix},
\]

(2.29)

say. The transmitted wave function on the right is thus

\[
|\Psi_{\text{trans}}\rangle = \frac{t_3 t_1}{2(1 - a^2)} e^{ikx} \left[ (1 + a)(\alpha + \beta) + (1 - a)(\alpha - \beta)i \right] \left[ (1 + a)(\alpha + \beta) - (1 - a)(\alpha - \beta)i \right].
\]

(2.30)

The transmission probability, that we call \(T_{\text{trans}}\), is given by

\[
T^{\text{trans}} = T_3 T_1 \frac{1 + |a|^2 + 2 \text{Re}(\alpha \beta^*) \text{Re}(a)}{|1 - a^2|^2}.
\]

(2.31)

In the absence of the PB, on the other hand, we have the fully coherent response \(T_{\text{coh}}\) of Eq. (2.21). The interference term \(2 \text{Re}(a)\) in Eq. (2.22) has been reduced, in Eq. (2.31), by a factor, whose magnitude \(|2 \text{Re}(\alpha \beta^*)| \leq 1\).

The effect of the PB, and of having measured the electron but not the PB, has thus been to decrease the magnitude of the interference term. We stress again that this effect is there even if the incident state is the pure state (2.29). In particular, for either pure state \(\alpha = 1, \beta = 0\), or \(\alpha = 0, \beta = 1\), we obtain, in the notation introduced right after Eq. (2.13)

\[
T^{\text{trans}} = T_3 T_1 \frac{1}{|1 - a^2|^2},
\]

(2.32)

where the interference term in the numerator is absent. This last result could be obtained directly from Eq. (2.14). For a mixture of these two states we thus obtain the same answer, i.e. \(T = T^1 = T^2\).

C. The Case \(m \to \infty\)

Let us inquire as to what kind of a PB would lead to the “classical” composition rule for the two elastic scatterers, i.e. to the equation

\[
T = \frac{T_1 T_3}{1 - R_1 R_3}.
\]

(2.33)

We shall see that for this to occur we need a PB with many states, i.e. \(m \to \infty\). We analyze two possibilities.

1. Choose

\[
(t_2)_{\mu \nu} = (t_2')_{\mu \nu} = \delta_{\nu, \mu - 1}.
\]

(2.34)

For finite \(m\) we use periodic boundary conditions, i.e. \(m + 1 \to 1\). But below we are interested in \(m \to \infty\). Assume that the electron is impinging from the left and the PB is initially, say, in its first state \(|1\rangle\). Each passage
of the electron through the PB will flip the latter to its next (orthogonal) state. Therefore, from the expansion (2.17), the transmitted wavefunction is:

$$|\Psi_{k_{1}\text{trans}}^{1}\rangle = e^{ikx}t_{3}t_{1}|[2] + a|4\rangle + a^{2}|6\rangle + \cdots$$ (2.35)

It is now clear that the transmission coefficient $T^{1}$ for the electron (without observing the PB) is given by Eq. (2.15).

2. We now discuss a model that allows varying the degree of dephasing in a continuous fashion, thus permitting the description of the crossover between the fully coherent response (2.20) and the fully incoherent, or classical, one (2.33).

We choose $t_{2} = t_{2}' = \exp(iH/2)$ where $H$ is an $m \times m$ Hermitian matrix. Its eigenvalues $\theta$ are chosen so that their density $mg(\theta)$ has width $\lambda$ (a trivial way to get that is to take a diagonal matrix and put the required spectrum by hand, then make an arbitrary unitary transformation). Obviously, the eigenvalues of $t_{2}'$ will be $\exp(i\theta)$, where $\theta$ is an eigenvalue of $H$. Assuming that the PB is in a mixed state and, thus, using Eq. (2.15), we have

$$T = T_{1}T_{3}\frac{1}{m} \sum_{r=1}^{m} \frac{1}{|1 - ae^{i\theta_{r}}|^{2}}.$$ (2.36)

which in the $m \to \infty$ limit can be written as

$$T = T_{1}T_{3}\int_{0}^{2\pi} \frac{g(\theta)d\theta}{|1 - ae^{i\theta}|^{2}}.$$ (2.37)

Rather then attempt to do the last integral exactly for a given $g(\theta)$, it is very instructive to expand, as in Eq. (2.17), each of the two factors in the denominator as a sum of powers such as $a^{k} \exp(ik\theta)$, and the complex conjugate, $(a^{*})^{k'} \exp(-ik'\theta)$. All the “diagonal” terms give the classical result, Eq. (2.33), as above. The mixed terms with $k \neq k'$ give the interference. With vanishing $\lambda$, we get the full interference terms. Otherwise the interference terms are killed once $|k - k'|\lambda > O(1)$. Thus $1/\lambda$ plays the role of the dephasing length, in units of the distance between the static scatterers.

It is possible, using the above, to analyze a number of interesting physical situations. Consider $a$ real and close to unity, i.e. $a = 1 - \delta$, with $0 < \delta \ll 1$ and $g(\theta)$ picked at $\theta = 0$. Take, for example, the peak of the resonant transmission for the fully coherent case, $\lambda = 0$. The height of the resonance is $\sim 1/\delta^{2}$. The relevant number of bounces (the number of significant terms in the series obtained upon expansion of $1/(1 - ae^{i\theta})$ needed to form the resonance) is $\sim 1/\delta$. Now introduce a finite $\lambda$. If $\lambda \ll \delta$, it will affect distant (i.e. further than $1/\delta$) terms in the series and thus will have no effect on the height of the resonance. Dephasing starts to “hurt” the peak once $\lambda \gg \delta$.

III. THE CONDUCTION PROBLEM IN THE PRESENCE OF A PHASE BREAKER IN LINEAR-RESPONSE.

In this section we discuss the conduction problem – a many-electron problem – in the presence of a PB from the standpoint of linear-response theory (LRT) (2.23) following the random phase approximation (RPA) scheme developed in Refs. (2.24, 2.23). The system to be studied has the geometry shown in Fig. 1: the constriction represents the sample, where we allow for the presence of a PB. As usual, the expanding horns represent the external leads that, in a laboratory setup, are attached to macroscopic bodies.

![Diagram of the conduction problem](image)

FIG. 1. The geometry for the electronic conduction problem studied in the text. The constriction represents the sample and the expanding horns, the external leads. Beyond the surfaces $S_{+}$ and $S_{-}$ the potential $\delta\phi_{\alpha}$ takes on constant values.

When no external voltage is applied, the whole system is in equilibrium and is described by the unperturbed Hamiltonian

$$H_{0} = \sum_{i} \left[ \frac{p_{i}^{2}}{2m} - e \sum_{l} \frac{e_{l}}{|r_{i} - R_{l}|} - e \sum_{j} \frac{e_{j}}{|r_{i} - R_{j}|} \right] + \sum_{i<i,j} \frac{e^{2}}{|r_{i} - r_{j}|} + \sum_{\mu=1}^{m} \sum_{\nu=1}^{m} |\mu\rangle \langle \nu| \sum_{\nu=1}^{m} V_{\mu\nu}(r_{i}) \langle \nu|.$$ (3.1)

Here, $-e$ is the electronic charge, $r_{i}$ the position variable of the $i$-th conduction electron, $R_{l}$ the (static) position of the $l$-th ion with positive charge $e_{l}$ (screened by the bound electrons), and $e_{l}$ and $R_{l}$ the charge and position of the $j$-th impurity, considered to be static. Thus $H_{0}$ contains the kinetic energy, the interaction of the electrons with the ions and the static impurities, as well as the electron-electron interaction, for which no approximation is assumed for the time being. It also contains, in the last two terms of Eq. (3.1), the intrinsic Hamiltonian of the PB and its interaction with the electrons, of the type introduced in the previous section for one electron. We do not consider a static magnetic field to be present, so that the problem is time-reversal invariant.

We denote by $\rho_{0}(r)$ and $\phi_{0}(r)$ the equilibrium charge density and potential, respectively, that satisfy Poisson’s equation
Application of an external voltage with frequency $\omega$ will cause a current density $J_0^\omega (r)$ in the system. It will also lead to a change in the charge density and in the potential. We denote these changes by $\delta \rho^\omega (r)$ and $\delta \phi^\omega (r)$ and emphasize that these are full changes, with respect to the equilibrium values $\rho_0(r)$ and $\phi_0(r)$. In the RPA approximation, which is employed in the present paper, there is no need to separate the full changes into external and induced parts. The essence of the RPA is to omit the electron-electron interactions from the Hamiltonian $H_0$ but to use, instead, the full potential $\delta \phi^\omega (r)$ (rather than the external one) in the formulation of LRT. The full potential is then determined self-consistently from the Poisson equation. We thus have the three following equations:

$$
\nabla^2 \phi_0(r) = -4\pi \rho_0(r).
$$

(3.2)

The Poisson equation (3.2) should be supplemented by boundary conditions. Sufficiently far inside the horns the local current density $J_0^\omega (r)$ becomes vanishingly small, so that these distant regions remain practically in equilibrium. This means that well inside the horns $\delta \phi^\omega (r)$ approaches constant values, $\delta \phi^\omega (-\infty)$ and $\delta \phi^\omega (+\infty)$. The difference

$$
\delta \phi^\omega (-\infty) - \delta \phi^\omega (+\infty) = V^\omega
$$

(3.9)

is the total potential drop on the sample system+horns ($V^\omega$ generally does not coincide with the external EMF, since some voltage drop can occur in other parts of the circuit, e.g., near the points where the external EMF source is connected to the horns). At the internal boundary of the system one should require zero current density normal to the boundary

$$
J_n(r_s) = 0,
$$

(3.10)

$r_s$ being a point on the internal boundary.

Solving (3.3), (3.4), (3.5) with the boundary conditions (3.9), (3.10) would enable one to determine the charge, current density and field distribution within the system. One could then compute the total current $I^\omega$ and, thus, the conductance $G^\omega = I^\omega /V^\omega$. This is a formidable problem. A great simplification, however, occurs in the dc limit, thanks to a result obtained in Ref. 15 that states that, in that limit and for fixed $r$, $r'$, the conductivity tensor is divergenceless; i.e.

$$
\partial_\beta \sigma^\omega_{\alpha\beta} (r, r') \to 0,
$$

(3.11)

The conductivity tensor relates current density to the electric field [rather than to the potential, as in equation (3.3)]; i.e.

$$
J_\alpha^\omega(r) = -\int d^3 r' \sigma^\omega_{\alpha\beta} (r, r') \partial_\beta \delta \phi^\omega (r'),
$$

(3.12)

and is given in terms of the current-current correlation function as

$$
\sigma^\omega_{\alpha\beta} (r, r') = \frac{1}{\hbar \omega} \int_0^\infty d\tau e^{i\omega\tau - \gamma \tau} \left\langle \left[ \tilde{j}_\alpha (r, \tau), \tilde{j}_\beta (r', 0) \right] \right\rangle_0
$$

$$
-\frac{e^2 n_0(r)}{im \omega} \delta(r-r') \delta_{\alpha\beta},
$$

(3.13)

$n_0(r)$ being the electron density in equilibrium. Integrating Eq. (3.12) by parts shows that, in the dc limit, the current density is insensitive to the full potential profile within the sample and depends only on the total potential drop between the two distant surfaces well inside the horns (Fig. 1):

$$
J_\alpha^{\omega \rightarrow 0}(r) = - \left[ \delta \phi^{\omega \rightarrow 0}(+\infty) \Gamma_\alpha^+ (r) + \delta \phi^{\omega \rightarrow 0}(-\infty) \Gamma_\alpha^- (r) \right],
$$

(3.14)

where

$$
\Gamma_\alpha^\pm (r) = \int_{S_L} dS_{\beta} \sigma^\omega_{\alpha\beta}^{-\omega \rightarrow 0} (r, r').
$$

(3.15)

This observation paves the way for a derivation of a Landauer formula from the LRT, and demonstrates that interactions, within RPA, do not affect the conductance. This conclusion, known for purely elastic scatterers, remains valid also in presence of the PB.

For zero temperature, and in the absence of PB, the answer is the well known one
\[ G = \frac{e^2}{\hbar} \sum_{ab} |t_{ab}|^2 , \] (3.16)

where \( t \) is the single-particle transmission matrix at the Fermi energy, from well inside the left horn to deep inside the right one.

When the system is described by the unperturbed Hamiltonian \( H_{00} \) of Eq. (3.8), the electrons can change the state of the PB through the e-PB interaction represented by the last term in Eq. (3.1): the coupling of the electrons with the PB has the consequence that the former are no longer independent. This results in a complicated structure for the \( N \)-electron eigenstates of \( H_{00} \), and the calculation of \( \sigma_{\alpha\beta}(r, \mathbf{r}') \) is, in principle, no longer feasible along the lines followed in the absence of the PB. In the next section we discuss a model for the e-PB interaction that does lead to a solution along similar lines. One might speculate on physical grounds that our final result, equation (4.34), should be valid under the following assumptions. One should first take the temperature to be very low, but much larger than the Kondo temperature, \( T_K \), due to the interaction of the PB with the electron gas. Furthermore, one should stay in the strict linear response regime, where for a finite distance \( L \) between the two reservoirs, the current satisfies:

\[ e/I \gg L/v_F. \] (3.17)

This assures that the separation in time between consecutive electrons participating in the transport is so large that the first electron reaches the downstream reservoir and thermalizes there before the next electron starts its journey. This can be expected to eliminate the electron-electron interaction mediated by the PB, which is a coherent second-order process.

**IV. A SOLUBLE MODEL**

We assume that \( H_{00} \) of Eq. (3.8) has such a structure that, in a suitable PB basis, labelled by \( \sigma, \sigma' \) below, it acquires the diagonal form

\[ \mathbf{T}^{\sigma\sigma'}_{00} = \left\{ \sum_i \left[ \frac{p_i^2}{2m} + U^\sigma(r_i) \right] + \Delta^\sigma \right\} \delta_{\sigma\sigma'} . \] (4.1)

Just as we did in Sec. II, we write the Hamiltonian in matrix form, where rows and columns, i.e. \( \sigma, \sigma' \), label the \( m \) states of the PB. The bar in \( \mathbf{T}^{\sigma\sigma'}_{00} \) indicates that the Hamiltonian is expressed in the new PB basis, that we shall call, for short, the D-basis (D for diagonal), as opposed to the original, or ND-basis.

In the ND basis, \( H_{00} \) is obtained from \( \mathbf{T}^{\sigma\sigma'}_{00} \) by means of a constant, real (in order to preserve reality of the Hamiltonian, and hence time-reversal invariance) orthogonal transformation \( O \) (an \( m \)-dimensional matrix), i.e.

\[ H_{00} = O \mathbf{T}^{\sigma\sigma'}_{00} O^T , \] (4.2)

or, in terms of its matrix elements

\[ H_{00}^{\mu\nu} = \sum_i \left[ \frac{p_i^2}{2m} \delta_{\mu\nu} + \sum_{\sigma=1}^m O^{\mu\sigma} U^\sigma(r_i) O^{\nu\sigma} \right] + \sum_{\sigma=1}^m O^{\mu\sigma} \Delta^\sigma O^{\nu\sigma} . \] (4.3)

We choose \( \Delta^\sigma \) constant, i.e. independent of \( \sigma \) (and hence we set it equal to zero), so as not to have constant terms in the off-diagonal matrix elements \( \mu \neq \nu \). In the language of Eq. (3.3), the energies \( E_{0\mu} \) of the PB states are degenerate and set equal to zero.

The Schrödinger equation in the D and ND-basis is

\[ \mathbf{H}_{00} \Psi = E \Psi , \] (4.4)

\[ H_{00} \Psi = E \Psi , \] (4.5)

respectively, with

\[ |\Psi\rangle = O |\overline{\Psi}\rangle . \] (4.6)

In the D-basis, the Hamiltonian \( \mathbf{T}^{\sigma\sigma'}_{00} \) can be written as

\[ \mathbf{T}^{\sigma\sigma'}_{00} = \sum_i \mathbf{T}^{\sigma\sigma'}_{00}(i) = \mathbf{T}^{\sigma\sigma'}_{00}(i) \delta_{\sigma\sigma'} , \] (4.7)

with

\[ \mathbf{T}^{\sigma\sigma'}_{00}(i) = \mathbf{T}^{\sigma\sigma'}_{00}(i) \delta_{\sigma\sigma'} \] (4.8)

and

\[ \mathbf{T}^{\sigma\sigma'}_{00}(i) = \frac{p_i^2}{2m} + U^\sigma(r_i) \] (4.9)

In the ND basis,

\[ H_{00}^{\mu\nu} = \sum_i H_{00}^{\mu\nu}(i) , \] (4.10)

with

\[ H_{00}^{\mu\nu}(i) = \frac{p_i^2}{2m} \delta_{\mu\nu} + U^{\mu\nu}(r_i) \] (4.11)

and

\[ U^{\mu\nu}(r_i) = \sum_{\sigma=1}^m O^{\mu\sigma} U^\sigma(r_i) O^{\nu\sigma} . \] (4.12)

If the matrix \( M^{\mu\sigma} = |O^{\mu\sigma}|^2 \) has nonzero determinant, we can find the \( U^\sigma(r_i) \)'s (\( \sigma = 1, \ldots, m \)) that reproduce any given set of diagonal potentials \( U^{\mu\nu}(r_i) \)'s (\( \mu = 1, \ldots, m \)); but then we are left with no freedom to select the off-diagonal interactions \( U^{\mu\nu}(r_i) \) (\( \mu \neq \nu \)), which become uniquely determined by the \( U^\sigma(r_i) \)'s. So, it is clear that the matrix elements \( U^{\mu\nu}(r_i) \) of Eq. (4.12) show strong correlations among themselves. These correlations make it possible to find a D-basis in which the Hamiltonian takes the form of Eqs. (4.7)-(4.9) and the problem breaks.
up into \( m \) independent single-particle ones: this is the feature that makes the problem soluble. Solving the Schrödinger equation (4.12) thus reduces to solving the \( m \) single-particle Schrödinger equations

\[
\left[ \frac{p^2}{2m} + U^\sigma(r_i) \right] \psi^\sigma_k(r_i) = \epsilon^\sigma_k \psi^\sigma_k(r_i), \quad \sigma = 1, \cdots, m.
\]  

(4.13)

We shall assume that none of the \( U^\sigma(r_i) \)'s admits bound states.

In the D-basis, the states

\[
|\Psi_k^\sigma\rangle = \begin{bmatrix}
0 \\
\vdots \\
0 \\
\vdots \\
0 \\
\end{bmatrix}
\]

(4.14)

with a nonzero value in the \( \sigma \)-th entry, form a complete set of orthonormalized (in a \( \delta \)-function sense) single-particle states, eigenfunctions of the single-particle Hamiltonian matrix (4.13).

In the D-basis, the \( S \) matrix associated with a scattering solution has the form

\[
S^{\sigma\sigma'} = S^\sigma \delta_{\sigma\sigma'}.
\]  

(4.15)

If the problem admits \( N \) open spatial channels, each \( S^\sigma \) is \( 2N \)-dimensional. In the ND-basis \( S \) takes the form

\[
S^{\mu\nu} = \sum_{\sigma=1}^{m} O^\mu\sigma S^\sigma O^{\nu\sigma}
\]  

(4.16)

and is \( 2mN \)-dimensional. The number of independent parameters associated with each unitary symmetric matrix \( S^\sigma \) is \( N(2N+1) \) and is thus \( mN(2N+1) \) for the total \( S \) and hence for \( S \). This makes it clear that the \( S \) matrices allowed by our soluble model do not have the "generic" structure used in some of the considerations of Sec. 4.1, but have a rather restricted one: in fact, a generic \( mN \)-dimensional \( S \)-matrix has a larger number, i.e. \( mN(2mN+1) \), of independent parameters.

For two particles, say, we have the states (not antisymmetrized yet), with \( \sigma = \pm 1 \)

\[
|\Psi^{\pm}_{k_1,k_2}\rangle = \begin{bmatrix}
\psi^{\pm}_{k_1}(r_1) \psi^{\pm}_{k_2}(r_2) \\
0 \\
\vdots \\
0 \\
\vdots \\
0 \\
\end{bmatrix}
\]  

(4.17)

To antisymmetrize we use a second-quantization language, so that for \( N \) electrons we have the states

\[
|\Psi^{\pm}_{k_1\cdots k_N}\rangle = (c^\sigma_{k_1})^\dagger \cdots (c^\sigma_{k_N})^\dagger |0\rangle,
\]  

(4.18)

with \((c^\sigma_{k})^\dagger\) creating one electron in state \( \psi^\sigma_k(r) \) and \( |0\rangle \) being the electron vacuum.

1. The conductivity tensor and the conductance

The conductivity tensor \( \sigma^\omega_{\alpha\beta}(r,r') \) is given in Eq. (4.13). The expectation value occurring in that equation has to be understood as

\[
\left\langle \left[ j_\alpha(r,\tau), j_\beta(r',0) \right] \right\rangle_{00} = \sum_{NM\sigma} P(NM;\sigma) \left\langle \left[ \tilde{j}_\alpha(r,\tau), \tilde{j}_\beta(r',0) \right] \right\rangle_{NM;\sigma}.
\]  

(4.19)

Here, \( |N;\sigma\rangle \) are the states of Eq. (4.18) in the D PB basis, \( N \) being the number of electrons and \( M \) an abbreviation for the configuration \( k_1, k_2, \cdots, k_N \). The states \( |NM;\sigma\rangle \) are eigenstates of the Hamiltonian \( \overline{H}_{00} \) of Eq. (4.7), with the energy \( E_{NM;\sigma} = \epsilon^\sigma_{k_1} + \cdots + \epsilon^\sigma_{k_N} \).

The current operator \( j_\alpha(r,\tau) \) in the interaction representation is given by

\[
\tilde{j}_\alpha(r,\tau) = e^{i\hat{\Pi}^\omega_{00}\tau} j_\alpha(r) e^{-i\hat{\Pi}^\omega_{00}\tau}.
\]  

(4.20)

In the D PB basis the Hamiltonian \( \overline{H}_{00} \) has the diagonal form given by Eq. (4.7)-(4.9), since the current operator \( j_\alpha(r) \) does not depend on the PB explicitly, \( \tilde{j}_\alpha(r,\tau) \) takes the diagonal form

\[
\tilde{j}^\sigma_{\alpha}(r,\tau) = \tilde{j}^\sigma_{\alpha}(r,\tau) \delta_{\sigma\sigma'},
\]  

(4.21)

where

\[
\tilde{j}^\sigma_{\alpha}(r,\tau) = e^{i\hat{\Pi}^\omega_{00}\tau} j_\alpha(r) e^{-i\hat{\Pi}^\omega_{00}\tau},
\]  

(4.22)

\( \overline{H}'_{00} \) being given by Eq. (4.7).

Definition (4.19) implies, as usual, that the density matrix is diagonal in the representation in which the Hamiltonian is diagonal, with diagonal elements \( P(NM;\sigma) \). Explicitly, \( P(NM;\sigma) \) is given by the grand-canonical ensemble (understanding now the labels \( N, M \) as the set of occupation numbers \( n_1, n_2, \cdots \) ) as
where the grand partition function is
\[ Z(\beta, \mu) = \sum_{\sigma=1}^{m} Z(\beta, \mu; \sigma), \] (4.24)
with
\[ Z(\beta, \mu; \sigma) = \sum_{n_i=0,1} e^{-\beta \sum_{i=1}^{\infty} n_{i}(\epsilon_{i}^{\sigma} - \mu)}. \] (4.25)

We define the conditional occupation probability
\[ P(n_1, n_2, \ldots | \sigma) = \frac{e^{-\beta \sum_{i=1}^{\infty} n_{i}(\epsilon_{i}^{\sigma} - \mu)}}{Z(\beta, \mu; \sigma)}, \] (4.26)
the condition being that the PB be precisely in the state \( \sigma \) of the D basis; its trace is 1. Then
\[ P(n_1, n_2, \ldots ; \sigma) = p(\sigma)P(n_1, n_2, \ldots | \sigma), \] (4.27)
where
\[ p(\sigma) = \frac{Z(\beta, \mu; \sigma)}{Z(\beta, \mu)} \] (4.28)
is the probability of finding the PB in state \( \sigma \), the temperature and chemical potential, not indicated explicitly, being \( \beta, \mu \). Of course we have
\[ \sum_{\sigma=1}^{m} p(\sigma) = 1, \] (4.29)
and so the trace of (4.27) is 1.

We can thus write the expectation value (4.19) as
\[ \left\langle \left[ j_{\alpha}(r, \tau), \bar{\rho}_{\beta}(r', 0) \right] \right\rangle_{00} = \sum_{\sigma=1}^{m} p(\sigma) \sum_{n_1, n_2, \ldots} P(n_1, n_2, \ldots | \sigma) \times \langle n_1, n_2, \ldots ; \sigma | \left[ \bar{\rho}_{\alpha}(r, \tau), \bar{\rho}_{\beta}^{\dagger}(r', 0) \right] | n_1, n_2, \ldots ; \sigma \rangle. \] (4.30)

Thus in the D PB basis the problem breaks up into \( m \) independent problems, for the Hamiltonians \( \mathcal{H}_{00}(i) \), \( \sigma = 1, \ldots, m \). The conductivity tensor \( \sigma_{\alpha, \beta}^{\nu, \mu}(r, r') \) of Eq. (3.13) can thus be written as
\[ \sigma_{\alpha, \beta}^{\nu, \mu}(r, r') = \sum_{\sigma=1}^{m} p(\sigma) \sigma_{\alpha, \beta}^{\nu, \sigma}(r, r'), \] (4.31)
where \( \sigma_{\alpha, \beta}^{\nu, \sigma}(r, r') \) is a conductivity tensor that can be expressed entirely in terms of single-electron Green’s functions\[3\] for the Hamiltonian \( \mathcal{H}_{00}(i) \).

We write the conductance \( G \) in terms of the “dimensionless conductance” \( g \) as
\[ G = \frac{e^2}{h} g. \] (4.32)
As \( \omega \to 0 \) and then the temperature \( \to 0 \), we find, for “spinless electrons”
\[ g = T \sum_{\sigma=1}^{m} p(\sigma) tr \left( \left[ \bar{T}^{\sigma} \right]^\dagger T^{\sigma} \right), \] (4.33)
\( \bar{T}^{\sigma} \) being the transmission matrix (an \( N \times N \) block of the matrix \( \mathcal{S}^{\sigma} \) of Eq. (4.15)) arising from the potential \( U^{\sigma}(r) \); the trace in the above equation is over spatial channels, as usual. Should \( U^{\sigma}(r) = U(r) \), i.e. independent of \( \sigma \) (and hence \( U^{\mu\nu}(r) = U(r)\delta_{\mu\nu} \) in any basis), the above formula would go over into the standard one. In the ND PB basis we finally find (\( a, b \) being spatial-channel indices)
\[ T = \sum_{\mu, \nu, \nu'} \sum_{a, b} \rho_{PB}^{\mu \nu} \left[ \omega_{ab}^{\nu \mu} \right]^* \omega_{ab}^{\nu' \nu} \] (4.34)
the trace being now over the spatial channels and the PB states. We have defined
\[ \rho_{PB}^{\mu \nu} = \sum_{\sigma} O^{\mu \sigma} p(\sigma) O^{\sigma \nu}. \] (4.35)
Eq. (4.34) has the structure of the standard Landauer formula, with an extra average over the PB states. This is also the structure of Eq. (2.13), that was obtained in the study of a single electron interacting with a PB with equal weights assigned to every PB state.

Should there be circumstances where the various \( p(\sigma) \) discussed above, at zero temperature, were all equal to \( 1/m \), we could write
\[ T = \frac{1}{m} \sum_{\mu, \nu=1}^{m} \sum_{a, b} |\omega_{ab}^{\nu \mu}|^2. \] (4.36)

We recall that it is for an e-PB interaction of the type described in Eq. (4.12) that
1) the many-electron Hamiltonian (with the e-e interaction switched off) breaks up, in the D basis, into \( m \) independent single-electron Hamiltonians, and hence
2) the conduction problem breaks up in a similar manner and is soluble in terms of single-electron quantities.
We also remind the reader that within our model the single-e-PB \( S \) matrix, and hence the transmission amplitudes appearing in Eq. (4.34), are not “generic”, but have a rather restricted structure.

The final answer, though, i.e. Eq. (4.34), is a very appealing one and is likely to be valid beyond the situation envisaged by the present model, i.e. to cases where
the e-PB $S$ matrix has a more general structure. However, in such a generic case it is not known to us under what approximations the conduction problem – even with the e-e Coulomb interaction switched off – could still be reduced to independent single-electron problems and thus expressed in terms of single-electron quantities. The best we can do at present is conjecture the validity of Eq. (4.34) for a generic e-PB $S$ matrix, under suitable approximations, as explained at the end of section III around equation (3.17).

V. A RANDOM-MATRIX MODEL FOR THE SCATTERING MATRIX

In the past, quantum electronic transport in mesoscopic systems has been described successfully in terms of ensembles of single-electron $S$ matrices (see, for instance, Refs. 6, 7, 11, 12, 16–18). In particular, it was shown that, in the absence of direct processes, quantum transport through classically chaotic cavities can be studied in terms of the invariant measure for the $S$ matrix, which is a precise mathematical formulation of the intuitive notion of “equal-a-priori-probabilities” in $S$-matrix space. For TRI systems, like the ones we are studying here, the invariant measure is also known as the Circular Orthogonal Ensemble (COE). The COE for $S$ matrices of dimensionality $2N$, $N$ being the number of open channels supported by the two leads attached to the cavity, gives, for the ensemble averaged (indicated by brackets $\langle \cdots \rangle$) spinless conductance and its variance,24

$$\langle T \rangle = \frac{N^2}{2N+1},$$  \hfill (5.1)

$$\text{var}(T) = \frac{N(N+1)^2}{(2N+1)^2(2N+3)},$$ \hfill (5.2)

respectively. The 1 in the denominator of Eq. (5.1) is the weak-localization correction (WLC).

We construct here an ensemble of $S$ matrices for the system consisting of a single electron and the PB, and, using the conductance formula obtained in the previous section, Eq. (4.34), we analyze the effect of the PB on the average and variance of the conductance.

Within the model we have been discussing for the e-PB system, we postulate $m$ independent COE’s for the $S$ matrices $S$ of Eq. (4.33). Ensemble averaging Eq. (4.34) we obtain

$$\langle T \rangle_m = \sum_{\sigma=1}^{m} p(\sigma) \langle \text{tr} \left[ (T^\dagger)^\dagger T \right] \rangle$$

$$= \sum_{\sigma=1}^{m} p(\sigma) \frac{N^2}{2N+1} = \frac{N^2}{2N+1},$$ \hfill (5.3)

the same result as in Eq. (5.1), in the absence of the PB. Thus the model is not generic enough to decrease the WLC. For the variance we obtain

$$[\text{var}(T)]_m = \sum_{\sigma=1}^{m} \langle p(\sigma)^2 \text{var} \left\{ \text{tr} \left[ (T^\dagger)^\dagger T \right] \right\} \rangle$$

$$= \frac{N(N+1)^2}{(2N+1)^2(2N+3)} \sum_{\sigma=1}^{m} \langle p(\sigma)^2 \rangle.$$ \hfill (5.4)

If one PB state $\sigma_0$ has probability 1 and all the others 0, $\sum_{\sigma=1}^{m} \langle p(\sigma)^2 \rangle = 1$. In the other extreme case of equiprobable PB states, $\sum_{\sigma=1}^{m} \langle p(\sigma)^2 \rangle = 1/m$. The conductance thus fluctuates less than in the absence of the PB, as expected.

Should the conjecture we made at the end of last section be true, and the result (4.34) be valid more generally, i.e. for a generic e-PB $S$ matrix, we could postulate a COE for the full $2mN$-dimensional $S$ matrix in the ND basis. We would then obtain, ensemble averaging Eq. (4.34)

$$\langle T \rangle_m = \sum_{\mu \nu \nu'} \sum_{ab} \rho^\mu\nu_{PB} \langle \left[ t_{ab}^\mu \right]^* t_{ab}^{\nu'} \rangle$$ \hfill (5.5)

From Ref. 12 we find

$$\langle \left[ t_{ab}^\mu \right]^* t_{ab}^{\nu'} \rangle = \frac{\delta_{\mu \nu}}{2mN+1}$$

and, since $\sum_{\nu} \rho^\mu\nu_{PB} = 1$, we finally obtain

$$\langle T \rangle_m = \frac{N^2}{2N+1}.$$ \hfill (5.6)

Now we observe that, as $m \to \infty$, the effect of the PB is to kill the WLC, as expected.

We find the variance of $T$ only in the simplified situation described by Eq. (4.36)

$$[\text{var}(T)]_m = \frac{1}{m^2} \text{var} \left[ \sum_{\mu \nu \nu'=1}^{m} \sum_{ab} |\rho_{ab}^\mu \nu|^2 \right].$$ \hfill (5.7)

The variance appearing on the right hand side of this last equation can be read form Eq. (4.12), with the replacement $N \Rightarrow mN$, to find

$$[\text{var}(T)]_m = \frac{N(N+\frac{1}{m})^2}{(2N+\frac{1}{m})^2(2N+\frac{4}{m})}.$$ \hfill (5.8)

For large $m$, conductance fluctuations are killed as well, as expected.
VI. CONCLUSIONS

We have discussed the quantum electronic conduction problem in a mesoscopic system, allowing for the presence of a phase breaker (PB) the system can interact with. The PB can exist in m quantum-mechanical states. It could represent an impurity with internal degrees of freedom, so that its state may change via the interaction with the electrons: for instance, it could be a magnetic impurity interacting with the electron spin. The PB might also represent the environment—for instance, the phonon field—whose state is allowed to change.

We first studied the problem of a single electron interacting with the PB, in order to investigate the effect of the latter in the interference terms that are there in the absence of the PB. We described the static, as well as the dynamical scatterers (the PB), in terms of their scattering or S matrices: this makes the discussion very intuitive, quite general, and amenable to the application of random-matrix models that have been developed in the past. We found an S-matrix formulation that is capable of describing the crossover from a purely coherent response to a classical, or incoherent one.

We then set out to study the conduction problem—a many-electron problem. The e-e interaction is treated in an RPA approximation. We could make the very general statement (that so far, to our knowledge, was known only in the absence of a PB) that in the dc limit one can give an explicit expression for the current in terms of the potential difference applied between the two reservoirs, the full potential profile not being needed. That expression involves the conductivity tensor \( \sigma^{\alpha\beta}(r,r') \) which, in the absence of the PB, can be calculated in terms of single-particle Green’s function and, eventually, single-particle transmission coefficients. In the presence of the PB, the e-e interaction induced by the PB has not allowed us to follow a similar path. That induced interaction is there even in the absence of the e-e Coulomb interaction, which, within RPA, was disposed of and replaced, in turn, by the solution of a self-consistent problem.

We proposed a model for the e-PB interaction that can be diagonalized and transformed into m single-particle problems. The conduction problem splits into m single-particle problems as well, and the final result for the conductance, given in Eq. (4.34), is like the standard one without a PB, except that now an extra trace over the m PB states appears. We should stress that, within the model, result (4.34) is not perturbative in the e-PB interaction strength. However, the model implies a single-electron-PB S matrix of a rather restricted form. For a “generic” single-electron-PB S matrix we do not even know under what approximations (that would imply disregarding the induced e-e interaction) the conduction problem can be reduced to a single-electron one. The result expressed in Eq. (4.34) is so intuitive, though, that we conjecture its validity under a more general e-PB S matrix, within some suitable approximation, as discussed at the end of section III around equation (3.17).

It would be instructive to solve this same conduction problem within the spirit of Landauer’s approach, just as in Ref. 1 and verify that one arrives at the result (4.34) under the special model used here and not in general. A suitable approximation for treating the more general problem might suggest itself in that approach. But this we have not succeeded to do so far.

Finally, a random-matrix model was setup for the description of the e-PB system, with possible applications to chaotic cavities: the effect of the PB on the conductance average and its fluctuations was analyzed. The limitations of the model became apparent in that study; in contrast, a generic e-PB S matrix was shown to give much more freedom in the description of the weak-localization correction and the conductance fluctuations.

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