The Extended Analysis On New Generalized Chaplygin Gas

WANG Jun*, WU Ya-Bo†, WANG Di, YANG Wei-Qiang
Department of Physics, Liaoning Normal University, Dalian 116029

We will extend the study of the new generalized Chaplygin gas (NGCG) based on [JCAP 0601(2006)003]. Concretely, we will not only discuss the change rates of the energy densities and the energy transfer of this model, but also perform the Om diagnostic to differentiate the ΛCDM model from the NGCG and the GCG models. Furthermore, in order to consider the influence of dark energy on the structure formation, we also present the evolution of growth index in this scenario with interaction.

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Recent observations of type Ia supernovae (SNe Ia)[1, 2] indicate that the expansion of the Universe is accelerating at the present time. From the observations of large scale structure (LSS)[3], we know that the Universe is spatially flat. The Wilkinson Microwave Anisotropy Probe (WMAP) indicates that our Universe components are as follows: usual baryon matter occupies about 4%, dark matter occupies about 23% and dark energy occupies about 73%. It is clear that the Universe is dominated by an exotic component with large negative pressure, referred to as dark energy. So many scientists believe the accelerating expansion of the Universe is due to the dark energy.

Based on this opinion many dark energy models have been proposed. The simplest form of dark energy is cosmological constant Λ which would encounter "fine-tuning" problem and "coincidence" problem. Other valid dark energy models are provided by scalar fields, such as: Quintessence[4, 5], which is introduced to solve the "coincidence" problem and characterized by the equation of state (EOS) \( w_{de} \) between -1 and -1/3 (namely, \( -1 < w_{de} < -1/3 \)); Phantom (ghost) field[6], which owns a negative kinetic energy and characterized by the EOS \( w_{de} \) less than -1 (namely, \( w_{de} < -1 \)); Tachyon field[7, 8]which can act as a source of dark energy depending upon the form of the tachyon potential, and so on. Other scenarios on dark energy include brane world[9], generalized Chaplygin gas[10], holographic dark energy[11], etc.

In the generalized Chaplygin gas (GCG) approach, dark energy and dark matter can be unified by using an exotic equation of state. This point can be easily seen from the fact that the GCG model behaves as a dust-like matter at early times and as a cosmological constant at late stage. Since the EOS of dark energy still can’t be determined exactly, the observational data show the EOS of dark energy is in the range of \((-1.38, -0.86)\)[12, 13], the GCG model is generalized to accommodate any possible X-type dark energy with constant \( w_{de} \). The new generalized Chaplygin gas (NGCG) as a dark energy model is proposed for unification of X-type dark energy and dark matter[14]. The EOS

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* E-mail: wangjun_3@126.com

† E-mail: ybwu61@163.com
of this model is $p = -\ddot{A}(a)/\rho^\alpha$, where $a$ is the scale factor and $\ddot{A}(a) = -w_{de} A a^{-3(1+w_{de})(1+\alpha)}$. We know that the NGCG model behaves as a dust-like matter at early times and as a cosmological constant at late stage. Also this model is a kind of the interacting XCDM model\cite{14}. Based on the Ref.\cite{14}, in this paper we will extend the analysis on the NGCG model by discussing the change rates of the energy densities for dark energy (DE) and dark matter (DM) as well as the energy transfer. Moreover, we perform the $Om$ diagnostic to differentiate the $\Lambda$CDM model from the NGCG and the GCG models. Furthermore, in order to consider the influence of DE on the structure formation in this scenario, we present the evolution of growth index in this model with interaction.

In the framework of FRW cosmology, the EOS of the NGCG model is described as\cite{14}

$$p_{NGCG} = -\frac{\ddot{A}(a)}{\rho_{NGCG}^\alpha},$$

(1)

where $\alpha$ is a real number and $\ddot{A}(a)$ is a function depending on the scale factor of the Universe. The form of the function $\ddot{A}(a)$ is:

$$\ddot{A}(a) = -w_{de} A a^{-3(1+w_{de})(1+\alpha)},$$

(2)

where $A$ is a positive constant and $w_{de}$, which should be taken as any possible value in the range (-1.38, -0.86), is a constant. We can see explicitly that when $w_{de} = -1$, the EOS can reduce to the GCG scenario, while when $\alpha = 0$ the XCDM model can be obtained again.

In the following discussions we suppose that in the NGCG model the fluid is made up of two components, one is the DE component marked as $\rho_{de}$, and the other is the DM component marked as $\rho_{dm}$, i.e.

$$\rho_{NGCG} = \rho_{de} + \rho_{dm},$$

(3)

$$p_{NGCG} = p_{de}.$$  

(4)

Then, the energy density can be expressed as\cite{14}

$$\rho_{NGCG} = \rho_{NGCG0} a^{-3[1 - A_s a^{-3w_{de}(1+\alpha)}]^{-\alpha}},$$

(5)

where $A_s = \frac{\Omega_{de0}}{\Omega_{m0}}$. The energy densities of the DE and the DM components can be respectively expressed as

$$\rho_{de} = \rho_{de0} a^{-3[1+w_{de}(1+\alpha)]} [1 - A_s + A_s a^{-3w_{de}(1+\alpha)}]^{-\alpha-1},$$

(6)

$$\rho_{dm} = \rho_{dm0} a^{-3} [1 - A_s + A_s a^{-3w_{de}(1+\alpha)}]^{-\alpha-1}.$$  

(7)

Below we study the change rates of the energy densities of DE and DM. First, we consider the case without interaction between them. By means of the continuity equation, the energy densities of DM and DE can be respectively obtained as follows:

$$\rho_{dm} = \rho_{dm0} (1 + z)^3,$$

(8)

$$\rho_{de} = \{\rho_{NGCG0}[1 - A_s + A_s (1 + z)^{3w_{de}(1+\alpha)}]^{-\alpha} - \rho_{dm0}\} (1 + z)^3.$$  

(9)
Thus, the change rates of the energy densities of DE and DM can be obtained:

\[
\frac{d\rho_{de}}{dz} = 3\rho_{NGCG0}M, \tag{10}
\]

\[
\frac{d\rho_{dm}}{dz} = 3\rho_{dm0}(1+z)^2, \tag{11}
\]

where 

\[
M = (1+z)^2[1-A_s + A_s(1+z)^{3w_{de}(1+\alpha)}]^{\frac{1}{1-\alpha}}(1 + \frac{w_{de}A_s(1+z)^{3w_{de}(1+\alpha)}}{1-A_s+A_s(1+z)^{3w_{de}(1+\alpha})}) \quad \text{and} \quad A_s = \frac{\Omega_{de0}}{1-\Omega_b} = 0.74 \quad (\text{the current parameters used in this paper are: } \Omega_{dm0} = 0.25, \Omega_{de0} = 0.7, \text{ and } \Omega_b = 0.05). \]

It is easy to see that the sign of the value of \(\frac{d\rho_{de}}{dz}\) lies on \(M\). Hence the evolutional trend of DE density can be discussed by the factor \(M\) as well.

\[
\begin{align*}
\text{FIG. 1: The evolutional trends of the factor } M \text{ and } \frac{d\rho_{dm}}{dz} \text{ with } z. \text{ Here we choose } A_s = 0.74, \alpha = 0.5, w_{de} = -1.2.
\end{align*}
\]

The evolutional trends of the factor \(M\) and \(\frac{d\rho_{dm}}{dz}\) are illustrated in Fig. 1, from which we can see that when \(z > 0\), the change rate of the energy density of DE decreases faster than that of DM as the redshift \(z\) becomes low. When \(z < 0\), it keeps decreasing but \(\frac{d\rho_{dm}}{dz}\) would approach to 0.

In what follows we will consider the NGCG model with interaction between DE with constant \(w_{de}\) and DM. According to the Eqs. (6) and (7), we can give the expressions of the change rates of the energy densities of DE and DM as follows:

\[
\frac{d\rho_{de}}{dz} = 3\rho_{de0}W, \tag{12}
\]

\[
\frac{d\rho_{dm}}{dz} = 3\rho_{dm0}S, \tag{13}
\]

where 

\[
W = (1+z)^{3w_{de}(1+\alpha)+2}[1-A_s + A_s(1+z)^{3w_{de}(1+\alpha)}]^{\frac{1}{1-\alpha}}(1 + \frac{w_{de}A_s(1+z)^{3w_{de}(1+\alpha)}}{1-A_s+A_s(1+z)^{3w_{de}(1+\alpha})}),
\]

\[
S = (1+z)^2[1-A_s + A_s(1+z)^{3w_{de}(1+\alpha)}]^{\frac{1}{1-\alpha}}(1 - \frac{\alpha w_{de}A_s(1+z)^{3w_{de}(1+\alpha)}}{1-A_s+A_s(1+z)^{3w_{de}(1+\alpha)}}). \quad \text{From Eq.}(12), \text{ it is easy to see that the sign of the value of } \frac{d\rho_{de}}{dz} \text{ depends on the factor } W. \text{ Fig.2 shows the relation between factor } W \text{ and the redshift } z.
\]
From Fig. 2, it is easy to see that as the redshift \( z \) becomes low the change rate of the energy density of DE is from increasing gradually to decreasing sharply. Specially, when \( z > 4 \) it would approach to 0. It arrives at the maximum when \( z = 0.53 \). While when \( z = 0.2 \) it vanishes. It follows that the point of \( z = 0.53 \) is a transformation point. Note that there is not any transformation point in the case without interaction.

From the above discussions, we see explicitly that there would exist energy transfer between DE and DM. Now, we consider a feasible interaction with an ansatz \( \Gamma = 3Hc^2\rho \) [15], where \( \rho \) is the total energy density and \( c^2 \) is a constant denoting the transfer strength. We suppose the components of the NGCG interact through the interaction term \( \Gamma \). The components of the NGCG respectively satisfy

\[
\dot{\rho}_{de} + 3H(1 + w_{de})\rho_{de} = -\Gamma, \tag{14}
\]

\[
\dot{\rho}_{dm} + 3H\rho_{dm} = \Gamma. \tag{15}
\]

Note that \( \Gamma > 0 \), which implies there is an energy transfer from DE to DM. By using Eqs. (7), (13) and (15) as well as \( \frac{dz}{dt} = -(1 + z)H \), we can obtain:

\[
\dot{\Gamma} = -9H\rho_{dm0}A_s w_{de} \alpha B, \tag{16}
\]

where \( B = (1 + z)^3w_{de}(1+\alpha)^2[1 - A_s + A_s(1 + z)^3w_{de}(1+\alpha)]^{\frac{1}{1+\alpha}} - \frac{(1+2\alpha)A_s w_{de}(1+z)^3w_{de}(1+\alpha)}{1-A_s+A_s(1+z)^3w_{de}(1+\alpha)} \). From Eq. (16), we can see that the evolution of \( \dot{\Gamma} \) can be described by the factor \( B \). The relation between factor \( B \) and the redshift \( z \) is shown in Fig. 3.

According to Fig. 3, we can see that when \( z > 0.38 \) the transfer direction of the energy flow is from DM to DE and the quantity of energy transfer is from increasing gradually to decreasing sharply as the redshift \( z \) becomes low. But when \(-1 < z < 0.38 \) the transfer direction of the energy flow just reverses and the quantity of energy transfer is from increasing to decreasing sharply, until approaches to 0 at \( z = -1 \). Clearly, the point of \( z = 0.38 \) is the transformation
point and the quantity of energy transfer doesn’t change any more at this point. Specially, when \( z = 0.78 \), the quantity of energy transfer from DM to DE reaches to the maximum. While when \( z = -0.02 \), it reaches to the maximum from DE to DM. It follows that the above discussions not only show the evolutionary laws of the energy transfer in the past and at present, but also predict the situation in the future.

Below, we will determine the present value of \( c^2 \). The ratio of the energy density can be defined as \( \dot{r} = \frac{\rho_{dm}}{\rho_{de}} \). Thus by means of Eqs. (6), (7), (14) and (15) the evolution equation of ratio \( r \) can be expressed as

\[
\dot{r} = \frac{\rho_{dm}}{\rho_{de}} \left( \frac{\dot{\rho}_{dm}}{\rho_{dm}} - \frac{\dot{\rho}_{de}}{\rho_{de}} \right) = 3H[r^2\left(\frac{1}{\Omega_{dm}} + \frac{1}{\Omega_{de}}\right) + w_{de}],
\]

from which, the following expression is easily obtained:

\[
c^2 = \left[ \frac{-w_{de}(1 + \alpha)}{(1 + z)H} + w_{de}\right]\left(\frac{1}{\Omega_{dm}} + \frac{1}{\Omega_{de}}\right)^{-1}.
\]

Hence we can obtain the corresponding present values of \( c^2 \) to the different values of \( w_{de} \) and \( \alpha \) shown in Tab. I.

| \( w_{de} \) | -0.8 | -1  | -1.2 | \( \alpha \) | 0.0 | 0.2 | 0.5 |
|-------------|------|-----|------|-------------|-----|-----|-----|
| \( c^2 \)   | -0.0193076 | -0.0242937 | -0.0292797 | \( c^2 \)   | -0.0242937 | -0.0242937 | -0.0242937 |

(a) \( \alpha = 0.5 \)

(b) \( w_{de} = -1 \)

TABLE I: The corresponding present values of \( c^2 \). The current parameters are taken to be \( z = 0 \), \( \Omega_{dm0} = 0.25 \), \( \Omega_{de0} = 0.7 \), \( H_0 = 70.5[13] \).

From Tab.I(b), it is easy to see that the values of \( c^2 \) don’t change with \( \alpha \) just as the case in the \( \Lambda \)CDM model.

Moreover, we will use a new geometrical diagnostic method, \( Om \) diagnostic[20], to differentiate the \( \Lambda \)CDM model...
from the NGCG and the GCG models. The $Om$ diagnostic is defined as follows

$$Om(x) = \frac{E^2(x) - 1}{x^3 - 1}, x = 1 + z,$$

(19)

where $E^2(x) = H(x)/H_0 = (1 - \Omega_{b0})x^3[1 - A_s + A_s x^{3 w_{de}(1+\alpha)}]^{\frac{1}{1+\alpha}} + \Omega_{b0}x^3$ for the NGCG model.

For the $\Lambda$CDM model, $Om(z) = \Omega_{om}$ is a constant. It provides a null test of cosmology constant. The benefit of $Om$ diagnostic is that the quantity $Om(x)$ can distinguish DE models with less dependence on matter density $\Omega_{m0}$ relative to the EOS of DE[20]. We can get the expression of $Om(x)$ in the NGCG model as

$$Om(x) = \frac{(1 - \Omega_{b0})x^3[1 - A_s + A_s x^{3 w_{de}(1+\alpha)}]^{\frac{1}{1+\alpha}} + \Omega_{b0}x^3 - 1}{x^3 - 1}$$

(20)

where $A_s = 0.74$. It follows that when taking $\alpha = 0.5, w_{de} = -1$, Eq.(20) can reduce into the $Om(x)$ of the GCG model, but when taking $\alpha = 0, w_{de} = -1$, it corresponds to the $Om(x)$ of the $\Lambda$CDM model. For the NGCG model, the curve of $Om(x)$ corresponding to $\alpha = 0.5, w_{de} = -1.2$ is plotted in Fig.4.

![Diagram](attachment:diagram.png)

FIG. 4: The $Om(x)$ evolution diagram. The current parameters are taken to be $\Omega_{dm0} = 0.25, \Omega_{de0} = 0.7, \Omega_{b0} = 0.05$ and $A_s = 0.74$.

The Fig.4 shows that when $x \geq 4$ the evolutionary trajectory of the NGCG model is similar to one of the GCG model, but in the recent period the both become very different. It follows that the both NGCG and GCG models are completely different from the $\Lambda$CDM model.

Furthermore, in order to study the influence of the NGCG model on the structure formation, now we discuss the growth index of the NGCG model. The growth index is an important quantity to test a model, which can be measured by the galaxy correlation function or the peculiar velocities. Its definition is as follows:

$$f \equiv \frac{\frac{d\ln \delta}{d\ln a}}{\frac{\delta}{\delta a}}$$

(21)
where \( \delta \equiv \frac{\delta \rho_m}{\rho_m} \) is the matter density contrast and \( a \) is the scale factor of the Universe.

According to the perturbation equation[16], the growth index \( f \) satisfies the following equation:

\[
f' + \frac{f^2}{a} + \left( \frac{2}{a} - 1 \right) f - \frac{3}{2a} \Omega_m = 0,
\]

where the prime denotes the derivation with respect to the scale factor \( a \). In the NGCG model, we take \( \Omega_m = \Omega_{dm} + \Omega_b \), the corresponding expression of \( \Omega_{dm} \) and \( \Omega_b \) reads[14]:

\[
\Omega_{dm} = \Omega_{dm0} E^{-2} a^{-3} \left[ 1 - A_s + A_s a^{-3 w_{de} (1 + \alpha)} \right]^{1/\alpha - 1},
\]

\[
\Omega_b = \Omega_{b0} E^{-2} a^{-3},
\]

where \( E = \left( (1 - \Omega_{b0}) a^{-3} \left[ 1 - A_s + A_s a^{-3 w_{de} (1 + \alpha)} \right]^{1/\alpha} + \Omega_{b0} a^{-3} \right)^{1/2} \) and \( A_s = 0.74 \).

The evolutionary trajectory of the growth index \( f \) for the NGCG model is plotted in Fig.5. By calculations, the value of \( f \) for the NGCG model is 0.592 at \( a = 0.87 \) (namely, \( z = 0.15 \)). From the 2dF galaxy redshift survey (2dFGRS)[17–19], we know that the observational value of \( f \) is \( f = 0.51 \pm 0.1 \) or \( f = 0.58 \pm 0.11 \) at the effective redshift of the survey \( z = 0.15 \). It means the theoretical value of growth index given by us is consistent with observation. In Fig.5, we also give the evolutionary trajectories of growth indices of the GCG model (namely, \( \alpha = 0.5, w_{de} = -1 \)) and the \( \Lambda \)CDM model (namely, \( \alpha = 0, w_{de} = -1 \)). By comparing these three evolutionary trajectories in Fig.5, we can find that at early times, the growth indices of these three models are the same; but near \( a = 0.09 \), the evolutionary trajectory of \( \Lambda \)CDM model would be different from the other two; while near \( a = 0.2 \), the evolutionary trajectory of the GCG model would deviate from one of the NGCG model.
In summary, we have extended the analysis on the new generalized Chaplygin gas (NGCG) model as the unification of DM and DE. Concretely, we have studied the evolutional trends of the energy densities, $\Omega_m$ diagnostic and the growth index in the NGCG model. By complicated calculations and analysis, we can give some comments as follows:

1. In the NGCG model without interaction between DM and DE, the change rate of the energy density of DE decreases gradually as the redshift $z$ becomes low, but it would be from increasing gradually to decreasing sharply in the case with interaction, while $z = 0.53$ is a transformation point.

2. For the NGCG model with interaction term $\Gamma = 3He^2\rho$, the evolutionary laws of the energy transfer have been discussed. Furthermore, we have also given the present values of $c^2$ when taking a fixed constant $\alpha$ (or $w_{de}$), but different values of $w_{de}$ (or $\alpha$). The results show that for a fixed $w_{de}$, the present values of $c^2$ don’t change with $\alpha$, which is as the same as one of the $\Lambda$CDM model.

3. We have performed the $\Omega_m$ diagnostic to the NGCG model and shown the discrimination among the NGCG, the GCG and the $\Lambda$CDM models.

4. The evolutionary trajectory of the growth index $f$ has also been illustrated for the NGCG model with interaction. We find the value of $f$ at $a = 0.87$ (i.e., $z = 0.15$) given by us agrees with the observational value of $f = 0.51 \pm 0.1$ or $f = 0.58 \pm 0.11$ at the effective redshift of the survey $z = 0.15$. By comparing the NGCG and the GCG models with the $\Lambda$CDM model, we have shown that at early times, the growth indices of these three models are the same. Near $a = 0.09$, the evolutionary trajectory of $\Lambda$CDM model would be different from ones of the other two. Near $a = 0.2$, the evolutionary trajectory of GCG model would deviate from one of the NGCG model.

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