Effective lagrangian description of top production and decay

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Abstract

We propose a rather general description of residual New Physics (NP) effects on the top quark couplings. These effects are described in terms of 20 gauge invariant $\dim = 6$ operators involving gauge and Higgs bosons as well as quarks of the third family. We compute their implications for the $\gamma t\bar{t}$, $Z t\bar{t}$ and $tbW$ vertices and study their observability in the process $e^- e^+ \rightarrow t \bar{t}$ with $t \rightarrow bW \rightarrow b\ell^+ \nu_\ell$. We present results for the integrated cross section, the angular distribution and various decay distribution and polarization asymmetries for NLC energies of $0.5 - 2 \, TeV$. Observability limits are discussed and interpreted in terms of the NP scales associated to each operator through the unitarity constraints. The general landscape of the residual NP effects in the heavy quark and bosonic sectors is also presented.

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1 Introduction

It is commonly hoped that the high value of the top mass may open a window towards understanding the mass generation mechanism. It is then important to look at the top quark interactions in a very accurate way, searching for possible departures from universality, which are somehow associated with the heavy $t$-mass; i.e. differences from the properties of the light quarks and leptons. This is particularly true, if no new particles are lying within the reach of the contemplate Colliders. Such top-mass effects are of course well known for certain Standard Model (SM) electroweak radiative corrections. They increase like $m_t^2$; as e.g. in the so-called $\delta \rho$ or $\epsilon_1$ parameters \[1\]. Our search for NP will then concentrate on whether there exist any additional effects, somehow related to the scalar sector and the large top-mass, $m_t$, which are beyond those expected in SM.

The most intriguing hint towards this kind of NP is provided by the present situation concerning the $Zb\bar{b}$ vertex. This vertex receives a well known SM contribution \[2\] proportional to $m_t^2$, which does not seem sufficient though, to explain the data. Indeed, the experimental results at LEP1 and at SLC suggest that the SM top effect in $\epsilon_1$ agrees with the top mass value found at Fermilab, while no corresponding agreement is observed for $\Gamma(Z \rightarrow b\bar{b})$ \[3\]. Thus, if the present experimental result on $\Gamma(Z \rightarrow b\bar{b})$ is correct, it probably indicates the appearance of a mechanism whose origin must lie beyond SM. Various types of ideas for this new physics are possible, partially stemming from the fact that the Left-versus-Right structure of the $Zb\bar{b}$ vertex is not yet completely established \[4, 5, 6\].

An additional measurement allowing to determine the asymmetry factor $A_b$ is required to clarify the $Z \rightarrow b\bar{b}$ situation. This can be achieved either by measuring the forward-backward asymmetry at LEP1, or more directly from the measurement at SLC of the polarized forward-backward asymmetry \[3\]. At present there exists some disagreement between these two measurements. But in any case, the data seem to suggest that if an NP effect is present, then it should predominantly be affecting the right-handed $Zb\bar{b}$ amplitude. The situation is further complicated by the observation of a (weaker) anomalous effect in the $Zc\bar{c}$ vertex, which cannot be associated to an obvious virtual top quark contribution and requires a more direct NP source affecting light quarks also \[7\].

We assume here, that in the foreseeable future, no new particles will be found, beyond those present in SM. In such a case, NP could only appear in the form of residual interactions generated at a very large scale; i.e. $\Lambda_{NP} \gg M_W$. It may turn out that these residual interactions stem from the scalar sector and affect only Higgs and its ”partners”; i.e. the fermions coupled most strongly to the scalar sector and the gauge bosons. Under these conditions, NP should be described by an effective lagrangian expressed in terms of $SU(3)_c \times SU(2) \times U(1)$ gauge invariant $dim = 6$ operators, involving the Higgs together with $W, Z, \gamma, $ the quarks of the third family and the gluon. To somewhat restrict the number of such operators, we impose the constraint that the quark depending operators should necessarily involve at least one $t_R$ field \[E\]. We do this motivated by the form of the SM Yukawa couplings in the limit where all fermion masses, except the top, are

\[1\]Higgs is an old particle in this sense.
neglected. In the present work we also impose $CP$ invariance for NP, and disregard operators which, (after the use of the equations of motion), would have rendered 4-quark operators involving leptons or light quarks of the first two families.

There exist 14 such $CP$-conserving ”top” operators which have been classified in [5]. In addition, there exist 6 $CP$-conserving purely bosonic ones, which are ”blind” with respect to the LEP1 Physics and have been studied in [8, 9]. The indirect constraints on these operators from the LEP1/SLC measurements (including those from $Z \to b\bar{b}$), have been established in [3] for the ”top” operators, and in [8, 9, 4] for the bosonic ones. These constraints appear to be quite mild, calling for a more detail study in a higher energy collider. Towards this aim, the observable signatures of the bosonic NP operators through the high energy processes $e^+e^- \to W^+W^-$ [10, 11], $e^+e^- \to HZ$ and $e^+e^- \to H\gamma$ [12, 13] have already been studied.

In the present work we are interested in the more direct tests of the above operators that will be provided by the real top production process $e^+e^- \to t\bar{t}$ and the decay $t \to Wb \to b\ell^+\nu$. Such a study should considerably improve the existing sensitivity limits on this kind of NP. Since we expect the NP effects to be rather small, it is sufficient for the calculation to restrict ourselves to the leading contributions. Thus, we either restrict ourselves to the tree level contribution, whenever this is non-vanishing or (if it vanishes) retain only the leading-log 1-loop effect, provided it is enhanced by a power of the large top mass, $m_t$. Under these conditions, box diagrams are never important in our studies. Thus, we just need to determine the NP effects on the $\gamma t\bar{t}$, $Zt\bar{t}$ and $tbW$ vertices.

The interesting physical quantities are the integrated cross section, the density matrix of the produced $t$ quark, and various angular distributions. We show how the density matrix elements can be measured through decay distributions with or without $e^\pm$-beam polarization. The sensitivities of the various observables to each operator are presented and the observability limits for the associated couplings are established. With the help of unitarity relations, these limits are translated into lower bounds on the scale of corresponding New Physics. Finally we draw the panorama of the knowledge that one can reach on the whole set of top and bosonic operators through these tests, as well as through the previous indirect ones.

The contents of the paper are the following: In Section 2 we enumerate the $dim = 6$ operators used to construct the effective NP lagrangian, and define their couplings and associated NP scales. In Section 3 we compute the corresponding general $e^+e^- \to t\bar{t}$ helicity amplitude and the top density matrix. Section 4 is devoted to the $t \to bW \to b\ell^+\nu$ decay of the produced top quark. In Section 5 we compute the SM 1-loop $m_t^2$-enhanced contributions to these amplitudes; and in Section 6 the leading NP contributions for all operators considered . The resulting panorama of residual NP effects is discussed in Section 7. Two appendices give details about the computations of the NP effects and the decay distributions and asymmetries.
2 The effective lagrangian.

The list of the \( \text{dim} = 6 \), \( SU(3)_c \times SU(2) \times U(1) \) gauge invariant and CP-conserving operators involving the third family of quarks with at least one \( t_R \) field, together with gauge and scalar boson fields, has been established in [5]. These operators are divided into two groups containing four and two quark fields respectively.

1) Four-quark operators

\[
\begin{align*}
O_{qt} &= (\bar{q}_Lt_R)(\bar{t}_Rq_L) , \\
O^{(8)}_{qt} &= (\bar{q}_L \gamma t_R)(\bar{t}_R \gamma q_L) , \\
O_{tt} &= \frac{1}{2}(\bar{t}_R \gamma_\mu t_R)(\bar{t}_R \gamma^\mu t_R) , \\
O_{tb} &= (\bar{t}_R \gamma_\mu t_R)(\bar{b}_R \gamma^\mu b_R) , \\
O^{(8)}_{tb} &= (\bar{t}_R \gamma_\mu \chi t_R)(\bar{b}_R \gamma^\mu \chi b_R) , \\
O_{qq} &= (\bar{t}_R \gamma_\mu t_R)(\bar{b}_R \gamma_\mu b_R) - (\bar{b}_R b_L)(\bar{t}_R t_L) - (\bar{b}_L t_R)(\bar{t}_L b_R) , \\
O^{(8)}_{qq} &= (\bar{t}_R \gamma_\mu \chi t_R)(\bar{b}_R \gamma_\mu \chi b_R) - (\bar{b}_R \chi t_L)(\bar{b}_L \chi b_R) - (\bar{b}_L \chi t_R)(\bar{t}_L \chi b_R) .
\end{align*}
\]

2) Two-quark operators.

\[
\begin{align*}
O_{t1} &= (\Phi^\dagger \Phi)(\bar{q}_Lt_R \tilde{\Phi} + \bar{t}_R \tilde{\Phi}^\dagger q_L) , \\
O_{t2} &= i \left[ (\Phi^\dagger D_\mu \Phi) - (D_\mu \Phi^\dagger) \right] (\bar{t}_R \gamma^\mu t_R) , \\
O_{t3} &= i (\Phi^\dagger D_\mu \Phi)(\bar{t}_R \gamma_\mu b_R) - i (D_\mu \Phi^\dagger \bar{\Phi})(\bar{b}_R \gamma_\mu t_R) , \\
O_{Dt} &= (\bar{q}_L D_\mu t_R)D^\mu \bar{\Phi} + D^\mu \bar{\Phi}^\dagger((D_\mu \bar{t}_R) q_L) , \\
O_{tW\Phi} &= (\bar{q}_L \sigma^{\mu\nu} \tau t_R)\bar{\Phi} \cdot \tilde{W}^{\mu\nu} + \bar{\Phi}^\dagger(\bar{t}_R \sigma^{\mu\nu} \tau q_L) \cdot \tilde{W}^{\mu\nu} , \\
O_{tB\Phi} &= (\bar{q}_L \sigma^{\mu\nu} t_R)\bar{\Phi} B_{\mu\nu} + \bar{\Phi}^\dagger(\bar{t}_R \sigma^{\mu\nu} q_L)B_{\mu\nu} , \\
O_{tG\Phi} &= \left[ (\bar{q}_L \sigma^{\mu\nu} \lambda a t_R)\bar{\Phi} + \bar{\Phi}^\dagger(\bar{t}_R \sigma^{\mu\nu} \lambda a q_L) \right] G^{a}_{\mu\nu} ,
\end{align*}
\]

where \( \lambda^a (a = 1, \ldots, 8) \) are the usual eight colour matrices.

\(^2\)These quark fields are of course understood as the weak-eigenstate fields. They are related to the fields creating or absorbing the mass-eigenstates through the usual unitary transformations leading to the CKM matrix.
In the preceding formulae the definitions

\[ \Phi = \left( \frac{i}{\sqrt{2}}(v + H - i\chi^3) \right), \]  

\[ D_\mu = (\partial_\mu + ig' YB_\mu + ig^2 \tau \cdot \vec{W}_\mu + ig s \lambda \cdot \vec{G}_\mu), \]

are used where \( v \approx 246 \text{ GeV} \), \( Y \) is the hypercharge of the field on which the covariant derivative acts, and \( \tau \) and \( \lambda \) are the isospin and colour matrices applicable whenever \( D_\mu \) acts on isodoublet fermions and quarks respectively. As already stated, in writing (1-14), we have used the equations of motion. If these later equations were not used, then two more operators are met which contain additional derivatives [14].

3) Bosonic operators.
In addition to the above fermionic \( \text{dim} = 6 \) operators, NP may also be hiding in purely bosonic ones. Provided CP invariance is imposed, this kind of NP is described by 11 \( \text{dim} = 6 \) purely bosonic operators first classified in [8]. Here, we retain only the six "blind" or "super-blind" ones [15], which are not severely constrained by the Z-peak experiments [16]. They are:

\[ O_W = \frac{1}{3!} \left( \vec{W}_\mu \nu \times \vec{W}_\nu \lambda \right) \cdot \vec{W}_\lambda \mu, \]

\[ O_{W\Phi} = i(D_\mu \Phi)\dagger \tau \cdot \vec{W}^{\mu\nu}(D_\nu \Phi), \]

\[ O_{B\Phi} = i(D_\mu \Phi)\dagger B^{\mu\nu}(D_\nu \Phi), \]

\[ O_{UW} = \frac{1}{v^2}((\Phi^\dagger \Phi - \frac{v^2}{2})(\Phi^\dagger \Phi - \frac{v^2}{2}) B^{\mu\nu} B_{\mu\nu}), \]

\[ O_{UB} = 4 \partial_\mu(\Phi^\dagger \Phi)\partial_\mu(\Phi^\dagger \Phi). \]

The remaining five operators (called \( O_{DW}, O_{DB}, O_{BW}, O_{\Phi_1}, O_{\Phi_3} \)) [8] are ignored here, since they are either too severely constrained, or they give no contribution, (up to the 1-loop level), to the processes we study.

The resulting effective lagrangian describing the residual NP interactions is written as

\[ \mathcal{L} = \mathcal{L}_t + \mathcal{L}_{bos}, \]

where the contribution from the 14 \( (i = 1...14) \) "top" operators is

\[ \mathcal{L}_t = \sum_i \frac{f_i}{m_i^2} O_i, \]
while the purely bosonic ones give

\[ \mathcal{L}_{\text{bos}} = \lambda_W \frac{g}{M_W^2} \mathcal{O}_W + f_W \frac{g}{2M_W^2} \mathcal{O}_W \Phi + f_B \frac{g'}{2M_W^2} \mathcal{O}_B \Phi + d \mathcal{O}_{UB} + \frac{d_B}{4} \mathcal{O}_{UB} + \frac{f_{\Phi^2}}{v^2} \mathcal{O}_{\Phi^2}. \]  

(25)

As a whole we have 20 independent operators that we shall occasionally globally label as \( \mathcal{O}_i \), with \( i = 1, \ldots, 20 \).

To each of the coupling constants \( f_i \) (or \( \lambda_W, d, d_B \)) appearing in this lagrangian, a corresponding New Physics scale \( \Lambda_{NP} \) is associated through the unitarity relations established in [18, 12, 14]. Obviously \( \Lambda_{NP} \) generally depends on the operator \( \mathcal{O}_i \) considered. Thus for the 6 purely bosonic operators we have found

\[ |\lambda_W| \simeq 19 \frac{M_W^2}{\Lambda_{NP}^2}, \quad |f_B| \simeq 98 \frac{M_W^2}{\Lambda_{NP}^2}, \quad |f_W| \simeq 31 \frac{M_W^2}{\Lambda_{NP}^2}, \]  

(26)

\[ d \simeq \frac{104.5 \left( \frac{M_W}{\Lambda_{NP}} \right)^2}{1 + 6.5 \left( \frac{M_W}{\Lambda_{NP}} \right)^2} \text{ for } d > 0, \]

\[ d \simeq -\frac{104.5 \left( \frac{M_W}{\Lambda_{NP}} \right)^2}{1 - 4 \left( \frac{M_W}{\Lambda_{NP}} \right)^2} \text{ for } d < 0, \]  

(27)

\[ d_B \simeq \frac{195.8 \left( \frac{M_W}{\Lambda_{NP}} \right)^2}{1 + 200 \left( \frac{M_W}{\Lambda_{NP}} \right)^2} \text{ for } d_B > 0, \]

\[ d_B \simeq -\frac{195.8 \left( \frac{M_W}{\Lambda_{NP}} \right)^2}{1 + 50 \left( \frac{M_W}{\Lambda_{NP}} \right)^2} \text{ for } d_B < 0, \]  

(28)

\[ |f_{\Phi^2}| \simeq 75 \frac{M_W^2}{\Lambda_{NP}^2}. \]  

(29)

On the other hand, for the 14 "top" operators, unitarity gives

\[ |f_{qt}| \simeq \frac{16\pi}{3} \left( \frac{m_t^2}{\Lambda_{NP}^2} \right), \]  

(30)

\[ |f_{qt}^{(8)}| \simeq \frac{9\pi}{\sqrt{2}} \left( \frac{m_t^2}{\Lambda_{NP}^2} \right), \]  

(31)

\[ |f_{tt}| \simeq 6\pi \left( \frac{m_t^2}{\Lambda_{NP}^2} \right), \]  

(32)

\(^4\)The expression for \( f_{\Phi^2} \) is only valid for \( \Lambda_{NP} \gg 3.7 \text{TeV} \). A more detail discussion is given in [12].

\(^5\)In the expression for \( \mathcal{O}_{L \Phi} \) we assumed \( \Lambda_{NP} \lesssim 10 \text{TeV} \). Our results are derived by considering four-body amplitudes at the tree approximation. This may not be adequate for \( \mathcal{O}_{11} \) which is given by the standard top Yukawa interaction multiplied by \( \Phi \Phi \). This problem is not further investigated though, since \( \mathcal{O}_{11} \) never contributes to the processes studied here.
\[ |f_{tb}| \simeq 8\pi \left( \frac{m_t^2}{\Lambda_{NP}^2} \right), \quad (33) \]
\[ |f_{tb}^{(8)}| \simeq \frac{9\pi}{2} \left( \frac{m_t^2}{\Lambda_{NP}^2} \right), \quad (34) \]
\[ |f_{qq}| \simeq \frac{32\pi}{7} \left( \frac{m_t^2}{\Lambda_{NP}^2} \right), \quad (35) \]
\[ |f_{qq}^{(8)}| \simeq 6\pi \left( \frac{m_t^2}{\Lambda_{NP}^2} \right), \quad (36) \]
\[ |f_{t1}| \simeq \frac{16\pi}{3\sqrt{2}} \left( \frac{m_t^2}{v\Lambda_{NP}} \right), \quad (37) \]
\[ |f_{t2}| \simeq 8\pi\sqrt{3} \left( \frac{m_t^2}{\Lambda_{NP}^2} \right), \quad (38) \]
\[ |f_{t3}| \simeq 8\pi\sqrt{6} \left( \frac{m_t^2}{\Lambda_{NP}^2} \right), \quad (39) \]
\[ |f_{D1}| \simeq 8.2 \left( \frac{m_t^2}{\Lambda_{NP}^2} \right), \quad (40) \]
\[ |f_{tW\Phi}| \simeq \frac{61.6}{\sqrt{1 + 645 \frac{m_t^2}{\Lambda_{NP}^2}}} \left( \frac{m_t^2}{\Lambda_{NP}^2} \right), \quad (41) \]
\[ |f_{tB\Phi}| \simeq \frac{61.6}{\sqrt{1 + 645 \frac{m_t^2}{\Lambda_{NP}^2}}} \left( \frac{m_t^2}{\Lambda_{NP}^2} \right), \quad (42) \]
\[ |f_{tG\Phi}| \simeq \frac{m_t^2\sqrt{\pi}}{v\Lambda_{NP}\sqrt{1 + \frac{2}{3}\alpha_s}}. \quad (43) \]

At present the most important constraints on these couplings arise from the Z-peak experiments at LEP1/SLC, [20, 16]. In a near future the process $e^+e^- \to W^+W^-$ at LEP2, is expected to give direct constraints on the bosonic operators in (17-19) [11, 10]. In addition, if the Higgs boson is light enough the processes $e^+e^- \to HZ$ and $e^+e^- \to H\gamma$ will also produce constraints on the 3 other bosonic operators (20-22) [12]. In Section 6, all these constraints will be presented together with the observability limits that could be derived from the $e^-e^+ \to t\bar{t}$ and $t \to bW$ processes.

3 The $e^-e^+ \to t\bar{t}$ amplitude

As it has been mentioned in the Introduction, box diagrams are never important for calculating the leading NP effects in $e^-e^+ \to t\bar{t}$. The amplitude has therefore a tree-level
structure with $\gamma$ and $Z$ exchange in the t-channel. Therefore, we only need to determine the $Vt\bar{t}$, $(V = \gamma, Z)$ vertex, whose most general CP-conserving form is

$$-ie^V_\mu J^\mu_V = -ie^V_\mu \frac{\bar{u}(p)}{2} [\gamma^\mu d^V_1(q^2) + (p - p')^\mu d^V_2(q^2)] v(p') ,$$  \hspace{1cm} (44)

where $e^V_\mu$ is the polarization of the vector boson V. The outgoing momenta $(p, p')$ refer to $(t, \bar{t})$ respectively and satisfy $q \equiv p + p'$. The normalizations are determined by $e_\gamma \equiv e$ and $e_Z \equiv e/(2s_Wc_W)$. The couplings $d^V_i$ are in general $q^2$ dependent form factors. The contributions to these from SM at tree level are

$$d^{1,SM}_1 = \frac{2}{3}, \quad d^{2,SM}_1 = g_{Vt} = \frac{1}{2} - \frac{4}{3}s_W^2, \quad d^{2,SM}_2 = -g_{At} = -\frac{1}{2}.$$

In addition, there exist SM contributions to these couplings at the 1-loop level, $d^{V,SM}_1$, whose leading large $m_t$ part is computed in Sect.5. Finally in Section 6 we calculate the leading NP contributions to $d^V_1$. For the operators $(O_{t2}, O_{Dt}, O_{W\Phi}, O_{tB\Phi})$, these arise at the tree-level. For $(O_{qt}, O_{q8}, O_{tt}, O_{tb}, O_{tCG\Phi})$ and the six purely bosonic operators $(O_{W}, O_{W\Phi}, O_{BB\Phi}, O_{UB}, O_{UB}, O_{q2})$ we need to go to the 1-loop in order to find a non-vanishing leading contribution which is also enhanced by a power of $m_t^2$. Finally, for the operators $O_{t6}^8, O_{qq}, O_{q8}, O_{t1}, O_{t3}$ we get no such leading NP contribution, up to the 1-loop order.

The $e^-e^+ \rightarrow t\bar{t}$ helicity amplitude is written as $F_{\lambda,\tau,\tau'}$, where $\lambda \equiv \lambda(e^-) = -\lambda'(e^+) = \pm1/2$ denote the $e^-$, $e^+$ helicities, while $\tau$ and $\tau'$ represent respectively the $t$ and $\bar{t}$ helicities. For completeness we also mention that the $(e^-, e^+)$ incoming momenta are denoted as $(k, k')$, while the $(t, \bar{t})$ outgoing momenta are $(p, p')$. Using the couplings defined in (44), we write

$$F_{\lambda,\tau,\tau'} = \sum_{V=\gamma, Z} 2\lambda e^2 \sqrt{s} (A_V - 2\lambda B_V) \left\{ d^V_1 [2m_t \sin \theta \delta_{\tau\tau'} + \sqrt{s} \cos \theta (\tau' - \tau) - 2\lambda \sqrt{s} \delta_{\tau,-\tau'}] - d^V_2 [2p^- \cos \theta \delta_{\tau,-\tau'} + 2\lambda (\tau - \tau')] - d^V_3 [4p^- \sin \theta \delta_{\tau\tau'}] \right\},$$ \hspace{1cm} (46)

with $A_\gamma = -1/s$, $A_Z = g_{Ve}/(4s_W^2c_W^2 D_Z)$, $B_\gamma = 0$, $B_Z = g_{Ac}/(4s_W^2c_W^2 D_Z)$ and $g_{Ve} = -1/2 + 2s_W^2$, $g_{Ac} = -1/2$, $D_Z = s - M_Z^2 + iM_Z \Gamma_Z$. In (46), $\theta$ is the $(e^-, t)$ scattering angle in the $(e^-, e^+)$ c.m. frame. The amplitude is normalized so that the unpolarized $e^-e^+ \rightarrow t\bar{t}$ differential cross section is given by

$$\frac{d\sigma(e^-e^+ \rightarrow t\bar{t})}{d\cos \theta} = \frac{3\beta_t}{128\pi s} \sum_{\lambda,\tau,\tau'} |F_{\lambda,\tau,\tau'}|^2,$$

where $\beta_t = (1 - 4m_t^2)^{1/2}$ and the colour factor has been included. We note that CP invariance implies $F_{\lambda,\tau,\tau'} = F_{\lambda,-\tau',-\tau}$. \hspace{1cm} (47)
which is of course satisfied by \((46)\). In fact, at the level of approximations used in constructing \((46)\), CPT implies also
\[
F_{\lambda,\tau,\tau'} = F^*_{\lambda,-\tau,-\tau'} ,
\]
which indicates that all helicity amplitudes must be real.

Because of CP and CPT symmetries, it turns out that the simple \(t\) (or \(\bar{t}\)) decay distribution contains all information that can be extracted from the amplitudes in \((46)\). Thus, nothing more can be learned by considering the combined decay distributions of \(t\) and \(\bar{t}\) simultaneously. It is, therefore, sufficient to consider only the simple spin density matrix of the produced \(t\) or \(\bar{t}\). For top-quark, this is
\[
\rho_{L,R}^{\tau_1\tau_2} = \sum_{\tau'} F_{\lambda,\tau_1,\tau'} F^*_{\lambda,\tau_2,\tau'} ,
\]
where \(L, R\) correspond respectively to \(\lambda \equiv \lambda(e^-) = -\lambda'(e^+) = \mp 1/2\). There are only six independent elements (real in our case), \(\rho_{L,R}^{++}, \rho_{L,R}^{--}\) and \(\rho_{L,R}^{+-} = \rho_{L,R}^{-+}\), which can be measured through the top production and decay distributions. Each \(\rho\) element has a typical angular distribution given in terms of the form \((1+\cos^2 \theta), \sin^2 \theta\) and \(\cos \theta\), producing symmetrical and asymmetrical \(\theta\)-distributions. Combining this with the decay angular information, various \(\rho\) elements can be isolated. Performing such measurements at a few \(e^-e^+\) energies, even for unpolarized beams, one can define a sufficient number of independent quantities that can be used to determine the six couplings \(d^V_i, V = \gamma, Z, i = 1, 3\). Many of these quantities, (and in particular those of interest here), are forward-backward asymmetries in the angular distribution of physical observables concerning the produced top-quark. QCD effects to these observables are probably small \([19, 22]\), and in any case they should be incorporated in our formalism in the future.

Electron beam polarization should provide an independent and maybe cleaner way to disentangle these couplings, through the separation of left-handed \((L \leftrightarrow \lambda = -1/2)\) and right-handed \((R \leftrightarrow \lambda = +1/2)\) contributions. Thus, in addition to the unpolarized quantities, we would then also have the \(L - R\) ones. This way, the information from the usual unpolarized \(L + R\) integrated cross section \(\sigma(e^-e^+ \rightarrow tt)\), will be augmented by the availability of also \(\sigma^L\) and \(\sigma^R\) allowing us to measure the integrated left-right asymmetry \(A_{LR}\) defined as \((\sigma^L - \sigma^R)/(\sigma^L + \sigma^R)\). Similarly, any forward-backward asymmetry constructed for the unpolarized \((L + R)\) case will be accompanied by the corresponding one in the polarized \((L - R)\) case. Details are given in Appendix B, where we thus define the six forward-backward asymmetries \(A_{FB}, A_{FB,pol}, H_{FB}, H_{FB,pol}, T_{FB}, T_{FB,pol}\).

\section{4 \(t \rightarrow Wb\) decay amplitudes and induced asymmetries.}

The \(t(p_t) \rightarrow W^+(p_W)b(p_b)\) decay, where \((p_t, p_W, p_b)\) are the related momenta, will be used to construct the asymmetries mentioned in the last paragraph of the previous
section which are sensitive to the NP couplings affecting the $e^- e^+ \to \ell \ell$, and to (a lesser extent) the ones determining $t \to bW$. To describe the NP effects in the $t$ decay, we write the general $t \to W^+b$ vertex in terms of four invariant couplings, related through CPT invariance to the other four invariant couplings for $\bar{t} \to W^-\bar{b}$. These couplings are given by

$$-i\epsilon^{\mu\nu}_{W^*} J^\mu_W = -i\frac{gV_{tb}^*}{2\sqrt{2}} \bar{u}_b(p_b)[\gamma^\mu d_1^W + \gamma^\mu\gamma^5 d_2^W + (p_t + p_b)^\mu d_3^W + (p_t + p_b)^\mu\gamma^5 d_4^W] u_t(p_t),$$

(51)

where $\epsilon^{\mu\nu}_{W^*}$ is the $W$ polarization vector and $V_{tb}^*$ is the appropriate CKM matrix element.

The couplings $d_i^W$ receive contributions from SM and NP. The tree level SM contribution is

$$d_1^{W,SM} = -d_2^{W,SM} = 1, \quad d_3^{W,SM} = d_4^{W,SM} = 0$$

(52)

The 1-loop $m_t^2$-enhanced SM contribution $d_i^{W,SM1}$ are computed in Sect. 5 and they also satisfy the relations

$$d_1^{W,SM} = -d_2^{W,SM}, \quad d_3^{W,SM} = d_4^{W,SM}$$

(53)

Finally the NP contributions to top decay are also given in Appendix B and collected in Sect. 6. Here we only note that the operators ($O_{i3}$, $O_{i7}$, $O_{iW\Phi}$) contribute already at the tree level, while $O_{qq}$, $O_{8q}$, $O_{qG\Phi}$, $O_{WW}$, $O_{B\Phi}$, $O_{UW}$, $O_{\Phi2}$ supply 1-loop "leading-log" contributions, enhanced by powers of $m_t^2$. The remaining $O_{qB}$, $O_{8q}$, $O_{B}$, $O_{tB}$, $O_{tW}$, $O_{W}$ and $O_{UB}$ give no such contribution to $t \to bW$, up to this order.

In Appendix B we give the explicit forms of the asymmetry observables, sensitive to the production couplings defined in (44), and the decay ones in (51). Since we expect the NP effects to be small, we are only interested in observables that are sensitive to the interferences between the NP and the tree-level SM effects. Below, we first comment on the asymmetries sensitive to the decay couplings and then on the production ones.

The $t \to bW^+ \to b l^+\nu$ observables which are interesting to measure are those sensitive to the interference between the NP and the tree-level SM effects. It turns out that these depend only on the combinations ($d_1^{W,NP} - d_2^{W,NP}$) and ($d_3^{W,NP} + d_4^{W,NP}$). The $d_i^{W}$ couplings have the peculiarity of leading only to a $W$, longitudinally polarized along the $t$ quark momentum, whereas $d_i^{W,1,2}$ contribute to both the transverse and longitudinal $W$'s. Because of this, only the combination ($d_3^{W,NP} + d_4^{W,NP}$) can contribute linearly to the asymmetries relevant for the top decay distribution. In Appendix B, two versions of such an asymmetry are given, referring to the angular distribution of the lepton coming from the semileptonic $t$ or $\bar{t}$ decay.

The other combination ($d_1^{W,NP} - d_2^{W,NP}$) cannot be seen at linear order through asymmetries. For a physical quantity sensitive to it, we have to look at the partial width $\Gamma(t \to Wb)$

$$\Gamma(t \to Wb) = \frac{G_F|V_{tb}|^2(m_t^2 - M_W^2)^2}{16\pi\sqrt{2}m_t^2}\left\{[(d_1^W)^2 + (d_2^W)^2](m_t^2 + 2M_W^2) + [(d_3^W)^2 + (d_4^W)^2](m_t^2 - M_W^2)^2 + 2[d_3^W d_1^W - d_4^W d_2^W]m_t(m_t^2 - M_W^2)\right\},$$

(54)
where it appears multiplied by the CKM matrix element also. Unfortunately, the width \( \Gamma(t \to Wb) \) cannot be directly measured to the necessary accuracy. Only indirectly can this be done, either using the fusion process \( W\gamma \to tb \) accessible at an \( e^-e^+ \) collider in the \( e\gamma \) mode (through laser backscattering); or using reactions like \( q\bar{q}l \to tb \) and \( Wg \to tb \) accessible at the Tevatron and LHC colliders respectively. For such a measurement, one can expect an accuracy of only about 20 to 30\% for \( (d_1^{W,NP} - d_2^{W,NP}) \) \([19, 20] \).

We next turn to the asymmetries sensitive to the production couplings defined in (14). For constructing them, we need a description of the angular distribution of top production and decay. Below, we only give expressions for completely longitudinally polarized beams. In such a case, the angular distribution for \( e^-e^+ \to \bar{t}t(t \to bW \to bl\nu) \) can be written as

\[
\frac{d\sigma_{L,R}}{d\cos\theta} = \frac{3\beta_t}{32\pi s} \sum_{\tau_1\tau_2\tau'} F_{\lambda\tau_1\tau'}F^*_{\lambda\tau_2\tau'}t_{\tau_1\tau_2} , \tag{55}
\]

where \( (L, R) \) correspond to \( \lambda = \lambda(e^-) = -\lambda(e^+) = \mp 1/2 \) respectively and \( t_{\tau_1\tau_2} \) is the top-quark decay matrix constructed in terms of the helicity amplitude \( M_\tau(t \to bW^+ \to bl^+\nu) \), with \( \tau \) being the top helicity. We thus have

\[
t_{\tau_1\tau_2} = \frac{(2\pi)^4}{2m_t\Gamma_t}\sum_{\text{spins}} M_{\tau_1}M^*_{\tau_2}d\Phi_3(bl\nu_t) , \tag{56}
\]

where \( \Gamma_t \) is the total top-width, \( \sum_{\text{spins}} \) means summation over the final \( (b, l^+, \nu_t) \) spins and \( d\Phi_3(bl\nu_t) \) is the usual 3-body phase space describing \( t \) decay in its rest frame, \([23] \).

It is convenient to express the 3-body phase space in terms of the Euler angles determining the \( t \)-decay plane. We start from the process \( e^- (k) e^+ (k') \to t(p)\bar{t}(p') \) in the center of mass frame, where with \( \theta \) we denote the \( (e^-, t) \) scattering angle. The \( t \)-quark rest frame (called \( t \)-frame hereafter) is then defined with its \( z \)-axis along the top-quark momentum; the \( x \)-axis is taken in the \( (t \bar{t}) \) production plane, so that the \( y \)-axis is perpendicular to it and along the direction of \( \vec{k} \times \vec{p} \) of the \( e^- \) and \( t \) momenta. In this \( t \)-frame we define the top decay plane through the Euler angles \( (\varphi_1, \theta_1, \psi_1) \) described in Appendix B. In addition, within the top decay plane we define \( \theta_t \) as the angle between the lepton momentum and the top momentum, after having boosted to the W-rest frame \([19] \). It is related to the \( l^+ \) energy in the \( t \)-frame by

\[
E_t = |\vec{p}_t| = \frac{m_t^2 + M_W^2 - \cos\theta_t(m_t^2 - M_W^2)}{4m_t} , \tag{57}
\]

where the \( (b, l^+) \) masses are neglected. In terms of the Euler angles, the 3-body phase space becomes \([25] \)

\[
\delta((p_t + p_\nu)^2 - M_W^2) d\Phi_3(bl\nu_t) \Rightarrow \frac{(m_t^2 - M_W^2)}{64m_t^2(2\pi)^9} d\varphi_1 d\cos\theta_1 d\psi_1 d\cos\theta_t , \tag{58}
\]

after including the constraint that the \( l\nu \) pair lies at the \( W \) mass shell. Like \( \rho \), the matrix \( t_{\tau_1\tau_2} \) also involves three real independent elements \( (\tau_1\tau_2) = (++) \), \((-\)) and \((+-)\).
They are explicitly written in terms of the above angles in Appendix B. Using these, we construct the three forward-backward asymmetries for the unpolarized case \((L + R)\) and another three for the polarized one \((L - R)\), which can be used to measure the production couplings in (14).

5 Leading \(m_t\)-enhanced SM contributions at 1-loop.

In the present section we study the 1-loop, \(m_t\)-enhanced SM contributions to \(e^-e^+ \rightarrow t\bar{t}\) and \(t \rightarrow bW\). Technically this means that we consider the large \(m_t\) limit of the 1-loop diagrams, keeping \(m_H/m_t\) and \(s/m_t^2 \equiv q^2/m_t^2\) finite. Such a study is useful for checking the possible appearance of any large \(m_t\) effect. It is also instructive for comparison with the corresponding NP effects. The relevant diagrams supplying such \(m_t^2\) enhancements consist of triangular vertex-diagrams for the \(\gamma t\bar{t}\), \(Zt\bar{t}\), \(Wtb\) vertices, and also of the \(t\) and \(b\) self-energy diagrams; involving exchanges of goldstone bosons and the physical Higgs. Diagrams involving gauge boson exchanges, or box diagrams, cannot generate \(m_t^2\)-enhancements. We have checked that these contributions to the form factors in (44) are gauge invariant and determine the complete SM large-\(q^2\), large-\(m_t\) effect in \(e^-e^+ \rightarrow t\bar{t}\) and \(t \rightarrow bW\). Note that we leave aside the gauge boson (\(\gamma, Z, W\)) self-energy contributions which are universal (i.e. not related to the \(t\bar{t}\) channel) and are taken into account in the usual renormalization procedure, [26]. For \(q^2 \gtrsim 4m_t^2\), the resulting 1-loop contributions to the form factors defined in (44) is given by

\[
d^1_{\gamma,SM}(q^2) = -C\left[\frac{8 + I_{se}}{3} + \frac{1}{3}I_0 - \frac{1}{6}J_1 + \frac{1}{3}(I_2 + I_{2H}) - \frac{2}{3}J_{4H}\right],
\]

\[
d^2_{\gamma,SM}(q^2) = -C\left[\frac{1}{3} + \frac{1}{3}I_0 + \frac{1}{6}J_1\right],
\]

\[
d^3_{\gamma,SM}(q^2) = \frac{2C}{3m_t}\left[\frac{1}{2}J_{2H} - J_0\right],
\]

\[
d^1_{Z,SM}(q^2) = C\left[-\frac{3}{4} + \frac{I_{se}}{3} + \frac{2s_W^2}{3}(8 + I_{se}) + \frac{2s_W^2}{3}I_0 + \frac{1}{2}(1 - \frac{2s_W^2}{3})J_1 - \frac{1}{4}(1 - \frac{8s_W^2}{3})(I_2 + I_{2H} - 2J_{4H})\right],
\]

\[
d^2_{Z,SM}(q^2) = C\left[\frac{3}{4} + \frac{I_{se}}{3} + \frac{2s_W^2}{3}I_0 - \frac{1}{2}(1 - \frac{2s_W^2}{3})J_1 + I_{1H}
\right.
\]

\[
\left. - \frac{1}{4}(I_2 + I_{2H}) + \frac{1}{2}J_{3H}\right],
\]

\[
d^3_{Z,SM}(q^2) = \frac{C}{m_t}\left[\frac{1}{4}(1 - \frac{8s_W^2}{3})J_{2H} - \frac{1}{4}(1 - \frac{4s_W^2}{3})J_0\right],
\]

with \(C = \frac{g^2m_t^2}{64\pi^2M_W^2}\), and

\[
I_{se} = -2\int_0^1 dx (1 - x) \ln[x^2 + \zeta(1 - x)] = 3
\]
$2\zeta \left(1 - \frac{\zeta}{4}\right) \ln(\zeta) - \zeta - (\zeta - 2)\sqrt{\zeta(\zeta - 4)} - ie \text{ Arcosh} \left(\frac{\sqrt{\zeta}}{2} - ie\right)$, \hspace{1em} (65)

$I_0 = 2 \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \ln((x_1 + x_2)(x_1 + x_2 - 1) - 4\eta x_1 x_2)$, \hspace{1em} (66)

$I_{1H} = 2 \int \int dx_1 dx_2 \ln((x_1 + x_2 - 1)^2 - 4\eta x_1 x_2 + \zeta x_1)$, \hspace{1em} (67)

$I_2 = 2 \int \int dx_1 dx_2 \ln((x_1 + x_2)^2 - 4\eta x_1 x_2)$, \hspace{1em} (68)

$I_{2H} = 2 \int \int dx_1 dx_2 \ln((x_1 + x_2)^2 - 4\eta x_1 x_2 + \zeta(1 - x_1 - x_2))$, \hspace{1em} (69)

$J_0 = 2 \int \int dx_1 dx_2 \frac{(x_1 + x_2)(x_1 + x_2 - 1)}{(x_1 + x_2)(x_1 + x_2 - 1) - 4\eta x_1 x_2}$, \hspace{1em} (70)

$J_1 = 2 \int \int dx_1 dx_2 \frac{(x_1 + x_2 - 1)}{(x_1 + x_2)(x_1 + x_2 - 1) - 4\eta x_1 x_2}$, \hspace{1em} (71)

$J_{2H} = 2 \int \int dx_1 dx_2 \left[\frac{(x_1 + x_2)(x_1 + x_2 - 2)}{(x_1 + x_2)^2 - 4\eta x_1 x_2 + \zeta(1 - x_1 - x_2)} + \frac{(x_1 + x_2)^2}{(x_1 + x_2)^2 - 4\eta x_1 x_2}\right]$, \hspace{1em} (72)

$J_{3H} = 2 \int \int dx_1 dx_2 \frac{2(1 - \frac{\zeta}{4}(x_1 + x_2 - 1))}{(x_1 + x_2)^2 - 4\eta x_1 x_2 + \zeta(1 - x_1 - x_2)}$, \hspace{1em} (73)

$J_{4H} = 2 \int \int dx_1 dx_2 \frac{2 - \frac{\zeta}{3}(x_1 + x_2 - 1)}{(x_1 + x_2)^2 - 4\eta x_1 x_2 + \zeta(1 - x_1 - x_2)}$, \hspace{1em} (74)

with $\eta = (q^2 + ie)/(4m_t^2)$ and $\zeta = m_H^2/m_t^2$. The double integration runs over the interval $[0, 1]$ for $x_1$, and $[0, 1 - x_1]$ for $x_2$. These integrals have been computed partially analytically and partially numerically.$^6$

It can be noted from these results that close to threshold (i.e. $q^2 \sim 4m_t^2$), there is a rather strong $m_t^2/m_H^2$ dependence arising from the triangle diagrams involving physical Higgs exchange. If we put $m_t \gg m_H$, then the SM contribution acquires infrared-type singularities, which are obviously related to the stability of scalar sector requiring $m_t \sim m_H$ for physically acceptable $m_t$ masses. [27].

In Fig.1 we plot (in units of the coefficient $C \approx 0.003$), these SM contributions to the six form factors, as functions of $\sqrt{q^2} \equiv (2E_e)$, for $m_H = 0.1$ TeV and $q^2 > 4m_t^2$. For $q^2 \gtrsim (0.5$ TeV)$^2$ we find from this figure an effect of the order of 1% for the vector $\gamma t\bar{t}$ coupling, and of the order of 3 per mille for the other vector and axial couplings. The "derivative" $d_3^{\gamma Z}$ couplings are somewhat weaker.

We next turn to the $m_t^2$-enhanced SM corrections to the $t \to bW$ decay. The resulting 1-loop contributions, expressed in terms of the decay couplings defined in (51), is

$$d_1^{W,SM} = -d_2^{W,SM} = -C \left[\frac{5}{4}I_{se} + \frac{1}{2}H_1\right] \hspace{1em} (75)$$

$^6$We thank N.D. Vlachos for his help in this numerical computation.
\[ d_{3}^{W,SM} = d_{4}^{W,SM} = - \frac{C}{2m_t} [1 + H_2] , \] (76)

where

\[ H_1 = 2 \int \int dx_1 dx_2 \ln[x_1(x_1 + x_2) - \zeta(x_1 + x_2 - 1)] , \] (77)

\[ H_2 = 2 \int \int dx_1 dx_2 \frac{x_1(x_1 + x_2 - 2)}{x_1(x_1 + x_2) - \zeta(x_1 + x_2 - 1)} . \] (78)

Fig.1: 1-loop SM contributions to the \( d_{j}^{f} \) and \( d_{j}^{Z} \) form factors defined in (44), as functions of \( \sqrt{q^2} = 2E_e \).

For \( m_H = 0.1 \, TeV \) these equations give

\[ d_{1}^{W,SM} = - d_{2}^{W,SM} = C(-0.92) , \] (79)

\[ d_{3}^{W,SM} = d_{4}^{W,SM} = C \left( \frac{0.0728}{2m_t} \right) . \] (80)

Comparing the definition of the various couplings given in (51), with the relations (75,76), we remark that the SM \( m_t^2 \)-enhanced couplings only affect the left-handed \( b_L \) field. This is also obvious from the structure of the relevant diagrams.

6 The NP effects.

In Appendix A we enumerate the relevant diagrams giving the leading NP contribution to \( e^-e^+ \rightarrow tt \) and \( t \rightarrow bW \), for each kind of operator. Since box diagrams are never important, the leading NP contribution to \( e^-e^+ \rightarrow tt \) can be expressed in terms of the NP contributions to the form factors introduced in (44). Here, \( s = q^2 \geq 4m_t^2 \) is understood. As already stated, the operators \( (\mathcal{O}_{t2}, \mathcal{O}_{Dt}, \mathcal{O}_{tW\phi}, \mathcal{O}_{tB\phi}) \) give tree-level contributions, which are

\[ d_{1}^{\gamma,NP}(s) = - \frac{4\sqrt{2}M_W}{e^2m_t} (s^2_{W} f_{tW\phi} + s_{W} c_{W} f_{tB\phi}) , \] (81)

\[ d_{3}^{\gamma,NP}(s) = \frac{2\sqrt{2}M_W}{e^2m_t^2} (s^2_{W} f_{tW\phi} + s_{W} c_{W} f_{tB\phi}) , \] (82)

\[ d_{1}^{Z,NP}(s) = \frac{2M_W}{g^2m_t^2} f_{t2} + \frac{8\sqrt{2}M_W}{g^2m_t} (-c_{W} f_{tW\phi} + s_{W} c_{W} f_{tB\phi}) , \] (83)
\[ d_2^{Z,\text{NP}}(s) = - \frac{2M_W^2}{g^2m_t^2} f_{12}, \quad (84) \]
\[ d_3^{Z,\text{NP}}(s) = - \frac{M_W}{\sqrt{2}gm_t^2} f_{Dt} + \frac{4\sqrt{2}M_W}{g^2m_t^2}(c_W^2 f_{tW\Phi} - s_Wc_W f_{tB}) \cdot (85) \]

We next turn to the operators contributing only at the 1-loop level. As explained above, we consider only the leading NP effect determined by the divergent part of the Feynman integrals, provided it is enhanced by some power of \( m_t^2 \). The contributions from the "top"-involving operators are then expressed in terms of
\[ F_i \equiv \frac{1}{16\pi^2} \ln \left( \frac{\Lambda^2}{\mu^2} \right) \frac{f_i}{m_t^2}, \quad (86) \]

with \( \Lambda \) being the divergent integral cutoff identified with the NP scale \( \Lambda_{NP} \), and \( \mu \sim \sqrt{s} \) being the scale where the effective coupling is measured. For the purely bosonic operators, due to their different normalization implied by (24,25), we should replace in (86) \( f_i/m_t^2 \rightarrow (\Lambda_W/M_W^2, \ f_W/M_W^2, \ f_B/M_B^2, \ d/v^2, \ d_B/v^2, \ f_{W2}/v^2) \) for \( (O_W, O_{W\Phi}, O_{B\Phi}, O_{UW}, O_{UB}, O_{\Phi_2}) \) respectively. We thus get
\[ d_1^{Z,\text{NP}}(s) = \frac{s}{6}Fqt + \frac{8s}{9}Fqt(s) - \frac{8s}{9}F_{ut} + \frac{s}{3}F_{tb} + \frac{256\sqrt{2}g_{s}m_{W}F_{tG\Phi}}{9g} + \frac{g^{2}s}{4}F_{W} - \frac{g^{2}m_{t}^{2}s}{32M_{W}^{2}}[F_{W\Phi} + F_{B\Phi}] - 2m_{t}^{2}F_{UV} - \frac{10m_{t}^{2}}{3}F_{UB}, \quad (87) \]
\[ d_2^{Z,\text{NP}}(s) = - \frac{s}{18}Fqt - \frac{8s}{27}Fqt(s) - \frac{8s}{9}F_{ut} + \frac{s}{3}F_{tb} - \frac{g^{2}s}{4}F_{W} + \frac{g^{2}m_{t}^{2}s}{32M_{W}^{2}}[F_{W\Phi} - 3F_{B\Phi}], \quad (88) \]
\[ d_3^{Z,\text{NP}}(s) = - \frac{128\sqrt{2}g_{s}M_{W}}{9g}F_{tG\Phi} + m_{t}F_{UV} + \frac{5m_{t}}{3}F_{UB}, \quad (89) \]
\[ d_1^{Z,\text{NP}}(s) = - \frac{ss_{W}^{2}}{3}Fqt - \frac{16ss_{W}^{2}}{9}Fqt(s) - 4 \left( m_{t}^{2} - \frac{4ss_{W}^{2}}{9} \right) F_{ut} - \frac{2ss_{W}^{2}}{3}F_{tb} + \frac{16\sqrt{2}g_{s}(3 - 8s_{W}^{2})m_{t}M_{W}}{9g}F_{tG\Phi} + \frac{ss_{W}^{2}g^{2}}{2}F_{W} - \frac{g^{2}m_{t}^{2}s}{16M_{W}^{2}}[c_{W}^{2}F_{W\Phi} - s_{W}^{2}F_{B\Phi}] - 4m_{t}^{2}c_{W}^{2}F_{UV} + \frac{20m_{t}^{2}s_{W}^{2}}{3}F_{UB} \quad (90) \]
\[ d_2^{Z,\text{NP}}(s) = - (m_{t}^{2} - \frac{ss_{W}^{2}}{9})Fqt - \left( \frac{16}{3} \right) \left( m_{t}^{2} - \frac{ss_{W}^{2}}{9} \right) Fqt(s) - 4(m_{t}^{2} - \frac{4ss_{W}^{2}}{9})F_{ut} - \frac{2ss_{W}^{2}}{3}F_{tb} - \frac{ss_{W}^{2}g^{2}}{2}F_{W} + \frac{g^{2}m_{t}^{2}s}{16M_{W}^{2}}[c_{W}^{2}F_{W\Phi} + 3s_{W}^{2}F_{B\Phi}] - 4m_{t}^{2}F_{\Phi_2}, \quad (91) \]

\(^7\text{We always use dimensional regularization.}\)
\[ d_Z(s) = - \frac{32\sqrt{2}g_s(3-8s_W^2)M_W}{9g} F_{tG\Phi} + 2m_t c^2 W F_{UB} - \frac{10m_t s_W^2}{3} F_{UB}. \] (92)

Similarly, for the \( t \to bW \) couplings defined in (51), the non vanishing tree level NP contributions are

\[ d_{W,NP}^{1} = \frac{2M_W^2}{g^2 m_t^2 f_3} - \frac{4\sqrt{2}M_W}{g^2 m_t} f_{iW\Phi}, \] (93)

\[ d_{W,NP}^{2} = \frac{2M_W^2}{g^2 m_t^2 f_3} + \frac{4\sqrt{2}M_W}{g^2 m_t} f_{iW\Phi}, \] (94)

\[ d_{W,NP}^{3} = \frac{4\sqrt{2}M_W}{g^2 m_t^2} f_{iW\Phi} - \frac{M_W}{g\sqrt{2} m_t^2} f_{Dt}, \] (95)

\[ d_{W,NP}^{4} = \frac{4\sqrt{2}M_W}{g^2 m_t^2} f_{iW\Phi} - \frac{M_W}{g\sqrt{2} m_t^2} f_{Dt}. \] (96)

Correspondingly, the non vanishing 1-loop \( m_t^2 \)-enhanced NP contributions (for the operators which do not have any tree-level ones) are

\[ d_{W,NP}^{1} = \frac{m_t^2}{2} F_{qq} + \frac{8m_t^2}{3} F_{qq(8)} + \frac{32\sqrt{2}m_t M_W g_s}{3g} F_{tG\Phi} + \frac{g^2 m_t^2 (13s_W^2 - 3)}{48c_W^2} F_{W\Phi} + \frac{g^2 m_t^2 s_W^2}{48c_W^2} F_{B\Phi} - 2m_t^2 F_{UB} + 2m_t^2 F_{tF_{2}}, \] (97)

\[ d_{W,NP}^{2} = \frac{m_t^2}{2} F_{qq} + \frac{8m_t^2}{3} F_{qq(8)} - \frac{32\sqrt{2}m_t M_W g_s}{3g} F_{tG\Phi} - \frac{g^2 m_t^2 (13s_W^2 - 3)}{48c_W^2} F_{W\Phi} - \frac{g^2 m_t^2 s_W^2}{48c_W^2} F_{B\Phi} + 2m_t^2 F_{UB} - 2m_t^2 F_{tF_{2}}, \] (98)

\[ d_{W,NP}^{3} = - \frac{m_t}{2} F_{qq} - \frac{8m_t}{3} F_{qq(8)} - \frac{32\sqrt{2}M_W g_s}{3g} F_{tG\Phi} - \frac{g^2 m_t (11s_W^2 - 6)}{24c_W} F_{W\Phi} - \frac{5g^2 m_t^2 s_W^2}{24c_W^2} F_{B\Phi} + 2m_t F_{UB}, \] (99)

\[ d_{W,NP}^{4} = \frac{m_t}{2} F_{qq} + \frac{8m_t}{3} F_{qq(8)} - \frac{32\sqrt{2}M_W g_s}{3g} F_{tG\Phi} - \frac{g^2 m_t (11s_W^2 - 6)}{24c_W} F_{W\Phi} - \frac{5g^2 m_t^2 s_W^2}{24c_W^2} F_{B\Phi} + 2m_t F_{UB}. \] (100)
We now review the NP effects of each operator on the various observables. First note that the operators $O_{tb}^{(8)}$ and $O_{tt}$ give no contribution (within our approximations) to either production or decay. The effects of the rest are illustrated in Table 1, where we give the NP couplings used and the implications for the $t \rightarrow bW$ decay observables. The effects on top production through $e^-e^+ \rightarrow t\bar{t}$ are presented in Figs.2-9c.

Fig.2: NP effects for $O_{qt}$ as seen in the unpolarized cross section $\sigma(e^-e^+ \rightarrow t\bar{t})$, as a function of the c.m. energy $2E_e$. The indicated value of the $f_{qt}$ coupling is chosen to create a 30% effect. Similar results are obtained for the contributing operators $O_{t2}$, $O_{Dt}$, $O_{tW\Phi}$, $O_{tB\Phi}$, $O_{qt}^{(8)}$, $O_{tt}$, $O_{tb}$, $O_{tG\Phi}$ and $O_{W}$, $O_{W\Phi}$, $O_{B\Phi}$, $O_{UW}$, $O_{UB}$ and $O_{\Phi2}$, using the NP couplings shown in the second column of Table 1.

\footnote{Note that in the figures, the signatures of $O_{qt}^{(8)}$ are never shown explicitly, since they are identical to those from $O_{qt}$, apart from an overall normalization factor of 16/3.}
Fig. 3: NP effects on the unpolarized forward-backward asymmetry in the differential cross section $A_{FB}(e^-e^+ \rightarrow t\bar{t})$, as function of the c.m. energy $2E_e$, for the NP couplings in Table 1. (a): 4-quark NP operators; $O_{tt}$ effect is similar to $O_{tb}$. (b): 2-quark NP operators; $O_{tB\Phi}$ gives similar effects to $O_{tW\Phi}$. (c): purely bosonic NP operators; the effects of $O_{W\Phi}$ and $O_{UB}$ are similar to those from $O_W$ and $O_{UW}$ respectively.
Fig. 4: NP effects on the unpolarized forward-backward asymmetry in the top average helicity $H_{FB}(e^- e^+ \rightarrow t\bar{t})$, as function of the c.m. energy $2E_e$, for the NP couplings in Table 1. (a): 4-quark NP operators. (b): 2-quark NP operators; the $\mathcal{O}_{tB\Phi}$ effect is similar to $1/3$ the one from $\mathcal{O}_{tW\Phi}$; while the effect from $\mathcal{O}_{Dt}$ is very small. (c): purely bosonic operators; $\mathcal{O}_{W\Phi}$ behaves similarly to $\mathcal{O}_{W}$; while $\mathcal{O}_{UB}$ gives a very small contribution.

Fig. 5: NP effects on the unpolarized forward-backward asymmetry in the top transverse polarization $T_{FB}(e^- e^+ \rightarrow t\bar{t})$, as function of the c.m. energy $2E_e$, for the NP couplings in Table 1. (a): 4-quark NP operators. (b1),(b2): 2-quark NP operators. (c): purely bosonic operators; $\mathcal{O}_{W\Phi}$ behaves similarly to $\mathcal{O}_{W}$; $\mathcal{O}_{WU}$ gives very small effect for $d = -5.94$, while for $d = 5.94$ the effect is similar to the one from $\mathcal{O}_{\Phi^2}$ for $f_{\Phi^2} = 11.9$.

Fig. 6: NP effects, for longitudinally polarized beams, on the Left-Right asymmetry $A_{LR}$, for the NP couplings in Table 1. (a): 4-quark NP operators; $\mathcal{O}_{tt}$ effect is similar to $\mathcal{O}_{tb}$. (b): 2-quark NP operators; $\mathcal{O}_{tG\phi}$ effect is very small. (c): purely bosonic NP operators; the effect of $\mathcal{O}_{W\Phi}$ is of similar magnitude but opposite sign to the one from $\mathcal{O}_{W}$; while the $\mathcal{O}_{\Phi^2}$ effect is very small.

Fig. 7: NP effects on the polarized forward-backward asymmetry in the differential cross section $A_{FB,pol}(e^- e^+ \rightarrow t\bar{t})$, as function of the c.m. energy $2E_e$, for the NP couplings in Table 1. (a): 4-quark NP operators; $\mathcal{O}_{tt}$ effect is similar to $\mathcal{O}_{tb}$. (b): 2-quark NP operators; $\mathcal{O}_{t2}$ effect is similar to $\mathcal{O}_{tG\Phi}$ give. (c): purely bosonic NP operators; $\mathcal{O}_{W\Phi}$ and $\mathcal{O}_{W}$ effects are of equal magnitude but of opposite sign to the $\mathcal{O}_{B\Phi}$ effect.

Fig. 8: NP effects on the polarized forward-backward asymmetry in the top average helicity $H_{FB,pol}(e^- e^+ \rightarrow t\bar{t})$, as function of the c.m. energy $2E_e$, for the NP couplings in Table 1. (a): 4-quark NP operators. (b): 2-quark NP operators; $\mathcal{O}_{tB\Phi}$ and $\mathcal{O}_{tW\Phi}$ behave similarly; $\mathcal{O}_{Dt}$ effect is very small. (c): purely bosonic operators; $\mathcal{O}_{UB}$ behaves similarly to $\mathcal{O}_{W}$; $\mathcal{O}_{W\Phi}$ effect is similar to $\mathcal{O}_{W}$.
Fig.9: NP effects on the polarized forward-backward asymmetry in the top transverse polarization $T_{FB,pol}(e^-e^+ \to t\bar{t})$, as function of the c.m. energy $2E_e$, for the NP couplings in Table 1. (a): 4-quark NP operators. (b): 2-quark NP operators; The size of the $\mathcal{O}_{tG\Phi}$ effect $\sim (\frac{1}{2} - \frac{1}{3})\mathcal{O}_{t2}$; while the effect from $\mathcal{O}_{tW\Phi}$ is a rough average of the $\mathcal{O}_{tB\Phi}$ and $\mathcal{O}_{Dt}$ ones. (c): purely bosonic operators; the effects of $\mathcal{O}_W$, $\mathcal{O}_{W\Phi}$ and $\mathcal{O}_{B\Phi}$ are not shown since they are small.

In order to make the production effects clearly visible in the figures, we have chosen the NP couplings such that the NP effect is about $\pm 30\%$ of the SM prediction on the integrated cross section. A corresponding choice with respect to the NP contribution to $\Gamma(t \to bW)$ has also been made for those operators which contribute only to decay and not to production. The third column in Table 1 identifies the operators giving non-vanishing contribution to top production either at the tree or at the 1-loop level. The rest of the columns in Table 1 describe the NP effects on $\Gamma(t \to bW)$ and the two forward-backward asymmetries constructed in Appendix B for the semileptonic top decay.
Table 1: NP couplings and effects on production and decay observables.

| $\mathcal{O}_i$ | $f_i, d, d_B, \lambda_W$ | $e^- e^+ \rightarrow t\bar{t}$ | $\Gamma(t \rightarrow bW)$ | $D_{FB}$ | $D_{FB}^2$ |
|----------------|-----------------------------|--------------------------------|---------------------------|----------|-------------|
| $SM$           |                             | no                            | 1.5581                    | 0.2210   | -0.5384     |
| $+\mathcal{O}_{qt}$ | (+, -)1.8                  | loop                          | No                        | No       | No          |
| $+\mathcal{O}_{tt}^{(8)}$ | (+, -)0.34                | loop                          | No                        | No       | No          |
| $+\mathcal{O}_{tb}^{(8)}$ | (+, -)0.27                | loop                          | No                        | No       | No          |
| $+\mathcal{O}_{t\ell}^{(8)}$ | (+, -)0.6                  | loop                          | No                        | No       | No          |
| $+\mathcal{O}_{qq}^{(8)}$ | (+, -)3                    | No                            | 1.5583                    | 0.2210   | -0.5384     |
| $+\mathcal{O}_{t_1}$ | No                          | No                            | 1.5644                    | 0.2210   | -0.5384, -0.5383 |
| $+\mathcal{O}_{t_2}$ | (+, -)1.12                 | tree                          | No                        | No       | No          |
| $+\mathcal{O}_{t_3}$ | (+, -)0.9                  | No                            | 2.1291                    | 0.2211   | -0.5385, -0.5388 |
| $+\mathcal{O}_{D_L}$ | (+, -)0.51                 | tree                          | 0.9373, 5.4581            | 0.2211   | -0.5385     |
| $+\mathcal{O}_{W \Phi}$ | (-, +)0.012               | tree                          | 1.6619, 1.4597            | 0.2407, 0.2030 | -0.6720, -0.4491 |
| $+\mathcal{O}_{B \Phi}$ | (-, +)0.099               | tree                          | No                        | No       | No          |
| $+\mathcal{O}_{tG \Phi}$ | (+, -)0.24                | loop                          | 1.5041, 1.6136            | 0.2117, 0.2313 | -0.4860, -0.6034 |
| $+\mathcal{O}_{W}$    | (+, -)0.40                 | loop                          | No                        | No       | No          |
| $+\mathcal{O}_{W \Phi}$ | (-, +)0.63                | loop                          | 1.5523, 1.5639            | 0.2219, 0.2202 | -0.5433, -0.5336 |
| $+\mathcal{O}_{tG \Phi}$ | (+, -)0.75                | loop                          | 1.5600, 1.5561            | 0.2207, 0.2214 | -0.5365, 0.5404 |
| $+\mathcal{O}_{lW \Phi}$ | (+, -)5.94                | loop                          | 1.6770, 1.4463            | 0.2437, 0.2022 | -0.6964, -0.4388 |
| $+\mathcal{O}_{W \Phi}$ | (-, +)5.35                | loop                          | No                        | No       | No          |
| $+\mathcal{O}_{W \Phi}$ | (-, +)11.9                | loop                          | 2.1235                    | 0.2210   | -0.5384     |

In Fig. 2 we give the NP results for the integrated unpolarized cross section $\sigma(e^- e^+ \rightarrow t\bar{t})$ for the $\mathcal{O}_{qt}$ case. Similar results would appear for all other operators, if the associated coupling constants take the values given in Table 1. The particular characteristics of each operator may then be studied by looking at the other observables appearing in the 4th-6th columns of Table 1 and in Figs. 3a-9c. In detail, for unpolarized beams, the forward-backward asymmetry $A_{FB}$ is illustrated in Figs. 3a,b,c, the asymmetry $H_{FB}$ in Figs. 4a,b,c, and the asymmetry $T_{FB}$ in Figs. 5a,b1,b2,c. Correspondingly, for longitudinally polarized $e^\pm$ beams, the asymmetry $A_{LR}$ is illustrated in Figs. 6a,b,c, while the forward-backward asymmetry $A_{FB,pol}$ is in Figs. 7a,b,c, the asymmetry $H_{FB,pol}$ in Figs. 8a,b,c and $T_{FB,pol}$ in Figs. 9a,b,c. Here, the "a"-figures refer to the 4-quark operators, the "b"-figures to the 2-quark ones, and the "c" to the bosonic operators. Occasionally in the figures, the results for some operators or observables almost coincide, for the couplings chosen above. Whenever this happens, it is just indicated in the figure caption.

The values for the coupling constants used in Table 1, are often unacceptably large,
either because of the existing indirect experimental constraints, or because they would imply, through unitarity, a very low NP scale. Nevertheless we used them in order to make the NP effects in the figures clearly visible.

The expected luminosity for a future linear collider is commonly taken as $80(s/TeV^2)$ $fb^{-1}$ per year. Since, the SM cross section for $e^+e^- \rightarrow t\bar{t}$ is about $170(TeV^2/s)$ $fb$, we expect a rate of more than $10^4$ events/year, implying a statistical accuracy of $\sim 1\%$ for the various physical quantities. The implied observability limits to various NP couplings are presented in Table 2, where we also give the present constraints [20]. In getting them, we have always assumed that only one NP operator acts at a time. Moreover, we have conservatively assumed a total (statistical + systematical) relative accuracy of 5% on the integrated cross section for unpolarized $e^\pm$ beams and an absolute 5% accuracy on the asymmetries defined in Appendix B. More precise numbers require of course detailed Monte Carlo analyses, taking into account the precise experimental conditions. Finally, using the unitarity relations presented in Section 2, we translate the observability limits on the various NP couplings, to bounds on the highest related NP scales $\Lambda_{NP}$, to which the specific measurements are sensitive. These bounds are also indicated in Table 2. In the last three columns we have indicated for each operator separately, the constraints established in ref.[5] and the expected LEP2 sensitivity. In all cases except for the $Z \rightarrow b\bar{b}$ case, the absolute value of the coupling constant is meant.

The following comments should now be made concerning the properties of the various operators and the observability limits presented in Table 2:

**Four-quark operators**

$O_{qt}$ and $O_{qt}^{(8)}$:
Both operators lead to the same effects (apart from an overall normalization factor $16/3$). They both contribute, at 1-loop, only to the vector and axial form factors $d_{1,2}^V$ and $d_{1,2}^A$. They do not contribute to the top decay. So their modifications of the SM predictions are always rather uniform as one can see on the figures. The observability limits obtained either from the integrated cross section or from $H_{FB}$, appear to be just marginally compatible with the LEP1 constraint from $Z \rightarrow b\bar{b}$, to which they also contribute at 1-loop, [3].

$O_{tt}$:
It induces (through 1-loop) a purely right-handed NP effect to the $\gamma t\bar{t}$ and $Zt\bar{t}$ couplings. There exist no $Z$-peak constraint on $O_{tt}$. The observability limit mainly comes from $H_{FB}$. No effect is generated in top decay.

$O_{tb}$:
Its effect is similar to the $O_{tt}$ one. However, it also gives a purely right-handed contribution to $Z \rightarrow b\bar{b}$, which is thus providing a LEP1 constraint. Its observability limit in Table 2, is just allowed by the present LEP1 results. No effect is generated in top decay.

$O_{tb}^{(8)}$:
This operator produces no effect in the processes studied here or in $Z$-peak physics.

$O_{qq}$ and $O_{qq}^{(8)}$:
They contribute (at the 1-loop level) to the $t \rightarrow bW$ decay couplings $d_{jW}^t$ giving a $\sigma_{\mu\nu}$-type contribution affecting only the right-handed $b_R$ field. They also give a $\sigma_{\mu\nu}$ contribution to
Z → b¯b. Both effects are too small to be observable for reasonable values of the coupling constants. They give no contribution to the γtt or Ztt vertices.

| Operator | $\sqrt{s} = 0.5$ TeV (Λ_{NP}) | $\sqrt{s} = 1$ TeV (Λ_{NP}) | $\sqrt{s} = 2$ TeV (Λ_{NP}) | Other constraints from $\epsilon_i$ | $Z \rightarrow b\bar{b}$ | LEP2 |
|----------|-------------------------------|-------------------------------|-------------------------------|---------------------------------|-----------------|------|
| $O_{qt}$ | 0.35(1.2)                     | 0.3(1.3)                      | 0.2(1.6)                      | —                              | $-0.3 \pm 0.2$ | —    |
| $O_{qt}^{(8)}$ | 0.07(1.3)                   | 0.06(1.4)                     | 0.04(1.8)                     | —                              | $-0.05 \pm 0.03$ | —    |
| $O_{tt}$ | 0.02(5.4)                     | 0.015(6.2)                    | 0.01(7.6)                     | —                              | —               | —    |
| $O_{tb}$ | 0.07(3.3)                     | 0.04(4.4)                     | 0.03(5.1)                     | —                              | $-0.3 \pm 0.2$ | —    |
| $O_{tb}^{(8)}$ | —                           | —                             | —                             | —                              | —               | —    |
| $O_{qq}$ | —                             | —                             | —                             | —                              | 38. ± 22.     | —    |
| $O_{qq}^{(8)}$ | —                           | —                             | —                             | —                              | 8. ± 5.       | —    |
| $O_{t1}$ | —                             | —                             | —                             | —                              | —               | —    |
| $O_{t2}$ | 0.011(11.)                    | 0.016(9.1)                    | 0.017(8.9)                    | 0.01                           | 0.3 ± 0.2     | —    |
| $O_{t3}$ | —                             | —                             | —                             | —                              | —               | —    |
| $O_{Dt}$ | 0.036(2.6)                    | 0.03(2.9)                     | 0.025(3.1)                    | 0.03                           | $-0.12 \pm 0.06$ | —    |
| $O_{W\Phi}$ | 0.002(30.5)                | 0.002(30.5)                   | 0.0015(35)                    | 0.014                          | —               | —    |
| $O_{B\Phi}$ | 0.0015(35.)                | 0.0015(35.)                   | 0.0013(38.)                   | 0.013                          | —               | —    |
| $O_{G\Phi}$ | 0.02(10.)                    | 0.03(6.9)                     | 0.075(2.8)                    | —                              | —               | —    |
| $O_{W}$ | 0.05(1.6)                     | 0.04(1.7)                     | 0.02(2.5)                     | —                              | 0.1             | —    |
| $O_{W\Phi}$ | 0.08(1.6)                    | 0.06(1.8)                     | 0.04(2.2)                     | —                              | 0.1             | —    |
| $O_{B\Phi}$ | 0.025(5.0)                   | 0.02(5.6)                     | 0.01(7.9)                     | —                              | 0.1             | —    |
| $O_{UW}$ | 0.5(1.2)                      | 0.8(0.9)                      | 1.6(0.65)                     | —                              | —               | 0.015 |
| $O_{UB}$ | 0.5(1.6)                      | 0.6(1.45)                     | 1.2(1.0)                      | —                              | —               | 0.05  |
| $O_{\phi_2}$ | 0.5(1.0)                    | 1.0(0.7)                      | 2.4(0.5)                      | —                              | —               | 0.01  |

Table 2: Sensitivity limits to "top" and "bosonic" operators in terms of NP couplings and related NP scales Λ_{NP} (TeV)

Two-quark operators.

$O_{t1}$: No effect is generated in top production or decay or in Z-peak physics.

$O_{t2}$: It gives a purely right-handed tree level contribution to the Ztt coupling. At the 1-loop level, it also contributes to $\epsilon_1$ and to the Zbâ over in a purely left-handed way. Present constraints from $\epsilon_1$ are marginal. There is no effect on the γtt and tbW vertices at tree level.

$O_{t3}$: At tree level, it produces a right-handed contribution to the tbW vertex. Its most impor-
tant constraint should come from $\Gamma(t \to bW)$. There exists no constraint from $Z$ peak physics.

$O_{Di}$:
At the tree level, it contributes a derivative coupling to the $Zt\bar{t}$ vertex, and has right-handed contribution to $tbW$. At the 1-loop level, it contributes to $\epsilon_1$ and to $Zb\bar{b}$ in a left-handed way. Present constraints from $Z$-peak are rather marginal. In the linear colliders, the dominant effect should come from $A_{FB, pol}(e^-e^+ \to t\bar{t})$.

$O_{tW\Phi}$:
It produces genuine tree level magnetic type $\sigma_{\mu\nu}$ couplings to the $\gamma t\bar{t}$, $Zt\bar{t}$ and $tbW$ vertices. The $tbW$ vertex has the additional characteristic that it only involves the left-handed $b_L$-field. $O_{tW\Phi}$ is presently constrained only by its 1-loop contribution to $\epsilon_3$. The observability limit in the linear colliders arises from the integrated production cross section, the top decay width and the decay asymmetries; (mainly the $D_{FB}^2$). This observability limit will supply only a very minor improvement to the present one from $\epsilon_3$. This operator could further be checked by looking for an enhancement in the decay $t \to WZb$, with the $Z$ decaying into lepton pairs. It can also contribute to $t \to WHb$, provided $H$ is sufficiently light.

$O_{tB\Phi}$:
It produces similar tree-level effects to the $\gamma t\bar{t}$ and $Zt\bar{t}$ vertices, but no contribution to $tbW$. The effect in the integrated cross section is similar to the one due to $O_{tW\Phi}$, but the effect on $\epsilon_3$ is weaker, leaving more chance for observability.

$O_{tG\Phi}$:
At the 1-loop level, it produces genuine magnetic type $\sigma_{\mu\nu}$ couplings to the $\gamma t\bar{t}$, $Zt\bar{t}$ and $tbW$ vertices. The $tbW$ vertex has the additional characteristic that it only involves the left-handed $b_L$-field. These properties are like those for $O_{tW\Phi}$; but appearing at 1-loop, rather than at tree level. As a result, there is now no contribution to $\epsilon_3$ at 1-loop. Future constraints to $O_{tG\Phi}$ from linear colliders should arise from studies of the integrated cross section, the decay width and the top decay asymmetries.

Bosonic operators.
The effects of these operators on the $\gamma t\bar{t}$, $Zt\bar{t}$ or $tbW$ vertices, arise only at the 1-loop level.

$O_{W}$:
It contributes only to the left-handed $\gamma t\bar{t}$ and $Zt\bar{t}$ form factors. A visible effect from them at a linear collider could appear in $H_{FB}$ and in the cross section. The needed value of the coupling falls just below the visibility domain of LEP2. No effect is generated in the decay.

$O_{W\Phi}$:
It contributes to the vector and axial form-factors for the $\gamma t\bar{t}$ and $Zt\bar{t}$ vertices. For the $tbW$ vertex, $O_{W\Phi}$ creates a left-handed and a derivative coupling, such that a left-handed $b_L$ field is only involved. The visible effects from these couplings at a linear collider are similar to those expected from $O_{W}$, and the same situation with respect to LEP2 is valid. The expected effects on $t \to bW$ seem to be below the observability level.
\(O_{B\Phi}\):
It induces the same type of couplings as \(O_{W\Phi}\). The sensitivity in \(H_{FB}\) is now somewhat better though; so that there exists the possibility of a visible effect from the \(\gamma t \bar{t}\) and \(Zt \bar{t}\) vertices, which will not be already excluded by LEP2. The effects on the \(t \rightarrow bW\) decay are still unobservable.

\(O_{UW}\):
As in the \(O_{tG\Phi}\) case, it produces genuine magnetic type \(\sigma_{\mu\nu}\) type couplings to the \(\gamma t \bar{t}, Zt \bar{t}\), and \(tbW\) vertices. The \(tbW\) vertex has the additional characteristic that it only involves a left-handed \(b_L\)-field. Note, that these same properties also arise in the \(O_{tW\Phi}\) case, where they are induced at the tree-level though. Another thing to note is that \(O_{UW}\) is very mildly constrained by \(Z\)-peak physics, which is also valid for \(O_{tG\Phi}\), but not true for \(O_{tW\Phi}\). A most distinctive signature discriminating \(O_{UW}\) from the other two operators, may be obtained by studying \(e^-e^+ \rightarrow ZH, \gamma H\) \([12]\).

\(O_{UB}\):
As far as the \(\gamma t \bar{t}\) and \(Zt \bar{t}\) vertices are concerned, the results are similar to the \(O_{UW}\) ones, but their ratio is different. For \(O_{UB}\) we have \(d_W^Z/d_W^Z = -2s_W^2\), while in the \(O_{UW}\) case we have instead \(d_W^Z/d_W^Z = -2c_W^2\). No effect appears in the \(t \rightarrow bW\) decay.

\(O_{\Phi^2}\):
This operator produces a purely axial \(Zt \bar{t}\) vertex, and a left-handed \(tbW\) one. There is no \(\gamma t \bar{t}\) vertex induced. Visible effects from \(Zt \bar{t}\) could be obtained by looking at \(H_{FB}\) and the integrated cross section. The study of \(\Gamma(t \rightarrow bW)\) should also help. Like the two previous operators, it could however be more strongly constrained by direct Higgs production.

7 Panorama of residual NP effects in the heavy quark and bosonic sectors

We have considered the possibility of anomalous top quark couplings induced by residual NP effects, described by twenty \(dim = 6\) gauge invariant operators. Fourteen of them involve the top quark, and the other six are purely bosonic. The couplings of these operators are associated to a NP scale through unitarity relations.

We have computed the effects of these operators in \(e^+e^- \rightarrow t \bar{t}\) and \(t \rightarrow bW\) decay. For \(e^+e^- \rightarrow t \bar{t}\), these effects are described in terms of six independent form factors for the general \(\gamma t \bar{t}\) and \(Zt \bar{t}\) vertices. Correspondingly, the \(t \rightarrow bW\) decay is described in terms of four couplings denoted as \(d_W^j\). The top quark density matrix can thus be expressed in terms of these form factors and couplings. We have shown how one can analyze this density matrix in order to get information on the possible forms of NP induced by the various operators. The extra information brought by polarized \(e^\pm\) beams, is also considered.

Thus, in addition to the integrated unpolarized cross section and the L-R asymmetry, it is possible for polarized beams to construct six different forward-backward asymmetries which should allow to disentangle the effects of the six form factors \(d_W^j, d_Z^j\). It is more difficult to disentangle the NP effects on the \(t \rightarrow bW\) decay, as no accurate measurement of \(\Gamma(t \rightarrow bW)\) is expected, and only one particular combination of decay couplings can easily
be measured through an asymmetry with respect to the final lepton, in a semileptonic
top decay.\footnote{The same is also true if a hadronic mode is considered.}

The consequences of the NP operators on the above form factors and $d_j^W$ couplings were
calculated to first order in the NP couplings. The calculation was done to the tree level,
whenever this gave a non-vanishing contribution. In case there was no such contribution,
we performed a calculation at the 1-loop level, keeping only the leading-log $m_t$-enhanced
part. Numerical illustrations have been given for the various observables, which reflect
the specific properties of each operator. We have then established the corresponding
observability limits for each operator in terms of the associated coupling constant and
identified the related NP scale. The results can be summarized as follows.

4-quark operators:
Among the seven 4-quark operators, four of them $O_{qt}$, $O_{qt}^{(8)}$, $O_{tt}$, $O_{tb}$, could give sizeable
1-loop effects in $e^+e^-\rightarrow tt$. $O_{tt}$ is not constrained by $Z$ peak physics, which means that $e^+e^-\rightarrow tt$ would provide a completely new test. The three other operators produce 1-loop
effects also in $Z\rightarrow b\bar{b}$, which have been studied in \cite{5}. The departure from SM presently
observed in $\Gamma(Z\rightarrow b\bar{b})$, if attributed to one of these three operators, could produce effects
which should be easily visible in the $O_{tb}$ case, but would only be marginally observable
for $O_{qt}$ and $O_{qt}^{(8)}$.

Concerning the remaining three 4-quark operators, we note that $O_{q\bar{q}}$ and $O_{q\bar{q}}^{(8)}$ are
constrained essentially only by $t\rightarrow bW$ \cite{5}, while $O_{tb}^{(8)}$ is not sensitive to $Z$-peak physics
or $e^-e^+\rightarrow t\bar{t}$, $t \rightarrow bW$.

2-quark operators:
Four of the seven 2-quark operators, $O_{t2}$, $O_{Dt}$, $O_{tW}\Phi$ and $O_{tB}\Phi$, produce tree-level effects
in $e^+e^-\rightarrow t\bar{t}$, while $O_{t3}$ produces a tree-level effect only in $t\rightarrow bW$. $O_{tG}\Phi$ contributes at
1-loop level to both production and decay. However the above four operators $O_{t2}$, $O_{Dt}$,
$O_{tW}\Phi$, $O_{tB}\Phi$ generate also some $\epsilon_j$ contribution to $Z$-peak physics. This seems to already
exclude an observable effect from $O_{t2}$ and $O_{Dt}$, but leaves some range for observability to
$O_{tW}\Phi$ and $O_{tB}\Phi$.

Concerning the rest of the 2-quark operators, we remark that $O_{t3}$ and $O_{tG}\Phi$ are
presently unconstrained; so $e^-e^+\rightarrow t\bar{t}$ and $t \rightarrow bW \rightarrow b\ell^+\nu$ would provided genuine
new tests of them. $O_{t1}$ is not observable through these processes though, and its study
requires $ttH$ production \cite{25}.

Bosonic operators:
All six bosonic operators contribute at 1-loop to $e^+e^-\rightarrow t\bar{t}$. Moreover, $O_{W}\Phi$, $O_{UW}$ and
$O_{\Phi2}$ could also significantly contribute to $t \rightarrow bW$. The operators $O_{W}$, $O_{W}\Phi$ and $O_{B}\Phi$
should have an observability level which would not be excluded by LEP2. Notice that
$O_{W}\Phi$ has a 1-loop effect in $Z\rightarrow b\bar{b}$ which could explain the observed anomaly there \cite{4}. In this case large effects should be observed in $tt$ production. However a direct study of
these operators in $e^+e^-\rightarrow W^+W^-$ at NLC should be even more stringent.

The three other bosonic operators involving Higgs field are almost unconstrained at
present. So nothing excludes their appearance. However if the Higgs mass is low enough
to allow for $e^+e^- \rightarrow HZ$ or $e^+e^- \rightarrow H\gamma$ at LEP2 and/or at NLC, then these processes would improve the sensitivity limits on these operators by 2 orders of magnitude.

In conclusion the process $e^+e^- \rightarrow t\bar{t}$ should bring essential information on residual NP effects affecting the heavy quark sector as well as the bosonic (gauge and scalar) sector. Its main interest is that it provides direct tests of the presence of genuine operators involving the third family of quarks. It could give hints about the origin of the anomalies recently observed in the $Zb\bar{b}$ couplings.

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Appendix A: New physics vertices generated by the effective lagrangian

It is easy to see that the leading-log $m_t^2$-enhanced NP contributions to $e^-e^+ \rightarrow t\bar{t}$ (up to the 1-loop order), come exclusively from vertex diagrams for the $\gamma \rightarrow t\bar{t}$ and $Z \rightarrow t\bar{t}$ vertices and from self-energies. The same is of course true for the diagrams affecting $t \rightarrow bW$. Thus, box diagrams never appear. Below we enumerate these contributions for the various operators.

Four-quark operators:
They contribute to the vertices $\gamma t\bar{t}$, $Zt\bar{t}$, $tbW$ through 1-loop diagrams involving a 4-quark interaction. This interaction can be read off the following extended expressions. Thus,

$$O_{qt} = (\bar{t}L \tau tR)(\bar{t}R \tau tL) + (\bar{b}L \tau tR)(\bar{b}R \tau tL)$$ (A.1)

contributes through $t$ and $b$ loops to the $\gamma t\bar{t}$ and $Zt\bar{t}$ vertices, but not to the $tbW$ one.

$$O_{qt}^{(8)} = (\bar{t}L \chi^\mu tR)(\bar{t}R \chi tL) + (\bar{b}L \chi^\mu tR)(\bar{b}R \chi tL)$$ (A.2)

contributes through a $b$ loop only to the $\gamma t\bar{t}$, $Zt\bar{t}$ vertices, but not to the $tbW$ one.

$$O_{tt} = \frac{1}{2}(\bar{t}R \gamma^\mu tR)(\bar{t}R \gamma^\mu tR)$$ (A.3)

contributes through $t$-loop to $\gamma t\bar{t}$, $Zt\bar{t}$ but not to $tbW$.

$$O_{tb} = (\bar{t}R \gamma^\mu tR)(\bar{b}R \gamma^\mu bR)$$ (A.4)

contributes through a $b$-loop to $\gamma t\bar{t}$, $Zt\bar{t}$ but not to $t \rightarrow bW$.

$$O_{tb}^{(8)} = (\bar{t}R \chi^\mu tR)(\bar{b}R \chi^\mu bR)$$ (A.5)

gives no contribution.

$$O_{qq} = (\bar{t}R tL)(\bar{b}R bL) + (\bar{t}L tR)(\bar{b}L bR) - (\bar{t}R bL)(\bar{b}R tL) - (\bar{b}L tR)(\bar{t}L bR)$$ (A.6)

contributes to the $tbW$ vertex, but not to $\gamma t\bar{t}$, $Zt\bar{t}$. Finally, the

$$O_{qq}^{(8)} = (\bar{t}R \chi^\mu tL)(\bar{b}R \chi bL) + (\bar{t}L \chi^\mu tR)(\bar{b}L \chi bR)$$

$$- (\bar{t}R \chi^\mu bL)(\bar{b}R \chi tL) - (\bar{b}L \chi^\mu tR)(\bar{t}L \chi bR)$$ (A.7)

contributions are obtained from the $O_{qq}$ ones, by multiplying by the factor $\frac{16}{3}$.

Two-quark operators:
Some of the operators in this class contribute already at the tree level, while others only at the 1-loop level. The later contributions arise from triangle diagrams for the $\gamma t\bar{t}$, $Zt\bar{t}$ and $tbW$ vertices, as well as from fermion self-energy ones. When an operator contributes at tree level, we do not care about its 1-loop contributions.
For the operator $O_{t_1}$, (after subtracting irrelevant contributions to the top mass), we get
\[
O_{t_1} = [\chi^+ \chi^- + vH + \frac{1}{2}(\chi^3 \chi^3 + H^2)] \left[ \frac{v + H}{\sqrt{2}} (\bar{t}t) + \frac{i}{\sqrt{2}} \chi^3 (\bar{t}\gamma^5 t) + i\chi^- (\bar{b}_L t_R) - i\chi^+ (\bar{t}_R b_L) \right], \tag{A.8}
\]
which gives no contribution to the amplitudes we are interested. For the $\gamma, Z \rightarrow t\bar{t}$ amplitudes, this comes about from the cancellation of the contributions from the vertex triangles involving $(ttH)$ and $(tH\chi^3)$ exchanges, and the $(tH)$-self-energy; while for the $t \rightarrow bW$ decay, the sum of the $(tH\chi^3)$ triangle and the $(tH)$-self-energy vanishes. The operator
\[
O_{t_2} = i(\bar{t}_L \gamma^\mu t_R) \left\{ (\chi^- \partial_\mu \chi^+ - \partial_\mu \chi^- \chi^+) + g(v + H)(\chi^- W_\mu^+ - \chi^+ W_\mu^-) 
- ig\chi^3 (\chi^- W_\mu^+ + \chi^+ W_\mu^-) + igZ(1 - 2s_W^2)Z_\mu \chi^+ \chi^- + 2ieA_\mu \chi^+ \chi^- 
- \frac{igZ}{2} \left[ (v + H)^2 + \chi^3 \partial_\mu H - i(v + H) \partial_\mu \chi^3 \right] \right\} \tag{A.9}
\]
contributes at tree level to $t\bar{t}$ production.
\[
O_{t_3} = i(\bar{t}_R \gamma^\mu b_R) \left\{ \frac{i}{\sqrt{2}} ((v + H - i\chi^3) \partial_\mu \chi^+ - \chi^+ \partial_\mu (H - i\chi^3)) + \frac{ig}{\sqrt{2}} \chi^+ W_\mu^- \chi^+ 
+ \frac{ig}{2\sqrt{2}} W_\mu^+(v + H - i\chi^3)^2 - \frac{gZc_W^2}{\sqrt{2}} (v + H - i\chi^3) Z_\mu \chi^+ 
- \frac{e}{\sqrt{2}} (v + H - i\chi^3) A_\mu \chi^+ \right\} 
- i(\bar{b}_R \gamma^\mu t_R) \left\{ \frac{-i}{\sqrt{2}} ((v + H + i\chi^3) \partial_\mu \chi^- - \chi^- \partial_\mu (H + i\chi^3)) - \frac{ig}{\sqrt{2}} \chi^- W_\mu^+ \chi^- 
- \frac{ig}{2\sqrt{2}} W_\mu^- (v + H + i\chi^3)^2 - \frac{gZc_W^2}{\sqrt{2}} (v + H + i\chi^3) Z_\mu \chi^- 
- \frac{e}{\sqrt{2}} (v + H + i\chi^3) A_\mu \chi^- \right\} \tag{A.10}
\]
has no effect on $t\bar{t}$ production, but contributes at tree level to the $t \rightarrow bW$ decay.
\[
O_{Dt} = \bar{t}_L \left[ \partial_\mu + ig \frac{2}{3} (-s_W Z_\mu + c_W A_\mu) + i\frac{g\sqrt{2}}{2} \vec{\chi} \cdot \vec{G}_\mu \right] t_R \cdot \left[ \frac{1}{\sqrt{2}} \partial_\mu (H + i\chi^3) 
+ \frac{ig}{2\sqrt{2}c_W} (v + H + i\chi^3) Z_\mu - \frac{g}{2} W_\mu^+ \chi^- \right] 
+ \bar{b}_L \left[ \partial_\mu + ig \frac{2}{3} (-s_W Z_\mu + c_W A_\mu) + i\frac{g\sqrt{2}}{2} \vec{\chi} \cdot \vec{G}_\mu \right] t_R \cdot \left[ \partial_\mu \chi^- 
+ \frac{g(1 - 2s_W^2)}{2c_W} Z_\mu \chi^- + eA_\mu \chi^- + \frac{ig}{\sqrt{2}} W_\mu^- (v + H + i\chi^3) \right] 
+ \bar{t}_R \left[ \partial_\mu - ig \frac{2}{3} (-s_W Z_\mu + c_W A_\mu) - i\frac{g\sqrt{2}}{2} \vec{\chi} \cdot \vec{G}_\mu \right] t_L \cdot \left[ \frac{1}{\sqrt{2}} \partial_\mu (H - i\chi^3) \right]
\]
\[-\frac{ig}{2\sqrt{2}c_W}(v + H - i\chi^3)Z_\mu - \frac{g}{\sqrt{2}W_\mu}\chi^+) + \]
\[\tilde{t}_R \left( G_\mu^\tau - \frac{ig}{3}(-s_W Z_\mu + c_W A_\mu) - i\frac{g}{2}\chi^+ \cdot G_\mu^\tau \right) b_L \left[ -i\partial_\mu\chi^+ \right]
+ g\left(1 - 2s_W^2\right)Z_\mu\chi^+ + A_\mu\chi^+ - \frac{ig}{\sqrt{2}W_\mu}(v + H - i\chi^3) \right] \text{(A.11)}
\]

contributes at tree level to both production and decay. The same is true for

\[O_{tW\Phi} = \left(c_W Z_\mu + s_W A_\mu\right)\left\{ \frac{1}{\sqrt{2}}(i\sigma^{\mu\nu}t)(v + H) + \frac{i}{\sqrt{2}}(\tilde{t}_R V_{\mu\nu} t_R)\chi^+ - (i\tilde{t}_R V_{\mu\nu} b_L)\chi^- \right\} \text{(A.12)}\]

while

\[O_{tB\Phi} = \left(-s_W Z_\mu + c_W A_\mu\right)\left\{ \frac{1}{\sqrt{2}}(i\sigma^{\mu\nu}t)(v + H) + \frac{i}{\sqrt{2}}(\tilde{t}_R V_{\mu\nu} t_R)\chi^+ - (i\tilde{t}_R V_{\mu\nu} b_L)\chi^- \right\} \text{(A.13)}\]

contributes at tree level only to \(t\bar{t}\) production. Finally

\[O_{tG\Phi} = \left\{ \frac{1}{\sqrt{2}}(i\sigma^{\mu\nu} t')(v + H) + \frac{i}{\sqrt{2}}(i\sigma^{\mu\nu} \lambda^a t_R)\chi^+ - (i\tilde{t}_R V_{\mu\nu} b_L)\chi^- \right\} \text{(A.14)}\]

contributes at 1-loop to production through the \((ttg)\) triangle and the \((tg)\) self-energy; and to \(t \rightarrow bW\) decay through the \((tbg)\) triangle and the \((tg)\) self-energy.

Bosonic operators:

Contributions in this class arise only at the 1-loop level, through triangle and self-energies diagrams. Thus, \(O_W\) (see (17)) contributes to \(e^-e^+ \rightarrow t\bar{t}\) production through the \((WWb)\) triangle\footnote{\(O_W\) does not produce \(m_t^2\) terms but it must be taken into consideration since the contribution is proportional to \(s\) which is larger than \(4m_t^2\) for the process under consideration.}, but gives no contribution to \(t \rightarrow bW\), since the sum of the \(m_t^2\)-enhanced parts of the \((tW\gamma)\) and \((tWZ)\) triangles vanishes. The operator \(O_{W\Phi}\) (see (18)) contributes to production through the \((tH\chi^3)\) and \((\chi^+\chi^-)\) triangles; and to decay through the \((t\chi^+\gamma)\), \((t\chi^+Z)\), \((b\chi^+\gamma)\) and \((b\chi^+Z)\) triangles. In a similar way \(O_{B\Phi}\) (see (19)) contributes to production through the \((tH\chi^3)\) and \((\chi^+\chi^-)\) triangles; and to decay through the \((t\chi^+\gamma)\), \((t\chi^+Z)\), \((b\chi^+\gamma)\) and \((b\chi^+Z)\) triangles. The operator \(O_{tW}\) (see (20)) contributes to production through the \((tH\gamma)\) and \((tHZ)\) triangles and to decay through \((tHW)\). Correspondingly, \(O_{UB}\) (see (21)) contributes to production through the \((tH\gamma)\) and \((tHZ)\) triangles, (like in the \(O_{tW}\) case), but gives no contribution to \(t \rightarrow bW\) decay. Finally \(O_{\Phi 2}\) (see (22)) induces a renormalization of the physical Higgs field at the tree level. This, in turn, gives contributions to production, through the \(ttH\)-triangle and the \(tH\)-self-energy, and to decay through the \(tH\)-self-energy.
Appendix B : Top decay distributions

As discussed in Section 4, it is convenient to express the 3-body phase space \( d\Phi_3(bl\nu_l) \) in terms of the Euler angles determining the \( t \)-decay plane. We start from the process \( e^- (k) e^+ (k') \rightarrow t(p) \bar{t}(p') \) in the center of mass frame, where the momenta are indicated in parentheses; and by \( \theta \) we denote the \( (e^-, t) \) scattering angle. The \( t \)-frame is defined with its \( z \)-axis along the top-quark momentum. The \( x \)-axis is taken in the \( (t \bar{t}) \) production plane, so that the \( y \)-axis is perpendicular to it and along the direction of \( \vec{k} \times \vec{p} \). In order to describe the decay-plane of the process \( t \rightarrow b(p_b) l^+(p_l) \nu_l(p_\nu) \), in the \( t \)-frame (with the momenta indicated in parentheses), we define the Euler rotation

\[
R_{\varphi_1 \vartheta_1 \psi_1} = 
\begin{pmatrix}
\cos \varphi_1 & -\sin \varphi_1 & 0 \\
\sin \varphi_1 & \cos \varphi_1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
\cos \vartheta_1 & 0 & \sin \vartheta_1 \\
0 & 1 & 0 \\
-\sin \vartheta_1 & 0 & \cos \vartheta_1
\end{pmatrix}
\begin{pmatrix}
\cos \psi_1 & -\sin \psi_1 & 0 \\
\sin \psi_1 & \cos \psi_1 & 0 \\
0 & 0 & 1
\end{pmatrix},
\tag{B.1}
\]

where \( (\varphi_1, \vartheta_1, \psi_1) \) satisfy \( 0 \leq \varphi_1, \psi_1 < 2\pi \), \( 0 \leq \vartheta_1 \leq \pi \). The meaning of these angles is given by remarking that the normal to the \( t \)-decay plane, with its orientation defined by \((\vec{p}_b \times \vec{p}_t)\), is given by

\[
\hat{n} = R_{\varphi_1 \vartheta_1 \psi_1} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \sin \vartheta_1 \cos \varphi_1 \\ \sin \vartheta_1 \sin \varphi_1 \\ \cos \vartheta_1 \end{pmatrix}.
\tag{B.2}
\]

Thus, \( \vartheta_1, \varphi_1 \) determine the \( \hat{n} \) orientation, while the \( b \) quark momentum in the \( t \)-rest frame is determined from \( \psi_1 \) through the relation

\[
\vec{p}_b = |\vec{p}_b| R_{\varphi_1 \vartheta_1 \psi_1} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = |\vec{p}_b| \begin{pmatrix} \cos \varphi_1 \cos \vartheta_1 \cos \psi_1 - \sin \varphi_1 \sin \psi_1 \\ \sin \varphi_1 \cos \vartheta_1 \cos \psi_1 + \cos \varphi_1 \sin \psi_1 \\ -\sin \vartheta_1 \cos \psi_1 \end{pmatrix}.
\tag{B.3}
\]

The corresponding expression for the \( l^+ \) momentum is obtained from (B.3) by substituting \( \vec{p}_b \rightarrow \vec{p}_l \) and \( \psi_1 \rightarrow \psi_1 + y_{12} \), where \( y_{12} \) is the angle between the \( b \) and \( l^+ \) momenta (in the \( t \) rest frame). To summarize, it is worthwhile to remark that the above Euler rotation moves the \( z \)-axis of the \( t \)-frame along the normal to the \( t \)-decay plane; while the \( x \)-axis is brought along the \( \vec{p}_b \) momentum, which of course lies within the decay plane.

Finally \( \theta_t \) is the angle between the lepton momentum and the top momentum in the \( W \)-rest frame, and it is related to the \( l^+ \) energy in the \( t \)-frame by

\[
E_l = |\vec{p}_l| = \frac{m_t^2 + M_W^2 - \cos \theta_t (m_t^2 - M_W^2)}{4m_t},
\tag{B.4}
\]

where the \((b, l^+)\) masses are neglected. Using these Euler angles, we obtain

\[
\delta((p_l + p_\nu)^2 - M_W^2) d\Phi_3(bl\nu_l) \Rightarrow \frac{(m_t^2 - M_W^2)}{64m_t^2(2\pi)^9} d\varphi_1 d\vartheta_1 d\psi_1 d \cos \theta_t,
\tag{B.5}
\]

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for the 3-body phase space in the case where the $\nu$-pair is at the $W$-mass shell \[23\].

The general expression of the differential cross section for $e^+e^- \to t\bar{t}$ with $t \to bW \to bl\nu_l$ and linearly polarized $[L(R)\ e^-]$ and $[R(L)\ e^+]$ beams, is written (compare (55,56)) as

$$
\frac{d\sigma^{L,R}}{d\cos\theta d\varphi_1 d\cos\vartheta d\psi_1 d\cos\theta_1} = \left( \frac{3\beta_t}{32(2\pi)^5 s} \right) \frac{G_F^2 M_W^3}{\Gamma_W \Gamma_t m_t} \left( \frac{m_t^2 - M_W^2}{4m_t} \right)^2 \rho^{L,R}_{\tau_1\tau_2} \cdot \mathcal{R}_{\tau_1\tau_2}, \quad (B.6)
$$

where $\beta_t = (1 - \frac{4m_t^2}{s})^{1/2}$. In (B.6), $\rho^{L,R}$ is the top density matrix defined in (50), while $\mathcal{R}$ is related to the top decay matrix $t_{\tau_1\tau_2}$ introduced in (56,58) by

$$
\frac{t_{\tau_1\tau_2}}{d\varphi_1 d\cos\vartheta d\psi_1 d\cos\theta_1} = \frac{G_F^2 M_W^3}{2(2\pi)^4 \Gamma_W \Gamma_t m_t} \left( \frac{m_t^2 - M_W^2}{4m_t} \right)^2 \mathcal{R}_{\tau_1\tau_2}. \quad (B.7)
$$

The $\rho$ matrix depends only on the $e^-e^+ \to t\bar{t}$-production and the angle $\theta$, (see (50)); while $\mathcal{R}$ depends on the three Euler angles $\varphi_1$, $\vartheta_1$, $\psi_1$, (defined in (B.1)) and on the $d_j^W$ couplings of (53) and the angle $\theta_t$. To simplify the expression for $\mathcal{R}$, we only keep terms linear in the NP and the 1-loop SM contributions to the couplings $d_j^W$, defining $\bar{d}_j^W \equiv d_j^{W,SM1} + d_j^{W,SM2}$. We thus get

$$
\rho^{L,R}_{\tau_1\tau_2} \cdot \mathcal{R}_{\tau_1\tau_2} = \frac{1}{2} (\rho_{++} + \rho_{--}) L^{L,R}(\mathcal{R}_{++} + \mathcal{R}_{--}) + \frac{1}{2} (\rho_{++} - \rho_{--}) L^{L,R}(\mathcal{R}_{++} - \mathcal{R}_{--})
\quad + \rho_{+-}^{L,R} (\mathcal{R}_{+-} + \mathcal{R}_{-+}), \quad (B.8)
$$

where

$$
\mathcal{R}_{++} + \mathcal{R}_{--} = (1 + \bar{d}_1^W - \bar{d}_2^W)[M_W^2 V_1 + m_t^2 V_2] + \left( \frac{m_t^2 - M_W^2}{m_t} \right) (\bar{d}_3^W + \bar{d}_4^W) m_t^2 V_2, \quad (B.9)
$$

$$
\mathcal{R}_{++} - \mathcal{R}_{--} = (1 + \bar{d}_1^W - \bar{d}_2^W)(m_t^2 V_4 - M_W^2 V_3 + 2m_t M_W V_5) + \left( \frac{m_t^2 - M_W^2}{m_t} \right) (\bar{d}_3^W + \bar{d}_4^W)[m_t^2 V_4 + m_t M_W V_5], \quad (B.10)
$$

$$
\mathcal{R}_{+-} + \mathcal{R}_{-+} = (1 + \bar{d}_1^W - \bar{d}_2^W)[M_W^2 V_6 - m_t^2 V_7 - 2m_t M_W V_8] - \left( \frac{m_t^2 - M_W^2}{m_t} \right) (\bar{d}_3^W + \bar{d}_4^W)[m_t^2 V_7 + m_t M_W V_8], \quad (B.11)
$$

and

$$
V_1 = (1 + \cos\theta_t)^2, \quad V_2 = \sin^2\theta_t, \quad (B.12)
$$

$$
V_3 = (1 + \cos\theta_t)^2 \sin\vartheta_1 \cos\psi_1, \quad V_4 = \sin^2\theta_t \sin\vartheta_1 \cos\psi_1, \quad (B.13)
$$

$$
V_5 = (1 + \cos\theta_t) \sin\theta_t \sin\vartheta_1 \sin\psi_1, \quad (B.14)
$$

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Using the angular dependence in (B.9-17) and constructing appropriate averages over \( \varphi_1, \vartheta_1 \) and \( \psi_1 \), it is possible to project quantities proportional to the \( \rho \) factors in each of the three terms of the r.h.s. in (B.8). More explicitly these quantities consist of products of the corresponding \( \rho \) elements, and of functions of \( \vartheta_1 \). Remember that the \( \rho \) elements depend only on \( \vartheta_1 \) and the NP couplings for \( \gamma t\bar{t}, Zt\bar{t} \). Thus, the subsequent construction of forward-backward asymmetries with respect to either \( \theta \) or \( \vartheta_1 \) respectively, allows the isolation of either the \( \rho \) factor or of the corresponding combination of the \( \bar{d}_j \) couplings. To do this we first describe the \( \rho \) elements entering the three terms in (B.8). For this, It is convenient to define for \( i = 1, 2, 3 \) (compare (44))

\[
d^L_i = d^L_i + \frac{1 - 2s_W^2}{4s_W^2c_W^2}\chi d^Z_i, \quad d^R_i = d^R_i - \frac{\chi}{2c_W^2}d^Z_i, \quad \text{(B.18)}
\]

where \( \chi \equiv s/(s - M_Z^2) \) and the Z width is neglected for \( s = q^2 > 4m_t^2 \). We then have

\[
(\rho_{++} + \rho_{-\cdot})^{L,R} = e^4 \sin^2 \theta \left( \frac{8m_t^2}{s} \right) \left[ d^L_i - \frac{2|\bar{p}|^2}{m_t}d^L_i \right]^2 \\
+ 2e^4(1 + \cos^2 \theta) \left[ (d^L_i)^2 + \frac{4|\bar{p}|^2}{s}(d^L_i)^2 \right] \mp e^4 \cos \theta \left( \frac{16|\bar{p}|}{\sqrt{s}} \right) d^L_i d^L_i, \quad \text{(B.19)}
\]

\[
(\rho_{++} - \rho_{-\cdot})^{L,R} = e^4 (1 + \cos^2 \theta) \left( \frac{8|\bar{p}|}{\sqrt{s}} \right) d^L_i d^L_i \\
\mp 4e^4 \cos \theta \left[ (d^L_i)^2 + \frac{4|\bar{p}|^2}{s}(d^L_i)^2 \right], \quad \text{(B.20)}
\]

\[
\rho_{+\cdot}^{L,R} = e^4 \sin \theta \left( \frac{4m_t}{\sqrt{s}} \right) \left[ d^L_i - \frac{2|\bar{p}|^2}{m_t}d^L_i \right] \left[ d^L_i - \frac{2|\bar{p}|}{\sqrt{s}} \cos \theta d^L_i \right], \quad \text{(B.21)}
\]

For unpolarized \( e^\pm \) beams, only the \( (L + R)/2 \) combination, like e.g. \( d\sigma^{unpol} = (d\sigma^L + d\sigma^R)/2 \) or \( (\rho^L + \rho^R)/2 \), is measurable through forward-backward asymmetries. There are three \( \rho \) and \( \mathcal{R} \) elements that can be studied this way. If longitudinal electron beam polarization is available, we can also consider the corresponding three \( (\rho^L - \rho^R) \) combinations and their forward-backward asymmetries.

Thus, by integrating both sides of (B.6) over \( d\varphi_1 d\cos \vartheta_1 d\psi_1 \), the first term in the r.h.s. of (B.8) is projected. Integrating also over \( \cos \vartheta_1 \), and constructing the forward-backward asymmetry with respect to the \( t\bar{t} \) production angle \( \theta \), allows the study of the NP effects

\[
V_6 = (1 + \cos \vartheta_1)^2(\cos \varphi_1 \cos \vartheta_1 \cos \psi_1 - \sin \varphi_1 \sin \psi_1), \quad \text{(B.15)}
\]

\[
V_7 = \sin^2 \vartheta_1(\cos \varphi_1 \cos \vartheta_1 \cos \psi_1 - \sin \varphi_1 \sin \psi_1), \quad \text{(B.16)}
\]

\[
V_8 = \sin \vartheta_1(1 + \cos \vartheta_1)(\cos \varphi_1 \cos \vartheta_1 \cos \psi_1 + \sin \varphi_1 \cos \psi_1), \quad \text{(B.17)}
\]
in \((\rho_{++} + \rho_{--})^{L,R}\). This asymmetry is of course the usual forward-backward asymmetry in the differential cross section for the top production through \(e^+e^- \to t\bar{t}\). We thus have

\[
A_{FB} = \frac{3\beta}{2} \frac{d^R d^R - d^L d^L}{(d^L)^2 + (d^R)^2 + \beta^2[(d^L)^2 - (d^R)^2] + \frac{2m_t^2}{s}(d^L - \frac{2|p|^2}{m_t} d^L)^2 + (d^R - \frac{2|p|^2}{m_t} d^R)^2}
\]

(B.22)

for the unpolarized case, while for the \(L - R\) one we have

\[
A_{FB,pol} = \frac{-3\beta}{2} \frac{d^R d^R + d^L d^L}{(d^L)^2 - (d^R)^2 + \beta^2[(d^L)^2 - (d^R)^2] + \frac{2m_t^2}{s}(d^L - \frac{2|p|^2}{m_t} d^L)^2 - (d^R - \frac{2|p|^2}{m_t} d^R)^2}
\]

(B.23)

The preceding method for constructing the forward-backward asymmetry is just given in order to emphasize its similarity to the methods for constructing the other asymmetries below. Consequently, by multiplying both sides of (B.6) by either \(\cos \psi_1\) or \(\sin \psi_1\) and integrating over \(d\varphi_1 d\cos \theta_1 d\psi_1\), the second term in the r.h.s. of (B.8) is projected. Integrating then over \(\cos \theta_1\), we construct the forward-backward asymmetry with respect to \(\theta\), for the quantity \((\rho_{++} - \rho_{--})^{L,R}\) controlling the angular distribution of the average helicity of the produced top-quark. Thus, in terms of the couplings defined in (43), the forward-backward asymmetry in the top-quark average helicity is

\[
H_{FB} = -\frac{3\{}{(d^L)^2 - (d^R)^2 + \beta^2[(d^L)^2 - (d^R)^2]}\{8\beta_1(d^L d^L + d^R d^R)}
\]

(B.24)

for the \(e^\pm\) unpolarized case, while for the \(L - R\) one we have

\[
H_{FB,pol} = -\frac{3\{}{(d^L)^2 + (d^R)^2 + \beta^2[(d^L)^2 + (d^R)^2]}\{8\beta_1(d^L d^L - d^R d^R)}
\]

(B.25)

Finally the third term in the r.h.s. of (B.8) is projected by multiplying both sides of (B.6) by quantities like any one of

\[
\cos \psi_1 \sin \varphi_1 \quad \sin \psi_1 \cos \varphi_1 \cos \theta_1 \quad \sin \psi_1 \sin \varphi_1 \quad \cos \psi_1 \cos \varphi_1 \cos \theta_1
\]

(B.26)

and integrating over \(d\varphi_1 d\cos \theta_1 d\psi_1\). The subsequent integration over \(\cos \theta_1\) allows the construction of the forward-backward asymmetry with respect to \(\theta\) for the quantity \(\rho^{L,R}_{++}\) controlling the angular distribution of the top average transverse polarization. Thus, for unpolarized \(e^\pm\) beams, the forward-backward asymmetry in the top transverse polarization is obtained, which is given by

\[
T_{FB} = \frac{4\beta}{3\pi} \frac{d^L d^L + d^R d^R - \frac{2|p|^2}{m_t} (d^L d^L + d^R d^R)}{(d^L)^2 - (d^R)^2 - \frac{2|p|^2}{m_t} (d^L d^L - d^R d^R)}
\]

(B.28)
while for polarized beams the $L - R$ case gives

$$T_{FB, pol} = -\frac{4\beta_t}{3\pi} \left[d_1^L d_2^L - d_1^R d_2^R - \left(\frac{2|p_t|^2}{m_t}\right)(d_3^L d_2^L - d_3^R d_2^R)\right].$$ \hspace{1cm} (B.29)

For any of the preceding three types of forward-backward asymmetries sensitive to the $t\bar{t}$ production couplings, we can construct corresponding asymmetries sensitive to the decay couplings $\bar{d}_j^W$. This is done in all cases by integrating at the last step over $\cos \theta$ (instead of over $\cos \theta_l$ done above) and constructing the forward-backward asymmetry with respect to $\theta_l$. As before, we always work to linear order in $\bar{d}_j^W$.

Thus, for the first case which led to (B.22,23) we get

$$D_{FB}^1 = \frac{-3M_W^2}{2(2M_W^2 + m_t^2)} \left[1 - \frac{m_t(m_t^2 - M_W^2)}{2M_W^2 + m_t^2}(\bar{d}_3^W + \bar{d}_4^W)\right].$$ \hspace{1cm} (B.30)

For the second case, we have already stated that the asymmetries (B.24,25) are obtained by using either $\cos \psi_1$ or $\sin \psi_1$ to project out the $p$ factor in the second term in the r.h.s. of (B.8). For the $\theta_l$ asymmetry though, these two projections give different asymmetries. Thus the asymmetry obtained through $\cos \psi_1$ is

$$D_{FB}^2 = \frac{-3M_W^2}{2(m_t^2 - 2M_W^2)} \left[1 - \frac{m_t(m_t^2 - M_W^2)}{m_t^2 - 2M_W^2}(\bar{d}_3^W + \bar{d}_4^W)\right],$$ \hspace{1cm} (B.31)

while the one obtained from $\sin \psi_1$ is independent of $\bar{d}_j^W$ and equal to

$$D_{FB}^3 = \frac{4}{3\pi}.$$ \hspace{1cm} (B.32)

Finally in the third case, we get $D_{FB}^3$ for the asymmetry obtained through the projector (B.27), and $D_{FB}^3$ for the asymmetry obtained through (B.26).

To linear order in the NP couplings, all these asymmetries can be expressed as a product of a factor describing the SM contribution, and another factor describing the NP correction. For this NP correction a tree level calculation is sufficient. Any QCD and 1-loop radiative corrections should in general be incorporated in the SM factor only. The QCD corrections have to some extent been studied in \cite{22,13} and have been found rather small. In any case this is something which we plan to do in the future. It is also interesting to remark that while the production asymmetries are sufficient to determine all $d_j^Z$ and $d_j^\gamma$ couplings even in the unpolarized case; this is not possible for the decay couplings. To linear order in the NP top-decay couplings, the above asymmetries are only sensitive to the combination $\bar{d}_3^W + \bar{d}_4^W$.

Finally we should also remark that the case where the $t$-quark decays hadronically, while $\bar{t} \rightarrow \bar{b}l^-\nu$, is very similar. Thus, if the orientation of the $\bar{t}$-rest frame is defined to be like the one obtained from the $t$-frame by rotating it by $180^\circ$ around the perpendicular to the $t\bar{t}$ production plane; and if the new Euler angles for the $\bar{t}$ decay plane are called
\((\varphi_2, \vartheta_2, \psi_2)\), and \(\theta_l\) is defined analogously; then all formulae in this Appendix remain the same, except of (B.12-17) where we should replace

\[
\varphi_1, \psi_1, \vartheta_1 \Rightarrow \varphi_2, \psi_2, -\vartheta_2 .
\]  

(B.33)

This way, all forward-backward asymmetries remain formally identical. Note though, that the definition of the top production angle \(\theta\) implies that (forward-backward) for \(\bar{t}\) means that we should subtract as

\[
\text{Backward}(\bar{t}) - \text{Forward}(\bar{t}) .
\]  

(B.34)
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