Testing and comparing tachyon inflation to single standard field inflation

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Abstract

We study the evolution of perturbations during the domination and decay of a massive particle species whose mass and decay rate are allowed to depend on the expectation value of a light scalar field. We specialize to the case where the light field is slow-rolling, showing that during a phase of inhomogeneous mass-domination and decay the isocurvature perturbation of the light field is converted into a curvature perturbation with an efficiency which is nine times larger than when the mass is fixed. We derive a condition on the annihilation and decay rates for the domination of the massive particles and we show that standard model particles cannot dominate the universe before nucleosynthesis. We also compare this mechanism with the curvaton model. Finally, observational signatures are discussed. A CDM isocurvature mode can be generated if the dark matter is produced out of equilibrium by both the inflaton and the massive particle species decay. Non-Gaussianities are present: they are chi-square deviations. However, they might be too small to be observable.

1 Introduction

The recent WMAP data[1] strongly supports the idea that the early universe underwent a phase of inflation. One typically considers an inflationary phase driven by the potential or
vacuum energy of a scalar field, the inflaton, whose dynamics is determined by the Klein-Gordon action. More recently, however, motivated by string theory, other non-standard scalar field actions have been used in cosmology. One particular type of field which has attracted attention is the tachyon \[ T \], whose action is of the Dirac-Born-Infeld form \[ S_T = \int d^4x \sqrt{-g} V(T) (1 + g^{\mu\nu} \partial_\mu T \partial_\nu T)^{1/2}, \quad \text{sign} \{g\} = (-, +, +, +), \quad (1.1) \]

where \( V(T) \) is its potential. According to Sen’s conjecture, in type II string theory the tachyon signals the instability of unstable and uncharged D-branes of tension \( \lambda = V(0) \).

In this context the positive potential \( V(T) \) is even and satisfy the properties \( V'(T > 0) < 0, \quad V(|T| \to \infty) \to 0 \).

Here (see also [4]) we take a phenomenological approach and study the inflationary predictions of a phase of inflation driven by a field \( T \) satisfying the action Eq. (1.1) and we assume that the potential satisfies the properties mentioned above. We call this tachyon inflation although the potential \( V(T) \) may not be particularly string inspired.

The questions we address here are: 1) Does tachyon inflation lead to the same predictions as standard single field inflation (SSFI)? 2) Can tachyon inflation already be ruled out by current observations? 3) Can we discriminate between tachyon inflation and SSFI in the light of new and planned future experiments? The answer to the first question is no: tachyon inflation leads to a deviation in one of the second order consistency relations. However, tachyon inflation cannot be ruled out at the moment, and its predictions are typically characteristic of small field or chaotic inflation. The answer to the final question is given in the last section.

2 Slow-roll predictions of tachyon inflation

In a homogeneous and isotropic background with scale factor \( a \), the tachyon field can be treated as a fluid with energy density and pressure \( \rho = V(T)/(1 - \dot{T}^2)^{1/2}, \quad P = w\rho = -(1 - \dot{T}^2)\rho \). For tachyon inflation, the basic condition for accelerated expansion is that \[ \ddot{a}/a = -\frac{1}{6M_{Pl}^2} (\rho + 3P) = \frac{1}{3M_{Pl}^2} \frac{V}{(1 - \dot{T}^2)^{1/2}} \left( 1 - \frac{3}{2} \dot{T}^2 \right) > 0 \Rightarrow \dot{T}^2 < \frac{2}{3}. \quad (2.1) \]

In order to study inflationary predictions we must define slow-roll parameters. Here we use the horizon-flow parameters[5] based on derivatives of \( H \) w.r.t. the number of \( e \)-folds \( N = \ln a \). This definition has the advantage of being independent of the field driving inflation and thus it is a natural choice to use in order to compare SSFI and tachyon inflation. We just need the first three parameters, which are

\[ \epsilon_1 = (3/2)\ddot{T}^2, \quad \epsilon_2 = \sqrt{2/3} \epsilon_1' / H = 2\ddot{H} / (H\dot{H}), \quad \epsilon_2 \epsilon_3 = \sqrt{2\epsilon_1 / 3} \epsilon_2' / H, \quad (2.2) \]

where ‘\( \cdot \)’ is the derivative w.r.t. \( T \). Thus as in SSFI, tachyon inflation is based upon the slow evolution of \( T \) in its potential \( V(T) \), with the slow-roll conditions \( \ddot{T} \ll 3H\dot{T} \) and \( T^2 \ll 1 \).

It is well known that during an inflationary phase, quantum vacuum fluctuations are stretched on scales larger than the horizon. There, they are frozen until they reenter the
horizon after inflation. Calculation of the spectra of scalar quantum fluctuations proceeds by defining a canonical variable which can be quantized with the standard methods. The straightforward generalization of the canonical variable to the case of a tachyon fluid is\[^6\]

\[ v_k = zM_{Pl}(\psi + H\frac{d\psi}{dT}) \]

where \( \psi \) is the Bardeen potential in conformal gauge and \( \delta T(t, x) \) is a linear perturbation around the homogeneous solution \( T(t) \). The pump field \( z \) is defined by \( z = \sqrt{3aT/(1 - T^2)^{1/2}} = -a\sqrt{2\epsilon_1}/w \). The equation derived from minimizing the action expanded to second order in \( v_k \) is\[^6\]

\[ \frac{d^2v_k}{d\tau^2} - (wk^2 + U(\tau))v_k = 0, \quad U(\tau) \equiv \frac{1}{z^2}\frac{d^2z}{d\tau^2}. \quad (2.3) \]

Note the factor of \(-w\), the speed of sound of the tachyon fluid, in front of \( k^2 \), absent in SSFI where the speed of sound is 1. Instead of computing directly \( U(\tau) \) in terms of the slow-roll parameters, we observe that in SSFI inflation the pump field is\[^7\] \( z_{SSFI} \equiv a\sqrt{2\epsilon_1} = z(1 - 2\epsilon_1/3)^{1/2} \), differing from \( z \) by a first order term in \( \epsilon_1 \). It follows that \( U = U_{SSFI} + a^2H^2\epsilon_1/2 + \text{order}(\epsilon_1^3) \), where we have used that \( dz/d\tau \simeq zaH \) at lowest order. Thus, the second order correction \( \propto \epsilon_1\epsilon_2 \) allows us to compute \( U \) up to second order in the slow-roll expansion from \( U_{SSFI} \) given in\[^5\].

Eq. (2.3) can be solved in terms of Hankel functions. After normalization to vacuum fluctuations for \( \sqrt{-wk/(aH)} \to \infty \), and on expanding the solution in the slow-roll parameters we find

\[ P_S^{1/2}(k) = \sqrt{\frac{k^2}{2\pi^2}} \left| \frac{v_k}{zM_{Pl}} \right| = \left. \frac{1 - (C + 1 - \alpha)\epsilon_1 - 1/2C\epsilon_2}{2\sqrt{2\pi}} \frac{H}{M_{Pl}\sqrt{\epsilon_1}} \right|_{k=aH}, \quad (2.4) \]

where \( C \) is a numerical constant. At this order in the slow-roll expansion, the parameter \( \alpha \) is the only difference between SSFI inflation and tachyon inflation. It vanishes in the first case but takes the value \( \alpha = 1/6 \) in the second. The spectrum of gravity waves \( P_T \) in tachyon inflation is exactly as in SSFI since in absence of anisotropic stress gravity waves are decoupled from matter.

A non-vanishing \( \alpha \) is responsible for second order deviations in one of the consistency relations, conditions on the observable parameters, thought to be distinctive of SSFI\[^8\]. Indeed, as expected, to lowest order in the slow-roll parameters \( r = P_T/P_S \), \( n = 1 + d\ln P_S(k)/d\ln k \), and \( n_T = d\ln P_T(k)/d\ln k \) are identical to those of SSFI, and the consistency relations are the same. However at higher order we find

\[ n_T = -(r/8)[1 - (1 - 2\alpha)r/16 + (1 - n)]. \quad (2.5) \]

This consistency relation is the next order version of \( n_T = -2\epsilon_1 = -r/8 \). There is a deviation from SSFI, represented by a nonvanishing \( \alpha \), which could in principle be a way of distinguishing tachyon inflation from SSFI. However, in order to see this deviation, \( n, r, \) and \( n_T \) should be known to a precision of \( \sim 10^{-3}r^2 \). The error on \( n_T \) for future Cosmic Microwave Background observations has been estimated by Song and Knox\[^9\]. Even for the largest possible values of \( r, r \sim 1 \), it is too large for the deviations predicted by the tachyon to be observable, so that Eq. (2.5) will be very difficult to test.
Figure 1: Models of tachyon inflation compared to the 2-dimensional likelihood contours (at 1σ and 2σ) on the \((n,r)\)-plane. The points represent the result of a random sampling from three different tachyonic potentials. In the text: \(V_a\), squares, blue; \(V_b\), circles, black; \(V_c\), stars, black; and \(V_d\), diamonds, red. We consider \(40 \leq N_s \leq 70\). The two dashed lines correspond to the limits between the three different regimes of inflation.

3 Models and comparison to WMAP data

We now study tachyon inflation for different potentials \(V(T)\) and extract \(n\), \(r\), and \(dn/d\ln k\). We follow the standard procedure: 1) for a given potential compute \(\epsilon_1\), \(\epsilon_2\), and \(\epsilon_2\epsilon_3\) as a function of \(T\); 2) estimate \(T_e\), the value of \(T\) at the end of inflation when \(\epsilon_1(T_e) = 1\); 3) calculate the number of e-foldings as a function of the field \(T\); 4) from \(\epsilon_1\), \(\epsilon_2\), and \(\epsilon_3\) calculate the observable parameters as a function of \(T\) and evaluate them at \(T_s = T(N_s)\), the value of \(T\) at which a length scale crosses the Hubble radius during inflation. By doing this we can draw some general properties of tachyon inflation and compare these with WMAP data. Results are shown in Fig. 1 using the likelihood contour from the analysis of S. Leach and A. Liddle [10].

We consider three class of potentials, inverse cosh potentials, \(V_a(T) = \lambda/\cosh(T/T_0)\), the exponential potential \(V_b(T) = \lambda e^{-T/T_b}\), the potential \(V_c(T) = \lambda[1 + (T/T_0)^4/27]e^{-T/T_b}\) and the inverse power-law \(V_d = \lambda/[1 + (T/T_0)^4]\). The first two are often considered in the string theory literature[11, 12] while the last two are not particularly string motivated but display interesting properties. It is useful to define the constant dimensionless ratio[13]
\( X_0^2 = \lambda T_0^2 / M_{Pl}^2 \) which appears in the slow-roll parameters derived from these potentials. Typically \( X_0 \gg 1 \) in order for the slow-roll conditions to be satisfied.

These potentials generally have a red spectrum of scalar perturbations with a negative and very small running of the scalar spectral index. For specific choices of potential such as \( V_a \), blue spectra can be obtained with very small \( r \). We divide the \((n, r)\)-plane into three regions of interest corresponding to 1) \( V'' \leq 0, \ 6\epsilon_1 \leq \epsilon_2; \) 2) \( 0 < V'' < \frac{V''}{V''} / V, \ 2\epsilon_1 < \epsilon_2 < 6\epsilon_1; \) and 3) \( \frac{V''}{V''} \leq V'' \), \( \epsilon_2 \leq 2\epsilon_1 \). It is interesting to compare the predictions of our potentials with the current data and to see whether it is possible to discriminate between SSFI and tachyon inflation. For \( V_a \) inflation takes place in both regimes 1 and 2. For a large set of parameters \( N^\ast \) and \( X_0 \) (excluding very small \( X_0 \)), the predictions are well inside the \( 2\sigma \) contour. There are non negligible gravity waves for large \( X_0 \), though for the range of \( N^\ast \) given above, \( r \lesssim 0.2 \). When \( X_0 \rightarrow \infty \), the predictions concentrate on the line \( \epsilon_2 = 2\epsilon_1 \) which are just those of potential \( V_b \). Potential \( V_c \) can occupy regimes 1, 2, and 3, and leads to a large contribution of gravity waves, although \( r \lesssim 0.2 \) in the region not excluded by current data. Potential \( V_d \) occupies much of the region of the inverse cosh potential as well as yielding blue spectra for negligible \( r \).

All the presented models seem to be consistent with the data. Hence, the first-year WMAP results are still too crude to constraint significantly the region of parameters. On the other hand, we still lack of information about the mechanism of reheating that could take place after tachyon inflation, leaving us with a large uncertainty on \( N^\ast \). Progress can be made by better estimating this particular parameter.

Our results point to the fact that it is difficult to distinguish between a model where inflation is driven by a Klein-Gordon scalar field or by some other field satisfying a non standard action. However, none of the potentials we have considered in our analysis lead to both a blue scalar spectral index and large gravity wave spectrum. Therefore for these potentials a large region in the \((n, r)\)-plane is not probed by tachyon inflation. This corresponds, in SSFI, to the region occupied by hybrid inflation. Detection of \( n > 1 \) and large \( r \), or of a large running of \( n \), can lead to the exclusion of tachyonic inflation.

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