Quark number scaling of $v_2$ in transverse kinetic energy and its implications for coalescence models

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We find that a simple extension of the coalescence model is sufficient to incorporate the perfect quark number scaling behavior of the elliptic flow in transverse kinetic energy, recently discovered by the PHENIX Collaboration. The flavor dependence of the elliptic flow can be consistently described in the low and intermediate $p_T$ if the transverse kinetic energy is conserved in the $2 \rightarrow 1$ or $3 \rightarrow 1$ parton coalescence process at the hadronization. Thus suggesting the quark coalescence as a possible hadronization mechanism at low $p_T$ as well.

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Quark recombination or coalescence models \cite{1, 2, 3, 4, 5} have been proposed to explain the large baryon to meson ratio \cite{6, 7} and the number of constituent quark scaling (NCQ-scaling) of the hadron $v_2$ in $p_T$ \cite{7, 8} seen in RHIC data. Since coalescence models explicitly assume partonic degree of freedom, their apparent, albeit qualitative, success in describing the data at intermediate $p_T$ is a strong support of the idea that the partonic matter has been created in Au-Au collisions at RHIC.

Coalescence models calculate the meson production via the convolution of Wigner functions for constituent quarks and the meson \cite{2, 3, 4, 5}:

$$\frac{d^3N}{dp^3} = \prod_{i=1}^{2} d^3x_i d^3p_i F_M(x_1, p_1, x_2, p_2) W_M(p, p_1, p_2, x_1, x_2)$$

(1)

where $F$ is the joint $q$ and $\bar{q}$ phase space distribution, $W_M$ is the meson Wigner function, which is typically approximated by

$$W_M = F_M(x_1 - x_2, p_1 - p_2) \delta(p_T - p_{T,1} - p_{T,2})$$

(2)

The $\delta$ function enforce the momentum conservation. It is straightforward to generalize these formula for baryons.

There are several implicit assumptions in coalescence models. Only valence quark degrees of freedom are assumed in the coalescence calculation, and the dynamic gluon contribution (appears as higher Fock state) \cite{9} could lead to a 20% correction to the NCQ-scaling. There is a unitarity problem at low $p_T$ \cite{2, 3, 4, 5} due the quadratical dependence of the meson yield on the number of quarks, although this can be reconciled when one take into a finite freeze out time and no quarks are used in more than one hadron \cite{10}. In addition, coalescence process reduce the entropy. This can be partially resolved by including resonance decay of unstable hadrons \cite{11}.

As Eq. (1) indicates, the level of NCQ-scaling depends on both the final state parton phase space distribution, described by $F_M$, and the hadronization process which is encoded in $W_M$. The constituent quark phase space distributions are driven by the space-time evolution of the partonic matter, which is typically described by hydrodynamics or parton cascade. Coalescence models explicitly assume that $F_M$ can be factorized into the single quark distribution function, $F_M = f(x_1, p_1)f(x_2, p_2)$, with identical anisotropic phase space distribution (only $v_2$ is considered here.)

$$f(x, p) = n(x, p) (1 + 2v_{2,q}(x, p)\cos(2\phi)) \cdot (3)$$

For a naive coalescence model where $q$ and $\bar{q}$ are assumed to be co-moving : $\Phi_M \propto \delta^3(x_1 - x_2) \delta^3(p_1 - p_2)$, the spatially averaged $v_2$ for quark and meson is \cite{12}:

$$\langle v_2,q \rangle = \frac{\int d^3x d\phi f(x, \frac{p}{2}) \cos(2\phi)}{\int d^3x d\phi f(x, \frac{p}{2})}$$

(4)

$$\langle n(x, \frac{p}{2})v_{2,q}(x, \frac{\phi}{2}) \rangle_x$$

$$\langle n(x, \frac{p}{2}) \rangle_x$$

$$\langle v_{2,M} \rangle = \frac{\int d^3x d\phi f^2(x, \frac{\phi}{2}) \cos(2\phi)}{\int d^3x d\phi f^2(x, \frac{\phi}{2})}$$

(5)

$$\langle n^2(x, \frac{p}{2})v_{2,q}(x, \frac{\phi}{2}) \rangle_x$$

$$\langle n^2(x, \frac{p}{2}) \rangle_x$$

$$\langle v_{2,q} \rangle = 2 \langle n(x, \frac{p}{2})v_{2,q}(x, \frac{\phi}{2}) \rangle_x$$

$$\langle n^2(x, \frac{p}{2}) \rangle_x$$

where $\langle A(x) \rangle_x = \int d^3x A(x)$. Because quark density $n(x, p)$ and parton anisotropy $v_{2,q}(x, p)$ could depend on quark local coordinate and momentum, NCQ-scaling in general does not follow as pointed in \cite{12, 13}. The space momentum correlation of the parton system before hadronization requires detailed study of the dynamical evolution of the system \cite{12, 14}.

The level of NCQ-scaling also depends on the modelling of hadronization process. It is known that the finite width of the hadron internal wave function can lead to 10-20% violation of the $v_2$ NCQ-scaling \cite{15}. In the extreme case where wave function is very wide, NCQ-scaling can be destroyed completely \cite{16}. Since coalescence deals with 2, 3 $\rightarrow 1$ processes, in general quarks or the coalesced hadron need to be off mass shell \cite{17}, and the interactions with the surrounding medium are
necessary to neutralize their virtuality. Thus the constituent quarks should be treated as quasi-particles and there is no reason a priori to assume the momentum conservation in coalescence process (Eq.2). Our goal is to study the consequences of modifying this assumption.

If coalescence is a relevant hadronization mechanism, it should be valid at all momentum. However, from the data point of view, NCQ-scaling of the $v_2$ can be preserved at low $p_T$, and the constant $Q$-scaling is not a universal curve. This is illustrated by Fig. 1, where the ratios of the scaled elliptic flow $(v_2/n)$ for various hadrons to a common polynomial fit are plotted as function of $p_T/n$ for light mesons such as pion and overshoots the $v_2$ values for heavy baryons such as $\bar{\Xi}$. Attempts have been made to reconcile the breaking of $v_2$ by including the resonance effect in the coalescence models. The agreement with data can be achieved qualitatively. We are not aware of any mechanisms within existing coalescence models that can accommodate the deviation of heavy baryons.

Recently it was point out by PHENIX Collaboration [20] that the constituent quark scaling of $v_2$ can be preserved at low $p_T$ if hadron transverse kinetic energy $K_{ET} = m_T - m$ instead of $p_T$ is used as a scaling variable. We duplicated the PHENIX finding in Fig. 2. The deviation from the universal $v_2(K_{ET}/n)/n$ scaling curve is very small, less than 10% in all cases. Motivated by this observation, we modify the momentum conservation relation into kinetic energy conservation relation in the coalescence formula Eq.2.

$$W_M = \Phi_M(x_1-x_2,p_1-p_2)\delta(m_T-m_{T,1}-m_{T,2}-m+m_1+m_2)$$

Without very detailed calculations, we can make the following remarks.

1. For the naive coalescence scenario, where only the co-moving $q$ and $\bar{q}$ recombine into hadron, assuming spatially uniform quark density $n(x,p) = n(p)$ or flow anisotropy $v_2,q(x,p) = v_2,q(p)$, we obtain the usual NCQ-scaling relation for meson and baryon [2, 3], except it is in $K_{ET}$, instead of $p_T$.

$$v_{2,M}(K_{ET}) = \frac{2v_{2,q}}{1 + 2v_{2,q}} \left| \frac{K_{ET}}{2} \right| \approx 2v_{2,q} \left| \frac{K_{ET}}{2} \right| \quad (7)$$

$$v_{2,B}(K_{ET}) = \frac{3v_{2,q} + 3v_{2,q}^3}{1 + 6v_{2,q}^2} \left| \frac{K_{ET}}{3} \right| \approx 3v_{2,q} \left| \frac{K_{ET}}{3} \right| \quad (8)$$

We have confirmed these relations through a simple Monte-Carlo simulation of the naive coalescence process, where $q$ or $\bar{q}$ are assumed to have mass of 350 MeV and an exponential spectra in $m_T$. In addition, similar to what is done in other recombination models, the transverse momentum of the combining quarks in x or y direction are required to be within 0.24 GeV/c of each other. In essence the coalescence formula Eq. 2 becomes

$$\frac{d^3N}{dp^3} = \int d^2p_1 d^2p_2 e^{-(m_{T,1}+m_{T,2})/A} \delta(K_{ET} - K_{ET,1} - K_{ET,2})$$

$$\Theta(\Delta_\rho - |p_{x,1} - p_{x,2}|)\Theta(\Delta_\rho - |p_{y,1} - p_{y,2}|) \quad (9)$$

where $A = 0.17$ GeV and $\Delta_\rho = 0.24$ GeV/c is the cut off of relative momentum between the two quarks. Since we focus on mid-rapidity, longitudinal momentum direction has been ignored in this study. Results of our calculations...
Each other. However, the resulting scaled

\[ \text{momentum conservation, because in general } m \neq m_1 + m_2. \]

This can be clearly seen by comparing the bottom panels of Fig.1 and Fig.2. Going from \( p_T/n \) to \( \text{KE}_T/n \), one effectively shift high \( p_T \) part of light meson \( v_2/n \) to the right and the low high \( p_T \) part of heavy baryon \( v_2/n \) to the left by an amount on the order of the hadron mass. However, the resulting scaled \( v_2 \) curves falls on top of each other.

3. In hydrodynamic model with Cooper-Fryer freeze out \([21]\), hadrons are statistically produced in the local co-moving frame with certain radial flow. At low \( p_T \), the elliptic flow should be proportional to \( p_T^2 \) according to the a general analytical expansion \([22]\). The Buda-Lund hydro model \([23]\) suggests a universal elliptic flow scaling \( : v_2 = I_1(w)/I_0(w), w \propto p_T^2/2m_T \), which, at low \( p_T \) (thus small \( w \)), reduce to \( v_2 \approx 0.5w \propto p_T^2/2m = \text{KE}_T \). We also arrive this relation through a schematic derivation \([24]\).

For any variable \( X \) and a linear relation between elliptic flow and \( X : v_2 \propto X, \text{NCQ-scaling in } X \) is a trivial additive artifact: \( v_2/n \propto X/n \). It was realized by \([13, 14]\) that either hydrodynamical mass ordering or NCQ-scaling of \( v_2 \) can results from quark coalescence. In our point of view, the coalescence condition \( \text{Eq.6} \) provide a natural reconciliation between the statistical hadronization and hadronization via coalescence. Since \( v_2 \) is proportional to \( \text{KE}_T \) at low \( p_T \), \( v_2 \) for various hadrons naturally fall on a universal curve in small \( \text{KE}_T \) independent of whether they are scaled by the number of constituent quark or not.

Due to the non-linear relation between \( p_T \) and \( \text{KE}_T \), at low or intermediate \( p_T \) region where mass effect is important, NCQ-scaling can not hold in both \( p_T \) and \( \text{KE}_T \) simultaneously. At \( p_T \gg m \), a schematic hydro calculation gives \( v_2 \approx \tanh(\xi p_T) \) \([25]\), where \( \xi \) depends on flow velocity. However, this relation gives neither NCQ-scaling in \( p_T \) nor NCQ-scaling in \( \text{KE}_T \).

A complete model should be able to describe not only elliptic flow but also the particle ratio, single particle spectra, etc. Our study is rather limited in this regard. We do not attempt to describe any observables other than elliptic flow. Various hadronic effects after quark coalescence, such as the hadronic re-scattering and resonance decay, are ignored in current study. Our discussion is focused on the physical consequences when quark coalescence is extended to low \( p_T \). We have demonstrated that results from a simple coalescence approach agree with the data fairly well in \( \text{KE}_T \). The deviation of the pion \( v_2/n \) from \( p_T/n \) scaling is naturally resolved when plotted in \( \text{KE}_T/n \) variable, without the need to rely on the resonance effect. The fact that RHIC data indicate strong flavor dependence of the \( v_2 \) and strong violation of the NCQ-scaling of the \( v_2 \) in \( p_T \) suggests that \( \text{KE}_T \) scaling of the \( v_2 \) at intermediate \( p_T \) is a rather non-trivial observation. It suggests that the quark coalescence might be an important hadronization mechanism at low \( p_T (p_T < 2 \text{ GeV/fc}) \).

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[24] For a thermal source and at \( p_T \ll m_T \), where \( v_T \) is the radial flow velocity,
\[
f(p, x) \propto \exp\left( -\frac{\gamma T (m_T - \vec{p} \cdot \vec{v}_T) \pm \mu}{T} \right) \propto \exp\left( -\frac{\gamma T m_T}{T} \right) \exp\left( -\frac{m (v(\phi))^2}{2T} \right)\]
we have ignored the \( \phi \) dependence of the \( T \) and \( \mu \). For this simple scenario, the elliptic flow would be,
\[
v_2 \approx \frac{m [(v)^2 - (\langle v \rangle)^2]}{8T} \approx \frac{p_T^2 \epsilon_v}{8mT} \]
\( \epsilon_v = \left( \langle v_x^2 \rangle - \langle v_y^2 \rangle / \langle v \rangle^2 \right) / \langle v \rangle^2 \) is the velocity anisotropy and should to first order be proportional to spacial anisotropy [21]: \( \epsilon_v = A \epsilon_s + O(p_T) \).