Continuous Creation of a Vortex in a Bose-Einstein Condensate with Hyperfine Spin $F = 2$

Mikko Möttönen¹, Naoki Matsumoto², Mikio Nakahara¹,² § and Tetsuo Ohmi³

Materials Physics Laboratory, Helsinki University of Technology, P.O. Box 2200
FIN-02015 HUT, Finland¹
Department of Physics, Kinki University, Higashi-Osaka 577-8502, Japan²
Department of Physics, Kyoto University, Kyoto 606-8502, Japan³

Abstract. It is shown that a vortex can be continuously created in a Bose-Einstein condensate with hyperfine spin $F = 2$ in a Ioffe-Pritchard trap by reversing the axial magnetic field adiabatically. It may be speculated that the condensate cannot be confined in the trap since the weak-field seeking state makes transitions to the neutral and the strong-field seeking states due to the degeneracy of these states along the vortex axis when the axial field vanishes. We have solved the Gross-Pitaevskii equation numerically with given external magnetic fields to show that this is not the case. It is shown that a considerable fraction of the condensate remains in the trap even when the axial field is reversed rather slowly. This scenario is also analysed in the presence of an optical plug along the vortex axis. Then the condensate remains within the $F_z = 2$ manifold, with respect to the local magnetic field, throughout the formation of a vortex and hence the loss of atoms does not take place.

§ To whom correspondence should be addressed (nakahara@math.kindai.ac.jp)
1. Introduction

It has been observed that the Bose-Einstein condensate (BEC) of alkali atom gas becomes superfluid [1, 2]. Superfluidity of this system is different from the previously known superfluid $^4$He in many aspects. For example, the BEC is a weak-coupling gas for which the Gross-Pitaevskii equation is applicable while superfluid $^4$He is a strong-coupling system. One of the most remarkable differences is that the BEC has spin degree of freedom originating from the hyperfine spin of the atom, and that this degree of freedom couples to external magnetic fields. Accordingly the order parameter of the condensate is also controlled at will by external magnetic fields. Superfluid $^3$He also has similar internal degrees of freedom, which, however, are rather difficult to control by external fields [3].

Taking advantage of this observation, we proposed a simple method to create a vortex in a BEC with the hyperfine spin $F = 1$ [4, 5]; a vortex-free BEC is intertwined topologically by manipulating the magnetic fields in the Ioffe-Pritchard trap to form a vortex with the winding number 2. This is achieved by reversing the axial magnetic field adiabatically while the planar quadrupole field is kept fixed.

In the present paper, a similar scenario is analysed for a BEC with $F = 2$, taking $^{87}$Rb as an example. The difference between the present case and that for $F = 1$ will be emphasised in our analysis. In the next section, we briefly review the order parameter of BEC with hyperfine spin $F = 2$ and the Gross-Pitaevskii (GP) equation which describes the time-evolution of the order parameter. In section 3, the ground state order parameter of the BEC in the weak-field seeking state confined in a harmonic potential is obtained. Then the time-evolution of the condensate, as the axial field is adiabatically reversed, is studied by solving the GP equation numerically. Cases with different reverse time are analysed to find the best possible reversing time for which the fraction of the remaining condensate in the vortex state is maximised. It is shown that the condensate in the end of this scenario has the winding number 4. In section 4, the GP equation is solved in the presence of an optical plug along the vortex axis. The BEC remains in the $F_z = 2$ weak-field seeking state, with respect to the local magnetic field, throughout the development, and hence no atoms will be lost during the formation of a vortex. Section 5 is devoted to conclusions and discussions.

2. Order Parameter of $F = 2$ BEC

2.1. General $F = 2$ condensate and Gross-Pitaevskii equation

Suppose a uniform magnetic field $B$ parallel to the $z$-axis is applied to a BEC of alkali atoms with the hyperfine spin $F = 2$. Then the hyperfine spin state of the atom is quantised along this axis; the eigenvalue $m$ of $F_z$ takes a value $-2 \leq m \leq +2$, where
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\[ F_z|m\rangle = m|m\rangle. \]

Let us introduce the following conventions:

\[
|2\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix},
|1\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix},
|0\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix},
|-1\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix},
|-2\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.
\]

The order parameter \(|\Psi\rangle\) is expanded in terms of \(|m\rangle\) as

\[ |\Psi\rangle = \sum_{m=-2}^{2} \Psi_m|m\rangle = (\Psi_2, \Psi_1, \Psi_0, \Psi_{-1}, \Psi_{-2})^T, \]

where \(T\) denotes the transpose.

The representation of the angular momentum operators \(F_k\) \((k = x, y, z)\) for \(F = 2\) is easily obtained from the well-known formulae:

\[
\langle 2, m|F_+|2, m'\rangle = \sqrt{(2-m)(3+m)}\delta_{m,m'+1},
\langle 2, m|F_-|2, m'\rangle = \sqrt{(2+m)(3-m)}\delta_{m,m'-1},
\langle 2, m|F_z|2, m'\rangle = m\delta_{m,m'},
\]

where \(F_\pm = F_x \pm iF_y\).

The dynamics of the condensate in the limit of zero temperature is given, within the mean field approximation, by the time-dependent Gross-Pitaevskii (GP) equation with spin degrees of freedom. This equation, obtained by Ciobanu, Yip and Ho [3] (see also [4]), is written in components \(\Psi_m\) as

\[
i\hbar \frac{\partial}{\partial t} \Psi_m = \left[ -\frac{\hbar^2}{2M} \nabla^2 + V(r) \right] \Psi_m + g_1 |\Psi|^2 \Psi_m + g_2 \left[ \Psi^\dagger (F_k)_{np} \Psi_p \right] (F_k)_{mq} \Psi_q
+ 5g_3 \Psi^\dagger (2m2n|00\rangle \langle 00|2p2q) \Psi_p \Psi_q + \frac{1}{2} \hbar \omega_{Lk} (F_k)_{mn} \Psi_n,
\]

where summations over \(k = x, y, z\) and \(-2 \leq n, p, q \leq 2\) are understood. Here, \(M\) is the mass of the atom and \(V(r)\) is the possible external potential. The Larmor frequency is defined as \(\hbar \omega_{Lk} = \gamma_\mu B_k\), where \(\gamma_\mu \simeq \mu_B\) is the gyromagnetic ratio of the atom and \(\mu_B\) the Bohr magneton. The interaction parameters are expressed in terms of the \(s\)-wave scattering length \(a_F\), \(F\) being the total hyperfine spin of the two-body scattering state, and are given by [3]

\[
g_1 = \frac{4\pi \hbar^2 4a_2 + 3a_4}{M} \frac{4a_2}{7},
g_2 = \frac{4\pi \hbar^2 a_2 - a_4}{M} \frac{a_2 - a_4}{7},
g_3 = \frac{4\pi \hbar^2}{M} \left( \frac{a_0 - a_4}{5} - \frac{2a_2 - 2a_4}{7} \right),
\]

where \(a_0 = 4.73\,\text{nm}, a_2 = 5.00\,\text{nm}\) and \(a_4 = 5.61\,\text{nm}\) for \(^{87}\text{Rb}\) atoms. This should be compared with \(F = 1\) BEC where there are only two types of scattering state and hence two interaction terms in the GP equation.
2.2. Weak-field seeking state

Suppose a strong magnetic field $B$ is applied along the $z$-axis. Then the components with $F_z = 1$ and 2 are in the weak-field seeking state (WFSS) while those with $F_z = -1$ and $-2$ are in the strong-field seeking state (SFSS). The presence of two WFSSs leads to an interesting two-component vortex that is not observed in $F = 1$ BEC as we see in the next section. The energy of the state with $F_z = 0$ is independent of the magnetic field and will be called the neutral state (NS) hereafter. Suppose a uniform condensate is in the state with $F_z = 2$. The order parameter of the condensate takes the form

$$|\Psi_0\rangle = f_0 (1, 0, 0, 0)^T$$

where $|f_0|^2$ is the number density of the condensate. Now let us consider a state which is quantised along an arbitrary local magnetic field

$$B(r) = B \begin{pmatrix} \sin \beta \cos \alpha \\ \sin \beta \sin \alpha \\ \cos \beta \end{pmatrix}. \quad (5)$$

Let $F_B \equiv B \cdot F / B$ be the projection of the hyperfine spin vector along the local magnetic field. The WFSS $|\Psi\rangle$ which satisfies $F_B |\Psi\rangle = +2 |\Psi\rangle$ is obtained by rotating $|\Psi_0\rangle$ by Euler angles $\alpha, \beta$ and $\gamma$ and is given by

$$|\Psi(r)\rangle = \exp(-i\alpha F_z) \exp(-\beta F_y) \exp(-i\gamma F_z) |\Psi_0\rangle$$

$$= f_0 e^{-2i\gamma} \begin{pmatrix} e^{-2i\alpha} \cos^4 \frac{\beta}{2} \\ 2e^{-i\alpha} \cos^3 \frac{\beta}{2} \sin \frac{\beta}{2} \\ \sqrt{6} \cos^2 \frac{\beta}{2} \sin^2 \frac{\beta}{2} \\ 2e^{i\alpha} \cos \frac{\beta}{2} \sin^3 \frac{\beta}{2} \\ e^{2i\alpha} \sin^4 \frac{\beta}{2} \end{pmatrix} \equiv f_0 |v\rangle. \quad (6)$$

The GP equation restricted within the WFSS is obtained by substituting Eq. (6) into Eq. (2), see the next section.

3. Vortex formation without optical plug

The formation of a vortex in the $F = 2$ condensate is analysed in this and the next sections. In the present section, we study the scenario without an optical plug along the vortex axis. Although some fraction of the condensate is lost from the trap in this scenario, the experimental setup will be much easier without introducing an optical plug. In fact, it will be shown below that a considerable amount of the condensate remains in the trap by properly choosing the time-dependence of the magnetic field.

3.1. Magnetic fields

Suppose a condensate is confined in a Ioffe-Pritchard trap. It is assumed that the trap is translationally invariant along the $z$ direction and rotationally invariant around the
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The quadrupole magnetic field of the trap takes the form

$$B_\perp(r) = B_\perp(r) \begin{pmatrix} \cos(-\phi) \\ \sin(-\phi) \\ 0 \end{pmatrix},$$

(7)

where $\phi$ is the polar angle. The magnitude $B_\perp(r)$ is proportional to the radial distance $r$ near the origin; $B_\perp(r) \sim B'_\perp r$. The uniform time-dependent field

$$B_z(t) = \begin{pmatrix} 0 \\ 0 \\ B_z(t) \end{pmatrix}$$

(8)

is also applied along the $z$-axis to prevent Majorana flips from taking place at $r \sim 0$ where $B_\perp$ vanishes. Now the total magnetic field is given by

$$B(r, t) = B_\perp(r) + B_z(t) = \begin{pmatrix} B_\perp(r) \cos(-\phi) \\ B_\perp(r) \sin(-\phi) \\ B_z(t) \end{pmatrix}.$$

(9)

Comparing this equation with Eq. (3), it is found that

$$\alpha = -\phi \quad \beta = \tan^{-1} \left[ \frac{B_\perp(r)}{B_z(t)} \right].$$

(10)

It has been shown in the previous works for $F = 1$ BEC that a vortex-free condensate in the beginning will end up with a condensate with a vortex of the winding number 2 if $B_z$ reverses its direction while $B_\perp$ is kept unchanged [4, 5]. We expect that the same magnetic field manipulation to lead to the vortex formation in a BEC with $F = 2$. The uniform axial field $B_z(t)$ must reverse its direction as

$$B_z(t) = \begin{cases} B_z(0) \left(1 - \frac{2t}{T}\right) & 0 \leq t \leq T \\ -B_z(0) & T < t. \end{cases}$$

(11)

to create a vortex along the $z$-axis. This gives a “twist” to the condensate leading to the formation of a vortex with winding number 4, see below.

Before we start the detailed analysis, it will be useful to outline the idea underlying our scenario. Suppose one has WFSS with $F_B = 2$ in the trap at $t = 0$. The magnetic field at $r \sim 0$ points $+z$ direction (i.e., $\beta \sim 0$) and hence the WFSS takes the form $\Psi_0$ of Eq. (4). Then the angle $\gamma$ must satisfy $\gamma = -\alpha$ for the BEC to be vortex-free, see Eq. (6). The field $B_z(t)$ vanishes at $t = T/2$, for which $\beta = \pi/2$, and the hyperfine spin is parallel to the quadrupole field $B_\perp$. Accordingly one must choose $\alpha = -\phi$ for this condition to be satisfied, see Eqs. (9) and (10). This also implies $\gamma = +\phi$. When the field $B_z$ is completely reversed at $t = T$, the magnetic field at $r \sim 0$ points down and hence $\beta \sim \pi$ there. Substituting $\alpha = -\gamma = \phi$ and $\beta = \pi$ into Eq. (3), one finds the order parameter at $t = T$;

$$|\Psi\rangle = f_0 e^{-2i\phi} (0, 0, 0, 0, e^{-2i\phi})^T,$$

(12)

which shows that a vortex with the winding number 4 has been created.
3.2. Initial state

Suppose a vortex-free BEC is confined in a Ioff-Pritchard trap, whose magnetic field takes the form (1) and that the condensate is in the eigenstate $F_B = 2$ with respect to the local magnetic field $B$ with $B_z = B_z(t)$. The condensate wave function is then obtained by solving the stationary state Gross-Pitaevskii equation. Substitution of Eq. (3) with $\alpha = -\gamma = -\phi$ into Eq. (2) yields

$$-\frac{\hbar^2}{2M} \nabla^2 (f_0 v_m) + (g_1 + 4g_2) f_0^3 v_m + \hbar \omega_L f_0 v_m = \mu f_0 v_m,$$

where we have put $\Psi_m \equiv f_0 v_m$. Note that the $g_3$ term vanishes identically for the present state. The condensate wave amplitude $f_0(r)$ is taken to be a real function without loss of generality. The eigenvalue $\mu$ is identified with the chemical potential. If one multiplies the above equation by $\{v_m\}^\dagger$ from the left and uses the identity $\sum_m |v_m|^2 = 1$ and other identities derived from this, one obtains the reduced GP equation for $f_0(r)$;

$$-\frac{\hbar^2}{2M} \left[ f''_0 + \frac{f'_0}{r} + (v^*_m \nabla^2 v_m) f_0 \right] + (g_1 + 4g_2) f_0^3 + \hbar \omega_L f_0 = \mu f_0,$$  

where

$$v^*_m \nabla^2 v_m = \left[ \frac{2}{r^2} (3 \cos^2 \beta - 5) \sin^2 \frac{\beta}{2} - \beta^2 \right]$$

comes from the rotation of the five-dimensional local orthonormal frame that defines the order parameter. The reduced GP equation looks similar to the ordinary scalar GP equation except that there is an extra term $\beta^2$ in $v^*_m \nabla^2 v_m$.

It is convenient to introduce the energy scale $\hbar \omega$ and the length scale $a_{\text{HO}}$ define by

$$\omega = \sqrt{\frac{\gamma \mu}{MB_z(0)B'_\perp}} \quad a_{\text{HO}} = \sqrt{\frac{\hbar}{M \omega}}.$$  

(15)

For a typical values $B_z(0) = 1\text{G}, B'_\perp = 200\text{G/cm}$ for $^{87}\text{Rb}$, one obtains $\hbar \omega \simeq 1.69 \cdot 10^{-24}\text{erg}$ and $a_{\text{HO}} \simeq 0.68\mu\text{m}$. After scaling all the physical quantities by these units, one obtains the dimensionless form of the reduced GP equation;

$$-\frac{1}{2} \left[ \tilde{f}_0'' + \frac{\tilde{f}_0'}{\tilde{r}} + \left[ \frac{2}{\tilde{r}^2} (3 \cos^2 \beta - 5) \sin^2 \frac{\beta}{2} - \beta^2 \right] \tilde{f}_0 \right] + (\tilde{g}_1 + 4\tilde{g}_2) \tilde{f}_0^3 + \tilde{\omega}_L \tilde{f}_0 = \tilde{\mu} \tilde{f}_0,$$  

where the dimensionless quantities are denoted with tilde. For example, $\tilde{r} = r/a_{\text{HO}}, \tilde{f}_0 = f_0 a_{\text{HO}}^{3/2}$ and $\tilde{g}_k = g_k/(-\hbar a_{\text{HO}}^3)$. The tilde will be dropped hereafter unless otherwise stated explicitly.

Figure 1 shows the ground state condensate wave function obtained by solving Eq. (16) numerically. We have chosen $f_0(r = 0) = 6$ which yields the central density $n_0 \sim 1.17 \cdot 10^{14}\text{cm}^{-3}$. This is roughly of the same order as that realised experimentally. The difference between the chemical potential and the Larmor energy at the origin is $\delta \mu = \mu - \omega_L = 3.95$, which amounts to $\delta \mu = 6.68 \cdot 10^{-24}\text{erg}$ in dimensionful form.
3.3. Time development

Now the time-dependent GP equation (4) is solved numerically with the initial condition $\Psi_m = f_0(r) v_m$ with $f_0$ been obtained in the previous subsection. We have introduced a tanh-shaped cutoff to mimic the loss of atoms from the trap; particles reach at $L \gg a_{HO}$ vanish from the system. We have made several choices of the reversing time $T$ and maximised the fraction of the condensate left in the trap in the final equilibrium state. The details of the algorithm are given in [5] and will not be repeated here.

Figure 2 shows the wave functions $|\Psi_m|$ for the choice $T/\tau = 1000$, where $\tau = 2\pi/\omega_L$ is the time scale set by the Larmor frequency at $t = 0$ and $r = 0$. One obtains $\tau \sim 7.14 \cdot 10^{-7}$sec for the parameters given in the previous subsection. The parameter $\tau$ is expected to be the measure of the adiabaticity. There are two weak-field seeking states possible for $F = 2$, those with $F_B = 2$ and $F_B = 1$. It turns out that the final vortex state is a mixture of these two states. When the axial field $B_z(t)$ vanishes at $t = T/2$, the gaps among WFSSs, SFSSs and NS disappear at $r = 0$ and the level crossing takes place there. Then the adiabatic assumption breaks down and some fraction of the condensate transforms into SFSSs and NS as well as $F_B = 1$ WFSS. Those components in SFSSs and NS eventually leave the trap and the final condensate is made of $F_B = 2$ and $F_B = 1$ components. It is a remarkable feature of the $F = 2$ BEC, compared to its $F = 1$ counterpart, that the vortex state thus created is mixture of these two WFSSs. The composite nature of the final vortex state is best revealed by projecting $|\Psi(r)|$ to $F_B = 1$ and 2 states. Let $|v\rangle$ be the vector defined in Eq. (6) and $|u\rangle = \exp(-i\alpha F_z) \exp(-\beta F_y) \exp(-i\gamma F_z)|1\rangle$. Then $\Pi_2(r) \equiv \langle u(r) | \Psi(r) \rangle$ and $\Pi_1(r) \equiv \langle u(r) | \Psi(r) \rangle$ depict the projected amplitudes of $|\Psi(r)\rangle$ to the local $F_B = 2$ and $F_B = 1$ state, respectively. These amplitudes are shown in Fig. 3 for $|\Psi(r)\rangle$ at $t = 10T$. It is interesting to note that the $F_B = 2$ component has a winding number 4 while $F_B = 1$ has 3.

The fraction of the condensate left in the trap at time $t$ has been plotted in Fig. 4...
Figure 2. Time dependence of the order parameter $|\Psi_m|$ for the reversing time $T/\tau = 1000$.

Figure 3. The projected amplitudes $|\Pi_2(r)|$ and $|\Pi_1(r)|$ obtained from the order parameter $|\Psi(r)|$ at $t = 10T$. 
for $T/\tau = 1000$. It should be noted that $\sim 2/5$ of the condensate is left in the trap when the system reaches an equilibrium at $t \gg T$.

Figure 5 shows the fraction of the condensate left in the trap in the equilibrium state at $t \gg T$ for various $T$. It can be seen from this figure that a considerable amount of the condensate is left in the trap for a wide variety of the reversing time $T$.

In the next section, we analyse the creation of a vortex in the presence of an optical plug along the centre of the system. It will be shown that the vortex thus created is purely made of $F_B = 2$ WFSS.

4. Vortex formation with optical plug

The loss of the condensate in the previous section takes place since the energy gaps among WFSSs, NS and SFSSs disappear at $r = 0$ when $B_z$ vanishes at $t = T/2$. One may introduce an optical plug along the vortex axis to prevent the condensate from entering this “dangerous” region. An optical plug may be simulated by a repulsive
Figure 6. Time dependence of the condensate amplitude $f_0$ in the presence of the optical plug. The reversing time is $T/\tau = 10000$. The condensate amplitudes at $t = 0$ and $t = T$ are almost degenerate.

The time-independent GP equation is given by

$$\frac{-1}{2} \left[ \ddot{f}_0 + \frac{\dot{f}_0}{r} + \left[ \frac{2}{r^2} (3 \cos^2 \beta - 5) \sin^2 \frac{\beta}{2} - \beta'^2 + V(r) \right] \ddot{f}_0 \right] + (g_1 + 4g_2) \dot{f}_0^3 + \omega_L \dot{f}_0 = \tilde{\mu} \tilde{f}_0$$

in dimensionless form, where $\tilde{V}(r) = V(r)/\hbar \omega$. The angle $\beta$ is given by $\beta(r) = \tan^{-1}[B_\perp(r)/B_z(r)]$. We will drop tilde from dimensionless quantities hereafter unless otherwise stated. The ground state condensate wave function is obtained by solving this equation numerically. We find the relative chemical potential $\delta \mu = \mu - \omega_L = 173$, which amounts to $\delta \mu = 2.92 \cdot 10^{-22}$ erg in dimensionful form, and the condensate wave function $f_0$ shown in Fig. 5.

The time-dependent GP equation

$$i \frac{\partial f_0}{\partial t} = -\frac{1}{2} \left[ f_0'' + \frac{f_0'}{r} + \left[ \frac{2}{r^2} (3 \cos^2 \beta - 5) \sin^2 \frac{\beta}{2} - \beta'^2 + V(r) \right] f_0 \right]$$

$$+ (g_1 + 4g_2) f_0^3 + \omega_L f_0$$

is solved with the initial wave function obtained above. Here $\beta = \beta(r,t) \equiv \tan^{-1}[B_\perp(r)/B_z(r)]$. It should be stressed again that the condensate remains within the $F_B = +2$ WFSS throughout the temporal evolution. Figure 7 shows the time-dependence of the components $\Psi_m$ for the choice $T/\tau = 10000$, namely $T = 7.14$ ms in dimensionful form. In contrast with the case without optical plug, the time-dependence
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Figure 7. The time dependence of the particle numbers in unit length \( N_m(t) = 2\pi \int |\Psi_m(r,t)|^2 r dr \) for \( T/\tau = 10000 \).

The order parameter is independent of the choice of \( T \) so far as \( T/\tau \) is large enough so that the adiabaticity is observed.

The vortex thus obtained has a region near the origin (\( r \sim 0 \)) where the condensate cannot approach due to the presence of the optical plug. The vortex current flows around a multiply connected region. This situation is analogous to the superconducting current flowing around a ring. It is natural to expect that a vortex without the optical plug may be obtained if one withdraws the optical plug after the persistent current is established at \( t = T \). (Note that the optical plug has been introduced to prevent Majorana flips at \( r \sim 0 \) at \( t \sim T/2 \). Accordingly the optical plug is not required anymore for \( t \geq T \).) Let us suppose that the optical plug is slowly turned off after \( t = T \) with the time constant \( t_0 \):

\[
V(r, t) = \begin{cases} 
V_0 \exp(-r^2/r_0^2) & 0 < t < T \\
V_0 \exp(-r^2/r_0^2) \exp[-(t - T)/t_0] & T < t.
\end{cases}
\]  

It is found that the condensate oscillates back and forth for small \( t_0 \). For sufficiently large \( t_0 \), however, the condensate smoothly rearranges itself to a vortex state without the optical plug. Figure 7 shows our numerical result for \( T/\tau = t_0/\tau = 10000 \), for which one still observes such oscillations.

A vortex with a winding number 4 is thus created without losing any atoms from the trap. It should be noted, however, that it is technically difficult, albeit not impossible \([8]\), to introduce an optical plug with a few microns of radius along the centre of the BEC whose radial dimension without optical plug is of the order of a few microns.

5. Conclusions and Discussions

The formation of a vortex in a BEC with \( F = 2 \) in a Ioffe-Pritchard trap has been considered by fully utilising the spinor degrees of freedom. It was shown that a vortex
with winding number 4 is created continuously from a condensate without a vortex, by simply reversing the axial magnetic field $B_z(t)$. This scenario has been studied with and without an optical plug at the centre of the vortex. Some amount of the BEC is lost from the trap in the absence of an optical plug while no atoms are lost in the presence of it. Our numerical analysis shows that there remains a considerable fraction of BEC even without the optical plug. The introduction of an optical plug in a trapped BEC is difficult, albeit not impossible.

Our vortex has a large winding number 4 and is expected to be unstable against decay into four singly quantised vortices in the absence of an optical plug. Whether a vortex with such a large winding number may be observable depends on how large the lifetime of the metastable state is compared to the trapping time of the BEC. Our preliminary analysis of the Bogoliubov equation suggests that the lifetime is of the order of 100ms and these highly-quantised vortices exist for a considerable duration of time.

We would like to thank Kazushige Machida and Takeshi Mizushima for discussions. One of the authors (MN) thanks Takuya Hirano, Ed Hinds and Malcolm Boshier for discussions. He also thanks Martti M. Salomaa for support and warm hospitality in the Materials Physics Laboratory at Helsinki University of Technology, Finland. MN’s work is partially supported by Grand-in-Aid from Ministry of Education, Culture, Sports, Science and Technology, Japan (Project Nos. 11640361 and 13135215).

**Note Added** — After we submitted our manuscript, the MIT group reported the formation of vortices according to the present scenario [9]. They employed hyperfine spin states $F = 1$ and $F = 2$ of $^{85}\text{Rb}$ and found that the vortex thus created had the winding number two in the former case while four in the latter case, in consistent with
our prediction. The vortex state has considerably long lifetime, at least 30ms after its formation, in spite of higher winding number, which suggests that these vortices are rather stable. The stability analysis of highly-quantised vortices is outside the scope of the present work and will be published elsewhere.

We are grateful to Aaron Leanhardt for informing us of their result.

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