A note on Dolby and Gull on radar time and the twin “paradox”

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Recently a suggestion has been made that standard textbook representations of hypersurfaces of simultaneity for the travelling twin in the twin “paradox” are incorrect. This suggestion is false: the standard textbooks are in agreement with a proper understanding of the relativity of simultaneity.

I. INTRODUCTION

It is common wisdom that there is nothing paradoxical about the twin “paradox”, and that appropriate attention to the unambiguous differences between the proper times of the travelling twin (whom we shall call “Barbara”) and the stay-at-home twin (“Alex”) resolves any air of paradox. Indeed, the reader may well feel exasperated that any work is still being done on this fallacious argument.

However, in a recent article in this journal, Dolby and Gull argue that the standard resolutions of the twin “paradox” incorrectly answer the question of how the travelling twin should assign times of occurrence to distant events; that is, how Barbara should represent her hypersurfaces of simultaneity. They claim that proper application of “radar time” to the accelerated twin allows us to sort out an unambiguous time of happening to each event, from Barbara’s perspective, and they go on to do just that.

I have no complaint with their mathematics (though see §IV). My query, rather, is with the point of their exercise. For correct attention to both the concept of simultaneity and to what it means for an observer to assign a “time of happening” to an event shows that there was no problem with the standard textbook resolutions in the first place—though perhaps it is true that the textbook authors were not aware of the facts that render their work unproblematic.

II. DOLBY AND GULL’S WORRY

Standard textbook resolutions of the twin paradox (in the instantaneous turnaround case) claim that the hypersurfaces of simultaneity for Barbara are as in Fig. 1. Regarding this figure, Dolby and Gull point out two things. Firstly they note that when Barbara instantaneously turns around, the points G and H are regarded as simultaneous by Barbara. They acknowledge that this problem is dissolved if we make the situation more physically realistic, and adopt a turnaround of extended duration, as in Fig. 2. (We do well to note that this move still leaves untouched the conceptual problem in the original, instantaneous case.) Secondly, they note that moving to an extended turnaround “cannot resolve the more serious problem... which occurs to Barbara’s left. Here her hypersurfaces of simultaneity are overlapping, and she assigns three times to every event!”

FIG. 1: A typical “textbook illustration” of the hypersurfaces of simultaneity of the travelling twin Barbara in the twin paradox. (Adapted from Dolby and Gull’s Figure 1.)

III. DEFUSING THE WORRY

Is the claim that more than one time of happening can be assigned to a single event such a patent absurdity as Dolby and Gull seem to regard it? I will now give a reinterpretation of Fig. 1 which will show that it is far from an absurdity. Let us call the point of turnaround T. Consider an observer, O₁, who travels inertially until T along the same path as Barbara, and whose worldline then sadly terminates. Consider another observer, O₂, who springs into existence at T, and shadows Barbara thereafter.

I take it that it is obvious that the hypersurfaces of simultaneity for O₁ and O₂ are as in Fig. 1 and that those hypersurfaces overlap without problem or absurdity. Indeed, it is a central fact about the relativistic
FIG. 2: Another typical “textbook illustration”, in which Barbara’s hypersurfaces of simultaneity “sweep around” from $EG$ to $FH$ during the period of turnaround (acceleration). (Adapted from Dolby and Gull’s Figure 2.)

theories that observers who are moving differently are in different frames of reference, and hence their partitions of events into simultaneity equivalence classes can differ quite radically. So it cannot be the mere fact that the hypersurfaces of simultaneity overlap that worries Dolby and Gull. It must rather be that it is possible that one observer, who happens to move differently at different times, can assign the same event to different equivalence classes under simultaneity, again at different times.

But, again, why should this be a problem? An absurd situation would be: if at one and the same time, an observer assigned the same event to two different simultaneity classes under simultaneity, again at different times. Perhaps Dolby and Gull are worried that a single, persisting observer cannot (without considerable mental problems) assign one and the same event to different simultaneity classes, even if the observer never does that at a single time. This seems an empty worry, for those differing assignments are precisely what we should expect in a relativistic scenario. Maybe Barbara is a determinedly pre-relativistic individual, in which case she may have some conceptual problems with the different assignments, perhaps because she still operates with a hidden Newtonian assumption that there is one universal time. But that cannot be at the root of Dolby and Gull’s worry, as they make no such assumption—see, however, §V.

Dolby and Gull go on to say the following:

If Barbara’s hypersurfaces of simultaneity at a certain time depend so sensitively on her instantaneous velocity as these diagrams suggest, then she would be forced to conclude that the distant planets swept backwards and forwards in time whenever she went dancing!

Perhaps if we can decipher this remark, we will get at the root of their worries.

As far as I can tell, their worry here is that, as Barbara’s instantaneous velocity changes from moment to moment, she will be forced to conclude that some events that are in her current subjective future (i.e. that lie within the future light cone of some event on her current hypersurface of simultaneity) were, at some point on her past worldline, judged to be in the past (i.e. lying within the past light cone of some event on her past (then-current) hypersurface of simultaneity). Of course, this is no absurdity: it has long been clear that the pre-theoretical concepts of “past” and “future” do not mesh perfectly with their relativized versions. Yet I cannot see anything more to Dolby and Gull’s worry, other that it is motivated by pre-theoretical intuitions about distant assignments of pastness and futurity to events, intuitions that should by now be seen as very doubtful in a relativistic universe.

One residual worry remains: What happens at $T$? In Fig. 1 it does seem that, at $T$, Barbara has two conflicting simultaneity assignments (this is related to Dolby and Gull’s first worry). The resolution is simple: we need to assign her at most one instantaneous frame of reference. The obvious one to choose is that at the instant $T$, she is counted as at rest: her frame of reference then yields a hypersurface of simultaneity that is orthogonal to Alex’s worldline, running horizontally across the page. This assignment has the virtue that it shows Fig. 1 to be the limit of Fig. 2 as the period of acceleration decreases in extension. Perhaps, though, we might think that at this instantaneous point, there is no sense to be made of the observer’s frame of reference, and hence perhaps we assign no hypersurface of simultaneity. This latter option has problems of its own, of course; but in principle, either choice serves to resolve the residual worry.

IV. THE CONVENTIONALITY OF SIMULTANEITY

On the basis of the above considerations, I see no force to the motivating remarks that Dolby and Gull provide, and hence I query whether their mathematical work...
needed to be performed.

Setting that issue aside, however, some interesting details emerge when one considers their positive proposal. They begin by defining the radar time of an event $e$ basically as follows: let $t_1$ be the (proper) time at which the observer sends a signal to $e$, and let $t_2$ be the (proper) time at which the observer receives a return signal from $e$. The radar time $\tau(e)$ of $e$ is defined in equation (1):

$$\tau(e) = t_1 + \frac{1}{2}(t_2 - t_1).$$

A hypersurface of simultaneity $\sigma_e$ is set of events with the same radar time ($\sigma_e = \{x : \tau(x) = \tau(e)\}$); it is obviously an equivalence class. This same relation of simultaneity is, as Dolby and Gull are well aware, Einstein's standard convention for simultaneity.

Quite a large body of work has sprung up concerning the status of this definition of simultaneity. Though they make passing reference to some of this work, Dolby and Gull do not engage more thoroughly with it. If they had, they would have noticed that while their radar time definition of simultaneity is an acceptable definition, it is by no means the only available option.

Debs and Redhead maintain that any definition of radar time is acceptable if it is compatible with the following:

$$\tau_e(e) = t_1 + \epsilon(t_2 - t_1) \quad (0 < \epsilon < 1).$$

They take it that any particular choice of value of $\epsilon$ in equation (2) is conventional; that is, not fixed by the physical facts, but rather by our conventional decision to use the term “simultaneous” to pick out an equivalence class under $\tau_e$. No contradiction with any physical fact is possible for any of these relations defined by different $\epsilon$ values, because only proper time has “objective status in special relativity.” If Debs and Redhead are right, then no special significance will attach to assignments of distant simultaneity at all: they are arbitrary and hence without physical importance. If that is true, then the purported conflict over assignments which so exercised Dolby and Gull is of even smaller significance than I made out above.

Of course, others have rushed to defend the Einsteinian convention, proving its unique adequacy if we set certain conditions on a plausible candidate for simultaneity. But those further conditions have been disputed too. Though I have no wish to defend it here, it seems that conventionalism about simultaneity remains an open possibility. If that possibility turns out to be true, then Dolby and Gull have done some excellent work defining a potential candidate simultaneity relation, but one that loses its importance once we see that any candidate relation, within wide bounds, will do. In particular, the textbook presentations of the hypersurfaces of simultaneity are perfectly good candidates for us to adopt as our convention.

V. FURTHER EXTERNAL CONSTRAINTS

However, it seems that Dolby and Gull’s motivation for proposing a unique foliation for any given observer does not rest on considerations intrinsic to special relativity. Rather, they have a particular application in mind, for which they regard unique foliation as crucial. The particular application is to fermionic particle creation in relativistic quantum field theory. Obviously if this application requires unique foliation, then these remarks about conventionality of simultaneity will be misapplied, since we are now in effect imposing additional physical constraints that suffice for uniqueness. Moreover, their particular proposal requires consistency of foliation for a given observer: so the textbook overlapping hypersurfaces at different times will also be incorrect.

One small concern about this approach is that we have strictly gone beyond the conceptual content of special relativity, and therefore that it is inappropriate to represent the Einsteinian convention as the only one possible for special relativity. Rather, the Einstein proposal is the only one that it is pragmatically appropriate to use when applying special relativity, given the further constraints imposed by our beliefs concerning what the actual world is like. But strictly speaking simultaneity may yet be conventional in standard special relativity without external constraints.

A bigger worry, however, is with the particular application they have in mind. The crucial point for Dolby and Gull is that, if we cannot establish a unique foliation for a given observer, certain features of the particle distribution for that observer will change. They argue that it is extremely odd indeed that a mere change in velocity (or worse, a merely conventional choice of simultaneity relation) could result in the appearance or disappearance of particles.

Unfortunately, it is now quite clear that there is no relativistic quantum theory of localizable particles. The most plausible response to this result, and indeed the standard response, is to argue that since different observers see different particle distributions, we should be anti-realists (or conventionalists!) about particle distributions. If this standard plausible response were accepted, that would undercut Dolby and Gull’s motivation for wanting a unique foliation of the spacetime to ensure reliable and constant particle distributions.

In sum, if we are irrealists about particle distributions, we have no motivation to demand unique foliations for given observers—and we have every reason to be irrealist about particle distributions, given the impossibility of giving an adequate characterization of local particles within relativistic quantum field theory, and moreover given that “RQFT itself shows how the illusion of localizable particles can arise, and how talk about localizable particles can be a useful fiction.” Insofar as we are prepared to deploy a particle number observable, we do so from the perspective of an instantaneous rest frame of an observer, and the foliation consequent upon that choice
VI. CONCLUSION

Dolby and Gull have given an elegant and precise example of how to apply the standard Einsteinian convention on assignments of distant simultaneity in the twin paradox case. It very usefully illustrates that the standard convention can be applied alike to accelerating and inertial observers, contrary to some of the received wisdom in the field.

However, their work was not strictly necessary. Firstly because there was no intuitive paradox or problem with the standard textbook presentations, as I showed in §III. Secondly, many people regard the definition of simultaneity as a conventional matter in any case, as discussed in §IV. Hence no acceptable choice for this convention can be criticized as mistaken on physical grounds, but only on grounds of usefulness for our purposes. Thirdly, even if we undermine the conventionality response by adding some further actual physical fact, that wouldn’t show the conceptual commitments of special relativity to be any different—moreover, I suspect that any such additional physical fact will not in actuality conflict with the standard representations of hypersurfaces of simultaneity (§V). Given these observations, I suggest, the standard textbook suggestion concerning what should be regarded as the hypersurfaces of simultaneity is at least as successful as Dolby and Gull’s more complicated alternative proposal.

Acknowledgments

Thanks to Hans Halvorson, Dave Baker, Steve Gull, and two anonymous reviewers for helpful comments, and to Luke Elson for the initial discussion which prompted my interest in this material.

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