Landau Hydrodynamics and RHIC Phenomena

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Abstract. The basic physical assumptions and results of Landau’s hydrodynamic model of particle production are reviewed. It is argued that these results have sufficient descriptive and predictive power in strong-interaction phenomenology, including recent RHIC data, to warrant a closer examination of the physical assumptions.

Keywords: hydrodynamics, heavy-ion collisions, charged-particle multiplicities, rapidity distributions, RHIC
PACS: 25.75.Dw

1. Introduction

The success of boost-invariant hydrodynamics at RHIC (e.g. [1]) to describe the systematics of elliptic flow is considered to be strong evidence that the strongly interacting system thermalizes early, coupling the dynamical evolution to initial-state geometry of the system. These calculations can, in principle, provide a means to extract the form of the equation of state of the thermalized system. This may provide insight into whether or not a quark-gluon plasma was in fact created in heavy ion collisions at RHIC.

It should not be forgotten, however, that hydrodynamics has a long history in the study of strongly-interacting systems. Coupled with the intense interest in statistical and thermal model calculations in the early 1950’s, spearheaded by Fermi and Landau [2, 3, 4], this led to a large body of strong interaction phenomenology, manifestly non boost-invariant, which was refined throughout the years [5, 6, 7, 8, 9, 10, 11]. It is the goal of this talk to review the central physical assumptions and predictions of the Landau hydrodynamical model (and its refinements), and to show its relevance to a wide range of results in a variety of strongly-interacting systems.

In the Landau-Fermi physical picture, the main physical assumptions were:

• The collision of two Lorentz-contracted hadrons or nuclei leads to full thermal-
ization in a volume of size $V m_p/\sqrt{s}$. This justifies the use of thermodynamics and establishes the system size and energy dependence.

- A massless blackbody EOS is assumed $p = \epsilon/3$. This allows the complete calculation of physical quantities.
- All chemical potentials (meson and baryon) are zero, which dramatically simplifies the mathematics.

The main results derived from these assumptions are:

- A universal formula for the produced entropy, determined mainly by the initial Lorentz contraction.
- Gaussian rapidity distributions, at least for particles produced several units away from the projectile rapidities.
- Thermal particle occupations determined by $T \sim m_\pi$.

2. Universal Entropy

Simply using the first law of thermodynamics and the blackbody EOS, Landau and Fermi both arrived at the same scaling formula for the multiplicity produced in a collision of two strongly interacting objects, $N_{ch} = \alpha S = K s^{1/4}$ \cite{2, 3, 4}. The agreement between this formula and a wide range of systems is shown in Fig. 1 with a constant of proportionality of $K=2.2$, which has been found to work with systems as diverse as $p+p$, $Au+Au$, $e^+e^-$ and $\nu+p$ \cite{5, 6, 7}.

It was always controversial to apply Landau’s reasoning to “small” systems, such as $e^+e^-$ annihilations. Beyond the usual arguments about falling far short of the thermodynamic limit, it has been argued that perturbative QCD (pQCD) already provides an excellent description of not just the multiplicity of produced hadrons as a function of energy, but the details of jet shapes (modulo hadronization corrections), rendering a “statistical” description superfluous (at best!) Despite this, it is interesting to observe that pQCD calculations (e.g. Ref. \cite{12}) and the Fermi-Landau formula agree within 10% over a wide range of energies, essentially up to where the LEP data ends, as shown in Fig. 1 (adapted from Ref. \cite{13}). This similarity is striking when one considers that pQCD implies an infinite mean-free-path for parton rescatterings, while the hydro description implies a negligible one for the (presumably partonic) degrees of freedom which thermalize.

The extension of the Fermi-Landau approach from a single $p+p$ collision to nuclei is surprisingly simple. Since the Lorentz contraction is not changed, the angular distributions are in principle similar to the smaller system \cite{1}. Then, provided that the interactions between the subvolumes of the system do not themselves open up new degrees of freedom, the entropy of the system as a whole will simply scale proportionally to the number of participating nucleons ($N_{part}$). This “$N_{part}$-scaling” has been observed for the total multiplicity (but not necessarily in any particular
region of phase space) in all collisions involving nuclei, from $p + A$ to $Au + Au$ collisions \[13, 14\].

3. Thermal Phenomenology and Hadrochemistry

In the Landau scenario, freezeout is not expected to occur immediately, as Fermi assumed, but rather when the temperature reaches the limit of the pion Compton wavelength $T = m_\pi$. This was based on a suggestion by Pomeranchuk \[15\] to avoid Fermi’s prediction that nucleons would outnumber pions by virtue of their larger statistical weight. This assumption leads to predictions for the relative population of various particle states similar to those made in the Hagedorn approach \[16, 17\].

A+A collisions clearly deviate from the Fermi-Landau formula at low energies. An obvious suspect is the phenomenon of baryon stopping, which is absent in $p+p$ collisions but is substantial in A+A \[18\]. If one puts back the $-\mu_B N_B$ term into the first law of thermodynamics, we immediately see how the presence of a conserved quantity associated with a substantial mass (i.e. the proton mass) will naturally suppress the total entropy: $S = (E + pV - \mu_B N_B)/T$. Using an existing thermal model code, Cleymans and Stankiewicz \[19\] calculated the entropy density as a function of $\sqrt{s}$. It rises to limiting value where $\mu_B \to 0$ and $T \to T_0$, the Hagedorn temperature. If we then assume that the total multiplicity scales linearly with the
4. Gaussian Rapidity Distributions

Up to this point, the discussions have only involved the total entropy, integrated over the full phase space. Landau was able to perform approximate calculations of the angular distributions by introducing hydrodynamic evolution using the standard equations of relativistic hydrodynamics $\partial_\mu T^{\mu\nu} = 0$ closed by the blackbody EOS \cite{3, 6}. These equations generically imply that the initial state entropy, which is produced in the process of thermalization, is distributed in rapidity space with a Gaussian form, the width determined by the initial Lorentz contraction. Including the total multiplicity formula to set the overall normalization, the full expression \cite{6} is:

$$\frac{dN}{dy} = \frac{K}{\sqrt{2\pi L}} s^{1/4} \exp\left(-\frac{y^2}{2L}\right).$$  \hspace{1cm} (1)

where $L = \frac{\sigma_y^2}{2} = (1/2) \ln(s/m_p^2) = \ln(\gamma)$. Already, one can see a connection between the shape of the distribution and the multiplicity, since from this definition $s^{1/4} \propto e^{L/2}$.

While the experimental status of Landau Gaussians vs. Feynman-Bjorken plateaus \cite{20, 21} in $dN/d\eta$ was ambiguous throughout the 1970’s \cite{5, 6}, despite strong evidence for the applicability of Landau’s formulas, the situation in A+A collisions was clarified rather quickly by the two large-acceptance RHIC experiments, PHOBOS and BRAHMS. PHOBOS quickly established that boost invariance was violated over a large rapidity range by inclusive measurements of $dN/d\eta$ over $|\eta| < 5.4$ \cite{22}. BRAHMS consolidated these observations by finding that the rapidity distributions of pions in $|y| < 3$ is Gaussian with a width parameter only 10% different from the Landau prediction \cite{23}. This led to a re-evaluation of all of the existing heavy ion data, where it was found that charged pion rapidity distributions all fell close to the Landau trend \cite{24, 25}.

5. Connections or Coincidences

While the Landau results seem describe, and even predict, several non-trivial results at RHIC, the formulas also encode features that are usually attributed to QCD, or its various approximations.

For example, measurements of $dN/d\eta$ in p+p and A+A collisions, boosted into the rest frame of one of the projectiles, showed the phenomenon of “limiting fragmentation” \cite{22}. This is energy-independence of the particle yields at a fixed rapidity distance from the projectile rapidity \cite{26}. One thing that has not typically
been appreciated in the context of the Landau approach is that limiting fragmentation seems to arise naturally from Equation (1): Transforming this distribution into the rest frame of one of the projectiles, \( y' = y - y_{\text{beam}} \), one finds that approximately:

\[
\frac{dN}{dy'} \propto \frac{1}{\sqrt{L}} \exp\left(-\frac{y'^2}{2L} - y'\right). \tag{2}
\]

Which only varies weakly with \( L \) for \( y' \sim 0 \). This seems too strong to be merely a coincidence, but it is not clear why limiting fragmentation, which implies \( x_F \) scaling, arises naturally from the Landau formulas, in which \( x_F \) does not seem to be a preferred variable [6].

Another interesting, and unexpected, connection can be made between the Landau expressions and the results from the CGC-based model of Kharzeev and Levin [27]. Once one fixes the peak \( dN/dy \) of both models, and then varies the energy, it appears that the width of \( dN/dy \) varies in a similar way as a function of beam energy, as shown in Fig. 4. It is interesting that the exponent extracted from HERA data, \( \lambda = 0.25 - 0.3 \), is surprisingly similar to the power seen in the Landau multiplicity formula. However, it is not clear why this similarity occurs, or how robust it is.

A final unexpected coincidence is seen in the transverse direction near \( y=0 \). Carruthers and Duong-van noticed that the \( p_T \) distribution of \( \pi^0 \)'s in \( p+p \) collisions was well described out to \( p_T = 10 \) \( \text{GeV} \) by a Gaussian distribution in transverse rapidity \( y_T = \frac{1}{2} \ln \left( \frac{m_T + p_T}{m_T - p_T} \right) \), with \( L \sim 0.51 \) [28]. While no derivation was given for this phenomenological description, which holds over 10 orders of magnitude,
an argument was made on a similar basis as for the Gaussian in the longitudinal direction. To see if this function continues to work well at RHIC energies, fits have been made to PHENIX $\pi^0$ data [29] and STAR inclusive charged data [30] from $p+p$ collisions, which are shown in Fig. 5. Reasonable agreement is found with the STAR data with $L = 0.56$, despite the combination of various particle species, and excellent agreement is found with the PHENIX data with $L = 0.54$, up to $p_T = 11$ GeV.

6. Conclusions

In conclusion, it is argued that the arguments made by Landau and collaborators in the mid-1950’s appear to have a surprising relevance for understanding RHIC phenomena. The results on the total multiplicity and the shape of the rapidity distributions hold in a robust way for a variety of systems, including $p+p$, $e^+e^-$ and Au+Au, perhaps providing a natural way to understand the apparent universality seen in the total number of charged particles. Several interesting connections are found between the Landau results and limiting fragmentation, CGC calculations, and even particle production at very high $p_T$. Understanding these connections may provide deeper insight into the strong interaction and the dynamical properties of strongly interacting systems.
Acknowledgments

The author would like to thank the organizers for an enjoyable and stimulating workshop - one that provided every possible opportunity to combine work and leisure. Special thanks to Mark Baker, Wit Busza, Dima Kharzeev, Jamie Nagle Gunther Roland, Gabor Veres and Bill Zajc for illuminating discussions.

References

1. P. F. Kolb and U. Heinz, arXiv:nucl-th/0305084.
2. E. Fermi, Prog. Theor. Phys. 5, 570 (1950).
3. L. D. Landau, Izv. Akad. Nauk Ser. Fiz. 17, 51 (1953).
4. S. Z. Belenkij and L. D. Landau, Nuovo Cim. Suppl. 3S10, 15 (1956) [Usp. Fiz. Nauk 56, 309 (1955)].
5. P. Carruthers and M. Doung-van, Phys. Rev. D 8, 859 (1973).
6. P. Carruthers, Annals N.Y.Acad.Sci. 229, 91 (1974).
7. P. A. Carruthers, LA-UR-81-2221 Presented at 5th High Energy Ion Study Conf., Berkeley, Calif., May 18-23, 1981.
8. F. Cooper, G. Frye and E. Schonberg, Phys. Rev. Lett. 32, 862 (1974).
9. F. Cooper and G. Frye, Phys. Rev. D 10, 186 (1974).
10. F. Cooper, G. Frye and E. Schonberg, Phys. Rev. D 11, 192 (1975).
11. O. V. Zhirov and E. V. Shuryak, Yad. Fiz. 21, 861 (1975).
12. A. H. Mueller, Nucl. Phys. B 213, 85 (1983).
13. B. B. Back et al., arXiv:nucl-ex/0301017.
14. R. Nouicer et al., arXiv:nucl-ex/0403033.
15. I. Y. Pomeranchuk, Dokl. Akad. Nauk Ser. Fiz. 78, 889 (1951).
16. J. Cleymans, Pramana 60, 787 (2003).
17. P. Braun-Munzinger, K. Redlich and J. Stachel, arXiv:nucl-th/0304013.
18. I. G. Bearden et al., arXiv:nucl-ex/0312023.
19. M. Stankiewicz, Honours Thesis, University of Cape Town (Advisor: Jean Cleymans) (2003).
20. R. P. Feynman, Phys. Rev. Lett. 23, 1415 (1969).
21. J. D. Bjorken, Phys. Rev. D 27, 140 (1983).
22. B. B. Back et al., Phys. Rev. Lett. 91, 052303 (2003).
23. I. G. Bearden et al., arXiv:nucl-ex/0403050.
24. M. Murray, arXiv:nucl-ex/0404007.
25. G. Roland, Quark Matter 2004.
26. J. Benecke, T. T. Chou, C. N. Yang and E. Yen, Phys. Rev. 188, 2159 (1969).
27. D. Kharzeev and E. Levin, Phys. Lett. B 523, 79 (2001).
28. D. V. Minh and P. Carruthers, Phys. Rev. Lett. 31, 133 (1973).
29. S. S. Adler et al., Phys. Rev. Lett. 91, 241803 (2003).
30. J. Adams et al., Phys. Rev. Lett. 91, 172302 (2003).