Waves in relativistic electron beam in low-density plasma

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Abstract. Waves in electron beam in low-density plasma are analyzed. The analysis is based on complete electrodynamics consideration. Dependencies of dispersion laws from system parameters are investigated. It is shown that when relativistic electron beam is passed through low-density plasma surface waves of two types may exist. The first type is a high frequency wave on a boundary between the beam and neutralization area and the second type wave is on the boundary between neutralization area and stationary plasma.

Key words: plasma, low-density plasma, ion focused regime, electron beam, wave generation, plasma instability.

Tasks of energy transformation of a high current relativistic electron beam to electromagnetic radiation are in the field of active investigation during recent years. Relativistic plasma RF electronics is focused on creation of high power RF amplifiers and generators [1, 2]. Plasma waveguides play role of decelerating systems. Low speed waves are used in linear particle wakefield accelerators where the correspondence between beam and electromagnetic wave speeds is necessary [3-5].

Waves in the beam – plasma system were investigated on the basis of hydro-electrodynamics approach. This method does not demand of additional defining of dielectric permittivity of a moving medium on the basis of particle dynamics equations. It also allows defining conditions of macroscopic balance of the loaded electron bunch in plasma. Similar problems were solved in [6], [7] within electrostatic approach.

In [8] steady and unstable surface electromagnetic waves on boundaries of relativistic plasma streams in the frequency range corresponding to positive values of dielectric permittivity were considered. In the same place existence of critical parameter was shown: the lowest mode number in cylindrical geometry corresponding to transition from slow waves to the fast. For an initial phase of development of hose instability of an electronic stream one more critical parameter – the cross wave size of the stream has been determined. In [9] surface electromagnetic waves on border of a relativistic electron beam in dense plasma in the frequency range corresponding to positive values of dielectric permeability were considered. As well as in case of the plasma stream, for the bunch in dense plasma critical parameters for instability development are the lowest mode number corresponding to transition from slow waves to fast and the cross wave size of the bunch.

As it will be shown below, unlike the bunch in dense plasma and from the plasma stream, in the rarefied plasma the dense bunch completely forces out electrons of plasma both from the beam volume, and from area of neutralization bigger than the beam radius. It leads to complication of dispersive properties of the beam-plasma system.
The hydrodynamic description of the cold multi component charged plasma [7] is based on the joint solution of Maxwell equations

\[
\begin{aligned}
\nabla \times E_\Sigma &= -\left(1/e\right)\left(\partial B_\Sigma / \partial t\right);
\n\nabla \times H_\Sigma &= \left(1/e\right)\left(\partial D_\Sigma / \partial t\right) + \left(4\pi/e\right) \sum_j e_j n_j \mathbf{v}_j \times \mathbf{E}_\Sigma;
\n\nabla B_\Sigma &= 0;
\n\nabla D_\Sigma &= 4\pi \sum_j n_j e_j.
\end{aligned}
\]

(1)

and hydrodynamics

\[
\begin{aligned}
\left(\partial / \partial t + \mathbf{v}_j \cdot \nabla\right) \mathbf{p}_j \Sigma &= e_j \left[\mathbf{E}_\Sigma + \left(1/c\right) \mathbf{v}_\Sigma \times \mathbf{B}_\Sigma\right];
\n\left(\partial n_j \Sigma / \partial t + \nabla \cdot \left(n_j \mathbf{v}_\Sigma\right)\right) &= 0,
\end{aligned}
\]

(2)

where \(E_\Sigma\), \(H_\Sigma\) are intensities of electric and magnetic fields respectively; \(c\) is the speed of light; \(D_\Sigma\), \(B_\Sigma\) are inductions of electric and magnetic fields respectively; \(e_j\), \(n_j\), \(\mathbf{v}_j\) are the charge, the concentration and the speed of a \(j\)-component of plasma respectively; \(\mathbf{p}_j \Sigma = m_j \gamma_j \mathbf{v}_\Sigma\) is the impulse of the \(j\)-component of plasma, \(m_j\) is the mass of the particle of this component, \(\gamma_j = \left(1 - \left|\mathbf{v}_\Sigma\right|^2 / c^2\right)^{-1/2}\) is its relativistic factor.

Systems of the equations of Maxwell and hydrodynamics (1), (2) have to be added with material relationships: \(B_\Sigma = H_\Sigma\); \(D_\Sigma = E_\Sigma\).

Let us consider the relativistic bunch of electrons in the motionless plasma environment with equilibrium concentration of particles \(n_i\). We will consider that the bunch represents a cylinder with radius \(R_b\) containing electrons moving along its axes (axis \(z\)) with the speed \(v_0 = \beta_0 c\) and the concentration \(n_0\). Density of the bunch exceeds plasma density. It is possible to consider that more density electron beam moves in a compensating ionic background, completely forcing out plasma electrons. The electrostatic force of interaction of electrons with the ionic background is counterbalanced by ponderomotive force from own magnetic field of the bunch: \(n_0 = n_i \gamma_0^2\), where \(\gamma_0 = \left(1 - \beta_0^2\right)^{-1/2}\). Out of the bunch the system (2) leads to a condition of an electric neutrality of the environment, from where follows that the bunch completely forces out plasma electrons from cylindrical area with the neutralization radius \(R_n\). Neutralization radius \(R_n\) is connected with bunch radius \(R_b\) by expression:

\[
R_n^2 = n_0 / n_i R_b^2 = \gamma_0^2 R_b^2.
\]

Out of neutralization area concentration of plasma electrons coincides with concentration of ions: \(n_1 = n_i\).

In case the current of \(I_b\) and the speed of electrons of \(v_0\) are set, and the density of the bunch exceeds the plasma density, there is the bunch self-focusing because of interaction of electrons to the ionic background. The established radius of the bunch can be determined as \(R_b^2 = I_b \left(\pi n_i e^2 \gamma_0^2\right)^{-1}\), where \(e\) is the elementary charge.

In relation to high-frequency electronic fluctuation the area of neutralization behaves as vacuum. Let the dielectric permittivity of the bunch is \(\varepsilon_2\) and of external plasma is \(\varepsilon_1\) (Figure 1). In works [10-
stability of the bunch in relation to its bend concerning the axis of the cylindrical ionic channel was investigated. We will analyze the bunch behavior at small disturbances of particle concentration in the cylinder.

Figure 1. The beam – rarefied plasma system geometry.

Having entered designations

\[
  k_{p0}^2 = 4\pi e^2 n_0 (mc^2)^{-1}; \quad k_{p1}^2 = 4\pi e^2 n_1 (mc^2)^{-1}; \quad k_{p2}^2 = 4\pi e^2 n_2 (mc^2)^{-1},
\]

we will receive for \( n_0 > n_1 \):

\[
  k_{p0}^2 = k_{p1}^2 \gamma_0^2, \quad R_n = \gamma_0 R_b, \quad R_b = \left(\frac{4eI_b}{m^2 c^2 V_0^2}\right)^{1/2}.
\]

Solutions of linearized Maxwell’s equations (1) and hydrodynamics (2) for the unlimited cylindrical bunch in the longitudinal direction we will look for in the cylindrical system of coordinates \((r, \theta, z)\) in the form:

\[
  \psi(r, \theta, z, t) = \psi(r)e^{i(\omega t + k_z z - \omega t)}
\]

where \( \omega \) is a circular frequency, \( k_z \) and \( v \) are longitudinal and azimuthal wave numbers.

For receiving the dispersion equation we will use the boundary conditions following from Maxwell’s equations:

\[
  n \times (E^{(2)} - E^{(1)}) = 0,
\]

\[
  n \times (H^{(2)} - H^{(1)}) = \frac{4\pi}{c} \lim_{\Delta r \to 0} \int_{-\Delta r}^{\Delta r} e_j \left( \vec{n}_j \mathbf{V}_j + n_j \mathbf{V}_j \right) dt
\]

where \( n \) is a normal to the bunch surface, \( E^{(2)}, E^{(1)}, H^{(2)}, H^{(1)} \) are intensity of electric and magnetic fields on both sides from the medium boundary, \( \Delta r \) is counted from the boundary in the direction of the normal.

Solving in systems of equations of Maxwell (1) and hydrodynamics (2), with boundary conditions on boundaries of the bunch and the area of neutralization we will receive the dispersion equation for electromagnetic waves in the system:

\[
  P^2 A_{Ih} A_{Kn} - P(A_{Kb} A_{Kn} + A_{Kb} A_{Kn}) + A_{Kb} A_{Kn} - 2PQ_o Q_n (S_{0Kb} - S_{0Ib}) (S_{0Kn} - S_{0In}) = 0
\]

where

\[
  S_1 = -\frac{K_v(kR_b \eta_1)}{\eta_0 K_v(kR_b \eta_1)}, \quad S_2 = \frac{I_v(kR_b \eta_2)}{\eta_0 I_v(kR_b \eta_2)}, \quad S_{0Kn} = -\frac{K_v(kR_b \eta_0)}{\eta_0 K_v(kR_b \eta_0)}, \quad S_{0In} = -\frac{I_v(kR_b \eta_0)}{\eta_0 I_v(kR_b \eta_0)},
\]

\[
  S_{0Kb} = \frac{K_v(kR_b \eta_0)}{\eta_0 K_v(kR_b \eta_0)}, \quad S_{0Ib} = \frac{I_v(kR_b \eta_0)}{\eta_0 I_v(kR_b \eta_0)}, \quad P = \frac{I_v(kR_b \eta_0) K_v(kR_b \eta_0)}{I_v(kR_b \eta_0) K_v(kR_b \eta_0)}, \quad Q_n = \frac{\eta_1}{kR_n} \left(\frac{1}{\eta_1^2} - \frac{1}{\eta_0^2}\right).
\]
The equation coincides with the dispersion equation for waves on the relativistic plasma stream in motionless plasma [8]. In ultra-relativistic limit $\gamma \to +\infty$ two variants are possible. The first is $\mathbb{R}n \to +\infty$, in this case the dispersion equation describes surface waves on borders between the relativistic plasma stream and vacuum. The second is $k_b \to 0$, thus the equation describes surface waves on border of vacuum and motionless plasma.

The following algorithm was developed for search of roots of the dispersive equation. On the first step of algorithm intervals of equation root isolation in the complex plane were determined. Two methods were applied: search of intervals of transition through zero of imaginary and real parts of the function value and search of minima of the absolute value of function. The found coordinates were substituted as initial approach for search of roots by the method of secants.

After search of possible roots their check by substitution of the received values in initial expression was made. If thus initial expression accepted the value which is not exceeding $1 \cdot 10^{-8}$, the decision that the root is found was made.

Search of roots was run at various initial values of parameters of the wave number $k_z$ and the bunch radius $R_b$. If the received value of the root slightly differed from the found values at other close $k_z$ and $R_b$, we have considered the decision steady. Therefore not separate points, but curves on figures below are of interest.

Dependences of roots of the dispersion equation on the bunch radius for three first modes of the system are presented in tables 1 ($n_1 = 10^{12}$ cm$^{-3}$) and 2 ($n_1 = 10^{14}$ cm$^{-3}$) at $k = 1$ cm$^{-1}$ ($f = 4.77$ GHz), $\beta = 0.9$. In the first and second lines of the tables real and imaginary parts of roots are shown in the dependence on bunch radius. Positions of roots on the complex plane are given in the third line of tables.

At $n_1 = 10^{11}$ cm$^{-3}$ lower there are no roots with $\text{Im}\ k_z \neq 0$ on zero mode. But on the first and the second modes there is a resonant convective instability of the bunch ($\text{Re}\ k_z \approx k$, $\text{Im}\ k_z \neq 0$) which takes place for all range of the bunch radius values (0.01 – 2.5 cm). Besides, there are slow modes of beam-plasma system with $\text{Re}\ k_z < k$, which are excited if the radius of the bunch exceeds some critical value.

At $n_1 = 10^{12}$ cm$^{-3}$ plasma frequency is less than the chosen frequency $k > k_p$ ($\omega > \omega_p$). In this case there is the resonant convective instability of the bunch on zero mode for which it is fair $\text{Re}\ k_z \approx k$, $\text{Im}\ k_z \neq 0$. At $\nu > 0$ slow modes of beam-plasma system are accruing with $\text{Re}\ k_z < k$, which are excited if the radius of the bunch exceeds some critical value.

Roots with $\text{Re}\ k_z \approx k$, $\text{Im}\ k_z \neq 0$ were obtained at $n_1 = 10^{13}$ cm$^{-3}$ on zero mode if the bunch radius is more than some critical value. As in the previous case there is the resonant convective instability of the bunch for those radius values and equilibrium concentration of particles of plasma.

At $n_1 = 10^{14}$ cm$^{-3}$ $k < k_p$ ($\omega < \omega_p$) on zero mode the resonant instability of the bunch practically does not arise. However accruing are waves with negative real part of $k_z$, that is the waves moving towards the bunch. At $\nu > 0$ slow modes are accruing at excess by the radius of the critical value and with real part of wave number, close to zero, that means emergence of absolute instability.

$$Q_b = \frac{\nu}{kR_b} \left( \frac{n}{n_0} - \frac{\bar{n}}{n_2} \right).$$

$$A_{ijb} = (S_{0ib} - \varepsilon_2 S_2)(S_{0jb} - S_2) - Q_b^2,$$

$$A_{ijn} = (S_{0in} - \varepsilon_1 S_1)(S_{0jn} - S_1) - Q_b^2.$$

$i, j = \{1, K\}$, $\bar{n} = \gamma^2 \left( (1 - \varepsilon_2 \beta^2)\eta + (\varepsilon_2 - 1)\beta \right)$, $\eta_0^2 = \eta^2 - 1$, $\eta_2^2 = \eta^2 - 1 - (\varepsilon_2 - 1)\gamma^2 (1 - \eta\beta)^2$, $\eta^2 = \eta^2 - \varepsilon_1$, $\eta = k_z / k$, $I_{\nu}(x)$ is the modified Bessel function, $K_{\nu}(x)$ is the Macdonald function.
At \( n_i = 10^{14} - 10^{15} \text{ cm}^{-3} \) there is the resonant convective instability of the bunch (\( \text{Re} \, k_z \approx k, \quad \text{Im} \, k_z \neq 0 \)) on the first and the second modes which is excited if the radius of the bunch exceeds some critical value.

**Table 1.** Roots of dispersion equation \( k_z \) vs bunch radius \( R_b \) at \( n_i = 10^{12} \text{ cm}^{-3} \).

| \( \nu = 0 \) | \( \nu = 1 \) | \( \nu = 2 \) |
|----------------|----------------|----------------|
| Re \( k_z \)   | Re \( k_z \)   | Re \( k_z \)   |
| Im \( k_z \)   | Im \( k_z \)   | Im \( k_z \)   |
| Re \( k_z \)   | Re \( k_z \)   | Re \( k_z \)   |

\( \nu \)
Table 2. Roots of dispersion equation $k_z$ vs bunch radius $R_b$ at $n_1 = 10^{14}$ cm$^{-3}$.

| $\nu = 0$ | $\nu = 1$ | $\nu = 2$ |
|-----------|-----------|-----------|
| $\text{Re } k_z$ | $\text{Re } k_z$ | $\text{Re } k_z$ |
| $\nu = 0$ | $\nu = 1$ | $\nu = 2$ |
| $\text{Im } k_z$ | $\text{Im } k_z$ | $\text{Im } k_z$ |
| $\nu = 0$ | $\nu = 1$ | $\nu = 2$ |

The analysis of the received dispersive equation shows that when passing the ultra relativistic bunch of electrons ($\beta = 0.999$) through the rarefied plasma there can be surface waves of two types: high-frequency on border of the relativistic bunch with area of neutralization and low-frequency on border of area of neutralization and motionless plasma. The first type corresponds to surface waves on border of the relativistic plasma stream with vacuum, the second – on border of the vacuum cylinder and motionless plasma.

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