One-Loop Electroweak Corrections for Polarized Møller Scattering at Different Renormalization Schemes and Conditions¹

A. G. Aleksejevs, S. G. Barkanova, A. N. Ilyichev, Yu. G. Kolomensky, and V. A. Zykunov

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Abstract — Using two different approaches, we perform detailed calculations of the one-loop (Next-to-Leading Order (NLO)) electroweak radiative corrections to the parity violating $e^+e^-\rightarrow e^+e^-\gamma$ scattering asymmetry. First approach, more classical, relies on calculations “by hand” with reasonable approximations, second approach relies on program packages FeynArts, FormCalc, LoopTools, and FORM. The detailed numerical analysis of the various contributions is provided for a wide range of energies relevant for the ultra-precise 11 GeV MOLLER experiment planned at the JLab, as well as future experiments at the International Linear Collider (ILC). The numerical results obtained within the on-shell renormalization scheme using two different sets of renormalization conditions are in excellent agreement. We also calculate the total NLO correction in the Constrained Differential Renormalization (CDR) scheme. Analysis of the results, along with the increasing experimental precision, shows that it is feasible that the corrections at the Next-to-Next-to-Leading Order (NNLO) level may be important for the next generation of experiments.

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1. INTRODUCTION

The Møller scattering measurements are not only one of the oldest and the best-established tools of modern physics, but also a clean, powerful probe of New Physics (NP) effects [1]. More recently, the significant interest to this process from both theoretical and experimental communities has been re-ignited by two precision experiments: E-158 [3] at SLAC, which made the first observation of parity violation in electron-electron scattering, and MOLLER experiment planned at JLab [5]. Both are dedicated to measuring the Parity-Violating (PV) asymmetry in the $e^+e^-\rightarrow e^+e^-\gamma$ scattering at low energies. The MOLLER experiment aims to measure the PV asymmetry in the scattering of longitudinally polarized electrons off unpolarized electrons with a combined statistical and systematic uncertainty of 36 parts per billion [5]. With the estimated systematic contribution of 1%, the measurement will still be statistics-limited, and further improvements in precision might be possible with additional running time.

The measurement of the electron’s weak charge $Q'_W$ with a 2.3% accuracy planned by MOLLER would yield the most precise single measurement of the weak mixing angle $\sin^2\theta_W$, with a fractional accuracy of 0.1%, at an average momentum transfer $Q^2 = 0.0056 \text{ GeV}^2$. At this precision, MOLLER can shed light on the discrepancy between the hadronic and leptonic determinations of $\sin^2\theta_W$ at the $Z$-boson pole. Furthermore, the difference between the values of $\sin^2\theta_W$ determined at the $Z$-boson pole and at low $Q^2$ is sensitive to the NP effects at TeV scales.

Before physics of interest can be extracted from the experimental data, radiative effects must be carefully treated. MOLLER’s stated precision goal is significantly more ambitious than that of its predecessor E-158, so very precise theoretical input for this measurement will be crucial. In spite of significant earlier theoretical effort dedicated to calculations of Electroweak Radiative Corrections (EWC) (see the early review papers [7] and [8], more recent [9] and [10], and numerous additional references in our paper [11]), we believe that a new level of accuracy is required for the next-generation, high-precision experiments. To match the expected experimental systematic uncertainty, it is desirable to keep the theoretical uncertainty due to the radiative corrections at or below the 0.1% level. Obviously, calculating large sets of one-loop Feynman diagrams by hand is a tedious task. Recently, program packages such as FeynArts [12], FormCalc [13], LoopTools [13] and FORM [14] have created the possibility of handling the substantial number of diagrams reasonably quickly, minimizing probability of human errors, and preventing the rapid error accumulation often unavoidable with purely numerical methods. One of the key features of the pre-
sent work is to compare, step by step, the complete one-loop set of EWC to the PV Møller scattering asymmetry calculated first by hand and then with FeynArts, FormCalc and LoopTools as base languages using two different renormalization conditions.

FeynArts is a Mathematica package which provides the generation and visualization of Feynman diagrams and amplitudes involving Standard Model particles. FormCalc, a Mathematica package, reads diagrams generated with FeynArts and evaluates amplitudes with the help of the program FORM in analytical form. FORM, a successor to SCHOONSCHIP [15], is a Symbolic Manipulation System which is also essential for our computer algebra-based method, and is used by FormCalc as a core program. LoopTools provides the many-point tensor coefficient functions and is used to numerically evaluate scalar and tensor one-loop integrals. In FormCalc, it is possible for the regularization to be done either by dimensional reduction or by the usual dimensional regularization scheme. After that, one may implement one of the two renormalization schemes (RS), the on-shell scheme or the Constrained Differential Renormalization (CDR) scheme. For calculations done at the one-loop level, the CDR scheme is equivalent to regularization done by dimensional reduction in the $\overline{MS}$ scheme with redefined scale $\log \overline{M}^2 = \log \mu^2 + 2$, where $\overline{M}$ and $\mu$ are the renormalization scales in the CDR and in the dimensional regularization method, respectively [13].

A complete automatization would limit the range of applications, so these packages are not “black box”; they require considerable human input on many stages. We call our approach based on FeynArts, FormCalc, LoopTools, and FORM “semi-automated”. On the other hand, these packages allow modifications to better suit specific projects. In [16], for example, we adopted FeynArts and FormCalc for the NLO calculations of the differential cross section in electron-nucleon scattering. In general, the results obtained with these packages can be presented in both analytical and numerical form. However, our equations for the EWC to the scattering asymmetry obtained with FeynArts and FormCalc are too lengthy and cumbersome, so we present only approximate equations obtained by hand. However, as we show in the numerical analysis section, the agreement between numerical results obtained with the two methods—“by hand” and semi-automatic—is excellent.

An additional way to ensure that our NLO EWC calculations are perfectly correct is the detailed comparison of results calculated with different renormalization conditions within the same scheme. Of course, the sum of all radiative corrections forming a full gauge-invariant set must be independent on the choice of renormalization conditions. Paper [17], for example, clearly demonstrated the cancellation of gauge dependencies in one-loop corrections from self-energies, tadpoles, vertex and box diagrams to physical amplitudes for four-fermion processes. However, the agreement between the results evaluated in different renormalization schemes can be guaranteed only if we take into account all orders of perturbation expansion, not just NLO. Since in this article we are only dealing with one-loop corrections, we do not expect the results produced in different schemes to be identical. In fact, the difference we see between the results obtained with the on-shell and CDR schemes indicates the need to consider higher-order corrections. A detailed discussion was given in [18], for example. The NNLO corrections will be our next task.

For now, we concentrate on achieving the best accuracy possible in one selected scheme and perform NLO calculations using two methods and two sets of renormalization conditions. For that, we choose the on-shell renormalization scheme with two different sets of renormalization conditions, the approach proposed by W. Hollik in [19] (see also [20]) and the approach suggested by A. Denner in [21]. For brevity, we will call Hollik’s renormalization conditions HRC and Denner’s conditions DRC. It is obvious that any renormalization scheme is required to meet physical conditions, although it is possible to vary renormalization conditions for the sake of simplicity of the problem and still keep the final gauge invariant results unchanged. As a result, contributions to the cross section coming from the different non-gauge invariant loop corrections (self-energies and vertex corrections) could vary greatly depending on the choice of renormalization conditions.

The first goal of this paper is to calculate the full set of one-loop EWC, both numerically with no simplifications using semi-automatic approach, and “by hand”, analytically in a compact asymptotic form [22–24], and compare the results. Our second goal is to present comparison and analysis of the various contributions to the cross-section asymmetry calculated within the HRC and DRC renormalization conditions. Our third aim is to calculate the total NLO corrections in the CDR scheme and estimate the importance of the NNLO corrections for such high-precision experiments as MOLLER.

The rest of the paper is organized as follows. In Section 2 we provide details of the basic notation, the lowest-order (Born or Leading Order (LO)) and NLO contributions to Møller scattering. The same section gives a short description of photon emission which is essential for removal of nonphysical parameters from regularized infrared divergent cross section. The details of the HRC and DRC renormalization conditions and a discussion of gauge invariance can be found in Section 3. Analysis of analytical and numerical results in the on-shell RS using HRC and DRC renormalization conditions is given in the beginning of Section 4. Later, in the same section, the CDR results are discussed. Section 5 includes the analysis of possi-
ble effects of an additional new-physics massive neutral boson on the observable asymmetry. Our conclusions and future plans are discussed in Section 6.

2. DEFINITIONS AND FRAMEWORK

In the Standard Model, the Born cross section for Möller scattering with the longitudinally-polarized electrons

\[ e^-(k_1) + e^-(p_1) \rightarrow e^-(k_2) + e^-(p_2), \tag{1} \]

can be represented in the form

\[ \sigma^0 = \frac{\pi \alpha^2}{s} \sum_{i,j=\gamma,Z} [\lambda_{-i,j}^2 (u^2 D_{ii} D_{jj} + \tilde{t}^2 D_{ii}^\dagger D_{jj}^\dagger) + \lambda_{+i,j}^2 (D_{ii}^\dagger + D_{jj}^\dagger)(D_{ii}^\dagger + D_{jj}^\dagger)], \tag{2} \]

where \( \sigma \equiv d\sigma/d\cos\theta \) and \( \theta \) is the scattering angle of the detected electron with momentum \( k_2 \) in the center of mass system of the initial electrons. The set of momenta of initial (\( k_1 \) and \( p_1 \)) and final (\( k_2 \) and \( p_2 \)) electrons (see Fig. 1) generates the standard set of Mandelstam variables,

\[ s = (k_1 + p_1)^2, \quad t = (k_1 - k_2)^2, \quad u = (k_2 - p_1)^2. \tag{3} \]

We neglect the electron mass \( m \) whenever possible and in particular when \( m^2 \ll s, -t, -u \).

A useful structure we employ in this paper is

\[ D^{rr} = \frac{1}{r - m^2_r} \]  

which depends on the Z-boson mass \( m_Z \) or on the photon mass \( m_\gamma \). The photon mass is set to zero everywhere with the exception of specially indicated cases where the photon mass is taken to be an infinitesimal parameter that regularizes an infrared divergence. In addition, we use the functions

\[ \lambda_{\pm}^{i,j} = \lambda_{1}^{i,j} \lambda_{1}^{\dagger} \pm \lambda_{2}^{i,j} \lambda_{2}^{\dagger}, \tag{5} \]

which are combinations of coupling constants and the degrees of polarizations \( p_{B(T)} \) of the electrons with momentum \( k_1(p_1) \) given by

\[ \lambda_{1}^{i,j} = \lambda_{1}^{i,j} - p_{B(T)} \lambda_{A}^{i,j}, \quad \lambda_{2}^{i,j} = \lambda_{1}^{i,j} - p_{B(T)} \lambda_{A}^{i,j}, \quad \lambda_{A}^{i,j} = \lambda_{A}^{i,j} + \lambda_{A}^{i,j}, \tag{6} \]

Here, vector and axial-vector parts of the couplings have the following structure

\[ v^v = 1, \quad a^v = 0, \quad \lambda^v = (I_e^v + 2s^2_W)/(2s_Wc_W), \quad a^a = I_e^a/(2s_Wc_W). \tag{7} \]

It should be recalled that \( I_e = -1/2 \) and \( s_W(c_W) \) are the sine (cosine) of the Weinberg mixing angle which is defined in terms of \( m_Z \) and \( m_W \) according to the rules of the Standard Model (SM): \( c_W = m_W/m_Z \) and \( s_W = \sqrt{1 - c_W^2} \). The electron degrees of polarization \( p_{B(T)} \) are labeled such that the subscripts \( L \) and \( R \) correspond to the values of \( p_{B(T)} \) = -1 and \( p_{B(T)} \) = +1 respectively. Here, the first subscript indicates the degree of polarization for the momentum \( k_1 \), while the second indicates the degree of polarization for the momentum \( p_1 \). Combining the degrees of electron beam polarizations, we can obtain four measurable cross sections. However by the virtue of the rotational invariance, two of them are identical: \( \sigma_{LL} = \sigma_{RL} \). The three polarization cross sections can be used to construct three independent asymmetries [25]. Of particular interest to us is the parity-violating asymmetry \( A_{LR} \) which is defined as follows

\[ A_{LR} = \frac{\sigma_{LL} + \sigma_{LR} - \sigma_{RL} - \sigma_{RR}}{\sigma_{LL} + \sigma_{LR} + \sigma_{RL} + \sigma_{RR}} = \frac{\sigma_{LL} - \sigma_{RR}}{\sigma_{LL} + 2\sigma_{LR} + \sigma_{RR}}. \tag{8} \]

This single-polarization asymmetry corresponding to the scattering of longitudinally polarized electrons on unpolarized electrons is proportional to the combination \( 1 - 4s_W^2 \), and is therefore highly sensitive to small changes in \( s_W \). That is why the asymmetry \( A_{LR} \) was used as the observable in E-158 and will be measured in the future MÖLLER experiment. At low energies and at Born level, the \( PV \) asymmetry \( A_{LR}^0 \) is given by

\[ A_{LR}^0 = \frac{s}{2m_w^2 + y^4 + (1 - y)^4} \left( \frac{1 - 4s_W^2}{s_W^2} \right), \quad y = -t/s. \tag{9} \]

The contribution of virtual particles (\( V \)-contribution) to the cross section of Möller scattering is described by the three classes of diagrams: boson self-energies (BSE) (they include \( \gamma \gamma \), \( \gamma Z \) and \( ZZ \) self-energies and are shown symbolically in Fig. 2a), vertex functions (Figs. 2b and 2c), and two-boson exchange diagrams (boxes) shown in Fig. 2d, 2e). In the on-shell
and CDR renormalization schemes there is no contribution from the electron self-energies. The corresponding cross section is given by the sum

$$\sigma^V = \sigma^{BSE} + \sigma^{\text{Vert}} + \sigma^{\text{Box}}. \quad (10)$$

The detailed expressions for all the terms in this sum were given in our recent paper [11].

Contributions coming from the vertex correction graphs (with a photon in the loop), as well as the $\gamma\gamma$ and $\gamma Z$ boxes suffer from the well-known infrared divergence. Regularization of this divergence can be done by giving the photon a small unphysical mass. Obviously, the final result should be free of unphysical parameters and hence such dependence has to be removed. That can be done if we consider additional contributions associated with photon emission diagrams (bremsstrahlung). The detailed description of this contribution is also given in [11]. The bremsstrahlung cross section can be broken down into two parts (soft and hard) as

$$\sigma^R = \sigma^{IR} + \sigma^{HR} \quad (11)$$

by separating the integration domain according to $k_0 < \omega$ and $k_0 > \omega$, where $k_0$ is the photon energy (in the reference frame co-moving with the center of mass of the primary electrons). The parameter $\omega$ corresponds to the maximum of the emitted soft-photon energy. First, we follow the methods of paper [26] to get a well-known result (see also [22–24, 27, 28]) for the soft-photon cross section (where $e$ is the base of the natural logarithm):

$$\sigma^{IR} = \frac{\alpha}{\pi} \left( 4 \log \frac{2\omega}{\lambda} \log \frac{tu}{em^2} - \log \frac{s}{em^2} + 1 - \frac{\pi^2}{3} + \log \frac{2u}{l} \right) \sigma^0. \quad (12)$$

Next, we sum the $IR$-terms of $V$- and $R$-contributions,

$$\sigma^C = \sigma^V + \sigma^{IR} \quad (13)$$

and get a result which is free of regularization parameter $\lambda$.

At this point let us continue with the discussion of the details of renormalization conditions we use in our calculations.

3. RENORMALIZATION CONDITIONS AND GAUGE INVARIANCE

To obtain the ultraviolet-finite result and render the parameters of the Standard Model real, we have to apply a renormalization procedure. For a gauge-invariant set, physical results should be invariant under different renormalization conditions. That is, although the contributions of the different types of diagrams can vary strongly for different renormalization conditions, the total impact of all one-loop virtual effects on observable quantities must remain independent. In other words, the contributions of separate self-energies and vertex correction functions strongly depend on the details of the renormalization conditions, and to properly account of the EWC they should be taken as one gauge-independent set. We will illustrate this for the case of the observable $A_{LR}$, which is especially sensitive to the renormalization conditions. In addition, we can verify that our results are correct by comparing the computer-based (DRC) and “by hand” (HRC) calculations. We now briefly describe our two chosen renormalization conditions, DRC and HRC, within the on-shell renormalization scheme.

Both use multiplicative renormalization constants, and as a result the electroweak Lagrangian, originally written in terms of bare parameters, is separated into a basic Lagrangian and a counterterm Lagrangian. The basic Lagrangian has the same form as the bare one, but depends on renormalized parameters and fields. The counterterm Lagrangian depends on renormalization constants of masses, charges and fields. Renormalization constants are fixed by the renormalization conditions, which are separated into two classes: the first determines the renormalization of the parameters, and the second fixes the renormalization of fields. The first class is related to physical observables at a given order of perturbation theory, and the second one is related to the Green’s functions and has no effect on calculations of $S$-matrix elements. Both approaches

![Fig. 2. One-loop $t$-channel diagrams for the Møller process. The circles represent the contributions of self-energies and vertex functions. The $u$-channel diagrams are obtained via the interchange $k_2 \rightarrow p_2$.](image-url)
use essentially the same renormalization conditions to fix the parameters of the SM in the following way:

\[
\text{Re} \hat{\Sigma}_T^W(m_W^2) = \text{Re} \hat{\Sigma}_T^{ZZ}(m_Z^2) = \text{Re} \hat{\Sigma}_T^{\gamma}(m_Z^2) = 0, \\
\hat{\Gamma}_\mu^{er}(k^2 = 0, \quad p^2 = m^2) = ie\gamma_\mu.
\]  

(14)

Here, \(\text{Re} \hat{\Sigma}_T^{ZZ}(m_{Z,W}^2)\) and \(\hat{\Gamma}_\mu^{er}(k^2 = 0, p^2 = m^2)\) are the real parts of the truncated, transverse renormalized boson self-energy and electron vertex correction graphs, respectively. The longitudinal parts of the boson self-energy make very small contributions and are not considered here. The first condition of Eq. (14) fixes the mass renormalization of the \(W, Z\)-bosons and fermions without quark mixing. The second condition fixes the renormalization of electric charge, and is derived from the Thomson limit when momentum transfer \(k^2 = 0\) and external electrons are on their mass shell. As for the renormalization conditions of the fields, both approaches are quite different. In HRC, field renormalization constants are determined from the following conditions:

\[
\hat{\Sigma}_T^{\gamma}(0) = 0, \quad \frac{\partial}{\partial k^2} \hat{\Sigma}_T^{\gamma}(0) = 0.
\]  

(15)

However, in the DRC renormalization conditions, the field renormalization is defined on-shell, as it was done for renormalization of the SM parameters. This explicitly introduces an additional set of conditions, besides Eqs. (14) and (15), which read:

\[
\text{Re} \hat{\Sigma}_T^{\gamma}(m_Z^2) = 0, \quad \frac{\partial}{\partial k^2} \hat{\Sigma}_T^{\gamma}(m_Z^2) = 0, \\
\frac{\partial}{\partial k^2} \hat{\Sigma}_T^{W}(m_W^2) = 0.
\]  

(16)

As a result, in DRC, renormalization constants for the fields of vector bosons are calculated in a relatively simple way, without the mass-renormalization constants are defined in a different way and depend on the mass-renormalization constants:

\[
\delta Z_W^{(H)} = -\frac{c_W}{s_W} \hat{\Sigma}_T^Z(0), \\
\delta Z_Z^{(H)} = \frac{c_W}{s_W} \hat{\Sigma}_T^Z(0) - 2s_W c_W Z^2 \left( \frac{m_Z^2 - m_W^2}{m_Z^2} \right), \\
\delta Z_Z^{(D)} = 2 \frac{m_Z^2}{m_Z^2} \left( \hat{\Sigma}_T^Z(0) - \frac{m_Z^2}{m_Z^2} \right), \\
\delta Z_T^{(D)} = \frac{\partial}{\partial k^2} \hat{\Sigma}_T^Z(0).
\]  

(17)

The presence of the mass renormalization constants in the field-renormalization Eq. (18) increases the values of the truncated and renormalized self-energy diagrams, and the dominant NLO contributions to the observable cross section come from these diagrams. In DRC, the mass renormalization constants appear in renormalization constants of the electroweak couplings, and hence we observe comparable contributions coming from both self-energies and vertex corrections. Of course, such a comparison has no physical meaning since neither self-energies nor vertex corrections represent a gauge-invariant set on their own. As is well known, only the sum of both groups is gauge invariant; later we show that both approaches give exactly the same results for the observable asymmetry. We would like to highlight that it is important to exercise caution when comparing separate contributions arising from the different renormalization conditions. This point is illustrated by Figs. 3, 4 and 5, where one can see various renormalized vector boson self-energies calculated with both DRC and HRC.

Let us note that the same renormalization conditions are imposed on the electromagnetic field. As we can see from Fig. 3, which shows the truncated and renormalized \(\gamma\gamma\) self-energies, there is no difference whatsoever between the two sets of conditions. The situation is quite different if we look at the results for the truncated and renormalized \(ZZ, \gamma Z\) and \(WW\) self-energies. In Figs. 4 and 5, we can see a substantial difference in the results obtained within the two sets of renormalization conditions, where the DRC set systematically leads to the self-energies being smaller in magnitude. As a result, with DRC, the self-energy contributions to Møller asymmetry are roughly a factor of two smaller in value compared to the values given by HRC. However, adding the DRC vertex corrections restores the total correction obtained with DRC to within 0.001% of the HRC result at all ener-
Now we are ready to proceed to the analysis of the results. Section 4.4.1. is done in a manner independent of the renormalization conditions, and all notations that follow below can be applied to both DRC and HRC of the on-shell scheme.

4. RESULTS AND ANALYSIS

4.1. On-Shell Renormalization

For the numerical analysis, we use $\alpha = 1/137.035999$, $m_W = 80.399$ GeV, and $m_Z = 91.1876$ GeV according to [29], take electron, muon, and $\tau$-lepton masses as $m_e = 0.510998910$ MeV, $m_\mu = 0.105658367$ GeV, $m_\tau = 1.77684$ GeV, and quark masses for loop contributions as $m_u = 0.06983$ GeV, $m_c = 1.2$ GeV, $m_t = 174$ GeV, $m_d = 0.06984$ GeV, $m_s = 0.15$ GeV, and $m_b = 4.6$ GeV. The light quark masses provide $\Delta \alpha_{\text{had}}^{(5)}(m_Z^2) = 0.02757$ [30], where

$$\Delta \alpha_{\text{had}}^{(5)}(s) = \frac{\alpha}{\pi} \sum_{f = u,d,s,c,b} Q_f^2 \left( \log \frac{s}{m_f^2} - \frac{5}{3} \right), \quad (19)$$

$Q_f$ is the electric charge of fermion $f$ in proton charge units $q_f$ ($q_f = \sqrt{4\pi\alpha}$). We can see that the use of the light quark masses as parameters regulated by the hadronic vacuum polarization is a better choice in this

Fig. 3. Truncated and renormalized $\gamma \gamma$ self-energies in both sets of renormalization conditions. The right graph shows the low-energy domain.

Fig. 4. Truncated and renormalized $ZZ$ and $WW$ self-energies in both sets of renormalization conditions.
is respectively. The main subject of the following analysis is the distribution to the observable will have to be measured more exactly) does not weak by nature, but is needed here to account for all corrections to the Born asymmetry as weak + σ_{ZSE}^{0} - σ_{0,0}^{0}. (20)

The operation $E_{LL} - E_{RR}$ under expression $E$ means $E_{LL} - E_{RR}$. Subscript 00 denotes the unpolarized cross section. The approximate equality is possible because $\sigma_{00}^{ZSE} / \sigma_{00}^{LL} - \sigma_{00}^{RR}$ is very small. The denominator of the last fraction is calculated directly from Eq. (2):

\[ \sigma_{LL}^{0} - \sigma_{RR}^{0} = 16\pi\alpha^{2} Z^{2} s (D^{Z} + D^{\bar{Z}}) (D^{Z} + D^{\bar{Z}}) \]

With simplifications, the numerator of Eq. (20) is

\[ \sigma_{ZSE}^{0} = \frac{\sigma_{LL}^{0} - \sigma_{RR}^{0}}{\sigma_{LL}^{0} - \sigma_{RR}^{0}} \]

\[ \approx -32\pi\alpha^{2} Z^{2} s \frac{D^{Z} + D^{\bar{Z}}}{m_{Z}^{2}} (D^{Z} + D^{\bar{Z}})(\Sigma^{Z}(t) + \Sigma^{Z}(u)). \]

**Fig. 5.** Truncated and renormalized $\gamma Z$ self-energies in both sets of renormalization conditions. The right graph shows the low-energy domain.
Finally,
\[ \delta_A^{\text{ZSE}} \approx \frac{\hat{\Sigma}_T(t) + \hat{\Sigma}_T(u)}{2m_Z^2} \]  
(23)

At small \( r \) (Eq. (4)) corresponding to \( E_{\text{lab}} = 11 \text{ GeV} \) and \( \theta = 90^\circ \), the corrections are \( \delta_A^{\text{ZSE}} \) (HRC) \( \approx 0.0309 \) vs. \( \delta_A^{\text{ZSE}} \) (DRC) \( \approx -0.0105 \).

Similarly, for \( \gamma ZSE \) contribution
\[ \sigma_{\gamma ZSE}^{\text{LL-RR}} \approx \frac{\pi \alpha^2}{s} D_S^{\gamma Z} \left[ D_t^{\gamma Z} \left( M_{\text{ev}}^{\gamma Z} + M_{\text{ev}}^{\gamma Z} \right) \right. \\
- D_u^{\gamma Z} \left( M_{\text{od}}^{\gamma Z} + M_{\text{od}}^{\gamma Z} \right) \right]_{\text{LL-RR}} + (t \leftrightarrow u) \\
\approx 16 \pi \alpha^2 \frac{Z}{m_Z} \left( D_t^{\gamma Z} + D_u^{\gamma Z} \right) \left( \frac{\hat{\Sigma}_T(t)}{t} + \frac{\hat{\Sigma}_T(u)}{u} \right). \]  
(24)

and
\[ \delta_A^{\gamma ZSE} \approx - \frac{1}{2 \sqrt{3}} \left( \frac{\hat{\Sigma}_T(t)}{t} + \frac{\hat{\Sigma}_T(u)}{u} \right). \]  
(25)

For \( \gamma ZSE \) at \( E_{\text{lab}} = 11 \text{ GeV} \) and \( \theta = 90^\circ \), the corrections are \( \delta_A^{\gamma ZSE} \) (HRC) \( \approx 0.6028 \) vs. \( \delta_A^{\gamma ZSE} \) (DRC) \( \approx -0.2909 \).

It is important to note that the deviation in \( \hat{\Sigma}_T \) has a dramatic impact on such a sensitive observable as \( A_{\text{LR}} \).

For example, the uncertainty in \( \hat{\Sigma}_T \) of 1% will result in a change in \( \delta_A^{\gamma ZSE} \) of up to 0.05.

All terms with properties of Eq. (20) contribute additively to the total correction, for example,
\[ \delta_A^{\gamma ZSE + ZSE} \approx \delta_A^{\gamma ZSE} + \delta_A^{ZSE}. \]  
(26)

We call such contributions additive. \( \gamma SE \) gives a non-additive and small contribution that we consider later.

4.1.2. Analysis of HV and box contribution to PV asymmetry. Starting with the \( \Lambda_2 \) contribution, which comes from the triangle diagrams with an additional massive boson, \( Z \) or \( W \), we get
\[ \delta_\Lambda_2 \approx \frac{\delta_\Lambda_2}{\sigma_{\text{LL-RR}}} \]  
(27)

The numerator, with some approximations, is
\[ \sigma_{\text{LL-RR}} \approx 8 \pi \frac{1}{2} \frac{Z}{a} s \left( D_t^{\gamma Z} + D_u^{\gamma Z} \right) \left( \frac{\Lambda_2(t, m_Z)}{t} + \frac{\Lambda_2(u, m_Z)}{u} \right), \]  
(28)

so the correction is proportional to \( \Lambda_2 \) in the following way:
\[ \delta_\Lambda_2 \approx \frac{3 \alpha m_Z^2}{4 \pi} \left( \frac{\Lambda_2(t, m_Z)}{t} + \frac{\Lambda_2(u, m_Z)}{u} \right). \]  
(29)

In HRC, we can simplify the result by using series expansion of \( \Lambda_2 \) at small \( t \):
\[ \Lambda_2(t, m_Z) = -\frac{t}{3m_Z} \left( 2 \log \frac{-t}{m_Z} - \frac{23}{6} \right) + O \left( \frac{t}{m_Z^2} \right), \]  
(30)

which gives
\[ \delta_\Lambda_2 \approx \frac{\alpha}{6 \pi} \left( \log \frac{t}{m_Z^2} - \frac{23}{6} \right). \]  
(31)

The numerical value obtained from above at \( E_{\text{lab}} = 11 \text{ GeV} \) and \( \theta = 90^\circ \) gives \( \delta_\Lambda_2 \approx 0.0125 \), which is in agreement with the exact (semi-automatic) numerical calculations.

The \( \Lambda_3 \) contribution, which represents the triangle diagrams with a 3-boson vertex, \( WW\gamma \) or \( WWZ \), is calculated in a similar way, so we present only the final result:
\[ \delta_\Lambda_3 \approx - \frac{3 \alpha m_Z^2}{32 \pi s W^2 v^2} \left( \frac{\Lambda_3(t, m_W)}{t} + \frac{\Lambda_3(u, m_W)}{u} \right). \]  
(32)

After simplifications and series expansion of \( \Lambda_3 \) at small \( t \),
\[ \Lambda_3(t, m_W) = -\frac{5t}{27m_W^2} + O \left( \frac{t}{m_W^2} \right), \]  
(33)

we find
\[ \delta_\Lambda_3 \approx \frac{5}{9 \pi} \left( \frac{1}{1 - 4 s_W^2} \right). \]  
(34)

Using Eq. (34) for \( E_{\text{lab}} = 11 \text{ GeV} \) and \( \theta = 90^\circ \), we obtain \( \delta_\Lambda_3 \approx 0.0118 \). Again, this approximate value calculated “by hand” is in a very good agreement with the exact result obtained with our second, computer-based approach.

The box part is UV-finite and does not require the renormalization procedure. We divide the box contribution into QED (\( \gamma \gamma \) - and \( \gamma Z \)-boxes) and a heavy-box part (HB = ZZ + WW):
\[ \sigma_{\text{Box}} = \sigma_{\text{QED}} + \sigma_{\text{HB}}. \]  
(35)

The types of boxes are shown in Figs. 2d, 2e. The IR-divergent QED-part of boxes (the first term in Eq. (35)) is described in detail both analytically and numerically in [11]. For the purely-weak part of the boxes (the second term), the equations are derived in the low-energy approximation.

The total weak correction to \( A_{\text{LR}} \) includes the HB cross section:
\[ \sigma_{\text{HB}} = \frac{\alpha}{s} \sum_{k = \gamma, Z} \left( B_{\text{ZZ}}^k + B_{\text{WW}}^k \right) + (t \leftrightarrow u), \]  
(36)
The Born asymmetry $A_{LR}^0$ and the structure of relative weak corrections to it for $E_{\text{lab}} = 11$ GeV at different $\theta$

| $\theta$ | 20   | 30   | 40   | 50   | 60   | 70   | 80   | 90   |
|---------|------|------|------|------|------|------|------|------|
| $A_{LR}^0$, ppb | 6.63 | 15.19 | 27.45 | 43.05 | 60.69 | 77.68 | 90.28 | 94.97 |
| $\gamma\gamma$-SE, DRC | -0.0043 | -0.0049 | -0.0054 | -0.0058 | -0.0062 | -0.0064 | -0.0066 | -0.0067 |
| $\gamma\gamma$-SE, HRC | -0.0043 | -0.0049 | -0.0054 | -0.0058 | -0.0062 | -0.0064 | -0.0066 | -0.0067 |
| $\gamma Z$-SE, DRC | -0.2919 | -0.2916 | -0.2914 | -0.2912 | -0.2911 | -0.2910 | -0.2909 | -0.2909 |
| $\gamma Z$-SE, HRC | -0.6051 | -0.6043 | -0.6042 | -0.6038 | -0.6034 | -0.6031 | -0.6028 | -0.6028 |
| ZZ-SE, DRC | -0.0105 | -0.0105 | -0.0105 | -0.0105 | -0.0105 | -0.0105 | -0.0105 | -0.0105 |
| ZZ-SE, HRC | 0.0309 | 0.0309 | 0.0309 | 0.0309 | 0.0309 | 0.0309 | 0.0309 | 0.0309 |
| HV, DRC | -0.2946 | -0.2633 | -0.2727 | -0.2703 | -0.2714 | -0.2712 | -0.2711 | -0.2711 |
| HV, HRC | -0.0015 | -0.0012 | -0.0010 | -0.0009 | -0.0008 | -0.0007 | -0.0007 | -0.0007 |
| ZZ-box, approx. | -0.013 | -0.013 | -0.013 | -0.013 | -0.013 | -0.013 | -0.013 | -0.013 |
| ZZ-box, exact | 0.0238 | 0.0238 | 0.0238 | 0.0239 | 0.0239 | 0.0238 | 0.0238 | 0.0238 |
| WW-box, approx. | 0.0238 | 0.0238 | 0.0238 | 0.0238 | 0.0238 | 0.0238 | 0.0238 | 0.0238 |
| WW-box, exact | 0.0238 | 0.0238 | 0.0238 | 0.0238 | 0.0238 | 0.0238 | 0.0238 | 0.0238 |
| total weak, DRC, exact | -0.5643 | -0.5430 | -0.5508 | -0.5489 | -0.5500 | -0.5495 | -0.5493 | -0.5493 |
| total weak, HRC, approx. | -0.5526 | -0.5514 | -0.5511 | -0.5505 | -0.5500 | -0.5496 | -0.5493 | -0.5493 |

where the expressions for $B_{ij}^k$ take a form

$$B_{ij}^k = D^k \lambda_\gamma \delta_{ij}^1 + (D^k + D^k) \lambda_\gamma \delta_{ij}^2,$$

$$B_{ww}^k = D^k \lambda_{\gamma\gamma} \delta_{ww}^1 + (D^k + D^k) \lambda_{\gamma\gamma} \delta_{ww}^2.$$  (37)

The combinations of the coupling constants are given in Eq. (5). Let us recall the coupling constants for the heavy boxes:

$$\nu^b = (\nu^Z)^2 + (a^Z)^2, \quad a^b = 2 \nu^Z a^Z,$$

$$\nu^c = a^c = 1/(4s_w^2).$$  (38)

At $s, |t|, |u| \ll m_Z^2$, the corrections $\delta_{(ij)}$ have a form:

$$\delta_{zz}^1 = \frac{3u^2}{2m_Z^2}, \quad \delta_{zz}^2 = -\frac{3s^2}{2m_Z^2},$$

$$\delta_{ww}^1 = \frac{2u^2}{m_w^2}, \quad \delta_{ww}^2 = \frac{s^2}{2m_w^2}.$$  (39)

At last, after simplification at small $t$, for the relative corrections to PV asymmetry coming from heavy boxes we find:

$$\delta_A^Z \approx -\frac{3}{2} |\bar{\nu}|, \quad \delta_A^W \approx \frac{\alpha}{4\pi s_w^2(1 - 4s_w^2)}.$$  (40)

The numerical values obtained from the equations above at $E_{\text{lab}} = 11$ GeV and $\theta = 90^\circ$ give $\delta_A^Z \approx -0.0013$ and $\delta_A^W \approx 0.0238$, which is once again in good agreement with the exact results evaluated with help of the FeynArts, FormCalc, LoopTools and Form program packages.

4.1.3. Numerical analysis on EWC to PV asymmetry. In the table below, we present the contributions to relative weak corrections calculated using two different approaches. In the first approach, we use approximate and compact expressions derived “by hand” with the application of HRC. In the second approach, we use computer-based analytical (FeynArts, FormCalc, and FORM) and when numerical (LoopTools) calculations, with DRC.

Table 1 demonstrates that the $\gamma\gamma$-SE contribution is small, non-additive and, as expected, is the same whether obtained in HRC or DRC. The $\gamma Z$-SE, ZZ-SE and HV contributions are rather sizeable, are all additive, and are different for HRC and DRC. The ZZ-box contribution is small, and the $WW$-box is dominant for the weak box correction. Both the ZZ-box and $WW$-box are additive and their sum is in excellent agreement regardless the method of calculations. The total relative weak correction is significant and in excellent agreement between the different methods. That confirms that we are dealing with a gauge-invariant set of graphs. The discrepancy between the two approaches is $\approx 0.0001$ at $\theta = 90^\circ$, but becomes larger with decreasing $\theta$.

In Fig. 6 we can see the relative weak corrections shown by solid line for DRC (exact) and dotted line for HRC (approximate). The dashed line shows the QED correction obtained by including soft bremsstrahlung to the Born asymmetry $A_{LR}^0$. We can see that for low energy region $1 < \sqrt{s} < 30$ GeV the results calculated
by the two methods are in excellent agreement. It is worth mentioning here that the semi-automated numerical calculations of boxes in the region of $\sqrt{s} \ll 1$ GeV suffer from the numerical instability due to Landau singularities. As for our approximated calculations, we have used the small-energy approximation with the expansion parameters taken as $r/f_0$ for energies $\sqrt{s} < 30$ GeV. In any case, for the 11 GeV relevant for the planned JLab experiment, the consistency of our calculations in both approaches is obvious, with a difference of $\sim 0.01\%$ or less. The dotted line for $\sqrt{s} > 500$ GeV on the Fig. 6 is obtained using HRC with the help of equations from [31], which used the high-energy approximation. We can see good agreement between our results for the high-energy region $\sqrt{s} > 500$ GeV which becomes better with energy increase. For $\sqrt{s} \geq 50$ GeV we have excellent agreement with the result of [27] if we use their SM parameters (see [11]). Furthermore, the relative QED correction (see Eq. 8 in [27] and dashed line in Fig. 6 here) is also in good qualitative and numerical agreement. In this case, we apply the same cut on the soft photon emission energy as in [27] ($\omega/\sqrt{s} = 0.05$). At the low-energy point corresponding to the E-158 experiment, and using our set of input parameters ($\alpha, m_W$ and $m_Z$) we find that $\delta^{\text{weak}} \approx -54\%$. If we translate our input parameters to the set $\alpha, G_F$ and $m_Z$ according to [27], we obtain good agreement with the result of [32].

4.2. Constrained Differential Renormalization

The CDR (Constrained Differential Renormalization) scheme, which provides renormalized expressions for Feynman graphs preserving the Ward identities, was introduced at the one-loop level in [33]. [34] expands on [33] to introduce the techniques for one-loop calculations in any renormalizable theory in four dimensions. The procedure has been implemented in FormCalc and LoopTools, which allows us to evaluate NLO EWC in CDR. Since our “scheme of choice” at the moment is on-shell, which is more suitable for calculating EWC beyond one-loop, we do not provide the same detailed analysis and step-by-step comparison between the two methods for CDR as we do for on-shell. The reason we evaluate NLO EWC in CDR is to obtain some indication of the size of the higher-order effects (NNLO and beyond) to see if there is enough motivation to do these very involved calculations in the future.

In Fig. 7, we can see the relative total correction to the unpolarized cross section versus $\sqrt{s}$ at $\theta = 90^\circ$. The filled circle corresponds to our predictions to the MOLLER experiment. Solid line corresponds to CDR and dotted line to on-shell RS.
between the on-shell and CDR results grows substantially as the weak correction becomes larger, it is clear that for an observable such as the PV asymmetry the difference between the on-shell and CDR schemes will be sizeable for the entire spectrum of energies \( \sqrt{s} < 2000 \text{ GeV} \). Because of that, we expect that the NNLO correction to the PV asymmetry may become important to PV precision physics in the future.

Figure 8 shows the relative weak (lower lines), and QED (upper lines) corrections to the Born asymmetry \( A_{LR}^0 \) versus \( \sqrt{s} \) at \( \theta = 90^\circ \). The difference is significant and is growing with increasing \( \sqrt{s} \). According to our calculations for \( E_{lab} = 11 \text{ GeV}, \omega = 0.05 \sqrt{s} \) and \( \theta = 90^\circ \), the total radiative correction to PV asymmetry is \(-69.8\%\) with on-shell and \(-58.5\%\) with CDR. The difference is not at all surprising. For E-158, for example, the one-loop weak corrections were found to be about \(-40\%\) in the \( \overline{MS} \) scheme [32] and about \(-50\%\) in the on-shell scheme [24, 28].

The physical, NLO-corrected asymmetries, computed in both on-shell and CDR schemes, are compared in Fig. 9. Here, for consistency with the \( \overline{MS} \) definition of the couplings to \( \mathcal{O}(\alpha) \) [35], we use \( \hat{s}_Z = \sin^2\hat{\theta}_W(M_Z) = 0.2313 [29] \) in the expression of the Born asymmetry. We find that the predictions for the physical PV asymmetry, computed to the same order in perturbation theory in two different schemes, differ by about 3%. The difference is an indication of the order of magnitude the higher-order, NNLO and beyond, terms.

The [28] estimated that the higher-order corrections are suppressed by \(-0.1\%\) relative to the one-loop result, possibly \(5\%\) in some cases, and thus are not significant source of uncertainty. However, we conclude that although the corrections at the NNLO level were not mandated by the previously achievable experimental precision, they may become important for the next generation of experiments.

5. EFFECT OF ADDITIONAL MASSIVE NEUTRAL BOSON

Let us now add a very simple NP assumption to our SM calculations and show how this NP contribution affects the observable asymmetry. The reason we want to do it in here is to investigate if the two complimentary methods we used in the previous sections, “by-hand” and semi-automated, can be applied in the NP domain. As we mention in the Introduction, FeynArts, FormCalc, LoopTools, and FORM are not “black box” programs and can be modified for specific projects, including adding the NP sector. As was already concluded in [36] and [37], the proposed MOLLER measurement could be influenced by radiative loop effects of new-physics particles. This type of calculation is out of scope of this paper, but we plan to provide a full estimation in our future publication. For
now, we assume that there is just one additional neutral boson (ANB), or $Z'$-boson, with the usual $V-A$ structure of interaction with fermions, vector(axial) coupling constants $\sqrt{s}(a^Z)$ and mass $m_{Z'}$. From the analysis done in the previous section, we can clearly see that in the low-energy region where $s, |t|, |u| \ll m_{Z}^2 < m_{Z'}^2$, contributions are mainly suppressed by propagator factors like $D^Z$. In this section, our goal is to analyze the contribution of $Z'$-Born and $ZZ'$-box diagrams to the observable scattering asymmetry for MOLLER experiment. The only significant contribution to the Born asymmetry comes from the interference terms from the $Z$ and photon diagrams. The relative correction to the Born asymmetry coming from $Z'$-boson is additive, and is given by

$$\delta^Z_A = \frac{\sqrt{s} \, \delta^2 \, m_{Z'}^2}{\sqrt{s} \, \delta^2 \, m_{Z}^2}. \quad (41)$$

According to [5], the goal of MOLLER is to measure the PV asymmetry to a precision of 2% (0.73 ppb). With this uncertainty, and assuming the identical coupling constants for $Z$ and $Z'$, it should be possible to detect ANB with a mass up to $m_{Z'} = \sqrt{m_{Z}^2/0.02} \approx 7 m_{Z}$. The sensitivity of MOLLER to $Z'$ increases if its parity-violating couplings are larger than those of $Z$, making the measurements of PV complementary to the direct searches at high energies.

The one-loop diagrams including ANB give significantly smaller contributions. As an example, let us consider $ZZ'$-box. As before, we perform our calculations by both approximate (“by-hand”) and exact (with FeynArts and FormCalc) methods, and get an excellent agreement. The expressions derived as a result of our approximate approach are presented below.

For the $ZZ'$-box contribution, the cross section can be expressed by the following short equation:

$$\sigma_{ZZ'-box} = \frac{3g^3}{s} \sum_{k=\gamma, Z} \left[ \lambda_{B_k}^{B_k} \left( D^k u \, u + D^k u \, l \right) \right] - 2 \lambda_{B_k} \left( D^k u \, u + D^k u \, l \right) + (Z \rightarrow Z'),$$ \quad (42)

where

$$L = \int_{0}^{1} \int_{0}^{1} dy \, m_{Z}^2 (1 - (1 - m_{Z}^2)(1 - z)) \approx \frac{1}{m_{Z} - m_{Z}^2} \log \frac{m_{Z}^2}{m_{Z}} \quad (43)$$

and the functions $\lambda_{B_k}$ are expressed through

$$\nu = \nu + \nu + \nu + \nu + \nu \nu \nu. \quad (44)$$

According to the methods described in Section 4.4.1, we can now calculate the relative correction to the observable asymmetry from the ANB contribution (i.e. from $ZZ'$-box) $\delta_{ANB}$ as:

$$\delta_{ANB} = \frac{6 \alpha m_{Z}^2 \nu \nu \nu \nu \nu \nu}{\pi \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \nu \n
6. CONCLUSIONS

In the presented work, we perform detailed calculations of the complete one-loop set of electroweak radiative corrections to the parity violating $e^- e^- \rightarrow e^- e^- (\gamma)$ scattering asymmetry both at low and high energies using the on-shell renormalization conditions proposed in [19] (see also [20]) and the conditions suggested in [21]. Although contributions from the self-energies and vertex diagrams calculated with the two sets of renormalization conditions differ significantly, our full gauge-invariant set still guarantees that the
total relative weak corrections are in excellent agreement for the two methods of calculation.

Obviously, it is important to exercise caution when comparing separate contributions arising from the different renormalization conditions unless these contributions form a gauge-invariant set (like boxes). Although this is a well-known fact in principle, it is still useful to demonstrate this in detail numerically for a specific example. We hope that our results illustrating the structure of relative weak corrections evaluated at different renormalization conditions will be of educational value to researchers working in this area.

In addition, we compare the asymptotic results obtained analytically, “by hand” (with HRC), with some approximations, and semi-automatically (with DRC), with no approximations required. As a result, we have a good agreement for the whole $0 < \sqrt{s} < 50$ GeV energy region. More specifically, for the kinematics relevant to the 11 GeV MOLLER experiment planned at JLab, our agreement between two approaches for the complete one-loop set of electroweak radiative corrections is better than 0.1%. We found no significant theoretical uncertainty coming from the largest possible source, the hadronic contributions to the vacuum polarization. The dependence on other uncertain input parameters, like the mass of the Higgs boson, is extremely weak and well below 0.1%. We conclude that the excellent agreement we obtained between the results calculated “by hand” and semi-automatically serves as a good illustration of opportunities offered by FeynArts, FormCalc, LoopTools, and FORM.

Considering the large size of the obtained radiative effects, it is obvious that the careful procedure for taking into account radiative correction is essential. Our plans include the construction of a Monte Carlo generator for the simulation of radiative events within Møller scattering to make our work directly useful to the experiment. Since we are now assured of the reliability of our calculations, we plan to base this Monte Carlo on the maximum-precision results from our semi-automatic approach.

Although making sure that the results obtained by two different approaches using two renormalization conditions are identical assures us that our NLO EWC calculations are error-free, it does not address the question of the size of NNLO corrections. The two-loop corrections are beyond the scope of this work, but we plan to address them in the future. One way to find some indication of the size of higher-order contributions is to compare physical observables computed to the same order in perturbation theory in different renormalization schemes. Our calculations in the on-shell and CDR schemes show that while the NLO terms differ by about 11%, the PV asymmetries differ by about 3%. At the level of precision of the future experiments such as MOLLER, higher-order corrections become important.

To see if the two complimentary approaches we successfully used for the SM calculations can be applied in the NP domain, we expanded FeynArts, FormCalc, LoopTools, and FORM to include an additional neutral boson ($Z$), calculated the relevant correction, and then obtained the same result by hand. Possible other contributions of new-physics particles to the Møller asymmetry still need to be investigated, and many of them can be included into the program packages mentioned above.

We believe that the future experiments at JLab and the ILC will mandate evaluation of the EWC beyond one loop. Once all the SM corrections are under control, it is worth considering NLO corrections including new-physics particles, starting with the Minimal Super Symmetric Model (MSSM). The most straightforward way to address these corrections is by employing the CDR scheme [33, 34] because the CDR approach can be easily expanded to MSSM. However, whether the CDR scheme will be applicable in evaluating the EWC at the NNLO level is still an open question. Our preliminary plan is to address the NNLO EWC with the on-shell scheme first, and if the effect is significant, stay with the same scheme for calculating contributions coming from the new-physics particles. The simple example of the ZZ-box we consider in Section 5 is gauge-invariant and is thus not affected by the choice of renormalization, but we have to be careful when choosing the scheme for our future work. Any suggestions from the community regarding the best approach to this task would be greatly appreciated.

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