Simple modelling of S-type NbO$_x$ locally active memristor

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Nanoscale S-type NbO$_x$ locally active memristors (LAMs) open up new opportunities in the brain-inspired neuromorphic computing. Simple yet accurate models for these memristors can provide benefits for designing related circuits and systems. Considering that the DC voltage–current plot of the NbO$_x$ LAM under current sweeping is composed of three regions, that is, high resistance region, negative differential resistance region, and low resistance region, a three-segment piecewise-linear method is applied to fit these three regions. Based on this developed relation of the voltage and current at DC, a simple model for the NbO$_x$ LAM is proposed. The parameters of the proposed model can be easily identified in terms of the quasi-static and dynamic electrical characteristics. A series of numerical simulations corroborate that the proposed model can accurately emulate the quasi-static voltage–current characteristics and oscillating behaviours of the NbO$_x$ LAM.

Introduction: The locally active memristor (LAM) refers to a memristor that exhibits negative differential resistance (NDR) for some values of voltage or current [1]. The LAM has numerous applications, such as periodic and chaotic oscillators [2–3], threshold logic devices [4], as well as biological neuronal dynamics [5].

The LAM can be classified into two categories: S-type (current-controlled) LAM and N-type (voltage-controlled) LAM, whose DC $V$–$I$ characteristics exhibit S-type and N-type NDR behaviours, respectively. The particular types of physical memristors, such as NbO$_x$, VO$_2$, and TaO$_x$ devices, belong to the S-type LAM [5-7]. As for the NbO$_x$ LAM, the NDR behaviour is caused by Joule heating originating from the temperature dependence of the Poole–Frenkel conduction mechanism [6]. The above finding leads to a physics-based model which can accurately mimic the electrical conduction that exhibits the S-type NDR in NbO$_x$ devices. However, the complex non-linearities in the physical model usually render mathematical analysis intractable.

In order to simplify the mathematical model and facilitate the research of the LAM system, a simple mathematical model for the S-type LAM was proposed in [8], but it failed to accurately match the $V$–$I$ characteristic of a specific physical memristor. Moreover, an efficient perturbation projection vector (PPV) modelling for the NbO$_x$ LAM-based oscillator was designed in [9], where the model method is only suitable for this peculiar oscillation circuit and its corresponding oscillatory network. Considering the limitation of the reported NbO$_x$ LAM models, we intend to design a simple accurate model for easy analysis.

In this work, a highly accurate physical model [6] and simple S-type NbO$_x$ LAM circuits are first given to analyse the memristor electrical characteristics. h.s is because the NbO$_x$ LAM has not been commercial. By three-segment piecewise-linear approximation of the DC $V$–$I$ curve, a simple modelling method for the S-type NbO$_x$ LAM is proposed. For verifying the accuracy of the proposed model, a comparison between the physical model-based simulation results and the proposed model estimates is discussed. Finally, a brief conclusion is drawn.

Physical model and memristor circuits: In [6], the physical model of the NbO$_x$ LAM is described as:

$$\begin{align*}
v &= R_{m0} i + R_{m0} \left( e^{v / q E_a} - 1 \right), \\
\frac{dT}{dt} &= \frac{R_m i^2 - T_m - T_{amb}}{C_{th}} - \frac{T_m - T_{amb}}{R_m C_{th}} \quad (1)
\end{align*}$$

Table 1. Configuration parameters for the physical model

| Parameter | Value |
|-----------|-------|
| $C_{th}$  | $2.5 \times 10^{-14}$ J·K$^{-1}$ |
| $E_a$     | 0.204 eV |
| $R_m$     | 47 Ω   |
| $R_{m0}$  | $1.5 \times 10^9$ K·W$^{-1}$ |
| $q$       | $1.6 \times 10^{-19}$ C |
| $t_{amb}$ | 19 nm  |
| $k$       | $1.38 \times 10^{-21}$ J·K$^{-1}$ |
| $\epsilon_0$ | 8.85 $\times 10^{-12}$ F·m$^{-1}$ |
| $\gamma$  | 45     |

Fig. 1 Three distinct NbO$_x$ LAM-based circuits: (a) Current excitation directly applied into the NbO$_x$ LAM, (b) voltage excitation directly applied into the NbO$_x$ LAM, (c) second-order NbO$_x$ LAM-based oscillator

Fig. 2 Quasi-static $V$–$I$ curves obtained by sweeping current and voltage

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where $v$ and $i$ are the voltage and current of the memristor; $R_{m0}$ is the memristance function; internal temperature $T_m$ denotes the state variable; $k$ is the Boltzmann constant; $E_a$ denotes the activation energy; $q$ is the electron charge; $E = v / t_{amb}$ describes the electric field in the device and $t_{amb}$ is the thickness of the threshold switching volume; $\epsilon_0$ and $\gamma$ are the vacuum permittivity and relative permittivity of the conducting filament; $R_0$ is a fitting parameter; $C_{th}$ and $R_m$ are the thermal capacitance and thermal resistance; $T_{amb}$ is the ambient temperature, i.e. 296.15 K.

The configuration parameters in [6] are used as a case study, as shown in Table 1.

Figure 1 shows three distinct NbO$_x$ LAM-based circuits. As for Figures 1a and 1b, the memristor is directly excited by a current source and a voltage source, respectively. Figure 1c depicts a simple NbO$_x$ LAM-based oscillator, where the DC voltage supply $V_1$ connected with a linear resistor $R$ can provide an appropriate current biasing for the memristor, and the capacitor $C$ implements the energy exchange with the memristor.

When the excitation waveforms in Figures 1a and 1b are slow-varying non-negative sinusoidal current signal ($f = 10$ Hz, $I_{pp} = 14$ mA) and voltage signal ($f = 10$ Hz, $V_{pp} = 1.1$ V), the resulted quasi-static $V$–$I$ curves of the memristor defined by (1) are shown in Figure 2, where the blue and red curves are obtained by sweeping current and voltage.

It can be seen from Figure 2 that there exists a difference between the two curves. As for the red one, it performs a threshold switching with the threshold voltage $V_1$ and its hold voltage is $V_2$. Besides, the memristance switches between the high-resistance state and the low-resistance state back and forth. Whereas, the blue one exhibits the S-type NDR characteristic and it can be separated into three parts: high resistance region OA, NDR region AB, and low resistance region BC. Observe that $I_1 = 0.78$ mA, $I_2 = 4.80$ mA, $V_1 = 1.04$ V, and $V_2 = 0.96$ V. Note that
at extreme-low frequency, the quasi-static $V-I$ plot of the S-type LAM under current testing is same with its DC $V-I$ plot [8].

Due to the NDR behaviour, the NbO$_2$ LAM-based circuit in Figure 1c can give rise to sustained oscillation [9]. This simple oscillator is a fundamental building block for brain-inspired neuromorphic computing, and the oscillating behaviours generated by the oscillator are used to carry out the parameter identification of the model in the following section.

**Simple modelling of the S-type NbO$_2$ LAM:** The S-type NDR which existed in the DC $V-I$ curve is the key and origin for the generation of oscillation, where the parameters $I_1, I_2, V_1$, and $V_2$ determine the window of oscillation. Hence, the DC $V-I$ characteristic is critical for the design of the S-type NbO$_2$ LAM-based oscillators.

Based on the aforementioned theoretical analysis, for simplicity, the DC $V-I$ plot of the NbO$_2$ LAM under current testing is approximated by three linear segments OA, AB, and BC, as depicted in Figure 3a, where the three linear segments have slopes:

- **Region OA:** $G_{oa} = 1/R_{oa}$ for $0 \leq I < I_1$ (2)
- **Region AB:** $G_{ab} = 1/R_{ab}$ for $I_1 \leq I \leq I_2$ (3)
- **Region BC:** $G_{bc} = 1/R_{bc}$ for $I_2 < I$ (4)

where $R_{oa}$, $R_{ab}$, and $R_{bc}$ denote the linearized high resistance, NDR, and low resistance, respectively. Because the response voltage is a single-valued function of the DC excitation current for the S-type LAM, the simple oscillator is based on the aforementioned theoretical analysis, for simplicity, the piecewise-linear DC $V-I$ plot is depicted in Figure 3b for illustrating the model, where the voltage and current axes are interchanged.

Figure 3 reveals that the curve is uniquely specified by five independent parameters: $I_1$, $I_2$, $V_1$, $V_2$, and $R_{oa}$. Note that $R_{oa} = V_1/I_1$ and $R_{bc} = (V_2 - V_1)/(I_2 - I_1)$. By using a three-segment piecewise-linear method, the following expression is used to mimic the DC $V-I$ characteristic of the S-type LAM

$$V = m + nl + k_1 |I - I_1| + k_2 |I - I_2| = g(I)$$

where $m$, $n$, $k_1$, and $k_2$ are fitting parameters.

In terms of the relation of voltage and current in the three distinct regions, (5) can be recast into the equations without absolute value signs as follows:

- **Region OA:** $g_{oa}(I) = (n - k_2)I + m + k_1I_1 + k_2I_2$ (6)
- **Region AB:** $g_{ab}(I) = (n + k_1 - k_2)I + m - k_1I_1 + k_2I_2$ (7)
- **Region BC:** $g_{bc}(I) = (n + k_1 + k_2)I + m - k_1I_1 - k_2I_2$ (8)

The voltage–current characteristics in the three linear segments OA, AB, and BC of Figure 3b can be described as:

$$V = g_{oa}(I) = R_{oa}I = (V_1/I_1)I$$

$$V = g_{ab}(I) = R_{ab}I = (V_2 - V_1)/(I_1 - I_2)$$

$$V = g_{bc}(I) = R_{bc}I = (V_2 - R_{bc}I_2)$$

By combining (6)–(8) with (9)–(11), one can obtain

$$n = 0.5 \left[ (V_1/I_1 + R_{oa}) \right], \quad k_1 = 0.5 \left[ (V_1 - V_2)/(I_1 - I_2) - V_1/I_1 \right]$$

$$k_2 = 0.5 \left[ R_{bc} - (V_2 - V_1)/(I_1 - I_2) \right], \quad m = -k_1I_1 - k_2I_2$$

Consequently, the values of the parameters $m$, $n$, $k_1$, and $k_2$ can be calculated based on $I_1$, $I_2$, $V_1$, $V_2$, and $R_{oa}$. Furthermore, the values of $I_1$, $I_2$, $V_1$, $V_2$, and the approximated $R_{bc}$ can be read based on the blue or red curve in Figure 2. In order to improve the accuracy, $R_{oa}$ will be further optimized in the subsequent analysis. As voltage excitation source is more easily available than current source in practical, the circuit topology in Figure 1b is adopted to obtain the quasi-static $V-I$ characteristic of the memristor and further identify the parameters in (5).

Considering that the S-type NbO$_2$ LAM is current-controlled and not an ideal memristor, a generic current-controlled memristor model is applied to describe it, namely,

$$\begin{align*}
  v &= R_{oa}(x)i \\
  \frac{dx}{dt} &= f(x, i)
\end{align*}$$

where $x$ is the state variable and $f$ expresses the formula for the time derivative of $x$.

As the memristance $R_{oa}$ only relies on $x$, a simple formula is

$$R_{oa}(x) = n + x$$

By setting the time derivative of the state $dc/dt$ to zero, the static $X$ for different DC input currents $I$ can be derived, and then the relation of voltage and current at DC can be obtained. In order to satisfy the piecewise-linear DC voltage–current characteristic in (5), the expression of $X$ should hold

$$X = (m + k_1 |I - I_1| + k_2 |I - I_2|)/I$$

Based on (15), the state equation can be written as:

$$\frac{dx}{dt} = \beta (m + k_1 |I - I_1| + k_2 |I - I_2| - x)$$

where $\beta$ is a critical parameter whose value reflects the rate of change of the variable $x$. After substituting (16) and (14) into (13), the calculated voltage-current relation at DC is equal to (5), which satisfies the design requirement.

Accordingly, a simple three-segment piecewise-linear model of the S-type NbO$_2$ LAM is represented as

$$\begin{align*}
  v &= (n + x)i \\
  \frac{dx}{dt} &= \beta (m + k_1 |I - I_1| + k_2 |I - I_2| - x)
\end{align*}$$

Equation (17) indicates that there are seven parameters in the proposed model of the NbO$_2$ LAM, which is simple in contrast with the existing models.

As the parameter $\beta$ determines the dynamic behaviours of the memristor, $\beta$ cannot be identified in terms of the quasi-static tests. Hence, dynamic characteristics of the memristor need to be recorded for parameter identification besides the quasi-static test. Here, the parameter $\beta$ and $R_{oa}$ are tuned through an optimization algorithm operating on test basis on the circuit in Figure 1c, where the circuit parameter configuration is $V_s = 5 \text{ V}, R = 1 \text{ k} \Omega$, and $C = 1 \text{ nF}$. As the oscillating frequency is a critical indicator for the oscillator design, the task regarded an error minimization of the predicted oscillating frequency. Here, the error is described via the relative root mean quadratic error (RRMQE) and is defined as:

$$e_f = \sqrt{\frac{(f_{sim} - f_{ref})^2}{f_{ref}^2}}$$

where $f_{sim}$ and $f_{ref}$ denote the oscillating frequency predicted by the proposed model and physical model of the NbO$_2$ LAM.
The optimization process achieves a proper minimization of the task function with $\beta = 1.65 \times 10^{11}$ and $R_m = 13.8 \Omega$. Based on the above optimization results and (12), the parameters in (17) are $I_1 = 0.78 \text{mA}$, $I_2 = 4.8 \text{mA}$, $m = 0.447$, $n = 672.3$, $k_1 = -674.9$, and $k_2 = 16.45$. The above analysis reveals that the parameter identification of the proposed simplified model of the memristor can be implemented based on the quasi-static and dynamic tests on the circuits in Figure 1b,c.

Modelling verification: In order to verify the practicability and accuracy of the proposed modelling method, a comparison between the physical model-based simulation results and the proposed model estimates regarding the circuits in Figure 1a–c. Here, the RRMQE for the variable $y$ is used to describe the error, i.e.,

$$e_y = \frac{1}{N} \sqrt{\sum_{k=1}^{N} (y_{\text{sim}}(k) - y_{\text{phy}}(k))^2}$$  \hspace{1cm} (19)

where $y_{\text{sim}}(k)$ and $y_{\text{phy}}(k)$ denote the $k_{th}$ sample data involving the memristor voltage and current predicted by the proposed model and the physical model, $\bar{y}$ denotes the time average operator. $N$ refers to the number of samples. Each memristor is characterized by the models in (1) and (17). All simulation calculations are conducted in the software LTSpice.

When the applied excitation signal in Figure 1a is $i(t) = (6.8 \sin(2\pi \times 10t) + 6.8) \text{mA}$, the blue and red curves in Figure 4a show the quasi-static $V$–$I$-plots predicted by the physical model and the proposed model, respectively. The physical model-predicted quasi-static $V$–$I$ characteristic (blue curve) and the proposed model-predicted one (red curve) under the same voltage sweeping is depicted in Figure 4b, where $v(t) = (0.55 \sin(2\pi \times 10t) + 0.55) \text{V}$. Observe that under the voltage and current sweepings, the proposed model-predicted quasi-static $V$–$I$ plots can match well with the physical model-based simulation results.

As for the simple memristor oscillator in Figure 1c, the simulation results predicted by the physical model (blue curves) and the proposed model (red curves) are depicted in Figure 5, where $V_{x} = 5 \text{V}$, $R = 1 \text{k}\Omega$, and $C = 1 \text{nF}$. The time-domain waveforms of the memristor voltage and current $i$ are shown in Figure 5a. Then the stable steady-state waveforms in plot a are further plotted in the $v$–$i$ plane in Figure 5b, where the arrowheads point out the motion direction of the trajectories. In this case, the operating points of the physical model and the proposed model-based memristor are $Q_1 (0.96 \text{V}, 4.04 \text{mA})$ and $Q_2 (0.98 \text{V}, 4.02 \text{mA})$ on the $v$–$i$ plane, which are located in the NDR region of Figure 4a and the oscillation may occur. As expected, the trajectories in Figure 5b exhibit limit cycles, which revolve around $Q_1$ and $Q_2$.

The RRMQEs of this work in terms of the quasi-static (threshold switching and S-type NDR) behaviours and dynamic behaviours are recorded in Table 2. These inevitable errors are caused by the non-linearity of each region, but they are within an allowable error range. Hence, the proposed model can reproduce the threshold switching and locally active (i.e. NDR) behaviours of the NbO$_2$ LAM, as well as the oscillating behaviours of the NbO$_2$, LAM-based circuit.

Conclusion: A simple model with seven parameters for the S-type NbO$_2$ LAM is proposed. The errors between the physical model-based simulation results and proposed model estimates regarding the three distinct circuits are within a maximum allowable RRMQE (i.e. $10^{-2}$), which manifests the accuracy of the proposed model. In contrast with the existing modelling methods, it has the advantages of simple expressions and high accuracy, which provide benefits for the research of the memristive system.

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### Table 2. RRMQEs of the proposed model

| Behaviour of the memristor | RRMQEs |
|----------------------------|--------|
| Threshold switching behaviour | $2.83 \times 10^{-4}$ |
| S-type NDR behaviour | $1.15 \times 10^{-4}$ |
| Dynamic behaviours of the oscillator | $2.01 \times 10^{-3}$ for current $1.62 \times 10^{-4}$ for voltage |