The unit of cubic matrices in modeling of micro-relief

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Abstract. The apparatus of cubic matrices allows us to give a classification of the contiguous cuboid. Invariant theory is used for this purpose. A cubic matrix is a generalization of the concept of a square or rectangular matrix. Increasing the dimension of a square matrix by one requires a new definition of the product of three-dimensional matrices. It is necessary to search for new invariants for the operator of parallel transfer of the coordinate system and the operator of rotation around the origin. The contiguous cuboid allows us to give a three-dimensional geometric model of the microrelief an order of magnitude more accurately than the contiguous paraboloid does. The geometrical structure of the contiguous cuboid is investigated by the method of sections. The curvature of the surface at the point of contact with the cuboid is estimated by the curvature tensor, which is analogous to the Riemann-Christoffel tensor. The curvature of the surface in the local area of the point of contact is estimated with certain assumptions. There is no more precise geometric object that defines the curvature of the surface.

1. Introduction

The final formation of the microrelief of the surface of parts occurs at the stage of finishing processing operations according to the regulated parameters. Regulation of parameters is due to the need to match the formed microrelief to the type of wear of the functional surface of the part during its operation. The need to take into account the three-dimensional characteristics of the microrelief is particularly relevant in the manufacture of parts, to the performance properties of which, in tribo-conjugations, high requirements are imposed. It is known, for example, that wear resistance, fatigue strength and other operational properties are largely determined by the shape of the surface microrelief, namely, the curvature of the peaks and troughs of micro-roughness. The part processing process is a single closed structure. One of the results of this process is the formed topography of the microrelief of the functional surface of the part.

Currently, each of the CAD models that describe a separate side of the part processing process is based on the provisions of the scientific discipline that directly studies the phenomena under consideration. There is no General approach in the analytical description of the microrelief formation process. The calculation of the forming surface of the tool does not calculate the topography of the microrelief. This is due to the lack
of sufficient information about the geometric structure of the microrelief as a three-dimensional image. For example: the method of statistical description of abrasive surfaces using Markov chain theory allows you to build only a two-dimensional model that includes cutting cycles along an idealized line in the cutting direction. This does not allow us to take into account the shape of the cutting edges in the direction perpendicular to the cutting speed vector and their position in relation to subsequent edges, which is necessary when explaining the process of removing material.

The fractal method is not quite suitable for geometric modeling of a microrelief, since the curvature of the local area for a fractal - "thick surface" is not equal to the Gauss curvature, which is calculated using the main curvatures. This is because for a fractal surface that has a tangent plane at a given point, there are no partial derivatives at the following points. In other methods: splines, Legendre, Koons, etc. For surface approximation and interpolation, use segments: a triangle or a square. The surface curvature of these segments, if we consider them to be local regions of a given point, is different from the natural curvature of the surface. Therefore, the accuracy of these methods in the analytical description of the microrelief and subsequent visualization of spatial scenes is not sufficient to represent the process of forming a microrelief as a single system with all its elements [1].

In order to improve the accuracy of the surface microgeometry estimation, you should additionally introduce geometric characteristics directly related to the surface curvature. None of the used geometrical models of the micro-relief does not contain similar geometric characteristics. For example: in one-dimensional models, the height of micro-dimensions is used as the main geometric characteristic. In two-dimensional and three-dimensional models, characteristics that do not sufficiently contain information about the curvature of the local section. Therefore, to date, there are no complete and well-founded three-dimensional geometric models of microrelief in the analytical description of the forming and processed surface. Thus, the development of an approach to modeling the process of surface microrelief formation, which allows determining a system of geometric characteristics containing sufficiently complete information for evaluating the microrelief, is relevant [2].

2. Modular-geometric approach to surface microrelief modeling

2.1. Theoretical justification of the module and a geometric modeling method of the microrelief

Classification of surfaces of complex shape from a geometric point of view cannot have a scientific basis. There are no General features in the surface structure [3]. The surface of a complex shape is structured on the basis of the modular principle, the method of structuring is determined by the problems of the theory of shape formation. The modular-geometric method used to solve these problems consists in approximating a local section of the surface by a contiguous paraboloid. The Riemann-Christoffel tensor is a geometric characteristic for estimating the curvature of a local area. The Analytical definition of a contiguous paraboloid as a second-order geometric image of contact with a given local area of the surface is determined from the Taylor series expansion. The Taylor series defines the geometric images of a higher order of contact: coboloid, quadronoid etc.

In general, the curvature of the surface at the point of contact is estimated by the angle of rotation of the vector transferred parallel to itself along a closed contour covering the point of contact and belonging to its local region, on the touching surface: paraboloid, cuboloid, etc.

The order of accuracy of determining the covariant differential that characterizes the change in the coordinates of the vector does not allow calculating the curvature of the quadroloid. The geometric structure of coboloid not investigated. In technical applications, we should limit ourselves to approximating the local area by a contiguous paraboloid, since we can postulate that the curvature of the surface at the point of contact is equal to the curvature of the contiguous paraboloid. In general, a discrete-defined frame surface can be approximated by a set of modules having a smooth "cross-linking", each of which is a contiguous
paraboloid of a certain type [4]. A frame discrete-defined surface and a surface with numerical marks belong to the same class of complex-shaped surface, the class of discrete-defined surface. The surface microrelief can be considered a surface with numerical marks. The modular principle used for describing the geometry of the frame.

2.2. Three-dimensional geometric model of a microrelief

Let us consider a method for constructing a three-dimensional geometric model of a surface microrelief by "gluing" (cross – linking) elementary curves of sites-infinities of squares or quadrilaterals, selected using special coordinate networks, in three-dimensional space.

The main task of constructing a three-dimensional geometric model of a surface with numerical marks is to calculate the parameters of a contiguous paraboloid in a local semi-geodesic coordinate system from the Gauss and Peterson – Codazzi equations [5]. A three-dimensional geometric model is a set of discrete surface sections of a certain type (plane, paraboloid, etc.) that approximate each local surface section with numerical marks.

The difference between a three-dimensional geometric model of a surface with numerical marks and other mathematical models of a complex surface is that each local section is reproduced with a higher degree of accuracy, since its curvature is preserved. In other well-known models, the curvature is treated as normal, i.e. the curvature of the curve obtained in the cross section of the surface by a plane perpendicular to the tangent plane and containing a given direction.

In a three-dimensional geometric model of a surface with numerical marks, the curvature is determined by the essential component $R_{1212}$ of the Riemann-Christoffel tensor, which gives the angle of rotation when an arbitrary vector is enclosed parallel to itself along a closed contour in the local region of a given point on the surface. The $R_{1212}$ component is related via the generalized Peterson-Codazzi equation to the parameters of a contiguous paraboloid approximating a local section of the surface with numerical marks [6].

To improve the accuracy of the approximation, you can choose a tensor associated with the parameters of the surface of a higher order of contact with the given local area: cuboid, quadrilateral..., n-loid. The surfaces that touch at this point with the surface with numerical marks, starting and ending with the n-loid, form a geometric series [7].

Thus, the problem of approximating a local area of the surface with numerical marks is related to the study of a geometric series in the theory of contact by tensor analysis methods. The paraboloid as a second-order surface was studied by methods of analytical geometry: sections and invariants. The importance of constructing a classification of local modules of a surface with numerical marks based on members of a geometric series of higher order than the second, and studying the geometric properties of these members is as follows.

First, as noted in the paper, due to the large variety of complex-shaped surfaces and methods for obtaining them, it is not possible to create a strict system for classifying complex-shaped surfaces at the macro level. Second, the classification of local surface modules with numerical marks allows you to recreate the geometry of the surface as a whole. Third, for modeling, you can choose as reference elements not a point or line, as it was done traditionally, but a surface defined analytically, which increases the accuracy of approximation according to the proposed calculation method. Fourth, the use of a geometric series as a universal tool makes it possible to describe both macro geometry and micro geometry of the surface. Thus, the geometric series that can be used as the basis for the classification of local surface modules is bounded by the third term. Geometric series – a series of geometric objects associated with the surface structure with numeric marks.

Classification of surface modules with numerical marks based on a geometric series is shown in table 1:
Table 1. Classification of surface modules with numerical marks based on a geometric series.

| Member number of the geometric series | Curvature tensor of a complex shape surface | Name of the approximating geometric object | Given the equation of the surface |
|---------------------------------------|---------------------------------------------|--------------------------------------------|----------------------------------|
| 1                                     | $R_{1212}=0$                                 | Tangent plane                              | $a_{144}x+a_{244}y+a_{344}z+a_{444}=0,$ $z=0$ |
| 2                                     | $R_{1212}>0$                                 | Elliptical paraboloid                       | $a_{114}x^2+a_{224}y^2+a_{324}z=0$ |
|                                       |                                             | Hyperbolic paraboloid                       | $a_{114}x^2+a_{224}y=0$ |
|                                       |                                             | Parabolic cylinder                          | $a_{111}x^3+3a_{114}x^2+a_{344}=0$ |
| 3                                     | $R_{1212}^*=0$                               | Soprikasalis-Xia coboloid                  | $a_{111}x^3+3a_{114}x^2+3a_{244}y^2+3a_{344}=0$ |
|                                       |                                             | Parabolic coboloid divergent                | $a_{111}x^3+3a_{114}x^2+3a_{244}y^2+3a_{344}=0$ |
|                                       |                                             | Tresurey coboloid                           | $a_{111}x^3+3a_{114}x^2+6a_{124}xy+3a_{344}z=0$ |

Here $R_{1212}$ and $R_{1212}^*$ in the table are the essential components, respectively, of the Riemann-kristoffel tensor and the analog of the Riemann-kristoffel tensor, which determines the curvature of the touching cuboloid at the point of contact with the surface with numerical marks. The tensor $R_{ijkl}$, where $i, j, k, l = 1, 2$ is defined similarly to the tensor $R_{ijkl}$, where $i, j, k, l = 1, 2$.

At the third term, the geometric series breaks off, this is due to the fact that it is impossible to determine the analogous Riemann – Christoffel tensor, which gives an idea of the curvature of the fourth term of the geometric series, the quadrirloid, at the point of contact with the surface with numerical marks. The latter circumstance is the result of determining the covariant differential as the main linear part of the increment of the tensor of a given field when it is transferred in parallel from an infinitely close point to a given point, i.e., determining it up to the first-order differential.

3. Cuboid and cubic matrix

3.1. Determination of the type of third-order algebraic curves by invariants based on the apparatus of cubic matrices

Unlike the definition of multiplication of cubic matrices given in his books, from what we propose here is that the multiplication of cubic matrices of square (flat) is defined in this paper in line with the fact that in the future when building classification ACTP, as well as surfaces of the third order are considered only the group of rotations and parallel transport coordinate system from the groups of affine transformations, not taking into account, for example, the group of homotheties, reflections, etc. The basics of the apparatus of multidimensional (spatial) matrices were developed by Prof. M. P. Sokolov 30 30, 31 In addition, the results obtained by M. P. Sokolov In general form in the analysis of affine transformations of plane curves of the
third order do not allow them to be used for the above problems. Therefore, it became necessary to adapt
the device of multidimensional matrices in order to simplify it and make it possible to use it in applied tasks
[8].

3.2. Basic definitions and operations of the cubic matrix apparatus

Definition. A spatial matrix is a structure of elements from the numeric field $K$ located on the surface and
inside a parallelepiped (figure 2.1). If $n = m = l$, then the matrix is called cubic, and $n$ is its order. In general,
a matrix is called a parallelepipedoid (with dimensions $n \times m \times l$) or an $n \times m \times l$ matrix. The numbers that make
up a matrix are called its elements. Let’s introduce notation. When the three-element designation, the first
subscript indicates the layer number (section), which is the element, the second row, third column, at the
intersection of which is the element in a fixed layer of a flat rectangular matrix.
Along with the matrix notation given in the definition, we will use the abbreviated notation:

$$
\|a_{ijk}\|(i = 1, n; j = 1, m; k = 1, l) \text{ or } A_{\text{cube}}.
$$

We will also use the notation of a matrix in the form of a geometric image with a single letter and a
symbol, for example, matrix:

$$
\text{A}.
$$

If $A$ is a cubic matrix of order $n = m = l = A_{\text{cube}}$.

We define one of the main operations on matrices – multiplication of a flat square matrix by a spatial
cubic matrix.

The operation of multiplying a flat square matrix by a spatial cubic matrix is defined by us in accordance
with the concept of a collapsed product of matrices $A$ and $B$ by indices, but taking into account that
multiplication of multidimensional matrices is equivalent to a chain of their elementary transformations, the
concept of which is closely related to two-dimensional matrices, which are known to be associated with
multidimensional matrices. Therefore, the operation of "fixing" the layer of the cubic matrix in the definition
of matrix multiplication is introduced not only as a mission, actually locking the layer, but also as the
operation that sets the layer of cubic matrix, which starts the multiplication by a square matrix according to
the known rule "row to column" [9].

The operation of multiplication is defined in such a way that it results in

$$
\|a_{ijk}\|(i = 1, n; j = 1, m; k = 1, l) \text{ or } A_{\text{cube}}.
$$

This definition of multiplication allows us not to consider the indices of the Cayley-Scott partition, but,
based on the intuition of three – dimensional space, to visually represent the product of matrices in the form
of a geometric image-a cube or a square, which greatly simplifies the study of cubic forms, on the basis of
which the classification of surface modules based on a touching cuboid is based [10].

As part of the definition of the collapsed product of multidimensional matrices as a chain of elementary
transformations of two-dimensional matrices, it is possible to determine the multiplication of a square matrix
by a cubic one based on the known apparatus of square matrices in a simpler way than given in.
The basic definitions and axiomatics of the apparatus of cubic matrices are given (figure 1).

**Figure 1.** To the definition of the matrix

3.3. Invariants of the left side of the General third-order surface equation with respect to the parallel transfer transformation

The invariants of the left side of the General third-order surface equation with respect to the parallel transport transformation are obtained from the equation:

\[ ZF(x, y, z) = X^T A_{cube} = 0. \]

where \( A_{cube} \) - cubic matrix of order \( n = 4 \):

\[ A_{cube} = \|a_{ijk}\|_{1}, a_{ii} = a_{ij} = a_{ji} \]

\[ a_{ijk} = a_{ikj} = \ldots = a_{kij}(i = \overline{1,4}; j = \overline{1,4}; k = \overline{1,4}), i \neq j; i \neq k; j \neq k. \]

\( ZF(x,y,z) \) - Equation of a complex surface in general form.

\( X \) column matrix in the coordinate system: \( x^i = x, x^j = y, x^l = z, x^l = l, \)
3.4. Invariants of the left side of the General equation of a complex surface with respect to the rotation transformation

The invariants of the left side of the General equation of a complex shape surface with respect to the rotation transformation are obtained from the equation:

\[ ZF(x', y', z', \zeta') = X' A' A_{cube} = a_{11}''(x')^3 + a_{22}''(y')^3 + a_{33}''(z')^3 + 3a_{12}''(x')^2y' + \\
+3a_{13}''(x')^2z' + 3a_{23}''(y')^2z' + 3a_{13}''x'(z')^2 + 3a_{23}''y'(z')^2 + \\
+6a_{123}''x'y'z' + 3a_{114}''(x')^2 + 3a_{224}''(y')^2 + 3a_{334}''(z')^2 + 6a_{1244}''x'y' + \\
+6a_{1344}''x'z' + 6a_{2344}''y'z' + 3a_{1444}''x' + 3a_{2444}''y' + 3a_{3444}''z' + a_{4444}'' = 0 \]

Where:

\[ a_{ijk} = a_{111}p_1^i p_1^j p_1^k + a_{222}p_2^i p_2^j p_2^k + a_{333}p_3^i p_3^j p_3^k + a_{112}(p_1^i p_2^j p_k^i) + \\
+ a_{113}(p_1^i p_3^j p_k^i) + a_{212}(p_2^i p_1^j p_k^i) + a_{223}(p_2^i p_3^j p_k^i) + a_{313}(p_3^i p_1^j p_k^i) + \\
+ a_{323}(p_3^i p_2^j p_k^i) + a_{123}(p_1^i p_2^j p_3^i) + a_{134}(p_1^i p_3^j p_4^i) + a_{234}(p_2^i p_3^j p_4^i) + \\
+ a_{344}(p_3^i p_4^j p_4^i) + a_{144}(p_1^i p_4^j p_4^i) + a_{244}(p_2^i p_4^j p_4^i) + a_{344}(p_3^i p_4^j p_4^i) + a_{444}(p_4^i p_4^j p_4^i) \]

\[ i, j, k = 1, 3 \]

\[ a_{st}'' = a_{114}p_1^i p_1^j p_1^k + a_{224}p_2^i p_2^j p_2^k + a_{334}p_3^i p_3^j p_3^k + a_{124}(p_1^i p_2^j p_3^k) + \\
+ a_{134}(p_1^i p_3^j p_2^k) + a_{234}(p_2^i p_3^j p_1^k) + a_{244}(p_2^i p_1^j p_3^k) + a_{344}(p_3^i p_1^j p_2^k) \]

\[ s, t = 1, 3; \]

\[ a_{m44}'' = a_{144}p_1^i + a_{244}p_2^i + a_{344}p_3^i, m = 1, 3; a_{444}'' = a_{444} \]

Coordinate matrix in the coordinate system \( x'y'z', t' = 1 \):

\[
X' = \begin{bmatrix}
x'_1 \\
x'_2 \\
x'_3 \\
x'_4 
\end{bmatrix}
\]

where: \( x'^1 = x' \); \( x'^2 = y' \); \( x'^3 = z' \); \( x'^4 = t' = 1 \).

\[
A' = \begin{bmatrix}
p_1^1 & p_1^2 & p_1^3 & 0 \\
p_2^1 & p_2^2 & p_2^3 & 0 \\
p_3^1 & p_3^2 & p_3^3 & 0 
\end{bmatrix}
\]

Matrix of unit vectors:

\[
\vec{e}_1 = \{p_1^1, p_1^2, p_1^3\}, \vec{e}_2 = \{p_2^1, p_2^2, p_2^3\}, \vec{e}_3 = \{p_3^1, p_3^2, p_3^3\}.
\]
4. Classification of a contiguous cuboid by invariants
The theory of invariants and the developed apparatus of cubic matrices allow us to determine the structure of a contiguous cuboid (table 2). This is important for the next step in geometric modeling of a complex surface. The microrelief is known to be structured on the basis of a contiguous paraboloid [11-13].

Table 2. Classification of a contiguous cuboid by invariants.

| Type of surface | The reduced SK equation in invariants | The name of the surface |
|-----------------|-------------------------------------|-------------------------|
| VIII            | $\pm \sqrt{1_{7,7}^{\text{II}} + (f_2 + f_3)^2} y^2 \pm$ | Parabolic cuboid diverging |
|                 | $\pm 3 \left[ \sqrt{1_{7,7}^{\text{II}} - 1_{7,7}^{\text{V}}} \right] y = 0$  |
| IX              | $-61_{7,7}^{\text{VY}} x y \pm 3 \sqrt{1_{7,7}^{\text{VY}} / 1_{7,7}^{\text{V}}} z = 0$  | Trident cuboid |
|                 | $\pm \sqrt{1_{7,7}^{\text{II}} + (f_2 + f_3)^2} x^3 + 3A_{7,7}^{\text{VII}} x^2 +$ |
|                 | $\pm 3 \left[ \sqrt{1_{7,7}^{\text{II}} - 1_{7,7}^{\text{V}}} \right] z = 0$  |

5. Setting up a holographic control (UGC)
Modern monitoring devices (figure 2) are designed in such a way that recording devices record parameter values from contour maps of an object. Polygonal maps are defined either with large errors, or over a fairly long time interval. It is not possible to control a hard-to-reach object—an abrasive grain moving in the part material. There is one way to expand the capabilities of monitoring devices and use the information obtained with their help to build three-dimensional models, the use of devices that study the holographic image of an object.

The principle of control of the devices in question is based on recent studies of the processes of obtaining a holographic image of an object in the visible and x-ray ranges. Devices of this series allow you to study processing processes not in a projection on a plane, but in space.

The technological installation for 3D holographic size control (figure 3-4) of complex mechanical engineering parts was originally conceived for a wide range of products of "conventional" industrial production of General engineering, as well as military products: shell casings, aviation ammunition, unguided and guided missiles, grenades, combat and warheads of guided missiles and ammunition. In addition, the problems of manufacturing machine parts and tools (dies) for the production of missiles and
ammunition are considered and positively resolved. The versatility of the proposed technological platform allows extending its application to the production of domestic civil products [14].

![Figure 2](image1.jpg)

**Figure 2.** Recording of a holographic image of a microrelief of a sample using the Leith-Upatnieks method.

![Figure 3](image2.jpg) ![Figure 4](image3.jpg)

**Figure 3.** The holographic image of a microrelief after finishing processing.  **Figure 4.** The holographic image of a microrelief after rough machining.

The modular-he metric approach to modeling a complex-shaped surface allows US to use UGC to determine the characteristics of this surface. In the case of a micro relief, this is the main curvature and height of the vertex of the contiguous paraboloid. In the case of a gas turbine blade feather, these are the geometric characteristics of an oblique helicoid.

6. **Conclusion**

The basic definitions and axioms the apparatus of cubic matrices, developed on the basis of spatial matrices, which allowed to develop the theory of module approximation of the surface complex forms of touching cuboidal (algebraic surfaces of the third order), analytical description which is the basis for the calculation of the shape of abrasive tools for processing the various modules of a surface of complex shape. The
proposed operations significantly simplify the study of cubic forms necessary for constructing a classification of surface modules based on a contiguous cuboid.

A classification of third-order algebraic curves on the plane is developed in accordance with the group of motions (five types) and a structural connection with the Newton classification (seven types) is established.

A classification of plane algebraic curves of the third order based on spatial matrices (of the third order) by invariants (five types) is developed, which allows us to proceed to the classification of third-order surfaces.

A classification of third-order planar algebraic curves based on spatial matrices (third order) by invariants (five types) is developed, which allows us to proceed to the classification of third-order surfaces.

A classification of third-order surfaces is developed based on coordinate transformations from the group of motions of the General third-order surface equation, which allowed us to establish the existence of nine types of third-order surfaces, only three of which represent a contiguous cuboloid. The resulting classification allows us to proceed to the classification of the module of a complex-shaped surface based on a contiguous cuboid [15].

A generalized classification of the modules of the microrelief surface is determined in accordance with the concept of a geometric series (based on a contiguous paraboloid and a contiguous cuboloid). Based on further research, a classification of the contiguous cuboloid by invariants is obtained, which allows using their structures for approximating surfaces by a contiguous cuboloid (similar to a contiguous paraboloid). The latter classification is also important because it allows using known methods to Express the coefficients of the reduced equation in terms of the components of the Riemann - Christoffel tensor.

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