Thermally driven classical Heisenberg model in 1D with a local time varying field

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Abstract. We study thermal transport in the one-dimensional classical Heisenberg model driven by boundary heat baths and in the presence of a local time varying magnetic field. We find that, in the steady state, the energy current shows thermal resonance as the frequency of the time-periodic forcing is varied. Even in the absence of a thermal bias a steady nonzero energy current can be induced in the system, whereas for the thermally driven system a current reversal can be achieved in the bulk by suitably tuning the system parameters. When the amplitude of the forcing field is increased the system exhibits multiple resonance peaks. Thermal resonance survives in the thermodynamic limit and their magnitude increases as the temperature of the system is decreased. We find that the resonance frequency is an intrinsic frequency of the model and is related to its spin wave dispersion spectrum. Finally we show that, similar to other generic force-driven systems, there is no thermal pumping despite the current reversal in the bulk of the system.

Keywords: transport processes/heat transfer (theory), heat conduction
1. Introduction

Low-dimensional thermal transport has been a topic of intense research interest in recent times. The reason for the sudden upsurge in this field is primarily two-fold. Firstly, low-dimensional systems have rich and intriguing transport properties and therefore contribute immensely in widening our theoretical understanding of the fundamental principles of transport, e.g., the necessary and sufficient criteria for the validity of Fourier law in thermally driven low-dimensional systems [1]–[3]. Although substantial progress has been made in the theoretical front, a comprehensive understanding is still lacking. The advancement in low-dimension experiments has also greatly motivated theoretical research since theoretical predictions can often be readily tested in laboratories nowadays. Secondly, studying thermal transport in low dimensions is of huge technological interest because of the recent breakthrough in nanoscale thermal devices which rely on the energy transport properties of phonons present in a system [4]. Similar to its electrical counterpart, theoretical designs for phononic devices such as thermal diodes, transistors, logic gates, memory devices, and phonon waveguides [5]–[9] have been proposed and some of them also experimentally realized recently. Thus, it has been possible to control and manipulate heat current, just as one would control electrical current in electronic devices [10]. These new exciting developments have induced a flurry of active theoretical and experimental research in this field at present.

Apart from phonons, spin waves (magnons) in magnetic systems have also turned out to be efficient energy carriers, and much experimental research is being undertaken to investigate the transport properties of magnetic systems. The classical Heisenberg model [11, 12] has been studied, both analytically and numerically, for several decades and has become a prototypical model for magnetic insulators. Although the transport properties of its quantum counterpart (spin-\(\frac{1}{2}\) quantum Heisenberg model) have been extensively studied [13]–[15], not much attention has been paid to the thermal transport properties of the classical Heisenberg model until recently [16, 18, 17, 19]. The thermal transport properties of the classical model are found to obey the Fourier law and this
diffusive energy transport is attributed to the nonlinear spin wave interactions of the system [16]. A deeper understanding of thermal properties of such spin systems is extremely desirable now since several new magnonic devices, such as memory elements, logic gates, switches, and waveguides are now being conceived by manipulating spin waves by external magnetic field in ferromagnetic materials [20, 21]. Transport in spin systems can also give rise to intriguing phenomenon such as the spin Seebeck effect, where a spin voltage is generated by a temperature gradient even across a ferromagnetic insulator and is due to the thermally excited spin wave interactions, about which very little is known [22, 23].

In this paper, we study the classical Heisenberg model in the presence of a time varying magnetic field that acts locally at one end of the one-dimensional system and a thermal bias is imposed by heat baths attached at the two boundaries. Eventually this force-driven system attains an oscillatory nonequilibrium steady state. In the steady state, the thermal current flowing through the system shows thermal resonance at a characteristic frequency of the external time-periodic forcing. The thermal current that flows through the driven system can be suitably manipulated, both in magnitude and direction, and for the thermally unbiased system a steady thermal current can be induced by proper tuning of the forcing frequency. The resonance frequency at which the current attains a maximum value is found to be independent of the system size, temperature and the boundary drive, and it changes when the spin interaction strength is altered. We explain the robustness of the resonance frequency by invoking the dispersion spectrum of the model. For some range of the model parameters, a multiresonance feature is observed in which more than one resonance peak appears in the frequency response of the system. The local external perturbation acts as an efficient probe to investigate the relevant spin modes responsible for transport of thermal energy in a magnetic system. Finally, we explicitly demonstrate that, although the direction of the current in the bulk can be reversed when the thermally driven system resonates, one cannot use this as a thermal pump [24]. Other models with local periodic forcing at one edge of the system that have been studied recently are the harmonic lattice [24], the Frenkel–Kontorova model [25, 24], the sine–Gordon model [26], the Fermi–Pasta–Ulam model [27, 28], the Klein–Gordon model [26, 29], and the discrete nonlinear Schrödinger system [30], showing many interesting features such as resonances, nonlinear delocalization, discrete breathers, bistability, and nonlinear supratransmission, which shed valuable light on the transport behavior of these systems. Since the classical Heisenberg model is directly accessible to experiments [31, 32] we believe such a study will be helpful for a better understanding of thermal transport in magnetic systems.

The remainder of the paper is organized as follows. We define the classical Heisenberg model in the presence of thermal drive and a local time varying magnetic field, and briefly discuss our numerical scheme in section 2. In section 3 we present our numerical results and investigate in detail the resonance feature seen in this system. We then demonstrate the absence of thermal pumping in this system, reconfirming the fact that thermal pumping is generically absent in such force-driven lattices. Finally, we conclude by summarizing our main results in section 4.

2. Model and numerical scheme

Consider classical Heisenberg spins \( \{ \vec{S}_i \} \) (three-dimensional unit vectors) on a one-dimensional regular lattice of length \( N \) (1 \( \leq i \leq N \)) with the Hamiltonian of the system

\[
H = -J \sum_{i=1}^{N-1} \vec{S}_i \cdot \vec{S}_{i+1} - H_0 \sum_{i=1}^{N} S_i^z - T_b \sum_{i=1}^{N} \beta (T_i - T) \vec{S}_i^z,
\]

where \( J \) is the exchange interaction between nearest neighbors, \( H_0 \) is the external magnetic field, \( T_b \) is the temperature of the heat bath, and \( \beta = 1/k_B T \) is the inverse temperature. The total energy of the system is

\[
E = \sum_{i=1}^{N} (\vec{S}_i^2/2J + H_0 S_i^z) + \sum_{i=1}^{N} \beta (T_i - T) \vec{S}_i^z.
\]
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Figure 1. Schematic diagram of the classical Heisenberg model in one dimension with two boundary heat baths at temperatures $T_l$ and $T_r$, together with a local time varying magnetic field $\vec{h}(t)$ that acts locally on one spin of the system.

Figure 1

The microscopic equation of motion for the spin vectors can be written as

$$\frac{d}{dt}\vec{S}_i = \vec{S}_i \times [\vec{B}_i + \vec{h}(t)\delta_{i1}]$$

(2)

where $\vec{B}_i = K(\vec{S}_{i-1} + \vec{S}_{i+1})$ is the local molecular field experienced by the spin at site $i$. Apart from the local forcing, there is also an overall thermal gradient across the system maintained by two heat baths attached at the two boundaries. This is implemented by introducing two additional spins at sites $i = 0$ and $N + 1$. The bonds between the pairs of spins $(\vec{S}_0, \vec{S}_1)$ and $(\vec{S}_N, \vec{S}_{N+1})$ at two opposite ends of the system behave as stochastic thermal baths. The left and right baths are in equilibrium at their respective temperatures, $T_l$ and $T_r$, and the bond energies, $E_l = -K\vec{S}_0 \cdot \vec{S}_1$ and $E_r = -K\vec{S}_N \cdot \vec{S}_{N+1}$, have a Boltzmann distribution. The interaction strength of the bath spins with the system is taken to be $K$, and therefore both $E_l$ and $E_r$ are bounded in the range $(-K, K)$. Thus the average energies of the baths are given by $\langle E_l \rangle = -K\mathcal{L}(K/T_l)$ and $\langle E_r \rangle = -K\mathcal{L}(K/T_r)$, with $\mathcal{L}(x) = \coth(x) - 1/x$ being the standard Langevin function.

We investigate the steady state transport properties of the Heisenberg model by numerically computing quantities, such as currents and energy profiles, using the discrete time odd even (DTOE) dynamics [17]. Briefly stated, the DTOE dynamics updates the spins belonging to the odd and even sublattices alternately using a spin precessional dynamics

$$\vec{S}_{i,t+1} = \left[\vec{S}\cos \phi + (\vec{S} \times \vec{B})\sin \phi + (\vec{S} \cdot \vec{B})\vec{B}(1 - \cos \phi)\right]_{i,t}$$

(3)

where $\vec{B}_i = \vec{B}_i/|\vec{B}_i|$, $\phi_i = |\vec{B}_i|\Delta t$ and $\Delta t$ is the integration time step. The above formula is also sometimes referred to as the rotation formula and holds for rotation of any finite magnitude [33]. Note that $\vec{B}_i$ in the above equation has to be replaced by the vector
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$\vec{B}_i + \vec{h}(t)$ for the first site $i = 1$. The advantages of using the DTOE dynamics are as follows. This dynamics is strictly energy conserving and maintains the length of the spin vectors naturally. The total spin conservation is also approximately maintained and is comparatively better than other conventional integration schemes [34]. Also it has been thoroughly verified that local thermal equilibrium is established in the thermally driven system (in the absence of forcing) when evolved using the DTOE dynamics independently of the value of the integration time step $\Delta t$.

Numerically the thermal baths are implemented in the following manner. The leftmost spin $\vec{S}_0$ is updated along with the even spins, whereas the rightmost spin $\vec{S}_{N+1}$ is updated along with the odd (even) spins if $N$ is even (odd). To update $\vec{S}_0$, first the bond energy $E_i$ between the spins $(\vec{S}_0, \vec{S}_1)$ is assigned randomly from a Boltzmann distribution corresponding to temperature $T_l$. Since during this update $\vec{S}_1$ is not modified (as it belongs to the odd sublattice) the spin $\vec{S}_0$ is reconstructed such that the bond energy between the updated spin $\vec{S}_0$ and the spin $\vec{S}_1$ becomes equal to the randomly assigned value of $E_i$ (which is refreshed from the Boltzmann distribution whenever $\vec{S}_0$ is updated). Similarly the rightmost spin $\vec{S}_{N+1}$ is also updated consistent with the temperature $T_r$. Thus one can set the two baths at a fixed average energy (or temperature) and a thermal current flows through the system if $T_l \neq T_r$. A thorough discussion of the DTOE scheme and numerical implementation of the thermal baths have been presented in [17].

The computation of the steady state thermal current is done as described in the following. Since the DTOE dynamics alternately updates only half of the spins (odd/even) but all the bond energies simultaneously, the energy of the bonds $E_i^e$ measured immediately after the update of odd spins is different from $E_i^o$ measured after the update of even spins, where $E_i = -K\vec{S}_i \cdot \vec{S}_{i+1} - \vec{h}(t) \cdot \vec{S}_i \delta_{i1}$. Clearly, the difference $E_i^e - E_i^o$ is a measure of the energy flowing through the $i$th bond in time $\Delta t$. Thus the thermal current (rate of flow of thermal energy) in the steady state is given by

$$j = \langle E_i^e - E_i^o \rangle / \Delta t \quad (4)$$

and the average energy of $i$th bond is $E_i = \frac{1}{2}(E_i^e + E_i^o)$. As already mentioned, the thermal current $j$ in the one-dimensional classical Heisenberg model obeys the Fourier law, i.e., $j \sim 1/N$. Let us define a total current $J \equiv jN$, which clearly is independent of the system size. In the following, we present our numerical results in terms of the total current $J$, since this will allow us to compare the thermal current of systems of different sizes.

3. Numerical results

3.1. Thermal resonance

For simulations, we choose the time varying magnetic field of the form $\vec{h}(t) \equiv (0, 0, A \sin(\omega t))$ and the boundary heat baths have average energy $\langle E_l \rangle = E_b$ and $\langle E_r \rangle = E_b + \Delta E$. Starting from a random initial configuration of spins, the system is evolved using the DTOE dynamics with integration time step $\Delta t = 0.25$. The spin system is first relaxed, typically for $\sim 10^6$–$10^7$ iterations, and thereafter the steady state quantities are computed for the next $\sim 10^7$–$10^8$ iterations, which is also averaged over several independent realizations (typically $\sim 1000$) of the initial spin configuration. In the absence of periodic
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Figure 2. Thermal resonance in the classical Heisenberg model: (a) thermal current $J$ shows thermal resonance for some value of the forcing frequency $\omega \approx 1.5$ where the $J(\omega)$ curve shows a peak. The low- and high-frequency behavior is same as that in the absence of any periodic forcing (shown as a dotted line); the parameters used are $N = 100, A = 1.0, E_b = -0.1, \Delta E = 0.015$. (Inset) The change of sign of the slope of the energy profile for $\omega = 1.5$ as compared to $\omega = 0.05, 20.0$ suggests current reversal in the bulk of the system.

Forcing, a current flows through the system in response to the imposed thermal bias. To fix the sign of the thermal current we set the following convention—a current flowing from a larger to a smaller site index $i$ is negative; in other words, a current from the right towards the left end of the lattice is taken to be negative and vice versa.

The steady state total current $J$ as a function of the forcing frequency $\omega$ is shown in figure 2. For small values of the forcing frequency the current flows through the system from the higher temperature end to the lower temperature end (hence negative according to our convention) and similarly for high frequencies, and the average value of the current is roughly the same in these two regimes. This can be understood from the fact that for very small frequencies the typical timescales of the system are much smaller than the forcing timescale and thus the system senses two opposite static forces, which amounts to no net forcing. In the other limit, the system fails to respond to the rapidly varying forcing and thus effectively experiences no external forcing. Thus, in the two asymptotic limits the frequency behavior is essentially the same, as can be clearly seen from figure 2.

However, in between these two frequency ranges the total energy current $J$ shows thermal resonance—the current attains a maximum value $J_m$, corresponding to a resonance frequency $\omega_m$. By properly choosing the bath temperatures, the current $J$ can even be made to change sign (here, from negative to a positive value). This implies that for a certain range of the forcing frequency the thermal current through the bulk of the system flows in the opposite direction, i.e., from the colder end to the hotter end.

To check that the thermal current indeed gets reversed for frequencies in the resonance region, we compute the energy profile of the system (see figure 2 (inset)) for three forcing
Figure 3. Thermal resonance survives in the thermodynamic limit, as can be understood from the $J(\omega)$ curve for system sizes $N = 50, 100$ and $200$, all other parameters remaining the same as figure 2. (Inset) The maximum current $J_m$ corresponding to $\omega = \omega_m$ saturates in the thermodynamic limit. The $J_m \sim N$ data obtained from simulation fits with the form $J_{m,\infty} - aN^{-\gamma}$ (shown as a broken line), where $J_{m,\infty} = 0.118$, $a = 0.633$, and $\gamma = 0.685$. 

frequencies belonging to the three frequency regimes. It is found that the energy profiles for small and large frequencies have the usual linear form connecting the two heat baths with a discontinuity at the forcing site. The energy profile for frequency in the resonance region $\omega = 1.5$, however, has an opposite slope in the bulk of the system. Thus, by merely tuning the forcing frequency, one can easily manipulate the magnitude as well as the direction of flow of the thermal current in the bulk of the system. Two more characteristic features of the observed thermal resonance are in order. Firstly, the resonance effect survives in the thermodynamic limit, which is evident from figure 3, where we have shown resonance for different system sizes. The maximum current $J_m$ as a function of the system size fits nicely with the functional form $J_m = J_{m,\infty} - aN^{-\gamma}$, where $J_{m,\infty}$ is the saturation value of maximum current in the thermodynamic limit, and $a, \gamma$ are fitting parameters. Thus the maximum current has a finite limiting value for a thermodynamically large system. This evidently shows that thermal resonance is an intrinsic feature of the system and not a finite size effect. Secondly, the resonance frequency $\omega_m$ seems to be completely independent of the system size, which again points to the fact that $\omega_m$ is an intrinsic frequency of the system.

If the temperature gradient is comparatively large (keeping all other parameters unchanged), the resonance phenomenon is still there but the current through the system now has a large negative value and tuning only the frequency cannot push the current to a positive value. As before, the energy profiles can be computed, which also validate the fact that the current, although it is amplified (the slope of the energy profile increases) does not get reversed (the slope does not change sign). This is shown in figure 4. However, increasing the amplitude can push the current to a positive value and current reversal can be achieved in the bulk of the system. Notice that there is a small shift in $\omega_m$ for larger
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Figure 4. (a) Thermal resonance with a larger energy gradient ($E_b = -0.2$ and $\Delta E = 0.175$) for different values of the forcing amplitude $A$. Increasing the amplitude increases the current and makes $J > 0$; (inset) energy profiles corresponding to $A = 1.0$, $2.0$ and $3.0$ for $N = 100$. The energy profile for $A = 1.0$ has a slope of opposite sign to that for $A = 2.0$ and $3.0$.

amplitude $A$. We will briefly discuss the frequency response for large forcing amplitudes at the end of this section.

We also investigate the effect of the temperature on the resonance feature. Let the temperatures of the two baths be $T_l = T$ and $T_r = T + \Delta T$, so the average temperature of the system can be taken to be $T$ ($\Delta T$ is small). For the same $\Delta T$ we find that the resonance effect is enhanced at a lower temperature and the current amplification is larger, as is shown in figure 5. To see how the maximum thermal current $J_m$ varies with temperature, we set $\omega = \omega_m \approx 1.5$ and measure $J_m$ for different bath temperatures, keeping the same temperature difference. The inset in figure 5 suggests that the maximum current $J_m$ is inversely proportional (approximately) to the temperature of the system. This can be understood from the fact that the external periodic forcing acts as an additional source of energy and alters the overall slope of the energy profiles by feeding in extra energy to the system at the forcing site. In figure 6(a) we display the local energy profile of the system for three different values of the bath energy $E_b$; these three curves can be collapsed by scaling the energy profiles with their respective average temperatures, as is shown in figure 6(b). So the slope of the energy profiles increases inversely with $T$ and therefore the current induced in the system also varies as $J \sim T^{-1}$.

To understand the role of the two energy sources driving the system away from equilibrium, namely the thermal baths and the external periodic forcing, we also look into the frequency response of the system in the absence of the thermal bias. We set the temperatures of the two baths as $T_1 = T_2 = T$, thus the system is an equilibrium system at temperature $T$ when there is no external forcing and does not carry a net current. The frequency response of this equilibrium system is shown in figure 7 along with the driven system for comparison. In this figure, out of the four curves, two of them correspond to (i) the driven system (filled data points) with forcing at the (a) colder end ($i = 1$) and
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Figure 5. Thermal current with frequency for different average energies of the bath $E_b = -0.1, -0.2$ and $-0.3$ and the same energy gradient $\Delta E = 0.015$. Resonance enhances as the temperature decreases. (Inset) Log–log plot of the maximum thermal current $J_m$ corresponding to $\omega = 1.5$ for $N = 500$.

Figure 6. (a) Energy profile $E(x)$ of the system for bath energies $E_b = -0.1, -0.2$ and $-0.3$ for $\Delta E = 0.015, \omega = 1.5$. (b) All the three profiles in (a) can be collapsed by scaling the energy axis as $E(x)T$.

(b) hotter end ($i = 1$ and heat baths are interchanged), and two for (ii) the equilibrium system (open data points) with forcing at (a) the end (we choose $i = 1$) and (b) the midpoint of the system ($i = N/2$).

In the absence of thermal drive, we find the same qualitative features as that of the driven system, except an overall shift of the $J \sim \omega$ curve (when forcing is at the end). Thus in the steady state a nonzero current flows even through the (thermally) unbiased system when the forcing is applied. However, as expected, there is no resonance when the forcing point is at the center of the equilibrium system and the total current $J(\omega)$ is zero since the system’s local energy profile remains symmetric about the site of forcing at all times (although each half of the lattice carries an equal and opposite nonzero current). Thus thermal resonance is purely the consequence of the local external forcing; thermal drive only sets the dc value of the current in the system. Due to the external forcing, the spin at the forcing site starts to oscillate and this causes its local energy to increase,
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Figure 7. Thermal current $J$ with forcing frequency $\omega$ for the thermally driven system and the thermally unbiased system. The filled data points are for the thermally driven system with forcing at the colder end and the hotter end marked as ‘C’ and ‘H’ respectively; $E_b = -0.1$ and $\Delta E = 0.015$. The open data points correspond to the unbiased system with forcing at one end and at the midpoint of the bulk marked as ‘E’ and ‘M’ respectively with $E_b = -0.1$.

which eventually leads to the overall modification of the system’s energy profile. The local average energy $E_f$ of the forcing site is maximum when the frequency of forcing is close to the spin wave frequency of the system (which is related to the typical response time of the spins) that contributes maximally to the energy current. The current $J$ exhibits thermal resonance at the same frequency at which the energy $E_f$ attains a maximum value, as can be seen from figure 8. Not only are the peak positions the same, but also the functional dependence of these two quantities with frequency, as can be seen from the collapse of the two curves (figure 8 (inset)).

In order to have a better insight into the underlying physical mechanism and to have a quantitative estimate of the resonance frequency $\omega_m$, we look into the dispersion relation of the model. The spin wave theory for this model in one dimension has been studied earlier [35, 36]. The general result that one obtains is that the dynamics of this nonlinear interacting model can be well described by the familiar ferromagnetic ($K > 0$) spin wave dispersion relation

$$\omega_q = 2K(1 - \cos q)$$

where the lattice spacing has been set to unity and $q$ is the wavevector [35]. However, the presence of finite amplitude nonlinear spin waves has also been predicted in this system and the dispersion relation gets slightly modified to $\omega_q = 2K \cos \theta_0 (1 - \cos q)$, where $0 \leq \theta_0 \leq \pi$ [36].

To verify if the resonance frequency $\omega_m$ is related to the typical spin wave modes $\omega_q$ of the system’s dispersion spectrum, we study the response of the equilibrium system for different values of the interaction strength $K$, keeping all other parameters unaltered. The frequency response of the systems with different $K$ values is shown in figure 9. Since

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Figure 8. The local average energy $E_f$ of the forcing site (here site $i = 1$) is shown with the current $J$ induced in the equilibrium system with the forcing frequency $\omega$. The energy $E_f$ has been shifted along the $y$-axis by an amount $E_b$, where $E_b = -0.1$ is the bath energy. (Inset) The two quantities, $E_f - E_b$ and $J$ (shown in the main figure), collapse on to each other when rescaled by their maximum values corresponding to $\omega = \omega_m$.

Figure 9. (a) Thermal current with frequency for different values of the interaction strength $K$. (b) All the curves in (a) can be collapsed by scaling the frequency as $\omega/K$. Here $E_b = -0.1$ and $N = 100$.

the typical value of the current $J$ is different for the different coupling strengths, we have scaled $J$ by its maximum value $J_m$ for the sake of better comparison. We find that the resonance frequency $\omega_m$ is different for systems with different $K$ values, as shown in figure 9(a). Furthermore, these resonance frequencies are proportional to the interaction strength, which is evident from the fact that all the different curves collapse on to one another when the frequency axis is scaled by their respective interaction constants $K$, and the rescaled resonance frequency $\omega_m/2K$ is roughly of the order of unity, in conformity with equation (5). This ensures that the resonance frequency $\omega_m$ is indeed related to the
Figure 10. (a) The thermal current $J$ for different average bath energies $E_b = -0.05, -0.1, -0.2$ and $-0.3$, with $\Delta E = 0.015$ and forcing amplitude $A = 5.0$ for a system size of $N = 100$. (b) The thermal current $J$ for different system sizes $N = 50, 100, 200$ and 500 with $A = 5.0$, $E_b = -0.1$ and with $\Delta E = 0.015$.

spin wave frequency $\omega_q$ and also explains its robustness to parameter changes, as we have seen in our numerical analysis; $\omega_m$ changes only when the interaction strength $K$ of the model is altered and is independent of other factors such as temperature, system size or even thermal drive.

An intuitive way of understanding the resonance phenomenon is by noting that the energy current $J$ should be roughly proportional to $\omega_q v_q$, where $v_q = d\omega_q/dq$ is the group velocity of the spin waves. Using the dispersion relation, the current can be expressed as $J \sim \omega_q \sqrt{1 - (1 - \omega_q)^2}$ (we have set $K = 1$ and ignored numerical factors). This expression for $J(\omega_q)$ has a similar functional form to the simulation results for $J(\omega)$ presented here (such as in figure 9). The spin wave frequency $\omega_q$ corresponding to the maximum value of the current $J_m$ can also be calculated and is found to be of order unity. Thus resonance occurs when the forcing frequency $\omega$ is resonant and close to the spin wave frequency corresponding to the maximum value of the current $J(\omega_q)$.

For certain parameter ranges, the thermal resonance described above can show multiresonance behavior in which the single resonance peak splits into two or more distinct resonance peaks. In figure 10 the thermal current $J$ is shown as a function of the forcing frequency $\omega$ for a relatively large forcing amplitude $A$. We find that for larger forcing amplitude $A = 5.0$, the central peak splits up in two peaks and the current $J$ corresponding to $\omega = \omega_m \approx 1.5$ is no longer the maximum; two other peaks appear on both sides of $\omega_m$. The multiresonance feature has the same parameter dependences as that of the single peak resonance i.e., it is more pronounced at lower temperature and it survives in the thermodynamic limit, as can be seen from figure 10. However, the number of resonance peaks seem to be independent of the average temperature and also the system size. Although for a linear system the number of such peaks should be equal to the system size [24] that is not the case here due to the inherent nonlinearity of the system. We have checked that this nonlinearity does not vanish, even for small systems at very low temperature, where our model has an effective harmonic description [17]. A more detailed study of the classical Heisenberg model is surely desirable to unravel the underlying physics of its multiresonance feature.
Figure 11. Computation of $\Delta n = n_l - n_r$ from a typical run using the DTOE dynamics for three sites $i = 1, N/2$ and $N$ in a system of size $N = 100$, $E_b = -0.1$, $\Delta E = 0.015$: (a) without periodic forcing, energy across all the sites flows towards the left; (b) with periodic forcing ($A = 1.0$, $\omega = 1.5$), energy across site $i = 1$ flows towards the left whereas, for sites $i = N/2$ and $N$, energy flows to the right. Thus both the heat baths absorb energy and there is no thermal pumping.

3.2. Absence of thermal pumping

Although the thermally driven system in resonance exhibits a reversed bulk current, there is no thermal pumping in this system. For a system to be a thermal pump, energy must be pumped from the colder heat bath and absorbed in the hotter heat bath, and as a consequence a reverse flow of energy occurs in the bulk of the system [24]. However, in our system (and other force-driven lattices in general), although the high temperature bath absorbs energy from the system and a reversed current flows through the bulk, the low-temperature heat bath does not pump energy into the bulk of the system. What actually happens is that the additional energy, drawn from the external periodic forcing, flows from the point of forcing (here $i = 1$) towards the two boundaries of the system, thus resulting in a reversed bulk current.

This can be clearly seen in our system by monitoring the energy flow across sites $i = 1, N/2$ and $N$. When the spin at a site $i$ is updated using the DTOE dynamics, the energy of the bonds connected to it, namely $E_{i-1}$ and $E_i$, are updated such that the sum $E_{i-1} + E_i$ remains the same before and after the update. In other words, a redistribution of the sum $E_{i-1} + E_i$ takes place while the $i$th spin is updated. Since our system is connected to stochastic thermal baths, this redistribution process is also stochastic in nature—the $(i-1)$th bond stochastically gains and loses energy and similarly for the $i$th bond. However, since there is an overall thermal gradient $(T_r - T_l)$, the total number of times energy flows in one particular direction (here, towards left since $T_r > T_l$) over a long time will obviously be more than that of the opposite direction, so that a steady energy current flows through the system. In the absence of the periodic forcing, the quantity $\Delta n = n_l - n_r$ is positive when measured for sites $i = 1, N/2$ and $N$, where $n_l$ ($n_r$) is the total number of times energy flows to the left (right) across a particular site. The results from a typical run, starting from a random initial condition of spins and using the DTOE dynamics, is shown in figure 11(a). Thus energy flows from the high-energy bath to the
low-energy bath, i.e., from the right to the left end of the system. However, when the
system is in the resonance region, $\Delta n$ for site $i = 1$ is positive, whereas it is negative for
sites $i = N/2$ and $N$. This shows that, while in resonance, energy flows from right to left
for $i = 1$, whereas for $i = N/2$ and the right end $i = N$, energy flows from left to right (see
figure 11(b)). Thus current flows towards the two boundaries from the point of periodic
perturbation in the bulk of the system. Evidently, there is no thermal pumping since both
the high-temperature and low-temperature baths absorb energy and thus the transport
of energy from the low- to high-temperature bath is absent.

4. Summary

To summarize, we report here an extensive numerical study of the one-dimensional
classical Heisenberg model in the presence of boundary drive and a local time varying
forcing. The thermal current that flows through the system shows resonance at some
characteristic frequency $\omega_m$ of the forcing for which the current attains a maximum
value. By properly tuning the boundary temperatures, we demonstrate that the energy
current flowing through the bulk of the system can be reversed for frequencies within
the resonance region. The magnitude of the current can also be controlled by tuning the
forcing amplitude $A$. This allows one to mechanically control the magnitude as well as the
direction of current in the system. This could be of use in nanoscale spin systems, where
a large thermal gradient is often not desirable.

We study the dependence of the thermal resonance on the parameters of the system.
It is found that the magnitude of resonance increases as the system size increases, and
survives in the thermodynamic limit. The maximum thermal current $J_m$, corresponding
to the resonance frequency $\omega_m$, saturates to a finite value for a thermodynamically large
system. Also decreasing the average temperature enhances the magnitude of resonance;
the maximum thermal current $J_m$ varies as $T^{-1}$, which can be explained by the increase
in the slope of the system’s energy profile with temperature as a consequence of the local
periodic forcing.

It is also found that thermal resonance can be obtained even in a system without
thermal drive and a nonzero current can be made to flow when the forcing is applied at
one end of the system. Thus resonance is a consequence of the periodic external forcing; the
thermal drive only produces an overall shift (along the $J$ axis) in the frequency response
of the system. The resonant frequency $\omega_m$ is quite robust and is found to be independent
of the system size, average temperature or the thermal drive. This led us speculate that
$\omega_m$ is related to some intrinsic frequency of the system. We find that, in conformity with
the system’s dispersion relation, $\omega_m$ changes only when the spin–spin interaction strength
$K$ is altered and the rescaled frequency $\omega_m / 2K$ is of the order of unity, as it should be.
The system also shows multiresonance, i.e. multiple peaks in $J(\omega)$, as the amplitude of
the external forcing is increased. The resonance and the multiresonance phenomenon can,
in general, be understood as follows. The external periodic forcing that is imposed on the
system acts as an additional source of energy for the system. When the forcing frequency
is resonant with the spin wave frequencies that largely contribute to the energy current,
the transfer of energy from the external perturbation to the system becomes large and
resonance peaks appear in the system’s frequency response. Resonance for small external
forcing amplitude (single resonance peak) is well described by the familiar spin wave

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Thermally driven classical Heisenberg model in 1D with a local time varying field dispersion. The multiresonance phenomenon is comparatively difficult to observe since the resonance peaks are smoothed out due to the inherent nonlinearity of the system.

Finally, we explicitly show using energy flow arguments that despite the reversal of the current in the bulk, this system fails to act as a thermal pump. This is consistent with the previous result that a force-driven lattice cannot direct thermal energy from the low-temperature heat bath to the high-temperature heat bath.

The classical model can be thought to be the infinitely large spin limit of the quantum Heisenberg model. This classical approximation of the quantum model is already seen to hold for systems with spin $s > 1$, for example, in Mn$^{2+}$ ($s = 5/2$), and thus is realizable in practice [31, 32]. As such, controlled laboratory experiments with model chemical compounds, which are nowadays routinely performed, can be used to test the theoretical predictions made in the classical Heisenberg model. Hopefully, such experimental studies will eventually lead us towards better heat control and management in future.

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References

[1] Bonetto F, Lebowitz J L and Rey-Bellet L, Fourier law: A challenge to theorists, 2000 Mathematical Physics 2000 ed A Fokas et al (London: Imperial College Press) pp 128–50
[2] Lepri S, Livi R and Politi A, 2003 Phys. Rep. 377 1
[3] Dhar A, 2008 Adv. Phys. 57 457
[4] Li N, Ren J, Wang L, Zhang G, Hänggi P and Li B, 2012 Rev. Mod. Phys. 84 1045–66
[5] Terraneo M, Peyrard M and Casati G, 2002 Phys. Rev. Lett. 88 094302
[6] Li B, Wang L and Casati G, 2004 Phys. Rev. Lett. 93 184301
[7] Wang L and Li B, 2005 Phys. Rev. Lett. 99 177208
[8] Valenzuela S O, 2013 Nature Mater. 11 199
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[30] Johansson M, Kopidakis G, Lepri S and Aubry S, 2009 Europhys. Lett. 86 10009
[31] De Jongh J L and Miedema A R, 1974 Adv. Phys. 23 1
[32] Steiner M, Villain J and Windsor C, 1976 Adv. Phys. 25 87
[33] Goldstein H, Poole C P and Safko J L, Classical Mechanics 3rd edn (Reading, MA: Addison-Wesley)
[34] Bagchi D, 2013 Phys. Rev. B 87 075133
[35] Kretzen H H, Mikeska H J and Patzak E, 1974 Z. Phys. 271 269
[36] Balakrishnan R and Dhamankar R, 1997 Phys. Lett. A 237 73