Research on variable probability assignment technique of genetic algorithm for WTA of ground target attacking

Guangyuan Fu, Chao Wang1, Daqiao Zhang, Ranhui Wang, Shujuan Zhang and Fei Tan
Xi'an Research Inst. of Hi-Tech, Hongqing Town, Xi'an, 710025, P. R. China

1Email: 1129154594@qq.com

Abstract. The targets and the use of weapons are complicated on attacking ground target, Besides, the weapon-target assignment (WTA) research is extremely difficult and lacking. For a timely and rational WTA scheme can not only help to seize fleeting war opportunities, but also optimize the use of weapon resources to maximize the battlefield benefits with minimum costs, this article builds a mathematical model and uses genetic algorithm (GA) to find the optimal WTA result. But the algorithm is difficult to prepare better initial population because of the random assignment of variables which results in undesirable optimization results. To overcome the challenge in initializing GA, this article puts forward a variable probability assignment technique. This technique first uses feasible solutions to determine the probability distribution of the variable values, then, uses probability assignment mode to replace the stochastic uniform assignment mode to initialize variables, and produces the initial population individual close to the optimal individual with certainty. The simulation calculation shows that the improved GA can effectively solve the WTA problem of the large scale ground targets attacking with better performance.

1. Introduction
WTA [1-5] is the key link of the operational command, which directly affects the operational process and outcome. It’s an important military issue for military powers to compete in. At present, WTA research has made great progress, but mostly regarding the study of air defense WTA (AD-WTA), which addresses air defense interception, whereas the ground targets attacking WTA (GTA-WTA) problem, which addresses ground targets, has rarely been studied. Compared with air defense interception, ground targets are diverse and suitable for many weaponry types, and the scale and complexity of the WTA problem are greater, requiring the use of more efficient algorithms. Using superior firepower against key enemy ground targets is an important means to quickly win a war. In recent local wars, the US military greatly weakened the enemy’s combat forces and won the war quickly by attacking and destroying the enemy’s key targets. The combating opportunities are fleeting, and weaponry resources are limited. Failure to allocate weaponry resources quickly and appropriately will not only waste weapons, but also spoil combat opportunities. Therefore, it is an extremely necessary and urgent work to study the GTA-WTA problem in depth and design an efficient optimization algorithm [6, 7].

Currently, the optimization algorithms for a GTA-WTA problem primarily include intelligent algorithms such as the GA [5], the particle swarm optimization (PSO) algorithm [8-10] and the bat algorithm [11]. For the GTA-WTA problem, Wang et al. [5] designed a variable value control method, which greatly improved computational efficiency by reasonably compressing the search space, and
ensured the quality of the solution by improving the mutation strategy. However, when the scale was too large, the optimization time was still long. Wang et al. [8] improved the particle initialization and weight coefficient selection method of the PSO algorithm, which improved the calculation efficiency and the quality of the solution. But it reduced the weight coefficient by a fixed number of iterations, without considering the scale of the problem and the quality of the solution at each stage. That makes it only applicable to the GTA-WTA problem of specific scale. Xia et al. [9], based on the PSO algorithm, introduced variable random decomposition strategy and cooperative co-evolution framework to solve the “curse of dimensionality” situation when solving large-scale GTA-WTA problem, which improved the optimization effect to a certain extent, but the result may fall into local optimum. Liu et al. [11] proposed an improved bat algorithm that combines the idea of niche elimination, which improved the quality of the solution. But it didn’t improve the computational efficiency effectively.

Comparing all kinds of solutions to the GTA-WTA problem, it is found that the method in reference [5] is simple and efficient, and has a better result. The reference [5] proposed a genetic algorithm-based variable value control (GA-VVC) method in which the concept of damage contribution is first defined; then, the damage contribution is used to restrain the amount of each type of weapon when striking each target, and the total weapon amount when striking a single target is minimized to decrease the variable initialization space and rationalize the variable value to expedite the search for feasible solutions. Then, the variation range of the variables is expanded, and the mutation strategy is changed to eliminate the problem of missing solution space caused by the narrowing of the variable value range and expedite the convergence of the algorithm. Based on the GA-VVC method, this paper designs a variable probability assignment technique. Compared with the GA-VVC method, the improved method can generate the initial population individual close to the optimal individuals and can effectively solve the poor optimization effect of large-scale GTA-WTA problems caused by random assignment of variables in reference [5], and further improve the computational efficiency and the quality of solution.

2. Model of GTA-WTA problem
In order to construct the model of GTA-WTA problem correctly, the following assumptions are made:

1) The damage probability of the weapon to the target is comprehensive damage probability, considering the weapon’s penetration probability, the target hit probability and the target damage probability.

2) The same type of weapon has the same comprehensive damage probability for similar targets.

3) Do not consider the order of the target strikes, and the maximum projection ability of a wave.

Based on the above assumptions, GTA-WTA problem can be briefly described as: There are \( N \)-type weapons to combat the \( M \)-kind targets, so that the average damage coefficient of the \( M \)-kind targets reaches \( C_1, C_2, \ldots, C_M \). The number of \( N \)-type weapons is \( N_1, N_2, \ldots, N_N \), the value of \( M \)-kind targets is \( V_1, V_2, \ldots, V_M \), the number of \( M \)-kind targets is \( M_1, M_2, \ldots, M_M \), and comprehensive damage probability of \( i \)-type weapon against \( j \)-kind target is \( p_{ij} \), \( i = 1, 2, \ldots, N \), \( j = 1, 2, \ldots, M \). The best goal of GTA-WTA is to meet target damage requirement with minimum weapon value consumption. If the number of \( i \)-type weapon acting on the \( k \)-th target of \( j \)-kind is \( m_{ijk} \), \( k = 1, 2, \ldots, M_j \), and value consumption of weapon is \( V_i \), then the model of GTA-WTA can be described as:

\[
\min V = \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{k=1}^{M_j} m_{ijk} \cdot V_i
\]  

3. Variable probability assignment technique of GA
The basic idea of GA [12,13] is derived from the genetic evolution of organisms. It simulates the evolution process of survival of the fittest in the natural biological population. It can effectively solve a variety of optimization problems [14], and its improved algorithm is also widely used to solve GTA-WTA problems [15,16]. The genetic evolution of biological population begins with the initial
population, and the pros and cons of the initial population affect the direction and speed of individual evolution directly. If the initial population is closer to the optimal individual, the individual will soon evolve into the optimal individual.

\[
\sum_{k=1}^{k=M_j} (1 - \prod_{i=1}^{j=N} (1 - p_{ij})^{m_{ik}}) (M_i)^{-1} \geq C_j
\]
\[
\sum_{j=1}^{j=M} \sum_{k=1}^{k=M} m_{ijk} \leq N_i
\]
\[
m_{ijk} \geq 0, \text{ and is an integer}
\]

The GA initializes the population by means of stochastic uniform assignment. It is difficult to generate the initial population individuals close to the optimal individuals. In order to generate the initial population individual close to the optimal individuals and solve the GTA-WTA problem quickly, a variable probability assignment technique is designed.

3.1. Feasibility analysis of variable probability assignment

Population initialization is an important part of GA. The closer an individual is to the optimal individual, the faster the algorithm search is, and the better the result. Population individuals contain multiple variables, and the essence of initialization is the assignment of variables to individuals. The proposed variable probability assignment technique first solves the probability distribution of the variable value by taking multiple feasible solutions as the sample, and takes some feasible solutions as the initial population individuals. Then according to the probability distribution, an individual is assigned to each initial population, which is called the alternate individual, and all the alternate individual constitute an alternate population. Finally, according to the variable probability distribution, the probability of the value of the initial individual and its alternate individual variables is calibrated. And compare the probability of the value of the same variable of the initial individual and its alternate individual variable, if the former is less than the latter, then the variable of the initial individual is reassigned, which assigns the value of the same variable of alternate individual.

The GA uses stochastic uniform assignment to take values from the variable range and assign the variables with equal probability, so that the population individuals can take any point in the search space with the equal probability, and can not produce the initial population individuals close to the optimal individuals with certainty. This assignment method is simple and effective for small-scale problems. However, for large-scale problems, because the search space is extremely large, and the optimal solution or even the feasible solution is only in a very small part of the search space, it is difficult to find better results quickly by iterative solution of the initial population generated by this method.

Comparing and analyzing the variables of the feasible solution and the individual of the optimal solution of GA, it can be found that the values of most variables are similar, and the values of other variables are also similar, indicating that the feasible solution contains a large amount of information to find the optimal solution. Analyze variables between the feasible solutions, and find that the values of the variables are similar, and the closer the value is to the value of the optimal solution, the more the number of variables with the same value. From the above two conclusions, it can be seen that the value of the feasible solution individual variable obeys a certain probability distribution, and the probability distribution can be solved by using multiple feasible solutions, and its probability density is larger near the value of the optimal individual variable. So the variable assigned by the probability distribution has a large probability to take the value close to the optimal solution individual variable.

The GA can iterate the feasible solution quickly. Although feasible solution individuals are relatively close to the optimal solution individuals, only some of the variables are similar or identical. With the optimization progresses, the number of variables with similar or identical values increases slowly, but the number of iterations required increases significantly. The variable probability assignment technique can quickly reassign some variables of feasible solution, and the reassignment of variables is closer to the value of the optimal individual variables, so that the value of more variables
can quickly approach the value of the optimal individual variables, and greatly improve the speed of the optimization algorithm.

3.2. Probability calculations

3.2.1. Probabilistic solution to the collection of individual samples. As the data for solving the probability distribution of variables, the sample individuals should be more or less close to the optimal solution individuals, so that the sample individuals are required to be feasible solutions. At the same time, sample individuals should be extensive, and the convergent sample individuals can only reflect the local information of the optimal solution individuals, which require that the sample individuals have certain differences.

For the requirement that all the sample individuals should be feasible solutions, the GA-VVC method can be used to iterate the feasible solutions rapidly. For the requirement that the sample individuals should have certain differences, the iteration can be stopped when the GA-VVC method iterates to the feasible solutions, and the optimal solution among the generated feasible solutions can be retained. Then repeat the process until enough sample individuals are collected. The collection process of sample individuals is shown in Figure 1.

![Figure 1. Collection flow chart of sample individuals.](image)

3.2.2. Probability distribution solution of individual variables. GTA-WTA problem is an integer programming problem. The individual is coded by the decimal coding method of GA. The collected sample individuals can be expressed as \( x_i = (x_{i1}, x_{i2}, ..., x_{ir}) \), \( i=1,2,3,... \) \( R \), \( R \) is the number of sample individuals, the experiment of \( R \) shows the value of \( R \) between 50 and 100 is suitable; \( r \) is the number of variables of an individual.

The value of the feasible individual variable follows a certain probability distribution, and the probability distribution can be calculated according to multiple sample individuals. The solution flow is:
1) The range of values of each variable in the statistical sample
If the maximum value of the \( J \)-th variable in the sample is \( x_j^* \), then \( x_j^* \) is:

\[
x_j^* = \max(x_{j1}, x_{j2}, \ldots, x_{jR})
\]

According to the value of \( x_j^* \), the value range \( k_j \) of the \( J \)-th variable is:

\[
k_j \in [0, x_j^*], \quad k_j \text{ is an integer}
\]  

2) Statistic the number of values taken by each variable within their range of values
Let the number of the \( J \)-th variable in the sample take the same value \( k_j \) as \( f(j, k_j) \). Let

\[
f(j, k_j) = 0, \quad i = 0, \text{ repeat the following steps until } i > R, \text{ then get the value } f(j, k_j^*) .
\]

\[
i = i + 1, \text{ if } x_j = k_j^*, \text{ then } f(j, k_j^*) = f(j, k_j^*) + 1 .
\]

3) Calculating the probability distribution of variables in the range of values
Let the probability of the value \( k_j^* \) of the \( J \)-th variable be \( p(j, k_j^*) \):

\[
p(j, k_j^*) = f(j, k_j^*) / R
\]

3.3. Initialization of population individuals
The specific process of population initialization is as follows:

1) Population initialization. Select \( R^* \) individuals randomly from the sample individuals as the initial population, expressed as \( y_1^{(1)} = (x_{j1}, x_{j2}, \ldots, x_{jR}) ; i = 1, 2, 3, \ldots, R^* ; \)

2) Construction of alternate population. From the probability distribution of variables, it can be seen that the probability of the value \( k_j^* \) of the \( J \)-th variable is \( p(j, k_j^*) \), the probability of the value in the interval \( [0, k_j^*] \) is \( \sum_{t=0}^{t=k_j^*-1} p(j, t) \), and the probability of the value in the interval \( (k_j^*, x_j^*] \) is \( \sum_{t=k_j^*+1}^{t=x_j^*-1} p(j, t) \). If the random number extracted from the interval \([0,1]\) is \( ran \), then the \( k_j^* \) value is:

\[
k_j^* = \begin{cases} 
0 \quad \text{when } ran \leq p(j, 0) \\
k_j^* \quad \text{when } \sum_{t=0}^{t=k_j^*-1} p(j, t) < ran \leq \sum_{t=0}^{t=k_j^*} p(j, t) 
\end{cases}
\]

Multiple use of formula (6) can generate the required alternate population \( y_1^{(2)} = (x_{j1}, x_{j2}, \ldots, x_{jR}) ; \)

3) Initial population reassignment. Although initial population individuals are feasible solution individuals, they are relatively close to the optimal solution individuals, but only some of them have similar or identical values. According to the variable probability distribution, some variables of the initial population individual can be quickly reassigned to make more variables close to the values of the optimal individual variable. The assignment of \( x_{ij} \) satisfies:

\[
x_{ij} = \begin{cases} 
x_{ij}^* \quad p(j, x_{ij}) \geq p(j, x_{ij}^*) \\
x_{ij}^* \quad p(j, x_{ij}) < p(j, x_{ij}^*) 
\end{cases}
\]

By using formula (7), the values of variables closer to the optimal individual in the initial population and its alternate population are assigned to the initial population individuals, resulting in a better initial population.

3.4 GTA-WTA problem solving process based on the improved GA
The flow chart of variable probability assignment technique applied to genetic algorithm to solve GTA-WTA problem is shown in Figure 2. The flow chart can be summarized as follows:
1) Solution of probability distribution of variables. Multiple feasible solutions are obtained by using the GA-VVC method, and the probabilistic distribution of variables can be calculated according to formula (5);

2) Population initialization. Multiple feasible solutions are randomly selected as the initial population, and the initial population can be reassigned by formula (7);

3) Algorithmic iteration. The improved GA in reference [5] is used for replication, crossover and mutation;

4) Iterative termination. Determine whether the target damage requirement is met, and determine whether the maximum fitness value of the population meets the requirement that there is no change within 100 generations. If both requirements are met, the iteration terminates; otherwise, proceed to step 3.

Figure 2. Flow chart of GTA-WTA problem solution.

4. Simulation and analysis of the GTA-WTA problem

4.1. GTA-WTA problem cases

For the typical GTA-WTA problem, four different cases were set based on the same background for comparative analysis.

Case background: A total of five different types of weapons are used, and the value of each type of weapon (unit: million), and the investment amount in each case is shown in Table 1. Combat a certain number of 6 types of ground targets, the target number in each case is shown in Table 2. The damage probability of each type of weapon for each type of target is shown in Table 3. The average damage coefficient of each type of the target is required to be 0.8, 0.8, 0.9, 0.85, 0.8 and 0.9. Then, try to solve the best weapon allocation scheme.
and “the time consumption” is the mean of the run times of 100 simulations. The weapon consumption value and the optimization time consumption of the GA-VVC method and the method in this paper are shown in Figure 3 and Figure 4.

**Table 1.** The value and total amount of all types of weapons.

| Weapon type | Total amount of weapons (unit: one weapon) | Value coefficient of weapon |
|-------------|-------------------------------------------|----------------------------|
|             | Case 1 | Case 2 | Case 3 | Case 4 |                          |
| W1          | 5      | 10     | 15     | 20     | 8                         |
| W2          | 5      | 5      | 10     | 15     | 7                         |
| W3          | 5      | 10     | 15     | 20     | 6                         |
| W4          | 10     | 10     | 20     | 25     | 5                         |
| W5          | 5      | 10     | 15     | 20     | 4                         |

**Table 2.** Target type and quantity.

| Target type | Target quantity (unit: one weapon) |
|-------------|-------------------------------------|
|             | Case 1 | Case 2 | Case 3 | Case 4 |
| T1          | 2      | 3      | 4      | 5      |
| T2          | 1      | 3      | 4      | 5      |
| T3          | 1      | 3      | 4      | 5      |
| T4          | 1      | 3      | 4      | 5      |
| T5          | 1      | 2      | 4      | 6      |
| T6          | 2      | 2      | 4      | 6      |

**Table 3.** Weapon’s damage probability to the target.

| Target type | W1 | W2 | W3 | W4 | W5 |
|-------------|----|----|----|----|----|
| T1          | 0.85 | 0.60 | 0.50 | 0.45 | 0.40 |
| T2          | 0.46 | 0.35 | 0.60 | 0.50 | 0.35 |
| T3          | 0.40 | 0.32 | 0.60 | 0.50 | 0.40 |
| T4          | 0.80 | 0.70 | 0.65 | 0.50 | 0.40 |
| T5          | 0.40 | 0.30 | 0.85 | 0.70 | 0.60 |
| T6          | 0.61 | 0.45 | 0.75 | 0.60 | 0.50 |

From Figure 3, it can be seen that the weapon consumption value optimized by this method is always less than that of the GA-VVC method. With the increase of the number of variables, the difference of the weapon consumption value optimized by the two methods also increases. It shows that the method in this paper is better than the GA-VVC method.

It can be seen from Figure 4 that with the increase of the number of variables, the optimization time of the GA-VVC method and the method in this paper increases, and the slope of the curve also increases. However, optimization time of the method in this paper is always less than that of the GA-VVC method, and the slope of the curve is always smaller than that of the GA-VVC method. It shows that the method in this paper takes less time than the GA-VVC method.

Based on the comparison and analysis, the improved GA proposed in this paper is correct and effective, which can further improve the computational efficiency and the quality of solution of GTA-WTA problem, and the larger the scale, the more obvious the advantages.
Figures 3 and 4 show the average weapon consumption value and the average time consumption of the two methods, but the robustness of the two methods cannot be verified and demonstrated. The robustness of the two methods can be measured here by the standard deviation of the weapon consumption value and the time consumption. For a set of values $y_1, y_2, \ldots, y_n$, the standard deviation $\sigma$ is:

$$\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (y_i - \frac{1}{n} \sum_{j=1}^{n} y_j)^2}$$

(8)

According to formula (8), the standard deviation of the weapon consumption value and the time consumption of the two methods for 100 experiments in the 4 cases can be calculated. The results are shown in Table 4.

**Table 4.** The standard deviation of the weapon consumption value and the time consumption.

| Case  | Method of reference [5] | The proposed method |
|-------|-------------------------|---------------------|
|       | The standard deviation of the weapon consumption value | The standard deviation of the time consumption | The standard deviation of the weapon consumption value | The standard deviation of the time consumption |
| Case 1 | 7.8 | 1.5 | 6.3 | 0.9 |
| Case 2 | 19.6 | 4.3 | 11.5 | 2.1 |
| Case 3 | 37.3 | 9.7 | 19.6 | 4.4 |
| Case 4 | 54.2 | 12.6 | 31.7 | 8.9 |

It can be seen from Table 4 that the standard deviation of the weapon consumption value and the time consumption are smaller than the GA-VVC method in reference [5], indicating that the proposed method has better robustness.

5. Conclusions
Aiming at the special problem of GTA-WTA, this paper designs a variable probability assignment technique, which can generate the initial population individuals close to the optimal individuals with
certainty, and effectively improve the computational efficiency and the quality of the solution, and have better robustness. Simulation experiments and comparative analysis show that this method has obvious advantages in computing efficiency and quality of solution for large-scale GTA-WTA problems, and the larger the scale, the more obvious the advantages.

Acknowledgments
This work was jointly supported by the National Natural Science Foundation for Young Scientists of China (Grant No. 61403397, 61202332), and the Natural Science Foundation of Shanxi Province, China (Grant No. 2015JM6313).

References
[1] Zhang J 2015 Research on weapon target assignment problem Proceedings of the 3rd China Conference on Command and Control 762-764
[2] Huang J 2016 Research on the Model of Weapon Target Assignment Wuhan Huazhong University of Science & Technology
[3] Xin B, Wang Y P and Chen J 2018 An efficient marginal-return-based constructive heuristic to solve the sensor-weapon-target assignment problem IEEE Transactions on Systems, Man, and Cybernetics: Systems 1-12
[4] Li Y, Kou Y X and Li Z W 2018 An Improved Nondominated Sorting Genetic Algorithm III Method for Solving Multiobjective Weapon-Target Assignment Part I: The Value of Fighter Combat International Journal of Aerospace Engineering
[5] Wang R H and Wang C 2016 Variable values control technology of genetic algorithm for wta of attacking ground target Acta Armamentarii 37 (10) 1889-1895
[6] Morris R D 2004 Weaponeering: Conventional Weapon System Effectiveness Reston: American Institute of Aeronautics and Astronautics INC
[7] Lee Z J, Su S F and Chou Y L 2003 Efficiently Solving General Weapon-Target Assignment Problem by Genetic Algorithms With Greed Eugenics IEEE Journal on Systems, Man, and Cybernetics-Bart B Cybernetics 33(1) 119-120
[8] Wang S H, Yang Q S, Wang R H et al 2017 Particle Swarm Optimization Based Weapon-target Assignment for Attacking Ground Targets Electronics Optics & Control 24 (3) 36-40
[9] Xia W, Liu X X, Fan Y T et al 2016 Weapon-target Assignment with an Improved Multi-objective Particle Swarm Optimization Algorithm Acta Armamentarii 37(11) 2085-2093
[10] Zhou D Y, Li X Y, Pan Q et al 2016 Multiobjective Weapon-Target Assignment Problem by Two-Stage Evolutionary Multiobjective Particle Swarm Optimization IEEE International Conference on Information and Automation (ICIA) 2016
[11] Liu S S, Xu R M and Pan J J 2017 Research on the Modeling and Solving of the Joint Long-range Strike Weapon Target Assignment Problem Based on the Niche Bat Algorithm Journal of Equipment Academy 28(2) 93-98
[12] Guo L, Chen G L,Wang X F, Zhuang Z Q et al. 2012 Genetic Algorithm and its Application Beijing City, Beijing Post and Telecommunications Press
[13] Wu A S, Garibay I 2002 The proportional genetic algorithm: gene expression in a genetic algorithm Genetic Programming and Evolvable Machines 3(2) 157-192
[14] Aytug H, Khouja M and Vergara F E 2003 Use of Genetic Algorithms to Solve Production and Operations Management Problems International Journal of Production Research 41(55) 3955-4009
[15] Yu L F, Liu J, Zhang W M et al 2016 Review of weapon-target assignment problem algorithm Mathematics in Practice and Theory 46(2) 26-32
[16] Wang J, Luo P C and Zhou J L 2017 A memetic algorithm for constrained weapon target assignment problems 2017 2nd IEEE International Conference on Computation Intelligence and Applications (ICCIJA) 182-188