Time-correlated excitation of particles during relaxation of Hamiltonian system

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Abstract. We investigate the long-time evolution of the Hamiltonian mean field model. Below the critical energy, high-energy particles are excited intermittently from a cluster. We show evidence that this excitation is not due to a thermal (random) hopping but a collective motion by calculating the probability distribution of the lifetime of the “fully-clustered” states. We find that the distribution obeys a power law whose index appears to depend on the energy density.

Chaotic dynamics in Hamiltonian systems with many degrees of freedom are interesting and important topics for understanding transient phenomena in nature. Such a system often leads to uniformly random states, i.e., thermal equilibrium, after long-time evolution. Uniform thermalization, however, is not the only future of chaotic motion in Hamiltonian systems. It is quite instructive to extract and formulate the dynamical motion which stationary statistical mechanics does not suffice to describe.

When we consider the ordered motion in a dissipative system, attractor concepts are important. To a Hamiltonian system, on the other hand, the attractor concept is not applicable. Nonetheless, this kind of the motion over several quasi-stationary states is often observed in a Hamiltonian system. For example, a one-dimensional self-gravitating system shows itinerant behavior between quasi-equilibria and transient states during long-time evolution \cite{1}. The transient state is a state where one particle bears the highest energy throughout the lifetime of the state, which is distinguished from the quasi-equilibrium state \cite{1}.

In this paper we show itinerant behavior between two states, “fully-clustered” and “excited” states, in the Hamiltonian mean field (HMF) model \cite{2} for which the Hamiltonian is

\begin{equation}
H = K + V = \sum_{i=1}^{N} \frac{p_i^2}{2} + \frac{1}{2N} \sum_{i,j=1}^{N} \left[1 - \cos(\theta_i - \theta_j)\right],
\end{equation}

where \(N\) is the number of the particles. The HMF model is a globally coupled pendulum system, and the equation of the motion can be expressed as that of a perturbed pendulum. One of the advantages of this model is that we can clearly distinguish the states; Using the notion of the separatrix, we introduce a distinction between two class of particles: high-energy (HEP) and low-energy (LEP) particles. HEP and LEP are defined by whether they move outside or inside the separatrix, respectively. Then, we define the “fully-clustered” and “excited” states as one without HEP or with HEP, respectively.
Below the critical temperature, it has been suggested, the HMF model shows an interesting phenomenon where the rotators form a drifting cluster [2]. Now we show the evidence that the excitation is not due to a thermal activation process. Here we calculate the probability distribution of the lifetime of the fully-clustered state. The lifetime is defined as the interval from the excitation of a HEP from a cluster to the absorption of the particle into the cluster again. If the transition process is simply random induced by thermal fluctuation, then the transition rate for each particle between the low-energy state (inside the separatrix) and high-energy state (outside the separatrix) will follow the Arrhenius-law $\gamma \equiv \exp(-\Delta E/k_BT)$, where $\Delta E$ is a energy difference between the states. The lifetime distribution of HEP in this case will be exponential with $(1 - \gamma)^t = \exp[\log(1 - \gamma)t]$.

Our results are shown in Figs. 1. The distribution is not exponential but, rather, obeys a power law, which indicates that the excitation of a particle from a cluster is not random but time correlated. Our discovery is quite interesting and important: The power-law distribution of the lifetime implies that, at the moment of ejection of an HEP, the system still ”remembers” when the previous HEP was swallowed into the cluster. One may think that the system tends to behave thermally as the increase of particle number, but this power law is observed even for large number of particles. The result may give new insight to the relaxation property of many-body systems.

It will be interesting to investigate the origin of the strong temporal correlation which gives the power-law behavior of the lifetime of HEP. Collective behavior of particles is one candidate and the understanding of its mechanism will cast a new light on the dynamics of many-particle systems.

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