Effects of Extra Dimensions on Unitarity and Higgs Boson Mass

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Abstract

We study the unitarity constraint on the two body Higgs boson elastic scattering in the presence of extra dimensions. The contributions from exchange of spin-2 and spin-0 Kaluza-Klein states can have large effect on the partial wave amplitude. Unitarity condition restrict the maximal allowed value for the ratio $r$ of the center of mass energy to the gravity scale to be less than one. Although the constraint on the standard Higgs boson mass for $r$ of order one is considerably relaxed, for small $r$ the constraint is similar to that in the Standard Model. The resulting bound on the Higgs boson mass is not dramatically altered if perturbative calculations are required to be valid up to the maximal allowed value for $r$.

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It has recently been proposed that gravitational effects can become large at a scale $M_S$ near the weak scale due to effects from extra dimensions [1,2], quite different from the traditional thought that gravitational effects only become large at the Planck scale $M_{Pl} \sim 10^{19}$ GeV. In this proposal the total space-time is $D = 4 + n$. The relation between the scale $M_S$ and the scale $M_{Pl}$, assuming all extra dimensions are compactified with the same size $R$, is given by $M_{Pl}^2 \sim R^n M_S^{2+n}$. With $M_S$ near a TeV and $n= 1$, $R$ would be too large. However, with $n$ larger than or equal to 2, the theory is not ruled out for $M_S \sim 1$ TeV. The lower bound for $M_S$ is constrained, typically, to be of order one TeV from present experimental data [3–7]. Future experiments will provide more stringent constraints [3–7].

There are many interesting phenomena due to the presence of extra dimensions [3–9]. In this paper we study effects from extra dimensions on the unitarity condition of partial wave amplitude in elastic two body Higgs boson scattering, and to study implications for the validity of perturbative calculations and for allowed Higgs boson mass.

In the minimal Standard Model (SM) there is a neutral Higgs boson $H$ resulting from spontaneous symmetry breaking of $SU(2)_L \times U(1)_Y$ to $U(1)_{em}$ due to the Higgs mechanism. The mechanism for spontaneous symmetry breaking is not well understood. There is no experimental evidence favoring any particular mechanism, such as the Higgs mechanism. The discovery of the Higgs boson and understanding of its properties are fundamentally important [10]. Many methods have been proposed to produce and to study the properties of Higgs bosons [10]. One of the most important issue is its mass. At present the lower bound on SM Higgs boson mass $m_H$ is set by LEP II to be 95.5 GeV at 95% C.L. [11]. There are many theoretical constraints on the Higgs boson masses. The constraint from unitarity of partial wave amplitudes of longitudinal gauge boson and/or Higgs boson scatterings provide some of the interesting upper bounds on the mass [12–15].

In the presence of extra dimensions there are additional contributions to gauge and/or Higgs boson scatterings due to exchanges of spin-2 and spin-0 graviton excitations (the KK states). The effects of these KK states can affect the partial wave amplitudes significantly if the ratio $r$ of the center of mass frame energy $\sqrt{s}$ and the gravity scale $M_S$ is close to or
larger than one. Unitarity condition for partial wave amplitude constrain $r$ to be less than one if perturbative calculations are valid. The allowed range for Higgs boson mass can also be different from that in the SM depending on the value of $r$. We find that effects from extra dimensions affect the Higgs boson scattering $HH \rightarrow HH$ the largest. In the following we will concentrate on this process, and will comment on other processes at the end.

In the SM, the scattering amplitude for the process $HH \rightarrow HH$ at tree level is given by

$$M_{SM}(s, t) = 3\sqrt{2}G_F m_H^2 (1 + \frac{3m_H^2}{s - m_H^2} + \frac{3m_H^2}{t - m_H^2} + \frac{3m_H^2}{u - m_H^2}),$$ (1)

where $s, t$ and $u = 4m_H^2 - s - t$ are the Mandelstam variables.

From the above expression, one obtains the $J=0$ partial wave amplitude $|a_{SM}^0|$.

$$|a_{SM}^0| = \frac{1}{16\pi} \left(\frac{4p_ip_f}{s}\right)^{1/2} \frac{1}{s - 4m_H^2} \int_{(s-4m_H^2)}^0 M_{SM}(s, t) dt$$

$$= \frac{G_F m_H^2}{8\sqrt{2}\pi} \sqrt{1 - \frac{4m_H^2}{s}} [3 + \frac{9m_H^2}{s - m_H^2} - \frac{18m_H^2}{s - 4m_H^2} \ln\left(\frac{s}{m_H^2} - 3\right)].$$ (2)

In the above $p_{i,f}$ are the momentum of the initial and final Higgs boson in the center of mass frame, respectively.

The Higgs boson mass is constrained if one requires the absolute value of $a_0$ not to violate the unitarity condition. There are many discussions of how to implement unitarity conditions. For our purpose of demonstrating possible large effects of extra dimensions, we use a weak condition $|a_0| < 1$ and work with tree level amplitude to obtain conservative bound. Applying this condition for $s \gg m_M^2$, one obtains

$$m_H^2 < \frac{8\sqrt{2}\pi}{3G_F} = 1010\text{GeV}. (3)$$

If $m_H$ is substantially less than the above bound, the magnitude of the amplitude is well within the bound everywhere.

With extra dimensions, there are new contributions to $HH \rightarrow HH$ due to exchange of KK states. Using the Feynmann rules derived in Ref [6], we obtain
\begin{align}
M_{NEW}(s, t) &= \kappa^2 \left\{ \frac{1}{s-m_H^2}[(2m_H^2-t)^2 + (2m_H^2-u)^2 - \frac{2}{3}(s-2m_H^2)^2 - \frac{4}{3}m_H^2 s] \\
&+ \frac{1}{t-m_H^2}[(2m_H^2-s)^2 + (2m_H^2-u)^2 - \frac{2}{3}(t-2m_H^2)^2 - \frac{4}{3}m_H^2 t] \\
&+ \frac{1}{u-m_H^2}[(2m_H^2-s)^2 + (2m_H^2-t)^2 - \frac{2}{3}(u-2m_H^2)^2 - \frac{4}{3}m_H^2 u] \right\} \\
&+ \kappa^2 \left\{ \frac{2(n-1)}{3(n+2)} \frac{(s+2m_H^2)^2}{s-m_H^2} + \frac{(t+2m_H^2)^2}{t-m_H^2} + \frac{(u+2m_H^2)^2}{u-m_H^2} \right\} .
\end{align}

The first and the second terms are due to exchanges of spin-2 and spin-0 KK states, respectively.

Summing over all intermediate KK states and projecting out the J=0 partial wave amplitude, we obtain

\begin{align}
a_0^{NEW} &= \sqrt{1 - \frac{4m_H^2}{s}} \left\{ \frac{2}{3(n+2)} + \frac{11s - 12m_H^2}{3nM_S^2} \\
&- \frac{2}{3(s-4m_H^2)} [M_S^2 Fn(4) + (6s - 8m_H^2) Fn(2) + \frac{6s(s-4m_H^2) + 16m_H^2}{M_S^2} Fn(0)] \right\} \\
&+ \frac{4(n-1)}{3(n+2)} \left\{ \frac{1}{2} (s+2m_H^2)^2 \frac{s^{n/2-1}}{M_S^{n+2}} (-i\pi + 2In(M_S/\sqrt{s})) + \frac{2}{n+2} + \frac{12m_H^2 - s}{nM_S^2} \\
&- \frac{2}{s-4m_H^2} [M_S^2 Fn(4) + 4m_H^2 Fn(2) + \frac{4m_H^4}{M_S^2} Fn(0)] \right\} \sqrt{1 - \frac{4m_H^2}{s}},
\end{align}

where

\begin{align}
In(x) &= \int_0^x \frac{y^{n-1}}{1 - y^2} dy, \\
Fn(\delta) &= \int_0^1 x^{n+\delta-1} \ln[(\frac{s-4m_H^2}{M_S^2}) + x^2] dx.
\end{align}

In the above we have used $\kappa^2 = 16\pi G_N = 16\pi (4\pi)^{n/2} \Gamma(n/2) R^{-n} M_S^{-(n+2)}$ as the convention for $M_S$.

In the expression for $a_0^{NEW}$ there are several constant terms which look dangerously large are all canceled by terms proportional to $Fn(4)$. In the large $M_S$ limit, $a_0^{NEW}$ is proportional to $1/M_S^4$ and approaches zero as $M_S$ goes to infinity with $s$ and $m_H$ kept finite. In this limit the theory reduces to the SM. However, when the ratio $\sqrt{s}/M_S$ approaches one, the real and imaginary parts of $a_0^{NEW}$ both become of order one and can violate the unitarity condition $|a_0| < 1$ even if $m_H$ is small. This indicates that the applicability of the effective theory, perturbatively, should be in the range $\sqrt{s} < M_S$. We remark that $a_0^{NEW}$ only becomes
sensitive to $m_H$ for small $M_S$. With sufficiently large $M_S$, $a_0^{NEW}$ by itself does not provide interesting information for $m_H$. However since the SM is sensitive to Higgs boson mass, the total contribution will provide information about $m_H$. To have a better understanding of the details we consider $|a_0|$ as a function of $r = \sqrt{s}/M_S$ and $m_H$ for four typical cases: a) $M_S = 5$ TeV and $n=2$; b) $M_S = 5$ TeV and $n=7$; c) $M_S = 100$ TeV and $n=2$; and d) $M_S = 100$ TeV and $n=7$, for illustrations. For cases c) and d), the results reduce to the SM if $\sqrt{s}$ is not too close to $M_S$, while for cases a) and b) the effects from extra dimensions can be large. The results are shown in Figs. 1 and 2.

From Figs. 1 and 2, we see that in all cases when $r$ approaches one, $a_0$ becomes large and can violate the unitarity condition setting the purterbative range (the maximal allowed range $r_{max}$ for $r$ where $|a_0| = 1$) of the theory to be: 0.81, 0.96, 0.81 and 0.96 for cases a), b), c) and d) with $m_H = 100$ GeV, respectively. With larger $m_H$, the allowed range for $r_{max}$ can be larger, as can be seen from Figs. 1 and 2, due to cancellation between contributions from the SM and extra dimensions, and the total $a_0$ is sensitive to $m_H$ as mentioned before. As long as $\sqrt{s}$ is much larger than $2m_H$, $r_{max}$ is not sensitive to $M_S$ but sensitive to the number of extra dimensions $n$. We have studied different value for $n$ up to 7 and find that in all cases, $r_{max}$ is constrained to be less than one, but varies from case to case. Violation of unitarity condition for $\sqrt{s} > M_S$ is not a big surprise because $M_S$ serves as a cut-off where gravity becomes strong. The calculation here provides a specific example. We stress, however, that results obtained using perturbative calculations with $\sqrt{s}$ close to $M_S$ are not reliable.

It has been shown [14] that in the SM at one loop level, for a given Higgs boson mass there is a critical scale $\sqrt{s_c}$ when $s$ is larger than $s_c$ the unitarity condition is violated ($s_c$ decreases as $m_H$ increases). With extra dimensions, the energy scale $s_{max}$ below which unitarity condition is reached earlier for small $M_S$ and $m_H$. If stronger requirement on $|a_0|$ is made as discussed in Ref. [14], the allowed $r_{max}$ will be smaller. The bound we obtained is a conservative one.

The amplitude $a_0$ is also sensitive to the Higgs boson mass $m_H$ as mentioned earlier. In
all cases the allowed range for the Higgs boson masses are altered, in some cases the change can be dramatical, such as case c), in certain ranges of \( r \). For small \( r \) the bound on the Higgs boson mass is similar to that in the SM. Higgs boson mass bound with smaller \( r \) is tighter than other ranges except \( r \) close to its maximal as can be seen from Figs. 1 and 2. The bound on Higgs boson mass must be the smallest in the whole valid range of \( r \) for perturbative calculation. Therefore the bound on the Higgs boson mass is not dramatically affect compared with SM case which is effectively determined with small value for \( r \) in our cases.

We also checked some other gauge and/or Higgs boson scattering processes. For example, the contribution to \( HH \to VV \) from extra dimensions in the limit neglecting the mass of the gauge boson \( V \), is given by

\[
a^{NEW}_0(HH \to VV) = \frac{1}{12} \frac{s^{n/2-1}}{M_S^{n+2}} \left[-i\pi + 2In(M_S/\sqrt{s})\right]s^2 \left(1 - \frac{4m_H^2}{s}\right)^{5/4}.
\]  

This is smaller than that for \( HH \to HH \). Unitarity consideration for \( HH \to HH \) obtain stronger constraint.

In conclusion we have shown that effects from extra dimensions can have large contributions to \( HH \to HH \). The range valid for perturbative calculations with extra dimensions is limited by the scale \( M_S \). Results obtained with \( \sqrt{s} > M_S \) are not reliable. Although Higgs boson mass bound can be drastically affected for certain ranges of \( \sqrt{s} \), the overall bound is not modified significantly if one requires perturbative calculation for \( HH \to HH \) to be valid up to maximal allowed value of \( r \).

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REFERENCES

[1] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B429, 263(1998); I. Antoniadis et al., Phys. Lett. B436, 257(1998); N. Arkani-Hamed, S. Dimopoulos and J. March-Russell, hep-ph/980124.

[2] I. Antoniadis, Phys. Lett. B246, 377(1990); R. Sundrum, hep-ph/9708329; G. Shiu and S.-H. Tye, Phys. Rev. D58, 106007(1998); hep-ph/9805157; Z. Kakushadze and S.-H. Tye, hep-ph/9809147; I. Antoniadis, et al., hep-ph/9804398.

[3] K. R. Dienes, E. Dudas and T. Gherghetta, Phys. Lett. B436, 55(1998); Nucl. Phys. B537, 47(1999); K.R. Dienes et al., hep-ph/9809406; N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Rev. D59, 086004(1999).

[4] G. Guidice, R. Rattazzi and J. Wells, Nucl. Phys. B544, 3(1999); E. Mirabelli, M. Perlestein and M. Peskin, Phys. Rev. Lett. 82, 2236(1999); J. Hewett, hep-ph/9811350; P. Mathews, S. Raychaudhuri and K. Sridhar, hep-ph/9811501, 9812486.

[5] T. Rizzo, hep-ph/9901203; 9902273; 9903475; 9904580; K. Agashe and N. Deshpande, hep-ph/9902263; K. Cheung and W.-Y. Keung, hep-ph/9903294; D. Atwood, S. Bar-Shalom and A. Soni, hep-ph/9903353; C. Balaze et al., hep-ph/9904220; H. Goldberg, hep-ph/9904318; H. Davoudiasl, hep-ph/9904425; Kingman Cheung, hep-ph/990460; 9904510; A. Gupta, N. Mondal and S. Raychaudhuri, hep-ph/9904234; G. Shiu, R. Shrock and H. Tye, hep-ph/9904262; K.-Y. Lee et al., hep-ph/9904355; 9905227; X.-G. He, hep-ph/9905295; Ithews, P. Poulose and K. Sridhar, hep-ph/9905395; I. Antoniadis, K. Benaki and M. Quiros, hep-ph/9905311; T. Han, D. Rainwater and D. Zeppenfeld, hep-ph/9905423.

[6] T. Han, J. Lykken and R.-J. Zhang, hep-ph/9811350.

[7] M. Graesser, hep-ph/9902310; P. Nath and M. Yamaguchi, hep-ph/9902323; M. Masip and A. Pomarol, hep-ph/9902467.
[8] N. Arkani-Hamed and S. Dimopoulos, hep-ph/9811353; Z. Berezhiani and G. Dvali, hep-ph/9811378; K. Dienes, E. Dudas and t. Gherghetta, hep-ph/9811428; N. Arkani-Hamed et al., hep-ph/9811448; A. Faraggi and M. Pospelov, hep-ph/9901299; M. Sakamoto, M. Tachibana and K. Takenaga, hep-ph/9902070; N. arkani-Hamed and M. Schmaltz, hep-ph/9903417; G. Dvali and A. Smirnov, hep-ph/9904211; L. Hall and C. Kolda, hep-ph/9904236; H.-C. Cheng, hep-ph/9904252; K. Yoshioka, hep-ph/9904433.

[9] S. Cullen and M. Perelstein, hep-ph/9903422; L. Hall and D. Smith, hep-ph/9904267; T, Banks, M. Dine and A. Nelson, hep-ph/9903019; S. Cline, hep-ph/9904493; N. Arkani-Hamed et al., hep-ph/9903224; A. Riotto, hep-ph/9904485; M. Maggiore and A. Riotto, hep-ph/9811089; A. Mazumbar, hep-ph/9902381; V. Barger et al., hep-ph/9905474.

[10] J. Gunion et al., Higgs Hunters’ Guide, Addison-Wesley, Reading, MA, 1990; J. Gunion, A. Stange and S. Willenbrock, hep-ph/9602238, in Electroweak Symmetry Breaking and New Physics at TEV, p23-145, Edited by T. L. Barklow.

[11] T. Greening, hep-ex/9903013.

[12] D. Dicus and V. Mathur, Phys. Rev. D7, 3111(1973).

[13] B. Lee, C. Quigg and H. Tacker, Phys. Rev. D16, 1519(1977).

[14] S. Dawson and S. Willenbrock, Phys. Rev. Lett. 62, 1232(1989); W. Marciano, G. Valencia and S. Willenbrock, Phys. Rev. D40, 1725(1989); L. Durand, J. Johnson and J. Lopez, Phys. Rev. Lett. 64, 1215(1990); Phys. Rev. D45, 3113(1992); L. Durand, P. Maher and K. Riesselmann, Phys. Rev. D48, 1084(1993).

[15] E. Lendvai, G. Pocsik and T. Torma, Mod. Phys. Lett. A6, 1195(1991); M. Seymour, Phys. Lett. B354, 409(1995).
FIG. 1. $|a_0|$ (vertical axis) as a function of $r = \sqrt{s}/M_S$ and $m_H$ (GeV) for $M_S = 5$ TeV with $n = 2$ and $n = 7$, respectively. Cases a) and b) are shown at left and right, respectively. The allowed parameter space with $|a_0| < 1$ are located at the left-lower corners indicated by the dented regions.

FIG. 2. The same as Fig. 1, but with $M_S = 100$ TeV. Cases c) and d) are shown at left and right, respectively. For $\sqrt{s}$ much smaller than $M_S$ ($r \ll 1$) the theory reduces to the SM.