Appendices for: Harvesting Full-Duplex Rate Gains in Cellular Networks with Half-Duplex User Terminals

Ahmad AlAmmouri, Hesham ElSawy, and Mohamed-Slim Alouini
Computer, Electrical, and Mathematical Sciences and Engineering (CEMSE) Division, King Abdullah University of Science and Technology (KAUST), Thuwal, Makkah Province, Saudi Arabia, Email: {ahmad.alammouri, hesham.elsawy, slim.alouini}@kaust.edu.sa

APPENDIX A

PROOF OF LEMMA 1

Starting with $L_{I_u}(s)$ which is given by,

$$L_{I_u}(s) \overset{(i)}{=} E \left[ \exp \left( -s \sum_{j \in \Psi_u} P_u h_j r_j^{-\eta} |I_u(\alpha)|^2 - s \sum_{j \in \Psi_d} P_d h_j r_j^{-\eta} |C_u(\alpha)|^2 - s\sigma_u^2(\alpha) \right) \right],$$

$$\overset{(ii)}{=} E \left[ \exp \left( \sum_{j \in \Psi_u} -sP_u h_j r_j^{-\eta} |I_u(\alpha)|^2 \right) \right] E \left[ \exp \left( \sum_{j \in \Psi_d} -sP_d h_j r_j^{-\eta} |C_u(\alpha)|^2 \right) \right] E \left[ \exp \left( -s\sigma_u^2(\alpha) \right) \right],$$

where (i) follows from [1, equation (15)] and (ii) from exploiting Assumption 2.

The first expectation in (1) represents the Laplace transform (LT) of the uplink (UL) to UL interference, which is denoted by $L_{I_{u-u}}(s)$. Due to the used UL power control and the closest BS association, an interference exclusion region exists around the receiving BS and is defined by $r_j < \left( \frac{P_u}{\rho} \right)^{\frac{1}{\eta}}$ [2]. Based on this, the LT can be evaluated as follows,

This document contains the appendices for the work in [1] which is submitted to 2016 IEEE International Conference on Communications (ICC).
\[ \mathcal{L}_{I_{u-d}}(s) = \mathbb{E} \left[ \exp \left( \sum_{j \in \Psi_d} -sP_d h_j r_j^{-\eta} |\mathcal{C}_d(\alpha)|^2 \mathbb{I} \left( r_j > \left( \frac{P_u}{\rho} \right)^{\frac{1}{\eta}} \right) \right) \right], \]

\[ \overset{(i)}{=} \mathbb{E}_{\tilde{\Psi}_d} \left[ \prod_{j \in \Psi_d} \mathbb{E}_{h_j, P_{u_j}} \left[ \exp \left( -sP_u h_j r_j^{-\eta} |\mathcal{I}_u(\alpha)|^2 \mathbb{I} \left( r_j > \left( \frac{P_u}{\rho} \right)^{\frac{1}{\eta}} \right) \right) \right] \right], \]

\[ \overset{(ii)}{=} \exp \left( -2\pi \lambda \mathbb{E}_{P_u} \left[ \int_0^\infty \mathbb{E}_{h} \left( 1 - \exp \left( -sP_u h r^{-\eta} |\mathcal{I}_u(\alpha)|^2 \right) \right) rdr \right] \right), \]

\[ \overset{(iii)}{=} \exp \left( -2\pi \lambda \frac{2}{\eta - 2} \mathbb{E}_{P_u} \left[ \frac{P_u^2}{2} |\mathcal{I}_u(\alpha)|^2 \right] _2F_1 \left[ 1, 1 - \frac{2}{\eta}, 2 - \frac{2}{\eta}, -\rho |\mathcal{I}_u(\alpha)|^2 s \right] \right), \]

where, (i) follows from the independence between \( \tilde{\Psi}_d, h_j, \) and \( P_{u_j} \), (ii) by exploiting Assumption 1 and by using the probability generation functional (PGFL) of PPP and (iii) by using the LT of \( h \) and by evaluating the integral. Equation (2) is the first exponential in [1, equation (22)].

The second expectation in equation (1) represents the LT of the interference from downlink (DL) on UL, which is denoted by \( \mathcal{L}_{I_{d-u}}(s) \). In this case, since no interference protection region exists around the receiving BS, and since the interfering BSs are distributed as PPP, it is also straightforward to find the expectation as in the previous analysis which results in,

\[ \mathcal{L}_{I_{d-u}}(s) = \mathbb{E} \left[ \exp \left( \sum_{j \in \Psi_u} -sP_u h_j r_j^{-\eta} |\mathcal{I}_u(\alpha)|^2 \right) \right], \]

\[ = \exp \left( -2\pi \lambda \int_0^\infty \mathbb{E}_{h} \left( 1 - \exp \left( -sP_u h r^{-\eta} |\mathcal{I}_u(\alpha)|^2 \right) \right) rdr \right), \]

\[ = \exp \left( -2\pi^2 \frac{\lambda}{\eta} \left( s |\mathcal{C}_u(\alpha)|^2 P_d \right)^{\frac{2}{\eta}} \csc \left( \frac{2\pi}{\eta} \right) \right). \]

Equation (3) is the second exponential in [1, equation (22)]. Last but not least, the third expectation represents the LT of the self interference (SI), by using [1, equation (16)] it results in \( U_3(s) \) given by [1, equation (25)].

Similar to \( \mathcal{L}_{I_u}(s) \), \( \mathcal{L}_{I_d}(s) \) can be expressed as,

\[ \mathcal{L}_{I_d}(s) = \mathbb{E} \left[ \exp \left( \sum_{j \in \Psi_d} -sP_d h_j r_j^{-\eta} |\mathcal{I}_d(\alpha)|^2 \right) \right] \mathbb{E} \left[ \exp \left( \sum_{j \in \Psi_u} -sP_u h_j r_j^{-\eta} |\mathcal{I}_u(\alpha)|^2 \right) \right] \mathbb{E} \left[ \exp \left( -s\sigma^2_{s_d}(\alpha) \right) \right]. \]
The first expectation in (4) represents the DL to DL interference, which is be denoted by $L_{I_{d-d}}(s)$. Due to the closest BS association, the interference protection region is defined by $r_j < r_o$, then by following similar steps as previously,

$$L_{I_{d-d}}(s) = \mathbb{E} \left[ \exp \left( \sum_{j \in \Psi_d} -sP_d h_j r_j^{-\eta} |\mathcal{I}_d(\alpha)|^2 \right) \right],$$

$$= \exp \left( -2\pi \lambda \int_{r_o}^{\infty} \mathbb{E}_h \left[ 1 - \exp \left( -sP_d hr^{-\eta} |\mathcal{I}_d(\alpha)|^2 \right) \right] r dr \right),$$

$$= \exp \left( \frac{-2\pi \lambda}{\eta - 2} r_o^{2-\eta} |\mathcal{I}_d(\alpha)|^2 s P_d \ _2 F_1 \left[ 1, 1 - \frac{2}{\eta}, 2 - \frac{2}{\eta}, -r_o^{-\eta} P_d |\mathcal{I}_d(\alpha)|^2 s \right] \right).$$

(5)

Equation (5) is the first exponential in [1, equation (21)]. The second expectation in (4) represents the UL interference on the DL. Similar to [3], we assume that the tagged UE is collocated with its serving BS. In the case of 2NT, the same interference protection region is defined as in the UL-UL interference. Since the expression is similar to the first expectation in (1) (only by changing $\mathcal{I}_u(\alpha)$ by $\mathcal{C}_d(\alpha)$) with the same interference protection region, it is straightforward to show that this expectation results in the second exponential in [1, equation (21)]. For 3NT, the intra-cell interference is excluded from the summation since it violates the defined interference protection region, so we have to obtain it separately. Let $P_{u_1}$, $h_{1-o}$, $r_{1-o}$, and $r_1$ denote the transmitted power of the interfering user, the channel gain between the two users, the distance between them and the distance between the interfering UE and the serving BS, respectively, then the LT of the interfering power can be expressed as,

$$\mathbb{E} \left[ e^{-sP_{u_1} h_{1-o} r_{1-o}^{-\eta} |\mathcal{C}_d(\alpha)|^2} \right] = \mathbb{E} \left[ e^{-s \rho r_1^\eta h_{1-o} (r_1^2 - 2r_o r_1 - 2 \cos(\theta))^{-\eta/2} |\mathcal{C}_d(\alpha)|^2} \right],$$

(6)

where the right hand side results by expressing $P_{u_1}$ as $\rho r_1^\eta$ and $r_{1-o}^2$ as $r_o^2 + r_1^2 - 2r_o r_1 \cos(\theta)$ by using the cosine rule, where $\theta$ is the uniformly distributed between 0 and $\pi$. Then $U_1(s, r_o)$ that is given by [1, equation (23)] can be obtained from (6) by averaging over $h_{1-o}$ which is exponentially distributed with unity mean. Last but not least, the third expectation represents the LT of the SI, by using [1, equation (17)] it results in $U_2(s)$ given by [1, equation (24)].

Finally, based on the closest BS association, the serving distance between the active UEs and their serving BSs which is given by [1, equation (25)] can be obtained from the null probability of the PPP as in [2] and based on it and on the UL power control, [1, equation (26)] can be easily obtained.
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