Gradients of Synchrotron Polarization: Tracing 3D Distribution of Magnetic Fields

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Abstract

We describe a new technique for probing galactic and extragalactic 2D and 3D magnetic field distribution using gradients of polarized synchrotron emission. The fluctuations of magnetic field are elongated along the ambient magnetic field. Therefore, the field variations are maximal perpendicular to the B-field. This allows tracing the B-field with synchrotron polarization gradients. We demonstrate that the Faraday depolarization allows us to map 3D B-field structure. The depolarization ensures that the polarization gradients sample the regions close to the observer with the sampling depth controlled by the frequency of radiation. We also analyze the B-field properties along the line of sight (LOS) by applying the gradient technique to the wavelength derivative of synchrotron polarization. This Synchrotron Derivative Polarization Gradients technique can recover the 3D vectors of the underlying B-fields. The new techniques are different from the Faraday tomography, as they provide a way to map the 3D distribution of B-field components perpendicular to the LOS. In addition, we find that the alignment of gradients of polarization with the synchrotron polarization can be used to separate the contribution of the foreground from the polarization of cosmological origin. We notice that the same alignment is also present for the dust polarization.

Key words: ISM: general – ISM: magnetic fields – ISM: structure – magnetohydrodynamics (MHD) – radio continuum: ISM

1. Introduction

Interstellar media (ISMs) of spiral galaxies are known to be both magnetized and turbulent (see Armstrong et al. 1995; Chepurnov & Lazarian 2010), with turbulent magnetic fields playing a critical role in many key astrophysical processes. These include star formation (see Mac Low & Klessen 2004; McKee & Ostriker 2007), the propagation and acceleration of cosmic rays (see Jokipii 1966; Yan & Lazarian 2008), and the regulation of heat and mass transfer between different ISM phases (see Draine 2009, for a list of the different ISM phases). In addition, the study of enigmatic cosmic microwave background (CMB) B-modes (Zaldarriaga & Seljak 1997) is impeded by the interference of fluctuations arising from the galactic magnetic field (Caldwell et al. 2017; Kandel et al. 2017a, 2017b). Thus, the detailed knowledge of the interstellar magnetic field structure is essential for both astrophysical and cosmological studies.

However, measuring the properties of the magnetic field is notoriously difficult. There are only a few techniques for studying the interstellar magnetic fields that are successful for probing magnetic fields in low-latitude diffuse ISM and molecular clouds. Zeeman splitting directly measures the strength of the line-of-sight (LOS) component of magnetic fields in molecular clouds (Crutcher 2012; Robishaw et al. 2015). Despite the long integration time employed to observe its weak effect, Zeeman measurements can only access the high-magnitude end of the interstellar magnetic fields in a mildly turbulent environment and can provide only limited morphology information. The morphology of the plane-of-sky component of magnetic fields can be obtained through starlight polarization (e.g., Pavel 2011) and polarized dust grain emissions (e.g., Ade et al. 2015). Besides the limitations or even failures arising from poorly controlled variations in radiation anisotropies, uncertain dust grain properties, and the effects of large optical depths (see Andersson et al. 2015), the polarization observations are in general difficult because the polarization percentage from telescope observations is comparable to the polarization arising from interstellar dust (Girart et al. 2015). The detection of magnetic fields in the high-latitude diffuse ISM is even more difficult, as it requires a higher sensitivity that is beyond the ability of present-day instruments.

Measurements of polarized synchrotron radiation and Faraday rotation (see Fletcher et al. 2011; Beck & Wielebinski 2013; Oppermann et al. 2015; Lenc et al. 2016; Van Eck et al. 2017) provide an important insight into the magnetic structure of the Milky Way and neighboring galaxies. More synchrotron data are becoming available with new instruments. For instance, the Low Frequency Array (LOFAR; see van Haarlem et al. 2013) provides insight into low-frequency synchrotron, for which Faraday depolarization may be important. The value of synchrotron as the source of information is only going to increase, as the Square Kilometer Array will provide detailed maps of diffuse emission with unprecedented resolution (see Johnston-Hollitt et al. 2015; Sun et al. 2015). This motivates us to study new ways in which the synchrotron data can be used.

In what follows we deal with synchrotron polarization gradients (SPGs; Gaensler et al. 2011; Burkhart et al. 2012; Iacobelli et al. 2014; Sun et al. 2014; Robitaille & Scaife 2015; Herron et al. 2017), but in this paper we use them in a way that is different from the earlier studies. This work continues our exploration of gradients of synchrotron polarization under the framework of the velocity gradient technique (VGT) as a means of tracing magnetic fields in different media. VGT was introduced in a series of recent papers (González-Casanova & Lazarian 2017; Yuen & Lazarian 2017a, 2017b; Lazarian & Yuen 2018) and is based on the modern understanding of MHD turbulence cascade (Goldreich & Sridhar 1995, hereafter GS95; see also Brandenburg & Lazarian 2013, for a review) and turbulent reconnection (Lazarian & Vishniac 1999, hereafter LV99; Eyink et al. 2011; see Lazarian et al. 2015, for a review).
In particular, it is important that velocity eddies are aligned with the surrounding local\(^1\) magnetic field (LV99, Cho & Vishniac 2000; Maron & Goldreich 2001). The VGT has proven to be a new promising way of studying the magnetic field in both galactic H\(_1\) and molecular clouds (see Yuen & Lazarian 2017b; Lazarian & Yuen 2018).

For Alfvénic turbulence, magnetic fluctuations and velocity fluctuations are symmetric. This opens ways for magnetic field tracing by studying the gradients of observables of magnetic fields, e.g., synchrotron radiations. Synchrotron intensities and polarized intensities resulting from Faraday rotation are the primary informants about magnetic field fluctuations.

The closest to the new technique that we introduce in this paper is the one in Lazarian et al. (2018b, hereafter LYL17), where we proposed magnetic field tracing with synchrotron intensities. Unlike the traditional technique of using synchrotron polarization, the new technique employed the synchrotron intensity gradients (SIGs) to trace the magnetic field in the ISM. The advantage of the SIGs compared to the tracing of the magnetic field using orientations of synchrotron polarization is that they do not require polarization studies and, more importantly, do not require the correction of the Faraday rotation effect. This technique was applied in LYL17 to the Planck synchrotron intensity maps, and the results were shown to be in good agreement with the Planck magnetic field maps obtained using synchrotron polarization.

The present paper studies the ways of getting 3D distribution of magnetic fields by SPGs. In previous works, we described how to use the galactic rotation curve (D. F. González-Casanova & A. Lazarian 2018, in preparation) and multimolecular tracers (Y. Hu et al. 2018, in preparation) to map the magnetic field in 3D (see Section 9). The present paper explores the depolarization arising from the Faraday rotation to map the 3D interstellar magnetic field structure. Our study focuses on describing two new techniques, namely, the SPG (Sections 5 and 7) technique and the one that employs gradients of the wavelength derivative of synchrotron polarization, i.e., synchrotron polarization derivative gradients (SPDGs; Section 6). We note that the aforementioned techniques are different from the Faraday tomography (Burn 1966; Brentjens & de Bruyn 2005; see Iacobelli et al. 2013; Jelíč et al. 2014, 2015; Van Eck et al. 2017, for recent observation in LOFAR polarimetric observations) in a few ways. First of all, SPGs are capable of studying the magnetic field structure perpendicular to the LOS. In addition, both SPGs and SPDGs map the distributions of magnetic field obtained in real spatial distance along the line of sight rather than in terms of Faraday polarization depths. It is also important that, compared to the Faraday tomography, our techniques do not require measurements at hundreds of frequencies\(^2\) in order to acquire a relatively accurate result.

For our treatment of the gradients the guidance is provided by the theory of polarized synchrotron fluctuations formulated in Lazarian & Pogosyan (2016, hereafter LP16). A number of predictions of LP16 were tested in the earlier papers (see Lee et al. 2016; Zhang et al. 2016). In our present paper, a few more LP16 predictions are tested (see Section 5 for the structure dependence of correlation and Appendix A for the power-law prediction of LP16).

In Section 2, we describe the fundamental principle of the gradient technique. In Section 3, we briefly describe the theoretical foundation of SPGs. In Section 4, we discuss the MHD simulations that we use to test our predictions. In Section 5, we discuss the behavior of SPGs in the case of random-field Faraday rotation. In Section 6, we discuss the estimation of Faraday rotation measure and the relation to the synchrotron polarization derivative with respect to squared wavelength. In Section 7 we discuss how the Faraday rotation effect due to a mean field along the LOS would change the properties of SPGs. In Section 8, we establish the recipe of constructing a 3D magnetic field from the gradient technique. We provide the discussion of our results in Section 9 and summarize in Section 10. In Appendix A, we show the statistical testing of LP16. In Appendix B, we illustrate that the properties of the Zeeman measure gradients (ZMGs) are similar to those of the Faraday rotation measure. In Appendix C, we discuss the possibilities of using gradients of dust polarizations, as we did in synchrotron polarizations. In Appendix D, we discuss our preliminary recipe on the 3D magnetic field based on SPDGs.

2. The Fundamentals of the Gradient Technique

As we mentioned earlier, physically one can imagine the strong MHD turbulence (GS95) as a superposition of anisotropic eddies aligned with the magnetic field. Since turbulent reconnection (LV99) enables unconstrained mixing motions in the direction perpendicular to the magnetic field, the gradients of eddy structures are thus perpendicular to the local magnetic field directions. As a result, the velocity and magnetic field gradients for Alfvénic turbulence trace the local direction of the magnetic field. In compressible MHD turbulence, numerical simulations confirmed that there exist three weakly interacting cascade modes: Alfvén, slow, and fast modes (see Cho & Lazarian 2002). The slow modes are passive, i.e., their scaling is imposed by the Alfvén modes (GS95, Lithwick & Goldreich 2001; Cho & Lazarian 2002, 2003). Therefore, both Alfvén and slow modes, i.e., the modes that contain most of the energy of the turbulent cascade, exhibit velocity and magnetic field gradients that trace magnetic field directions.

Formally GS95 theory provides an adequate description of turbulence for sub-Alfvénic motion, i.e., corresponding to Alfvén Mach number \(M_\text{A} = V_L/V_\text{A}\) equal to unity. Here \(V_L\) is the turbulent injection velocity at the scale \(L\), and \(V_\text{A}\) is the Alfvén velocity. However, the theory was generalized in LV99 for \(M_\text{A} < 1\), where it is shown that the relations similar to those in GS95 are true for the turbulent motions at scales less than \(L M_\text{A}^2\). For larger scales the turbulence is weak (LV99; Galtier et al. 2000), but the fluctuations are still perpendicular to the magnetic field.

The super-Alfvénic turbulence, i.e., \(M_\text{A} > 1\), presents a more complicated case from the point of view of gradient studies. The motions at the scales larger than \(L M_\text{A}^3\) are marginally constrained by magnetic field, and therefore the
turbulence is essentially hydrodynamic (see Lazarian 2006). However, at scales less than the aforementioned scale the GS95 scalings are applicable. Spatial filtering of the large-scale contributions can help to reveal the magnetic field structure in this case (see Lazarian & Yuen 2018).

The theory of synchrotron polarization fluctuations, formulated in LP16, predicts that the scaling laws of polarization fluctuations can be related to the statistics of underlying fluctuations in a rather nontrivial way. Therefore, it is important to formulate general criteria when the gradients of measured fluctuations represent the underlying magnetic field structure.

The statistical properties of gradients reflect the statistical properties of the underlying anisotropic turbulence. There are two criteria for the gradient technique (for both velocity and magnetic fluctuations) to reliably trace magnetic field structure:

1. Fluctuations at the small scale are anisotropic, and the anisotropy (characterized by parallel and perpendicular dimensions of eddies $l_p$ and $l_\perp$) is with respect to the direction of the magnetic field local to the eddies.

2. The contribution from small-scale fluctuations dominates over that of the large scales.

The first condition is satisfied as a result of the properties of the MHD turbulence that we mentioned above, i.e., velocities and magnetic field structures arising from Alfvén and slow modes (see Cho & Lazarian 2003) are aligned with the local magnetic field directions.

For example, consider an anisotropic scaling of the correlation function of an observed measure $X(l_x, l_y) \sim l_x^{-p} l_y^{-q}$ and the anisotropy scaling $l_x \sim l_y^{1/2}$. Statistically, the field-perpendicular gradients of $X$ are $\nabla X = \nabla X^{\perp} = \nabla X^{\perp} \sim l_x^{-q} l_y^{-1} l_x^{-1} \sim l_x^{-q}$, $l_y^{-q}$, where $q < 1$ or parallel ($q > 1$) to the local magnetic field, and $p$ determines whether the smallest scale dominates ($p < 1$) or not ($p > 1$). In the case of GS95, it corresponds to $p = 1/3$ and $q = 2/3$, indicating that both conditions are satisfied for the 3D local measurements.

Readers should be careful that the statistics of magnetized turbulence change when the corresponding turbulent variables (e.g., density $\rho$, velocity $v$) are projected along the LOS. The reason behind this is that the local measurements of anisotropy are not available after compression along the LOS, and thus the LOS-averaged anisotropy of turbulence is determined by the anisotropy at the largest scale (see Esquivel & Lazarian 2005), i.e., the “observed” $q$ is zero after the LOS averaging. This change of $q$, however, would not violate the conditions for gradients to be maximally perpendicular to the magnetic field. For the change of $p$ after projection, it is easy to demonstrate that the 2D spectrum of turbulence obtained by projecting the fluctuations from 3D has the same spectral index of $-11/3$ (see Lazarian & Pogosyan 2000). The relation between the spectral slope of correlations and the slope of the turbulence power spectrum in 2D in this situation is $-11/3 + 2 = -5/3$, where $p$ is the dimensionality of the space. Therefore, the observed 2D fluctuations scale as $l_2^{-5/6}$ (i.e., $p = 5/6$). The gradients’ anisotropy scales as $l_2^{-1/6}$ (i.e., $p = 1/6$). This means that the contribution of the smallest scales is dominant in projected observables. These fluctuations are aligned with the local magnetic field, and the observed measure reflects the LOS-averaged magnetic field.

In our later sections, the anisotropy conditions are automatically satisfied since the GS95 scaling applies to our numerical work. As a result, we only have to examine whether the smallest-scale contribution dominates in the observed correlation or structure function scaling.

### 3. Theoretical Expectation of Gradients of Polarsynchrotron Radiation

#### 3.1. Polarsynchrotron Radiation in the Presence of Faraday Rotation

To characterize the fluctuations of the synchrotron polarization, one can use different combinations of the Stokes parameters (see Lazarian & Pogosyan 2012). In this paper, we follow the approach in LP16 and focus on the measure of the linear polarization $P$, which is

$$P = Q + iU,$$  \hspace{1cm} (1)

where $Q$ and $U$ are the Stokes parameters.

We consider an extended synchrotron region, where both synchrotron emission and Faraday rotation are taking place simultaneously. Special cases, e.g., the regions of synchrotron radiation being separated from those of Faraday rotation (see the analytical description in LP16), can be analyzed easily following the same treatment we have below. The polarization of the synchrotron emission at the source is characterized by the polarized intensity density $P_i(X, z)$, where $X$ is the 2D plane-of-sky vector and $z$ is the distance along the LOS. The polarized intensity detected by an observer in direction $X$ at wavelength $\lambda$ is given by

$$P_i(X, \lambda^2) = \int_0^L dz P_i(X, z) e^{2iXz}. \hspace{1cm} (2)$$

The region is extended up to the scale $L$, and the Faraday rotation measure $\Phi(z)$ is given by (see Brentjens & de Bruyn 2005)

$$\Phi(z) = 0.81 \int_0^z n_e T e^{-H_z} d\ell \text{ rad m}^{-2}, \hspace{1cm} (3)$$

where $H_z$ is the strength of the LOS component of the magnetic field in $\mu G$, $n_e T$ is the thermal electron density, and the distance $z$ is measured in parsecs.

For our present paper, following the convention in LP16, we ignore the wavelength dependences of synchrotron polarization $P_i$ arising from the cosmic-ray spectrum. This can be accomplished, for example, by determining the wavelength scaling from the intensity measurements. Thus, in what follows, $P_i(X, z)$ at the source is treated as wavelength independent, while the observed polarization $P(X, \lambda^2)$ contains the wavelength dependence that arises from the Faraday rotation effect.

The Stokes parameters at the source (see Equation (2)) depend on the cosmic-ray index $\gamma$. However, Lazarian & Pogosyan (2012) show that only the amplitudes of Stokes parameters are scaled up with respect to the cosmic-ray index and the spatial variations of the Stokes parameters are similar to...
fluctuations of magnetic fields perpendicular to the LOS and the Faraday effect. We use the LP16 relations to explore when the gradients measure the local magnetic field, as well as to define the extent of the regions over which we collect polarized signal.

The SPGs contains the contributions from both $Q$ and $U$. In LP16, the discussion on the statistics of complex cross-correlation functions is limited to its real part $R((P(r)P^*(r + r')) = (Q(r)Q(r + r')) + U(r)U(r + r'))$. In this work, we also study the contributions of both $Q$ and $U$, as well as their quadratic combination, i.e., the phase-independent polarized intensity $P = (Q^2 + U^2)^{1/2}$, which can be studied similarly to SIGs. In Section 5.2, we will briefly discuss the properties of the cross-intensity $X = QU$, which is similar to the imaginary part of the cross-correlation function.

We also explore fluctuations arising from the Faraday rotation effect in this paper. They are proportional (see Equation (3)) to the LOS integral of fluctuations of $n_e T B_0$, where $n_e T$ is thermal electron density and $B_0$ is the LOS component of the magnetic field. Mathematically, the product $n_e T B_0$ behaves similarly to the density times the parallel component of velocity, i.e., $\rho v_\parallel$, which enters the expressions for velocity centroids (see Esquivel & Lazarian 2005). Our study of velocity centroid gradients (VCGs) in González-Casanova & Lazarian (2017, hereafter GL17), Yuen & Lazarian (2017a, hereafter YL17a), and Yuen & Lazarian (2017b, hereafter YL17b) demonstrated that they are perpendicular to the local magnetic field. Thus, we expect the FRGs to behave similarly.

Compared to the SIGs, the most promising effect related to the SPGs and SPDGs is the Faraday depolarization, i.e., only the regions close to the observer contribute to the measured polarization, while unpolarized radiation is coming from more distant regions. This effect, analytically studied in LP16, provides the wavelength-dependent LOS depth for probing magnetic fields (see Section 8), which makes it possible to employ polarization gradients to study the 3D magnetic field structure. Figure 1 illustrates how the effective length scale is defined pictorially. Consider the situation in which the observer records the synchrotron emission from the volume on the left and its LOS is parallel to the horizontal axis. We only study the case where the effective depth $L_{\text{eff}}$, i.e., the distance whose polarized radiation is collected effectively, is less than the total thickness $L$ of the emitting volume, i.e., $L_{\text{eff}}/L < 1$. In other words, only a part of the volume close to the observer contributes to the measured $Q$ and $U$. The emission coming from distances larger than $L_{\text{eff}}$ from the observer is depolarized, as we illustrated in Figure 1. We refer to $L_{\text{eff}}$ as the effective length scale over which we probe the magnetic field with polarized synchrotron.

3.3. Effect of Density Fluctuations

One should keep in mind that density is an indirect measure of the turbulence statistics. For example, the density structures can be distorted by shocks, which can make the interpretation of density gradients in terms of magnetic field rather than ambiguous (YL17b). Therefore, it is important to understand whether the fluctuations arising from the Faraday effect or from synchrotron emissivity dominate the fluctuations of polarized intensities. In most cases, the fluctuations of synchrotron emissivity are dominated by magnetic field fluctuations because the fluctuations of relativistic electron density are negligible at small scales. At the same time, the fluctuations of

**Figure 1.** Pictorial illustration showing how the effective length scale is defined. In the case of random magnetic field the square root of dispersion enters the expression of $L_{\text{eff}}$ instead of $B_{\text{LOS}}$ (see also Equation (7)).

the case $\gamma = 2$. Therefore, for our gradient study, which deals with the spatial variations of magnetic fields, it is sufficient to study only the $\gamma = 2$ case, which means that the synchrotron emissivity is proportional to the squared component of the magnetic field perpendicular to the LOS. By expressing the $Q$ and $U$ Stokes parameters at the source, we obtain

$$Q(X, z) \propto p n_e (H_e^2(z) - H_\parallel^2(z)), \quad (4)$$

$$U(X, z) \propto p n_e 2 H_e(z) H_\parallel(z), \quad (5)$$

where $p$ is the polarization fraction, which is to be assumed constant, and $n_e$ is the relativistic electron density. The definitions of the Stokes parameters above correspond to the synchrotron intensity at the source $I(X, z) \propto H_e^2(z) + H_\parallel^2(z)$.

3.2. Pictorial Description of Polarization Gradients

The SIGs that we introduced in Lazarian et al. (2018b) have a lot of similarities with the polarization gradients that we deal with in this paper. The most important effect that the polarization brings is related to the ability to use the effect of Faraday depolarization to constrain the region that is being sampled (see Figure 1). The Faraday depolarization depth is defined in LP16 for both the turbulent and regular magnetic field. An additional advantage of the gradients of polarization is to sample the magnetic field structure with the Faraday rotation fluctuations. This information is complementary to what can be obtained with the SIGs and, combined with the Faraday depolarization effect, provides the ability to restore the 3D magnetic field structure.

The three gradients we investigate in the current work are the SPGs (Section 5) and SPDGs (Section 6). We also briefly discuss a variant of SPDGs, which is called Faraday rotation gradients (FRGs; Section 6).

LP16 describes how the statistics of fluctuations of the polarization $P$ and its wavelength derivative $dP/d\lambda^2$ depend on
Faraday rotation are affected by both the thermal electron density and magnetic field fluctuations. The study in LP16 provides us with theoretical guidance when one of the other effect dominates the fluctuations.

4. Numerical Simulations Employed for Testing Our Predictions

4.1. MHD Turbulence Simulations

The numerical 3D MHD simulations of turbulence are performed using a single-fluid, operator-split, staggered-grid MHD Eulerian code ZEUS-MP/3D (Hayes et al. 2006) to set up a 3D, uniform, isothermal turbulent medium. We use a range of Alfvénic and sonic Mach numbers, i.e., $M_A = V_L/V_A$ and $M_s = V_s/V_c$, where $V_L$ is the injection velocity, while $V_A$ and $V_c$ are the Alfvén and sonic velocities, respectively. The numerical parameters are listed in Table 1 in sequence of ascending values of media magnetization given by $\beta$. The domain $M_A < M_s$ corresponds to the simulations with magnetic pressure larger than the thermal pressure, i.e., plasma $\beta/2 = V_L^2/V_A^2 < 1$, while the domain $M_A > M_s$ corresponds to the $\beta/2 > 1$. For instance, Ms0.4Ma0.04 corresponds to $M_s \approx 0.4$ and $M_A \approx 0.04$. In this study we focus on sub- and trans-Alfvénic cases and leave the discussion of super-Alfvénic simulations for our future publications.

| Model        | $M_s$ | $M_A$ | $\beta \approx 2\left(\frac{M_s}{M_A}\right)^2$ |
|--------------|-------|-------|----------------------------------|
| Ms0.2Ma0.02  | 0.2   | 0.02  | 0.02                             |
| Ms0.4Ma0.04  | 0.4   | 0.04  | 0.02                             |
| Ms0.8Ma0.08  | 0.8   | 0.08  | 0.02                             |
| Ms1.6Ma0.16  | 1.6   | 0.16  | 0.02                             |
| Ms3.2Ma0.32  | 3.2   | 0.32  | 0.02                             |
| Ms6.4Ma0.64  | 6.4   | 0.64  | 0.02                             |
| Ms0.2Ma0.07  | 0.2   | 0.07  | 0.22                             |
| Ms0.4Ma0.13  | 0.4   | 0.13  | 0.22                             |
| Ms0.8Ma0.26  | 0.8   | 0.26  | 0.22                             |
| Ms1.6Ma0.53  | 1.6   | 0.53  | 0.22                             |
| Ms0.13Ma0.4  | 0.13  | 0.4   | 18                               |
| Ms0.20Ma0.66 | 0.20  | 0.66  | 18                               |
| Ms0.26Ma0.8  | 0.26  | 0.8   | 18                               |
| Ms0.9Ma0.4   | 0.04  | 0.4   | 200                              |
| Ms0.08Ma0.8  | 0.08  | 0.8   | 200                              |
| Ms0.2Ma2.0   | 0.2   | 2.0   | 200                              |

Note. The magnetic criticality $\Phi = 2\pi G^{1/2}\rho L/B$ is set to be 2 for all simulation data. Resolutions of them are all 480$^3$.

4.2. Calculating Synchrotron Polarization, Gradients, and Alignment Measure

Synchrotron radiation. In this work, we are interested in scalings and do not keep the numerical pre-factors. In Section 5 we investigate multifrequency synchrotron maps, namely, position–position–frequency (PPF) cubes (see LP16).

Block averaging for gradient calculations. Gradients of polarization are calculated by taking the values of polarization in the neighboring points and dividing them over the distances between the points following the recipe of YL17a. In this work, we focus on the smallest-scale contribution (see Section 2), as we did in Lazarian & Yuen (2018, hereafter LY18). YL17b provided the explicit expressions for block-averaging and error-estimating procedures, which we will follow in this paper.

Alignment measure (AM). To quantify how good two vector fields are aligned, we employed the alignment measure that is introduced in analogy with the grain alignment studies (see Lazarian 2007):

$$\text{AM} = 2\langle \cos^2 \theta_r \rangle - 1$$

(see GL17, YL17a), with a range of $[-1, 1]$ measuring the relative alignment between rotated gradients and magnetic fields, where $\theta_r$ is the relative angle between any two vector fields. A perfect alignment gives AM = 1, whereas random orientations generate AM = 0. We shall use AM to quantify the alignments of polarization gradients with respect to magnetic field.

5. SPGs: Random Magnetic Field along the LOS

For our studies the invariant quadratic measure $|P| = \sqrt{Q^2 + U^2}$ is used (see Section 5.3). However, to gain intuition with the properties of polarized synchrotron, we explore the properties of $Q$ and $U$ Stokes components. To get a theoretical guidance for our present study, we will use the analytical calculations of correlation functions of polarized radiation in LP16. Notice that gradients are not equivalent to the correlation function, but statistically there is a correspondence between the structure functions and the gradients (Section 2). When comparing the direction of gradients with respect to magnetic field, we would consistently refer to LP16 (as we do in Sections 5.3, 5.4, as well as in Appendix A.3). The various parameters that the problem depends on are listed in Table 2.

Before analyzing the SPGs quantitatively, we would like to list various regimes of synchrotron polarization study described analytically in LP16. In the case of sub-Alfvénic turbulence, the source term $P_i$ is dominated by the mean field rather than the fluctuating one. The two regimes, (1) strong and (2) weak Faraday rotation, depend on whether the ratio of the scale that is sampled by polarization to the size of the emitting region, i.e., $L_{\text{eff}}/L_s$ is smaller (strong) or larger (weak) than unity:

$$\frac{L_{\text{eff}}}{L_s} \sim \frac{1}{\frac{1}{L_s} \frac{1}{\rho \phi}}$$

(7)

where $\phi = \max(\sqrt{2}\sigma_\phi, \bar{\phi})$, with $\sigma_\phi$ the dispersion of random magnetic field,

$$\sigma_\phi^2 = \langle \Delta(n_e)H_z^2 \rangle$$

$$= \bar{\Pi}_T^2 \langle \Delta n_e \rangle^2 + \bar{n}_e^2 \langle \Delta H_z^2 \rangle + \langle \Delta n_e \rangle \langle \Delta H_z \rangle$$

(8)

where $\bar{\phi}$ is the mean Faraday rotation measure density,

$$\bar{\phi} \propto \langle n_e H_z \rangle = \pi_L \bar{\Pi}_T,$$

(9)

where $\Delta n_e$ and $\Delta H_z$ are the fluctuations of the thermal electron density and the magnetic field.

6 In general, different scales carry information about different physical processes. Therefore, gradients calculated at intermediate scales can contain important complementary information. If one is interested in gradients on larger scales, then filtering of the contributions from the small scales is appropriate.
Table 2
Parameters that the Synchrotron Polarization Statistics Depends on

| Parameter | Meaning | First Appearance |
|-----------|---------|------------------|
| \( \lambda \) | Wavelength of observations | Equation (2) |
| \( \nu \) | Frequency of the observation | Section 5.2 |
| \( L \) | LOS extent of the emitting region | Equation (2) |
| \( R \) | Separation between LOSs | Equation (11) |
| \( r_0 \) | Correlation length for Faraday rotation measure density | Section 5.4 |
| \( r_i \) | Correlation length for polarization at the source | Equation (12) |
| \( \mathcal{L}_Q \) | Distance of 1 rad revolution by mean Faraday rotation | Section 5.4 |
| \( \mathcal{L}_{\sigma_Q} \) | Distance of 1 rad revolution by random Faraday rotation | Section 5.3 |
| \( {\mathcal{L}}_{\text{eff}} \) | The smallest of \( \mathcal{L}_{\sigma_Q} \) and \( \mathcal{L}_Q \) | Equation (7) |

Scales:

| Spectral indices: | | |
|-------------------|-------------------|-------------------|
| \( m_Q \) | Correlation index for Faraday RM density | Equation (11) |
| \( m_i \) | Correlation index for polarization at the source | Equation (12) |

Basic statistical:

| | | |
|-------------------|-------------------|-------------------|
| \( \bar{\phi} \) | Mean Faraday RM density | Equation (2) |
| \( \sigma_{\phi} \) | rms Faraday RM density fluctuation | Equation (8) |
| \( \vec{P}_i \) | Mean polarization at the source | Equation (9) |
| \( \sigma_i \) | rms polarization fluctuation at the source | Equation (11) |

Note. Modified from LP16.

Note that \( \mathcal{L}_{\text{eff}} \) is a frequency-dependent parameter (Equation (7)). If we have measurements from two different frequencies, we would have the Stokes parameters containing information of magnetic field directions from different LOS depths. The differences between these parameters will therefore provide a measure of magnetic field between the two depths. As a result, if multiple frequency measurements are available, a 3D magnetic field distribution can be revealed by repeatedly calculating the difference of Stokes parameters obtained with the neighboring frequency measurements.

We would also like to address how the change of \(|P|\) would depend on the regimes that we are going to study. To briefly characterize this, we plot the \(|P|\) and \( \mathcal{L}_{\text{eff}}/L \) versus the frequency \( \nu \) in Figure 2. We see that in the regime of \( \mathcal{L}_{\text{eff}}/L \sim 1 \), \(|P|\) changes the most. It is also interesting that when the frequency \( \nu \) drops below some value, \(|P|\) remains constant and significantly above zero. We recognize this as a numerical artifact of our cubes having finite resolution. We explain in Appendix A.3 how this effect arises from the finite number of points along the LOS. Readers should note that our simulation resolution restricts us to synthesizing ultralow synchrotron emissions. Therefore, we only investigate synchrotron emission structures in the frequency range of 100 MHz–100 GHz. For example, in Figure 2 one can see that the amplitude of polarized synchrotron emission is constant (corresponding to the numerical noise regime discussed in the Appendix A) when frequency is lower than 100 MHz. However, the central idea of our method is to utilize the effective length \( \mathcal{L}_{\text{eff}} \) (Equation (7)) to estimate the magnetic field directions for a slice in a certain LOS depth. Since the nature of the Faraday rotation effect is the same for both low and high radio frequencies, we expect that the same technique rooted in the LP16 analytical study can work for low radio frequencies.

5.1. Effect of Telescope Resolution

Based on the theory in LP16, the following equation is the constraint of the telescope resolution \( \Delta R \) if one wants to acquire the correct and unbiased statistics of synchrotron polarization:

\[
1/r_i < \lambda^2 \max (\sigma_{\phi}, \bar{\phi}) < 1/\Delta R, \tag{10}
\]

where \( r_i \) is the correlation length for polarization, and \( \sigma_{\phi} \) and \( \bar{\phi} \) characterize the Faraday rotation effect for random and regular magnetic field, respectively (see further explanation in Table 2). Naturally, this also sets a limitation for the gradient technique. The same criterion also applies to the numerical analysis we perform in the present paper. For instance, a numerical artifact emerges when the inequality (Equation (10)) is violated (see Appendix A.3).

5.2. SPGs: Competition between Q and U

We would like to give a visual illustration on how the Faraday rotation from the strong and weak regimes changes the structure of both \( Q \) and \( U \). Figure 3 shows the structures of \( Q \) and \( U \) maps in relative scales for three different regimes of Faraday rotation from our cube Ms1.6Ma0.53, which is a sub-Alfvénic cube. The dispersion for the quantity \( \sigma_{\phi} \) is 1.42 in numerical cube Ms1.6Ma0.53. We select three frequencies \( \nu = 100 \text{ MHz}, 1 \text{ GHz}, \text{ and } 10 \text{ GHz for our case study, which corresponds to } \mathcal{L}_{\text{eff}}/L \sim 0.0068, 0.68, \text{ and } 68, \text{ respectively. For this cube, the mean magnetic field is pointing to the right and has minimal oscillations vertically. Therefore, we expect a dominance of } Q \text{ over } U. \text{ For the best presentation of Figure 3, we adjusted the color bar to relative scales so that most structures are seen visually. As gradients identify the orientation of intensity structures, we show that these structures in Figure 3. One can see that in the case of } \nu = 100 \text{ MHz both } Q \text{ and } U \text{ maps are being dominated by noise. We recognize this as a numerical artifact} \)

\[
\text{For any practical studies these are the scales determined by the telescope resolution. However, one may intentionally degrade the resolution to study the physical processes at different scales.} \]
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Figure 2. Variation of amplitude of \( P \) and \( L_{\text{eff}}/L \) vs. the frequency \( \nu \).

Figure 3. Illustration of \( Q \) and \( U \) maps in different frequencies.

(see Appendix A.3). As the frequency increases, the noise-like structures are less prominent. For our case, the structures of \( Q \) maps are aligned with the mean magnetic field direction, while those for \( U \) tend to be perpendicular to the magnetic field. We shall see how the competing effects between \( Q \) and \( U \) in \(|P| = (Q^2 + U^2)^{1/2}\) change the gradient orientation. We also reported the ratio between \( \langle U \rangle \) and \( \langle Q \rangle \) with respect to frequency. In fact, \( \langle U \rangle / \langle Q \rangle \sim 1 \) when \( \nu = 1 \) GHz, whereas it is \( \sim 0.016 \) when \( \nu = 10 \) GHz. Note that the properties of \(|P| = (Q^2 + U^2)^{1/2}\) and \( X = QU \) are related to the relative importance of \( Q \) and \( U \).

5.3. SPGs for Weak Faraday Rotation Effect

We start with the simple case of weak Faraday rotation. This situation corresponds to the case when the wavelength of radiation is sufficiently short that Faraday rotation can be disregarded. In this situation, the mathematical structure of Equation (2) is similar to that of unpolarized synchrotron intensities. Therefore, it is not surprising that we get a structure of the SPGs similar to that of the SIGs.

The theoretical expectation of LP16 suggests that the weak Faraday rotation case, i.e., when the correlation scale of the synchrotron emission correlation\(^8\) is less than the depolarization scale, i.e., \( r_i < L_{\sigma_i} \), has a only a weak effect on the correlation of observed polarization. As we cannot obtain \( r_i \) beforehand, we enforce our weak rotation case to be \( L < L_{\sigma_i} \). This ensures that the ratio in Equation (7) is greater than unity and \( r_i < L_{\sigma_i} \). The calculations in LP16 provide

\[
\langle |P(X_1) - P(X_2)|^2 \rangle \propto \sigma_i^2 R^{1+m},
\]

where \( L_{\text{eff}} > L \),

\[
(11)
\]

where \( \bar{m} \equiv \min(1, m) \), \( \sigma_i^2 \) is the rms polarization fluctuation at the source, and \( m \) is the correlation index of the polarization at the source. From Section 2 we know that the power-law dependence of \( R \) satisfies the expected \( p < 1 \) requirement\(^9\) (see Section 2).

From the principle in Section 2 we confirm that gradients in the weak Faraday rotation case are able to trace the projected magnetic field. LP16 showed that the anisotropy arising from a \( \sigma_i \propto H^2 \) term in Equation (2) can be measured with synchrotron intensities. As the spatial anisotropy part is not altered in Equation (11), we predict that the direction of the SPG is similar to that of SIGs.

The prediction of the directional similarity of SPGs and SIGs in the weak Faraday rotation case is tested by applying the block-averaging recipe of YL17a to the polarization \(|P| = (Q^2 + U^2)^{1/2}\), as well as to \( X = QU \). This is illustrated in Figure 4, where the SIGs and the SPGs calculated for the weak rotation case (\( \nu = 2 \) GHz) are compared for the projected magnetic field in the data cube Ms0.08Ma0.8. One can already see the structural similarity of the SIGs and SPGs in Figure 4. When Faraday rotation is weak, the polarized intensity is dominated by the Stokes \( U \) in our synthetic cubes. Gradients from the cross-intensity \( X = QU \) trace magnetic fields much worse compared to \( |P| \) (see Figure 4).

The advantage of phase independence for \( |P| \) is very useful when we reconstruct the 3D magnetic field distribution in Section 8. The utility of cross-intensity \( X \) for the gradient analysis will be explored elsewhere. Unless otherwise stated, in what follows we deal with \(|P| \).

5.4. SPGs for Strong Faraday Rotation Effect

The strong Faraday rotation case opens a unique path for 3D magnetic field studies. Figure 5 illustrates how the SPGs are aligned compared with the projected magnetic field. The left

\[
\text{This is essentially the correlation scale of magnetic field fluctuations.}
\]

\[
\text{The correlation function should be } \sim R^{-1+\bar{m}}, \text{ in which } R \text{ has a power of } 1+\bar{m}, \text{ which for the case of Kolmogorov-type scaling, i.e., } m = 2/3, \text{ provides the fluctuation } \delta b \sim R^{1/3} \text{ with the gradients } \sim R^{1/3}, \text{ i.e., the correlation function is the most sensitive to the fluctuations at the smallest scales following Section 2.}
\]
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panel of Figure 5 illustrates the SPGs of $|P|$ in the case of strong Faraday rotation corresponding to the $\nu = 1$ GHz. In the right panel of Figure 5 we show the map with weak Faraday rotation. One can visually see both the striations along the magnetic field lines and the 90° rotated gradients being aligned with the magnetic field directions.

Below we explain why the strong Faraday rotation setting is so advantageous from the point of view of tomographic studies of magnetic fields. Indeed, we will show that the scales for the decorrelation of the polarized signal arise from the rotation by random (scale $\mathcal{L}_a$) and regular (scale $\mathcal{L}_b$) magnetic fields (see Table 2).

For strong rotation, we cannot replace $r_i$ by $L$ as described in Section 5.3 because the polarization is not sampled through the entire volume. For instance, consider the dominance of turbulent Faraday decorrelation, i.e., $\mathcal{L}_a < \mathcal{L}_b$. The strong regime $\mathcal{L}_b \ll \nu$, and therefore the polarized radiation is sampling only a small part of the nearby synchrotron eddy. The result for the polarization correlation that is obtained in LP16 for this regime is

$$\langle P(X_1)P^*(X_2) \rangle \propto \zeta_1(R, 0) r_i^{-1} R^{-(1+\tilde{m}_\nu)}$$

where $\tilde{m}_\nu = \min(1, m_\nu)$. Again, from Section 2, we see that the requirement for small-scale gradients to dominate is satisfied. For small separations $R \ll r_\nu$, the correlation function of synchrotron emissivity at the source is Equation (12), which is nearly constant, and therefore the observed fluctuations of $|P|$ are dominated by the Faraday effect (see Appendix A for the illustration).

At scales less than the correlation scale of Faraday rotation $r_\nu$ (see Table 2) the gradients are dominated by the effects of Faraday rotation. As the correlation function of polarization fluctuations increases with the decreasing of the scales, the gradients of the polarization fluctuations are dominated by the smallest-scale fluctuations. Therefore, we expect the gradients of $P(X)$ to be perpendicular to the plane-of-sky magnetic field projection within the distance $\mathcal{L}_b \equiv (\lambda \phi)^{-1}$ (see Table 2) from the observer. It is important that, by changing the wavelength of the radiation $\lambda$, one can change the thickness of the volume inside which SPGs can trace the magnetic field.

5.5. Alignment Measure in Different Regimes
and Numerical Artifacts

We analyze the change of AM with respect to projected magnetic field directions before block averaging for various $\nu$. Figure 6 shows a plot of the alignment measure with respect to the frequencies when the gradients are smoothed by a Gaussian kernel (see LYLC17) of $\sigma = 2$ pixels.

When $\nu \gtrsim 6.8$ GHz, the change of alignment measure is infinitesimal and shows the same features as the SIGs, which approximately corresponds to the weak Faraday rotation regime. As the frequency decreases, the striations seen in Figure 5 get to be dominant, and this brings a higher AM than that in raw SIGs with its peak at $\nu = 3.8$ GHz. As the frequency continues to decrease, the AM quickly drops to zero owing to the dominance of noise-like structures already seen in Figure 5. As a result, gradients in these very low frequency synchrotron maps are isotropic and incapable of tracing the magnetic field. We mark the approximate range for the two regimes we discussed above in Figure 6. The peak we recognized in Figure 6 is helpful for construction of the 3D
magnetic field, because the corresponding frequency that has a peak in the AM-frequency plot (Figure 6) is related to the effective screening length $L_{\text{eff}}$.

### 6. The Faraday Rotation Measure and the Synchrotron Polarization Derivative Gradients

From the theoretical study in LP16, we know that the fluctuations of $|P|$ are mostly coming from the variations of synchrotron emissivity at the source rather than the fluctuations from Faraday rotation. To study the fluctuations of Faraday rotation, LP16 proposed to use $dP/d\lambda^2$ measures. In the case that synchrotron emission and Faraday rotation regions are spatially distinct, the aforementioned measure is directly proportional to the Faraday rotation measure $\sim \int n_e B_\| dz$. A important relation was found in LP16 for the spatial correlations of $dP/d\lambda^2$, when the volumes of synchrotron emission and Faraday rotation coincide. The relative importance of the Faraday rotation fluctuations and synchrotron emission fluctuations depends on the separation of points at which the correlation is calculated (LP16). In terms of gradients of $dP/d\lambda^2$, this means that one can probe the anisotropy of the two types of fluctuations by calculating the gradients at different scales. The latter can be achieved by calculating the gradients not using the adjacent points, but taking the points separated by a given distance. Such a study may provide valuable additional information. For instance, the differences between the synchrotron emission and the gradients that are influenced by the Faraday effect can give insight into the distribution of the rotation measure.

We start with the simplest case. The Faraday rotation measure can be available when the polarized signal passes through the Faraday rotating screen. This is a valid approximation when the synchrotron emission and Faraday rotation regions are separated. As we mentioned earlier, the structure of Equation (3) is very similar to the velocity centroid, i.e., $C \propto \int d\rho(z) v_r(z)$, where $z$ is the LOS axis. Therefore, we expect that the FRGs have similar properties to the VCGs in, e.g., YL17a. In left panel of Figure 7 the 90° rotated FRGs are compared to the projected magnetic field for the synthetic observations obtained with the cube Ms1.6Ma0.53. To compare with the weighted FRGs (see below), the background map and the gradients are both based on the absolute value of the rotation measure, even though the distribution of gradients is independent of signs of the map. The alignment measure for the FRGs is significant ($AM = 0.67$), which means that the FRGs are capable of successfully tracing magnetic fields.\(^\text{10}\)

The FRG is a limiting case of long wavelengths for the SPDGs, which can be used also in situations when the Faraday fluctuations and synchrotron emission arise from the same volume. The SPDGs are gradients of the polarization derivative $dP/d\lambda^2$, whose correlations

$$\left\langle \frac{dP(X_1)}{d\lambda^2} \frac{dP^*(X_2)}{d\lambda^2} \right\rangle$$

were studied in LP16. In the case of the strong mean field dominating over the random field along the LOS, LP16 predicted that

$$\left\langle \frac{dP(X_1)}{d\lambda^2} \frac{dP^*(X_2)}{d\lambda^2} \right\rangle \propto R^{1+\tilde{m}_\phi}$$

where $\tilde{m}_\phi = \min(m, 1)$ (see Table 2). This is suggestive that the SPDGs, i.e., the gradients of $dP(X)/d\lambda^2$, are arising from the fluctuation of Faraday rotation: there is a power-law dependence on the correlation function (Equation (14)) to the Faraday rotation measure correlation index.

The situation is more complex in the case of weak mean magnetic field: the correlations of intrinsic synchrotron polarization at the source $P$, and the fluctuations of the Faraday rotation $\Delta(n_e, H_r)$ enter symmetrically in the correlation of the polarization derivatives. The dominant asymptotic contribution depends on both the correlation scale of the two types of correlations and their indices (see LP16). Assuming that $r_\phi < r_r$ and $m_\phi < m$, the structure function of the correlation derivatives obtained in LP16 is

$$\left\langle \left( \frac{dP(X_1)}{d\lambda^2} - \frac{dP(X_2)}{d\lambda^2} \right)^2 \right\rangle \propto \sigma_1^2 \sigma_2^2 L^2(\tau_r/L)^m R^{1+m_\phi}/r_\phi^{m_\phi}$$

where $R < r_\phi$, $m_\phi < m$, suggesting again the dominance of the Faraday fluctuations in the SPDGs.

In reality, obtaining the rotation measure in the form of Equation (3) is usually difficult. Therefore, we would be more interested in the so-called polarization-weighted Faraday measure.

---

\(^{10}\) We expect that the alignment can be increased further if the additional techniques that we tested for velocity gradients, e.g., angle constraint and moving window (see LY18), are applied.
rotation measure $\hat{\Phi}_P$:
\[ \hat{\Phi}_P = -i \frac{d \log (P)}{d \lambda^2} = \frac{\int_0^L dz \Phi(X, z) P_1(X, z) e^{i \lambda \Phi(X, z)}}{\int_0^L dz P_1(X, z) e^{i \lambda \Phi(X, z)}}, \]
which is simply the log-derivative of Equation (2). As we described above, we would be more interested in the modulus of the structure of the weighted measure. For simplicity, we take $\lambda \to 0$ (i.e., $\nu \to \infty$) to suppress the Faraday rotation effect. The right panel of Figure 7 shows $\hat{\Phi}$ for the same cube Ms1.6Ma0.53 by taking the differences on the $(Q, U)$ maps from $\nu = 10$ GHz with $\Delta \nu = 100$ MHz ($\Delta \lambda^2 = c^2 / f^3 \Delta f = 9 \times 10^{-6}$ m), i.e.,
\[ \Delta Q = Q(\lambda = 10.1 \text{ GHz}) - Q(\lambda = 10 \text{ GHz}) \]
\[ \Delta U = U(\lambda = 10.1 \text{ GHz}) - U(\lambda = 10 \text{ GHz}). \]
The difference of the polarized intensity is then $|\hat{\Phi}| = \sqrt{\Delta Q^2 + \Delta U^2}$. One can observe that both the structure of the map of $|\hat{\Phi}|$ and the gradients of $|\hat{\Phi}|$ behave similarly to the true rotation measure (Equation (3)). This means that the studies with high enough $\nu$ can deliver the Faraday rotation measure from the synchrotron-emitting volume. Moreover, the alignment measure of the gradients of $|\hat{\Phi}|$ is relatively high ($\text{AM} = 0.57$), which is suggestive that $|\hat{\Phi}|$ is also a good measure of magnetic field direction for the case of high-frequency radiation (i.e., small $\lambda$).

7. SPGs: Faraday Rotation by Mean Field

A particular regime in which strong Faraday rotation is induced by a component of the mean magnetic field along the LOS was studied in LP16. In this section, we explore what this regime means to the gradient technique on tracing the magnetic field.

In the strong Faraday rotation case, if the mean field dominates the Faraday rotation $L_{\phi} > L_\phi$ (see Table 2), LP16 predicts that under the condition of $L_{\phi} < \eta$ we have
\[ \langle P(X_1) P^*(X_2) \rangle \sim L \ L_{\phi} \xi(R, 0), \]
where $\lambda^2 \eta_{\phi} > 1$. (18)

We test this by artificially putting an LOS component of magnetic field with $B_{\text{LOS}} = 1$ and $\sigma_\phi = 0.2$. The field is applied as an external agent and is not self-consistent with the simulations. However, it serves the purpose of illustrating the effect that we discuss in this section. The mean-field component $\bar{\phi}$ is then larger than the oscillating part and is dominant in the calculation of effective length scale. We then pick two frequencies 1.5 and 15 GHz, which correspond to $L_{\text{eff}} / L = 1.02$ and 102, respectively. Figure 8 shows the gradients in the two regimes and also the underlying intensities.

The gradient analysis of $|P|$ in this section is similar to Section 5, so we shall directly show the results from all three cases. Figure 8 illustrates the gradients for the three regimes. One can see a similar trend for alignment measure to spike when $L_{\text{eff}} / L$ is on the order of unity (i.e., $\nu = 1.5$ GHz). The dependence of AM on frequency is very similar for the mean-field case to the one in Figure 5 for the random-field case. Therefore, in what follows we shall unify the treatment for both the random field and the mean-field Faraday rotation effect.

Note that even the presence of the small random field changes the amplitude in Equation (18). It is easy to see that for $L > L_{\phi}$, it is $L_{\phi}$ that should substitute $L$ in Equation (18). The change of the amplitude does not change the properties of gradients, however.

8. Toward Constructing a 3D Map of Magnetic Fields Using SPGs

Aside from the average magnetic field along the LOS, the special properties of the Faraday depolarization effect allow one to extract the information of magnetic field morphology along the LOS and also potentially the 3D structure of the magnetic field. The Faraday depolarization effect limits sampling of the magnetic fields within the boundary $L_{\text{eff}}$ from the observer. In this section we illustrate the ability of the technique, showing that by changing the wavelength one can trace changes of the magnetic field direction that take place along the LOS. Much more, however, can be done with the technique, and in Appendix D we discuss the possibility of obtaining the true 3D structure of magnetic field with the polarization gradients.

We start by summarizing the properties of SPGs that we found. First of all, in both weak and strong Faraday rotation regimes, the SPGs trace magnetic fields well. More importantly, in the strong Faraday rotation regime, the SPGs sample magnetic field up to a depth of $\sim L_{\text{eff}}$. It is important that within the volume limited by $L_{\text{eff}}$, the directions traced by SPGs, unlike the polarization directions, are not distorted by the Faraday rotation effect. As $L_{\text{eff}}$ depends on the wavelength $\lambda$, by changing $\lambda$ one can sample the magnetic field at different distances. As a result, the 3D distribution of the plane-of-sky magnetic field components can be obtained.11

We illustrate the capabilities of the SPGs using a toy model of magnetic field distribution. The SPGs sample the averaged magnetic field direction for depths $z \in [0, L_{\text{eff}}]$. To test this numerically, we set up an artificial case based on the numerical cube Ms1.6Ma0.53 by dividing it along the LOS into three pieces. We rotate the density and magnetic field structures in the second piece for 30° but keep the other two pieces unchanged. We shall refer to the altered cube as the partially rotated cube.

We then apply the Faraday rotation module for $\nu = 0.85, 1.2$, and 7 GHz, similar to what we did in Section 5, which corresponds to $L_{\text{eff}} / L \sim 0.33, 0.66$, and 22.34. From Figure 9, we clearly see that the gradient structure depends on the relative scale $L_{\text{eff}} / L$. For example, in the left panel of Figure 9 only the closest 1/3 of the magnetic field information from the partially rotated cube is effectively projected onto $Q$ and $U$. Therefore, the gradients from $|P|$ are quite uniformly pointing to the horizontal axis. As the

11 Similar to the SIGs, it is possible to get the amplitude of magnetic field perpendicular to the LOS using the synchrotron intensities. Thus, not only the direction of the magnetic field vector but also its amplitude is available.
frequency increases to 1.2 GHz (Figure 9), the second part of the rotated cube, which contains the 30° rotated fields, contributes to the Stokes parameters. As a result, the rotation of the vectors is observed. However, the weighting of gradients that are farther from the observer is generally smaller than that of those that are closer to the observer. Therefore, we do not expect the 30° rotated structure to change the intensity structure much. Due to the fact that gradients are linearly added along the LOS, we can see from the middle panel of Figure 9 that obvious structures are found oriented 30° away from the horizontal axis. This can be explained by the fact that the resultant SPGs are the linear combinations of sliced gradients along and 30° away from the horizontal axis. In the right panel of Figure 9 we see the case when $L_{\text{eff}}/L \gg 1$. This means that the 30° part is fully contributing to the $Q, U$. As a result, the direction of the gradients is like the average of the horizontal and 30° rotated gradients.

We note that additional information can be obtained by calculating the SIGs of the difference of the total minus the polarized intensities. This measure tests the magnetic field in the region $[L_{\text{eff}}, L]$. This can be complementary to the SPGs within the studies of 3D magnetic field structure. We include a preliminary recipe using SPDGs in Appendix D, but a detailed study will be provided elsewhere.

9. Discussion

9.1. Combining Synchrotron Gradients and Polarization Measurement

9.1.1. Synchrotron Intensity Gradients + Polarization Measurements

In our earlier paper (LYLC17), the SIGs have been shown to be a reliable technique for tracing magnetic field. Unlike tracing magnetic fields with polarization directions, the SIGs are not subject to Faraday rotation and therefore do not require multiple frequency measurements to compensate for the Faraday effect. An additional advantage is that measuring intensity is easier than polarization.

Combining measurements of polarization with the SIGs provides several ways to obtain synergy of the two measurements. First of all, combining the magnetic field obtained with synchrotron polarization and the SIGs, it is possible to increase the reliability of magnetic field tracing. In terms of CMB studies, the information from the SIGs can be used as a prior for establishing the synchrotron polarization directions.\(^{12}\) Combining the SIGs and the polarization is very advantageous, as we do not expect the gradients of CMB intensity to correlate with the directions of synchrotron polarization. Second, the SIGs can help to correct the maps of polarization measured with just one frequency for the Faraday rotation effect. The great advantage of this approach is that no additional measurements are necessary: the same $Q$ and $U$ data will be analyzed in two synergistic ways. The SIGs require block averaging (YL17a), which decreases the resolution of the map. If the Faraday rotation arises from the nearby galactic regions, the decrease of the resolution is not a problem, and the polarization map can be nicely corrected for the Faraday rotation. Moreover, as a result of this correction, a map of the Faraday rotations can be obtained. This map can not only inform us about the parallel to LOS magnetic field strength; to such a map one can apply the FRG technique that we described in this paper to get additional information about the magnetic field directions, which can be compared to that obtained with the polarization map. The variations in the directions obtained by the three techniques provide the information about the thermal electron densities that affect the FRGs.

9.1.2. SPGs + Polarization Measurements

All that we have discussed in terms of combining the SIGs and the polarization measurements is applicable to combining the SPGs with the direct synchrotron polarization measurements for $L_{\text{eff}} > L$. Indeed, for this setting one can use the SPGs for (1) improving the accuracy of synchrotron polarization measurements, (2) distinguishing the polarization from synchrotron from the CMB polarization,\(^{13}\) and (3) providing Faraday rotation maps that can be analyzed to get additional information, e.g., about the parallel to the LOS component of the magnetic field and the distribution of thermal plasmas along the LOS.

The case of $L_{\text{eff}} < L$ provides a new direction for studying magnetic fields in 3D. Depending on the correlation properties of the magnetic field at the scales sampled by gradients, the regions sampled by the SPGs and synchrotron polarized intensities may coincide or may not coincide (see LP16). In the former case, point

\(^{12}\) In LY18 we have discussed the possibility of using velocity gradients calculated with H1 data for this purpose.

\(^{13}\) The use of SPGs may be advantageous compared to the SIGs, as the former are not affected by the gradients from unpolarized radiation.
“c” from the section above is applicable, but for the probing of magnetic fields and thermal plasmas not through the entire volume, but up to the boundary given by \( L_{\text{eff}} \). In the latter case, the difference opens ways for getting additional magnetic field information. We shall elaborate these possibilities elsewhere.

9.2. Studying 3D Magnetic Field Distribution

9.2.1. 3D Structure with Synchrotron Polarization

As soon as \( L_{\text{eff}} \) is less than \( L \), the information about the 3D magnetic field distribution becomes available with the SPGs and the SPDGs. Comparing the results obtained with two techniques, one can get insight into the distribution of thermal electrons. This also allows us to focus studies of magnetic field on particular regions with higher ionization. For instance, if there is an \( \text{H} \text{II} \) region along the LOS, by comparing the SPG and the SPDG measures, one can potentially get insight into the magnetic field structure of the \( \text{H} \text{II} \) region. We intend to try this approach elsewhere.

The value of \( L_{\text{eff}} \) depends on the Faraday rotation turbulent dispersion or the Faraday rotation depolarization arising from the regular magnetic field. As we get more information about the magnetized turbulent ISM, we get better correspondence between \( L_{\text{eff}} \) and the LOS distribution of the magnetic field available through the two aforementioned gradient techniques. For instance, the ways of studying Faraday dispersion using correlations of the synchrotron polarization \( P \) and its derivative \( dP/d\lambda^2 \) are described in LP17.

9.2.2. Synergy to the Faraday Tomography Method

The technique that we proposed in this paper is complementary to the technique of Faraday tomography proposed by Burn (1966). The latter technique is aimed at obtaining the information of intrinsic polarization \( P_i \) as a function of Faraday depth (see Brentjens & de Bruyn 2005). The method created by Burn (1966) treated Equation (2) as a Fourier transform of the Faraday dispersion function \( F(\phi) = P_i dz/d\phi \) with the variable \( \phi \propto \int dznH_z \), which is the Faraday depth. With conditions for resolution and the effective length-scale condition Equation (7) being satisfied, one can perform the inverse Fourier transform on \( \lambda^2 \) and obtain a 3D structure of the Faraday dispersion function.

A significant advantage of our technique (Appendix D.1) compared to the Faraday tomography is that our technique does not require so many measurements.\(^{14}\)

A serious limitation of the Faraday tomography is related to its treatment of turbulent magnetic fields. In general, the LOS component of the magnetic field in turbulence is expected to change its sign. As a result, the Faraday depth \( \Phi \) is also changing its sign along the LOS, i.e., in the case of field reversals the estimation of the LOS magnetic field through Faraday depth becomes ambiguous. This presents a fundamental problem\(^{15}\) for the Faraday tomography. Our gradient approach does not have these sorts of limitations and is intended for studying realistic turbulent magnetic fields. The turbulent magnetic field, in fact, provides the real-space 3D boundary, which allows one to study the magnetic field up to the boundary \( L_{\text{eff}} \), the actual extent of which is varied by changing the wavelength.

We believe that the synergy of the two techniques can be valuable in the future. First of all, we see ways of improving the Faraday tomography. Simple considerations suggest, for instance, that, having a PPF rotation cube, one can calculate gradients within this cube, and these gradients should be perpendicular to the mean magnetic field for the slice of the cube. We plan to explore these possibilities elsewhere.

9.3. Toward a Universal Gradient Technique

9.3.1. General Description

The techniques in this paper that use polarization gradients should be viewed as a part of the gradient technique that employs a modern understanding of MHD turbulence. At first, gradients of velocities are with the VCGs (Gonzalez-Casanova & Lazarian 2017; Yuen & Lazarian 2017a, 2017b) based on the fact that velocity eddies are elongated along local magnetic field directions (GS95). Later applications of the VGT include the velocity channel gradients (VChGs) and reduced velocity centroid gradients, which are also explored in LY18.

In a separate development, the gradients of the synchrotron emission (SIGs; LYL17) are shown to be capable of tracing magnetic field directions. In the present paper we introduced a few new gradient measures dealing with the synchrotron polarization. The SPGs are mostly sensitive to the polarization fluctuations arising from the variations of synchrotron emissivity, while SPDGs are more focused on the polarization fluctuations arising from the Faraday rotation. For the Faraday rotation screen, the SPDGs transfer to the FRGs.

The list above does not exhaust the useful gradient measures. For instance, in Appendix B we discuss the Zeeman gradients, which are very similar in properties to the FRGs. In an earlier discussion of Faraday tomography, we suggested that gradients can be a useful tracer of perpendicular to the LOS magnetic field direction within this technique. Moreover, applying the block averaging that we developed initially within the VCG approach in YL17a to the density gradients (IGs) we introduced in YL17b, a new technique of IGs becomes available for the magnetic field studies.

9.3.2. Shared Foundations and Shared Procedures and Synergy

All these gradient techniques are based on the same property of anisotropic MHD turbulence, i.e., turbulent eddies elongated along the local magnetic field, and use similar procedures for their calculation. The close relation of different gradient techniques is easy to understand. Indeed, magnetic and velocity fluctuations enter the expressions for Alfvenic turbulence in a symmetric way. This is not generally true about density fluctuations. The studies in Beresnyak et al. (2005) and Kowal et al. (2007) show that for high sonic Mach number \( M_s \) the density clumps with a relatively low filling factor dominate both the spectral slope and its anisotropy. At the same time, turbulent density filling most of the volume exhibits the GS95 scaling, with density passively transferred by the turbulent velocity field. This opens a way to tracing magnetic fields using density gradients, provided that we can identify and remove the volumes of the clumps. This is the essence of our IG (see YL17b) technique, which has its unique complementary features. Indeed, the IGs can be used together with velocity gradients in

\(^{14}\) The Nyquist criterion for reconstructing the Faraday dispersion would be on the order of 100. Potentially if one considers the sparse properties of \( \lambda^2 \), the number of measurements may be reduced and sampled through the compressive sampling theory (Li et al. 2011). We explore this method of study elsewhere.

\(^{15}\) One thing that Faraday tomography relies on is that \( \lambda^2 \) is not dependent on the sign of \( B \) and is constructed by each frequency map, providing a reliable LOS distance determination method compared to the Faraday tomography distance through Faraday depth.
order to identify shocks and regions of gravitational collapse. In some cases when the spectral resolution is not adequate or even not available (as in the case of dust emission), the IGs themselves can be used to get a rough picture of magnetic field distribution.

We note that the IG technique should be distinguished from the Histograms of Relative Orientation (HRO) technique in Soler et al. (2013). In the latter technique, the statistical correlation between the relative orientation of density gradients and magnetic field is sought as a function of column density. Unlike the IGs, the HRO is unable to trace magnetic fields and shocks spatially. HRO and the IGs provide different types of information and are complementary. Indeed, the IGs use the block averaging and are able to trace the magnetic field.

Therefore, advances in one technique entail advances in other techniques. For instance, new procedures of averaging, e.g., the moving window procedure and angle constraint procedure, have been tested for the VChGs in LY18. However, these procedures are also applicable in improving the accuracy in other gradient techniques. Similarly, we showed numerically in LY17 that it is important to filter out the contribution of large-scale eddies to trace magnetic fields with SIGs in super-Alfvénic turbulence. We expect that the same filtering allows other types of gradients to trace the magnetic field in super-Alfvén turbulent media.

We also would like to stress that the machinery developed for one gradient technique is usually applicable to other gradient techniques. For instance, the block averaging, originally suggested for the VCGs (YL17a), was later used for other techniques, including synchrotron-based ones. Similarly, more recently discussed techniques of measuring Alfvén Mach number $M_A$ by analyzing the distribution function of the gradient directions within a block (Lazarian et al. 2018a) can be used to obtain $M_A$ using the synchrotron-based techniques, i.e., SIGs, SPGs, and SPDGs. A significant advantage of the new techniques that we introduced in this paper, namely, the SPGs and the SPDGs, is that the distribution of $M_A$ can be obtained in 3D.

By analyzing the gradient measures from different tracers, one can also study the magnetic field structures and other physical properties from different regimes of the same region. For instance, both total line emission and dust emission can be used for calculating the IGs. Combining different measures, one can get additional information. For example, combining various measures for velocity gradients and the IGs, one can identify shocks in diffuse media and regions of gravitational collapse in molecular clouds (YL17b, LY18).

9.4. Gradients and the Search of Cosmological B-modes

9.4.1. Polarization Gradient of the Polarization Alignment Effect

Polarization of the galactic foreground is the most serious contaminant for studying B-mode polarization. Our study indicates that there is a very important difference between the properties of the gradients of polarization related to the two types of emission. In particular, the gradients of the foreground polarization are expected to be well aligned with the direction of the foreground polarized emission.\(^{16}\)

The property of the alignment of gradients with the polarization is a universal property of polarized foregrounds, and it is correct not only for the synchrotron foreground contamination. For instance, we discuss in Appendix C that the gradients of polarization from aligned dust are expected to be aligned with the dust polarization directions.

The effect of the “Gradients of Polarization Alignment” opens a new attractive possibility of separating the foreground polarization from that of the cosmological origin. The presence of the aforementioned effect in the data indicated the presence of the foreground contamination. The structure of the polarization gradients that are directionally correlated with polarization can be used as a prior in separating the polarized contamination of both synchrotron and dust from the polarized CMB. We will discuss the detailed procedure of the corresponding analysis elsewhere.

9.4.2. Synergy with the Velocity Gradients

In addition, the analysis of velocity gradients can provide the expected directions of magnetic fields using H1 radiation.\(^{17}\) This is similar to the suggestion in Clark et al. (2014, 2015) of using the filamentary structures in velocity channel maps in order to get predictions of the polarization arising from the dust at high galactic latitudes. We believe that the aforementioned filamentary structures are the result of velocity crowding and are related to the velocity gradients. In our collaborative work with S. Clark we are going to provide a comparison of the two approaches. We note that Clark (2018) introduced a measure of magnetic tangling that can be obtained through the analysis of the filamentary pattern. In Lazarian et al. (2018a) we related the dispersion of the gradients with $M_A$, which is a measure of magnetic disorder within MHD turbulence theory. In fact, in the same paper we related also the dispersion of dust polarization degree to $M_A$. This provides a way of not only predicting the direction of the galactic dust polarization but also predicting the expected degree of dust alignment.

9.4.3. Gradients in ISM and Star Formation Studies

A classical picture of the ISM includes the coexistence of different phases (see Draine 2011). These phases are magnetized with the synchrotron emission arising from hot and warm ISM phases. To understand the connection of magnetic fields in the different phases, it is good to use different tracers. Gradients provide such a possibility. For instance, velocity gradients can trace magnetic fields using various emission and absorption lines. In YL17b and LY18 the possibility of studying star formation using velocity gradients and the combination of the velocity and intensity gradients have been explored.

It is very important that gradient techniques provide an extremely promising way of studying the 3D magnetic field structure of the diffuse media. In this paper we have discussed the 3D studies for SPGs. For velocity gradients the 3D information can also be obtained (LY18). For instance, by employing the Milky Way rotation curve, it is possible to determine magnetic field directions as those change along the LOS, obtain magnetic fields in high-velocity H1 clouds, study magnetic fields in molecular clouds that are in galactic

\(^{16}\) This statement becomes less accurate if the foreground fluctuation arises from the super-Alfvénic turbulence. In this situation, the difference of the additive properties of gradients and the Stokes parameters can induce deviations. However, both theoretical arguments and the analysis of observational data testify in favor of sub-Alfvénic turbulence at high galactic latitudes (see Kandel et al. 2018). For such a turbulence our present study testifies that a very good alignment between the polarization and gradient direction should be present.

\(^{17}\) Dust and gas are expected to be well mixed (see Draine 2011). Therefore, the magnetic field revealed with the gradients is the same magnetic field that aligns the dust, and the intensity of the dust emission is proportional to the 21 cm emission at high latitudes.
disks and occlude each other, etc. The first study of the distribution of magnetic fields in HI disks was performed in D. F. González-Casanova & A. Lazarian (2018, in preparation), with the results successfully tested with the starlight polarization from the stars to which distances were known. In addition, the use of multiple chemical tracers provides a promising way of probing 3D magnetic field structure of molecular clouds from the edges (e.g., using CO, CN) to the core (e.g., using NH2). The synchrotron-based technique also provides the 3D information, but by using the wavelength dependence of the polarization decorrelation length. This opens new horizons for the 3D galactic magnetic field studies.

9.5. New Opportunities for Magnetic Field Studies

We are currently just scratching the surface of a very rich subject. We expect that further studies of gradients can provide us with a lot of new information about magnetic field structure, as well as properties of turbulent ISM. The synergy of different gradient techniques is still something that has not been properly explored. A great advantage of the gradient technique is that the unique information can be obtained even with the existing data sets.

9.5.1. Studying 3D Magnetic Field Structure of Distant Objects

In the previous section (Section 8), we see that there is a possibility of obtaining the 3D magnetic field structures in the Milky Way through the use of SPGs. In fact, it is possible to show that the same 3D mapping is possible for the external galaxies. With the recent advancement of interferometry, we can also measure synchrotron radiation in nearby galaxies. The Faraday rotation induced by the Milky Way ISM acts as a distorting screen for the synchrotron polarization direction. However, it does not change the value of total polarization arising from an external galaxy. As a result, unlike the direction of polarization, polarization gradients are not affected by the Milky Way Faraday rotation and can trace magnetic fields effectively.

As the observational wavelength changes, the Faraday screening effect that we addressed in Section 3.2 becomes also applicable to extragalactic objects and supernova remnants. As a result, the recipe that we addressed in Section 8 is also applicable to these objects. This can bring the studies of synchrotron-emitting objects to a new level.

9.5.2. Synergy with Other Approaches

We should stress that while gradients provide their own information on magnetic field structure, an important synergy between the gradients and polarization exists and should be utilized. We briefly discussed the possible use of the SPGs and SIGs together with polarization. Similarly, the velocity gradients can be used together with dust polarization in order to identify the regions of gravitational collapse within molecular clouds. While our study in LY18 shows that this can be done with velocity gradients alone, the additional information obtained with polarimetry is very helpful.\(^{18}\)

We discussed earlier that the HRO technique (Soler et al. 2013) is different from the IG technique that applies our approaches to density fluctuations. The IGs that trace both magnetic fields and shocks and their joint use with HRO are analyzed in K. H. Yuen & A. Lazarian (2018, in preparation).

The gradient technique is also complementary to other techniques of magnetic field studies, and they can potentially be used together for extracting other physical conditions. For instance, the anisotropic behavior from the theoretical prediction of magnetized turbulence theory illuminates the correlation function anisotropy method (Lazarian et al. 2002) in studying coarse magnetic field structure. The recipe is then elaborated and thoughtfully tested in subsequent papers (Esquivel & Lazarian 2005; Heyer et al. 2008). We have shown that such correlations correctly trace the mean magnetic field but fail to trace the detailed magnetic field structure that is attainable through the use of gradients (see LY17). In the present paper we trace magnetic fields using the gradients of polarized synchrotron intensities.

The relation between the polarization gradient amplitudes and the sonic Mach number was studied in Gaensler et al. (2011) and Burkhart et al. (2012). It was shown there that one can find the sonic Mach number \(M_s\) by using these distributions. Our present study suggests that by using the depolarization effect one can provide such a study in 3D. Moreover, as we discussed earlier, the approach that we successfully tried in Lazarian et al. (2018a) by relating the dispersions of velocity gradients to the Alfvénic Mach number \(M_A\) is expected to be also applicable to the SPGs. As a result, apart from tracing the 3D distribution of magnetic fields, the synchrotron gradients can provide the 3D distribution of \(M_s\) and \(M_A\), which is very valuable for describing many astrophysical processes, including the propagation and acceleration of cosmic rays (see Lazarian & Yan 2014) and star formation (see Burkhart 2018).

10. Summary

In this paper we present a new way of tracing the galactic magnetic field using synchrotron polarization. The theoretical foundations of our new technique are rooted both in the modern understanding of the nature of turbulent MHD cascade (GS95; LV99) and in the theory of synchrotron polarization fluctuations (LP16). Our study is focused on exploring the properties of two measures that are available from observations, the SPGs and the SPDGs. We explore how the directions obtained with these measures are related to the directions of the underlying magnetic fields. We consider the effect of Faraday depolarization and study the cases that this effect arises both from the turbulent and from the mean field. Our main results can be briefly summarized as follows:

1. Both gradient measures, the SPGs and the SPDGs, are perpendicular to the direction of the magnetic field, and they allow us to trace magnetic fields in the regime of both weak and strong Faraday rotation. The direction of the gradients traces the magnetic field and does not require correcting for the effect of Faraday rotation.
2. 3D mapping of magnetic fields with the SPGs and the SPDGs can be obtained, as the wavelength-dependent Faraday depolarization constrains the depth over which the polarized signal is collected.
3. As the polarized radiation intensity provides the magnetic field strength, SPGs and SPDGs provide not only the direction of the magnetic field in the plane of the sky but also its amplitude. Moreover, the full 3D vector of the magnetic field can be obtained if the SPDG technique is employed.
4. The SPGs are complementary to other techniques of magnetic field studies, e.g., our technique that makes use of velocity gradients. The approaches developed for the

\(^{18}\) Our preliminary analysis shows that such regions of gravitational collapse do not constitute a significant part of the molecular clouds that we analyze. This contradicts the theories that attribute the nonthermal line broadening observed in molecular gas to the consequence of the gravitational collapse.
VGT, e.g., measurement of Alfvén Mach number using the velocity gradient distribution, are applicable to the polarization gradients.

5. The SPGs and SPDGs open new ways to analyze data for ISM research in the Milky Way and other galaxies, as well as for magnetic field studies in the intercluster medium.

6. The present study indicates additional types of magnetic field studies, e.g., using the gradients of Faraday rotation measure, dust polarization gradients (DPGs), and ZMGs.

7. In terms of CMB polarization studies, in particular, in connection with the search of the elusive B-modes, our study reveals an important effect of the alignment of gradient polarization and the polarization direction that is not altered by Faraday rotation, which is expected for both the synchrotron and dust polarization and makes the foreground polarization different from that of cosmological origin. This opens new ways of separating the two types of polarization.

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Appendix A

Classification of the Faraday Rotation Regimes in LP16

Here we study numerically how the turbulence statistics of three regimes described in our paper is related to the theoretical predictions in LP16. This can be viewed as the numerical testing of the LP16 results that we employ in our paper. In particular, we are most interested in the strong Faraday rotation regime described in LP16.

A.1. Weak Regime

We study the structure function for both parallel and perpendicular parts of the cube. Figure 10 shows the second-order statistical (structural/correlation) functions with respect to the real-space scale. The left panel of Figure 10 is the weak rotation regime has an expected dependence of $5/3$ in the structure function. Right: the strong regime exhibits the LP16 predicted slope of $-1/3$ in terms of a correlation function.

A.2. Strong Regime

The strong Faraday rotation regime is contributed to by both the source term $P_i$ and the Faraday rotation term $\Phi$ in Equation (2). The relative importance of the two terms affects the behavior of both gradients and correlation functions.
Therefore, we want to test how the relative contributions from the two parts alter the statistics of polarized synchrotron emissions. Figure 11 shows how $\Pi$ and $\Phi$ affect the polarized maps. To test this, we set one of them to be constant while changing the other. For the constant source term case (left panel of Figure 11), the structure is basically parallel to the mean magnetic field direction (horizontal axis). This is in agreement that the gradients of Faraday rotation trace the magnetic field (see Section 6). For the constant Faraday rotation case (middle panel of Figure 11), the structures are more filamentary compared to the constant source term case. This feature is retained in the physical case containing both effects (right panel of Figure 11), which also exhibits a general alignment toward the mean magnetic field.

The reduced correlation function (correlation function minus the minimum of it) for the strong Faraday rotation regime is shown in Figure 10. The dependence of the correlation function on the 2D separation between the points is $R^{-1/3}$, which is consistent with the case of intermediate asymptotics obtained in LP16, namely, when $r_\phi > L_{\text{eff}}$, the correlation function

$$
\langle P(X_1)P^*(X_2) \rangle \propto r_\phi^{-m_\phi/2} R^{-m_i/2}.
$$

In our case $m_\phi = 2/3$, which means an asymptotic of slope $-1/3$, as shown in Figure 11.

### A.3. Numerical Artifact from Discrete Faraday Rotation along the LOS

We observe numerical artifacts when the ratio of length scales $L_{\text{eff}}/L$ is comparable to the grid size $\Delta z$ of the numerical cube. The striking effect of low-frequency synthetic synchrotron emission is that it retains the features of the large-scale structure, as shown in Figure 12. In contrast to Figure 2, if $\Delta z \ll 1$, the structure of Equation (2) looks like the following: Assuming that $\theta$ is a uniform random variable in $[0, 2\pi]$, Equation (2) becomes

$$
P_{\lambda \gg \lambda_{\text{cut}}} \sim \int dz P_i e^{i\theta},
$$

and then the continuous distribution of $P_i$ will guarantee that the amplitude of $P$ drops to zero. However, when there are discrete Faraday sources along the LOS, Equation (20) becomes

$$
P_{\lambda \gg \lambda_{\text{cut}}} \sim \int dz P_i e^{i\theta} \delta\left(\sin\left(\frac{\pi z}{\Delta z}\right)\right),
$$

where $\delta$ is the Dirac delta function. Notice that the resultant $|P|$ is not zero when $\lambda$ is big. Pictorially we can understand the addition in Equation (21) as illustrated in Figure 13. While the phase is random, the amplitude from the largest vectors is retained, which is exactly what the structure in Equation (2) referred to. If the amplitude is set to be constant, the mixing of random phase constant vectors is equivalent to the random walk around the origin. In terms of the polarized intensity amplitude, the varying case reflects how the largest $P_i$ contributes to the map.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Figure11.png}
\caption{Synthetic (obtained with 3D MHD) synchrotron polarization maps. Left: with constant $P_i$; middle: constant Faraday rotation term $\Phi$; right: both $P_i$ and $\Phi$ vary. Frequency at $\mu = 1$ GHz, cube Ms1.6Ma0.53.}
\end{figure}
Appendix B
Gradients of Zeeman Measurements

The interstellar Zeeman observations provide the direct measurement of the strength parallel to the LOS component of the magnetic field. The first-order energy difference measured due to the Zeeman effect is

\[ E^{(1)}_Z = \mu_B g_J B_{\parallel} m_J. \]  

(22)

In special circumstances Zeeman measurement can be available over a patch of the sky. In this situation, the structure of the signal is similar to that of the Faraday rotation. Thus, we expect to have the ZMGs directed perpendicular to the plane-of-sky magnetic field. In numerical simulation, we mimic the effect of the Zeeman effect and assume that the quantum quantities \((\mu_B, g_J, m_J)\) are constant. Figure 14 illustrates this effect; the alignment measure is 0.72.

Appendix C
Gradients of Dust Polarization

The observed 2D projection of the emitting volume has the following polarized emissivity (see Kandel et al. 2018):

\[ \epsilon_{\text{pol}} = \epsilon_Q + i\epsilon_U \sim f_j(H)(H_e + iH_o)^2, \]  

(23)
where $H_s$, $H_f$ are sky-projected magnetic field components for galactic synchrotron $f_{\text{synch}} \approx \text{const}$ and for galactic dust $f_{\text{dust}} = H^{-2}$. Because of the structural similarity of the expressions, one should expect that the DPGs, i.e., gradients of $P_{\text{dust}}$, should have properties similar to the SPGs, i.e., gradients of $P$ that we study in this paper. For the frequencies of dust emission, the Faraday effect is negligible, as a rule.

The valuable property of DPGs is that they are expected to be aligned perpendicular to the magnetic field. This property provides a way of distinguishing this type of polarization from the polarization of cosmological origin and the polarization from scattered light. Elsewhere we will discuss the procedure of combining dust emission gradients (or gas density gradients, e.g., obtained via H I emission) and DPGs in order to separate dust polarization from other sources of polarization, e.g., from the polarization of cosmological origin.

**Appendix D**

Using Polarization Gradients for Studying 3D Magnetic Field Distribution

**D.1. A Brief Description of the Procedure**

The discussion of SPDGs in Section 6 was focused on using the FRGs as a synergistic way of studying projected magnetic field direction together with SPGs. From Section 8, SPGs represent the cumulative projection of magnetic field structures for $L_{\text{eff}} < L$. Since gradients are linearly added along the LOS, it is natural to consider the differences of the SPGs from two frequencies $\lambda_{1,2}$, where $\Delta \lambda = \lambda_2 - \lambda_1 \ll \lambda_1$. The difference of SPGs over the change of $\lambda^2$ contains the information of magnetic field projection between $[L_{\text{eff}}(\lambda_2), L_{\text{eff}}(\lambda_1)]$ (see Equation (7)). In the limit of $\Delta \lambda \rightarrow 0$ the difference presents the SPDG measure. We shall denote $L_i = L_{\text{eff}}(\lambda)$ for the remaining part of this section.

We pictorially illustrate our method in Figure 15. According to LP16, only a distance smaller than $L_{\text{eff}}$ contributes to the polarization measurements taken by the observer, i.e., to Stokes parameters $Q, U$. The “1-radian condition” was introduced in LP16 as the condition for decorrelation, and this defines $L_{\text{eff}}$.

\[
\lambda^2 \Phi = \lambda^2 0.81 \int_0^{L_{\text{eff}}} dz_\perp H_z \approx 1. \tag{24}
\]

The objective in this subsection is to explore how to get the magnetic field morphology, both parallel and perpendicular to the LOS in a slice of $\Delta L \ll L$. The latter condition is satisfied when $\Delta L / L \sim (\lambda^2 \phi)/(\lambda^2 \phi) \ll 1$. The Faraday depolarization effect allows us to write, for a particular $\lambda$,

\[
P(\lambda) = \int_0^{\infty} dz_\perp P_i(X, z) e^{2\lambda^2 \Phi(z, \lambda)} 
\approx \int_0^{L_{\text{eff}}(\lambda)} dz_\perp P_i(X, z) e^{2\lambda^2 \Phi(z, \lambda)}. \tag{25}
\]

The spatial gradients of Equation (25) are just SPGs and provide the cumulative contribution of $P_i$ plus the Faraday rotation factor along the LOS.

The polarization equations for both $\lambda$ values (Equation (2)) are not that different if $\Delta \nu = c / \lambda^2 \Delta \lambda$ is small. Following the discussion in Section 8, we split Equation (2) into two parts and assume that the noise-like part ($z > L_{\text{eff}}$) does not contribute to the average of the polarization map. Thus, the difference of $P(\lambda_1)$ and $P(\lambda_2)$ is simply

\[
\Delta P \approx \int_{L_2}^{L_1} dz_\perp P_i(X, z) e^{2\lambda^2 \Phi(z, \lambda)}. \tag{26}
\]

The plane-of-sky direction of magnetic field between $l \in [L_2, L_1]$ is then given by

\[
\nabla \frac{d|P|}{d \lambda^2} \sim \lambda^{-2} \nabla |P(\lambda_1) - P(\lambda_2)|. \tag{27}
\]

To get the full 3D vector of magnetic field, we also need to obtain the inclination angle of magnetic field relative to the LOS direction, which requires an estimation of magnetic field strength in between $[L_2, L_1]$. From LP16 we know that the quantity $dP / d\lambda^2$ provides the estimate of the rotation measure, which is linearly proportional to the magnetic field strength:

\[
B_z \sim \Delta L /(n_e) \int_{L_2}^{L_1} dz_\perp H_z \sim \frac{4\pi (P_2 - P_1)}{(\Delta L/(n_e))(\lambda_2 - \lambda_1)}. \tag{28}
\]

The strength of the magnetic field component perpendicular to the LOS $B_{\text{POS}}$ can be obtained from the polarized synchrotron intensity, i.e., $B_{\text{POS}} \sim \sqrt{\delta P / \Delta L}$, where $\delta P$ is...
the difference of polarized synchrotron intensity arising from the \([L_2, L_1]\) slice.

The inclination angle of the magnetic field relative to the LOS direction can be approximated by

\[
\tan \phi = \frac{B_{\text{POS}}}{B_z}.
\]

The 3D magnetic field can then be constructed when the plane-of-sky gradient orientation \(\theta_{\text{grad}}\), inclination \(f\), and total field strength \(B_{\text{tot}} = (B_{\text{POS}}^2 + B_2^2)^{1/2}\) are available. By using different \(\lambda_{1,2}\) with small enough \(\Delta \lambda\), the 3D field structure can be obtained.\(^{19}\)

D.2. Tracing the Distribution of Plane-of-sky Magnetic Fields in a Slice

We test our recipe (Appendix D.1) in the random-field case, which corresponds to the calculation we did in Section 5. We pick \(\nu = 1\) GHz with \(\Delta \nu = 10\) MHz, which corresponds to the case of \(\Delta \lambda = 0.003\) m, \(L_{\text{eff}} = 4.9\), and \(\Delta L = 0.21\) (10 slices over 480 spatial resolution for Ms1.6Ma0.53). The condition \(\Delta L/L \ll 1\) is satisfied, and therefore the polarization gradients can be computed readily following the treatment in Section 6 and Appendix D.1. The resultant \(d|P|/d\lambda^2\) map and its gradients (SPDGs, in red) are shown in Figure 16. To compare with polarization, we compute the projected magnetic field from the 3D magnetic field data with LOS deepness of \([L_{\text{eff}} - \Delta L, L_{\text{eff}}]\). Numerically we compute the effective Stokes parameters \((Q_{\Delta \lambda}, U_{\Delta \lambda})\) by

\[
Q_{\Delta \lambda} \sim \int_{L_{\text{eff}} - \Delta \lambda}^{L_{\text{eff}}} dz Q(X, z)
\]

\[
U_{\Delta \lambda} \sim \int_{L_{\text{eff}} - \Delta \lambda}^{L_{\text{eff}}} dz U(X, z).
\]

The effective polarization angle, \(\theta_{\Delta \lambda}\), reflects the average magnetic field orientations in the deepness of \([L_{\text{eff}} - \Delta L, L_{\text{eff}}]\). The alignment measure between SPDGs and the effective polarizations in Figure 16 is \(AM = 0.73\).

We have shown that we can trace the magnetic field using the polarization gradients as \(L_{\text{eff}} < L\). It is easy to see that one cannot use the direction of synchrotron polarization to do the same job. For instance, Figure 17 shows the orientation histogram (see YL17a for a formal definition) of both polarization angles computed through differences of Stokes parameter (left) and the SPDG orientation (right). One can see that there is a preferred direction for gradient angle orientation inside a block but not for the polarization. Therefore, we conclude that the gradients provide a unique way of evaluating magnetic field structure that has significant advantages compared to the traditional magnetic field tracing using synchrotron polarization directions.

---

Figure 16. Sliced SPDGs (rotated 90°; red) compared with the sliced projected magnetic field (blue) overlaid on the \(d|P|/d\lambda^2\) when \(\nu = 1\) GHz. The alignment measure is 0.73.

\(^{19}\) The advantage of using the modulus \(|P|\) over the cross-intensity \(X\) (see Section 5) is evident, since there is an additional phase factor for the complex polarization difference (Equation 26). Considering carefully the rotation of \(Q, U\) due to this phase factor, one shall realize that the modulus \(|P|\) acquires no additional factor, while that for cross-intensity \(X\) has a factor in the form of \(e^{2\pi i \lambda_0 \Delta \lambda}\).
D.3. 3D Construction

As we discussed above, we can use data corresponding to different \((L_{\text{eff}}, \Delta L_{\text{eff}})\) to reconstruct the 3D magnetic field structure. To reduce the complexity, we would use the mean-field results from a 15-slice irregular \(\Delta \nu\) PPF cube ranging from 1.011 Hz to 1.153 GHz. The sample PPF cube would then provide 14 slices of magnetic field with length scale from \(L = 4.56\) to \(L = 4.66\) with equal \(\Delta L\) spacing of 0.1. We listed our choice of frequencies in Table 3 for the reader’s reference. The spacing corresponds to about 5 out of 480 slices in our cube with a total LOS depth of 10 pc \((5/480 \times 10 \sim 0.104\) pc). Therefore, we selected the appropriate slices from the original cube with Stokes-parameter addition within the slice to obtain the five-slice POS component and taking a five-slice mean for the LOS component. Under this construction, we can compute the 3D magnetic field by gradients following the recipe in Appendix D.1 and compare it with the mean 3D orientation from the five-slice component. The difference between the mean inclination is about \(2^\circ\).

Figure 18 shows the 3D magnetic field constructed by Appendix D.1 compared to the real 3D structure from the same cube M51.6Ma0.53. For the sake of comparison, we show only the first layer of the vectors for readers to compare. The 3D morphology predicted by the recipe is close to what is in the original data. We also compare the two 3D vector field with its

### Table 3

| \(\nu\) (GHz) | \(\Delta \nu\) (MHz) | \(L_{\text{eff}}\) (pc) |
|---------------|---------------------|---------------------|
| 1.011         | 11                  | 4.56                |
| 1.022         | 11                  | 4.66                |
| 1.032         | 11                  | 4.76                |
| 1.042         | 11                  | 4.86                |
| 1.053         | 10                  | 4.96                |
| 1.064         | 10                  | 5.06                |
| 1.074         | 10                  | 5.16                |
| 1.084         | 10                  | 5.26                |
| 1.094         | 10                  | 5.36                |
| 1.104         | 10                  | 5.46                |
| 1.114         | 10                  | 5.56                |
| 1.124         | 10                  | 5.66                |
| 1.134         | 10                  | 5.76                |
| 1.143         | 10                  | 5.86                |
| 1.153         | 10                  | 5.96                |

![Figure 17](image_url)

**Figure 17.** Angle orientation histograms for polarization angle computed through the difference of Stokes parameters (left) and the SPDG orientation (right). The zero degree corresponds to the horizontal axis. While gradients provide the direction of magnetic field, the polarization is affected by Faraday rotation and does not represent the magnetic field.

![Figure 18](image_url)

**Figure 18.** 3D magnetic field morphology constructed from the real slices spaced \(\Delta L = 0.1\) (blue) and from the re-construction of the magnetic field employing our recipe (red). The vectors are clearly aligned to each other.
dot product, which gives the cosine of the relative angle between them. Figure 19 shows the plot of the relative angle. The peak is at around 15°, which means that the recipe from Appendix D.1 is promising as the first step of tracing the 3D magnetic field. Further improvements of the technique will be discussed elsewhere.

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Figure 19. 3D relative angle scatter plot obtained by taking the dot product on the unit vectors of the B-field predicted by the recipe (Appendix D.1) and the real field.

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