Using Atomic Clocks to Detect Gravitational Waves

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Atomic clocks have recently reached a fractional timing precision of $\lesssim 10^{-18}$. We point out that an array of atomic clocks, distributed along the Earth’s orbit around the Sun, will have the sensitivity needed to detect the time dilation effect of mHz gravitational waves (GWs), such as those emitted by supermassive black hole binaries at cosmological distances. Simultaneous measurement of clock-rates at different phases of a passing GW provides an attractive alternative to the interferometric detection of temporal variations in distance between test masses separated by less than a GW wavelength, currently envisioned for the eLISA mission.

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Introduction. Over the past year, the precision of optical lattice clocks has advanced dramatically, to a fractional timing precision of (\(\Delta t/t\) \(\sim 10^{-18}\)), with prospects for a future improvement by two additional orders of magnitude through the use of other atoms \textsuperscript{[3,4]}. Here, we point out that the new regime of timing precision made accessible by atomic clocks overlaps with the expected amplitudes of time dilation and compression due to the passage of gravitational waves (GWs). The standard time-dilation effect for a clock at some distance from a black hole, would be modulated by the periodic change in this distance due to the orbital motion in a binary black hole system. Quantitatively, compact binaries of supermassive black holes \textsuperscript{[4,5]} at cosmological distances produce a periodic modulation of the time-time component of the metric (in a Newtonian gauge) at a level of

\[
 h_{00} \approx 9 \times 10^{-18} \left( \frac{D_L}{\text{Gpc}} \right)^{-1} \left( \frac{M_z}{10^6 M_\odot} \right)^{5/3} \left( \frac{f}{\text{mHz}} \right)^{2/3},
\]

where \(D_L(z)\) is the luminosity distance for a source at redshift \(z\), \(M_z \equiv (1+z)(M_1 M_2)^{3/5}/(M_1 + M_2)^{1/5}\) with \(M_1\) and \(M_2\) being the masses of the binary members which are assumed here to be on a circular orbit, and \(f = (\omega/2\pi)\) is the observed (redshifted) GW frequency (obtaining its maximum value based on the orbital frequency at the binary’s innermost stable circular orbit).\textsuperscript{1}

Such black hole binaries are a natural consequence of galaxy mergers \textsuperscript{[7]}. Around the Milky Way’s supermassive black hole, GWs with a similar amplitude and frequency can be emitted by orbiting close-in stars \textsuperscript{[8]}. Such sources constitute the primary targets for the future eLISA space observatory \textsuperscript{[9]}, which aims to detect GWs in the frequency range of 0.1-100 mHz and is planned for launch in two decades. eLISA is designed as a laser interferometer that will record the phase shift introduced by a passing GW as the wave induces a change in the space-time curvature, and hence a change in light travel time between its three test masses, which are separated from each other by \(10^6\) km (less than the GW wavelength). Here, we consider the alternative approach of detecting the differential time dilation experienced by clocks located at different phases of a passing GW. For this purpose, we propose using an array of atomic clocks.

Atomic clocks in space. We envision a set of small orbiting units (at a minimum two of them) equipped with atomic clocks, and a primary spacecraft between them, as illustrated in Fig. 1. For simplicity, the clocks are assumed to be distributed along the circular orbit of the Earth around the Sun at an orbital radius of 8.3 light-minutes (1AU \(\equiv 1.5 \times 10^8\) km), since this configuration minimizes the kinetic energy requirements for launch. The typical distances between units will thus be \(\sim 10^8\) km. Additional units will naturally improve the sensitivity and directional angular resolution.

A passing GW characterized by a metric perturbation \(h_{\mu\nu} e^{i(kr - \omega t)}\) will induce periodic variations (over a period \(2\pi/\omega = 2\pi/kc\)) in the ticking rate of each clock \textsuperscript{[10]}. The amplitude of the timing variation would be \(\frac{1}{2} h_{00}\) (since time is dilated by \((1 + h_{00})^{-1/2}\)). For each pair of clocks, the relative phase of the periodic variation \(\Delta \phi = k \Delta r\) will be dictated by the projection of the difference in clock position vectors \(\Delta r\) on the GW wavevector \(k\) (which defines the GW propagation direction). The largest timing phase difference would be realized between clocks separated by half a wavelength, \(\frac{1}{2} \lambda = 1\text{AU}(f/1\text{mHz})^{-1}\), corresponding to a timing phase difference \(\Delta \phi = \pi\). Measurements of the \(N(N-1)\) phase differences \(\{\Delta \phi_{i,j}\}_{i,j=(1,2,...,N)}\) for a set of \(N\) clocks would allow to localize the GW source position on the sky as \(-k\).

To communicate the periodic time dilation or compression signal associated with a GW, the clock in each unit should drive an optical-frequency (\(f_{\text{laser}} \sim 10^{15}\) Hz) laser, pointed at the primary spacecraft. The required signal-

\textsuperscript{1}In this paper, we adopt for pedagogical reasons a Newtonian gauge which is commonly used to describe the time-dilation effect due to stationary gravity, as measured in the Pound-Rebka experiment \textsuperscript{[2]}. In this gauge, an oscillating perturbation in the time-time component of the metric, \(h_{00}\), would trigger periodic variation in the Pound-Rebka time dilation and a mismatch between the ticking rate of clocks separated apart.
to-noise ratio is not dictated by the precision for distance measurements through laser phase shifts, but rather by what is needed to maintain an optical phase lock between the different units such that the ticking rate of their clocks can be compared at a high precision. Techniques for remote optical clock comparison had been developed over the past decade [11, 12] and are conceptually different from the interferometric technique currently envisioned for eLISA. In the absence of phase noise in space, the distance noise level of $\sim 5 \times 10^{-8} \text{ cm}/\sqrt{\text{Hz}}$ expected from the shot noise of a 1 Watt laser detected by telescopes of 30 cm diameter across a 1-2AU path length, would be more than sufficient to phase lock a single optical cycle. The needed laser is available and already in use. The primary spacecraft will interfere the laser signals from the two daughter clocks to produce a beat signal with a frequency $v_{\text{beat}} \sim h_{\text{GW}}/h_{\text{laser}}$, i.e. with a period of $\lesssim 10^3$ s, comparable to the period of the GW, and hence detectable.

**Discussion**

The proposed method is fundamentally different from the current interferometric methods which underline the design of Advanced-LIGO [13] and eLISA. In GW interferometry, the GW wavelength is considerably longer than the interferometer arm length, and at any given moment one is essentially measuring the roughly-uniform space curvature in the region between the test masses by means of the light-travel time. Half a GW period later, the change in light-travel time reveals the passage of the GW. In our clock-timing method, the separation between clocks is of order half a GW wavelength and one measures the simultaneous difference in clock rates between the low-curvature and high-curvature phases of the GW (which again flips after half a GW period). The change in the light travel time between the clocks due to the GW averages out over a GW wavelength, and thus amounts to a higher order correction to the beat frequency, which can be neglected.

Our proposal is to measure timing variations rather than distance variations. Our detection scheme is not concerned with laser phase variations that are induced by distance variations between free-floating units, but rather with the change in the rate at which clocks are ticking relative to each other. The lasers are locked to the clocks so as to keep their timing stability at a high level.

The precision achieved by optical lattice clocks scales inversely with the square root of the integration time and limits the clock-timing method to low-frequency GWs with $f \lesssim 1 \text{ mHz}$, allowing integration for $\sim f^{-1} \gtrsim 10^3 \text{ s}$. This, in turn, requires clocks at $\sim 1 \text{ AU}$ separations, as considered here. Secular trends (due to slow orbital variations in the gravitational effect of the Sun or planets, the energy loss of the Sun, or the phase shift due to the solar wind) over the short period of the measurement ($\lesssim 1 \text{ hour}$) can be separated from the periodic GW signal in the frequency range of interest here. Furthermore, for GW signals lasting weeks or months, the orbital motion will scan the ecliptic, improving the source localization accuracy. Solar oscillations at mHz frequency (p-mode and especially g-mode) could also be detected by the proposed array.

As indicated by Eq. 1, the timing precision of atomic clocks is sufficient to detect a GW from a supermassive black hole binary at cosmological distances within one GW period. The signal-to-noise ratio (SNR) of GW detection will improve in proportion to the square-root of the number of wave cycles being observed. Therefore, to achieve SNR=1, the fractional timing precision per cycle could be worse than the GW amplitude by the square-root of the number of wave cycles observed.

So far we assumed a monochromatic GW signal and ignored the slow drift in the GW frequency (so-called chirp) due to the inspiral of the supermassive black hole binary. The lifetime of tight binaries is in fact limited by the rate of GW energy loss [14]. For a circular binary orbit, the fractional frequency increase during an infinitesimal observing time $\Delta t_{\text{obs}}$ is given by [1],

$$\frac{\Delta f}{f} = 0.3 \left( \frac{M_z}{10^6 M_\odot} \right)^{5/3} \left( \frac{f}{\text{mHz}} \right)^{8/3} \left( \frac{\Delta t_{\text{obs}}}{1 \text{ hour}} \right).$$

The temporal rise in frequency $f$ (chirp) of the GW signal can also be detected through clock timing to provide a second constraint on the values of $M_z$ and $D_L$ in addition to Eq. 1. Since there are $\gtrsim 10^{12}$ galactic mergers within a Hubble time in the observable volume of the universe, the duty cycle of detectable signals (event rate times lifetime) could be high [15, 16].

The use of a large number of uncorrelated clocks, $N$, in every clock unit, can improve the timing precision by a factor of $1/\sqrt{N}$. This may become another advantage of the proposed method, as atomic clocks become progressively small, low-weight and inexpensive. If the clocks are
quantum entangled, the timing precision could improve as $1/N$ and inversely with integration time [17].

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