Multijet rates in $e^+e^-$ annihilation: perturbation theory versus LEP data

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Abstract

We show that the next-to-leading order perturbative prediction, matched with the next-to-leading logarithmic approximation for predicting both two-, three- and four-jet rates using the Durham jet-clustering algorithm, in the $0.001 < y_{\text{cut}} < 0.1$ range gives a very accurate description of the data obtained at the Large Electron Positron collider. This information can be utilized either for simultaneous measurement of the strong coupling and the QCD colour charges, or for improving the QCD background prediction in new particle searches at LEP2.
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1. INTRODUCTION

Jet production rates provide one of the most intuitive ways to study the underlying parton structure of hadronic events. However, lacking the necessary theoretical accuracy, for measuring the strong coupling $\alpha_s$, only the differential $y_3$ distributions were used so far, where $y_3$ is the event shape variable measuring the value of the jet resolution parameter $y_{\text{cut}}$ for which the jet multiplicity of a given event changes from three-jet to two-jet. The new next-to-leading order results for four-jet rates $^{[1–3]}$ raise the possibility of using LEP four-jet data for $\alpha_s$ precision measurements.

In electron-positron annihilation the widely known Durham $^3$ jet-clustering algorithm has become an indispensable tool for classifying multihadron final states into jets. This jet algorithm has the advantages of relatively small hadronization corrections and of the possibility of resumming large logarithms near the edge of the phase space (small $y_{\text{cut}}$ region), thus extending the validity of the perturbative prediction. Recently a new, Cambridge, jet-algorithm was proposed; it has similar resummation properties, but smaller hadronization corrections for mean jet multiplicities $^5$. More detailed studies showed, however, that the small hadronization corrections found for the Cambridge algorithm in the study of the mean jet rate are due to cancellations among corrections for the individual jet production rates. Apart from the very small values of the resolution parameter, $y_{\text{cut}} < 10^{-3.2}$, the Durham clustering shows, for the individual rates, comparably small (for $y_{\text{cut}} > 10^{-2}$), or even much smaller, hadronization corrections $^6$. We will be using data with $y_{\text{cut}} > 10^{-3}$; therefore, in this talk we consider only multijet rates obtained using the Durham algorithm.

2. THE THEORETICAL DESCRIPTION

The $n$-jet rates are defined as the ratio of the $n$-jet cross section to the total hadronic cross section, and at next-to-leading order these take the general form

$$R_n = \frac{\sigma_{n-jet}}{\sigma_{tot}} = \frac{\eta(\mu)^{n-2} B_n(y_{\text{cut}})}{\eta(\mu)^{n-1} \left\{ (n-2)B_n(y_{\text{cut}})\beta_0 \ln(x_\mu) + C_n(y_{\text{cut}}) \right\}}.$$  

In this equation $\eta(\mu) = \alpha_s(\mu)C_F/2\pi$, $x_\mu = \mu/\sqrt{s}$, where $\mu$ is the renormalization scale and $\sqrt{s}$ is the total c.m. energy. The functions $B_n$.
and $C_n$ are independent of the renormalization scale, $B_n$ is the Born approximation and $C_n$ is the radiative correction. These functions in the case of the Durham clustering for $n = 2$ and $3$ were calculated based upon the ERT matrix elements [6] and for $n = 4$ they were obtained in Refs. [12] based upon the one-loop matrix elements of Ref. [2]. We use the two-loop expression for the running coupling,

$$\eta(\mu) = \frac{\eta(M_Z)}{w(\mu, M_Z)} \left( 1 - \frac{\beta_0}{\beta} \eta(M_Z) \ln(w(\mu, M_Z)) \right),$$

with

$$w(q, q_0) = 1 - \beta_0 \eta(q_0) \ln \left( \frac{q_0}{q} \right),$$

$$\beta_0 = \frac{11}{3} x - \frac{4}{3} y_t,$$

$$\beta_1 = \frac{17}{3} x^2 - 2 y_t - \frac{10}{3} x y_t$$

($x = C_A/C_F$ and $y_t = T_R N_t/C_F = N_t/2C_F$). All theoretical predictions in this contribution were obtained for five light-quark flavours at the $Z^0$ peak with $M_Z = 91.187$ GeV, $\Gamma_Z = 2.49$ GeV, $\sin^2 \theta_W = 0.23$ and $\alpha_s(M_Z) = 0.118$.

Multijet fractions decrease very rapidly with increasing resolution parameter $y_{cut}$. Consequently, most of the available multijet data are at small $y_{cut}$. It is well known that for small values of $y_{cut}$ the fixed order perturbative prediction is not reliable, because the expansion parameter $(\alpha_s/2\pi) \ln^2 y_{cut}$ logarithmically enhances the higher-order corrections. For instance, $(\alpha_s/2\pi) \ln^2 0.01 \approx 0.4$. Thus, one has to perform the all-order resummation of the leading and next-to-leading logarithmic (NLL) contributions. This resummation is possible for the Durham algorithm using the coherent branching formalism [6]. The two-, three- and four-jet rates in the NLL approximation are given in terms of the NLL emission probabilities $\Gamma_i(Q, q)$ [6] which have the following form:

$$\Gamma_q(Q, q) =$$

$$\frac{4\eta(q)}{q} \left[ \left( 1 + \eta(q) K \right) \ln \frac{Q}{q} - \frac{3}{4} \right],$$

$$\Gamma_g(Q, q) =$$

$$\frac{4x \eta(q)}{q} \left[ \left( 1 + \eta(q) K \right) \ln \frac{Q}{q} - \frac{11}{12} \right],$$

$$\Gamma_f(Q, q) = \frac{4w \eta(q)}{3 q}.$$  

We relate the $\eta(q)$ strong coupling appearing in the emission probabilities to the strong coupling at the relevant renormalization scale, $\eta(\mu)$, according to the one-loop formula

$$\eta(q) = \frac{\eta(\mu)}{w(q, \mu)},$$

where $w(q, q_0)$ was defined in Eq. (6), and we use Eq. (6) for expressing $\eta(\mu)$ in terms of $\eta(M_Z)$. We could also use a two-loop formula for $\eta(q)$, but the result would differ only in subleading logarithms. However, we take into account a certain part of subleading soft logarithms with the inclusion of the $K$ term. The $K$ coefficient is renormalization-scheme dependent. In the \overline{MS} scheme it is given by

$$K = \left( \frac{67}{18} - \frac{\pi^2}{6} \right) x - \frac{10}{9} y_t.$$  

The result of this resummation together with its renormalization-scale dependence in the case of four-jet rates was studied in Ref. [6], where we found that the fixed-order and the NLL approximations differ significantly. One expects that for large values of $y_{cut}$ the former, and for small values of $y_{cut}$ the latter is the reliable description; therefore, the two results have to be matched.

The Durham multijet rates can be resummed at leading and NLL order, but they do not satisfy a simple exponentiation [10] (except for the two-jet rate). For an observable that does not exponentiate, the viable matching scheme is the R-matching [6], which we use according to the following formula:

$$R_n^{\text{match}} = R_n^{\text{NLL}} +$$

$$\left[ \eta^{n-2} \left( B_n - B_n^{\text{NLL}} \right) + \eta^{n-1} \left( C_n - C_n^{\text{NLL}} \right) \right],$$

where $B_n^{\text{NLL}}$ and $C_n^{\text{NLL}}$ are the coefficients in the expansion of $R_n^{\text{NLL}}$ as in Eq. (6).
3. RESULTS

In Figs. 1, 2 and 3, we show the theoretical prediction at the various levels of approximation: in fixed-order perturbation theory at the Born level (LO), at next-to-leading order (NLO), resummed and R-matched prediction (NLO+NLL), and improved resummed and R-matched prediction (NLO+NLL+K), for the two-, three- and four-jet rates respectively. Also shown are the multijet rates measured by the ALEPH collaboration at the $Z^0$ peak \([11]\) corrected to the parton level using the PYTHIA Monte Carlo \([12]\). We used bin-by-bin correction and the consistency of the correction was checked by using the HERWIG Monte Carlo \([13]\). The two programs gave the same correction factors within statistical errors. The errors on the data are the scaled statistical errors of the published hadron level data, and we did not include any systematic experimental error, or the error due to the hadron-to-parton correction. In the lower parts of the plots we show the relative difference \((\text{data-theory})/\text{theory}\), where theory means the ‘NLO+NLL+K’ prediction and we also indicated the renormalization scale dependence.

Figures 1–3 deserve several remarks. First of all, we see that the inclusion of the radiative corrections improves the fixed-order description of the data, using the natural scale \(x_\mu = 1\) for larger values of \(y_{\text{cut}}\). Secondly, the importance of resummation in the small \(y_{\text{cut}}\) region is clearly seen, but it is still not sufficient to describe the data at the natural scale, neglected subleading terms are still important. On the other hand, the improved resummation seems to take into account just the right amount of subleading terms; this makes the agreement between data and theory almost perfect over the whole \(y_{\text{cut}}\) region, as can be seen from the lower part of the plots. (In the case of four-jet rate, for \(y_{\text{cut}} > 10^{-1.7}\), \(\delta_4\) falls outside the \(\pm 3\%\) band, one should keep in mind that in this region i) the renormalization scale dependence is relatively large and ii) the number of events is very small; therefore the statistical errors of the data and that of the hadron-to-parton correction are very large.)

We found remarkably small scale dependence
for the ‘NLO+NLL+K’ predictions in the region $y_{\text{cut}} > 10^{-3}$. This feature, however, should be taken with care. On the one hand at any artificial narrowing of the scale-dependence bands, e.g. at a crossover point, the bands almost certainly do not represent the size of the truncation error at that point. On the other, the improvement, obtained by including the two-loop coefficient $K$, affects NNLL terms, but there are other contributions of the same order that are not taken into account (e.g. next-to-leading order running of $\alpha_s$ and other dynamical effects). The scale dependence of the ‘NLO+NLL+K’ result would consistently be under control only after the inclusion of the complete set of NNLL terms. One expects however, that the scale dependence of the ‘NLO+NLL’ prediction is an upper bound for the scale dependence of the perturbative prediction, with subleading logarithms taken into account completely. Therefore, we may use our ‘NLO+NLL+K’ prediction for QCD tests, for instance, for measuring the strong coupling $\alpha_s$ with the condition that we estimate the systematic theoretical uncertainty due to the scale dependence from the scale dependence of the ‘NLO+NLL’ prediction (obtained from varying the scale $\mu$ between 0.5 and 2 as standard choice). The result of such a fit is given in Table 1, where the central value was obtained using the ‘NLO+NLL+K’ result with $\mu = 1$ and the error represents

- the statistical error on the data,
- the systematic error due to changing the fit range (the whole range shown in Figs. 2 and 3 was chosen) by one bin at both ends in both directions,
- the error due to the use of different Monte Carlo programs (PYTHIA and HERWIG) for calculating the hadronization corrections,
- and the error due to the variation of renormalization scale as described above, all added in quadrature. This error is strongly dominated by the scale uncertainty. Of course, we could not include the systematic experimental error. Also, in Table 1 we show the result of the fit when the renormalization scale is left as a free parameter. It is remarkable that, in the case of the three-jet rate, the natural scale $\mu = 1$ is very close to the scale giving the smallest $\chi^2$/d.o.f. ($x_\mu = 0.92$). On the other hand, in the case of the four-jet rate, the fitted scale is still somewhat lower than the natural scale. In our interpretation, this is due to the importance of the still neglected subleading terms. Indeed, we could also fit the four-jet data with the ‘NLO+NLL’ prediction, but with a very low scale ($x_\mu \simeq 0.2$) and a slightly higher value of the strong coupling ($\alpha_s(M_Z) = 0.121$).

4. CONCLUSION

In this talk we studied the perturbative description of two-, three- and four-jet rates produced at LEP, obtained using the Durham clustering algorithm. We found that the best theoretical approximation that is currently available gives a remarkably precise account of the data. In a previous publication [14], we found that the angular correlations defined on four-jet events are also well described (within ±5%) by perturbative QCD.
Table 1
Results of $\alpha_s(M_Z)$ fits with fixed and fitted renormalization scale using multijet data obtained by the ALEPH collaboration at LEP1.

| $\alpha_s(M_Z)$ | $R_3$ | $R_4$ | $R_3 \& R_4$ |
|-----------------|-------|-------|----------------|
| $x_\mu$ fixed   | 0.116 | 0.1182| 0.1175         |
| $\chi^2$/d.o.f. | 5.9/17| 29.2/12| 54.4/29        |

| statistical errors | ±0.0004 | ±0.0003 | ±0.0002 |
| fit range          | ±0.0002 | ±0.0002 | ±0.0001 |
| hadroniz.          | ±0.0008 | ±0.0005 | ±0.0001 |
| ren. scale         | ±0.0015 | ±0.0022 | ±0.0017 |
| total              | ±0.0018 | ±0.0023 | ±0.0018 |

| $\alpha_s(M_Z)$ | 0.116 | 0.1176 | 0.1173 |
| $x_\mu$ fitted   | 0.92  | 0.64   | 0.7    |
| $\chi^2$/d.o.f.  | 5.5/17| 10.4/12| 29.6/29|

These observations suggest that the same procedure should provide an accurate prediction of the multijet backgrounds encountered in new particle searches and $W$-mass measurements at LEP2.

We also performed a measurement of the strong coupling based upon multijet rates. We found $\alpha_s(M_Z) = 0.1173 \pm 0.0018$, where the error includes the statistical and theoretical ones, but not the systematic experimental ones.

These results were produced in part by a partonic Monte Carlo program called DEBRECEN [15], which is based upon the dipole formalism [16] and can be used for the calculation of QCD radiative corrections to the differential cross section of any kind of infrared-safe three- and four-jet observable in electron-positron annihilation.

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