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Mathematical assessment of the dynamics of novel coronavirus infection with treatment: A fractional study

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In this paper, a mathematical model is formulated to study the transmission dynamics of the novel coronavirus infection under the effect of treatment. The compartmental model is firstly formulated using a system of nonlinear ordinary differential equations. Then, with the help of Caputo operator, the model is reformulated in order to obtain deeper insights into disease dynamics. The basic mathematical features of the time fractional model are rigorously presented. The nonlinear least square procedure is implemented in order to parameterize the model using COVID-19 cumulative cases in Saudi Arabia for the selected time period. The important threshold parameter called the basic reproduction number is evaluated based on the estimated parameters and is found \( R_0 \approx 1.60 \). The fractional Lyapunov approach is used to prove the global stability of the model around the disease free equilibrium point. Moreover, the model in Caputo sense is solved numerically via an efficient numerical scheme known as the fractional Adams-Bashforth-Molten approach. Finally, the model is simulated to present the graphical impact of memory index and various intervention strategies such as social-distancing, disinfection of the virus from environment and treatment rate on the pandemic peaks. This study emphasizes the important role of various scenarios in these intervention strategies in curtailing the burden of COVID-19.

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1. Introduction

Pandemics are considered among those climate-induced challenges which have been witnessed throughout the history of human civilization. Plague, Measles, Spanish flu, Cholera, HIV AIDS are some of the previous pandemics which have drawn drastic impacts across the globe. Recently, the whole world is facing a new challenge in terms of a novel coronavirus disease (or COVID-19). A new coronavirus was first identified as the cause of a COVID-19 outbreak that appeared in China in December 2019. The novel virus named the severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) caused COVID-19. The diseases such as the common cold, Middle East respiratory syndrome (MERS) and severe acute respiratory syndrome (SARS) are caused by the family of coronaviruses. Although various aspects of this novel infectious disease are not yet known and need to be investigated. The current main spreading sources are either with direct contact with infected person or through airborne transmission. A person in close contact within about 6 feet to an infected person can catch the virus. The virus may also be transmitted through the respiratory droplets of an infected person while coughing, sneezing, or even talking [1,2]. The COVID-19 infection can also be transferred by touching a surface contaminated with the virus. The symptoms of COVID-19 vary from person to person and can range from very mild to severe. Some people may have very mild or only a few symptoms. The common signs or symptoms of this novel infection include fever, cough, or tiredness, etc. The loss of taste or smell is an early sign of this infection whereas some other symptoms are shortness or difficulty in breathing, sore throat, headache, chest pain, etc. Globally, as on 08 February 2021, there have been more than 10 million confirmed COVID-19 cases including 2,316,389 deaths, reported to WHO. A total of more than 7.5 million cases are totally recovered [1,2].

The variable dynamics of the ongoing pandemic of COVID-19 is a huge challenge for the whole world. Various approaches have been adopted in this regard. The mathematical modeling approach is considered the language of nature and is one of the reliable tools to provide a better interpretation of a disease dynamics which altimetry supports the public health authorities in decision-making. Additionally, an appropriate mathematical model can be used effectively to predict disease transmission and possible con-
trol. Mostly, the mathematical models of epidemics have been formulated with the help of differential equations which may be ordinary, fuzzy, partial, delay or stochastic in nature. Recently, a rich literature on mathematical modeling can be found addressing various aspects of the novel COVID-19 dynamics. Saif and Khan studied the impact of various controlling measures for the COVID-19 outbreak in Pakistan [3]. Additionally, in [3] the authors presented some useful results regarding optimal control analysis to provide the best controlling strategies. A new mathematical model is formulated in [4] in order to assess the impact of lockdown in the Nigerians population during the ongoing COVID-19 pandemic. A mathematical modeling approach with optimal cost-effective analysis is carried out in [5,6] to investigate an appropriate controlling strategy for the infection in various regions. The dynamics of COVID-19 utilizing a stochastic transmission model have been investigated in [7]. A new epidemic model coupled with the application reinforcement learning control theory, is presented in order to address the issues regarding the available vaccine effective usage and distribution among the population [8].

Mathematical model formulation via the fractional order differential operator provides a better understanding of a real world problem and gains much attention in the last few years to model complex phenomena. The fractional (i.e., non-integer) order derivatives possess the fading memory effects and capture the crossover behavior shown by various physical and biological processes. The well-known differential operators with fractional order include the classical Caputo with power law index [9], Caputo-Fabrizio with exponential kernel [10] and Atangana-Baleanu-Caputo (ABC) with Mittag-Leffler kernel [11]. The application of fractional operator can be found in formulating real-world problems of science and engineering especially in epidemiology [12-17]. Numerous mathematical models based on non-integer order differential systems have been proposed recently to analyze the dynamical behavior of the novel coronavirus disease. A COVID-19 mathematical model with non-singular fractional order operators is formulated and analyzed in [18]. The authors in [18] utilized two different fractional operators in order to present a detailed and deeper insights into the disease dynamics. The application of fuzzy fractional derivatives with both singular and non-singular kernels can be found in [19]. An efficient numerical study of a fractional COVID-19 epidemic model with the help of Legendre spectral approach is presented in [20]. The formulation and analysis of an epidemic model interpreting the COVID-19 is studied in [21]. The impact treatment and some non-pharmaceutical interventions on the curtailling of COVID-19 have been explored via a fractional compartmental model in [22]. Additionally, the authors in [22] used real data from Pakistan to validate the theoretical results and model prediction. The dynamics of novel mathematical model based on the Caputo operator coupled with real data from Wuhan China was studied in [23]. An extensive analysis both theoretically and numerically of a fractional COVID-19 mathematical model was carried out in [14]. The authors in [14] used three different fractional order operators to analyze the dynamics of novel COVID-19. Recently, a modified fractional modeling approach has been adopted to investigate the impact of various control measures and viral transmission on the dynamics of COVID-19 outbreak in Saudi Arabia [24].

Keeping the previous literature in mind, the goal of the current study is to construct a transmission model with time fractional derivative in Caputo sense for the assessment of treatment and other controlling parameters on COVID-19 incidence. The utilization of fractional operator in the model is due to the fact that the spread of infectious diseases has memory dependency and sometimes cross-over patterns. Additionally, a fractional order model provides more deeper insights of the disease dynamics in compression of the integer-order transmission models. To formulate the model we incorporate the pre-symptomatic and treatment classes in the previous similar studies. Moreover, to better explore the dynamics and impact of treatment we extend the classical integer order COVID-19 model to fractional order in the Caputo sense. Additionally, the incidence data from the Kingdom of Saudi Arabia (KSA) for a specific period are taken to estimate the model parameters. We start with the basic definition regarding the present study.

2. Basic results

We recall some basics definitions and results relevant to the present study.

**Definition 1.** Let \( y(t) \in C^\alpha \) be function. The fractional derivative of Caputo type with a given order \( \rho \) (where in \( n - 1 < \rho \leq n \) and \( n \in \mathbb{N} \)) is showed as follows: [9];

\[
\text{D}_t^\alpha y(t) = \frac{1}{\Gamma(n-\rho)} \int_0^t \frac{y^{(\nu)}(\xi)}{(t-\xi)^{\rho-n+1}} d\xi.
\]

Clearly, \( \text{D}_t^\alpha y(t) \) approaches to \( y'(t) \) as \( \rho \to 1 \).

**Definition 2.** The integral relevant to the above power law kernel derivative is defined by Podlubny [9]:

\[
\text{I}_t^\alpha y(t) = \frac{1}{\Gamma(\rho)} \int_0^t y(\xi)(t-\xi)^{\rho-1} d\xi.
\]

**Definition 3.** The ABC fractional operator with \( \rho \) (0 \leq \rho \leq 1) is defined as follows [11]:

\[
\text{ABC}_t^\alpha y(t) = \frac{\Gamma(1+\rho)}{\Gamma(1-\rho)} \int_0^t y(\xi)E_{\rho}\left[\frac{-\rho(t-\xi)^\rho}{1-\rho}\right] d\xi.
\]

The uniform global stability of a system with Caputo type derivative via the fractional Lyapunov approach is presented in following theorem established in [25].

**Theorem 1.** Suppose \( \partial^\alpha \) be an equilibrium point of the system \( \text{D}_t^\alpha x(t) = f(t, x(t)) \), and \( \Psi \in \mathbb{R}^n \) denote the domains that contains \( x^* \). further, let \( \mathcal{L} : [0, \infty) \times \Gamma \to \mathbb{R} \) be a continuously differentiable function satisfying

\[
M_1(x) \leq \mathcal{L}(t, x(t)) \leq M_2(x).
\]

and

\[
\text{D}_t^\alpha \mathcal{L}(t, x(t)) \leq -M_3(x).
\]

\( \forall \ 0 < \alpha < 1 \) and \( x \in \Gamma \), where \( M_1(x), M_2(x) \) and \( M_3(x) \) show the continuous positive definite functions upon \( \Gamma \). Then, \( x^* \) known to the uniformly asymptotically stable equilibrium of the system under consideration.

3. Description of the COVID-19 transmission model

This section initially presents the description of the mathematical model with integer-order differential system to describe the COVID-19 dynamics. The classical model is reformulated via the Caputo operator. For the model formulation, the total human population size denoted by \( N(t) \) at any time instant \( t \) is divided into seven mutually-exclusive sub-groups of population. These sub-population groups are the susceptible-\( S(t) \), exposed who are early or newly-infected individuals and incubating the infection \( E_1(t) \), pre-symptomatic infectious people who are about to complete the period of incubation \( E_2(t) \), infected with clinical symptoms of disease \( I(t) \), infected who have not developed symptoms of disease \( A(t) \), hospitalized or under treatment individuals \( T(t) \) and the recovered/removed one are shown by \( R(t) \). Thus the total population can be expressed as

\[ N = S + E_1 + E_2 + I + A + T + R. \]
It is revealed that the effects of environmental factors play a considerable role in the transmission dynamics of the ongoing outbreak of COVID-19. Therefore, we also consider the concentration of the COVID-19 in the environment is denoted by $E_n(t)$. The details of state variables are given in Table 1.

Few of the main assumptions considered in the mathematical model formulation are:

(i) Homogeneous mixing which means that the population under consideration is well-mixed and each individual is equally-likely to mix with each other.

(ii) People who are in the early-exposed class $E_1$ and are in the incubation period cannot transmit the virus.

(iii) The individuals in $E_2$ are close to surviving the incubation period and are infectious i.e., capable of shedding the virus.

(iv) The individuals in $I$ class who have fully clinical symptoms have the ability to transmit the infection.

(v) The individuals in $A$ class are also capable in the disease transmission.

(vi) The virus may be transmitted through environmental routes i.e. contaminated surfaces etc.

(vii) The death due to COVID-19 is assumed in asymptomatic and hospitalized/treated individuals.

(viii) The individuals in $E_2$, $I$, and $A$ are releasing the virus into environment/surfaces.

(ix) The natural recovery from infection provides permanent immunity against reinfection.

(x) The state variables used in the modeling procedure are assumed to be everywhere differentiable, i.e., these are a smooth function of $t$.

With the above assumptions, we design the following mathematical model based on the dynamical transmission of COVID-19:

$$
\begin{align*}
\frac{dS}{dt} &= \Pi - (\beta_1 E_2 + \beta I + \beta_A A + \beta_e E_n) \frac{S}{N} - \mu S, \\
\frac{dE_1}{dt} &= (\beta_1 E_2 + \beta I + \beta_A A + \beta_e E_n) \frac{S}{N} - (\sigma_1 + \mu) E_1, \\
\frac{dE_2}{dt} &= \sigma_1 E_1 - (\sigma_2 + \mu) E_2, \\
\frac{dI}{dt} &= \sigma_2 (1 - \tau) E_2 - (\mu + \xi_1 + \xi_2) I, \\
\frac{dA}{dt} &= \tau \xi_2 E_2 - (\mu + \xi_2) A, \\
\frac{dT}{dt} &= \xi_1 I - (\mu + \xi_2 + \xi_3) T, \\
\frac{dR}{dt} &= \delta_1 I + \delta_2 A + \delta_3 T - \mu R, \\
\frac{dE_n}{dt} &= \nu_1 E_2 + \nu_2 I + \nu_3 A - \nu E_n, \\
\frac{dD}{dt} &= \xi_1 I + \xi_2 T.
\end{align*}
$$

The non-negative initial conditions (ICs) corresponding to the model (6) are describe as follows:

$$
S(0) = S_0 \geq 0, \quad E_1(0) = E_{10} \geq 0, \quad E_2(0) = E_{20} \geq 0, \quad I(0) = I_0 \geq 0, \quad A(0) = A_0 \geq 0, \quad T(0) = T_0 \geq 0, \quad R(0) = R_0 \geq 0, \quad E_n(0) = E_{n0} \geq 0.
$$

The parameters $\Pi$ and $\mu$ in the mathematical model (6) denote the birth and natural mortality rates respectively. The transmission rates of the COVID-19 infection to susceptible individuals by pre-symptomatic, asymptomatic and symptomatically-infected individuals are denoted by $\beta_1$, $\beta_2$ and $\beta_3$ respectively, where it is assumed that $\beta_1 \neq \beta_2 \neq \beta_3$. The parameter $\beta_e$ denotes the transmission rate of disease due to the virus concentration in the contaminated surfaces in the environment. The transmission from the $E_1$ to $E_2$ is shown by the expression $\sigma_1$. The incubation period of the pre-symtomatic individuals is denoted by $\sigma_2$ and after which a fraction $\tau$, $(0 < \tau < 1)$ with no (or mild) symptoms moves to the asymptomatic infected class $A(t)$ and the remaining who develops the symptoms enters to $I(t)$. The death rates due to the COVID-19 infection in the symptomatic and hospitalized or under treatment individuals are $\delta_1$ and $\delta_2$ respectively. The symptomatically-infected individuals are treated (or hospitalized) at the rate $\xi$. The individuals in the compartment $I(E_2)\cap A$ recover from disease at a rate $\delta_1(\delta_2)\delta_3)$. The parameters $\nu_i$ for $i = 1, 2, 3$, measure the rates at which the individuals in $E_2$, $I$ and $A$ classes respectively contribute to the infection to the environmental reservoir. The symbol $\nu$ is the rate of removing of virus from the contaminated reservoir. The last equation in the model (6) denoted by $\frac{dE_n}{dt}$ is used for the dynamics of population who decanted from COVID-19.

For the sake of convenience let us denote

$$
\lambda(t) = (\frac{\beta_1 E_2 + \beta I + \beta_A A + \beta_e E_n}{N}) \frac{S}{N} - \mu S, \quad b_1 = (\sigma_1 + \mu), \quad b_2 = (\sigma_2 + \mu), \quad b_3 = (\mu + \xi_1 + \xi_2 + \xi_3), \quad b_4 = (\mu + \delta_1) \quad b_5 = (\mu + \xi_2 + \xi_3).
$$

Thus, the model (6) can be redesigned in the following simple form:

$$
\begin{align*}
\frac{dS}{dt} &= \Pi - (\beta_1 E_2 + \beta I + \beta_A A + \beta_e E_n) \frac{S}{N} - \mu S, \\
\frac{dE_1}{dt} &= (\beta_1 E_2 + \beta I + \beta_A A + \beta_e E_n) \frac{S}{N} - (\sigma_1 + \mu) E_1, \\
\frac{dE_2}{dt} &= \sigma_1 E_1 - (\sigma_2 + \mu) E_2, \\
\frac{dI}{dt} &= \sigma_2 (1 - \tau) E_2 - (\mu + \xi_1 + \xi_2) I, \\
\frac{dA}{dt} &= \tau \xi_2 E_2 - (\mu + \xi_2) A, \\
\frac{dT}{dt} &= \xi_1 I - (\mu + \xi_2 + \xi_3) T, \\
\frac{dR}{dt} &= \delta_1 I + \delta_2 A + \delta_3 T - \mu R, \\
\frac{dE_n}{dt} &= \nu_1 E_2 + \nu_2 I + \nu_3 A - \nu E_n, \\
\frac{dD}{dt} &= \xi_1 I + \xi_2 T.
\end{align*}
$$

3.1. Data fitting and parameter estimation process

The purpose of this part is to present the model fitting and estimation procedure of the parameters involved in the system (6). The cumulative recorded confirmed cases in the population of KSA (from March 1 to the end of August 2020 depicted in Fig. 1) are used for the estimation of parameters. The least square regression technique is implemented which is based on minimizing the sum of the square differences between each reported infected data point and the corresponding simulated data point from the COVID-
In addition, the birth rate $\Pi$ and the natural death rate $\mu$ are estimated from the literature [26]. The estimated and fitted numerical values of the parameters involved in the COVID-19 epidemic model are tabulated in Table 2 while the resulting fitting curve of the model to the observed data is depicted in Fig. 2 which shows a good fit. Based on the estimated parameters values from Table 2, the reproductive number is obtained as $R_0 \approx 1.60$. The initial values of state variables considered for simulation results are as $S(0) = 34811870$, $E_1(0) = 2000$, $E_2(0) = 300$, $I(0) = 1$, $A(0) = T(0) = R(0) = 0$ and $E_n(0) = 30000$.

4. Formulation of COVID-19 model in Caputo case

The epidemic models with classical integer order derivative have their importance to explore the transmission dynamics of a disease. However, due to the additional features of non-integer order differential operators, a mathematical model formulated via the fractional order are more suitable and provides a better way to study the transmission dynamics of a disease under consideration. In this section, we extend the COVID-19 transmission model presented in (8) to fractional order with the help of classical Ca-
puto operator in order to achieve a generalized model. To obtain the fractional model, the system (8) can be reproduced in terms of integral as:

\[
\begin{align*}
\frac{ds}{dt} &= \int_{0}^{t} k(t - s') \left[ \Pi - (\beta_s E_2 + \beta I + \beta_{A} A + \beta_{\nu} E_\nu) \frac{S}{N} - \mu S \right] ds', \\
\frac{dE_1}{dt} &= \int_{0}^{t} k(t - s') \left[ (\beta_s E_2 + \beta I + \beta_{A} A + \beta_{\nu} E_\nu) \frac{S}{N} - b_1 E_1 \right] ds', \\
\frac{dE_2}{dt} &= \int_{0}^{t} k(t - s') \left[ \sigma_1 E_1 - b_2 E_2 \right] ds', \\
\frac{dI}{dt} &= \int_{0}^{t} k(t - s') \left[ \sigma_2 (1 - \tau) E_2 - b_1 I \right] ds', \\
\frac{dA}{dt} &= \int_{0}^{t} k(t - s') \left[ \tau \sigma_2 E_2 - b_2 A \right] ds', \\
\frac{dT}{dt} &= \int_{0}^{t} k(t - s') \left[ \frac{\theta_1 - \theta_2}{\theta_1} T - b_3 T \right] ds', \\
\frac{dE_\nu}{dt} &= \int_{0}^{t} k(t - s') \left[ \delta_1 I + \delta_2 A + \delta_3 T - \mu R \right] ds', \\
\frac{dR}{dt} &= \int_{0}^{t} k(t - s') \left[ v_1 E_2 + v_2 A + v_3 E_\nu \right] ds'.
\end{align*}
\]

where, the expression \(k(t - t')\) shows the time-dependent kernel. Further,

\[
k(t - t') = \frac{(t - t')^{\alpha - 2}}{\Gamma(\alpha - 1)}.
\]

Putting the value of kernel in Eq. (9), and then the application of a Caputo operator with order \(\alpha\), we arrived

\[
\begin{align*}
\mathcal{D}^\alpha_{t_0} \left[ \frac{ds}{dt} \right] &= \mathcal{D}^\alpha_{t_0} \left[ \int_{0}^{t} k(t - s') \left[ \Pi - (\beta_s E_2 + \beta I + \beta_{A} A + \beta_{\nu} E_\nu) \frac{S}{N} - \mu S \right] ds' \right], \\
\mathcal{D}^\alpha_{t_0} \left[ \frac{dE_1}{dt} \right] &= \mathcal{D}^\alpha_{t_0} \left[ \int_{0}^{t} k(t - s') \left[ (\beta_s E_2 + \beta I + \beta_{A} A + \beta_{\nu} E_\nu) \frac{S}{N} - b_1 E_1 \right] ds' \right], \\
\mathcal{D}^\alpha_{t_0} \left[ \frac{dE_2}{dt} \right] &= \mathcal{D}^\alpha_{t_0} \left[ \int_{0}^{t} k(t - s') \left[ \sigma_1 E_1 - b_2 E_2 \right] ds' \right], \\
\mathcal{D}^\alpha_{t_0} \left[ \frac{dI}{dt} \right] &= \mathcal{D}^\alpha_{t_0} \left[ \int_{0}^{t} k(t - s') \left[ \sigma_2 (1 - \tau) E_2 - b_1 I \right] ds' \right], \\
\mathcal{D}^\alpha_{t_0} \left[ \frac{dA}{dt} \right] &= \mathcal{D}^\alpha_{t_0} \left[ \int_{0}^{t} k(t - s') \left[ \tau \sigma_2 E_2 - b_2 A \right] ds' \right], \\
\mathcal{D}^\alpha_{t_0} \left[ \frac{dT}{dt} \right] &= \mathcal{D}^\alpha_{t_0} \left[ \int_{0}^{t} k(t - s') \left[ \frac{\theta_1 - \theta_2}{\theta_1} T - b_3 T \right] ds' \right], \\
\mathcal{D}^\alpha_{t_0} \left[ \frac{dE_\nu}{dt} \right] &= \mathcal{D}^\alpha_{t_0} \left[ \int_{0}^{t} k(t - s') \left[ \delta_1 I + \delta_2 A + \delta_3 T - \mu R \right] ds' \right], \\
\mathcal{D}^\alpha_{t_0} \left[ \frac{dR}{dt} \right] &= \mathcal{D}^\alpha_{t_0} \left[ \int_{0}^{t} k(t - s') \left[ v_1 E_2 + v_2 A + v_3 E_\nu \right] ds' \right].
\end{align*}
\]

Thus we finally leads to the following system describing the fractional COVID-19 mathematical model:

\[
\begin{align*}
\mathcal{D}^\alpha_{t_0} S &= \frac{\Pi}{N} - (\beta_s E_2 + \beta I + \beta_{A} A + \beta_{\nu} E_\nu) \frac{S}{N} - \mu S, \\
\mathcal{D}^\alpha_{t_0} E_1 &= (\beta_s E_2 + \beta I + \beta_{A} A + \beta_{\nu} E_\nu) \frac{S}{N} - b_1 E_1, \\
\mathcal{D}^\alpha_{t_0} E_2 &= \sigma_1 E_1 - b_2 E_2, \\
\mathcal{D}^\alpha_{t_0} I &= \sigma_2 (1 - \tau) E_2 - b_1 I, \\
\mathcal{D}^\alpha_{t_0} A &= \tau \sigma_2 E_2 - b_2 A, \\
\mathcal{D}^\alpha_{t_0} T &= \frac{\theta_1 - \theta_2}{\theta_1} T - b_3 T, \\
\mathcal{D}^\alpha_{t_0} E_\nu &= v_1 E_2 + v_2 A + v_3 E_\nu - v_4 E_\nu,
\end{align*}
\]

subject to ICs (7). The Caputo derivative in the system (12) is shown by the notation \(\mathcal{D}^\alpha_{t_0}\), where \(\alpha \in (0, 1]\) denotes the arbitrary order.

5. Theoretical analysis of the model

In the present section, some of the basic properties including the biologically-feasible region, basic reproduction number, stability of analysis, etc of the Caputo COVID-19 epidemic model as shown in (12) are provided. We proceed as follow:

The following set is considered as the feasible region for the dynamics of the fractional COVID-19 model (12):

\[
\Xi \subset \mathbb{R}^4_+,
\]

such that

\[
\Xi = \left\{ (S(t), E_1(t), E_2(t), I(t), A(t), T(t), R(t)) \in \mathbb{R}^7_+ : N(t) \leq \frac{\Pi}{\mu}, \right. \\
\left. E_n(t) \in \mathbb{R}^4_+: \frac{\mu}{\nu} v_1 + v_2 + v_3 \right\}.
\]

Lemma 1. The region \(\Xi \subset \mathbb{R}^4_+\), is positively invariant for the model (12) with non-negative ICs in \(\mathbb{R}^4_+\).

Proof. The total dynamics of the population is obtained by adding the respective equations in the model (12)

\[
\mathcal{D}^\alpha_{t_0} N(t) = \mathcal{D}^\alpha_{t_0} S(t) + \mathcal{D}^\alpha_{t_0} E_1(t) + \mathcal{D}^\alpha_{t_0} E_2(t) + \mathcal{D}^\alpha_{t_0} I(t) + \mathcal{D}^\alpha_{t_0} A(t) + \mathcal{D}^\alpha_{t_0} T(t) + \mathcal{D}^\alpha_{t_0} R(t).
\]

We lead to the following

\[
\mathcal{D}^\alpha_{t_0} N(t) = \frac{\Pi}{N} - \xi_1 I(t) - \xi_2 T(t) - \mu N(t) \leq \Pi - \mu N(t).
\]
The application of Laplace on the above inequality leads to the following expression

\[ N(s) \leq \frac{\Pi}{s(s^\alpha + \mu)} - N(0) \frac{s^{\alpha - 1}}{s^\alpha + \mu}. \]

Moreover, applying inverse Laplace, leads the following

\[ N(t) \leq N(0)E_{\alpha,1}(-\mu t^\alpha) + \Pi t^\alpha E_{\alpha,\alpha+1}(-\mu t^\alpha), \]

where the Mittag-Leffler function describe in terms of the infinite power series as follows:

\[ E_{\alpha,\beta}(z) = \sum_0^\infty \frac{z^k}{\Gamma(\alpha k + \beta)}, \]

with the following property

\[ L[t^\beta E_{\alpha,\beta}(\pm \alpha t^\alpha)] = \frac{s^{\alpha - \beta}}{s^\alpha + \alpha}. \]

Therefore, we deduced that \( N(t) \) converges to \( \frac{\Pi}{\mu} \) as \( t \to \infty \) and for all \( t > 0 \) and in a result the solutions of the model with non-negative ICs in \( \Xi \) remain in \( \Xi \). □

5.1. Existence and positivity of the solutions

This section presents the existence as well as the positivity of the model solution.

For this purpose the approach in [27] is utilized. To proceeds we first recall the following theorem.

**Lemma 2.** [27] Let \( \mathcal{F}(t) \in C[m_1, m_2] \) and \( \mathcal{F}^{(\alpha)}_{m_1} \) \( \mathcal{F}(t) \in C(m_1, m_2) \), then the following interpretation is observed

\[ \mathcal{F}(t) = \mathcal{F}(m_1) + \frac{1}{\Gamma(\alpha)} \mathcal{F}^{(\alpha)}_{m_1}(\zeta)(t - m_1)^\alpha, \]

such that \( m_1 \leq \zeta \leq t, \forall t \in (m_1, m_2]. \)
Fig. 5. Influence of $\beta_A$ (disease transmission coefficient due to $A$) and $\xi$ (treatment rate of symptomatic COVID-19 individuals) on $R_0$ with corresponding contour plot.

Fig. 6. Influence of $\beta_{en}$ (disease transmission coefficient due to $E_{en}$) and $\xi$ (treatment rate of symptomatic COVID-19 individuals) on $R_0$ with corresponding contour plot.

Corollary 1. [27] Suppose that $F(t) \in C[m_1, m_2]$ and $\frac{\mathcal{C}}{m_1} D^\alpha_t F(t) \in C[m_1, m_2]$, where $\alpha \in (0, 1]$, then if

(i) $\frac{\mathcal{C}}{m_1} D^\alpha_t F(t) \geq 0, \forall \ t \in (m_1, m_2)$, then $F(t)$ is non-decreasing.

(ii) $\frac{\mathcal{C}}{m_1} D^\alpha_t F(t) \leq 0, \forall \ t \in (m_1, m_2)$, then $F(t)$ is non-increasing.

The following result is presented for the aforementioned properties of the solutions of the Caputo COVID-19 model (12).

Theorem 2. The fractional COVID-19 epidemic model (12) in Caputo sense possess a unique solution. Additionally, the solution will be non-negative.

Proof. Making use of the respective results in [28], the existence of the solution can be shown easily. Moreover, the uniqueness of the solution can be obtained by utilizing the result proven in remark 3.2 [28]. Finally, for non-negative solution to show, it is required that on each hyperplane bounding the positive orthant, the vector field point $R^8$.

From the system (12), we follow that

$\frac{\mathcal{C}}{m_1} D^\alpha_t S(t)|_{S=0} = \Pi \geq 0,$

$\frac{\mathcal{C}}{m_1} D^\alpha_t E_1(t)|_{E_1=0} = \lambda(t)S \geq 0,$

$\frac{\mathcal{C}}{m_1} D^\alpha_t E_2(t)|_{E_2=0} = \sigma_1 E_1 \geq 0,$

$\frac{\mathcal{C}}{m_1} D^\alpha_t A(t)|_{A=0} = \tau \sigma_2 E_2 \geq 0,$

$\frac{\mathcal{C}}{m_1} D^\alpha_t T(t)|_{T=0} = \xi I \geq 0,$

$\frac{\mathcal{C}}{m_1} D^\alpha_t R(t)|_{R=0} = \delta_1 I + \delta_2 A + \delta_3 T \geq 0,$
Furthermore, $Z$ by equilibrium threshold solutions thus, Fig. 7. Impact of $\beta_\nu$ (disease transmission coefficient due to $A$) and $\nu_1$ (environmental virus concentration due to pre-symptomatic individuals) on $R_0$ with corresponding contour plot.

\[ e^{D}E_i(t)|_{t=0} = v_1E_2 + v_2I + v_3A \geq 0. \]

Thus, following the aforementioned results it proves that all the solutions will remain in $\mathbb{R}^8_+$, for all $t \geq 0$.

5.2. The basic reproductive number

The basic reproduction number commonly express by $R_0$ is threshold parameter and plays an important role to predict the dynamics of a disease under consideration. The well-known next generation matrix approach is utilized to obtain $R_0$. The disease free equilibrium (DFE) of the COVID-19 epidemic model (12) denoted by $Z_0$ and given as follows:

\[ Z_0 = \left( \frac{\prod}{\mu} 0, 0, 0, 0, 0, 0, 0 \right). \]

Furthermore, to evaluate the basic reproduction number $R_0$, the necessary matrices obtained by taking only the infected classes in (12) are:

\[
F = \begin{pmatrix}
0 & \beta_\nu & \beta_\lambda & 0 & \beta_\mu \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{pmatrix},
\]

\[
V = \begin{pmatrix}
b_1 & 0 & 0 & 0 & 0 & 0 \\
-\sigma_1 & b_2 & 0 & 0 & 0 & 0 \\
0 & -\sigma_2 & b_3 & 0 & 0 & 0 \\
0 & 0 & -\xi & b_4 & 0 & 0 \\
0 & 0 & 0 & -\nu_1 & b_5 & 0 \\
0 & 0 & 0 & 0 & -\nu_2 & \nu \\
\end{pmatrix}.
\]

The evaluated basic reproduction number is finally as follows:

\[
R_0 = \frac{\sigma_1 \beta_\nu + \sigma_1 \sigma_2 (1-\tau) \beta_\lambda + \sigma_1 \sigma_2 \tau \beta_\mu + \sigma_1 (b_4 \nu_2 \sigma_2 (1-\tau) + b_1 v_3 \sigma_2 \tau + b_2 b_4 \nu_1) \beta_\mu}{b_1 b_2 b_3}.
\]

5.3. Global stability of DFE via Lyapunov approach

Global asymptotic stability (GAS) about the DFE $Z_0$ of the model (12) is shown by utilizing the Layapunov function approach in fractional case. The following theorem addresses the desire result.

**Theorem 3.** If $R_0 < 1$, then DFE of the model in Caputo case described in (12) is GAS.

**Proof.** Consider the appropriate Lyapunov function

\[ \bar{\theta}(E_1, E_2, I, A, E_0) = \kappa_1 E_1(t) + \kappa_2 E_2(t) + \kappa_3 I(t) + \kappa_4 A(t) + \kappa_5 E_0(t), \]

where, the unknown coefficients $\kappa_i > 0$, for $i = 1, \ldots, 5$. The time Caputo fractional derivative of above function is as follows:

\[ e^{D}E_i(t)|_{t=0} = v_1E_2 + v_2I + v_3A \geq 0. \]
Fig. 8. Simulations of the fractional COVID-19 epidemic model (12) for various values of fractional order $\alpha$. 
Utilizing the corresponding equations of the (12), we leads to the following

\[ \frac{\partial \Theta(E_1, E_2, I, A, E_n)}{\partial t} = \alpha \left( \beta_1 E_2 + \beta I + \beta A + \beta_n E_n \right) \frac{S}{R} - b_1 E_1 + a_2 \left( \sigma_1 E_1 - b_2 E_2 \right) \]

\[ + a_3 \left( \sigma_2 (1 - \tau) E_2 - b_3 f \right) + a_4 \left( \tau \sigma_2 E_2 - b_4 A \right) \]

\[ + a_5 \left( v_1 E_2 + v_2 I + v_3 A - v E_0 \right) \]

\[ \leq a_1 \left( \beta_1 E_2 + \beta I + \beta A + \beta_n E_n \right) - b_1 E_1 + a_2 \left( \sigma_1 E_1 - b_2 E_2 \right) \]

\[ + a_3 \left( \sigma_2 (1 - \tau) E_2 - b_3 f \right) + a_4 \left( \tau \sigma_2 E_2 - b_4 A \right) \]

\[ + a_5 \left( v_1 E_2 + v_2 I + v_3 A - v E_0 \right) \text{ as } S \leq N. \]

Now choosing the unknown constants values as

\[ a_1 = \alpha_1, \quad a_2 = b_1, \quad a_3 = \frac{\sigma_1 v_1 \beta_1 + v_2 \sigma_1 \beta_n}{b_2 v}, \]

\[ a_4 = \frac{\sigma_1 \beta_1 v + \sigma_1 \beta_n v_3}{b_4 v}, \quad \text{and} \quad a_5 = \frac{\beta_n \sigma_1}{v}. \]

and then after some simplification, we have,

\[ \frac{\partial \Theta(E_1, E_2, I, A, E_n)}{\partial t} \leq b_2 \left( R_0 - 1 \right) E_2. \]

It is clear that when \( R_0 < 1 \) then \( \frac{\partial \Theta(E_1, E_2, I, A, E_n)}{\partial t} \) is negative,? and hence following Theorem 1 established in [25] the DFE \( E_0 \) is GAS in the region \( \Sigma \). \( \square \)

### 6. Interpretations of \( R_0 \) versus model parameters

This section presents the influence of the model parameters on the dynamics of the most important threshold quantity i.e., basic reproduction number \( R_0 \). The resulting graphical interpretations are shown in Figs. 3–7. We also depict the corresponding contour plots in each case. Fig. 3 presents the behavior of
\( \mathcal{R}_0 \) versus the disease transmission rate \( \beta_a \) and treatment rate of symptomatic COVID-19 infected individuals \( \xi \). This graphical interpretation demonstrates that the basic reproduction number effectively decreases to a value less than 1 with a decrease in \( \beta_a \) and an increase in treatment rate \( \xi \). The dynamics of \( \mathcal{R}_0 \) versus the transmission rate \( \beta_t \) and the treatment rate \( \xi \) is depicted in Fig. 4. The influence of \( \beta_A \) (disease transmission coefficient due to asymptomatic individuals \( A \)) and \( \xi \) (treatment rate of symptomatic COVID-19 individuals) on \( \mathcal{R}_0 \) with corresponding contour plot is shown in Fig. 5. The combined effect of the disease transmission rate due to environmental or contaminated surfaces \( \beta_e \), and treatment rate \( \xi \) is analyzed in Fig. 6. Finally, we analyzed the impact of \( \beta_A \) (disease transmission coefficient due to \( A \)) and \( \nu_1 \) (environmental virus concentration due to pre-symptomatic individuals) on \( \mathcal{R}_0 \) in Fig. 7(a) with the corresponding contour plot in Fig. 7(b).

7. Numerical results of the fractional model

After the theoretical analysis of the COVID-19 epidemic model in the Caputo sense, we present the numerical analysis in this section. The model (12) is first solved numerically using an efficient iterative scheme called the generalized fractional Adams-Bashforth-molten approach [29] recently used in [21]. The resulting numerical scheme and the values given in Table 2 for model parameters are utilized to carry out the simulation results for various model parameters and memory index \( \alpha \). The iterative solution is obtained in the following subsection.

7.1. Numerical scheme

This subsection presents a brief numerical procedure for the iterative solution of the COVID-19 transmission model in Caputo sense (12). The fractional Adams-Bashforth-molten is applied for this purpose. In order to obtain the desire scheme, the system (12) can be redesigned in the following problem:

\[
\begin{align*}
&\mathcal{D}^{\alpha}_t \mathbf{u}(t) = \mathcal{G} \left( t, \mathbf{u}(t) \right), \quad 0 < t < T, \\
&\mathbf{u}^{(p)}(0) = \mathbf{u}_{0}^{(p)}, \quad p = 0, 1, \ldots, v, \quad v = [\alpha].
\end{align*}
\]  

where, \( \mathbf{u} = (S, E_1, E_2, I, A, T, R, E_n) \in \mathbb{R}^8 \), and the function \( \mathcal{G}(t, \mathbf{u}(t)) \) expresses a continuous real valued vector function. Utilizing the integral in Caputo case, the above problem (13) can be converted
to the following form:

\[
\mathbf{u}(t) = \sum_{p=0}^{n-1} \mathbf{u}^{(p)} \frac{t^p}{p!} + \frac{1}{\Gamma(\alpha)} \int_0^t (t-x)^{\alpha-1} G(x, \mathbf{u}(x)) \mathrm{d}x.
\] (14)

In order to perform the integration involved in (14), a uniform grid on [0, T] with step size \( h = \frac{T}{n} \), \( n \in \mathbb{N} \), where \( t_0 = nh \), \( n = 0, 1, \ldots, N \) is considered. Thus the system described in (12) can be written as follows:

\[
S_{s+1}(t) = S_0 + \frac{h^\nu}{\Gamma(\alpha + 2)} \left[ \Pi - (\beta_1 + \beta_0 A + \beta_0^2) \frac{S_{p1}}{N} - \mu S_{1} \right] + \frac{h^\nu}{\Gamma(\alpha + 2)} \sum_{j=0}^{n} b_{s+1} \left\{ \Pi - (\beta_1 E_{j2} + \beta_0 A_1 + \beta_0 E_{s0}) \frac{S_{j1}}{N} - \mu S_{j1} \right\}.
\]

\[
E_{s+1}(t) = E_0 + \frac{h^\nu}{\Gamma(\alpha + 2)} \left[ (\beta_0 A + \beta_0 A^2) \frac{S_{p1}}{N} - \mu E_{1} \right] + \frac{h^\nu}{\Gamma(\alpha + 2)} \sum_{j=0}^{n} b_{s+1} \left\{ \beta_0 A_1 + \beta_1 + \beta_0 A_1 + \beta_0 E_{s0} \frac{S_{j1}}{N} - \mu E_{j1} \right\}.
\]

\[
E_{s+1}(t) = E_0 + \frac{h^\nu}{\Gamma(\alpha + 2)} \left[ \sigma_2 E_{2}^p - b_1 E_{2} \right] + \frac{h^\nu}{\Gamma(\alpha + 2)} \sum_{j=0}^{n} b_{s+1} \left\{ \sigma_2 (1 - \tau) E_{2}^p - b_1 E_{2} \right\}.
\]

\[
A_{s+1}(t) = A_0 + \frac{h^\nu}{\Gamma(\alpha + 2)} \left[ \tau \sigma_2 E_{2}^p - b_2 A_1 \right] + \frac{h^\nu}{\Gamma(\alpha + 2)} \sum_{j=0}^{n} b_{s+1} \left\{ \tau \sigma_2 E_{2}^p - b_2 A_1 \right\}.
\]

\[
T_{s+1}(t) = T_0 + \frac{h^\nu}{\Gamma(\alpha + 2)} \left[ 1 - \tau \sigma_2 E_{2}^p - b_2 F_3 \right] + \frac{h^\nu}{\Gamma(\alpha + 2)} \sum_{j=0}^{n} b_{s+1} \left\{ 1 - \tau \sigma_2 E_{2}^p - b_2 F_3 \right\}.
\]

\[
R_{s+1}(t) = R_0 + \frac{h^\nu}{\Gamma(\alpha + 2)} \left[ \delta_1 F_2 + \delta_2 A_1 + \delta_3 T_3 - \mu R_3 \right] + \frac{h^\nu}{\Gamma(\alpha + 2)} \sum_{j=0}^{n} b_{s+1} \left\{ \delta_1 F_2 + \delta_2 A_1 + \delta_3 T_3 - \mu R_3 \right\}.
\]

\[
E_{n,s+1}(t) = E_{n0} + \frac{h^\nu}{\Gamma(\alpha + 2)} \left[ v_1 E_{2}^p + v_2 F_3 + v_3 A_1 - v_4 E_{2} \right] + \frac{h^\nu}{\Gamma(\alpha + 2)} \sum_{j=0}^{n} b_{s+1} \left\{ v_1 E_{2}^p + v_2 F_3 + v_3 A_1 - v_4 E_{2} \right\}.
\]

where,

\[
S_{S+1}(t) = S_0 + \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{n} \sigma_2 E_{2}^p \left[ \Pi - (\beta_1 E_{j2} + \beta_0 A_1 + \beta_0 E_{s0}) \frac{S_{j1}}{N} - \mu S_{j1} \right].
\]

Fig. 11. Simulations showing the dynamics of the cumulative asymptomatic COVID-19 cases for various levels (mild, moderate, and strict) of social-distancing strategies and (a) \( \alpha = 1 \), (b) \( \alpha = 0.95 \), (c) \( \alpha = 0.90 \), (d) \( \alpha = 0.80 \).
Further, we have in the above expressions

$$b_{j,n+1} = \begin{cases} \alpha^{j+1} - (n - \alpha)(n + 1)^j, & j = 0 \\ (n - j + 2)^{j+1} - (n - j)^{j+1} - 2(n - j + 1)^{j+1}, & 1 \leq j \leq n, \\ 1, & j = n + 1. \end{cases}$$

and

$$\theta_{j,n+1} = \frac{h^\alpha}{\alpha} [(n - j + 1)^n (n - j)^n], \quad 0 \leq j \leq n.$$

7.2. Simulation results

This section aims to simulate the COVID-19 epidemic model in Caputo sense formulated in the system (12) using the iterative scheme developed in the previous section. The simulation process is carried out with the help of baseline values of the parameters tabulated in Table 2 and for various values of memory index $\alpha \in [0, 1]$. We mainly analyzed the impact of memory index $\alpha$, social-distancing (to its baseline, mild, moderate and strict levels), and for various values of $\nu$ and $\lambda$. Particularly, in the simulation results the mild (or low), moderate, and strict (high) levels of social-distancing correspond to a 10%, 30%, and 40% reduction, respectively, in the rates of the parameters (i.e., $\beta_c$, $\beta_I$, $\beta_A$, $\beta_{E_1}$), in relation to their baseline values shown in Table 2. Firstly, the dynamics of the various population classes for five different values of $\alpha \in (0, 1]$ is depicted in Fig. 8(a–i).

Secondly, the impact of different social-distancing intervention compliance levels for the cumulative cases of pre-symptomatic $E_2$, and...
symptomatic and asymptomatic COVID-19 individuals A, is shown in Figs. 9–11 respectively. These simulation results are performed for four values of the fractional order \( \alpha \). The same interpretation is also carried for the total death cases as shown in Fig. 12(a–d). In each sub-plot of Fig. 9, the blue curve shows the dynamics of \( E_2(t) \) class if the social-distancing compliance is kept at its baseline level while the last curve (green) depicts the dynamics pre-symptomatic individuals when strict social distancing interventions are taken in consideration. These graphical results (as shown in Fig. 9), reveal a dramatic reduction in the cumulative pre-symptomatic individuals over time with variation in social distancing interventions from mild to strict levels. Additionally, it is further observed that reduction is slightly faster and the peaks of curves occur over longer period of time for smaller value of \( \alpha \) (longer memory index) as can be seen in Fig. 9(b–d). Similarly, the impact of various social-distancing compliance levels upon the cumulative newly symptomatic and asymptomatic COVID-19 infected cases is analyzed in Figs. 10 and 11 respectively. A significant reduction is observed in these population with the increase in social distancing interventions from mild, moderate to strict level to its current baseline. Finally, we also analyzed the impact of variations in social-distancing strategies over the recorded cumulative COVID-induced mortality cases. One can observe that the cumulative mortality cases are dramatically decreased if strict social distancing is implemented as can be seen in Fig. 12. This interpretation is carried out for various of \( \alpha \) in order to display the impact of memory index on the disease incidence.

Thirdly, the dynamics of the pre-symptomatic, symptomatic, asymptomatic COVID-19 individuals and the COVID-induced death cases for different removal rates of the viruses from the environment reservoir (i.e., \( \nu \)) is depicted in Figs. 13–16 respectively. In each Figure, the solid blue curve shows the graphical result when the parameter \( \nu \) is kept to its current baseline level in Table 2. It is observed that the curves of the cumulative infected cases dramatically decrease if the parameter \( \nu \) is increased to %50 to its baseline values as can be seen in Figs. 13–15(a–c) respectively. Furthermore,
Conclusions

Despite the high promising improvement in effective and safe anti-COVID vaccines as well as other treatments, still, the main strategies to minimize the disease incidence are the use of non-pharmaceutical interventions. The mathematical models are a useful tool to explore the dynamics of a disease outbreak and to suggest the possible preventions for it. In this paper, we developed a novel mathematical model in order to assess the potential community-wide impact of various control strategies in a community. The model was first formulated with the help of classical integer-order differential system. The fractional order operator in Caputo sense is then applied to extend the proposed model. The Caputo operator is used for the sake to analyze the impact of memory effects on the disease transmission. The basic mathematical analysis of the Caputo epidemic model is carried out to gain insight into the dynamical features of the disease. The global stability of the disease free equilibrium point is established via the fractional Lyapunov approach. The model is fitted to the reported incidence data of COVID-19 in the Kingdom of Saudi Arabia for selected period of time which results in the estimated values of model parameters. The important threshold parameter called the basic reproduction number is evaluated theoretically as well as numerically and is found \( R_0 \approx 1.60 \). Moreover, the model is solved numerically using an efficient iterative scheme. The estimated and fitted parameters are utilized to carry out the simulation results of the model for various senecios. The model is simulated for various social-distancing levels (mild, moderate and high/strict) and its impact on the COVID-19 infected cases is shown graphically. Our simulation results showed that with strict social-distancing the pandemic peaks are significantly reduced to a more realistically-
Fig. 15. Simulations describing the impact of various rates of $\nu$ (removal rate of virus from revisors or surfaces) on the cumulative asymptomatic COVID-19 cases where, (a) $\alpha = 1$, (b) $\alpha = 0.95$, (c) $\alpha = 0.90$. 
Fig. 16. Graphical results showing the impact of various rates of $\nu$ (removal rate of virus from revisors or surfaces) on the cumulative death cases due COVID-19 where, (a) $\alpha = 1$, (b) $\alpha = 0.95$, (c) $\alpha = 0.90$. 

(a) 

(b) 

(c)
attainable level. Furthermore, we also carried the simulation results showing the impact of various levels of viral removal rate (ν) from the environment or surfaces on the disease incidence. The computational results revealed that the pandemic can be reduced significantly with increase in ν. In conclusion, the present study shows that proper social distancing strategies and disinfection spry to reduce the environment viral transmission is necessary eliminating COVID-19 not only in the Kingdom of Saudi Arabia but in the whole world. In the future, the present model can be reformulated via fractional operators with non-singular and non-local kernels.

Declaration of Competing Interest

We declare that there does not exist any conflict of interest regarding this paper.

CRediT authorship contribution statement

Xuan Liu: Conceptualization, Methodology, Software, Data curation, Writing – review & editing, Supervision, Formal analysis. Saif Ullah: Conceptualization, Writing – original draft, Software, Investigation. Ahmed Alshehri: Writing – review & editing, Supervision, Validation, Visualization. Mohamed Altanji: Supervision, Methodology, Investigation, Visualization, Formal analysis, Writing – review & editing.

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