Dynamic response of footbridges with tuned mass dampers

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Abstract. The increase of vibration problems in modern footbridges shows that footbridges should no longer be designed for static loads only. Not only natural frequencies but also damping properties and pedestrian loading determine the dynamic response of footbridges and design tools should consider all of these factors. Footbridge vibrations don’t cause usually structural problems, but if the vibration behaviour does not satisfy the comfort criteria, changes in the design or damping devices could be considered. The most popular external damping devices are viscous dampers and tuned mass dampers (TMD). The paper presents the basic principles of optimal TMD configuration and design procedure. The efficiency of TMD is demonstrated on the example of a footbridge prone to vibrations induced by pedestrians. It is shown that if the TMD is tuned quite precisely the reduction of accelerations can be very significant.

1. Introduction

Modern footbridges are very often lightweight and flexible structures, where the first natural frequencies of vibration may fall close to dominant frequencies of the dynamic excitation due to walking or running. Such bridges are susceptible to vertical as well as to horizontal vibrations leading to a resonant response characterized by high levels of vibration and a dynamic design is necessary.

The increase of vibration problems in modern footbridges shows that footbridges should no longer be designed for static loads only. But fulfilling the natural frequency requirements (given in many codes) restricts footbridge design: very flexible, lightweight structures, such as stress ribbon bridges, cable-stayed or suspension bridges may not satisfy these requirements. Moreover not only natural frequencies but also damping properties and pedestrian loading altogether determine the dynamic response. Design tools should consider all of these factors.

Although footbridge vibrations don’t cause usually structural problems, they can induce some uncomfortable sensation, and so many codes establish maximum acceptable values of acceleration. Provided that the vibration behaviour due to expected pedestrian traffic is checked with dynamic calculations and satisfies the required comfort, any type of footbridge can be designed and constructed. If the vibration behaviour does not satisfy some comfort criteria, changes in the design or damping devices could be considered.

2. Proposed design procedure

The design procedure should satisfy two slightly contradictory requirements: to be simple and accurate. The proposed design procedure should contain the following steps (cf. [5]):
• Determination of fundamental frequency limit values in horizontal and vertical directions \( (f_{v,\text{limit}}, f_{h,\text{limit}}) \) above which the vibration limit state is satisfied. The proposed limit value for vertical fundamental frequency is \( f_{v,\text{limit}} = 5 \) Hz, the limit value for horizontal fundamental frequency \( f_{h,\text{limit}} = 2.5 \) Hz.

• Evaluation of footbridge natural frequencies in vertical and horizontal direction \( (f_v, f_h) \).

• Comparison of footbridge natural frequencies to the limit values.

If the frequencies are higher than the limit values the serviceability limit state in relation to vibration is satisfied, otherwise the verification procedure will be continued according to the following steps:

• Determination of the acceleration limit values \( (a_{v,\text{limit}}, a_{h,\text{limit}}) \) to ensure pedestrian comfort. The proposed limit values for acceleration in vertical and horizontal directions are \( a_{v,\text{limit}} = 0.7 \) m/s\(^2\) and \( a_{h,\text{limit}} = 0.2 \) m/s\(^2\), respectively. These limit values are suggested in most of the codes.

• Determination of pedestrian loads (determination of load models). Three different types of human motion are commonly considered to model the dynamic loads applied by pedestrians, namely walking, running and rhythmic jumping. The pacing frequency \( f_p \) and the pedestrian forward speed \( v_p \) are two parameters that play a fundamental role in terms of the characterisation of the excitation.

• Evaluation of maximum accelerations \( (a_v, a_h) \) of footbridge for each load model.

• Comparison of footbridge maximum accelerations and limit accelerations.

If the accelerations are higher than the limit values, changes the vibration characteristics of the footbridge (natural frequencies) or damping devices will be considered and the design procedure is repeated.

3. Tuned mass damping

To avoid undesirable vibrations of the structure it is a good idea to install tuned mass dampers (TMDs) on the footbridge to dissipate the energy from one or more modes. A TMD is often a much more lucrative solution when compared to changing the natural frequencies of the structure.

The theory of how a TMD works, and how to determine the optimal characteristics are summarized in this chapter. The idealized system of a TMD is shown in Figure 1. This configuration is characterized by a mass \( m_d \) attached to the main structure, having the modal mass \( m_0 \) by a spring having the stiffness \( k_d \) and a viscous damper with the parameter \( c_d \). The frequency of the structure is characterised via the modal stiffness \( k_0 \) and the internal damping of the structure is \( c_0 \). When the structure vibrates, a relative motion of the attached mass is generated. The velocity of this relative motion enables the damper to dissipate energy from the system and thereby comprehend large response while the structure is exposed to a harmonic force close to its natural frequency.

![Figure 1. Idealized system of a TMD.](image)
3.1. Optimal damping configuration

When installing a TMD the following steps should be considered to obtain the optimal damper properties:

- The position of the damper must be selected, so it can perform optimally.
- The frequency of the damper must be tuned.
- The optimal damping level must be determined.

The TMD must be designed to prevent undesirable response due to specific mode. The energy dissipated by the TMD is proportional to the relative velocity of the deck. As the relative velocity is related to the maximum displacement, the TMD should be placed to the point, where the vibration mode has the largest modal displacement.

TMDs are normally tuned so that the two peaks of the damped system frequency response curve have the same dynamic amplification, when expressed in terms of displacements.

If the damping is too low, undesirable dynamic amplification will occur near the two undamped resonance frequencies because the dampers dissipate too little energy. On the contrary, if the damping is too high, the relative motion of the damper will be too low resulting in a configuration where damper and structure acts as one unit.

3.2. Design procedure

The design procedure may be as follows (cf. [1]):

1. Choice of TMD mass $m_d$, based on the ratio $\mu$ to the structural modal mass $m_s$ ($\mu = m_d / m_s$). Typical values of the mass ratio can range from 0.01 to 0.05.
2. Calculation of optimum TMD frequency ratio, expressed by the ratio $\delta$ between the TMD’s frequency $f_d$ and the system’s frequency $f_s$ ($\delta = f_d / f_s$)

$$\delta_{opt} = \frac{1}{1 + \mu}.$$  

3. Calculation of optimum TMD damping ratio $\xi_{opt}$

$$\xi_{opt} = \sqrt{\frac{3\mu}{8(1 + \mu)^3}}.$$  

4. Calculation of the TMD constants:

- Natural frequency
  $$f_d = \frac{f_s}{1 + \mu};$$  

- Stiffness
  $$k_d = \left(2\pi f_d\right)^2 m_d;$$  

- Damping
  $$c_d = 2\sqrt{k_dm_d\xi_{opt}};$$  

Important remarks on TMD design:

- The frequency of the TMD shall be tuned quite precisely.
- The compliance with the optimum damping is less important.
- TMDs are most effective when the damping of the structure is low.
- It is not worth increasing the mass ratio too much, for large mass ratios, the amplitude of the TMD oscillations is reduced.
• The exact tuning of the TMD occurs experimentally; therefore great care should be paid to construction details.
• The performance of a TMD is extremely sensitive to frequency de-tuning, which can occur as consequence of slight frequency changes associated with pedestrian loads or with modifications within the structure during its lifetime; therefore it is of interest to evaluate the TMD efficiency for an estimated range of frequencies.

4. Example - Footbridge with TMD
The footbridge is a simply supported beam made of lightweight concrete. The dimensions of the beam were chosen to make it prone to vibrations induced by pedestrians. A TMD is mounted at the midspan.

Parameters of the footbridge:
- Mass: \( m_s = 5300 \text{ kg} \)
- Stiffness: \( k_s = 870 \text{ kN/m} \)
- Damping ratio: \( \xi_s = 0.015 \)
- Natural frequency: \( f_s = 2.04 \text{ Hz} \)

Parameters of the TMD:
- Mass: \( m_d = 250 \text{ kg} \)
- Mass ratio: \( \mu = m_d / m_s = 0.047 \)
- Natural frequency (cf. Eq.(3)): \( f_d = f_s / (1 + \mu) = 1.95 \text{ Hz} \)
- Stiffness (cf. Eq.(4)): \( k_d = \left( \frac{2 \pi f_d^2}{2} \right) m_d = 37.5 \text{ kN/m} \)
- Damping ratio (cf. Eq.(2)): \( \xi_{opt} = \frac{3 \mu}{8(1 + \mu)^{3/2}} = 0.124 \)
- Damping (cf. Eq.(5)): \( c_d = 2\sqrt{k_d m_d \xi_{opt}} = 759 \text{ kg/s} \)

One 95 kg-heavy person crosses the footbridge with a frequency of 2 Hz, which is approximately equal to the first natural frequency of the bridge. The length of the step is 0.70 m. The excitation force is given in Figure 2. Other types of analytical force models can be found in the literature: time-domain models (deterministic and probabilistic force models) and frequency-domain models – for a detailed review cf. [2] and e.g. [4]. The suitable model of mutual interaction between human gait and elastic bridge has been developed in [3].

![Figure 2. Excitation force.](image-url)
Figure 3 shows the time history of the displacement at midspan with locked TMD, acceleration is shown in Figure 4. The results with free TMD are given in Figure 5 and Figure 6.

The effect of the TMD can be easily seen in Fig. 3 and Fig. 5 for displacement time histories and in Figures 4 and 6 for acceleration time history. The maximum acceleration at midspan reduced from 1.6 m/s\(^2\) (locked TMD) to 0.35 m/s\(^2\) (free TMD), which corresponds to a permissible value 0.7 m/s\(^2\).

![Figure 3. Displacement at midspan with locked TMD.](image)

![Figure 4. Acceleration at midspan with locked TMD.](image)

![Figure 5. Displacement at midspan with free TMD.](image)
5. Conclusions
Not only natural frequencies but also damping properties and pedestrian loading determine the dynamic response of footbridges and design tools should consider all of these factors. Footbridge vibrations don’t cause usually structural problems, but if the vibration behaviour does not satisfy the comfort criteria, changes in the design or damping devices could be considered.

The control of the vibration response in a footbridge implies the introduction of modifications, which can include variation of the mass, frequency or structural damping. For an already constructed structure, the simplest approach is based on the increase of the structural damping, which can be usually achieved either by implementation of external damping devices. The most popular of these are viscous dampers and TMDs.

The paper presents the basic principles of optimal TMD configuration and design procedure. The efficiency of TMD is demonstrated on the example of a footbridge prone to vibrations induced by pedestrians. It has been shown that if the TMD is tuned quite precisely (especially its frequency) the reduction of accelerations can be very significant.

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References
[1] Research Fund for Coal and Steel 2007 Human induced Vibrations of Steel Structures Design of Footbridges
[2] Živanović S, Pavic A and Reynolds P 2001 Vibration serviceability of footbridges under human-induced excitation: a literature review Journal of Sound and Vibration 279 pp 1-74
[3] Máca J, Valášek M, Machač J and Vampola T 2010 Dynamic Interaction between Human Gait and Footbridges Proc The Tenth Int. Conf. on Engineering Computational Technology (Valencia) ed B H V Topping, J M Adam et al (Stirlingshire, Scotland: Civil-Comp Press) CDROM pp 1-13
[4] Bachman H and Ammann W 1987 Vibrations in Structures Induced by Man and Machines (IABSE Structural Engineering Document 3e)
[5] Granič I Š 2015 Serviceability verification of pedestrian bridges under pedestrian loading Tehnički vjesnik 22 pp 527-537