Quantum noise with exchange and tunnelling: predictions for a two-particle scattering experiment with time-dependent oscillatory potentials

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Abstract. Quantum noise with exchange and tunnelling is studied within time-dependent wave packets. A novel expression for the quantum noise of two identical particles injected simultaneously from opposite sides of a tunnelling barrier is presented. Such quantum noise expression provides a physical (non-spurious) explanation for the experimental detection of two electrons at the same side under static potentials. Numerical simulations of the two-particle scattering probabilities in a double barrier potential with an oscillatory well are performed. The dependence of the quantum noise on the electron energy and oscillatory frequency is analysed. The peculiar behaviour of the dependence of the quantum noise on such parameters is proposed as a test about the soundness of this novel quantum noise expression, for either static or oscillatory potentials.

Keywords: mesoscopic systems (theory), current fluctuations, quantum transport
1. Introduction

In general, the own definition of transport implies movement, variations in time. However, because solving time-dependent transport models entails a large computational burden \[1\], there are many examples of time-independent models that provide accurate predictions about transport phenomena. Specifically, in quantum transport, a normal and extended approximation is to substitute the intrinsic time dependence of the states involved in the computations by time-independent ones \[2–7\]. The success of such time-independent models to solve many quantum transport phenomena in a comfortable way is unquestionable. However, can we always remove the explicit time dependence of transport models? \[8–15\] The answer is not simple at all.

In this paper, we analyse if such time dependence of the states is relevant or not in a type of Hong–Ou–Mandel (HOM) experiment \[16, 17\] with tunnelling and exchange. The HOM experiment can be summarized as follows. Two identical electrons are injected simultaneously from two different inputs and after scattering on an electron beam splitter they are measured at two different outputs. The correlation between the detection of the two outputs is measured depending on the injection delay. From these correlation values one can directly obtain quantum noise, i.e. the fluctuations in the number of detected electrons in the output. In particular, we focus on a situation quite close to the experiment mentioned above, but where the scattering region is a double barrier potential with an oscillatory quantum well in a one-dimensional (1D) system, see figure 1. In particular, we focus on the case where there is no delay in the injection among both electrons. Then, in principle it is expected that quantum noise is fully suppressed due to the Pauli principle, which states that two electrons cannot be at the same place with the same state \[18\]. As a consequence, it is expected that each electron will be located at a different output with no (zero frequency) current fluctuations. However, our numerical results and experiments in these types of HOM systems show that, even if quantum noise is reduced it is not completely suppressed, indicating the non-zero probability of detecting simultaneously two electrons on the same side.
Most time-independent models developed for quantum transport are based on the reasonable assumption that the energy width of the wave packet is small enough so that the transmission coefficient is essentially constant on the wave packet energy range. We have shown in a previous paper [19] that, when the above assumption is not fulfilled, the probability of having two electrons in the same place is different from zero, even when the Fermi nature of particles is explicitly considered. In this paper we develop a novel expression of the quantum noise in this two-particle scattering process, which takes into account these unexpected probabilities. We will show that this expression gives a fundamental unavoidable reason for the quantum noise unexpected enhancement, which was observed in [16]. Let us note that other HOM experiments, e.g. see [17], obtain an experimental noise value very close to zero for simultaneous detection. These results are also compatible with our general noise expression developed in this paper, and they correspond to scenarios where the initial energy width of the (Leviton) wave packet is small compared with the energy variations of the barrier transmission coefficient, i.e. a time-independent model is enough to capture the main physics.

Finally, with the situation described in figure 1, we propose an experiment with oscillatory potentials that is able to test the signature of our explanation over other possible causes of the unexpected experimental results in this type of HOM systems (for example, decoherence [20], spurious effects [21], time delay [22] or interaction between different modes [23, 24]).

This paper is organized as follows. In section 2 we discuss the two-particle scattering and how it is possible to measure in the same position two initially identical electrons after interaction with the time-dependent potential barrier. In section 3 we explain how quantum noise is computed taking into account these new two-particle scattering possibilities. We named these probabilities ‘new’ because a zero probability for this process is predicted by the Landauer–Büttiker formalism [3, 5]. In section 4 we explain a procedure which will be able to test the soundness of our proposal, through a time-dependent oscillatory double barrier potential. Finally, in section 5 we conclude.
2. Two-particle scattering

2.1. The system

We consider an oscillatory potential barrier system with two identical electrons, initially located on each side of the barrier on the same side, but with opposite momentum. We solve the two-particle time-dependent Schrödinger equation,

$$i\hbar \frac{\partial \Phi}{\partial t} = \left[ -\frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial x_1^2} - \frac{\hbar^2}{2m^*} \frac{\partial^2}{\partial x_2^2} + V(x_1, t) + V(x_2, t) \right] \Phi,$$

where $m^*$ is the effective electron mass and $V(x_i, t)$ takes into account the one-particle interaction between one electron and the time-dependent tunnelling barrier depicted in figure 1. The exchange interaction is introduced in equation (1) in the shape of the initial wave function $\Phi(x_1, x_2, t_0)$. The anti-symmetrical many-particle wave function for two electrons is:

$$\Phi(x_1, x_2, t_0) = \frac{\phi_a(x_1, t_0)\phi_b(x_2, t_0) - \phi_b(x_2, t_0)\phi_a(x_1, t_0)}{\sqrt{2}}.$$

After the interaction with the barrier, in addition to the usual scattering probabilities where both electrons are found on each side of the barrier (figures 2(a) and (b)), it is also possible to find both of them on the same side of the barrier, i.e. both on the left or both on the right-hand side (figures 2(c) and (d)).

The only requirement to get the novel probabilities is that the initial states ($\phi_a$ and $\phi_b$) are described by localized (normalizable) wave functions of finite energy width $\Delta E$ (not by the scattering states [3, 5] with $\Delta E = 0$, i.e. infinitely extended in space) and with a transmission coefficient of the barrier that is not constant in the wave packet energy range $[E - \Delta E/2, E + \Delta E/2]$ of our wave packet, as mentioned in the introduction. These new final scenarios are not in contradiction with the Pauli principle, as remarked in the introduction. In our case, they can occur because, after the interaction with the barrier, the transmitted and reflected wave functions of the electrons suffer a different evolution and therefore they do not overlap, or at least not completely, allowing the possibility of being in the same place (figures 2(c) and (d)) with the same energy, but with different spatial shape. For further discussion, see [19].

2.2. The new probabilities

These new probabilities are reflected in equations (3) and (4) (proved and derived in [19]). The former describes the probability of two electrons being, both, on the left-hand side of the barrier:

$$P_{LL} = \int_{-\infty}^{0} dx_1 \int_{-\infty}^{0} dx_2 \ |\Phi|^2 = R_a T_b - |I_{a,b}^r|^2.$$

The latter provides the probability of both electrons being located on the right-hand side:
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Finally, the probability of one particle on each side is:

$$P_{RR} = T_a R_b - |I_{a,b}^{r,t}|^2.$$  

Finally, the probability of one particle on each side is:

$$P_{CR} = R_a R_b + T_a T_b + 2|I_{a,b}^{r,t}|^2.$$  

In these expressions, $R_i$ and $T_i$, with $i = a, b$, are the single-particle reflection and transmission coefficient of the $i$-wave packet. $I_{a,b}^{r,t}$ is the important overlapping term among the different wave packets after the interaction with the barrier:

$$I_{a,b}^{r,t} = \int_{-\infty}^{0} dx \, \phi_a^r(x, t_1) \phi_b^s(x, t_1),$$

where $\phi_a^r$ and $\phi_b^s$ are the reflected component of the initial state $\phi_a$ and the transmitted component of the initial state $\phi_b$, respectively. The time $t_1$ can be any time (after the scattering with the barrier) large enough so that the probability presence in the barrier region remains negligible.

Figure 2. Two identically injected wave packets from the left $x_a$ and from the right $x_b$ of a scattering barrier. The solid (red) regions represent the barriers. (a) and (b) each particle is detected on a different side of the barrier at final time $t_1$ when the interaction with the barrier has almost finished. (c) and (d) both particles are detected on the same side of the barrier.

$$\int_{-\infty}^{0}$$

$$\phi_a^r(x, t_1) \phi_b^s(x, t_1)$$

$$d x$$

$$\phi_a^r(x, t_1) \phi_b^s(x, t_1)$$

$$d x$$

where $\phi_a^r$ and $\phi_b^s$ are the reflected component of the initial state $\phi_a$ and the transmitted component of the initial state $\phi_b$, respectively. The time $t_1$ can be any time (after the scattering with the barrier) large enough so that the probability presence in the barrier region remains negligible.

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Depending on the contribution of the overlapping term in equation (6), the probabilities (3)-(5) can achieve two different particular limits:

- The Landauer–Büttiker results [5] are recovered when $|I_{a,b}^{r,t}|^2$ is equal to $RT$ and therefore $P_{RR} = P_{LL} = 0$, and $P_{ER} = 1$. Thus, there is no possibility of finding both particles in the same place. This limit is achieved when the wave packets are spatially large and similar (not identical) to infinitely extended scattering states.

- The results for a distinguishable particle are obtained when $I_{a,b}^{r,t} = 0$, i.e. the reflected wave packet and the transmitted one are orthogonal. Then, $P_{RR} = T_a R_b$, $P_{RR} = T_a R_b$ and $P_{ER} = R_a R_b + T_a T_b$. This case is achieved when the wave packets, after the interaction with the barrier, are very different. For example, this occurs in the case of a double barrier, when both electrons have the resonant energy, but one is transmitted and the other is reflected.

3. Quantum noise with the new probabilities

In this section, after having developed the new two-particle scattering probabilities $P_{RR}$, $P_{LL}$ and $P_{ER}$, we compute the quantum noise formula with the new possibilities (figures 2(c) and (d)) described above. The noise will be computed under the following approximations:

- We assume that the noise contribution comes from one and two-particle processes. In detail, we consider (i) one electron injected from the left and none from the right, (ii) one from the right and none from the left, and (iii) one from left and one from right. These restrictions are reasonable for standard devices working at room temperature. The extension to many-particle processes at any temperature will be detailed in a future work [25].

- We treat classically the interaction between electrons and the quasi-electrostatic potential barrier. In other words, we neglect the photonic nature of the electromagnetic field because the frequencies and the electric field we are dealing with are such that the number of available photons is large enough [26].

- In order to simplify the final noise expression, we consider a symmetric system, therefore we will assume that $T_a = T_b$, $R_a = R_b$ and $P_{RR} = P_{LL}$. The generalization to non-symmetrical scenarios can be done straightforwardly.

- We assume that the measurement of the current is done at zero or low frequencies, where displacement currents can be neglected because their contributions are zero after time averaging, i.e. the current is only particle current.
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Under these approximations, noise can be computed from the knowledge of the number \( N \) of transmitted particles through the barrier during the time \( t_d \):

\[
\langle S \rangle = \lim_{t_d \to \infty} 2q^2 \frac{\langle N^2 \rangle_{t_d} - \langle N \rangle_{t_d}^2}{t_d}.
\]

We define \( \langle N \rangle_{t_d} = \sum_{N=-\infty}^{N=\infty} P(N)N \) and \( \langle N^2 \rangle_{t_d} = \sum_{N=-\infty}^{N=\infty} P(N)N^2 \), where \( P(N) \) is the probability of \( N \) particles being transmitted from the left to the right reservoir. The probabilities \( P(N) \) are computed from the direct solution of the two-particle Schrödinger equation including exchange interaction (equation (1)) and taking into account the Fermi–Dirac occupation function \( f_i \), with \( i = a, b \); see Table 1.

Now, one can determine the noise \( \langle S \rangle \), which, due to the new \( P(N) \), is related to \( \mathcal{P}_{LL} \) and \( \mathcal{P}_{RR} \):

\[
\langle S \rangle = \frac{4q^2}{\hbar} [T f_a(1-f_b) + f_b(1-f_a)] + T(1-T)(f_a-f_b)^2 + 2P_{LL} f_a f_b.
\]

Expression (8) contains the usual Landauer–Büttiker formalism noise expression in the case where \( \mathcal{P}_{RR} = \mathcal{P}_{LL} = 0 \). However, we have shown that generally, \( \mathcal{P}_{LL} \neq 0 \) and quantum noise is increased. In the limit of the distinguishable particles behaviour, the classical noise results (understood as a noise due to tunnelling of electrons without considering exchange interaction among them) are recovered (\( \mathcal{P}_{LL} = RT \)) and electrons behave like classical particles. In general, the results predicted by equation (8) lie among the Landauer–Büttiker formalism and the classical results.

Table 1. Probability \( P(N) \) (upper) that \( N \) (lower) electrons are transmitted from the left to right reservoir during the time interval \( t_d \). \( f_i \) is the Fermi distribution \( (i = a, b) \) and \( T_i \) and \( R_i \) are the \( i \)-wave packet single particle transmission and reflection coefficients.

| \( b \) | \( a \) | Injection Transmitted | Injection Reflected | No injection |
|--------|--------|----------------------|--------------------|-------------|
|        |        | \( \mathcal{P}_{LL} f_a f_b \) | \( \mathcal{P}_{LL} f_a f_b \) | \( T(1-f_a)f_b \) |
|        |        | 0                    | -1                 | -1          |
|        |        | \( \mathcal{P}_{RR} f_a f_b \) | \( \frac{\mathcal{P}_{RR}}{2} f_a f_b \) | \( R(1-f_a)f_b \) |
|        |        | 1                    | 0                  | 0           |
|        |        | \( T f_a(1-f_b) \) | \( R f_a(1-f_b) \) | \( (1-f_a)(1-f_b) \) |
|        |        | 1                    | 0                  | 0           |
4. Numerical results for the oscillatory proposed experiment

As mentioned in section 1, there is a HOM experiment [16] where two identical electrons are injected simultaneously from two inputs and measured in two outputs. In that experiment, it is found out that the possibility of measuring both electrons on the same side is not zero, as is usually expected. Apart from [23, 24], which explain this result because of the interaction between different Landau levels in the inner channels, other explanations appeal for decoherence [20], spurious results [21] and time delay in the injection [22].

With the approach explained previously in sections 2 and 3, alternatively, we give a fundamental (non-spurious) reason for the experimental results. Due to the time and energy-dependence evolution of the electron wave packet, after the interaction with the barrier, the reflected and transmitted components of the wave packets do not overlap completely. Thus, there is no reason to expect that they cannot be detected in the same place (according to equations (3) and (4)) because their states are different and then the exclusion Pauli principle does not apply. The reader can find more details in [19].

As has been already exposed, in this work we propose an experiment with oscillating potentials which would be able to test the reliability of our explanation for the unexpected noise results. We analyse the case where two electrons are injected simultaneously with the same energy from both sides of a double barrier at the same distance from the barrier. This double barrier system has a time-dependent well (see figure 1), which oscillates periodically according to expression $V_w = \frac{V_0}{2} \sin(\omega t)$. In order to increase the visualization of these new probabilities, we will consider the injection of electrons whose energies are close to the (first) resonant energy of the double barrier. For these energies, the transmission coefficient has a sharp energy dependence so that the reflected and transmitted wave packets become almost orthogonal, $|I_{ab}|^2 \approx 0$. This resonant scenario is selected to maximize the condition that the transmission coefficient is not constant in the wave packet energy range $[E - \Delta E/2, E + \Delta E/2]$. See further details in [19]. Then, the new probabilities (3) and (4) become more relevant.

We can reasonably expect that, by changing the potential level of the well inside the barrier, the resonant energy of the double barrier will change accordingly, while the electron approaches the barrier region. This will cause that for some electrons that in the time-independent case were not resonant, and therefore their probability of finding both at same place was low, will be resonant, increasing enormously the probabilities of finding both of them on the same side of the barrier.

In our proposed experiment we inject artificially two electrons, one injected from each side of the barrier simultaneously. Artificially means that the standard injection from the mesoscopic contacts at thermal equilibrium is not pertinent in this experiment. Then, in equation (8), the temperature is not relevant and $f_a = f_b = 1$. No thermal noise has to be considered in this type of experiment with artificial injection. Under these considerations, the only term which survives in equation (8) is the last one, which contains the new probability $P_{CC}$ of finding both particles on the same side. Therefore, in our particular scenario, the computation of $P_{CC}$ provides directly, apart from a constant factor, the quantum noise in equation (8).
We performed simulations for the experiment as a function of the energy of the injected electrons and also of the oscillation frequency of the bottom potential of the quantum well. We chose as the initial state for the electron wave functions \( \phi_{0a} \) and \( \phi_{0b} \) a Gaussian function
\[
\phi = \frac{2}{\sqrt{\pi \sigma}} e^{-\frac{(x-x_0)^2}{\sigma^2}},
\]
whose initial position is \( x_0 = 175 \) nm far from the centre of the barrier, dispersion \( \sigma = 50 \) nm and initial central momentum \( k_0 = \sqrt{2mE}/\hbar \). The barriers are 0.4 eV high, with thickness 1.0 nm and the quantum well length is 5.2 nm. We emphasize that any other localized wave function can be chosen without modifying qualitatively the results discussed in this work, as far as the transmission coefficient is not constant on the energy range of the wave packet \([E - \Delta E/2, E + \Delta E/2]\).

This result is seen in figures 3 and 4, where we plot the overlapping term \( I_{ab}^{r,t} \) (equation (6) and noise \( \langle S \rangle \) (equation (8)) respectively, as a function of the energy of the electrons and also of the oscillatory frequency of the well. The results corresponding to the static case (no oscillatory well) are seen at a frequency equal to zero. In this static situation, the resonant energy is \( E_r = 0.073 eV \), and we appreciate in figure 3 that at this energy value, the overlapping is minimum, and in figure 4 that the noise achieves its maximum value.

In figure 3 (figure 4), we observe that when we switch on the oscillation, the minimum (maximum in figure 4) value for the overlapping \( I_{ab}^{r,t} \) (noise \( \langle S \rangle \) in figure 4) moves. Therefore, as we expected, resonant energies could be found for other energies even if in the static case they were not. Moreover, the position of the maximum values of \( \langle S \rangle \) in figure 4 for different frequencies and energies can be easily understood. In our scenario, the applied potential energy at the well is:
\[
V(t) = V_0 \sin(\omega t + \phi_0),
\]
where we have defined \( \phi_0 \) as the time that the electron takes to arrive at the double barrier. We consider that the electron transit time \( \tau \), i.e. the time spent by the electron in the double barrier, is much shorter than the inverse of the frequencies we are dealing with. Taking that \( \omega(t - t_b) \ll 1 \) for \( t_b < t < t_b + \tau \), we can provide a Taylor approximation to expand equation (10) as:
\[
V(t) = \frac{V_0}{2} \{ \cos(\omega t_b) \cos(\omega(t - t_b)) + \cos(\omega t_b) \sin(\omega(t - t_b)) \}
\approx \frac{V_0}{2} \left[ \sin(\omega t_b) \left( 1 - \frac{\omega^2(t - t_b)^2}{2} \right) + \cos(\omega t_b) \omega(t - t_b) \right] \approx \frac{V_0}{2} \sin(\omega t_b).
\]
Therefore, the electron perceives a ‘static’ potential when it is inside the barrier and no harmonics should be taken into account [29]. Finally, a valid approximation for the resonant energy is:
\[
E_r = E_{r0} + \frac{V_0}{2} \sin(\omega t_b),
\]
where \( E_{r0} \) is the resonant energy when there is no oscillation in the well and \( t_b \) the time that the electron takes to arrive to the barrier. This time \( t_b \) is the ratio between the
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Figure 3. The overlapping term $|I_{a,b}^{|2}|$ is plotted as a function of the frequency of the oscillation well and also as a function of the central energy of the injected electrons. We see that for certain values, the overlapping is almost zero, corresponding to the resonant energies.

Figure 4. Noise is plotted as a function of the frequency of the oscillation well and also as a function of the central energy of the injected electrons. We appreciate how there is a line where maximum values are achieved. In the blue solid line, we see the maximum values expected for the noise from equation (13), the data fit accurately.

distance $x_0$ from the place where the injection takes place until the barrier, and the velocity of the electron $v_e$:

$$t_b = \frac{x_0}{v_e} = \frac{x_0 m_e^2}{\sqrt{2 m E_r}}. \tag{12}$$

From equations (11) and (12) one realizes that the frequency for the maximum probabilities as a function of the resonant energy is:

$$w(E_r) = \sqrt{\frac{2 m E_r}{x_0 m^4}} \arcsin\left(\frac{2(E_r - E_{r0})}{V_b}\right). \tag{13}$$
which is in perfect agreement with the results observed in figure 4. There, we see that the peak values for the noise at each energy move accordingly to equation (13), which is plotted in the frequency–energy plane with a (blue) solid line.

Therefore, the simulations performed with sinusoidal potentials provide a clear behaviour: when we move to higher frequencies, the quantum noise will be increased as we move to higher energies and will achieve a maximum at the new resonant energy. The experiment can be modified and include other behaviours when changing the potential according to another expression. For instance, \( V_w = -\frac{\hbar}{2} \sin(\omega t) \), in this case the resonant energy will decrease as the frequency increases (mathematically, the negative sign in front of the sinusoidal signal can also be introduced as a negative frequency in the plot figure 4). In this work, we propose that this very particular behaviour of the maximum of the quantum noise \( \langle S \rangle_{\text{max}} \) in the frequency–energy plane can be used as a test of our novel physical explanation of non-zero correlations in this type of HOM experiments with exchange and tunnelling. The experimental confirmation of these predictions will, in fact, give support for the need of using time-dependent states when modelling quantum noise in such experiments even for static (DC) conditions.

5. Conclusions

Motivated by the Hong–Ou–Mandel kind experiment on quantum noise performed with electrons [16], we analyse a similar two-particle scattering scenario with exchange and tunnelling. We inject two identical electrons with opposite momenta in a double barrier potential with an oscillatory well. Then, because of the different evolution suffered by the transmitted and reflected wave packet, we prove that two electrons can be found in the same place after the interaction with the barrier with probabilities given by equations (3) and (4). These new probabilities lead to a novel quantum noise expression (equation (8)). We remark that this formula contains two particular and interesting limits: the quantum Landauer–Büttiker noise expression and the distinguishable classical noise result.

Finally, we perform numerical simulations for oscillatory potentials which can certify the soundness of the new probabilities explained above. We compute the new quantum noise as a function of the energy of the initial electrons and of the frequency of the oscillatory well. We show in figure 4 that the maximum value of noise changes in energy as the frequency changes according to equation (13). We propose to reproduce these type of HOM experiment with an oscillatory well. Then, the satisfactory test on the experimental agreement of the maximum of the quantum noise with equation (13) (or similar ones depending on the condition of the experiment) will, in fact, conclude that in order to extract all the phenomenology in the scattering phenomena, the time-dependent evolution and localized nature of the electron cannot be neglected.

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