Spectral Detrended Fluctuation Analysis and Its Application to Heart Rate Variability Assessment

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(Dated: 5th June 2005)

Abstract

Detrend fluctuation analysis (DFA) has become a choice method for effective analysis of a broad variety of nonstationary signals. We show in the present article that, provided the nonstationary fluctuations occur at a large enough time scale, an alternative approach can be obtained by using the Fourier series of the signal. More specifically, signal reconstructions considering Fourier series with increasing number of higher spectral components are subtracted from the signal, while the dispersion of such a difference is calculated. The slope of the loglog representation of the dispersions in terms of the time scale (reciprocal of the frequency) is calculated and used for the characterization of the signal. The detrend action in this methodology is performed by the early incorporation of the low frequency spectral components in the signal representation. The application of the spectral DFA to the analysis of heart rate variability data has yielded results which are similar to those obtained by traditional DFA. Because of the direct relationship with the spectral content of the analyzed signal, the spectral DFA may be used as a complementary resource for characterization and analysis of some types of nonstationary signals.

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I. INTRODUCTION

Several natural signals are characterized by nonstationarity, i.e. the variation of its statistical description along time. Introduced by Peng and collaborators [1], detrend fluctuation analysis — DFA — has proven to be particularly effective for coping with nonstationary signals, providing results which tend to be more robust than previous methods such as those based on the Hurst coefficient. One of the interesting features of DFA is its ability to reveal the fractal nature (e.g. [2, 3]) of one-dimensional signals. A particularly impacting application of DFA has been the characterization of heart rate variability (HRV) (e.g. [4]), as estimated from sequences of interbeat intervals (RRi). The traditional approach to such a problem involves normalizing the interbeat sequence so as to achieve null mean and unit variance, integrating the resulting sequence, dividing it into boxes of fixed size, obtaining linear fit (or higher polynomial approximations) inside each box, and estimating the standard deviation of the difference between the sequence and linear fit. The absolute value of the inclination of the loglog curve yields the $\alpha$ parameter, which has been shown to be useful for applications to several analyses. Note that all logarithms in this article are natural logarithms.

Many interesting results have been obtained by DFA analysis of RRi sequences, including the fact that the coefficient $\alpha$ provides good sensitivity to the age of the analyzed individuals, with younger subjects tending to present smaller values of $\alpha$ (e.g. [4, 5, 6, 7]). This effect is to a great extent a consequence of the fact that the heart rate of youngsters tend to exhibit higher small scale variability, which populates the lefthand side of the loglog curve, therefore decreasing the value of $\alpha$. At the same time, complementary studies have shown that the effect of exercise in humans tend to lead to a reduction of the value of $\alpha_1$, namely the slope of the loglog curve considering 4 to 11 heatbeats [8]. As with the aging effect, such a reduced slope is also a consequence of the enhancement of small scale information in the interbeat interval signals. Additional works dealing with the effect of exercise and physical fitness on heart dynamics include but are not limited to [9, 10, 11, 12].

While traditional DFA analysis has consolidated itself as an important method for analysing heart dynamics, it is also interesting to consider modifications of the primary approach which can lead to eventual complementation of interpretation or improvements. For instance, Willson and collaborators [13, 14] have shown that the $\alpha$ coefficient in DFA
can be related to the frequency-weighted Fourier spectrum of the original signal. The present work reports on a modified DFA method where the piecewise partition and fitting of the original signal is replaced by its respective Fourier series with increasing number of coefficients. In addition to its inherent simplicity – it only involves calculating the fast Fourier transform and standard deviations, the proposed methodology bears a direct relationship with the progressive approximation of the signal by its Fourier series. The potential of the proposed DFA variant is illustrated with respect to human interbeat interval analysis involving young and older subjects submitted or not to regular exercises. It is shown that the spectral DFA leads to results which are similar to those obtained by using the traditional DFA regarding the effects of age and physical conditioning.

This article starts by briefly revising the traditional DFA, describing the spectral DFA, and illustrating its application in the characterization of interbeat interval analysis by using the traditional and spectral DFA methods.

II. TRADITIONAL DFA

Typically, ECG is recorded during a fixed total period of time, and the interbeat intervals are extracted afterwards (e.g. by considering the signal peaks). The interbeat intervals values are represented as a discrete sequence $r$, such that $r(k)$ corresponds to the $k$-th interbeat interval, with $k = 1, 2, \ldots, N$. Let $\langle r \rangle$ and $\sigma_r$ stand for the average and standard deviation of the sequence $r(k)$. A normalized sequence $s(k)$ is obtained through the transformation

$$s(k) = \frac{r(k) - \langle r \rangle}{\sigma_r} \quad (1)$$

It can be shown that $s(k)$ has null average and unit variance. The sequence $s(k)$ is then divided into boxes of size $L = L_{\text{min}}, \ldots, L_{\text{max}}$ and linear interpolation is performed within each box considering several box sizes. The difference $d(k)$ between $s(k)$ and the obtained regression is calculated for each box. The standard deviation $sd(L)$ of the difference sequence $d(k)$ is estimated, and a loglog curve is obtained for $sd(L)$ in terms of $L$. In case the loglog curve corresponds to a straight line, the original sequence $r(k)$ is understood to present fractal structure, with the absolute value of the inclination of the loglog curve, called $\alpha$, providing a parameter which has shown to be particularly useful for characterization of the heart rate variability, especially regarding the age of the individuals [4], as well as the effect
FIG. 1: The RRi time series for a sedentary (a) and active (b) older subjects. The respective loglog curves are shown in (c) and (d). The values of $\alpha$ (i.e. the curve slope) obtained for these curves were 0.98 and 0.94, respectively.

of aerobic training [8].

Figure 1 illustrates two RRi signals typical of a sedentary (a) and active older subjects (b) as well as the respective loglog curves (c-d), considering box sizes ranging from 4 to 11 heartbeats.

A. Spectral DFA

The Fourier series of the normalized interbeat interval signal $s(k)$ can be calculated in matrix form as

$$ S = W s $$

where

$$ W = [w_{i,j}], w_{i,j} = \exp{-2\pi\sqrt{-1} ij/N} $$

$$ i, j = 0, 1, \ldots, N - 1 $$

The reconstruction $s_m(k)$ of $s(k)$ considering the $m$ first Fourier coefficients (i.e. $0, 1, \ldots, m - 1$) can be obtained as
\[ s_m = QS \]  \hspace{1cm} (4)

where

\[ Q = [q_{i,j}], \]  \hspace{1cm} (5)

\[ q_{i,j} = \begin{cases} \exp\left\{2\pi\sqrt{-1} \frac{ij}{N}\right\} & i = 0, 1, \ldots, N - 1; \ j = 0, 1, \ldots, m - 1 \\ 0 & i = 0, 1, \ldots, N - 1; \ j = m, \ldots, N - 1 \end{cases} \]

Note that this equation corresponds to the Fourier series of the signal considering \( m \) spectral components, where the quantity \( u = ij \) is proportional to the frequency. The difference between the original signal and its reconstruction is therefore given as \( d(k) = s(k) - s_m(k) \), whose standard deviation is henceforth expressed as \( sd(m) \).

The spectral DFA proposed in this work involves calculating \( sd(m) \) for several values of \( m \), plotting the curve of \( \log(sd(m)) \) in terms of \( \log(1/m) = -\log(m) \), and obtaining the slope \( \gamma \), which has a similar role as the parameter \( \alpha \) in the traditional DFA. Note that as the values of \( m \) are proportional to the Fourier series frequencies, the quantity \( 1/m \) is proportional to the period of the sinusoidal functions in the Fourier kernel. Thus, \( 1/m \) can be understood as a time scale parameter (analogous to the box size in the traditional DFA) underlying the modified DFA analysis.

The detrending effect of the above described approach is illustrated in Figure 2. The original signal \( s_0(k) \) in (a) is contaminated with a triangular trend function \( tr(k) \), yielding the signal \( s(k) \) to be analyzed (b). Because of the slow variation of such a trend function, the Fourier power spectrum of \( s(k) \) can be separated into two groups, one with lower frequencies containing the trend signal, at the left hand side in (c), and the other corresponding to the original signal \( s_0 \), at the righthand side in (c). Provided the original signal and trend contamination are reasonably well-separated along the spectral decomposition, the trend effect can be removed by the above difference procedure before the signal of interest start influencing the slope \( m \). In other words, the dispersion of the difference signal becomes independent of the trend function.

Figures 3(a) and (b) illustrate the Fourier power spectrum for the same sedentary and active older individuals in Figure 1. Note that the interbeat dynamics of the active older, whose power spectrum is shown in (b), involves a more intense higher frequency composition. The respective loglog curves obtained by the spectral DFA are illustrated in (c) and (d),
FIG. 2: An original signal of interest \( s_0(k) \) (a), its contamination by a triangular function (b), and the respective Fourier power spectrum (c).

respectively. It is clear from these curves that physical exercise has as effect the reduction of the inclination of the loglog curve, indicating that regular physical activity tends to add small scale detail (i.e. high frequency components) to the interbeat dynamics. Note that the second curve in Figure 3 is not well fitted by the linear regression line. This is a consequence of the fact that the Fourier series approximations considered in the modified DFA methodology contain all coefficients ranging from 0 to \( m \), implying faster convergence to the original interbeat signal than that which could be obtained by using polynomial piecewise fitting. Note the substantially higher inclination (slope) of the curve obtained for the sedentary subject, reflecting its lack of high frequency composition.
FIG. 3: The Fourier spectrum for the same sedentary (a) and active (b) older subjects considered in Figure 1. The respective loglog curves, shown in (c) and (d), have slopes of 0.87 and 0.34, respectively.

III. INTERBEAT INTERVAL ANALYSIS

In order to illustrate the performance and features of the Fourier DFA, this method has been applied to a database of 40 interbeat interval sequences.

The RRi database includes measurements obtained at the same time of a day from 40 male individuals divided into four classes according to age and fitness (i.e., sedentary or active), all with good health as confirmed by extensive clinical, physical and laboratory examinations detailed in previous studies [15]. The individual included in the active classes had been through regular physical activities over a long period of time (at least 15 years in the case of the older group). On the day of experiments, after further medical examinations, the subjects remained at rest in supine position during 20 minutes and then ECG was recorded for 15 minutes. The ECG and heart rate were acquired through a one-channel heart monitor (ECAFIX TC500, Sp, Brazil) and processed by using an analog-digital converter Lab.PC+ (National Instruments, Co., Austin, TX, USA), interfaced to a personal computer. After analog to digital conversion, the R-R interval (ms) was calculated on a beat-to-beat basis by using a customised software (SP, Brazil) [16]. At least 5 minutes of ECG recording, characterized by the highest stability, were considered for the DFA investigations in the current work.
FIG. 4: The scatterplot (a) obtained by considering the slope of the loglog curves produced by the traditional and spectral DFA methodologies. Density probability functions for the four types of interbeat data obtained by normal fit of the measurements obtained by using the traditional (b) and spectral (c) DFA.

After normal transformation by using Equation 1, each RRi signal was processed by both traditional and spectral DFA. The intervals considered for calculating the loglog slopes were from 4 to 11 beats in the traditional DFA and from -4 to -2 (in log scale) in the spectral method. The scatterplot obtained by considering these two measurements for each analyzed human individual is shown in Figure 4 (a). The normal density functions fitted for each of the four classes considering each of the slopes is given in (b) and (c). It is clear from (a) that the results obtained by the traditional and spectral DFA methods are highly correlated, while the inspection of (b) and (c) indicates that similar separations between the several classes are obtained by the two methodologies. In both cases, the sedentary individuals yielded loglog curve slope which is higher than those for the active subjects.
IV. CONCLUDING REMARKS

This article has reported on a spectral variation of the detrend fluctuation analysis, DFA. Provided the trend effects are slow varying in time, the suggested methodology allows the detrend of higher frequency components of the signal of interest. The slope of the loglog curve of the dispersion against time scale has been shown to have an interpretation analogous to the $\alpha$ coefficient in the traditional DFA. The behavior of the spectral DFA has been illustrated with respect to interbeat analysis considering four types of individuals, yielding discrimination results which are closely correlated with those obtained by traditional DFA. Because of its spectral nature, the new type of DFA may be of interest for some applications in which one want to have a more direct comparison of relationship with the spectral content of the signals.

Acknowledgments

Luciano da F. Costa thanks FAPESP (proc. 99/12765-2), CNPq (proc. 3082231/03-1) and Human Frontier for financial support. Ester da Silva thanks FAPESP (proc. 01/07427-2) and Aparecida Maria Catai thanks CNPq (proc. 478799/2003-9).

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