Intermediated Implementation

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Abstract

Many real-world problems such as sales and healthcare regulation involve a principal, multiple intermediaries and agents with hidden characteristics. In these problems, intermediaries compete through offering menus of multifaceted consumption bundles to agents, whereas the principal is limited to regulating sub-aspects of the sold bundles by legal, informational and administrative barriers. We study how the principal can implement through intermediaries any social choice rule that is incentive compatible and individually rational for agents in quasi-linear environments. When intermediaries have private values, intermediated implementation can be achieved through per-unit fee schedule that allows intermediaries to break even under the target social choice rule. When intermediaries have interdependent values, per-unit fee schedules cannot generally be used to achieve implementation, whereas regulating the distribution over sub-aspects can under general conditions about the target allocation.

Keywords: implementation; intermediaries; adverse selection; market structure.

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1 Introduction

Many real-world problems such as sales and healthcare regulation involve a principal, multiple intermediaries and agents with hidden characteristics. In these problems, intermediaries compete through offering menus of multifaceted consumption bundles to agents, whereas the principal is limited to regulating sub-aspects of the sold bundles by legal, informational and administrative barriers. For instance, while a manufacturer controls the product supply, she cannot influence the prices that retailers charge to customers with hidden tastes. Under market-based healthcare systems, many government policies, such as coverage plan subsidies, impose only partial regulations on the insurance products that private companies sell to patients with hidden risk types.

This paper examines the possibility of implementing through intermediaries any social choice rule that is incentive compatible and individually rational for agents in quasi-linear environments. To formalize the above described implementation friction, we study a game where competing intermediaries offer menus of consumption bundles to agents, taking the principal’s policy on sold consumption goods as given. A sub-game perfect equilibrium induced by a policy achieves intermediated implementation if agents obtain target consumption bundles and intermediaries break even on the equilibrium path. Most of our results concern when and how we can achieve intermediated implementation in every sub-game perfect equilibrium.

We consider two kinds of policies: per-unit fee schedules and distribution regulations. The effectiveness of these policies depends on whether intermediaries have private values or interdependent values. In the first case, intermediaries’ payoffs do not depend directly on agents’ hidden characteristics, and implementation can be achieved through a per-unit fee schedule that allows intermediaries to break even under the target social choice rule. In the second case, per-unit fee schedules cannot generally be used to achieve implementation, whereas regulating the distribution of sold consumption goods can under general conditions about the target allocation.

To illustrate, consider the problem between a car manufacturer, multiple dealers, and a continuum of customers whose hidden taste for car quality is high with 20 percent chance and low with 80 percent chance. The manufacturer cares more than maximizing the profit from car sales (e.g., brand building), and her target quality-

\footnote{In 1980, the Sherman Act’s complete ban of vertical price fixing became effective again, and retail price maintenance has been illegal in the U.S. since then.}
price rule is incentive compatible and individually rational for customers. In lieu of
the Sherman Act, she gives dealers the freedom to set car prices while maintaining
the control over the product supply. Her policy instruments include: (1) invoice-price
schedules that charge different per-unit fees for different kinds of sold cars, and (2)
quota schemes mandating that dealers sell a certain variety of cars.

When the profit from car sales does not depend directly on customer’s hidden
taste, charging the target level of profit from serving each car fulfills the manufacturer’s purpose. Under the assumption of private values, a car makes a profit if and only if it is sold above the target level of price. Meanwhile, incentive compatibility implies that each customer prefers his target bundle to that of other customers, let along bundles that are profitable to dealers. Thus, the competition between dealers drives profit to zero and sustains the target quality-price rule in an equilibrium.

When the profit from car sales depends directly on customer’s taste (e.g., high-
taste customers are more difficult to serve than low-taste customers), invoice-price
schedules do not generally work for the same reason as that given by Rothschild and Stiglitz (1976). Instead, consider a quota scheme mandating that each dealer sell 20 percent of high-quality cars and 80 percent of low-quality cars. In order to meet this requirement, the only way that dealers can deviate from serving target-quality cars to customers is to sell high-quality cars to low-taste customers and low-quality cars to high-taste customers. However, since this permuted quality rule is decreasing in customer’s taste, it cannot be part of any incentive compatible allocation and cannot arise as a result of dealers’ profitable deviations. Thus in equilibrium, dealers offer target-quality cars to customers and competition drives prices to target levels.

The above described findings generalize to settings where agents have multi-
dimensional hidden characteristics. There distribution regulation achieves intermedi-
ated implementation in every sub-game perfect equilibrium if and only if permuting
target consumption goods yields a distinct total utility of consumption among agents.
In single-dimensional environments, the above described condition is automatically
satisfied if agents’ utilities satisfy the single-crossing property. In multi-dimensional
environments, the profiles of agents’ utilities ruled out by this condition are negligible
compared to those that can be attained under incentive compatible allocations.
1.1 Related Literature

Equilibrium in competitive environments  Our result in the case of private values can be understood by drawing analogies with classical welfare theorems. In an augmented economy where target consumption bundles represent intermediaries’ production frontier, incentive compatibility implies that the target social choice rule is Pareto-efficient and can be sustained by the Bertrand competition between intermediaries. An antecedent to this result appears in Fagart (1996), showing how efficiency can be attained in laissez-faire economies where the payoffs of competitive intermediaries are independent of agents’ hidden characteristics.

Since Rothschild and Stiglitz (1976), economists have long recognized that interdependent values, or adverse selection, may lead to equilibrium non-existence or inefficiency in competitive environments. Various methods to tackle these challenges have then been developed, including: Prescott and Townsend (1984) and Bisin and Gottardi (2006), which examine the outcomes of price-taking behaviors in general equilibrium economies; Miyazaki (1977), Wilson (1977), Riley (1979), Netzer and Scheuer (2014), and the references therein, where the solution concepts allow players to anticipate other people’s reactions to their contractual offers; Guerrieri et al. (2010), which uses search and match frictions to restore equilibrium existence; and Attar et al. (2011), which demonstrates the usefulness of latent contracts for deterring cream-skimming deviations when contracting is non-exclusive.

The current paper differs from the above described ones in several aspects. First, we examine policy interventions that implement any incentive compatible and individually rational allocation through intermediaries rather than to characterizing outcomes of laissez-faire economies. Second, we allow intermediaries to specify all aspects of the bundle, impose no limit on the menu size and adopt a standard solution concept, which all tend to make it harder, not easier, to establish equilibrium existence. Stantcheva (2014) studies a related problem, namely how the government can achieve redistribution when the employment contract signed between firms and workers can only be partially regulated. However, the author adopts Miyazaki’s (1977) solution concept and focuses on the comparison between linear and non-linear taxations in various informational environments.

Cyclic monotonicity  Rochet (1987) shows that in screening problems where the agent has quasi-linear utilities and multi-dimensional hidden hidden characteristics, a con-
sumption rule is implementable if and only if it is cyclically monotone. [Rahman (2011)] points out that in Rochet’s (1987) setting, a consumption rule is cyclically monotone if and only if every permutation of it is weakly unprofitable for the agent. In our setting, distribution regulation achieves intermediated implementation if and only if every permutation of the target consumption rule is not implementable. Combining this result with that of Rahman (2011) yields our key condition, namely permutation yields a distinct total utility of consumption among agents.

2 Model

2.1 Setup

Primitives There is a principal, a finite number $I \geq 2$ of intermediaries and a unit mass of infinitesimal agents. Agents’ hidden characteristics, denoted by $\theta$, are distributed on a finite set $\Theta$ according to a probability mass function $P_\theta$. A bundle $(x, y)$ consists of a consumption good $x$ and a price $y$ that belongs to compact and connected set $X \subset \mathbb{R}^d$ and $Y \subset \mathbb{R}$, respectively. Agents have single-unit demand for bundles and quasi-linear utilities $u(x, y, \theta) = v(x, \theta) - y$, whereas a constant-returns-to-scale technology yields a profit $\pi(x, y, \theta)$ from serving bundle $(x, y)$ to type $\theta$ agent. The functions $u$ and $\pi$ are continuous, and the function $\pi$ is increasing in $y$.

Let $\emptyset$ denote the null bundle that yields zero reservation utilities to inactive players. Assume without loss of generality that $\emptyset \in X \times Y$.

Target social choice rule The target social choice rule can be any deterministic mapping $(\hat{x}, \hat{y}) : \Theta \rightarrow X \times Y$ that satisfies agents’ incentive compatibility and individual rationality constraints, i.e., for all $\theta$,

\[
 u(\hat{x}(\theta), \hat{y}(\theta), \theta) \geq u(\hat{x}(\theta'), \hat{y}(\theta'), \theta) \quad \forall \theta',
\]

and

\[
 u(\hat{x}(\theta), \hat{y}(\theta), \theta) \geq 0,
\]

and it is taken as given throughout the analysis. Let $\hat{x}(\Theta)$, $\hat{y}(\Theta)$ and $(\hat{x}, \hat{y})(\Theta)$ denote the image of $\Theta$ under the mapping $\hat{x}$, $\hat{y}$ and $(\hat{x}, \hat{y})$, respectively. For each $x \in \hat{x}(\Theta)$,
Let $\hat{y}(x)$ be the unique $y \in \hat{g}(\Theta)$ such that $(x, y) \in (\hat{x}, \hat{y})(\Theta)$. and let

$$\hat{\pi}(x) = E_{\theta} [\pi(x, \hat{y}(x), \theta) | (\hat{x}, \hat{y})(\theta) = (x, \hat{y}(x))] \quad (2.1)$$

be the expected profit from serving bundle $(x, \hat{y}(x))$ to its target agents.

**Intermediated implementation**  If the principal owns the technology for serving agents, then she can simply dictate the menu $\{(\hat{x}(\theta), \hat{y}(\theta)) : \theta \in \Theta\}$ and let agents self-select. Now suppose, instead, that the above described technology is owned by intermediaries, whereas the principal is limited to regulating intermediaries’ sold consumption goods by frictions such as the ones described in Section II. The current paper examines when and how the principal can implement the target social choice rule in these situations, especially when her goal is different than maximizing intermediaries’ profits.

Formally, let $\mu_i$ denote the measure on $X \times Y \times \Theta$ that is induced by intermediary $i$’s sold bundles, and let $\nu_i$ be the measure on $X$ that is induced by $\mu_i$. A policy $\psi : \Delta(X) \to \mathbb{R}$ maps measures on sold consumption goods to real-valued fees. Based on $\nu_i$, the principal charges intermediary $i$ a fee $\psi(\nu_i)$, leaving the latter with the following net profit:

$$\int_{(x,y,\theta)} \pi(x, y, \theta) d\mu_i - \psi(\nu_i). \quad (2.2)$$

Time evolves as follows:

1. the principal commits to a policy $\psi$;
2. intermediaries simultaneously propose menus of consumption bundles to agents, i.e., $\sigma_i \in 2^{X \times Y}$, $i = 1, \cdots, I$;
3. each agent selects a bundle from $\bigcup_i \sigma_i \cup \{0\}$;
4. intermediaries pay fees to the principal.

Our solution concept is sub-game perfect equilibrium (or equilibrium for short). Below is our notion of implementation:

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2 Uniqueness is easy to establish: if there exist $y, y' \in \hat{g}(\Theta)$ such that $y < y'$ and $(x, y), (x, y') \in (\hat{x}, \hat{y})(\Theta)$, then all agents prefer $(x, y)$ to $(x, y')$ and hence $(x, y') \notin (\hat{x}, \hat{y})(\Theta)$, a contradiction.
Definition 1. A sub-game perfect equilibrium induced by a policy $\psi$ achieves intermediated implementation if agents consume target bundles and intermediaries break even on the equilibrium path.

We have results on both partial and full implementations.

2.2 Candidate policies

Our candidate policies are: per-unit fee schedules and distribution regulations. For every sold bundle $(x, y)$, a per-unit fee schedule charges a fee $t(x)$ if $x \in \hat{x}(\Theta)$, as well as a big penalty (denoted by “$+\infty$”) if $x \notin \hat{x}(\Theta)$. The penalty term will never be invoked if the principal controls the product supply as in our leading example, and it is feasible for government entities that can determine the legality of sold goods as in the case of insurance sales. Much attention will be given to a fee schedule that charges the profit from serving each consumption good to its target agents at the target level of price, i.e., $t(x) = \hat{\pi}(x)$ for every $x \in \hat{x}(\Theta)$. Under this fee schedule, the total charge to intermediary $i$ is

$$\hat{\psi}_{per-unit}(\nu_i) = \int_{x \in \hat{x}(\Theta)} \hat{\pi}(x)d\nu_i$$  \hspace{1cm} (2.3)

By contrast, distribution regulation imposes aggregate-level restrictions on sold consumption goods. Much attention will be given to the following policy, which implements the above fee schedule if an intermediary’s sold bundles match the probability measure $\hat{P}_x$ on $X$ induced by the target social choice rule, and charges a penalty otherwise, i.e.,

$$\hat{\psi}_{distr}(\nu_i) = \begin{cases} \hat{\psi}_{per-unit}(\nu_i) & \text{if } \frac{\nu_i}{\int_{x \in \hat{x}(\Theta)}d\nu_i} = \hat{P}_x, \\ +\infty & \text{otherwise.} \end{cases}$$  \hspace{1cm} (2.4)

In the case where any $x \notin \hat{x}(\Theta)$ is infeasible for intermediaries (e.g., for the above described reasons), $\hat{\psi}_{per-unit}$ is the weakest regulation that potentially fulfills our purpose, because intermediaries pay exactly $\hat{\psi}_{per-unit}(\nu_i)$ in every equilibrium that achieves intermediated implementation under any policy. Meanwhile, $\hat{\psi}_{per-unit}$ deters all detectable deviations by intermediaries and hence constitutes the strongest kind of the policies that potentially fulfills our purpose.
3 Main results

The effectiveness of per-unit fee schedules and distribution regulations depends on whether intermediaries have private values or interdependent values. The profit from serving agents doesn’t depend on the latter’s types in the first case, and the opposite is true in the second case:

Definition 2. Intermediaries have private values if $\pi(x, y, \theta)$ is invariant with $\theta$ for all $(x, y)$, and they have interdependent values otherwise.

The remainder of this section is devoted to understanding which policies are effective under what conditions.

3.1 Private values

Our first theorem demonstrates the effectiveness of $\hat{\psi}_{\text{per-unit}}$ when intermediaries have private values:

Theorem 1. Suppose intermediaries have private values. Then under $\hat{\psi}_{\text{per-unit}},$

(i) there exists a sub-game perfect equilibrium where $\sigma^*_i = \{(\hat{x}(\theta), \hat{y}(\theta)) : \theta \in \Theta\}$ for $i = 1, \ldots, I$ and each agent $\theta$ consumes $(\hat{x}(\theta), \hat{y}(\theta))$ on the equilibrium path;

(ii) if, in addition, $\min_{y \in Y} \pi(x, y) < \hat{\pi}(x)$ for all $x \in X$, then all sub-game perfect equilibria achieve intermediated implementation if and only if no agent is indifferent between his bundle and any other bundle under the target social choice rule.

Part (i) of Theorem 1 prescribes an equilibrium that achieves intermediated implementation. Since $\hat{\psi}_{\text{per-unit}}$ allows intermediaries to break even under the target allocation, the assumption of private values implies that every consumption good yields a profit if and only if it is sold above the target level of price. Meanwhile, incentive compatibility implies that each agent weakly prefers his target bundle to that of other agents, let alone bundles that are profitable to intermediaries. Thus no intermediary can make a profit when other intermediaries charge the target levels of prices, and Part (i) is immediate.

Part (ii) of Theorem 1 gives sufficient and necessary conditions for all sub-game perfect equilibria induced by $\hat{\psi}_{\text{per-unit}}$ to achieve intermediated implementation. A
key step in the proof, stated in Lemma 1 of Appendix A.1, shows that all equilibria
induced by \( \hat{\psi}_{\text{per-unit}} \) are payoff-equivalent, in that all sold consumption goods are
traded at the target level of price, all agents obtain the target levels of utilities and
all intermediaries break even. Thus if an equilibrium fails to achieve implementation,
then some agent’s target consumption good is not traded on the equilibrium path.
But then this agent must be indifferent under the target allocation, because otherwise
any intermediary can offer his target bundle at a slight discount and make a profit.
Part (ii) is then immediate.

3.2 Interdependent values

3.2.1 Per-unit fee schedule

As illustrated below, per-unit fee schedules cannot be generally used to achieve inter-
mediated implementation in the presence of interdependent values:

Example 1. \( \Theta = \{ \theta_1, \theta_2 \} \) and \( \pi(x, y, \theta_2) > \pi(x, y, \theta_1) \) for all \((x, y)\). The target
allocation \((\hat{x}_i, \hat{y}_i), i = 1, 2 \) is such that \( \hat{y}_1 > \min Y \), \((\text{IC}_{\theta_2}) \) is binding and \((\text{IC}_{\theta_1}) \) is
slack. Below we argue that no equilibrium induced by \( \hat{\psi}_{\text{per-unit}} \) achieves intermediated
implementation. In Appendix A.2 we prove the same negative result for all per-unit
fee schedules.

Under the assumption that intermediaries prefer to serve type \( \theta_2 \) agents than type
\( \theta_1 \) agents, other things being equal, we have that

\[
\pi(\hat{x}_1, \hat{y}_2, \theta_2) > \hat{\psi}_{\text{per-unit}}(\hat{x}_1) \triangleq \pi(\hat{x}_1, \hat{y}_1, \theta_1).
\]

Now suppose, to the contrary, that an equilibrium induced by \( \hat{\psi}_{\text{per-unit}} \) achieves
intermediated implementation. Consider a deviation by intermediary \( i \) that adds
\((\hat{x}_1, \hat{y}_1 - \epsilon)\) to its menu, where \( \epsilon \) is a small positive number. By assumption, the new
bundle is preferred by both types of agents to their equilibrium bundles. Further, it
raises intermediary \( i \)'s profit by

\[
\rho \pi(\hat{x}_1, \hat{y}_1, \theta_2) + (1 - \rho) \pi(\hat{x}_1, \hat{y}_1, \theta_1) - \hat{\psi}_{\text{per-unit}}(\hat{x}_1)
= \rho [\pi(\hat{x}_1, \hat{y}_1, \theta_2) - \pi(\hat{x}_1, \hat{y}_1, \theta_1)]
> 0,
\]
where $\rho$ denotes the population of type $\theta_2$ agents in the economy. This leads to a contradiction.

### 3.2.2 Distribution regulation

The negative result of the previous section calls for the use of aggregate-level regulations. In this section, we first construct an equilibrium that achieves intermediated implementation under $\hat{\psi}_{\text{distr}}$, and then give sufficient and necessary conditions for all equilibria induced by $\hat{\psi}_{\text{distr}}$ to achieve intermediated implementation.

A few definitions before we go into detail. Definitions 3 and 4 are standard, and Definition 3 should not be confused with Definition 1:

**Definition 3.** A consumption rule $x : \Theta \rightarrow X$ is implementable if there exists a transfer rule $y : \Theta \rightarrow Y$ such that $(x, y) : \Theta \rightarrow X \times Y$ is incentive compatible for agents.

**Definition 4.** A bijection $\pi : \Theta \rightarrow \Theta$ constitutes a cyclic permutation of $\Theta$ if $\theta \rightarrow \pi(\theta) \rightarrow \pi \circ \pi(\theta) \rightarrow \cdots$ forms a $|\Theta|$-cycle.

**Definition 5.** Permuting a consumption rule $x : \Theta \rightarrow X$ yields a distinct total utility of consumption among agents (hereafter, DU) if for any $\Theta' \subset \Theta$ such that $x(\theta) \neq x(\theta') \forall \theta, \theta' \in \Theta'$ and any cyclic permutation $\pi : \Theta' \rightarrow \Theta'$, we have that

$$\sum_{\theta \in \Theta'} v(x(\theta), \theta) \neq \sum_{\theta \in \Theta'} v(x(\pi(\theta)), \theta).$$

(DU)

**Theorem 2.** Under $\hat{\psi}_{\text{distr}},$

(i) there exists a sub-game perfect equilibrium where $\sigma^*_i = \{(\hat{x}(\theta), \hat{y}(\theta)) : \theta \in \Theta\}$ for $i = 1, \cdots, I$ and each agent $\theta$ consumes $(\hat{x}(\theta), \hat{y}(\theta))$ on the equilibrium path;

(ii) all sub-game perfect equilibria achieve intermediated implementation if and only if $\hat{x} : \Theta \rightarrow X$ satisfies (DU).

**Part (i)** Under $\hat{\psi}_{\text{distr}}$, the only way that intermediaries can make profits is to permute target consumption goods among agents. If these deviations are deemed to be unprofitable for intermediaries, then all agents obtain their target consumption goods, and competition drives prices to target levels.
Thus the problem boils down to showing that intermediaries cannot profit from permuting target consumption goods among agents. The next example helps build intuition; the formal proof is relegated to Appendix B.2.

**Example 2.** There are two types of agents $\theta_1$ and $\theta_2$, each obtains a distinct consumption good $\hat{x}_i$, $i = 1, 2$, under the target allocation. Since the target consumption rule is implementable, it satisfies Rochet’s (1987) cyclic monotonicity:

$$v(\hat{x}_1, \theta_1) + v(\hat{x}_2, \theta_2) \geq v(\hat{x}_1, \theta_2) + v(\hat{x}_2, \theta_1).$$

Meanwhile, if permuting consumption goods among agents yields a new implementable rule, then applying cyclic monotonicity again yields

$$v(\hat{x}_2, \theta_1) + v(\hat{x}_1, \theta_2) \geq v(\hat{x}_1, \theta_2) + v(\hat{x}_2, \theta_1).$$

Combining these inequalities shows that any permuted consumption rule is not implementable and hence cannot be part of any profitable deviation for intermediaries if $\hat{x}$ satisfies (DU):

$$v(\hat{x}_1, \theta_1) + v(\hat{x}_2, \theta_2) \neq v(\hat{x}_1, \theta_2) + v(\hat{x}_2, \theta_1).$$

Finally, if (DU) is violated, then agents must be indifferent under all incentive compatible allocations that comprise $\hat{x}_1$ and $\hat{x}_2$. If agents break ties in favor of their target consumption goods off equilibrium path, then any permuted consumption rule cannot arise off equilibrium path, let alone part of any profitable deviation for intermediaries.

**Part (ii)** The “if” direction is immediate. To establish the “only if” direction, we construct – through exploiting agents’ tie-breaking rules – bad equilibrium in which indifferent agents swap target consumption goods on the equilibrium path. Unlike in the case of private values, here we may lose payoff equivalence, in that not all equilibrium outcomes induced by $\hat{\psi}_{\text{dist}}$ yield the same levels of payoffs to players. Readers are referred to Appendix B.3 for such a counterexample.


4 Discussions

\[ \textbf{(DU)} \] The condition \textbf{(DU)} stipulates that permuting target consumption goods among agents must not yield new implementable rules, and it rules out cycles of agents who are mutually indifferent between each others’ bundles under all permutations of the target consumption rule. As illustrated in Appendix \[ \text{B.4} \] this condition can be easily satisfied in both single- and multi-dimensional environments.

**Coarse distribution regulation** In reality, the principal may not enforce the exact distribution regulation due to administrative and informational barriers. In these situations, consider instead the following coarse distribution regulation:

\[
\tilde{\psi}_{\text{distr}}(\nu_i) = \begin{cases} 
\hat{\psi}_{\text{per-unit}}(\nu_i) & \text{if } \left\| \frac{\nu_i}{\int_{x \in \Theta} d\nu_i} - \hat{P}_x \right\| < \epsilon, \\
+\infty & \text{otherwise},
\end{cases}
\]  

(4.1)

where \[ \| \cdot \| \] denotes the sup-norm.

**Corollary 1.** Suppose agents of the same type behave the same. Then Theorem 2 holds true under \( \tilde{\psi}_{\text{distr}} \) when \( \epsilon \) is small.

**Proof.** Take \( \epsilon \ll \min_{\theta \in \Theta} P_\theta(\theta) \). Repeating the proof of Theorem 2 step by step gives the desired result. \( \blacksquare \)

**Modeling choice** The restriction to policies of form \( \psi : \Delta(X) \to \mathbb{R} \) is a direct translation of the implementation friction we have in mind into math, and comparing the outcomes achievable by these policies with those in the benchmark case of direct implementation sheds light on how much we can gain through using mechanisms with richer message spaces.

As discussed in Section 1.1 our intermediaries can propose menus of consumption bundles and therefore possess more freedom than their counterparts as in many existing studies on adverse selection. Though we do not consider the competing mechanisms studied in common agency games, e.g., Epstein and Peters (1999) and Martimort and Stole (2002), it is easy to show that Part (i) of Theorems 1 and 2 on partial implementation holds true even if the contract between intermediaries and agents can have more dimensions than consumption goods and prices.
Relaxing Assumptions  The assumption of quasi-linearity enables us to invoke Rochet’s (1987) cyclic monotonicity, whereas that of finite types helps us avoid infinite cycles in the proof. These assumptions can be relaxed to different extents, depending on whether intermediaries have private or interdependent values. Specifically, Theorem 1 requires only that agents’ utilities are decreasing and no assumption about the type space, whereas Theorem 2 can be generalized to CARA utility functions in the application to insurance sales. See Appendices A and C for further details.

Future work  So far we have considered two extreme policies and one intermediary market structure, leaving the analysis of monopolistic intermediary to the online appendix. In the future, we hope to consider more intermediate cases and to obtain characterizations of the second-best policy when the condition for achieving intermediated implementation fails to hold.

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Appendices

A Omitted materials from Section 3.1

A.1 Proof of Theorem

Part (i): Let $\sigma^*_j = \{ (\hat{x}(\theta'), \hat{y}(\theta)) : \theta \in \Theta \}$ for all $j \neq i$ as in Theorem and write $u(x, y, \theta) = v(x, \theta) - y$ for ease. Suppose, to the contrary, there exists $(\hat{x}(\theta'), y) \in \sigma^*_i$ such that $u(\hat{x}(\theta'), y, \theta) \geq u(\hat{x}(\theta), \hat{y}(\theta), \theta)$ for some $\theta$, $\pi(\hat{x}(\theta'), y) \geq \hat{\pi}(\hat{x}(\theta')) \triangleq$
\[ \pi (\hat{x}(\theta'), \hat{y}(\theta')) \], and one of these inequalities is strict. Assume w.l.o.g. that it is 
\[ u (\hat{x}(\theta'), y, \theta) > u (\hat{x}(\theta), \hat{y}(\theta), \theta). \]

By incentive compatibility, we have that 
\[ u (\hat{x}(\theta'), y, \theta) \geq u (\hat{x}(\theta'), \hat{y}(\theta'), \theta) \]
and hence that 
\[ u (\hat{x}(\theta'), y, \theta) > u (\hat{x}(\theta'), \hat{y}(\theta'), \theta). \]
Since \( u \) is decreasing in \( y \) and \( \pi \) is increasing in \( y \), it follows that 
\[ y < \hat{y}(\theta') \]
and hence that 
\[ \hat{\pi}(\hat{x}(\theta'), y, \theta) \leq \pi (\hat{x}(\theta'), \hat{y}(\theta')) \]
\[ \hat{\pi}(\hat{x}(\theta')) \], a contradiction.

Part (ii): The next lemma is useful:

**Lemma 1.** Suppose intermediaries have private values and \( \min_{y \in Y} \pi(x, y) < \hat{\pi}(x) \)
for all \( x \in X \). Then in every sub-game perfect equilibrium induced by \( \hat{\psi}_{\text{per-unit}} \), all sold consumption goods are traded at the target levels of prices, all agents obtain the target levels of utilities and all intermediaries break even.

**Proof.** Take any equilibrium induced by \( \hat{\psi}_{\text{per-unit}} \). Denote the set of sold bundles by \( A \), the set of bundles sold by intermediary \( i \) by \( A_i \), and the measure of agents who consume bundles from set \( S \subset A \) by \( \mu(S) \).

We first demonstrate that any positive measure of sold bundles yields zero profit. If the contrary is true, then let \( B \) be any subset of \( A \) such that \( \mu(B) > 0 \) and \( \inf_{(x, y) \in B} \{\pi(x, y) - \hat{\pi}(x)\} \triangleq \delta > 0 \). Consider two cases:

**Case 1** \( \mu(B \setminus A_i) > 0 \) for some \( i \). In this case, intermediary \( i \) can add \( B \setminus A_i \) to its menu and make a profit, a contradiction.

**Case 2** \( \mu(B \setminus A_i) = 0 \) for all \( i = 1, \ldots, I \). In this case, consider a deviation by any intermediary \( j \) that undercut the \( y \)-dimension of every bundle in \( B \) by a small amount \( \epsilon > 0 \). This deviation is feasible for intermediary \( j \) under the assumption that \( \min_y \pi(x, y) < \hat{\pi}(x) \) for all \( x \). It has two effects: (1) it helps \( j \) attract all agents who used to consume bundles from \( B \cap A_{-j} \), and the resulting gain is bounded below by \( \mu(B \cap A_{-j}) \cdot \delta \); (2) it undercut the price that \( j \) charges its own customers, and the resulting loss is of \( O(\epsilon) \) by the assumption of private values. The net change in \( j \)'s payoff is positive when \( \epsilon \) is small, a contradiction.

We next show that all agents obtain the target levels of utilities. Indeed, if the equilibrium utility of a positive measure of agents in \( \Theta' \subset \Theta \) falls short of its target level, then any intermediary can add \( \{\hat{x}(\theta), \hat{y}(\theta) + \epsilon : \theta \in \Theta'\} \) to its menu and make a profit when \( \epsilon \) is small but positive, a contradiction. \( \square \)
We now complete the proof of Part (ii). From Lemma 1, it follows that if an equilibrium induced by \( \hat{\psi}_{\text{per-unit}} \) fails to achieve implementation, then the only explanation is that some agent \( \theta \)'s target consumption good \( \hat{x}(\theta) \) is not traded on the equilibrium path. But then agent \( \theta \) must be indifferent between his own bundle and some else’s bundle under the target allocation, because otherwise any intermediary can add \((\hat{x}(\theta), \hat{y}(\theta) - \epsilon)\) to its menu and make a profit, a contradiction. Part (ii) is then immediate.

A.2 Completing Example 1

Take any other per-unit fee schedule than \( \hat{\psi}_{\text{per-unit}} \). From the discussion in Section 3.2.1, it follows that in any equilibrium that achieves intermediated implementation, intermediaries must incur a loss (resp. make a profit) from serving bundle \( b \) (resp. \( b' \)) to its target agents in order to break even on average. Consider two cases:

**Case 1** \( b = (\hat{x}_1, \hat{y}_1) \) and \( b' = (\hat{x}_2, \hat{y}_2) \). Take any intermediary \( i \) who serves \( b \) to type \( \theta_1 \) agents and notice the following contradiction. On the one hand, if \( i \) is the only intermediary who serves \( b \) to type \( \theta_1 \) agents, then any other active intermediary must be serving \( b' \) to type \( \theta_2 \) agents and making a profit, suggesting that \( i \) can make a profit, too, by offering \((\hat{x}_2, \hat{y}_2 - \epsilon)\) to type \( \theta_2 \) agents. On the other hand, if another intermediary \( j \neq i \) is serving \( b \) to type \( \theta_1 \) agents, too, then \( i \) can drop \( b \) from its menu and save the loss.

**Case 2** \( b = (\hat{x}_2, \hat{y}_2) \) and \( b' = (\hat{x}_1, \hat{y}_1) \). Take any intermediary \( i \) who serves \( b \) to type \( \theta_2 \) agents and notice the following contradiction. On the one hand, if \( i \) is the only intermediary who serves \( b \) to type \( \theta_2 \) agents, then any other active intermediary must be serving \( b' \) to type \( \theta_1 \) agents and making a profit, suggesting that \( i \) can make a profit, too, by offering \((\hat{x}_1, \hat{y}_1 - \epsilon)\) to both types of agents. On the other hand, if another intermediary \( j \neq i \) is serving \( b \) to type \( \theta_2 \) agents, too, then \( i \) can drop \( b \) from its menu and save the loss.
B Omitted materials from Section 3.2.2

B.1 A useful lemma

Lemma 2. Let $x : \Theta \to X$ be an implementable consumption rule that satisfies (DU). Then for any $\Theta' \subset \Theta$ such that $x(\theta) \neq x(\theta') \ \forall \theta, \theta' \in \Theta'$ and any cyclic permutation $\pi$ of $\Theta'$, $x \circ \pi : \Theta' \to X$ is not implementable among agents in $\Theta'$.

Proof. Let $x : \Theta \to X$ and $\Theta' \subset \Theta$ be as above. For ease, write $\Theta' = \{\theta_1, \cdots, \theta_m\}$ and $x(\theta_i) = x_i$ for $i = 1, \cdots, m$. By assumption, there exists a transfer rule $y : \Theta' \to \mathbb{R}$ such that

$$v(x_2, \theta_1) - y_2 \leq v(x_1, \theta_1) - y_1,$$
$$v(x_3, \theta_2) - y_3 \leq v(x_2, \theta_2) - y_2,$$
$$\cdots$$
$$v(x_1, \theta_m) - y_1 \leq v(x_m, \theta_m) - y_m.$$  

Summing over these inequalities yields $\sum_{i=1}^{m} v(x_{i+1}, \theta_i) \leq \sum_{i=1}^{m} v(x_i, \theta_i)$. Meanwhile, if $(x_2, \cdots, x_m, x_1)$ is implementable among $(\theta_1, \cdots, \theta_m)$, then there exists a transfer rule $y' : \Theta' \to \mathbb{R}$ such that

$$v(x_2, \theta_1) - y'_2 \geq v(x_1, \theta_1) - y'_1,$$
$$v(x_3, \theta_2) - y'_3 \geq v(x_2, \theta_2) - y'_2,$$
$$\cdots$$
$$v(x_1, \theta_m) - y'_1 \geq v(x_m, \theta_m) - y'_m.$$  

Summing over these inequalities yields $\sum_{i=1}^{m} v(x_{i+1}, \theta_i) \geq \sum_{i=1}^{m} v(x_i, \theta_i)$. Thus, a sufficient condition for $(x_2, \cdots, x_m, x_1)$ to be not implementable among $(\theta_1, \cdots, \theta_m)$ is $\sum_{i=1}^{m} v(x_{i+1}, \theta_i) \neq \sum_{i=1}^{m} v(x_i, \theta_i)$. Repeating the above argument for all $\Theta' \subset \Theta$ and all cyclic permutations of $\Theta'$ gives the desired result. 

B.2 Proof of Theorem 2 (i)

Take any strategy profile $(\sigma_i, \sigma^*_j)$ where $\sigma^*_j = \{(\hat{x}(\theta), \hat{y}(\theta)) : \theta \in \Theta\}$ for all $j \neq i$. Let $x^j : \Theta \to X$ be the consumption correspondence induced by intermediary $j$’s sold
bundles. For any \( x, x' \in \hat{x}(\Theta) \) such that \( \hat{x}(\theta) = x \) and \( x' \in x^j(\theta) \) for some \( \theta \) and \( j \), we say that \( x \) (resp. \( x' \)) is an immediate predecessor (resp. immediate successor) of \( x' \) (resp. \( x \)), and write \( x \rightarrow x' \). Let \( y^j(x) \) denote the price charged by intermediary \( j \) for \( x \in \hat{x}(\Theta) \). By assumption, \( y^j(x) = \hat{y}(x) \) for all \( j \neq i \), where \( \hat{y}(x) \) is the unique price \( y \in Y \) such that \((x, y) \in (\hat{x}, \hat{y})(\Theta)\).

Consider the graph generated by \((\sigma_i, \sigma^*_i)\). For each \( x \in x^i(\Theta) \), only three situations can happen: (1) \( x \) has no immediate predecessor or successor, (2) \( x \) is part of a cycle, and (3) \( x \) is part of a chain. Below we go through these cases one by one:

**Case 1**  In this case, we must have \( y^i(x) = \hat{y}(x) \), as the remaining possibilities are ruled out as follows:

- If \( y^i(x) > \hat{y}(x) \), then no agent will purchase \( x \) from \( i \), a contradiction.
- If \( y^i(x) < \hat{y}(x) \), then all agents with target consumption good \( x \) will purchase \( x \) from \( i \). In order to satisfy the distribution requirement, \( i \) must charge \( y^i(x') \leq \hat{y}(x') \) for all other \( x' \in \hat{x}(\Theta) \) and incur a loss, a contradiction.

**Case 2**  This case is ruled out either by \( \{\text{DU}\} \) or by agents’ tie-breaking rules (off equilibrium path). Specifically, for any cycle \( x \rightarrow x' \rightarrow \cdots \), let \( \theta, \theta', \cdots \) be any sequence of agents that consumes (i) \( x, x', \cdots \) under the target social choice rule, and (ii) \( x', x'', \cdots \) now:

- If \( \hat{x} : \Theta \rightarrow X \) satisfies \( \{\text{DU}\} \), then the new consumption rule that assigns \( x' \) to \( \theta, x'' \) to \( \theta' \), so on and so forth, cannot be part of any incentive compatible allocation, let alone the result of intermediary \( i \)'s profitable deviation.
- If \( \hat{x} : \Theta \rightarrow X \) violates \( \{\text{DU}\} \), then the inequalities in the proof of Lemma 2 are all binding, suggesting that \( \theta \) is indifferent between \( x \) and \( x' \), \( \theta' \) is indifferent between \( x' \) and \( x'' \), so on and so forth, under any incentive compatible allocation. If agents break ties in favor of their target consumption goods, then the cycle \( x \rightarrow x' \rightarrow \cdots \) cannot arise in the first place, let alone the result of intermediary \( i \)'s profitable deviation.

**Case 3**  This case is impossible, too. Take any chain with end node \( x' \triangleq \hat{x}(\theta') \), and let \( x'' \triangleq \hat{x}(\theta'') \) be an immediate predecessor of \( x' \). Notice three things:
1. Incentive compatibility implies that \( v(x', \theta''') - \hat{y}(x') \geq v(x', \theta''') - \hat{y}(x') \);

2. \( x'' \to x' \) implies that \( v(x'', \theta''') - \hat{y}(x'') \leq v(x', \theta''') - y^i(x') \);

3. The definition of end node means that type \( \theta' \) agents consume \( x' \).

Combining these observations yields \( y^i(x') = \hat{y}(x') \), as the remaining possibilities can be ruled out as follows:

- If \( y^i(x') > \hat{y}(x') \), then \( v(x', \theta''') - y^i(x') < v(x', \theta''') - \hat{y}(x') \) and hence type \( \theta''' \) agents will not purchase \((x', y^i(x'))\), a contradiction.

- If \( y^i(x') < \hat{y}(x') \), then the very fact that \( x' \) is an end node implies that type \( \theta' \) agents obtain \( x' \) from intermediary \( i \). This observation, together with the assumption that \( x'' \to x' \), implies that \( i \) violates the distribution requirement, a contradiction.

Repeating this argument for every \( x \) along the chain yields \( y^i(x) = \hat{y}(x) \) for all \( x \in x^i(\Theta) \).

**B.3 Proof of Theorem 2 (ii)**

**The “if” direction** Let \( x^i \) denote the consumption correspondence induced by intermediary \( i \)'s sold bundles. In any equilibrium, we must have \( \bigcup_{i=1}^I x^i(\Theta) = \hat{x}(\Theta) \), because otherwise either some intermediary is violating the distribution requirement or all intermediaries are inactive, a contradiction.

Take the graph generated by agents’ consumption choices. Clearly, this graph contains no cycle because of (DU), and it contains no chain because otherwise some intermediary violates the distribution requirement and would prefer to exit the market instead. Thus the bundle consumed by each type \( \theta \) agent takes the form of \((\hat{x}(\theta), y(\theta))\), and the competition between intermediaries drives \( y(\theta) \) to \( \hat{y}(\theta) \) for all \( \theta \).

**The “only if” direction** Let \( \Theta' \) denote a typical subset of \( \Theta \) whereby \( \hat{x}(\theta) \neq \hat{x}(\theta') \) for all \( \theta, \theta' \in \Theta' \) and \( \sum_{\theta \in \Theta'} v(\hat{x}(\theta), \theta) = \sum_{\theta \in \Theta'} v(\hat{x}(\pi(\theta)), \theta) \) for some cyclic permutation \( \pi : \Theta' \to \Theta' \). In what follows we will call \( \Theta' \) an indifference cycle. By definition, (i) \( \hat{x} \) violates (DU) if and only if indifference cycle exists, and (ii) any implementable consumption correspondence that differs from the target consumption

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rule and satisfies the distribution requirement can be obtained from permuting target consumption goods along indifference cycles.

Take any such consumption correspondence and let intermediaries compete profit to zero. In order to sustain this outcome in equilibrium, we assume that agents adopt the same tie-breaking rule on and off equilibrium path. Then off equilibrium path, it follows from Point (ii) that agents must obtain the same consumption goods as they do on the equilibrium path in order for the deviating intermediary to meet the distribution requirement. But then the deviation is unprofitable, because profit has already been competed away on the equilibrium path.

**Payoff disparity** Below we give an example where the target consumption rule violates (DU) and not all equilibria induced by $\hat{\psi}_{distr}$ are payoff-equivalent:

**Example 2** (Continued). Suppose $\pi(x, y, \theta) = y - c(x, \theta)$, the target consumption rule violates (DU) and each type of agents consists of half the population. For ease, write $u_{ij} = u(\hat{x}_i, \theta_j)$ and $c_{ij} = c(\hat{x}_i, \theta_j)$ for $i, j = 1, 2$, and normalize $\pi(\hat{x}_i, \hat{y}_i, \theta_i)$ to zero for $i = 1, 2$.

Consider an outcome that assigns $(\hat{x}_2, y_2)$ to type $\theta_1$ agents and $(\hat{x}_1, y_1)$ to type $\theta_2$ agents. To sustain this outcome in equilibrium, we must satisfy agents’ incentive compatibility constraints and intermediaries’ zero-profit condition:

$$\begin{cases} y_1 - y_2 = u_{12} - u_{22} = u_{11} - u_{21}, \\ y_1 - c_{12} + y_2 - c_{21} = 0. \end{cases}$$

Denote the solution to this system of equations by $(y^*_1, y^*_2)$.

Now suppose an intermediary unilaterally deviates from the above described outcome. In order to make a profit, the deviator must offer $(\hat{x}_1, y'_1)$ to type $\theta_1$ agents and $(\hat{x}_2, y'_2)$ to type $\theta_2$ agents, where

$$\begin{cases} y'_1 - y'_2 = u_{12} - u_{22} = u_{11} - u_{21}, \\ y'_1 - c_{11} + y'_2 - c_{22} \geq 0. \end{cases}$$

In the case where $c_{11} + c_{22} \gg c_{21} + c_{12}$, each agent $\theta_i$ strictly prefer $(\hat{x}_{-i}, y^*_i)$ to all bundles of form $(\hat{x}_i, y'_i)$, where $(y'_1, y'_2)$ satisfies the second system of inequalities. Thus the deviation is unprofitable, and the above described outcome can be attained
in an equilibrium. The result on payoff disparity is then immediate.

B.4 DU

As illustrated below, the condition (DU) can be easily satisfied in both single- and multi-dimensional environments:

Example 3. If \( X, \Theta \subset \mathbb{R} \) and \( v(x, \theta) \) is supermodular in \((x, \theta)\), then any implementable consumption rule is non-decreasing in agent’s type (Milgrom and Shannon (1994)) and thus satisfies (DU).

Example 4. \( \Theta = \{\theta_1, \ldots, \theta_N\} \). Denote a typical consumption rule by \((x_1, \ldots, x_N)\), where \( x_i \) is the consumption good of agent \( \theta_i \). Let \( V \) be an \( N \times N \) matrix whose \( ij^{th} \) entry is agent \( \theta_j \)’s utility from consuming \( x_i \). Let \( \mathbb{I} \) denote the diagonal matrix and \( \Pi \) a typical permutation matrix, both are of order \( N \).

By definition, the utility matrices ruled out by (DU) belong to a finite union of subspaces of \( \mathbb{R}^{N \times N} \):

\[
\bigcup_{\Pi \neq \mathbb{I}} \{V: (\mathbb{I} - \Pi) \cdot V = 0\}.
\]

By Rochet (1987), the set of utility matrices that can be attained under incentive compatible allocations is given by a finite intersection of half-spaces of \( \mathbb{R}^{N \times N} \):

\[
\bigcap_{\Pi \neq \mathbb{I}} \{V: (\mathbb{I} - \Pi) \cdot V \geq 0\}.
\]

The first set is negligible compared to the second one.

C Insurance sales

This section investigates an extension to insurance sales. Suppose the principal is the government and faces a continuum of patients with hidden risk types \( \theta = (\theta_1, \ldots, \theta_d) \) drawn from \( \Theta = \Delta^d \). Type \( \theta \) patient’s endowment is a random variable equal to \( e_s \in \mathbb{R} \) with probability \( \theta_s, s = 1, \ldots, d \). An insurance policy \((x, y)\) consists of a state-contingent consumption plan \( x = (x_1, \ldots, x_d) \in \mathbb{R}^d \) and a premium \( y \in \mathbb{R} \). It yields an expected utility \( u(x, y, \theta) = \sum_{s=1}^d \theta_s \cdot v(x_s - y) \) to type \( \theta \) patients and an expected profit \( \pi(x, y, \theta) = y - \sum_{s=1}^d \theta_s \cdot (x_s - e_s) \) to the provider.
The target social choice rule maximizes an (unspecified) welfare function, subject to patients’ (IC) and (IR) constraints. Under single-payer systems, the government can simply propose the menu of target policies and let patients self-select. Under market-based systems, however, many governmental policies, such as coverage plan subsidies, impose only partial regulations on the insurance policies sold by private companies. The next corollary demonstrates the effectiveness of \( \hat{\psi}_{\text{distr}} \), which resembles newly introduced rulings by the Affordable Care Act mandating that all participating companies in the health insurance exchange offer a variety of plans and penalizing companies for selling too many low coverage plans.

**Corollary 2.** Suppose \( v(c) = -\frac{1}{\lambda} \exp(-\lambda c) \) for some \( \lambda > 0 \). Then under \( \hat{\psi}_{\text{distr}} \),

1. there exists a sub-game perfect equilibrium where \( \sigma^*_i = \{(\hat{x}(\theta), \hat{y}(\theta)) : \theta \in \Theta \} \) for \( i = 1, \ldots, I \) and each type \( \theta \) patient consumes \( (\hat{x}(\theta), \hat{y}(\theta)) \) on the equilibrium path;

2. all sub-game perfect equilibria achieve intermediated implementation if and only if \( \hat{x} : \Theta \to X \) satisfies \( (DU) \) in a quasi-linear economy where agents’ utilities are given by \( \lambda^{-1} \log \left( -\sum_{s=1}^{d} \theta_s \exp(-\lambda x_s) \right) - y \).

**Proof.** Rewrite \( (IC_\theta) \) as \( \tilde{v}(x(\theta), \theta) - y(\theta) \leq \tilde{v}(x(\theta'), \theta) - y(\theta') \), where \( \tilde{v}(x, \theta) \triangleq \lambda^{-1} \log \left( -\sum_{s=1}^{d} \theta_s \exp(-\lambda x_s) \right) \). Plugging this into the proofs of Lemma 2 and Theorem 2 gives the desired result. 

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\(^3\)Jean Folger, “How to Choose Between Bronze, Silver, Gold and Platinum Health Insurance Plans,” *Forbes*, October 1, 2013.

\(^4\)“Explaining Health Care Reform: Risk Adjustment, Reinsurance, and Risk Corridors,” *The Kaiser Family Foundation*, January 22, 2014.