A Non-Chiral Extension of the Standard Model with Mirror Fermions

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Abstract

The difficulties of defining chiral gauge theories non-perturbatively suggest a vector-like extension of the standard model with three mirror fermion families. Some phenomenological implications of such an extension are discussed.

Introduction

The electroweak sector of the standard model is based on a chiral gauge theory with local chiral symmetry. Since the electroweak gauge couplings are weak, renormalized perturbation theory is an appropriate framework in most applications. Nevertheless, there are some questions where the non-abelian non-perturbative nature of the SU(2) coupling becomes relevant. For instance, this is the case for the electroweak phase transition where perturbation theory is plagued by severe infrared problems and also for some instanton induced multiparticle processes in high energy scattering. This makes a non-perturbative lattice formulation of chiral gauge theories not just a matter of principle but also an important practical question.

It is a remarkable fact that, in contrast to vector-like gauge theories as QCD where the lattice formulation is easy and elegant, the non-perturbative lattice formulation of chiral gauge theories is very difficult, if not impossible. After many years of struggle one can conjecture that beyond the perturbative framework no chiral gauge theories exist! The assumptions, which are the basis for this conjecture, are:

- quantum field theories are defined as limits of regularized cut-off theories;
- the infinite cut-off limit exists and is independent of the choice of the regularization;
- there exists an explicitly gauge invariant local action;

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• the formulation can be given by a Euclidean path integral satisfying reflection positivity, which implies unitarity after Wick rotation to Minkowski space.

Under these, rather natural, assumptions all attempts for constructing a chiral gauge theory seem to fail.

Relaxing some of these assumptions allows for lattice formulations which have a chance to give a consistent mathematical framework. Recently favored examples are: the “Rome-approach” [1] where gauge fixing is assumed similarly to perturbation theory, the “two-lattice approach” [2] where fermions are assumed to move on a much finer lattice than the gauge fields and the “overlap formalism” [3] where the path integral is abandoned and the fermion determinant is defined by the overlap of ground states of five-dimensional auxiliary hamiltonians. Particularly striking is the impossibility to find an explicitly gauge invariant formulation: all these approaches have to break local gauge symmetry on the lattice and hope to be able to enforce gauge symmetry restoration in the continuum limit. This is in sharp contrast to vector-like gauge symmetries and to the very nature of local gauge invariance which arises due to the independence of internal symmetry orientations in different space-time points.

A gauge invariant lattice formulation of an extension of the standard model becomes possible if one assumes the doubling of the physical fermion spectrum by mirror fermions [4]. This implies that, besides the three “left-handed” fermion families, three “right-handed” mirror fermion families exist [5]. The existence of pairs of fermion-mirror-fermion families make the gauge symmetry “vector-like” and allows for explicit local gauge invariance.

The mirror fermion families have not been experimentally observed up to now and may only exist if their masses are high enough and their mixings with the observed light fermions are small enough. Some possible deviations from the standard model, observed in recent years, might have to do with direct or indirect effects of the mirror fermions. Examples are possible small violations of the universality of gauge couplings [6] and/or the excess of high $Q^2$ events at HERA, which can be caused by a new strong interaction of mirror fermions at the TeV-scale [7]. In spite of some possible positive signals, an overall detailed study of the phenomenological viability of the vector-like extension of the standard model is still missing. Nevertheless, first investigations have already been carried out at the one-loop perturbative level [8]. In the present talk I shall point out a few recently discussed possibilities within the mirror fermion framework.

**Mirror fermions: mixing schemes**

In order to illustrate the possible mixing between fermions and their mirror fermion partners, let us consider the mass matrix of a fermion-mirror-fermion pair on the chiral basis
\[(\overline{\psi}_R, \overline{\psi}_L, \overline{\chi}_R, \overline{\chi}_L) \otimes (\psi_L, \psi_R, \chi_L, \chi_R)\): \\
\[M = \begin{pmatrix}
\mu_\psi & 0 & \mu_R & 0 \\
0 & \mu_\psi & 0 & \mu_L \\
\mu_L & 0 & \mu_\chi & 0 \\
0 & \mu_R & 0 & \mu_\chi
\end{pmatrix} , \] (1)

Here \(\mu_{(L,R)}\) are the fermion-mirror-fermion mixing mass parameters, and the diagonal elements are produced by spontaneous symmetry breaking:

\[\mu_\psi = G_\psi v, \quad \mu_\chi = G_\chi v , \] (2)

with the Yukawa-couplings \(G_\psi, G_\chi\) and the vacuum expectation value of the Higgs scalar field \(v\).

For \(\mu_R \neq \mu_L\) the mass matrix \(M\) is not symmetric, hence one has to diagonalize \(M^T M\) by \(O^T_{(LR)} M^T M O_{(LR)}\), and \(M M^T\) by \(O^T_{(RL)} M M^T O_{(RL)}\), where

\[O_{(LR)} = \begin{pmatrix}
\cos \alpha_L & 0 & \sin \alpha_L & 0 \\
0 & \cos \alpha_R & 0 & \sin \alpha_R \\
-\sin \alpha_L & 0 & \cos \alpha_L & 0 \\
0 & -\sin \alpha_R & 0 & \cos \alpha_R
\end{pmatrix}, \] (3)

and \(O_{(RL)}\) is obtained by exchanging the indices \(R \leftrightarrow L\). The rotation angles of the left-handed, respectively, right-handed components satisfy

\[\tan(2\alpha_L) = \frac{2(\mu_\chi \mu_L + \mu_\psi \mu_R)}{\mu_\chi^2 + \mu_\psi^2 - \mu_\chi^2 - \mu_R^2} , \quad \tan(2\alpha_R) = \frac{2(\mu_\chi \mu_R + \mu_\psi \mu_L)}{\mu_\chi^2 + \mu_R^2 - \mu_\psi^2 - \mu_L^2} . \] (4)

The two (positive) mass-squared eigenvalues are given by

\[\mu^2_{1,2} = \frac{1}{2} \left\{ \mu_\chi^2 + \mu_\psi^2 + \mu_L^2 + \mu_R^2 \right. + \left. \left[(\mu_\chi^2 - \mu_\psi^2)^2 + (\mu_L^2 - \mu_R^2)^2\right] \right. \\
\left. + (\mu_L^2 - \mu_R^2)^2 + 2(\mu_\chi^2 + \mu_\psi^2)(\mu_L^2 + \mu_R^2) + 8\mu_\chi \mu_\psi \mu_L \mu_R \right\}^{1/2} . \] (5)

For \(\mu_\psi, \mu_L, \mu_R \ll \mu_\chi\) there is a light state with \(\mu_1 = O(\mu_\psi, \mu_L, \mu_R)\) and a heavy state with \(\mu_2 = O(\mu_\chi)\). In general, both the light and heavy states are mixtures of the original fermion and mirror fermion.

In case of three mirror pairs of fermion families the diagonalization of the mass matrix is in principle similar but, of course, more complicated \[\]. The non-observation of mirror fermion pair production at LEP implies that the masses of the mirror fermions, including massive mirror neutrinos, have to be above \(\simeq 90\) GeV. The mixing angles among fermions and mirror fermions have to be small, in order to avoid an excessive violation of universality in \(W^-\) and \(Z\)-boson couplings. The limits on the mixing angles are stronger for the first fermion family than for other families, and stronger for leptons than for quarks \[\].
Leptoquarks and new strong interactions

If the mirror families are very heavy then the renormalization of the Yukawa couplings implies strong interactions at a relatively nearby energy scale above the electroweak scale. (See the bounds on mirror fermion masses following from the requirement of perturbative unification \[1\]).

Let us denote, as usual, the gauge couplings for SU(3), SU(2) and U(1), respectively, by \( g_3, g_2 \) and \( g' \). The Yukawa couplings for light fermions are \( G_{\psi} \) and for mirror fermions \( G_{\chi} \), whereas the quartic scalar coupling is denoted by \( \lambda \). The renormalization group equations for these couplings are, for simplicity, at one-loop level with equal Yukawa couplings for \( \nu \)-, \( l \)-, \( u \)- and \( d \)-type fermions:

\[
\frac{dg_3}{dt} = -\frac{3g_3^3}{16\pi^2}, \quad \frac{dg_2}{dt} = \frac{5g_2^3}{96\pi^2}, \quad \frac{dg'}{dt} = \frac{81g'^3}{96\pi^2},
\]

\[
\frac{dG_{\psi\nu}}{dt} = \frac{G_{\psi\nu}}{16\pi^2} \left\{ -\frac{9}{4}g_2^2 - \frac{3}{4}g'^2 + \frac{9}{2}G_{\psi\nu}^2 + \frac{3}{2}G_{\chi}^2 \right\},
\]

\[
+3 \left( G_{\chi}^2 + G_{\psi\chi}^2 \right) + 9 \left( G_{\psi\nu}^2 + G_{\psi\chi}^2 + G_{\chi}^2 \right),
\]

\[
\frac{dG_{\psi\ell}}{dt} = \frac{G_{\psi\ell}}{16\pi^2} \left\{ -\frac{9}{4}g_2^2 - \frac{15}{4}g'^2 + \frac{9}{2}G_{\psi\ell}^2 + \frac{3}{2}G_{\psi\nu}^2 \right\},
\]

\[
+3 \left( G_{\psi\nu}^2 + G_{\psi\ell}^2 \right) + 9 \left( G_{\psi\nu}^2 + G_{\psi\ell}^2 + G_{\chi}^2 \right),
\]

\[
\frac{dG_{\psi u}}{dt} = \frac{G_{\psi u}}{16\pi^2} \left\{ -\frac{9}{4}g_3^2 - \frac{17}{2}g'^2 + \frac{21}{2}G_{\psi u}^2 + \frac{15}{2}G_{\psi d}^2 \right\},
\]

\[
+3 \left( G_{\psi\nu}^2 + G_{\psi\ell}^2 + G_{\psi\chi}^2 + G_{\psi\chi}^2 \right) + 9 \left( G_{\psi\nu}^2 + G_{\psi\ell}^2 \right),
\]

\[
\frac{dG_{\psi d}}{dt} = \frac{G_{\psi d}}{16\pi^2} \left\{ -\frac{9}{4}g_3^2 - \frac{5}{12}g'^2 + \frac{21}{2}G_{\psi d}^2 + \frac{15}{2}G_{\psi u}^2 \right\},
\]

\[
+3 \left( G_{\psi\ell}^2 + G_{\psi\ell}^2 \right) + 9 \left( G_{\psi\nu}^2 + G_{\psi\ell}^2 \right),
\]

\[
\frac{dG_{\chi}}{dt} = \{ \psi \leftrightarrow \chi \},
\]

\[
\frac{d\lambda}{dt} = \frac{1}{16\pi^2} \left\{ 4\lambda^2 - \lambda \left( g_2^2 + 3g'^2 \right) + \frac{27}{4}g_2^4 + \frac{9}{2}g_2^2g'^2 + \frac{9}{4}g'^4 \right\},
\]

\[
+12\lambda \left( G_{\psi\nu}^2 + G_{\psi\ell}^2 + G_{\psi\chi}^2 + G_{\psi\chi}^2 \right) + 36\lambda \left( G_{\psi\nu}^2 + G_{\psi\ell}^2 + G_{\chi}^2 + G_{\chi}^2 \right)
\]

\[
-36 \left( G_{\psi\nu}^4 + G_{\psi\ell}^4 + G_{\psi\chi}^4 + G_{\psi\chi}^4 \right) - 108 \left( G_{\psi\nu}^4 + G_{\psi\ell}^4 + G_{\psi\chi}^4 + G_{\psi\chi}^4 \right). \tag{6}
\]

The variable \( t \) is defined as usual by \( t \equiv \log \mu \). The assumed degeneracy of \( \nu \)-, \( l \)-, \( u \)- and \( d \)-type fermions in the three mirror pairs of fermion families does not change the qualitative behaviour, because the Yukawa-couplings are in any case dominated by the heavy mirror fermions. Not even the separate inclusion of a heavy top quark has a strong influence. A typical behaviour of the couplings as a function of \( t \) is shown by figure \[2\]. As one can see, for the given initial conditions corresponding to mirror fermion masses of about 200 GeV, the quartic and Yukawa couplings diverge at an energy scale about a factor of 10 higher than the initial (electroweak) scale.
Starting values at t=0:

\begin{align*}
  g_s &= 1.2, \quad g_\omega = 0.7, \quad g^* = 0.4, \quad \lambda = 5.5 \\
  G_{\text{in}} &= 0.01, \quad G_{\text{in}} = 0.01, \quad G_{\text{in}} = 0.01, \quad G_{\text{in}} = 0.01 \\
  g_s &= 1.2, \quad g_s = 1.2, \quad g_s = 1.2, \quad g_s = 1.2
\end{align*}

Figure 1: Numerical solution of the renormalization group equations for gauge-, quartic- and Yukawa-couplings. The initial values are shown in the left upper corner.

The divergence of the Higgs scalar couplings signals the existence of new strong interactions in the TeV energy range. A simple scheme within the vector-like extension of the standard model, based on a strong U(1) interaction, has been considered recently in [7]. A strong U(1) interaction has been first proposed for the explanation of the high \( Q^2 \) HERA anomaly in ref. [10]. Scenarios with strongly interacting non-trivial U(1) gauge theories are supported by recent lattice investigations showing non-trivial scaling behaviour in the continuum limit [11]. (For further references see this paper.)

Within the vector-like extension of the standard model the nature of the new (vector-like) strong interactions, which eventually produce leptoquark bound states, is quite arbitrary. Besides U(1), another interesting possibility is to assume that the strong interaction at the TeV scale involves the SU(2)\(_R\) gauge group, which is originally not gauged. (Note that SU(2)\(_R\) with three mirror pairs of families is not asymptotically free. Asymptotic freedom holds if the SU(2)\(_R\)'s for the three families are gauged separately as SU(2)\(_R^{\otimes 3}\).) In this case
leptoquarks can appear both in neutral current and charged current $e^+p$ processes. In fact, assuming strong OZI-rule for the couplings of leptoquarks to their constituents (as in [7]), the dominant effective four-fermion couplings relevant for $e^+d$-scattering are:

$$
\mathcal{L} \simeq \alpha_L^{(l)^2} (\bar{\tau}_L d_R) \left[ (\bar{\tau}_R u_L) + (\bar{d}_R e_L) \right] \\
+ \alpha_L^{(q)^2} (\bar{\tau}_R d_L) \left[ (\bar{\tau}_L u_R) + (\bar{d}_L e_R) \right] + \mathcal{O}(\alpha_L^{(l)} \alpha_L^{(q)}) .
$$

(7)

The omitted off-diagonal terms proportional to $\alpha_L^{(l)} \alpha_L^{(q)}$ are strongly constrained by low energy data [12]. Assuming that the product $\alpha_L^{(l)} \alpha_L^{(q)}$ is very small and one of the diagonal terms shown in eq. (7) dominates, the low energy constraints are avoided. The consequence of eq. (7) is that the decay branching ratio of the letoquark in $e^+d$-scattering to neutral current and charged current final states is roughly the same.

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