Updated values of running quark and lepton masses at GUT scale in SM, 2HDM and MSSM

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Updated values of running quark and lepton masses at GUT (Grand unified theories) scales are important for fermion mass model building, and to calculate neutrino masses, in GUTs. We present their values at GUT scales, in SM, MSSM and 2HDM theories, using the latest values of running quark and lepton masses.

I. INTRODUCTION

Gauge theories are very attractive theories to explain the origin of all interactions among the fundamental particles. Standard model (SM) is a gauge theory based on group $SU(2)_L \times U(1)_Y \times SU(3)_C (G_{213})$. In SM, all fundamental particles get their masses via the celebrated Higgs mechanism. One of the major goals of current research in experimental and theoretical high energy physics is to understand the origin of all fermion masses and mixings, including those of neutrinos. Although SM has been very successful in explaining many of the observed experimental results, some questions remain unanswered in it. Gauge hierarchy problem, unification of gauge couplings, neutrino masses, origin of baryon asymmetry of the Universe (BAU), being the most important ones. Some of these problems can be circumvented if we consider two higgs doublet model (2HDM), minimal supersymmetric standard model (MSSM) [1], and GUTs. Although problem of gauge hierarchy is not solved by 2HDM, unification of gauge couplings is possible at GUT scales, in MSSM, and also after embedding them in non-SUSY GUTs like SO(10) [2]. Very recently [3], we have shown unification of the three gauge couplings $\alpha_{1Y}$ (for $U(1)_Y$), $\alpha_{2L}$ (for $SU(2)_L$), and $\alpha_{2C}$ (for $SU(3)_C$) in non-SUSY SM with additional flavor symmetries, and also estimated limits on proton life time.

It is now a well established fact that neutrinos have mass, and mix with each other and oscillate to other flavors. We know that in SM, neutrinos masses can not be explained, and hence we need to go to theories beyond standard model (BSM). One of the most promising theories, to explain small neutrino masses, is the grand unified theory (GUT), like SO(10), in which all the fermions, including the right handed (RH) neutrino, are present in a single 16-dimensional representation. These theories require running masses and mixings of quarks and charged leptons at GUT scales, for calculating neutrino masses. In theories based upon quark-lepton unification, like L-R symmetric $SU(2)_L \times SU(2)_R \times SU(4)_C$ group, these values are also required at intermediate scales. Unification of fundamental forces is based upon gauge symmetries which contain the standard model with fermions in the fundamental representations. Thus, the explanation of fermion masses and mixings must emerge from a successful unified gauge theory. And hence, the running fermion masses are required to build underlying textures and models for existence of appropriate unified theory.

Values of running masses of quarks and charged leptons at higher scales in SM, 2HDM and MSSM are available in literature [4]. They have been used quite extensively, by many researchers, e.g. in

- [5], [6], for constructing neutrino masses
- [7], for studying structures of unified theories
- [8], to study type II seesaw dominance in Non-SUSY and split SUSY SO(10) theory
- [9], for study of SO(10) models, to explain fermion masses and mixing angles, including neutrino masses.
- [10], for study of inverse seesaw in NonSUSY SO(10) theories

But, in all these works, older values from [4] have been used, and new data for fermion masses are available, for using as input at lower scales. The aim of present work is, to update these values, and fill the gap. We have used latest data for masses and couplings from PDG [11]. Conversion of $\bar{\text{MS}}$ to DR scheme is done using formulas given in [12], and top quark mass is taken from [13]. Following the analysis of [4], we use RGEs for Yukawa couplings, gauge

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couplings and VEVs separately, and calculate running values of fermion masses at GUT scale. These values at other intermediate scales, calculation of neutrino masses using them, will be presented elsewhere \[14\].

The paper has been organized as follows. In Section 2, we give a pedagogical discussion on fermion masses. Section 3 contains methodology of, how to run fermion masses from one energy scale to another. In Section 3, our new results, on updated values of running fermion masses, at GUT scale $2 \times 10^{16}$ GeV, have been presented. Discussions and conclusions have been given in Section 5.

II. A PEDAGOGICAL DISCUSSION ON FERMION MASSES

Now, we will have a pedagogical discussion on fermion masses. According to quantum field theory (QFT), the "bare" masses in the Lagrangian are infinite for all particles, but divergent loop contributions to the propagator cancel them out to give finite "dressed" masses. This is called renormalization. These dressed particle masses are actually measured in experiments. So in the case of an electron, for example, the experimentally measured electron mass is an input parameter to the theory, and according to QFT, the bare electron mass must be infinite, but the mass "runs" from infinity at very small length scales, to a constant at very large length scales ("IR fixed point"). So this IR-limit value is the same as the experimentally measured value.

We know that quarks are confined, and free quarks cannot be observed experimentally. This short distance confinement is believed to be because of nonperturbative effects, and is associated with the scale $\Lambda_{QCD} \sim 2 \text{ GeV}^2$. At energies greater than $\Lambda_{QCD}$, the QCD is perturbative. Since free quarks do not exist at energy scales less than $\Lambda_{QCD}$ (also called infrared (IR) limit), mass for them is not well defined. Hence quark masses are scale dependent, and they are often defined at a energy scale. The scale dependent quark masses are called 'current' or 'running' quark mass, and they are renormalization scheme dependent. But equivalence of these renormalization scheme-dependent quark masses can be established with renormalization group equations (RGEs). The 'constituent' quark mass is believed to be roughly the mass that contributes to observed mass of hadron, for example. Nonrelativistic quark models use constituent quark masses, the constituent mass of up and down quarks are $\sim 350 \text{ MeV}$.

For quarks masses also 'running' takes place, but instead of converging to a constant, they diverge at the energy scale $\Lambda_{QCD}$. They become infinite at a much smaller length scale. This makes perfect sense because quarks are confined into hadrons and can’t be observed macroscopically. The masses given in PDG \[11\] are the values of the 'running' masses at some energy scale greater than (length scale smaller than) $\Lambda_{QCD}$, defined in some specific renormalization scheme.

III. RUNNING OF MASSES AND COUPLINGS USING RGES

In the renormalization theories, where the Yukawa couplings and the VEVs run separately \[15\]-\[23\], the Dirac mass of a fermion can be defined as

$$M_i(\mu) = Y_i(\mu)v_i(\mu).$$

Here, $M_i(\mu)$ is the Dirac mass of the $i$-type fermion, $Y_i(\mu)$ is corresponding Yukawa coupling, and $v_i(\mu)$ is the running VEV (Vacuum expectation value), at the scale $\mu$. In these scenarios, the Yukawa couplings and VEVs run separately, independent of each other. Many authors have used these \[15\]-\[23\], see \[4\] (Das, Parida) for a complete discussion.

The relevant terms of the Lagrangian, for masses of fermions, in SM, can be written as:

$$L = \bar{q}_L Y_U \phi u_R + \bar{q}_L Y_D \phi d_R + \bar{l}_L \Phi e_R + h.c.$$  \hspace{1cm} (2)

Here, $\phi$ is the higgs particle, $v(\mu)$ its running VEV at scale $\mu$, $q_L$ is the left handed quark doublet, $u_R$ is the right handed quark, $d_R$ is the right handed quark, $l_L$ is the left handed lepton doublet, and $e_R$ is the right handed electron. Since in SM, no right handed neutrinos are present, there is no term in the Lagrangian for the neutrino mass. Similarly, for 2HDM and MSSM, this can be written as:

$$L = \bar{q}_L Y_U \phi_U u_R + \bar{q}_L Y_D \phi_D d_R + \bar{l}_L \Phi e_R + h.c.$$  \hspace{1cm} (3)

Here,

$$< \phi_U^0 > = v_U(\mu) = v(\mu)sin\beta, < \phi_D^0 > = v_D(\mu) = v(\mu)cos\beta$$  \hspace{1cm} (4)
Now, we write the RGEs for running of Yukawa and gauge couplings, for the SM, 2HDM, and MSSM, along with their RG coefficients. They have been given in [4], but we present them here for the sake of completeness only. The one-loop RGEs for Yukawa couplings, for SM, MSSM and 2HDM, can be written as [15]-[18], [24]-[26].

\[
16\pi^2 \frac{dY_U}{dt} = \left[ TR(3Y_U Y_U^\dagger + 3aY_D Y_D^\dagger + aY_E Y_E^\dagger) \right. \\
+ \left. \frac{3}{2} (bY_U Y_U^\dagger + cY_D Y_D^\dagger) - \sum_i C_i^{(u)} g_i^2 Y_U \right] v
\]

(6)

\[
16\pi^2 \frac{dY_D}{dt} = \left[ TR(3aY_U Y_U^\dagger + 3Y_D Y_D^\dagger + \nu E Y_E^\dagger) \right. \\
+ \left. \frac{3}{2} (bY_D Y_D^\dagger + cY_U Y_U^\dagger) - \sum_i C_i^{(d)} g_i^2 Y_D \right] v
\]

(7)

\[
16\pi^2 \frac{dY_E}{dt} = \left[ TR(3aY_U Y_U^\dagger + 3Y_D Y_D^\dagger + \nu E Y_E^\dagger) \right. \\
+ \left. \frac{3}{2} bY_E Y_E^\dagger - \sum_i C_i^{(e)} g_i^2 Y_E \right] v
\]

(8)

The RGEs for the VEV in SM, up to 2-loop have been derived using wave-function renormalisation of the scalar field [15-16, 18-19, 21-22], and the 1-loop equation is

\[
16\pi^2 \frac{dv}{dt} = \left[ \sum_i C_i^{(v)} g_i^2 - TR(3Y_U Y_U^\dagger + 3Y_D Y_D^\dagger + \nu E Y_E^\dagger) \right] v
\]

(9)

Here, \( t = \ln \mu \). The RGEs for \( v_a (a = u, d) \) in the 2HDM up to 1-loop and MSSM up to 2-loops are available in [15-18, 20]. The 1-loop equations in both theories are

\[
16\pi^2 \frac{dv_a}{dt} = \left[ \sum_i C_i^{(v)} g_i^2 - TR(3Y_U Y_U^\dagger + 3Y_D Y_D^\dagger + \nu E Y_E^\dagger) \right] v_u
\]

(10)

\[
16\pi^2 \frac{dv_a}{dt} = \left[ \sum_i C_i^{(v)} g_i^2 - TR(3Y_D Y_D^\dagger + \nu E Y_E^\dagger) \right] v_d
\]

(11)

The RGE for the gauge couplings for the three models are

\[
16\pi^2 \frac{dg_i}{dt} = b_i g_i^3
\]

(12)

2-loop contributions are available in literature [15-18, 21-26], and we use them from Das, Parida [4].

Using above RGEs, we run the values of fermion masses, from low scale \( M_Z \) to higher scale \( 2 \times 10^{16} \) GeV. The input values of running fermion masses at \( M_Z \) have been taken from PDG [11], and [12]. Our results have been presented in next section.

IV. RESULTS

The new results of our computations have been presented in Tables (I-VI). We have presented comparisons of all our results with older values (Das, Parida, EPCJ 2001). We have used mass of the Higgs to be 125 GeV. It can be noted that from a recent global analysis [27], mass of the Higgs boson has been expected to be around this value. The scale of supersymmetry breaking, \( M_S = 1 \) Tev has been used. It is worth mentioning here that some signatures
of SUSY have been detected at LHC in third family of fermions \[28\]. The pole mass of top quark is used from PDG \[11\], to be \(m_t = 172.9 \pm 0.6 \pm 0.9 \) GeV. This is first converted to running mass \(m_t(M_Z) = 172.1 \pm 0.6 \pm 0.9\), as described in Xing et al \[12\]. This value is used for SM and 2HDM. Then, for MSSM only, we convert this running value \(m_t(M_Z)\) to DR(dimensional regularization) scheme value, by using Eq. (22) of Xing et. al \[12\], and find this to be \(m_t(M_Z)_{DR} = 169.9 \pm 0.6 \pm 0.9\). The latest PDG value \(1/\alpha(M_Z) = 128.91\) and \(\alpha_s(M_Z) = 0.1189 \pm 0.0020\) are used in our analysis.

### A. Running fermion masses in SM at GUT scale = \(2 \times 10^{16}\) GeV

#### TABLE-II COMPARISON OF FERMION MASSES IN SM, 2-LOOP

| Fermion | Mass (This work) \[MeV\] | Mass (Das, Parida) \[MeV\] |
|---------|--------------------------|--------------------------|
| \(m_u\) | \(0.4565^{+0.1492}_{-0.1485}\) | \(0.8351^{+0.1696}_{-0.1700}\) |
| \(m_c\) | \(0.2225^{+0.0586}_{-0.0580}\) | \(0.2426^{+0.0235}_{-0.0234}\) |
| \(m_t\) | \(70.5188^{+0.9585}_{-0.9479}\) | \(75.4348^{+9.9643}_{-8.5401}\) |
| \(m_d\) | \(1.0773^{+0.4474}_{-0.4561}\) | \(1.7372^{+0.4846}_{-0.5266}\) |
| \(m_s\) | \(20.4323^{+5.7159}_{-5.4912}\) | \(34.5971^{+4.8857}_{-5.1071}\) |
| \(m_b\) | \(0.9321^{+0.0166}_{-0.0172}\) | \(0.9574^{+0.0037}_{-0.0045}\) |
| \(m_e\) | \(0.4413 \pm 0.0003\) | \(0.4413 \pm 0.0001\) |
| \(m_\mu\) | \(93.116 \mp 0.017\) | \(93.1431^{+0.0136}_{-0.0010}\) |
| \(m_\tau\) | \(1.6109 \mp 0.0003\) | \(1.583^{+0.0009}_{-0.0005}\) |

### B. Running fermion masses in MSSM at GUT scale = \(2 \times 10^{16}\) GeV

#### TABLE-III COMPARISON OF MASSES IN MSSM, 2-LOOP, T\(\tan\beta = 10\)

| fermion | mass(this work) \[MeV\] | mass (Das, Parida) \[MeV\] |
|---------|--------------------------|--------------------------|
| \(m_u\) | \(0.3961^{+0.1505}_{-0.1281}\) | \(0.7238^{+0.1365}_{-0.1467}\) |
| \(m_c\) | \(0.1930^{+0.0245}_{-0.0245}\) | \(0.2103^{+0.0190}_{-0.0212}\) |
| \(m_t\) | \(71.8083^{+1.2940}_{-1.0349}\) | \(82.4333^{+2.5067}_{-2.6761}\) |
| \(m_d\) | \(0.9316^{+0.3859}_{-0.3798}\) | \(1.5036^{+0.4235}_{-0.3204}\) |
| \(m_s\) | \(17.6702^{+8.9233}_{-1.9506}\) | \(29.9454^{+3.9001}_{-1.5444}\) |
| \(m_b\) | \(0.9898^{+0.0259}_{-0.0250}\) | \(1.0630^{+0.1414}_{-0.0865}\) |
| \(m_e\) | \(0.3585^{+0.0001}_{-0.0001}\) | \(0.3585 \pm 0.0003\) |
| \(m_\mu\) | \(75.639 \mp 0.0003\) | \(75.6712^{+0.0078}_{-0.0001}\) |
| \(m_\tau\) | \(1.3146^{+0.0004}_{-0.0004}\) | \(1.2922^{+0.0012}_{-0.0012}\) |
TABLE-IV COMPARISON OF MASSES IN MSSM, 2-LOOP, TANβ=55

| fermion | mass (this work)   | mass (Das, Parida)  |
|---------|--------------------|---------------------|
| m_u     | 0.3963±0.1506 MeV  | 0.7244±0.1219 MeV  |
| m_c     | 0.1932±0.0240 GeV  | 0.2105±0.0151 GeV  |
| m_t     | 80.447±2.9128 GeV  | 95.1486±69.2836 GeV|
| m_d     | 0.9284±0.3836 MeV  | 1.4967±0.4157 MeV  |
| m_s     | 17.6097±4.8972 MeV | 29.8135±4.4967 MeV |
| m_b     | 1.2424±0.0626 GeV  | 1.4167±0.4803 GeV  |
| m_e     | 0.3569±0.0001 MeV  | 0.3565±0.0001 MeV  |
| m_μ     | 75.3570±0.0034 MeV | 75.2938±0.0110 MeV |
| m_τ     | 1.6459±0.0112 GeV  | 1.6292±0.0143 GeV  |

C. Running fermion masses in 2HDM at GUT scale = 2 × 10^16 GeV

TABLE-V COMPARISON OF MASSES IN 2HDM, 1-LOOP, TANβ=10

| fermion | mass (this work)   | mass (Das, Parida)  |
|---------|--------------------|---------------------|
| m_u     | 0.4776±0.0290 MeV  | 0.8749±0.1701 MeV  |
| m_c     | 0.2328±0.0290 MeV  | 0.2542±0.0295 MeV  |
| m_t     | 74.1053±1.1944 MeV | 79.6373±1.124 MeV |
| m_d     | 1.1274±0.4572 MeV  | 1.8204±0.505 MeV  |
| m_s     | 21.3821±0.9000 MeV | 36.2544±5.083 MeV |
| m_b     | 1.1615±0.0929 GeV  | 1.2309±0.0826 GeV |
| m_e     | 0.4407±0.0001 MeV  | 0.4407±0.0001 MeV |
| m_μ     | 92.9898±0.0119 MeV | 93.0197±0.0122 MeV|
| m_τ     | 1.61277±0.0003 GeV | 1.5851±0.0005 GeV |

TABLE-VI COMPARISON OF MASSES IN 2HDM, 1-LOOP, TANβ=55

| fermion | mass (this work)   | mass (Das, Parida)  |
|---------|--------------------|---------------------|
| m_u     | 0.4776±0.0246 MeV  | 0.8749±0.1701 MeV  |
| m_c     | 0.2328±0.0290 MeV  | 0.2542±0.0295 MeV  |
| m_t     | 77.3752±1.4741 MeV | 83.9317±10.326 GeV|
| m_d     | 1.1274±0.4572 MeV  | 1.8204±0.505 MeV  |
| m_s     | 21.3836±0.9000 MeV | 36.2584±5.083 MeV |
| m_b     | 1.3053±0.0409 GeV  | 1.4128±0.1162 GeV |
| m_e     | 0.4407±0.0001 MeV  | 0.4407±0.0001 MeV |
| m_μ     | 93.0222±0.011 MeV  | 93.0536±0.0116 MeV|
| m_τ     | 1.8138±0.0053 GeV  | 1.7851±0.0107 GeV |

V. DISCUSSIONS AND CONCLUSIONS

We have presented updated values of running fermion masses in SM, 2HDM and MSSM at GUT scale, at tan β = 10 and 55, using 2-loop RGEs for SM and MSSM, and 1-loop RGEs for the 2HDM. It can be seen from our results (Tables I-VI ) that these new values of fermion masses are quiet different from their older counterparts (Das, Parida [4]). They can be used for calculation of neutrino masses in GUTs at higher scales, as well as for building of theories for fermion mass models. Here, we would like to mention that we have verified our calculations, by reproducing the values reported in Das, Parida [4]. Also, our values are different from values reported in Xing. et. al. [12]. This is because we have used a different scheme for running fermion masses from low scale to GUT scale. We have used RGEs for
Yukawa couplings and VEVs separately. As discussed in text, fermion masses using this scheme have been used extensively in literature. Hence the results presented in this paper are very important. The values of running fermion masses at other intermediate scales, and calculation of neutrino masses using them will be presented elsewhere.

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