Exploring the performance of thin-film superconducting multilayers as kinetic inductance detectors for low-frequency detection

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Abstract
We have solved numerically the diffusive Usadel equations that describe the spatially varying superconducting proximity effect in Ti–Al thin-film bi- and trilayers with thickness values that are suitable for kinetic inductance detectors (KIDs) to operate as photon detectors with detection thresholds in the frequency range of 50–90 GHz. Using Nam’s extension of the Mattis–Bardeen calculation of the superconductor complex conductivity, we show how to calculate the surface impedance for the spatially varying case, and hence the surface impedance quality factor. In addition, we calculate energy-and spatially-averaged quasiparticle lifetimes at temperatures well-below the transition temperature and compare to calculation in Al. Our results for the pair-breaking threshold demonstrate differences between bilayers and trilayers with the same total film thicknesses. We also predict high quality factors and long multilayer-averaged quasiparticle recombination times compared to thin-film Al. Our calculations give a route for designing KIDs to operate in this scientifically-important frequency regime.

Keywords: kinetic inductance detectors, superconducting proximity effect, surface impedance, recombination time, Usadel equations

1. Introduction

Kinetic inductance detectors (KIDs) are thin-film superconducting devices, typically ~100 nm thick, that can be configured as ultra-sensitive detectors for astronomical observations across the electromagnetic spectrum [1]. They are readily fabricated by conventional ultra-high vacuum deposition techniques and can be patterned using optical lithography. KID operating temperatures are low (T ~ 1 K or below) giving excellent performance in terms of energy resolution when used as microcalorimeters, or noise equivalent power when used as bolometers. Achieving ultimate performance and low noise requires that the operating temperature satisfies T/Tc < 0.15, where Tc is the superconducting transition temperature. Low operating temperatures ensure sufficiently long quasiparticle recombination times τr such that the quasiparticle generation-recombination noise is minimized and signal-to-noise is maximized [2–6]. In practice the minimum operating temperature Tb is determined by available cryogenic technology, so that Tb ~ 60 mK using an adiabatic demagnetization refrigerator (ADR), or a dilution refrigerator, has become a practical lower-bound for experiments sited in remote high-altitude terrestrial observatories or on satellites. The photon detection threshold of a KID is determined by the requirement that the photon has sufficient energy to break a Cooper pair hν ≥ 2Δg, where h is Planck’s constant, ν is the frequency of the detected radiation, and Δg is the superconducting energy gap. Each broken pair creates two excess quasiparticles, and the timescale for the recombination of the excess back into pairs, whilst emitting phonons, is determined by τr. The effect of pair-breaking can be detected by configuring the superconductor into a thin-film L–C resonator that is readout by a low-frequency microwave probe (typically 1–10 GHz) close to the circuit resonant frequency f0. A change in the number of pairs, changes
the superconductor complex conductivity $\sigma$, surface impedance $Z_s$, quality factor $Q$, and $f_s$ can be observed in the through-transmission of the probe circuit. The advantage of the scheme is that as few as two coaxial lines are required to connect from the room temperature to $T_c$ to readout an array. The readout readily lends itself to frequency domain multiplexing. High detector $Q$ values are required to achieve high responsivity, high sensitivity, and high multiplexing factors [1, 2].

There exists a particular challenge in the design of KIDs as low-frequency detectors for Cosmic Microwave Background observations at $\sim 70$–$120$ GHz from the ground, or on a satellite, [7, 8] for measurement of low red-shift CO lines at around $100$–$110$ GHz, [9, 10] or for measurements of $O_2$ rotation lines at $50$–$60$ GHz for atmospheric profiling [11–13]. At the detection threshold frequency $f_s = 2\Delta_k/h$, the details of the photon-Cooper pair interaction matrix (the so-called case II interaction) mean that the interaction probability is vanishingly-small at low temperatures and increases towards the normal-state value only slowly as a function of $\nu$ [14]. Using the Bardeen–Cooper–Schrieffer (BCS) result $\Delta_k = 1.76k_B T_c$ where $k_B$ is Boltzmann’s constant, the constraints on $T_c$ determined by bath temperature and pair-breaking threshold can be parameterized as $7 T_c \lesssim \tau_c \lesssim \kappa^{-1}$. $\kappa$ depends on geometry, such that, for direct absorption, $\kappa \approx 14$ mK GHz$^{-1}$ at threshold, reducing to $\approx 10$ mK GHz$^{-1}$ if a margin of 25% is adopted to achieve a pair-breaking efficiency of order 30% of the normal-state value (see figure 6 later). There are few elemental superconductors with the required $T_c$ ($\sim 500$–$900$ mK) that satisfy these constraints. For example, $\text{Al}$ with $T_c = 1.2$ K has a detection threshold of 88 GHz. Current solutions to this problem have been to use alloy superconductors, such as $\text{AlMn}$, [15] or reactively sputtered materials such as $\text{TiN}$, [16] such that $T_c$ and $\Delta_k$ can be adjusted to the application. But these sub-stoichiometric compounds have shown wide variations in their properties even in a single deposition, [17] and the current preferred solution is to use multilayers consisting of pure $\text{Ti}$ and stoichiometric $\text{TiN}$ [18, 19]. For transition edge sensors where it is necessary to tune $T_c$, the most-common technological solution is a metallic bi- or trilayer where the properties of a superconducting film $S$ are modified by the superconducting proximity effect due to its close electronic contact to a lower-gap superconducting or normal metal $S'$. Cooper pairs diffuse from $S$ into $S'$, changing the overall superconducting properties of the composite. Catalano et al. [8] have recently explored this approach in the context of $\text{Al}$/$\text{Ti}$ KIDs. Their prediction for the energy gap of the bilayer used Cooper’s model [20] to calculate the weighted-average electron-phonon coupling constant $\lambda_0 N (N_0$ is the single-spin density of electron states at the Fermi energy $V$ is the interaction potential). The usual BCS self-consistency expression was then used to solve for $T_c$ [14] and hence $\Delta_q$. We will refer to this approach as the ‘weighted-average model’.

A full analysis of a multilayer resonator for use as a KID requires a proper account of the spatial variation of its superconducting properties as a function of position through the film, $x$. The best approach to this problem is to solve the one-dimensional diffusive Usadel equations with appropriate boundary conditions [21–25]. This is the approach we use here. For simplicity we concentrate on $\text{Ti}$–$\text{Al}$ bilayers and $\text{Al}$–$\text{Ti}$–$\text{Al}$ trilayers although the method would apply for any material combinations that can be described by BCS superconductivity. We compare the results of these proximity structures with that of $\text{Al}$, since the properties of $\text{Al}$ are well characterized, both as a BCS superconductor, and as a mature KID system (e.g. NIKA) [26, 27].

In section 2 we outline the theoretical basis of the model. This allows us to calculate the spatial variation of the superconducting order parameter $\Delta(x)$, the quasiparticle and pair DoS, and the superconducting energy gap $\Delta_q$. We show how to calculate the complex conductivity $\sigma(x)$ and the surface impedance $Z_s$ taking account of spatially varying film properties. We discuss how to take account of the ordering of a multilayer with respect to the incident field. From $Z_s$ we calculate the surface impedance quality factor $Q_s$. We calculate the position and energy dependent quasiparticle recombination time $\tau(x, E)$ ($E$ is the energy) and introduce expressions for energy- and multilayer-averaging of $\tau_q$. Throughout these calculations we take full account of the spatial variation of the material parameters $T_c$, $N_0$, normal-state conductivity $\sigma_n$, and characteristic quasiparticle recombination time $\tau_0$ for dissimilar materials. Whilst some of these equations have been described previously, here we present a consistent formulation using a common framework. This is essential and means that the model described reduces to more usual expressions in the homogeneous BCS limit.

Section 3 gives details of the numerical method and material parameters and we discuss the temperature scaling used to ensure that calculations of the properties of multilayers with different $\nu_q$ are done in comparable temperature regimes. In section 4 we show some of the most important outcomes of the modelling. The quasiparticle and pair DoS calculated here deviate significantly from the homogeneous (BCS) case—this deviation is a characteristic result for any multilayer. Section 5 discusses and summarizes the work. Taking proper account of spatial variation not only leads to predictions for multilayer performance for $Q$ and $\tau_q$ that are surprising at first sight, but also predicts significant benefits from using multilayers as detectors operating below 90 GHz.

2. Theory

2.1. Usadel equation and self-consistency

Our analysis of multilayers is based on Usadel’s model for the generalized proximity effect in the dirty-limit. Layer boundaries within multilayer structures function as scattering centres and ensure that the behaviour of the structure is governed by dirty-limit equations [22]. The one-dimensional Usadel equation states that [21–24]

$$iD_0 \frac{\partial}{\partial x} \left( \sigma \frac{\partial}{\partial x} \hat{G} \right) = [i\omega_n \sigma_0 + \hat{\Delta}(x), \hat{G}]$$  \hspace{1cm} (1)
The matrices are defined such that
\[
\tilde{G}(x, \hbar \omega_n) = \begin{pmatrix} G & F \\ F^* & -G \end{pmatrix}
\quad \text{and} \quad
\Delta(x) = \begin{pmatrix} 0 & \Delta(x) \\ \Delta^*(x) & 0 \end{pmatrix},
\]
where \( \hbar = \hbar/2\pi \), subscript \( S \) denotes the layer of the material, \( \omega_n = 2\pi k_B T \left( n + \frac{1}{2} \right) / \hbar \) are the Matsubara frequencies, \( n = 0, 1, 2, \ldots \), \( \sigma \) is the Pauli matrix, \( \Delta(x) \) is the superconducting order parameter, and \( [a, b] = ab - ba \) is the commutator. \( D_\xi = 2\pi \xi^2 / k_B T \) is the diffusion coefficient, where \( \xi \) is the coherence length. \( G \) and \( F \) are the normal and anomalous Green’s functions and the normalization conditions are \( \tilde{G}^2 = 1 \), and \( G^2 + FF^* = 1 \). The analytic continuation of the Green’s functions into the continuous domain of quasiparticle energies \( E \) is achieved by \( \omega_n \to -iE \) in equation (1).

In the \( \theta \)-parameterization, which automatically satisfies the normalization conditions, we have
\[
\tilde{G}(x, \hbar \omega_n) = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix},
\]
where \( \theta = \theta(x, \hbar \omega_n) \). This yields the \( \theta \)-parameterized Usadel equation
\[
\frac{d^2 \theta}{dx^2} = \hbar \omega_n \sin \theta - \Delta(x) \cos \theta. \tag{2}
\]
Equation (2) needs to be solved along with the superconducting self-consistency equation
\[
\Delta(x) \ln \left( \frac{T_c}{T} \right) - 2\pi k_B T \sum_{\omega_n > 0} \left( \frac{\Delta(x)}{\hbar \omega_n} - \sin \theta \right) = 0, \tag{3}
\]
subject to appropriate boundary conditions (BCs) introduced in [28]. At the open boundaries of the \( S \) or \( S' \) layers of bilayer \( S'-S \) or trilayer \( S-S'-S \) structures, the BCs are
\[
\frac{\partial \theta_{S,S'}}{\partial x} = 0. \tag{4}
\]
At the \( S-S' \) interfaces,
\[
\frac{1}{\rho_s} \frac{\partial \theta_s}{\partial x} = \frac{1}{\rho_{s'}} \frac{\partial \theta_{s'}}{\partial x}, \tag{5}
\]
\[
\gamma_{B, S S'} \frac{\partial \theta_s}{\partial x} = \sin(\theta_{s'} - \theta_s). \tag{6}
\]
At the \( S'-S \) interfaces,
\[
\gamma_{B, S} \xi_{S} \frac{\partial \theta_{s'}}{\partial x} = \sin(\theta_s - \theta_{s'}). \tag{7}
\]
Here \( \gamma_{B, S} = R_B / \left( \rho_{S'} \xi_S \right) \) is a measure of boundary resistivity, \( \rho_{S', S} \) are the normal-state resistivities of \( S, S' \) layer and \( R_B \) is the product of the boundary resistance between the \( S-S' \) layers and its area.

Finally, the quasiparticle density of states is given by \( N(x, E) = N_0(x) Q(x, E) \), where \( Q(x, E) = \text{Re}[\cos \theta(x, E)] \), and \( N_0(x) \) is the position-dependent single-spin density of electron states. We define the pair-breaking threshold \( 2\Delta_0 \) for the proximity structure as twice the energy for which the DoS becomes appreciably non-zero \((N/N_0 > \delta)\), where \( \delta \) is the numerical precision.

Figure 1. Transmission line representation of a multilayer as a combination of layers of thickness \( dx \), terminated at vacuum of impedance \( Z_0 \).

2.2. Nam’s formulation for conductivities

Nam’s equations [29] are a generalization of the Mattis–Bardeen [30] theory into strong-coupling and impure superconductors. The real and imaginary parts of the complex conductivity \( \sigma = \sigma_1 - j\sigma_2 \) can be expressed as
\[
\frac{\sigma_1(\nu)}{\sigma_N} = \frac{1}{h \nu} \int_{\Delta_0 - h\nu}^{\Delta_0} dE \ g_1(E, h\nu + E), \tag{8}
\]
\[
\times \left[ 1 - 2f(E + h\nu, T) \right] + \frac{2}{h \nu} \int_{\Delta_0}^\infty dE \ g_2(E, h\nu + E),
\]
\[
\frac{\sigma_2(\nu)}{\sigma_N} = \frac{1}{h \nu} \int_{\Delta_0 - h\nu}^{\Delta_0} dE \ g_2(E, h\nu + E) \tag{9}
\]
\[
\times \left[ 1 - 2f(E + h\nu, T) \right] + \frac{1}{h \nu} \int_{\Delta_0}^\infty dE \ g_2(E, h\nu + E) \times \left[ 1 - 2f(E + h\nu, T) \right] + g_2(h\nu + E) \left[ 1 - 2f(E, T) \right],
\]
where the lower limit of the first integral in equation (9) refers to the larger of the two energies \( \Delta_0 - h\nu \) and \( -\Delta_0 \), and \( f(E, T) \) is the Fermi distribution function. The coherence factors \( g_{1,2} \) are given by
\[
g_1(E, h\nu + E) = Q(E)Q(h\nu + E) + P(E)P(h\nu + E),
\]
\[
g_2(E, h\nu + E) = -\bar{Q}(E)Q(h\nu + E) - \bar{P}(E)P(h\nu + E),
\]
where \( Q(E), \bar{Q}(E), P(E), \bar{P}(E) \), are the generalized quasiparticle and pair DoS
\[
Q(E) + j\bar{Q}(E) = \text{Re}[\cos \theta(E) + j\text{Im}[\cos \theta(E)],
\]
\[
P(E) + j\bar{P}(E) = \text{Re}[\text{Im}[\cos \theta(E)] + j\text{Im}[-j \sin \theta(E)].
\]

2.3. Surface impedance, transfer matrices and quality factor

In order to calculate \( Z_0 \), we use a transmission line model where the multilayer is sub-divided into a combination of thin layers [31] each represented by an equivalent circuit of series impedance \( dZ = j2\pi \mu \eta_i dx \) and shunt admittance \( dG = \sigma(\nu, x) dx \), terminating at vacuum, as shown in
The Bloch vector with the surface impedance quality factor beyond the scope of this work. Our calculations of electromagnetic coupling and coplanar waveguide, lumped-element KID etc including electron-phonon coupling time constant becomes very slow at low temperatures for low energy quasiparticles. A full solution for strong non-equilibrium conditions, such as high loading or strong pulses, requires solving the self-consistency equation with the kinetic equations describing quasiparticle and phonon coupling.

2.4. Multilayer-averaged quasiparticle recombination times

We use the low-energy expression for quasiparticle recombination time given by [33] and the material-characteristic electron-phonon coupling time constant $\tau_0$ as described by Kaplan et al [34]. We ignore quasiparticle scattering, which becomes very slow at low temperatures for low energy quasiparticles [34]. The recombination lifetime is given by

$$\tau_0(x) = \frac{1}{(k_B T_x) [1 - f(E, T)]} \int_{E+\Delta E}^{\infty} d\Omega \Omega^2 \times \left[ n(\Omega, T) + 1 \right] f(\Omega - E, T) \times \left( Q(x, \Omega - E) + \frac{\Delta(x)}{E} P(x, \Omega - E) \right),$$

where $\Omega$ is the phonon energy and $n(\Omega, T)$ is the Bose distribution. We note here the change in sign within the final bracket of equation (11) compared with equation (7d) of [33]. This ensures that $\tau_0$ here reduces to Kaplan’s result (equation (8), [34]) with the appropriate expressions for $Q$ and $P$ for homogeneous BCS superconductors.

Assuming thermal equilibrium, we use $Q(x, E)$ and $f(E, T)$ as weights to obtain the energy-averaged recombination time as a function of position

$$\left\langle \tau(x) \right\rangle_E = \frac{\int_{\Delta E}^{\infty} \tau_0(x, E) N_0(x) Q(x, E) f(E, T) dE}{\int_{\Delta E}^{\infty} N_0(x) Q(x, E) f(E, T) dE}.$$  

Applying weighted averaging over both energy and position, the multilayer-averaged quasiparticle recombination time is obtained,

$$\left\langle \tau_E \right\rangle = \frac{\int_{\Delta E}^{\infty} \tau_0(x, E) N_0(x) Q(x, E) f(E, T) dE}{\int_{\Delta E}^{\infty} N_0(x) Q(x, E) f(E, T) dE}.$$  

For thin films at low temperatures where $\left\langle \tau_0(x, E) \right\rangle \gg \frac{2}{\Delta E_{\text{Al},\text{Ti}}/2 D_{\text{Al},\text{Ti}}}$, equation (13) represents the best-estimate of the overall quasiparticle recombination time, including the effect of quasiparticle trapping, provided the number of excess quasiparticles is small such that recombination with thermal quasiparticles dominates. A full solution for strong non-equilibrium conditions, such as high loading or strong pulses, requires solving the self-consistency equation with the kinetic equations describing quasiparticle and phonon coupling.

3. Methodology

The Usadel equation (2) and the self-consistency equation (3) are solved iteratively, with appropriate BCS, for Ti–Al bilayers and Al–Ti–Al trilayers with the geometry shown in figure 2, at $T = 0.1$ K and $\gamma_{B,\text{Al}} = 0.01$ or $\gamma_{B,\text{Al}} = 100$ to represent both high and low-transmission interfaces. Our solver uses MATLAB’s multipoint boundary value problem numerical solver. The solver computes the residuals using geometry-specific boundary function handles, to ensure that the solution satisfies equations (4)–(7). The detailed steps of our implementation are as follows:

(i) A uniform trial $\Delta(x) = (\Delta_{\text{Al}} + \Delta_{\text{Ti}})/2$ is used to solve for $\theta(x, \omega_0)$, using equation (2) with maximum $n = 125$. $\theta(x, \omega_0)$ is then used to solve for $\Delta(x)$ using equation (3). The solution then updates $\Delta(x)$.

(ii) An iterative loop of step (i) continues until convergence is achieved in $\Delta(x)$, such that the relative change in
\( \Delta(x) \) between the final two iterations is less than 0.001%.

(iii) The Usadel equation is then solved once using \( \Delta(x) \)
from (ii) in the domain of continuous quasiparticle energy \( E \) to obtain \( \theta(x, E) \). This is done by applying
\( \hbar \omega_n \rightarrow -iE \) to equation (2).

(iv) Other properties, i.e. \( \sigma_{\perp}, Z_{\perp}, Q_{\perp}, \) and \( \tau_\perp \), are calculated from \( \theta(x, E) \) using equations described in section 2.

A list of basic physical parameters used in this paper can be found in table 1. Data for superconducting coherence lengths are calculated using the dirty limit expression, \( \xi = \xi_0 l/3 \), from [22], where \( \xi_0 \) is the BCS coherence length, and \( l \) is the electron mean free path. Calculation of \( \xi \) is supplemented by Al residual resistivity ratio (RRR\(_{\text{Al}}\)) from [39], and Ti mean free path at 300 K \( (l_{\text{Ti},300} \text{K}) \) from [40].

### 3.1. Scaling the temperature

We scale the temperature of calculation for both \( Q \) and \( \tau_\perp \) to remove the effect of reducing \( \nu_\perp \). Reducing the temperature of the calculation removes the effect of a reduced \( \Delta_e \) to first-order. The principal temperature dependence of both \( Q \) and \( \tau_\perp \) arises from the Fermi functions in equations (8), (9) and (12) leading to a close-to \( \exp(-\Delta/k_B T) \) dependence at a fixed \( T \) in both cases. Scaling \( T \) removes this effect, giving comparable operating regimes but it also means that in practice lower operating regimes but it also means that in practice lower temperatures are attainable.

### 4. Results

#### 4.1. Density of states gap and pair-breaking threshold

Figure 3 shows the normalized DoS for 5 Al–Ti–Al trilayers with Ti thicknesses, (a)–(e), 25, 50, 100, 150, and 200 nm respectively, at the open Al boundaries (solid lines) and at the centre of the Ti layer (dashed lines). The total Al thickness is 50 nm. The DoS is BCS-like for the thinnest Ti layer (trace (a)) due to both \( d_{\text{Al}} \) and \( d_{\text{Ti}} \) being much smaller than their respective coherence lengths. The DoS broadens for thicker Ti films. For very thick Ti layers (not shown) the DoS at the Ti layer tends to the BCS DoS for Ti, whereas the DoS at the edge of the Al broadens further and does not vary to any BCS limit. The broadening behaviour highlights the need for a rigorous model beyond BCS-like DoS approximations. In this numerical study, we use \( \delta = 0.01 \) to determine the pair-breaking threshold \( 2\Delta_e \). For clarity we have only indicated this for trace (e). The inset shows the DoS at different positions within the 25 nm–100 nm–25 nm multilayer. Trace (i) shows the DoS at the centre of Ti layer, (ii) the DoS at the Ti-side of the internal Ti–Al interface, (iii) the DoS at the external Al boundary. The DoS does not vary greatly with position in the Al layer. The high Al normal-state conductivity, compared to Ti, restricts the extent of variation of DoS in the Al layer. The DoS at different positions along the
same device share a common energy gap because all films are relatively thin \((d \sim \xi)\).

Figure 4 shows the DoS for 5 Ti–Al bilayers of Al thicknesses, (a) to (e), 25, 45, 80, 105 and 130 nm respectively at the open Al boundaries (solid lines) and at the open Ti boundaries (dashed lines). Ti thicknesses for the bilayers were chosen to give the same pair-breaking thresholds as the trilayers. A key difference between bi- and trilayers with equal \(\Delta_s\) is the reduction in the DoS in the Al layer near \(\Delta_s\) for bilayers. At large values of \(d_{\text{Ti}}\), the reduction is more pronounced as individual Al layer thicknesses (instead of total Al layer thickness) becomes more important to the DoS. This reduction in the DoS has immediate impact on the thermal quasiparticle density \(N_0(E) = 4 \int N_0(Q(x), E) f(E, T) dE\) in the Al and likewise changes \(\sigma\) and \(\tau_i\).

Figure 5 shows the pair-breaking threshold \(2\Delta_s\) for both bi- and trilayers because the total Al thickness is fixed: the detail imposed by the geometry is lost. It can be seen from all of (a)–(c) that the proximity effect is a long range diffusive process that extends over much longer distances than the component coherence lengths.

The inset shows the DoS for a trilayer of 25 nm–100 nm–25 nm geometry at the centre of the Ti layer (red line) and the outer boundary of the Al layer (blue line), calculated for \(\gamma_{\text{BAI}} = 100\). In this case, where the film coupling is weak, the DoS across the multilayer shows more structure with two relatively broad peaks, the first somewhat greater than \(\Delta_s\) and the second \(\Delta_s\) close to \(\Delta_{\text{Al}}\). The DoS at the centre of the Ti increases significantly from zero at \(\Delta_s\) and shows a small increase at \(\Delta_2\). The DoS at the edge of the Al increases slowly from zero at \(\Delta_s\) and shows a large increase at \(\Delta_2\). For very low-transmission boundaries \(\gamma_{\text{BAI}} \rightarrow \infty\), the DoS at the edge of the Ti increases exclusively at \(\Delta_2 = \Delta_{\text{Al}}\), the proximity coupling disappears in this case and we find a BCS-like DoS for the individual layers.

The overall effect on KID performance as a result of these spatially varying DoS with \(d_{\text{Ti}}\) is not obvious at first sight. In the next sections we explore the impacts on \(Z_\nu\), \(Q\) and \(\tau_i\) for bi- and trilayers.

4.2. Conductivity, impedance and quality factor

Figure 6 shows normalized conductivities \(\sigma / \sigma_N\) as a function of \(\nu\) for two trilayers with (a) \(d_{\text{Ti}} = 25\), and (b) 200 nm. The total Al thickness is 50 nm. Full lines show \(\sigma_1\) and dashed lines \(\sigma_2\). The black lines show calculations for Al for comparison. In all cases \(T = 0.1\) K and \(\gamma_{\text{BAI}} = 0.01\) and \(\Delta_{\text{Al}} = 50\) nm. Also indicated is \(\nu_{\text{B}}\) for the 25 nm–200 nm–25 nm trilayer. The ratio \(\sigma_1 / \sigma_N\) is identical to the superconducting to normal-state photon
Figure 6. Normalized real (solid line) and imaginary (dashed line) conductivities for two trilayers, (a) red line, \(d_{T_1} = 25\) nm, (b) blue line, \(d_{T_1} = 200\) nm. The black lines are calculated for Al, \(T = 0.1\) K, and \(\gamma_{Al,Al} = 0.01\). The total Al thickness is 50 nm. The inset shows the variation of \(2\Delta_g\) as a function of \(d_{T_1}\).

Figure 7. Frequency dependence of \(R_s\) (solid line) and \(X_s\) (dashed line) for two trilayers: (a) red line, \(d_{T_1} = 25\) nm, (b) blue line, \(d_{T_1} = 200\) nm. \(T = 0.1\) K and \(\gamma_{Al,Al} = 0.01\) and \(d_{Al} = 25\) nm. The inset shows the variation of \(R_s\) (solid lines) and \(X_s\) (dashed lines) for a bilayer with \(d_{T_1} = 130\) nm, at frequencies close to \(v_k = 61\) GHz. The blue lines indicate incidence on the Ti surface, the red lines indicate incidence on the Al surface.

Figure 8. Frequency dependence of \(Q_s\) for trilayers with varying \(d_{T_1}\). (a) Red line, \(d_{T_1} = 25\) nm, (b) blue line, \(d_{T_1} = 200\) nm. \(T = 0.1\) K, \(\gamma_{Al,Al} = 0.01\), with total Al thickness of 50 nm. The inset shows the variation of \(Q_s\) with \(\nu = 3\) GHz as a function of \(v_k\): (red solid line) trilayers, (blue solid line) bilayers with incidence on the Al surface, (blue dashed line) bilayers with incidence on the Ti surface. The temperature of the calculation is scaled such that \(T = 170\) mK \(\times \left(\Delta_g/\Delta_{Al}\right)\) for all multilayers.

In order to show the effect on the detection sensitivity of a KID fabricated from these bi- and trilayers, in figure 8 we plot \(Q_s\), at scaled temperatures in the range of 100 mK for the lowest gap multilayers up to 170 mK for Al. \(Q_s\) increases by a factor \(\approx 2\) as \(d_{T_1}\) increases from 25 to 200 nm. Changing the thickness of an Al KID by the same amount would change \(Q_s\) by a factor \(\approx 3\). From the point-of-view of KID operation this begins to suggest that there is little penalty in device sensitivity even having engineered the reduced threshold.

The inset of figure 8 shows \(Q_s\) at \(\nu = 3\) GHz representing a typical KID readout frequency, as a function of threshold \(v_k\). The solid blue line shows \(Q_s\) for bilayers with the field incident on the Al surface, the dashed blue line shows \(Q_s\) for absorption ratio [14]. \(\sigma_i/\sigma_N\) has a threshold at \(v_k = 2\Delta_g/h\) and shows a slow increase above threshold. As a result, although Al KIDs have \(v_k = 88\) GHz, they only operate efficiently above 100 GHz or even 120 GHz [8]. We find that changes in the normalized \(\sigma_i/\sigma_N\) are small as a function of position within a particular bi- or trilayer, and thus we have not shown these results. This is a result of the highly-transmissive interfaces. However, the absolute values of \(\sigma\) are very different in the Al and Ti layers due to the very different values of \(\sigma_N\).
bilayers with field incident on the Ti surface. In both cases as \( \nu_g \) is reduced \( Q \) increases due to the increase in total thickness but note that the geometry—in this case the ordering with respect to the incident field—also affects \( Q \). The effect of increasing \( d_{Ti} \) is more significant when the field interacts from the Ti side. The solid red line shows \( Q \) for Al–Ti–Al bilayers. At high thresholds \( \nu_g \sim 70 \text{ GHz} \), corresponding to thin Ti layers, \( Q \), for the trilayer lies between the bilayer \( Q \)-values. The increase of the trilayer \( Q \), compared to bilayers at low \( \nu_g \), can be understood from figure 5: a thicker Ti layer is required in trilayers to achieve the same \( \nu_g \). Two points are important. \( Q \), for multilayer resonators with low \( \nu_g \) remain highly-suitable for KIDs. The ordering of a bilayer with respect to the field is also important to maximizing \( Q \). This emphasizes the need for full electromagnetic modelling of the field distributions.

4.3. Quasiparticle lifetimes in multilayers

Here we show results of calculations of energy- and multi-layer-averaged recombination times \( \langle \tau_E \rangle \) and \( \langle \tau_{E,x} \rangle \), respectively. For all multilayers, the temperature is scaled such that \( T = 170 \text{ mK} \times (\Delta_g/\Delta_{AI}) \).

Figure 9 shows \( \langle \tau_E \rangle \) as a function of position for trilayers with five Ti thicknesses (a)–(e) 25, 50, 100, 150 and 200 nm respectively and, for comparison, calculation for Al. For the thinnest trilayer, figure 9 (a) \( \langle \tau_E \rangle \) is close to the calculated value for Al. For thicker trilayers \( \langle \tau_E \rangle \) increases in both the Al and Ti layers. This arises from the reducing quasiparticle density of states in the Al at low energies as \( d_{Ti} \) increases (see figure 3), and the combined effect of the reduced lower limit of the integral in equation (11) and the \( \Omega^2 \) dependence of the phonon density of states factor within that integral. The situation is similar for bilayers.

Figure 10 shows \( \langle \tau_E \rangle \) as a function of position for bilayers for five Ti thicknesses (a)–(e) 25, 45, 80, 105 and 130 nm respectively and \( d_{AI} = 50 \text{ nm} \). The Ti thicknesses are chosen so that \( \nu_g \), for (a)–(e) in figure 10 are the same as (a)–(e) in figure 9. Although similar, in detail we see that the bilayer recombination times are now longer in both the Ti and Al compared to trilayers with the same \( \nu_g \). This arises from the reduced DoS in the Al in bilayers compared to trilayers with the same \( \nu_g \), noted earlier in section 4.1. This emphasizes the importance of a full calculation of the spatially-dependent DoS.

The inset of figure 9 shows \( \langle \tau_{E,x} \rangle \) as a function of \( d_{Ti} \) for bilayers and trilayers. As expected from the main plots, \( \langle \tau_{E,x} \rangle \), increases with \( d_{Ti} \) and for bilayers the increase is more rapid. The behaviour here can be understood from figure 5, \( \Delta_g \) changes more quickly with \( d_{Ti} \) for bilayers than trilayers. The detailed differences arise from the different \( d_{Ti} \) required to achieve a given \( \nu_g \) in the two instances, the differing spatial variations in the DoS in Al and Ti, (in particular the reduced Al DoS in a bilayer having the same \( \nu_g \)—compare figures 3 and 4), and also the effect of the weighting with \( N_g \) and \( d_{Ti} \), used in the calculation of \( \langle \tau_{E,x} \rangle \) in equation (13).

The inset of figure 10 shows \( \langle \tau_{E,x} \rangle \) as a function of the experimental requirement \( \nu_g \) for both bilayers and trilayers. We find that bilayers and trilayers with the same \( \nu_g \) have very
similar multilayer-averaged recombination times, the bilayer being of order 20% longer, so that both behave comparably. The choice between them can be made on the basis of other considerations—ease of deposition or material processing or film thickness requirements for example. We have not included the effect of phonon-trapping in these calculations [42]. For thicker multilayers we would expect phonon-trapping to provide an additional enhancement of the effective quasiparticle lifetime.

5. Discussion and conclusions

We have described a full analysis of multilayer resonators suitable for KIDs based on the diffusive Usadel equations with appropriate boundary conditions that takes proper account of the spatial variation of the superconducting properties as a function position in the film. We calculate the spatial variation of the superconducting order parameter $\Delta(x)$, the quasiparticle and pair DoS, the superconducting energy gap $\Delta_g$, the complex conductivity $\sigma$, and energy- and multilayer-averaged $\tau$ for Al–Ti multilayers. We account for the spatial variation of $T_c, N_0$, normal-state conductivity $\sigma_N$, and characteristic quasiparticle lifetime $\tau_0$. We have also described how to calculate the surface impedance $Z_s$ including varying film properties and the ordering of the multilayer with respect to the incident fields. Our predictions of $\Delta_g$ differ significantly from earlier predictions that were based on the weighted-average model, and indicate differences between bilayers and trilayers with the same total film thicknesses.

The calculated quasiparticle and pair densities of states for the Al–Ti multilayers considered here deviate significantly from the homogeneous (BCS) case. We predict high quality factors $Q$, and long multilayer-averaged quasiparticle recombination times $\langle \tau_{r,E} \rangle$ compared to thin-film Al. Following [2], we define a figure-of-merit for multilayers such that $F(\nu) \propto Q_s \langle \tau_{r,E} \rangle \Theta(\nu - \nu_g)$, where $\Theta$ is the Heaviside step function, giving appropriate weight to the required detection threshold. Even without evaluation, considering values of $Q$ and $\langle \tau_{r,E} \rangle$, shown in figures 8 and 10, values of $F(\nu)$ for Al–Ti multilayers would be comparable to or better than those of Al films whilst achieving the required $\nu_g$. The multilayer structures described in this study are excellent candidates for high-sensitivity, multiplexible KIDs with targeted low-frequency detection thresholds.

Our work demands a programme of theoretical and experimental investigation of proximity-effect resonators for KID applications. For the first time, full electromagnetic modelling of multilayer resonators is possible. We find that full consideration of the field distribution is essential, particularly if bilayers are to be used. We find that in terms of detector sensitivity, there should be little difference in the performance of bi- and trilayers provided the geometry is properly considered. We have also identified relatively simple experimental measurements that would confirm our predictions of multilayer behaviour. Demonstrating high $Q$ and long $\tau$ in Al–Ti multilayers is critical for KIDs based on multilayers to become the preferred solution for sub-100 GHz detection. Further work is required to optimize device geometry and material combinations for particular applications. Overall, we have described a coherent approach to calculating the properties of superconducting multilayers for use as KIDs for experimentally important applications at frequencies in the range 50–100 GHz.

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References

[1] Zmuidzinas J 2012 Annu. Rev. Condens. Matter Phys. 3 169
[2] Leduc H G et al 2010 Appl. Phys. Lett. 97 102509
[3] Doyle S, Maukhopf P, Naylon J, Porch A and Duncombe C 2008 J. Low Temp. Phys. 151 530
[4] Barends R et al 2008 J. Low Temp. Phys. 151 518
[5] Janssen R M J et al 2013 Appl. Phys. Lett. 103 203503
[6] de Visser P J, Baselmans J J A, Bueno J, Llobhart N and Klapwijk T M 2014 Nat. Commun. 5 3130
[7] Planck HiFi Core Team 2011 Astron. Astrophys. 536 A4
[8] Catalano A et al 2015 Astron. Astrophys. 580 A15
[9] Ciccone C et al 2012 Astron. Astrophys. 543 A99
[10] Thomas C N 2014 arXiv:1401.4395
[11] Mahloul J et al 2015 Q. J. R. Meteorol. Soc. 141 3268
[12] Aires F et al 2015 J. Geophys. Res. Atmos. 120 11334
[13] Turner E C et al 2016 Atmos. Meas. Technol. 9 5461
[14] Tinkham M 1994 Introduction to Superconductivity 2nd edn (New York: McGraw-Hill)
[15] Jones G 2017 Appl. Phys. Lett. 110 222601
[16] Coiffard G et al 2016 J. Low Temp. Phys. 184 654
[17] Vissers M R et al 2013 Thin Solid Films 548 485
[18] Vissers M R et al 2013 Appl. Phys. Lett. 102 232603
[19] Giachero A et al 2014 J. Low Temp. Phys. 176 155
[20] Cooper L N 1961 Phys. Rev. Lett. 6 68954
[21] Usadel K D 1970 Phys. Rev. Lett. 25 507
[22] Brammertz G et al 2001 Physica C 350 227
[23] Golubov A A, Kupriyanov M Y and Il’ichev E 2004 Rev. Mod. Phys. 76 411
[24] Vesenko A S, Golubov A A, Kupriyanov M Y and Weides M 2008 Phys. Rev. B 77 134507
[25] Wang G et al 2017 IEEE Trans. Appl. Supercond. 25 2100405
[26] Vardulakis G 2007 Superconducting kinetic inductance detectors: theory, simulations, and experiments PhD Thesis University of Cambridge
[27] Monfardini A et al 2011 Astrophys. J. Suppl. Ser. 194 24
[28] Kuprianov M Y and Lukichev V F 1988 Sov. Phys.—JETP 67 1163
[29] Nam S B 1967 Phys. Rev. 156 487
[30] Mattis D and Bardeen J 1958 Phys. Rev. 111 412
[31] Kerr A R 1999 Surface impedance of superconductors and normal conductors in EM simulators MMA Memo 21 1–17
[32] Chang K 1994 Microwave Solid-state Circuits and Applications (New York: Wiley)
[33] Golubov A A et al 1994 Phys. Rev. B 49 12953
[34] Kaplan S B et al 1976 Phys. Rev. B 14 4854
[35] Chang J-J and Scalapino D J 1978 J. Low Temp. Phys. 31 1
[36] Gladstone G, Jensen M A and Schrieffer J R 1969 Superconductivity in the transition metals: theory and measurement *Superconductivity* ed R D Parks (New York: Marcel Dekker) pp 665–816

[37] Fujii G et al 2012 *J. Low Temp. Phys.* 167 815

[38] Parlato L et al 2005 *Supercond. Sci. Technol.* 18 1244

[39] Fukuda D et al 2007 *IEEE Trans. Appl. Supercond.* 17 259

[40] Reale C 1973 *Rev. Bras. Fis.* 3 431

[41] de Visser P J et al 2014 *Phys. Rev. Lett.* 112 047004

[42] Rothwarf A and Taylor B N 1967 *Phys. Rev. Lett.* 19 27