Baryon moments in a QCD-based model

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We have analyzed the theory of the baryon magnetic moments in the (approximate) QCD setting suggested by Brambilla et al. By modifying their derivation of the $qqq$ interaction and wave equation for baryons, we derived expressions for the baryon moments in terms of the underlying quark moments, including the first corrections associated with the binding of the quarks in baryons. The results, which hold in the “quenched approximation” in which the contributions of virtual quark pairs are neglected, fail to describe the measured moments, with typical errors on the order of 7%. We conclude from our analysis that the quenched approximation is at fault, and that the baryon moments give a sensitive test of that standard approximation in the lattice and analytic approaches to QCD.

1 Introduction

The magnetic moments of the stable baryons are very well measured, and their theoretical calculation gives a sensitive test of our understanding of baryon structure in quantum chromodynamics (QCD). Although the basic pattern and approximate magnitudes of the moments can be explained using the nonrelativistic quark model, the deviation of the moments from the quark-model pattern has not been explained dynamically despite many attempts. We discuss that problem here from the point of view of nonperturbative QCD. In particular, we have analyzed the theory of the moments in the approximate QCD setting developed in the work of Brambilla et al. [1], who used a Wilson-loop approach to study the $qqq$ bound state problem. By modifying their derivation of the $qqq$ interaction and wave equation, we have derived expressions for the baryon moments in terms of the underlying quark moments, including the first corrections associated with the binding of the quarks in baryons. Our results hold in the “quenched” approximation in which there are no internal quark loops embedded in the world sheet swept out by the Wilson lines joining the valence quarks, and no pairs associated with the valence lines. The corrections to the moments are

* Presented by L. Durand at the Como Conference on Quark Confinement and the Hadron Spectrum II, June, 1996
of relative order $\langle V \rangle / m_q$, where $V$ is a typical component of the binding potential and $m_q$ is a constituent quark mass, and are potentially large enough to explain the deviations of the measured moments from the quark-model values.

To test the theory, we have constructed variational wave functions for the baryons using the interactions derived by Brambilla et al. \cite{1}, including all spin and orbital configurations possible for $J^P = \frac{1}{2}^+$ and internal and total orbital angular momenta $L \leq 2$, and used them to calculate the moments. The effects of excited orbital contributions to the moments are negligible, as expected. The new contributions tend to cancel, and the remnants do not have the correct pattern to explain the discrepancies between theory and experiment.

The approximations underlying the Wilson-line analysis are known. Since of these, only the quenched approximation has a direct effect on the spin dependent terms with which we are dealing, we conclude that the use of the quenched approximation is responsible for the deviations of theory from experiment, and that the moment problem provides a sensitive test of this standard approximation in lattice and analytic QCD.

In a world-sheet picture, the inclusion of internal quark loops which describe meson emission and absorption by the baryon would allow the introduction of internal orbital angular momentum and spin, and would affect the moments. We are now investigating meson-loop contributions using approximations suggested by the world-sheet picture and chiral perturbation theory. In the following sections, we sketch our derivations and calculations, and justify our conclusions. A more detailed discussion will be given elsewhere.

2 Baryon moments in QCD

2.1 The problem

The simple, nonrelativistic quark model gives a qualitatively good description of the baryon moments. Under the assumption that each baryon is composed of three valence or constituent quarks in a state with all internal orbital angular momenta equal to zero, the moments are given by expectation values of the spin moment operators

$$\mu_{QM} = \sum_q \mu_q \sigma_q,$$  \hspace{1cm} (1)

leading to the standard expressions

$$\mu_p = \frac{1}{3}(4\mu_u - \mu_d), \ldots, \quad \mu_q = \frac{e_q}{2m_q}.$$  \hspace{1cm} (2)

The masses in the quark moments are clearly effective masses, and can be treated as free parameters in attempting to fit the data. A least-squares fit to the measured octet moments, taken with equal weights, is shown in Table 1. The moments are given in nuclear magnetons (nm). The pattern of the signs of
the quark model moments agrees with observation, while the root-mean-square
deviation of theory from experiment is 0.14 nuclear magnetons, or about 9%
of the average magnitudes of the moments. Agreement at this level can be
regarded as an outstanding success of the quark model, but the deviations also
give a very sensitive test of baryon structure: there is presently no completely
successful first-principles theory of the moments.

Table 1: Quark model fit to the magnetic moments in the baryon octet. All
moments are given in nuclear magnetons.

| Baryon | Experiment | Quark Model | $\Delta \mu$ |
|--------|------------|-------------|-------------|
| $p$    | 2.793      | 2.728       | 0.065       |
| $n$    | -1.913     | -1.819      | -0.094      |
| $\Sigma^+$ | 2.458 ± 0.010 | 2.639       | -0.181      |
| $\Sigma^-$ | -1.160 ± 0.025 | -0.999      | -0.161      |
| $|\Sigma^0 \to \Lambda\gamma|$ | 1.61 ± 0.08 | 1.575       | -0.03       |
| $\Lambda$ | -0.613 ± 0.004 | -0.642      | 0.029       |
| $\Xi^0$ | -1.250 ± 0.014 | -1.462      | 0.212       |
| $\Xi^-$ | -0.651 ± 0.025 | -0.553      | -0.098      |

2.2 The Wilson-loop approach to the baryon moments

Our approach to the baryon moment problem is based on the work of Brambilla
et al. [1], who derived the interaction potential and wave equation for the
valence quarks in a baryon from QCD using a Wilson-line construction. The
basic idea is to construct a Green’s function for the propagation of a gauge-
invariant combination of quarks joined by path ordered Wilson-line factors

$$U = P \exp \left( ig \int A_g \cdot dx \right), \quad (3)$$

where $A_g$ is the color gauge field. The Wilson lines sweep out a three-sheeted
world sheet of the form shown in Fig. 1 as the quarks move from their initial to
their final configurations.

By making an expansion in powers of $1/m_q$ and considering only forward
propagation of the quarks in time using the Foldy-Wouthuysen approximation,
Brambilla et al. are able to derive a Hamiltonian and Schrödinger equation
for the quarks, with an interaction which involves an average over the gauge
field. That average is performed using the minimal surface approximation in
which fluctuations in the world sheet are ignored, and the geometry is chosen
to minimize the total area of the world sheet per Wilson, subject to the motion
of the quarks. The short-distance QCD interactions are taken into account
explicitly.
The result of this construction is an effective Hamiltonian to be used in a semirelativistic Schrödinger equation $H \Psi = e \Psi$,

$$
H = \sum_i \sqrt{p_i^2 + m_i^2} + \sigma(r_1 + r_2 + r_3) - \sum_{i<j} \frac{2 \alpha_s}{3 r_{ij}}
$$

$$
- \frac{1}{2m_1 r_1} s_1 \cdot (r_1 \times p_1) + \frac{1}{3m_1} s_1 \cdot [(r_{12} \times p_1) \frac{\alpha_s}{r_{12}^2} + (r_{13} \times p_1) \frac{\alpha_s}{r_{13}^2}]
$$

$$
- \frac{2}{3} \frac{1}{m_1 m_2 r_{12}^2} s_1 \cdot r_{12} \times p_2 - \frac{1}{3} \frac{1}{m_1 m_2 r_{13}^2} s_1 \cdot r_{13} \times p_3 + \cdots.
$$

Here $r_{ij} = x_i - x_j$ is the separation of quarks $i$ and $j$, $r_i$ is the distance of quark $i$ from point at which the sum $r_1 + r_2 + r_3$ is minimized, and $p_i$ and $s_i$ are the momentum and spin operators for quark $i$. The parameter $\sigma$ is a “string tension” which specifies the strength of the long range confining interaction, and $\alpha_s$ is the strong coupling. The terms hidden in the ellipsis include tensor and spin-singlet interactions which will not play a role in the analysis of corrections to the moment operator, and the terms that result from permutations of the particle labels. The full Hamiltonian is given in [1]. This Hamiltonian, including the terms omitted here, gives a good description of the baryon spectrum as shown by Carlson, Kogut, and Pandharipande [2] and by Capstick and Isgur [3], who proposed it on the basis of reasonable physical arguments, but did not give formal derivations from QCD.

The presence of the quark momenta $p_i$ in the Thomas-type spin-dependent interactions in Eq. (4) suggests that new contributions to the magnetic moment operator could arise in a complete theory through the minimal substitution

$$
p_i \rightarrow p_i - e_i A_{em}(x_i),
$$

Figure 1: World sheet picture for the structure of a baryon.
with \( A_{em}(x_i) \) the electromagnetic vector potential associated with an external magnetic field. However, this point is obscured in the work of Brambilla \textit{et al.} by the transformations they make to express the equations for the Green's function in Wilson-loop form. We have therefore repeated their derivation of the valence-quark Hamiltonian in Eq. (4), replacing the SU(3)_c color gauge field \( gA_g \) by the full SU(3)_c \times U(1)_{em} gauge field \( gA_g + e_q A_{em} \) at the beginning. By then reorganizing the calculation of the three-quark Hamiltonian to keep \( e_q A_{em} \) explicit throughout, and then expanding to first order in \( A_{em} \) with

\[
A_{em} = \frac{1}{2} B \times x_q, \quad B = \text{constant},
\]

we can identify the modified magnetic moment operator through the relation

\[
\Delta H = -\mu \cdot B.
\]

The new moment operator, \( \mu = \mu_{QM} + \Delta \mu \), involves the leading corrections to the quark-model operator associated with the binding interactions. \( \Delta \mu \) can, in fact, be read off from the terms in Eq. (4) which depend on both the quark spins and momenta by making the minimal substitution in Eq. (5). For example, the term which depends on \( s_1 \cdot r_{12} \times p_1 \) gives an extra contribution

\[
\frac{\epsilon_1}{6m_1^2} x_1 \times (s_1 \times r_{12}) \frac{\alpha_s}{r_{12}}
\]

to \( \mu_1 \). There are also possible orbital contributions to the moments because the Hamiltonian mixes states with nonzero orbital angular momenta with the ground state. These have the standard form to the accuracy we need.

The baryon moments are now given by expressions of the form

\[
\mu = \sum_{i=1}^{3} \mu_i \langle \sigma_{i,z} \rangle (1 + \delta_i) + \sum_{i=1}^{3} \mu_i \langle L_{i,z} \rangle,
\]

where we have quantized along \( B \), taken along the \( z \) axis. The expectation values are to be calculated in the baryon ground states. The correction terms \( \delta_i \) from the new operators are given for the \( L = 0 \) baryons other than the \( \Lambda \) by

\[
\begin{align*}
\delta_i &= \frac{3\epsilon_0 + \epsilon_1}{2m_1} + \frac{\epsilon_2}{\epsilon_1 m_3} - \frac{\Delta_0 + \Delta_1}{2m_1}, \quad i = 1, 2, \\
\delta_3 &= \frac{\epsilon_2}{m_3} + \frac{2\epsilon_1}{\epsilon_3 m_1} - \frac{\Delta_2}{m_3},
\end{align*}
\]

where the \( \epsilon \)'s and \( \Delta \)'s are ground state radial matrix elements,

\[
\begin{align*}
\epsilon_0 &= \left\langle \frac{2\alpha_s}{3} \right\rangle, \quad \epsilon_1 = \left\langle \frac{2}{3} \alpha_s r_{23} \cdot z_2 \right\rangle, \quad \epsilon_2 = \left\langle \frac{2}{3} \alpha_s r_{31} \cdot z_3 \right\rangle, \\
\Delta_0 &= \left\langle \frac{\sigma r_{12}}{12} \right\rangle, \quad \Delta_1 = \left\langle \frac{\sigma r_{23} \cdot z_2}{6r_{23}} \right\rangle, \quad \Delta_2 = \left\langle \frac{\sigma r_{31} \cdot z_3}{6r_{31}} \right\rangle.
\end{align*}
\]
The identical quarks in these baryons are labelled 1 and 2, the unlike quark, 3. In writing these results, we have made the approximation $r_1 + r_2 + r_3 \approx \frac{1}{3}(r_{12} + r_{23} + r_{31})$, known to be reasonably accurate for the ground state baryons [2], and used the corresponding Thomas spin interaction. The result for the $\Lambda$ is similar.

### 2.3 Tests of the model

Rough estimates of the matrix elements above suggest that $\epsilon_i/m_l \approx 0.05$ and that $\Delta_i/m_l \approx 0.3$ for the light-quark masses used in Refs. 2 and 3. Binding effects are therefore potentially large, and are different for different quarks and baryons. To obtain a quantitative test of these effects and reliable estimates of the orbital contributions to the moments, expected to be small, we have performed a detailed analysis of the baryon wavefunctions using the Hamiltonian given in [1]. We use Jacobi-type internal coordinates $\rho, \lambda$, with $\rho = x_1 - x_2$, and $\lambda = R_{12} - x_3$, where $R_{ij}$ is the coordinate of the center of mass of quarks $i$ and $j$. The most general $j = \frac{1}{2}^+$ baryon wave function for quarks 1 and 2 identical and orbital angular momenta $L_\rho, L_\lambda, L \leq 2$ has the form

$$\psi_{1,2,m} = \left[ a_0 \psi_0 \mathbf{1} + ib_0 \psi_b (\sigma_1 - \sigma_2) \cdot \rho \times \lambda + c_0 \psi_c t_{12}(\rho) + d_0 \psi_d (\sigma_1 - \sigma_2) \cdot \sigma_3 \rho \cdot \lambda \right] \chi_{l,m}^{S_{12}=1}.$$  \hspace{1cm} (12)

Here $t_{12}$ is the usual tensor operator

$$t_{12}(\mathbf{x}) = 3\sigma_1 \cdot \mathbf{x} \sigma_2 \cdot \mathbf{x} - \sigma_1 \cdot \sigma_2 \mathbf{x}^2,$$  \hspace{1cm} (13)

and $\chi_{l,m}^{S_{12}=1}$ is the standard three-particle spinor for $S_{12} = 1, j = \frac{1}{2}, j_3 = m$. The scalar functions $\psi_i = \psi_i(\rho^2, \lambda^2)$ are normalized together with the accompanying spin operators. The constant coefficients $a_0, \ldots, e_0$, normalized to unity, give the fractions of the various component states in $\psi$. With this form of the wave function, we can use trace methods for the spins to calculate such quantities as $\langle H \rangle$ and $\langle \mu \rangle$.

We have made variational calculations of the energies and approximate wave functions of the ground state baryons and their first excited states using the Hamiltonian in [1]. The information on the excited states allows us to calculate the coefficients $b_0, \ldots, e_0$ perturbatively. While the Hamiltonian does mix orbitally excited states into the $L_\rho = L_\lambda = L = 0$ quark-model ground state, the coefficients are very small, ranging from essentially zero to about 0.02 depending on the baryon. Because these coefficients only appear quadratically in the baryon moments, the orbital contributions to the moments are completely negligible.

The radial matrix elements $\epsilon_i$ and $\Delta_i$ defined in Eq. (11) are significant, with the $\epsilon$’s ranging from 10 to 20 MeV, and the $\Delta$’s from 40 to 70 MeV. A new fit to the moment data using the expressions in Eqs. (11) and (12), with the quark
masses allowed to vary, gives a small improvement in the fitted moments, with
the root-mean-square deviation from the measured moments decreasing from
0.14 nm for the quark model to 0.10 nm. A secondary effect of the corrections
is to change the fitted quark masses or moments significantly, with the effective
quark masses decreasing, or the moments increasing.

We have concluded from this exercise first, that the finer details of baryon
structure are not yet described correctly in the present QCD-based model, and
second, that the baryon magnetic moments provide a very sensitive check on
the theory.

3 Possible improvements in the theory

We begin the discussion of possible improvements in the theory of the moments
by recalling the approximations used in the construction given by Brambilla et al. [1] and in our derivation of the moment operators:

(i) the $1/m_q$ expansion;

(ii) the minimal surface approximation;

(iii) forward propagation of the quarks in time;

(iv) and the quenched approximation.

Of these approximations, only (iii) and (iv) are likely to affect the moments
significantly.

The $1/m_q$ expansion is to be interpreted as an expansion in constituent
quark masses. It can be resummed in the kinetic terms in the Hamiltonian as in
Eq. (4), and leads elsewhere to the appearance of effective inverse masses $1/m_i$
which need not be the same as the kinematic masses, but represent averages of
quantities such as $1/E_i = 1/\sqrt{p_i^2 + m_i^2}$. Since there is no new spin dependence
involved, and we have treated the quark masses as free parameters in fitting the
moments, relativistic corrections of this sort are unlikely to change our results.

The average over the color gauge field $A_g$ in the minimal surface approxi-
imation neglects fluctuations, and minimizes the surface energy of the world
sheet to obtain the approximate Hamiltonian. Since the color fields do not
carry charge, they do not contribute internal currents in the baryon, and an
improved treatment of the averaging would presumably not change the moment
operator directly. While it could change the functional form of the Hamiltonian
somewhat, the quantitative success of the model in describing baryon spectra
suggests that the changes would not be large, and their effect on the moments
through the $\epsilon$ and $\Delta$ parameters would be minimal.

In contrast, the remaining two approximations affect the moment operator
directly. Internal quark loops can contribute circulating currents in the baryon.
These are omitted in the quenched approximation. In addition, the forward
propagation of the quarks in time inherent in the use of the Foldy-Wouthuysen approximation eliminates quark pair effects connected with the valence lines in Fig. 1. We believe that this general quenched picture underlies the difficulties with the model, and that it will be necessary to include pair effects to obtain a precise dynamical description of the moments.

One effect of internal quark loops can be seen in Fig. 2. In this figure, we suppose that a quark loop is embedded in the minimal world sheet of the baryon. The effect is to give a meson state and a new baryon in a world-sheet picture of meson emission and absorption. An external magnetic field interacting with the system will see a meson current as well as the baryon current, and the moment will be modified. Meson currents were invoked in the past in attempts to explain the full anomalous moments of the nucleons. Here we are only concerned with presumably small corrections to the quark-model moments.

One approach to the calculation of meson loop effects is through chiral perturbation theory. This has a long history, and has been studied recently in the context of baryon moments by a number of authors [4, 5, 6]. The relevant diagrams for the interaction of the baryons with the pseudo Goldstone bosons of the chiral theory are shown in Fig. 3. The work of Jenkins et al. [4] uses heavy baryon chiral perturbation theory [7], and studies the effect on the moments of the “nonanalytic terms” in the symmetry breaking parameter $m_s$. The

![Figure 2: Quark loops embedded in the world sheet give meson-baryon states.](image)

![Figure 3: Goldstone-boson diagrams which contribution to the magnetic moments.](image)
analytic terms in $m_s$ are ambiguous because the inevitable appearance of new
couplings at each order in the chiral expansion, and are ignored. The results
of Jenkins et al. are not encouraging as they stand, but we have found errors
in some of the coupling factors given in the published paper. Luty et al. concentrate on a simultaneous expansion in $1/N_c$ and $m_s$, and obtain interesting
sum rules for the moments but little dynamical information. Finally, Bos et al. consider the moments from the point of view of flavor SU(3) breaking in
the baryon octet using a different chiral counting than that usually used, and
obtain a very successful parametrization for the moments. However, this model
is again nondynamical, and has seven parameters to describe eight measured
moments. In fact, a fundamental problem with the chiral expansion from our
perspective is that it simply parametrizes the moments with an expansion consistent with QCD, but does not control the higher order terms in what appears
to be a slowly convergent series.

We are presently investigating meson loop effects using a somewhat different
approach suggested by the world-sheet picture. The baryon appears in this
picture as an extended object which must absorb the recoil momentum in the
emission of a meson. We would therefore expect wave function effects (form
factors) to be important for high meson momenta, and to supply a natural
cutoff for loop graphs. The wave function appears naturally when the process
is viewed using “old fashioned” instead of Feynman perturbation theory, as
indicated in Fig. energy denominators and vertex functions combine to give the $B'M$ component of the wave function for a baryon $B$, or, with different
time ordering, the $BM$ component of the wave function of $B'$. Since the wave
functions are expected to be fairly soft, with characteristic momenta below the
chiral cutoff of $\sim 1$ GeV, they can be expected to combine a number of terms in
the chiral expansion in a way that is dynamically accessible through approximate
models derived from QCD such as that of Brambilla et al. The non-point
character of the meson-baryon vertex in spacetime indicated in Fig. contrasts
sharply with the point vertex used in chiral perturbation theory, Fig. and is

Figure 4: (a) Appearance of Bethe-Salpeter wave functions in the diagrams. (b) Schematic reminder of the distributed nature of the meson-baryon vertex.
also likely to play a role. We have reached the same conclusions by studying the exact sideways dispersion relations for the moments given by Bincer [8]. The problem there is in extracting the quark-model moments.

We note finally that a different aspect of symmetry breaking, the suppression of strange-quark loops through mass effects, appears as a natural possibility in a world-sheet picture. Note that this is not the same as the suppression of kaon loops considered by other authors; see, e.g., [4] and the references given there.

4 Conclusions

On the basis of the work sketched above, we have concluded that the magnetic moments of the baryons give a sensitive test of baryon structure and of approximations in QCD. In particular, the quenched approximation, while reasonably successful when used in the calculation of baryon and meson spectra, appears to fail for the moments. The accurate calculation of the moments by lattice methods will presumably involve going beyond that approximation, and will provide a precision test of the methods used.

We find also that the world-sheet picture of baryon structure gives useful insights into the moments problem, and provides a new point of view which could be developed further. Problems which need further study within this approach to QCD include the following:

(i) establishing the connection to, and the relevance of, the chiral limit;

(ii) the development of methods to incorporate loop effects which build in the extended structure of the baryons;

(iii) and the possible usefulness of string theory methods in the calculation of loop effects.

Acknowledgments

The authors have benefited from conversations with Dr. Nora Brambilla, and appreciate her interest in this work, and her organization of the Como Conference. This research was supported in part by the U.S. Department of Energy under Grant No. DE-FG02-95ER40896, and in part by the University of Wisconsin Graduate School with funds from the Wisconsin Alumni Research Foundation.

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