Exploring Hidden-charm and hidden-strange Hexaquarks states from Lattice QCD

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Based on five different ensembles of newly-generated (2+1)-flavor configurations with the pion mass around $m_\pi \approx (140-310)$ MeV, we present a lattice analysis of hidden-charm and hidden strange hexaquarks with the quark content $usc\bar{d}\bar{s}\bar{c}$. The correlation matrix of two types of operators with $I^{PC} = 0^{++}, 1^{--}$ and $1^{++}$ are simulated to extract the masses of hexaquark candidates which are then extrapolated to the physical pion mass and the continuum limit. Results indicate that masses of the ground states are below the $\Xi_c\Xi_c$ threshold and provide a characteristic signal for the experimental discovery of hexaquark candidates. This may enrich the versatile structures of multiquarks and is an indispensable step to decipher the nonperturbative nature of fundamental interactions of quarks and gluons.

Introduction: The spectrum of hadron excitations discovered at experimental facilities around the world manifests the fundamental interactions of elementary quarks and gluons, governed by the quantum gauge field theory of QCD. Understanding the complex emergent phenomena of this field theory has captivated the attention of theoretical physicists in the last decades. To date one of the perennial problems in hadron physics is to establish the existence of exotic hadrons that defy the quark-antiquark interpretation for mesons and three-quark assignment for baryons [15]. Candidates of such exotic hadrons, including tetraquark and pentaquark states, have been recently discovered and confirmed in various experimental measurements [18][25][33][35][56]. These experimental progress give us strong confidence for the existence of hexaquark states.

The proposition of six quarks as a single hadron structure was first presented in 1964 [26], and a renowned realization is deuteron. In addition, the $d^*(2380)$ resonance, reported by CELSIUS/WASA and WASA-at-COSY collaborations [27][31], is widely believed to be a dibaryon. Until now, a lot of explorations of hexaquarks with different quark constituents have been conducted in theory, such as heavy dibaryons (qqqqQQ) [32][33], doubly-heavy dibaryons (qqQQQQ) [34][35], fully light dibaryons [36][39], and full heavy dibaryons [40]. In view of the ab-initio framework–lattice QCD, the major challenges include not only the accurate extraction of the binding energy (E.g.,[41]), but also the complicated contraction involving many quarks with the same flavor.

On the other hand, hadrons composed of three quarks and three-anti-quarks belong to another category of hexaquarks. Other than light ones, hidden-charm and hidden-bottom hexaquarks are of special interest since heavy quarks have much larger masses and thus are more easily distinguished from ordinary mesons. Investigating this type of hexaquarks through lattice QCD is even harder than the dibaryons, due to the mixing with the three meson states.

In this work, we show a lattice QCD investigation of the hadronic structures containing three quarks and three antiquarks, both using the bayron-anti-bayron type and three-meson type interpolation fields with $I^{PC} = 1(0^{++}), 1^{++}, 0^{--}, 1^{--}$ and $1^{++}$. To minimize the impact of disconnected diagrams, we have opted for a quark composition of $usc\bar{d}\bar{s}\bar{c}$ in the case of isospin $I = 1$. In this case, the annihilation diagrams of charm and strange quarks may contribute, but their contribution is expected to be suppressed by the OZI rule and therefore are ignored in the first-step study. After making the chiral and continuum extrapolation, we find that the spectrum of hexaquarks which are below the $\Xi_c\Xi_c$ threshold, and two ground states are close to the three-meson thresholds. This feature is consistent with the result from the chromo-magnetic interaction model which also finds a large binding energy for the hexaquark states with certain quantum numbers [50], but quite different from the model predictions which focus on the near-threshold structure of $\Xi_c\Xi_c$ only. This interesting observation can be further examined by more theoretical analyses and validated by future experimental measurements.

Theoretical Framework: A most powerful method to systematically tackle the nonperturbative strong interaction is Lattice QCD [41], in which the quark and
ghon fields are discretized on a space-time grid of finite size, allowing numerical computation by averaging over large numbers of possible field configurations generated by Monte-Carlo. In particular, from the time dependence of correlation functions calculated in this way, one can determine a discrete spectrum of various hadrons. Thus Lattice QCD provides a first-principles technique for explorations of quantities of interest, such as spectrum, scattering phase, and radiative transitions.

To determine the mass spectrum, one firstly needs to construct appropriate interpolating operators with definite symmetries. We use the baryon-anti-baryon type (denoted by superscript A) and three-meson type (denoted by superscript B) interpolation fields to construct our correlation function matrix. Therefore, for the hexaquarks, one can construct the interpolating operators with quantum numbers $0^{++}, 0^{-+}, 1^{++}, 1^{--}$ as

\[ O^{A}(x) = e^{abc} e^{def} [u_{a}^{T} C \gamma_{5} s_{b}^{T}] \bar{d}_{d} C \gamma_{5} s_{c}^{T} \times [\bar{c}_{f} c_{e}] (x), \]
\[ O^{B}(x) = [\bar{c}_{f} c_{e}] (x) \times [\bar{c}_{f} c_{e}] (x), \]
\[ O^{A}_{t}(x) = e^{abc} e^{def} [u_{a}^{T} C \gamma_{5} s_{b}^{T}] \bar{d}_{d} C \gamma_{5} s_{c}^{T} \times [\bar{c}_{f} c_{e}] (x), \]
\[ O^{B}_{t}(x) = [\bar{c}_{f} c_{e}] (x) \times [\bar{c}_{f} c_{e}] (x). \]

Here $C = i \gamma_{2} \gamma_{4}$ is the charge conjugation matrix and $a, ..., f$ are color indices. The operators at the source on the Coulomb gauge fixed configuration would be the following two kinds:

\[ O^{A}_{s}(t) = \sum_{y_{1}, i=1,6} e^{abc} e^{def} [u_{a}^{T} (y_{1}) C \gamma_{5} s_{b}^{T} (y_{2})] \times [\bar{c}_{f} c_{e}] (x), \]
\[ O^{B}_{s}(t) = [\bar{c}_{f} c_{e}] (x) \times [\bar{c}_{f} c_{e}] (x). \]

where $\gamma_{x}$ can be $1, \gamma_{0}, \gamma_{1}, \gamma_{i}$ corresponding to quantum numbers $0^{++}, 0^{-+}, 1^{++}, 1^{--}$ and all six positions are integrated separately, as we are using the Coulomb wall source. The operators at the sink are using only one position integration. It should be noticed that there are various potential operators that can be used, and a comprehensive treatment should take into account all these operators, and simulate the corresponding correlation functions. In this work, we have opted for baryon-anti-baryon interpolating operators in the form of $\Xi^{+} \Xi_{c}^{-}$, and include the corresponding three-meson interpolating operators represented by $KK_{c}$ of quantum numbers $0^{+}+(KK_{c})/\psi$ of quantum numbers $1^{+-}$ and so on which might give large contributions to the correlation functions.

Then the determination of the mass spectrum proceeds from the calculation of correlation functions matrices between this operator and its hermitian conjugate at Euclidean times $t$ and $0$ of the form

\[ C_{i}^{\alpha\beta}(t) = \langle 0 | O_{i}^{\alpha}(t) C_{i}^{\beta}(0) | 0 \rangle, \]

where $i$ labels operators with the four different quantum numbers, and $\alpha, \beta$ can be either $A$ or $B$. For each quantum number, we evaluate a $2 \times 2$ correlation matrix and then we solve the equation for the generalized eigenvalue problem (GEVP)

\[ C(t) v_{n}(t, t_{0}) = \lambda(t) v_{n}(t, t_{0}), \]

where $t_{0}$ is a reference time slice, $\lambda$ is the eigenvalue of the matrix $C(t_{0})^{-1}C(t)$ and $v_{n}$ being the eigenvectors correspondingly. Normally one chooses $t_{0}$ large enough and the signal is good and stable. The parameter $t_{0}$ is tunable and one could optimize the calculation by choosing $t_{0}$ such that the correlation matrix is dominated by the desired eigenvalues at that particular $t_{0}$ (preferring a larger $t_{0}$) with an acceptable signal-to-noise ratio (preferring a smaller $t_{0}$). The energy eigenvalues for the system are then obtained by diagonalizing the matrix $C(t_{0})^{-1}C(t)$ or $(C(t)C(t_{0})^{-1})$. The eigenvalues of the matrix have the usual exponential decay behavior as described by Eq. (6) and therefore the exact energy $E_{n}$ can be extracted from the effective mass plateau of the eigenvalue $\lambda_{n}$.

\[ \lambda_{n}(t, t_{0}) = e^{-E_{n}(t-t_{0})} + \mathcal{O}(e^{-|\delta E|(t-t_{0})}) \]

where $\delta E$ is the energy gap between $E_{n+1}$ and $E_{n}$. Including only correlation functions projected to zero momentum, we have $E_{n} = M_{n}$, which yields the ground state. The effective masses can be obtained from two-state fits of the eigenvalues or the plateau fit of the effective masses.

**Lattice Simulation:** We employ $(2+1)$-flavor Wilson clover fermion gauge configurations generated with the lattice spacings $a = 0.054$fm, $0.080$fm, $0.108$fm. A first analysis of $\Xi_{c} \rightarrow \Xi$ form factors using two ensembles (C08P30S and C11P29S) has been conducted in Ref. [48], and predictions on partial widths for semileptonic $\Xi_{c}$ decays were used in the experimental background simulation by Belle collaboration [49]. Tab. I shows the parameters of these configurations. The pion masses and the lattice spacings are given in units of MeV and fm, respectively. The bare strange quark mass is determined such that the mass of $\eta_{s}$ is around 700 MeV [57, 58], and the bare charm quark mass is tuned to accommodate the spin-average value of the $J/\psi$ and $\eta_{c}$ masses.
where $m_{\pi}$ and $J/\psi$ mesons are at different momenta, and obtained the energy from two-state fits of the correlations. Some of the results are shown in Fig. 1. The upper panel of Fig. 1 shows the dispersion relation for $\pi$ on the ensemble C06P30S where six momenta are chosen. Then the dispersion relation is parametrized as

$$E^2 = m^2 + c_2 p^2 + c_3 p^4 a^2,$$

where $c_2$ and $c_3$ are parameters to be extracted through a fit. Deviations of $c_2$ from unity and $c_3$ from zero characterize the discretization errors. As one can see from Fig. 1, all lattice results can be well described by Eq. 9 with a reasonable $\chi^2/d.o.f$. The results $c_{2,\pi} = 1.123(69)$, $c_{2,\eta} = 1.064(47)$, $c_{2,J/\psi} = 0.954(45)$ are consistent with the square of the speed of light, while the $c_3$ parameters are all close to 0. From the results, one can notice that the dispersion relations for the ordinary mesons with $u/d, s, c$ quarks are well-respected on these lattice configurations after the continuum extrapolation.

**Numerical Results for Hexaquarks:** The focus of this work is the states with $I = 1, J^{PC} = 0^{+}, 0^{++}, 1^{--}, 1^{++}$. In the calculation of the two-point correlation functions matrix in Eq. (4), we have used $399 \times 20, 451 \times 48, 203 \times 48, 653 \times 40$ and $136 \times 80$ “configurations×loop-t” for C11P29S, C11P22M, C11P14L, C08P30S and C06P30S ensembles, respectively. To extract the mass for hexaquark states, we adopt the two-state parametrization for the eigenvalues obtained through diagonalizing the $2 \times 2$ matrix element:

$$\lambda(t, t_0) = e^{-M_H(t-t_0)}[1 + \Delta c \times e^{-\Delta E(t-t_0)}].$$

where $\Delta c, M_H$ and $\Delta E$ are parameters to be determined through a correlated fit of the lattice data. $M_H$ is the lowest-lying hexaquark state and $\Delta E$ corresponds to the relative mass gap of the excited state. It is necessary to stress that the $\Delta E$ is an effective energy reflecting the contributions from all possible higher states.

| Hadron | $K$ | $D$ | $D_s$ | $\Lambda$ |
|--------|-----|-----|-------|----------|
| Lattice | 0.4869(41) | 1.867(576) | 1.9766(65) | 1.074(48) |
| Exp. | 0.4937/0.4976 | 1.864/1.870 | 1.968 | 1.115 |
| Hadron | $\Xi$ | $\Omega$ | $\Lambda_c$ | $\Xi_c$ |
| Lattice | 1.354(22) | 1.699(42) | 2.348(59) | 2.438(68) |
| Exp. | 1.314 | 1.672 | 2.286 | 2.468 |
| Hadron | $\eta_c$ | $J/\psi$ |
| Lattice | 3.0041(20) | 3.097(24) |
| Exp. | 2.9839 | 3.0969 |

**TABLE I.** Parameters of the 2+1 flavor Wilson clover fermion ensembles used in this calculation. The pion masses and the lattice spacings are given in units of MeV, and fm, respectively.

| Ensemble | $\beta$ | $L^3 \times T$ | $a$ | $\kappa_l$ | $m_{\pi}$ | $\kappa_s$ | $\kappa_c$ |
|----------|--------|----------------|-----|---------|---------|---------|---------|
| C11P29S | 6.2 | $24^3 \times 72$ | 0.108 | 0.1343 | 284(2) | 0.1327 | 0.1117 |
| C11P22M | 6.2 | $32^3 \times 64$ | 0.108 | 0.1344 | 221.8(7) | 0.1326 | 0.1116 |
| C11P14L | 6.2 | $48^3 \times 96$ | 0.108 | 0.1345 | 135.1(6) | 0.1327 | 0.1116 |
| C08P30S | 6.41 | $32^3 \times 96$ | 0.080 | 0.1326 | 296.8(9) | 0.1316 | 0.1181 |
| C06P30S | 6.72 | $48^3 \times 144$ | 0.054 | 0.1311 | 312(1) | 0.1305 | 0.1227 |

In addition, we explore the mass-energy dispersion relations for the ordinary mesons to ensure that all the lattice discretization errors are under control. We have generated the two-point correlation functions for the $\pi$, $\eta$, and $J/\psi$ mesons at different momenta, and obtained the energy from two-state fits of the correlations.

**TABLE II.** Mass (in the unit of GeV) for the ordinary mesons. With the five ensembles of configurations, we have extracted the mass from the analysis of two-point correlation functions and extrapolated to the continuum and physical pion mass limit, and the errors are statistical. The experimental data are taken from Particle Data Group [47].
Results with the five ensembles for the masses are collected in Fig. 3. To accommodate the effects caused by the nonphysical pion mass and discretization, we also perform a simultaneous extrapolation of the masses for $m_{\pi}$ and lattice spacing $a$ using the parametrization as Eq. (7). The fit plots are shown in Fig. 4, with a reasonable $\chi^2$ and the final results of the hexaquark states at the physical pion mass are shown in Tab. III.

![Image](image-url)

**FIG. 1.** The dispersion relation for $\pi$ on the ensemble C06P30S (upper panel) and for $J/\psi$ on the three ensembles C11P29S, C08P30S, and C06P30S (lower panel).

| $I(J^{PC})$ | $1(0^{-+})$ | $1(1^{--})$ | $1(0^{++})$ | $1(1^{++})$ |
|-------------|-------------|-------------|-------------|-------------|
| mass(GeV)   | 3.865(44)   | 3.960(49)   | 4.12(13)    | 4.273(95)   |

**TABLE III.** The obtained hexaquark masses (in units of GeV) for the ground states at the physical pion mass after the chiral and continuum extrapolation.

*Discussions:* Our final results of the hexaquarks states with quantum numbers $0^{-+}, 1^{--}, 0^{++}, 1^{++}$ at the physical $m_{\pi}$ and at continuum limit are shown in the following Tab. III. It is interesting to notice that the threshold of $\Xi^{+}_{u} - \Xi^{-}_{u}$ is about 4.938 GeV [47], while the results for the hexaquark states are below this threshold by around 700 to 1000 MeV. This feature is consistent with the result from the chromo-magnetic interaction model which also finds a large binding energy for the hexaquark states with certain quantum numbers [50]. But it should be noticed that a more conclusive statement requests more studies by including more channel effects. In addition the experimental mass summation of three mesons $K^{+}, K^{-}$ and $\eta_{c}$ is 3.971 GeV [47], and according to Tab. III the mass for the $1(0^{++})$ state is slightly below this threshold. We note that the threshold of three mesons $\pi^{+}, \eta$ and $\eta_{c}$ is 3.667 GeV and the one of three mesons $\pi^{+}, \eta'$ and $\eta_{c}$ is 4.077 GeV. The mass for the $1(0^{++})$ state is above the threshold of three mesons $\pi^{+}, \eta$ and $\eta_{c}$, it is also likely that the lowest state of hexaquark shows signals of a three-meson state $\pi\eta\eta_c$ with relative nonzero angular momenta. Since in this analysis the disconnected diagrams are not considered, the constituent $\bar{u}u/dd$ of $\eta(\eta')$ states within the spectrum is likely to be suppressed. As a result, the particle combination of the lowest state with $J^{PC} = 0^{+-}$ might be $\pi\eta\eta_c$, since the unphysical $\eta_c$ dominates the contribution to the connected strange quark diagram in the pseudoscalar channel. An experimental search in this energy would be very helpful to clarify this finding. Theoretically more extensive studies on the hexaquark spectra by including more interpolating operators are required to further clarify the properties of the hexaquarks. We defer these studies to the future.

*Conclusions:* Based on the newly generated (2+1)-flavor configurations with the pion mass around $m_{\pi}\simeq 140-310$ MeV, we have presented a first lattice sim-
ulation of hidden-charm and hidden-strange hexaquark states with the quark content \( \bar{u}\bar{c}\bar{d}\bar{s}\bar{d}\bar{c} \). Four different quantum numbers are assigned for the hexaquarks and the corresponding mass spectrum is derived. We have also extrapolated the results both to the physical pion mass and continuum limit.

These results are helpful towards the search for such types of exotic states in the future experiments.

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FIG. 3. The obtained masses of different quantum numbers on the five ensembles. The horizontal axis \( 0^{-+}, 1^{--}, 0^{++}, 1^{++} \) are correspond to the corresponding ground states.

Appendix A: extrapolation of hadrons

In this appendix, we have collected the masses for the \( K, D, D_s, \Lambda, \Xi, \Omega, \Lambda_c, \Xi_c, \eta_c \) and \( J/\psi \) on the five ensembles. After the continuum and physical pion extrapolation, shown in Fig.5, the results for these hadrons are collected in Tab. II.
FIG. 4. The continuum and physical pion mass extrapolation for the mass of hexaquark states.

Appendix B: Effective mass on other ensembles

Effective masses for hexaquarks on the four different quantum numbers on C08P30S, C11P14L, C11P22M, and C11P29S are shown in Fig 6-9.

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FIG. 5. Extrapolation for the mass for the ordinary hadrons: $K$, $D$, $D_s$, $\Lambda$, $\Xi$, $\Omega$, $\Lambda_c$, $\Xi_c$, $\eta_c$ and $J/\psi$.
FIG. 6. Effective mass of the ground states for hexaquarks on the four different quantum numbers on different ensembles.
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