A Pricing Mechanism Which Implements Allocation in Shannon Formula of Home Cellular Wireless Communication

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Abstract. According to the requirements of resource management and interference suppression in-home cellular Wireless Communication, take the ratio of effective capacity to power consumption as the optimization objective, the Shannon formula was taken into account. A pricing mechanism that implements a rate allocation in Nash Equilibrium point in unicast provisioning was presented. The operation mechanism of this mechanism is described in detail, compared with previous similar mechanisms, the mechanism more able to meet the actual needs of communication.

1. Introduction

Wireless relay network has been taken as a solution to set up communications between disconnected users who are unable to share information due to the long distance or obstruction reliably. Instead, a processed version of the data sent from one or more transmitters to one or more receivers was forwarded by a relay. In this paper, an energy-efficient distributed relay networking structure where the spatial diversity of distributed multiple-input multiple-output systems is proposed. This distributed relay network consists of several single-antenna relay nodes which are spatially distributed to receive transmissions from many users through an uplink, while each of them performs a simple quantization operation to perform a quantized version of the combination of the received signals through a downlink to a receiver.

Current scheduling schemes in multi-relay wireless networks mainly focus on single-source wireless networks with the same link status. Furthermore, the sequential-forward scheduling scheme is usually used, and the transmission efficiency is comparatively low. To solve this problem, a priority scheduling scheme based on random linear network coding is proposed. In different transmission stages, the feedback information is generated according to the packets accepting state or the linear relation among the encoding vectors. The number of effective packets of the corresponding relay node is calculated. In the condition of different link status, the effective information of each relay node and the link transmission reliability is taken into consideration comprehensively to generate the priority index and complete scheduling. This scheme can realize cooperation transmission in multi-relays for multi-sources information. When the link status difference is noticeable, the optimal forwarding node and the path can be adaptively chosen to improve the information transmission efficiency. According to the simulation results, this scheme can effectively improve network throughput and reduce the number of retransmission compared with the traditional scheduling schemes for single-source wireless networks.
For balancing the energy consumption of nodes in wireless sensor networks, and prolonging the network lifetime, a WSN routing algorithm was proposed for the transmission of data between remote nodes. The minimum energy routing was selected according to Stojmenović's theory, the data transmission power between nodes could get a minimum value in some distance. In this distance, the nodes sent data directly, and if the distance between two nodes was over this distance, the node transmitted the data through the intermediate nodes. The minimum energy node for routing was selected to achieve the purpose of the energy balance of nodes. Most of the existing literature is concerning with the former theory—the realization theory and has formed many kinds of models. Most of them are inspired by Kelly [1]. The features of Kelly were illustrated as following: resources is assumed to be owned by the network, which are bundled together in certain form of services, and are allocated to the users by their own algorithm; the network sets up resource prices which generating service prices one by one; and each user claim his own service price which is used for calculating an allocation that would maximize his net utility. In [2,3], the cases were illustrated where the users behave strategically, the resource allocations calculated by Kelly's Mechanism performed a specific "efficiency loss." Particularly [3] given the evidence presented in the case where each user bids on individual network resources, a lower bound on the efficiency loss would be finally presented. Kelly's model has been proved to achieve the allocation in which the sum of the users' gross utility function was maximized over the Nash equilibrium messages [4,5,6]. Nevertheless, there lies a problem in Kelly that the Nash point can be reached only when all the users in the network follow the allocation given by the whole system and do not have a single mind to get more rate from the network. Generally speaking, there doesn’t exist the best mechanism which meets all these conditions, neither the allocation which maximizes the sum of the users' net utilities and satisfies the budget balance in dominant strategies. However, some mechanisms could still work functionally in the public goods economies, now called Groves-Ledyard mechanisms [7].

The rest of the paper is organized as follows: in Section II, the model of the network resource allocation problem is formulated. The tax function is briefly presented to reach the Nash Equilibrium of the network and prove that it meets some properties in Section III. In Section IV, some definitions are illustrated that the proofs in Section III are reasonable and effective.

2. Problem Formulation
Consider multiple source-destination pairs that exist in a network where many dedicated relays that can help forward the date for each source-destination pair. Here, the source-destination pair is defined as a user. We apply $M$ to indicate the set of user pairs, $N$ to indicate the set of relay stations, and $L$ to indicate the set of all the possible links in the network system. One relay can transmit data through multiple pairs under the relay's power constraint, and one user can apply many relays for its data transmission. Here we assume that the users' transmission rate is variable, and each user needs a least transmission rate to obtain his data transmission. The upper bound to the capacity of a link was calculated in Shannon's theory, in bits per second (bps), as a function of the available bandwidth and the signal-to-noise ratio. The theorem can be stated as:

$$C = B \times \log2(1 + S/N)$$

where $C$ is the achievable channel capacity, $B$ is the bandwidth of the line, $S$ is the average signal power, and $N$ is the average noise power.

The signal-to-noise ratio $(S/N)$ is usually expressed in decibels (dB) given by the formula: $10 \times \log10(S/N)$. So for example a signal-to-noise ratio of 1000 is commonly expressed as: $10 \times \log10(1000) = 30$ dB.

Here is a graph showing the relationship between $C/B$ and $S/N$ (in dB):

Let $P_m$ denote the trasmot power at source $m \in \{1,2,3 \ldots M\}$ and $G_{s_m d_m}$ the channel gain between the source to the destination for user $m$. Then the signal-to-noise ratio (SNR) that results from direct transmission from the source $k$ to its destination in one time slot can be expressed by
where $\sigma^2$ is the noise variance. The rate at the destination of direct transmission is

$$R_{sm.dm} = W \log_2 (1 + \Gamma_{sm.dm})$$

where $W$ is the bandwidth of the channel used by user $m$. Without loss of generality, assumptions can be made that the noises in different channels are different, and the stabilities of all the channels are guaranteed over time.

In the next time slot, assuming $X_{sm}$ is the broadcast signal from source $m$, the received signal at relay $n \in \{1, 2, 3 \ldots N\}$ from the source $m$ is

$$Y_{sm.r_n} = \sqrt{P_{sm} G_{sm.dm}} X_{sm} + \eta_{sm,r_n}$$

where $G_{sm.dm}$ is the channel gain between the source and the destination for user $m$, and $\eta_{sm,r_n} \sim N(0, \sigma^2)$. Relay $n$ amplifies $Y_{sm,r_n}$ and forwards the message to destination. The received signal is written as

$$Y_{r_n.dm} = \sqrt{P_{r_n.dm} G_{r_n.dm}} X_{r_n.dm} + \eta_{r_n.dm}$$

Where

$$X_{r_n.dm} = \frac{Y_{sm.r_n}}{|Y_{sm.r_n}|}$$

is the transmitted signal from relay $n$ to the destination $m$ which is normalized to have the unit energy, $P_{r_n.dm}$ is the transmit power at relay $n$ and $G_{r_n.dm}$ is the channel gain between relay $n$ to the destination of user $m$.

Substituting the expressions all above, is can be written as:

$$Y_{r_n.dm} = \frac{\sqrt{P_{r_n.dm} G_{r_n.dm}} \sqrt{P_{sm} G_{sm.dm}} X_{sm} + \eta_{sm.dm}}{\sqrt{P_{sm} G_{sm.dm} + \sigma^2}} + \eta_{r_n.dm}$$

Using it, the relay SNR for source $m$ helped by relay $n$ is given by

$$\Gamma_{sm.r_n.dm} = \frac{P_{sm} G_{sm.dm} G_{r_n.dm}}{\sigma^2 (P_{sm} G_{sm.dm} + P_{r_n.dm} G_{r_n.dm} + \sigma^2)}$$

The rate at the destination $m$ by maximal ratio combining with relay $n$ is

$$R_{sm.r_n.dm} = \frac{1}{2} W \log_2 (1 + \Gamma_{sm.dm} + \Gamma_{sm.r_n.dm})$$

If the user can receive many relays’ help at one slot, then the rate of user $m$ can be expressed as the following:

$$R_m = \frac{1}{2} W \log_2 (1 + \Gamma_{sm.dm} + \sum_{n \in S(m)} \Gamma_{sm.r_n.dm})$$

where 1/2 is based on the fact that cooperative transmission uses half of the resources.

The main problem we are solving is how to efficiently allocate the power at the relays to reach the maximum point while each user is believed to reach its “own” maximum point. The transmit power $P_{sm}$ is assumed to be a constant. We can formulate the problem obeying all the conditions beyond:

$$\max \sum_{m \in M} \ln(R_m - R_{m.min})$$
\[ R_m \geq R_{m,\text{min}}, \text{for all users} \]
\[ \sum_{m \in M} P_{r_n,d_m} \leq P_{r_n,d} \text{, for all relays } n \in N \]
\[ P_{r_n,d_m} \geq 0, \text{for all users} \]

\[ R_m \leq R_{s_m,r_n,d_m} = \frac{1}{2} W \log_2 \left( 1 + \Gamma_{s_m,d_m} + \sum_{n \in \mathcal{N}(k)} \Gamma_{s_m,r_n,d_m} \right), \text{for all users} \]

where \( R_{m,\text{min}} \) is the minimal rate for user \( m \). For each user, the allocated rate should be no less than \( R_{m,\text{min}} \) to guarantee its transmission. The second constraint indicates that the power relay allocates to user \( m \) should not be larger than the max power the relay can provide itself, which is a known constant. The last constraint comes from the expression of \( R_m \), and changed into inequality without loss of generality.

In the following of the passage, we will use \( P \) to refer to the problem constructed here.

### 3. A Pricing Mechanism in Practice

In this chapter, we will adapt the pricing mechanism into solving the previous problem. Here, we will use constant \( c \) to express \( P_{r_n,d} \) and \( x_i (i \in M) \) to express \( P_{r_n,d,n} \). Also, we would use \( l(n) \) to indicate the users that ask the \( n^{th} \) \( (n \in N) \) relay for helping to transmit the data. We should know here that for user \( i, P_{s_m} \) is a constant, and what the user can do is to broadcast his desired bandwidth he wants to get from the relay stations.

First, we consider the problem constructed beyond multi-links transmission.

1) **Pricing Mechanism**: Assume that the available bandwidth would be assigned to each one of the users to maximize their own individual revenue.

Consider the following pricing mechanism:

**Messages**: Each user service provider transmits a message \( (x_i, p_i) \) to the links, where \( x_i \) represents the desired amount of bandwidth, and \( p_i \) represents the valuation of each user of the “price” per unit per bandwidth.

Links provide a message \( (p_{-i}, d_i, \gamma) \) where

\[ p_{-i} = \frac{\sum_{j \in M} p_j}{N-1} \]  

is the average of the other users’ price per unit of bandwidth, \( \gamma \) is the positive constant, while

\[ d_i = \sum_{j \in M} (x_j) - c \]

is the excessed demand when user’s demand is eliminated.

User \( i \) just broadcast its desired bandwidth from the relay to its service provider and receives a message from its service provider.

**Allocation**: Given an equilibrium set of messages

\[ (x^*, p^*) \triangleq ((x_1^*, p_1^*), (x_2^*, p_2^*), (x_3^*, p_3^*) \ldots (x_N^*, p_N^*)) \]

each user \( i \) is allocated the requested bandwidth \( x_i^* \) and is taxed as follows:

\[ t_i(x^*, p^*) = \sum_{l \in \mathcal{N}(n)} \left( x_i^* - \frac{W_i c}{\sum_{l \in \mathcal{N}(n)} W_i} \times p_{-i} \right) + \left( \frac{p_l}{p_{-i}(x_i^* + c)} - \frac{\sum_{j \in \mathcal{N}(n)} (x_j^* - c)}{\gamma} \right) + x_i^*(c, \gamma) \]  

Where

\[ c_1 = \frac{W_l P_{s_m} G_{s_m,r_n}}{2 \ln(2) \sigma^2 (1 + \Gamma_{s_m,d_m} + \Gamma_{s_m,r_n,d_m})} \]

\[ c_2 = \frac{P_{s_m} G_{s_m,r_n}}{G_{s_m,d_m}}, \text{for user } m \in l(n) \]
\[ \chi^{(x^*,c,y)}_+ \simeq \left( \max \left\{ 0, \frac{\sum_{j \in M} (x_j - c)}{y} \right\} \right)^2 \]

Here, the expression of \( t_i(x^*, p^*) \) is a formula simply for the case of only one relay. Considering the multi-relays case, the taxed formula \( t_i(x^*, p^*) \) would change into the following:

\[
t_i(x^*, p^*) \simeq \left( x^*_i - \frac{W_i c}{\sum_{l \in (n)} W_l} \right) \times p_{-i} + \left( \frac{p_i c_1 c_2}{p_{-i} (x^*_i + c_2)^2} \frac{\sum_{j \in (l)} (x_j - c)}{y} + \chi^{(x^*,c,y)}_+ \right) \]

The meaning of the constants \( c_1, c_2 \) and the variable \( \chi^{(x^*,c,y)}_+ \) will be shown in the later sections.

2) Implementation: Here we will try to prove that even the users behave strategically in the network system using the taxed function mentioned above, the equilibrium allocation generated by the network system are still efficient.

**Theorem:** The mechanism illustrated in the section above is applied in Nash Equilibria problem \( P \).

**Proof:** The proof of this theorem is presented in the later sections.

In the following lemma, the Nash Equilibrium of the network system generates is this section satisfies voluntary participation property. In fact, it is proved that each user prefers taking part in setting the Nash Equilibrium allocations more than no participating in the allocation process.

**Lemma:** The mechanism presented in the section above meets the voluntary participation.

**Proof:** For each one for the user \( j \in M \), fix their demand bandwidth and price except user \( i \).

\[
\max_{x_i, p_i} u_i(x_i) - \left( x^*_i - \frac{W_i c}{\sum_{l \in (n)} W_l} \right) \times p_{-i} + \left( \frac{p_i c_1 c_2}{p_{-i} (x^*_i + c_2)^2} \frac{\sum_{j \in (l)} (x_j - c)}{y} + \chi^{(x^*,c,y)}_+ \right) \geq 0
\]

\[
\max_{p_i} u_i(0) - \left( 0 - \frac{W_i c}{\sum_{l \in (n)} W_l} \right) \times p_{-i} + \left( \frac{p_i c_1 c_2}{p_{-i} (x^*_i + c_2)^2} \frac{\sum_{j \in (l)} (x_j - c)}{y} + \chi^{(x^*,c,y)}_+ \right) \geq 0
\]

The series of inequalities above shows that for user \( i \in M \) in the system for any Nash Equilibrium the voluntary participation condition holds under the condition that for arbitrarily fixed demands and prices of the users in the system except the user \( i \).

**Discussion:**

In this section, a mechanism is proposed which implements a rate allocation to the problem \( P \) in Nash Equilibrium. In construction of the mechanism, the key point of it is the tax function which meets properties as follows:

1) For each user, he proposes the price only dependent on the message he receives from his service provider, and the price is actually the most efficient price he can propose to the system.

2) All the prices that each user in the system propose is just the allocation at the Nash Equilibrium dependent on the information on the last slot.

3) At equilibrium, the capacity is constrained.

4) At equilibrium, the allocation designed by the system is efficient.

Before proving whether the allocation satisfies all the properties above, here we will first analyse the component in tax function.

Note that:

\[
t_i(x, p) \simeq I + II
\]

Where

\[
I \simeq \left( x^*_i - \frac{W_i c}{\sum_{l \in (n)} W_l} \right) \times p_{-i}
\]

\[
II \simeq \frac{p_i c_1 c_2}{p_{-i} (x^*_i + c_2)^2} \frac{\sum_{j \in (l)} (x_j - c)}{y} + \chi^{(x^*,c,y)}_+
\]
is to determine how much “money” the user \( i \) should pay/receive regarding the prices users \( j \in M \) except \( i \) for the bandwidth he should buy/sell, while \( II \) balances the terms generated by the other terms in the utility function of user \( i \) to reach the Nash Equilibria point proposed before (and this has been proved in The later sections) and to ensure the capacity constraints are satisfied.

To optimize the solution of problem \( P \), the LaGrange function would be included first as:

\[
\Lambda(x, y, \lambda, \varphi) = U_i(x_i, y_i) - \lambda \left( \sum_{i \in N} x_i - c \right) - \varphi \sum_{i \in N} y_i
\]

An optimal allocation \((x^*, p^*) = \{(x_i, p_i^*)\}_{i \in N}\) the necessary and sufficient Karush-Kuhn-Tuker (KKT) conditions for optimality are:

\[
\frac{\partial}{\partial x_i} \Lambda(x, y, \lambda, \varphi) = \frac{\partial}{\partial x_i} U_i(x_i^*) - \lambda = 0, \forall i \in N
\]

\[
\frac{\partial}{\partial y_i} \Lambda(x, y, \lambda, \varphi) = 1 - \varphi = 0, \forall i \in N
\]

\[
\lambda \left( \sum_{i \in N} x_i^* - c \right) = 0
\]

\[
\varphi \sum_{i \in N} y_i = 0
\]

where \( \lambda \) and \( \varphi \) are the Lagrange multipliers for the capacity and budget constraints.

Substituting equation (21) into (23), the KKT conditions can be reduced to:

\[
\frac{\partial}{\partial x_i} U_i(x_i^*) = 0, \forall i \in N
\]

\[
\lambda \left( \sum_{i \in N} x_i^* - c \right) = 0
\]

\[
\sum_{i \in N} y_i = 0
\]

Where (24) present that at optimality the marginal utility equals the LaGrange multiplier \( \lambda \), (25) claims the capability constraint, and (26) states that the allocation must meet budget balanced condition.

Then we will start investigating the Nash equilibrium generated by our, and checking whether it follows the equations (24) to (26).

Before we start, note that for every user \( i \in N \), there will be a fixed \( p_{-i}, c \) and \( r_{-i} \). The user \( i \) picks \( x_i \) to maximize his utility function.

\[
\max_{x_i, p_i} u_i(x_i) = \left( x_i - \frac{wc}{\sum_{i \in N} w_i} p_i - \frac{c}{\sum_{i \in N} (x_j - c)} + \chi^*(x, c, y) \right)
\]

At a Nash Equilibrium message, equation (27) is at maximum by user \( i \). Assuming \((\tilde{x}, \tilde{p}) = (\tilde{x}_i, \tilde{p}_i)_{i \in N}\) is a Nash Equilibrium message.

\[
U_i(x_i) = R_{sm, r_{n, d_{m}}} = \frac{1}{2} \log_2 (1 + \Gamma_{sm, d_{m}} + \Gamma_{sm, r_{n, d_{m}}}), \text{as defined, } x_i \equiv P_{r_{n, d_{m}}}
\]

\[
\frac{\partial}{\partial x_i} U_i(x_i) = \frac{W}{2 \ln(2) (1 + \Gamma_{sm, d_{m}} + \Gamma_{sm, r_{n, d_{m}}})^2} \left[ \frac{P_{sm} G_{sm, r_{n, d_{m}}} - P_{r_{n, d_{m}}}}{P_{sm} G_{sm, r_{n, d_{m}}} + P_{r_{n, d_{m}}} G_{r_{n, d_{m}}} + \sigma^2} \right]
\]

Here, let:
We can get the following formula:

\[
\frac{\partial}{\partial x} u_i(x_i) = c_1 \cdot \frac{c_2}{(x_i+c_2)^2}
\]  

(29)

The equation (28) is from the original problem, and equation (28) is deviated by the equation (28). By the first-order condition for each user, we will have the following:

\[
\frac{\partial}{\partial x} u_i(\tilde{x}_i) + \lambda - \tilde{p}_i - \frac{c_1 c_2}{(\tilde{x}_i+c_2)^2} \frac{\gamma}{\tilde{x}_i+c_2} \sum_{j \in E} (\tilde{x}_j) - c = 0
\]

(30)

Equation (30) is deviated from equation (27), and equation (31) is one of the problem’s fundamental assumptions which implies that the power of one relay should all be used to helping the users to transmit the data. So, the equation (30) can be changed into:

\[
\frac{c_1 c_2}{(\tilde{x}_i+c_2)^2} + \lambda - \tilde{p}_i - \tilde{p}_i - \frac{c_1 c_2}{(\tilde{x}_i+c_2)^2} \frac{\gamma}{\tilde{x}_i+c_2} \sum_{j \in E} (\tilde{x}_j) - c = 0
\]

From this equation, we have:

\[
\tilde{p}_i = \tilde{p}_i, \quad \lambda = \tilde{p}_i
\]

Then:

\[
\lambda = \tilde{p}_i = \tilde{p}_i = p
\]

And equation (32) is the strong enough evidence that the taxed equation meets the first-order condition of the KKT conditions (24).

From the original taxed equation, we have:

\[
\sum_{i \in E} (\tilde{x}_i - \frac{w_i c}{\sum_{i \in E} (w_i)}) \times \tilde{p}_i = (\sum_{i \in E} (\tilde{x}_i) - c) \times \tilde{p}_i = \lambda (\sum_{i \in E} (\tilde{x}_i) - c) - c
\]

(33)

where

\[
\sum_{i \in E} (\tilde{x}_i) - c = 0
\]

(34)

As a result, the taxed function meets the equation (25).

To prove the last condition of the KKT conditions:

\[
\sum_{i \in E} Y_i = \sum_{i \in E} \left[ (\tilde{x}_i - \frac{w_i c}{\sum_{i \in E} (w_i)}) \times p_i \right] = \sum_{i \in E} \left( \frac{\tilde{p}_i}{\tilde{p}_i} \right) \frac{c_1 c_2}{(\tilde{x}_i+c_2)^2} \frac{\gamma}{\tilde{x}_i+c_2} \sum_{j \in E} (\tilde{x}_j) - c + \lambda (\tilde{x}_i) - c
\]

(35)

\[
= \sum_{i \in E} \left( \frac{c_1 c_2}{(\tilde{x}_i+c_2)^2} \frac{\gamma}{\tilde{x}_i+c_2} \sum_{j \in E} (\tilde{x}_j) - c + \lambda (\tilde{x}_i) - c \right)
\]

\[
= \sum_{i \in E} \left( \frac{c_1 c_2}{(\tilde{x}_i+c_2)^2} \frac{\gamma}{\tilde{x}_i+c_2} \right) + \sum_{i \in E} \lambda (\tilde{x}_i) - c
\]

\[
= 0
\]
which proves that \( \sum_{i \in l(n)} y_i = 0 \), and all the KKT conditions are met.

The proof above is just for a single-link system, for the multi-links system the proof will change, but they are still very similar, so we don’t need to give the detailed proof here.

**Remark:** We will find that as soon as the system reaches the Nash Equilibrium point for all users \( i \in M \), we will have (i): \( p_{l,i(n)} = p_{j,l(n)} \), and (ii): \( p_{l,i(n)} = p_{\bar{i}l(n)} \).

Here are the properties of this network system:

**Property 1:**
Note that at the equilibrium, only the \( I \) term of the tax function plays the role of how much the user \( i \) should pay to the whole system (or we could say, links), and there is not a \( p_I \) in the term, which means that how much one user should pay is only dependent on the prices announced by other users in the system on the last time slot. As a result, the only way the user allowed to do is to play to role of a price taker and request the amount of bandwidth he desires.

**Property 2:**
There are many methods to construct a taxed function satisfying all properties above [8]. Particularly, for each user \( i \in M \), term \( II \) of the taxed function can be generated as a sum of a series of functions that all meet the properties.

### 4. Theorem Properties

In this section, we will define what is “efficiency” as well as other kinds of properties of the outcomers generated by resource allocation mechanisms. In detail, they are (i) satisfy voluntary participation, (ii) feasible and (iii) efficient. These are well-established properties in the mathematics economics community.

**Definition 4.1:** We call an allocation vector \((x, t) \triangleq ((x_1, t_1), (x_2, t_2), \ldots, (x_N, t_N))\) for problem \( P \).

Let \((x^*, t^*) \triangleq ((x_1^*, t_1^*), (x_2^*, t_2^*) \ldots, (x_N^*, t_N^*))\) define an output efficient and balanced allocation. On any feasible allocation, we will have:

\[
\sum_{i \in N} U_i(x^*, t^*) = \sum_{i \in N} U_i(x_i^*) + \sum_{I \in \mathcal{I}} t_i^* \\
= \sum_{i \in N} U_i(x_i^*) + \sum_{I \in \mathcal{I}} t(I)_i^* + \sum_{I \in VI} t(II)^*_i \\
\geq \sum_{v \in V} U_v(x_v) + \sum_{I \in VI} t(I)_i + \sum_{I \in VI} t(II)
\]

where \( II = III + IV \) which partly means the two terms in the taxed function.

At the equilibrium point \( I \geq V \) since the definition of output efficient and III \( \geq VI, VI \geq VII \) for the property of the taxed function and the definition of balanced, which proves the efficiency of \((x^*, t^*)\).

Proof that an efficient allocation has to be balanced is also needed. Assuming by contradiction that an efficient allocation exists for the problem \( P \): \((x^*, t^*) \triangleq ((x_1^*, t_1^*), (x_2^*, t_2^*) \ldots, (x_N^*, t_N^*))\), which is not balanced, then \( \sum_{i \in N} t_i^* = -c < 0 \).

Consider allocations \((\bar{x}, \bar{t}) \triangleq ((\bar{x}_1, \bar{t}_1), (\bar{x}_2, \bar{t}_2), \ldots, (\bar{x}_N, \bar{t}_N)) = ((x_1^*, t_1^* + c), (x_2^*, t_2^*) \ldots, (x_N^*, t_N^*))\).

Note that this allocation satisfies all the constraints of the problem \( P \), so it is feasible. Then

\[
\sum_{i \in N} U_i(x_1^*, t_1^*) = \sum_{i \in N} U_i(x_1^* + t_1^*) = U_1(x_1^*) + t_1^* + \sum_{2 \leq i \leq N} U_i(x_i^* + t_i^*) \\
\leq U_1(x_1^*) + t_1^* + \sum_{2 \leq i \leq N} U_i(x_i^* + t_i^*) \\
\leq \sum_{i \in N} U_i(\bar{x}_i, \bar{t}_i)
\]

(36)
This contradicts the assumption that \((x, t)\) is efficient, which proves that the allocation \((x, t)\) is efficient.

We are now proving that efficient allocation has to be output efficient. Let 
\[
(x^*, t^*) = (x_1^*, t_1^*), (x_2^*, t_2^*) \ldots (x_N^*, t_N^*)
\]

We need to show that 
\[
\sum_{i \in N} U_i(x_i^*) \geq \sum_{i \in N} U_i(x_i)
\]

Note that if \((x, t)\) is feasible, so it can be written as \((x, t^*)\). Then by the efficiency of \((w^*, t^*)\), along with the fact that efficient allocations have been balanced, we will have:
\[
\sum_{i \in N} U_i(x_i^*) = \sum_{i \in N}(U_i(x_i^*) + t_i^*) \geq \sum_{i \in N}(U_i(x_i) + t_i) = \sum_{i \in N} U_i(x_i)
\]

which proves our result.

**Definition 4.2**: A feasible allocation \((x, t)\) is designed to be satisfied, if 
\[
U_i(x_i, t_i) \geq 0, \forall i \in N.
\]

The interpretation of feasible allocation is defined that it must work under the physical constraints of the resource allocation problem. The interpretation of voluntary participation is that no user participating in the resource allocation process will become worse off at the end of the allocation process (i.e. each user’s utility will be always greater or equal to zero in the allocation process).

Here some different efficiency criteria will be presented for allocation processes.

**Definition 4.3**: Assuming a feasible allocation
\[
(x^*, t^*) = (x_1^*, t_1^*), (x_2^*, t_2^*) \ldots (x_N^*, t_N^*)
\]

We have
\[
\sum_{i \in N} U_i(x_i^*, t_i^*) \geq \sum_{i \in N} U_i(x_i, t_i)
\]

**Definition 4.4**: A feasible allocation \((x^*, t^*)\) to problem \(P\) is said to be Pareto efficient if there does not exist another feasible allocation \((x, y)\), such that 
\[
U_i(x_i, t_i) \geq U_i(x_i^*, t_i^*), \forall i \in N
\]

**Definition 4.5**: A feasible allocation \((x^*, y^*)\) is called output efficient if for any other feasible allocations \((x, t)\) we will have:
\[
\sum_{i \in N} U_i(x_i^*) \geq \sum_{i \in N} U_i(x_i)
\]

First note that in general, efficient allocations would imply Pareto efficient allocations.

5. **Discussion**

In this section, some of the assumptions and issues which appear throughout the whole passage will be discussed.

**Interpretation of \(\gamma\)**: In determining the tax of the one user should pay to the whole network system, we note that there exists a \(\gamma\) in the allocation to the problem \(P\). This \(\gamma\) is a constant and is an amount generated by the relay links. If the amount of \(\gamma\) is large, then the prices of the users \(i \in l(n)\) will approach to the desired Nash Equilibria point very fast. On the other hand, the prices of users given a too small amount of \(\gamma\) will approach to the desired point slowly and may oscillate around it.

The optimal value for \(\gamma\) would maximize the rate of convergence of the mechanism, which is affected by some factors: (i) the topology of the network; (ii) the second derivative of the utility functions. [8]

**Interpretation of \(p_l(n)\)**: We note that the prices of all the users in the links are generated by the users’ service providers. We design the mechanism like this is because that if the users can declare the prices themselves, there will create another kind of “strategic behaviour” like this: fix all the demand and
prices declared by the users in the link $l(n)$ except the user $i$, if the user $i$ is just a strategic user, what he probably do is to declare the price as large as he can to get more bandwidth and just pay at the price that generated by the other users’ prices, which means one may use the right of the declaration of the prices to get as more demand as he can and pay little for it. This situation will surely break the balance of the whole system.

**Interpretation of $\chi^+(x^*,e,y)$**: The only reason of introducing $\chi^+(x^*,e,y)$ into the term II of the taxed function for eliminating the possible undesirable equilibria where: (i) for some $l \in L$ all user’s $p_l$ are equal to 0; (ii) the excess demand on the link $l$ is never negative.

**Not covering the network cost**: In the model this passage constructs, we only assume that there are finite links, users, and relays in it, with finite bandwidth. And the objective of this passage is to find out an allocation that can maximize the sum of the utilities of the users in the network according to the conditions of the network itself. As we all can see, in the formulation we didn’t consider the cost incurred by the network for providing the bandwidth allocation.

**6. Conclusions**

Based on the resource management and interference suppression needs of home cellular wireless communication, the goal is to keep the wireless networking system well balanced with the highest energy efficiency at the participation of relay stations. This study optimized the ratio of effective capacity to power consumption, a pricing mechanism which implements a rate allocation in Nash Equilibrium point in a unicast provisioning was presented. we improve the pricing mechanism in the literature [8,9,10] by applying formal Shannon formula in it and reconstruct the utility function to let it more close to the communication world. This mechanism has the following problem to solve: (i) generating efficient allocation that maximize both the sum of net utilities; (ii) balancing the budget.

Through the study, a transmission rate distribution pattern at the Nash equilibrium point is obtained to better meet the actual needs of communication.

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