Some Important Aspects of Source Location Protection in Globally Attacked Sensor Networks

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Abstract—In the problem of location anonymity of the events exposed to a global eavesdropper, we highlight and analyze some aspects that are missing in the prior work, which is especially relevant for the quality of secure sensing in delay-intolerant applications monitoring rare and spatially sparse events, and deployed as large wireless sensor networks with single data collector. We propose an efficient scheme for generating fake network traffic to disguise the real event notification. The efficiency of the scheme that provides statistical source location anonymity is achieved by partitioning network nodes randomly into several dummy source groups. Members of the same group collectively emulate both temporal and spatial distribution of the event. Under such dummy-traffic framework of the source anonymity protection, we aim to better model the global eavesdropper, especially her way of using statistical tests to detect the real event, and to present the quality of the location protection relative to the adversary’s strength. In addition, our approach aims to reduce the per-event work spent to generate the fake traffic while, most importantly, providing a guaranteed latency in reporting the event. The latency is controlled by decoupling the routing from the fake-traffic schedule. A good dummy source group design also provides a robust protection of event bursts. This is achieved at the expense of the significant overhead as the number of dummy source groups must be increased to the reciprocal value of the false alarm parameter used in the statistical test. We believe that the proposed source anonymity protection strategy, and the evaluation framework, are well justified by the abundance of applications that monitor a rare event with known temporal statistics, and uniform spatial distribution.

I. INTRODUCTION

Privacy issues are an important aspect of monitoring applications in wireless sensor networks (WSNs). A recent survey of state-of-the-art research on privacy protection in WSNs [5], among other problems, reviews strategies to protect the object observed by a WSN node, referred to as source, from the global eavesdropper (Eve) [5], which can infer the location of the object based on the expected behavior. Consequently, Eve attempts to from the content. In addition, Eve, despite being so powerful and omnipresent, needs to stay invisible. We define outage as the event when, following an eavesdropped transmission, Eve reveals itself by taking actions based on the false suspicion that an event occurred. The actions may involve physical presence of the attacker or her faculties, in order to capture or destroy the object. Hence, a false-alarm presents a risk of personal exposure and liability.

The adversary gathers the source of the transmission based on the change in the traffic pattern; a conspicuous case would be when a node starts transmitting after a prolonged period of inactivity in the WSN. For many event-reporting applications, despite the fact that the attacker cannot learn the details from the message content, inferring the contextual information, i.e. whether, when and where a concerned event has happened, may be enough to jeopardize monitored resources. Intuitively, the persistent dummy (fake) traffic is the only way to obfuscate the events, and the formal proof for it is available in [5]. Dummy packets follow a predefined schedule, aligned with the expected timeline of real packets, so that Eve cannot observe the change. To better explain the intricacies of this approach, especially in light of the existing research, we next introduce two models of monitored phenomena. Let us first define the application delay as the delay in event reporting. In both scenarios, the duration of time is relative to the application delay constraint, which is a known value. We assume that the WSN is divided into cells, such that each sensor node monitors a unique cell and that the events are occurring in a uniform manner over time and space. An active event is any event that is not reported to the sink yet.

1) Scenario A: Frequent and Dense Events: This model describes monitoring of a physical phenomena that creates on average one event per cell over one cycle, a relatively short period of time, whose duration is larger but of the same order of magnitude as the application latency constraint. In other words, in any cycle, there are many active events in the network.

2) Scenario B: Rare and Spatially Sparse Events: In this model, the events are rare and isolated. For example, if the allowed application delay is in minutes, the expected interval between events is measured in hours or days. They are spatially sparse: we assume that there is at most one active event at a time. In fact, the examples of monitoring applications at the beginning of [5A] all represent the scenario B. No single node can statistically emulate the spatial and temporal characteristics of the events in this model. In addition, by observing a single node for the duration on the order of the application delay Eve can not reliably deduce deviation from the expected behavior. Consequently, Eve attempts to
observe abnormalities in the network-wide traffic pattern. The anonymity protection scheme described here implements the traffic pattern in a decentralized manner, so that the occurrence of real events does not cause observable abnormalities.

B. Solution Outline

The uniform spatial/time distribution of events guides naturally the baseline scenario for the dummy traffic: all cells in the network send dummy messages at a constant rate regardless of whether a real event has occurred or not. That means that an event would have to wait to be reported on average for half of the inter-transmission interval. However, since the traffic in the network always keeps the same pattern, it effectively defeats any traffic analysis techniques. The main problem with dummy traffic is immediately obvious from the basic scenario: limiting the reporting delay calls for a high-rate fake traffic, which is not only expensive but may quickly burn out the network.

Our approach stipulates the importance of knowing the event’s temporal dynamics, in terms of the described scenarios; it allows us to design energy-efficient protection strategies. It is natural to assume that the expected frequency of events will be known to everyone (both the attacker and the network architect), given that we design the monitoring application for a particular physical phenomena, and that the easiest characterization of a random process is through its first moment, or the moment’s estimate. Next, among the all-positive probability distributions with a given expected value, exponential distribution has the highest entropy. Hence, assuming that inter-event times follow an exponential distribution of the estimated mean leads to a good and justifiable model of the event’s randomness. In terms of traffic overhead/energy-consumption and interference, the optimal design would force each node to transmit as rarely as possible, and that would be in exponentially distributed intervals of the expected duration exactly equal to the expected time between real events; smaller intervals generate more traffic and therefore cost more, while larger ones create too few opportunities for embedding the real-traffic, especially under delay constraints.

The second major underpinning of our approach is the network-centric view of the problem. The following aspects of the problem are looked at from both the event’s and the network’s perspective:

- Event is characterized as a spatio-temporal process over the whole network area.
- Event-reporting delay includes routing latency.
- Fake-traffic shaping is a decentralized process, collaboratively maintained by all nodes.
- Energy consumption per-event of the protection strategy is equally split among network nodes, and substantially decreased with respect to the baseline strategy due to nodes’ collaboration.

Finally, our source-anonymity protection scheme aims to achieve statistical event unobservability. The absolute protection under baseline strategy is not applicable to delay-sensitive applications, such is the majority of event monitoring in WSNs. Secondly, we do not adjust the timing of real events as in [7], to make the event pass the statistical test under the test parameters assumed to be used by Eve. Instead, we make the event pass the test with the same probability as the dummy transmissions, making it statistically indistinguishable.

C. Existing Research

There are a couple of papers that study the WSN source anonymity and, in our opinion, provide relatively efficient solutions only under scenario A [5], [7], [8]. At the expense of huge traffic overhead, a practical tradeoff between security and latency is proposed in [7]. The paper proposes to decrease the delay in event notification by having every WSN node maintain a random schedule sampled from the same (exponential) distribution of inter-transmission intervals. The improved latency in event notifications with respect to the baseline approach is achieved without making the real traffic observable by Eve. This approach assumes that the attacker knows the defense strategy of the WSN, and that he will use a state-of-the-art statistical test to distinguish the real event from the fake one. The rationale is that fake traffic, even if random, is designed to follow a distribution, while the real events may not. According to [7], the event sources should run the same test, and adjust the time of the real event to pass the test. Here, the event sources test only the intervals between own transmissions, as each node strives to maintain exponentiality of its own intervals. The test is assumed to be corrected for the finite sizes [2] by both Eve and the sources. The paper compares two varieties of this approach: one in which the real event embeds itself by waiting until next scheduled transmission (ProbRate), and another one (FitProbRate), when it waits as little as needed to pass the goodness-of-fit test for the exponential distribution, inferred from the previous transmissions. The latter approach results in smaller delay, but requires the correction of the schedule: the next scheduled fake event should be canceled, and the rest of the schedule is also verified so that the test failure at the next (dummy) transmission does not point out to the preceding disturbance. For monitoring WSN applications described in scenario A, where the spatio-temporal frequency of the events justifies frequent transmissions from every node, and hence, nodes create their own fake traffic, correcting the schedule can be implemented as an extension of the dummy schedule protocol. Let us note here that [2] does not explicitly state that the exponential distribution of fake events is designed to emulate the expected frequency of actual events, although it can be inferred from the values used in the simulation. The paper does not assume any particular event distribution, and, consequently, does not evaluate the performance with respect to overhead per event.

We further observe that the scheme proposed in [2] defines delay as the time between the event occurrence and the source’s transmission, which holds only for WSN applications in which the sink is one hop away from any source. If the packet is delivered to the sink in a hop-by-hop manner, the latency includes another random part due to summation of the exponentially distributed delays associated with such
transmission schedule of each relay. We refer to this additional delay as the **publishing route (PR) latency**. When the expected value $\lambda$ of the inter-transmission times is the same at each node, as in [7], and designed to imitate a relatively rare event pattern, for the source-sink route of $h$ hops, the PR latency becomes an Erlang-distributed random variable with mean $h\lambda$. From this point of view too, Scenario $B$ requires a modified approach to source anonymity.

To decrease the overhead of the dummy protection scheme in [7], in [8] the same authors propose a WSN with several proxy nodes, which pick up transmissions from surrounding nodes, and filter out the dummy packets. Apart from requiring mitigating solutions, frequent dummy traffic inevitably leads to interference, which additionally increases the PR latency.

In summary, an important missing point in the existing research is the analysis of the overhead per event, especially in WSNs with rare and spatially sparse events. Under such a scenario ($B$), let us scrutinize the approach in [7], where each node generates its own fake traffic. As explained, the overhead-optimal design would require each node to transmit with an average rate equal to the expected frequency of real events. For the cases simulated in [7], the mean of dummy message intervals is $20s$, and real events arrive according to a Poisson process with the rate changing from $1/20$ to $1/100$. Their protection scheme achieves the average latency of less than 1s. If we replace seconds with hours, having in mind events that happen once a day, or once a couple of days, the delay of one hour does not seem to be acceptable. Additionally, under the overhead-optimal design, the PR latency for rare events is prohibitive, even for applications that are not delay-sensitive.

The next section briefly introduces our solution to this problem. Section [III] describes and analyzes the decentralized algorithm that implements the proposed solution. Section [IV] presents the simulations for some of the most realistic and important event dynamics, and demonstrates the superiority of our approach in protecting the anonymity of such a source. In section [V] we discuss statistical tests considered in our approach. Finally, in [VI] we conclude.

**II. Our Approach**

**A. System Model**

We have a static WSN of $n$ nodes. There is one static sink collecting event notifications from all nodes. The monitoring application is delay sensitive: the time between the event occurrence and the sink’s notification must be smaller than $\Delta$. We assume that monitored events have Poisson temporal distribution of a known rate $\lambda = 1/\mu$, and uniform spatial distribution over the area of network deployment. Hence, the time between the events is distributed according to an exponential distribution $\zeta_{\mu}$ (of expected value $\mu >> \Delta$). The source is assumed to transmit a burst of packets, all describing the event. For simplicity, we first analyze the burst of unit length (one packet). In addition, we separately analyze the existence of outliers in the event’s distribution.

**B. Problem Formulation**

We already established that the only way to confuse Eve is through persistent network-wide transmissions. Simultaneously, as hop-by-hop is prevalent data transfer model in WSNs, and distance to a sink may be considerable, to satisfy the application latency constraints, we need to decouple routing from the fake traffic schedule by allowing immediate relaying of event notifications as opposed to piggybacking them on the existing fake transmissions. However, as such a route may be backtraced to the source, a similar routing path should be emulated from each fake source (see Figure [I]). Given the mentioned constraints, we set the goal to achieve statistically strong source anonymity through methods that optimize energy and delay [5]. We hereby propose a pattern of fake traffic that scales well with network size, and satisfies application-latency constraints, while protecting the source location up to a given significance level, defined under the strong statistical tests available to Eve [4], [12]. To confuse the attacker, we propose to replicate the spatio-temporal process through the following mechanisms:

- **Source Emulation:** a subset of $d$ nodes regularly wakes up to act as dummy sources. As a result, any real event is covered by $d$ dummy sources, which we refer to as dummy population. To explain what it means for an event to be covered, we introduce a time interval, dubbed round, whose duration is equal to the expected inter-event time. Hence, the length of a round is $\mu$. Covering a source implies the expected existence of fake transmissions in the same round in which the event occurs. To engage all nodes equally, we may divide the network in $d$ groups and assign one representative of the group to a distinct round. Each group will maintain a schedule that emulates the event distribution $\zeta_{\mu}$. Due to the size of the dummy population, the probability distribution of the intervals between any two consecutive dummy events is $\zeta_{\mu/d}$ (exponential of expected value $\mu/d$). Such cooperative shaping of the fake traffic in order to emulate a sufficiently dense Poisson distribution is amenable to distributed implementation, which is thoroughly explained in Section [III]. It also allows us to uniformly distribute fake activity across the network area. However, as the attacker overhears every transmission that occurs in the network, and may integrate all recorded temporal data into one global network activity timeline, it is judicious to ignore for a moment the decentralized implementation. Instead, we look at the global timeline as if it was produced by a single source sampling the values of event inter-arrivals from the distribution $\zeta_{\mu/d}$. The joint empirical distribution of inter-transmission times, created by extending the fake schedule with the immediate (un-delayed) transmissions of real events, is, based on the transmissions’ independence, $\zeta_{\mu/(d+1)}$, i.e. exponential with the expected value $\mu/(d + 1)$. For sufficiently large $d$, $\zeta_{\mu/(d+1)}$ does not diverge perceptibly from the distribution on the global timeline of fake events $\zeta_{\mu/d}$. 


• Route Emulation: each source (dummy or real) forwards the packet along a predetermined route towards the sink (see Figure 1). The inter-transmission time between relays is constant and significantly shorter than $\Delta t$ (and, consequently, orders of magnitude smaller than $\mu$, as opposed to $\tau$ where it is tied to the inter-transmission time of dummy sources). When the real source starts transmitting without delay, the application latency is equal to the routing delay, which is now decoupled from the time dynamics of the fake traffic, and can be further optimized by minimizing the number of hops. This can be achieved by implementing shortest-path routing, or by increasing the transmission range of the relays through the usage of advanced channel codes, while keeping the transmission power constant.

• Knowing the Attacker’s Detection Methods: Eve is assumed to be able to estimate the distribution of recorded transmission times. The estimated distribution is used as a reference point in the real-event detection strategy that involves a statistical test. The basic idea of those tests is to evaluate the distance between the distribution of the sample data and a specified probability distribution, which is the one that has been estimated based on a sufficiently large data set. Alternatively, the test evaluates if the current sample comes from the same distribution as the previously evaluated ones. If the distance is statistically significant, where the significance level is derived from a parameter of the test, which also defines the percentage FA of false alarms, it is decided that data do not follow this distribution. In Section V we describe in more detail the testing strategies that Eve may opt for.

As the latency issue is decoupled from the fake traffic design, we seek to determine the minimal dummy population size needed to secure a given statistical anonymity, hence minimizing the overhead. For a WSN of size $n$, we define $W_n$ as per-event and per-round energy consumption of the source anonymity mechanism. We express $W_n$ in terms of the number of packet-forwarding hops, where we upper-bound the length of the publishing route by $h = O \left( \frac{\sqrt{n}}{\log n} \right)$, assuming the transmission range necessary for a connected network of size $n$ [10]. Hence, the cost of each fake source transmission will be of the same order. With $d$ fake sources covering each real event, $W_n = O \left( (d + 1) \frac{\sqrt{n}}{\log n} \right)$, which demonstrates the importance of optimizing $d$, as the source anonymity calls for a sufficiently large $d$.

### III. DECENTRALIZED GENERATION OF FAKE TRAFFIC

#### A. Baseline Decentralized Algorithm

In Section II we highlighted the importance of cooperative and distributed shaping of the fake traffic that should result in sufficiently dense Poisson distribution of dummy transmissions. Let us now propose the realization of such a decentralized system. We first establish the baseline for the decentralized implementation. As the dummy population covering each event needs to include on average $d$ fake sources, we define $n$ epoch of duration $T = \frac{\mu n}{2}$ in which each network node will get to be a dummy once. Then, we let each node draw a time instant to transmit in this epoch by sampling uniform distribution $U(0, T)$. The causality of transmissions will arrange all node samples in increasing order, resulting in exponential distribution $\zeta_{\mu/d}$ of inter-transmission times. The procedure can be extended to the consecutive epochs, so that in the $i$th epoch nodes draw their transmission times from $U((i - 1)T, iT)$. Note that the particular uniform distribution range does not overlap with the ranges of distributions pertaining to other epochs. The collective empirical distribution of transmission times is the distribution of almost independent disjoint events, and therefore it approximates the Poissonian distribution. The independence of transmission events is broken only on the boundary of the epochs, as with each new epoch the nodes sample from a uniform distribution of different disjoint range. Hence, the distribution of the interval $Z$ between the first event in the new epoch and the last event in the previous epoch is not exponential.

$$Z = T + U - V = T + \min \{X_1, \cdots, X_n\} - \max \{X_1, \cdots, X_n\}.$$  \hspace{1cm} (1)

$$F_U(u) = P(U \leq u) = 1 - P(U > u) = 1 - \prod_{i=1}^{n} P(X_i > u) = 1 - \prod_{i=1}^{n} (1 - P(X_i \leq u)) = 1 - \prod_{i=1}^{n} (1 - F_X(u)) = 1 - (1 - F_X(u))^n.$$  \hspace{1cm} (2)

The probability distribution for $U$ is

$$f_U(u) = n(1 - F_X(u))^{n-1} f_X(u) = \frac{d}{\mu} \left(1 - \frac{u}{T}\right)^{n-1}, \text{ for } T \geq u \geq 0.$$  \hspace{1cm} (3)

![Fig. 1. Dummy population of size $d$ accompanies an event; nodes have been assigned the round index $j$ at the initialization, and index assignment sometimes includes extra effort to create spatial clusters of nodes within dummy population, to further confuse Eve (look at the nodes delimited by a dashed-blue ellipse).](image-url)
and o.w. 0, which is for sufficiently large $n$ clearly exponential distribution of expected value $\frac{\mu}{d}$

$$f_U(u) = \frac{1}{d} e^{-\frac{u}{\mu}}. \quad (4)$$

Further,

$$F_V(v) = \Pr(V \leq v) = \sum_{i=1}^{n} \Pr(X_i \leq v) = \prod_{i=1}^{n} F_X(v) = F_X^N(v). \quad (5)$$

$$f_V(v) = n F_X^{-1}(v) f_X(v) = \frac{d}{\mu} \left( \frac{v}{T} \right)^{n-1}, \quad (6)$$

for $T \geq v \geq 0$. For sufficiently large $n$

$$f_V(v) = \frac{d}{\mu} e^{-\frac{T-u}{\mu}}. \quad (7)$$

For $W = T - V$, $f_W(w) = \frac{d}{\mu} e^{-\frac{T-u}{\mu}}$, where now

$$Z = U + W. \quad (8)$$

The random variable $Z$ has a range $[0, 2T]$, and, from $(8)$, its probability distribution is Erlang, with the shape parameter of 2, denoted $\epsilon_{\epsilon(2, \mu)}$:

$$f_Z(z) = \int_{0}^{T} f_U(\tau) f_W(z - \tau) d\tau$$

$$= \int_{0}^{z} \frac{d}{\mu} e^{-\frac{T-u}{\mu}} \frac{d}{\mu} e^{-\frac{z-u}{\mu}} d\tau = z d^2 e^{-\frac{T-z}{\mu}}. \quad (9)$$

As the probability of an inter-epoch sample in the collection of test samples of size $t$, where $t < n$, is $\frac{1}{d}$ or 0, the average test-failure probability will be at most $\frac{1}{d} FA_E + \frac{t-1}{t} FA$, where $FA_E$ denotes failure probability when Erlang-distributed samples are tested on exponentiality. Even though the distribution of $Z$ differs from $\epsilon_{\epsilon(2, \mu)}$, we observe that the baseline decentralized implementation will pass the exponentiality test for any reasonable size of $t$. Such a distribution satisfies our needs for real-event obfuscation. However, for other reasons (explained later in this section), related to cases when the outliers of the real-event temporal distribution coincide with the spatial correlation of events, we propose the following realization of the dummy traffic, dubbed group implementation.

**B. Group Algorithm**

In the initialization, the WSN nodes are divided into $d$ groups of size $\lceil n/d \rceil$, and every node in the group is assigned an index $i$, denoting the round in which the node will cover the source. Hence, the $k$th round, where $k \equiv i \mod \lceil n/d \rceil$, $i \in \{1, \lceil n/d \rceil\}$, will have a dummy population of $d$ nodes belonging to different groups. A group schedule is created by letting the $i$th member select the specific time instant to transmit a fake message, sampled from the uniform distribution $U((i-1)\mu, i\mu)$. Such an algorithm is amenable to distributed implementation, since each node can independently measure time and keep count of the current round. Once the round index corresponds to node’s index modulo group size, the node draws a sample from the pertaining distribution, and determines its transmission time. As the independence of transmission events is broken only on the boundary of the rounds, the distribution of the interval $Z$ between the first event in the new round and the last event in the previous round is not exponential. Once again, $Z = U + W$, where $W = \mu - V$, $V = \max \{X_1, \cdots, X_d\}$, and $U = \min \{X_1, \cdots, X_d\}$.

$$f_U(u) = 1 - (1 - F_X(u))^d, \quad \text{and} \quad (10)$$

and o.w. 0, which is, for sufficiently large $d$, exponential distribution of expected value $\frac{\mu}{d}$. Now, following a derivation similar to $(5)$, we obtain

$$f_Z(z) = z d^2 \mu^2 e^{-\frac{T-z}{\mu}} = \epsilon_{\epsilon(2, \mu)}, \quad (13)$$

for sufficiently large $d$. Hence, with group implementation $Z$ follows the same Erlang distribution $\epsilon_{\epsilon(2, \mu)}$ as in the baseline decentralized implementation. However, to be able to state that the test sample of size $t < n$, which includes inter-round intervals with probability $1/d$, will be statistically indistinguishable from the sample of exponentially distributed intervals, we need to impose a stricter requirement for the value of $d$. The average test-failure probability will be $\frac{1}{d} FA_E + \frac{d-1}{d} FA$. Upper-bounding the failure probability $FA_E$ (when Erlang samples are tested on exponentiality) with one, we obtain that $d$ should be at least $\frac{1}{FA}$. If the spatially-uniform events do follow the distribution $\sigma_n$ in time, this may unnecessarily increase the per-event overhead with respect to the baseline.

**1) Spatial Correlation:** However, if we have a more complex event distribution, by selecting the group algorithm we are able to not only render the event’s temporal characteristics indistinguishable, but also to obfuscate spatial correlation. For example, a time-burst of events (temporally correlated observations), illustrated in Figure 2(C), may coincide with the spatial correlation of the same events. The time burst can be handled in both decentralized algorithms by a careful design of $d$, as we will illustrate in Section [IV]. The group implementation can be leveraged to disguise the spatial patterns by regularly (in each round) assigning a sufficiently large part of the dummy population to random adjacent nodes (such as nodes within the dashed-blue ellipse in Figure 1], while the rest of the dummy population will be uniformly scattered across the network. If the average spatial burst of real events is of size $b < d$, then in the $i$th round one of $d$ groups may be the principal one, whose representative will therefore select the closest $b$ unscheduled
nodes to be representatives of additional $b$ groups, and the rest of the dummy population is selected randomly. In the rest of the paper we will focus on the group implementation.

IV. Simulations

When designing the simulations, we dismissed the possibility that Eve would test the schedule of any single node, since, in our scenario, inter-transmission times per-node are large with respect to $\mu$ (inter-event times), and, hence, it takes a lot of time to record a reasonable test sample. Our primary goal was to demonstrate the influence of the dummy population size $d$ to the statistical properties of the network-wide transmission schedule, both in the absence of real events, and under different stochastic models for real events. For sufficiently large $d$, which is still much smaller than $n$, our simulations show that the insertion of events does not statistically change the time axis. Therefore, by running the statistical tests, Eve does not obtain any additional information that would help her capture the monitored object, even if the time of transmission of a real source is not delayed. With the existing work [7], an adjustment delay is added, and another mechanism may be needed to fix the sample mean affected by the adjustments, to delude Eve’s sequential analysis tests, such as SPRT [12].

Performing the Anderson-Darling (A-D) test [4], which is a powerful test for exponentiality [3, 9], on the samples drawn from an exponential distribution results in a percentage of failures, which represent false alarms. The percentage of false alarms is a random variable whose mean corresponds to the false-alarm parameter of the test (also referred to as the significance level), denoted by $\alpha$. Due to randomness, over certain sets of test samples this percentage will fluctuate around the value of the parameter provided by the test. Our

Fig. 2. (A) Illustration of our group algorithm for fake-traffic generation, where we randomly form $d$ groups of nodes to cooperatively create the schedule by sampling a series of uniform distributions. (B) Large enough $d$ renders small divergence from the Poisson distribution statistically irrelevant. Bottom axis: for sufficiently large $d$, mixing several "group" schedules and the timeline of the real events (red stars) into a global transmission schedule observed by Eve, statistically indistinguishable from Poisson schedule. (C) Exponential distribution of events disturbed by a burst of events with small interarrival times, called pauses.

Fig. 3. The effect of the sample size on the False Alarm (FA) trend (FA as a function of the round number, each round containing $d$ samples): (A) For samples drawn from an exponential distribution and the sample size $d=10$ at round 1 FA's mean stays constant. (B) For uniformly sampled transmission times, the sample size equal to $d$ includes samples mostly from a single round, hence, exponential samples, and FA's mean is constant as in (A) (C) For uniform sampling and the sample size $d=100$, all samples at round 1 for $d=100$ FA achieves close approximation with exponential distribution across the rounds, as opposed to $d=10$ that does so only within one round (D) At each new round we test 200 preceding samples testing strategy monitors the rate of test failures to evaluate if it behaves as expected for the exponential distribution. Certain outliers do not perceptibly change the rate of failures, but more frequent or consistent divergence from the exponential distribution of samples will have a visible effect on the monitored rate.

If $d$ is not sufficiently large, inter-transmission times in group implementation would not follow exponential distribution, and goodness-of-fit tests of exponentiality [4] will fail in a much larger percentage of cases than the value of the test’s FA parameter. For small $d$, the test results are also sensitive to the sample size: smaller samples that span over just one round will most likely pass the test since ordered uniform samples
in any range \([(i-1)\mu, i\mu]\) produce intervals described by the exponential distribution \(\zeta_{\mu/d}\). This is illustrated in Figure 3(b). Large number of samples would include many rounds, with \(1/d\) portion of samples not belonging to the exponential distribution. Figure 3 compares the pure exponential samples whose false alarm statistics are independent of the sample size (pane (a)), with the samples obtained using our decentralized algorithm for \(d = 10\) and \(d = 100\), when the tested sample size was \(d\) in each round (b), \(di\) in the \(i\)th round (c), or 200 constantly (d). The FA statistics are plotted as a function of the round number \(i\), although the sample size was changing with the round index as \(di\) only in subfigure (c). In this case, we see that for sufficiently large \(d\) of 100 our algorithm based on uniform sampling achieves close approximation of the exponential distribution as it exhibits the same FA percentage across the rounds, as opposed to \(d = 10\) which stays exponential only within one round. In subfigure (d) at each new round we test 200 preceding samples, which for \(d = 10\) can be only collected after 20 rounds (notice the slope in the upper graph for the first 20 rounds). The same sample size of 200 was used to test the influence the inserted real events had on the percentage of test failures, illustrated in Figures 3(b) and 3(c). As our decentralized algorithm for small dummy populations does not approximate exponential distribution sufficiently well, it is also more sensitive to the disturbances due to insertion of real events. When \(d\) is small (10), interleaving the real event schedule into the fake events timeline even improves the results of the A-D test. Although this phenomenon can be explained by the decreased contribution of Erlang samples (relative to exponential), we omit the detailed explanation due to space limitations.

As explained in the previous section, for larger \(d\), the exponential distribution \(\zeta_{\mu/d}\) is statistically indistinguishable from the distribution of intervals produced by the group algorithm. Accordingly, the false alarm frequency obtained with decentralized sampling of ordered uniform distributions is exactly what we obtained testing pure exponential samples. The mean of the distribution is \(\mu/d\), which is also sufficiently small to allow imperceptible interleaving of the real event

![Figure 4](image1.png)

![Figure 5](image2.png)
samples.

We also test the proposed location-anonymity protection for a more general event distribution, i.e. when the empirical distribution is disturbed with respect to the estimated one, either by spurious outliers sampled from some $\zeta_{\mu/q}$, $q > 1$, or by the bursts of events whose triggers are separated by exponentially distributed intervals according to $\zeta_{\mu}$, as illustrated in Figure 2(C). The latter case is of most interest to us, as it represents a common scenario in which a whole cluster of events, usually observed by several sensors, is related to a single object. Temporally, the burst is a set of events whose inter-event times, referred to as pauses, are much smaller than one round, and whose size is a considerable fraction of the test sample size. Such clusters may occur as a result of inherent physical dynamics of the observed process. We assume that the spatial correlation among events in the burst does not exist, but we offer a solution for the opposite case as well (Figure 1).

We now provide more detail about the simulations presented in Figures 5 and 6. We start with a pre-designed schedule where a timeline for fake events is determined by drawing $\sigma d$ samples from exponential distribution $\zeta_{\mu/d}$, where $\sigma = 80$ is the time-window we observe, measured in rounds. We perform the following experiments with such a fake-event schedule, where the plots in Figures 5 and 6 illustrate the average behavior over 250 runs, based on different, independently drawn fake and real timelines:

- Adding real events that follow the estimated distribution $\zeta_{\mu}$: As we insert events into the fake schedule, we perform the A-D test on the sample which includes the event itself and 200 previous events (if accumulated by the time of the event occurrence).
- Adding real events that follow the estimated timeline $\zeta_{\mu}$, perturbed in 20% of the cases, picked at random, by samples taken from an exponential probability of smaller expected value: We perform the same A-D test as above for each real event, where the expected value of the disturbing distribution is $\mu/1000$, $\mu/100$, $\mu/10$ (in Fig. 5(b), (c), (d), respectively), for $d = 10$, and $d = 100$.
- Adding real events that follow the estimated timeline $\zeta_{\mu}$, with an exception of one burst of close samples (having small intra-burst intervals, or pauses): We perform the same A-D test as above for each real event, where the values for the pause duration are $\mu/1000$, $\mu/100$, $\mu/10$ (in Fig. 4(a), (b), (c)). Over all runs of the experiment, the burst position is kept constant (in the same round), to highlight the impact on any single run.

All these experiments are repeated when a timeline for fake events is generated by drawing $\sigma d$ samples using the baseline decentralized algorithm, and finally when the fake event schedule is created by the group decentralized method. It is important to highlight that obtained false alarm statistics, in the case of sufficiently large $d$, were the same for all 3 mechanisms of fake traffic generation. Hence, our approximation performs well for properly designed values of $d$. Figure 5 illustrate the importance of the proper design, as a pause duration which is smaller than network-level inter-transmission time $\frac{\mu}{d}$ causes considerable increase of the FA rate if the burst is long enough.

V. DISCUSSION OF STATISTICAL TESTS

A. Tests of Exponentiality

We base our analysis on the assumption of a single statistical test, namely Anderson-Darling test for exponentiality. Apart from the motives of simplicity, and the existence of a reference that employs the same test [7], we here provide additional arguments for such an approach. The goodness-of-fit (GOF) of a statistical model describes how well it fits a set of observations. Measures of goodness of fit typically depict the discrepancy between observed values and the values expected under the hypothesized probability distribution $F(x)$. The classical GOF test is Pearson’s $\chi^2$ , which is good only for discrete distributions. EDF tests refer to Empirical Distribution Function, denoted $F_n(x)$, and defined as the relative number of samples (out of $n$) that are smaller or equal to $x$. EDF tests define different statistics, based on a measure of the discrepancy between $F_n(x)$ and the probability distribution in question.

In [2] five of the leading EDF statistics, including the statistic $A^2$ defined under A-D test, are examined in three important situations: when the hypothesized distribution $F(x)$ is completely specified, when $F(x)$ is normal, and finally, the case that concerns us, when $F(x)$ is exponential, with the expected value to be estimated. The paper shows that, when used properly, EDF tests have much higher powers than previously reported, and those of $A^2$ are comparable, and appear to be highly correlated with Shapiro-Wilk regression test statistics, that were previously considered superior in power, although difficult to compute. In fact, all the examined tests are shown to be competitive in terms of power. In particular, $A^2$ seem to be especially powerful to detect divergence in the mean. The results published in [2] are very important as EDF statistics are easily calculated. The two facts represent the central argument for using Anderson-Darling test as a model for Eve’s approach to statistical inference.

However, for the A-D test of exponentiality when the distribution parameter is unknown, we ought to calculate an estimate of the expected value $\mu$. Maximum likelihood estimate of the parameter $\nu$ is exactly the mean of the samples. While this estimate is the most likely reconstruction of the true parameter, it is only an estimate, and, hence, the more data points are included, the better the estimate will be. The confidence interval of how good the estimate is can be exactly calculated based on the number of samples [11].

Hence, when testing the most recently recorded transmission intervals, Eve will have to consider a large enough sample (observation window) in order to secure sufficient confidence in the hypothesized distribution $F(x)$. Moreover, an account must be taken of the sample size in adjusting either the test-statistic or its critical values for desired $\alpha$. If the value of the statistic is found to be larger than the critical value (also called the significance point), the test claims that the sample is not from the exponential distribution. The asymptotic critical
values have been calculated theoretically in [1]. For finite \( n \), significance points are difficult to find theoretically and have been found through Monte-Carlo simulations by the same author. The A-D significance points for various values of \( \alpha \), and several sample sizes, are given in Table 2 of [2], and are utilized in our implementation of the A-D test.

B. Utilization of Tests in Our Protection Scheme

We design fake traffic so that the test-failure statistics do not change if exponentially distributed real events are added to the schedule. Given the average interval between real events \( \mu \), this is achieved if \( \nu = \frac{\mu}{d} \), and \( d \) is sufficiently large. For the reasons stated above, we assume that Eve will work with larger sample sizes. We tested with samples of length 200, event though for exponentially distributed real events without outliers times shorter samples work equally well. When interpreting the results of statistical tests in the context of our protection scheme, it is of extreme importance to model the testing strategy used by Eve. A certain percentage of test failures, on the order of false-alarm parameter, will occur for perfectly exponential test samples, and over time we will also observe some variance in the rate of those failures. We refer to this rate as the false-alarm rate. The same rate of failures quantifies the probability of Eve’s false decision that the real event has happened. We design the fake traffic to achieve the statistical indistinguishability of the traffic pattern from a Poisson process (i.e. exponential distribution of intervals), which is achieved when the rate of failures produces by testing the global traffic has the same dynamics as the false-alarm rate. Only if an unexpected amount of outliers is present in the real event distribution, the failure rate will increase above the false-alarm level. Eve sets its FA parameter (\( \alpha \)) based on its own measure of how important it is not to detect an event when there is none. If it is not important, Eve may set the FA parameter high in order to improve the probability of detection. In the opposite case, she will set the FA parameter low. In both cases our protection mechanism will successfully counteract, as the fake-traffic design did not depend on the \( \alpha \) parameter used in the attacker’s tests, but instead the empirical false-alarm statistics were used as a reference to detect the divergence from exponentiality.

In addition, by relying upon the test results, which may be false, Eve risks to be discovered, hence creating an outage of the global attack. Eve may opt for a different strategy: if making a decision based on false alarm creates an outage of the attack, Eve will not use the immediate result of the test to make a decision. Instead, Eve will look at the trends of failure statistics, trying to detect consistent divergence that drives the percentage of failures above the relaxed significance level, based on the empirical rate of failures (see Figure 5). This detection procedure is efficient in detecting outliers, such as a burst of events which deviates from the distribution accounted for in the event model or/and when the set of event transmissions are correlated in space.

VI. Conclusion

The proposed decentralized implementations of the fake traffic provide desired statistical source anonymity with the minimal overhead and a delay that depends only on the efficiency of packet routing. Simultaneously, they utilize the network resources in a balanced and fair way, and provide flexibility necessary to handle different temporal and spatial profiles of the event process. By designing only one parameter, the size of the dummy population \( d \), according to the known statistical characteristic of the observed process we achieve such flexibility. In this paper, we discussed the minimal dummy population for several statistical models of the event. The minimal value of \( d \) depends on the implementation, as the deployment of group implementation requires \( d \) to be at least \( \frac{\mu}{\nu} \). Uniform spatial distribution of events does not call for the group implementation, and this constraint on \( d \) does not hold. However, \( d \) needs to be large enough to render \( \zeta_{Sp}/(d+1) \) statistically indistinguishable. Finally, if the event process include occasional bursts, the minimal value of \( d \) is defined by the ratio of the expected inter-burst time \( \mu \), and the expected intra-burst time (i.e. pause). Detailed analysis is omitted due to space constraints.

Our future work is to formalize a metric for the goodness of a WSN source anonymity scheme that includes the Eve’s outage probability and her work needed to collect statistically relevant samples. By including the adversary’s work and vulnerabilities, we aim to better model a global eavesdropper, and to present the quality of the source anonymity protection as relative to the adversary’s strength. In addition, the goodness metric should include the statistically guaranteed anonymity level, the work spent to obfuscate the events, and the latency guarantees by the proposed algorithm.

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