Subspace Clustering of Very Sparse High-Dimensional Data

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**Abstract**—In this paper we consider the problem of clustering collections of very short texts using subspace clustering. This problem arises in many applications such as product categorisation, fraud detection, and sentiment analysis. The main challenge lies in the fact that the vectorial representation of short texts is both high-dimensional, due to the large number of unique terms in the corpus, and extremely sparse, as each text contains a very small number of words with no repetition. We propose a new, simple subspace clustering algorithm that relies on linear algebra to cluster such datasets. Experimental results on identifying product categories from product names obtained from the US Amazon website indicate that the algorithm can be competitive against state-of-the-art clustering algorithms.

**Index Terms**—Subspace clustering, Principal angles, High-dimensionality, Short texts.

I. INTRODUCTION

In recent years, there has been an increasing need to understand and analyse the huge volumes of text data that have become available on different platforms. For example, Amazon may wish to automate their product categorisation based on product names and descriptions, Twitter may wish to utilise automatic online policing to identify sensitive and non-sensitive Tweets, etc. The lack of labels for the vast majority of such texts makes this an unsupervised learning problem.

In this work, we study the problem of clustering collections of very short texts. Short length has two important implications. First, in each “document” each word is effectively observed once. Second, the vast majority of pairs of texts have no words in common. These properties pose challenges for established text mining algorithms, as well as for statistical methods that employ generative models, such as the Latent Dirichlet Allocation [1], which require long texts to achieve reliable parameter estimates.

In this setting it is sensible to argue that texts sharing even a few number of common words are very similar to each other. Therefore associating clusters with linear combinations of the features (i.e. linear subspaces) is reasonable.

Subspace clustering refers to a set of methods that aim to identify clusters defined in linear and/or affine subspaces of the full-dimensional data. Such methods can be categorised into four classes: algebraic, statistical, iterative and spectral. Algebraic methods rely on matrix factorisation [3] or polynomial algebra [4] to identify subspaces. A highly cited method from this class is the robust subspace segmentation by low rank representation [5] (LRR). LRR relies on the idea that observations from the same cluster can be expressed by the same set of bases vectors, and thus as linear combinations of each other. It first builds a similarity matrix for the data through solving an optimisation problem based on the low rank data representation, and then applies spectral clustering.

Statistical methods impose explicit assumptions about the data generating process for each cluster. A probabilistic model is estimated based on the principles of maximum likelihood. A prominent method from this class is the mixture of probabilistic principal component analysers [6], which models each cluster as a multivariate Gaussian distribution.

Iterative methods refine the cluster assignment and the estimated subspaces to optimise an objective function. Projective $k$-means [7] (PKM) is such an extension of the classic $k$-means algorithm. PKM aims to minimise the root mean square error between each observation and its projection onto the corresponding low-dimensional subspace. This is achieved by alternating between computing the cluster centroids and updating the cluster assignment. In PKM the centroid of each cluster is the mean of the data projected onto a linear subspace defined by the principal components vectors.

Spectral (clustering)-based methods construct a similarity matrix that is representative of how close each pair of data
objects are, and then apply standard spectral clustering. The success of such methods critically depends on the choice of the similarity measure. Sparse Subspace Clustering [8] (SSC) estimates the similarity matrix by solving an optimisation problem which aims to express each observation as a linear combination of the other observations. The coefficients of the optimal combination are used in the similarity matrix.

In this paper we propose a new, simple subspace clustering algorithm, motivated by the characteristics of short texts. The algorithm first identifies subspaces that contain few but very similar observations. Then an appropriate dissimilarity measure is used to merge these subspaces into meaningful clusters. We apply the algorithm on a dataset of product names obtained from Amazon website and make available by the The Billion Prices Project [9], and show that its performance is competitive with state-of-the-art (subspace) clustering algorithms.

The rest of the paper is organised as follows. Section II presents the methodology, and the comparative evaluation of the proposed algorithm is provided in Section III. Conclusions and future research directions are discussed in Section IV.

II. METHODOLOGY

We obtain vectorial representation for the $N$ product names through the well established Term Frequency–Inverse Document Frequency (TF-IDF) [2] approach. Since each text is very short, and different texts contain different words, the TF-IDF matrix, $X \in \mathbb{R}^{P \times N}$, is sparse and high-dimensional. For the specific dataset we consider, each product name consists of a very small number of words (with effectively no repetition), and the vast majority of pairs of product names have no words in common.

In subspace clustering, each observation is assumed to lie on (or close to) a relatively low-dimensional subspace. A $d_k$-dimensional linear subspace, $S_k \subset \mathbb{R}^P$ is defined as,

$$S_k = \{ x \in \mathbb{R}^P : x = U_k y \},$$

where $U \in \mathbb{R}^{P \times d_k}$ is an orthonormal matrix defining the basis of the subspace, and $y \in \mathbb{R}^{d_k}$ is the representation of $x$ in terms of the column vectors of $U_k$. The goal of subspace clustering is to identify the $K$ subspaces, and assign each observation to the appropriate subspace. In the context of our problem, features of $X$ correspond to unique words. It is therefore sensible to assume that texts that share a combination of words are similar to each other.

The first step in the proposed approach is to transform the TF-IDF matrix, $X$, into its reduced row echelon form [10], by applying the well known Gauss-Jordan elimination. In this process a sequence of row operations are performed to bring $X$ into a form that satisfies:

1) the leftmost non-zero entry of each row is 1;
2) the leftmost non-zero entry of each row is the only non-zero entry in the corresponding column;
3) for any two different leftmost non-zero entries, one located in row $i$, column $j$ and the other in row $s$, column $t$. If $s > i$, then $t > j$;
4) rows in which every entry is zero are beneath all rows with non-zero entries.

Let $X_{ref}$ denote the reduced row echelon form of $X$. The columns of $X_{ref}$ that have a single non-zero element are called pivot columns. The first column of $X_{ref}$ is always a pivot column. Moreover, column $j > 1$ is a pivot column, if and only if the $j$-th column of $X$ cannot be expressed as a linear combination of the previous columns (i.e. columns $1, \ldots, j-1$). Let $j$ be a non-pivot column of $X_{ref}$. The non-zero elements in this column specify the coefficients of the linear combination of the previous pivot columns that yield the $j$-th column vector.

Since observations that can be written as linear combinations of each other belong to the same linear subspace, $X_{ref}$ provides valuable information to identify clusters spanning different subspaces [11]. A simple approach to identify subsets of observations that belong to the same linear subspace through $X_{ref}$ is the following. Define the matrix $Y \in \{0,1\}^{P \times N}$ as $Y(i,j) = 1(X_{ref}(i,j) \neq 0)$, where 1(·) is the indicator function that returns one if its argument is true and zero otherwise. Then the adjacency matrix, $A = Y^\top Y$, defines a graph, $G(A)$, whose connected components are subsets of observations that can be expressed as linear combinations of each other.

For the problem of clustering very short texts the graph $G(A)$ has a very large number of connected components, many of which consist of a single observation. Texts that belong to the same connected component are very similar, and hence this partitioning is very accurate in terms of purity [12], but it is of no practical use since it completely fails to capture broader groups. Figure I provides the histogram of the number of observations in each connected component of $G(A)$ for the Amazon product names dataset. As the figure shows, the vast majority of connected components contain less than ten observations, while the mode of this distribution is at one.

![Fig. 1: Histogram of the number of observations in each subspace identified through the reduced row echelon form of the TF-IDF matrix.](image-url)
To form meaningful clusters in such datasets, we need an appropriate measure of dissimilarity that would allow us to merge the previously identified subspaces. In this work, we utilise the concept of principal angles, first introduced in [13].

**Definition II.1 (Principal Angles).** Let \( S_i \) and \( S_j \) be two linear subspaces of an inner product space with \( 1 \leq \dim S_i = d_i \leq \dim S_j = d_j \). The principal angles,

\[
0 \leq \theta_1 \leq \theta_2 \leq \ldots \leq \theta_{d_i} \leq \pi/2,
\]

between \( S_i \) and \( S_j \) can be defined recursively for \( k = 1, \ldots, d_i \) as,

\[
\cos(\theta_k) = \max_{u \in S_i, v \in S_j} \cos(u^T v) = u_k^T v_k,
\]

subject to,

\[
\|u\| = \|v\| = 1, \quad \text{and} \quad u^T u_m = 0, \quad v^T v_m = 0, \quad \text{for} \quad 0 < m < k.
\]

Applying Principal Component Analysis (PCA) to the subset of observations assigned to each connected component of \( G(A) \), one readily obtains an orthonormal basis for each subspace. Let the columns of matrices \( Q_{S_i} \in \mathbb{R}^{P \times d_i} \) and \( Q_{S_j} \in \mathbb{R}^{P \times d_j} \) constitute orthonormal bases for two linear subspaces \( S_i \) and \( S_j \), respectively. The principal angles between \( S_i \) and \( S_j \) can be obtained from the singular value decomposition, \( Q_{S_i}^T Q_{S_j} = Y ZZ^T \), as follows

\[
\theta_k = \arccos(\Sigma(k, k)), \quad i \in \{1, \ldots, d_i\}. \tag{1}
\]

Principal angles ignore the difference in dimensionality between the two subspaces, which for our purposes is very important. To accommodate for this, we assume that \( S_i \) and \( S_j \) have maximum dissimilarity along the dimensions \((d_j - d_i)\). Thus we define the dissimilarity between two linear subspaces, \( S_i \) and \( S_j \) as,

\[
D(i, j) = \frac{1}{d_j} \left( d_j - d_i + \sum_{i=1}^{d_i} (1 - \cos(\theta_i)) \right),
\]

\[
= 1 - \frac{1}{d_j} \sum_{i=1}^{d_i} \cos(\theta_i). \tag{2}
\]

To obtain the final set of \( K \) clusters we apply the spectral clustering algorithm of Ng et al. [14] using \( D \) as the dissimilarity matrix. This spectral clustering algorithm uses the Gaussian kernel on pairwise distances / dissimilarities, as such its performance depends on the choice of the bandwidth parameter. In this work we use the local scaling rule proposed in [15],

\[
W(i, j) = \exp \left\{ -\frac{D(i, j)^2}{s_i s_j} \right\}, \tag{3}
\]

where \( s_i \) (\( s_j \)) is the dissimilarity of the \( i \)-th (\( j \)-th) observation to its \( k \)-th nearest neighbour. All the observations allocated to a given subspace are assigned to the same cluster label as the subspace. Algorithm [1] outlines the steps of the proposed approach.

### Algorithm 1: Minimum Angle Clustering (MAC)

| Input | TF-IDF matrix \( X \in \mathbb{R}^{P \times N} \); Number of clusters \( K \) |
|-------|-------------------------------------------------|
| Output | Cluster assignment \( C \in \{1, \ldots, K\} \) |

1. Compute Reduced Row Echelon Form: \( X_{\text{ref}} = \text{rref}(X) \)
2. Define matrix \( Y \) through \( Y(i, j) = 1(X_{\text{ref}}(i, j) \neq 0) \)
3. Construct graph: \( G \) from adjacency matrix \( A = Y^T Y \)
4. Compute connected components of \( G \): \( \{c_1, \ldots, c_{n_c}\} \)
5. for \( i = 1 \) to \( n_c \) do
   6. Apply PCA to \( X(:,c_i) \) to obtain orthonormal basis for \( i \)-th subspace \( Q_i \in \mathbb{R}^{P \times d_i} \)
   7. for \( j = 1 \) to \( i - 1 \) do
      8. Estimate dissimilarity with previous subspaces, \( D(i, j) \) through Eq. (2)
   end
11. Apply Spectral Clustering on \( W \) defined in Eq. (3) to obtain cluster assignment of subspaces
12. To all the observations in each connected component of \( G(A) \) assign the same cluster label as that of the associated subspace

### III. Experimental Results

In this section, we compare the performance of our proposed method against state-of-the-art subspace, and standard clustering algorithms on the task of clustering Amazon product names dataset [9]. This dataset contains five broad product categories: Electronics, Home and appliances, Mix, Office products, and Pharmacy and Health. We use the standard TF-IDF format to represent the product names. The resulting TF-IDF matrix has 2921 observations and and 2106 features/unique words.

We compare the performance of MAC with the following clustering algorithms: Sparse Subspace Clustering [8] (SCC), Low Rank Representation [5] (LRR), Projective k-Means Clustering [7] (PKM), Spectral Clustering [14] (SC), and Principal Component Divisive Partitioning [16] (PDDP).

SCC, LRR and PKM, are state-of-the-art subspace clustering algorithms. PDDP is included as it has been developed for the purpose of partitioning documents that have been embedded in high-dimensional Euclidean space. SC is a generic clustering methodology that has been successfully applied on numerous high-dimensional applications, most notably image segmentation [17]. A further reason for including this algorithm in the performance comparison is that MAC employs SC at its last step. Thus we need to investigate first, whether the performance of our algorithm is attributable to SC; and second whether the information from the connected components of \( G(A) \) suffices to correctly identify the clusters in this dataset (and hence the next step of defining dissimilarity based on principal angles is not worthwhile). For completeness we also consider Latent Dirichlet Allocation [1] (LDA) which has been widely applied in text mining.
| Method | MAC | SSC | LRR | PKM | SC(X) | SC(A) | LDA | PDDP |
|--------|-----|-----|-----|-----|-------|-------|-----|------|
| Purity | 0.742 | 0.219 | 0.510 | 0.591 | 0.512 | 0.519 | 0.510 | 0.578 |
| NMI    | 0.328 | 0.032 | 0.041 | 0.218 | 0.022 | 0.052 | 0.021 | 0.084 |
| ARI    | 0.251 | 0.025 | -0.023 | 0.191 | 0.000 | -0.024 | 0.011 | 0.065 |
| Runtime | 137.672 | 421.231 | 3050.412 | 148.141 | 6.652 | 96.688 | 4.624 | 15.713 |

TABLE I: Clustering performance and runtime comparison (in seconds) on US Amazon web-scraped dataset.

We assess performance through three external cluster evaluation measures: Purity [12], Normalised Mutual Information [18] (NMI), and Adjusted Rand Index [19] (ARI). For all three measures higher values indicate superior performance in the sense that cluster labels are in better agreement with the actual cluster assignment. Purity and NMI assume values in [0, 1], while the adjusted Rand index is in [−1, 1]. Table I reports the performance of all algorithms on our dataset. As the table shows, MAC outperforms the other methods with respect to all three measures. It is important to note that the performance of MAC is substantially better than that of the two SC variants, the first using the original TF-IDF data representation (column SC(X) in the table), and the second using as similarity matrix the adjacency matrix A obtained after transforming the matrix into the reduced row echelon form. The second best performing method is PKM, while the purity scores for PDDP and the two SC variants are comparable to that of PKM. With the exception of PKM, MAC achieves an improvement of an order of magnitude compared to all other algorithms with respect to NMI and ARI.

IV. CONCLUSIONS

We proposed a new, simple algorithm for subspace clustering that is effective in clustering collections of very short texts. The algorithm is designed to exploit the properties of the very sparse and high-dimensional TF-IDF representation of such datasets. It first identifies low-dimensional linear subspaces that contain small clusters of texts that share common words. To merge these into meaningful clusters we use principal angles to quantify the dissimilarity between linear subspaces, which in the present context correspond to combinations of words. Experimental results on a dataset of product names show that this simple approach compares favourably with standard and subspace clustering methods.

In future work, we aim to develop approaches to correctly identify the hierarchical structure of product categories. We also aim to investigate active learning approaches to assist the cluster validation process.

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