Gigahertz frequency comb offset stabilization based on supercontinuum generation in silicon nitride waveguides

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Abstract: Silicon nitride (Si3N4) waveguides represent a novel photonic platform that is ideally suited for energy efficient and ultrabroadband nonlinear interactions from the visible to the mid-infrared. Chip-based supercontinuum generation in Si3N4 offers a path towards a fully-integrated and highly compact comb source for sensing and time-and-frequency metrology applications. We demonstrate the first successful frequency comb offset stabilization that utilizes a Si3N4 waveguide for octave-spanning supercontinuum generation and achieve the lowest integrated residual phase noise of any diode-pumped gigahertz laser comb to date. In addition, we perform a direct comparison to a standard silica photonic crystal fiber (PCF) using the same ultrafast solid-state laser oscillator operating at 1 µm. We identify the minimal role of Raman scattering in Si3N4 as a key benefit that allows to overcome the fundamental limitations of silica fibers set by Raman-induced self-frequency shift.

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References and links

1. L.-S. Ma, Z. Bi, A. Bartels, L. Robertsson, M. Zucco, R. S. Windeler, G. Wilpers, C. Oates, L. Hollberg, and S. A. Diddams, “Optical frequency synthesis and comparison with uncertainty at the 10(-19) level,” Science 303(5665), 1843–1845 (2004).
2. H. R. Telle, G. Steinmeyer, A. E. Dunlop, J. Stenger, D. H. Sutter, and U. Keller, “Carrier-envelope offset phase control: A novel concept for absolute optical frequency measurement and ultrashort pulse generation,” Appl. Phys. B 69(4), 327–332 (1999).
3. D. J. Jones, S. A. Diddams, J. K. Ranka, A. Stenzl, R. S. Windeler, J. L. Hall, and S. T. Cundiff, “Carrier-envelope phase control of femtosecond mode-locked lasers and direct optical frequency synthesis,” Science 288(5466), 635–639 (2000).
4. A. Apolonski, A. Poppe, G. Tempea, C. Spielmann, T. Udem, R. Holzwarth, T. W. Hänsch, and F. Krausz, “Controlling the phase evolution of few-cycle light pulses,” Phys. Rev. Lett. 85(4), 740–743 (2000).
5. T. Udem, R. Holzwarth, and T. W. Hänsch, “Optical frequency metrology,” Nature 416(6877), 233–237 (2002).
6. J. Stenger, H. Schnatz, C. Tamm, and H. R. Telle, “Ultraprecise measurement of optical frequency ratios,” Phys. Rev. Lett. 88(7), 073601 (2002).
7. A. Bartels, D. Heinecke, and S. A. Diddams, “10-GHz self-referenced optical frequency comb,” Science 326(5953), 681 (2009).
8. I. Hartl, H. A. McKay, R. Thapa, B. K. Thomas, L. Dong, and M. E. Fermann, “GHz Yb-femtosecond-fiber laser frequency comb,” in Conference on Lasers and Electro-Optics/International Quantum Electronics Conference(Optical Society of America, Baltimore, Maryland, 2009), p. CMN1.
9. D. Chao, M. Sander, G. Chang, J. Morse, J. Cox, G. Petrich, L. Kolodziejski, F. Kaertner, and E. Ippen, “Self-referenced erbium fiber laser frequency comb at a GHz repetition rate,” in Optical Fiber Communication Conference(Optical Society of America, Los Angeles, California, 2012), p. OW1C.2.

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10. S. Pekarek, T. Südmeier, S. Lecomte, S. Kandermann, J. M. Dudley, and U. Keller, “Self-referenceable frequency comb from a gigahertz diode-pumped solid-state laser,” Opt. Express 19(17), 16491–16497 (2011).

11. K. Ickes, R. E. Saperstein, N. Ali, and Y. Fainman, “Thermal and Kerr nonlinear properties of plasma-deposited silicon nitride/silicon dioxide waveguides,” Opt. Express 16(17), 12987–12994 (2008).

12. U. Keller, “Recent developments in compact ultrafast lasers,” Nature 424(6950), 831–838 (2003).

13. A. S. Mayer, A. Klenner, A. R. Johnson, K. Luke, M. R. E. Lamont, Y. Okawachi, M. Lipson, A. L. Gaeta, and U. Keller, “Frequency comb offset detection using supercontinuum generation in silicon nitride waveguides,” Opt. Express 23(12), 15440–15451 (2015).

14. A. R. Johnson, A. S. Mayer, A. Klenner, K. Luke, E. S. Lamb, M. R. E. Lamont, C. Joshi, Y. Okawachi, F. W. Wise, M. Lipson, U. Keller, and A. L. Gaeta, “Octave-spanning coherent supercontinuum generation in a silicon nitride waveguide,” Opt. Lett. 40(21), 5117–5120 (2015).

15. V. R. Almeida, C. A. Barrios, R. R. Panepucci, and M. Lipson, “All-optical control of light on a silicon chip,” Nature 431(7012), 1081–1084 (2004).

16. S. Schilt and T. Südmeier, “Carrier-envelope offset stabilized ultrafast diode-pumped solid-state lasers,” Appl. Sci. 5(4), 787–816 (2015).

17. D. Milam, “Review and assessment of measured values of the nonlinear refractive-index coefficient of fused silica,” Appl. Opt. 37(3), 546–550 (1998).

18. X. Jiang, N. Y. Joly, M. A. Finger, F. Babic, G. K. L. Wong, J. C. Travers, and P. S. J. Russell, “Deep-ultraviolet to mid-infrared supercontinuum generated in solid-core ZBLAN photonic crystal fibre,” Nat. Photonics 9(2), 133–139 (2015).

19. D. Lorenc, M. Aranyosiova, R. Buczyński, R. Stepień, I. Bugar, A. Vincze, and D. Velic, “Nonlinear refractive index of multicomponent glasses designed for fabrication of photonic crystal fibers,” Appl. Phys. B 93(2-3), 531–538 (2008).

20. G. Sobon, M. Klimeczak, J. Sotor, K. Krzempek, D. Pysz, R. Stepień, T. Martynkien, K. M. Abramski, and R. Buczyński, “Infrared supercontinuum generation in soft-glass photonic crystal fibers pumped at 1560 nm,” Opt. Mater. Expr. 4(1), 17–15 (2014).

21. E. Yousef, M. Hotzel, and C. Rüssel, “Linear and non-linear refractive indices of tellurite glasses in the system TeO2–WO3–ZnF2,” J. Non-Cryst. Solids 342(1-3), 82–88 (2004).

22. B. J. Eggleton, B. Luther-Davies, and K. Richardson, “Chalcogenide photonics,” Nat. Photonics 5, 141–148 (2011).

23. P. B. Corkum, P. P. Ho, R. R. Alfano, and J. T. Manassah, “Generation of infrared supercontinuum covering 3-14 microm in dielectrics and semiconductors,” Opt. Lett. 10(12), 624–626 (1985).

24. U. D. Dave, C. Ciret, S.-P. Gorza, S. Combrie, A. De Rossi, F. Raineri, G. Roelkens, and B. Kuyken, “Dispersive-wave-based octave-spanning supercontinuum generation in InGaP membrane waveguides on a silicon substrate,” Opt. Lett. 40(15), 3584–3587 (2015).

25. R. DeSalvo, A. A. Said, D. J. Hagan, E. W. VanStryland, and M. Sheik-Bahae, “Infrared to ultraviolet measurements of two-photon absorption and n2 in wide bandgap solids,” IEEE J. Quantum Electron. 32(8), 1324–1333 (1996).

26. C. R. Phillips, C. Langrock, J. S. Pelc, M. M. Fejer, J. Jiang, M. E. Ferrmann, and I. Hartl, “Supercontinuum generation in a silicon nanowire,” Opt. Lett. 36(19), 3912–3914 (2011).

27. H. Guo, X. Zeng, B. Zhou, and M. Bache, “Few-cycle solitons and supercontinuum generation with cascaded quadratic nonlinearities in unpoled lithium niobate ridge waveguides,” Opt. Lett. 39(5), 1105–1108 (2014).

28. D. J. Mosa, R. Morandotti, A. L. Gaeta, and M. Lipson, “New CMOS-compatible platforms based on silicon nitride and Hydex for nonlinear optics,” Nat. Photonics 7(8), 597–607 (2013).

29. R. Soref, “Mid-infrared photonics in silicon and germanium,” Nat. Photonics 4(8), 495–497 (2010).

30. B. Kuyken, T. Ideguchi, S. Holznner, M. Yan, T. W. Hänsch, J. Van Campenhout, P. Verheyen, S. Coen, F. Leo, R. Baets, G. Roelkens, and N. Piqué, “An octave-spanning mid-infrared frequency comb generated in a silicon nanophotonic wire waveguide,” Nat. Commun. 6, 6310 (2015).

31. R. K. W. Lau, M. R. E. Lamont, A. G. Griffith, Y. Okawachi, M. Lipson, and A. L. Gaeta, “Octave-spanning mid-infrared supercontinuum generation in silicon nanowaveguides,” Opt. Lett. 39(15), 4518–4521 (2014).

32. N. Singh, D. D. Hudson, Y. Yu, C. Grillot, S. D. Jackson, A. Casas-Bedoya, A. Read, P. Atanackovic, S. G. Duvall, S. Palomba, B. Luther-Davies, S. Madden, D. J. Moss, and B. J. Eggleton, “Midinfrared supercontinuum generation from 2 to 6 µm in a silicon nanowire,” Optica 2(9), 797–802 (2015).

33. M. Foster, K. Moll, and A. Gaeta, “Optimal waveguide dimensions for nonlinear interactions,” Opt. Express 12(13), 2880–2887 (2004).

34. D. Marcuse, “Loss Analysis of Single-Mode Fiber Splices,” Bell Syst. Tech. J. 56(5), 703–718 (1977).

35. U. Keller, K. J. Weingarten, F. X. Kärtner, D. Kopf, B. Braun, I. D. Jung, R. Fluck, C. Höninger, N. Matuschek, and J. Aus der Au, “Semiconductor saturable absorber mirrors (SESAMs) for femtosecond to nanosecond pulse generation in solid-state lasers,” IEEE J. Sel. Top. Quantum Electron. 2(3), 435–453 (1996).

36. J. Petit, P. Goldner, and B. Viana, “Laser emission with low quantum defect in Yb: CaGdAlO4.,” Opt. Lett. 31(11), 1345–1347 (2005).

37. A. Klenner, S. Schilt, T. Südmeier, and U. Keller, “Gigahertz frequency comb from a diode-pumped solid-state laser,” Opt. Express 22(25), 31008–31019 (2014).

38. A. Klenner, M. Holling, and U. Keller, “High peak power gigahertz Yb:CALGO laser,” Opt. Express 22(10), 11884–11891 (2014).
1. Introduction

Optical frequency combs consist of ultrafine optical lines that are equally spaced in the frequency domain with an uncertainty down to the $10^{-19}$ level [1]. Over the past 15 years, the generation and stabilization of such combs [2–4] has enabled control of the cycles of light, which represents a major achievement for laser research and high precision optical metrology [5,6]. The generated comb can be fully stabilized by phase-locking both the spacing of the comb lines and the comb offset, which is also known as the carrier envelope offset (CEO). While stabilizing the comb line spacing of a frequency comb generated by a modelocked laser requires the stabilization of the pulse repetition rate, i.e. by simply acting on the position of a cavity mirror, the detection and stabilization of the CEO is more challenging. Self-referenced comb offset stabilization is one of the preferred schemes, as it allows for phase locking without a highly-stable external optical reference. For the most common self-referencing technique based on $f$-to-$2f$ interferometry, optical comb spectra spanning at least one octave are required [2]. Such ultrabroadband spectra are typically generated through the process of supercontinuum generation (SCG). Traditionally, SCG is done at MHz pulse repetition rates by propagating a modelocked femtosecond laser pulse through a highly nonlinear silica fiber. Such systems are designed to be embedded in specialized laboratories for experiments that require operation at medium to high pulse energies. However, the size and cost of these setups currently prevents wide-spread comb applications outside of laboratories. For many applications in spectroscopy and time-and-frequency metrology, low-energy frequency combs in the 1 to 10 GHz regime are highly desirable [7–10]. Such gigahertz combs typically generate lower peak intensities and therefore would greatly benefit from more efficient SCG. The strength of the nonlinear interactions leading to spectral broadening of a pulse propagating inside a waveguide is determined by the nonlinear phase shift $\gamma PL$, where $P$ is the power and $L$ is the waveguide length. The nonlinear parameter $\gamma$ is defined as

$$\gamma = \frac{2\pi n_2}{A_{\text{eff}} \lambda}$$

(1)

where $n_2$ is the material nonlinear index, $\lambda$ is the vacuum wavelength, and $A_{\text{eff}}$ is the effective area of the mode propagating inside the waveguide. For a given length $L$, i.e. fixed accumulated dispersion, lowering the pulse energy required to obtain an octave-spanning coherent spectrum must be compensated for by increasing the value of $\gamma$. An obvious first approach to obtaining a high value of $\gamma$ is thus to choose a waveguide core material with a high $n_2$. Fig. 1(a) gives an overview of the material platforms that have been used to demonstrate octave-spanning SCG. Si$_3$N$_4$ is a highly promising material for nonlinear optics and frequency comb generation since it offers a high nonlinear index of $2.4 \times 10^{-15}$ cm$^2$/W and a broad transparency range from visible wavelengths to the mid-IR [11].
Ultrafast diode-pumped solid-state laser oscillators [12] have most recently produced ultrabroadband supercontinua (SC) using chip-based silicon nitride (Si$_3$N$_4$) photonic waveguides [13, 14]. Multi-octave-spanning coherent spectra have been obtained with an order of magnitude less pulse energy as a direct result of the higher nonlinearity provided by these silicon-based photonics platform [15] compared to silica fibers. In comparison to materials with similar nonlinear indices, such as lithium niobate, lead-bismuth-silicate and tellurite glasses, Si$_3$N$_4$ has the additional benefit of being compatible with the complementary metal-oxide-semiconductor (CMOS) fabrication techniques, which are widely used in industry. Compact and robust Si$_3$N$_4$ waveguides can be produced at low costs on wafer scale using standard deposition and lithography methods. Simple dispersion engineering by adjusting the geometrical dimensions of the waveguide allows for designing dispersion profiles with more than one zero-dispersion-wavelengths (ZDW’s) to achieve highly coherent octave-spanning spectra.

![Fig. 1](image_url)

**Fig. 1.** (a) Overview of materials and their nonlinear indices $n_2$ which have been used to generate octave-spanning spectra [11, 17–32] The transparency region of each material is indicated. (b) Exemplary representation of the influence of mode-confinement on the nonlinear parameter. The optimal physical core area that yields the maximal $\gamma$ as a function of wavelength was determined using the relation described in Foster et al. [33] for 3 different platforms. The $\gamma$ values divided by the respective material nonlinearity $n_2$ of the actual oxide-clad Si$_3$N$_4$ and the silica PCF used for the experimental results presented in this paper are marked with stars.

Foster et al. [33] provides design guidelines for a maximum value of $\gamma$ as a function of the wavelength and the linear refractive indices of core and cladding. The best result is achieved for the smallest possible core that still enables confinement of the fundamental mode without a significant amount of power being guided in the low-nonlinearity cladding. Figure 1(b) shows the influence of linear core/cladding index contrast on the nonlinear parameter $\gamma$ as a function of wavelength for 3 exemplary cases: the extreme case of a silica PCF, i.e. corresponding to a silica nanotaper core fully surrounded by air, a Si$_3$N$_4$ with a silica cladding, and an air-clad Si$_3$N$_4$-nanotaper. The optimal core size and the resulting maximum $\gamma' = \gamma/n_2$ value are depicted assuming a Gaussian-like fundamental beam profile, for which the effective area is $A_{\text{eff}} = \pi w^2$ with $w$ denoting the mode radius. The relation between the physical core radius and the radius of the mode is estimated using the Marcuse formula [34]. For comparison, the normalized values $\gamma'$ of the Si$_3$N$_4$/SiO$_2$ waveguide presented in this paper and the photonic crystal fiber (PCF) NL-3.2-945 (NKT Photonics) are included. It can be seen that for the Si$_3$N$_4$ waveguide, the value is close to the theoretical optimum, whereas for the commercial PCF it lies an order of magnitude below a silica nanotaper. This comparably low nonlinearity is a result of an air-hole patterned cladding which is far from providing the index contrast one would obtain in the case of a fragile, rod-type silica nanotaper. Thus the 140-times higher nonlinear parameter $\gamma$ of the Si$_3$N$_4$-waveguide presented in this paper compared to the PCF (3.25 W$^{-1}$m$^{-1}$ vs. 0.023 W$^{-1}$m$^{-1}$) originates from an order of magnitude difference in both the nonlinear material index $n_2$, and the mode confinement.
In this paper we demonstrate the first self-referenced frequency comb offset stabilization based on SCG in Si$_3$N$_4$ waveguides and show that this platform is particularly suitable for low-energy and low-noise stabilized frequency comb generation. We use a SESAM-modelocked [35] diode-pumped Yb:CALGO [36] solid-state laser with a center wavelength of ~1 µm [37, 38] and achieve the lowest integrated residual phase noise of any diode-pumped gigahertz laser comb to date.

Moreover, we numerically investigate the coherence of supercontinua generated in Si$_3$N$_4$ compared to silica fibers. Standard photonic crystal fibers (PCF’s) for wavelengths > 1 µm often rely on the Raman effect to provide sufficient spectral broadening. We show that the coherence of the Raman self-frequency shifted spectral components rapidly degrades with increasing nonlinearity $\gamma$, which prevents coherent SCG and thus CEO beat note detection at low pulse energies. The Raman effect in Si$_3$N$_4$ waveguides is regarded to be very weak (see [39] supplementary material). The lack of significant Raman gain in single-pass Si$_3$N$_4$ waveguides represents a clear advantage over silica fibers for highly efficient frequency comb generation, since it allows for generating coherent supercontinua with higher nonlinearity and lower energies.

2. Supercontinuum generation in Si$_3$N$_4$ waveguides versus silica PCF

The Si$_3$N$_4$ waveguide used in our experiments is designed to provide anomalous group velocity dispersion surrounding the 1-µm pump wavelength (Figs. 2(a)-2(e)). The dispersion of the Si$_3$N$_4$ waveguide is customized by engineering the contribution of the waveguide dispersion using a waveguide cross section of about 690 × 880 nm. For our short pump pulses (<100 fs), coherent SCG occurs through higher-order soliton compression and the emission of short- and long-wavelength dispersive waves (DW’s) [40, 41]. The waveguide length is chosen to be 7.5 mm to cut off propagation after DW emission to allow for the broadest spectral bandwidth and to avoid any degradation of coherence. The spectral evolution with propagation distance for this waveguide is shown in Fig. 3(a) for a coupled pulse energy of 26 pJ, with an input pulse centered at 1055 nm, FWHM bandwidth of 18 nm, and a slight positive chirp (800 fs$^2$), corresponding to the experimental parameters provided by our gigahertz diode-pumped solid-state laser oscillator.

For comparison, an example of SCG in a commercially available PCF (NKT NL3.2-945, one ZDW at 945 nm) is shown in Fig. 3(b), using the same pulse duration and chirp as mentioned above, however with 377 pJ of coupled pulse energy. In contrast to the SC generated in the Si$_3$N$_4$ waveguide, the spectrum at the soliton fission point is not yet suitable for f-to-2f interferometry. A fiber length of 1 m is necessary until the ejected soliton has shifted to sufficiently long wavelengths via the Raman self-frequency shift and shorter wavelengths are accessed via cross phase modulation (XPM) of the shifted soliton and the DW. For the experimental input parameters mentioned so far, the first-order coherence is close to unity over the whole spectral bandwidth for both platforms (Figs. 5(c) and 5(d)). However, in order to obtain the same spectral span with as little energy as in the case of the Si$_3$N$_4$ waveguide, either the fiber length or the nonlinear parameter of the silica fiber would have to be increased accordingly (i.e. via tapering of the cross section).

In the following, we show that such an increase of the nonlinear parameter in a silica fiber yields to a deterioration of the coherence as a result of the Raman effect. For this purpose, we numerically solve the general nonlinear Schrödinger equation (GNLSE) [40, 42] for a scan of nonlinear parameters $\gamma' = K \times \gamma$ and pulse energies $E'_p = E_p / K$, where $K$ is increased from 1 to 40 in steps of 4. We keep the dispersion profile unaltered and maintain a constant soliton order (i.e. $N = 3$) by decreasing the input power accordingly, so that the spectral width and shape of the supercontinuum remains the same.
Fig. 2. Si$_3$N$_4$ photonic waveguide chip. (a) A Si$_3$N$_4$ waveguide with a rectangular cross-section is sandwiched in a SiO$_2$ layer on the buried oxide of a Si substrate. Optimized light coupling with comparably low losses of $< 8$ dB is achieved using inverse tapers at the input and output. The inverse tapers have a length of 100 µm and a 3-µm gap between the end of the taper and the facet [43]. (b) The photograph shows a typical Si$_3$N$_4$ chip that incorporates 7 waveguides of various lengths. An SEM image of the rectangular cross-section is shown in (c). (d) The fundamental guided mode which is guided in the waveguide was simulated with a finite element solver. The waveguide bending radii are kept $>100$ um to avoid additional dispersion effects. The resulting dispersion curve featuring two zero-dispersion wavelengths (ZDW’s) is shown in (e).

The GNLSE is solved with a split-step Fourier-transform algorithm [42]. The nonlinear step is done with a second-order Runge-Kutta method as published in Dudley et al. [40]. Both stimulated Raman as well as spontaneous Raman effects are included. The latter are thermally induced and depend on the Bose-Einstein distribution of vibrational modes in the material at a given temperature. The effect is independent of the input power, but scales with the nonlinearity as $\sqrt{\gamma}$.

We observe a clear degradation of the coherence of the generated SC, which is particularly pronounced for the spectral components around the Raman-shifted soliton and even stronger for its XPM wave at short wavelength (Fig. 3(c)). The noise processes that may deteriorate the coherence can be divided into three main components; technical laser noise, input shot noise and spontaneous Raman scattering. While the first one is discarded in these simulations, the input shot noise is accounted for with one photon per mode. The spontaneous Raman process describes photon-phonon scattering at the silica crystal. In Fig. 3(d) the coherence values at the soliton (1360 nm) and the XPM wave (680 nm) is depicted for different scaling factors $K$. A scaling factor of $K = 40$ leads to a drop in coherence for the PCF below 80% at 1360 nm (green area) and 60% at 680 nm (purple area). Essentially the same coherence degradation is observed when the shot noise is turned off and only spontaneous Raman effects are taken into account (green and purple circles). In the case of pure input shot noise (green and purple crosses), i.e. spontaneous Raman scattering switched off, only a slight influence is noticeable.

This coherence degradation becomes even more pronounced for higher soliton orders (i.e. $N = 7$). It becomes dominant whenever the Raman self-frequency shift has a large...
contribution to the SCG process in a highly nonlinear silica fiber. Especially in the wavelength range from 1 to 1.5 µm, most silica PCF’s provide a single ZDW and the Raman self-frequency shift is needed to extend the spectrum to longer wavelengths. A platform that does not exhibit significant Raman gain, as it is the case for Si$_3$N$_4$, represents a clear advantage for achieving highly coherent supercontinua at very low energies. In general, the highest coherence is usually achieved close to the soliton fission point. Designing the platform such as to provide an octave-spanning spectrum at the fission point thus ultimately optimizes the quality of the CEO beat note detected by $f$-to-$2f$ interferometry.

![Image](https://example.com/image.png)

Fig. 3. Spectral evolution of the SC and the loss of coherence due to the Raman effect. (a) The SC in Si$_3$N$_4$ is generated within 7.5 mm by dispersive wave (DW) emission at long and short wavelengths. (b) The photonic crystal fiber (PCF) initially generates a DW at short wavelengths. During the 1-m propagation length a Raman-induced soliton self-frequency shift and cross-phase modulation (XPM) between soliton and DW generates the required bandwidth. (c) A potentially high nonlinearity in the PCF (high scaling factor $K$) causes a significant degradation of the coherence of the frequency-shifted soliton and XPM wave. (d) The coherence for the PCF drops below 80% at 1360 nm (green area) and 60% at 680 nm (purple area) for $K = 40$. The coherence loss is mainly caused by the spontaneous Raman response in the PCF. Simulations with Raman but without shot noise are shown as purple and green circles. Simulations with shot noise only (green crosses: 1360 nm; purple crosses: 680 nm) show much less degradation.

3. Self-referenced frequency comb offset detection

The ultrafast laser used for comparing the performance of both SCG platforms is a SESAM-modelocked diode-pumped Yb:CALGO [36] laser that generates pulses with 64-fs duration at a pulse repetition rate of 1.025 GHz and can provide an average output power as high as 1.8 W at a center wavelength of 1055 nm described in more details in [37]. Without any additional pulse amplification or nonlinear compression, the beam is either coupled into the silica PCF or the Si$_3$N$_4$ waveguide (Fig. 4).

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In order to detect the carrier-envelope-offset frequency \(f_{\text{CEO}}\) of our laser, we use a Michelson-type \(f\)-to-2\(f\) interferometer, where light around 1360 nm is doubled in a periodically poled lithium niobate (PPLN) crystal and overlapped in space and time with the spectral components at 680 nm. The generated beat signal is recorded by an avalanche photodiode (APD). The CEO beat notes and corresponding optical spectra obtained with 36 pJ (377 pJ) of coupled pulse energy in the Si\(_3\)N\(_4\) waveguide (silica PCF), are shown in Figs. 5(a)-5(d). Our numerical simulations are in very good agreement with the experimental spectra and show that the SC of both platforms are highly coherent. We note that the Si\(_3\)N\(_4\)-based spectrum yields 500 nm more bandwidth and features less amplitude modulation especially in the spectral parts used for the CEO beat note detection. The beat note detection for a self-referenced stabilization requires a high degree of coherence but also sufficient temporal overlap of both spectral components.

While the absolute signal-to-noise ratio (SNR) of the CEO beats also depends on factors such as the amount of light sent onto the APD and the resolution bandwidth of the microwave spectrum analyzer, the difference in signal-to-signal ratio between the CEO beat signals and the repetition rate signal mainly originates from the spectral shape and coherence of the SC spectra, \textit{i.e.} the distribution of optical power and noise over the wavelength range of interest, and thus is an inherent property of the SCG platform. Balanced optical power levels in both interferometer arms result in the highest signal-to-signal ratio. Although the pulse repetition rate signal is filtered out in the CEO-stabilization feedback loop, maximizing this signal-to-signal ratio has an implication on the achievable SNR of the CEO signal as it allows for reaching the same SNR (of the CEO signal) with less optical power on the APD and thus avoiding a higher noise floor and saturation effects.

An important improvement of the free-running CEO signal was achieved by pumping the modelocked laser with a laser-diode that has an integrated volume holographic grating (VHG). The grating stabilizes and reduces the spectral bandwidth of the spatially multimode pump diode (Lissotschenko Mikrooptik GmbH). The passive pump wavelength stabilization reduces the linewidth of the free-running \(f_{\text{CEO}}\) by one order of magnitude (Fig. 6).
Fig. 5. Detection of the comb offset frequency ($f_{CEO}$) of a 1.025-GHz SESAM modelocked diode-pumped solid-state laser using either (a) a Si$_3$N$_4$ waveguide or (b) a photonic crystal fiber (PCF) under the same laser operating conditions (input pulse spectrum centered at 1055 nm, FWHM 18 nm) and using the identical f-to-$2f$ interferometer with coupled pulse energies of 36 pJ and 377 pJ, respectively. SC spectrum generated in (c) Si$_3$N$_4$ and (d) the PCF. Both Si$_3$N$_4$ and PCF platforms provide fully coherent octave-spanning SC, however the Si$_3$N$_4$ waveguide provides 500 nm more bandwidth than the PCF. The SC generated in Si$_3$N$_4$ features much less amplitude modulation, which is highly desirable for applications. Simulations are shown in black, while colors denote experimental results. The indicated spectral portions at 1360 nm and 680 nm are used for f-to-$2f$ detection. Although both platforms provide similar CEO signal-to-noise ratios, the signal contrast to the repetition rate at 1.025 GHz is improved by 15 dB for Si$_3$N$_4$. RBW: resolution bandwidth, OSA: optical spectrum analyzer, FTIR: Fourier transform infrared spectrometer.

Fig. 6. Influence of integrated volume holographic grating (VHG) stabilization of the pump wavelength: The FWHM of the free-running CEO beat note is reduced from about 1.7 MHz to about 170 kHz for the same pump diode model with an integrated VHG.
4. Carrier envelope offset stabilization of a compact low-noise frequency comb

We use the generated strong CEO beat signal to perform the first frequency comb offset stabilization based on SCG in Si₃N₄. The linewidth of the free-running CEO scales with the pulse repetition rate: any jitter of the phase slip per cavity roundtrip, $\Delta \phi_{CEO}(t)$, is multiplied by the repetition rate to yield the CEO frequency according to the relation $f_{CEO}(t) = (\Delta \phi_{CEO}(t) / 2\pi) \times f_{rep}$ [2, 45]. As a result, the stabilization of combs at gigahertz repetition rate is more challenging. The VHG-stabilized pump laser significantly lowers the required bandwidth of the feedback electronics compared to previous results [37]. The comb-offset of the gigahertz source laser was phase-locked to an RF reference source (Agilent PSG Analog Signal Generator E8257D) using an analog feedback loop acting on the current of the pump diode. The feedback electronics consist of a phase detector (Menlo DXD200), whose output is filtered by a PID-controller (Vescent D2-125) and subsequently converted into a current signal using a homebuilt Voltage-to-Current (V-I) converter. The free-running and locked CEO beat note at 73.9 MHz are shown in Fig. 7(a). In Fig. 7(b), the 200-Hz-span at 1-Hz resolution bandwith (microwave spectrum analyzer limit) shows the strong coherent peak with minor power line noise at 50 Hz.

![Fig. 7. Stabilized comb offset frequency ($f_{CEO}$) based on SC generated in Si₃N₄ waveguide. (a) Free-running (blue) and stabilized $f_{CEO}$ showing a strong coherent peak at the reference frequency of 73.9 MHz (red) recorded with a microwave spectrum analyzer (MSA). Stabilization feedback loop resonances are visible at around 150 kHz. (b) Magnification of coherent peak at 1 Hz resolution bandwidth shows only minor line-noise at 50 Hz. (c) The frequency noise power spectral density (PSD) of the stabilized comb offset frequency measured with a signal source analyzer (Agilent E5052B) drops integrally below the $\beta$-separation line which is a necessary requirement for a tight phase lock [46]. (d) The integrated residual phase noise (PN) is 304 mrad [1 Hz, 5 MHz].](image)

The noise analysis of the phase-locked comb-offset frequency reveals a drastic drop of the in-loop CEO frequency noise upon onset of the feedback loop. The residual frequency noise power spectral density (PSD) lies integrally below the $\beta$-separation line (Fig. 7(c)), which is a necessary requirement for a tight phase-lock [46]. The integrated phase noise amounts to 304 mrad [1 Hz, 5 MHz] (Fig. 7(d)). To our knowledge, this is the lowest reported value from any gigahertz diode-pumped laser comb to date. The use of a VHG for pump wavelength...
stabilization has a major impact in achieving this low integrated phase noise. The lock is stable for several hours and the experiment was repeated several times with the same performance over a time-span of several months. We have repeated the locking experiment with the CEO beat note based on the PCF without a noticeable difference in the noise performance. As reported in Klenner et al. [37], a more sophisticated high-bandwidth analog electronic feedback loop, which acts on the current of the multimode pump diode, is also capable of stabilizing the CEO-beat notes with a larger FWHM and a lower SNR (i.e., as obtained without pump wavelength stabilization).

5. Conclusion

We have shown the first self-referenced comb offset stabilization of a modelocked laser based on SCG in Si$_3$N$_4$ waveguides. The stabilized SESAM modelocked laser provides, to the best of our knowledge, the lowest integrated residual phase noise of any diode-pumped gigahertz laser comb to date. The combination of highly coherent SCG and passive wavelength stabilization of the diode-laser pump leads to this superior performance. The energy requirement for octave-spanning SCG is reduced by more than an order of magnitude compared to a commercial silica PCF. Moreover, the core/cladding composition of Si$_3$N$_4$ and SiO$_2$ allows for high confinement while preserving the possibility to engineer the nonlinear parameter $\gamma$ as well as the dispersion profile for various laser sources without the need for fragile suspended nanotapers.

We show that the absence of a significant Raman gain is a key benefit for coherent SCG at low energies and a high nonlinear parameter. The experimentally generated SC spectrum in Si$_3$N$_4$ also exhibits much less amplitude modulations. The compatibility of Si$_3$N$_4$ waveguides with CMOS technology opens new avenues for stabilized compact frequency combs. We can envision an ultra-compact chip-device for integrating all three necessary stages for self-referencing: SCG, $f$-to-$2f$ interferometer with detection and control electronics. Such a chip-device would greatly reduce the cost and complexity and increase the robustness of stabilized optical frequency combs and further promote their use outside of the laboratory environment.

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