Abstract. Since agricultural cooperatives have developed rapidly under the farmland transfer policy in China, they play an important role in the new operation pattern for China’s fresh agricultural product supply chains. To enhance current agricultural supply chains’ stability, we consider a three-level (farmer-cooperative-retailer) fresh agricultural product supply chain, conduct quantitative analysis of the impact of the quantity flexibility contract, and compare the impact of the relational contract with that of the quantity flexibility contract on the freshness and the profit. Our results show that a suitable relational contract can improve the freshness and increase the profit of the three-level supply chain, but cannot fully guarantee its stability. Furthermore, the government’s subsidy policy can improve the relational contract stability of the fresh agricultural product supply chain by providing the protection price contract mechanism of the agricultural product and the cold chain facility subsidy contract.

1. Introduction. Fresh agricultural products have many characteristics like freshness, perishable and price instability, which have great impacts on supply chain operation. So fresh agricultural product supply chains have always attracted much attention from global scholars. Many research works suggest that the establishment
of appropriate coordination mechanisms is an important way to preserve the stability of the fresh agricultural product supply chain. These coordination mechanisms incorporate interest incentives, risk-sharing, and revenue sharing contracts. Specifically, one can use the incentive mechanism to improve the contractual stability of fresh products [6], the risk-sharing mechanism between producers and sellers [2, 13], and the coordination mechanism of the fresh agricultural product supply chain from the perspective of revenue sharing contract to promote the stability of its operation [22, 8].

On the other hand, the relational contract is also the key to affect the operational efficiency of the fresh agricultural product supply chain. We can reduce the transaction cost of production and operation through developing a supply chain contract relationship [23]. Assets specificity, reciprocal investment and trust affect the governance structure and the relational contract process [19]. Various models have been developed to analyze the contract stability of an agricultural product supply chain [21, 11, 20].

Moreover, the freshness is also important to affect the stability of fresh agricultural product supply chain, since the freshness of agricultural products will directly affect the utility of consumers, and further affect the demand for products, market prices and contract stability. Consumers will change demand according to the degree of deterioration of agricultural products [9]. Many researchers have studied the effect of freshness. For example, Herbon et al. [5] constructed a quadratic function to characterize the effect of freshness on consumer utility. Qin et al. [10] constructed a joint inventory management model to improve the freshness of agricultural products. Wang et al. [12] studied the effect of freshness on the profit of two-stage fresh agricultural product supply chain and presented the corresponding incentive mechanism. Chen et al. [3] considered the influence of freshness on the profit of fresh product, and constructed a dynamic pricing model considering freshness. Hou et al. [7] verified the impact of logistics service providers’ efforts on the freshness and quantity loss of agricultural products. Yang et al. [18] analyzed the incentive mechanism of fresh agricultural product supply chain from the perspective of revenue sharing contract. Therefore, how to design a suitable incentive mechanism to ensure the freshness is critical to the operation of fresh product supply chains.

In practice, the construction of cold chain facilities is the main measure to guarantee the freshness of agricultural products, and it is also conducive to maintain the relationship stability. Cold chain facilities have a significant effect on improving the freshness and the profit of the agricultural product supply chain, but different control methods [15, 17, 14, 4] have different impacts on the stability of the relational contract [16]. Therefore, how to build a suitable cold chain facility in a multi-level fresh agricultural product supply chain to improve the stability is another main issue which needs to be further explored. At present, the government has provided a subsidy policy for agricultural production and cold chain facilities construction in China. How to implement subsidies methods will have an important impact on the stability of the relational contract of the fresh product supply chain.

This paper considers a three-level (farmer-cooperative-retailer) fresh agricultural product supply chain, and quantitatively analyzes the impact of the quantity flexibility contract and the relational contract on the freshness and the profit. On this basis, we study the impact of different models of China’s government subsidy contract on stability of the three-level fresh agricultural product supply chain. This
paper’s main contributions are as follows: (1) Most of the existing works on the stability of the supply chain relationship of fresh agricultural products are based on the perspective of two-level supply chain. Under the background of farmland transfer, a large number of agricultural cooperatives have emerged in China, and they play an essential role to improve the operational efficiency and contract stability of the fresh agricultural product supply chain. The study of contract stability under the new three-level supply chain framework is more suitable for the current agricultural product supply chain in China’s actual operation and development trend. (2) We analyze various subsidy models in the three-level fresh agricultural product supply chain, which will affect the design of the incentive mechanism and the distribution of benefits among members. (3) We study the impact of demand fluctuations caused by the changes in agricultural product freshness. In the past research on the stability of fresh agricultural product supply chains, much attention for impact on the stability of relational contracts was only paid to asset specificity, uncertainty, reciprocal investment and trust. Especially, there is little research on the impact of different government subsidy mechanisms on the stability of three-level fresh agricultural product supply chains.

2. Preliminaries. We need notations in Table 1 and some assumptions.

**Assumption 1.** In the operation of three-level fresh product supply chain, only a single cooperative is considered to form a long-term trading relationship with multiple farmers, and retailers can only form long-term trading relationships with cooperatives.
Assumption 2. We assume that the fresh product’s value to the retailer is greater than the external market price, that is, \( V_x > P_x \). At the same time, the price of the fresh product of the external market is increasing step by step, that is, \( P_x > P_c > P_0 \).

Assumption 3. Assume that the farmers, the cooperative and the company chase their maximum profits. In this situation, the relational contract helps supply chain operations, and improves the overall supply chain profit.

By using the market demand function \([1]\) the demand for the fresh agricultural products \( D \) is affected by the price \( P \) and the freshness \( \theta \) and can be written as

\[
D = D(P, \theta) = \alpha P^{-k} \theta
\]

where \( \alpha \) is a measure of the size of the market and \( k > 1 \) is the market demand price elasticity. So, the demand of a contract signed by the farmer is \( D_0 = D(P_0, \theta_1) = \alpha P_0^{-k} \theta_1 \), and the demand of a contract signed by the cooperative is \( D_x = D(P_x, \theta_1, \theta_2) = \alpha P_x^{-k} \theta_1 \theta_2 \). The variable cost for the farmer in order to preserve a high degree of fresh product is \( C_a = C_a(\rho, \theta_1) = \rho \theta_1^2 / 2 \), where \( \rho \) is the elasticity of cost function and the corresponding variable cost for the cooperative is \( C_b = C_b(\rho, \theta_2) = \rho \theta_2^2 / 2 \). Obviously, \( C_a(\rho, \theta_1) \) and \( C_b(\rho, \theta_2) \) conform to general economic assumptions.

During the operation process of the three-level fresh product supply chain, the trading partner reaches the demand \( D_0 \) and signs the contract at the wholesale price \( P_0 \). Due to the lag of the execution contract, both parties in the performance period will adjust the actual transaction volume with the current demand of the market, called the quantity flexibility contract. In order to reduce the consumption as much as possible, improve the overall profit of the supply chain and maintain a stable cooperative relationship, we normally propose the compensation plan for the freshness loss of agricultural products. Therefore, we establish a relational contract. When signing the contract the cooperative provides the payment plan to the farmer as

\[
W_c = P_0 D_0 + P_c (D_c - D_0) + \eta_1 (B_t) D_0 \theta_1 + C_t
\]

(1)

The payment plan provided by the retailer to the cooperative is

\[
W_c = P_0 D_0 + P_c (D_c - D_0) + \eta_2 (B_t) D_0 \theta_2 + C_t
\]

(2)

where \( \eta_1 \) and \( \eta_2 \) are incentive coefficients for the quality of the agricultural products, and \( C_t \) is the extra transaction costs. Under the condition of mutual performance, \( C_t = 0 \), and defaults cause extra transaction costs \( C_{t1}, C_{t2}, \) and \( C_{t3} \) for the farmer, the cooperative and retailer, respectively \( (C_{t1} > C_{t2} > C_{t3}) \). \( P_0 D_0 \) represents the amount that the cooperative should pay during the initial order period. \( P_c D_c \) represents the amount that the retailer should pay during the initial order period.

The negotiation ability of members is inversely proportional to the cost of re-transaction. In other words, if the transaction cost is low, the negotiation bargaining power will be strong. Therefore, the relationship between the negotiation ability and the transaction cost can be expressed as \( B_t = k / C_t \), where \( k \) is the elastic coefficient and \( k > 0 \). In the absence of signed contracts, the revenue for farmers is \( P_c D_c \), its production cost is \( C_0 D_x / (\theta_1 \theta_2) \) the cost of preservation is \( C_a \) and the re-transaction cost is \( C_{t1} \). Then, without signing a contract, the farmer profit will be

\[
\pi_f = P_c D_c - C_0 D_x / (\theta_1 \theta_2) - C_a - C_{t1}
\]

(3)
The profits of the cooperative and the retailer can be obtained as
\[
\pi_1 = V_x D_x - P_c D_c - C_t3
\]
(5)

So, the overall profit of the supply chain without signing a contract is
\[
\pi_{sx1} = \pi_{f1} + \pi_{c1} + \pi_{e1}
\]
(6)

If the farmer and the cooperative have signed a contract, and the cooperative and the retailer have also signed a contract, then their re-transaction costs are \(C_t = 0\)
The profit of the farmer is the payment plan provided by the cooperative minus the production cost and the freshkeeping cost. Therefore, the profit of the farmer when signing the contract is
\[
\pi_{f2} = W_c - C_0 \frac{D_x}{\theta_1 \theta_2} - C_a
\]
(7)
The cooperative profit is the payment plan provided by the retailer minus the cooperative expenditure plan and preservation cost, i.e., the profit of the cooperative when signing the contract is
\[
\pi_{c2} = W_c - W_c - C_b
\]
(8)

When signing the contract, the retailer’s profit is retailer’s revenue minus the spending plan
\[
\pi_{e2} = V_x D_x - W_c
\]
(9)

Therefore, the profit of the supply chain is
\[
\pi_{sx2} = \pi_{f2} + \pi_{c2} + \pi_{e2}
\]
(10)

3. Quantity flexibility contracts and relational contracts.

3.1. Freshness and profits for quantity flexibility contracts. According to (3) when the farmer fails to sign a contract with the cooperative, the profit of farmer depends on the sales income and the cost incurred in the production process. The sales income depends on the current market price and the current demand. The cost depends on the farmer’s production cost, the cost of maintaining freshness and transaction cost. Substituting \(D_x = \alpha P_x^{-k} \theta_1 \theta_2, D_c = \alpha P_c^{-k} \theta_1,\) and \(C_a = \rho \theta_1^2 / 2\) into (3), we obtain the profit of the farmer
\[
\pi_{f1} = P_c D_c - C_0 \frac{D_c}{\theta_1 \theta_2} - C_a - C_{t1} = P_c \alpha P_c^{-k} \theta_1 - C_0 \alpha P_x^{-k} - \frac{1}{2} \rho \theta_1^2 - C_{t1}
\]
(11)

Letting \(\partial \pi_{f1} / \partial \theta_1 = P_c \alpha P_c^{-k} - \rho \theta_1 = 0,\) and \(\partial \pi_{f1}^2 / \partial \theta_1^2 = -\rho < 0,\) then the freshness has a maximum value. We conclude that when the agricultural product’s freshness is \(\theta_1^* = P_c \alpha P_c^{-k} / \rho,\) the farmer obtains the maximum profit. On the other hand, when the flexibility contract is not signed, the cooperative obtains the maximum profit when
\[
\pi_{c1} (\theta_1^*) = P_x D_x - P_c D_c - C_b - C_{t2} = P_x \alpha P_x^{-k} \theta_1 \theta_2 - P_c \alpha P_c^{-k} \theta_1 - \frac{1}{2} \rho \theta_2^2 - C_{t1}
\]
(12)
When \(\partial \pi_{c1} (\theta_1^*) / \partial \theta_2 = P_c \alpha P_c^{-k} P_x \alpha P_x^{-k} / \rho - \rho \theta_2 = 0,\) and \(\partial \pi_{c1}^2 / \partial \theta_2^2 = -\rho < 0,\) then the freshness \(\theta_2^*\) has maximum value. So, when the agricultural product’s freshness provided by the cooperative is \(\theta_2^* = P_c \alpha P_c^{-k} P_x \alpha P_x^{-k} / \rho^2,\) the cooperative obtains
the maximum profit. Substituting $\theta^*_1, \theta^*_2$ into the retailer’s profit function (5), the retailer’s profit will be

$$\pi_{e1}(\theta^*_1, \theta^*_2) = (V_x - P_x) \alpha P_x^{-k} k^\alpha \rho \theta^*_1 \theta^*_2$$

Therefore, when the flexible quantity contract is not signed, the maximum profit of the entire fresh agricultural product supply chain is

$$\pi_{sx1} = V_x D_x - C_0 D_x \theta^*_1 \theta^*_2 - C_a - C_b - C_{t1} - C_{t2} - C_{t3}$$

In practice, because the farmer has asymmetric information, his bargaining power is weak and the re-transaction cost is high. In the case of the fresh product is not added to incentive amounts, the farmer will keep the product fresh according to the above principle of profit maximization. If the farmer signs a quantity flexibility contract with the cooperative, the cooperative signs with the retailer, the transaction is carried out according to the quantity flexible contract and the market demand. In the case of signing a flexible quantity contract, if the farmer and the cooperative are all trading according to the quantity flexibility contract, the framer and the cooperative have no transaction costs. For this case, the overall profit of the supply chain is

$$\pi_{sx2} = V_x D_x - C_0 D_x \theta^*_1 \theta^*_2 - C_a - C_b$$

It can be seen from (13) and (14) that $\pi_{sx2} - \pi_{sx1} = C_{t1} + C_{t2} + C_{t3} \geq 0$, which indicates that the farmers, the cooperatives and the retailers have the increased overall profit of the supply chain when signing the flexibility contract.

3.2. Freshness and profits for the relational contracts. The overall profit of three-level fresh agricultural product supply chain improves after quantity flexibility contract has been signed. However, there is no incentive or influence on the freshness. Thus, based on the flexibility contract of supply chain partners, the appropriate incentive mechanism is introduced to form the relational contract. A relational contract refers to the retailer and the cooperative gives the corresponding incentives to the partners according to the freshness of the traded agricultural products, which can increase the profit of the partners while increasing the degree of freshness of agricultural products, and can enhance the stability of the fresh agricultural product supply chain.

**Proposition 1.** In the operation of three-level fresh agricultural product supply chain, the incentive contract is formed according to the freshness of agricultural products, and there is a certain range of incentive amount, which can increase the freshness of the agricultural products while increasing the profit of the partners. It could make the contracts achieve “self-discipline, self-execution”, thereby improving the stability of the relational contract.

**Proof.** Let $\theta_{1x}, \theta_{2x}$ be the agricultural product freshness provided by the farmers and the cooperatives. In the relational contract, the cooperatives and the retailers respectively give the incentives to the farmers and the cooperatives in order to improve the freshness of agricultural products. At the same time, the profits of the farmers and the cooperatives increase, respectively. When the freshness of agricultural product is giving the incentives, the profit of farmer is $\pi_{f2} = W_c -$
when the contract is signed, we substitute the freshness of agricultural product
\[ \theta_{1x} = (A + B) / \rho \] and \[ \theta_{2x} = (A + B) C / \rho^2 \] into the farmer’s profit function \[ \pi_{f2} = W_c - C_0 D_x / (\theta_1 \theta_2) - C_a \] to get
\[ W_c = P_0 D_0 + P_c (D_c - D_0) + \eta_1 (B_t) D_0 \theta_1 + C_t \text{ and } C_t = 0 \]

Therefore, the profit of the farmer when signing the relational contract is
\[ \pi_{f2} (\theta_{1x}, \theta_{2x}) = \left[ \eta_1 (B_t) \alpha P_0^{-k} - \frac{1}{2} \rho \right] \theta_{1x}^2 - \left[ P_0 \alpha P_0^{-k} + P_c \alpha \left( P_c^{-k} - P_0^{-k} \right) \right] \theta_{1x} - C_0 \alpha P_x^{-k} \]

When the contract is not signed, we substitute the freshness of agricultural products
\[ \theta_1 = P_c \alpha P_c^{-k} / \rho \] and \[ \theta_2 = P_c \alpha P_c^{-k} P_x^{-k} / \rho^2 \] into (3) and obtain
\[ \pi_{f1} (\theta_{1x}) = \frac{B^2}{2 \rho} - C_0 \alpha P_x^{-k} - C_{t1}, \text{ and } \pi_{f2} > \pi_{f1} \]

So
\[ \eta_1 > \frac{B^2}{2 (A + B)} - \frac{A - B}{2} + \frac{P_c \alpha P_0^{-k} - C_{t1} \rho}{A + B} \]
Substituting $\theta_{1e}$ and $\theta_{2e}$ into $\pi_{e2}$, substituting $\theta_{1r}$ and $\theta_{2r}$ into $\pi_{e1}$, and considering $\pi_{e2} > \pi_{e1}$, we obtain

$$\eta_2 < (V_x - P_x) \alpha P_x \frac{A + B}{\rho} + \frac{\rho (P_x - P_c) \alpha P_x^{-k}}{P_c \alpha P_x^{-k}} - B^2 \frac{(V_x - P_x) \alpha P_x^{-k}}{\rho (A + B)} + \frac{C_{13} \rho^2}{C (A + B)}$$

Substituting $\theta_{1e}$ and $\theta_{2e}$ into $\pi_{e2}$, substituting $\theta_{1r}$ and $\theta_{2r}$ into $\pi_{e1}$, and considering $\pi_{e2} > \pi_{e1}$, we obtain

$$\eta_2 > \frac{(A + B) \rho}{C} - \frac{3 (A + B) C}{2 \rho} + \frac{C B^2}{(A + B) \rho} - \frac{(A + B) B^2}{C \rho} - \frac{C B^2}{2 (A + B) \rho} + \frac{\eta_1 \rho}{C} - \frac{C_{12} \rho^2}{C (A + B)}$$

Therefore, when $\eta_1 \in [M, N]$ and $\eta_2 \in [P, Q]$, we have

$$M = \frac{2 B^2}{2 (A + B)} - P_0 \alpha P_0^{-k} + \frac{A - B}{2} + P_0 \alpha P_0^{-k} - \frac{C_{11} \rho}{A + B}$$

$$Q = (V_x - P_x) \alpha P_x \frac{A + B}{\rho} + \frac{\rho (P_x - P_c) \alpha P_x^{-k}}{P_c \alpha P_x^{-k}} - B^2 \frac{(V_x - P_x) \alpha P_x^{-k}}{\rho (A + B)} + \frac{C_{13} \rho^2}{\rho (A + B)}$$

$$P = \frac{(A + B) \rho}{C} - \frac{3 (A + B) C}{2 \rho} + \frac{C B^2}{(A + B) \rho} - \frac{(A + B) B^2}{C \rho} - \frac{C B^2}{2 (A + B) \rho} + \frac{M \rho}{C} + \frac{C_{12} \rho^2}{C (A + B)}$$

$$N = Q \frac{A + B}{B} - (C + A) + \frac{C^2 (A + B)}{\rho^2} - \frac{C^2 B^2}{\rho^2 (A + B)} + \frac{B^2}{A + B} + 2 \rho^2 (A + B)^2 + \frac{C_{12} \rho^2}{A + B}$$

Therefore, when the incentive coefficient given by the cooperative $\eta_1$ between $[M, N]$ and the cooperative incentive coefficient $\eta_2$ between $[P, Q]$, the farmer, the cooperative, and the retailer can obtain more profits than before, the relational contract can achieve “self-discipline, self-execution”.

From above discussions, we can see that the relational contract based on the quantity flexibility contract can realize the self-coordination of the supply chain as a whole within a certain incentive coefficient. However, due to the unstable supply of fresh agricultural products and high price fluctuations, there are opportunistic tendencies at all levels of enterprises, which may lead to the occurrence of relational contract breach sometimes. The penalty mechanism for breaching contracts and the construction of special assets for cold chain facilities are important ways to guarantee the stability. However, there are also difficulties to solve these problems, such as high illegal execution costs and large disparity in negotiation capabilities. Furthermore, we try to explore how to solve the stability of relational contract from the perspective of government subsidies.

4. Government subsidy mechanism effect.

4.1. Relational contract stability between farmers and cooperatives. In the relational contract between the farmer and the cooperative, we assume that any party should require to pay a liquidated damages after default is $\gamma$, if the cooperative directly purchases agricultural products from the market and the profits are greater than those purchased from the agreed farmers, the cooperatives will have a tendency to default. Therefore, when $V_x D_x - P_c D_c - C_b - C_{122} - \gamma P_c > W_x - W_c - C_b$, i.e., $P_c < \{(V_x - P_c) D_c - (P_c - P_b) D_b - C_{122}\}/\gamma$, the cooperative will default.

**Proposition 2.** Government provides a suitable protection price contract, and the policy can enhance the stability between the farmer and the cooperative base on the relation contract.
Proof. The government may provide $P_t$ as a protection price policy. If the cooperative finds that the price of fresh product in the spot market is lower than the contract price, government requires the cooperative to purchase fresh agricultural products from the farmers at the protection price and to provide corresponding financial subsidies to the cooperatives. The specific protection price and subsidy amount are $P_t = (c_1 - P_c) D_c - (c_2 - P_0) D_0 - C_{12})/\gamma$ and $D_0 (P_t - P_c)$. Under this subsidy contract, the cooperatives have no opportunistic tendency to default. The farmers concern that under the protection of the price mechanism, although the price change can affect the profits of farmers, but it does not affect the endogenous variable $\theta$ yet. The freshness of agricultural products is only related to external factors such as the corrosion resistance and natural conditions of the agricultural products and the efforts of the production and transportation parties. After the acquisition with the government’s protection price $P_t$, the profit of the farmer becomes

$$\pi'_{f2}(\theta_{1x}, \theta_{2x}) = W_c - C_0 \theta_{1x} \theta_{2x} - \alpha,$$

where

$$W_c = P_0 D_0 + \alpha (D_c - D_0) + \eta_1 (B_t) D_0 \theta_1 + C_t$$

available to

$$\pi'_{f2}(\theta_{1x}, \theta_{2x}) = \left( \eta_1 (B_t) \alpha P_0 - \frac{1}{\gamma} \right) \theta_{1x} - \left( \frac{P_0 - \alpha P_0 + P_t \alpha (P_c - P_0^k)}{\gamma} \right) \theta_{1x} - C_0 \alpha P_x^k$$

Then, we have

$$\pi'_{f2}(\theta_{1x}, \theta_{2x}) - \pi'_{f2}(\theta_{1x}, \theta_{2x}) = (P_t - P_c) \alpha (P_c - P_0^k) > 0$$

It can be concluded that the implementation of the minimum protection price contract to purchase agricultural products increases the profit of farmers. Therefore, under the protection price contract, the profits of farmers are increased; on the other hand, the opportunism of cooperative defaults is avoided, and the stability of the relational contract between farmers and cooperatives is enhanced.

\[\square\]

4.2. Relational contract stability between cooperatives with retailers. If the government subsidizes the cooperative, the cooperative invests in lower cold chain facilities than retailers, the cooperative’s negotiating ability $B_t$ is improved, and the relational contract tends to be stable. Conversely, if the retailer is subsidized, the negotiation ability between the cooperative and the retailer will further expand, which will lead to the retailer’s opportunistic tendency.

Proposition 3. In the case of a joint venture between the retailer and the cooperative to build a cold chain facility, the government provides appropriate funding subsidies for the cooperative’s cold chain facilities. Existence of optimal freshness incentive coefficient can improve the stability of cooperative and retailer while improving the freshness of agricultural products.

Proof. Let $D_x$ be the retailer’s need. According to the situation of agricultural product consumption, the quantity of agricultural products provided by the cooperative is $D_x/\theta_{2x}$. The ratio of agricultural products that cooperative and retailers need to refrigerate is $D_x/\theta_{2x}$. Assume that the cost of cold chain facility construction is $C_s$, funded by the amount of agricultural products that need to be refrigerated by both parties, that is cooperatives funded $C_s/(1 + \theta_{2x})$ retailer funded $\theta_{2x} C_s/(1 + \theta_{2x})$
The excitation coefficient can be obtained in this range of conditions and subsidies for the contribution ratio

$$\begin{align*}
\pi'_{c2}(\theta_1x,\theta_2x) &> \pi_{c1}(\theta_1,\theta_2) \\
\pi'_{c2}(\theta_1x,\theta_2x) &> \pi_{c1}(\theta_1,\theta_2)
\end{align*}$$

(22)

The specific expansion is

$$\begin{align*}
\pi_{c3}(\theta_1x,\theta_2x) &= P_x(D_x - D_c) + D_0(P_c - P_0) + \eta_2(B_t) D_c \theta_2x \\
&- \eta_1(B_t) D_0 \theta_1x - \frac{1}{2} \rho \theta_2x^2 - \frac{1}{1 + \theta_2x} C_s + a D_c \\
\pi_{c1}(\theta_1,\theta_2) &= P_x \alpha P_x^{-k} \theta_1 \theta_2 - P_c \alpha P_c^{-k} \theta_1 - \frac{1}{2} \rho \theta_1^2 - C_{t2}
\end{align*}$$

$$\begin{align*}
\pi'_{c3}(\theta_1x,\theta_2x) &= (V_x - P_x) D_x + (P_x - P_c) D_c + \eta_2(B_t) D_c \theta_2x - \frac{\theta_2x}{1 + \theta_2x} C_s \\
\pi_{c1}(\theta_1,\theta_2) &= (V_x - P_x) \alpha P_x^{-k} \theta_1 \theta_2 - C_{t3}
\end{align*}$$

(23)

Then,

$$\begin{align*}
\theta_1 &= \frac{B}{\rho}, \theta_2 = \frac{BC}{\rho^2}, \theta_1x = \frac{A + B}{\rho}, \theta_2x = \frac{(A + B)C}{\rho^2}
\end{align*}$$

are applied to (23) then it can be obtained that

$$\begin{align*}
\pi_{c3}(\theta_1x,\theta_2x) &= \left[\frac{(P_x \alpha P_x^{-k})^2}{2\rho} + \eta_2(B_t) \alpha P_c^{-k} P_x \alpha P_x^{-k} \rho - \eta_1(B_t) \alpha P_0^{-k}\right] \frac{(A + B)^2}{\rho^2} \\
&+ \left[\alpha P_0^{-k}(P_c - P_0) - P_x \alpha P_c^{-k} + \left(a - \frac{1}{1 + \theta_2x} \mu\right) \alpha P_c^{-k}\right] A + B \rho. \\
\pi_{c1}(\theta_1,\theta_2) &= \frac{B^2 (P_x \alpha P_x^{-k})^2}{2\rho^3} - \frac{B}{\rho} - C_{t2}
\end{align*}$$

$$\begin{align*}
\pi'_{c2}(\theta_1x,\theta_2x) &= P_x \alpha P_x^{-k} [(V_x - P_x) \alpha P_x^{-k} + \eta_2(B_t) \alpha P_c^{-k} \frac{(A + B)^2}{\rho^2}] \\
&+ \alpha P_c^{-k} \left[(P_x - P_c) - \frac{\theta_2x}{1 + \theta_2x} \mu\right] \frac{A + B}{\rho} \\
\pi_{c1}(\theta_1,\theta_2) &= \frac{B^2 P_x \alpha P_x^{-k} (V_x - P_x) \alpha P_x^{-k}}{\rho^3} - C_{t3}
\end{align*}$$

Substituting them into the equation (22) gives the range of excitation coefficients

$$\begin{align*}
\eta'_2(B_t) &> [M + P_0 - P_c] \frac{\rho^2}{(A + B)C} \left(\frac{P_0}{P_c}\right)^{-k} - P_x A^2 + 2AB \left(\frac{P_x}{P_c}\right)^{-k} \\
&+ \frac{1}{\alpha P_c^{-k}} \left[A^2 C + 2ABC + \frac{C_{t2} (A + B)^2 C}{2(A + B)^2}\right] + \frac{\rho^2}{(A + B)C} \left[(P_x - a) - \frac{P_c}{A + B}\right]
\end{align*}$$

$$\begin{align*}
\eta'_2(B_t) &< \frac{C_{t3} \rho^3}{\alpha P_c^{-k} (A + B)C} + (V_x - P_x) \left(\frac{P_x}{P_c}\right)^{-k} + \frac{(P_x - P_c) \rho^2}{(A + B)C} - \frac{\mu \rho^2}{\rho^2 + (A + B)C}
\end{align*}$$

Get the incentive coefficient $\eta'_2 \in [X, Y]$, where

$$\begin{align*}
X &= [M + P_0 - P_c] \frac{\rho^2}{(A + B)C} \left(\frac{P_0}{P_c}\right)^{-k} - P_x A^2 + 2AB \left(\frac{P_x}{P_c}\right)^{-k} \\
&+ \frac{1}{\alpha P_c^{-k}} \left[A^2 C + 2ABC + \frac{C_{t2} (A + B)^2 C}{2(A + B)^2}\right] + \frac{\rho^2}{(A + B)C} \left[(P_x - a) - \frac{P_c}{A + B}\right]
\end{align*}$$

$$\begin{align*}
Y &= \frac{C_{t3} \rho^3}{\alpha P_c^{-k} (A + B)C} + (V_x - P_x) \left(\frac{P_x}{P_c}\right)^{-k} + \frac{(P_x - P_c) \rho^2}{(A + B)C} - \frac{\mu \rho^2}{\rho^2 + (A + B)C}
\end{align*}$$
Figure 1. The relationship between re-transaction costs and incentive coefficients of cooperatives and retailers

\[ Y = \frac{C_{t3} \rho^2}{\alpha P_c^k (A + B) C} + (V_x - P_x) \left( \frac{P_x}{P_c} \right)^{-k} + \frac{(P_x - P_c) \rho^2}{(A + B) C} - \frac{\mu \rho^2}{\rho^2 + (A + B) C} \]

The incentive coefficient \( \eta \) is proportional to the negotiation ability \( B_t \), and the negotiation \( B_t \) is inversely proportional to the re-transaction coefficient \( C_t \). So, it can be obtained indirectly the relationship between \( \eta \) and \( C_t \). Obtained \( X \) and \( Y \) from the above, it can be seen that \( \eta^*_t \) should be one function with \( C_{t2} \) and \( C_{t3} \), it will change with \( C_{t2} \) and \( C_{t3} \). And it has a positive correlation with \( C_{t3} \) and a negative correlation with \( C_{t2} \). Therefore, we can get \( \eta^*_t (C_{t2}), \eta^*_t (C_{t3}) \) function image as shown in Figure 1, which shows that there must be an intersection \( C^*_t \) gets the optimal incentive coefficient \( \eta^* \) then the company offers the best incentive coefficient, cooperatives provide optimal fresh degree of agricultural products, and cooperatives and retailers do not appear default status, which enhances the stability of cooperative relationships with retailer contracts.

5. Case analysis. This section uses data to validate data models and relevant conclusions from theoretical discussions. Assume an apple’s three-level fresh produce supply chain, the relevant parameters are set as follows: The unit cost of the farmer \( C_0 = 1 \) Yuan, farmers and cooperatives signed contractual price \( P_0 = 2.5 \) Yuan, the contract price signed by the retailer and the cooperative \( P_c = 3.3 \) Yuan, the spot market price at the time of performance of the contract \( P_x = 5.5 \) Yuan, the market value of fresh agricultural products \( V_x = 7 \) Yuan, the market demand price elasticity coefficient \( k = 1.3 \), the market size factor \( \alpha = 20000 \), the cost factor \( \rho = 15000 \), the farmers re-transaction costs \( C_{t1} = 2000 \) Yuan, the cooperative re-transaction cost \( C_{t2} = 1600 \) Yuan, the retailer re-transaction cost \( C_{t3} = 1200 \) Yuan, the cooperative default amount \( \gamma = 15000 \) Yuan, the cold chain facility unit cost \( \mu = 0.2 \) Yuan, and the government unit subsidized price \( a = 1.8 \) Yuan.
Table 2. Profits of members of the supply chain under different contractual relationships

|                | \(\pi_f\)     | \(\pi_c\)   | \(\pi_e\)   | \(\pi_{sx}\)   |
|----------------|--------------|-------------|-------------|----------------|
| No contract    | 1720.33      | 1071.20     | 2921.24     | 5712.77        |
| Flexibility contract | 1942.14      | 1221.72     | 13752.03    | 16915.89       |
| Relational contract | 2702.69      | 3949.91     | 14867.88    | 21520.48       |
| Government subsidy  | 5034.03      | 10221.25    | 14555.91    | 29811.19       |

On this basis, the freshness of agricultural products \(\theta_1^* = P_c \alpha P_c^{-k}/\rho = 0.9319\)

\(\theta_2^* = P_c \alpha P_x^{-k} P_x \alpha P_x^{-k}/\rho^2 = 0.7451\).

The variable cost of maintaining the freshness of the farmer is \(C_a = \rho \theta_2^*/2 = 4163.82\).

We use MATLAB R2014a to solve the problem, and obtain the profit situation including without signing the contract, signing the flexible contract, signing relational contract and using government subsidy.

The following can be observed from Table 2: (1) As the contractual relationship enhances, the total profit of the supply chain gradually increases. Similarly, the change in contractual relationship, the variance of profits among farmers, cooperatives and retailers is gradually decreasing, and the distribution of profits is gradually averaged. It also shows that the negotiating power of farmers, cooperatives and retailers tends to be average, and stability of supply chain gradually enhanced. (2) The incentive coefficient introduced in the relational contract take values, within a given range \(\eta_1 = 0.95\), and \(\eta_1 = 0.92\). By getting the profits of the members of the supply chain, it can be clearly seen that the profit gaps between the farmers, the cooperatives and the retailers are further reduced. (3) However, it is not difficult to see that the farmers and the cooperative have always been in a weak position in the contractual relationship. Therefore, in order to further improve the stability of the supply chain and enhance the negotiation power of the farmers and the cooperative, the existing government subsidy mechanism is considered. First, the government can introduce the minimum purchase price, in the case analysis, at the time \(P_t = 4.2 > P_c\), the stability of the cooperative and the farmer relational contract can be guaranteed to protect the interests of the farmers. Secondly, for the cold chain facilities jointly established by the “cooperative-retailer”, the government can subsidize the cooperative according to its scale. According to the example, it also narrowed the gap in negotiation power between cooperatives and retailers, the stability of supply chain relational contracts was significantly enhanced.

6. Conclusion. This paper analyzes the stability of the farmer-cooperative-retailer three-level fresh agricultural product supply chain from the aspects of different contracts and government subsidies. The main conclusions include the following:

(1) Under the three-level organization mode of “farmer-cooperative-retailer”, regardless of whether the flexibility contract is signed or not, it exists the same optimal agricultural product freshness. But since the flexible contract is formed, a stable trading relationship is formed, which can effectively reduce the re-transaction cost of the farmers and the cooperative. Therefore, the overall profit and efficiency of the supply chain under contractual conditions are better than not signing flexible contracts.

(2) In practice, due to the perishable of fresh agricultural products, supply uncertainty and price changes, the profits of supply chain cannot be maximized when there is no incentive for the freshness of agricultural products. Constructing the
incentive mechanism of freshness and forming a relational contract based on freshness, it can not only improve the optimal freshness level, but also increase the overall profit of the supply chain.

(3) Due to the differences in bargaining power among supply chain members, coupled with the influence of real factors such as unstable supply and large price fluctuations, thus the constraint strength of the overall stability of the chain is still weak if the relational contract of incentive mechanism is simply introduced. Therefore, based on the investigation of the government subsidy policy and the negotiating ability of the fresh agricultural product supply chain members, the paper discusses two subsidy models in order to improve relational contract stability. In the “farmer-cooperative” stage, the government issued a protection price policy, which not only can effectively protect the interests of farmers, but also further maintain the stability of the relational contract. At the same time, in the “cooperative-retailer” stage, by encouraging collaboration to build cold chain facilities and providing a reasonable range of financial subsidies for cooperatives, the optimal existence of incentive coefficient is proved. The incentive coefficient maximizes the freshness of agricultural products, increases their income and improves the stability of the relational contract.

The main conclusions of this paper are drawn under the relevant assumptions and only the traditional channel trading mode of agricultural products is considered. In fact, there are multiple operating modes and incentive mechanisms in the three-level fresh agricultural product supply chain, and the relevant conclusions may not be universal. On the other hand, Internet transactions are becoming more and more popular, and online B2C transactions of fresh product have been widely used. Therefore, the online and offline dual-channel supply chain coordination of fresh product can be taken as the next research direction.

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