Exponential $H_{\infty}$ synchronization of non-fragile sampled-data controlled complex dynamical networks with random coupling and time varying delay

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ABSTRACT
This paper investigates the problem of non-fragile sampled-data control for synchronization of complex dynamical networks with randomly coupling and time varying delay under exponential $H_{\infty}$ approach. By adopting an appropriate Lyapunov Krasovskii functional (LKF) and taking into consideration full information on the sampling pattern, free-matrix based integral and Wirtinger inequalities are explored leading to the establishment of sufficient conditions to guarantee the exponential $H_{\infty}$ synchronization stability and disturbance attenuation of the closed loop network, with a designed non-fragile controller under all randomly admissible gain variations. The results are presented in terms of Linear matrix inequalities (LMIs), which can effectively be solved by some available softwares. Finally, two simulated results are demonstrated to show the effectiveness and less conservativeness of our proposed scheme.

1. Introduction
In the past few decades, a lot of attention have been devoted to the study of complex dynamical networks (CDNs), which have massive applications in areas of science and engineering such as Internet, biological networks, World Wide Web, epidemic spreading networks, social networks and electric power grids (Dorato, 1998; Hu, Wang, Gao, & Stergioulas, 2012; Kwon, Park, & Lee, 2008; Lakshmanan, Mathiyalagan, Park, Sakthivel, & Rihan, 2014; Li & Chen, 2004; Li, Zhang, Hu, & Nie, 2011; Liu, Guo, Park, & Lee, 2018; Seuret & Gouaisbaut, 2013; Strogatz, 2001; Su & Shen, 2015; Wu, Park, Su, Song, & Chu, 2012; Zeng, He, Wu, & She, 2015; Zhang, He, & Wu, 2010; Zhang, Wu, She, & He, 2005). Many of these networks have shown some complexities with regard to their topologies and dynamical characteristics concerning the network nodes and their coupled units. CDNs are represented by a large interconnecting of nodes, which comprise of nonlinear dynamical systems. There are some significant and important phenomena in CDNs, which can be described by coupled ordinary differential equations such as synchronization, spatiotemporal chaos, and self-organization (Wu et al., 2012). Synchronization as one of the fundamental and significant phenomena of CDNs, have in recent times received much attention amongst most researchers, because of its broad applications in the fields of mathematics, engineering, traffic systems and computer science (Rakkiyappan, Latha, & Sivarancani, 2017; Shi, Yang, Wang, Zhong, & Wang, 2018; Yin et al., 2016). The essence of synchronization phenomenon in CDNs, is to allow all coupled nodes or some selected nodes in the network to approach the trajectory of a target node. To ensure the synchronization of CDNs, many control schemes have been deeply studied and proposed, where adaptive pinning control was studied in Guo, Pan, and Nian (2016) and Astrom and Wittenmark (1994), whereby the authors utilized pinning adaptive control strategy to ensure the control of cluster synchronization problems of their proposed network. In Yang, Cao, and Lu (2012), the authors studied synchronization of complex networks with the hybrid adaptive and impulsive control methods. With recent advancement in digital measurement, high quality computers, communication networks and intelligent instruments, continuous-time controllers are been replaced with digital controllers (Lee, Park, Kwon, & Sakthivel, 2017). Furthermore, these have resulted in improved reliability, accuracy and better stability performance. The need for sampled-data control happens to be another important control strategy which have received much attention in the study of CDNs (Lee, Wu, & Park, 2012; Li et al., 2011; Liu et al., 2018; Shen, Wang, & Liu, 2012; Wang, Zhang, & Wang, 2015; Yucel,
Ali Syed, Gunasekaran, & Arik, 2017; Zhang et al., 2010; Zheng, Zhang, Zhong, Wang, & Shi, 2017). The sampled-data control system is a continuous plant connected to discrete - time controller where digital -to- analog (D/A) and analog-to-digital (A/D) components are used. The sampled-data control schemes have many advantages, amongst them are but not limited to easy implementation, reduction of overall size, low cost of maintenances, amount of transmitted information greatly reduced and above all, increased efficiency in bandwidth usage (Li et al., 2011; Rakkiyappan et al., 2017; Yucel et al., 2017). Additionally, choosing a suitable sampling period is an important factor to be considered in sampled-data control.

Interestingly, it is an established fact that, a larger sampling period will lead to lower communication channel occupying, fewer actuation of controller and less frequent signal transmission which are very desirable (Li et al., 2011; Wu et al., 2012). The input delay approach is one of the most widely studied methods adopted in the analysis and synthesis of sampled data systems where the sampling holder is modeled as delay control inputs (Fridman, 2010). In fact, it is of great essence to ascertain how well will such systems be influenced by a designed sampled-data controller and also, to what extent will the desired performance be achieved? In real world applications, time delays are inevitable in most physical systems (Li, Dong, Han, Hou, & Li, 2017; Lu & Chen, 2004; Park, Kwon, Park, Lee, & Cha, 2012; Yang, Dong, Wang, Ren, & Alsaadi, 2016), therefore it is important to be considered in the investigation of synchronization of CDNs under the exponential $H_{\infty}$ approach. The existence of time delays which might occur as a result of, limited information channels and large-scale interconnected complex networks could lead to undesired oscillation, instability and poor performance of the CDNs (Lakshmanan et al., 2014; Liang, Wu, & Chen, 2016; Zeng et al., 2015).

That notwithstanding, modeling errors, disturbance inputs and parameter perturbations or uncertainties do occur in CDNs, which contribute to the degradation and poor performance of the network. It is important to investigate such randomly coupling and time-varying delay CDN under exponential $H_{\infty}$ synchronization with non-fragile sampled-data controller, which is very critical and important in theory and practical applications (Dong et al., 2017). In the past few years, many researchers devoted quality time and resources to the study of non-fragile control and filter implementation design problems with some applications in control and communication field such as altitude control of satellites, missile control, chemical process control and macroeconomic system control (Li et al., 2017; Li, Deng, & Peng, 2012). The aim was to design controllers for some given systems such that, they become insensitive to some amount of errors and deviations with regards to their gains, environmental effects, modeling errors and uncertainties. It is therefore significant for designed controllers to tolerate some level of uncertainties in their parameters. In Hou, Dong, Wang, Ren, and Alsaadi (2016), a non-fragile state estimator is designed for admissible gain variations. In addition, the study of sampled-data control, which implies the successful updating of signal transmitted from the sampler to the controller and then to the zero-order hold (ZOH) at instant of $t_k$ can experience some form of a constant signal transmission delay as investigated in Liu et al. (2018), Lee et al. (2012) and Liu and Fridman (2012). Based on the reasons above, and as our first motivation in this paper, it is prudent and beneficial to investigate the exponential $H_{\infty}$ synchronization of non-fragile sampled data control for randomly coupling and time varying delay of CDNs. Furthermore, in practical implementation of controller design, there are some uncertainties which might show up resulting from analog-to-digital and digital-to-analog conversions, round off errors in numerical computations and deterioration of system components. These occurrences have the tendency of deteriorating the performance or causing instability of the closed-loop systems hence, our second motivation to study and design a non-fragile sampled-data controller with norm bounded uncertainties which incorporates a constant signal transmission delay for the CDNs under exponential $H_{\infty}$ synchronization (Lee et al., 2012; Wu et al., 2012). The main contributions of this work are: 1) dealing with the CDNs in terms of the randomly occurring coupling, time varying delay, external disturbances and nonlinearities. 2) addressing the fragilities of the sampled-data controller in the presence of constant signal transmission delay. 3) designing and implementing the proposed controller to guarantee synchronization of the CDNs under an unstable circumstance. 4) Two solved numerical examples which indicated less conservativeness in our results compared with some existing simulation results (Li et al., 2011; Liu et al., 2018; Su & Shen, 2015; Wu et al., 2012; Yang, Shu, Zhong, & Wang, 2016).

The rest of the paper is organized as follows: In Section 2, the exponential $H_{\infty}$ synchronization problem of CDNs with randomly coupling and time-varying delays is formulated. A synchronization criterion for the exponential $H_{\infty}$ problem is derived with non-fragile sampled-data control in Section 3. Section 4 gives the simulation results of two numerical examples and the effectiveness of our proposed approach. Finally, conclusions are given in Section 5.

**Notation:** Throughout this paper the notations are standard. ‘I’ stands for identity matrix with appropriate
The symmetric matrices $X$ and $Y$, the notation $c$ indicates the symmetric elements of the symmetric matrix; the non-linear dynamics of nodes.

$$R_m \times n$$ denotes the $n$-dimensional Euclidean space; $R^{m \times n}$ is the set of all $m \times n$ real matrices; $*$ indicates the symmetric elements of the symmetric matrix; the coupling matrix which also represents the network topology.

$2. Problem formulation and preliminaries$

Consider the following CDNs comprising of $N$ identical coupled nodes with each node being an $n$-dimensional dynamical system:

$$\begin{align*}
\dot{x}_i(t) &= f(x_i(t)) + (1 - \delta_1(t)) \sum_{j=1}^{N} F_{ij} \Gamma_1 x_j(t) \\
+ \delta_1(t) \sum_{j=1}^{N} F_{ij} \Gamma_2 x_j(t - \gamma(t)) + u_i(t) + D \varpi_i(t), \\
\hat{p}_i(t) &= C x_i(t), \quad i = 1, 2, \ldots, N.
\end{align*}$$

(1)

where $x_i(t) \in \mathbb{R}^n$, $\hat{p}_i(t) \in \mathbb{R}^l$ and $u_i(t) \in \mathbb{R}^n$ are the state vectors, the corresponding outputs and the control input respectively of node $i$, $f(x_i(t)) = [f_1(x_i(t)), f_2(x_i(t)), \ldots, f_n(x_i(t))]^T$ is a nonlinear vector valued function describing the non-linear dynamics of nodes. $\Gamma_1, \Gamma_2 \in \mathbb{R}^{n \times n}$ are constant inner coupling matrices, $F = (F_{ij})_{N \times N}$ is the outer-coupling matrix which also represent the network topology, $C \in \mathbb{R}^{n \times n}$, and $D \in \mathbb{R}^{n \times k}$. If there is a connection between node $i$ and node $j$ ($i \neq j$), then $F_{ij} = 1$, otherwise $F_{ij} = 0$ ($i \neq j$). $\varpi_i(t) \in \mathbb{R}^k$ is the external disturbance which belongs to $\mathcal{L}_2[0, \infty)$. The diagonal elements of matrix $F$ are defined by

$$F_{ii} = - \sum_{j=1, j \neq i}^{N} F_{ij}, \quad i = 1, 2, \ldots, N.$$  

(2)

The scalar $\gamma(t)$ is taken as the time-varying delay which satisfies

$$0 \leq \gamma(t) \leq \gamma, \quad \dot{\gamma}(t) \leq \nu$$

(3)

where $\gamma > 0$ and $\nu > 0$ are known constants. $\delta_1(t) \in \mathbb{R}$ denotes a stochastic variable, which in the form of a Bernoulli’s distribution sequences and defined by

$$\delta_1(t) = \begin{cases} 
1 & : \text{delayed information exchange occurs,} \\
0 & : \text{delayed information exchange does not occur}
\end{cases}$$

Below indicates the probability of stochastic variable $\delta_1(t)$:

$$Pr(\delta_1(t) = 1) = \delta_1, \quad Pr(\delta_1(t) = 0) = 1 - \delta_1,$$  

(5)

with $\delta_1 \in [0, 1]$ being a known constant. Also, $E(\delta_1(t))$ is known to be the expectation of $\delta_1(t)$, therefore, we have

$$E(\delta_1(t) - \delta_1) = 0, \quad E((\delta_1(t) - \delta_1)^2) = \delta_1(1 - \delta_1)$$

(6)

**Assumption 2.1:** Let $f(\bullet) : \mathbb{R}^n \rightarrow \mathbb{R}^n$ be a continuous vector-valued function which satisfies the following sector-bounded condition:

$$[f(x) - f(y) - Q_1(x - y)]^T [f(x) - f(y) - Q_2(x - y)] \leq 0,$$

(7)

where $Q_1$ and $Q_2$ are known constant matrices of appropriate dimensions.

The nonlinear function in (7) is very general which entails the well known Lipschitz condition as a special case. It is assumed that $s(t) \in \mathbb{R}^n$ is the state trajectory of the unforced isolated node which satisfies $\dot{s} = f(s(t))$ with $\dot{s} = Cs(t)$ as the output of the unforced isolated node $s(t)$. Then, the error vector becomes $\delta_i(t) = x_i(t) - s(t)$, whereby the synchronization error dynamics of (1) is obtained as follows:

$$\begin{align*}
\dot{\delta}_i(t) &= g(\delta_i(t)) + (1 - \delta_1(t)) \sum_{j=1}^{N} F_{ij} \Gamma_1 \delta_j(t) \\
+ \delta_1(t) \sum_{j=1}^{N} F_{ij} \Gamma_2 \delta_j(t - \gamma(t)) + u_i(t) + D \varpi_i(t) \\
i = 1, 2, \ldots, N
\end{align*}$$

(8)

where $g(\delta_i(t)) = f(x_i(t)) - f(s(t))$. Then the output error between $\hat{p}_i(t)$ and $\hat{q}(t)$ has the following form:

$$\begin{align*}
\hat{\delta}_i(t) &= \hat{p}_i(t) - \hat{q}(t), \quad i = 1, 2, \ldots, N. \\
\hat{\delta}_i(t) &= C \delta_i(t)
\end{align*}$$

(9)

The control signal for the synchronization is generated by utilizing the Zero-Order-Hold (ZOH) function with a sequence holding times

$$0 = t_0 < t_1 < \cdots < t_k < \cdots, \quad \lim_{k \to +\infty} t_k = +\infty.$$  

(10)

with the sampling interval defined as

$$t_{k+1} - t_k = d_k \leq d,$$  

(11)

for all $k \geq 0$, $d > 0$ is the largest sampling interval. The sampling is non-periodic, although it is assumed to be
bounded. Considering the error dynamics (8), we adopt the design of a memory set of sampled-data state feedback controller in the form

\[
\dot{u}_i(t) = u_{id}(t_k) = (K_{i} + \alpha(t) \Delta K_{i}(t)) \delta_i(t_k - i),
\]

\[
t_k \leq t < t_{k+1}, \quad i = 1, 2, \ldots, N
\]

(12)

where \( u_{id}(\cdot) \) is a discrete-time control signal, \( t_k \) represents the sampling instant of \( i \)th node and \( K_{i} \) is the appropriate feedback control gain matrix to be estimated, and \( \epsilon \) is the constant signal transmission delay. \( \alpha(t) \) is the stochastic variable used to indicate the randomly occurring controller fluctuations, which follows the Bernoulli distributed sequence taking on values as stated in Hu et al. (2012):

\[
\alpha(t) = \begin{cases} 
1 & : \text{controller fluctuation occurs}, \\
0 & : \text{controller fluctuation does not occur} 
\end{cases}
\]

(13)

Below indicates the probability and expectation of the stochastic variable \( \alpha(t) \):

\[
Pr[\alpha(t) = 1] = \alpha_1, \quad Pr[\alpha(t) = 0] = 1 - \alpha_1,
\]

(14)

with \( \alpha_1 \in [0, 1] \) being a known constant.

Let \( E[\alpha(t)] \) be the expectation of \( \alpha(t) \), therefore we can have

\[
E[\alpha(t) - \alpha_1] = 0, \quad E[(\alpha(t) - \alpha_1)^2] = \alpha_1 (1 - \alpha_1)
\]

(15)

The perturbation phenomenon is considered in the controller design as uncertainties in the form \( \Delta K_{i}(t) \) being the controller gain fluctuations. \( \Delta K_{i}(t) \) has the following representations:

\[
\Delta K_{i}(t) = B_i \Delta(t) L_{i}
\]

(16)

where \( B_i \) and \( L_{i} \) are known appropriately dimensioned constant matrices and \( \Delta(t) \) being an unknown matrix function which satisfies the condition

\[
\Delta^T(t) \Delta(t) \leq I.
\]

(17)

Now, consider the state feedback controller (12) in the error dynamics (8), which becomes:

\[
\dot{\hat{\theta}}_i(t) = \hat{g}(\hat{\theta}_i(t)) + (1 - \delta_1(t)) \sum_{j=1}^{N} F_{ij} \Gamma_1 \hat{\theta}_j(t)
\]

\[
+ \delta_1(t) \sum_{j=1}^{N} F_{ij} \Gamma_2 \hat{\theta}_j(t - \gamma(t)) + K_i \hat{\theta}_i(t - m(t))
\]

\[
+ \alpha_1 \Delta K_{1i}(t) \hat{\theta}_i(t - m(t)) + (\alpha(t) - \alpha_1)
\]

\[
\times \Delta K_{1i}(t) \hat{\theta}_i(t - m(t)) + \hat{D} \omega(t), \quad i = 1, 2, \ldots, N
\]

\[
\hat{p}_i(t) = \tilde{\zeta} \hat{\theta}_i(t)
\]

(18)

where \( m(t) = t - t_k + \epsilon, \ t_k \leq t < t_{k+1}, \epsilon \leq m(t) < t_{k+1} - t_k + \epsilon \leq \bar{m}, \hat{m}(t) \leq \omega < 1 \).

It is obvious that, (18) can be represented in the kro-\( \text{necker} \) form as:

\[
\dot{\hat{\theta}}(t) = \hat{g}(\hat{\theta}(t)) + (1 - \delta_1(t))(F \otimes \Gamma_1) \hat{\theta}(t) + \delta_1(t)
\]

\[
\times (F \otimes \Gamma_2) \hat{\theta}(t - \gamma(t)) + K \hat{\theta}(t - m(t))
\]

\[
+ \alpha_1 \Delta \zeta(t) + (\alpha(t) - \alpha_1) \Delta \zeta(t) + \hat{D} \omega(t)
\]

(19)

\[
\hat{p}(t) = \tilde{\zeta} \theta(t)
\]

where \( K = \text{diag}(K_{11}, K_{12}, \ldots, K_{1N}) \), \( B = \text{diag}(B_1, B_2, \ldots, B_N) \), \( L_1 = \text{diag}(L_{11}, L_{12}, \ldots, L_{1N}) \), \( \tilde{\zeta} = (I \otimes \Delta(t) = \Delta(t) 
\]

\( L_1 \hat{\theta}(t - m(t)), \bar{D} = \text{diag}(D, D, \ldots, D), \Delta(t) = \text{diag}(\Delta(t), \Delta(t), \ldots, \Delta(t)) \)

\[
\hat{g}(\hat{\theta}(t)) = \begin{bmatrix} f(x_1(t)) - f(s(t)) \\
f(x_2(t)) - f(s(t)) \\
\vdots \\
f(x_N(t)) - f(s(t)) \end{bmatrix}, \quad \omega(t) = \begin{bmatrix} \omega_1(t) \\
\omega_2(t) \\
\vdots \\
\omega_N(t) \end{bmatrix}
\]

\[
\hat{\theta}(t) = \begin{bmatrix} \dot{\theta}_1(t) \\
\dot{\theta}_2(t) \\
\vdots \\
\dot{\theta}_N(t) \end{bmatrix}, \quad \hat{\theta}(t - m(t)) = \begin{bmatrix} \dot{\theta}_1(t - m(t)) \\
\dot{\theta}_2(t - m(t)) \\
\vdots \\
\dot{\theta}_N(t - m(t)) \end{bmatrix}
\]

Remark 2.1: The occurrence of controller gain fluctuations might result from actuator degradations, hence, the need for readjustment of the controller gains during implementations (Dorato, 1998). These parameter uncertainties are unavoidable which would eventually affect the stability and performance of the network if not handled well. The non-fragile sampled-data control is considered to ensure synchronization of the CDNs with such parameter perturbations or uncertainties.

We now present the following definitions and lemmas to help derive the main results in this paper.

**Definition 2.1 (Li et al., 2011):** The CDNs (1) is exponentially synchronized if the closed-loop error dynamics (19) is exponentially stable, when there exist two constants \( \alpha > 0 \) and \( \beta > 0 \), with \( \sigma(t) = 0 \) such that, the following condition holds:

\[
\mathbb{E}[\| \dot{\theta}(t) \|^2] \leq \beta e^{-\alpha t} \sup_{\max|\gamma|,|\phi| \leq \phi \leq 0} \mathbb{E}[\| \dot{\theta}(\phi) \|, \| \dot{\theta}(\phi) \|^2].
\]

(20)

**Definition 2.2 (Lakshmishnan et al., 2014):** The CDNs(1) is said to be exponentially \( H_\infty \) synchronized when \( \sigma(t) = 0 \) holds in (20) under a zero initial condition, if there exists
\[ S > 0 \text{ and a scalar } \varrho > 0 \text{ such that} \]
\[ J = \int_0^\infty e^{2as} (\dot{p}^T(s) \ddot{p}(s) - \varrho^2 Y^T(s) \ddot{y}(s)) \, ds < 0 \quad (21) \]
where a non-zero \( \sigma(t) \in L[0, \infty) \) and the parameter \( \varrho \) is known as the \( H_{\infty} \) norm bound or the disturbance attenuation level.

**Lemma 2.1 (Free-matrix based inequality Zeng et al., 2015):** Let \( \vartheta(t) \) be a differentiable signal in \( \bar{a}, \bar{b} \) \( \rightarrow \mathbb{R}^n. \)

For a given symmetric positive matrices \( \bar{\mathbf{R}} \in \mathbb{R}^{n \times n} \) with \( \bar{R}_1, \bar{R}_2 \in \mathbb{R}^{3n \times 3n} \) and any matrices \( \bar{R}_3 \in \mathbb{R}^{3n \times 3n} \), and

\[
\begin{bmatrix}
\bar{R}_{11} \\
\bar{R}_{12} \\
\bar{R}_{13} \\
\bar{R}_{21} \\
\bar{R}_{22} \\
\bar{R}_{23}
\end{bmatrix}
\] satisfying

\[ \Psi = \begin{bmatrix} \bar{R}_1 & \bar{R}_2 \\ \bar{R}_3 & \bar{R}_4 \end{bmatrix} > 0, \]
the following inequality holds:

\[ -\int_0^{\bar{b}} \vartheta^T(s) \dot{R} \vartheta(s) \, ds \leq \Psi^T \begin{bmatrix} \Gamma + \Pi \end{bmatrix} \Psi \]

where, \( \Gamma = (\bar{\vartheta} - \vartheta)(\bar{R}_1 + \bar{R}_2 \bar{R}_1), \quad \Pi = \text{Sym}(\bar{R}_1 \bar{R}_1) \), \( \bar{R}_1 = [l, -l, 0] \), \( \bar{R}_3 = [-l, -l, 2l] \), \( \Psi = [\vartheta^T(\bar{\vartheta}), \vartheta^T(\vartheta), 1/(\bar{\vartheta} - \vartheta)] \int_0^{\bar{b}} \vartheta^T(s) \, ds \).

**Lemma 2.2 (Seuret & Gouaisbaut, 2013):** Let \( \bar{\mathbf{R}} \) be a positive symmetric definite matrix and for continuously differentiable function \( \vartheta(t) \) in \( \bar{a}, \bar{b} \) \( \rightarrow \mathbb{R}^n \), the following inequality is established:

\[ -\bar{R} \dot{\vartheta}(b) - \vartheta(\vartheta(t)) - 3\Delta \bar{R} \Delta \]

where \( \Delta = \vartheta(\bar{b}) + \vartheta(\bar{a}) - 2/(\bar{b} - \bar{a}) \int_0^{\bar{a}} \vartheta(s) \, ds \).

**Lemma 2.3 (Kwon et al., 2008):** For any real matrices \( \bar{X}, \bar{Y} \) and \( Q > 0 \), all having appropriate dimensions, the following inequality holds:

\[ 2\bar{X}^T \bar{Y} \leq \bar{X}^T Q \bar{X} + \bar{Y}^T Q^{-1} \bar{Y} \]

**Remark 2.2:** It is important to note that, the free matrices in the inequality of Lemma 2.1 gives a better relaxation in deriving the stability criteria which ensures the achievement of tight bound. It is easy to arrive at some well established integral inequalities as special cases of Lemma 2.1. An instance is letting \( \bar{R}_1 = [Y^T, 0]^T, \bar{R}_2 = 0, \bar{R}_3 = \text{diag}(\vartheta(t), 0, 0, 0), \bar{R}_4 = 0, \) and \( \bar{R}_5 = 0, \) this reduces the above lemma to the case in Zhang et al. (2005, Lemma 2). In another situation, let \( \bar{R}_1 = 1/(\bar{\vartheta} - \vartheta)(-\bar{R}, \bar{R}, 0)^T, \bar{R}_2 = \bar{R}/(\bar{\vartheta} - \vartheta)(-\bar{R}, \bar{R}, -2\bar{R}), \bar{R}_3 = \bar{R}^{-1} \bar{R}_1 \bar{R}_2, \) and \( \bar{R}_4 = \bar{R}_3 \bar{R}_2^{-1} \bar{R}_3^{-1}, \) the integral inequality becomes the well known Wirtinger integral inequality (Seuret & Gouaisbaut, 2013).

### 3. Main results

In this section, we first establish sufficient conditions, which allow the error dynamic system (19) of the CDN’s to be exponentially synchronized with \( \sigma(t) \neq 0 \) under the designed non-fragile sampled data control for the CDN(1). For the sake brevity in our presentation of the main results, we first denote

\[ \bar{\Delta}(t) = \Delta(t)L_1 \dot{\vartheta}(t - m(t)) \quad \bar{Q}_1 = \frac{(l \otimes Q_1)^T(l \otimes Q_2)}{2} + \frac{(l \otimes Q_1)^T(l \otimes Q_1)}{2} \]

\[ \bar{Q}_2 = -\frac{(l \otimes Q_1)^T(l \otimes Q_2)^T}{2} \]

To ensure simplicity of matrix representation, we use \( e_i^T = \frac{l - i}{\sqrt{l(l+1)}}, 0, 0, \ldots, 0 \) \( (i = 1, \ldots, 15) \) to represent block entry matrices.

\[ \bar{X}_1 = \begin{bmatrix} 0^T \end{bmatrix}, \quad \bar{X}_{11} = [l, -l, 0], \quad \bar{X}_2 = \begin{bmatrix} e_1^T \\ e_2^T \\ e_3^T \end{bmatrix} \]

\[ \bar{X}_{12} = [-l, -l, 2l] \]

\[ \bar{Y}_1(t) = \begin{bmatrix} \dot{\vartheta}^T(t), \dot{\vartheta}^T(t), \dot{\vartheta}^T(t - \gamma(t)), \dot{\vartheta}^T(t - \gamma) \end{bmatrix}, \]

\[ \dot{\vartheta}^T(t - m(t)), \dot{\vartheta}^T(t - m), \frac{1}{\gamma(t)} \int_{t - \gamma(t)}^t \dot{\vartheta}^T(s) \, ds, \]

\[ \frac{1}{\gamma(t) - \gamma(t)} \int_{t - \gamma(t)}^{t - \gamma(t)} \dot{\vartheta}^T(s) \, ds, \dot{\vartheta}^T(t - m(t)), \dot{\vartheta}^T(t - m), \frac{1}{\gamma(t)} \int_{t - \gamma(t)}^t \dot{\vartheta}^T(s) \, ds, \dot{\vartheta}^T(t - k), \int_{t - k}^t \dot{\vartheta}^T(s) \, ds, \dot{\vartheta}^T(t - k), \frac{1}{t - k} \int_{k}^t \dot{\vartheta}^T(s) \, ds, \overline{\sigma(t)} \]

\[ \Psi = (e_1^T + (1 - \delta_1)(F \otimes \Gamma_1))e_1 + \delta_1(F \otimes \Gamma_2)e_3 + K_1 \delta_2 + D \delta_{15} - e_2 \]

\[ \mathcal{M}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ \mathcal{M}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]
Theorem 3.1: For given scalars \( \gamma > 0, \nu > 0, \iota \geq 0, \delta_1 \in [0,1], \alpha_1 \in [0,1], \omega < 1, \sigma(i(1,2)), \Omega, \tau, \theta, m \) and \( \bar{e} \), the error dynamic system (19) is exponentially synchronized with a decay rate of \( \alpha \) under \( H_\infty \) norm for all \( d_k \leq d \), if there exist matrices \( \mathcal{P} \), \( \Omega_i \), \( \tilde{\mathcal{S}} \), \( \tilde{\mathcal{S}}_j \), \( \mathcal{S} > 0 \), \( Q > 0 \), \( \mathcal{V} = \begin{bmatrix} \mathcal{V}_1 & \mathcal{V}_{12} \\ \mathcal{V}_{12}^T & \mathcal{V}_{22} \end{bmatrix} > 0 \), symmetrical matrices \( \mathcal{X}_1, \mathcal{X}_3, W_1, W_3, X_1, X_2, X_5 \) and any matrices 

\[
\mathcal{X}_2, \tilde{N}_1, \tilde{N}_2, W_2, \mathcal{M}_1, \tilde{\mathcal{M}}_2, \tilde{\mathcal{M}}_2^*, \begin{bmatrix} \mathcal{X}_1 & \mathcal{X}_2 \\ \mathcal{X}_3 & \mathcal{X}_4 \end{bmatrix} > 0, \begin{bmatrix} W_1 & W_2 & \mathcal{M}_1 \\ \mathcal{M}_2 & \mathcal{M}_2^* \end{bmatrix} > 0,
\]

where

\[
\begin{aligned}
\Phi_1 &= e_1(2\alpha \gamma + \sum_{i=1}^{m} \Omega_i + Z_1 - Z_3 - \frac{X_1 + X_1^T}{2} - 2\bar{e}\tilde{Q}_1 - \frac{2\bar{e}}{2m\alpha} \tilde{\mathcal{S}}_2) e_1^T + e_1\mathcal{P} e_1^T + e_2\mathcal{P} e_2^T \\
&+ e_2(e_{2\alpha} - \frac{1}{2\alpha} \tilde{\mathcal{S}}_1 + \tilde{m}\tilde{\mathcal{S}}_2 + \frac{\iota}{2\alpha}(e_{2\alpha} - 1)Z_3 + \frac{d - \iota}{2\alpha}(e_{2\alpha} - e_{2\alpha})Z_4) e_1^T - (1 - \nu)e_{2\alpha} e_3 e_3^T \\
&- e_{2\alpha} e_4 e_4^T - e_{2\alpha} e_5 e_5^T + e_5 \tilde{Q}_1 + \frac{1}{3} \tilde{Q}_1^T e_1^T - e_{2\alpha} e_6 e_6^T - e_{2\alpha} e_7 e_7^T + e_7 \tilde{Q}_1 + \frac{1}{3} \tilde{Q}_1^T e_1^T - e_{2\alpha} e_8 e_8^T - e_{2\alpha} e_9 e_9^T + e_9 \tilde{Q}_1 + \frac{1}{3} \tilde{Q}_1^T e_1^T - e_{2\alpha} e_{10} e_{10}^T - e_{2\alpha} e_{11} e_{11}^T - e_{2\alpha} e_{12} e_{12}^T - e_{2\alpha} e_{13} e_{13}^T \\
&- e_{2\alpha} e_{14} e_{14}^T - e_{2\alpha} e_{15} e_{15}^T + e_{16} \tilde{Q}_1 + \frac{1}{3} \tilde{Q}_1^T e_1^T - e_{2\alpha} e_{16} e_{16}^T - e_{2\alpha} e_{17} e_{17}^T + e_{17} \tilde{Q}_1 + \frac{1}{3} \tilde{Q}_1^T e_1^T - e_{2\alpha} e_{18} e_{18}^T - e_{2\alpha} e_{19} e_{19}^T + e_{19} \tilde{Q}_1 + \frac{1}{3} \tilde{Q}_1^T e_1^T - e_{2\alpha} e_{20} e_{20}^T - e_{2\alpha} e_{21} e_{21}^T + e_{21} \tilde{Q}_1 + \frac{1}{3} \tilde{Q}_1^T e_1^T - e_{2\alpha} e_{22} e_{22}^T \\
&- e_{2\alpha} e_{23} e_{23}^T - e_{2\alpha} e_{24} e_{24}^T + e_{24} \tilde{Q}_1 + \frac{1}{3} \tilde{Q}_1^T e_1^T - e_{2\alpha} e_{25} e_{25}^T - e_{2\alpha} e_{26} e_{26}^T + e_{26} \tilde{Q}_1 + \frac{1}{3} \tilde{Q}_1^T e_1^T - e_{2\alpha} e_{27} e_{27}^T - e_{2\alpha} e_{28} e_{28}^T + e_{28} \tilde{Q}_1 + \frac{1}{3} \tilde{Q}_1^T e_1^T - e_{2\alpha} e_{29} e_{29}^T - e_{2\alpha} e_{30} e_{30}^T + e_{30} \tilde{Q}_1 + \frac{1}{3} \tilde{Q}_1^T e_1^T - e_{2\alpha} e_{31} e_{31}^T - e_{2\alpha} e_{32} e_{32}^T + e_{32} \tilde{Q}_1 + \frac{1}{3} \tilde{Q}_1^T e_1^T - e_{2\alpha} e_{33} e_{33}^T - e_{2\alpha} e_{34} e_{34}^T + e_{34} \tilde{Q}_1 + \frac{1}{3} \tilde{Q}_1^T e_1^T
\end{aligned}
\]

\[
\mathcal{F}_3 = e_2 \tilde{\mathcal{V}}_{11} e_2^T + e_1 \tilde{\mathcal{V}}_{12} e_1^T + \text{Sym}(e_2 \tilde{\mathcal{V}}_{12} e_1^T)
\]

\[
\mathcal{F}_4 = e_1 \tilde{X}_1 e_1^T + e_2 \tilde{X}_1 e_1^T + e_1 \tilde{X}_2 e_1^T + e_1 \tilde{X}_2 e_1^T + e_1 \tilde{X}_2 e_1^T + e_2 (-X_2 + X_2) e_1^T
\]

Moreover, the desired control gain matrix is given by

\[
K_1 = \tilde{F}^{-1}\tilde{G}.
\]
**Proof:** Consider the following Lyapunov-Krasovskii function (LKF) for the error system (19):

\[ V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t) + V_6(t) + V_7(t) \]

\[ V_1(t) = e^{2at} \vartheta^T(T) P \vartheta(t) \]

\[ V_2(t) = \int_{t-\gamma}^{t} e^{2as} \vartheta^T(s) \Omega_1 \vartheta(s) \ ds + \int_{t-m}^{t} e^{2as} \vartheta^T(s) \Omega_2 \vartheta(s) \ ds \]

\[ V_3(t) = \int_{t-\gamma}^{t} e^{2as} \vartheta^T(s) \Omega_3 \vartheta(s) \ ds + \int_{t-m}^{t} e^{2as} \vartheta^T(s) \Omega_4 \vartheta(s) \ ds \]

\[ V_4(t) = \int_{t-\gamma}^{t} \int_{t-\lambda}^{t} e^{2as} \vartheta^T(s) \vartheta(s) \ ds \ dx + \int_{t-m}^{t} (m - t + s) e^{2as} \vartheta^T(s) \vartheta(s) \ ds \]

\[ V_5(t) = \int_{t-\gamma}^{t} \int_{t-d}^{t} e^{2as} \vartheta^T(s) \vartheta(s) \ ds \ dx + (t - i) \int_{t-m}^{t} e^{2as} \vartheta^T(s) \vartheta(s) \ ds \]

\[ V_6(t) = (d - (t - t_k)) \int_{t_k}^{t} e^{2as} \left[ \frac{\vartheta(t) - \vartheta(t_k)}{\vartheta(t_k)} \right] \left[ \begin{array}{c} X_1 + X_1^T \ 0 \end{array} \right] \ ds \]

\[ V_7(t) = (d - (t - t_k))e^{2at} \left[ \int_{t_k}^{t} \vartheta(s) \ ds \right] \left[ \begin{array}{c} X_1 + X_1^T \ 0 \end{array} \right] \]

Now considering the infinitesimal operator \( \mathcal{L} \) of \( V(t) \), which is defined as follows:

\[ \mathcal{L}V(t) = \lim_{\Delta \to 0^+} \frac{1}{\Delta} \left[ \mathbb{E}[V(t + \Delta)|t] - V(t) \right] \]

where

\[ \mathbb{E}[\mathcal{L}V(t)] = \mathbb{E}[2axe^{2at} \vartheta^T(T) P \vartheta(t) + 2e^{2at} \vartheta^T(t) P \vartheta(t)] \]

\[ \mathbb{E}[\mathcal{L}V_2(t)] = \mathbb{E}[e^{2at} \vartheta^T(t) \Omega_1 \vartheta(t) - e^{2at} \vartheta^T(t - \gamma) \vartheta(t - \gamma) \Omega_1 \vartheta(t - \gamma) + e^{2at} \vartheta^T(t) \Omega_2 \vartheta(t) - e^{2at} \vartheta^T(t - \bar{m}) \bar{\vartheta}(t - \bar{m})] \]

\[ \mathbb{E}[\mathcal{L}V_3(t)] = \mathbb{E}[e^{2at} \vartheta^T(t) \Omega_3 \vartheta(t) - (1 - \vartheta^T(t)) e^{2at} \vartheta^T(t - \gamma) \Omega_4 \vartheta(t - \gamma) + e^{2at} \vartheta^T(t) \Omega_4 \vartheta(t) - (1 - \bar{m}(t)) e^{2at} \vartheta^T(t - \bar{m}) \bar{\vartheta}(t - \bar{m})] \]

\[ \mathbb{E}[\mathcal{L}V_4(t)] = \mathbb{E}[e^{2at} \vartheta^T(t) \Omega_5 \vartheta(t) - (1 - \vartheta^T(t)) e^{2at} \vartheta^T(t - \gamma) \Omega_5 \vartheta(t - \gamma) - (1 - \omega) \vartheta^T(t - m(t)) e^{2at} \vartheta(t - m(t))] \]

where
From Lemma 2.1, we shall have the integral part of (29) and (30) will result in

\[
-\int_{t-\gamma}^{t} \hat{\theta}^T(s) \Xi(s) \hat{\theta}(s) \, ds = -\int_{t-\gamma}^{t} \hat{\theta}^T(s) \Xi(s) \hat{\theta}(s) \, ds - \int_{t-\gamma}^{t} \hat{\theta}^T(s) \Xi(s) \hat{\theta}(s) \, ds
\]

\[
\leq \tilde{\eta}(t) \left[ \gamma \tilde{Z}_1(\bar{X}_1 + \frac{1}{3} \tilde{X}_3) \tilde{Z}_1 + \gamma \tilde{s}_2(W_1 + \frac{1}{3} W_3) \tilde{Z}_2 + \text{Sym} \{ \tilde{N}_1 \tilde{N}_1 \tilde{Z}_1 \} + \tilde{\bar{N}}_2 \tilde{N}_2 \tilde{Z}_2 + \tilde{s}_2 \tilde{M}_1 \tilde{Z}_2 + \tilde{s}_2 \tilde{M}_2 \tilde{Z}_2 \right] \tilde{\eta}(t)
\]

\[
(30)
\]

\[
E[\mathcal{L}_4(t)] \leq E \left[ e^{2\alpha t} \left( \hat{\theta}^T(t) \left( \frac{e^{2\alpha \gamma} - 1}{2\alpha} \right) \Xi \hat{\theta}(t) + \tilde{\eta}(t) (\gamma \tilde{Z}_1(\bar{X}_1 + \frac{1}{3} \tilde{X}_3) \tilde{Z}_1 + \gamma \tilde{s}_2(W_1 + \frac{1}{3} W_3) \tilde{Z}_2 + \text{Sym} \{ \tilde{N}_1 \tilde{N}_1 \tilde{Z}_1 \} + \tilde{s}_2 \tilde{M}_1 \tilde{Z}_2 + \tilde{s}_2 \tilde{M}_2 \tilde{Z}_2 \} \tilde{\eta}(t) \right)
\]

\[
= e^{2\alpha t} E \left[ \hat{\theta}^T(t) Z_1 \hat{\theta}(t) - e^{2\alpha (t-\gamma)} \hat{\theta}^T(t) Z_1 \hat{\theta}(t - \gamma) + e^{2\alpha (t-\gamma)} \hat{\theta}^T(t - \gamma) Z_2 \hat{\theta}(t) \right]
\]

\[
- e^{2\alpha (t-\gamma)} \hat{\theta}^T(t - \gamma) Z_2 \hat{\theta}(t) + d \int_{t-\gamma}^{t} e^{2\alpha (t-\gamma)} \hat{\theta}^T(t) Z_4 \hat{\theta}(t) \, ds \leq E \left[ e^{2\alpha t} \left( \hat{\theta}^T(t) Z_1 \hat{\theta}(t) + \hat{\theta}^T(t) (e^{-2\alpha (Z_2 - Z_1)}) \hat{\theta}^T(t - \gamma) - e^{-2\alpha d} \hat{\theta}^T(t - \gamma) Z_2 \hat{\theta}(t) \right) \right]
\]

\[
= e^{2\alpha t} E \left[ \hat{\theta}^T(t) Z_1 \hat{\theta}(t) + \hat{\theta}^T(t + \lambda) (e^{-2\alpha (Z_2 - Z_1)}) \hat{\theta}^T(t - \gamma) - e^{-2\alpha d} \hat{\theta}^T(t - \gamma) Z_2 \hat{\theta}(t) \right]
\]

\[
+ d \int_{t-\gamma}^{t} e^{-2\alpha d} \hat{\theta}^T(t) Z_4 \hat{\theta}(t) \, ds \leq e^{2\alpha t} E \left[ \hat{\theta}^T(t) Z_1 \hat{\theta}(t) + \hat{\theta}^T(t + \lambda) (e^{-2\alpha (Z_2 - Z_1)}) \hat{\theta}^T(t - \gamma) - e^{-2\alpha d} \hat{\theta}^T(t - \gamma) Z_2 \hat{\theta}(t) \right]
\]

\[
- d \int_{t-\gamma}^{t} e^{-2\alpha d} \hat{\theta}^T(t) Z_4 \hat{\theta}(t) \, ds \leq e^{2\alpha t} E \left[ \hat{\theta}^T(t) Z_1 \hat{\theta}(t) + \hat{\theta}^T(t - \gamma) (e^{-2\alpha (Z_2 - Z_1)}) \hat{\theta}^T(t - \gamma) - e^{-2\alpha d} \hat{\theta}^T(t - \gamma) Z_2 \hat{\theta}(t) \right]
\]

\[
- d \int_{t-\gamma}^{t} e^{-2\alpha d} \hat{\theta}^T(t) Z_4 \hat{\theta}(t) \, ds \leq e^{2\alpha t} E \left[ \hat{\theta}^T(t) Z_1 \hat{\theta}(t) + \hat{\theta}^T(t) (e^{-2\alpha (Z_2 - Z_1)}) \hat{\theta}^T(t - \gamma) - e^{-2\alpha d} \hat{\theta}^T(t - \gamma) Z_2 \hat{\theta}(t) \right]
\]

\[
- d \int_{t-\gamma}^{t} e^{-2\alpha d} \hat{\theta}^T(t) Z_4 \hat{\theta}(t) \, ds \leq e^{2\alpha t} E \left[ \hat{\theta}^T(t) Z_1 \hat{\theta}(t) + \hat{\theta}^T(t - \gamma) (e^{-2\alpha (Z_2 - Z_1)}) \hat{\theta}^T(t - \gamma) - e^{-2\alpha d} \hat{\theta}^T(t - \gamma) Z_2 \hat{\theta}(t) \right]
\]

\[
- d \int_{t-\gamma}^{t} e^{-2\alpha d} \hat{\theta}^T(t) Z_4 \hat{\theta}(t) \, ds \leq e^{2\alpha t} E \left[ \hat{\theta}^T(t) Z_1 \hat{\theta}(t) + \hat{\theta}^T(t) (e^{-2\alpha (Z_2 - Z_1)}) \hat{\theta}^T(t - \gamma) - e^{-2\alpha d} \hat{\theta}^T(t - \gamma) Z_2 \hat{\theta}(t) \right]
\]

\[
- d \int_{t-\gamma}^{t} e^{-2\alpha d} \hat{\theta}^T(t) Z_4 \hat{\theta}(t) \, ds \leq e^{2\alpha t} E \left[ \hat{\theta}^T(t) Z_1 \hat{\theta}(t) + \hat{\theta}^T(t) (e^{-2\alpha (Z_2 - Z_1)}) \hat{\theta}^T(t - \gamma) - e^{-2\alpha d} \hat{\theta}^T(t - \gamma) Z_2 \hat{\theta}(t) \right]
\]

\[
- d \int_{t-\gamma}^{t} e^{-2\alpha d} \hat{\theta}^T(t) Z_4 \hat{\theta}(t) \, ds \leq e^{2\alpha t} E \left[ \hat{\theta}^T(t) Z_1 \hat{\theta}(t) + \hat{\theta}^T(t) (e^{-2\alpha (Z_2 - Z_1)}) \hat{\theta}^T(t - \gamma) - e^{-2\alpha d} \hat{\theta}^T(t - \gamma) Z_2 \hat{\theta}(t) \right]
\]

\[
- d \int_{t-\gamma}^{t} e^{-2\alpha d} \hat{\theta}^T(t) Z_4 \hat{\theta}(t) \, ds \leq e^{2\alpha t} E \left[ \hat{\theta}^T(t) Z_1 \hat{\theta}(t) + \hat{\theta}^T(t) (e^{-2\alpha (Z_2 - Z_1)}) \hat{\theta}^T(t - \gamma) - e^{-2\alpha d} \hat{\theta}^T(t - \gamma) Z_2 \hat{\theta}(t) \right]
\]
Consider Lemma 2.2 and the integral term of (33)

\[
\begin{align*}
\int_{t_k}^{t} \dot{\theta}^T(s) \dot{\vartheta}(s) \, ds & \leq \frac{1}{t - t_k} \eta(t) \left( M_1^T V_{11} M_1 - (t - t_k) F_{11} \right) \\
& + 3 M_2^T V_{11} M_2 \eta(t)
\end{align*}
\]

(34)

For appropriately dimensioned matrices \( F_i, (i = 1, 2) \), it is easy to derive the following

\[
\begin{align*}
\frac{1}{t - t_k} (V_{11} M_1 - (t - t_k) F_{11}) V_{11}^{-1} (V_{11}) & \times M_1 - (t - t_k) F_1 \geq 0 \\
\frac{1}{t - t_k} (V_{11} M_2 - (t - t_k) F_{22}) V_{11}^{-1} (V_{11}) & \times M_2 - (t - t_k) F_2 \geq 0
\end{align*}
\]

(35)

(36)

where the following inequalities hold

\[
\begin{align*}
- \frac{1}{t - t_k} M_1^T V_{11} M_1 & \leq - F_1^T M_1 - M_1^T F_1 + (t - t_k) \\
& \times F_1^T V_{11}^{-1} F_1,
\end{align*}
\]

(37)

\[
\begin{align*}
- \frac{1}{t - t_k} M_2^T V_{11} M_2 & \leq - F_2^T M_2 - M_2^T F_2 + (t - t_k) \\
& \times F_2^T V_{11}^{-1} F_2,
\end{align*}
\]

(38)

Equation (33) is rewritten as

\[
\begin{align*}
\mathbb{E}[L_6(t)] & \leq e^{2at} \mathbb{E} \left[ e^{-2ad} \left( \eta^T(t)(t - t_k) (F_1^T V_{11}^{-1} F_1 + 3 F_2^T V_{11}^{-1} F_2 - F_1^T M_1 - M_1^T F_1 - 3 F_2^T M_2 \\
- 3 M_2^T F_2) \eta(t) + 2 \theta^T(t) V_{12} \vartheta(t_k) + \theta^T(t_k) (V_{12} + V_{12}^T (t - t_k) V_{22}) \vartheta(t_k) \\
+ (d - (t - t_k)) (t - t_k) F_{11} \vartheta(t) + 2 \theta^T(t) V_{12} \vartheta(t_k) + \theta^T(t_k) V_{22} \vartheta(t_k) \right) \right]
\end{align*}
\]

(39)
For a matrix $\bar{F}$ with appropriate dimensions and any given scalars $\alpha_1$ and $\alpha_2$, we derive from (19):

$$2 e^{2at} \mathbb{E}[(\sigma_1 \vartheta^T(t) \bar{F}) + \vartheta^T(t) \bar{F} + \sigma_2 \vartheta^T(t-m(t)) \bar{F}] [\bar{g}(\vartheta(t)) + (1 - \delta_1)(F \otimes \Gamma_1) \vartheta(t) + \delta_1 (F \otimes \Gamma_2)]$$

$$\times \vartheta(t - \gamma(t)) + K \vartheta(t - m(t)) + \alpha_1 B \bar{\Delta} \vartheta(t) + \bar{D} \vartheta(t) - \hat{\vartheta}(t))$$

$$\leq 2 e^{2at} \mathbb{E}[(\sigma_1 \vartheta^T(t) + \vartheta^T(t) + \sigma_2 \vartheta^T(t-m(t)) [\bar{F}] [\bar{g}(\vartheta(t)) + (1 - \delta_1)(F \otimes \Gamma_1) \vartheta(t) + \delta_1 (F \otimes \Gamma_2)]$$

$$\times \vartheta(t - \gamma(t)) + K \vartheta(t - m(t)) + \bar{D} \vartheta(t) + \hat{\vartheta}(t)] + e^{2at} \mathbb{E}[(\sigma_1 \vartheta^T(t) + \vartheta^T(t) + \sigma_2 \vartheta^T(t-m(t))]$$

$$\times \bar{F} B Q^{-1} B^T \sigma_1 \vartheta^T(t) + \vartheta^T(t) + \sigma_2 \vartheta^T(t-m(t))] + \vartheta^T(t-m(t)) L_1^T Q \vartheta(t - m(t))]$$

$$= 0. \quad (41)$$

Based on (7), with a scalar $\bar{r}$, it follows that

$$\bar{r} e^{2at} [\bar{g}(\vartheta(t)) - Q_1 \vartheta(t)]^T [\bar{g}(\vartheta(t)) - Q_2 \vartheta(t)] \leq 0 \quad (42)$$

also, it is equivalently represented as

$$- \bar{r} e^{2at} \left[ \begin{array}{l} \vartheta(t) \\ \bar{Q}_1 \\ \bar{Q}_2 \\ \bar{g}(\vartheta(t)) \end{array} \right] \geq 0 \quad (43)$$

Thus, adding the left-hand side of (41)–(43) to $\mathbb{E}[(\mathcal{L} V(t))]$, we have for $t \in [t_k, t_{k+1})$

$$\mathbb{E} [e^{2at} \mathcal{S}^T(t) \mathcal{S}(t) - \vartheta^2 e^{2at} \sigma \vartheta^T(t) \mathcal{S} + \mathcal{L} V(t)]$$

$$\leq e^{2at} \mathcal{S}^T(t) \mathcal{L} \mathcal{L}^T(t) \mathcal{S}(t) \leq \mathcal{L} V(t) \quad (44)$$

where,

$$\Phi = \Phi_1 + (d - (t - t_k)) (\Phi_2 + \Phi_3) + (t - t_k) e^{-2at}$$

$$\mathcal{F}_1^T \mathcal{V}_{11}^{-1} \mathcal{F}_1 + e^{-2at} 3 \mathcal{F}_2^T \mathcal{V}_{11}^{-1} \mathcal{F}_2 - \Phi_2$$

$$+ \Phi_3 Q^{-1} \Phi_3^T.$$

Hence, $\Phi$ is a convex combination of $t - t_k$ and $d - (t - t_k)$, therefore $\Phi < 0$, if and only if (23) with the following inequalities hold

$$\Phi_1 + (d - (t - t_k)) (\Phi_4 + \Phi_3) + (t - t_k) e^{-2at}$$

$$\mathcal{F}_1^T \mathcal{V}_{11}^{-1} \mathcal{F}_1 + e^{-2at} 3 \mathcal{F}_2^T \mathcal{V}_{11}^{-1} \mathcal{F}_2 - \Phi_2$$

$$+ \Phi_3 Q^{-1} \Phi_3^T < 0. \quad (45)$$

$$\Phi_1 + (d (\Phi_4 + \Phi_3) + \Phi_3 Q^{-1} \Phi_3^T < 0. \quad (46)$$

By considering the Schur complement, (45) and (46) are equal to (23), and (24) respectively.

$$\mathbb{E}[(\mathcal{L} V(t))] < 0, \quad \forall t \in [t_k, t_{k+1}). \quad (47)$$

It would be realized that, the terms containing $t - t_k$ and $t_{k+1} - t$ vanishes before $t_k$ and after $t_k$, that is $V_j(0) = 0(i = 6, 7)$, and by considering the generalized Itô's formula,

$$\mathbb{E}[(\mathcal{L} V(t))] - \mathbb{E}[(\mathcal{L} V(0))] = \int_0^t \mathbb{E}[(\mathcal{L} S)] ds \leq 0,$$

$$\mathbb{E}[(\mathcal{L} V(t))] \leq \mathbb{E}[(\mathcal{L} V(t_k))] \leq \mathbb{E}[(\mathcal{L} V(t_{k-1}))] \leq \cdots$$

$$\leq \mathbb{E}[(\mathcal{L} V(0))] , \quad t \geq 0. \quad (48)$$
Moreover, we can derive the following

\[ E\{V(0)\} = \vartheta^T(0)\varphi(0) + \int_{-\gamma}^{0} e^{2\alpha t}\vartheta^T(s)\Omega_1\vartheta(s)\,ds + \int_{-\bar{\lambda}}^{0} e^{2\alpha t}\vartheta^T(s)\Omega_2\vartheta(s)\,ds \]

\[ + \int_{-\gamma}^{0} e^{2\alpha t}\vartheta^T(s)\Omega_3\vartheta(s)\,ds + \int_{-\gamma}^{0} e^{2\alpha t}\vartheta^T(s)\Omega_4\vartheta(s)\,ds + \int_{-\gamma}^{0} \int_{-\gamma}^{0} e^{2\alpha s(-\lambda)}\vartheta^T(s)\Xi_1\vartheta(s)\,ds\,d\lambda. \]

\[ + (d - i)\int_{-d}^{-i} \int_{-\gamma}^{0} e^{2\alpha(s-\lambda)}\vartheta^T(s)\Xi_2\vartheta(s)\,ds\,d\lambda. \]

\[ \leq \lambda_{\text{max}}(\mathcal{P})\|\varphi(0)\|^2 + \frac{1 - e^{-2\alpha \gamma}}{2\alpha} (\lambda_{\text{max}}(\Omega_1) + \lambda_{\text{max}}(\Omega_3)) \sup_{-\gamma \leq \theta \leq 0} \|\vartheta(\theta)\|^2 \]

\[ + \frac{1 - e^{-2\alpha \bar{\lambda}}}{2\alpha} (\lambda_{\text{max}}(\Omega_2) + \lambda_{\text{max}}(\Omega_4)) \sup_{-\bar{\lambda} \leq \theta \leq 0} \|\vartheta(\theta)\|^2 \]

\[ + \frac{e^{2\alpha \gamma} + 2\alpha \gamma - 1}{4\alpha^2} \lambda_{\text{max}}(\Xi_1) \sup_{-\gamma \leq \theta \leq 0} \|\vartheta(\theta)\|^2 + \frac{1 - e^{-2\alpha i}}{2\alpha} \lambda_{\text{max}}(Z_1) \]

\[ \times \sup_{-1 \leq \theta \leq 0} \|\vartheta(\theta)\|^2 + \frac{e^{-2\alpha i} - e^{-2\alpha d}}{2\alpha} \lambda_{\text{max}}(Z_2) \sup_{-d \leq \theta \leq 0} \|\vartheta(\theta)\|^2 + \frac{ie^{2\alpha i} + 2\alpha^2 - i}{4\alpha^2} \lambda_{\text{max}}(Z_3) \]

\[ \times \sup_{-d \leq \theta \leq 0} \|\vartheta(\theta)\|^2 + (d - i)\frac{e^{2\alpha d} - e^{2\alpha i} - 2\alpha(d - i)}{4\alpha^2} \lambda_{\text{max}}(Z_4) \sup_{-d \leq \theta \leq 0} \|\vartheta(\theta)\|^2 \]  \Tag{49}

Likewise, from \(V_1(t)\), it can be established that

\[ V(t) \geq e^{2\alpha t}\lambda_{\text{min}}(\mathcal{P}) \|\vartheta(t)\|^2. \] \Tag{50}

Therefore, considering Equation \(48\) and \(50\), it follows that:

\[ E\|\vartheta(t)\| \leq e^{-\alpha t} \sqrt{\frac{\kappa}{\lambda_{\text{min}}(\mathcal{P})}} \sup_{-\gamma \leq \theta \leq 0} \|\vartheta(\theta)\|, \|\vartheta(\theta)\| \]

\[ E\|\vartheta(\theta)\|, \|\dot{\vartheta}(\theta)\| \]  \Tag{51}

where

\[ \kappa = \lambda_{\text{max}}(\mathcal{P}) + \frac{1 - e^{-2\alpha \gamma}}{2\alpha} (\lambda_{\text{max}}(\Omega_1) + \lambda_{\text{max}}(\Omega_3)) \]

\[ + \frac{1 - e^{-2\alpha \bar{\lambda}}}{2\alpha} (\lambda_{\text{max}}(\Omega_2) + \lambda_{\text{max}}(\Omega_4)) \]

\[ + \frac{e^{2\alpha \gamma} + 2\alpha \gamma - 1}{4\alpha^2} \lambda_{\text{max}}(\Xi_1) \]

\[ + \frac{1 - e^{-2\alpha i}}{2\alpha} \lambda_{\text{max}}(Z_1) + \frac{e^{-2\alpha i} - e^{-2\alpha d}}{2\alpha} \lambda_{\text{max}}(Z_2) \]

\[ + \frac{ie^{2\alpha i} + 2\alpha^2 - i}{4\alpha^2} \lambda_{\text{max}}(Z_3) \lambda_{\text{max}}(Z_4) \]

\[ \times (d - i)\frac{e^{2\alpha d} - e^{2\alpha i} - 2\alpha(d - i)}{4\alpha^2} . \]  \Tag{52}

According to Definition \(2.1\), we can infer from \(51\) that, the error dynamics \(19\) is exponentially synchronized with the decay rate \(\alpha\). This completes the proof. \Halmos

**Remark 3.1:** Considering an appropriate LKF which is very important in the derivation of less conservative results. Based on the time-dependent LKF approach, the involvement of \(V_6(t)\) and \(V_7(t)\) which in our case contain \(t_k\) and \(t_{k+1}\) terms ensured the full use of the available information pertaining to the actual sampling formation. Hence an improvement in the conservativeness of our proposed results. In the cases of Zhang et al. \(2010\) and Wu et al. \(2012\), such terms were not included, hence our Theorem 3.1 is deemed more efficient and practical.

**Remark 3.2:** Our Theorem 3.1, provide a new synchronization criterion for \(1\). It is obvious that, our given results have been formulated by LMIs, can be readily verified using already existing tools such as Matlab LMI tools box. The LMIs in our case, also took into consideration transmission delays in the non-fragile sampled-data controller design and exponential decay rate \(\alpha\).

When \(\varphi(t) = 0\), the exponential synchronization of the CDNs \(19\) will reduce to the following error dynamical
system,
\[
\dot{\theta}(t) = \ddot{g}(\theta(t)) + (1 - \delta_1(t))(F \otimes \Gamma_1) \dot{\theta}(t) + \delta_1(t) (F \otimes \Gamma_2) \dot{\theta}(t - \gamma(t)) + K_1 \dot{\theta}(t - m(t)) + \alpha_1 \theta \Delta(t) + (\alpha(t) - \alpha_1) \theta \Delta(t)
\]
\[
\hat{\rho}(t) = \hat{C}_d(t).
\]

Based on the LKF given at (25) and Theorem 3.1, we have the following Theorem 3.2 on the exponential synchronization of non-fragile sampled data controlled CDN with random coupling and time-varying delay. For simplicity of presentation of results, the following notations are used:
\[
\hat{e}_i^T = [\hat{e}_1^T, \ldots, \hat{e}_{14}^T](i = 1, \ldots, 14)\text{ to represent block entry matrices.}
\]
\[
\bar{\hat{e}}_1 = \begin{bmatrix} \hat{e}_1^T \\ \vdots \\ \hat{e}_{14}^T \end{bmatrix}, \quad \bar{\hat{e}}_{11} = [l, -l, 0], \quad \bar{\hat{e}}_2 = \begin{bmatrix} \hat{e}_2^T \\ \vdots \\ \hat{e}_{14}^T \end{bmatrix},
\]
\[
\bar{\hat{e}}_{12} = [-l, -l, 2l]
\]
\[
\hat{\theta}^T(t) = [\hat{\theta}^T(t), \hat{\theta}^T(t - \gamma(t)), \hat{\theta}^T(t - \gamma),
\]
\[
\frac{1}{\gamma - \gamma(t)} \int_{t - \gamma(t)}^{t} \hat{\theta}^T(s) \, ds,
\]
\[
\frac{1}{t - t_k} \int_{t_k}^{t} \hat{\theta}^T(s) \, ds \hat{\theta}^T(t_k),
\]
\[
\Psi = (e_{13} + (1 - \delta_1)(F \otimes \Gamma_1)e_1 + \delta_1(F \otimes \Gamma_2)e_3 + K_1 e_5 - e_2)
\]
\[
\tilde{M}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}
\]
\[
\tilde{M}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]

**Theorem 3.2:** For given scalars \( \gamma > 0, \nu > 0, t \geq 0, \delta_1 \in [0, 1], \alpha_1 \in [0, 1], \omega < 1, \alpha(i = 1, 2), \tau, \bar{m}, \text{ and } \bar{e}, \) the error dynamic system (19) is exponentially synchronized with the decay rate \( \alpha \) for all \( d_k \leq d, \) if there exist matrices \( P > 0, \Omega_i > 0(i = 1, 2, 3, 4), \Sigma_j > 0(j = 1, 2), \) \( Z_k > 0(k = 1, 2, 3, 4), Q > 0, \) \( V = \begin{bmatrix} V_{11} & V_{12} \\ V_{12}^T & V_{22} \end{bmatrix} > 0, \) symmetrical matrices \( \bar{X}_1, \bar{X}_2, \bar{W}_1, \bar{W}_3, \bar{X}_1, \bar{X}_2, \bar{X}_5 \) and any matrices \( \bar{X}_2, \bar{N}_1, \bar{N}_2, \bar{W}_2, \bar{M}_1, \bar{M}_2, \)
\[
\begin{bmatrix} \bar{X}_1 & \bar{X}_2 & \bar{N}_1 \\ \bar{X}_3 & \bar{N}_2 & \bar{N}_1 \end{bmatrix} > 0, \quad \begin{bmatrix} W_1 & W_2 & \bar{M}_1 \\ \bar{W}_2 & \bar{M}_2 & \bar{M}_1 \end{bmatrix} > 0,
\]
\[
\begin{bmatrix} X_1 + X_2^T \\ -X_1 + X_2 \\ \bar{X}_2 + \bar{X}_1^T \end{bmatrix} > 0,
\]
\[
\tilde{F}, \tilde{F}_1 \text{ and } \tilde{F}_2 \text{ with appropriate dimensions such that}
\]
\[
\begin{bmatrix} \tilde{\Omega}_1 - d \tilde{\Omega}_2 & d \tilde{F}_1^T & 3d \tilde{F}_2 \\ * & -d e^{2ad} \tilde{V}_{11} & 0 \\ * & * & -3d e^{2ad} \tilde{V}_{11} \end{bmatrix} < 0,
\]
\[
\begin{bmatrix} \tilde{\Omega}_1 + d(\tilde{\Omega}_2 + \tilde{\Omega}_4) & \tilde{\alpha} \tilde{\Omega}_5 \\ * & -\tilde{\alpha} Q \end{bmatrix} < 0.
\]

where,
\[
\tilde{\Omega}_1 = e_1 \left( 2\alpha \tau \rho + \sum_{i=1}^{4} \Omega_i + Z_1 - Z_3 - X_1 + X_2^T \right) - \bar{\tau} \tilde{Q}_1 - e^{-2ad} \tilde{v}_2 e_1^T + e_1 \tau P e_1^T + e_2 \tau P e_1^T
\]
\[
+ e_2 \left( e^{2ad} - 1 \right) \tilde{Q}_1 + \bar{m} \tilde{Q}_2 + \frac{e_2}{2\alpha} \left( e^{2ad} - 1 \right) Z_3 + \frac{d}{2\alpha} \left( e^{2ad} - e^{2ad} \right) Z_4 \right) e_1^T - (1 - \nu) e^{-2ad} e_1 Z_3 e_1^T
\]
\[
- e^{-2ad} e_1 Z_1 e_1^T + e_5 \alpha L_1 ^T Q L_1 e_5^T - e^{-2ad} \left( 1 - \omega \right) e_3 Q_4 e_5^T + e^{-2ad} \tilde{v}_2 e_2 e_5^T - e^{-2ad} e_6 Q_3 Z_3^T e_5^T
\]
\[
+ \frac{e_2}{2\alpha} \tilde{Q}_1 + \frac{e_3}{2\alpha} \tilde{Q}_2 - e_2 \tilde{Q}_2 e_5 + e_2 \tilde{v}_1 e_5^T + e_2 \tilde{v}_2 e_5^T + e_2 \tilde{v}_3 e_5^T + e_2 \tilde{v}_4 e_5^T
\]
\[
+ e_9 (e^{-2ad} (Z_2 - Z_1) - Z_3 - Z_4) e_9^T - e^{-2ad} e_{10} Z_2 e_{10}^T + e_9 Z_4 e_9^T + e_{10} Z_4 e_9^T - e_{10} Z_4 e_9^T
\]
Furthermore, the desired control gain matrix is given by $K_1 = \hat{F}^{-1} \hat{G}$.

Remark 3.3: When $\delta_1(t) = 1$ is considered in (53) with no disturbances ($\sigma(t) = 0$) and transmission delays as well as absence of randomness in the controller gain uncertainties, with similar system considered in Wu et al. (2012), Liu et al. (2018) and Su and Shen (2015).

4. Numerical examples

This section presents two numerical examples to illustrate the validity and effectiveness of the proposed method in this paper.

Example 4.1: Consider the CDNs with three nodes (1) in our situation. The outer coupling matrix $F = (F_{ij})_{3 \times 3}$ with $F = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$.

Case (1): Given that, the inner coupling matrices $\Gamma_1 = 0$ and $\Gamma_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$ with the random coupling probability $\delta_1 = 1$. The nonlinear function $f(\cdot)$ is taken as $f(x_i(t)) = \begin{bmatrix} -0.5x_1 + \tanh(0.2x_1) + 0.2x_2 \\ 0.95x_2 - \tanh(0.75x_2) \end{bmatrix}$, $i = 1, 2, 3$.

which can be found that, $f(\cdot)$ satisfies (7) with $Q_1 = \begin{bmatrix} -0.5 & 0.2 \\ 0 & 0.95 \end{bmatrix}$, $Q_2 = \begin{bmatrix} -0.3 & 0.2 \\ 0 & 0.2 \end{bmatrix}$.

The uncertainties matrices for the controller design are as follows:

$B_1 = \begin{bmatrix} 0.4 \\ 0 \\ 0 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0.5 \\ 0 \\ 0 \end{bmatrix}$,

$L_{11} = \begin{bmatrix} 0.3 \\ 0 \\ 0 \end{bmatrix}$, $L_{12} = \begin{bmatrix} 0.2 \\ 0 \\ 0 \end{bmatrix}$, $L_{13} = \begin{bmatrix} 0.1 \\ 0 \\ 0 \end{bmatrix}$,

$\Delta(t) = \begin{bmatrix} \sin(t) & 0 \\ 0 & \cos(t) \end{bmatrix}$

Choosing $\alpha_1 = 0.3, \alpha_2 = 0.4, \gamma(t) = m(t) = 0.2 + 0.05(10 \sin(t))$, accordingly by simple calculations, we have $\gamma = 0.25, m = 0.25, \nu = 0.5, \omega = 0.5, \alpha_1 = 0$, the transmission delay in the controller $i = 0$. By applying Theorem 3.2, the system under consideration is similar to that in Li et al. (2011), Liu et al. (2018) and Wu et al. (2012), when $c = 1$, it is considered as a special case in our paper. In this example, our maximum sampling period $d = 1.2151$ which is larger and less conservative as compared with the others in Table 1, where our results showed 5.04% improvement of the sampling period over that of Yang, Shu et al., (2016), 16.50% in

| Method           | $\delta_1 = 1$ and $\sigma(t) = 0$ | $d$  | % improvement by Theorem 3.2 |
|------------------|-----------------------------------|------|-----------------------------|
| Li et al. (2011) | 0.1082                           | 118.03 |                              |
| Yang, Shu et al. (2016) | 0.10428                        | 16.50  |                              |
| Theorem 3.2      | 1.2151                           | 16.50  |                              |
| Wu et al. (2012) | 0.5409                           | 124.64 |                              |
the case of Liu et al. (2018), 118.03% increment in the case of Wu et al. (2012) and 124.64% improvement in the case of Li et al. (2011) having the exponential decay rate ($\alpha$) set at 0.3 in our simulations. If one considers a sampling period $d \in (1.1564, 1.2151]$, then with exception of our proposed scheme, all the others as stated will not be able to achieve the desired error synchronization. Therefore, our results is less conservative and effective. Now, utilizing the Matlab LMI toolbox with $\gamma_1 = 0$, $\gamma_2 = 0$ and the above given uncertainties parameters, the LMI (54) and (55) are solved with a maximum sampling period of 1.2151 which achieved feasible solutions. The corresponding non-fragile controller gain matrices obtained are as follows:

$$K_{11} = \begin{bmatrix} -0.6457 & -0.0051 \\ -0.0054 & -0.6588 \end{bmatrix},$$

$$K_{12} = \begin{bmatrix} -0.6444 & -0.0047 \\ -0.0050 & -0.6562 \end{bmatrix},$$

$$K_{13} = \begin{bmatrix} -0.7059 & -0.0006 \\ -0.0008 & -0.6994 \end{bmatrix}.$$

In this simulations, the initial values for the error dynamic system are $x_1(0) = [3, -2]^T$, $x_2(0) = [2, 5]^T$, $x_3(0) = [-5, 6]^T$ and $s(0) = [3, 2]^T$. Figure 2 shows the error synchronization of the CDNs (19) with our control design inputs. The results shows that, the error synchronization converges to the target trajectory (zero). The trajectories for the error dynamic system without the control inputs and the control inputs into the system are shown in Figures 1 and 3 respectively.

Case (2): Given that the coupling matrices are

$$\Gamma_1 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix},$$

$$F = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

and $D = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.
Figure 3. The state trajectories of control inputs with $d = 1.2151$ and $\alpha_1 = 1$ in Example 4.1 (case 1).

with the random coupling probability $\delta_1 = 0.5$, external disturbance of $\sigma(t) = 2/(1 + t^2)$ into the system. We also set the following parameters as $\sigma_1 = 0.3$, $\sigma_2 = 0.4$, $\gamma = 0.25$, $m = 0.25$, $\nu = 0.5$, $\omega = 0.5$, $\alpha_1 = 1$, the delay in the controller, $\iota = 0.05$. According to Theorem 3.1 with sampling interval $d = 0.465$ and setting $\rho = 0.3$, we achieved the following controller gain parameters:

$$K_{11} = \begin{bmatrix} -0.7635 & -0.0016 \\ -0.0017 & -0.7659 \end{bmatrix},$$

$$K_{12} = \begin{bmatrix} -0.7591 & -0.0016 \\ -0.0017 & -0.7618 \end{bmatrix},$$

$$K_{13} = \begin{bmatrix} -0.7537 & -0.0011 \\ -0.0011 & -0.7587 \end{bmatrix}.$$

Example 4.2: Chua’s circuit is considered in this example as an unforced isolated node of CDNs (Hu et al., 2012), which is represented by the following equations:

$$\dot{\vartheta}_1(t) = \tilde{\sigma}_1(-\vartheta_1(t) + \vartheta_2(t) - \rho(\vartheta_1(t))),$$

$$\dot{\vartheta}_2(t) = \vartheta_1(t) - \vartheta_2(t) + \vartheta_3(t),$$

$$\dot{\vartheta}_3(t) = -\tilde{\sigma}_2 \vartheta_2(t),$$

where $\tilde{\sigma}_1 = 10$, $\tilde{\sigma}_2 = 14.87$, $\rho(\vartheta_1(t)) = 0.68 \vartheta_1 + \frac{1}{2} (-1.27 - 0.68)(|\vartheta_1 + 1| - |\vartheta_1 - 1|)$. It can be calculated from Equation (7)

$$Q_1 = \begin{bmatrix} 2.7 & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -14.87 & 0 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} -3.2 & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -14.87 & 0 \end{bmatrix}.$$

The outer coupling matrix $F = (F)_{N \times N}$ is taken as

$$F = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix},$$

and the inner coupling matrices are also assumed to be $\Gamma_1 = 0$ and

$$\Gamma_2 = \begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 0.9 & 0 \\ 0 & 0 & 0.9 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

We assume the non existence of uncertainties in the controller design ($\alpha_1 = 0$), with other given parameters as $\alpha = 0.3$, $\bar{\tau} = 0.07$, $\alpha_1 = 2$, $\alpha_2 = 3$ and the time-varying delay chosen as $\gamma(t) = 0.03 + 0.01 \sin(t)$ which implies $\gamma = 0.04$ and $\nu = 0.01$ by some calculations. By solving Theorem 3.2, the maximum sampling period $d$ is 0.1858, which showed less conservativeness as compared with other results in Table 2. The resulting nonfragile sampled data controller gains are shown in Figure 4:

$$K_{11} = \begin{bmatrix} -5.3066 & 0.3097 & 0.2543 \\ 0.0950 & -5.1166 & -0.1433 \\ 0.2864 & -0.4719 & -5.4942 \end{bmatrix}.$$

Table 2. Maximum sampling period $d$ for various sampled control in Example 4.2 for $\delta_1 = 1$ and $\sigma(t) = 0.$

| Method                  | $d$   | % improvement by Theorem 3.2 |
|------------------------|-------|-----------------------------|
| Wu et al. (2012)       | 0.0711| 161.32                      |
| Su and Shen (2015)     | 0.1327| 40.01                       |
| Liu et al. (2018)      | 0.1580| 17.60                       |
| Theorem 3.2            | 0.1858|                             |
Figure 4. The state trajectories of error system with control inputs in Example 4.1 (Case 2).

Figure 5. The state trajectories of error system with control inputs in Example 4.2.

Figure 6. The state trajectories of control inputs in Example 4.2.
Simulating our results to show the effectiveness of our approach, have the initial values of the dynamical networks set to $x_1(0) = [2, -3, 5]^T$, $x_2(0) = [5, -7, 1]^T$, $x_3(0) = [2, -2, 4]^T$, and $s(0) = [1, 0, -2]^T$. The response curves are depicted in Figures 5, 6, and 7 which indicate the error synchronization with control inputs, the control inputs response and finally, the error system without control inputs respectively. From the above simulations, it can be concluded that our designed nonfragile sampled-data controller design is effective, less conservative and will also guarantee the synchronization CDNs with randomly occurring perturbations in the controller.

5. Conclusion

The exponential $H_{\infty}$ synchronization of randomly coupling CDNs with time-varying delay, under non-fragile sampled-data control have been investigated. In this work, the time-varying Lyapunov Krasovskii functional is formulated in which the well known free-matrix based and Wirtinger inequalities have been used in the derivations of less conservativeness of our results as compared with other existing ones. Finally, two examples have been simulated to illustrate the feasibilities and effectiveness of our proposed scheme.

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