Damage detection in plate-like structures using High-Order mode shape derivatives

Mohamed Abdel-Basset Abdo
Associate professor, Civil Engineering Department, Assiut University, Assiut 71516, Egypt.
mohd_abdo2002@yahoo.co.uk
doi:10.6088/ijcser.002020300

ABSTRACT

This paper investigates the application and reliability of using high-order mode shape derivatives especially, the fourth derivative in damage detection of plate-like structures. Numerical analyses have been carried out for low- and high-order mode shapes of simply supported and cantilever steel plate models. Six scenarios of damages are studied for plate models to represent different damage characteristics. The influence of artificial noise on the damage identification using changes in fourth derivative of mode shapes has been investigated. Based on the numerical studies, it is shown that the fourth derivative of mode shape is promising in detecting and locating structural damage in plate-like structures since it is localized at the damage locations even for a small amount of damage using only one of the mode shapes. Both low- and high-order mode shapes give successful results. Also, damage indices of the fourth derivatives give smoother localization and consequently better damage identification than those of curvature of mode shapes. Furthermore, using high-order modes (which can be measured by advanced sensors) does not improve the results of damage identification using the fourth derivative. Unfortunately, damage detection using changes in fourth derivative of mode shapes is sensitive to measurement noise.

Keywords: Mode shape derivative, high order modes, damage detection, plate-like structures and measurement noise.

1. Introduction

Over the past few decades, major advances have been realized in the fields of structural dynamics and experimental modal analysis. Indeed, knowledge of structural dynamic characteristics allows one to diagnose vibration problems, to evaluate the effects of different loading conditions, to examine the effects of perturbations in structural properties and to control the behaviour of the structure. So, precise and detailed knowledge of the dynamic characteristics of structures has become increasingly important in recent years (Doebling et al., 1996). In linear vibration-based damage detection, the basic concept is that global modal parameters (resonant frequencies, mode shapes and modal damping) are functions of the physical properties of the structure (mass, damping and stiffness). Therefore, changes in the physical properties will cause changes in the modal characteristics and the measured response of the structure (Ewins, 1984).

For more than four decades, significant work has been done in the formulation of vibration-based damage detection algorithms. Salawu (1997) gave a literature review of the state of the art of damage detection using changes in natural frequency which can be quickly conducted and are often more reliable than mode shapes. Pandey et al. (1991) demonstrated that changes in mode shape curvature could be a good indicator of damage. Zhao and Dewolf (1999)
Damage detection in plate-like structures using High-Order mode shape derivatives
Mohamed Abdel-Basset Abdo

presented a theoretical sensitivity study comparing the use of natural frequencies, mode shapes and modal flexibility for structural damage detection. The results demonstrate that modal flexibility is more likely to indicate damage than either the other two. Pandey and Biswas (1994) proposed a damage localization method based on directly examining the changes in the measured modal flexibility of a beam structure. Also, Pandey and Biswas (1995) presented an experimental verification of locating damage in structures using flexibility difference method. Abdo and Hori (2002) used the changes in rotation of mode shapes in detecting damage. Lu et al. (2002) pointed out that Pandey's method is difficult to locate multiple damages, and they recommended the modal flexibility curvature for multiple damage localization due to its high sensitivity to closely distributed structural damages. Wu and Law (2004) extended the application of uniform load surface curvature (ULSC) to plate structures and showed that the ULS has much less truncation effect and is less sensitive to experimental errors. Furthermore, Abdo (2004) made a comparison study between the curvature of mode shape and the ULSC. He showed that the mode shape curvature can pinpoint damage locations even with one of the lower mode shapes of the structure and it does not require the mode frequency. Also, the mode shape curvature has an advantage that it can be measured directly without approximation, which will improve the results of damage identification. Applications of damage identification to existing structures are investigated by many researchers, (Saito and Yokota, 1996; Quan and Weiguo, 1998; and Huang et al., 1999).

Indeed, almost all studies on using damage characteristics to detect damage have been done using low-order mode shapes and frequencies. This is because high-order mode shapes and frequencies were difficult to be identified using ordinary sensors. However, major advances have been realized recently in the field of structural dynamics and mechanical vibration measurements, e.g., using Laser Doppler Vibrometer (LDV) and Scanning Laser Doppler Vibrometer (SLDV) which possess extremely high resolution and able to measure in wide frequency range. So, high-order modes can be identified and measured easily. Characteristics for typical LDV device and its applications can be found in (Ewins, 2000; and Siringoringo and Fujino, 2006, respectively). Thus, Abdo and Abdel-Naiem (2005) studied the reliability of using high-order mode shapes in structural damage detection. They showed that the curvature of both low- and high-order mode shapes give successful results and it does not matter to use curvature of low or high-order mode shapes in damage identification but the important thing is the accuracy of measurements in vibration testing.

Whalen et al. (2006) investigated the fourth derivative of mode shapes in detecting damage and its application to the I-40 bridge. They found that the method showed excellent accuracy in locating stiffness reductions in beam-like structures while using a small number of measured modal displacement points. Ismail et al. (2006) used mode shape fourth derivative divided by the mode shape displacement data (local stiffness indicator) to detect damage in RC beams. They found that the local stiffness indicator is good at locating damage in RC beams but it produced poor results in the vicinity of supports. Whalen (2008) investigated the behaviour of high order mode shape derivatives in damaged, beam-like structures. All high order modal derivative discontinuities display strong localization under the assumption of beam-like vibrations. To date there has not been a study reported in the technical literature that directly extends the application of high order modal derivative discontinuities to detect damage in plate-like structures which is related to engineering applications in the construction of bridges, cranes, ships, etc. The numerical results reported in this paper attempt to fill this void in the study of damage detection methods.

International Journal of Civil and Structural Engineering
Volume 2 Issue 3 2012
The objective of this paper is to investigate the sensitivity of high-order mode shape derivatives, especially the fourth derivative to existence of damage in plate-like structures. Both analytical and numerical investigations of the effect of damage on mode shape derivatives are carried out. Both low- and high-order mode shapes are thoroughly studied. A careful numerical study is carried out by using the finite element method to analyze dynamic behaviour of damaged structural members. To demonstrate the results, two steel plate models are investigated, a cantilever plate (fixed, free, free, free) and a simply supported plate (simply, free, simply, free). The influence of artificial noise due to inevitable errors in measurements on the damage identification using changes in fourth derivative of mode shapes has been investigated.

2. Theoretical Background

2.1 Beam Element

2.1.1 Curvature of Mode Shape

For simplicity, Abdo (2002) considers a beam element to examine the above observations. The beam is simply supported, and has the stiffness $K=EI$ with $E$ and $I$ being the Young’s modulus and the second moment of the cross section, respectively. A narrow zone of damage is assumed at $x_d - d/2 < x < x_d + d/2$ with the stiffness $K - \Delta K$ with $d/L << 1$. By approximating $M$ as $K\kappa^0(x_d)$, where $M$ and $\kappa$ stand for the bending moment and curvature, Abdo (2002) found that the curvature at the intersection suffers a jump which can be evaluated as

$$[\kappa] \approx \left( \frac{K}{K - \Delta K} \right) - 1 \left| \kappa^0(x_d) \right|. \tag{1}$$

Since $d$ is small, this jump leads to a spike in the curvature of mode shape, its width is $d$ and its height is related to $\Delta K$ and $\kappa^0$; the spike becomes sharper as $d$ decreases and the height increases as $\Delta K$ increases. So, the mode shape curvature is a good damage indicator in detecting the location and the magnitude of beam damage(s).

2.1.2 Fourth Derivative of Mode Shape

The concept of using fourth derivatives of mode shapes in damage detection of beams was first used by Whalen et al. (2006) who used the Euler-Bernoulli beam model. They assumed that the shearing deformations, rotational inertia and axial effects to be negligible. They assumed that the stiffness $EI$ of the beam can vary with position $x$, the governing equation of motion for undamped free vibration of the beam can be expressed as:

$$\frac{\partial^2}{\partial x^2} \left[ EI(x) \frac{\partial^2 y}{\partial x^2} \right] + \rho A(x) \frac{\partial^2 y}{\partial t^2} = 0, \tag{2}$$

where $\rho A(x)$ is the linear mass density of the structure. With standard separation of variables argument and solving for the fourth derivative they got:

$$\psi_n^{(4)}(x) = \frac{\omega_n^2 \rho A(x)}{EI(x)} \psi_n(x) - 2 \frac{EI^{(1)}(x)}{EI(x)} \psi_n^{(3)}(x) - \frac{EI^{(2)}(x)}{EI(x)} \psi_n^{(2)}(x). \tag{3}$$

where $\omega_n(x)$ is the natural frequency of vibration for $n$th mode shape $\psi_n'(x)$. Whalen et al. (2006) showed that if damage causes a change in $EI(x)$, the terms involving derivatives of
EI(x) can have large values due to the localized nature of this change. Therefore, if there is damage in a structure, large discontinuities in the magnitude of the fourth derivative of mode shape will increase sharply at the location of damage. Hence, it is suitable for locating damage in beams.

2.2. Thin Plates

For simplicity, a thin plate (Kirchhoff plate) with a small transverse (out of plane) displacement w is considered. The governing equation of motion for undamped free vibration of the Kirchhoff plate can be expressed as:

\[ \nabla^2 D \nabla^2 w + \rho h \ddot{w} = 0, \quad (4) \]

where \( \rho h \) is the mass density per unit area, over dots indicate differentiation to time t and \( \nabla^2 \) is the differential operator (Laplacian differential) defined in Cartesian coordinate (rectangular plates) as follows:

\[ \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \quad (5) \]

and D is the bending/flexural rigidity of the plate as follows:

\[ D = \frac{Eh^3}{12(1-v^2)}, \quad (6) \]

in which E is the Young’s modulus, \( v \) is Poisson’s ratio of the plate material and h is the thickness of plate. Indeed, Equation (4) for free vibration of the Kirchhoff plate is similar to Equation (2) for free vibration of the beam element. Thus, it is well expected that the fourth derivative of mode shapes are good damage indicators for a plate member as well. We will examine this in detail, using the numerical computation in the next sections.

3. Analytical Model

As a practical example, the model chosen for this study is a steel plate (Abdo and Abdel-Naiem, 2005). Since the stress concentration in the plate often occurs at the supported edges, the study of the dynamic stress concentration of a plate suffers from failure at its connection(s) is of general significance. So, damage is mainly introduced at the supported edge(s) to represent the failure, e.g., in a welded or bolted connection of a structure. A further study is carried out to investigate the effect of damage in some elements of the plate models. In this case, the change in the stiffness due to damage is modelled as a reduction in the Young’s modulus (E) of one or two elements.

The dimensions of the plate are 50\( \times \)50\( \times \)0.625 [cm] and the mechanical properties of the plate are; Young's modulus, \( E=210 \) [GN/m²], Poisson's ratio, \( v=0.3 \), and the density, \( \rho=7,850 \) [kg/m³]. Figure 1 illustrates the finite element model of the steel plate. The pre-damage and post-damage modal parameters were calculated numerically using the software package MARC/Mentat (2010). Four-node shell element with six degrees of freedom (DOF) per node, three translations and three rotations \( (U_x, U_y, U_z, \theta_x, \theta_y, \theta_z) \), are used. The convergence of the natural frequencies and displacement mode shapes were checked via comparing different meshes and the finite element model consists of 40\( \times \)40 elements, and 1681 nodes (Abdo and Hori, 2002).

Two examples are studied; a cantilever and a simply supported plate, as shown in figure 1. The first example is a plate which is simply supported (only the three translations, \( U_x, U_y, U_z, \)).
are restrained) at two edges, at \( X=0 \) and \( X=50 \) [cm] and free otherwise. The second example is a cantilever plate which is clamped (all six DOFs are restrained) at one edge, at \( X=0 \) [cm] and free otherwise (Abdo and Abdel-Naiem, 2005). Six scenarios of damages are considered in the analysis. In the first four scenarios, damages are mainly modelled as a part of free boundary at the damaged regions of the connection(s) of the plate, i.e., the damage is represented by free six DOFs at the damaged nodes of both simply and cantilever plates. The degree of damage (damage extent) is related to the ratio between the length of the damaged region and the total length of the supported edge. Table 1 shows the damage characteristics of the first four scenarios of damage of the steel plate model. Indeed, two scenarios of damage are studied for each example to represent not only different locations but also different extents.

Table 1 shows the damage characteristics of the first four scenarios of damage of the steel plate model. Indeed, two scenarios of damage are studied for each example to represent not only different locations but also different extents.

Two additional scenarios of damages are investigated on the cantilever plate model. In this case, the change in the stiffness due to damage was modelled as a reduction in the Young’s modulus (E) of one or more elements, (see Equation (6)). In scenario-V, the damage was modelled as a reduction in the Young’s modulus, \( \Delta E = 5\% \), for element 1561, but in scenario VI the damage was modelled as a reduction in the Young’s modulus, \( \Delta E = 5\% \), for elements 1561 and 1600, simultaneously. Indeed, these two additional scenarios of damage represent small amount of damage at the free edge of the cantilever plate, i.e., at the region of less stress concentration. Table 2 shows the damage characteristics of scenarios V and VI of the steel plate model (Abdo and Abdel-Naiem, 2005).

The damage of the model in this study is assumed to affect only the stiffness matrix but not the inertia matrix in the eigenproblem formulation. This assumption is consistent with those used by many researchers, e.g., (Pandey et al., 1991). The displacement in the Z-direction (\( U_z \)), is only considered in the analysis. In-plane displacements, \( U_x, U_y \), of the plate model are neglected because they are much smaller compared with \( U_z \) displacement in thin plates. Rotations are not taken into consideration because of the difficulty in measuring them accurately. The displacement mode shape is normalized with respect to the square root of the sum of squares (SRSS) of the obtained displacement. The importance of applying a data normalization procedure is that false-positive indications of damage are minimized (Abdo and Abdel-Naiem, 2005). Fourth derivatives of mode shapes are calculated by using the normalized displacements in Z-direction.

**Figure 1:** Finite element model of the steel plate: (a) Simply supported plate; (b) cantilever plate
Damage detection in plate-like structures using High-Order mode shape derivatives

Mohamed Abdel-Basset Abdo

Table (1): Damage characteristics at the boundaries of the plate model

| Damage characteristics | Simply supported plate (Simply, free, simply, free) | Cantilever plate (Fixed, free, free, free) |
|------------------------|----------------------------------------------------|-------------------------------------------|
| Scenario-I             | Scenario-II                                        | Scenario-III                             | Scenario-IV                             |
| Number of damage(s)    | 2                                                   | 2                                         | 1                                         | 2                                         |
| Location(s)            | Node 1 and nodes 39, 40, 41 (One edge only)       | Node 1 and nodes 1679, 1680, 1681 (Two edges) | Nodes 20, 21, 22 (Middle of the edge) | Nodes 1, 2, 40, 41 (Ends of the edge) |
| Damage extent          | 2.5% + 7.5%                                        | 2.5% + 7.5%                              | 10%                                      | 5% + 5%                                  |

Table (2): Damage characteristics at some elements of the cantilever plate model

| Damage characteristics | Cantilever plate (Fixed, free, free, free) |
|------------------------|-------------------------------------------|
| Scenario-V             | Scenario-VI                              |
| Number of damages      | 1                                         | 2                                         |
| Location               | Element 1561                              | Elements (1561+1600)                      |
| Damage extent          | ∆E = 5%                                  | ∆E = (5%+5%)                             |

4. Results and Discussion

The frequencies and mode shapes for the intact and damaged plates are studied for twenty modes of the above-mentioned two examples, simply supported and cantilever plates. As mentioned before, four scenarios of damage of these two examples are considered to represent different damage extents and multiple damage locations at the supported edge(s). Furthermore, two additional scenarios of damages are considered, as a 5% reduction in the Young's modulus (E), of one or two elements at the free edge of the cantilever plate to represent small amount of damage.

4.1 Fourth Derivative of Mode Shape

Two components of the fourth derivative of mode shapes in Z-direction, \(\frac{\partial^4 U_z}{\partial x^4}\) and \(\frac{\partial^4 U_z}{\partial y^4}\) are calculated of the normalized displacement mode shapes at each node by using a central difference approximation. For the sake of generality, the following parameter of the fourth derivative of mode shapes is considered;

\[
D(I) = \frac{\partial^4 U_z}{\partial x^4} + \frac{\partial^4 U_z}{\partial y^4}.
\]  

Actually, the absolute value of the parameter, \(D(I)\) in Equation (7), is used in this study.

For comparison study, the absolute differences of the parameter of fourth derivative of mode shapes at the \(i\) th node of the \(j\) th mode, \(\beta_{ij}\), are normalized according to the algorithm which is developed by Stubbs et al. (1995),

\[
C_{ij} = \frac{\beta_{ij} - \mu_{\beta ij}}{\sigma_{\beta ij}},
\]  

International Journal of Civil and Structural Engineering
Volume 2 Issue 3 2012

806
where, $C_{ij}$ is the damage index of the fourth derivative, $\mu_{\beta j}$ and $\sigma_{\beta j}$ are respectively, the mean and standard deviation of the absolute differences of the parameter of fourth derivative for the $j$th mode shape. The damaged location is that at which the damage indices have values $C_{ij} \geq 2$. It should be mentioned that the points that have damage indices $C_{ij} \geq 2$, indicate that the probability of false alarm (Pfa) is just 0.0228 (Abdo, 2004).

### 4.1.1 Low-Order Mode Shapes

For low-order mode shapes, the results of the first five mode shapes are investigated. The damage indices of the fourth derivative of the second mode shape between the intact and the damaged plates for scenario-I to scenario-VI are plotted in figure 2 (a) to figure 2 (f), respectively. It is shown that two damages of different locations and different extents (the ratio of length is 1:3) are successfully identified for the simply supported plate as shown in figure 2 (a) and figure 2 (b). Also, one-damage or two-damage locations with the same extent introduced to the clamped edge of the cantilever plate are identified as shown in figure 2 (c) and figure 2 (d). The damage can be accurately located even for a small amount of damage (5 percent loss in the Young’s modulus in one or two elements) at the far free edge of the cantilever plate model as shown in figure 2 (e) and figure 2 (f). It is important to mention that the false positive of the damage index of the fourth derivative of the second mode shape of scenario III (shown in figure 2 (c)) is interpreted by the fact that the second mode is a twisting mode and the node line of the second mode shape passes through the damage location. On the other hand, the changes of the natural frequencies of the first mode shape of scenarios V and VI are zero (Abdo and Hori, 2002), so the location of damage can not be distinguished from noise. This is the reason that the first mode shape is not used for comparison in figure 2.

The damage indices of the fourth derivative of the 4th mode shape between the intact and the damaged plates for scenario-I to scenario-VI are plotted in figure 3 (a) to figure 3 (f), respectively. It is shown that all scenarios of damages can be accurately located using the 4th mode shape. Similar results are obtained for the 3rd and 5th modes. The above results show that using the fourth derivative of mode shapes is promising in structural damage identification. Damages of various levels of severity can be located accurately, using only one of the low-order mode shapes.
Damage detection in plate-like structures using High-Order mode shape derivatives
Mohamed Abdel-Basset Abdo

Figure 2: Damage indices of the fourth derivative of the 2nd mode shape between the intact and damaged plate models

4.1.2 High-Order Mode Shapes

For high-order mode shapes, the results of the 16th to 20th mode shapes are investigated. The damage indices of the fourth derivative of the 16th mode shape between the intact and the damaged plates for scenario-I to scenario-VI are illustrated in figure 4 (a) to figure 4 (f), respectively. This figure demonstrates that the differences of the mode shape fourth derivative are localized at the damaged regions for the 16th mode for all scenarios of damage; one location, two locations with the same damage extent and multiple damage locations with different damage extents. Also, the damage can be accurately located even for a small amount of damage (5 percent loss in the Young’s modulus in one or two elements) at the far free edge of the cantilever plate. Indeed, similar results are obtained for other high-order mode shapes. The results show the generality and capability of using the changes in mode shape fourth derivative to detect and pinpoint different types of damage. These results are in agreement with the conclusions of Section 2.1.2 and Section 2.2.

Scenario-I

Scenario-II
Damage detection in plate-like structures using High-Order mode shape derivatives
Mohamed Abdel-Basset Abdo

Comparing the results in figure 3 with those in figure 4 it is shown that the parameter of fourth derivatives are effective and reliable in detecting and locating structural damage for both low-order mode shapes (figure 3) as well as high-order mode shapes (figure 4). Thus, it is not needed to measure high-order mode shapes (which can be measured only by advanced sensors) for structural damage detection. However, low-order mode shapes (which can be easily measured by ordinary sensors) can give excellent results for damage identification using the fourth derivative of mode shape. So, fourth derivatives of mode shapes are promising in detecting and locating structural damage in plate-like structures as well as beam-like structures.

4.2 Fourth Derivative versus Curvature of Mode Shape

Indeed, two components of the curvature of mode shapes in Z-direction, $\frac{\partial^2 U_z}{\partial x^2}$ and $\frac{\partial^2 U_z}{\partial y^2}$ are calculated at each node by using a central difference approximation using the normalized displacement mode shapes. For the sake of generality, the following parameter of the curvature of mode shapes is considered;
Damage detection in plate-like structures using High-Order mode shape derivatives
Mohamed Abdel-Basset Abdo

\[
C(I) = \frac{\partial^2 U_\cdot}{\partial x^2} + \frac{\partial^2 U_\cdot}{\partial y^2}.
\]

Indeed, the absolute value of the first parameter, \(C(I)\) in Equation (9), is used in this study. The damage index of the mode shape curvature is calculated as in Equation (8).

To show the advantages of the fourth derivative of mode shapes, it is important to compare the results of damage detection using the fourth derivative of mode shapes obtained in this study and the curvature of mode shapes for low- and high order mode shapes. Figure 5 (a to f) plot the damage indices of the curvature of the 4th mode shape between the intact and the damaged plates for scenario-I to scenario-VI, respectively. Also, figure 6 plots the damage indices of the curvature of the 16th mode shape between the intact and the damaged plates for the pre-mentioned six scenarios of damages. Indeed, figure 5 shows that the damage indices of curvature of low-order mode shapes are reliable in damage identification for all scenarios of damage except some false positives in scenario-V. On the other hand, figure 6 shows that the damage indices of curvature of high-order mode shapes are less sensitive to damage identification for all scenarios of damage because of the false positives in all scenarios of damage but the damage can be located and distinguished.

Comparing figure 5 with figure 3 and figure 6 with figure 4, it is clear that the damage indices of the fourth derivatives give smoother localization and consequently better damage identification than those of curvature of mode shapes. It is important to mention that the fourth derivatives of mode shapes in this study are calculated approximately using the central difference approximation. The central difference approximation is used twice to calculate the curvature of curvature of mode shapes. It is well expected that if the curvature or the fourth derivatives of mode shapes are measured directly (e.g., using optical fibre sensors), they will give better results for both low- and high-order mode shapes. Also, it is worth to mention that

\[
Y
\]
the 4\textsuperscript{th} and 16\textsuperscript{th} modes are used for the comparison study so as not to repeat the results of the curvature of mode shapes obtained by Abdo and Abdel-Naiem (2005) where the 3\textsuperscript{rd} and 20\textsuperscript{th} modes are used.

\textbf{Figure 5:} Damage indices of the curvature of the 4th mode shape between the intact and damaged plate models
4.3 Effect of Noise on Fourth Derivative of Mode Shape

In section 4.1 it is found that the changes in fourth derivative of mode shapes are able to fairly locate the damage when presented with noise free information. In actual practice, fourth derivatives are indirectly measured and data are contaminated with noise. Measurement error may be attributed to sensors, accuracy of measurements, and environmental conditions. So, the effect of artificial measurement noise on the damage localization method is considered. The displacement of mode shapes of the baseline state of the plate model is considered as noise free. However, random noise is inflicted as 5 and 10 percent of the root mean square of the original results, to simulate the measured error to the damaged plate only. The coefficient of variation, $\delta = \sigma/\mu$, ($\sigma$ is the standard deviation of data and $\mu$ is the mean) is considered to be constant for the noise and the original data. The pseudo random numbers with a normal distribution is generated using MATHEMATICA package, (2008). Noise is incorporated into the displacement measurements as absolute error, i.e., an error term is added to the base "error free" measurements.

Figure 6: Damage indices of the curvature of the 16th mode shape between the intact and damaged plate models
Damage detection in plate-like structures using High-Order mode shape derivatives
Mohamed Abdel-Basset Abdo

Figure 7: Damage indices of the fourth derivative of the 4th mode shape between the intact and damaged plate models with 5% noise

Figure 7 (a to e) plots the damage indices of the fourth derivative of the 4th mode shape between the intact (noise free) and the damaged plates (with 5 percent normal distribution measurement noise) for scenario-I to scenario-VI, respectively. Also, figure 8 (a to e) plots the damage indices of the fourth derivative of the 16th mode shape between the intact (noise free) and the damaged plates (with 5 percent normal distribution measurement noise) for the pre-mentioned six scenarios of damages.

Comparing figure 7 with figure 3, it can be easily seen that existence of noise leads to small values of damage indices as well as a degradation in detecting damage using changes in fourth derivative of mode shapes. For the first four scenarios of damage, although false positives occur, spikes at the damaged nodes have occurred and the damage can be located accurately. However, for damage scenarios V and VI where the damage is very small and simulated as a 5 percent loss in Young’s modulus of one and two elements respectively, it is found that none of the first five modes is able to indicate the damage location(s). It should be mentioned that similar results are obtained for other low order mode shapes.
Damage detection in plate-like structures using High-Order mode shape derivatives
Mohamed Abdel-Basset Abdo

Figure 8: Damage indices of the fourth derivative of the 16th mode shape between the intact and damaged plate models with 5% noise

Comparing figure 8 with figure 4, it can also be easily seen that existence of noise leads to a degradation in detecting damage using changes in fourth derivative of mode shapes. For the first two scenarios of damage, however, the damage can be located accurately due to less false positives. For damage scenarios III and IV, the damage indices cannot identify the damage location from false positives. For damage scenarios V and VI, false positives mask damage locations completely so that none of the five higher modes is able to indicate the damage location(s). It should be mentioned that when using 10 percent noise, the damage indices can locate only the more severe damage in scenarios-I and II but cannot locate the smaller damage. The above results are attributed to the fact that the forth derivatives are estimated using central difference approximation two times to calculate the curvature of mode shapes. The obtained results prove that using changes in fourth derivative of mode shapes in damage detection is sensitive to measurement noise.

5. Conclusions

This paper investigates the application and reliability of using high-order mode shape derivatives especially, the fourth derivative in damage detection of plate-like structures. Numerical analyses have been carried out on low- and high-order mode shapes of simply supported and cantilever steel plate models. Six scenarios of damage are studied for plate models to represent different damage characteristics. Based on the numerical studies on plate-like structures, the following conclusions are noted:

1) Fourth derivative of mode shape is a good damage indicator since it is sharply localized at the damage locations even for a small amount of damage using only one of the low- or high-order mode shapes.

2) Damage indices of the fourth derivatives give smoother localization and consequently better damage identification than those of curvature of mode shapes.
3) Using high-order modes (which can be measured by advanced sensors) does not improve the results of damage identification using the modal fourth derivatives. So, in damage identification using modal fourth derivative it does not matter to use low or high-order mode shapes but the important thing is the accuracy of measurements in vibration testing.

4) Damage detection using changes in fourth derivative of mode shapes is sensitive to measurement noise due to using central difference approximation two times to calculate the curvature of curvature of mode shapes.

Indeed, it is well expected that if fourth derivatives of mode shapes are measured directly without approximation (using e.g., optical fibre sensors), the fourth derivatives of mode shapes will give more stable and reliable results of damage identification of plate-like structures.

6. References

1. Abdo, M. A.-B. and Hori, M. (2002), A Numerical Study of Structural Damage Detection Using Changes in the Rotation of Mode Shapes, Journal of Sound and Vibration, 251(2), pp 227-239.

2. Abdo, M. A.-B. (2004), Comparative Study of Curvature Techniques Used in Damage Identification, International Conference on Structural & Geotechnical Engineering and Construction Technology, IC-SGECT ’04 , Mansoura, Egypt, paper No. 66, pp 73-86, 23-25.

3. Abdo, M. A.-B. (2002), Structural Health Monitoring Using Changes In Dynamic Characteristics, Ph. D. thesis, The University of Tokyo, Japan, 172 pp.

4. Abdo, M. A.-B. and Abdel-Naiem, M. A. (2005), Reliability of Using High-Order Mode Shapes In Structural Damage Detection, Journal of Engineering Sciences, Assiut University, 33(6), pp 2051-2068.

5. Doebling, S. W., Farrar, C. R., Prime, M. P., and Shevitz, D. W. (1996), Damage Identification And Health Monitoring Of Structural and Mechanical Systems From Changes In Their Vibration Characteristics, A Literature review, LA-13070-MS.

6. Ewins, D. J. (1984), Modal Testing: Theory and Practice, Wiley, New York, 313 pp.

7. Ewins, D. J. (2000), Modal Testing: Theory, Practice and Application, Research Studies Press, England, 562 pp.

8. Huang, C. S., Yang, Y. B., Lu, L. Y., and Chen, C. H. (1999), Dynamic Testing and System Identification of a Multi-Span Highway Bridge, Journal of Earthquake Engineering and Structural Dynamics, 28, pp 857-878.

9. Ismail, Z., Abdul Razak, H. and Abdul Rahman, A.G. (2006), Determination of Damage Location in RC Beams using Mode Shape Derivatives, Journal of Engineering Structures, 28, pp 1566-1573.

10. Lu, Q., Ren, G. and Zhao, Y. (2002), Multiple Damage Location with Flexibility Curvature and Relative Frequency Change for Beam Structures, Journal of Sound and Vibration, 253(5), pp 1101-1114.

11. MARC Analysis Research Corporation (2010), Volumes; A, B and C and Mentat User’s Guide, Version 2010.
12. MATHEMATICA version 7, (2008), STEPHEN WOLFRAM,

13. Pandey, A. K., Biswas, M., and Samman, M. M. (1991) Damage Detection from Changes in Curvature Mode Shapes, Journal of Sound and Vibration, 145(2), pp 321-332.

14. Pandey, A. K. and Biswas, M. (1994), Damage Detection in Structures using Changes in Flexibility, Journal of Sound and Vibration, 169(1), pp 3-17.

15. Pandey, A. K. and Biswas, M. (1995), Experimental Verification of Flexibility Difference Method for Locating Damage in Structures, Journal of Sound and Vibration, 184(2), pp 311-328.

16. Quan, Q. and Weiguo, Z. (1998), Damage Detection of Suspension Bridges, Proc. 16th International Modal Analysis Conference (IMAC), California, USA, Feb. 2-5, 1998, 2, pp 945-951.

17. Saito, T. and Yokota, H. (1996), Evaluation of Dynamic Characteristics of High-Rise Buildings using System Identification Techniques, Journal of Wind Engineering and Industrial Aerodynamics, 59, pp 299-307.

18. Salawu, O. S. (1997), Detection of Structural Damage through Changes in Frequency, A review, Journal of Engineering Structures, 19(9), pp 718-723.

19. Siringoringo, D. M. and Fujino, Y. (2006), Experimental Study of Laser Doppler Vibrometer and Ambient Vibration for Vibration-Based Damage Detection, Journal of Engineering Structures, 28, pp 1803-1815.

20. Stubbs, N., Kim, J.-T., and Farrar, C. R. (1995), Field Verification of a Nondestructive Damage Localization and Severity Estimation Algorithm, 13th International Modal Analysis Conference, (IMAC), Society for Experimental Mechanics, USA, pp 210-218.

21. Whalen, T. M., Liu, J. and Gauthier, J. F. (2006), Application of the Higher Order Derivative Discontinuity Method To The I-40 Bridge Damage Detection Problem, Proceedings of the 4th World Conference on Structural Control and Monitoring, (4WCSCM-138), San Diego, California, 11-13 July, 2006.

22. Whalen, T. M. (2008), The Behaviour of Higher Order Mode Shape Derivatives in Damaged, Beam-Like Structures, Journal of Sound and Vibration, 309(3-5), pp 426-464.

23. Wu, D. and Law, S. S. (2004), Damage Localization in Plate Structures from Uniform Load Surface Curvature, Journal of Sound and Vibration, 276(1-2), pp 227-244.

24. Zhao, J. and Dewolf, J. T. (1999), Sensitivity Study for Vibrational Parameters Used in Damage Detection, Journal of Structural Engineering, ASCE, 125(4), pp 410-416.