Largely deformed states of $^{13}$B

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The excited states of $^{13}$B were studied with a method of antisymmetrized molecular dynamics (AMD). The theoretical results suggest that the intruder states with large deformations construct the rotational bands, $K^\pi = 3/2^-$ and $K^\pi = 1/2^+$, starting from 5 MeV and 8 MeV, respectively. The neutron structure of the $K^\pi = 3/2^-$ is analogous to the intruder ground state of $^{12}$Be. In the predicted $K^\pi = 1/2^+$ band, we found very exotic structure with a proton intruder configuration. This proton intruder state has a larger deformation than superdeformation. The band-head $1/2^+$ state is assigned to the $1/2^+$ (4.83 MeV), which was experimentally suggested to be the proton intruder state because of the strong production via the proton-transfer to the $^{12}$Be($0^+$) state in the $^4$He($^{12}$Be,$^{13}$B$\gamma$)X experiments.

I. INTRODUCTION

In the recent progress of experimental and theoretical researches of unstable nuclei, various exotic phenomena have been discovered. As well known, one of the attractive subjects in neutron-rich nuclei is the breaking of neutron magic number such as $N = 8$ and $N = 20$ suggested in neutron-rich $p$-shell and $sd$-shell nuclei. The breaking of the $N = 8$ shell in $^{11}$Be has been well known for a long time because of the parity inversion of the ground state, $J^\pi = 1/2^+$. For $^{12}$Be, various experimental researches have been recently achieved $[1, 2, 3]$ to study the properties of the ground band, and the breaking of $N = 8$ shell closure has been established. Concerning the mechanism of the $N = 8$ shell breaking, the recent observation $[8]$ of the significant $d$-wave component in the $^{12}$Be ground state is the direct probe for the deformation, which should be one of the essential factors for the breaking in Be isotopes suggested in theoretical calculations $[4, 5, 6, 7, 8]$. In the theoretical side, many kinds of microscopic calculations have been performed to investigate neutron-rich nuclei with the breaking shell closure. In case of neutron-rich Be isotopes, various properties have been successfully described by many groups from a point of view of cluster (see references in Ref.$[8]$). As a result, it is considered that molecular orbital structure with large deformation is a key for the breaking of $N = 8$ magic number in neutron-rich Be.

In the molecular orbital picture, Be isotopes are described by 2 $\alpha$ clusters and valence neutrons which occupy the molecular orbitals formed around the $2\alpha$ core $[5, 6, 7, 10]$. The molecular orbitals are given by a linear combination of $p$-orbitals around the $\alpha$ clusters, and they are associated with the orbitals in two-center shell model $[12]$. In the spherical limit, the negative-parity $\pi$-orbitals are the lowest for the valence neutrons. On the other hand, with the development of the $2\alpha$ clustering, so-called $\sigma$ orbital, which is the longitudinal positive-parity orbital, comes down because its kinetic energy decreases. Finally, the inversion of the $\pi$ and $\sigma$ orbitals occurs in a system with well-developed $2\alpha$ clustering. The inversion of the molecular orbitals corresponds to the parity inversion of the $'p'$ and $'sd'$ orbitals because the $\pi$ and $\sigma$ orbitals in the molecular orbital model are associated with the $p$ and $sd$ orbitals in the deformed shell model, respectively. The intruder state of $^{12}$Be is written by the configuration with two neutrons in the $\sigma$-orbital. If the $2\alpha$ structure develops enough in $^{12}$Be, the intruder state may become the ground state. This necessarily causes the significant mixture of $d$-wave component, which is consistent with the recent measurements $[3]$. Moreover, most of the low-lying states in neutron-rich Be isotopes can be well described by the molecular orbital structure $[3, 5, 6, 7, 10, 11, 13, 14, 15, 16, 17]$, and a variety of cluster states has been predicted in excited states of $^{11}$Be and $^{12}$Be $[5, 6, 7, 8, 11, 13, 16, 19]$. These facts indicate that cluster aspect is one of essential features in light unstable nuclei as well as in light stable nuclei. In particular, cluster structure is favored in neutron-rich Be, where a variety of exotic structure arises due to the the formation of 2 $\alpha$ clusters and the molecular orbitals.

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Let us turn to structure of B isotopes, which have a proton number $Z = 5$ larger by one than Be isotopes. In contrast to the situation of $^{12}$Be, $^{13}$B is considered to have the normal ground state with a neutron $p$-shell closed configuration. Comparing with the intruder configuration of the ground state in $^{12}$Be, this shows that the $N = 8$ magic number is restored in $^{13}$B due to the additional proton. Thus, the additional proton gives drastic structure change of the ground state, however, it is still natural to expect that intruder states may appear in the excited states of $^{13}$B. If a cluster structure can develop also in B as well as Be, the intruder states may lie in low excitation energy region, the ground state, however, it is still natural to expect that intruder states may appear in the excited states of $^{13}$B. Due to the additional proton $g_{9/2}$, the $^{13}$B is most spherical among B isotopes, while developed cluster structure has been predicted in further neutron-rich B like $^{15}$B and $^{17}$B. These results suggest a trend of two-center cluster structure in $Z = 5$ systems.

In this paper, we investigated deformed states of $^{13}$B by performing microscopic calculations of the ground and excited states of $^{13}$B. We focused on their cluster aspect. In particular, the molecular orbital structure is our major interest. In the present study we applied an AMD method, which is known to be a powerful approach to investigate cluster structure of unstable nuclei$^{[20, 21, 22]}$. The present method is the same as that applied to the studies of $^{10}$Be,$^{11}$Be, and $^{12}$Be$^{[6, 7, 15]}$. Namely, we performed variation after spin-parity projection within the framework of AMD$^{[23]}$. This method has been proved to successfully describe various properties of the ground and excited states of Be isotopes.

The paper is organized as follows. In the next section, we briefly explain the theoretical method of the present work. Results and discussions are given in III. Finally, we give a summary in IV.

II. FORMULATION

We performed energy variation after spin parity projection (VAP) within the AMD model space, as was done in the previous studies$^{[7, 15, 23]}$. The detailed formulation of the AMD method for nuclear structure study is described in$^{[20, 21, 22, 23]}$. In particular, the formulation of the present calculations is basically the same as that described in$^{[7, 15]}$.

An AMD wave function is a Slater determinant of Gaussian wave packets,

$$\Phi_{\text{AMD}}(\mathbf{Z}) = \frac{1}{\sqrt{A!}} A \{ \phi_1, \phi_2, \ldots, \phi_A \},$$

(1)

where the $i$th single-particle wave function is written by a product of spatial($\phi$), intrinsic spin($\chi$) and isospin($\tau$) wave functions as,

$$\phi_i = \phi_{\mathbf{x}_i, \chi_i \tau_i},$$

(2)

$$\phi_{\mathbf{x}_i}(\mathbf{r}_j) \propto \exp\{-\nu(\mathbf{r}_j - \mathbf{x}_i/\sqrt{\nu})^2\},$$

(3)

$$\chi_i = \left(\frac{1}{2} + \xi_i\right)\chi_{1} + \left(\frac{1}{2} - \xi_i\right)\chi_{-1}.$$  

(4)

$\phi_{\mathbf{x}_i}$ and $\chi_i$ are spatial and spin functions, and $\tau_i$ is iso-spin function which is fixed to be up(proton) or down(neutron). We used a width parameter $\nu = 0.18$ fm$^{-2}$, which is chosen to be the optimum value for $^{13}$B. Accordingly, an AMD wave function is expressed by a set of variational parameters, $\mathbf{Z} = \{ \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_A, \xi_1, \xi_2, \ldots, \xi_A \}$.

For the lowest $J^\pi$ state, we varied the parameters $\mathbf{x}_i$ and $\xi_i (i = 1 \sim A)$ to minimize the energy expectation value of the Hamiltonian, $\langle \Phi | H | \Phi \rangle / \langle \Phi | \Phi \rangle$, for the spin-parity projected AMD wave function; $\Phi = P_{MK}^{J^\pi} \Phi_{\text{AMD}}(\mathbf{Z})$. Here, $P_{MK}^{J^\pi}$ is the spin-parity projection operator. Then we obtained the optimum solution of the parameter set; $\mathbf{Z}^{J^\pi}_i$ for the lowest $J^\pi$ state. The solution $\mathbf{Z}^{J^\pi}_n$ for the $n$th $J^\pi$ state are calculated by varying $\mathbf{Z}$ so as to minimize the energy of the orthogonal component to the lower states.

After the VAP calculations for the $J^\pi_n$ states with respect to various $J$, $n$ and $\pi = \pm$, we obtained the optimum intrinsic wave functions, $\Phi_{\text{AMD}}(\mathbf{Z}^{J^\pi}_n)$, which approximately describe the corresponding $J^\pi_n$ states. After the VAP, we superposed the spin-parity eigen wave functions projected from all the obtained AMD wave functions. Namely, we determined the final wave functions for the $J^\pi_n$ states as,

$$|J^\pi_n\rangle = \sum_{i,K} c^{J^\pi_n}_{i,K} (K, J_i, \pi_i, k_i) |P_{MK}^{J^\pi} \Phi_{\text{AMD}}(\mathbf{Z}^{J^\pi}_i)\rangle.$$  

(5)
III. RESULTS

We adopted the same effective nuclear interaction as that used in the study of Be isotopes [3, 4], which consists of the central force, the spin-orbit force and the Coulomb force. The Majorana parameters in the MVI force are \( m = 0.65 \), and the strengths of the spin-orbit force are \( u_{11} = -u_{11} = 3700 \) MeV. The VAP calculations of AMD using these interactions reproduce well the breaking of neutron magic number \( N = 8 \) in \( 11^\text{Be} \) and \( 12^\text{Be} \). With this interaction the calculated binding energies of \( 12^\text{Be} \) and \( 13^\text{Be} \) are 61.9 MeV and 76.4 MeV, which underestimate the experimental values, 68.6 MeV and 84.5 MeV, respectively, however, we adopted this parametrization because the energy levels of the excited states in \( 10^\text{Be} \), \( 11^\text{Be} \) and \( 12^\text{Be} \) are reasonably reproduced.

The basis AMD wave functions were obtained by the VAP for the ground and excited states of \( 13^\text{Be} \). The number of the basis AMD wave functions in the present calculations are 23. The initial wave function in the energy variation was randomly chosen for \( J \leq 5/2 \) states. For \( J \geq 7/2 \) states, we started the variational calculation from the initial wave function projected from the obtained wave function \( \Phi_{\text{AMD}}(Z^J_{K^{1+}\pi}) \) for the \( J' \leq 5/2 \) states. These independent AMD wave functions were superposed to calculate the final wave functions.

A. Energies and deformation

\( 13^\text{B} \) is a nucleus with neutron magic number \( N = 8 \), and its ground state is the \( 3/2^- \) with normal configuration of \( p \)-shell closure. Above the \( 3/2^- \) ground state of \( 13^\text{B} \), it is experimentally known that many states exist in the excitation energy \( E_x \geq 3.5 \) MeV region with high level density. Unfortunately, spins and parities of most of these states are unknown. Recently, the state at 4.83 MeV has been assigned to be a \( 1/2^+ \) state by \( ^4\text{He}[^{12}\text{Be},^{13}\text{Be}]X \) experiments [20]. Because of its strong production via proton-transfer to the \( ^{12}\text{Be}(0^+) \) state, this excited state is suggested to be a proton intruder state.

The calculated energy levels of the negative- and positive-parity states of \( 13^\text{B} \) are shown in Fig. 1. In addition to the ground \( 3/2^- \) state, we obtained many excited states with various \( J^\pi \) in the region \( E_x \geq 4 \) MeV. These states may correspond to the observed levels in this energy region. In the excited states, we found three largely deformed bands, \( K^\pi = 3/2^- \), \( K^\pi = 1/2^+ \) and \( K^\pi = 1/2^- \) (solid lines). These bands are composed of intruder states or well-developed cluster states. In particular, the band-head state \( 1/2^+_1 \) of the \( K^\pi = 1/2^+ \) is the proton intruder state with a large deformation, and hence this should be assigned to the experimental \( 1/2^+ \) (4.83 MeV) state. The \( K^\pi = 1/2^- \) band was obtained by the spin-parity projection and the diagonalization of the obtained wave functions, though the VAP calculations were not performed for the corresponding \( J^\pi \) states. This band is dominantly the \( \alpha-^9\text{Li} \) cluster state. The intrinsic structures of these deformed states are discussed later in detail. As for other excited states (disconnected filled circles), intrinsic deformation of the major AMD wave function \( \Phi_{\text{AMD}}(Z^J_{K^{1+}\pi}) \), which dominates the final wave functions \( |J^\pi_n\rangle \), is small or as large as normal deformation at most. These excited states are regarded to be dominated by \( 0\hbar\omega \) or and neutron \( 1\hbar\omega \) configurations.

Figure 2 shows density distribution and deformation parameters of the major AMD wave functions, \( \Phi_{\text{AMD}}(Z_{1/2^-}^J) \), \( \Phi_{\text{AMD}}(Z_{3/2^-}^J) \), \( \Phi_{\text{AMD}}(Z_{1/2^+}^J) \), \( \Phi_{\text{AMD}}(Z_{1/2^-}^J) \), \( \Phi_{\text{AMD}}(Z_{1/2^+}^J) \), \( \Phi_{\text{AMD}}(Z_{3/2^-}^J) \), \( \Phi_{\text{AMD}}(Z_{3/2^+}^J) \), \( \Phi_{\text{AMD}}(Z_{3/2^-}^J) \), \( \Phi_{\text{AMD}}(Z_{3/2^+}^J) \), \( \Phi_{\text{AMD}}(Z_{5/2^+}^J) \), \( \Phi_{\text{AMD}}(Z_{5/2^-}^J) \), \( \Phi_{\text{AMD}}(Z_{7/2^+}^J) \), \( \Phi_{\text{AMD}}(Z_{7/2^-}^J) \), which were obtained by the VAP for the corresponding \( J^\pi \) states. The ground state \( (3/2^-) \) has the most spherical shape (Fig. 2b), due to the neutron \( p \)-shell closure. This is consistent with the previous work by AMD [20]. In the \( 1/2^- \) state (Fig. 2a), a three-center cluster core structure appears. The core clusters are an \( \alpha \) with two valence neutrons, a triton and an \( \alpha \). This state is approximately regarded as the \( SU(3) \)-limit cluster state though the spatial cluster development is somehow contained. The similar three-center cluster structure is found also in the \( 5/2^- \) state. In the \( 3/2^- \) and the \( 5/2^- \) states (Fig. 2c and d), we found remarkably deformed structure with developed cluster cores. These states are the members of the \( K^\pi = 3/2^- \) band which starts from \( E_x = 5 \) MeV. It is interesting that such a largely deformed band appears only 5 MeV above the ground state, even though this nucleus has neutron magic number \( N = 8 \). The \( J^\pi = 5/2^+ \) state of this band is on the yrast line. Moreover, it is striking that a further large deformation arises in the \( 1/2^+ \) state (Fig. 2c), which is the band-head state of the \( K^\pi = 1/2^+ \) band. The deformation \( \beta = 0.74 \) of this state exceeds the value for superdeformation and is close to the value 0.9 for hyperdeformation. In the \( 7/2^- \) state, we obtained the well-developed cluster structure like \( ^9\text{Li}+\alpha \) (Fig. 2f). In the final wave functions after the superposition, these two components of the largely deformed state (Fig. 2c) and the \( ^9\text{Li}+\alpha \) cluster state (Fig. 2f) constitute the rotational band, \( K^\pi = 1/2^+ \), with a mixing of them. The \( |1/2^+_1\rangle \), \( |3/2^+_1\rangle \), \( |5/2^+_1\rangle \) states are dominated by \( P_{KM}^{1/2^+}\Phi_{\text{AMD}}(Z_{1/2^+}^J) \) in about 90%, 65% and 60%, respectively. On the other hand, the \( |7/2^+_1\rangle \) and \( |11/2^+_1\rangle \) contains major percentage of \( P_{KM}^{1/2^+}\Phi_{\text{AMD}}(Z_{3/2^+}^J) \) with remarkable \(^9\text{Li}+\alpha \) cluster structure, while other states in the \( K^\pi = 1/2^+ \) band are the mixture of these two components. It indicates that the weak-coupling cluster feature is enhanced in high spin region of the \( K^\pi = 1/2^+ \) band. The negative-parity states \( P_{KM}^{1/2^-}\Phi_{\text{AMD}}(Z_{3/2^-}^J) \) projected from the \(^9\text{Li}+\alpha \) cluster structure
construct the $K^\pi = 1/2^-$ band.

![Graph](image)

**FIG. 1:** Excitation energies of the negative- and positive-parity states of $^{13}$B calculated by superposition of the basis wave functions. Filled circles are the energies of the $J_n^\pi$ states, for which the VAP calculations were done. Open circles are the energies of the $J_n^\pi$ states, which were obtained by diagonalization of Hamiltonian by superposing wave functions, but the VAP calculations were not performed for the corresponding $J_n^\pi$ states.

### B. Cluster feature in the $K^\pi = 3/2^-$ and $K^\pi = 1/2^+$

In this subsection, we discuss cluster features of the deformed bands, $K^\pi = 3/2^-$ and $K^\pi = 1/2^+$, by analyzing the single-particle orbitals in the intrinsic states. Hereafter, we mainly analyze the major AMD wave functions, $\Phi_{AMD}(Z_1^{5/2^-})$, $\Phi_{AMD}(Z_1^{1/2^+})$ and $\Phi_{AMD}(Z_3^{7/2^+})$ obtained by VAP for the $J_n^\pi = 5/2_1^-(K^\pi = 3/2^-)$, $J_n^\pi = 1/2_1^+(K^\pi = 1/2^+)$ and $J_n^\pi = 7/2_3^+(K^\pi = 1/2^-)$ states, and compare them with that for the intruder ground state $J_n^\pi = 0_1^+$ of $^{12}$Be.

First, we give single-particle energies in Fig. 3. In these deformed states, the level structure of the single-particle wave functions shows a feature of the $2\alpha + p + 4n$ structure rather than that of spherical shell structure. That is to say, the lowest four proton orbitals and the lowest four neutron orbitals form the $2\alpha$ core, while the higher orbitals correspond to one valence proton and four valence neutrons.

Next, we illustrate the density distribution of the single-particle wave functions for the valence nucleons in Fig. 4. The total densities of protons and neutrons are also given as well as total matter densities in the figure. Generally speaking, the matter densities show two-center structures in all these deformed states. However, the behavior of the valence nucleons are different among these three states, $J_n^\pi = 5/2_1^-(K^\pi = 3/2^-)$, $J_n^\pi = 1/2_1^+(K^\pi = 1/2^+)$ and $J_n^\pi = 7/2_3^+(K^\pi = 1/2^-)$. 


In the $J^\pi = 5/2^- (K^\pi = 3/2^-)$ state (Fig. 3(a)), two valence neutrons occupy an approximately positive-parity orbital, and the other two neutrons and a proton are in orbitals with dominant negative-parity component. Since the negative- and positive-parity orbitals of the valence nucleons are associated with the $p$-orbitals and $sd$-orbitals, respectively, we can roughly describe the states in the $K^\pi = 3/2^-$ band by the neutron $2\hbar\omega$ excited configurations. Let us turn to the molecular orbital features. The positive-parity orbital of the last two neutrons is largely deformed and has nodes along the $2\alpha$ direction (longitudinal axis). This orbital well corresponds to the so-called $\sigma$ orbital in the molecular orbital picture \[5, 8, 9, 10, 11\]. It has been already known that the $\sigma$-like orbital of valence nucleons appear in various Be isotopes (see references in \[8\]). In the AMD study \[7\], it has been revealed that the ground state of $^{12}\text{Be}$ is dominated by the intruder state with two neutrons in the $\sigma$ orbital, which reduce kinetic energy due to the developed $2\alpha$-core structure. In Fig. 3(4), density distribution and single-particle orbitals of $^{12}\text{Be}(0^+_1)$ are shown. As seen in the figure, the last two neutrons in the $^{12}\text{Be}(0^+_1)$ state occupy the $\sigma$ orbital. The point is that the neutron structure of the $^{13}\text{B}(K^\pi = 3/2^-)$ band is very similar to that of the $^{12}\text{Be}(0^+_1)$. Therefore, we conclude that the $K^\pi = 3/2^-$ is the band of the intruder neutron $2\hbar\omega$ states, and interpreted as $^{12}\text{Be}(0^+_1)+p$, where the $^{12}\text{Be}(0^+_1)$ has the intruder configuration and the additional proton strongly couples to the deformed core. It is also interesting that the additional proton in the normal $p$-shell affects a change of the deformation of total density, which results in the smaller deformation of the $^{13}\text{B}(K^\pi = 3/2^-)$ than the $^{12}\text{Be}(0^+_1)$. In the band-head $1/2^+_1$ state of the $K^\pi = 1/2^+$ band (Fig. 3(b)), $1\hbar\omega$ excitation occurs in the proton shell. Namely, the last proton occupy a $\sigma$-like orbital, which is quite similar to that of the highest neutron orbital in the $K^\pi = 3/2^-$ band and also that in the $^{12}\text{Be}(0^+_1)$. This is extraordinary configuration because excitations are naively expected in the neutron side in case of neutron-rich nuclei. It can be understood by the lowering mechanism of the $\sigma$ orbital due to the developed two-center structure. In the two-center shell model \[12\], the energy of the $\sigma$ orbital comes down with the increase of two-center distance $d$. On the other hand, the negative-parity levels ($\pi$) originating in the $p$ orbitals split and four of the negative-parity levels go up as the two-center distance increases. As a result, the inversion of the positive-parity $\sigma$ orbital and the negative-parity $\pi$ orbitals occurs. Finally, in the large distance region $d \sim 5 - 6$ fm, the $\sigma$ orbital becomes the fifth orbital which is the lowest one for valence nucleons around the $2\alpha$ core. This situation is realized in the $1/2^+_1$ state, where the two-center structure with the distance $d \sim 5$ fm was obtained in the present calculation. It is also associated with the hyperdeformation, where the declined positive-parity orbital becomes the fifth lowest orbital in the Nilsson’s deformed shell model. In fact, the deformation $\beta = 0.74$ of the $1/2^+_1$ state exceeds
the value $\beta = 0.6$ for the superdeformation. On the other hand, four valence neutrons are localized in one side of the two-center structure. It is contrast to the molecular orbital feature of the valence proton, which is moving around the whole system. It seems that spatial correlation of four valence neutrons is so strong that form a $p_{3/2}$ shell closure. As a result, totally 8 neutrons separate into two groups consisting of 2 and 6 neutrons with a weak-coupling feature. We here give a comment on similarity of the neutron structure between the $1/2^+$ and $5/2^-$ states. Comparing the neutron structure of the $1/2^+$ with that of the $5/2^-$, the profile of the total neutron density of the $1/2^+$ is similar to that of the $5/2^-$ described by the neutron $2\hbar \omega$ configuration. Therefore, we can propose an alternative interpretation in a mean-field picture that the $1/2^+$ state is roughly described by the $3\hbar \omega$ configuration with the proton $1\hbar \omega$ and the neutron $2\hbar \omega$ excitations. In the present calculation, we suggest that such the exotic state, the proton intruder state of neutron-rich nuclei, may exist as the lowest $1/2^+$ state at $E_x = 8$ MeV. Recently, the state at 4.83 MeV has been assigned to be a $1/2^+$ state by $^4\text{He}(^{12}\text{Be},^{13}\text{B})\gamma$ experiments [20]. Because of strong production via the proton-transfer to the $^{12}\text{Be}(0^+)$ state and analysis of angular dependence, Ota et al. suggested this $1/2^+$ state to be a proton intruder state. The present prediction of the proton intruder configuration in the $1/2^+$ state is consistent with this observation though the theoretical excitation energy of the present calculation is slightly higher than the experimental value.

In the $7/2^+$ state in the $K^\pi = 1/2^+$ band, all the valence proton and the valence four neutrons are localized around one of the $\alpha$ cores(Fig. 4(c)). It indicates that the molecular orbital aspect disappears, while the weak-coupling feature of $\alpha-^9\text{Li}$ clustering is enhanced. Because of the spatial localization, the orbitals of the valence nucleons have no definite parity. In this case, the unnatural parity $'+'$ of the total system is carried by the parity asymmetric $\alpha-^9\text{Li}$ structure. It is in different situation from the $1/2^+$ state with the intruder proton configuration, where the parity $'+'$ originates in the proton $1\hbar \omega$ excitation of the single-particle orbital from negative to positive one. As mentioned before, after the superposition of the AMD wave functions, the proton intruder state and the $\alpha+^9\text{Li}$ cluster state are mixed to each other to contribute the structure change depending on $J$ in the $K^\pi = 1/2^+$ band. The former component is dominant in the low spin states, while the latter component is significant in high spin states.

![FIG. 3: Single-particle energies in the $5/2^-$, $1/2^+$ and $7/2^+$ states. The energies are calculated by diagonalizing Hartree-Fock-like single-particle Hamiltonian within the single AMD wave function [14] which dominates the $|J^+_n\rangle$ state.](image)

**IV. SUMMARY**

The excited states of $^{13}\text{B}$ were studied with a method of antisymmetrized molecular dynamics(AMD). We obtained the largely deformed states which construct the rotational bands, $K^\pi = 3/2^-$, $K^\pi = 1/2^+$, and $K^\pi = 1/2^-$. In these deformed states, we found various kinds of cluster aspect including molecular orbital features in the two-center structure.
The $K^\pi = 3/2^-$ band is the deformed one with molecular orbital structure. This band is described by the intruder neutron $2\hbar\omega$ configuration, and interpreted as $^{12}\text{Be}(0^+_1)+p$, where the $^{12}\text{Be}(0^+_1)$ has the intruder configuration and the additional proton strongly couples to the deformed $^{12}\text{Be}$ core. The excited neutron orbital is regarded as the $\sigma$ orbital in the molecular orbital picture. Experimentally, there is a report which has been suggested the excited state at 10 MeV to be a high spin state of the neutron $2\hbar\omega$ configuration [25], which would be a member of this $K^\pi = 3/2^-$ band.

We found the proton intruder structure in the $K^\pi = 1/2^+$ band with remarkably large deformation. The deformation $\beta = 0.74$ of the band-head $1/2^+_1$ state is larger than that for the superdeformation. In the molecular orbital picture, the last proton is described by the $\sigma$ orbital. This is very exotic state with the proton intruder configuration in neutron-rich nuclei. In the present calculations, it was suggested that the $K^\pi = 1/2^+$ band starts from the lowest $1/2^+$ state at $E_x = 8$ MeV. We assigned this state to the recently observed $1/2^+$ state at 4.83 MeV, which has been suggested to be the proton intruder state by Ota et al. in the $^4\text{He}(^{12}\text{Be},^{13}\text{B}\gamma)X$ experiments. In the high spin states of the $K^\pi = 1/2^+$ band, $\alpha+^9\text{Li}$-like cluster structure well develops.

It is striking that such the deformed states with highly excited configurations or well-developed cluster states appear in the energy region compatible to the normal excited states, even though $^{13}\text{B}$ has the neutron-shell closure in the ground state. Especially the molecular orbital, $\sigma$, is formed in the deformed states of $^{13}\text{B}$ as well as neutron-rich Be isotopes. The cluster aspect of the deformed states in $^{13}\text{B}$ can be understood in natural extension of cluster structure of Be isotopes. That is to say, the formation of the $2\alpha$-cluster core and the role of the valence nucleons(one proton and four neutrons) are key in the largely deformed states of $^{13}\text{B}$. It is challenging to investigate possible exotic structure with clustering in excited states of further neutron-rich B isotopes like $^{15}\text{B}$ and $^{17}\text{B}$.

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[1] H. Iwasaki et al., Phys. Lett. B —bf 481, 7 (2000).
[2] A. Navin et al., Phys. Rev. Lett. 85, 266 (2000).
[3] S. D. Pain et al., Phys. Rev. Lett. 96, 032502 (2006).
[4] T. Otsuka and N. Fukushima, Phys. Rep. 264, 297 (1996).
[5] N. Itagaki and S. Okabe, Phys. Rev. C 61, 044306 (2000); N. Itagaki, S. Okabe and K. Ikeda, Phys. Rev. C 62, 034301 (2000).
[6] Y. Kanada-En’yo and H. Horiuchi, Phys. Rev. C 66, 024305 (2002).
[7] Y. Kanada-En’yo and H. Horiuchi, Phys. Rev. C 68, 014319 (2003).
[8] W. von Oertzen, M. Freer and Y. Kanada-En’yo, Phys. Rep. 432, 43 (2006).
[9] S. Okabe, Y. Abe and H. Tanaka, Prog. Theor. Phys. 57, 866 (1977).
[10] M. Seya, M. Kohno, and S. Nagata, Prog. Theor. Phys. 65, 204 (1981).
[11] W. von Oertzen, Z. Phys. A 354, 37 (1996); 357, 355(1997).
[12] D. Scharnweber, U. Mosel, andW. Greiner, Phys. Rev. Lett. 24, 601 (1970).
[13] K. Arai, Y. Ogawa, Y. Suzuki and K. Varga, Phys. Rev. C 54, 132 (1996).
[14] A. Doté, H. Horiuchi, and Y. Kanada-En’yo, Phys. Rev. C 56, 1844 (1997).
[15] Y. Kanada-En’yo, H. Horiuchi and A. Doté, Phys. Rev. C 60, 064304(1999).
[16] Y. Ogawa, K. Arai, Y. Suzuki, and K. Varga, Nucl. Phys. A673 122 (2000).
[17] M. Ito, K. Kato and K. Ikeda, Phys. Lett. B 588, 43 (2004).
[18] M. Ito and Y. Sakuragi, Phys. Rev. C 62, 064310 (2000).
[19] P. Descouvemont and D. Baye, Phys. Lett. B 505, 71(2001).
[20] Y. Kanada-En’yo, H. Horiuchi and A. Ono, Phys. Rev. C 52, 628 (1995); Y. Kanada-En’yo and H. Horiuchi, Phys. Rev. C 52, 647 (1995).
[21] Y. Kanada-En’yo and H. Horiuchi, Prog. Theor. Phys. Suppl.142, 205 (2001).
[22] Y. Kanada-En’yo, M. Kimura and H. Horiuchi, Comptes rendus Physique Vol.4, 497 (2003).
[23] Y. Kanada-En’yo, Phys. Rev. Lett. 81, 5291 (1998).
[24] Y. Kanada-En’yo and H. Horiuchi Phys. Rev. C 55, 2860 (1997).
[25] R. Kalpakchieva et al., Eur. Phys. J. A 7, 451 (2000).
[26] Ota et al., private communication, JPS meeting, March 2007, Tokyo, Japan.
[27] Y. Kanada-En’yo, Prog. Theor. Phys. 117, 655 (2007).