Bearing friction effect on cup anemometer performance modelling

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Abstract. In the present work, the effect of the friction forces at bearings on cup anemometer performance is studied. The study is based on the classical analytical approach to cup anemometer performance (2-cup model), used in the analysis by Schrenk (1929) and Wyngaard (1981). The friction torque dependence on temperature was modelled using exponential functions fitted to the experimental results from RISØ report #1348 by Pedersen (2003). Results indicate a logical poorer performance (in terms of a lower rotation speed at the same wind velocity), with an increase of the friction. However, this decrease of the performance is affected by the aerodynamic characteristics of the cups. More precisely, results indicate that the effect of the friction is modified depending on the ratio between the maximum value of the aerodynamic drag coefficient (at 0º yaw angle) and the minimum one (at 180º yaw angle). This reveals as a possible way to increase the efficiency of the cup anemometer rotors. Besides, if the friction torque is included in the equations, a noticeable deviation of the rotation rate (0.5–1% with regard to the expected rotation rate without considering friction) is found for low temperatures.

1. Introduction
The cup anemometer is the most popular wind speed sensor in both the wind energy sector and meteorology. Its great accuracy, robustness, reliability and costs (both the instrumentation and its calibration) make this sensor unbeatable in the aforementioned sector when compared to other more modern and sophisticated technologies (LIDAR, SODAR, sonic).

However, the cup anemometer has some well-known drawbacks. Firstly, the need of recalibrations after a certain period of service in the field can be mentioned [1–3]. Besides, it’s use is restricted to non-extreme weather conditions such as icing [4–6]. Additionally, the cup anemometer has a very specific problem called overspeeding, which is the overestimation of the wind speed in turbulent flows due to the cup/rotor aerodynamics. The difference between the cups aerodynamic force coefficient depending on the yaw angle (i.e., the cups’ position regarding the local flow), makes the rotor to respond faster to accelerations of the wind speed than to decelerations, the average velocity measured by the instrument being then overestimated [7,8].

In the past years, a quite deep research on the cup anemometer performance has been carried out at the IDR/UPM Institute [9,10]. The aim of the present paper is to describe the last results of this research line, regarding the effect of the friction on the instrument’s performance.
2. Modelling cup anemometers’ performance

The equation that defines the performance of a cup anemometer is the following [11]:

$$I \frac{d\omega}{dt} = Q_A - Q_F,$$

where $I$ is the moment of inertia of the rotor, $Q_A$ is the aerodynamic torque, and $Q_F$ is the frictional torque that depends on the air temperature, $T$, and the rotation speed, $\omega$ [12,13]:

$$Q_F = F_0(T) + F_1(T)\omega + F_2(T)\omega^2.$$  

The aerodynamic torque is traditionally defined by the aerodynamic forces on the cups, this force being applied to the centre of each cup [14]:

$$Q_A = \frac{1}{2} \rho S_c R_{rc} V_r^2 (\theta) c_N(\alpha(\theta)) + \frac{1}{2} \rho S_c R_{rc} V_r^2 (\theta + 120^\circ) c_N(\alpha(\theta + 120^\circ)) + \frac{1}{2} \rho S_c R_{rc} V_r^2 (\theta + 240^\circ) c_N(\alpha(\theta + 240^\circ)),$$

where $V_r$ is the wind speed relative to the cups, $c_N$ is the aerodynamic normal force coefficient, $\alpha$ is the local wind flow direction with respect to the cups whereas $\theta$ is the angle of the rotor with respect to a reference line, $R_{rc}$ is the distance of the cups’ centres to the cup anemometer rotation axis, and $S_c$ is the front area of the cups. The aerodynamic normal force coefficient, $c_N$, is normally based on static aerodynamic testing of cups (see Figure 1-Left) which consists in measurements taken by means of a wind tunnel testing campaign where the cups were placed successively in fixed positions regarding the wind flow. However, the cups are rotating when they are considered as part of a cup anemometer rotor. Therefore, the wind flow direction regarding the cups, $\alpha$, should not be confused with the angular position of the cups, $\theta$ (see Figure 1-Right). Besides, it should be also taken into account that the wind speed relative to the cups, $V_r$, depends on both, the wind speed, $V$, and the rotation speed of the cups’ centres, $\omega R_{rc}$ (see the vector composition in Figure 1-Right [14–16]):

$$V_r(\theta) = \sqrt{V^2 + (\omega R_{rc})^2 - 2V\omega R_{rc} \cos(\theta)}.$$  

Additionally, the following equation relates the wind direction angle, $\alpha$, and the position angle $\theta$ [14–16]

$$\tan(\alpha) = \frac{K \sin(\theta)}{K \cos(\theta) - 1},$$

where the anemometer factor $K$ is the ratio of the wind speed, $V$, to the rotation speed of the cups’ centres, $\omega R_{rc}$:

$$K = \frac{V}{\omega R_{rc}}.$$  

The stationary solution of equation (1) implies a constant rotation speed, $\omega$, although the rotor will accelerate/decelerate 3 times per turn due to its 3-cup configuration. Therefore, in absence of friction, any turbulence effect or change in the wind flow velocity, the rotation speed can be defined as [14]:

$$\omega(t) = \omega_0 + \sum_{n=1}^{\infty} \omega_n \sin(3n\omega_0 t + \phi_n),$$

which, bearing in mind the magnitude of the terms on the right side, can be simplified to the following equation:

$$\omega(t) = \omega_0 + \omega_1 \sin(3\omega_0 t + \phi_1).$$
2.1. The 2-cup model
If equation (1) is averaged along one turn and the friction torque is assumed to be much smaller than the aerodynamic torque, that is, \( Q_F \ll Q_A \), the averaged aerodynamic force on each cup is zero, which, considered the model shown in the graph from Figure 1, it implies the force equilibrium between two positions, \( \alpha = 0^\circ \) and \( \alpha = 180^\circ \):

\[
0 = \frac{1}{2} \rho (V - \omega R_c)^2 R_c S_c c_{D1} - \frac{1}{2} \rho (V + \omega R_c)^2 R_c S_c c_{D2},
\]

where \( c_{D1} \) and \( c_{D2} \) are the aerodynamic normal force coefficients for \( \alpha = 0^\circ \) and \( \alpha = 180^\circ \), respectively. The above equation has the solution:

\[
\omega_0 R_c / V = \frac{1}{K_0} = \frac{\sqrt{c_{D1}} - \sqrt{c_{D2}}}{\sqrt{c_{D1}} + \sqrt{c_{D2}}} = \frac{1 - \sqrt{c_{D2}/c_{D1}}}{1 + \sqrt{c_{D2}/c_{D1}}} = 1 - \frac{\lambda}{1 + \lambda},
\]

which gives a quite accurate value of the average rotation speed of the anemometer’s rotor, \( \omega_0 \), as a function of \( \lambda \) the root of the coefficients \( c_{D1} \) and \( c_{D2} \) ratio.
2.2. Modelling the friction forces

The coefficients of the friction torque $F_0$, $F_1$ and $F_2$ (equation (2)) of a cup anemometer (RISØ P2546) have been measured by Pedersen [17]. These friction torque coefficients are plotted in Figure 2. The following equation is proposed to fit the experimental results:

$$F_i = a_i + b_i \exp \left( -\frac{T}{T_i^*} \right).$$  \hspace{1cm} (11)

where $a_i$, $b_i$, and $T_i^*$ are the parameters corresponding to friction torque coefficient $F_i$, and $T$ is the temperature. The coefficients corresponding to these fittings, are shown in Table 1 and in Figure 2. Additionally, other family of friction torque coefficients $F_0$, $F_1$ and $F_2$, have been proposed in [13] for Classcup anemometers such as the RISØ P2546. These results have been included in Figure 2 for a comparison with the first ones.

![Figure 2](image-url)
Table 1. Parameters in equation (11), $a$, $b$, and $T^*$, fitted to the experimental measurements of the friction torque coefficients from equation (2), $F_0$, $F_1$, and $F_2$, [17]. See also Figure 1. The RMSE of these fittings is also included in the table.

|       | $a$         | $b$         | $T^*$      | RMSE     |
|-------|-------------|-------------|------------|----------|
| $F_0$ | 9.4012·10^{-6} | 8.9018·10^{-6} | 6.3364     | 1.4823·10^{-5} |
| $F_1$ | 1.8284·10^{-7} | 4.6052·10^{-7} | 7.7126     | 3.1649·10^{-7} |
| $F_2$ | 3.3679·10^{-10} | -1.1545·10^{-9} | 5.9967     | 2.7377·10^{-9} |

3. Results

In this section, the effect of the friction is considered to analyze its influence on the rotation speed of the anemometer’s rotor, $\omega$.

3.1. Analysis based on the 2-cup model

If equation (1) is averaged along one turn of the anemometer’s rotor and, taking into account the 2-cup model, the following equation can be derived:

$$
\frac{1}{3} \frac{Q_F}{\rho V^2 R_n S_e} = c_F \left( \frac{1}{K} \right)^2 c_{d1} - \left( \frac{1 + \frac{1}{\lambda}}{\lambda} \right)^2 c_{d2} .
$$

(12)

If the rotation speed is assumed to be slightly modified due to the friction torque:

$$
\omega = \omega_0 + \Delta \omega = \omega_0 + \omega_\varepsilon \varepsilon ,
$$

(13)

where $\varepsilon<<1$. Additionally, the anemometer factor is defined as:

$$
\frac{1}{K} = \frac{1}{K_0} + \frac{\Delta \omega R_m}{V} = \frac{1}{K_0} + \frac{\varepsilon}{K_0} .
$$

(14)

Going back to equation (12), and taking into account equation (14):

$$
\frac{1}{3} \frac{Q_F}{\rho V^2 R_n S_e} = \frac{c_F}{3} \left( \frac{1}{K_0} - \frac{\varepsilon}{K_0} \right)^2 c_{d1} - \left( \frac{1 + \frac{1}{\lambda}}{\lambda} \right)^2 c_{d2} .
$$

(15)

If the friction is considered a minor effect $c_F \sim \varepsilon$.

$$
\frac{c_F}{3} = -2 \left( \frac{1}{K_0} \right) \varepsilon c_{d1} - 2 \left( \frac{1 + \frac{1}{\lambda}}{\lambda} \right) \varepsilon c_{d2} ,
$$

(16)

$$
\frac{c_F}{3} = -4 \varepsilon c_{d1} \lambda \left( \frac{1 - \lambda}{1 + \lambda} \right) ,
$$

(17)

$$
\varepsilon = \frac{\Delta \omega}{\omega_0} = -\frac{c_F}{12} \frac{1}{\lambda} \left( \frac{1 + \lambda}{1 - \lambda} \right) = -\xi c_F .
$$

(18)
Figure 3. Coefficient $\xi$ that multiplied the friction coefficient $c_F$ in equation (23), as a function of drag coefficient ratio $\lambda^2$, for $c_{D1} = 1.8, 2.0, \text{and } 2.3$.

Equation (18) indicates that friction reduces the average rotation speed of the cup anemometer, as expected, this reduction being dependent of the coefficients $c_{D1}$ and $c_{D2}$. In Figure 3, the variable $\xi$ that multiplies the friction coefficient, $c_F$, in the above equation is plotted as a function of $c_{D2}$ to $c_{D1}$ ratio (that is, $\lambda^2$), for different values of $c_{D1}$. The results plotted in the graph of Figure 3 were calculated for $c_{D1} = 2.3, 2.0 \text{ and } 1.8$, and $c_{D2} \in [0.7, 1.1]$. It can be observed that the effect of the friction on the rotation speed, $\omega$, decreases for larger values of the aerodynamic drag at $\alpha = 0^\circ$, $c_{D1}$, and lower ratios of $c_{D2}$ to $c_{D1}$.

3.2. Effect of the friction

Bearing in mind the results from Section 2.2, the RISØ P2546 cup anemometer friction coefficients corresponding to $T = -10^\circ \text{C}, 0^\circ \text{C}, 10^\circ \text{C}, 20^\circ \text{C} \text{ and } 30^\circ \text{C}$ have been calculated and included in Table 2. Considering the geometry of the anemometer’s rotor, the average calibration coefficients for the model P2546A [18] and the drag coefficients $c_{D1}$ and $c_{D2}$, it is possible to calculate the friction coefficient, $c_F$, and the effect of the friction on the rotation rate, $\varepsilon$ (equation (18)), at a given wind speed, $V$.

In Figure 4, the effect of the friction on the RISØ P2546A cup anemometer’s rotation rate, $\varepsilon$, is plotted in relation to the temperature, $T$, for wind speeds $V = 4, 10 \text{ and } 16 \text{ m} \cdot \text{s}^{-1}$. For these calculations the selected values of the cups’ aerodynamics were $c_{D1} = 2.0$ and $c_{D2} = 0.8$ ($\xi = 0.2926$).

The results obtained reveal that for low wind speeds and low temperatures there might be a quite noticeable deviation (around 1%) from the foreseen cup anemometer’s rotation rate.
Table 2. RISO P2546 cup anemometer friction torque coefficients from equation (2), \( F_0, F_1 \) and \( F_2 \), calculated from the fittings of equation (11) to the testing results [17].

| \( T \) [°C] | \( F_0 \) \( \times 10^{-5} \) | \( F_1 \) \( \times 10^{-7} \) | \( F_2 \) \( \times 10^{-9} \) |
|-----------|---------------------|---------------------|---------------------|
| -10       | 5.25392             | 1.86685             | -5.78141            |
| 0         | 1.83018             | 6.43363             | -8.17751            |
| 10        | 1.12369             | 3.08779             | 1.18920             |
| 20        | 9.77905             | 2.17281             | 2.95675             |
| 30        | 9.47822             | 1.92259             | 3.29030             |

Figure 4. Percentage deviation of the rotation rate, \(-\varepsilon\), estimated for a RISO P2546A cup anemometer as a function of temperature, \( T \), at three different wind speeds, \( V = 4, 10 \) and \( 16 \) m·s\(^{-1}\).

4. Conclusions
The most relevant conclusions of the work described in this paper are:

- The friction torque has been included as an independent term in a cup anemometer analytical model.
- The friction torque has been modelled as a function of the temperature and the anemometer’s rotation rate, by fitting a mathematical expression to experimental results.
- The effect of the friction torque might be noticeable (0.5-1%) at low wind speeds and low temperatures of service.

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