Direct search limits on the Littlest Higgs model

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Abstract

Recent direct searches for new massive particles place constraints on the free parameters of the Littlest Higgs model. Depending on the choice of model free parameters, the direct search limit on the global symmetry breaking scale $f$ can range from as low as a few hundred GeV to in excess of 4.5 TeV. The most stringent constraints are from exclusion of the $A_H$ using high-mass dilepton resonance searches. The $Z_H$ provides the best constraint in parameter regions where the $A_H$ decouples from leptons. Current top pair resonance data approach but do not yet reach a useful limit in the anomaly-cancelling case, but do provide a constraint for a limited range of parameters in other cases. A neutral gauge boson is shown to be undetectable in dilepton resonances for a significant range of parameter space due to decoupling from Standard Model leptons, providing a counterexample to broad claims that a new neutral gauge boson (sometimes generically referred to as a $Z'$) is ruled out to a high mass scale.

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I. INTRODUCTION

The Standard Model of particle physics continues to meet all experimental tests, and yet studies of the structure of the model suggest that it is incomplete. One well-known difficulty is that the mass of the Higgs boson, an essential participant in the standard description of electroweak interactions, receives loop corrections to its mass (squared) that are quadratic in the loop momenta. The largest correction is due to the top quark loop, with smaller contributions coming from loops of the electroweak gauge bosons and of the Higgs boson itself. These corrections offset a tree-level mass term for the Higgs boson. At a high energy scale, the loop contributions become large, and require a precise near-cancellation by the tree-level mass term in order to achieve an effective Higgs boson mass on the order of 100 GeV, as required by fits to precision electroweak parameters [1]. The existence of such a near-cancellation is highly suggestive of an as-yet unidentified partial or broken symmetry. This symmetry, evident at high energy scales, would be the cause of a cancellation which, at low energy scales, appears only as an apparent coincidence.

In supersymmetric models, the problem of quadratic Higgs mass divergences is resolved by the introduction of an opposite-statistics partner for each particle in the Standard Model. Recently, extended symmetry groups have been found which can contain the $SU(2)_L \otimes U(1)_Y$ electroweak gauge group of the Standard Model as well as additional structure that provides for quadratic divergence cancellation between same-statistics particles. These are known as “Little Higgs” models [2]. The Little Higgs models realize a long-standing conception of the Higgs as a pseudo-Goldstone boson [3].

The “Little Higgs mechanism” embodied in these models provides an alternative to supersymmetry for resolving the Higgs mass problem. The Little Higgs models remove the need for finely-tuned cancellations to beyond about 10 TeV, largely beyond the reach of current electroweak constraints on new strong interactions. However, Little Higgs models typically leave an uncancelled logarithmic mass contribution, which requires additional new contributions at some high scale to preserve a small Higgs mass. Consequently we need not, and in fact cannot, view a Little Higgs model as complete, but must keep in mind that at sufficiently high energies there will be additional contributions from the unidentified complete theory, the so-called “ultraviolet completion” [4].

A minimum requirement for any proposed extension to the Standard Model is that it conform to known experimental constraints. In practice this means that it must not violate the current experimental limits on the properties of known particles, including fits to the free parameters of the electroweak theory, and the current search limits for new particles. Applying this requirement to
Table I: New particles in the Littlest Higgs model \[11\][12], where \( m_w = \frac{g_w}{2}, m_z = \frac{g_z}{2c_w} \)

the Little Higgs models is considerably complicated by the incomplete nature of the theories. We may find that the identified contributions of a certain model violate, say, the known experimental limits on the mass of the \( Z^0 \) gauge boson, and yet not be able to determine whether this violation may be offset by contributions from the ultraviolet completion. Considerable effort has been made to determine the true constraints to Little Higgs models from precision electroweak measurements, using a variety of approaches \[5, 6, 7, 8, 9\]. In this work, a complementary effort is made to constrain the Littlest Higgs model using direct search results.

In Section II of this paper, the Littlest Higgs model is briefly described, primarily to introduce notation. Section III presents the main results of the paper, including a scan for the lightest new particles in various regions of parameter space, the full mass-dependent partial decay widths (at tree level) of the most-often lightest particle, the \( A_H \), and the combined results of various constraints on the symmetry-breaking scale \( f \). Section IV provides some conclusions.

II. THE LITTLEST HIGGS MODEL

The prototype Little Higgs model is the “Littlest Higgs” \[10\]. This model is presented in detail, with Feynman rules and some phenomenological results, in \[11\]. Two expressions for masses are corrected in \[12\]. In this section I will sketch the structure of the model, and introduce the notation that will be used.

The enlarged electroweak space of the Littlest Higgs model is the symmetric tensor representation of \( SU(5) \), which is broken at a scale \( f \) by a tensor vacuum expectation value to the coset \( SU(5)/SO(5) \). The result is \( 24 - 10 = 14 \) Goldstone bosons, which are parameterized by a \( 5 \times 5 \)

\[ \begin{array}{|c|c|c|}
\hline
\text{Particles} & \text{Spin} & \text{Mass Squared} \\
\hline
\Phi^0, \Phi^+, \Phi^-, \Phi^{++}, \Phi^{--} & 0 & \frac{2m_H^2 f^2}{v^4 (1 - (\frac{f}{2})^2)} \\
T, T^c & \frac{1}{2} & \frac{m^2}{v^2} (\lambda_1 \lambda_2 f)^2 \\
A_H & 1 & m^2 \frac{2 s_w^2 (\frac{f^2}{2 c_w^2 v^2} - 1)}{5 c_w^2} \\
Z_H & 1 & m_w^2 (\frac{f^2}{2 c_w^2 v^2} - 1) \\
W_H^+, W_H^- & 1 & m_w^2 (\frac{f^2}{5 c_w^2 v^2} - 1) \\
\hline
\end{array} \]

In fact not all such studies agree. One study\[7\] concludes, for example at \( f = 3 \text{ TeV} \), that only a tightly constrained region of parameter space with \( c > 0.9 \) is allowed, while \[8\] finds that a wide range of parameter space is allowed, all with \( c < 0.85 \).
non-linear sigma field $\Sigma(x)$. Four of these Goldstone bosons will become the complex Higgs doublet of the Standard Model, four will become the longitudinal modes of four new massive gauge bosons, and the remaining six will form a massive complex scalar triplet. This scalar triplet has a positive mass squared, which we require \[11\], only so long as its vacuum expectation value $v'$ satisfies (See Table I):

$$v' < \frac{v^2}{4f} \approx \frac{0.015 \text{ TeV}^2}{f}$$

where $v = 246$ GeV is the Standard Model electroweak scale. In this work $v'$ enters only in the determination of the lightest new particle and is set to 2 GeV, which satisfies the above relation as long as $f$ is less than 7.5 TeV.

The $SU(5)$ group contains an $SU(2)_1 \otimes U(1)_1 \otimes SU(2)_2 \otimes U(1)_2$ subgroup, which is gauged,\(^2\) with gauge couplings $g_1, g'_1, g_2, \text{ and } g'_2$, respectively. The result is eight gauge bosons, which after diagonalization to mass eigenstates are the gauge bosons of the Standard Model, here labeled $A_L, Z_L, W^\pm_L$, and new gauge bosons with masses of order $f$, labeled $A_H, Z_H, W^\pm_H$. The mass diagonalization can be described in terms of the mixing angles:

$$c = \cos \theta = \frac{g_1}{\sqrt{g_1^2 + g_2^2}} = \sqrt{1 - s^2}$$

and

$$c' = \cos \theta' = \frac{g'_1}{\sqrt{g'_1^2 + g'_2^2}} = \sqrt{1 - s'^2}. \tag{3}$$

The allowed range of mixing angles is bounded by the perturbative limits

$$g_1, g_2, g'_1, g'_2 < 4\pi. \tag{4}$$

With the $SU(5)$ represented in this way, quadratic divergences of Higgs mass due to Standard Model gauge boson loops are cancelled by opposite-sign contributions from the new massive gauge bosons, leaving a logarithmic leading contribution term.

\(^2\) A variant of the Littlest Higgs model omits the gauging of one of the $U(1)$ groups, which is acceptable on the grounds that the contribution to quadratic divergence by the photon loop is numerically small. This variant is increasingly favored in the literature, since direct search limits would be greatly eased, along with constraints from electroweak precision data, at the cost of an additional ad hoc assumption. The ungauged $U(1)$ group is then expressed as an additional scalar with interesting phenomenology \[13\].
TABLE II: Free parameters of the Littlest Higgs model [11], with typical values used in this study. The parameters $c$ and $c'$ are bounded by perturbative limits.

To address the largest contribution to the quadratic divergence, that of the top quark loop, a new pair of massive top-like fermions $\tilde{t}$, $\tilde{t}^C$ is introduced. A row vector with $\tilde{t}$ is constructed as $\chi = (b_3 t_3 \tilde{t})$, where $(t_3 b_3)$ is the third-generation quark weak doublet. The coupling of $\chi$ to the Higgs boson is mediated by the Yukawa terms in the Lagrangian:

\[
\frac{1}{2} \lambda_1 f \epsilon_{ijk} \epsilon_{xy} \chi_i \Sigma_{jx} \Sigma_{ky} t_3^C \chi + \lambda_2 f \tilde{t}^C + \text{higher corrections} \tag{5}
\]

where $i, j, k$ are summed over 1, 2, 3 and $x, y$ are summed over 4, 5, so as to pick the Higgs doublet out of $\Sigma(x)$. The second term sets the overall mass scale of the new fermions to $f$. After mass diagonalization in this sector, the Standard Model top and bottom quarks emerge along with new massive top partners, which cancel the leading quadratic divergences.

Because the Standard Model top quark mass is known, the parameters $\lambda_1$ and $\lambda_2$ are constrained. The constraint works out to be:

\[
\frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2} \approx \left( \frac{v}{m_t} \right)^2 . \tag{6}
\]

Unless otherwise indicated, $\lambda_1$ is set to 1, with $\lambda_2$ set as required to produce the top mass $m_t = 175$ GeV.

The requirement that the Yukawa terms be gauge-invariant restricts the $U(1)_1$ and $U(1)_2$ “hypercharge” assignments of the fermions, such that the sum for each fermion equals the Standard Model hypercharge [11]. The remaining freedom of assignment is limited to one parameter for the quarks, $y_u$, and one for the leptons, $y_e$. This freedom is eliminated if we require that all anomalies are cancelled in the theory, resulting in the fixed values $y_u = -\frac{2}{3}$ and $y_e = \frac{2}{3}$. For the top quark, these assignments are fixed by the Yukawa terms above [9], but in general anomaly cancellation is not strictly required since this theory is explicitly not complete, and additional anomaly-cancelling
contributions could arise from unidentified higher-energy contributions. The result is a proliferation of more or less unmotivated assignment choices. Motivation for specific choices can be found in the various high-energy completions of the theory, in which all anomalies should cancel. In this work the anomaly-cancelling choices will be made in all cases for third-generation fermions, and for all fermions unless noted.

The free parameters of the Littlest Higgs model and typical values used in this study are summarized in Table II.

III. DIRECT SEARCH LIMITS

To determine the direct-search exclusion limits for the Littlest Higgs model, as a function of the free parameters of the theory, it is useful to begin by identifying the lightest new particle, since lighter particles are generally easier to produce and therefore to observe, provided that they do not have suppressed couplings to Standard Model particles. As has been widely noted in the literature, the $A_H$ is the lightest new particle for most parameter choices. A detailed scan of the $f, c, c'$ parameter space shows that the $A_H$ is always the lightest particle for $c' > 0.12$. For $c' < 0.12$, there are parameter choices for which the $\Phi$ and for which the degenerate $W_H, Z_H$ are the lightest new particles. The direct search limits on the Littlest Higgs model will be found from applicable
searches for new massive particles. Since all the new particles of the Littlest Higgs model have masses proportional to \( f \) in leading order, I will use \( f \) as a convenient common measure of the lower exclusion limits on the model.

The mass of the \( A_H \) is displayed in Figure 1 as a function of \( c' \) for several values of \( f \). It scales with \( f \), and is strongly dependent on \( c' \). The current Tevatron run II search reach for narrow resonances similar to \( A_H \) is about 900 GeV [14], so current direct search data will be able to rule out much of parameter space at \( f = 3 \) TeV and below, but less parameter space above \( f = 3 \) TeV.

Since the \( A_H \) is in most of parameter space the lightest new particle, only its decays to Standard Model particles will be considered. For \( c' < 0.12 \), decays to other new Littlest Higgs model particles, such as \( A_H \rightarrow W_H + W_L \), are possible for some parameter choices and would need to be included in a more detailed study of this portion of parameter space.

The \( A_H \) can decay to Standard Model fermion pairs with partial width:

\[
\Gamma(A_H \rightarrow f\bar{f}) = \frac{C M_A}{12\pi} (1 - 4 \frac{M_f^2}{M_A^2})^{\frac{3}{2}} (g_v^2 (1 + 2 \frac{M_f^2}{M_A^2}) + g_a^2 (1 - 4 \frac{M_f^2}{M_A^2}))
\]

where \( C \) is the appropriate color factor for the fermions, \( M_A \) is the \( A_H \) mass, \( M_f \) is the fermion mass, and \( g_v \) and \( g_a \) are the appropriate vector and axial couplings of the \( A_H \) to the fermions, as catalogued in [11].

The decay width of a particle affects how and to what extent it is experimentally detectable, since production of particles with very large decay widths may be difficult to distinguish from background processes. In the case of a supposed new particle, it is useful to characterize the decay width as a function of the unknown new particle mass. For the \( A_H \), the decays to fermion pairs are to leading order manifestly linear in \( A_H \) mass.

The \( A_H \) can also decay to Standard Model \( W \) bosons with partial width:

\[
\Gamma(A_H \rightarrow W^+W^-) = g_{WWA}^2 \frac{1}{192\pi} M_A \left( \frac{M_A}{M_W} \right)^4 \left( 1 - 4 \frac{M_W^2}{M_A^2} \right)^{\frac{3}{2}} \left( 1 + 20 \frac{M_W^2}{M_A^2} + 12 \frac{M_W^4}{M_A^4} \right)
\]

where \( g_{WWA} \) is the triple boson vertex factor and \( M_W \) is the mass of the Standard Model \( W \).

Note that the mass of the \( A_H \) is proportional to \( f \), so linearity in \( f \) means linearity in \( M_A \). The \( A_H W^+W^- \) coupling constant is:

\[
g_{AWW} = \frac{5 e c_w v^2}{2 s_w^2 f^2 s' c' (c'^2 - s'^2)}
\]
FIG. 2: The ratio of the total decay width of the $A_H$ to its mass in the Littlest Higgs model, calculated for $f = 3000$ GeV and $c = 0.1$. The ratio is only very weakly dependent on $f$ and $c$, as long as the $A_H$ is above the top pair production threshold, because the decay width is linear in the mass. Only decays to Standard Model particles are included, so the curve represents a lower limit for $c' < 0.12$. Higher-order terms in the Sigma field expansion are not included and may significantly modify the large width regions.

so decays to $W_L$ pairs are suppressed by four powers of $f$ in the coupling constant squared, which offset four of the five powers of $f$ in the partial width (from the powers of $M_A$), so the partial width to $W$s is also linear in $M_A$.  

Decays are also possible to a $Z^0$ boson and Higgs boson:

$$\Gamma(A_H \to Z^0 H) = \frac{g^2 \cot^2(2\theta')}{{96\pi}} \left( \frac{M_Z^2}{M_A^2} + \left( 1 + \frac{M_Z^2}{M_A^2} - \frac{M_H^2}{M_A^2} \right)^2 \right)$$

(10)

where $|\vec{P}|$ is the momentum of one of the outgoing decay products:

$$|\vec{P}| = \frac{M_A}{2} \sqrt{1 + \frac{M_Z^4}{M_A^4} + \frac{M_H^4}{M_A^4} - 2\frac{M_Z^2}{M_A^2} - 2\frac{M_H^2}{M_A^2} - 2\frac{M_Z^2 M_H^2}{M_A^4}}.$$  

(11)

In a previous work [15], the possibility of decays to $W$ pairs was neglected due to an apparent belief that the highly suppressed coupling would make the partial width negligible. That is in fact not the case, as is shown here: Kinematic factors (the polarization sums) in the amplitude restore the partial width to linearity in $M_A$. That work also has a typographically incorrect expression for their kinematic factor $\lambda$. 

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FIG. 3: Some branching ratios for the decays of the Littlest Higgs $A_H$, for various choices of the free parameters $(y_u, y_e, \lambda_1)$. The $l^+l^-$ branching ratio is for each of $e^+e^-, \mu^+\mu^-, \tau^+\tau^-$. The choice of $\lambda_1 = 0$ will not correctly recreate the top quark mass; this choice was included to allow comparison to a previous study.

In the large $M_A$ limit ($|\vec{P}| \to \frac{M_A}{2}$), decays to $Z^0H$ are also linear in $M_A$. The dominant decays are therefore all linear in $M_A$ and so the total width is as well. Note that in the limit of massless decay products, the partial widths to $W^+W^-$ pairs and to $Z^0H$ are equal, as they must be in accordance with the Goldstone boson equivalence theorem. However, at sufficiently low $A_H$ masses, where the decay product masses are not negligible, the partial widths can be quite different.

There would also be, for sufficient $A_H$ mass, the quartic decays $A_H \to Z^0HH$, $A_H \to \gamma W^+W^-$, and $A_H \to Z^0W^+W^-$. These decays are all suppressed by a four powers of the coupling constants, and so would likely be rare. They are not considered further here.

The resulting total width-to-mass ratio of the $A_H$ is presented in Figure 2. Depending on the value of the mixing angle $c'$, the $A_H$ can range from very wide, with a total width in excess of the mass, to extraordinarily narrow. For example, at $c' = 0.1$, the total width is 568 GeV at a mass of 2425 GeV, but for $c' = 0.5$, the total width is only 1.0 GeV at a mass of 556 GeV. In general, the very large widths occur for large ($c' \gtrsim 0.9$) and small ($c' \lesssim 0.15$) cosines of the $\theta'$ mixing angle where, as we have seen, the $A_H$ mass becomes large. It may be that higher-order terms in the Sigma field would significantly modify the width in the large-width regions; this possibility is not
explored here. For very large widths, the Breit-Wigner propagator would need a fully momentum-
dependent width, but that is not done here since the large $A_H$ masses where this would occur
are beyond current search limits in any case. The region of very narrow width ($0.5 \lesssim c' \lesssim 0.7$)
ocurs because in this region, the coupling to fermions, and so the largest contributions to the
total width, approach zero, as will be considered in the discussion of branching ratios below. The
essential point is that for almost all of the range of $c'$, $\Gamma_{Tot} \ll M_A$, so we can find exclusion limits
from generic searches for narrow resonances.

The branching ratios of the $A_H$ decay are shown in Figure 3 for four sets of choices of the
hypercharge and Yukawa parameters. For the $y_u = -\frac{2}{5}, y_e = \frac{3}{5}$ hypercharge assignments, which
cancel all Standard Model anomalies, the decays to dileptons $A_H \to e^+e^-$ and $A_H \to \mu^+\mu^-$ are
dominant for more than half of the range of $c'$. The dilepton decays are in fact large for all values of $c'$ and all four studied combinations of the parameters $y_u, y_e, \lambda_1$ except, in three cases, for a narrow
region around $c' = 0.63$. This “leptophobic” range of parameters is due to a special cancellation
of terms in the vector and axial couplings of the $A_H$ to the leptons [11]:

$$g_v(A_H \bar{l}l) = \frac{g'}{2s'c'}(2y_e - \frac{9}{5} + \frac{3}{2}c'^2),$$
$$g_a(A_H \bar{l}l) = \frac{g'}{2s'c'}(-\frac{1}{5} + \frac{1}{2}c'^2).$$

(12)

Inspection of the couplings to all Standard Model fermions (other than top) shows that they, too,
vanish at this special angle, as they must in order to maintain anomaly cancellation. To constrain
this region we can consider decays to top pairs, or to $W$ bosons. Couplings to top pairs do not
vanish at this same angle because of the presence of an additional coupling term originating in the
mixing of the top quark with the new fermion $\tilde{t}$:

$$g_v(A_H \bar{t}t) = \frac{g'}{2s'c'}(2y_u + \frac{17}{15} - \frac{5}{6}c'^2 - \frac{1}{2}c'^2 - \frac{1}{5}\lambda_1^2 + \lambda_2^2),$$
$$g_a(A_H \bar{t}t) = \frac{g'}{2s'c'}(-\frac{1}{5} - \frac{1}{2}c'^2 - \frac{1}{5}\lambda_1^2 + \lambda_2^2).$$

(13)

The most recent dilepton resonance results are from CDF [16], in which non-model specific
searches using 450 pb$^{-1}$ of integrated luminosity from Run II of the Tevatron are presented in
terms of the generic $U(1)$ gauge boson families of Carena, Daleo, Dobrescu, and Tait (CDDT) [17].
This latest experimental result, as it is presented, is somewhat less useful in constraining the
Littlest Higgs model than prior studies, which presented generic cross-section limits. One reason
for this is that the CDDT framework defines its gauge boson families so as to achieve full anomaly
cancellation, which does not occur for effective theories with incomplete anomaly cancellation, as can be the case here. For the special case of complete anomaly cancellation, we can map the vector and axial couplings of the $A_H$ to the chiral couplings of CDDT by subtracting and adding, and find that the $A_H$ is a member of the CDDT family $q + xu$, with $x = 4$. For the couplings to $u\bar{u}$, for example, we find the left-handed coupling to be:

$$g_z q_L = g_v - g_a = \frac{g'}{2s'c'} (2y_u + \frac{14}{15} - \frac{1}{3} c'^2) = \frac{g'}{2s'c'} (\frac{2}{15} - \frac{1}{3} c'^2)$$

(14)

which, for $q + 4u$, is

$$= \frac{1}{3} g_z$$

(15)

so

$$g_z = \frac{3g'}{2s'c'} (\frac{2}{15} - \frac{1}{3} c'^2).$$

(16)

 Likewise, for the right-handed coupling:

$$g_z u_R = g_v + g_a = \frac{g'}{2s'c'} (2y_u + \frac{4}{3} - \frac{4}{3} c'^2) = \frac{4}{3} g_z.$$  

(17)

The remaining $A_H$ couplings are consistent with this identification. Note that the identification with any CDDT family will fail unless we make the anomaly-cancelling choices $y_u = -\frac{2}{5}$, $y_e = \frac{3}{5}$,
FIG. 5: The direct search limits on $f$ for the Littlest Higgs Model, with $\cot \theta = 0.3$ on the left, and $\cot \theta = 1.0$ on the right, for the anomaly-cancelling choice of free parameters $y_u = -\frac{2}{5}$, $y_e = \frac{3}{5}$. Regions below the points and curves are excluded. The six points are limits from a search specific to the CDDT framework [16]. The $Z_H$ curves are developed from search data also presented in [16]. The $A_H$ curves are developed from data presented in [14].

and that the other new neutral gauge boson in the Littlest Higgs model, the $Z_H$, does not conform to any CDDT family, because it is not a $U(1)$ gauge boson. In these cases constraints could be found from the limits on generic $c_u$ and $c_d$ couplings, as also defined by CDDT [17].

Numerical values of $|g_z|$ are presented in Figure 4. The absolute value is shown since there is an inconsequential sign change in the expression for $g_z$ at $c' = 0.63$. CDF presents results for three discrete values of the coupling: $g_z = 0.03, 0.05,$ and $0.10$. It is therefore possible to constrain the Littlest Higgs model at a discrete set of six values of $c'$. Those limits are presented in Figure 5 as a set of six values of $f$, one for each available value of $c'$. These limits range as high as $f = 4700$ GeV. From these six points we find a suggestion of the limits for the full range of $c'$. A presentation by CDF of limits at a larger and denser set of $g_z$ values would permit a more detailed limit on $f$, for the anomaly-cancelling case.

A fuller picture can be formed by using generic search results for resonances in dileptons, as done in a previous study using Tevatron Run I data and a fixed excluded event rate for all $A_H$ masses [5]. The most recent data set is from CDF [14] for 200 pb$^{-1}$ of integrated luminosity from Run II of the Tevatron. Generic resonance searches for particles of spin 0,1, and 2, with masses
FIG. 6: The cross section $\sigma(p\bar{p} \to A_H \to ll)$ in the Littlest Higgs model, for several choices of the parameters $(y_u, y_e, \lambda_1)$, and symmetry-breaking scale $f = 3, 4,$ and $5$ TeV. The small jags in some curves are artifacts of the Monte Carlo calculation.

from 150 to 900 GeV, are presented as 95% confidence limits on $\sigma(p\bar{p} \to X) \cdot BR(X \to ll)$. Observed limits for spin 1 particles range from 21 fb to 490 fb.

General limits on the Littlest Higgs model can be found by calculating the cross section $\sigma(p\bar{p} \to A_H \to ll)$ and comparing to the search limits on narrow spin-1 resonances for various values of $f$, and so various $M_A$. The cross-sections themselves for a few values of $f$ and other free parameters are presented in Figure 6. These cross-sections are calculated at Tevatron Run II energy $\sqrt{s} = 1.96$ TeV, at leading order with a K-factor correction:

$$K = 1 + \frac{4}{3} \frac{\alpha_s(Q^2)}{2\pi} \left(1 + \frac{4}{3} \pi^2 \right)$$

applied as a function of $Q^2$ over the resonance, and using the CTEQ6L parton distribution functions \[18\]. The cross-section is relatively large around $c' = 0.4$, and $c' = 0.9$, so we will find the most stringent limits near these values. The cross-section becomes vanishingly small for large and small $c'$ because the $A_H$ mass becomes too large for significant production at Tevatron energies. Around $c' = 0.63$, the cross-section vanishes due to decoupling from leptons and quarks.

The results of comparing the computed cross-sections to the CDF search limits are presented in Figure 6. The overall symmetry-breaking scale $f$ is found to be excluded up to as much as 4.5 TeV, in the anomaly-cancelling case, for some values of $c'$. For other values of $c'$, the model is
FIG. 7: The cross section $\sigma(p\bar{p} \rightarrow A_H \rightarrow t\bar{t})$ in the Littlest Higgs model, for several choices of the parameters $(y_u, y_e, \lambda_1)$. The small jags in some curves are artifacts of the Monte Carlo calculation.

not constrained, due to either decoupling or to a large $A_H$ mass. The calculated exclusion curves are somewhat below the exclusion points provided by CDF’s CDDT-specific analysis, as would be expected since the calculated curves are based on less than half of the integrated luminosity used to generate the CDDT-specific points. This relationship provides a cross-check on the independent Monte Carlo simulations of the $A_H$ cross section described here with that done by CDF. It is not possible to seek to constrain the Littlest Higgs model much further in the high mass regions of parameter space with Tevatron data.

To attempt to constrain the Littlest Higgs model in the decoupling range around $c' = 0.63$, we can look to the current limits on resonances in top pair production by DØ and CDF [19]. The DØ results, for 680 pb$^{-1}$ of integrated luminosity of Run II data, are the most useful for this purpose since generic narrow resonance search limits are presented. The DØ study assumes a neutral boson width $\Gamma = 0.012M$, with 95% confidence limit exclusions ranging from 5.08 pb at a mass of 350 GeV to a minimum of 0.88 pb at 650 GeV, and then increasing to 1.43 pb at 1000 GeV. These limits are considerably larger than the equivalent results for leptons, so we cannot expect as strong a constraint overall. The comparable cross section $\sigma(p\bar{p} \rightarrow A_H \rightarrow t\bar{t})$ is calculated as previously for the cross-section to leptons, and is presented in Figure 7. For the anomaly-cancelling case, the calculated cross-section is at most 0.54 pb, so unfortunately the current experimental limits on
resonances in top pair production do not quite provide a constraint. For other choices of the free parameters the top pair resonance limits do provide a constraint for a limited range of $c'$.

The remaining hope for constraining the decoupling range from $A_H$ decays falls to the $W_L^+W_L^-$ decay mode. No experimental results on resonances in $W$ pair production are available, so we must turn to the total $W_L^+W_L^-$ production cross section. The most current result in provided by CDF [20], with an observed cross-section of $13.6 \pm 2.3$(stat) $\pm 1.6$(sys) $\pm 1.2$(lum) pb. Since the entire cross-section of $A_H$ production at the Tevatron is at most 4 pb [11], the current limits on $W$ pair production are in fact not able to provide a constraint for the Littlest Higgs model.

Having exhausted the likely signals for the $A_H$, we turn to one of the next-lightest particles, the $Z_H$. CDF has presented a set of model-specific search limits for the Littlest Higgs $Z_H$ for a set of values of cot $\theta$ [16]. These limits can be converted to limits on the scale $f$, and are displayed in Figure 5. The $Z_H$ results provide an exclusion limit for all values of $c'$, filling in the $A_H$ decoupling and high mass regions, although at a much lower exclusion limit.\footnote{To the extent that an incorrect expression for the $Z_H$ mass has been used in these CDF limits, they may require reconsideration. My own studies of the effect of correcting the $Z_H$ mass expression suggest that any changes of limits due to this specific correction are likely to be small.}

**IV. CONCLUSIONS**

In this work I have made a fairly thorough study of the direct limits on the Littlest Higgs model provided by current searches for new particles, for a range of free parameters, although primarily focused on the anomaly-canceling case. Recent experimental results permit some constraint in all cases. For a limited range of parameters, lower limits on the symmetry-breaking scale can range as high as 4.7 TeV.

The possibility of detection of new particles, and the best means to do so, depends critically on the width of the associated resonance. Wide resonances can be difficult to detect, and so determining the width behavior is important. The survey of dominant Standard Model decays of the $A_H$ presented here, with all final masses included, has found that all dominant contributions have partial widths proportional to the $A_H$ mass. Large widths do occur for large ($c' \gtrsim 0.9$) and small ($c' \lesssim 0.15$) cosines of the $U(1)_1 \otimes U(1)_2$ mixing angle $c'$, and in these regions contributions from non-Standard Model decays, and higher-order contributions from the $\Sigma(x)$ field may become important.

As can be seen in Figures 5 and 6, a low-mass $A_H$ neutral gauge boson can be present and yet
undetectable in dilepton studies for a significant range of parameter space, due to decoupling from Standard Model leptons. The $A_H$ provides a striking counterexample to broad claims that a new neutral gauge boson (sometimes generically referred to as a $Z'$) is ruled out to a high mass scale.

Some observations can be made on recent presentations of experimental results. Searches for individual model-specific new particles are very useful where applicable. However, in a model such as the Littlest Higgs, with a number of free parameters, carefully described searches for generic narrow resonances, as is done in [14], are also useful. Classifications of new particles such as the CDDT framework can play a useful role, too, although this study provides examples of the limitations of such a system, since it is not meant to comprehensively covers the full range of possible new neutral gauge bosons. In the Littlest Higgs model, the $A_H$ boson in the non-anomaly cancelling case, and the $Z_H$ boson, cannot be placed in the CDDT framework. Limits can be found from constraints on the generic couplings $c_u$ and $c_d$ in all these cases, however.

In this study most of the meaningful constraints were obtained from dilepton resonance studies. As the Tevatron integrated luminosity builds over the next few years, we can look forward to dilepton results providing increasingly strong constraints or, in a more exciting possibility, beginning to characterize an actual high mass resonance.

Top quark resonances do not yet provide a constraint in most cases, but, for the anomaly-cancelling case, the current search limits are less than two times greater than the maximum calculated cross-section. It appears that at higher integrated luminosity, the search for resonances in top pair production at the Tevatron may well be able to provide a constraint in this case, so further studies in this area would be helpful. If feasible, searches for resonances in $W$ pair production would also be quite useful.

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