Phase Diagram of (Proto)Neutron Star Matter in an Extended Bag Model

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Abstract. We illustrate the consequence of the simultaneous restoration of chiral symmetry and deconfinement at finite temperature under neutron star conditions in an extended bag model, vBag, which we developed in earlier work.

1. Introduction
Currently, no consistent approach exists to describe medium properties of hadrons and mesons at the level of quarks and gluons at high density. Hence, the deconfinement phase transition is usually constructed from a given hadronic equation of state (EoS) with hadrons and mesons as the fundamental degrees of freedom and an independently computed quark matter EoS, although few attempts exist to improve this situation (c.f. Refs. [1, 2]). A general review of recent developments concerning the EoS in astrophysical applications can be found in Ref. [3].

The novel quark matter EoS vBag was introduced recently [4] with particular focus on applications in astrophysics; vBag explicitly accounts for dynamical chiral symmetry breaking (DχSB) and repulsive vector interactions. The latter property is essential for the stiffening of the EoS towards high densities in order to yield maximum neutron star masses in agreement with the current constraint of about 2 M⊙ [5, 6] (the previously quoted value in Ref. [6] was recently revised to 1.928 ± 0.017 M⊙ [7]). In order to ensure chiral symmetry restoration and (de)confinement to coincide, which is usually not provided for the ‘standard’ Maxwell or Gibbs approach for the phase transition, vBag mimics (de)confinement via a phenomenological correction to the EoS. It is determined by the hadronic EoS chosen for the construction of the phase transition.

More recently, vbag was extended to finite temperatures and arbitrary isospin asymmetry to study consequences for the phase diagram [8]. In the following we review the basic foundation of vBag and present the phase diagram of matter under (proto)neutron star conditions.

2. vBag - a Bag Model Extension
We briefly sketch the derivation of vBag [4] as a particular approximation of QCD’s in-medium Dyson-Schwinger equations (DSE), which initially contains the standard Nambu–Jona-Lasinio (NJL) model (c.f. [9, 10, 11]) as a particular limiting case. When chiral symmetry is restored
it behaves in analogy to the thermodynamic bag model (tdBag). However, vBag accounts for vector repulsion in which aspect it extends beyond the tdBag model.

In general, the in-medium, dressed-quark propagator maintains the structure of a free, relativistic Fermion propagator,

\[ S(p^2, \tilde{p}_4)^{-1} = i\gamma^\nu \tilde{p}_\nu A(p^2, \tilde{p}_4) + i\gamma_4 \tilde{p}_4 C(p^2, \tilde{p}_4) + B(p^2, \tilde{p}_4), \]

with \( \tilde{p}_4 = p_4 + i\mu \). Evidently, the gap functions \( A, B, \) and \( C \) account for non-ideal behaviour due to interactions. Unlike for vacuum studies, the gaps are complex valued and divergences only. E.g., infra-red (IR) cutoff schemes can remove unphysical implications [14].

In order to obtain the propagator one solves the following gap equation,

\[ S(p^2, \tilde{p}_4)^{-1} = i\gamma^\nu \tilde{p}_\nu + i\gamma_4 \tilde{p}_4 + m + \Sigma(p^2, \tilde{p}_4), \]

\[ \Sigma(p^2, \tilde{p}_4) = \int \frac{d^4q}{(2\pi)^4} g^2(\mu) D_{\rho\sigma}(q-p, \mu) \frac{\lambda^\sigma}{2} \gamma_\rho S(q^2, \tilde{q}_4) \Gamma_\sigma(q, p, \mu), \]

where \( m \) is the bare mass, \( D_{\rho\sigma}(k, \mu) \) is the dressed-gluon propagator and \( \Gamma_\sigma(q, p, \mu) \) is the dressed-quark-gluon vertex. Naturally, at the level of the self energy \( \Sigma(p^2, \tilde{p}_4) \) approximations can be made in order to simplify the gap equations. In all following discussions we impose:

\[ \Gamma_\sigma(q, p) = \frac{1}{2} \lambda^\nu \gamma_\sigma, \]

for the vertex and thus define a rainbow gap equation, which is the leading-order in a systematic, symmetry-preserving DSE truncation scheme [12, 13].

The NJL model is understood as the result of a contact interaction in configuration space provided by the gluon propagator. Transformed into momentum space this reads as a constant, momentum-independent coupling. The model is ultraviolet-divergent if no regularization is performed. Here, we perform a hard cut-off in the ultra-violet (UV) and express the effective gluon propagator as follows,

\[ g^2 D^{\rho\sigma}(p - q) = \frac{1}{m_G^2} \Theta(\Lambda^2 - q^2) \delta^{\rho\sigma}. \]

The Heaviside function \( \Theta \) provides a 3-momentum cutoff for space-like momenta \( p^2 > \Lambda^2 \). This is sufficient to regularize all UV divergences inherent to \( \Sigma(p^2, \tilde{p}_4) \). Different regularization procedures are available and in fact the regularization scheme does not have to affect UV divergences only. E.g., infra-red (IR) cutoff schemes can remove unphysical implications [14]. However, the chosen hard cut-off scheme reproduces standard NJL model results and allows to match them to tdBAG, i.e. to describe quarks as quasi ideal gas of Fermions. \( m_G \) is a gluon mass scale which in this model simply defines the coupling strength. These approximations are sufficient to write the gap equations. For the A-gap follows the trivial, medium independent solution, \( A = 1 \). The remaining gap equations take the following form,

\[ B_p = m + \frac{16N_c}{9m_G^2} \int \frac{d^4q}{(2\pi)^4} \frac{B_q}{q^2 A_q^2 + \tilde{q}_4^2 C_q^2 + B_q^2}, \]

\[ \tilde{p}_4^2 C_p = \tilde{p}_4^2 + \frac{8N_c}{9m_G^2} \int \frac{d^4q}{(2\pi)^4} \frac{\tilde{p}_4 \tilde{q}_4 C_q}{q^2 A_q^2 + \tilde{q}_4^2 C_q^2 + B_q^2}. \]

The integrals do not explicitly depend on the external momentum \( p \) and consequently, both gap solutions are constant at any given \( \mu \). Both equations can be recast as follows,

\[ B = m + \frac{4N_c}{9m_G^2} n_4(\mu^*, B), \]

\[ \mu = \mu^* + \frac{2N_c}{9m_G^2} n_4(\mu^*, B), \]
in terms of the single-flavor scalar and vector densities, $n_s$ and $n_v$, of an ideal spin-degenerate Fermi gas,

\begin{align}
  n_s &= 2 \sum_{\pm} \int_{\Lambda} \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{B \left( \frac{1}{2} - \frac{1}{1 + \exp(E^\pm / T)} \right)}{1 + \exp(E^\pm / T)}, \\
  n_v &= 2 \sum_{\pm} \int_{\Lambda} \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{\mp 1}{1 + \exp(E^\pm / T)},
\end{align}

(9)

with $E^2 = \mathbf{p}^2 + B^2$ and $E^\pm = E \pm \mu^*$. The merit of the NJL model is the ability to describe chiral symmetry breaking as the formation of a scalar condensate, and chiral symmetry restoration as the melting of the same. It should be kept in mind though, that it is the scalar density which requires UV regularization and in that sense chiral symmetry breaking can be considered as the most sensitive part of the model.

The next information that Eqs. (9) and (10) provide is well known; the NJL model describes quarks as quasi-ideal particles with corresponding quasi particle poles. However, confinement is not accounted for. In Ref. [4] it has been pointed out how this can be understood as a reason for the fact that NJL models typically provide a larger bag constant than, e.g., the MIT-bag model. The NJL model does not 'bind'. Adding binding energy per volume can lead to interesting consequences for the phase diagram, as illustrated in Ref. [8]. It will be further discussed in the following.

### 3. Phase Diagram

In order to reproduce the behavior of the NJL model in the chirally restored phase in good approximation we subtract a chiral bag constant $B_{X,f}$ from the pressure for each flavor independently,

\begin{equation}
  P_f(\mu_f) = P_{\text{kin}}^{\text{FG},f}(\mu_f^*) + \frac{K_v}{2} n_{\text{FG},f}^{2}(\mu_f^*) - B_{X,f}.
\end{equation}

(11)

The second term corresponds to the vector condensate, with $K_v$ being related to the vector current-current interaction coupling constant, which in combination with the renormalisation of the chemical potential causes an effective stiffening of the EoS with increasing density. This term is not accounted for in the standard tdBag model. By construction, chiral symmetry is restored for $P_f(\mu_f) > 0$. The critical chiral chemical potential $\mu_f^*$ for a single flavor is then defined by $P_f(\mu_f^*) = 0$. For the two-flavor case we redefine the chiral threshold condition to hold for the total light quark pressure instead for each single flavor independently,

\begin{equation}
  \sum_{f=u,d} P_f(T, \mu_{X,f}) = 0.
\end{equation}

(12)

This procedure aims to avoid sequential deconfinement of different quark flavors. We want our model to account for chiral symmetry breaking and the confinement transition simultaneously at $\mu_{B,X}$. To do this the critical chemical potential for deconfinement, $\mu_{B,dc}$, has to match $\mu_{B,X}$. This is achieved by exploiting that the total pressure of the system is fixed by the previous equations only up to a constant $B_{dc}$,

\begin{equation}
  P^Q = \sum_f P_f(T, \mu_f) + B_{dc}.
\end{equation}

(13)

Introducing $B_{dc}$ thus enables us to ensure that the pressure of hadronic and quark matter equals at the critical chemical potential $\mu_{B,X}$ for the chiral transition. In order to match both pressures
one easily finds the value of $B_{dc} = P^H(\mu_{B,\chi})$ at given $T$ and $\mu_C$. Note, the baryon chemical potential and the charge chemical potential are defined as follows:

$$\mu_B = \mu_u + 2\mu_d, \quad \mu_C = \mu_u - \mu_d.$$  

(14, 15)

At given $T$ and $\mu_C$ we denote our critical chemical potentials for $D\chi$SB and confinement as $\mu_{B,\chi}$ and $\mu_{B,dc}$ respectively. As shown in detail in Ref. [8], $B_{dc}$ depends on the nuclear EoS and thus becomes medium dependent. According to our prescription for the phase transition it varies with temperature and isospin asymmetry, i.e. $B_{dc} \to B_{dc}(T, \mu_C)$, while we keep it constant with respect to $\mu_B$. This results in additional contributions to the charge number and entropy density which are derivative terms with respect to the pressure. We will not discuss any of these quantities in the following and rather focus on the effect of simultaneous $D\chi$SB and deconfinement on the phase diagram of matter under (proto)neutron star conditions, viz. $\beta$-equilibrium, electric charge neutrality and finite temperature.

The yellow region in Fig. 1 corresponds to the result obtained for a model without an additional bag constant which accounts for confinement effects. At $T = 0$ the coexistence region between $\rho_0$ (onset density of the phase transition) and $\rho_\chi$ (onset density of pure quark matter) reproduces exactly NJL model results. However, we treat $B_\chi$ as a constant and hence the agreement will be less accurate with increasing temperature as we neglect the melting of the scalar condensate.

As typical for NJL type models, the transition occurs at rather high densities. Diquark condensates can lower this density [18]. Under the assumption of simultaneous $D\chi$SB and deconfinement the onset density of quark matter is solely defined by the chiral restoration and, for vBag, the chiral bag constant. The corresponding density follows the right side edge of the blue region in Fig 1. Without the aforementioned assumption this line would change for a different nuclear EoS. In our approach, this is not the case. The nuclear EoS alters the onset of the coexistence region only. We further note, that the transition density in our approach is significantly smaller than for the tdBag model with a bag constant that is adjusted to reproduce NJL model results (yellow region). This behavior of vBag favors the existence of quark matter in the cores of neutron stars.

In Ref. [4] it has been further shown that due to the incorporated vector interaction stable hybrid neutron star configuration of at least 2 $M_\odot$ can be obtained. With vBag we provide a
novel quark-matter EoS for a broad spectrum of applications in astrophysics, e.g., simulations of neutron star mergers [19], core collapse supernovae [20, 21, 22] and protoneutron star cooling [23], where commonly the simple tdBag-model has been employed. However, the classical and in astrophysics studies commonly employed tdBag model does not account for chiral physics and without perturbative corrections violates the $2 M_{\odot}$ neutron star mass constraint due to missing repulsive interactions. Here, vBag provides solutions that cure both problems without losing the simplicity and flexibility of tdBag.

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