Composite Higgs models based on the chiral symmetries of “theory space” can produce Higgs bosons with masses of order 100 GeV from underlying strong dynamics at scales up to 10 TeV without fine tuning. This talk argues that flavor-violating interactions generically arising from underlying flavor dynamics constrain the Higgs compositeness scale to be $\sim 75$ GeV, implying that significant fine-tuning is required. Bounds from CP violation and weak isospin violation are also discussed.

1. Introduction

The Standard Higgs Model employs a fundamental scalar doublet to break the electroweak symmetry and provide fermion masses. Well-known difficulties, including the hierarchy problem and the triviality problem, imply that the Standard Higgs Model is just a low-energy effective theory.

Suppose that the Higgs field $\phi$ is actually a composite state arising from underlying strong dynamics at a higher energy scale, $\Lambda$. We can estimate the sizes of operators involving $\phi$ in the low-energy effective theory using dimensional analysis. A theory with light scalar particles in a single symmetry-group representation depends on two parameters: $\Lambda$, the scale of the underlying physics, and $f$, the analog of $f_\pi$ in QCD. Our estimates of the low-energy effects of the underlying physics will depend on $\kappa \equiv \Lambda/f$.

Regardless of the precise nature of the underlying strongly-interacting physics that produces $\phi$, there must be flavor dynamics at a scale $\tilde{\Lambda}$ that gives rise to the different Yukawa couplings of the Higgs boson to
ordinary fermions. If this flavor dynamics arises from gauge interactions it will generally cause flavor-changing neutral currents (as in ETC models). Similarly, there are likely to be couplings that violate CP and weak isospin. This talk reviews the constraints which FCNC, CP-violation, and weak-isospin violation place on composite Higgs models and applies the limits to models developed under the rubric of “theory space”.

2. Composite Higgs Phenomenology

2.1. Flavor

Quark Yukawa couplings arise from flavor physics coupling the left-handed doublets \( \psi_L \) and right-handed singlets \( q_R \) to the strongly-interacting constituents of the composite Higgs doublet. If these new flavor interactions are gauge interactions with gauge coupling \( g \) and gauge boson mass \( M \), dimensional analysis estimates the resulting Yukawa coupling is of order \( g^2 M^2 \). To produce a quark mass \( m_q \), the Yukawa coupling must equal \( \sqrt{2} m_q/v \) where \( v \approx 246 \text{ GeV} \). This implies

\[
\Lambda \gtrsim \frac{M}{g} \sqrt{2 \kappa m_q / v} .
\] (1)

If experiment sets a lower limit on \( M/g \), eqn. (1) gives a lower bound on \( \Lambda \).

Consider the interactions responsible for the c-quark mass. Through Cabibbo mixing, these interactions must couple to the u-quark as well:

\[
L_{\text{eff}} = - (\cos \theta_L^c \sin \theta_L^c)^2 \frac{g^2}{M^2} \overline{(\gamma^\mu u_L)} (\gamma^\mu \gamma^\nu u_L) - (\cos \theta_R^c \sin \theta_R^c)^2 \frac{g^2}{M^2} \overline{(\gamma^\mu u_R)} (\gamma^\mu \gamma^\nu u_R) - 2 \cos \theta_L^c \sin \theta_R^c \cos \theta_R^c \sin \theta_R^c \frac{g^2}{M^2} \overline{(\gamma^\mu u_L)} (\gamma^\mu \gamma^\nu u_R) ,
\] (2)

where \( g \) and \( M \) are of the same order as those which produce the c-quark Yukawa coupling, and \( \theta_L^c, \theta_R^c \) relate the gauge and mass eigenstates.

The color-singlet products of currents in eqn. (2) contribute to \( D \)-meson mixing. The left-handed or right-handed current-current operators yield

\[
\left( \frac{M}{g} \right)_{LL,RR} \gtrsim f_D \left( \frac{2 m_D B_D}{3 \Delta m_D} \right)^{1/2} \cos \theta_L^c \sin \theta_R^c \approx 225 \text{ TeV} ,
\] (3)
where $\Delta m_D \lesssim 4.6 \times 10^{-11}$ MeV $^{10}$, and $f_D \sqrt{B_D} = 0.2$ GeV $^{11}$, $\theta_{L,R} \approx \theta_C$. A bound$^5$ on the scale of the underlying dynamics follows from eqn. (1):

$$\Lambda \gtrsim 21 \text{TeV} \sqrt{\kappa \left( \frac{m_c}{1.5 \text{GeV}} \right)},$$

(4)

so that $\Lambda \gtrsim 74$ TeV for $\kappa \approx 4\pi$. The LR product of color-singlet currents gives a weaker bound than eqn. (4). The LR product of color-octet currents, $L_{eff} = -2 \cos \theta_c L \sin \theta_c L \cos \theta_c R \sin \theta_c R g^2 M_2 (\phi_L^\dagger D^\mu \phi_L) (\phi_R^\dagger D^\mu \phi_R)$, gives a stronger bound$^5$:

$$\Lambda \gtrsim 53 \text{TeV} \sqrt{\kappa \left( \frac{1.5 \text{GeV}}{m_c} \right)},$$

(6)

Analogous bounds on $\Lambda$ can be derived from neutral Kaon mixing. However, because $m_s \ll m_c$, while the $d-s$ and $u-c$ mixings are expected to be of comparable size, these bounds on $\Lambda$ are weaker than (4)$^9$.

2.2. Isospin

Weak-isospin violation is a key issue in composite Higgs models$^{9,13,14,15}$. The standard one-doublet Higgs model has an accidental custodial isospin symmetry$^{16}$, which implies $\rho \approx 1$. While all operators of dimension $\leq 4$ automatically respect custodial symmetry, terms of higher dimension that arise from the underlying physics at scale $\Lambda$ in general will not.

The leading custodial-symmetry violating operator

$$\frac{\kappa^2}{\Lambda^2} (\phi^\dagger D^\mu \phi)(\phi^\dagger D^\mu \phi)$$

(7)

gives rise to a contribution to the $\rho$ parameter$^{13}$

$$\Delta \rho = -O(\kappa^2 \frac{v^2}{\Lambda^2}).$$

(8)

The limit $|\Delta \rho| \lesssim 0.4\%$ implies $\Lambda \gtrsim 4 \text{TeV} \cdot \kappa$.

2.3. CP Violation

In the absence of additional superweak interactions to give rise to CP-violation in K-mixing ($\varepsilon$), the flavor interactions responsible for the $s$-quark Yukawa couplings must do so. This yields strong bounds on $\Lambda$. Recalling

$$\text{Re} \varepsilon \approx \frac{\text{Im} M_{12}}{2 \Delta M} \lesssim 1.65 \times 10^{-3},$$

(9)
and assuming that there are phases of order 1 in the $\Delta S = 2$ operators analogous to those shown in eqn. (2), we find

$$\Lambda \sim 120 \text{ TeV} \sqrt{\frac{m_s}{200 \text{ MeV}}}.$$  \hfill (10)

3. Composite Higgs Bosons from Theory Space

A set of “theory space” composite Higgs models can be represented as an $N \times N$ toroidal lattice of “sites” connected by “links”, using “moose” or “quiver” notation \textsuperscript{12}. Each site except (1, 1) represents a gauged $SU(3)$ group, while the links represent non-linear sigma fields transforming as $(N, \bar{N})$’s under the adjacent groups. At the site (1, 1), only the $SU(2) \times U(1)$ subgroup of an $SU(3)$ global symmetry is gauged. For simplicity, we will assume the gauge couplings of the $SU(3)$ gauge groups are the same for every site (except (1, 1)). Calling the “pion decay constant” of the chiral-symmetry-breaking dynamics $f$, dimensional analysis\textsuperscript{2} then implies that the scale $\Lambda$ of the underlying high-energy dynamics which gives rise to this theory is $\Lambda \lesssim 4\pi f$.

The $2N^2$ Goldstone bosons of the chiral symmetry breaking dynamics are incorporated into the sigma-model fields. As described in\textsuperscript{6,7}, $N^2 - 1$ sets of Goldstone bosons are eaten, $N^2 - 1$ get mass from “plaquette operators” which explicitly break the chiral symmetries, and two sets which are uniform in the ‘u’ or ‘v’ directions, along the lattice axes, remain massless in the very low-energy theory: Both the $\pi_u$ and $\pi_v$ fields contain $SU(2) \times U(1)$ doublet scalars $\phi_u$ and $\phi_v$ with the quantum numbers of the Higgs boson. A negative mass-squared for one or both Higgs bosons may be introduced either through a symmetry-breaking plaquette operator at the site (1, 1)\textsuperscript{6} or through the effect of coupling the Higgs bosons to the top-quark\textsuperscript{7}. In either case, the resulting mass-squared of the Higgs is $|m_h|^2 \sim \frac{\Lambda^2}{s}$. 

4. Constraints on Theory Space

4.1. Flavour and CP

Because the light quarks and leptons transform under the $SU(2) \times U(1)$ gauge interactions at a site in theory space\textsuperscript{6,7}, Yukawa couplings of these fermions to the composite Higgs bosons are generated. The FCNC and CP-violation limits derived in Section 2 therefore apply. Because the composite Higgs bosons are delocalized over the $N^2$ sites of theory space, the lower
bound on $\Lambda$ is a factor of $\sqrt{N}$ stronger. From D-meson mixing, we have

$$\Lambda \gtrsim 21 \text{ TeV} \sqrt{\kappa N \left( \frac{m_c}{1.5 \text{ GeV}} \right)},$$

so that $\Lambda \gtrsim \sqrt{N} \cdot 74 \text{ TeV}$ for $\kappa = 4\pi$. From CP-violation ($\epsilon$), we have

$$\Lambda \gtrsim 120 \text{ TeV} \sqrt{\kappa N \left( \frac{m_s}{200 \text{ MeV}} \right)},$$

meaning $\Lambda \gtrsim \sqrt{N} \cdot 425 \text{ TeV}$ for $\kappa = 4\pi$.

A significant advantage of theory space models is supposed to be their ability to produce a light Higgs without fine-tuning. We must check how compatible this is with the FCNC and CP-violation constraints above.

The most important corrections to the Higgs boson masses arise from the interactions added to give rise to the top-quark mass. The fermion loop Coleman-Weinberg contribution to the Higgs mass-squared is of order

$$|\delta m^2_H| \approx \frac{N_c y_t^2 M^2}{16\pi^2} \approx \frac{N_c y_t^2}{(16\pi^2)^2} \Lambda^2,$$

where $N_c = 3$ accounts for color. In this case, the absence of fine-tuning ($\delta m^2_H/m^2_H < 1$) implies

$$\Lambda \lesssim \frac{16\pi^2 \sqrt{\lambda v}}{N_c y_t^2 N} \approx \frac{22 \text{ TeV} \sqrt{\lambda}}{N}.$$

Comparing eqs. (14) and (11) we see that remaining consistent with the low-energy constraints makes fine-tuning inevitable for large $N$. Even for the smallest $N$, some fine-tuning will be required. For example, for $N = 2$ ($N = \sqrt{2}$) fine-tuning on the order of 1% (3%) is required by the bound on D-meson mixing. If the bound from CP violation (10) must also be satisfied, the fine-tuning required is of order 0.04% (0.09%).

5. Weak Isospin Violation

The kinetic energy terms for the light composite Higgses include isospin-violating interactions

$$L_{kin} \supset -\frac{1}{6N f^2} \left[ (\partial_{\mu} \phi_u^\dagger \phi_u)^2 - (\partial_{\mu} \phi_u^\dagger u \phi_u^\dagger \phi_u) (\phi_u^\dagger \partial^\mu \phi_u + \phi_u^\dagger \phi_u^\dagger \phi_u) \right] + u \leftrightarrow v.$$

The resulting contribution to the $\rho$ parameter is

$$\Delta \rho^* = \alpha \Delta T = \frac{\nu^2}{4N^2 f^2} \left( 1 - \frac{\sin^2 2\beta}{2} \right).$$
Current limits derived from precision electroweak observables require that $\Delta T \lesssim 0.5$ at 95% confidence level for a Higgs mass less than 500 GeV. The bound in eqn. 16 implies that

$$\Lambda \simeq 4\pi f \gtrsim \frac{25 \text{ TeV}}{N} \left(1 - \frac{\sin^2 2\beta}{2}\right)^{1/2}.$$  

(17)

Comparison with eqn. 14 shows that the underlying strong dynamics cannot be at energies $\ll 10$ TeV, even if the high-energy theory contains approximate flavor and CP symmetries that avoid the limits of eqns. (4, 10).

6. Discussion

Theory space models propose to provide a naturally light composite Higgs boson without relying on approximate symmetries of the high-energy underlying strong dynamics. This talk argues that the low-energy structure of composite Higgs models does not automatically make them invulnerable to constraints from FCNC, CP-violation, or weak-isospin violation. Assumptions about symmetries of the underlying dynamics are required (see, e.g., discussion in ref. 5).

For theory space models based on an $N \times N$ toroidal lattice, the lower limit from FCNC on the scale of strong dynamics is $\Lambda \geq 74 \text{ TeV}\sqrt{N}$, implying a minimum bound of 105 TeV. However, if fine-tuning of the higgs mass is to be avoided in such models, $\Lambda \leq 22 \text{ TeV}\sqrt{N}$/N; preventing FCNC then leads to fine-tuning at the level of $10/N^3\%$. The lower limits on $\Lambda$ from weak isospin violation are weaker than those from FCNC (but hard to avoid), while those from CP-violation can be much stronger.

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