Some Subordinating Results for Classes of Functions Defined by Sălăgean Type $q$ Derivative Operator

Mohamed K. Aouf, Adela O. Mostafa

Abstract. In this paper, we investigate several interesting some subordination results for classes of analytic functions defined by the Sălăgean type $q$-derivative operator.

1. Introduction

The class of analytic functions of the form:

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \quad (z \in \mathbb{U} = \{z \in \mathbb{C}, |z| < 1\}),$$

is denoted by $\mathcal{A}$. Also, denote by $\mathcal{K}$ the subclass of functions $f \in \mathcal{A}$ which are convex in $\mathbb{U}$. For functions $f$ given by (1) and $g \in \mathcal{A}$ given by $g(z) = z + \sum_{k=2}^{\infty} b_k z^k$, the Hadamard product (or convolution) of $f$ and $g$ is defined by

$$(f \ast g)(z) = z + \sum_{k=2}^{\infty} a_k b_k z^k = (g \ast f)(z).$$

If $f$ and $g$ are analytic functions in $\mathbb{U}$, we say that $f$ is subordinate to $g$ ($f \prec g$) if there exists an analytic function $w$, with $w(0) = 0$ and $|w(z)| < 1$, $z \in \mathbb{U}$, such that $f(z) = g(w(z))$. Furthermore, if $g$ is univalent in $\mathbb{U}$, then (see [10] and [23]):

$$f(z) < g(z) \quad (z \in \mathbb{U}) \iff f(0) = g(0) \text{ and } f(\mathbb{U}) \subset g(\mathbb{U}).$$

A function $f(z) \in \mathcal{A}$ is in the class $\mathcal{UCV}(\alpha, \beta)$ of uniformly convex functions of order $\alpha (-1 \leq \alpha < 1)$ and type $\beta \geq 0$ if it satisfies

$$\Re \left\{ 1 + \frac{zf''(z)}{f'(z)} - \alpha \right\} \geq \beta \left| \frac{zf''(z)}{f'(z)} \right|,$$

(2)

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Email addresses: mkaouf127@yahoo.com (Mohamed K. Aouf), adelaeg254@yahoo.com (Adela O. Mostafa)
and is in the corresponding class $SP(\alpha, \beta)$ of uniformly starlike of order $\alpha(-1 \leq \alpha < 1)$ and type $\beta \geq 0$ if it satisfies

$$\text{Re}\left\{\frac{zf'(z)}{f(z)} - \alpha\right\} \geq \beta\left|\frac{zf'(z)}{f(z)} - 1\right|,$$

where the classes $SP(\alpha, \beta)$ and $UCV(\alpha, \beta)$ were introduced and studied by ([2], [9], [17] and [29]).

From (2) and (3), we have

$$f(z) \in UCV(\alpha, \beta) \iff zf'(z) \in SP(\alpha, \beta).$$

We note that:

i) $UCV(0, 1) = UCV$ is the class of uniformly convex functions introduced and studied by Goodman [13];

ii) $UCV(\alpha, 1) = UCV(\alpha), SP(\alpha, 1) = SP(\alpha)$ and $SP(0, 1) = SP$ (see [25]);

iii) $UCV(0, \beta) = \beta - UCV$ and $SP(0, \beta) = \beta - SP$ (see [16], [18] and [19]).

One of the most common applications in number theory, especially in the theory of partitions is using the q-series or polynomials has already observed in the monograph by Srivastava and Karlsson [35, pp. 350-351] and, more recently, in a survey-cum-expository review article by Srivastava [31] on the widespread usages of the q-analysis including in geometric function theory of complex analysis, our investigation here is believed to present another advance in the subject of the ($q$-) calculus.

We now present a brief expository overview of the classical ($q$-) analysis and the Salagean operator which will be used in this paper.

**Definition 1.1.** For $q \in (0, 1)$, the $q$-number $[i]_q$ is defined by

$$[i]_q = \begin{cases} \frac{1 - q^i}{1 - q} & i \in \mathbb{C} \\ \sum_{k=0}^{n-1} q^k = 1 + q + q^2 \ldots & i = n \in \mathbb{N} = \{1, 2, \ldots\}. \end{cases}$$

We see that $[i]_q = \frac{1 - q^i}{1 - q} \to i$ as $q \to 1 -$.

**Definition 1.2.** For $q \in (0, 1)$, the $q$-derivative of $f \in \mathcal{U}$, is given by (see [20] [21], [22], [27], [28], [30], [31], [32], [34], [36], [37], [38], [40] and [41])

$$D_qf(z) = \begin{cases} \frac{f(g(z))}{g(0)}, & z \neq 0 \\ f'(0), & z = 0. \end{cases}$$

From Definitions 1 and 2, note that (see [36])

$$\lim_{q \to 1} D_qf(z) = \lim_{q \to 1} \frac{f(z) - f(gz)}{(1-q)z} = f'(z),$$

for a function $f$ which is differentiable in a given subset of $\mathbb{C}$. It is readily deduced from (1) and (4) that

$$D_qf(z) = 1 + \sum_{k=2}^{\infty} [k]_q a_k z^{k-1}.$$

**Definition 1.3.** For $f \in \mathcal{U}$, Govindaraj and Sivashubramanian [14] (see also [24]) defined the Salagean $q$-derivative operator by

$$D_q^0f(z) = f(z),$$

$$D_q^1f(z) = zD_qf(z),$$

$$D_q^nf(z) = zD_q(D_q^{n-1}f(z)), n \in \mathbb{N}.$$
It is easy to have
\[ D_q^n f(z) = z + \sum_{k=2}^{\infty} [k]_q^n a_k z^k (n \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}). \] (6)

We see that
\[ \lim_{q \to 1^-} D_q^n f(z) = D^n f(z) = z + \sum_{k=2}^{\infty} k^q a_k z^k (n \in \mathbb{N}_0), \]
where the differential operator \( D^n \) was introduced and studied by Salagean [26] (see also Aouf and Srivastava [7]).

**Definition 1.4.** For \(-1 \leq \alpha < 1\), \(\beta \geq 0\), \(0 < q < 1\), \(n \in \mathbb{N}_0\), \(f(z) \in \mathfrak{A}\) of the form (1), \(z \in \mathbb{U}\), let \(S_n(\alpha, \beta, q)\) be the subclass of \(\mathfrak{A}\) consisting of functions satisfying

\[
\text{Re} \left\{ \frac{zD_q(zD_q^n f(z))}{D_q^n f(z)} - \alpha \right\} > \beta \left| \frac{zD_q(zD_q^n f(z))}{D_q^n f(z)} - 1 \right|.
\] (7)

and \(C_n(\alpha, \beta, q)\) be the subclass of \(\mathfrak{A}\) consisting of functions satisfying

\[
\text{Re} \left\{ \frac{D_q(zD_q^n f(z))}{D_q^n f(z)} - \alpha \right\} > \beta \left| \frac{D_q(zD_q^n f(z))}{D_q^n f(z)} - 1 \right|.
\] (8)

It follows from (7) and (8) that

\[ D_q^n f(z) \in C_n(\alpha, \beta, q) \iff zD_q(zD_q^n f(z)) \in S_n(\alpha, \beta, q). \] (9)

Note that:

i) \(S_n(\alpha, \beta, q) = S(\alpha, \beta, q) = \left\{ f \in \mathfrak{A} : \text{Re} \left\{ \frac{zD_q f(z)}{f(z)} - \alpha \right\} > \beta \left| \frac{zD_q f(z)}{f(z)} - 1 \right| \right\}\)

and

\[ S(\alpha, 0, q) = S(\alpha, q) = \text{Re} \left\{ \frac{zD_q f(z)}{f(z)} \right\} > \alpha (0 \leq \alpha < 1); \]

ii) \(C_n(\alpha, \beta, q) = C(\alpha, \beta, q)\)

\[ = \left\{ f \in \mathfrak{A} : \text{Re} \left\{ \frac{D_q(zD_q f(z))}{D_q f(z)} - \alpha \right\} > \beta \left| \frac{D_q(zD_q f(z))}{D_q f(z)} - 1 \right| \right\}\]

and

\[ C(\alpha, 0, q) = C(\alpha, q) = \text{Re} \left\{ \frac{D_q(zD_q f(z))}{D_q f(z)} \right\} > \alpha (0 \leq \alpha < 1); \]

iii) \(\lim_{q \to 1^-} S_n(\alpha, \beta, q) = S_n(\alpha, \beta)\)

\[ = \left\{ f \in \mathfrak{A} : \text{Re} \left\{ \frac{z(D^n f(z))'}{D^n f(z)} - \alpha \right\} > \beta \left| \frac{z(D^n f(z))'}{D^n f(z)} - 1 \right| \right\}; \]

iv) \(\lim_{q \to 1^-} C_n(\alpha, \beta, q) = C_n(\alpha, \beta)\)

\[ = \left\{ f \in \mathfrak{A} : \text{Re} \left\{ 1 + \frac{z(D^n f(z))''}{(D^n f(z))'} - \alpha \right\} > \beta \left| \frac{z(D^n f(z))''}{(D^n f(z))'} \right| \right\}; \]

v) \(\lim_{q \to 1^-} S(\alpha, \beta, q) = SP(\alpha, \beta)\) and \(\lim_{q \to 1^-} C(\alpha, \beta, q) = UCV(\alpha, \beta). \)
2. Main Results

Throughout this paper unless otherwise mentioned, we assume that $-1 \leq \alpha < 1$, $\beta \geq 0$, $q \in (0, 1)$, $n \in \mathbb{N}$, $f(z) \in \mathcal{A}$ of the form (1) and $z \in \mathbb{U}$.

To prove our main result we need the following definition and lemma.

**Definition 2.1.** [42]. The sequence $\{c_k\}_{k=1}^{\infty}$ of complex numbers is said to be a subordinating factor sequence if whenever $f(z)$ of the form (1) analytic, univalent and convex in $\mathbb{U}$, we have

$$\sum_{k=1}^{\infty} c_k a_k z^k < f(z) \quad (a_1 = 1).$$

**Lemma 2.2.** [42]. The sequence $\{c_k\}_{k=1}^{\infty}$ is a subordinating factor sequence if and only if

$$\text{Re} \left\{ 1 + 2 \sum_{k=1}^{\infty} c_k z^k \right\} > 0.$$

We now prove the following lemmas.

**Lemma 2.3.** If $f(z)$ satisfies the following inequality

$$\sum_{k=2}^{\infty} \left| k \right| \left| (1 + \beta) - (\alpha + \beta) \right| \left| k \right| a_k \leq 1 - \alpha, \quad (10)$$

then, $f(z) \in S_n(\alpha, \beta, q)$.

**Proof.** Making use of (10), it is suffices to prove that

$$\beta \left| \frac{zD_q(D_q^n f(z))}{D_q^n f(z)} - 1 \right| - \text{Re} \left\{ \frac{zD_q(D_q^n f(z))}{D_q^n f(z)} - 1 \right\} < 1 - \alpha.$$

We have

$$\beta \left| \frac{zD_q(D_q^n f(z))}{D_q^n f(z)} - 1 \right| - \text{Re} \left\{ \frac{zD_q(D_q^n f(z))}{D_q^n f(z)} - 1 \right\} \leq (1 + \beta) \left| \frac{zD_q(D_q^n f(z))}{D_q^n f(z)} - 1 \right| = (1 + \beta) \left| \frac{\sum_{k=2}^{\infty} (\left| k \right| - 1) a_k z^{k-1}}{1 + \sum_{k=2}^{\infty} \left| k \right| a_k z^{k-1}} \right| \leq (1 + \beta) \frac{\sum_{k=2}^{\infty} (\left| k \right| - 1) \left| k \right| \left| a_k \right| z^{k-1}}{1 - \sum_{k=2}^{\infty} \left| k \right| \left| a_k \right| z^{k-1}}.$$
Since \( a \) is a subordinating factor sequence, with
Thus by Definition 1, the subordination result (12) will hold true if the sequence

The constant factor

\[ \frac{[2]_q(1 + \beta) - (\alpha + \beta) [2]_q^\alpha}{2[[2]_q(1 + \beta) - (\alpha + \beta) [2]_q^\alpha + (1 - \alpha)]} \]

in (12) cannot be replaced by a large one.

Proof. Let \( f(z) \in \mathcal{S}_n'(\alpha, \beta, q) \) and \( g(z) \in \mathcal{K} \). Then

\[ (f * g)(z) < g(z) \]

and

\[ \text{Re}(f(z)) > -\frac{[2]_q(1 + \beta) - (\alpha + \beta) [2]_q^\alpha + (1 - \alpha)}{[2]_q(1 + \beta) - (\alpha + \beta) [2]_q^\alpha}. \]

Thus by Definition 1, the subordination result (12) will hold true if the sequence

\[ \left\{ \frac{[2]_q(1 + \beta) - (\alpha + \beta) [2]_q^\alpha}{2[[2]_q(1 + \beta) - (\alpha + \beta) [2]_q^\alpha + (1 - \alpha)]} a_k \right\}_{k=1}^\infty \]

is a subordinating factor sequence, with \( a_1 = 1 \). In view of Lemma 1, this is equivalent to

\[ \text{Re} \left\{ 1 + \sum_{k=1}^{\infty} \frac{[2]_q(1 + \beta) - (\alpha + \beta) [2]_q^\alpha}{[[2]_q(1 + \beta) - (\alpha + \beta) [2]_q^\alpha + (1 - \alpha)] a_k z^k} \right\} > 0. \]

Since

\[ \Phi(k) = [k]_q(1 + \beta) - (\alpha + \beta) [k]_q^\alpha (k \geq 2; \beta \geq 0; -1 \leq \alpha < 1, n \in \mathbb{N}_0; 0 < q < 1) \]
is an increasing function of $k$, then, when $|z| = r < 1$, we have

$$
\text{Re}\left\{ 1 + \sum_{k=1}^{\infty} \frac{[2]_q(1 + \beta) - (\alpha + \beta)[2]_q^n}{[2]_q(1 + \beta) - (\alpha + \beta)[2]_q^n + (1 - \alpha) q_k z^k} \right\} = \text{Re}\left\{ 1 + \sum_{k=2}^{\infty} \frac{[2]_q(1 + \beta) - (\alpha + \beta)[2]_q^n}{[2]_q(1 + \beta) - (\alpha + \beta)[2]_q^n + (1 - \alpha)} \sum_{k=2}^{\infty} \frac{[2]_q(1 + \beta) - (\alpha + \beta)[2]_q^n z^k}{[2]_q(1 + \beta) - (\alpha + \beta)[2]_q^n + (1 - \alpha)} \right\} \\
\geq 1 - \frac{[2]_q(1 + \beta) - (\alpha + \beta)[2]_q^n}{[2]_q(1 + \beta) - (\alpha + \beta)[2]_q^n + (1 - \alpha)} r - \frac{[2]_q(1 + \beta) - (\alpha + \beta)[2]_q^n}{[2]_q(1 + \beta) - (\alpha + \beta)[2]_q^n + (1 - \alpha)} \sum_{k=2}^{\infty} \frac{[2]_q(1 + \beta) - (\alpha + \beta)[2]_q^n}{[2]_q(1 + \beta) - (\alpha + \beta)[2]_q^n + (1 - \alpha)} r - \frac{[2]_q(1 + \beta) - (\alpha + \beta)[2]_q^n}{[2]_q(1 + \beta) - (\alpha + \beta)[2]_q^n + (1 - \alpha)} r
\\= 1 - r > 0,
$$

where we also used the assertion (10) of Lemma 2. Thus (16) holds in $\mathcal{U}$ and also, the subordination result (12) asserted by Theorem 1. The inequality (13) follows from (12) by taking the convex function $U$ where we also used the assertion (10) of Lemma 2. Thus (16) holds in

$$
\text{consider the function } f(z) = z(1 - z)^{-1} = z + \sum_{k=2}^{\infty} z^k. \text{ To prove the sharpness of the constant}
\begin{equation}
[2]_q(1 + \beta) - (\alpha + \beta)[2]_q^n
\frac{1 - \alpha}{2([2]_q(1 + \beta) - (\alpha + \beta)[2]_q^n + (1 - \alpha))] z^2.
\end{equation}
\end{equation}

Thus, from (12), we have

$$
\frac{[2]_q(1 + \beta) - (\alpha + \beta)[2]_q^n}{2([2]_q(1 + \beta) - (\alpha + \beta)[2]_q^n + (1 - \alpha))] f(z) < \frac{z}{1-z} (z \in \mathcal{U}).
\end{equation}
\end{equation}

Moreover, it can easily be verified for the function $f_0(z)$ that

$$
\min_{|z|=r} \left\{ \text{Re} \left( \frac{[2]_q(1 + \beta) - (\alpha + \beta)[2]_q^n}{2([2]_q(1 + \beta) - (\alpha + \beta)[2]_q^n + (1 - \alpha))] f_0(z) \right) \right\} = \frac{1}{2}.
$$

This shows that the constant

$$
\frac{[2]_q(1 + \beta) - (\alpha + \beta)[2]_q^n}{2([2]_q(1 + \beta) - (\alpha + \beta)[2]_q^n + (1 - \alpha))}
$$

is the best possible, which completes the proof.

Similarly, we can prove the following theorem for the class $C_n^\alpha(\alpha, \beta, q)$. **Theorem **
Theorem 2.6. Let \( f(z) \in C^\alpha(\alpha, \beta, q) \) and \( g(z) \in \mathcal{K} \). Then

\[
\frac{[2]_n(1 + \beta) - (\alpha + \beta)[2]_n[2]_n^\alpha}{2[[2]_n(1 + \beta) - (\alpha + \beta)][2]_n[2]_n^\alpha + (1 - \alpha)]}(f \ast g)(z) < g(z)
\]

(19)

and

\[
\text{Re}(f(z)) > -\frac{[2]_n(1 + \beta) - (\alpha + \beta)[2]_n[2]_n^\alpha + (1 - \alpha)}{[2]_n(1 + \beta) - (\alpha + \beta)[2]_n[2]_n^\alpha + (1 - \alpha)}.
\]

(20)

The constant factor

\[
\frac{[2]_n(1 + \beta) - (\alpha + \beta)[2]_n[2]_n^\alpha}{2[[2]_n(1 + \beta) - (\alpha + \beta)][2]_n[2]_n^\alpha + (1 - \alpha)]}
\]

in (19) cannot be replaced by a large one.

Putting \( n = 0 \) in Theorems 1 and 2, respectively, we have

Corollary 2.7. Let \( f(z) \in S^\alpha(\alpha, \beta, q) \) and satisfies the condition

\[
\sum_{k=2}^{\infty} [k]_n(1 + \beta) - (\alpha + \beta) |a_k| \leq 1 - \alpha,
\]

then

\[
\frac{[2]_n(1 + \beta) - (\alpha + \beta)}{2[[2]_n(1 + \beta) - (\alpha + \beta) + (1 - \alpha)]}(f \ast g)(z) < g(z), g \in \mathcal{K}
\]

and

\[
\text{Re}(f(z)) > -\frac{[2]_n(1 + \beta) - (\alpha + \beta) + (1 - \alpha)}{[2]_n(1 + \beta) - (\alpha + \beta) + (1 - \alpha)}.
\]

The constant factor

\[
\frac{[2]_n(1 + \beta) - (\alpha + \beta)}{2[[2]_n(1 + \beta) - (\alpha + \beta) + (1 - \alpha)]}
\]

is the best estimate.

Corollary 2.8. Let \( f(z) \in C^\alpha(\alpha, \beta, q) \) and satisfies the condition

\[
\sum_{k=2}^{\infty} [k]_n[k]_n(1 + \beta) - (\alpha + \beta) |a_k| \leq 1 - \alpha,
\]

then

\[
\frac{[2]_n[2]_n(1 + \beta) - (\alpha + \beta)}{2[[2]_n[2]_n(1 + \beta) - (\alpha + \beta) + (1 - \alpha)]}(f \ast g)(z) < g(z), g \in \mathcal{K}
\]

and

\[
\text{Re}(f(z)) > -\frac{[2]_n[2]_n(1 + \beta) - (\alpha + \beta) + (1 - \alpha)}{[2]_n[2]_n(1 + \beta) - (\alpha + \beta) + (1 - \alpha)}.
\]

The constant factor

\[
\frac{[2]_n[2]_n(1 + \beta) - (\alpha + \beta)}{2[[2]_n[2]_n(1 + \beta) - (\alpha + \beta) + (1 - \alpha)]}
\]

is the best estimate.

Putting \( \beta = 0 \) in Corollaries 1 and 2, respectively, we have
Corollary 2.9. Let \( f(z) \in S^*(\alpha, q) \) and satisfies the condition

\[
\sum_{k=2}^{\infty} ([k]_q - \alpha) |a_k| \leq 1 - \alpha,
\]

then

\[
\left( \frac{[2]_q - \alpha}{2([2]_q + 1 - 2\alpha)} \right)(f \ast g)(z) < g(z), g \in \mathcal{K}
\]

and

\[
\text{Re}(f(z)) > -\frac{[2]_q + 1 - 2\alpha}{[2]_q - \alpha}.
\]

The constant factor \( \frac{[2]_q - \alpha}{2([2]_q + 1 - 2\alpha)} \) is the best estimate.

Corollary 2.10. Let \( f(z) \in C^*(\alpha, q) \) and satisfies the condition

\[
\sum_{k=2}^{\infty} [k]_q ([k]_q - \alpha) |a_k| \leq 1 - \alpha,
\]

then

\[
\left( \frac{[2]_q ([2]_q - \alpha)}{2([2]_q([2]_q - \alpha) + 1 - \alpha)} \right)(f \ast g)(z) < g(z), g \in \mathcal{K}
\]

and

\[
\text{Re}(f(z)) > -\frac{2([2]_q([2]_q - \alpha) + 1 - \alpha)}{[2]_q([2]_q - \alpha)}.
\]

The constant factor \( \frac{[2]_q ([2]_q - \alpha)}{2([2]_q([2]_q - \alpha) + 1 - \alpha)} \) is the best estimate.

Remark 2.11. i) Letting \( q \to 1^- \) in Theorems 1 and 2, respectively, we have the results obtained by Aouf and Mostafa [4, Corollaries 2.6 and 2.10, respectively];

ii) Letting \( q \to 1^- \) in Corollaries 1 and 2, respectively, we have the results obtained by Frasin [12, Corollaries, 2.2 and 2.5, respectively];

iii) Letting \( q \to 1^- \) in Corollaries 3 and 4, respectively, we have the results obtained by Frasin [12, Corollaries, 2.3 and 2.6, respectively].

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