Research Article

Statistical Analysis of COVID-19 Data for Three Different Regions in the Kingdom of Saudi Arabia: Using a New Two-Parameter Statistical Model

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1. Introduction

In recent years, many various of statisticians have been attracted by create new families of distributions for example; exponentiated generalized-G in [1], logarithmic-X family of distributions [2], sine-G in [3], odd Perks-G in [4], odd Lindley-G in [5], truncated Cauchy power-G in [6], truncated Cauchy power Weibull-G-G in [7], Topp-Leone-G in [8], odd Nadarajah-Haghighi-G in [9], the Marshall–Olkin alpha power-G in [10], T-X generator studied in [11], type I half-logistic Burr X-G in [12], KM transformation family in [13], (DUS) transformation family in [14], arc sine exponentiated-X family in [15], Marshall-Olkin odd Burr III-G family in [16], among others.

Reference [17] investigates the half-logistic-G (HL-G) family, a novel family of continuous distributions with an additional shape parameter $\theta > 0$. The HL-G cumulative distribution function (cdf) is supplied via

$$F(z; \theta, \omega) = \frac{1 - [1 - G(z; \omega)]^{\theta}}{1 + [1 - G(z; \omega)]^{\theta}}, \quad z \in \mathbb{R}, \theta > 0.$$  \hspace{1cm} (1)

The HL-G family’s density function (pdf) is described as

$$f(z; \theta, \omega) = \frac{2\theta g(z; \omega)[1 - G(z; \omega)]^{\theta-1}}{[1 + [1 - G(z; \omega)]^{\theta}]^2}, \quad z \in \mathbb{R}, \theta > 0.$$  \hspace{1cm} (2)
Table 1: Numerical values of Mos for the HLIMExp model for $\beta = 3$ different values of parameter $\theta$.

| $\theta$ | $E(Z)$ | $E(Z^2)$ | $E(Z^3)$ | $E(Z^4)$ | $H$ | $\sigma^2$ | SK | KU | CV |
|----------|--------|----------|----------|----------|-----|------------|----|----|----|
| 4        | 0.452  | 9.951    | 1.702    | 0.290    | 1.006| 6.582      | 2.992| 1.709| 1.192|
| 4.5      | 0.425  | 8.335    | 1.513    | 0.227    | 1.052| 4.469      | 2.341| 1.486| 1.122|
| 5        | 0.404  | 7.293    | 1.370    | 0.186    | 1.092| 3.275      | 1.915| 1.322| 1.066|
| 5.5      | 0.387  | 6.568    | 1.256    | 0.156    | 1.130| 2.531      | 1.616| 1.197| 1.020|
| 6        | 0.372  | 6.036    | 1.163    | 0.134    | 1.164| 2.034      | 1.398| 1.098| 0.982|
| 6.5      | 0.360  | 5.630    | 1.086    | 0.117    | 1.195| 1.684      | 1.231| 1.018| 0.949|
| 7        | 0.349  | 5.310    | 1.019    | 0.103    | 1.224| 1.427      | 1.101| 0.952| 0.921|
| 7.5      | 0.340  | 5.052    | 0.961    | 0.093    | 1.251| 1.232      | 0.996| 0.896| 0.896|
| 8        | 0.331  | 4.840    | 0.910    | 0.084    | 1.276| 1.080      | 0.910| 0.848| 0.874|
| 8.5      | 0.324  | 4.662    | 0.865    | 0.077    | 1.300| 0.959      | 0.839| 0.807| 0.855|

respectively. A random variable $(Rv)Z$ has pdf (2) which would be specified as $Z \sim HL - G(z; \omega)$.

Reference [18] presented the moment exponential (MExp) model by allocating weight to the exponential (Exp) model. They established that the MExp distribution is more adaptable than the Exp model. The cdf and pdf files are available.

\[
G(t; \beta) = 1 - \left(1 + \frac{t}{\beta}\right)e^{-(t/\beta)}, \quad t > 0, \quad (3)
\]

\[
g(t, \beta) = \frac{t}{\beta^2}e^{-(t/\beta)}, \quad t > 0, \quad (4)
\]

respectively, where $\beta > 0$ is a scale parameter.

The inverse MExp (IMExp) distribution was presented in reference [19], and it is produced by utilizing the R.v $Z = 1/T$, where $T$ is as follows (4). The cdf and pdf files in the IMExp distribution are specified as

\[
G(z; \beta) = \left(1 + \frac{\beta}{z}\right)e^{-(\beta/z)}, \quad z > 0, \quad \beta > 0, \quad (5)
\]

\[
g(z; \beta) = \frac{\beta^2}{z^2}e^{-(\beta/z)}, \quad z > 0, \quad \beta > 0.
\]

In this research, we propose an extension of the IMExp model, which is built using the HL-G family and the IMExp model, known as the half-logistic inverse moment exponential (HLIMExp) distribution.

The aim goal of this article can be considered in the following items:

(i) To introduce a new two-parameter lifetime model which is called the HLIMExp

(ii) The new model is very flexible, and the pdf can take different shapes such as unimodal, right skewness, and heavy tail. Also, the hr faç can be increasing, upside-down, and J-shaped

(iii) Many numerical values of the moments are calculated in Table 1. And we can note from it that ($a$)

When $\beta = 3$ and $\theta$ is increasing, then the numerical values of $E(Z), E(Z^2), E(Z^3), E(Z^4)$, variance($\sigma^2$), skewness (SK), and kurtosis (KU) are decreasing but the numerical values of harmonic mean ($H$) are increasing

(iv) The simulation study is carried out to assess the behavior of parameters, and the numerical results are mentioned in Tables 2–5. From these tables, we can note that when the value of $n$ is increased, the value of $\Omega$ and $\Omega^4$ is decreased

(v) Three separate sets of COVID-19 data from Al Bahah, Al Madinah Al Munawarah, and Riyadh are utilized to test the HLIMExp model's applicability. The HLIMExp model is compared to several other well-known distributions. Using several analytical criteria, the results show that the HLIMExp distribution produces promising outcomes in terms of flexibility

The following is an outline of the remainder of this article: Section 2 discusses the construction of the HLIMExp...
In Section 6, we investigated the potentiality of the HLIMExp method. Section 5 employs Monte Carlo simulation techniques. Estimation using the maximum likelihood (ML) estimation order statistics, moments, moment generating function, and distribution, including the linear representation of HLIMExp pdf, moments (Mo), the harmonic mean (MoGF), and conditional moment (CoMo). We discussed certain HLIMExp distribution features in this part, including linear representation of HLIMExp pdf, and get cdf and pdf.

\[
F(z; \beta, \theta) = \frac{1 - \left[ 1 - (1 + (\beta/z)) e^{-(\beta/z)} \right]^\theta}{1 + \left[ 1 - (1 + (\beta/z)) e^{-(\beta/z)} \right]^\theta}, \quad z > 0, \beta, \theta > 0.
\]  

\[
f(z; \beta, \theta) = \frac{2\theta (\beta^2/z^2) e^{-(\beta/z)} [1 - (1 + (\beta/z)) e^{-(\beta/z)}]^\theta - 1}{\left[ 1 + (1 + (\beta/z)) e^{-(\beta/z)} \right]^\theta}, \quad z > 0, \theta > 0.
\]

The survival function (sf) is provided by

\[
F(z; \beta, \theta) = \frac{2 [1 - (1 + (\beta/z)) e^{-(\beta/z)}]^{\theta - 1}}{1 + [1 - (1 + (\beta/z)) e^{-(\beta/z)}]^{\theta - 1}}, \quad z > 0, \beta, \theta > 0.
\]

The failure rate and reversed hrf for the HLIMExp are calculated as follows:

\[
h(z; \beta, \theta) = \frac{\theta (\beta^2/z^2) e^{-(\beta/z)} [1 - (1 + (\beta/z)) e^{-(\beta/z)}]^\theta}{\left[ 1 - (1 + (\beta/z)) e^{-(\beta/z)} \right]^{\theta - 1}},
\]

\[
\tau(z; \beta, \theta) = \frac{2\theta (\beta^2/z^2) e^{-(\beta/z)} [1 - (1 + (\beta/z)) e^{-(\beta/z)}]^\theta - 1}{\left[ 1 - (1 + (\beta/z)) e^{-(\beta/z)} \right]^{\theta - 1}}.
\]

Different shapes of the pdf and hrf of HLIMExp with different parameter values are mentioned in Figures 1 and 2.

### 3. Statistical Properties

We discussed certain HLIMExp distribution features in this part, including linear representation of HLIMExp pdf, moments (Mo), the harmonic mean (H), moment generating function (MoGF), and conditional moment (CoMo).

#### 3.1. Linear Representation

A linear form of the pdf and cdf is offered in this part to introduce statistical properties of the HLIMExp distribution. Using the following binomial expansion,

\[
(1 + z)^{-m} = \sum_{i=0}^{\infty} (-1)^i \binom{m + i - 1}{i} z^i,
\]

where \(|z| < 1\) and \(b\) is a positive real noninteger. By applying (10) in the next term, we get

\[
\left[ 1 + \left( 1 + \left( \frac{\beta}{z} \right) e^{-(\beta/z)} \right) \right]^{-1} = \sum_{i=0}^{\infty} (-1)^i \left( 1 + \left( \frac{\beta}{z} \right) e^{-(\beta/z)} \right)^i.
\]
Table 5: MLEs, $\Omega_1$, $\Omega_2$, $\Omega_3$, and $\Omega_4$ of HLIMExp model for $\beta = 1.5$ and $\theta = 1.2$.

| $n$ | MLEs | $\Omega_1$ | $\Omega_2$ | $\Omega_3$ | $\Omega_4$ |
|-----|------|------------|------------|------------|------------|
| 30  |      | 1.687      | 0.277      | 1.010      | 2.363      |
| 50  |      | 1.239      | 0.043      | 0.853      | 1.626      |
| 100 |      | 1.526      | 0.045      | 1.070      | 1.982      |
| 300 |      | 1.225      | 0.014      | 1.909      | 0.592      |
| 400 |      | 1.529      | 0.032      | 1.206      | 1.852      |
| 500 |      | 1.218      | 0.012      | 0.999      | 1.418      |

Inserting the previous equation in (7), we have

$$f(z; \beta, \theta) = 2\theta \beta^2 \sum_{j=0}^{\infty} (-1)^j (1 + z)^{3-\beta(-\theta z)} \cdot \left[ 1 - \left(1 + \frac{\beta}{z}\right)^{\theta(i+1)-1} \right] \cdot \left[ 1 - \left(1 + \frac{\beta}{z}\right)^{\theta(i+1)-1} \right].$$  \hspace{1cm} (12)

Again, applying the general binomial theorem, we get

$$\left[ 1 - \left(1 + \frac{\beta}{z}\right)^{\theta(i+1)-1} \right] = \sum_{j=0}^{\infty} (-1)^j \left(\theta(i+1)-1\right) \left(1 + \frac{\beta}{z}\right)^j e^{-j(\beta z)},$$  \hspace{1cm} (13)

Inserting the previous equation in (7), we have

$$f(z; \beta, \theta) = 2\theta \beta^2 \sum_{j=0}^{\infty} (-1)^j (1 + z)^{3-\beta(-\theta z)} \left(\theta(i+1)-1\right) \cdot \left(1 + \frac{\beta}{z}\right)^j \cdot z^{-3} e^{-(\beta z)^j}.$$

Again, using the binomial expansion, we get

$$f(z; \beta, \theta) = \sum_{k=0}^{\infty} S_k z^{-k-3} e^{-(\beta z)^j},$$  \hspace{1cm} (15)

where

$$S_k = 2\theta \beta^{k+2} \sum_{j=0}^{\infty} (-1)^j (1 + 1) \left(\theta(i+1)-1\right) \cdot \left(1 + \frac{\beta}{z}\right)^j \cdot z^{-3} e^{-(\beta z)^j}.$$

3.2. Moments. The $r^{th}$ Mo of the HLIMExp distribution are discussed in this subsection. Moments are essential in any statistical study, but especially in applications, it can be used to investigate the main properties and qualities of a distribution (e.g., tendency, dispersion, skewness, and kurtosis). The $r^{th}$ Mo of Z denoted by $\mu_r$ may be calculated using (8).

$$\mu_r = E(Z^r) = \sum_{k=0}^{\infty} S_k \int_0^{\infty} z^{r-k-3} e^{-(\beta(j+1)/z)} dz,$$

then,

$$\mu_r = \sum_{k=0}^{\infty} S_k (\beta(j+1))^{-r-k-2} \Gamma(k + r + 2).$$  \hspace{1cm} (17)

The $r^{th}$ inverse Mo of Z denoted by $\mu_r^-$ may be calculated using (8).

$$\mu_r^- = E(Z^{r-1}) = \sum_{k=0}^{\infty} S_k \int_0^{\infty} z^{r-k-3} e^{-(\beta(j+1)/z)} dz,$$

then,

$$\mu_r^- = \sum_{k=0}^{\infty} S_k (\beta(j+1))^{-r-k-2} \Gamma(k + r + 2).$$  \hspace{1cm} (18)

The harmonic mean of Z is given by

$$H = E\left(\frac{1}{Z}\right) = \sum_{k=0}^{\infty} S_k \int_0^{\infty} z^{-k-4} e^{-(\beta(j+1)/z)} dz,$$

then,

$$\mu_r = \sum_{k=0}^{\infty} S_k (\beta(j+1))^{-k-3} \Gamma(r + 3).$$  \hspace{1cm} (19)

MoGFs are useful for several reasons, one of which is their application to analysis of sums of random variables. The MoGF of $ZM_f(t)$ is deduced from (7) as

$$M_f(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!} \mu_r = \sum_{r=0}^{\infty} \sum_{k=0}^{\infty} S_k t^r \Gamma(k + 2)(\beta(j+1))^{-r-k-2}.$$

Numerical values for specific values of parameters of the first four ordinary Mos, $E(Z)$, $E(Z^2)$, $E(Z^3)$, $E(Z^4)$, variance ($\sigma^2$), skewness (SK), and kurtosis (KU) of the HLIMExp model are reported in Table 1.
Figure 1: Different shapes of pdf for the HLIMExp model.
Figure 2: Different shapes of hrf for the HLIMExp model.
upper incomplete Mo say \( \eta_i(t) \) could be expressed with

\[
\eta_i(t) = \int_0^\infty z^{i} f(z; \beta, \theta) dz = \sum_{k=0}^{\infty} S_k \int_0^\infty z^{i-k-3} e^{-\left(\beta \frac{j+1}{t}\right)} dz
\]

\[
= \sum_{k=0}^{\infty} S_k \beta^{j+1} \Gamma \left( k-s+2, \frac{\beta (j+1)}{t} \right).
\]

Similarly, the \( s \)th lower incomplete Mo function is provided through

\[
\phi_i(t) = \int_0^t z^{i} f(z; \beta, \theta) dz = \sum_{k=0}^{\infty} S_k \int_0^t z^{i-k-3} e^{-\left(\beta \frac{j+1}{t}\right)} dz
\]

\[
= \sum_{k=0}^{\infty} S_k \beta^{j+1} \Gamma \left( k-s+2, \frac{\beta (j+1)}{t} \right).
\]

4. Method of Maximum Likelihood

Let \( z_1, z_2, \ldots, z_n \) be a random sample of size \( n \) from the HLI-MExp model with two parameters \( \beta \) and \( \theta \); the log-likelihood function is

\[
L = n \log (2\theta) - 2n \log \beta - 3 \sum_{i=1}^n z_i - \sum_{i=1}^n \frac{\beta}{z_i}
\]

\[
+ (\theta - 1) \sum_{i=1}^n \log [G_i] - 2 \sum_{i=1}^n \log \left[ 1 + [G_i]^\theta \right].
\]

(26)

For calculation MLE estimation, we need partial derivatives of \( L(Z | \beta, \theta) \) by parameters

\[
\frac{\partial \log L}{\partial \beta} = -\frac{2n}{\beta} - \sum_{i=1}^n \frac{V_i}{G_i} + (\theta - 1) \sum_{i=1}^n \frac{V_i G_i}{G_i^{\theta - 1}}
\]

\[
- 2 \sum_{i=1}^n \frac{\theta V_i G_i^{\theta - 1}}{1 + [G_i]^{\theta}},
\]

\[
\frac{\partial \log L}{\partial \theta} = n + \sum_{i=1}^n \log [G_i] - 2 \sum_{i=1}^n \frac{[G_i]^{\theta} \ln [G_i]}{1 + [G_i]^{\theta}},
\]

(27)

where \( G_i = 1 - (1 + (\beta z_i)) e^{-\left(\beta z_i\right)} \) and \( V_i = \partial G_i / \partial \beta = (\beta / (z_i)) e^{-\left(\beta z_i\right)} \). As result, estimations of the parameters can be found \( \hat{\beta}_{MLE} \) and \( \hat{\theta}_{MLE} \) the solution of the two equations \( \partial L / \partial \beta = 0 \) and \( \partial L / \partial \theta = 0 \) by using software Mathematica (9).

5. Simulation Results

A simulation result is included in this section to analyze the behavior of estimators in the presence of complete samples by using the Newton-Raphson iteration method and by using Mathematica (8) software. Mean square errors (MSE), lower and upper bound of confidence interval (CI), and average length of confidence interval are computed using Mathematica 9. The accompanying algorithm is constructed in the next part:

Table 6: Al Bahah, Al Madinah Al Munawarah, and Riyadh Regions, coronavirus cases (COVID-19).

| Year   | Month | Al Bahah | Al Madinah Al Munawarah | Riyadh |
|--------|-------|----------|-------------------------|--------|
| 2021   | Jan   | 85       | 281                     | 1994   |
| 2021   | Feb   | 213      | 273                     | 4524   |
| 2021   | Mar   | 78       | 475                     | 5612   |
| 2021   | Apr   | 227      | 1001                    | 12038  |
| 2021   | May   | 409      | 2266                    | 10458  |
| 2021   | Jun   | 541      | 2167                    | 7593   |
| 2021   | Jul   | 772      | 1860                    | 8747   |
| 2021   | Aug   | 292      | 1050                    | 3856   |
| 2021   | Sep   | 32       | 193                     | 760    |
| 2021   | Oct   | 7        | 89                      | 549    |
| 2021   | Nov   | 6        | 73                      | 401    |
| 2021   | Dec   | 55       | 341                     | 2541   |
| 2022   | Jan   | 1430     | 8607                    | 44169  |
| 2022   | Feb   | 644      | 2477                    | 19641  |
| 2022   | Mar   | 77       | 460                     | 1612   |
| 2022   | Apr   | 49       | 423                     | 691    |
| 2022   | May   | 22       | 163                     | 170    |

Table 7: Some descriptive analysis of the data.

|       | Al Bahah     | Al Madinah | Al Munawarah | Riyadh |
|-------|--------------|------------|--------------|--------|
| N     | 17           | 17         | 17           |        |
| Mean  | 290.529      | 1305.824   | 7373.882     |        |
| Median| 85           | 460        | 3856         |        |
| Skewness| 1.982      | 3.108      | 2.756        |        |
| Kurtosis| 4.327      | 10.927     | 8.65         |        |
| Range | 1424         | 8534       | 43999        |        |
| Min   | 6            | 73         | 170          |        |
| Max   | 1430         | 8607       | 44169        |        |
| Sum   | 4939         | 22199      | 125356       |        |

(i) 5000 RS of size \( n = 30, 50, 100, 300, 400, \) and 500 are generated from the HLI-MExp model

(ii) The parameters’ exact values are chosen

(iii) The ML estimates (MLEs), \( \Omega_1, \Omega_2, \Omega_3, \) and \( \Omega_4 \) for selected values of parameters are computed

(iv) Tables 2–5 provide the numerical outputs based on the entire data set

6. Applications

This section concerned with three important real data sets. The data called Saudi Arabia Coronavirus cases (COVID-19) situation in Al Bahah, Al Madinah Al Munawarah and Riyadh regions from January 2022 to May 2022.
The three data sets were obtained from the following electronic address: https://datasource.kapsarc.org/explore/dataset/saudi-arabia-coronavirus-disease-COVID-19-situation/. The data sets are reported in Table 6. The descriptive analysis of the three data sets is reported in Table 7.

Here, in this section, the three data sets mentioned below are examined to demonstrate how the HLIMExp distribution outperforms alternative models, comparing the new model to some models, namely, type II Topp-Leone inverse Rayleigh (TIITOLIR) distribution by [20], half-logistic inverse Rayleigh (HLOIR) distribution by [21], beta transmuted Lindley (BT-Li) distribution by [22], the transmuted modified Weibull (TMW) distribution by [23], and the weighted Lindley (W-Li) distribution by [24]. We calculate the model parameters’ MLEs and standard errors (SEs). To evaluate distribution

### Table 8: Numerical values of MLEs, SEs, \( V_1 \), \( V_2 \), \( V_3 \), \( V_4 \), \( V_5 \), and \( V_6 \) tests for the first data set.

| Distributions | \( \alpha \) | MLE and SE | \( \beta \) | \( \theta \) | \( \lambda \) | \( V_1 \) | \( V_2 \) | \( V_3 \) | \( V_4 \) | \( V_5 \) | \( V_6 \) |
|---------------|-------------|------------|----------|----------|--------|--------|--------|--------|--------|--------|--------|
| HLIMExp       | 24.214      | 0.336      | (9.688)  | (0.081)  | 231.459| 232.317| 229.92 | 231.625| 0.167  | 0.732  |
| TIITOLIR      | 6.626       | 0.196      | (1.828)  | (0.051)  | 236.208| 237.065| 234.669| 236.373| 0.244  | 0.265  |
| HLOIR         | 8.739       | 0.272      | (2.643)  | (0.059)  | 233.253| 234.11 | 231.714| 233.419| 0.204  | 0.48   |
| W-Li          | 0.088       | 0.004      | (0.078)  | (0.001)  | 232.468| 233.326| 230.929| 232.634| 0.275  | 0.153  |
| BT-Li         | 0.010       | 0.320      | (0.017)  | (0.568)  | 232.376| 235.709| 229.297| 232.707| 0.181  | 0.631  |
| TMW           | 0.230       | 0.0072     | (0.140)  | (0.0002) | 235.812| 241.267| 231.965| 236.226| 0.243  | 0.27   |
| ILBE          | 0.010       | 0.320      | (0.017)  | (0.568)  | 232.376| 235.709| 229.297| 232.707| 0.181  | 0.631  |
| LBE           | 40.23       | 0.496      | (0.002)  | (1.037)  | 285.722| 287.095| 284.949| 284.903| 0.358  | 0.026  |

### Table 9: Numerical values of MLEs, SEs, \( V_1 \), \( V_2 \), \( V_3 \), \( V_4 \), \( V_5 \), and \( V_6 \) tests for the second data set.

| Distributions | \( \alpha \) | MLE and SE | \( \beta \) | \( \theta \) | \( \lambda \) | \( V_1 \) | \( V_2 \) | \( V_3 \) | \( V_4 \) | \( V_5 \) | \( V_6 \) |
|---------------|-------------|------------|----------|----------|--------|--------|--------|--------|--------|--------|--------|
| HLIMExp       | 292.561     | 0.520      | (103.158)| (0.138)  | 276.46 | 277.317| 274.921| 276.626| 0.118  | 0.972  |
| TIITOLIR      | 89.906      | 0.311      | (20.808)| (0.085)  | 278.671| 279.528| 277.132| 278.837| 0.163  | 0.755  |
| HLOIR         | 114.890     | 0.412      | (29.837)| (0.095)  | 277.112| 277.969| 275.573| 277.278| 0.125  | 0.954  |
| W-Li          | 0.053       | 0.0008     | (0.075) | (0.0002) | 282.778| 283.635| 281.239| 282.943| 0.288  | 0.119  |
| BT-Li         | 0.001       | 0.496      | (0.002) | (0.726)  | 284.572| 287.095| 284.949| 284.903| 0.358  | 0.026  |
| TMW           | 0.519       | 0.0006     | (0.400) | (0.0002) | 286.033| 291.488| 282.185| 286.447| 0.239  | 0.286  |
| ILBE          | 0.001       | 0.496      | (0.002) | (0.726)  | 284.572| 287.095| 284.949| 284.903| 0.358  | 0.026  |
| LBE           | 652.912     | 0.520      | (111.973)| 0.0006   | 294.272| 294.539| 293.503| 294.355| 0.272  | 0.16   |
models, we use criteria such as the $V_1$, $V_2$, $V_3$, $V_4$, $V_5$, and $V_6$ tests. In contrast, the wider distribution relates to smaller $V_1$, $V_2$, $V_3$, $V_4$, and $V_5$ and the highest value of $V_6$. The MLEs of the eight fitted models and their SEs and the numerical values of $V_1$, $V_2$, $V_3$, $V_4$, $V_5$, and $V_6$ for the three data sets are presented in Tables 8–10. We find that the HLIMExp distribution with two parameters provides a better fit than seven models. It has the smallest values of $V_1$, $V_2$, $V_3$, $V_4$, and $V_5$ and the greatest value of $V_6$ among those considered here. Moreover, the plots of empirical cdf, empirical pdf, and PP plots of our competitive model for the three data sets are displayed in Figures 3–5, respectively. The HLIMExp model clearly gives the best overall fit and so may be picked as the most appropriate model for explaining data.
7. Conclusion

We propose a novel two-parameter distribution called the half-logistic inverted moment exponential distribution in this research. HLIMExp’s pdf may be written as a linear combination of IMExp densities. We compute explicit formulas for several of its statistical features, such as HLIMExp pdf linear representation, OS, Moms, MoGF, and CoMo. The greatest likelihood estimate is investigated. The accuracy and performance of estimations are evaluated using simulation results. Three separate sets of COVID-19 data from Al Bahah, Al Madinah Al Munawarah, and Riyadh are utilized to test the HLIMExp model’s applicability. The HLIMExp model is compared to several other well-known distributions. Using several analytical criteria, the results show that the HLIMExp distribution produces promising outcomes in terms of flexibility. In the future works, we can use the new suggested model in many works such as (a) using it to study the statistical inference of the suggested model under different censored schemes, (b) using it to study the statistical inference of the suggested model under different ranked set sampling, (c) accelerated lifetime test can be studied for the new model, and (d) the statistical inference of stress strength model for the new suggested model can be studied.
Data Availability

All data are mentioned in this article.

Conflicts of Interest

The authors declare no conflict of interest.

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