Unbiased Learning to Rank: Online or Offline?

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ABSTRACT
How to obtain an unbiased ranking model by learning to rank with biased user feedback is an important research question for IR. Existing work on unbiased learning to rank (ULTR) can be broadly categorized into two groups – the studies on unbiased learning algorithms with logged data, namely the offline unbiased learning, and the studies on unbiased parameters estimation with real-time user interactions, namely the online learning to rank. While their definitions of ‘unbiasness’ are different, these two types of ULTR algorithms share the same goal – to find the best models that rank documents based on their intrinsic relevance or utility. However, most studies on offline and online unbiased learning to rank are carried in parallel without detailed comparisons on their background theories and empirical performance. In this paper, we formalize the task of unbiased learning to rank and show that existing algorithms for offline unbiased learning and online learning to rank are just the two sides of the same coin. We evaluate six state-of-the-art ULTR algorithms and find that most of them can be used in both offline settings and online environments with or without minor modifications. Further, we analyze how different offline and online learning paradigms would affect the theoretical foundation and empirical effectiveness of each algorithm on both synthetic and real search data. Our findings could provide important insights and guideline for choosing and deploying ULTR algorithms in practice.

CCS CONCEPTS
- Information systems → Learning to rank.

KEYWORDS
Learning to rank, unbiased learning, online learning

1 INTRODUCTION
The study of learning to rank with implicit user feedback such as click data has received considerable attention in both academia and industry [24]. On the one hand, by collecting data from user interactions, we can better capture the true utility of each document [46] and create large-scale training data for ranking optimization without extensive human annotations [25]. On the other hand, learning to rank directly with implicit user feedback could suffer from the intrinsic noise and bias in user interactions (e.g., position bias [15]). Thus, how to learn an unbiased learning to rank models from biased user feedback is thus an important question for the IR community.

Existing research on unbiased learning to rank (ULTR) algorithms can be broadly categorized into two groups. The first group focuses on creating robust learning algorithms that protect ranking models from inheriting data bias when training with observed data [5, 19, 27, 47]. Because these types of algorithms can work with search logs or historical data, they are often referred to as the offline learning methods, or, in most literature, the standard unbiased learning to rank algorithms. The second group focuses on designing an interactive learning process so that we can collect unbiased feedback or estimate unbiased gradients for the training of ranking models [38, 45, 48]. These types of methods require real-time interactions with end users on ranking results in each learning step, so they are often referred to as the online learning algorithms, or, in most literature, the online learning to rank algorithms.

While both groups of methods have been widely studied under the topic of unbiased learning to rank, their definitions of ‘unbiasness’ are slightly different. In the studies of standard ULTR algorithms (with offline data), unbiasedness usually refers to the ability of an algorithm in terms of removing the effect of data bias in the training of a ranking model [26]. The concept of unbiasedness in online learning to rank, on the other hand, emphasizes more on whether an algorithm can help a model converge to the best ranking model for a particular task [44]. Despite these differences, both offline ULTR algorithms and online learning to rank share a single goal for ranking optimization, that is to find the best model that ranks query-document pairs according to their intrinsic relevance.

Then the question is: are offline learning and the online learning just two sides of the same coin for unbiased learning to rank? The
answer looks to be "no" considering their different motivations and definitions. However, after examining the formulations of several algorithms, we observe that almost all ULTR algorithms in offline learning can be directly applied to online learning, and some methods in online learning can be directly used on offline data without or with minor modifications. Does this mean that a good unbiased learning to rank algorithm could be used both in offline settings and online settings? Or are there any properties that make an algorithm suitable for offline learning or online learning? Unfortunately, most research on offline unbiased learning to rank and online learning to rank are carried in parallel without comparisons. Some recent studies tried to compare the empirical performance of existing ULTR algorithms proposed separately in offline learning and online learning environments [20], but they are restricted by the stereotype that "offline" methods can only be used offline and ignores the discussion of the connections between offline unbiased learning and online learning to rank in theory.

In this paper, we conduct a comprehensive analysis on six state-of-the-art unbiased learning to rank algorithms from two families – the counterfactual learning family and the bandit learning family – and discuss their characteristics in both offline and online learning. Specifically, we focus on two research questions:

RQ1: What are the theoretical differences and connections between ULTR algorithms proposed for offline learning and online learning?
RQ2: How do learning paradigms affect the empirical effectiveness and robustness of ULTR algorithms?

To answer these questions, we develop a unified mathematical framework for unbiased learning to rank and discover that previous ULTR algorithms proposed for offline and online learning are tackling the same problem from two perspectives. Also, from our empirical studies with synthetic and real click data, we find that different ULTR algorithms have different sensitivities to learning paradigms. For example, the performance of counterfactual learning algorithms are stable in both offline and online learning paradigms, while the effectiveness and robustness of bandit learning algorithms vary significantly from case to case. These findings provide both theoretical insights to the problem of unbiased learning to rank and practical guidelines to the deployment of ULTR algorithms.

The rest of the paper is organized as the followings. Section 2 formally formulates the problem of unbiased learning to rank. Section 3 introduces the theoretical background of existing ULTR algorithms. Section 4 discusses how to deploy each ULTR algorithm in different learning paradigms, and Section 5 shows our empirical experiments. Finally, we discuss the related work in Section 6 and summarize our findings to provide guidance for the future use of ULTR algorithms in Section 7.

### 2 PROBLEM DEFINITIONS

In this section, we mathematically formalize the problem of unbiased learning to rank. A summary of the notations used in this paper is shown in Table 1.

| q, d, πq | A query q and a ranked list (πq) of documents d for q. |
| f, θ, φ | A ranking function f parameterized by θ and an examination propensity model φ. |
| L, l, Δ | The global loss function (L) of a ranking model, its local loss (l) on a query, and the loss (Δ) on a specific document in the query. |
| o, r, c | Bernoulli variables that represent whether a document is observed (o), perceived as relevant (r) and clicked (c). |

As discussed in previous studies [35], the goal of learning to rank is to learn a ranking function \( f_\theta \) which takes the feature vector of document \( d \) as input and produces a ranking score \( f_\theta(d) \) so that ranking documents by \( f_\theta(d) \) would result in the same ranked list as ranking documents by their intrinsic relevance, which we refer to as \( r \), to the query. Formally, let \( L \) be a loss function of \( f_\theta \), then learning to rank is to find the best \( \theta \) that

\[
\theta^* = \arg \min_\theta L(\theta) = \arg \min_\theta \int_q l(f_\theta, r_q) \, dP(q)
\]

where \( l(f_\theta, r_q) \) is the local ranking loss of each query session.

In noisy feedback environments such as Web search [24] and e-commerce search [39], relevance labels \( r \) are often inaccessible or unavailable. Instead, noisy user feedback that correlates to result relevance, e.g. user clicks, can be easily collected in large scale. Similarly, we could conduct learning to rank with noisy user feedback and get the optimal ranking model as

\[
\theta^* = \arg \min_\theta L'(\theta) = \arg \min_\theta \int_q \mathbb{E}_{\pi_q}[l'(f_\theta, c_{\pi_q})] \, dP(q, \pi_q)
\]

where \( \pi_q \) is the ranked list displayed in the session of \( q \), and \( c \) is the noisy feedback collected from users.

Then, we can define the task of unbiased learning to rank as

**Definition 2.1.** Given a loss function \( L \), unbiased learning to rank is to find a function \( L' \) so that \( L(\theta^*) = L(\theta^{'*}) \).

Mathematically, we easily derive two theorems for the definition of unbiased loss function in unbiased learning to rank as:

**Theorem 2.2.** \( L' \) is unbiased if for any \( \theta, L'(\theta) = L(\theta) \).

**Theorem 2.3.** \( L' \) is unbiased if for any \((\theta, \theta')\), \( L'(\theta') \geq L'(\theta^{'*}) \Rightarrow L(\theta') \geq L(\theta^{'*}) \).

Based on these theorems, in this paper, we conduct analysis on six representative and state-of-the-art unbiased learning to rank algorithms in both offline and online settings from two perspectives: theoretical foundations and practical deployment.

### 3 THEORETICAL FOUNDATIONS

In this section, we discuss the theory behind existing unbiased learning to rank algorithms and how they achieve or approximate unbiasedness as defined in Definition 2.1. For simplicity, we focus on the scenario of Web search where \( r \) is the true relevance of each document and \( c \) is the click behavior of search users. We assume that each document will be click (i.e., \( c_d = 1 \)) if and only if the user has examined the document (i.e., \( o_d = 1 \)) and the document is relevant (i.e., \( r_d = 1 \)) [40]. In other words, we have

\[
P(c_d = 1) = P(o_d = 1) \cdot P(r_d = 1)
\]

Also, as most ranking metrics (e.g. MAP, NDCG [21], ERR [12]) only concern about the position of relevant documents, we assume...
that
\[ l(f_\theta, r) = \sum_{d,r_d=1} \Delta(d,r_d|f_\theta) \]

where \( \Delta(d,r_d|f_\theta) \) is a function that computes the individual loss on each relevant document for a ranking model \( f_\theta \).

Broadly speaking, existing unbiased learning to rank algorithms can be categorized into two groups. The first group focuses on achieving Theorem 2.2 by directly removing the inherent bias from user feedback in the computation of ranking loss, while the second group focuses on achieving Theorem 2.3 by manipulating the process of user feedback collection to estimate unbiased gradients for model optimization. Formally, in Eq. (2), the first group tries to solve unbiased learning to rank by developing new ranking loss function \( l'(f_\theta,c_{\pi_q}) \), while the second group aims to tackle the problem by controlling the distribution of \( P(q,\pi_q) \). Based on different motivations, we have two families of unbiased learning to rank algorithms designed under those two groups—the counterfactual learning family and the bandit learning family.

### 3.1 Counterfactual Learning Family

The idea of counterfactual learning is to remove the effect of data bias in the computation of ranking loss so that the model trained with biased data (i.e., clicks) would converge to the model trained unbiased labels (i.e., the relevance of a document). Specifically, optimization in counterfactual learning can be simplified as

\[ \mathcal{L}'(\theta) = \int_q \int_{\pi_q} \mathbb{E}_{c_{\pi_q}}[l'(f_\theta,c_{\pi_q})]dP(q,\pi_q) = \int_q \mathbb{E}_{d}[l'(f_\theta,c)]dP(q) \quad (5) \]

where we use \( c \) to represent the clicks observed in each search session. Because it doesn’t concern about the distribution of how results are displayed in each query (i.e., \( \pi_q \)), counterfactual learning naturally suits the need of learning to rank with historical data where the distribution of the logging systems may not be available. Indeed, most studies on counterfactual unbiased learning to rank focus on the use of search logs and offline learning [1, 5, 27, 46, 47].

Based on Eq. (5), we introduce four algorithms in the rest of this section, which are the Inverse Propensity Weighting model [27, 46], the Regression-based EM model [47], the Dual Learning Algorithm [5], and the Pairwise Debiasing model [19].

#### 3.1.1 Inverse Propensity Weighting

Inverse Propensity Weighting (IPW) is one of the first ULTR algorithms proposed under the framework of counterfactual learning [27, 46]. The basic idea of IPW is to revise the computation of \( l(f_\theta, r) \) with user feedback data \( c \) as

\[ l'(f_\theta, c) = l_{IPW}(f_\theta, c) = \sum_{d,c_d=1} \frac{\Delta(d,c_d|f_\theta)}{P(\alpha_d = 1)} \quad (6) \]

where \( P(\alpha_d = 1) \) is the probability of document \( d \) being examined in the search session. Joachims et al. [27] proves that the expectation of the \( l_{IPW}(f_\theta,c) \) is equal to \( l(f_\theta,r) \) as

\[
\mathbb{E}_o[l_{IPW}(f_\theta,c)] = \mathbb{E}_o\left[ \sum_{d,c_d=1} \frac{\Delta(d,c_d|f_\theta)}{P(\alpha_d = 1)} \right] \\
= \mathbb{E}_o\left[ \sum_{d,r_d=1} \alpha_d \cdot \frac{\Delta(d,r_d|f_\theta)}{P(\alpha_d = 1)} \right] \\
= \sum_{d,r_d=1} P(\alpha_d = 1) \cdot \frac{\Delta(d,r_d|f_\theta)}{P(\alpha_d = 1)} \\
= \sum_{d,r_d=1} \Delta(d,r_d|f_\theta) = l(f_\theta,r)
\]

Thus, according to Theorem 2.2, IPW is theoretically principled for unbiased learning to rank.

The key of IPW is the estimation of examination propensity (i.e., \( P(\alpha_d = 1) \)). Assuming that the examination of documents only depends on their positions in ranked lists, Wang et al. [46] and Joachims et al. [27] conducted online result randomization to estimate \( P(\alpha_d = 1) \). Though harmful to user experience, this method creates an unbiased estimation of \( P(\alpha_d = 1) \) in theory [5, 47], which consequentially guarantees the unbiasness of IPW.

#### 3.1.2 Regression EM

To avoid hurting user experience with online result randomization, Wang et al. [47] propose to unify the training of ranking models and the estimation of examination propensity with a graphic model and an EM algorithm, which we refer to as the Regression EM model (REM). Based on Eq. (3), REM computes the likelihood of observed clicks for each query \( q \) as

\[ \log P(c) = \sum_d c_d \log(P(\alpha_d = 1) \cdot P(r_d = 1)) + (1 - c_d) \log(1 - P(\alpha_d = 1) \cdot P(r_d = 1)) \]

where \( c_d \) is observed from search logs or online user interactions, and \( \alpha_d \) and \( r_d \) are latent variables. Specifically, \( P(r_d = 1) \) is computed based on the ranking function \( f_\theta \) as

\[ P(r_d = 1) = \frac{1}{1 + \exp(-f_\theta(d))} \]

Using standard EM algorithms, we can estimate \( \alpha_d \) and \( f_\theta(d) \) based on observed click logs and Eq. (8). The estimation is guaranteed (by EM [10]) to be unbiased for the pointwise loss function \( l \) like

\[ l(f_\theta, r) = -\sum_d r_d \log(P(r_d = 1)) + (1 - r_d) \log(P(r_d = 1)) \]

Therefore, REM is unbiased under Theorem 2.2.

#### 3.1.3 Dual Learning Algorithm

Proposed with REM in parallel, Dual Learning Algorithm (DLA) [5] also tries to conduct unbiased learning to rank without result randomization. The key observation in DLA is that the positions of \( \alpha_d \) and \( r_d \) are interchangeable in Eq. (3), which means that, in theory, the counterfactual learning of IPW can be applied on both directions simultaneously. Specifically, Ai et al. [5] propose to treat the estimation of examination propensity as a dual problem of learning to rank and learn a propensity...
model $\phi$ by optimizing an inverted relevance weighting (IRW) as

$$l_{IRW}(\phi, c) = \sum_{d, c_d=1} \Delta(d, c_d|\phi) P(r_d = 1)$$  \hspace{1cm} (9)

Suppose that the expectation of $P(r)$ on each position (over all sessions) is unchanged, then we have similar induction to Eq. (7) as

$$\mathbb{E}_r[l_{IRW}(\phi, c)] = \mathbb{E}_r \left[ \sum_{d, c_d=1} \frac{\Delta(d, c_d|\phi)}{P(r_d = 1)} \right]$$

where $\phi$ is the unbiased estimation of examination propensity model $\phi$. Thus, if we model $P(\phi, c)$ with $f_0(\phi)$ and $\phi_d$, DLA can conduct unbiased learning for both $f_0$ and $\phi$ simultaneously.

### 3.1.4 Pairwise Debiasing

The Pairwise Debiasing (PairD) model is a variation of DLA with pairwise ranking losses proposed by Hu et al. [19]. It conducts learning to rank with IPW and estimates examination propensity models together with the ranking models. The differences between DLA and PairD are two-fold. First, PairD is specifically designed for pairwise learning to rank. Second, PairD tries to consider unclicked documents in its learning process by assuming that

$$P(c_d = 0) = t \cdot P(r_d = 0)$$  \hspace{1cm} (11)

and computes the IPW version of $l'(f_0, \phi, c)$ as

$$l'(f_0, \phi, c) = l_{pairD}(f_0, \phi, c) = \sum_{d^+, d^-} \Delta(f_0, d^+, d^-) P(\phi_d = 1) \cdot t$$  \hspace{1cm} (12)

Due to the limited space, we ignore the derivations of optimization loss for $P(\phi_d = 1)$ and $t$ in this paper.

Obviously, PairD is not theoretically unbiased because its assumption in Eq. (11) contradicts to the basic assumption of clicks in Eq. (3). While its theoretical foundation is controversial, its empirical performance is not bad as it can alleviate click bias in some degrees in its pairwise loss and counterfactual process.

### 3.2 Bandit Learning Family

Bandit learning aims to collect real-time user feedback in controlled environments so that we can explain and analyze the observed data and update models accordingly. In unbiased learning to rank, this means estimating unbiased parameter gradients from click data by controlling the displayed ranked lists for each query in each session. Ranking optimization in bandit learning can be reformulated as

$$L'(\theta) = \int_{\pi_q} \int_{\phi} \mathbb{E}_{c_\pi_q} [l(f_0, c_{\pi_q})] dP(\pi_q)$$  \hspace{1cm} (13)

where we simply replace $r$ with $c_{\pi_q}$ in Eq. (1) and manipulate $P(\pi_q)$ to achieve unbiasedness. In other words, the goal of unbiased bandit learning to rank is to find $P(\pi_q)$ so that for any $(\theta, \theta')$:

$$\int \mathbb{E}_{c_{\pi_q}} [l(f_0, c_{\pi_q})] dP(\pi_q) \geq \int \mathbb{E}_{c_{\pi'}} [l(f_0, c_{\pi'})] dP(\pi_q) \Rightarrow l(f_0, r) \geq l(f_0, r')$$

As bandit learning can reuse $l(f_0, r)$ on click data, algorithms under this family are easier to design and analyze [26]. However, because they require the control of $P(\pi_q)$, bandit learning algorithms are often used in online environments, which is why they are commonly referred to as the online learning to rank algorithms. Next, we briefly discuss a classic and a state-of-the-art algorithm in the bandit learning family, which are the Dueling Bandit Gradient Descent and the Pairwise Differentiable Gradient Descent algorithm.

#### 3.2.1 Dueling Bandit Gradient Descent

Proposed by Yue and Joachims [48], the Dueling Bandit Gradient Descent (DBGD) algorithm optimizes $f_0$ with three steps:

- Sample (usually uniformly) $\theta'$ given $\theta$ so that $f_0$ and $f_{\theta'}$ produce different ranked lists $\pi_\theta$ and $\pi_{\theta'}$, respectively.
- Show $\pi_\theta$ and $\pi_{\theta'}$ (directly or interleaved) to real users to collect clicks and compute $l(f_0, c_{\pi_\theta})$ and $l(f_{\theta'}, c_{\pi_{\theta'}})$.
- Let $\theta = \theta'$ if $\mathbb{E}_{c_{\pi_\theta}} [l(f_0, c_{\pi_\theta})] < \mathbb{E}_{c_{\pi_{\theta'}}} [l(f_0, c_{\pi_{\theta'}})]$.

By repeating these steps, DBGD can gradually improve $f_0$ and achieve unbiased learning to rank in online environments.

The theoretical foundation of DBGD is established on the fact that, when both examined by users, relevant documents are more observed, which makes it stochastically sampled according to $f_0$ and the Plackett-Luce model as

$$P(\pi_q|q) = \prod_{i=1}^{\left|\pi_q\right|} \frac{\exp(f_0(d_i))}{\sum_{j=1}^{\left|\pi_q\right|} \exp(f_0(d_j))}$$  \hspace{1cm} (15)

where $d_i$ is the $i$th document in $\pi_q$. Assuming that users always read $\pi_q$ sequentially [15], PDGD treats $l(f_0, c)$ as the sum of pairwise losses $\Delta(f_0, d_i, d_j)$ over document pair as

$$\Delta(f_0, c_{\pi_{\pi_q}}) = \sum_{d_i, d_j, j < i, \sum_{c_{d_j}=1, c_{d_j}=0} \rho(d_i, d_j, \pi_q) \cdot \Delta(f_0, d_i, d_j)$$  \hspace{1cm} (16)

where $d_i$ is a clicked document behind $d_j$ in $\pi_q$, and $\rho(d_i, d_j, \pi_q)$ is

$$\rho(d_i, d_j, \pi_q) = \frac{P(\pi_q|d_i, d_j|q) + P(\pi_q|d_j, d_i|q)}{P(\pi_q|q)}$$  \hspace{1cm} (17)

where $\pi_q(d_i, d_j)$ is the ranking of documents after reversing the position of $d_i$ and $d_j$ in $\pi_q$. While the mathematical proof is verbose [38], it is easy to give an intuitive example showing the unbiasedness of PDGD. For example, suppose that there is a query with only 2 candidate documents, i.e.
which randomly shuffle the positions of all documents before showing the results. Joachims et al. [27] propose to conduct online result randomization, using the clicks on the documents to the users. In this case, counterfactual learning can be applied on offline data. The main difference compared to the stochastic online paradigm is that counterfactual learning are essentially the two sides of the same problem from two different perspectives. The former focuses on the design of the ranking function \( f_\theta \) to achieve Theorem 2.2, while the latter tries to manipulate \( P(q, \pi_q) \) to achieve Theorem 2.3. From this perspective, the theories behind counterfactual learning and bandit learning are essentially the two sides of the same coin. The main reason why the studies of unbiased learning to rank with counterfactual learning are often conducted separately from those with bandit learning (which are more often referred to as online learning to rank) is that counterfactual learning can be applied on offline data while bandit learning can only be used in online environments.

In fact, the connection between counterfactual learning and bandit learning is stronger than it is appeared to be. For example, to estimate examination propensity for IPW, Wang et al. [46] and Joachims et al. [27] propose to conduct online result randomization, which randomly shuffle the positions of all documents before showing them to the users. In this case, \( P(\pi_q|q) = 1/|Q_q| \) is a uniform distribution over the universal set of possible ranked lists \( Q_q \). If we use the clicks on the 2nd position from online result randomization to compute pairwise loss on documents \( d_a \) and \( d_b \), then we have

\[
l'(f_\theta, c)|\pi_q \mid (\theta) = \int_{\pi_q} P(c\mid \pi_q) dP(\pi_q|q) = P(c|\pi_q) dP(\pi_q|q) = 1/|Q_q|
\]

where \( \pi_q^d \) is the ranked list where \( d_a \) is at the \( i \)-th position, and \( P(c|\pi_q^d = 1) \) is the probability of \( d_a \) being clicked when it is at the \( i \)-th position. Comparing Eq (18) and (19), we can see that bandit learning is essentially an online result randomization with controlled prior distributions of document ranking. If we use online randomization to estimate examination propensity, we have unbiased counterfactual learning algorithms; if we use online randomization to estimate relative document relevance, we have bandit learning algorithms.

4 PRACTICAL DEPLOYMENTS

In this section, we discuss the deployments of existing unbiased learning to rank algorithms in practice. Specifically, we analyze how and why different offline and online learning paradigms would affect the effectiveness of algorithms in the counterfactual learning family and the bandit learning family.

4.1 Offline or Online

Although the research on ULTR in offline settings and online settings has mostly been carried out in parallel, there is, to the best of our knowledge, no study that illustrates why offline ULTR methods cannot be used in online learning and why online ULTR algorithms are not applicable on offline data. In fact, we observe that almost all algorithms in the counterfactual learning family can be applied in online learning environments, and some methods in the bandit learning family are doable on offline data.

In this paper, we focus our analysis on three types of learning paradigms. The first one, which we refer to as the offline paradigm (Off), is a classic setting where we train a ranking function \( f_\theta \) based on the click logs collected from an existing system. In this paradigm, both the displayed ranked list \( (\pi_q) \) and the clicks on it \( (c|\pi_q) \) are fixed and observed in advance. The second one, which we refer to as the stochastic online paradigm (OnS), is an online setting where \( \pi_q \) is dynamically sampled with the Plackett-Luce model in Eq. (15) according to the current state of \( f_\theta \), and \( c_\pi \) is updated based on \( c_\pi \) collected online. The third setting, which we refer to as the deterministic online paradigm (OnD), is same to the stochastic online paradigm except that \( \pi_q \) is created by ranking documents with \( f_\theta \) directly.

Usually, algorithms in the counterfactual learning family can be applied in both offline and online settings because they have no requirement on \( \pi_q \). We could easily create three variations for each counterfactual learning to rank algorithm. For example, we have IPWOff, IPWOnS, and IPWOnD for IPW with the offline learning, the stochastic online learning, and the deterministic online learning. Similarly, we also have three variations for REM, DLA, and PairD.
4.2 Effect of Learning Paradigms

The comparison of offline learning paradigms and online learning paradigms has received considerable attention in the studies of learning to rank [35]. Empirically, it is widely believed that, given same types of ranking functions, online learning paradigms are likely to produce more effective ranking models than offline learning as the former can alleviate the problem of selection bias [46] and collect direct feedback on the current state of the ranking function [38]. In contrast, offline learning often has less parameter variance than online learning [44] and much less cost on user experience and system development in practice. In ULTR, however, the effect of learning paradigms varies from algorithms to algorithms. A illustration of how different learning paradigms (i.e., Off, OnS, and OnD) affect the theoretical foundations and empirical effectiveness of each unbiased learning to rank algorithms is shown in Figure 1.

In counterfactual learning, algorithms like IPW and REM have no assumption on the distribution of $\pi_q$, so the change of learning paradigms will have no effect on their effectiveness in theory. For DLA, however, things are more complicated. Because the proof in Eq. (10) relies on the assumption that $E_r[r_d] = P(r_d = 1)$ for each result position, the unbiasedness of DLA is not guaranteed when the backend model of $\pi_q$ keeps changing. However, when $f_\theta$ is almost converged or the learning rate is small enough, $E_r[r_d]$ would still be equal or similar to $P(r_d = 1)$ in stochastic or deterministic online learning. Thus, applying online learning to DLA may hurt its robustness in training but still achieve good results in the end.\(^1\)

In bandit learning, most algorithms are not applicable to or not theoretically principled in offline learning paradigms because $\pi_q$ is fixed in offline data and most bandit learning algorithms require the control of $\pi_q$ in order to achieve unbiasedness. Also, even in online settings, the effectiveness of bandit learning algorithms could be significantly affected by the sampling strategies of $\pi_q$. For instance, while DBGD is theoretically principled in both stochastic online learning and deterministic online learning environments, it generally explores less rankings in deterministic online learning and have larger model variance. While PDGD doesn’t involve any result interleaving in the training process, it is only theoretically guaranteed to find the unbiased ranking model when $\pi_q$ is strictly sampled based on $f_\theta$ in stochastic manners. Any disturbance on the distribution of $\pi_q$ (e.g., change from OnS to OnD) would hurt the performance of PDGD.

5 EMPIRICAL EXPERIMENTS

In this section, we present our empirical analysis on ULTR algorithms. Specifically, we conduct experiments on both synthetic and real click data from Web search to evaluate the effect of learning paradigms and the performance of existing ULTR algorithms.

5.1 Experiments with Synthetic Data

To fully test ULTR algorithms with different learning paradigms, we conducted experiments using synthetic click data derived from Yahoo! Learning to Rank Collection (set 1)\(^2\). This dataset contains 29,921 queries and 701k documents sampled from a commercial English search engine. Each query-document pair is represented with 700 features and annotated with 5-level relevance judgments from 0 (i.e., irrelevant) to 4 (i.e. perfectly relevant). Due to privacy concerns, no click data is released on this dataset. Therefore, we simulate click data following the methodology used by previous studies [5, 19, 27]. Specifically, we sampled the probability of examination on a document $d$ as

$$P(d) = P(a_i = 1) = \nu_i^\eta$$

where $i$ is the position of $d$ in the displayed ranked list $\pi_q$, $\nu_i$ is the examination probability on $i$ estimated by eye-tracking studies [27], and $\eta$ is a hyper-parameter that controls the severity of position bias. We sampled the probability of $d$ being perceived as relevant as

$$P(r_d = 1) = \epsilon + (1 - \epsilon) \frac{2^y - 1}{2^z - 1}$$

where $y$ is the annotated relevance label of $d$ and $\epsilon$ is a hyper-parameter controlling the probability of noisy clicks. Then, we simulated user behavior by generating synthetic clicks according to Eq. (3). For simplicity, we fixed $\eta = 1.0$ and $\epsilon = 0.1$ unless stated otherwise. It is worth noting that, in this paper, all experiments are conducted in environments with selection bias [46], which means that users can only see and click on the top 10 retrieved results for each query. We ignore the analysis of unbiased learning to rank without selection bias [20] as it is unrealistic in practice.

5.1.1 Model setup and evaluation. Our goal is to conduct a fair comparison of the unbiasedness of different ULTR algorithms with different learning paradigms. Therefore, we use a single type of ranking models for all ULTR algorithms discussed in this paper. Specifically, the ranking model we used is a multiple-layer perceptron network (MLP) with non-linear ELU activation functions. It has three hidden layers (with 512, 256, and 128 neurons) and batch normalization on each layer before activation. We implement the

\(^1\)We ignore the discussion of PairD here because it is not theoretically principled.

\(^2\)http://webscope.sandbox.yahoo.com
local loss function \( l \) as pairwise cross entropy loss [7] except for REM (which uses a sigmoid loss in the EM algorithm) [47] and DLA (which uses a softmax loss for dual learning) [5]. We used online EM [10] for EM algorithms and tuned learning rates from 0.01 to 0.05 for each ULTR algorithm. We set batch size as 256 and trained each algorithm for 10k steps. For offline learning, we created a synthetic production model by training a Ranking SVM model [23] with 1% data randomly sampled from the original training set. The production model is used to generate the ranked lists in offline click logs. For IPW, we conducted a separate online result randomization experiments with click simulation to estimate the inverse propensity weights. We created an unbiased learning to rank toolbox \(<\text{anonymous}>\) that include all the algorithms and experiment settings reported in this paper. The \(<\text{anonymous}>\)-toolbox will be released after the publication of this paper.

For evaluation, we trained and tested all models with the predefined training, validation, and test data in the Yahoo! dataset and used two standard ranking metrics – the normalized Discounted Cumulative Gain (nDCG) [21] and the Expected Reciprocal Rank (ERR) [12]. The former is constructed based on the theory of information gain while the later is built based on the model of user satisfaction in Web search. Ranking models are selected according to their nDCG@10 on the validation data in training. Each experiment are repeated for 5 times, and we compute the average metric value on top 3 and 10 results. Significant test is conducted based on the Fisher randomization test [42] with \( p \leq 0.05 \).

### 5.2 How do learning paradigms affect algorithm effectiveness?

Previous studies [20, 35] argue that online learning can help ranking algorithms explore a greater parameter space than offline learning, especially when users can only see and click a limited number of results in each query (i.e., selection bias). Thus, it is believed that

**H1:** *Online learning is better than offline learning for unbiased learning to rank in environments with selection bias.*

To test this hypothesis, we trained each ULTR algorithm with both online and offline learning paradigms and summarize the results in Table 2. Algorithms in Table 2 are grouped into three categories: (1) the counterfactual learning family, which includes IPW, REM, DLA, and PairD; (2) the bandit learning family, which includes DBGD and PDGD; (3) the production model (Prod.) used to generate offline click logs and the naive algorithm (NA) that directly trains ranking models with user clicks. For simplicity, we only show the significant test results with respect to NA with offline learning (NAoff) in Table 2. However, it’s worth noting that any differences larger or equal to 0.002 are statistically significant.

As shown in Table 2, IPW, DLA, and PDGD achieve the best performance among all the algorithms tested in our experiments. DLA and IPW achieve the highest nDCG@10 in offline learning and deterministic online learning, while PDGD performed the best in stochastic online learning. However, when comparing the best results of offline learning and online learning, we do not observe any significant differences. The performance of the best offline model (i.e., DLAoff) is mostly the same to the performance of the production model (Prod.) is mostly the same to the performance of the

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**Figure 2:** Effect of production models for offline learning. The best online model (i.e., PDGDQEs). This contradicts the hypothesis that online learning should be better than offline learning in environments with selection bias.

To validate this observation, we further manipulate the production model in offline learning to check whether the performance of the logging systems would affect the performance of ULTR algorithms. Specifically, we change the size of the training data for the production model and plot the results in Figure 2. The x-axis of Figure 2 represents the proportion of sampled training data for the production model in the original training data of Yahoo! dataset, and the y-axis is the nDCG@10 of each algorithm in offline learning. As depicted in Figure 2, the performance of the production model (Prod.) increases when the size of the sampled training data increases. Also, we observed that the performance of NA is positively correlated to the performance of the production system. In contrast, the nDCGs of IPW, DLA and REM are stable despite the change of production models. This indicates that the high performance of IPW and DLA in Table 2 is not a coincidence.

In fact, the phenomenon that offline unbiased learning to rank can achieve the same performance of online unbiased learning to rank is not surprising. As shown in Eq. (5), the unbiasedness of counterfactual learning algorithms does not concern about the distribution of the displayed ranked list \( \pi_q \). Theoretically speaking, methods such as IPW and DLA are guaranteed to find the best unbiased ranking models no matter how \( \pi_q \) is created, with or without selection bias. As shown in Table 2 and Figure 2, we indeed observe no significant difference between the best offline model and the best online model. Therefore, at least when the logging system of offline data is reasonable good (e.g., Prod. in Figure 2), we can reject the hypothesis H1.

### 5.3 How do learning paradigms affect algorithm robustness?

As discussed in Section 3 and 4, the unbiasedness of most counterfactual learning algorithms is independent from the distribution of displayed ranked list (i.e., \( P(\pi_q|q) \)), and the effectiveness of bandit learning algorithms is invariant to the inherited click bias in ranking loss \( l(f_0, c) \). Therefore, we have the following hypotheses:
Table 2: Comparison of unbiased learning-to-rank (ULTR) algorithms with different learning paradigms on Yahoo! LETOR data. Significant improvements or degradations with respect to the offline version of NA are indicated with +/-.

| Offline Learning (Off) | Stochastic Online Learning (OnS) | Deterministic Online Learning (OnD) |
|-----------------------|---------------------------------|-------------------------------------|
|                       | ERR@3  | nDCG@3  | ERR@10 | nDCG@10 | ERR@3  | nDCG@3  | ERR@10 | nDCG@10 | ERR@3  | nDCG@3  | ERR@10 | nDCG@10 |
| IPW                   | 0.423* | 0.651*  | 0.462* | 0.755*  | 0.426* | 0.686*  | 0.463* | 0.757*  | 0.425* | 0.681*  | 0.462* | 0.749*  |
| REM                   | 0.422* | 0.674*  | 0.459* | 0.741*  | 0.424* | 0.676*  | 0.461* | 0.745*  | 0.425* | 0.678*  | 0.462* | 0.747*  |
| DLA                   | 0.427* | 0.686*  | 0.464* | 0.754*  | 0.423* | 0.680*  | 0.461* | 0.750*  | 0.424* | 0.679*  | 0.461* | 0.749*  |
| PairD                 | 0.413  | 0.662*  | 0.451  | 0.737   | 0.423* | 0.679*  | 0.460* | 0.749*  | 0.415  | 0.653*  | 0.453  | 0.725*  |
| PDGD                  | 0.286* | 0.392*  | 0.263  | 0.533   | -      | -       | -      | -       | -      | -       | -      | -       |
| DBGD                  | -      | -       | -      | -       | 0.320  | 0.561   | 0.368  | 0.672   | 0.271  | 0.506   | 0.524  | 0.632   |
| IPW                   | 0.377  | 0.616   | 0.418  | 0.705   | -      | -       | -      | -       | -      | -       | -      | -       |
| REM                   | 0.377  | 0.616   | 0.418  | 0.705   | -      | -       | -      | -       | -      | -       | -      | -       |
| DLA                   | 0.377  | 0.616   | 0.418  | 0.705   | -      | -       | -      | -       | -      | -       | -      | -       |
| PDGD                  | 0.377  | 0.616   | 0.418  | 0.705   | -      | -       | -      | -       | -      | -       | -      | -       |
| NA                    | 0.414  | 0.664   | 0.452  | 0.738   | 0.421* | 0.670*  | 0.458* | 0.740*  | 0.417* | 0.662*  | 0.455* | 0.734*  |

**Figure 3:** Test performance with respect to training steps.

**H2:** The performance of counterfactual learning algorithms are invariant to learning paradigms (i.e., offline, stochastic online, or deterministic online), while the performance of bandit learning algorithms are sensitive to learning paradigms.

**H3:** With proper learning paradigms, bandit learning is more robust to variable click bias than counterfactual learning.

For the validation of H2, we report the performance of each ULTR algorithm with different learning paradigms in Table 2. As shown in the table, the performance of IPW, REM, and DLA is similar in offline learning, stochastic online learning, and deterministic online learning. Their differences in different learning paradigms are less than 1% in terms of nDCG and ERR. In contrast, we observe huge performance differences between the DBGD and PDGD using different learning paradigms. For example, the performance of PDGD in stochastic online learning is 42% and 4% better than PDGD in offline learning and deterministic online learning, respectively.

Figure 3 further plots the test performance of different algorithms in the training process. As we can see in the figure, when the number of training steps increases, the learning curves of IPW, DLA, and REM with offline, stochastic online, and deterministic online learning are smooth and similar to each other. The learning curve of PDGD with stochastic online learning (PDGD_{onS}) is also extremely low (i.e., η = 0.2) or high (i.e., η = 2.0). This supports the hypothesis that, with proper learning paradigms, bandit learning algorithms are more robust to variable click bias (H3).
5.4 Experiments with Real Data

In this section, we want to shed some lights on the actual performance of different ULTR algorithms on real click data. Due to the limit of our experiment resources, we are prohibited to do any types of online learning or online result randomization on real web search engines, so we focus on analyzing the performance of ULTR algorithms in offline settings. Specifically, we conduct offline experiments with the Tiangong dataset. Tiangong dataset contains 3 million search sessions sampled from real search engine traffic as well as the top 10 documents and clicks. Each query-document pair is represented with 33 standard ranking features extracted based on data harvested from multiple ranking functions. There are also studies that have hundreds of features (much more than those released in the original rankings of documents). Because the original rankings of documents are represented with 33 standard ranking features extracted based on the term statistics, BM25, and language modeling scores on urls, titles, and document content. Also, Tiangong provides a separate test set with 100 queries and corresponding top 100 documents with 5-level relevance annotations for evaluation purpose. To the best of our knowledge, this is the only public dataset that contains both user click data and human annotated relevance judgments.

5.4.1 Model setup and evaluation. Most settings of ranking models and loss functions are same to our experiments on synthetic data. However, because the ranking features in Tiangong are highly limited, the performance of a ranking model could be severely affected by parameter initialization. To guarantee the fairness of algorithm comparisons and the reproducibility of the experiments, we initialize all model parameters with a constant (i.e., 0.001). Also, we reduced the hidden layer sizes of MLP to 64 and 32, and tuned the learning rate from 0.0001 to 0.005. We used NDCG as our ranking metrics and reported the best test performance of each model after training. Numbers are averaged from 10 repeated experiments to guarantee their credibility. We ignore IPW and DBGD in this experiment because they require online result manipulations for estimating examination propensity or unbiased relevance signals.

5.4.2 Results. Our experiment results on the Tiangong dataset are summarized in Table 3. Here we report both the mean and the standard deviation of the 10 repeated runs for each algorithm. As shown in the table, our experiment results on Tiangong are quite different from those on the synthetic data. First, the performance of the naive algorithm (NA) that trains the ranking model with clicks directly is highly competitive. As discussed previously, training ranking models with biased clicks are essentially optimizing the original rankings of documents. Because the original rankings of documents in Tiangong are created with the production systems that have hundreds of features (much more than those released in the dataset), optimizing the original rankings would already produce very good results. Second, on Tiangong, REM performed the worst among all ULTR algorithms. Because the released features in Tiangong are simple text-matching features with limited expressive power, estimating relevance with pointwise loss functions using the 33 features on Tiangong is much more risky than using the 700 production features on the Yahoo! dataset. Thus, it is reasonable to observe that REM, which uses the sigmoid pointwise loss, obtained worse performance than other models that use pairwise loss.

Similar to those observed on the synthetic data, DLA achieved the best performance among all ULTR algorithms tested in our experiments. Also, it is the only model that outperforms NA. Different methods can be broadly classified into two families: the counterfactual learning family that originally adopts an offline learning paradigm and the bandit learning family that usually requires large-scale data with annotated relevance labels that are expensive and time-consuming to collect.

Table 3: Comparison of unbiased learning-to-rank (ULTR) algorithms on Tiangong data. Numbers are shown with their standard deviation in 10 repeated runs (i.e., ±x).

| ULTR Algorithms | nDCG@3 | nDCG@5 | nDCG@10 |
|-----------------|--------|--------|---------|
| REM             | 0.437 ± 0.011 | 0.439 ± 0.010 | 0.452 ± 0.004 |
| DLA             | 0.466 ± 0.002 | 0.458 ± 0.002 | 0.471 ± 0.001 |
| PairD           | 0.448 ± 0.009 | 0.453 ± 0.005 | 0.465 ± 0.002 |
| PDGD            | 0.430 ± 0.004 | 0.443 ± 0.003 | 0.464 ± 0.001 |
| NA              | 0.464 ± 0.004 | 0.455 ± 0.002 | 0.467 ± 0.001 |

from PairD and PDGD, DLA is theoretically guaranteed to find the unbiased ranking model in offline learning, and this advantage in theory has been reflected in the empirical experiments.

6 RELATED WORK

Learning to rank refers to the machine learning techniques for training a ranking model [32]. According to their loss functions [35], learning to rank algorithms can be categorized as pointwise [33], pairwise [7, 22], or listwise [4, 8, 9] approaches. While proven effective in improving the ranking performance [11], training learning-to-rank models usually requires large-scale data with annotated relevance labels that are expensive and time-consuming to collect.

To solve this problem, IR researchers have tried to leverage the implicit feedback from user behavior as an alternative data source for training ranking models. However, implicit feedback such as user clicks is noisy and affected by different kinds of biases [25, 29, 37, 43], e.g., the ranking position has a strong influence on where users click [24]. Thus, many studies have investigated how to extract unbiased and reliable relevance signals from biased click signals. For example, Joachims [22] proposed to treat clicks as preferences between clicked and skipped documents; Richardson et al. [40] formalize the Examination Hypothesis (Eq. 3) to model the position bias in the ranked list of ads. Accordingly, a series of click models (e.g. [12, 15, 16, 37, 43], also see [14]) have been proposed to model the examination probability and infer accurate relevance feedback from user clicks. Nonetheless, for reliable inference, click models usually require that the same query-document pair appears multiple times [36], making these approaches invalid for tail queries and many retrieval tasks (e.g. email search).

Another group of approaches, which is the focus of this study, tried to directly train an unbiased ranking model with biased user feedback. We refer to these approaches as the unbiased learning to rank (ULTR) approaches. As mentioned earlier, existing ULTR methods can be broadly classified into two families: the counterfactual learning family that originally adopts an offline learning paradigm and the bandit learning family that usually associate with an online learning paradigm. The key of counterfactual learning algorithms is the Inverse Propensity Weighting (IPW) [27, 46] and the estimation of examination propensity [5, 47]. Other than those introduced in Section 3.1, there are counterfactual learning algorithms that derive propensity estimation from online interleaving [27] or intervention data harvested from multiple ranking functions [3]. There are also studies on applying IPW to different behavioral biases such as the trust bias [2] and the recency bias [13]. The core of the bandit learning algorithms is the estimation of unbiased model gradients from online result manipulation and user feedback. There is extensive research on extending DBGD (in Section 3.2) with different result
exploration strategies [41, 45, 49, 50] and variance reduction techniques [44]. There are also algorithms developed independently with DBGD that combine click models [15, 18] with online bandit learning [28, 30, 31, 34, 50]. However, these click-model based methods usually estimate the utility of ranked documents on a per-query basis, which make them converge slower and less practical than other online algorithms such as PDGD (in Section 3.2).

While separate studies and tutorials on offline ULTR [6] and online ULTR [17] have been presented recently, to the best of our knowledge, there is no comprehensive analysis on their theoretical connections and differences. Jägerman et al. [20] tried to compare counterfactual learning algorithms and bandit learning algorithms empirically, but they are confined by the stereotype that the former must be used offline and thus provided limited insights in theory. Our study in this paper is timely and important for the understanding and applications of ULTR in practice.

7 CONCLUSION

In this paper, we discuss the differences and connections between unbiased learning to rank algorithms proposed in offline settings and online settings. We show that the existing studies on offline unbiased learning algorithms (i.e., the counterfactual learning family) and the online learning to rank algorithms (i.e., the bandit learning family) are essentially solving the same problem from two theoretical perspectives. We formally evaluate six state-of-the-art unbiased learning to rank algorithms and show how different learning paradigms would affect the theoretical foundations and empirical effectiveness of each algorithm.

As demonstrated in Section 3 and 5, the unbiasedness of counterfactual learning algorithms are invariant to the distribution of the displayed results while the unbiasedness of bandit learning algorithms are more robust to the variance of click bias. Whether these properties benefit or hurt their applications in practice varies from cases to cases. For example, when user satisfaction is not sensitive to the quality of result ranking and we have full control over the result pages, bandit learning algorithms may produce more robust ranking models than counterfactual learning. However, in search engines where no single model can fully control the final displayed ranked lists (e.g., the result pages of most commercial search engines are the combination of ads and organic results produced by ranked lists (e.g., the result pages of most commercial search engines)), counterfactual learning is preferable to bandit learning because the effectiveness of the later is extremely sensitive to the final distribution of the displayed result lists.

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