Laboratory observation of a nonlinear interaction between shear Alfvén waves

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Abstract

An experimental investigation of nonlinear interactions between shear Alfvén waves in a laboratory plasma is presented. Two Alfvén waves, generated by a resonant cavity, are observed to beat together, driving a low frequency nonlinear pseudo-mode at the beat frequency. The pseudo-mode then scatters the Alfvén waves, generating a series of sidebands. The observed interaction is very strong, with the normalized amplitude of the driven pseudo-mode comparable to the normalized magnetic field amplitude ($\delta B/B$) of the interacting Alfvén waves.

PACS numbers: 52.35.Bj, 52.35.Mw
The Alfvén wave is the fundamental low-frequency normal mode of a magnetized plasma and is a ubiquitous feature of space plasmas (e.g., the auroral ionosphere and the solar wind) and laboratory plasmas (e.g., tokamaks and linear devices). The linear properties of these waves have recently been explored in detailed laboratory experiments. However, the nonlinear behavior of Alfvén waves has not been investigated in the laboratory. From a weak turbulence point of view, nonlinear interactions between Alfvén waves are responsible for the cascade of energy in magnetohydrodynamic (MHD) turbulence. In the incompressible MHD approximation an anisotropic energy cascade results from interactions between counter-propagating shear Alfvén waves. In MHD turbulence theories, interactions are generally assumed to be local in wavenumber space and density fluctuations are assumed to play only a passive role in the cascade. However, nonlocal interactions between shear waves can generate beat-wave driven density fluctuations. This mechanism is essential in decay instabilities such as the parametric and modulational decay instabilities, where the pump and daughter Alfvén waves beat together to drive an ion acoustic wave or a nonlinear pseudo-mode, respectively. In addition, density fluctuations are an integral part of Alfvén waves with small perpendicular scale (dispersive kinetic or inertial Alfvén waves) and can therefore become active participants in the cascade as it approaches the dissipation scale. Density fluctuations generated at small scale can scatter large scale Alfvén waves, and could therefore influence the cascade at larger scales.

In this Letter, an observation of a nonlinear interaction between shear Alfvén waves in a laboratory plasma is presented. Large amplitude shear Alfvén waves are generated using a resonant cavity. In circumstances where two waves are simultaneously emitted by the cavity, production of sideband waves and low frequency fluctuations at the sideband separation frequency is observed. The interaction is identified as a beat-wave interaction between co-propagating shear waves, where a nonlinear pseudo-mode is driven at the beat frequency. The psuedo-mode then scatters the Alfvén waves, generating a spectrum of sidebands. The amplitude of the driven pseudo-mode is substantial, much larger than would be predicted by simple magnetohydrodynamic (MHD) theory.

The experiments were performed in the upgraded Large Plasma Device (LAPD), which is part of the Basic Plasma Science Facility (BaPSF) at UCLA. LAPD is an 18 m long, 1 m diameter cylindrical vacuum chamber, surrounded by 90 magnetic field coils. Pulsed
plasmas (∼10 ms in duration) are created at a repetition rate of 1 Hz using a barium oxide coated nickel cathode source. Typical plasma parameters are $n_e \sim 1 \times 10^{12}$ cm$^{-3}$, $T_e \sim 6$ eV, $T_i \sim 1$ eV, and $B < 2kG$. The experiments were performed using helium as a working gas. LAPD plasmas have values of $\beta = 2\mu_e n k_B (T_e + T_i)/B^2$ comparable to the electron-to-ion mass ratio, $\beta \sim m_e/M_i$ (typical $\beta$ values range from $5 \times 10^{-5}$ to $1 \times 10^{-3}$). In these experiments, $\beta \gtrsim m_e/M_i$, and the electron thermal speed is therefore larger than the Alfvén speed ($v_{th,e} > v_A$). The ion sound gyroradius, $\rho_s$, in these experiments is $\sim 0.5 - 1.5$ cm.

Large amplitude shear Alfvén waves are generated using the Alfvén wave MASER [19, 20]. The nickel cathode and semi-transparent molybdenum anode of the plasma source in LAPD define a resonant cavity from which spontaneous shear Alfvén wave MASER emission at $f \sim 0.6f_{ci}$ is observed, with $k_\perp \rho_s$ typically $0.3 - 0.5$. The emitted shear waves are eigenmodes of the cylindrical LAPD plasma column, with $m = 0$ or $m = 1$ azimuthal mode number. The amplitude of the waves can be as large as $\delta B/B \gtrsim 1\%$ and can be controlled through changing the plasma source discharge current. Emission of the waves typically begins early in the LAPD discharge, as the source region current is ramping up and plasma parameters and profiles are evolving. During this early phase the MASER emission is observed to “mode hop,” where a sudden transition in the frequency of emission is observed [20]. The top panel of Figure 1 shows the FFT (fast Fourier transform) power spectrum versus time for magnetic field fluctuations measured in the main LAPD plasma column (∼7 m from the source region) along with a line plot of the spectrum at $t = 8$ms (helium discharge, 700G, $f_{ci} = 266$ kHz). Two mode hops are apparent during the early evolution of the magnetic field spectrum, and the second hop is identified through mode structure measurements as a transition from $m = 0$ to $m = 1$. After the mode hop, the $m = 0$ mode does not completely disappear, but persists at a much lower level as is apparent in the accompanying line plot. An additional frequency is observed in the spectrum, located above the $m = 1$ frequency by the difference frequency between the $m = 0$ and $m = 1$ modes. This mode appears to be a sideband of the primary $m = 1$ mode which could be generated by a weak nonlinear interaction between the $m = 0$ and $m = 1$ mode.

It was found that much stronger sideband generation is observed during high-current discharges when the length of the main LAPD plasma column is shortened to 10 m by terminating the plasma with an electrically-floating aluminum plate. The bottom panel of Figure 1 shows FFT power spectra of measured magnetic field ($B$) and Langmuir probe
FIG. 1: Top panel: typical power spectrum of shear Alfvén waves during MASER emission. Bottom: $B$ and $I_{\text{sat}}$ power spectra during MASER emission with the floating plate closed.

measured ion saturation current ($I_{\text{sat}} \propto n_e \sqrt{T_e}$) during MASER emission with the floating plate closed. The magnetic spectrum is considerably more complicated in this case, showing fluctuations at a number of discrete frequencies and an additional mode hop (which is likely due to a change in radial mode structure). In the steady state phase of the discharge ($t > 6$ ms), there are two strong modes present (identified as $m = 0$ and $m = 1$), surrounded by a number of sidebands. Simultaneously, low frequency $I_{\text{sat}}$ fluctuations are observed at the sideband separation frequency and harmonics. These observations are consistent with the following scenario: (1) Simultaneous emission of two large amplitude shear modes ($m = 1$ and $m = 0$) from the cavity occurs. (2) The two shear waves beat together, driving a low frequency fluctuation at the beat frequency. (3) The low frequency fluctuation then scatters the incident shear waves, leading to the generation of the observed sidebands. Our conjecture is that termination of the high discharge current plasma with the floating plate induces simultaneous spontaneous emission of large amplitude $m = 0$ and $m = 1$ modes, leading to the proposed scenario.

In order to conclusively determine the nature of the sideband generation, the capability to externally drive the resonant cavity was developed. To excite the cavity, oscillating currents...
are driven between the anode and cathode using external power supplies. Figure 2(a) shows the cavity response (magnitude of the emitted shear wave, measured in the main plasma column) as a function of drive frequency (normalized to $f_{ci}$) for cases with the floating plate open and closed. The resonant nature of the cavity emission is clear, as is the modification in the properties of the cavity upon closing the floating plate. The external drive couples primarily to the $m = 0$ mode, but the additional peak observed with a closed floating plate corresponds to the $m = 1$ mode frequency, and may indicate coupling to that mode. The cavity was then operated so that the spontaneous emission of only one mode was observed ($m = 1$), and a second mode ($m = 0$) was externally excited, allowing an investigation of the interaction between the two shear waves with a variable separation frequency. These experiments were conducted with the floating plate both open and closed. Lower discharge currents were necessary to limit the spontaneous cavity emission to a single mode with the plate closed.

Figure 2(b) shows the magnetic power spectrum versus drive frequency during a frequency separation scan with the floating plate closed. The spontaneous emission ($m = 1$) is fixed during the scan (the horizontal feature at $f \sim 0.66f_{ci}$) while the frequency of the driven mode is changed. The production of sidebands, in particular the first upper sideband, is evident over a wide range of frequency separations between the driven ($m = 0$) and spontaneous ($m = 1$) waves. The change in sideband amplitude versus frequency separation is primarily due to changes in the externally driven mode amplitude with frequency (due to the $Q$ of the cavity). Figure 2(c) shows a line plot of the magnetic power spectrum when the drive frequency is at the peak of the $m = 0$ resonance, $f_{drive} = 0.61f_{ci}$. A number of sidebands around the driven ($m = 0$) and spontaneous ($m = 1$) shear waves are visible. Figure 2(d) shows the $I_{sat}$ power spectrum at the same drive frequency, which exhibits fluctuations at the sideband separation frequency and harmonics. Figure 2(e) and (f) show $B$ and $I_{sat}$ spectra during driven-spontaneous interaction with the floating plate open. Shifts in the absolute frequency and frequency separation of the $m = 0$ and $m = 1$ modes are evident relative to the closed floating plate case. Aside from these differences in the linear cavity response, the observed spectra are quite similar to those observed in the closed floating plate case: magnetic sidebands are evident, as are low frequency $I_{sat}$ fluctuations at the sideband separation frequency. This suggests that the primary role of the closed floating plate in the data shown in Figure 1 is to modify the cavity properties so that two large amplitude shear
wave modes are emitted simultaneously. The mechanism by which the floating plate causes this multi-mode emission is not yet understood, but it is known that plasma parameters and profiles are altered relative to the non-terminated case. The presence of the floating plate also allows for the possibility of reflection of the Alfvén waves and therefore the development of a counter-propagating population of waves. In addition, the closed floating plate and the anode define a second resonant cavity which can support shear Alfvén wave modes. The qualitative similarity between the closed and open floating plate cases shown in Figure 2 suggests that the second cavity and the associated reflected shear waves are not essential for the interaction, and that instead it is more likely an interaction between co-propagating shear waves emitted by the cavity.

![Figure 2](image_url)

**FIG. 2:** (a) Cavity response as a function of normalized external drive frequency. (b) Magnetic power spectrum due to interacting shear waves during a frequency separation scan. For drive frequency on the $m = 0$ resonance: (c) Magnetic power spectrum and (d) ion saturation current spectrum with the floating plate closed. (e) Magnetic power spectrum and (f) $I_{\text{sat}}$ power spectrum with the floating plate open.

Figure shows the structure of the fluctuations during an interaction between driven and spontaneously emitted cavity shear modes. Due to the shot-to-shot phase variation in
the spontaneous emission, cross-correlation techniques were used to determine the structure of the interacting modes. Two probes were used, one fixed at one spatial location (to provide a phase reference) while the second was moved shot-to-shot to 961 positions in the plane perpendicular to the background field by computer control. The two probes were separated axially (along the magnetic field) by approximately 1 m. The driven MASER wave has an $m = 0$ structure, with a hollow amplitude profile and a single current channel. The spontaneous MASER exhibits primarily an $m = 1$ structure, with two counter-rotating current channels and a peaked amplitude profile between the two. The structure of the upper sideband has some similarity to the spontaneous $m = 1$ emission, but is more concentrated in the periphery, which may be consistent with $m > 1$ mode content. The pattern of the $I_{\text{sat}}$ fluctuation is more centrally localized, and is not readily identifiable as any single azimuthal mode number.

![Diagram](image)

FIG. 3: Structure of the driven MASER, spontaneous MASER, upper sideband, and low frequency $I_{\text{sat}}$ fluctuation in the plane perpendicular to $B$.

Figure 4 shows raw $B$ and $I_{\text{sat}}$ fluctuation measurements during an interaction experiment where both shear waves are driven with the floating plate open (i.e. there is no spontaneous emission from the cavity). In this case both modes are $m = 0$ and each driven slightly off resonance to achieve the desired frequency separation. The raw data shows that the beat wave interaction also occurs in this case, indicating that the interaction is not dependent
on the presence of two different azimuthal modes. Simultaneous measurements of $I_{\text{sat}}$ at two axial locations separated by 2.88 m are shown from which a parallel phase velocity for the low frequency fluctuation can be computed. The average parallel phase velocity in the region of constant plasma density is measured to be $294 \pm 35$ km/s, where the Alfvén speed is $\sim 550$ km/s and the ion acoustic speed is $\sim 13$ km/s. The computed phase velocity is consistent with three-wave matching rules ($\omega_1 + \omega_2 = \omega_3$ and $\vec{k}_1 + \vec{k}_2 = \vec{k}_3$) using the kinetic shear Alfvén wave dispersion relation, $k_{\parallel}^2 = \omega^2 (1 - \omega^2/\omega_{ci}^2 + k_{\perp}^2 \rho_s^2)$, where $k_{\perp} \rho_s$ is taken from a spatial fit of the shear wave eigenmode structure to be $\sim 0.38$ for the two interacting Alfvén waves \cite{20}. The low frequency fluctuation does not correspond to a linear plasma wave (e.g. an ion acoustic wave) and is instead a beat-wave driven nonlinear perturbation or psuedo-mode. While the observations are not consistent with a decay instability, it should be pointed out that the modulational instability involves a similar set of modes: in this instability, the pump shear wave decays into a forward-propagating sideband wave and a pseudo-mode at the sideband separation frequency.

![FIG. 4: Top: raw magnetic pick-up loop signal during beat wave experiment. Bottom: simultaneous $I_{\text{sat}}$ measurements at two different axial locations.](image)

The amplitude of the $I_{\text{sat}}$ fluctuation is observed to scale approximately bilinearly with the amplitude of the two interacting shear waves. The normalized amplitude is quite large with $\delta I_{\text{sat}}/I_{\text{sat}} \sim 1 - 10\% \gtrsim \delta B/B$, substantially larger than the normalized density perturbation which would be predicted by simple ideal MHD theory ($\delta n/n \sim (\delta B/B)^2$). In
addition to density, \( I_{\text{sat}} \) is sensitive to electron temperature and to any population of fast electrons (those with energies greater than the negative probe bias \( \sim 65V \)), and therefore fluctuations in these quantities might explain the magnitude of the observed \( I_{\text{sat}} \) signal. However, preliminary microwave interferometer measurements indicate that there are significant line-average density fluctuations associated with the psuedo-mode, and that it is therefore largely a fluctuation in density. Future work will focus on measurements to more accurately determine the magnitude of \( \delta n/n \) in the psuedo-mode and compare it with more comprehensive theoretical predictions.

In summary, a nonlinear beat-wave interaction between shear Alfvén waves has been observed. Two resonant-cavity-produced, co-propagating shear waves are observed to beat together, resulting in a low frequency fluctuation at the beat frequency and the subsequent creation of Alfvénic sidebands. The low frequency fluctuation is identified as a nonlinearly driven psuedo-mode generated by the beat between the two co-propagating shear waves. Counter-propagating interactions will be explored in future experiments, where beat-wave driven ion acoustic waves may be possible.

The authors would like to thank S. C. Cowley, W. Gekelman, J. E. Maggs, and G. J. Morales for invaluable discussions. This work was completed using the Basic Plasma Science Facility at UCLA, which is funded by DOE and NSF. This work was supported by DOE grant DE-FG02-02ER54688 and by DOE Fusion Science Center Cooperative Agreement DE-FC02-04ER54785.

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