Measurement matrix design for colocated MIMO radar imaging based on Compressive Sensing

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Abstract. Compressive Sensing (CS) is recently used in the colocated MIMO radar to reconstruct the high resolution range profiles (HRRPs) of the target. By exploiting the joint-block sparsity of the HRRPs, the reconstruction result is improved significantly. In this paper, a compressive measurement matrix is designed to reduce the amount of data. An objective for measurement matrix optimization is presented by considering the block sparsity of the HRRPs, and the Weighted Coherence Minimization algorithm is used to optimize the measurement matrix. Simulation results show that the optimized measurement matrix is superior to the Gaussians randomly measurement matrix which doesn’t utilize the block sparsity of the HRRPs.

1. Introduction

Radar imaging for aerospace targets is useful for both civil and military use. The well-known inverse synthetic aperture radar (ISAR) imaging technique [1] suffers from the complex motion compensation. Thus, it is hardly applied to the case of maneuvering target imaging. Multiple-input multiple-output (MIMO) radar [2] is a new structure radar, which may be used for imaging. Multiple independent signals are transmitted by multiple transmitters and the target echoes are received by multiple receivers. A large virtual aperture is obtained after signal processing. So the high resolution is possible even via the short coherent processing interval (CPI). In [3,4], the MIMO technique is utilized to improve the imaging result of ISAR with the short CPI. The two or three dimension (2D or 3D) images are obtained based on MIMO radar by only transmitting one pulse waveform in [5,6]. In these papers, the multiple waveforms are supposed to be orthogonal perfectly. However, the waveforms in the same frequency are hardly fully orthogonal in practice. The mutual interference of the multiple waveforms will seriously degrade the quality of the target image. In [7-9], some new orthogonal waveforms, such as the short-term shift orthogonal waveform and orthogonal-frequency-division multiplexing (OFDM) chirp waveform, are proposed to improve the imaging quality. However, the results are still unsatisfactory if further processing like digital beam forming (DBF) is not used.

Compressive Sensing (CS) is a new theory which utilizes the sparsity of signal and is able to reconstruct the signal only with the partial samplings. Because the radar target is usually composed of a few strong scatterers, so the target is sparse. In recent years, CS is used in MIMO radar to improve the imaging result and reduce the samplings. In [10,11], an integrated spare model is built for MIMO radar imaging. By using the target sparsity, a good target image is obtained when the waveforms are not orthogonal. In the integrated spare model, the target image is reshaped into a vector and then be reconstructed. When the image dimension is large, huge memory spaces and computation time are required. In [12], a sequential spare model is built for MIMO radar imaging. Therein, the high
resolution range profiles (HRRPs) of the target are reconstructed firstly and the 2D image is obtained later. The quality of the reconstructed HRRPs is vitally important for the final result. In the paper, the block and joint sparsity of the HRRPs are explored and a joint-block spare recovery algorithm is proposed to enhance the reconstruction quality of the HRRPs. As a result, a satisfying 2D image is obtained.

Generally speaking, CS is composed of two parts, i.e., compressive measurement and sparse reconstruction. All the CS-related papers above just use the ‘sparse reconstruction’. By using the compressive measurement, the amount of data for MIMO imaging can be reduced. Gaussian random measurement matrix (GRMM) is usually used in CS recovery because GRMM is incoherent with any basis matrix in high probability. Although the GRMM typically satisfies the mutual incoherence property (MIP), which guarantees sparse signal recovery, it is not necessarily the best matrix for the sparse signal which has the special sparse structure. In [13], a Weighted Coherence Minimization (WCM) method is presented to design the measurement matrix for the block sparsity. In this paper, the problem of measurement matrix design for MIMO radar imaging is studied based on the sparse model in [12]. The block sparsity of the HRRPs is considered and the WCM method is introduced to design the measurement matrix for MIMO radar imaging. Simulation results show that the designed measurement matrix produces better imaging result than the GRMM.

2. MIMO radar imaging model

In this Section, a 1D linear array in the colocated MIMO radar is considered. The array configuration has $M$ transmitters and $N$ receivers. The inter-element distance of transmitters is $Nd$ and that of receivers is $d$. Let $\phi_m$ be the $m$th baseband transmit signal vector which has the length $L$. Divide the range scope for imaging into $K$ grids. Define the range kernel matrix of the $k$th range grid as

$$T^k = \begin{bmatrix} 0 & \ldots & 0 \\ I_{L \times L} \\ 0 & \ldots & 0 \end{bmatrix} \in \mathbb{C}^{(K+L) \times L}.$$  

And then the transmit waveforms matrix can be denoted as

$$\Phi = [\Phi_0, \ldots, \Phi_k, \ldots, \Phi_{K-1}] \in \mathbb{C}^{(K+L) \times MK}, \text{ where } \Phi_k = T^k \cdot [\phi_0, \ldots, \phi_m, \ldots, \phi_{M-1}] \in \mathbb{C}^{(K+L) \times M}.$$  

According to [12], the target echo can be expressed in a matrix form as

$$Y = \Phi \Theta$$  

(1)

where $Y = [y_0, \ldots, y_n, \ldots, y_{N-1}] \in \mathbb{C}^{(K+L) \times N}$ is the receive signal matrix and $y_n$ is the receive signal vector at the $n$th receiver; $\Theta = [\bar{\theta}_0, \ldots, \bar{\theta}_n, \ldots, \bar{\theta}_{N-1}] \in \mathbb{C}^{MK \times N}$ is the target range profiles, where

$$\bar{\theta}_n = [\rho_{0,n}^T, \ldots, \rho_{m,n}^T, \ldots, \rho_{(K-1)n,n}^T] \in \mathbb{C}^{MK \times 1}$$

is the target range profile corresponding to the $n$th receiver, $\rho_{k,n} = [\rho_{k,0n}, \ldots, \rho_{k,mn}, \ldots, \rho_{k,(M-1)n}] \in \mathbb{C}^{M \times 1}$, where $\rho_{k,mn}$ denotes the complex scattering coefficient of $k$th range grid in the $m$th transmit and $n$th receive channel.

Considering the step of compressive measurement in CS, a measurement $\Psi \in \mathbb{C}^{D \times (K+L)}$, where $D$ is the number of compressive measurement and $D < (K + L)$, is used to compress the target echo. And then the compressed echo is expressed as

$$Y' = \Psi Y = \Psi \Phi \Theta$$  

(2)

In order to image the target, it should recover the target range profile $\Theta$ from the compressed echo $Y'$. It is proved in [12] that $\Theta$ holds the joint sparsity, i.e., the indexes of nonzero elements are not changing among different $\bar{\theta}_n$ ($n = 0, 1, \ldots, N-1$) and nonzero elements of $\bar{\theta}_n$ occur in clusters. $\rho_{k,nn}$ is the $k$th block and the block length is the transmit waveform number $M$. Therefore, $\Theta$ can be solved by the following optimization criterion...
\[
\min_{\Theta} \|\Theta\|_{F,p}, \quad \text{s.t.} \|Y' - \Psi\Phi\Theta\|_p < \varepsilon
\]  
(3)

where \(\varepsilon\) denotes the noise level and \(\|\cdot\|_p\) is a mixed matrix norm which is defined as \[12]\]

\[
\|\Theta\|_{F,p} = \left\|\rho_{\omega}\right\|_p = \left(\sum_{k=1}^{\rho} \left(\left\|\rho_{\omega}\right\|_p\right)^\frac{1}{p}\right)^\frac{1}{1/p}, \quad 0 \leq p \leq 1.
\]

3. Measurement matrix design via WCM

In order to recover \(\Theta\) from \(Y'\) exactly, the mutual coherence of the columns of matrix \(\Psi\Phi\) should be small enough. Usually, the mutual coherence is ensured by minimizing the Frobenius norm \(\|\Psi\Phi\|_{F,1}\). When the transmit waveforms matrix \(\Phi\) is fixed, the measurement matrix \(\Psi\) can be designed by solving the minimization problem. However, the former criterion doesn’t consider the block sparsity of \(\Theta\), so the resultant \(\Psi\) may be not the best one for \(\Theta\) and will not produce the perfect result. In the following, a criterion which takes the block sparsity into account \[13]\] is used. Let \(G = (\Psi\Phi)^\top \Psi\Phi \in \mathbb{C}^{MK \times MK}\). According to the block structure of \(\Theta\), \(G\) can be seen as a group of \(K \times K\) sub-blocks and each sub-block has the size of \(M \times M\). Define a total interblock coherence

\[
\mu_{\alpha} = \sum_{j=1}^{K} \sum_{i=1}^{K} \left|G[i,j]\right|^2
\]

(4)

where \(G[i,j]\) denotes the \((i,j)\)th sub-block of \(G\), \(i = 1, \cdots, K; j = 1, \cdots, K\). Define a total subblock coherence

\[
v' = \sum_j \left|G[j,j]\right|^2 - \sum_m \left(G_m^m\right)^2
\]

(5)

where \(G_m^m\) denotes the \(m\)th diagonal entry of \(G\), \(m = 1, \cdots, MK\). Further define a normalization penalty

\[
\eta = \sum_m \left(G_m^m - 1\right)^2
\]

(6)

Therefore, a criterion for measurement matrix design is shown as

\[
\min_{\Psi} \frac{1}{2} \eta + (1-\alpha)\mu_{\alpha} + \alpha v'
\]

(7)

where \(\alpha \in (0,1)\) is a parameter which is to balance the total interblock coherence and the total subblock coherence.

In the next, the WCM algorithm is used to solve the above minimization problem. Let

\[
f(G) = \frac{1}{2} \eta(G) + (1-\alpha)\mu_{\alpha}(G) + \alpha v'(G) = \frac{1}{2} \left[ u_{\alpha}(G) \right] + (1-\alpha) \left[ u_{\mu}(G) \right] + \alpha \left[ u_v(G) \right]
\]

(8)

where

\[
\begin{align*}
\alpha_{\alpha}(G)[i,j] &= \begin{cases} G[i,j] - 1, & i = j, a = b \\
0, & \text{else}
\end{cases}, & \alpha_{\mu}(G)[i,j] &= \begin{cases} G[i,j], & i \neq j \\
0, & \text{else}
\end{cases}, \\
\alpha_{v}(G)[i,j] &= \begin{cases} G[i,j], & i = j, a \neq b \\
0, & \text{else}
\end{cases}, & \alpha_{v}(G)[i,j] \text{ denotes the } (a,b)\text{th entry of } G[i,j].
\end{align*}
\]

(8) is further expressed as
$$f(G) = \frac{1}{2} \| G - h_G(G) \|_F^2 + (1-\alpha) \| G - h_G(G) \|_F^2 + \alpha \| G - h_G(G) \|_F^2$$

(9)

where

$$h_G(G)[i,j] = \begin{cases} 1, & i = j, a = b \\ G[i,j], & \text{else} \end{cases}$$

and

$$h_{\mu}(G)[i,j] = \begin{cases} 0, & i = j, a \neq b \\ G[i,j], & \text{else} \end{cases}$$

A iteration process can be used to solve (9). In the $\gamma$th iteration, the objective is expressed as

$$g(G, G^{(\gamma)}) = \frac{1}{2} \| G - h_G(G^{(\gamma)}) \|_F^2 + (1-\alpha) \| G - h_G(G^{(\gamma)}) \|_F^2 + \alpha \| G - h_G(G^{(\gamma)}) \|_F^2$$

(10)

where $G^{(\gamma)} = \Phi^{(\gamma)} \Psi^{(\gamma)} \Phi$ , $\Psi^{(\gamma)}$ is the measurement matrix from the previous iteration.

It is proved in [13] that the solution of the problem $\min_{\Psi^{(\gamma)}} g(G, G^{(\gamma)})$ is

$$\Psi^{(\gamma+1)} = A_0 D \Psi^{(\gamma)}$$

(11)

where $U A U^*$ is the eigenvalue decomposition of $\Phi \Phi^*$, $A_0$ and $V_0$ is the top $D$ eigenvalues and the corresponding $D$ eigenvectors of $A^{0.5} U^* \Phi h(G^{(\gamma)}) \Phi^* A^{0.5}$, and

$$h_i(\gamma) = \frac{2}{3} \left[ \frac{1}{2} h_i(\gamma) + (1-\alpha) h_\mu(\gamma) + \alpha h_\eta(\gamma) \right]$$

(12)

In the initialization, let $\Psi^{(0)} = [I_D, 0], A^{0.5} U^*$ and $\gamma = 0$. Update $G^{(\gamma)}$ and $\Psi^{(\gamma)}$ circularly until the convergence condition is met and the final measurement matrix is obtained.

4. Simulation result

In the experiment, a linear array with 4-transmitters and 20-receivers is considered. The inter-element distances of receivers and transmitters are 3 m and 60 m, respectively. Four polyphase coded signals with code length of 450 are transmitted. The bandwidth is 600 MHz and the center frequency is 10 GHz. A target within the range scope of 50 m is used and the distance between the target and the radar is 5 km. The length of the received target echo is 650. In the next, the measurement matrix with the size of $520 \times 650$ is used to compress the echo. The algorithm with $\alpha=0.9$ is conducted with 20 iterations and the measurement matrix is designed. A Gaussians randomly measurement matrix is used for comparison. The grammar matrices $G = (\Psi \Phi)^* \Psi \Phi$ (with the zero diagonal elements) of the Gaussians randomly measurement matrix and the designed measurement matrix are shown in Fig. 1 and Fig. 2, respectively. To see more clearly, the portions (0~50 rows/columns) of the grammar matrices are also shown. It can be seen the designed measurement matrix has the lower total subblock coherence since we set $\alpha=0.9$. But the Gaussians randomly measurement matrix doesn’t weight the coherence of the block.
Figure 1. Grammar matrix of the Gaussians randomly measurement matrix: (a) the whole of the matrix; (b) the portion of the matrix.

Figure 2. Grammar matrix of the designed measurement matrix: (a) the whole of the matrix; (b) the portion of the matrix.

The JB-SL0 algorithm \cite{12} is used to reconstruct the range profiles of the target and then the 2D image is obtained via the SL0 algorithm. The imaging results of the Gaussians randomly measurement matrix and the designed measurement matrix are shown in Fig. 3 and Fig. 4, respectively. The image contrast (IC) and image entropy (IE) of Fig. 3(b) are IC =8.6302, IE =6.9034. And the image indicators of Fig. 4(b) are IC =8.8685, IE =6.8853. The larger IC value and the lower IE value represent the better image quality. Therefore, the designed measurement matrix results in a better imaging result in comparison with the Gaussians randomly measurement matrix.

Figure 3. Imaging result via the Gaussians randomly measurement matrix: (a) range profiles; (b) 2D image.
5. Conclusion

Measurement matrix is designed for colocated MIMO radar imaging. The block sparsity, but not just the sparsity, of the target range profiles is considered when designing the measurement matrix. Simulation result shows the better imaging result is obtained by using the designed measurement matrix in comparison with the Gaussians randomly measurement matrix. However, the improvement is slight. Further work will be done on designing transmitting waveform and measurement matrix jointly, which is expected to improve the performance further.

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