The contribution of off-shell gluons to the longitudinal structure function $F_L$

A.V. Kotikov

Bogoliubov Laboratory of Theoretical Physics
Joint Institute for Nuclear Research
141980 Dubna, Russia

A.V. Lipatov

Department of Physics
Lomonosov Moscow State University
119899 Moscow, Russia

N.P. Zotov

Skobeltsyn Institute of Nuclear Physics
Lomonosov Moscow State University
119992 Moscow, Russia

Abstract

We present results for the structure function $F_L$ for a gluon target having nonzero transverse momentum square at order $\alpha_s$. The results of double convolution (with respect to Bjorken variable $x$ and the transverse momentum) of the perturbative part and the unintegrated gluon densities are compared with recent experimental data for $F_L$ at low $x$ values and with predictions of other approaches.

PACS number(s): 13.60.Hb, 12.38.Bx, 13.15.Dk
1 Introduction

The basic information on the internal structure of nucleons is extracted from the process of deep inelastic (lepton-hadron) scattering (DIS). Its differential cross-section has the form:

\[ \frac{d^2 \sigma}{dx dy} = \frac{2\pi \alpha_s^2}{x Q^4} \left[(1 - y + y^2/2) F_2(x, Q^2) - \left(y^2/2\right) F_L(x, Q^2)\right], \]

where \( F_2(x, Q^2) \) and \( F_L(x, Q^2) \) are the transverse and longitudinal structure functions (SF), respectively, \( q^\mu \) and \( p^\mu \) are the photon and the hadron 4-momentums and \( x = Q^2/(2pq) \) with \( Q^2 = -q^2 > 0 \).

The longitudinal SF \( F_L(x, Q^2) \) is a very sensitive QCD characteristic because it is equal to zero in the parton model with spin \(-1/2\) partons. Unfortunately, essentially at small values of \( x \), the experimental extraction of \( F_L \) data is required a rather cumbersome procedure (see [1, 2], for example). Moreover, the perturbative QCD leads to some controversial results in the case of SF \( F_L \). The next-to-leading order (NLO) corrections to the longitudinal coefficient function, which are large and negative at small \( x \) [3, 4], need a resummation procedure that leads to coupling constant scale higher essentially then \( Q^2 \) (see [6, 4, 7]).

Recently there have been important new data [13]-[18] of the longitudinal SF \( F_L \), which have probed the small-\( x \) region down to \( x \sim 10^{-2} \). Moreover, the SF \( F_L \) can be related at small \( x \) with SF \( F_2 \) and the derivation \( dF_2/d\ln(Q^2) \) (see [19]-[21]). In this way most precise predictions based on data of \( F_2 \) and \( dF_2/d\ln(Q^2) \) (see [15] and references therein) can be obtained for \( F_L \). These predictions can be considered as indirect ‘experimental data’ for \( F_L \).

In this paper for analysis of the above data we will use so called \( k_T \)-factorization approach [22, 23, 24] based on BFKL dynamics [12] (see also recent review [11] and references therein). In the framework of the \( k_T \)-factorization approach, a study of the longitudinal SF \( F_L \) has been done firstly in Ref. [25], where small \( x \) asymptotics of \( F_L \) has been evaluated using the BFKL results for the Mellin transform of the unintegrated gluon distribution and the longitudinal Wilson coefficient functions has been calculated analytically for the full perturbative series at asymptotically small \( x \) values. Since we want to analyze \( F_L \) data in a broader range at small \( x \), we will use parameterizations of the unintegrated gluon distribution function \( \Phi_g(x, k_{T}^2) \) (see Section 3).

The unintegrated gluon distribution \( \Phi_g(x, k_{T}^2) \) (\( f_g \) is the (integrated) gluon distribution in proton multiplied by \( x \) and \( k_T \) is the transverse part of gluon 4-momentum \( k^\mu \))

\[ f_g(x, Q^2) = \int_{k_T^2}^{Q^2} dk_{T}^2 \Phi_g(x, k_{T}^2) \quad \text{(hereafter } k^2 = -k_{T}^2) \]  

(1)

is the basic dynamical quantity in the \( k_T \)-factorization approach [4]. It satisfies the BFKL equation [12].

---

1 Without a resummation the NLO approximation of SF \( F_L \) can be negative at low \( x \) and quite low \( Q^2 \) values (see [4, 5]).

2 Note that at low \( x \) a similar property has been observed also in the approaches [6, 7, 10] (see recent review [11] and discussions therein), which based on Balitsky-Fadin-Kuraev-Lipatov (BFKL) dynamics [12], where the leading \( \ln(1/x) \) contributions are summed.

3 In our previous analysis [24] we shown that the property \( k^2 = -k_{T}^2 \) leads to the equality of the Bjorken \( x \) value in the standard renormalization-group approach and in the Sudakov one.
Notice that the integral is divergent at lower limit (at least, for some parameterizations of $\Phi_g(x, k^2_\perp)$) and so it leads to the necessity to consider the difference $f_g(x, Q^2) - f_g(x, Q^2_0)$ with some nonzero $Q^2_0$ (see discussions in [26]), i.e.

$$f_g(x, Q^2) = f_g(x, Q^2_0) + \int_{Q^2_0}^{Q^2} dk^2_\perp \Phi_g(x, k^2_\perp) \quad (2)$$

Then, in the $k_T$-factorization the SF $F_{2,L}(x, Q^2)$ are driven at small $x$ primarily by gluons and are related in the following way to the unintegrated distribution $\Phi_g(x, k^2_\perp)$:

$$F_{2,L}(x, Q^2) = \int_x^1 \frac{dz}{z} \int dk^2_\perp \sum_{i=u,d,s,c} e^2_i \cdot \tilde{C}^{g}_{2,L}(x/z, Q^2, m^2_i, k^2_\perp) \Phi_g(z, k^2_\perp), \quad (3)$$

where $e^2_i$ are charge squares of active quarks.

The functions $\tilde{C}^{g}_{2,L}(x, Q^2, m^2_i, k^2_\perp)$ can be regarded as SF of the off-shell gluons with virtuality $k^2_\perp$ (hereafter we call them as hard structure functions [4]). They are described by the sum of the quark boxes (and crossed boxes) diagram contribution to the photon-gluon interaction (see Fig. 1).

The purpose of the paper is to give predictions for the longitudinal SF $F_L(x, Q^2)$ based on the calculations of the hard SF $\tilde{C}^{g}_{2,L}(x, Q^2, m^2_i, k^2_\perp)$, given in our previous study [26], and several parameterizations of unintegrated gluon distributions (see [11] and references therein).

It is instructive to note that the diagrams shown in Fig. 1. are similar to those of the photon-photon scattering process. The corresponding QED contributions have been calculated many years ago in Ref. [27] (see also the beautiful review in Ref. [28]). Our results have been calculated independently and they are in full agreement with Ref. [27]. Moreover, our results are in agreement with the corresponding integral representations for $\tilde{C}^{g}_{2,L}$, given in [22, 23] and numerically with results of [23]. However, we hope that our formulas which are given in a simpler form could be useful for others. This simpler form for the hard SF $C^{g}_{2,L}$ comes from using the relation between results based on non-sense, transverse and longitudinal gluon polarizations (see Eq. (13) below) observed in [26] for gauge-invariant sets of diagrams.

The structure of this paper is as follows: in Section 2 we present the basic formulae of our approach. Section 3 contains the relations between SF $F_{L}$ and $F_2$ and the derivative $dF_2/d\ln Q^2$, obtained in [13, 21, 21] in the framework of Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) approach [30] (i.e. in the collinear approximation: $k^2_\perp = 0$). In Section 4 we give the predictions for the structure function $F_L$ for three cases of unintegrated gluon distributions.

## 2 Basic formulae

To begin with, we review shortly results of Ref. [26] needed below in our investigations.

The hadron part of the DIS spin-average lepton-hadron cross section can be represented in the form [4]

$$F_{\mu\nu} = e_{\mu\nu}(q) \ F_L(x, Q^2) + d_{\mu\nu}(q, p) \ F_2(x, Q^2), \quad (4)$$

by analogy with similar relations between cross-sections and hard cross-sections.

Hereafter we consider only one-photon exchange approximation.
where
\[ e_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \quad \text{and} \quad d_{\mu\nu}(q, p) = -\left[ g_{\mu\nu} + 2x_B \frac{(p_\mu q_\nu + p_\nu q_\mu)}{q^2} + p_\mu p_\nu \frac{4x_B^2}{q^2} \right] \]

2.1 Feynman-gauge gluon polarization

As it has been shown in Ref. [26], it is very convenient to consider, as the first approximation, gluons having polarization tensor (hereafter the indices \( \alpha \) and \( \beta \) are connected with gluons and \( \mu \) and \( \nu \) are connected with photons)

\[ \hat{P}^{\alpha\beta} = -g^{\alpha\beta} \]  

(5)

The tensor corresponds is equal to the standard choice of polarization matrix in the framework of collinear approximation. In a sense the case of polarization is equal to the standard DIS suggestions about parton properties, excepting their off-shell property. The polarization (5) gives the main contribution to the polarization tensor we are interested in (see below)

\[ \hat{P}_{BFKL}^{\alpha\beta} = \frac{k_\perp^\alpha k_\perp^\beta}{k_\perp^2}, \]  

(6)

which comes from the high energy (or \( k_T \)) factorization prescription [22, 23, 24].

Contracting the photon projectors (connected with photon indices of diagrams on Fig.1.)

\[ \hat{P}^{(1)}_{\mu\nu} = -\frac{1}{2} g_{\mu\nu} \quad \text{and} \quad \hat{P}^{(2)}_{\mu\nu} = 4z^2 \frac{k_\mu k_\nu}{Q^2}, \]

(here \( z = Q^2/(2pq) \) is the corresponding Bjorken variable on parton level) with the parton tensor \( F^p_{\mu\nu} \):

\[ F^p_{\mu\nu} = e_{\mu\nu}(q) \ F^p_L(z, Q^2) + d_{\mu\nu}(q, k) \ F^p_2(z, Q^2), \]  

(7)

we obtain at the parton level (i.e. for off-shell gluons having momentum \( k_\mu \)), when \( \hat{C}^g_{2,L}(z) \sim F^p_2(z, Q^2) \),

\[ \beta^2 \cdot \hat{C}^g_2(x) = \mathcal{K} \left[ \frac{f^{(1)}}{2\beta^2} + \frac{3}{2\beta^2} \cdot f^{(2)} \right] \]  

(8)

\[ \beta^2 \cdot \hat{C}^g_L(x) = \mathcal{K} \left[ 4bx^2 f^{(1)} + \frac{1+2bx^2}{\beta^2} \cdot f^{(2)} \right] = \mathcal{K} \cdot f^{(2)} + 4bx^2 \beta^2 \cdot C^g_2, \]  

(9)

---

6In principle, we can use here more general cases of polarization tensor (for example, one is based on Landau or unitary gauge). The difference between them and (5) is \( k_\alpha \) and/or \( k_\beta \) and, hence, it leads to zero contributions because Feynman diagrams on Fig.1 are gauge invariant.

7We would like to note that the BFKL-like polarization tensor (5) is a particular case of so-called nonsense polarization of the particles in \( t \)-channel. The nonsense polarization makes the main contributions for cross sections in \( s \)-channel at \( s \to \infty \) (see, for example, Ref. [21] and references therein). The limit \( s \to \infty \) corresponds to the small values of Bjorken variable \( x \), that is just the range of our study.

8The hard SF \( \hat{C}^g_{2,L} \) do not depend on the type of target, so we can replace \( z \to x \) below.
where the normalization factor \( \mathcal{K} = a_s(Q^2) \cdot x \),

\[
\hat{P}^{(i)}_{\mu\nu} F_{\mu\nu} = \mathcal{K} \cdot f^{(i)}, \ i = 1, 2
\]

and \( a_s(Q^2) = \alpha_s(Q^2)/(4\pi), \ \bar{\beta}^2 = 1 - 4bx^2, \ b = -k^2/Q^2 \equiv k^2_\perp/Q^2 > 0, \ a = m^2/Q^2 \).

Applying the projectors \( \hat{P}^{(i)}_{\mu\nu} \) to Feynman diagrams displayed in Fig.1, we obtain the following results

\[
f^{(1)} = -2\beta \left[ 1 - \left( 1 - 2x(1+b-2a) \cdot [1-x(1+b+2a)] \right) \right] \cdot f_1
\]

\[
+ \ (2a - b)(1 - 2a)x^2 \cdot f_2,
\]

(10)

\[
f^{(2)} = 8x \cdot \beta \left[ (1 - (1+b)x) - 2x \left( bx(1-(1+b)x)(1+b-2a) + a\bar{\beta}^2 \right) \right] \cdot f_1
\]

\[
+ \ bx^2(1 - (1+b)x)(2a - b) \cdot f_2,
\]

(11)

where

\[
\beta^2 = 1 - \frac{4ax}{(1 - (1+b)x)}
\]

(12)

and

\[
f_1 = \frac{1}{\beta \beta} \cdot \ln \frac{1 + \beta \bar{\beta}}{1 - \beta \bar{\beta}}, \quad f_2 = -\frac{4}{1 - \beta^2 \bar{\beta}^2}
\]

2.2 BFKL-like gluon polarization

Now we take into account the BFKL-like gluon polarization (13). As we shown in [26], the projector \( \hat{P}^{\alpha\beta}_{BFKL} \) can be represented as

\[
\hat{P}^{\alpha\beta}_{BFKL} = -\frac{1}{2\beta^4} \left[ \beta^2 g^{\alpha\beta} - 12bx^2 \frac{q^\alpha q^\beta}{Q^2} \right]
\]

(13)

In the previous subsection we have already presented the contributions to hard SF using the first term in the brackets of the r.h.s. of (13). Repeating the above calculations with the projector \( \sim q^\alpha q^\beta \), we obtain the total contributions to hard SF which can be represented as the following shift of the results in Eqs.(8) - (11):

\[
\hat{C}\!\!\!\!_2^q(x) \to \hat{C}\!\!\!\!_2^{q, BFKL}(x), \ \hat{C}\!\!\!\!_L^q(x) \to \hat{C}\!\!\!\!_L^{q, BFKL}(x);
\]

\[
f^{(1)} \to f^{(1)}_{BFKL} = \frac{1}{\beta^4} \left[ \beta^2 f^{(1)} - 3bx^2 \bar{f}^{(1)} \right]
\]

\[
f^{(2)} \to f^{(2)}_{BFKL} = \frac{1}{\beta^4} \left[ \beta^2 f^{(2)} - 3bx^2 \bar{f}^{(2)} \right],
\]

(14)
where

\[
\tilde{f}^{(1)} = -\beta \left[ \frac{1 - x(1 + b)}{x} - 2 \left( x(1 - x(1 + b))(1 + b - 2a) + a\tilde{\beta}^2 \right) \cdot f_1 \right.
\]

\[- x(1 - x(1 + b))(1 - 2a) \cdot f_2 \right], \\
\tilde{f}^{(2)} = 4 \cdot \beta (1 - (1 + b)x)^2 \left[ 2 - (1 + 2bx^2) \cdot f_1 - bx^2 \cdot f_2 \right].
\] (15)

Notice that the general formulae are needed only to evaluate the charm contribution to structure functions \(F_2\) and \(F_L\), i.e. \(F_{2c}\) and \(F_{Lc}\). To evaluate the corresponding light-quark contributions, i.e. \(F_{Ll}\), we can use \(m^2 = 0\) limit of above formulae.

### 2.3 The case \(m^2 = 0\)

When \(m^2 = 0\) the hard SF \(\hat{C}_{kj}^g(x)\) are defined by \(f^{(1)}\), \(f^{(2)}\), \(\tilde{f}^{(1)}\) and \(\tilde{f}^{(2)}\) (as in Eqs. (8), (9) and (14)) which can be represented as

\[
f^{(1)} = -2 \left[ 2 - \left( 1 - 2x(1 + b) + 2x^2(1 + b)^2 \right) \cdot L(\tilde{\beta}) \right],
\] (17)

\[
f^{(2)} = 8x(1 + b)(1 - (1 + b)x) \left[ 1 - 2bx^2 \cdot L(\tilde{\beta}) \right],
\] (18)

\[
\tilde{f}^{(1)} = - \frac{(1 + b)(1 - x(1 + b))}{bx} \left[ 1 - 2bx^2 \cdot L(\tilde{\beta}) \right] = - \frac{1}{8bx^2} f^{(2)},
\] (19)

\[
\tilde{f}^{(2)} = 4(1 - x(1 + b))^2 \left[ 3 - (1 + 2bx^2) \cdot L(\tilde{\beta}) \right]
\] (20)

and, thus, (see Eq. (14))

\[
\tilde{\beta}^4 \cdot f_{BFKL}^{(1)} = (-2) (1 - x(1 + b)) \left[ 2 \left( 1 - 2x(1 + b) + \frac{x^2(1 - b)^2}{1 - x(1 + b)} \right) \right.
\]

\[- \left( 1 - x(1 + b) - 4x^3b(1 + b) + \frac{x^2(1 - b)^2}{1 - x(1 + b)} \right) \cdot L(\tilde{\beta}) \],
\] (21)

\[
\tilde{\beta}^4 \cdot f_{BFKL}^{(2)} = 8x(1 - x(1 + b)) \left[ 1 + b - 18bx(1 - x(1 + b)) \right.
\]

\[+ 2bx \left( 3 - 4x(1 + b) + 6bx^2(1 - x(1 + b)) \right) \cdot L(\tilde{\beta}) \],
\] (22)

where

\[L(\tilde{\beta}) = \frac{1}{\tilde{\beta}} \cdot \ln \frac{1 + \tilde{\beta}}{1 - \tilde{\beta}}\]
3 Relations between $F_L$, $F_2$ and derivation of $F_2$ in the case of collinear approximation

Another information about the SF $F_L$ can be obtained in the collinear approximation (i.e. when $k^2 = 0$) in the following way.

In the framework of perturbative QCD, there is the possibility to connecting $F_L$ to $F_2$ and its derivation $dF_2/d\ln Q^2$ due the fact that at small $x$ the DIS structure functions depend only on two independent functions: the gluon distribution and singlet quark one (the nonsinglet quark density is negligible at small $x$), which in turn can be expressed in terms of measurable SF $F_2$ and its derivation $dF_2/d\ln Q^2$.

In this way, by analogy with the case of the gluon distribution function (see [32, 33] and references therein), the behavior of $F_L(x, Q^2)$ has been studied in [19]-[21], using the HERA data [34, 35] and the method [36] of replacement of the Mellin convolution by ordinary products. Thus, the small $x$ behavior of the SF $F_L(x, Q^2)$ can be well considered as new small $x$ 'experimental data' of $F_L$. The relations can be violated by nonperturbative corrections like higher twist ones (see [38, 39]), which can be large exactly in the case of SF $F_L$ [25, 40].

Because $k_T$-factorization approach is one of popular nonperturbative approaches used at small $x$, it is very useful to compare its predictions with the results of [19]-[21] based on the relations between SF $F_L(x, Q^2)$, $F_2(x, Q^2)$ and $dF_2(x, Q^2)/d\ln Q^2$. It is the main purpose of the study.

The $k_T$-factorization approach relates strongly to Regge-like behavior of parton distributions. So, we restrict our investigations to SF and parton distributions at the following form (hereafter $a = q, g$):  

\[
f_a(x, Q^2) \sim F_2(x, Q^2) \sim x^{-\delta(Q^2)}
\]

Note that really the slopes of the sea quark and gluon distributions: $\delta_q$ and $\delta_g$, respectively, and the slope $\delta_{F_2}$ of SF $F_2$ are little different. The slopes have a familiar property $\delta_q < \delta_{F_2} < \delta_g$ (see Refs. [11]-[16] and references therein). We will neglect, however, this difference and use in our investigations the experimental values of $\delta(Q^2) \equiv \delta_{F_2}(Q^2)$ extracted by H1 Collaboration [14] (see [11] and references therein). We note that the $Q^2$-dependence is in very good agreement with perturbative QCD at $Q^2 \geq 2$ GeV$^2$ (see [18]). Moreover, the values of the slope $\delta(Q^2)$ are in agreement with recent phenomenological studies (see, for example, [8]) incorporating the next-to-leading corrections [49] (see also [50]) in the framework of BFKL approach.

Thus, assuming the Regge-like behavior [23] for the gluon distribution and $F_2(x, Q^2)$ at $x^{-\delta} \gg 1$ and using the NLO approximation for collinear coefficient functions and anomalous

9 The method is based on previous investigations [1, 2].

10 Now the preliminary ZEUS data for the slope $d\ln F_2/d\ln(1/x)$ are available as some points on Figs. 8 and 9 in Ref. [17]. Moreover, the new preliminary H1 points have been presented on the Workshop DIS2002 (see [17]). Both the new points are shown quite similar properties to compare with H1 data [11]. Unfortunately, tables of the ZEUS data and the new H1 data are unavailable yet and, so, the points cannot be used here.
dimensions of Wilson operators, the following results for $F_L(x, Q^2)$ have been obtained in [20]:

$$F_L(x, Q^2) = -2 \frac{B_{qg}^{0,1+\delta}(1 + a_s(Q^2)R_{qg}^{0,1+\delta})}{\gamma_{qq}^{(0),1+\delta} + \gamma_{gg}^{(1),1+\delta} a_s(Q^2)} \left[ \frac{dF_2(x, Q^2)}{d\ln Q^2} \right] + \frac{a_s(Q^2)}{2} \left( \frac{B_{qg}^{0,1+\delta}}{B_{qg}^{1,1+\delta}} (\gamma_{qq}^{(0),1+\delta} - \gamma_{gg}^{(0),1+\delta}) \right) F_2(x, Q^2) + O(a_s^2, x^{2-\delta}, \alpha x^{1-\delta}), \quad (24)$$

where

$$\overline{\gamma}_{qg}^{(1),\eta} = \gamma_{qg}^{(1),\eta} + 2 \gamma_{qg}^{(1),\eta} + 2 \gamma_{gq}^{(0),\eta} (2\beta_0 + \gamma_{gg}^{(0),\eta} - \gamma_{qq}^{(0),\eta}), \quad \overline{R}_{L}^{\eta} = R_{qg}^{\eta} - 2 \gamma_{qg}^{(0),\eta} \frac{B_{qg}^{0,\eta}}{B_{qg}^{0,0}}.$$

Here $\overline{\gamma}_{qa}^{(1),\eta}$ and $\overline{R}_L^{\eta}$ ($a = q, g$) are the combinations [1] of the 'anomalous dimensions' of Wilson operators $\gamma_{qa}^{(0),\eta} = a_s\gamma_{qa}^{(0),\eta} + a_s^2 \gamma_{qa}^{(1),\eta} + O(a_s^3)$ and 'Wilson coefficients' $a_s B_{n}^{\eta}(1 + a_s R_{L}^{\eta}) + O(a_s^3)$ and $a_s B_{n}^{\eta} + O(a_s^2)$ with the 'moment' argument $\eta$ (i.e., the combinations of the functions which can be obtained by analytical continuation of the corresponding anomalous dimensions and coefficient functions from integer values $n$ of their argument to non-integer ones $\eta$).

Note that, in principle, any term like $\sim 1/(n + m)$ ($m = 0, 1, 2, \ldots$) which comes to the corresponding combinations of the anomalous dimensions and coefficient functions: $\overline{\gamma}_{qa}^{(1),\eta}$ and $\overline{R}_L^{\eta}$, should contribute to Eq. (24) in the following form (after replacement of Mellin convolutions by usual products in the DGLAP equations (see [36])):

$$\frac{1}{1 + \delta + m} \left( 1 + \frac{\Gamma(2 + \delta + m)\Gamma(1 + \nu)}{\Gamma(2 + \delta + m + 1 + \nu)} \right) x^{1+\delta+m}, \quad (25)$$

where the value of $\nu$ comes (see [31, 32, 53]) from asymptotics of parton distributions $f_a(x)$ at $x \to 1$: $f_a \sim (1 - x)^{\nu_a}$, and $11 \nu \approx 4$ from quark account rules [54]. The additional term $\sim x^{1+\delta+m}$ in Eq. (25) is important only for $m = -1$ case (i.e. for the singular parts $\sim 1/(n - 1)$ of the corresponding anomalous dimensions and coefficient functions) and quite small values of $\delta$ (i.e. for $x^\delta \sim Const$).

Thus, excepting the case when $m = -1$ and $x^\delta \sim Const$, we can replace (25) by its first term $1/(1 + \delta + m)$, i.e. our variables $\overline{\gamma}_{qa}^{(1),\eta}$ and $\overline{R}_L^{\eta}$ are just the combinations of corresponding anomalous dimensions and coefficient functions at $n = 1 + \delta$. When $m = -1$ and $x^\delta \sim Const$ at the considered small $x$ range, we should replace the term $1/(n - 1)$, if it was exist in the variables $\overline{\gamma}_{qa}^{(1),\eta}$ and $\overline{R}_L^{\eta}$ ($a = q, g$), by the following term:

$$\frac{1}{\delta} = \frac{1}{\delta} \left[ 1 - \frac{\Gamma(1 - \delta)\Gamma(1 + \nu)}{\Gamma(1 - \delta + \nu)} x^\delta \right], \quad (26)$$

Note also that the $1/\delta$ coincides approximately with $1/\delta$ when $\delta \neq 0$ and $x \to 0$. However, at $\delta \to 0$, the value of $1/\delta$ is not singular:

$$\frac{1}{\delta} \to \ln \left( \frac{1}{x} \right) - \left[ \Psi(1 + \nu) - \Psi(1) \right], \quad (27)$$

11Because we consider here $F_2(x, Q^2)$ but not the singlet quark distribution in the corresponding DGLAP equations [34].
12In our formula [24] we have our interest mostly to gluons, so we can apply $\nu = \nu_q \approx 4$ below.
So, the Eq. (24) together with well-known expressions of anomalous dimensions $\gamma_{ab}^{(0),n}$ and $\gamma_{ab}^{(1),n}$ ($a, b = q, g$) and coefficient functions $B_2^{a,n}$ and $R_L^a,n$ ($a = q, g$) (see [53] and [56, 57], respectively, and references therein) gives a possibility to extract $S_F$ fixed from the agreement these parameterizations with the exact values of $\delta$. The calculations are based on precise experimental data of $S_F$ and $\gamma$ and $\delta = 0$ because they are obtained at quite large $x$.

For concrete $\delta$-values, the Eq. (24) simplifies essentially (see [20]). For example, for $\delta = 0.3$ we obtain (for the number of active quarks $f = 4$ and $\overline{MS}$ scheme):

$$F_L(x, Q^2) = \frac{0.84}{1 + 59.3 a_s(Q^2)} \left[ \frac{dF_2(0.48x, Q^2)}{d \ln Q^2} + 3.59 a_s(Q^2)F_2(0.48x, Q^2) \right] + O(a_s^2 x^{2-\delta}, a_s x^{-1-\delta})$$

(28)

At arbitrary $\delta$ values, in real applications it is very useful to simplify Eq. (24) as follows. We keep the exact $\delta$-dependence only for the leading order terms, which are very simple. In the NLO corrections we extract the terms $\sim 1/\delta$, which are changed strongly when $0 \leq \delta \leq 1$, and parameterize the rest terms in the form: $a_i + b_i \delta + c_i \delta^2$. The coefficients $a_i$, $b_i$, $c_i$ are fixed from the agreement these parameterizations with the exact values of $\bar{T}_{qa}^{(1),n}$ and $\bar{T}_{Lq}^{(1),n}$ at $\delta = 0$, 0.3 and 0.5. These exact values can be found in Refs. [33, 20, 21].

Then, the approximate representation of Eq. (24) of for arbitrary $\delta$ value has the form:

$$F_L(x, Q^2) = \frac{r(1 + \delta)(\xi(\delta))^\delta}{(1 + 30 a_s(Q^2)[1/\delta - 116/45 \rho_1(\delta)])} \left[ \frac{dF_2(x \xi(\delta), Q^2)}{d \ln Q^2} + \frac{8}{3} \rho_2(\delta) a_s(Q^2)F_2(x \xi(\delta), Q^2) \right]$$

$$+ O(a_s^2, a_s x, x^2),$$

(29)

where

$$r(\delta) = \frac{4 \delta}{2 + \delta + \delta^2}, \quad \xi(\delta) = \frac{r(\delta)}{r(1 + \delta)},$$

$$\rho_1(\delta) = 1 + \delta + \delta^2/4, \quad \rho_2(\delta) = 1 - 2.39 \delta + 2.69 \delta^2,$$

(30)

4 Comparison with $F_L$ experimental data

With the help of the results obtained in the previous Section we have analyzed experimental data for SF $F_L$ from H1 [13] and [18], NMC [28], CCFR [39] and [40], BCDMS [21] collaborations [22]. Note, that we do not correct CCFR data [39] and [40] which has been obtained in $\nu N$ processes because the terms $\sim x \cdot m^2/Q^2$, which are different in $\mu N$ and $\nu N$ processes, are not so strong at low $x$.

We calculate the SF $F_L$ as the sum of two types of contributions: charm quark one $F_L^c$ and light quark one $F_L^l$:

$$F_L = F_L^l + F_L^c$$

(31)

\[\text{Sometimes there are experimental data for the ratio } R = \sigma_L/\sigma_T, \text{ which can be recalculated for the SF } F_L \text{ because } F_L = F_2 R/(1 + R).\]

14 We do not use experimental data of $R$ from SLAC [22, 23], EM [24] and CDHSW [25] Collaborations because they are obtained at quite large $x$ values.
We use the expression (3) for the calculation of the both SF $F^l_L$ and $F^c_L$ in the following form (here $a_c = m_c^2/Q^2$):

$$F^l_L(x, Q^2) = \sum_{f=u,d,s} e_f^2 \left[ \int_x^1 \frac{dz}{z} \hat{C}^g_{L,BFKL}(x, Q^2, 0) z f_g(z, Q_0^2) + \right. $$

$$+ \frac{2}{z} \int_{z_{min}^{(i)}}^{z_{max}^{(i)}} dz \int_{k_{min}^{(i)}}^{k_{max}^{(i)}} dk_{\perp}^{2} \hat{C}^g_{L,BFKL}(x, Q^2, k_{\perp}^2) \Phi(z, k_{\perp}^2, Q_0^2) \right],$$

(32)

$$F^c_L(x, Q^2) = e_c^2 \int_x^1 \frac{dz}{z} \hat{C}^g_{L,BFKL}(x, m_c^2, Q^2, 0) z f_g(z, Q_0^2) + $$

$$+ \frac{2}{z} \int_{z_{min}^{(i)}}^{z_{max}^{(i)}} dz \int_{k_{min}^{(i)}}^{k_{max}^{(i)}} dk_{\perp}^{2} \hat{C}^g_{L,BFKL}(x, m_c^2, Q^2, k_{\perp}^2) \Phi(z, k_{\perp}^2, Q_0^2) \right],$$

(33)

where $\hat{C}^g_{L,BFKL}(x_B, Q^2, m_c^2, k_{\perp}^2)$ are given by Eqs. (1) and (14).

The integration limits in the expression (33) have the following values:

$$z_{min}^{(1)} = x(1 + 4a_c + \frac{Q_0^2}{Q^2}), \quad z_{max}^{(1)} = 2x(1 + 2a_c);$$

$$k_{min}^{(2)} = Q_0^2, \quad k_{max}^{(2)} = \left( \frac{z}{x} - (1 + 4a_c) \right) Q^2;$$

$$z_{min}^{(2)} = 2x(1 + 2a_c), \quad z_{max}^{(2)} = 1;$$

$$k_{min}^{(2)} = Q_0^2, \quad k_{max}^{(2)} = Q^2;$$

(34)

The ranges of integration correspond to positive values of square roots in expressions (11), (12), (14) and (16) and also should obey to kinematical restriction ($z \leq (1 + 4a_c + b)^{-1}$) following from condition $\beta^2 \geq 0$ (see Eqs. (14)-(12)). In Eq. (32) the ranges (34) are used at $a_c = 0$.

In Fig. 2 we show the SF $F_L$ as a function $x$ for different values of $Q^2$ in comparison with H1 experimental data sets: old one of [13] (black triangles), last year one of [15] (black squares) and new preliminary one of [15] (black circles) and also with NMC [58] (white triangles), CCFR [59] (white circles) and BCDMS [61] data (white squares). For comparison with these data we present the results of the calculation with three different parameterizations for the unintegrated gluon distribution $\Phi(x, k_{\perp}^2, Q_0^2)$ at $Q_0^2 = 4 \text{ GeV}^2$. All of them: Kwiecinski-Martin-Stasto (KMS) one [72] and Blumlein (JB) one [68] and Golec-Biernat and Wusthoff (GBW), have been used already in our previous work [26] and reviewed there.

There are several other popular parameterizations (see, for example, Kimber-Martin-Ryskin (KMR) [70] and Jung-Salam (JS) [71]), which are not used in our study mostly because of technical difficulties. Note that all above parameterizations give quite similar results excepting, perhaps, the contributions from the small $k_{\perp}^2$-range: $k_{\perp}^2 \leq 1 \text{ GeV}^2$ (see Ref. [11] and references therein). Because we use $Q_0^2 = 4 \text{ GeV}^2$ in the study of SF $F_L$, our results depend very slightly on the the small $k_{\perp}^2$-range of the parameterizations. In the case JB, GBW and KMS sets this observation is supported below by our results and we expect that the application of KMR and JS sets should not strongly change our results.

The differences observed between the curves 2, 3 and 4 are due to the different behavior of the unintegrated gluon distribution as function $x$ and $k_{\perp}$. We see that the SF $F_L$ obtained
in the $k_T$-factorization approach with KMS and JB parameterizations is close each other \[^{15}\] and higher than the SF obtained in the pure perturbative QCD with the GRV gluon density at the leading order approximation. Otherwise, the $k_T$–factorization approach with GBW parameterization is very close to pure QCD predictions \[^{16}\] it should be so because GBW model has deviations from perturbative QCD only at quite low $Q^2$ values. Thus, the predictions of perturbative QCD and ones based on $k_T$–factorization approach are in agreement each other and with all data within modern experimental uncertainties. So, a possible high values of high-twist corrections to SF $F_L$ predicted in \[^{10}\] can be important only at low $Q^2$ values: $Q^2 \leq Q^2_0 = 4 \text{ GeV}^2$.

Fig. 3 is similar to Fig. 2 with one excepting: we add 'experimental data' obtained using the relation between SF $F_L(x, Q^2)$, $F_2(x, Q^2)$ and $dF_2(x, Q^2)/d\ln Q^2$ (see Section 3) as black stars. Because the corresponding data for SF $F_2(x, Q^2)$ and $dF_2(x, Q^2)/d\ln Q^2$ essentially more precise (see \[^{15}\]) to compare with the preliminary data \[^{18}\] for $F_L$, the 'experimental data' have strongly suppressed uncertainties. As it is shown on Fig. 3 there are very good agreement between the new preliminary data \[^{18}\], the 'experimental data' and predictions of perturbative QCD and $k_T$–factorization approach.

To estimate of the value of charm mass effect, we recalculate SF $F_L^c$ also in massless approximation similar to (32). In Fig. 4 we show importance of exact $m_c$-dependence in hard SF of $F_L^c$ to compare with its massless approximation, where we should have $F_L^c(m_c = 0)/F_L(m_c = 0) = 2/5$. As it is possible to see on Fig. 4, the ratio $F_L^c/F_L$ goes to massless limit $2/5$ only at asymptotically large $Q^2$ values.

5 Conclusions

We have applied in the framework of $k_T$-factorization approach the results of the calculation of the perturbative parts for the structure functions $F_L$ and $F_T^c$ for a gluon target having nonzero momentum square, in the process of photon-gluon fusion to the analysis of present data for the structure function $F_L$ \[^{17}\]. The analysis has been performed with several parameterizations of unintegrated gluon distributions, for comparison. We have found good agreement between all existing experimental data, the predictions for $F_L$ obtained from the relation between SF $F_L(x, Q^2)$, $F_2(x, Q^2)$ and $dF_2(x, Q^2)/d\ln Q^2$ and the results obtained in the framework of perturbative QCD and ones based on $k_T$–factorization approach with the three different parameterizations of unintegrated gluon distributions.

We note that it could be also very useful to evaluate the SF $F_2$ itself \[^{18}\] and the derivatives of $F_2$ respect to the logarithms of $1/x$ and $Q^2$ with our expressions using the unintegrated gluons. We are considering to present this work and also the predictions for the ratio $R = \sigma_L/\sigma_T$ in a forthcoming article.

\[^{15}\]Note that very similar results have been obtained also for Ryskin-Shabelsky parameterization \[^{72}\] (see \[^{73}\]).

\[^{16}\]This fact is evident also from quite large value of $Q^2_0 = 4 \text{ GeV}^2$ chosen here.

\[^{17}\]In Ref \[^{26}\] we have also obtained quite large contribution of SF $F_L^c$ at low $x$ and high $Q^2$ ($Q^2 \geq 30 \text{ GeV}^2$).

\[^{18}\]A study of the SF $F_2$ in the framework of $k_T$-factorization has been already done in Ref. \[^{74}\].
The consideration of the SF $F_2$ in the framework of the leading-twist approximation of perturbative QCD (i.e. for “pure” perturbative QCD) leads to very good agreement (see Ref. [45] and references therein) with the HERA data at low $x$ and $Q^2 \geq 1.5$ GeV$^2$. The agreement improves at lower $Q^2$ when higher twist terms are taken into account in Ref. [38, 39]. As it has been studied in Refs. [45, 38], the SF $F_2$ at low $Q^2$ is sensitive to the small-$x$ behavior of quark distributions. Thus, our future analysis of $F_2$ in broad $Q^2$ range in the framework of $k_T$-factorization approach should require the incorporation of parameterizations of unintegrated quark densities, introduced recently (see Ref. [70, 11] and references therein).

Acknowledgements

We are grateful to Professor Catani for useful discussions and comments.

The study is supported in part by the RFBR grant 02-02-17513. One of the authors (A.V.K.) is supported in part by Heisenberg-Landau program and INTAS grant N366. N.P.Z. also acknowledge the support of Royal Swedish Academy of Sciences.

References

[1] A.M. Cooper-Sarkar, G. Ingelman, K.R. Long, R.G. Roberts and D.H. Saxon, Z. Phys. C39 (1988) 281;

[2] L. Bauer, O. Frixione, and M. Kneesch, in Proc. of the Int. Workshop on Future Physics on HERA, Hamburg, DESY (1996), p.77 [hep-ex/9609017].

[3] S. Keller, M. Miramontes, G. Parente, J. Sanchez-Guillen, and O.A. Sampayo, Phys. Lett. B270 (1990) 61;
L.H. Orr and W.J. Stirling, Phys. Rev. Lett. B66 (1991) 1673;
E. Berger and R. Meng, Phys. Lett. B304 (1993) 318.

[4] A.V. Kotikov, JETP Lett. 59 (1994) 1; Phys. Lett. B338 (1994) 349.

[5] R.S. Thorne, in: Proc. of the Int. Workshop on Deep Inelastic Scattering (2002), Cracow.

[6] Yu.L. Dokshitzer, D.V. Shirkov, Z. Phys. C67 (1995) 449;

[7] W.K. Wong, Phys. Rev. D54 (1996) 1094.

[8] S.J. Brodsky, V.S. Fadin, V.T. Kim, L.N. Lipatov and G.B. Pivovarov, JETP. Lett. 70 (1999) 155;
V.T. Kim, L.N. Lipatov and G.B. Pivovarov, in: Proceedings of the VIIIth Blois Workshop at IHEP, Protvino, Russia, 1999 (IIITAP-99-013, hep-ph/9911228); in: Proceedings of the Symposium on Multiparticle Dynamics (ISMD99), Providence, Rhode Island, 1999 (IIITAP-99-014, hep-ph/9911242).

[9] S.J. Brodsky, V.S. Fadin, V.T. Kim, L.N. Lipatov and G.B. Pivovarov, in: Proceedings of the PHOTON2001, Ascona, Switzerland, 2001 (CERN-TH/2001-341, SLAC-PUB-9069, hep-ph/0111390);
[10] M. Ciafaloni, D. Colferai and G.P. Salam, Phys. Rev. D60 (1999) 114036; JHEP 07 (2000) 054; 
R.S. Thorne, Phys. Lett. B474 (2000) 372; Phys. Rev. D60 (1999) 054031; D64 (2001) 
074005; 
G. Altarelli, R.D. Ball and S. Forte, Nucl. Phys. B621 (2002) 359.

[11] Bo Andersson et al., [hep-ph/0204113].

[12] L.N. Lipatov, Sov. J. Nucl. Phys. 23 (1976) 338; 
E.A. Kuraev, L.N. Lipatov and V.S. Fadin, Sov. Phys. JETP 44 (1976) 443, 45 (1977) 
199; 
Ya.Ya. Balitzki and L.N. Lipatov, Sov. J. Nucl. Phys. 28 (1978) 822; 
L.N. Lipatov, Sov. Phys. JETP 63 (1986) 904.

[13] H1 Collab.: S. Aid et al., Phys.Lett. B393 (1997) 452.

[14] R.S. Thorne, Phys.Lett. B418 (1998) 371.

[15] H1 Collab.: S. Adloff et al., Eur. Phys. J. C21 (2001) 33

[16] H1 Collab.: D. Eckstein, in Proc. Int. Workshop on Deep Inelastic Scattering, (2001), 
Bologna; 
H1 Collab.: M.Klein, in Proc. of the 9th Int. Workshop on Deep Inelastic Scattering, 
DIS 2001 (2001), Bologna.

[17] H1 Collab.: T. Lastovicka, in Proc. of the 10th Int. Workshop on Deep Inelastic Scat- 
tering, DIS 2002 (2002), Cracow; 
H1 Collab.: J. Gayler, in Proc. of the 10th Int. Workshop on Deep Inelastic Scattering, 
DIS 2002 (2002), Cracow.

[18] H1 Collab., N. Gogitidze, J. Phys. G28 (2002) 751 (hep-ph/0201047)

[19] A.V. Kotikov, JETP 80 (1995) 979.

[20] A.V. Kotikov and G. Parente, Mod. Phys. Lett. A12 (1997) 963; in Proc. Int. Work- 
shop on Deep Inelastic Scattering and Related Phenomena (1996), Rome, p. 237 (hep- 
ph/9608409);

[21] A.V. Kotikov and G. Parente, JETP 85 (1997) 17; [hep-ph/9609439].

[22] S. Catani, M. Ciafaloni and F. Hautmann, Phys. Lett. B242 (1990) 97; Nucl. Phys. 
B366 (1991) 135; Nucl. Phys. B (Proc.Suppl.) 29A (1992) 182; Preprint CERN - 
TH.6398/92, in Proceeding of the Workshop on Physics at HERA (Hamburg, 1991), 
v.2, p.690;

[23] J.C. Collins and R.K. Ellis, Nucl. Phys. B360 (1991) 3.

[24] E.M. Levin, M.G. Ryskin, Yu.M. Shabelskii and A.G. Shuvaev, Sov. J. Nucl. Phys. 53 
(1991) 657.
[25] S. Catani and F. Hautmann, Nucl. Phys. B427 (1994) 475; S. Catani, Preprint DFF 254-7-96 [hep-ph/9608310].

[26] A.V. Kotikov, A. V. Lipatov, G. Parente and N.P. Zotov, Preprint US-FT/7-01 [hep-ph/0107133]; in Proc. of the XVIth International Workshop “High Energy Physics and Quantum Field Theory“ (2001), Moscow.

[27] V.N. Baier, V.S. Fadin and V.A. Khose, Zh.Eksp.Teor.Fiz. 50 (1966) 156 [Sov. J. JETP 23 (1966) 104]; V.N. Baier, V.M. Katkov and V.S. Fadin, Relativistic electron radiation, (Moscow, Atomizdat, 1973) (in Russian); V.G. Zima, Yad. Fiz. 16 (1972) 1051 [Sov. J. Nucl. Phys. 16 (1973) 580].

[28] V.M. Budnev, I.F. Ginsburg, G.V. Meledin and V.G. Serbo, Phys. Rept. 15 (1975) 181.

[29] G. Bottazzi, G. Marchesini, G.P. Salam, and M. Scorletti, JHEP 9812 (1998) 011.

[30] V.N. Gribov and L.N. Lipatov, Sov. J. Nucl. Phys. 15 (1972) 438, 15 (1972) 675; L.N. Lipatov, Sov. J. Nucl. Phys. 20 (1975) 94; G. Altarelli and G. Parisi, Nucl. Phys. B126 (1977) 298; Yu.L. Dokshitzer, Sov. Phys. JETP 46 (1977) 641.

[31] E.A. Kuraev and L.N. Lipatov, Yad. Fiz. 16 (1972) 1060 [Sov. J. Nucl. Phys. 16 (1973) 584].

[32] K. Prytz, Phys.Lett. B311 (1993) 286.

[33] A.V. Kotikov, JETP Lett. 59 (1994) 667; A.V. Kotikov and G. Parente, Phys.Lett. B379 (1996) 195.

[34] H1 Collab., T. Ahmed et al., Nucl. Phys. B439 (1995) 471.

[35] ZEUS Collab., M. Derrick et al., Z. Phys. C65 (1995) 379.

[36] A.V. Kotikov, Phys. Atom. Nucl. 57 (1994) 133; Phys. Rev. D49 (1994) 5746.

[37] F. Martin, Phys. Rev. D19 (1979) 1382; C. Lopez and F.I. Yndurain, Nucl. Phys. B171 (1980) 231; B183 (1981) 157.

[38] A.V. Kotikov and G. Parente, in Proc. Int. Seminar Relativistic Nuclear Physics and Quantum Chromodynamics (2000), Dubna [hep-ph/0012299]; in Proc. of the 9th Int. Workshop on Deep Inelastic Scattering, DIS 2001 (2001), Bologna [hep-ph/0106173].

[39] V.G. Krivokhijine and A.V. Kotikov, JINR preprint E2-2001-190 [hep-ph/0108224]; in: Proc. of the XVIth International Workshop “High Energy Physics and Quantum Field Theory“ (2001), Moscow [hep-ph/0206221]; in: Proc. of the Int. Workshop “Renormalization Group 2002“ (2002), High Tatras, Slovakia [hep-ph/0207222].

[40] J. Bartels, K.Golec-Biernat, and K. Peters, Eur.Phys.J. C17 (2000) 121; J. Bartels, in: Proc. of the Int. Workshop on Deep Inelastic Scattering (2002), Cracow.
[41] H1 Collab.: C. Adloff et al., Phys. Lett. B520 (2001) 183.

[42] A.D. Martin, W.S. Stirling and R.G. Roberts, Phys.Lett. B387 (1996) 419.

[43] M. Gluck, E. Reya and A. Vogt, Eur. Phys. J C5 (1998) 461.

[44] A.D. Martin, W.S. Stirling, R.G. Roberts and R.S. Thorne, Eur. Phys. J C23 (2002) 73;
CTEQ Collab.: J. Pumplin et al., Preprint MSU-HEP-011101 (hep-ph/0201195).

[45] A.V. Kotikov and G. Parente, Nucl. Phys. B549 (1999) 242; Nucl. Phys. (Proc. Suppl.) 99A (2001) 196 (hep-ph/0010352); in Proc. of the Int. Conference PQFT98 (1998), Dubna (hep-ph/9810223); in Proc. of the 8th Int. Workshop on Deep Inelastic Scattering, DIS 2000 (2000), Liverpool, p. 198 (hep-ph/0006197).

[46] A.V. Kotikov, Mod. Phys. Lett. A 11 (1996) 103; Phys. At. Nucl. 59 (1996) 2137.

[47] ZEUS Collab., B. Surrow, talk given at the International Europhysics Conference on High Energy Physics, July 2001 (hep-ph/0201025).

[48] A.V. Kotikov and G. Parente, Preprint US-FT/3-02 (hep-ph/0207276).

[49] V.N. Fadin and L.N. Lipatov, Phys. Lett. B429 (1998) 127;
M. Ciafaloni and G. Camici, Phys. Lett. B430 (1998) 349.

[50] A.V. Kotikov and L.N. Lipatov, Nucl. Phys. B582 (2000) 19; in: Proc. of the XXXV Winter School, Repino, S’Peterburg, 2001 (hep-ph/0112346).

[51] A.V. Kotikov, Phys. Atom. Nucl. 56 (1993) 1276.

[52] V.I. Vovk, A.V. Kotikov, and S.I. Maximov, Theor. Math. Phys. 84 (1990) 744;
A.V. Kotikov, S.I. Maximov, and I.S. Parobij, Theor. Math. Phys. 111 (1997) 442.

[53] L. L. Jenkovszky, A. V. Kotikov and F. Paccanoni, Sov. J. Nucl. Phys. 55 (1992) 1224; JETP Lett. 58 (1993) 163; Phys. Lett. B 314 (1993) 421.

[54] V.A. Matveev, R.M. Muradian and A.N. Tavkhelidze, Lett. Nuovo Cim. 7 (1973) 719;
S.J. Brodsky and G.R. Farrar, Phys. Rev. Lett. 31 (1973) 1153;
S.J. Brodsky, J. Ellis, E. Cardi, M. Karliner and M.A. Samuel, Phys. Rev. D56 (1997) 6980.

[55] E.G. Floratos, C. Kounnas, and R. Lacage, Nucl. Phys. B192 (1981) 417.

[56] D.I. Kazakov and A.V. Kotikov, Theor.Math.Phys. 73 (1987) 1264; Nucl.Phys. B307 (1988) 721; E: B345 (1990) 299.

[57] D.I. Kazakov and A.V. Kotikov, Phys.Lett. B291 (1992) 171.

[58] NM Collab., M. Arneodo et al., Nucl. Phys. B483 (1997) 3; A.J. Milsztajn, in Proc. of the 4th Int. Workshop on Deep Inelastic Scattering, DIS96 (1996), Rome, p.220.
[59] U.K. Yang et al., J. Phys. G22 (1996) 775;  
      A. Bodek, in Proc. of the 4th Int. Workshop on Deep Inelastic Scattering, DIS96 (1996), Rome, p.213;  
      A. Bodek, S. Rock, and U.K. Yang, Univ. Rochester preprint, UR-1355, 1995.

[60] CCFR/NuTeV Collab.: U.K. Yang et al., Phys.Rev.Lett. 87 (2001) 251802;  
      CCFR/NuTeV Collab.: A. Bodek, in Proc. of the 9th Int. Workshop on Deep Inelastic Scattering, DIS 2001 (2001), Bologna (hep-ex/00105087).

[61] BCDMS Collab., A.C. Benvenuti et al., Phys. Lett. B223 (1989) 485; B237 (1990) 592.

[62] SLAC Collab., L.W. Whitlow et al., Phys. Lett. B250 (1990) 193.

[63] E140X Collab., L.H. Tao et al., Z. Phys. C70 (1996) 387.

[64] EM Collab., J.J. Aubert et al., Nucl. Phys. B259 (1985) 189; B293 (1987) 740.

[65] CDHJSW Collab., P. Berge et al., Z. Phys. C49 (1991) 187.

[66] M.G. Ryskin and Yu.M. Shabelski, Z. Phys. C61 (1994) 517; C66 (1995) 151.

[67] J. Kwiecinski, A.D. Martin and A. Stasto, Phys.Rev. D56 (1997) 3991.

[68] J. Blumlein, Preprints DESY 95-121 (hep-ph/9506403), DESY 95-125 (hep-ph/9506446).

[69] K. Golec-Biernat and M Wusthoff, Phys.Rev. D59 (1999) 014017; D60 (1999) 014015, 114023.

[70] M.A. Kimber, A.D. Martin, and M.G. Ryskin, Phys. Rev. D63 (2001) 114027.

[71] H. Jung and G. Salam, Eur.Phys.J. C19 (2001) 351; H. Jung, hep-ph/9908497

[72] M.G. Ryskin and Yu.M. Shabelski, Z. Phys. C61 (1994) 517; C66 (1995) 151.

[73] A.V. Kotikov, A. V. Lipatov, G. Parente and N.P. Zotov, in Proc. of the International School “Heavy Quark Physics” (2002), Dubna.

[74] H. Jung, Nucl. Phys. (Proc. Suppl.) 79 (1999) 429.
Figure captions

**Fig. 1** The diagrams contributing to $T_{\mu\nu}$ for a gluon target. They should be multiplied by a factor of 2 because of the opposite direction of the fermion loop. The diagram (a) should be also doubled because of crossing symmetry.

**Fig. 2** The structure function $F_L(x, Q^2)$ as a function of $x$ for different values of $Q^2$ compared to experimental data. The H1 data: the first 1997 ones [13], new 2001 ones [15] and preliminary ones [18] are shown as black triangles, circles and squares, respectively. The data of NM [58], CCFR [60] and BCDMS [61] Collaborations are shown as white triangles, circles and squares, respectively. Curves 1, 2, 3 and 4 correspond to SF obtained in the perturbative QCD with the GRV [43] gluon density at the leading order approximation and to SF obtained in the $k_T$ factorization approach with JB (at $Q^2_0 = 4 \text{ GeV}^2$) [68], Kwiecinski-Martin-Stasto (KMS) and GBW [69] parametrizations of unintegrated gluon distribution.

**Fig. 3** The structure function $F_L(x, Q^2)$ as a function of $x$ for different values of $Q^2$. To compare with Fig. 2 'experimental data' (see [19]-[21] and Section 3) are added as black stars. The 'experimental data' values depend mostly on the derivative $dF_2(x, Q^2)/d\ln Q^2$, which data are known at little different $Q^2$ values (see [15]). So, the 'experimental data' obtained at 12, 15, 20, 25 and 35 GeV$^2$ are presented here at 13.4, 15.3, 22.4, 29.6 and 39.7 GeV$^2$, respectively.

**Fig. 4** The ratio $F_{cL}(x, Q^2)/F_L(x, Q^2)$ as a function of $x$ for different values of $Q^2$. 
Figure 1:
Figure 2
Figure 3
Figure 4