Local adjacency metric dimension of sun graph and stacked book graph

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Abstract. A graph is a mathematical system consisting of a non-empty set of nodes and a set of empty sides. One of the topics to be studied in graph theory is the metric dimension. Application in the metric dimension is the navigation robot system on a path. Robot moves from one vertex to another vertex in the field by minimizing the errors that occur in translating the instructions (code) obtained from the vertices of that location. To move the robot must give different instructions (code). In order for the robot to move efficiently, the robot must be fast to translate the code of the nodes of the location it passes. so that the location vertex has a minimum distance. However, if the robot must move with the vertex location on a very large field, so the robot can not detect because the distance is too far.[6] In this case, the robot can determine its position by utilizing location vertices based on adjacency. The problem is to find the minimum cardinality of the required location vertex, and where to put, so that the robot can determine its location. The solution to this problem is the dimension of adjacency metric and adjacency metric bases. Rodriguez-Velzquez and Fernau combine the adjacency metric dimensions with local metric dimensions, thus becoming the local adjacency metric dimension. In the local adjacency metric dimension each vertex in the graph may have the same adjacency representation as the terms of the vertices. To obtain the local metric dimension of values in the graph of the Sun and the stacked book graph is used the construction method by considering the representation of each adjacent vertex of the graph.

1. Introduction
The concept of metric dimension was first introduced by F. Harary, et all [3] in On the metric dimension of a graph. Application in the metric dimension is the navigation robot. Research about local metric dimension of graph have done by Okamoto et al.[7], Then, Rodriguez and Fernau [9], continued their research that is about local adjacency metric dimension of corona graphs. Their research is developing of the concept about adjacency metric dimension of graphs that has introduced by Jannesari and Omoomi [4]. Farthes before. The local adjacency metric dimension is that each vertex on a graph may have the same adjacency representation as the terms neither of the adjacency vertex. Motivated by results in [3], [7], [4], and [9], this paper Local Adjacency Metric dimension of Sun Graph and Stacked Book Graph

2. Preliminaries
This section presents about some definitions and notions that are using in this research. These concepts are taken from [1]. Let $G = (V(G), E(G))$ is a simple, finite and connected graph with a set of vertices $V(G)$ and set of edges $E(G)$. Distance between two vertices $u$ and $v$ on graph
$G$ is the shortest path length from $u$ to $v$ on graph $G$, denoted by $d(u, v) = d(v, u)$. If the $u$ and $v$ vertex are not connected, then $d(u, v)$ is undefined and listed as $d(u, v) = \infty$. [8]

Two adjacent vertices $v$ and $w_k$ will be write $v \sim w_k$ and two vertices $v$ and $w_k$ that is not adjacent with $v \sim w_k$. The distance between two vertices $v$ and $w_k$ in $G$, $d(v, w_k)$ is the length of shortest path joining $v$ and $w_k$. The adjacency distance between $v$ and $w_k$ denoted by $d_2(v, w_k)$ and defines by [9]

$$d_2(u, w_k) = \begin{cases} 
0, & \text{if } v = w_k \\
1, & \text{if } v \sim w_k \\
2, & \text{if } v \approx w_k
\end{cases}$$

Let $W = \{w_1, w_2, ..., w_k\} \subseteq V(G)$ be in order set of vertices and $v$ is a vertex in $G$ The adjacency representation of $v$ with respect to $W$ is the ordered $k$-tuple :

$$r_2(v | W) = (d_2(v, w_1), d_2(v, w_2), d_2(v, w_3), ..., d_2(v, k))$$

$W$ is called a local adjacency resolving set of $G$ if a pair of adjacent distinct vertex in $G$ have different adjacency representations. A minimum local adjacency resolving set for $G$ is local adjacency metric basis of $G$. Adjacency metric dimension for $G$, denoted by $dim_{A,l}(G)$ is the cardinality of vertices in local adjacency metric basis for $G$. Sun graph is defined by a graph cycle $C_n$ corona Complement graph Complete $\bar{C}_m$. In this research for writing the notation of the Sun graph is denoted $M_{n,m}$, where $M_{n,m} = C_n \circ \bar{K}_m$ with $n \geq 3$ and $m \geq 1$.[5] Stacked Book Graph is defined by a cartesian $K_{1,n} \times P_m$ with $n \geq 2$, $m \geq 2$. In this research for writing the notation of the stacked book graph is denoted $B_{n,m}$.[2]

3. Main Results

In the following, we present some useful results on the local adjacency metric dimension of sun graph and stacked book graph.

3.1. Local adjacency metric dimension of sun graph

Let $W \subseteq V(M_{n,m})$ be lokal adjacency metric basis. Let sun graph with order $n \geq 2$ and $m \geq 2$.

**Theorem 3.1** Let $G \cong M_{n,m}$ with $n \geq 3$ and $m \geq 1$, then

$$dim_{A,l}(G) = \begin{cases} 
2, & \text{if } n = 3 \\
2 + \frac{n - 4}{3}, & \text{if } n > 3
\end{cases}$$

**Proof 3.2** For $n = 3$, Let $W = \{v_1, v_2\} \subseteq V(G)$. We will show that $W$ is a local adjacency resolving set of $G$. The local adjacency of vertices from $V(G) - W$ are as follows :

$r(v_1 | W) = (0, 1)$, $r(v_{1m} | W) = (1, 2)$

$r(v_2 | W) = (1, 0)$, $r(v_{2m} | W) = (2, 1)$

$r(v_3 | W) = (1, 1)$, $r(v_{3m} | W) = (2, 2)$

As we see that all of the adjacency representation of adjacent vertices are distinct. So $W = \{v_1, v_2\} \subseteq V(G)$ is a local adjacency resolving set for $G$.

The cardinality of $W$, $|W| = 2$ is minimum because if $|W| < 2$ have $r(v_2 | W) = r(v_3 | W)$ and $r(v_2 | W) = r(v_{2m} | W)$. Without loss of generality, so we get $dim_{A,l}(G) = 2$ for $n = 3$.

For $n > 3$. Let $W = \{v_1, v_2, v_3, ..., v_{2 + \frac{n - 4}{3}}\} \subseteq V(G)$. All of the adjacency representation of adjacent vertices are distinct. So $W \subseteq V(G)$ is a local adjacency resolving
set for $G$. The cardinality of $W$, $|W| = \left\lfloor \frac{2 + \frac{n-4}{3}}{2} \right\rfloor$ is minimum because $|W| < \left\lfloor \frac{2 + \frac{n-4}{3}}{2} \right\rfloor$ have $r(v_{n-1}|W) = r(v_{n-1}|m|W)$.

Without loss of generality, so we get $\dim_{A,l}(G) = \left\lfloor \frac{2 + \frac{n-4}{3}}{2} \right\rfloor$ for $n > 3$.

3.2. Local adjacency metric dimension of stacked book graph

Let $W \subseteq V(B_{n,m})$ be a local adjacency metric basis. Let stacked book graph with order $n \geq 2$ and $m \geq 2$.

**Theorem 3.3** Let $G \cong B_{n,m}$ with $n \geq 2$ and $m \geq 2$, then

$$\dim_{A,l}(G) = \begin{cases} \frac{m}{2}, & \text{if } m \text{ even} \\ \frac{m-1}{2}, & \text{if } m \text{ odd} \end{cases}$$

**Proof 3.4** For $n \geq 2$, $m \geq 2$, and $m$ even we will be shown $\dim_{A,l}(G) = \frac{m}{2}$. To determine local adjacency metric dimension of stacked book graph, then we will look for upper bound and lower bound of stacked book graph.

a) To determine of upper bound

Let $W_1 = \{v_1, v_3, v_5, ..., v_{2p-1}\}$, $p = 1, 2, ..., \frac{m}{2}$ so that $W_1$ has a different local adjacency distance representation for each vertex in the stacked book graph. $W_1$ is local adjacency resolving set, $|W_1| = \frac{m}{2}$. $W_1$ is local adjacency resolving set but not a local adjacency metric basis because is still a $W_1$ that has a minimum of cardinality. so $\dim_{A,l}(G) \leq \frac{m}{2}$

b) To determine of lower bound

Let $W_2 = \{v_1, v_3, v_5, v_7, ..., v_{2p-1}\}$, $p = 1, 2, ..., (\frac{m}{2} - 1)$. $|W_2| = \frac{m}{2} - 1$. Because $r(v_m|W_2) = r(v_{m-1}|W_2)$ so $W_2$ is not local adjacency resolving set. Two adjacency vertices has some representation. So determine of lower bound is $\frac{m}{2} \leq \dim_{A,l}(G)$. Without loss of generality, we have $\frac{m}{2} \leq \dim_{A,l}(G) \leq \frac{m}{2}$. So it proved that so it proved that $\dim_{A,l}(G) = \frac{m}{2}$

For $n \geq 2$, $m \geq 2$, and $m$ odd we will be shown $\dim_{A,l}(G) = \frac{m-1}{2}$. To determine local adjacency metric dimension of stacked book graph, then we will look for upper bound and lower bound of stacked book graph.

a) To determine of upper bound

Let $W_1 = \{v_2, v_4, v_6, v_8, ..., v_{2p}\}$, $p = 1, 2, ..., \frac{m-1}{2}$ so that $W_1$ has a different local adjacency distance representation for each vertex in the stacked book graph. $W_1$ is local adjacency resolving set, $|W_1| = \frac{m-1}{2}$. $W_1$ is local adjacency resolving set but not a local adjacency metric basis because is still a $W_1$ that has a minimum of cardinality. so $\dim_{A,l}(G) \leq \frac{m-1}{2}$

b) To determine of lower bound

Let $W_2 = \{v_2, v_4, v_6, v_8, ..., v_{2p}\}$, $p = 1, 2, ..., (\frac{m-1}{2} - 1)$. $|W_2| = \frac{m-1}{2} - 1$. Because $r(v_m|W_2) = r(v_{m-1}|W_2)$ and $r(v_{m-1}|W_2) = r(v_{m-2}|W_2)$ so $W_2$ is not local adjacency resolving set. Two adjacency vertices has some representation. So determine of lower bound is $\frac{m-1}{2} \leq \dim_{A,l}(G)$. Without loss of generality, we have $\frac{m-1}{2} \leq \dim_{A,l}(G) \leq \frac{m-1}{2}$. So it proved that so it proved that $\dim_{A,l}(G) = \frac{m-1}{2}$
4. Conclusion
In this paper we have given result the local adjacency metric dimension of sun graph and stacked book graph. The result shows that the local adjacency metric dimension of sun graph as follows:
we have we have \( \dim_{A,l}(G) = 2 \) if \( n = 3 \) and \( \dim_{A,l}(G) = \left\lfloor \frac{2 + \frac{n - 4}{3}}{2} \right\rfloor \) if \( n > 3 \) and \( \dim_{A,l}(G) = \frac{m}{2} \) if \( m \) even and \( \dim_{A,l}(G) = \frac{m - 1}{2} \) if \( m \) odd. Based on our research, we have maximum adjacency distance to determine local adjacency metric basis is 3 vertices.

5. References
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