Optimizations with Intelligent Reflecting Surfaces (IRSs) in 6G Wireless Networks: Power Control, Quality of Service, Max-Min Fair Beamforming for Unicast, Broadcast, and Multicast with Multi-antenna Mobile Users and Multiple IRSs

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Abstract—Intelligent reflecting surfaces (IRSs) have received much attention recently and are envisioned to promote 6G communication networks. In this paper, we formulate optimization problems for power control under quality of service (QoS) and max-min fair QoS under three kinds of traffic patterns from a base station (BS) to mobile users (MUs): unicast, broadcast, and multicast. The optimizations are achieved by jointly designing the transmit beamforming of the BS and the phase shift matrix of the IRS. For power control under QoS, existing IRS studies in the literature address only the unicast setting, whereas no IRS work has considered max-min fair QoS. Furthermore, we extend our above optimization studies to the novel settings of multi-antenna mobile users or multiple intelligent reflecting surfaces. For all the above optimizations, we provide detailed analyses to propose efficient algorithms. To summarize, our paper presents a comprehensive study of power control, QoS, and fairness in wireless networks enhanced by IRSs.

Index Terms—Intelligent reflecting surfaces, 6G communications, power control, quality of service, max-min fair design, wireless networks.

I. INTRODUCTION

Intelligent reflecting surfaces (IRSs) and IRS-aided communications. An intelligent reflecting surface (IRS), or simply an intelligent surface, can intelligently control the wireless environment to improve signal strength received at the destination. This is vastly different from prior techniques which improve wireless communications via optimizations at the sender or receiver. Specifically, an IRS consists of many IRS units, each of which can reflect the incident signal at a reconfigurable angle. In such IRS-aided communications, the wireless signal travels from the source to the IRS, is optimized at the IRS, and then travels from the IRS to the destination. Such communication method is particularly useful when the source and destination such as a base station (BS) and a mobile user (MU) have a weak wireless channel in between due to obstacles or poor environmental conditions, or they do not have direct line of sights.

Because of the ability to configure wireless environments, IRSs are envisioned by many experts in wireless communications to play an important role in 6G networks. In November 2018, the Japanese mobile operator NTT DoCoMo and a startup MetaWave demonstrated the use of IRS-like technology for assisting wireless communications in 28GHz band [1]. IRSs have been compared with the massive MIMO technology used in 5G communications. IRSs reflect wireless signals and hence consume little power, whereas massive MIMO transmits signals and needs much more power [2].

Problems studied in this paper: Various optimizations in IRS-aided communications. In this bold paper, we investigate how to jointly design the transmit beamforming of the BS and the phase shift matrix of the IRS, for the two optimization problems of power control under quality of service (QoS) and max-min fair QoS, under various traffic models from the BS to MUs including unicast, broadcast, and multicast, in consideration of constraints of the phase shift matrix (e.g., with or without amplitude attenuation, continuous or discrete phase shifts), with extensions to multi-antenna mobile users or/and multiple IRSs. We characterize the QoS for an MU by the received signal-to-interference-plus-noise ratio (SINR) at the MU.

Contributions. The contributions of this paper are summarized as follows:

1) We formulate optimization problems for power control under QoS and max-min fair QoS under three kinds of traffic patterns from the BS to the MUs: unicast, broadcast, and multicast. The former optimization problem is addressed only in the unicast setting by existing IRS studies [3–5], whereas no IRS work has considered the latter problem.
2) Furthermore, we extend our optimization problems to consider multi-antenna mobile users or/and multiple IRSs, where such settings are novel in their own rights.
3) For all the optimizations discussed above, we present detailed analyses to propose efficient algorithms.

Organization of this paper. Section II presents the communication models. In Section III, we formulate the optimization
problems. The analysis and algorithms for solving the problems are elaborated in Section [V]. In Section [VI] we survey related studies. Finally, Section [VII] concludes the paper.

Notation. Scalars are denoted by italic letters, while vectors and matrices are denoted by bold-face lower-case and upper-case letters, respectively. \( C \) denotes the set of all complex numbers. For a matrix \( M \), its transpose and conjugate transpose are denoted by \( M^T \) and \( M^H \), while \( M_{i,j} \) (if not defined in other ways) means the element in the \( i \)th row and \( j \)th column of \( M \). For a vector \( x \), its transpose, conjugate transpose, and Euclidean norm are denoted by \( x^T \), \( x^H \), and \( \| x \| \), while \( x_i \) (if not defined in other ways) means the \( i \)th element of \( x \).

II. COMMUNICATION MODELS

We now present the IRS-aided wireless communication models.

In a typical system which we study, there are a base station (BS) with \( M \) antennas, an intelligent reflecting surface (IRS) with \( N \) IRS units, and \( K \) single-antenna mobile users (MUs) numbered from 1 to \( K \). We will be clear when the system is extended to the cases of multi-antenna MUs or/and multiple IRSs.

We will discuss three kinds of traffic patterns from the BS to the MUs:

- **unicast**, where the BS sends an independent data stream to each MU.
- **broadcast**, where the BS sends the same data stream to all \( K \) MUs, and
- **multicast**, where \( K \) MUs are divided into \( g \) groups \( G_1, G_2, \ldots, G_g \), and the BS sends an independent data stream to each group.

Since unicast and broadcast can be seen as special cases of multicast, we focus on multicast below, where \( k \) denotes the group index and \( i \) denotes the MU index; i.e., \( k \in \{1, \ldots, g\} \) and \( i \in G_k \). When multicast reduces to broadcast, there is only one group and \( i \) still denotes the MU index. When multicast reduces to unicast, each MU is a group and \( k \) denotes the MU index.

We define the following notation for the wireless channels. Let \( H_{b,r} \in \mathbb{C}^{N \times M} \) be the channel from the BS to the IRS. For MU \( i \in G_k \) with \( k \in \{1, \ldots, g\} \), we define \( h_{i,t}^H \in \mathbb{C}^{1 \times N} \) as the channel from the IRS to the \( i \)th MU, and define \( h_{b,i}^H \in \mathbb{C}^{1 \times M} \) as the downlink channel from the BS to the \( i \)th MU. In the above notation, the subscript "b" represents the BS, whereas the subscript "r" signifies IRS. When we extend one IRS to multiple IRSs (say \( L \)), the subscript "r" will be replaced by the IRS index \( l \in \{1, \ldots, L\} \) to denote the channel associated with the \( l \)th IRS; i.e., \( H_{b,l} \), and \( h_{l,t}^H \) will be changed to \( H_{b,l,t} \) and \( h_{l,t}^H \). When we extend single-antenna MUs to multi-antenna MUs, we will add a subscript \( q \) after the subscript \( i \) in the channel notation to denote the channel associated with MU \( i \)'s \( q \)th antenna (note \( q \in \{1, \ldots, Q_i\} \) if MU \( i \) has \( Q_i \) antennas). This means that in the case of multi-antenna MUs and one IRS, \( h_{i,t}^H \) and \( h_{b,i}^H \) will be replaced by \( h_{i,t,1}^H \) and \( h_{b,i,1}^H \) while in the case of multi-antenna MUs and \( L \) IRSs, \( h_{i,t}^H \) and \( h_{b,i}^H \) will be replaced by \( h_{i,t,1,q}^H \) and \( h_{b,i,1,q}^H \).

We now focus back on the case of single-antenna MUs and one IRS. In an IRS with \( N \) IRS units, for the \( n \)th IRS unit with \( n \in \{1, \ldots, N\} \), we let \( \beta_n \) be its amplitude change factor and \( \theta_n \) be its phase shift to the incident signal. Then we define the reflection coefficient matrix \( \Phi \) as follows:

\[
\Phi := \text{diag}(\beta_1 e^{j\theta_1}, \ldots, \beta_N e^{j\theta_N}),
\]

which means an \( N \times N \) diagonal matrix with the diagonal elements being \( \beta_1 e^{j\theta_1}, \ldots, \beta_N e^{j\theta_N} \).

If the IRS units change only phases of the incident signals but do not change their amplitudes, then \( \beta_n = 1 \) for \( n \in \{1, \ldots, N\} \), and each diagonal element of \( \Phi \) lies in the complex unit circle, so that

\[
\Phi := \text{diag}(e^{j\theta_1}, \ldots, e^{j\theta_N}).
\]

Clearly, the reflection coefficient matrix in Eq. (1) is a generalization of phase shift matrix Eq. (2). Most studies \([6]-[10]\) in the literature to date have assumed \( \beta_n = 1 \) for \( n \in \{1, \ldots, N\} \), with few work \([11]\) considering \( \beta_n \leq 1 \) (i.e., amplitude attenuation is possible). For simplicity, we use reflection coefficient matrix and phase shift matrix interchangeably and they both can be in the form of Eq. (1).

About the values that the phase shifts \( \theta_n |_{n \in \{1, \ldots, N\}} \) can take, the two simple variants are the continuous and discrete models below. In the continuous case, each of \( \theta_n |_{n \in \{1, \ldots, N\}} \) can take any value in \([0, 2\pi]\), as in \([3], [4]\). In the discrete case, each of \( \theta_n |_{n \in \{1, \ldots, N\}} \) can only take predefined discrete values; e.g., \( \tau \) discrete values equally spaced on a circle for some positive integer \( \tau \): \( \{0, \frac{2\pi}{\tau}, \ldots, \frac{2\pi(\tau-1)}{\tau}\} \), as in \([5], [12]\).

We define \( h_i^H(\Phi) \in \mathbb{C}^{1 \times M} \) by

\[
h_i^H(\Phi) := h_{i,t}^H(\Phi) H_{b,r} + h_{b,i}^H,
\]

so that \( h_i^H(\Phi) \) means the overall downlink channel to MU \( i \) by combining the direct channel with the indirect channels via all IRS units.

Let \( w_x \in \mathbb{C}^{M \times 1} \) be the BS transmit beamforming for group \( G_k \) with \( k \in \{1, \ldots, g\} \). For BS’s signal \( s_k \) for group \( G_k \), when it arrives at MU \( i \) of group \( G_k \), the received signal at MU \( i \) is given by \( s_k h_i^H(\Phi) w_x \). The interference at MU \( i \) consists of signals intended for other groups \( j \in \{1, \ldots, g\} \setminus \{k\} \) and is given by \( \sum_{j \in \{1, \ldots, g\} \setminus \{k\}} s_j h_i^H(\Phi) w_j \). We consider that signals are normalized to unit power, so that the signal-to-interference-plus-noise ratio (SINR) at MU \( i \) is given by

\[
\text{SINR}_i = \sum_{j \in \{1, \ldots, g\} \setminus \{k\}} \frac{|h_i^H(\Phi) w_k|^2}{|h_i^H(\Phi) w_j|^2 + \sigma_i^2},
\]

where \( \sigma_i^2 \) denotes the additive white Gaussian noise’s power spectral density at MU \( i \).

We consider that the BS controls the IRS and obtains the channel information \( h_{i,t}^H, H_{b,r}, h_{b,i}^H \) at the channel estimation stage. For example, we can consider a time-division duplexing (TDD) protocol for the uplink and downlink, and exploit channel reciprocity to acquire the channel state information.
After getting $h_{i}^{H}$, $H_{b,i}$, $h_{b,i}$ and other parameters from the mobile users, the goal of the BS is to design its transmit beamforming $W$ and the IRS’s phase shift matrix $\Phi$ for an optimization problem such as power control under QoS and max-min fair QoS discussed below. Afterwards, the BS will set the transmit beamforming as the obtained $W$, and remotely set the IRS phase shift as the obtained $\Phi$.

In this paper, we focus on the following two optimization problems: power control under QoS, and max-min fair QoS. We briefly discuss them below and will present more details in Section III.

**Power control under QoS.** The total power consumed by the BS to transmit the signals to all $g$ groups is given by

$$\sum_{k=1}^{g} \|w_{k}\|^2, \quad (5)$$

where the operation $\| \cdot \|$ denotes the Euclidean norm.

Power control under QoS means minimizing the BS’s total power consumption in (5) subject to the constraint that SINR, in Eq. (4) is at least some predefined requirement $\gamma_{i}$, for MU index $i \in G_{k}$ with group index $k \in \{1, \ldots, g\}$.

**Max-min fair QoS.** In the optimization problem of max-min fair QoS, similar to the seminal work [12] by Karipidis et al., we consider that the received SINR of each MU is scaled by a predetermined factor $1/\gamma_{i}$ for a positive real constant $\gamma_{i}$, to model possibly different grades of services. Then the minimum scaled SINRs among all MUs is given by

$$\min_{k \in \{1, \ldots, g\}} \min_{i \in G_{k}} \frac{\text{SINR}_{i}}{\gamma_{i}}, \quad (6)$$

where $\text{SINR}_{i}$ is given by Eq. (4).

In max-min fair QoS, the problem is to maximize the term in (5) subject to the constraint that the BS’s total power consumption

$$\sum_{k=1}^{g} \|w_{k}\|^2 \quad (5)$$

is at most some value $P$. Maximizing (6) is more general than the problem of maximizing the minimum SINR among all MUs, since the former reduces to the latter in the special case of equal $\gamma_{i}$ for all $i$.

**III. Optimization Problems**

In this section, we elaborate the following two optimization problems which have been briefly discussed in the previous section:

- **power control under QoS**, and
- **max-min fair QoS**.

The optimizations are done by jointly designing the transmit beamforming of the BS and the phase shift matrix of the IRS.

As already noted in the previous section, we discuss three kinds of traffic patterns from the BS to the MUs:

- **unicast**, where the BS sends an independent data stream to each MU,
- **broadcast**, where the BS sends the same data stream to all $K$ MUs, and
- **multicast**, where $K$ MUs are divided into $g$ groups $G_{1}, G_{2}, \ldots, G_{g}$, and the BS sends an independent data stream to each group.

The combination of the two optimization problems and the three traffic patterns induce six settings.

For the above six settings, we further have the following variants:

- the reflection coefficient matrix $\Phi$ can be in the form of Eq. (1) or (2) (i.e., with or without amplitude attenuation), where the phase shifts $\theta_{n}|_{n \in \{1, \ldots, N\}}$ can further be continuous or discrete,
- an MU can have single antenna or multiple antennas,
- the system can have one IRS of $N$ IRS units, or $L$ IRSs comprising $N_{1}, \ldots, N_{L}$ IRS units.

Below, we first discuss optimization problems for the multicast traffic with single-antenna MUs and one IRS, which will imply the corresponding problems for unicast and broadcast since unicast and broadcast can be seen as special cases of multicast. Later, we extend the problems to the cases of multi-antenna MUs or/and multiple IRSs.

**A. Power control under QoS**

For power control under QoS, we first present the multicast setting and then reduce it to the unicast and broadcast cases.

**Multicast.** We have defined the notation for the multicast in Section II. For the multicast traffic, power control under QoS means minimizing the BS’s total power consumption

$$\sum_{k=1}^{g} \|w_{k}\|^2 \quad (5)$$

subject to the constraint that SINR, in Eq. (4) is at least some predefined requirement $\gamma_{i}$, for MU index $i \in G_{k}$ with group index $k \in \{1, \ldots, g\}$. Hence, power control under QoS for multicast traffic is given by the following optimization problem:

\[(P1): \min_{W,\Phi} \sum_{k=1}^{g} \|w_{k}\|^2 \quad (7a)\]

\[\text{s.t.} \sum_{j \in \{1, \ldots, g\} \setminus \{k\}} \frac{|h_{i}^{H}(\Phi)w_{k}|^2}{|h_{i}^{H}(\Phi)w_{j}|^2 + \sigma_{t}^2} \geq \gamma_{i}, \quad (7b)\]

\[\forall k \in \{1, \ldots, g\}, \forall i \in G_{k}, \quad \text{Constraints on } \Phi. \quad (7c)\]

**Unicast.** When multicast reduces to unicast, each MU is a group, so the number $g$ of groups is $K$, and $k$ denotes the MU index. Then Problem (P1) becomes

\[(P2): \min_{W,\Phi} \sum_{k=1}^{K} \|w_{k}\|^2 \quad (8a)\]

\[\text{s.t.} \sum_{j \in \{1, \ldots, K\} \setminus \{k\}} \frac{|h_{i}^{H}(\Phi)w_{k}|^2}{|h_{i}^{H}(\Phi)w_{j}|^2 + \sigma_{k}^2} \geq \gamma_{k}, \quad (8b)\]

\[\forall k \in \{1, \ldots, K\}, \quad \text{Constraints on } \Phi. \quad (8c)\]

Recent studies by Wu and Zhang [3–5] have addressed Problem (P2) with the constraints on $\Phi$ of (8c) given in the form of Eq. (2) (i.e., $\Phi = \text{diag}(e^{j\theta_{1}}, \ldots, e^{j\theta_{N}})$). In [3, 4], each of $\theta_{n}|_{n \in \{1, \ldots, N\}}$ can take any value in $[0, 2\pi)$. In contrast, in [5], each of $\theta_{n}|_{n \in \{1, \ldots, N\}}$ can only take the following $\tau$
discrete values equally spaced on a circle for some positive integer $\tau$: $\{0, 2\pi \tau, \ldots, 2\pi(r-\tau)\}$.

**Broadcast.** When multicast reduces to broadcast, there is only one group $G_1$, so that $g = 1$ and $G_1 = \{1, \ldots, K\}$. Then Problem (P1) becomes

\begin{equation}
(P3): \min_{w, \Phi} \|w\|^2 \tag{9a}
\end{equation}

s.t. \[
\frac{|h^H(\Phi)w|^2}{\sigma_i^2} \geq \gamma_i, \, \forall i \in \{1, \ldots, K\}, \tag{9b}
\]

Constrains on $\Phi$.

We summarize Problems (P1)–(P3) in Table I on Page 6.

**Extensions to multi-antenna MUs or/and multiple IRSs.** The above Problems (P1)–(P3) consider single-antenna MUs and one IRS. We now extend the problems to the cases of multi-antenna MUs or/and multiple IRSs.

- **Multi-antenna MUs and one IRS.** When MU $i$ has $Q_i$ antennas for $i \in G_k$ with $k \in \{1, \ldots, g\}$, with $q \in \{1, \ldots, Q_i\}$ indexing antennas of MU $i$, we define $h_{i,q}^H \in \mathbb{C}^{1 \times N}$ as the channel from the BS to the $q$th antenna of MU $i$, and define $h_{b,i,q}^H \in \mathbb{C}^{1 \times M}$ as the downlink channel from the BS to the $q$th antenna of MU $i$. Then we define $h_{i,q}(\Phi)$ as follows to represent the overall downlink channel to MU $i$’s $q$th antenna by combining the direct channel with the indirect channels via all IRS units:

\[
h_{i,q}(\Phi) := h_{b,i,q}^H + \sum_{k=1}^{L} h_{b,i,q}^H \Phi_k H_{b,k}, \tag{10}
\]

Then at MU $i$, the power of the received signal associated with MU group $k$ is given by

\[
|w^H H_i(\Phi)w_k|^2, \tag{11}
\]

where we define $H_i(\Phi)$ as follows for notational simplicity:

\[
H_i(\Phi) := \sum_{q=1}^{Q_i} h_{i,q}(\Phi)h_{i,q}^H(\Phi). \tag{12}
\]

Similar to (11), at MU $i$, the power of the received interference associated with MU group $j \in \{1, \ldots, g\} \setminus \{k\}$ is given by $|w^H H_j(\Phi)w_j|^2$. Then

- replacing $|h_{i,q}^H(\Phi)w|^2$ and $|h_{i,q}^H(\Phi)w_j|^2$ of Problem (P1) by $|w^H H_i(\Phi)w_k|^2$ and $|w^H H_i(\Phi)w_j|^2$ of Problem (P2) by $|w^H H_k(\Phi)w_k|^2$ and $|w^H H_k(\Phi)w_j|^2$ of Problem (P3) by $|w^H H_j(\Phi)w_j|^2$.

we obtain the corresponding optimization problems respectively with multi-antenna MUs and one IRS. They are denoted by Problems (P1-MA)–(P3-MA) and presented in Table III on Page 8 where “MA” means multi-antenna.

- **Single-antenna MUs and multiple IRSs.** When there are $L$ IRSs comprising $N_1, \ldots, N_L$ IRS units, we define for $\ell \in \{1, \ldots, L\}$ that $H_{b,\ell} \in \mathbb{C}^{N_i \times M}$ represents the channel from the BS to the $\ell$th IRS, and $h_{i,\ell}^H \in \mathbb{C}^{1 \times N_i}$ represents the channel from the $\ell$th IRS to the $i$th MU, for MU $i \in G_k$ with $k \in \{1, \ldots, g\}$. Then with the phase shift matrices of the $L$ IRSs denoted by $\Phi_1, \ldots, \Phi_L$, we define $h_{i,\ell}^H(\Phi_1, \ldots, \Phi_L) := \sum_{\ell=1}^{L} h_{i,\ell}^H(\Phi_\ell) H_{b,\ell}$ as follows to represent the overall downlink channel to MU $i$ by combining the direct channel with the indirect channels via all IRSs:

\[
h_{i,\ell}(\Phi_1, \ldots, \Phi_L) := h_{b,i,\ell}^H + \sum_{\ell=1}^{L} h_{b,i,\ell}^H \Phi_\ell H_{b,\ell}. \tag{13}
\]

Furthermore, we define

\[
H_i(\Phi_1, \ldots, \Phi_L) := \sum_{q=1}^{Q_i} h_{i,q}(\Phi_1, \ldots, \Phi_L) h_{i,q}^H(\Phi_1, \ldots, \Phi_L). \tag{15}
\]

Then

- replacing $|w^H H_i(\Phi)w_k|^2$ and $|w^H H_i(\Phi)w_j|^2$ of Problem (P1) by $|w^H H_1(\Phi)w_k|^2$ and $|w^H H_1(\Phi)w_j|^2$ of Problem (P2) by $|w^H H_k(\Phi)w_k|^2$ and $|w^H H_k(\Phi)w_j|^2$ of Problem (P3) by $|w^H H_j(\Phi)w_j|^2$.\]
max \quad P

model possibly different grades of services. For the multicast total power consumption we consider that the received SINR of each MU

is a predetermined factor

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and hence can be removed from the optimizations, Problems (P4)–(P6) become maximizing the minimum SINR in the system.

We summarize Problems (P4)–(P6) in Table IV on Page 6.

B. Max-min fair QoS

Similar to Section III-A for max-min fair QoS here, we first present the multicast setting and then reduce it to the unicast and broadcast cases.

Multicast. We have defined the notation for multicast in Section II. Similar to the seminal work [12] by Karipidis et al., we consider that the received SINR of each MU i is scaled by a predetermined factor \(1/\gamma_i\) for a positive real constant \(\gamma_i\), to model possibly different grades of services. For the multicast traffic, max-min fair QoS means maximizing the minimum scaled SINRs among all MUs in \(G\) subject to that the BS’s total power consumption \(\sum_{k=1}^g \|w_k\|^2\) in (5) is at most some value \(P\). Then we obtain the following optimization problem:

(P4): \[
\text{maximize} \quad \min_{w, \Phi} \quad \min_{k \in \{1, \ldots, g\}} \quad \frac{|h_k^H(\Phi)w_k|^2}{\gamma_i} \\
\text{subject to} \quad \sum_{j \in \{1, \ldots, g\} \setminus \{k\}} |h_j^H(\Phi)w_j|^2 + \sigma_i^2 \\
\text{and also replacing the constraints on } \Phi \text{ in Eq. (7a) or (7c) by the constraints on } \Phi_1, \ldots, \Phi_L, \text{ we obtain the corresponding optimization problems respectively with multi-antenna MUs and multiple IRSs. They are denoted by Problems (P1-MA-MR)–(P3-MA-MR) and presented in Table IV on Page 9.}
\]

When multicast reduces to unicast, each MU is a group, so the number \(g\) of groups is \(K\), and \(k\) denotes the MU index. Then Problem (P4) becomes

(P5): \[
\text{maximize} \quad \min_{w, \Phi} \quad \min_{k \in \{1, \ldots, K\}} \quad \frac{|h_k^H(\Phi)w_k|^2}{\gamma_k} \\
\text{subject to} \quad \sum_{j \in \{1, \ldots, K\} \setminus \{k\}} |h_j^H(\Phi)w_j|^2 + \sigma_k^2 \\
\text{and constraints on } \Phi.
\]

Unicast. When multicast reduces to unicast, each MU is a group, so the number \(g\) of groups is \(K\), and \(k\) denotes the MU index. Then Problem (P4) becomes

(P6): \[
\text{maximize} \quad \min_{w, \Phi} \quad \min_{i \in \{1, \ldots, K\}} \quad \frac{|h_i^H(\Phi)w_i|^2}{\gamma_i \sigma_i^2} \\
\text{subject to} \quad \|w\|^2 \leq P, \\
\text{and constraints on } \Phi.
\]

Broadcast. When multicast reduces to broadcast, there is only one group \(G_1\), so that \(g = 1\) and \(G_1 = \{1, \ldots, K\}\). Then Problem (P4) becomes

Similar to the multicast case above, for the unicast (resp., broadcast) setting, in Eq. (17a) (resp., (18a), the received SINR of MU \(k\) (resp., MU \(i\)) is scaled by a predetermined factor \(1/\gamma_k\) (resp., MU \(1/\gamma_i\)), to model possibly different grades of services [12].

In the special case where all the \(\gamma\) factors are the same and hence can be removed from the optimizations, Problems (P4)–(P6) become maximizing the minimum SINR in the system. We summarize Problems (P4)–(P6) in Table IV on Page 6.

Extensions to multi-antenna MUs or/and multiple IRSs. The above Problems (P4)–(P6) consider single-antenna MUs and one IRS. We now extend the problems to the cases of multi-antenna MUs or/and multiple IRSs. Modifying Problems (P4)–(P6) in a way similar to that of modifying Problems (P1)–(P3) in Section III-A we have the following:

• Under multi-antenna MUs and one IRS, we modify Problems (P4)–(P6) to Problems (P4-MA)–(P6-MA), which are presented in Table II on Page 7.

• Under single-antenna MUs and multiple IRSs, we modify Problems (P4)–(P6) to Problems (P4-MA)–(P6-MA), which are presented in Table III on Page 8.

• Under multi-antenna MUs and multiple IRSs, we modify Problems (P4)–(P6) to Problems (P4-MA-MA)–(P6-MA-MA), which are presented in Table IV on Page 9.

IV. SOLVING THE OPTIMIZATION PROBLEMS

A. Solutions to Problems (P1)–(P3)

We now propose algorithms to solve Problems (P1)–(P3). Since Problems (P2) and (P3) can be seen as special cases of Problem (P1), we first focus on solving Problem (P1), and then apply the solutions to Problems (P2) and (P3).

We use the idea of alternating optimization, which is widely used in multivariate optimization. Specifically, we will optimize \(W\) and \(\Phi\) alternatively to solve Problem (P1). Below, we discuss the optimization of \(W\) given \(\Phi\) and the optimization of \(\Phi\) given \(W\).

Optimizing \(W\) given \(\Phi\). Given some \(\Phi\) satisfying the constraints in (7a), the goal of optimizing \(W\) for Problem (P1) is finding \(W\) to minimize the objective function in Eq. (7a) subject to the constraint in Eq. (7b).

We use the idea of exchanging variables and convert Problem (P1) to a form that is easier to analyze. Specifically, we define

\[ X_k := w_k w_k^H, \quad \forall k \in \{1, \ldots, g\}. \]

Then the objective function in (3) of Problem (P1) is given by

\[ \|w_k\|^2 = w_k^H w_k = \text{trace}(w_k w_k^H) = \text{trace}(X_k). \]

To express the constraint in Inequality (7b) of Problem (P1), we note

\[ |h_k^H(\Phi)w_k|^2 = h_k^H(\Phi)w_k w_k^H h_k(\Phi) = \text{trace}(w_k w_k^H h_k(\Phi) h_k^H(\Phi)) = \text{trace}(X_k H_k(\Phi)), \]

where

\[ X_k := w_k w_k^H, \quad \forall k \in \{1, \ldots, g\}. \]

Then the objective function in (3) of Problem (P1) is given by

\[ \text{trace}(X_k) = \text{trace}(w_k w_k^H h_k(\Phi) h_k^H(\Phi)) = \text{trace}(X_k H_k(\Phi)), \]

where

\[ X_k := w_k w_k^H, \quad \forall k \in \{1, \ldots, g\}. \]

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\[ X_k := w_k w_k^H, \quad \forall k \in \{1, \ldots, g\}. \]
### Problem (P1):

**Power control under QoS**

| Traffic Problem | Multicast | Unicast | Broadcast |
|-----------------|-----------|---------|-----------|
| (P1):           | \[
\begin{align*}
\min_{W, \Phi} & \quad \sum_{k=1}^{g} \|w_k\|^2 \\
\text{s.t.} & \quad \sum_{j \in \{1, \ldots, g\} \setminus \{k\}} |h_i^H(\Phi)w_j|^2 + \sigma_i^2 \geq \gamma_i, \\
& \quad \forall k \in \{1, \ldots, g\}, \forall i \in \mathcal{G}_k,
\end{align*}
\] |
| Constraints on $\Phi$. | | | |

| Max-min fair QoS | \[
\begin{align*}
\max_{W, \Phi} & \quad \min_{k \in \{1, \ldots, g\}} \min_{i \in \mathcal{G}_k} \gamma_i \sum_{j \in \{1, \ldots, g\} \setminus \{k\}} |h_i^H(\Phi)w_j|^2 + \sigma_i^2 \\
\text{s.t.} & \quad \sum_{k=1}^{g} \|w_k\|^2 \leq P, \\
& \quad \forall k \in \{1, \ldots, g\}, \forall i \in \mathcal{G}_k,
\end{align*}
\] |

*Table I (with single-antenna mobile users and one intelligent reflecting surface):* Optimizing the transmit beamforming $W$ (or $w$) of the base station (BS) and the phase shift matrix $\Phi$ of the intelligent reflecting surface (IRS) comprising $N$ IRS units for power control under QoS and max-min fair QoS under various downlink traffic patterns (i.e., unicast, broadcast, and multicast) from a multi-antenna base station to $K$ single-antenna mobile users (MUs). In the unicast case, the BS sends an independent data stream to each MU. In the broadcast case, the BS sends the same data stream to all $K$ MUs. In the multicast case, $K$ MUs are divided into $g$ groups $\mathcal{G}_1, \mathcal{G}_2, \ldots, \mathcal{G}_g$, and the BS sends an independent data stream to each group. The notation $h_i^H(\Phi)$ in the table means $h_{b,k}^H + h_{i,k}^H \Phi H_{b,i}$, meaning the overall downlink channel to MU $k$ by combining the direct channel with the indirect channels via all IRS units. Similarly, $h_i^H(\Phi)$ in the table means $h_{b,i}^H + h_{i,i}^H \Phi H_{b,i}$. The notation $P$ denotes the maximal power consumed by the BS.

where we define

$$H_i(\Phi) := h_i(\Phi)h_i^H(\Phi). \quad (22)$$

Replacing $k$ by $j$ in Eq. (21), we also have $|h_i^H(\Phi)w_j|^2 = \text{trace}(X_kH_i(\Phi))$. Using this and Eq. (21), we express the expression in Eq. (20) of Problem (P1) as

$$\sum_{j \in \{1, \ldots, g\} \setminus \{k\}} \text{trace}(X_kH_i(\Phi)) + \sigma_i^2 \geq \gamma_i. \quad (23)$$

In addition, the definition of $X_k$ in Eq. (19) implies that $X_k$ is semi-definite and has rank one. Combining this with Eq. (20) and Inequality (23), the problem of optimizing $W$ given $\Phi$ for Problem (P1) is given by

(P1a): \[
\begin{align*}
\min_{\{X_k\}_k^g} & \quad \sum_{k=1}^{g} \text{trace}(X_k) \\
\text{s.t.} & \quad \gamma_i \sigma_i^2 + \gamma_i \sum_{j \in \{1, \ldots, g\} \setminus \{k\}} \text{trace}(X_jH_i(\Phi)) \\
& \quad \forall k \in \{1, \ldots, g\}, \forall i \in \mathcal{G}_k,
\end{align*}
\] \quad (24a)

The only non-convex part in Problem (P1a) is the rank constraint in Eq. (24d). Hence, we adopt semidefinite relaxation (SDR) and drop Eq. (24d) to obtain a semidefinite programming problem. After a candidate solution is obtained, appro-
| Traffic Problem | Multicast | Unicast | Broadcast |
|----------------|-----------|---------|-----------|
| (P1-MA): \[
\min_{\Phi} \sum_{k=1}^{g} \|w_k\|^2 \\
\text{s.t.} \sum_{j \in \{1,\ldots,g\}\setminus\{k\}} w_j^H H_k(\Phi) w_j + \sigma_i^2 \geq \gamma_i, \\
\forall k \in \{1,\ldots,g\}, \forall i \in \mathcal{G}_k, \\
\text{Constraints on } \Phi.
\]| (P2-MA): \[
\min_{\Phi} \sum_{k=1}^{g} \|w_k\|^2 \\
\text{s.t.} \sum_{j \in \{1,\ldots,g\}\setminus\{k\}} w_j^H H_k(\Phi) w_j + \sigma_i^2 \geq \gamma_i, \\
\forall k \in \{1,\ldots,g\}, \forall i \in \mathcal{G}_k, \\
\text{Constraints on } \Phi.
\]| (P3-MA): \[
\min_{w,\Phi} \|w\|^2 \\
\text{s.t.} \sum_{j \in \{1,\ldots,g\}\setminus\{k\}} (w_j^H H_k(\Phi) w_j + \sigma_i^2) \geq \gamma_i, \\
\forall k \in \{1,\ldots,g\}, \forall i \in \mathcal{G}_k, \\
\text{Constraints on } \Phi.
\] |

Max-min fair QoS

(P4-MA): \[
\max_{\Phi} \min_{k \in \{1,\ldots,g\}} \min_{w_k} \left[ \sum_{j \in \{1,\ldots,g\}\setminus\{k\}} w_j^H H_k(\Phi) w_j + \sigma_i^2 \right] \\
\text{s.t.} \sum_{k=1}^{g} \|w_k\|^2 \leq P, \\
\text{Constraints on } \Phi.
\]

(P5-MA): \[
\max_{\Phi} \min_{k \in \{1,\ldots,g\}} \min_{w_k} \left[ \sum_{j \in \{1,\ldots,g\}\setminus\{k\}} w_j^H H_k(\Phi) w_j + \sigma_i^2 \right] \\
\text{s.t.} \sum_{k=1}^{g} \|w_k\|^2 \leq P, \\
\text{Constraints on } \Phi.
\]

(P6-MA): \[
\max_{w,\Phi} \min_{k \in \{1,\ldots,g\}} \min_{w_k} \left[ \sum_{j \in \{1,\ldots,g\}\setminus\{k\}} w_j^H H_k(\Phi) w_j + \sigma_i^2 \right] \\
\text{s.t.} \|w\|^2 \leq P, \\
\text{Constraints on } \Phi.
\]

Table II (with multi-antenna mobile users and one intelligent reflecting surface): Optimizing the transmit beamforming \(W\) (or \(w\)) of the base station (BS) and the phase shift matrix \(\Phi\) of the intelligent reflecting surface (IRS) comprising \(N\) IRS units for power control under QoS and max-min fair QoS under various downlink traffic patterns (i.e., unicast, broadcast, and multicast) from a multi-antenna base station to \(K\) multi-antenna mobile users (MUs), where MU \(i\) has \(Q_i\) antennas for \(i \in \mathcal{G}_k\) with \(k \in \{1,\ldots,g\}\). In the unicast case, the BS sends an independent data stream to each MU. In the broadcast case, the BS sends the same data stream to all \(K\) MUs. In the multicast case, \(K\) MUs are divided into \(g\) groups \(\mathcal{G}_1, \mathcal{G}_2, \ldots, \mathcal{G}_g\), and the BS sends an independent data stream to each group. The notation \(H_k(\Phi)\) in the table means \(\sum_{q=1}^{Q_k} h_{k,q}(\Phi) h_{k,q}^H(\Phi)\), where \(h_{k,q}(\Phi)\) denotes \(h_{b,k,q}^H + h_{r,k,q}^H \Phi_{b,r}\) and means the overall downlink channel to MU \(k\)’s antenna by combining the direct channel with the indirect channels via all IRS units. Similarly, \(H_i(\Phi)\) in the table means \(\sum_{q=1}^{Q_i} h_{i,q}(\Phi) h_{i,q}^H(\Phi)\), where \(h_{i,q}(\Phi)\) denotes \(h_{b,i,q}^H + h_{r,i,q}^H \Phi_{b,r}\). The notation \(P\) denotes the maximal power consumed by the BS.

Finding \(\Phi\) given \(W\). Given \(W\), Problem (P1) becomes the following feasibility check problem of finding \(\Phi\):

(P1b): Find \(\Phi\) \hspace{2cm} (25a) \hspace{2cm} \text{s.t.} \sum_{j \in \{1,\ldots,g\}\setminus\{k\}} |h_j^H(\Phi) w_j|^2 + \sigma_i^2 \geq \gamma_i, \\
\forall k \in \{1,\ldots,g\}, \forall i \in \mathcal{G}_k, \\
\text{Constraints on } \Phi. \hspace{2cm} (25b)

To solve Problem (P1b), we start with analyzing the constraint in Inequality (25b). To this end, we recall the definition of \(h_i^H(\Phi)\) in Eq. (3) to obtain \(h_i^H(\Phi) w_k\) as \(h_{r,i}^H \Phi_{b,i} w_k + h_{b,i}^H w_k\).

If the constraint on \(\Phi\) in Eq. (25c) is Eq. (1) (i.e., \(\Phi := \text{diag}(\beta_1 e^{j\theta_1}, \ldots, \beta_N e^{j\theta_N})\)), we find it convenient to define \(\Phi := [\beta_1 e^{j\theta_1}, \ldots, \beta_N e^{j\theta_N}]^H\). \hspace{2cm} (26)

Then we change variables to have \(h_i^H(\Phi) w_k = h_{r,i}^H \Phi_{b,i} w_k + h_{b,i}^H w_k = \Phi^H a_i(w_k) + b_i(w_k)\), \hspace{2cm} (27)

where we define \(a_i(w_k) := \text{diag}(h_{r,i}^H) \Phi_{b,i} w_k\), \hspace{2cm} (28)

and complex numbers \(b_i(w_k)\) by \hspace{2cm} (29)

From Eq. (27), we further compute \(|h_i^H(\Phi) w_k|^2\) which
appears in Inequality (25b):

\[ |h_k^H(\Phi)w_k|^2 = (\phi^H a_k(w_k) + b_k(w_k))(a_k^H(w_k)\phi + b_k^H(w_k)) = \phi^H a_k(w_k) a_k^H(w_k)\phi + \phi^H a_k(w_k) b_k^H(w_k) + b_k(w_k) a_k^H(w_k)\phi + b_k(w_k) b_k^H(w_k) = [\phi^H, 1] A_k(w_k) \begin{bmatrix} \phi^H \\ 1 \end{bmatrix} + b_k(w_k) b_k^H(w_k), \] (30)

where we define

\[ A_k(w_k) := \begin{bmatrix} a_k(w_k)a_k^H(w_k), & a_k(w_k)b_k^H(w_k) \\ b_k(w_k)a_k^H(w_k), & 0 \end{bmatrix}. \] (31)

Replacing \( k \) by \( j \) in Eq. (30), we also have

\[ |h_j^H(\Phi)w_j|^2 = [\phi^H, 1] A_j(w_j) \begin{bmatrix} \phi^H \\ 1 \end{bmatrix} + b_j(w_j) b_j^H(w_j). \] (32)

in Inequality (25b) of Problem (P1b) as

\[ \gamma_i \left( \sum_{j \in \{1, \ldots, g\} \setminus \{k\}} |h_j^H(\Phi)w_j|^2 + \sigma_i^2 \right) \geq \gamma_i \] (34)

We introduce an auxiliary variable \( t \), which is a complex number satisfying \( |t| = 1 \), and define

\[ v := t \begin{bmatrix} \phi^H \\ 1 \end{bmatrix} = \begin{bmatrix} \phi^H \\ t \end{bmatrix}. \] (35)

Then with \( |t| = 1 \), Inequality (34) is equivalent to

\[ v^H A_j(w_j)v + b_j(w_j) b_j^H(w_k) \geq \gamma_i \left( \sigma_i^2 + \sum_{j \in \{1, \ldots, g\} \setminus \{k\}} v^H A_j(w_j)v + b_j(w_j) b_j^H(w_j) \right). \] (36)
Multicast
Unicast
Broadcast

Power control under QoS

(P1-MA-MR):
\[
\min_{W, \Phi_1, \ldots, \Phi_L} \sum_{k=1}^{g} \|w_k\|^2 \\
\text{s.t.} \quad \sum_{j \in \{1, \ldots, g\} \setminus \{k\}} w_j^H H_{ij} \Phi_l w_j + \sigma_i^2 \geq \gamma_i, \\
\forall k \in \{1, \ldots, g\}, \forall i \in G_k, \\
\text{Constraints on } \Phi_1, \ldots, \Phi_L.
\]

(QoS under fair QoS Max-min)

(P2-MA-MR):
\[
\min_{W, \Phi} \sum_{k=1}^{K} \|w_k\|^2 \\
\text{s.t.} \quad \sum_{j \in \{1, \ldots, \gamma\} \setminus \{k\}} w_j^H H_{ij} \Phi w_j + \sigma_i^2 \geq \gamma_i, \\
\forall k \in \{1, \ldots, K\}, \\
\text{Constraints on } \Phi_1, \ldots, \Phi_L.
\]

(Max-min fair QoS)

(P4-MA-MR):
\[
\min_{W, \Phi_1, \ldots, \Phi_L} w_i^H H_{ij} \Phi_l w_i \\
\text{s.t.} \quad \sum_{k=1}^{g} \|w_k\|^2 \leq P, \\
\text{Constraints on } \Phi_1, \ldots, \Phi_L.
\]

(P5-MA-MR):
\[
\max_{W, \Phi_1, \ldots, \Phi_L} w_i^H H_{ij} \Phi_l w_i \\
\text{s.t.} \quad \sum_{k=1}^{g} \|w_k\|^2 \leq P, \\
\text{Constraints on } \Phi_1, \ldots, \Phi_L.
\]

(P6-MA-MR):
\[
\max_{W, \Phi_1, \ldots, \Phi_L} w_i^H H_{ij} \Phi_l w_i \\
\text{s.t.} \quad \|w_i\|^2 \leq P, \\
\text{Constraints on } \Phi_1, \ldots, \Phi_L.
\]

Table IV (with multi-antenna mobile users and multiple intelligent reflecting surfaces): Optimizing the transmit beamforming \(W\) (or \(w\)) of the base station (BS) and the phase shift matrix \(\Phi_1, \ldots, \Phi_L\) of \(L\) intelligent reflecting surfaces (IRSs) comprising \(N_1, \ldots, N_L\). IRS units for power control under QoS and max-min fair QoS under various downlink traffic patterns (i.e., unicast, broadcast, and multicast) from a multi-antenna base station to \(K\) multi-antenna mobile users (MUs), where MU \(i\) has \(Q_k\) antennas for \(i \in G_k\) with \(k \in \{1, \ldots, g\}\). In the unicast case, the BS sends an independent data stream to each MU. In the broadcast case, the BS sends the same data stream to all \(K\) MUs. In the multicast case, \(K\) MUs are divided into \(g\) groups \(G_1, G_2, \ldots, G_g\), and the BS sends an independent data stream to each group. The notation \(H_{ij}(\Phi_1, \ldots, \Phi_L)\) in the table means \(\sum_{q=1}^{Q_k} h_{k,q}(\Phi_1, \ldots, \Phi_L) h_{k,q}^H(\Phi_1, \ldots, \Phi_L)\), where \(h_{k,q}(\Phi_1, \ldots, \Phi_L)\) denotes \(h_{k,q} + \sum_{\ell=1}^{L} h_{k,q,\ell} \Phi_{\ell} h_{b,\ell}\) and means the overall downlink channel to MU \(i\) by combining the direct channel with the indirect channels via all IRSs. Similarly, \(H_{ij}(\Phi_1, \ldots, \Phi_L)\) in the table means \(\sum_{q=1}^{Q_k} h_{i,q}(\Phi_1, \ldots, \Phi_L) h_{i,q}^H(\Phi_1, \ldots, \Phi_L)\), where \(h_{i,q}(\Phi_1, \ldots, \Phi_L)\) denotes \(h_{i,q} + \sum_{\ell=1}^{L} h_{i,q,\ell} \Phi_{\ell} h_{b,\ell}\). The notation \(P\) denotes the maximal power consumed by the BS.

We further define
\[
V := vv^H. 
\]
If the constraint on \(\Phi\) in Eq. (26c) is in the form of Eq. (1) (i.e., \(\Phi := \text{diag} (\beta_1 e^{j\theta_1}, \ldots, \beta_N e^{j\theta_N})\)) so that \(\phi := [\beta_1 e^{j\theta_1}, \ldots, \beta_N e^{j\theta_N}]^H\) from Eq. (25), then the diagonal elements of \(V\) are given by \(V_{n,n} = \beta_n e^{j\theta_n}\), \(\beta_n e^{j\theta_n} = \beta_n^2\) for \(n \in \{1, \ldots, N\}\) and \(V_{N+1,N+1} = t \cdot t^H = 1\). Moreover, \(V\) is semi-definite and has rank one.

Using Eq. (27), we write Inequality (36) as
\[
\text{trace}(A_i(w_k)V) + b_i(w_k) b_i^H(w_k) \\
\geq \gamma_i \left\{ \sigma_i^2 + \sum_{j \in \{1, \ldots, g\} \setminus \{k\}} \left[ \text{trace}(A_j(w_j)V) + b_j(w_j) b_j^H(w_j) \right] \right\}. 
\]
(38)

From the above discussion, we convert Problem (P1b) into
\[
\text{(P1c) : Find } V, \quad \text{s.t.} \quad \text{trace}(A_i(w_k)V) + b_i(w_k) b_i^H(w_k) \\
\geq \gamma_i \left\{ \sigma_i^2 + \sum_{j \in \{1, \ldots, g\} \setminus \{k\}} \left[ \text{trace}(A_j(w_j)V) + b_j(w_j) b_j^H(w_j) \right] \right\}, 
\]
(39a)

The above constraint in Eq. (39c) is for the case where all \(\beta_n\) are predefined constants. A special case of particular interest is the case of all \(\beta_n\) being 1 so that Eq. (39c) and Eq. (39d) can together be written as \(V_{n,n} = 1\) for \(n \in \{1, \ldots, n+1\}\).

If each \(\beta_n\) can take any value in \([0, 1]\), then Eq. (39c) can be replaced by \(V_{n,n} \in [0, 1]\) for \(n \in \{1, \ldots, N\}\). Similarly, we may consider the most general case where some \(\beta_n\) are
the candidate solution into a solution which satisfies the rank constraint in Eq. (39f). Hence, we adopt semidefinite relaxation (SDR) and drop Eq. (39f) to obtain the following semidefinite programming Problem (P1d):

(P1d) : Find $V$  
\[ \text{s.t. } \begin{align*}
    & \text{trace}(A_i(w_k)V) + b_i(w_k)(w_k) \\
    & \geq \gamma_i \left\{ \sigma_i^2 + \sum_{j \in \{1, \ldots, g\} \setminus \{k\}} \left[ \text{trace}(A_i(w_j)V) + b_i(w_j)(w_j) \right] \right\}, \\
    & \forall k \in \{1, \ldots, g\}, \forall i \in G_k, \\
    & V_{n,n} = \beta_n^2, \forall n \in \{1, \ldots, N\}, \\
    & V_{N+1,N+1} = 1, \\
    & V \succeq 0.
\end{align*} \]  
(40b)

The only non-convex part in Problem (P1c) is the rank constraint in Eq. (39f). Hence, we adopt semidefinite relaxation (SDR) and drop Eq. (39f) to obtain the following semidefinite programming Problem (P1d):

(P1d) : Find $V$  
\[ \text{s.t. } \begin{align*}
    & \text{trace}(A_i(w_k)V) + b_i(w_k)(w_k) \\
    & \geq \alpha_i + \gamma_i \left\{ \sigma_i^2 + \sum_{j \in \{1, \ldots, g\} \setminus \{k\}} \left[ \text{trace}(A_i(w_j)V) + b_i(w_j)(w_j) \right] \right\}, \\
    & \forall k \in \{1, \ldots, g\}, \forall i \in G_k, \\
    & V_{n,n} = \beta_n^2, \forall n \in \{1, \ldots, N\}, \\
    & V_{N+1,N+1} = 1, \\
    & V \succeq 0.
\end{align*} \]  
(41b)

The quantity $\alpha_i$ can be understood as MU $i$'s “SINR residual” in the phase shift optimization [4].

After a candidate solution $V$ is obtained by solving Problem (P1d) or Problem (P1d'), appropriate post-processing such as Gaussian randomization [13] can be applied to convert the candidate solution into a solution which satisfies the rank constraint in Eq. (39f).

Combining the above discussion of alternatively optimizing $W$ and $\Phi$, we present our method to solve Problem (P1) as Algorithm 1 via alternating optimization to find $W$ and $\Phi$ for Problem (P1), which generalizes Problems (P2) and (P3).

Algorithm 1
\begin{enumerate}
    \item Initialize $\Phi$ as some initial (e.g., randomly generated) $\Phi^{(0)} := \text{diag}(\tilde{\beta}_1 e^{\tilde{r}_1}, \ldots, \tilde{\beta}_N e^{\tilde{r}_N})$ which satisfies the constraints on $\Phi$;
    \item Set the iteration number $r \leftarrow 1$;
    \item while do
        \begin{enumerate}
            \item The “while” loop will end if Line 8 or 20 is executed.
            \item Given $\Phi$ as $\Phi^{(r-1)}$, use methods of Karipidis et al. [12] or other papers to solve Problem (P1a) and post-process the obtained $\{X_k\}_{k=1}^{g}$ to set $W$ as some $W^{(r)} := [w_1^{(r)}, \ldots, w_g^{(r)}]$;
            \item Compute the object function value $f^{(r)} \leftarrow \sum_{k=1}^{g} ||w_k^{(r)}||^2$;
            \item if $r \geq 2$ then
                \begin{enumerate}
                    \item if $1 - \frac{f^{(r)}}{f^{(r-1)}}$ denoting the relative difference between the object function values in consecutive iterations $r - 1$ and $r$ is small then
                        \begin{enumerate}
                            \item Break;
                        \end{enumerate}
                \end{enumerate}
            \item end if
        \end{enumerate}
    \end{enumerate}
\end{enumerate}

\begin{algorithm}
\begin{enumerate}
    \item Generate a random vector $v_z^{(r)}$ from a circular-symmetric complex Gaussian distribution $\mathcal{CN}(0, I_{N+1})$ with zero mean and covariance matrix $I_{N+1}$ (the identity matrix with size $N + 1$);
    \item Compute $v_z^{(r)} \leftarrow U \Lambda^{\frac{1}{2}} z^{(r)}$;
    \item With $u_{N+1}$ being the $(N + 1)$th element of $v_z^{(r)}$, take the first $N$ elements of $v_z^{(r)}$ to form a vector $w_z^{(r)}$;
    \item Scale each component of $w_z^{(r)}$ independently to obtain $\phi_z^{(r)}$ such that the $n$th element of $\phi_z^{(r)}$ has magnitude $\beta_n$; i.e., with $w_z^{(r)}$ represented by $[w_1, \ldots, w_N]^T$, compute $\phi_z^{(r)} \leftarrow \left[ \frac{\beta_1 w_1}{|w_1|}, \ldots, \frac{\beta_N w_N}{|w_N|} \right]^T$;
\end{enumerate}
\end{algorithm}


B. Solutions to Problems (P4)--(P6)

We now propose algorithms to solve Problems (P4)--(P6). Since Problems (P5) and (P6) can be seen as special cases of Problem (P4), we first focus on solving Problem (P4), and then apply the solutions to Problems (P5) and (P6).

We introduce an auxiliary variable \( t \) and convert Problem (P4) of Eq. (10) into the following equivalent Problem (P4a):

\[
\begin{align*}
\text{(P4a): } & \max_{W, \Phi, t} t \\
& \text{s.t. } h_i^H(\Phi)w_k^2 \geq t, \\
& \quad \gamma_i \sum_{j \in \{1, \ldots, g\}\setminus\{k\}} |h_j^H(\Phi)w_j|^2 + \sigma_i^2, \\
& \quad \forall k \in \{1, \ldots, g\}, \forall i \in \mathcal{G}_k, \\
& \quad \sum_{k=1}^g \|w_k\|^2 \leq P, \\
& \quad \text{Constraints on } \Phi, \\
& \quad t \geq 0.
\end{align*}
\]

We use the idea of alternating optimization, which is widely used in multivariate optimization. Specifically, we perform the following optimizations alternatively to solve Problem (P4): optimizing \( W \) and \( t \) given \( \Phi \), and finding \( \Phi \) given \( W \) and \( t \). The details are presented below.

Optimizing \( W \) and \( t \) given \( \Phi \). Given some \( \Phi \) satisfying the constraints in (42d), Problem (P4a) means finding \( W \) and \( t \) to maximize \( t \) subject to the constraints in (42b) (42c) (42d).

We define \( X_k \) and \( H_i(\Phi) \) according to Eq. (19) and (22), i.e.,

\[
\begin{align*}
X_k & := w_k w_k^H, \\
H_i(\Phi) & := h_i(\Phi) h_i^H(\Phi).
\end{align*}
\]

Replacing \( k \) by \( j \) in Eq. (43), we also have expressions for \( H_j(\Phi) \). Then similar to the process of writing Inequality (7b) of Problem (P1) as Inequality (44b) of Problem (P1a), we write Inequality (42b) of Problem (P4a) as Inequality (45b) below. Then given \( \Phi \), Problem (P4a) becomes the following Problem (P4b):

\[
\begin{align*}
\text{(P4b): } & \max_{\{X_k\}_{k=1}^t} t \\
& \text{s.t. } \text{trace}(X_k H_i(\Phi)) \geq t \gamma_i \sigma_i^2 + t \gamma_i \sum_{j \in \{1, \ldots, g\}\setminus\{k\}} \text{trace}(X_j H_j(\Phi)), \\
& \quad \forall k \in \{1, \ldots, g\}, \forall i \in \mathcal{G}_k, \\
& \quad \sum_{k=1}^g \text{trace}(X_k) \leq P, \\
& \quad X_k \succeq 0, \forall k \in \{1, \ldots, g\}, \\
& \quad \text{rank}(X_k) = 1, \forall k \in \{1, \ldots, g\}, \\
& \quad t \geq 0.
\end{align*}
\]

A problem similar to Problem (P4b) has been used to solved by Karipidis et al. [12], where the notation of Problem \( F_r \) is used to denote the problem after dropping Eq. (45c). Hence, we can apply methods of [12] to solve Problem (P4b).

Finding \( \Phi \) given \( W \) and \( t \). Given \( W \) and \( t \), Problem (P4) becomes the following feasibility check problem of finding \( \Phi \):

\[
\begin{align*}
\text{(P4d): } & \text{Find } \Phi \\
& \text{s.t. } \sum_{j \in \{1, \ldots, g\}\setminus\{k\}} |h_j^H(\Phi)w_j|^2 + \sigma_i^2 \geq t \gamma_i, \\
& \quad \forall k \in \{1, \ldots, g\}, \forall i \in \mathcal{G}_k, \\
& \quad \text{Constraints on } \Phi,
\end{align*}
\]

The only difference between Problem (P4d) of Eq. (46) and Problem (P1b) of Eq. (25) is that the right hand side in Inequality (46b) of Problem (P4d) has \( t \gamma_i \), whereas the right hand side in Inequality (25b) of Problem (P1b) has \( \gamma_i \). Hence, we can apply the discussed approach of solving Problem (P1b) to solve Problem (P4d). Specifically, if the constraint on \( \Phi \) in Eq. (25c) is Eq. (1) (i.e., \( \Phi := \text{diag}(\beta_1 e^{j \theta_1}, \ldots, \beta_N e^{j \theta_N}) \)), we define \( \phi, b_i(w_k) \), and \( A_i(w_k) \) according to Eq. (26) (29) and (31). Then in a way similar to the derivation leading to Inequality (34), we write the constraint in Eq. (46b) of Problem (P4d) as

\[
\begin{align*}
& \phi^H, \quad 1 \ A_i(w_k) \begin{bmatrix} \phi \\ 1 \end{bmatrix} + b_i(w_k) b_i^H(w_k) \\
& \quad \geq t \gamma_i \left\{ \sigma_i^2 + \sum_{j \in \{1, \ldots, g\}\setminus\{k\}} \left[ \phi^H, \quad 1 \ A_i(w_j) \begin{bmatrix} \phi \\ 1 \end{bmatrix} + b_i(w_j) b_i^H(w_j) \right] \right\},
\end{align*}
\]

Then defining \( V \) according to (37), similar to the process of formulating Problem (P1c) of Eq. (59), we can convert
Problem (P4d) into the following Problem (P4e):

\[(P4e): \text{Find } V \quad \text{s.t.} \quad \begin{align*}
\text{trace}(A_i(w_k)V) + b_i(w_k) & b_i^H(w_k) \\
& \geq \tau_i \left( \sigma_i^2 + \sum_{j \in \{1, \ldots, g\}\setminus \{k\}} \text{trace}(A_i(w_j)V) + b_i(w_j) b_i^H(w_j) \right) \quad \quad \quad \quad \text{for } i \in \{1, \ldots, g\}\setminus \{k\}.
\end{align*}
\]

(48a)

The only non-convex part in Problem (P4e) is the rank constraint in Eq. (48e). Hence, we adopt semidefinite relaxation (SDR) and drop Eq. (48e) to obtain a semidefinite programming problem. The discussion is similar to that for Problem (P1c) of Eq. (39). Similar to the practice of replacing (P1d) by (P1d'), we can also replace (P4e) by (P4'e) below which may find better $\Phi$ and hence $V$ to accelerate the alternating optimization process:

\[(P4'e): \max_{V, \alpha} \sum_{k=1}^{g} \sum_{i \in G_k} \alpha_i \quad \text{s.t.} \quad \begin{align*}
\text{trace}(A_i(w_k)V) + b_i(w_k) & b_i^H(w_k) \\
& \geq \alpha_i + \tau_i \left( \sigma_i^2 + \sum_{j \in \{1, \ldots, g\}\setminus \{k\}} \text{trace}(A_i(w_j)V) + b_i(w_j) b_i^H(w_j) \right) \quad \quad \quad \quad \text{for } i \in \{1, \ldots, g\}\setminus \{k\}.
\end{align*}
\]

(49a)

Algorithm 2 via alternating optimization to find $W$ and $\Phi$ for Problem (P4), which generalizes Problems (P5) and (P6).

1. Initialize $\Phi$ as some initial (e.g., randomly generated) $\Phi^{(0)} := \text{diag}(\beta_1 e^{j\theta_1}, \ldots, \beta_N e^{j\theta_N})$ which satisfies the constraints on $\Phi$;
2. Set the iteration number $r \leftarrow 1$;
3. while $1$ do
   \{The “while” loop will end if Line 7 or 19 is executed.\}
   \{Comment: Optimizing $W$ and $t$ given $\Phi$:\
4. Given $\Phi$ as $\Phi^{(r-1)}$, use methods of Karipidis et al. [12] or other papers to solve Problem (P4b) in Eq. (45), and post-process the solution to set $W$ as some $W^{(r)} := [w_1^{(r)}, \ldots, w_g^{(r)}]$ and set $t$ as some $t^{(r)}$;
5. if $r \geq 2$ then
6. \quad if $\frac{E^{(r)}}{E^{(r-1)}} - 1$ denoting the relative difference between the object function values in consecutive iterations $r - 1$ and $r$ is small then
7. \quad \quad break;
8. \quad else
9. \quad \quad if
10. \quad \quad \{Finding $\Phi$ given $W$ and $t$:\
11. \quad \quad \quad Given $W$ as $W^{(r)}$ and $t$ as $t^{(r)}$, solve Problem (P4e) in Eq. (48) or Problem (P4'e) in Eq. (49), and denote the obtained $V$ as $V^{(r)}_{SDR}$;
12. \quad \quad \quad \{Comment: Gaussian randomization:\
13. \quad \quad \quad \quad Perform the eigenvalue decomposition on $V^{(r)}_{SDR}$ to obtain a unitary matrix $U$ and a diagonal matrix $\Lambda$ such that $V^{(r)}_{SDR} = U \Lambda^{1/2} H$;
14. \quad \quad \quad \quad for $z$ from $1$ to some sufficiently large $Z$ do
15. \quad \quad \quad \quad \quad Generate a random vector $v_z^{(r)}$ from a circularly-symmetric complex Gaussian distribution $\mathcal{CN}(0, I_{N+1})$ with zero mean and covariance matrix $I_{N+1}$ (the identity matrix with size $N + 1$);
16. \quad \quad \quad \quad \quad Compute $u_z^{(r)} := U \Lambda^{1/2} v_z^{(r)}$;
17. \quad \quad \quad \quad \quad With $u_z^{(r)}$ being the $(N+1)$th element of $u_z^{(r)}$, take the first $N$ elements of $u_z^{(r)}$ to form a vector $w_z^{(r)}$;
18. \quad \quad \quad \quad \quad \text{Scale each component of } w_z^{(r)} \text{ independently to obtain } \phi_z^{(r)} \text{ such that the } n\text{th element of } \phi_z^{(r)} \text{ has magnitude } \beta_n; \text{ i.e., with } w_z^{(r)} \text{ represented by } [w_1, \ldots, w_N]^T, \text{ compute } \phi_z^{(r)} := \left[ \beta_1 w_1^{(r)}, \ldots, \beta_N w_N^{(r)} \right]^T\};
19. \quad \quad \quad \quad end for
20. \quad \quad \quad if for $z \in \{1,2, \ldots, Z\}$, there is no $\phi_z^{(r)}$ to ensure Eq. (47) after setting $\phi$ as $\phi_z^{(r)}$ then
21. \quad \quad \quad \quad break;
22. \quad \quad else
23. \quad \quad \quad \quad Select one $\phi_z^{(r)}$ according to some ordering among those ensuring Eq. (47) and with $\phi_z^{(r)}$ denoting the selected one;
24. \quad \quad \quad \quad Map $\phi_z^{(r)}$ to some $\Phi_z^{(r)}$ to satisfy the constraint on $\phi$ (e.g., discrete element values);
25. \quad \quad \quad \quad Set $\Phi \leftarrow \Phi_z^{(r)}$ for $\Phi_z^{(r)} := \text{diag}\left(\Phi_z^{(r)}\right)$, and denote such $\Phi$ as $\Phi^{(r)}$ for notation convenience;
26. \quad \quad end if
27. \quad else
28. \quad \quad update the iteration number $r \leftarrow r + 1$;
29. \quad end while
we have discussed in Section IV-A. More specifically, since Problem (P1-MA-MR) is in the most general form, we start with elaborating its solution below.

We restate Problem (P1-MA-MR) given in Table IV of Page 9 as follows:

\[
\begin{align*}
(P1-MA-MR): \\
\min_{W, \Phi_1, \ldots, \Phi_L} & \quad \sum_{k=1}^{g} \|w_k\|^2 \quad (50a) \\
\text{s.t.} & \quad \sum_{j \in \{1, \ldots, g\} \setminus \{k\}} w_k^H H_i(\Phi_1, \ldots, \Phi_L) w_j + \sigma_i^2 \geq \gamma_i, \\
& \quad \forall k \in \{1, \ldots, g\}, \forall i \in G_k,
\end{align*}
\]

We use the idea of alternating optimization. Specifically, we will optimize \(\Phi_1, \ldots, \Phi_L\) and \(W\) alternatively to solve Problem (P1-MA-MR-a). Below, we discuss the optimization of \(W\) given \(\Phi_1, \ldots, \Phi_L\) and the optimization of \(\Phi_1, \ldots, \Phi_L\) given \(W\).

Optimizing \(W\) given \(\Phi_1, \ldots, \Phi_L\). Given some \(\Phi_1, \ldots, \Phi_L\) satisfying the constraints in Eq. (50c), Problem (P1-MA-MR) means finding \(W\) given \(\Phi_1, \ldots, \Phi_L\) to minimize the objective function \(\sum_{k=1}^{g} \|w_k\|^2\) in (50a) subject to the constraints in (50b).

As in Eq. (19), we define \(X_k\) as follows:

\[
X_k := w_k w_k^H, \quad \forall k \in \{1, \ldots, g\}. \quad (51)
\]

Similar to the process of writing Inequality (7b) of Problem (P1) as Inequality (24b) of Problem (P1a), we write Inequality (50b) of Problem (P1-MA-MR-a) as Inequality (52b) below. Then optimizing \(W\) given \(\Phi_1, \ldots, \Phi_L\) for Problem (P1-MA-MR) becomes solving \(\{X_k\}^g_{k=1}\) for the following Problem (P1-MA-MR-a):

\[
(P1-MA-MR-a): \\
\min_{\{X_k\}^g_{k=1}} & \quad \sum_{k=1}^{g} \text{trace}(X_k) \quad (52a) \\
\text{s.t.} & \quad \text{trace}(X_k H_i(\Phi_1, \ldots, \Phi_L)) \geq \gamma_i \sigma_i^2 + \gamma_i \sum_{j \in \{1, \ldots, g\} \setminus \{k\}} \text{trace}(X_j H_i(\Phi_1, \ldots, \Phi_L)), \\
& \quad \forall k \in \{1, \ldots, g\}, \forall i \in G_k, \\
& \quad \text{rank}(X_k) = 1, \forall k \in \{1, \ldots, g\} \quad (52d)
\]

The only non-convex part in Problem (P1-MA-MR-a) is the rank constraint in Eq. (52d). Hence, we adopt semidefinite relaxation (SDR) and drop Eq. (52d) to obtain a semidefinite programming problem. After a candidate solution is obtained, appropriate post-processing such as Gaussian randomization [13] (see also randA, randB, and randC coined by [14]) is applied to convert the candidate solution into a solution which satisfies the rank constraint. The above method has been used to solve a problem similar to Problem (P1-MA-MR-a) by Karipidis et al. [12], where the notation of Problem \(Q\) is used. Hence, we can apply methods of [12] to solve Problem (P1-MA-MR-a).

**Finding \(\Phi_1, \ldots, \Phi_L\) given \(W\).** Given \(W\), Problem (P1-MA-MR) becomes the following feasibility check problem of finding \(\Phi_1, \ldots, \Phi_L\):

\[
(P1-MA-MR-b): \\
\text{Find} \quad \Phi_1, \ldots, \Phi_L \quad (53a) \\
\text{s.t.} & \quad \sum_{j \in \{1, \ldots, g\} \setminus \{k\}} w_k^H H_i(\Phi_1, \ldots, \Phi_L) w_j + \sigma_i^2 \geq \gamma_i, \\
& \quad \forall k \in \{1, \ldots, g\}, \forall i \in G_k, \\
& \quad \text{Constraints on} \quad \Phi_1, \ldots, \Phi_L. \quad (53c)
\]

Recall from the caption of Table IV on Page 9 that \(H_i(\Phi_1, \ldots, \Phi_L)\) appearing in Inequality (53b) is defined as

\[
H_i(\Phi_1, \ldots, \Phi_L) := \sum_{q=1}^{Q_i} h_{i,q}(\Phi_1, \ldots, \Phi_L) h_{i,q}^H(\Phi_1, \ldots, \Phi_L), \quad (54)
\]

for

\[
h_{i,q}(\Phi_1, \ldots, \Phi_L) := h_{b,i,q}^H + \sum_{\ell=1}^{L} h_{\ell,i,q}^H \Phi_{b,\ell}. \quad (55)
\]

Then \(w_k^H H_i(\Phi_1, \ldots, \Phi_L) w_k\) appearing in Inequality (53b) of Problem (P1-MA-MR-b) is given by

\[
w_k^H H_i(\Phi_1, \ldots, \Phi_L) w_k = \sum_{q=1}^{Q_i} w_k^H h_{i,q}(\Phi_1, \ldots, \Phi_L) h_{i,q}^H(\Phi_1, \ldots, \Phi_L) w_k. \quad (56)
\]

Below we analyze \(h_{i,q}^H(\Phi_1, \ldots, \Phi_L) w_k\) which appears in Eq. (55).

If the constraint on each \(\Phi_{\ell}\) is in the form of Eq. (1); i.e.,

\[
\Phi_{\ell} := \text{diag}(\beta_{\ell,1} e^{i \theta_{\ell,1}}, \ldots, \beta_{\ell,N_i} e^{i \theta_{\ell,N_i}}), \quad \forall \ell \in \{1, \ldots, L\},
\]

then we define

\[
\Phi_{\ell} := [\beta_{\ell,1} e^{i \theta_{\ell,1}}, \ldots, \beta_{\ell,N_i} e^{i \theta_{\ell,N_i}}]^H, \quad \forall \ell \in \{1, \ldots, L\},
\]

and change variables in Eq. (55) to obtain for \(\ell \in \{1, \ldots, L\}, k \in \{1, \ldots, g\}, i \in G_k\) and \(q \in \{1, \ldots, Q_i\}\) that

\[
h_{i,q}^H(\Phi_1, \ldots, \Phi_L) w_k = b_{i,q}(w_k) + \sum_{\ell=1}^{L} \phi_{\ell,i,q}(w_k), \quad (59)
\]

where we define \(a_{\ell,i,q}(w_k) \in \mathbb{C}^{N_i \times 1}\) by

\[
a_{\ell,i,q}(w_k) := \text{diag}(h_{\ell,i,q}^H) \Phi_{b,\ell} w_k. \quad (60)
\]
and complex numbers \( b_{i,q}(w_k) \) by
\[
 b_{i,q}(w_k) := h_{b,i,q}^H w_k. \tag{61}
\]

Applying Eq. \( 59 \) to Eq. \( 56 \), we have
\[
\sum_{q=1}^{Q_i} \left\{ b_{i,q}(w_k) b_{i,q}^H (w_k) \right\} \geq \gamma_i \left\{ \sigma_i^2 + \sum_{j \in \{1, \ldots, g\} \setminus \{k\}} \sum_{q=1}^{Q_j} \left\{ b_{j,q}(w_j) b_{j,q}^H (w_j) \right\} \right\}. \tag{65}
\]

Then with \( |t| = 1 \), Inequality \( 65 \) is equivalent to
\[
\sum_{q=1}^{Q_i} \left\{ v^H A_{i,q}(w_k)v + b_{i,q}(w_k) b_{i,q}^H (w_k) \right\} \geq \gamma_i \left\{ \sigma_i^2 + \sum_{j \in \{1, \ldots, g\} \setminus \{k\}} \sum_{q=1}^{Q_j} \left\{ v^H A_{j,q}(w_j)v + b_{j,q}(w_j) b_{j,q}^H (w_j) \right\} \right\}. \tag{67}
\]

We further define
\[
 V := vv^H, \tag{68}
\]

Clearly, \( V \) is semi-definite and has rank one. For \( \Phi \), in the form of Eq. \( 53b \), the diagonal elements of \( V \) are as follows:
\[
 V_{(\sum_{j=1}^{L} N_j), (\sum_{j=1}^{L} N_j)} := (\beta_{x,y} e^{i \theta_{x,y}})^H \beta_{x,y} e^{i \theta_{x-y}} = \beta_{x,y}^2, \tag{69}
\]

and
\[
 V_{(\sum_{j=1}^{L} N_j), (\sum_{j=1}^{L} N_j) - t} = t^H t = 1. \tag{70}
\]

Using Eq. \( 68 \), we write Eq. \( 67 \) as
\[
\sum_{q=1}^{Q_i} \left\{ \text{trace}(A_{i,q}(w_k) V) + b_{i,q}(w_k) b_{i,q}^H (w_k) \right\} \geq \gamma_i \left\{ \sigma_i^2 + \sum_{j \in \{1, \ldots, g\} \setminus \{k\}} \sum_{q=1}^{Q_j} \left\{ \text{trace}(A_{j,q}(w_j) V) + b_{j,q}(w_j) b_{j,q}^H (w_j) \right\} \right\}. \tag{71}
\]

From the above discussion, we convert Problem (P1-MA-
MR-b) of Eq. (53) into

(P1-MA-MR-c):

Find $V$

s.t. \[
\sum_{q=1}^{Q_1} \left\{ \text{trace}(A_{i,q}(w_k)V) + b_{i,q}(w_k)b_{i,q}^H(w_k) \right\} \geq \gamma_i \left\{ \sigma_i^2 + \right\} \sum_{j \in \{1,...,g\} \backslash \{k\}} \sum_{q=1}^{Q_1} \left\{ \text{trace}(A_{i,q}(w_j)V) + b_{i,q}(w_j)b_{i,q}^H(w_j) \right\},
\]

\forall k \in \{1,...,g\}, \forall i \in G_k,

V_{(y+\sum_{\ell=0}^{r-1} N_i),(y+\sum_{\ell=0}^{r-1} N_i)} = \beta_{x,y}^2,

\forall x \in \{1,...,L\}, \forall y \in \{1,...,N_x\},

V_{(1+\sum_{\ell=1}^{L} N_i),(1+\sum_{\ell=1}^{L} N_i)} = 1,

V \geq 0,

\text{rank}(V) = 1.

The above constraint in Eq. (72c) is for the case where all $\beta_{x,y} \mid x \in \{1,...,L\}$ are predefined constants. A special case of $y \in \{1,...,N_y\}$ particular interest is the case of all $\beta_{x,y} \mid x \in \{1,...,L\}$, being 1 so that Eq. (72c) and Eq. (72d) can together be written as $V_{n,n}$ = 1 for $n \in \{1,...,1 + \sum_{\ell=1}^{L} N_i\}$. If each $\beta_{x,y} \mid x \in \{1,...,L\}$, can take any value in $[0,1]$, then Eq. (72c) can be replaced by $V_{(y+\sum_{\ell=0}^{r-1} N_i),(y+\sum_{\ell=0}^{r-1} N_i)} \in [0,1]$ for $x \in \{1,...,L\}$ and $y \in \{1,...,N_x\}$. Similarly, we may consider the most general case where some $\beta_{x,y}$ are predefined constants while other $\beta_{x,y}$ can vary.

The only non-convex part in Problem (P1-MA-MR-c) is the rank constraint in Eq. (72f). Hence, we adopt semidefinite relaxation (SDR) and drop Eq. (72f) to obtain the following semidefinite programming Problem (P1-MA-MR-d):

(P1-MA-MR-d):

Find $V$

s.t. \[
\sum_{q=1}^{Q_1} \left\{ \text{trace}(A_{i,q}(w_k)V) + b_{i,q}(w_k)b_{i,q}^H(w_k) \right\} \geq \gamma_i \left\{ \sigma_i^2 + \right\} \sum_{j \in \{1,...,g\} \backslash \{k\}} \sum_{q=1}^{Q_1} \left\{ \text{trace}(A_{i,q}(w_j)V) + b_{i,q}(w_j)b_{i,q}^H(w_j) \right\},
\]

\forall k \in \{1,...,g\}, \forall i \in G_k,

V_{(y+\sum_{\ell=0}^{r-1} N_i),(y+\sum_{\ell=0}^{r-1} N_i)} = \beta_{x,y}^2,

\forall x \in \{1,...,L\}, \forall y \in \{1,...,N_x\},

V_{(1+\sum_{\ell=1}^{L} N_i),(1+\sum_{\ell=1}^{L} N_i)} = 1,

V \geq 0.

Problem (P1-MA-MR-d) belongs to semidefinite programming and can be solved efficiently [13]. Moreover, Problem (P1-MA-MR-d) can be replaced by Problem (P1-MA-MR-d') below which may find better $\Phi$ and hence $V$ to accelerate the alternating optimization process:

(P1-MA-MR-d'):

\[
\max_{V,\alpha} \sum_{k=1}^{g} \sum_{i \in G_k} \alpha_i \left( \sum_{q=1}^{Q_1} \left\{ \text{trace}(A_{i,q}(w_k)V) + b_{i,q}(w_k)b_{i,q}^H(w_k) \right\} \right)
\]

s.t. \[
\sum_{q=1}^{Q_1} \left\{ \text{trace}(A_{i,q}(w_k)V) + b_{i,q}(w_k)b_{i,q}^H(w_k) \right\} \geq \alpha_i \left( \sigma_i^2 + \right) \sum_{j \in \{1,...,g\} \backslash \{k\}} \sum_{q=1}^{Q_1} \left\{ \text{trace}(A_{i,q}(w_j)V) + b_{i,q}(w_j)b_{i,q}^H(w_j) \right\},
\]

\forall k \in \{1,...,g\}, \forall i \in G_k,

V_{(y+\sum_{\ell=0}^{r-1} N_i),(y+\sum_{\ell=0}^{r-1} N_i)} = \beta_{x,y}^2,

\forall x \in \{1,...,L\}, \forall y \in \{1,...,N_x\},

V_{(1+\sum_{\ell=1}^{L} N_i),(1+\sum_{\ell=1}^{L} N_i)} = 1,

V \geq 0,

\alpha_i \geq 0, \forall k \in \{1,...,g\}, \forall i \in G_k.

After a candidate solution $V$ is obtained by solving Problem (P1-MA-MR-d) or Problem (P1-MA-MR-d'), appropriate post-processing such as Gaussian randomization [13] can be applied to convert the candidate solution into a solution which satisfies the rank constraint in Eq. (39).

Combining the above discussion of alternatively optimizing $W$ and $\Phi$, we present our method to solve Problem (P1-MA-MR) as Algorithm 5.

D. Solutions to Problems (P4-MA)–(P6-MA), (P4-MR)–(P6-MR), and (P4-MA-MR)–(P6-MA-MR)

As Problems (P4-MA)–(P6-MA), (P4-MR)–(P6-MR), and (P4-MA-MR)–(P6-MA-MR) are generalizations of Problems (P4)–(P6), we will solve the former problems in ways similar to the solutions for the latter problems, which we have discussed in Section V-B. More specifically, since Problem (P4-MA-MR) is in the most general form, we start with elaborating its solution below.

We restate Problem (P4-MA-MR) given in Table IV of
Algorithm 3 via alternating optimization to find $\Phi_1, \ldots, \Phi_L$ and $W$ for Problem (P1-MA-MR), which generalizes Problems (P2-MA-MR) (P3-MA-MR), (P1-MA)–(P3-MA), (P1-MR)–(P3-MR), and (P1)–(P3).

1: Initialize $\Phi_\ell$ for $\ell \in \{1, \ldots, L\}$ as some initial (e.g., randomly generated) $\Phi_\ell^{(0)} := \text{diag}(\beta_{\ell,1}e^{j\theta_{\ell,1}}, \ldots, \beta_{\ell,N_{\ell}}e^{j\theta_{\ell,N_{\ell}}})$ which satisfies the constraints on $\Phi_\ell$;
2: Set the iteration number $r \leftarrow 1$;
3: while 1 do
   4: \{The “while” loop will end if Line 8 or 20 is executed.\}
   5:  \{Comment: Optimizing $W$ given $\Phi_1, \ldots, \Phi_L$.\}
   6:  Given $\Phi_1, \ldots, \Phi_L$ as $\Phi_1^{(r-1)}, \ldots, \Phi_L^{(r-1)}$, use methods of Karipidis et al. \cite{12} or other papers to solve Problem (P1-MA-MR-a) in Eq. (62), and post-process the obtained $\{X_k\}_{g=1}^q$ to set $W$ as some $W^{(r)} := [w_1^{(r)}, \ldots, w_g^{(r)}]$;
   7:  Compute the objective function value $f^{(r)} \leftarrow \sum_{k=1}^q \|w_k^{(r)}\|^2$;
   8:  if $r \geq 2$ then
      9: \{if $1 - \frac{f^{(r)}}{f^{(r-1)}}$, denoting the relative difference between the objective function values in consecutive iterations $r - 1$ and $r$ is small then\}
   10:   break;
   11: end if
   12: \{Finding $\Phi_1, \ldots, \Phi_L$ given $W$:\}
   13:  Given $W$ as $W^{(r)}$, solve Problem (P1-MA-MR-d) in Eq. (73) or Problem (P1-MA-MR-d’) in Eq. (74), and denote the obtained $V$ as $V^{(r)}$;
   14: \{Comment: Gaussian randomization:\}
   15:  Perform the eigenvalue decomposition on $V^{(r)}$ to obtain a unitary matrix $U$ and a diagonal matrix $\Lambda$ such that $V^{(r)}_{\text{SDR}} = U \Lambda U^H$;
   16: for $z$ from 1 to some sufficiently large $Z$ do
      17:  Generate a random vector $v_z^{(r)}$ from a circularly-symmetric complex Gaussian distribution $CN(0, I_{1+\sum_{\ell=1}^L N_{\ell}})$ with zero mean and covariance matrix $I_{1+\sum_{\ell=1}^L N_{\ell}}$ (the identity matrix with size $1 + \sum_{\ell=1}^L N_{\ell}$);
      18:  Compute $v_z^{(r)} \leftarrow U \Lambda^{\frac{1}{2}} r_z^{(r)}$;
      19:  With $v_1^{(r)}$, $\ldots$, $v_z^{(r)}$, the $(1 + \sum_{\ell=1}^L N_{\ell})$th element of $v_z^{(r)}$, take the first $\sum_{\ell=1}^L N_{\ell}$ elements of $\frac{v_z^{(r)}}{v_1^{(r)}}$ to form a vector $w_z^{(r)}$;
      20:  Scale each component of $w_z^{(r)}$ independently to obtain $\phi_z^{(r)}$ such that the $(y + \sum_{\ell=0}^{z-1} N_{\ell})$th element of $\phi_z^{(r)}$ has magnitude $\beta_{x,y}$, for $x \in \{1, \ldots, L\}$ and $y \in \{1, \ldots, N_z\}$;
   21: end for
   22: if for $z \in \{1, 2, \ldots, Z\}$, there is no $\phi_z^{(r)}$ to ensure Eq. (65) after setting $\phi$ as $\phi_z^{(r)}$ then
      23:   break;
   24: else
      25:  Select one $\phi_z^{(r)}$ according to some ordering among those ensuring Eq. (65) and denote the selected one by $\phi_z^{(r)}$;
      26:  Map $\phi_z^{(r)}$ to some $\phi^{(r)}$ to satisfy the constraint on $\phi$ (e.g., discrete element values);
   27: end if
   28: for $\ell \in \{1, 2, \ldots, L\}$, set $\Phi_\ell$ as $\Phi_\ell^{(r)} := \text{diag}\left((\phi_\ell^{(r)})^H\right)$, where $\phi_\ell^{(r)}|_{\ell \in \{1, 2, \ldots, L\}}$ are defined such that $\phi^{(r)} = \begin{bmatrix} \phi_1^{(r)} \\ \vdots \\ \phi_L^{(r)} \end{bmatrix}$;
   29: \{end while\}
Problem (P4-MA-MR-a):

\[
\begin{align*}
&\text{max} \quad w, \Phi_1, \ldots, \Phi_L, t \\
&\text{s.t.} \quad \sum_{k=1}^{g} \| w_k \|^2 \leq P, \\
&\quad \text{Constraints on } \Phi_1, \ldots, \Phi_L.
\end{align*}
\]

We introduce an auxiliary variable \( t \) and convert Problem (P4) into the following equivalent Problem (P4-MA-MR-a):

\[
\begin{align*}
&\text{(P4-MA-MR-a):} \\
&\quad \text{max} \quad W, \Phi_1, \ldots, \Phi_L, t \\
&\quad \text{s.t.} \quad \sum_{k=1}^{g} \| w_k \|^2 \leq P, \\
&\quad \quad \text{Constraints on } \Phi_1, \ldots, \Phi_L, \\
&\quad t \geq 0.
\end{align*}
\]

We use the idea of alternating optimization, which is widely used in multivariate optimization. Specifically, we perform the following optimizations alternatively to solve Problem (P4-MA-MR-a): optimizing \( W \) and \( t \) given \( \Phi \), and finding \( \Phi \) given \( W \) and \( t \). The details are presented below.

**Optimizing \( W \) and \( t \) given \( \Phi_1, \ldots, \Phi_L \).** Given some \( \Phi_1, \ldots, \Phi_L \) satisfying the constraints in Eq. (76d), Problem (P4-MA-MR) means finding \( W \) and \( t \) given \( \Phi_1, \ldots, \Phi_L \) to maximize \( t \) subject to the constraints in Eqs. (76b), (76c), and (76e).

We define \( X_k \) according to Eq. (19); i.e.,

\[
X_k := w_k w_k^H, \quad \forall k \in \{1, \ldots, g\}.
\]

Then similar to the process of writing Inequality (75b) of Problem (P1) as Inequality (24b) of Problem (P1a), we write Inequality (76b) of Problem (P4-MA-MR-a) as Inequality (78b) below. Then optimizing \( W \) given \( \Phi_1, \ldots, \Phi_L \) for Problem (P4-MA-MR-a) becomes solving \( \{ X_k \}_{k=1}^{g} \) for the following Problem (P4-MA-MR-b):

\[
\begin{align*}
&\text{(P4-MA-MR-b):} \\
&\quad \text{max} \quad t \\
&\quad \text{s.t.} \quad \text{trace}(X_k H_i(\Phi_1, \ldots, \Phi_L)) \geq t \gamma_i \sigma_i^2 + \text{trace}(X_j H_i(\Phi_1, \ldots, \Phi_L)), \\
&\quad \forall k \in \{1, \ldots, g\}, \forall i \in G_k, \\
&\quad \sum_{k=1}^{g} \text{trace}(X_k) \leq P, \\
&\quad X_k \succeq 0, \forall k \in \{1, \ldots, g\}, \\
&\quad \text{rank}(X_k) = 1, \forall k \in \{1, \ldots, g\}, \\
&\quad t \geq 0.
\end{align*}
\]

A problem similar to Problem (P4-MA-MR-b) has been used to solve by Karipidis et al. [12], where the notation of Problem \( f_\epsilon \) is used to denote the problem after dropping Eq. (76c). Hence, we can apply methods of [12] to solve Problem (P4-MA-MR-b).

**Finding \( \Phi_1, \ldots, \Phi_L \) given \( W \) and \( t \).** Given \( W \) and \( t \), Problem (P4-MA-MR) becomes the following feasibility check problem of finding \( \Phi_1, \ldots, \Phi_L \):

\[
\begin{align*}
&\text{(P4-MA-MR-c):} \\
&\quad \text{Find} \quad \Phi_1, \ldots, \Phi_L \\
&\quad \text{s.t.} \quad \sum_{j \in \{1, \ldots, g\} \setminus \{k\}} w_j^H H_i(\Phi_1, \ldots, \Phi_L) w_j + \sigma_i^2 \geq t \gamma_i, \\
&\quad \forall k \in \{1, \ldots, g\}, \forall i \in G_k, \\
&\quad \text{Constraints on } \Phi_1, \ldots, \Phi_L.
\end{align*}
\]

The only difference between Problem (P4-MA-MR-c) of Eq. (79) and Problem (P1-MA-MR-b) of Eq. (53) is that the right hand side in Inequality (79b) of Problem (P4-MA-MR-c) has \( t \gamma_i \), whereas the right hand side in Inequality (53b) of Problem (P1-MA-MR-b) has \( \gamma_i \). Hence, we can apply the discussed approach of solving Problem (P1-MA-MR-b) to solve Problem (P4-MA-MR-c). Specifically, if the constraints on \( \Phi_1 \) in Eq. (24c) are in the form of Eq. (57) (i.e., \( \Phi_1 := \text{diag}(\beta_1 e^{j \theta_1}, \ldots, \beta_L e^{j \theta_L}) \) for \( \ell \in \{1, \ldots, L\} \)), we define \( \Phi_1, \Phi_2, \ldots, \Phi_L \) according to Eq. (58) (59) (61) (63) and (64). Then in a similar way to the derivation leading to Inequality (65), we write the constraint in Eq. (79b) of Problem (P4-MA-MR-c) as

\[
\begin{align*}
&\sum_{q=1}^{Q_i} \left\{ \phi^H, 1 \right\} A_{i, q}(w_k) \left\{ \phi^H, 1 \right\} + b_{i, q}(w_k) b_{i, q}^H(w_k) \\
&\geq t \gamma_i \left\{ \sigma_i^2 + \sum_{j \in \{1, \ldots, g\} \setminus \{k\}} \sum_{q=1}^{Q_i} \left\{ \phi^H, 1 \right\} A_{i, q}(w_j) \left\{ \phi^H, 1 \right\} + b_{i, q}(w_j) b_{i, q}^H(w_j) \right\}. 
\end{align*}
\]
Then defining \( V \) according to Eq. (68) with \( v \) defined in Eq. (66), similar to the process of formulating Problem (P1-MA-MR-c) of Eq. (72), we can convert Problem (P4-MA-MR-c) into the following Problem (P4-MA-MR-d):

\[
\text{(P4-MA-MR-d):}
\]

Find \( V \)

\[
\begin{align*}
\sum_{q=1}^{Q_i} \left\{ \text{trace}(A_{i,q}(w_k)V) + b_{i,q}(w_k) b_{i,q}^H(w_k) \right\} \\
&\geq t \gamma_i \left\{ \sigma_i^2 + \right\} \\
&\sum_{j \in \{1, \ldots, g\} \setminus \{k\}} \sum_{q=1}^{Q_i} \left\{ \text{trace}(A_{i,q}(w_j)V) + b_{i,q}(w_j) b_{i,q}^H(w_j) \right\}.
\end{align*}
\]

\[
(81a)
\]

subject to

\[
\forall k \in \{1, \ldots, g\}, \forall i \in \mathcal{G}_k, \\
V \geq 0, \\
\text{rank}(V) = 1.
\]

\[
\text{(81b)}
\]

The above constraint in Eq. (81c) is for the case where all \( \beta_{x,y} | x \in \{1, \ldots, L\} \) are predefined constants. A special case of particular interest is the case of all \( \beta_{x,y} | x \in \{1, \ldots, L\} \) being 1 so that Eq. (81c) and Eq. (81d) can together be written as \( V_{n,n} = 1 \) for \( n \in \{1, \ldots, 1 + \sum_{\ell=1}^{L} N_{\ell} \} \) if all \( \beta_{x,y} | x \in \{1, \ldots, L\} \) can take any value in \([0, 1] \), then Eq. (81a) can be replaced by \( V \left( y+\sum_{\ell=1}^{r-1} N_{\ell}, y+\sum_{\ell=1}^{r-1} N_{\ell} \right) \in [0, 1] \) for \( x \in \{1, \ldots, L\} \) and \( y \in \{1, \ldots, N_x\} \). Similarly, we may consider the most general case where some \( \beta_{x,y} \) are predefined constants while other \( \beta_{x,y} \) can vary.

The only non-convex part in Problem (P4-MA-MR-d) is the rank constraint in Eq. (81f). Hence, we adopt semidefinite relaxation (SDR) and drop Eq. (81f) to obtain the following semidefinite programming Problem (P4-MA-MR-e):

\[
\text{(P4-MA-MR-e):}
\]

Find \( V \)

\[
\begin{align*}
\sum_{q=1}^{Q_i} \left\{ \text{trace}(A_{i,q}(w_k)V) + b_{i,q}(w_k) b_{i,q}^H(w_k) \right\} \\
&\geq t \gamma_i \left\{ \sigma_i^2 + \right\} \\
&\sum_{j \in \{1, \ldots, g\} \setminus \{k\}} \sum_{q=1}^{Q_i} \left\{ \text{trace}(A_{i,q}(w_j)V) + b_{i,q}(w_j) b_{i,q}^H(w_j) \right\}.
\end{align*}
\]

\[
(82a)
\]

subject to

\[
\forall k \in \{1, \ldots, g\}, \forall i \in \mathcal{G}_k, \\
V \left( y+\sum_{\ell=0}^{r-1} N_{\ell}, y+\sum_{\ell=0}^{r-1} N_{\ell} \right) \in [0, 1], \\
\forall x \in \{1, \ldots, L\}, \forall y \in \{1, \ldots, N_x\}, \\
V \left( 1 + \sum_{\ell=1}^{L} N_{\ell}, 1 + \sum_{\ell=1}^{L} N_{\ell} \right) = 1, \\
V \geq 0.
\]

\[
\text{(82b)}
\]

Problem (P4-MA-MR-e) belongs to semidefinite programming and can be solved efficiently [13]. Moreover, Problem (P4-MA-MR-e) can be replaced by Problem (P4-MA-MR-e') below which may find better \( \Phi \) and hence \( V \) to accelerate the alternating optimization process:

\[
\text{(P4-MA-MR-e'):}
\]

\[
\begin{align*}
\max_{V,\alpha} \sum_{k=1}^{g} \sum_{i \in \mathcal{G}_k} \alpha_i \\
\sum_{q=1}^{Q_i} \left\{ \text{trace}(A_{i,q}(w_k)V) + b_{i,q}(w_k) b_{i,q}^H(w_k) \right\} \\
&\geq \alpha_i + t \gamma_i \left\{ \sigma_i^2 + \right\} \\
&\sum_{j \in \{1, \ldots, g\} \setminus \{k\}} \sum_{q=1}^{Q_i} \left\{ \text{trace}(A_{i,q}(w_j)V) + b_{i,q}(w_j) b_{i,q}^H(w_j) \right\}.
\end{align*}
\]

\[
(83a)
\]

subject to

\[
\forall k \in \{1, \ldots, g\}, \forall i \in \mathcal{G}_k, \\
V \left( y+\sum_{\ell=0}^{r-1} N_{\ell}, y+\sum_{\ell=0}^{r-1} N_{\ell} \right) \in [0, 1], \\
\forall x \in \{1, \ldots, L\}, \forall y \in \{1, \ldots, N_x\}, \\
V \left( 1 + \sum_{\ell=1}^{L} N_{\ell}, 1 + \sum_{\ell=1}^{L} N_{\ell} \right) = 1, \\
V \geq 0, \\
\alpha_i \geq 0, \forall k \in \{1, \ldots, g\}, \forall i \in \mathcal{G}_k.
\]

\[
\text{(83b)}
\]

After a candidate solution \( V \) is obtained by solving Problem (P4-MA-MR-e) or Problem (P4-MA-MR-e'), appropriate post-processing such as Gaussian randomization [13] can be applied to convert the candidate solution into a solution which satisfies the rank constraint in Eq. (81f).

Combining the above discussion of alternating optimization, we present our method to solve Problem (P4-MA-MR) as Algorithm 4.
Algorithm 4 via alternating optimization to find $W$ and $\Phi_1, \ldots, \Phi_L$ for Problem (P4-MA-MR), which generalizes Problems (P5-MA-MR) (P6-MA-MR), (P4-MA)–(P6-MA), (P4-MR)–(P6-MR), and (P4)–(P6).

1: Initialize $\Phi_\ell$ for $\ell \in \{1, \ldots, L\}$ as some initial (e.g., randomly generated) $\Phi_\ell^{(0)} := \text{diag}(\beta_{1,1}e^{j\theta_{1,1}}, \ldots, \beta_{L,N_\ell}e^{j\theta_{L,N_\ell}})$ which satisfies the constraints on $\Phi_\ell$;
2: Set the iteration number $r \leftarrow 1$;
3: while 1 do
   4:   Given $\Phi_1, \ldots, \Phi_L$ as $\Phi_\ell^{(r-1)}, \ldots, \Phi_L^{(r-1)}$, use methods of Karipidis et al. \cite{karipidis99} or other papers to solve Problem (P4-MA-MR-b) in Eq. (78), and post-process the obtained $\{X_k\}_{k=1}^g$ and $t$ to set $W$ as some $W^{(r)} := \{w_1^{(r)}, \ldots, w_g^{(r)}\}$ and set $t$ as some $t^{(r)}$;
5:   if $r \geq 2$ then
6:      if $\|t^{(r)} - t^{(r-1)}\|_1 = 1$ denoting the relative difference between the object function values in consecutive iterations $r - 1$ and $r$ is small then
7:         break;
8:   end if
9: end if
10: Given $W$ as $W^{(r)}$ and $t$ as $t^{(r)}$, solve Problem (P4-MA-MR-c) in Eq. (82) or Problem (P4-MA-MR-c') in Eq. (83), and denote the obtained $V$ as $V^{(r)}$;
11: Perform the eigenvalue decomposition on $V^{(r)}_{\text{SDR}}$ to obtain a unitary matrix $U$ and a diagonal matrix $\Lambda$ such that $V^{(r)}_{\text{SDR}} = U\Lambda U^H$;
12: for $z$ from 1 to some sufficiently large $Z$ do
13:   Generate a random vector $r_z^{(r)}$ from a circularly-symmetric complex Gaussian distribution $\mathcal{CN}(0, I_{1+\sum_{\ell=1}^L N_\ell})$ with zero mean and covariance matrix $I_{1+\sum_{\ell=1}^L N_\ell}$ (the identity matrix with size $1 + \sum_{\ell=1}^L N_\ell$);
14:   Compute $v_z^{(r)} \leftarrow U\Lambda^{1/2}r_z^{(r)}$;
15:   With $v_1, \ldots, v_{\sum_{\ell=1}^L N_\ell}$ being the $(1 + \sum_{\ell=1}^L N_\ell)^{th}$ element of $v_z^{(r)}$, take the first $\sum_{\ell=1}^L N_\ell$ elements of $\frac{v_z^{(r)}}{v_1, \ldots, v_{\sum_{\ell=1}^L N_\ell}}$ to form a vector $w_z^{(r)}$;
16:   Scale each component of $w_z^{(r)}$ independently to obtain $\phi_z^{(r)}$ such that the $(y + \sum_{\ell=0}^{z-1} N_\ell)$th element of $\phi_z^{(r)}$ has magnitude $\beta_{x,y}$, for $x \in \{1, \ldots, L\}$ and $y \in \{1, \ldots, N_z\}$;
17: end for
18: if for $z \in \{1, 2, \ldots, Z\}$, there is no $\phi_z^{(r)}$ to ensure Eq. (80) after setting $\phi$ as $\phi_z^{(r)}$ then
19:   break;
20: else
21:   Select one $\phi_z^{(r)}$ according to some ordering among those ensuring Eq. (80) and denote the selected one by $\phi_z^{(r)}$;
22:   Map $\phi_z^{(r)}$ to some $\phi^{(r)}$ to satisfy the constraint on $\phi$ (e.g., discrete element values);
23:   For $\ell \in \{1, 2, \ldots, L\}$, set $\Phi_\ell$ as $\Phi_\ell^{(r)} := \text{diag}\{\phi_\ell^{(r)}\}_{\ell=1}^L$, where $\phi_\ell^{(r)}{\ell\in\{1,2,\ldots,L\}}$ are defined such that $\phi^{(r)} = \left[\begin{array}{c} \phi_1^{(r)} \\ \vdots \\ \phi_L^{(r)} \end{array} \right]$;
24: end if
25: Update the iteration number $r \leftarrow r + 1$;
26: end while
V. RELATED WORK

We discuss both related studies in wireless communications aided by IRSs and those without IRSs.

**IRS-aided wireless communications.** Since IRSs can be controlled to reflect incident wireless signals in a desired way, IRS-aided communications have recently received much attention in the literature. The studies include analyses of data rates, optimizations of power or spectral efficiency, and channel estimation. In these studies, IRSs are also referred to as large intelligent surface, reconﬁgurable intelligent surface, software-deﬁned surface, and passive intelligent mirrors. Interested readers can refer to surveys of IRS-aided communications.

For IRS-aided communications between a base station and mobile users, downlinks are investigated in and uplinks are studied in. In particular, for power control under QoS, Problem (P2) for the unicast setting has been addressed by Wu and Zhang, while uplinks are studied in. Previous studies on power control and QoS for wireless communications without IRSs. Power control and QoS for wireless communications without IRSs have been investigated extensively in the literature. We now discuss some representative studies. First, for power control under QoS, the unicast setting is considered by and the broadcast setting is studied by. Second, for max-min fair QoS, the unicast setting is considered by and the broadcast setting is studied by.

**Previous studies on power control and QoS for wireless communications without IRSs.** Power control and QoS for wireless communications without IRSs have been investigated extensively in the literature. We now discuss some representative studies. First, for power control under QoS, the unicast setting is considered by and the broadcast setting is studied by. Second, for max-min fair QoS, the unicast setting is considered by and the broadcast setting is studied by. We also consider the novel settings of multi-antenna mobile users on/and multiple intelligent reﬂecting surfaces. For all the optimizations discussed above, we present detailed analyses to propose efﬁcient algorithms. There are many future research directions to investigate. We list some as follows: 1) discussing the NP-hardness of our formulated optimization problems, 2) proving approximation bounds of our proposed algorithms, 3) conducting extensive experiments to validate our theoretical analyses and algorithms, 4) extending the models to take into account channel estimation errors and mobilities of MU s or/and LISs, 5) extending current studies of data transfer to wireless power transfer, 6) applying our analyses to other optimization problems such as those for weighted sum-rate or weighted power transfer.

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