Dynamics of the wakefield of a multi-petawatt, femtosecond laser pulse in a configuration with ultrarelativistic electrons

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Abstract – The wakefield excitation in an unmagnetized plasma by a multi-petawatt, femtosecond, pancake-shaped laser pulse is described both analytically and numerically in the regime with ultrarelativistic electron jitter velocities, when the plasma electrons are almost expelled from the pulse region. This is done, for the first time, in fluid theory, using a novel mathematical model that does not break down for very intense pump strengths, in contrast to the standard approach that uses the laser field envelope and the ponderomotive guiding center averaging. A three-timescale description is introduced, with the intermediate scale associated with the nonlinear phase of the electromagnetic wave and with the bending of its wave front. The evolution of the pulse and of its electrostatic waves are studied by the numerical solution in a two-dimensional geometry, with the spot diameter ∼ 100 μm. It has revealed that the nonlocal plasma response stretches very short pulses and that those with the length of 1–2 laser wavelengths, favored by the analytic estimates obtained in the local limit, are unstable. The optimum initial pulse length exceeds 1.5–2 μm.

Introduction. – Tajima and Dawson proposed [1] in 1979 to accelerate charged particles by large amplitude electron density waves, propagating through underdense plasma in the wake of an intense laser pulse. Plasma wakes can sustain electrostatic fields of several GV/cm, 10^3 times above the electric breakdown in conventional accelerators, enabling the construction of low cost, miniature laser-plasma accelerators (LPAs) [2]. Most powerful LPA systems at present time, or planned for the near future [3], include the Nd:Glass lasers with wavelength λ = 1.06 μm, pulse duration T = 300–500fs and power P ≲ 1PW, and the facilities using Ti:Sapphire technology, λ = 0.65–1.1 μm, having shorter pulses, T = 25–60fs, and the power P = 0.1–1PW. So far, the maximum electron beam energy achieved in LPAs is ≳ 1 GeV [4]. Fundamental limitation is set by the pump depletion. To produce a 10 GeV electron bunch with a charge of 1 nC, holding 10 J of kinetic energy, with a laser to particle beam efficiency 1–10%, laser energy of 100–1000 J is needed, i.e. P = 40–400PW, if the pulse duration is ∼ 25 fs.

In the terminology introduced in ref. [5], an LPA is said to be operating in the moderate intensity regime (MIR) or in the strong intensity regime (SIR) when the electron quiver motion is mildly relativistic, p⊥ < m0c or ultra-relativistic, p⊥ ≥ m0c, respectively (here p⊥ = eE⊥/ω and ω and E⊥ are the angular frequency and the amplitude of the laser electric field). Simple scaling [5] reveals that in the SIR the electron density perturbation is comparable to the plasma density, i.e. that the ponderomotive force entirely routs plasma electrons from the pulse area and leaves a wake of immobile, positively charged ions. For a spheroidal laser pulse, whose length and width are comparable to the plasma length, L∥ ∼ L⊥ ∼ 2πc/ωpe, the threshold for the complete expulsion of the electrons was estimated to be p⊥ ≥ 4m0c [6]. The phenomenological theory [7] found that such plasma cavity, or bubble, develops when the nonlinearities are sufficiently strong to produce a plasma wave breaking after the first oscillation. 3D PIC simulations [8] confirmed the existence of the bubble regime in LPA and showed that a bubble can
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trap background electrons and accelerate them, with a monoenergetic spectrum. Spheroidal bubbles are inherently electromagnetic [2], since the wake is encircled by a sheath of relativistic electrons (return current) exerting a Lorentz force on electrons. The balance of the Lorentz, Coulomb and ponderomotive forces determines the size of a 3D bubble. For self-similar 3D pulses, the optimum wake generation [9,10] occurs for the pulse length $a 3D$ bubble. For self-similar 3D pulses, the optimum wake Coulomb and ponderomotive forces determines the size of the Lorentz force on electrons. The balance of the Lorentz, emtically electromagnetic [2], since the wake is encircled by a monoenergetic spectrum. Spheroidal bubbles are inher-
d needs to be focussed to a diameter $d_s \sim 60 \mu m$, comparable to the pulse length.

In this letter, a theoretical study of the ultrarelativistic regime and SIR (ultrarelativistic regime) in terms of laser intensity $I = wc$, where $w$ is the energy density of the e.m. (electromagnetic) wave, $w = e_0 E_{L,0}^2 / 2$, we note that lasers with $\lambda \lesssim 1 \mu m$ have $I \sim \left( e_0 / c \right) (2 \pi m_0 c^2 / e) \lambda^2 \sim 10^{18} W/cm^2$ in the MIR and $I \gg 10^{18} W/cm^3$ in the SIR. The diameter $d_s$ of the laser spot is estimated from $I = 4 \pi \sigma_{\perp}^2$, which in the SIR yields $d_s \approx 305 \lambda \sqrt{P/(10^{15} W)}$. Thus, a 100 TW class Ti:Sapphire laser needs to be focussed to a diameter $d_s \approx 60 \mu m$, comparable to the pulse length.

In this letter, a theoretical study of the ultrarelativistic regime in the laser wakefield generation is for the first time carried out in fluid theory. We consider the propagation, in a cold unmagnetized plasma, of a pancake-shaped, ultrashort laser pulse, with energy $\sim 100 J$ and intensity $I \sim 0.5 \times 10^{20} W/cm^2$. Such multi-petawatt pulse achieves an ultrarelativistic intensity that strongly perturbs the electron density (causing almost complete expulsion) even if focussed to a large spot, $d_s \sim 100 \mu m \ll 305 \lambda \sqrt{P/(10^{15} W)} \sim 1000 \mu m$, much bigger than the length of a 25 fs pulse ($L \sim 7.5 \mu m$). Some authors argue that a “pancake” shape is beneficial for LPA [11], because a tight laser spot $d_s \sim 10 \mu m$ gives an acceleration length of only a few mm (estimated as twice the Rayleigh length), restricting the electrons’ energy gain. Although a suitably preformed plasma and the nonlinear self-guiding may enable a tightly focussed laser pulse to propagate well beyond 2–3 Rayleigh lengths [10,12,13], its strong radial electric field expels most electrons, permitting only a few to be trapped and accelerated by a 3D potential [11].

A SIR involves vastly different scalings in the core and at the pulse edges. The core is almost devoid of electrons and the e.m. pulse practically propagates in vacuum, featuring linear properties. The nonlinear self-organization [5,11] occurs at the edges, which are in a MIR. For laser intensities $I \sim 10^{20} W/cm^2$ we employ new model equations that describe both the SIR core and the MIR edges. This is not possible in the classical two-timescale description of a slowly varying envelope of the laser pulse. We develop a three-timescale description, with an intermediate timescale associated with the intensity-dependent phase of the e.m. pulse. In SIR, the nonlinear phase is resolved within the WKB (Wentzel-Kramers-Brillouin) approximation. The phase introduces new nonlinearities in the wave equation that suppress, in the core, both the nonlocal nonlinearity and the dispersion of the e.m. wave. Our 2D numerical result reveals that, as the core of the pulse runs at a higher group velocity than the leading edge, an initial steepening of the pulse’s front edge takes place, which is known to occur in the absence of nonlocality (i.e. spatial dispersion) in the plasma response [14]. Soon, the latter produce an effective mixing of the rapid core of the pulse with its slow front edge, pushing it forward and producing a frontward stretching of the laser pulse. Remarkably, the stretched pulse propagates in the plasma for several mm, consistent with the results of self-injection experiments [15].

Mathematical model. – Due to their extreme complexity, analytic studies of the laser-plasma interaction with intensities suitable for LPA have been attempted only for quasi-1D, pancake-shaped pulses, using the “quasi-static” approximation and a cold-fluid description, see the classical papers [11,16–18] and references therein. Recently in a mildly relativistic regime, the evolution of the plasma wake and of the laser pulse (depletion, frequency red-shifting) was satisfactorily described using a reduced wave equation and a quasistatic plasma response [19], with a good agreement with full Maxwell-fluid results. Such fluid calculations provided an additional insight into purely particle phenomena, e.g., by establishing the appropriate thresholds for the electron trapping and wave breaking, and showed that the electron dephasing (rather than laser depletion) limits the LPA’s energy gain. Following these works, considering an unmagnetized plasma, assuming $\nabla \perp \ll \partial / \partial z$, and taking that the solution is slowly varying in the frame that moves with velocity $u_\perp \vec{e}_x$, we derived our system wave equation + Poisson’s equation, (1), (2). The derivation is given in [5]. We note that eqs. (1) and (2) are valid also for ultrarelativistic electrons, $p_{\perp,0} \gg m_0 c$. Being affected by the return electron current, a wake is inately electromagnetic, but for sufficiently broad pulses, both pancake-shaped [5,19] and spherical [20], the electromagnetic effects are weak and we consider the wake as purely electrostatic.

Using the normalizations $\vec{p} \rightarrow \vec{p} / m_0 c$, $\vec{v} \rightarrow \vec{v} / c$, $\phi \rightarrow -e \phi / m_0 c^2$, $\vec{A} \rightarrow -\vec{e} \vec{A} / m_0 c$, $u \rightarrow u / c$, $t \rightarrow \omega_{pe} t$, $\vec{r} \rightarrow (\omega_{pe} / c)(\vec{r} - c_0 ut)$, our basic equations take the form

$$\left[ \frac{\partial^2}{\partial t^2} - 2u \frac{\partial}{\partial t} \frac{\partial}{\partial z} - \left( 1 - u^2 \right) \frac{\partial^2}{\partial z^2} - \nabla^2 \perp + \frac{1}{1 - \phi} \right] \vec{A}_\perp =$$

$$- \left( \frac{\partial}{\partial t} - u \frac{\partial}{\partial z} \right) \nabla \perp \phi,$$

$$\frac{\partial^2 \phi}{\partial z^2} = \frac{(\phi - 1)^2 - 1 - \vec{A}_\perp^2}{2(\phi - 1)^2},$$

where $\vec{A}_\perp^2 = \vec{A}_\perp \cdot \vec{A}_\perp$. Remarkably, eqs. (1) and (2) describe, beyond the ponderomotive guiding center approximation [21], the spatio-temporal evolution of an e.m. pulse of arbitrary intensity, interacting with a Langmuir wave via nonlocal nonlinearities arising from relativistic effects. Although the PIC algorithms are now

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getting very fast, it is argued [22] that fluid models for the plasma response may still be useful since the simulations of the next generation of LPA experiments (meter scale, 10 GeV) will increase the computational requirements around 1000-fold. The performance of LPA simulations is vastly improved using the ponderomotive guiding center averaging and modeling the envelope evolution of the laser field rather than the field itself [23]. However, such procedure breaks down for very intense pump strengths [22]. We avoid the breakdown using a three-timescale procedure, and seek the solution of eq. (1) as the sum of slow and rapid components, allowing the phase of the latter to evolve on an intermediate scale. We take \( \vec{A}_\perp = \vec{A}_\perp^{(0)} + \vec{A}_\perp \), where \( \vec{A}_\perp^{(0)} \) is the vector potential of the self-generated quasistationary magnetic field and \( \vec{A}_\perp \) is associated with the electromagnetic wave of the laser, \( \vec{A}_\perp^{(0)} = \vec{A}_\perp(0, \vec{r}, t) \). This is identified as the slow and intermediate scales. The dimensionless dispersion relation is

\[
\frac{\omega}{\vec{A}} = \kappa^2 \left[ \left( \nabla \phi \right)^2 + 2 \left( \partial \phi / \partial t_1 - (\partial \phi / \partial t_1)^2 \right) \right]^{1/2}. \tag{5}
\]

These permit us to rewrite eqs. (1) and (2) as

\[
2 \Re \left\{ e^{\frac{i}{2} \phi(t_1, \vec{r}_1) - \frac{1}{2} \kappa z} \left[ \alpha \vec{A}_\perp - 2i \epsilon \left( \frac{\partial \vec{A}_\perp}{\partial t_1} - \frac{\partial \vec{A}_\perp}{\partial t_1} \right) \right] \right\} = \epsilon \left( \frac{\partial^2}{\partial t_1^2} - \vec{u} \frac{\partial}{\partial t_1} - \vec{u} \frac{\partial}{\partial t_1} \right) \nabla \phi - \epsilon \left( \frac{\partial^2}{\partial t_1^2} - \vec{u} \frac{\partial}{\partial t_1} - \vec{u} \frac{\partial}{\partial t_1} \right) \vec{A}_\perp^{(0)}, \tag{6}
\]

where \( \alpha = \kappa^2 (\phi - (1 - \phi)) - i \left( \nabla_\perp \phi - \partial_\perp^2 \phi / \partial t_1^2 \right) \) and

\[
\vec{u}_k = c_\perp \left( \partial / \partial x_k \right) + c_\perp \left( \partial / \partial y_k \right) + c_\perp \left( \partial / \partial z_k \right), \quad k = 1, 2.
\]

The right-hand side of the wave equation (6) is slowly varying in space and time and cannot be resonant with the high-frequency oscillations on the left-hand side. The slow vector potential, \( \vec{A}_\perp^{(0)} \), comes from the quasistationary magnetic field generated in the laser-plasma interaction, for whose accurate description one needs to include also the kinetic effects that are responsible, e.g., for the off-diagonal terms in the stress tensor for electrons [24,25], for the return electron current [26-28], etc. As the derivation of our equations is based on a cold and unmagnetized plasma model [11,17,18], they are valid only when the right-hand side of eq. (6) is negligible. A scaling analysis of eqs. (6), (7) shows that the slow vector potential \( \vec{A}_\perp^{(0)} \) and the self-generated magnetic field can be neglected if

\[
\max (\phi, |\vec{A}_\perp^{(0)}|) < (e \nabla_\perp^{-1}) \partial / \partial t_2. \tag{8}
\]

As \( \kappa(\phi) \) is a localized, well-behaved function of its argument the left-hand side of eq. (5) varies on the same spatial and temporal scales as the wake potential \( \phi(t_1, \vec{r}_1) \), see eqs. (2) and (7). In other words, the function \( \kappa^2(\phi) \) is adopted to be a slowly varying function of the spatial variables \( \vec{r}_1 \) and a very slowly varying function of the variable \( t_1 \). Then, the fundamental solution for the phase \( \varphi \) obtained from eq. (5) is slowly varying with \( t_1 \), viz., \( \partial \varphi / \partial t_1 \ll 1 \). Using the new variables \( \vec{r}_1 - \vec{r}_1 \) and \( \tau = t_1 - \int_{\vec{r}_1}^{\vec{r}_1} d \vec{r} \cdot \nabla \phi (\nabla \varphi)^{-2} \), eq. (5) becomes

\[
(\nabla \varphi)^2 - \kappa^2(\phi) = \mathcal{O}(\epsilon^4), \tag{9}
\]

where derivatives with respect to the retarded time \( \tau \) appear only in small terms, of order \( \mathcal{O}(\epsilon^4) \). Consistently with the stationary, 1D approximation used in the derivation of the Poisson’s equation (2), which is accurate to \( \epsilon^2 \), the right-hand side of eq. (9) can be neglected.

Particularly simple is the case of a circularly polarized laser wave, \( \vec{A}_\perp = (\vec{A}_\perp + \sqrt{2})(\vec{e}_\perp \pm i \vec{e}_\perp) \), when we have \( \vec{A}_\perp^2 = |\vec{A}_\perp|^2 \), i.e., the second harmonic is absent. Similarly, for a linearly polarized wave, we will neglect the second harmonic, whose contribution is nonresonant, and use \( |\vec{A}_\perp|^2 \approx |\vec{A}_\perp|^2 \), where \( \vec{A}_\perp = \vec{A}_\perp(0, \vec{r}) \). Now, with the accuracy to \( \epsilon^2 \), our basic system of equations reduces to

\[
[\alpha_{Re}(\phi) + i \alpha_{Im}(\phi)] A_\perp - 2i \epsilon^2 \partial A_\perp / \partial t_2 \tag{10}
\]

\[
-2i \epsilon (\nabla \varphi \cdot \nabla_\perp) A_\perp - \epsilon^2 \nabla_\perp^2 A_\perp = 0,
\]

\[
2 \partial \partial \nabla_\perp 2 - 1 - \left(1 + |A_\perp|^2 \right) / (\phi - 1), \tag{11}
\]

\[
(\nabla \varphi)^2 - \kappa^2(\phi), \tag{12}
\]

where (12) is an eikonal equation (the geometrical optics’ limit), while \( \alpha_{Re} \) and \( \alpha_{Im} \) are the real and imaginary parts of \( \alpha \), respectively. With the accuracy to \( \epsilon^2 \) we have

\[
\alpha_{Re}(\phi) = \phi (1 - \phi) + \kappa^2(\phi), \quad \alpha_{Im}(\phi) = -\nabla_\perp \varphi. \tag{13}
\]

We adopt an auxiliary function \( \kappa(\phi) \) which, asymptotically, in the MIR and SIR, reduces to the limits elucidated in [5]. In the MIR, \( |\phi| \sim \epsilon^2 \), a Schrödinger equation with nonlocal cubic nonlinearity is to be recovered, which is
realized within the scaling $\alpha_{Re}(\phi) \sim \kappa(\phi) \sim \varphi \sim \mathcal{O}(\epsilon^2)$, viz.

$$[2i \omega \partial/\partial t + \omega^{-2} \partial^2/\partial z^2 + \nabla^2 \perp - \phi] A_{L_\perp} = 0,$$  \hspace{1cm} (14)

$$2 \left( \partial^2/\partial z^2 + 1 \right) \phi = -|A_{L_\perp}|^2.$$  \hspace{1cm} (15)

In the ultrarelativistic regime (i.e. SIR), $1 \ll \phi \lesssim 1/\epsilon$ (for $\phi > 1/\epsilon$ one may not neglect the self-generated magnetic field), we have $\phi = -|A_{L_\perp}|$ and $\nabla \phi = 1$ and the wave equation describes an e.m. wave propagating in vacuum, 

$$(2i \omega \partial/\partial t + \nabla^2 \perp) A_{L_\perp} = -A_{L_\perp}/\phi \to 0,$$  \hspace{1cm} (16)

while $\phi$ is found from $2 \partial^2 \phi/\partial z^2 = 1 - |A_{L_\perp}|^2/\phi^2$. These are realized when $\alpha_{Re} \to 1/\phi \to -\epsilon$ and $\kappa \to 1$. Close to the edges of an ultrarelativistic laser pulse, in the region where $\phi \sim \mathcal{O}(1)$, $\alpha_{Re}(\phi)$ needs to be sufficiently small, so that the nonlinear term $\alpha A_{L_\perp}$ has the same scaling as the linear terms in the wave equation. We adopt a simple expression $\alpha_{Re}(\phi) = \phi(1 + \phi^2)/(1 - \phi)^2$, and consequently:

$$\kappa(\phi) = -[\phi/(1 - \phi)] \left[ 1 + 2/(1 - \phi)^2 \right].$$  \hspace{1cm} (17)

**Numerical results.** – For an efficient LPA, one needs to find, within the technical constraints of the available lasers, the parameters that optimize the system’s performance. This can be tedious due to the large number of parameters involved (e.g., the initial state of the pulse alone is determined by three parameters — phase, length, and amplitude). To start an optimization, first we make an “educated guess” about the most stable length of the laser pulse, by finding an analytic stationary solution of eqs. (10)–(12) and (17) under simplified, albeit unphysical, conditions. For this, we neglect all transverse effects, $\partial/\partial x = \partial/\partial y = 0$, and the nonlocality effects in Poisson’s equation (11), viz. $(\partial/\partial z_2 - \epsilon u \partial/\partial t_2)\phi \to 0$.

The wave equation is then readily solved in the form $A_{L_\perp} = a_L(t_2, z_2) \exp[\i\delta k (z_2 - \delta \omega t_2 - \varphi(t_1, z_1))]$, where $a_L$, $\delta k$ and $\delta \omega$ are purely real quantities and the subscript $L$ denotes a local solution. Separating real and imaginary parts of eq. (10), and after some algebra, we have

$$a_L^{-1} \left( \partial^2 a_L/\partial z_2^2 \right) + \epsilon^{-2} \left[ 1 - (1 + a_L^2)^{-1/2} \right] = \delta k^2,$$  \hspace{1cm} (18)

which is easily integrated by quadratures

$$z_2 - \delta k t_2 = \left( \epsilon/\sqrt{2} \right) \int da_L \left[ c_1 a_L^2 + (1 + a_L^2)^{1/2} - 1 \right]^{-1/2}.$$  \hspace{1cm} (19)

Here $c_1 \equiv (\epsilon^2/2)(\delta k^2 - 2 \delta \omega - 1/\epsilon^2)$ is arbitrary constant and the maximum value of $a_L$ is $a_{max} = (1/c_1)\sqrt{1 + 2c_1}$. A numerical integration of eq. (19), with a typical LPA experimental value $\epsilon = 1/12$ and with the maximum laser vector potential $a_{max} = 14.12$, that is within the ultrarelativistic regime determined by $a \geq 4$ [6], reveals a typical bell-shaped profile. Its $e$-folding length $L_{||e,f}$, i.e. the separation between the points where the intensity is reduced by the factor $e$, $a_L^2(\ln_{||e,f}/2) = a_{max}^2/\epsilon$, is given by $L_{||e,f} = 0.65$, or $L_{||e,f} = 1.1 \mu m$ in non-scaled variables. This value of $L_{||e,f}$ is much shorter than that realized in the existing laser systems. Furthermore, it is close to a laser wavelength and the resulting nonlocal effects will be strong. Thus, the local solution (19) is not a good “zeroth estimate” for a stable pulse in a realistic experiment. For this reason, we numerically solve eqs. (10)–(12) and (17) in 2D (with $\partial/\partial y_2 = 0$), adopting laser parameters that are somewhat different from (19), but may become available in a near future. We employed a laser pulse with $60\%$ amplitude and twice the length of the local solution (19), i.e. used the initial condition with r.m.s. width $L_{rms} = 150 \mu m$, laser energy $E \approx 100 J$ and power $P \approx 20 PW$. We assumed a Gaussian transverse profile, $A_{L_\perp}(x_2, z_2, 0) = 0.6 a_L(z_2/L_\perp) \exp(\i \delta k z_2) \exp(-x_2^2/2L_\perp^2)$, where $a_L$ is given by (19) and...
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Fig. 3: (Color online) The nonlinear phase $\varphi(x_2,z_2,t_2)$ of the laser pulse displayed in fig. 1.

we adopt $\delta k = 0.5$, $L_x = 7.5$, and $L_z = 1.6$ (i.e. pulse duration $\sim 5.5$ ps). We consider an initially quiescent plasma $\phi(x_2,z_2,0) = \varphi(x_2,z_2,0) = 0$. The solution is shown in figs. 1–3, obtained on a standard PC by an “off the shelf” numerical package, that became numerically unstable at $t_{2,\text{max}} \sim 1.1$ (or $10^{-11}$ s in physical units). During this time the pulse traveled for 3 mm, which is comparable to the lengths of self-injection experiments and simulations [15,29]. Neither a transverse contraction nor the folding of the pancake pulse to a V-shape, known in the MIR [5,11], was observed during this time. Due to a stretching in the forward direction, occurring mostly in the central part of the pulse where the amplitude was the largest, the laser amplitude dropped by 40%, to $|A_{\text{max}}| \sim 5$, which is still within the ultrarelativistic regime, $|A|_\text{a} \geq 4$ [6]. Inside the pulse, the electron density was almost zero and the laser light practically propagated in a vacuum, i.e. the core of the pulse propagated with the speed of light and the nonlinear effects were weak. The nonlocality produced an effective mixing with the front edge of the pulse (that tends to propagate with the group velocity), which is pushed forward by the core. An oscillating wake with a large first potential minimum $|\phi|_{\text{max}} \gtrsim 1$ developed by $t_2 = 0.5$. The nonlinear phase $\varphi$ evolved simultaneously with the wake and produced a sizable bending of the laser wave front.

Conclusions. – In this letter we have studied, using a semi-analytic hydrodynamic description, a strong intensity regime of the propagation of pancake-shaped laser pulses through an unmagnetized plasma, with specifications that may be realized in the next generation of LPA experiments. We derive novel model equations, based on a three-timescale description, that account for the evolution of the nonlinear phase of the laser wave. At very large laser intensities this gives a smooth transition to a nondispersive e.m. wave and the saturation of the nonlocal nonlinearity. These equations are solved numerically, on a standard PC, in the regime when the ultrarelativistic electrons are almost expelled by the radiation pressure of a femtosecond laser pulse focussed to a $\gtrsim 100 \mu$m spot. We could follow the pulse along a 3 mm path, which is comparable with the plasma dimensions in self-injection experiments and simulations [2,15,29]. Practically no transverse self-focussing and filamentation have been observed for such broad pulse, but its Rleigh length is sufficiently long to allow for an efficient electron acceleration without self-guiding. The plasma wake, whose peak potential is $|\phi| \sim 1$, preserves its length $(\sim \lambda_p)$. The nonlocality effects stretch the laser pulse (2–3)-fold in the forward direction. In the mildly relativistic (MIR) regime, a moderate stretching has been observed for pulses that, initially, were sufficiently shorter than the plasma length $\lambda_p$ [5,11]. Conversely, MIR pulses that are $\gtrsim \lambda_p$, undergo longitudinal compression into a “laser piston” [30] and for them a mild stretching may be desirable [29], because it compensates the nonlinear red-shift and delays the formation of the “piston”, which reduces the dark current.

Our calculations have been performed for a laser energy $\sim 100$ J per pulse with duration $T \gtrsim 5$ fs, providing the power of several tens of petawatts and the intensity $I \sim 10^{20}$ W/cm$^2$. Lasers with such energy specifications are planned for the near future, mostly in the Nd:Glass technology. The Vulcan upgrade [31,32] will have a new laser beamline with 300 J in 30 fs (10 PW), that can be focussed to $10^{22}$ W/cm$^2$. Extreme Light Infrastructure (ELI) [33] will produce in its second section, planned for a later phase, a 10 PW beamline compressed to 130 fs, providing a power density $I > 10^{23}$ W/cm$^2$. Ultrastrong Ti:Sapphire lasers have also been planned, such as Astra-Gemini [31], a dual beam upgrade to a PW class of the existing Astra facility that will supply $10^{22}$ W/cm$^2$ on target. Each beam will have 0.5 PW, i.e. 15 J compressed to 30 fs. Further compression to $\sim 5$ fs may be achieved by photon deceleration and thin plasma lenses [34,35], and the transverse filamentation can be stabilized by a periodic plasma-vacuum structure [36]. Our semi-analytic fluid theory may be a valuable tool for the predictions and the analyses of LPA experiments in the ultrarelativistic regime with these lasers, focussed to a spot $\gtrsim 100 \mu$m, for which it can provide an estimate for the accelerating wakefield and its dynamics. While the oversimplified local model preferred very short (single oscillation) laser pulses, the observed stretching implied that an optimized LPA system requires a longer pulse. Kinetic effects, e.g., plasma wave breaking, trapping of resonant particles and their subsequent acceleration, are not included in the present analysis. They are subject of our study in progress, to be presented later.

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