Massively parallel ultrafast random bit generation with a chip-scale laser

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Random numbers are widely used for information security, cryptography, stochastic modeling, and quantum simulation. The technical challenges for physical generation of true random numbers are speed and scalability. Here we demonstrate a simple yet powerful method for ultrafast generation of many random bit streams with a single laser diode. We propose spatio-temporal interference of many lasing modes in a specially designed cavity as a new paradigm for greatly accelerated random bit generation. The spontaneous emission noise guarantees true randomness of the generated bits. We achieve a total bit rate of the order 100 Tb/s, which is two orders of magnitude higher than the fastest physical random bit generators reported to date. Our scalable approach is robust, compact, energy efficient, and will have a broad impact on secure communication and high-performance computation.

The performance and reliability of our digital networked society rely on the ability to generate large quantities of randomness. The ever-increasing demand to improve the security of digital information has shifted the random number generation from relying solely on pseudo-random algorithms to employing physical entropy sources. Ultrafast physical random number generators are key devices for achieving ultimate performance and reliability in communication and computation systems. Optoelectronic techniques have been developed for high-speed random bit generation [1,29]. One prominent example of photonic random number generators are chaotic semiconductor lasers that feature fast (sub-nanosecond) dynamics and large (tens of GHz bandwidth) [9,29]. By coupling several lasers to further increase the bandwidth and employing post-processing schemes to extract more bits in analog-to-digital conversion (ADC), the random bit generation (RBG) rate is pushed to 1 Tb/s [21,24,29]. However, the intrinsic time scales of lasing instabilities [30] impose an ultimate limit on the entropy generation rate. A further increase in the RBG rate requires a different physical process with inherently faster dynamics.

Concurrently, parallel RBG schemes can greatly enhance the generation rate and the scalability by producing many bit streams simultaneously. In the spatial domain, parallel generation of physical random numbers was realized by sampling two-dimensional laser speckle patterns created by a moving diffuser or a vibrating multimode fiber [31,32]. However, the generation rates are low (Mb/s) due to the inherently long mechanical timescales. Chaotic broad-area semiconductor lasers were investigated for high-speed parallel-RBG [33]. However, correlations of intensity fluctuations at different spatial locations impede the generation of independent parallel bit streams. Spectral demultiplexing of amplified spontaneous emission [6,34] or heterodyning chaotic laser emission [17] are employed for parallel RBG with rates up to sub-Tb/s per channel. So far, spectral-domain parallel RBG has been demonstrated with less than 10 channels. A significant increase in the number of channels is technically challenging.

Here we propose and demonstrate a method that not only greatly enhances the random bit rate in a single channel but also provides hundreds of channels for simultaneous generation of independent bit streams. We employ the ultrafast (picosecond) spatio-temporal interference of many lasing modes to accelerate emission intensity fluctuations in space and time, so as to massively produce ultrafast random bit streams in parallel. This is achieved by tailoring the geometry of a broad-area semiconductor laser to vastly increase the number of transverse lasing modes and suppress dynamical instabilities.

FIG. 1. Schematic of a chip-scale parallel random bit generator. (A) Sketch of a many-mode electrically-pumped semiconductor laser with curved end facets. Its emission is measured by two arrays of photodetectors for parallel RBG. (B) Top-view SEM image of a fabricated laser with cavity length $L = 400 \, \mu m$, width $W = 282 \, \mu m$, and the radius of end facet $R = 230 \, \mu m$. (C) Sketch of our experimental setup. The lasing emission at one end facet is imaged onto a streak camera that records the temporal fluctuation of the emission intensities at many spatial locations simultaneously.
FIG. 2. Suppressing spatio-temporal correlations of the lasing emission. (A) A wide-stripe edge-emitting semiconductor laser has a cavity formed by two planar end facets. It supports low-order transverse modes with the typical spatial profile shown. (B) Measured emission intensity $I(x, t)$ at one end facet of a 100 $\mu$m-wide, 1000 $\mu$m-long GaAs/AlGaAs QW laser featuring filamentation and irregular pulsations. The electric current injected to the stripe is 600 mA, where the lasing threshold current is 320 mA. (C) The spatio-temporal correlation function $C(\Delta x, \Delta t)$ of the emission intensity in (B), averaged over 10 measurements, reveals non-local spatial and temporal correlations. (D) Our laser cavity has curved end facets and supports high-order transverse modes. The spatial intensity distribution of a high-order transverse mode is plotted as an example. (E) The measured spatio-temporal trace of the lasing emission from the cavity shown in Fig. 1B is free of the micron-sized filaments and GHz oscillations that are seen in (B). The electric current is 800 mA, twice the lasing threshold current. (F) The spatio-temporal correlation function $C(\Delta x, \Delta t)$ of the emission intensity in (E), averaged over 10 measurements, shows the complete elimination of long-range spatio-temporal correlations.

like filamentation. Specifically, we design a chip-scale laser diode (Fig. 2A) to maximize the number of lasing modes and make their frequency spacings incommensurate, so that their interference patterns are complex and aperiodic. The spontaneous emission adds stochastic noise that ensures the intensity fluctuations are unpredictable and non-reproducible. In addition to the ultrafast bit rate for a single spatial channel, our laser provides several hundreds of independent spatial channels for parallel RBG. In a proof-of-concept demonstration, we attain a cumulative bit rate on the order of 100 Tb/s, representing a two orders-of-magnitude improvement over existing methods. Our in-depth analysis reveals the physical origin underlying the massive entropy creation by our device.

ELIMINATION OF LONG-RANGE SPATIO-TEMPORAL CORRELATIONS

To provide a large number of spatial channels for parallel RBG, the laser must support many transverse modes lasing simultaneously. A conventional broad-area edge-emitting semiconductor laser has a stripe geometry with two flat end facets (Fig. 2A). Typically, lasing occurs only in the lower order transverse modes because the higher order ones experience stronger diffraction losses and less gain. Furthermore, modulational instabilities induced by nonlinear interactions of the lasing modes with the gain material entail irregular pulsation and filamentation [35, 36]. As shown in Fig. 2B, the emission from a wide-stripe GaAs quantum well (QW) laser is spatially concentrated at multiple locations, forming wire-like streaks called filaments. They originate from carrier depletion in a transverse region of high lasing intensity, which leads to a local increase in the refractive index. The ensuing lensing effect causes self-focusing of the optical field, which evolves into a filament with transverse size of several microns. The filaments are inherently unstable, and their intensities oscillate irregularly on sub-nanosecond time scale. The spatio-temporal correlation function of the intensity fluctuations $C(\Delta x, \Delta t)$ (see materials and methods) in Fig. 2C features a complicated yet reproducible structure, revealing non-local correlations in space and time. These long-range temporal correlation degrades the quality of random bits generated at a spatial location. The long-range spatial correlation is detrimental to parallel RBG, as the random bit streams generated at different locations are not completely independent. Since the spatio-temporal scales of these self-organized structures are determined by intrinsic properties of the active gain medium [30], they fundamentally limit the parallel RBG rate.

To achieve massively parallel ultrafast RBG, we must greatly increase the number of transverse lasing modes and
strongly suppress the long-range spatio-temporal correlations of the emission intensity. To this end, we tailor the cavity geometry, increasing the width and curving the end facets as shown in Fig. 1B. High-order transverse modes are effectively confined inside such a cavity (Fig. 2D), and their spatial overlap with the injected carriers is increased by shaping the top metal contact (Fig. 1B). Fine tuning of the facet curvature maximizes the number of transverse lasing modes. The high-order transverse lasing modes have transverse wavelength $\lambda_t \sim 1 \, \mu m$, which renders the size of spatial holes burnt by them smaller than the typical filament width of a few microns. The resulting refractive index variations on such small spatial scales disrupt lensing and self-focusing effect from low-order transverse modes, thus preventing filamentation. Consequently, long-range spatio-temporal correlations disappear in $C(\Delta x, \Delta t)$ (Fig. 2F). The dramatically shortened correlation lengths in space and time pave the ground for a significant increase in the number of independent spatial channels for parallel RBG in addition to a great enhancement of the RBG rate of every individual spatial channel.

**SPATIO-TEMPORAL INTERFERENCE OF LASING MODES**

In the absence of lasing instabilities, the temporal fluctuations of the emission intensity are caused by the interference of lasing modes with different frequencies. The characteristic time scale for such fluctuations is inversely proportional to the spectral width of total emission. A typical width of 1.3 nm results in temporal fluctuations on the scale of 1 ps (see supplementary text). Such fluctuations are observed in the spatio-temporal trace of the emission intensity at one facet in Fig. 3A. All lasing modes have constant, albeit random phases within their coherence time of about 10 ns (the inverse of the individual lasing mode linewidth), and they interfere to create spatio-temporal speckle. The temporal speckle grain size, estimated from the full width at half maximum (FWHM) of $C(\Delta x, \Delta t)$ in time (Fig. 3B), is 2.8 ps, which is limited by the temporal resolution of the streak camera (see supplementary text).

The ultrafast temporal fluctuations of lasing intensities lead to an extremely broad power spectrum (Fig. 3C). The radio-frequency (RF) power is at least an order-of-magnitude higher than the background noise. The standard bandwidth, which contains 80% of the power, is 315 GHz. To confirm that such broadband dynamics is created by the interference of lasing modes, we numerically simulate many-mode interference and calculate the power spectrum (see supplementary text). The simulated spectrum is even broader with the standard bandwidth of 632 GHz. After accounting for the temporal resolution of the streak camera (≈ 1.2 ps), the simulated RF spectrum matches the measured one, confirming that the ultra-broad power spectrum results from the interference of many transverse and longitudinal modes.

The number of independent spatial channels for parallel RBG depends on the spatial correlation length of the lasing emission. In the absence of non-local correlations, the local correlation length is on the order of the spatial speckle grain size. Its value, obtained from the FWHM of $C(\Delta x, \Delta t)$ in space (Fig. 3B), is 1.5 µm. This length is limited by the spatial resolution of imaging optics and streak camera. Ignoring the finite experimental resolution, the simulation gives a correlation length of 0.5 µm, which is half of the transverse wavelength of the highest-order transverse lasing modes (see supplementary text). Thanks to the extremely short spatial correlation length, hundreds of independent spatial channels are available for parallel RBG.

To ensure the spatio-temporal interference pattern never repeats itself, we tailor the laser cavity geometry to make the transverse mode spacing incommensurate to the longitudinal mode spacing (free spectral range). Furthermore, the spontaneous emission constantly feeds noise of quantum mechanical origin into the lasing modes, randomizing their phases on the
scale of one coherence time. By harvesting the quantum noise, the intensity fluctuations are truly unpredictable and irreproducible in space and time.

RANDOM BIT GENERATION

We generate a random bit stream in a single channel from the time trace of the lasing emission intensity at one spatial position (Fig. 4A). Faster temporal sampling will give a higher bit rate, but adjacent bits are correlated if the sampling period $\tau$ becomes shorter than the correlation time of the fluctuations. We choose an optimal sampling period of $\tau = 2.44$ ps (Fig. 4A), giving the highest bit rate with negligible correlation (see supplementary text). The probability density of the sampled intensity $I_n$ in Fig. 4B exhibits an exponential tail, which is a hallmark of Rayleigh speckle statistics. As the asymptotic probability distribution would yield biased bits, we subtract adjacent samples $\Delta I_n = I_{n+1} - I_n$, yielding a symmetric probability distribution (Fig. 4C) [10,16]. The sampled intensities are transformed to $N_{\text{digit}} = 4$ bits by binning $\Delta I_n$ into $2^{N_{\text{digit}}} = 16$ values. The bin size is larger than the range of the background noise, hence the randomness originates purely from the laser emission. Among the four bits, two least significant bits (LSBs) are taken, doubling the bit rate from the sampling rate. To remove any residual correlation, we perform exclusive OR (XOR) on the bit stream with its copy delayed by 30.5 ps. The correlation of adjacent bits is reduced to $10^{-3}$ for $\Delta t \geq 1.22$ ps (Fig. 4D), hence the single-channel bit rate is 820 Gb/s.

For spatial multiplexing, a shorter distance $\Delta x$ between adjacent channels corresponds to a larger number of spatial channels. However, correlated bit streams will be produced if $\Delta x$ is shorter than the spatial correlation length of lasing emission. To find the optimal $\Delta x$, we calculate the mutual information between a pair of bit streams (see materials and methods). As seen in Fig. 4F, less information is shared between two channels if they are farther apart. With $\Delta x \geq 0.75 \mu m$, the mutual information drops to the value of uncorrelated bit streams from different lasers. We set $\Delta x = 0.5 \mu m$, at which the mutual information falls to $10^{-5}$ bit.

Experimentally we collect only a part of the spatial channels due to the limited efficiency of the collecting optics. The parts of the curved facet away from the center are out of focus when imaged onto the streak camera, resulting in a lower signal-to-noise ratio. Thus, we sample only the emission from the central part of width 121 $\mu m$ where the intensity fluctuations have sufficient dynamic range for high-quality RBG. The number of spatial channels is 243, which is about half of the total channels possible with complete collection of emission.

To evaluate the quality of the generated random bits, we apply the NIST SP 800-22 test suite. The parallel bit streams with a cumulative rate of 200 Tb/s pass all the statistics tests (see supplementary text).
ENTROPY GENERATION RATE AND SPATIAL DEGREES OF FREEDOM

The high RBG rate indicates a huge amount of entropy created by our laser. To identify its physical origin, we consider a simple model including only the interference of transverse and longitudinal lasing modes and the spontaneous emission noise (see materials and methods). Using the Cohen-Procaccia algorithm [1] [39], we calculate the entropy rate $h_{CP}$ for a bit stream generated from the simulated intensity fluctuations of a single spatial channel. Fig. 5A shows the convergence of $h_{CP}$ for different embedding dimension $d$. Stochastic fluctuations of the emission intensity result in a linear increase of $h_{CP}$ with the number of digits $N_{digit}$. The interference of a large number of lasing modes and the spontaneous emission noise both contribute to entropy generation (see supplementary text). In Fig. 5A, $h_{CP}$ reaches the information theoretical limit $h_0$ (see materials and methods) [1], indicating the maximal possible bit rate for a single channel is achieved.

To find how many independent spatial channels are available for parallel RBG, we investigate the effective spatial degrees of freedom of the lasing emission pattern. Intuitively, the number of spatial degrees of freedom in the total intensity pattern is expected to be $2M$, where $M$ is the number of transverse lasing modes, and the factor 2 stems from the independent degrees of freedom in the amplitude and phase of the field of a single mode (see materials and methods). However, gain competition and saturation will make the mode amplitudes uneven, effectively reducing the spatial degrees of freedom. Applying the Karhunen-Loeve decomposition to the simulated intensity pattern $I(x, t)$, we compute the Shannon entropy to obtain the complexity $H$ as a function of the number of transverse modes $M$ (see materials and methods) [40]. In Fig. 5B, the number of effective degrees of freedom $2^H$ grows linearly with $M$, but with a slope smaller than 2.

By maximizing $M$ with our cavity design, the maximum number of spatial channels is available for parallel RBG. After the ADC of the emission intensity, only two LSBs are kept to further reduce the spatial correlation length, and the number of independent channels is thus increased by a factor of 4 (see supplementary text). If all the emission will be collected, our laser, supporting several hundred transverse lasing modes, can produce independent bit streams from $\sim 1000$ spatial channels.

DISCUSSION AND CONCLUSIONS

We combine the classical wave-effect of many-mode interference and the quantum mechanical process of spontaneous emission to achieve ultrafast parallel true RBG. By suppressing the modulational instabilities in a broad-area semiconductor laser, we are able to utilize the spatio-temporal interference of many transverse and longitudinal modes to greatly enhance the RBG rate. The incommensurate frequency spacings of lasing modes and the spontaneous emission noise guarantee the random bit streams are aperiodic and irreproducible. Increasing the number of transverse lasing modes not only increases the number of spatial channels, but also enhances the RBG rate in each individual channel.

In the proof-of-concept experiment, we demonstrate parallel RBG in 243 channels with 820 Gb/s per channel. Both the single-channel RBG rate and the number of spatial channels are limited by the resolution and efficiency of our experimental apparatus. Improving the temporal resolution of photodetection can boost the single-channel RBG rate to 1 Tb/s even with 1-bit ADC. Increasing the spatial resolution of optical imaging will raise the number of channels to 1000. The cumulative bit rate will then exceed 1 Pb/s.

To create a compact system for parallel RBG, two arrays of fast photodetectors can be integrated with the laser in a single chip (Fig. 1A). The PIN photodiodes can be fabricated in the same GaAs/AlGaAs QW wafer, and reverse biased to detect the laser emission. Thanks to the high quantum efficiency, the emission power reaches 0.1~1 mW per channel (see supplementary text), which can be detected by the photodiode without a built-in amplifier. Even though the photodiodes will not be fast enough to fully resolve the temporal intensity fluctuations, the spatial multiplexing will enhance the RBG rate with a large number of channels.

Compared to the existing RBG schemes, our method, based on a single laser diode without feedback or injection, is extremely simple yet highly efficient. The performance is robust against fabrication imperfections such as surface roughness of the laser facets. We note that in the current experiment the random bit streams are generated by a computer through off-line post-processing. Real-time streaming of random bits to a computer may be realized by conducting the post-processing “on the fly” with FPGA boards [25, 27]. Such an implementation will enable a wide range of applications of our ultra-fast, compact, robust and energy-efficient RBG.
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Author contributions. K.K. conducted experiment and data analysis, and prepared the manuscript; S.B. built the experimental setup and guided data analysis; Y.Z. and Q.W. did sample fabrication; S.G. and O.H. guided the analysis of semiconductor setup and guided data analysis; Y.Z. and Q.W. did sample fabrication; S.G. and O.H. guided the analysis of semiconductor setup and guided data analysis; Y.Z. and Q.W. did sample fabrication.

Competing interests. The authors declare no competing financial interests.

Data and materials availability. All data presented in this Article and the supplementary materials are available on reasonable request to H.C.

MATERIALS AND METHODS

Device fabrication

We fabricate edge-emitting semiconductor lasers with a commercial GaAs/AlGaAs quantum well epitwafer (Q-Photonics QEWLD-808). The cavities with curved facets are defined by photolithography and etched by an inductively coupled plasma reactive ion etcher. The etch depth is 4 μm. The top metal contacts are fabricated by photolithography, Ti/Au deposition, and lift-off (see Ref. [37] for details). The wide- stripe lasers with planar facets in Fig. 2a are made by wafer cleaving.
Device testing

The diode laser is mounted on a copper plate. A tungsten needle (Quater Research H-20242) is placed on the top contact for electric current injection. To reduce sample heating, we use a diode driver (DEI Scientific, PCX-7401) to generate 0.6 µs-long current pulses at a repetition rate of 1 Hz.

The lasing emission on one end facet is imaged by a 20× microscope objective (NA = 0.4) and a lens (focal length 150 mm) onto the entrance slit of a streak camera (Hamamatsu C5680). The fast sweep unit (MS6760) of the streak camera records the spatio-temporal traces of emission intensity. To exclude the initial transient dynamics, only the latter part of the 0.6 µs-long emission pulse is measured. Due to the finite temporal measurement range of the streak camera, we sequentially combine thousands of streak images from consecutive pulses to obtain microsecond-long time traces. This process does not increase the entropy generation rate according to our numerical simulation detailed in supplementary text.

Spatio-temporal cross-correlation

We calculate the correlation function \( C(\Delta x, \Delta t) \) of the spatio-temporal emission intensity pattern \( I(x,t) \),

\[
C(\Delta x, \Delta t) = \frac{\langle \delta I(x_0 + \Delta x, t + \Delta t) \delta I(x_0, t) \rangle_t}{\sqrt{\langle \delta I^2(x_0 + \Delta x, t) \rangle_t \langle \delta I^2(x_0, t) \rangle_t}} \tag{S1}
\]

where \( \delta I(x,t) = I(x,t) - \langle I(x,t) \rangle_t \) represents the temporal fluctuation of emission intensity at the transverse position \( x \) on the end facet, and \( x_0 \) denotes the center of the facet \( [56] \).

Mutual information

We calculate the mutual information between a pair of bit streams, \( Y_x \) and \( Y_{x+\Delta x} \), generated from emission intensities at locations \( x \) and \( x + \Delta x \),

\[
h(\Delta x) = \left( \sum_{Y} p(Y_x, Y_{x+\Delta x}) \log_2 \frac{p(Y_x, Y_{x+\Delta x})}{p(Y_x)p(Y_{x+\Delta x})} \right)_{x} \tag{S2}
\]

Here \( p(Y_x) \) is the probability density function (PDF) of a random bit stream \( Y_x \), and \( p(Y_x, Y_{x+\Delta x}) \) is the joint PDF of the random bit streams \( Y_x \) and \( Y_{x+\Delta x} \).

Numerical modeling

The cavity resonances are calculated with the eigenmode solver module of COMSOL Multiphysics. To investigate the mode competition for optical gain, we calculate the lasing modes with the single pole approximation steady-state ab-initio lasing theory (SPA-SALT) \([41]\). To maximize the number of transverse lasing modes, we optimize the cavity geometry, the ratio of cavity length to width \( L/W = 1.41 \), and the radius of the end facets \( R/L = 0.58 \) \([37]\). The total emitted field is a sum of fields in numerous transverse and longitudinal modes with frequencies within the GaAs QW gain spectrum. The intensity pattern at one facet can be written as

\[
I(x,t) = \left| \sum_{m=0}^{M-1} \sum_{q} A_{m,q} e^{i\phi_{m,q}(t)} \psi_m(x) e^{i2\pi\nu_{m,q} t} \right|^2, \tag{S3}
\]

where \( \nu_{m,q} \) is the frequency of a mode with the transverse index \( m \) and longitudinal index \( q \), \( \psi_m(x) \) represents its transverse field profile on the end facet, and \( A_{m,q} \) and \( \phi_{m,q} \) denote its global amplitude and phase. Due to spontaneous emission, \( \phi_{m,q}(t) \) fluctuates in time, following a Wiener process,

\[
\Delta \phi(t) = \phi(t + \Delta t) - \phi(t) = \sqrt{2\pi \delta \nu \Delta t} Z(t), \tag{S4}
\]

where \( \delta \nu \) is the lasing mode linewidth, \( \Delta t \) is a discrete time step, and \( Z(t) \) is a normal-distributed random number with a standard deviation of 1. For each mode, the initial phase \( \phi(t = 0) \) is randomly chosen in the interval \([0,2\pi]\) with a uniform probability density.

Entropy rate calculation

Using the Cohen-Procaccia algorithm \([1]\), we compute the entropy rate \( h_{CP} \) as a function of the bin size \( \epsilon \) for intensity digitization and the temporal sampling period \( \tau \). For a time trace of emission intensity \( I(t) \) at a single position of the laser facet, we construct \( d \)-dimensional data sets by introducing time delays: \( I_1 = I(t), I_2 = I(t + \tau), ..., I_d = I(t + (d-1)\tau) \). Then we randomly select \( N \) reference points in the \( d \)-dimensional space. For each reference point \( j \), we compute \( f_j(\epsilon) \), the fraction of other points within a \( d \)-dimensional box of width \( \epsilon \). The \( d \)-dimensional pattern entropy estimate is given by

\[
H_d(\epsilon, \tau) = -\frac{1}{N} \sum_{j=1}^{N} \log_2[f_j(\epsilon)]. \tag{S5}
\]

The Cohen-Procaccia entropy rate estimate is then obtained by

\[
h_{CP}(\epsilon, \tau, d) = \tau^{-1}[H_d(\epsilon, \tau) - H_{d-1}(\epsilon, \tau)]. \tag{S6}
\]

Here \( \epsilon = (I_{\text{max}} - I_{\text{min}})/2^{N_{\text{digit}}} \), with \( I_{\text{max}} \) and \( I_{\text{min}} \) being the maximum and minimum intensities.

In Fig. 3, \( h_{CP} \) is plotted as a function of \( N_{\text{digit}} \) for different \( d \). The time trace of emission intensity in a single channel is numerically calculated with the cavity parameters identical to the experimental ones (\( L = 600 \mu m, W = 424 \mu m, R = 345 \mu m \)). There are 8 longitudinal mode groups within the emission spectrum. The number of transverse is \( M = 200 \). The optical linewidth of each mode is \( 345 \mu m \). The temporal range is 2.5 µs, yielding a time trace of 10⁶ samples. To be consistent with the experimental data, the intensity value is rounded to an integer number with a mean of 60. The range of intensity
Karhunen-Loeve decomposition

We perform a Karhunen-Loeve decomposition of the simulated lasing intensity pattern \( I(x, t) \) in a cavity of length \( L = 40 \, \mu m \), width \( W = 28.2 \, \mu m \), radius of end facets \( R = 23 \, \mu m \) and refractive index \( n = 3.37 \). The mode frequencies \( \omega_{m,q} \) and spatial field profiles \( \psi_{m,q}(x) \) are obtained from the COMSOL calculation of cavity resonances. The amplitudes \( A_{m,q} \) of individual lasing modes are calculated with the SPA-SALT. Their phases \( \phi_{m,q} \) are random numbers in the range of \([0, 2\pi]\).

From the intensity fluctuation \( \delta I(x, t) = I(x, t) - \langle I(x, t) \rangle \), the spatial covariance matrix \( C_{ab} = \langle \delta I(x_a, t) \delta I(x_b, t) \rangle \) is constructed and its eigenvalues \( \lambda_\alpha \) are computed \([44]\). \( \lambda_\alpha \) is sorted from high to low with the index \( \alpha \), and it reflects the weight of the corresponding eigenmode in \( I(x, t) \). The value of \( \lambda_\alpha \) has a sudden drop at \( \alpha = 2M \), where \( M \) is the number of transverse lasing modes, confirming the spatial degrees of freedom is \( 2M \).

We quantify the spatial complexity by the Shannon entropy of the eigenvalues \([40]\)

\[
H = - \sum_\alpha p_\alpha \log_2 p_\alpha, \quad (S8)
\]

where \( p_\alpha = \lambda_\alpha / \sum \lambda_\alpha \) is the normalized eigenvalue. The effective degree of freedom \( 2^H \) is less than \( 2M \) due to unequal contributions of individual eigenmodes to \( I(x, t) \).

**SUPPLEMENTARY TEXT**

A. Device characterization

The many-mode semiconductor lasers fabricated for parallel random bit generation (RBG) have a cavity length of \( L = 600 \, \mu m \), a transverse width of \( W = 424 \, \mu m \), and the radius of the curved facets is \( R = 345 \, \mu m \). A measured LI curve is plotted in Fig. S1A. The lasing threshold current is 570 mA. At a pump current of 1200 mA, the power of lasing emission collected by the objective lens (NA = 0.4) from one end of the cavity is 93 mW. The collection efficiency is estimated to be about 20\%, with the divergence angle of the lasing emission, the transmission and numerical aperture of the objective lens, and the collection from a single facet of the cavity all taken into account. Thus the total emission power is about 470 mW, which corresponds to a quantum efficiency of 0.74 W/A. We record the far-field speckle pattern created by the output beam passing through a diffuser and calculate its intensity contrast \([37]\). The number of transverse lasing modes \( M \) estimated from the measured speckle contrast of 0.07 is about 200. Hence the emission power per transverse mode is on the order of 1 mW.

Figure S1B shows the measured emission spectrum at two times of the lasing threshold integrated over a 0.6 \( \mu s \)-long pulse. Individual lasing modes cannot be resolved, as the wavelength separation between adjacent modes is smaller than the resolution of the spectrometer. The full-width-at-half-maximum (FWHM) of the emission spectrum is 1.3 nm. The spectrum \( S(\lambda) \) is fitted by a Gaussian function centered at \( \lambda_0 = 800 \, nm \) with a standard deviation of 0.55 nm. The free spectral range (longitudinal mode spacing) of the 600 \( \mu m \)-long cavity is \( \lambda_{FSR} = \lambda_0^2 / 2nL = 0.16 \, nm \), where the effective refractive index \( n = 3.37 \) is calculated from optical confinement in the direction perpendicular to the cavity plane for transverse electric (TE) polarization, i.e., electric field parallel to the cavity plane. Hence the emission spectrum \( S(\lambda) \) contains 8 longitudinal modal groups within its FWHM.

B. Numerical modeling

1. Cavity resonances

We calculate the resonances of the passive cavity with the COMSOL eigenfrequency solver module. Since the laser emission is purely TE polarized, we compute only TE-polarized modes which are the solutions of the scalar Helmholtz equation

\[
[\nabla^2 + k^2 n^2(x, y)] H_z(x, y) = 0 \quad (S9)
\]

with outgoing wave boundary conditions, where \( k \) is the free-space wave number, \( n \) is the effective refractive index, and \( H_z \) is the z-component of the magnetic field. As light can escape
in the transverse direction, absorptive boundary conditions are imposed on the lateral sides of the cavity using perfectly matched layers. Since numerical modeling of a cavity as large as the fabricated lasers \((L = 600 \, \mu m, W = 424 \, \mu m)\) is too computationally expensive, we instead simulate a smaller cavity of the same geometry \((L = 40.0 \, \mu m, W = 28.2 \, \mu m)\). We calculate the resonant modes with wavelengths around 800 nm which is similar to the measured laser wavelength (Fig. S1B). However, since the smaller cavity has a larger mode spacing, we extend the wavelength range of simulation to 795–805 nm in order to increase the number of modes, covering 4 free spectral ranges.

Figure S2A shows the quality \((Q)\) factors and wavelengths of the passive cavity modes. The transverse mode numbers are indicated. In Fig. S2B, the modes are arranged in terms of their transverse and longitudinal mode indices. Although its size is relatively small, the simulated cavity exhibits about 50 transverse modes. The number of transverse modes scales linearly with the cavity size \([37]\).

### 2. Lasing modes

Mode competition for optical gain tends to reduce the number of transverse lasing modes. We calculate the lasing modes using the single pole approximation steady-state \(ab\)-\initio laser theory (SPA-SALT) \([41, 45, 46]\), taking gain saturation into full account. We assume a spatially uniform distribution of the pump and a flat gain spectrum within the wavelength range of 795–805 nm.

The modes that lase in the simulation are marked by red squares in Fig. S2A. When the pump is two times the lasing threshold, the number of transverse lasing modes \(M\) is 46. Hence, most of the transverse modes in the cavity can lase in spite of gain competition.

3. Many-mode interference

With spatio-temporal instabilities and filamentation suppressed in our laser, the lasing modes correspond to the high-\(Q\) resonances of the passive cavity. To simulate a large cavity of size equal to the fabricated one \((L = 600 \, \mu m)\), we compute the mode frequencies \(v_{m,q} = c/\Delta \lambda_{m,q}\) with the analytical expression for Hermite-Gaussian modes,

\[
\nu_{m,q} = \frac{c}{2nL} \left[ q + \frac{1}{\pi} \left( m + \frac{1}{2} \right) \arccos(g) \right] \tag{S10}
\]

where \(g = 1 - L/R\) is the cavity stability parameter, which is \(g = -0.74\) for the fabricated laser.

Summing over all lasing modes, the emission intensity at one spatial position, e.g., the center of the end facet \(x_0\), is

\[
I(t) = \left| \sum_{m=0}^{M-1} \sum_{q} A_{m,q} e^{i(2\pi v_{m,q}t + \phi_{m,q}(t))} \right|^2 \tag{S11}
\]

with \(A_{m,q}\) approximated by \(\sqrt{S(\lambda_{m,q})}\), where \(S(\lambda_{m,q})\) is the fit of the measured emission spectrum in Fig. S1B. The total number of transverse modes is \(M = 200\) as in the experiments \([37]\).

To account for the spontaneous emission, we introduce a stochastic fluctuation to the phase \(\phi_{m,q}(t)\) of each lasing mode \([47]\) (see supplementary text). The mode linewidth is set to \(\delta \nu = 100\, MHz\), which is typical for a GaAs/AlGaAs QW edge-emitting laser \([48, 49]\). We set the discrete time step \(\Delta t\) to 0.1 ps. As shown in Fig. S3A, the phase of each mode undergoes a random walk. The optical spectrum of a single lasing mode, calculated via the temporal Fourier transform of its field, exhibits a Lorentzian-shaped line with FWHM equal to \(\delta \nu\) as shown in Fig. S3B.
4. Single-channel dynamics

Figure S4A shows a portion of the simulated time trace \( I(t) \) for a single spatial channel. The sampling period is set to 0.5 ps, corresponding to the pixel size of our streak camera. We compute the temporal correlation function for \( I(t) \) defined as

\[
C(\Delta t) = \frac{\langle \delta I(t) \delta I(t + \Delta t) \rangle_t}{\langle \delta I^2(t) \rangle_t},
\]

where \( \delta I(t) = I(t) - \langle I(t) \rangle_t \) is the intensity fluctuation around the mean \( \langle I(t) \rangle_t \). As shown in Fig. S4B, the half-width-at-half-maximum (HWHM) of the correlation function is 0.5 ps. This is smaller than the measured correlation width (HWHM) of 1.4 ps due to the finite temporal resolution of the streak camera.

To take into account the temporal resolution, we convolve the simulated time trace with a temporal point spread function (PSF). The PSF of the streak camera is approximated by a Lorentzian function with a FWHM of 1.2 ps. The convolution smoothes the time trace (Fig. S4A), and increases the temporal correlation width to 1.4 ps, in agreement with the experimental value (Fig. S4B).

5. Power spectra

To simulate the radio frequency (RF) spectra shown in Fig. 3C of the main text, we calculate the Fourier transform of the simulated intensity \( I(t) \). The time trace is 500 ps long, and the sampling period is 0.5 ps. The power spectra are averaged over 10 time traces with different phase noise to be consistent with the measurements.

To understand how the broad RF spectrum is formed, we vary the number of transverse modes \( M \) while keeping the number of longitudinal mode groups constant. This corresponds to changing the width of a laser cavity while keeping its length fixed. With only the fundamental transverse mode \( M = 1 \) (Fig. S5A), the RF spectrum features multiple peaks separated by the longitudinal mode spacing (free spectral range) \( \Delta \nu_{FSR} = c/2nL \). With increasing \( M \), each peak becomes a group of peaks that originate from the beating of transverse modes. For the cavity with \( g = -0.74 \) considered here, the transverse mode spacing is incommensurate with the free spectral range because \( \arccos(g)/\pi \) in Eq. S10 is an irrational number. As a result, the additional beating frequencies that appear when increasing the number of transverse modes eventually fill the entire frequency range. With \( M = 200 \) transverse modes in Fig. S5C, the power spectrum becomes continuous and featureless, in agreement with the experimental data in Fig. 3C of the main text. The dense packing of the power spectrum increases the entropy generation as will be shown in the next subsection.

6. Entropy rate

To understand how the number of transverse modes \( M \) and the spontaneous emission noise affect entropy generation, we vary \( M \) and turn on/off the phase noise when calculating the entropy rate. The number of longitudinal mode groups...
FIG. S6. Cohen-Procacia entropy rate of simulated time traces. (A) A portion of the simulated intensity time trace $I(t)$ with only the fundamental transverse modes ($M = 1$), and without spontaneous emission noise. (B) $I(t)$ with $M = 3$ and without spontaneous emission noise. (C) $I(t)$ with $M = 3$ and with spontaneous emission noise ($\delta \nu = 100$ MHz). (D) Entropy rate $h_{CP}$ for the time traces in (A)-(C) with embedding dimension $d = 2$. The sampling period $\tau$ is 2.5 ps.

is kept fixed at 8 as before. The length of the simulated time traces is 2.5 ps. With only one transverse mode and without spontaneous emission noise, the intensity trace is periodic in time (Fig. S6A). The periodic modulation results from the temporal beating of the longitudinal modes with equal frequency spacing. Fig. S6D shows the Cohen-Procacia entropy rate $h_{CP}$ versus the number of digits $N_{digit}$ for embedding dimension $d = 2$. As $d$ increases, $h_{CP}$ will go down and eventually diminish (not shown). With $M = 3$ transverse modes, the intensity trace in Fig. S6B exhibits more complex and aperiodic modulations. Since the transverse and longitudinal mode spacings are incommensurate, the temporal beating of 24 modes will produce an intensity trace that will never repeat itself. Consequently, $h_{CP}$ increases in Fig. S6B. Adding the spontaneous emission noise (Fig. S6C) further contributes to entropy generation as can be seen in the increased $h_{CP}$ in Fig. S6D.

C. Random bit generation and testing

1. Temporal sampling rate

The black line in Fig. S7A is the emission intensity at one spatial position of the laser facet that is recorded by the streak camera. The temporal pixel size of the streak camera corresponds to 0.49 ps, and the sampling points are denoted by black dots. The temporal correlation function (Eq. S12) for this trace in Fig. S7B reveals significant correlations among neighboring sampling points. To create independent bits, we must choose a longer sampling period $\tau$, which reduces the sampling rate and hence bit generation rate, however.

In order to find the optimal sampling period, we calculate the Cohen-Procacia entropy generation rate per sample for the experimental bit streams generated with different $\tau$. The sampled intensity is binned into $2^{N_{digit}}$ equally spaced intervals to create $N_{digit}$ bits. As shown in Fig. S7C, the entropy rate per sample first increases with the sampling period $\tau$, then levels off for $\tau \geq 2.44$ ps. This trend is similar for different $N_{digit}$. Hence we set the sampling period to be 5 times the streak camera pixel size, $\tau = 2.44$ ps, to extract the maximal entropy per sample with the fastest possible sampling rate.

2. Suppressing temporal correlations

As seen in Fig. S7B, even after choosing a sampling period of $\tau = 2.44$ ps, the correlation of adjacent samples is not completely eliminated. RBG demands negligible correlation between successive bits. Digitization and post-processing play a crucial role in removing the remaining correlation. During the analog-to-digital conversion (ADC), the measured intensity is transformed to $N_{digit} = 4$ bits, and only the two least significant bits (LSBs) are kept. In Fig. S8, we compare the temporal correlation function (Eq. S12) of the bit stream to that of the original intensity trace. For long time lags, keeping only the LSBs reduces the correlation to $10^{-3}$, which is the lower limit given by the finite length of the bit stream (1000 samples). For short time lags (Fig. S8B), the correlation for the LSBs decays more rapidly, shortening the correlation time.

To further reduce the short-term correlation, we conduct XOR of the bit stream and its delayed copy. We choose a relatively long delay of 30.5 ps because a short delay may not be sufficient to remove all correlation. As shown in Fig. S8B, the correlation between the nearest neighbors is reduced by more
the facet is imaged onto the streak camera, its two edges are away from the object plane (marked by the black dotted line in Fig. S9A) and are thus out of focus. Moreover, the field of view (FoV) is only about 260 μm (marked by the red dashed line in Fig. S9A), which is narrower than the facet width W = 424 μm. For RBG, we use only the spatial channels around the middle of the facet. More specifically, bit streams are extracted from the central 121 μm-wide segment where the emission intensity is above 70% of the maximal intensity (indicated by the vertical dashed blue line in Fig. S9B). The channels with higher mean intensity have stronger temporal fluctuations, thus more entropy can be extracted.

In Fig. S9B, we compare the measured emission profile to the ideal one obtained from numerical simulation. Using SPASAL [46], we calculate the lasing modes in two small cavities of lengths L = 20, 40 μm with the same geometry as the laser cavities in the experiment. After rescaling, the calculated intensity distribution on the curved facet is identical for the two cavities. Hence, the emission profile is universal and scales linearly with the cavity size. The FWHM of the measured profile is about half of the simulated one. Hence, only half of the available spatial channels are used for random bit generation in the current experiment.

3. Number of spatial channels

Not all spatial channels are accessible in the current experiment because of the limited collection efficiency of the optics. The measured intensity of lasing emission on one curved facet decreases rapidly with distance from the center of the facet (red curve in Fig. S9B). As the central part of the facet is imaged onto the streak camera, its two edges are away from the object plane (marked by the black dotted line in Fig. S9A) and are thus out of focus. Moreover, the field of view (FoV) is only about 260 μm (marked by the red dashed line in Fig. S9A), which is narrower than the facet width W = 424 μm. For RBG, we use only the spatial channels around the middle of the facet. More specifically, bit streams are extracted from the central 121 μm-wide segment where the emission intensity is above 70% of the maximal intensity (indicated by the vertical dashed blue line in Fig. S9B). The channels with higher mean intensity have stronger temporal fluctuations, thus more entropy can be extracted.

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4. Spatial correlation length

The spacing between independent spatial channels is determined by the spatial correlation length of the emission intensity. To find its relation to the transverse wavelength of the lasing modes, we simulate the spatio-temporal intensity pattern of the lasing emission from a small cavity with L = 40 μm, W = 28.2 μm and R = 23 μm (see materials and methods). The number of transversal lasing modes M at a pump level of two times the lasing threshold is 46 (see Fig. S2). Fig. S10A is a portion of the spatio-temporal intensity pattern at one cavity facet with full spatial resolution. In the experiment, the numerical aperture (NA) of the imaging optics is 0.4, and the spatial resolution of the streak camera is about 0.5 μm. The convolution with the spatial point spread functions (PSF) of imaging optics and streak camera enlarges the spatial speckle grains (Fig. S10B). In Fig. S10C, we show the correlation of the intensity fluctuations $\delta I(x, t) = I(x, t) - \langle I(x, t) \rangle_t$ at spatial locations separated by $\Delta x$,

$$C(\Delta x) = \frac{\langle \delta I(x, t) \delta I(x + \Delta x, t) \rangle_t}{\sqrt{\langle \delta I^2(x, t) \rangle_t \langle \delta I^2(x + \Delta x, t) \rangle_t}}$$

(S13)

The FWHM of $C(\Delta x)$ is 0.25 μm for the fully resolved pattern in Fig. S10A. The average speckle grain size (FWHM) is 0.5 μm, which is close to half of the transverse wavelength of the highest-order transverse lasing mode. The limited spatial resolution of imaging optics and streak camera enlarges the speckle grain size to 1.5 μm, thus reducing the number of independent channels.
Digitization and post-processing reduce the spatial correlation length, similarly to the suppression of temporal correlation in Fig. S8. In Fig. 4F of the main text, the mutual information of bit streams from two spatial channels becomes negligible when their separation $\Delta x$ exceeds 0.75 $\mu$m. However, the original intensity pattern in Fig. 3 of the main text shows a spatial correlation extending over a distance of 1.4 $\mu$m. In Fig. S11 we compare the mutual information between two bit streams produced experimentally for three different cases: (i) thresholding $N_{\text{digit}} = 1$, the simplest bit-extraction scheme; (ii) keeping 2 LSBs from analog-to-bit conversion with $N_{\text{digit}} = 4$; (iii) conducting XOR of (ii) with its delayed copy. In comparison to (i), the mutual information in (ii) is reduced by more than one order of magnitude and becomes short-ranged. A further reduction of the magnitude and spatial correlation length of the mutual information is seen in (iii). When the channel separation exceeds 0.75 $\mu$m, the mutual information decreases to a constant value of $10^{-6}$. This value is inversely proportional to the length of the bit stream (Fig. S11B). This scaling indicates that the residual mutual information results from the finite bit stream length.

Since the temporal measurement range of our streak camera is limited, it is impossible to measure a long continuous time trace. Instead we make separate measurements and combine the time traces. Since this process could potentially increase the randomness, we check numerically whether the entropy generation rate is changed by it.

Using Eq. S11 to simulate the many-mode interference, we obtain a 2.5 $\mu$s-long time trace of the emission intensity in a
FIG. S13. **NIST SP800-22 statistical test results.** (A) Pass proportions and (B) composite P-values of 15 statistical tests. Multiple dots in (B) represent several subtests for each statistical test. The red lines denote the criteria for a qualified random bit generator. As a test input, we use 243 bit streams obtained by spatial demultiplexing of the emission from a single laser.

We validate the quality of the random bit streams in Fig. 4 of the main text using the NIST SP800-22 Random Bit Generator test suite [50]. The suite consists of 15 different statistical tests, some of which include subtests. Each test returns p-values. When the p-value exceeds a significance level of \( \alpha = 0.01 \), the bit stream is considered as random. For \( k \) bit streams, we examine if they pass or fail for each statistical test. The pass proportion should be within \( (1 - \alpha) \pm 3\sqrt{\alpha(1 - \alpha)}/k \). For the test, we use \( k = 243 \) bit streams generated from different spatial channels in parallel. As shown in Fig. S13A, the pass proportions are all above the criterion indicated by the red line.

For a good random bit generator, the p-values from the \( k \) bit streams should be uniformly distributed. The composite P-value (p-value of the p-values) is a measure of uniformity of the p-values. The distribution of p-values is considered as uniform when the composite P-value is larger than a significance level of \( 10^{-4} \). Figure S13B shows that the composite P-values of all subtests are above the significance level (indicated by the red line). The test results verify the high quality of the random bits generated from our device.