Implications of Fritzsch-like lepton mass matrices

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Abstract

Using seesaw mechanism and Fritzsch-like texture 6 zero and 5 zero lepton Dirac mass matrices, detailed predictions for cases pertaining to normal/inverted hierarchy as well as degenerate scenario of neutrino masses have been carried out. All the cases considered here pertaining to inverted hierarchy and degenerate scenario of neutrino masses are ruled out by the existing data. For the normal hierarchy cases, the lower limit of $m_{\nu_1}$ and of $s_{13}$ as well as the range of Dirac-like CP violating phase $\delta$ would have implications for the texture 6 zero and texture 5 zero cases considered here.

In the last few years, apart from establishing the hypothesis of neutrino oscillations, impressive advances have been made in understanding the phenomenology of neutrino oscillations through solar neutrino experiments \cite{1}, atmospheric neutrino experiments \cite{2}, reactor based experiments \cite{3} and accelerator based experiments \cite{4}. At present, one of the key issues in the context of neutrino oscillation phenomenology is to understand the pattern of neutrino masses and mixings which seems to be vastly different from that of quark masses and mixings. In fact, in the case of quarks the masses and mixing angles show distinct hierarchy, whereas in the case of neutrinos the two mixing angles governing solar and atmospheric neutrino oscillations look to be rather large and may even be maximal. The third angle is very small compared to these and at present only its upper limit is known. Similarly, at present there is no consensus about neutrino masses which may show normal/inverted hierarchy or may even be degenerate. The situation becomes further complicated when one realizes that neutrino masses are much smaller than lepton and quark masses.

In the context of quark masses, it may be noted that texture specific mass matrices \cite{5, 6} seem to be very helpful in understanding the pattern of quark mixings and CP violation. This has motivated several attempts \cite{7}, in the flavor as well as the non flavor basis, to consider texture specific lepton mass matrices for explaining the pattern of
neutrino masses and mixings. In the absence of sufficient amount of data regarding
neutrino masses and mixing angles, it would require a very careful scrutiny of all possible
textures to find viable structures which are compatible with data and theoretical ideas so
that these be kept in mind while formulating mass matrices at the GUT (Grand Unified
Theories) scale. In this context, using seesaw mechanism as well as normal hierarchy of
neutrinos, Fukugita, Tanimoto and Yanagida \[8\] have carried out an interesting analysis
of Fritzsch-like texture 6 zero mass matrices \[9\]. It may be noted that when small neutrino
masses are sought to be explained through seesaw mechanism \[10\] given by
\[
M_\nu = -M_{\nu D}^T (M_R)^{-1} M_{\nu D},
\]
(1)

where \(M_{\nu D}\) and \(M_R\) are respectively the Dirac neutrino mass matrix and the right-handed
Majorana neutrino mass matrix, then the predictions are quite different when texture is
imposed on \(M_{\nu D}\) or \(M_\nu\). For the normal hierarchy of neutrino masses, while Fukugita \textit{et al.} \[8\] have imposed texture 6 zero structure on \(M_{\nu D}\), Xing \textit{et al.} \[11\] have considered
several possible texture specific structures for \(M_\nu\). These attempts also use parallel texture
structures for neutrinos and charged lepton mass matrices, compatible with specific models
of GUTs \[12\] as well as these could be obtained using considerations of Abelian family
symmetries \[12\]. In the absence of any clear signals from the data regarding the structure
of mass matrices, it becomes desirable to carry out detailed and exhaustive studies related
to any particular texture of lepton mass matrices.

Using seesaw mechanism and imposing Fritzsch-like texture structure on Dirac neu-
trino mass matrices, with charged leptons having Fritzsch-like texture structure as well
as being in the flavor basis, the purpose of the present communication is to investigate
large number of distinct possibilities of texture 6 zero and 5 zero mass matrices for nor-
mal/inverted hierarchy as well as degenerate scenario of neutrino masses. Further, detailed
dependence of mixing angles on the lightest neutrino mass as well as the parameter space
available to the phases of mass matrices have also been investigated for texture 6 zero as
well as for texture 5 zero cases. Furthermore, several phenomenological quantities such
as Jarlskog’s rephasing invariant parameter \(J\), the CP violating Dirac-like phase \(\delta\) and
the effective neutrino mass \(\langle m_{ee} \rangle\), related to neutrinoless double beta decay \((\beta\beta)_{0\nu}\), have
also been calculated for different cases.

To begin with, we summarize the most recent (August 2006) 3\(\sigma\) values of the neutrino
mass and mixing parameters \[13\],
\[
\Delta m_{12}^2 = (7.1 - 8.9) \times 10^{-5} \text{ eV}^2, \quad \Delta m_{23}^2 = (2.0 - 3.2) \times 10^{-3} \text{ eV}^2,
\]
(2)
\[
\sin^2 \theta_{12} = 0.24 - 0.40, \quad \sin^2 \theta_{23} = 0.34 - 0.68, \quad \sin^2 \theta_{13} \leq 0.040.
\]
(3)

To define the various texture specific cases considered here, we begin with the modified
Fritzsch-like matrices, e.g.,

\[
M_l = \begin{pmatrix}
0 & A_l & 0 \\
A_l^* & D_l & B_l \\
0 & B_l^* & C_l
\end{pmatrix}, \quad M_{\nu D} = \begin{pmatrix}
0 & A_\nu & 0 \\
A_\nu^* & D_\nu & B_\nu \\
0 & B_\nu^* & C_\nu
\end{pmatrix},
\]

(4)

$M_l$ and $M_{\nu D}$ respectively corresponding to Dirac-like charged lepton and neutrino mass matrices. Both the matrices are texture 2 zero type with $A_{l(\nu)} = |A_{l(\nu)}|e^{i\alpha_{l(\nu)}}$ and $B_{l(\nu)} = |B_{l(\nu)}|e^{i\beta_{l(\nu)}}$, in case these are symmetric then $A_{l(\nu)}^*$ and $B_{l(\nu)}^*$ should be replaced by $A_{l(\nu)}$ and $B_{l(\nu)}$, as well as $C_{l(\nu)}$ and $D_{l(\nu)}$ should respectively be defined as $C_{l(\nu)} = |C_{l(\nu)}|e^{i\gamma_{l(\nu)}}$ and $D_{l(\nu)} = |D_{l(\nu)}|e^{i\delta_{l(\nu)}}$. The matrices considered by Fukugita et al. are of symmetric kind and can be obtained from the above mentioned matrices by taking both $D_l$ and $D_\nu$ to be zero, which reduces the matrices $M_l$ and $M_{\nu D}$ to texture 3 zero type. Texture 5 zero matrices can be obtained by taking either $D_l = 0$ and $D_\nu \neq 0$ or $D_\nu = 0$ and $D_l \neq 0$, thereby, giving rise to two possible cases of texture 5 zero matrices, referred to as texture 5 zero $D_l = 0$ case pertaining to $M_l$ texture 3 zero type and $M_{\nu D}$ texture 2 zero type and texture 5 zero $D_\nu = 0$ case pertaining to $M_l$ texture 2 zero type and $M_{\nu D}$ texture 3 zero type.

To fix the notations and conventions as well as to facilitate the understanding of inverted hierarchy case and its relationship to the normal hierarchy case, we detail the essentials of formalism connecting the mass matrix to the neutrino mixing matrix. The mass matrices $M_l$ and $M_{\nu D}$ given in equation (4), for hermitian as well as symmetric case, can be exactly diagonalized, details of hermitian case can be looked up in our earlier work [1], the symmetric case can similarly be worked out. To facilitate diagonalization, the mass matrix $M_k$, where $k = l, \nu D$, can be expressed as

\[
M_k = Q_k M_k^T P_k,
\]

(5)

where $M_k^T$ is a real symmetric matrix with real eigenvalues and $Q_k$ and $P_k$ are diagonal phase matrices, for the hermitian case $Q_k = P_k^{\dagger}$. In general, the real matrix $M_k^\dagger$ is diagonalized by the orthogonal transformation $O_k$, e.g.,

\[
M_k^{\dagger\text{diag}} = (Q_k O_k \xi_k)^\dagger M_k (P_k^{\dagger} O_k),
\]

(6)

wherein, to facilitate the construction of diagonalizing transformations for different hierarchies, we have introduced $\xi_k$ defined as $\text{diag}(1, e^{i\pi}, 1)$ for the case of normal hierarchy and as $\text{diag}(1, e^{i\pi}, e^{i\pi})$ for the case of inverted hierarchy.

The case of leptons is fairly straightforward, whereas in the case of neutrinos, the diagonalizing transformation is hierarchy specific as well as requires some fine tuning of the phases of the right handed neutrino mass matrix $M_R$. To clarify this point further, the matrix $M_\nu$, given in equation (1), can be expressed as

\[
M_\nu = -P_\nu D O_\nu D M_\nu^{\dagger \text{diag}} \xi_{\nu D} O_\nu^T Q_\nu^T (M_R)^{-1} Q_\nu D O_\nu D \xi_{\nu D} M_\nu^{\dagger \text{diag}} O_\nu^T P_\nu D,
\]

(7)
wherein, assuming fine tuning, the phase matrices $Q^T_{\nu D}$ and $Q_{\nu D}$ along with $-M_{R}$ can be taken as $m_{R}\ \text{diag}(1,1,1)$ as well as using the unitarity of $\xi_{\nu D}$ and orthogonality of $O_{\nu D}$, the above equation can be expressed as

$$M_{\nu} = P_{\nu D}O_{\nu D}(M_{\nu D}^{\text{diag}})^2(O_{\nu D}^T P_{\nu D})^{-1}.$$  \hspace{1cm} (8)

The lepton mixing matrix in terms of the matrices used for diagonalizing the mass matrices $M_{l}$ and $M_{\nu}$, which can be obtained respectively from equations (3) and (8), is expressed as

$$U = (Q_{l}O_{l}\xi_{l})^\dagger(P_{\nu D}O_{\nu D}).$$  \hspace{1cm} (9)

Eliminating the phase matrix $\xi_{l}$ by redefinition of the charged lepton phases, the above equation becomes

$$U = O_{l}^\dagger Q_{l}P_{\nu D}O_{\nu D},$$  \hspace{1cm} (10)

where $Q_{l}P_{\nu D}$, without loss of generality, can be taken as $(e^{i\phi_{1}}, 1, e^{i\phi_{2}})$, $\phi_{1}$ and $\phi_{2}$ being related to the phases of mass matrices and can be treated as free parameters.

For making the manuscript self-contained as well as to understand the relationship between diagonalizing transformations for different hierarchies of neutrino masses and for the charged lepton case, we present here the essentials of these transformations. To begin with, we first consider the general diagonalizing transformation $O_{k}$, whose first element can be written as

$$O_{k}(11) = \sqrt{\frac{m_{2}m_{3}(m_{3} + m_{2} - D_{l(\nu)})}{(m_{1} + m_{2} + m_{3} - D_{l(\nu)})(m_{1} - m_{3})(m_{1} - m_{2})}},$$  \hspace{1cm} (11)

where $m_{1}, m_{2}, m_{3}$ are eigenvalues of $M_{k}$. In the case of charged leptons, because of the hierarchy $m_{e} \ll m_{\mu} \ll m_{\tau}$, the mass eigenstates can be approximated respectively to the flavor eigenstates, as has been considered by several authors [8,11]. In this approximation, $m_{l1} \simeq m_{e}, m_{l2} \simeq m_{\mu}$ and $m_{l3} \simeq m_{\tau}$, one can obtain the first element of the matrix $O_{l}$ from the above element, equation (11), by replacing $m_{1}, m_{2}, m_{3}$ by $m_{e}, -m_{\mu}, m_{\tau}$, e.g.,

$$O_{l}(11) = \sqrt{\frac{m_{\mu}m_{\tau}(m_{\tau} - m_{\mu} - D_{l})}{(m_{e} - m_{\mu} + m_{\tau} - D_{l})(m_{\tau} - m_{e})(m_{e} + m_{\mu})}}.$$  \hspace{1cm} (12)

Equation (11) can also be used to obtain the first element of diagonalizing transformation for Majorana neutrinos, assuming normal hierarchy, defined as $m_{\nu_{1}} < m_{\nu_{2}} \ll m_{\nu_{3}}$, and also valid for the degenerate case defined as $m_{\nu_{1}} \lesssim m_{\nu_{2}} \sim m_{\nu_{3}}$, by replacing $m_{1}, m_{2}, m_{3}$ by $\sqrt{m_{\nu_{1}}m_{R}}, -\sqrt{m_{\nu_{2}}m_{R}}, \sqrt{m_{\nu_{3}}m_{R}}$, e.g.,

$$O_{\nu}(11) = \sqrt{\frac{\sqrt{m_{\nu_{2}}}\sqrt{m_{\nu_{1}}}(\sqrt{m_{\nu_{3}}} - \sqrt{m_{\nu_{2}}} - D_{\nu})}{(\sqrt{m_{\nu_{1}}} - \sqrt{m_{\nu_{2}}} + \sqrt{m_{\nu_{3}}} - D_{\nu})(\sqrt{m_{\nu_{3}}} - \sqrt{m_{\nu_{1}}})(\sqrt{m_{\nu_{1}}} + \sqrt{m_{\nu_{2}}})}},$$  \hspace{1cm} (13)
where \( m_{\nu_1}, m_{\nu_2} \) and \( m_{\nu_3} \) are neutrino masses. The parameter \( D_{\nu} \) is to be divided by \( \sqrt{m_R} \), however as \( D_{\nu} \) is arbitrary therefore we retain it as it is.

In the same manner, one can obtain the elements of diagonalizing transformation for the inverted hierarchy case, defined as \( m_{\nu_3} < m_{\nu_1} < m_{\nu_2} \), by replacing \( m_1, m_2, m_3 \) in equation (11) with \( \sqrt{m_{\nu_1} m_R}, -\sqrt{m_{\nu_2} m_R}, -\sqrt{m_{\nu_3} m_R} \), e.g.,

\[
O_{\nu}(11) = \frac{\sqrt{m_{\nu_2}}\sqrt{m_{\nu_3}}(D_{\nu} + \sqrt{m_{\nu_2} + \sqrt{m_{\nu_3}}})}{(-\sqrt{m_{\nu_1}} + \sqrt{m_{\nu_2}} + \sqrt{m_{\nu_3}} + D_{\nu})(\sqrt{m_{\nu_1}} + \sqrt{m_{\nu_3}})(\sqrt{m_{\nu_2}} + \sqrt{m_{\nu_3}})}.
\]

(14)

The other elements of diagonalizing transformations in the case of neutrinos as well as charged leptons can similarly be found.

Assuming neutrinos to be Majorana-like, we have carried out detailed calculations pertaining to texture 6 zero as well as two possible cases of texture 5 zero lepton mass matrices, e.g., \( D_l = 0 \) case and \( D_{\nu} = 0 \) case. Corresponding to each of these cases, we have considered three possibilities of neutrino masses having normal/ inverted hierarchy or being degenerate. In addition to these 9 possibilities, we have also considered those cases when the charged leptons are in the flavor basis. These possibilities sum up to 18, however, the texture 5 zero \( D_{\nu} = 0 \) case with charged leptons in the flavor basis reduces to the similar texture 6 zero case, hence the 18 possibilities reduce to 15 distinct cases.

Before discussing the results, we would like to mention some of the details pertaining to various inputs. The masses and mixing angles, used in the analysis, have been constrained by data given in equations (2) and (3). For the purpose of calculations, we have taken the lightest neutrino mass, the phases \( \phi_1, \phi_2 \) and \( D_{l,\nu} \) as free parameters, the other two masses are constrained by \( \Delta m^2_{12} = m_{\nu_2}^2 - m_{\nu_1}^2 \) and \( \Delta m^2_{23} = m_{\nu_3}^2 - m_{\nu_2}^2 \) in the normal hierarchy case and by \( \Delta m^2_{23} = m_{\nu_2}^2 - m_{\nu_3}^2 \) in the inverted hierarchy case. It may be noted that lightest neutrino mass corresponds to \( m_{\nu_1} \) for the normal hierarchy case and to \( m_{\nu_3} \) for the inverted hierarchy case. In the case of normal hierarchy, the explored range for \( m_{\nu_1} \) is taken to be 0.0001 eV – 1.0 eV, which is essentially governed by the mixing angle \( s_{12} \), related to the ratio \( \frac{m_{\nu_1}}{m_{\nu_2}} \). For the inverted hierarchy case also we have taken the same range for \( m_{\nu_3} \) as our conclusions remain unaffected even if the range is extended further. In the absence of any constraint on the phases, \( \phi_1 \) and \( \phi_2 \) have been given full variation from 0 to 2\( \pi \). Although \( D_{l,\nu} \) are free parameters, however, they have been constrained such that diagonalizing transformations, \( O_l \) and \( O_{\nu} \), always remain real, implying \( D_l < m_{\nu_3} - m_{\nu_1} \) whereas \( D_{\nu} < \sqrt{m_{\nu_3}} - \sqrt{m_{\nu_2}} \) for normal hierarchy and \( D_{\nu} < \sqrt{m_{\nu_1}} - \sqrt{m_{\nu_2}} \) for inverted hierarchy.

Out of all the cases considered here, we first discuss those where \( M_l \) is taken to be texture specific being the most general ones. We begin with the cases pertaining to inverted hierarchy or when neutrino masses are degenerate. Interestingly, we find that all the cases pertaining to inverted hierarchy and degenerate scenario of neutrino masses seem to be ruled out. For the texture 6 zero case, in figure (1), by giving full variations to other parameters, we have plotted the mixing angle \( s_{23} \) against the lightest neutrino mass. The dotted lines and the dot-dashed lines depict the limits obtained assuming normal and inverted hierarchy respectively, the solid horizontal lines show the
3σ limits of $s_{23}$ given in equation (3). It is clear from the figure that inverted hierarchy is ruled out by the experimental limits on $s_{23}$. We arrive at similar conclusions in case we plot the corresponding figures for $s_{12}$ and $s_{13}$. Also from this figure, one can easily check that degenerate scenario characterized by either $m_{\nu_1} \lesssim m_{\nu_2} \sim m_{\nu_3} \sim 0.1$ eV or $m_{\nu_3} \sim m_{\nu_1} \lesssim m_{\nu_2} \sim 0.1$ eV is clearly ruled out.

For texture 5 zero cases, we first discuss the case when $D_l = 0$ and $D_\nu \neq 0$. Primarily to facilitate comparison with texture 6 zero case, in figure (2) we have plotted $s_{23}$ against the lightest neutrino mass for both normal and inverted hierarchy for a particular value of $D_\nu = \sqrt{m_{\nu_3}}$. Interestingly, we find texture 5 zero $D_l = 0$ case shows a big change in the behaviour of $s_{23}$ versus the lightest neutrino mass as compared to the texture 6 zero case shown in figure (1). A closer look at figure (2) reveals that the region pertaining to inverted hierarchy, depicted by dot-dashed lines, shows an overlap with the experimental limits on $s_{23}$, depicted by solid horizontal lines, around the region when neutrino masses are almost degenerate. This suggests that in case the degenerate scenario is ruled out inverted hierarchy is also ruled out. To this end as well as for extending our results to other allowed values of $D_\nu$, in figure (3) we have plotted allowed parameter space for the three mixing angles in the $D_\nu$–lightest neutrino mass plane, for texture 5 zero $D_l = 0$ case. A closer look at the figure shows that the allowed parameter spaces of the three mixing angles show an overlap when $D_\nu \sim 0$, which leads to the present texture 6 zero case, wherein degenerate scenario has already been ruled out. The above analysis from figure (3) clearly indicates that inverted hierarchy as well as degenerate scenario is ruled out for texture 5 zero $D_l = 0$ case, not only for $D_\nu = \sqrt{m_{\nu_3}}$ but also for its other acceptable values. Coming to the texture 5 zero $D_l = 0$ and $D_\nu \neq 0$ case, a plot of $s_{23}$ against the lightest neutrino mass is very similar to figure (1) pertaining to the texture 6 zero case, therefore we have not presented it here. By similar arguments, this case is also ruled out for inverted hierarchy as well as for degenerate scenario.

Interestingly, we find that even if we give wider variations to all the parameters, all possible cases considered here pertaining to inverted hierarchy and degenerate scenario are ruled out. It may also be added that in the case when charged leptons are in the flavor basis, the mixing matrix becomes much more simplified and one can easily check that cases pertaining to inverted hierarchy as well as degenerate scenario for the texture 6 zero and 5 zero mass matrices are ruled out. Further, for the sake of completion, we have also investigated the cases when $M_\nu$ is texture specific or neutrinos are Dirac-like and find that inverted hierarchy and degenerate scenario are again ruled out, details regarding these have been not included here.

After ruling out the cases pertaining to inverted hierarchy and degenerate scenario, we now discuss the normal hierarchy cases. For texture 6 zero as well as two cases of texture 5 zero mass matrices, in table (1) we have presented the viable ranges of neutrino masses, mixing angle $s_{13}$, Jarlskog’s rephasing invariant parameter $J$, CP violating phase $\delta$ and effective neutrino mass $\langle m_{ee} \rangle$ related to neutrinoless double beta decay $(\beta\beta)_0$. The parameter $J$ can be calculated by using its expression given in [8], whereas $\delta$ can be determined from $J = s_{12} s_{23} s_{13} c_{12} c_{23} c_{13}^2 \sin \delta$ where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$, for $i, j = 1, 2, 3$. The effective Majorana mass, measured in $(\beta\beta)_0$ decay experiment, is
Given as

\[ \langle m_{ee} \rangle = m_{\nu_1} U_{e1}^2 + m_{\nu_2} U_{e2}^2 + m_{\nu_3} U_{e3}^2. \] (15)

Considering first the texture 6 zero case, the possibility when charged leptons are in flavor basis is completely ruled out, therefore the results presented in table (I) correspond to the case when \( M_l \) is considered texture specific. As can be checked from table (I), the presently calculated values of parameters \( m_{\nu_1}, s_{13}, J \) and \( \langle m_{ee} \rangle \), found by using the latest data, are well within the ranges obtained by Fukugita et al., which are given as \( m_{\nu_1} = 0.0004 - 0.0030, s_{13} = 0.04 - 0.20, J \leq 0.025 \) and \( \langle m_{ee} \rangle = 0.002 - 0.007 \). Also, from the table, one finds the lower limit on \( s_{13} = 0.066 \), therefore a measurement of \( s_{13} \) would have implications for this case. Similarly, a measurement of effective mass \( \langle m_{ee} \rangle \), through the \((\beta\beta)_{0\nu}\) decay experiments, would also have implications for these kind of mass matrices. Besides the above mentioned parameters, we have also considered the implications of \( s_{13} \) on the phases \( \phi_1 \) and \( \phi_2 \). To this end, in figure (4) we have drawn the contours for \( s_{13} \) in \( \phi_1 - \phi_2 \) plane. From the figure it is clear that \( s_{13} \) plays an important role in constraining the phases, in particular, we find that if lower limit of \( s_{13} \) is on the higher side, then \( \phi_1 \) is restricted to I or IV quadrant.

Coming to the texture 5 zero cases, to begin with we consider the \( D_l = 0 \) case. Interestingly, results are obtained for both the possibilities of \( M_l \) having Fritzsch-like structure as well as \( M_l \) being in the flavor basis. When \( M_l \) is assumed to have Fritzsch-like structure, one would like to emphasize a few points. A general look at the table reveals that the possibility of \( D_\nu \neq 0 \) considerably affects the viable range of \( m_{\nu_1} \), particularly its lower limit. Similarly, the lower limit of \( s_{13} \) is pushed higher. This can be easily understood by noting that \( s_{13} \) is more sensitive to variations in \( D_\nu \) than variations in \( D_l \). Further, the lower limit of \( s_{13} \) is pushed higher as the upper limit of \( m_{\nu_1} \) now becomes somewhat lower as compared to the 6 zero case. Also, it may be of interest to construct the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix [14] which we find as

\[ U = \begin{pmatrix} 0.7898 - 0.8571 & 0.5035 - 0.5971 & 0.0761 - 0.1600 \\ 0.1845 - 0.4413 & 0.5349 - 0.7459 & 0.5725 - 0.8135 \\ 0.3546 - 0.5615 & 0.3926 - 0.6689 & 0.5652 - 0.8107 \end{pmatrix}. \] (16)

When \( M_l \) is considered in the flavor basis, we get a very narrow range of masses, \( m_{\nu_1} \sim 0.00063, m_{\nu_2} = 0.0086 - 0.0088 \) and \( m_{\nu_3} = 0.0534 - 0.0546 \), for which 5 zero matrices are viable. Also for this case, \( s_{13} \) is almost near its upper experimental limit, therefore, lowering down of \( s_{13} \) value would almost rule out this case.

Considering the texture 5 zero \( D_\nu = 0 \) case, we note that when \( M_l \) is considered in the flavor basis, we do not find any viable solution, however when it has Fritzsch-like structure there are a few important observations. The range of \( m_{\nu_1} \) gets extended as compared to the 6 zero case, whereas compared to the texture 5 zero \( D_l = 0 \) case, both the lower and upper limits of \( m_{\nu_1} \) have higher values. Interestingly, this case has the widest \( s_{13} \) range among all the cases considered here. The PMNS matrix corresponding to this case does not show any major variation compared to the earlier case, except that the ranges of some of the elements like \( U_{\mu 1}, U_{\mu 2}, U_{\tau 1} \) and \( U_{\tau 2} \) become little wider. This can be understood
when one realizes that $D_l$ can take much wider variation compared to $D_\nu$.

A general look at the table reveals several interesting points. It immediately brings out the fact that the value of $\langle m_{ee} \rangle$, a measure of $(\beta \beta)_{0\nu}$ decay, has more or less the same range for all the cases. This can be understood through equation (15) from which one finds that the major contribution to $\langle m_{ee} \rangle$ is given by the term proportional to $m_{\nu_2}$ as the first term gets suppressed by the small value of $m_{\nu_1}$ whereas the third term gets suppressed by the small value of $U_{e3}^2$. Also, it must be noted that the calculated values of $\langle m_{ee} \rangle$ are much less compared to the present limits of $\langle m_{ee} \rangle$ [14], therefore, these do not have any implications for texture 6 zero and texture 5 zero cases. However, the future experiments with considerably higher sensitivities, aiming to measure $\langle m_{ee} \rangle \approx 3.6 \times 10^{-2}$ eV (MOON [16]) and $\langle m_{ee} \rangle \approx 2.7 \times 10^{-2}$ eV (CUORE [17]), would have implications on the cases considered here.

In the absence of any definite information about $J$ as well as $\delta$, we find that the ranges corresponding to different cases are in agreement with other similar calculations, however, it is interesting to note that the ranges of $J$ and $\delta$ for the texture 5 zero $D_\nu = 0$ case are much wider than the other two cases. This, perhaps, is not due to any single factor, rather it is due to almost equal contribution of several terms in the case of Majorana neutrinos.

To summarize, using seesaw mechanism and Fritzsch-like texture 6 zero and 5 zero lepton Dirac mass matrices, detailed predictions for 15 distinct possible cases pertaining to normal/inverted hierarchy as well as degenerate scenario of neutrino masses have been carried out. Interestingly, all the presently considered cases pertaining to inverted hierarchy and degenerate scenario seem to be ruled out. Further, inverted hierarchy and degenerate scenario are also ruled out when $M_l$ and $M_\nu$ have Fritzsch-like textures.

In the normal hierarchy cases, when the charged lepton mass matrix $M_l$ is assumed to be in flavor basis, the texture 6 zero and the texture 5 zero $D_\nu = 0$ case are again ruled out. For the viable texture 6 zero and 5 zero cases, we find the lower limits of $m_{\nu_1}$ and $s_{13}$ would have implications for the texture specific cases considered here. Interestingly, the lower limits of $s_{13}$ for the texture 5 zero $D_l = 0$ and $D_\nu = 0$ cases show an appreciable difference. Further, the phase $\phi_1$ seems to have strong dependence on the $s_{13}$ value for texture 6 zero as well as texture 5 zero mass matrices. Similarly, the Dirac-like CP violating phase $\delta$ shows very interesting behaviour, e.g., the texture 6 zero case and the texture 5 zero $D_l = 0$ case allow the range $0^\circ - 50^\circ$ whereas, the texture 5 zero $D_\nu = 0$ case allows comparatively a larger range $0^\circ - 90^\circ$. The restricted range of $\delta$, in spite of full variation to phases $\phi_1$ and $\phi_2$, seems to be due to texture structure, hence, any information about $\delta$ would have important implications. The different cases of texture 6 zero and texture 5 zero matrices do not show any divergence for the value of effective mass $\langle m_{ee} \rangle$.

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Table 1: Calculated ranges for neutrino mass and mixing parameters obtained by varying $\phi_1$ and $\phi_2$ from 0 to $2\pi$ for the normal hierarchy case. Inputs have been defined in the text. All masses are in eV.

![Table 1](image1)

Figure 1: Plots showing variation of mixing angle $s_{23}$ with lightest neutrino mass for texture 6 zero case. The dotted lines and the dot-dashed lines depict the limits obtained assuming normal and inverted hierarchy respectively, the solid horizontal lines show the $3\sigma$ limits of $s_{23}$ given in equation (3).
Figure 2: Plots showing variation of mixing angle $s_{23}$ with lightest neutrino mass for texture 5 zero $D_l = 0$ case, with a value $D_\nu = \sqrt{m_{\nu_3}}$. The representations of the curves remain the same as in figure (I).

Figure 3: Plots showing allowed parameter space for the three mixing angles in the $D_\nu$–lightest neutrino mass plane, for texture 5 zero $D_l = 0$ case for the inverted hierarchy, with $D_\nu$ being varied from 0 to a value such that $D_\nu < \sqrt{m_{\nu_1}} - \sqrt{m_{\nu_3}}$. Dotted lines depict allowed parameter space for $s_{12}$, dot-dashed lines depict allowed parameter space for $s_{23}$ and solid lines depict allowed parameter space for $s_{13}$. 
Figure 4: The contours of $s_{13}$ in $\phi_1 - \phi_2$ plane for 6 zero matrices for the normal hierarchy case.