INFLUENCE OF THE SOMMERFELD CORRECTIONS TO THE ELECTRIC POLARIZABILITY OF THE PION

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Abstract

The valence-quark contribution to the electric polarizability of the charged pion in a semirelativistic description is shown to be smaller than its nonrelativistic limit.

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1 Electric Polarizability: Probe of the Pion’s Internal Structure

Exposing a physical system composed of electrically charged constituents to an external electric field \( E \) induces the electric dipole moment

\[ d = 4\pi \kappa E; \]

the constant of proportionality herein, the electric polarizability \( \kappa \), measures the system’s rigidity under deformation and represents a clue to the internal structure of composite systems. Chiral perturbation theory, the low-energy limit of quantum chromodynamics, makes a precise unambiguous prediction for the electric polarizability \( \kappa_{\pi^\pm} \) of the charged pion \( \pi^\pm \). Consequently, \( \kappa_{\pi^\pm} \) forms a crucial test of quantum chromodynamics.\(^2\) Experiment studies \( \kappa_{\pi^\pm} \) by reactions based on Compton scattering

\[ \gamma + \pi^\pm \rightarrow \gamma + \pi^\pm \]

or crossed processes. Our weighted average of the few existing experimental measurements of the electric polarizability of the charged pion reads\(^2\)

\[ \kappa_{\pi^\pm}^{\text{exp}} = (4.3 \pm 1.2) \times 10^{-4} \text{ fm}^3. \]

The mean value of the theoretical predictions of all quantum-field-theoretic descriptions of the mesons, for the electric polarizability of the charged pion,

\[ \kappa_{\pi^\pm}^{\text{QFT}} = 5.5 \times 2.3^{+1}_{-1} \times 10^{-4} \text{ fm}^3, \]

is in reasonable agreement with experiment\(^2\).

2 Electric Polarizabilities of Mesons within Quark Potential Models

Quark potential models of mesons are, in principle, only capable to describe the contributions of the valence quarks to the meson’s electric polarizability. In a naïve static nonrelativistic potential model this quark-core contribution accounts for only a fraction 1/80 of the charged-pion electric polarizability\(^3\).

\(^1\)Using the Heaviside–Lorentz system of units for electromagnetic quantities, the unit of electric charge, \( e \), is related to the electromagnetic fine structure constant \( \alpha \) by

\[ \alpha \equiv e^2/4\pi \simeq 1/137. \]

\(^2\)The implications of chiral symmetry for the pion’s electromagnetic polarizabilities are reviewed in Ref. [1].
We examine the effects of kinematics by relaxing the tight nonrelativistic bound towards more reliable semirelativistic treatments of mesons viewed as constituent quark-antiquark bound states, with a Hamiltonian

\[ H = H_0 + W, \]

where the bound state is described by the unperturbed Hamiltonian \( H_0 \), its interaction with the external electric field \( E \) is encoded in a perturbation \( W \). For equal-mass bound-state constituents with electric charges \( q_1, q_2 \) located at \( r_1, r_2 \), this residual dipole interaction reads

\[ W = \frac{1}{2} (q_2 - q_1) e (r_1 - r_2) \cdot E. \]

The semirelativistic Hamiltonian \( H_0 \) governing the dynamics of two quarks with mass \( m \), relative momentum \( p \), and relative distance \( r \) reads, in their center-of-momentum frame,

\[ H_0 = 2 \sqrt{p^2 + m^2} + V(r). \]

The strong interactions between the quarks are described by a static central potential \( V(r) \). We analyze the change of the quark-model prediction for the charged-pion electric polarizability brought about when taking into account the Sommerfeld corrections

\[ \Delta E = -\frac{1}{2} d \cdot E = -\frac{\langle \phi | W^2 | \phi \rangle^2}{\langle \phi | [W, H_0] W | \phi \rangle}. \]

Assuming, w.l.o.g., \( E \) to be parallel to the \( z \)-axis of the employed coordinate frame allows to read off the electric polarizability \( \kappa \) of a meson consisting of equal-mass quarks from the contribution of order \( E^2 \) to \( \Delta E \) (for details, and the generalization to the case of quarks of unequal masses, consult Ref. [2]):

\[ \kappa = \frac{\alpha}{18} (q_2 - q_1)^2 \frac{\langle \phi | r^2 | \phi \rangle^2}{\langle \phi | [z, H_0] z | \phi \rangle}. \]

Assuming the ground state of the meson under study to be described by a real wave function \( \phi(p) \), \( p \equiv |p| \), the electric polarizability \( \kappa \) of this meson is thus determined by the magnitude of the expectation values \( \langle \phi | r^2 | \phi \rangle \) and [2]

\[ \langle \phi | [z, H_0] z | \phi \rangle = \frac{1}{3} \int d^3 p \frac{2 p^2 + 3 m^2}{(p^2 + m^2)^{3/2}} \phi^2(p) \leq \langle \phi | [z, H_0] z | \phi \rangle_{NR} = \frac{1}{18} \frac{1}{m}. \]

As indicated, the latter is bounded from above by its nonrelativistic limit.
3 Representative Results, Brief Discussion and Conclusion

The lowest-energy eigenfunction $\phi(p)$ of the semirelativistic Hamiltonian $H_0$ is obtained variationally with a 25-dimensional trial space spanned by basis functions involving generalized Laguerre polynomials [4]. Table 1 compares for some typical interquark potentials $V(r)$ the electric polarizability $\kappa_{\pi^\pm}$ of the charged pion computed from relativistic and nonrelativistic kinematics.

Table 1: Ratio of the electric polarizability of the charged pion $\pi^\pm$ in various nonrelativistic ($\kappa_{\text{NR}}$) and relativistic ($\kappa$) potential models, for the canonical light-quark constituent mass $m = 0.336$ GeV and “reasonable” interquark potential parameters (cf. Ref. [5]).

| potential  | harmonic oscillator | Coulomb | linear   | funnel      |
|------------|---------------------|---------|----------|-------------|
| $V(r)$     | $0.5r^2$            | $-\frac{0.413}{r}$ | $0.15r$  | $-\frac{0.413}{r} + 0.15r$ |
| $\kappa_{\text{NR}}/\kappa$ | 1.02 | 1.08 | 1.2 | 1.36 |
| $|\delta|$ | $3 \times 10^{-3}$ | $2 \times 10^{-4}$ | $6 \times 10^{-10}$ | $6 \times 10^{-5}$ |

The accuracy of approximate eigenstates $|\varphi\rangle$ of Hamiltonians

$$H = T + V$$

involving kinetic terms $T(p)$ and potentials $V(x)$ is quantified, according to a criterion based on the relativistic virial theorem [6], by the nonzero value of [7]

$$\delta \equiv 1 - \frac{\langle \varphi | x \cdot \frac{\partial}{\partial x} V(x) | \varphi \rangle}{\langle \varphi | p \cdot \frac{\partial}{\partial p} T(p) | \varphi \rangle}.$$  

The very tiny values of $|\delta|$ in Table 1 give us great confidence in our results.

From Table 1, we conclude that the Sommerfeld corrections diminish the quark-core contribution to the pion electric polarizability. Similar findings [2] are expected when including also relativistic corrections to the interaction [8].
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