Quantum entanglement
at all distances

ICTP, Trieste
June 14, 2022

Subir Sachdev

Talk online: sachdev.physics.harvard.edu
Quantum statistical mechanics of black holes and strange metals

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Foundations by Boltzmann
Statistical interpretation of entropy (1870)

\[ S = k_B \log W \]

Density of quantum states \( D(E) = \exp(S(E)/k_B) \)

Ludwig Boltzmann
20 February 1844, Vienna - September 5, 1906, Trieste
Quantum Entanglement
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Einstein, Podolsky, Rosen (1935)
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Measurement of one electron instantaneously determines the state of the other electron very far away.
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Quantum Entanglement

Einstein, Podolsky, Rosen (1935)

Measurement of one electron instantaneously determines the state of the other electron very far away.

Spooky action at a distance!
I cannot seriously believe in it because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at a distance.

Albert Einstein to Max Born, 3 March 1947
Quantum black holes and holography
Black Holes

Objects so dense that light is gravitationally bound to them.

Horizon radius \( R = \frac{2GM}{c^2} \)

\( G \) Newton’s constant, \( c \) velocity of light, \( M \) mass of black hole

For \( M = \) earth’s mass, \( R \approx 9 \text{ mm}! \)
Quantum Entanglement across a black hole horizon
Quantum Entanglement across a black hole horizon

Black hole horizon
Quantum Entanglement across a black hole horizon
Quantum Entanglement across a black hole horizon

Hawking (1975) used other arguments to show that black hole horizons have a temperature. (The entanglement reasoning: to an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.)
From Hawking, we learn that...

- Black holes have an entropy and a temperature:
  \[ T = \frac{\hbar c^3}{8\pi GMk_B} \]

- The entropy is proportional to their surface area:
  \[ S = \frac{Ak_Bc^3}{4G\hbar} \]
We need one more important fact…

- Black holes have an entropy and a temperature
  \[ T = \frac{\hbar c^3}{8\pi G M k_B} \]

- The entropy is proportional to their surface area
  \[ S = \frac{A k_B c^3}{4G\hbar} \]

- They reach thermal equilibrium at the fastest possible rate
  \[ \sim \frac{\hbar}{k_B T} \]
Black Holes Obey Information-Emission Limits

G. Carullo, D. Laghi, J. Veitch, W. Del Pozzo, Phys. Rev. Lett. 126, 161102 (2021)

An analysis of the gravitational waves emitted from black hole mergers confirms that black holes are the fastest known information dissipaters.

Gravity wave observations of 8 different black holes show a relaxation time

$$\tau \sim \frac{8\pi GM}{c^3} = \frac{\hbar}{k_B T}$$

$$H = \frac{1}{\pi} \frac{\hbar/\tau}{k_B T}$$
One more important fact...

- Black holes have an entropy and a temperature \( T = \frac{\hbar c^3}{8\pi GM k_B} \)

- The entropy is proportional to their surface area \( S = \frac{A k_B c^3}{4G \hbar} \)

- They reach thermal equilibrium at the fastest possible rate \( \sim \frac{\hbar}{k_B T} \)!
A quantum computer simulating a black hole must have:

- Number of ‘qubits’ proportional to the surface area
- Maximal, long-range quantum entanglement between the qubits

Spooky action at a distance!
Questions and Answers

Can we find a quantum simulation of a black hole whose $D(E)$ matches the Bekenstein-Hawking entropy?
Questions and Answers

Can we find a quantum simulation of a black hole whose $D(E)$ matches the Bekenstein-Hawking entropy? Yes, for charged black holes:

\[
D(E) \approx \exp \left( A_0 c^3 / 4 \right) \sim G \cosh \left( \pi \frac{A_3}{2} / c^2 \right) \sim 2 G \sqrt{1 / 2 A}
\]

where $A_0$ is the horizon area at $T = 0$. There is no degeneracy, but an exponentially small level spacing down to the ground state.

In more recent work, the SYK quantum simulation has also consistently described the evolution of the entropy for a black hole past the Page time.
Questions and Answers

Can we find a quantum simulation of a black hole whose $D(E)$ matches the Bekenstein-Hawking entropy? Yes, for charged black holes:

- With sufficient low energy supersymmetry, string theory yields:

  \[ D(E) = \exp \left( \frac{Ac^3}{4\hbar G} \right) \delta(E) \]
  
  \[ + \theta(E - \Delta) f(E - \Delta) + \ldots \]

There are exponentially many degenerate BPS ground states, and an energy gap $\Delta$ above the ground state.
Can we find a quantum simulation of a black hole whose $D(E)$ matches the Bekenstein-Hawking entropy? Yes, for charged black holes:

- For generic black holes in 3+1 dimensions, the SYK model yields:

$$D(E) \sim \exp \left( \frac{A_0 c^3}{4\hbar G} \right) \sinh \left( \left[ \frac{\sqrt{\pi} A_0^{3/2} c^2}{\hbar^2 G} E \right]^{1/2} \right)$$

where $A_0$ is the horizon area at $T = 0$. There is no degeneracy, but an exponentially small level spacing down to the ground state.
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Statistical interpretation of entropy (1870)

\[ S = k_B \log W \]

Density of quantum states 
\[ D(E) = \exp\left(\frac{S(E)}{k_B}\right) \]

Ludwig Boltzmann
20 February 1844, Vienna - September 5, 1906, Trieste
Boltzmann equation (1872)
Dilute classical gas

Molecular chaos: successive collisions are statistically independent

\[
\frac{\partial f_p}{\partial t} + \mathbf{F} \cdot \nabla_p f_p = C[f]
\]

\[
C[f] \propto \int_{p_1,2,3} \cdots [f_{p_1} f_{p_2} f_{p_3}]
\]

Ludwig Boltzmann
20 February 1844, Vienna -
September 5, 1906, Trieste
Quantum Boltzmann equation (Landau)
Dense gas of electrons

Neglects quantum interference (entanglement) between successive collisions

\[ \frac{\partial f_p}{\partial t} + F \cdot \nabla_p f_p = C[f] \]

\[ C[f] \propto \int_{p_1,2,3} \cdots [f_p f_{p_1} (1 - f_{p_2})(1 - f_{p_3}) \]
\[ - f_{p_2} f_{p_3} (1 - f_p)(1 - f_{p_1})] \]

Ludwig Boltzmann
20 February 1844, Vienna -
September 5, 1906, Trieste
Quantum theory of electrons: ordinary metals and strange metals
Each copper atom donates its outermost electron. These electrons move freely throughout the crystal and carry current.
Flow of electrons described by Boltzmann equation $\Rightarrow$
typical scattering time $\tau \sim 1/T^2$, resistivity $\rho(T) = \rho(0) + AT^2$

The time $\tau$ is much longer than a limiting ‘Planckian time’ $\frac{\hbar}{k_B T}$.

The long scattering time implies that individual electrons are well-defined.

The motion of electrons is ‘ballistic’ or ‘integrable’ up to the long time $\tau$, after which it is chaotic.
Strange Metal

Temperature (K)

AF insulator

Superconductor

Hole doping, $p$

$p_c$

$T^*$

$T_N$

$T_c$

Cu$^{2+}$, Cu$^{3+}$

O$^{2-}$

Y$^{3+}$

Ba$^{2+}$

3.8227 Å

11.6802 Å
FIG. 2 Measurement of the diffusion constant (a) and compressibility ((a)-inset) for a gas of ultra-cold 6Li atoms in an optical lattice, realizing a two-dimensional Fermi-Hubbard model with $U/t' > 7.5$ at a density $n_0 = 0.825$. (b) Reconstructed 'resistivity' using Einstein-Sutherland relation. Grey horizontal dashed line represents the estimated MIR value. Theoretical calculations using DMFT (in green) and the finite-$T$ Lanczos method (in blue) are shown; the band representation indicates estimated error bars. Adapted from (Brown et al., 2019).

\[ A = \frac{d\rho}{dT} \]

FIG. 3 Examples of temperature linear resistivity extending over a wide range of temperature scales in (a) hole-doped La$_{2-x}$Sr$_x$CuO$_4$ (LSCO) near optimal doping (adapted from (Giraldo-Gallo et al., 2018)), and (b) magic-angle twisted bilayer graphene (MATBG) near $\nu = \pi$, relative to charge neutrality, $\nu = 0$ (adapted from (Jaoui et al., 2021)). In LSCO, $T_{coh}$ can be inferred to be much lower than any characteristic energy scales by turning on a magnetic field and accounting for the finite magnetoresistance ((a)-top inset); the variation of the slope (A) on hole-doping is shown in (a)-bottom inset. In MATBG, the linearity for a range of dopings near $\nu = \pi$ ((b)-inset) persists down to $\sim 40$ mK. Both family of materials also display a Planckian form of $\rho$ associated with intermediate energy scales (and consistent with ARPES and ADMR) is used, rather than the $LSCO$: Giraldo-Gallo et al. 2018

MATBG: Jaoui et al. 2021
Linear-in temperature resistivity from an isotropic Planckian scattering rate

G. Grissonnanche, Y. Fang, A. Legros, S. Verret, F. Laliberté, C. Collignon, J. Zhou, D. Graf, P. Goddard, L. Taillefer, B. J. Ramshaw

Current flow without quasiparticles

\[
\hbar/\tau = \alpha k_B T
\]

\[
\alpha = 1.2 \pm 0.4
\]
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\[ \frac{\hbar}{\tau} = \alpha k_B T \]

\[ \alpha = 1.2 \pm 0.4 \]

Current flow without quasiparticles
Questions

• Needed: A theory for current flow in a ‘strange metal’ with an entangled soup of electrons.

• Needed: theory for collision time in resistivity $\sim \frac{\hbar}{(k_B T)}$.

• Needed: theory for the appearance of superconductivity in such a ‘strange metal’.
Sachdev-Ye-Kitaev Model
A solvable model of multi-particle entanglement which accounts for quantum interference between successive collisions:

leading to a metal with no particle-like excitations
August Kekule, theory of the benzene molecule, 1865
Kekule’s spooky dream

Here Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail.
Kekule’s spooky dream

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Here Kekulé spoke of the creation of the theory. He said that he had discovered the ring shape of the benzene molecule after having a reverie or day-dream of a snake seizing its own tail.*

![Diagram of benzene molecule]

\[
\begin{align*}
\text{ouroboros} & = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle
\end{align*}
\]

*Wikipedia
By Krutika Haraniya

SNAPSHOT HISTORIES

You May bring alive the many stories that make India and get our readers access to the best Live History India is a first of its kind digital platform aimed at helping you Rediscover India all the way from Boston, USA.

In the Footsteps of Ram

The rest of the board was covered with pictures of animals, flowers and people. Cosmological elements, with upper regions depicting divine beings and the heavens. Interestingly, the reason the game pivoted around pure luck was because it was in keeping with the Jain philosophical notion – emphasizing the ideas of fate and destiny.

Interestingly, the reason the game pivoted around pure luck was because it was in keeping with the Jain philosophical notion – emphasizing the ideas of fate and destiny. This was in contrast to other ancient games such as Chaturanga, Leela, and Gyan Chaupar that had to do with religious philosophies? In the original, it served as a lesson in morality. Playing this game wasn't just about winning or losing, but finding out how close you were, to liberation from the bondage of passions. The game was symbolic of a man's journey in the path of good is much more difficult to tread, than a path of sins.

The ladders represented virtues while the snakes represented vices. The ladder was called the parampada soppa, which means an eagle's nest on the side of a cliff, indicating a lofty place. The last square represented either a God or heaven meaning you have attained liberation. In the game, the ladders represented virtues such as faith, generosity, humility and wisdom. For example, the last square represented either a God or heaven meaning you have attained liberation. It is believed to have been invented by Jain monks to promote the concept of liberation.

The Snakes and Ladders board game today, there wasn't any standardization then. The most variations – a Hindu and a very rare, Sufi Muslim version. The Pahari style of the game could run up to 360 squares. Meanwhile, there also developed other 'philosophical' games. Ancient Ladders and Snakes is one of those. It is believed to have been invented by Jain monks to promote the concept of liberation.

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The history of Snakes & Ladders goes back around 1000 years to 10th century CE where it was probably created by Jain monks to promote the concept of liberation from the bondage of passions. The game was symbolic of a man's journey in the path of good is much more difficult to tread, than a path of sins.

In 1943, it was rebranded as Chutes and Ladders which taught kids about good and bad deeds.

My spooky dream*

*Not true
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

Pick a set of random positions
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

Place electrons randomly on some sites
The SYK model

Place electrons randomly on some sites

$U_{11,12;5,14}$
The SYK model

Entangle electrons pairwise randomly

$U_{11,12;5,14}$

Sachdev, Ye (1993); Kitaev (2015)
The SYK model

Entangle electrons pairwise randomly

Sachdev, Ye (1993); Kitaev (2015)
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

Entangle electrons pairwise randomly

$U_{4,5;11,18}$

Entangle electrons pairwise randomly
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$U_{14, 19; 1, 13}$

Entangle electrons pairwise randomly
The SYK model

Entangle electrons pairwise randomly

$U_{14,19;1,13}$

Sachdev, Ye (1993); Kitaev (2015)
The SYK model

$U_{9,18;5,15}$

Entangle electrons pairwise randomly

Sachdev, Ye (1993); Kitaev (2015)
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

$U_{9,18;5,15}$

Entangle electrons pairwise randomly
The SYK model

Sachdev, Ye (1993); Kitaev (2015)

Entangle electrons pairwise randomly

$U_{6,8;4,14}$
The SYK model

Entangle electrons pairwise randomly

$U_{6,8;4,14}$

Sachdev, Ye (1993); Kitaev (2015)
The Sachdev-Ye-Kitaev (SYK) model

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large $N$ limit; T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. 53, 385 (1981))

$$H = \frac{1}{(2N)^3/2} \sum_{\alpha, \beta, \gamma, \delta=1}^{N} U_{\alpha \beta; \gamma \delta} c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta - \mu \sum_{\alpha} c_\alpha^\dagger c_\alpha$$

$$c_\alpha c_\beta + c_\beta c_\alpha = 0, \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha \beta}$$

$$Q = \frac{1}{N} \sum_{\alpha} c_\alpha^\dagger c_\alpha$$

$U_{\alpha \beta; \gamma \delta}$ are independent random variables with $\overline{U_{\alpha \beta; \gamma \delta}} = 0$ and $|U_{\alpha \beta; \gamma \delta}|^2 = U^2$

$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL 70, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX 5, 041025 (2015)
Complex SYK model

Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations in the large $N$ limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$

$$G(\tau = 0^-) = Q.$$  

Conformal solution at $\mu = 0$, $G(\tau) \sim \frac{\text{sgn}(\tau)}{\sqrt{|\tau|}}$.

S. Sachdev and J. Ye,
PRL 70, 3339 (1993)
The SYK model

- Planckian time dynamics without quasiparticles with a relaxation time $\sim \hbar/(k_B T)$ when $k_B T \ll U$.

---

D. Chowdhury, A. Georges, O. Parcollet, S. Sachdev, arXiv: 2109.05037, review article
The SYK model

- Planckian time dynamics without quasiparticles with a relaxation time \( \sim \hbar/(k_B T) \) when \( k_B T \ll U \).

- There is an extensive entropy as \( T \to 0 \) (\( \lim_{T \to 0} \lim_{N \to \infty} S/N \neq 0 \)); however, the ground state is not extensively degenerate.
The SYK model

\[ D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue} \]

Many-body density of states
The SYK model

\[ D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue} \]

\[ D(E) \sim e^{S(E)} \]

\[ = e^{Ns_0 + \sqrt{2N\gamma E}} \]

\[ S(T \to 0) = N(s_0 + \gamma T) \]

Many-body density of states
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\[ s_0 = 0.464848 \ldots \]

A. Georges, O. Parcollet, and S. Sachdev,
PRB 63, 134406 (2001)

Many-body density of states
The SYK model

\[ D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue} \]

\[ D(E) \sim e^{S(E)} = e^{Ns_0 + \sqrt{2N\gamma E}} \]

\[ S(T \to 0) = N(s_0 + \gamma T) \]

No particle-like decomposition: wavefunctions change chaotically from one state to the next.

\[ D(E) \sim 2e^{Ns_0} \sqrt{2N\gamma E} \]

\[ s_0 = 0.464848 \ldots \]

A. Georges, O. Parcollet, and S. Sachdev, PRB 63, 134406 (2001)

Maldacena, Stanford (2016); Bagrets et al. (2017); Cotler et al. (2017); Kitaev, Suh (2017); Stanford, Witten (2017)
The SYK model

- Planckian time dynamics without quasiparticles with a relaxation time $\sim \hbar/(k_B T)$ when $k_B T \ll U$.

- There is an extensive entropy as $T \to 0$ ($\lim_{T \to 0} \lim_{N \to \infty} S/N \neq 0$); however, the ground state is not extensively degenerate.
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- Planckian time dynamics without quasiparticles with a relaxation time $\sim \hbar/(k_B T)$ when $k_B T \ll U$.

- There is an extensive entropy as $T \to 0$ ($\lim_{T \to 0} \lim_{N \to \infty} S/N \neq 0$); however, the ground state is not extensively degenerate.

- The $D(E)$ is determined by a time-reparameterization $\tau \to f(\tau)$ mode (similar to the graviton being fluctuations of the spacetime metric), and a phase mode $\phi(\tau)$:

\[
Z_{SYK} = e^{Ns_0} \int \mathcal{D}f \mathcal{D}\phi \exp \left( -\frac{1}{\hbar} \int_0^{\hbar/(k_B T)} d\tau \mathcal{L}_{SYK}[f, \phi] \right)
\]
From the SYK model to charged black holes
Bohr-Sommerfeld semiclassical quantum theory of a black hole in $d$ spatial dimensions

$$Z = \int \mathcal{D}g_{\mu\nu}\mathcal{D}a_\mu \exp\left(-\frac{1}{\hbar} \int d^d x \int_0^{\hbar/(k_B T)} d\tau \sqrt{g} \mathcal{L}_d[g_{\mu\nu}, a_\mu]\right)$$

$g_{\mu\nu} \Rightarrow$ spacetime metric, $g = \det(g_{\mu\nu})$

$a_\mu \Rightarrow$ Electromagnetic gauge field

$\mathcal{L}_d \Rightarrow$ Classical Einstein-Maxwell action

$\mathcal{U}(t) = \exp(-i\mathcal{H}t/\hbar) \iff Z = \text{Tr} \exp(-\mathcal{H}/(k_B T))$
Bohr-Sommerfeld semiclassical quantum theory of a black hole in $d$ spatial dimensions

$$
Z = \int \mathcal{D}g_{\mu\nu}\mathcal{D}a_{\mu} \exp \left( -\frac{1}{\hbar} \int d^d x \int_{0}^{\hbar/(k_B T)} d\tau \sqrt{g} \mathcal{L}_d[g_{\mu\nu}, a_{\mu}] \right)
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- $g_{\mu\nu}$ \Rightarrow spacetime metric, \quad $g = \text{det}(g_{\mu\nu})$
- $a_{\mu}$ \Rightarrow Electromagnetic gauge field
- $\mathcal{L}_d$ \Rightarrow Classical Einstein-Maxwell action

$$
\mathcal{U}(t) = \exp \left( -i\mathcal{H}t/\hbar \right) \quad \Leftrightarrow \quad Z = \text{Tr} \exp \left( -\mathcal{H}/(k_B T) \right)
$$

- Spacetime geometry of a black hole in imaginary time $\tau$
- Evaluate path integral at black hole saddle point

G. W. Gibbons and S. W. Hawking, Phys. Rev. D 15, 2752 (1977).
Reissner-Nordstrom black hole of Einstein-Maxwell theory

Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS$_2$) at low energies!
Reissner-Nordstrom black hole of Einstein-Maxwell theory

Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS$_2$) at low energies!

Is there a mapping to a quantum system with Planckian dynamics in 0+1 dimensions?
Yes!

The low energy theory of a charged black hole is exactly the low energy theory of time reparameterizations of the SYK model.
Quantum theory of charged black holes

The near-horizon 1+1 dimensional theory of a charged black hole

\[ Z_Q = e^{S_H(0)} \int \mathcal{D}g_{\mu\nu} \mathcal{D}a_\mu \exp \left( -\frac{1}{\hbar} \int d\zeta \int_0^{\hbar/(k_B T)} d\tau \sqrt{g} \mathcal{L}_{JT}[g_{\mu\nu}, a_\mu] \right) \]
Quantum theory of charged black holes
The near-horizon 1+1 dimensional theory of a charged black hole

\[ Z_Q = e^{S_{H(0)}} \int Dg_{\mu\nu} Da_\mu \exp \left( -\frac{1}{\hbar} \int d\zeta \int_0^{\hbar/(k_B T)} d\tau \sqrt{g} \mathcal{L}_{JT}[g_{\mu\nu}, a_\mu] \right) \]

\[ \Downarrow \]

\[ Z_{SYK} = e^{N s_0} \int Df D\phi \exp \left( -\frac{1}{\hbar} \int_0^{\hbar/(k_B T)} d\tau \mathcal{L}_{SYK}[f, \phi] \right) \]

after relating the boundary component of \( g_{\mu\nu} \) to \( f \), and the boundary value of \( a_\tau \) to \( \phi \).

Sachdev (2010); Kitaev (2015); Sachdev (2015); Maldacena, Stanford, Yang (2016); Moitra, Trivedi, Vishal (2018); Gaikwad, Joshi, Mandal, Wadia (2018); Sachdev (2019; Iliesiu, Turiaci (2020)
Quantum theory of charged black holes

The near-horizon 1+1 dimensional theory of a charged black hole

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\[ Z_{SYK} = e^{N s_0} \int \mathcal{D}f \mathcal{D}\phi \exp \left( -\frac{1}{\hbar} \int_0^{\hbar/(k_B T)} d\tau \mathcal{L}_{SYK}[f, \phi] \right) \]

after relating the boundary component of \( g_{\mu\nu} \) to \( f \), and the boundary value of \( a_\tau \) to \( \phi \).

\[ \frac{S(T)}{k_B} = \frac{1}{\hbar G} \left( \frac{A_0 c^3}{4} + \frac{\sqrt{\pi} A_0^{3/2}}{2} \frac{c^2}{k_B T} \frac{k_B T}{\hbar} \right) \]

where \( A_0 \) is the area of the black hole horizon at \( T = 0 \).
Quantum theory of charged black holes

The near-horizon 1+1 dimensional theory of a charged black hole

\[ Z_Q = e^{S_H(0)} \int \mathcal{D}g_{\mu\nu} \mathcal{D}a_\mu \exp \left( -\frac{1}{\hbar} \int d\zeta \int_0^{\hbar/(k_BT)} d\tau \sqrt{g} \mathcal{L}_{JT}[g_{\mu\nu}, a_\mu] \right) \]

\[ \uparrow \]

\[ Z_{SYK} = e^{N s_0} \int \mathcal{D}f \mathcal{D}\phi \exp \left( -\frac{1}{\hbar} \int_0^{\hbar/(k_BT)} d\tau \mathcal{L}_{SYK}[f, \phi] \right) \]

after relating the boundary component of \( g_{\mu\nu} \) to \( f \), and the boundary value of \( a_\tau \) to \( \phi \).

\[ \frac{S(T)}{k_B} = \frac{1}{\hbar G} \left( \frac{A_0 c^3}{4} + \frac{\sqrt{\pi} A_0^{3/2} c^2}{2} \frac{k_B T}{\hbar} \right) - \frac{3}{2} \ln \left( \frac{c^5/\hbar G}{k_B T/\hbar} \right) + \ldots . \]

where \( A_0 \) is the area of the black hole horizon at \( T = 0 \).
Quantum theory of charged black holes

\[ D(E) \sim \exp \left( \frac{A_0 c^3}{4 \hbar G} \right) \sinh \left( \sqrt{\frac{\pi}{A_0^3/2} c^2} \frac{E}{\hbar^2 G} \right)^{1/2} \]

Same lower energy coarse-grained density of states in a model of interacting (fermionic) qubits with a discrete spectrum!
Yukawa-SYK models and strange metals
Yukawa-SYK models

\[ H = \sum_{ij} t_{ij} \psi_i^\dagger \psi_j + \sum_{\ell} \frac{1}{2} (\pi_\ell^2 + \omega_\ell^2 \phi_\ell^2) + \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell \]

Leads to fully self-consistent Migdal-Eliashberg equations
\[ \Sigma_\psi \sim g^2 G_\psi G_\phi, \quad \Sigma_\phi \sim g^2 G_\psi G_\psi \]

in a SYK-like large \( N \) limit.

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Yukawa-SYK models

\[ \mathcal{H} = -\mu \sum_{i} \psi_i^\dagger \psi_i + \sum_{\ell} \frac{1}{2} (\pi_{\ell}^2 + \omega_0^2 \phi_{\ell}^2) + \frac{1}{N} \sum_{i,j,\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_{\ell} \]

with \( g_{ij\ell} \) independent random numbers with zero mean. The large \( N \) saddle point equations are

\[
\begin{align*}
G(i\omega_n) &= \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} , \\
D(i\omega_n) &= \frac{1}{\omega_n^2 + \omega_0^2 - \Pi(i\omega_n)} , \\
\Sigma(\tau) &= g^2 G(\tau) D(\tau) , \\
\Pi(\tau) &= -g^2 G(\tau) G(-\tau)
\end{align*}
\]

Make the low frequency ansatz

\[
G(i\omega) \sim -i \text{sgn}(\omega) |\omega|^{-(1-2\Delta)} , \\
D(i\omega) \sim |\omega|^{1-4\Delta} , \\
\frac{1}{4} < \Delta < \frac{1}{2}
\]

A consistent solution exists for

\[
\frac{4\Delta - 1}{2(2\Delta - 1)[\sec(2\pi\Delta) - 1]} = 1 , \\
\Delta = 0.42037 \ldots
\]

I. Esterlis and J. Schmalian, PRB 100, 115132 (2019)
See also Yuxuan Wang, PRL 124, 017002 (2020)
Fermi surface coupled to a critical boson

Occupied states
\[ \varepsilon(k) < 0 \]

Empty states
\[ \varepsilon(k) > 0 \]

\( \psi \)

\( k_x \)
\( k_y \)

\[ S = \mathbb{Z}^3 \times (\oplus \mu)^2 + \ldots \]

a critical boson \( \phi \)

- Nematic order
- Ferromagnetic order
- Transverse component of abelian or non-abelian gauge field
Fermi surface coupled to a critical boson

\[ \varepsilon(k) < 0 \] Occupied states
\[ \varepsilon(k) > 0 \] Empty states

\[ \psi \]

\[ g \int d^2r d\tau \psi^\dagger(r, \tau)\psi(r, \tau)\phi(r, \tau) \] “Yukawa” coupling

Yields a non-Fermi liquid without quasiparticles, but with zero resistivity (due to boson “drag”)!
Fermi surface coupled to a critical boson

Occupied states \( \varepsilon(k) < 0 \)

Empty states \( \varepsilon(k) > 0 \)

a critical boson \( \phi \)
- Nematic order
- Ferromagnetic order
- Transverse component of abelian or non-abelian gauge field

"Yukawa" coupling:
\[
\frac{g_{ij\ell}}{N} \int d^2r d\tau \psi_i^\dagger(r, \tau) \psi_j(r, \tau) \phi_\ell(r, \tau)
\]

Random couplings in flavor space lead to large \( N \) theory of a strange metal, with zero resistivity
Fermi surface coupled to a critical boson

Occupied states \( \varepsilon(k) < 0 \)

Empty states \( \varepsilon(k) > 0 \)

\[
\int d^2r d\tau \left[ \frac{g_{ij\ell}}{N} + \frac{g'_{ij\ell}(r)}{N} \right] \psi_i^\dagger(r, \tau) \psi_j(r, \tau) \phi_l(r, \tau)
\]

Random couplings in flavor and position space leads to large \( N \) theory of a strange metal, with linear-\( T \) resistivity

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv: 2203.04990
Summary

- SYK: a solvable model without particle-like excitations, exhibiting thermalization and many-body chaos in a time of order $\hbar/(k_B T)$, independent of microscopic energy scales.

- Low energy theory of time reparameterizations is the theory of the boundary graviton in 1+1 dimensional quantum gravity on AdS$_2$.

- The semiclassical entropy of Einstein gravity is reproduced by a unitary quantum system with a discrete spectrum. Further work along these lines has led to progress on the Page curve describing the time evolution of the entropy of an evaporating black hole.

- Linear $\alpha$-resistivity arises from spatially random interactions in a two-dimensional quantum-critical metal.
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- The density of states of a charged black holes in Einstein gravity is reproduced by a unitary quantum system with a discrete spectrum. Further work along these lines has led to progress on the Page curve describing the time evolution of the entropy of an evaporating black hole.
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