Background-metric Independent Formulation of 4D Quantum Gravity

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(Revised Version)

Abstract

Using the background-metric independence for the traceless mode as well as the conformal mode, 4D quantum gravity is described as a quantum field theory defined on a non-dynamical background-metric. The measure then induces an action with 4 derivatives. So we think that 4-th order gravity is essential and the Einstein-Hilbert term should be treated like a mass term. We introduce the dimensionless self-coupling constant $t$ for the traceless mode. In this paper we study a model where the measure can be evaluated in the limit $t \to 0$ exactly, using the background-metric independence for the conformal mode. The $t$-dependence of the measure is determined perturbatively using the background-metric independence for the traceless mode.

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1 Introduction

Four dimensional quantum gravity [1]–[2] is one of the most interesting issues left in the developments of quantum field theory. The big problem in 4D quantum gravity is that the naive perturbation theory breaks down. On the other hand it is believed that quantum gravity in two dimensions is a well-defined quantum field theory [3]–[7]. Certain formulations of 2D quantum gravity have been solved exactly [3, 4]. This success in two dimensions have inspired many ideas on quantum gravity. Based on such ideas conformal mode dynamics in 4 dimensions have been studied by Antoniadis, Mazur and Mottola [18, 19, 20]. In this paper, we develope these ideas further, and re-investigate four dimensional quantum gravity including the traceless mode.

One of the most important idea to define quantum gravity in the generally coordinate invariant way is the background-metric independence. The original expression of quantum gravity defined by the functional integration over the dynamical metric is trivially invariant under any change of non-dynamical background-metric. But, when the functional measures are re-expressed by ones defined on the background-metric, the background-metric independence gives strong constraints on the theory.

The background-metric independence includes conformal invariance, which is just the key ingredient to solve 2D quantum gravity exactly [4]. As stressed in ref. the conformal invariance is purely quantum symmetry realized just when gravity is quantized, which does not always require the classical theory to be conformally invariant. Furthermore this idea is independent of dimensions. Naively, it is difficult to imagine that conformally invariant theory is not well-defined. Therefore we think that even in 4 dimensions quantum gravity is well-defined if we formulate it in the background-metric independent way. In two dimensions it is enough to consider the conformal invariance [4, 5], while in four dimensions it is necessary to consider the background-metric independence for the traceless mode as well as the conformal mode.

In four dimensions the measure induces an action with 4 derivatives. So we think that the 4-th order action is rather natural in 4 dimensions [4, 5] and the Einstein-Hilbert action should be treated like a mass term [3–11], where the square of mass is the inverse of the gravitational coupling constant. The classical limit is then given in the large mass limit.

The aim of this paper is to give a proper definition of 4D quantum grav-
ity. In the next section we give general arguments about background-metric independence before going to concrete calculations. In Sect.3 we discuss the induced action for the conformal mode in general cases. We here pay attention to the special property of D-th order operators in D dimensions \[15\]. The argument of \(D = 4\) is essentially used when we evaluate the measure for gravitational fields. After giving some remarks on the measures of matter fields in Sect.4, we evaluate the measure of gravitational field in Sect.5. We then introduce the dimensionless self-coupling constant \(t\) for the traceless mode and consider the perturbation theory on \(t\) \[17\]. The conformal mode is treated in a non-perturbative way. We discuss a model where the measure can be evaluated exactly in the \(t \to 0\) limit. The model in the limit essentially corresponds to the one studied by Antoniadis, Mazur and Mottola \[19\] though their treatment of the \(R^2\)-term is different from ours. To evaluate the \(t\)-dependence we give an ansatz based on the background-metric independence for the traceless mode. It is solved in self-consistent manner. In Sect.6 we give some comments on scaling operators in 4 dimensions.

2 General Arguments

Quantum gravity is defined by the functional integral over the metric field as follows:

\[
Z = \int \left[ g^{-1}dg \right]_g \left[ df \right]_g \exp \left[ -I_{CL}(f,g) \right],
\]

where \(g\) is the metric field restricted to \(g_{\mu \nu} = g_{\nu \mu}\) and \(\text{vol(diff.)}\) is the gauge volume for diffeomorphism. \(f\) is a matter field discussed in Sect.4. In this paper we consider scalar and gauge fields. The functional measure for integration over the metric is defined by

\[
< \delta g, \delta g >_g = \int d^Dx \sqrt{g} g^{\alpha \beta} g^{\gamma \delta} (\delta g_{\alpha \gamma} \delta g_{\beta \delta} + u \delta g_{\alpha \beta} \delta g_{\gamma \delta}) ,
\]

where \(u > -1/D\) by positive definiteness of the norm. This definition is rather symbolic because the measure depends on the dynamical variables \(g\) explicitly. The aim of this paper is to rewrite the measure as one defined on the non-dynamical background-metric as in usual field theories.

Decompose the metric into the conformal mode \(\phi\), the traceless mode \(h\) and the background-metric \(\hat{g}\) as follows:

\[
g_{\mu \nu} = e^{2\phi} \hat{g}_{\mu \nu}
\]
\[ \bar{g}_{\mu\nu} = (\hat{g} e^h)_{\mu\nu} = \hat{g}_{\mu\lambda}(\delta_{\lambda\nu}^h + h^\lambda_{\nu} + \frac{1}{2} (h^2)_{\lambda\nu} + \cdots) . \] (2.4)

where \( tr(h) = h^\mu_{\mu} = 0 \). An arbitrary variation of the metric is given by

\[ \delta g_{\mu\nu} = 2 \delta \phi g_{\mu\nu} + g_{\mu\lambda}(e^{-h} \delta e^h)^\lambda_{\nu} . \] (2.5)

Since \( tr(e^{-h} \delta e^h) = \int_0^1 ds \ tr(e^{-sh} \delta h e^{sh}) = 0 \), the variation of the conformal mode and that of the traceless mode are orthogonal in the functional space defined by the norm (2.2). Therefore the measure of metric can be decomposed as

\[ \frac{[g^{-1} dg]_{\bar{g}}}{\text{vol(diff.)}} = \frac{[d\phi]_{\bar{g}}[e^{-h} de^h]_{\bar{g}}}{\text{vol(diff.)}} , \] (2.6)

where the norms for the conformal mode and the traceless mode are defined respectively by

\[ <\delta \phi, \delta \phi>_g = \int d^D x \sqrt{g} (\delta \phi)^2 , \] (2.7)

\[ <\delta h, \delta h>_g = \int d^D x \sqrt{g} \ tr(e^{-h} \delta e^h)^2 . \] (2.8)

Let us rewrite the functional measures defined on the dynamical metric \( g \) into those defined on the non-dynamical background-metric \( \hat{g} \) as in usual quantum field theories. First consider conformal mode dependence of the measures. The partition function will be equivalently expressed as

\[ Z = \int \frac{[d\phi]_{\bar{g}}[e^{-h} de^h]_{\bar{g}}[df]_{\bar{g}}}{\text{vol(diff.)}} \exp\left[ -S(\phi, \bar{g}) - I_{CL}(f, \bar{g}) \right] , \] (2.9)

where \( S \) is the action for the conformal mode induced from the measures. It is worth making some remarks on this expression. The first is that we here do not give any change for the classical action. Namely, the induced action is purely the contribution from the measures. The second is that the measures of metric fields are defined on the background metric \( \hat{g} \) because of \( \det \bar{g} = \det \hat{g} \), while for matter fields they in general depend on the traceless mode explicitly so that they are defined on the metric \( \bar{g} \).

Originally the partition function is defined by the metric \( g = e^{2\phi} \bar{g} \) so that the theory should be invariant under the simultaneous changes [14]:

\[ \bar{g} \to e^{2\omega} \bar{g} , \quad \phi \to \phi - \omega . \] (2.10)
In order that the theory is invariant under these changes, the action $S$ should in general satisfy the following transformation law:

$$S(\phi - \omega, e^{2\omega} \bar{g}) = S(\phi, \bar{g}) - R(\omega, \phi, \bar{g}) .$$  \hspace{1cm} (2.11)

The measure is then transformed as

$$[d\phi]_{e^{2\omega} \bar{g}} [e^{-h} d e^h]_{e^{2\omega} \bar{g}} [df]_{e^{2\omega} \bar{g}} = [d\phi]_{\bar{g}} [e^{-h} d e^h]_{\bar{g}} [df]_{\bar{g}} \exp[-R(\omega, \phi, \bar{g})] .$$  \hspace{1cm} (2.12)

Here note that the measure $[d\phi]_{\bar{g}}$ is invariant under a local shift $\phi \to \phi - \omega$. Because of this property the invariance under the changes (2.10) means the invariance under the conformal change of the background: $\bar{g} \to e^{2\omega} \bar{g}$.

In this paper we consider the case of $R(\omega, \phi, \bar{g}) = S(\omega, \bar{g})$, which is called the Wess-Zumino condition [21]. We make some comments on this particular case in the context of scalar field. Explicit form of such an action is given in the next section.

Next consider the background-metric independence for the traceless mode. The theory should be invariant under the simultaneous changes

$$\hat{g} \to \hat{g} e^b, \quad e^h \to e^{-b} e^h ,$$  \hspace{1cm} (2.13)

where $tr(b) = 0$, which preserves the combination $\bar{g} = \hat{g} e^h$. The measure for the matter field can be rewritten in the form

$$[df]_{\bar{g}} = [df]_{\hat{g}} e^{-W(e^h, \hat{g})} ,$$  \hspace{1cm} (2.14)

where the induced action for the traceless mode should satisfy the Wess-Zumino condition [21]

$$W(e^{-b} e^h, \hat{g} e^h) = W(e^h, \hat{g}) - W(e^h, \hat{g}) .$$  \hspace{1cm} (2.15)

The explicit form of $W$ is discussed in Sect.4. Note that the measure $[e^{-h} d e^h]_{\bar{g}}$ is left-invariant under the change $e^h \to e^{-b} e^h$ so that the theory becomes invariant under the change of the background: $\hat{g} \to \hat{g} e^b$. Thus the theory becomes invariant under any change of the background-metric. This is reasonable because the background-metric is quite artificial so that the theory should be independent of how to choose the background-metric.
3 D-th Order Operators in D Dimensions

Before evaluating the measure of gravitational field, we discuss the general cases first. Consider $N$ scalar fields $\varphi_A$ ($A = 1, \ldots, N$), which have an action with $2n$-th derivatives in D dimensions:

$$I(\varphi, g) = \frac{1}{2(4\pi)^{D/2}} \int d^Dx \sqrt{g} \varphi_A D^{(n)}_{AB} \varphi_B ,$$

where $D^{(n)}_{AB} = (-\Box)^n \delta_{AB} + \Pi_{AB}$ is a covariant operator. $\Pi_{AB}$ is a lower-derivative matrix operator and $\Box = \nabla^\mu \nabla_\mu$. Let us calculate the induced action $S(\phi, \bar{g})$ defined by the relation

$$\int [d\varphi] g e^{-I(\varphi, g)} = e^{-S(\phi, \bar{g})} \int [d\varphi] \bar{g} e^{-I(\phi, g)} ,$$

where the functional measure of l.h.s. is defined by

$$<\delta \phi, \delta \phi> = \int d^Dx \sqrt{\bar{g}} \delta \varphi_A \delta \varphi_A .$$

The measure of r.h.s. is defined by replacing the determinant of metric $\sqrt{g}$ into $\sqrt{\bar{g}}$, while note that the action $I$ of r.h.s. depends on $g$, not on $\bar{g}$. Therefore the argument can be applied to a “non-conformally” invariant theory also.

From the definition (3.2), the variation of the induced action for the conformal mode is given by

$$\delta \phi S(\phi, \bar{g}) = -\frac{n}{2} \text{Tr} \left( \delta \phi D^{(n)} e^{-\epsilon D^{(n)}} \right) + \frac{1}{2} \text{Tr} \left( \delta \varphi D^{(n)} + \delta K D^{(n)} \right) ,$$

where $D^{(n)} = e^{D\phi} D^{(n)}$ is a non-covariant operator and $\epsilon = 1/L^{2n}$. Here $L \to \infty$ is a cutoff. The variation of the $2n$-th order operator can be written in the form $\delta_{\phi} D^{(n)} = -2n \delta \phi D^{(n)} + \delta K$, where $\delta K$ depends on the details of lower derivative terms. The variation of $\delta^{(n)}$ is given by $\delta \varphi \delta^{(n)} = (D - 2n) \delta \phi D^{(n)} + e^{D\phi} \delta K$. Using these variations we get the following expression:

$$\delta \phi S(\phi, \bar{g}) = -n \text{Tr} \left( \delta \phi e^{-\epsilon D^{(n)}} \right) + \frac{1}{2} \text{Tr} \left( \delta K D^{(n)} - \frac{1}{2} \text{Tr} \left( e^{D\phi} \delta K D^{(n)} \right) \right) ,$$

$$= -n \text{Tr} \left( \delta \phi e^{-\epsilon D^{(n)}} \right) + (n - D/2) \text{Tr} \left( \delta \phi e^{-\epsilon D^{(n)}} \right) . \quad (3.5)$$
The last equality is proved by using the relation between the Green functions: 
\[ <x|D^{(n)-1}|x'>_\bar{g} = <x|D^{(n)-1}|x>_g \] such that

\[
Tr \left( e^{D\phi} \delta K D^{(n)-1} \right) = tr \int d^D x \sqrt{\bar{g}} e^{D\phi} \delta K <x|D^{(n)-1}|x>_g \]

\[
= tr \int d^D x \sqrt{\bar{g}} \delta K <x|D^{(n)-1}|x>_g = Tr \left( \delta K D^{(n)-1} \right) , \tag{3.6}
\]

where \( tr \) takes over the indices \( A, B \).

For \( D = 2n \), the expression is simplified. The non-covariant part vanishes so that the variation of the induced action is written by using the covariant quantity \( \mathcal{H}^{(n)}(x, \epsilon) = <x|e^{-\epsilon D^{(n)}}|x>_g \). Furthermore, in this case, the induced action \( S(\phi, \bar{g}) \) satisfies the Wess-Zumino condition. It is proved in the following. Let us apply the simultaneous changes (2.10) to both sides of the definition (3.2). The l.h.s. is invariant under the changes so that we obtain the following relation:

\[
e^{-S(\phi - \omega, e^{2\omega} \bar{g})} \int [d\varphi] e^{2\omega} \bar{g} e^{-I(\varphi, \bar{g})} = e^{-S(\phi, \bar{g})} \int [d\varphi] \bar{g} e^{-I(\varphi, \bar{g})} . \tag{3.7}
\]

Now define the action \( R(\omega, \phi, \bar{g}) \) by the relation

\[
\int [d\varphi] e^{2\omega} \bar{g} e^{-I(\varphi, \bar{g})} = e^{-R(\omega, \phi, \bar{g})} \int [d\varphi] \bar{g} e^{-I(\varphi, \bar{g})} . \tag{3.8}
\]

Then we obtain the general relation (2.11). Next consider the variation of \( R(\omega, \phi, \bar{g}) \) w.r.t. the conformal mode \( \phi \), which is given by

\[
\delta_\phi R(\omega, \phi, \bar{g}) = -\delta_\phi \log \det^{-1/2} D^{(n)}(\omega) + \delta_\phi \log \det^{-1/2} D^{(n)} , \tag{3.9}
\]

where \( D^{(n)}(\omega) = e^{-D\omega} D^{(n)}(n) \) and \( D^{(n)} \) has been defined before. As in the same way discussed above we obtain the following expression:

\[
\delta_\phi R(\omega, \phi, \bar{g}) = (D/2 - n) \left[ Tr \left( \delta_\phi e^{-\epsilon D^{(n)}} \right) - Tr \left( \delta_\phi e^{-\epsilon D^{(n)}} \right) \right] , \tag{3.10}
\]

where we use the relation between the Green functions: 
\[ <x|D^{(n)-1}(\omega)|x'>_{e^{2\omega} \bar{g}} = <x|D^{(n)-1}|x'>_\bar{g} \] . Therefore, in the case of \( D = 2n \), the action is independent of \( \phi \) such that \( R(\omega, \phi, \bar{g}) = R(\omega, \bar{g}) \). From the condition at \( \phi = \omega \), the action \( R(\omega, \bar{g}) \) is nothing but \( S(\omega, \bar{g}) \). Thus we proved that the induced action \( S(\phi, \bar{g}) \) of \( D = 2n \) defined by the relation (3.2) satisfies the Wess-Zumino condition.
In two dimensions consider the usual second order operator. It is well-known that the finite term of the heat kernel expansion for $H^{(1)}(x,\epsilon)$ is given by the scalar curvature. Thus the integrated action is given by the Liouville action even though the classical theory is not conformally invariant [15].

In four dimensions we must consider the 4-th order operator. The induced action is then given by integrating the covariant quantity $H^{(2)}(x,\epsilon)$ over the conformal mode. From the general argument by Duff [22], such a quantity, or what is called trace anomaly depends only on two constants $a$ and $b$ so that the induced action is given by

$$S(\phi, \bar{g}) = -\frac{1}{(4\pi)^2} \int d^4x \int_0^\phi \delta \phi \sqrt{\bar{g}} \tilde{a}^{(2)}$$

$$= \frac{1}{(4\pi)^2} \int d^4x \int_0^\phi \delta \phi \sqrt{\bar{g}} \left[ a F + \frac{2}{3} \Box R + b G \right], \quad (3.11)$$

where $a^{(2)}$ is the finite term of $H^{(2)}(x,\epsilon)$ defined in eq. (B.1). The constant of integration is determined by the condition $S(\phi = 0, \bar{g}) = 0$ because both sides of functional integrations are equivalent at $\phi = 0$. $F$ and $G$ are the square of Weyl tensor and Euler density respectively:

$$F = R_{\mu\nu\lambda\sigma} R^{\mu\nu\lambda\sigma} - 2 R_{\mu\nu} R^{\mu\nu} + \frac{1}{3} R^2, \quad (3.12)$$

$$G = R_{\mu\nu\lambda\sigma} R^{\mu\nu\lambda\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2. \quad (3.13)$$

The quantities $F$, $G$ and $\Box R$ are separately integrable w.r.t. the conformal mode [23]. It is useful to consider the following combination [23, 18, 19]:

$$G - \frac{2}{3} \Box R = e^{-4\phi} \left( 4 \Delta_4 \phi + \bar{G} - \frac{2}{3} \Box \bar{R} \right), \quad (3.14)$$

where $\Delta_4$ is the conformally covariant 4-th order operator defined by

$$\Delta_4 = \Box^2 + 2 R^{\mu\nu} \nabla_\mu \nabla_\nu - \frac{2}{3} \Box R + \frac{1}{3} (\nabla^{\mu} R) \nabla_\mu \quad (3.15)$$

which satisfies $\Delta_4 = e^{-4\phi} \bar{\Delta}_4$. The induced action then becomes

$$S(\phi, \bar{g}) = \frac{1}{(4\pi)^2} \int d^4x \sqrt{\bar{g}} \left[ a F + 2 b \phi \bar{G} + b \left( \bar{G} - \frac{2}{3} \Box \bar{R} \right) \phi \right]$$

$$- \frac{1}{(4\pi)^2} \frac{a + b}{18} \int d^4x \left( \sqrt{g R^2} - \sqrt{\bar{g} \bar{R}^2} \right). \quad (3.16)$$
This action really satisfies the Wess-Zumino condition, which can be generally proved, if the integrand \( a_2^{(n)} \) is integrable as well as covariant, as follows:

\[
S(\phi - \omega, e^{2\omega} \bar{g}) = -\frac{1}{(4\pi)^2} \int d^4x \int_0^{\phi - \omega} \delta \sigma \sqrt{\bar{g}} e^{4(\sigma + \omega)} a_2^{(n)} |_{g = e^{2(\sigma + \omega)} \bar{g}} = -\frac{1}{(4\pi)^2} \int d^4x \int_0^\phi \delta \sigma \sqrt{\bar{g}} e^{4\sigma} a_2^{(n)} |_{g = e^{2\sigma} \bar{g}} = S(\phi, \bar{g}) - S(\omega, \bar{g}). \tag{3.17}
\]

The last equality is proved by dividing the integral region \([\omega, \phi] \) into \([0, \phi] - [0, \omega] \). In the above case the first term rather trivially satisfies the Wess-Zumino condition. In the second term the \( \sqrt{g} R^2 \)-term itself does not satisfy the Wess-Zumino condition, but the above combination \( \sqrt{g} R^2 - \sqrt{\bar{g}} R^2 \) satisfies it.

The results of this section are very important when we discuss the contributions from the measures of gravity in Sect. 5.

### 4 The Measures of Matter Fields

In this section we briefly discuss matter field contributions to the induced action. Matter field actions are constructed with at most second order derivatives of fields. As discussed in the previous section, such fields are rather special in 4 dimensions. We make some comments on the measures of scalar field and gauge field.

#### 4.1 Scalar fields

Let us consider scalar field coupled to the curvature as follows:

\[
I_S(X, g) = \frac{1}{2} \frac{1}{(4\pi)^2} \int d^4x \sqrt{\bar{g}}(g^{\mu\nu} \partial_\mu X \partial_\nu X + \xi R X^2) . \tag{4.1}
\]

From arguments of the previous section, the variation of the induced action becomes a non-covariant form in this case. We now do not know whether such an integrand is integrable or not. Even if integrable, the integrated action does not satisfy the Wess-Zumino condition so that the theory becomes more complicated. So we only consider the conformally coupled scalar field with \( \xi = 1/6 \), which is described as \( I_{CS} \).
Instead of the relation (3.2), we use the following one:

\[
\int [dX]_g e^{-I_{CS}(X,g)} = e^{-\Gamma(\phi,\bar{g})} \int [dX]_\bar{g} e^{-I_{CS}(X,\bar{g})}.
\] (4.2)

The difference is that the action \(I_{CS}\) of r.h.s. is defined on the metric \(\bar{g}\), not on
the metric \(g\) so that the variation of \(\Gamma\) is simply given in the form
\[
\delta \phi \Gamma(\phi,\bar{g}) = \frac{-1}{4\pi^2} \int d^4x \sqrt{g} g^{\mu\lambda} g^{\nu\sigma} F_{\mu\nu} F_{\lambda\sigma},
\]
where \(F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu\). Gauge theory is classically
conformally invariant in 4 dimensions which is described as
\(I_{A}(A_\mu, g) = I_{A}(A_\mu, \bar{g})\), where the gauge field is not rescaled. The measure of gauge field
is defined by the norm
\[
<\delta A, \delta A>_{g} = \int d^4x \sqrt{g} g^{\mu\nu} \delta A_\mu \delta A_\nu.
\] (4.6)

Unlike the case of scalar field it depends on both the conformal mode and the
traceless mode.

\[\text{Note that the conformal invariance of scalar field in 4 dimensions is described by rescaling the scalar field as well as the metric as } I_{CS}(X,g) = I_{CS}(\hat{X},\hat{g}), \text{ where } \hat{X} = e^\phi X. \text{ Thus}
\]
\[
\int [dX]_g e^{-I_{CS}(X,g)} \neq \int [dX]_\bar{g} e^{-I_{CS}(X,\bar{g})} = \int [d\hat{X}]_{\bar{g}} e^{-I_{CS}(\hat{X},\bar{g})}.
\] (4.3)

4.2 Gauge fields

In this subsection we consider abelian gauge fields defined by the action

\[
I_{A}(A_\mu, g) = \frac{1}{4(4\pi)^2} \int d^4x \sqrt{g} g^{\mu\lambda} g^{\nu\sigma} F_{\mu\nu} F_{\lambda\sigma},
\] (4.5)

where \(F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu\). Gauge theory is classically
conformally invariant in 4 dimensions which is described as \(I_{A}(A_\mu, g) = I_{A}(A_\mu, \bar{g})\), where the gauge field is not rescaled. The measure of gauge field
is defined by the norm
\[
<\delta A, \delta A>_{g} = \int d^4x \sqrt{g} g^{\mu\nu} \delta A_\mu \delta A_\nu.
\] (4.6)

Unlike the case of scalar field it depends on both the conformal mode and the
traceless mode.
As for the conformal mode, it is well known that when we rewrite the measure on \( g \) into the one on \( \bar{g} \), we obtain the induced action (3.16) with the coefficients \( a_A = -\frac{N_A}{10}, \quad b_A = \frac{31N_A}{180} \), (4.7)

where \( N_A \) is the number of gauge fields.

Now, consider the induced action for the traceless mode defined by

\[
[dA_\mu]_\bar{g} = [dA_\mu]_\hat{g} e^{-W(e^h,\hat{g})} .
\]

Apply the simultaneous changes (2.13) in both sides of (4.8). The measure of l.h.s. (and also the induced action for the conformal mode and the classical gauge action) is invariant under the changes, while the r.h.s. becomes

\[
[dA_\mu]_\hat{g} e^{-W(e^{-b}e^h,\hat{g} e^b)} = [dA_\mu]_\bar{g} e^{-W(e^h,\hat{g})-W(e^{-b}e^h,\hat{g} e^b)} ,
\]

where we use the relation (4.8) again with \( h \) replaced with \( b \). The r.h.s. of (4.9) should become the original form so that \( W \) should satisfy the Wess-Zumino condition (2.15), which can be rewritten in more familiar form by introducing the one form \( (V_\mu)^{\alpha}_\beta = \hat{g}^{\alpha \lambda} \partial_\mu \hat{g}_{\lambda \beta} \) and notations \( H = e^h \) and \( B = e^b \) as follows:

\[
W(B^{-1}H, V_\mu^B) = W(H, V_\mu) - W(B, V_\mu) \quad (4.10)
\]

where

\[
V_\mu^B = (\hat{g} B)^{-1} \partial_\mu (\hat{g} B) = B^{-1}V_\mu B + B^{-1} \partial_\mu B . \quad (4.11)
\]

The solution of the Wess-Zumino condition is well-known \([21]\), which is given by

\[
W(H, V_\mu) = \zeta \int_0^1 ds \int d^4x \, tr(h \, G(V_\mu^s)) \quad (4.12)
\]

where \( G(V_\mu) \) is the non-abelian anomaly of the one form \( V_\mu \) and

\[
V_\mu^s = e^{-sh}V_\mu e^{sh} + e^{-sh} \partial_\mu e^{sh} . \quad (4.13)
\]

Thus what remains to do would be to determine the overall coefficient \( \zeta \), which we do not discuss anymore in this paper.
5 The Measures of Gravitational Fields

In this section we consider the measures of conformal and traceless modes of gravity. Henceforth we introduce the dimensionless self-coupling constant $t$ for the traceless mode in the way \[17\]
\[
\bar{g}_{\mu\nu} = (\hat{g}e^{th})_{\mu\nu}.
\] (5.1)
The classical action for the conformal mode is given by the $R^2$-action and that for the traceless mode is the Weyl action divided by the square of the coupling $t$, which is
\[
I_G = \frac{1}{(4\pi)^2} \int d^4x \sqrt{g} \left( \frac{1}{t^2} F + cR^2 - m^2 R + \Lambda \right),
\] (5.2)
where $m^2$ is the inverse of gravitational constant and $\Lambda$ is the cosmological constant. In the flat background the 4 derivative parts of the Lagrangian have the form $\frac{1}{2}tr(h\partial^4h) + 36c\phi\partial^4\phi + o(t)$. The presence of the Einstein-Hilbert term now gives rise to the tachyon problem at the classical level for $c > 0$ and $\Lambda = 0$ \[6\]–\[11\], but in the quantum theory the kinetic term of conformal mode is induced from the measures and also we consider $\Lambda \neq 0$ case so that such a problem will disappear. The question of unitarity still remains to be clarified \[6\]–\[11\]. We here only stress that the theory is unitary at the low energy and we cannot avoid the 4-th order action to ensure the background-metric independence.

The coefficient $c$ is in general arbitrary, but for technical reasons it is determined to be a special value later.

5.1 The induced action in the $t \to 0$ limit

5.1.1 Traceless mode

As a first approximation we consider the $t \to 0$ limit. The metric $\bar{g}$ then reduces to the background metric $\hat{g}$ so that the matter field, the conformal mode and the traceless mode are decoupled each other. So we can evaluate the contributions from the measures exactly. This approximation is nothing but the one adopted in \[19\] though our management of the $R^2$-terms in eqs. \(5.1\) and \(5.2\) are different from theirs. The difference affects $t$-dependent contributions discussed in Sect.5.2.
We first calculate the induced action from the measure of traceless mode. At the $t \to 0$ limit the measure (2.8) divided by $t$ reduces to $[dh]_{g'}$ defined by the norm $< \delta h, \delta h >_{g'} = \int \sqrt{g'} tr(\delta h)^2$, where $g'_{\mu\nu} = e^{2\phi} \hat{g}_{\mu\nu}$. The action now becomes

$$I^{(0)}_G(h, g') = \frac{1}{(4\pi)^2} \int d^4x \sqrt{g'} \left( \frac{1}{2} h_{\mu\nu} T(g')^{\mu\nu}_{\lambda\sigma} h^{\lambda\sigma} + cR'^2 - m^2 R' + \Lambda \right),$$

(5.3)

where $h_{\mu\nu} = g'_{\mu\lambda} h^{\lambda\nu}$. To justify one-loop calculations we discard the linear term of $h$ in the expansion of classical action by imposing the constraints $R'_{\mu\nu} = \frac{1}{4} g'_{\mu\nu} R'$ and $\nabla'_{\mu} R' = 0$. The induced action is calculated using the quantity $\mathcal{H}^{(2)}(x, \epsilon)$ for the operator $T$, or one loop divergence of $\det T$.

To calculate the coefficients $a$ and $b$ in (3.16) we have to fix the gauge. The Lagrangian for the traceless part is described in the form

$$\frac{1}{2} \sqrt{g'} h_{\mu\nu} T(g')^{\mu\nu}_{\lambda\sigma} h^{\lambda\sigma} = \frac{1}{2} \sqrt{g'} h_{\mu\nu} T^{NS}(g')^{\mu\nu}_{\lambda\sigma} h^{\lambda\sigma} + \chi^\mu N(g')_{\mu\nu} \chi^\nu,$$

(5.4)

where $\chi^\mu = \nabla'_{\lambda} h^{\mu\lambda}$. The nonsingular operator $T^{NS}$ and $N$ are defined by eqs. (A.10) and (A.11). According to the standard procedure for the 4-th order operators [1, 9, 11] we adopt the gauge-fixing condition $\chi^\mu = 0$ and gauge-fixing term such that the action $I_G + I_{F\times}$ is the only non-singular action of $T^{NS}$. Applying the general coordinate transformation $\delta h_{\mu\nu} = \nabla_{\mu} \xi^{\nu} + \nabla_{\nu} \xi^{\mu} - \frac{1}{2} (\nabla_{\lambda} \xi^{\lambda}) g_{\mu\nu}$ to the gauge-fixing condition we obtain the ghost Lagrangian $\sqrt{g'} \psi^\mu M_{GH}(g')_{\mu\nu} \psi^\nu$ with

$$M_{GH}(g')_{\mu\nu} = \Box' g'_{\mu\nu} + \frac{1}{2} \nabla'_{\mu} \nabla'_{\nu} + R'_{\mu\nu}. $$

(5.5)

Then the contribution from the measure of traceless mode can be derived by calculating the quantity

$$\delta\phi S(\phi, \hat{g}) = -\delta\phi \log \frac{\det^{1/2} N(g') \det M_{GH}(g')}{\det^{1/2} T^{NS}(g')} \big|_{\text{kernel part}}$$

$$= -\frac{1}{(4\pi)^2} \int d^4x \delta\phi \sqrt{g'} \left( a_2^{(2)}(T^{NS}) - a_2^{(1)}(N) - 2a_2^{(1)}(M_{GH}) \right),$$

(5.6)

where $a_2^{(n)}$ is defined in eq. (B.1). Using the formulae (B.10) and (B.12), we obtain the following quantities:

$$a_2^{(2)}(T^{NS}) = \frac{21}{10} R'_{\mu\nu\lambda\sigma} R'^{\mu\nu\lambda\sigma} + \frac{29}{40} R'^2,$$

(5.7)
\[ a_2^{(1)}(N) = -\frac{11}{180} R'_{\mu\nu\lambda\sigma} R'^{\mu\nu\lambda\sigma} + \frac{161}{120} R'^2, \quad (5.8) \]

\[ a_2^{(1)}(M_{GH}) = -\frac{11}{180} R'_{\mu\nu\lambda\sigma} R'^{\mu\nu\lambda\sigma} + \frac{11}{45} R'^2. \quad (5.9) \]

The combinations \( F' \) and \( G' \) are now described in the forms \( R'_{\mu\nu\lambda\sigma} R'^{\mu\nu\lambda\sigma} - \frac{1}{6} R'^2 \) and \( R'_{\mu\nu\lambda\sigma} R'^{\mu\nu\lambda\sigma} \) respectively. So we can determine the coefficients \( a \) and \( b \) of the induced action, which are given by [9, 10]

\[ a_h = -\frac{199}{30}, \quad b_h = \frac{87}{20}. \quad (5.10) \]

5.1.2 Conformal mode

As in the two dimensional cases [14, 15], we assume that the contribution from the measure of conformal mode is given in the form (3.16). The coefficients \( a_\phi \) and \( b_\phi \) are determined in a self-consistent way. Consider the conformal change of the background metric

\[ \hat{g}_{\mu\nu} \rightarrow \hat{g}^{(\omega)}_{\mu\nu} = e^{2\omega} \hat{g}_{\mu\nu}. \quad (5.11) \]

We then obtain the partition function

\[ Z(\hat{g}(\omega)) = \int [d\phi]\hat{g}(\omega)[dh]\hat{g}(\omega)[dX]\hat{g}(\omega)[dA]\hat{g}(\omega) \exp \left[ -\mathcal{I}^{(0)}(X, A, h, \phi; \hat{g}(\omega)) \right] \quad (5.12) \]

and

\[ \mathcal{I}^{(0)}(X, A, h, \phi; \hat{g}(\omega)) = S(\phi, \hat{g}(\omega)) + I_{CS}(X, \hat{g}(\omega)) + I_A(A, \hat{g}(\omega)) + I_G^{(0)}(h, e^{2\phi} \hat{g}(\omega)). \quad (5.13) \]

The coefficients of the induced action are given by \( a = a_X + a_A + a_h + a_\phi \) and \( b = b_X + b_A + b_h + b_\phi \). The \( \omega \)-dependence of the measures for \( X \), \( A_\mu \) and \( h^{\mu\nu} \) can be obtained by repeating the previous calculations with \( \phi \) replaced with \( \omega \), which are given by the action \( S(\omega, \hat{g}) \) with the coefficients (4.4), (4.7) and (5.11) respectively.

The contribution from the conformal mode is calculated by using the definition of the partition function above. The Einstein-Hilbert and the cosmological terms have dimensional parameters so that, when the 4-th order term exists, these terms do not contribute to the 4-th order induced action (3.11). To justify calculations we have to set the linear term of \( \phi \) vanishing.
To do this, however, we have to relate \( F(\omega) \) and \( G(\omega) \) so that we cannot determine the coefficients \( a \) and \( b \) because for lack of information. Therefore we neglect the \( \phi^2 \)-term of the total action. It can be carried out by canceling out the \( R^2 \)-terms from the classical action and the induced action by taking the value

\[
c = \frac{1}{18}(a + b) .
\]  

(5.14)

In this case we only calculate the quantity

\[
\delta_\omega S(\omega, \hat{g}) = -\delta_\omega \log \det^{-1/2} \Delta_4(\omega) = -\frac{1}{(4\pi)^2} \int d^4x \delta_\omega \sqrt{\hat{g}(\omega)} a_2^2(\Delta_4^{(\omega)}) .
\]  

(5.15)

Using the formula (B.10) and the definition of \( \Delta_4^{(3.15)} \), we obtain

\[
a_2^2(\Delta_4^{(\omega)}) = \frac{1}{90} \hat{R}_{\mu\nu\lambda\sigma} \hat{R}^{\mu\nu\lambda\sigma}_{(\omega)} + \frac{1}{90} \hat{R}_2^{(\omega)}. 
\]

(5.16)

From this we get the values of coefficients in the induced action [19]:

\[
a_\phi = \frac{1}{15}, \quad b_\phi = -\frac{7}{90} .
\]

(5.17)

### 5.1.3 Background-metric independence at the \( t \to 0 \) limit

Combining the results calculated before we can extract the \( \omega \)-dependence of the measure in the partition function (5.12). We thus obtain the expression

\[
Z(\hat{g}(\omega)) = \int [d\phi][dh][dX][dA] \exp \left[ -S(\omega, \hat{g}) - I^{(0)}(X^{\omega}, A, h, \phi; \hat{g}(\omega)) \right],
\]

(5.18)

where \( X^{\omega} = e^{-\omega}X \) such that \( I_{CS}(X^{\omega}, \hat{g}(\omega)) = I_{CS}(X, \hat{g}) \). The coefficients for the induced action \( S(\omega, \hat{g}) \) and also \( S(\phi, \hat{g}) \) in the action \( I^{(0)} \) are given by

\[
a = -\frac{N_x}{120} - \frac{N_A}{10} - \frac{199}{30} + \frac{1}{15},
\]

\[
b = \frac{N_x}{360} + \frac{31N_A}{180} + \frac{87}{20} - \frac{7}{90} .
\]

(5.19)

(5.20)

Since the measure \( [d\phi] \) is now invariant under a local shift, it turns out that, changing the variable as \( \phi \to \phi - \omega \) and using the Wess-Zumino condition \( S(\phi - \omega; \hat{g}(\omega)) + S(\omega, \hat{g}) = S(\phi, \hat{g}) \), the partition function goes back to the original form defined on the metric \( \hat{g} \). Thus we proved \( Z(\hat{g}(\omega)) = Z(\hat{g}) \) in the \( t \to 0 \) limit.
5.2 The induced action for $t \neq 0$

The background-metric independence for the traceless mode indicates that the $t$-dependence of the induced action, apart from $W(e^h, \hat{g})$ \footnote{This action does not affect the later calculations.}, should appear in the combination of the metric $\hat{g} = \hat{g}^{th}$ because the measure defined on the background metric itself is invariant under the simultaneous changes for the traceless mode (2.13). Now we assume the $t$-dependence of the partition function in the following form:

$$Z = \int \frac{[d\phi]_{\hat{g}}[1_t e^{-th} d\theta t]}{\text{vol(gauge)}} \exp \left[ -\mathcal{I}(X, A, \phi; \hat{g}) \right], \quad (5.21)$$

where the total action is

$$\mathcal{I}(X, A, \phi; \hat{g}) = \frac{1}{(4\pi)^2} \int d^4x \sqrt{\hat{g}} \left[ 2b(t)\phi \Delta_4 \phi + a(t)\bar{F}\phi + b(t) \left( \bar{G} - \frac{2}{3} \Box \bar{R} \right) \phi \right. \\
+ \frac{1}{t^2} \bar{F} + \frac{1}{18} (a(t) + b(t))\bar{R}^2 \left. \right] + \frac{1}{(4\pi)^2} \int d^4x \sqrt{\hat{g}} (-m^2 \bar{R} + \Lambda) + I_{CS}(X, \hat{g}) + I_A(A, \hat{g}), \quad (5.22)$$

where $a(t) = \sum_n a_n t^{2n}$ and $b(t) = \sum_n b_n t^{2n}$ with $a_0 = a$ and $b_0 = b$ given in (5.19) and (5.20) respectively. The coefficient in front of the classical $R^2$-action is now defined by the $t$-dependent value $c(t) = \frac{1}{18} (a(t) + b(t))$ so that the $R^2$-terms cancel out. Here note that the $\bar{R}^2$-term in (3.16) remains in the action.

Let us consider the conformal change of the background-metric (5.11). The $\omega$-dependences of the measures are now calculated as perturbations in $t$. The contributions from matter fields have already been calculated in Sect.3. The gravitational contributions are evaluated using the total action defined above. Expanding the action up to the $t^2$-order, the quadratic terms in fields is given by

$$\mathcal{I}_2(\hat{g}(\omega)) = \frac{1}{(4\pi)^2} \int d^4x \sqrt{\hat{g}(\omega)} \left[ \frac{1}{2} h_{\mu\nu} \{ T^{NS}(\hat{g}(\omega))_{\mu\nu,\lambda\sigma} + c t^2 \hat{R}(\omega) L(\hat{g}(\omega))_{\mu\nu,\lambda\sigma} \} h^{\lambda\sigma} \right. \\
+ 2(b + b_1 t^2) \phi \Delta_4^{(4)} \phi - 4(a + b)t \phi \hat{R}_\mu^{\mu\lambda\sigma} \Delta_\lambda^{(4)} \hat{\nabla}_\sigma^{(4)} h_{\mu\nu} \left. \right]$$

This action does not affect the later calculations.
expression (5.24) disappears in the gauge-fixed action

\[ \xi \text{ responds to take the gauge-fixing condition} \]

Thus we take the gauge-fixing term

\[ \text{where} \]

become diagonal. To do this we rewrite the action

\[ \text{term of} \]

\[ \delta g = 1 \]

\[ \mu \]

\[ \nu \]

\[ \lambda \]

\[ \sigma \]

\[ \text{reduces to the form} \]

\[ \hat{\Delta}^{(w)} \]

The ghost action. The kinetic term of the ghost Lagrangian is now given in

\[ \text{the second order operator} L^{\mu \nu \chi \sigma} \text{is defined by (A.8).} \]

The conditions \( \hat{R}^{(w)} = \frac{4}{3} \hat{g}_{\mu \nu} \hat{R}^{(w)} \) and \( \hat{\nabla}_{(w)} \hat{R}^{(w)} = 0 \) are imposed for the linear term of \( h \) to vanish. Under the conditions, the quartic operator \( \hat{\Delta}^{(w)} \) reduces to the form

\[ \hat{\Delta}^{(w)} \]

The gauge-fixing term is defined such that the highest derivative terms become diagonal. To do this we rewrite the action \( \mathcal{I}_2 \) in the form

\[ \frac{1}{(4\pi)^2} \int d^4 x \sqrt{g(\omega)} \left\{ T^{NS}(\hat{g}(\omega))^{\mu \nu \chi \sigma} + ct^2 \hat{R}(\hat{g}(\omega))^{\mu \nu \chi \sigma} \right\} h^{\lambda \sigma} \]

\[ + 2(b + bt^2) \phi \hat{\Delta}^{(w)} \phi + \frac{b^2 t^2}{6} (\phi \Box (\omega) \phi - \hat{R}(\omega) \phi) \]

\[ -4(a + bt^2) \phi \hat{R}^{\mu \nu \chi \sigma} \hat{\nabla}_\lambda (\omega) \hat{\nabla}_\sigma (\omega) h_{\mu \nu} - \frac{1}{3} (a + 2b) t \phi \hat{R}^{(w)} \hat{\nabla}_\mu (\omega) \hat{\nabla}_\nu (\omega) h_{\mu \nu} \]

\[ \left\{ \nabla^\nu (\omega) \psi + \frac{b t}{2} \hat{\nabla}_\mu (\omega) \phi \right\} \mathcal{N}(\hat{g}(\omega))_{\mu \nu} \left( \chi^\nu (\omega) + \frac{b t}{2} \hat{\nabla}_\nu (\omega) \phi \right) \right], \]

where \( \chi^\mu (\omega) = \hat{\nabla}_\mu (\omega) h^{\lambda \sigma} \) and

\[ \mathcal{N}(\hat{g}(\omega))_{\mu \nu} = N(\hat{g}(\omega))_{\mu \nu} + ct^2 ( - \hat{\nabla}_\mu (\omega) \hat{\nabla}_\nu (\omega) + \hat{R}(\omega) \hat{g}_{\mu \nu} ) \]  

Thus we take the gauge-fixing term \( \mathcal{I}_{FIX} \) such that the last term of the expression (5.24) disappears in the gauge-fixed action \( \mathcal{I}_2 + \mathcal{I}_{FIX} \). This corresponds to take the gauge-fixing condition \( \chi^\mu (\omega) + \frac{b t}{2} \hat{\nabla}_\mu (\omega) \phi = 0 \). The general coordinate transformation \( \delta \hat{g}_{\mu \nu} = g_{\mu \lambda} \nabla_\nu \xi^\lambda + g_{\nu \lambda} \nabla_\mu \xi^\lambda \) is expressed as

\[ \delta \phi = \frac{1}{4} \hat{\nabla}_\lambda (\omega) \xi^\lambda + \xi^\lambda \hat{\nabla}_\lambda (\omega) \phi, \]

\[ \frac{t}{2} \hat{h}_\nu (\omega) \xi^\mu + \hat{\nabla}_\nu (\omega) \xi^\mu - \frac{1}{2} \delta^\mu_\nu \hat{\nabla}_\lambda (\omega) \xi^\lambda + t \xi^\lambda \hat{\nabla}_\lambda (\omega) h_{\mu \nu} \]

\[ + \frac{t}{2} h^\lambda (\omega) (\hat{\nabla}_\nu (\omega) \xi^\lambda = \hat{\nabla}_\nu (\omega) \xi^\lambda + \xi^\lambda \hat{\nabla}_\nu (\omega) h_{\mu \nu} \]

\[ + \frac{t}{2} h^\lambda (\omega) (\hat{\nabla}_\nu (\omega) \xi^\lambda - \hat{\nabla}_\nu (\omega) \xi^\lambda ) + \frac{t}{2} h^\lambda (\omega) \xi^\mu \]

where \( \xi^\mu = \hat{g}^{(w)}(\omega) \xi^\lambda \). Applying it to the gauge-fixing condition we can obtain the ghost action. The kinetic term of the ghost Lagrangian is now given in
the form $\sqrt{g(\omega)}\psi^\mu \mathcal{M}_{GH} (\hat{g}(\omega))_{\mu\nu} \psi^\nu$ with

$$\mathcal{M}_{GH} (\hat{g}(\omega))_{\mu\nu} = M_{GH} (\hat{g}(\omega))_{\mu\nu} + \frac{bt^2}{8} \hat{\nabla}_\mu (\omega) \hat{\nabla}_\nu (\omega) .$$  \hfill (5.28)

Changing the normalization as $\phi' = (4b + 4bt^2 + \frac{b^2}{3}t^2)^{1/2}\phi$, we then obtain the following expression:

$$\mathcal{I}_2 + \mathcal{I}_{FIX} = \frac{1}{2(4\pi)^2} \int d^4 x \sqrt{g(\omega)} (\phi', h_{\mu\nu}) \mathcal{K} \left( \frac{\phi'}{h^{\lambda\sigma}} \right),$$  \hfill (5.29)

where

$$\mathcal{K} = \begin{pmatrix} \Box (\omega) & 0 \\ 0 & \Box (\omega) \delta^\mu_\lambda \delta^\nu_\sigma \end{pmatrix} + \begin{pmatrix} A & 0 \\ 0 & C_{\lambda\sigma}^{\mu\nu} \end{pmatrix} + \begin{pmatrix} 0 & B_{\lambda\sigma}^{\mu\nu} \\ B_{\lambda\sigma}^{\mu\nu} & 0 \end{pmatrix}$$  \hfill (5.30)

and

$$A = -\left(\frac{1}{6} + \frac{5}{72}bt^2\right) \hat{R}(\omega) \Box (\omega),$$

$$C_{\lambda\sigma}^{\mu\nu} = T^{NS} (\hat{g}(\omega))^{\mu\nu}_{\lambda\sigma} + ct^2 \hat{R}(\omega) L (\hat{g}(\omega))^{\mu\nu}_{\lambda\sigma},$$

$$B_{\lambda\sigma}^{\mu\nu} = -\frac{t}{2\sqrt{b}} \left\{ 4(a + b) \hat{R}(\omega)^{\mu\nu}_{\lambda\sigma} \hat{\nabla}_\lambda (\omega) \hat{\nabla}_\sigma (\omega) + \frac{1}{3} (a + 2b) \hat{R}(\omega) \hat{\nabla}^{\mu} (\omega) \hat{\nabla}^{\nu} (\omega) \right\},$$  \hfill (5.31)

where the prime on $T^{NS}$ stands for removing the $\Box (\omega)$ term.

To derive the $\omega$-dependence of the measure for gravity we have to evaluate the quantity

$$\delta_\omega S(\omega, \hat{g}) = -\delta_\omega \log \frac{\det^{1/2} \mathcal{N}(\hat{g}(\omega)) \det \mathcal{M}_{GH}(\hat{g}(\omega))}{\det^{1/2} \mathcal{K}(\hat{g}(\omega))} \bigg|_{\text{kernel part}}$$  \hfill (5.32)

$$= -\frac{1}{(4\pi)^2} \int d^4 x \delta_\omega \sqrt{g(\omega)} (a_2^{(2)}(\mathcal{K}) - a_2^{(1)}(\mathcal{N}) - 2a_2^{(1)}(\mathcal{M}_{GH})),$$

Using the generalized Schwinger-DeWitt technique [12, 9] summarized in appendix B, we first calculate the divergent part of $\log \det \mathcal{K} = Tr \log \mathcal{K}$, which is expanded in inverse powers of derivatives as

$$-Tr \log \mathcal{K} = \gamma(A) + \gamma(C) + \gamma(B),$$  \hfill (5.33)
\[ \gamma(A) = -2Tr \log □(ω) - Tr(A \frac{1}{□(ω)}) + \frac{1}{2}Tr(A^2 \frac{1}{□(ω)}) \]
\[ \gamma(C) = -2Tr \log(□(ω)I) - Tr(C \frac{1}{□(ω)}) + \frac{1}{2}Tr(C^2 \frac{1}{□(ω)}) \]
\[ \gamma(B) = Tr(B^2 \frac{1}{□(ω)}) , \]

where I = \( \delta^\mu_\lambda \delta^\nu_\sigma - \frac{1}{2} (\delta^\mu_\lambda \delta^\nu_\sigma + \delta^\mu_\sigma \delta^\nu_\lambda) \) and Tr includes the trace over the indices \( \mu, \nu \).

The contribution to the induced action from the diagonal part of the conformal mode is calculated as (2.2)
\[ a_2^2(2^2(A)) = a_2^2(\hat{\Delta}_ω^4), \]
where the \( t^4 \)-term is neglected. It turns out that the \( t^2 \)-correction does not appear in this part.

For the diagonal part of the traceless mode we obtain the following quantity:
\[ a_2^2(B) = a_2^2(TNS(\hat{g}(ω))) = 6et^2\hat{R}^2(ω) . \]
The off-diagonal part is calculated by using the formula (B.7), which gives the \( t^2 \)-order contribution
\[ a_2^2(2^2(B)) = t^2 \left( \frac{(a + b)^2}{4b} R_\mu^\omega \lambda^\sigma R_\mu^\nu^\lambda^\sigma - \frac{2a^2 + 4ab + b^2}{48b} \hat{R}^2(ω) \right) . \]

Thus \( a_2^2(K) \) is given by summing up the results from the operators A, B and C.

The ghost parts are calculated as \( a_2^2(\mathcal{N}) = a_2^2(N(\hat{g}(ω))) - \frac{5}{2}ct^2\hat{R}^2(ω) \) and also \( a_2^2(M_{GH}) = a_2^2(M_{GH}(\hat{g}(ω))) - \frac{1}{72}bt^2\hat{R}^2(ω) \). Combining the above results we finally obtain the \( t^2 \)-dependent part of \( a_2^2(K) - a_2^0(\mathcal{N}) - 2a_2^0(M_{GH}) \) in the form
\[ t^2 \left( \frac{(a + b)^2}{4b} R_\mu^\omega \lambda^\sigma R_\mu^\nu^\lambda^\sigma - \frac{6a^2 + 40ab + 27b^2}{144b} \hat{R}^2(ω) \right) . \]

Now, we can determine the coefficients \( a_1 \) and \( b_1 \) from the above results. We finally obtain the induced action \( S(ω, \hat{g}) \) with the coefficients
\[ a_1 = -\frac{6a^2 + 40ab + 27b^2}{24b} , \]
\[ b_1 = \frac{7}{6}a + \frac{7}{8}b . \]
Here note that the measures of matter fields do not give the contributions directly to the coefficients $a_1$ and $b_1$, which contribute indirectly to them through the values $a$ and $b$ given by (5.19) and (5.20).

The background-metric independence for the traceless mode indicates that the induced action should be in the form $S(\omega, \bar{g})$ if it includes the interaction terms. As discussed in the previous subsection this $\omega$-dependence can be removed by changing the field: $\phi \to \phi - \omega$ so that the partition function goes back to the original one defined on $\bar{g}$ (5.21) provided $a_1$ and $b_1$ are given by (5.37) and (5.38). In summary we get the coefficients in the action (5.22) as

$$a(t) = -\frac{N_X}{120} - \frac{N_A}{10} - \frac{197}{30} + t^2 \frac{13N_X^2 + 1412N_XN_A + 36988N_X - 7428N_A^2 + 635656N_A + 16011772}{2880(N_X + 62N_A + 1538)},$$

$$b(t) = \frac{N_X}{360} + \frac{31N_A}{180} + \frac{769}{180} - t^2 \left( \frac{7N_X}{960} - \frac{49N_A}{1440} + \frac{1883}{480} \right).$$

6 Discussions on Scaling Operators

In this paper we proposed the background-metric independent formulation of 4D quantum gravity. A model of 4D quantum gravity was described as a quantum field theory defined on the background-metric (5.21) with the coefficients (5.39, 5.40) by solving the ansatz of the background-metric independence (5.22) in the self-consistent manner.

The problem of renormalizability still remains to be solved, but we think that if the diffeomorphism invariance ensures renormalizability, our model will be renormalizable because we can easily show that the background-metric independence really ensures the diffeomorphism invariance in quantum level [28].

The rest of this section is devoted to discuss scaling operators in 4D quantum gravity. The cosmological constant and the Einstein-Hilbert terms are the lower-derivative operators with conformal charges. As in two dimensions, such operators will receive corrections like

$$\Lambda \int d^4x \sqrt{\bar{g}} e^{\alpha(t)\phi}$$

(6.1)
for the cosmological constant and

\[-m^2 \int d^4x \sqrt{g} e^{\beta(t)\phi} \left( \tilde{R} + \gamma(t) \nabla^\mu \phi \nabla_\mu \phi \right)\]  \hspace{1cm} (6.2)

for the Einstein-Hilbert term.

Henceforth we take the flat background-metric for simplicity, though in perturbation theory we should choose a background-metric such that the approximation is well defined. At least up to the \(t^2\)-order, we can use the argument on the scaling operators in refs. [18, 20]. As for the cosmological constant operator, the conformal charge is classically given by \(\alpha(t) = \text{dim } [\Lambda] = 4\). In quantum theory it receives a correction as \(\alpha(t) = 4 + \gamma_\Lambda\). The anomalous dimension is now calculated using the gauge-fixed action in the form \(\gamma_\Lambda = \frac{\alpha(t)^2}{2b'(t)}\), where \(b'(t) = b(t) + \frac{\nu^2 + 4}{12}\), so that the quadratic equation is obtained. Solving the equation, we get the following value:

\[\alpha(t) = 2b'(t) \left( 1 - \sqrt{1 - \frac{4}{b'(t)}} \right),\]  \hspace{1cm} (6.3)

where the solution such that \(\alpha(t) \to 4\) at the classical limit \(b'(t) \to \infty\) is chosen. Similarly for the Einstein-Hilbert term we obtain

\[\beta(t) = 2b'(t) \left( 1 - \sqrt{1 - \frac{2}{b'(t)}} \right).\]  \hspace{1cm} (6.4)

Physically the cosmological constant should be real so that we obtain the condition \(b'(t) \geq 4\) for 4D quantum gravity to exist. Recently the evidence of a smooth phase in 4D simplicial quantum gravity coupled to \(U(1)\) gauge theories is reported in the numerical simulations [26], which suggests that \(b'(t) < 4\) for \(N_A = 0\), but \(b'(t) > 4\) for \(N_A \geq 1\). Naively, comparing with our result we obtain the value for the coupling constant: \(0.11 < t^2 < 0.20\). This seems to indicate that the perturbation of \(t\) on the flat background-metric is not so bad.

Finally we give a comment on the consistency of our calculations. Consider a constant shift of conformal field: \(\phi \to \phi + \eta\). Then the mass scales are rescaled as \(\Lambda \to \Lambda e^{\alpha(t)\eta}\) and \(m^2 \to m^2 e^{\beta(t)\eta}\), and the Weyl action with the coefficient \(\eta a(t)\) is induced as well. This extra Weyl action, however, gives at most \(t^4\)-corrections to the coefficients \(a(t)\) and \(b(t)\) of the induced action so that our results are self-consistent at least up to the \(t^2\)-order.
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Appendix

A Some Important Formulae

The conformal mode dependence of curvatures is given by

\begin{align}
R &= e^{-2\phi} (R - 6 \Box \phi - 6 \nabla^\mu \phi \nabla_\mu \phi), \\
R_{\mu\nu} &= \hat{R}_{\mu\nu} - 2 \nabla_\mu \nabla_\nu \phi + 2 \hat{\nabla}_\mu \phi \hat{\nabla}_\nu \phi - \hat{g}_{\mu\nu} (2 \nabla^\lambda \phi \nabla_\lambda \phi + \Box \phi),
\end{align}

where \( \hat{g}_{\mu\nu} = e^{2\phi} \hat{g}_{\mu\nu} \). The square of the Weyl tensor \( F \) defined by eq.(3.12) is conformally covariant: \( F = e^{-4\phi} \hat{F} \). The Euler density \( G \) defined by eq.(3.13) is a total derivative, which is proved by using the Riemann identity [2]

\begin{align}
R^{\alpha\beta\gamma} R^\nu_{\alpha\beta\gamma} - 2 R^{\mu\alpha\nu\beta} R_{\alpha\beta} - 2 R^{\mu\alpha} R^\nu_{\alpha} + R^{\mu\nu} R = \frac{1}{4} \hat{g}^{\mu\nu} G.
\end{align}

The curvature is expanded w.r.t. the traceless mode as

\begin{align}
\bar{R} &= \hat{R} - \hat{R}_{\mu\nu} h^{\mu\nu} + \hat{\nabla}_\mu \hat{\nabla}_\nu h^{\mu\nu} - \frac{1}{4} \hat{\nabla}^\lambda h^\mu_{\nu} \hat{\nabla}_\lambda h^\nu_{\mu} \\
&+ \frac{1}{2} \hat{R}^\sigma_{\mu\nu} h^\lambda_{\sigma} h^{\mu\nu} + \frac{1}{2} \hat{\nabla}^\nu h^{\mu}_{\mu} \hat{\nabla}_\lambda h^{\lambda}_{\mu} - \hat{\nabla}_\mu (h^\mu_{\nu} \hat{\nabla}^\lambda h^{\nu}_{\lambda}) + \cdots,
\end{align}

where \( \hat{g}_{\mu\nu} = (\hat{g} e^h)_{\mu\nu} \) and \( h_{\mu\nu} = \hat{g}_{\mu\lambda} h^{\lambda}_{\nu} \).

Under the conditions \( \hat{R}_{\mu\nu} = \frac{1}{4} \hat{g}_{\mu\nu} \hat{R} \) and \( \hat{\nabla}_\mu \hat{R} = 0 \), the 4-th order actions are expanded in the followings. The functions \( \bar{G} \) and \( \bar{F} \) are given by

\begin{align}
\bar{G} &= \hat{G} - 4 \hat{R}^{\mu\lambda\sigma\rho} \hat{\nabla}_\lambda \hat{\nabla}_\sigma h_{\mu\rho} + \cdots, \\
\bar{F} &= \hat{F} - 4 \hat{R}^{\mu\lambda\sigma\rho} \hat{\nabla}_\lambda \hat{\nabla}_\sigma h_{\mu\rho} - \frac{1}{3} \hat{R} \hat{\nabla}^\mu \hat{\nabla}^\nu h_{\mu\nu} + \cdots.
\end{align}
Here note that the linear terms of \( h \) can be written in total derivative ones because \( \hat{\nabla}^\mu \hat{R}_{\mu \alpha \beta \gamma} = \hat{\nabla}_\beta \hat{R}_{\alpha \gamma} - \hat{\nabla}_\gamma \hat{R}_{\alpha \beta} = 0 \). The \( \hat{R}^2 \)-action is

\[
\int d^4x \sqrt{\hat{g}} \hat{R}^2 = \int d^4x \sqrt{\hat{g}} \left( \hat{R}^2 + \frac{1}{2} h_{\mu \nu} \hat{R} L^\mu \nu \lambda \sigma (\hat{g}) h^{\lambda \sigma} \right) - \chi^\mu \hat{\nabla}_\mu \hat{\nabla}_\nu \chi^\nu + \hat{R} \chi^\mu \chi_\mu + \cdots \tag{A.7}
\]

where \( \chi^\mu = \hat{\nabla}^\lambda h_{\lambda}^\mu \) and

\[
L^\mu \nu \lambda \sigma (\hat{g}) = \hat{\Box} \delta^\mu_{(\lambda} \delta^\nu_{\sigma)} + 2 \hat{R} \chi_{\lambda}^\mu \chi_{\sigma}^\nu . \tag{A.8}
\]

The Weyl action becomes

\[
\int d^4x \sqrt{g} F = 2 \int d^4x \sqrt{\hat{g}} \left( \hat{R}^\mu \nu \hat{R}_{\mu \nu} - \frac{1}{3} \hat{R}^2 \right) = \int d^4x \sqrt{\hat{g}} \left[ -\frac{1}{6} \hat{R}^2 + \frac{1}{2} h_{\mu \nu} T^{NS}(\hat{g})^\mu \nu \lambda \sigma \chi^\lambda \chi^\sigma + \chi^\mu \chi^\nu \right] + \cdots \tag{A.9}
\]

up to the Euler number. The nonsingular operators \( T^{NS} \) and \( N \) are defined by

\[
T^{NS}(\hat{g})^\mu \nu \lambda \sigma = \hat{\Box} \delta^\mu_{(\lambda} \delta^\nu_{\sigma)} - \frac{1}{6} \hat{R} \hat{\Box} \delta^\mu_{(\lambda} \delta^\nu_{\sigma)} + 4 \hat{R} \chi^\mu \chi^\nu \tag{A.10}
\]

and

\[
N(\hat{g})_{\mu \nu} = \hat{\Box} \hat{g}_{\mu \nu} - \frac{1}{3} \hat{\nabla}_\mu \hat{\nabla}_\nu - \frac{11}{12} \hat{R} \hat{g}_{\mu \nu} . \tag{A.11}
\]

### B Generalized Schwinger-DeWitt Technique

Expand the covariant quantity \( \mathcal{H}^{(n)}(x, s) = \langle x | e^{-sD^{(n)}} | x \rangle \) in 4 dimensions for \( n = 1, 2 \) as

\[
\mathcal{H}^{(n)}(x, s) = \frac{1}{(4\pi)^2} \frac{1}{ns^n} \left( a_0^{(n)} + a_1^{(n)} s + a_2^{(n)} s^2 + \cdots \right) . \tag{B.1}
\]
The $s$-independent term is given by $a_2^{(n)}$ for $n = 1, 2$. On the other hand the divergence is described by using the quantity $a_2^{(n)}$ as follows:

$$ - Tr \log D^{(n)} = \frac{\log L^2}{(4\pi)^2} \int d^4x \sqrt{g} \ a_2^{(n)}. \quad (B.2) $$

Thus we must calculate the $\log L^2$ divergences to determine the coefficients $a_2^{(n)}$ up to the $\Box R$-term.

Let us first consider the 4-th order operator in 4 dimensions defined by eq.(3.1) with $n = 2$. It is here expressed as

$$ D = \Box^2 I + \Pi, \quad (B.3) $$

where $I$ is the identity operator for the indices $A, B$ and $\Pi$ is at most second order matrix operator. Then one gets the following expression [12]:

$$ - Tr \log D = -2Tr \log I - Tr \left( \Pi \frac{1}{\Box^2} \right) + \frac{1}{2} Tr \left( \Pi^2 \frac{1}{\Box^4} \right). \quad (B.4) $$

The r.h.s. is calculated by using the universal functional trace formulae [12]

$$ Tr \log \Box |_{div} = -\frac{\log L^2}{(4\pi)^2} \int d^4x \sqrt{g} \ tr \left[ \left( \frac{1}{180} R_{\mu\nu,\lambda\sigma} R_{\mu\nu}^{\lambda\sigma} - \frac{1}{180} R_{\mu\nu} R_{\mu\nu}^{\lambda\sigma} + \frac{1}{12} R_{\mu\nu} R_{\mu\nu}^{\lambda\sigma} \right) \right] (B.5) $$

and

$$ \nabla_\mu \nabla_\nu \frac{1}{\Box^2} \delta(y, x)|_{y=x} = \frac{\log L^2}{(4\pi)^2 \sqrt{g}} \left[ \left( \frac{1}{6} R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) I + \frac{1}{2} R_{\mu\nu} R_{\mu\nu} \right], (B.6) $$

$$ \nabla_\mu \nabla_\nu \nabla_\lambda \nabla_\sigma \frac{1}{\Box^4} \delta(y, x)|_{y=x} = \frac{\log L^2}{(4\pi)^2 \sqrt{g}} \left[ \frac{1}{24} \left( g_{\mu\nu} g_{\lambda\sigma} + g_{\mu\lambda} g_{\nu\sigma} + g_{\mu\sigma} g_{\nu\lambda} \right) \right], (B.7) $$

where $(R_{\mu\nu})^A_B$ is defined by

$$ [\nabla_\mu, \nabla_\nu] \varphi^A = (R_{\mu\nu})^A_B \varphi^B. \quad (B.8) $$

It is useful to consider the following general form to evaluate the diagonal parts of gravity sector in Sect.5:

$$ D = \Box^2 I + X^{\alpha\beta} \nabla_\alpha \nabla_\beta + Y^\alpha \nabla_\alpha + Z, \quad (B.9) $$

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where $X^{\alpha\beta} = X^{\beta\alpha}$. Using the formulae listed above one gets the following expression for the divergent part \[9, 12\]:

$$- Tr \log D |^{\text{div}} = \frac{\log L^2}{(4\pi)^2} \int d^4x \sqrt{g} \ tr \left[ \frac{1}{90} \left( R_{\mu\nu\lambda\sigma} R^{\mu\nu\lambda\sigma} - R_{\mu\nu} R^{\mu\nu} \right) I \right. $$

$$+ \frac{1}{6} R_{\alpha\beta} R^{\alpha\beta} + \frac{1}{36} R^2 I - Z - \frac{1}{6} R_{\alpha\beta} X^{\alpha\beta} $$

$$\left. + \frac{1}{12} RX + \frac{1}{48} X^2 + \frac{1}{24} X_{\alpha\beta} X^{\alpha\beta} \right], \quad (B.10)$$

where $X = X^{\alpha}$.

In the case of $\varphi_A = h_{\mu\nu}$, the quantities $X, Y$ and $Z$ can be read from the expressions in appendix A and $(\mathcal{R}_{\alpha\beta})^{\mu\nu}_{\lambda\sigma} = R^{\mu\nu}_{\lambda\sigma} \delta^\nu_\sigma + R^{\nu}_{\sigma\alpha\beta} \delta^\mu_\lambda$. In calculating the trace for the indices $A = (\mu \nu)$, one has to take the traceless condition for $h_{\mu\nu}$ into account. It is carried out by replacing $h_{\mu\nu}$ with $H_{\mu\nu}$ defined by the relation $h_{\mu\nu} = H_{\mu\nu} - \frac{1}{4} g_{\mu\nu} H^\lambda_\lambda$, or replacing the operator $(X^{\alpha\beta})^{\mu\nu}_{\lambda\sigma}$, for example, with $(I_H X_{\alpha\beta})^{\mu\nu}_{\lambda\sigma}$, where $(I_H)^{\mu\nu}_{\lambda\sigma} = \delta^{\mu}_{\lambda} \delta^{\nu}_{\sigma} - \frac{1}{4} g^{\mu\nu} g_{\lambda\sigma}$ and $I^2_H = I_H$.

To evaluate the ghost parts it is useful to consider the following second order operator:

$$Q_{\mu\nu} = \Box g_{\mu\nu} - \lambda \nabla_\mu \nabla_\nu + Z_{\mu\nu}. \quad (B.11)$$

The divergent part is given by the formula \[9, 12\]

$$- Tr \log Q |^{\text{div}} = \frac{\log L^2}{(4\pi)^2} \int d^4x \sqrt{g} \ \frac{1}{3} \left[ - \frac{11}{60} \left( R_{\mu\nu\lambda\sigma} R^{\mu\nu\lambda\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \right) \right. $$

$$+ \left( \frac{\gamma^2}{8} + \gamma - \frac{4}{5} \right) R_{\mu\nu} R^{\mu\nu} + \left( \frac{\gamma^2}{16} + \frac{\gamma}{4} + \frac{7}{20} \right) R^2 $$

$$+ \left( \frac{\gamma^2}{4} + \gamma \right) R_{\mu\nu} Z^{\mu\nu} + \left( \frac{\gamma^2}{8} + \frac{3}{4} \gamma + \frac{3}{2} \right) Z_{\mu\nu} Z^{\mu\nu} $$

$$\left. + \left( \frac{\gamma^2}{8} + \frac{\gamma}{4} + \frac{1}{2} \right) R Z + \frac{\gamma^2}{16} Z^2 \right], \quad (B.12)$$

where $\gamma = \lambda/(1 - \lambda)$ and $Z = Z^{\mu}_{\mu}$.

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