Holographic Indeterminacy, Uncertainty and Noise

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A theory is developed to describe the nonlocal effect of spacetime quantization on position measurements transverse to macroscopic separations. Spacetime quantum states close to a classical null trajectory are approximated by plane wavefunctions of Planck wavelength \( l_P \) reference beams; these are used to connect transverse position operators at macroscopically separated events. Transverse positions of events with null spacetime separation, but separated by macroscopic spatial distance \( L \), are shown to be quantum conjugate observables, leading to holographic indeterminacy and a new uncertainty principle, a lower bound on the standard deviation of relative transverse position \( \Delta x_\perp > \sqrt{l_P L} \) or angular orientation \( \Delta \theta > \sqrt{l_P L} \). The resulting limit on the number of independent degrees of freedom is shown to agree quantitatively with holographic covariant entropy bounds derived from black hole physics and string theory. The theory predicts a universal “holographic noise” of spacetime, appearing as shear perturbations with a frequency-independent power spectral density \( S_H = l_P/c \), or in equivalent metric perturbation units, \( k_{H,\text{rms}} \approx \sqrt{l_P/c} = 2.3 \times 10^{-22}/\sqrt{\text{Hz}} \). If this description of holographic phenomenology is valid, interferometers with current technology could undertake direct quantitative studies of quantum gravity.

INTRODUCTION

In standard field theory, quantum particles and fields propagate and interact on an unquantized, classical spacetime manifold. Their behavior is described in terms of quantum operators that are functions of spacetime position. This idealized model entirely neglects the quantum degrees of freedom of spacetime itself. In reality, spacetime is widely thought to be a quantum system whose apparent classical properties, including the fundamental and invariant causal structure defined by null paths (such as light rays), emerge as a limit of a more fundamental quantum system.

Longstanding approaches to quantum gravity (as in ref. [2]) based, like field theory, on local quantization analysis, suggest that effects of spacetime quantization only become important at the Planck scale, \( l_P = \sqrt{\hbar G_N/c^3} = 1.616 \times 10^{-33}\text{cm} \), and average out on larger scales or lower energies. However, a reasonable alternative hypothesis is that positions on the spacetime manifold, like any other classical quantities, are defined by quantum observable operators. In that case, emergent spacetime will ultimately be described by introducing a fundamental quantum system prior to the assignment of spacetime position observables. In an extended, emergent “spacetime made of waves”, nonlocal measurements, involving comparison of observables at widely separated spacetime events, can show new behavior due to Planck scale quantization that is not predicted in field theory.

Indeed, general arguments[3, 4], based on the idea of the spacetime metric emerging from a wavefunction at Planck resolution, suggest a new quantum behavior of spacetime, called holographic indeterminacy, that may expose its quantum degrees of freedom to direct experimental investigation. The wavefunction of a spacetime null path connecting two events is modeled as a plane wave of Planck wavelength, with uncertain orientation in the transverse directions. Positions transverse to null trajectories are defined by quantum observables relative to hypothetical reference beams of Planck-wavelength radiation that define the metric. This general and simple idea is only an effective theory of a new quantum behavior, rather than a fundamental theory, but is sufficient to estimate the nonlocal, macroscopic physical phenomenon of holographic indeterminacy: nonlocal measurements of transverse positions display nonclassical quantum indeterminacy on scales much larger than the Planck length. This paper develops more fully the theory of holographic indeterminacy of spacetime and its observable consequences.

Independent motivation for holographic indeterminacy comes from the fact that the resulting decomposition of spacetime eigenstates, while not in agreement with field theory, does agree with holographic behavior of spacetime degrees of freedom estimated from the physics of black hole evaporation and string theory[5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22]. Holographic properties, including the scaling of entropy with area instead of volume and nonlocality of quantum correlations, have long been in puzzling contradiction to field theory. Holographic indeterminacy accounts for these effects, and also defines the spacetime states more explicitly than string theory.

Whereas string theory has not had a clearly developed classical limit to show the character of the effects of holography on observable phenomena viewed from inside a nearly-flat spacetime, the concrete hypothesis of holographic indeterminacy—which can be regarded as a particular hypothesis for how holographic encoding works in nature—results in definite predictions of observable physical effects. The theory presented here describes one way holographic behavior could appear to observers in a nearly flat spacetime like our local environment.

Holographic indeterminacy is thus distinguished from previous treatments of holographic effects by being a more
specific hypothesis with a more concrete predicted phenomenology. In other respects holographic indeterminacy appears to be a rather conservative implementation of holography. Quantum mechanical unitarity is preserved, as are local symmetries; the only new effects are nonlocal. Moreover, the nonlocal effects are localized in angle; the new correlations and holographic noise grow gradually from events along null trajectories and stay close to the classical trajectories. No new particles, symmetries, or dimensions are predicted, and indeed no new connection is made to physical fields; the new behavior is entirely geometrical and entirely nonlocal. Even Planck’s constant cancels out of the final results aside from one fundamental length, the Planck length. It remains to be seen how to apply these ideas in highly curved spacetimes such as the regions near black hole or cosmological event horizons; the analysis here deals only with nearly flat spacetimes.

It is interesting that the most promising experimental probes of this Planck-scale physics do not use Planck energy particles directly. Instead, the probes are macroscopic elements of laboratory-scale interferometers—proof masses whose own wavefunctions are spatially narrow and remain so even under a precise position measurement, so that particles directly. Instead, the probes are macroscopic elements of laboratory-scale interferometers—proof masses whose own wavefunctions are spatially narrow and remain so even under a precise position measurement, so that

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Holographic indeterminacy results from diffraction of the fundamental Planck scale waves over macroscopic distances. A classical trajectory is specified by the orientation of a plane wave (propagating on the z axis, say) and a transverse position \((x_\perp, y_\perp)\) in the plane of the wave. However, specifying a transverse position creates an uncertainty in transverse momentum, and therefore in distant transverse position on the same axis. Thus position observables at widely separated points in an emergent spacetime become conjugate quantum operators with the usual associated indeterminacy and uncertainty. An extended spacetime region emerging from Planck energy waves has a substantial quantum uncertainty, far larger than the Planck length.

The central hypothesis is that the position of any body in spacetime is defined by some quantum observable operator, \(\hat{x}\). It operates on quantum states of spacetime that connect measurements made at widely separated events, normally connected by assuming a classical metric. The following theory describes a candidate form of the connection between events on null trajectories, by reference to classical null trajectories.

Consider an operator \(\hat{x}_\perp\) that measures position along an axis \(x\) perpendicular to a classical null trajectory connecting events 1 and 2. At time \(t_1\) a measurement is made of body 1 and at time \(t_2\) a measurement is made of body 2. We can write a quantum-mechanical amplitude for obtaining a particular pair of position results as

\[
\langle x_\perp 1, x_\perp 2 \rangle = \langle \psi_{\text{body}2} | \hat{x}_\perp 2 | \psi_{\text{spacetime}} \rangle \langle \psi_{\text{spacetime}} | \hat{x}_\perp 1 | \psi_{\text{body}1} \rangle.
\]

(1)

The bodies in question may be proof masses in an interferometer, other observer apparatus, black holes, or elementary particles. The states of the bodies and the spacetime are labeled by their wavefunctions \(\psi\). In particular, the wavefunctions in the states \(|\psi_{\text{body}1}\rangle, |\psi_{\text{body}2}\rangle\) refer to the usual quantum mechanical wavefunctions referring to a local, classical spacetime position. This formula represents coherent combinations of spacetimes and positions leading to a given quantum-mechanical amplitude for the specified overall result.

The analysis below develops a simple model for the effects of the connecting spacetime, \(|\psi_{\text{spacetime}}\rangle\). In the case where \(\psi_{\text{body}1}\) and \(\psi_{\text{body}2}\) are very narrow—such as macroscopic proof masses in an interferometer—the structure of \(\psi_{\text{spacetime}}\) still leads to indeterminacy in position. The usual procedure in quantum mechanics and field theory assumes a classical spacetime, so that the state \(|\psi_{\text{spacetime}}\rangle \langle \psi_{\text{spacetime}} | \hat{x}_\perp 1 | \psi_{\text{body}1} \rangle\) is represented by a delta-function of deviation from the classical null trajectory from \(x_\perp 1\). The amplitude for body 2 to be at a particular position in that case is just given by the wavefunction of the body \(\psi_{\text{body}2}\), and indeed this idea defines the meaning of position wavefunction. When \(\psi_{\text{body}2}\) is very narrow, as in a macroscopic body rather than a particle, the position is classical. Here we focus on the new effects created by including the width of \(\psi_{\text{spacetime}}\).

The measurement process summarized in Eq. (1) does not specify a particular form of interaction, but comprises an operational definition of position. The relative position amplitude \(\langle x_\perp 1, x_\perp 2 \rangle\) replaces the definite classical positions of bodies or particles in their interactions with other bodies or particles. The difference is only noticed in nonlocal comparisons of positions. For those, the final amplitude now depends on the spacetime wavefunction connecting widely separated events, and has a width dominated by the overall envelope of \(\psi_{\text{spacetime}}\). No new locally detectable interactions or effective fields are predicted on either a classical or quantum level. Instead, there is a loss of precision and determinacy that increases with the scale over which a measurement is made.
The state $|\psi_{\text{spacetime}}\rangle$ can be decomposed in the usual way,

$$|\psi_{\text{spacetime}}\rangle = \sum a_i |\psi_i\rangle.$$  \hfill (2)

As usual in quantum mechanics, an observation fixes the state to be one of those components at the moment of observation, when it must be in an eigenstate of the observable operator. The measurement of an observable is a process of correlation that fixes a branch of the wavefunction along a future null cone from the observation event. The arguments here suggest a simple and general physical motivation for a particular kind of decomposition in nearly flat spacetime: the spacetime eigenstates are a set of approximately plane wavefunctions. Transverse localization leads to an indeterminacy of orientation for those wavefunctions.

We choose to decompose the spacetime wavefunction using classical null trajectories as reference states, since these define a frame-independent invariant causal structure. Consider a spacetime reference particle chosen to have Planck energy in a particular reference frame (again, not a transformation property expected of a regular particle or field). Its wavefunction will be identified with a particular eigenstate of $\psi_{\text{spacetime}}$. Its classical null trajectory is along the $z$ axis in flat spacetime, with $x = y = 0$, and momentum $p_0 = \hbar/l_P$. In the $z$ direction its quantum wavefunction is a plane wave,

$$\psi \propto \exp[i(t-z)/l_P]$$ \hfill (3)

However, specifying a classical trajectory to measure spatial positions (via Eq. 1) requires localization in the transverse directions, $x_\perp$ and $y_\perp$. As a result the orientation of the wave and therefore the classical trajectory are subject to quantum indeterminacy. Then the measured transverse positions of any pairs of bodies as emergently defined in Eq. (1) will have wavefunction widths relative to this classical null trajectory determined not mainly by their locally defined wavefunctions $\psi_{\text{body}}$, but that of the reference particle representing the spacetime wavefunction connecting them, $\psi_{\text{spacetime}}$.

At time $t_1$ the reference particle obeys the usual Heisenberg commutation relation between momentum and position operators along the transverse $x$-axis,

$$[\hat{x}_\perp(t_1), \hat{p}_\perp(t_1)] = -i\hbar,$$ \hfill (4)

where $x_\perp, p_\perp$ refer to any axis chosen perpendicular to the null trajectory connecting events 1 and 2. Thus the spacetime wavefunction is a plane wave in $z$ but has an uncertain orientation associated with localized measurements in the $x_\perp$ direction.

Consider the reference particle at time $t_2$ in a particular frame. (Recall that this is a null trajectory so the actual spacetime interval between these events vanishes.) The transverse momentum $p_\perp(t_1)$ of the particle at event 1 is related to its transverse position displacement at event 2 by the angular deflection,

$$p_\perp(t_1)/p_0 = p_\perp(t_1)l_P/\hbar = x_\perp(t_2)/(t_2-t_1).$$ \hfill (5)

Combining equations (4) and (5) yields a commutation relation between transverse position operators,

$$[\hat{x}_\perp(t_1), \hat{x}_\perp(t_2)] = -il_P(t_2-t_1).$$ \hfill (6)

This formula specifies in terms of observable operators the effect of the intervening spacetime operator, $|\psi_{\text{spacetime}}\rangle\langle\psi_{\text{spacetime}}|$, in Eq. (1). Transverse spatial positions thus become conjugate variables subject to quantum indeterminacy.

It is interesting that even though Eq. (6) is a quantum commutation relation, Planck’s constant $\hbar$ does not appear explicitly. Once the conjugate quantum variables are both spatial positions, $\hbar$ is no longer needed, as it is when relating position and momentum operators: the quantum uncertainty is a purely geometrical effect and depends only on the scale of a system compared to the fundamental length $l_P$. The actual Planck constant only appears implicitly in connection with other units (and with other fields via $G_N$) through the usual definition of $l_P$, which according to our conjecture plays the role of a minimum length for the fundamental theory in any frame.

It should be emphasized that the transverse momentum here does not refer to an interaction with a particular particle. Rather, it is the transverse momentum associated with the specification of any classical trajectory. Although we have used particle/wave duality to describe a reference particle, this is just a device for estimating the effects of the spacetime wavefunction, corresponding to departures from a definite classical null path. The plane wave is coherent in the transverse direction: nearby paths share almost the same value of deviation $x_\perp$ from a local reference trajectory. Thus the indeterminacy cannot be detected locally even in the transverse direction. Clearly this transverse coherence of an effective spacetime wavefunction cannot extend indefinitely but for consistency must decohere slowly with transverse separation, as discussed below.
**HOLOGRAPHIC UNCERTAINTY OF POSITION AND ANGLE**

In the usual way, the indeterminacy (Eq. [6]) yields a Heisenberg uncertainty relation,

\[ \Delta x_\perp(t_1)\Delta x_\perp(t_2) > l_p(t_2 - t_1)/2, \]  

(7)

where \( \Delta x_\perp(t_1), \Delta x_\perp(t_2) \) denote the standard deviations at events 1 and 2 of the distributions of position measurements of any body or particle, describing quantum-gravitational departures from the classical null ray connecting them. The standard deviation \( \Delta x_\perp \) of the difference in relative transverse positions is then given by \( \Delta x_\perp^2 = \Delta x_\perp(t_1)^2 + \Delta x_\perp(t_2)^2 \); it has a minimum value when \( \Delta x_\perp(t_1) = \Delta x_\perp(t_2) \), so

\[ \Delta x_\perp^2 > l_p L, \]  

(8)

This has a minimum value when \( \Delta x_\perp(t_1) = \Delta x_\perp(t_2) \), and

\[ \Delta x_\perp(t_1)^2 = \Delta x_\perp(t_2)^2 > l_p(t_2 - t_1)/2, \]  

(10)

so we have a lower bound for standard deviation in the distribution of angles, an uncertainty principle for the angular orientation of any null ray that applies independently to each transverse axis:

\[ \Delta \theta_x > \sqrt{l_p/L}, \quad \Delta \theta_y > \sqrt{l_p/L}. \]  

(11)

Note that the angular uncertainty actually increases with smaller \( L \). Indeed, one view of emergent spacetime suggested by holographic indeterminacy is that a classical spatial direction is actually ill defined at the Planck scale and only becomes well defined after many Planck lengths of propagation.

The quantum deflections of neighboring null trajectories are not drawn from independent distributions. Although the orientation is indeterminate, the same orientation of the plane wave from event 1, and the same transverse displacement, are shared coherently by all particles on nearby null trajectories. Local measurements of transverse positions are not limited in precision; they are all shared the same same spacetime eigenstate, with local classical spacetime behavior, as required by agreement with local physics. In Eq. [1], the operator \( |\psi_{\text{spacetime}}\rangle\langle \psi_{\text{spacetime}}| \) operates coherently on measurements of any and all bodies near the same events.

Spacetime states of nearby null trajectories do however gradually decohere with larger separation. Consider two nearby null rays from event 1, terminating “at the same time” at distance \( L \), with transverse positions separated by a classical distance \( \delta x_\perp \). This relative transverse position is indeterminate and uncertain with standard deviation,

\[ \Delta(\delta x_\perp) > \left( \frac{\delta x_\perp}{L} \right) \sqrt{l_p \delta x_\perp} = \left( \frac{\delta x_\perp}{L} \right)^{3/2} \sqrt{l_p L}. \]  

(12)

One way to see this is to consider the effect of spacetime indeterminacy on classical spacelike surfaces of simultaneity, which also are defined by the emergent metric. A ray connecting bodies at the classical separation \( \delta x_\perp \) has its own transverse uncertainty \( \simeq \sqrt{l_p \delta x_\perp} \), with a corresponding indeterminacy in the relative distances along the direction to event 1. For consistency the emergent metric must have a corresponding small uncertainty in the orientation of the constant-time surface, and therefore in the difference \( \delta x_\perp \) between positions assigned to those events.

Similarly, consider two nearby “parallel” null rays with transverse separation \( \delta x_\perp(t_1) \); the uncertainty in their relative angular orientation after propagating distance \( L \) is

\[ \Delta \theta(\delta x_\perp) > \left( \frac{\delta x_\perp}{L} \right)^{3/2} \sqrt{l_p/L}. \]  

(13)
Holographic uncertainty adds quantum noise to the parallel postulate of Euclidean space; at this level it is indeter-
minate whether or not rays are parallel.

The decoherence leads to detectable quantum position noise, as discussed below. It can be measured by an inter-
ferometer that compares the relative transverse positions of two widely separated bodies, at a time corresponding to
a null separation of measurement events, from another body at a large transverse distance.

SPACETIME QUANTUM DEGREES OF FREEDOM

The uncertainty in orientations or angles translates into a limit on the number of distinguishable angular orientations
of plane-wave modes of any quantum field, and therefore a limit on the total number of quantum degrees of freedom
more severe than that in standard field theory with a cutoff at the Planck scale. Taking both transverse directions
into account, and counting states by assuming Nyquist sampling on a sphere (that is, states or degrees of freedom
separated by two standard deviations in each transverse direction $x$ and $y$), the number of distinguishable orientations
for rays of length $L$ is given by

$$N = \frac{4\pi L^2}{2(\Delta x_{\perp 1}^2 + \Delta x_{\perp 2}^2)^{1/2} \times 2(\Delta y_{\perp 1}^2 + \Delta y_{\perp 2}^2)^{1/2}}.$$  (14)

As above, from Eq. (7), in both $x$ and $y$ directions the uncertainty is minimized for $\Delta x_{\perp 1} = \Delta x_{\perp 2} = \sqrt{l_P L/2}$; the maximum number of distinguishable directions in a sphere of radius $L$ is thus

$$N_\theta(L) < \frac{4\pi}{4\Delta \theta_x \Delta \theta_y} = \pi L/l_P.$$  (15)

We can take this number as a bound on the number of distinguishable wavevector directions for field modes confined
to the volume. In field theory, the number of distinguishable wavevector directions in the same volume is much larger,
$\approx 4\pi (L/l_P)^2$. The different behavior shows the effect of holographic blurring at macroscopic distances: in emergent
spacetime, extended field states lack independence they have in field theory.

The number of quantum degrees of freedom up to the Planck scale is $\approx L/l_P$ field modes per direction, so the
number of degrees of freedom is bounded by

$$S \approx N_\theta(L) L/l_P < \pi (L/l_P)^2.$$  (16)

This maximum number of degrees of freedom agrees (up to a numerical factor of the order of unity, depending on
the exact nature of the Planck cutoff) with the maximum number of degrees of freedom allowed by covariant entropy
bounds. Although the derivation has been different, the physical connection is clear: for example, the transverse
envelopes of the spatial wavefunctions of these spacetime states resemble those corresponding to particles evaporating
from a black hole of Schwarzschild radius $\approx \Delta x_{\perp 1}(L)$ into flat space, one process used to estimate covariant entropy
bounds. Those particles must experience a similar “blurring” in their transverse positions far away, otherwise their
states would contain more information (and observables could reveal more data) than available in the black hole that
produces them, a violation of quantum unitary evolution.

Although this picture seems radical, it is also a rather straightforward expression, in terms of behavior in 3-space, of
the effects of holographic bounds on spacetime degrees of freedom. The conjectured holographic behavior of quantum
gravity has long been supported by arguments based on black hole physics and string theory, in spite of its apparent contradiction with features of field theory such as locality. The detailed quantitative agreement is the main reason to adopt $l_P$ as the fundamental length for this theory, and suggests that the decomposition into transversely-localized plane waves approximates a complete decomposition of the quantum
degrees of freedom of nearly-flat spacetime.

OBSERVABLE HOLOGRAPHIC SHEAR NOISE

This implementation of holography has physical implications for observable real-world behavior. Transverse posi-
tions at separation $\approx L$ in any frame, measured from a distance $\approx L$, have an uncertainty $\Delta x_{\perp} > \sqrt{l_P L}$, which is
$> \sqrt{L/l_P}$ times larger than the local resolution of Planck scale field theory. Similarly, the relative orientation of null
geodesics of length $L$ and separated by $L$ has an irreducible angular uncertainty $\Delta \theta > \sqrt{l_P L}$ in each orthogonal
The metric perturbation of a gravitational wave with + or though we can use the same technology to detect both types of perturbation and the same units to describe both.

branch of the wavefunction corresponding to a classical spacetime. indeterminate. The measured radial distances between the bodies change accordingly since they inhabit a common

However, after a decoherence time, roughly a light travel time around the triangle, the relative positions are again indeterminate. The measured radial distances between the bodies change accordingly since they inhabit a common branch of the wavefunction corresponding to a classical spacetime.

The transverse spatial character of holographic shear noise distinguishes it from gravitational wave strain, even though we can use the same technology to detect both types of perturbation and the same units to describe both. The metric perturbation of a gravitational wave with + or × polarization, propagating on the z axis, has a 3-space dependence in the transverse-traceless (TT) gauge,

\[ h^+_{ij} = h^+ \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad h^\times_{ij} = h^\times \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \]  

(Here the indices \( i, j = 1, 2, 3 \) correspond to \( x, y, z \). The full description of the wave multiplies these by the usual wave dependence, \( e^{-i(kz - \omega t)} \).) By contrast the apparent metric perturbations of \( x \)- and \( y \)-polarized shear modes propagating on the \( z \) axis have a 3-space dependence,

\[ h^x_{ij} = h^x_H \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \quad h^y_{ij} = h^y_H \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}. \]  

The components of spatial displacement are

\[ \Delta x^j = \frac{1}{2} h_{jk} x^k, \]  

so as expected the shear perturbation corresponds to the displacement caused by a change in the orientation of \( z \). Classically there is no physical significance to shear perturbations of flat space since they can be removed by coordinate transformation. However, holographic indeterminacy adds a quantum stochastic element to transverse positions, so even flat space has an apparent gaussian superposition of these perturbations with random phases and orientations. It can be characterized by the power spectral density of the metric perturbation amplitude \( h_H \) as function of frequency.

The spectrum of the holographic noise is universal and depends only on the one scale in the system, \( l_P \). The power spectral density (mean square dimensionless metric perturbation per frequency interval) is independent of frequency, given by

\[ S_H \simeq l_P/c. \]  

In units similar to those used to describe equivalent metric strain noise in gravitational wave detectors, the rms amplitude of holographic noise in terms of equivalent metric shear is

\[ h_{H,\text{rms}} = S_H^{1/2} \simeq \sqrt{l_P/c} = 2.3 \times 10^{-22}/\sqrt{\text{Hz}}. \]  

This noise level is comparable to that already achieved for measurements of tensor waves in science runs of operational interferometers such as LIGO. Holographic noise should therefore be measurable with similar technology. However,
it can only be detected in a system designed with a layout capable of detecting transverse positions at events with macroscopic spatial separation.

Like gravitational waves, these perturbations cannot be detected in a purely local experiment. However, unlike the tensor modes of physical gravitational waves (Eq. 17), holographic perturbations in position are transverse to spatial separation (Eq. 18). They can never be detected, as gravitational waves can, by purely radial measurements in one direction. (Otherwise it would be possible to extract energy from the holographic noise, which is impossible.) They can however be detected by comparing the results of measurements of different paths. The experiment must not only be nonlocal but must also compare transverse positions at events with null separation, widely separated in space (in the experiment frame). For example, a Michelson interferometer like LIGO compares transverse distances in two arms, but only at one place, so it does not detect holographic noise. The indeterminacy produced by the holographic shear noise can however be detected in gravitational-wave detector interferometers of appropriate geometry. Purely radial measurements of distances along some baselines can be arranged to measure transverse positions associated with other baselines at wide separations. Some of the experimental options for achieving this are discussed in [4]. They include triangular configurations, such as LISA. Holographic noise for example leads to random variations in the total pathlength in a circuit that can be detected in a Sagnac-type interferometer, even if it is synthesized from three separate radial distance measurements [4, 24, 25, 26, 27, 28]. The coherence estimate above (Eq. 12) suggests that the detectable noise is limited by the shortest side of the triangle.

Holographic indeterminacy appears to be a new feature of emergent quantum spacetime with no physical counterpart in classical gravity. In perturbations of classical flat spacetime these shear modes are simply gauge artifacts; they carry neither energy nor information, and have no observable effects. This feature partially explains why quantized shear modes have not previously played a more prominent role in quantized gravity in field theory. Holographic noise indeed does not carry energy or information, but it does appear to create an observable nonlocal effect associated with quantum indeterminacy in the branching of the spacetime metric.

Strange as holographic noise seems, it is hard to reject out of hand the idea that there must be some nonlocal indeterminacy in position, without some equally profound modification of quantum mechanics (e.g., [29]). The noise is a direct consequence of information limits and in that sense is more general than the particular indeterminacy model described here. The holographic entropy bounds (if valid) require that there must be fundamental limits on the dimension of Hilbert space and therefore a limit on the overall precision of position measurements, creating an effective granularity much coarser than the Planck resolution implied by field theory. These must be manifested in some form of added effective noise to measurements by an apparatus capable of measuring positions to sufficient accuracy in a two-dimensional plane. Consider again a right triangle: without holographic indeterminacy, it would be possible to measure the positions of the vertex events to Planck precision, but holography does not allow enough information in the spacetime to specify such a large number of distinguishable positions with all the independence allowed by field theory. The limited number of independently distinguishable positions or angular orientations necessarily translates into a sampling noise in observables. If holographic bounds are implemented by holographic indeterminacy, they lead inevitably to holographic noise [4]; other implementations of holography may produce less conspicuous effects, but they necessarily imply the same information deficit.

Although no new particles are predicted from this theory, indeterminacy might measurably affect the behavior of some systems even on a microscopic scale. Nonlocality, relative to Planck distances, applies even on a subatomic scale. Spatially extended, coherent quantum states sample a spacetime that has holographic quantum variations in its geometry. For example, the background classical spacetime for an extended state such as an atomic nucleus, with a typical length scale $\approx 10^{20} l_p$, has angular uncertainty along each axis of $\Delta \theta \approx 10^{-10}$; for atomic or molecular states, $\Delta \theta \approx 10^{-13}$. Although symmetries in some atomic and nuclear coherent eigenstates are tested more precisely than this, spatially extended coherent eigenstates that would be coherent in classical spacetime should remain so in the presence of holographic spacetime indeterminacy; the intermediate states $|\psi_{\text{spacetime}}\rangle\langle \psi_{\text{spacetime}}|$ in Eq. (1) do not introduce noise if the whole system is in a coherent eigenstate since no measurement is made collapsing the spacetime to a definite eigenstate of transverse position. On the other hand, fields interact on a fluctuating geometry, which might allow coherent mixing or other observable effects in systems where the Hamiltonian has some dependence on transverse position. It is worth considering whether some microscopic quantum systems could be found or constructed with a configuration that has measurable consequences of holographic indeterminacy.

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