Investigation and identification of the spectral composition of noise in the accelerometer channel of the navigation module

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Abstract. The paper studies and measures the characteristics of random error components of the output signal in the accelerometer channel of an inertial navigation module, in which the MMA7331LT accelerometer is used. The spectral composition of the random components of the errors of the accelerometer was estimated using the Allan method of aeration. The calculated values of the coefficients characterizing the intensity of various types of noise are obtained.

1. Introduction

Theoretical and experimental studies related to the synthesis of simulation models of inertial navigation systems (INS), inertial sensors and command-measuring devices are intensively conducted abroad and in Russia. The main problem in the design of simulation models is the accounting and modeling of random component errors. The complexity of solving this problem is due to the fact that the structure and nature of the noise components have different power spectral densities, correlation times, and intensities. Therefore, the identification of individual noise components in the measurement channel of an inertial sensor is not an easy task for modeling, since the intensity of the noise components may depend on the conditions of experimental measurements. Traditionally, mathematical methods based on the calculation of correlation functions and power spectral densities are used to describe and simulate random processes [1].

Recently, however, the Allan method of variation has been widely used to assess the spectral composition of noise.

The aim of this work is to evaluate the spectral composition of random error components of the output signal of the MMA7331LT accelerometer to determine their contribution to the resulting error. An estimate of the intensities of the individual noise components can be used to implement the transfer function of the forming digital filter of the accelerometer channel of the INM in order to increase the accuracy of the simulation model of the accelerometer.

2. MMA7331L Output Accuracy

The main problem of navigation systems operating for a long time in the autonomous mode is the presence of an error that accumulates over time. The error of the output signals of the accelerometer can be represented as an expression:

\[ \varepsilon = \varepsilon_s + \tilde{\varepsilon}, \]  

where \( \varepsilon_s \) – is a systematic error; \( \tilde{\varepsilon} \) – is a random error.
The value of the systematic error remains constant or naturally changing during repeated measurements and linear acceleration conversion. The error of the second type varies randomly during repeated measurements and linear acceleration conversion.

The occurrence of a systematic error in the MEMS accelerometer is due to the following factors:

- random error associated with the instability of the internal microelectronic components of the circuit;
- presence of natural frequencies of the micromechanical design of the sensitive pendulum element;
- influence of disturbing forces or moments on the moving parts of the pendulum of the accelerometer.

Systematic error consists of the main and additional component. The latter is related to the sensitivity of the accelerometer to additional external factors (magnetic field, heating rate, vibration, pressure, etc.).

The main component of the systematic error of the accelerometer will differ for different instances of the same brand of accelerometer. This component is related to the temperature and heating rate of the accelerometer. The systematic error can be taken into account using the experiment and the subsequent approximation of the obtained data.

Correction of random errors is a more complicated procedure, since the effect of this type of error varies randomly from experiment to experiment. Earlier in [2] and [3], the authors developed various versions of the simulation model of the accelerometer.

They present methods for calibrating the standard deviation of noise and zero drift of the accelerometer in the form of polynomial temperature dependences. It was assumed that a long drift is a systematic component of the error, and the variable component of the drift and random noise contribute to the random component of the error. In these works, the random component of the error was represented as an integral quantity, using the assumption that the intensity of the resulting noise is distributed according to the normal law. Its assessment in a wide temperature range makes it possible to increase the accuracy of the simulation model up to 4%.

However, it is desirable to evaluate the spectral composition of noise to further improve the accuracy of the simulation model. This is due to the fact that the amplitude and spectral composition of the individual noise components depend on the operating time of the INM, changing even at the same temperature.

In this work, the identification of individual noise components is carried out, which will improve the accuracy of the accelerometer model by developing a digital INM digital filter, which reduces the amplitude of the integral noise of the INM.

3. Analysis of the main types of noise
According to the literature [4, 5], seven noise components can be distinguished in the inertial sensor channel.

3.1. Quantization noise
The nature of this type of noise is due to the discrete nature of the sensors. The appearance of this type of noise is explained by the small difference between the amplitudes of the analog signal and the amplitudes of the points selected taking into account the resolution of the ADC. In this regard, the results of the digital signal will be slightly different from the original analog signal. Depending on the type of ADC, quantization noise can occur when rounding to a certain bit or when dropping the least significant bits. In this case, part of the information of the original analog signal is lost. This type of noise is characterized by a short correlation time. This means that it can be filtered by a band-pass filter. An error of this type does not accumulate over time, but is associated only with the loss of part of the original information of the analog signal.
3.2. Velocity Random Walk
This noise is described by a Wiener random process. It represents the component of additive white noise, which manifests itself in the deviation of the linear velocity values from the expected randomly. This noise component affects the result of acceleration integration. As a result, the standard deviation (SD) of the noise voltage increases with increasing time:

$$\sigma_v(t) = \sigma_w \cdot \sqrt{T_0 \cdot t},$$  \hspace{1cm} (2)

where
- $\sigma_v(t)$ – velocity random walk SD;
- $T_0$ – sampling period;
- $t$ – time;
- $\sigma_w$ – Standard deviation of white noise in the accelerometer channel.

This type of noise is described by a spectrum of white noise with a short correlation time. Most sources of this type of noise are eliminated by mathematical modeling or by creating filter.

3.3. Bias Instability
Zero instability is caused by noise in the electronic components of the inertial sensors taking and processing information. This type of noise is also called flicker noise (pink low-frequency noise) with a spectral density of $1/f$. This type of noise manifests occurs in almost all components of electronic equipment, having a significant effect at low frequencies. However, with increasing frequency, its intensity decreases.

3.4. Rate Random Walk
This type of noise is a random departure of the rate of change in the values of the sensors. For example, an accelerometer has random acceleration drift. At large time intervals (several hours), a slow monotonic deviation of the measured value is observed. This error is characterized by a very long correlation time. To evaluate this error, it is necessary to remove a significant amount of data from the sensors. However, over long periods of time, the data from the sensor are subject to the influence of environmental conditions, which may affect the accuracy of the estimate of this error.

3.5. Rate Ramp
This type of error is a deterministic deviation of sensor readings at significant intervals.

3.6. Sinusoidal noise
Sinusoidal noise is a noise generated by vibrations in the sensor components. The principle of operation of any MEMS sensors is based on the measurement of vibration/rotations and linear accelerations of the device components. This is due to the noise of the sensor output signal by a sinusoidal component directly related to the resonant frequency of its components.

3.7. Correlation noise (Markov noise)
This type of noise can occur in any electronic circuits where components with cRC and LC-chains are present. The correlation function of the form is used to describe it:

$$K(\tau) = D \cdot e^{-\mu |\tau|},$$  \hspace{1cm} (3)

where $D$ – noise dispersion; $\mu$ – attenuation coefficient.

4. Allan variation method
Allan variation method is used to evaluate and identify the various noise components, based on the calculation of a two-sample variance [4] (variance of adjacent data pairs):

$$\sigma_A^2(\tau) = \frac{1}{2(N-1)} \cdot \sum_{k=1}^{N-1} (A_{k+1}(\tau) - A_k(\tau))^2,$$  \hspace{1cm} (4)

where
\[ A_k(\tau) \] – average value of the acceleration signal of the k-th number of the discrete;
\[ N \] – the number of parts of the partition of the time domain;
\[ \tau = n \cdot \tau_0 \] – time interval of averaged data;
\[ \tau_0 \] – ADC sampling time (1/1200 for ADS1281).

In numerical calculations, a modified Allan variation formula is usually used.

\[
\sigma^2_A(n \cdot \tau_0) = \frac{1}{2(L-2-N) \cdot (n \tau_0)^2} \cdot \sum_{j=1}^{L-2} (V_{j+2n} - 2 \cdot V_{j+n} + V_j)^2 ,
\] (5)

where
\[ n = 1,2,3... \ n_{max}, \text{ with } n_{max} \leq \frac{L-1}{2} ; \]
\[ L \] – total number of samples;
\[ j \] – number of samples used in linear acceleration integration;
\[ V_k(\tau) \] and \[ V_{k+1}(\tau) \] – averaged linear velocity signals received from the accelerometer. These speed signals are determined by the formulas:

\[
V_k(\tau) = \frac{1}{\tau} \cdot \int_{t_k}^{t_k + \tau} A(t) \, dt ; \quad V_{k+1}(\tau) = \frac{1}{\tau} \cdot \int_{t_{k+1}}^{t_{k+1} + \tau} A(t) \, dt ,
\] (6)

Here \( A(t) \) - output signal from the accelerometer.

Based on the basic types of noise components known for inertial meters [4, 5], we can write an expression approximating the Allan variation \( \sigma^2_A(\tau) \) by the polynomial \( p^2_A(\tau) \) in the following form:

\[
\sigma^2_A(\tau) \approx p^2_A(\tau) = R^2 \cdot \frac{\tau}{2} + K^2 \cdot \frac{\tau}{3} + B^2 \cdot \frac{2}{\pi} \cdot \ln(2) + N^2 \cdot \frac{1}{\tau} + Q^2 \cdot \frac{3}{\tau^2} ,
\] (7)

Expression (7) includes 5 main sources of noise. Here, the values of the coefficients \( R, K, B, N, Q \) of polynomial (7) characterize the intensity of individual noise components of the meter output signal: noise of continuous drift, random acceleration change, instability of zero bias, random velocity wander and quantization noise, respectively.

Sometimes, two more terms related to sinusoidal noise and correlation noise are added to the polynomial approximation (7). In this case, these noise components can be a priori not included in expression (7) due to the use of an effective sinc-filter in the delta-sigma ADS1281.

5. Method for assessing noise intensity

The numerical values of the coefficients in equation (7) are determined by the least squares method (LSM), which ensures the maximum coincidence of the approximating function \( p^2_A(\tau) \) with experimental \( \sigma^2_A(\tau) \). The approximation (7) is taken as a basis.

Let us set:

\[
\sigma^2_A(\tau) = C_1 \cdot \tau^2 + C_2 \cdot \tau + C_3 + C_4 \cdot \tau^{-1} + C_5 \cdot \tau^{-2} ,
\]

where

\[
C_1 = \frac{R^2}{2} ; \quad C_2 = \frac{K^2}{3} ; \quad C_3 = \frac{2 \cdot \ln(2)}{\pi} \cdot B^2 ; \quad C_4 = N^2 ; \quad C_5 = 3 \cdot Q^2 .
\]

We make the matrix equation:

\[
\begin{pmatrix}
a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\
a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\
a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\
a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\
a_{51} & a_{52} & a_{53} & a_{54} & a_{55}
\end{pmatrix}
\begin{pmatrix}
C_1 \\
C_2 \\
C_3 \\
C_4 \\
C_5
\end{pmatrix}
= 
\begin{pmatrix}
Q_1 \\
Q_2 \\
Q_3 \\
Q_4 \\
Q_5
\end{pmatrix}
\] (8)

We use a minimum of functionality:

\[ F^2(C_1,C_2,C_3,C_4,C_5) \rightarrow \min \text{ or } \frac{\partial F^2}{\partial x_i} = 0; \ i = 1..5. \]
The matrix of coefficients and the column vector are determined analytically:

\[
\begin{pmatrix}
a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\
a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\
a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\
a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\
a_{51} & a_{52} & a_{53} & a_{54} & a_{55}
\end{pmatrix}
\cdot
\begin{pmatrix}
\Sigma_{i=1}^{N} t_i^4 & \Sigma_{i=1}^{N} t_i^3 & \Sigma_{i=1}^{N} t_i^2 & \Sigma_{i=1}^{N} t_i & N \\
\Sigma_{i=1}^{N} t_i^3 & \Sigma_{i=1}^{N} t_i^2 & \Sigma_{i=1}^{N} t_i & N & \Sigma_{i=1}^{N} t_i^{-1} \\
\Sigma_{i=1}^{N} t_i^2 & \Sigma_{i=1}^{N} t_i & N & \Sigma_{i=1}^{N} t_i^{-1} & \Sigma_{i=1}^{N} t_i^{-2} \\
\Sigma_{i=1}^{N} t_i & N & \Sigma_{i=1}^{N} t_i^{-1} & \Sigma_{i=1}^{N} t_i^{-2} & \Sigma_{i=1}^{N} t_i^{-3} \\
N & \Sigma_{i=1}^{N} t_i^{-1} & \Sigma_{i=1}^{N} t_i^{-2} & \Sigma_{i=1}^{N} t_i^{-3} & \Sigma_{i=1}^{N} t_i^{-4}
\end{pmatrix}
\]

(9)

Column vector of coefficients:

\[
\begin{pmatrix}
b_1 \\
b_2 \\
b_3 \\
b_4 \\
b_5
\end{pmatrix}
= \begin{pmatrix}
\Sigma_{i=1}^{17} t_i^2 \cdot \sigma_{K_i}^2 \\
\Sigma_{i=1}^{17} t_i^2 \cdot \sigma_{K_i}^2 \\
\Sigma_{i=1}^{17} t_i^{-1} \cdot \sigma_{A_i}^2 \\
\Sigma_{i=1}^{17} t_i^{-2} \cdot \sigma_{A_i}^2 \\
\Sigma_{i=1}^{17} t_i^{-3} \cdot \sigma_{A_i}^2
\end{pmatrix},
\]

(10)

where \( t_i \) – ADC sampling time.

The graphs of the dependences of the Allan standard deviation, obtained as a result of experiment, on the averaging time \( \tau \) for each of the three axes of the MMA7331 accelerometer are shown in the figure. The results of a numerical evaluation of the values of the coefficients \( R, K, B, N, Q \), characterizing the corresponding noise intensities are given in the table 1.

**Table 1. Values of noise intensity factors.**

| Measuring axes | Noise intensity factors |
|----------------|------------------------|
|                | \[|R| \cdot 10^{-8}, \frac{M}{C^3}\] | \[|K| \cdot 10^{-5}, \frac{M}{C^{5/2}}\] | \[|B| \cdot 10^{-4}, \frac{M}{C^2}\] | \[|N| \cdot 10^{-4}, \frac{M}{C^{3/2}}\] | \[|Q| \cdot 10^{-4}, \frac{M}{C}\] |
| Axis X         | 4.2                    | 3.6                    | 0.55                   | 1.05                   | 2.25                   |
| Axis Y         | 3.7                    | 2.3                    | 0.39                   | 0.94                   | 1.82                   |
| Axis Z         | 5.9                    | 3.3                    | 0.48                   | 1.03                   | 2.12                   |

The table defines the values of the coefficients \( R, K, B, N, Q \). Their comparison with the passport characteristics for this accelerometer showed the following:

- the value of the \( Q \) coefficient for each axis is of the order of \( 10^{-4} \), which significantly exceeds the unit price of the least significant bit of the accelerometer, set by the developer of the INM with a high-bit ADC at \( \approx 5.5 \cdot 10^{-8} \) m/s and indicates an increased noise level of the meter;
- the estimate of the coefficient \( N \) for each axis is also about \( 10^{-4} \), which shows the high intensity of the indicated type of noise for the accelerometer channel;
- the estimation of coefficient \( B \) is about \( (0.4-0.5) \cdot 10^{-4} \) m/s². This estimate can be considered stable, since the deviation from launch to launch of the accelerometer (RMS) does not exceed \( 2.1 \cdot 10^{-4} \) m/s²;
- the coefficients \( K \) and \( R \) are of the order of \( 10^{-6} \) and \( 10^{-8} \), respectively, which indicates a low intensity of these types of noise. The difference in the experimental data for each of the three measuring axes of the accelerometer indicates the predominance of different types of noise in the measurement channels. In addition, it can be seen from Figure 1 below that the Allan curves obtained along the X and Z axes have additional high-frequency noise. Identification of the nature of this noise source requires a separate study.
Figure 1. Allan RMS dependence $\sigma_A(\tau)$ on averaging time $\tau$.

6. Conclusion

The spectral composition and intensity of various components of the noise of the MMA7331LT accelerometer were estimated using the Allan variation method. The difference between Allan curves for different axes of sensitivity of the accelerometer is revealed, which is associated both with the dominance of various components of the analyzed noise sources and with the presence of additional noise sources for each axis. Separate types of noise have been found that require detailed study to obtain a more accurate stochastic model.

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