A $D=4$ $N=1$ Orbifold of Type I Strings

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Abstract

We consider the propagation of Type I open superstrings on orbifolds with four non-compact dimensions and $N = 1$ supersymmetry. In this paper, we concentrate on a non-trivial $\mathbb{Z}_2 \times \mathbb{Z}_2$ example. We show that consistency conditions, arising from tadpole cancellation and algebraic sources, require the existence of three sets of Dirichlet 5-branes. We discuss fully the enhancements of the spectrum when these 5-branes intersect. An amusing attribute of these models is the importance of the tree-level (in Type I language) superpotential to the consistent relationship between Higgsing and the motions of 5-branes.

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1 Introduction

The solution of \( N = 1 \) field theories in four dimensions relies heavily on understanding different limits of moduli space. One certainly expects that such a tool will play a significant role in string theories with \( N = 1 \) supersymmetry as well. Strong-weak coupling duality of string theory allows us to control the strong coupling regime of some string theories in terms of other weakly coupled ones. One such example is Type I/heterotic duality; one of the first steps towards solving the low energy dynamics of the \( SO(32) \) heterotic string theory is to understand Type I compactifications with \( N = 1 \) supersymmetry. In this paper we will discuss a simple but non-trivial compactification of Type I strings to four dimensions.

There are other motivations for studying four dimensional compactifications of the Type I theory. One aspect is a potential importance of this technology for phenomenology: for example, one finds matter fields which transform as bilinears of the fundamental representation in vast quantities (open strings have two ends). We may also hope to find, within a general Type II orientifold framework, constructions with chiral fermions, although the model discussed here does not have this property. Lastly, the couplings in Type I theory (or open string sectors in general) may behave quite differently than expected from perturbative heterotic string theory, alleviating some unpleasant general features of heterotic string theory model building\(^1\).

Another motivation is the study of solitons in string theory. Free or solvable conformal field theories that admit solitons or other non-perturbative objects provide the most reliable avenue for discussing the properties of these objects.

Type I superstrings compactified to six dimensions on a K3 orbifold have been considered in several recent papers. In Ref.\(^2\), the worldsheet consistency conditions were studied. It was found that for consistent open string propagation, there must be 32 parallel 5-branes,\(^3\) as well as the 32 9-branes found in 10-dimensional theory\(^4\). Furthermore, spacetime anomaly constraints were studied in Ref.\(^5\), and it was found that parts of the gauge groups found in Ref.\(^2\) were in fact broken through a modification of the Green-Schwarz mechanism. Other six-dimensional orientifolds have been dis-
In this paper, we begin a study of orientifolds with four non-compact dimensions. Here we discuss Type I superstrings compactified on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$, where the two $\mathbb{Z}_2$'s act as follows on the compact coordinates:

$$
R_1 : \begin{align*}
X^{6,7,8,9} &\rightarrow -X^{6,7,8,9} \\
X^{4,5} &\rightarrow +X^{4,5}
\end{align*}$$

$$
R_2 : \begin{align*}
X^{4,5,8,9} &\rightarrow -X^{4,5,8,9} \\
X^{6,7} &\rightarrow +X^{6,7}
\end{align*}
$$

The orbifold group also includes the projection $R_3 = R_1 R_2$ which acts on the 4, 5, 6, 7 coordinates. Since we consider Type I strings, we will consider the worldsheet orientation reversal $\Omega$ as well. Thus the full orientifold group contains the elements $\{ 1, R_i, \Omega, \Omega R_i \}, i = 1, 2, 3$.

This is a seemingly complicated orbifold to begin with, but it is perhaps the simplest non-trivial extension of Ref. [2] to four dimensions, and as we will see has some very interesting new features. We construct here a consistent Type I compactification and study the tree-level superpotential and Higgsing phenomena in some detail. Many details of this and other models will be left for further publications. In particular, there are many interesting non-perturbative aspects to be explored.

In the next section, we discuss some general aspects of the spectrum of these theories. In the following section, we will give some details of the worldsheet consistency conditions following from tadpole cancellation. We will find that three orthogonal sets of 32 5-branes will be necessary, in addition to 32 9-branes. This configuration preserves $N = 1$ supersymmetry in four dimensions and leads to a rich variety of phenomena associated with 5-brane morphology. The remaining supersymmetry may be demonstrated as follows. First, on the worldsheet, we take the action of $R_1$ and $R_2$ to be

$$
R_1 = \exp (i\pi (J_{67} + J_{89})) \\
R_2 = \exp (i\pi (J_{45} + J_{89}))
$$

when acting on any worldsheet field. Thus the worldsheet supercharge left
invariant by these operations is given by

\[ e^{-\phi/2} e^{i(H_0 + H_1 + H_2 + H_3 - H_4)/2} \] (1.3)

One could worry that this does not look invariant under \( R_3 \) if defined similarly to (1.2); however, the identification \( R_3 = R_1 R_2 \) leads to:

\[ R_3 = e^{2\pi i J_{89}} \exp(i\pi (J_{45} + J_{67})) \] (1.4)

The non-trivial \( J_{89} \) factor makes the supercharge invariant. From the spacetime point of view, we see the same result: the \( \Omega \) projection leaves \( Q + \tilde{Q} \) invariant. The \( R_i \) projection leaves \( Q + R_i \tilde{Q} \); only those components with \( R_i \)-eigenvalues equal to +1 correspond to unbroken supersymmetries. Since \( R_3 = R_1 R_2 \), there are only two such independent conditions and thus the supersymmetry is reduced by a factor of 1/8 from what it would have been in the Type IIB case (which would give \( N = 8 \) in \( D = 4 \)).

In Section 3, we discuss consistency conditions arising from the cancellation of unphysical Ramond tadpoles. In Section 4, we discuss further constraints and derive the spectrum of the theory in different configurations of 5-branes. Finally, in the last sections, we derive the tree-level superpotential and demonstrate its importance to T-duality and the correspondence between the motion of 5-branes and Higgsing.

## 2 The Orientifold Group and the Spectrum

We use throughout the notation of Ref. [2]. The orientifold group \( G = G_1 + \Omega G_2 \) acts on open string states as:

\[ g : |\psi, ij\rangle \rightarrow (\gamma_g)_{ii'} |g : \psi, i'j'\rangle \left(\gamma_g^{-1}\right)_{j'j} \] (2.1)

\[ \Omega h : |\psi, ij\rangle \rightarrow (\gamma_{\Omega h})_{ii'} |\Omega h : \psi, j'i'\rangle \left(\gamma_{\Omega h}^{-1}\right)_{j'j} \]

for \( g \in G_1 \) and \( h \in G_2 \). A great deal of effort will go into obtaining a consistent representation for the various matrices \( \gamma \). These must form a (projective) representation of the orientifold group and must pass several additional algebraic tests. First, operator products relate sectors to one another, and (2.1) must be consistent with this. Second, the calculation of
unphysical tadpoles will place further constraints. We will find that these constraints taken together essentially determine the $\gamma$'s completely. We begin with some remarks on the structure of the spectrum.

2.1 Closed String Spectrum

The untwisted states are formed out of:

\[
\begin{align*}
R_1 & \quad R_2 & \quad R_3 & \quad SO(2)_{ST} \\
\psi_{-1/2}^\mu |0 \rangle & + & + & + & \pm 1 \\
\psi_{1/2}^{1,5} |0 \rangle & + & - & - & 2 \times 0 \\
\psi_{-1/2}^{6,7} |0 \rangle & - & + & - & 2 \times 0 \\
\psi_{-1/2}^{8,9} |0 \rangle & - & - & + & 2 \times 0 \\
\end{align*}
\]

(2.2)

We have listed the transformation of each state under the $R_i$, as well as its representation under the spacetime Lorentz group. Because of $\Omega$, we must symmetrize in the NS-NS sector, and antisymmetrize in the R-R sector. We thus get the spectrum (here $m_1 = 4, 5, m_2 = 6, 7$, etc.):

\[
\begin{align*}
NSNS & : \quad \psi_{-1/2}^{(\mu)} |0 \rangle_L \otimes \bar{\psi}_{-1/2}^{(\nu)} |0 \rangle_R & \quad \pm 1/2 & \oplus & \pm 1/2 \\
\psi_{-1/2}^{(m_i)} |0 \rangle_L \otimes \bar{\psi}_{-1/2}^{(n_i)} |0 \rangle_R & \quad \pm 1/2 & \oplus & \pm 1/2 \\
R & : \quad |s_1 s_2 s_3 s_4 \rangle & \quad \pm 1/2 & \oplus & \pm 1/2 \\
\end{align*}
\]

(2.3)

where $\Psi_{\alpha \pm \pm \pm}$ labels the Ramond ground state with helicity $\alpha$ and $R_1(R_2)$ numbers $\pm(\pm')$. The $\Omega$ projection symmetrizes the R-NS states given here with those of the NS-R sector. Thus the untwisted closed string sector consists of the gravity multiplet, the dilaton chiral multiplet and six chiral multiplets associated with the torus.

Next, we have states from the twisted sectors. Consider first the sector
twisted by $R_1$. The massless states are formed from:

|       | $R_1$ | $R_2$ | $SO(2)_{ST}$ |
|-------|-------|-------|---------------|
| NS    | $s_3 s_4$; $s_3 = -s_4 + i(2s_4)$ | +1/2 | 0             |
| R     | $s_1 s_2$; $s_1 = -s_2 + i(2s_2)$ | -1/2 | 0             |

We thus get the states:

\[
\begin{align*}
NSNS & : \frac{1}{\sqrt{2}} (|+-\rangle \otimes |-+\rangle + |+-\rangle \otimes |+-\rangle) & 1/2 \\
RR & : \frac{1}{\sqrt{2}} (|+-\rangle \otimes |+-\rangle - |+-\rangle \otimes |+-\rangle) & -1/2 \\
RNS & : |+-\rangle \otimes |+-\rangle & 0 \\
\end{align*}
\]

where again, R-NS should be symmetrized with NS-R. We see then that we get one chiral multiplet per twisted sector; this is the blowing up mode for the corresponding fixed point. Each of the $R_i$ has 16 fixed ‘points’ (actually complex lines), so there are a total of 48 such chiral multiplets.

### 2.2 Open string states

Consider first the 99-sector. In the NS sector, there are states of the form $\psi^{|0,ab\rangle \lambda_{ab}}$ and $\psi^{|0,ab\rangle \lambda_{ab}}$. The Chan-Paton matrices satisfy:

\[\lambda^{(0)} = +\gamma_{R_1,9} \lambda^{(0)} \gamma_{R_1,9}^{-1}; \lambda^{(0)} = +\gamma_{R_2,9} \lambda^{(0)} \gamma_{R_2,9}^{-1}; \lambda^{(0)} = -\gamma_{R_2,9} \lambda^{(0)} T \gamma_{R_2,9}; \lambda^{(0)} = -\gamma_{R_2,9} \lambda^{(0)} T \gamma_{R_2,9}^{-1}\]

\[\lambda^{(1)} = +\gamma_{R_1,9} \lambda^{(1)} \gamma_{R_1,9}^{-1}; \lambda^{(1)} = -\gamma_{R_2,9} \lambda^{(1)} \gamma_{R_2,9}^{-1}; \lambda^{(1)} = -\gamma_{R_2,9} \lambda^{(1)} T \gamma_{R_2,9}; \lambda^{(1)} = -\gamma_{R_2,9} \lambda^{(1)} T \gamma_{R_2,9}^{-1}\]

\[\lambda^{(2)} = -\gamma_{R_1,9} \lambda^{(2)} \gamma_{R_1,9}^{-1}; \lambda^{(2)} = +\gamma_{R_2,9} \lambda^{(2)} \gamma_{R_2,9}^{-1}; \lambda^{(2)} = +\gamma_{R_2,9} \lambda^{(2)} T \gamma_{R_2,9}; \lambda^{(2)} = +\gamma_{R_2,9} \lambda^{(2)} T \gamma_{R_2,9}^{-1}\]

\[\lambda^{(3)} = -\gamma_{R_1,9} \lambda^{(3)} \gamma_{R_1,9}^{-1}; \lambda^{(3)} = -\gamma_{R_2,9} \lambda^{(3)} \gamma_{R_2,9}^{-1}; \lambda^{(3)} = -\gamma_{R_2,9} \lambda^{(3)} T \gamma_{R_2,9}; \lambda^{(3)} = -\gamma_{R_2,9} \lambda^{(3)} T \gamma_{R_2,9}^{-1}\]

As indicated above, there will be three sets of 5-branes, which we will refer to as $5_i$-branes; the $5_i$-brane fills 4-dimensional spacetime plus the $i^{th}$ $T^2$ spanned by $X^{m_i}$. For 5-branes at fixed points, the $5_i$-sector will satisfy constraints similar to those of the 99-sector, except for a sign change in the $\Omega$-transformation for the $\psi^{m_j}$ states ($j \neq i$).

Now consider moving the $5_i$-brane away from the $R_i$ fixed point. A general configuration is shown (for $i = 1$) in Fig. 1. Each 5-brane has three images generically, and thus there can be at most eight together. In this case, $R_i$ relates states to those of an image. $\Omega$ acts as $-1$ (as well as transposing) on the six orthogonal dimensions and as $+1$ on the other four.
There are also (complex) axes on which the 5$_i$-brane is at a fixed point of $R_j$ $(j \neq i)$. As is clear from the figure, this corresponds to two of the images approaching each other across the axis and there will then be extra massless states arising from strings stretching from one brane to an image.

Now let us discuss 5$_i$9-states and 5$_i$5$_j$-states more carefully. The 5$_i$9 states are much as in Ref. [2]. Consider a 5$_1$9-state; since $X_{6,7,8,9}$ satisfy Neumann boundary conditions on one end and Dirichlet on the other, these $X$’s will have 1/2-integer modings. The corresponding fermions have integer modings and thus the vacuum forms a representation of the corresponding zero-mode Clifford algebra. The NS state is\footnote{Note that the form of the supercharge (1.3) implies that there will be some sign changes in the GSO projection in the 5$_i$9 sector.}

$$5_19: \ |s_3s_4, ij\rangle \lambda_{ij}, \ s_3 = -s_4,$$

constituting two real bosons (per element of $\lambda$). Away from the fixed points, there are no constraints, apart from the GSO. The $\Omega$-projection relates 5$_i$9 to 95$_i$ and the $R_i$-projection relates one state to its image. The $R_j$-projections $(j \neq i)$ also map to an image. Now, if the 5$_1$-brane is at a fixed point of $R_1$, then $\lambda$ is restricted by $\lambda = \gamma_{R1,I} \lambda \gamma^{-1}_{R1,I}$. $R_j$ $(j \neq 1)$ flip the sign of $X^{4,5}$; since the lowest lying states have no dependence on $X^{4,5}$ (e.g., momentum in these...
directions would cost energy), \( \lambda \) is further restricted:

\[
\lambda = (2s_4 i) \gamma_{R2} \lambda \gamma_{R2}^{-1}
\]

\[
\lambda = (2s_4 i) \gamma_{R3} \lambda \gamma_{R3}^{-1}
\]

where the phases are deduced from eqs. (1.2, 1.4).

5,5-states have some similarities to the 59 states. Consider for definiteness the 5153 state. A generically positioned 53-brane appears as in Figs. 2. Clearly there can be massless 5153-states if the 53-brane in Figures 2 overlaps the 51-brane, i.e., they have the same \( X^{6,7} \), as in Figure 3. In this case, the states will be of the form \( |s_2 s_4, ij) \lambda_{ij} \). There will be no restriction, apart from GSO \( (s_2 = -s_4) \) when the branes are at generic points, as \( R_i \) will map states to images. When symmetries are enhanced, for example, if
the $5_1$-brane intersects the 89-axis, the states arrange themselves in representations of the enhanced symmetry. There will be additional restrictions from the $R_i$, but the discussion of these restrictions is more efficiently left for later sections. We turn now to the worldsheet consistency conditions and the calculation of tadpoles.

3 Worldsheet Consistency and Tadpole cancellation

We again use the notation of Ref. [2] where applicable. We denote the volumes of the three 2-tori as $V_i$, and write $v_i = V_i/4\pi^2\alpha'$. The volume of spacetime is denoted $V_4$, $v_4 = V_4/(4\pi^2\alpha')^2$. The cancellation of tadpoles for unphysical states is a collaboration amongst the Klein bottle, the Möbius strip and the cylinder. We discuss each topology in turn.

3.1 The Klein Bottle

We compute the closed string trace

$$KB : \text{Tr} \; \frac{\Omega}{2} \left( \frac{1 + R_1}{2} \right) \left( \frac{1 + R_2}{2} \right) \left( \frac{1 + (-1)^F}{2} \right) q^{L_o + \tilde{L}_o} \quad (3.1)$$

Note that the orbifold part expands out to $\frac{1}{4}(1 + R_1 + R_2 + R_3)$. We have $L_o = N_L + \frac{1}{2} \alpha' p_L^2 - 1/2$ with $N_L = \sum r \alpha_{-r} \cdot \alpha_r + \sum r \psi_{-r} \cdot \psi_r$, etc. and $p_{L,R}^2 = (m_i/r_i \pm n_i/r_i/\alpha')^2$. $\Omega$ acts trivially on $m$ and flips the sign of $n$. A projection $R$ changes the sign of both (the appropriate) $m$ and $n$. The momentum integration gives $(4\pi^2\alpha' t)^{-2}$. $\Omega$ correlates left movers with right movers and we have

$$\Omega|0\rangle_{NS-NS} = -|0\rangle_{NS-NS}$$

$$(-1)^F|0\rangle_{NS-NS} = -|0\rangle_{NS-NS} \quad (3.2)$$

$$\Omega|0\rangle_{R-R} = -|0\rangle_{R-R}$$

$$(-1)^F|0\rangle_{R-R} = \pm|0\rangle_{R-R}$$

Since $\Omega\psi\Omega^{-1} = \bar{\psi}$, $\Omega$ acts as $-(-1)^{F_L}$ on all closed string states. So in the untwisted sector we get contributions (“$\eta_3 = -$” in the language of Ref. [4]):

$$\Omega(-1)^F : \left[ \frac{\alpha}{\bar{f}_1(q)} \right]^8 \prod_{j=1}^3 M_j$$

$$\Omega R_i(-1)^F : \left[ \frac{\alpha}{\bar{f}_1(q)} \right]^8 M_i \prod_{j \neq i} W_j \quad (3.3)$$
from the NS-NS sector. (Here $q = e^{-2\pi t}$). This is to be multiplied by $\frac{1}{16} v_4 \int \frac{dt}{2t}$. We have defined the momentum and winding factors for the $j^{th}$ torus

$$M_j = \left( \sum_n e^{-\pi n^2 t/v_j} \right)^2$$

$$W_j = \left( \sum_m e^{-\pi m^2 t/v_j} \right)^2.$$  

The R-R sector gives the same result with $f_3^8$ replaced by $-f_2^8$, with the contributions coming from $\Omega$ and $\Omega R_i$ (those with $(-1)^F$ cancel, because of the action on the R-R vacua). After appropriate rescalings and resumptions, we arrive at:

$$v_4 \int_0^\infty \frac{dt}{2t^2} \left[ \frac{f_2(e^{-\pi /2t})}{f_1(e^{-\pi t})} \right]^8 \left[ \prod_i v_i \tilde{M}_i + \sum_j v_j \tilde{M}_j \prod_{i \neq j} \frac{\tilde{W}_i}{v_i} \right] (3.5)$$

where

$$\tilde{M}_i = \left( \sum_s e^{-\pi s^2 t/v_i} \right)^2$$

$$\tilde{W}_i = \left( \sum_s e^{-\pi s^2 t v_i} \right)^2.$$ 

Asymptotically ($t \to 0$), we get ($t = 1/4\ell$):

$$32 v_4 \int d\ell \left( \frac{v_1 v_2 v_3 + v_1}{v_2 v_3} + \frac{v_2}{v_3 v_1} + \frac{v_3}{v_1 v_2} \right). (3.7)$$

Thus, the Klein bottle gives a tadpole for a 10-form potential, as well as for 3 different 6-forms, each proportional to the appropriate T-dual volume element. The three different 6-forms appearing here will ultimately be responsible for the inclusion of three different Dirichlet 5-branes.

### 3.2 The Möbius Strip

Here we evaluate

$$MS : \ Tr_{NS-R} \frac{\Omega}{2} \left( \frac{1 + R_1}{2} \right) \left( \frac{1 + R_2}{2} \right) \left( \frac{1 + (-1)^F}{2} \right) q^L$$

$$q^L$$
over 99- and 5,5,- states (the $\eta_3 = -$ condition of Ref. [4] corresponds to NS states here). We have

$$L_o = \alpha' p^2 + \left[ \sum r \alpha_{-r} \cdot \alpha_r + \sum r \psi_{-r} \cdot \psi_r - 1/2 \right] + \begin{cases} \frac{m^2}{\alpha'} r^2 \quad &99 \\ \frac{n^2}{\alpha'} r^2 \quad &55 \end{cases}$$

(3.9)

and the non-compact momentum integration now gives $(8\pi^2 t\alpha')^{-2}$.

$\Omega$ acts on oscillators as:

$$\begin{align*}
\alpha_r &\to \pm e^{i\pi r} \alpha_r \\
\psi_r &\to \pm e^{i\pi r} \psi_r
\end{align*}$$

(3.10)

and as $e^{i\pi/2}$ on the open string vacuum (plus the action on Chan-Paton).

Consider first the 99-sector. The oscillator sums involving $R_i$ turn out to cancel; the remaining terms can be written in the form $i q^{1/2} [f_2 f_4 / f_1 f_3 (q)]^8$. This result is to be multiplied by $\frac{\nu}{16} \int \frac{dt}{(2\pi)^{1/2}} q^{-1/2}$, times the Chan-Paton and momentum state sums, which are $i \text{Tr} \gamma^F_{\Omega,9} \gamma^D_{\Omega,9} \prod_i M'_i$. We define

$$M'_j = \left( \sum_n e^{-2\pi t n^2 / v_j} \right)^2$$

(3.11)

$$W'_j = \left( \sum_m e^{-2\pi t m^2 v_j} \right)^2.$$

In the 5,5,-sector we find that the NS fermions give

$$\begin{align*}
\Omega : & \Pi_r (1 + e^{i\pi r} q^r)^4 \Pi_r (1 - e^{i\pi r} q^r)^4 \\
\Omega(-1)^F : & -\Pi_r (1 - e^{i\pi r} q^r)^4 \Pi_r (1 + e^{i\pi r} q^r)^4 \\
\Omega R_i : & \Pi_r (1 + e^{i\pi r} q^r)^8 \\
\Omega R_i(-1)^F : & -\Pi_r (1 - e^{i\pi r} q^r)^8 \\
\Omega R_j : & \Pi_r (1 + e^{i\pi r} q^r)^4 \Pi_r (1 - e^{i\pi r} q^r)^4 \\
\Omega R_j(-1)^F : & -\Pi_r (1 - e^{i\pi r} q^r)^4 \Pi_r (1 + e^{i\pi r} q^r)^4
\end{align*}$$

(3.12)

for $r \in \mathbb{Z} + 1/2$. The two different terms above are from Neumann and Dirichlet boundary conditions respectively. All but the $R_i$ terms cancel, and those are the same as the 99- contribution; the boson contribution here is also the same as the $\Omega$ terms from the 99-sector.
The total result for the Möbius strip then is

\[-\frac{v_4}{16} \int \frac{dt}{(2t)^3} \left[ \frac{f_2(q)f_4(q)}{f_1(q)f_3(q)} \right]^8 \left\{ \text{Tr} \gamma_{\Omega,9}^{-1} \gamma_{\Omega,9} \prod_i M_i' + \sum_i \text{Tr} \gamma_{\Omega R_i,5}^{-1} \gamma_{\Omega R_i,5} M_i' \prod_{j \neq i} W_j \right\} \]

(3.13)

Defining

\[ \tilde{M}_i' = \left( \sum_s e^{-\pi v_s s^2 / 2t} \right)^2 \]

(3.14)

\[ \tilde{W}_i' = \left( \sum_s e^{-\pi s^2 / 2tv_i} \right)^2, \]

we obtain, by rescaling and resummation

\[-\frac{v_4}{64} \int \frac{dt}{t^2} \left[ \frac{f_2(q)f_4(q)}{f_1(q)f_3(q)} \right]^8 \times \left\{ \text{Tr} \gamma_{\Omega,9}^{-1} \gamma_{\Omega,9} \prod_i v_i \tilde{M}_i' + \sum_i \text{Tr} \gamma_{\Omega R_i,5}^{-1} \gamma_{\Omega R_i,5} v_i \tilde{M}_i' \prod_{j \neq i} \frac{\tilde{W}_j'}{v_j} \right\} \]

(3.15)

Finally, as \( t \to 0 \), we find (\( t = 1/8 \ell \))

\[-2v_4 \int d\ell \left[ v_1v_2v_3 \text{Tr} \gamma_{\Omega,9}^{-1} \gamma_{\Omega,9} + \sum_i v_i \prod_{j \neq i} \frac{1}{v_j} \text{Tr} \gamma_{\Omega R_i,5}^{-1} \gamma_{\Omega R_i,5} \right] \]

(3.16)

### 3.3 The Annulus

Here we evaluate

\[ Cy^l : \text{Tr} NS-R \frac{1}{2} \left( \frac{1 + R_1}{2} \right) \left( \frac{1 + R_2}{2} \right) \left( \frac{1 + (-1)^F}{2} \right) q^{L_o} \]

(3.17)

over 99-, 5_5j_5-, 5_9- and 95_9- states (\( \forall i, j \)). (\( \eta_3 = - \) here corresponds to dropping the \((-1)^F\) terms). The Ramond sector carries an overall minus sign, which is the usual field theory sign for fermion loops.

The 99-sector: the fermions give:

\[ 1(NS) : \prod_r \left( 1 + q^r - 1/2 \right)^8 \]

(3.18)

\[ 1(R) : -16 \prod_r (1 + q^r)^8 \]

where \( r \in \mathbb{Z} \). Including the bosons, we then find that the oscillators contribute \( q^{1/2} \left[ f_4(\sqrt{q})/f_1(\sqrt{q}) \right]^8 \). The Chan-Paton factors are \( \text{Tr} \gamma_{1,9} \cdot \text{Tr} \gamma_{1,9}^{-1} \).
and the momentum modes give $\Pi_i M'_i$. So we have so far

$$
99: \quad \frac{v_4}{16} \int \frac{dt}{(2\pi)^3} \left[ f_4 f_1 (\sqrt{q}) \right]^8 \left\{ \text{Tr} \; \gamma_{1,9} \cdot \text{Tr} \; \gamma_{1,9}^{-1} \Pi_i M'_i \right\} \\
\rightarrow \frac{1}{32} v_4 \int d\ell \; n_9^2 \; v_1 v_2 v_3
$$

with $n_9$ the number of 9-branes. The $R_i$-operators in the trace give:

$$
R_i(NS) : \quad \Pi_r \left( 1 + q^r - 1/2 \right)^4 \left( 1 - q^r - 1/2 \right)^4
$$

$$
R_i(R) : \quad 0
$$

and the bosons give $\Pi_r (1 - q^r)^{-4} (1 + q^r)^{-4}$. The result is $4[f_3 f_4 / f_1 f_2 (\sqrt{q})]^4$. Chan-Paton give $\text{Tr} \; \gamma_{R_i,9} \cdot \text{Tr} \; \gamma_{R_i,9}^{-1}$ and there will be a factor of $M'_i$ from windings. The result is

$$
99: \quad \frac{v_4}{16} \int \frac{dt}{(2\pi)^3} \left[ f_4 f_1 (\sqrt{q}) \right]^4 \left\{ 4 \sum_i M'_i \text{Tr} \; \gamma_{R_i,9} \cdot \text{Tr} \; \gamma_{R_i,9}^{-1} \right\} \\
\rightarrow \frac{1}{32} v_4 \int d\ell \; \sum_i v_i \text{Tr} \; \gamma_{R_i,9} \cdot \text{Tr} \; \gamma_{R_i,9}^{-1}.
$$

The 5,5-sector: we have 4 NN and 4 DD bosons. We find contributions:

$$
1: \quad NS : \quad \Pi_r \left( 1 + q^r - 1/2 \right)^8 \\
R : \quad -16 \Pi_r (1 + q^r)^8 \\
Bos : \quad \Pi_r (1 - q^r)^{-8}
$$

$$
R_j : \quad NS : \quad \Pi_r \left( 1 + q^r - 1/2 \right)^4 \left( 1 - q^r - 1/2 \right)^4 \\
R : \quad 0 \\
Bos : \quad \Pi_r (1 - q^r)^{-4} (1 + q^r)^{-4}
$$

Thus, the oscillators contribute for the unit operator $[f_4 / f_1 (\sqrt{q})]^8$ and for the $R_i$-insertions $+4[f_3 f_4 / f_1 f_2 (\sqrt{q})]^4$. The unit operator piece gets multiplied by

$$
\sum_i M'_i \sum_{a,b \in 5_i} (\gamma_{1,5})_{a} (\gamma_{1,5}^{-1})_{b} \prod_{m,j \neq i} \sum_{w} \exp(-t(2\pi w r_j + X^m_a - X^m_b)^2/2\pi\alpha').
$$

The $R_i$ operator pieces get just $\sum_i M'_i \sum_{f=1}^{16} \text{Tr} \; \gamma_{R_i,5_1} \cdot \text{Tr} \; \gamma_{R_i,5_1}^{-1}$.

In the 5,5-sector ($i \neq j$) one has 4 ND bosons, 2 NN and 2 DD. For the unit operator, NS gives $4 \Pi_r (1 + q^r - 1/2)^4 (1 + q^r)^4$ but the Ramond sector is equal and opposite. For $R_i$ insertions, the NS sector gives zero (because of cancellation between vacua). The Ramond sector contributes only for $\epsilon_{ijk} R_k$.

In this case, bosons give $\Pi_r (1 + q^r - 1/2)^{-4} (1 - q^r)^{-4}$ and the Ramond fermions give $-4 \Pi_r (1 - q^r - 1/2)^4 (1 + q^r)^4$. The product is $-\left[f_2 f_4 / f_1 f_3 (\sqrt{q})\right]^4$.

The 5,9-sector is almost identical to the above case. The only sector which contributes is the Ramond $R_i$ term, and the oscillator parts again give $-\left[f_2 f_4 / f_1 f_3 (\sqrt{q})\right]^4$.
3.4 The Full Tadpole

For brevity, let us drop terms proportional to $\text{Tr} \gamma_R$, as these are all zero as we will see in the next section. The tadpole is then given by:

$$\frac{1}{32} v_4 f \int d\ell \left\{ v_1 v_2 v_3 \left( 32^2 - 64 \text{Tr} \gamma^T_{\Omega,9}\gamma^{-1}_{\Omega,9} + n_9^2 \right) + \sum_i v_i \prod_{j \neq i} \frac{1}{v_j} \left( 32^2 - 64 \text{Tr} \gamma^T_{\Omega R_i,5_i}\gamma^{-1}_{\Omega R_i,5_i} + n_{5_i}^2 \right) \right\} \quad (3.23)$$

Tadpole cancellation then implies

$$\gamma^T_{\Omega,9} = +\gamma_{\Omega,9}, \quad (3.24)$$

$$\gamma^T_{\Omega R_i,5_i} = +\gamma_{R_i,\Omega,5_i}, \quad (3.25)$$

and a total of 32 9-branes plus 32 of each of three types of 5-brane.

4 Chan-Paton Representation of $\Omega, R_i$

The basic orientifold group is defined, in part, by the following generators and relations:

$$(\Omega R_i)^2 = 1, \quad \Omega^2 = 1, \quad (4.1)$$

$$(\Omega R_i)(\Omega R_j)(\Omega R_k) = 1, \quad (\Omega R_i)\Omega(\Omega R_i)\Omega = 1. \quad (4.2)$$

for $j \neq k \neq i$. We would like the $\gamma$-matrices, in conjunction with the action of the orientifold group on the bulk CFT, to form a projective representation of the orientifold group\footnote{Although one can perhaps relax this condition.}. The bulk CFT contribution is easy to calculate, as explained in \cite{[2]}. leaving us with what might be considered strange phase differences between different sectors.

The relations (4.1),(4.2) imply the following relations

$$\gamma_{\Omega R_i}\gamma^{-1}_{\Omega R_i} T = c_i(s), \quad \gamma_{\Omega}\gamma^{-1}_{\Omega} T = c(s) \quad (4.3)$$

$$\gamma_{\Omega R_i}\gamma^{-1}_{\Omega R_j} T \gamma_{\Omega R_i}\gamma^{-1}_{\Omega R_j} T = \rho_{ijk}(s), \gamma_{\Omega R_i}\gamma^{-1}_{\Omega R_i} T \gamma_{\Omega R_i}\gamma^{-1}_{\Omega R_i} T = \rho_i(s) \quad (4.4)$$

where the factors on the RHS differ from sector to sector (the sectors are labeled by $s$).
We will analyze in some detail a few of these phases, and just list the others. We will argue that in addition to (3.24), (3.25) we must have cocycles of the following form

\[ \gamma^T_{\Omega,5_i} = -\gamma_{\Omega,5_i}, \quad (4.5) \]

\[ \gamma^T_{R_i\Omega,9} = -\gamma_{R_i\Omega,9}, \quad \gamma^T_{R_i\Omega,5_j} = -\gamma_{R_i\Omega,5_j}, \quad (4.6) \]

To show this, we need to examine all the mixed sectors, and in fact the problem is over-determined.

Let us begin with \( \gamma_{\Omega} \). The method is explained in Refs. [2], [8]. Consider first the product of two vertex operators in the 95\(_i\) sector. We obtain, in either the 99- or the 5\(_i\)5\(_i\)-sector, the product of two complex fermions, both of which are either NN or DD. Each of these rotates by a phase \( \pm i \) under \( \Omega \), and so the two contribute \(-1\) to the action of \( \Omega \) in the 99 or 5\(_i\)5\(_i\) sectors. This corresponds to \( \Omega^2 = -1 \) on the oscillators of the 95\(_i\) sector and hence we obtain[2] the \(-1\) in eq. (4.5). Note that this is consistent also with the 5\(_i\)5\(_j\)-sector: taking the product of two such vertex operators we obtain, in either the 5\(_i\)5\(_i\)- or 5\(_j\)5\(_j\)-sector, a product of two complex fermions, one DD and the other NN. This leads to the same phase for all \( i \), as in (4.5).

To show the consistency of (4.6), we need to look at \( R_i\Omega \) in each of four sectors. Again we consider the product of two vertex operators of a given sector.

- **95\(_i\):** Taking the product of two such operators, we obtain, in the 99- or 5\(_i\)5\(_i\)-sector, the product of two fermions as discussed above. Both change sign under \( R_i \), giving a net \(-1\) sign between eq. (3.25) and the first of eqs. (4.6).

- **5\(_i\)5\(_j\):** Here, only one of the two fermions will change sign under \( R_i \), giving a net \(-1\) sign between eq. (3.25) and the second of eqs. (4.6).

- **95\(_j\) \((i \neq j)\):** In the 99 (5\(_j\)5\(_j\)) we obtain the product of two complex NN(DD) fermions; one of them is flipped by \( R_i \) giving a relative phase of +1 between the 9- and 5\(_j\)-sectors, consistent with eq. (4.6).
• $5_j 5_k$ ($j, k \neq i$): In the $5_j 5_j$ ($5_k 5_k$) we obtain the product of an NN and a DD fermion; both are flipped by $R_i$ giving a relative phase of +1 between the $5_j$- and $5_k$-sectors, consistent with eq. (4.6).

Thus the conditions (4.5),(4.6) are implied by (3.24),(3.25) and are, fortunately, self consistent.

We still need to check the other relations that define the orientifold group, and determine the relative phase between different sectors. The phase differences can be read from the explicit solution in the next section, which is unique up to unitary transformations on the Chan-Paton factors. At this stage we will suffice in presenting all the relevant information in the following table.

|         | $\Omega^2$ | $R_1^2$ | $R_2^2$ | $R_3^2$ | $R_1 R_2 R_3$ |
|---------|------------|---------|---------|---------|---------------|
| 99      | +1         | +1      | +1      | +1      | +1            |
| $5_i 5_i$ | +1         | +1      | +1      | +1      | +1            |
| 95_1    | −1         | +1      | −1      | −1      | −1            |
| 95_2    | −1         | −1      | +1      | −1      | −1            |
| 95_3    | −1         | −1      | −1      | +1      | +1            |
| $5_1 5_2$ | +1         | −1      | −1      | +1      | +1            |
| $5_3 5_1$ | +1         | −1      | +1      | −1      | −1            |
| $5_2 5_3$ | +1         | +1      | −1      | −1      | −1            |

We have again looked at mixed sectors. As before this determines the difference in the phases between the different unmixed sectors, and as before the problem is over-determined. To obtain the phase difference between, say, some $5_i 5_i$ sector and, say, 99 sector, pick the appropriate line in the table and multiply the appropriate phases in the row.

### 4.1 A definite choice

Let us now make a definite choice for the $\gamma$’s. Define

$$M_i = \begin{cases} 
( 0 & I ) , 
( -\varepsilon & 0 ) , 
( 0 & \varepsilon ) 
\end{cases}, \quad (4.7)$$

$$D = \begin{pmatrix} 0 & -\varepsilon \\ \varepsilon & 0 \end{pmatrix} \quad (4.8)$$

where $\varepsilon = i \sigma_2$ and each block is understood to be a direct product with the $k$-dimensional identity matrix, $k \leq 32$ being the number of coincident $p$-branes.
of the appropriate type. These matrices satisfy $M_i^2 = -I$, $M_i M_j = \epsilon_{ij k} M_k$ and $M_i^T = M_i^{-1}$. It is useful to also define

$$N_i \equiv D M_i = \left\{ \begin{pmatrix} \varepsilon & 0 \\ 0 & \varepsilon \end{pmatrix}, \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \right\}.$$

We will work in an enlarged matrix notation consisting of blocks corresponding to 9, 51, 52, 53-branes. It is convenient to define cocycles $C_1 = \text{diag}(I, -I, I, I)$, etc., and $C_0 = -C_1 C_2 C_3 = \text{diag}(-I, I, I, I)$.

Collecting the results of the previous section, we have:

$$\gamma^T_\Omega = -C_0 \gamma_\Omega$$

(4.9)

$$\gamma^T_{R,\Omega} = -C_i \gamma_{R,\Omega}$$

(4.10)

and

$$\gamma_{R,\Omega} \gamma^T_{R,\Omega} \gamma_{R,\Omega} \gamma^T_\Omega = C_0 C_3 \epsilon_{ijk} I \quad \text{(for } i \neq j \neq k)$$

(4.11)

$$\gamma_{R,\Omega} \gamma^T_{R,\Omega} \gamma_{R,\Omega} \gamma^T_\Omega = C_0 C_i I$$

(4.12)

$$\gamma_{R,\Omega} \gamma_{R,\Omega} \gamma_{R,\Omega} \gamma_{R,\Omega} = C_0 C_3 \epsilon_{ijk} I$$

(4.13)

$$\gamma_{R,\Omega} \gamma_{R,\Omega} \gamma_{R,\Omega} = C_0 C_i I$$

(4.14)

There are many choices consistent with the above constraints. A particular example, consisting entirely of orthogonal matrices is

| $\gamma_\Omega$ | $\gamma_{R,\Omega}$ | $\gamma_{R,\Omega}$ | $\gamma_{R,\Omega}$ | $\gamma_{R,\Omega}$ | $\gamma_{R,\Omega}$ | $\gamma_{R,\Omega}$ |
|---|---|---|---|---|---|---|
| 9 | $I$ | $M_1$ | $M_2$ | $M_3$ | $-M_1$ | $-M_2$ | $-M_3$ |
| 51 | $N_1$ | $-D$ | $M_2$ | $M_3$ | $-M_1$ | $N_3$ | $-N_2$ |
| 52 | $N_1$ | $M_3$ | $-D$ | $M_2$ | $-N_2$ | $-M_1$ | $N_3$ |
| 53 | $N_1$ | $M_2$ | $M_3$ | $D$ | $N_3$ | $-N_2$ | $M_1$ |

This choice satisfies $\gamma_{R_i} = C_0 \gamma_{R,\Omega} \gamma_\Omega$. and

$$\gamma^T_{R_i} = C_0 C_i \gamma_{R_i}$$

(4.15)

In fact the choice of matrices given in the table is essentially unique: one can show that the available gauge freedoms allow one to put any consistent choice in this form. This is not a trivial statement; a-priori we can choose the factors $\rho_{ijk}, \rho_i$ in the 99 sector to be whatever we want (and then they are determined in the other sectors) but this does not seem to be the case. If we start with any values for these constants other then the ones that correspond to this solution we cannot solve all the constraints.
5 The Spectrum

In this section we examine the spectrum and interactions dictated by the algebraic and tadpole cancellation conditions found above. The conditions on Chan-Paton matrices of individual states were outlined in Section 2.2. We consider each sector in turn.

5.1 99-sector

We have \( \gamma_{\Omega, 9} = I \), \( \gamma_{R_i, 9} = -M_i \). Thus from eqs. (2.6), the 99-gauge boson Chan-Paton factor \( \lambda^{(0)} \) is antisymmetric and satisfies \( \lambda^{(0)} = -M_i \lambda^{(0)} M_i \). The solution to these conditions is (\( A = \text{antisym.}, S = \text{sym.} \)):

\[
\lambda^{(0)} = \begin{pmatrix}
A & S_1 & S_2 & S_3 \\
-S_1 & A & S_3 & -S_2 \\
-S_2 & -S_3 & A & S_1 \\
-S_3 & S_2 & -S_1 & A
\end{pmatrix}
\]

which is the adjoint representation of \( Sp(8) \). This may be broken to smaller groups by the addition of Wilson lines.

The matter fields \( \lambda^{(i)} \) are

\[
\lambda^{(1)} = \begin{pmatrix}
A_1 & A_2 & S & A_3 \\
A_2 & -A_1 & -A_3 & S \\
-S & -A_3 & A_1 & A_2 \\
A_3 & S & A_2 & -A_1
\end{pmatrix} ; \quad \lambda^{(2)} = \begin{pmatrix}
A_3 & S & A_1 & A_2 \\
-S & A_3 & A_2 & -A_1 \\
A_1 & A_2 & -A_3 & -S \\
A_2 & -A_1 & S & -A_3
\end{pmatrix} ;
\]

\[
\lambda^{(3)} = \begin{pmatrix}
A_1 & A_2 & A_3 & S \\
A_2 & -A_1 & -S & A_3 \\
A_3 & S & -A_1 & -A_2 \\
-S & A_3 & -A_2 & A_1
\end{pmatrix} \quad (5.1)
\]

These are six bosons (giving 3 chiral multiplets) in the \( \mathbf{E} = 120 \) of \( Sp(8) \).

5.2 5i, 5\(_i\)-sector

First take the 5\(_i\) which are at fixed points of \( R_i \). For example, for \( i = 1 \), we have, from Section 2.2, the conditions

\[
\lambda^{(0)T} = N_1 \lambda^{(0)} N_1; \quad \lambda^{(0)} = -M_1 \lambda^{(0)} M_1; \quad \lambda^{(0)} = +N_2,3 \lambda^{(0)} N_{2,3}
\]
It is convenient to define \( \bar{\lambda}^{(A)} = N_1 \lambda^{(A)} \); if 4\( k \) 5\( i \)-branes are at the same fixed point of \( R_i \), we obtain an \( Sp(k) \) gauge group, with
\[
\bar{\lambda}^{(0)} = \begin{pmatrix} S & 0 \\ 0 & S \end{pmatrix}.
\]

where the blocks are 2\( k \)-dimensional (\( k \leq 8 \)).

Applying the conditions stated in Section 2.2, we find that \( \bar{\lambda}^{(1)} \) is symmetric, and \( \bar{\lambda}^{(2,3)} \) antisymmetric. Solutions are:
\[
\bar{\lambda}^{(i)} = \begin{cases} 
\begin{pmatrix} 0 & A \\ -A & 0 \end{pmatrix}, & \begin{pmatrix} A & 0 \\ 0 & -A \end{pmatrix}, & \begin{pmatrix} 0 & A \\ A & 0 \end{pmatrix} 
\end{cases}
\]

Thus we again get 3 chiral multiplets in the block. For all 5\( i \)-branes at the same fixed point of \( R_i \), we see that the spectrum is completely T-dual.

Now consider moving 5-branes away from the fixed points. First suppose we move the branes away from the fixed point of \( R_i \) along the fixed direction of \( R_j \) (refer again to Fig. 1). For definiteness, consider a 5\( i \)-brane on the 6, 7-axis (i.e. along the fixed line of \( R_2 \)). \( R_1 \) and \( R_3 \) map a brane to its image and so place no restrictions. Thus we have the conditions:
\[
\lambda^{(0,1)T} = +N_1 \lambda^{(0,1)} N_1; \quad \lambda^{(2,3)T} = -N_1 \lambda^{(2,3)} N_1 \quad (5.4)
\]
\[
\lambda^{(0,2)} = +N_3 \lambda^{(0,2)} N_3; \quad \lambda^{(1,3)} = -N_3 \lambda^{(1,3)} N_3 \quad (5.5)
\]

With \( \bar{\lambda}^{(A)} \) defined as above, \( \bar{\lambda}^{(0,1)} \) are symmetric and \( \bar{\lambda}^{(2,3)} \) antisymmetric. If, as above, we started with 4\( k \) 5\( i \)-branes at the fixed point, we end up with at most 2\( k \) 5\( i \)-branes along the 67-axis (their images accounting for the other 2\( k \)). We then find
\[
\bar{\lambda}^{(0)} = \begin{pmatrix} S_1 & S_2 \\ S_2 & S_1 \end{pmatrix}; \quad \bar{\lambda}^{(1)} = \begin{pmatrix} S & A \\ -A & -S \end{pmatrix}; \quad (5.6)
\]
\[
\bar{\lambda}^{(2)} = \begin{pmatrix} A_1 & A_2 \\ A_2 & A_1 \end{pmatrix}; \quad \bar{\lambda}^{(3)} = \begin{pmatrix} A & S \\ -S & -A \end{pmatrix}; \quad (5.7)
\]

where the blocks of these matrices are at most \( k \times k \)-dimensional. \( \bar{\lambda}^{(0)} \) is the adjoint of \( Sp(k/2) \times Sp(k/2) \). Note the difference in form between \( \bar{\lambda}^{(2)} \) and \( \bar{\lambda}^{(1,3)} \); this is because the motion of 5\( i \)-branes along the \( R_2 \)-fixed line is T-dual to turning on the \( \lambda_2 \) of the 99-sector. We expect then that \( \lambda^{(2)} \) is special, being identified with the Higgs mode. Each of the \( Sp(k/2) \) thus has matter in
Note that these fit appropriately into vector and hypermultiplets of $N = 2$ supersymmetry. As we will see later, a detailed understanding of this spectrum in terms of Higgsing of the previous case relies crucially on the presence of the tree-level superpotential. Note also that this all occurs only for even $k$ (only a group of eight 5-branes may move off of a fixed point); thus the moduli space breaks up into disconnected sectors, in a fashion similar to that of Ref. [2].

If we move the 5-branes away from fixed planes entirely, there are three images, and the $R_i$ give no condition on states. In the 55-sector, $\Omega$ gives (for 5 branes) $\lambda^{(0,1)} = N_1 \lambda^{(0,1)T} N_1$ and $\lambda^{(2,3)} = -N_1 \lambda^{(2,3)T} N_1$. Proceeding as above, we may define $\lambda^{(A)} = N_1 \tilde{\lambda}^{(A)}$; the $\lambda^{(0,1)}$ will be symmetric and the $\tilde{\lambda}^{(2,3)}$ antisymmetric, but otherwise unconstrained. We thus find an $Sp(k/2)$ gauge group, with matter in $\square + 2 \times \square$. In Higgsing from $Sp(k/2) \times Sp(k/2)$, the $\square$ of one group is eaten, while the $\square$'s, as we discuss later, are made massive through the superpotential. Note that this spectrum can be arranged into multiplets of $N = 2$ supersymmetry. The operators which appear in the (renormalizable) superpotential are also consistent with the enhanced supersymmetry. In fact we do not expect this to be an exact symmetry; it merely reflects the fact that isolated 5-branes act like those of Ref. [2].

### 5.3 95- and 55-sectors

Let us now check 95 and 55 states. If we have $4k_9$ 9-branes and $4k_{5_1}$ 51-branes at a fixed point of $R_1$, we find projections

$$\lambda = -M_1 \lambda M_1$$  \hfill (5.8)

$$\lambda = -\eta M_2 \lambda N_3 = +\eta M_3 \lambda N_2$$  \hfill (5.9)

where $\eta = (2s_4)i$, as in eq. (2.8). This has solution:

$$\lambda = \begin{pmatrix} m_1 & m_2 & m_3 & m_4 \\ -\eta m_1 & -\eta m_2 & \eta m_3 & \eta m_4 \\ -m_3 & -m_4 & m_1 & m_2 \\ -\eta m_3 & -\eta m_4 & -\eta m_1 & -\eta m_2 \end{pmatrix}$$

where $m_i$ are arbitrary $k_9 \times k_{5_1}$ matrices. This gives one chiral superfield in the $(2k_9, 2k_{5_1}) = (\square, \square)$ of $Sp(k_9) \times Sp(k_{5_1})$. 

\[19\]
If the 51-brane is moved out along the fixed line of $R_2$, we have only the constraint from $R_2$: $\lambda = -\eta M_2 \lambda N_3$. This will transform as $(\Box, \Box, 1) \oplus (\Box, 1, \Box)$ of the $Sp(k_9) \times Sp(k_{51}/2) \times Sp(k_{51}/2)$ gauge group.

If we move the 5-branes away from fixed planes entirely, there are no constraints, and again we find $(\Box, \Box, \Box)$ of the unbroken gauge group. Note that since the corresponding Higgsing does not give vevs to these fields, it must be that the extra fields are made massive by the superpotential. Indeed this will be found in the next section.

As discussed in Section 2, when two different 5-branes intersect one another, there are massless $5_i, 5_j$ states. These will transform in the $(\Box, \Box)$ of the appropriate groups. Again, the dynamics of these states depend crucially on the existence of a superpotential, and so we turn to that now.

6 The Superpotential

We have indicated in several places above the importance of the tree-level superpotential in the proper understanding of the symmetry breaking phenomena discussed above. We study this in some detail in this section, concentrating on couplings of open string modes only, which are relevant for the discussion of Higgsing.

The matter multiplets of open string states organize themselves into chiral multiplets $z_i = \phi^{2m_i} + i\phi^{2m_i-1}$, one per $T^2$. In addition, in the 59 and 515j sectors, there are states which we will label simply by $z$, with a superscript to identify the appropriate sector. The superpotential should be invariant under automorphisms of the orientifold group.

We consider here only the renormalizable part of the superpotential connecting open string states. We will determine this by consideration of three-point disc amplitudes involving two Ramond states and one NS state. If such an amplitude is non-vanishing, it can of course be interpreted as a term in the superpotential. We can take vertex operators in the canonical ghost pictures:

$$V_{-1/2}^{(j)} = u^{(j)}_{\alpha} S^\alpha e^{-\phi/2} e^{i\bar{w}_j \cdot \bar{H} e^{ik \cdot X}}$$
$$V_{-1}^{(j)} = e^{-\phi} e^{i\bar{v}_j \cdot \bar{H} e^{ik \cdot X}}$$

In order for a term $W = z_i z_j z_k$ to exist, we must have $\bar{w}_i + \bar{w}_j + \bar{v}_k = 0$. 

20
Traces over Chan-Paton factors will be understood. We identify the $w_i$ via their transformation properties under the $R_j$.

- **99**: In the 99 sector, we have $\vec{w}_1 = (\pm \frac{1}{2}, \mp \frac{1}{2}, \pm \frac{1}{2})$, $\vec{w}_2 = (\mp \frac{1}{2}, \pm \frac{1}{2}, \mp \frac{1}{2})$, $\vec{w}_3 = (\pm \frac{1}{2}, \mp \frac{1}{2}, \pm \frac{1}{2})$, $\vec{v}_1 = (\pm 1, 0, 0)$, $\vec{v}_2 = (0, \pm 1, 0)$, and $\vec{v}_3 = (0, 0, \mp 1)$. We deduce that there is a term

$$W = z_{99}^{1} z_{99}^{2} z_{99}^{3}.$$

- **5_19**: The Ramond state has $\vec{w}_1 = (\pm \frac{1}{2}, 0, 0)$. There are thus additional terms of the form

$$W = z_{i}^{99} z_{95}^{i} z_{5}^{9}.$$

- **5_i5_i**: The vectors will be similar to those of the 99-sector. Thus we expect

$$W = z_{i}^{5_1} z_{i}^{5_2} z_{i}^{5_3} + z_{i}^{5_1} z_{i}^{5_9} z_{5}^{i}.$$

- **5_i5_j**: When 5-branes cross, there will be additional massless states, and there can be couplings of the form

$$W = \epsilon_{ijk} z_{i}^{5_1} z_{j}^{5_1} z_{k}^{5_1} + z_{5_1}^{5_1} z_{5_2} z_{5_3} z_{5_1} + z_{5_1}^{5_1} z_{5_9} z_{5}^{i}.$$

### 7 Further Comments on Higgsing

With an understanding of the superpotential, we can return to the discussion of Higgsing related to moving 5-branes away from fixed points. Consider again 32 5_1-branes at a fixed point of $R_1$. Moving some of these away is T-dual to turning on Wilson lines in the 9-brane sector, or equivalently, to giving a vev to one of the antisymmetric tensor fields. Thus we expect that moving 5-branes should have a low energy description in terms of Higgsing. Consider moving $2k_5$ 5_1-branes out along the fixed line of $R_2$. This corresponds to turning on the antisymmetric tensor field $z_{5_1}^{5_1}$, and as we have seen in previous sections, the gauge group is broken to $Sp(8 - k_5/2) \times Sp(k_5/2) \times Sp(k_5/2)$, the first factor arising from those branes still at the $R_1$ fixed point. As far as
$z_5^{51}$ is concerned, the physics is simple: some of its components are eaten, and the remaining components appear as in eq. (5.7). Certain components of the other $5_15_1$ modes are given mass through the superpotential

$$W = \langle z_2^{5151} \rangle z_1^{5151} z_3^{5151}$$

leading to the remaining fields in eqs. (5.6), (5.7). As well, if there are massless $5_15_j$ ($j \neq 1$) modes, parts of these will be given mass by the superpotential as well. This is completely sensible, and the situation is displayed in Figure 4. The vev that we have been discussing corresponds to moving the $5_1$-brane vertically, leaving the $5_3$-brane in place, which clearly gives mass to those open string modes stretching between the two.

Note however that if the two different 5-branes are moved together up the 67-axis in Figure 4, we should see from the superpotential that the $5_15_3$-state is not made massive. Indeed, this situation corresponds to turning on both $z_2^{5151}$ and $z_2^{5353}$. The relevant terms in the superpotential are

$$W = \left( \langle z_2^{5151} \rangle - \langle z_2^{5353} \rangle \right) z_2^{5153} z_2^{5351}$$

When the two vevs are turned on equally, the $5_15_3$-states are not lifted.

It is clear from these examples that the motion of Dirichlet 5-branes is completely equivalent to Higgsing provided the superpotential is taken into account.
8 Conclusions

In this paper, we have presented a four-dimensional model with $N = 1$ supersymmetry which is an orbifold of Type I superstrings, and have concentrated on some of the technology that goes into such a construction. A fully consistent picture emerges if three types of Dirichlet 5-branes are included, all of which fill the four space-time dimensions. We have confined ourselves to a tree-level analysis of (what are from the low energy point of view), Higgsing phenomena. Clearly many outstanding issues remain particularly those related to non-perturbative issues and duality; we will return to these in a future publication\[7\].

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