Potential regime for heavy quarks dynamics and Lorentz nature of confinement

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Abstract

Propagation of the heavy quark in the field of a static antiquark source is studied in the framework of effective Dirac equation. The model of QCD vacuum is described by bilocal gluonic correlators. In the heavy quark limit the effective interaction is reduced to the potential one with 5/6 Lorentz scalar and 1/6 Lorentz vector linear confinement, while spin–orbit term is in agreement with Eichten–Feinberg–Gromes results. New spin–independent corrections to the leading confining regime are identified, which arise due to the nonlocality of the interaction in time direction and quark Zitterbewegung.

A lot of evidence exists that the non-abelian nature of Yang-Mills QCD leads to the confinement of colour charges. Apart from purely theoretical considerations, the main bulk of data on confinement comes from the lattice QCD simulations and from the phenomenology of hadronic spectra. The lattice calculations firmly establish the linear rising force between two static colour sources, while the hadronic masses are most successfully described with the effective $q\bar{q}$ potential which is the sum of linear and Coulomb forces. Complementary to these facts is the idea that QCD at large distances is a string theory, and linear potential between heavy constituents is a manifestation of the string–type dynamics.

In general, the dynamics governed by QCD should be nonlocal; nevertheless, it is natural to assume that in the heavy quark limit it is reduced to the nonrelativistic local potential acting between quark and antiquark which is supplied by subleading $O(1/m^2)$ corrections. Among these corrections an important role is played by spin-dependent forces which define the spin splittings of heavy quarkonia and serve as a testing ground for various theoretical approaches.

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The most consistent derivation of the spin-dependent potentials was performed in the framework of the Wilson loop approach [1, 2], where the potentials were expressed in terms of expectation values of gluonic field insertions into a Wilson loop corresponding to the propagation of $q\bar{q}$ pair, and fundamental relations between static and spin-dependent potentials were established [2].

One of the challenging problems is the long range part of the spin-orbit force

$$V_{SO}(r) = \left( \frac{\vec{\sigma} q \vec{l}_q}{4m_q^2} - \frac{\vec{\sigma} q \vec{l}_q}{4m_q^2} \right) \left( \frac{1}{r} \frac{\partial \varepsilon}{\partial r} + \frac{2}{r} \frac{\partial V_1}{\partial r} \right) - \frac{1}{2m_q m_{\bar{q}}} \left( \vec{\sigma} q \vec{l}_q - \vec{\sigma} q \vec{l}_q \right) \frac{1}{r} \frac{\partial V_2}{\partial r},$$

which is sensitive to the Lorentz nature of the static confining interaction $\varepsilon(r)$. If it is a local Lorentz scalar potential, one has

$$V_1 = -\varepsilon \quad V_2 = 0,$$

while for the time component of the Lorentz vector it is

$$V_1 = 0 \quad V_2 = \varepsilon.$$

Phenomenological analysis of heavy quarkonia spectra clearly prefers possibility (2), but it was shown within Vacuum Background Correlators Method [3-6] and in the framework of the Coulomb gauge QCD Hamiltonian [7] that relation (2) is respected at large distances without ad hoc assumption of scalar confinement. It was demonstrated in [3-6] that nonrelativistic reduction of the long range interaction is not the whole story, and additional contributions to the spin–orbit force exist, which are due to the nonlocality of the QCD generated interaction.

The aim of the present paper is to study systematically the nonlocality corrections to the effective long range interaction for heavy quark. We have found a new constraint on the parameters of the interaction, which appears to be crucial for the selfconsistent potential–type dynamics of heavy quarks. We adopt the approach suggested recently in [8] and, in a more simple version, in [9], that allows, as a byproduct, to establish explicitly the Lorentz nature of confinement. Our derivation is restricted to the case of a heavy quark propagating in the field of an infinitely heavy antiquark colour source, and is only the first step towards defining the full dynamics of heavy quarkonia.

The starting point of approach [8] is the Green function $S_{q\bar{Q}}$ for the $q\bar{Q}$ system, written in the Euclidean space as

$$S_{q\bar{Q}}(x, y) = \frac{1}{N_C} \int D\psi D\psi^+ DA_\mu \exp \left\{ -\frac{1}{4} \int d^4xF_{\mu \nu}^a - \int d^4x \psi^+ (-i\partial - im - \hat{A}) \psi \right\} \times$$

$$\times \psi^+(x) S_Q(x, y) \psi(y),$$

where $S_Q(x, y)$ is the propagator of the static antiquark placed at the origin. To consider the one-body limit it is convenient to choose the modified Fock–Schwinger gauge [10]

$$A_4(x_4, \bar{0}) = 0, \quad \bar{x} \bar{A}(x_4, \bar{x}) = 0,$$
in which $S_Q(x, y)$ is simply

$$S_Q(x, y) = i \frac{1 - \gamma^4}{2} \theta(x_4 - y_4) e^{-M(x_4 - y_4)} + i \frac{1 + \gamma^4}{2} \theta(y_4 - x_4) e^{-M(y_4 - x_4)}. \quad (6)$$

Integration over gluonic field $A_\mu$ in (4) can be performed with the result

$$S_{qQ}(x, y) = \frac{1}{N_C} \int D\psi D\bar{\psi} \exp \left\{ - \int d^4x L_{eff}(\psi, \bar{\psi}^+) \right\} \bar{\psi}^+(x) S_Q(x, y) \psi(y), \quad (7)$$

where $L_{eff}(\psi, \bar{\psi}^+)$ is the effective quark Lagrangian:

$$\int d^4x L_{eff}(\psi, \bar{\psi}^+) = \int d^4x \bar{\psi}^{\alpha_1}(x) \gamma_\mu \psi^{\alpha}(x) < A^{\alpha}_{\mu} > +$$

$$+ \frac{1}{2} \int d^4x_1 d^4x_2 \bar{\psi}^{\alpha_1}(x_1) \gamma_\mu_1 \psi^{\beta_1}(x_1) \psi^{\alpha_2}(x_2) \gamma_\mu_2 \psi^{\beta_2}(x_2) < A^{\alpha_1}_{\mu_1} A^{\alpha_2}_{\mu_2} > + \ldots, \quad (8)$$

where all $\alpha$-s and $\beta$-s are fundamental colour indeces, and the irreducible correlators $< A^{\alpha}_{\mu_1} (x_1) \ldots A^{\alpha_n}_{\mu_n} (x_n) >$ of all orders should enter. The first one, $< A^{\alpha}_{\mu} >$, vanishes due to the gauge and Lorentz invariances, and in what follows we keep only bilocal correlator $< A^{\alpha}_{\mu}(x) A^{\beta}_{\nu}(y) > \equiv K^{\alpha\gamma}_{\mu\beta\delta}(x, y)$ and disregard the contributions of higher correlators.

Using gauge invariance of the vacuum one has

$$K^{\alpha\gamma}_{\mu\beta\delta}(x, y) = (\lambda_{\alpha})^{\alpha}_{\beta} (\lambda_{\beta})^{\gamma}_{\delta} K^{ab}_{\mu\nu}(x, y) = 2 (\lambda_{\alpha})^{\alpha}_{\beta} (\lambda_{\gamma})^{\gamma}_{\delta} K_{\mu\nu}(x, y), \quad (9)$$

$$K_{\mu\nu}(x, y) = \frac{1}{2(N_C^2 - 1)} K^{aa}_{\mu\nu}(x, y),$$

and, because of the relation $(\lambda_{\alpha})^{\alpha}_{\beta} (\lambda_{\gamma})^{\gamma}_{\delta} = \frac{1}{2} \delta^{\alpha}_{\beta} \delta^{\gamma}_{\delta} - \frac{1}{2N_C} \delta^{\alpha}_{\beta} \delta^{\gamma}_{\delta}$, expression (8) takes the form

$$\int d^4x L_{eff}(\psi, \bar{\psi}^+) = \int d^4x \bar{\psi}^{\alpha}(x) (-i\bar{\partial} - im) \psi^{\alpha}(x) +$$

$$+ \frac{1}{2} \int d^4x d^4y \bar{\psi}^{\alpha}(x) \gamma_\mu \psi^{\beta}(x) \psi^{\gamma}(y) \gamma_\nu \psi^{\alpha}(y) K_{\mu\nu}(x, y) \quad (10)$$

in the limit $N_C \rightarrow \infty$, yielding the Schwinger–Dyson equation

$$(-i\partial_x - im) S(x, y) - i \int d^4z M(x, z) S(z, y) = \delta^{(4)}(x - y) \quad (11)$$

with the self–energy part $M(x, z)$ given by

$$- iM(x, z) = K_{\mu\nu}(x, z) \gamma_\mu S(x, z) \gamma_\nu, \quad (12)$$

where $S(x, y) = \frac{1}{N_C} < \psi^{\beta}(x) \psi^{\beta}(y) >$ is the colour trace of the quark Green function. As in gauge (3) Green function (8) of the static source is unity in the colour space, quantity $S(x, y)$ completely defines propagation of the colourless $q\bar{Q}$ object.
Gauge condition \((5)\) can be rewritten as

\[
A^a_i(x_4, \vec{x}) = \int_0^1 F^a_{i4}(x_4, \alpha \vec{x}) d\alpha,
\]

\[
A^a_i(x_4, \vec{x}) = \int_0^1 \alpha x_k F^a_{ki}(x_4, \alpha \vec{x}) d\alpha, \quad i = 1, 2, 3,
\]

so the average \(K_{\mu\nu}\) can be expressed in terms of field strength correlator \(<F^a_{\mu\nu}(x)F^b_{\lambda\rho}(y)>\), for which we use the parametrization [3, 4]:

\[
< F^a_{\mu\nu}(x)F^b_{\lambda\rho}(y) > = \frac{\delta^{ab}}{N_C^2 - 1} D(x - y)(\delta_{\mu\lambda}\delta_{\nu\rho} - \delta_{\mu\rho}\delta_{\nu\lambda}) + \Delta^{(1)},
\]

where the second term \(\Delta^{(1)}\) is a full derivative and does not contribute to the confinement.

As we are interested only in long range force, we consider only the term proportional to \(D(x - y)\) in \((13)\), which, in contrast to \(\Delta^{(1)}\), contributes to the area law with the string tension

\[
\sigma = 2 \int_0^\infty d\tau \int_0^\infty d\lambda D(\tau, \lambda).
\]

Function \(D(u_4, |\vec{u}|)\) is actually a function of \(u_4^2 + \vec{u}^2\) due to Lorentz invariance, but in our apparently non-invariant treatment we keep dependences on \(|\vec{u}|\) and \(|u_4|\) separately as in \((14)\).

Finally, for average \(K_{\mu\nu}(x, y)\) one has \((\tau = x_4 - y_4)\):

\[
K_{44}(\tau, \vec{x}, \vec{y}) = (\vec{x}\vec{y}) \int_0^1 d\alpha f_0 \int_0^1 d\beta D(\tau, |\alpha \vec{x} - \beta \vec{y}|),
\]

\[
K_{4i}(\tau, \vec{x}, \vec{y}) = K_{i4}(\tau, \vec{x}, \vec{y}) = 0,
\]

\[
K_{ik}(\tau, \vec{x}, \vec{y}) = ((\vec{x}\vec{y})\delta_{ik} - y_k x_i) \int_0^1 \alpha d\alpha \int_0^1 d\beta D(\tau, |\alpha \vec{x} - \beta \vec{y}|).
\]

The system of equations \((11), (12)\) may be rewritten in terms of wave functions as

\[
(-i\hat{\partial}_x - im)\psi(x) + \int d^4z K_{\mu\nu}(x, z) \gamma_{\mu} S(x, z) \gamma_{\nu} \psi(z) = 0.
\]

This equation is essentially nonlinear, since the eigenfunctions \(\psi_n\) enter the spectral representation for \(S(x, z)\). In the heavy quark limit we solve it perturbatively, substituting the free Green function \(S_0(x, z)\) into the self–energy part. The resulting linear equation was considered in [8, 9].

As correlators \((13)\) are defined in the Euclidean space, it is convenient to formulate the eigenvalues problem for equation \((18)\) in the Euclidean space too and perform the Wick rotation to the Minkowski one afterwards arriving at the Dirac-type equation

\[
(\hat{\alpha} \gamma + \gamma_0 m + \gamma_0 \gamma_4) \psi = E \psi,
\]
with operator $\hat{M}$ given by

$$\hat{M}(\vec{x}, \vec{z}) = -i \int_0^\infty d\tau K_{\mu\nu}(\tau, \vec{x}, \vec{z}) \int \frac{d^3 k}{(2\pi)^3} e^{i\vec{k}(\vec{x} - \vec{z})} \times$$

$$\times \left\{ \gamma_\mu - \frac{i\gamma_4}{2} \right\} \left\{ \begin{array}{l}
\gamma_\mu \left( 1 + \left( \varepsilon_0 - \frac{\vec{k}^2}{2m} \right) \tau \right) + i\gamma_\mu \left( \begin{array}{c}
\frac{1 + \gamma_4}{2} \gamma_\nu e^{-\varepsilon_0 \tau} + \gamma_\mu \frac{1 - \gamma_4}{2} \gamma_\nu e^{-2m\tau} + \\
n\gamma_\mu \left( \frac{\vec{k} \cdot \vec{\gamma}}{2m} - \frac{i\vec{k}^2}{4m^2} \right) \gamma_\nu \left( 1 + e^{-2m\tau} \right),
\end{array} \right.
\end{array} \right\},$$

where $\varepsilon = \varepsilon(\vec{k}) = \sqrt{\vec{k}^2 + m^2}$ and $\gamma$-matrices are Euclidean ones ($\gamma_4 E = \gamma_0 M$, $\vec{\gamma} E = -i\vec{\gamma} M$).

Operator $\hat{M}$ is nonlocal both in space and time, and our strategy is to find the leading local limit, to establish the first order nonlocal corrections and to check if they are small. To this end we approximately rewrite the expression in the curly brackets in (20) in the form

$$i\gamma_\mu \left( 1 + \left( \varepsilon_0 - \frac{\vec{k}^2}{2m} \right) \tau \right) + i\gamma_\mu \left( \begin{array}{c}
\frac{1 + \gamma_4}{2} \gamma_\nu e^{-\varepsilon_0 \tau} + \frac{1 - \gamma_4}{2} \gamma_\nu e^{-2m\tau} + \\
n\gamma_\mu \left( \frac{\vec{k} \cdot \vec{\gamma}}{2m} - \frac{i\vec{k}^2}{4m^2} \right) \gamma_\nu \left( 1 + e^{-2m\tau} \right),
\end{array} \right\},$$

introducing the quarks binding energy $\varepsilon_0 = E - m$.

The leading local confining interaction is obtained after omitting the terms proportional to $\varepsilon_0$, $\vec{\gamma} \vec{k}$ and $\vec{k}^2$ in (21), and yields

$$\hat{M}_0(\vec{x}, \vec{z}) = \delta^{(3)}(\vec{x} - \vec{z}) \int_0^\infty d\tau K_{\mu\nu}(\tau, \vec{x}, \vec{x}) \left\{ \gamma_\mu \frac{1 + \gamma_4}{2} \gamma_\nu + \gamma_\mu \frac{1 - \gamma_4}{2} \gamma_\nu e^{-2m\tau} \right\} =$$

$$= \delta^{(3)}(\vec{x} - \vec{z}) V_{conf}(\vec{x}).$$

It is relevant now to comment on the Lorentz nature of confinement and to clarify some related confusions. The underlying interaction is bilinear in vector vertices, as it is clearly seen from the effective Lagrangian (8): everywhere including resulting expression (22) the interaction contains $\gamma_\mu \ldots \gamma_\nu$ product. As it will be shown below, for heavy quarks this structure actually reduces to $\gamma_0 \ldots \gamma_0$ product, and we agree at this point with the statement made in [7]: the interaction is time–like vector one. However, this almost trivial observation does not straightforwardly help to answer another question, which is usually put in connection with the problem of the Lorentz structure: the effective local confining interaction $\hat{M}_0(\vec{x}, \vec{z})$ can be proportional either to unity or to $\gamma_0$, and it is added either to the mass term (scalar confinement) or to the energy term (vector confinement) in the effective Dirac equation (19). The answer to this question depends on the behaviour of the function $K_{\mu\nu}$. 

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The correlator $D(u_4, |\vec{u}|)$ should decrease in all directions in the Euclidean space so that the string tension (16) were finite, and the Dirac structure of the confining potential depends on the correlation length $T_g$ which governs this decrease ($r = |\vec{x}|$):

\[
V_{\text{conf}}(r) = r(1 + \gamma_4) \int_0^\infty d\tau \int_0^r d\lambda D(\tau, \lambda) \left\{ 1 - \frac{\lambda}{r} + \left( \frac{2}{3} - \frac{\lambda}{r} + \frac{\lambda^3}{3r^3} \right) e^{-2m\tau} \right\} +
\]

\[
+ r(1 - \gamma_4) \int_0^\infty d\tau \int_0^r d\lambda D(\tau, \lambda) \left\{ \frac{2}{3} - \frac{\lambda}{r} + \frac{\lambda^3}{3r^3} + \left( 1 - \frac{\lambda}{r} \right) e^{-2m\tau} \right\} \tag{23}
\]

with large distance ($r \gg T_g$) behaviour

\[
V_{\text{conf}}(r) = \left( \frac{5}{6} + \frac{1}{6} \gamma_4 \right) \sigma r + O(\sigma r m T_g) \tag{24}
\]

for $m T_g \gg 1$, and

\[
V_{\text{conf}}(r) = \frac{5}{3} \sigma r + O(\sigma r m T_g) \tag{25}
\]

for $m T_g \ll 1$. Both regimes were established in [8, 9]. In what follows we demonstrate that only the regime $m T_g \gg 1$ is selfconsistent.

Now we consider various corrections to the leading interaction (23). The terms proportional to $(\vec{\gamma}\vec{k})$ in (21) are calculated by means of integration by parts and give

\[
\hat{M}_{\vec{k}} = \delta^{(3)}(\vec{x} - \vec{z}) V_{\vec{k}}(\vec{x}), \tag{26}
\]

\[
V_{\vec{k}}(\vec{x}) = -\frac{i}{2m} \int_0^\infty d\tau (1 + e^{-2m\tau}) \left\{ \gamma_\mu \gamma_i \gamma_\nu K_{\mu\nu}(\tau, \vec{x}, \vec{x}) \partial_i - i \gamma_\mu \gamma_i \gamma_\nu \partial_{\vec{z}_i} K_{\mu\nu}(\tau, \vec{x}, \vec{z}) |_{\vec{z} = \vec{x}} \right\} =
\]

\[
= \frac{1}{m} \int_0^\infty d\tau (1 + e^{-2m\tau}) \int_0^r d\lambda D(\tau, \lambda) \left\{ (r - \lambda)i(\vec{\gamma}\vec{p}) + \left( \frac{2}{3} - \lambda + \frac{\lambda^3}{3r^2} \right) i(\vec{\gamma}\vec{n})(\vec{n}\vec{p}) + \right\}
\]

\[
+ \left( \frac{3}{2} - \frac{\lambda}{r} \right) (\vec{n})^2 \right\},
\]

\[
\vec{n} = \frac{\vec{x}}{r}, \quad \vec{p} = -i \frac{\partial}{\partial \vec{x}}.
\]

Problems start with the terms proportional to $\vec{k}^2$ in (21). To bring these terms into local form one should integrate by parts twice:

\[
\hat{M}_{\vec{k}^2} = \delta^{(3)}(\vec{x} - \vec{z}) V_{\vec{k}^2}(\vec{x}), \tag{27}
\]

\[
V_{\vec{k}^2} = \frac{1}{4m^2} \int_0^\infty d\tau \gamma_\mu (1 + e^{-2m\tau} + (1 + \gamma_4) m\tau) \gamma_\nu \left\{ \frac{\partial^2}{\partial \vec{z}_i^2} K_{\mu\nu}(\tau, \vec{x}, \vec{z}) |_{\vec{z} = \vec{x}} +
\]

\[
+ \frac{1}{2} \frac{\partial^2}{\partial \vec{z}_i \partial \vec{z}_j} K_{\mu\nu}(\tau, \vec{x}, \vec{z}) |_{\vec{z} = \vec{x}} + \right\},
\]

\[
\frac{\partial}{\partial \vec{z}_i} K_{\mu\nu}(\tau, \vec{x}, \vec{z}) |_{\vec{z} = \vec{x}} + \right\}.
\]
\[
+2i \frac{\partial}{\partial z_i} K_{\mu \nu}(\tau, \vec{x}, \vec{z}) \big|_{\vec{z} = \vec{x}} \hat{p}_i - K_{\mu \nu}(\tau, \vec{x}, \vec{x}) \hat{p}_i^2 \bigg) ,
\]
and only momentum dependent terms can be expressed in terms of integrals of the function \(D(\tau, \lambda)\):

\[
V_{\vec{k}z}(\vec{x}) = \frac{1}{m^2} \int_0^\infty d\tau \int_0^r d\lambda D(\tau, \lambda) \left\{ (1 + e^{-2m\tau} + (1 + \gamma_4)m\tau) \left[ -\frac{1}{2}(r - \lambda)\hat{p}^2 + \frac{i}{2}(\vec{n} \hat{p}) \right] + (1 + e^{-2m\tau} + (1 - \gamma_4)m\tau) \left[ -\frac{1}{2} \left( \frac{2}{3}r - \lambda + \frac{\lambda^3}{3r^3} \right) \hat{p}^2 + \frac{i}{3} \left( 1 - \frac{\lambda^3}{r^3} \right) (\vec{n} \hat{p}) \right] \right\} + V_{\vec{k}z} \text{(momentum independent)},
\]

\(\hat{l}_i = \varepsilon_{ijk} x_j \hat{p}_k\).

Moreover, the momentum independent contribution appears to diverge in the \(T_g \to 0\) limit. Indeed,

\[
V_{\vec{k}z} \text{(momentum independent)} = \frac{1}{2m^2} \int_0^\infty d\tau (1 + e^{-2m\tau} + (1 + \gamma_4)m\tau) \times \int_0^r d\lambda \left\{ -\frac{1}{r} + \frac{2\lambda}{r^2} + \frac{1}{3} \left( 1 - \frac{\lambda}{r} \right) \left( 1 + \frac{\lambda}{2r} + \frac{\lambda^2}{r^2} \right) \left( \frac{2}{\lambda} \frac{\partial}{\partial \lambda} + \frac{\partial^2}{\partial \lambda^2} \right) \right\} D(\tau, \lambda) + \frac{1}{2m^2} \int_0^\infty d\tau (1 + e^{-2m\tau} + (1 - \gamma_4)m\tau) \times \int_0^r d\lambda \left\{ -\frac{2}{3r} \left( 1 - \frac{\lambda}{r} \right) \left( 1 - 2\frac{\lambda}{r} - 2\frac{\lambda^3}{r^3} \right) + \frac{2}{5} r \left( 1 - \frac{\lambda}{r} \right)^2 \left( 1 - \frac{\lambda}{2r} + \frac{\lambda^2}{2r^2} + \frac{\lambda^3}{4r^3} \right) \left( \frac{2}{\lambda} \frac{\partial}{\partial \lambda} + \frac{\partial^2}{\partial \lambda^2} \right) \right\} D(\tau, \lambda),
\]
and for large \(r \ (r \gg T_g)\) one has the following asymptotics for expression (29):

\[
V_{\vec{k}z} \text{(momentum independent)} = \begin{cases} 
(\alpha_+ (1 + \gamma_4) + \alpha_- (1 - \gamma_4)) \frac{\sigma r}{m T_g}, & m T_g \gg 1 \\
\beta \frac{\sigma r}{m^2 T_g^2}, & m T_g \ll 1,
\end{cases}
\]
where \(\alpha_+, \alpha_-\) and \(\beta\) are some coefficients of order unity depending on the explicit form of function \(D(\tau, \lambda)\). It is clear from expression (29) and asymptotics (30) that for the case \(m T_g \gg 1\) \(V_{\vec{k}z}\) is indeed a correction to the leading regime (23), (24). For \(m T_g \ll 1\) this correction is larger than the leading interaction, and we conclude in such a way that the embarrassing regime (25) is not potential, and, moreover, for such light quarks one should turn to the full system of equations (11) and (12).
The term proportional to the binding energy is simply

\[ V_{\varepsilon}(\vec{x}) = \varepsilon_0 \int_0^\infty d\tau K_{\mu\nu}(\tau, \vec{x}, \vec{x}) \left\{ \gamma_\mu \frac{1 + \gamma_4}{2} \gamma_\nu + \gamma_\mu \frac{1 - \gamma_4}{2} \gamma_\nu e^{-m\tau} \right\}, \]

and does not cause many additional problems, it is small comparing to the confinement term if \( \varepsilon_0 T_g \ll 1 \), as it was already found in [9].

The effective Schroedinger equation is obtained from Dirac equation (19) by the standard Foldy–Wouthuysen (FW) reduction, and displays a lot of pleasant features for \( mT_g \gg 1 \). For the upper Dirac component of the quark wave function the resulting Hamiltonian is

\[ H_{FW} = m + \frac{\vec{p}^2}{2m} + \varepsilon_E(r) + V_{LS}(r) + \varepsilon_M(r) + V_{SI}(r), \]

where \( \varepsilon_E(r) \) is the standard confining interaction

\[ \varepsilon_E(r) = 2 \int_0^\infty d\tau \int_0^r d\lambda D(\tau, \lambda) (r - \lambda), \]

and

\[ V_{LS}(r) = -\frac{\vec{\sigma} \vec{l}}{4m^2 r} \int_0^\infty d\tau \int_0^r d\lambda D(\tau, \lambda) \left( 1 - \frac{2\lambda}{r} \right). \]

Expressions (33) and (34) were obtained from equation (19) in [9], and coincide with the ones given by the Vacuum Background Correlators Method [3-6]. As it was shown in [4], form (34) for the spin–orbit force is equivalent to the form obtained by Gromes [2] in terms of Wilson loop expectations.

As for \( r \gg T_g \)

\[ \varepsilon_E = \sigma r \]

and

\[ V_{LS} = -\frac{\vec{\sigma} \vec{l}}{4m^2 r}, \]

these expressions mimic scalar confinement [2] at large distances. Nevertheless, confining interaction (24) is not a scalar one, at large distances it is 5/6 scalar and 1/6 vector, and is due to the electric correlator \( K_{44} \) only as one should expect for the nonrelativistic particle.

There are three sources for the spin–orbit force (34). One piece comes from the FW reduction of confinement (23), it is purely magnetic and gives

\[ V^{(0)}_{LS} = -\frac{\vec{\sigma} \vec{l}}{3m^2 r} \int_0^\infty d\tau \int_0^r d\lambda D(\tau, \lambda) \left( 1 - \frac{\lambda^3}{r^3} \right) \bigg|_{r \gg T_g} \approx -\frac{\vec{\sigma} \vec{l}}{6m^2 r}. \]
The second one stems from the FW reduction of $\bar{\gamma}k$ term and contains both electric and magnetic contributions yielding

$$V_{LS}^{(1)} = \frac{\sigma T}{6m^2r} \int_0^\infty d\tau \int_0^r d\lambda D(\tau, \lambda) \left(1 + \frac{3\lambda^2}{r^2} - \frac{\lambda^3}{r^3}\right) \approx \frac{\sigma l}{12m^2r}.$$ 

The third piece is from $\bar{k}^2$ term. It is purely magnetic and its share is

$$V_{LS}^{(2)} = -\frac{\sigma l}{3m^2r} \int_0^\infty d\tau \int_0^r d\lambda D(\tau, \lambda) \left(1 - \frac{3\lambda}{2r} + \frac{\lambda^3}{2r^3}\right) \approx -\frac{\sigma l}{6m^2r}.$$ 

Magnetic confinement

$$\varepsilon_M(r) = 2r \int_0^\infty d\tau e^{-2mT_g} \int_0^r d\lambda D(\tau, \lambda) \left(\frac{2}{3} - \frac{\lambda}{r} + \frac{\lambda^3}{3r^3}\right),$$

behave as $\sigma r/mT_g$ at large distances and it is suppressed for large $mT_g$.

There are, of course, spin–independent corrections of the standard Darwin term type, which are $O\left(\frac{\sigma m^2r}{m^2T_g}\right)$ at large distances. Another correction is given by electric momentum dependent part of $V_{\bar{k}^2}$, which is $O\left(\frac{\sigma T_g}{mr}\right)$, and the binding energy correction (31) contributes at the same order of magnitude. Nevertheless, the main spin–independent correction comes from the electric part of $V_{\bar{k}^2}$ (momentum independent):

$$V_{SI}(r) = \frac{1}{m} \int_0^\infty d\tau \int_0^r d\lambda \left\{-\frac{1}{r} + \frac{2\lambda}{r^2} + \frac{r}{3} \left(1 - \frac{\lambda}{r}\right) \left(1 + \frac{\lambda}{2r} + \frac{\lambda^2}{r^2}\right) \left(\frac{2}{\lambda} \frac{\partial}{\partial \lambda} + \frac{\partial^2}{\partial \lambda^2}\right)\right\} D(\tau, \lambda),$$

which behaves as $\sigma r/mT_g$ at large distances, competing with magnetic confinement (35).

The results of the suggested approach reproduce well–known formulas for leading confinement (33) and spin–orbit (34) forces. Our derivation does not appeal to the Wilson loop approach and effective QCD string at large distances, but, as model (15) for the QCD vacuum is compatible with the area law, the salient features are the same. A kind of a string is developed connecting quark and antiquark, and this string is the minimal string of the Vacuum Background Correlators Method [4-6], or the flux tube with gluonic degrees of freedom in the ground state [7, 11], as far as we neglect the $\Delta^{(1)}$ contributions to the field strength correlator (35). Nevertheless, as we deal with full Dirac Hamiltonian (18), where both nonlocality in time direction and Zitterbewegung are included, we are able to disclose new important corrections (35) and (36).

We have demonstrated that, apart from naive nonrelativistic condition $m \gg \sqrt{\sigma}$ and more sophisticated condition $\varepsilon_0T_g \ll 1$ [4], another requirement, $mT_g \gg 1$, is needed for the potential-type description of heavy quark dynamics to be valid. To what extent
this requirement is indeed new? In the $N_C \to \infty$ limit correlators (15) are given by the pure Yang–Mills theory, with single nonperturbative mass scale. On the other hand, model (15) contains two dimensional parameters, correlation length $T_g$ and $D(0)$, which is proportional to the gluonic condensate and is related to the string tension $\sigma$ via equation (16). So it is not surprising that two phenomenological quantities, $\sqrt{\sigma}$ and $T_g^{-1}$, should be of the same order of magnitude, and indeed they are. The commonly accepted value of the string tension gives $\sqrt{\sigma} \approx 0.4GeV$, while lattice measurements \cite{12} give for the correlation length value $T_g^{-1} \sim 1GeV$, so the requirement $mT_g \gg 1$ does not bring drastic changes into our understanding of heavy quark dynamics. Much more interesting are new corrections (35) and (36), which are potentially large and even at large distances depend on the explicit form of vacuum correlation function $D$.

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