An Analytical Method for Tensile-Test Curves of Slacked Multifilament Yarn

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Abstract

In conventional tensile tests for textile materials, a pretension is applied to the specimens to obtain a credible sample length. However, when we wear a comfortable garment, many yarns in it are not stretched. So, we consider that the transition behavior of textile materials from relaxed to low-stress conditions is important in evaluating the comfort of a garment. As a preliminary study, we developed an analytical method for tensile-test results of slacked yarn, where we applied the Savitzky-Golay approach to compute the second derivative-graphs of the original curves and fitted them with two beta distribution probability density functions using the least-squares method. In the experiment, we measured the tensile-test curves of an untwisted polyester multifilament yarn with intentional slack. Our results show that the second-derivative graph is well summarized by our model using the least-squares method. As well, the overall load-extension properties, including slack length of each specimen, also can be analyzed objectively in our method.

Key Words: Slacked yarn, Multifilament yarn, Tensile properties, Beta distribution, Savitzky-Golay method, Second-derivative graph of tensile-test curves

1. Introduction

Tensile tests are important experiments for evaluating textile materials. So, many researchers have developed testing methods to obtain detailed information about particular materials [1]. Young’s modulus and the breaking stress and strain values are considered to be important parameters for characterizing yarns and textiles, whose performance is evaluated by tensile tests according to the standards set by the International Organization for Standardization (ISO), the Japan Industrial Standards (JIS), and so on. These standards request applying pretension to the specimens to avoid fluctuation of data caused by length fluctuations of the specimens.

Garments need to be comfortable as well as durable, and for this purpose, they are designed to enable an easy movement of the human body. Under such conditions, the yarns in the garment are not stretched but relaxed. Therefore, we considered that the tensile behavior of slacked yarn is more important than the breaking properties to discuss the comfort of garments. Recently, based on similar ideas, the tensile behavior of yarn under low-stress conditions was reported [2].

In this study, the tensile properties of slacked polyester multifilament yarn are discussed. When we perform tensile tests with a slacked yarn, we obtain a graph comprising a horizontal part that corresponds to the specimen’s slack. To analyze this type of data, we applied the Savitzky-Golay method [3-5] and a beta distribution.

2. Theoretical discussion

2.1 Differentiation of tensile-test curve

Figure 1 shows a conceptual diagram of a tensile-test curve of slacked yarn. In the figure, the horizontal axis corresponds to the displacement of the crosshead of the testing machine. In the upper graph of Figure 1, the vertical axis corresponds to the tensile load. In the first place, due to the slack of the yarn, no tensile load is detected. Next, the yarn is straightened and begins to resist the stretching; hence, the tensile load starts increasing. Thereafter, in the case of a ductile material, the slope of the curve becomes gentler at the yield point. Finally, the yarn breaks. Focusing on the slope of the curve, we can divide the tensile-test curve into four zones, as follows: Zone 0 is the first flat part corresponding to the slack of the yarn. In Zone 1, the slope of the curve increases with increasing crosshead displacement. Then, in Zone 2, the slope of the curve decreases due to the yielding of the material. Finally, in Zone 3, the slope of the curve remains almost constant until the yarn breaks.

In the above discussion, we divided the curve based on the slope of the curve. This implies that the first-derivative value of the curve may make this discussion clearer and/or more concrete. The middle
chart of Figure 1 shows the first-derivative graph of the tensile-test curve. Using this figure, we can directly read the slope of the tensile-test curve. The first-derivative value increases in Zone 1 and decreases in Zone 2. At the boundary of Zones 1 and 2, it shows the maximum slope value corresponding to Young’s modulus of the specimen.

By differentiating the original curve, the situation becomes a bit easier to understand but, the obtained graph is not simple yet. Thus, we differentiate once again and think about the second-order-derivative value. The lower chart of Figure 1 shows the second-order-derivative graph, which has a very simple profile, that is, there are two successive peaks, one is positive and the other is negative. We considered that these peaks can be modeled using an adequate method.

2.2 The Savitzky-Golay method

Differentiation is a very important step to realize the above idea. However, it is not avoidable to include some noise in the real data—and this noise significantly obstructs the differentiation of the data signals. So, in the process of differentiation, we applied the Savitzky-Golay (SG) method [3] where, the derivative value is estimated in the following way: First, a section including an odd number of data points is selected, as shown in Figure 2. Next, the data within the section is fitted to a polynomial, as shown in the lower of Figure 2. Finally, the derivative value of the polynomial is used as the estimated derivative value of the original data. To obtain the second-order-derivative value, the order of the fitting polynomial must be equal to or greater than two. On the other hand, in the case that the order of polynomial is too high, the estimation accuracy of the derivative values may deteriorate due to overfitting.

In this study, we considered a polynomial order of two and a number of data points in the data section of 101. Since the sampling interval is 0.1 mm in the experiment, the length of the data section corresponds to a 1-cm crosshead displacement.

2.3 Beta distribution

Equation 1 gives the probability density function of the beta distribution, \( f_B(u; \alpha, \beta) \):

\[
f_B(u; \alpha, \beta) = \frac{u^{\alpha-1}(1-u)^{\beta-1}}{B(\alpha, \beta)} \quad (0 \leq u \leq 1) \tag{1}
\]
where, \( u \) is a random variable of the beta distribution, \( \alpha \) and \( \beta \) are shape parameters of the beta distribution, and \( B(\alpha, \beta) \) is the beta function. Figure 3 shows some examples of this distribution, where it can be seen that the profile of the distribution changes upon changing the shape-parameters’ combination. In this study, we applied Equation 1 to fit the peak shape of the second-order-derivative curve in the following method.

### 2.4 Peak fitting

Equation 2 shows the model function, \( f(x) \), we used in fitting the beta distribution to the second-order-derivative curve of the measured data:

\[
f(x) = \begin{cases} 
0 & 0 \leq x \leq x_1 \\
\frac{s_1 s_2 f_{\beta} \left( \frac{x - x_1}{x_2 - x_1}, \alpha_1, \beta_1 \right)}{s_2 f_{\beta} \left( \frac{x - x_1}{x_3 - x_2}, \alpha_2, \beta_2 \right)} & x_1 \leq x \leq x_2 \\
\frac{s_2 f_{\beta} \left( \frac{x - x_2}{x_3 - x_2}, \alpha_2, \beta_2 \right)}{0} & x_2 \leq x \leq x_3 \\
\end{cases}
\]

where \( x_1, x_2, \) and \( x_3 \) correspond to the boundaries of the zones shown in Figure 1, \( \alpha_1, \beta_1, \alpha_2, \) and \( \beta_2 \) are the distribution shape parameters, and \( s_1 \) and \( s_2 \) are the scale factors. Note that \( s_2 \) has a negative value.

By using the solver feature of Microsoft Excel, we fitted Equation 2 to the second-order-derivative curve of the data obtained by the SG method using the least-squares approach.

### 3. Experiment

#### 3.1 Sample

An untwisted polyester multifilament yarn (15tex, 36filaments) was used for the experiment.

#### 3.2 Testing method

A universal tensile testing machine (Autocom AC-1kN-CM, Baldwin Corp.) was employed in the experiment. The initial distance of the clamping jaws was 300 mm. The crosshead speed was 300 mm/min. In our experiment, the specimens were attached without pretension, and sufficient yarn slack was intentionally secured. The number of tests was 30. For the purpose of comparison, we also performed 30 tensile tests with pretension, according to JIS.

#### 3.3 Data analysis

Since the sample material is an untwisted multifilament yarn, it breaks in multiple steps. However, we focus on the tensile behavior of the yarn under low-stress conditions. Thus, in our data analysis,

![Graph showing tensile load vs. crosshead displacement](image)

**Fig. 4** An example of the measured and analyzed data. Top: measured load-extension curve. Middle: second-derivative graph of the load-extension curve. Bottom: model graph fit to the second-derivative graph.
the data after maximum load were ignored.

The second derivative of the experimental data was estimated by the SG method, and the calculation of the SG method was performed using Microsoft Excel (Microsoft). To obtain the second-order-derivative value, we selected the polynomial order two.

4. Results and discussion

4.1 Graph example

Figure 4 shows an example of the results. The upper graph shows the measured tensile-test curve. (actually, the data involving some noise), the middle graph shows the second-order-derivative of the measured data, estimated by the SG method. As can be seen

| Specimen number | Maximum load (N) | Crosshead displacement at maximum load (mm) | Estimated parameters |
|-----------------|-----------------|------------------------------------------|---------------------|
|                 |                 | $x_1$  | $x_2$  | $x_3$  | $a_1$ | $a_2$ | $a_3$ | $s_1$ | $s_2$ |
| 1               | 4.83            | 151.5 | 43.1   | 55.6   | 68.2  | 5.25  | 2.79  | 1.70  | 2.86  | 0.0133 | -0.0111 |
| 2               | 5.27            | 142.6 | 37.1   | 47.4   | 59.6  | 3.37  | 2.35  | 2.03  | 3.29  | 0.0160 | -0.0106 |
| 3               | 3.78            | 84.8  | 26.0   | 35.3   | 48.5  | 3.47  | 2.49  | 1.94  | 4.21  | 0.0192 | -0.0113 |
| 4               | 3.82            | 91.4  | 32.9   | 42.7   | 56.7  | 3.38  | 2.48  | 1.95  | 3.92  | 0.0166 | -0.0096 |
| 5               | 5.31            | 108.6 | 11.0   | 20.2   | 33.5  | 3.35  | 2.41  | 2.03  | 4.43  | 0.0195 | -0.0112 |
| 6               | 5.07            | 119.7 | 26.0   | 34.9   | 48.2  | 3.31  | 2.41  | 2.04  | 4.72  | 0.0206 | -0.0116 |
| 7               | 3.56            | 85.6  | 23.3   | 32.1   | 45.5  | 3.30  | 2.41  | 2.04  | 4.81  | 0.0211 | -0.0118 |
| 8               | 5.44            | 114.7 | 17.7   | 27.2   | 40.8  | 3.39  | 2.44  | 2.02  | 4.36  | 0.0184 | -0.0105 |
| 9               | 5.06            | 117.7 | 27.6   | 36.9   | 50.6  | 3.37  | 2.44  | 1.99  | 4.37  | 0.0189 | -0.0108 |
| 10              | 4.99            | 106.5 | 25.8   | 34.8   | 47.7  | 3.38  | 2.45  | 1.95  | 4.20  | 0.0201 | -0.0119 |
| 11              | 4.83            | 174.8 | 72.8   | 82.3   | 96.5  | 3.32  | 2.44  | 1.92  | 4.00  | 0.0173 | -0.0099 |
| 12              | 5.33            | 140.2 | 40.4   | 49.8   | 63.4  | 3.45  | 2.45  | 1.99  | 4.34  | 0.0185 | -0.0110 |
| 13              | 4.97            | 165.7 | 62.6   | 72.0   | 86.3  | 3.39  | 2.46  | 1.96  | 4.33  | 0.0180 | -0.0102 |
| 14              | 4.90            | 145.5 | 41.6   | 51.2   | 64.9  | 3.44  | 2.41  | 2.06  | 4.52  | 0.0181 | -0.0108 |
| 15              | 5.34            | 103.8 | 9.11   | 18.2   | 30.7  | 3.26  | 2.39  | 2.02  | 4.12  | 0.0197 | -0.0119 |
| 16              | 4.98            | 135.4 | 33.5   | 42.6   | 55.5  | 3.24  | 2.40  | 2.00  | 4.06  | 0.0191 | -0.0115 |
| 17              | 3.87            | 85.6  | 25.4   | 34.5   | 47.5  | 3.32  | 2.42  | 2.01  | 4.29  | 0.0194 | -0.0115 |
| 18              | 5.03            | 107.5 | 21.5   | 30.6   | 43.7  | 3.31  | 2.41  | 2.01  | 4.39  | 0.0197 | -0.0115 |
| 19              | 4.97            | 114.8 | 21.9   | 30.9   | 44.1  | 3.33  | 2.41  | 2.01  | 4.46  | 0.0200 | -0.0115 |
| 20              | 5.10            | 112.1 | 23.6   | 32.7   | 45.9  | 3.37  | 2.42  | 2.01  | 4.47  | 0.0201 | -0.0116 |
| 21              | 4.81            | 142.2 | 46.3   | 55.5   | 69.3  | 3.31  | 2.43  | 1.98  | 4.24  | 0.0185 | -0.0106 |
| 22              | 4.70            | 144.1 | 40.4   | 49.8   | 63.6  | 3.38  | 2.42  | 2.03  | 4.50  | 0.0187 | -0.0108 |
| 23              | 5.28            | 130.7 | 37.6   | 46.8   | 60.4  | 3.35  | 2.43  | 2.01  | 4.33  | 0.0189 | -0.0110 |
| 24              | 4.73            | 163.8 | 60.5   | 69.9   | 84.2  | 3.34  | 2.45  | 1.94  | 4.19  | 0.0174 | -0.0098 |
| 25              | 3.68            | 113.8 | 52.4   | 61.8   | 76.2  | 3.44  | 2.46  | 1.97  | 4.44  | 0.0179 | -0.0101 |
| 26              | 4.67            | 115.4 | 38.0   | 47.1   | 60.6  | 3.24  | 2.38  | 2.05  | 4.45  | 0.0193 | -0.0111 |
| 27              | 5.40            | 131.0 | 28.9   | 38.2   | 51.7  | 3.29  | 2.40  | 2.05  | 4.33  | 0.0185 | -0.0108 |
| 28              | 4.64            | 142.3 | 52.2   | 61.5   | 75.9  | 3.25  | 2.40  | 1.95  | 4.21  | 0.0175 | -0.0097 |
| 29              | 5.21            | 149.6 | 48.7   | 58.1   | 72.2  | 3.28  | 2.42  | 1.99  | 4.27  | 0.0176 | -0.0099 |
| 30              | 3.67            | 107.6 | 42.3   | 51.9   | 66.2  | 3.43  | 2.45  | 2.02  | 4.35  | 0.0173 | -0.0100 |

Average 4.77 3.41 2.44 1.99 4.25 0.0185 -0.0109
St. dev.* 0.576 0.353 0.073 0.066 0.374 0.00154 0.000708
CV**, % 12.1 10.4 3.0 3.3 8.80 8.30 -6.52

*St. dev.: Standard deviation. **CV: Coefficient of variation.
in the figure, the second-order-derivative curve comprises two successive peaks, one positive and the other negative, similar to the abovementioned discussion. The lower graph of Figure 4 shows the model function curve fitted by the least-squares method. As shown in this graph, the peaks are well summarized by our model.

4.2 Estimated parameters

Table 1 lists the analysis results derived from curve fitting. In this table, \( x_1 \) is the boundary value for Zones 0 and 1, as shown in Figure 1. In other words, this parameter represents the amount of slack in each test. Thus, we can estimate the real specimen length using the total jaw distance (300 mm) and \( x_1 \). As the slack length is not controlled in our experiment, the average is meaningless. Similarly, \( x_2 \) is the boundary value for Zones 1 and 2. It corresponds to the maximum slope of the original curve. Generally, some skill is required to read Young's modulus from the obtained chart. In contrast, with our method, we can clearly find the point of maximum slope. Additionally, it is much more difficult to determine the end of the yielding zone in chart reading, whereas in our method, we can obtain that as \( x_3 \).

The curve profiles of the peaks are also provided in Table 1. The parameters \( \alpha_1 \) and \( \beta_1 \) correspond to the first peak of the second-derivative graph while \( \alpha_2 \) and \( \beta_2 \) correspond to the second peak. Roughly speaking, the left-side curve profile of the beta distribution probability density function is determined by \( \alpha \) while the right-side curve is ruled by \( \beta \). The first shape parameter, \( \alpha_1 \), is always greater than two. This fact implies that the left side of the curve does not intersect the \( x \) axis but touches it. Similar results are obtained with the last shape parameter, \( \beta_2 \), which corresponds to the right side of the second peak. In our model, \( \beta_1 \) and \( \alpha_2 \) correspond to the curve profile around the connecting point of the peaks. We considered that the value of these parameters is close to two because the curve intersects the \( x \) axis.

4.3 Comparison with the conventional method

Figure 5 shows graphs obtained by conventional pretension method and slacked method. The graph of pretension method is offset horizontally for space saving. As shown in the figure, the profiles of obtained curves are similar and significant difference is not found.

The maximum load and the strain at maximum load of two methods are also compared. Table 2 shows the comparison results. As shown in this table, the average of the maximum load measured by the slacked test is slightly smaller than that determined by the conventional method. However, the result of the \( t \)-test shows that they are not significantly different.

To compare the extension at maximum load, the amount of slack should be considered in our method. Thus, we assumed that the real specimen length was the total jaw distance and estimated the slack derived as \( x_1 \) in Table 1. With this total, we calculated the strain of maximum load in the slacked test. Recalling Table 2, the strain values at maximum load are also not significantly different.

As mentioned above, our approach and conventional pretension method offer similar results. Our model assumes test material is

| Test method | Pretension | Slacked | Result of \( t \)-test |
|-------------|------------|---------|----------------------|
| Maximum load, N | Average | 4.97 | 4.77 | Not significantly different |
| | Standard deviation | 0.18 | 0.58 | |
| | Coefficient of variation, % | 3.6 | 12.1 | |
| Strain at maximum load, % | Average | 27.0 | 26.6 | |
| | Standard deviation | 2.4 | 4.7 | Not significantly different |
| | Coefficient of variation, % | 9.0 | 17.7 | |
5. Conclusions

In this study, we developed an analytical method for tensile-test curves of slacked yarn, which is considered to be important in textile material study. The second derivative of tensile-test curve is well summarized by our model based on the beta distribution. The second-derivative curve is obtained from the measured tensile-test curve of slacked yarn by the Savitzky-Goaly method. The results show that the measured data is well summarized by our model.

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