Quantum phase transition (QPT) has been actively studied in condensed matter physics, because it occurs between emergent quantum phases[1]. In particular, superfluid– and superconductor–insulator (SI) transitions in superconducting films[2] and ultracold atoms in optical potentials[3, 4] are typical examples of QPTs. In SI-QPTs, while superfluid phases are unique and well understood, there are various possibilities for insulating phases, because they are determined by competition between quantum fluctuations, interparticle correlation, and external potential. In spatially periodic systems such as bosons in optical lattices, the insulating phase is Mott insulator. In disordered metals and atoms in disordered potentials, Anderson insulator and Bose glass are the candidates for insulating phases. In this Letter, we propose that helium films offer a new example of QPT between a superfluid and a novel insulating phase, which has an energy gap and a finite compressibility.

Helium (bosonic $^4$He and fermionic $^3$He) films formed on solid substrates by adsorption undergo various QPTs between competing phases by changing coverage $n$. We found anomalous elastic phenomena in bosonic $^4$He and fermionic $^3$He films adsorbed on a glass substrate. The films stiffen under AC strain at low temperature with an excess dissipation. The onset temperature of the stiffening decreases to 0 K as $n$ approaches a critical coverage $n_c$. The elastic anomaly is explained by thermal activation of helium atoms from the localized to extended states with a distributed energy gap. We determine for the first time the energy band structure of helium films from elasticity. The ground states of $^4$He and $^3$He are identical gapped and compressible, which are possibly self-organized Mott insulator or Mott glass.

Elastic Anomaly of Helium Films at a Quantum Phase Transition

T. Makiuchi,1 M. Tagai,1 Y. Nago,1 D. Takahashi,2 and K. Shirahama1

1Department of Physics, Keio University, Yokohama 223-8522, Japan
2Center for Liberal Arts and Sciences, Ashikaga University, Ashikaga 326-8558, Japan

(Dated: August 23, 2018)

Helium films show various quantum phases that undergo quantum phase transitions by changing coverage $n$. We found anomalous elastic phenomena in bosonic $^4$He and fermionic $^3$He films adsorbed on a glass substrate. The films stiffen under AC strain at low temperature with an excess dissipation. The onset temperature of the stiffening decreases to 0 K as $n$ approaches a critical coverage $n_c$. The elastic anomaly is explained by thermal activation of helium atoms from the localized to extended states with a distributed energy gap. We determine for the first time the energy band structure of helium films from elasticity. The ground states of $^4$He and $^3$He at $n < n_c$ are identically gapped and compressible, which are possibly self-organized Mott insulator or Mott glass.

PACS numbers: 05.30.Rt, 64.70.Tg, 67.25.bh, 67.25.dj, 67.30.ej, 68.60.Bs

The most important feature of $^4$He films on disordered substrates is that there is only one “quantum critical coverage” $n_c$. Films at $n < n_c$ are considered to be in an insulating phase, meaning that a SI-QPT occurs at $n_c$. We emphasize that $^4$He on disordered substrates realizes an ideal SI-QPT. On graphite, $^4$He superfluidity and $^3$He magnetism are strongly influenced by corrugation from substrate. It is rather surprising that superfluid transition of $^4$He on Mylar shows a perfect agreement with the BKT theory[11], while $^4$He on graphite does not[13].

At $n < n_c$, helium adatoms are localized on substrate. The localization was first attributed to strong attraction between helium and substrate. This conjecture was supported by the fact that $n_c$ scales with the depths of potential of different substrates[7]. Fisher et al. proposed that many-body effects of correlation and disorder make $^4$He film at $n < n_c$ localized to be a Bose glass, which is characterized by no gap and finite compressibility[14]. But no evidence for the Bose glass of $^4$He film was found experimentally[10]. We have found anomalous behavior in elasticity of helium films, an important property that is related to a compressibility of ground state. The ground state at $n < n_c$ is found to be a gapped many-body state such as Mott insulator or Mott glass[15], which has intermediate properties between Mott insulator and Bose glass.

Contrary to $^4$He, studies of $^3$He films on disordered substrates were few. Since $^3$He films show no superfluidity and the heat capacity is dominated by a contribution from nuclear spins[19], critical coverage $n_c$ was not identified for $^3$He. Also in $^3$He films, we have observed elastic anomaly identical to that of $^4$He films. The critical coverage $n_c$ is identified for the first time for $^3$He.

We have measured elasticity of helium films using a torsional oscillator (TO) shown in Fig. 1(c). The TO consists of a BeCu torsion rod containing a cylindrical porous Gelsil glass sample (17 mm in length, 5.4 mm in diameter) and a metal dummy bob. The porous Gelsil
The surface area is 166 m$^2$. The TO is mounted on a dilution refrigerator, and operated at the resonant temperature (c) Cross-sectional view of the TO. The uppermost part has interconnected pores, which acts as a substrate[17]. Energy gap $\Delta$ obtained from fitting (see text) is also shown. Arrows indicate critical coverage $n_c$.

In Fig. (a) and (b), we show raw data for $f$ from 0 to 23 $\mu$mol/m$^2$ for $^4$He, and to 20 $\mu$mol/m$^2$ for $^3$He, respectively. The data at $n = 0$ is referred to as the background[15]. Helium adsorption increases $f$ in the entire temperature range. TO frequency is given by $f = (1/2\pi)\sqrt{k/I}$, where $k$ is a torsion constant (stiffness) of the rod and $I$ is a moment of inertia of the dummy bob. As $I$ is constant, the increase in $f$ indicates that helium adsorption stiffens the rod.

At each coverage, $f$ increases as $T$ decreases more rapidly than the background does. The gradual increase in $f$ suggests a crossover of a softening of helium film to a stiff state, not a first order phase transition. The dissipation $Q^{-1}$ has a peak at a temperature where the slope of $f$ is the largest, and its position decreases with increasing $n$. At lower $T$, $f$ tends to saturate and $Q^{-1}$ decreases. These behaviors of $f(T)$ and $Q^{-1}(T)$ are qualitatively the same for $^4$He and $^3$He films.

We show that the observed increase in $f$ is originated from change in elasticity of helium adatoms. If helium film stiffens, the apparent shear modulus of Gelsil substrate $G_g$ will increase, i.e. $G_g \rightarrow G_g + \delta G_g$. Helium adsorption also increases the apparent density of Gelsil, i.e. $\rho_g \rightarrow \rho_g + \delta \rho_g$. We have computed the $\delta f - \delta G_g$ and $\delta f - \delta \rho_g$ relations by FEM simulations[19]. Linear relations $2\delta f/f_0 = 0.197\delta G_g/G_{g0}$ and $2\delta f/f_0 = -1.33 \times 10^{-4} \delta \rho_g/\rho_{g0}$ are obtained. Even if 20 $\mu$mol/m$^2$ of $^4$He were decoupled from the oscillation, $f$ would increase 2 mHz, which is nearly two orders of magnitude smaller than the observed increment $\delta f \sim 50$ mHz. Therefore, the increase in TO frequency is neither due to superfluidity nor to slippage of helium films, but is originated from stiffening of helium films.

The dependencies of $f$ and $Q^{-1}$ are typical of a relaxational crossover between a soft state at high $T$ and a stiff state at low $T$ under AC stress applied to a substrate-He system. Assuming that the relaxation is caused essentially by adsorbed helium, the relaxational contribution to $f$ and $Q^{-1}$ is obtained by subtraction of the background. Figure 2 shows a normalized frequency shift $2\delta f/f_0$ and excess dissipation $\delta Q^{-1}$ for $^4$He at $n = 18$ $\mu$mol/m$^2$, where $\delta f$ is obtained by subtracting the background and a constant so as to set $\delta f = 0$ at 1.0 K, and $\delta Q^{-1} = Q^{-1} - Q_{0}^{-1}$, where $Q_0$ is the background. The dissipation $\delta Q^{-1}$ becomes negative below 0.1 K, meaning that helium adsorption decreases the internal loss of glass. The peak temperature $T_p$ of $\delta Q^{-1}$ smoothly approaches 0 K with a concave curvature, as shown in Fig. (d).

The relaxational crossover is explained by a thermal activation process of helium adatoms between two discrete energy bands[10, 13]. At $T = 0$, helium atoms are localized and forms an energy band. At finite temperatures, the localized atoms are thermally excited to another band of extended states separated by an energy gap. The excited atoms move freely on the substrate, and act as a normal fluid. We analyze $2\delta f/f_0$ and $\delta Q^{-1}$ by dynamic response functions for anelastic relaxation[20].
Nonlinear fittings give ∆ relaxation. The gap is fitted by a power law

\[ \Delta = \Delta_0 \left(1 - n/n_c\right)^a. \]  

Nonlinear fittings give ∆0/kB = 23.9 K, n_c = 23.0 μmol/m² and a = 1.32 for ⁴He, and 36.5 K, 19.8 μmol/m² and 1.80 for ³He, respectively.

The fact that ∆ smoothly decreases to zero as \( n \to n_c \) indicates that the energy band also smoothly changes with \( n \). We propose an energy band in Fig. 3(a). The localized states are completely filled at \( T = 0 \), and its uppermost edge is determined by \( n \). Atoms in the localized states contribute to the elasticity. On the other hand, the extended states are empty at \( T = 0 \), and their lowermost edge, \( \mu_0 \), has no dependence on \( n \). At high \( T \), helium atoms are thermally excited to the extended states, resulting a softening. At \( n \geq n_c \), the gap is closed, and helium atoms can enter the extended states even at \( T = 0 \). ³He atoms condensed in the extended states show superfluidity. This scenario was first discussed by Crowell et al. in a heat capacity study of ⁴He films.[10]

The band for each \( n \) is analogous to that of intrinsic semiconductor. The chemical potential \( \mu \) at \( T = 0 \) is a function of \( n \) and is located at the middle of the gap, so that

\[ \mu(n) = \mu_0 - \Delta(n)/2. \]
The 2D compressibility is, by definition,

\[ \kappa_{2D\mu} = \left( N_A n^2 \frac{\partial \mu}{\partial n} \right)^{-1}, \tag{7} \]

where \( N_A \) is the Avogadro constant (\( N_A n \) is 2D number density of atoms). We refer to the 2D compressibility obtained from \( \mu(n) \) as \( \kappa_{2D\mu} \). From Eqs. \( \ref{7} \), \( \ref{8} \) and \( \ref{9} \),

\[ \kappa_{2D\mu} = \frac{2nc}{\alpha N_A n^2 \Delta G |1 - n/n_c|^{1 - a}}. \tag{8} \]

The results are drawn in Fig. \( \ref{b} \) with solid curves.

The 2D compressibility is also obtained by directly comparing the observed frequency increment \( \delta f(n) \) with the FEM result, 2\( f/f_0 = 0.197 \delta G_k/G_{g0} \). Here \( \delta f(n) \) is frequency increment from \( f(n_c) \) at the lowest temperature. \( G_{g0} = 7.38 \) GPa from our ultrasound study \( \ref{18} \). With a general relation \( K = \lambda + (2/3)G \), where \( K \) is Young’s modulus and \( \lambda \) is Lame constant, an effective 3D compressibility of helium film \( \kappa \) is \( \kappa^{-1} = \delta K \approx (2/3)\delta G \). It is converted to the 2D compressibility by \( \kappa_{2D} = \kappa/d \), where \( d = v_{ilm} n \) is mean film thickness and \( v_{ilm} \) is molar volume of helium film. Since \( v_{ilm} \) is unknown, we employ \( v \) of liquid helium at 0 bar, which might be larger than \( v_{ilm} \). Combining these equations,

\[ \kappa_{2D} = \frac{0.148 f_0}{\delta f(n) G_{g0} v n}. \tag{9} \]

In Fig. \( \ref{b} \), we plot \( \kappa_{2D} \) obtained from Eq. \( \ref{9} \). The overall agreement between \( \kappa_{2D\mu} \) and \( \kappa_{2D} \) from \( \delta f \) definitely assures the proposed band.

In both \( ^4 \)He and \( ^3 \)He, \( \kappa_{2D} \) first decreases, makes a plateau, and finally shows divergent behavior as \( n \) approaches \( n_c \). The similarity in \( ^4 \)He and \( ^3 \)He is revealed also by a “unified” phase diagram shown in Fig. \( \ref{4} \). The peak temperatures, \( T_p \)'s, of \( ^4 \)He and \( ^3 \)He as a function of \( n/n_c \) nearly collapse onto each other, except that the curvatures differ. Figures \( \ref{1}(d) \) and \( \ref{4} \) show that \( \Delta(n) \) obeys a power law \( \Delta \propto |n - n_c|^\alpha \) with \( \alpha > 1 \). A symmetry may exist between the critical exponent of \( \Delta \) and that of superfluid \( T_c \) of \( ^4 \)He films, in which \( T_c \propto (n - n_c)^w \) with \( w > 1 \) \( \ref{10} \).

Our finding is that \( ^4 \)He and \( ^3 \)He films at \( n < n_c \) are identically gapped and compressible irrespectively of quantum statistics. These features do not strictly agree with the properties of Bose glass (BG; gapped, compressible, for \( ^4 \)He\( \ref{14} \)), Mott insulator (MI; gapped, incompressible) or Mott glass (MG; single-particle gap, incompressible) \( \ref{15} \).

We propose, however, that the localized helium films are MI or MG in the realistic situation. One may expect the following scenario: Helium atoms are first adsorbed on some particularly deep adsorption sites, so as to weaken randomness. Additional helium atoms are adsorbed on the weakened potential surface, and self-organize a nearly spatially periodic 2D MI or MG with an \( n \)-dependent lattice constant. The self-organization of sites allows a finite compressibility. The gapped localized state which terminates at a certain coverage \( n_c \) has been observed in helium films on various substrates, such as Vycor, Hectorite (2D flat substrate), FSM (1D pores), and zeolites \( \ref{22} \). This suggests that the gapped localized MI or MG ubiquitously exists, irrespectively of substrate randomness and dimensionality.

As to \( ^4 \)He, our result does not reject possibility of BG in the vicinity of \( n = n_c \), where the gap is almost closed and the compressibility significantly increases. Theories predict BG between MI and superfluid in the presence of moderate disorder \( \ref{14} \). The previous experiment discussed a quantum critical behavior of possible BG near \( n_c \) \( \ref{10} \). In our system, BG can exist at about \( \Delta < n < 23 \mu \text{mol/m}^2 \), and a corresponding Fermi glass can occur in \( ^3 \)He at \( 19 < n < 20 \mu \text{mol/m}^2 \). Recently, QPTs among MI, MG, BG and Bose-Einstein condensate is realized in a quantum magnet \( \ref{22} \). Elusive localized phases become increasingly under experimental survey. Helium films in disordered substrates can open a new perspective of QPTs for advantage of variable correlation and quantum statistics.

In summary, we have discovered that the localized \( ^4 \)He and \( ^3 \)He films show an identical elastic anomaly. The anomaly is explained by thermal activation of helium atoms from the localized to extended states with a distributed energy gap, which decreases as the film approaches the critical coverage \( n_c \). Both the localized \( ^4 \)He and \( ^3 \)He are gapped and compressible, suggesting that the ground state is self-organized Mott insulator or Mott glass. Future studies in the vicinity of \( n_c \) at lower temperatures will unveil the nature of the QPT.
We thank M. Kobayashi, T. Ohtsuki and A. J. Beekman for useful discussions. This work was supported by JSPS KAKENHI Grant Number JP17H02925.

[1] S. Sachdev, *Quantum Phase Transitions*, 2nd ed. (Cambridge University Press, 2011).
[2] V. Dobrosavljevic, N. Trivedi, and J. V. James M., eds., “Conductor-insulator quantum phase transitions,” (Oxford University Press, 2012).
[3] M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, Nature 415, 39 (2002).
[4] I. Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. 80, 885 (2008).
[5] G. Zimmerli, G. Mistura, and M. H. W. Chan, Physical Review Letters 68, 60 (1992).
[6] A. Casey, H. Patel, J. Nyéki, B. P. Cowan, and J. Saunders, Phys. Rev. Lett. 90, 115301 (2003).
[7] M. Neumann, J. Nyéki, B. Cowan, and J. Saunders, Science 317, 1356 (2007).
[8] J. Nyéki, A. Phillis, A. Ho, D. Lee, P. Coleman, J. Parpia, B. Cowan, and J. Saunders, Nature Physics 13, 455 (2017).
[9] G. A. Csáthy, J. D. Reppy, and M. H. W. Chan, Phys. Rev. Lett. 91, 235301 (2003).
[10] P. A. Crowell, F. W. Van Keuls, and J. D. Reppy, Phys. Rev. Lett. 75, 1106 (1995); Phys. Rev. B 55, 12620 (1997).
[11] D. J. Bishop and J. D. Reppy, Phys. Rev. Lett. 40, 1727 (1978); Phys. Rev. B 22, 5171 (1980).
[12] J. D. Reppy, J. Low Temp. Phys. 87, 205 (1992).
[13] P. A. Crowell and J. D. Reppy, Physical Review B 53, 2701 (1996).
[14] M. P. A. Fisher, P. B. Weichman, G. Grinstein, and D. S. Fisher, Phys. Rev. B 40, 546 (1989).
[15] T. Giamarchi, P. L. Doussal, and E. Orignac, Phys. Rev. B 64, 245119 (2001).
[16] A. Golov and F. Pobell, Phys. Rev. B 53, 12647 (1996).
[17] K. Yamamoto, H. Nakashima, Y. Shibayama, and K. Shirahama, Phys. Rev. Lett. 93, 075302 (2004).
[18] See Supplemental Material at [URL will be inserted by publisher] for details of the experimental method, the analysis including FEM simulation, and previous torsional oscillator studies.
[19] R. H. Tait and J. D. Reppy, Phys. Rev. B 20, 997 (1979).
[20] A. S. Nowick and B. S. Berry, *Anelastic Relaxation in Crystalline Solids* (Academic Press, New York and London, 1972).
[21] M. Boninsegni, J. Low Temp. Phys. 159, 441 (2010).
[22] N. Wada, T. Matsushita, M. Hieda, and R. Toda, Journal of Low Temperature Physics 157, 324 (2000).
[23] R. Yu, L. Yin, N. S. Sullivan, J. S. Xia, C. Huan, A. Paduan-Filho, N. F. Oliveira Jr, S. Haas, A. Steppke, C. F. Miclea, F. Weickert, R. Movshovich, E.-D. Mun, B. L. Scott, V. S. Zapf, and T. Roscilde, Nature 489, 379 (2012).
Supplemental Material for
“Elastic Anomaly of Helium Films at a Quantum Phase Transition”

T. Makiuchi,¹ M. Tagai,¹ Y. Nago,¹ D. Takahashi,² and K. Shirahama¹
¹Department of Physics, Keio University, Yokohama 223-8522, Japan
²Center for Liberal Arts and Sciences, Ashikaga University, Ashikaga 326-8558, Japan
(Dated: August 23, 2018)

EXPERIMENTAL

Torsional oscillator and experimental method

The porous Gelsil glass sample is cylindrical, 17 mm in length and 5.4 mm in diameter. A surface area, measured by N₂ adsorption and analyzed with BET method, is 166 m² (454 m²/g). A pore diameter distribution, analyzed with BJH method, has a peak at 3.9 nm, which is slightly larger than a nominal value 2.5 nm.

The torsional oscillator (TO) we have employed is shown in Fig. S1. The TO consists of a beryllium copper (BeCu) torsion rod containing the Gelsil and a dummy bob. The Gelsil is glued into the BeCu tube (6.0 and 5.5 mm in outer and inner diameter, respectively) with epoxy (Stycast 1266). The TO was mounted on a mechanical isolator consisting of a massive copper platform (70 mm φ, 30 mm thick) and a copper rod (5 mm φ, 30 mm long). Two brass electrodes, which are for drive and pickup, are located on the platform so as to form two parallel plate capacitors with the flat faces of the dummy bob. The whole setup was mounted under a mixing chamber of a dilution refrigerator (Cryoconcept Inc.). Sample temperature was measured using a RuO₂ thermometer (below 43 mK) and a calibrated germanium thermometer (43 mK–5 K) on the platform.

We first performed the measurement for ⁴He films, then warmed up the setup to room temperature to get rid of ⁴He, thereafter made the measurement for ³He. We used G1 grade ⁴He gas with impurity concentration less than 5 × 10⁻⁷, and ³He gas with nominal purity 99.95 %.

The TO oscillates electrostatically at the resonant frequency using a loop circuit. Bias voltage 200 V dc was applied at the dummy bob electrode. Drive voltage was 1.5 V square pulse of a width of 50 μs. In this condition, the strain applied to Gelsil rod is estimated to be 1.6 × 10⁻⁷, and the maximal velocity at the Gelsil rim is estimated to be 15 μm/s. We confirmed that the oscillation amplitude is linear to the drive voltage around this condition.

We measured Q of the TO by ringdown at a temperature and used the fact that Q is proportional to the amplitude R of the pick-up current; Q = aR where a is constant. Thermal cycles, however, may change a in practice. When the drive voltage is cut off, R decreases exponentially with time; R = R₀e⁻⁴/or where τ is a relaxation time. τ was measured before the warming and after the cooling at the lowest temperature for each coverage. Using a relation Q = πfτ, we examined that a was constant during each run. For the ⁴He run, a = 2.48 × 10¹³ Å⁻¹, and for the ³He run, a = 2.06 × 10¹³ Å⁻¹.

Determination of the background

We refer to the temperature dependencies of f and Q⁻¹ without helium (n = 0) as the background. The resonant frequencies of the TO of n = 0 at 1.0 K were f₀ = 860.822 Hz for ⁴He and f₀ = 860.145 Hz for ³He run. A slight difference is due to thermal cycle. Fig. S2 shows the resonant frequency f (subtracting a constant) and the energy dissipation Q⁻¹ of n = 0 for ⁴He and ³He runs as a function of temperature. The frequency was found to be linear in log(T/K) at T > 20 mK. We fit f by

\[
f(T, n = 0) = \sum_{i=0}^{1} a_i \log(T/K)^i.
\]  

The fitting results are shown in Fig. S2. At T ≤ 30 mK f shows a maximum and decrease with lowering T, probably by the effect of tunneling two-level systems (TLS) in glass. We assume that f is constant below 30 mK because the number of data are not sufficient to fit the T dependence.
For $^4$He films of coverage $n < 15 \mu\text{mol/m}^2$, the annealing was done at 5 K for 5 hours. At $16 \leq n \leq 26 \mu\text{mol/m}^2$, it was done at 1.1 K for more than 14 hours. For $^3$He films, we annealed at 5 K for 5 hours for all coverages. For 16 $\mu\text{mol/m}^2$ of $^3$He, we first annealed at 1.1 K for 12 hours as in the case of $^4$He film. However, this condition was not sufficient because the frequency and dissipation were almost the same as those of previous $n = 15 \mu\text{mol/m}^2$ data. This indicates that $^3$He atoms in the extended state are less mobile than $^4$He atoms. We finally found that the annealing at 5 K for 5 hours was sufficient for $^3$He.

## Background subtraction and fittings

We define the frequency shift by

$$\delta f \equiv f(T, n) - f(1 \text{K}, n) - [f(1 \text{K}, n) - f(1 \text{K}, 0)].$$  \hspace{1cm} (3)

By this definition, we have omitted the $T$-dependent background and the small frequency increments which were seen for all coverages at high temperatures. This omission was necessary for the fitting of data to the response function described in the main text, and did not give influence to the analysis described in the main text, except that the 2D compressibility of helium film does not show divergent behavior in reality.

We also define the excess energy dissipation by

$$\delta Q^{-1} \equiv Q^{-1}(T, n) - Q^{-1}(T, n = 0).$$  \hspace{1cm} (4)

The background-subtracted data of four coverages of $^4$He and $^3$He are shown with the results of fitting in Fig. S2 and S3, respectively. The peak temperature $T_p$ is obtained from these $\delta Q^{-1}$ data.

### Finite Element Method

In this work, simulations with finite element method (FEM) were quite essential to lead to the conclusion that the frequency increase observed in the current TO and in standard TOs (see below) is entirely originated from the stiffening of helium films. Here we describe the detail of FEM simulations.

By FEM, we have computed how much the resonant frequency of the current TO changes by stiffening of the Gelsil glass after helium adsorption, and how much $f$ changes by weighting of Gelsil. We treat the Gelsil rod as a continuous material. The Young’s modulus and the Poisson’s ratio are $E = 17.1$ GPa and $\nu = 0.155$ measured by an ultrasound measurement (unpublished). The calculated resonant frequency by FEM was $f_0 = 962$ Hz for the present TO, which is larger than the measured value $f_0 = 860$ Hz. The origin of this difference in $f_0$ is not known. One possible reason is that the inhomogeneity of silica structure in the porous glass sample, which

![Graph showing resonant frequency and dissipation vs temperature](image-url)
is not taken into account in the FEM, might affect the resonant frequency in reality. The reduced resonant frequency shift $2\delta f/f_0$, however, is a good quantity to compare the measured value to the FEM result.

We calculated $f$ when the shear modulus $G_\text{s}$ and the density $\rho_\text{s}$ changes by a small factor. The changes in $f$ are well fitted by linear functions as shown in Fig. S5. The results are

$$2\delta f/f_0 = 0.197 \frac{\delta G_\text{s}}{G_{\text{g}0}}. \quad (5)$$

FIG. S3. $2\delta f/f_0$ and $\delta Q^{-1}$ for $^4\text{He}$ films at (a) 15.0, (b) 17.0, (c) 20.0, and (d) 22.0 $\mu$mol/m$^2$. Solid lines are the results of fitting to the complex response functions with a log-normal distributed energy gap (see main text).

FIG. S4. $2\delta f/f_0$ and $\delta Q^{-1}$ for $^3\text{He}$ films at (a) 10.0, (b) 15.0, (c) 17.0, and (d) 18.0 $\mu$mol/m$^2$. Solid lines are the results of fitting to the complex response functions with a log-normal distributed energy gap (see main text).

FIG. S5. Calculated frequency shift due to changes in the density $\rho_\text{s}$ and in the shear modulus $G_\text{s}$ of the Gelsil sample by FEM simulation. A possible maximal value of $2\delta f/f_0$ in the case of full-pore $^4\text{He}$ is indicated with an arrow. Inset is a false color picture indicating the movement of parts of our TO.

and

$$2\delta f/f_0 = -1.33 \times 10^{-4} \frac{\delta \rho_\text{s}}{\rho_{\text{g}0}}. \quad (6)$$

The effective density change in the Gelsil due to helium adsorption is given by

$$\frac{\delta \rho_\text{s}}{\rho_{\text{g}0}} = \frac{m n A S}{m_{\text{g}0}} \leq \frac{\rho_{\text{liq}}}{\rho_{\text{g}0}}. \quad (7)$$

where $m$ is mass of a helium atom, $n$ the coverage, $N_A$ Avogadro constant and $\rho_{\text{liq}}$ density of bulk liquid helium. $S = 166$ m$^2$, $m_{\text{g}0} = 0.371$ g, $p = 0.54$, and $\rho_{\text{g}0} = 0.954$ g/cm$^3$ are the surface area, the mass, the porosity, and the density of the Gelsil, respectively. For example, $n_c = 23.0$ $\mu$mol/m$^2$ of $^4\text{He}$ film gives $\delta \rho_\text{s}/\rho_{\text{g}0} = 0.041$ and $2\delta f/f_0 = -5.5 \times 10^{-6}$. Even if the pores are filled with liquid $^4\text{He}$, it gives $\delta \rho_\text{s}/\rho_{\text{g}0} = \rho_{\text{liq}}/\rho_{\text{g}0} = 0.08$ and $2\delta f/f_0 = -1 \times 10^{-5}$. Therefore the measured frequency increments $2\delta f/f_0 \sim 1 \times 10^{-4}$ cannot be explained by decoupling due to slippage, superfluid, or “supersolid” transition, but is explained by changes in elasticity by $\delta G_\text{s}/G_{\text{g}0} \sim 0.0005$.

**INTERPRETATION OF STANDARD TORSIONAL OSCILLATOR EXPERIMENTS**

Our direct elasticity measurement has been motivated by the observation of frequency shift and excess dissipation in two studies using “standard” TOs. Here we briefly discuss the results and interpretation.
The experiments have been done with two TOs, which we refer to as TO1 and TO2, shown in Fig. S6. Each of the TOs contained a disk sample of porous Gelsil glass inside a torsion bob. In TO1, we glued all the faces of the glass sample to the wall by Stycast 1266 epoxy. The epoxy penetrated to the hole of the torsion rod was carefully removed by inserting a drill. On the other hand, in TO2, there was an open space between the porous glass and one side of the wall of the bob, at which the torsion rod is attached (actually the torsion rod and the bob wall is made of a single piece of BeCu).

We performed measurements of $f$ and $Q^{-1}$ of TO1 and TO2 with adsorbed $^4$He at $6 < n < 35$ $\mu$mol/m$^2$. In TO1, we observed an increase in $f$ accompanied by a dissipation peak at $n < 22$ $\mu$mol/m$^2$, as in the case of the present work. Fig. S7 shows a result of fitting to the observations, which are converted to the normalized frequency shift $2\delta f/f_0$ and excess dissipation $\delta Q^{-1}$. We see that the overall $T$ dependencies of TO1 are identical to the results of the present TO, in which the Gelsil sample is located in torsion rod. The fitting of the data to the complex response functions described in the main text works well. We confirmed that in TO1 the dependence of the behaviors on $n$ and the obtained fitting parameters such as energy gap $\Delta$ are also identical to the present TO. At $n > n_c$, ordinary superfluid transitions were observed as an increase in $f$ below $T_c$, which is shown in Fig. 4 in the main text.

In TO2, however, such elastic anomalies below $n_c$ were not observed, while the superfluid transition was seen at $n > n_c$ as in TO1. We have found that the absence of elastic anomalies in TO2 is originated from the existence of open space between a face of porous glass disk and the wall of the TO cell near the torsion rod. We calculated the change in resonant frequency when the shear modulus of glass inside the TO bob increases, assuming the structures of TO1 and TO2 in FEM simulations. The results are shown in Fig. S8. When the shear modulus of glass inside TO1 increases 5 percent, $f$ increases about 60 mHz, while it increases only 2 mHz in TO2.

We interpret these results as follows: In realistic TO made of metal, the torsion bob is not rigid, and the resonant frequency of the fundamental torsion mode is determined not only by the shear modulus of the torsion rod but also by the shear modulus of the torsion bob, which consists of porous glass, BeCu enclosure and adsorbed helium in our experiments. This non-ideal nature of TO has been established by studies of apparent supersolidity of bulk solid $^4$He using TOs with many different designs. In particular, it has been realized as the Maris effect that the stiffness of the part of TO near the torsion rod has a large contribution to resonant frequency. The presence of elastic anomalies in TO1 and its absence in TO2 may be explained by the difference in the amount of adsorbed helium.
be a manifestation of the Maris effect. In TO2, the stiffening of porous glass sample by helium adsorption will hardly contribute to the total torsion constant by the existence of open space inside the bob. We emphasize that this effect would be revealed only by FEM simulations, because it is difficult to calculate analytically the resonant frequency of a realistic TO with complicated structure and composites of different materials.

[1] H. J. Maris, Phys. Rev. B 86, 020502 (2012).