LARES/WEBER-SAT, frame-dragging and fundamental physics

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Abstract

After a brief introduction on the scientific objectives of the LARES/WEBER-SAT satellite we present the recent measurement of the Lense-Thirring effect using the nodes of the LAGEOS and LAGEOS 2 satellites and using the Earth gravity model EIGENGRACE02S obtained by the GRACE space mission, we also include some determination of the rate of change of the lowest order Earth’s even zonal harmonics. Finally, we describe an interesting possibility of testing the Brane-World unified theory of fundamental interactions by the use of a specially designed LARES/WEBER-SAT satellite.

1 Introduction

The scientific objectives of the WEBER-SAT satellite are:

(1) High precision tests of Einstein’s theory of general relativity, in particular, (1a) a ~ 1% measurement of the frame-dragging effect due to the angular momentum of a body, i.e., the Lense-Thirring effect, and test of the Earth’s gravitomagnetic field. The Lense-Thirring effect \[ \Omega \] is a tiny shift of the orbit of a test-particle. Frame-dragging, gravitomagnetic field and Lense-Thirring effect are theoretical predictions of Einstein’s theory of general relativity (see, e.g., ref. [3]). (1b) A High precision test \[ \delta \] of large distance infrared modification of gravity recently proposed by Dvali [5] to explain the dark-energy problem (a study will be required to precisely assess the achievable level of accuracy), see below. An improved high precision test of the inverse square law for very weak-field gravity and improved test of the equivalence principle (see the Italian Space Agency LARES phase A study [6]). A \(~10^{-3}\) measurement of the gravito-electric general relativistic perigee precession of the WEBER-SAT and a high precision measurement of the corresponding combination of the PPN (Parametrized-Post-Newtonian) parameters \[ \beta \] and \[ \gamma \] [7, 8]. This test will be achieved in the field of Earth
with a range of about 10000 km, more accurate tests of \( \beta \) and \( \gamma \) are achieved in the field of Sun but with a much longer range. The PPN parameters \( \beta \) and \( \gamma \) test Einstein’s theory of gravitation versus other metric theories of gravitation. (1d) Other tests of general relativity and gravitation (see LARES phase A study \[6\]).

(2) Measurements and improved determinations in geodesy and geodynamics

Frame-dragging may be thought of as an aspect of the Einstein’s principle of equivalence stating that, in a sufficiently small neighbourhood of a spacetime point, the effects of gravitation are not observable inside a freely falling frame, i.e., inside the so-called ”Einstein elevator” \[3\]. The basic aspect of the equivalence principle is the equality of inertial and gravitational mass, which is one of the best tested principles of physics, measured to date with an accuracy of about \(10^{-13}\). However, the axes of a freely falling frame, where the equivalence principle holds, are not fixed relative to ”distant inertial space”, i.e. with respect to ”distant fixed stars”, but they are ”dragged” by any moving mass. For example, they are dragged by a rotating mass; this is the ”dragging of inertial frames” or ”frame-dragging” as Einstein called it in 1913. In the near future the Gravity Probe B (GP-B) mission will try to measure frame-dragging, with unprecedented accuracy, on small super-conducting gyroscopes (the axes of the frames where the equivalence principle holds) orbiting around the Earth. GP-B will collect data, over a period of about one year only, that will then be analyzed to measure frame-dragging. However, the WEBER-SAT will collect data for a period of virtually hundreds of years (being a totally passive satellite with a very small orbital decay). These data could then be analyzed again in the future using the future improved gravitational models, in order to obtain much improved tests of frame-dragging and of other gravitational effects.

The orbit of a test-particle, such as a small satellite, is also a kind of gyroscope. Indeed, two of the orbital elements of a test-particle behave as ”gyroscopes”: the node and the pericenter (neglecting all the other perturbations).

Frame-dragging, gravitomagnetic field and Lense-Thirring effect have been described in several papers and studies, see for example ref. \[3\] and the ASI LARES phase A study \[6\].

Here we just point out that the Gravity Probe B experiment will try to measure the gravitomagnetic effect generated by the angular momentum of the Earth on a gyroscope, whereas the WEBER-SAT should measure the Earth’s angular momentum effect on the orbit of a test particle. In some
alternative theories the two effects may be different as in the case of a non-metric theory with asymmetric connection, such as the Cartan theory with torsion, that may affect in a different way the orbit of a test-particle and of a gyroscope (see ref. [3]).

2 A recent measurement of the Lense-Thirring effect using the LAGEOS satellites

Let us briefly report a recent measurement of the Lense-Thirring effect on the two Earth satellites LAGEOS and LAGEOS 2 [9]. We measured the Earth frame-dragging to be 99% of the value predicted by general relativity; the uncertainty of this measurement was 5% including all the known errors and 10% allowing for underestimated and unknown error sources.

Recently, by analysing the uncertainties in the spherical harmonic coefficients of the recent Earth gravity model EIGEN-GRACE02S obtained by the NASA space mission GRACE [10, 11], we found that the only relevant uncertainty in the orbit of the LAGEOS satellites [12], comparing it with the magnitude of the Lense-Thirring effect, is the one, \( \delta J_2 \), in the Earth quadrupole moment, \( J_2 \), which describes the Earth oblateness. In the EIGEN-GRACE02S model, the relative uncertainty \( \delta J_2 / J_2 \) is about \( 10^{-7} \). This uncertainty corresponds on the orbits of the LAGEOS satellites to a shift of the node larger than a few times the Lense-Thirring effect. However, the orbital uncertainty due to all the other harmonics is only a few percent of the general relativity shift. Therefore, in order to eliminate the orbital uncertainty due to \( \delta J_2 \) and in order to solve for the Lense-Thirring effect, it is necessary and sufficient to use only two observables. The two orbital observables we have analyzed are the two nodes of the LAGEOS satellites [13, 14, 15]. After modelling all the orbital perturbations, apart from the Lense-Thirring effect, we are able to predict the LAGEOS satellites’ orbit with an error (root-mean-square of the residuals) of about 3 cm for a 15 day arc, corresponding to about fraction of a half millisecond of arc at the LAGEOS satellites altitude. The Lense-Thirring effect is in contrast 31 milliarcsec/yr on the LAGEOS node and 31.5 milliarcsec/yr on the LAGEOS 2 node, as calculated by the Lense-Thirring formula. The residual (calculated minus observed) nodal rate of the LAGEOS satellites, \( \dot{\Omega}_{\text{residual}} \), is therefore:

\[
(\text{residual nodal rate}) = (\text{nodal rate from } \delta J_2 \text{ error}) + (\text{nodal rate from other } \delta J_{2n} \text{ errors}) + (\text{Lense-Thirring effect}) + (\text{other smaller modelling errors}),
\]

where the \( \delta J_{2n} \) are the errors in the Earth even zonal harmonic coefficients, \( J_{2n} \), of degree \( 2n \). We can then solve for the Lense-Thirring effect the sys-
tem of the two observed residual nodal rates for the Lense-Thirring effect and simultaneously eliminate the error due to the $\delta J_{2n}$ uncertainty. The maximum error in the combination of the residuals due to the $\delta J_{2n}$ is 4% of the Lense-Thirring effect.

In ref. [9] is reported the analysis (using the orbital estimator GEODYN [16]) of nearly eleven years of laser-ranging data, from January 1993 to December 2003, corresponding to about one million of normal points, i.e., to about 100 million laser ranging observations from more than 50 ILRS stations distributed all over the world [17].

In [9] we analyze the observed residuals of the nodal longitudes of the LAGEOS satellites, combined according to the formula to cancel the $\delta J_2$ uncertainty. The best fit line through the raw residuals (one-parameter fit) has a slope of 47.4 milliarcsec/yr; the root-mean-square of these post-fit residuals is 15 milliarcsec. The residuals after removal of six main frequencies, corresponding to a thirteen-parameter fit with a secular trend plus phase and amplitude of six main signals with periods of 1044, 905, 281, 569, 111 and 284.5 days, have a secular trend is 47.9 milliarcsec/yr, however the root-mean-square of these post-fit residuals is 6 milliarcsec only. The Lense-Thirring effect predicted by general relativity for the combination of the LAGEOS nodal longitudes amounts to 48.2 milliarcsec/yr. Therefore, corresponding to the thirteen parameter fit, the observed Lense-Thirring effect is 47.9 milliarcsec/yr, corresponding to 99% of the general relativistic prediction. In conclusion, this analysis confirms the Einstein’s theory predictions of frame-dragging and Lense-Thirring effect [9]. The total uncertainty of our measurement is, including systematic errors, 5% of the Lense-Thirring effect and 10% allowing for underestimated and unknown error sources. For example, if we consider the time-independent gravitational error (root-sum-square) to be three times larger we get a corresponding error of 9% and a total uncertainty of less than 10%.

3 Some geodetic results using the LAGEOS satellites and the EIGENGRACE02S model

Using the method of analysis of about 11 years of satellite ranging observations reported in the previous section, in addition to the accurate determination of the Lense-Thirring effect, an anomalous variation in the Earth gravity field since 1998 was observed [18] that was clearly identified as an anomalous increase in the Earth quadrupole moment. The trend in the nodal longitudes of both satellites distinctly showed a variation in the Earth
gravity field since 1998. This effect was proved \cite{18} to be due to an increase in the $J_2$ coefficient, indeed, combining the node residuals according to the formula to eliminate the $J_2$ perturbation only, the effect disappeared. The anomalous trend observed using EIGEN-GRACE02S was also accurately reproduced using the previous EGM96 Earth gravity model and the recent EIGEN-2 model due to the CHAMP satellite. This result confirms the measured relative increase of $J_2$ of the order of $10^{-11}$ that was recently reported \cite{19}. The Earth mass redistribution associated with this phenomenon is so far not clearly understood.

It is important to stress that together with the measurement of the Lense-Thirring effect, it was also measured the effect of the variations of $J_2$, $J_4$ and $J_6$ on the nodes of the LAGEOS satellites. In \cite{20} it is indeed reported an effective (i.e. including the effect of the higher even zonal harmonics) value of $\dot{J}_4^{\text{Effective}} \approx -1.5 \times 10^{-11}$. In the EIGENGRACE02S model, obtained by the GRACE mission only, the Earth gravity field was measured during the period 2002-2003. Corrections due to $\dot{J}_2$ and $\dot{J}_4$ were then applied to this 2002-2003 measurement in order to obtain a gravity field model antecedent to 2002-2003. These values of $\dot{J}_2$ and $\dot{J}_4$ used by the GFZ team are $\dot{J}_2 = -2.6 \times 10^{-11}$ and $\dot{J}_4 = -1.41 \times 10^{-11}$ and they were measured on the basis of completely independent 30-year observations before 2002. The only constraint (based on the validity of the GRACE measurements) is that the $\dot{J}_4$ correction applied to $J_4$, must of course produce in 2002-2003 the same value of $J_4$ that was measured by GRACE in 2002-2003, at least within the EIGENGRACE02S uncertainties in $J_4$. In ref. \cite{20} are reported the orbital analyses using the orbital estimator GEODYN with and without a contribution of $\dot{J}_4 = -1.41 \times 10^{-11}$. First, it is important to stress that in the case of not applying this $\dot{J}_4$ correction to the orbital analysis, it can be clearly seen, by visual inspection, a hump in the combined residuals. Indeed, the effect of the time variation $\dot{J}_4$ shows up as a quadratic effect in the cumulative nodal longitude of the LAGEOS satellites, therefore the combined residuals of LAGEOS and LAGEOS 2 were fitted with a parabola, together with a straight line and with the main periodic terms. Then, by fitting the raw residuals obtained without any $\dot{J}_4$, it was measured a $\dot{J}_4^{\text{Effective}} \approx -1.5 \times 10^{-11}$, which includes the effect of $\dot{J}_6$ and of higher even zonal harmonics on the LAGEOS satellites. On other hand, in the analysis of the combined residuals obtained with the EIGENGRACE02S correction of $\dot{J}_4 = -1.41 \times 10^{-11}$, it was measured a $\dot{J}_4^{\text{Effective}}$ of less than $-0.1 \times 10^{-11}$, in complete agreement with the previous case. It is finally important to stress that this small value of the unmodelled quadratic effects in our nodal combination due to
the unmodelled $\dot{J}_{2n}$ effects (with $2n \geq 4$) corresponds to a change in the measured value of the Lense-Thirring effect of about 1%. In other words using the value of $\dot{J}_4^{Effective} = -1.5 \times 10^{-11}$ for the LAGEOS satellites that we obtained from fitting the combined residuals (which is about 6% larger than the value $\dot{J}_4 = -1.41 \times 10^{-11}$ given in the EIGENGRACE02S model) resulted in a change of the measured value of frame-dragging by about 1% only with respect to the case of using $\dot{J}_4 = -1.41 \times 10^{-11}$; in conclusion this 1% variation fully agrees with the error analysis given in [9].

4 On the possibility of testing Brane-World theories with WEBER-SAT/LARES

Let us now briefly describe the possibility of probing some recently proposed modifications of gravity using the Runge-Lenz vector, i.e., the perigee of WEBER-SAT [4].

In Newtonian mechanics, the orbital angular momentum of a satellite and its nodal line, the intersection of its orbital plane with the equatorial plane of the central body, maintain a constant direction relative to “distant inertial space” for a motion under a central force. The Runge-Lenz vector, joining the focus and the pericenter of the orbit of a satellite, has also a constant direction relative to “distant inertial space” for a motion under a central force dependent on the inverse of the squared distance from the central body. Using the technique of laser-ranging with retro-reflectors to send back the short laser pulses, to this date we are able to measure distances with a precision of a few cm to a point on the Moon and of a few millimeters to a small artificial satellite. The instantaneous position of the LAGEOS satellites can be measured with an uncertainty of a few millimeters and their orbits, with semi-major axes $a_{LAGEOS} \approx 12270$ km and $a_{LAGEOS II} \approx 12210$ km, can be predicted, over 15 day periods, with a root-mean-square of the range residuals of a few cm. This uncertainty in the calculated orbits of the LAGEOS satellites is due to errors in modelling their orbital perturbations and, in particular, in modelling the deviations from spherical symmetry of the Earth’s gravity field, described by a spherical harmonics expansion of the Earth’s potential. However, to date, the terrestrial gravity field is determined with impressive accuracy, in particular with the dedicated satellites CHAMP and especially GRACE [10]. Regarding the perigee, the observable quantity is $e a \dot{\omega}$, where $e$ is the orbital eccentricity of the satellite and, $\omega$, the argument of perigee, that is the angle on its orbital plane measuring the departure of the satellite perigee from the equatorial plane of Earth.
Therefore, we can increase the measurement precision by considering orbits with larger eccentricities.

Motivated by the cosmological dark energy problem, Dvali recently proposed string theories leading, among other things, to weak field modifications of gravity [3]. One of the interesting observational consequences of the large distance infrared modification of gravity pointed out by Dvali is the anomalous shift of the pericenter of a test particle. The anomalous perihelion precession predicted by this gravity modification for the Moon perigee is:

$$\delta \phi = -\left[\frac{3\pi \sqrt{2}/4}{(r_c r_g^{1/2})}\right] \frac{r^{3/2}}{r_c} \text{rad/orbit}$$

(1)

Where, \(r_g = 0.886\,\text{cm}\) is the gravitational radius of the Earth, \(r\) is the Earth-satellite distance and \(r_c = 6\,\text{Gpc}\) is the gravity modification parameter that gives the observed galaxies acceleration without dark energy [3]. \(\delta \phi = 1.4 \cdot 10^{-12}\,\text{rad/orbit}\) for the Moon.

Therefore in the case of the WEBER-SAT satellite with a semimajor axis of about 12270 km this effect would amount to 0.04 milliarcsec/yr only.

Since this effect of infrared gravity modification is proportional to the 3/2 power of the semi-major axis and however the number of orbits per year goes as the -3/2 power of the semi-major axis, in terms of radians per year the perigee shift is the same for both the Moon and WEBER-SAT, i.e. \(1.9 \cdot 10^{-11}\,\text{rad/yr}\). Therefore, we simply need to consider what can be gained, or lost, with the use of WEBER-SAT versus the Moon. In regard to the measurement precision, the ranging precision is very roughly proportional to the range distance, i.e. is a few cm for the Moon and a few millimeters for the WEBER-SAT, then since the shift of the perigee at the satellite altitude is \(1.9 \cdot 10^{-11}\,\text{rad/yr}\) times the semi-major axis, the ratio of ranging precision to the effect to be measured is roughly the same for both the Moon and WEBER-SAT, even though slightly more favorable for the Moon. However, since the recovery of the perigee shift is proportional to the eccentricity of the satellite, we could orbit the WEBER-SAT satellite with a much larger eccentricity than the one of the Moon and therefore we could make the measurement of the perigee shift of the WEBER-SAT more precise than the one of the Moon.

However, critical are the systematic errors acting on the WEBER-SAT:

(a) the impact of the modelling uncertainties in the gravitational perturbations is critical for the WEBER-SAT satellite indeed the zonal harmonics of the Earth gravity field produce a perigee shift that is a function of the inverse powers of the semi-major axis, \(a: 1/a^{(2n+3/2)}\), for each even zonal
harmonic coefficient $J_{2n}$ with $n$ integer. However, considering the improvements in the future Earth gravity models from the mission CHAMP, GRACE and GOCE, the future uncertainty in the perigee shift due to gravitational perturbations should drastically decrease.

There is also an interesting possibility to choose a different orbit for WEBER-SAT, not at 70 degrees of inclination, but for example with the special inclination of about 63.4 degrees, (of the type of the Molniya orbits) at which a satellite would have a null perigee shift due to the Earth’s quadrupole moment $J_2$. In this way we would be able to cancel the major part of the $J_{2n}$ uncertainties to measure the perigee, but we will lose some accuracy in the measurement of the Lense-Thirring effect on the node, even though the use of LAGEOS and LAGEOS II, together with WEBER-SAT, and the future improvements in the accuracy of the Earth’s gravity models should not make this Lense-Thirring measurement much worst than a 1 % measurement (this possibility has to be further investigated) and furthermore, in this case, the Lense-Thirring effect could be measured using the WEBER-SAT perigee.

The impact of the modelling uncertainties in the non-gravitational perturbations is also critical for the WEBER-SAT satellite indeed the crucial factor is the cross-sectional area to mass ratio. In other words the acceleration produced by a non-gravitational force (such as radiation pressure) acting on a satellite is proportional to its cross sectional area and inversely proportional to its mass. Then this ratio is roughly proportional to the inverse of the radius of the considered satellites. Then for the Moon is very small whereas for WEBER-SAT may be a critical source of error.

These forces could be reduced by (a) a much denser and larger satellite than the one of the original proposal (although by increasing the cost of the mission); (b) the non-gravitational perturbations of the perigee could also be reduced through the use of a much more eccentric orbit and (c) the mismodelling of the radiation pressure perturbations could be reduced by special optical and thermal tests performed on the WEBER-SAT satellite and through the measurement of its spin axis and rotational rate, once in orbit. Finally, we also mention the possibility of a drag free system, that implies acceleration sensors and propulsion systems on board, or of a spherical satellite made of a material transparent to most of the radiation; similar spherical retro-reflectors have been tested by the Russian space agency.

On September 2004 at a LARES INFN-meeting in Padova (with B. Bertotti, M. Cerdonio, F. Morselli, A. Riotto, I.C., A. Paolozzi, S. Dell’Agnello, L. Iorio and others.), I.C. pointed out the need to use a very large eccentricity and a higher orbit of LARES in the attempt to measure the Brane-World
effects on its orbit. In this way, with a larger value of the eccentricity, $e$, the non-gravitational effects on its perigee would be reduced. Contemporarily, using a higher orbit all the effects due to $J_{2n}$ with $2n \geq 4$ would be drastically reduced. At that meeting I.C. proposed, as an example, an orbit with semimajor axis of 36000 km. A very large value of the eccentricity is critical to minimize the non-gravitational perturbations on the perigee, on other hand a large value of the semimajor axis with a suitable value of $e$ would reduce the $\delta J_{2n}$ effects on the perigee and make the errors due to the $\delta J_{2n}$, with $2n \geq 4$, smaller than the Brane-World effect on the LARES perigee.

The suitable orbit for LARES is currently under study, indeed as pointed out in ref. [21] a higher orbit would imply the need of a larger satellite in order for LARES to be visible by the laser ranging stations, however this will imply a higher cost for the satellite and for its launch.

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