Novel $CP$-violating Effects in $B$ decays from Charged-Higgs in a Two-Higgs Doublet Model for the Top Quark

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Abstract

We explore charged-Higgs $CP$-violating effects in a specific type III two-Higgs doublet model which is theoretically attractive as it accommodates the large mass of the top quark in a natural fashion. Two new $CP$-violating phases arise from the right-handed up quark sector. We consider $CP$ violation in both neutral and charged $B$ decays. Some of the important findings are as follows.
1) Large direct-$CP$ asymmetry is found to be possible for $B^\pm \to \psi/J K^\pm$.
2) Sizable $D\bar{D}$ mixing effect at the percent level is found to be admissible despite the stringent constraints from the data on $K\bar{K}$ mixing, $b \to s\gamma$ and $B^- \to \tau\bar{\nu}$ decays.
3) A simple but distinctive $CP$ asymmetry pattern emerges in decays of $B_d$ and $B_s$ mesons, including $B_d \to \psi/J K_S$, $D^+D^-$, and $B_s \to D_s^+D_s^-$, $\psi \eta/\eta'$, $\psi/J K_S$.
4) The effect of $D\bar{D}$ mixing on the $CP$ asymmetry in $B^\pm \to D/\bar{D}K^\pm$ and on the extraction of the angle $\gamma$ of the unitarity triangle from such decays can be significant.

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I. INTRODUCTION

One of the main programs at the upcoming B factories is to measure the size of CP violation in as many $B$ decay modes as possible so as to establish the pattern of CP violation among various $B$ decays. This then may allow for an experimental test not only of the standard model (SM) Cabibbo-Kobayashi-Maskawa (CKM) paradigm for CP violation, but also many extensions of the SM that often contain new sources of CP violation.

In this paper, we continue to study the distinctive phenomenological implications of a two Higgs doublet model for the top quark that is designed to take into account the large mass of the top quark in a natural fashion. This model contains flavor violation and new sources of $CP$ violation in the charged Higgs sector. In this top-quark two-Higgs doublet model (T2HDM) first introduced in Ref. [3], the top quark is assigned a special status by coupling it to one Higgs doublet that gets a large vacuum expectation value (VEV), whereas all the other quarks are coupled only to the other Higgs doublet whose VEV is much smaller. This arrangement of Yukawa interactions is motivated by the mass hierarchy between the top and the other quarks, and the T2HDM can be considered as a special case of the general 2HDM (type III) [4]. The unique predictions of the model for the $CP$ asymmetries in both neutral and charged $B$ decays should allow for many experimental tests at the $B$-factories.

One notable feature about type III 2HDMs is the fact that natural flavor conservation (NFC) [5] is not imposed on the Yukawa interactions as is done in models I and II of 2HDM [4]. However, the assumption of NFC is more of a convenience than a necessity [4], and this is especially true for the top quark as at the moment there is no experimental data that require it. Relaxing the assumption of NFC leads to many interesting phenomenological implications [4]. As a result, three distinctive features arise in this T2HDM which are absent in models with NFC. Firstly, there are new CP-violating phases (in the charged Higgs sector) besides the CKM phase. These new phases come from the unitary diagonalization matrix acting on the right-handed (RH) up-type quarks. Secondly, some charged Higgs Yukawa couplings are greatly enhanced by the large ratio of the two Higgs VEVs denoted by $\tan \beta$. This
is the case, for example, in $H^+ c_R q_L (q = d, s, b)$ and $H^+ u_R b_L$, whereas these couplings are suppressed by $1/\tan \beta$ in models with NFC. These two features have important implications for $K \bar{K}$ and $D \bar{D}$ mixing, as well as for CP violation in $B$ decays. Finally, flavor changing neutral Higgs (FCNH) couplings exist among the up-type quarks but not the down-type quarks, and could contribute, for example, to $D \bar{D}$ mixing at tree level $^3$.

It is the first two aspects of the model that we wish to concentrate on in this work. An immediate consequence of these two features is the resulting complex tree-level $b \to c$ decay amplitudes. This distinguishes it from many popular models with new physics at the loop level, and it has important implications both for neutral $B$ decays to $CP$ eigenstates and for direct $CP$ violation in charged $B$ decays. In a previous note $^2$, we have highlighted the implications of the model for the $CP$ asymmetry in the “gold-plated” mode $B \to \psi/J K_S$ and found that the asymmetry could take very different values from the SM expectation. In this work, we would like to extend our previous analysis in two ways.

First, we will perform a systematic analysis of the $CP$ asymmetries in the various $B_d$ and $B_s$ decay channels, including, for example, $B_d \to \psi/J K_S$, $D^+ D^-$, $\pi \pi$, and $B_s \to D_s^+ D_s^-$, $\psi \eta/\eta'$, $\psi/J K_S$. A simple and distinctive pattern of $CP$ asymmetry emerges from our study. As a result, studies of the $CP$ asymmetries in a few $B$ decay modes may be utilized to extract information on both the angle $\beta_{\text{CKM}}$ of the CKM Unitarity Triangle (UT) and one new $CP$-violating phase of this model. With more measurements, one can confirm or rule out the model via consistency checks.

Second, on account of the new $CP$ phase in the Higgs-mediated decay amplitudes, we investigate in the T2HDM direct $CP$-violating effects in charged $B$ decays, including $B^\pm \to \psi/J K^\pm$ and $B^\pm \to D/\bar{D} K^\pm$. The former mode could exhibit a sizable partial rate asymmetry (PRA) if the strong phase difference is not small. This mode is of special interest

$^\dagger$To avoid confusion with the $\beta = \tan^{-1} v_2/v_1$ associated with the ratio of Higgs VEVs in the 2HDM, we use $\beta_{\text{CKM}}$ to denote one of the angles of the the CKM unitarity triangle.
due to its experimental cleanliness and high branching ratio. The $B^\pm \to D/\bar{D}K^\pm$ mode is of interest for the measurement of the angle $\gamma$ of the unitarity triangle [7] within the SM where the $D\bar{D}$ mixing effect is negligible. One interesting implication of the T2HDM is that the $D\bar{D}$ mixing can be significantly enhanced so that $x_D \equiv \Delta m_D/\Gamma_D = \mathcal{O}(10^{-2})$. Improvements in the existing bound on $x_D$ may therefore be very worthwhile. This possible enhancement arises because the $D\bar{D}$ mass difference is quite sensitive to the poorly constrained $u_R - t_R$ mixing of the right-handed sector, and a large mass splitting due to charged Higgs can occur. This large $D\bar{D}$ mixing can in turn strongly modify the direct $CP$ asymmetry in $B^\pm \to D/\bar{D}K^\pm$. The implication for the extraction of $\gamma$ within the T2HDM will be discussed.

The outline of the paper is as follows: the model is introduced in section II. The most stringent experimental constraints from $K\bar{K}$ mixing, $b \to s\gamma$ and $B \to \tau\bar{\nu}$ decays are presented in section III. In section IV, we explore the pattern of $CP$ asymmetries in neutral $B_d$ and $B_s$ decays. Section V is devoted to the study of direct $CP$ violation in charged $B$ meson decays in the T2HDM, first in $B^\pm \to \psi/JK^\pm$, then in $B^\pm \to D/\bar{D}K^\pm$. We conclude with some discussion in section VI.

II. THE MODEL

The fact that the top quark mass is of order the weak scale and is much larger than the other five quarks is suggestive of a different origin for its mass than the other five. Many attempts have been made along this direction, including the dynamical top-condensation model [8] and the top-color model [9]. As an alternative to the dynamical models of the top mass and electroweak symmetry breaking, a top-quark 2HDM (T2HDM) was proposed [3] to accommodate the top mass and the weak scale through a separate Higgs doublet than the one that is responsible for the masses of the other quarks and all the charged leptons. The T2HDM can be viewed as an effective low energy parameterization through the Yukawa interactions of some high energy dynamics which generates both the top mass and the
weak scale. The two scalar doublets could be composite, as in top-color models, and the Yukawa interactions could be the residual effect of some higher energy four-Fermi operators. Indeed, some similarities can be noted in the effective low energy flavor physics between dynamical top models and certain Model III 2HDMs including the T2HDM.

The Yukawa interaction of the T2HDM can be simply written as follows,

\[
\mathcal{L}_Y = -\overline{L}_L \phi_1 E_R - \overline{Q}_L \phi_1 F d_R - \overline{Q}_L \tilde{\phi}_1 G^1 u_R - \overline{Q}_L \tilde{\phi}_2 G^2 u_R + \text{H.c.},
\]

where the two Higgs doublets are denoted by \( \phi_i \) with \( \tilde{\phi}_i = i \sigma^2 \phi_i^\ast \) \( i = 1, 2 \), and where the 3 \times 3 Yukawa matrices \( E, F \) and \( G \) give masses respectively to the charged leptons, the down and up type quarks; \( 1^{(1)} \equiv \text{diag}(1,1,0) \) and \( 1^{(2)} \equiv \text{diag}(0,0,1) \) are two orthogonal projection operators onto the first two and the third families respectively, and \( Q_L \) and \( L_L \) are the usual left-handed quark and lepton doublets. The heaviness of the top quark arises as a result of the much larger VEV of \( \phi_2 \) to which no other quark couples. As a result, one notable feature about this model is that the ratio of the two Higgs VEVs, \( \tan \beta = v_2/v_1 \), is required to be large so that the Yukawa couplings of the top and bottom quarks are similar in magnitude. We will take \( \tan \beta \geq 10 \) in the following analysis.

This assignment of Yukawa couplings is not in line with the notion of NFC, and this leads to two tree level FCNH interactions, \( h \bar{t} c \) and \( h \bar{t} u \), both of which are not constrained by present data. Some phenomenological studies of the neutral Higgs sector, including the effect on \( \bar{D}D \) mixing, can be found in [3]. On the other hand, only little attention has been paid to the charged Higgs sector [4], and it is this aspect of the model on which we would like to focus.

The charged Higgs Yukawa couplings can be obtained as,

\[
\mathcal{L}_Y^C = \frac{g}{\sqrt{2} m_W} \left\{ -\overline{\tau}_L V M_D d_R \left[ G^+ - \tan \beta H^+ \right] + \overline{\tau}_R M_U V d_L \left[ G^+ - \tan \beta H^+ \right] + \overline{\tau}_R \Sigma^\dagger V d_L \left[ \tan \beta + \cot \beta \right] H^+ + \text{H.c.} \right\},
\]

where \( G^\pm \) and \( H^\pm \) represent the would-be Goldstone bosons and the physical charged Higgs bosons, respectively, and \( M_U \) and \( M_D \) are the diagonal up- and down-type mass matrices.
Here $\Sigma \equiv M_U U_R^\dagger 1^{(2)} U_R$, where $U_R$ denotes the unitary rotation of the RH up-type quarks from gauge to mass eigenstates.

One feature that distinguishes the T2HDM from 2HDM’s with NFC (models I and II) is the presence of the unitarity matrix $U_R$ contained in the $\Sigma$ matrix. This gives rise to two new $CP$-violating phases as well as non-standard Yukawa couplings. The unitary matrix $U_R$ can, in general, be parameterized as:

$$U_R = \begin{pmatrix}
\cos \phi & -\sin \phi & 0 \\
\sin \phi & \cos \phi & 0 \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
1 & 0 & 0 \\
0 & \sqrt{1 - |\epsilon_{ct}\xi|^2} & -\epsilon_{ct}\xi^* \\
0 & \epsilon_{ct}\xi & \sqrt{1 - |\epsilon_{ct}\xi|^2}
\end{pmatrix} \begin{pmatrix}
\sqrt{1 - |\epsilon_{ct}\xi'|^2} & 0 & -\epsilon_{ct}\xi'^* \\
0 & 1 & 0 \\
\epsilon_{ct}\xi' & 0 & \sqrt{1 - |\epsilon_{ct}\xi'|^2}
\end{pmatrix},$$

where $\epsilon_{ct} \equiv m_c/m_t$ and where $\xi$ and $\xi'$ are complex numbers with $|\xi'| < |\xi| = O(1)$. Note that the form of $\Sigma$ is independent of the 1-2 rotation and depends only on the two unknown complex parameters $\xi = |\xi|e^{-i\delta}$ and $\xi' = |\xi'|e^{-i\delta'}$.

Table 1: Comparison of the magnitudes of charged Higgs Yukawa couplings in the T2HDM and Models I and II 2HDM for large $\tan \beta$.

| Vertices       | T2HDM                  | Models I and II 2HDM |
|----------------|------------------------|----------------------|
| $\tau_R b_L H^+$ | $\simeq m_c \xi V_{tb} \tan \beta$ | $\gg m_c V_{tb} \cot \beta$ |
| $\bar{c}_R q_L H^+$ ($q = d, s$) | $\simeq m_c \tan \beta (-V_{cq} + V_{tq} \xi^*)$ | $\gg m_c V_{cq} \cot \beta$ |
| $\bar{u}_R b_L H^+$ | $\simeq m_c \xi'' V_{tb} \tan \beta$ | $\gg m_u V_{ub} \cot \beta$ |
| $\bar{t}_R q_L H^+$ ($q = d, s, b$) | $\sim m_t V_{tq} \cot \beta$ | $\sim m_t V_{tq} \cot \beta$ |

The phenomenologically important Yukawa couplings are listed in Table 1. Several remarks can now be made concerning Table 1. In comparison to the popular 2HDM (model II) which is realized, e.g. in the supersymmetric extension of the SM, the T2HDM contains rather large $\tau_R b_L H^+$, $\tau_R b_L H^+$, $\tau_R s_L H^+$, and $\tau_R d_L H^+$ couplings, all enhanced by the large $\tan \beta$. In particular, the anomalously large $\tau_R b_L H^+$ coupling is directly proportional to $m_t V_{tb} \tan \beta$ and to the $u_R - t_R$ mixing parameter $\epsilon_{ct}\xi'^*$, whereas in model II it depends
on $m_u V_{ub} \cot \beta$ which is completely negligible. The unknown mixing parameter $\xi'$ can be
constrained from the experimental data on $B^- \rightarrow \tau \bar{\nu}$ decay and $D \bar{D}$ mixing. Similarly, the
potentially large charm quark Yukawa interactions can be expected to affect in a significant
way the $K \bar{K}$ system and the $CP$ asymmetries in $B$ decays. By contrast, the charged Higgs
collection to $B \bar{B}$ mixing is negligible due to the $1/ \tan \beta$ suppression of the top quark
Yukawa coupling.

We now turn to a detailed analysis of the most stringent experimental constraints from
$K \bar{K}$, $b \rightarrow s \gamma$, and $B^- \rightarrow \tau \bar{\nu}$. For the numerical estimates, we will assume $|\xi| = 1$ unless
otherwise stated.

III. EXPERIMENTAL CONSTRAINTS

A. Constraints from $K \bar{K}$ mixing

The $\Delta S = 2$ effective Hamiltonian receives contributions from box diagrams with virtual
$W$ and $H$ bosons and up-type quarks. The short distance contribution to the $K_L - K_S$ mass
difference $\Delta m_K$ in the standard model mainly comes from the $c$ quark. In the 2HDM that we
are considering, the new contribution is dominated by the $HHcc$ box diagram for $\tan \beta > 10$.
The leading term in the $\Delta S = 2$ effective Hamiltonian from charged Higgs exchange can be
expressed as

$$H_{\Delta S=2}^{\chi} \sim \frac{G_F}{16\pi^2} \lambda_1^2 \eta_1 \frac{m_c^4 \tan^4 \beta}{m_H^2} (\bar{d}_L \gamma_\mu s_L)^2 + \text{H.c.} . \quad (4)$$

where $\eta_1$ is the short-distance QCD correction factor, and $\lambda_c = V_{cs} V_{cd}^* - \xi V_{td}^* V_{cs} - \xi^* V_{ts} V_{cd}^* + |\xi|^2 V_{ts} V_{td}^*$. The last three terms in $\lambda_c$ are suppressed by $\lambda^2$ ($\lambda = |V_{us}| = 0.22$) relative to
the first term and can be neglected in the mass difference $\Delta m_K$. By way of contrast, these
terms are essential for the $CP$-violation parameter $\epsilon_K$.

The charged Higgs contribution to $\Delta m_K$ can be easily obtained from Eq. (4) by setting
$\lambda_c \rightarrow V_{cs} V_{cd}^*$. The total short distance contribution to $\Delta m_K$ is given by [2],

1
\[(\Delta m_K)_{SD} = \frac{G_F^2 f_K^2 B_K m_K \lambda_c^2}{6\pi^2} \times \left( m_c^2 \eta_1 + m_c^4 \frac{\tan^4 \beta}{4m_H^2} \eta'_1 \right) \tag{5}\]

where the first term is from the SM and the second from Higgs exchange, \( f_K = 160 \text{ MeV} \) is the kaon decay constant, \( B_K = 0.87 \pm 0.14 \) \cite{11, 12} is the bag factor, and \( \eta_1 = 1.38 \pm 0.53 \) \cite{13} and \( \eta'_1 \) are the QCD corrections to the two box diagrams. The SM top quark contribution is a few percent of the charm quark contribution and is not included in the above equation. Similarly the contribution from \( WHcc \) and other box diagrams is negligible in the large \( \tan \beta \) limit and is not considered.

Because of the large uncertainties in \( B_K, \eta_1, m_c \) and in the long distance contribution, we have used the method described in \cite{11} to derive the constraints on the model. The QCD correction factor \( \eta'_1 \) is unknown, and we simply assign it the value of \( \eta_1 \) and allow them to vary independently within their 1\( \sigma \) ranges in our error analysis. Assuming the long distance effect to be 30\% \cite{14} of \( \Delta m_K \), we get

\[ m_H / \tan^2 \beta > 0.48 \text{ GeV} \tag{6} \]

for \( \tan \beta > 10 \) and at the 95\% C.L.. Note that the \( \tan^4 \beta \) enhancement in \( \Delta m_K \) leads to a severe lower bound on the Higgs mass for large \( \tan \beta \). This is a unique feature of the T2HDM.

Similar to \( \Delta m_K \), the \( CP \)-violating parameter \( \epsilon_K \) also receives a significant contribution from charged Higgs exchange, which is again dominated by the \( HHcc \) box diagram due to its \( \tan^4 \beta \) dependence. However, the physics of \( \epsilon_K \) differs from that of \( \Delta m_K \) in three important aspects. First, unlike \( \Delta m_K \), \( \epsilon_K \) is short-distance dominated and is theoretically under better control. Second, as the leading, first term of \( \lambda_c \) is real, the sub-leading, 2nd and 3rd terms now become important for CP violation. In fact, the charged Higgs contribution to \( \epsilon_K \) is directly proportional to the \( c_R - t_R \) mixing parameter \( \xi \), whereas \( \Delta m_K \) is independent of \( \xi \) to a good approximation. Third, within the SM, \( \epsilon_K \) depends to a large extent on the top quark box diagram, and also on the \( ct \) and \( cc \) box diagrams to a lesser extent. In contrast, the short distance contribution to \( \Delta m_K \) is predominantly due to the charm quark.
The SM expression for $\epsilon_K$ is well understood and can be found, for example, in [13]. The dominant Higgs contribution can be obtained from Eq. (4),

$$\epsilon_K^H = e^{i\frac{\pi}{4}} C_\lambda B_K A \lambda^4 \eta_1 \sqrt{\rho^2 + \eta^2} \sin(\gamma + \delta)|\xi| \frac{(m_c \tan \beta)^4}{4m_W^2 m_H^2}$$

(7)

where $A = 0.82 \pm 0.04$, $\rho$, and $\eta$ are the CKM parameters in the Wolfenstein parameterization [15], $\gamma \equiv \tan^{-1} \eta/\rho$ is one of the angles of the unitarity triangle, and $C_\epsilon = \frac{G_F^2 f_K^2 m_K^6}{6\sqrt{2} \pi^2} = 3.78 \times 10^4$.

As $\gamma$ is basically a free parameter in this model, we can obtain bounds on the parameter $Y \equiv \sin(\gamma + \delta)|\xi| \left(\frac{\tan \beta}{20}\right)^4 \left(\frac{200 \text{ GeV}}{m_H}\right)^2$ for any given value of $\gamma$ by allowing $\sqrt{\rho^2 + \eta^2}$ to vary within its 1$\sigma$ uncertainties derived from $b \to u e \nu$, which receives negligible contribution from Higgs exchange. It is interesting to note that charged Higgs exchange can be solely responsible for $\epsilon_K$ if the CKM matrix is real (i.e. $\gamma = 0^\circ$). Using the method of [11] for error analysis, we obtain at the 95% C.L. $0.08 < Y < 0.39$ for the case of a real CKM matrix. If we assume that $\gamma$ takes its SM central value of $68^\circ$ [11], the constraint becomes $-0.085 < Y < 0.08$. And for $\gamma = -45^\circ$, we get the bound $0.14 < Y < 0.65$. Note that unlike $\Delta m_K$, the $\epsilon_K$ constraint in the $m_H - \tan \beta$ plane depends on $|\xi|$ and $\delta$, and it could be more stringent than the $\Delta m_K$ bound of Eq. (3) [2].

**B. The $b \to s \gamma$ decay rate**

We now turn to the constraints from $b$ decays. The inclusive radiative decay $b \to s \gamma$ has been studied in detail in [2]. The main result of that analysis can be summarized as follows.

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‡As will be discussed later, the CDF result on $a_{\psi K_S}$ [16] tends to disfavor a large, negative $\gamma$. 

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FIG. 1. The constraint from the $b \to s\gamma$ rate on the $m_H - \tan \beta$ plane of the T2HDM for $|\xi| = 1.0$ and with different values of the phase $\delta$. The value of $\delta$ increases from the bottom curve to the top curve. The area below each curve is excluded.

As in Model II 2HDM, a strong constraint is imposed from $b \to s\gamma$ on the allowed parameter space of the T2HDM. However, the form of the constraint differs significantly. In model II, a lower bound of about 370 GeV can be placed on the charged Higgs mass, independent of $\tan \beta$ [17]. A similar situation does not occur for the T2HDM due to the nonstandard Higgs Yukawa couplings and the presence of the new $CP$-violating phase $\delta$. In the T2HDM, the charged Higgs amplitude could interfere either constructively or destructively with the SM amplitude depending on the new phase $\delta$. Consequently, the lower limit on the charged Higgs mass shows a strong dependence on both $\tan \beta$ and the phase $\delta$, and a much lower Higgs mass is still allowed.

Neglecting the charged-Higgs induced scalar operator $(\bar{c}_R b_L)(\bar{s}_L c_R)$, we can derive a bound on the ratio $R = B(b \to X_s\gamma)/B(b \to X_c\ell\bar{\nu})$ by combining the theoretical and experimental uncertainties [2]:

$$0.0012 \leq R^{\text{theory}} \leq 0.0046,$$

(8)
where \( R^{\text{theory}} \) denotes the SM plus charged Higgs contribution. The constraint on the \( m_H - \tan \beta \) plane depends on both \( |\xi| \) and its phase \( \delta \), and is shown in Fig. 1.

C. \( B^- \to \tau \bar{\nu} \)

As noted in section II, the decay \( B^- \to \tau \bar{\nu} \) could receive its dominant contribution from charge Higgs exchange on account of the anomalously large \( H^+ \bar{u}_R b_L \) coupling. Although the current experimental limit on \( BR(B^- \to \tau \bar{\nu}) \) is about one order of magnitude above the SM prediction, it already starts to constrain the T2HDM in a nontrivial way.

The leading term of the effective Hamiltonian from charged Higgs exchange that mediates this decay is given by

\[
H^{H}_{\text{eff}} = 2\sqrt{2} G_F V_{tb} \frac{m_c \tan^2 \beta \xi'^*}{m^2_H} (\bar{u}_R b_L) (\bar{\tau}_R \nu_L) + \text{H.c.} \tag{9}
\]

Assuming that the charged-Higgs-exchange contribution dominates over the SM \( W^- \) exchange, one can obtain the branching ratio as,

\[
BR(B^- \to \tau \bar{\nu}) = \frac{\tau_B}{8\pi} \frac{G_F^2 f_B^2 m^2 \tau m_B}{m^2_H} \left( 1 - \frac{m^2_c}{m^2_B} \right)^2 \left( \frac{m^2_H m_c |\xi'| \tan^2 \beta}{m^2_H (m_b + m_u)} \right)^2 \]

\[
= 4.6 \times 10^{-2} \left( \frac{f_B}{200 \text{ MeV}} \right)^2 \left( \frac{10 \text{ GeV}}{m_H/\tan \beta} \right)^4 |\xi'|^2 \tag{10}
\]

where \( \tau_B \) is the \( B \) lifetime, and we have used \( f_B \simeq 200 \text{ MeV} \) \cite{12} for the \( B^- \) decay constant, \( m_c = 1.3 \text{ GeV} \), and \( m_b = 4.5 \text{ GeV} \). The present experimental limit \cite{18} \( BR(B^- \to \tau \bar{\nu}) < 5.7 \times 10^{-4} \) then translates into an upper bound on the \( u_R - t_R \) mixing parameter \( \xi' \),

\[
|\xi'| < \left( \frac{m_H/\text{GeV}}{30 \tan \beta} \right)^2 \tag{11}
\]

D. \( D \bar{D} \) mixing

The SM effect on \( D \bar{D} \) mixing is vanishingly small, estimated to be \( \Delta m_D \sim \mathcal{O}(10^{-16}) \text{ GeV} \) \cite{19}. This is three orders of magnitude below the current experimental limit of \( \Delta m_D < \)
1.6 \times 10^{-13} \text{ GeV} \[15\], which translates into an upper bound on the mixing parameter \( x_D \equiv \Delta m_D / \Gamma_D < 0.1 \). As will be shown below, one loop charged Higgs exchange in the T2HDM could contribute to \( x_D \) at the few percent level.

We note first that there exists in the T2HDM \( \Delta C = 2 \) four-quark operators from tree level neutral Higgs exchange. This effect, however, is greatly suppressed by the RH mixings \( |\epsilon^2_c \xi \xi' |^2 \propto m_c^4 / m_t^4 \) \[3\].

\[
\Delta m_D \left| \phi^0 \right| \propto G_F (\tan \beta)^2 m_c^6 / m_t^2 m_h^2 .
\]

(12)

In comparison, the one-loop box diagram with internal charged Higgs and \( b \) quarks is enhanced by the anomalously large Yukawa couplings \( H^+ \bar{c}_R b_L \) and \( H^+ \bar{u}_R b_L \), both of which suffer no CKM suppression and grow as \( \tan \beta \). This feature differs significantly from the SM box diagram where the virtual \( b \) quark effect, being highly Cabibbo-suppressed, is negligible compared to that of the \( s \) and \( d \) quarks. Simple power counting then gives for the \( HHbb \) box diagram,

\[
\Delta m_D \left| H^+ \right| \propto G_F^2 (\tan \beta)^4 m_c^4 / m_H^2 .
\]

(13)

This effect could be three orders of magnitude above the tree-level neutral Higgs contribution for the same choice of the mixing parameters, and can generate a sizable \( x_D \) near the threshold of experimental discovery.

The calculation of the one-loop box diagram can be greatly simplified by setting the external momenta of the \( c \) and \( u \) quarks to zero. Neglecting the short-distance QCD correction, the effective Hamiltonian can be obtained as,

\[
H_H^{\Delta C=2} \simeq \frac{G_F^2}{16 \pi^2} (\xi \xi'^* \gamma_R \gamma_\mu \gamma_L \gamma_\mu \gamma_R \gamma_L) \propto H.c. ,
\]

(14)

where we have taken \( V_{tb} = 1 \). By use of the vacuum saturation approximation, one has

\[
x_D \simeq \frac{G_F^2}{6 \pi^2} |\xi \xi'^*|^2 \frac{m_D^4 \tan^4 \beta}{4 m_H^2 \Gamma_D} \frac{m_t^4}{m_h^2} \left( \frac{1 \text{ GeV}}{m_H / \tan^2 \beta} \right)^2
\]

\[
\simeq 7.5% \times |\xi \xi'^*|^2 \left( \frac{1 \text{ GeV}}{m_H / \tan^2 \beta} \right)^2
\]

\[
< 7.5% \times |\xi|^2 \left( \frac{m_H}{900 \text{ GeV}} \right)^2
\]
where we have taken $f_D \simeq 0.2$ GeV \cite{12}, and the $B \to \tau\nu$ bound (Eq. (11)) has been imposed in the last step. Numerically, one has $x_D \leq 2\%|\xi|^2$ for $m_H = 450$ GeV and $\tan\beta = 30$, and $x_D \leq 6\%|\xi|^2$ for $m_H = 800$ GeV and $\tan\beta = 40$. Both are consistent with the $\Delta m_K$ and $b \to s\gamma$ constraints. The $\epsilon_K$ constraint can also be satisfied by choosing the phases of $\gamma$ and $\delta$, whereas $x_D$ is independent of the phases. Thus, in this model $x_D$ can be of order a few percent. Continual experimental improvements over the existing bound of 0.1 \cite{18} are therefore strongly encouraged.

The neutral-meson mixing parameter $p/q$ \cite{18} is also modified in the T2HDM. To a good approximation, it is given by

$$\frac{p_D}{q_D} = e^{-i2\theta_D}, \tag{16}$$

where $\theta_D = \arg(\xi) - \arg(\xi')$ is the mixing phase. Recall that the mixing phase is zero in the SM. Finally, we note that the parameter $y_D = \Delta\Gamma/\Gamma_D$ is very small in the T2HDM, as the $D/\bar{D}$ decay amplitudes are unaffected by the Higgs exchange.

**IV. CP ASYMMETRY IN NEUTRAL B DECAYS TO CP EIGENSTATES**

**A. General Framework**

The non-standard Higgs interactions which we have been discussing could have significant effects on the time-dependent $CP$-asymmetry

$$a(t) = \frac{\Gamma(B^0(t) \to f) - \Gamma(B^0(t) \to \bar{f})}{\Gamma(B^0(t) \to f) + \Gamma(B^0(t) \to \bar{f})}, \tag{17}$$

where $\Gamma(B^0(t) \to f)$ ($\Gamma(B^0(t) \to \bar{f})$) represents the time-dependent probability for a state tagged as a $B^0$ ($\bar{B}^0$) at time $t = 0$ to decay into the final state $f$ ($\bar{f}$) at time $t$. We will first consider the case when the final state is a $CP$ eigenstate, and thus $\bar{f} = f$.

It is convenient to introduce the re-phasing invariant quantity \cite{18},

$$\tilde{\lambda} = \frac{q \bar{A}}{p A} \tag{18}$$
where \( A \equiv < f | B > \) and \( \overline{A} \equiv < f | \overline{B} > \) denote the \( B \) and \( \overline{B} \) decay amplitudes. Neglecting the small \( CP \)-violating effect in \( B \overline{B} \) mixing, the neutral \( B \) meson mixing parameter can be written as a pure phase: \( q/p = e^{-i2\phi_M} \) with \( \phi_M \) the mixing phase. The time-dependent \( CP \) asymmetry can now be expressed in terms of \( \tilde{\lambda} \),

\[
a(t) = \frac{(1 - |\tilde{\lambda}|^2) \cos(\Delta Mt) - 2\text{Im}(\tilde{\lambda}) \sin(\Delta Mt)}{1 + |\lambda|^2}.
\] (19)

For certain channels without direct \( CP \) violation, i.e. \( |\overline{A}/A| = 1 \), we can write \( A/\overline{A} = f \text{}_{CP} e^{-2i\phi_D} \), with \( \phi_D \) the phase of the decay amplitude and \( f \text{}_{CP} = \pm 1 \) the \( CP \) eigenvalue of the final state \( f \). The \( CP \) asymmetry then takes the simple form

\[
a(t) = -\text{Im}(\tilde{\lambda}) \sin(\Delta Mt) = f \text{}_{CP} \sin 2(\phi_D + \phi_M) \sin(\Delta Mt) .
\] (20)

As we have noted, \( B \overline{B} \) mixing receives a very small contribution from charged Higgs exchange for moderate values of \( \xi \) after imposing the \( K \overline{K} \) constraints. We therefore only need to consider the charged Higgs effect on the decay amplitudes. The tree-level Hamiltonian for the transition \( b \rightarrow c\overline{c}q \) \((q = s, d)\) has a simple form in the large \( \tan \beta \) limit [2],

\[
\mathcal{H}_{\text{eff}} \simeq 2\sqrt{2} G_F V_{cb} V_{cq}^* \left[ \overline{c}_L \gamma_\mu b_L \overline{q}_L \gamma^\mu c_L + 2\zeta e^{i\delta} \overline{c}_R b_L \overline{q}_L c_R \right] + \text{H.c.},
\] (21)

where

\[
\zeta e^{i\delta} = \frac{1}{2 V_{cb}} \left( \frac{m_c \tan \beta}{m_H} \right)^2 \xi^*,
\] (22)

with \( \zeta \) taken to be real and positive. The charged Higgs contribution, though suppressed by small Yukawa couplings, is enhanced by the CKM factor \( V_{tb}/V_{cb} \simeq 25 \) relative to the \( W \)-exchange amplitude. As a result, \( \zeta \) as large as 0.2 is allowed by data [2]. This could significantly modify the SM predictions for the \( CP \) asymmetries in a variety of \( B \) decay channels.

The evaluation of the Higgs amplitude can be greatly simplified by assuming factorization, which in this instance may be qualitatively reasonable due to the presence of a heavy quark in the initial and in the final state. The total amplitude can then be written as
\[ \mathcal{A} \equiv \mathcal{A}(B^0 \rightarrow f) \simeq \mathcal{A}_{SM} \left[ 1 - \zeta e^{-i\delta} \right]. \]  

(23) 

The ratio of the $B$ and $\bar{B}$ decay amplitudes is 

\[ \frac{\mathcal{A}}{\overline{\mathcal{A}}} = \frac{\mathcal{A}_{SM}}{\overline{\mathcal{A}}_{SM}} \exp(-2i\theta), \]  

(24) 

with the new phase angle given by 

\[ \tan \theta = \frac{\zeta \sin \delta}{1 - \zeta \cos \delta}. \]  

(25) 

The allowed range for $\theta$ could be of order ten degrees for $\xi$ of order one [4]. 

Therefore, in the T2HDM, the $CP$ asymmetries for neutral $B$ decays to $CP$ eigenstates can be expressed in the general form, 

\[ a(t) = f_{CP} \sin 2(\varphi_{SM} + \varphi_{D} + \theta) \sin(\Delta M t), \]  

(26) 

where direct $CP$-violating effect has been neglected. The task for $CP$ asymmetry study in the T2HDM is thus reduced to the evaluation of $\theta$. Using Eq. (26), we now examine the $CP$ asymmetry pattern in $B_d$ and $B_s$ decays. 

\section*{B. $b \rightarrow c\bar{c}s$} 

In the SM, the quark transition $b \rightarrow c\bar{c}s$ proceeds mainly through tree-level $W$ exchange. Even including the penguin contribution, the amplitude for $b \rightarrow c\bar{c}s$ is proportional to a single CKM phase $\arg(V_{cb}V_{cs}^*)$ to a very good approximation. Direct $CP$ violation is therefore absent in this process and $|\mathcal{A}| = |\mathcal{A}|$ in the SM. The inclusion of charged Higgs does not alter this equality on account of Eq.(24). Furthermore, being suppressed by $1/\tan^4 \beta$, the charged Higgs effect on $B\bar{B}$ mixing is negligible if $|\xi| \leq 1$. Eq.(26) is therefore applicable to the $CP$ study of this quark transition in both $B_d$ and $B_s$ decays.

\begin{itemize} 
  \item[(i)] $B_d$ decays: $B_d \rightarrow \psi K_S$ 
  \end{itemize} 

In the SM, one of the most studied and cleanest observables is the $CP$ asymmetry in $B_d \rightarrow \psi K_S$. As the final state is $CP$-odd ($f_{CP} = -1$), one has from Eq.(26)
\[
a(t)_{\text{SM}} = -\sin 2\beta_{\text{CKM}} \sin(\Delta M t),
\]  

(27)

where \(\beta_{\text{CKM}} \equiv \arg (-V_{cd}^* V_{cb}/V_{td}^* V_{tb})\). The current SM fit gives \(\sin 2\beta_{\text{CKM}} = 0.75 \pm 0.10\) \cite{11}. Recently, the CDF collaboration has reported a preliminary measurement of the \(CP\) asymmetry in \(B_d \to \psi K_S\): \(a_{\psi K_S} = 0.79^{+0.41}_{-0.44}\) \cite{16}. Within the SM, this simply implies \(\sin 2\beta_{\text{CKM}} = 0.79^{+0.44}_{-0.44}\). Though not precise, this measurement can already be used to put constraints on some models of new physics \cite{20}.

In the presence of charged Higgs interactions, the \(CP\) asymmetry no longer measures the CKM angle \(\beta_{\text{CKM}}\). Instead, it is given by

\[
a(t) = -\sin 2(\beta_{\text{CKM}} + \theta) \sin(\Delta M t),
\]

(28)

and the coefficient \(\sin 2(\beta_{\text{CKM}} + \theta)\) could take values quite different from the SM prediction \cite{2}. This can lead to a distinct signature for new physics. As CDF and other experiments improve the measurement of \(a_{\psi K_S}\), they would put important constraints on the parameters of the T2HDM. Indeed, the current CDF measurement \cite{16} disfavors a large, negative \(\gamma\) for the T2HDM, as can be seen from Fig. 2.

\(\text{(ii) } B_s \text{ decays: } B_s \to D_s^+ D_s^- \text{ and } B_s \to \psi \eta/\eta'\)

One of the notable features of the \(B_s\) system is that the \(\Delta M\) mixing rate is much larger than that of its \(B_d\) counterpart. In the SM, this is mostly due to the \(|V_{ts}/V_{td}|^2\) enhancement factor. Present experimental data give a lower limit of \cite{18}

\[
\Delta M_s > 9.1 \, \text{ps}^{-1} \quad (x_s > 14).
\]

(29)

Time-integrated \(CP\) asymmetry will be strongly suppressed by a factor of \(x_s/(1 + x_s^2)\) and will be extremely difficult to measure in the near future. Time-dependent \(CP\) asymmetry will, on the other hand, require excellent time resolution of the detector, and is also likely to be very difficult. For definiteness, we will consider time-dependent asymmetry only.

Among the exclusive channels available to \(B_s\) (\(B_s\)) decays through \(b \to c \bar{c} s\) (\(b \to c \bar{c} s\)), \(B_s \to \psi \phi\) has a relatively large branching ratio. Its final state, however, is not a \(CP\)
eigenstate and $CP$ asymmetry analysis requires information about the angular distribution of its final state. On the other hand, $B_s \to \psi \eta'/\eta'$ and $B_s \to D_s^+ D_s^-$ decays are quite simple to analyze. As for $B_d \to \psi K_S$, Eq. (26) is applicable in these cases.

It is easily seen that in the Wolfenstein parameterization $\phi_D \simeq \phi_M \simeq 0$ in the SM. Therefore, the SM does not give rise to any $CP$ asymmetry,

$$a(t)_{SM} \simeq 0.$$ (30)

This singles out $B_s$ decays as unique probes for new sources of $CP$ violation.

As noted before, in the T2HDM with $|\xi| \sim 1$, charged Higgs contribution to $B_s \bar{B}_s$ mixing is negligible compared to the SM contribution. The decay phase is simply given by $\theta$. Since $f_{CP} = 1$ for both $B_s \to D_s^+ D_s^-$ and $B_s \to \psi \eta'/\eta'$, one has from Eq.(26),

$$a(t) = \sin(2\theta) \sin(\Delta M_s t).$$ (31)

The size of the $CP$ asymmetry depends on the CKM phase $\gamma$ and the Higgs phase $\delta$, and could be of order tens of percents, as shown in Figs. 2.

C. $b \to c\bar{c}d$

Compared to $b \to c\bar{s}s$, the $b \to c\bar{c}d$ transition is not as clean. This is due to the fact that the penguin contribution, though loop-suppressed, suffers no CKM suppression relative to the tree amplitude. As a first approximation, we will neglect the pure penguin amplitude and take $|\mathcal{A}| = |\mathcal{A}|$. As for $b \to c\bar{s}s$ transition, this relation remains valid in the presence of charged Higgs exchange in the factorization approach.

(i) $B_d$ decays: $B_d \to D^+ D^-$

The final state of the decay $B_d \to D^+ D^-$ consists of both $I = 0$ and $I = 1$ amplitudes. However, to the extent that the pure penguin contribution can be neglected, the decay amplitude has a single weak phase $\arg(V_{cb}V_{cd}^*)$, and we can take $|\mathcal{A}| = |\mathcal{A}|$ and use Eq. (26) to obtain the $CP$ asymmetry,
$$a(t)_{\text{SM}} = \sin 2\beta_{\text{CKM}} \sin(\Delta M t) ,$$

(32)

where we have used $f_{CP} = 1$.

The inclusion of charged Higgs interactions introduces a new weak phase in the decay amplitude, and one generally expects direct $CP$ violation to occur from interference between the two isospin amplitudes. However, if the direct $CP$-violating effect is small, the $CP$ asymmetry then takes the simple form,

$$a(t) = \sin 2(\beta_{\text{CKM}} + \theta) \sin(\Delta M t) ,$$

(33)

similar to the decay $B_d \to \psi K_S$. The amplitude of the asymmetry $\sin 2(\beta_{\text{CKM}} + \theta)$ can take a wide range of values, distinctive from the SM expectation (see Fig. 4).

(ii) $B_s$ decays: $B_s \to \psi K_S$

Compared to the $B_d \to D^+ D^-$ decay, $B_s \to \psi K_S$ is a cleaner mode for $CP$ study for the following reason. The final state is a pure $I = 1/2$ state with orbital angular momentum $l = 1$, and the penguin contribution is expected to be small due to both loop and color suppressions. As a result, direct $CP$ violation is suppressed in the SM. As for $B_s \to \psi \eta/\eta'$, the SM prediction for the $CP$ asymmetry in the interference between decays with and without mixing is approximately zero,

$$a(t)_{\text{SM}} \simeq 0 .$$

(34)

In the T2HDM with $|\xi| \sim 1$, charged Higgs affects only the decay but not the mixing amplitude. Similar to the case $B_s \to \psi \eta/\eta'$, the $CP$ asymmetry is simply given by

$$a(t) = -\sin 2\theta \sin(\Delta M_s t) ,$$

(35)

where we have used $f_{CP} = -1$. The amplitude $\sin 2\theta$ could be of order tens of percent, in sharp contrast to the SM prediction.
FIG. 2. Allowed regions (shaded) of the CP asymmetries \( \sin 2\theta \) and \( \sin 2(\beta_{\text{CKM}} + \theta) \) in the T2HDM (taking \( \xi = e^{-i\delta} \)) for three representative choices of \( \gamma \): (a) \( \gamma = 68^\circ \) is a best fit value \(^{[11]}\) of the SM, (b) \( \gamma = 0^\circ \) corresponds to a real CKM matrix, and (c) \( \gamma = -45^\circ \). The top horizontal line in (a) is for the SM assuming a best fit \( \sin 2\beta_{\text{CKM}} = 0.75 \) \(^{[11]}\). The most stringent constraints from \( \Delta m_K, \epsilon_K \), and \( b \rightarrow s\gamma \) have been imposed. Note that if the CDF central value for \( a_{\psi K_S} \) \(^{[16]}\) persists with improved measurements, the scenario with a large and negative \( \gamma \) (scenario (c)) will be ruled out.

D. Other \( B \)-decays

In this subsection, we discuss neutral \( B \) decays involving the \( u \) quark Yukawa couplings \( H^+ \bar{u}_R q_L \ (q = d, s, b) \). These couplings depend on \( \xi' \), the mixing parameter between the first and the third families in \( U_R \). The size of \( \xi' \) is constrained by the \( B \rightarrow \tau \nu \) rate as given by Eq. \(^{[11]}\). There are six charged-current four-quark operators for \( b \) decays involving the \( u \) quark, \( b \rightarrow c \bar{u}d, b \rightarrow u \bar{c}d, b \rightarrow c \bar{u}s, b \rightarrow u \bar{c}s, b \rightarrow u \bar{u}d, b \rightarrow u \bar{u}s \). In the T2HDM, only \( b \rightarrow u \bar{c}s, b \rightarrow u \bar{c}d, \) and \( b \rightarrow u \bar{u}s \) could receive sizable charged-Higgs contributions compared to their corresponding SM amplitudes. The phenomenological implications can be summarized as follows:

- \( b \rightarrow u \bar{u}d \) and \( b \rightarrow d \bar{d}d \): The charged Higgs effect is small after imposing the \( \xi' \) constraint. The decay \( B_d \rightarrow \pi \pi \) remains as in the SM, and one may use isospin
analysis [21] to extract the UT angle $\alpha$.

• $b \to u\bar{u}s$ and $b \to d\bar{d}s$: Here, the SM amplitudes are dominated by penguin contributions [22]. In the T2HDM, even though tree-level Higgs-change effect is negligible, charged-Higgs-mediated penguin contributions could become appreciable. Consequently, the procedure to extract the angle $\gamma$ from the $B^\pm \to \pi K, \pi\pi$ decays [23] may be modified.

• $b \to c\bar{u}d$ and $b \to u\bar{c}d$: Both decays are free from penguin contributions. In the SM, the latter amplitude is Cabibbo-suppressed by $\mathcal{O}(\lambda^2)$ with respect to the former, and $CP$ asymmetry measurements in the hadronic decays $B_d \to D_{CP}\pi$ and $B_d \to D_{CP}\rho$ provide a clean way to extract the angle $\beta_{CKM}$. On the other hand, the decay $B_s \to D_{CP}K_S$ is expected in the SM to have an approximately zero $CP$ asymmetry. Charged-Higgs exchange in the T2HDM has a negligible effect on $b \to c\bar{u}d$, whereas its contribution to the Cabibbo-suppressed decay $b \to u\bar{c}d$ can be sizable but still smaller than the SM one. As a result, the $CP$ asymmetries remain the same as the SM prediction.

• $b \to u\bar{c}s$ and $b \to c\bar{u}s$: These transitions can mediate the charged $B^\pm \to K^\pm D/\bar{D}$ decays, which may provide a clean way within the SM to extract the angle $\gamma$ [7]. Charged-Higgs-exchange contribution in the T2HDM will be discussed in detail in the next section on direct $CP$ violation.

Some main results of our analysis concerning $B_d$ and $B_s$ decays are summarized in Table 2.
Table 2: $CP$ asymmetries for neutral $B$ decays to $CP$ eigenstates in the SM and in the T2HDM. For $B_s \to \rho K_S$, the pure-penguin contribution may be important \cite{22}. As a result, the tree- and pure-penguin-amplitudes of different weak (and possibly strong) phases are competitive, and the $CP$ asymmetry may not be simply related to the CKM phase. The overall sign of the asymmetry in $B_d \to D_{CP}\rho$ depends on the $CP$ properties of $D_{CP}$.

| quark transitions | $B_d$ decays | $B_s$ decays |
|-------------------|--------------|--------------|
|                   | final states | final states | SM | T2HDM | SM | T2HDM |
| $b \to c\bar{c}s$ | $\psi K_S$  | $\psi K_S$  | $- \sin 2\beta_{CKM}$ | $- \sin 2(\beta_{CKM} + \theta)$ | $D_s^+D_s^-$, $\psi\eta/\eta'$ | 0 | $\sin 2\theta$ |
| $b \to c\bar{c}d$ | $D^+D^-$    | $\sin 2\beta_{CKM}$ | $\sin 2(\beta_{CKM} + \theta)$ | $\psi K_S$ | 0 | $- \sin 2\theta$ |
| $b \to c\bar{u}d$ | $D_{CP}\rho$ | $\pm \sin 2\beta_{CKM}$ | $\pm \sin 2\beta_{CKM}$ | $D_{CP}K_S$ | 0 | 0 |
| $b \to u\bar{u}d$ | $\pi\pi$    | $- \sin 2\alpha$ | $- \sin 2\alpha$ | $\rho K_S$ | competing | competing |

V. DIRECT CP VIOLATION IN B DECAYS

A. Direct CP violation in $B^\pm \to \psi/JK^\pm$

In this subsection, we address the use of charged $B$ decays to search for direct CP violation. This requires a difference in the strong phases associated with the $W$- and $H$-mediated decay amplitudes leading to the exclusive final state(s) of interest. As an illustration, we will focus our attention on the decay $B^\pm \to \psi/JK^\pm$. There are several features about this mode which make it interesting to study both experimentally and theoretically. The $B^\pm \to \psi/JK^\pm$ decays proceed through the quark decay $b \to c\bar{c}s$ and its conjugate process. In the SM, the weak phase associated with the decay amplitude is vanishingly small; therefore, the SM predicts a zero rate asymmetry. Second, it is clean to measure experimentally. Third, it has a large branching ratio of $\sim 10^{-3}$ \cite{18}. These features make the experimental measurement very worthwhile in the search for new sources of $CP$ violation. However, as we can not reliably compute the hadronic matrix elements of either the SM
current-current four-Fermi operator or the charged-Higgs-induced scalar operator, the relative strong phase between the $W$- and $H$-mediated amplitudes remains largely unknown. Therefore, no reliable predictions can be made about the size of the asymmetry.

Formally, the amplitude for $B^+ \to \psi/JK^+$ can be written as

$$\mathcal{A} = \mathcal{A}_{SM}[1 - \zeta e^{-i\delta} e^{-i\phi_s}].$$

(36)

where $\delta$ and $\phi_s$ are the weak and strong phase difference respectively. It then follows that the amplitude for $B^- \to \psi/JK^-$ is given as

$$\bar{\mathcal{A}} = \mathcal{A}_{SM}[1 - \zeta e^{i\delta} e^{-i\phi_s}].$$

(37)

The CP-violating partial rate asymmetry (PRA) can be expressed as

$$a_{CP} = \frac{\Gamma(B^+ \to K^+\psi) - \Gamma(B^- \to K^-\psi)}{\Gamma(B^+ \to K^+\psi) + \Gamma(B^- \to K^-\psi)} = \frac{2\zeta \sin \delta \sin \phi_s}{1 + \zeta^2 - 2\zeta \cos \delta \cos \phi_s}.$$ 

(38)

As $a_{CP}$ is directly proportional to $\zeta$ when $\zeta$ is small, it is crucial to determine how large $\zeta$ can be after imposing the experimental constraints. From section III, we know that the most stringent constraints on $\zeta$ come from the $b \to s\gamma$ rate, the $K_L-K_S$ mass difference $\Delta m_K$, and $\epsilon_K$. For $|\xi| = 1$, the combined constraints from $\Delta m_K$ (see Eq.(39)) and $b \to s\gamma$ (see Fig. 1 of Ref. [2]) imply an upper bound of

$$\zeta \leq 0.2.$$ 

(39)

As can be seen from Fig. 1, this limit is saturated for $\delta \leq 45^\circ$, and becomes more stringent as the phase $\delta$ increases all the way to $180^\circ$. Imposing the $\epsilon_K$ constraint may generally make $\zeta$ even smaller, depending on the CKM phase and $\delta$ (see Fig. 1 of Ref. [2] for an illustration). Suffice it to say that after taking all the data into account, $a_{CP}$ could be of order 10\% in the T2HDM unless the strong phase $\phi_s$ is much suppressed, as is illustrated in Fig. 3.
FIG. 3. Direct CP asymmetry with two representative values for the strong phase: $\phi_s = 90^\circ$ (solid lines) and $\phi_s = 30^\circ$ (broken lines). (a) is the contour plot for $a_{CP}/\sin\phi_s$ with contour levels 0.1, 0.2, and 0.3 respectively, (b) is the maximum value $a_{CP}/\sin\phi_s$ takes as a function of $\zeta$.

Recall that in the SM, the PRA has to come from the interference between the tree diagram and a loop diagram with internal $u$-quark which gives the required weak phase difference, $\text{Arg}[V_{ub}V_{us}^*/V_{cb}V_{cs}^*]$. Consequently, the rate asymmetry in the SM is both loop- and Cabibbo-suppressed and is expected to be vanishingly small. In the T2HDM, on the other hand, the CP asymmetry arises from the interference between two tree diagrams, and may only be suppressed by the relative strength of the two amplitudes, $\zeta$. As a result, rate asymmetry at the 10% level can be naturally expected with $\zeta \sim 0.1$, at least in those exclusive channels where the strong phase difference is not too suppressed.

We remark, in passing, that the CPT theorem does not impose any particular restriction on a specific channel (such as $\psi/JK$), materializing from $c\bar{c}s$ as there are many other channels originating from the same quark transition. Indeed, in the next subsection, we will discuss a similar case where large direct CP asymmetry is expected, based purely on the SM, in exclusive channels emerging from quark-level interference between two tree graphs.

To estimate the number of $B$’s needed in order to measure a 10% asymmetry at the $2\sigma$ level, we assume the detector efficiency to be 25%. Then for the needed number of $B$’s we find $\frac{2^2}{10^{-3.6}\times0.12\times0.12\times0.25} \simeq 1.3 \times 10^7$. In deducing this number, we have used $\text{BR}(\psi/J \rightarrow e^+e^-, \mu^+\mu^-) = 12\%$. This estimate suggests that such an analysis may be doable at various
facilities. Indeed, the existing data sample ($\sim 10^7$ $B$'s) at CLEO can already be used for this important search \cite{24}.

It is worth pointing out that the possibility of sizable direct CP violating effect in $B^\pm \to \psi/JK^\pm$ in the T2HDM does not arise in 2HDM's with NFC (Models I and II) where the $W$ and $H$ exchange amplitudes have the same weak phase. Note also that large CP asymmetries may also be expected in charged $B$ decays through $b \to c\bar{c}d$ and its conjugate process, for example, in $B^\pm \to \psi/J\pi^\pm$ (the SM expectation for the $CP$ asymmetry in this mode is small). However, the branching ratio for this decay is Cabibbo-suppressed by a factor of 20 compared to that of $B^\pm \to \psi/JK^\pm$.

Before leaving this subsection, we want to emphasize that there are many exclusive channels available through the $\bar{b} \to c\bar{c}s$ transition, e.g. $\psi K^+, \psi' K^+, \psi K^+\pi\pi, \bar{D}D_s^+, D\bar{D}K^+, \cdots$, that may offer ample opportunities for sizable strong phase difference ($\phi_s$) for certain modes. It is thus important that experimental searches for direct $CP$ violation in charged $B$ decays include as many of these modes as possible and not be restricted to the $B^\pm \to \psi/JK^\pm$ channel only.

**B. CP violation in $B^\pm \to D/\bar{D}K^\pm$ and the extraction of $\gamma$**

The decays $B^+ \to K^+D$ and $B^+ \to K^+\bar{D}$ proceed respectively through the transitions $\bar{b} \to \bar{u}c\bar{s}$ and $\bar{b} \to \bar{c}u\bar{s}$. Although the decay amplitude for $B^+ \to K^+D$ is both color- and Cabibbo-suppressed relative to $B^+ \to K^+\bar{D}$, the amplitudes for decays to certain common final states via $B^+ \to K^+D[\to f]$ and $B^+ \to K^+\bar{D}[\to f]$ can be expected to be comparable. This is due to the fact that $D$ decays into $f$ through the Cabibbo-allowed channel $c \to s\bar{d}u$ and $\bar{D}$ decays through the doubly-Cabibbo-suppressed channel $\bar{c} \to \bar{s}\bar{d}u$ into the same final state $f$. Examples of such common final states of the $D/\bar{D}$ decay include $f = K^-\pi^+, K^-\pi^+\pi^0, K^-\rho^+, K^-a_1^+, K'^-\pi^+$, etc. Atwood, Dunietz, and Soni (ADS) \cite{7} observed that within the SM the large interference effect between the two decay chains may give rise to $O(1)$ partial rate asymmetries (PRA) between the $B^+$ and $B^-$ decays,
\[ a_{Kf} \equiv \frac{\Gamma(B^+ \to K^+[f]D) - \Gamma(B^- \to K^-[\bar{f}]D)}{\Gamma(B^+ \to K^+[f]D) + \Gamma(B^- \to K^-[\bar{f}]D)} . \]  

Furthermore, Ref. [7] suggests that the angle \( \gamma \) of the unitarity triangle can be extracted from a study involving a minimum of two different final states \( f \). In this section, we examine the charged Higgs effect on the direct CP asymmetry and on the extraction of \( \gamma \) using the \( B^\pm \to D/\bar{D}K^\pm \) decays.

In the T2HDM, due to the small Yukawa couplings, \( H^+ u(1 \pm \gamma_5)q \ (q = d, s) \), the charged Higgs amplitudes for \( c \to s\bar{d}u \) and \( \bar{c} \to \bar{s}\bar{d}u \) are negligible compared to the corresponding SM amplitudes. The hadronic decays \( D(\bar{D}) \to f \) thus remain as in the SM. Because of the same suppression, the Higgs contribution to \( B^+ \to K^+\bar{D} \) is at most a few percent of the SM amplitude and can also be safely neglected. On the other hand, the charged Higgs amplitude for the color- and Cabibbo-suppressed decay \( B^+ \to K^+D \) could in principle be sizable as it involves the potentially large Yukawa’s \( H^+ u_R b_L \) and \( \bar{H}^+ e_R s_L \). However, the present experimental limit on the \( B^+ \to \tau^+\nu \) rate already places an upper bound of about 25% on the decay-amplitude ratio \( |A_H/A_W| \). For simplicity of the analysis, we will first ignore the charged-Higgs contribution to the decay \( B^+ \to K^+D \).

In the SM, \( D\bar{D} \) mixing is vanishingly small. As discussed in Sect. III D, however, the \( D\bar{D} \) mixing parameter, \( x_D \equiv \Delta m_D/\Gamma_D \), could be as large as a few percent in the T2HDM. This will in turn affect the direct CP asymmetry in \( B^+ \to K^+[f]D \) and \( B^- \to K^-[\bar{f}]D \) decays, as well as the standard procedure to extract \( \gamma \) from these decays. Note that in 2HDM’s with NFC (Models I and II), charged Higgs effect on \( D\bar{D} \) mixing and on the relevant \( B \) and \( D \) decays is small, and the ADS method [7] to extract \( \gamma \) remains unmodified. We now analyze in the T2HDM this new effect due to \( D\bar{D} \) mixing.

Recall that in the absence of \( D\bar{D} \) mixing, as in the SM, the total decay amplitude for \( B^+ \) can be written as [5]

\[ A_{ADS} = A_{B^+ \to K^+D}A_{D \to f} + A_{B^+ \to K^+\bar{D}}A_{\bar{D} \to f} . \]  

In the SM, we can parameterize \( A_{B^+ \to K^+D}/A_{B^+ \to K^+D} = r_B e^{i\gamma} e^{i\Delta_B} \), and \( A_{D \to f}/A_{D \to f} = \ldots \).
$-r_D e^{i \Delta_D}$, with $\Delta_B$ and $\Delta_D$ denoting the strong phase differences in the $B$ and $D$ decay amplitudes respectively. The amplitude ratios can be estimated up to uncertainties in the hadronic matrix elements, $r_D \sim |(V_{cs} V_{us}^{*})/(V_{cs} V_{ud})| \sim 0.05$ for $D$ decays, and $r_B \sim |(V_{ub} V_{cs})/(V_{cb} V_{us})| |a_2/a_1| \sim 0.08$, where we have used $|V_{ub}/V_{cb}| \sim 0.08$ and $|a_2/a_1| \sim 0.21$ [25] accounts for the color suppression factor. As we will include the effect due to $D\bar{D}$ mixing, we cannot use the $D$ and $\bar{D}$ decay branching ratios to extract $r_D$.

The partial rate asymmetry can be easily obtained as,

$$a_{Kf,SM} = \frac{2 r_B r_D \sin \gamma \sin(\Delta_B - \Delta_D)}{r_B^2 + r_D^2 - 2 r_B r_D \cos \gamma \cos(\Delta_B - \Delta_D)}.$$  \hspace{1cm} (42)

Note that a non-zero strong phase difference is required for the asymmetry and for the extraction of $\gamma$, and the asymmetry need not be small.

Current data set an upper limit on the $D\bar{D}$ mixing parameter $x_D < 0.1$, and this allows us to approximate the $D\bar{D}$ mixing effect by keeping terms up to the quadratic order in $x_D$. $D\bar{D}$ mixing thus leads to one additional contribution to the $B$ decay amplitude: $B^+ \rightarrow K^+ \bar{D}$, followed by the subsequent oscillation $\bar{D} \rightarrow D$ with $D$ decaying to the Cabibbo-allowed final state, e.g. $f = K^- \pi^+$. Neglecting the other $D\bar{D}$-mixing-induced decay path which involves both the color-suppressed $B$ decay and the doubly-Cabibbo-suppressed $D$ decay, this new contribution can be written as [26]

$$A_{D\bar{D}} = A_{B^+ \rightarrow K^+ D} \frac{p_D}{q_D} g_-(t) A_{D \rightarrow f}. \hspace{1cm} (43)$$

Here up to quadratic order in $x_D$, $g_-(t) \simeq \frac{i}{2} x_D \tau_D e^{-\tau_D/2}$ with $\tau_D = \Gamma_D t$, and $\frac{p_D}{q_D} = e^{-i2\theta_D}$ with the $D\bar{D}$ mixing phase $\theta_D = \text{arg}(\xi) - \text{arg}(\xi')$ in the T2HDM.

The time-integrated $B$ decay width including the $D\bar{D}$ mixing effect is then given by

$$\Gamma(B^+ \rightarrow K^+[\bar{f}]_D) \sim r_B^2 + r_D^2 - 2 r_B r_D \cos(\gamma + \Delta_B - \Delta_D)$$

$$+ x_D r_B \sin(\gamma + 2 \theta_D + \Delta_B) - x_D r_D \sin(2 \theta_D + \Delta_D) + x_D^2/2 \hspace{1cm} (44)$$

$$\Gamma(B^- \rightarrow K^-[\bar{f}]_D) \sim r_B^2 + r_D^2 - 2 r_B r_D \cos(\Delta_B - \Delta_D - \gamma)$$

$$+ x_D r_B \sin(\Delta_B - \gamma - 2 \theta_D) - x_D r_D \sin(\Delta_D - 2 \theta_D) + x_D^2/2, \hspace{1cm} (45)$$

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where the $x_D r_B$ and $x_D r_D$ terms come from the interferences between $A_{DD}$ and the color-suppressed $B$-decay amplitude and the doubly-Cabibbo-suppressed $D$-decay amplitude respectively. The $CP$ asymmetry can then be obtained as,

$$a_{Kf} = \frac{2 r_B r_D \sin \gamma \sin(\Delta_B - \Delta_D) + x_D r_B \sin(\gamma + 2\theta_D) \cos \Delta_B - x_D r_D \sin 2\theta_D \cos \Delta_D}{r_B^2 + r_D^2 - 2 r_B r_D \cos \gamma \cos(\Delta_B - \Delta_D) + \Gamma_{x_D}} \tag{46}$$

where $\Gamma_{x_D} = x_D r_B \cos(\gamma + 2\theta_D) \sin \Delta_B - x_D r_D \cos 2\theta_D \sin \Delta_D + x_D^2 / 2$.

It is interesting to note that whereas a sizable strong phase difference is essential for a large rate asymmetry in the SM, this is no longer necessary with a non-zero $x_D$. While in general there is no reason for both strong phases ($\Delta_B$ and $\Delta_D$) to be small, for illustration, let us consider the $CP$ asymmetry in the limit that they are, i.e., for simplicity, we set $\Delta_B = \Delta_D = 0$, which may be a good approximation for certain decay channels. Then

$$a_{Kf} \rightarrow \frac{x_D r_B \sin(\gamma + 2\theta_D) - x_D r_D \sin 2\theta_D}{r_B^2 + r_D^2 - 2 r_B r_D \cos \gamma + x_D^2 / 2} \tag{47}$$

This is the asymmetry due to $D\bar{D}$ mixing. For $x_D$ of a few percent, $a_{Kf}$ could be of order tens of percent, comparable to the expected asymmetry within the SM. In other words, the presence of $D\bar{D}$ mixing could affect the ADS method to extract $\gamma$ in a significant way. This $D\bar{D}$ mixing-induced asymmetry $a_{Kf}$ is shown in Fig. 4.

Given the many decay channels available for both $D$ and $\bar{D}$, it is highly unlikely that $\Delta_D$ should be small for all the modes. For illustrative purpose, we also consider the case of $\Delta_B = 0^\circ$ and $\Delta_D = 30^\circ$, and evaluate the effect of $D\bar{D}$ mixing on the PRA, $a_{Kf}$. The numerical results are presented in Fig. 5. One again observes a sizeable effect due to $D\bar{D}$ mixing.

Figs. 4 and 5 illustrate the effect of $D\bar{D}$ mixing on two classes of decays with respectively small and large strong phase differences. For $x_D$ of order a few percent, one arrives at a no-lose theorem that large $CP$ asymmetries are expected for both cases. However, it seems that the angle $\gamma$ can not be extracted in a simple manner from such decays in the presence of a sizeable $D\bar{D}$ mixing.
FIG. 4. \( \bar{D}D \)-mixing-induced direct \( CP \) asymmetry \( a_{Kf} \) as given by Eq. (47), assuming the strong phase differences \( \Delta_B \) and \( \Delta_D \) to be zero. We have taken \( r_B = 0.08 \), \( r_D = 0.05 \), and \( \gamma = 60^\circ \) for the numerical analysis. Shown in (a) is the contour plot for \( a_{Kf} \) with contour levels 0.1, 0.3, 0.5 (solid lines) and -0.1, -0.3, -0.5 (dashed lines). The solid and dashed lines in (b) denote respectively the maximum and the minimum values which \( a_{Kf} \) can take as a function of \( x_D \).

FIG. 5. The effect of \( D\bar{D} \) mixing on the PRA \( a_{Kf} \) for \( \Delta_B = 0^\circ \) and \( \Delta_D = 30^\circ \). We have used \( r_B = 0.08 \), \( r_D = 0.05 \), and \( \gamma = 60^\circ \) for the numerical analysis. Shown in (a) is the contour plot for \( a_{Kf} \) with contour levels \(-0.8\), \(-0.5\), \(-0.2\), and 0.3. The two solid lines in (b) represent respectively the maximum and the minimum values which \( a_{Kf} \) can take as a function of \( x_D \). The broken line in (b) shows the value of \( a_{Kf} \) in the absence of \( D\bar{D} \) mixing as expected in the SM.
VI. CONCLUSION

As has been shown in this work, the phenomenology of the T2HDM differs from that of Models I and II 2HDM (and the SM) in many interesting ways. These include their different effects on $K\bar{K}$, $D\bar{D}$, and $B\bar{B}$ mixings, on charged-current decays, and on $CP$-violation in both neutral and charged $B$ decays. We have examined the $CP$-violating phenomenology associated with the anomalous charged-Higgs Yukawa couplings in the T2HDM. The effects of the two new $CP$-violation parameters of the model, $\xi$ and $\xi'$, nicely separate, with the latter mainly affecting processes that involve $D\bar{D}$ mixing. As a result, some clean predictions for neutral $B$ decays can be made within the T2HDM as presented in Table 2. The $\xi$-related $CP$ angle $\theta$ can be directly measured from the $B_s \to D_s^+D_s^-$, $\psi\eta'/\eta'$, or $\psi K_S$ decay without SM pollution. On the other hand, the $\alpha$ and $\beta_{CKM}$ angles of the unitarity triangle may be extracted from $CP$ asymmetry measurements in the decays $B_d \to \pi\pi$ and $B_d \to D_{CP}\rho/\pi$ respectively without charged-Higgs contamination. Based on the information on $\theta$ and $\beta_{CKM}$, a cross check on the model can then be made by taking into account the $CP$ asymmetry measurement in the decays $B_d \to \psi K_S$ and/or $B_d \to D^+D^-$. Besides $CP$ asymmetries in neutral $B$ decays, the model also gives rise to generically large $CP$-violating effects in charged $B$ decays. For example, the partial rate asymmetry in $B^\pm \to K^\pm\psi$ can be of order ten percent if the strong phase difference is not too much suppressed. The model can also lead to a sizable $D\bar{D}$ mixing effect, depending on the mixing in the right-handed up quark sector. This in turn could strongly modify the direct $CP$ asymmetry in $B^\pm \to D/\bar{D}K^\pm$, and thus the procedure to extract the CKM angle $\gamma$ from this process. On the other hand, as in most charged-Higgs models of $CP$ violation, the direct $CP$-asymmetry in the radiative decay $b \to s\gamma$ is found to be tiny [27].

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Note Added in Proof:

Since this manuscript was sent for publication, the CLEO Collaboration has published results on their first search for direct $CP$ violation in $B^\pm \rightarrow K^\pm \psi/J$ [See G. Bonvicini et al., Phys. Rev. Lett. 84, 5940 (2000).].
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