Generation of Axion-Like Couplings via Quantum Corrections in a Lorentz Violating Background

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Abstract

Light pseudoscalars, or axion like particles (ALPs), are much studied due to their potential relevance to the fields of particle physics, astrophysics and cosmology. The most relevant coupling of ALPs from the viewpoint of current experimental searches is to the photon: in this work, we study the generation of this coupling as an effect of quantum corrections, originated from an underlying Lorentz violating background. Most interestingly, we show that the interaction so generated turns out to be Lorentz invariant, thus mimicking the standard ALPs coupling to the photon that is considered in the experiments. This consideration implies that violations of spacetime symmetries, much studied as possible consequences of physics in very high energy scales, might infiltrate in other realms of physics in unsuspected ways. Additionally, we conjecture that a similar mechanism can also generate Lorentz invariant couplings involving scalar particles and photons, playing a possible role in the phenomenology of Higgs bosons.

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I. INTRODUCTION

Theoretical motivations for the introduction of light pseudoscalars in the Standard Model ranges from solving specific technical issues in the theory of strong interactions to more general questions such as the search for dark matter candidates (for a theoretical introduction and some experimental developments, see for example [1, 2]). The search for these light pseudoscalars has been an active field of experimental work, usually by looking for the interaction between these particles and the photon. Our main purpose in this work is to show that this particular interaction can be generated from an underlying Lorentz violating background. More precisely, we will assume the existence of a massive fermionic field with Lorentz violating (LV) interactions of a specific form, and show that the resulting low energy effective Lagrangian contains Lorentz invariant (LI) interactions whose intensity are functions of LV parameters. We will argue that these LI terms are the dominant ones in the low energy phenomenology of pseudoscalars interacting with photons, thus proposing a scenario where the most relevant effects of the LV are actually “standard” LI interactions. In this way the experiments for searching light pseudoscalars would also be able to probe the specific setup of LV that we consider – conversely, our result suggests that, contrary to common belief, an underlying LV could be relevant to the study of typically LI phenomena. This finding is relevant in pointing out that possible violations of spacetime symmetries could permeate other areas of physics, with possible consequences in contexts where such violations are not typically considered relevant.

The axion is a well studied example of light pseudoscalar appearing as a pseudo-Nambu-Goldstone boson from the breaking of an anomalous global chiral $U(1)$ symmetry, the Peccei-Quinn (PQ) symmetry, and leading to a dynamical solution for the strong CP problem in QCD [3–5]. Experiments for searching the axion [6], both direct and via astrophysical observations, are being performed since the 1980’s, and a class of “invisible axion”-type models [7–10] are still phenomenologically viable. More concretely, consider a pseudoscalar field $\phi$ coupled to a charged fermion field $\psi$ according to the term $i\phi \bar{\psi} \gamma_5 \psi$. Assuming that the fermionic field is heavy it can be integrated out, and the low energy effective Lagrangian describing the pseudoscalar particles interacting with photons turns out to be

$$L_{\text{eff}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \frac{m_{\phi}^2}{2} \phi^2 - \frac{g_{\phi\gamma}}{4} \phi F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad (1)$$

where $g_{\phi\gamma}$ is a coupling constant with inverse of mass dimension, $m_{\phi}$ is the axion mass, $F_{\mu\nu}$ is the electromagnetic field-strength, and $\tilde{F}^{\mu\nu} = \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$ its dual. In axion models there is a specific
relation between $m_\phi$ and $g_{\phi\gamma}$, both related to the axion decay constant $f_a$ which is assumedly much higher than the electroweak scale. Indeed \[ \text{[11]} \]

$$m_\phi = \frac{z^{1/2}}{1 + z} \frac{f_\pi m_\pi}{f_a} ; \quad g_{\phi\gamma} = \frac{\alpha}{2 \pi f_a} \left( r - \frac{24 + z}{31 + z} \right),$$

(2)

where $m_\pi$ and $f_\pi$ are the pion mass and decay constant, $z$ is the ratio of the masses of up and down quarks, $\alpha$ is the QCD coupling constant, and $r$ is the ratio of the electromagnetic and color anomalies of the axial current associated with PQ symmetry. Despite the ratio $r$ being model dependent to some degree, this relation is a typical signature of the light pseudoscalar related to the PQ mechanism, which is conventionally denoted the axion.

More generically, particle excitations of very light pseudoscalar fields having interactions like the one in Eq. (1) are denoted as axion-like-particles (ALPs), and they are not necessarily connected to the properties of the QCD vacuum. For example, compactifications in string theory have been shown to produce light pseudoscalars \[ \text{[12]} \] (see also \[ \text{[13]} \] and references therein).

The last term in Eq. (1) is the lowest dimensional operator for the pseudoscalar field interaction with the electromagnetic field. Therefore, the interaction

$$\mathcal{L}_{\text{int}} = -\frac{g_{\phi\gamma}}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} = g_{\phi\gamma} \phi \vec{E} \cdot \vec{B},$$

(3)

is the most relevant one in the low energy regime. It leads to the remarkable phenomenon of ALP-photon oscillation: even with feeble couplings with the usual matter, an hypothetical ALP of very low mass can oscillate into a detectable photon when passing through a magnetic field \[ \text{[6]} \]. Combining the up to date experimental results and the constraints from energy loss in stars \[ \text{[14]} \], the interaction in Eq. (3) is now excluded in an impressive range of values $g_{\phi\gamma} \gtrsim 10^{-10} \text{GeV}^{-1}$ for a wide range of mass $m_\phi$ in the sub-eV scale \[ \text{[11]} \]. Also, new proposals for searching ALPs whose couplings with photons could be directly tested up to $g_{\phi\gamma} \sim 10^{-11} - 10^{-12} \text{GeV}^{-1}$ are underway \[ \text{[15, 16]} \].

For all of this the interaction in Eq. (3) is presently one of the most important proposals of new physics relevant to the low energy regime, originated from some theory associated to a high mass scale parameter $M$, such that $g_{\phi\gamma} \propto 1/M$. The models from which Eq. (3) is derived are generically Lorentz invariant (LI). In this work, we follow this perspective, but relaxing the condition of Lorentz invariance: we consider some Lorentz violating new physics at a very high energy scale, and look for effects which are relevant to a very low energy phenomenology. We will show a specific setup of LV that can generate the interaction in Eq. (3) as its dominant effect. The LV is assumed here to occur due to the existence of constant vectors $b^\mu$, $d^\mu$ in the interaction of the pseudoscalar and electromagnetic fields with the heavy fermion, which after being integrated out will lead to $g_{\phi\gamma} \propto b \cdot d$. 

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Concerning theories which include LV, a framework for the incorporation of a possible LV within our current understanding of the elementary interactions, called Standard Model Extension (SME), has been used to provide an environment for testing the validity of relativistic symmetry in different phenomena \[17–19\]. In essence, one incorporates in the Standard Model Lagrangian all possible covariant, renormalizable and gauge invariant terms that involve constant background tensors. These are assumed to appear from a more fundamental theory at very high energy scales – via a spontaneous symmetry breaking that gives a non vanishing vev for some tensorial field, for instance \[20\]. One example is the so-called Carrol-Field-Jackiw (CFJ) term \( k_\mu \epsilon^{\mu \nu \rho \sigma} A_\nu \partial_\rho A_\sigma \) \[21\], which leads to modified propagation of light waves in vacuum and – since these effects are not seen in our current experiments – to strong bounds on the value of the Lorentz violating vector \( k_\mu \). The CFJ term can be induced by quantum corrections, starting from the Lorentz breaking coupling \( b_\mu \bar{\psi} \gamma_5 \gamma^\mu \psi \) and integrating out the fermionic degrees of freedom, thus providing a connection between the \( b_\mu \) and the vector \( k_\mu \); it allows the translation of constraints from the photon sector to other sectors of the SME, strongly constraining these models. This mechanism have been intensively studied in recent years (see \[22\] and references therein, and also \[23–25\]), in part due to the appearance of regularization dependent integrals in the calculations, an aspect that will also appear in our model. However, it must be pointed out that while the CFJ term is quadratic, directly modifying the photon propagation in vacuum, in our context the LV affects only the interactions of the photon with the ALP field.

The SME have been extended to include gravitational interactions \[26\] and, more recently, its extension to higher dimensional operators is being worked out \[27, 28\]. Our model is also in some sense an extension of the SME, since its starting point is a model containing LV interactions involving Standard Model fields and two new ingredients: the light pseudoscalar, whose low energy phenomenology we shall be interested in unveiling, and a massive fermion whose integration in the effective action will provide the translation of the very high energy LV to a low energy LI effect.

This work is organized as follows. The model we shall be dealing with is presented in Section II involving a new massive fermion and a pseudoscalar fields coupled via LV interactions to the gauge fields of the Standard Model. In Section III we calculate the corrections to the effective action generated after integration of the heavy fermion, and show that the dominant operator generated by the LV in the low energy regime is LI, and reproduces the standard ALP-photon interaction. We comment on possible consequences of this result for ALP phenomenology in Section IV. Finally, Section V contains our conclusions and some final remarks.
II. THE MODEL

We now show explicitly how the low energy effective interaction between the pseudoscalar and the electromagnetic fields can be generated from a model for high energy physics which has LV interactions. Let us take into account a single charged vectorial fermionic field $\psi \sim (1, 1)$, and the pseudoscalar field $\phi \sim (1, 0)$, where the numbers in parenthesis means the transformation under the Standard Model gauge group $SU(2)_L \otimes U(1)_Y$. Besides the minimal coupling with the hypercharge $U(1)_Y$ gauge field $B_\mu$, it is assumed that $\psi$ has a LV interaction $B_{\mu\nu} d^\nu \overline{\psi} \gamma^\mu \psi$, where $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$. Also, the pseudoscalar field $\phi$ enters via a LV Yukawa coupling $\overline{\psi} \gamma^5 b \phi \psi$. The starting Lagrangian is therefore

$$L = \overline{\psi} \left[ i \frac{\partial}{\partial t} - m - \gamma^\mu (g' B_\mu + B_{\mu\nu} d^\nu) - \gamma^5 b \phi \right] \psi,$$

where the fermion mass $m$ is supposedly higher than the electroweak scale, actually high enough to justify the integration of its degrees of freedom. Also, $g'$ is the gauge coupling constant, and $b^\mu$, $d^\mu$ are constant vectors leading to LV; it is worth mentioning that $b^\mu$, $d^\mu$ have different mass dimensions. The model (4) is an extension of the one used earlier in [23, 24, 29] for studies of the perturbative generation of the aether term; in [25] its version involving only the non minimal interaction was used for the same purpose.

We shall not be interested in investigating the origin of the LV terms, but rather to show that from Eq. (4) we can generate the interaction in Eq. (3), after the field $\psi$ is integrated out. Notice that we do not have to consider the $\phi$ kinetic term in our calculations, that means the mechanism we shall describe is completely independent of the mass of pseudoscalar, which can be taken to be as small as necessary to fit experimental constraints.

The one-loop correction to the effective action of the gauge field $B_\mu$ can be expressed as usual in terms of a functional trace,

$$S_{\text{eff}} [B] = -i \text{Tr} \ln \left( i \partial - m - \gamma^\mu (g' B_\mu + B_{\mu\nu} d^\nu) - \gamma^5 b \phi \right),$$

where

$$\tilde{B}_\mu = g' B_\mu + B_{\mu\nu} d^\nu.$$  

This effective action can be expanded in the following power series,

$$S_{\text{eff}} [B] = i \text{Tr} \sum_{n=1}^{\infty} \frac{1}{n} \left[ \frac{1}{i \partial - m} \left( \gamma^\mu \tilde{B}_\mu + \gamma^5 b \phi \right) \right]^n.$$
From Eq. (7) we will obtain a myriad of effective interactions involving the pseudoscalar and
gauge fields. On general grounds, only the lowest dimensional ones are expected to contribute to
the low energy phenomenology we are interested in, since the others will involve additional powers
of the large fermion mass \( m \) in the denominator. We will argue, in the following Section, that the
dominant interactions generated in the ALP-photon sector are LI, and reproduce in form Eq. (3).

III. GENERATION OF ALPS-PHOTON INTERACTION

First we show how to extract the ALP-photon interaction vertex from Eq (7), after that we will
justify the fact that this interaction is indeed the dominant one in the low energy regime. We start
by isolating in Eq. (7) the contributions of the second order in \( \tilde{B}^{\mu} \) and first order in \( \phi \),
\[
S_{\text{eff}} [B] \supset i \text{Tr} \left[ \frac{1}{i \partial - m} \gamma^{\mu} \tilde{B}_{\mu} \frac{1}{i \partial - m} \gamma^{\nu} \tilde{B}_{\nu} \frac{1}{i \partial - m} \gamma_{5} b \phi \right],
\]
where the cyclic property of the trace have been used. Taking into account Eq. (6), after Fourier
transform, one obtains
\[
S_{\text{eff}} [B] \supset g' d^{\mu} b_{\sigma} \int \frac{d^{4} k}{(2\pi)^{4}} \frac{d^{4} q}{(2\pi)^{4}} \tilde{\phi} \left( q \right) B_{\mu} \left( k \right) B_{\nu \rho} \left( -k - q \right) \Pi^{\mu \nu \sigma} \left( k, q \right),
\]
where the one-loop correction to the vacuum polarization reads
\[
\Pi^{\mu \nu \sigma} \left( k, q \right) = \text{tr} \left( \frac{d^{4} p}{(2\pi)^{4}} \gamma^{\mu} S \left( p - k - q \right) \gamma^{\nu} S \left( p \right) \gamma_{5} \gamma^{\sigma} S \left( p - q \right) \right) + \text{finite terms}.
\]
Here, \( S(p) = (\not{p} - m)^{-1} \) is the free Fermion propagator. Eq. (10) corresponds to the sum of
triangle graphs depicted in Fig. 1: it will prove convenient to calculate it by expanding around
\( q = 0 \). Noticing that \( \Pi^{\mu \nu \sigma} \left( k, q \right) \) is defined by a potentially linearly divergent integral, we can write that
\[
\Pi^{\mu \nu \sigma} \left( k, q \right) = \Pi^{\mu \nu \sigma} \left( k, 0 \right) + q_{\sigma} \partial_{q_{\sigma}} \Pi^{\mu \nu \sigma} \left( k, q \right) |_{q=0} + \text{finite terms}.
\]
The integrals contributing to \( \Pi^{\mu \nu \sigma} \left( k, 0 \right) \) turn out to be rather similar to the ones considered
in the perturbative generation of the CFJ term as studied, for example, in [30], with a difference
of momentum routing that amounts to a shift in the integration momentum \( p \rightarrow p - k \). Since
these integrals are linearly divergent, such a shift would produce a finite shift in the result of the
integrals, i.e.,
\[
\Pi^{\mu \nu \sigma} \left( k, 0 \right) = \tilde{\Pi}^{\mu \nu \sigma} \left( k \right) + C^{\alpha \beta \gamma} \varepsilon^{\mu \nu \sigma} \partial_{\rho} k_{\rho},
\]
where $C'''$ is a finite constant, and $\Pi^{\mu\nu\sigma}(k)$ is the polarization tensor appearing in [30]. The Feynman integral involved in $\Pi^{\mu\nu\sigma}(k)$ turns out to be finite and ambiguous upon regularization, but also proportional to $\varepsilon^{\mu\nu\sigma\rho} k_\rho$. We can therefore conclude that

$$\Pi^{\mu\nu\sigma}(k,0) = C'' \varepsilon^{\mu\nu\sigma\rho} k_\rho,$$

(13)

where the exact value of the finite coefficient $C''$ depends on the regularization scheme and the way the integral is manipulated. This ambiguity in the evaluation of triangle graphs is well known, playing a key role in the study of anomalies in chiral gauge theories, for example. In that context, ambiguous integrals are fixed by imposing a physical requirement, such as the conservation of the electric current. Different physical requirements have also been considered in the calculation of the CFJ term (see for example [31–33]), but none of these results apply directly to our case. We remind that the CFJ term is a quadratic term in the effective photon Lagrangian, while we are studying the generation of an interaction term involving photons and pseudoscalars.

In summary, from Eqs. (10) and (13), we can state that

$$\Pi^{\mu\nu\sigma}(k,q) = C'' \varepsilon^{\mu\nu\sigma\rho} k_\rho + q_\sigma \frac{\partial}{\partial q_\sigma} \Pi^{\mu\nu\sigma}(k,q)|_{q=0} + \text{finite terms}.$$  

(14)

The first term in Eq. (14) is finite and ambiguous; the second one could be at the most logarithmically divergent by power counting, but after explicit calculation it turns out to be finite. In fact,
it can be shown that

\[ q \sigma \frac{\partial}{\partial q \sigma} \Pi^{\mu \nu \rho}_{(k,q)}|_{q=0} = \frac{C''}{2} \varepsilon^{\mu \nu \rho \sigma} q_{\rho} \]

\[ + D \left( k^\mu \varepsilon^{\nu \sigma \rho \theta} q_{\theta} k_{\theta} + (\mu \leftrightarrow \nu) \right) + E \varepsilon^{\mu \nu \rho \sigma} k_{\rho} (k \cdot q) , \]  

(15)

where again one obtains a contribution from the same finite (yet ambiguous) Feynman integrand that contributed to the first term in Eq. (14). Thus the quantum corrections to the photon effective action in our theory are completely finite.

Focusing at the moment on the terms proportional to \( \varepsilon^{\mu \nu \sigma \rho} q_{\rho} \) in Eqs. (14) and (15), one can go back to Eq. (9), thus obtaining

\[ S_{\text{eff}}[B] \supset C' g' f_{\lambda} d_{\lambda} B_{\rho \mu} B_{\kappa \nu} . \]  

(16)

After electroweak symmetry breaking, the \( B_{\mu} \) field rotates into a superposition of the photon \( A_{\mu} \) and the neutral weak gauge boson \( Z_{\mu} \),

\[ B_{\mu} = \sin \theta Z_{\mu} + \cos \theta A_{\mu} , \]  

(17)

and \( g' \) can also be related to the electric charge \( g' = e / \cos \theta \). Since we are ultimately interested in the consequences of the interaction in Eq. (15) in the very low energy regime, contributions involving the \( Z_{\mu} \) field (which is very heavy) can be disregarded. The relevant effective interaction thus generated is

\[ S_{\text{photon}}[A] \supset C e \phi \varepsilon^{\rho \mu \nu \lambda} b_{\lambda} d_{\rho} F_{\mu \nu} F_{\kappa \upsilon} . \]  

(18)

To recognize in (18) an expression similar to (3), one has to note that

\[ \varepsilon^{\rho \mu \nu \lambda} b_{\lambda} d_{\rho} F_{\mu \nu} F_{\kappa \upsilon} = 2 (b \cdot d) \vec{E} \cdot \vec{B} , \]  

(19)

therefore

\[ S_{\text{photon}}[A] \supset 2 C e (b \cdot d) \phi \vec{E} \cdot \vec{B} . \]  

(20)

This is the main result of this paper. We obtain with the mechanism described in the previous paragraphs a term of the same form as Eq. (3), which is the most relevant interaction involving the pseudo-scalar, from the point of view of current experimental searches.

We can now argue that the interaction in Eq. (20) is indeed the dominant one in the low energy regime. It will be essential the assumption that the fermion mass \( m \) in Eq. (3) is very high: notice
however that the resulting low energy interaction (20) is independent of the mass scale \( m \), differently from what happens in LI theories as discussed after Eq. (1), in which the coupling \( g_{\phi \gamma} \sim 1/f_a \) (\( f_a \) being the relevant large mass scale in that case). The mass independence of Eq. (20) means that we can consider \( m \) as large as necessary for the following arguments to hold, without modifying the intensity of the generated ALP-photon interaction.

Consider for example the second and third terms in the right hand side of Eq. (15). These could induce higher derivatives operators with nontrivial dependence on the directions of the LV parameters \( b^\mu, d^\mu \). The coefficient \( E \), whose explicit form is

\[
E = \frac{i}{2\pi^2} \int_0^1 dz \int_0^{1-z} dx \frac{4x^3 - 5x^2 + x}{x(x-1)k^2 + m^2}
- \frac{i}{2\pi^2} \int_0^1 dz \int_0^{1-z} dx \frac{k^2x^3(x-1)^2 - m^2x(x^2-1)}{[x(x-1)k^2 + m^2]^2},
\]

(21)

would correspond to an interaction proportional to \( \partial^\mu \phi (b_\sigma \partial_\sigma F^{\nu\sigma}) (d^\rho F_{\nu\rho}) \) in the low energy effective action. The integrals over the Feynman parameters can be explicitly performed and, assuming \( m^2 \gg k^2 \), we verify that

\[
E \sim -\frac{i}{12\pi^2 m^2},
\]

(22)

meaning the corresponding higher derivative term is strongly suppressed in the low energy regime. In the same limit, it can be shown that

\[
D = \frac{i}{2\pi^2} \int_0^1 dz \int_0^{1-z} dx \frac{x(x-1)}{x(x-1)k^2 + m^2} \sim -\frac{i}{24\pi^2 m^2}.
\]

(23)

The same should happen to the remaining terms in Eq. (14): they should represent higher derivative terms, strongly suppressed by powers of the inverse mass of the fermion, which is assumedly very high.

IV. IMPLICATION FOR ALPS PHENOMENOLOGY

ALPs are very light particles, and therefore the only relevant operators in the phenomenology of these particles are the dominant ones in the very low energy regime. We have argued, in the previous section, that in this model the lowest dimensional, dominant operator is the one given in Eq. (20). It is surprising to notice that even with the model starting with explicit Lorentz violation, this interaction depends only on the scalar \( b \cdot d = b^\mu d_\mu \) [36]. That means the underlying LV background involving the vectors \( b_\mu, d_\mu \) that we considered generated a Lorentz invariant interaction, which mimics precisely the effects of the ALPs interactions studied in the Lorentz preserving context. We
conclude that whenever the kind of LV that we considered as the initial input for our calculations turns out to happen in nature, the effective ALP-photon coupling that are measured by experiments is actually

\[ g^{(\text{eff})}_{\phi\gamma} \phi F_{\mu\nu} \tilde{F}^{\mu\nu} = \left( g^{(LI)}_{\phi\gamma} + 2C e (b \cdot d) \right) \phi F_{\mu\nu} \tilde{F}^{\mu\nu}, \]

where \( g^{(LI)}_{\phi\gamma} \) represents the contribution generated by other – Lorentz invariant – interactions (such as the ones due to the Peccei-Quinn mechanism for solving the strong CP problem, described in Section I). One might even say that Lorentz violation could represent an alternative mechanism to generate such couplings, that is, even if \( g^{(LI)}_{\phi\gamma} = 0 \), one could have the essential signals of the presence of an ALP-photon interaction induced only by the LV.

Since the interaction in Eq. (20) is LI, one could not resort to the usual signatures of LV theories for imposing constraints on the LV parameters \( b^\mu \), \( d^\mu \), such as a dependence on the orientation of the laboratory to these fixed vectors. However, if a QCD axion happens to be found, and the specific details of the embedding of the PQ chiral symmetry in the Standard Model are fixed, then small deviations from the mass and coupling relation derived from Eq. (2) could be attributed to the effects of the interaction in Eq. (20), as can be seen in Eq. (24). Conversely, bounds on the LV parameters \( b^\mu \), \( d^\mu \) could be drawn from a precise experimental verification of the QCD axion predictions.

The appearance of the ambiguous constant \( C \) in (20) brings into question the technical puzzle involved in its calculation. Assuming, for the sake of the argument, \( C \sim 1 \), one roughly obtains from (20) and (3) the experimental constraint

\[ e (b \cdot d) \lesssim 10^{-10}\text{GeV}^{-1}. \]

(25)

We are unaware of independent experimental constraints on the Lorentz violating couplings \( d^\nu F_{\mu\nu} \tilde{\psi} \gamma^\mu \psi \) and \( \phi \tilde{\psi} \gamma_5 b \psi \), but if one of these could be found, we could put an experimental constraint on the other one. Notice however that Eq. (20) is of second order in LV parameters, so we might not be able to put very stringent bounds on \( b_\mu \) or \( d_\mu \) based only on Eq. (25). This is a question that deserves further study since it connects the ambiguous constant \( C \) with an experimentally observable quantity, namely the effective coupling in Eq. (20). We expect that the ambiguity in the calculation of \( C \) should be fixed by physical constraints, maybe of the experimental nature.
V. CONCLUSIONS AND PERSPECTIVES

In this work, we have shown that a model with LV interaction in the high energy regime can induce, in the low energy limit, LI operators that can be relevant to ALPs phenomenology. Our construction is based on the assumption of the existence of a very massive fermion field, with specific LV interactions, which upon integration will provide suppression factors of inverse of its mass to all LV operators in the photon effective action, except for the leading operator in Eq. (20), which happens to be LI. Thus, we proposed a novel way to seek for possible consequences of LV occurring in some fundamental high energy theory, that is, looking at LI effects that could in principle be seen in current experimental searches for light pseudoscalars.

In summary, the connection between Lorentz violation and the phenomenology of axion-like particles can have very interesting consequences both for the searches of possible violations of standard spacetime symmetries, and for the search of the elusive light pseudoscalars that could solve many theoretical puzzles in our current understanding of the universe.

To finish, one may wonder whether other relevant LI operators could be induced by and underlying LV dynamics, maybe a LI operator relevant to the Higgs phenomenology. We may consider the following model,

\[ \mathcal{L} = \bar{\psi} \left[ i \partial - m - i \gamma_\mu \left( g' B_\mu + u^\nu \epsilon_{\mu\nu\lambda\rho} B^{\lambda\rho} \right) - \gamma_5 \phi' \right] \psi, \]  

(26)

instead of Eq. (6), where \( u^\nu \) is a LV vector and now \( v_\mu \) is a LV pseudovector, and \( \phi' \) a real scalar field. Since \( \psi \) is assumed to be a singlet under the gauge symmetries of the Standard Model, it cannot directly couple to the usual Higgs doublet field \( H \). However, if there is a \( H-\phi' \) interaction leading to a mixing after spontaneous symmetry breaking, such that \( \phi' = \langle \phi' \rangle + \cos \phi + \sin \phi \ h \), with \( \langle \phi' \rangle \) the \( \phi' \) vacuum expectation value and the unprimed fields being mass eigenstates, then the Higgs field \( h \) ends up coupled to \( \psi \). An example of interaction term leading to such a mixing is \( H^\dagger H \phi'^2 \). From Eq. (26), by repeating the steps described in the previous paragraphs, one would arrive at

\[ S_{\text{eff}} [B] \supset 2 C g' F_{\rho\mu} \epsilon^{\rho\mu\lambda\nu} v_\lambda \epsilon_{\nu\kappa\sigma\tau} u^\kappa \phi' F^{\sigma\tau}. \]  

(27)

In this way one generates scalar couplings to the photon of the general form

\[ \mathcal{L}_{h\gamma} = c_1 \phi' (u^\mu F_{\mu\nu}) (v_\alpha F^{\alpha\nu}) + c_2 (u \cdot v) \phi' F^{\mu\nu} F_{\mu\nu}. \]  

(28)

The first term is sensitive to the direction of the Lorentz breaking vectors \( u_\mu, v_\mu \), as can be made
explicit by rewriting it as

\[ c_1 \varphi' \left[ - (u^0v^0) \vec{E}^2 + (u^0) \vec{v} \cdot (\vec{E} \times \vec{B}) + (v^0) \vec{u} \cdot (\vec{E} \times \vec{B}) + \left( \vec{E} \cdot \vec{u} \right) \left( \vec{E} \cdot \vec{v} \right) + \vec{u} \cdot \left[ \vec{B} \times \left( \vec{B} \times \vec{v} \right) \right] \right]. \]

(29)

From this interaction, a typically Lorentz violating phenomenology could be studied, namely, the dependence of physical measurements on the direction of the vectors \( v_\mu, u_\mu \), which by assumption are fixed in some preferred cosmological Lorentz frame.

The second term in Eq. (28), however, depend on the scalar \( u \cdot v \), and with the mixing there appears the interaction \( c_2 (u \cdot v) \sin \alpha h F_{\mu\nu} F^{\mu\nu} \), which is identical in form to the effective couplings in the Standard Model responsible for the diphoton Higgs boson decay. This decay was one of the main channels for the observation of the resonance at 126 GeV recently discovered at the LHC \[34, 35\]. Being second order in the LV parameters, this correction should be small enough for not to bring any obvious inconsistency with the observations, however we might speculate whether LV could induce some small deviations from the predictions of the Standard Model that could be still measurable in this or some other process.

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The classification of $b^\mu d_\mu$ as a scalar might require some clarification. It is important, when considering Lorentz violating extensions of the Standard Model, to distinguish between observer and particle Lorentz transformations. Under the first of these, all Lorentz indices transform covariantly, while under the second, indices of fields, derivatives and gamma matrices transform as usual, while $b^\mu$ and $d_\mu$ remain constant. Therefore, $b^\mu d_\mu$ is a genuine Lorentz scalar under observer Lorentz transformation because of the covariant contraction of indices, while it remain constant under particle Lorentz transformation because of the constancy of the $b^\mu$ and $d_\mu$ independently. Either way, the expression $b^\mu d_\mu$ is invariant, which justifies our slight abuse of nomenclature in simply calling it a scalar.