The $W$ boson mass $(m_W)$ is a key parameter of the standard model (SM), constraining the mass of the unobserved Higgs boson. Using Tevatron $p\bar{p}$ collision data from 1992-1995, the CDF and DØ collaborations measured $m_W$ to $\delta m_W = 59$ MeV. The ongoing Tevatron Run 2 has produced a factor of 5 more collisions, promising a significant reduction in $\delta m_W$. CDF has analyzed the first $\approx 200$ pb$^{-1}$ of Run 2 data and determined its $\delta m_W$ to be 76 MeV.

1 Introduction

The SM describes all non-gravitational interactions in terms of an $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge symmetry. Non-zero particle masses arise from the breaking of the $SU(2)_L \times U(1)_Y$ electroweak symmetry via the Higgs mechanism. The Higgs boson is the last unobserved SM particle, and the measured electroweak parameters severely constrain its mass $(m_H)$. The constraint can be obtained from the radiative correction $\Delta r$ to the $W$ boson mass $(m_W)$

$$m_W^2 = \frac{\pi \alpha_{EM}}{\sqrt{2} G_F (1 - m_Z^2/m_W^2) (1 - \Delta r)}.$$  

(1)

The correction $\Delta r \approx 0.67\%$ results predominantly from Higgs and $t\bar{b}$ loops in the $W$ boson propagator. Because of the precise measurements of the parameters $\alpha_{EM}$ ($\delta \alpha_{EM}/\alpha_{EM} = 0.014\%$ at $Q^2 = m_Z^2$), $G_F$ ($\delta G_F/G_F = 0.0009\%$), and $m_Z^2$ ($\delta m_Z^2/m_Z^2 = 0.004\%$), the uncertainties on $m_t$ and $m_W$ dominate the uncertainty on the inferred $m_H$. To obtain equal $\chi^2$ contributions in a fit to $m_H$, the relation $\delta m_W = 0.007 \delta m_t$ must hold. For the Run 1 $\delta m_t$ of 4.3 GeV, the required $\delta m_W$ is 30 MeV, close to $\delta m_W(\text{world}) = 34$ MeV. The impending Run 2 top mass measurements will significantly reduce $\delta m_t$, making $\delta m_W$ reduction of primary importance.

The study of ongoing Run 2 $p\bar{p}$ collisions at the Tevatron will achieve this goal. With 2 fb$^{-1}$ of data, the CDF and DØ collaborations expect to complete measurements with $\delta m_W$ between
40 MeV$^3$ and 50 MeV$^4$. Combining with the measurement from LEP ($\delta m_W = 42$ MeV) and the Run 2 $\delta m_t \approx 2$ GeV will result in $\delta m_H/m_H \approx 30\%$.$^5$

The CDF and DØ collaborations are currently analyzing Run 2 data, with DØ finalizing its event selection and precision calorimeter calibration, and CDF performing necessary cross-checks to its full analysis with $\approx 200$ pb$^{-1}$ of data. The CDF collaboration has determined the $W$ boson mass uncertainty associated with these data to be 76 MeV.

2 Measuring the $W$ Boson Mass at the Tevatron

The $m_W$ measurement in $p\bar{p}$ data uses $s$-channel resonant $W$ bosons with leptonic decays. The transverse momentum of the decay $e$ or $\mu$ ($p_T^l$) can be measured with high precision and thus provides the bulk of the mass information. Additional information comes from the decay $\nu$ transverse momentum ($p_T^\nu$), which is inferred from the measured energy imbalance in the event. Since the lepton energy is well measured, the dominant uncertainty on $p_T^\nu$ comes from measuring the hadrons recoiling against the produced $W$ boson. Because the $Z$ boson has a similar mass and production mechanism to the $W$ boson, events with $Z$ bosons can be used to calibrate and model the detector response to hadronic activity.

The best statistical power for measuring $m_W$ is obtained by combining $p_T^l$ and $p_T^\nu$ into the transverse mass, defined as:

$$m_T = \sqrt{2p_T^l p_T^\nu (1 - \cos(\Delta \phi))}. \tag{2}$$

The transverse mass ignores the unmeasured $\nu$ momentum along the beam direction ($\hat{z}$). This distribution has a peak at $m_W$ (if we neglect detector resolution and final-state photon radiation) and a long tail below $m_W$, corresponding to events with $p_T^\nu \neq 0$.

3 Run 2 CDF $W$ Boson Mass Measurement

The relevant components of the CDF detector for the $m_W$ measurement are a large open-cell drift chamber immersed in a 1.4 T magnetic field, surrounded by a lead-scintillator sampling calorimeter. Because of the similar resolutions and acceptances for 40 GeV $e$ and $\mu$, the combination of the two channels nearly doubles the effective statistics for the $m_W$ measurement.

The CDF strategy for the measurement proceeds as follows: Model $W$ boson production and decay; calibrate track momentum using high-statistics resonances; calibrate calorimeter energy using $e$ tracks from $W$ boson decays; model hadronic response and resolution; estimate backgrounds; and fit the transverse mass distribution to obtain $m_W$.

3.1 Event Generation

There are two important components of $W$ boson production for measuring $m_W$: the fractional momenta of $u$ and $d$ quarks inside the proton, and the $W$ $p_T$. The $u$ and $d$ momenta determine $p_z^W$, which affects the $m_T$ distribution. The $u$ and $d$ fractional momenta are constrained from global fits to high-energy data and embodied in parton distribution functions (PDFs) independently parametrized by the CTEQ and MRST collaborations. Using a CTEQ prescription for obtaining PDF uncertainties, the CDF collaboration has estimated $\delta m_W(PDF) = 15$ MeV.

The $W$ boson $p_T$ distribution is predicted by an event generator (RESBOS) that combines a QCD next-to-leading-log calculation with three non-perturbative parameters fit from high energy data. The dominant constraint on these parameters comes from the $Z$ boson $p_T$ measurement in Run 1. The generator and detector simulation predict the observed Run 2 $Z$ boson $p_T$ spectrum well (Fig. 1). The uncertainty on the RESBOS parameters results in $\delta m_W(p_T^W) = 13$ MeV.
In the $W$ decay, the most important effect for the $W$ mass measurement is the radiation of a $\gamma$ from a final-state $l^\pm$. This radiation results in a reduced $l^\pm$ momentum, potentially affecting the inferred mass of the $W$ boson. CDF bases its simulation of final-state radiation on a QED next-to-leading order event generator (WGRAD). Effects from initial-state radiation, interference, and higher-order terms are not simulated, resulting in a 20 (15) MeV uncertainty for the $m_W$ measurement in the $\mu$ ($e$) channel.

3.2 Track Momentum Calibration

A charged particle’s momentum is measured through its observed curvature in the tracker. Since the momentum is inversely proportional to curvature, the momentum scale is measured as a function of the mean inverse momentum of $J/\psi$ muons and fit to a line. The line has zero slope, verifying the applicability of the extracted scale to $W$ boson decays.

To improve momentum resolution, muon tracks from $W$ and $Z$ decays use the beam position as a point in the track fit. This constraint cannot be applied to $J/\psi$ decays since they can be separated from the beam line. Instead, $\Upsilon$ decays are used to verify that the beam constraint produces no bias on the momentum calibration. A systematic uncertainty of 15 MeV accounts for the observed difference in scale. Including the uncertainty due to tracker alignment, CDF estimates an uncertainty of $\delta m_W(p_T \text{ scale}) = 25$ MeV.

3.3 Calorimeter Energy Calibration

Given the momentum calibration, electron tracks from $W$ decays are used to calibrate the electromagnetic calorimeter. The calorimeter energy is scaled such that the ratio of energy to track momentum ($E/p$) is equal to 1. To correct for an energy-dependent scale, the $E/p$ distribution is fit as a function of electron $E_T$ and a correction applied.

The significant amount of material in the silicon detector inside the tracker affects the position of the $E/p$ peak. An uncertainty on the amount of material translates into an uncertainty on the measured $E$ scale. The fraction of events in the region $1.19 < E/p < 1.85$ is a measure of the material. The extent to which this region is not well modelled results in a 55 MeV uncertainty on the $W$ mass. This uncertainty dominates the total $\delta m_W(E \text{ scale})$ of 70 MeV.

3.4 Hadronic Recoil Measurement and Simulation

The hadronic recoil energy is measured by vectorially summing all the energy in the calorimeter, excluding that contributed by the $l$. The detector response to the hadronic energy is defined as

\[
\text{events} = \frac{\text{events}}{\text{GeV}}
\]

CDF Run 2 Preliminary

\[
\chi^2/\text{dof} = 64/58
\]

Figure 1: Left: The $Z$ boson $p_T$ spectrum in CDF Run 2 $Z \to \mu\mu$ data (points) compared to the spectrum generated with RESBOS (solid). Right: The $m_T$ distribution for $W$ boson decays to $\mu\nu$. In the $W$ decay, the most important effect for the $W$ mass measurement is the radiation of a $\gamma$ from a final-state $l^\pm$. This radiation results in a reduced $l^\pm$ momentum, potentially affecting the inferred mass of the $W$ boson. CDF bases its simulation of final-state radiation on a QED next-to-leading order event generator (WGRAD). Effects from initial-state radiation, interference, and higher-order terms are not simulated, resulting in a 20 (15) MeV uncertainty for the $m_W$ measurement in the $\mu$ ($e$) channel.
Table 1: The uncertainties on the W boson mass measurement in MeV/c^2 using 0.2 fb^{-1} of Run 2 CDF data. The CDF Run 1B uncertainties are shown for comparison.

| Systematic Uncertainty                  | Electrons (Run 1B) | Muons (Run 1B) |
|-----------------------------------------|--------------------|----------------|
| Production and Decay Model              | 30 (30)            | 30 (30)        |
| Lepton E Scale and Resolution           | 70 (80)            | 30 (87)        |
| Recoil Scale and Resolution             | 50 (37)            | 50 (35)        |
| Backgrounds                             | 20 (5)             | 20 (25)        |
| Statistics                              | 45 (65)            | 50 (100)       |
| **Total**                               | **105 (110)**      | **85 (140)**   |

\[ R = \frac{u_{\text{meas}}}{u_{\text{true}}} \], where \( u_{\text{true}} \) is the recoil energy of the W boson. The response is measured using \( Z \rightarrow ll \), since the \( l \) is measured more precisely than the hadronic energy.

The hadronic energy resolution is modelled as having a component from the underlying event (independent of recoil) and a component from the recoiling hadrons. The model parameters are tuned using the resolution of \( Z \rightarrow ll \) along the axis bisecting the leptons. This axis is the least susceptible to fluctuations in \( l \) energy. The recoil response and resolution uncertainty on the W mass is 50 MeV, of which 37 MeV is due to the model of the underlying energy resolution.

### 3.5 Backgrounds

The backgrounds common to the \( W \rightarrow e\nu \) and \( W \rightarrow \mu\nu \) samples are: \( Z \rightarrow ll \), where one \( l \) is not reconstructed; \( W \rightarrow \tau\nu \rightarrow l3\nu \); and dijet production, with one hadronic jet misreconstructed as an \( l \). In addition, the \( \mu \) sample includes background from cosmic rays and decays in flight. The \( W \) and \( Z \) backgrounds are estimated using Monte Carlo. The dijet background estimation uses events with significant energy surrounding the \( l \) to enhance hadronic background and obtain a background \( \mathcal{E}_T \)-distribution. The data \( \mathcal{E}_T \)-distribution is then fit using the \( W \) and jet distributions as input. The cosmic ray background is determined using track hit timing information and the decay-in-flight background estimated by fitting the \( \Delta \phi(l, \mathcal{E}_T) \) distribution to a combination of \( W \) and decay-in-flight distributions. These estimates result in \( \delta m_W(\text{background}) = 20 \text{ MeV} \).

### 3.6 Mass Fit and Systematics

Given the energy calibrations, recoil model, and background estimation, the \( m_T \) distribution is fit for the \( e \) and \( \mu \) channels. The predicted line shape agrees with that of the data (Fig. 1). The central value is blinded while CDF cross-checks the analysis with independent data sets and simulation. Combining the two channels (Table 1) results in \( \delta m_W = 76 \text{ MeV} \).

### References

1. S. Eidelman et al., Phys. Lett. B 592, 1 (2004).
2. G. Azuelos et al., hep-ph/0003275 (2000).
3. R. Blair et al., Fermilab-Pub-96-390-E (1996)
4. S. Abachi et al., Fermilab-Pub-96-357-E (1996).
5. J. Erls and P. Langacker, hep-ph/9809352 (1998).
6. CTEQ Collaboration, J. Pumplin et al., J. High Energy Phys. 7, 12 (2002).
7. A. Martin, R.G. Roberts, W.J. Stirling, R.S. Thorne, Eur. Phys. J. C 28, 455 (2003).
8. C. Balazs and C.P. Yuan, Phys. Rev. D 56, 5558 (1997).
9. T. Affolder et al., Phys. Rev. D 63, 032003 (2001).