Abstract

High luminosity Super B/Flavor factories, near and on top of the Υ resonances, allow for a detailed investigation of CP-violation in τ physics. In particular, bounds on the τ electric dipole moment can be obtained from CP-odd observables. We perform an independent analysis from other low and high energy data. For polarized electron beam a CP-odd asymmetry, associated to the normal polarization term, can be used to set stringent bounds on the τ electric dipole moment.

1 Introduction

The standard model (SM) describes with high accuracy most of the particle physics experiments \cite{sm}. However, the first clue to physics beyond the SM has been found in neutrino physics \cite{nu}. This opens the possibility for new phenomena related to CP violation physics, particularly in the leptonic sector. The time reversal odd electric dipole moment (EDM) for leptons, specially the electron and the muon, has been extensively investigated and strong limits were measured \cite{edm}:

\[ d_\gamma^e = (0.07 \pm 0.07) \times 10^{-26} \text{ecm} \]  

(1)

Present bounds for the τ lepton EDM are much lower than for the electron or muon case \cite{ra}:

\[ -0.22 \text{ecm} < Re(d_\gamma^\tau) \times 10^{16} < 0.45 \text{ecm} \quad (95\% \text{ C.L.}). \]  

(2)
The EDM effective operator flips chirality and, therefore, the \( \tau \) lepton physics is expected to be more sensitive to contributions coming from new physics. In the SM the CP-violation is introduced by the CKM mechanism. There the EDM and weak-EDM (\textit{i.e.} the T-odd diagonal coupling with the Z) are generated at very high order in the coupling constant. This opens a way to test many models: CP-odd observables related to EDM would give no appreciable effect from the SM and any experimental signal should be identified with beyond the SM physics where the EDM can be generated at 1-loop. In refs. [4,5] the \( \tau \) weak-EDM has been studied in CP-odd observables at high energies through linear polarizations and spin-spin correlations [6,7]. In ref. [8] the sensitivity to the weak-EDM in spin-spin correlation observables was studied for tau-charm-factories with polarized electrons. EDM limits for the \( \tau \), from CP-even observables such as total cross sections or decay widths, have also been considered in [9,10,11]. The limit in Eq. (2) was found by the BELLE Collaboration measuring CP-odd spin correlation observables. Most of the statistics for the \( \tau \) pair production was dominated in the past by LEP, but the high luminosity of the B factories and their upgrades have nowadays the largest \( \tau \) pair samples. In the future, the data will be dominated by the proposed Super B/Flavor factories [12]. These facilities may also have the possibility of polarized beams. In this paper we present new CP-odd observables, related to the \( \tau \) pairs produced at low energies with polarized beams, that may lead to competitive results with the present bounds for the \( \tau \) EDM.

2 Effective Lagrangian

We parametrize the low energy new physics effects by an effective Lagrangian built with the SM particle spectrum containing higher dimension gauge invariant operators suppressed by the scale of new physics, \( \Lambda \) [13]. The leading order EDM and weak-EDM Lagrangian for CP violation has only two dimension six operators [14] that contribute:

\[
\mathcal{L}_{\text{eff}} = i\alpha_B \mathcal{O}_B + i\alpha_W \mathcal{O}_W + \text{h.c.} \tag{3}
\]

where \( \alpha_B \) and \( \alpha_W \) are real couplings and the operators are defined as follows:

\[
\mathcal{O}_B = \frac{g'}{2\Lambda^2} \bar{\nu}_L \bar{\tau}_L \sigma_{\mu\nu} \tau_R B^{\mu\nu} \, , \quad \mathcal{O}_W = \frac{g}{2\Lambda^2} \bar{\nu}_L \bar{\tau}_L \phi \sigma_{\mu\nu} \tau_R W^{\mu\nu} \, . \tag{4}
\]

Here \( L_L = (\nu_L, \tau_L) \) is the \( \tau \) leptonic doublet, \( \phi \) is the Higgs doublet, \( B^{\mu\nu} \) and \( W^{\mu\nu} \) are the U(1)\(_Y\) and SU(2)\(_L\) field strength tensors, and \( g' \) and \( g \) are the gauge couplings. After spontaneous symmetry breaking the interactions in Eq. (4) produce the usual EDM effective operators:
\[ \mathcal{L}^{\gamma, Z}_{\text{eff}} = -i d_{\gamma}^{\tau} \bar{\tau} \sigma_{\mu\nu} \gamma^5 \tau F^{\mu
u} - i d_{Z}^{\tau} \bar{\tau} \sigma_{\mu\nu} \gamma^5 \tau Z^{\mu
u} \] (5)

where \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) and \( Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu \) are the abelian field strength tensors of the photon and \( Z \) gauge boson and \( d_{\gamma}^\tau \) and \( d_{Z}^\tau \) are the electric and weak-electric dipole moments, respectively. Note that as long as \(|q^2| \ll \Lambda^2\) there is no need to distinguish between the new physics contribution to the EDM form factors and the EDM dipole moment.

The \( e^+ e^- \rightarrow \tau^+ \tau^- \) cross section has contributions coming from the SM and the terms in Eq.(5). Tree level contributions come from \( \gamma \) or \( \Upsilon \) exchange in the s-channel and are shown in Fig.(1). Other contributions coming from diagrams where at least one photon line is substituted by a \( Z \) are suppressed by powers of \((q^2/M_Z^2)\).

As stated in the introduction the CKM contributions to CP-odd observables are far below the present experimental sensitivity. The bounds on the EDM that one may get are just the ones coming from new physics.

![Diagrams](image)

Fig. 1. Diagrams (a) direct \( \gamma \) exchange (b) \( \Upsilon \) production (c) EDM in \( \gamma \) exchange (d) EDM at the \( \Upsilon \)-peak.

## 3 Low energy polarized beams and the EDM.

For longitudinally polarized electrons the \( \tau \)-EDM modifies the angular distribution for the \( e^+ e^- \rightarrow \tau^+ (s_+) \tau^- (s_-) \) cross section. The normal-to the scattering plane- polarization \((P_N)\) of each \( \tau \) is the only linear component which is \( T \)-odd. For CP-conserving interactions, the CP-even term \((s_+ + s_-)_N\) of the normal polarization only gets contribution through the combined effect of both an helicity-flip transition and the presence of absorptive parts, which are both suppressed in the SM. For a CP-violating interaction, such as an EDM, the \((s_+ - s_-)_N\) CP-odd term gets a non-vanishing value without the
need of absorptive parts. $P_N$ is also even under parity ($P$) so any observable sensitive to the EDM will need in addition a $P$-odd contribution. In our case this comes from the longitudinally polarized electrons. We use the notation of references [14,15,16]. The $\mathbf{s}^\pm$ are the $\tau^\pm$ spin vectors in the $\tau^\pm$ rest system, $s^\pm = (0, s^\pm_x, s^\pm_y, s^\pm_z)$. Polarization along the directions $x, y, z$ are called transverse (T), normal (N) and longitudinal (L), respectively. Let us first consider the s-channel $\tau$-pair production (diagrams (a) and (c) in Fig.(1)). We assume that the $\tau$ production plane and direction of flight can be fully reconstructed, which is the case for both $\tau$’s decaying semileptonically [17]. The spin dependent part of the differential cross section for $\tau$ pair production with polarized electrons with helicity $\lambda$ is:

$$\frac{d\sigma^S}{d\Omega_{\tau^-}}|_\lambda = \frac{\alpha^2}{16}\frac{s}{\beta}\{\lambda [(s_- + s_+)xX_+ + (s_- + s_+)zZ_+ + (s_- - s_+)yY_-] + (s_- - s_+)xX_- + (s_- - s_+)zZ_-\} \tag{6}$$

where

$$X_+ = \frac{1}{\gamma} \sin \theta_{\tau^-}, \quad X_- = -\frac{1}{2} \sin(2\theta) \frac{2m_\tau}{e} \text{Im} \left\{d_\tau^I\right\},$$

$$Z_+ = + \cos \theta_{\tau^-}, \quad Z_- = -\frac{1}{\gamma} \sin^2 \theta \frac{2m_\tau}{e} \text{Im} \left\{d_\tau^I\right\},$$

$$Y_- = \gamma/P \sin \theta_{\tau^-} \frac{2m_\tau}{e} \text{Re} \left\{d_\tau^I\right\}. \tag{7}$$

and $\alpha$ is the fine structure constant, $s = q^2$ is the squared CM energy and $\gamma = \sqrt{s/2m_\tau}$, $\beta = (1 - 1/\gamma^2)^{1/2}$ are the dilation factor and $\tau$ velocity, respectively. Eq. (7) shows that the $\tau$-EDM is the leading contribution to the Normal Polarization.

The cross section for the process $e^+e^- (pol) \rightarrow \gamma \rightarrow \tau^+\tau^- \rightarrow h^+\bar{\nu}_\tau h^-\nu_\tau$ can be written as [18]:

$$d\sigma \left( e^+e^- \rightarrow \gamma \rightarrow \tau^+\tau^- \rightarrow h^+\bar{\nu}_\tau h^-\nu_\tau \right) |_\lambda = 4 \left. d\sigma \left( e^+e^- \rightarrow \tau^+ (\bar{n}^*_+ \tau^- (\bar{n}^*_-) \right) \right|_\lambda \times Br(\tau^+ \rightarrow h^+\bar{\nu}_\tau) Br(\tau^- \rightarrow h^-\nu_\tau) \frac{d\Omega_{h^+}}{4\pi} \frac{d\Omega_{h^-}}{4\pi} \tag{8}$$

with

$$\bar{n}^*_\pm = \mp \alpha_\pm \frac{\bar{q}^*_\pm}{|\bar{q}^*_\pm|} = \mp \alpha_\pm (\sin \theta^*_\pm \cos \phi_\pm, \sin \theta^*_\pm \sin \phi_\pm, \cos \theta^*_\pm) \tag{9}$$

and $\theta_{\tau^-}$ is the center of mass angle of the $\tau^-$ with respect to the electron, $\phi_\pm$ and $\theta^*_\pm$ are the azimuthal and polar angles of the produced hadrons $h^\pm (\bar{q}^*_\pm)$ in
the \( \tau^\pm \) rest frame (the * means that the quantity is given in the \( \tau \) rest frame). If we integrate over the \( \tau^- \) angular variables then all the information on the \( Z_+ \) and \( X_- \) terms of the cross section is eliminated:

\[
d^4 \sigma^S |_{\lambda} = \frac{\pi^2 \alpha^2 \beta}{4s} \left( \frac{\alpha^2}{\beta} \right) BTH(\tau^+ \rightarrow h^+ \bar{\nu}_\tau) BTH(\tau^- \rightarrow h^- \nu_\tau) \frac{d\Omega_{h^+}}{4\pi} \frac{d\Omega_{h^-}}{4\pi} \times \left\{ \frac{\lambda}{\gamma} \left[ (n^*_x)^2 + (n^*_y)^2 \right] + \lambda \beta \left[ (n^*_x)^2 - (n^*_y)^2 \right] \frac{2m_\tau}{e} \text{Re} \{d_\tau^\gamma\} \right\} + \frac{4}{3\gamma} \left[ (n^*_x)^2 - (n^*_y)^2 \right] \frac{2m_\tau}{e} \text{Im} \{d_\tau^\gamma\}\right\} \quad (10)
\]

Subtracting for different helicities leaves only the real part of the \( \tau \)-EDM:

\[
d^2 \sigma^S |_{Pol(e^-)} = d^2 \sigma^S |_{\lambda=1} - d^2 \sigma^S |_{\lambda=-1} \quad (12)
\]

Keeping only azimuthal angles and integrating all other variables one gets:

\[
\left. \frac{d^2 \sigma^S}{d\phi_-d\phi_+} \right|_{Pol(e^-)} = \frac{\pi \alpha^2 \beta}{32s} BTH(\tau^+ \rightarrow h^+ \bar{\nu}_\tau) BTH(\tau^- \rightarrow h^- \nu_\tau) \times \left\{ \frac{1}{\gamma} \left[ (\alpha_-) \cos \phi_- - (\alpha_+) \cos \phi_+ \right] + \beta \gamma \left[ (\alpha_-) \cos \phi_- - (\alpha_+) \cos \phi_+ \right] \frac{2m_\tau}{e} \text{Re} \{d_\tau^\gamma\} \right\} \quad (14)
\]

We can now define the azimuthal asymmetry as:

\[
A^-_N = \frac{\sigma^\tau_- - \sigma^\tau_+}{\sigma} = \alpha_\pm \frac{3\pi \gamma \beta}{8(3-\beta^2)} \frac{2m_\tau}{e} \text{Re} \{d_\tau^\gamma\} \quad (15)
\]

where

\[
\sigma^\tau_- = \int_0^{2\pi} d\phi_- \left[ \int_0^{2\pi} d\phi_+ \frac{d^2 \sigma^S}{d\phi_-d\phi_+} \right]_{Pol(e^-)} = Br(\tau^+ \rightarrow h^+ \bar{\nu}_\tau) Br(\tau^- \rightarrow h^- \nu_\tau) \frac{1}{8} \frac{2m_\tau}{e} \text{Re} \{d_\tau^\gamma\} \quad (16)
\]

\[
\sigma^\tau_+ = \int_0^{2\pi} d\phi_+ \left[ \int_0^{2\pi} d\phi_- \frac{d^2 \sigma^S}{d\phi_-d\phi_+} \right]_{Pol(e^-)} = -Br(\tau^+ \rightarrow h^+ \bar{\nu}_\tau) Br(\tau^- \rightarrow h^- \nu_\tau) \frac{1}{8} \frac{2m_\tau}{e} \text{Re} \{d_\tau^\gamma\} \quad (17)
\]
All other terms in the cross section are eliminated when we integrate in this way. Notice that this integration procedure does not erase suppressed contributions coming from the CP-even term of the Normal Polarization. To eliminate spurious CP-even terms we define a CP-odd observable by summing up the asymmetry in Eq.(15) for $\tau^+$ and for $\tau^-$

$$A_N^{CP} = \frac{1}{2} \left( A_N^+ + A_N^- \right) = \alpha_h \frac{3\pi\gamma\beta}{8(3 - \beta^2)} \frac{2m_\tau}{e} \Re \{ d_\tau \}$$  \hspace{1cm} (18)$$

The $\gamma-Z$ interference has been considered in ref.[16] at $q^2 = (10 GeV)^2$; this contribution is suppressed by a factor of the order of $10^{-6}$. This is two orders of magnitude below the expected sensitivity for the asymmetries. In any case these terms do not contribute to the CP-odd $A_N^{CP}$ of Eq.(18).

All these ideas can be applied for $e^+e^-$ collisions at the $\Upsilon$ peak where the $\tau$ pair production is mediated by the $\Upsilon$ resonances: $e^+e^- \rightarrow \Upsilon \rightarrow \tau^-\tau^-$. In this case the resonant diagrams (b) and (d) of Fig.(1) dominate the process on the $\Upsilon$ peaks. This has been extensively discussed in ref.[19]. The main result is that the $\tau$ pair production at the $\Upsilon$ peak introduces the same $\tau$ polarization matrix terms as the direct production with $\gamma$ exchange (diagrams (a) and (c)) except for a the overall multiplicative factor $|H(s)|^2$ in the cross section:

$$H(M_\Upsilon^2) = -i \frac{3}{\alpha} Br \left( \Upsilon \rightarrow e^+e^- \right)$$  \hspace{1cm} (19)$$

Besides, at the $\Upsilon$ peak, the interference of diagrams (a) and (d) plus the interference of diagrams (b) and (c) is exactly zero and so it is the interference of diagrams (a) and (b). Thus, the only contributions proportional to the EDM come with the interference of diagrams (b) and (d), while diagram (b) squared gives the leading contribution to the cross section. Finally we obtain no changes in the asymmetries we have already computed: the only difference is in the value of the resonant production cross section at the $\Upsilon$ peak that is multiplied by the overall factor $|H(M_\Upsilon^2)|^2$.

4 Bounds on the EDM

Let us discuss the $\tau$-EDM bounds that can be set by measuring this observable. We assume a set of integrated luminosities for high statistics $B$/Flavor factories. We also consider the decay channels $\pi^{\pm} \bar{\nu}_\tau$ or $\rho^{\pm} \bar{\nu}_\tau$ (i.e. $h_1$, $h_2 = \pi$, $\rho$) for the traced $\tau^{\pm}$, while we sum up over $\pi^{\pm} \nu_\tau$ and $\rho^{\pm} \nu_\tau$ hadronic decay channels for the non traced $\tau^{\pm}$.

The bounds for the $\tau$-EDM that can be set in different scenarios are:
\[ |Re \{d_\tau^e\} | \leq 4.4 \times 10^{-19} \text{ emc}, \text{ Babar + Belle at } 2ab^{-1} \]
\[ |Re \{d_\tau^\gamma\} | \leq 1.6 \times 10^{-19} \text{ emc}, \text{ SuperB/Flavor factory, 1 yr running, } 15ab^{-1} \]
\[ |Re \{d_\tau^\tau\} | \leq 7.2 \times 10^{-20} \text{ emc}, \text{ SuperB/Flavor factory, 5 yrs running, } 75ab^{-1} \]

These limits improve the PDG ones of Eq. [2] by two orders of magnitude.

To conclude, we have shown that low energy data allows a clear separation of the effects coming from the electromagnetic EDM, the weak EDM and interference effects. Polarized electron beams open the possibility to put bounds on the \( \tau \) EDM coming from single \( \tau \) polarization observables. These observables allow for an independent analysis of the EDM bounds from what has been done with other high and low energy data. The new bounds may by two orders of magnitude below the PDG limits.

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**References**

[1] W.-M. Yao et al., Journal of Physics G 33, (2006) 1.

[2] K. Inami et al. [BELLE Collaboration], Phys. Lett. B551 (2003) 16.

[3] Y. Fukuda et al. [Super-Kamiokande Collaboration], Phys. Rev. Lett. 81 (1998) 1562.

[4] J. Bernabéu, G.A. González-Sprinberg and J. Vidal, Phys. Lett. B326 (1994) 168.

[5] W. Bernreuther, U. Low, J. P. Ma and O. Nachtmann, Z. Phys. C 43, 117 (1989).

[6] M. Acciarri et al. [L3 Collaboration], Phys. Lett. B434 (1998) 169.

[7] D. Buskulic et al. [ALEPH Collaboration], Phys. Lett. B 346, 371 (1995); K. Ackerstaff et al. [OPAL Collaboration], Z. Phys. C 74, 403 (1997); H. Albrecht et al. [ARGUS Collaboration], Phys. Lett. B485 (2000) 37.

[8] B. Ananthanarayan and S.D. Rindani, Phys. Rev. D51 (1995) 5996.

[9] F. del Aguila and M. Sher, Phys. Lett. B252 (1990) 116.
[10] J.A. Grifols and A. Mendez, Phys. Lett. B255 (1991) 611 and Erratum Phys. Lett. B259 (1991) 512.

[11] R. Escribano and E. Masso, Phys. Lett. 395 (1997) 369.

[12] See for example [http://www.lnf.infn.it/conference/superb06/] and [http://www-conf.slac.stanford.edu/superb/]

[13] W. Buchmuller and D. Wyler, Nucl. Phys. B268 (1986) 621; C.N. Leung, S.T. Love and S. Rao, Zeit. für Physik C31 (1986) 433; M. Bilenky and A. Santamaria, Nucl. Phys. B420 (1994) 47.

[14] G.A. González-Sprinberg, A. Santamaria, J. Vidal, Nucl. Phys. B582 (2000) 3.

[15] J. Bernabéu, G.A. González-Sprinberg, M. Tung and J. Vidal, Nucl. Phys. B436 (1995) 474.

[16] J. Bernabéu, G.A. González-Sprinberg and J. Vidal, Nucl. Phys. B763 (2007) 283.

[17] J. H. Kuhn, Phys. Lett. B 313 (1993) 458.

[18] Y.S. Tsai Phys. Rev. D4 (1971) 2821.

[19] J. Bernabéu, G. A. González-Sprinberg and J. Vidal, Nucl. Phys. B 701 (2004) 87.