The Effects of Diffusion of Information on Epidemic Spread
A Multilayer Approach

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Abstract

In this work, the aim is to study the spread of a contagious disease and information on a multilayer social system. The main idea is to find a criterion under which the adoption of the spreading information block or suppress the epidemic spread. A two-layer network is the base of the model. The first layer describes the direct contact interactions while the second layer is the information propagation layer. Both layers consist of the same nodes. The society consists of five different categories of individuals: Susceptibles, infective, recovered, vaccinated and precautioned. Initially, only one infected individual starts transmitting the infection. Direct contact interactions spread the infection to the susceptibles. The information spreads through the second layer. The SIR model is employed for the infection spread while the Bass equation models the adoption of information. The control parameters of the competition between the spread of information and spread of disease are the topology and the density of connectivity. The topology of the information layer is a scale-free network with increasing density of edges. In the contact layer, regular and scale-free networks with the same average degree per node used interchangeably. The observation is that increasing complexity of the contact network reduces the role of individual awareness. If the contact layer consists of networks with limited range connections, or the edges sparser than the information network spread of information plays a significant role in controlling the epidemics. Social Networks, Multilayer networks, Epidemic, SIR model, Diffusion of Information, Bass Model.
1 introduction

Social interactions are complicated relations of competing interests. In this sense, the social systems are complex systems. The modelling of the social phenomena requires a good understanding of the real interaction patterns and the dynamics among the members of the society. In the last 20 years the interest on the complex networks has intensified. Recent realistic models of real-world complex systems [1, 2] has improved the understanding of a large variety of complex social interactions. Nevertheless, as the knowledge changes new aspects of the social interactions enter the modelling considerations. As the societies become more technology oriented, new channels of communication and interaction rapidly changed the structure and the topology of the interaction networks. Previously designed single layer real-world networks left their place to multilayer networks: A developing social phenomenon finds its reflections on the other layers of social networks as different types of interactions. The spread of contagious disease is an excellent example of this situation. In the contact layer, the interactions of the individuals result in the spread of infection while in the second layer, the information on the contagiousness of the disease motivates the individuals to take preventive measures. Considering the severeness of both human and economy wise results of an epidemic, the importance of the more in-depth understanding of the role of inter-woven social networks during an epidemic spread becomes more apparent.

The mathematical models of diffusion of contagious diseases have a long history, starting early 20th century. The early models [3, 4] are aggregate models. The early models have rapidly evolved to agent-based models and lately also incorporated the structure of the underlying social networks [5, 6]. The epidemic studies on networks [7, 8, 9, 10] are not only useful and limited to the spread of contagious diseases within the human societies. A large variety of complex systems, such as physical, engineering, technological, and information networks exhibit similar diffusion of malicious agents [11, 12, 13, 14]. Complex networks are potent tools to describe spreading phenomena in both human societies and the other real-life problems. Never the less the spreading phenomena among the human societies have more elements than a single complex network. As a simple example, the traveling individuals change the dynamics of spreading infections. Similarly, information networks and social networks affect the dynamic of spreading.
Hence recently the models of spreading the infections are extended from single complex network to multilayer networks. Multilayer networks [15, 16, 17] are composed of several layers of complex structures in which the same node may have multiple channels of interactions.

In this paper, the focus is on the topic of spreading of contagious disease while the nodes communicate on the severeness of the epidemic. A two-layer network is employed. The first layer is the network of contact interactions where the epidemic spreads, while the interaction on the second layer spreads the information. The individual gain awareness by the information gathering from the information network.

The dynamics of epidemic spreading is one of the hottest research topics in complex network science. The most commonly encountered contagious diseases are suitably modeled by the susceptible-infected-recovered (SIR), susceptible-infected-susceptible (SIS) and susceptible-infected (SI) Epidemiology models. Different spreading mechanisms and epidemic control strategies are introduced for all three types of epidemiology models on complex networks [5, 18, 19, 6]. In fighting the infectious diseases, best prevention strategy is the immunization. The immunization of the whole population is not a possibly realizable challenge [23]. Hence various strategies of immunization which may be effective in the prevention of further spreading the infection are introduced. Random immunization and targetted (selected) immunization are the methods which aim to block the spreading paths of the contagion. The efficiency of the immunization is greater if one can select highly connected nodes. Such a selection requires prior knowledge of the whole network. Another immunization strategy is the acquaintance immunization in which the selection of highly connected nodes is naturally realized [18, 20, 19, 21, 22].

An other approach to the efficient immunization is awareness motivated immunization: the informed individuals decide to take precautions. The most effective element of the decision-making process is the word-of-mouth. The word-of-mouth immediately recalls one-to-one interaction. In the real-life, the contact networks are only the small part of the interaction network. In the modern societies, most of the information comes from the virtual-communication networks. In this sense, the word-of-mouth is all trustable one-to-one correspondences. The human element of the immunization strategies constitutes spread of information and decision-making processes. The multiple networks widen the understanding of epidemic and epidemic control methods by introducing multiple spread mechanisms. The best example
is the disease spreading on the contact network during the diffusion of information on another. The information creates the awareness of which is essential to control the epidemic spread \cite{24, 25, 26} hence, the competition between the awareness and the disease spreading may rise to an epidemic threshold \cite{27, 28, 29}.

In the multilayer networks the same nodes, the constituents of the society, are shared by different layers of the network. Each layer has different edge topologies and dynamics. The multilayer networks capture correct interaction structures between the nodes since an individual in a society may have different kinds of interactions such as business relations, social environment, connections through social media. Hence each is in direct contact with some members of the society while communicating with some others on a virtual network of friends. On online social networks, the information propagates between the nodes through friendship connections which may be entirely different from the contact network of the nodes.

In the proposed model, two interacting networks constitute the base of the artificial society. The first network is the contact network in which contact interactions result in the spread of contagious disease. At this layer, SIR model governs the dynamics of the diffusion of contagion. The second layer, information spread layer, connects the same nodes with a different connectivity pattern. Not all of the informed individuals act upon the received knowledge. There is an adoption process after which the individual reacts. The Bass model \cite{30} governs the information adoption process. The Bass model originally was introduced to describe the adoption process of a new product. Despite its simplicity of the model is still successful to explain the diffusion of new ideas, information and it is commonly used in marketing studies. The main success of the Bass Model is due to the well represented social behavior of the individuals. This classification is based on Roger’s seminal work \cite{31, 32} on the diffusion of innovation. Bass Model assumes to types of individuals. The first type accepts the new idea as soon as it is introduced. The second one is the majority of the population who like to see the benefit of adaption of the new information. In this work the Bass model \cite{30} sets the dynamics of information spread. In the multilayer approach to disease spread, adoption of the information may lead to the adoption of a method of prevention of the disease.
2 The Model

The model consists of $N$ nodes which accommodate $N$ interacting individuals. A two-layer multiplex network, which has common nodes but different connectivity pattern, connects individuals with each other. Two sets of parameters, each set indicating the state of the individual in the corresponding layer, identify the individuals. Hence the $i^{th}$ node is represented by $X_i[S_{\text{Layer}1}, S_{\text{Layer}2}]$. The first layer is the contact layer where the infection spreads. The contact layer state parameter, $S_{\text{Layer}1}$ has five values: Susceptible $S$, infected $I$, recovered $R$, vaccinated $V$ and the letter $P$, indicate the individual who has taken precautionary measures. The individuals who take precautionary measures remain susceptible, with a reduced probability of interactions with their neighbors. The information spreads on the second, virtual, layer. The awareness parameter, $S_{\text{Layer}2}$ takes only two values aware (informed) $AW$ and non-aware (uninformed) $NA$.

2.1 Interactions

Initially all nodes are initialized as susceptible, $S_{\text{Layer}1} = S$, and non-aware, $S_{\text{Layer}2} = NA$. The infection spread from only one randomly chosen node $S_{\text{Layer}1} = I$. An infected individual automatically becomes aware, $S_{\text{Layer}2} = AW$. Both contamination and information spread start from this single node. The infection spreads in the first layer by the contact interactions. SIR model dynamics,

$$\frac{dS(t)}{dt} = -\beta I(t) S(t)$$
$$\frac{dI(t)}{dt} = \beta I(t) S(t) - \gamma I(t)$$
$$\frac{dR(t)}{dt} = \gamma I(t)$$

(1)

where $S(t)$, $I(t)$, and $R(t)$ are the number of susceptible, infected and removed individuals at time $t$. The SIR model has two free parameters, $\beta$ and $\gamma$. The parameter, $\beta$, represents an average rate of encounters between the infected and susceptible individuals. The second parameter, $\gamma$ is the rate of recovery per unit time. The recovered infected individuals gain immunity.
For the agent-based simulation model, SIR model dynamics is implemented as probabilistic interactions among the members of the society. At each time step, randomly selected nodes interact with the neighbors at the contact layer and spread information on the virtual network. The rules are: If a susceptible or a precautioned individual interacts with an infected neighbor, become infected ($S$ and $P \rightarrow I$) with probability $\beta$. Recovered, $R$ and vaccinated, $V$ individuals are not affected from an infected member of the society. The precautioned individuals, $P$ avoid interaction with individuals in any state with probability $Prb$.

The information layer serves for two purposes: Spread and adoption of the information on the disease. For both of these processes Bass model is suitable since the model parametrize the human behavior for the adoption processes. The original form of the Bass model assumes two different type of individuals. The first group is the innovators who adopt a new idea immediately after being informed. The second group is the imitators who want to see the results of the accepting the new idea by observing the results on already adopted individuals. A new idea starts to diffuse through innovators. After a certain number of initial adopters, imitators are the main driving force in the spread of information. In the classical form the Bass equation,

$$\frac{dA_W(t)}{dt} = (p + \frac{q}{N}A_W(t))(N - A_W(t))$$

where, $p$ and $q$ are innovation and imitation parameters, $N$ and $A_W$ are the total number and the number of aware individuals. Here, the innovation parameter best understood as the probability of adoption of new information immediately after being informed. The imitations parameter is related to the probability of adoption after observing the experiences of the neighbors (Word-of-mouth). Informed individuals transmit the information to their neighbors through their connections on the second layer. When an individual receives the information evaluates the information. According to the dynamics defined by the Bass equation, the information is adopted or not. In the agent-based approach, the information adoption takes the following form:

$$X_i[S, NA] = X_i[S, AW]$$

if $p > r$ or

$$X_i[S, NA] = X_i[S, AW]$$

else if $\frac{q}{NN} \times NN_{AW} > r$
here, $N_N$ and $N_{AW}$ are the number of nearest neighbors and number of aware neighbors respectively. If the information is adopted the awareness state is set to aware, $S_{layer_2} = AW$. Once an individual is informed remains informed, but only once sends the message to all neighbors after adopting the information.

The individuals take precursory measures according to their attitudes. Two precursory measures are vaccination and reducing the probability of interactions. If the susceptible individual is vaccinated ($V$), gain immunity. If an individual takes a precursory measure of reducing the number of interactions does not gain immunity. They remain susceptible, but their interaction probability is reduced. The interaction at the information layer leads to the adoption of information and decision of a precursory action.

If an individual is susceptible and informed, $X_i[S, AW]$, may take precaution or may prefer vaccination.

\[
\text{if } X_i[S, AW] = \begin{cases} 
X_i[P, AW] & \text{if } Prb > r \\
X_i[V, AW] & \text{if } (1 - Prb) > r 
\end{cases}
\]  

(4)

The unaffected and aware individuals may be in two states: Vaccinated, $V$, or precaution is taken, $P$ in which case the probability of interactions of the individual changes. At each time step a randomly choosen individual interact with a randomly choosen neighbor. The individual and its neighbor can be in any of the five states. Unles the interaction is between a susceptible and contaminated, interacting individuals do not change state. There are two types of susceptibles: $S$ and $P$ state individuals. For $S$ state each interaction with a infective individual spread contamination. For the individuals who are in the $P$ state, the individual does not interact at every time step even if they are choosen. Their interactions are limited with a probability $p_{interaction}$ which represent the prevention effort of the $P$-state individuals. Interaction probability is kept constant as $p_{interaction} = 0.25$, only one fourth of the encounters ends with a physical contact. If a $P$ state individual interact with an $I$ state individual change state.

### 2.2 The multiplex network

Two interconnected networks, one for contact interactions and the second one for the spread of information carry the social interactions. Both sys-
tems share the same nodes with different intra-layer connectivity structures. The proliferation of contagious disease progresses on the contact network. The contact network layer has two alternative network structures: Regular two-dimensional lattices with periodic boundary conditions and scale-free networks. The underlying network structure is scale-free for the information layer. This choice is due to the similarities between the scale-free and the real-world social network structures.

Both regular and scale-free networks are used as the contact layer. In the regular network case, periodic boundary condition with simple square \((k = 4)\) and triangular \((k = 6)\) lattices are used to test the effects of connectivity. Two different scale-free networks with the same average connectivity \((< k > = 4 \text{ and } 6)\) per node are tested on the contact interaction layer. Barabási-Albert network algorithm is used to generate the scale-free networks. In this algorithm, the number of seed nodes, \(m\), guarantees the average number of undirected edges, \(< k > = 2 \times m\). Changing the number of seed nodes controls the density of the number of connections, the degree of the node. The degree distribution of the nodes affects the spread of the information and the contagious disease. On the information layer, only scale-free networks are used. The networks with a wide range of average degree distributions are obtained by using the Barabasi-Albert algorithm for the information layer. The effects of information spread on the spread of contagious disease are tested by using lattices in the range of \(< k > = 4 \text{ to } 20\). The relation between the connectivity structure of two layers and the speed of the disease spread is the subject of the next section.

\section*{3 Results and Discussions}

An artificial society of \(N = 10000\) inhabitants, each occupying a node on a multiplex network, are the constituents of the simulation system. The connectivity of the nodes is two-fold. The first layer of the multiplex network is the contact network where individuals interact with each other through direct contact interactions. Hence, the contact layer provides a media for the transmission of contagious disease. The second layer is the information layer, through which the information spread via virtual contacts. The conditions and the speed of the spread of news and infection are a function of both topology and the average degree of the nodes. The contact layer consists of both regular lattice and scale-free networks while for the information spread
layer, only scale-free networks with varying average degree per node are used. The presented results are the averages of 100 simulations each starting from a statistically independent initial configuration. The creation of an initial configuration consists of creation of multiplex network, initializing both contact and information layer state parameters of each node. Iterations continued until the stationary configurations are reached. The required time duration varies according to the topology and the density of the links of the contact layer. For regular lattices, approximately 250 time steps are observed to be sufficient. Barabási-Albert network provides a faster transmitting media. The system reach the stable configurations after only 50 time steps. During the simulation all parameters, apart from the lattice parameters are kept fixed to compare the effects of the lattice topology. The contact layer parameters which controls the spread of contagious disease, the infection transmission, $\beta$ and recovery, $\gamma$ parameters of the SIR model are kept constant for all networks. The transmission and recovery parameters are $\beta = 1$ and $\gamma = 0.2$ respectively. The information adoption is controled by the Bass equation parameters, $p$ and $q$. Individuals who adopt an information immediately after being informed are rare. The majority adopt after observing the results of first hand experiances. The values of innovation and imitation parameters are assumed to be similar to those of the average values obtained from the marketing studies. From marketing the average ranges are $0.001 < p < 0.1$ and $0.1 < q < 0.5$ for innovation and imitation parameters respectively. In this work, the fixed values of $p = 0.05$ and $q = 0.35$ are employed for all lattices.

In the societies, the direct contact networks usually have relatively small average degree per node. Hence in the contact layer, the average degree per node is limited to $< k > = 4$ and $< k > = 6$. The regular networks are 2-dimensional simple square ($k = 4$) and triangular ($k = 6$) lattices with periodic boundary conditions, while the scale-free networks are generated by using Barabási-Albert algorithm with initial sites of 2 and 3 which corresponds to the average degree, $< k > = 4$ and 6. The information spread layer is expected to have denser connections between the nodes. Hence, undirected scale-free networks with increasing density of the edges are generated by using Barabási-Albert algorithm. The average degree, $< k >$, per node is the control parameter of the spreads on the different information networks.

Figure 1 shows the spread of contagious disease on regular and scale-free networks without the contribution of information layer. Two different topologies, with the same average degree per node, are square and triangular
lattices and $m = 2$ and $m = 3$ Barabási-Albert networks respectively. The comparison between the figures 1(a) and 1(b) shows that the spread of infection on the contact layer, almost three times faster on the scale-free network than the corresponding regular network for the same transmission and recovery parameters. Since the transmission parameter is high, disease spread among the population, but the peak values of the number of infected individuals are differ depending on the topology. In the scale-free network case, almost half of the population is contaminated at the peak of the infection spread. In the regular lattice case, the peak value of the number of infected remains around 10% of the total population. The increasing number of average connections per node pronounces the difference. Its significance becomes more apparent when the availability of the work-force and continuity of the social system is considered. Hence reducing the peak value of the number of infected through mass media and social media plays a crucial role in the continuity of the social systems.

Constructing multiplex networks to study the effects of the information spread requires comparing multiple information networks which have the same contact layer network. As the first set of examples, a simple square lattice and a set of scale-free networks with the progressively increasing number
of edges are taken as the topologies of contact and information networks respectively. Figure 2 shows the effects of increasing number of communication links. Usually, in the social systems, the contact networks are local interactions. Hence, diffusion takes more time than real-world networks. In this first model, increasing number of second layer links speeds up the spread of information. If the individuals absorb and use the information in the correct way by taking precautions or getting vaccinated, the disappearance or at least control of the contamination is possible. Figures 2(a), 2(b), 2(c) and 2(d) show the effects of the increasing number of links on the spread of contamination. As explained on the section 2.1, the contamination spread according to the dynamics of the SIR model with an infection transmission rate, $\beta = 1$. If a susceptible contacts with an infective gets contaminated. Each informs all neighbors when infected. Adoption of the information is a process governed by the dynamics provided by the Bass equation. A small percentage of the individuals (innovators) immediately adopt the information and take a precaution. Others, collect information from the neighboring nodes before making a decision. Both innovators and imitators have two choices as far as the precautions are concerned: getting vaccinated and avoiding contacts with the neighbors. In this work, the first assumption is that only 20% of the informed individuals choose vaccination. The rest prefers to keep away from any contact interaction. A second assumption is that on the 75% of the occasions susceptibles can save themselves from contamination by avoiding direct contact.

Figure 3 show the changes in the number of susceptibles, infected, recovered, vaccinated and precautioned for a constant speed of contamination spread, the peak of the number of infected individuals decreases with increasing density of the information links. Figure 3(a) shows the changes in the number of susceptibles with as a function of time and number of initial sites, $m$ (Average degree, $< k > = 2 \cdot m$ ). For small $m$, all individuals get infected. As the number of initial sites approaches to $m = 10$, over 40% of all susceptibles remain unaffected from the contamination which reduces the number of recovered (Figure 3(c)). Similarly, the peak of the number of infected individuals decreases rapidly with the increasing number of information links (Figure 3(b)). The number of vaccinated remains rather small compared with the number of precaution. Figure 3(d) shows the changes in the number of vaccinated (Below) and the number of precautioned (Above) respect to the changes in the number of connections in the information layer.

This effect manifests itself more profoundly in scale-free networks in the
Figure 2: Contagious disease spread on simple square lattice, $k = 4$, while the information spreads on scale-free network topology with increasing connectivity.
Figure 3: The effect of increasing connectivity of the information network on the infection spread. The number of initial sites of the information layer changes from 0 (network consists of only the contact layer) to 10.
contact layer case. When the contact layer is in scale-free topology, the speed of transmission of infection is high comparing with the regular networks. Therefore, the peak of the number of infected is higher in the scale-free contact network concerning regular networks. Figures 1(a) and 1(b) show the differences in speed and the scale of the contamination between lattices and scale-free networks with equal average degree per node. The topology of the information layer plays a very important role in reducing both the total number of infected individuals and the peak in the number of infected individuals. Figure 4 show the effect of the increasing density of information links while contact network is also scale-free with an average degree of 4 per node. The real-world networks, due to complex connectivity structure, speed up the spreading phenomena. Figures 4(a), 4(b), 4(c), 4(d) show the effect of the density of information layer connections for fixed average degree in the contact layer. Increasing the number of average degree decrease the peak of the number of infected. Two effects contribute to the decrease in the infectives, vaccination and awareness. Informed individuals either get vaccinated and gain immunity or avoid direct contacts with neighbors. As the number of communication links increase, the number of aware individuals increase which result in reducing the number of infected. Comparison of the figures 4(a), 4(b), 4(c), 4(d) indicate that the main contribution in the prevention of the epidemic spread comes from the group of individuals who try to avoid direct contacts with the neighbours. This group of individuals increase as the number of infected individuals increase. Their peak is just before the peak of the number of infected individuals which prevent further contamination. As the number of links of the information layer increase, the peak of informed individuals increases with a further suppression on the spread of infection. The contribution on the prevention of the vaccinated does not grow with the same rate.

When the contact layer becomes denser, the propagation of the infection is very fast. Hence spread of information to prevent further spreads of the illness is less effective. Figure 5 shows the effect of information spread while the contact layer has scale-free topology with average degree per node is 6.

Figure 6 summarize the results of the model. The effect of the density of links on the information layer is observed on two different contact network topologies, regular and scale-free networks with equal average degrees per node. The figures 6(a) and 6(b) show the percentages of recovered (dash-dot), susceptible (dashed), precautioned (solid) and vaccinated (dashed) individuals after the ending the spread of infection. In fact, the bottom line shows
Figure 4: Spread of contagious disease in a society with scale-free multilayer network topology. Contact layer has single initial sites configuration, $m = 2$ while the number of initial sites of the information layer changes, $m = 2, 4, 6$ and 10.
Figure 5: Spread of contagious disease in multi-layer network with scale-free network topology. The same as figure 4 only the contact layer has denser connectivity structure, \( m = 3 \).
Figure 6: Aftermath of the epidemic. The susceptible, infected, recovered, vaccinated and precautioned population versus the number of initial sites of the information layer. Constant contact layer parameter is fixed to $k = 4$ for regular network (a) and $m = 2$ for scale-free network (b).
no infected individuals. Figure 6(a) indicate that as the average number
of links of the information layer increase the number of infected individuals
\( (R) \) decrease to almost 40% of the population which indicate that the total
number of healthy (vaccinated or uninfected) reach up to 60%. The situation
changes slightly in the case of scale-free contact layer with \(< k > = 4\). Figure 6(b)
show that with increasing information spread the percentage of
the recovered individuals goes down to only 60% of the population. Total
percentage of the non effected individuals are almost 40%. The difference
in the spread speed of the infection and information can explain this drastic
difference between two layer topologies on the scale-free networks.

4 Discussions and Conclusions

Recent analytical and simulation models indicate that the epidemic spreading
on physical contact networks ignite the spread of awareness. The awareness
of the individuals, in turn, suppress the disease spreading. In this work,
the discussion is the relation between the epidemic spread and the effect
of individual awareness. In the proposed model society, the individual interacts
through a two-layer multiplex network, physical contact and information
spreading layers. The common nodes are affected by both the infection and
information spreading in different layers. The dynamics of infection and in-
formation spreads are controlled by the SIR and Bass models respectively.
Adoption of information changes the attitude of the individuals; awareness
diffusion creates a group of self-protected individuals. In this model, two
types of self-protection are considered. Vaccination is the ultimate immu-
nization method for most of the viral infections. Nevertheless, vaccination
requires some effort, time and expenditure. Hence, the first assumption is
that only 20% of the population consider vaccination. The rest try to avoid
contacts with neighbors. The price of not being vaccinated is the that the
precautions, apart from the vaccination provide only partial protection. As
a second assumption, protection level of 75% is used to change the character-
istics of the infection spread. Different topologies of contact and information
networks embed different diffusion dynamics. Even the same topology with
increasing number of an average degree changes the spread rates of informa-
tion and contamination. The effect of awareness on suppressing the infection
spread makes its impact if the contact network diffusion speed is less than
the spreading speed of information. The individual responce to an epidemic
situation exhibit similarities but also vary from the adoption of an innovation. In the epidemic case there exists an immediate danger to the well being of the individual. Identification of the individual response parameters, by using the real data, may improve the epidemic prevention efforts considerably.

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\[ S_m = 2 \]
\[ I_m = 2 \]
\[ R_m = 2 \]
Time

Population Percentage

- $S_m = 3$
- $I_m = 3$
- $R_m = 3$
