We have performed strip line ferromagnetic resonance on weak stripe domains in permalloy layers. We used Thiele equation of motion to interpret our experimental data: the lowest frequency mode corresponds to the motion of vortices inside the layer, its frequency is evaluated through the relation 
\[ \omega = \frac{k}{G} \]
where \( G \) is the gyrovector magnitude and \( k \) is a stiffness which mainly depends on demagnetizing effects.

1. Introduction
In the last decade, many experimental and numerical studies dealt with dynamics of non saturated magnetic films, lines and dots [1,2,3]. Measurements were performed using various techniques such as ferromagnetic resonance using cavities [1] or strip lines [2] and Brillouin light scattering [3]. One of these interesting problems on non uniform magnetization dynamics is the resonances of weak stripe domains (periodic configuration of vortices with alternate magnetization circulation). These resonances have been experimentally and numerically studied. The numerical simulations prove that the lowest frequencies modes are localized at the center of the vortices or at the center of regions near the surfaces where the magnetization direction drastically changes [4]. In this paper, we propose to explain our experimental results on permalloy films obtained through a strip line resonance device, by using the Thiele equation of motion for magnetic configurations such as vortices or regions near the surface.

2. Experiments
We studied a permalloy film elaborated by sputtering [5]. It was observed through magnetic force microscopy (MFM). The MFM images clearly evidenced weak stripe domains. This configuration appears if the perpendicular magnetic anisotropy energy \( (K) \) is lower than the demagnetizing energy \( (2\pi M^2) \) and if the film thickness is higher than the critical thickness \( (D_c) \) which depends on the exchange \( (A) \) and on the anisotropy energy \( (K) \): e.g. if \( K \ll 2\pi M^2 \) then \( D_c = 2\pi \sqrt{A/K} \).

The studied sample is characterized by the following parameters: thickness \( D = 450 \times 10^{-7} \text{cm} \), \( 4\pi M = 10^4 \text{ G} \), \( \frac{2\pi}{k} = 3 \times 10^{-3} \text{ GHz/Oe} \), \( A = 10^{-6} \text{ erg/cm} \), \( K = 8 \times 10^4 \text{ erg/cm}^3 \). For this sample, \( 2\pi M^2 = 4 \times 10^6 \text{ erg/cm}^3 \) is much greater than \( K \) thus \( D_c = 2\pi \sqrt{A/K} = 222 \times 10^{-7} \text{cm} \). The sample thickness is about twice as high as the critical thickness.

The magnetization dynamics was studied by a strip line device. Eigenfrequencies were measured versus the magnetic field applied in the direction of the stripes. Figure 1 displays the variation of the 2 lowest frequencies which concern motion of localized regions in the film as
proved by numerical calculations [4]. When the applied field is greater than the saturating field (about 100 Oe), only one mode is observable: it is the uniform mode, its frequency is obtained by the relation \( \omega = \gamma \sqrt{H(H + 4\pi M)} \); this expression is valid in the case of vanishing anisotropy.

For the studied sample submitted to a field \( H = 100 \) Oe, the frequency is about 3 GHz.

![Figure 1. Lowest frequencies vs applied field, dots are experimental data, lines correspond to an approximate calculation](image)

3. Interpretation

We study the magnetization oscillation in a non saturated magnetic film where the static magnetization is organized into weak stripe domains. Numerical calculations [4] demonstrated that the lowest frequency modes are localized in areas where the magnetization direction drastically changes. That is the reason why we attempt to calculate these frequencies using Thiele equation of motion [6]: each area is considered as an object that oscillates around its equilibrium position. Thiele equation reads (neglecting dissipation) :

\[
\vec{G} \times \vec{V} + \vec{F} = 0
\]

where \( \vec{G} = \frac{1}{7M^2} \int \int \left( \left( \dot{\vec{M}} \cdot (\frac{\partial}{\partial y} \vec{M} \times \frac{\partial}{\partial z} \vec{M}) \right) \dot{\vec{e}}_x + \left( \dot{\vec{M}} \cdot (\frac{\partial}{\partial z} \vec{M} \times \frac{\partial}{\partial x} \vec{M}) \right) \dot{\vec{e}}_y + \left( \dot{\vec{M}} \cdot (\frac{\partial}{\partial x} \vec{M} \times \frac{\partial}{\partial y} \vec{M}) \right) \dot{\vec{e}}_z \right) dx dy dz
\]

\( \vec{V} \) is the speed of the center of the object and \( \vec{F} \) is the force applied to the object. This equation is obtained by assuming that a configuration moves without distortion: \( \vec{M}(\vec{r}, t) = \vec{M}(\vec{r} - \vec{R}(t)) \) where \( \vec{R}(t) \) is the position of the center of the configuration. Thus \( \frac{d}{dt} \vec{M} = -\vec{V} \cdot \nabla \vec{M} \) is large where \( \nabla \vec{M} \) is large. In the Thiele framework, the magnetization mainly oscillates in the vortices or between closure areas.

3.1. Description of the vibrations

Weak stripe domains are a periodic configuration (figure 2). For low \( \frac{K}{2\pi M} \) materials, this pattern exhibits an aspect similar to the Landau-Lifshitz type which reduces for thin film as piled-up vortices with alternate magnetization circulation (figure 3). One period can be regarded as 2 vortices (3,4) surrounded by 4 boundary objects (1,2,5,6) between closure areas.

At the center of each object, the magnetization is parallel to the z axis. At a distance \( r \) from the center, the magnetization makes an angle \( \theta \) with respect to the z axis; this angle only depends on \( r \). The magnetization component in the x y plane makes an angle \( \phi \) with respect to x axis; this angle only depends on the angle \( \alpha \) defining the position in the x y plane \((x = r \cos \alpha, y = r \sin \alpha)\). We have \( \phi_1 = \phi_6 = -\alpha, \phi_2 = \phi_5 = \pi - \alpha, \phi_3 = \alpha + \pi/2, \phi_4 = \alpha - \pi/2 \).

Each object is characterized by a gyrovector \( \vec{G} = \frac{M}{L} (\cos(\theta_{\text{max}}) - 1) (\phi(\alpha_{\text{max}}) - \phi(\alpha_{\text{min}})) L \vec{e}_z \) where \( L \) is the length of a stripe.
Consequently the 2 vortices have the same gyrovector $\vec{G}_3 = \vec{G}_4 = G\vec{e}_z$ with $G = (\cos(\theta_{max}) - 1)2\pi M/L$ and the 4 objects between closure areas have the same gyrovector $\vec{G}_1 = \vec{G}_2 = \vec{G}_5 = \vec{G}_6 = G'\vec{e}_z$ with $G' = (1 - \cos(\theta_{max}))2\pi M L$.

The 2 vortices are surrounded by 4 equivalent objects, thus we assume that for $p=3,4$, the force $\vec{F}_p$ derives from the energy $\frac{1}{2}k(x_p^2 + y_p^2)$ where $(x_p, y_p)$ is the displacement of the center of the object $p$. $k$ is the restoring force constant given by the second derivative of the potential part of the energy.

The situation of the object 1,2,5,6 is not symmetric, thus we assume that for $p=1,2,5,6$, the force $\vec{F}_p$ derives from the energy $\frac{1}{2}(k'x_p^2 + k'y_p^2)$ where $(x_p, y_p)$ is the displacement of the center of the object $p$. The constants $k'$ and $k''$ are different because of the asymmetric environment.

We consider oscillatory displacements: $x_p = X_p \cos(\omega_p t)$, $y_p = Y_p \cos(\omega_p t + \varphi_p)$. The 2 vortices have the same gyrovector and are submitted to the same force thus $X_3 = X_4 = X$, $Y_3 = Y_4 = Y$, $\omega_3 = \omega_4 = \omega$, $\varphi_3 = \varphi_4 = \varphi$.

Thiele equation yields $-G\omega = k$, $\omega = -\frac{2}{2\pi}, X = Y$

The 4 boundary objects have the same gyrovector and are submitted to the same force thus for $p=1,2,5,6$, $X_p = X'$, $Y_p = Y'$, $\omega_p = \omega'$, $\varphi_p = \varphi'$.

Thiele equation yields $G'\omega' = \sqrt{k'k''}$, $\omega' = \frac{2}{2\pi}, X'\sqrt{k''} = Y'\sqrt{k''}$

### 3.2. Zero Field Frequencies

Measurements at zero field give $\frac{\omega}{2\pi} = 1$ GHz and $\frac{\omega'}{2\pi} = 1.5$ GHz. The studied sample is characterized by the following parameters: thickness $D = 450 \times 10^{-7}$ cm, $4\pi M = 10^4$ G, $\frac{B}{2\pi} = 3 \times 10^{-3}$ GHz/Oe, $A = 10^{-6}$ erg/cm, $K = 8 \times 10^4$ erg/cm$^3$. We deduce $\frac{2}{\gamma} = 330$ Oe, $\frac{\omega'}{2\pi} = 300$ Oe, and assuming $\theta_{max} = \frac{\pi}{2}$ we deduce $k/L = 1.7 \times 10^6$ erg/cm$^3$ and $\sqrt{k'k''}/L = 1.3 \times 10^6$ erg/cm$^3$.

These values of $k/L$ and $\sqrt{k'k''}/L$ have the same order of magnitude as the demagnetizing energy $2\pi M^2 = 4 \times 10^6$ erg/cm$^3$, while they are much greater than the anisotropy energy $K= 8 \times 10^4$ erg/cm$^3$ or than the wall energy $\sqrt{AK}/D = 6.2 \times 10^3$ erg/cm$^3$ or than the exchange energy $A/D^2 = 490$ erg/cm$^3$. We conclude that the origin of the force $\vec{F}_p$ is the demagnetizing field induced by the displacement of the objects. This is illustrated in the case of the displacement of vortex 4 along the x axis on figure 4.

This displacement induces positive magnetic charges on the upper diagonals and negative magnetic charges on the lower diagonals: $\sigma = \vec{n}_{1-2}.(\vec{M}_1 - \vec{M}_2) = 2M \delta (\cos \psi)$ where $\psi$ is the angle between $\vec{n}_{1-2}$ and $\vec{M}_1$ thus $\sigma = -\sqrt{2}M \delta \psi$; moreover for the upper diagonals $\delta \psi = -x\sqrt{2}/D$ and for the lower diagonals $\delta \psi = x\sqrt{2}/D$. These charges induce a demagnetizing field $H_d \sim Mx/D$ from the upper side to the lower side. The induced variation of energy reads

![Figure 2. Weak stripe domain](image)

![Figure 3. Model](image)
\[ \frac{1}{2} MH_d S_{left} L - \frac{1}{2} MH_d S_{right} L \] where \( S_{left} \) is the area of the left triangle and \( S_{right} \) is the area of the right triangle thus \( S_{left} - S_{right} \sim Dx \), thus this variation of energy is proportional to \( M^2 x^2 L \). We deduce that \( k/L \sim M^2 \). This calculation provides an estimation of \( k/L \) in agreement with the measurements. A similar description is possible to estimate \( \mu = L \).

### 3.3. Frequency dependence on applied field

The variation of the frequencies versus the applied field \( H \) can be explained in the following way: \( H_d \) and the magnetization component in the section are proportional to \( \sin(\theta_{\text{max}}) \) thus \( k/L \sim \sin^2(\theta_{\text{max}}) \) and \( \sqrt{\kappa k' L} \sim \sin^2(\theta_{\text{max}}) \) while \( G \sim 1 - \cos(\theta_{\text{max}}) \) and \( G' \sim 1 - \cos(\theta_{\text{max}}) \).

Thus \( \omega \) and \( \omega' \) are proportional to \( \frac{\sin^2(\theta_{\text{max}})}{1 - \cos(\theta_{\text{max}})} = 1 + \cos(\theta_{\text{max}}) \). While the applied field increases, \( \theta_{\text{max}} \) decreases thus \( \omega \) and \( \omega' \) increase. This approach agrees with the measurements up to 60 Oe (see figure 1) : frequencies are calculated through the expression \( \omega = \omega(0)(1 + \cos(\theta_{\text{max}})) \) with \( \theta_{\text{max}} = \frac{\pi}{2}(1 - \frac{H}{H_{\text{sat}}}) \) where \( H_{\text{sat}} \) is the saturating field i.e. \( H_{\text{sat}} = 100 \) Oe. The discrepancy between the calculations and the measurements around the saturating field are related to the variation of the ratio period/thickness occurring near the saturation. This distortion modifies \( k, k' \) and \( k'' \). Consequently they do not vary like \( \sin^2(\theta_{\text{max}}) \) in the vicinity of the saturation.

### 4. Conclusion

We studied permalloy films elaborated by sputtering. MFM images of these films clearly evidence weak stripe domains. We performed ferromagnetic resonance using a strip line device on these non saturated films. For low fields, we evidenced several eigen modes. These modes involve the motion of vortices or of configurations where the magnetization direction drastically changes. We used a description based on Thiele equation to give account of the experimental results.

### 5. References

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