Study of spin-density matrix in exclusive electroproduction of $\omega$ meson at HERMES

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Abstract. Exclusive electroproduction of $\omega$ mesons on unpolarized hydrogen and deuterium targets is studied in the kinematic region of $Q^2 > 1.0 \text{ GeV}^2$, $3.0 \text{ GeV} < W < 6.3 \text{ GeV}$, and $-t' < 0.2 \text{ GeV}^2$. The data were accumulated with the HERMES forward spectrometer during the 1996-2007 running period using the 27.6 GeV longitudinally polarized electron or positron beam of HERA. Spin-density matrix elements are presented in projections of $Q^2$ or $-t'$. Violation of $s$-channel helicity conservation is observed for some of these elements. A sizable contribution from unnatural-parity-exchange amplitudes is established for special combinations of spin-density matrix elements. The determination of the virtual-photon longitudinal-to-transverse cross-section ratio reveals that a dominant part of the cross section arises from transversely polarized photons. Good agreement is found between the HERMES proton data and results of a pQCD-inspired Goloskokov-Kroll model that includes pion-pole contributions.

1. Introduction
Exclusive electroproduction of vector mesons on nucleons gives information both on the reaction mechanisms and on the nucleon structure [1, 2]. The electroproduction at high energies can be considered to consist of three subprocesses: i) the incident lepton emits a virtual photon $\gamma^*$, which dissociates into a quark-antiquark pair; ii) this $q\bar{q}$ pair interacts strongly with the nucleon; iii) the observed vector meson is formed from the scattered $q\bar{q}$ pair. In Regge phenomenology, the interaction of the $q\bar{q}$ pair with the nucleon proceeds through the exchanges of a pomeron or/and exchanges of other reggeons (e.g., $\rho, f_2, \pi, a_1, \ldots$). If the quantum numbers of the particle lying on the Regge trajectory are $J^P = 0^+, 1^-$, etc., the process is denoted Natural Parity Exchange (NPE). Alternatively, the case of $J^P = 0^-, 1^+$, etc. corresponds to Unnatural Parity Exchange (UPE). In perturbative quantum chromodynamics (pQCD), the interaction of the $q\bar{q}$ pair with the nucleon proceeds via two-gluon exchange or $q\bar{q}$ exchange, where the former corresponds to the pomeron exchange and the latter to the exchange of other reggeons. In the framework of pQCD valid at large photon virtuality $Q^2$ and high photon-nucleon center-of-mass (CM) energy $W$, the nucleon structure can also be studied through hard exclusive meson production as the process amplitude contains Generalized Parton Distributions (GPDs) [3, 4, 5]. The amplitude $F_{00}$ of longitudinal vector meson production by longitudinal virtual photons is proven to factorize rigorously into a perturbative calculable part that describes hard scattering of partons and two soft parts [5, 6]. The soft parts contain GPDs and a meson distribution amplitude. At high $Q^2$, $W$ and for leading twist, the GPDs $H^f$ and $E^f$ are sufficient to describe exclusive vector-meson production on a nucleon, where $f$ denotes a quark of flavor $f$ or a gluon. These GPDs are related.
to the total angular momentum carried by the parton in the nucleon [7]. In the Goloskokov-Kroll (GK) model (see [8] and references therein) the validity of factorization is assumed for other amplitudes in addition to $F_{00}$ and this assumption is justified with a good description of the existing data. Recently, Spin-Density Matrix Elements (SDMEs) were studied by CLAS [9] for exclusive $\omega$ electroproduction at $1.6$ GeV$^2 < Q^2 < 5.2$ GeV$^2$ and the contribution of the $\pi$-reggeon was found to be dominant. The presented HERMES data can also be well described in the GK model if the pion exchange is included into account.

2. Formalism

The $\omega$ meson is produced and decays in the following reactions:

$$e + p \rightarrow e + p + \omega, \quad \omega \rightarrow \pi^+ + \pi^- + \pi^0, \quad \pi^0 \rightarrow 2\gamma$$

with a branching ratio $Br = 89.1\%$ for the $\omega$ decay. The angular distribution of the three final-state pions depends on the SDMEs. In the present paper, the formalism proposed in Ref. [10] for SDMEs $r_{\lambda_{\omega}\lambda_V}^{\alpha}$ is used. The vector-meson spin-density matrix is expressed in terms of a set of nine matrices $r_{\lambda_{\omega}\lambda_V}^{\alpha}$ related to various photon polarization states: transversely polarized photon ($\alpha=0,...,3$) corresponding to the photon helicity $\lambda_\gamma = \pm 1$, longitudinally polarized photon ($\alpha=4$) for which $\lambda_\gamma = 0$, and terms describing their interference ($\alpha=5,...,8$) [10]. Here, $\lambda_V$ and $\lambda_V'$ denote the helicities of the vector meson. When contributions of transverse and longitudinal photons cannot be separated, the SDMEs $r_{\lambda_{\omega}\lambda_V'}^{\alpha}$ being the linear combination of $r_{\lambda_{\omega}\lambda_V}^{\alpha}$ and $r_{\lambda_{\omega}\lambda_V'}^{\alpha}$ are customarily used (see details in [10, 11]).

The vector-meson SDMEs $r_{\lambda_{\omega}\lambda_V'}^{\alpha}$ can be expressed in terms of helicity amplitudes $F_{\lambda_{\omega}\lambda_V\lambda_N\lambda_N'}$. These CM amplitudes of the process $\gamma^* N \rightarrow VN$ describe the transition of a virtual photon with helicity $\lambda_\gamma$ to a vector meson (V) with helicity $\lambda_V$, while $\lambda_N$ and $\lambda_N'$ are the helicities of the nucleon (N) in the initial and final states, respectively. Helicity amplitudes depend on $W$, $Q^2$, and $t' = t - t_{\text{min}}$, where $t$ is the Mandelstam variable and $-t_{\text{min}}$ represents the smallest kinematically allowed value of $-t$ at fixed virtual-photon energy and $Q^2$.

Any helicity amplitude can be decomposed into a sum of a NPE amplitude $T$ and an UPE amplitude $U$: $F_{\lambda_{\omega}\lambda_V\lambda_N\lambda_N'} = T_{\lambda_{\omega}\lambda_V\lambda_N\lambda_N'} + U_{\lambda_{\omega}\lambda_V\lambda_N\lambda_N'}$, for details see Refs. [10, 11]. In the one reggeon exchange approximation, NPE amplitudes are due to exchanges with pomeron, $p$, $f_2$, $a_2$ etc. reggeons while UPE amplitudes are amplitudes of exchanges with $\pi$, $a_1$ etc. reggeons. For brevity, the nucleon helicity indexes will be omitted if this does not lead to misunderstanding.

The SDMEs in exclusive $\omega$ electroproduction are determined using the processes in Eqs. (1). They are fitted as parameters of the angular distribution $W_{U+L}(\Phi, \phi, \cos \Theta)$ to the experimental distribution of the three pions originating from the $\omega$ decay. The angular distribution $W_{U+L}(\Phi, \phi, \cos \Theta)$ is decomposed into $W_U$ containing 15 “unpolarized” SDMEs and $W_L$ determined with 8 “polarized” SDMEs: $W_{U+L}(\Phi, \phi, \cos \Theta) = W_U(\Phi, \phi, \cos \Theta) + P_\parallel W_L(\Phi, \phi, \cos \Theta)$, where $P_\parallel$ is a longitudinal beam polarization. The “polarized” SDMEs can be extracted only from data collected with a longitudinally polarized beam.

The angle between the $\omega$ production plane and the lepton scattering plane in the CM system is denoted $\Phi$. The variables $\Theta$ and $\phi$ are respectively the polar and azimuthal angles of the unit vector $\hat{n}$ normal to the decay plane in the $\omega$-meson rest frame. For more details see [12].

3. Data Analysis

The data were accumulated with the HERMES spectrometer during the running period of 1996 to 2007 using the 27.6 GeV longitudinally polarized electron or positron beam of HERA, and gaseous hydrogen or deuterium targets. The HERMES forward spectrometer is described in Ref. [13]. The spectrometer permitted a precise measurement of charged-particle momenta,
Figure 1. Breit-Wigner fit (solid line) of $\pi^+\pi^-\pi^0$ invariant mass distributions after application of all criteria to select $\omega$ mesons produced exclusively from proton (top) and from deuteron (bottom). The dashed line represents the PDG value of the $\omega$ mass.

with a resolution of 1.5%. A separation of leptons was achieved with an average efficiency of 98% and a hadron contamination below 1%.

The following requirements were applied for event selection:

i) Exactly two oppositely charged hadrons and one lepton with the same charge as the beam lepton are identified.

ii) A $\pi^0$ meson that is reconstructed from two calorimeter clusters is selected requiring the two-photon invariant mass to be in the interval $0.11 \text{ GeV} < M(\gamma\gamma) < 0.16 \text{ GeV}$.

iii) The three-pion invariant mass is required to obey $0.71 \text{ GeV} \leq M(\pi^+\pi^-\pi^0) \leq 0.87 \text{ GeV}$.

iv) The scattered-lepton momentum lies above $3.5 \text{ GeV}$.

v) Taking into account the spectrometer resolution, the missing energy $\Delta E$ has to lie in the interval $-1.0 \text{ GeV} < \Delta E < 0.8 \text{ GeV}$. Here, $\Delta E = M_p^2 - M_X^2$, with $M_p$ being the proton mass and $M_X^2 = (p + q - p_{\pi^+} - p_{\pi^-} - p_{\pi^0})^2$ the missing mass squared, where $p$, $q$, $p_{\pi^+}$, $p_{\pi^-}$, and $p_{\pi^0}$ are the four-momenta of target nucleon, virtual photon, and each of the three pions respectively.

vi) The constraints $Q^2 > 1.0 \text{ GeV}^2$, $6.3 \text{ GeV} > W > 3 \text{ GeV}$, $-t' < 0.2 \text{ GeV}^2$ are applied.

After application of all these constraints, the proton sample contains 2260 and the deuteron sample 1332 events of exclusively produced $\omega$ mesons. These data samples are referred to in the following as data in the “entire kinematic region”. The invariant-mass distributions for exclusively produced $\omega$ mesons are shown in Fig. 1.

The distributions of missing energy $\Delta E$, shown in Fig. 2, exhibit clearly visible exclusive peaks. The shaded histograms represent semi-inclusive deep-inelastic scattering (SIDIS) background obtained from a PYTHIA Monte Carlo simulation that is normalised to the data in the region of $2 \text{ GeV} < \Delta E < 20 \text{ GeV}$. The simulation is used to determine the fraction of background under the exclusive peak, which is calculated as the ratio of number of background events to the total number of events. It amounts to about 20% for the entire kinematic region and increases from 16% to 26% with increasing $-t'$.

4. Results

The results on SDMEs are obtained using an unbinned maximum likelihood method. All details of the fit procedure and taking into account the background corrections are described in Ref. [12].
The SDMEs of the $\omega$ and $\rho^0$ mesons for the entire kinematic region are shown in Fig. 3. These SDMEs are divided into five classes corresponding to various helicity transitions $\lambda_{n} \rightarrow \lambda_{v}$. The main terms in the expressions of class-A SDMEs correspond to the transitions from longitudinal virtual photons to longitudinal vector mesons, $\gamma^*_L \rightarrow V_L$, and from transverse virtual photons to transverse vector mesons, $\gamma^*_T \rightarrow V_T$. The dominant terms of class B correspond to the interference of these two transitions. The main terms of class-C, D, and E SDMEs are proportional to small amplitudes of $\gamma^*_T \rightarrow V_L$, $\gamma^*_L \rightarrow V_T$, and $\gamma^*_T \rightarrow V_{-T}$ transitions respectively.

The SDMEs for the proton and deuteron data are found to be consistent with each other within their total uncertainties. In Fig. 3, the uncertainties of the polarized SDMEs are larger than those of the unpolarized SDMEs because the mean polarization $<|P_{0}|> \approx 40\%$ and in the equation for the angular distribution they are multiplied by the small factor $|P_{0}|/\sqrt{1-\epsilon} \approx 0.2$. Here, $\epsilon$ is the ratio of fluxes of the longitudinally and transversely polarized virtual photons.

The linear combination of the class-A and class-B SDMEs which are to be zero, if $S$-Channel Helicity Conservation (SCHC) approximation is valid, are really zero within experimental uncertainty. Indeed, the proton data yield $r_{1-1}^1 + \text{Im}(r_{1-1}^2) = -0.004 \pm 0.038 \pm 0.015$, $\text{Re}(r_{10}^5) + \text{Im}(r_{10}^b) = -0.024 \pm 0.013 \pm 0.004$, $\text{Im}(r_{10}^5) - \text{Re}(r_{10}^8) = -0.000 \pm 0.100 \pm 0.018$, and the deuteron data give $r_{1-1}^1 + \text{Im}(r_{1-1}^2) = 0.033 \pm 0.049 \pm 0.016$, $\text{Re}(r_{10}^5) + \text{Im}(r_{10}^b) = 0.001 \pm 0.016 \pm 0.005$, $\text{Im}(r_{10}^5) - \text{Re}(r_{10}^8) = 0.104 \pm 0.110 \pm 0.023$. All SDMEs of classes C, D, and E are to be zero in the SCHC approximation. The SDME $r_{0}^0$ violates SCHC since it is nonzero as is seen from Fig. 3.

The comparison of $\omega$ and $\rho^0$ [11] SDMEs is shown in Fig. 3. As is seen the SDMEs $r_{1-1}^1$ and $\text{Im}(r_{1-1}^2)$ have opposite sign for $\omega$ and $\rho^0$ mesons. The explanation follows from expressions of these SDMEs in terms of the amplitudes given in [10, 11]: $|T_{11}|^2 > |U_{11}|^2$ for the $\rho^0$ meson, while $|U_{11}|^2 > |T_{11}|^2$ for the $\omega$ meson. Moreover, the contribution of the UPE amplitude $U_{11}$ dominates in the cross section of the $\omega$-meson production while its contribution for the $\rho^0$-meson is small [11]. As the simplest consequence of the UPE dominance, the values of the SDME combinations $u_1$, $u_2$, and $u_3$ are unexpectedly large for the $\omega$ meson as is seen from Fig. 4. They are well described in the GK model [8] if the pion exchange is included into consideration as follows from a comparison of solid and dashed curves in Fig. 4. The quantities $u_1$, $u_2$, and $u_3$ are: $u_1 = 1 - r_{00}^{104} + 2r_{1-1}^{104} - 2r_{11}^{104} - 2r_{1-1}^{104}$, $u_2 = r_{11}^{11} + r_{1-1}^{11}$, and $u_3 = r_{11}^{11} + r_{1-1}^{11}$.

Figure 2. The $\Delta E$ distributions of $\omega$ mesons produced in the entire kinematic region and three kinematic bins in $-\ell'$ are compared with SIDIS $\Delta E$ distributions from PYTHIA (shaded area). The vertical dashed line denotes the upper limit of the exclusive region.
Figure 3. Comparison of 23 SDMEs in exclusive $\omega$ and $\rho^0$ [11] electroproduction at HERMES for the entire kinematic region. The SDMEs for $\omega$ are extracted in the region with $\langle Q^2 \rangle = 2.42$ GeV$^2$, $\langle W \rangle = 4.8$ GeV, $\langle -t' \rangle = 0.080$ GeV$^2$ while for $\rho^0$ production $\langle Q^2 \rangle = 1.95$ GeV$^2$, $\langle W \rangle = 4.8$ GeV, and $\langle -t' \rangle = 0.13$ GeV$^2$. The inner error bars represent the statistical uncertainties, while the outer ones indicate the statistical and systematic uncertainties added in quadrature. Unpolarized (polarized) SDMEs are displayed in the unshaded (shaded) areas.

Figure 4. The $Q^2$ and $-t'$ dependences of $u_1$, $u_2$, and $u_3$. The open symbols represent the values over the entire kinematic region. Solid (dashed) curves are obtained in the GK model when the pion exchange is (not) taken into account. Other notations are as in Fig. 3.
Figure 5. The $Q^2$ (top) and $-t'$ (bottom) dependence of the longitudinal-to-transverse virtual-photon differential cross-section ratio for exclusive $\omega$ and $\rho^0$ electroproduction at HERMES, where the $-t'$ bin is [0.0-0.2] GeV$^2$ for $\omega$ meson and [0.0-0.4] GeV$^2$ for $\rho^0$ meson [11]. The triangle symbols represent the value of $R$ in the entire kinematic region. Otherwise as for Fig. 4.

The differential cross-section ratio, $R = \frac{d\sigma_L(\gamma^*_L \rightarrow V)}{d\sigma_T(\gamma^*_T \rightarrow V)}$ of longitudinal to transverse photons is experimentally determined from the measured SDME $r_{00}^{04}$ using the equation [11] $R \approx r_{00}^{04}/[\epsilon(1 - r_{00}^{04})]$. This relation is exact in the case of SCHC. A comparison of the ratio for the $\omega$ and $\rho^0$ meson shows that the former is by a factor of about 3 smaller than the latter. This fact is a manifestation of the larger fractional contribution of pion exchange for the case of the $\omega$ meson than for the $\rho^0$ meson. Indeed, the formula $R \approx |T_{00}/T_{11}|^2/(1 + |U_{11}/T_{11}|^2)$ valid for rough estimations shows that since $|U_{11}/T_{11}|^2 > 1$ for $\omega$ while $|U_{11}/T_{11}|^2 < 1$ for $\rho^0$ mesons the ratio $R$ should be smaller for the $\omega$ meson if $|T_{00}/T_{11}|^2$ are approximately equal to each other for these two mesons. The crucial importance of the pion pole contribution is illustrated in Fig. 5 by a comparison of calculations in the GK model, in which this contribution is taken into account and ignored. The GK model appears to fully account for the $Q^2$ and $t'$ dependences of $R$ and shows good agreement with the data only if the pion exchange is taken into account.

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