The radiation properties of an accretion disk with a non-zero torque on its inner edge

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Abstract

The structure of the inner edge of the accretion disk around a black hole can be altered, if the matter inside the marginally stable orbit is magnetically connected to the disk. In this case, a non-zero torque is exerted on its inner edge, and the accretion efficiency $\epsilon$ can be much higher than that in the standard accretion disk model. We explore the radiation properties of an accretion disk at its sonic point around a black hole with a time-steady torque exerted on the inner edge of the disk. The local structure of the accretion flow at the sonic point is investigated in the frame of general relativity. It is found that the accretion flow will be optically thin at its sonic point for most cases, if the additional accretion efficiency $\delta \epsilon$ caused by the torque is as high as $\sim 10\%$. The results imply that the variable torque may trigger transitions of the flow between different accretion types.

Key words: accretion, accretion disks–black hole physics–radiation mechanisms: general–galaxies: nuclei

1. Introduction

Most work on accretion disks around black holes are on the assumption that there is no stress at the inner edge of the disk (Shakura & Sunyaev 1973). The inner edge of the disk should occur very close to the radius of the marginally stable orbit $r_{\text{ms}}$, since the plunging matter in the region of unstable orbits rapidly becomes causally disconnected from the disk. Recently, Krolik (1999) questioned this assumption and suggested that the matter inside the marginally stable orbit can remain magnetically connected to the disk. If this is the case, an additional torque would be exerted on the inner edge of the disk. Gammie (1999) has shown that the torque enhances the amount of energy released in the disk. The energy of the matter in the plunging region is extracted and then released in the disk. The extracted energy is transported outwards by the stresses in the disk. The torque alters the disk structure, and then the radiation spectrum of the disk (Agol & Krolik 2000, hereafter AK00). In their calculations, AK00 assumed that the circular motion of the matter in the disk is Keplerian, and the released gravitational energy of the accreting matter is balanced by the radiation locally, i.e., no radial energy advection has been considered in their calculations.

In this work, we investigate the local structure of the accretion disk at the sonic point with a non-zero torque exerted on its inner edge. The radiation properties of the disk at its sonic point is explored. In our calculations, the radial energy advection in the flow is considered, and the circular velocity of the flow is not limited to the Keplerian value.

2. Model

AK00 assumed that the circular motion of the matter in the disk is Keplerian instead of considering the radial force balance, and the inner edge of the disk is therefore set at $r_{\text{ms}}$ manually. They further assumed that the generated energy is dissipated locally in the disk, i.e., the radial energy advection has not been considered in their work. The general relativistic effects are included in their calculations, as done by Novikov & Thorne (1973).

For a transonic accretion flow around a black hole, we take the radius of sonic point $r_s$ as the inner edge of the disk. We assume that a time-steady torque is exerted on the inner edge of the disk, as done by AK00 on the assumption of a Keplerian disk (and also see Paczynski (2000) and Afshordi & Paczynski (2002)). We include the angular and radial momentum equations in our calculations and the circular velocity of the flow is not limited to the Keplerian value. Our calculations are in the frame of general relativity, and the radial energy advection in the flow is included in the energy equation. The local structure of the flow at the sonic point is therefore available. The torque may be variable and extended to a region of the disk outside the radius of sonic point. For simplicity, we restrict our calculations to the simplest case as AK00, i.e., a time-steady torque is exerted on the inner edge of the disk.

The magnetic field plays an important role in angular momentum transfer of the matter in the disk. The value of viscosity $\alpha$ can be determined by the magnetohydrodynamical processes taking place in the accretion flow (Hawley 2000). The value of $\alpha$ of the flow may vary with time and radius. The angular momentum transfer in the disk caused by any specific magnetohydrodynamical processes may be described approxi-
mately by the parameter $\alpha$, though the precise value of $\alpha$ and its variation is unavailable unless the detailed magnetohydrodynamical processes in the flow are included in the numerical simulations. The angular momentum transfer of the matter in the flow caused by the magnetohydrodynamical processes in the flow may correspond to a certain value of $\alpha$, and the torque exerted on the inner edge of the disk is governed by the same magnetohydrodynamical processes. So, the values of $\alpha$ and the torque are determined simultaneously by the magnetohydrodynamical processes taking place in the disk. In order to avoid being involved in the complicated magnetohydrodynamical processes taking place in the disk. In order to avoid being involved in the complicated magnetohydrodynamical processes taking place in the flow, we adopt $\alpha$-viscosity and another independent parameter $\delta \nu$ to describe the angular momentum transfer of the matter in the flow and the torque on the inner edge of the disk, respectively. This may induce some inconsistency in our calculations, but we can explore the general radiation properties of the disk varying $\alpha$ for a given torque. This will not change our main conclusions on the radiation properties of the disk (see further discussion in Sect. 5).

For a disk with an additional torque exerted on its inner edge, the viscosity $\nu$ at the sonic point is

$$\nu(r_s) = \nu_0(r_s) + \delta \nu,$$  

where $\nu_0(r_s)$ is the viscosity at the sonic point without an additional torque, $\delta \nu$ is caused by the additional torque exerted on the inner edge of the disk.

The main goal of this work is to explore the local structure and radiation properties of the disk at its sonic point. The radiation properties are mainly governed by the radiative processes and the energy dissipated in the flow. The $\alpha$-description on the angular momentum transfer in the disk is therefore sufficiently good for present investigation, though the exact value of $\alpha$ is unknown. In this work, we focus on the local structure of the disk and the radiation properties at the sonic point of the flow. We assume that $\alpha$-viscosity is valid in the flow at the sonic point. The variation of $\alpha$ along radius may not affect our main results of the radiation properties, since we focus on the local structure of the disk at the sonic point.

### 3. Equations of accretion flows in Kerr geometry

Many authors have included general relativistic effects in their models of accretion disks around black holes (e.g., Novikov & Thorne 1973; Lu 1985; Abramowicz et al. 1996; Peitz & Appl 1997; Gammie & Popham 1998; Popham & Gammie 1998; Mannomoto 2000; etc.). In this work, we mainly adopt the equations presented by Abramowicz et al. (1996, hereafter A96). One may refer to A96 for details.

The metric of a Kerr black hole on the equatorial plane takes the form given by Novikov & Thorne (1973) ($G = c = 1$),

$$ds^2 = -\frac{r^2 \Delta}{A} + \frac{A}{r^2} (d\varphi - \omega dt)^2 + \frac{r^2}{\Delta} dr^2 + dz^2,$$  

where

$$\Delta = r^2 - 2Mr + a^2,$$  

$$A = r^4 + r^2a^2 + 2Mr a^2,$$  

and

$$\omega = \frac{2Mar}{A}. $$  

Here $M$ is the black hole mass and $a$ is the specific angular momentum of the Kerr black hole.

The Lorentz factor $\gamma$ is defined by

$$\gamma = \frac{1}{\sqrt{1 - (v(r))^2 - (v^2(r))^2}},$$  

where

$$v(r) = \tilde{R} \tilde{\Omega}.$$  

The quantities $\tilde{R}$ and $\tilde{\Omega}$ are defined by

$$\tilde{R}^2 = \frac{A^2}{r^4 \Delta}, \quad \tilde{\Omega} = \Omega - \omega.$$  

The radial velocity component $V$ is given by

$$\frac{V}{\sqrt{1 - V^2}} = \gamma v(r) = \frac{r}{\tilde{R}^2} g_{rr}^{1/2}.$$  

Combining Eqs. (6)-(9), we have

$$\gamma^2 = \left(1 - \frac{1}{\Omega^2 R^2}\right) \left(1 - \frac{1}{1 - V^2}\right),$$  

and

$$V = \frac{v(r)}{\sqrt{1 - \Omega^2 R^2}}.$$  

The angular momentum conservation is

$$\frac{M}{2\pi} \frac{d l}{dr} + \frac{1}{r} \frac{d}{dr} \left(\Sigma \nu A^{3/2} \frac{\Delta^{1/2} \gamma^3}{r^4} \frac{d \Omega}{dr}\right) = 0, \quad (12)$$

where

$$l = \gamma \left(\frac{A^{3/2}}{r^3 \Delta^{1/2}}\right) \tilde{\Omega}.$$  

is the specific angular momentum per unit mass, and the term of the angular momentum carried by the vertical flux of radiation has been neglected (A96).

Integrating Eq. (12), we have

$$\frac{M}{2\pi} (l - l_0) = -\Sigma \nu A^{3/2} \frac{\Delta^{1/2} \gamma^3}{r^4} \frac{d \Omega}{dr} \bigg|_{r = r_s}, \quad (14)$$

where $l_0$ is the integral constant. We assume that an additional torque is exerted on the inner edge of the disk: $r = r_s$, and $r_s$ is the radius of sonic point. At the sonic point, the viscosity $\nu(r_s)$ is described by Eq. (1). The additional accretion efficiency caused by the torque is $\delta \nu$. We can obtain a relation at $r = r_s$:

$$\delta \nu \frac{M}{2\pi \Omega} = -\Sigma \nu A^{3/2} \frac{\Delta^{1/2} \gamma^3}{r^4} \frac{d \Omega}{dr} \bigg|_{r = r_s}. \quad (15)$$

In the absence of an additional torque, the viscosity $\nu_0(r_s)$ at the sonic point is very low, namely zero-torque approximation. In this work, we consider the cases with significant additional torques on the inner edges of the disks, so $\delta \nu \gg \nu_0$ is usually satisfied at the sonic point. For simplicity, we use an approximation: $\nu(r_s) \simeq \delta \nu$ in our calculations. We further assume that $\alpha$-viscosity is valid even at the sonic point. The value of $\alpha$ at the sonic point may be different from that in the region outside the radius of sonic point.
The radial component of the momentum conservation is
\[ \frac{V}{1 - V^2} \frac{dV}{dr} = \frac{A}{r} - \frac{1}{\Sigma} \frac{dP}{dr}, \tag{16} \]
where
\[ A = \frac{MA}{r^3 \Delta \Omega K \Omega K} (\Omega - \Omega K) (\Omega - \Omega K^+), \tag{17} \]
and \( P = 2H \rho p \) is the vertical integrated pressure.

The angular velocities of the corotating and counterrotating Keplerian orbits are
\[ \Omega K^+ = \pm \frac{M^{1/2}}{r^{3/2} \pm aM^{1/2}}. \tag{18} \]

The surface energy generation rate \( F^+ \) and the cooling rate \( F^- \) are
\[ F^+ = \nu \frac{\Sigma A^2}{r^3} \frac{\gamma}{\rho} \frac{d\Omega}{dr} \tag{19} ^2, \]
and
\[ F^- = \frac{16 \sigma T^4}{3 \pi}, \tag{20} \]
respectively. Here we only consider the optically thick case. The energy equation can be written as
\[ F^+ = F^+ - F^-. \tag{21} \]

The equation of state is
\[ \rho = \frac{1}{3} \frac{aT^4 + \rho kT}{\mu m_{\text{H}}}, \tag{22} \]
where the ratio of the specific heats of the gas \( \gamma = 5/3 \) has been adopted. The magnetic pressure has been neglected, since we restrict our calculations to the optically thick case. For the optically thick case, the radiation processes are not relevant to the magnetic pressure.

Using Eq. (22), the entropy gradient can be calculated by
\[ T dS = \frac{\rho}{\rho} \left( 12 - \frac{21}{\beta} \right) d\ln T - (4 - 3\beta) d\ln \rho \tag{23} , \]
where \( \beta = \rho_k / \rho \).

The advection cooling rate due to the radial motion of the gas is
\[ F^+ = \frac{M}{2 \pi r} \frac{\rho}{T} \frac{dS}{dr} \]
\[ = \frac{M}{2 \pi r} \frac{\rho}{T} \left[ 12 - \frac{21}{\beta} \right] d\ln T - (4 - 3\beta) d\ln \rho \tag{24} . \]

The vertical force balance gives
\[ \frac{p}{\rho H^2} = \frac{\gamma^2 M}{r^3} \left[ \frac{(r^2 + a^2)^2 + 2\Delta a^2}{(r^2 + a^2)^2 - \Delta a^2} \right]. \tag{25} \]

Substituting Eqs. (22), (24) and (25) into Eq. (16), we can rewrite the radial motion equation as
\[ \frac{dV}{dr} = \frac{N}{D} (1 - V^2), \tag{26} \]
where
\[ D = V - \frac{72 - 51 \beta - 9 \beta^2}{56 - 45 \beta - 3 \beta^2} \frac{c^2}{V^2}, \tag{27} \]
and
\[ N = \frac{A}{r} + \frac{72 - 51 \beta - 9 \beta^2}{56 - 45 \beta - 3 \beta^2} \frac{\beta^2}{2 \pi r} \frac{d}{dr} \ln \Delta^{1/2} \]
\[ + \frac{8 + 3 \beta + 3 \beta^2}{56 - 45 \beta - 3 \beta^2} \frac{\gamma^2}{\sigma} \frac{M}{r^3} \left[ \frac{(r^2 + a^2)^2 + 2\Delta a^2}{(r^2 + a^2)^2 - \Delta a^2} \right] \]
\[ + \frac{16 - 12 \beta}{56 - 45 \beta - 3 \beta^2} \frac{2 \pi r F^+}{M}. \tag{28} \]

The sonic point is defined by the condition
\[ D = N = 0, \tag{29} \]
and we have the radial velocity of the flow at the sonic point:
\[ V_s = \left( \frac{72 - 51 \beta - 9 \beta^2}{56 - 45 \beta - 3 \beta^2} \right)^{1/2} c_s. \tag{30} \]

The standard \( \omega \)-viscosity
\[ \nu = \frac{2}{3} \alpha c_s H, \tag{31} \]
is adopted in this work.

Substituting Eqs. (15), (19), (20) and (24) into Eqs. (28) and (29), the physical quantities \( \rho_k \) and \( T_k \) of the flow at the sonic point are available, if the parameters \( M, M, a, r_s, \alpha, \delta \epsilon \) and \( \Omega_s \) are specified.

The effective optical depth in the vertical direction is
\[ \tau_{\text{eff}} = 0.5 \Sigma \sqrt{(\kappa_{\text{es}} + \kappa_{\text{H}}) m_{\text{H}}}, \tag{32} \]
where \( \kappa_{\text{es}} \) and \( \kappa_{\text{H}} \) are the electron scattering opacity and free-free opacity, respectively.

4. Results

In principle, the set of equations describing the accretion flow around a black hole can be integrated with suitable outer boundary conditions. The solution is required to satisfy the regularity condition (29) at the sonic point \( r = r_s \) for a given \( \delta \epsilon \). The specific angular momentum \( l_s \) at the sonic point and the radius of sonic point \( r_s \) are simultaneously given by the numerical solution. In the case of \( \delta \epsilon = 0, l_s \approx l_0 \). The main numerical difficulties in solving the set of equations would be the particular value of \( l_s \) which should satisfy the regularity condition (29)(see A96 for the cases of \( l_s \approx l_0 \)).

In this work, we search all possible solutions in the parameter plane \( r_s - l_s \), instead of integrating the equations with outer boundary conditions numerically. Using this method, we can study the local properties of the disk at the sonic point. All possible solutions satisfying condition (29) can be found. The advantage of this approach is that all physical solutions satisfying both the outer boundary conditions and the inner regularity condition (29) will be included in the solutions found in this way, though some solutions we find would be unphysical.

As discussed in Sect. 3, we only consider the optically thick case (see Eq. (20)). So, all optically thick solutions should satisfy \( \tau_{\text{eff}} > 1 \) at the sonic point.

In this work, we focus on the geometrically thin accretion disk, all calculations are therefore performed for accretion rates...
Fig. 1. The parameter plane $r_s - l_s$ for all possible solutions with different values of $\delta \epsilon$ (labeled near the curves). The parameters: $M = 10^9 M_\odot$, $a = 0.95$ and $\dot{M}/\dot{M}_{\text{Edd}} = 0.001$ are adopted. The parameters in the left side of the curves are for optically thick solutions, i.e., $\tau_{\text{eff}} > 1$. The radius of the black hole horizon $r_h$, radius of the minimal bound circular orbit $r_{\text{mb}}$ and radius of the marginally stable orbits $r_{\text{ms}}$ are marked in the figure. The upper panel is for $\alpha = 1$, while the lower panel is for $\alpha = 0.1$.

Fig. 2. The same as Fig. 1, but for $\dot{M}/\dot{M}_{\text{Edd}} = 0.1$.

Fig. 3. The same as Fig. 1, but for a Schwarzschild black hole ($a = 0$).

Fig. 4. The same as Fig. 2, but for a Schwarzschild black hole ($a = 0$).
5. Discussion of the results

The physical, global solutions for optically thick flows are available by integrating a set of equations with suitable inner and outer boundary conditions. The problem is that the search for such a parameter plane for all physical global solutions with all possible different inner and outer boundary conditions would be very difficult. The parameter plane \( r_s - l_s \) for physical, global solutions will be included in the present parameter plane \( r_s - l_s \), but the global solution will set a more strict constraint on the existence of optically thick flows. So, the conclusion is tight that no physical, optically thick disk solution at the sonic point will be present outside the parameter plane \( r_s - l_s \) obtained here, though the possibility cannot be ruled out that the flow is optically thin at the sonic point while it becomes optically thick at a large radius outside the sonic point.

It is found that the location of the sonic point for an optically thick accretion flow moves towards the horizon of the hole with the increase of \( \delta \epsilon \). For high \( \delta \epsilon \), no optically thick accretion flow at the sonic point is present. A higher \( \delta \epsilon \) means that much energy of the plunging matter inside the marginally stable orbit is extracted to the disk, and the disk is heated to a higher temperature. The free-free opacity \( \kappa_f \) decreases with the increase of temperature. The disk then becomes optically thin.

Comparing the results for different values of \( \alpha \), we find that the accretion flow with a low \( \alpha \) will be optically thin even for a low \( \delta \epsilon \), i.e., the optically thick flows at the sonic point with low \( \alpha \) are present only for low \( \delta \epsilon \) cases. A high \( \alpha \) makes the energy extracted from the plunging matter inside the inner edge of the disk transported outwards efficiently. Therefore, only a small fraction of the energy is released locally at the inner edge of the disk, and the flow can still remain optically thick for a relatively high \( \delta \epsilon \). For the lower \( \alpha \) case, much energy is released locally and the disk is heated to a higher temperature, and the disk becomes optically thin at the sonic point. For a high accretion rate \( \dot{m} = \dot{M}/\dot{M}_{\text{Edd}} \), the surface density of the disk is so high that the disk is optically thin even for a relatively high \( \delta \epsilon \) (compare results in Figs 1-4 for different values of \( \dot{m} \)).

The sonic point of the accretion flow usually locates at the radius close to the marginally stable orbit, since the matter will plunge to the hole rapidly inside \( r_{\text{ms}} \). The radius of sonic point is in principle available by integrating the set of the equations with suitable outer and inner boundary conditions. In the present work, we only consider the inner boundary condition at the sonic point, and only the range of the radius of sonic point is given. From Figs. 5 and 6, we find that \( \delta \epsilon \leq 0.07 \) should be satisfied for an optically thick flow at the sonic point around a
Kerr black hole ($a = 0.95$), if $r_s \geq r_{mb}$ is assumed (see Fig. 5). For a Schwarzschild black hole, $\delta \epsilon \leq 0.02$ should be satisfied (see Fig. 6). In our calculations, we found that the results are insensitive to the black hole mass.

Recently, the XMM-Newton observation of the Seyfert 1 galaxy MCG–6-30-15 reveals an extremely broad and red-shifted Fe Kα line (Wilms et al. 2001). The explanation on the observed spectrum requires the disk to have a very steep emissivity profile with an index around 4.3-5.0. A possible explanation is provided by the magnetic connection between the inner region of the disk and the plunging matter inside the inner edge of the disk (Wilms 2001; Krolik 1999; AK00). Another possible explanation is based on the magnetic connection between a rotating black hole and a disk (Wilms 2001; Li 2002a,b). However, it is also suggested that the origin of the broad Fe Kα line can be explained in the frame of an illuminated relativistic accretion disk (Martocchia, Matt & Karas 2002). AK00 suggested that the locally generated surface energy flux scales as $r^{-7/2}$ at large $r$ in the limit of infinite efficiency or zero accretion rate in their model, rather than that scales as $r^{-3}$ in the standard thin disk model. This is helpful to explain the required steep emissivity profile. In this work, we have investigated the local structure and radiation properties of the disk at the sonic point. The global solution to this problem could be tested against the observed broad fluorescent Fe Kα line in MCG–6-30-15, which will be given in our future work.

In this work, we find that the radiation properties of the disk at the sonic point are sensitive to the torque exerted on the inner edge of the disk. The torque may probably be variable (e.g., Armitage, Reynold & Chiang 2001; Hawley 2001; Hawley & Krolik 2001, 2002; Reynold & Armitage 2001; etc.). Our results imply that the variable torque may trigger transitions of the flow between optically thick and optically thin accretion types. The global solutions are necessary to attack this problem, which is beyond the scope of the present work, and it will be reported in our future work.

In present calculations, we have assumed that the torque is exerted on the inner edge of the disk, as done by AK00. If the torque is exerted on the extended region outside the inner edge of the disk, the disk can be optically thick at the sonic point even for a relatively higher $\delta \epsilon$ than that reported here.

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