The magic numbers of equal spheres on triply periodic minimal surfaces

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Regular structures of equal spheres on the triply periodic minimal surfaces known as primitive (P), gyroid (G) and diamond (D) surfaces are investigated by using Monte Carlo simulations of hard spheres undergoing the Alder transition. Remarkably, there exist magic numbers producing the regular structures, which are simply explained by means of hexagulation numbers defined as \( H = h^2 + k^2 - hk \), in analogy with the Caspar and Klug’s triangulation numbers, \( T = h^2 + k^2 + hk \) for icosahedral viruses, [1] where \( h \) and \( k \) are equal to nonnegative integers. Understanding the significance of symmetry of the surfaces, the total number of spheres per cubic unit cell \( N \) is represented by \( N = 8H, 16H, \) and \( 32H \) for P-, G- and D-surfaces, respectively. Accordingly, these arrangements are analyzed in terms of space groups, equivalent positions (Wyckoff positions), and polygonal-tiling representations. The key is that there is only a limited number of efficient physical design possible even on the triply periodic minimal surfaces. [2, 3]

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