Geometrical synthesis of four-bar gear train with related gears

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Abstract. The method of optimal geometric synthesis of four-bar cylindrical gear with related gears with one external and one internal gearing is proposed. The method is based on a new calculation system that increases the number of synthesis output parameters to six, which makes it possible to considerably expand the domain of existence of the synthesis problem solution with the simultaneous synthesis of two gearing. The software implementing the proposed technique is described. To solve the problem of nonlinear mathematical programming the library of Optimization Toolbox functions included in Matlab is used. Examples of calculations are given.

1. Introduction

The first step in the design of any gear is a geometric synthesis. At this stage, the basic geometric parameters of the gears are determined to ensure the functionality of the transmission and the necessary values of quality characteristics. The main parameters of the spur gears are: the number of teeth \( z_1, z_2 \), addendum modification coefficients \( x_1, x_2 \), module \( m \). The transmission must meet a number of constraints and provide acceptable values of quality characteristics: transverse contact ratio, pressure angle etc. Thus, this problem relates to multiparametric multicriteria problems. The number of teeth of gears are determined by the necessary speed ratio, and the module – by strength calculation. In geometric calculations, the module is a scale factor and does not affect the values of quality characteristics. Therefore, the output parameters of the synthesis are only the addendum modification coefficients \( x_1, x_2 \). This makes it particularly important to investigate the existence of a solution to the synthesis problem, to construct the domain of existence of the synthesis problem solution, and to find an optimal or rational solution.

The most common method of solving the problem of geometric synthesis of cylindrical gears with involute tooth profile is the method of limiting contours [1-3]. The method is based on the construction of the domain of existence of the solution of the synthesis problem, bounded by the lines of certain values of the transmission quality characteristics, and the cut-off lines in which the execution of the synthesis constraints is guaranteed. The method is simple, visual, included in the existing standards. The very large number of variants of the number of teeth and the large nomenclature of the tools do not allow you to graphically represent all possible variants. For example, GOST 9323-79 [4] contains more than a hundred variants of pinion type cutters, differing values of modules and the number of teeth. Besides, wear of tools results in displacement of boundary lines of limiting contours. This makes it impossible to graphically represent all possible variants. Therefore, combined limiting contours uniting several variants [5-6] are constructed. This is a drawback of the method, since it requires the use of interpolation in the solution of a particular task and can exclude from consideration the acceptable solutions.
The use of computer technology made it possible to automate the process of building limiting contours, build special limiting contours for each case and significantly expand the number of conditions used in their construction. Various software variants have been developed that allow to build limiting contours interactively [7], to build dynamic limiting contours, changing the boundary values of the quality characteristics [8-9], also taking into account additional quality characteristics: contact strength, specific slip, etc. [10], and taking into account the features of plastic gears [11].

The location and size of the domain of existence of the geometric synthesis problem solution is significantly influenced by the choice of the calculation system. The geometric calculation of the gear can be guided by the use of a specific standard tool, including taking into account its wear, and the use of impersonal standard tool. The second variant of calculation narrows possibilities of synthesis and does not allow to define values of all geometrical parameters.

The basic system of geometric calculation implements constant standard bottom clearances in the working gearing. The change of bottom clearances, which is allowed by the current standards for both external and internal gearing, allows to expand the possibilities of geometric synthesis. In fact, this approach increases the number of the synthesis output parameters relating to them the tip circles diameters of the gears or the associated bottom clearance coefficients. The recommended calculation system [1, 12] provides for the calculation constant, but non-standard bottom clearances, constant tooth depth, constant ratio of tooth depth to the bottom clearance, constant tooth thickness on the tip circle and a number of others. These calculation systems allow to change the location of domain of existence of the synthesis problem solution, but introduce additional constraints.

Sidorov at al. [13] proposed to refer to the synthesis output parameters bottom clearances $c_{12}, c_{21}$ linking the center distance $a_{w12}$, the tip diameters $d_{a1}, d_{a2}$ and the root diameters $d_{f1}, d_{f2}$ of the meshing gears:

for external gearing

$\begin{align*}
  c_{12} &= a_{w12} - 0.5(d_{a1} + d_{f2}), \\
  c_{21} &= a_{w12} - 0.5(d_{a2} + d_{f1})
\end{align*}$

(1)

for internal gearing

$\begin{align*}
  c_{12} &= 0.5(d_{f2} - d_{a1}) - a_{w12}, \\
  c_{21} &= 0.5(d_{a2} - d_{f1}) - a_{w12}.
\end{align*}$

(2)

Sidorov at al. [14] shown that such a system of calculation for one of the most complex problems of geometric synthesis – the synthesis of internal gearing with a small difference in the number of teeth – allows several times to expand the domain of existence of the synthesis problem solution.

The solution of the synthesis problem is much more complicated in the geometric synthesis of four-bar gear, in which one of the gears simultaneously participates in two gearing. This is due to the fact that the number of synthesis conditions increases several times, and the number of synthesis output parameters increases from two to three. In addition, with a separate synthesis of two gearing, geometric parameters of the idler can be different, which requires the solution of two interrelated tasks.

Bolotovskii at al. [1] offered to build the combined limiting contours for two gearing with use of one of the systems of calculation specified above. This approach is very time-consuming, requires a number of successive attempts and does not guarantee the determination of the entire possible domain of existence of the synthesis problem solution.

Silchenko at al. [15-16] proposed to use a three-dimensional limiting contour built in the coordinate system $x_1, x_2, x_3$ (the addendum modification coefficients of the three gears). Such a limiting contour quite clearly reflects the relationship of the parameters, but to solve specific problems it is necessary to use the projection of the contour on the plane.

Egorova at al. [17-18] proposed to use a calculation system for the idler, which provides a constant tooth depth at any addendum modification coefficients. Built on this basis, the combined limiting contour is used similar to the conventional limiting contour of the three-bar transmission.
In all of the above-described geometric synthesis studies, the number of the synthesis output parameters is limited by the addendum modification coefficients. The tip diameters of the gears are determined by the adopted calculation system. This limits the possibilities of synthesis. It is obvious that the variation of addendum modification coefficients as well as in the three-bar transmission will expand the scope of the solution of the problem of synthesis of four-bar gear.

The aim of the article is to develop a mathematical model and software for the geometric synthesis of four-bar cylindrical gear with related gears with an increased number of output synthesis parameters.

2. Formulation of problem
Four-bar straight-tooth involute transmission with one external and one internal gearing is considered (figure 1).

![Figure 1. Transmission scheme](image)

The parameters of the basic rack correspond to GOST 13755-2015 [19] and ISO 53:1998 [20] (the type of the basic rack tooth profile – A): pressure angle $\alpha = 20^\circ$; addendum factor $h_a^* = 1$; dedendum factor $h_f^* = 1$, fillet radius of basic rack $\rho_f^* = 0.38$; common depth of standard basic rack and mating standard basic root tooth $h_w^* = 2$; clearance coefficient $c^* = 0.25$. There is no tooth profile modification.

When calculating the geometric parameters and quality characteristics of the transmission are used common assumptions: do not take into account the deformation of the transmission elements and manufacturing and assembly errors. Gears with both external and internal teeth are cut by the generating method with a pinion type cutter according to GOST 9323-79 [4]. Center distances are not given.

3. Theoretical part
Geometric parameters of gears are calculated as given: module $m$; numbers of teeth $z_1$, $z_2$, $z_3$; the addendum modification coefficients $x_1$, $x_2$, $x_3$; the clearance coefficients $c_{12}^*$, $c_{21}^*$, $c_{23}^*$, $c_{32}^*$. Geometrical parameters of external and internal gearing, as well as parameters in generation gearings with pinion type cutters, are determined by the known formulas according to GOST 16532-60 [21] and GOST 19274-73 [22].

The root diameters of gears, which are cut with using pinion type cutters,
\[
\begin{align*}
    d_{f1} &= 2a_{w01} - d_{a0}; \\
    d_{f2} &= 2a_{w02} - d_{a0}; \\
    d_{f3} &= 2a_{w03} + d_{a0},
\end{align*}
\]
where \( a_{w01}, a_{w02}, a_{w03} \) are center distances in generation gearings; \( d_{a0} \) is the root diameter of the pinion type cutter.

The tip diameters, taking into account (1), (2), must satisfy the dependencies,

\[
\begin{align*}
    d_{a1} &= 2a_{w12} - d_{f1} - 2c^*_1m; \\
    d_{a2} &= 2a_{w12} - d_{f1} - 2c^*_2m; \\
    d_{a3} &= d_{f2} + 2a_{w23} + 2c^*_3m.
\end{align*}
\]

(3)

Because the idler gear at the same time is in two gearing: external and internal, it should satisfy the condition

\[
2a_{w12} - d_{f1} - 2c^*_1m = d_{f3} - 2a_{w23} - 2c^*_3m.
\]

(4)

This condition leads to a decrease in the number of output parameters of the synthesis to six, and the clearance coefficient \( c^*_3 \) is determined by equation (4).

In order to synthesize functional transmission it is necessary to perform additional conditions: absence of teeth interference, absence of teeth undercutting, absence of teeth pointing submitted in the form of constraints for the synthesis. The quality of the gearing is characterized by transverse contact ratios \( \alpha_{w12}, \alpha_{w23} \) and the working pressure angles \( \alpha_{w12}, \alpha_{w23} \). The corresponding dependences for checking the constrains of synthesis and calculating the values of quality characteristics are given in [1, 5, 6].

4. Algorithm and software development

Since the selected quality characteristics of the gear are contradictory, the strategy of additive compensation of contradictions of criteria [23-24] is chosen to solve the problem of optimization synthesis. The additive objective function is taken as

\[
F(x_1, x_2, x_3, c_{12}, c_{21}, c_{32}) = k_1 \frac{1}{\epsilon_{\alpha_{w12}}} + k_2 \frac{1}{\epsilon_{\alpha_{w23}}} + k_3 k'_3 \alpha_{w12} + k'_4 k'_4 \alpha_{w23},
\]

where \( k_1, k_2, k_3, k_4 \) are weighting coefficients, the values of which are assigned depending on the importance of the criteria in solving a particular problem, and \( k_1 + k_2 + k_3 + k_4 = 1; \ k'_3, k'_4 = 2.3 \) – normalizing factors. To solve the problem of nonlinear mathematical programming, the library of Optimization Toolbox functions included in Matlab was used. It allows to find the extremum of the objective function with restrictions in the form of equality and inequality, as well as restrictions imposed on the values of optimized parameters (output parameters of synthesis). User interaction with the program is organized using the graphical user interface (GUI).

The main window of the program is shown in figure 2.

The program allows you to enter the initial data of the gears and the tool, the area of permissible values of the optimized parameters, the values of the weight coefficients in the dialog mode or from a prepared Microsoft Excel table. In the field of "Optimization" displays the optimal values of the output parameters of the synthesis, and in the field of "Restrictions" – the values of quality characteristics and reserves on the constraints of the synthesis.
5. Results and discussions
A number of transmission variants were calculated to verify the functionality and test the proposed calculation system and software. In table 1 comparison of the results of calculation of transmission parameters for the optimal variant and variants in which the values of the components of the output parameters vector of the synthesis were changed (compared with the optimal) by 5-20 %.

Table 1. Software testing.

| Parameter | Optimal | Variants |
|-----------|---------|----------|
|           |         | 1        | 2        | 3        | 4        |
| $x_1$     | 0.0374  | 0.0300   | 0.0450   | 0.0410   | 0.4500   |
| $x_2$     | 0       | 0.0100   | 0.0100   | 0.0200   | 0.0500   |
| $x_3$     | 0       | 0.0150   | 0.0240   | 0.0600   | 0.1000   |
| $c_{12}$  | 0.2000  | 0.2500   | 0.2300   | 0.2700   | 0.3000   |
| $c_{21}$  | 0.2000  | 0.2200   | 0.2800   | 0.3000   | 0.2500   |
| $c_{23}$  | 0.1884  | 0.2101   | 0.2703   | 0.2932   | 0.2459   |
| $c_{32}$  | 0.4302  | 0.4100   | 0.4200   | 0.4360   | 0.4500   |
| $e_{a12}$ | 1.6047  | 1.5609   | 1.5262   | 1.4871   | 1.4945   |
| $e_{a23}$ | 1.7409  | 1.7536   | 1.6979   | 1.6534   | 1.6549   |
| $\alpha_{w12}$, deg. | 20.2830 | 20.3022  | 20.4130  | 20.4570  | 20.7023  |
| $\alpha_{w23}$, deg. | 20.0030 | 20.0342  | 20.0953  | 20.2698  | 20.3360  |
| Constraint satisfaction | Yes | No | No | No | No |
| $F(x_1, x_2, x_3, c_{12}, c_{21}, c_{32})$ | 0.7030 | 0.7075 | 0.7170 | 0.7200 | 0.7300 |

Table 2 shows the influence of the weight coefficients on the choice of the optimal variant.
Table 2. The influence of weight coefficients.

| Parameter | Weight coefficients |
|-----------|----------------------|
| $k_1 = 0.25; k_2 = 0.25; k_3 = 0.25$ | $k_1 = 0.4; k_2 = 0.4; k_3 = 0.45$ | $k_1 = 0.05; k_2 = 0.05; k_3 = 0.05$ |
| $x_1$ | 0.0374 | 0.0374 | 0.0374 | 0.0374 |
| $x_2$ | 0 | 0 | 0 | 0.5542 |
| $x_3$ | 0.2000 | 0.2000 | 0.2000 | 0.2000 |
| $c_{12}$ | 0.1884 | 0.1884 | 0.2275 | 0.3094 |
| $c_{32}$ | 0.4302 | 0.4301 | 0.2472 | 0.4705 |
| $e_{a12}$ | 1.6047 | 1.6047 | 1.6047 | 1.4795 |
| $e_{a23}$ | 1.7490 | 1.7490 | 1.7491 | 1.4790 |
| $\alpha_{w12}, \text{ deg.}$ | 20.2830 | 20.2830 | 20.2830 | 23.7245 |
| $\alpha_{w23}, \text{ deg.}$ | 20.0030 | 20.0003 | 25.1330 | 15.0000 |

Initial data for calculations: module $m = 2.5$; number of teeth of gears $z_1 = 15; z_2 = 26; z_3 = 72$. For cutting gears with external teeth used pinion type cutter 2536-109, gear with internal teeth – pinion type cutter 2530-0168 according to GOST 9323-79 [4].

6. Conclusions

The proposed method of geometric synthesis of four-bar gear, based on the new calculation system, allows to increase the number of output parameters of the synthesis from three to six, to expand the domain of existence of the synthesis problem solution, to quickly choose the most optimal variant of the gear drive, ensuring the fulfillment of all constraints of the synthesis and obtaining the set values of quality characteristics corresponding to the specific problem to be solved. Both the technique and the software can be easily expanded by including in their composition additional constraints and geometric and kinematic parameters used in the calculation of the gear drive for strength.

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