Decentralized Tracking of Large-Scale Stochastic Nonlinear Systems via Output-Feedback

JIABAO GU, HUI WANG, (Member, IEEE), AND WUQUAN LI, (Senior Member, IEEE)
School of Mathematics and Statistics Science, Ludong University, Yantai 264025, China
Corresponding author: Hui Wang (csgyz@126.com)

This work was supported in part by the Shandong Province Higher Educational Excellent Youth Innovation Team, China, under Grant 2019KJN017; and in part by the Shandong Provincial Natural Science Foundation for Distinguished Young Scholars, China, under Grant ZR2019JQ22.

ABSTRACT This paper investigates the decentralized output-tracking problem for a class of high-order stochastic nonlinear systems (SNSs). Compared with the existing results, we consider more practical and more general systems, i.e., the system is large-scale and linearization parts may have unstable modes. An output-feedback tracking controller is designed based on a decentralized high-gain homogeneous domination technique. By using advanced stochastic analysis methods, we show that the output tracking errors can be made arbitrarily small while all the states of the closed-loop system remain to be bounded in probability. Finally, the effectiveness of the output-feedback tracking controller is demonstrated by a simulation example.

INDEX TERMS Output-feedback, tracking, large-scale stochastic nonlinear systems.

I. INTRODUCTION

In practical engineering, with the wide application of stochastic control [1], [2], the study of stochastic systems has attracted more and more researchers’ attention [3]–[8]. The design includes state-feedback control and output-feedback control. For state-feedback, [9] focuses on the cooperative control problem of multiple SNSs perturbed by second-order moment processes in a directed topology. Reference [10] studies a class of high-order with stochastic inverse dynamics. Subsequently, [11] discusses output-constrained stochastic systems with low-order and high-order nonlinear and high-order nonlinear and stochastic inverse dynamics. Reference [12] considers the case where the diffusion term and the drift term are unknown parameters for stochastic systems with strict feedback. However, due to the internal variable attribute of the state variable, the state cannot be directly measured, which limits the physical composition of the state-feedback. The basic way to solve this problem is to introduce state reconstruction or state estimation, and use the reconstructed state or estimated state as feedback variables to form state feedback. Furthermore, scholars use an observer to investigate the output-feedback, e.g., [13] is the first to study the output-feedback global stabilization problem of SNSs. [14] studies output-feedback stabilization of SNSs with unknown covariance. Reference [15] considers strict-feedback systems with sensor uncertainties. In addition, for the decentralized output-feedback stabilization problem, [16] investigates SNSs with three types of uncertainties, including nonlinear uncertain interactions, parametric uncertainties, and stochastic inverse dynamics. Reference [17] discusses high-order SNSs.

Besides the stabilization results above, as demonstrated by [18], output tracking is widely used in the military, navigation, and other fields. Recently, [19], [20] discuss the output tracking problem of high-order SNSs with benchmark mechanical systems and stationary Markovian switching respectively. It should be pointed out that [19], [20] only consider the state-feedback output tracking problem, which requires all states of the system to be available. However, as we have previously analyzed, some state information is not always available, so some scholars study the output-feedback tracking control of SNSs [21]–[24]. Specifically, [21] studies adaptive output-feedback tracking control with dynamic uncertainties and unmeasured states. In [23], the output-feedback tracking problem with unstable
linearization is studied. Unfortunately, there are few research results on the design of decentralized output-feedback tracking controller for high-order SNSs.

Based on these discussions, we aim to resolve the decentralized output-feedback tracking for high-order SNSs with unstable linearization. The main contributions and characteristics of this paper are two-fold:

(1) The system model we take into account is more applicable than the existing results [18]–[24]. Different from the previous results [18]–[20], we consider unmeasurable states. Unlike previous studies in [21]–[24], we investigate the decentralized control system. Decentralized systems with unmeasurable states make the controller design process more complicated and difficult. More advanced stochastic analysis techniques are needed.

(2) In order to deal with the problem of output-feedback tracking control for large-scale high-order SNSs, a new stochastic high gain homogeneous control design method is proposed. Different from centralized systems [13]–[15], [21]–[25], the stability analysis of decentralized systems is more challenging, which is another contribution of this paper.

The rest of this paper is listed as follows. The problem is formulated in Section II. In Section III, an output-tracking controller is designed. Section IV is the stability analysis. A simulation is given in Section V. The conclusions are collected in Section VI.

Notations: $R^n$ denotes the n-dimensional space and the set of nonnegative real numbers is represented by $R^+$, $R^+_{n \times n} = \{ q \in R^n: q > 0 \}$ and $X$ denotes the matrix or vector, its transpose is represented by $X^T$. $|X|$ denotes the Euclidean norm of a vector $X$. When $X$ is square, $\text{Tr}[X]$ denotes its trace. For $A \in R^{m \times n}$, $|A|$ denotes the Frobenius norm $|A| = (\sum_{i=1}^{n} \sum_{j=1}^{m} A_{ij}^2)^{1/2}$ and $|A|_\infty = \max_{1 \leq i \leq n} \{ \sum_{j=1}^{m} |A_{ij}| \}$. The set of all functions with continuous $r$th partial derivatives is represented by $C^r$. Let $C_1^{2,1}(R^2 \times R^+ \times S; R^+)$ represent all nonnegative functions $V$ on $R^2 \times R^+ \times S$ which are $C^2$ in $x$ and $C^1$ in $t$.

II. PROBLEM FORMULATION

Consider the following interconnected large-scale high-order SNSs:

\[ d\xi_1 = \left( f_{i1}(\xi) + g_{i1}(\xi) \right) dt + g_{i1}(\xi) d\omega, \]
\[ d\xi_2 = \left( f_{i2}(\xi) + g_{i2}(\xi) \right) dt + g_{i2}(\xi) d\omega, \]
\[ \vdots \]
\[ d\xi_n = \left( f_{in}(\xi) + g_{in}(\xi) \right) dt + g_{in}(\xi) d\omega, \]
\[ \gamma_i = \xi_i - \gamma_i(t), \]

where $\xi = (\xi_1^T, \cdots, \xi_m^T)^T \in R^{mn}$, $\xi = (\xi_1, \cdots, \xi_n)^T \in R^n$, $\gamma_i \in R$ and $u_i \in R$, $i = 1, \cdots, m$, are the system and subsystem state, output and input, respectively. $\xi_1, \cdots, \xi_m$ are unmeasurable. $\gamma_i(t)$ is the reference signal to be tracked. $\omega$ is an $r$-dimensional standard Wiener process defined on a probability space $(\Omega, \mathcal{F}, P)$, with $\Omega$ being a sample space, $\mathcal{F}$ being a filtration, and $P$ being a probability measure. For $i = 1, \cdots, m, j = 1, \cdots, n, p_{ij} \in R^+_{n \times n} = \{ q \in R^n: q \geq 1 \}$ and $q$ is a ratio of odd integers, the functions $f_{ij} : R^{mn} \rightarrow R$ and $g_{ij} : R^{mn} \rightarrow R^n$, are assumed to be $C^1$, vanishing at the origin.

The following assumptions are made on system (1).

Assumption 1: There are constants $b, c \geq 0$, and $\tau \geq 0$ for $i = 1, \cdots, m, j = 1, \cdots, n$, such that

\[ |f_{ij}(\xi)| \leq b \sum_{k=1}^{m} \sum_{l=1}^{n} |\xi|^{(2\epsilon_j+\tau)/(2\epsilon_k)} + c, \]
\[ |g_{ij}(\xi)| \leq b \sum_{k=1}^{m} \sum_{l=1}^{n} |\xi|^{(2\epsilon_j+\tau)/(2\epsilon_k)} + c, \]

where $\epsilon_1 = 1$, $\epsilon_j + \tau = p_{ij} \epsilon_{i,j+1}$. Let $\epsilon_0 = \max_{1 \leq j \leq n} |\epsilon_j|$, and $a_i = \frac{\epsilon_i}{\epsilon_0}$, $i = 1, \cdots, m, j = 1, \cdots, n$. Furthermore, for fixed $i$, one of the following conditions should be satisfied:

1. $\epsilon_i + \tau \geq \epsilon_i$, if $a_{ij} = 1$ or $a_{ij} \geq 2$ for all $i = j, \cdots, n$.
2. $\epsilon_i + \tau \geq 2\epsilon_i$, otherwise.

Assumption 2: The reference signal $\gamma_i(t)$ is continuously differentiable. Moreover, there exists a known constant $M > 0$, such that $|\gamma_i(t)| + |\dot{\gamma}_i(t)| \leq M, \forall t \in [0, +\infty)$.

Assumption 3: For $i = 1, \cdots, m, j = 1, \cdots, n, p_{ij} = p_j$, where $p_j \in R^+_{n \times n}$.

Remark 1: In Assumption 1, it should be emphasized that compared with [17], the constant term $c$ is added, which guarantees the existence of additive bounded disturbances in the drift and diffusion terms. Compared with [21]–[23], we study large-scale SNSs with cross terms shown in Assumption 1. New design schemes should be developed.

For system (1), our objective is to design an output-feedback controller such that the output tracking errors can be made arbitrarily small while all the states of the closed-loop system remain to be bounded in probability.

III. OUTPUT-TRACKING CONTROLLER DESIGN

The design process is divided into two steps:

- Firstly, we analyze the nominal system of (1);
- Secondly, by adopting homogeneous domination method, we design an output-tracking controller for system (1).

A. NOMINAL SYSTEM ANALYSIS

In this part, we study the nominal system of (1) for $i = 1, \cdots, m$.

\[ d\xi_j = \xi_{j+1} dt, j = 1, \cdots, n-1, \]
\[ d\xi_n = \xi_{n} dt, \]
\[ \gamma_i = \xi_i(t). \]

For system (2), from [17], we get the observer

\[ \hat{\xi}_1 = -l_{1,k} \xi_{n-1} + \xi_{0,k-1}, \]
\[ \hat{\xi}_k = (\eta_k + l_{1,k-1} \xi_{n-k-1}) \xi_{n-k-1} + \xi_{k-1}, \]

and the output-feedback controller

\[ u(\hat{\xi}) = -a_{i,m} \xi_{i,m} + a_{i,n-1} \xi_{i,n-1} + \cdots + a_{i,n-1} \xi_{i,n-1} + \xi_{k-1}, \]

\[ + a_{i,n-1} \xi_{i,n-2} \cdots + a_{i,1} \xi_{i,1} \xi_{i+1} + \xi_{i,n}, \]

\[ + a_{i,n-1} \xi_{i,n-2} \cdots + a_{i,1} \xi_{i,1} \xi_{i+1} + \xi_{i,n}, \]
where $\tilde{\xi}_{i1} = \xi_{i1}$ and $l_{ik-1}$, $k = 2, \ldots, n$, are constant gains, $\tilde{\xi}_{i} = (\tilde{\xi}_{i1}, \tilde{\xi}_{i2}, \ldots, \tilde{\xi}_{in})^T$, and $\alpha_{i1}, \ldots, \alpha_{in}$ are constants.

Defining $\mathbf{Z}_i = (\xi_{i1}, \ldots, \xi_{in}, \eta_{i2}, \ldots, \eta_{in})^T$, the closed-loop system can be written as

$$d \mathbf{Z}_i = E_i(\mathbf{Z}_i)dt$$

$$= \left( \xi_{i1}^p, \ldots, \xi_{in}^p, v_i^f, f_{i1} + 1, \ldots, f_{i2n-1} \right)^T dt, \quad (5)$$

where $f_{ij} = -l_{ij-n} \tilde{\xi}_{i1-n}^{p_{ij-n}}$, $j = n + 1, \ldots, 2n - 1$.

For the system (5), we choose the Lyapunov function candidate as

$$V_i = V_{in}(\xi_{i1}, \ldots, \xi_{in}) + U_i(\eta_{i2}, \ldots, \eta_{in}),$$

$$V_{in} = \sum_{j=1}^n \int_{\xi_{i1}}^{\xi_{i1}^p} \frac{|\sigma_j/\epsilon_{i1} - \xi_j|^{4\alpha_{i1}-1}}{\alpha_{i1}} ds,$$

$$U_i = \sum_{j=2}^n \int_{\eta_{i2}}^{\eta_{i2}^p} \frac{|\sigma_j/\epsilon_{i2} - \eta_j|^{4\alpha_{i2}-1}}{\alpha_{i2}} ds,$$

where $\tilde{\xi}_{i1}^p = \alpha_{i1}^{\epsilon_{i1}}/\epsilon_{i1}^{\alpha_{i1}}$, $\zeta_{ij} = \xi_{i1}^{\epsilon_{i1}}/\alpha_{i1}$ and $\gamma_{ij} = \eta_{i2}^{\epsilon_{i2}}/\alpha_{i2}$.

By using [17], we can obtain the following result.

**Lemma 1**: By using $V(\mathbf{Z}_i)$ defined in (6), for system (5), we have

1. $V(\mathbf{Z}_i)$ is positive definite and homogeneous of degree $4/\sigma - \tau$ with dilation $\Delta = (\epsilon_{i1}, \epsilon_{i2}, \ldots, \epsilon_{in})$ for $\xi_{i1}, \ldots, \xi_{in}$

2. $$\frac{\partial V_i}{\partial \mathbf{Z}_i} E_i(\mathbf{Z}_i) \leq -C_0 ||\mathbf{Z}_i||^{4/\sigma},$$

where $||\mathbf{Z}_i||_\Delta = (\sum_{j=1}^n |\xi_j|^{2/\epsilon_j} + \sum_{j=2}^n |\eta_j|^{2/\epsilon_{j-1}})^{1/2}$.

**B. OUTPUT TRACKING CONTROLLER DESIGN**

Define

$$\Theta_i = y_i, \quad \Theta_{ii} = \xi_{ij}, \quad i = 1, \ldots, m, \quad j = 2, \ldots, n - 1. \quad (7)$$

By using (7), system (1) can be rewritten as

$$d \Theta_{ij} = \left( \Theta_{i1}^p + T_{ij}(\Theta_i, y_{ir}) \right) dt + T_{ij}(\Theta_i, y_{ir}) d\omega, \quad j = 1, \ldots, n - 1,$$

$$d \Theta_{in} = \left( U_i^{in} + \tilde{f}_{in}(\Theta_{in}, y_{ir}) \right) dt + \tilde{g}_{in}(\Theta_{in}, y_{ir}) d\omega, \quad y_i = \Theta_i,$$  

where

$$\tilde{f}_{1}(\Theta_{i1}, y_{ir}) = f_{i1}(\Theta_{i1} + y_{ir}) - \dot{y}_{ir},$$

$$\tilde{g}_{1}(\Theta_{i1}, y_{ir}) = g_{i1}(\Theta_{i1} + y_{ir}),$$

$$\tilde{f}_{ij}(\Theta_{ij}, y_{ir}) = f_{ij}(\Theta_{ij} + y_{ir}, \Theta_{i2}, \ldots, \Theta_{ij}),$$

$$\tilde{g}_{ij}(\Theta_{ij}, y_{ir}) = g_{ij}(\Theta_{ij} + y_{ir}, \Theta_{i2}, \ldots, \Theta_{ij}), \quad i = 1, \ldots, m, \quad j = 2, \ldots, n. \quad (9)$$

By (9) and using Assumptions 1-2, there are two constants $b_1, c_1 > 0$ such that

$$\tilde{f}_{ij}(\Theta_{ij}, y_{ir}) \leq b_1 \sum_{j=1}^m \sum_{k=1}^l |\Theta_{jk}|^{(\epsilon_{ij} + \tau)/\epsilon_{jk} + c_1}, \quad \tilde{g}_{ij}(\Theta_{ij}, y_{ir}) \leq b_1 \sum_{j=1}^m \sum_{k=1}^l |\Theta_{jk}|^{(2\epsilon_{ij} + \tau)/2\epsilon_{jk} + c_1}. \quad (10)$$

Introduce the change of coordinates

$$\tilde{\xi}_{i1} = \Theta_{i1},$$

$$\tilde{\xi}_{ij} = \Theta_{ij}/L_{ij},$$

$$\tilde{u}_i^{in} = \frac{U_i^{in} - \tilde{f}_{in}(\tilde{\Theta}_{i1}, y_{ir})}{L_{ij}^{in}}, \quad i = 1, \ldots, m, \quad j = 2, \ldots, n, \quad (11)$$

where $u_i = 0, \quad u_{ij} = (\tilde{\Theta}_{ij}, \tilde{\Theta}_{i2}, \ldots, \tilde{\Theta}_{i1}), \quad$ and $L > 1$ is a constant. By using (11), (8) can be rewritten as

$$d \tilde{\xi}_{ij} = \left( \tilde{L}_{ij} \tilde{\xi}_{ij+1} + \tilde{f}_{ij}(\tilde{\Theta}_{ij}, y_{ir}) \right) dt + \tilde{g}_{ij}(\tilde{\Theta}_{ij}, y_{ir}) \frac{L_{ij}}{L_{ij}^{in}} d\omega, \quad j = 1, \ldots, n - 1,$$

$$d \tilde{\xi}_{in} = \left( \tilde{L}_{ij} \tilde{u}_i^{in} + \tilde{f}_{in}(\tilde{\Theta}_{i1}, y_{ir}) \right) dt + \tilde{g}_{in}(\tilde{\Theta}_{i1}, y_{ir}) \frac{L_{ij}}{L_{ij}^{in}} d\omega, \quad y_i = \tilde{\Theta}_{i1}. \quad (12)$$

Next, we design the homogeneous observer as

$$\tilde{\eta}_{ik} = -L_{ik} \tilde{\xi}_{i1}^{p_{ik-1}},$$

$$\tilde{\xi}_{ik} = (\tilde{\eta}_{ik} + l_{ik-1} \tilde{\xi}_{i1}^{p_{ik-1}})^{\epsilon_{ik-1}}/\epsilon_{ik-1}, \quad k = 2, \ldots, n, \quad (13)$$

and the output-feedback controller

$$u_i(\tilde{\xi}_i) = -\alpha_{i1}^{\epsilon_{i1}}/\epsilon_{i1}^{\alpha_{i1}} + \alpha_{i1-1}^{\epsilon_{i1-1}}/\epsilon_{i1-1}^{\alpha_{i1-1}} + \cdots + \alpha_{i1-1}^{\epsilon_{i1-1}}/\epsilon_{i1-1}^{\alpha_{i1-1}} \tilde{\Theta}_{i1}^{\epsilon_{i1}}/\epsilon_{i1}^{\alpha_{i1}}. \quad (14)$$

The $i$th closed-loop subsystem (12)-(14) can be rewritten as

$$d \mathbf{Z}_i = LE_i(\mathbf{Z}_i)dt + F_i(\mathbf{Z}_i)dt + G_i(\mathbf{Z}_i)d\omega, \quad (15)$$

where

$$\mathbf{Z}_i = (\mathbf{Z}_{i1}, \ldots, \mathbf{Z}_{im})^T,$$

$$\mathbf{Z}_i = (\xi_{i1}, \ldots, \xi_{in}, \eta_{i2}, \ldots, \eta_{in})^T,$$

$$F_i(\mathbf{Z}_i) = \left( \tilde{f}_{1i}(\tilde{\Theta}_{i1}, y_{ir}), \ldots, \tilde{f}_{imi}(\tilde{\Theta}_{i1}, y_{ir}) \right)^T,$$

$$G_i(\mathbf{Z}_i) = \left( \tilde{g}_{1i}(\tilde{\Theta}_{i1}, y_{ir}), \ldots, \tilde{g}_{imi}(\tilde{\Theta}_{i1}, y_{ir}) \right)^T.$$
C. STABILITY ANALYSIS

In this part, for the $i$th closed-loop subsystem, we first give two crucial lemmas, which is useful to deal with the Hessian terms. Then, we present the main results of the stability analysis.

Lemma 2: If Assumption 1 is satisfied, there is a positive constant $\beta_{0i}$ such that

$$\frac{4l\sigma}{8\sigma - \tau - 2\epsilon_{ij}} \leq 1 - \beta_{0i}, i = 1, \ldots, m, j = 1, \ldots, n. \quad (17)$$

Proof: By choosing $l$ and $\sigma$, we can get that

$$l\sigma \geq \epsilon_{ij} + \tau, j = 1, \ldots, n,$$

which with $\epsilon_{ij} > 0$ yields

$$8l\sigma = 4l\sigma + 4l\sigma \geq 4l\sigma + 4\epsilon_{ij} + 4\tau > 4l\sigma + 2\epsilon_{ij} + \tau,$$

which yields

$$0 < \frac{4l\sigma}{8\sigma - \tau - 2\epsilon_{ij}} < 1, j = 1, \ldots, n. \quad (19)$$

From (19), we can choose arbitrarily constant $\beta_{0i} > 0$ satisfying

$$0 < \beta_{0i} \leq 1 - \max_{1 \leq j \leq n} \left\{ \frac{4l\sigma}{8\sigma - \tau - 2\epsilon_{ij}} \right\},$$

so (17) is true, and the proof is thus done.

The following lemma uses homogeneous theory to estimate the $i$th gradient term and Hessian term.

Lemma 3: If Assumptions 1-2 hold, then the following conclusions are satisfied:

$$\left| \frac{\partial V_i}{\partial \Xi} F_i(\Xi) \right| \leq (c_0 L^{1-\gamma_0} + 1) \| \Xi \|_{4\sigma}^4 \beta_1,$$

$$(20)$$

$$\frac{1}{2} \text{Tr} \left\{ G_i \frac{\partial^2 V_i}{\partial Z_i^2} G_i^T \right\} \leq n^2 (L^{1-\gamma_0} + 2L^{-\beta_{0i}}) \| \Xi \|_{4\sigma}^4 + \beta_{12},\quad (21)$$

where $c_0, \gamma_0, \bar{\gamma}_0, \beta_0, \beta_1, \beta_2 > 0$ are constants.

Proof: First, we estimate the $i$th gradient terms: proof of (20).

By using (10) and $L > 1$, with [26, Lemma 2], we have

$$\frac{\tilde{f}_i(\tilde{\eta}_{ij}, y_{ij})}{L_{ij}} \leq b_1 L^{-\epsilon_{ij} + 1} \frac{\left| \xi_{sk} \right|_{ijsk}}{L_{ij}} + c_1 L^{-\epsilon_{ij}}$$

$$= b_1 \sum_{k=1}^{m} \left| \frac{\xi_{sk}}{L_{ij}} \right|_{ijsk}^{\epsilon_{ij} + \tau} + c_1 L^{-\epsilon_{ij}}. \quad (22)$$

By the definition of $\epsilon_{ij}$ and $\nu_{ij}$, with Assumption 3, we have

$$\epsilon_{sk} = \nu_{sk} - \frac{1}{\eta_{sk} \cdots \eta_{sk,k-1}}, \epsilon_{ij} \eta_{sk} - \epsilon_{sk} \nu_{ij} < 0. \quad (23)$$

From (23) and $L > 1$, the power of the gain $L$ can be estimated as

$$\left( \frac{\epsilon_{ij} + \tau}{\nu_{ij}} \right)_{sk} - \epsilon_{ij} \leq 1 - \gamma_{sk}, \quad (24)$$

where $\gamma_{sk} = \frac{1}{\rho_{sk}^{ijsk} \eta_{sk,k-1}} > 0$. By choosing $\gamma_0 = \min_{i,j,s,k} \gamma_{sk}$, with $L > 1$ and (22), we get

$$\left| \frac{\tilde{f}_i(\tilde{\eta}_{ij}, y_{ij})}{L_{ij}} \right| \leq b_1 L^{-\epsilon_{ij}} \frac{\sum_{j=1}^{m} \sum_{k=1}^{l} \left| \xi_{sk} \right|_{ijsk}^{\epsilon_{ij} + \tau}}{L_{ij}} + c_1 L^{-\epsilon_{ij}}$$

$$\leq \tilde{b}_1 L^{-\epsilon_{ij}} \Xi_{\| \Xi \|_{4\sigma}^4}^{\epsilon_{ij} + \tau} + c_1 L^{-\epsilon_{ij}}, \quad (25)$$

where $\tilde{b}_1$ is a positive constant. By using [26, Lemma 2] and (1), we obtain that $\frac{\partial V_i}{\partial \Xi}$ is homogeneous of degree $4l\sigma - \tau - \epsilon_{ij}$.

By using (25) and [26, Lemma 1], with Young’s inequality, we get

$$\left| \frac{\partial V_i}{\partial \Xi} F_i(\Xi) \right| \leq c_0 L^{1-\gamma_0} \Xi_{\| \Xi \|_{4\sigma}^4} + c_1 \sum_{j=1}^{n} L^{-\epsilon_{ij}} \left| \frac{\partial V_i}{\partial \Xi} \right|,$$

where $c_0 > 0$ is a constant.

By using [26, Lemma 2] and Young’s inequality, we obtain

$$c_1 \sum_{j=1}^{n} L^{-\epsilon_{ij}} \left| \frac{\partial V_i}{\partial \Xi} \right| \leq \tilde{c}_1 \sum_{j=1}^{n} \Xi_{\| \Xi \|_{4\sigma}^4}^{\epsilon_{ij} + \tau} + \beta_{12} L^{-4l_{4i} \epsilon_{ij} + (\epsilon_{ij} + \tau)} \Xi_{\| \Xi \|_{4\sigma}^4}^{\epsilon_{ij} + \tau},$$

where $\tilde{c}_1 > 0$ a constant and $\beta_{ij} = \frac{\gamma_{sk} + \lambda_{ij}}{4l\sigma}$.

Substituting (27) into (26) yields

$$\left| \frac{\partial V_i}{\partial \Xi} F_i(\Xi) \right| \leq (c_0 L^{1-\gamma_0} + 1) \Xi_{\| \Xi \|_{4\sigma}^4}^{\epsilon_{ij} + \tau} + \beta_{11}, \quad (28)$$

where

$$\beta_{11} = \sum_{j=1}^{n} \beta_{ij} L^{-4l_{4i} \epsilon_{ij} + (\epsilon_{ij} + \tau)}. \quad (29)$$

Then, we estimate the $i$th Hessian terms: proof of (21).

Similarly, using (10), we get

$$\frac{\tilde{g}_{il}(\tilde{\eta}_{ij}, y_{ij})}{L_{ij}} \leq \tilde{b}_1 L^{1-2-\gamma_1} \Xi_{\| \Xi \|_{4\sigma}^4}^{\epsilon_{ij} + \tau/2} + c_1 L^{-\epsilon_{ij}}, \quad (30)$$

where $\tilde{\gamma}_1$, $\tilde{b}_1 > 0$ are constants.

Let $G_i$ denote the $j$th column of $G$, $j = 1, \ldots, n$. From (30) and [26, Lemma 1], we have

$$\left| G_j^T G_k \right| = \frac{\tilde{g}_{ij}^T(\tilde{\eta}_{ij}, y_{ij})}{L_{ij}} \frac{\tilde{g}_{ik}^T(\tilde{\eta}_{ik}, y_{ik})}{L_{ik}} \Xi_{\| \Xi \|_{4\sigma}^4}^{\epsilon_{ij} + \tau/2} + c_1 L^{-\epsilon_{ij}}.$$
\[ \begin{align*}
&\leq \beta_1^2 L^{1-\gamma_2} \|\Xi\|_\Delta^{4\sigma} + \tau \sum_{j,k=1}^n \beta_{ik} L^{-4\sigma(\gamma_1+i_k)/(\tau/2+\epsilon_k)} \\
&\quad + \sum_{j,k=1}^n \beta_{ik} L^{-4\sigma(\gamma_1+i_k)/(\tau/2+\epsilon_k)} \\
&\quad \leq n^2 L^{1-\beta_0} \|\Xi\|_\Delta^{4\sigma} + \sum_{j,k=1}^n \beta_{ik} L^{-4\sigma(\gamma_1+i_k)/(\tau/2+\epsilon_k)},
\end{align*} \]

where \( \gamma_2 > 0 \) is constant.

From [26, Lemma 2] and 1, we know that \( \frac{\partial^2 V_j}{\partial \xi_2^2} \) is homogeneous of degree \( 4\sigma - \tau - \epsilon_j - \epsilon_k \). By using the definition of \( G_i(\Xi) \), [26, Lemma 1] and \( |A|_\infty \leq \sqrt{\tau} |A| \) is an \( r \)-dimensional square matrix, we can obtain that

\[
\frac{1}{2} \text{Tr} \left\{ G_i \frac{\partial^2 V_i}{\partial \xi_2^2} G^T_i \right\} \leq \frac{1}{2} \left( \frac{\partial^2 V_i}{\partial \xi_2^2} \right)_{i \in \Delta} \leq \frac{1}{2} \sum_{i=1}^n \left( \frac{\partial^2 V_i}{\partial \xi_2^2} \right)_{i \in \Delta} \| G_i \|_{i \in \Delta},
\]

For the first term at the end of (32), denoting \( \tilde{\gamma}_0 = \tilde{\gamma}_1 + \tilde{\gamma}_2 \), we get

\[ \delta_1 \sum_{j,k=1}^n L^{1-\tilde{\gamma}_1-\tilde{\gamma}_2} \|\Xi\|_\Delta^{4\sigma} \leq n^2 L^{1-\tilde{\gamma}_0} \|\Xi\|_\Delta^{4\sigma}. \]

For the second term at the end of (32), with Lemma 2 and Young’s inequality, we get

\[ \delta_2 \sum_{j,k=1}^n L^{1/2-\tilde{\gamma}_1} \|\Xi\|_\Delta^{4\sigma} \leq n^2 L^{1/2-\tilde{\gamma}_0} \|\Xi\|_\Delta^{4\sigma} \]

where \( \Theta = 4\sigma - \tau - \epsilon_j - \epsilon_k \).

Substituting (33) into (32) yields

\[ \frac{1}{2} \text{Tr} \left\{ G_i \frac{\partial^2 V_i}{\partial \xi_2^2} G^T_i \right\} \leq n^2 (L^{1-\tilde{\gamma}_0} + 2L^{1-\beta_0} + 1) \|\Xi\|_\Delta^{4\sigma} + \beta_{2}, \]

where

\[ \beta_{2} = \sum_{j,k=1}^n \beta_{ik} L^{-4\sigma(\gamma_1+i_k)/(\tau/2+\epsilon_k)}. \]
\[
+ \sum_{j,k=1}^{n} \beta_{j,k} L^{-8 \sigma \varepsilon_{(j+k)}} / (r+2 \varepsilon_{j,k}) \\
+ \sum_{j,k=1}^{n} \beta_{j,k} 4 L^{-4 \sigma \varepsilon_{(j+k)}} / (r+\varepsilon_{j}+\varepsilon_{k}).
\]

Now, we are in a position to state the main result of this paper.

**Theorem 1:** If Assumptions 1-2 hold for the stochastic high-order SNs (1). By using the change of coordinates (7) and (11), the observer (13), and the output-feedback controller (14), the practical output-feedback tracking problem for system (1) is solved.

**Proof:** Consider the whole closed-loop system (16). By using Itô formula, Lemmas 1 and 3, we get

\[
LV|_{(16)} = L \sum_{i=1}^{m} \frac{\partial V_i}{\partial \Xi_i} E_i(\Xi) + \sum_{i=1}^{m} \frac{\partial V_i}{\partial \Xi_i} F_i(\Xi) \\
+ \frac{1}{2} \sum_{i=1}^{m} \text{Tr}(G_i(\Xi) \frac{\partial^2 V_i}{\partial \Xi_i^2} G_i^T(\Xi)) \\
\leq -L \sum_{i=1}^{m} \xi_0 ||\Xi||^{4 \sigma}_\Delta + \beta \\
- L \sum_{i=1}^{m} \left( -c_0 L^{-\gamma_0} - L^{-1} \right) ||\Xi||^{4 \sigma}_\Delta \\
+ L \sum_{i=1}^{m} \left( L^{-\gamma_0} + 2 L^{-\beta_0} + L^{-1} \right) ||\Xi||^{4 \sigma}_\Delta \\
\leq -L \left( c_0 - c_0 L^{-\gamma_0} - L^{-1} \right) ||\Xi||^{4 \sigma}_\Delta + \beta \\
+ L \sum_{i=1}^{m} \left( L^{-\gamma_0} + 2 L^{-\beta_0} + L^{-1} \right) ||\Xi||^{4 \sigma}_\Delta,
\]

where \( c_0, c_0, \beta_0 \) and \( \beta = \sum_{i=1}^{m} (\beta_1 + \beta_2) \) are positive constants.

Since \( L > 0 \), we choose a sufficient large \( L \) such that

\[
\tilde{c}_0 = c_0 - c_0 L^{-\gamma_0} - L^{-1} \\
- n^2 (L^{-\gamma_1} + 2 L^{-\beta_0} + L^{-1}) > 0.
\]

By using (38) and (39), we get

\[
LV|_{(16)} \leq -L \tilde{c}_0 ||\Xi||^{4 \sigma}_\Delta + \beta.
\]

From [26, Lemma 2] and 1, there are positive constants \( \tilde{\delta}_1 \) and \( \tilde{\delta}_2 \) such that

\[
\tilde{\delta}_1 ||\Xi||^{4 \sigma}_\Delta - \tau \leq V(\Xi) \leq \tilde{\delta}_2 ||\Xi||^{4 \sigma}_\Delta - \tau.
\]

By using (41), with Young’s inequality, we get

\[
\tilde{\delta}_2 ||\Xi||^{4 \sigma}_\Delta - \tau \leq L ||\Xi||^{4 \sigma}_\Delta + \frac{\tau}{4 \sigma} \tilde{\delta}_2^{4 \sigma} \tau^{4 \sigma}_\Delta / \tau \\
\cdot \left( L \frac{4 \sigma}{4 \sigma - \tau} \right)^{-\delta \sigma \tau / \tau},
\]

which means that

\[
-L ||\Xi||^{4 \sigma}_\Delta \leq -\tilde{\delta}_2 ||\Xi||^{4 \sigma}_\Delta - \tau + \frac{\tau}{4 \sigma} \tilde{\delta}_2^{4 \sigma} \tau^{4 \sigma}_\Delta / \tau.
\]

Substituting (42) into (40), with (41), we have

\[
LV|_{(16)} \leq -\tilde{\delta}_2 \tilde{c}_0 ||\Xi||^{4 \sigma}_\Delta - \tau + \beta \leq -\tilde{c}_0 V + \tilde{\beta},
\]

where

\[
\tilde{\beta} = \tilde{c}_0 \frac{\tau}{4 \sigma} \delta \tau^{4 \sigma}_\Delta / \tau \left( 4 \sigma - \tau \right)^{-\delta \sigma \tau / \tau} + \beta.
\]

By using (43) and [16, Th.1], we obtain that the whole closed-loop system (16) has an almost surely unique solution on \([0, \infty)\) and all the states of the closed-loop system are bounded in probability.

Let

\[
\sigma_l = \inf\{t : t \geq t_0, ||\Xi(t)|| \geq l\}, \forall l > 0.
\]

When \( t \geq t_0 \), choose \( t_1 = \min\{\sigma_l, t\} \). We can obtain that bounded \( ||\Xi(t)|| \) on interval \([t_0, t_1] \) a.s., which means that \( V(\Xi) \) is bounded on \([t_0, t_1] \) a.s. By using (43), we get that \( LV \) is bounded in the interval \([t_0, t_1] \) a.s. Thus by using [27, Lemma 1.9], we can get

\[
E(e^{\tilde{c}_0 V(\Xi(t))}) \\
\leq e^{\tilde{c}_0 V(\Xi(t_0))} + E \int_{t_0}^{t} e^{\tilde{\beta} V(\Xi(s))} ds \\
+ \tilde{c}_0 E \int_{t_0}^{t} e^{\tilde{c}_0 V(\Xi(s))} ds.
\]

Note that \( \lim_{l \to \infty} \sigma_l = \infty \). Then, letting \( l \to \infty \), with (44), we get

\[
e^{\tilde{c}_0 V(\Xi(t))} \leq e^{\tilde{c}_0 V(\Xi(t_0))} + \tilde{c}_0 E \int_{t_0}^{t} e^{\tilde{\beta} V(\Xi(s))} ds \\
+ \tilde{c}_0 E \int_{t_0}^{t} e^{\tilde{c}_0 V(\Xi(s))} ds,
\]

which together with (43) implies

\[
e^{\tilde{c}_0 V(\Xi(t))} \leq e^{\tilde{c}_0 V(\Xi(t_0))} + \frac{\tilde{\beta}}{\tilde{c}_0} e^{\tilde{\beta} t_0} - \frac{\tilde{\beta}}{\tilde{c}_0} e^{\tilde{c}_0 t_0},
\]

or equivalently.

\[
EV(\Xi(t)) \leq e^{-\tilde{c}_0 (t-t_0)} EV(\Xi(t_0)) + \tilde{\beta} e^{\tilde{\beta} t_0} - \tilde{\beta} e^{\tilde{c}_0 t_0},
\]

By the definition of \( \tilde{\beta} \) and tuning the gain \( L \) that \( \tilde{\beta} \) can be made any small, which with (47) yields that \( EV(\Xi(t)) \) can be tuned to be any small. Therefore, noting \( \Xi_i = (\xi_{i1}, \xi_{i2}, \ldots, \xi_{in}, \eta_{i1}, \ldots, \eta_{in}) \), we can obtain that for initial value \( x(t_0) \) and arbitrarily given \( \varepsilon \), there exists a sufficient large \( L \) and finite-time \( T(x(t_0), \varepsilon) \) and such that

\[
E|\xi_{it}(t) - y_{ir}(t)| < \varepsilon, \forall t > T(x(t_0), \varepsilon).
\]

The proof is thus done.
Remark 2: Different from the analysis results of existing centralized systems [21]–[24] (i.e. \( m = 1 \)), our analysis method is more general, because it is suitable for decentralized systems and is also effective for centralized systems. Unlike the deterministic systems [28], [29], by using \( H_\infty \) formula, the stochastic system will produce more nonlinear terms, which increases the difficulty of analyzing the stability. To solve this problem, we use homogeneous technology, which can be found in (20) and (21). Different from the decentralized system [30], it is not necessary to estimate the boundary of diffusion and drift term step by step, which simplifies the design process and improves the design efficiency, see (28) and (32).

IV. A SIMULATION EXAMPLE

In this part, a simulation example is given to show the availability of the control method.

Consider the following system

\[
\begin{align*}
    d\xi_{11} &= \left( \frac{7}{12} + 0.1 \right) dt + 0.2d\omega, \\
    d\xi_{12} &= (u_1 + \xi_{12}\xi_{22}) dt + \frac{1}{3}\xi_{12}\sin \xi_{22}d\omega, \\
    y_1 &= \xi_{11} - \frac{1}{1+t}, \\
    d\xi_{21} &= \left( \xi_{22}^7 + 1 \right) dt + 0.5d\omega, \\
    d\xi_{22} &= (u_2 + \xi_{22}) dt + \xi_{22}^2d\omega, \\
    y_2 &= \xi_{21} - \frac{1}{1+t},
\end{align*}
\]

(49)

where \( \rho_{11} = \rho_{21} = \frac{2}{7} \) and \( \rho_{12} = \rho_{22} = \frac{5}{7} \). By choosing \( \tau = 6, \epsilon_{11} = \epsilon_{21} = 1, \epsilon_{12} = \epsilon_{22} = 3 \) and \( \gamma_i(t) = \gamma_{11}(t) = \gamma_{22}(t) = \frac{1}{1+t} \). Obviously, Assumptions 1-2 are satisfied. We choose \( \sigma_1 = \sigma_2 = 3 \) and \( \rho = 9 \).

By introducing the coordinates

\[
\begin{align*}
    \hat{\xi}_{11} &= \xi_{11} - \frac{1}{1+t}, \quad \hat{\xi}_{12} = \frac{\xi_{12}}{L^{3/7}}, \quad \nu_{1}^{5/3} = \frac{u_1^{1/3}}{L^{10/7}}, \\
    \hat{\xi}_{21} &= \xi_{21} - \frac{1}{1+t}, \quad \hat{\xi}_{22} = \frac{\xi_{22}}{L^{3/7}}, \quad \nu_{2}^{5/3} = \frac{u_2^{1/3}}{L^{10/7}},
\end{align*}
\]

(50)

where \( L > 1 \) is a constant, system (49) can be written as

\[
\begin{align*}
    d\hat{\xi}_{11} &= \left( L\xi_{12}^7 + 0.1 + \frac{1}{1+t} \right) dt + 0.2d\omega, \\
    d\hat{\xi}_{12} &= \left( L\nu_{1}^{5/3} + L^7\xi_{12}\xi_{22} \right) dt + \frac{1}{3}\xi_{12}\sin \left( L^7\xi_{22} \right) d\omega, \\
    y_1 &= \hat{\xi}_{11}, \\
    d\hat{\xi}_{21} &= \left( L^7\xi_{22}^7 + 1 + \frac{1}{1+t} \right) dt + 0.5d\omega, \\
    d\hat{\xi}_{22} &= \left( L^2\nu_{2}^{5/3} + L^7\xi_{22} \right) dt + L^7\xi_{22}^2d\omega, \\
    y_2 &= \hat{\xi}_{21},
\end{align*}
\]

(51)

By following the design procedure in Section III, we have

\[
\begin{align*}
    \hat{n}_{12} &= -L\xi_{12}^7, \\
    \hat{\xi}_{12} &= \left( \eta_{12} + L\xi_{11} \right)^3, \\
    v_1(\hat{\xi}_{11}, \hat{\xi}_{12}) &= -\alpha_{12}(\hat{\xi}_{12} + \alpha_{11}\hat{\xi}_{11})^3, \\
    \hat{\eta}_{22} &= -L\xi_{22}^7, \\
    \hat{\xi}_{22} &= \left( \eta_{22} + L\xi_{21} \right)^3, \\
    v_2(\hat{\xi}_{21}, \hat{\xi}_{22}) &= -\alpha_{22}(\hat{\xi}_{22} + \alpha_{21}\hat{\xi}_{21})^3.
\end{align*}
\]

(52)

For simulation, we select \( L = 3, \xi_{11} = \xi_{21} = 4, \alpha_{11} = 4.5, \alpha_{12} = 3.3, \alpha_{21} = 3, \alpha_{22} = 3.2, \) and the initial conditions as \( \xi_{11}(0) = 3, \xi_{12}(0) = -1, \xi_{21}(0) = -1, \xi_{22}(0) = 1, \eta_{12}(0) = -1, \eta_{22}(0) = 1 \). We can obtain FIGURE 1, which illustrates that the signals \((\xi_{11}, \xi_{21})\) converge to the reference

![FIGURE 1. Responses of tracking during the 0 – 10s.](image1)

![FIGURE 2. Response of tracking errors during the 0 – 10s.](image2)

![FIGURE 3. Response of controllers of closed-loop system (49)–(52).](image3)
signal $y_r(t)$. Meanwhile, we obtain FIGURE 2, FIGURE 3, and FIGURE 4, which respectively depicts that the signals of tracking errors, controllers, and observation errors of the closed-loop system (49)-(52) converge to zero, which means that our constructed observers and output-feedback tracking controllers are efficient.

**Remark 3:** It should be pointed out that our method can not only deal with the case that the drift term and diffusion term are functions, but also deal with the case that they are constants, as shown in (49). This is one of the advantages of this paper.

**V. CONCLUSION**

We investigate the decentralized output-feedback tracking problem for a class of large-scale high-order SNSs in this paper. Specifically, by developing a decentralized high-gain homogeneous domination technique, we design an output-feedback tracking controller. By using advanced stochastic analysis methods, we show that the expectation of the tracking error can be made arbitrarily small while all the states of the closed-loop system remain to be bounded in probability.

There are many related problems to be considered, such as how to extend the result to more general systems [31]–[33].

**REFERENCES**

[1] X. R. Mao, *Stochastic Differential Equations and Applications*. Sawston, U.K.: Horwood, 1997.

[2] P. Protter, *Stochastic Integration and Differential Equations*. Berlin, Germany: Springer, 2013.

[3] M. Krstic and M. Bement, “Nonovershooting control of strict-feedback nonlinear systems,” *IEEE Trans. Autom. Control*, vol. 51, no. 12, pp. 1938–1943, Dec. 2006.

[4] Z.-J. Wu, X.-J. Xie, P. Shi, and Y.-Q. Xia, “Backstepping controller design for a class of stochastic nonlinear systems with Markovian switching,” *Automatica*, vol. 45, no. 4, pp. 997–1004, 2009.

[5] Y. Li, L. Liu, and G. Feng, “Robust adaptive output feedback control to a class of non-triangular stochastic nonlinear systems,” *Automatica*, vol. 89, pp. 325–332, Mar. 2018.

[6] M. Jiang, X.-J. Xie, and K. Zhang, “Finite-time stabilization of stochastic high-order nonlinear systems with FT-SISS inverse dynamics,” *IEEE Trans. Autom. Control*, vol. 64, no. 1, pp. 313–320, Jan. 2019.

[7] W. Li and M. Krstic, “Mean-nonovershooting control of stochastic nonlinear systems,” *IEEE Trans. Autom. Control*, vol. 66, no. 12, pp. 5756–5771, Dec. 2021.

[8] H. Li, W. Li, and J. Gu, “Decentralized stabilization of large-scale stochastic nonlinear systems with time-varying powers,” *Appl. Math. Comput.*, vol. 418, Apr. 2022, Art. no. 126787.

[9] W. Li, L. Liu, and G. Feng, “Cooperative control of multiple nonlinear benchmark systems perturbed by second-order moment processes,” *IEEE Trans. Cybern.*, vol. 50, no. 3, pp. 902–910, Mar. 2020.

[10] X.-J. Xie and J. Tian, “State-feedback stabilization for high-order stochastic nonlinear systems with stochastic inverse dynamics,” *Int. J. Robust Nonlinear Control*, vol. 17, no. 14, pp. 1343–1362, 2007.

[11] R.-H. Cui and X.-J. Xie, “Finite-time stabilization of output-constrained stochastic high-order nonlinear systems with high-order and low-order nonlinearities,” *Automatica*, vol. 136, Feb. 2022, Art. no. 110085.

[12] W. Li and M. Krstic, “Stochastic adaptive nonlinear control with filterless least squares,” *IEEE Trans. Autom. Control*, vol. 66, no. 9, pp. 3893–3905, Sep. 2021.

[13] H. Deng and M. Krstic, “Output-feedback stochastic nonlinear stabilization,” *IEEE Trans. Autom. Control*, vol. 44, no. 2, pp. 328–333, Feb. 1999.

[14] H. Deng and M. Krstic, “Output-feedback stabilization of stochastic nonlinear systems driven by noise of unknown covariance,” *Syst. Control Lett.*, vol. 39, no. 3, pp. 173–182, Mar. 2000.

[15] W. Li, X. Yao, and M. Krstic, “Adaptive-gain observer-based stabilization of stochastic strict-feedback systems with sensor uncertainty,” *Automatica*, vol. 120, Oct. 2020, Art. no. 109112.

[16] S.-J. Liu, J.-F. Zhang, and Z.-P. Jiang, “Decentralized adaptive output-feedback stabilization for large-scale stochastic nonlinear systems,” *Automatica*, vol. 43, no. 2, pp. 238–251, 2007.

[17] W. Li, X. Wei, and S. Zhang, “Decentralised output-feedback control for high-order stochastic non-linear systems,” *IET Control Theory Appl.*, vol. 6, no. 6, pp. 838–846, Apr. 2012.

[18] H.-B. Ji and H.-S. Xi, “Adaptive output-feedback tracking of stochastic nonlinear systems,” *IEEE Trans. Autom. Control*, vol. 51, no. 2, pp. 355–360, Feb. 2006.

[19] X.-J. Xie and N. Duan, “Output tracking of high-order stochastic nonlinear systems with application to benchmark mechanical system,” *IEEE Trans. Autom. Control*, vol. 55, no. 5, pp. 1197–1202, May 2010.

[20] W. Q. Li and Z. J. Wu, “Output tracking of stochastic high-order nonlinear systems with Markovian switching,” *IEEE Trans. Autom. Control*, vol. 58, no. 6, pp. 1585–1590, Jun. 2013.

[21] T. Zhang and X. Xia, “Adaptive output feedback control of stochastic nonlinear systems with dynamic uncertainties,” *Int. J. Robust Nonlinear Control*, vol. 25, no. 9, pp. 1282–1300, Jun. 2015.

[22] H.-S. Yan, Y.-Q. Han, and Q.-M. Sun, “Optimal output-feedback tracking of SISO stochastic nonlinear systems using multi-dimensional Taylor network,” *Trans. Inst. Meas. Control*, vol. 40, no. 10, pp. 3049–3058, Jun. 2018.

[23] W. Li, L. Liu, and G. Feng, “Output tracking of stochastic nonlinear systems with unstable state feedback,” *Int. J. Robust Nonlinear Control*, vol. 28, no. 2, pp. 466–477, Jan. 2018.

[24] W. Li, L. Liu, and G. Feng, “Distributed output-feedback tracking of multiple nonlinear systems with unmeasurable states,” *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 51, no. 1, pp. 477–486, Jan. 2021.

[25] W. Li and M. Krstic, “Prescribed-time output-feedback control of stochastic nonlinear systems,” *IEEE Trans. Autom. Control*, early access, Feb. 14, 2022, doi: 10.1109/TAC.2022.3151587.

[26] I. Polendo, C. I. Qian, and C. B. Schrader, “Homogeneous domination and the decentralized control problem for nonlinear system stabilization,” in *Advances in Statistical Control, Algebraic Systems Theory, and Dynamic Systems Characteristics*. Boston, MA, USA: Springer, 2008, pp. 257–280.

[27] X. R. Mao and C. G. Yuan, *Stochastic Differential Equations With Markovian Switching*. London, U.K.: Imperial College Press, 2006.

[28] C. Qian, “A homogeneous domination approach for global output feedback stabilization of a class of nonlinear systems,” in *Proc. Amer. Control Conf.*, 2005, pp. 4708–4715.

[29] Q. Gong and C. Qian, “Global practical tracking of a class of nonlinear systems by output feedback,” *Automatica*, vol. 43, no. 1, pp. 184–189, Jan. 2007.

[30] Y.-Q. Han and H.-S. Yan, “Observer-based multi-dimensional Taylor network decentralised adaptive tracking control of large-scale stochastic nonlinear systems,” *Int. J. Control*, vol. 93, no. 7, pp. 1605–1618, Jul. 2020.
JIABAO GU received the B.S. degree in information and computing science from the Shandong University of Technology, China, in 2020. He is currently pursuing the M.S. degree with the School of Mathematics and Statistics Science, Ludong University, China. His research interest includes the control of high-order stochastic systems.

HUI WANG (Member, IEEE) received the M.S. degree in operational research and control theory from Ludong University, China, in 2015. She is currently a Lecturer with Ludong University. Her research interest includes distributed control of multi-agent systems.

WUQUAN LI (Senior Member, IEEE) received the Ph.D. degree from the College of Information Science and Engineering, Northeastern University, China, in 2011. From 2012 to 2014, he carried out the Postdoctoral Research with the Institute of Systems Science, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, China. Since January 2011, he has been with the School of Mathematics and Statistics Science, Ludong University, where he is currently a Professor. He is a Young Taishan Scholar in China. He was a Visiting Scholar at the University of California, San Diego, USA. His research interests include stochastic nonlinear systems control and identification of nonlinear systems. He serves as an Associate Editor for two international journals: Systems and Control Letters and Asian Journal of Control.