Constitutive modeling and FE implementation for anisotropic hardening under proportional loading conditions

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Abstract. Anisotropic hardening response (or evolution of yield surface) is an important issue for numerical modeling of sheet metal forming and springback. Lee et al. (2017) recently introduced an improved constitutive model based on the Stoughton and Yoon (2009)’s equation, called the S-Y2009 model in this paper, in order to capture the anisotropic hardening in proportional loading conditions. The Lee et al. (2017)’s model was built by coupling the S-Y2009 model and a non-quadratic model to control the curvature of the yield fitting for more accurate prediction of the yield surface. The Lee et al. (2017)’s model (called the coupled model in this paper) showed good agreements with the measured data. However, in the aforementioned paper, a simulation study for sheet metal forming process with the coupled model was not reported. This paper presents the coupled model in two points of view. The first is the ability of the coupled model to capture the evolution of the yield surface. The other is the performance of the coupled model to describe the anisotropic hardening in a bulge test simulation. Predicting the anisotropic hardening including the biaxial stress state is important to follow the measured data. For the simulation, the coupled model was implemented into Vectorized User MATerial interface (VUMAT) of ABAQUS. The Yld2000-2d model was also incorporated in the comparison because the Yld2000-2d model has been showing good agreements with the initial anisotropy. The results of this study show that capturing the anisotropic hardening is important and the presented approach can be a good model in the sheet metal forming simulation under the proportional loading conditions.

1. Introduction
Prediction of yield surface is an important issue in sheet metal forming simulations. Since plastic hardening differs with respect to the texture of material, yield surface evolves as plastic strain increases. Researchers have proposed some numerical models to capture the evolution of yield surface (or anisotropic hardening) [1-5]. These models are based on an interpolation method at discrete levels of plastic strain. Though these models are effective and have made improvements in capturing anisotropic hardening, they are strongly affected by the interpolation function. In addition, they need a complex transformation process. Stoughton and Yoon [6] introduced an alternative model, called S-Y 2009 model in this paper, which does not need an interpolation and optimization at the discrete levels of plastic strain. The S-Y 2009 model is based on the normalized Hill’s (1948) function [7] and can explicitly incorporate four stress hardening data in $0^\circ$, $45^\circ$, $90^\circ$ to the rolling direction (RD) and the equal biaxial (EB) condition with a hardening function. Despite this advantage, the S-Y 2009 model cannot control its curvature of yield surface. In order to improve this weakness of the S-Y 2009 model, Lee et al., [8] introduced an improved model with coupling of the S-Y 2009 model with a non-quadratic yield function in order to change the curvature of yield surface; this model is called the coupled model in this paper. In this paper, the yield surface fitting of the coupled model is compared with the measured data to show the ability of the coupled model. The target materials are MP980, AA5182-O, AA6022-T43 and 718AT sheets. Then, a bulge test simulation is conducted with the materials. Since the pole of the bulge test undergoes a biaxial stress state, accounting for the anisotropic hardening including the biaxial stress state is important to follow the experimental data. Yld200-2d model [9] is incorporated in the study for describing the initial yield surface data. The study shows that the coupled model can follow the measured data excellently in the bulge test simulation.

2. Numerical model

The S-Y 2009 model under the plane stress condition is described as below [6]:

$$f_{S-Y2009}(\sigma, \bar{\varepsilon}^p) = \left[ \frac{\sigma_{11}}{\bar{\sigma}_0(\bar{\varepsilon}^p)} - \frac{\sigma_{22}}{\bar{\sigma}_{90}(\bar{\varepsilon}^p)} \right] (\sigma_{11} - \sigma_{22}) + \frac{\sigma_{12}^2}{\bar{\sigma}_{12}^2(\bar{\varepsilon}^p)} + \frac{4\sigma_{12}}{\bar{\sigma}_{12}^2(\bar{\varepsilon}^p)} + \frac{\sigma_{11} - \sigma_{22}}{\bar{\sigma}_{12}^2(\bar{\varepsilon}^p)},$$ (1)

where

\begin{equation}
\begin{cases}
\text{Elastic deformation} & \text{if } f(\sigma, \bar{\varepsilon}^p) < 1 \\
\text{Elastoplastic deformation} & \text{if } f(\sigma, \bar{\varepsilon}^p) = 1
\end{cases}
\end{equation}

\(\sigma_{11} \), \(\sigma_{22}\) and \(\sigma_{12}\) are stress components. \(\bar{\sigma}_0(\bar{\varepsilon}^p)\), \(\bar{\sigma}_{90}(\bar{\varepsilon}^p)\), \(\bar{\sigma}_{12}(\bar{\varepsilon}^p)\) and \(\bar{\sigma}_{12}(\bar{\varepsilon}^p)\) are hardening functions at $0^\circ$, $45^\circ$, $90^\circ$ from the RD and EB condition, respectively. Though each hardening function has different parameters according to a loading direction, the equation form is the same. This paper employs a modified Hockett-Sherby hardening functions as below:

$$\bar{\sigma}(\bar{\varepsilon}^p) = A - B \exp \left( -C \left( \bar{\varepsilon}^p \right)^{1/2} \right) + D \bar{\varepsilon}^p,$$ (2)

where \(A\), \(B\), \(C\), \(D\), and \(D\) are the model constants of the modified Hockett-Sherby model and they have different values with respect to a loading condition. The values of the model constants for test materials are summarized in Table 1. Since the S-Y 2009 model is based on a quadratic form, Lee et al. [8] made coupling of the S-Y 2009 model with a non-quadratic function as below:

$$F_{Coup}(\sigma, \bar{\varepsilon}^p) = \left[ f_{S-Y2009}(\sigma, \bar{\varepsilon}^p) \cdot f_{Non-quad}(\sigma) \right]^{1/2},$$ (3)

where

$$f_{Non-quad}(\sigma) = \frac{I}{2}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_1\sigma_3 - \sigma_2\sigma_3).$$
\( F_{\text{Coup}}(\sigma, \varepsilon^p) \) is the coupled model and \( f_{\text{Non-quad}}(\sigma) \) is a non-quadratic function to determine curvature of the whole model, \( F_{\text{Coup}}(\sigma, \varepsilon^p) \). \( \sigma_1 \) and \( \sigma_\parallel \) are the principal stresses. This model employs a non-associated flow rule to determine the direction of plastic strain and the plastic potential function employed is given below:

\[
\tilde{\sigma}_p(\sigma) = \sqrt{\sigma_{11}^2 + \lambda_p \sigma_{22}^2 - 2\nu_p \sigma_{11} \sigma_{22} + 2\rho_p \sigma_{12} \sigma_{12}},
\]

where \( \lambda_p = \frac{(r_\sigma + l)(r_\sigma + l)}{r_\nu(r_\nu + l)} \), \( \nu_p = \frac{r_\nu}{r_\nu + l} \), and \( \rho_p = \frac{r_\nu(r_\nu + l)}{r_\nu(r_\nu + l) + 2r_\sigma} \).

\( r_\sigma \), \( r_\nu \), and \( r_\rho \) are \( r \)-values and the values of \( \lambda_p \), \( \nu_p \), and \( \rho_p \) are summarized in Table 2. This model was implemented into Vectorized User MATerial interface (VUMAT) of ABAQUS and used for the bulge test simulation.

### Table 1. Hardening parameters

| Material   | Condition | A (MPa) | B (MPa) | C   | b   | D (MPa) | r-values |
|------------|-----------|---------|---------|-----|-----|---------|----------|
| 718AT      | 0°        | 529.54  | 318.34  | 9.79| 1.00| 6.17    | 1.830    |
|            | 45°       | 520.33  | 300.91  | 10.17| 1.00| 0.86    | 2.294    |
|            | 90°       | 516.09  | 299.58  | 10.43| 1.00| 0.44    | 2.517    |
|            | EB        | 708.38  | 470.50  | 10.04| 1.00| 2.57    | 0.803    |
| MP980      | 0°        | 1011.98 | 371.15  | 52.99| 0.79| 1114.29 | 0.810    |
|            | 45°       | 999.57  | 400.75  | 61.88| 0.81| 1290.46 | 0.995    |
|            | 90°       | 1028.35 | 411.03  | 77.22| 0.84| 1197.45 | 1.058    |
|            | EB        | 1090.72 | 507.75  | 18.77| 0.64| 435.47  | 0.977    |
| AA6022-T43 | 0°        | 338.86  | 202.29  | 10.38| 1.00| 0.42    | 1.029    |
|            | 45°       | 335.22  | 199.10  | 8.99 | 1.00| 0.38    | 0.532    |
|            | 90°       | 322.13  | 193.64  | 9.20 | 1.00| 0.00    | 0.728    |
|            | EB        | 362.06  | 233.21  | 7.35 | 1.00| 2.55    | 1.149    |
| AA5182-O   | 0°        | 366.75  | 250.97  | 11.18| 1.00| 0.23    | 0.957    |
|            | 45°       | 358.44  | 246.79  | 9.74 | 1.00| 0.67    | 0.934    |
|            | 90°       | 362.39  | 248.11  | 9.98 | 1.00| 0.00    | 1.058    |
|            | EB        | 432.33  | 307.11  | 6.32 | 1.00| 8.36    | 0.948    |

### Table 2. Plastic potential parameters

|          | 718AT | MP980 | AA6022-T43 | AA5182-O |
|----------|-------|-------|------------|----------|
| \( \lambda_p \) | 1.2038 | 0.8705 | 0.9036 | 0.9512 |
| \( \nu_p \)  | 0.5071 | 0.4475 | 0.6466 | 0.4890 |
| \( \rho_p \)  | 1.2275 | 1.4583 | 1.7051 | 1.3956 |
Table 3. Parameters of Yld2000-2d model

| Materials | Yld2000-2d parameters |
|-----------|------------------------|
|           | $\alpha_1$ | $\alpha_2$ | $\alpha_3$ | $\alpha_4$ | $\alpha_5$ | $\alpha_6$ | $\alpha_7$ | $n$   |
| 718AT     | 1.0434     | 1.0291     | 0.8678     | 0.8857     | 0.9243     | 0.8411     | 1.0175     | 6    |
| MP980     | 0.8943     | 1.1598     | 1.200      | 1.0489     | 1.0561     | 1.1770     | 1.0747     | 6    |
| AA6022-T43| 0.9699     | 1.0761     | 1.0339     | 1.0700     | 1.0287     | 1.1312     | 0.9640     | 8    |
| AA5182-O  | 0.9546     | 1.0335     | 0.8260     | 0.9706     | 0.9732     | 0.8425     | 1.0113     | 8    |

3. Results and discussions

Fig. 1 compares the yield surface fitting of the coupled and Yld2000-2d models with the measured data. The parameter values of the Yld2000-2d model are shown in Table 3. The Yld2000-2d and coupled models have very similar initial yield surfaces in all of the materials, as shown in Fig. 1(a-d). Since the coupled model is able to change the curvature, this model can have the similar shape with the initial yield fitting of the Yld2000-2d model in all of the cases. However, as plastic strain increases, the error of the Yld2000-2d model increases especially at the EB stress in each material. On the other hand, the coupled model can capture all of the measured data by following the evolution of the yield surface. The maximum difference between two models occurs at the EB stress in the four cases.

Figure 1. Yield surface fitting of the Yld2000-2d and coupled models
A bulge test simulation was conducted as shown in Fig. 2. The bulge test is one of widely used material tests. In addition, since the two models have the largest difference at the EB condition in each case, analyzing the stress state at the pole of the bulge test is a good study. Fig. 2 shows a quarter model of the bulge test. The radius of the die is 70mm and the thickness of the sheet is 1mm. The simulation was conducted with ABAQUS explicit analysis with the VUMAT. Fig. 3 shows the stress-strain curves of the models at the pole and the blue triangles present the measured data. The measured data came from the Stoughton and Yoon’s study [6]. The coupled model has so good agreements with the measured data. However, the Yld2000-2d model cannot follow the measured data because the model dose not describe the evolution of the yield surface. In the 718AT sheet, the difference between two models is almost 200MPa and it is going to affect the fracture prediction with a stress-based forming limit. These results show that capturing the evolution of yield surface is very important in sheet metal forming simulations and the coupled model captures the measured data very well in the proportional loading cases.

Figure 2. Quarter model of the bulge test
4. Conclusion

This work presents the coupled model in two points of view. The details are summarized as below:

(1) Since the coupled model can control the curvature, this model has almost the same initial yield shape with the prediction of the Yld2000-2d model in all of the test materials.

(2) The ability of the coupled model to describe the evolution of yield surface leads to good agreements with the measured yield data at every level of plastic deformation. On the other hand, the Yld2000-2d model leads to big errors as plastic strain increases. The maximum difference between two models is generated at the EB stress condition. This difference is going to have an effect on sheet metal forming simulation.

(3) The coupled model can follow the measured data of the stress-strain curve in the bulge test while the Yld2000-2d model cannot. This difference will affect prediction of fracture with a stress-based forming limit.

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