On the deterministic and stochastic model applications to typhoid fever disease dynamics

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Abstract: A deterministic and stochastic model depicting typhoid fever spread in human host community is considered. The deterministic model is analyzed and found to be positively invariant, well – posed and meaningful in the sense of typhoid infection spread. Using the typhoid fever – absent equilibrium points, the control basic reproduction threshold ($R_{typ}$) of the model is obtained and the estimation yields $R_{typ} \approx 0.744$. This value implies that controls of vaccination, screening and treatment is effective enough to drive $R_{typ}$ below unity, that is, the level of infection is not effective to cause a typhoid epidemic. Also, the deterministic model is changed into a stochastic model system and the mean and variance of the stochastic model are obtained. The existence and uniqueness of the stochastic model is discussed and the corresponding graphical description of the impact of some parameters against is plotted, while the stochastic simulations are done using the Euler - Maruyama numerical scheme coded in matlab to describe the behavior of each sub - population of human individuals of the model system.

Keywords: Typhoid fever; Vaccination; $R_{typ}$; Existence and uniqueness; Vaccination; Screening; Treatment; Euler - Maruyama.

1. Introduction

Typhoid fever is a debilitating disease that is life - threatening to the global world, is caused by the bacterium Salmonella Typhi. This disease is transmitted through contaminated food and water. It is estimated according to World Health Organization (WHO) that 11 - 20 million individuals become sick from typhoid and mortality cases due to typhoid fever in 128, 000 - 161,000 humans is recorded every year [1]. The clinical symptoms associated to typhoid fever infection includes, fever, fatigue, headache, nausea, abdominal pain, diarrhea, rash, constipation and death respectively. Typhoid fever is treated with antibiotics as well as taking timely vaccination to prevent the infection. Environmental
sanitation is also essential to decontaminate infected environmental sources like water and food sources. Mathematical models are good predictive tools in depicting the evolution, transmission, and likely elimination of diseases [2]. Several works have been done on the derivation of deterministic and stochastic models to describe typhoid transmission. Adetunde [4, 5], studied a mathematical model to predict the prevalence of typhoid in Kassena – Nankan region in Ghana, while Nkemnole [3], formulated a model to describe typhoid fever in Gbagada hospital in Lagos state of Nigeria and suggest possible control policies. Also, Lauria et al. [6], described the impact of reducing public spending in reducing typhoid infection using a mathematical model, while Mushayabasa [7], Watson and Edmund [8], investigated the effect of timely vaccination in minimizing typhoid fever using mathematical model, and Adeboye and Haruna [9] studied the coinfection of typhoid and malaria and how forms of healthy interventions are applied to stem the disease spread. Moreover, the existence and uniqueness of typhoid fever stochastic model was described by Omame et al. [10], while Makinde, Getachew and Tilahun [11] studied the optimal control as cost effective strategy of a direct and indirect transmission of typhoid fever control. In addition, Stephen and Nkuba [12], formulated a mathematical model of typhoid fever with the effect education, vaccination and treatment as healthy interventions in minimizing typhoid disease in human host population, see also, Aji, Aldila [13], Handari, Moathloadi and Gosalamang [14]. Other works on typhoid disease models includes Karundithu, Kimathi and Osman [15] and Jegede [16]. Different from what others have considered, this work proposes the implementation of deterministic and stochastic applications to typhoid disease dynamics in human host. Parameters and variables describing the effect of vaccinations of susceptible births and immigrants, screening and treatment of carriers and infected individuals are incorporated into the model build-up. The paper is arranged into sections. Section 2 present the deterministic implementation of the model and analysis as well as the computation of . Section 3 involves the formulation of the stochastic model and the computation of mean and variance of the stochastic model and the existence and uniqueness of the stochastic model. Also, Section 4 shows the illustrations of graphs and explanations of the numerical results while Section 5 involves the summary of the work.

2. Deterministic Model Implementation

Here the model is grouped into sub – population of the total human host population denoted by $N_p(t)$ where the following are denoted as; Susceptible humans $S_p(t)$; Vaccinated humans $V_c(t)$; Infected humans $I_r(t)$; Carrier humans $C_r(t)$; Treated humans $T_r(t)$ and Recovered humans $R_r(t)$ at time $t > 0$, so that $N_p(t) = S_p(t) + V_c(t) + I_r(t) + C_r(t) + T_r(t) + R_r(t)$. The definitions of parameters are displayed in Table 1. The assumptions guiding the model implementation are that, there is birth and death rate, vaccinations of susceptible births and influx of immigrants are considered. There is recovery through treatment and loss of immunity after recovery, while vaccination wanes overtime. Putting together the assumptions coupled with the definition of parameters in Table 1 leads to a system of first order ordinary differential equations given by:
Fig 1. Flow diagram of typhoid epidemic interactions among human individuals

\[
\begin{align*}
\frac{dS_p}{dt} &= (1-\rho)\Lambda - (\alpha_1 I_f + \mu)S_p + (1-\kappa)\zeta + \tau V_c + e_o R_r, \\
\frac{dV_c}{dt} &= \rho\Lambda + \kappa\zeta - (\tau + \mu)V_c, \\
\frac{dI_f}{dt} &= \alpha_1 S_p I_f - (\beta + \eta_1 + \sigma + \mu)I_f, \\
\frac{dC_r}{dt} &= \beta I_f - (1-\delta)\alpha_2 C_r - \mu C_r, \\
\frac{dT_r}{dt} &= \eta_1 I_f - \alpha_2 \delta C_r - (\mu + \eta_2)T_r, \\
\frac{dR_r}{dt} &= \eta_2 T_r - (\mu + e_o)R_r.
\end{align*}
\]

(1)

Subject to the initial conditions \( S_p(0) \geq 0, V_c(0) \geq 0, I_f(0) \geq 0, C_r(0) \geq 0, T_r(0) \geq 0, R_r(0) \geq 0. \)

Table 1. Parameter descriptions of the model describing Typhoid dynamics

| Parameters | Descriptions                           | Values    | Sources |
|------------|----------------------------------------|-----------|---------|
| \( \Lambda \) | Recruitment of susceptible births      | 0.83681   | [13]    |
| \( \rho \)   | Fraction of vaccinated susceptible     | 0.9       | [8]     |
| \( \alpha_1 \) | Effective infectious contact rate      | 0.0002    | [13]    |
| \( \zeta \)   | Infected immigrants                    | 0.027     | Assumed |
| \( \mu \)    | Natural death rate                     | 0.02041   | [17]    |
| \( \kappa \)  | Fraction of vaccinated immigrants      | 0.09      | [13]    |
| \( \tau \)   | Vaccination waning rate                | 0.33      | Assumed |
| \( e_o \)    | Loss of immunity                       | 0.33      | [15]    |
| \( \beta \)  | Transition rate from infected to carrier | 0.5      | [14]    |
2.1. Analysis of the Model and Computation of $R_{typh}$

Theorem 1: A region $\Omega$ exist where the solution space $(S_p, V_c, I_f, C_r, T_r, R_r)$ is invariant positively and well - posed.

\textit{proof:} Given the space $(S_p, V_c, I_f, C_r, T_r, R_r)$ with positive initial starts $S_p(0) \geq 0, V_c(0) \geq 0, I_f(0) \geq 0, C_r(0) \geq 0, T_r(0) \geq 0, R_r(0) \geq 0$.

Then $N_p(S_p, V_c, I_f, C_r, T_r, R_r) = S_p(t) + V_c(t) + I_f(t) + C_r(t) + T_r(t) + R_r(t)$. (2)

Adding up the whole system of $N_p$ along the derivative of the model system (1) yields

$N_p = (1 - \rho)\Lambda + (1 - \kappa)\zeta - (S_p + V_c + I_f + C_r + T_r + R_r)\mu$ (3)

So that

$N_p = (1 - \rho)\Lambda + (1 - \kappa)\zeta - N_p\mu$ (4)

Solving (4) yields

$N_p \leq \frac{(1 - \rho)\Lambda + (1 - \kappa)\zeta}{\mu} (1 - \exp(-\mu t)) + N_p(S_p + V_c + I_f + C_r + T_r + R_r)\exp(-\mu t)$ (5)

As $t \to \infty$ in (5), taking the limit of both sides of (5) yields

$N_p \leq \frac{(1 - \rho)\Lambda + (1 - \kappa)\zeta}{\mu}$ (6)

From (6), all solutions stays and remain in the region $\Omega$ and are non – negative. The positive invariant region exist and given by

$\Omega = \{(S_p, V_c, I_f, C_r, T_r, R_r) \in \mathbb{R}^6^+ : N_p \leq \frac{(1 - \rho)\Lambda + (1 - \kappa)\zeta}{\mu}\}$. (7)

Hence, model system (1) is positively invariant, well - posed and meaningful in the sense of typhoid dynamics. The trivial static solution does since recruitment of birth of susceptible term and influx of
immigrants terms exist. Therefore the typhoid - free equilibrium solutions i.e., when the system is free of typhoid infection is given by

\[
(S_p, V_c, I_f, C_r, T_r, R_r) = \left(\frac{(1-\rho)A+(1-\kappa)\xi}{\mu}, \frac{\rho A + \kappa \xi}{\tau + \mu}, 0, 0, 0, 0\right).
\] (8)

The basic reproduction number \((R_{\text{typ}})\) of this model system is the rate of average secondary cases of typhoid generated when a typical typhoid infected individual is admitted into the susceptible population of humans in the course of their infection. The next generation matrix method is used to compute for \(R_{\text{typ}}\), see [12, 15]. The two typhoid infected class \(I_f\) and \(C_r\) is linearized around the typhoid - free equilibrium solution to give \(R_{\text{typ}}\) of model system (1) as

\[
R_{\text{typ}} = \frac{\alpha_3 A (1-\rho) + (1-\kappa)\xi}{\mu (\beta + \eta_1 + \sigma + \mu)}
\] (9)

In (9), \(R_{\text{typ}}\) is called a vaccination controlled basic reproduction number, where births of susceptible recruited and influx of immigrants are vaccinated, where \(\rho > 1 - \frac{1}{R_{\text{typ}}}\) and \(\kappa > 1 - \frac{1}{R_{\text{typ}}}\) leads to a herd immunity level. The parameter \(\alpha_3\) denoted typhoid transmission rate per single infective, while each infectious individual spends on average \(\frac{1}{\beta + \eta_1}\) time units in their class. The period of typhoid infection is minimized due to natural and typhoid related mortality. In this work, \(R_{\text{typ}} \approx 0.744\), which is less than unity. However, \(R_{\text{typ}}\) can be greater than unity, if vaccination wanes overtime and there is non - compliance to other public health measures.

3. Transformation of the Deterministic Model (1) into a Stochastic Model.

Here, the interest is changing the parameters involved in model system (1) into random variables, where the randomness is incorporated into the stochastic model. The stochastic model consists of the drift or deterministic parts and diffusion or stochastic parts. In order to obtain the mean and the variance of the model, the transition probabilities obtained in model system (1) is presented in Table 2.

| Changes | Probabilities | Event |
|---------|---------------|-------|
| \([1 0 0 0 0 0]^T\) | \(P_1 = \Delta \Delta t\) | Birth of a susceptible |
| \([1 0 0 0 0 0]^T\) | \(P_2 = \zeta \Delta t\) | Influx immigrants |
| \([-1 0 1 0 0 0]^T\) | \(P_3 = a_1 S_p I_f \Delta t\) | Effective infectious contact rate |
| \([-1 1 0 0 0 0]^T\) | \(P_4 = \kappa \xi \Delta t\) | Vaccinated immigrants |
| \([-1 1 0 0 0 0]^T\) | \(P_5 = \rho A \Delta t\) | Vaccinated susceptible births |
| \([-1 0 0 0 0 0]^T\) | \(P_6 = \mu S_p \Delta t\) | Natural death of susceptible |
| \([1 -1 0 0 0 0]^T\) | \(P_7 = \tau V_c \Delta t\) | Vaccination waning |
| \([0 -1 0 0 0 0]^T\) | \(P_8 = \mu V_c \Delta t\) | Natural death of vaccinated |
| \([0 0 -1 1 0 0]^T\) | \(P_9 = \beta I_f \Delta t\) | Transition of infected to carrier class |
| \([0 0 -1 0 1 0]^T\) | \(P_{10} = \eta I_f \Delta t\) | Transition of infected to treated class |
| \([0 0 -1 0 0 0]^T\) | \(P_{11} = \sigma C_r \Delta t\) | Death due to typhoid infection |
| \([0 0 -1 0 0 0]^T\) | \(P_{12} = \mu I_f \Delta t\) | Natural death of infected individuals |
| \([0 0 0 -1 1 0]^T\) | \(P_{13} = \delta \epsilon C_r \Delta t\) | Treated carriers |
| \([0 0 0 -1 0 0]^T\) | \(P_{14} = \alpha C_r \Delta t\) | Screened carriers |
3.1. Mean and Variance of the Stochastic Model

Following the method of Allen [4], the stochastic model equations are given by

$$d\mathbf{X} = \mathbf{f}(t, \mathbf{X}(t))dt + \mathbf{G}(t, \mathbf{X}(t))d\mathbf{W}(t)$$

$$\mathbf{X}(0) = [X_1(0), X_2(0), X_3(0), X_4(0), X_5(0), X_6(0)]^T$$

Where the drift vector is defined as

$$\mathbf{f} = \sum_{j=1}^{19} p_j \mathbf{A}_j$$

Where $\mathbf{A}_j$ and $p_j$ denote the random changes and transition probabilities respectively. Applying (10) and (11) to the model system (1) yields the drift vector $\mathbf{f}$ given by

$$\mathbf{f} = \begin{pmatrix}
(1-\rho)\Lambda - (\alpha_1 I_f + \mu)S_p + (1-\kappa)\xi + \tau V_c + e_o R_r, \\
\rho\Lambda + k\xi - (\tau + \mu)V_c, \\
\alpha_1 S_p I_f - (\beta + \eta_1 + \sigma + \mu)I_f \\
\beta I_f - (1-\delta)\alpha_2 C_r - \mu C_r \\
\eta_1 I_f - \alpha_2 \delta C_r - (\mu + \eta_2)T_r, \\
\eta_2 T_r - (\mu + e_o)R_r.
\end{pmatrix}$$

The diffusion matrix $\mathbf{G}$ has the entries $\lambda_{i,j} p_j^{1/2}$, where $\lambda_{i,j}$ and $p_j$ ($i = 1, \ldots, 6$, $j = 1, \ldots, 19$) are the components of the random changes and transition probabilities respectively.

Also,

$$\mathbf{W}(t) = [W_1(t), W_2(t), W_3(t), W_4(t), W_5(t), W_6(t), W_7(t), W_8(t), W_9(t), W_{10}(t), W_{11}(t), W_{12}(t), W_{13}(t), W_{14}(t), W_{15}(t), W_{16}(t), W_{17}(t), W_{18}(t), W_{19}(t)]^T$$

Where (13) is a vector of nineteen independent Wiener processes.

Considering $\mathbf{f}(t, \mathbf{X}(t))$ as the drift part (mean) and $\mathbf{G}(t, \mathbf{X}(t))$ as the diffusion part (variance) where

$$\mathbf{f}(t, \mathbf{X}(t)) = \frac{E\Delta x}{\Delta t} \quad \text{and} \quad G(t, \mathbf{X}(t)) = V^{1/2} = \sqrt{\frac{E[\Delta x \Delta x^T]}{\Delta t}}$$

Then $E\Delta x = \sum_{i=1}^{19} p_i (\Delta x_i) \Delta t$, so that
\[ E[\Delta x] = \begin{bmatrix} P_1 + P_2 - P_3 + P_4 - P_5 + P_6 + P_7 + P_{19} \\
+ P_5 - P_7 - P_8 \\
- P_9 - P_{10} - P_{11} - P_{12} \\
+ P_9 - P_{13} - P_{14} - P_{15} \\
+ P_{10} + P_{13} - P_{16} - P_{17} \\
- P_{16} - P_{18} - P_{19} \end{bmatrix} \]

(14)

And \( E[\Delta x \Delta x^T] = \sum_{i=1}^{19} p_i (\Delta x_i) \Delta t^2 \Delta \), where

\[
E[\Delta x \Delta x^T] = \begin{bmatrix} P_1 + P_2 + P_6 \\
+ P_8 \\
P_{11} + P_{12} \\
P_{14} + P_{15} \\
P_{17} \\
P_{18} \end{bmatrix}
\]

(15)

From (12) – (16), the transformed stochastic model equations are given by:

\[
\begin{align*}
\dot{S}_p &= (1 - \rho)\Lambda - (\alpha_1 I_1 + \mu) S_p + (1 - \kappa) \xi + \tau \gamma + e_o R_r + \sqrt{P_1 + P_2 - P_3 - P_4 - P_5 - P_6 + P_7 + P_{19} W_1 + \\
&& \sqrt{P_1 W_1 + \sqrt{P_2 W_2 + \sqrt{P_3 W_3}}} \\
\dot{V}_c &= \rho \Lambda + \kappa \Lambda - (\tau + \mu) V_c + \sqrt{P_4 + P_5} - P_7 + P_8 W_2 + \sqrt{P_8 W_6}, \\
\dot{I}_r &= \alpha_1 S_p - (1 - \delta) \alpha_2 C_r - \mu C_r + \sqrt{P_9 + P_{13} - P_{14} - P_{15} W_4 + \sqrt{P_{14} W_{14}}} + \sqrt{P_{15} W_{15}}, \\
\dot{C}_r &= \beta I_r - (1 - \delta) \alpha_2 C_r - \mu C_r + \sqrt{P_9 + P_{13} - P_{14} - P_{15} W_4 + \sqrt{P_{14} W_{14}}} + \sqrt{P_{15} W_{15}}, \\
\dot{T}_r &= \eta_1 I_r - \alpha_2 \delta C_r - (\tau + \mu) T_r + \sqrt{P_{10} + P_{13} - P_{16} - P_{17} W_5 + \sqrt{P_{17} W_7}}, \\
\dot{R}_r &= \eta_2 T_r - (\mu + e_o) R_r + \sqrt{P_{16} - P_{15} - P_{19} W_6 + \sqrt{P_{18} W_{18}}}.
\end{align*}
\]

Subject to the initial conditions \( S_p(0) \geq 0, \ V_c(0) \geq 0, \ I_r(0) \geq 0, \ C_r(0) \geq 0, \ T_r(0) \geq 0, \ R_r(0) \geq 0 \).

3.2 Existence and Uniqueness of the stochastic model (16)

Assuming that the coefficients in the following system of differential equations

\[
dX^i_t = a_i(t, X_t) dt + \sum_{i=1}^{n} \sum_{j=1}^{m} b_{ij} (t, X_t) dW_t^j
\]

(17)

where

\[ X_t = (X^1_t, X^2_t, \ldots, X^n_t)^T \]

(18)

And

\[ W_t = (W^{1}_t, W^{2}_t, \ldots, W^{m}_t)^T \]

(19)

Then \( a_i(t, X_t) \) is a \( n \)-dimensional vectors with entries \( a_{ij}(t, x) \) and \( b_{ij}(t, x) \) where a \( n \times m \) matrix with entries \( b_{ij}(t, x) \) satisfy the following Lipschitz and growth conditions for some constant \( k < \infty \), and for all \( t \in \mathbb{R} \), and \( x, y \in \mathbb{R}^n \) with the following
\[ \|a_i(t, x) - a_i(t, y)\| \leq k\|x - y\|, \]
\[ \|b_{ij}(t, x) - b_{ij}(t, y)\| \leq k\|x - y\|, \]
\[ \|a_i(t, x)\| \leq k\|x\|, \]
\[ \|b_{ij}(t, x)\| \leq k\|x\|, \]
\[ \|b\| = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{m} b_{ij}(x)^2}, \]
\[ \|a\| = \sqrt{\sum_{i=1}^{n} a_i(x)^2}. \]

(20)

Then for each \( x \in \mathbb{R}^n \) there is a unique solution to the system of stochastic differential equations (16) such that \( X = x \). The following changes are made from (16) so that

\[ f_1 = (1-p)\Lambda - (\alpha_1 I_f + \mu)S_p + (1-\kappa)\zeta + \tau V_c + e_0 R_r, \]
\[ f_2 = p\Lambda + \kappa \zeta - (\tau + \mu)V_c, \]
\[ f_3 = \alpha_2 S_p I_f - (\beta + \eta_1 + \sigma + \mu)I_f \]
\[ f_4 = \beta I_f - (1-\delta)\alpha_2 C_r - \mu C_r \]
\[ f_5 = \eta_1 I_f - \alpha_2 \delta C_r - (\mu + \eta_2)T_r, \]
\[ f_6 = \eta_2 T_r - (\mu + e_0)R_r. \]

(21)

Then there exists a constant \( M > 0 \) such that

\[ \left| \frac{\partial f_1}{\partial p} \right| \leq M, \left| \frac{\partial f_1}{\partial \tau} \right| \leq M, \left| \frac{\partial f_1}{\partial V_c} \right| \leq M, \left| \frac{\partial f_1}{\partial I_f} \right| = 0, \left| \frac{\partial f_1}{\partial I_r} \right| = 0, \left| \frac{\partial f_1}{\partial e_0} \right| \leq M, \]

(22)

\[ \left| \frac{\partial f_2}{\partial p} \right| = \left| \frac{\partial f_2}{\partial \tau} \right| = \left| \frac{\partial f_2}{\partial V_c} \right| = \left| \frac{\partial f_2}{\partial I_f} \right| = 0, \left| \frac{\partial f_2}{\partial I_r} \right| = \left| \tau + \mu \right| \leq M, \]

(23)

\[ \left| \frac{\partial f_3}{\partial p} \right| = \left| \frac{\partial f_3}{\partial \tau} \right| = \left| \frac{\partial f_3}{\partial V_c} \right| = \left| \frac{\partial f_3}{\partial I_f} \right| = 0, \left| \frac{\partial f_3}{\partial I_r} \right| = \left| \beta \right| \leq M, \left| \frac{\partial f_3}{\partial C_r} \right| = \left| (1-\delta)\alpha_2 + \mu \right| \leq M, \]

(24)

\[ \left| \frac{\partial f_4}{\partial p} \right| = \left| \frac{\partial f_4}{\partial \tau} \right| = \left| \frac{\partial f_4}{\partial V_c} \right| = \left| \frac{\partial f_4}{\partial I_f} \right| = 0, \left| \frac{\partial f_4}{\partial I_r} \right| = \left| \eta_1 \right| \leq M, \left| \frac{\partial f_4}{\partial C_r} \right| = \left| \alpha_2 \delta \right| \leq M, \left| \frac{\partial f_4}{\partial T_r} \right| = \left| \mu + \eta_2 \right| \leq M \]

(25)

\[ \left| \frac{\partial f_5}{\partial p} \right| = \left| \frac{\partial f_5}{\partial \tau} \right| = \left| \frac{\partial f_5}{\partial V_c} \right| = \left| \frac{\partial f_5}{\partial I_f} \right| = 0, \left| \frac{\partial f_5}{\partial I_r} \right| = \left| \eta_2 \right| \leq M, \left| \frac{\partial f_5}{\partial C_r} \right| = \left| \alpha_2 \right| \leq M, \left| \frac{\partial f_5}{\partial T_r} \right| = \left| \mu + \eta_2 \right| \leq M \]

(26)
The elements of the diffusion matrix are continuously differentiable. Therefore for stochastic differential equation describing typhoid dynamics in (16), we obtain

\[ \|f\| = \sqrt{\sum_{i=1}^{6} f_i(x)^2} \quad \text{and} \quad \|G\| = \sqrt{\sum_{i=1}^{6} \sum_{j=1}^{19} b_{ij}(x)^2}. \]  

Both \(\|f\|\) and \(\|G\|\) are continuously differentiable and hence satisfy the Lipschitz condition. Since the norms exist, they are bounded. The drift and the diffusion matrices are therefore bounded. Hence, they satisfy the conditions for existence and uniqueness of solution.

4. Numerical Simulations and Discussion of Results.

In this section, the behavior of the parameters involved in the computation of \(R_{typ}\) \((R_{typ} \approx 0.7944)\) of the deterministic model is analyzed is shown in Figs. 2 - 9. The parameter values used in the course of simulation and sources can be seen in Table 1. The following initial starts were adopted, \(S_p = 70, V_c = 30, I_p = 20, C_r = 40, T_r = 25\). The numerical value of \(R_{typ} \approx 0.7944\) was plotted against the parameter values involved in the computation of \(R_{typ}\). Also, the stochastic Euler-Maruyama scheme via matlab was employed to solve the stochastic model equations in (16), and the results of each sub-group of human host population was plotted against time, which is taken to be 15 days.

Fig. 2: Impact of \(\alpha_1\) on \(R_{typ}\).  
Fig. 3: impact of \(\Lambda\) on \(R_{typ}\).

Fig. 2 describes the impact of the parameter \(\alpha_1(0.0020)\) on \(R_{typ}\). The sharp rise of the curve, denote that \(R_{typ}\) is likely to increase above unity when effective infectious contact occur regularly between typhoid infected and susceptible individuals. To minimize typhoid infections, Healthy measures has to be implemented to keep \(R_{typ} < 1\). Fig. 3 depict that, as birth rate increases, susceptibility to typhoid infection increases which is likely to increase \(R_{typ}\) unless control measure of vaccination is administered to susceptible births and immigrants before being infected with typhoid fever disease.
Fig. 4 describes the impact of natural death rate. The gradual decline shows that infections can be minimized by natural death and $R_{typ}$ threshold is kept below 1. Fig. 5 describes the effect of $\kappa(0.9)$ which denote the gradual decline of the vaccination of proportion of immigrants that timely vaccination is a potent strategy in reducing and probably eliminating typhoid in immigrants.

Fig. 4 describes the impact of natural death rate. The gradual decline shows that infections can be minimized by natural death and $R_{typ}$ threshold is kept below 1. Fig. 5 describes the effect of $\kappa(0.9)$ which denote the gradual decline of the vaccination of proportion of immigrants that timely vaccination is a potent strategy in reducing and probably eliminating typhoid in immigrants.

Fig. 6: Impact of $\zeta$ on $R_{typ}$

Fig. 7: Impact of $\rho$ on $R_{typ}$.
The impact of $\zeta$ is seen on $R_{typ}$. $R_{typ}$ could be greater than 1, if in the event of steady influx of immigrants, vaccines are not applied timely or not readily available. But with the decline as time decreases, prompt vaccination of immigrants is essential to combat typhoid fever. Fig. 7 describes the effect of timely vaccination of susceptible births. The sharp decline shows that early administration of vaccination is effective in controlling $R_{typ}$ and with consistent vaccination, elimination of typhoid fever in human host community is certain.

![Fig 7: Effect of Timely Vaccination of Susceptible Births](image1)

![Fig 8: Impact of $\eta_1$ on $R_{typ}$](image2)

![Fig 9: Impact of $\sigma$ on $R_{typ}$](image3)

Fig. 8 describes the impact of $\eta_1$. It shows that compliance to timely and effective treatment of typhoid infected individuals is effective in keeping $R_{typ} < 1$. Hence, drugs like antibiotics are needed to treat the disease. Fig. 9 describes the gradual decline of the impact of $\sigma$ against $R_{typ}$. This shows that death due to typhoid is capable of minimizing typhoid, but due to the typhoid related deaths, controls are important to combat and eliminate typhoid while keeping the basic threshold $R_{typ}$ low.
Fig. 10: Solution of $S_p$ against time (days)

Fig. 11: Solution of $V_c$ against time (days)

Fig. 10 shows that between 5 - 15 days, susceptible sub-group decreases and become gradually infected, especially if vaccines are not applied on time or not available. Also, the gradual decline may be due to certain individuals who refused to be available for vaccine administration, or factors of death due to natural causes. Fig. 11 shows that vaccination of susceptible births and immigrants are effective in controlling typhoid disease. The steady decline shows that between 9 - 15 days, typhoid fever will be lessened. Therefore vaccination programmes must be made available in the human host community.

Fig. 12: Solution of $I_f$ against time (days)

Fig. 13: Solution of $C_r$ against time (days)

Fig. 12 shows that the infection level rose from 20 to about 45 individuals in 5 days, but it suddenly declines due to adherence to screening and treatment of infected individuals. Also, the sudden decline occurs due to death related to typhoid. The decline in behavior of Fig. 13 depict a decrease in the
carrier sub-population within 15 days as time increases. The declining is due to screening and treatment against typhoid infection.

Fig. 14: Solution of $T_r$ against time (days)  

Fig. 15: Solution of $R_r$ against time (days).

Fig. 14 shows the gradual rise of treated individuals within 15 days. Administration of antibiotics is effective in stemming the infection of typhoid fever in human host. Fig. 15 displays the behavior of the solution of recovered human populations. The recovered population increases due to the response of the treated individuals to treatment against salmonella typhi.

5. Summary

A deterministic and stochastic model describing the transmission of typhoid is considered. The deterministic Model is shown to be well-posed and meaningful in the sense of typhoid epidemic. The estimated $R_{typ} \approx 0.7944$ reveal that the vaccination, treatment and screening are effective forms of interventions strategies, which lessens $R_{typ}$ below unity, otherwise the disease becomes a full blown epidemic. The existence and uniqueness of the stochastic model system is analyzed and the mean and variance is obtained. Simulations carried out in this work reveal that the intervention strategies of vaccination of susceptible births, screening and treatment of carriers and infected individuals adopted in this work is effective in minimizing typhoid infection in human host community.

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