Localized instabilities at conifolds

Angel M. Uranga

Dpto. de Física Teórica C-XI and Instituto de Física Teórica C-XVI
Universidad Autónoma de Madrid, 28049 Madrid, Spain

Abstract

We consider the M-theory lifts of configurations of type IIA D6-branes intersecting at angles. In supersymmetry preserving cases, the lifts correspond to special holonomy geometries, like conifolds and $G_2$ holonomy singularities. Transitions in which D6-branes approach and recombine lift to topology changing transition in these geometries. In some instances non-supersymmetric configurations can be reliably lifted, leading to the same topological manifolds, but endowed with non-supersymmetric metrics. In these cases the phase transitions are driven dynamically, due to instabilities localized at the singularities. Even though in non-compact setups the instabilities relax to infinity, in compact situations there exist nearby minima where the instabilities disappear and the decay reaches a well-defined (in general supersymmetric) endpoint.

\footnote{Angel.Uranga@uam.es}
1 Introduction

Recently new insights into the dynamics of non-supersymmetric string configurations have been achieved by studying localized instabilities. For instance, open string tachyons localized on D-brane world-volumes (see e.g. [1]), and closed string tachyons at twisted sectors of non-supersymmetric orbifolds [2, 3, 4, 5].

In this paper we study a new (but somewhat related) kind of instabilities, localized at certain non-supersymmetric singularities, obtained as U-duals of brane configurations (basically, as M-theory lifts of type IIA intersecting D6-brane configurations; a related kind of lifts has been recently studied in [8]). The instabilities arise in the singular world-sheet CFT regime, a fact which makes their analysis difficult. In particular, standard tools like use of D-brane probes [2], mirror symmetry [4], etc. are not valid. Instead, we rely on the intuition gained from the U-dual D-brane system, plus analysis of the energetics justified by use of BPS formulae.

Our singularities contain a set of continuous parameters (not dynamical moduli) connecting them to special holonomy metrics, for instance conifold singularities. The susy cases contain moduli which parametrize smoothing of the singularities, and/or topology changing transitions. Our main result is that in the non-supersymmetric situation these processes are dynamically triggered by the localized instability. In non-compact situations, the dynamical smoothing of the singularity is reminiscent of [2]. In compact setups [1], however, our localized instabilities present new features, namely they can reach a stable point at finite distance, and they lead to no reduction in spacetime dimension.

The paper is organized as follows: In Section 2 we describe configurations of D6-branes intersecting over five dimensions, and their M-theory lift given by conifold singularities. In Sections 2.2, 2.3 we argue that the configurations suffer an instability, and propose a dynamical conifold transition as the natural relaxation process. Section 3 contains similar discussions for other D6-brane intersections, involving a dynamical topology changing flop of 2-spheres (Section 3.1) and a dynamical resolution of codimension seven singularities, of $G_2$ holonomy in the supersymmetric case (Section 3.2). Section 4 contains our final comments. Appendix A reviews the construction of recombined special lagrangian cycles [12], and appendix B discusses topological obstructions to phase transitions from the D6-brane and M-theory viewpoints.

1See [9] for a similar kind of localized tachyons in fluxbrane backgrounds. For some results on not localized closed tachyons, see e.g. [10].

2Configurations of intersecting D6-branes in compact manifolds have been studied in [3, 10, 11].
2 Two angle system

2.1 System of D6-branes intersecting over 5d

In this section we consider the dynamics of two D6-branes intersecting over a five-dimensional subspace of their world-volume. Without loss of generality, the geometry of the configuration is

$$\text{D6} \ 0 \ 1 \ 2 \ 3 \ 4 \times [6 \ 7]_{\theta_1} [8 \ 9]_{\theta_2}$$

$$\text{D6}' \ 0 \ 1 \ 2 \ 3 \ 4 \times [6 \ 7]_{\theta_1'} [8 \ 9]_{\theta_2'}$$ (2.1)

meaning that the D6-brane spans the directions 01234 times a line at angle $\theta_1$ with the $x^6$ axis in the $\mathbb{R}^2$ parametrized by 67, times a line at angle $\theta_2$ with the $x^8$ axis in $(\mathbb{R}^2)^{89}$. Analogously for the D6'-brane.

We will also consider compact examples with 6789 parametrizing a $T^4$, taken rectangular for simplicity $(T^2)^{67} \times (T^2)^{89}$. The angles are then defined in terms of the torus radii $R_i$ and the wrapping numbers $(n_6, n_7, n_8, n_9)$, which moreover specify the homology class of the D6-brane worldvolumes

$$[\Pi_{D6}] = (n_6 [a_1] + n_7 [b_1]) \otimes (n_8 [a_2] + n_9 [b_2])$$ (2.2)

The system has two branches: i) Since the direction 5 is transverse to both branes, they may be separated in that direction, their distance being controlled by the vev of a tree-level modulus $\rho$ (with radiative potential generated in non-supersymmetric situations, see below) neutral under the D6-brane gauge symmetries. We call this the Coulomb branch; ii) At the origin of the Coulomb branch, i.e. for intersecting branes, charged massless or tachyonic scalars $\phi$ arise at their intersection. Vevs for these fields parametrize a Higgs branch where both branes recombine into a single smooth one.

Let us define $\Delta \theta_i = \theta_i - \theta_i'$. The configuration is supersymmetric when the $SO(4)$ rotation relating the two 2-planes in $(\mathbb{R}^4)_{6789}$ is within $SU(2)$ [13], that is

$$\Delta \theta_1 \pm \Delta \theta_2 = 0 , \text{ for some choice of sign}$$ (2.3)

and if so preserves 1/4 of the supersymmetries. In such situation the above two branches are degenerate and the fields $\rho$ and $\phi$ are exact moduli (even non-perturbatively, due to the eight unbroken susys). For any non-zero vev for $\phi$, the system corresponds to a recombined D6-brane wrapped on a supersymmetric 2-cycle in the coordinates 6789. In the compact case its homology class is the sum of the original homology classes (2.2) $[\Pi_{\text{tot}}] = [\Pi_{D6}] + [\Pi_{D6'}]$; in the non-compact case, its asymptotic form is
that of the original intersecting 2-planes in $\mathbb{R}^4$. This 2-cycle is special lagrangian in the complex structure natural in our splitting $T^2 \times T^2$. In a different complex structure it is a holomorphic 2-cycle in the right homology class.

In other cases, $\Delta \theta_1 \pm \Delta \theta_2 \neq 0$ for both signs, all supersymmetries are broken, and moduli space is lifted. For instance, for $0 \leq \Delta \theta_i \leq \pi$, the lightest scalar fields arising at the intersection of branes at the origin of the Coulomb branch have masses

$$\alpha' M^2 = \frac{1}{2\pi} (\Delta \theta_1 - \Delta \theta_2) ; \quad \alpha' M^2 = \frac{1}{2\pi} (-\Delta \theta_1 + \Delta \theta_2) \quad (2.4)$$

In any non-supersymmetric situation, $\Delta \theta_1 - \Delta \theta_2 \neq 0$, exactly one of these complex scalars is tachyonic. In the supersymmetric case, both get massless and combine with fermions to fill out a hypermultiplet.

The tachyon at the intersection triggers the dynamical recombination of the D6-branes. Since the tachyon is localized, the decay proceeds via an expanding shell which leaves the recombined configuration behind. In the non-compact case, a simple picture of the shell at any finite time is as follows. Outside a 3-ball of finite extent the D6-branes are unperturbed and span two 2-planes (minus two disks whose interior is inside the shell) at angles; inside the 3-ball the recombined cycle is a special lagrangian 2-cycle with boundary given by two disks glued to the outside solution at the shell. The existence and explicit construction of this configuration is provided in [12], see appendix A. The difference in tension dynamically pushes the joining shell to infinity. The process can be regarded as triggered by the intersecting tachyon for small $\phi$ vevs, and by the Dirac-Born-Infeld action (which tends to minimize the volume of the recombined cycle) deep in the Higgs branch.

In the non-compact case the process proceeds to infinity. In the compact case, the D6-brane ends up wrapping the minimal volume supersymmetric 2-cycle in the class $[\Pi_{tot}]$. This configuration can be regarded as the minimum of the tachyon potential (possibly multi-tachyon potential, if there are multiple intersections [1]).

Concerning the Coulomb branch, two D6-branes at angles suffer a mutual attraction generated by tree-level exchange of closed string fields, or equivalently by a one-loop running of open string states. In any event, a potential is developed for $\rho$ which pushes the system towards the origin of the Coulomb branch. Hence, two D6-branes initially deep in the Coulomb branch dynamically tend to approach and suffer a transition to a Higgs branch, where they recombine. This process has been described in [14] (in a

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3In compact examples, there may be obstructions to such recombinations, for instance in cases with a single intersection point, see appendix B. In the remainder of the paper we consider the relevant transitions to be allowed.
particular application as a model for hybrid inflation; see [15] for an earlier proposal to use Coulomb to Higgs transitions in brane models as hybrid inflation scenarios).

2.2 Energetics in the M-theory lift

Since the M-theory lifts of systems of D6-branes are purely geometrical, we would expect the above system to provide interesting dynamical phenomena involving purely gravitational systems. Our purpose in this section is to support the existence of the above Coulomb to Higgs transition in the IIA strong coupling regime (M-theory supergravity regime), and analyze its geometrical interpretation in M-theory.

The M-theory lift of \( n \) D6-branes is given by \( \mathbb{R}^7 \times X_4 \) where \( X_4 \) is an \( n \)-center Taub-NUT geometry, with metric given by

\[
\begin{align*}
    ds_{TN}^2 &= V d\bar{r}^2 + V^{-1} (d\psi + \vec{\omega} \cdot d\bar{r})^2 \\
    V &= \frac{1}{R} + \sum_{i=1}^{n} \frac{1}{|\bar{r} - \bar{r}_i|} ; \quad \vec{\nabla} \times \vec{\omega} = \vec{\nabla} V
\end{align*}
\]

The metric describes an \( S^1 \) fibration over \( \mathbb{R}^3 \), parametrized by \( \bar{r} \), with fiber degenerating over the locations \( \bar{r}_i \), and having asymptotic constant radius \( R \). In the limit of large asymptotic radius (equivalently, in the near core region), the constant term in \( V \) drops, and the metric becomes asymptotically conical, an ALE geometry.

The naive M-theory lift of two D6-branes intersecting as in section 2.1 corresponds to a six-dimensional geometry looking like two intersecting Taub-NUT fibrations. The full-fledged metric of these systems have not been studied, but their topology is relatively clear. As we will discuss below (see Section 2.3) in the supersymmetric situation the constraints from supersymmetry are enough to fix the metric in the infinite asymptotic radius regime (or near core limit) to be the conifold metric. Hence, we expect the dynamics of the system in non-supersymmetric situations to teach us about dynamics of non-supersymmetric metric deformations of the conifold.

The first question that arises is how one may extrapolate to strong coupling the picture we obtained studying D6-brane systems at weak coupling. A reliable way to do so is to use BPS formulae for the tension of the system, in situations where supersymmetry is good enough to prevent strong corrections from appearing.

Let us consider the compactification of M-theory on a rectangular 5-torus, \( T^5 = T^2 \times T^2 \times S^1 \), parametrized by 67, 89, 10 respectively. Consider a state given by the superposition of two far-away Kaluza-Klein monopoles associated to the \( S^1 \) direction 10. One of them spans the directions 01234 and wraps the cycles \((n_6, n_7)\) and \((n_8, n_9)\) in \( T^2 \times T^2 \); the second spans 01234 and wraps the cycles \((n'_6, n'_7)\) and \((n'_8, n'_9)\). Both are
separated by a large distance along the direction 5. Due to factorization, for distances much larger than the M-theory circle the interactions between the two objects are negligible and the state is reliably represented by a superposition of two Taub-NUT geometries. In non-compact space the metric would be roughly speaking of the kind

$$ds = d\vec{x}_{01234}^2 + (V + V') d\vec{r}^2 + (V + V')^{-1} (d\psi + \vec{\omega} \cdot d\Delta \vec{r} + \vec{\omega}' \cdot d\Delta \vec{r}')^2$$

where

$$V = \frac{1}{2R} + \frac{1}{|\Delta \vec{r}|}; \quad \nabla \times_3 \vec{\omega} = \nabla V$$

where

$$\Delta \vec{r} = (x_5 - x_0, -\sin \theta_1 x_6 + \cos \theta_1 x_7, -\sin \theta_2 x_8 + \cos \theta_2 x_9)$$

is the distance of a point to the TN center, and where $\times_3$ is vector product acting in the 3-dimensional space transverse to the TN center. Analogously for quantities $V'$, $\omega'$, $\times'_3$ associated to the second Kaluza-Klein monopole.

The above metric does not solve the equations of motion, but is approximately correct for large separation $x_0 - x'_0 \gg$. This is as expected for weakly interacting KK monopoles.

For well separated objects, the total tension $Z_{\text{tot}}$ of the resulting five-dimensional object is just the sum of the individual KK monopole tensions. For future convenience, we define $q_i = R_i n_i$ and obtain

$$Z_{\text{total}} = Z_{q_i} + Z_{q'_i} \quad (2.8)$$

where

$$Z_{q_i} = T_{KK} (q_6^2 + q_7^2)^{1/2} (q_8^2 + q_9^2)^{1/2} \quad (2.9)$$

and analogously for primed quantities. Here $T_{KK} = M_{P}^9 R_{10}^2$ is the KK monopole tension.

The state of KK monopoles at angles is generically non-supersymmetric, and does not saturate the BPS bound. Gravitational and electromagnetic interactions between two separated KK monopoles with differently oriented world-volume generically do not cancel and lead to an attractive interaction, pulling them towards the origin of the Coulomb branch.

Let us compute the BPS bound for a state with the above charges. Define the independent charges $q_{ij} = q_i q_j$, $\tilde{q}_{ij} = q_{ij} + q'_{ij}$, associated to a basis of homology cycles in $T^4$. The BPS bound can be analyzed as in e.g. [10] and follows from the maximal
eigenvalue of the central charge matrix, which for states with only (arbitrary) KK monopole charges reads.

\[ Z = T_{KK} R_m R_n q_{mn} \Gamma^{mn} \]  

(2.10)

where \( \Gamma^m \) are Dirac matrices. In our case

\[ Z = T_{KK} (q_{68} \Gamma^{68} + q_{69} \Gamma^{69} + q_{78} \Gamma^{78} + q_{79} \Gamma^{97}) \]  

(2.11)

Since the total charges do not satisfy the quadratic constraint \( \tilde{q}_{68} \tilde{q}_{79} = \tilde{q}_{69} \tilde{q}_{78} \) (except for the trivial case of collinear charges, namely parallel branes) the BPS saturating state is at most 1/4 supersymmetric. The bound (for some assumed signs for the \( \tilde{q}_{mn} \)) is

\[ Z = T_{KK} \left[ (\tilde{q}_{78} \pm \tilde{q}_{69})^2 + (\tilde{q}_{79} \mp \tilde{q}_{68})^2 \right]^{1/2} \]  

(2.12)

In terms of the angles \( \theta_i, \theta'_i \)

\[ q_6 = (q_6^2 + q_7^2)^{1/2} \cos \theta_1 ; \quad q_8 = (q_8^2 + q_9^2)^{1/2} \cos \theta_2 \]  

(2.13)

\[ q_7 = (q_6^2 + q_7^2)^{1/2} \sin \theta_1 ; \quad q_9 = (q_8^2 + q_9^2)^{1/2} \sin \theta_2 \]  

(2.14)

(and analogously for primed charges) and after some algebra, the BPS bound can be recast as

\[ Z = [ (Z_{q_i} + Z_{q_i'})^2 - 4 Z_{q_i} Z_{q'_i} \sin^2(\delta/2) ]^{1/2} \]  

(2.15)

where \( \delta = \Delta \theta_1 - \Delta \theta_2 \).

The order parameter \( \delta \) vanishes when the relative angles define an \( SU(2) \) rotation, in which case the BPS bound is \( Z_{q_i} + Z_{q'_i} \) and is saturated by the original two KK monopole configuration, which is supersymmetric. For non-zero \( \delta \), the BPS bound is smaller, and the initial state is non-supersymmetric. At small M-theory radius, the bound is saturated by a KK monopole wrapped in the holomorphic curve in the class \([\Pi_{\text{tot}}]\) in \( T^4 \). Since this state is supersymmetric, it persists even at large M-theory radius. In the supergravity regime it should look like a Taub-NUT space fibered over the 2-cycle. An ‘adiabatic’ ansatz for the metric, by fibering the TN metric over the 2-cycle, would be reliable for \( R_i \gg R_{10} \) (curvature radii for the 2-cycle much larger than the \( S_1 \) compactification length).

Hence the original state of two well separated KK monopoles is unstable against decay to the 1/4 BPS state which corresponds to the lift of a D6-branes wrapped on
the recombined 2-cycle. On the Coulomb branch the instability is driven by the long-distance interaction between well separated KK monopoles. At Planckian distances, a local instability must develop in the region near the intersection of the two Taub-NUT cores, and triggers the transition to the final recombined state. In the following section we describe the local process mediating this decay to a Higgs branch.

2.3 The dynamical conifold transition

One particular advantage of the system we are studying is that it contains a set of continuous parameters connecting it to a supersymmetric situation. In particular, one may study the lift to M-theory of the transition from the Coulomb to the Higgs branch in the supersymmetric configuration, and then perturb it mildly by a small change in the torus radii violating the $SU(2)$ relation between angles. In this fashion, the main effect of lack of supersymmetry is that the transition is dynamically triggered, instead of taking place along flat directions in moduli space.

In the supersymmetric situation, the phase transition in which D6-branes at angles approach and recombine lifts to a topology changing transition between two distinct Calabi-Yau threefolds. More concretely, the intersections lift to conifold singularities, and the phase transition maps to a conifold transition from the small resolution to the deformation phase. Let us describe this story.

Consider two D6-branes intersecting over 01234 and spanning two 2-planes in 6789 in a supersymmetric fashion. By choosing suitable complex coordinates $z, w$, the locus in $\mathbb{R}^4$ wrapped by the D6-branes may be written $zw = 0$. Now recall that the M-theory lift of a D6-brane may be written as a complex manifold as $xy = v$; namely, a fibration of $\mathbb{C}^*$’s (parametrized by $x, y$) over $\mathbb{C}$ (parametrized by $v$) with fiber degenerating over $v = 0$ (the location of the Taub-NUT center). Extending to our case, the M-theory lift of two D6-branes intersecting in $SU(2)$ angles can be described (as a complex manifold) as a $\mathbb{C}^*$ fibration over $\mathbb{C}^2$ (parametrized by $z, w$) degenerating over $zw = 0$. The total space is the submanifold $xy - zw = 0$ in $\mathbb{C}^4$. This is the description of the conifold singularity as a complex manifold\footnote{See [17] for a similar derivation. A different but related derivation starts with intersecting NS-fivebranes (a system U-dual to intersecting D6-branes) and obtains a conifold geometry by applying T-duality [18, 19].}. Finally, supersymmetry of the configuration guarantees the threefold is endowed with the Calabi-Yau metric.

The Coulomb branch in which the D6-branes separate corresponds to the small resolution phase of the conifold, where the singularity is replaced by a 2-sphere. In the
Figure 1: Schematic picture of the lift of configurations of D6-branes intersecting at two angles on the Coulomb (a) and Higgs (b) branch. They are related by a topology changing conifold transition. We have highlighted the non-trivial two- and three-spheres in these geometries, which are obtained as $S^1$ fibrations over a segment (a) and a disk (b) on the base.

Lift of two D6-branes on the Coulomb branch, the 2-sphere is clearly visible as the $S^1$ fibration over a segment joining the centers of the two Taub-NUTs on the base, see Fig 1a. On the Higgs branch, where the intersecting D6-branes recombine and wrap a single smooth 2-cycle, the system lifts to a $\mathbb{C}^*$ fibration degenerating over the complex curve $zw = \varepsilon$, with $\varepsilon$ parametrizing the Higgs branch. The complex equation for the threefold is $zw - xy = \varepsilon$, namely the deformed conifold, where the singularity is replaced by an $S^3$. In the lift of the recombined D6-brane, the $S^3$ is visible as the $S^1$ fibration over a disk on the base, bounded by a non-trivial circle in the 2-cycle [Fig 1b].

Hence the transition from the Coulomb to the Higgs branch in which D6-branes approach and recombine lifts to the familiar conifold transition, in which a resolved conifold shrinks its two-sphere and instead a three-sphere grows [20, 21]. The transition is a Higgs mechanism triggered by a vev for a state given by an M2-brane wrapped on the vanishing 2-cycle. This state is the lift of the hypermultiplet arising at the intersection of D6-branes.

Performing a mild perturbation of the above system by a small change of angles away from the $SU(2)$ relation leads to presumably small corrections to the above picture. The topology of the resulting lift is unchanged, but the asymptotic behaviour of the lift (in the non-compact setup) forces the metric to be non-supersymmetric.

The main effect is that the conifold transition is driven by a dynamical mechanism which tends to minimize the energy and make the total tension of the system approach

\[ \text{If } \varepsilon = r^2e^{i\theta}, \text{ the circle in } z_1z_2 = \varepsilon \text{ is given by defining } z_1 = e^{i\theta/2}(x_1 + ix_2), z_2 = e^{i\theta/2}(x_1 - ix_2), \text{ and taking } x_i \text{ real in } x_1^2 + x_2^2 = r^2. \]
Figure 2: Phase transition in the M-theory lift of intersecting D6-branes a) in the small circle limit (weakly coupled IIA) and b) in the large circle regime. Even though the nature of the instability is not fully understood (lies beyond supergravity due to small cycles) it certainly mediates a dynamical conifold transition transforming an initially large 2-cycle into a finally large 3-cycle.

the BPS bound. Deep in the ‘Coulomb branch’ the gravitational dynamics between the Taub-NUT cores leads to a dynamical shrinking of the 2-sphere. The nature of the instability arising near the origin in the Coulomb branch is less clear; a putative tachyonic nature of the wrapped M2-brane state, as naively extrapolated from the weak coupling IIA limit, may not be the right answer.

In any event, some M-theory mechanism triggers the transition to the Higgs branch. Further indirect evidence for this is the fact that, deep in the Higgs branch, for curvature lengths on the base cycle much larger than $R_{10}$, the dynamics is governed by the effective action for KK monopoles wrapped on the recombined cycle. Since it becomes mainly the Dirac-Born-Infeld action (see e.g. [22, 23]), it tends to minimize the wrapped volume, which corresponds to continuing the recombination. The complete situation is depicted in Figure 2.

We conclude this section giving two intuitive arguments to understand why the non-supersymmetric conifold geometry likes to deform dynamically. First one can resort to an analysis of the effective field theory for the $U(1)$ gauge field arising from integrating the 3-form over the vanishing 2-cycle, and the charged ‘hypermultiplet’ obtained from the M2-brane wrapped on it. In the supersymmetric situation, the conifold transition is a Coulomb to Higgs transition in this field theory (we understand additional conifolds to be present in compact setups, see appendix B), via flat directions. The D6-brane angle rotation breaking supersymmetry breaks the $SU(2)$ R-symmetry of the system, hence it naturally corresponds to turning on a Fayet-Iliopoulos term in the conifold field theory. This naturally triggers a dynamical transition into the Higgs branch.

A second argument relies on partial results from T-duality between (unfortunately
smeared) NS-brane configurations and conifolds. The conifold metric \[20\] has the structure

\[
ds^2 = dr^2 + r^2 [A (d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2 + d\theta_2^2 + \sin^2 \theta_2 d\phi_2^2) + \\
+ B (d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2)^2]
\]

In relating this to T-dual intersecting NS-branes, ref \[19\] suggest replacing the Maurer-Cartan forms of \(S^2\) to those of \(R^2\) \(d\theta_1, \sin \theta_1 d\phi_1 \rightarrow dx_6, dx_7\) (and analogously for \(\theta_2, \phi_2\)) leading to

\[
ds^2 = dr^2 + r^2 [A (dx_6^2 + dx_7^2 + dx_8^2 + dx_9^2) + B (d\psi + x_6dx_7 + x_8dx_9)^2] \quad (2.17)
\]

Intuitively, the non-supersymmetric conifold corresponds to a geometry with off-diagonal metric in 6789 space, leading to roughly speaking

\[
ds^2 = dr^2 + r^2 \left[ A (dx_6^2 + dx_7^2 + dx_8^2 + dx_9^2 + \alpha dx_6 dx_8 + \beta dx_7 dx_9) + \\
+ B (d\psi + x_6dx_7 + x_8dx_9)^2 \right] \quad (2.18)
\]

For \(SU(2)\) angles, which amounts to \(\alpha = -\beta\), the metric is equivalent to (2.17) via a coordinate change. For non-\(SU(2)\) angles, susy is broken even in the asymptotic region. The metric must relax towards a susy metric with some asymptotic off-diagonal metric in 6789. This is provided by a deformed conifold metric \[20, 24\], which can be written (using \(R^2\) Maurer-Cartan forms) as \[26\]

\[
ds^2 = dr^2 + [C(r) (dx_6^2 + dx_7^2 + dx_8^2 + dx_9^2) + D(r) (dx_6, dx_7) \cdot R(\psi) \cdot (dx_8, dx_9)^T + \\
+E(r) (d\psi + x_6dx_7 + x_8dx_9)]
\]

where \(R(\psi)\) is a \(2 \times 2\) rotation matrix with angle \(\psi\). Hence the non-supersymmetric conifold likes to develop a deformed conifold at its tip, since it is the Calabi-Yau metric with more similar asymptotic behaviour \[1\]. Even though the argument is suggestive, a detailed match of features like the \(\psi\) dependence in the off-diagonal metric in 6789 would require being able to treat localized sources in the T-dual NS-brane configuration.

### 2.4 Discussion of compact models

In several respects the above instabilities are similar to those arising from closed string twisted tachyons at non-supersymmetric orbifold singularities \[2\]. Namely, our instabilities are localized at singular points, they signal the dynamical resolution of the

\[\text{T-duality of deformed conifolds and recombined branes has appeared in} \quad [23, 23, 27] \]

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singularity, and for large resolutions are triggered by gravitational interactions (while at short distances they have a stringy/ M-theory origin). Also, in non-compact setups the dynamical smoothing of singularities proceeds to infinity in both cases, with an expanding shell of energy separating the (possibly partially) smoothed region from the still non-supersymmetric asymptotic one.

In this section we would like to point out that nevertheless the behaviour of both instabilities seems qualitatively different in compact setups. It has been argued from different viewpoints \cite{2,4} that Zamolodchikov’s c-theorem implies that in compact setups condensation of closed string twisted tachyons must lead to a reduction of the number of spacetime dimensions. Such an argument is clearly not applicable to our instabilities, since they arise in M-theory or in string theory in the regime where the world-sheet CFT description breaks down (namely at Strominger’s conifold point \cite{28}, and analogs).

In fact, it is easy to cook up configurations of D6-branes in compact setups which lead to compact threefolds with instabilities of the kind discussed above, and which relax to a supersymmetric situations without loss of spacetime dimension. A simple and controllable situation is to start with D6-branes wrapped on factorizable special lagrangian 2-cycles on $T^2 \times T^2$, and slightly change the $T^2$ complex structures. The D6-branes suffer a slight recombination after which they wrap a recombined 2-cycle, which is special lagrangian in the new complex structure. From the viewpoint of M-theory, the lift after the complex structure deformation is a threefold with a number of conifold singularities of the above non-supersymmetric kind. Their corresponding instabilities trigger a dynamical deformation of the conifolds which ends at finite distance in the deformation parameters $\theta$.

Namely, the field controlling the 3-cycle size reaches a minimum of its potential. In this sense, the configurations we have studied are the simplest cases of localized instabilities with no open string origin which have non-trivial minima from the unstable potential at finite distance. We hope our results, although qualitative, are inspiring in the search for other situations with these features.

\footnote{As discussed in appendix \ref{app:b}, one must require models with several intersections, in order to avoid topological obstructions to the transition.}
3 One- and three-angle systems

In this section we perform a similar analysis in other systems of D6-branes at angles. Even though we encounter some new features, the main ideas are analogous, so our discussion is more sketchy.

3.1 One angle system

Consider a system of two D6-branes intersecting over a six-dimensional subspace of their volumes. The geometry is as follows

\[
\begin{align*}
\text{D6} & : 0 \ 1 \ 2 \ 3 \ 4 \times 6 \times [8 \ 9]_\theta \\
\text{D6'} & : 0 \ 1 \ 2 \ 3 \ 4 \times 6 \times [8 \ 9]_{\theta'}
\end{align*}
\]

Contrary to Section 2, this system is non-supersymmetric for any non-zero value for \(\Delta \theta = \theta - \theta'\). The open string sector at the intersection always contains a tachyon, with mass

\[
\alpha' M^2 = -\frac{1}{2\pi} \Delta \theta
\]

which triggers the recombination of the intersecting D6-branes. In a non-compact setup the recombination proceeds to infinity, while for D6-branes wrapped on 1-cycles \((n_8, n_9), (n'_8, n'_9)\) on \(T^2\), the recombined D6-brane ends up wrapping the cycle \((n_8 + n'_8, n_9 + n'_9)\), the minimal volume cycle in its homology class. The final state is 1/2 BPS.

By separating the branes in the transverse directions 5, 7, the weakly interacting D6-branes can be reliably lifted to M-theory as a five-dimensional metric given by a superposition of two Taub-NUT spaces. In this regime, in a compact setup, the 6d tension of the system is the addition of the individual tensions

\[
Z_{\text{total}} = Z_{n_8,n_9} + Z_{n'_8,n'_9} = T_{KK} \left[ \left[ (n_8 R_8)^2 + (n_9 R_9)^2 \right]^{1/2} + \left[ (n'_8 R_8)^2 + (n'_9 R_9)^2 \right]^{1/2} \right]
\]

This is always larger than the BPS bound for a state with those charges, which can be readily computed to be

\[
Z_{\text{BPS}} = T_{KK} \left[ \left( n_8 + n'_8 \right)^2 R_8^2 + \left( n_9 + n'_9 \right)^2 R_9^2 \right]^{1/2}
\]

The BPS bound is saturated by a Taub-NUT wrapped on the 1-cycle \((n_8 + n'_8, n_9 + n'_9)\). Therefore, the system of two Taub-NUT’s well separated in the directions 57 is unstable against decay to a lower energy configuration.
We propose this decay to occur in a manner similar to Section 2. Namely, far in the Coulomb branch gravitational interactions between the Taub-NUT cores leads to their approach until reaching Planckian distances. At this stage an instability develops, localized near the core intersections. This instability triggers the recombination of their cores, ending up in a geometry which is a Taub-NUT $S^1$ fibration degenerating over the 1-cycle $(n_8 + n_8', n_9 + n_9')$.

Interestingly enough, the process represents the decay of a non-supersymmetric five-dimensional non-trivial geometry into a supersymmetric factorized geometry of the form $X_4 \times S^1$, where $X_4$ is the final Taub-NUT geometry, and the $S^1$ spans the direction transverse to $(n_8 + n_8', n_9 + n_9')$ in $T^2$.

From the local viewpoint, the geometrical transition is a topology changing transition, in which a 2-sphere present in the Coulomb branch shrinks, and a new topologically different 2-sphere grows in the Higgs branch, see Fig 3.

### 3.2 Three angle case

Consider the case of two D6-branes intersecting over a four-dimensional subspace of their volume. The geometry is

\[
\begin{align*}
D6 & : 0 1 2 3 \,[ \, 4 \, 5 \,]_{\theta_1} \,[ \, 6 \, 7 \,]_{\theta_2} \,[ \, 8 \, 9 \,]_{\theta_3} \\
D6' & : 0 1 2 3 \,[ \, 4 \, 5 \,]_{\theta_1'} \,[ \, 6 \, 7 \,]_{\theta_2'} \,[ \, 8 \, 9 \,]_{\theta_3'}
\end{align*}
\]

(3.4)

The configuration is supersymmetric when the $SO(6)$ rotation between the two spanned 3-planes lies within an $SU(3)$ subgroup, i.e.

\[
\Delta \theta_1 \pm \Delta \theta_2 \pm \Delta \theta_3 = 0 \quad \text{for some choice of signs.} \quad (3.5)
\]
The intersection contains a chiral multiplet (arising from D6-D6′ open strings) whose scalar component vev parametrizes recombination of 3-cycles into a single smooth special lagrangian 3-cycle. In contrast with previous situations, there is no Coulomb branch for this system, due to the absence of overall transverse dimensions.

In non-supersymmetric situations this complex scalar may be tachyonic or non-tachyonic. Assuming $0 \leq \Delta \theta_i \leq \pi$, the lightest scalars have masses

$$\alpha' M^2 = \frac{1}{2\pi}(-\Delta \theta_1 + \Delta \theta_2 + \Delta \theta_3)$$

$$\alpha' M^2 = \frac{1}{2\pi}(\Delta \theta_1 - \Delta \theta_2 + \Delta \theta_3)$$

$$\alpha' M^2 = \frac{1}{2\pi}(\Delta \theta_1 + \Delta \theta_2 - \Delta \theta_3)$$

$$\alpha' M^2 = -\frac{1}{2\pi}(-\pi - \Delta \theta_1 - \Delta \theta_2 + \Delta \theta_3)$$

In angle space, the non-tachyonic range lies within a tetrahedron introduced in [29], see [30] for further details.

For angles in the tachyonic range, there exist manifolds with smaller volume (in fact special lagrangian manifolds) and same asymptotic behaviour [12]. Hence, the tachyon triggers decay to a single D6-brane wrapped on such 3-manifold, which is a 1/8 BPS state. In non-compact setups the recombination proceeds to infinity, via the familiar shell expansion process, while in compact ones it stops at the minimal volume cycle in its homology class.

For non-tachyonic angles, such smaller volume 3-cycles do not exist, hence the configuration remains non-supersymmetric, but stable against small perturbations. In compact setups, however, non-perturbative global rearrangements of D6-branes may allow decay of non-tachyonic configurations [6].

Let us now describe the lift of these systems. As in Section 2.2 it is convenient to discuss the supersymmetric case first. This lift must correspond [31] to a $G_2$-holonomy seven-dimensional singularity. In the near core regime, or infinite asymptotic radius limit, the metric (and its relation to intersecting D6-branes) has been explicitly discussed in [32] (see also [33]), which moreover shows that the resolution of the singularity corresponds to the D6-brane recombination.

Concerning the non-supersymmetric case, it is possible to perform a BPS analysis

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8In fact it is easy to describe a domain wall which interpolates between two such (meta)stable minima. It is given by a D8-brane spanning 012456789, and at $x^3 = 0$. Consider a configuration of (semi-infinite) D6-branes wrapped on 3-cycles $[\Pi_a]$ and spanning 012 and $x^3 < 0$, and ending on the D8-brane; and a similar configuration of D6-branes wrapped on cycles $[\Pi'_a]$ and spanning 012 and $x_3 > 0$. Nucleation of these D8-$\overline{D}8$ domain walls (or expansion of a spherical D8-brane) mediates the decay of metastable vacua, such as those in [11]. For supersymmetric cases [1, 11] the D8-brane domain wall is BPS, and separates different $N = 1$ susy configurations.
by considering objects wrapped on 3-cycles on $T^6$. However, a first difficulty is that in the absence of Coulomb branch there is not enough protection against strong corrections in lifting the initial intersecting D6-brane configuration.

For the sake of the argument we could assume that the lift gives some kind of intersecting Taub-NUT metric, namely a non-$G_2$ metric in the topology of the singularity in $\mathbb{P}^2$. Being non-supersymmetric, this geometry does not saturate the BPS bound. In the tachyonic range of angles, the BPS bound is however saturated by the state which gives the lift of the recombined D6-brane system (which is BPS, and hence must exist in M-theory). Hence the singularity is unstable against dynamical smoothing, which drives the configuration into the Higgs branch, where it eventually looks like a Taub-NUT fibration over the recombined 3-cycle. In the non-tachyonic range of angles, there is no guarantee on the existence of BPS saturating objects, hence the stability of the singularity is unclear (namely, it may remain stable, or decay into some other different non-BPS geometry, stable at large $R_{10}$).

## 4 Conclusions

In this note we have discussed the M-theory lift of the dynamics of diverse intersecting D6-brane systems. They provide interesting dynamical processes of purely gravitational systems in string / M-theory. Given the difficult regime where such processes take place, our main tool has been an analysis of energetics using BPS formulae. Several questions concerning the explicit description of the geometries involved are beyond our tools.

A possible improvement in this respect could be provided by studying the supergravity solutions for intersecting/recombined NS-brane systems, and using T-duality to uncover the intersecting/recombined Taub-NUT metrics in our discussions. Unfortunately, most metrics for intersecting brane systems in the literature involve smeared sources, whereas our purposes would require localized source solutions. These have become available more recently and we may expect some progress in this direction.

Clearly many generalizations of our results are possible, in particular involving singularities resulting from lifts of more D6-branes (for instance, threefold singularities $xy = z^n w^m$ from two bunches of $n, m$ D6-branes, etc), or involving D6-branes and O6-planes. We hope these and other examples are helpful in extending our picture of

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9 Given the absence of overall transverse dimensions, cancellation of charge in the compact setup should be required for consistency; we will consider our two-brane system to be part of a larger set, and center on the dynamics of a particular instability, essentially unaffected by the rest.
condensation of instabilities associated to non-supersymmetric singularities.

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A Recombined special lagrangian cycles

Following [12], we review the construction of special lagrangian 2-cycle mediating the recombination of two intersecting 2-planes. For the $n$-cycle case, see [12]. The 2-cycles considered are of the form

$$(z_1(t, \varphi), z_2(t, \varphi)) = (\cos \varphi z_1(t), \sin \varphi z_2(t))$$

(A.1)

with $z_i$ giving coordinates in $\mathbb{R}^4 = \mathbb{R}^2 \times \mathbb{R}^2$, and $\phi, t$ parametrizing the 2-cycle. The fact that the 2-cycle is special lagrangian follows from

$$\frac{dz_1}{dt} = iz_2; \quad \frac{dz_2}{dt} = iz_1$$

(A.2)

The initial boundary condition is taken $z_i(t = 0) = c_i \in \mathbb{R}$.

For any given $t = t_0$, the region $|t| \leq |t_0|$, $\varphi$ arbitrary, defines a special lagrangian 2-cycle, with boundary two circles $z_i(t = \pm t_0, \varphi)$ lying within two 2-planes at relative angles $\Delta \theta_i = 2\text{Arg}(z_i(t_0))$. These angles are in general not in $SU(2)$ relation, and moreover for any set of angles in non-$SU(2)$ relation there exists $t_0$ such that $\Delta \theta_i = 2\text{Arg}(z_i(t_0))$.

The region provides a recombined special lagrangian 2-cycle with a recombination size parametrized by the $c_i$, and whose boundary can be exactly glued to two 2-planes intersecting at non-susy angles. Hence, given two D6-branes at non-supersymmetric angles, the above 2-regions provide recombined 2-cycles mediating the tachyon condensation. The decay proceeds via a family of 2-cycles, with larger and larger $c_i$, and glued to the 2-planes at larger and larger radii, in a shell expansion picture. The expansion of the shell is triggered by the difference in tension between the supersymmetric interior region and the non-susy exterior.
Figure 4: Intersecting D6-branes provide a skeleton picture for certain Calabi-Yau threefolds with conifold singularities. The shaded area provides the skeleton picture for a 3-chain defining a homology relation between 2-spheres A, C, in the small resolution phase. This homology relation allows a conifold transition involving the corresponding nodes.

B Kähler constraints in conifold transitions from D6-branes

There is an interesting constraint [20, 35] on the possible conifold transitions that can be undergone by a compact Calabi-Yau with conifold singularities. Given a set of conifold singularities, the deformation phase is only possible if there are homology relations among the 2-spheres which shrink in the singular limit.

In particular, this implies that a compact Calabi-Yau cannot undergo a transition at a single conifold point. This is argued as follows: starting in the resolution phase, the small 2-sphere has a compact dual 4-cycle intersecting the 2-cycle. After the transition to the putative deformation phase, the 4-cycle would become a 4-chain with boundary the 3-sphere. Hence the 3-sphere would be homologically trivial, and would allow for no modulus to parametrize the branch. On the other hand, with for instance two conifold points with homologically related shrinking 2-spheres, the dual 4-cycle intersects both; in the deformation phase it becomes a 4-chain defining a homological relation between the two 3-spheres, hence allowing for one complex parameter moduli space. In general the dimension of the moduli space of deformations is given by the number of independent such homology relations.

This kind of constraint should also arise in Calabi-Yau manifolds obtained as lifts of configurations of intersecting D6-branes wrapped on (supersymmetric) 2-cycles. We would like to show that in fact they are easily obtained in terms of the latter. Indeed,
Figure 5: The 3-chain $\Sigma_3$ mentioned in figure 4 is obtained by fibering the M-theory circle over the region here depicted. Notice that the 2-spheres at the two locations B are glued onto each other, and do not belong to the boundary of $\Sigma_3$.

the existence of the deformation phase involving a number of conifold singularities corresponds to entering the Higgs branch in which a number of D6-brane intersections are smoothed out by recombination. This is the Higgs branch of a gauge field theory with eight supercharges, living on the non-compact piece of the D6-brane world-volumes. Considering e.g. two D6-branes with $N$ intersections, it is a $U(1)$ gauge theory with $N$ charged hypermultiplets (this $U(1)$ is the difference of the D6-brane world-volume $U(1)$’s, and all hypers have the same charge). The Higgs branch is absent for $N = 1$, while for higher $N$ there is an $N - 1$ (quaternionic) dimensional Higgs branch, as follows from the F-term and D-term equations

$$Q_1 \tilde{Q}_1 + \ldots Q_N \tilde{Q}_N = 0$$
$$|Q_1|^2 + \ldots |Q_N|^2 = |\tilde{Q}_1|^2 + \ldots |\tilde{Q}_N|^2 = 0$$ (B.1)

where $(Q_i, \tilde{Q}_i)$ denotes the $i^{th}$ hypermultiplet. Particular mesonic directions in the Higgs branch are given by the vevs $Q_i = \tilde{Q}_j = v, i \neq j$, with other fields set to zero.

The D6-brane / gauge field theory analysis then suggests that in the M-theory lift the corresponding threefolds have homology relations among the 2-cycles at the conifold singularities associated to hypermultiplets getting a vev. In fact, the corresponding 3-chains are easily constructed using the D6-brane picture. In figure 4a we have depicted an example with three intersections, in the singular conifold limit, while in figure 4b separation of D6-branes in $x^5$ leads to a small resolution of the conifolds. The 2-spheres arise from fibering the M-theory circle over segments in $x^5$ and at locations A, B, C in 67 (and at the intersection in 89). A 3-chain $\Sigma_3$ defining a homology relation between $^{10}$In terms of D6-brane geometry, the existence of Higgs branch corresponds to the existence of a recombined Slag 2-cycle in the correct homology class. In a T-dual version (see e.g. [30, 31]) it is related to the existence of certain gauge field connection carrying the appropriate Chern classes.
the 2-spheres at A, C, is obtained by fibering the M-theory circle over a two-dimensional region whose projections are given by the shaded area. An improved picture of this region is provided in figure 5 (notice that the 2-sphere B is not a boundary of Σ3). This homology relation allows the conifold transition associated to the mesonic brane involving the hypers at intersections A, C.

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