problem in an extended gauge mediation
supersymmetry breaking

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Abstract

We study the problem and the radiative electroweak symmetry breaking in an extended
gauge mediation supersymmetry breaking (GMSB) model, in which the messenger fields
are assumed to couple to the different singlet fields due to the discrete symmetry. Since
the spectrum of superpartners is modified, the constraint from the problem can be
relaxed in comparison with the ordinary GMSB at least from the viewpoint of radiative
symmetry breaking. We study the consistency of the values of $\tan \beta$ and $B$ with the radiative
electroweak symmetry breaking and also the mass spectrum of superpartners. We present
such a concrete example in a supersymmetric SU(5) unified model which is constructed
from a direct product gauge structure by imposing the doublet-triplet splitting.

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1 Introduction

Supersymmetry is now considered to be the most promising candidate for the solution of the gauge hierarchy problem. Although we have no direct evidence of the supersymmetry still now, the unification shown by the gauge couplings in the minimal supersymmetric standard model (MSSM) may indirectly reveal its signal. In the supersymmetric models the most important subject is to clarify the supersymmetry breaking mechanism in the observable world. Flavor changing neutral current processes severely constrain the scenario for the supersymmetry breaking. From this point of view the gauge mediation supersymmetry breaking (GMSB) [1]-[7] seems to be prominent since the mediation is performed in the flavor blind way by the standard model gauge interaction.

In the ordinary minimal GMSB scenario [2]-[6], the messenger fields \(q'; \cdot\) and \(q'; \cdot\) which come from the vectorlike chiral superfields \(5 + 5 \) of \(SU(5)\) are considered to have couplings in the superpotential such as

\[
W_{GMSB} = q\bar{q}q + \cdot S '; \cdot
\]

(1)

where \(S \) is a singlet chiral super field. Both the scalar component \(S \) and its F-term \(F_S \) are assumed to get vacuum expectation values (VEVs) through the couplings to the gauginos and the scalar superpartners are respectively produced by the one-loop and two-loop effects through the couplings in eq. (1). These masses are characterized by

\[hF_S = hS,\]

and then \(B\) is considered to be in the range \(20-100 \) TeV.

The chiral super field \(S\) is usually considered not to have a direct coupling to the doublet Higgs chiral super fields \(H_1 \) and \(H_2 \) in the superpotential, although it can be an origin for both the term and the bilinear soft supersymmetry breaking parameter \(B\). The reason is that the relation \(B \) induced from such a coupling makes \(B \) too large for the electroweak symmetry breaking under the assumption \(B = 0 \) (100) GeV. On the other hand, if we assume that \(B\) has a suitable value for the electroweak symmetry breaking, the resulting small cannot satisfy the potential minimum condition. Since \(B\) takes a large value as mentioned above, it makes both \(B\) and \(B\) difficult to take suitable values for the radiative symmetry breaking [2, 3, 4, 7]. Even if there are no such a coupling in

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1 We will use the same notation for the scalar component as its chiral super field.

2 In the ordinary GMSB scenario the potential minimum condition requires \(B < \) as seen later.
the superpotential, and \( B \) can be produced radiatively picking up the supersymmetry breaking effect and \( B = \) is generally satisfied \([8]\). This suggests that the electroweak symmetry breaking cannot be induced radiatively also in this case. Thus, it is usually considered that the terms should have another independent origin. This requires us to introduce new additional fields for this purpose. A lot of models of this kind have been proposed by now \([2,3,4,7,8]\).

In this paper we show that an extended GM SB model proposed here may make ease the difficulty of the problem in comparison with the ordinary GM SB at least from the viewpoint of the radiative symmetry breaking. In this model we can relax the constraint on \( B \). The consistency of such a scenario with the radiative electroweak symmetry breaking is studied in some detail by solving numerically the renormalization group equations (RGEs). Its phenomenological results are also discussed. We present its concrete example in the deconstructed SU(5) unified model. In this model the superpotential is suitably arranged by the discrete symmetry which is introduced to resolve the doublet-triplet Higgs degeneracy \([9]\) in the basis of the direct product gauge structure.

## 2 Soft SUSY breaking and problem

We extend the superpotential \((1)\) for the messenger fields in such a way that the messenger fields \( q; q \) and \( \tilde{q}; \tilde{q} \) couple to the different singlet chiral super fields \( S_1 \) and \( S_2 \) which are assumed to have couplings to the hidden sector where the supersymmetry is supposed to be broken. This can happen incidentally as a result of a suitable discrete symmetry as we will see it later in an explicit example. Thus the couplings of messenger fields are expressed as

\[
W_{\text{GM SB}}^0 = q S_1 qq + \tilde{q} S_2 \tilde{q} \quad (2)
\]

If we assume that both \( S \) and \( F \) get the VEVs, the gaugino masses and the soft scalar masses are generated through the one-loop and two-loop diagrams, respectively, as in the ordinary case. However, the mass formulas are modified from the ordinary ones since the messenger fields \( q; q \) and \( \tilde{q}; \tilde{q} \) couple to the different singlet fields \( S \).

The mass formulas of the superpartners in this type of model have been discussed in \([10]\). Under the ordinary assumption such as \( hF \) \( i \) \( q; q \) \( H \) \( i^2 \) \([2]\), the mass formulas take a very simple form. The masses \( M_k \) of the gauginos \( k \) of the MSSM gauge group
can be written in the form as

\[ M_3 = \frac{3}{4}; \quad M_2 = \frac{2}{4}; \quad M_1 = \frac{1}{4} \]

(3)

where \( r = g_R^2 = 4 \) and \( = h_F S \). The soft scalar masses \( m_F^2 \) can be written as

\[ m_F^2 = 2 C_3 \frac{3}{4} + 2 \frac{2}{3} Y \frac{2}{4} \frac{1}{2} j_1 j_2 + 2 C_2 \frac{2}{4} \frac{2}{4} \frac{2}{2} j_1 j_2 j_2 \]

(4)

where \( C_3 = 4 = 3 \) and 0 for the SU(3) triplet and singlet fields, and \( C_2 = 3 = 4 \) and 0 for the SU(2) doublet and singlet fields, respectively. The hypercharge \( Y \) is expressed as \( Y = 2 ( Q + Y ) \).

These formulas can give a rather different mass spectrum for the gauginos and the scalar superpartners in comparison with the ordinary GM SB scenario. The spectrum depends on the value of \( 2 = 1 \). In fact, if we assume \( 1 = 1 < 2 \), the mass difference between the color singlet fields and the colored fields tends to be smaller in comparison with the one in the ordinary scenario at least in the supersymmetry breaking scale. As an example, we take \( 1 = 60 \text{ TeV} \) and \( 2 = 150 \text{ TeV} \) to show a typical spectrum of the superpartners at the supersymmetry breaking scale. The resulting spectrum is

\[ M_3' = 415 \text{ GeV}; \quad M_2' = 418 \text{ GeV}; \quad M_1' = 166 \text{ GeV}; \quad m_Q' = 851 \text{ GeV}; \]
\[ m_U' = 690 \text{ GeV}; \quad m_D' = 682 \text{ GeV}; \quad m_L' = 520 \text{ GeV}; \quad m_E' = 195 \text{ GeV}; \]
\[ m_1 = m_2' = 520 \text{ GeV}; \]

(5)

where \( m_1 \) and \( m_2 \) are masses of the Higgs scalars that couple with the fields in the down and up sectors of quarks and leptons, respectively. These masses are somewhat affected by the running effect based on the renormalization group equations (RGEs), although the running region depends on the values of \( 1 \) and \( 2 \). As discussed in [6], in the minimal GM SB model the soft supersymmetry breaking \( A_f \) parameters can also be expected to be induced through the radiative correction in such a way as

\[ A_f' = A_f ( ) + M_2 ( ) 185 + 0.34 h_f^2 + \]

(6)

where we should omit a term of \( h_t \) except for the top sector \( ( f = t ) \). Thus, even if \( A_f ( ) = 0 \) is assumed, we can expect \( A_f \) to be generated through this effect.\(^3\)

\(^3\)In the present study we assume \( A_f ( ) = 0 \). The soft supersymmetry breaking parameters \( B = \) is also known to follow the similar radiative correction to eq. (6) and the phenomenological studies have been done [6, 11, 12]. However, we will discuss the origin of \( B ( ) \) in the following.
As mentioned in the introduction, the values of $S$ and $B$ are crucial for the electroweak symmetry breaking. Here we examine the effect of the introduction of a coupling $S_1 H_1 H_2$ in the superpotential. It can give a contribution to both $S$ and $B$ terms in the form as

$$S = h S_1 i; \quad B = h F_{S_1 i};$$

Unfortunately, as in the ordinary case the problematic relation $B = 0$ is satisfied also in this case. However, this relation does not eventually rule out the possibility for the radiative symmetry breaking in the present case. This is very different from the ordinary GM SB.

An important aspect of the problem in the GM SB is crucially related to the radiative electroweak symmetry breaking. In order to see this, we study the well-known conditions for the radiative electroweak symmetry breaking. In the MSSM the minimization conditions of the tree-level scalar potential are written as

$$\sin 2 \beta = \frac{2B}{m_1^2 + m_2^2 + 2 m_t^2};$$

$$m_2^2 = \frac{2m_1^2 - 2m_2^2 \tan^2 \beta}{\tan^2 \beta};$$

where we assume that $S$ and $B$ are real for simplicity. In these equations the Higgs scalar masses $m_1^2$ and $m_2^2$ should be improved into the values at the weak scale by using the RGEs. If we take account of the dominant one-loop contributions, they can be written as [6]

$$m_1^2 (M_W) \; m_2^2 (M_W) \; \left\{ \begin{array}{l}
\frac{3}{2} m_2^2 (M_W) \; \frac{2 M_W^2}{2 \beta(\tan^2 \beta)} \; \frac{1}{1} \; \frac{1}{22} M_W^2 (\tan^2 \beta) \; \frac{1}{\tan^2 \beta} \; \frac{1}{1} \\
\frac{h_t^2}{8 \; m_t^2 \ln \frac{M_W}{m_t}} \;
\end{array} \right\};$$

where $h_t$ and $m_t$ represent the top Yukawa coupling constant and the stop mass. They are approximated by the values at $\mu$. The masses of the gauginos and scalar superpartners at the supersymmetry breaking scale are determined by eqs. (3) and (4).

We first remind the situation for the radiative symmetry breaking in the ordinary GM SB case $(S = B = 0)$ by checking a condition $m_1^2 + m_2^2 + 2 m_t^2 > 2 B j$ which is obtained from the condition (8) and is also required by the vacuum stability. Inserting eq. (10) into this inequality, we find that this necessary condition can be approximately written as

$$\frac{3}{2} m_1^2 + 5 \frac{2}{4} m_2^2 + \frac{1}{6} m_t^2 \; 2 \; \frac{4 h_t^2}{3 \; \tan^2 \beta} \; \frac{2}{2} \; \ln \frac{6}{3} \; p_6 \; \frac{1}{1} \; \frac{2}{2} \; (B j) \; ^2 > \frac{2}{2} \; (B j) \; ^2;$$

(11)
It is easy to find that the condition (11) is never satisfied unless $h_t$ takes an unacceptably small value in the case of $B\ j>2$, which is caused by the relation $B = \ldots$ because of $\ldots$. Thus, we need to consider an additional origin of to make the condition $B < 2$ be satisfied. This is the well-known result in the ordinary GM SB scenario [4, 7]. This fact might make us consider that the condition $m_1^2 + m_2^2 + 2^2 > 2B\ j$ cannot be satisfied in the GM SB model without the new origin for as far as the undesirable relation $B = \ldots$ exists. In the present model, however, $1\ 1$ is not generally supposed to be equal to $2$. This feature can give us a new possibility with regard to the electroweak symmetry breaking even under the existence of the relation $B = \ldots$ if $1 < 2$ is satisfied and then the spectrum of superpartners is m odi ed.

In order to see this, it is useful to note that the factor $(3 = 2)^2\ln((P\ 6 = 3))$ in eq. (11) should be m odi ed into an approximated factor

$$1 + \frac{16}{9}\frac{3}{2}^2\frac{1}{2}^2\frac{2}{1}^2\ln\left(\frac{P\ 6}{6}\right)\frac{1}{2}\left(\frac{2}{3}(1 = 2)^2 + 9\frac{2}{3} = 16\right)^{1-2}$$

(12)

in the present extended GM SB. This is caused by the change in the formulas of the soft scalar masses. We nd that $m_1^2 + m_2^2 + 2^2 > 2B\ j$ is satis ed as far as $1 < 2$ even in the case of $h_t'$ 1 and $B\ j > 2$. The same change related to the radiative correction due to the top Yukawa coupling tends to make the allowed value of $\tan \beta$ smaller than the one in the ordinary GM SB scenario with the additional contribution [6, 12, 4]. This can be found from eq. (9). Moreover, the same equation suggests that there appears an upper bound of $2 = 1$ if we impose the lower bound for $\tan \beta$. For example, if we require $\tan \beta > 2$, we nd that $2 = 1 < 3.5$ should be satis ed.

The more accurate analysis on this aspect can be done numerically by using the one-loop RGEs in the MSSM. For this purpose we can transform the conditions (8) and (9) into the formulas for $^2$ such as

$$\tan^2\left(m_1^2, m_2^2\right)\tan 2 + m_2^2\sin 2^2;$$

$$\tan^2\left(1, m_2^2\right)\frac{1}{2}m_1^2\frac{1}{2}m_2^2;$$

(13)

where we use $B = 1$. To estimate these formulas we take the following procedure.

The gauge and Yukawa coupling constants are evolved from the gauge coupling uni cation

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4In this paper we assume that $1, 2$ and $\beta$ are positive.
Fig. 1 The values of $2A$ and $2B$ predicted by the conditions for the radiative symmetry breaking in the case of $B = 1$. We can find the solutions as the crossing points of $2A$ and $2B$.

scale to the weak scale. The soft supersymmetry breaking parameters are introduced at scale and evolved to the weak scale. We use the weak scale values of $m_1^2$ and $m_2^2$ obtained in this way and also the value of $\tan\beta$ which is determined by the top quark mass and the value of top Yukawa coupling obtained from the RGEs. In Fig. 1 we plot each value of $2A$ and $2B$ in the $(\tan\beta_1, 2)$ plane for $\tan\beta_1 = 60$ TeV and several values of $\tan\beta$. $2A$ takes very small values of $0(1)$ GeV and the smaller $\tan\beta$ realizes the larger value of $2A$. $2B$ is very sensitive to the value of $\tan\beta_1 = 1$ in comparison with $2A$. From this figure we can find that there are solutions in the region such as $\tan\beta_1 < 3$ if we impose $\tan\beta > 2.4$ which corresponds to the constraint from the neutral Higgs boson search. However, if we take the larger value for $\tan\beta_1$, we can obtain the solutions for the larger values of $\tan\beta_1$.

Although the radiative symmetry breaking can be found to occur just within the framework without adding any fields, we need to impose other phenomenological constraints for the scenario to be realistic. Since the absolute values of $\tan\beta_1$ and $\tan\beta_2$ are directly constrained by the experimental bounds for the masses of the gluino and the neutralino, we find that these values should be in the range

$$\tan\beta_1 > 20 \text{ TeV}; \quad \tan\beta_2 > 10^2 \text{ GeV} : \quad (14)$$

This means that the present solution to the problem requires other contribution to the term to overcome the constraint from the neutralino mass bound. However, it is useful
to note that the situation on the origin of the term is not the same as the ordinary case. Since $j<2$ is not required in this case unlike the ordinary GM SB, the constraint imposed from the radiative symmetry breaking on the additional contribution $0$ to the term can be expected to be sufficiently weaken.

In order to study this aspect, we study the radiative symmetry breaking and the spectrum of the superpartners by using the one-loop RGEs. In this study we need only modify the parameter into $\sim = + 0$, where $\sim$ and $B$ are defined by eq. (7). Since in the present soft supersymmetry breaking scheme there are four parameters and we take them as $\sim$ and $0$ in addition to $i,j$, we can predict the spectrum of the superpartners in a rather restrictive way through this study. As the phenomenological constraints, we impose the experimental mass bounds for the superpartners and also require both the color and electromagnetic charge not to be broken. Under these conditions we search the allowed parameter region in the case of $j = 60$ TeV. In Fig. 2 we give scatter plots for each value of $\sim$ and $B$ for the solutions of the radiative symmetry breaking at each value of $2 = 1$. In this figure we can see that there are the solutions with $j > \sim$ for the $2 = 1 > 2$ region, although the solutions are restricted into the ones with $j < \sim$ for $2 = 1 < 2$. Here the ordinary GM SB should be noted to correspond to $2 = 1 = 1$. It should be also noted that the parameter is allowed to be smaller than the one in the ordinary GM SB.

We give the spectrum of the superpartners obtained in the same analysis for the case of $j = 60$ TeV in Fig. 3. On the lightest chargino and neutralino by combining Figs. 2 and 3 we can find that they are dominated by the gaugino in the region $2 = 1 < 2$ and they change into the Higgsino dominated one in the region $2 = 1 > 2$. The next lightest superpartner is always the neutralino as far as $2 = 1 > 1$ is assumed. The CP-even neutral Higgs boson mass slightly decreases when $2 = 1$ increases. This follows the behavior of the stop mass. Although the neutral Higgs boson mass is almost equal to the experimental bound for this $j$ value, it can be larger by taking $1$ larger. The difference of the mass spectrum of superpartners from the one in the ordinary GM SB becomes clear in the larger $2 = 1$ region. In that region the mass gap between the colored gauge and the color singlet fields becomes smaller and by using this feature we might distinguish the present model from the ordinary one.

Finally we briefly comment on other features which are not mentioned before. Since
Fig. 2. The relation between ~ and B, B =~ in the solutions for the radiative symmetry breaking conditions. The ~ and B instanton is represented by the GeV unit.

the soft masses of two Higgs fields H_1 and H_2 are same at the supersymmetry breaking scale, the radiative symmetry breaking predicts the relatively small value of tan \( \beta \) such as 2.5 \( \pm \) 7.5. Although tan \( \beta > 10 \) is possible, it needs the fine tuning of parameters. There can appear an interesting feature for the coupling unification scale in the case of \( \beta = \frac{1}{1} > 1 \). The unification scale of the coupling constants of SU(3) and SU(2) can be pushed up to the higher scale depending on the values of \( \frac{1}{1} \).

This aspect comes from the fact that the SU(2) nonsinglet superpartners decouple earlier than others. However, we need further study whether the large shift of unification scale can be consistent with the radiative symmetry breaking.

3 A deconstructed SUSY SU(5) model

In this section we consider a model which can realize the extended GMSB discussed in the previous section. As such an interesting example, we propose a unified SU(5) model with a direct product gauge structure such as \( G = SU(5)^0 \times SU(5)^0 \) and a global discrete symmetry F which commutes with this gauge symmetry [10]. A field content of the model is listed in Table 1. They are composed of bifundamental superfields \( (5; 5) \).

\(^5\)The similar possibility has been discussed in other context in [13].
Fig. 3 Mass spectrum of the superpartners for the parameter sets which satisfy the radiative symmetry breaking conditions and various phenomenological constraints. The lightest one for each superpartner is shown except for the gauginos.

and \( (5; 5) \), an adjoint Higgs chiral superfield \((1; 24)\), three sets of chiral superfields \((10; 1) + 5(5; 1)\) which correspond to three generations of quarks and leptons, a set of chiral superfield \(H(5; 1) + H^c(1; 5)\) which contains Higgs doublets, and also a set of chiral superfield \((5; 1) + (1; 5)\) in order to cancel the gauge anomaly induced by the above contents. We additionally introduce several singlet chiral superfields \(S\) and \(N\).

In order to induce the symmetry breaking at the high energy scale we introduce a superpotential such as

\[
W_1 = M \, \text{Tr}(1 \ 2) + \frac{1}{2} M \, \text{Tr}^2 + \text{Tr} \ 1 \ 2 + \frac{1}{3} \ 3 : \quad (15)
\]

As it is shown in [10], it is easily found that the scalar potential derived from this \(W_1\) has a nontrivial minimum which is realized at

\[
= M^* \, \text{diag} (2 ; 2 ; 3 ; 3) ; \quad 1 = \ ; \quad 2 = \frac{1}{M} \ \frac{M}{M^*} : \quad (16)
\]

where \(M^*\) is defined as \(M^* = M = \). There is no D-term contribution to \(V\) from these vacuum expectation values (VEVs) in eq. (16) and then the supersymmetry is conserved at this stage. The remaining symmetries and a parameter can be determined by assuming that the model is constructed based on the suitable deconstruction [10].

We consider that the theory space of the model is represented by the moose diagram which is composed of the \(n\) sites \(Q_i\) placed on the vertices of an \(n\)-polygon and one site
on its center P of this polygon [14]. We assign SU(5) on the site P and SU(5) on each site Q\(_i\) and also put a bifundamental chiral super field on each link from P to Q\(_i\). On each link from Q\(_i\) to Q\(_{i+1}\) we put the adjoint Higgs chiral super field of SU(5)\(^0\). Here we introduce an equivalence relation only for the boundary points of the polygon by the 2 =n rotation and we identify this Z\(_n\) symmetry with the above mentioned discrete symmetry F. This makes us consider the reduced theory space composed of only three sites P, Q\(_1\), and Q\(_2\), in which the field contents become equivalent to the one given in Table 1.

Under these settings we can find the following interesting results [14, 10]. First, in this model the symmetry G\(\rightarrow\) F breaks down into H\(\rightarrow\) F\(^0\) by considering the vacuum defined by eq. (16). Here H = SU(3) \(\times\) SU(2) \(\times\) U(1) is a subgroup of the diagonal sum SU(5) of G and also a discrete symmetry F\(^0\) is a diagonal subgroup of F\(^+\) = G\(_{(1)}\)\(^0\) where G\(_{(1)}\)\(^0\) is a discrete subgroup of a hypercharge in SU(5)\(^0\). Since the definition of F\(^0\) contains the discrete subgroup of U(1)\(^0\) in SU(5)\(^0\) as its component, every field which has a nontrivial transformation property with respect to SU(5)\(^0\) can have different charges. This feature makes it possible to split the doublet Higgs fields from the colored Higgs fields and also forbid the messenger fields to couple with the same singlet chiral super fields. We assign the charges of F\(^0\) for every field as shown in Table 1. The second result is that the parameter is fixed through an equation \(2 + 1 M = 0\).

In order to fix the discrete symmetry F\(^0\) we impose the following conditions on F\(^0\) to satisfy various phenomenological constraints, as we have done it in [10].

(i) Each term in the superpotential \(W_1\) should exist and this requirement imposes the conditions:
\[
+ + 2(c+d) = 0; \quad + + 3(c+d) = 0; \quad \text{(17)}
\]

(ii) To realize the doublet-triplet splitting, only the color triplet Higgs chiral super fields \(H_3\) and \(H_3'\) except for the ordinary doublet Higgs chiral super fields \(H_2\) and \(H_2'\) should become massive. Thus the Yukawa coupling \(\lambda H_2 H_2'\) should be forbidden although \(\lambda H_3 H_3'\) is allowed. This gives the conditions such as
\[
+ + 3(a+c) \neq 0; \quad + + + 2(a+c) = 0; \quad \text{(18)}
\]

(iii) Yukawa couplings of quarks and leptons, that is, \(10_{10} H_2\) and \(10_{10} H_2'\) should exist. This requires
\[
2 + = 0; \quad + + + 3(a+c) = 0; \quad \text{(19)}
\]

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(iv) The fields and should be massless at the G breaking scale and play the role of the messenger fields of the supersymmetry breaking which is assumed to occur in the S sector. These require both the absence of and the existence of the coupling . These conditions can be written as

\[ + + + 2(b+d) \neq 0; \quad + + 3(b+d) \neq 0; \]
\[ + + + 2(b+d) = 0; \quad + + + 3(b+d) = 0; \quad (20) \]

(v) The neutrino should be massive and the proton should be stable. This means that \( \frac{5}{1} \) \( H_2 \) and \( \frac{1}{2} \) \( \tilde{H}_1 \) should exist and \( \frac{1}{2} \) \( H_5 \) and \( \frac{3}{10} \) \( \tilde{H}_5 \) should be forbidden [14]. These require

\[ + + = 0; \quad 2 = 0; \quad 2 \neq 0; \quad 3 + \neq 0; \quad (21) \]

(vi) The gauge invariant bare mass terms of the fields such as \( \frac{5}{1} H, H, \tilde{H} \) should be

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Table 1 Charge assignment of the discrete symmetry \( F^0 \) for the chiral super fields.

|              | \( F \) (G rep.) | \( F^0 \) |
|--------------|------------------|-----------|
|              | \( F \)          | \( F^0 \) |
|              | \( F \)          | \( F^0 \) |
| Quarks/Leptons | \( \frac{3}{10} (10;1) \) | \( F \) |
| \( j = 1 \) \( 3 \) | \( \frac{3}{5} (5;1) \) | \( F^0 \) |
| \( 1 \) \( 1 \) \( 1 \) |
| Higgs fields  | \( H (5;1) \)     | \( F \) |
| \( H' (1;5) \) | \( + 2a \)        | \( F^0 \) |
| \( 2 \) \( 1 \) \( 3 \) |
| Messenger fields | \( (5;1) \)     | \( F \) |
| \( (1;5) \) | \( + 2b \)        | \( F^0 \) |
| \( 2 \) \( 1 \) \( 3 \) |
| Bifundamental field | \( 1 (5;5) \)     | \( F \) |
| \( + 2c \) | \( F^0 \) |
| \( 2 \) \( 1 \) \( 3 \) |
| Adjoint Higgs field | \( (1;24) \) | \( F \) |
| \( 0 \) | \( F^0 \) |
| Singletons    | \( S_1 (1;1) \) | \( F \) |
| \( S_2 (1;1) \) | \( ! \) | \( F^0 \) |
| \( N (1;1) \) | \( ! \) |

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**Table 1** Charge assignment of the discrete symmetry \( F^0 \) for the chiral super fields.
forbidden. These conditions are summarized as,

\[ + \epsilon 0; \quad + \epsilon 0; \quad + 2(a + b) \epsilon 0; \quad + 3(a + b) \epsilon 0: \quad (22) \]

Here we additionally assume that both origin of and \( B \) is in the Higgs coupling with \( S_1 \) in order to embed our scenario discussed in the previous section into this unified model. For this realization we introduce a term \( _1S_1H_2H_2 \) in the superpotential.\(^7\) The condition for the existence of such a term can be written as

\[ + + + 3(a + c) = 0: \quad (23) \]

Every condition above should be understood up to modulus \( n \) when we take \( F^0 = Z_n \).

We can easily find an example of the consistent solution for these constraints (17) \{ (23). In order to show its existence concretely, we give an example here. If we take \( F^0 = Z_{20} \), these condition can be satisfied under the charge assignment,\(^8\)

\[ \begin{array}{lllll}
  & = & 1; & = & = c = 2; & = & = a = b = 3; \\
  & & = 5; & d = 6; & = 8; & = 10: \\
\end{array} \quad (24) \]

It should be noted that the different singlet fields \( S_{1,2} \) are generally required for the couplings to \( Q \) and \( L \), which play a role of messengers of the supersymmetry breaking. This feature incidentally comes from the introduction of the direct product gauge structure motivated to realize the doublet-triplet splitting, which requires the \( F^0 \) charges of \( \) and 

to satisfy

\[ = 5(\phi + d) \not\equiv 0 \pmod{n}: \quad (25) \]

We can now consider the physics at the scale after the symmetry breaking due to the VEVs in eq. (16). The massless degrees of freedom are composed of the contents of the MSSM and the fields \((\eta';l)\) and \((\eta';t)\) which come from \((1;5)\) and \((5;1)\) and also

\(^6\)We cannot forbid the bare mass terms of the singlet chiral superfields completely based on the discrete symmetry \( F^0 \) alone. Although we might need additional symmetry to prohibit \( \), we do not discuss it further here and we only assume that they have no bare mass.

\(^7\)If we make the Higgs doublets couple to \( S_2 \) instead of \( S_1 \), seem not to be large enough to satisfy the radiative symmetry breaking condition.

\(^8\)We have not taken account of the anomaly of \( F^0 \) here. Although this anomaly cancellation might require the introduction of new fields and impose the additional constraints on the charges, it does not affect the result of the present study of the model.
the singlet fields $S_{1,2}$. Thus we can expect the successful gauge coupling unification for these field contents in the similar way to the M SSM. Under the in position of the discrete symmetry $F^0$, the superpotential for these fields can be written as

$$W_2 = h_1 \frac{1}{10} 10H_2 + \frac{h_2}{M} 10 \frac{1}{5} S_{1,2} H_2 + \frac{h_3}{M} S_{1,2} H_2$$

$$+ \frac{h_4}{1} 5H_2 + \frac{h_5}{M} 1 \frac{1}{2} S_{1,2}$$

$$+ \frac{1}{M} 2S_{1,2} q + \frac{2}{M} S_{1,2} \gamma$$

(26)

where M is the effective unification scale. We use the usual notation in the M SSM for the doublet Higgs fields such as $H_1$, $H_2$ and $H_2$. The several terms can be suppressed by the additional factors $1_{12} \ h_{12} = M$ coming from the VEVs $h_{1,2}$ given in eq. (16) since each term is controlled by the discrete symmetry $F^0$. This feature makes several terms phenomenologically favorable. For example, the second term in the first line which includes the M SSM relevant terms seem to be favorable to explain the hierarchy between the masses of top and bottom quarks for the various values of $\tan \beta$. The mass hierarchy between the top quark and the bottom quark requires that $12$ should be $O(10^2)$ or larger. This feature also causes the favorable effects on the second line which is relevant to the neutrino masses. In fact, if the VEVs $h_{1,2}$ take the suitable values so as to be $12 = O(10^2)$, the right-handed neutrinos $1$ can have the mass of $O(10^{13})$ GeV which is suitable to explain the experiment data for the solar and atmospheric neutrinos.

In the last line of eq. (26), as we expected, $q; \ q$ and $\gamma; \ \gamma$ couple with the different singlet fields $S_{1,2}$. Thus the messenger sector assumed in the previous section is realized. The last term in the first line can be an origin of the and B term since both the scalar component and $F$-component of $S_1$ are assumed to get the VEVs. Both and B in eq. (7) can be induced by taking $1 = h_{11}$. In fact, if we assume

$$1 = O(10^2); \ h_{S_1} = 0 (10^5) \text{ GeV}; \ h_{F_1} = 0 (10^3) \text{ GeV}^2;$$

(27)

we can consistently obtain an suitable values of $B$ and for the radiative electroweak symmetry breaking which has been discussed in the previous section. To satisfy the neutralino mass bound, however, we need to introduce an additional origin for $0$. If we can introduce such an origin as $0 = O(100) \text{ GeV}$, the radiative symmetry breaking condition is expected to be easily satisfied based on the analysis in the previous section. The new origin may be given by the non renormalizable couplings among the Higgs chiral super fields.
and the singlet chiral super field $N$ whose scalar potential has a negative curvature due to the Kähler potential interaction \cite{3,7}. We consider the following terms in the effective Lagrangian,

$$
Z \quad S^0_1 S^0_1 N^0 N + \frac{Z}{M} \frac{1}{2} \frac{p^2}{2p^m} N^{m+3} + \frac{1}{M} \frac{q+1}{2} \frac{q}{2} \frac{1}{n+1} \frac{q}{n+1} \frac{1}{H_1 H_2} + h_c + (28)
$$

where each term should be determined by the discrete symmetry presented in Table 1. From these terms the additional contribution to the $t$ terms is yielded at the tree-level as

$$
\begin{align}
0 &= \frac{1}{(n+2)(m+3)^2} \frac{1}{p+2} \frac{q+1}{p+1} \frac{q+1}{n+1} \frac{q}{n+1} \frac{1}{F_S} \frac{n+1}{M} \frac{1}{n+1} + h_c \quad (29)
\end{align}
$$

If $2n = m + 1$ is satisfied \cite{3,7}, the magnitude of this term is approximately expressed as $\frac{1}{p+2} \frac{q+1}{p+1} \frac{q+1}{n+1} \frac{q}{n+1} \frac{1}{F_S} \frac{n+1}{M} \frac{1}{n+1}$. Therefore, we can obtain $0$ of $O(100)$ GeV by assuming $p = q = 0$ and taking account of eq. (27). Since $B$ is not generated at the tree-level along with this term, the dominant $B$ comes from $S^0_1 S^1 H_1 H_2$ in eq. (26) and then our result for the radiative symmetry breaking obtained in the previous section can be directly applicable to this model. It should be also noted that the effective Lagrangian (28) with $p = q = 0$ can be constructed on the basis of the discrete symmetry $F^0$ by defining the charge of $N$ as $= 5$ in the case of $n = 1$ and $m = 1$.

Finally we should comment on the relation to the mass eigenvalues and the mixings of quarks and leptons. We would like to stress that the existence of the suppression factor $1$ is favorable for the explanation of the masses of quarks and leptons as mentioned below eq. (26). The value of $1$ is constrained by the masses of a bottom quark and a lepton. If we impose $\tan \beta > 2$ which is required by the neutral Higgs boson mass constraint, $\beta > 10^2$ should be satisfied. This is consistent with the condition given in eq. (27) and also with the neutrino oscillation data. If we introduce Frogatt-Nielsen flavor $U(1)$ symmetry into this model along the line of \cite{15}, the qualitatively satisfactory mass eigenvalues and mixing angles for quarks and leptons are expected to be derived. We will discuss this subject in other place.

4 Summary

We have investigated the problem and the radiative symmetry breaking in the extended GM SB scenario, which can be derived, for example, from the supersymmetric
uni ed SU(5) model with the doublet-triplet splitting. The model may be constructed through the deconstruction by extending the gauge structure into the direct product group SU(5)$^0$ $SU(5)^0$. The low energy spectrum is the one of the MSSM with the additional chiral super elds which can play a role of messengers in the GMSB. The discrete symmetry forces the color triplet and color singlet messengers to couple to the different singlet chiral super elds whose scalar and auxiliary components are assumed to get the VEVs due to the hidden sector dynamics.

In such a model the direct coupling between the doublet Higgs elds and the one of these singlet elds is allowed but suppressed due to this discrete symmetry. This coupling can give the origin of both $\Delta$ and $B$ terms. Since the model has two scales which are relevant to the supersymmetry breaking and the superpartner masses depend on both of them, the induced $\Delta$ and $B$ can be consistent with the radiative electroweak symmetry breaking. This aspect is largely different from the ordinary minimal GMSB scenario and it may present a new solution for the problem in the GMSB scenario at least from the viewpoint of the radiative symmetry breaking. However, to make the model consistent with the experimental bounds for the masses of superpartners, it seems to be required to introduce the additional contribution to the $\Delta$ term. Some interesting features different from the ordinary GMSB appear in the spectrum of the superpartners. The mass difference between the colored and color singlet superpartners tends to be smaller in comparison with the ordinary GMSB scenario and also the gaugino masses become non-universal generally. The next lightest superparticle can be always the neutralino. The gauge coupling unification scale may be pushed upwards somewhat.

Further phenomenological study of this kind of model seems to be worthy since it is constructed on the basis of the reasonable motivation to solve the doublet-triplet splitting problem in the grand unied model.

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