Research Article
CR-NOMA Networks over Nakagami-\(m\) Fading: CSI Imperfection Perspective

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There are high demands on both massive connections and high spectrum efficiency, and the cognitive radio-based nonorthogonal multiple access (CR-NOMA) system is developed to satisfy such demand. The unreal situation of CR-NOMA is considering the perfect channel state information (CSI) in receivers. This paper indicates impacts of imperfect CSI on outage and throughput performance. In particular, we focus on performance of the secondary network related to the imperfect CSI, and we derive closed-form expressions of outage probability and throughput for the downlink in such a CR-NOMA system. Particularly, a general form of Nakagami-\(m\) fading channel is adopted to examine the impact of fading on the performance of the CR-NOMA system. As the main achievement, we conduct extensive simulations and provide analyses to demonstrate the outage performance of the CR-NOMA system with CSI imperfections.

1. Introduction

Considering as a promising multiple access technique, non-orthogonal multiple access (NOMA) has been proposed due to the higher traffic adaptation and optimized spectral efficiency (SE) [1]. By sharing frequency/code/time resources, the NOMA-based networks serve multiple users allocated various power allocation (PA) coefficients. In particular, the users with weak channel qualities are required more power compared with the users possessing stronger channel conditions. The strong users benefit from successive interference cancellation (SIC) to detect their own messages [1]. By introducing a cooperative diversity gain, cooperative NOMA (co-NOMA) has been studied in various scenarios to highlight the performance of NOMA [2–5]. To enhance the reliability performance, a co-NOMA scheme was investigated in [5], in which relay nodes are assigned as the users with better channel conditions (namely, NOMA-strong users).

To further improve the SE, an underlay cognitive radio (CR) network is considered as another technique that allows secondary users (SUs) to transmit only if they do not result in reducing performance of primary users (PUs) related to harmful interference [6]. To address some of the 5G challenges including spectrum efficiency and massive connectivity, it is necessary to study the combination of NOMA with CR networks while maintaining reliable transmission of primary and secondary users. In various circumstances of the underlay CR-NOMA developed in [7–10], a superimposed signal is sent by the secondary transmitter to the secondary NOMA destination users. The authors concluded that if the PA factors and target rates are accurately chosen, the performance of NOMA users can be enhanced.

The outage performance of CR NOMA systems has been investigated extensively in the literature [11–15]. Particularly, Nakagami-\(m\) fading channels are adopted in suitable systems such as [11, 15]. The authors in [16] studied overlay CR-inspired NOMA by introducing expressions of the outage probability. A similar scenario was performed in [17]; however, different from [16], only the best secondary user is chosen to forward the signal to the primary user. To assist in transmission between secondary NOMA users and a secondary base station (SBS), many relays are deployed [18].
In such a system, it is beneficial as the selected relay with the strongest link with the SBS is required to forward signals to the secondary receivers [18]. However, these papers assumed that channel state information (CSI) is available and exact estimation at the receivers. In practice, imperfect CSI occurs as the main reason of degraded performance. It motivates us to study the new system performance analysis for CR-NOMA over Nakagami-\(m\) fading. In particular, we evaluate numerous new challenges when compared to previous works. These main challenges are summarized as follows.

(i) A cooperative scheme needs to be deployed in the context of NOMA transmission. Such a NOMA scheme has been recognized as a crucial approach to further improve spectral efficiency in CR-NOMA, especially for users with poor channel condition

(ii) Spectrum sharing is required at the secondary network in CR-NOMA; then, transmit power constraint is necessary since the SUs are permitted to reuse the licensed spectrum bands belonging to the PU. Spectrum sharing performs well under the conditions that the interference to the PU is guaranteed to be less than an assigned threshold

(iii) Outage performance of CR-NOMA network depends on the estimated channels at receiver. However, imperfect CSI make crucial impacts on SNR and then outage probability becomes worse.

To address these challenges, we propose the CR-NOMA under a scenario of imperfect CSI by considering the Nakagami-\(m\) fading channel model. The proposed system in this research is challenging and more realistic compared with [19]. On the comprehensiveness, we implement our proposed method through simulations. We summarize the main contributions of this research as follows.

(i) It need be improved performance of far NOMA users in CR-NOMA. Due to poor channel condition, the far user requires to assign a higher power allocation factor. Particularly, the secondary network is affected by interference from the primary nodes; then, relaying scheme should be implemented to achieve expected system improvement.

(ii) Impacts of interference among two networks in CR-NOMA are considered. Degraded performance can be accepted under the limitation of interference satisfied. In particular, outage probability and throughput are the two most important metrics and we derive the concrete expressions of these metrics under the constraints of transmit power and separated target rates of NOMA users.

(iii) Simulation results show the performance of the proposed CR-NOMA model using Nakagami-\(m\) fading channel models with respect to different variable conditions. We mainly focus on the trade-off between the system outage performance and other characteristic parameters such as level of CSI imperfections, total transmit power, and target rates. The results illustrate that the system outage behavior has a satisfied value within a certain range.

2. System Model

We examine a general NOMA model as a system model shown in Figure 1 wherein a downlink of CR-NOMA scenario. The relay is necessary to serve two NOMA users \(U_1\) and \(U_2\) which can operate in the secondary network (SN) containing secondary source (BS) intends and relay. Moreover, the relay operates in a decode-and-forward (DF) mode. It is noted that such SN deals with interference to the primary transmitter (PT) and primary destinations (PD) are operated in the primary network (PN). The transmit source BS in the SN is limited by transmit power of the PN. It is assumed that all nodes are equipped single antenna and operated in half-duplex mode. The Nakagami-\(m\) fading channel coefficients are adopted in the considered system with parameter \(m\). In such a CR-NOMA system, we denote wireless channels from node \(a\) to node \(b\) as \(g_{ab}\). In the second hop of the SN, we denote \(g_1\) and \(g_2\) denote the links from relay to user \(U_1\) and \(U_2\), respectively. To guarantee NOMA fairness characteristic, let \(a_1\) and \(a_2\) be the power allocation factors. It is noted that we can assume that \(a_1 > a_2\) with \(a_1 + a_2 = 1\) [20, 21]. \(n_R, n_1, n_2\) are denoted as Additive White Gaussian Noise (AWGN) with variance of \(N_0\).

In a real scenario, the imperfect CSI-related channels are known at the receivers; it is given by [22]

\[
g_j = \hat{g}_j + \epsilon_j, \tag{1}\]

where \(j \in \{SR, 1, 2\}\), \(\hat{g}_j\) represents the estimated channel factor, and \(\epsilon_j\) stands for the channel estimation error. Moreover, \(IP \sim \text{CN}(0, N_0\mu)\) is denoted as the interference from the PN to SN, and \(\mu\) is the scaling coefficient of \(IP\). The CSI of primary transmitters is not available at the secondary receivers as reported in [19].

To enable functions of CR in the context of the CR-NOMA system, the transmit power of secondary node \(Q\) is restricted as [19] \(P_Q \leq \min \left( I_{th}/g_{QP}^2, P_Q \right), Q \in \{S, R\}\). We denote \(P_Q\) as the maximum average allowed transmit power, with \(I_{th}\) denoted as the interference temperature constraint (ITC) at node PD.

There are two phases in transmission at the SN. The received signal at the relay \(R\) in the first phase is computed by

\[
y_{SR} = \left( \hat{g}_{SR} + \epsilon_{SR} \right) \sqrt{P_S} \left( \sqrt{a_1 x_1} + \sqrt{a_2 x_2} \right) + \left[ I_P + n_R \right]
= \hat{g}_{SR} \sqrt{P_S} \left( \sqrt{a_1 x_1} + \sqrt{a_2 x_2} \right) + \epsilon_{SR} \sqrt{P_S} \left( \sqrt{a_1 x_1} + \sqrt{a_2 x_2} \right) + \left[ I_P + n_R \right]. \tag{2}\]

effective noise

To further evaluate performance, the signal-to-interference-plus-noise ratio (SINR) and signal-to-noise ratio (SNR) before and after performing SIC can be obtained to
In the second phase, the received signals in $R_1$,  $R_2 = R_{1,2}$ are given as the interference link

$$
\gamma_{iSR} = \frac{P_s |g_{iSR}|^2 a_i}{\rho_s |e_{SR}|^2 |\rho_s + |e_{SR}|^2 + \mu + 1},
$$

where $\rho_s = P_s/N_0$ is defined as the transmit SNR at the BS.

In the second phase, the received signals in $U_{i, 1, 2}$, can be expressed by

$$
\gamma_i = (\hat{g}_i + e_i) \sqrt{P_R} (\sqrt{a_1 x_1 + \sqrt{a_2 x_2}) + I_p + n_i = \gamma_{iSR} \sqrt{P_R} (\sqrt{a_1 x_1 + \sqrt{a_2 x_2}) + I_p + n_i.
$$

Considering signal at $U_1$, the SINR to detect its signal $x_1$ is given by

$$
\gamma_{U_1,1} = \frac{|g_{U_1}|^2 a_1 \rho_R}{|g_{U_1}|^2 |a_2 \rho_R + |\rho_R |^2|e_{U_1}|^2 + \mu + 1}.
$$

Similarly, to detect signal $x_1, x_2$, we compute the corresponding SINR at user $U_2$ as

$$
\gamma_{U_2,2} = \frac{|g_{U_2}|^2 a_1 \rho_R}{|g_{U_2}|^2 |a_2 \rho_R + |\rho_R |^2|e_{U_2}|^2 + \mu + 1}.
$$

3. Performance Analysis

To compute the outage probability (OP) for users $U_i$, these functions are necessary, and they include the probability density function (PDF) and the cumulative distribution function (CDF) of dedicated Nakagami-$m$ channel, respectively, as [15]

$$
f_{|\gamma_i|}(x) = \frac{x^{m_i-1} e^{-x/\Omega_i}}{\Gamma(m_i)(\Omega_i)^{m_i}},
$$

$$
F_{|\gamma_i|}(x) = 1 - \frac{x \{(m_i, x/\Omega_i) \gamma_i \}^{m_i}}{\Gamma(m_i)} = 1 - e^{-x/\Omega_i} \sum_{i=0}^{m_i-1} \frac{x^i}{i!},
$$

where $\Gamma(a) = \int_0^\infty x^{a-1} e^{-x} dx$, $y(a, x) = \int_0^\infty x^{a-1} e^{-x} dx$, and $\Gamma(a, x) = \int_0^x x^{a-1} e^{-x} dx$ represent for the Gamma function, the lower and upper incomplete Gamma function, respectively ([23], Equations 8.310.1, 8.350.1, and 8.350.2).

It is noted that the PDF of $|e_j|^2$ is given as $f_{|e_j|^2}(x) = x^{m_j-1} e^{-x/\Omega_j}/(\Gamma(m_j)(\Omega_j)^{m_j})$, in which $\Omega = \lambda/m$, $m$, and $\lambda$ stand for the fading severity factor and mean, respectively. Regarding Nakagami fading, we also assume $m$ is an integer number.

3.1. Outage Probability of User $U_i$. The OP is determined by SNR, i.e., $\gamma_{th_i} = 2^{R_{th_i}} - 1$ with $R_{th_i}$ being the target rate at user $U_{i, 1, 2}$, and its unit is BPCU (BPCU is short for bit per channel use). Then, it can be computed the OP of $U_1$ as [19]

$$
P_{out} = 1 - \text{Pr} \left( \min \left( \gamma_{U_{1,1}} \gamma_{U_{1,2}}, \gamma_{U_{2,1}} \right) > \gamma_{th_1} \right) = 1 - \text{Pr} (\gamma_{R_1} > \gamma_{th_1}) \text{Pr} (\gamma_{U_{1,1}} > \gamma_{th_1}) \text{Pr} (\gamma_{U_{1,2}} > \gamma_{th_1}).
$$

Lemma. The first term of (11) $\Psi_1$ is formulated by

$$
\Psi_1 = \sum_{i_{SR}=0}^{m_{SR} - 1} \sum_{k_{SR}=0}^{m_{SR} - 1} \left( m_{SR} + i_{SR} - k_{SR} - 1 \right) \frac{\Theta_{k_{SR}} (\Omega_{SR} \Theta_{i_{SR}} \Theta_{k_{SR}}) e^{\beta_{SR}/\Theta_{SR}}}{\Theta_{k_{SR}} (\Theta_{i_{SR}} \Theta_{k_{SR}})}
$$

$$
+ \sum_{i_{SR}=0}^{m_{SR} - 1} \sum_{k_{SR}=0}^{m_{SR} - 1} \left( m_{SR} + i_{SR} - k_{SR} - 1 \right) \frac{\beta_{SR}^{m_{SR} - i_{SR} - k_{SR}} \Omega_{SR}^{m_{SR} - i_{SR} - k_{SR}} \times \Gamma \left( m_{SR} + k_{SR} \left( \beta_{SR} \Omega_{SR} + \Theta_{SR} \right) \right) \left( \frac{\Theta_{SR}}{\Theta_{SR}} \right) \left( \Theta_{SR} \right)^{m_{SR} - i_{SR} - k_{SR}}}
$$

$$
\times \left( \beta_{SR} + \Omega_{SR} \right)^{-m_{SR} - k_{SR}}.
$$

Proof. See the appendix.
Based on (7), we can write \( \Psi_2 \) as

\[
\Psi_2 = \Pr \left( \left| g_{y_{1}^2} \right|^2 a_1 r_{k_2} + p_{SR} + |e_{y_{1}}|^2 \right) > \mu + 1 \right) \\
= \Pr \left( \left| g_{y_{1}^2} \right|^2 a_1 r_{k_2} + p_{SR} + |e_{y_{1}}|^2 \right) > \mu + 1 \right) \\
+ \Pr \left( \left| g_{y_{1}^2} \right|^2 a_1 r_{k_2} + p_{SR} + |e_{y_{1}}|^2 \right) > \mu + 1 \right) \\
\Gamma \left( \frac{m_{SR} + 1}{\Omega_{SR}} \right) \left( \Omega_{SR} \right)^{m_{SR}} \\
\times \left( \beta_{2} + \frac{\Omega_{SR}}{\Omega_{SR}^2} \right)^{-m_{SR} - 1}.
\]

(13)

With the help of (9), (10), and some transform, it can be expressed as

\[
\Psi_2 = \sum_{i=0}^{m_{SR}-1} \sum_{k=0}^{m_{SR}-1} \left( m_{SR} + i_2 - k_2 - 1 \right) \\
\times \Gamma \left( \frac{m_{SR} + 1}{\Omega_{SR}} \right) \left( \Omega_{SR} \right)^{m_{SR}} \\
\times \left( \beta_{2} + \frac{\Omega_{SR}}{\Omega_{SR}^2} \right)^{-m_{SR} - 1}.
\]

(14)

where \( \Omega = \Omega_{SR}/\Omega_{x}. \) Similarly, putting (6) into (11) and the close form of \( \Psi_3 \) can be obtained as

\[
\Psi_3 = \sum_{i=0}^{m_{SR}-1} \sum_{k=0}^{m_{SR}-1} \left( m_{SR} + i_2 - k_2 - 1 \right) \\
\times \Gamma \left( \frac{m_{SR} + 1}{\Omega_{SR}} \right) \left( \Omega_{SR} \right)^{m_{SR}} \\
\times \left( \beta_{2} + \frac{\Omega_{SR}}{\Omega_{SR}^2} \right)^{-m_{SR} - 1}.
\]

(15)

Putting the result of Lemma (12), (14), and (15) into (11), the OP of \( x_1 \) is written as

\[
P_{\text{out}, 1} = \left\{ \begin{array}{ll}
1 - \Psi_1 \times \Psi_2 \times \Psi_3, & \text{if } \gamma_{th1} < \frac{a_1}{a_2} \\
1, & \text{otherwise.}
\end{array} \right.
\]

(16)

3.2. Outage Probability of \( x_2 \). Similarly, it can be obtained outage probability for \( x_2 \) as

\[
P_{\text{out}, 2} = 1 - \Pr \left( \min \left( \gamma_{R_{2x}U_{21}}, \gamma_{U_{12}} \right) > \gamma_{th2} \right) \\
= 1 - \Pr \left( \gamma_{R_{2x}}, \gamma_{U_{12}} \gamma_{th2} \right) \Pr \left( \gamma_{U_{12}} > \gamma_{th2} \right). \quad (17)
\]

Then, with the (4), \( \Psi_4 \) can be expressed as

\[
P_{\text{out}, 2} = \Pr \left( \frac{\left| g_{\gamma_{SR}^2} \right|^2 a_2 r_{y_{2}}}{|e_{y_{2}}|^2} > \mu + 1 \right) \\
= \Pr \left( \frac{\left| g_{\gamma_{SR}^2} \right|^2 a_2 r_{y_{2}}}{|e_{y_{2}}|^2} > \mu + 1 \right) \\
+ \Pr \left( \frac{\left| g_{\gamma_{SR}^2} \right|^2 a_2 r_{y_{2}}}{|e_{y_{2}}|^2} > \mu + 1 \right) \\
\Gamma \left( \frac{m_{SR} + 1}{\Omega_{SR}} \right) \left( \Omega_{SR} \right)^{m_{SR}} \\
\times \left( \beta_{2} + \frac{\Omega_{SR}}{\Omega_{SR}^2} \right)^{-m_{SR} - 1}.
\]

(19)

where \( \beta_{2} = \gamma_{th2}/a_2 \). Then, \( \Psi_2 \) is written as

\[
P_{\text{out}, 2} = \sum_{i=0}^{m_{SR}-1} \sum_{k=0}^{m_{SR}-1} \left( m_{SR} + i_2 - k_2 - 1 \right) \\
\times \Gamma \left( \frac{m_{SR} + 1}{\Omega_{SR}} \right) \left( \Omega_{SR} \right)^{m_{SR}} \\
\times \left( \beta_{2} + \frac{\Omega_{SR}}{\Omega_{SR}^2} \right)^{-m_{SR} - 1}.
\]

(20)
Finally, the closed-form outage probability of $x_2$ is formulated as

$$P_{\text{out}}^2 = 1 - \Psi_4 \times \Psi_5. \quad (21)$$

Moreover, we can write the incomplete Gamma function as

$$\gamma(a, bx) \xrightarrow{x \to 0} (bx)^a. \quad (22)$$

3.3. **Asymptotic Analysis.** To obtain the insight about CR-NOMA systems, the asymptotic outage probability is presented in high SNR regimes $\rho = \rho_S = \rho_R \to \infty$ based on the outage probability. In high SNR, we use $e^{-x} \approx 1 - x$.

Base on analysis, the asymptotic outage probability of $x_1$ is obtained as

$$P_{\text{out,as}}^1 = 1 - \Psi_1^{\infty} \times \Psi_2^{\infty} \times \Psi_3^{\infty}, \quad (23)$$

**Figure 2:** The OP of two users versus transmit SNR with $m = 2$ and $\lambda_e = 0.01$.

**Figure 3:** Outage performance versus transmit $\rho_1$ varying $\rho$ with $m = 2$ and $\lambda_e = 0.01$. 

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Base on analysis, the asymptotic outage probability of $x_1$ is obtained as

$$P_{\text{out,as}}^1 = 1 - \Psi_1^{\infty} \times \Psi_2^{\infty} \times \Psi_3^{\infty}, \quad (23)$$
where $\Psi_1^{\alpha}$, $\Psi_2^{\alpha}$, and $\Psi_3^{\alpha}$ are expressed, respectively, as

$$
\Psi_1^{\alpha} = \sum_{i=0}^{n-1} \left( m_{i-1} + i - k_{\alpha} - 1 \right) \phi_{k_{\alpha}} \left( \frac{1 - \frac{\beta_\Theta}{\Omega_2} \frac{\rho_1}{\Omega_{m_{\alpha}}} }{\Theta_1} \right)^{m_{\alpha} - 1} \times \left( \frac{\Omega_2}{\Theta_1} \right)^{m_{\alpha} - i} \times \left( \frac{\Omega_{m_{\alpha}}}{\Theta_{m_{\alpha}}} \right)^{m_{\alpha} - i}
$$

$$
\Psi_2^{\alpha} = \sum_{i=0}^{n-1} \left( m_{i-1} + i - k_{\alpha} - 1 \right) \phi_{k_{\alpha}} \left( \frac{1 - \frac{\beta_\Theta}{\Omega_2} \frac{\rho_1}{\Omega_{m_{\alpha}}} }{\Theta_1} \right)^{m_{\alpha} - 1} \times \left( \frac{\Omega_2}{\Theta_1} \right)^{m_{\alpha} - i} \times \left( \frac{\Omega_{m_{\alpha}}}{\Theta_{m_{\alpha}}} \right)^{m_{\alpha} - i} \times \left( \frac{\Omega_{m_{\alpha}}}{\Theta_{m_{\alpha}}} \right)^{m_{\alpha} - i}
$$

$$
\Psi_3^{\alpha} = \sum_{i=0}^{n-1} \left( m_{i-1} + i - k_{\alpha} - 1 \right) \phi_{k_{\alpha}} \left( \frac{1 - \frac{\beta_\Theta}{\Omega_2} \frac{\rho_1}{\Omega_{m_{\alpha}}} }{\Theta_1} \right)^{m_{\alpha} - 1} \times \left( \frac{\Omega_2}{\Theta_1} \right)^{m_{\alpha} - i} \times \left( \frac{\Omega_{m_{\alpha}}}{\Theta_{m_{\alpha}}} \right)^{m_{\alpha} - i}
$$

Similarly, the asymptotic outage probability of $x_2$ is expressed as

$$
P_{out,\alpha} = 1 - \Psi_4^{\alpha} \times \Psi_5^{\alpha},
$$

where

$$
\Psi_4^{\alpha} = \sum_{i=0}^{n-1} \left( m_{i-1} + i - k_{\alpha} - 1 \right) \phi_{k_{\alpha}} \left( \frac{1 - \frac{\beta_\Theta}{\Omega_2} \frac{\rho_1}{\Omega_{m_{\alpha}}} }{\Theta_1} \right)^{m_{\alpha} - 1} \times \left( \frac{\Omega_2}{\Theta_1} \right)^{m_{\alpha} - i} \times \left( \frac{\Omega_{m_{\alpha}}}{\Theta_{m_{\alpha}}} \right)^{m_{\alpha} - i}
$$

$$
\Psi_5^{\alpha} = \sum_{i=0}^{n-1} \left( m_{i-1} + i - k_{\alpha} - 1 \right) \phi_{k_{\alpha}} \left( \frac{1 - \frac{\beta_\Theta}{\Omega_2} \frac{\rho_1}{\Omega_{m_{\alpha}}} }{\Theta_1} \right)^{m_{\alpha} - 1} \times \left( \frac{\Omega_2}{\Theta_1} \right)^{m_{\alpha} - i} \times \left( \frac{\Omega_{m_{\alpha}}}{\Theta_{m_{\alpha}}} \right)^{m_{\alpha} - i}
$$

3.4. Consideration on Throughput. The achieved outage probability leads to computations on throughput to indicate the ability of transmission at fixed target rates. As an extra metric, overall throughput can be given as

$$
T_{total} = (1 - P_{out}^1) R_{th1} + (1 - P_{out}^2) R_{th2}.
$$

4. Simulation Result

The main parameters in simulation are provided in detail. First, the fixed power allocation factors are assigned for two NOMA users, $a_1 = 0.8$, $a_2 = 0.2$. In addition, we set channel gains as $A_{\delta} = 1$, $\lambda_{SP} = \lambda_{RP} = 0.1$, $\lambda_1 = 1$, $\lambda_2 = 2$, $R_{th} = R_{th1} = R_{th2} = 1$ (BPCU), $\lambda_{\delta} = \lambda_{SP} = \lambda_1 = \lambda_2 = 1$, $m = m_{\delta} = m_{SP} = m_{RP} = 0.1$, and $\rho_1 = 10 dB$ except for specific cases.

In Figure 2, we consider the OP versus transmit SNR at the SN. Different OP values are seen for two users at the SN due to different power allocation factors assigned to $U_i$. As the main achievement, the exact matching can be seen between analytical results and simulation results. Secondly, the OP of two users are better than the OMA case in the whole range of SNR $\rho$, which confirms the benefit of the CR-NOMA model. In addition, the OP can be improved as increasing transmit SNR at the BS, which confirms the necessity of improving the quality of transmit signal.

In Figure 3, two cases of $\rho$ which makes impacts on the OP of two users versus the transmit SNR at secondary source are studied. It can be seen clearly that $\rho = 30 dB$ is a better case. The main reason is that $\rho = 30 dB$ leads to higher SINR and then OP will be improved.

Figure 4 plots OP of two users in different fading parameters; $m = 1$, $m = 3$, and $m = 3$ are the better outage behavior. It can be seen clearly that $m = 3$ leads to a better channel which plays an important role to improve the performance of destinations. The main reason is that main expressions of SINR depend on channel gains. Therefore, higher channel gains result in higher SINR and the corresponding outage performance can be enhanced. When $m = 3$, such outage performance meets saturation lines sooner for the case of $m = 1$, which confirms that the outage performance can not be adjusted by increasing SNR at the base station at high SNR region.
Moreover, other trend of OP is similar as previous cases. In Figure 5, the OP decreases significantly for the case of $R_{th} = 0.5$ and $\lambda_e = 0.01$. Two factors make influence on OP performance which is confirmed in this figure. It is important to examine OP under the impact of imperfect CSI. Therefore, $\lambda_e$ related to the level of imperfect CSI is controlling parameter to target best outage performance for two users. The lower target rate $R_{th}$ is required at lower CSI imperfection CSI that provides better outage performance. It can be observed the lowest case is the OP of user $U_1$ that corresponds to $\lambda_e = 0.01$ and $R_{th} = 0.5$.

Regarding impacts of power allocation factors on OP performance, Figure 6 requires we perform simulation for case of $a_1 > 0.5$. Figure 5 shows the different trends of OPs for two users $U_i$ when we change power factor as $0.5 < a_1 < 1$. Looking at case $a_1 < 0.7$, the OP of the user $U_2$ is better than that of $U_1$. However, opposite trend happens as $a_1 > 0.7$. The main reason is that power level allocated to each user is strictly related to the OP.

To look on throughput performance, $m$ fading parameter makes influence on the performance of two users of such a CR-NOMA, all observations as Figure 7. It can be observed
Figure 6: The OP versus power allocation factor $a_1$ with $m = 2$ and $\lambda_e = 0.01$.

Figure 7: Throughput of system versus transmit SNR at BS with varying $\lambda_e$ and $m$.

Figure 8: Throughput of system versus $R_{th}$ with varying $\rho = \rho_1$. 
that the highest throughput occurs at $\lambda_c = 0.01$ and $m = 3$. The ceiling throughput is also seen at a high region on transmit SNR as SNR is greater than 25 dB. Interestingly, the existence of $R_{th}$ corresponds to maximal throughput as simulation reported in Figure 8.

5. Conclusions

In this paper, we have studied the impact of imperfect CSI on the CR-NOMA as a possible realistic scenario. Two users are evaluated in terms of outage performance to show advantages of a developed CR-NOMA system wherein NOMA users cooperate with the relay nodes. In particular, we derived the closed-form expressions of outage probabilities and throughput to show performance gaps among two NOMA users. Our simulation analysis confirms that the target rates and power allocation factors are main impacts on these metrics. The further simulation results show that CR-NOMA yields better outage performance once we limit imperfect CSI at low level and proper power allocation factor achieved.

Appendix

Proof of Lemma

Next, we further examine the outage event at the BS for $x_2$ and it can be formulated by the following.

It can be recalled that

$$
\Xi_2 = \Pr \left( \frac{|g\wedge_{SR}|^2 a_1 \rho_s}{|g\wedge_{SR}|^2 a_1 \rho_s + |e_{SR}|^2 \rho_s + 1} > \gamma_{th}, \rho_s < \frac{\rho_1}{|g_{SR}|^2} \right) 
+ \Pr \left( \frac{|g\wedge_{SR}|^2 a_1 \rho_s}{|g\wedge_{SR}|^2 a_1 \rho_s + |e_{SR}|^2 \rho_s + 1} > \gamma_{th}, \rho_s > \frac{\rho_1}{|g_{SR}|^2} \right),
$$

(A.1)

where $\rho_s = P_s/N_{SR}$, $\rho_0 = P_0/N_{SR}$, and $\rho_1 = I_{th}/N_0$. Then, we determine the first and second term of (A.1) which are denoted as $C_1$ and $C_2$, respectively. Firstly, $C_1$ can be rewritten as

$$
C_1 = \Pr \left( |g\wedge_{SR}|^2 > \gamma_{th} |e_{SR}|^2 + \gamma_{th} |e_{SR}|^2 + 1 \right) \Pr \left( \rho_s < \frac{\rho_1}{|g_{SR}|^2} \right),
$$

(A.2)

where $\gamma_{th} = I_{th}/a_1 - I_{th} \rho_s$ and $\Theta = \mu + 1/\rho_s$.

Then, the first term of $C_1$ is denoted as $C_{11}$ and it can be obtained as

$$
C_{11} = \sum_{i=0}^{m-1} \sum_{k=0}^{k_{SR}} \frac{(m_{SR} + i_{SR} - k_{SR} - 1)}{(m_{SR} - 1)} \times \Omega_{SR}^{m_{SR}} \Omega_{SR}^{i_{SR} - k_{SR} - 1} \rho_1^{i_{SR}} e^{-\gamma_{th} \rho_s / \Gamma(m_{SR})},
$$

(A.4)

where $\Omega_{SR} = \Omega_{SR}/\Omega_{SR}$. Then, the second term (A.2) is so-called as $C_{12}$. With the help of (10), $C_{12}$ can be given as

$$
C_{12} = 1 - \frac{\Gamma(m_{SR}, \rho_1/\rho_s \Omega_{SR})}{\Gamma(m_{SR})}.
$$

(A.5)

The second component is computed as

$$
C_2 = \Pr \left( |g\wedge_{SR}|^2 > \gamma_{th} |e_{SR}|^2 + \gamma_{th} |e_{SR}|^2 + 1 \right) \Pr \left( \rho_s > \frac{\rho_1}{|g_{SR}|^2} \right).
$$

(A.6)

in which $\Theta = (\mu + 1)/\rho_s$. Similarly, $C_2$ is rewritten as

$$
C_2 = \sum_{i=0}^{m-1} \sum_{k=0}^{k_{SR}} \frac{(m_{SR} + i_{SR} - k_{SR} - 1)}{(m_{SR} - 1)} \times \Omega_{SR}^{m_{SR}} \Omega_{SR}^{i_{SR} - k_{SR} - 1} \rho_1^{i_{SR}} e^{-\gamma_{th} \rho_s / \Gamma(m_{SR})},
$$

(A.7)

Putting (A.4), (A.5), and (A.7) into (A.1), we can obtain (12). It completes the proof.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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