Comment on non-Gaussianity in hybrid inflation

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In the literature there have been incompatible estimates for the amount of non-Gaussianity in hybrid inflation. In this note we point out the sources for the discrepancies and show that the results for the amount of non-Gaussianity in hybrid inflation obtained by two different methods, namely, perturbing Einstein equation to second order and the separate universe approach, indeed are compatible. This provides confidence in the methods themselves and in the actual computation of non-Gaussianities.

Introduction. Recently there has been considerable interest in the possible non-Gaussian component of the cosmological perturbations. Non-Gaussian perturbations in hybrid inflation were first estimated using consistent second order perturbation theory in [1], where the formalism of Acquaviva et al. [2] was used. The approach is to perturb the metric and matter sides of the Einstein equation, and use the resulting equations to obtain the dynamics of required quantities. These quantities are then used to find out the second order curvature perturbation during inflation.

This method leads to a set of equations and, even after simplifying (although motivated and not too constraining) assumptions, the final expression for the second order curvature is complicated, containing several nonlocal and time-integrated terms. An order of magnitude of the result was estimated already in the original study [1]. It was later re-estimated in [3], where the result seemed to disagree with the earlier estimates and implies an insignificant contribution from the transverse field, \( \sigma \), in hybrid inflation.

A completely different method, the \( \delta N \) formalism or the separate universe approach (see e.g. [4]), for computing the second order curvature perturbation in hybrid inflation was employed by Lyth and Rodríguez in [5]. The result disagrees with the earlier estimates and implies an insignificant contribution from the transverse field, \( \sigma \), in hybrid inflation.

There is a discrepancy between the results obtained with the two different methods, thus raising doubts on the validity of the methods. However, we argue that in both [1] and [3] the time evolution of certain quantities is not properly taken into account. In this brief comment we present a re-estimate of the original expression for the second order curvature perturbation from the \( \sigma \) field. We show that when all the time evolutions are correctly taken into account the order of magnitude estimate does indeed agree with the result obtained with the separate universe approach. The second order curvature seems to be proportional to the slow roll parameters. Such a small curvature alone would make the resultant non-Gaussianity unobservable, but according to [3] there is a further suppression of the nonlinearity parameter in this particular model due to the uncorrelated nature of the \( \sigma \) non-Gaussianities.

Original computation. The original estimation of the non-Gaussianity was obtained in [1] by extending the formalism of [2] for two scalar fields. The approach is to expand the perturbations of the metric and the matter, which consists of two scalar fields, up to second order. These perturbed quantities are then used to write Einstein equations to second order. The curvature perturbation, defined for one scalar field in the first order as \( \mathcal{R} = \psi + H \dot{\phi} \), is written for two scalar fields and expanded to second order; here \( \psi \) is metric perturbation, \( H = \dot{a}/a \) is the Hubble parameter, \( \delta \phi \) is the inflaton perturbation, and \( \psi \) is the time derivative of the background value of the inflaton field. The Einstein equations are then used to obtain the evolution of \( \mathcal{R} \) to second order, which in turn is then used to estimate the amount of non-Gaussianity.

The analytic calculation of the evolution equations in the case of two scalar fields becomes complicated. To alleviate these difficulties two simplifying assumptions are made in [1], namely, it is assumed that the background value \( \sigma_0 = 0 \) and that the potential does not have any terms linear in \( \sigma \). The latter assumption means that \( \sigma_0 = 0 \) indeed is a local minimum. The assumptions are well motivated and not too constraining, and they clearly apply to many other models in addition to hybrid inflation, whose potential is

\[
V = V_0 - \frac{m^2}{2} \sigma^2 + \frac{1}{2} \sigma^4 + \frac{m^2}{2} \varphi^2 + \frac{g^2}{2} \sigma^2 \varphi^2.
\]

In fact, at least some number of \( e \)-folds after horizon exit of the relevant scales, the form of the potential can be taken to be

\[
V = V_0 + \frac{m^2}{2} \sigma^2 + \frac{m^2}{2} \varphi^2,
\]

where \( m_\sigma \) and \( m_\varphi \) are the effective masses of \( \sigma \) and \( \varphi \), respectively.

The two constraints cause the second field, \( \sigma \), to completely decouple from the first order Einstein equations. Its behaviour in the first order is only governed by the Klein-Gordon equation. At the second order, the contribution of \( \sigma \) becomes completely additive, i.e. in the evolution equations there are no \( \varphi \sigma \)-mixing terms. This enables one to make use of the result of [2] for the inflaton contribution in [1]. Therefore, what is new in [1] is the contribution coming from the transverse field \( \sigma \).

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1 This is during inflation. Complete second order perturbation theory connecting inflationary perturbations to the observed CMB anisotropies has so far not been developed.
The expression obtained in the original paper [1] for the contribution of the $\sigma$ field perturbations to the second order curvature perturbation is rather complicated, containing several nonlocal terms and several time integrated terms. It reads

$$\mathcal{R}^{(2)}_{\sigma} = \frac{1}{\epsilon H M_P^2} \left\{ \int \left[ 6H \Delta^{-1} \partial_i \left( \delta \sigma_i \partial^i \delta \sigma_1 \right) + 4\Delta^{-1} \partial_i \left( \delta \sigma_i \partial^i \delta \sigma_1 \right)^* - 2(\delta \sigma_1)^2 + m_{\sigma}^2 (\delta \sigma_1)^2 \right. + (\epsilon - \eta) 6H \Delta^{-2} \partial_i \left( \partial_i \partial^i \delta \sigma_1 \partial^i \delta \sigma_1 \right)^* + (\epsilon - \eta) H \Delta^{-2} \partial_i \partial^i \left( \partial_i \delta \sigma_1 \partial^i \delta \sigma_1 \right)^* - 3\Delta^{-2} \partial_i \left( \partial_i \partial^i \delta \sigma_1 \partial^i \delta \sigma_1 \right)^* - \frac{1}{2} \Delta^{-2} \partial_i \partial^i \left( \partial_i \delta \sigma_1 \partial^i \delta \sigma_1 \right)^* \right. \] \nonumber 

### Equation (1)

$$\left. \left. - \frac{1}{2} \Delta^{-2} \partial_i \partial^i \left( \partial_i \delta \sigma_1 \partial^i \delta \sigma_1 \right)^* + \frac{1}{2} \Delta^{-2} \partial_i \partial^i \left( \partial_i \delta \sigma_1 \partial^i \delta \sigma_1 \right)^* + \frac{3\epsilon H}{2} \Delta^{-2} \partial_i \left( \partial_i \delta \sigma_1 \partial^i \delta \sigma_1 \right)^* \right] dt \right. - \Delta^{-1} \partial_i \left( \delta \sigma_1 \partial^i \delta \sigma_1 \right)^* \right\},$$

where $\Delta^{-1}$ is the inverse Laplacian, $M_P \equiv (8\pi G_N)^{-1/2}$ is the reduced Planck mass, $\delta \sigma_1$ is the first order perturbation of the transverse scalar field $\sigma$, and $\epsilon \equiv M_P^2 \left( \frac{1}{2} \frac{\partial V}{\partial \phi} \right)^2$ and $\eta \equiv M_P \frac{1}{2} \frac{\partial^2 V}{\partial \sigma^2}$ are slow roll parameters; dot denotes derivative with respect to time.

In [1] the slow roll solution $\delta \sigma_1 \propto e^{-m_{\sigma}^2 t/3H}$ was used to obtain an estimate for the time derivative $\left| \delta \sigma_1 \right| \sim m_{\sigma}^2 \left| \delta \sigma_1 \right|$. (Double time derivatives were estimated by $d^2/dt^2 \sim (m_{\sigma}^2/H)^2$, but actually one should use equation of motion to get rid of them). Since both fields, $\phi$ and $\sigma$, are effectively massless, the relation $\left| \delta \sigma_1 \right| \sim \left| \delta \phi \right|$ for the first order perturbations was used to approximate

$$\left| \frac{H}{\phi} \delta \sigma_1 \right| \sim \left| \frac{H}{\phi} \delta \phi_1 \right| \equiv \left| \mathcal{R}^{(1)} \right|.$$  \nonumber$$

### Equation (2)

In the first order Einstein equations $\sigma$ is completely decoupled, and the situation is essentially that of a single field inflation. Therefore, $\mathcal{R}^{(1)}$ stays constant outside horizon.

Only order of magnitude estimate was pursued and cancelling spatial derivative operators were neglected, e.g. $|\Delta^{-1}\partial_i \mathcal{R}^{(1)}| / \partial^i \mathcal{R}^{(1)}| \sim |\mathcal{R}^{(1)}|^2$. For the estimation of the time integral the quantities $H$, $\epsilon$, $\eta$, $m_{\sigma}$ and $\delta \sigma_1$ in the integrand were taken to be constants. The original result in [1] for the estimate of the second order perturbation due to $\sigma$ reads

$$\mathcal{R}^{(2)}_{\sigma} \sim O(\epsilon, \eta, m_{\sigma}^2 / H^2) \left| \mathcal{R}^{(1)} \right|^2.$$  \nonumber$$

### Equation (3)

Since $\eta_{\sigma} \equiv M_P^2 \frac{\partial^2 V}{\partial \sigma^2} \approx m_{\sigma}^2 / H^2$, the entire coefficient is of the order slow roll parameters.

Re-estimate by Lyth and Rodríguez. Later Lyth and Rodríguez [2] made a re-estimation of Eq. (1) by inserting initial conditions and writing the equation as a definite integral. They, however, used the same estimates, Eq. (2) (with $\mathcal{R}^{(1)} \sim$ const) and $|\delta \sigma_1| \sim m_{\sigma}^2 / |\delta \sigma_1|$, as the original study [1]. Similarly, they also assumed $H$, $m_{\sigma}$, and $\epsilon$ to be constants, ending up with

$$\mathcal{R}^{(2)}(t) - \mathcal{R}^{(2)}(t_i) = \frac{1}{\epsilon H M_P^2} \int_{t_i}^t \left[ 6H \Delta^{-1} \partial_i \left( \delta \sigma_i \partial^i \delta \sigma_1 \right)^* + m_{\sigma}^2 (\delta \sigma_1)^2 \right] dt \sim \Delta N \frac{m_{\sigma}^2}{H^2} \left| \mathcal{R}^{(1)} \right|^2.$$  \nonumber$$

### Equation (4)

Since $\Delta N$ is the number of e-folds, which can very well be $\sim 60$, this result implies a much larger $\sigma$ contribution to the second order curvature perturbation than in the original study.

**Separate universe approach.** In addition to the cosmological perturbation theory approach [1], there also exists a recent computation [3] of the second order curvature perturbation in hybrid inflation using the separate universe approach.

The general idea of the separate universe approach (see e.g. [4] for a concise description) is to consider each point in space as being surrounded by a homogeneous FRW universe. Each point then has its own expansion parameter $N$, i.e. local number of e-folds, independent of the value of the expansion parameter (or any quantity) in other points. This expansion parameter depends on the values of relevant quantities, such as unperturbed scalar fields, at that point. The complete, inhomogeneous, behaviour of the universe is obtained when all the separately treated points are patched together.

The curvature perturbation, $\zeta$, is in [3] defined by

$$g_{ij} = a^2(t) e^{-2\gamma(t, x)} \delta_{ij}(t, x),$$  \nonumber$$

where, within inflationary context, $\gamma_{ij}$ contains the tensor perturbation which we do not consider here, (see [3] for more details). Up to second order in scalar field perturbations ($\delta \phi_i = \delta \phi_0(t, x)$) $\zeta$ is obtained from

$$\zeta(t, x) = \sum_i N_i(t) \delta \phi_i + \frac{1}{2} \sum_{ij} N_{ij}(t) \delta \phi_i \delta \phi_j,$$  \nonumber$$

### Equation (6)

where $N_i \equiv \partial N_i / \partial \delta \phi_i$ and $N_{ij} \equiv \partial^2 N / \partial \delta \phi_i \partial \delta \phi_j$.

Adapting the notation $V = V_0(1 + \frac{1}{2} \eta \varphi^2 + \frac{1}{2} \eta \sigma^2)$ for the potential, the curvature perturbation reads

$$\zeta = \frac{\partial \varphi}{\eta \varphi} \frac{\eta}{2} \left( \frac{\partial \varphi}{\eta \varphi} \right)^2 + \frac{\eta_{\sigma}}{2} e^{2\Delta N} \left( \frac{\partial \sigma}{\eta \sigma} \right)^2.$$  \nonumber$$

### Equation (7)

The fields $\varphi$ and $\sigma$ are assumed to be massless and their perturbations, $\delta \varphi$ and $\delta \sigma$, are assumed to have the same spectrum $\left( \frac{1}{2} \right)^2$ in [3]. Therefore, we can set

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2 They used a different definition for the second order curvature, but that is irrelevant here.
\[ |\zeta_1| \sim |\frac{\delta \varphi}{\eta_\sigma}| \sim |\frac{\delta \varphi}{\eta_\sigma}| \] and estimate the last term in Eq. (11), i.e. the contribution of \( \sigma \) to the second order curvature perturbation as

\[ \zeta_{\sigma, \varphi} \sim O(\eta_\sigma) e^{2\Delta N(\eta_\sigma - \eta_\sigma^*)} |\zeta_1|^2 . \tag{8} \]

**Source for the discrepancy.** The seeming discrepancy between the two different methods basically comes down to articles 1, and 3, or more precisely, assuming \( \epsilon \) to be constant, and to the assumption in Eq. (12), i.e.

\[ |\delta \sigma_1(t)| \sim |\delta \varphi_1(t)| , \tag{9} \]

which is not generally justified. Eq. (9) only holds immediately after horizon exit (\( t = t_i \)), when the amplitude of the perturbation of any effectively massless field \( \zeta \) is \( |\zeta| \sim H \).

Using the slow roll equations (outside horizon) we obtain

\[ \begin{align*}
\delta \sigma_1(t) &= \delta \sigma_1(t_i) e^{-\eta_\sigma \Delta N}, \\
\delta \varphi_1(t) &= \delta \varphi_1(t_i) e^{-\eta_\sigma \Delta N}, \\
\varphi_0(t) &= \varphi_0(t_i) e^{-\eta_\sigma \Delta N}, \tag{10} \end{align*} \]

where \( \Delta N = H \Delta t \) is the number of e-folds since \( t_i \); thus, we obtain \( \delta \sigma_1 = -\eta_\sigma H \delta \sigma_1, \delta \varphi_1 = -\eta_\sigma H \delta \varphi_1, \) and \( \varphi_0 = -\eta_\sigma H \varphi_0 \). Since \( \epsilon \sim 2\eta_\sigma \), we can also readily write

\[ \epsilon(t) = \epsilon_i e^{-2\eta_\sigma \Delta N}, \tag{11} \]

where we have denoted \( \epsilon_i \equiv \epsilon(t_i) \).

Now, it is immediately clear that \(|R^{(1)}| = |H \delta \varphi_1/\varphi_0| \) stays constant, but one also sees that \( \Delta N \) e-folds after horizon exit

\[ |H \frac{\delta \sigma_1}{\varphi_0}| \sim |R^{(1)}| = \frac{\delta \sigma_1}{\delta \varphi_1} \sim e^{2\Delta N(\eta_\sigma - \eta_\sigma^*)}. \tag{12} \]

For the estimation of the time integral in Eq. (11) the important point is that one may not move \( 1/\epsilon \) and \( \delta \sigma_1 \) into and out of the time integral.

**Re-estimate of non-Gaussianity.** Now we present a re-estimate of Eq. (11) using the time evolutions expressed in Eqs. (10) and (11). First, we notice that

\[ 6H \Delta^{-1} \partial_t (\delta \sigma_1 \partial^i \delta \sigma_1) + 2 \Delta^{-1} \partial_t (\delta \sigma_1 \partial_i \delta \sigma_1)^* - (\delta \sigma_1)^2 + m^2_\sigma (\delta \sigma_1)^2 = 2 \Delta^{-1} \partial_t \left[ (3H \delta \sigma_1 + \delta \sigma_1 + m^2_\sigma \delta \sigma_1) \partial^i \delta \sigma_1 \right] = 0 , \tag{13} \]

since \( 3H \delta \sigma_1 + \delta \sigma_1 + m^2_\sigma \delta \sigma_1 = 0 \) outside horizon. Thus, Eq. (11) can be written

\[ R^{(2)}_\sigma = \frac{1}{\epsilon H M_P^2} \left\{ \int^t \left[ 2 \Delta^{-1} \partial_t (\delta \varphi_1^i \partial^j \delta \sigma_1)^* - (\delta \sigma_1)^2 + 2H(\epsilon - \eta_\sigma) \gamma_\sigma - \gamma_\sigma^* \right] dt - \Delta^{-1} \partial_t (\delta \varphi_1^i \partial^j \delta \sigma_1) + \gamma_\sigma + \epsilon \gamma_\sigma \right\} \]

\[ = \frac{1}{\epsilon H M_P^2} \left\{ \int^t \left[ - (\delta \sigma_1)^2 + 2H \epsilon \gamma_\sigma \right] dt + \Delta^{-1} \partial_t (\delta \varphi_1^i \partial^j \delta \sigma_1) + H(\epsilon - 2\eta_\sigma) \gamma_\sigma \right\} , \tag{14} \]

where we have denoted

\[ \gamma_\sigma = 3 \Delta^{-2} \partial_t (\partial_k \partial^k \delta \sigma_1 \partial^i \delta \sigma_1) + \frac{1}{2} \Delta^{-2} \partial_t (\partial_k \partial^k \delta \sigma_1 \partial^i \delta \sigma_1) . \tag{15} \]

The last step is due to time derivatives within the time integral (\( \eta_\sigma \) and \( H \) are assumed to be constants).

Eq. (13) is used to get rid of the first term in Eq. (11), \[ \int 6H \Delta^{-1} \partial_t (\delta \varphi_1^i \partial^j \delta \sigma_1) dt \], which would give too large a contribution to the estimate. Note that the order of magnitude estimate does not take into account possible cancellations and, therefore, provides only an upper limit. However, the cancellations can be treated explicitly, as is done here.

For estimation purposes we also adopt the potential used in 3, \( V = V_0(1 + \frac{1}{2} \eta_\sigma^2 + \frac{1}{2} \eta_\sigma^2) \). The slow roll parameters \( \eta_\sigma \) and \( \eta_\sigma^* \) are constants, and we also set \( H \) to be constant. The time evolutions of \( \delta \sigma_1, \delta \varphi_1, \varphi_0, \) and \( \epsilon \) are given by Eqs. (10) and (11).

Since the order of magnitude estimate anyway gives an upper limit, and since \( \epsilon \leq \epsilon_i \) for any time \( t \geq t_i \), we replace \( \epsilon \) with \( \epsilon_i \) except in the factor \( \frac{1}{2} \). We again neglect cancelling orders of spatial derivative operators, and put \( \delta \sigma_1 = -\eta_\sigma H \delta \sigma_1 \). The estimate for the second order curvature perturbation, Eq. (11), thus becomes

\[ R^{(2)}_\sigma \sim \frac{1}{\epsilon H M_P^2} \left\{ \int^t \left[ \eta_\sigma^2 H^2 |\delta \sigma_1|^2 + \epsilon_i \eta_\sigma H |\delta \sigma_1|^2 \right] dt + \eta_\sigma H |\delta \sigma_1|^2 + \epsilon_i H |\delta \sigma_1|^2 + \epsilon H |\delta \sigma_1|^2 \right\} \]

\[ \sim \frac{1}{\epsilon H M_P^2} \left\{ O(\epsilon_i, \eta_\sigma) \int^t \eta_\sigma H^2 |\delta \sigma_1|^2 dt + O(\epsilon_i, \eta_\sigma, \eta_\sigma) H |\delta \sigma_1|^2 \right\} . \tag{16} \]

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3 We use the indefinite integral here, instead of definite one with initial conditions, since the initial second order curvature, or at least initial non-Gaussianity, is supposedly small enough to be safely neglected 3.

4 Because of this, both terms in \( \gamma_\sigma \) are effectively the same.
We have now two terms to evaluate, namely

$$\frac{1}{\epsilon HM_\sigma^2} H |\delta \sigma_1|^2 = \frac{|\delta \sigma_1(t_\epsilon)|^2}{\epsilon_\sigma M_\sigma^2} e^{2\Delta N(\eta-\eta_\sigma)}$$

$$= \left| \frac{H \delta \sigma_1(t_\epsilon)}{\phi(t_\epsilon)} \right|^2 e^{2\Delta N(\eta-\eta_\sigma)}$$

$$= e^{2\Delta N(\eta-\eta_\sigma)} |R^{(1)}|^2 , \quad (17)$$

and

$$\frac{1}{\epsilon HM_\sigma^2} \int H^2 |\delta \sigma_1|^2 dt$$

$$= \frac{|\delta \sigma_1(t_\epsilon)|^2}{\epsilon M_\sigma^2} \int e^{-2\eta N} dN$$

$$\sim \frac{|\delta \sigma_1(t_\epsilon)|^2}{\epsilon M_\sigma^2} e^{-2\eta_\sigma \Delta N} = e^{2\Delta N(\eta-\eta_\sigma)} |R^{(1)}|^2 . \quad (18)$$

Therefore, our final estimate reads

$$R^{(2)}_\sigma \sim O(\epsilon, \eta, \eta_\sigma) e^{2\Delta N(\eta-\eta_\sigma)} |R^{(1)}|^2 . \quad (19)$$

Comparing to the estimate in Eq. (9) one sees that the two results are of the same form and of the same order. Instead of $O(\epsilon, \eta, \eta_\sigma)$ Eq. (9) only has a factor $O(\eta_\sigma)$. Here we have, however, provided only an order of magnitude estimate and there still is a possibility for cancellations which may have been overlooked. It may be worth pointing out that if the nonlocal terms are discarded in Eq. (12) and only the first term is estimated, we obtain an $O(\eta_\sigma)$ coefficient only.

**Discussion.** The maximum number of e-folds for the observable scales is $\sim 60$ and observational limits for the spectral index of the curvature perturbation require $|\eta| \lesssim 0.01$. It is therefore unlikely that the exponential factor $e^{2\Delta N(\eta-\eta_\sigma)}$ would provide any significant enhancement, and the overall factor of the order slow roll parameters gives the magnitude of the result.

The quantity measured in CMB experiments is the nonlinearity parameter $f_{NL}$, which is related to $R^{(2)}$, but according to Eq. (9) it is highly suppressed in this scenario. The projected sensitivity (using first order perturbation theory) for an ideal experiment is no better than $f_{NL} \sim 1$ even including polarisation. It was later realized by Vernizzi [8] that the the second order curvature perturbation defined in [2] has an artificial time evolution $\dot{R}^{(2)} \sim (2\epsilon - \eta)R^{(1)}$. However, even taking this small effect into account, it seems safe to say that the non-Gaussianity produced in the hybrid scenario by the $\sigma$ field seems to be too small to be observed unless some key aspects of the scenario are changed.

There are still conceptual and practical problems with computing non-Gaussianities. The second order theory is not yet well established and the connection between theoretical calculations and CMB observations is far from complete. One problem is that the usual scalar-vector-tensor decomposition of the perturbations is inherently non-local [9]. These non-localities do not appear in the first order, but in the second order equations there are terms like $\Delta^{-1}(\partial_1 g \partial_0 g)$ and $\Delta^{-1}(g \Delta g)$, where $g$ represents a generic perturbation. The physical interpretation of these terms is not clear.

Lyth and Rodríguez apply the separate universe approach in $\mathbb{R}$ to compute the second order perturbations, or non-Gaussianity, in various scenarios including hybrid inflation. The formalism they use does not produce nonlocal terms involving the inverse Laplacian $\Delta^{-1}$, and they state that in “...such terms must cancel if correctly evaluated.” Needless to say, this is quite a strong claim. It is actually not certain whether the separate universe approach is completely correct when expanded to second order in perturbations. Indeed, in [10] it was stated that any nonlinear interaction introduces mode-mode couplings which undermine the separate universe picture. As an example, recent studies of non-Gaussianities in preheating [10] demonstrate that these mode-mode couplings seem to be important.

Despite all the problems and ambiguities in the different second order formalisms it is comforting that the two different approaches discussed here can now be seen to produce the same result for the second order curvature perturbation and, therefore, for the amount of non-Gaussianity in hybrid inflation.

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