Local-Duality QCD Sum Rules for Pseudoscalar-Meson Form Factors

Irina Balakireva*, Wolfgang Lucha† and Dmitri Melikhov†,**

*SINP, Moscow State University, 119991 Moscow, Russia
†Institute for High Energy Physics, Austrian Academy of Sciences, Nikolsdorfergasse 18, A-1050 Vienna, Austria
**Faculty of Physics, University of Vienna, Boltzmanngasse 5, A-1090 Vienna, Austria

Abstract. We scrutinize recent findings on the charged-pion elastic form factor and the form factors entering in neutral-meson-to-photon transition amplitudes within the framework of QCD sum rules.

Keywords: pseudoscalar meson, pion, η meson, η′ meson, hadronic properties, elastic form factor, meson–photon transition form factor, quantum chromodynamics, QCD sum rule, local-duality limit

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MOTIVATION FOR REHASHING SOME RATHER OLD STORIES

QCD sum rules relate observable hadronic properties to the parameters of QCD, the QFT responsible for the formation of the bound states, by evaluating appropriate correlators of interpolating currents on the hadron level and on the level of quarks and gluons, the QCD degrees of freedom. All nonlocal products of currents may be expressed as series of local operators by Wilson’s operator product expansion; as a consequence of this at QCD level correlators obtain perturbative and nonperturbative contributions, the latter involving the universal vacuum condensates. Borel transformations to new variables—called the Borel parameters—serve to suppress impacts of both excitations and continuum, and to remove existing subtraction terms. By rephrasing the perturbative parts of QCD-level correlators as dispersion integrals over spectral densities, our ignorance about higher hadronic states can be hidden by invoking the concept of quark–hadron duality: Beyond certain effective thresholds, all the perturbative QCD contributions are assumed to cancel those of hadron excitations and continuum. For infinitely large Borel (mass) parameters, all contributions of nonperturbative QCD vanish and one ends up with QCD sum rules in the limit of local duality (LD); these LD sum rules constitute famous tools for the analysis of form factors. We adopt this approach to revise anew [1, 2] recent dubious findings for the charged-pion elastic form factor and the form factor governing the neutral-pion–γ transition π0 → γγ∗.

DISPERSSIVE QCD SUM RULES IN LOCAL-DUALITY LIMIT [3]

The dependence of both form factors $F(Q^2)$ on the involved momentum transfer squared $Q^2 \geq 0$ may be extracted from two LD sum rules satisfied by three-current correlators, of one vector and two axialvector currents for the charged-pion’s elastic form factor $F_\pi(Q^2)$ or of one axialvector and two vector currents for the neutral-pion’s transition form factor...
$F_{\pi\gamma}(Q^2)$, involving exclusively perturbative spectral densities $\Delta(s_1, s_2, Q^2)$ and $\sigma(s, Q^2)$, respectively, as well as the (weak) decay constant $f_\pi$ of the charged pion, $f_\pi = 130$ MeV:

$$F_\pi(Q^2) = \frac{1}{f_\pi} \int_0^{s_{\text{eff}}(Q^2)} ds_1 \int_0^{s_{\text{eff}}(Q^2)} ds_2 \Delta(s_1, s_2, Q^2), \quad F_{\pi\gamma}(Q^2) = \frac{1}{f_\pi} \int_0^{s_{\text{eff}}(Q^2)} ds \sigma(s, Q^2).$$

Any nonperturbative dynamics is encoded in the effective thresholds $s_{\text{eff}}(Q^2)$ or $\bar{s}_{\text{eff}}(Q^2)$. As power series in the strong coupling $\alpha_s$, the spectral densities are known up to $O(\alpha_s)$ or two-loop accuracy [4]. Factorization theorems for hard form factors entail, as asymptotic form-factor behaviour, $Q^2 F_\pi(Q^2) \to 8\pi \alpha_s(Q^2) f_\pi^2$ and $Q^2 F_{\pi\gamma}(Q^2) \to \sqrt{2} f_\pi$ for $Q^2 \to \infty$ [5]. This feature is reproduced by the LD sum rules if the effective thresholds behave like

$$\lim_{Q^2 \to \infty} s_{\text{eff}}(Q^2) = \lim_{Q^2 \to \infty} \bar{s}_{\text{eff}}(Q^2) = 4\pi^2 f_\pi^2 \approx 0.671 \text{ GeV}^2.$$

The formulation of reliable criteria for fixing effective thresholds is highly nontrivial [6]: $s_{\text{eff}}(Q^2)$ and $\bar{s}_{\text{eff}}(Q^2)$ won’t be equal neither to these asymptotes nor to each other at finite $Q^2$ [7]. In the simplest LD model [3] they are approximated at moderate but not too small $Q^2$ by their asymptotes: $s_{\text{eff}}(Q^2) = \bar{s}_{\text{eff}}(Q^2) = 4\pi^2 f_\pi^2$. For clarification and quantification of our concerns, let’s introduce the notion of an equivalent effective threshold, defined by requiring that one’s sum rule for a form factor, equipped with this quantity as its effective threshold, reproduces the experimental data or a particular theoretical prediction exactly. The accuracy of any sum-rule approach can be estimated from quantum mechanics: there all form factors can be found exactly by numerical solution [8] of Schrödinger equations.

**CHARGED-PION ELASTIC FORM FACTOR [1]**

Although the pion belongs to the best-studied meson states, some of its properties are not sufficiently well understood. The qualitative behaviour of its elastic form factor, $F_\pi(Q^2)$, for momentum transfers squared $Q^2 \approx 5–50$ GeV$^2$ gives rise to, or triggers, a controversy between theory and experiment [9] (Fig. 1). Inspecting the present status by adopting our equivalent effective thresholds, we find that our exact effective threshold, calculated back from the available experimental data [9] approaches our (marginally more sophisticated)

![Figure 1](image-url)
parametrization [1] of the effective threshold $s_{\text{eff}}(Q^2)$, interpolating between its LD limit and its value at $Q^2 = 0$, given by normalization, at rather low $Q^2$ (Fig. 2, left). In contrast, recent theoretical analyses [10] apparently miss local duality up to $Q^2 \approx 20$ GeV$^2$ (Fig. 2, right). So we conclude that, in the region $Q^2 = 20–50$ GeV$^2$, sizable deviations of $F_\pi(Q^2)$ from the LD expectations, as predicted by some analyses [10], seem to be rather unlikely.

**FORM FACTORS FOR $\eta$, $\eta'$ MESON TRANSITIONS $\eta^{(i)} \rightarrow \gamma \gamma^{*}$ [2]**

The *flavour structure* of the isoscalar mesons $\eta$, $\eta'$ is a linear combination of $\bar{u}u$, $\bar{d}d$, and $\bar{s}s$. The non-ideal mixing of $\eta$, $\eta'$ is reflected by their transition form factors $F_i(\eta, \eta')\gamma(Q^2)$ receiving non-strange and strange contributions. We obtain for the transition form factors of both $\eta$ and $\eta'$ the anticipated agreement between LD and experiment [11, 12] (Fig. 3).

**FORM FACTOR FOR NEUTRAL-PION TRANSITION $\pi^0 \rightarrow \gamma \gamma^{*}$ [2]**

Interestingly, some of the measurements [11, 13, 14] of the neutral pion’s transition form factor $F_{\pi\gamma}(Q^2)$ evince a rapid growth with $Q^2$ that still awaits an explanation (Fig. 4, left). However, a recent Belle measurement [14] definitely contradicts the BABAR findings for $F_{\pi\gamma}(Q^2)$. We conclude that the BABAR results must be taken with due care; see also [15].
FIGURE 4. Form factor $F_{\pi\gamma}(Q^2)$ for the $\pi^0$ transition $\pi^0 \rightarrow \gamma \gamma^*$: the theoretically unexpected and rather surprising deviation of the BABAR data [13] from the expectations of LD (left panel) causes our equivalent effective threshold $s_{\text{eff}}(Q^2)$ to rise linearly without any sign of caring about the LD prediction (right panel).

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