Relativistic corrections to the Sellmeier equation allow derivation of Demjanov’s formula

Peter C. Morris

Transputer Systems, PO Box 544, Marden, Australia 5070

Abstract

A recent paper by V.V. Demjanov in Physics Letters A reported a formula that relates the magnitude of Michelson interferometer fringe shifts to refractive index and absolute velocity. We show that relativistic corrections to the Sellmeier equation allow an alternative derivation of the formula.

*Electronic address: ptr.mrrs@optusnet.com.au
In Sellmeier dispersion theory, a material is treated as a collection of atoms whose negative electron clouds are displaced from the positive nucleus by the oscillating electric fields of the light beam. The oscillating dipoles resonate at a specific frequency (or wavelength), so the dielectric response is modeled as one or more Lorentz oscillators. For $m$ oscillators the Sellmeier equation may be written.

$$n^2 = 1 + \sum_{i=1}^{m} \frac{A_i \lambda^2}{\lambda^2 - \lambda_i^2}$$  \hspace{1cm} (1)

Here $\lambda$ is the wavelength of the incident light in vacuum. But if the vacuum is physical and our frame of reference undergoes length contraction due to motion relative to that vacuum, then $\lambda$ for that direction will be proportionally shorter than in our frame of reference. To correct for that effect, $\lambda$ in the above formula might need to replaced with $\lambda/\gamma$, where $\gamma$ is the Lorentz gamma factor.

This possibility is explored below. To simplify calculations we will assume that one oscillator dominates the contribution to $n$. We can then use the following one term form of the Sellmeier equation,

$$n^2 = 1 + \frac{B \lambda^2}{\lambda^2 - C}$$  \hspace{1cm} (2)

where, if oscillator $j$ is the dominant one,

$$C = \lambda_j^2$$

and

$$B = B_j = \sum_{i=1}^{m} \frac{\lambda_i^2 - \lambda_j^2}{\lambda^2 - \lambda_i^2} A_i$$  \hspace{1cm} (3)

accounts for contributions to $n$ from all the $A_i$, at the specific wavelength of $\lambda$.

We will then find that replacing $\lambda$ by $\lambda/\gamma$ and making one further change, allows derivation of Demjanov’s formula.
II. DERIVATION

A. Replacing $\lambda$ with $\lambda/\gamma$

Let a Michelson interferometer containing an optical medium be moving with absolute velocity $v$ in the direction of its parallel arm.

We hypothesize that the refractive index for a light path through the medium is given by the following formula, derived from (2) above,

$$
n = \sqrt{1 + \frac{B (\lambda/\gamma_p)^2}{(\lambda/\gamma_p)^2 - C}}
$$

where

- $\gamma_p = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ and,

- $v_p$ is the component of absolute velocity that is parallel to the light path

Solving for $\lambda$ gives the inverse relation,

$$
\lambda = \sqrt{\frac{C\gamma_p}{1 - \frac{B}{n^2 - 1}}}
$$

Let the refractive index for the orthogonal arm for light of wavelength $\lambda_0$ be $n$.

Then since $v_p$ for the orthogonal arm is zero, equation (5) gives,

$$
\lambda_0 = \sqrt{\frac{C}{1 - \frac{B}{n^2 - 1}}}
$$

Then using equation (4), the refractive index for the parallel arm, moving with absolute velocity $v_p$ must be,

$$
n_p = \sqrt{1 + \frac{B\lambda_0^2}{\lambda_0^2 - C\gamma_p^2}}
$$
\[
\sqrt{1 + \frac{B}{1 - \frac{v^2}{c^2} \gamma_p^2}}
\]  \tag{7}

Substituting \(v_p = v\) and also for \(\lambda_0\) and \(\gamma_p\) gives,

\[
n_p = \sqrt{1 + \frac{B \left(1 - \frac{v^2}{c^2}\right)}{B \left(\frac{1}{n^2} - \frac{v^2}{c^2}\right)}}
\]  \tag{8}

For \(v \ll c\) Taylor expansion then gives,

\[
n_p = n - \frac{(n^2 - 1)(B + 1 - n^2)}{2Bn} \frac{v^2}{c^2} + O(v^4)
\]  \tag{9}

As \(n\) and \(n_p\) are effective refractive indexes in the frame of the interferometer, we can now calculate travel times for return light paths as follows.

For the orthogonal arm,

\[
T_o = \frac{2L}{c/n} = \frac{2Ln}{c}
\]

For the parallel arm,

\[
T_p = \frac{2L}{c/n_p} = \frac{2Ln_p}{c}
\]

giving a travel time difference of

\[
\Delta T = T_o - T_p = \frac{2L}{c} (n - n_p)
\]

\[
= \frac{L}{c} \left(\frac{(n^2 - 1)(B + 1 - n^2)}{Bn} \frac{v^2}{c^2}\right) + O(v^4)
\]  \tag{10}

which using \(\epsilon = n^2\) and \(\Delta \epsilon = n^2 - 1\) can be re-expressed as,

\[
\Delta T = \frac{v^2}{c^2} \frac{L}{c\sqrt{\epsilon}} \frac{\Delta \epsilon (B - \Delta \epsilon)}{B}
\]  \tag{11}

When \(B \approx 1\) this approximates Demjanov’s formula (9) in \([1]\), which is,

\[
\Delta T = \frac{v^2}{c^2} \frac{L}{c\sqrt{\epsilon}} \Delta \epsilon (1 - \Delta \epsilon)
\]  \tag{12}
B. Derivation of a closer variant

The above steps obtained a variant of Demjanov’s formula. However substituting the Sellmeier coefficients for water provided in Table 1 of [5] into equations (1) and (3), showed that the value of $B$ would be less than the value of $n^2 - 1 = \Delta \varepsilon$ for wavelengths of light [9] used by Demjanov with light paths through water. So formula (11) predicts negative values of $\Delta T$ for water, while Demjanov's formula predicts positive values. The experimentally determined values shown in Figure 1 of [1] are positive, so for the case of water, formula (11) is falsified.

This prompted an effort to obtain a closer variant and version 5 of this paper describes the obtaining of,

$$\Delta T = \frac{v^2}{c^2} \frac{L}{c\sqrt{\varepsilon}} \frac{\Delta \varepsilon(1 - \Delta \varepsilon)}{B}$$  \hspace{1cm} (13)

However, even this is not close enough because for gases [7], B is orders of magnitude smaller than 1, so predictions for $\Delta T$ obtained using (13) would be much larger than observed values [1, 2, 8].

So another attempt was made, but as the next section shows, this resulted in derivation of Demjanov’s formula itself.

C. Derivation of Demjanov’s formula

This section reports how Demjanov’s formula was obtained by considering hypothetical effects of absolute motion on the orthogonal arm, in addition to those assumed for the parallel arm.

For non-polarized light, the dipoles within the material would oscillate in a plane orthogonal to the light beam and the direction of absolute motion would also lie in that plane. The amplitude and frequency of oscillations parallel with the direction of absolute motion might be affected and those effects might distribute to oscillations in other directions within the plane as well.
Lacking knowledge of such effects, a simple approach was to experiment with applying different kinds of correction until a satisfactory formula for $\Delta T$ was obtained. Eventually the following was found,

$$n = \sqrt{1 + \frac{Bg_o^2 \lambda^2}{\lambda^2 - C \gamma_o^2 g_o^2}}$$  \hspace{1cm} (14)

where

- $g_o = 1 + (1 - B) (\gamma_o - 1)$ is hypothesized to account for effects of orthogonal absolute motion,
- $\gamma_o = \frac{1}{\sqrt{1 - \frac{v_o^2}{c^2}}}$, and,
- $v_o$ is the component of absolute velocity that is orthogonal to the light path.

The above definition of $g_o$ has the following features

- When $v_o = 0$, $g_o = 1$
- For $v_o \ll c$, $g_o^2 = 1 + (1 - B) \frac{v_o^2}{c^2} + O(v^4)$

Since the quantity $Bg_o^2$ is related to oscillation amplitude, this is equivalent to a hypothesis that absolute motion causes a negative feedback effect when $B > 1$ and a positive feedback effect when $B < 1$.

*Continuing the derivation*

Let the refractive index for the medium at rest in the physical vacuum for light of wavelength $\lambda_0$ be $n$.

Then since $v_o$ and $v_p$ are zero, solving (14) for $\lambda$ gives,

$$\lambda_0 = \sqrt{\frac{C - \frac{B}{n^2 - 1}}{1 - \frac{B}{n^2 - 1}}}$$  \hspace{1cm} (15)
Then for the orthogonal arm, substituting $\lambda = \lambda_0$ and $\gamma_p = 1$ (since $v_p = 0$) in equation (14), gives the refractive index as,

$$n_o = \sqrt{1 + \frac{B g_o^2}{1 - \left(1 - \frac{B}{n^2-1}\right) g_o^2}}$$

For $v_o = v \ll c$, we can substitute $g_o^2 \approx 1 + (1 - B) \frac{v^2}{c^2}$ which gives,

$$n_o = \sqrt{1 + \frac{B \left(1 + (1 - B) \frac{v^2}{c^2}\right)}{1 - \left(1 - \frac{B}{n^2-1}\right) \left(1 + (1 - B) \frac{v^2}{c^2}\right)}}$$

and Taylor expansion then gives,

$$n_o = n - \frac{(n^2 - 1)^2(B-1)}{2 B n} \frac{v^2}{c^2} + O(v^4) \quad (16)$$

For the parallel arm using $v_o = 0$ and $v_p = v$ we once again obtain equation (9) viz.,

$$n_p = n - \frac{(n^2 - 1)(B + 1 - n^2)}{2 B n} \frac{v^2}{c^2} + O(v^4) \quad (17)$$

Using the same logic as before,

$$T_o = \frac{2L}{c/n_o} = \frac{2Ln_o}{c}$$

$$T_p = \frac{2L}{c/n_p} = \frac{2Ln_p}{c}$$

giving travel time difference,

$$\Delta T = T_o - T_p = \frac{2L}{c} (n_o - n_p)$$

$$= \frac{2L}{c} \left( \left( n - \frac{(n^2 - 1)^2(B-1)}{2 B n} \frac{v^2}{c^2} \right) - \left( n - \frac{(n^2 - 1)(B + 1 - n^2)}{2 B n} \frac{v^2}{c^2} \right) \right) + O(v^4)$$

$$= \frac{L}{c} \left( \frac{(n^2 - 1)(-Bn^2 + B + n^2 - 1)}{Bn} + \frac{(n^2 - 1)(B + 1 - n^2)}{Bn} \right) \frac{v^2}{c^2}$$
\[ \Delta T = \frac{\nu^2}{c^2} \frac{L}{c\sqrt{\epsilon}} \Delta \epsilon (1 - \Delta \epsilon) \]  

III. CONCLUSION

The above has shown that relativistic corrections to the Sellmeier equation allow derivation of Demjanov’s formula.

However it should be noted that the corrections presented here are not the only possible ones. So it remains to be seen if they are valid or whether better alternatives will be found.

Acknowledgments

The author is grateful to Dr. V.P. Dmitriyev for helpful comments.

[1] V.V. Demjanov, “Physical interpretation of the fringe shift measured on Michelson interferometer in optical media”, Phys.Lett. A, 374, Issue 9, 1110-1112 (2010)

[2] V.V. Demjanov, “What and how does a Michelson interferometer measure?”, arXiv:1003.2899v1 [physics.gen-ph]

[3] V.P. Dmitriyev, “Absolute motion determined from Michelson-type experiments in optical media”, arXiv:1002.1619v3 [physics.gen-ph]

[4] W. Sellmeier, Ann. Phys. Chem. 143, 271 (1871)

An overview of dispersion is available in, http://www2.ph.ed.ac.uk/~wjh/teaching/optics/documents/dispersion.pdf

[5] Y. Coello et al., “Group-velocity dispersion measurements of water, seawater, and ocular components using multiphoton intrapulse interference phase scan”, Applied Optics Vol. 46, No. 35 (2007)
[6] I.H. Malitson, “Interspecimen comparison of the refractive index of fused silica”, Journal of the Optical Society of America, 55, 1205-1209, 1965.

[7] U. Griesmann and J. Burnett, “Refractivity of nitrogen gas in the vacuum ultraviolet”, Opt. Lett. 24, 1699-1701 (1999)

[8] D.C. Miller, “The Ether-Drift Experiment and the Determination of the Absolute Motion of the Earth”, Rev. Mod. Phys. 5, 203-242(1933)

[9] Eg. For light of wavelength 900 nm through water, $B = 0.755745$ while $\Delta \epsilon = 0.760544$