A study is made of the scattering of two large composite projectiles, such as heavy ions, which are initially prepared in a pure quantum state. It is shown that the quantum field theoretic evolution equation for this system, under certain conditions, goes over in form to the master equation of classical statistical mechanics. Thus, the statistical mechanical description of heavy ion collision is viewed as an implied outcome of the Correspondence Principle, which states that in the limit of large quantum number, quantum dynamics goes over to classical dynamics. This hypothesis is explored within the master equation transcription with particular focus on the quark-gluon formation scenario.

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I. INTRODUCTION

In the study of heavy ion collisions, a fundamental question has been brought to the fore about the substructure of matter beyond the observed hadronic resonances. Originally, Hagedorn showed that in hadron collisions, the hypothesis of an intermediate gas like state composed of hadronic resonances, the hadron gas, could describe the $p_{\perp}$-distribution of produced secondaries \[1\]. The advent of QCD and its success through the parton model, eventually brought forth the conjecture that fundamental quarks and gluons may be capable of existing in a plasma phase at higher temperatures and densities to the hadron gas \[2\, 3\, 4\]. To test this idea, collisions between heavy nuclei have been a central source of experimental input \[5\], with at present the Relativistic Heavy Ion Collider (RHIC) operating at 200 GeV/nucleon \[6\] and in the near future A Large Ion Collider Experiment (ALICE) at LHC to operate at 5.5 TeV/nucleon \[7\]. So far experimental data seems to rule out the hadron gas phase in very high energy heavy nuclei collisions, and the data is also inconclusive about the quark-gluon plasma phase \[8\, 9\]. Thus all parts of the theoretical picture have not fit as yet, but there is suggestive evidence of a richer particle-like substructure beyond the conventional hadron gas. There are several differing ideas on the properties of this intermediate state. However there is a unifying hypothesis, that heavy ion collision can produce large transient states, which are comprised of constituents intermediate to the final asymptotic state of hadrons, and the evolution of such states can be described within a non-equilibrium statistical mechanical framework. Moreover the hypothesis is applied to individual scattering events and is not simply a heuristic model for describing an ensemble average of events.

This paper develops the large transient state hypothesis of heavy ion collision as a specific concept, independent of the particulars of the constituents, such as quark-gluon plasma or etc... The essential step to understand is how a pure quantum field theory process can be interpreted within a statistical mechanical description. For this we identify a formal link between conventional quantum field theoretic scattering and nonequilibrium statistical mechanics, by determining the conditions necessary to transcribe pure quantum evolution of a scattering process into the fundamental evolution equation of nonequilibrium statistical mechanics, the master equation. This transcription has been understood for a long time for general quantum mechanical systems \[8\, 9\, 10\]. We will recast these ideas for scattering processes. There is one previous effort we are aware of in \[11\] that has addressed the similar problem as in this paper and we will attempt to place that work in context with our own as we proceed.

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Two clarifications are needed about the large transient state hypothesis. First the treatment in this paper does not pertain to the properties of statistical density matrices. Our considerations are primarily focused on pure quantum states. We are asking in what way they can be described in thermodynamic and statistical mechanical terms. This distinction is important in the heavy ion problem since it would be an unnatural expectation that asymptotic final state properties, such as enhanced particle content of certain species, was a result of the statistical uncertainties in ones sample of initial states. As a second clarification, our considerations are not related to issues in quantum measurement theory. All we are saying is that in the limit of large quantum number, a particular equation with a classical statistical mechanical interpretation will, to a good approximation, reproduce the evolution of the system, which in rigor is given by the quantum equations of motion.

A yet more primary aim of this paper is to examine the purpose and formal interpretation of transients states in a scattering process. The issue here is that within the standard formalism of scattering theory [12], the treatment for heavy ions or for example electrons, is fundamentally not much different. In both cases the problem so posed is given an initial states, $i$, on the in-state manifold, $|i,\text{in}\rangle$, what is its projection on a final state, $f$, on the out-state manifold $|f,\text{out}\rangle$. This is a time independent formulation of scattering. The solution of the problem requires determining the change of basis matrix between the $|\text{in}\rangle$ and $|\text{out}\rangle$ basis, thus the S-matrix,

$$S_{fi} \equiv \langle f,\text{out}|i,\text{in}\rangle. \quad (1)$$

The S-matrix contains the maximum information that can be obtained from any high energy scattering experiment done to date, since all such experiments only measure the asymptotic state before and after the collision.

The standard procedure for computing the S-matrix is through quantum field theory where two equivalent approaches are available, the time dependent and time independent formalisms. For scattering between simple systems such as electrons or other fundamental particles, the time independent formalism typically is used. For the heavy ion problem, due to the complexity of the problem the S-matrix language is formally never used, and the models used to describe the process rely on a time evolution picture. The underpinnings of such models within formal scattering theory are not well understood. The other purpose of this paper is to explore this direction.

For addressing both problems outlined above, a central framework is developed for large transient states in heavy ion collision relying on a three step procedure of projection, contraction and evolution. In the projection step, formal criteria are specified for selecting events that emerge from large transient states, since not all final states in a heavy ion collision necessarily originate from such intermediate states. Associated with this step is isolating such states within the scattering theory formalism, so that they can be studied independent of other processes occurring in the complete scattering event. In the contraction step, the description of the transient state is addressed. Complete theoretical knowledge about the transient state is unnecessary, since the resolution to which this state will be measured in the experiment is grossly imperfect. Specifically, in scattering experiments, the transient state is indirectly measured through the particle spectrum of a large multiparticle final state. The measurement of only certain gross properties of such final states are of interest, and so the transients states from which they emerge also need only be described up to some limited accuracy. The contraction step is meant to temper the theoretical description to a level adequate for the experimental measurement. The combination of the projection and contraction steps yield a well prescribed identification and description of the transient state. Finally the evolution step addresses the dynamical time development of the transient state that has been modelled by the previous two steps.

The paper is organized as follows. In Sect. II the time dependent and independent scattering formalisms of quantum field theory are reviewed. A general definition of transient states is given in Subsect. III, and the time dependent formalism is then reexpressed in a form that is convenient for treating transient states. In Sect. III the idea of contracted description of dynamics is developed and then applied to the heavy ion collision problem. In Sect. IV the large transient state hypothesis of heavy ion collision is presented in the context of the ideas developed in the preceding sections. In Sect. V a master equation is presented that describes the evolution of two colliding heavy ions; the key significance of this result, based on the ideas of contracted description from earlier sections is, this master equation is describing the evolution of a pure quantum state. In Sect. VI the phenomenology of heavy ion collision is reviewed and then related to our formal construction from the previous sections. Finally Sect. VII is the Conclusion. There are also two Appendices with some supporting material for constructing the central result of this paper, the master equation.
II. FORMAL SCATTERING THEORY INVOLVING LARGE TRANSIENT STATES

As discussed in the Introduction, the formation of large transient states in collision events may allow making predictions about aspects of the asymptotic final states to which they evolve. In this Section a formalism is developed for treating large transient states in scattering. Such a formalism must encompass three details. First, since transient states inherently are spacetime concepts, they must be described within the time-dependent formalism of scattering. Second, the outcome of any scattering process in principle can be described in terms of the time independent formalism of scattering, since information is needed only about the asymptotic initial and final states and the S-matrix supplies the relevant transition amplitude. As such there must exist an interpretation of large transient state formation during scattering, in terms of the time-independent formalism. Third, only for some subclass of all scattering events, the transient state description may be relevant and so a projection operator is needed to separate out such events.

To develop the above three points, the outline of this section is as follows. The time-dependent scattering formalism first will be reviewed within the context of the collision of two incoming heavy ion wavepackets. Then this approach will be related to the time-independent scattering formalism. With these basic points in place, focus will turn to large transient states. A projection operator will be defined to formally separate final states that evolved from large intermediate transient states. With this, the transient state formally now can be isolated. At this point, it is a simple matter to apply the time evolution operator to evolve the transient state. Furthermore, the projection step along with invoking basic assumptions about large transient states allows deducing some properties of the S-matrix elements for events with large transient states.

A. Time Dependent Scattering Theory

Let

\[ |\phi_a^k \rangle \equiv c_a^\dagger (k)|0\rangle \]

(2)
denote the single particle eigenstate of the full Hamiltonian with three momentum \(k\), for species \(a\), and with energy \(E_a^k\), where \(H\) is the full Hamiltonian of the theory. The left hand side of Eq. (2) defines the operator \(c_a^\dagger (k)\) on the right hand side and \(|0\rangle\) is the true vacuum of the theory. Let the initial state of the two incoming relativistic heavy ions \((a = h_j, j = 1, 2)\) at \(t_i \ll 0\) be two wavepackets centered at coordinates \(x_1 = (0, 0, z_1), x_2 = (0, 0, -z_2)\) with momenta \(k_1 = (0, 0, k_1), k_2 = (0, 0, k_2)\) respectively. The wavepackets are initially far apart, so that \(|x_1 - x_2| \gg \Delta x_1 + \Delta x_2\), where \(\Delta x_j\) is the spread in wavepacket \(j\). The wavepackets are prepared such that their classical trajectories collide at \(t = 0\), so that \(z_1 = z_2 \equiv z_i\) and \(c_{ti} \equiv -z_i\). This initial state evolves in the Schrodinger representation as

\[ |I(t)\rangle = \exp[-iH(t-t_i)]|I(t_i)\rangle. \]

(3)

For \(|I(t_i)\rangle\) to represent a incoming state of two particles means that for \(t_i \to -\infty\)

\[ \langle I(t_i)|O(x)|I(t_i)\rangle \to -\infty \langle I_1(t_i)|O(x)|I_1(t_i)\rangle + \langle I_2(t_i)|O(x)|I_2(t_i)\rangle, \]

(4)

where \(O(x)\) is a local operator representing any observable and

\[ |I_j(t_i)\rangle = \int d^3k f^{x_j-k_j}(k)c^\dagger_{h_j}(k)|0\rangle e^{-ik_j t_i}, \]

(5)

where \(f^{x_j-k_j}(k)\) is a smearing function, which in coordinate space is peaked about \(x_j\) and in momentum space is peaked about \(k_j\). Thus for appropriate choice of smearing functions that localize the two incoming particles far apart, as specified above, the initial state is

\[ |\gamma_{h_1(x_1,k_1),h_2(x_2,k_2)}(t_i)\rangle = \int d^3k \int d^3k f^{x_1-k_1}(k)f^{x_2,k_2}(k')c^\dagger_{h_1}(k)c^\dagger_{h_2}(k')|0\rangle e^{-i(E_{h_1}(k) + E_{h_2}(k'))|t_i}. \]

(6)

1 For notational convenience, we will assume that both incoming particles and all outgoing particles are distinguishable. This could be extended to treat the statistics of indistinguishable particles.
The superscript SE denotes the basis of product “single-particle eigenstates”. In an interacting theory, such product states are not eigenstates. However if the theory describes scattering, then such states must be approximate eigenstates when all the particles are far apart. This statement will be further clarified in the next subsection.

Let the final state after the collision at \( t_f \gg 0 \) be some \( n \)-particle state of species \( a_1 \ldots a_n \) with outgoing wavepackets that are widely spaced. The details for representing the final state are the same as the initial state, and we obtain

\[
\langle F_{\{x,q\}}^{\text{SE}}(t_f) | = \prod_{i=1}^{n} d^3q_i E_{a_1}(q_1) \ldots E_{a_n}(q_n) f^{x_1,q_1}(q'_1) \ldots f^{x_n,q_n}(q'_n) e^{i \sum_{j=1}^{n} E_{a_j}(q'_j)t_f}, \quad (7)
\]

with \( \{ q \} \equiv \{ q_1 \ldots q_n \} \). The amplitude \( A_{fi} \) to go from the initial state at time \( t_i \) to the final state Eq. (7) at time \( t_f \) is

\[
A_{fi} = \langle F_{\{x,q\}}^{\text{SE}}(t_f) \langle \exp[-iH(t_f - t_i)] \rangle f_{\{x_1,k_1\},\ldots,\{x_2,k_2\}}^{\text{SE}}(t_i) \rangle_{t_i \rightarrow -\infty, t_f \rightarrow \infty}. \quad (8)
\]

### B. Time Independent Scattering Theory

The time independent formulation of scattering is obtained from the time dependent one by first reexpressing \( |I^{\text{SE}}(t_i)\rangle \) and \( \langle F^{\text{SE}}(t_f) \rangle \) in an eigenstate basis. There are two eigenstate bases that are particularly convenient, the in and out basis. They both satisfy the Schrödinger equation

\[
H_{\psi}^{\text{in(out)}}(\{k\}) = E_{\psi}(\{k\}) \psi^{\text{in(out)}}(\{k\}) \quad \text{(9)}
\]

but have different boundary conditions. The boundary conditions for the in (out) states are such that they describe incoming (outgoing) particles at large negative (positive) time. The incoming states are defined as follows. Consider the case of two incoming heavy ions \( h_1, h_2 \), with \( \psi^{\text{in},h_1,h_2}_{\{k_1,k_2\}} \) their eigenstate in the in-basis. Let

\[
|I^{\text{IE}}(t_i)\rangle = \int d^3k d^3k' f^{x_1,k_1}(k) f^{x_2,k_2}(k') \psi^{\text{in},h_1,h_2}_{\{k_1,k_2\}} e^{-i(E_{h_1}(k)+E_{h_2}(k'))t_i}, \quad (10)
\]

where the superscript IE denotes that the expansion basis is the “in-eigenstates” and the smearing functions \( f^{x,k}(k') \) are the same as in eq. (6). The statement that \( \{ \psi^{\text{in}} \} \) are in-eigenstates means, in addition to Eq. (9),

\[
\langle \xi | f^{\text{IE}}_{\{k_1,k_2\}}(t_i) \rangle \rightarrow_{t_i \rightarrow -\infty} \langle \xi | f^{\text{SE}}_{\{k_1,k_2\}}(t_i) \rangle, \quad (11)
\]

where \( |\xi\rangle \) is any arbitrary fixed state in the Hilbert space. Similarly for out-eigenstates, let

\[
\langle F^{\text{OE}}_{\{q\}}(t_f) | = \prod_{i=1}^{n} d^3q'_i f^{x_1,q_1}(q'_1) \ldots f^{x_n,q_n}(q'_n) \psi^{\text{out},a_1,a_2,\ldots,a_n}_{\{q_1'\ldots,q_n'\}} e^{i \sum_{j=1}^{n} E_{a_j}(q'_j)t_f}, \quad (12)
\]

where OE denotes the basis of “out-eigenstates”. In analogy with Eq. (11), in this case the condition is

\[
\langle F^{\text{OE}}_{\{q\}}(t_f) | \xi \rangle_{t_f \rightarrow \infty} \rightarrow \langle F^{\text{SE}}_{\{q\}}(t_f) | \xi \rangle. \quad (13)
\]

The scattering amplitude Eq. (8) in terms of the in-out basis becomes

\[
A_{fi} = \int d^3q'_1 f^{x_1,q_1}(q'_1) \int d^3k'_1 d^3k'_2 f^{x_1,k_1}(k'_1) f^{x_2,k_2}(k'_2) S_{\{q,a\},\{k,h\}}, \quad (14)
\]

where

\[
S_{\{q,a\},\{k,h\}} = \langle \psi^{\text{out}}_{\{q\}} | \psi^{\text{in}}_{\{h\}} \rangle \langle \{ k \} | (15)
\]

is the S-matrix. This completes the demonstration of relating the time dependent and independent scattering formalisms in quantum field theory.
C. Transient State Description of Scattering

In this Subsection the time-dependent scattering formalism is reexpressed to allow a convenient description of states that are intermediate to the asymptotic incoming and outgoing states, which hereafter will be referred to as transient states. The construction below is purely kinematic, thus is nonspecific to the nature of the transient states. However, to help fix ideas, this construction will be developed in the context of the heavy ion collision scenario.

1. Definition - Projection

According to quantum mechanics, measuring a system in a state $\langle F \rangle$ at time $t_F$ results in the filtering or reduction of the wavefunction $\Psi(t_F)$ at time $t_F$. Quantum dynamics implies that an appropriate filter or projection operator $P^t_F$ can be applied to $\Psi(t)$ at any time $t < t_F$ such that the projected state at $t$, $\psi_{P^t_F}(t)$, will evolve into $|F\rangle$ at $t_F$, with predicted amplitude $\langle F | \psi(t_F) \rangle$. Specifically let

$$ P^t_F \equiv e^{iH(t_F-t)}|F\rangle\langle F|e^{-iH(t_F-t)}, $$

then

$$ \langle F | e^{-iH(t_F-t)} | P^t_F \Psi(t) \rangle \equiv \langle F | e^{-iH(t_F-t)} | \Psi_{P^t_F}(t) \rangle $$

$$ = \langle F | \Psi(t_F) \rangle $$

and

$$ \langle F' | e^{-iH(t_F-t)} | P^t_F \Psi(t) \rangle = 0, $$

if $\langle F' | F \rangle = 0$.

A final state $\langle F \rangle$ at time $t_F$ will be said to have been composed of $N$-transient states at time $t < t_F$ with respect to a explicit basis $\{\xi\}$ if $\psi_{P^t_F}(t)$ factorizes into an $N$-product state of the form

$$ \Psi_{P^t_F}(t) = \prod_{j=1}^N \Psi^\xi_{T,F_j}(t). $$

The time interval $\Delta t_{P^t_F}$ over which a state $\Psi^\xi_{T,F_j}$ retains a factorized form from the rest of the state vector defines its lifetime.

Note that the decomposition of transient state Eq. (20) is dependent on the choice of basis $\{\xi\}$. Also note that by this definition, the state $\langle F \rangle$ is not restricted to the asymptotic manifold of final states. $\langle F \rangle$ may characterize an intermediate configuration, thus permitting identification of different stages in the evolution of a transient state.

Aside from the lifetime $\Delta t$, transient states are described with the standard set of operator observables, energy, momentum internal charge and so on. By large transient state, we always mean a large many-body transient state. To identify a transient state as large requires the existence of a number density operator $\hat{N}_C^\xi(x)$ that acts on some subspace $\{\xi'\}$ of the $\{\xi\}$-basis. A transient state $\Psi_T$ is large if the number density of $\xi'$-quanta is sufficiently large within a sufficiently large volume $V$,

$$ \langle \Psi_T | \hat{N}_C^\xi(x) | \Psi_T \rangle - \langle 0 | \hat{N}_C^\xi(x) | 0 \rangle > n^\text{min}_T \quad \text{for} \quad x \in V > V^\text{min}_T, $$

where $|0\rangle$ is the vacuum state. The specific values of $n^\text{min}_T$ and $V^\text{min}_T$ depend on the specific dynamical theory.

2. Description

A description of transient states requires dividing the collision process into a pre, during and post collision time period. In the pre-collision period, the two incoming wavepackets are mutually non-interacting, thus are well described by $|F^SE(t)\rangle$ up to the time $t = t^{\circ}_c < 0$. Here $t^{\circ}_c$ is defined as the time when the rms spread
of both incoming wavepackets first make contact. The post-collision period begins as \( t_c^+ > 0 \) when the rms spread of the two wavepackets would no longer overlap had the two wavepackets never interacted. If both wavepackets are symmetrical about their respective centers then \( t_c^+ = |t_c^-| \). The transient states are formed during the collision time period \( t_c^- < t < t_c^+ \) and evolve during the post-collision period as isolated localized systems i.e. blobs. The formation can be of any number of isolated transient states. Generically, in heavy ion collision three transient states are hypothesized, one in the central region (\( \sim 3 \) units of central rapidity) and two "fireballs" in the right and left fragmentation regions respectively \[13\]. During the post-collision period for each transient state \( j \), there is a hadronization time \( t_{h_j} > t_c^+ \). For \( t > t_{h_j} \) the state \( j \) is simply described in terms of the asymptotic single-particle eigenstate basis \( |F^{SE}_{j}(t > t_{h_j})\rangle \).

3. Formalism

In terms of time-dependent scattering theory, the transient state description is expressed as follows. For \( t < t_c^- \) as said above the state is \( |F^{SE}(t)\rangle \). The transient state begins, by definition, at \( t_c^- \) starting with the first stage, the formation state. During the formation period, \( t_c^- > t > t_c^- \), the state vector evolves as

\[
\psi_T(t) \equiv e^{-iH(t-t_c^-)}|F^{SE}(t_c^-)\rangle.
\]

Hard interactions occur predominately in the formation period. Anytime after \( t_c^- \) the formation state in general may separate into multiple transient states. Although no separation will be sharp, it is convenient to express approximately separate states as such. The phenomenological nature of the transient state approach reflects in the lack of a prescriptive procedure for implementing the separations. Hadron phenomenology indicates that collisions typically separate spatially into three regions, central (C) and a left (L) and right (R) fragmentation region. For a final state \( |F^{SE}| \) that evolved from an intermediate large transient state, during the formation period there should be a single transient state. At the end of the formation period, \( t_c^+ \), this transient state should spatially separate approximately into these three components. In any basis \( \{\xi\} \) in which the elementary excitations describing the transient state are local, the separated state can be expressed in the factorized form

\[
e^{-iH(t_c^+ - t_c^-)}|F^{SE}(t_c^-)\rangle = |\psi_C^{\{\xi\}}(t_c^-)\rangle |\psi_L^{\{\xi\}}(t_c^-)\rangle |\psi_R^{\{\xi\}}(t_c^-)\rangle,
\]

where the superscript \( I \) denotes the incoming asymptotic state from which this transient state evolved. QCD effects will smear this sharp separation, but an approximate separation is justified if the transient states are sufficiently large. Assuming this approximation, for \( t > t_c^+ \) the evolution of each transient state in Eq. (22) is independent

\[
|\psi_j^{\{\xi\}}(t > t_c^+)\rangle = e^{-iH(t-t_c^+)}|\psi_j^{\{\xi\}}(t_c^+)\rangle,
\]

where \( J = C, L, R \). Define

\[
A_{\xi,\psi_j^{\{\xi\}}}(t - t_c^+) = \langle \xi e^{-iH(t-t_c^+)}|\psi_j^{\{\xi\}}(t_c^+)\rangle
\]

as the probability amplitude for the transient state \( J \) at \( t_c^+ \) to be in state \( \{\xi\} \) at \( t \). At the end of the transient state period \( t_{h_j} \), each of the three spatially separated transient state will hadronize independently. The final asymptotic state can be factorized into its C, R and L regions as \( \langle F^{SE} \rangle = \langle F_L F_C F_R \rangle \), with amplitude to go into a particular final state \( \langle F^{SE}_j(t_{h_j})\rangle \),

\[
A_{\psi_j^{\{\xi\}},\psi_j^{\{\xi\}}}(t - t_c^+)_{|t-t_c^-| \rightarrow \infty} = \langle F^{SE}_j(t) e^{-iH(t-t_c^+)}|\psi_j^{\{\xi\}}(t_c^+)\rangle_{t \rightarrow \infty}.
\]

For scattering process that are well described by a transient state time-dependent approach in a basis \( \{\xi\} \), there corresponds in the time-independent approach a factorized form for the respective S-matrix elements in the same basis, \[2\]

\[
|I(t_c^-)\rangle = |H\rangle \times |J\rangle,
\]

\[2\] It is worthwhile to compare the transient state description above to the one in hard scattering \[13\]. For hard scattering, the parton model hypothesis implies a separation at \( t_c^- \) as a two component transient state
\[ S_{FI} = \prod_{J=C,L,R} A_{F_J,\psi_J(t)}. \] (28)

### III. CONTRACTED QUANTUM DYNAMICS

Once the large transient state has been isolated with the projection operator of Subsect. [11C], focus turns to the description of the state itself. Recall a motivation for considering events with large transient states is that due to their large size, their dynamics may have simplifying features and at the same time such a state could dominate many features of the final asymptotic states. A central point here is that complete detail about the large transient state is not necessary, but rather a contracted description suffices. In this Section, such a contracted description for large transient states is obtained. This will start first with a elementary discussion of contracted quantum dynamics, with specific emphasis on the implication of the large number limit.

#### A. Definition - Contraction

Almost any measurement in physics results in a contracted description of that system. This is both because not all observables of the system are measured and because of the ones that are, the measurement uncertainties exceed the intrinsic quantum mechanical uncertainties. To a given measurement, all theoretical descriptions of the system in question are equally fundamental if they can correctly predict the values of the specific measured observables within their measured uncertainties. The most fundamental theory is the one that correctly predicts all observables of any system and within the intrinsic uncertainties set by quantum mechanics. Less fundamental theories are derived consequences of this most fundamental theory in which some information has been removed. These less fundamental theories are called contracted descriptions. They form a hierarchy in which one contracted description is derived from a higher one and so on up to the most fundamental theory.

In quantum mechanics a physical process is generally analyzed by the following sequence of steps. The measurement is described through a set of operator observables \( \{ \hat{O}_j \} \), which in general are measured at various times \( t_l \), from which the experimental data emerges as expectation values \( \{ O_j(t_l) \} \) with measurement uncertainties \( \{ \Delta O_j(t_l) \} \). The objective then is to explain this experimental data from theory. In quantum mechanics, this implies determining the wavefunction \( \psi(t) \) of the system. Any wavefunction solution \( \psi \) would be considered adequate, if all \( \{ O_{th}^j(t_l) \} \) agree with \( \{ O_{exp}^j(t_l) \} \), where

\[ O_{th}^j(t_l) = \langle \psi(t_l) | \hat{O}_j | \psi(t_l) \rangle, \] (29)

and within the measurement uncertainties \( \{ \Delta O_j(t_l) \} \), so

\[ |O_{th}^j(t_l) - O_{exp}^j(t_l)| \leq \Delta O_j(t_l). \] (30)

The most fundamental description would require determining all the energy eigenstates \( \{ \Psi_n \} \) of the system from which the system wavefunction in general would be expressed as

\[ \psi(t) = \sum_n a_n \phi(E_n) \exp(-iE_n t), \] (31)

where \( |H \rangle \) is the partonic state that participates in the hard interaction and \( |J \rangle \) is the state of the two beam jets. For this case, pQCD can be used to determine the validity of the separation. It is well know from pQCD that an initial state of the form eq (27) rapidly loses its factorized form due to soft and collinear gluon exchange between \( |H \rangle \) and \( |J \rangle \), which leads to large infrared corrections. However detailed calculation from pQCD shows that these divergences cancel for the probability amplitude squared once it is summed over a specific class of final states. This expectation of pQCD has been convincingly verified by experiment in particular for inclusive hard scattering. Thus the transient state ansatz Eq. (27) is good for extracting inclusive information about hard scattering. In contrast, as will be seen, the transient state hypothesis of heavy ion collision addresses individual events.
with the expansion coefficients \( \{ a_n \} \) fixed from experimental data. If measurements at some initial time \( t_0 \) are done of a complete set of operator observables and up to minimum quantum mechanical uncertainty, then all expansion coefficients \( \{ a_n \} \) could be precisely determined. On the other hand, if the measurement is less complete, then the state can not be precisely specified. In this case the wavefunction \( \psi(t) \) only can be approximately known, which means either the expansion coefficients can be determined only up to some uncertainty \( \{ \Delta a_n \} \), or the states of the expansion basis in Eq. (31) are only approximate energy eigenstates, resolved up to measurement uncertainty \( \Delta U \), or a combination of both.

In this Section an explicit contracted description will be obtained for the case of large transient states in heavy ion collisions. For this, the notion of “similar state” will be useful. Consider a wavefunction expanded at some time \( t_i \) in an arbitrary basis \( \{ |\alpha_i\rangle \} \)

\[
\psi = \int \sum_j c_j |\alpha_j\rangle.
\]  

(32)

Two states \( |\alpha_1\rangle, \ |\alpha_2\rangle \) are defined to be similar at time \( t_i \) with respect to a specified set of observables \( \{ \hat{O} \} \) if conditions Eqs. (30) remain valid at this time \( t_i \) after the replacement \( |\alpha_1\rangle \rightarrow |\alpha_2\rangle \) or \( |\alpha_2\rangle \rightarrow |\alpha_1\rangle \) in Eq. (32). Thus the specified set of operators and measurement uncertainties partition the Hilbert space into cells of similar states at each time \( t_i \). The degree of partitioning depends on the choice of Hilbert space basis. A poor choice results in very few similar states per cell, whereas a good choice results in a large number of similar states per cell.

B. Relevance to Heavy Ion Collision

The formation of large transient states in a heavy ion collision requires a contracted description both for theoretical and experimental reasons. In regards the former, the unknown confinement mechanism relinquishes ability to compute many details about the transient state. Even without compounding this problem, the complexity of such large states only can be understood, in practice, by approximation methods. Here the first implementation of contraction arises through use of the quasiparticle concept. This concept plays a central role since a tacit assumption is that the main properties of the large transient state are controlled by a large density of nearly free particle-like states. The quark-gluon plasma picture of the large transient state is a common example of application of the quasiparticle concept. However as discussed in sections to come, a more general description underlies this.

From the perspective of experiment, a contracted description also is required, since a large transient state leads to a large multiparticle final state. Although in principle such states can be measured with precise resolution, limited only by quantum mechanical uncertainties, in practice this is out of the question. Moreover, extracting information about only the transient state from the complete final state data, intertwines experimental measurement with theory. Ultimately a middleground is sought, where information from experiment and theory are in mutual balance. The large transient hypothesis is testable since it falls within this middleground. Its underlying premise is that the large number limit dominates the dynamics, in which case theoretical methods are available to treat it and at the same time the size of the state should leave a identifiable signature in the final state data.

Thus observables measured in the contracted description minimally will include total energy \( U \) and particle number, which expressed in thermodynamic language can be termed entropy \( S \). The volume of the transient state also could be measured \[17\], from which combining with \( U \), a temperature \( T \) can be assigned. The measurements of energy and entropy also may be further resolved into bins of different momentum, particle species, etc... of the respective particles.

---

3 In this paper energy is denoted in a few different ways, depending in what context it is used. The quantum mechanical Hamiltonian is denoted as \( \hat{H} \) and the n-th energy eigenstate is denoted as \( E_n \). On the other hand, energy expectation values, such as those giving total energy or energy within some specified region etc... will be denoted as \( U \); this choice follows a convention for energy often used in thermodynamics and statistical mechanics \[13, 16\], and so carries the right suggestive meaning also for this paper.
1. Role of time-energy uncertainty in measurement

The concept of a transient state implies temporal evolution with observations of the state at successive intervals of times. However in a scattering experiment, observations only occur twice, at asymptotic times before, \( t_i \), and after, \( t_f \), the collision, so that transient state formation during a collision process is a notion that can be tested only indirectly. The putative transient state must be modelled, with the predictions that each model yields only at asymptotic large time testable with experiment. This clearly leaves considerable room for degeneracy amongst candidate models, and so speaks to a central difficulty of the transient state problem of heavy ion collision. Nevertheless, within the mathematical construct, the transient state takes on a dynamical reality, and all models that correctly predict the outcome of the collision experiment are viable dynamical scenarios during the intermediate time period.

The first step in determining models for the transient state is specifying the maximum energy resolution to which the actual state is measured, since any model need only be valid up to this resolution. If the lifetime of the transient state is \( \Delta t_{tr} \), then the uncertainty principle dictates that the minimum energy uncertainty to which the state can be determined is \( \delta U \sim 1/\Delta t_{tr} \). If a measurement of the state were made to such a refined resolution, then comparison to theory would require knowing the complete quantum mechanical wavefunction of the state. In a heavy ion collision experiment, even if in principle the final state were measured with perfect resolution, it still would not imply the same holds for the transient state. This is because the transient state cannot be measured in isolation, since it is surrounded by many other processes that comprise the entire collision event. The final state that is observed in the detectors contains a vast mixture of information about all these processes. Separating out the portions due to the transient state would introduce a measurement uncertainty in the energy \( \Delta U \gg \delta U \). The strength of the large transient state hypothesis is that for energy resolution \( \Delta U \gg \delta U \), but still \( U > \Delta U \), the large number limit dictates the properties of the transient state, so that neither perfect experimental nor theoretical knowledge about the collision process is necessary. Thus models of putative transient states must be accurate only to within the measurement uncertainty \( \Delta U \), with dynamical evolution in the model treated only in time steps of size \( \Delta t \sim h/\Delta U \).

To quantify the above inequalities, consider the evolution of states formed in the central region in a heavy ion collision. It is believed that for events in which large transient states formation is possible in the central region \([18, 19]\), the total energy of such states is \( U \approx (5 - 1000)\text{GeV} \) and \( \Delta t_{tr} \approx (10 - 1000)\text{fm}/c \). Since in a collision experiment information about the transient state must be inferred from the final state particle spectrum, the minimum measurement uncertainty of the transient state energy can be set as that of a typical minijet \( \Delta U \gtrsim 1\text{GeV} \). For these values, it implies \( \delta U \approx h/\Delta t_{tr} \approx (10^{-2} - 10^{-4})\text{GeV} \ll \Delta U \approx 1\text{GeV} \) and the evolution time step must be \( \Delta t \gtrsim 0.2\text{fm}/c \).

C. Contracted description of large transient states

This subsection formulates the contracted description of large transient states with specific reference to heavy ion collision. Consider a set of operator observables \( \{\hat{O}_j\} \) that are experimentally measured in the final state, with measurement uncertainties \( \{\Delta O_j\} \). Amongst the observables would be the total energy \( U \), entropy \( S \), perhaps also local versions of these operators in smaller volume regions of the putative transient state, and other observables. All the observables are macroscopic in that the intrinsic quantum mechanical uncertainties

\[
\delta O_j \equiv [(\psi|\hat{O}_j^2|\psi) - (\psi|\hat{O}_j|\psi)²]^{1/2}, \quad (33)
\]

are smaller than the measurement uncertainties \( \Delta O_j \),

\[
\Delta O_j \Delta O_l \gg \delta O_j \delta O_l, \quad (34)
\]

for all \( j, l \). Within our mathematical construct of the transient state, we wish to model evolution in time steps of \( \Delta t \gtrsim h/\Delta U \) with operator expectation values resolved at each time step up to the measurement uncertainties \( \{\Delta O_j\} \).
1. Slowly varying operators

Recalling the uncertainty principle relation

$$\delta U \delta O_j \gtrsim \langle \psi | [\hat{H}, \hat{O}_j] | \psi \rangle \approx \langle \psi | \hat{O}_j | \psi \rangle \rangle,$$  \hspace{1cm} (35)

Eq. (34) implies

$$\Delta U \Delta O_j \gg \langle \psi | \dot{\hat{O}}_j | \psi \rangle,$$  \hspace{1cm} (36)

where \(\psi\) is any arbitrary states. Also, the observation time interval \(\Delta t\) is dictated through the uncertainty principle relation

$$\Delta U \Delta t \approx \hbar.$$  \hspace{1cm} (37)

Thus for measurement time intervals larger than the uncertainty principle lower bound, but not in excess to Eq. (36) it implies

$$\Delta U \Delta O_j \approx \Delta O_j \hbar / \Delta t,$$  \hspace{1cm} (38)

so that from Eq. (36)

$$\Delta O_j \gg \langle \psi | \dot{\hat{O}}_j | \psi \rangle / \hbar,$$  \hspace{1cm} (39)

for any state \(\psi\). Operators satisfying Eq. (39) are called “slowly varying” in [8]. In terms of Eq. (32) if all operators used in partitioning similar states are slowly varying, then it implies this partitioning remains valid over several measurement time intervals. For a partitioning of states based on such a set of slowly varying operator observables, in Sect. V it will be shown that the state vector obeys an evolution equation that is classical in form. There are several details related to the above construction requiring comment and they will be addressed in the three subsections that follow.

2. Approximate diagonalization

Amongst the operator observables, the energy operator has a special role, since energy will be conserved. The measurement of the system’s energy at value \(U_N\) with uncertainty \(\Delta U\) separates the energy eigenstates into shells \(N\). If energy is the only measured operator then each shell \(N\) has a set of approximate energy eigenstates \(\{\psi_{Nn}\}\) such that

$$\langle \psi_{Nn} | \hat{H} | \psi_{N'n'} \rangle = [U_N + O(\Delta U)] \delta_{NN'} \delta_{nn'} + O(\ll \Delta U).$$  \hspace{1cm} (40)

Here the magnitude of the off-diagonal elements \(O(\ll \Delta U)\) are determined by the conditions Eqs. (33) and (34) which implies for any \(m, \sum_{n,n\neq m} |H_{nn}|^2 \equiv \delta U^2 \ll \Delta U^2.

Now consider another operator \(\hat{O}_1\). Due to relations Eqs. (33) and (35), up to measurement uncertainty, \(\hat{H}\) and \(\hat{O}_1\) commute and so can be simultaneously approximately diagonalized as follows. A diagonalization in general would require all the approximate eigenstates \(\{\Psi_{Nn}\}\) from all the shells \(N\), but first it will be shown that the diagonalization done here can be achieved in each shell \(N\) with only the approximate eigenvectors in that particular shell. From Eq. (35) choosing \(\psi = a\Psi_{Nn} + b\Psi_{N'm}\) where for normalization \(|a|^2 + |b|^2 = 1\), consider a particular matrix element

$$(\hat{H} \hat{O}_1 - \hat{O}_1 \hat{H})_{N'n',Nn} \approx (E_{N'n'} - E_{Nn}) \hat{O}_{1N'n',Nn} \sim \delta \hat{O}_1 \delta U.$$  \hspace{1cm} (41)

If \(N \neq N'\), so different energy shells separated by order \(k\Delta U\) with \(k > 1\), then

$$|E_{N'n'} - E_{Nn}| > k\Delta U,$$  \hspace{1cm} (42)

which when combined with Eq. (11), implies

$$\frac{\hat{O}_{1N'n',Nn}}{\delta \hat{O}_1} < \frac{\delta \hat{O}_1 \delta U}{k\Delta U \delta \hat{O}_1} < \frac{\delta \hat{O}_1 \delta U}{\Delta U \delta \hat{O}_1} \ll 1.$$  \hspace{1cm} (43)
This shows that all off-diagonal matrix elements of \( \hat{O}_1 \) outside shell \( N \) are tiny, with those in increasingly distant energy shells getting increasingly smaller. So to a good approximation, all states outside the shell \( N \) can be ignored and amongst the states, \( \{ \psi_{N,n} \} \), in a given shell \( N \), a new orthonormal basis \( \{ \phi_{N_1,n}^{(N)} \} \) can be obtained by a unitary transformation

\[
\phi_{N_1,n}^{(N)} = \sum_{p \in N} \Psi_{N,p}(\Psi_{N,p}|\phi_{N_1,n}^{(N)}).
\]  

In the basis \( \{ \phi_{N_1,n}^{(N)} \} \) and in a given cell \( NN_1 \), \( \hat{H} \) is approximately diagonal with eigenvalue \( U_N \) and \( \hat{O}_1 \) is approximately diagonal with eigenvalue \( O_{N_1}^{N} \).

In particular, for the new basis \( \{ \phi_{N_1,n}^{(N)} \} \), for \( \hat{H} \) it still holds

\[
\langle \phi_{N_1,n}^{(N)'}|\hat{H}|\phi_{N_1,n}^{(N)} \rangle = \langle H_N + O(\Delta U)\rangle \delta_{N,N'} \delta_{N_1,N_1'} \delta_{n,n'} + O(\ll \Delta U),
\]  

and now for \( \hat{O}_1 \)

\[
\langle \phi_{N_1,n}^{(N)}|\hat{O}_1|\phi_{N_1,n}^{(N)'} \rangle = \langle O_1^N + O(\Delta O_1) \rangle \delta_{N,N'} \delta_{N_1,N_1'} \delta_{n,n'} + O(\ll \Delta O_1),
\]  

where the magnitude of the off-diagonal elements of \( \hat{O}_1 \) are determined as for the case of energy from Eqs. (33) and (34) to be for any \( m \),

\[
\sqrt{\sum_{n,n\neq m} O_{mn}^2} \ll \Delta O_1.
\]

In this new basis Eq. (44), with respect to the operators \( \hat{H} \) and \( \hat{O}_1 \), all states \( \phi_{N_1,n}^{(N)} \) in a given cell \( NN_1 \) are similar. Finally, note that in the \( \{ \phi_{N_1,n}^{(N)} \} \) basis, \( \hat{H} \) also has an ordering on its off-diagonal elements from conditions analogous to Eqs. (41) - (43). In particular for sufficiently distant eigenstates (i.e. sufficiently large \( k > 1 \))

\[
|O_{1m} - O_{1n}| \approx k \Delta O_1,
\]

from Eq. (45)

\[
(HO_1 - O_1H)_{mn} \approx (O_{1m} - O_{1n})H_{mn} \sim \delta O_1 \delta U,
\]

which implies

\[
\frac{H_{mn}}{\Delta U} < \frac{\delta O_1 \delta U}{k \Delta O_1 \Delta U} < \frac{\delta O_1 \delta U}{\Delta O_1 \Delta U} \ll 1.
\]

Another observable \( \hat{O}_2 \) can now be considered. Due to Eq. (34) \( \hat{H} \), \( \hat{O}_1 \), and \( \hat{O}_2 \) can be approximately mutually diagonalized. In particular the energy relation Eq. (45) applies equally to any operator \( \hat{O}_j \) with the same consequence as found in Eqs. (41) - (43) for \( \hat{O}_1 \). Next consider matrix elements of \( \hat{O}_2 \) between two different \( O_1 \)-cells \( NN_1 \) and \( NN_1' \). The same treatment applies here as leading to Eq. (44), to give

\[
O_{2N_1;N_1'N_1N_1'} \ll \Delta O_2 \Delta O_1 \ll 1, \quad \text{when } N_1 \neq N_1'.
\]

Thus the basis states \( \{ \phi_{N_1,n}^{(N)} \} \) can be transformed to a new orthonormal basis in which \( \hat{O}_2 \) is approximately diagonal in cells \( N_2 \) between which expectation values are separated by at least \( O(\Delta O_2) \),

\[
\phi_{N_1,n}^{(N)} = \sum_{p \in N N_1} \phi_{N_1,p}^{(N)}|\phi_{N_1,N_2m}^{(N)}\rangle,
\]

such that

\[
\langle \phi_{N_1,N_2m}^{(N)}|\hat{O}_2|\phi_{N_1,N_2m'}^{(N)} \rangle = \langle O_{2N_1,N_1}^N + O(\Delta O_2) \rangle \delta_{NN'} \delta_{N_1,N_1'} \delta_{N_2,N_2'} \delta_{m,m'} + O(\ll \Delta O_2).
\]

This procedure of approximate diagonalization can be implemented for all the operator observables under consideration \( O_3, O_4, \ldots, O_5 \). At the end, each energy shell \( N \) is subdivided into cells \( J \) with the original approximate energy eigenfunctions \( \{ \psi_{N,n} \} \) transformed to a new orthonormal basis \( \{ \phi_{J,n}^{(N)} \} \) such that

\[
\langle \phi_{J,n}^{(N)}|\hat{O}_j|\phi_{J,m}^{(N)} \rangle = \langle O_{J,n}^N + O(\Delta O_j) \rangle \delta_{NN'} \delta_{J,J'} \delta_{m,m'} + O(\ll \Delta O_j).
\]

If thought of as matrices, the operators \( \hat{O}_j \) in this basis are block diagonal, with each block corresponding to a \( J \)-cell and with small nonzero matrix elements connecting different blocks.
3. Large number limit

Consider a sequence of state vectors \{\Psi_j\} in Eq. (32) for which the number density \(N(x)\) in some region \(x \in V\) Eq. (21) increases, and in turn for an appropriate choice of basis states \{|\alpha\}\} the number of states in each cell increases. Suppose the matrix element

\[\langle \Psi_j | \hat{O} | \Psi_j \rangle, \tag{54}\]

is computed for this sequence, where \(\hat{O}\) is any one of the observables in the contracted description. If there is no a priori correlation amongst the states \(|\alpha\rangle\) of the state vector, as the number of components becomes large, cross terms in Eq. (54) will cancel due to the random phases of the expansion coefficients \(c_\alpha\), whereas the diagonal terms will not cancel. Thus

\[\langle \Psi_j | O | \Psi_j \rangle = \int \sum_{\alpha \beta} O_{\alpha \beta} c_\alpha c_\beta \tag{55}\]

\[\rightarrow \int \sum_\alpha O_{\alpha \alpha} |c_\alpha|^2, \tag{56}\]

where \(O_{\alpha \beta} \equiv \langle \alpha | O | \beta \rangle\).

This property is interpreted as a general outcome of the Correspondence Principle. The Correspondence Principle states that for a fixed measurement resolution, as the quantum numbers of the observed system increase, the classical limit is reached. One example of its application is the hydrogen atom, which has energy levels \(E_n \propto 1/n^2\) with the difference between adjacent levels \(\Delta E_n \propto (n + 1)^{-2} - n^{-2}\). The classical limit is when \(\Delta E_n \propto 1/n^3\), which occurs for large \(n\) when the leading quantum corrections \(\Delta E_n^{\mathrm{qu}} \propto n^{-4}\) becomes negligible. For a given measuring apparatus that is measuring the highly excited hydrogen atom, as \(n\) increases, eventually the resolution of the apparatus can no longer discern the \(n^{-4}\) quantum correction. At that point, one would say the classical limit of the hydrogen atom has been reached up to the measurement resolution of that apparatus. In a many-body system with no induced mechanism for attaining coherence, as the energy increases, it implies either particles are being added to the system and/or the particles in the system are being further excited. In both cases, the quantum numbers that are getting large are the particle occupancies of the excited energy levels. A measuring apparatus that probes the energetic particles in the system will become increasingly less sensitive to quantum effects amongst the energetic particles as its detectors of fixed resolution are forced to receive a greater influx of particles. This is the essence of the classical limit for a large many-body system. In Sect. V it will be shown that in this limit, quantum evolution can be expressed in a form that resembles classical evolution.

4. Transient state basis

The previous subsections have developed the concept of contracted descriptions and shown explicitly how to obtain them. This subsection suggests both an appropriate Hilbert space basis for the large transient state problem of heavy ion collision as well as a contracted description within this basis. In particular, following on the quark-gluon Fock space basis of the quark-gluon plasma hypothesis, here we consider this idea in its generalization as simply a Fock space of quasi-free particles which exist during the lifetime of the transient state. No assumption is required here that the underlying fundamental theory, here QCD, is weakly interaction. For strongly interacting dense systems it is possible that more complicated excitations can emerge, which amongst themselves interact with moderate or weak strength. This quasiparticle picture \[20, 21\] is successfully applied to strongly interacting many-body electron systems. In the present context, if a plasma-like state is to emerge during the course of a heavy ion collision, then almost by definition such a hypothesis implies that there are particle-like excitations, whether quarks and gluons or something else, existing during the lifetime of the plasma phase, which do not interact so strongly that they lose their individual particle identities. Thus implicit in a plasma hypothesis is also the assumption that some sort of quasiparticle excitations emerge for some time period just after the collision. Of course in addition to these quasiparticle excitations which define the properties of the plasma, in general there would be other strongly interacting phenomenon. To incorporate this basic picture, the large transient state Hilbert space basis \{\xi\}
is constructed as a tensor product of transient states \( \{ \tau \} \) and core states \( \{ c \} \),

\[
\{ \xi \} = \{ \tau \} \times \{ c \}.
\]  

(57)

Any state \( |\tau \rangle \) in \( \{ \tau \} \) by definition is orthogonal to any state \( |c \rangle \) in \( \{ c \} \),

\[
\langle c | \tau \rangle = 0, \quad \forall \ |\tau \rangle \in \{ \tau \}, \; |c \rangle \in \{ c \}.
\]  

(58)

The subspace \( \{ \tau \} \) is a Fock space built from the set of creation/destruction operators \( \{ \tau^\dagger_{\alpha}(k, \tau_{\alpha}(k)) \} \), where \( k \) is the momentum and \( \alpha \) are the internal quantum numbers of the transons. Additional restrictions may also be placed on this transon Fock space. For example, momentum regions in which the transons are strongly interacting, and so not well represented by the quasiparticle approximation could be eliminated, such as the low momentum region in confining theories. In [11], a separation of the Hilbert space is also made similar to here, into what they call the “parton” and “gluonic” subspaces, which have close analogy to our transon and core subspaces respectively.

The reference state for the transon Fock space is defined by the condition

\[
\tau_{\alpha}(k)|\text{ref}\rangle = 0, \quad \forall \ \tau_{\alpha}(k) \in \{ \tau_{\alpha} \},
\]  

(59)

and the states

\[
\tau_{\alpha}^\dagger(k)|\text{ref}\rangle
\]  

(60)

define the excitations of single nearly-free particles with internal quantum numbers \( \alpha \), three-momentum \( k \), and energy \( E_{\alpha}(k) = \sqrt{k^2 + m_{\alpha}^2} \). These nearly-free particles that comprise the subspace \( \{ \tau \} \) will be called transons. The core states \( |c \rangle \) are composed of all states necessary to complete the transient state basis \( \{ \xi \} \). Note that by definition, any state in \( \{ c \} \) can serve as the reference state \( |\text{ref}\rangle \) for the transon Fock-space. Our definition of the transon basis is not specific to transons being weakly interacting, although as they are particle like excitations, by definition their interactions are moderate enough to preserve this property. The idea here is similar to that for the parton subspace in [11], except in that work a more extreme condition is imposed that interactions between the parton excitations are completely switched off.

The contracted description of Subsect. 31 in the transient state basis is as follows. Recall the evolution of the transient state wavefunction Eq. 31 is considered under the contracted description Eq. 30, and one seeks an appropriate set of approximate eigenstates. In the transient state basis, we hypothesize the approximate eigenstates at energy \( E \) up to uncertainty \( \Delta E \) are expressed as

\[
\phi_{\alpha,\beta}(E_{\alpha} + E_{\beta}) \leftrightarrow \prod_{j=1}^{M} \tau_{\alpha_{j} \beta_{j}}^\dagger(k_{j})|\text{core}_{\beta}\rangle \quad \text{(general)},
\]  

(61)

where \( \{ \alpha \} \equiv \{ \alpha_{1} \ldots \alpha_{M} \} \) are the internal quantum numbers of the M-transons in this state, \( E_{\{\alpha\}} = \sum_{j=1}^{M} \sqrt{k_{j}^2 + m_{\alpha_{j}}^2} \), and \( E_{\beta} \) is the energy associated with the core state. Note that the energy \( E_{\alpha} \) associated with a state \( \phi_{\{\alpha\},\beta} \) is only that of the transons. This represents the energy from the state that is available for a large many-body transient state. It is also useful to identify two particular types of states in this basis

\[
\prod_{j=1}^{M} \tau_{\alpha_{j}}^\dagger(k_{j})|0\rangle_{\perp} \equiv |k_{\alpha_{1}} \ldots k_{\alpha_{M}}\rangle_{\perp} = \{|\{k, \alpha\}^{M}\rangle \text{ (pure transons)}
\]  

(62)

and

\[
|\text{core}\rangle \quad \text{(pure core)}.
\]  

(63)

Here \( |0\rangle_{\perp} \) is the true vacuum of the theory, \( |0\rangle_{\text{true}} \), except orthogonalized with respect to all pure transon states

\[
|0\rangle_{\perp} = N \left[ |0\rangle_{\text{true}} - \sum_{M=0}^{\infty} \sum_{\{\alpha\}} \int d^{3}k_{j} |\{k, \alpha\}^{M}\rangle_{\perp} \langle \{k, \alpha\}^{M}|0\rangle_{\text{true}} \right],
\]  

(64)
where \( N \) is a normalization constant. From the definition of similar states given below Eq. 32, similarity in the transon basis is defined to mean small separation in all quantum numbers: occupancy of each species, momentum distribution, and other internal quantum numbers \( \alpha \). Depending on the type of measurement that is done, similarity may also place conditions on the core states.

### IV. LARGE TRANSIENT STATE HYPOTHESIS OF HEAVY ION COLLISION

Based on the concepts of projection and contraction developed in the last two sections and combining that with the implications of the large number limit, the large transient state hypothesis of heavy ion collision now is stated. Recall in the time dependent description, the two heavy ion wavepackets begin to overlap at \( t \sim t_c^- \). The nearly-free particles that comprise the transient state must have the pseudo-fock space representation Eq. 61 in the collision wavefunction \( \Psi^{SE}(t) \) for \( t \leq t_c^- \). At the moment of collision, the components of \( \Psi^{SE}(t_c^-) \) with large occupancy of nearly-free particles are associated in spacetime to the formation of a dense many-particle region at the site of the collision. The probability for creation of such a region is determined by the probability amplitudes of the components of the wavefunction having high occupancy of transons. From Sec. III C 3 recall that for fixed measurement resolution, the larger the state, the greater its tendency toward classical behavior. The large transient state hypothesis in the context of heavy ion collision implies the existence of transon states of adequately high occupancy, such that for measurement resolutions of interest, their behavior is classical. To classical characteristics of the large transient state correspond properties of \( \Psi^{SE} \): To entropy \( S \) corresponds occupancy number, to temperature \( T \) corresponds energy density and to energy \( U \) corresponds the sum of the free particle energies of the nearly-free particles. Included here is the spatial volume occupied by the transient state, by combining \( T \) and \( U \).

The wavefunction description of heavy ion collision, relied upon heavily in this paper, clearly has limited practical applicability in dynamical calculations due to its complexity. However, reference to wavefunctions is advantageous in developing the central theme of this paper, which is to understand how classical behavior emerges in the intrinsically quantum process of heavy ion collision. Furthermore, the use of wavefunctions may be helpful in assessing probabilities and sizes of putative large transient states, especially in relative terms, such as p-p versus A-A or between ions of different nucleon numbers.

For example consider the following model. Let the incoming p-p wavefunction just before the collision at \( t_c^- \) be approximated by the product form

\[
\Psi^{SE}_{pp}(t_c^-; (0, k), (0, -k)) = \Psi_{pp}^+(t_c^-; (0, k))\Psi_{pp}^-(t_c^-; (0, -k))
\]

where the expression \((0, k) \equiv (x, k)\) denotes that the center of the proton p wavepacket at \( t = 0 \) is at \( x = 0 \) and the peak momentum along the collision axis is \( k \). The superscripts +,− denote the direction of the protons along the collision axis. Each wavefunction has the quasi-Fock space expansion in terms of transon and core states of the form

\[
\psi_p(t; x, k) = \sum_{M,\{\alpha\}} \sum_j \int_{\infty} [dk]^M c_j(\{k, \alpha\}^M; t)\{k, \alpha\}^M, core_j),
\]

where the shorthand notation used here and below is

\[
\int_{\infty} [dk]^M = \int_{-\infty}^{\infty} \prod_j d^3 k_j
\]

and

\[
\sum_{M,\{\alpha\}} = \sum_{M=0}^{\infty} \sum_{\alpha_1...\alpha_M}.
\]

Thus for the heavy ion collision state, approximate it as the product state of \( A^- \)-moving and \( A^- \)-moving nucleons with bound state effects within each ion represented only through assigning \( A_c \) nucleons as central and \( A_p \) nucleons as peripheral with \( A = A_c + A_p \). Thus the heavy ion A-A collision wavefunction is approximated as

\[
\Psi^{SE}_{AA}(t_c^-; (0, Aq), (0, -Aq)) = \Psi_{A_p}^+(t_c^-; (0, A_pq))\Psi_{A_c}^+(t_c^-; (0, A_cq))\Psi_{A_p}^-(t_c^-; (0, A_pq))\Psi_{A_c}^-(t_c^-; (0, A_cq))
\]
with

$$\Psi_{A_c}(t^-; (0, q)) = \prod_{j=1}^{A_c} \Psi_{p_j}^\pm (t^-; (x_j, q)), \quad (70)$$

\(a = \pm, l = p, c\) and \(x_j\) the position of nucleon \(p_j\) at time \(t = 0\).

Assume that a central collision is necessary to create a large transient state in the central region when the two heavy ions collide. Thus in what follows the heavy ion collision is assumed to involve \(\Psi_{A_c}^+ \Psi_{A_c}^-\) with \(\Psi_{A_c}^-\) passive. \(A_c\) here is treated as a phenomenological parameter.

This model can demonstrate tendency toward the large number limit, thus based on Subsect. III C 3 the classical limit, as \(A\) increases. For example, consider transient state formation in the central region. The first step in the time history of such a process requires a multiple collision event in which several quanta are stripped-off both incoming heavy ions and deposited into the central region. The combinatoric probability for a multiple collision event of this sort increases with \(A_c\). Suppose the initial collision between the two heavy ions involves some type of quanta, such as quarks and gluons, partons, hadronic resonances etc... The precise nature of these quanta does not need to be specified for this analysis. Simply suppose that \(p(\sqrt{s})\) is the probability that two such quanta at CM energy \(\sqrt{s}\) collide and deposit all their energy into the central region. Then the probability \(P_{2m}(n)\) that out of \(m\) impinging quanta from both directions a collision with \(n\)-pairs occurs in which the energy is deposited in the the central regions would be

$$P_{2m}(n) = N_m(p) \left( \frac{m!}{(m-n)!n!} \right)^2 n! p^n, \quad (71)$$

where \(N_m(p)\) is a normalization constant such that \(\sum_{m=0}^{\infty} P_{2m} = 1\). As an example, suppose the colliding quanta under consideration were nucleons. Let \(p(\sqrt{s})\) be the probability in a nucleon-nucleon collision at CM energy \(\sqrt{s}\) that energy deposition in the central region is above some pre-specified density. Then in a \(A-A\) collision, \(A_c\) nucleons in each impinging nuclei can in a central collision deposit their energy in the same region, and the probability for this would be from Eq. (71),

$$P_{2A_c}(A_c) = N_{A_c}(p) p^{A_c}. \quad (72)$$

Thus for \(p \ll 1\), events with the maximum deposition of energy, and so the largest transient states, would have a highly suppressed probability, in fact by several orders of magnitude, to the typical events. For example at RHIC there are somewhere around 5 billion events recorded in total and so for Au-Au collisions \((A = 197)\) based on the above formula for \(p \approx 0.9\) there would have been less than one high collision event where all the 197 nucleons collided. This is a crude estimate but it demonstrates how suppressed the very high collision events are, yet these event fluctuations may be the most interesting for studying properties of large transient states.

V. DERIVATION OF THE MASTER EQUATION

This section obtains the evolution equation for the large transient state. This equation will express the evolution of a single pure quantum mechanical state up to an accuracy adequate for measurement at the specified macroscopic resolutions \(\{\Delta O_j\}\). The large transient state by definition has a high density of particles Eq. (21), so that within a resolution band there are many quantum mechanical states. Due to this, it will emerge that the evolution equation for a pure quantum mechanical state, representing a typical large transient state, in general will have a classical form, in particular the master equation.

Our derivation of this master equation follows Van Kampen [5] (see Appendix A for a review), except that we are converting his reasoning, which addressed condensed matter systems, to the scattering problem. There are several differences between these two types of systems. First, in the scattering problem the large transient state exists only for a finite duration of time, whereas for the condensed matter systems considered by Van Kampen [5], this time interval effectively was infinite. Second, in the scattering problem, the transient state of interest is not in isolation. Thus the entire scattering system can not be subject to the master equation. Third, the elementary excitations of condensed matter systems generally are well understood, since either they are closely associated with asymptotic particles such as electrons or with lattice vibrations such as phonons. In contrast in the scattering problem, since the elementary constituents of QCD, quarks and gluons, confine, the elementary excitations are unclear.

The purpose of Sects. II and III was to clarify all three of these problems. Points one and two were addressed through the time-energy considerations in Subsect. III B 1 and the definition of projection operations in Subsect. III C 1. To address point three the problem was divided into two parts. First the
quasiparticles were defined in very general terms via the transons of Subsect. III C. As a second step, specific identification of transons with QCD excitations can be made. This will be addressed in Sect. VI.

The derivation that follows considers the simplest case in which the state vector has an expansion in the pure transon basis Eq. (62), with transons of scalar, spinor or vector types. It would be straightforward to extend this derivation to include the mixed basis of transon and core states. The derivation below is done in a conventional k-space basis for the transons. The same derivation applies with minor notational changes if the kinematic specification of the transons is in a mixed (x, k)-basis (see Appendix B) or in a basis of rapidity and transverse momenta. Turning to the derivation, let

\[ c(\{k, \alpha\}^M; t) = c(k_1, \alpha_1; \ldots; k_M, \alpha_M; t) \]  

(72)

denote the probability amplitude for the state with \( M \) transons of types \( \alpha_1, \ldots, \alpha_M \) and respective momentum for \( j \)-th transon, \( k_j \). A transon type is characterized by all internal quantum numbers: baryon number, spin, polarization, isospin, and for nonasymptotic transons color, flavor, etc. The normalization condition for the expansion coefficients is

\[ \sum_{M,\{\alpha\}} \int_0^\infty [dk]^M |c(\{k, \alpha\}^M, t)|^2 = 1, \]  

(73)

where the shorthand notation is defined in Eqs. (67) and (68).

A contracted description of the dynamics will now be constructed. To illustrate this procedure, we hypothesize that the similar states of this contracted description will be those states within small intervals of momentum to each other and with some partitioning of the discrete quantum numbers. The contraction over momenta is equivalent to a course graining. In particular, this will require dividing the momentum interval into discrete cells with widths \( \Delta k \equiv (\Delta k_x, \Delta k_y, \Delta k_z) \) in the three spatial directions with the cells centered at points \( k^m \equiv (m_x, m_y, m_z) \Delta k \) for integers \( (m_x, m_y, m_z) \) and with each integer ranging from \(-\infty \) to \( \infty \). In addition, the evolution will be examined in discrete time intervals \( \Delta t \ll \Delta t_{tr} \), where \( \Delta t_{tr} \) is defined in Subsect. III B.1 as the lifetime of the transient state. Accordingly the probability coefficients will be smeared over a time interval \( \Delta t \).

Denote the course grained probability coefficients as

\[ P(\{km, \bar{\alpha}\}^M; t) \equiv P(k_1, \bar{\alpha}_1; \ldots, k_M, \bar{\alpha}_M; t). \]  

(74)

Explicitly

\[ P(\{km, \bar{\alpha}\}^M; t) = \frac{1}{\Delta t} \int_{t-\Delta t}^t dt' \sum_{\{\alpha\} \in \{\bar{\alpha}\}} \int_{\Delta m} [dk]^M |c(\{k, \alpha\}^M, t')|^2, \]  

(75)

where

\[ \int_{\Delta m} [dk]^M \equiv \prod_{j=1}^M \left( \int_{k_j^{m_j} + \Delta k}^{k_j^{m_j} - \Delta k} dk_j \right)^2. \]  

(76)

Note that

\[ \sum_M \sum_{\{km\}} \sum_{\{\bar{\alpha}\}} P(\{km, \bar{\alpha}\}^M; t) = \sum_{M=0}^\infty \sum_{\{\alpha\}} \int_0^\infty [dk]^M |c(\{k, \alpha\}^M; t)|^2 = 1. \]  

(77)

From the fundamental dynamics, we seek the evolution equation for the probability coefficients \( \{P(\{km, \bar{\alpha}\}^M; t)\} \). Furthermore we want to know their evolution between small finite time steps \( \Delta t \ll \Delta t_{tr} \).

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4 The momentum representation used here assumes the probability to find two identical particles with exactly the same momentum has zero measure. For highly degenerate systems, like a Bose condensate, an occupancy number basis is preferable to this momentum basis. Our derivation to follow could be converted to an occupancy number basis.
and not for infinitesimal time differences. Let the initial state of the system, at say the time in Subsect. 3.2 designated to be just before the collision $t = t_0^-$, be given as $c((k, \alpha)^M; t_0^-)$. After a time $t > t_0^-$, the amplitude coefficients of the system will be

$$c((k, \alpha)^M; t) = \sum_{M', \{\alpha'\}} \int_{\Delta t} [dk]^M \sum_r \langle(k, \alpha)^M | e^{-iE_s(t-t_0^-)} | r \rangle \langle(k', \alpha')^{M'} | c((k', \alpha')^{M'}; t_0^-),$$

(78)

where

$$\sum_r |r\rangle \langle r| = 1$$

(79)

is an intermediate sum in the energy representation, with this notation including the possibility of both discrete and continuous spectra. Note that this is the exact quantum evolution of the system and for $\Delta t$ is an intermediate sum in the energy representation, with this notation including the possibility of both discrete and continuous spectra. Note that this is the exact quantum evolution of the system and for example no perturbative assumption of weak coupling is used.

From this we obtain the evolution of the course grained coefficients to be

$$P((k^m, \bar{\alpha})^M; t) = \sum_{M', \{\alpha'\}} \sum_{\{\alpha\}} \int_{\Delta t} [dk]^M \int_{\Delta t} [dk']^M' \int_{\Delta t} [dk'']^M'' \sum_r \sum_{s} \langle(k, \alpha)^M | (k', \alpha')^{M'}; (k'', \alpha'')^{M''} \rangle_{t-t_0^-} c((k', \alpha')^{M'}; t_0^-) c^*((k'', \alpha'')^{M''}; t_0^-),$$

(80)

where

$$\left[ (k, \alpha)^M | (k', \alpha')^{M'}; (k'', \alpha'')^{M''} \right]_t = \frac{1}{\Delta t} \int_{t}^{t+\Delta t} dt' \sum_r \sum_{s} \langle(k, \alpha)^M | r \rangle \langle(k', \alpha')^{M'} \rangle_{t-t_0^-} c((k', \alpha')^{M'}; t_0^-) c^*((k'', \alpha'')^{M''}; t_0^-).$$

(81)

Similar to the arguments in Subsect. 3.3 in the limit that the system size is large, there will be many cross terms in Eq. (80) and the relative phases amongst the amplitude coefficients will be random initially and thus during evolution. Therefore the cross terms will tend to cancel. On the other hand, the diagonal terms are all positive, and thus will always add. This is the first element of Van Kampen’s theory. From this we get

$$P((k^m, \bar{\alpha})^M; t) = \sum_{M', \{\alpha'\}} \sum_{\{\alpha\}} \int_{\Delta t} [dk]^M \int_{\Delta t} [dk']^M' \int_{\Delta t} [dk'']^M'' \sum_r \sum_{s} \langle(k, \alpha)^M | (k', \alpha')^{M'}; (k'', \alpha'')^{M''} \rangle_{t-t_0^-} c((k', \alpha')^{M'}; t_0^-) c^*((k'', \alpha'')^{M''}; t_0^-).$$

(82)

The second observation of Van Kampen was that within a course grained cell $\Delta_M^m$, both factors in Eq. (80) vary independently. This implies

$$P((k^m, \bar{\alpha})^M; t) = \sum_{M', \{\alpha'\}} \left( \mathcal{D} \left[ (k', \alpha')^{M'}; (k'', \alpha'')^{M''} \right]_{t-t_0^-} \right) \sum_{M'} \sum_{\{\alpha'\}} \left( \mathcal{D} \left[ (k', \alpha')^{M'}; (k'', \alpha'')^{M''} \right]_{t-t_0^-} \right) P((k^m, \bar{\alpha})^M; t).$$

(83)

$^5$ The point can be more clearly illustrated with a discrete sum. Consider the sets of N positive numbers $\{a_j\}$ and $\{b_j\}$. Let $a = \sum_{j=1}^{N} a_j$ and $b = \sum_{j=1}^{N} b_j$. Define $\epsilon_j^a$ and $\epsilon_j^b$ such that $a_j = a + \epsilon_j^a$ and $b_j = b + \epsilon_j^b$. This implies by definition that $\sum_{j=1}^{N} \epsilon_j^a = \sum_{j=1}^{N} \epsilon_j^b = 0$. Then we have $\sum_{j=1}^{N} a_j b_j = \sum_{j=1}^{N} (a_j + \epsilon_j^a)(b_j + \epsilon_j^b) = Na + \sum_{j=1}^{N} \epsilon_j^a \epsilon_j^b \equiv N a b$. The last step follows since the second term involving the sum of $\epsilon_j^a \epsilon_j^b$ will have both positive and negative contributions. When N is large these will tend to cancel amongst each other.
For convenience, converting the notation now to a single index, the evolution equation for the probability coefficients becomes

\[ \int_{\Delta \{m\}} \frac{dk}{\Delta k} = \prod_{j=1}^{M} \int_{k_{j_x} - \Delta k}^{k_{j_x} + \Delta k} \frac{dk_{j_x}}{\Delta k} \int_{k_{j_y} - \Delta k}^{k_{j_y} + \Delta k} \frac{dk_{j_y}}{\Delta k} \int_{k_{j_z} - \Delta k}^{k_{j_z} + \Delta k} \frac{dk_{j_z}}{\Delta k} \]  

(84)

and

\[ \left\{ \left\{ \{k^m, \bar{\alpha}\}^M \| \{k^{m'}, \bar{\alpha}'\}^M \| \{k^{m''}, \bar{\alpha}''\}^M \right\}_{\bar{\alpha}} \right\} = \sum_{\{\bar{\alpha}\} \in \{\bar{\alpha}\}} \int_{\Delta \{m\}} [dk] \int_{\Delta \{m'\}} [dk'] \int_{\Delta \{m''\}} [dk''] \left\{ \left\{ k^M \| k'^M \| k''M \right\} = \delta_{M,M'} \delta_{\{\bar{\alpha}\} \{\bar{\alpha}'\} \{\bar{\alpha}''\}} \right. \]

(85)

Note the above is a statement about the dynamics and not the initial conditions.

We can now obtain the evolution equation for the course grained probabilities. For this we are interested in time steps \( \Delta t \) and not for infinitesimal time differences. For small \( \Delta t \) upon temporal averaging and course graining one obtains the general form

\[ \left\{ \left\{ \{k^m, \bar{\alpha}\}^M \| \{k^{m'}, \bar{\alpha}'\}^M \| \{k^{m''}, \bar{\alpha}''\}^M \right\}_{\bar{\alpha}} \right\} = \delta_{M,M'} \delta_{\{\bar{\alpha}\} \{\bar{\alpha}'\} \{\bar{\alpha}''\}} + \Delta t W \left( \left\{ \left\{ k^m, \bar{\alpha}\right\}^M \| \{k^{m'}, \bar{\alpha}'\}^M \| \{k^{m''}, \bar{\alpha}''\}^M \right\}_{\bar{\alpha}} \right). \]

For convenience, converting the notation now to a single index,

\[ (M, \{\bar{\alpha}\}, \{m\}) \rightarrow J \]

(86)

the evolution equation for the probability coefficients becomes

\[ \frac{dP_J(t)}{dt} = \frac{P_J(t + \Delta t) - P_J(t)}{\Delta t} = \sum_{J'} W_{J,J'} P_{J'}(t). \]

(87)

Above, we have used Eq. (83) to express \( P_J(t + \Delta t) \) in terms of the set \( \{P_J(t)\} \). To arrive at the above form, one should also note that \( P_{J'}(t) \) is expressed in the form of Eq. (75). In Eq. (87) \( W_{J,J'} \) denotes the transition probability between states \( J \) and \( J' \) and \( P_J(t) \) denotes the occupation probability of state \( J \).

Since

\[ \sum_{M} \sum_{\{m\}} \sum_{\{\bar{\alpha}\}} \left\{ \left\{ \{k^m, \bar{\alpha}\}^M \| \{k^{m'}, \bar{\alpha}'\}^M \| \{k^{m''}, \bar{\alpha}''\}^M \right\}_{\bar{\alpha}} \right\} = 1, \]

this implies \( \sum_{J} W_{J,J'} = 0 \). From this we arrive at

\[ \frac{dP_J(t)}{dt} = \sum_{J'} [W_{J,J'} P_{J'}(t) - W_{J',J} P_J(t)], \]

(88)

which is the master equation.

Let us comment on some aspects of Eq. (88). The equilibrium distribution is determined by the solution of

\[ \sum_{J'} \left[ (W_{J,J'} P_{J'}(t) - W_{J',J} P_J(t)) \right] = 0. \]

(89)

Also in equilibrium, by the equipartition hypothesis it follows that,

\[ \frac{P_J(t)}{P_{J'}(t)} = \frac{G_J}{G_{J'}}, \]

(90)

where \( G_J \) is the total multiplicity of the states in the cell \( J \). The proof of detailed balance follows from time reversal symmetry of the underlying dynamics. Let \( T(\{k, \alpha\}) \) be the time reversed state of \( \{k, \alpha\} \). Then it follows from microscopic dynamics that,

\[ \{k', \alpha'; t | \{k, \alpha\}, 0\} = \langle T(\{k, \alpha\}); t | T(\{k', \alpha'\}), 0 \rangle. \]

(91)
Since a coarse grained cell always contains pairs of state related by T, it means that
\[ W_{J,J'}G_{J'} = W_{J,J'}G_J. \]  
Eqs. (90) and (92) give the necessary ingredients to obtain the detailed balance relation
\[ W_{J,J'}P_{J'} = W_{J,J'}P_{J'}, \]
where there is no summation on the repeated indices.

Let us now address when the canonical thermal distribution can be applied to describe the transient state. Recall that the canonical distribution describes a system in equilibrium when it is in contact with an external heat bath. In contrast, the transient state is an isolated system at some fixed energy. An ensemble respecting such an isolated system in equilibrium consists then of all states \( r \) within a small interval \( \Delta U \) about the given energy \( U_r \) having equal probability,
\[ P_r = \begin{cases} C & \text{if } U_r - \Delta U < U < U_r + \Delta U \\ 0 & \text{otherwise.} \end{cases} \]  
The canonical distribution can be applied to such a microcanonical ensemble Eq. (94) for describing some small part of the whole system. For example suppose we separate the cell \( J \) into a small portion denoted by \( J_S \) and the remainder \( J_R \), \( J_S \cup J_R = J \) and \( J_S \cap J_R = 0 \). We can obtain probability coefficients exclusively for \( J_S \) by summing over all configurations of \( J_R \), so \( P_{J_S} = \sum_{J_R} P_J \). The cells \( J_S \) are no longer closed systems at some fixed energy, since energy can flow from \( R \) into \( S \). Then provided \( R \) is very large compared to \( S \), by the usual system-reservoir arguments \[16, 22\], in equilibrium the \( P_S \) coefficients will satisfy the canonical distribution. For example if our cells were in x-space, we could ask what is the equilibrium distribution of some small volume of the complete transient state, and in equilibrium it would obey the canonical distribution. Aside from the equilibrium distribution, the master equation permits very general characterization of the statistical state; an example of some other possibilities which may be interesting to consider in the context of the heavy ion transient states are \[23\]. An examination of how pure quantum evolution in systems with a large number of degrees of freedom develops into a statistical mechanical behavior has been studied for general systems in \[24\]. There are similarities between that work and ours here, such as similar conditions are considered on diagonal versus off-diagonal matrix elements. Moreover our conclusions are similar to \[24\] although we obtain a general evolution equation, the master equation, that can treat a variety of statistical behavior whereas in \[24\] the focus is only on understanding the equilibrium limit.

This Section examined the evolution equation of a pure quantum state which is measured with macroscopic uncertainties, meaning resolution widths in which a large number of quantum states are indistinguishable. For such a case, it was shown above that the evolution equation for the state vector has a classical form, in particular the master equation. This formal association with the master equation implies all properties of this equation in its application to nonequilibrium statistical systems carry over. In particular, recall the master equation keeps account of all transitions between the states of a statistical system, and thus is the fundamental equation for describing the nonequilibrium dynamics of the given system. Moreover, other contracted descriptions, further down in the hierarchy can be obtained from it, such as the Boltzmann equation and the hydrodynamic equation \[25\]. As one detail, rather than the k-space basis for the transon excitations, as used in this Section, another possibility is a basis of phase space excitations, as described in Appendix B, for which the resulting master equation would express transitions in phase space. Such an equation formally would be analogous to that for a classical gas, for which considerable information about master equation solutions already exist and could be readily utilized. Alternatively the transons could be described in terms of rapidity in the longitudinal direction and two transverse momenta. Such a choice of kinematic variables would be a more appropriate starting point for deriving the Bjorken hydrodynamical model \[18\] from a more fundamental level using the master equation transcription of this paper.

In closing this Section, an important distinction is emphasized here between pure quantum states versus statistical ensembles. A pure quantum state contains information about one single scattering event. In contrast a statistical ensemble contains information over some type of summation over several events, each represented by a pure state. The Wigner distribution \[26, 27\] is an approach to examine evolution equations of statistical ensembles of quantum mechanical pure states. However, the origin of the master equation derived here has no relation to the Wigner distribution. The justification for classical arguments, such as quark-gluon plasma formation, in predicting outcomes of a scattering experiment, must be established on individual events and that is what our derivation of the master equation has accomplished.
VI. HEAVY ION COLLISION SCENARIO

This section reviews the phenomenology of transient state formation during heavy ion collision and then explores how it could be studied using the methods developed in this paper. Although the heavy ion collision scenario was mainly formulated with the quark-gluon plasma formation scenario in mind, after our review in Subsect VI A the picture is reexamined in the more general terms developed in the previous sections.

A. Phenomenology of heavy ion collision

The basic scenario for large transient state formation in heavy ion collision was formulated in \cite{18, 19}, in terms of a spacetime picture and this will now be reviewed. The process of interest is $A - A$ collisions at ultra-relativistic energy. The events of specific interest for exhibiting statistical mechanical behavior are central collisions; such collisions can be identified by an enhanced production of grey-prongs \cite{28}. Alternatively at RHIC central collisions are characterized by a combination of both minimum number of particles in the zero-degree calorimeters (ZDC) and the largest event multiplicities \cite{29}. In spacetime, central collisions are described to occur at impact parameter $b$ less than the range of the nuclear force $b < 1$fm. In the center-of-momentum frame, both relativistic nuclei appear Lorentz contracted along their longitudinal direction of motion, so that at extremely high energies, the collision is likened to that of two discs or “pancake-like” projectiles \cite{18, 30}. The spacetime picture provides a center-of-momentum per nucleon energy scale, implicit to the Lorentz contraction of $E_{CM}^0 \approx 7 - 70$GeV, above which the longitudinal contraction is smaller than the characteristic hadron scale $\approx 0.1 - 1$fm. Phenomenologically at center-of-momentum energies above $E_{CM}^0$, it is expected that the collision will result in two fragmentation regions approximately separated from a central region.

The heavy ion experiments of interest for quark-gluon plasma formation are at large nucleon number $A \geq 100$ and at center-of-momentum energies above $E_{CM}^0$ per nucleon, since such cases are believed to yield the highest probability for realizing a large energy density central region. Heavy ion facilities meeting these specifications were initially the CERN SPS collider where $\sqrt{s} \approx 20$GeV per nucleon, at present is RHIC where $\sqrt{s} = 200$GeV/nucleon and in the near future will be ALICE where $\sqrt{s} = 5.5$TeV/nucleon. The spacetime evolution picture in this energy regime is as follows. The colliding nuclei are rather transparent to each other. However the low momentum components will interact strongly and some portion may come to rest at $t \approx t_{c}^{+}$ in the central region about $z = 0$. This scraped-off debris will continue to interact with constituents of high longitudinal momentum in the receding nuclei after $t > t_{c}^{+}$. These interactions will occur after a time $\tau \approx 1$fm/c in the rest frame of the colliding constituents. This implies the higher the longitudinal momentum, the later they interact and the further away from the collision point $z \approx 0$ they are deposited. One phenomenological fact from pp-collisions that is carried over for A-A collisions is the presence of particle distribution in the central region that is uniform in rapidity. Combining this with the above spacetime picture, it implies new matter of increasingly larger rapidity is deposited at the edges of the central region, just behind the receding nuclei, and when looked within the rest frame of this matter, the energy density is constant. In addition, the two separating nuclei may \textquotedblleft heat-up\textquotedblright as they pass through each other and this could lead to baryon rich fireballs in the two fragmentation regions \cite{13} (but see also \cite{31}).

The above picture supplies the initial conditions on the matter in the central region. To treat the subsequent evolution, another phenomenological input from pp-collisions is applied, that there is little long range correlation in rapidity. This transcribes in spatial terms to mean interactions amongst particles produced in the central region only extending a small distance $\Delta z \approx O(1)$fm along the collision axis. Thus each slice of matter in the central region of thickness $\Delta z \approx O(1)$fm evolves primarily independently except for some interactions with the nearest neighbor slices. Since matter first is deposited at $z \approx 0$, the centermost slice begins its evolution first, with slices increasingly further away from $z = 0$ beginning evolution at increasingly later time. Initially the particles deposited in the central region have a large distribution of longitudinal momentum, whereas the transverse momentum distribution is random. This leads to a longitudinally expanding central region with initially no transverse expansion. The quark-gluon formation scenario is focused on the intermediate time period just after the central and two receding nuclei have approximately separated from each other, at $t \geq t_{c}^{+}$ and before final state hadronic particle production has occurred at $t < t_{b}$. The time when the central region and two receding nuclei separate into three independent regions is estimated as $t_{c}^{+} \approx 1$fm/c, where we have set the initial time of impact as $t_{c}^{-} \approx -1$fm/c.
The evolution of the central region, in particular, is modelled as an isentropic hydrodynamic expansion \cite{18,32}. In this picture, the phenomenologically motivated assumption based on \( pp \) collisions is that the particle distribution in the central region just after impact, say at \( t = t_c^+ \), is uniformly distributed in rapidity sets the initial conditions for the subsequent evolution of the central region. In particular, at \( t = t_c^+ \), the particles in the central region, which corresponds to a small region at say around \( x = 0 \), have a large dispersion in velocities, with the longitudinal velocity gradient being large, whereas the transverse velocity dispersion is random. 

Estimates \cite{18,19} of the central region place its volume as \( V \approx (5 - 100) \text{fm}^3 \) and energy density as \( \epsilon \approx (1 - 10) \text{GeV/fm}^3 \). This leads to a total energy in the central region of \( U \approx (5 - 1000) \text{GeV} \). The central region is pictured to expand for a duration of time in a plasma-like state, before hadronization begins at \( t_h \). Estimates give \( t_h/t_c^+ \approx 10 - 1000 \), which implies the plasma state extends for a time interval of \( \Delta t_e \approx (10 - 1000) \text{fm}/c \). Due to the high energy density of the plasma, the collision time for its particles is estimated as \( \tau_{\text{collision}} \approx (0.1 - 1) \text{fm}/c \), whereas the expansion time scale of the central region, since it is effectively a 1-d expansion, is \( V/V \sim t \approx 1 \text{fm}/c \). From comparing these two scales, it is assumed the local dynamics within the plasma are only weakly affected by the system’s expansion.

This scenario now will be interpreted in terms of the formalism developed in the previous sections. The projection step of Subsect. \( \text{II}C.1 \) is accomplished here by going to sufficiently high collisions energies, \( \sqrt{s} > E_{\text{CM}}^{(0)} \), where the final state particle production in the central region are well separated from those in the two fragmentation regions. The large transient states of interest are believed to form for a selected set of events in the central region, which are distinguished through additional conditions placed on the final state particle spectrum. To specify these conditions, the contraction step is necessary. For this the full set of operator observables and measurement uncertainties must be specified and then, as discussed in Subsect. \( \text{III}B.1 \), theoretical models of detail and accuracy consistent with experimental resolution must be formulated. Sect. \( \text{III} \) explained the purpose and methodology of the contraction step and finished-off in Subsect. \( \text{II}C.3 \) by introducing the transon model, which is a generalized basis and contracted description for the heavy ion problem. Finally Sect. \( \text{IV} \) determined the evolution equation for a quantum state vector expressed in terms of the transon basis. Specifically, relying on the large number limit, it was demonstrated there how the quantum evolution equation reduces in form to the master equation of nonequilibrium statistical mechanics. The initial conditions for evolving the transient state must be constrained by the specifications of the above reviewed phenomenological model. With this, the master equation then can dynamically determine how the state evolves, whether towards thermalization or some other statistical state.

B. Identifying transon candidates

What remains and will now be discussed is the specifications of the excitations that comprise the transons. In broader terms, if a large manybody transient state formed during a heavy ion collision, it would rapidly evolve through several stages in which several types of excitations may exist. The appropriate question is does any particular manybody excitation dominate during the transient period sufficiently to leave some imprint on the final state? The central step in answering this question is to determine the types of QCD excitations that can be the transons of a large transient state during a heavy ion collision. Some of the possibilities are as follows:

**Transons as hadrons** - This is the benchmark example which form the constituents of the hadron gas transient state \( \text{I} \). Since hadrons and hadronic resonances are asymptotic states, their properties are unambiguous. Therefore, statistical mechanics is applicable to treating the hadron gas with few assumptions.

**Transons as partons** - The parton model is used to treat quark-ghron interactions in perturbative QCD high energy collision problems, where both incoming hadrons exchange only a few of these particles. Thus the parton model is applicable for short lived, small transient states with very high energy densities. On the other hand, in the large transient state heavy ion collision problem, both incoming heavy ions exchange many constituent particles. This problem addresses moderately high energy densities distributed over a large spatial volume. The only similarity between both problems is that individual collisions between the quarks and gluons is at adequately high energies for asymptotic freedom to be applicable \( \text{III} \). Aside from this, both problems differ substantially. Although probabilistic distributions appear in pQCD problems, their interpretation is completely different from those associated with the master equation of Sect. \( \text{V} \). In pQCD, the probabilistic distribution functions for the quarks and gluons arise from an averaging over a certain class of events, which exploits unitarity to eliminate infrared divergences. In contrast, in the large transient state
problem, only one single event is treated, and the cancellation of any infrared divergences must arise from
within that single system.

Transons as energetic $|p| \geq T$ quarks and gluons - This appears to be the democratic hopeful. The
picture is that of a high temperature gas of quarks and gluons, which due to asymptotic freedom will
interact weakly. Two questions are generally raised about quark-gluon plasma. The first question pertains
to the existence of such a phase in QCD. The second question is whether proper conditions can be realized
during a heavy ion collision, to produce such a phase. The first question has been studied extensively by both
lattice simulations \cite{3} and various analytical approaches \cite{2}. The evidence suggests that QCD undergoes a
phase transition from a gas of hadrons to a gas of quarks and gluons at a temperature $T_c = 190$ MeV. The
properties of the quark-gluon plasma have been examined considerably \cite{1, 5, 12, 19, 36}, with particular
emphasis on a gauge invariant description. The second question is much less clearly understood, since there
are a few complicating factors to this simple picture. First, a perturbative treatment of such a state leads
to infrared divergences \cite{37}, which generically are expected in any theory with a massless gauge boson.
Optimistically this is a technical problem of infrared summation. Considerable effort has gone to study this
\cite{38}. Second, a transient state of energetic quarks and gluons that is produced in the laboratory is immersed
within the true vacuum of QCD $|0\rangle_{\text{true}}$. However perturbative high temperature QCD calculations on the
thermodynamics of quarks and gluons are performed with respect to the perturbative vacuum of QCD
$|0\rangle_{\text{pert}}$. Is this theoretical idealization well representative of the actual process? In the transon model, this
question equates to whether $|0\rangle_{\text{true}}$ and $|0\rangle_{\perp}$ are ”approximately” the same, where $|0\rangle_{\perp}$ from Eq. (64) is
the true vacuum orthogonalized with respective all energetic quark-gluon states that primarily comprise the
large transient state. If affirmative, it means the true vacuum does not contain many energetic quarks and
gluons and so $|0\rangle_{\perp}$ should be approximately stationary. The transon model can also be approximated by
replacing $|0\rangle_{\perp}$ with $|0\rangle_{\text{pert}}$. This replacement is valid if the energetic quarks and gluons of the transient state
interact minimally with the low energy components of the true vacuum. High temperature perturbative
QCD in conjunction with the bag model \cite{39} follows this set of assumptions with the additional requirement
that there is a global shift in energy $E_{\text{pert}} - E_{\text{true}} = B$. The final dilemma in interpreting quarks and
gluons as the transons is that the picture of a color singlet transient state represented in a Fock space of
quarks and gluons is misleading since the gluon field operator $A^{\alpha \mu}(x)$ does not transform in color space as an
octet. At the level of thermodynamics this does not appear to be a big problem, since the differences are not
significant between a color unrestricted versus color-singlet projected summation of the partition function
\cite{40}, yet out-of-equilibrium this could be a more important issue.

Transons as manybody quasiparticle - The essential question in realizing quark-gluon plasma is whether
color can flow over extended regions. If that occurs, it implies a liberation of color degrees of freedom, which
is the signature of the new thermodynamic state. For this to occur, it is not mandatory that the gas of
nearly-free particles be quarks and gluons. The effective dynamics that most conveniently characterizes
color flow may require collective variables of the entire system as opposed to the fundamental quark-gluon
operators. In analogy to condensed matter physics systems, these collective variables generically can be
called quasiparticles.

In condensed matter systems a plasma oscillation is a collective longitudinal excitation of the elementary
particles and one quantum of this excitation is called a plasmon. Finite temperature perturbative QCD
calculations indicate a similar collective effect amongst the gluons \cite{41}. The calculation of the finite tem-
perature gluon self-energy show a nonvanishing longitudinal and transverse gluon mass. This modifies the
gluon dispersion at low momentum from $E = |p|$ to $E = \sqrt{p^2 + m_g^2 + O(p^2)}$. Here $m_g \sim$ MeV is the
induced gluon mass or synonymously the plasmon frequency. A color singlet collective density fluctuation
that propagates through the plasma at sound velocity has also been considered in \cite{42} and identified as the
hydrodynamic phonon.

Transons as elementary color singlet clusters - We can regard hadrons and liberated quarks and gluons as
two extreme ends of the range of possible excitations in the large transient state. It is possible that the for
the large transient state created in heavy ion collision, a mixture of both color singlet and color liberated
excitations may co-exist. In statistical systems, it is common that the long range order in the system is in
a certain phase, but there is short range order with properties of an entirely different phase. For example,
for the Ising model in the high-temperature phase, there is no long range order, since globally the spins are
decorrelated. Nevertheless, locally correlations amongst nearby spins still exist, with this short range
order being increasingly more pronounced as the critical point is approached from above. In analogy, the
large transient states produced in heavy ion collision may not have adequately high energy density and a
sufficiently large volume to totally eliminate local color correlation. In this case singlet clusters will
emerge. Their density may be quite low, so that just a few small color singlet clusters are immersed in an otherwise sea of liberated quarks and gluons. On the other hand, the whole phase may be dominated by color singlet regions of size bigger than hadron scales but smaller than the volume of the transient state.

The importance of neutralizing color in the large transient state appears from another direction, which further motivates the presence of color singlet clusters. Due to confinement in QCD, all energy eigenstates of the theory are color singlet. Since evolution of the scattering process could be described in Eq. (75) with an energy eigenstate basis, it means in principle the system can be described with only color singlet excitations. Moreover, assuming an adequate separation between the central and fragmentation regions, the transient state in the central region, described by the projection Eq. (21) would be color singlet, and so by similar reasoning, its excitations also could be described with only color singlet states. An immediate consequence of these considerations is the transient state basis in Subsect. II C 4 should be restricted to color singlet states. There are a few ways in which this could be realized. One possibility is the transon and core states independently are both color singlets. A second possibility is the transons, most likely high energy quarks and gluons, be regarded in a color unrestricted Fock space, where the core states appropriately neutralize color. The interpretation is that an arbitrary color unrestricted state of energetic quarks and gluons will neutralize itself through low energy components, which are represented through the core states. If it is then assumed that the low energy components have no other affect on the development of the large transient state, then they can be dropped. With further assumptions, The color unrestricted quarks-gluon states can be constructed over a convenient reference state such as \(|0\rangle_{\text{pert}}\). Finally, a third possibility is the individual transon excitations themselves are color singlet. If the transon excitations were hadrons, that would be one example of this. However, it is possible that more elementary color singlet clusters of just a few correlated quarks and gluons, could emerge over the time scale of the transient state. A color singlet quark-gluon Hilbert state also can be constructed, in which color singlet composite operators are in general products of quarks and gluons

$$O_{f_1\ldots f_M\bar{f}_1\ldots\bar{f}_N}(\mathbf{R}) = \int d^3r_1\ldots d^3r_{M+N} \delta^{(3)}(\mathbf{R} - \sum_{j=1}^{M+N} \frac{r_j}{N+M}) F(r_1\ldots r_{M+N}) \prod_{j=1}^{M} \psi^{f_j}_{a_j b_j}(r_j) \prod_{j=1}^{N} \bar{\psi}^{\bar{f}_j}_{b_j w_j}(r_{j+M}) \Gamma_{\mu_1\ldots\mu_n}^{\nu_1\ldots\nu_n} w_{1} w_{M} \ldots w_{N} \prod_{i=1}^{N} U^{a_i b_j}(r_i, r_{j+M}),$$

where \(\Gamma_{\mu_1\ldots\mu_n}^{\nu_1\ldots\nu_n}\) is a Dirac tensor associated with the \(M\) fermions of flavor \(f_j\) and \(N\) anti-fermions of flavor \(\bar{f}_j\) and \(U^{ab}(r, r')\) are the gauge field link matrices. With these operators, the product state

$$\prod_{j=1}^{n} O(R_j |\rangle |\rangle)_{\perp}$$

are not orthogonal, although if their centers \(R_j\) are sufficiently far apart, they should be approximately orthogonal.

C. Examples of models

In order to do a concrete calculation with the methodology developed in this paper, what is needed first is transon and core states to be identified and a Hamiltonian specified which describes their interactions. With this in place, matrix elements, etc... required for the master equation in Sect. V can then be computed. One also would need to calculate the eigenstates and eigenfunctions for this system, which in most practical cases would have to be done perturbatively.

The ultimate goal would be to start with the full QCD Hilbert space of states and then identify the transon excitations that would be relevant during the transient state created during the heavy ion collision, with some possibilities listed in Subsect. VII B. With this identification, the full QCD Hilbert space is then decomposed into the transon and core sectors and an effective Hamiltonian is obtained that describes all interactions. With this done, one can proceed with construction of the master equation following Sect. V.

However this is too ambitious for the time being. A more immediate application of the methods in this paper would be to provide a more complete dynamical description of some of the hypothesized models of the
transient state such as the hadron gas etc... For the case of the hadron gas, rather than just computing thermodynamic quantities, a dynamical nonequilibrium treatment is possible. For example, one could consider the situation where the primary transon states are the $\pi$, $\rho$ and $a_1$ mesons. These mesons are considered important in the production of dileptons and photons in the hadron gas [43]. One could start with the effective Lagrangian for this system [43], and compute in perturbation theory the energy eigenstates, eigenvalues and matrix elements needed in Eq. (81) from which the master equation is constructed. Finally one needs to specify an initial state, when these excitations become important in the transient state, from which to start evolution of the master equation. This might be specified through considerations of hydrodynamical models [33] and quark and gluon parton distributions [35, 44].

In a similar way, the nonequilibrium dynamics of other possible transon excitations can be examined via the master equation. For example, strange quark production could be examined in chiral models [45]. Alternatively, effective theories of quarks and gluons [46] could be examined as simplified versions of a complete QCD treatment of quark-gluon plasma.

VII. CONCLUSION

The purpose of this paper was to surmise the first principles origin of a classical statistical mechanical description of multi-hadron production in heavy ion collision. Some special focus was placed on quark-gluon plasma formation, due to its widespread interest. As we have seen, realizing a statistical mechanical description requires a straightforward application of fundamental and well known concepts about the large number limit and randomness. The master equation, which is the final result of this transcription, lends itself to a large variety of statistical mechanical behavior, of which the often studied case of thermalization is just one special case.

The transcription made in this paper was for a single pure quantum state of two incoming heavy ions. This is not the familiar starting point for treatments of heavy ion collision. Normally a density matrix formalism is used [27, 47], due to the belief that is the only practical means of treating such a complex system. However, in principle one should be able to construct a pure quantum state of two incoming nuclei, and studies its evolution in a manner no different from say electron-electron scattering. Furthermore if one is making predictions about final state observables such as particle yield and type, they must have their explanation at the level of pure quantum dynamics. This has been one motivation for the considerations in this paper.

This paper addressed a similar problem to that in [11], which is under what conditions does a pure quantum state behaves in a classical manner, with application to the heavy ion collision problem. Our decomposition of the full Hilbert space into transons and core states is analogous to the parton and gluonic subspaces respectively identified in [11], with the diagonalization in the partonic subspace in [11] similar but more extreme to our approximate diagonalization in the transon subspace. Where this paper goes beyond [11] is in addressing time evolution. In [11] the development stops at the point where the decomposition into their parton and gluonic subspaces is made, in which by construction observables in the partonic subspace are diagonal, i.e. the Schmidt decomposition. As stated there, such a construction in general can only be made at one time slice. In order for their decomposition to sustain over time the perspective adopted in [11] was this is a model specific issue depending on the particular nature of the interactions in the fundamental Hamiltonian. In particular it depends on whether in the particular theory one is examining certain states, called "pointer" states, can be identified which in essence would preserve the Schmidt decomposition over time. Thus based on the perspective in [11], since it is dynamics specific, the conclusion is that in general the Schmidt decomposition will not remain valid over time. The key step where this paper deviates from [11] is that nowhere in [11] are the facts exploited that they are examining a system with a large number of degrees of freedom and that ultimately a contracted description of the system will be made. What we observe in this paper is that in the large number limit, a search for exact pointer states from the fundamental Hamiltonian is not needed. In the large number limit, the conditions on the interactions can be relaxed considerably to only that they are not very strong, and in such a case, the large number limit will dominate over any off-diagonal correlations thus realizing an evolution that is effectively classical. The main result in this paper was in Sect. III C which demonstrated how this approximate diagonalization occurs in the large number limit. In the language of [11], our results basically say that there are a class of states that effectively can act as pointer states and identifying this class depends on the specified degree of resolution to which the system
is measured. Thus in general the set of pointer states is not unique, but rather a whole range of states can emerge which within the needed resolution of the measurement can behave effectively as the pointer states. It also should be noted here that although there are many treatments of classical limits to quantum mechanics via path integral approaches, our derivation in Sect. III C as an extension and completion of that started by Van Kampen in [8], is one of the few that demonstrates the classical limit within a completely canonical approach; independent of our application to the heavy ion collision problem, this is another significant result of this paper.

Historically, thermodynamic approaches to hadron scattering in the "old" school were initiated by Fermi [48] and developed also by Landau [49]. In more recent times Carruthers and collaborators [50, 51] have developed these ideas to a more general nonequilibrium statistical mechanical approach. In this "new" school of thought, the change in perspective has been that there need not be anything less rigorous about using statistical mechanical variables as the fundamental modes of description [30]. Although theorist have studied more seriously the predictive content of statistical kinetic theory to scattering processes, at present little unification with conventional quantum mechanical approaches to scattering has been made. This paper has attempted to shed some light in this direction. For one thing, this paper provides a general framework in terms of the threefold projection, contraction, evolution, scheme, which converts a quantum mechanical scattering problem involving a large composite system into a form that resembles a nonequilibrium statistical mechanical problem.

Our purpose here was to convey an alternative mode of thinking, that ultimately encapsulates a more general methodology. In doing this, the treatment in this paper has only captured a small essence of the problem and this after considerable simplification. To briefly summarize, in our approach, first, time was separated into three sections, initial, final, and interacting. Ascribing an initial and final interaction time implicitly relies on a description with wave-packets. We then froze the initial state at the onset of the interacting time period. Due to the largeness of the initial system, we assumed randomness amongst the expansion coefficients when expressed in terms of the quasiparticle basis and assumed this held throughout evolution in the interacting time period. In the limit that this approximation is valid, dynamics took a form of classical probabilistic evolution. How valid is this limit in a real world problem? There obviously is not complete randomness in the initial state, since a great part of it propagates into the final state. How should one describe the initial state as an expansion of weakly interacting quasiparticles plus a coherent core? An approach was suggested in Subsect. III C 4 in terms of what we called core states and generalized particle like excitations termed transons. How universal is such a separation for different heavy nuclei?

Although the answers to such questions will be hard to answer, these questions are demonstrative of a rich variety of physics made available by a statistical mechanical description of multi-hadron production. In as much as high-energy hadron scattering is a manybody problem, the high energy collision of two heavy ions is a many-manybody problem. The path via statistical mechanics toward its understanding could be summarized as an attempt to replace s, t, and u with S, T, and U.

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Appendix A

In this appendix Van Kampen’s theory is reviewed. Given a Hamiltonian, $\hat{H}$, Van Kampen first separates its eigenstates into groups. Each group is specified by a central energy $U_j$ and a resolution $\Delta U_j$. The resolution is fixed by the experimental apparatus. He then classifies suitable linear combinations of eigenstates within an energy band, into equivalence classes. To implement the classification, he introduces the set of all operator observables that the experimental apparatus measures. This set includes the Hamiltonian, $\hat{H}$, and all other observed operators $\{\hat{O}_1, ..., \hat{O}_j\}$. The resolution of all observables $R = \{\Delta U; \Delta \hat{O}_1, \Delta \hat{O}_2, ..., \Delta \hat{O}_j\}$
are set by the experimental apparatus. All states within the resolution band $R$ and with central values $C_l \equiv \{U_l; O_{1l}, ..., O_{nl}\}$ are placed in equivalence class $G_l$.

To obtain the set of states in $G_l$, Van Kampen starts with the set of eigenstates within the band $U_l\pm \Delta U$ and proceeds constructively as follows. He forms linear combinations from these states, so as to diagonalize the operator $\hat{O}_l$ up to resolution $\Delta O_l$. Within each band $O_{1l}\pm \Delta O_l$, he repeats his "partial diagonalization" procedure with respect to the operator $\hat{O}_2$ and so on for the rest. In the end he ends up with certain linear combinations amongst the eigenstates in the specified resolution band. These states are grouped into the equivalence classes $G_l$. The states in each equivalence class $G_l$, up to observation, are identical. All systems measured with resolution $R$ about central values $C_l$ have a Hilbert space expansion solely with respect to the states in $G_l$.

The energy operator played a special role in that the system at all times remained within its initially specified energy band. The problem of dynamics to solve here was, given an initial state with coarse grained expectation values $\{O_{1l}, \cdots, O_{nl}\}$, what are the expectation values at a future time $t$. The macroscopic nature of the problem enters because the observables were specified within resolution bands much bigger than the minimal values set by quantum dynamics. Under these specifications, Van Kampen attempted to derive an evolution equation.

It is helpful to state the above in a slightly different way also. The initial state is first specified to lie in an invariant subspace of the complete Hilbert space, based on the resolution widths of all conserved observables. Temporal evolution is therefore restricted to lie on the so determined manifold of states. The description of the state vector is, however, further described with respect to a set of nonconserved observables. It is the temporal evolution of these quantities that constitutes the dynamical, nonstationary aspect of the problem.

In relativistic scattering processes, in addition to energy, the conserved observables are the total three-momenta, the angular momentum and discrete quantum numbers, such as baryon number etc... The nonconserved "observables" in the construction of the main text were the number density operators of the quasiparticle states.

Having specified the initial state within equivalence classes, $G_l$, rather than immediately introduce a statistical density matrix, Van Kampen followed a slightly different approach, which retains closer contact to the underlying quantum dynamics. Van Kampen constructed a pure state by taking an arbitrary linear combination of states in $G_l$ as the initial state. Since the expansion coefficients have no specified correlations amongst each other for the states in $G_l$, in the limit the number of states, $N_l$ in $G_l$ gets bigger he showed how dynamical evolution took the form of a master equation. We followed this reasoning in our explicit construction in Sect. [V]. With this slight chance in viewpoint, Van Kampen was able to demonstrate a matter of principle which is often obscured in treatments that start on the onset with a statistical density matrix. In particular, he found a set of conditions under which quantum dynamics goes over into a form that resembles nonequilibrium statistical dynamics.

Appendix B

An alternative to the k-space transon excitations used in Sect. [V] is phase space localized excitations. These are defined as

$$\tau^\dagger_{\alpha}(x_r, p_s) = \int d^3k \delta_{g(x_r, p_s)}(k) \tau^\dagger_{\alpha}(k), \quad (97)$$

where $\tau^\dagger_{\alpha}(k)$ is a creation operator for a transon with three-momentum $k$ and with all other quantum numbers represented through $\alpha$. The set $\{g(x_r, p_s)(k)\}$ is a complete set of phase space centered orthogonal functions with centers $(x_r, p_s)$ defined on an integral lattice of spacing $a$ and $b$ as $(x_r, p_s) = (ra, sb)$. Some examples from wavelet theory would be the generalized Wannier functions [52] and the Gabor functions [53, 54]. Note that the phase space points form a discrete not continuum set, with the volume of each cell governed by the uncertainty principle.

With the choice of transon states in Eq. (97), the continuum integrals in Eq. (13) and the equations that follow are all replaced by discrete sums over the phase space points $\{x_r, p_s\}$. The contraction procedure in this case would group states in nearby phase space points into similar states, along with additional conditions imposed by grouping of the other quantum numbers $\alpha$. Thus, when the state vector expansion in this wavelet basis satisfies the conditions for the master equation evolution, these enlarged phase cells will correspond...
to classical statistical cells. With the master equation expressing transitions amongst statistical cells, the problem formally becomes equivalent to that of a classical gas of particles. For this case, considerable information is available about treating master equations. In particular, following standard lines now from classical statistical mechanics, the Boltzmann equation can be derived from Eq. [33], using for example Siegert’s construction [55]. Then from the Boltzmann equation, the hydrodynamic equations can be obtained by known methods [29].

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