High refractive index dielectric spheres present remarkable light-scattering properties in the spectral range dominated by dipolar modes. However, most of these properties are absent for larger spheres under plane wave illumination. Here, a proposal to unravel these dipolar regimes for larger particles under the illumination of a pure dipolar field is presented. This type of illumination ensures that the scattering response of the sphere is purely dipolar. In this scenario, it is shown that Kerker conditions are not only related to duality symmetry and a strong backward-to-forward asymmetric light-scattering, but also to the appearance of non-radiating sources: the so-called hybrid anapoles. Finally, it is shown that all the above-mentioned scattering features under dipolar illumination are reproducible with an experimentally accessible tightly-focused Gaussian beam.

1. Introduction

High refractive index (HRI) particles have received increasing interest during the past decade as building blocks of all-dielectric metamaterials and optical devices. Their unique optical properties are mostly linked with the excitation of single dipolar modes. Nevertheless, for microsized objects, these interesting dipolar responses are hindered due to the contribution from higher multipolar orders under plane wave (PW) illumination.

The optical properties of HRI subwavelength objects include strong backward-to-forward asymmetric scattering. When the dipolar electric and magnetic polarizabilities are identical, at the first Kerker condition, the emergence of the zero optical backscattering condition is satisfied for nanospheres. This condition also preserves the helicity of the incident beam, which is defined as the projection of the total angular momentum onto the wave’s linear momentum direction. Helicity conservation implies the restoration of an internal symmetry of electromagnetism: duality symmetry. Interestingly, the zero optical backscattering condition and duality restoration are linked and fully determined by the asymmetry parameter ($g$-parameter) in the dipolar regime. This magnitude has been widely employed in standard scattering theory to describe the directionality of the electromagnetic fields emitted by particles under study. As a matter of fact, and at the first Kerker condition, helicity conservation and $g$-parameter maximization ($g = 0.5$) occur simultaneously, leading to the absence of backscattered light for nanospheres in the dipolar regime.

The $g$-parameter may also be used as a signature of pure electric (or magnetic) HRI dipolar scatterers, as these cases would necessarily lead to symmetrical scattering, namely, $g = 0$. In this scenario, the amplitude of the electric (or magnetic) dipolar mode may vanish, and hence, dipolar scattered light vanishes accordingly. In this case, it is still possible to have internal electromagnetic energy, which are often referred to as anapoles. These particular optical states, often referred to as anapoles, have attracted attention in a wide range of different fields of physics. However, it is still challenging to obtain anapoles for the most studied phenomenon in the scattering of light: a homogeneous sphere under PW illumination. This difficulty arises from the inevitable contribution of higher multipolar orders in the scattering of a sphere. Nevertheless, several efforts have been made in this direction, and anapoles have been observed for other geometries such as HRI nanodisks, nanowires, or core–shells under PW illumination. Further, tightly-focused radially polarized beams, which do not excite the magnetic multipole components, have been presented as a possible approach to unveil anapoles in HRI spheres in the limit of small particle. However, the simultaneous suppression of the electric and magnetic dipolar scattering efficiencies for larger spheres, at the so-called hybrid anapole, is still a matter of research.

In this work, we show that dipolar spectral regimes can be induced in nano- and microsized dielectric spheres by illuminating with a pure dipolar field (PDF). This illumination does not excite higher multipolar orders, and hence, in analogy to HRI spheres in the limit of small particle, the scattering can be tuned from almost zero forward to perfect zero backscattering.
An intrinsic advantage of this type of illumination is that it allows us to find these light-scattering features for different size parameters. Also, we unravel the so-called hybrid anapole for several nano- and microsized dielectric spheres behaving dipolarly, unveiling a hidden connection with the first Kerker condition. As a result, we refer to these particular non-radiating states, characterized by zero scattering and an enhanced internal electromagnetic energy, as Kerker anapoles. Finally, we show that experimentally accessible tightly-focused Gaussian beams (GB) mimic the above-mentioned light-scattering properties under PDF illumination, which strongly motivates future experimental verification.

2. Results and Discussion

The selective excitation of electric and magnetic dipolar modes by PDF has been shown in a previous work[33] and can be explicitly derived from the multipole expansion of a circularly polarized PW.

\[
\frac{E^{(PW)}}{E_0} = \frac{\hat{x} + i\hat{y}}{\sqrt{2}} e^{i\delta_k} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} C^{(PW)}_{\ell m} \Psi^{(\ell)}_{\ell m}
\]  

(1)

Here \(C^{(PW)}_{\ell m}\) are the PW coefficients and \(\Psi^{(\ell)}_{\ell m}\) the vector spherical wavefunctions (VSWFs).[30] Let us recall that these VSWFs are simultaneous eigenvectors of the square of the total angular momentum, \(J^2\), the \(z\) component of the total angular momentum, \(J_z\), and the helicity operator for monochromatic waves, \(\Lambda\), with eigenvalues \(\ell(e^{+1}), m_z,\) and \(\sigma\), respectively.

At this point, we consider a homogeneous dielectric sphere of radius \(a\) and refractive index contrast \(\epsilon\) centred at the origin \((r = 0)\). As the chosen system is rotationally symmetric around the \(OZ\) axis, \(m_z\) is preserved in the scattering process, while the conservation of helicity, \(\sigma\), crucially depends on the number of multipoles involved in the scattering process.[38] Due to a fundamental property of the Mie coefficients,[38,39] preservation of helicity can only occur for lossless pure-multipolar regions, that is, spectral regimes that can be fully described by just one multipolar order \(\ell\). This fact restricts duality symmetry restoration to small dipolar particles under PW illumination.

To address this issue, we propose for the incident illumination, the use of a PDF, \(\langle E(\Psi) \rangle\), which presents a well-defined \(J_z, J_x,\) and \(\Lambda\) with eigenvalues \(\ell(e^{+1}), m_z,\) and \(\sigma\), respectively. As shown in ref. [33], this is a sectoral and propagating illumination that, by construction, only excites electric and magnetic dipoles. Notice that this phenomenon stems from the fact that, for particles placed at the focus, the only multipoles that can be excited in the particle must also be present in the incident beam.[40,41]

To get some insight into the relevance of illuminating with a PDF, let us consider the \(g\)-parameter, which encodes the optical response of a sphere via the interference between the electric and magnetic Mie coefficients.[13] As previously mentioned, a PDF can induce only a dipolar response, and hence, the \(g\)-parameter, which is constructed from the scattered Poynting vector

\[
g = \frac{\langle \cos \theta \rangle}{\langle S \cdot \hat{r} \rangle} = \frac{\int_{0}^{\pi} \langle S \cdot \hat{r} \rangle \cos \theta \, d\Omega}{\int_{0}^{\pi} \langle S \cdot \hat{r} \rangle \, d\Omega}
\]  

(2)

is strictly identical to the expression derived for small dipolar particles under PW illumination.[35] In both scenarios, the \(g\)-parameter reads as

\[
g = \frac{\text{Re} \{a b^*_f\}}{|a|^2 + |b|^2} = \frac{\sin a_1 \sin b_1 \cos (a_1 - b_1)}{\sin^2 a_1 + \sin^2 b_1}
\]  

(3)

where \(a_1 = i \sin \alpha_1 e^{i\phi_1}\) and \(b_1 = i \sin \beta_1 e^{-i\phi_1}\) denote the electric and magnetic Mie coefficients, respectively.[13]

\[
a_f = \frac{mS_x(mx)S'_y(x) - S_y(x)S'_x(mx)}{mS_x(mx)C'_y(x) - C_y(x)S'_x(mx)}
\]  

(4)

and

\[
b_f = \frac{S_y(mx)S'_x(x) - mS_x(mx)S'_y(x)}{S'_x(mx)C_y(x) - mC_x(mx)S'_y(x)}
\]  

(5)

Here \(S(x) = \sqrt{\frac{2}{\pi}} J_{\ell+\frac{1}{2}}(z)\) and \(C(x) = \sqrt{\frac{2}{\pi}} H_{\ell+\frac{1}{2}}(z)\) denote the Riccati-Bessel functions.

Moreover, the expectation value of the helicity, \(\langle \Lambda \rangle\), and the ratio between the back and forward scattering cross sections, \(\sigma_f/\sigma_t\), are linked and fully determined by the \(g\)-parameter under a dipolar excitation,[12], namely,

\[
\frac{\sigma_f}{\sigma_t} = \frac{1 - \langle \Lambda \rangle}{\Gamma + \langle \Lambda \rangle}
\]  

(6)

Notice that these relations, first brought to the physical scene for small dipolar particles under PW illumination,[14] can now be extended to arbitrary sized dielectric spheres illuminated by a PDF. As a matter of fact, a map of \(g\) for a dielectric sphere as a function of the refractive contrast index \(m\) and the size parameter \(\gamma = mx = mka\), see Figure 1a, reveals successive first Kerker conditions \((g = 0.5)\) even for low refractive index dielectric spheres. These first Kerker conditions give rise to \(\langle \Lambda \rangle = 2g = +1\) and, hence, to the restoration of the electromagnetic duality symmetry under PDF illumination. This phenomenon can be inferred from Figure 1b, where the zero optical backscattering is also presented. Notice that under PW illumination, the conservation of helicity requires small dipolar particles, for example, \(\gamma < 4\).

Moreover, an interesting phenomenon that is unveiled under PDF illumination is the emergence of pure electric (or magnetic) scattering regimes for \(\sin \beta_1 = 0\) (or \(\sin \alpha_1 = 0\)). Note that both conditions give rise to \(g = 0\), according to Equation (3). In order to infer each scattering regime in Figure 1a, we distinguish in Figure 2a the three solutions that lead to \(g = 0\). These correspond to the destructive interference of the electric and magnetic Mie coefficients, namely, \(\alpha_1 = \beta_1 = \pm \pi/2\) (green line) or to the above mentioned pure electric (\(\sin \beta_1 = 0\)) or magnetic (\(\sin \alpha_1 = 0\)) scattering regimes in orange and blue lines, respectively. Particularly, when these latter two lines cross each other, zero scattering points appear under dipolar excitation, that is, the scattering.
cross section \(\sigma_{\text{tot}} \propto (\sin^2 \alpha_1 + \sin^2 \beta_1) = 0\), and, hence, the sphere is effectively invisible to the incoming light. These singularities, which are visible in Figure 1a, have been predicted in a recent theoretical work under the name of hybrid anapole modes.\(^{34}\) In that work, the solutions of the so-called hybrid anapoles are found by solving transcendental equations that require numerical methods. However, it is straightforward to notice that these anapoles arise analytically by imposing the first Kerker condition with zero scattering, that is, \(S'_m(x) = 0\) with \(S'_s(x) = 0\) or \(S'_s(mx) = 0\) with \(S_s(x) = 0\). Note that both conditions lead to \(|a_1| = |b_1| = 0\), according to Equations (4) and (5).

To get a deeper understanding of these non-radiating anapoles, we have depicted the g-parameter as a function of the dipolar electric and magnetic scattering phase-shifts (see Equation (3)) in Figure 2b. As it can be inferred, the anapole emerges, under PDF illumination, when the condition \(\alpha_1 = \beta_1\) is met (see the vertical black line). Strikingly, this optical invisibility condition presents a direct connection with both Kerker conditions and light transport phenomena in spheres: at the first Kerker condition \((\alpha_1 = \beta_1)\) the g-parameter is maximized, \(g = 0.5\) (intense red color) and, then, the zero optical backscattering condition is met. On the other hand, when \(\alpha_1 = -\beta_1\), the generalized second Kerker condition (GSKC) is satisfied (intense blue color) and, then, the optimal backward scattering condition is guaranteed.\(^{35}\) Notice that achieving these optical responses (either first Kerker condition or GSKC) in the vicinity of these anapoles not only changes the directionality of the scattered light drastically but also the transport mean free path\(^{42,43}\) and the dipolar optical forces,\(^{44}\) as \(g\) can flip from \(g = 0.5\) (duality) to \(g \approx -0.5\) (nearly anti-duality\(^{11}\)) within a small perturbation of the scattering phase-shifts. Figure 1b summarizes the physics behind these phenomena at \(y \approx 11\) where we depict the abrupt change of the scattered helicity of a lossless sphere with \(m = 2.4\). In this line, it is crucial to notice that these non-radiating anapoles cannot be found for lossy spheres since \(a_1 \neq b_1\) for all \(\ell\) in the presence of absorption or gain, as mathematically demonstrated in ref. \[39\]. For all the reasons mentioned above, we honestly believe that “Kerker anapole” is a name that more precisely describes the origin of this type of anapoles. On physical grounds, non-radiating anapoles cannot emerge for lossy systems such as plasmonic particles or HRI nanostructures in the visible spectral range\(^{45}\) since an object that absorbs must radiate. As a matter of fact and in Figure 3a, we confirm that the Kerker anapole state is not reached for a 200 nm Silicon (Si) sphere in the visible spectral range under PDF illumination due to absorptive effects. In contrast, in the lossless scenario \((\mathcal{G}(m) = 0)\) illustrated in Figure 3b, we can
clearly observe the Kerker anapole at $\lambda \approx 460$ nm in this ideal non-absorbing scenario.

Hitherto, we have discussed the optical properties of homogeneous dielectric spheres under a PDF excitation. However, even though these sectoral and propagating beams fulfill Maxwell’s equations, its experimental implementation is not standard yet. In this regard, we alternatively propose the use of tightly-focused Gaussian beams\(^{[46-53]}\) whose multipolar decomposition has been extensively analyzed. Specifically, we apply the method presented in ref. \([50]\), in which the multipolar decomposition is determined by the beam coefficients $C_m^n$. The independent control of both angular momentum and polarization, allows for the generation of a GB with well-defined $m = \sigma = 1$. In this case, the relative weight of each multipolar order with respect to the dipolar order can be highly minimized when the beam is tightly focused at the center of the object. As a matter of fact, by using commercial data from high numerical apertures and focal length, namely, NA = 0.9, 0.95, 1, and $f = 1.8$ mm, and fixing the optimal ratio between focal length and beam waist, $f/\omega_0 \approx 0.9$, the relative weights can be straightforwardly obtained. These are shown in Figure 4a, where a tightly focused GB is also presented. This figure shows that for higher NA values, the dipolar character of the beam is strengthened, both in absolute and relative terms.

The scattering efficiency of a sphere illuminated by a tightly focused GB with well-defined helicity $i^9$\(^a\) is given by:

$$Q_{GB}^{m,n} = \sum_{\ell} \frac{(Q_{\ell} + 1)}{\pi} |C_m^n|(|\sin^2 \alpha_n + \sin^2 \beta_n)| $$

where the value of the beam shape function $C_m^n$ heavily decreases as the multipolar order $\ell$ is increased. By using the computed relative weights for, for example, NA = 0.95, shown in Figure 4a, we calculate in Figure 4b the scattering efficiency as a function of the contrast index $m$ for a fixed size parameter of $x = 4.49$. As it can be inferred, the scattering efficiency practically vanishes for several refractive indexes, namely, $Q_{GB}^{m,n} \approx 9 \times 10^{-1}$, for $m = 1.72, 2.43, 3.13, 3.83$. These are examples of non-radiating Kerker anapoles that give rise to the singularities of the g-parameter under PDF illumination, according to Figure 1a. However, under PW illumination (see Figure 4c), Kerker anapoles cannot be characterized as non-radiating sources since higher multipolar orders predominantly contribute to the scattering optical response of the spheres.

As a further verification of the existence of non-radiating sources under the considered tightly-focused GB with NA = 0.95, we also compute the internal electromagnetic energy normalized with respect to the GB energy in the absence of particle. The internal electromagnetic energy is calculated from the internal fields\(^{[50]}\), $E_{int}^\rho$, and $H_{int}^\rho$, using the internal Mie coefficients \(^{[13]}\). Therefore, the ratio between internal, $W_{int}$, and incident, $W$, energies in the particle’s volume, $V$, reads as:

$$\frac{W_{int}}{W} = \epsilon_i \int \left( |E_{int}^\rho|^2 + Z^2 |H_{int}^\rho|^2 \right) dV$$

where $\epsilon_i = m_{\rho}^\rho$, $Z = \sqrt{\mu_0/\epsilon_0} = (1/\epsilon_0)^{1/2} = Z_0/m_{\rho}^\rho$, and $m_{\rho}^\rho$ is the refractive index of the particle. As it can be concluded from Figure 5, the scattering efficiency is practically suppressed at the Kerker anapoles while the normalized internal electromagnetic is considerably enhanced. This phenomenon confirms the existence of non-radiating induced sources in the depicted spheres. Importantly, these are natural materials in the visible spectral range: the Kerker anapole in Figure 5a arises for a diamond sphere embedded in air with a radius of 380 nm and $\lambda_0 = 532$ nm.
nm. The other Kerker anapole in Figure 5b could correspond to an aluminium arsenide (AlAs) sphere embedded in air with a radius of 430 nm and $\lambda_0 = 604$ nm. Interestingly, this Kerker anapole also corresponds to a HRI germanium (Ge) microsphere embedded in water with a radius of 1025 nm in the telecom spectral range, namely, $\lambda_0 = 1900$ nm.

Finally, let us comment on an overlooked aspect regarding the Kerker anapoles and the Cartesian decomposition into the so-called electric, magnetic, and toroidal dipole moments. These are only meaningful in the long-wave approximation\cite{16}, which is only valid for scatterers in the limit of small particle, namely, $x \ll 1$. However, as we have shown, Kerker anapoles arise for relatively large values of $x$, well beyond the small particle limit. See for instance Figures 1, 4, and 5. As a result, the traditional Cartesian decomposition is not suitable for the Kerker anapoles analysis, and an exact decomposition must be considered.\cite{16}

3. Conclusion

In conclusion, we have shown that dipolar spectral regimes can be unveiled for nano- and microsized dielectric spheres under PDF illumination. As a result, we have demonstrated that unexplored Kerker conditions can lead to backward-to-forward asymmetric scattering, or equivalently, from dual to anti-dual behavior even for microsized spheres. Moreover, we have shown that non-radiating Kerker anapoles emerge for several microsized dielectric spheres behaving dipolarly, opening new insights into optical invisibility. Finally, we have also demonstrated that Gaussian beams tightly-focused by commercial microscopic objectives could mimic the scattering features produced by these PDFs. These findings could drive the implementation of new experiments beyond the actual physical picture, which is mostly restricted to HRI materials in the limit of small particle.

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Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

Research data are not shared.

Keywords

optical anapoles, kerker conditions, mie scattering

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