Contact interaction of a two-layer package of nanobeams package of Bernoulli-Euler nanobeams with gap between them

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Abstract. In this work, mathematical model of contact interaction of two parallel Bernoulli-Euler nanobeams with a small gap between them is built on the basis of a modified couple stress theory. Contact interaction is considered in accordance with B.Ya. Cantor's theory. The influence of size-dependent coefficient on contact interaction and the nature of the oscillations have been studied. Wavelet analysis, methods of the qualitative theory of differential equations, methods for analyzing the sign of the largest Lyapunov's exponent are used to solve and analyze.

1. Introduction
Mechanical multilayer systems of micro- and nanoscale beams with gaps between them are components of modern devices used in various fields: mechanical engineering, instrument making, aerospace industry, medicine and gyroscopy. Micro and nano-sized beams are subject to external influences of a different nature, which has a significant impact on the accuracy and reliability of the entire device during operation. The gap between the elements of the system often leads to chaotic oscillations of entire structure even if there is a slight deflection. The behavior of the system upon contact interaction of its elements with each other is an urgent issue. Works [1, 2] are devoted to oscillatory regime as a result of contact interaction of beams. Classical solid mechanics is not able to consider size-dependent behavior of a multilayer micro- and nanosystem due to the lack of a parameter for scaling effect. Today, there are various theories that allow modeling scale effects in the continuum. It is worth to mention nonlocal, modified couple stress, surface and gradient theory of elasticity. A variety of scientific studies on various topics associated with contact interaction of the elements of a multilayer system with a gap between them (scaling theory; external influencing factors; contact interaction models) suggest the importance of the issue, its versatility and many areas for the application of systems.

In works [3-6], mathematical models of size-dependent and geometrically nonlinear beams, plates and shells were built. However, contact interaction of nanobeams has not been studied in them.

2. Problem formulation
In the work offers mathematical model of a two-layer nanosystem. It consists of two parallel nanobeams, coupled through boundary conditions under transverse load. The drawing of the beam structure is shown in figure 1.

Considered beam structure is represented as a two-dimensional region of the space \( \mathbb{R}^2 \) in a Cartesian rectangular coordinate system: a midline is drawn in the body of nanobeam 1, OX axis directed from left to right along the midline, OZ axis goes down, perpendicularly to OX axis. In the indicated coordinate system, a package of two beams is a two-dimensional region \( \Omega = \{ x \in [0, a]; h \leq z \leq h_1 + 3h \}, 0 \leq i \leq \infty \) (see figure 1).

The following hypotheses and assumptions were used to build mathematical model of contact interaction of two nanobeams: nanobeams are single-layer, isotropic, elastic and obey Hooke's law; the longitudinal size of nanobeams significantly exceeds their transverse dimensions of unit thickness; contact pressure is taken according to the model of Cantor B.Ya. [7]; normal stresses on sites that are parallel to the axis are negligible; geometric nonlinearity is determined in the form of T. Karman [8]; size-dependent effects are determined by a modified couple stress theory proposed by Young [9]. To build a model of contact interaction of nanoscale beams according to the model of Kantor B.Ya., it is necessary to introduce the term \((-1)^i K(w_i - w_z - h)\) \(\Psi\), where number of nanobeams are denoted by \(i = 1, 2\) and function \(\Psi\) is determined by the following formula \(\Psi = \frac{1}{2}[1 + \text{sign}(w_i - h_i - w_z)]\), i.e. \(\Psi = 1\), if there is coupling between nanobeams \(- w_i > w_z + h_i\), otherwise coupling is absent. Stiffness coefficient of transverse compression of the structure in coupling areas is denoted by \(K\) and the gap between nanobeams is denoted by \(h_i\).

The equations of motion of nanobeams, considering the first-order hypothesis, boundary and initial conditions are derived from the Hamilton-Ostrogradsky energy principle. The equations of motion of a two-layer package of Bernoulli - Eulerian nanobeams in displacements considering energy dissipation are written in the following form:
As an example, we consider a beam nanostructure with the following parameters: length $a = 200$ nm, thickness $2h = 4$ nm, elastic modulus $E = 1.7233 \times 10^{-15} kg/s/nm^2$, Poisson's ratio $\nu = 0.3$, clearance $h_k = 0.04 nm$.

Desired system (1) is reduced to a dimensionless form using the following relations:

$$
\frac{w}{2h} = \frac{w}{\bar{w}} = \frac{ua}{(2h)^2}, \quad \bar{x} = \frac{x}{a}, \quad \lambda = \frac{a}{(2h)^{1/2}}, \quad \bar{q} = \frac{q}{2(1+\nu)^2} \left(\frac{a^2(1-\nu^2)}{(2h)^4E}\right),
$$

$$
\bar{t} = \frac{t}{\tau}, \quad \tau = \frac{a}{c}, \quad c = \sqrt{\frac{Eg}{(1-\nu^2)\rho}}, \quad \bar{\epsilon} = \frac{\epsilon}{c}.
$$

Here $g$ is the acceleration of gravity, $\rho$ is the density.

Rigid fixing of both ends of the nanobeams of was chosen as boundary conditions:

$$
\dot{w}_1(0,t) = \dot{w}_1(1,t) = u_1(0,t) = u_1(1,t) = \frac{\partial w_1(0,t)}{\partial x} = \frac{\partial w_1(1,t)}{\partial x} = 0, i = 1, 2
$$

and zero initial conditions.

Alternating transverse load distributed over the surface affects upper nanobeam:

$$
q = q_0 \sin(\omega t),
$$

3. Numerical experiment

As an example, we consider a beam nanostructure with the following parameters: length $a = 200$ nm, thickness $2h = 4$ nm, elastic modulus $E = 1.7233 \times 10^{-15} kg/s/nm^2$, Poisson's ratio $\nu = 0.3$, clearance $h_k = 0.04 nm$.

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and zero initial conditions.

Alternating transverse load distributed over the surface affects upper nanobeam:

$$
q = q_0 \sin(\omega t),
$$
Where $q_0$ is amplitude, $\omega_p = 5.1$ is frequency of forcing vibrations.

As a result of contact interaction of nanobeams, their complex vibrations are chaotic. Under size-dependent coefficient $\gamma_2 = 0.1$, the upper nanobeam oscillates at the dominant frequencies: $\omega_p = 5.1$, $\omega_1 = 0.85$, $\omega_2 = 2\omega_1 = 1.7$, $\omega_3 = 3.1$ (table 1).

| Table 1. Dynamic characteristics of nanobeams for $\gamma_2 = 0.1$ |
|---------------------------------------------------------------|
| Nanobeam 1 | phase portraits | Fourier power spectra | the 2D Morlet wavelet spectra |
| ![phase portraits](image1) | ![Fourier power spectra](image2) | ![the 2D Morlet wavelet spectra](image3) |

When size-dependent coefficient $\gamma_2 = 0.3$, upper nanobeam oscillates at the dominant frequencies: $\omega_p = 5.1$, $\omega_1 = 0.95$, $\omega_2 = 2.05$, $\omega_3 = 3.95$ (table 2).

| Table 2. Dynamic characteristics of nanobeams for $\gamma_2 = 0.3$ |
|---------------------------------------------------------------|
| Nanobeam 1 | phase portraits | Fourier power spectra | the 2D Morlet wavelet spectra |
| ![phase portraits](image4) | ![Fourier power spectra](image5) | ![the 2D Morlet wavelet spectra](image6) |
4. Conclusion

Work offers mathematical model of contact interaction of a two-layer nanosystem, which consists of two parallel nanobeams, interconnected via boundary conditions, under the effect of a transverse load. As a result of the contact interaction of nanobeams, their complex vibrations become chaotic. The size-dependent coefficient affects the dominant frequencies at oscillations occur.

Acknowledgments

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