Bad Metals from Fluctuating Density Waves

Blaise Goutéraux

NORDITA, Stockholm

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Acknowledgments

Based on
‘Bad Metals from Fluctuating Density Waves’, [arXiv:1612.04381], and
‘Hydrodynamic transport in phase-disordered charge density wave states’,
[arXiv:1702.05104]
together with Luca Delacrétaz, Sean Hartnoll and Anna Karlsson

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Central motivation: bad metallic transport

Two experimental challenges for theorists [Hussey, Takenaka & Takagi’04]:

- $T$-linear resistivity violating the MIR bound: no quasiparticles

$$\ell k_F \gtrsim \hbar \quad \Rightarrow \quad \rho \equiv \sigma^{-1} = \frac{m}{ne^2 \tau_{tr}} \lesssim \rho_{MIR} \sim 150 \, \mu\Omega.\text{cm}$$

- Optical conductivity: far IR peak ($\sim 10^2 \text{cm}^{-1}$) moving off axis as $T$ increases to room temperature.
Planckian dynamics

\[ \rho = \frac{m}{ne^2\tau_{tr}} \sim T \quad \Rightarrow \quad \tau_{tr} = \tau_P \equiv \frac{\hbar}{k_B T} \]

[Bruin et al, Science 339 804 (2013)]
The Planckian timescale

**Universal scale** in all systems at finite temperature which follows from dimensional analysis

\[
[\hbar] = J \cdot s, \quad [k_B] = J \cdot K^{-1}, \quad [T] = K \quad \Rightarrow \quad \tau_P = \frac{\hbar}{k_B T}
\]

In strongly-coupled, quantum systems, expected to be the **fastest equilibration time** allowed by Nature and Quantum Mechanics [Sachdev,Zaanen]. At room temperature

\[
\tau_P \sim 25 \text{fs}
\]
Off-axis peaks in optical conductivity data (1)

\( \text{Bi}_2\text{Sr}_2\text{CuO}_6 \)
[PRB 55 14152 (1997)]

\( \text{Ca}_2\text{RuO}_3 \)
[PRB 66 041104 (2002)]

\( \text{La}_{1.9}\text{Sr}_{0.1}\text{CuO}_4 \)
[Phil Mag 84 2847 (2004)]

\( \text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta} \)
[J. of Phy: Cond Mat 19 125208 (2007)]

\( \text{Na}_{0.7}\text{CoO}_2 \)
[PRL 93 237007 (2004)]

\( \text{Ti}_2\text{Ba}_2\text{CuO}_6+\delta \)
[PRB 51 3312 (1995)]
Off-axis peaks in optical conductivity data (2)

La$_2$126
[PRB 67 134526 (2003)]

$\theta$-(BEDT-TTF)$_2$I$_3$ (a)
[PRL 95 227801 (2005)]

$\theta$-(BEDT-TTF)$_2$I$_3$ (c)
[PRL 95 227801 (2005)]

YBa$_2$(Cu$_{1-x}$Zn$_x$)$_3$O$_{7-\delta}$
[PRB 57 081 (1998)]

V$_2$O$_3$
[PRL 75 105 (1995)]

LiV$_2$O$_4$
[PRL 99 167402 (2007)]
Planckian dynamics in the optical conductivity

\[\hbar \omega_{\text{peak}} \sim k_B T, \quad \hbar \Delta \omega \sim k_B T,\]
- These observations suggest that **Planckian dynamics** is a defining feature of both **ac and dc transport** in bad metals.

- Planckian dynamics also emerge in the **low energy effective description** of strongly-coupled (holographic) quantum matter.

- **Universal** low energy effective theory?
I will offer a theory based on hydrodynamics and spontaneous translation symmetry breaking which
- leads to **small dc conductivities**;
- accounts for the **far IR off-axis peak** in $\sigma(\omega)$;
- naturally **relates** the dc and ac transport timescales.

**Disclaimer**: effective low energy theory of transport, not a microscopic theory.
Spontaneous translation symmetry breaking

![Diagram showing the phase diagram for spontaneous translation symmetry breaking.]

- **Temperature, $T$ (K)**
  - $T_N$, $T^*$
  - Pseudogap
  - Strange metal
  - Charge order
  - Fermi liquid

- **Hole doping, $p$**
  - $p_{\text{min}}$, $p_{c1}$, $p_{c2}$, $p_{\text{max}}$

- **Co concentration, $x$**
  - $T_{\text{SDW}}$, $T_c$
  - SDW, SC

- **Pressure, $P$ (kbar)**
  - $T_{\text{SDW}}$, $T_c$
  - SDW, SC
  - (TMTSF)$_2$PF$_6$
Hydrodynamics

Short-lived quasiparticles: conserved quantities are more fundamental for late-time transport

\[ \partial_t \epsilon + \vec{\nabla}\vec{\pi} = 0 \]
\[ \partial_t \pi^i + \nabla_k \tau^{ik} = 0 \]
\[ \partial_t \rho + \vec{\nabla} j = 0 \]

Hydrodynamics: long wavelength description of the system

[credit: Beekman et al’16]
We also wish to include a CDW:

\[ \rho(x) = \rho_0 \cos [Qx + \phi(x, t)] \]

The phase \( \phi(x, t) \) is a new dof coming from the SSB of translations (Goldstone): ‘phonon’ of the electronic crystal.
Constitutive relation for the current and the Goldstone

\[ j = nev + \ldots, \quad \dot{\phi} = v + \ldots \]

Standard procedure to extract retarded Green’s functions [Kadanoff & Martin’63].

**Weak disorder**: finite momentum lifetime \(1/\Gamma_\pi\)

\[ \partial_t \pi^i + \nabla_k \tau^{ik} = -\Gamma_\pi^i \]

and pins the Goldstone \(\phi\) with a small mass \(k_0\):

\[ f = \frac{\kappa}{2} \phi \left( -\partial^2 + k_0^2 \right) \phi \]
Conductivity of a pinned CDW [Grüner’88]

- Conductivity

\[ \sigma = \frac{ne^2}{m} \frac{-i\omega}{(-i\omega)(\Gamma_\pi - i\omega) + \omega_0^2} \]

- Peak at \( \omega_0 = k_0 \sqrt{\kappa/\chi_{\pi\pi}} \),
  width \( \Gamma_\pi \).

- **Dc insulator** due to Galilean invariance.
We wish to describe conducting CDWs. Two mechanisms

1. Relax Galilean symmetry;
2. Introduce phase disordering by mobile dislocations.
Modified constitutive relation for the current

\[ j = qv - \sigma_o \nabla \mu + \ldots, \quad \dot{\phi} = v + \ldots \]

\( \sigma_o \) is a **diffusive** transport coefficient encoding charge transport **without momentum drag**.

Conductivity

\[ \sigma = \sigma_o + \frac{q^2}{\chi_{\pi\pi}} \frac{-i\omega}{(-i\omega)(\Gamma_\pi - i\omega) + \omega_o^2} \]

Non-zero dc conductivity

\[ \sigma_{dc} = \sigma_o + O(\Gamma_\pi) \]

Can be **small** even for weak momentum relaxation: **bad metal**.
However, recall that $\omega_{peak}, \Delta \omega \sim O(1/\tau_P)$: quantum!

Quantum fluctuating cdws in underdoped cuprates [Kivelson et al’03].

Quantum fluctuating cdws in the bad metallic regime?
In 2d, crystals can **melt by proliferation of topological defects** in the crystalline structure [Nelson & Halperin’79].

At \( T = 0 \): quantum melting [Kivelson et al’98, Beekman et al’16].

The phase gets disordered (\( \sim \) BKT) at a rate \( \Omega \): **flow of mobile dislocations**, ‘flux-flow’ formula [arXiv:1702.05104].
Now the conductivity reads

\[
\sigma = \frac{ne^2}{m} \frac{(\Omega - i\omega)}{(\Omega - i\omega)(\Gamma_\pi - i\omega) + \omega_o^2}, \quad \sigma_{dc} = \frac{ne^2}{m} \frac{1}{\Gamma_{CDW}}
\]

\[
\Gamma_{CDW} = \Gamma_\pi + \frac{\omega_o^2}{\Omega}
\]

New transport inverse timescale, **non-zero** even if \(\Gamma_\pi \sim 0\).

**Off-axis peak** for sufficiently small \(\Omega\) or large pinning \(\omega_o\)

\[
\omega_o \geq \frac{\Omega^3}{\Gamma_\pi + 2\Omega}
\]
Bad metallic transport from fluctuating CDWs

- Neglect momentum relaxation $\Gamma_\pi \ll \omega_0, \Omega$:

$$\sigma_{dc} = \frac{n e^2 \Omega}{m \omega_0^2}$$

- The width and position of the peak are controlled by $\Omega, \omega_0$. The data shows $\Omega \sim \omega_0 \sim k_B T/\hbar$

$$\Rightarrow \rho_{dc} = \frac{1}{\sigma_{dc}} \sim \frac{m}{n e^2} \frac{k_B T}{\hbar}$$

$T$-linear resistivity!

- Hydrodynamics of fluctuating CDWs provide a natural mechanism whereby the ac and dc conductivities are controlled by the same Planckian timescale.
Experimental signatures: spectrum
Experimental signatures: spatially resolved conductivity

\[ \sigma_1(\omega, k) \]

\[ \omega \]
Resistivity upturns from fluctuating cdws

\[ \rho = \frac{m}{ne^2} \Gamma_{CDW}, \quad \Gamma_{CDW} = \Gamma_\pi + \frac{\omega_o^2}{\Omega} \]

An upturn occurs as \( \Omega \) decreases and phase fluctuations dominate \( \Gamma_{CDW} \): relation to underdoped cuprates and static charge order?

Violation of the Wiedeman-Franz law: \( \rho / T \sim 1/\Omega \gg L_o \).
Some open questions

- Typical frequency scales of order $T$: at the edge of validity of hydrodynamics $\omega \ll T$.

- The role played by the Planckian timescale is indicative of quantum criticality \cite{Sachdev}: quantum critical computation.

- Work in progress: use Gauge/Gravity duality to compute non-hydrodynamic transport in a metallic phase with spontaneously broken translation symmetry.