THE JONES POLYNOMIAL OF AN ALMOST ALTERNATING LINK

Dean D. Spyropoulos (despyropoulos@vassar.edu)
Vassar College [Mentor: Adam Lowrance]

Abstract of Report Talk: The Jones polynomial is a knot invariant discovered by Vaughan Jones in 1984. A fundamental open question concerning the Jones polynomial is the conjecture that the Jones polynomial detects the unknot; that is, if the Jones polynomial of a knot $V_K(t)$ is the same as the Jones polynomial $V_\bigcirc(t)$ of the unknot $\bigcirc$, then $K$ is the unknot $\bigcirc$. This Jones unknotting conjecture is known to hold for many families of knots (e.g. alternating, adequate, semi-adequate), but remains open in general. For each $\ell \geq 2$, Eliahou, Kauffman, and Thistlethwaite found infinite families of $\ell$-component links whose Jones polynomial is the same as the $\ell$-component unlink.

A link is almost alternating if it has a diagram that can be transformed into an alternating diagram via one crossing change. We show that the Jones unknotting conjecture holds for almost alternating links. Furthermore, we show that no almost alternating link with $\ell$ components has the same Jones polynomial as the $\ell$-component unlink. Our results follow from new formulas for the first and last two coefficients of the Jones polynomial of an almost alternating link. These formulas are obtained from a relationship between the Tutte polynomial and Jones polynomial for alternating links.

Received: July 22, 2018