Unequal mass binary black hole plunges and gravitational recoil

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Abstract

We present results from fully nonlinear simulations of unequal mass binary black holes plunging from close separations well inside the innermost stable circular orbit with mass ratios \( q \equiv M_1/M_2 = \{1, 0.85, 0.78, 0.55, 0.32\} \), or equivalently, with reduced mass parameters \( \eta \equiv M_1 M_2 / (M_1 + M_2)^2 = \{0.25, 0.248, 0.246, 0.229, 0.183\} \). For each case, the initial binary orbital parameters are chosen from the Cook–Baumgarte equal-mass ISCO configuration. We show waveforms of the dominant \( \ell = 2, 3 \) modes and compute estimates of energy and angular momentum radiated. For the plunges from the close separations considered, we measure kick velocities from gravitational radiation recoil in the range 25–82 km s\(^{-1}\). Due to the initial close separations our kick velocity estimates should be understood as a lower bound. The close configurations considered are also likely to contain significant eccentricities influencing the recoil velocity.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

The Laser Interferometer Space Antenna (LISA) will offer a unique look at merging supermassive black holes (BHs) [3, 4]. When galaxies collide, the BHs at the centre of each galaxy [5, 6] will merge and radiate gravitational waves. In a generic merger situation, the colliding BHs will have different masses and spins; thus they will radiate gravitational waves anisotropically. This anisotropic radiation will carry both net angular and linear momenta [7]. The linear momentum emitted implies a recoil of, or kick to, the BH product of the merger. A large enough recoil may cause the BH resulting from the merger to be strongly offset from the centre of the galaxy or potentially even kicked out of small galaxies [8, 9]. Such a scenario would have a significant impact on the standard picture of merger tree history of galaxies.
A complete investigation of the gravitational recoil from unequal mass BH mergers must include the contributions to the kick from the inspiral, merger and ringdown stages of the life of the binary. Given the tremendous progress that numerical relativity has recently made in simulating binary black hole (BBH) systems \cite{10–15}, fully numerical investigations can now be used to investigate gravitational recoil. Recent numerical studies \cite{36, 18} as well as post-Newtonian estimates \cite{16, 17} suggest that most of the kick gets accumulated during the plunge. Motivated by these observations, we carried out a series of unequal mass BBH simulations, using initial data that correspond to BHs plunging from innermost stable circular orbit (ISCO), i.e. initial separations of $2.34M$. In particular, we focus on the dominant $\ell = 2, 3$ modes and study how the energy radiated in these modes changes as the mass ratio is varied. Additionally, we calculate the energy and angular momentum radiated, and from the gravitational radiation we estimate kick velocities in the range $25–82$ km s$^{-1}$ acquired by the final BH.

Soon after the completion of our study, Baker \textit{et al} \cite{36} studied a much further separated system and Gonzales \textit{et al} \cite{18} published fully relativistic kick estimates from BHs with initial separation of $7M$ for a broad range of mass ratios, from $q = M_1/M_2 = 1$ to $q = 0.25$, or equivalently $\eta = q/(1 + q)^2$ from 0.25 to 0.16. They estimated a maximum kick of 175.70 km s$^{-1}$ for a mass ratio of $\eta = 0.195$. Using close-limit approximation techniques Sopuerta \textit{et al} investigated gravitational wave recoil and also the effect of small eccentricities \cite{1, 2}.

1.1. Initial data

The initial data sets are constructed via the puncture method using a spectral code \cite{19, 20}. The essence of this method is to solve the Hamiltonian constraint for the conformal factor $\phi$. The initial three-metric is conformally flat, maximally sliced, and the extrinsic curvature is given by the Bowen–York solution to the momentum constraint. The conformal factor $\phi$ is used to set the initial lapse as $\alpha = \phi^{-2}$ \cite{13, 21}, while the initial shift is $\beta^i = 0$.

For the equal mass setup, we evolve the so-called QC-0 initial data set \cite{22}. This is intended to represent a quasi-circular configuration of inspiralling puncture BBHs at the ISCO. QC-0 data have been used as the starting point by other studies \cite{11, 13, 14}. The BHs in QC-0 perform about half of an orbit prior to merging \cite{13, 14}; that is, QC-0 looks more like a plunge/grazing collision. The intersection of the event horizon with the initial slice, however, has the topology of two separate spheres \cite{11}.

The puncture BBH data of the Bowen–York type are defined by the bare masses $m_{1,2}$ of the BHs, their coordinate locations $C_{1,2}$, assumed to be along the $x$-axis in the $xy$-plane, and their linear momenta $P_{1,2}$, pointing along the $y$-axis. In the construction of the initial data, we vary $m_2 = \{1, 1.2, 1.3, 2, 4\}m_1$ while keeping $m_1$ fixed to its QC-0 value of 0.453. We also keep fixed to the QC-0 values the puncture coordinate separation $d = |C_1 - C_2| = 2.34$ and the momentum parameters $P_{1,2} = \pm 0.333$, which means that the angular momentum value, $J = 0.779$, also remains unchanged. Note that the total ADM mass, $M_{\text{ADM}}$, of the configurations does change as do $J$ and $P$ when given in ADM mass units. Due to this setup, our initial data of unequal mass BBH systems do not obey the quasi-circular orbit condition of minimal binding energy \cite{23}. The present work is aimed at investigating the effects of varying, in the QC-0 setup, the bare mass parameter only. The motivation behind this choice was to study gravitational recoil starting from the ISCO plunge with the simplest parameter exploration. There is strong indication that the kick imparted to the BH that has resulted from the merger is dominated by the gravitational recoil during the plunge from ISCO \cite{16}. We are currently extending the present study and investigating both unequal mass BBH mergers...
with initial separations outside of ISCO in quasi-circular orbit and plunge configurations with post-Newtonian orbital parameters [16].

Table 1 summarizes the parameters in our simulations. We list the total mass $M = M_1 + M_2$, the mass ratio parameter $\eta = M_1 M_2 / M^2$ where $M_{1,2}$ are the irreducible masses of the BHs computed from their individual apparent horizon (AH) areas, as well as angular momentum $J$ and momentum parameter $P$ in terms of the reduced mass, $\mu = M_1 M_2 / M$.

Table 1 also provides the time $t_{\text{AH}}$ in $M_{\text{ADM}}$ units when a common AH is first found. For QC-0 the orbital period of the equal mass case is estimated as $t = 37.4 M_{\text{ADM}}$. The drop in the time to merger $t_{\text{AH}}$ in our results should not be interpreted as ‘unequal mass binaries merge faster’. The effect is due to our approach in which the angular momentum of the configuration decreases as the bare mass ratio is decreased. Ideally one would like to compare initial data sets which are far separated and have the same orbital frequency. It is possible that, for our cases with $q \leq 0.55$, a common event horizon already exists in the initial data slice, and therefore we are evolving a single, distorted BH.

1.2. Methods

The evolutions were carried out using a code that solves the BSSN 3+1 formulation of Einstein’s equations [24–26]. We use the ‘moving punctures’ approach without excision [13, 14]. The code was produced by the Kranc code generation package [27] and uses the Cactus infrastructure. The simulations were performed using fourth-order accurate centred finite differencing, except for the advection terms which were one-sided and second order accurate. The temporal updating is carried out with a three-step iterative Crank–Nicholson scheme with a Courant factor of 0.25. Tests in the waveform for the equal-mass setup using resolutions $h = \{1/16, 1/20, 1/32\}$ produced convergence slightly below second order. Mesh refinement in the code is provided by Carpet [28], and tracking of AHs is handled by AHFinderDirect [29].

The gauge conditions used were modified versions of the 1+log lapse and $\Gamma$-driver shifts. Specifically, the lapse $\alpha$ was evolved using $\partial_t \alpha = -2\alpha K$, where $K$ is the trace of the extrinsic curvature. On the other hand, the shift vector was obtained from [14]: $\partial_t \beta^i = F B^i$ and $\partial_t B^i = \delta_i \ddot{\Gamma}^i - \beta_j \partial_j \ddot{\Gamma}^i - \xi B^i$ with $\xi$ a constant dissipative parameter and $F = 3\alpha / 4$, which guarantees that the asymptotic gauge speed associated with the longitudinal shift components is equal to the speed of light. The evolutions were started with $\beta^i$ and $B^i = 0$. The advection term $\beta^i \partial_j \ddot{\Gamma}^i$ removes certain zero-speed modes of the system as analysed in [30]. The parameter $\xi$ can be used to tune the rate of horizon expansion over the course of the evolution; large values lead to faster horizon growth. We have used values in the range $2 \leq \xi \leq 5$ and
Figure 1. The irreducible mass of the apparent horizon as a function of time for different mass ratios $q$.

We have not found any instabilities in this range. For the runs reported in this paper, we used $\xi = 4$.

Our computational domain consisted of fixed 2:1 mesh refinements with five levels. The finest grid spanned $-2 \leq x, y \leq 2$ and $0 \leq z \leq 2$, with the coarsest $-96 \leq x, y \leq 96$ and $0 \leq z \leq 96$ (we use bitant symmetry in the $z$ direction). All the refinement levels except for the coarsest have the same number of grid points. We have run the unequal BBH models at two different resolutions, $h = 1/16, 1/20$.

1.3. Results

Figure 1 shows the irreducible mass of the AHs as a function of time, where $M_{\text{irr}} = \sqrt{A/16\pi}$ and $A$ is the AH area. In all simulations with $q > 0.32$, we are able to track $M_{\text{irr}}$ very accurately for both of the individual merging BHs, as well as the resulting BH, over the entire course of the simulation. In the case of $q = 0.32$, we cannot track the smaller individual irreducible mass very accurately before merger. There is also a spurious drift in the common apparent horizon mass beyond the initial ADM mass content of the spacetime but the error only grows about 2% over $t = 50M_{\text{ADM}}$. Note that the smaller the value of $q$, the earlier the merger as indicated by $t_{\text{AH}}$ in table 1. Post-merger, the individual AHs lose their meaning as apparent horizons since by definition the AH denotes the outermost marginally trapped surfaces. The individual surfaces remain marginally trapped.

Figure 2 shows snapshots of the horizons every $4.6M_{\text{ADM}}$ before merger and at $t = \{40, 80, 105\}M_{\text{ADM}}$ after merger for the $q = 0.78$ case. The other cases are qualitatively similar. Note how the initial common AH has an asymmetric peanut shape due to the unequal masses. Soon after it appears, the common AH becomes spherical, as the dynamical gauges drive the coordinates toward those of a single BH. At that moment, the common AH begins to drift slowly away from the origin. The last AH snapshot in figure 2 was taken at $t = 105M_{\text{ADM}}$. The evident drift in the coordinate location of the common AH provides a hint that a kick is generated as a consequence of gravitational recoil.

For waveform extraction, we compute Zerilli modes $\psi_{\ell m}$ using the Abrahams and Price convention [31]. Formally, the method assumes a spherically symmetric background. Simulations of QC-0 data [13, 14] have produced rotating BHs with the Kerr parameter $a \sim 0.7$. The main effect of using Zerilli extraction in spacetimes with significant angular momentum content will be a spurious amplitude offset in the waveform [32, 33]. Figure 3
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Figure 2. Snapshots of the AH location in the $xy$-plane for the case $q = 0.78$. The larger BH is on the left moving toward the bottom. The snapshots are taken every $4.6M_{\text{ADM}}$ prior to merger and at $t = \{40, 80, 105\}M_{\text{ADM}}$ after merger. The first common AH is found at $t = 9.9M_{\text{ADM}}$. The trajectories of the AH centroids are also shown. The common AH moves to the right and slightly upward after merger.

Table 2. Estimates of radiated energy $E_{\text{rad}}$ (total), $E_0$ ($m = 0$ mode) and $E_2$ ($m = 2$ mode) as a % of the total initial ADM energy. Radiated angular momentum $J_{\text{rad}}$ as a percentage of the initial angular momentum. Kick velocities $V$ from gravitational recoil. The errors are the differences between the $h = 1/16$ and $1/20$ resolution results multiplied by 2.

| $q$  | $E_{\text{rad}}$ | $E_0$  | $E_2$  | $J_{\text{rad}}$ | $V$ (km s$^{-1}$) |
|------|------------------|--------|--------|-------------------|------------------|
| 1.00 | 2.7 ± 0.8        | 0.05 ± 0.1 | 1.3 ± 0.4 | 15 ± 6 | 1 ± 2 |
| 0.85 | 1.7 ± 0.2        | 0.038 ± 0.004 | 0.78 ± 0.02 | 10 ± 0.8 | 50 ± 20 |
| 0.78 | 1.1 ± 0.8        | 0.06 ± 0.04 | 0.55 ± 0.02 | 7.4 ± 0.8 | 72 ± 40 |
| 0.55 | 0.4 ± 0.2        | 0.011 ± 0.006 | 0.16 ± 0.06 | 2.6 ± 0.6 | 90 ± 50 |
| 0.32 | 0.05             | 0.001    | 0.024   | 0.4     | 31     |

displays the dominant modes ($\ell = 2, m = 2$ and $\ell = 3, m = 3$) for the different $q$ simulations. The extraction surface is located at $r = 15$ with the outer boundary at $r = 96$. Note in figure 3 that the $\ell = 2, m = 2$ mode simply decreases in amplitude as the $q$ ratio is decreased. This is mostly due to the initial data approaching that of a single distorted BH. The $\ell = 3, m = 3$ mode is smallest for the equal mass case (where it should be 0) and for the $q = 0.32$ case. Around $t = 50M_{\text{ADM}}$, the lower amplitude waveforms become affected by outer boundary effects. The strongly dominant $\ell = 2, m = 2$ mode remains accurate until the end of the simulation except for the $q = 0.32$ case where even the dominant mode is affected by the boundary by $t = 60M_{\text{ADM}}$.

From the extracted modes, table 2 summarizes estimates of the energy $E_{\text{rad}}$ and angular momentum $J_{\text{rad}}$ emitted as a percentage of the total energy and angular momentum as well as the recoil velocities $V$ in km s$^{-1}$ [34]. We compute the radiated energy from [31]

$$\frac{dE_{\text{rad}}}{dt} = \frac{1}{32\pi} \sum_{\ell,m} \left[ \frac{d\psi_{\ell m}^+}{dt} \right]^2 + \left| \psi_{\ell m}^\times \right|^2,$$
Figure 3. The dominant modes ($\ell = 2, m = 2$ and $\ell = 3, m = 3$) of the real part of the Zerilli function $\psi_{\ell m}$ against time for the different $q$ ratios. The waveforms were extracted at $r = 15$. The $\ell = 2, m = 2$ mode decreases in amplitude with decreasing $q$ while the $\ell = 3, m = 3$ mode increases and then decreases again.

The radiated angular momentum via

$$\frac{dJ_{\text{rad}}}{dt} = \frac{1}{32\pi} \sum_{\ell,m} \text{Im} \left[ \frac{d\psi_{\ell m}^+}{dt} (\psi_{\ell m}^+) + \psi_{\ell m}^+ \int_{-\infty}^t (\psi_{\ell m}^+) \, dt' \right]$$

and the recoil velocity from

$$\frac{dP^k}{d\tau} = \frac{\rho^2}{16\pi} \int \left[ \left( \frac{dh_+}{d\tau} \right)^2 + \left( \frac{dh_\times}{d\tau} \right)^2 \right] n^k \, d\Omega.$$

Here $-2 Y^{\ell m}(\theta, \phi)$ denotes the spin-weight $-2$ tensor spherical harmonics. The kick velocity is obtained from $MV^k = \int P^k \, d\tau$ with $M$ the total mass of the binary. The reported recoil velocity is $|V|$.

As expected, the radiated energy and angular momentum are strongly dominated by the $\ell = 2, m = 2$ mode. The energy and angular momentum radiated for the $q = 1$ case are in good agreement with previous work [13, 14]. The numbers reported in the table are
computed from the $h = 1/20$ simulations. The errors are the difference between the quantities computed from the $h = \{1/16, 1/20\}$ resolution simulations multiplied by 2. The Richardson error estimate for our resolutions and second-order convergence is 16/9 and 2 is used as a conservative bound. We do not report errors for the $q = 0.32$ because the $h = 1/16$ simulation has insufficient resolution to compute the waveform. With increasing resolution the estimates of $E_{\text{rad}}$ and $J_{\text{rad}}$ did increase, so assuming convergence, one would expect the continuum estimates for radiated energies and angular momenta to be slightly higher than those reported here.

In order to compare the gravitational wave content of different simulations, table 2 also presents the energy emitted in the $m = 0$ and $m = 2$ modes, i.e. $E_0$ and $E_2$ respectively, as a percentage of the total initial ADM energy. The $\ell = 2, m = 0$ waveform in the $q = 1$ case changes quite strongly between the $h = 1/20$ and $h = 1/16$ simulations, resulting in large error bars on $E_0$ and $V$ (which should be zero for the equal mass case). Since the effect is not as strong for the $\ell = 2, m = 2$ mode, the dominant mode for $E$ and $J_{\text{rad}}$, the error bars are smaller. The $\ell = 2, m = 2$ mode dominates for a pure inspiral, whereas the $\ell = 2, m = 0$ mode dominates for a plunge, so we would expect a larger $E_2/E_0$ ratio for inspirals than for plunges. It is reassuring that differences in the radiated energies of these modes exist; however, due to the construction of the initial data sequence, these differences do not reflect a change of $q$ only. It will be interesting to see how this changes with further separated, truly inspiralling configurations starting from outside ISCO.

The recoil velocities, $V$, reported in table 1 are consistent with those obtained from head-on collisions [37], mixed numerical-perturbative inspiral mergers [35] and recent fully relativistic results at larger separations [36, 18]. For reference, kicks from gravitational recoil of relevance to galactic BH merger scenarios [9] have been recently estimated to second post-Newtonian order [16]. The kicks were found to be dominated by the plunge phase and could reach speeds larger than 100 km s$^{-1}$ for $0.1 \leq \eta \leq 0.24$ or $1/8 \leq q \leq 2/3$. Recent work using the effective one-body approach gives lower kick velocities of at most 74 km s$^{-1}$ at $\eta = 0.2$ or $q = 0.38$ [17].

In figure 4 we show the recoil velocity accumulated as a function of time. The close detector radius manifests itself in the non-zero initial offset of the recoil velocity of the $q = 0.32$ model. The inspiral contribution is notably absent from these models due to their
close separation (cf [36, 18]). Also note that the initial data contain an initial feature that we cannot remove as it is already overlaid with the merger signal. In further separation models as in [18] the initial feature shows up in the weaker inspiral part and can therefore easily be removed. Figure 5 shows our recoil velocities as a function of the reduced mass parameter $\eta$, including kick estimates found by others [16, 17, 35, 36]. Our $q = 0.85, \eta = 0.248$ and $q = 0.78, \eta = 0.246$ cases are comparable to the estimates from Blanchet et al [16]. The reason for the agreement is that these cases are closer to the QC-0 equal mass case. Thus the initial data setup is not too far from ISCO and quasi-circularity. The case $q = 0.55, \eta = 0.229$ yields the largest recoil velocity, $\sim 82$ km s$^{-1}$, also consistent with Blanchet et al [16] who find a peak in the recoil around $q = 0.4, \eta = 0.204$. Recently Baker et al [36] reported a recoil velocity of 105 km s$^{-1}$ for $q = 0.67, \eta = 0.24$, a value larger than our kicks. The difference is mostly due to the larger initial separation of their BHs. Our smallest kick is obtained in the $q = 0.32, \eta = 0.183$ case. This is likely a significant underestimation since we are probably dealing with a single, distorted BH.

2. Conclusions

We have shown results from a series of unequal mass BBH inspiral simulations. The kick velocities spanned a range of 25–82 km s$^{-1}$. Our kick velocity estimates should be understood as a lower bound because the initial separations are smaller than those normally used (for example in [16]) for the plunge phase. The observed drop in the time to merger $t_{\text{AH}}$ in our results comes from our approach in which the angular momentum of the configuration decreases as the bare mass ratio is decreased. This approach introduces significant uncertainty in the recoil extracted and in particular it is possible that for our cases with $q \leq 0.55$, a common event horizon already exists in the initial data slice; and, therefore, we are evolving a single, distorted BH. Ideally one would like to compare initial data sets which are far separated and have the same orbital frequency. We would also like to point out that these close configurations contain significant eccentricities which are known to influence the recoil velocities. For small eccentricities $e \leq 0.1$ [2] shows $V_R \propto (1 + e)$. More on equal mass inspirals and eccentricities can be found in [38, 39].
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In addition to the gravitational recoil, we estimated the radiated energy and angular momentum, paying particular attention to the energy radiated independently in the $m = 0, 2$ modes. We also monitored the irreducible masses during the simulations, providing a good indicator that the near-zone physics was accurately evolved. A more detailed comparison of the dependence of the waveforms on the mass ratio from inspirals starting outside the ISCO is currently under investigation. A strong dependence of the waveform on the binary parameters would facilitate their estimation, but at the same time it would hinder initial detection efforts as many templates would be required. If waveforms, on the other hand, do not vary significantly with mass ratios, the search effort could be easier at the expense of accurate parameter estimation.

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