Higgs-Boson Couplings Beyond the Standard Model

Martin B Einhorn\textsuperscript{a,b}, José Wudka\textsuperscript{c}

\textsuperscript{a}Kavli Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106-4030
\textsuperscript{b}Michigan Center for Theoretical Physics, University of Michigan, Ann Arbor, MI 48109
\textsuperscript{c}Department of Physics and Astronomy, University of California, Riverside, CA 92521-0413

Abstract

The implications for Higgs decays of potential new physics beyond the Standard Model (BSM) are considered in the context of effective field theory, assuming perturbative decoupling. Using existing data to restrict which dimension-six operators can arise, it is shown that, given the existing experimental constraints, only a small number of operators can affect the decays of the Higgs: those that may be potentially-tree-generated (PTG) and modify the Higgs-fermion couplings, or those that may be loop-generated (LG) that modify the Higgs couplings to $\gamma\gamma$, $Z\gamma$ and $GG$. Implications for specific branching ratios are given in terms of the coefficients of various dimension-six operators. In such a scenario, the ratios $\Gamma(H \rightarrow WW^*)/\Gamma(H \rightarrow ZZ^*)$ and $\Gamma(H \rightarrow W\ell\nu)/\Gamma(H \rightarrow Z\ell\ell)$ equal to their standard model values to an accuracy of $O(1\%)$ or less.

Keywords: Higgs boson couplings, Beyond Standard Model, Effective field theory

1. Introduction

The observation of a new particle by two detectors \cite{1,2} at the LHC has offered a candidate for the long-sought Higgs boson of the Standard Model (SM). Within errors, the observed properties are consistent with the Higgs boson of the SM insofar as its production rate and branching ratios are concerned, coming primarily from data in the $WW^*$, $ZZ^*$, and $\gamma\gamma$ decay...
modes. There is some supporting evidence from enhancements in TeVatron data [3], primarily from $b\bar{b}$ decays. So far, the evidence is consistent with SM expectations, with the exception of the rate in the $\gamma\gamma$-channel, which apparently exceeds SM estimates in one of the experiments [1].

Naturally, a primary goal of further experiments is to determine whether the couplings of the Higgs to weak bosons $g_{HWW}$, $g_{HZZ}$ and the couplings to fermions agree with the SM expectations. These studies may be informed by theoretical expectations, and many papers have been written (for recent summaries see e.g. [4]) about the implications of models that include particles beyond the SM (BSM). With a mass $m_H \approx 125$ GeV, the Higgs would appear to have a small enough self-coupling for perturbation theory to be reliable. In that case, there are two possibilities for the additional particles in such models: either (1) they involve new “light” particles of a mass comparable to or lighter than the Higgs boson, as, for example, in models having two Higgs doublets, including some supersymmetric model [4] in which most or all of their mass derives from the electroweak scale as do SM particles, or (2) all new particles are more massive than $m_H$, deriving their mass from some new underlying scale. An example would be softly-broken supersymmetric models with super-renormalizable couplings large compared to the weak scale. In the absence of the observation of a new particle, it can be difficult to decide in which situation we find ourselves.

The question is, what can be inferred from deviations of experimental data from SM expectations of the properties of the observed scalar? In the former case, there tend to be rather large deviations from the SM, arising from mixing angles between two or more multiplets. Typically, couplings of a Higgs boson differ already at tree level by factors such as $\tan \beta$, the ratio of vacuum expectation values of different doublets. In the latter case, one may perform a model-independent analysis using a generic effective Lagrangian approach, taking into account that the first corrections to the SM can be described in terms of higher-dimensional operators (HDO). A large number of publications have appeared recently that discuss various aspects of this approach; it would require a lengthy review to cite all the papers that have been written on this, and such a list would be out of date by the time of this publication; for a representative sample see [7].

A related question is, if there are no other particles discovered and no

\[^2\text{Some recent fits of such models to LHC data are [5, 6].}\]
deviations from the SM observed, what conclusions can be drawn, given the level of accuracy of the experiments? As much as possible, one would like to draw model-independent conclusions, although that may be very difficult in the near term.

Some years ago, the authors [8] and others [9, 10] performed such analyses, both for weakly-interacting, decoupling scenarios and for strongly interacting models [11, 12]. In this paper, we shall assume the underlying physics is decoupling and weakly coupled, at least to a good approximation. What this means in practice is that the particles that we call “light,” such as the top quark, predominantly get their masses as a result of spontaneous breaking of $SU(2) \otimes U(1)$. We assume that the “heavy” particles get their masses primarily via some other mechanism, although they may also receive electroweak contributions. For example, this would be the case if the scale of supersymmetry-breaking were large compared to the weak scale, giving some superpartners parametrically large masses.

In phenomenological studies of deviations from the SM, it has been advocated that, to fit experimental events involving the production and decay of a single Higgs boson, one employ an effective Lagrangian of the form

$$L_{eff} = \frac{H}{v} \left[ 2c_W M_W^2 W^- W^+ \mu + c_Z M_Z^2 Z^\mu Z_{\mu}^\nu + c_t m_t t \bar{t} + c_b m_b b \bar{b} + c_\tau m_\tau \tau \bar{\tau} \right] + \frac{H}{3\pi v} \left[ c_\gamma \frac{2}{3} G_{\mu\nu}^2 + c_g \frac{\alpha_S}{4} G_{\mu\nu}^2 \right].$$

(1) (2)

This effective Lagrangian is presumed to describe interactions in the so-called unitary-gauge, where the Higgs doublet is of the form $\phi = (v + H)/\sqrt{2}(0, 1)^T$ (and $v = \sqrt{2} \langle \phi \rangle \approx 246$ GeV.) In the SM, the interactions described in eq. (2) arise at the one-loop level, whereas those in eq. (1) arise already in tree approximation. To leading order, all the coefficients $c_k = 1$, which, given present experimental accuracies, is sufficient, although further radiative corrections can be included if necessary.

Although the decay rates are unambiguous in the Standard Model, unless we know the form of the BSM Lagrangian, one cannot blithely use eqs. (1) and (2) as an effective Lagrangian without acknowledging other implications of such a choice. Because of the equivalence theorem, which we will review in the next section, the form of an effective Lagrangian involves a certain degree

---

3See, eg., the reviews in refs. [6, 13].
of arbitrariness. The particular choice of operators does not affect expectations for S-matrix elements, but the associated Green’s functions may be very different. At the very least, these additional assumptions need to be spelled out in detail. The expressions eqs. (1), (2), although not gauge-invariant, are intended to be used in unitary gauge. Some decays can be well approximated by treating all particles on-shell, so that, e.g., $c_b, c_\tau, c_\gamma$ may be defined with all three particles on-mass-shell, other parameters such as $c_W, c_Z, c_t$ represent coupling constants that, for kinematical reasons, cannot be determined experimentally. As an example, the $H \to WW^*$ mode represents $H \to W\ell\nu\ell$ (where $\ell = e, \mu$), which receives a contribution from a virtual $W$ exchange, but others as well, and these depend on the operator basis being used. Extracting limits on coefficients such as $c_W$ requires a complete calculation that includes all relevant contributions to insure the results are independent of the operator basis and fully gauge invariant.

In fact, among other results below, we shall show that modifications to the SM couplings from new physics $c_W, c_Z$ are negligible within foreseeable experimental errors (except for a possible common normalization effect that does not contribute to the branching ratios), unless there are other light particles whose masses arise primarily, if not solely, from electroweak symmetry-breaking. In that case, the form of eq. (1) is unsuitable as a starting point for fitting or interpreting experimental data. No conclusions can be drawn from it without making presumptions about other operators and processes involving the Higgs boson.

2. Some features of effective Lagrangians

By now, the language of effective field theory has become familiar [14, 15, 16, 17], especially to researchers studying physics BSM. We will review it here only to the extent that we need to establish notation and to summarize some features of the approach; the details of our general approach are provided in a companion paper [18].

We imagine a theory where the heavy scale $\Lambda$ is assumed too large for the corresponding excitations to be directly produced; their virtual effects, however, may be observable. Assuming also that the heavy physics decouples then implies that at scales below $\Lambda$ the effective action can be expanded in a power series in $\Lambda$ (multiplied by logarithmic corrections) where all each power multiplies a local operator, and those terms that grow with $\Lambda$ can be absorbed in a renormalization of the low-energy parameters. After this
renormalization the effective action takes the form

\[ S_{\text{eff}} = \int d^4x L_{\text{eff}} \quad L_{\text{eff}} = L_0 + \sum_{n \geq 5} \frac{1}{\Lambda^n} \sum_i f_i^{(n)} O_i^{(n)}, \tag{3} \]

where we assumed the underlying physics is weakly coupled\(^4\). The coefficients \(f_i\) encode all the details of the underlying model and can therefore be used to parameterize all possible types of heavy physics; in general they can also depend logarithmically on \(\Lambda\). For the case being considered here, \(L_0\) corresponds to the full SM Lagrangian, in which case there is a single dimension-five \(^{19}\) operator that violates lepton number by two units and generates a neutrino Majorana mass. Aside from this, dimension-six operators then represent the leading virtual new physics effects resulting from any weakly-coupled, decoupling heavy particles.

The number of dimension-six operators is large (\(\sim 100\)), but not all need be included in calculations since all low-energy effects can be parametrized by the coefficients of a reduced set of operators we refer to as a basis. This is a set of (dimension-six) operators \(\{O_a\}\) (henceforth we drop the superscript \((6)\)) with the property that any other operator \(O\) obeys the relation

\[ O - \sum_a \kappa_a O_a = \sum_\phi U_\phi \frac{\delta S_0}{\delta \phi}, \tag{4} \]

where the \(\kappa_a\) are appropriately chosen constants; \(\phi\), a generic light field; \(\delta S_0/\delta \phi\), the classical equations of motion; and \(U_\phi\), local operators. Each term in eq. (4) is gauge- and Lorentz-invariant. We will say that the combination \(O - \sum \kappa_a O_a\) vanishes “on-shell,” and that \(O\) is equivalent to \(\sum \kappa_a O_a\). In addition, we demand that no linear combination of basis elements vanishes on shell.

To establish some terminology, note that the HDO form a vector space. An equivalence relation produces a unique partition of a vector space into equivalence classes. A basis will have one operator from each equivalence class. A minimal basis choice for dimension-six operators is presented in \(^{20}\). While the preceding is a familiar construction, in \(^{18}\), we put forward an improved method of choosing basis operators with reference how these operators

\(^4\) Although \(L_{\text{eff}}\) must be Hermitian, it is not always most expedient to make each term in the sum Hermitian; \(eg.,\) in the SM, the Yukawa couplings are an illustration. In such cases, each term is implicitly accompanied by its Hermitian conjugate.
may arise in extensions of the SM. An *extension* of $L_0$ is any model containing heavy particles that reduces to $L_0$ for operators of dimension four or less at scales below some threshold $\Lambda$. An extended model may also be referred to as an *embedding* of $L_0$.

In general, HDO may be identified as either potentially-tree-generated (PTG) or loop-generated (LG) \[8, 18\]. A PTG operator $O^{PTG}$ is one for which an extension can be found in which it arises from a tree-diagram. An LG operator $O^{LG}$ is one that (1) cannot emerge from a tree-graph in any embedding of $L_0$ and (2) can arise from loop-graphs.\[5\] This is a useful distinction because LG operators have coefficients that are typically suppressed by a factor $\sim 1/(4\pi)^2n$ relative to PTG operators, where $n$ is the number of loops. Having potentially larger coefficients, the PTG operators may be more sensitive to new physics effects.

It is important to note that whether an operator is LG or PTG is a property of the heavy physics, while an equivalence relation of the form eq. (4) is a property of the light theory. An equivalence class of operators may contain only PTG-operators, only LG-operators, or both kinds. It is helpful to identify this property of each equivalence class and, in cases when a class contains both kinds of operators, to choose a basis operator from among the PTG-operators. This provides the most conservative approach to interpreting experimental data, whether providing limits on or evidence for BSM physics effects. The reason is that the parametrization covers the widest class of heavy physics theories: those that generate the operators in question at tree-level, as well as those that may generate them only through loop corrections. In \[18\], we delineated the equivalence relations for the SM and analyzed the basis chosen in \[20\]. We identified those that were LG, those that were PTG, and in cases where an equivalence class contained both types, showed that the basis chosen in \[20\] were in fact PTG operators, as required. This then is good basis for studying physics BSM. In the remainder of this paper, we indicate how this may be applied to Higgs production and decay.

---

5This classification can be made either without restriction on the embeddings of $L_0$ or, if one is interested in a restricted set of embeddings, from extended models respecting some additional local or global symmetry. For example, the limits on violation of baryon- or lepton-number suggest that such operators may be ignored for analyzing LHC data, regardless of whether they are PTG or LG.
3. New physics contributions to Higgs decay

Limiting our attention to baryon- and lepton-conserving operators involving only SM fields (e.g. no right-handed neutrinos), and adopting the basis \( \{ O_a \} \) of dimension-six operators given in ref \[20\], the effective Lagrangian takes the form

\[
\mathcal{L}_{\text{eff}} = \mathcal{L}_{SM} + \sum_a \frac{f_a}{\Lambda^2} O_a + \cdots,
\]

where the ellipsis denote operators of dimension > 6. We need only PTG operators that contribute to the various Higgs decay channels, except for the \( \gamma\gamma \), \( \gamma Z \), and \( GG \) final states which occur at one-loop in the SM and in all extensions of the SM. These therefore, invite comparison with one-LG corrections to new physics. As discussed above, the operators included describe the leading deviation from the SM, whereas those we neglect will be too small to be observed, at least in present experiments. In the following we will assume that the coefficients \( f_a \) are real.

We find that the PTG operators contributing to the Higgs decay channels measured at the LHC are also involved in other well-measured process, namely, \( Z \) and \( W \) lepton decays and custodial symmetry violations associated with the oblique \( T \) (or \( \rho \)) parameter. Current data indicate that deviation from the SM in these processes lie below the level of 0.1%-1%, so that the contributions to the corresponding operators to Higgs decays can be neglected given the current precision in that decay.

The PTG operator basis contributing to the measured Higgs decays can be separated into 3 classes:

1. Pure Higgs operators:

\[
O_{\partial\phi} = \frac{1}{2}(\partial_\mu |\phi|^2)^2, \quad O_\phi = |\phi|^6.
\]

Ignoring self-interactions of the scalar field, the effect of \( O_{\partial\phi} \) will be to change the normalization of the Higgs field after symmetry breaking.

---

6 Although we will use the basis of \[20\], we prefer to denote the operators as \( O_a \) rather than \( Q_a \). For convenience, we reproduce them in Appendix A.

7 For those operators \( O_a \) that are Hermitian, \( f_a \) are necessarily real; for others, including the ones relevant in Higgs decays, it is equivalent to assuming that CP violation is unimportant for these applications.

8 In \[20\], \( O_{\partial\phi} \) is replaced by \( -O_{\phi\partial} \), which is the same after integration by parts.
Indeed, in unitary gauge,

$$\phi = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v + h \end{array} \right),$$  

(7)

(with $v = \sqrt{2} \langle \phi \rangle \simeq 246$ GeV,) so we get

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{f_{\partial \phi}}{\Lambda^2} O_{\partial \phi} + \cdots \approx \frac{1}{2} (1 + \epsilon f_{\partial \phi}) (\partial h)^2 + \cdots,$$  

(8)

where $\epsilon \equiv v^2 / \Lambda^2$. Thus, the canonically normalized Higgs field will be

$$H = \sqrt{1 + f_{\partial \phi} \epsilon} \ h \approx \left(1 + \frac{1}{2} f_{\partial \phi} \epsilon \right) h.$$  

(9)

This modifies all processes involving a single Higgs boson in the same way, replacing the SM Higgs field $h$ by $H$.

The operator $O_\phi$ also modifies the parameters in the Higgs sector, such as the vacuum expectation value $v$, but these effects can be absorbed in finite renormalizations of $\mathcal{L}_0$ with no observable effects. In that case, $O_\phi$ could only be distinguished through Higgs self-coupling effects [21, 22] which, at present, are experimentally out of reach. In contrast the rescaling (9) does have (potential) observable consequences.

2. Operators modifying Higgs couplings to $W$ and $Z$:

These include basis operators of the type $X^2 \phi^2$, called $O_{\phi X}$, $O_{\phi X}$, $O_{\phi WB}$ and $O_{\phi \bar{W}B}$ (see Appendix). All of these are LG operators and will be neglected in first approximation. That means that the Higgs coupling to $ZZ$ and $WW$ may be assumed to be SM to within about 0.1−1%. This important result is analogous to our earlier result concerning BSM corrections to triple-gauge-boson couplings [18, 8].

The basis we employ also contains an operator of the type $\phi^4 D^2$:

$$O_{\phi D} \equiv |\phi^D|$,  

(10)

This operator would generate a mass shift for the $Z$ and produce a change in the $\rho$-parameter [20] or, equivalently, the so-called oblique $T$ parameter [21] from its SM value, specifically

$$\delta T = -\frac{1}{\alpha} \epsilon f_{\phi D}.$$  

(10)
where, in obtaining this relation, we ignored the effective-operator contributions to the Fermi constant \([28]\). Current experimental constraints give \(|\delta T| \lesssim 0.1\), implying that, even though \(O_{\phi D}\) may affect Higgs decays (specifically, the \(H \to ZZ^*\) mode\(^{10}\)) these effects will be too small to be observed, given the experimental precision achievable at the LHC.

3. Higgs and Gauge Boson Couplings to Fermions:

These include all the operators of the types \(\psi^2 \phi^2 D\), called \((O_{\phi\psi})_{pr}\) and \((O_{\phi ud})_{pr}\), and \(\psi^2 X \phi\), called \((O_{\psi X})_{pr}\), for any fermion \(\psi\), where \(p, r\) are family indices. The former are all PTG operators, but the latter are all LG. Therefore, in first approximation, we will neglect \(O_{\psi X}\).

Limits on flavor-changing neutral currents\(^{11}\) suggest that the thresholds for generation-changing operators \((O_{\phi\psi})_{pr}\) in the coupling of the \(Z\) are very high, so we may assume that \(p = r\), but this still leaves the \(Z\) couplings to the 3 families, \((O_{\phi\psi})_{pp}\) for each type of fermion \(\psi = \{\ell, e, q, u, d\}\). Many of these are already precisely determined\(^{23}\), especially for \(\psi =\) leptons for all 3 generations, from LEP measurements. The \(Z\)-lepton couplings are measured to at least 1% and agree with the SM predictions to that precision, so their potential contributions to Higgs decay widths will lie in this range. These effects are of order \(f \epsilon\) and correspond to a scale \(\Lambda > 2.5\) TeV when \(f \sim 1\). For quarks, they are similar experimentally constrained for all flavors except for the \(t\)-quark. Although these operators break custodial symmetry and therefore would change \(|\delta T|\), their lowest order contributions are in one-loop corrections and so are not strongly constrained by the experimental value of \(|T|\). To improve on existing limits significantly seems to be beyond the reach of a hadron collider such as LHC.

The operator \((O_{\phi ud})_{pr}\) would modify couplings of the \(W\) in ways that would affect both family-changing couplings as well as Higgs decays. These couplings of the \(W\) to the first two generations have been well-studied, and there are even constraints from \(t \to bW\). Once again, this would also contribute to \(|\delta T|\), but only at one loop order, and, given

\(^9\)See the review by Erler and Langacker in \([23]\).
\(^{10}\)See comments on this decay mode in next section.
\(^{11}\)See the review by Ceccucci, Ligeti, and Sakai in \([23]\).
that the top quark only makes a contribution of about 0.2, a bound on the order of 10% gives no significant constraint on the corresponding $\epsilon f_{\psi u d}$. The operator $O^{(3)}_{\psi \ell}$ potentially modifies the $W\ell\nu$ coupling, but since measurements agree with the Standard Model to a precision below 1%, the effective operator modifications to these couplings should lie in this range; as with the $Z$ case their contributions to Higgs decays can be ignored.

This case also includes corrections to the Yukawa couplings of the Higgs, which would affect the fermion masses as well as Higgs decays

$$(O_{e\psi})_{pr} = |\varphi|^2 \bar{r}_p e_r \varphi, \quad (O_{u\psi})_{pr} = |\varphi|^2 \bar{q}_p u_r \varphi, \quad (O_{d\psi})_{pr} = |\varphi|^2 \bar{q}_p d_r \varphi, \quad (11)$$

where the fermions carry generational indices. If one simply replaces the Higgs field by its vacuum expectation value, the contributions of these operators cannot be distinguished from the SM contribution. Therefore, the GIM mechanism will continue to work, as well as tests of CKM unitarity\footnote{See the review by Blucher and Marciano in \cite{23}.}. However, they affect the Higgs decays to fermion pairs differently than SM Yukawa couplings, since these are cubic in the scalar doublet whereas Yukawa couplings are linear. Thus, comparing the Higgs decay rates to the fermion masses may provide a good test for the presence of these operators. At present, it has not yet been determined whether the SM couplings to the observed Higgs are proportional to the mass, so there is no constraint on the operator coefficients $f_{\psi \varphi}$.

Even with much increased precision, the projected limits on the effective operator coefficients may be harder to interpret (e.g. extracting limits on the scale of new physics) than one might think, depending on the nature of the underlying theory. In the SM, the vanishing of a fermion mass give rise to an enhanced chiral symmetry. If that were a property of the HDO as well, as is commonly believed to be the case, then the coefficient of these operators ought to be proportional to at least the first power of the corresponding Yukawa coupling. Thus, even fairly inaccurate measurements that limit the size of these kinds of corrections to Higgs decay would be a good indicator of whether this hypothesis is correct.

4. Loop-generated operators:

In most applications, operators that are necessarily generated by heavy-particle loops are disregarded as being subdominant, or because their...
effects are expected to be difficult to detect experimentally. The rare decays $H \rightarrow \gamma\gamma, Z\gamma, GG$ (where $G$ represents a gluon) present exceptions since the SM contributions are themselves loop-generated [29, 30]. In particular, the experimental precision for $H \rightarrow \gamma\gamma$ may eventually reach the level needed to detect or set limits on deviations generated by the heavy physics. These deviations are generated by (i) modifications of the vertices involved in the SM loops (such as the $Ht\bar{t}$ coupling) and/or (ii) contributions from operators generated by heavy-physics loops. The former are listed above, the latter are the following:

\[
O_{\phi X} = \frac{1}{2}|\phi|^2 X_{\mu\nu} X^{\mu\nu}, \quad X = \{G^A, W^I, B\}; \quad O_{WB} = (\phi^\dagger \tau^I \phi) B^{\mu\nu} W^I_{\mu\nu}.
\]

There are also operators involving the dual tensors $\tilde{X}_{\mu\nu}$, but these do not interfere with the SM amplitudes, and so generate contributions of order $\Lambda^{-4}$ or smaller. The coefficients of these operators are suppressed by a loop factor $1/(16\pi^2)$; each field $X$ is also accompanied by the corresponding gauge coupling constant since gauge fields couple universally.

Summarizing: in the following, given the precision anticipated for LHC experiments studying Higgs decays, we will ignore potential effects of the operator $O_{\phi D}$ and those of type $\psi^2 \phi^2 D$, viz., $O_{\phi\psi}$ and $O_{\phi ud}$. It is worth noting that these same operators generate “contact” vertices of the form $HZee$ and $HWe\nu$ that could contribute to $H \rightarrow WW^*, ZZ^*$ decays, so these too can be ignored.

Thus, the only dimension-six operators to be retained for Higgs decays at LHC are $O_{\phi\phi}$ and $O_{\psi\phi}$. Recall that in the SM, the same transformation that diagonalizes the fermion masses will diagonalize the Higgs couplings, so there remain no (quark) flavor-changing couplings. When the operators $(O_{\psi\phi})_{pr}$ are introduced however, the diagonalization of the fermion mass matrix will not, in general, remove the flavor-changing Higgs couplings. The corresponding amplitudes, however, will be suppressed by a factor of $\epsilon$, and therefore, the decay rates by $\epsilon^2$. Concerning the flavor-diagonal operators, we will assume that their coefficients are proportional to the corresponding fermion masses, so that quantities of the form $f_{\psi\phi}/m_{\psi}$ have a finite limit as $m_{\psi} \rightarrow 0$. This naturality assumption also makes these effects much harder to observe.
4. Implications for LHC experiments

We now determine the manner in which new heavy, decoupling physics can affect these decays while taking into consideration the limitations from existing data, as described above.

The operators \((\mathcal{O}_{\psi \phi})_{pp}, \psi = e_p, u_p, d_p\), together with \(\mathcal{O}_{\partial \phi}\), modify the determination of the Yukawa couplings, which are not yet much experimentally constrained, as well as the Higgs decays into fermions as follows:

\[
\Gamma(H \to \bar{\psi}\psi) = \kappa_{\psi}^2 \Gamma_{SM}(H \to \bar{\psi}\psi); \quad \kappa_{\psi}^2 = \left(1 - f_{\partial \phi} \epsilon + \frac{\sqrt{2} \mu}{m_{\psi}} f_{\psi \phi} \epsilon\right).
\]  

Before proceeding further, some comments are in order concerning the decays referred to by the CMS and ATLAS collaborations as \(H \to ZZ^*\) and \(H \to WW^*\). These are a shorthand for the average of the leptonic final states in which the virtual \(Z^*\) decays into either \(e^+e^-\) or \(\mu^+\mu^-\) and in which the \(W^*\) decays into either \(e\nu_e\) or \(\mu\nu_\mu\). If we consider the actual S-matrix elements for these processes, we find, \(eg.,\) the decay to \(a\ Z\ plus\ a\ lepton\ pair\) receives contributions from 3 diagrams:

The vertices here are intended to include the sum of the SM couplings and the corresponding PTG dimension-six operators. The first has a \(Z\) internal line and, in addition to the SM contributions, is affected by \(\mathcal{O}_{\partial \phi}\). The third graph involves a contact \(HZee\) or \(HZ\mu\mu\) interaction, which, we have argued in the previous section, can be neglected.

The second diagram has an \(e\) or \(\mu\) internal line. As emphasized earlier, the first two diagrams are not gauge invariant in general, not even in the SM. However, that gauge dependence is associated with the nonzero fermion mass, and the Yukawa couplings of the \(e\) and \(\mu\) make a tiny contribution to these decays. We further assumed that the coefficients \(f_{\psi \phi}\) were proportional to the fermion mass. If we set the mass zero, then the second diagram vanishes, and
the first becomes gauge-invariant. Stated otherwise, in the limit of vanishing fermion mass, both the vector and the axial-vector currents are conserved (the latter in the Goldstone mode.)

Whether the preceding arguments remain true for the $\tau$-lepton remains to be seen, but the decay $Z \rightarrow \tau\tau$ agrees with the SM to the same accuracy as for decays to $ee$ and $\mu\mu$.

For whatever reasons, whether because of this approximate chiral symmetry or because their threshold $\Lambda$ is very large, the HDO’s that could give rise to $\delta T$ give negligible corrections as well.

As a result, only the effects from $\mathcal{O}_{\partial\phi}$ remain unconstrained, so we obtain

$$\Gamma(H \rightarrow ZZ^*) = \kappa_Z^2 \Gamma_{SM}(H \rightarrow ZZ^*); \quad \kappa_Z^2 = (1 - f_{\partial\phi} \epsilon).$$  \hspace{1cm} (14)

A similar discussion applies to the $H \rightarrow WW^*$ mode: the effects generated by possible deviations from the SM in the $W\ell\nu$ couplings are well below the current experimental precision to which this decay mode is measured. So we find

$$\Gamma(H \rightarrow WW^*) = \kappa_W^2 \Gamma_{SM}(H \rightarrow WW^*); \quad \kappa_W^2 = (1 - f_{\partial\phi} \epsilon).$$  \hspace{1cm} (15)

The expected modification for both of these decay widths are the same because the contributions from $\mathcal{O}_{\partial\phi}$ respect custodial symmetry, so the ratio $\Gamma(H \rightarrow ZZ^*)/\Gamma(H \rightarrow WW^*)$ equals their Standard Model value to an accuracy of 1% or less. If, on the contrary, this ratio is observed to differ markedly from 1, it is likely there will be other particles whose mass scale are also generated primarily by electroweak symmetry-breaking, as occur in models with more than one Higgs doublet and in supersymmetric models.

Finally, we consider briefly three rare but important decays:

1. The $H \rightarrow \gamma\gamma$ mode receives contributions both from tree-level modifications to the $Htt$ and $HWW$ couplings, as well as from the loop-induced effective operators $\mathcal{O}_{\phi X}$ for $X = W^I, B$ in (12). In order to display explicitly the loop nature of these operators and including the fact that gauge bosons couple universally, we will write

$$f_{\phi W} = \frac{g^2}{16\pi^2} \tilde{f}_W,$$  \hspace{0.5cm} $$f_{\phi B} = \frac{g'^2}{16\pi^2} \tilde{f}_B,$$  \hspace{1cm} (16)

\footnotetext{13}{In obtaining the numbers below, we will substitute the following values for the top and Higgs masses and for the SM vacuum expectation value: $m_t = 173.5$ GeV, $m_H = 125$ GeV, $v = 246.22$ GeV.}
so that these contributions to the effective Lagrangian become
\[ L_{\text{eff-loop}}^{(\gamma\gamma)} = \frac{1}{\Lambda^2} \left( f_{\phi W} O_{\phi W} + f_{\phi B} O_{\phi B} \right) = \frac{\epsilon}{v} \frac{\alpha}{4\pi} \tilde{f}_{\gamma\gamma} \frac{1}{2} H F_{\mu\nu} F^{\mu\nu}, \quad (17) \]

where \( \tilde{f}_{\gamma\gamma} = \tilde{f}_{W} + \tilde{f}_{B} \). Using the standard expressions for the top and W loop contributions [31], we find
\[ \Gamma(H \rightarrow \gamma\gamma) = \kappa_{\gamma\gamma}^2 \Gamma_{SM}(H \rightarrow \gamma\gamma) ; \quad \kappa_{\gamma\gamma}^2 = 1 - f_{\partial\phi}\epsilon + 0.30\tilde{f}_{\gamma\gamma}\epsilon + 0.28f_{t\phi}\epsilon. \quad (18) \]

2. The \( H \rightarrow Z\gamma \) mode also receives contributions from possible effective operator modifications of the \( Ht\bar{t} \) and \( HWW \) vertices, as well as from \( O_{\phi X} X = W^I, B \) and \( O_{WB} \) in (12) that generate
\[ L_{1\text{loop}}^{(Z\gamma)} = \frac{eg}{16\pi^2} \frac{v}{\Lambda^2} \tilde{f}_{Z\gamma} F_{\mu\nu} Z^{\mu\nu}, \quad (19) \]

where
\[ \tilde{f}_{Z\gamma} = \frac{16\pi^2}{eg} \left[ \frac{1}{2} (f_{\phi W} - f_{\phi B}) s_{2w} - f_{WB} c_{2w} \right], \quad (20) \]

where \( s_{2w} (c_{2w}) \) denotes the sine (cosine) of twice the weak-mixing angle. Using the known expressions for the loop factors we find
\[ \Gamma(H \rightarrow Z\gamma) = \kappa_{Z\gamma}^2 \Gamma_{SM}(H \rightarrow Z\gamma) ; \quad \kappa_{Z\gamma}^2 = 1 - f_{\partial\phi}\epsilon + 1.82\tilde{f}_{Z\gamma}\epsilon + 1.46f_{t\phi}\epsilon. \quad (21) \]

3. Finally, the \( H \rightarrow GG \) mode receives contributions from \( O_{t\phi} \) as well as from \( O_{\phi G} \) in (12). Writing the coefficient of the latter as \( f_{\phi G} = g_s^2 \tilde{f}_{GG}/(16\pi^2) \), where \( g_s \) is the SU(3) color gauge coupling constant,
\[ \Gamma(H \rightarrow GG) = \kappa_{GG}^2 \Gamma_{SM}(H \rightarrow GG) ; \quad \kappa_{GG}^2 = 1 - f_{\partial\phi}\epsilon + 2.91\tilde{f}_{GG}\epsilon + 4f_{t\phi}\epsilon. \quad (22) \]

This mode can potentially receive significant radiative corrections, however, explicit evaluation show that these are large only for \( m_H > 2m_t \) [34], which is not the case. (Radiative corrections to the lighter quark modes are large, however, all contributions to the width from light quarks are suppressed by a factor \( (m_q/v)^2 \), and can be ignored.)

4.1. Branching ratios and production cross section

For \( m_H \sim 125 \) GeV, the main decays of the SM Higgs are into the \( b\bar{b} \) (58%) and \( WW^* \) (21%) channels, but the \( GG \) (9%), \( \tau\tau \) (6%), \( cc \) (3%) and \( ZZ^* \) (3%) are also significant
\[ \Gamma(H) = \kappa_H^2 \Gamma_{SM}(H) ; \quad \kappa_H^2 = 1 - f_{\partial\phi}\epsilon + \beta\epsilon, \quad (23) \]
where, using the Standard Model values available at \[35\],

\[
\beta = \sum_{\psi=b,c,\tau} f_{\psi\phi} \frac{\sqrt{2} v B(H \to \bar{\psi}\psi)}{m_\psi} + (2.91 \tilde{f}_{GG} + 4 f_{t\phi}) B(H \to GG) \\
= 43.115 f_{b\phi} + 7.947 f_{c\phi} + 12.385 f_{\tau\phi} + 0.343 f_{t\phi} + 0.249 \tilde{f}_{GG}.
\]

Then, for any final state $\xi$, the branching ratio $B$ is related to the SM ratio $B_{SM}$ as

\[
B(H \to \xi) = \frac{\kappa^2_\xi}{\kappa^2_H} B_{SM}(H \to \xi).
\]

Specifically,

\[
B(H \to \bar{\psi}\psi) = \left(1 + \frac{\sqrt{2} v}{m_\psi} f_{\psi\phi}\epsilon - \beta \epsilon\right) B_{SM}(H \to \bar{\psi}\psi),
\]

\[
B(H \to Z\ell\ell) = (1 - \beta \epsilon) B_{SM}(H \to Z\ell\ell),
\]

\[
B(H \to W\ell\nu) = (1 - \beta \epsilon) B_{SM}(H \to W\ell\nu),
\]

\[
B(H \to \gamma\gamma) = \left(1 + 0.30 \tilde{f}_{\gamma\gamma}\epsilon + 0.28 f_{t\phi}\epsilon - \beta \epsilon\right) B_{SM}(H \to \gamma\gamma),
\]

\[
B(H \to Z\gamma) = \left(1 + 1.82 \tilde{f}_{Z\gamma}\epsilon + 1.46 f_{t\phi}\epsilon - \beta \epsilon\right) B_{SM}(H \to Z\gamma),
\]

\[
B(H \to GG) = \left(1 + 2.91 \tilde{f}_{GG}\epsilon + 4 f_{t\phi}\epsilon - \beta \epsilon\right) B_{SM}(H \to GG).
\]

Note that the ratio $B(H \to Z\ell\ell)/B(H \to W\ell\nu)$ is expected to have deviations below 1% from the SM value. Should this prove not to be the case, it would provide another strong indication of the presence of other light particles that affect these decays.

Although the decay mode $H \to GG$ has not been measured, the main contributions to the production cross section is in fact the inverse process of gluon fusion. Hence, to a good approximation we have

\[
\sigma^{prod} \simeq \kappa_{GG}^2 \sigma_{SM}^{prod}.
\]

That might be probed, although it suffers from the usual difficulties in determining the absolute normalization of a cross section.

The above new physics corrections are of order $\epsilon$, which for $\Lambda > 1$ TeV is smaller than 0.1. This provides a measure of the precision that LHC experiments will need to reach in order to probe physics at this scale (though sometimes a precision of 10% might suffice, depending on how large the coefficients $f_i$ and the numerical coefficients multiplying them are). None of
the relevant experiments have (yet) reached this level, in fact, current experimental precision in these decays allows only the exclusion of new physics at scales that have already been probed directly.

5. Conclusion

The purpose of this paper was to extend the analysis of ref. [8] to include couplings of the SM Higgs, especially those relevant to LHC measurements, in a model-independent fashion, taking into account the existing constraints on breaking of the custodial $SU(2)$ symmetry of the Higgs sector of the SM. Implications for specific branching ratios were given in terms of the coefficients of various dimension-six operators. The expressions presented show that any deviation of the couplings $g_{HWW}$ or $g_{HZZ}$ from the SM, at a level of accuracy observable by LHC experiments ($\sim 10\%$ or higher), can be explained only by having $\epsilon \sim 1$, which corresponds to new physics at a scale below 1 TeV. If the ATLAS enhancement in the $\gamma\gamma$ mode is verified, then this is strengthened considerably; it seems as if only new physics around the electroweak scale could account for this effect. Thus, we have sharpened the contrast between models that modify the SM by introduction of some higher mass scale and those, for example, having more than one Higgs doublet contributing to the weak scale vacuum expectation value $v$; the current anomaly in the photon mode would then belong to the second possibility. As the data improve, it will be exciting to observe how BSM physics is first manifested. We hope this analysis provides another tool by which this conclusion may be hastened.

Acknowledgement

The research of one of us (MBE) was supported in part by the National Science Foundation under Grant No. NSF PHY11-25915.
Appendix A. Dimension-Six Basis Operators for the SM\textsuperscript{14}

| $X^3$ (LG) | $\varphi^6$ and $\varphi^4D^2$ (PTG) | $\psi^2\varphi^3$ (PTG) |
|-----------------|--------------------------------------|-----------------------------|
| $O_G$ | $f^{ABC}G_{\mu}^A G_{\nu}^B G_{\rho}^C$ | $O_{\varphi}$ | $(\varphi^\dagger\varphi)^3$ | $O_{\varphi}^{-}$ | $(\varphi^\dagger\varphi)(\bar{I}_p e, \varphi)$ |
| $\tilde{O}_G$ | $f^{ABC}\tilde{G}_{\mu}^A G_{\nu}^B G_{\rho}^C$ | $O_{\psi}$ | $(\varphi^\dagger\varphi)(\varphi^\dagger\varphi)$ | $O_{\psi}^{-}$ | $(\varphi^\dagger\varphi)(\bar{q}_mu, \varphi)$ |
| $O_W$ | $\varepsilon^{IJK} W^I_{\mu} W^J_{\nu} W^K_{\rho}$ | $O_{\varphi D}$ | $(\varphi^\dagger D_\mu \varphi) (\varphi^\dagger D_\mu \varphi)$ | $O_{\varphi D}^{-}$ | $(\varphi^\dagger\varphi)(\bar{q}_mu, \varphi)$ |
| $O_{\tilde{W}}$ | $\varepsilon^{IJK} \tilde{W}^I_{\mu} W^J_{\nu} W^K_{\rho}$ | | | |

| $X^2\varphi^2$ (LG) | $\psi^2X\varphi$ (LG) | $\psi^2\varphi^2D$ (PTG) |
|-----------------|--------------------------------------|-----------------------------|
| $O_{\varphi G}$ | $\varphi^\dagger \varphi G_{\mu}^A G_{\nu}^A$ | $O_{\varphi W}$ | $(\bar{l}_\mu \sigma^{\mu\nu} e) \tau^I \varphi W_{\mu\nu}^I$ | $O_{\varphi}^{(1)}$ | $(\varphi^\dagger \hat{D}_\mu \varphi)(\bar{l}_\mu \gamma^\mu l_r)$ |
| $\varphi G$ | $\varphi^\dagger \varphi \tilde{G}_{\mu}^A G_{\nu}^A$ | $O_{\varphi B}$ | $(\bar{B}_{\mu} \sigma_{\mu\nu} e) \tau^I \varphi B_{\mu\nu}$ | $O_{\varphi}^{(3)}$ | $(\varphi^\dagger \hat{D}_\mu \varphi)(\bar{l}_\mu \gamma^\mu l_r)$ |
| $O_{\varphi W}$ | $\varphi^\dagger \varphi W_{\mu}^I W_{\nu}^I$ | $O_{\varphi B}$ | $(\bar{l}_\mu \sigma_{\mu\nu} e) \tau^I \varphi B_{\mu\nu}$ | $O_{\varphi}^{(1)}$ | $(\varphi^\dagger \hat{D}_\mu \varphi)(\bar{B}_{\mu} \gamma^\mu q_r)$ |
| $O_{\tilde{W}}$ | $\varphi^\dagger \varphi \tilde{W}_{\mu}^I W_{\nu}^I$ | $O_{\varphi B}$ | $(\bar{B}_{\mu} \sigma_{\mu\nu} e) \tau^I \varphi B_{\mu\nu}$ | $O_{\varphi}^{(3)}$ | $(\varphi^\dagger \hat{D}_\mu \varphi)(\bar{B}_{\mu} \gamma^\mu q_r)$ |
| $O_{\varphi B}$ | $\varphi^\dagger \varphi B_{\mu\nu} B_{\mu\nu}$ | $O_{\varphi B}$ | $(\bar{B}_{\mu} \sigma_{\mu\nu} e) \tau^I \varphi B_{\mu\nu}$ | $O_{\varphi}^{(1)}$ | $(\varphi^\dagger \hat{D}_\mu \varphi)(\bar{B}_{\mu} \gamma^\mu q_r)$ |
| $O_{\tilde{W}}$ | $\varphi^\dagger \varphi \tilde{W}_{\mu\nu} B_{\mu\nu}$ | $O_{\varphi B}$ | $(\bar{B}_{\mu} \sigma_{\mu\nu} e) \tau^I \varphi B_{\mu\nu}$ | $O_{\varphi}^{(3)}$ | $(\varphi^\dagger \hat{D}_\mu \varphi)(\bar{B}_{\mu} \gamma^\mu q_r)$ |

Table A.1: Dimension-six operators other than the four-fermion ones.

\textsuperscript{14}These tables are taken from \[20\], by permission of the authors. We changed the operator names from \(Q\) to \(O\) to conform to the present conventions.
All are PTG.

| $\mathbf{(LL)(LL)}$ | $\mathbf{(RR)(RR)}$ | $\mathbf{(LL)(RR)}$ |
|----------------------|----------------------|----------------------|
| $O_{ll}$             | $\bar{1}_p \gamma_{\mu} l_r (\bar{l}_s \gamma_{\mu} l_l)$ | $\bar{1}_p \gamma_{\mu} l_r (\bar{e}_s \gamma_{\mu} e_l)$ | $\bar{1}_p \gamma_{\mu} l_r (\bar{e}_s \gamma_{\mu} e_l)$ |
| $O^{(1)}_{qq}$       | $\bar{q}_p \gamma_{\mu} q_r (\bar{q}_s \gamma_{\mu} q_l)$ | $\bar{u}_p \gamma_{\mu} u_r (\bar{u}_s \gamma_{\mu} u_l)$ | $\bar{u}_p \gamma_{\mu} u_r (\bar{u}_s \gamma_{\mu} u_l)$ |
| $O^{(3)}_{qq}$       | $(\bar{q}_p \gamma_{\mu} \tau^I q_r (\bar{q}_s \gamma_{\mu} \tau^I q_l)$ | $(\bar{d}_p \gamma_{\mu} d_r (\bar{d}_s \gamma_{\mu} d_l)$ | $(\bar{d}_p \gamma_{\mu} d_r (\bar{d}_s \gamma_{\mu} d_l)$ |
| $O^{(1)}_{1q}$       | $(\bar{1}_p \gamma_{\mu} l_r (\bar{q}_s \gamma_{\mu} q_l)$ | $(\bar{e}_p \gamma_{\mu} e_r (\bar{u}_s \gamma_{\mu} u_l)$ | $(\bar{e}_p \gamma_{\mu} e_r (\bar{u}_s \gamma_{\mu} u_l)$ |
| $O^{(3)}_{1q}$       | $(\bar{1}_p \gamma_{\mu} \tau^I l_r (\bar{q}_s \gamma_{\mu} \tau^I q_l)$ | $(\bar{e}_p \gamma_{\mu} e_r (\bar{d}_s \gamma_{\mu} d_l)$ | $(\bar{e}_p \gamma_{\mu} e_r (\bar{d}_s \gamma_{\mu} d_l)$ |
| $O_{ud}$             | $(\bar{u}_p \gamma_{\mu} u_r (\bar{d}_s \gamma_{\mu} d_l)$ | $(\bar{e}_p \gamma_{\mu} e_r (\bar{d}_s \gamma_{\mu} d_l)$ | $(\bar{e}_p \gamma_{\mu} e_r (\bar{d}_s \gamma_{\mu} d_l)$ |
| $O^{(1)}_{ud}$       | $(\bar{u}_p \gamma_{\mu} u_r (\bar{d}_s \gamma_{\mu} d_l)$ | $(\bar{e}_p \gamma_{\mu} e_r (\bar{d}_s \gamma_{\mu} d_l)$ | $(\bar{e}_p \gamma_{\mu} e_r (\bar{d}_s \gamma_{\mu} d_l)$ |
| $O^{(8)}_{ud}$       | $(\bar{u}_p \gamma_{\mu} T^A u_r (\bar{d}_s \gamma_{\mu} T^A d_l)$ | $(\bar{e}_p \gamma_{\mu} e_r (\bar{d}_s \gamma_{\mu} d_l)$ | $(\bar{e}_p \gamma_{\mu} e_r (\bar{d}_s \gamma_{\mu} d_l)$ |

Table A.2: Four-fermion operators conserving baryon number.

\[
\begin{align*}
\mathbf{(LR)(RL)} & \text{ and } \mathbf{(LR)(LR)} \\
O_{ledq} & = (\bar{l}_p e_r (d_s q_l) (\bar{d}_s q_l) \\
O^{(1)}_{quqd} & = (\bar{q}_p u_r (\bar{q}_s d_l) (\bar{q}_s d_l) \\
O^{(8)}_{quqd} & = (\bar{q}_p T^A u_r (\bar{q}_s T^A d_l) (\bar{q}_s T^A d_l) \\
O^{(1)}_{lequ} & = (\bar{l}_p e_r (\bar{q}_s u_l) (\bar{q}_s u_l) \\
O^{(3)}_{lequ} & = (\bar{l}_p \sigma_{\mu \nu} e_r (\bar{q}_s \sigma_{\mu \nu} u_l) (\bar{q}_s \sigma_{\mu \nu} u_l) \\
\end{align*}
\]
References

[1] G. Aad et al. [ATLAS Collaboration], Phys. Lett. B 716 (2012) 1
[arXiv:1207.7214 [hep-ex]].

[2] S. Chatrchyan et al. [CMS Collaboration], Phys. Lett. B 716 (2012) 30
[arXiv:1207.7235 [hep-ex]].

[3] Tevatron New Physics Higgs Working Group and CDF and D0 Collaborations,
arXiv:1207.0449 [hep-ex].

[4] M. Flechl et al. [CMS Collaboration], arXiv:1307.4589 [hep-ex].
M. Baak, et al. (GFitter group), Eur. Phys. J. C 72 (2012) 2003
[arXiv:1107.0975 [hep-ph]].
A. Djouadi, J. Phys. Conf. Ser. 447 (2013) 012002.

[5] P. M. Ferreira, et al., arXiv:1211.3131 [hep-ph].

[6] G. Belanger et al., arXiv:1212.5244 [hep-ph].

[7] A. Pomarol and F. Riva, arXiv:1308.2803 [hep-ph]. H. Mebane, et al.
arXiv:1306.3380 [hep-ph]. G. Isidori, A. V. Manohar and M. Trott,
arXiv:1305.0663 [hep-ph]. A. Hayreter and G. Valencia, arXiv:1304.6976
[hep-ph]. B. Dumont, S. Fichet and G. von Gersdorff, JHEP 1307
(2013) 065 [arXiv:1304.3369 [hep-ph]]. R. Contino, et al. JHEP
1307 (2013) 035 [arXiv:1303.3876 [hep-ph]]. J. Elias-Miro, et al.
arXiv:1302.5661 [hep-ph]. T. Corbett, et al. Phys. Rev. D 86 (2012)
075013 [arXiv:1207.1344 [hep-ph]]. G. Cacciapaglia, et al. JHEP
1303 (2013) 029 [arXiv:1210.8120 [hep-ph]]. G. Passarino, Nucl. Phys. B 868
(2013) 416 [arXiv:1209.5538 [hep-ph]]. E. Masso and V. Sanz, Phys. Rev.
D 87 (2013) 033001 [arXiv:1211.1320 [hep-ph]].

[8] C. Arzt, M. B. Einhorn and J. Wudka, Nucl. Phys. B 433 (1995) 41
[hep-ph/9405214].

[9] A. De Rujula, et al., Nucl. Phys. B 384 (1992) 3.

[10] K. Hagiwara, et al., Phys. Rev. D 48 (1993) 2182.

[11] For reviews, see J. Ellison and J. Wudka, Ann. Rev. Nucl. Part. Sci. 48
(1998) 33 [hep-ph/9804322].
A. Pich, Rept. Prog. Phys. 58 (1995) 563 [hep-ph/9502366].
[12] A. Pich, I. Rosell and J. J. Sanz-Cillero, Phys. Rev. Lett. 110 (2013) 181801 [arXiv:1212.6769 [hep-ph]].
R. Alonso, et al. Phys. Lett. B 722 (2013) 330 [arXiv:1212.3305 [hep-ph]].

[13] F. Zwirner, Theory Summary, XLVIIIth Rencontres de Moriond, Electroweak Interactions and Unified Theories, 9 March 2013.

[14] H. Georgi, Ann. Rev. Nucl. Part. Sci. 43 (1993) 209.

[15] S. Weinberg, Cambridge, UK: Univ. Pr. (1996) 489 p

[16] L. Alvarez-Gaume and M. A. Vazquez-Mozo, Lect. Notes Phys. 839 (2012) 1.

[17] A. Zee, “Quantum field theory in a nutshell,” (2nd ed.), Princeton, NJ: Princeton Univ. Pr. (2010) 576 p

[18] M. B Einhorn and J. Wudka, arXiv:1307.0478 [hep-ph].

[19] S. Weinberg, Phys. Rev. Lett. 43 (1979) 1566.

[20] B. Grzadkowski et al., JHEP 1010 (2010) 085 [arXiv:1008.4884 [hep-ph]].

[21] F. Bonnet et al., Phys. Rev. D 85 (2012) 035016 [arXiv:1105.5140 [hep-ph]].

[22] F. Bonnet, et al., Phys. Rev. D 86 (2012) 093014 [arXiv:1207.4599 [hep-ph]].

[23] J. Beringer et al. [Particle Data Group Collaboration], Phys. Rev. D 86 (2012) 010001.

[24] W. Buchmuller and D. Wyler, Nucl. Phys. B 268 (1986) 621.

[25] M. B. Einhorn, “The Standard Model Higgs Boson,” Amsterdam, Netherlands: North-Holland (1991) 390 p.

[26] See, eg., M. B. Einhorn, D. R. T. Jones and M. J. G. Veltman, Nucl. Phys. B 191 (1981) 146, and references therein.

[27] M. E. Peskin and T. Takeuchi, Phys. Rev. D 46 (1992) 381.
[28] For details, see G. Sanchez-Colon and J. Wudka, Phys. Lett. B 432 (1998) 383 [hep-ph/9805366].

[29] J. R. Ellis, M. K. Gaillard and D. V. Nanopoulos, Nucl. Phys. B 106 (1976) 292.

[30] F. Wilczek, Phys. Rev. Lett. 39 (1977) 1304.

[31] M. A. Shifman et al., Sov. J. Nucl. Phys. 30 (1979) 711 [Yad. Fiz. 30 (1979) 1368]. For a pedagogical review see: L. B. Okun, “Leptons And Quarks,” (Amsterdam, Netherlands: North-holland: 1982).

[32] [CMS Collaboration], CMS-PAS-HIG-13-005.

[33] [ATLAS Collaboration], ATLAS-CONF-2013-034.

[34] M. Spira et al., Nucl. Phys. B 453 (1995) 17 [hep-ph/9504378].

[35] https://twiki.cern.ch/twiki/bin/view/LHCPhysics/CERNYellowReportPageBR2