On the issue of calculating the force factors acting on the working bodies of a single rotor screw compressor

V Pronin, D Zhignovskaya, A Minikaev and D Yezhep

Faculty of Cryogenic engineering, ITMO University, Lomonosova st., 9, St. Petersburg, 191002, Russia

E-mail: diana.zhignovskaya@gmail.com

Abstract. The subject of research is a single rotor screw compressor. For the proper compressor design, the forces and moments must be calculated. The cutter’s teeth have a circular shape, therefore, the calculation is reduced to the determination of a torque. This power calculation is based on the law of pressure change in closed cavities of a single rotor screw compressor depending on the angle of a screw rotation and the law of change in the horizontal projection area of a cutter’s tooth meshed with a screw, depending on the angle of a cutter rotation. The term "flat closed figure" in our task means the horizontal projection of the disk cutter’s tooth part meshed with a screw, which takes the form of a triangle or trapezoid. The work shows a horizontal projection of the engagement rotor screw with a disk cutter. To determine the torque acting on the central screw, it is necessary to calculate the circumferential forces acting on the screw and the shoulders of the application forces. The calculation method described above allows one with a sufficient degree of accuracy to determine the forces and moments acting on the working bodies of a screw single-rotor compressor, and therefore, to calculate the deformation of the parts of a screw single-rotor compressor.

1. Introduction

For the correct design of a screw single-rotor compressor (SSC), it is necessary to be able to calculate the forces and moments acting on its working bodies. The main efforts arising in the working part of the machine are due to the pressure of the compressing substance. The closed working cavity (Figure 1) is limited by the cavity of a screw 1, the cylindrical part of a housing 2 and the tooth of a cutter 3. The total force $F_{n_1}$, acting on a cavity of screw can be decomposed into the radial $F_{r_1}$ and circumferential $F_{t_1}$ force. Due to the symmetry of the working parts of the screw rotor compressor, the radial forces are compensated, and the circumferential forces create torque $M_1$. A radial force $F_{r_2}$ will act on the cylindrical section 2, causing tensile stresses in it [1,2]. A force $F_{a_1}$ directed parallel to the axis of rotation cutter and causing a corresponding bending moment will act on the cutter tooth. The listed forces are defined as the product of the excess pressure in the cavity $\Delta P_a$ by the area of the corresponding projection. In addition to the indicated forces, an axial force $F_{a_2}$ due to the difference in discharge and suction pressures will act on the end surface of the screw. However, in most cases, the screw rotor compressor is designed so that the cavity formed by a housing and the end surface on the discharge side is connected to the suction cavity to unload a screw from action of an axial force $F_{a_2}$. 
2. Formulation of the problem

Thus, the task of power calculation of a screw single-rotor compressor is reduced, first of all, to the determination of forces $F_{a1}$ and $F_{t1}$, that are equal in magnitude. To determine them, it is necessary to know the law of pressure change in the closed cavities of the compressor, depending on the angle of rotation of the screw, to know the law of changing the horizontal projection area of the tooth of the cutter meshed with the screw, depending on the angle of rotation of the cutter [3]. When changing the horizontal projection area of tooth cutter meshed with a screw, depending on the angle of cutter rotation [3].

In order to graphically represent the law of change in closed cavities of aerospace defense, we use:

$$\frac{p_s}{P(\alpha)} = \left[\frac{V - V(\alpha)}{V}\right]^m$$  \hspace{1cm} (1)

where $m$ - an average polytropic indicator; $p_s$ - a suction pressure, Pa; $P(\alpha)$ - a pressure of the working medium in screw cavities depending on the rotation angle, Pa; $V$ - a volume of the helical groove, m$^3$; $V(\alpha)$ - a part of the volume of a screw groove of the screw described by tooth cutter meshed with screw, depending on the angle of screw rotation, m$^3$.

It follows from (1) that

$$\alpha = \frac{p_s V^m}{V - V(\alpha)}$$

where $\alpha$ - an angle of screw rotation: $\alpha = \beta / i$; $\beta$ - an angle of rotation of a disk cutter, deg; $i$ - gear ratio determined by the ratio of the screw number $z_1$ to the number of the disk cutter $z_2$.

The pressure of the working medium in the cavity of a screw, depending on an angle of the cutter’s rotation is found from:

$$p(\beta) = \left[\frac{p_s V^m}{V - V(\beta)}\right]^m.$$  \hspace{1cm} (2)

The volume of the screw groove of the screw is determined by the method based on the Gulden theorem, which assumes that the volume of the body obtained by rotating some flat closed figure
about an axis lying in the plane of this figure and not intersecting it is equal to the product of the area of this figure by the length of the arc described by it center of gravity \[4,5\].

3. Materials and methods

The term "flat closed figure" in our task means the horizontal projection of a tooth disk cutter meshed with a screw, which takes the form of a triangle or trapezoid. The volume cavity of the screw is defined as the sum of the volumes \( V = V_1 + V_2 \), where \( V_1 \) – a volume of the part of the screw groove described by a flat closed figure in the form of a triangle, \( m^3 \); \( V_2 \) – the part volume of screw groove described by a flat closed figure in the form of a trapezoid, \( m^3 \).

Volume \( V_2 \) is determined:

\[
V_2 = A_1 + (\beta_{1k} - \beta_{1n}) - A_2 \ln \left( \frac{\beta_{1k} + \beta_{1n}}{\beta_{1k} - \beta_{1n}} \right) - A_3 (\sin \beta_{1k} - \sin \beta_{1n}).
\] (3)

There \( \beta_{1k} = \arccos \frac{D_{2n}}{D_{e2}} - \gamma \) – an angular gearing parameter, deg; \( \gamma = \arcsin \frac{b}{D_{e2}} \) – a half of the angular width of the tooth disk cutter, deg; \( D_{2n} \) – a diameter of the initial circumference of the disk cutter, m (Fig.2); \( D_{e2} \) – an outer diameter of the disk cutter, m; \( b \) – a width of the tooth of the disk cutter, \( m \); \( A_1 = \frac{D_{e2}}{41} (D_{e1} + D_{2n}) \), \( A_2 = \frac{b}{41} (D_{2n}D_{e1} + \frac{D_{2n}^2}{2} + \frac{b^2}{6}) \), \( A_3 = \frac{b}{81} (D_o^2 - \frac{b^3}{3}) \) - geometric constants, \( m^3 \). \( D_o = D_{e2} \cdot \cos \gamma \) - a diameter of the inscribed circle, m; \( \beta_{1n} \) – an angle of the beginning compression, that is, the angle at which the screw groove of the screw is cut off from the compressor suction cavity by the tooth of the disk cutter; \( |\beta_{1n}| = |\beta’| - |\beta_1| \), where \( \beta’ = \arccos \frac{D_{2n}}{D} \), \( \beta_1 = \arcsin \frac{b}{D} \) - angular parameters of engagement, deg; \( D = (D_{2n} + 4l)^{1/2} \) – a diameter of inscribed circle, m; \( l \) – a half the length of a chopped part of a screw [6].

![Figure 2. The horizontal projection of the engagement of the rotor screw with a disk cutter](image)
In this reference frame, the values of the angles $\beta', \beta_1, \beta_{1n}$ are negative therefore, the angle $\beta_{1n}$ in (3) should be substituted with a minus sign.

Volume $V_1$ is calculated using:

$$V_1 = \beta_1(\beta_{2k} - \beta_{1k}) - \beta_2 \frac{\tan \beta_{2k}}{\tan \beta_{1k}} + \frac{\sin \beta_{2k}}{\sin \beta_{1k}} + \beta_4 \frac{\cos \beta_{2k}}{\cos \beta_{1k}} + \beta_5 \frac{\tan \beta_{2k}}{\tan \beta_{1k}} - \beta_6 \frac{\tan (\beta_{2k}/2 + \pi/4)}{\tan (\beta_{1k}/2 + \pi/4)} - \beta_7 (\sin \beta_{2k} - \sin \beta_{1k}) - \beta_8 (\cos \beta_{2k} - \cos \beta_{1k}),$$

where $\beta_{2k} = \beta_{1k} + 2\gamma$ – an angular gearing parameter, deg:

- $\beta_1 = (D_0 b/8i)(D_{e1} + D_{2n})$
- $\beta_2 = (D_0 b/8i)(D_{2n} D_{e1} + D_{2n}^2/2 + D_0^2/6)$
- $\beta_3 = (D_0/16i)(D_{e1} + D_{2n})$
- $\beta_4 = (b^2/16i)(D_{e1} + D_{2n})$
- $\beta_5 = (D_{2n}^2/16i)(D_{e1} + D_{2n}/3)$
- $\beta_6 = (b/8i)(D_{2n} D_{e1} + D_{2n}^2/2 + b^2/6)$
- $\beta_7 = (b/16i)(D_0^2 + b^2/3)$
- $\beta_8 = (D_0/16i)(D_0^2/3 + b^2)$

geometric constants of screw single compressor, m³

4. Discussion and results

As a result of calculations carried out according to (2), the law of pressure change in the closed cavities of the screw single-rotor compressor in Figure 3.

![Figure 3. The law of pressure change in the closed cavities of a screw single-rotor compressor](image)

The internal compression pressure $P(\alpha)$ by the condition coincides with the discharge pressure $P_n$. The axial force acting on the disk cutter is found by:

$$F_{a1} = \sum_{i=1}^{n} F_{ai1},$$

where $F_{a1}$ is a force acting on one of a teeth disk cutter meshed with a screw, N; $n$ – the number of teeth disk cutter engaged with a screw [7,8].
The force acting on one tooth of disk cutter is determined by \( F_{a1i} = S(\beta)[P(\beta) - P_s] \), where \( S(\beta) \) – horizontal projection area of the tooth disk cutter meshed with a screw, depending on the angle rotation of the cutter, \( m^2 \).

The value \( S(\beta) \) in the range interval of angles \( \beta_1 \ll \beta \ll \beta_{1k} \) is determined by:

\[
S(\beta) = \frac{D_o b}{2} \left( 1 - \frac{D_{2n}}{D_o \cos \beta} \right),
\]

and in interval \( \beta_{1k} \ll \beta \ll \beta_{2k} \) by

\[
S(\beta) = \frac{D_o b \operatorname{ctg} \beta}{8} - \frac{D_o D_{2n}}{4 \sin \beta} + \frac{D_o b}{4} + \frac{b^2}{8} \tan \beta + \frac{D_{2n}^2}{8 \sin \beta \cos \beta} - \frac{D_{2n} b}{4 \cos \beta}.
\]

Graphically, the results of calculations carried out by (5) and (6) are presented in Figure 4.

5. Conclusion

To determine the torque \( M_1 \) acting on the screw rotor, it is necessary to calculate the circumferential forces \( F_{t1} \) acting on the screw and the moments of the application of forces \( L \) [9].

\[
M_1 = n \left[ F_{t1} L_{1(\beta)} + F_{t2} L_{2(\beta)} + \ldots + F_{tk} L_{1k(\beta)} \right],
\]

where \( n \) – number of disk cutters; \( L_i(\beta) \) – moment of the force \( F_{ti} \), \( m \). The moment of the force \( F_{t1i} \) is calculated by

\[
L_i(\beta) = \frac{D_{e1} + D_{2n}}{2} - r_i(\beta),
\]

where \( r_i(\beta) \) – moment of force’s application \( F_{a1i} \) relative to the axis of a cutter rotation, \( m \).

\[
r_i(\beta) = \left( y_c^2 + x_c^2 + \frac{D_{2n}^2}{4} - D_{e2} \sqrt{y_c^2 + x_c^2} \cos \left[ \arccos \frac{y_c}{\sqrt{y_c^2 + x_c^2}} - \arcsin \frac{b}{D_{e2}} \right] \right)^{1/2},
\]

by \( \beta_{1k} \ll \beta \ll \beta_{2k}, \beta_{2k} = \beta_k + 2\gamma \).

\[
r_i(\beta) = \left( \frac{b^2 \sin \beta}{6 D_o (\cos \beta - D_{2n}/D_o)} \right)^2 + \left[ \frac{D_o}{4} + \frac{D_{2n}}{4 \cos \beta} \right]^2 - \frac{b^2 \cos^2 \beta}{12 D_o \cos \beta (\cos \beta - D_{2n}/D_o)} \right)^{1/2},
\]

by \( \beta_{1h} \ll \beta \ll \beta_{1k} \).
There:

\[ y_c = \frac{a(f + c)}{2(f + c + a)}; a = \frac{D_o}{2} - \frac{D_{zn}}{2\cos \beta} + \frac{b}{2}\tan \beta, \]

\[ x_c = \frac{f(a + c)}{2(f + c + a)}; c = \left( a^2 + f^2 \right)^{1/2}; f = \frac{ac\tan \beta}{2}. \]

where \( y_c, x_c \) – the coordinates in center of gravity of the tooth disk cutter horizontal projection meshed with the screw; \( a, f, c \) – are the lengths sides of the tooth disk cutter horizontal projection meshed with a triangle and meshed with the screw [10,11].

The calculation method described above allows one to determine with sufficient accuracy the forces and moments acting on the working parts of a single rotor screw compressor, and therefore, to calculate the deformation of the screw single rotor compressor parts, determine the safe gaps in the compressor working part and provide the necessary strength and durability of the most critical screw single rotor compressor components.

References
[1] Sakun I A 1970 Screw compressors (Leningrad: Mechanical Engineering)
[2] Pronin V A 1978 “Investigation of a screw compressor with self-unloading support nodes”, PhD thesis, Leningrad.
[3] Pronin VA, Kuznetsov YL, Zhignovskaiia DV 2018 Features of designing screw compressors for the oil and gas industry AIP Conference Proceedings 2007 030017
[4] Stosic N, Smith I K, Kovacevic A and Zhang W M 2005 An Investigation of Liquid Injection in Refrigeration Screw Compressors, ICCR the 5th International Conference on Compressors and Refrigeration, Dalian.
[5] Heinz P. Bloch A 2006 Practical Guide to Compressor Technology, 2nd Edition «WILEY» pp 4-30 ISBN: 978-0-471-92952-9.
[6] Fleming J, Tang Y and Cook G 1998 The helical twin screw compressor. Part 1: development, applications and competitive position. Proc. Instn. Mech. Engrs., Part C, Journal of Mechanical Engineering Science 212 pp 355–367.
[7] Tian F, Tao K and Shao J 2010 The Research on Meshing Pair Profile of Single-Screw Compressor, IEEE Computer Society, Wuhan, China.
[8] Tang, Y 1995 Computer aided design of twin screw compressors. PhD thesis, University of Strathclyde, Scotland.
[9] Xing Z 2000 Screw Compressors: Theory, Design and Application, (in Chinese), China Machine Press, Beijing, China.
[10] Excell, J 2013 The rise of additive manufacturing The Engineer. Retrieved 31–56
[11] Yerezhep D, Tukmakova A, Fomin V, Masalimov A, Asach A, Novotelnova A and Baranov A 2018 J. Phys.: Conf. Ser. 1015 032151. DOI: 10.1088/1742-6596/1015/3/032151.