UHE neutrinos: higher twists, scales, saturation

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Abstract. It is shown that in the ultra-high energy neutrino interactions the higher twist corrections brought about by the non-conservation of the top-bottom current dramatically change the longitudinal structure function, $F_L$. To the Double Leading Log Approximation simple and numerically accurate formulas for $F_L$ and $\sigma^{\nu N}$ are derived.

1 What is UHE ?

Neutrinos coming from active galactic nuclei, gamma ray bursts [1] and emerging in more speculative scenarios like breakdown of Lorentz invariance and decays of super-massive particles have rather hard spectrum extending beyond $E_\nu \sim 10^{11}$ GeV [2]. These Ultra-High Energy (UHE) neutrinos probe the gluon density in the target nucleon at very small values of Bjorken $x$ thus providing an opportunity of doing small-$x$ physics in a new kinematical domain. The properties of the neutrino-nucleon total cross section $\sigma^{\nu N}(E_\nu)$ at $E_\nu$ above $10^8$ GeV were analyzed by many [3].

2 Scales - prodotti tipici

The overall hardness scale of the process $\nu N \rightarrow \mu X$ is usually estimated as

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Indeed, to the Double Leading Log Approximation (DLLA)

\[ \sigma^{\nu N} \propto \int dQ^2 \left( \frac{m_W^2}{m_W^2 + Q^2} \right)^2 \exp \sqrt{C \log(1/x) \log \log Q^2} \]  

and the origin of Eq.(1) becomes evident. This observation entails (is based on) the smallness of the characteristic value of Bjorken \( x \) which is \( x \sim \frac{m_W^2}{2m_N E_\nu} \).

### 3 When top enters the game

The above estimate, \( Q^2 \sim m_W^2 \), is not unreasonable only for light flavor currents. The top-bottom current needs special care. The phenomenon of Charged Current Non-Conservation (CCNC) pushes the hardness scale up to \( \sim m_t^2 \) [4].

### 4 \( F_L \) as a carrier of CCNC effects

Weak currents are not conserved. But in what way? For longitudinal/scalar W-boson the transition vertex \( W_L \rightarrow t \bar{b} \) is \( \propto \varepsilon_L^\mu J_\mu \propto \partial_\mu J_\mu \propto m_t \pm m_\bar{b} \). Therefore, the observable quantity \( F_L \propto \varepsilon_L^\mu T^{\mu\nu} \varepsilon_L^\nu \) called the longitudinal structure function provides a measure of the CCNC effect. Here \( T^{\mu\nu} \) represents the imaginary part of the forward scattering Compton amplitude. The longitudinal component of the \( \nu N \) total cross section is proportional to \( F_L \).

### 5 \( F_L \) and \( \kappa \)-factorization

The gauge invariant sum of diagrams like that shown in 1 results in

\[ \frac{dF_L(x, Q^2)}{dz d^2k} = \frac{Q^2}{4\pi^3} \int \frac{d^2\kappa}{\kappa^4 \alpha_s(q^2)\mathcal{F}(x, \kappa^2)} (V_S + A_S + V_P + A_P), \]

where \( \mathcal{F} \) is un-integrated gluon density, \( \kappa \) - gluon momentum, \( z, k \) - Sudakov’s variables of t-quark. We find it convenient to separate contributions of the light cone Fock states \( |tb \rangle \) with angular momentum \( L = 0 \) (S-wave) and \( L = 1 \) (P-wave). The appearance of the P-wave component is the manifestation of the CCNC.
6 Higher twists. P-wave: $|W\rangle \rightarrow |t\bar{b}, L = 1\rangle$

Normally, the transition of the light cone scalar $W$-boson into the P-wave $q\bar{q}'$-state is suppressed by the factor $m_W^2/Q^2$ [4]. However, in the case at issue $m_t^2 \equiv m_t^2 \gg Q^2 \sim m_W^2$ and, consequently, there is no suppression at all. Upon the azimuthal averaging

$$\langle V_P(m_t, m_b) \rangle \simeq \frac{(m_t - m_b)^2 \kappa^2(k^4 + \varepsilon^4)}{Q^2(k^2 + \varepsilon^2)^4}$$

and $\langle A_P(m_t, m_b) \rangle = (g_A/g_V)^2 V_P(m_t, -m_b)$, where $\varepsilon^2 = z(1-z)Q^2 + (1-z)m_t^2 + zm_b^2$.

In the soft gluon approximation, $\kappa^2 \ll k^2 + \varepsilon^2$, and the P-wave component of

$$F_L = F_L^S + F_L^P$$

is dominated by highly asymmetric configurations with [5]

$$z \sim 1 - \frac{m_b^2}{m_t^2 + Q^2}.$$ 

Therefore,

$$F_L^P(x, Q^2) \simeq \frac{m_t^2}{m_t^2 + Q^2} \int_{m_b^2}^{m_t^2} \frac{d\varepsilon^2}{\varepsilon^2} \frac{\alpha_S(\varepsilon^2)}{3\pi} G(x, \varepsilon^2)$$

Note, the factor $m_t^2/(m_t^2 + Q^2)$ emerges here as a property of the transition vertex $W \rightarrow t\bar{b}$ rather than the property of the interaction of the light cone $t\bar{b}$-dipole with the target [5].

7 S-wave: $|W\rangle \rightarrow |t\bar{b}, L = 0\rangle$

Once again for soft gluons the azimuthal averaging leads to

$$\langle V_S(m_t, m_b) \rangle \simeq \frac{1}{Q^2} \left(2Q^2 z(1-z) + (m_t - m_b) [(1-z)m_t - zm_b] \right)^2 \frac{2\kappa^2 k^2}{(k^2 + \varepsilon^2)^4}$$
and \((A_S(m_t, m_b)) = (g_A/g_V)^2 V_S(m_t, -m_b)\). The S-wave term in (5) integrated over \(k\) has approximately uniform \(z\)-distribution. Then the DLLA estimate is as follows

\[
F^S_L(x, Q^2) \simeq \frac{2\alpha_S(\bar{\varepsilon}^2)}{3\pi} G(x, \bar{\varepsilon}^2),
\]

where \(\bar{\varepsilon}^2 \simeq (Q^2 + 2m_t^2)/4\).

### 8 Numerical estimates

To DLLA the CCNC contribution to \(\sigma^{\nu N}\) with the gluon density \(G(x, k^2)\) from [6] is estimated as \(\sigma^{CCNC}_{\nu N} \simeq 0.43 \times 10^{-31} \text{ cm}^2\) for \(E_\nu = 10^{12} \text{ GeV}\). We neglected here the contribution of hard gluons to the proton longitudinal structure function. Therefore, the DLLA gives the lower estimate for \(F_L\).

For comparison, the frequently used massless approximation gives at \(E_\nu = 10^{12} \text{ GeV}\) the cross section \(\sigma^{\nu N}\) that for different gluon densities varies in the range [7]

\[
0.2 \times 10^{-31} \text{ cm}^2 < \sigma^{\nu N} < 1.5 \times 10^{-31} \text{ cm}^2
\]

### 9 Scales and saturation

At small-\(x\) the unitarity/saturation effect enters the game [8, 9]. In massless approximation the unitarity correction to \(\sigma^{\nu N}\) was found to be a 50 per cent effect [10]. In particular, it was shown that the unitarity effect turns \(\sigma^{\nu N}_{CC} \simeq 1 \times 10^{-31} \text{ cm}^2\) at \(E_\nu = 10^{12} \text{ GeV}\) into \(\sigma^{\nu N}_{CC} \simeq 0.5 \times 10^{-31} \text{ cm}^2\). The strength of the unitarity/saturation effect depends on the hardness scale of the process, the first higher twist correction is estimated as [11]

\[
\sim \frac{\alpha_S(Q^2) G(x, Q^2)}{Q^2} \frac{1}{\pi R^2}
\]

The CCNC hardness scale, \(m_t^2\), is much “harder” than the hardness scale for the light flavor currents. The latter is \(\lessapprox m_W^2\). Thus we conclude that the unitarity affects strongly the light quark contribution to \(\sigma^{\nu N}\) but leaves the CCNC term intact.

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