Adjoint charge in electromagnetic field

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Abstract — Making use of the octonion operator, the electromagnetic field generates an adjoint field theoretically. The source of adjoint field includes the adjoint charge and the adjoint current. The adjoint charge has an impact on the gravitational mass and the mass distribution in the electromagnetic field with its adjoint field, and causes further the predictions to departure slightly from the conservation of mass. The inferences can explain why the adjoint charge will influence the mass distribution in the gravitational field and electromagnetic field of celestial bodies. And then the adjoint charge can be considered as one kind of candidate for the dark matter.

1. INTRODUCTION

The mass and the 'missing mass' both are crucial physical conceptions for various field theories. There is only the conception of mass in the existing electromagnetic field and gravitational field, which are described with the vectorial quantity. But this perspective can not explain why there is the 'missing mass' [1, 2] in the universe up to now.

The algebra of quaternions [3] was first used by J. C. Maxwell to describe the electromagnetic field. While the octonions [4] can be used to demonstrate the characteristics of gravitational field and electromagnetic field, including the conservation of mass etc. The related theoretical inferences are only dealt with the quaternion operator but the octonion operator [5]. In the octonion space, the operator should be extended from the quaternion operator to the octonion operator.

Making use of the octonion operator, the electromagnetic field demonstrated by the octonions will generate an adjoint field. The source of adjoint field includes the adjoint charge and adjoint current. The adjoint charge and its movement can not be observed by usual experiments. However, when the adjoint charge is combined with the ordinary charge to become the charged particles, their movements will be accompanied by some mechanical or electric effects.

The electromagnetic field and its adjoint field both can be demonstrated by the quaternions, although they are quite different from each other indeed. With the property of octonions, we find that the adjoint charge has an influence on the conservation of mass in the electromagnetic field. The adjoint charge takes part in the gravitational interaction and impacts the mass distribution, and then can be considered as one kind of candidate for dark matter [6, 7].

2. OCTONION TRANSFORMATION

The quaternion spaces can be used to describe the electromagnetic field or the gravitational field. Meanwhile, two quaternion spaces can combine together to become an octonion space. The latter can be used to demonstrate the electromagnetic field and gravitational field simultaneously.

The quaternion space can be considered as the two-dimensional complex space, and the octonion space as the two-dimensional quaternion space. In the quaternion space for the gravitational field, the basis vector is \( \mathbb{E}_g = (1, i_1, i_2, i_3) \), and the radius vector is \( \mathbb{R}_g = (r_0, r_1, r_2, r_3) \), with the velocity \( \mathbb{V}_g = (v_0, v_1, v_2, v_3) \). For the electromagnetic field, the basis vector is \( \mathbb{E}_e = (I_0, I_1, I_2, I_3) \), the radius vector is \( \mathbb{R}_e = (R_0, R_1, R_2, R_3) \), with the velocity \( \mathbb{V}_e = (V_0, V_1, V_2, V_3) \).

The \( \mathbb{E}_e \) is independent of the \( \mathbb{E}_g \), with \( \mathbb{E}_e = \mathbb{E}_g \circ I_0 \). The \( \circ \) denotes the octonion multiplication. The \( \mathbb{E}_g \) and \( \mathbb{E}_e \) can be combined together to become the basis vector \( \mathbb{E} \) of the octonion space.

\[
\mathbb{E} = \mathbb{E}_g + \mathbb{E}_e = (1, i_1, i_2, i_3, I_0, I_1, I_2, I_3)
\]

In the octonion space, the radius vector \( \mathbb{R} \) is
\[
\mathbb{R} = \Sigma(r_i i_i) + \Sigma(R_i I_i),
\]

and the velocity \( \mathbb{V} \) is
\[
\mathbb{V} = \Sigma(v_i i_i) + \Sigma(V_i I_i).
\]
where, \( r_0 = v_0 t; \) \( v_0 \) is the speed of light, \( t \) is the time. \( i = 0, 1, 2, 3, \ j = 1, 2, 3, \ 0_i = 1.\)

In some special cases, one ordinary mass \( m \) can be combined with one ordinary charge \( q \) to become an ordinary particle, such as the proton and electron etc. And then, we can measure their various characteristics, and have following relation.

\[ R_i I_i = r_i I_0 \ ; \ V_i I_i = v_i I_0 . \] (4)

The octonion quantity \( \mathbb{D}(d_0, d_1, d_2, d_3, D_0, D_1, D_2, D_3) \) is defined as follows.

\[ \mathbb{D} = d_0 + \Sigma(d_j i_j) + \Sigma(D_i I_i) \] (5)

where, \( d_i \) and \( D_i \) are all real.

When the coordinate system is transformed into the other, the physical quantity \( \mathbb{D} \) will be transformed into one new octonion \( \mathbb{D}'(d'_0, d'_1, d'_2, d'_3, D'_0, D'_1, D'_2, D'_3) \).

\[ \mathbb{D}' = K^* \circ \mathbb{D} \circ K \] (6)

where, \( K \) is one octonion, and \( K^* \circ K = 1; \ * \) denotes the conjugate of octonion.

If the \( d_0 \) does not take part in the above transformation, it satisfies the following relation.

\[ d_0 = d'_0 \] (7)

In the above equation, the scalar part \( d_0 \) is preserved during the octonion coordinates are transforming. Some scalar invariants of electromagnetic field will be obtained from this characteristics of the octonion.

| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
|---|---|---|---|---|---|---|
| \( \mathbb{I}_0 \) | \( \mathbb{I}_1 \) | \( \mathbb{I}_0 \) | \( \mathbb{I}_1 \) | \( \mathbb{I}_2 \) | \( \mathbb{I}_3 \) |
| \( \mathbb{I}_0 \) | \( \mathbb{I}_1 \) | \( \mathbb{I}_2 \) | \( \mathbb{I}_3 \) |
| \( \mathbb{I}_0 \) | \( \mathbb{I}_1 \) | \( \mathbb{I}_2 \) | \( \mathbb{I}_3 \) |

3. ELECTROMAGNETIC FIELD

By means of the octonion operator, the electromagnetic field will generate an adjoint field in the octonion space. The adjoint field is derived from the electromagnetic field potential. The source of adjoint field includes the adjoint charge and adjoint electric current. In case of the electromagnetic field is accompanied by the adjoint field, the adjoint charge has an influence on the gravitational mass, and then causes some mechanical or electric effects. As a result, the adjoint charge may be considered as one kind of candidate for the dark matter.

The electromagnetic field potential is

\[ A_e = \Sigma(A_i I_i) . \] (8)

In the electromagnetic field, the field strength \( \mathbb{B}_e = \Sigma(b_{ei} I_i) + \Sigma(B_{ei} I_i) \) consists of the electromagnetic strength \( \mathbb{B}_{eg} \) and adjoint strength \( \mathbb{B}_{ee} \).

\[ \mathbb{B}_e = \Delta \circ A_e = \mathbb{B}_{eg} + \mathbb{B}_{ee} \] (9)

where, \( \mathbb{B}_{ee} = \Sigma(b_{ei} I_i), \mathbb{B}_{eg} = \Sigma(B_{ei} I_i); \ \Delta = \Sigma I_i (\partial/\partial r_i) + \Sigma I_i (\partial/\partial R_i). \)

In the above, we choose the following gauge conditions to simplify succeeding calculation.

\[ \partial A_0/\partial r_0 - \Sigma(\partial A_j/\partial r_j) = 0 , \ \partial A_0/\partial R_0 + \Sigma(\partial A_j/\partial R_j) = 0 . \] (10)
The adjoint field strength $\mathbb{B}_{eg}$ in Eq.(9) includes two components, $\mathbf{g}_e = (g_{e01}, g_{e02}, g_{e03})$ and $\mathbf{b}_e = (g_{e23}, g_{e31}, g_{e12})$.

$$\mathbf{g}_e/v_0 = i_1(\partial_0 A_1 - \partial_1 A_0) + i_2(\partial_0 A_2 - \partial_2 A_0) + i_3(\partial_0 A_3 - \partial_3 A_0)$$

$$\mathbf{b}_e = i_1(\partial_3 A_2 - \partial_2 A_3) + i_2(\partial_3 A_1 - \partial_1 A_3) + i_3(\partial_2 A_1 - \partial_1 A_2)$$

Meanwhile, the electromagnetic field strength $\mathbb{B}_{eg}$ involves two parts, $\mathbf{E}_e = (E_{e01}, E_{e02}, E_{e03})$ and $\mathbf{B}_e = (B_{e23}, B_{e31}, B_{e12})$.

$$\mathbf{E}_e/v_0 = I_1(\partial_0 A_1 + \partial_1 A_0) + I_2(\partial_0 A_2 + \partial_2 A_0) + I_3(\partial_0 A_3 + \partial_3 A_0)$$

$$\mathbf{B}_e = I_1(\partial_3 A_2 - \partial_2 A_3) + I_2(\partial_3 A_1 - \partial_1 A_3) + I_3(\partial_2 A_1 - \partial_1 A_2)$$

The electric current density $S_{eg} = qV_g \circ \mathbf{I}_0$ is the source of electromagnetic field, and its adjoint current density $S_{e} = qV_g$ is that of adjoint field. And they can be combined together to become the field source $S_e$. In the octonion space, the electromagnetic source $S_e$ can be defined from the electromagnetic field strength $\mathbb{B} = k_b \mathbb{B}_e$.

$$\mu \mathbb{S} = -(\mathbb{B}/v_0 + \mathbb{B})^* \circ \mathbb{B} = k_b(\mu_{ee} S_{ee} + \mu_{eg} S_{eg}) - \mathbb{B}^* \circ \mathbb{B}/v_0$$

where, $k_b^2 = \mu_{gg}/\mu_{eg}; \mu_{gg}, \mu_{ee}$, and $\mu_{eg}$ are the coefficients. $q$ is the adjoint charge.

The $\mathbb{B}^* \circ \mathbb{B}/(2\mu_{gg})$ is the field energy density.

$$\mathbb{B}^* \circ \mathbb{B}/\mu_{gg} = \mathbb{B}^* \circ \mathbb{B}_e/\mu_{eg}$$

The above means that the electromagnetic field and its adjoint field both make a contribution to the gravitational mass in the octonion space.

4. CONSERVATION OF MASS

In the electromagnetic field and its adjoint field, for one charged particle with inertial massless, the linear momentum density $\mathbb{P} = \mu S/\mu_{gg}$ is written as

$$\mathbb{P} = \hat{m} v_0 + \Sigma(M_q V_i \circ \mathbf{I}_0) + \Sigma(M_e v_i \hat{i}_i).$$

where, $\hat{m} = -(\mathbb{B}^* \circ \mathbb{B}/\mu_{gg})/v_0^2$; $M_q = qk_b \mu_{eg}/\mu_{gg}; M_e = qk_b \mu_{ee}/\mu_{gg}$.

The above means that the gravitational mass density ($\hat{m} + M_e$) is changed with all kinds of field strengths in the electromagnetic field and its adjoint field. From Eq.(6), we have one linear momentum density, $\mathbb{P}(p_0', p_1', p_2', p_3', P_0', P_1', P_2', P_3')$, when the octonion coordinate system is rotated. And we obtain the invariant equation from Eqs.(7) and (16).

$$(\hat{m} + M_e) v_0 = (\hat{m}' + M_e') v_0'$$

Under Eqs.(3), (7), and (17), we find the gravitational mass density ($\hat{m} + M_e$) remains unchanged.

$$\hat{m} + M_e = \hat{m}' + M_e'$$

The above means that the gravitational mass density ($\hat{m} + M_e$) will keep unchanged, under the octonion coordinate transformation in Eq.(6) in the electromagnetic field and its adjoint field.

5. CONCLUSION

In the octonion space, the electromagnetic field described by the octonion operator will generate an adjoint field. In some cases, the electromagnetic field will be accompanied by its adjoint field. And that the source of adjoint field will impact the mass distribution and the conservation of mass in the electromagnetic field and gravitational field, especially in the universe.

In the electromagnetic field with its adjoint field, the adjoint field exerts an influence on the gravity in two aspects. On the one hand, the adjoint charge presents to the quaternion space for the gravitational field, and is one part of the gravitational mass. The adjoint charge is similar to the mass, and possesses the characteristics of gravity. On the other hand, the gravitational mass density is changed with the electromagnetic strength and adjoint field strength. The gravitational mass takes part in the gravitational interaction, so that the adjoint field strength will effect the gravity. It states that the conservation of mass will be changed with the adjoint field strength and
adjoint charge. Therefore the adjoint charge can be considered as one kind of dark matter in the astronomy related to the electromagnetic field.

It should be noted that the study of adjoint charge in the electromagnetic field and its adjoint field examined only one simple case with very weak field strength in the electromagnetic field with its adjoint field. Despite its preliminary character, this study can clearly indicate the field strength of electromagnetic field and its adjoint field have an influence on the gravity and the conservation of mass. For the future studies, the related investigation will concentrate on only the predictions of the gravity fluctuation and the mass distribution, in the strong adjoint field strength of electromagnetic field with its adjoint field.

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