Classification method of production system equipment importance based on grey correlation interval AHP-Entropy method

Binbin XU*,**, Ziyue WANG*, Wei LUO* and Tianqi MA*
*Key Laboratory of CNC Equipment Reliability, Ministry of Education, School of Mechanical and Aerospace Engineering, Jilin University, Changchun, Jilin 130022, People’s Republic of China
E-mail: luoweicn@jlu.edu.cn
**Sino-German College of Intelligent Manufacturing, Shenzhen Technology University, Shenzhen, Guangdong 518118, People’s Republic of China

Abstract
In order to better implement maintenance strategies according to equipment levels, this paper proposes a method based on the grey correlation interval Analytic hierarchy process(AHP)-Entropy method to divide the importance of production system equipment. First, a three-level importance evaluation index system for the production system equipment is established considering the characteristics of the production system in five aspects of operating rate, reliability, maintainability, economy and detectability. Next, the interval number is combined with the AHP and the Entropy method to obtain the interval AHP and the interval Entropy method, and the two methods are combined into an interval AHP-Entropy method to determine the combined weight of each evaluation index. Combined with the evaluated equipment, the grey correlation comprehensive evaluation analysis is performed, the grey correlation degree is calculated and ranked, and the equipment is finally divided by the ABC classification method. Finally, the method is applied to the equipment division of the actual production system. Compared with the interval AHP and interval Entropy method, the evaluation difference rates are reduced by 20% and 15%, respectively, which verifies the effectiveness of the method and provides an accurate and scientific method for equipment division.

Keywords: Equipment division, Interval number, Analytic hierarchy process(AHP), Entropy method, Grey correlation comprehensive evaluation analysis

1. Introduction

The production system has been adopted by more and more manufacturing enterprises due to its advantages of high output, good product quality, and labor cost saving. Equipment is an important part of the production system, but because of its variety and complex structure, any equipment failure may cause the shutdown of the entire production system, it is not reasonable to adopt the same maintenance strategy for all equipment. Therefore, it is of great significance to divide the production system equipment first and then formulate different maintenance strategies according to the classification level. A scientific and effective method to divide the importance of production system equipment is the primary problem to be solved.

Multi-index comprehensive evaluation is a common method to solve the equipment division. According to different weight determination methods, it can be divided into three categories: the first category is subjective weighting method, such as AHP, Delphi method, fuzzy comprehensive evaluation method, etc. Nguyen, H. (2009) applied AHP to the risk estimation of the ship system and used the pairwise comparison method to deal with it. Cioca, L. I. et al. (2019) applied AHP to the selection of five-axis CNC machining centers equipped on the production system to determine the best model. Lee, A. H. I. et al. (2010) used the fuzzy Delphi method to select the influencing factors of new equipment technology transfer, and provided a basis for enterprises to purchase new equipment. Li, Y. et al. (2017) combined the fuzzy comprehensive evaluation method and the AHP to evaluate the operation level of energy-intensive equipment, and verified the effectiveness of the method through a case. This kind of method is greatly influenced by human factors.
and has a strong subjective judgment color. The second category is objective weighting method, such as principal component analysis, Entropy method, TOPSIS method, etc. Banguero, E. et al. (2020) used principal component analysis (PCA) to diagnose the health status of a battery energy storage system, established a PCA model and determined the causes that affect it. Liu, F. et al. (2017) established a large commercial building fire risk evaluation system, calculated the weight of each index by the Entropy method, and finally calculated the fire risk rating value of the evaluation system. Liu, J. et al. (2018) adopted the fuzzy TOPSIS method similar to ideal solution to calculate the overall risk levels of electric vehicle charging infrastructure projects, and verified the feasibility and effectiveness of the method through a comparative analysis and a sensitivity analysis. This kind of method depends on enough sample data to easily ignore the relative importance of experts to different indicators. The third category is the combination weighting method. Wu, G. D. (2017), Wang, J. S. (2017) and An, Z. (2018) all use the AHP and Entropy method to determine the subjective and objective weights of evaluation indexes, and then adopt the combination weighting method to determine the comprehensive weight of indexes, which overcomes the shortcomings of the single method and makes the evaluation results more accurate. Shaverdi, M. (2016) and Zyoud, S. H. (2016) both adopt the fuzzy AHP to determine the weight of evaluation indexes, and the fuzzy TOPSIS method to rank the evaluated schemes according to the evaluation and preference of experts. This kind of method can make the experts’ weighting of different attributes reach the unity of subjective and objective.

When experts face incomplete information and complicated problems, interval numbers are used to describe them. The interval number represents a kind of ambiguity and uncertainty of itself. It can be expressed in the form of interval in practical problems, which is more in system with objective reality and human thinking. Ahn, B. S. (2017) and Yan, X. H. (2015) both use the interval AHP to calculate interval weights by comparing the schemes and failure factors. Zhang, Q. (2018) and Zhao, H. (2017) both adopt the interval Entropy method to calculate the interval weight of evaluation indexes, which can better reflect the continuity of weight values. The current researches all use a single method and interval number. Therefore, it is more significant to propose a weight determination method that takes into account the interval AHP and interval Entropy method.

This paper proposes a production system equipment division method that can comprehensively consider the advantages and disadvantages of subjective and objective weighting methods, and can accurately reflect the actual situation of decision information. First, a three-level evaluation index system for the importance of production system equipment is established. Secondly, an interval AHP-Entropy method is proposed to determine the combined weight of each evaluation index, then conduct the grey correlation comprehensive evaluation analysis and the importance division for the evaluated equipment. Finally, a production system is taken as an example to divide the equipment.

2. Establishing the importance evaluation index system for the production system equipment

Determining the evaluation index is the core part of establishing the importance evaluation index system for the production system equipment. Scientific, objective, and reasonable evaluation index has a crucial impact on the accuracy of the evaluation results. According to the principles of comprehensiveness, objectivity and representativeness proposed by Wan, N. F. (2013), taking full account of all aspects of the evaluation purpose, the characteristics of the production system and the factors affecting the evaluation, as well as on-site inspections of the production system, a large number of data collection and related expert consultation, referring to the evaluation index proposed by Li, K. C. (2018), Tang, Y. (2017), Wang, X. J. (2017) and Yoon, Y. G. (2019), the factors that have a great influence on the production system equipment importance are selected as the final construction elements of the index system, and the three-level importance evaluation index system for the production system equipment is established as shown in Table 1. Take the importance of the production system equipment as the highest level $U$, select five aspects of operating rate, reliability, maintainability, economy and detectability as the middle level (the first-level evaluation index $U_i (i = 1, 2, \cdots, m)$), and then divide the first-level evaluation index of the middle level into 15 lowest level indexes (the second-level evaluation index $U_{ij} (i = 1, 2, \cdots, n; j = 1, 2, \cdots, n_i)$).
Table 1 Three-level importance evaluation index system for the production system equipment.

| Importance of the production system $U$ | Operating rate $U_1$ | Utilization rate of equipment $U_{11}$ |
|-----------------------------------------|----------------------|---------------------------------------|
|                                         |                      | Downtime and quality loss $U_{12}$    |
| Reliability $U_2$                       |                      | Failure impact on human and environment $U_{21}$ |
|                                         |                      | Failure impact on system function $U_{22}$ |
|                                         |                      | Importance in operation $U_{23}$       |
|                                         |                      | Failure frequency $U_{24}$             |
| Maintainability $U_3$                   |                      | Difficulty of inspection $U_{31}$      |
|                                         |                      | Difficulty of maintenance $U_{32}$     |
|                                         |                      | Repair skill of maintenance personnel $U_{33}$ |
|                                         |                      | Supply of spare parts $U_{34}$         |
| Economy $U_4$                           |                      | Inspection cost $U_{41}$               |
|                                         |                      | Maintenance cost $U_{42}$              |
|                                         |                      | Downtime cost $U_{43}$                 |
|                                         |                      | Equipment renewal cost $U_{44}$        |
| Detectability $U_5$                     |                      | Monitorability $U_{51}$               |

3. Determining index weights for interval AHP-Entropy method

This paper proposes an interval AHP-Entropy method to calculate the combination weight of evaluation indexes. The interval AHP-Entropy method is a weight determination method that combines interval numbers with AHP and Entropy method into interval AHP and interval Entropy method, and then combines the two methods. Interval weights are calculated through the interval number matrix and weight vector, and a distance function is introduced to obtain the combined weight values of the first-level and the second-level evaluation indexes, respectively.

3.1 Determining index weights for interval AHP

(1) Constructing the judgment matrix for interval AHP

Each element in the judgment matrix of interval AHP is an interval number. In the judgment matrix, $u_{ij} = [1,1]$, $u_{ij} = [a_{ij}, b_{ij}]$, $u_{ji} = 1/u_{ij} = [1/b_{ij}, 1/a_{ij}]$ are the upper and lower limits of the judgment results, indicating the relative importance between the two indexes. The value of the judgment matrix generally adopts the 1-9 scale method established by Professor Satty, T. L. (1977) as shown in Table 2. The 1-9 scale method uses nine numbers between 1 and 9 and their reciprocal as the value of the judgment element to scale the relative importance of each index to form a judgment matrix. This method can evaluate the importance of the indexes, check and maintain the consistency of the judgment process. It is a very commonly-used method. Many papers also use the 1-9 scale method, such as Beynon, M. (2002), Zhang, Z. Y. (2009), or sometimes use the shorter 1-5 scale method, such as Wang, G. Q. (2009). Invite $k$ experts to independently judge interval numbers of the relative importance between indexes, take $a_{ij} = \frac{1}{k} \sum_{k=1}^{k} a_{ij}^{(k)}$, $b_{ij} = \frac{1}{k} \sum_{k=1}^{k} b_{ij}^{(k)}$, in which $1/9 \leq a_{ij} \leq b_{ij} \leq 9$, and the interval judgment matrix is shown in Table 3.
| Number | Value | Meaning                      |
|--------|-------|------------------------------|
| 1      | 1     | \(i\) and \(j\) are equally important |
| 2      | 3     | \(i\) is slightly more important than \(j\) |
| 3      | 5     | \(i\) is obviously more important than \(j\) |
| 4      | 7     | \(i\) is much more important than \(j\) |
| 5      | 9     | \(i\) is extremely more important than \(j\) |
| 6      | 1/3   | \(i\) is slightly less important than \(j\) |
| 7      | 1/5   | \(i\) is obviously less important than \(j\) |
| 8      | 1/7   | \(i\) is much less important than \(j\) |
| 9      | 1/9   | \(i\) is extremely less important than \(j\) |
| 10     | \(2, 4, 6, 8, 1/2, 1/4, 1/6, 1/8\) | the intermediate value |

### Table 3 Interval judgment matrix.

| Index | \(U_1\) | \(U_2\) | \(\ldots\) | \(U_n\) |
|-------|---------|---------|-------------|---------|
| \(U_1\) | 1,1     | \(a_{12}, b_{12}\) | \(\ldots\) | \(a_{1n}, b_{1n}\) |
| \(U_2\) | \(1/b_{12}, 1/a_{12}\) | 1,1    | \(\ldots\) | \(a_{2n}, b_{2n}\) |
| \(\ldots\) | \(\ldots\) | \(\ldots\) | \(\ldots\) | \(\ldots\) |
| \(U_n\) | \(1/b_{1n}, 1/a_{1n}\) | \(1/b_{2n}, 1/a_{2n}\) | \(\ldots\) | 1,1 |

### (2) Determining the range of index weights

The interval logarithm least square method proposed by Wei, C. P. (1996) is used for \(A\) and \(B\) to obtain the normalized weight vectors \(w_l = [w_{a_1}, w_{a_2}, \ldots, w_{a_n}]^T\) and \(w_u = [w_{b_1}, w_{b_2}, \ldots, w_{b_n}]^T\), respectively:

\[
w_l = \left[ \prod_{i=1}^{n} \frac{a_{i,j} a_{k}}{a_{k_j} a_{i}} \right]^{1/2n}, k = 1, 2, \ldots, n; \quad w_u = \left[ \prod_{i=1}^{n} \frac{b_{i,j} b_{k}}{b_{k_j} b_{i}} \right]^{1/2n}, k = 1, 2, \ldots, n
\]

Calculated by \(A = (a_{ij})_{n \times n}, B = (b_{ij})_{n \times n}\):

\[
k_j = \sqrt{\frac{\sum_{i=1}^{n} \frac{1}{b_{ij}}}{\sum_{i=1}^{n} a_{ij}}}, \quad k_u = \sqrt{\frac{\sum_{i=1}^{n} \frac{1}{a_{ij}}}{\sum_{i=1}^{n} b_{ij}}}
\]

in which \(k_l\) and \(k_u\) are all positive real numbers satisfying \(0 < k_l w_l < k_u w_u\).

The index weight range of interval AHP is:

\[
w_{AHP} = [w_l, w_u] = [k_l w_l, k_u w_u]
\]

### 3.2 Determining index weights for interval Entropy method

#### (1) Normalizing the indexes

Because the measurement units of the indexes are not uniform, the matrices \(A = (a_{ij})_{n \times n}\) and \(B = (b_{ij})_{n \times n}\) should be standardized according to the following equation to obtain the normalized interval judgment matrices \(A' = (a'_{ij})_{n \times n}\) and \(B' = (b'_{ij})_{n \times n}\). For positive indexes,

\[
\begin{align*}
A' & = \left[ \frac{a_{ij}}{\sum_{i=1}^{n} a_{ij}} \right]_{n \times n} \\
B' & = \left[ \frac{b_{ij}}{\sum_{i=1}^{n} b_{ij}} \right]_{n \times n}
\end{align*}
\]
\[ a_{ij} = \frac{a_j - \min \{a_{i1}, a_{i2}, \ldots, a_{in}\}}{\max \{a_{i1}, a_{i2}, \ldots, a_{in}\} - \min \{a_{i1}, a_{i2}, \ldots, a_{in}\}}, \]
\[ b_{ij} = \frac{b_j - \min \{b_{i1}, b_{i2}, \ldots, b_{in}\}}{\max \{b_{i1}, b_{i2}, \ldots, b_{in}\} - \min \{b_{i1}, b_{i2}, \ldots, b_{in}\}} \]  
(4)

For negative indexes,
\[ a_{ij} = \frac{\max \{a_{i1}, a_{i2}, \ldots, a_{in}\} - a_j}{\max \{a_{i1}, a_{i2}, \ldots, a_{in}\} - \min \{a_{i1}, a_{i2}, \ldots, a_{in}\}}, \]
\[ b_{ij} = \frac{\max \{b_{i1}, b_{i2}, \ldots, b_{in}\} - b_j}{\max \{b_{i1}, b_{i2}, \ldots, b_{in}\} - \min \{b_{i1}, b_{i2}, \ldots, b_{in}\}} \]  
(5)

(2) Determining the information Entropy weights

Calculate the proportion of the \( j \)-th value in the lower and upper limits under the \( i \)-th index according to equation (6),
\[ g_j = \frac{a_j}{\sum_{j=1}^{n} a_j}, \quad h_j = \frac{b_j}{\sum_{j=1}^{n} b_j} \]
(6)

The information Entropy values of the lower and upper limits of the \( i \)-th index are:
\[ x_i = -\frac{1}{\ln n} \sum_{j=1}^{n} g_{ij} \ln g_{ij}, \quad y_i = -\frac{1}{\ln n} \sum_{j=1}^{n} h_{ij} \ln h_{ij} \]
(7)

If \( g_{ij} = 0 \) or \( h_{ij} = 0 \), it is recorded as \( x_i = 0 \) or \( y_i = 0 \).

According to equation (8), the information Entropy weights of the upper and lower limits of each evaluation index can be obtained as:
\[ w_x = \frac{1 - x_i}{n - \sum_{i=1}^{n} x_i}, \quad w_y = \frac{1 - y_i}{n - \sum_{i=1}^{n} y_i} \]
(8)

The index weight range of interval Entropy method is:
\[ w_{EM} = [w_{x1}, w_{y1}] \]
(9)

3.3 Determining index combination weights for interval AHP-Entropy method

In this paper, the subjective and objective weight ranges of indexes are calculated by interval AHP and interval Entropy method, respectively \( w_{AHP} = [w_a, w_b] \), \( w_{EM} = [w_{x1}, w_{y1}] \), and the combined weight interval ranges are recorded as \( w_i = [w_{x_i}, w_{y_i}] \).
\[ w_{x_i} = \alpha_i w_{x_1} + \beta_i w_{x_1}, \quad w_{y_i} = \alpha_2 w_{y_1} + \beta_2 w_{y_1} \]
(10)
in which \( \alpha_i (i = 1, 2) \) and \( \beta_i (i = 1, 2) \) are the weight distribution coefficients of interval AHP and interval Entropy method, respectively, and \( \alpha_i + \beta_i = 1 \).

There is a certain difference between the weight values obtained by interval AHP and interval Entropy method, the distance function adopted by Wu, M. Y. (2015) is introduced. The distance function has good geometric characteristics and can calculate the overall distance between the two methods, which reflects the difference between the two methods.
based on the upper and lower limits of the interval.

\[ \gamma_1(w_a, w_x) = \sqrt{\frac{1}{2} \sum_{i=1}^{m} (w_{ai} - w_{xi})^2}, \quad \gamma_2(w_b, w_y) = \sqrt{\frac{1}{2} \sum_{i=1}^{m} (w_{bi} - w_{yi})^2} \]  

(11)

in which \( \gamma_i (i=1,2) \) is the distance function.

Ensure that the difference degree of the distribution coefficients \( \alpha_i \) and \( \beta_i \) is equal to the distance function, it is necessary to satisfy the following requirements:

\[ \left[ \gamma_1(w_a, w_x) \right]^2 = (\alpha_i - \beta_i)^2, \quad \left[ \gamma_2(w_b, w_y) \right]^2 = (\alpha_i - \beta_i)^2 \]  

(12)

After determining the values of \( \alpha_i \) and \( \beta_i \), substitute them into equation (10) to calculate the upper and lower limits of the weight, and then take the intermediate value of the interval as the combined weight \( W_i \).

\[ W_i = \frac{w_i^- + w_i^+}{2} \]  

(13)

\( W_i \) is the index combination weight determined by the interval AHP-Entropy weight method.

4. Grey correlation comprehensive evaluation analysis

The grey correlation comprehensive evaluation analysis is a method to analyze the correlation degree or similarity degree between various elements in the system. Zhao, X. et al. (2016) established a multi-objective evaluation model of equipment maintenance support efficiency based on the grey correlation analysis method, and conducted in-depth research on it. This paper will establish a comprehensive evaluation method based on the interval AHP-Entropy method and grey correlation analysis. Determine the comparison sequence and the reference sequence, and calculate the grey correlation coefficient of each evaluation index of the comparison sequence and the reference sequence. Combined with the combination weight values of the first-level and second-level evaluation indexes, the grey correlation degree of the highest-level \( U \) of the evaluated equipment can be obtained. According to the grey correlation degree of the importance evaluation \( U \), all the evaluated equipment will be sorted and divided to achieve the overall goal of dividing the importance of production system equipment.

(1) Determining the equipment importance level table

This paper divides the scoring standard into five levels: {higher, high, medium, low, lower}, and assigns 9, 7, 5, 3, and 1 points according to the importance of the equipment. When the importance degree is higher than the maximum value, between two adjacent importance degrees, or lower than the minimum value, the scores are respectively assigned to 10, 8, 6, 4, 2, and 0, as shown in Table 4.

| Level | Description |
|-------|-------------|
| higher | Once the equipment fails, it will face long-term shutdown and maintenance, huger cost loss, and have a greater impact on the production system. It is generally difficult to identify in advance and requires special precautions and controls. |
| high | Once the equipment fails, it will face long-term shutdown and maintenance, huge cost loss, and have a great impact on the production system. |
| medium | Once the equipment fails, it will cause cost loss and have a certain impact on the production system, but it can be repaired in a short time. |
| low | Once the equipment fails, it will have a little impact on the production system, low cost loss, and it can be repaired in a short time. |
| lower | Once the equipment fails, it will have a less impact on the production system. |

(2) Determining the evaluation sample matrix

Assume that \( k \) experts scored the \( s \)-th equipment according to the evaluation indexes \( U_{ij}(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n) \).
and the scoring standard of the importance level table, and the average value is taken as the original data of the $s$-th evaluated equipment in the $U_{ij}$-th evaluation index. The data information is arranged into a matrix form, and the evaluation sample matrix of the evaluated equipment is recorded as $D_{U_i}$.

$$D_{U_i} = \begin{bmatrix} d_{i1}^1 & d_{i2}^1 & \cdots & d_{in}^1 \\ d_{i1}^2 & d_{i2}^2 & \cdots & d_{in}^2 \\ \vdots & \vdots & \ddots & \vdots \\ d_{i1}^s & d_{i2}^s & \cdots & d_{in}^s \end{bmatrix}$$  \hspace{1cm} (14)

In which, $d_{ij}^s$ is the original data of the $s$-th evaluated equipment in the $U_{ij}$-th evaluation index.

(3) Determining the optimal reference sequence

Assume the optimal reference sequence is:

$$d_{ij}^* = [d_{i1}^*, d_{i2}^*, \cdots, d_{in}^*]$$  \hspace{1cm} (15)

in which $d_{ij}^*$ is the optimal value of the $U_{ij}$-th evaluation index. If the evaluation index $U_{ij}$ is a positive index, the maximum value of the index among all evaluated equipment is taken; If the evaluation index $U_{ij}$ is a negative index, the minimum value among all evaluated equipment is taken.

After the optimal reference sequence is determined, the reference evaluation matrix $D_{U_i}^*$ can be constructed.

$$D_{U_i}^* = \begin{bmatrix} d_{i1}^* & d_{i2}^* & \cdots & d_{in}^* \\ d_{i1}^1 & d_{i2}^1 & \cdots & d_{in}^1 \\ \vdots & \vdots & \ddots & \vdots \\ d_{i1}^s & d_{i2}^s & \cdots & d_{in}^s \end{bmatrix}$$  \hspace{1cm} (16)

(4) Normalizing the evaluation matrix

Because the evaluation indexes usually have different dimensions and orders of magnitude, the original data need to be normalized and converted into values within the range of $[0,1]$. The matrix $D_{U_i}^*$ is normalized by equation (17).

For positive and negative indexes are:

$$c_{ij}^s = \frac{d_{ij}^* - \min d_{ij}^s}{\max d_{ij}^s - \min d_{ij}^s}, c_{ij}^s = \frac{\max d_{ij}^s - d_{ij}^*}{\max d_{ij}^s - \min d_{ij}^s}$$  \hspace{1cm} (17)

in which $\min d_{ij}^s$ and $\max d_{ij}^s$ are the minimum and maximum values of the $s$-th evaluated equipment in the $U_{ij}$-th evaluation index, respectively.

The normalized evaluation matrix is:

$$C_{U_i} = \begin{bmatrix} c_{i1}^* & c_{i2}^* & \cdots & c_{in}^* \\ c_{i1}^l & c_{i2}^l & \cdots & c_{in}^l \\ \vdots & \vdots & \ddots & \vdots \\ c_{i1}^s & c_{i2}^s & \cdots & c_{in}^s \end{bmatrix}$$  \hspace{1cm} (18)

(5) Determining the grey correlation coefficient

According to the grey system theory, $C_{U_i} = [c_{i1}, c_{i2}, \cdots, c_{in}]$ is taken as the reference sequence and $C_{U_i}^* = [c_{i1}^*, c_{i2}^*, \cdots, c_{in}^*]$ is taken as the comparison sequence. The grey correlation coefficient $\xi_{ij}$ between the $U_{ij}$-th evaluation index and the $U_{ij}$-th optimal evaluation index of the $s$-th evaluated equipment is calculated according to equation (19). The grey correlation coefficient reflects the similarity degree between the reference sequence and comparison sequence. The larger the grey correlation coefficient, the closer the comparison sequence to the reference
sequence.

\[
\xi_{ij}^s = \frac{\min_{y \in S} \min_{s \in S} |c_{ij}^s - c_{ij}^y| + \rho \max_{y \in S} \max_{s \in S} |c_{ij}^s - c_{ij}^y|}{|c_{ij}^s - c_{ij}^y| + \rho \max_{y \in S} \max_{s \in S} |c_{ij}^s - c_{ij}^y|}
\]  

(19)

In which, \( \rho \) is the resolution coefficient, \( \rho \in [0, 1] \), generally \( \rho = 0.5 \).

The correlation matrix of the evaluation index is:

\[
E_{U_i} = \begin{bmatrix}
\xi_{11}^1 & \xi_{12}^1 & \cdots & \xi_{1n_i}^1 \\
\xi_{21}^1 & \xi_{22}^1 & \cdots & \xi_{2n_i}^1 \\
\vdots & \vdots & \ddots & \vdots \\
\xi_{m_i}^1 & \xi_{m_i}^2 & \cdots & \xi_{m_i}^n
\end{bmatrix}
\]  

(20)

(6) Determining the grey correlation degree of the middle level \( U_i \)

The grey correlation degree of \( U_i \) is calculated according to equation (21).

\[
R_{U_i} = W_{U_i} \cdot E_{U_i}^T
\]  

(21)

In which, \( W_{U_i} \) is the combination weight vector of the lowest level evaluation index.

(7) Determining the grey correlation degree of the highest level \( U \)

The grey correlation degree of \( U \) is calculated according to equation (22). The larger the \( R_i \), the greater the correlation between the comparison sequence and the reference sequence on the evaluation index. The higher the evaluation result of the evaluated equipment, and the higher the classification level. On the contrary, the correlation degree between the two is smaller, the final evaluation result is lower and the classification level is lower.

\[
R_i = W_U \cdot \begin{bmatrix} R_{U_1}, R_{U_2}, \cdots, R_{U_n} \end{bmatrix}
\]  

(22)

In which, \( W_U \) is the combination weight vector of the middle level evaluation index.

Finally, the evaluated equipment is ranked according to the size of the grey correlation degree and classified by the Activity Based Classification proposed by the management scientist Dickie, H. F.. Assume that the top 20\% of equipment is defined as important equipment, 20\%-50\% of equipment is defined as secondary equipment, and the remaining equipment is defined as general equipment. Therefore, the division of the production system equipment has been completed.

**5. An example of the equipment division for a production system**

An automobile engine crankshaft production system is composed of 20 processing machines with numbers \( m_1-m_{20} \) connected in series. In this paper, the comprehensive evaluation method of the interval AHP-entropy method and the grey correlation analysis is used to obtain the combination weight of the equipment and the grey correlation degree of the highest level \( U \). All the evaluated equipment is sorted and divided, and the production system equipment is divided into important equipment, secondary equipment and general equipment, which achieves the goal of dividing the equipment importance of the production system. The production system equipment layout and the equipment name are shown in Figure 1, and the production system site operation diagram and the crankshaft component diagram are shown in Figure 2.
5.1 Determining index weights for interval AHP-Entropy method

According to the influencing factors of each evaluation index, a judgment matrix is constructed based on the evaluation opinions of five experts, and the combined weight is determined according to the calculation steps of the interval AHP-Entropy method. The results are shown in Tables 5 to 9 below.

| Table 5 Judgment matrix of the production system equipment importance $U$. |
|----------------|----------------|----------------|----------------|----------------|----------------|
| $U$            | $U_1$          | $U_2$          | $U_3$          | $U_4$          | $U_5$          | $W_U$         |
| $U_1$          | [1,1]          | [3,5]          | [4,6]          | [2,3]          | [5,6]          | 0.2114        |
| $U_2$          | $[1/5,1/3]$    | [1,1]          | [5/2,9/2]      | [1/3,1]        | [1/4,1/3]      | 0.1973        |
| $U_3$          | $[1/6,1/4]$    | $[2/9,2/5]$    | [1,1]          | $[1/5,1/3]$    | [5,7]          | 0.2454        |
| $U_4$          | $[1/3,1/2]$    | [1,3]          | [3,5]          | [1,1]          | $[7/2,5]$      | 0.1687        |
| $U_5$          | $[1/6,1/5]$    | [3,4]          | $[1/7,1/5]$    | $[1/5,2/7]$    | [1,1]          | 0.1772        |

| Table 6 Judgment matrix of operating rate $U_1$. |
|----------------|----------------|----------------|----------------|----------------|
| $U$            | $U_{11}$       | $U_{12}$       | $U_{13}$       | $W_{U_1}$      |
| $U_{11}$       | [1,1]          | $[3/4,3/2]$    |               | 0.5147         |
| $U_{12}$       | $[2/3,4/3]$    | [1,1]          |               | 0.4853         |
5.2 Grey correlation comprehensive evaluation analysis of production system equipment

This paper organizes 5 experts to score 20 evaluated equipment according to the evaluation indexes \(U_{ij}\) and the scoring standard of the importance level table, and takes the average value as the original data. The evaluation sample matrices of the evaluated equipment are:

\[
D_{V_1}^T = \begin{bmatrix}
1.8 & 2.8 & 3.2 & 3 & 3.6 & 5.4 & 4 & 2.6 & 3 & 4.8 & 3 & 3.6 & 4.8 & 4.6 & 3 & 3.2 & 3.6 & 3.2 & 2.2 & 2.6 \\
2.8 & 2.8 & 3.8 & 1.8 & 3.4 & 3.4 & 3.6 & 2.8 & 1.6 & 3.4 & 2.6 & 3.4 & 3.4 & 4 & 2 & 3.8 & 3.6 & 3 & 3 & 2.8
\end{bmatrix}
\]

\[
D_{V_2}^T = \begin{bmatrix}
3.2 & 3.8 & 4.4 & 4 & 3.6 & 2.6 & 4.4 & 3.6 & 2.8 & 4.4 & 3 & 2.6 & 4 & 2.6 & 4.2 & 2.4 & 3.8 & 4.6 & 3.2 \\
3.8 & 3.4 & 3.4 & 4.4 & 3.6 & 4.2 & 1.6 & 5.8 & 3.4 & 3 & 3 & 4 & 4.2 & 4.6 & 1.8 & 3 & 4.6 & 2.6 \\
1.8 & 2.6 & 5.8 & 4 & 4.8 & 4 & 3.8 & 2.2 & 4 & 3.4 & 3.8 & 3.8 & 1.8 & 4.6 & 4.2 & 3.2 & 2.6 & 3.6 & 4.2 & 3.6 \\
3.2 & 5.8 & 3.8 & 4.4 & 5 & 4 & 3.6 & 3.6 & 5 & 2.6 & 5.4 & 4.4 & 3.8 & 3.4 & 3 & 3 & 2.2 & 2.4 & 3 & 4 & 3.6
\end{bmatrix}
\]

\[
D_{V_3}^T = \begin{bmatrix}
2.6 & 3 & 4.6 & 3.6 & 3 & 3.6 & 3.6 & 4.4 & 3 & 3.2 & 4.4 & 2.6 & 4.2 & 3.4 & 2.4 & 3.8 & 4.6 & 3.6 \\
3.6 & 3 & 3.4 & 3.4 & 4.4 & 5 & 2.8 & 5.2 & 3 & 4.4 & 4 & 2.6 & 3.8 & 3.8 & 2 & 2.4 & 2 & 3.4 & 3 & 2.4 & 3.4 \\
2.6 & 3.4 & 3.6 & 2.8 & 4.2 & 3.8 & 4.4 & 1.6 & 4.8 & 2.2 & 3.6 & 4 & 3.4 & 4.6 & 3.8 & 3.2 & 3.6 & 3.2 & 2.2 & 3.6 \\
3 & 3 & 3.2 & 3.6 & 4 & 4 & 4.4 & 1.4 & 4.6 & 4 & 4.2 & 4.2 & 3.8 & 4.2 & 4.6 & 3 & 1.8 & 3 & 3 & 2.6 & 2.8
\end{bmatrix}
\]

\[
D_{V_4}^T = \begin{bmatrix}
3.2 & 4.2 & 3.2 & 3.6 & 4.6 & 3.4 & 1.8 & 1.4 & 4 & 4 & 3.4 & 4 & 3.8 & 3.8 & 4.6 & 4.2 & 1.8 & 1.6 & 3.2 & 2.8 \\
6 & 5.6 & 1.6 & 3.6 & 5.2 & 2.6 & 3.2 & 2.6 & 2.4 & 4.8 & 2.6 & 5.4 & 3.8 & 3.4 & 3.6 & 3.4 & 1.8 & 2.8 & 3.2 & 2 \\
2.2 & 2 & 4.2 & 3.2 & 5.6 & 3.4 & 3.6 & 2.8 & 3 & 3 & 3.4 & 4.8 & 3.6 & 4 & 5 & 4 & 2.6 & 3.6 & 3.6 & 3.4 \\
4.4 & 4.2 & 4.8 & 3.4 & 5 & 4 & 2.4 & 3.2 & 4 & 4 & 2.4 & 4.6 & 3 & 3 & 3.4 & 2.4 & 2.8 & 3.4 & 3.6 & 3.6
\end{bmatrix}
\]

\[
D_{V_5}^T = \begin{bmatrix}
2.6 & 2.4 & 2.6 & 4.4 & 4 & 3.4 & 2.8 & 1.6 & 2 & 3 & 4.6 & 4 & 3.8 & 3.4 & 3.2 & 2.2 & 2 & 3.6 & 2.8 & 3 \end{bmatrix}
\]

The optimal reference sequence is:

\[
d_{v_i}^* = [5.4 \ 1.6], d_{v_2}^* = [2.4 \ 1.6 \ 5.8 \ 2.2], d_{v_3}^* = [1.8 \ 2 \ 4.8 \ 4.6], d_{v_4}^* = [1.4 \ 1.6 \ 2 \ 2.4], d_{v_5}^* = [4.6]
\]

According to equation (17), the evaluation matrix \(D_{v_i}^T\) is normalized to obtain the matrix \(C_{v_i}\); According to equation (19), the grey correlation coefficient \(\xi_{ij}\) between the \(U_{ij}\)-th evaluation index and \(U_{ij}\)-th optimal evaluation index of the \(s\)-th evaluated equipment is calculated, and the correlation matrix of each index is obtained as follows:
According to equation (21), the grey correlation degree of the evaluation index $U_i$ is calculated as:

$$R_{U_i} = W_{U_i} \cdot E_{U_i}^T = [0.4142, 0.4532, 0.4029, 0.6366, 0.4515, 0.7088, 0.4715, 0.4441, 0.4880, 0.5969, 0.5423, 0.5937, 0.5856, 0.5881, 0.4880, 0.5953, 0.6153, 0.4620, 0.5218]$$

According to equation (22), the grey correlation degree of the evaluation index $U$ is calculated as:

$$R_i = W_U \cdot \left[ R_{U_1}, R_{U_2}, \ldots, R_{U_n} \right] = [0.4861, 0.5076, 0.5294, 0.6275, 0.5543, 0.5842, 0.5816, 0.4828, 0.5828, 0.5551, 0.5969, 0.5423, 0.5937, 0.5856, 0.5881, 0.4880, 0.5953, 0.6153, 0.4620, 0.5218]$$

The evaluated equipment is ranked according to the size of the grey correlation degree and classified by the ABC classification method. According to the calculation results, the equipment numbered 4, 11, 17, 18 are defined as the important equipment, recorded as A, the equipment numbered 6, 7, 9, 13, 14, 15 are defined as the secondary equipment, recorded as B, and the remaining are defined as the general equipment, recorded as C.

5.3 Comparative analysis

In order to verify the accuracy and scientific of the evaluation results of the grey correlation interval AHP-Entropy method proposed in this paper, the same data is used to compare the method with the interval AHP and interval Entropy method, respectively. Table 10 shows the grey correlation degree and classification of the evaluated equipment. The equipment number $m_i$ is taken as the abscissa, and the grey correlation degree $R_i$ obtained by the three methods for evaluating the equipment is taken as the ordinate. The evaluation result chart is shown in Figure 2.

| $E_{U_1}^T$ | $E_{U_2}^T$ | $E_{U_3}^T$ | $E_{U_4}^T$ |
|-------------|-------------|-------------|-------------|
| [0.3 0.4 0.5 0.4 0.5 1 0.6 0.4 0.4 0.8 0.4 0.5 0.8 0.7 0.4 0.5 0.5 0.4 0.4] | [0.6 1 0.8 1 0.6 0.5 0.8 0.4 0.5 0.7 0.4 0.6 0.8 0.4 0.8 0.4 1 0.4 0.3 0.6] | [0.7 0.6 0.4 0.5 0.6 0.5 0.6 0.3 0.4 0.6 0.4 0.4 0.7 0.5 0.6 0.8 0.8 0.1 0.5 0.5] | [0.5 0.5 0.4 0.5 0.6 0.4 0.7 0.3 0.7 0.3 0.6 0.4 0.4 0.7 0.5 0.7 0.5 0.5 0.1 0.5 0.5 0.5 0.4 0.4 0.8 0.4 0.4 0.3 0.4 0.4 0.4 0.4 0.3 0.4 0.8 0.1 0.4 0.4 0.4 0.4 0.4 0.4 0.4 0.4 0.3 0.4 0.4 0.4 0.4 0.4 0.4 0.4 0.4 0.4 0.4 0.4 0.4 0.4]|

(Xu, Wang, Luo and Ma, Journal of Advanced Mechanical Design, Systems, and Manufacturing, Vol.14, No.6 (2020))
Table 10 Grey correlation degree and classification.

| \( m_i \) | Interval AHP | Interval Entropy method | Interval AHP-Entropy method |
|---------|--------------|-------------------------|-----------------------------|
| 1       | 0.4775(C)    | 0.4883(C)               | 0.4861(C)                   |
| 2       | 0.4989(C)    | 0.5076(C)               | 0.5076(C)                   |
| 3       | 0.4887(C)    | 0.5450(C)               | 0.5294(C)                   |
| 4       | 0.6125(A)    | 0.6482(A)               | 0.6275(A)                   |
| 5       | 0.4740(C)    | 0.6024(B)               | 0.5543(C)                   |
| 6       | 0.6207(A)    | 0.5667(C)               | 0.5842(B)                   |
| 7       | 0.5577(B)    | 0.5893(B)               | 0.5816(B)                   |
| 8       | 0.5205(C)    | 0.4567(C)               | 0.4828(C)                   |
| 9       | 0.6216(A)    | 0.5604(C)               | 0.5828(B)                   |
| 10      | 0.5527(B)    | 0.5463(C)               | 0.5551(C)                   |
| 11      | 0.5380(C)    | 0.6480(A)               | 0.5969(A)                   |
| 12      | 0.4751(C)    | 0.5854(B)               | 0.5423(C)                   |
| 13      | 0.5640(B)    | 0.6141121(B)            | 0.5937(B)                   |
| 14      | 0.5417(B)    | 0.6141127(A)            | 0.5856(B)                   |
| 15      | 0.5564(B)    | 0.5993(B)               | 0.5881(B)                   |
| 16      | 0.4529(C)    | 0.4925(C)               | 0.4880(C)                   |
| 17      | 0.5900(A)    | 0.6028(B)               | 0.5953(A)                   |
| 18      | 0.5742(B)    | 0.6249(A)               | 0.6153(A)                   |
| 19      | 0.4566(C)    | 0.4614(C)               | 0.4620(C)                   |
| 20      | 0.5076(C)    | 0.5274(C)               | 0.5218(C)                   |

Fig. 3 Equipment evaluation results.

It can be seen from Figure 3 that the interval AHP and interval Entropy method have a large difference in the comprehensive evaluation value of the same equipment. This is because the interval AHP has higher subjectivity and is greatly affected by human factors. The interval Entropy method determines the weights based on the information provided by the data and uses objective data, but the results ignore the relative importance of experts on different evaluation index. Therefore, the interval AHP-Entropy method proposed in this paper comprehensively considers the effects of subjective and objective factors, so that the index evaluation reaches a unified subjective and objective. The evaluation result of the equipment lies between the interval AHP and interval Entropy method.

Comparing the results of equipment division by the three methods in Table 10, it can be seen that the evaluation results have some differences. The interval AHP and interval Entropy method yield different results for equipment 5, 6,
9, 10, 11, 12, 14, 17, and 18, and the evaluation difference rate is 45%. The interval AHP and interval AHP-Entropy method have different evaluation results on equipment 6, 9, 10, 11, 18, and the evaluation difference rate is 25%. The interval Entropy method and interval AHP-Entropy method have different evaluation results for equipment 5, 6, 9, 12, 14, and 17, with a difference rate of 30%. Compared with the other two methods, the method proposed in this paper has reduced the evaluation difference rate by 20% and 15% respectively, which effectively reduces the degree of difference in evaluation results.

6. Conclusion

(1) Aiming at the characteristics of the production system in five aspects of operating rate, reliability, maintainability, economy and detectability, a three-level importance evaluation index system for the production system equipment is established. This paper proposes the interval AHP-Entropy method, which uses the interval AHP and interval Entropy method to obtain interval weights, and then combines the two methods to obtain the comprehensive weight of the importance evaluation index, which reaches the unity of subjective and objective.

(2) Combining the grey correlation analysis and the combined weights obtained from the interval AHP-Entropy method to comprehensively evaluate the production system equipment, ranking the evaluated equipment according to the grey correlation degree, and completing the equipment division by the ABC classification method. It makes full use of all basic data to improve the scientific and accuracy of evaluation results.

(3) The grey correlation interval AHP-Entropy method is applied to an example of a production system. The results of equipment importance division are compared with the interval AHP and interval Entropy method, and the evaluation difference rates are reduced by 20% and 15% respectively, which verify the scientific and effective of the proposed method.

Acknowledgement

This research is supported by National Science and Technology Major Project of China (Grant No. 2014ZX04015031) and Science and Technology Development Plan Project of Jilin Province (Grant No. 201805200068JH).

References

Ahn, B. S., The analytic hierarchy process with interval preference statements, Omega, Vol.67 (2017), pp.177-185.
An, Z. and Song, L. R., Research on the role of AHP-entropy method in the identification and evaluation of China tariff source risk, Journal of Intelligent and Fuzzy Systems, Vol.34, No.2 (2018), pp.1053-1060.
Banguero, E., Correcher, A., Pérez-Navarro, Á. et al., Diagnosis of a battery energy storage system based on principal component analysis, Renewable Energy, Vol.146 (2020), pp.2438-2449.
Beynon, M., An analysis of distributions of priority values from alternative comparison scales within AHP, European Journal of Operational Research, Vol.140, No.1 (2002), pp.104-117.
Cioca, L. I., Breaz, R. E., and Racz, S. G., Reducing the risks during the purchase of five-Axis CNC machining centers using AHP method and fuzzy systems, Sustainability, Vol.11, No.2 (2019), pp.1-23.
Lee, A. H. I., Wang, W. M. and Lin, T. Y., An evaluation framework for technology transfer of new equipment in high technology industry, Technological Forecasting & Social Change, Vol.77, No.1 (2010), pp.135-150.
Li, K. C., Jin, L., Jin, R. N. et al., Comprehensive evaluation index system and weight grey correlation analysis of electric energy substitution project, AIP Conference Proceedings, Vol.2036, No.1 (2018), DOI:10.1063/1.5075645.
Li, Y., Sun, Z. D., Han, L. et al., Fuzzy comprehensive evaluation method for energy management systems based on an Internet of things, IEEE Access, Vol.5 (2017), pp.21312-21322.
Liu, F., Zhao, S. Z., Weng, M. C., et al., Fire risk assessment for large-scale commercial buildings based on structure entropy weight method, Safety Science, Vol.94 (2017), pp.26-40.
Liu, J. and Wei, Q., Risk evaluation of electric vehicle charging infrastructure public-private partnership projects in China using fuzzy TOPSIS, Journal of Cleaner Production, Vol.189 (2018), pp.211-222.
Nguyen, H., The application of the ahp method in ship system risk estimation, Polish Maritime Research, Vol.16, No.1 (2009), pp.78-82.
Satty, T. L., A scaling method for priorities in hierarchical structures, Journal of Mathematical Psychology, Vol.15, No.3 (1977), pp.234-281.

Shaverdi, M., Ramezani, I., Tahmasebi, R. et al., Combining fuzzy AHP and fuzzy TOPSIS with financial ratios to design a novel performance evaluation model, International Journal of Fuzzy Systems, Vol.18, No.2 (2016), pp.248-262.

Tang, Y., Liu, Q. Y., Jing, J. J. et al., A framework for identification of maintenance significant items in reliability centered maintenance, Energy, Vol.118 (2017), pp.1295-1303.

Wan, N. F., Ji, X. Y., Jiang, J. X. et al., A methodological approach to assess the combined reduction of chemical pesticides and chemical fertilizers for low-carbon agriculture, Ecological Indicators, Vol.24 (2013), pp.344-352.

Wang, G. Q., Qin, L., Li, G. X. et al., Landfill site selection using spatial information technologies and AHP: A case study in Beijing, China, Journal of Environmental Management, Vol.90, No.8 (2009), pp.2414-2421.

Wang, J. S., Diao, M. Z. and Yue, K. H., Optimization on pinch point temperature difference of ORC system based on AHP-Entropy method, Energy, Vol.141 (2017), pp.97-107.

Wang, X. J., Chen, X. Y., Yu, K. et al., Construction of smart distribution grid efficiency evaluation index system, 2017 IEEE Conference on Energy Internet and Energy System Integration (EI2) (2017), pp.1-4.

Wei, C. P. and Hou, C. J., Three new priority methods and their comparison in the uncertain type of AHP, Journal of Qufu Normal University, Vol.22, No.2 (1996), pp.25-31 (in Chinese).

Wu, G. D., Duan, K. F., Zuo, J. et al., Integrated sustainability assessment of public rental housing community based on a hybrid method of AHP-Entropy weight and cloud model, Sustainability, Vol.9, No.4 (2017), pp.603-627.

Wu, M. Y., Yang, K., Liu, L. et al., Reclaimed water assessment based on gray situation decision and combination weighting method, Journal of Water Resources and Water Engineering, Vol.33, No.8 (2015), pp.33-36 (in Chinese).

Yan, X. H. and Liu, C. X., Interval analytic hierarchy process application to fault diagnosis of electronic control engine, Road Transport, No.11 (2015), pp.99-103 (in Chinese).

Yoon, Y. G., Yu, S. W., Hyung, J. P. et al., A study on the reliability of equipment system through case-study on the manufacture of machinery/electronic equipment using practical QRM (quality, reliability, maintenance) process and evaluation index, Microelectronics Reliability, Vol.100-101 (2019), DOI:10.1016/j.microrel.2019.113411.

Zhang, Q., Fang, Z. G. and Tao, L. Y., Generalized interval grey entropy weight distribution for combining cases indicators, System Engineering Theory and Practice, Vol.38, No.8 (2018), pp.2057-2067.

Zhang, Z. Y., Liu, X. B. and Yang, S. L., A note on the 1-9 scale and index scale in AHP, Cutting-Edge Research Topics on Multiple Criteria Decision Making, Vol.35 (2009), pp.630-634.

Zhao, H., You, J. X. and Liu, H. C., Failure mode and effect analysis using MULTIMOORA method with continuous weighted entropy under interval-valued intuitionistic fuzzy environment, Soft Computing, Vol.21, No.18 (2017), pp.5355-5367.

Zhao, X., Guo, M. and Ruan, Y. J., Study on the equipment maintenance support efficiency evaluation based on the gray correlation analysis method, Advances in Energy, Environment and Materials Science, (2016), pp.327-330.

Zyoud, S. H., Kaufmann, L. G., Shaheen, H. et al., A framework for water loss management in developing countries under fuzzy environment: Integration of Fuzzy AHP with Fuzzy TOPSIS, Expert Systems with Applications, Vol.61 (2016), pp.86-105.