PERTURBATIVE ANALYSIS OF THE MSSM ELECTROWEAK PHASE TRANSITION

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Abstract
Light stops can give a strong first order electroweak phase transition in the Minimal Supersymmetric Standard Model (MSSM) satisfying one of the necessary conditions for electroweak baryogenesis. I discuss the region of parameter space where this occurs using a perturbative analysis of the Higgs effective potential. Two-loop QCD corrections associated with stop loops are crucial to open this baryogenesis window, which corresponds to $2.25 \lesssim \tan \beta \lesssim 3.6$, one stop not much heavier than the top quark, $m_A > 120 \text{ GeV}$ and a light Higgs boson with $m_h < 85 \text{ GeV}$. This region will be explored by LEP II very soon.

June 1997
Light stops can give a strong first order electroweak phase transition in the Minimal Supersymmetric Standard Model (MSSM) satisfying one of the necessary conditions for electroweak baryogenesis. I discuss the region of parameter space where this occurs using a perturbative analysis of the Higgs effective potential. Two-loop QCD corrections associated with stop loops are crucial to open this baryogenesis window, which corresponds to \( 2.25 \lesssim \tan \beta \lesssim 3.6 \), one stop not much heavier than the top quark, \( m_A > 120 \text{ GeV} \) and a light Higgs boson with \( m_h < 85 \text{ GeV} \). This region will be explored by LEP II very soon.

The study of the properties of the electroweak phase transition (and in particular of its strength) in the MSSM is motivated by the possibility of electroweak baryogenesis (which cannot take place in the minimal Standard Model). A necessary condition for a successful baryogenesis at the electroweak phase transition is that this transition is strong enough to suppress sphaleron reactions in the broken phase, preventing the erasure of the created baryon asymmetry. In this talk, I address the question of whether this constraining condition can be met in some region of the MSSM parameter space (improving thus over the situation in the SM). The emphasis here will be on the qualitative aspects rather than in the quantitative details, which can be found in refs. [1].

To a great extent, the prospects for a strong transition depend on the details of the \( T = 0 \) Higgs potential. The MSSM Higgs sector contains two Higgs doublets \( H_{1,2} \) of opposite hypercharge. Electroweak breaking is described by a Higgs potential which is a function of two fields \( \varphi_{1,2} \) (\( \sim \text{Re} H_{1,2}^0 \)). In the broken phase, the Higgs spectrum consists of two neutral scalars \( h^0, H^0 \) (\( m_h < m_H \)), one pseudoscalar \( A^0 \) and a charged Higgs pair \( H^\pm \) and, at tree-level, the properties of the Higgs sector are determined by \( \tan \beta = \langle H_2^0 \rangle / \langle H_1^0 \rangle = v_2 / v_1 \) and \( m_A \) (with \( v_2^2 + v_2^2 = (246 \text{ GeV})^2 \) fixed by the gauge boson masses).

In the \( (\varphi_1, \varphi_2) \) plane, the minimum of the \( T = 0 \) potential lies along the direction determined by \( \tan \beta \). When \( m_A \gg m_Z \), the mass eigenstate \( h^0 \) (which always has a mass controlled by the electroweak scale) is aligned with that breaking direction while the orthogonal \( H^0 \) direction has mass \( m_H \approx m_A \). In that case, the low-energy potential reduces to a one-dimensional SM-like potential \( V(h^0) \). The same is true for the temperature dependent potential.
as long as $T \ll m_H$. For $m_A \sim m_Z$, $h^0$ forms some angle with the breaking direction and the full two Higgs potential should be considered. This picture is not affected qualitatively by radiative corrections which, however, change sizably the quantitative properties of the Higgs sector. The main effect is that the mass along the breaking direction is

$$m_\beta^2 = m_Z^2 \cos^2 2\beta + \frac{3}{2\pi^2} \frac{m_t^4}{v^2} \log \frac{m_{t_1} m_{t_2}}{m_t^2},$$  \hspace{1cm} (1)$$

where the second term comes from top-stop (with masses $m_t = h_t \varphi_2$ and $m_{t_1,2}$ respectively) loops. The mass of $h^0$ satisfies $m_h \leq m_\beta$.

The temperature dependent effective potential can be computed up to two-loops in perturbation theory (with resummation of important thermal effects) using standard techniques\textsuperscript{3}, and the electroweak transition at $T_c$ studied. To a good approximation, this transition proceeds along a fixed direction $\varphi_2/\varphi_1 \sim \tan \beta_{T_c} = v_2(T_c)/v_1(T_c)$. The numerical analysis shows that $\tan \beta_{T_c} \gtrsim \tan \beta$ ($\gg$ for small $m_A$ and $\simeq$ for large $m_A$). The transition strength, as measured by $v(T_c)/T_c$, is controlled by the quartic Higgs coupling along the direction of breaking

$$\frac{v(T_c)}{T_c} \sim \frac{1}{\lambda_{\beta T_c}},$$  \hspace{1cm} (2)$$

and $\lambda_{\beta T_c}$ is directly proportional to $m_\beta^2$ (given by Eq. (1) with $\beta \rightarrow \beta_{T_c}$) which, in general, is not the mass of any physical Higgs boson. The condition to avoid the erasure of baryons by sphalerons right after the transition is $v(T_c)/T_c > 1$.

We can now describe how $v/T_c$ depends on the $(m_A, \tan \beta)$ parameters. At large $m_A$ ($\gg m_Z, T_c$), $\beta_{T_c} \simeq \beta$ and $m_\beta^2 \simeq m_h^4$. From the $\tan \beta$ dependence of Eq. (1) we conclude that $v/T_c$ is larger for $\tan \beta \simeq 1$ (which minimizes $m_h$). For small $m_A$ ($\sim m_Z, T_c$), $\beta_{T_c} \gg \beta$ and larger values of $m_\beta^2$ are probed. In this case, having a light $h^0$ does not help to increase $v/T_c$ because $h^0$ lies along a direction which is not the breaking direction. We conclude that, for fixed $\tan \beta$, $v/T_c$ is smaller for smaller $m_A$ and the region where the transition is stronger is that of large $m_A$ and small $\tan \beta$. In that region, $h^0$ (and its potential) is SM-like. This means that, without extra SUSY contributions to the potential, the SM result $v/T_c \ll 1$ for $m_h > 70.7$ GeV (LEP II limit\textsuperscript{5}) is recovered. Thus, the two-doublet structure of the potential, being constrained by Supersymmetry, is of no help to strengthen the transition.

The important SUSY effects that can change this situation are those of the stops, which, being bosons, with a large number of degrees of freedom and a large coupling ($\sim h_t$) to the Higgs fields can affect $v/T_c$ sizably through
contributions of infrared origin to the potential. The field-dependent masses of the stops are of the form (neglecting $\tilde{t}_L - \tilde{t}_R$ mixing which tends to weaken the transition and $D$ terms for simplicity)

$$m_t^2(\varphi, T) \sim \tilde{m}^2 + h_t^2 \varphi_2^2 + \Pi_t(T),$$

where $\tilde{m}$ ($m_{Q,U}$ for $\tilde{t}_{L,R}$ respectively) are soft-susy breaking masses and $\Pi_t \sim g^2 T^2$ is a thermal contribution to the effective stop masses from interactions with the surrounding plasma. In the absence of $\tilde{m}^2$ and $\Pi_t$, static stop modes contribute to the 1-loop potential a term $-T h_t^2 \varphi^3$ which enhances $v/T_c$ significantly. At two-loops, the same modes, in a $t - \tilde{t} - gluon$ setting-sun diagram, contribute $-h_t^2 g_s^2 T^2 \varphi^2 \log \varphi^2$ with a similar enhancement effect on $v/T_c$. Non-zero $\tilde{m}^2$ and $\Pi_t$ screen this ideal behaviour reducing the effect, so that the transition is stronger for smaller $m_Q^2$ and $m_U^2$. A lower limit on $m_Q^2$ can be obtained requiring that the contribution of the $(\tilde{t}, \tilde{b})_L$ doublet to the $\rho$ parameter stays within experimental limits. There is no such constraint on $m_U^2$, so that we set it to zero. Negative values of $m_Q^2$ are associated with the existence of a dangerous color breaking minimum of the full scalar potential along the stop direction. Modest negative values of $m_U^2$ are however still allowed cosmologically and can further increase $v/T_c$. Loops of stop static modes have no infrared problems because $\tilde{m}^2$ and $\Pi_t$ cut-off infrared divergences. However, they can reappear if $m_Q^2 + \Pi_t \sim 0$, casting some doubt on the reliability of the perturbative analysis of the $m_U^2 < 0$ region. To be on the safe side I restrict my discussion to the $m_U^2 \gtrsim 0$ region. For analyses of the $m_U^2 < 0$ region, see refs. [8].

Figure 1 contains the results for $v/T_c$ from an analysis of the effective potential up to two-loops in resummed perturbation theory with $M_t = 175$ GeV, $m_Q = 250$ GeV, $m_U = 0$ and zero stop mixing. It shows how $v/T_c$ can be larger than 1 for low values of $\tan \beta$ (solid lines) and large enough $m_A$. Very low $\tan \beta$ corresponds to a light $h^0$ (lines of constant $m_h$ are dashed), which is experimentally excluded. Imposing the LEP limit ($m_h \gtrsim 70$ GeV) we find a window for baryogenesis which has $2.25 \simeq \tan \beta \simeq 3.6$, $m_A > 120$ GeV and $m_h < 85$ GeV. In addition, one of the stops should be not much heavier than $M_t$ (values slightly bigger are still allowed). Comparison of this with 1-loop results shows the importance of the two-loop corrections to achieve $v/T_c > 1$. The large size of two-loop contributions is not a symptom of the breaking of the perturbative expansion because QCD corrections appear first at that order. Similar 2-loop corrections were present in the SM for top loops, but there no enhancement of $v/T_c$ was present due to the fermionic nature of the top quark ($\delta V \sim h_t^2 \varphi^2 T^2$ in that case).

In the SM, the resummed perturbative analysis of the electroweak phase
transition along the same lines is not reliable for values of the Higgs mass larger than $m_W$. One can estimate the loop expansion parameter (e.g. in the vicinity of the broken minimum) as $\epsilon_{SM} \sim g^2 T / m_W(\varphi,T) \sim \lambda / g^2$ (in the SM, the transition is driven by gauge boson loops. $\epsilon_{SM}$ is the cost of an extra $W$ loop). To have $\epsilon_{SM} < 1$ requires $m_h \lesssim m_W$. In the MSSM, when stops drive the transition, the cost of an extra stop loop is $h_t^2 T / m_t(\varphi,T) \sim \lambda / h_t^2$ and we see that our analysis is expected to be reliable for larger values of $m_h$.

In particular, the window found for baryogenesis falls within the region where our loop expansion is under control.

Interesting alternative approaches to the study of the MSSM electroweak phase transition have been followed using 3d reduced effective theories plus lattice simulations (see talks by M. Laine and J. Cline) with qualitatively similar results. However, these analyses are less reliable precisely in the small $m_U^2$ region, which is the interesting one for baryogenesis. Clearly, more work
along these lines would be desirable.

To conclude, if stops are close to the Fermi scale (as originally motivated by Supersymmetry) they drive the electroweak phase transition and can give \( v/T_c > 1 \) as a necessary ingredient for electroweak baryogenesis. QCD radiative effects are crucial to this point. Nevertheless, the available parameter space is very constrained. In particular, the condition \( m_h < 85 \text{ GeV} \) will be challenged by LEP II very soon. Let us hope SUSY gets lucky once more.

Acknowledgments

I thank B. de Carlos for an enjoyable collaboration on the topic presented. This work was supported by the U.S. Department of Energy Grant No. DOE-EY-76-02-3071.

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