PINOCCHIO and the hierarchical build-up of dark matter haloes

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Accepted 0000 December 1. Received 0000 December 1; in original form 0000 October 1

ABSTRACT

We study the ability of PINOCCHIO (PINpointing Orbit-Crossing Collapsed Hierarchical Objects) to predict the merging histories of dark matter (DM) haloes, comparing the PINOCCHIO predictions with the results of two large N-body simulations run from the same set of initial conditions. We focus our attention on quantities most relevant to galaxy formation and large-scale structure studies. PINOCCHIO is able to predict the statistics of merger trees with a typical accuracy of 20 per cent. Its validity extends to higher-order moments of the distribution of progenitors. The agreement is valid also at the object-by-object level, with 70-90 per cent of the progenitors cleanly recognised when the parent halo is cleanly recognised itself. Predictions are presented also for quantities that are usually not reproduced by semi-analytic codes, such as the two-point correlation function of the progenitors of massive haloes and the distribution of initial orbital parameters of merging haloes. For the accuracy of the prediction and for the facility with which merger histories are produced, PINOCCHIO provides a means to generate catalogues of DM haloes which is extremely competitive to large-scale N-body simulations, making it a suitable tool for galaxy formation and large-scale structure studies.

Key words: galaxies: haloes – galaxies: formation – galaxies: clustering – cosmology: theory – dark matter

1 INTRODUCTION

In the Hierarchical Clustering Scenario, structure in the Universe forms from the aggregation and merging of smaller subunits. This theoretical picture is now substantiated, at least on a qualitative level, by a wealth of observations of the high-redshift (\(z \sim 3 - 5\)) Universe. In the most commonly discussed scenario, the ‘Cold Dark Matter’ (CDM) one, hierarchical clustering is driven by the gravitational collapse of DM fluctuations, while the visible astrophysical objects are generated from baryons falling into the DM haloes (see, e.g., White & Rees 1978). Thus the process of formation and evolution of DM haloes is of fundamental importance for understanding the properties of galaxies or galaxy clusters.

The formation of DM haloes involves highly non-linear dynamical processes which can not be followed analytically. To face this problem it is necessary to resort to numerical N-body simulations. Besides this time-consuming method, one can use also analytical approximations that are able to predict with fair accuracy some relevant quantities related to the assembly of DM haloes. Moreover, the analytic methods help to shed light on the complex gravitational problem of hierarchical clustering. The pioneers of the analytical approach were Press & Schechter (1974; hereafter PS) who derived an expression for the mass function of DM haloes. This was found to give a fair approximation of the N-body results (Efstathiou et al. 1988; see for a review Monaco 1998). The PS approach was extended by Bond et al. (1991; see also Peacock & Heavens 1990; Bower 1991; Lacey & Cole 1993) (Extended PS formalism, hereafter EPS), who fixed a normalization problem of the original PS work. The EPS model can be used to predict also some properties of DM haloes, such as their formation time, survival time and merger rate. These predictions were tested against numerical simulations, again with success, by Lacey & Cole (1994). The EPS formalism has recently become a standard tool to construct synthetic catalogues of DM haloes for galaxy formation programs (see, e.g., Kauffmann, White & Guiderdoni 1993; Somerville & Primack 1999; Cole et al. 2000).

However, recent work with larger N-body simulations has revealed significant discrepancies between PS and EPS predictions and numerical results. The PS mass function has been shown to underpredict the number of massive haloes and over-predict the number of low-mass ones (Gelb...
& Bertschinger 1994, Governato et al. 1999; Jenkins et al. 2001; Bode et al. 2001). Similar discrepancies were observed in the reconstruction of the conditional mass function, i.e. the number density of haloes bound to flow into a parent halo of given mass at a subsequent time * (Somerville & Kolatt 1999; Sheth & Lemson 1999b). The EPS formalism is also affected by limitations, in that it does not give full information on the spatial distribution of haloes (Catelan et al. 1998; Jing 1998, 1999; Porciani, Catelan & Lacey 1999), and by inconsistencies in the use of smoothing filters and in the construction of merger trees (Somerville & Kolatt 1999; Sheth & Lemson 1999b; Cole et al. 2001). Attempts to improve this formalism, or to develop alternative ones, were reviewed by Monaco (1998). A more recent and successful extension is due to Sheth & Tormen (1999) and Sheth, Mo & Tormen (2001); their model improves significantly the fit of the mass function and the extension of dynamics to ellipsoidal collapse (EPS is based on linear theory), but does not remove the inconsistencies of the EPS approach and does not provide spatial information of haloes either. This method has also been applied to build random realizations of the merging histories of DM haloes (Sheth & Tormen 2001), but it does not provide a significant improvement respect to the standard merger trees.

Recently, we have presented a new algorithm, called PINOCCHIO (PINpointing Orbit-Crossing Collapsed Hierarchical Objects), to generate synthetic catalogues of DM haloes with known mass, position, peculiar velocity, merger history and angular momentum (Monaco et al. 2001, hereafter paper I; Monaco, Theuns & Taffoni 2001, hereafter paper II). In contrast to EPS, PINOCCHIO is able both to reproduce statistical quantities, such as the mass or two-point correlation function of haloes, and to reproduce haloes on an point-by-point basis.

In this paper, we investigate in detail how well PINOCCHIO is able to recover the merger histories (or merger trees) of DM haloes. We compare the PINOCCHIO code with numerical N-body simulations and with the analytical estimates of the EPS theory. We examine the ability of PINOCCHIO to reconstruct the main statistical properties of the merger trees, extending the analysis to predict the correlation function and the initial orbital parameters of merging haloes.

Section 2 gives a brief description of the PINOCCHIO code with special attention to the extraction of the merger trees. In Section 3 we compare the statistical properties of the distribution of DM haloes at different redshifts given by PINOCCHIO with the results of numerical N-body simulations. Section 4 is dedicated to the study of the spatial distribution of the haloes that will form cluster sized objects at the present time. Section 5 shows the ability of PINOCCHIO to predict the impact parameters of merging haloes. The conclusions are reported in Section 6.

2 MERGER TREES FROM PINOCCHIO

The PINOCCHIO code was presented in paper I and described in full detail in paper II. Here we give only a brief description of the code, necessary to discuss the procedure used to extract the merger trees.

For a given cosmological background model and a power spectrum of fluctuations, a Gaussian linear density contrast field \( \delta_l \) (i.e. linearly extrapolated to \( z = 0 \)) is generated on a cubic grid, in a way much similar to what is usually done to generate initial condition for N-body simulations. The linear density contrast \( \delta_l \) is smoothed repeatedly with Gaussian filters of FWHM \( R \), where \( R \) takes values that are equally spaced (\( \sim 20 \) smoothing radii usually give an adequate sampling). For each point \( q \) of the Lagrangian (initial) coordinate and for each smoothing radius \( R \), the collapse time (i.e. the time at which the particle is predicted to enter a high-density, multi-stream region) is computed using Lagrangian Perturbation Theory (hereafter LPT; see e.g. Bouchet 1996, Buchert 1996, Catelan 1995) and its ellipsoidal truncation (Monaco 1997). Technically, the collapse time is defined as the instant of Orbit Crossing (OC); this definition is discussed at length in paper II (see also Monaco 1994, 1997a). For each particle only the earliest collapse time is recorded which amounts to recording the field

\[
F_{\text{max}}(q) \equiv \max_{R} \left( \frac{1}{b(t_c)} \right),
\]

Here \( b(t) \) is the linear growing mode (see Padmanabhan 1993; Monaco 1998) and \( t_c \) is the OC-collapse time. Notice that in an Einstein-de Sitter Universe \( F_{\text{max}} = (1 + z_{\text{max}}) \), where \( z_{\text{max}} \) is the largest collapse redshift at which the particle collapses.†

Besides \( F_{\text{max}} \), we record also the smoothing radius \( R_{\text{max}} \) for which the maximum in equation (1) is reached, and the velocity of the particle at collapse time as given by the Zel’dovich approximation (1970) (which is the first-order term of LPT),

\[
v_{\text{max}} = -b(t) \nabla \phi(q, R_{\text{max}})
\]

(in units of comoving displacement), where \( \phi(q, R_{\text{max}}) \) is the rescaled peculiar gravitational potential (smoothed at \( R_{\text{max}} \)), which obeys the Poisson equation \( \nabla^2 \phi(q) = \delta_l \). All differentiations and convolutions are performed using fast Fourier transforms.

The collapsed medium is then “fragmented” into isolated objects using an algorithm designed to mimic the accretion and merger events of hierarchical collapse. Collapsed particles may belong to relaxed haloes or to lower-density filaments. At the instant a particle is deemed to collapse, we decide which halo, if any, it accreted onto. The candidates haloes are those that already contain one Lagrangian neighbour of the particle (on the initial grid \( q \) of Lagrangian positions, the six particles nearest to a given one are its “Lagrangian neighbours”). The particle will accrete onto the halo if its distance (in the Eulerian space at the collapse time) from the centre of mass of the candidate halo is

* In the following, the ‘final’ haloes at \( z = 0 \) (or occasionally at higher redshift) will be called parent, while the higher-redshift haloes that flow into the parent will be called progenitors.

† Taking the largest redshift (or \( F \)-value) of collapse is analogous to considering the largest collapse radius, as done in the EPS formalism.
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Figure 1. Conditional mass functions in the ΛCDM case for parent haloes identified at \( z = 0 \). The mass threshold is fixed at \( M_{\text{th}} = 7.6 \times 10^{10} \, M_\odot \) (10 particles), the redshift increases from left to right and covers the values: \( z = 1, 2, 4 \). The mass of the parent halo increases from top to bottom, the adopted values are: \( M_0 = 5.0 \times 10^{12} M_\odot \), \( 3.0 \times 10^{13} M_\odot \) and \( 2.0 \times 10^{14} M_\odot \). The points represent the simulation data while the solid lines are the prediction of PINOCCHIO; the dashed lines are the analytical predictions of the EPS formalism.

Figure 2 shows the mass functions for different mass thresholds, \( M_{\text{th}} \), and redshifts, \( z \), as indicated in the figure. The mass functions are expressed in logarithmic scale, and the points represent the simulation data while the solid lines are the prediction of PINOCCHIO; the dashed lines are the analytical predictions of the EPS formalism.

The fragmentation algorithm requires the introduction of free parameters, which are analogous to those required by any clump-finding algorithm applied to N-body simulations. These parameters specify the level of overdensity at which the halo is defined, or are introduced to fix resolution effects. They are discussed in paper II (to which we refer for all details) and chosen by requiring the fit of the mass function of haloes selected with the friends-of-friends (FOF) algorithm with linking length 0.2 times the mean inter-particle distance at \( z = 0 \). Here we use for the parameters the values found in paper II.

To give a taste of the speed of PINOCCHIO with respect to simulations, a \( 256^3 \) particles realization needs about 6 hours on a Pentium III 450MHz computer, with a RAM requirement of about 512 Mb. The corresponding simulation required a week on 10 processor parallel computer. Statistical quantities like the mass function or the two-point correlation function are reproduced with a typical error of \( \sim 10 \) per cent or smaller. At the object-by-object level, the accuracy of PINOCCHIO depends on the degree of non-linearity that

\( R_N = N^{1/3} \) in grid unit, where \( N \) is the number of particles in the halo), otherwise they are catalogued as filaments. If a particle has more than one candidate halo, we check whether these haloes should merge. The merging condition is very similar to the accretion one: two groups merge if their distance in the Lagrangian space is smaller than a fraction of the size of the largest halo. If a particle is a local maximum of \( z_{\text{max}} \) it is considered as the seed of a new halo. Finally, filament particles are accreted onto a halo when they neighbour in the Lagrangian space an accreting particle; this is done to mimic accretion of filaments onto haloes.
Figure 2. Same as in Fig. 1 but for the SCDM case. The mass threshold is $M_{th} = 1.49 \times 10^{13} M_\odot$ (10 particles).
function. This is to test the effect of the degrading of the agreement at small masses. In general, at least 30 particles are necessary to identify reliably a halo both in the simulations and in PINOCCHIO, so we consider a threshold mass of 30 particles for the other statistical analysis.

The merger trees for the FOF haloes at final time $z_0$ are constructed as follows. Progenitors are defined as those haloes that at the higher redshift $z$ contain some of the particles of the parent halo at $z_0$. As noted by some authors (see e.g. Somerville et al. 2000), some particles that are located in a progenitor are not included later into the parent. This reflects the actual dynamics of the haloes that suffer stripping and evaporation events, and make the progenitor identification process more ambiguous. We then adopt two simple rules:

(i) if a parent halo contains less than 90 per cent of the mass of all its progenitors at redshift $z$, then it is excluded from the analysis (this happens in a few percent of cases);

(ii) we assign to the progenitor the mass of all its particles that will flow in to the parent at $z_0$.

In this way we force mass conservation in the merger tree and reject some extreme cases when the progenitor is strongly affected by these ‘evaporation’ effects.

Due to the limited number of available outputs, the merger trees obtained from our simulations are very coarsely-grained in time. This highlights one of the advantages of using a code like PINOCCHIO to produce the merger trees. In fact, in PINOCCHIO we follow the merging of haloes in real time, and then we can link each progenitor to its parent after each merging event, while in the simulations (where haloes are identified after the run) it is necessary to analyse and cross-correlate a large number of outputs to follow the merger histories. In other words, the generation of the merger trees is by far less expensive (in term of CPU time, disk space and human labour) in PINOCCHIO than in a simulation; in fact PINOCCHIO automatically compute the merging history of haloes and it does not need any further analysis.

### 3.2 Progenitor mass function

The progenitor (conditional) mass function $dN(M, z|M_0, z_0)/dM$ is the number density of progenitors of mass $M$ at redshift $z$ that merge to form the parent $M_0$ at redshift $z_0$. An estimate of this quantity based on the PS formalism was found by Bower (1991), while Bond et al. (1991) gave the basis for computing it in the EPS formalism (as done by Lacey & Cole 1993).

Let $\delta(z)$ the critical density for a spherical perturbation to collapse at redshift $z$ and $\Lambda(M) = \sigma^2(M)$ the variance of the initial density field when smoothed over regions that contain on average a mass $M$. The fraction of mass of the parent halo that was in the progenitors of mass $M$ is:

$$\frac{\delta(z)}{\Lambda(M) - \Lambda(M_0)} = \frac{\Lambda(M_0)}{\Lambda(M)}$$

The PINOCCHIO conditional mass function and that obtained from the simulations are computed by averaging over a mass interval around log $M_0$ of 0.01 dex.

In Fig. 1 and Fig. 2 we compare the conditional mass functions obtained from the EPS formalism, PINOCCHIO and the simulations for the CDM and the SCDM case respectively. The bottom panels of Fig. 1 show for the CDM case the results for a cluster-sized parent at $M_0 = 2 \times 10^{14} M_\odot$, the case of haloes corresponding to small groups ($M_0 = 0.025, 0.5, 0.75, 1, 2, 3, 4, 5$).

### Parameters of the considered numerical simulations

| Cosmology | $\Omega_0$ | $\Omega_\Lambda$ | $h_0$ | $\sigma_8$ | $n$ | $L_{\text{box}}$ | $N_p$ | $M_{\text{part}}$ | output redshifts |
|-----------|-----------|----------------|------|-----------|----|----------------|------|-----------------|-----------------|
| $\Lambda$CDM | 0.3 | 0.7 | 0.65 | 0.9 | 1. | 100 Mpc/h | 256$^3$ | $7.64 \times 10^{10} M_\odot$ | 0, 0.25, 0.5, 0.75, 1, 2, 3, 4, 5 |
| SCDM | 1. | 0. | 0.5 | 1. | 1. | 500 Mpc/h | 360$^3$ | $1.49 \times 10^{12} M_\odot$ | 0, 0.43, 1.13, 1.86 |

Figure 3. The distribution of the mass of the largest progenitor $M_1$ for the $\Lambda$CDM case with mass threshold $M_{\text{th}} = 2.3 \times 10^{11} M_\odot$ (30 particles). The histograms are the PINOCCHIO predictions and the points connected with solid lines are the simulations'. The quantity plotted on the upper part of each box is the mean of the distribution of the mass ratio of the second largest progenitor $M_2$ to the first largest progenitor $M_1$ versus the mass ratio of the largest progenitor to the parent halo. The solid line is the PINOCCHIO result and the dashed lines show its 1σ variance. The points with error bar are the simulation data.
3.3 Higher-order analysis of the progenitor distribution

We evaluate the distribution of the mass of the largest progenitor $M_1$ (i.e. the most massive halo that flows into the parent) for each of the parent haloes analysed before. The histograms on Fig. 3 and Fig. 4 show the distribution of the mass of the larger progenitor normalized to the parent mass, $M_1/M_0$, predicted by PINOCCHIO for the ΛCDM and SCDM case (in the following the mass threshold is always set to 30 particles). The symbols connected with lines denote the corresponding simulation results. The agreement between the numerical experiment and PINOCCHIO is very good. Both the mean value and the width of the distribution are reproduced with good accuracy at all redshifts.

The distribution of $M_1/M_0$ provides also a hint on the formation time of the parent. In fact, the standard definition of formation time for a halo of mass $M_0$ is the epoch at which the size of its largest progenitor first becomes greater than $M_0/2$. So we assume as the average formation redshift for a parent halo of mass $M_0$ the time at which the peak of the distribution $M_1/M_0$ is at one half. The good agreement of PINOCCHIO with the simulations can thus be extended also to the halo formation times. For instance Fig. 4 suggests that, in this SCDM cosmology, a halo of $1 \times 10^{13} M_\odot$ forms at $z \sim 0.43$ or later. Notice that a more detailed analysis of formation times is hampered by the small number of simulation outputs available.

In the upper part of the plots of Fig. 3 and Fig. 4 the distribution of $M_2/M_1$ (the ratio of the second largest progenitor and largest ones) given $M_1/M_0$ is shown. The points are the mean value of the distribution and the error bars are the corresponding $1\sigma$ variance, both measured in the simulations. The solid lines and the dashed lines are the same quantities predicted by PINOCCHIO. Again the agreement is very good.

The results reported in this section and in the previous one suggest that the merging histories of haloes produced by PINOCCHIO reproduce with very good accuracy the statistical properties of the masses of those extracted from numerical simulations. We notice that the EPS based algorithms to produce merger trees (Lacey & Cole 1994, Somerville & Kolatt 1999, Sheth & Lemson 1999, Coles et al. 2000) are by construction forced to reproduce the EPS analytical distributions, and they suffer of the same discrepancy noted for the EPS analytical prediction (Fig. 1 and 2).

3.4 The progenitors in number

In this section we analyse the statistical properties of the distribution of the number of progenitors of a halo of mass $M_0$.

In Fig. 5 and Fig. 6 we show the probability $P(N, M_0)$ that a halo of mass $M_0$ has $N$ progenitors. The average of these distribution gives (with suitable normalisation) the integral of the conditional mass function to the threshold mass, and is dominated by the more numerous small-mass objects.

The histograms show the distribution of the number of progenitors evaluated from PINOCCHIO for different parent masses and redshifts. The filled symbols connected with lines are the distribution extracted from the simulation. We notice that PINOCCHIO reproduces fairly well the distributions also for the more massive haloes and at all redshifts. The ability of PINOCCHIO in predicting the distribution of the number of progenitors can be quantified by comparing the first and second moments measured in the simulations with their values predicted by PINOCCHIO. In fig 7 we show the average $\mu_1$ and the rescaled variance $\mu_2/\mu_1$ as a function of the parent halo mass for different redshifts. The points connected with solid lines are the PINOCCHIO prediction and the open symbols are the values measured from the simulations. The dashed lines are the EPS analytical prediction for $\mu_1$ computed by integrating equation (4).
Figure 5. Probability that an halo $M_0$ at $z = 0$ has $N$ progenitors for the $\Lambda$CDM case. The threshold mass is $M_{\text{th}} = 2.3 \times 10^{11} M_\odot$ (30 particles). The points connected with solid lines represent the simulation data while the histograms are the prediction of PINOCCHIO. The vertical lines are the EPS analytical prediction for the mean of the distribution.

Figure 6. Same as in Fig. 5 for the SCDM case. The threshold mass is $M_{\text{th}} = 1.3 \times 10^{14} M_\odot$ (30 particles).
Figure 7. The first two moments of the distribution of the number of progenitors $P(N, M_0)$ as a function of the parent mass $M_0$. The left plots show the ΛCDM case at redshift $z = 1$ and 2. The threshold mass is $M_{th} = 2.3 \times 10^{11} M_\odot$ (30 particles) and we plot the mean (squares) and the rescaled variance (circle s) up to $M_0 = 1000 M_{th}$. The solid lines with open symbols are the PINOCCHIO results and the filled symbols are the simulation data. The dashed line is the EPS analytical prediction for the mean. The right plots show the SCDM case at redshift $z = 0.43$ and $z = 1.13$. The threshold mass is $M_{th} = 1.3 \times 10^{14} M_\odot$ (30 particles), and we plot the mean and the rescaled variance up to $M_0 = 50 M_{th}$.

Note that for arbitrary initial conditions the EPS formalism cannot analytically evaluate the higher moments of the distribution.

The agreement between PINOCCHIO and the simulations varies from the 5 per cent of the ΛCDM to the 10 per cent of the SCDM case but it does not depend on the redshift. Again PINOCCHIO is found to improve with respect to EPS. In particular, at low redshift the EPS predictions underestimate the mean value by a factor that ranges from 20 per cent to 30 per cent. Our results can be compared to those shown by Sheth & Lemson (1999b) and Somerville et al. (2000) for EPS based merger trees and with Sheth & Tormen (2001) who elaborate an excursion set model based on ellipsoidal collapse. In general, PINOCCHIO reproduces the statistical properties of progenitor distributions with better accuracy than the other methods. It is remarkable that the tests based on parent haloes with different mass ranges give very similar results, reproducing the simulations with a comparable accuracy.

3.5 Object-by-object comparison

We finally test the degree of agreement between PINOCCHIO and the simulations at the object-by-object level for the number of progenitors that are cleanly reconstructed. In paper I and paper II a pair of haloes coming from the two catalogues (PINOCCHIO and FOF) were defined as cleanly assigned to each other if they overlapped in the Lagrangian space for at least 30 per cent of their volume and no other object overlapped with either of them to a higher degree. The cleanly assigned haloes were shown to overlap on average at the 60-70 per cent over all redshifts. The fraction of cleanly assigned haloes was found to depend on the degree of non linearity reached by the system, decreasing from almost 100 per cent to 70 per cent, at worst, at later times. This is due to the lower accuracy of the Zel’довich approximation in predicting the displacements as the density field becomes more and more non-linear (see paper II).

We now quantify the number of PINOCCHIO progenitors that are cleanly assigned to FOF progenitors for each cleanly assigned parent halo. For this analysis we restrict ourselves to the ΛCDM case, which gives a wider mass range but a higher level of non-linearity. In Fig. 8 we show, for the parents that are cleanly identified, the fraction $f_p$ of cleanly assigned progenitors as a function of the parent mass. The three lines correspond to the different redshifts. The tree plots on the right are the scatter plots of $f_p$ as function of the progenitor mass $M$ normalized to the parent mass $M_0$, for the parent haloes of mass $10^{11} M_\odot < M_0 < 10^{15} M_\odot$; the redshift increases from top to bottom.

Figure 8. Fraction $f_p$ of cleanly assigned progenitors for redshift $z = 1, 2$ and $4$. The left hand side represents the fraction of cleanly assigned progenitors as function of the parent mass. The three lines correspond to the different redshifts. The tree plots on the right are the scatter plots of $f_p$ as function of the progenitor mass $M$ normalized to the parent mass $M_0$, for the parent haloes of mass $10^{11} M_\odot < M_0 < 10^{15} M_\odot$; the redshift increases from top to bottom.

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$z = 1$. This is mainly due to the small progenitors, as the right panels of Fig. 8 show. The progenitors which carry a mass of less that $\sim 20$ per cent are those that are worst reconstructed. We conclude that PINOCCHIO is able to reconstruct correctly the main branches of the merger trees, while secondary branches, especially present in the larger haloes, are reconstructed in a noisier way.

4 THE SPATIAL PROPERTIES OF MERGING HALOES

One remarkable limit of EPS is the lack of spatial information for the haloes. Several authors (Mo & White 1996; Mo, Jing & White 1997; Catelan et al. 1998; Porciani et al. 1998) found approximate analytical expressions for the bias of haloes of fixed mass, i.e. for the ratio between the two-point correlation function of haloes and that of the underlying matter field. Such analytical estimates have been found to agree with the results of simulations to within $\sim 40$ per cent (Mo & White 1996; Jing 1998; Porciani, Catelan & Lacey 1999; Sheth, Mo & Tormen 2000; Colberg et al. 2001). In this approach it is not possible to know how the bias changes for haloes with different merger histories. This piece of information is precious to produce predictions on the bias of galaxies of different types, that typically have different merger histories.

As shown in paper I and II, PINOCCHIO haloes have the same correlation length $r_0$ as FOF haloes to within 10 per cent error. Having knowledge of both merger histories and halo positions, PINOCCHIO can provide information on the relation between clustering and merging. To show this, we select PINOCCHIO and FOF haloes in the $\Lambda$CDM cosmology at $z = 0$ with masses greater than $10^{14} \, M_\odot$. We check their merging histories at $z = 1, 2$ and 4, and we evaluate the the two-point correlation functions for their progenitors. In Figure 9 the solid lines represent the two-point correlation function of progenitors, $\xi_p(r)$, evaluated in PINOCCHIO compared with the same quantity measured in the simulation. The plots show that PINOCCHIO reproduces such correlation functions to within $\sim 20$ per cent error.

We also compare this function with the average correlation function, $\bar{\xi}_h(r)$, at the same redshifts. It is apparent that PINOCCHIO reproduces correctly the larger clustering amplitude of haloes that flow into cluster sized one. The bias between the two halo populations is defined as: $b^2(r, z) = \xi_p(r)/\bar{\xi}_h(r)$. We compare in the bottom row of plots in Fig. 9 the bias measured in the simulation with PINOCCHIO results. The bias is recovered to within $\sim 20$ per cent and the scale dependence is correctly reproduced.
5 ORBITAL PARAMETERS OF THE MERGING HALOES

In the hierarchical clustering scenario a merging event between two or more haloes corresponds to the loss of identity of the single primitive units which merge to form a new halo. However, high-resolution N-body simulations show that the dynamical evolution after an encounter is more complicated than this idealized picture: the haloes may retain their identity, and become substructure of the new system (Moore, Katz & Lake 1996; Tormen 1997; Ghigna et al. 1998; Tormen, Diaferio & Syer 1998). Indeed, this is in line with the same evidence of galaxies within galaxy groups or clusters. The life of these substructures is affected by the various dynamical effects that contribute to their disruption. The dynamical friction force drives the satellites towards the center of mass of the system where they can merge with the central object or among themselves. While a satellite orbits inside the main halo, the tidal forces exerted by the background induce its evaporation and reduce its mass (Gnedin & Ostriker 1997; Gnedin, Hernquist & Ostriker 1999; Taylor & Babul 2000; Taffoni et al. 2001, Taffoni et. al 2001 in prep).

The evolution of substructure is one of the crucial points to modeling galaxy formation. A most important aspect of this is the prediction of the initial orbital parameters, i.e. the energy and the angular momentum of the orbit of the satellites infalling into the main halo. The large-scale simulations as those we use in the present paper lack enough resolution to address such processes. High-resolution simulation are necessary to describe the evolution of satellites (Tormen 1997; Ghigna et al. 1998), at the cost of simulating one cluster at a time. It is then useful to resort again to analytic modeling of the dynamical friction and tidal stripping (Chandrasekhar 1943; see e.g., Binney & Tremaine 1987, Lacey & Cole 1993; van den Bosch et al. 1999, Colpi, Mayer & Governato 2000). These analytical models require knowledge of the initial orbital parameters. In the semi-analytical, EPS-based codes for galaxy formation, these parameters are in general Monte-Carlo extracted from some distributions obtained from high-resolution simulations (Tormen 1997; Ghigna et al. 1998).

Within the PINOCCHIO code, it is possible to predict the impact parameters of the merging satellites, as the infall velocities and the relative distances are known. Notice that this calculation is analogous to that of angular momentum of haloes presented in paper II. Given the impact (Zel’dovich) velocity \( \Delta v \) and the relative distance \( \Delta r \) the angular momentum and the energy are computed as:

\[
J = \Delta r \wedge \Delta v \quad (5)
\]

\[
E = \frac{1}{2} (\Delta \mathbf{v})^2 + \phi(|\Delta r|). \quad (6)
\]

\( \phi(|\Delta r|) \) can be evaluated as the gravitational potential of a point mass which touches the external layer of a spherical halo of mass \( M \): \( \phi(|r|) = GM/|r| \). The linear growth of the relative velocity is stopped at a physical time equal to a half of the merging time (see paper II).

To study the ability of PINOCCHIO in predicting the orbital parameters of DM substructures we compute the distribution of the orbital parameters of the satellites that merge with a halo of mass \( M = 2 \times 10^{14} \, M_\odot \). We express the angular momentum and the energy for unit mass in terms of the circularity \( \epsilon \equiv J/E \), where \( J(E) = V_c \, r_c(E) \) is the angular momentum and \( r_c(E) \) is the radius of the circular orbit with the same energy (see eg., van den Bosch et al. 1999). To do that we assume a Navarro, Frank & White (1996) density profile for the main halo and we calculate the associated potential energy profile \( \phi_{NFW}(R) \) and the associated circular velocity profile \( V_c(R) \) (see e.g. Navarro, Frank & White 1996; Klypin et al. 1999, Taffoni et al. in prep). We consider a particular combination of the orbital parameters

\[
\Theta_{orb} = \epsilon^{0.78} \left( \frac{r_c(E)}{R_{vir}} \right)^2, \quad (7)
\]

introduced by Cole et. al (2001). They use the Tormen (1997) results to derive the distribution for the \( \Theta_{orb} \) factor and they find that this distribution can be fitted with a log normal function with mean value \( \langle \log_{10}(\Theta_{orb}) \rangle = -0.14 \) and dispersion \( \delta = (\langle \log_{10}(\Theta_{orb}) \rangle)^2 = 0.26 \).

The result of our analysis are presented in Fig. 10, we compare the distribution of the \( \Theta_{orb} \) factor measured in PINOCCHIO (histogram) with the theoretical fit derived by Cole et. al (2001) (solid line). We note that the distribution measured form PINOCCHIO reproduce with good accuracy the log normal function. The average value derived by our analysis is \( \langle \log_{10}(\Theta_{orb}) \rangle = -0.18 \) and the dispersion is \( \delta = (\langle \log_{10}(\Theta_{orb}) \rangle)^2 = 0.23 \).

6 CONCLUSIONS

We have tested the predictions of the PINOCCHIO code, presented in paper I and paper II, regarding the hierarchical nature of halo formation, with particular attention to those
aspects that are mostly relevant for galaxy formation. We have compared the results of PINOCCHIO with those of two large N-body simulations (ΛCDM and SCDM cosmologies) drawing the following conclusions:

1. The merger histories of the PINOCCHIO haloes resemble closely those found applying the FOF algorithm to the N-body simulations. The agreement is valid at the statistical level for groups of at least 30 particles (good results are obtained even for haloes of 10 particles).

2. Statistical quantities like the conditional mass function, the distribution of the largest progenitor, the ratio of the second largest to largest progenitors, and the higher moments of the progenitor distributions are recovered with a typical accuracy of ∼20 per cent.

3. The agreement is good also at the object-by-object level, as PINOCCHIO cleanly reproduces ∼70 per cent of the progenitors when parent haloes are cleanly recognised themselves. The agreement slowly degrades with time.

4. The increased noise recovered in the object-by-object agreement, as time progresses and non-linearity grows, does not influence the accuracy of the predictions in a statistical sense.

5. The correlation function of higher-redshift haloes that are progenitors of lower-redshift massive haloes is correctly reproduced to within an accuracy of ∼10 per cent in r0. The scale dependent bias of these with respect to the total halo population is also reproduced to within an accuracy of 20 per cent or better.

6. PINOCCHIO gives an estimate of the initial orbital parameters of merging haloes as well, which is found in reasonable agreement with available results from high-resolution N-body simulations.

Compared with the widely used EPS approach (Bond et al. 1991; Bower 1991; Lacey & Cole 1993), PINOCCHIO improves in most respects.

1. The fit of the statistical quantities achieved by PINOCCHIO is much better than the PS and EPS estimates, which show discrepancies up to a factor of 2 (see Governato et al. 2000; Jenkins et al. 2001; Bode et al. 2001; Somerville et al. 1999; Sheth & Lemson 1999b, Bagla et al. 1997).

2. The validity of PINOCCHIO extends to the object-by-object level, in contrast to EPS (Bond et al. 1991; White 1996).

3. PINOCCHIO is not affected by the inconsistencies of the EPS approach, that can be corrected only by means of heuristic recipes (Somerville & Kolatt 1999; Sheth & Lemson 1999; Cole et al. 2001).

4. PINOCCHIO provides much more useful information on the haloes, such as positions, velocities and angular momenta at initial orbital parameters at merger; at the same time it is not more computational demanding than an EPS based code to generate the merging histories of haloes.

These results confirm the validity of PINOCCHIO as a fast and flexible tool to study galaxy formation or to generate catalogues of galaxies or galaxy clusters, suitable for large-scale structure studies. In fact PINOCCHIO reproduces, in a much quicker way and to a very good level of accuracy, all the information that can be obtained from a large-scale N-body simulation with pure dark matter, without needing all the post processing necessary to obtain the merger trees of haloes.

PINOCCHIO is available at http://www.daut.univ.trieste.it/~pinocchio.

ACKNOWLEDGMENTS

We thank Stefano Borgani, Monica Colpi, Fabio Governato, Lucio Mayer and Cristiano Porciani for useful discussions.

REFERENCES

Barden J. M., Bond J. R., Kaiser N., Szalay A. S., 1986, ApJ, 304, 15
Binney, J., Tremaine, S. 1987, Galactic Dynamics (Princeton: Princeton University Press)
Bode P., Bahcall N.A., Ford E.B., Ostriker J.P., 2001, ApJ, 551, 15B
Bond J. R., Cole S., Elstathiu G., Kaiser N., 1991, ApJ, 379, 440
Bouchet F., 1996, in Dark Matter in the Universe, ed. S. Bonometto et al. IOS, Amsterdam
Buchert T., 1996, in Dark Matter in the Universe, ed. S. Bonometto et al. IOS, Amsterdam
Bower R., 1991, MNRAS, 248, 332
Catelan P., 1995, MNRAS, 276, 115
Catelan P., Luchin, F., Matarrese, S., Porciani, C., 1998, MNRAS, 297, 692
Chandrasekhar, S. 1943, ApJ, 97, 255
Cole S., Lacey C. G., Baugh C. M., Frenk C. S., 2000, MNRAS, 319, 168
Colberg, J. M., White, S.D.M., Yoshida, N., et al, 2000, MNRAS, 319, 209
Colpi, M., Mayer, L., Governato, F., 1999, ApJ, 525, 720
Couchman, H., Thomas, P., Pearce, F., 1995, ApJ, 452, 797
Elstathiu G.P., Frenk C.S., White S. D. M., Davis M., 1998, MNRAS, 235, 715
Gelb J. M., Bertchinger E., 1994, ApJ, 436, 467
Ghigna, S., Moore, B., Governato, F., Lake, G., Quinn, T., Stadel, J. 1998, MNRAS, 300, 146
Gnedin, O. Y., Ostriker, J., 1997, ApJ, 474, 223
Gnedin, O. Y., Hernquist, L., Ostriker, J., P, 1999, ApJ, 514, 109
Gnedin, O. Y., Lee, H., M., Ostriker, J., P., 1999, ApJ, 522, 935
Governato, F., Babul, A., Quinn, T., Tozzi, P., Baugh, C. M., Katz, N., Lake, G. 1999, MNRAS, 307, 949
Jenkins A., Frenk C.S., White S. D. M., Colberg J.M., Cole S., Evrard A.E., Couchman H.M.P., Yoshida N., 2001, MNRAS, 321, 372
Jing, 1998, ApJ, 503,9
Jing, 1999, ApJ, 515,45
Kauffmann, G., White, S. D. M., Guiderdoni, B. 1993, MNRAS, 264, 201
Klypin, A., Gottlber, S., Kravtsov, A., V., Khokholov, A., M., 1999, ApJ, 516, 530
Lacey, C., Cole, S. 1993, MNRAS, 262, 627
Lacey, C., Cole S., 1994, MNRAS, 271, 676
Mo H. J., Jing, White S. D. M., 1997, MNRAS, 284, 189
Mo H. J., White S. D. M., 1996, MNRAS, 282, 347
Monaco, P., 1995, ApJ, 447, 23
Monaco, P., 1997, MNRAS, 287, 753
Monaco, P., 1998, Fund. Cosm. Phys., 19, 153
Monaco, P., Theuns T., Taffoni G., Governato F., Quinn T., Stadel J., 2001, ApJ, in press
Monaco, P., Theuns T., Taffoni G., 2001, MNRAS, submitted
Moore, B., Katz, N., Lake, G. 1996, ApJ, 457, 455
Navarro, J. F., Frenk, C. S., White, S. D. M. 1996, ApJ, 462, 563
Padmanabham, T., 1993, Structure Formation in the Universe, Cambridge University Press
Peacock, J. A., Heavens, A. F., 1990, MNRAS, 243, 133
Porciani, C., Catelan, P., Lacey, C., 1999, ApJ, 513, 99
Porciani, C., Matarrese, S., Lucchin, F., Catelan, P., MNRAS, 298, 109
Press, W. H., Schechter, P. 1974, ApJ, 187, 425
Sheth R. K., Lemson G., 1999a, MNRAS, 304, 767
Sheth R. K., Lemson G., 1999b, MNRAS, 305, 946
Sheth R. K., Mo H. J., Tormen G., 2001, MNRAS, 323, 1
Sheth R. K., Tormen G., 1999, MNRAS, 308, 119
Sheth R. K., Tormen G., 2001, MNRAS in press, astro-ph/0105113
Somerville, R. S., Kolatt, T. S. 1999, MNRAS, 305, 1
Somerville, R. S., Lemson, G., Kolatt, T. S., Dekel, A., 2000, MNRAS, 316, 479
Somerville, R. S., Primack, J. R. 1999, MNRAS, 310, 1087
Taffoni, G., Mayer, L., Colpi, M., Governato, F., 2001, astro-ph/0109029
Tormen, G. 1997 , MNRAS, 290, 411
Tormen, G., Diaferio, A., Seyer, 1998 , MNRAS, 299, 728
White, S. D. M., Rees, M. J. 1978, MNRAS, 183, 341
van den Bosch, F., Lewis, G., Lake, G., Stadel, J., 1999, ApJ 515, 50
Zel’dovich YA. B., 1970, Astrofizika, 6, 319 (translated in Astrophysics, 6, 164 [1973])

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