The power spectrum of the residual rotation curve velocity as a probe of past mergers

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ABSTRACT

According to the ΛCDM cosmological framework, galaxies underwent multiple mergers in their history. In this paper we propose to use the power spectrum of the residual fluctuations of the rotation curve velocity as a probe of past mergers. The proposition relies on the assertion that mergers are expected to induce large scale flows and in case of major mergers shocks are induced as well. Instabilities of the large scale flows and shocks could generate a large scale turbulence whose size is comparable to the galactic disk dimensions. We develop expressions relating underlying turbulence spectral function to the observational power spectrum of the residual of the rotation curve velocity. This relation can be used to test whether turbulence exists in a given galaxy. The method is applied to the regular spiral galaxy NGC3198 with the conclusion that it underwent a minor merger about 7 Gyr ago.

Key words: galaxies: general, galaxies: kinematics and dynamics, turbulence, ISM: kinematics and dynamics, galaxies: mergers, galaxies: individual (NGC3198)

1 INTRODUCTION

The standard ΛCDM cosmology implies that galaxies grow during their history by accretion and mergers (Robertson et al. 2006; Kaviraj et al. 2009; Stewart et al. 2009). We propose to use the power spectrum of the residual fluctuations of the rotation curve velocity as a probe of past mergers. The proposition relies on the plausible assumption that mergers induce large scale flows, and in the case of major mergers shocks are induced as well. The large scale flows and shocks can generate a turbulence, in the ISM. The physical parameters of the ISM make the generation of turbulence quite easy once an instability and an energy source are available and indeed turbulence in the ISM is quite ubiquitous (Elmegreen & Scalo 2004). The instabilities related to mergers are e.g. the Kelvin-Helmholtz instability and the Richtmyer-Meshkov instability due to the shock acceleration (Mikaelian 1990; Graham & Zhang 2000).

The spatial scale of the generated turbulence is expected to be comparable to the galactic disk size. Such a large scale turbulence is different from local turbulence generated by e.g. supernovae (scale of the order of 10 pc), super shells resulting from chains of supernovae (scale of the order of 100 pc), or winds from early type stars (scale of the order of 100–1000 pc). Moreover, a spatial scale comparable to a galactic size implies for turbulent rms velocities of the order of 10 km/s, a turbulence life time of a few Gyr. For minor to medium mergers (the most common) the turbulence is expected to be subsonic and so its life time is even longer. The long life time of the turbulence implies that it can be detected also in nearby galaxies. Thus, the turbulence serves as a fossil evidence of the past merger.

If such primordial turbulence exists, it is expected to be superimposed on the rotation velocity. Hence, part (or all) of the fluctuating residual of the observational rotation velocity may be identified with this primordial turbulence. Hints for the existence of such a turbulence are found in observations of radial velocities in disks of galaxies (Beauvais & Bothun 1999; Wong, Blitz, & Bosma 2004; Elson, de Blok, & Kraan-Korteweg 2010; Trachternach et al. 2008) and the observation of Begeman (1989) that the residual of the rotation velocity as function of the azimuthal angle along a ring in the galaxy plane, is oscillatory. The above velocities are of the order of 5–10 km/s.

The theoretical consideration and the observational hints motivate one to compute the observational power spectrum of the fluctuating residuals of the rotation velocity. The residuals depends of course on the definition of the mean rotation curve. To avoid this ambiguity we shall consider only galaxies with a flat rotation curve over a sizable part of the radial extent. The residuals are defined with respect to the

* I thank the Department of Astronomy and Astrophysics, Tel Aviv University, for the hospitality while on Sabbatical Leave from Afeka College.
constant mean value of the rotation velocity. The resulting observational power spectrum can be used to distinguish between random residuals (or local small size fluctuations) and fluctuations which are a manifestation of a galactic scale turbulence. If the latter is revealed, it means that it has been generated by a process with a spatial coherence scale comparable to that of the galaxy. A merger or close passage by another galaxy is the natural candidate.

We derive the relation between the power spectrum of the observational residual rotation velocity, and the underlying 3D turbulence spectrum. This relation can be applied to determine the turbulence 3D spectral function from the observational power spectrum. In addition, the latter supplies an estimate for the line of sight depth of the turbulent region.

The method is demonstrated for the rotation curve of the spiral galaxy NGC3198 and leads to the conclusion that in spite its ordered shape, this galaxy has experienced a minor merger about 7 Gyr ago.

2 POWER SPECTRUM OF THE RESIDUAL ROTATION VELOCITY CURVES

To test whether a large scale turbulence exists in a given galaxy one needs the relation between the power spectrum of the observational residuals and the spectral function of the underlying 3D turbulence. The observational rotation curve is usually obtained by use of the tilted ring model [Roestad et al. 1974]. We show that its power spectrum gives essentially the same results as a power spectrum of the residual Position – Velocity (PV) curve along a given axis on the galactic disk. The advantage is that the rotation velocity at a given radius, is based on many measurements at this radius, along the ring circumference and so the precision is better than that of the PV data which is based on a single measurement for each radius.

2.1 Turbulence Spectrum

The 3D spectral function of the turbulent velocity, \( \Phi(\kappa) \) is defined in terms of the 2-point correlation of the turbulent velocity field

\[
< \vec{v}(\vec{r}) \cdot \vec{v}(\vec{r} + \vec{r}') >
\]

where the angular brackets denote ensemble averaging (Lesiurs 1997). In practice, the ergodic assumption is invoked and the ensemble average is replaced by space, area or line averages. For homogeneous turbulence the two point correlation is a function of the separation between the two points, so that

\[
\Phi(\kappa) = \frac{1}{(2\pi)^3} \int < \vec{v}(\vec{r}) \cdot \vec{v}(\vec{r} + \vec{r}') > e^{i\kappa \cdot \vec{r}} d^3r
\]

the inverse transform yields

\[
< \vec{v}(\vec{r}) \cdot \vec{v}(\vec{r} + \vec{r}') > = \int \Phi(\kappa) e^{-i\kappa \cdot \vec{r}} d^3k
\]

In the homogeneous and isotropic case \( \Phi \) depends only on the absolute value of the wave number and it is useful to introduce the turbulence energy spectrum \( E(k) \) and the turbulent velocity spectral function \( F(k) = 2E(k) \) so that

\[
\Phi(\kappa) = \Phi(k) = \frac{F(k)}{4\pi k^2}, \quad |k| = |\kappa|
\]

The measured radial velocity at each position in the plane of the sky, is an intensity-weighted average of the velocity contributed by the emitting gas along the line of sight. A simplifying assumption of homogeneity translates this into a line of sight average of the velocity.

In the following two subsections we assume that a 3D large scale turbulence exists and derive the relations between the observational power spectrum of the PV and rotation curve data and the underlying 3D turbulence. The simplifying assumptions of homogeneity and isotropy are used.

2.2 The Power Spectrum of Position Velocity Data

Let us consider first the power spectrum of a Position-Velocity (PV) data, namely the rotation velocity as function of position along a given axis of the galaxy disk. This power spectrum is defined as Fourier transform of the two point correlation of the observed residual velocity. Denoting the axis as \( y = 0 \) and the line of sight as \( z \), and the residual observed rotation velocity as \( u \), the power spectrum is

\[
P_{pv}(q) = \frac{1}{2\pi} \int e^{iqx} u(x + x', 0, z) u(x', 0, z') dx dx'
\]

Using the isotropy assumption and Eq. 2 one finds

\[
< u(x + x', 0, z) u(x', 0, z') > = \frac{1}{3} \int \Phi(\kappa) e^{-i(kxz + kzz')} dk^3k
\]

which when inserted in Eq. 4 leads to

\[
P_{pv}(q) = \frac{1}{3} \int \Phi(q, k_y, k_z) e^{i(k_x(z - z'))} dk_y dk_z dz dz'
\]

and Integrations over \( z, z' \) yield

\[
P_{pv}(q, D) = \frac{1}{3} \int \Phi(q, k_y, k_z) \left( \frac{\sin(k_z D/2)}{(k_z D/2)} \right)^2 dk_y dk_z
\]

with \( D \), the depth along the line of sight of the turbulent region. In the case of a disk galaxy \( D \cos i = 2H \) with \( H \) denoting the scale height of the disk and \( i \) the inclination angle.

For an homogeneous and isotropic turbulence, the 3D turbulence spectral function is a power law of the absolute value of the wavenumber: \( \Phi(q, k_y, k_z) = A(q^2 + k_y^2 + k_z^2)^{-m} \).

In the incompressible subsonic case the spectrum is the Kolmogorov spectrum with \( m = 11/6 \) (Lesiurs 1973). For compressible supersonic turbulence \( m = 2 \) (Passot et al. 1998). The steeper slope is due to the fact that a fraction of the turbulent kinetic energy density at a given wavenumber is converted to compression work decreasing the rate of energy transfer to the larger wavenumbers.

Integration over \( k_y \) from \( -\infty \) to \( \infty \) yields

\[
P_{pv}(q, D) = B(m) \int_{-\infty}^{\infty} (q^2 + k_z^2)^{-(m + 1/2)} \left( \frac{\sin(k_z D/2)}{(k_z D/2)} \right)^2 dk_z
\]

where \( B(m) \) is a constant depending on \( m \) and \( D \). One can further express the power spectrum in the form

\[
P_{pv}(q, D) = B(m) \int_{-\infty}^{\infty} (qD/2)^2 + \eta^2 \left( \frac{\sin^2 \eta}{\eta^2} \right) d\eta
\]

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The power spectrum is a power law with index $2 - 2m$ for $qD/2 << 1$ and $1 - 2m$ for $qD/2 >> 1$. This effect of the change of the power law index was found observationally by Elmegreen, Kim, & Staveley-Smith (2001) for the $H_1$ power spectrum of the LMC, by Dutta et al. (2009) for the $H_1$ power spectrum of NGC1058 and recently by Block et al. (2010) for the infrared power spectrum of the LMC, and by Contini & Goldman (2010) for both the velocity and mid infrared power spectra of the shocked nebulae near the turbulent Galactic Center. The results of the last two references are based on the Spitzer data.

2.3 The Power Spectrum of the Rotation Curve

The rotation curve is obtained by use of the tilted ring model (Rogstad, Lockhart, & Wright 1974) in which the disk is approximated as a superposition of concentric rings or annuli. The observed velocity field is then used to derive for each ring a rotation velocity, an inclination angle and a position angle. The rotation curve velocity as function of radius obtained in this way is more accurate than the PV curve, since the value at each radius is derived from multiple observational values.

This motivates a computation of the power spectrum of the residual fluctuations of the rotation curve and its use to test for the existence of an underlying 3D turbulence. For random fluctuations the power spectrum is expected to be independent of the wavenumber. If however, the power spectrum varies in an ordered manner with the wavenumber it is possible that it reflects an underlying turbulence.

As in the case of the PV power spectrum there is the need to obtain the relation between the observationally computed power spectrum and the underlying turbulence spectral function. In the case of the rotation curve this task seems a priori more difficult than the PV case since the rotation velocity is fitted to the observed line of sight velocities along the ring circumference and not measured directly along a given line in the plane of the disk. For simplicity the fit along the ring is represented as a simple average over its circumference, so that

$$P(q) = \frac{1}{(2\pi)^3} \int e^{iqr} \Phi(k, z) \eta \eta' \, dk \, dq \, dz$$

As in the PV case, here too use of Eq. 2 and assuming isotropy of the turbulence imply

$$P(q) = \frac{1}{3} \int (q^2 + k^2)^{m-1} \sin^2(\eta D/2) \, dk \, dq$$

where $k = \sqrt{k^2 + \eta^2}$ is the wavenumber in the plane of the sky and $\alpha$ denotes its azimuthal angle. In the integration along the ring circumference $\alpha$ spans an interval of $2\pi$ and so do $\alpha$ and $\alpha'$. One gets,

$$P(q) = \frac{3}{(2\pi)^3} \int (q^2 + k^2)^{m-1} \sin^2(\eta D/2) \, dk \, dq$$

Figure 1. $P(q)$ in dimensionless units, as function of $qD/2$ for $m = 11/6$

$$P(q) = \frac{1}{3} \int \Phi(k, z) e^{iqr} \eta D/2 \, dk \, dq$$

with $J_0$ denoting the zero order Bessel function of the first kind.

For an underlying 3D turbulence characterized by a power law spectral function $\Phi(k, z) = A(k^2 + k_z^2)^{−m}$, the integrations on $r$ and $k$ yield

$$P(q) = B_1(m) \int (q^2 + k^2)^{m-1/2} \sin^2(\eta D/2) \, dk$$

where $B_1(m)$ is a constant depending on the index $m$ and $D$ is the depth along the line of sight. The analytic integrations over $r$ and $k$ leading to Eq. 12 were done using the Mathematica 7 (2010) software. One can further express the power spectrum in the form

$$P(q) = B_2(m) \int (qD/2)^m \sin^2(\eta D/2) \, d\eta$$

Note that the observational power spectrum of the residual velocity curve has the same relation with respect to the underlying 3D turbulence as does the observational PV power spectrum. This result should not come as a surprise since the rotation curve can be regarded as an averaging of PV data along many axes. As a result, the power spectrum of its residual fluctuations is an average of the PV power spectra and hence share the same relation with the spectrum of the underlying 3D turbulence. As noted above, it is expected to be more accurate than the PV power spectrum.

Fig. 1 displays the observational power spectrum as function of $qD/2$ for an underlying 3D Kolmogorov turbulence characterized by $m = 11/6$. For spatial scales much larger than the depth along the line of sight, the index of the power spectrum is $-5/3$ and for spatial scales much smaller than the depth it approaches $-8/3$. The transition between the two regimes is at $kD/2 \approx 1.4$ and there the logarithmic slope is $-13/6$.

2.4 The Observational Power Spectrum of the Rotation Curve of NGC9183

The proposed method is demonstrated by applying it to NGC9183. The latter appears to be a rather ordered...
galaxy, although some deviations from regularity were noted (Bosma 1981; van Albada et al. 1985; Begeman 1989). The \( H_I \) column densities do not exceed \( 5 \times 10^{20} \text{cm}^{-2} \) so the optical depth is rather small. The data presented in Table 2, of Begeman (1989) show a very flat rotation curve for radial distances in the range \( 1.5' - 11' \) corresponding to \( 4.1 \text{kpc} - 29.9 \text{kpc} \). This is the range of radial distances to which the proposed test is applied.

The residuals (with respect to the mean value) are plotted in Fig. 2 as function of radial distance in units of arcminutes. Up to \( 3' \), the radial distances are spaced by \( 0.25' \) and later by \( 0.5' \). Since the computation of power spectra requires even spacings, we omitted the data points at positions \( 1.75', 2.25', 2.57' \) thus there remain overall 20 evenly spaced data points. Note that the uncertainties are \( (2 - 3) \text{km/s} \); substantially smaller than the \( 15.5 \text{km/s} \) uncertainty in a single measurement (Begeman 1989). This arises from the use of many measurements along the ring and demonstrates the advantage of using a rotation curve rather than a PV data.

The residual velocity shown in Fig. 2 does seem to exhibit fluctuations, as function of radial distance. But one has yet to test whether these fluctuations do indeed represent a large scale turbulence.

The Mathematica (2010) software was used to compute the power spectrum of the residual rotation velocity as function of the wavenumber \( q = 2\pi/l \), with \( l \) denoting the corresponding spatial scale. Because of the cyclic nature of the discrete Fourier transform, the power spectrum is obtained for relative wavenumbers in the range \( 1 - 11 \) with \( q = 1 \) corresponding \( l_0 = 25.8 \text{kpc} \), the largest spatial scale of the turbulence and \( q = 11 \) corresponds to \( 2.35 \text{kpc} \), which is about twice the radial spacing between the observations used in the present analysis.

To obtain the error bars for the observational power spectrum we performed \( 10^3 \) simulations of “observational” velocity sets. At each position a velocity was randomly chosen from a normal distribution with a mean equal to the observational value at this position and a standard deviation equal to the observational uncertainty at this position.

For each such set, the power spectrum was computed and subsequently the standard deviations, at each wavenumber, of the logarithm of the simulated power spectra. These standard deviations were taken as the uncertainties in the logarithm of the observational power spectrum of the residual rotation velocity.

The standard deviation of the observational residual rotation velocity of Fig. 2 is \( 3.6 \text{km/s} \). Hence, if it indeed represents a turbulence the latter must be subsonic (in the isotropic case \( v_{\text{turb}} = \sqrt{33.6} = 6.2 \text{ km/s} \)). The theoretical fit is thus chosen to be a turbulence spectral function with \( m = 11/6 \). The depth \( D \), along the line of sight and the overall normalization of the function given by Eq. (13) to minimize the reduced \( \chi^2 \) value.

The observational power spectrum of the residual rotation curve velocity is shown in Fig. 3 together with two such fits. The lower one is the best fit with \( \chi^2 = 0.43 \). It has a depth along the line of sight \( D = 7.31 \text{kpc} \) translated into a scale height for neutral hydrogen \( H_{HI} = 1130 \text{pc} \). The law value of \( \chi^2 \) suggests that the uncertainties in Begeman (1989) are probably overestimated.

This analysis implies that the residuals are consistent with being a manifestation of a subsonic turbulence with a largest scale of \( 25.8 \text{ pc} \), a turbulent velocity of \( 3.6 \text{km/s} \), and a lifetime scale of the order of \( 7 \text{Gyr} \).

About 75% of the \( \chi^2 \) value is contributed by the point at relative wavenumber \( q = 7 \) corresponding to a scale of about \( 3.7 \text{kpc} \). The upper curve in Fig. 3 is the best fit to the power spectrum with this point excluded. It has a \( \chi^2 = 0.11 \), and a depth \( D = 3.78 \text{kpc} \) corresponding to a scale height \( H_{HI} = 584 \text{pc} \). This latter is more consistent with observational values for disk galaxies (Bagetakos et al. 2010). For each of the two fits there is a correlation between the normalization of the fit and the value of \( D \), resulting in a flat dependence of \( \chi^2 \) on \( D \). Thus, in each case the calculated \( D \) and the corresponding scale height vary in a range of about \( \pm20\% \).
3 DISCUSSION

This paper puts forward the idea that analyzing the residual of the rotation curve velocities could test for the existence of a galactic-scale turbulence. If such a turbulence exits, the large spatial scale implies a lifetime of the order of a few Gyr. These two features point to a primordial merger as the natural generating mechanism. The test relies on the fact that random residuals of the velocity curve would have produce a flat power spectrum in sharp contrast with known turbulence spectra that exhibit a power law dependence on wavenumber, for wide ranges of the latter.

In this paper the method is proposed for galaxies exhibiting an observational rotation curve which is flat over sizable range of radial distances. This limitation avoids ambiguity as to the definition of what constitutes the residual velocity curve. In any case, if the derived observational power spectrum can be identified with turbulence spectrum, the issue of the definition of the residual is a posteriori solved.

We derived a semi-analytic relation between the observational power spectrum and the spectral function of the underlying 3D turbulence. We proved that the relation is of the same form for the observational spectra calculated from a position velocity data and from the rotation curve. The latter is of course advantageous since at each radial distance it represents a fit along a ring and not a single value. The method is capable of determining the depth along the line of sight of turbulent gas and hence the disk scale height.

The method has been demonstrated for the rotation curve of NGC3198. The computed power spectrum fits very well a Kolmogorov spectrum with a logarithmic slope of $-5/3$ for the largest scales and changing to $-8/3$ for the small scales. The turbulent rms velocity is about $6 \text{ km/s}$; the lifetime is of the order of $7 \text{ Gyr}$ and the implied scale height about $580$ pc. For this galaxy, the observational power spectrum spans a decade in wavenumbers. Clearly, an higher spatial resolution would have enabled a wider wavenumber range, impropve the numerical fit of the depth and the scale height, and lend more credibility to the conclusions. But even so, the case for an underlying 3D turbulence in NGC3198 seems quite good.

The method can be used as an observational tool for finding how common were mergers. It easy to identify merges in cases of galaxies with a disturbed shape. These mergers are usually major mergers. The majority of the cosmological mergers are expected to be minor mergers that don’t have a sharp morphological imprint. The present method enables detecting turbulence also for minor mergers and for galaxies that appear regular, as in the case of NGC3198. The long lifetime of the turbulence enables using it as a fossil evidence for primordial mergers.

It would be interesting to look for turbulence in additional galaxies and specifically, to test for a correlation between the disk thickness and the turbulent rms velocity.

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ACKNOWLEDGMENTS

I thank D. Cheluche and S. Sadeh for discussions, and H. Goldman for thorough reading and comments regarding the manuscript. The research was supported by the Afeka College research committee.