Fusion reaction of a weakly-bound nucleus with a deformed target

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We discuss the role of deformation of the target nucleus in the fusion reaction of the $^{15}$C + $^{232}$Th system at energies around the Coulomb barrier, for which $^{15}$C is a well-known one-neutron halo nucleus. To this end, we construct the potential between $^{15}$C and $^{232}$Th with the double folding procedure, assuming that the projectile nucleus is composed of the core nucleus, $^{14}$C, and a valance neutron. By taking into account the halo nature of the projectile nucleus as well as the deformation of the target nucleus, we simultaneously reproduce the fusion cross sections for the $^{15}$C + $^{232}$Th and the $^{15}$C + $^{209}$Bi systems. Our calculation indicates that the net effect of the breakup and the transfer channels is small for this system.

I. INTRODUCTION

One of the most important discoveries in nuclei near the neutron dripline is the halo phenomenon [1, 2]. It is characterized by a spatially extended density distribution, originated from the weakly-bound property of neutron-rich nuclei. Starting from $^{11}$Li [2], several other halo nuclei have also been observed successively. For example, $^6$He, $^{14}$Be, and $^{17}$B are regarded as two-neutron halo nuclei, while $^{11}$Be, $^{15}$C and $^{19}$C are categorized as one-neutron halo nuclei [3, 4]. Recently, heavier halo nuclei, such as $^{19}$C [5], $^{25}$C [6], $^{31}$Ne [7], and $^{37}$Mg [8] have also been found at Radioactive Ion Beam Facility (RIBF) in RIKEN.

Fusion reactions of halo nuclei have attracted lots of attention [9-19]. It is generally known that fusion cross sections at energies around the Coulomb barrier are sensitive to the structure of colliding nuclei [20-24], and it is thus likely that the halo structure significantly affects fusion reactions, both in a static and a dynamical ways. With the development of the radio-isotope technology, a large number of experimental data for fusion of halo nuclei have been accumulated. For instance, fusion cross sections for the $^{11}$Li + $^{208}$Pb [25], $^6$He + $^{238}$U [26], $^6$He + $^{209}$Bi [27, 28], $^{11}$Be + $^{209}$Bi [29], and $^{15}$C + $^{232}$Th [30] systems have been reported.

Interestingly, it has been reported that fusion cross sections for the $^6$He + $^{238}$U system [26] do not show any significant influence of the halo structure of $^6$He albeit that $^6$He is a well-known halo nucleus. This is in contrast to fusion cross sections for the $^{11}$Li + $^{208}$Pb [25] system, which show an enhancement with respect to the fusion cross sections for the $^9$Li + $^{208}$Pb system. The $^6$He + $^{209}$Bi system also shows a similar trend as in the $^{11}$Li + $^{208}$Pb system [28]. In the case one-neutron halo nuclei, cross sections for the $^{11}$Be+$^{209}$Bi system are reported to be similar to those for the $^{16}$Be+$^{209}$Bi system [29]. Origins for this apparent difference among these systems have not yet been understood completely, even though the fissile nature of the $^{238}$U may play some role.

In this regard, it is interesting to notice that $^{238}$U is a well deformed nucleus while $^{208}$Pb and $^{209}$Bi are spherical nuclei. The aim of this paper is to investigate the role of deformation of the target nucleus in fusion of a halo nucleus. To this end, we shall discuss the fusion reaction of the $^{15}$C+$^{232}$Th system. The $^{15}$C nucleus is a one-neutron halo nucleus [4], and its structure is much simpler than the structure of the two-neutron halo nuclei $^{11}$Li and $^6$He. The $^{15}$C+$^{232}$Th system thus provides an ideal opportunity to disentangle the deformation and the halo effects. Moreover, $^{15}$C is heavier than $^{11}$Li and $^6$He, and more significant effects of the target deformation can be expected for the $^{15}$C+$^{232}$Th system as compared to the $^{11}$Li, $^6$He+$^{238}$U systems. Notice that the previous calculation for this system used a very simple spectator model and did not take into account the halo structure of $^{15}$C [30]. It has yet to be clarified how much the measured fusion enhancement can be accounted for by taking into account the halo structure of $^{15}$C.

The paper is organized as follows. In Sec. II we first analyze the fusion of the $^{14}$C + $^{232}$Th system by using a deformed Woods-Saxon potential. In Sec. III we analyze the fusion of the $^{15}$C + $^{232}$Th system and discuss the role of the halo structure and the deformation effect. To this end, we construct the potential between $^{15}$C and $^{232}$Th with the double folding formalism accounting for the halo structure of the $^{15}$C nucleus. We finally summarize the

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II. FUSION REACTION OF THE $^{14}$C+$^{232}$Th SYSTEM

Before we discuss the fusion cross sections for the $^{15}$C+$^{232}$Th system, we first analyze the $^{14}$C+$^{232}$Th system. In order to take into account the deformation effect of the target nucleus $^{232}$Th, we employ a deformed Woods-Saxon (WS) potential for the relative motion between the target and the projectile nuclei $^{22,31}$:

$$V_{14C-T}(r, \theta) = \frac{V_0}{1 + \exp\left(\left(\frac{r - R_0 - R_T}{\sum \lambda \beta_{AT} Y_{0\lambda} (\theta)}\right)/a\right)},$$

where $V_0$, $R_0$, and $a$ are the depth, the radius, and the diffuseness parameters, respectively. $R_T$ and $\beta_{AT}$ are the radius and the deformation parameters of the target nucleus $^{232}$Th. The Coulomb potential also has a deformed form given by $^{22,31}$

$$V_C(r, \theta) = \frac{Z_P Z_T e^2}{r} + \frac{3Z_P Z_T e^2 R_T^2}{5 r^3} \left( \beta_{2T} + \frac{2}{7} \sqrt{\frac{5}{\pi}} \beta_{2T} \right) Y_{20}(\theta),$$

$$+ \frac{3Z_P Z_T e^2 R_T^4}{9 r^5} \left( \beta_{4T} + \frac{9}{7} \sqrt{\frac{5}{\pi}} \beta_{2T} \right) Y_{40}(\theta)$$

with the second order in the quadrupole deformation parameter, $\beta_{2T}$, and the first order in the hexadecapole deformation parameter, $\beta_{4T}$. $Z_P$ and $Z_T$ are the atomic number of the projectile and the target nuclei, respectively. Fusion is simulated with the incoming wave number of the projectile and the target nuclei, respectively. Fusion is simulated with the incoming wave number of the projectile and the target nuclei, respectively. Here, the radius parameter $r$ is defined as $R_0 = r_0 (A_P^{1/3} + A_T^{1/3})$, where $A_P$ and $A_T$ are the mass numbers of the projectile and the target nuclei, respectively. The resultant barrier height, $V_b$, the barrier position, $R_b$, and the barrier curvature, $\hbar\Omega$ are also shown.

| $V_0$ (MeV) | $r_0$ (fm) | $a$ (fm) | $V_b$ (MeV) | $R_b$ (fm) | $\hbar\Omega$ (MeV) |
|------------|-----------|---------|-------------|-----------|------------------|
| 90.00      | 1.179     | 0.654   | 60.34       | 12.17     | 4.53             |

FIG. 1. Fusion cross sections for the $^{14}$C + $^{232}$Th system. The dashed line denotes cross sections in the absence of the deformation effect of the target nucleus $^{232}$Th while the solid line is obtained by taking into account the deformation effect with a deformed Woods-Saxon potential. The experimental data are taken from Ref. $^{34}$.

III. FUSION REACTION OF THE $^{15}$C+$^{232}$Th SYSTEM

A. The internuclear potential

Let us now discuss the fusion reaction of the $^{15}$C + $^{232}$Th system. We first construct the potential between $^{15}$C and $^{232}$Th taking into account the deformation effect of the target nucleus as well as the halo structure of the projectile. To this end, we employ the double folding approach and construct the potential as

$$V_{15C-T}(r; r_d) = \int dr_p \int dr_T \rho_p(r_p) \rho_T(r_T; r_d) \times V_{NN}(r - r_p + r_T),$$

where $V_{NN}$ is an effective nucleon-nucleon interaction, while $\rho_p(r_p)$ and $\rho_T(r_T; r_d)$ are the density profiles for the projectile and the target nuclei, respectively. Here, the density of the deformed target is defined with respect to the orientation angle, $r_d$, in the space fixed frame. In this paper, we employ the deformed Woods-Saxon density given by

$$\rho_T(r; r_d) = \frac{\rho_0}{1 + \exp\left[\left(-c(1 + \sum \lambda \beta_{AT} Y_{0\lambda}(\theta_{rd}))\right)/z\right]},$$

where $\theta_{rd}$ is the angle between $r$ and $r_d$, and $\beta_{AT}$ are the deformation parameters. We take $c=6.851$ fm, $z=0.518$.

TABLE I. The depth, $V_0$, the radius, $r_0$, and the diffuseness, $a$, parameters for the deformed Woods-Saxon potential for the $^{14}$C + $^{232}$Th reaction. Here, the radius parameter $r$ is defined as $R_0 = r_0 (A_P^{1/3} + A_T^{1/3})$, where $A_P$ and $A_T$ are the mass numbers of the projectile and the target nuclei, respectively. The resultant barrier height, $V_b$, the barrier position, $R_b$, and the barrier curvature, $\hbar\Omega$ are also shown.
where the energy and the length are given in units of MeV and fm, respectively. Notice that this interaction also includes the knock-on exchange effect in the zero-range approximation.

To evaluate the double folding potential, the target density \( v \) is expanded as

\[
\rho_T(r; r_d) = \sum_{\lambda} \rho_{T\lambda}(r) Y_{\lambda0}(\theta_{rd})
\]

(6)

\[
= \sum_{\lambda,\mu} \rho_{T\lambda}(r) \sqrt{\frac{4\pi}{2\lambda+1}} Y_{\lambda\mu}(\hat{r}) Y^*_{\lambda\mu}(\hat{r}_d).
\]

(7)

Substituting this into Eq. (10), one obtains the potential in a form of

\[
V_{15C-T}(r; r_{def}) = \sum_{\lambda,\mu} V_{\lambda}(r) \sqrt{\frac{4\pi}{2\lambda+1}} Y_{\lambda\mu}(\hat{r}) Y^*_{\lambda\mu}(\hat{r}_d),
\]

(8)

with

\[
V_{\lambda}(r) = \int d\mathbf{r}_p \int d\mathbf{r}_T \rho_n(r_p) \rho_{T\lambda}(r_T) V_{NN}(r - r_p + r_T).
\]

(9)

In the isocentrifugal approximation, one then sets \( r_d = 0 \) and finally obtains

\[
V_{15C-T}(r, \theta) = \sum_{\lambda} V_{\lambda}(r) Y_{\lambda0}(\theta).
\]

(10)

We assume that the projectile nucleus \( ^{15}\text{C} \) takes the two-body structure, with the spherical core nucleus \( ^{14}\text{C} \) and a valence neutron. The density of the projectile is then given as

\[
\rho_p(r) = \rho_c(r) + \rho_n(r),
\]

(11)

where \( \rho_c(r) \) and \( \rho_n(r) \) are the density for the core nucleus and the valence neutron, respectively. If one uses this density, the folding potential of Eq. (6) is also separated into two parts:

\[
V_{15C-T}(r, \theta) = V_{14C-T}(r, \theta) + V_{n-T}(r, \theta).
\]

(12)

For simplicity, we replace the interaction between the core and the target nuclei, \( V_{14C-T}(r, \theta) \), by the deformed Woods-Saxon potential determined in the previous section.

For the density for the valence neutron, we construct it using a \( 2s_{1/2} \) neutron wave function in a Woods-Saxon potential as

\[
\rho_n(r) = \frac{1}{4\pi} \left[ R_{2s_{1/2}}(r) \right]^2,
\]

(13)

where \( R_{2s_{1/2}}(r) \) is the radial part of the wave function. To this end, we use the Woods-Saxon potential with set C in Ref. [30], which reproduces the empirical neutron separation energy for this state, \( E_{2s_{1/2}} = -1.21 \) MeV. Figure 2 shows the projectile density thus obtained. The blue dashed line shows the density for the core nucleus, \( ^{14}\text{C} \), while the red dot-dashed line denotes the valence neutron density. For the description of the core density, we use the modified harmonic-oscillator model, whose parameters can be found in Ref. [37]. One can see that the valence neutron density has a long tail, reflecting the halo structure of the \( ^{15}\text{C} \) nucleus.

![FIG. 2. The density distribution of the \( ^{15}\text{C} \) nucleus (the solid line). The dashed and the dot-dashed lines denote the contribution of the core nucleus and the valence neutron, respectively.](image)

The solid lines in Fig. 2 shows the neutron-target potential obtained with the double folding procedure. The top, the middle, and the bottom panels show the monopole, the quadrupole, and the hexadecapole components, respectively. In order to discuss properties of these potentials, we fit them with a Woods-Saxon function and its first derivative. That is,

\[
V_{\lambda}(r) = \frac{-V_0}{1 + \exp \left[ \left( r - R_0 \right)/a \right]} \quad (\lambda = 0),
\]

(14)

\[
= \frac{-V_0 \exp \left[ \left( r - R_0 \right)/a \right]}{\left(1 + \exp \left[ \left( r - R_0 \right)/a \right] \right)^2} \quad (\lambda = 2, 4),
\]

(15)

The results of the fitting are shown in Fig. 3 by the dashed lines (see Table II for the parameters). Since the region around the position of the Coulomb barrier region is most important for fusion cross sections, the fitting are performed mainly in the surface region, \( r > 8 \) fm. In Fig. 3 one can see that the folding potential can be well fitted with the Woods-Saxon function. The hexadecapole component, \( V_4(r) \), has some deviation from the Woods-Saxon function, but its contribution to the total potential is much smaller than the monopole and the quadrupole components. We also find that the contribution of \( \lambda = 6 \), that is, \( V_6(r) \), is negligible, with a small depth size of about \( -0.1 \) MeV. Note that the Coulomb barrier parameters, obtained by setting the deformation parameters \( \beta_{T\lambda} \) to be zero in the folding procedure, are \( V_6 = 58.81 \) MeV.
I shows the total potential (that is, the sum of the nuclear and the Coulomb potentials). The red dashed lines show fits with the Woods-Saxon function. The top, the middle, and the bottom panels are for \( V_0(r) \), \( V_2(r) \), and \( V_4(r) \) in Eq. (7), respectively. Note that the case of \( \theta = 0^\circ \) corresponds to \( \theta = 90^\circ \). The solid and the dashed lines are for the \( 14C + 232Th \) system and the \( 15C + 232Th \) system, respectively. The black solid lines show the results of the double folding potential, while the red dashed lines show fits with the Woods-Saxon function.

\[ V_0 = 10.5923 \text{ MeV}, \quad R_0 = 7.981 \text{ fm}, \quad a = 1.572 \text{ fm} \]

\[ V_2 = 28.756 \text{ MeV}, \quad R_0 = 7.080 \text{ fm}, \quad a = 1.890 \text{ fm} \]

\[ V_4 = 4.398 \text{ MeV}, \quad R_0 = 8.131 \text{ fm}, \quad a = 1.944 \text{ fm} \]

Let us now calculate fusion cross sections for the \( 15C + 232Th \) system using the potential constructed in the previous subsection. For simplicity, we include up to \( \lambda = 4 \) in the multipole expansion of the double folding potential between the valence neutron in \( 15C \) and the target nucleus.

Figure 4 presents a comparison between the calculated fusion cross sections and the experimental data. The red circles and the blue stars show the experimental data for the \( 15C + 232Th \) and \( 14C + 232Th \) systems, respectively. The violet dashed curve shows the results for the \( 14C + 232Th \) system, which is the same as the solid line in Fig. 3. The green dot-dashed curve shows the results for the \( 15C + 232Th \) system, obtained using the potential which is simply scaled from that for the \( 14C + 232Th \) system. That is, the radius parameter in the Woods-Saxon potential for the \( 14C + 232Th \) system is changed from \( r_0 (14^{1/3} + 232^{1/3}) \) to \( r_0 (15^{1/3} + 232^{1/3}) \). This calculation does not take into account the weakly bound nature.
of the $^{15}$C projectile, and corresponds to the calculation presented in Ref. [50]. In fact, the dashed and the dot-dashed lines show similar fusion cross sections to each other, as has been argued in Ref. [50]. Even though this calculation reproduces the experimental data at energies above the Coulomb barrier, $E_{\text{c.m.}} \geq 60$ MeV, it considerably underestimates fusion cross sections in the energy region below the Coulomb barrier.

Fusion cross sections evaluated with the halo nature of the $^{15}$C nucleus are shown by the black solid line. One can clearly see that this calculation successfully reproduces the two lowest data points below the Coulomb barrier, even though it slightly overestimates the fusion cross sections around $E_{\text{c.m.}} = 60$ MeV. We emphasize that this calculation takes into account neither the breakup of $^{15}$C nor the transfer channels of the valence neutron. Given that the experimental data are reproduced without these two effect act in a opposite way so that their net effect on fusion cross sections is small. It remains an open problem whether this finding is due to the presence of the strong coupling to the rotational excitations of the target nucleus. In this regard, it would be useful to investigate systematically fusion of a halo nucleus with deformed target nuclei. A comparison of the $^{15}$C + $^{232}$Th system to the $^{15}$C + $^{238}$U and $^{15}$C+$^{208}$Pb systems might also provide useful information.

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IV. SUMMARY AND CONCLUSION

We have calculated cross sections for $^{15}$C+$^{232}$Th system, for which $^{15}$C is a well-known one-neutron halo nucleus while $^{232}$Th is a well deformed nucleus. Without considering explicitly couplings to the breakup and the transfer channels, we have evaluated the cross sections within the double folding formalism taking into account the halo structure of $^{15}$C and the deformation of the target nucleus. We have shown that our calculation reproduces simultaneously the experimental fusion cross sections for the $^{15}$C+$^{232}$Th and $^{14}$C+$^{232}$Th systems. This clearly indicates that both the deformation effect and the halo structure play an important role in this reaction.

We conclude that the effect of the coupling to the breakup and the transfer channels is small for this system, even though we cannot exclude a possibility that these two effect act in a opposite way so that their net effect on fusion cross sections is small. It remains an open problem whether this finding is due to the presence of the strong coupling to the rotational excitations of the target nucleus. In this regard, it would be useful to investigate systematically fusion of a halo nucleus with deformed target nuclei. A comparison of the $^{15}$C + $^{232}$Th system to the $^{15}$C + $^{238}$U and $^{15}$C+$^{208}$Pb systems might also provide useful information.

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