Gluon induced contributions to $Z\gamma$ production at hadron colliders

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Abstract

We study the contribution of gluon induced partonic subprocesses to $Z\gamma$ pair production at hadron colliders. These processes contribute only at next-to-next-to-leading order but are potentially enhanced by two factors of the gluon parton densities. However, we find that their contribution is modest and that next-to-leading order calculations give reliable predictions.
1 Introduction

The substantially increased number of $Z\gamma$ pairs that are expected to be produced in Run II at the Tevatron will allow an improvement to previous studies \[1, 2, 3\] of $Z\gamma$-pair production at hadron colliders. Accordingly, the $ZZ\gamma$ and $Z\gamma\gamma$ couplings, which are absent in the Standard Model, can be investigated in more detail. Assuming no deviations from the Standard Model are found, tighter limits on such anomalous couplings will be possible.

In order to fully exploit the experimental data it is important to have theoretical predictions of sufficient accuracy. Next-to-leading order cross sections for this process have been calculated quite some time ago \[4\]. Subsequently, the theoretical predictions have been improved by taking into account the leptonic decay of the $Z$-boson, anomalous couplings and full leptonic correlations \[5, 6, 7, 8\].

For some distributions, the next-to-leading order corrections have been found to be very large. In particular in the large transverse momentum region, which is interesting for investigating anomalous couplings, the corrections can increase the theoretical prediction of the cross section by several 100%. The reason for this large corrections is that at NLO new partonic processes contribute. Whereas the tree-level calculation only incorporates the partonic process $q\bar{q} \rightarrow Z\gamma \rightarrow \ell\bar{\ell}\gamma$, at next-to-leading order also processes with a gluon in the initial state such as $gq \rightarrow Z\gamma q \rightarrow \ell\bar{\ell}q\gamma$ contribute. The large gluon density at low $x$ overcomes the suppression by an additional factor of $\alpha_s$. Thus, even though this contribution is formally next-to-leading order, numerically it can be as important as the leading-order term.

This immediately leads to the question whether at NNLO a similar effect occurs. Higher-order corrections to partonic processes that are already present at NLO are not expected to lead to large corrections. However, at NNLO a new class of partonic processes have to be taken into account, namely processes with two gluons in the initial state. Such processes are suppressed by two factors of $\alpha_s$ but are potentially enhanced by two factors of the gluon parton distribution function. If there is a similar effect as at NLO, this could lead to corrections as important as the Born term.

This question has been studied for $WZ$ and $W\gamma$ production \[9, 10\]. For these processes it has been found that the gluon induced NNLO corrections are negligible. However, there is an important difference in $Z\gamma$ production with respect to $WZ$, $W\gamma$ production. In fact, there are generally two classes of contributions for vector-boson pair production with two gluons in the initial state. There are tree-level processes $gg \rightarrow VVq\bar{q}$ and loop processes $gg \rightarrow VV$, where $V$ denotes some vector boson. In the case of $WZ$ and $W\gamma$ the loop processes vanish due to charge conservation but for $Z\gamma$ this is not the case. The study in \[9, 10\] showed that the tree-level processes result in a very small contribution for $WZ$ and $W\gamma$. While it is natural to expect that this is also the case for $Z\gamma$ production, for this process there remains the question of how important the gluon induced loop corrections are.

The loop induced process $gg \rightarrow Z\gamma$ has been studied previously \[11, 12, 13\], where
it has been found that these contributions can be quite large but are not dominant.

This work extends the previous analyses in that both classes of gluon induced corrections to \( Z\gamma \) production are included and anomalous \( ZZ\gamma \) and \( Z\gamma\gamma \) couplings as well as full leptonic correlations are taken into account. Even though the gluon induced correction is substantially more important for \( Z\gamma \) production than for \( W\gamma, WZ \) it can still be safely neglected. In fact, not only are the contributions from tree-level processes tiny (as in the \( W\gamma, WZ \) case) but also the loop processes are smaller than anticipated. The latter effect is entirely due to a change in the parton distribution function used in our analysis compared to [12, 13].

2 Calculation

Our aim is to compute the part of the NNLO corrections to \( Z\gamma \) pair production that is enhanced by two factors of the gluon parton distribution function. This consists of two parts. Firstly, we have to compute the tree-level amplitudes \( gg \to q\bar{q}Z\gamma \to q\bar{q}\ell\bar{\ell}\gamma \) and integrate over the quark-antiquark final state phase space. Secondly, we have to evaluate the loop diagrams \( gg \to Z\gamma \to \ell\bar{\ell}\gamma \).

The contributions of the tree-level diagrams \( gg \to q\bar{q}Z\gamma \to q\bar{q}\ell\bar{\ell}\gamma \) can be computed in a very similar manner as for \( gg \to q\bar{q}W\gamma \), presented in [9]. This is a double bremsstrahlung NNLO contribution and in general is difficult to integrate over the phase space. However, due to the simple structure of the soft and collinear limits of the corresponding matrix elements, the integration over the singular regions of the phase space is much simpler than for a general NNLO calculation. In fact, as discussed in [9], the only singularities we are concerned with are single and double initial state collinear singularities. They come from the region of phase space where the two incoming gluons independently split into a quark-antiquark pair.

In order to perform the integration over phase space we use a generalization of the subtraction method presented in [14]. The singularities that arise upon integration over the region of phase space with the final state (anti)quark collinear to the incoming gluons are absorbed into the parton distribution functions. It should be noted that this contribution is factorization scheme dependent. Since we use parton densities obtained in the \( \overline{MS} \)-scheme we evaluate the partonic cross section in this scheme. We should also mention that the factorization scheme dependence is beyond the approximation we make since it is not enhanced by two factors of the gluon density.

The tree-level diagrams for \( gg \to q\bar{q}VV \) have been computed previously [15]. We included anomalous couplings \( ZZ\gamma \) and \( Z\gamma\gamma \). They are incorporated by using the \( Z(\alpha)(q_1)\gamma(\beta)(q_2)Z(\mu)(p) \) vertex [16, 17]

\[
\Gamma_{Z\gamma Z}(q_1, q_2, p) = \frac{i(p^2 - q_1^2)}{M_Z^2} \left( h_1^Z (q_2^\mu g^{\alpha\beta} - q_2^\alpha g^{\mu\beta}) + \frac{h_2^Z}{M_Z^2} p^\alpha (p \cdot q_2 g^{\mu\beta} - q_2^\mu p^\beta) \right.
\]

\[
- h_3^Z \varepsilon^{\alpha\beta\nu\rho} q_2_{\nu} - \frac{h_4^Z}{M_Z^2} \varepsilon^{\mu\beta\nu\rho} p^\alpha p_{\nu} q_{2\rho} \right) \tag{1}
\]
where it should be noted that the Z-boson with momentum $p$ is off-shell and the other two gauge bosons are on-shell. The vertex for $Z_{\alpha}(q_1)\gamma\beta(q_2)\gamma\mu(p)$ is as in eq. (1), but with $q_1^2 \to 0$ and $h_i^Z \to h_i^Z$.

The inclusion of anomalous couplings also necessitates the recalculation of the gluon induced loop diagrams. First of all, there are box diagrams. They have been calculated some time ago [11, 12, 13] but these results were obtained summing over helicities and did not include the decay of the Z-boson. However, since the axial coupling of the Z does not contribute, the amplitude can also be extracted from (the $N_F$-part of) the QCD amplitudes $gg \to q\bar{q}\gamma$ [18, 19]. In fact we simply have to change the gluon that decays into the $q\bar{q}$ pair into a $Z$. This can be done by picking the relevant subamplitudes of the $gg \to q\bar{q}\gamma$ process and changing the couplings accordingly.

In addition to the box diagrams there are triangle diagrams with a triple $ZZ\gamma$ or $Z\gamma\gamma$ vertex. These diagrams are only present for anomalous couplings. Furthermore, due to Yang’s theorem [20] this contribution is only non-vanishing because of off-shell effects. It should also be stressed, that the vertex of eq. (1) does not include all possible anomalous couplings. The vertex in eq. (1) does not include terms $\sim p^\mu$ since it is assumed that the gauge bosons couple to a conserved current. However, in our situation this is not the case and additional anomalous couplings are possible [21]. There is one additional CP conserving $(p\mu q^\alpha q^\beta \varepsilon_{\sigma\alpha\beta})$ and two additional CP violating couplings $(p\mu g_{\alpha\beta}$ and $p\mu(p-q_2)_{\sigma}(p-q_1)_{\beta})$. In our calculation we have not implemented these additional anomalous couplings, since — given the small size of the gluon induced corrections — it is very unlikely that useful information about these couplings can be extracted.

The effects of the top quark are taken into account by performing an expansion in $1/m_t$ and neglecting terms of order $(1/m_t)^4$. This approach is motivated by assuming that the partonic center of mass energy $\sqrt{s} < m_t$. Even though this is not generally true at the LHC, the fact that the gluon distribution is strongly peaked towards small values of $x$, makes this a reasonably good approximation.

To deal with the problem of photon isolation, we will use the procedure introduced by Frixione [22], i.e. we reject all events unless the transverse hadronic momentum deposited in a cone of size $R_0$ around the momentum of the photon fulfills the following condition

$$\sum_i p_{Ti} \theta(R-R_{i\gamma}) \leq p_T\gamma \left( \frac{1-\cos R}{1-\cos R_0} \right),$$

for all $R \leq R_0$, where the ‘distance’ in pseudorapidity and azimuthal angle is defined by $R_{i\gamma} = \sqrt{(\eta_i - \eta_{\gamma})^2 + (\phi_i - \phi_{\gamma})^2}$.

3 Numerical Results

For the numerical results presented in this section we follow closely the analysis presented in [9]. We use the MRST 2001 parton distribution functions [23] with the
one-loop expression for the coupling constant \( \alpha_s(M_Z) = 0.119 \). Strictly speaking, for the \( gg \) induced processes we should use a NNLO parton distribution. However, it is not expected that a change to the not yet fully available NNLO distributions would alter our conclusions. The factorization and renormalization scales are fixed to \( \mu_F = \mu_R = \sqrt{M_Z^2 + (p_T^γ)^2} \).

The mass of the \( Z \) has been set to \( M_Z = 91.187 \) GeV. For the couplings of the vector bosons with the quarks we use \( \alpha = \alpha(M_Z) = 1/128 \) whereas for the photon coupling we use \( \alpha = 1/137 \). Note that we do not include the branching ratios for the decay of the \( Z \) into leptons.

For our numerical results we use a set of standard cuts. Charged leptons are required to have \( p_T > 20 \) GeV and \( \eta < 2.5 \). The photon transverse momentum cut we use is \( p_T^γ > 20 \) GeV, while for the isolation prescription in eq. (2) we set \( R_0 = 1 \).

Since the potential enhancement due to the gluon densities is larger for increasing center of mass energy, we restrict ourselves to studying the situation at the LHC. Thus, we consider \( Zγ \) production in proton–proton collision at an energy \( \sqrt{s} = 14 \) TeV.

The canonical quantity we use to investigate the importance of the gluon induced corrections is the transverse momentum of the photon. In Figure 1 we show the contribution of the various initial partonic states to \( d\sigma/dp_T^γ \). The solid line shows the contribution at NLO of the initial state \( q\bar{q} \). This is the dominant part and differs only

![Figure 1: The contribution to the \( p_T^γ \) distribution in \( Zγ \) production at the LHC separated into the various partonic initial states: \( q\bar{q} \) (solid line), \( gq \) (dashed) and \( gg \). The gluon induced contribution is separated into the loop and bremsstrahlung part.](image-url)
slightly from the tree-level result. The $gq$ (and $g\bar{q}$) initial states enter at NLO and can result in a correction as big as 70% which is shown as the dashed line. Finally, there are the $gg$ induced processes. We split the result into the loop part (short dashed) and the tree-level bremsstrahlung part (dotted). The latter is negative and very small. This is very similar to what has been found for $WZ$ and $W\gamma$ production \cite{9}. The loop part is substantially bigger but still does not exceed 5%. Note that differs quite a bit from what has been found in previous analyses \cite{12, 13}. We checked that this is entirely due to an update in the gluon distribution function. In fact, our results agree with those in \cite{12, 13} if we use the same parton distribution functions.

The situation does not change if anomalous couplings are added. The relative importance of the gluon induced processes is still small. Thus, there is no hope of gaining any information about non-standard anomalous couplings that in principle contribute to this process.

We also investigated other quantities and the picture remains the same. The loop corrections have more or less the expected size but are never very important. The reason for the smallness of the bremsstrahlung corrections is the following: in the important region of small partonic center of mass energy $\sqrt{s}$, corresponding to small $x$, the partonic cross section $d\hat{\sigma}$ is small and turns negative. Only for increasing $\sqrt{s}$ we find that $d\hat{\sigma}$ is of the expected size, i.e. suppressed by two orders of $\alpha_s$. However, in this region $x$ is not small anymore and, thus, there is no enhancement due to the gluon distribution. This might well be related to the very simple structure of the singularities of the partonic cross section. For a process with a more complicated structure, e.g. involving $t$-channel gluon exchange, the bremsstrahlung corrections could easily be significantly higher.

4 Conclusions

We studied the contribution of the partonic processes $gg \rightarrow gqZ\gamma$ and $gg \rightarrow Z\gamma$ to the production of $Z\gamma$ pairs at hadron colliders. These contributions are enhanced by two factors of the large gluon density. This could potentially overcome the $O(\alpha_s^2)$ suppression. However, we found that under no circumstances are these contributions particularly important. In fact, they are even substantially smaller than anticipated from previous analyses \cite{12, 13}, a change that is due to using updated parton distribution functions.

In summary, the next-to-leading order cross section for $Z\gamma$ production provide us with a reasonably precise theoretical prediction. The large NLO corrections due to the opening of a new partonic channel with $qg$ initial states are taken into account. Higher order corrections to these partonic processes are expected to be well under control and the new partonic channels that open at NNLO, namely $gg$ initial state processes, do not result in large corrections.
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