TOPICS IN STRING UNIFICATION

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ABSTRACT

I discuss several aspects of strings as unified theories. After recalling the difficulties of the simplest supersymmetric grand unification schemes I emphasize the distinct features of string unification. An important role in constraining the effective low energy physics from strings is played by duality symmetries. The discussed topics include the unification of coupling constants (computation of $\sin^2 \theta_W$ and $\alpha_s$ at the weak scale), supersymmetry breaking through gaugino condensation, and properties of the induced SUSY-breaking soft terms. I remark that departures from universality in the soft terms are (in contrast to the minimal SUSY model) generically expected.

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1. Strings as unified theories

The recent LEP precision measurements of the Standard Model (SM) gauge-coupling parameters have confirmed [1] the remarkable agreement with the expectations from supersymmetric grand unification [2], [3]. If taken seriously, this agreement would suggest the existence of a supersymmetric GUT like $SU(5)$ or $SO(10)$ beyond an energy scale $M_X \equiv 10^{16}$ GeV.

While the prediction for $\sin^2 \theta_W$ in these models works remarkably well, one must recall, however, that these theories have several important theoretical problems. Perhaps the more severe one is that of understanding the huge mass splitting between the Higgs doublets of the theory and the colour triplet chiral fields which come along in any GUT scheme (the doublet–triplet splitting problem). It is not just that we do not understand why this splitting occurs; the worst problem is that the mechanisms considered up to now destroy the hierarchy of masses, i.e. give, either at tree level or in loops, a huge mass to the standard Higgs doublets. The simplest example of this fact is just the minimal SUSY-$SU(5)$ model with SUSY-breaking soft terms. There you get doublet-triplet splitting by fine-tuning of the couplings $\lambda$ and $\mu$ in the superpotential $\lambda \phi_{24} H_5 H_\bar{5} + \mu H_5 H_\bar{5}$. However once SUSY is broken you get soft SUSY-breaking couplings $A m \lambda \phi_{24} H_5 H_\bar{5} + B m \mu H_5 H_\bar{5}$, where $A, B, m$ are independent SUSY-breaking parameters. Now the fine-tuning that guaranteed the masslessness of the Higgs doublets in the superpotential is spoiled by the contribution of the soft terms which, for generic $A, B$, gives a huge mass to the Higgses. This is not just a problem of the minimal model but appears to be quite generic in SUSY-GUTs, although it sometimes appears only at one loop or two loops [4]. The origin of the problem is that soft susy-breaking terms spoil the hierarchy whenever there are fields that couple both to the doublets and other supermassive fields. Apart from this generic problem, it is also well known that usual GUTs have difficulties with their predictions for quark-lepton masses of the first two generations. Furthermore, the breaking of the GUT symmetry generally requires huge Higgs representations, especially if one wants to get consistent quark-lepton mass spectra.
In view of the problems reviewed above, one should take the success of standard SUSY-GUTs with a grain of salt. There are further theoretical reasons to believe that such schemes are too naive to be true. To start with, the standard coupling of these theories to $N = 1$ supergravity (required in order to obtain the appropriate soft terms in a natural way) leads to a non-renormalizable theory. Although below the Planck mass one can deal with a renormalizable effective Lagrangian, eventually one will have to face the problem of a non-renormalizable quantum gravity.

Four-dimensional supersymmetric strings are obvious candidates for unified theories since they provide us, in principle, with the first finite theories of all interactions, including gravity. However, string theory is not a model for unification, it is an alternative to field theory itself. Below the string scale ($\simeq M_{\text{Planck}}$) one expects to describe the low-energy physics in terms of some standard field theory, presumably some sort of $N = 1$ supergravity Lagrangian coupled to the standard model (or some of its gauge extensions). The actual challenge is trying to see what constraints (if any) should obey such a Lagrangian because of its stringy origin. Asking for a model-independent test of string theory is perhaps too much in the same sense as asking for a model-independent test of field theory (without any reference to the actual interactions realized in nature) is itself hard! But one can easily see that the building of string versions of the standard model or of possible extensions is quite constrained: string model building has its own rules. Symmetries such as superconformal and modular invariance on the worldsheet as well as target-space duality symmetries substantially constrain the particle content and interactions of the string models.

There are several techniques to build 4-D strings, and typically a given model may be constructed in several different ways [5]. Several examples of 4-D string theories with three generations of quarks and leptons (plus extra exotics) exist in the literature [6], [7], [8], [9]. Very often (maybe always [10]) one can derive the four-dimensional string starting from the 10-D heterotic string and compactifying the 6 extra dimensions into an unobservable space with overall size $R \equiv 1/10^{18} \text{GeV}^{-1}$. The massless sector of the theory contains the standard pure $N = 1$ supergravity particles, plus gauge particles and matter fields. There are essentially three types of massless matter fields in the massless sector: 1)
Charged matter fields $\phi_\alpha$. These include the standard quark, lepton, and Higgs superfields. ii) The moduli fields $T_i$. These are massless fields, which are singlets under the observed gauge interactions. Some of them are associated to the size and shape of the compactifying manifold. In particular, the real part of one of them (usually denoted $T$) is related to the compactification radius ($\text{Re}T = R^2$).

iii) The dilaton field $S$. This is a singlet field whose real part is related to the tree-level gauge coupling constant ($\text{Re}S = 1/g^2$). One of the specific features of 4-D strings is that coupling constants are in fact fields whose vacuum expectation values correspond to the measured values. The couplings depend on the dilaton field $S$ as well as on the moduli fields $T_i$. Unfortunately these fields have a vanishing scalar potential in perturbation theory and hence we need a certain knowledge of the non-perturbative effects which may eventually determine $\langle S \rangle$ and $\langle T_i \rangle$. However, we may still get relationships amongst different coupling constants assuming that eventually the dynamics will fix those vev’s (some recent ideas about how this may happen are discussed below).

One important point to remark is that, in string theory, gauge interactions are necessarily unified even in the absence of a unification gauge group such as $SU(5)$. Thus, for instance, if one builds an $SU(3) \times SU(2) \times U(1)$ string, the gauge coupling constants are unified with the Newton coupling constant as follows [11]:

$$
g_1^2k_1 = g_2^2k_2 = g_3^2k_3 = \frac{4\pi}{\alpha'}G_{\text{Newton}}, \tag{1}
$$

where $k_i$ are the Kac-Moody levels of the $U(1)$, $SU(2)$ and $SU(3)$ factors and $\alpha'$ is the inverse of the string tension squared. For the case of $SU(2)$ and $SU(3)$, the levels are integer numbers. For the case of $U(1)$, the level $k_1$ is just a normalization factor of the hypercharge, and it is a rational number in the specific models constructed up to now. Most of the string models constructed up to now have non-Abelian Kac-Moody levels $= 1$. Concerning $k_1$, many models have in fact $k_1 > 5/3$. The boundary condition of standard GUTs is obtained for $k_3 = k_2 = 3/5k_1$, but other possibilities are in general allowed in a non-unified $SU(3) \times SU(2) \times U(1)$ string model.
2. Duality symmetries

It has been realized in the last three years that in generic 4-D strings the spectrum and interactions are invariant under certain discrete infinite groups called duality symmetries [12]. Those are transformations in the space of the moduli $T_i$ which also induce $T_i$-dependent transformations in the rest of the massless chiral fields. In the case of the overall modulus $T \equiv R^2 + i\eta$, the discrete group has two types of generators: i) One that relates small and large $R^2$ and ii) a sort of discrete Peccei-Quinn symmetry under which $T \rightarrow T + i$. The first symmetry is remarkable because it tells us that there is a minimum physical scale below which the physics will be just equivalent. The prototype duality symmetry is target space modular invariance which is present in orbifold-like 4-dimensional strings. This symmetry acting on the overall modulus $T$ is given by the class of transformations

$$T \rightarrow \frac{aT - ib}{icT + d}; \quad a, b, c, d \in \mathbb{Z},$$

$$ad - bc = 1. \quad (2)$$

This corresponds to the discrete infinite group $SL(2, \mathbb{Z})$ generated by $T \rightarrow 1/T$ and $T \rightarrow T + i$. It has been shown that this is a symmetry of the string spectrum and interactions order by order in perturbation theory [13] on the string coupling constant ($S$). In what follows we are going to concentrate on this specific example of duality symmetry, which is the one relevant for one of the largest known classes of 4-D strings, that of Abelian $Z_N$ [14],[6] and $Z_N \times Z_M$ [15] orbifolds.

Since the string theory is invariant under this symmetry, the effective low-energy Lagrangian will also be invariant under it. In the case of an $N = 1$ supersymmetric model, the Lagrangian will be determined by three functions: the Kahler potential $K(\phi_\alpha, T_i, S)$, the superpotential $W$, and the gauge kinetic function $f$. Considering, for simplicity, the dependence on the overall modulus $T$, it is well known that the tree-level Kahler potential for orbifold-type 4-D strings has the form [16], [17], [18]:

$$K(\phi_\alpha, T, S) = -\log(S + S^*) - 3\log(T + T^*) + \sum_j (T + T^*)^{n_j} \phi_j^* \phi_j \quad (3)$$

to first order in the $\phi$'s. The $n_j$'s are integers ($n_j = -1, -2$ for untwisted and
twisted matter fields. Those values are increased or decreased in units depending on the number of twisted oscillators involved in the vertex [19], [20]). Since the $S, T$ fields have no superpotential (order by order in perturbation theory), one can see that the complete superpotential will be a holomorphic function of $\phi_\alpha$ and $T, W(\phi_\alpha, T)$. The complete Lagrangian depends on $K$ and $W$ only through the combination

$$G \equiv K + \log|W|^2. \quad (4)$$

One can check that $G$ is invariant under the modular transformation (2) provided the matter fields $\phi_j$ and the superpotential $W(\phi_j, T)$ transform like

$$\phi_j \rightarrow \phi_j (icT + d)^{n_j},$$

$$W(\phi_j, T) \rightarrow W(\phi_j, T) \delta(icT + d)^{-3}, \quad (5)$$

where $\delta$ is a phase which is irrelevant for our purposes. One says that the superpotential transforms as a modular form of weight $-3$ and the matter field $\phi_j$ transforms as a modular form of weight $n_j$.

Apart from its theoretical importance, duality symmetries are important because the effective low-energy Lagrangian has to respect this symmetry at the tree level and in loops. This invariance has been checked explicitly at the tree level [21] in orbifold compactifications and also in some one-loop computations [22], [23]. More importantly, there are reasons to believe that the symmetry is also respected by non perturbative effects, i.e. it may be broken spontaneously but not explicitly [24], [25]. This has important consequences since e.g., if some non-perturbative superpotential is generated, it will have to be a modular form of weight $-3$. Since the type of modular forms available is very limited, one can use this information to guess the form of the possible non-perturbative effects [25] (see below). One can also impose one-loop invariance under modular transformations, i.e. cancellations of duality anomalies, in order to constrain the possible massless multiplets present in a given model [20]. Thus, for example, it can be seen that one cannot possibly build a $Z_3$ or $Z_4$ model in which the only massless fields with non-vanishing quantum numbers under the standard model are those of the minimal SUSY model [20]. Such a model would necessarily have duality anomalies. Owing to these facts, duality symmetries are important ingredients to
trace the stringy origin of a given low-energy effective field theory. We will now show the importance of these symmetries in several different phenomenological contexts.

3. String unification of coupling constants

As we mentioned above, gauge coupling constants are unified in string theory even in the absence of a unification group such as $SU(5)$. However, the Planck scale boundary conditions depend on the values of the $k_i$’s. If we have a string with some sort of grand unification structure, one expects to have $k_3 = k_2 = 3/5k_1$, leading to the standard GUTs boundary conditions. Nevertheless those boundary conditions are also possible in direct $SU(3) \times SU(2) \times U(1)$ strings ($k_3 = k_2 = 1$ is very common in specific models whereas getting $k_1 = 5/3$ is typically non-trivial). The actual success of the standard GUTs boundary conditions makes it advisable to construct strings with that property.

Unlike GUTs, in which one computes the unification mass in terms of the crossing point of two running coupling constants, in strings we do know the string unification scale since it is related to the Planck mass in a known way. In the $\overline{MS}$ scheme one finds [26], [27] $M_{\text{string}} = 0.7 \times g_{\text{string}} \times 10^{18}$ GeV. Below this scale the couplings are renormalized in the usual way [28]. The one-loop running gauge coupling constant of a gauge group $G_a$ is of the following form:

$$\frac{1}{g_a^2(\mu)} = \frac{k_a}{g_{\text{string}}^2} + \frac{b_a}{16\pi^2} \log \frac{M_{\text{string}}^2}{\mu^2} + \Delta_a. \quad (6)$$

Here $b_a = -3C(G_a) + \sum_j T(R_j)$ is the $N = 1$ $\beta$-function coefficient and $\Delta_a$ represent possible threshold effects at the unification scale. Now, since we do know the unification scale, by running coupling constants down to low energies one can compute not only $\sin^2 \theta_W(M_W)$ but some other gauge coupling, e.g. $\alpha_3(M_W)$. If one assumes the particle content of the minimal SUSY-SM and the big desert hypothesis, one gets (neglecting threshold effects) [29], [30]:

$$\sin^2 \theta_W(M_W) = 0.218 \quad ; \quad \alpha_3(M_W) = 0.20. \quad (7)$$

Both numbers are several standard deviations away from the measured values ($\sin^2 \theta_W^{\text{exp}} = 0.233 \pm 0.0008$, $\alpha_3^{\text{exp}} = 0.115 \pm 0.007$). There are three main
possibilities to explain this disagreement (apart from forgetting about strings, which is obviously foolish): i) There is an intermediate scale $M_X \sim 10^{16}$ GeV at which a GUT symmetry like $SU(5)$ or $SO(10)$ is at work. This has the very same problems of the standard SUSY-GUTs that we discussed above. It also requires the use of higher Kac-Moody levels ($k_5 \geq 2$), which is both technically complicated and phenomenologically problematic [31], [32]. ii) There are additional massless chiral particles on top of the ones of the minimal SUSY-model. This was considered in ref. [33] and more recently, in the context of strings, in [30]. iii) We stick to the minimal low-energy content of the standard model, but the string threshold effects $\Delta_a$ are important and correct for the disagreement with experiment [29]. Notice that the string threshold gives potentially large corrections since it involves an infinite tower of massive states.

The second possibility is perfectly possible, although one will eventually have to explain how and why the required extra particles (and no others) remained light. From the point of view of pursuing minimal things first, the third possibility would be more attractive. In the field theory case Weinberg provided a long time ago a formula [34], [35] to compute the threshold effects in usual GUTs (in the $\overline{MS}$ scheme):}

\[ \Delta_{FT}^a = \frac{1}{2} T^a(V) + 4 T^a(F) \log\left(\frac{M_X}{M_F}\right) + \frac{1}{2} T^a(S) \log\left(\frac{M_X}{M_S}\right), \]

\[ (8) \]

where $V, F, S$ refer to vector, fermion, and scalar heavy particles, respectively, and $T^a$ are the quadratic Casimirs of those particles. As we can see, in order to compute the threshold effects we need to know two things: i) The complete spectrum of heavy particles and ii) their quantum numbers with respect to the unbroken gauge group. If we want to do something similar in string theory we will thus need a detailed knowledge of the supermassive spectrum (i.e. the complete one-loop partition function). Of course, this will be something very model dependent in general. Such a computation was worked out in the case of orbifold 4-D strings in ref [26]. These corrections are small, as is usually the case in field theory. However, in certain situations (when there are massive modes whose mass explicitly depends on the compactification radius $R^2$), the threshold string corrections may be very large. This comes about because the gauge kinetic
function \( f \) (whose real part equals \( 1/g^2 \)) gets dependent on the moduli \( T_i \) at one loop (it is just = \( k_a S \) at the tree level). This fact was already realized a long time ago, using scale-invariance arguments which lead one to expect (for large \( R^2 \)) one-loop-corrected \( f \)-functions [36]:

\[
f^a = k_a S + \epsilon_a T
\]  

(9)

, where \( \epsilon_a \) are small group-dependent coefficients. It is clear that for sufficiently large \( R^2 = ReT \) such loop corrections can be very important. More recently these corrections have been computed (for all \( T \) values) in the case of orbifold 4-D strings, yielding the result [22]

\[
f^a = k_a S + \frac{b'_a}{8\pi^2} \log(\eta^2(T)) ,
\]  

(10)

where \( \eta(T) \) is the Dedekind function and \( b'_a \) is given by [19], [37]

\[
b'_a = 3C(G_a) - \sum_j T(R_j)(3 + 2n_j) ,
\]  

(11)

where \( C(G_a) \) is the quadratic Casimir in the adjoint (e.g. = \( N \) in \( SU(N) \)) and \( n_j \) are the modular weights of the matter fields. The sum in (11) runs over all \textit{massless} fields with \( G_a \) quantum numbers. The large \( T \) expansion \( \eta(T) \simeq e^{-\pi T}/(1 - e^{-2\pi T} + ...) \) and in such a limit we recover eq. (9) with \( \epsilon_a \simeq -b'_a/(48\pi) \). The one-loop \( T \)-dependent piece in (10) may be understood as a threshold correction due to the string massive modes with a \( T \)-dependent mass. One thus finds for the string threshold corrections

\[
\Delta_a(T, \bar{T}) = \frac{1}{16\pi^2}b'_a \log(T_R |\eta(T)|^4) ,
\]  

(12)

where \( T_R = T + \bar{T} = 2R^2 \). The extra piece, \( \log T_R \), is in fact related to the massless fields. This extra dependence on \( ReT \) originates in one-loop graphs involving only massless fields and it is there [37],[19] because of the Kahler invariance of the \( N = 1 \) supergravity Lagrangian and also because of the tree-level explicit \( T \) dependence of the matter kinetic terms in eq. (3). It is worth noticing that \( \Delta_a \)
is explicitly invariant [25] under the modular transformations (2). Indeed, the Dedekind function is known to transform like $|\eta(T)|^4 \rightarrow |icT + d|^2|\eta(T)|^4$, and this is cancelled by the transformation of $T_R$. Thus the modular one-loop non-invariance of the massless sector is cancelled by the contribution coming from the heavy modes.

Anyway, it is clear that a term like that in eqn. (12) can give rise to substantial contributions and represent the leading threshold corrections for sufficiently large $T$ [29]. Moreover these corrections only require knowledge about the massless sector of the theory (i.e. the $b'_a$’s). This is in contrast with the field theory case of eq.(8), in which full knowledge of the massive sector of the theory is required. These corrections can thus be computed in terms of the (known) quantum numbers of the massless particles and their (unknown, but restricted [20]) modular weights $n_j$. (The origin of this fact is that massless and massive sectors of the theory are connected by duality and that the massive contribution needs to have a certain form in order to cancel the duality non-invariance of the massless sector one-loop contribution.) Now, taking different linear combinations of coupling constants one arrives at the results [29]

$$\sin^2 \theta_W(\mu) = \frac{k_2}{k_1 + k_2} - \frac{k_1}{k_1 + k_2} \frac{\alpha_e(\mu)}{4\pi} \left( A \log\left(\frac{M_{\text{string}}^2}{\mu^2}\right) - A' \log(T_R|\eta(T)|^4) \right),$$

where $A$ is given by $A \equiv \frac{k_2}{k_1}b_1 - b_2$ and $A'$ has the same expression after replacing $b_i \rightarrow b'_i$; $\alpha_e$ is the fine structure constant evaluated at a low-energy scale $\mu$ (e.g. $\mu = M_Z$). In an analogous way one can compute the low-energy value of the strong interactions fine-structure constant $\alpha_s$

$$\frac{1}{\alpha_s(\mu)} = \frac{k_3}{(k_1 + k_2)} \left( \frac{1}{\alpha_e(\mu)} - \frac{1}{4\pi} B \log\left(\frac{M_{\text{string}}^2}{\mu^2}\right) - \frac{1}{4\pi} B' \log(T_R|\eta(T)|^4) \right),$$

where $B \equiv b_1 + b_2 - \frac{(k_1 + k_2)}{k_3}b_3$ and $B'$ has the same expression after replacing $b_i \rightarrow b'_i$. Taking the standard values $k_3 = k_2 = 3/5k_1$ and setting $A' = B' = 0$, one recovers the results in eq. (7). On the other hand one can see if the measured values can be accommodated by choosing appropriate values for $A', B'$ (i.e. modular weights $n_j$) and $T_R = R^2$. One can eliminate the $T$-dependence
from eqs. (13) and (14). Defining $\gamma = B'/A'$ one finds that reasonable results can be obtained for $\sin^2 \theta_W$ and $\alpha_s$ provided [29]

$$\gamma = \frac{5}{3} \alpha_e \left( \frac{1/\alpha_s^0 - 1/\alpha_s^{exp} (\mu)}{(\sin^2 \theta_W^0 - \sin^2 \theta_W^{exp} (\mu))} \right),$$  \hspace{1cm} (15)

where $\sin^2 \theta_W^0$ and $\alpha_s^0$ are given by (7). Allowing for experimental errors one finds the numerical constraint $2.2 \leq \gamma \equiv B'/A' \leq 4.0$. In addition, in order for the threshold corrections to have the correct sign one needs $A' > 0, B' > 0$. If all these conditions are met, there is always a value of $R^2$ such that one gets good agreement with experiment (of course, one has to check that $R^2$ is not so large that the threshold corrections are too big and even dominate the tree-level coupling constant!). Equation (15) translates through eqn. (11) into a constraint on the modular weights $n_j$ of physical quark, lepton, and Higgs superfields which can be used to constrain specific 4-D string models (see ref. [20]). The reader can check that a possible satisfactory solution is obtained if, for instance, $R^2 \sim 16$ (in string units) and one has family-independent modular weights $n_Q = n_D = -1; n_U = -2; n_L = n_E = -3$ and $n_H + n_{\bar{H}} = -5$. Many other possibilities exist depending on the available modular weights in each orbifold model. This latter point is crucial and one can see that many possibilities can be ruled out in this way [20]. For example, if only the overall modulus $T$ is considered, the minimal scenario discussed above is not possible for any $Z_N$ orbifold (the situation changes if the three different $T_j$ planes and/or larger Kac-Moody levels are considered) [20].

The conclusion of the above analysis is that string threshold effects can make the predictions for $\sin^2 \theta_W$ and $\alpha_s$ fit to experimental results for moderately large values of $R^2$, provided an orbifold with the minimal particle content and appropriate modular weights exist (of course, extra particles without standard model quantum numbers are always allowed). The other obvious alternative [33], [30], is having a particular set of extra massless matter fields. But even in this case the threshold corrections could be important and it could well be that both things were present in the actual case chosen by nature [20]. Notice equations (13), (14) are valid for arbitrary low-energy particle content, and it is likely that both
a modification of the massless particle content and the contribution of large $T$-dependent threshold corrections were required to fit the data without introducing new intermediate mass scales in the theory [20].

4. Duality-invariant supersymmetry breaking

Another relevant problem in which duality seems to play an important role is that of supersymmetry breaking in string models. Specific 4-D string models very often come with extra “hidden” gauge interactions which couple to the observed particles only gravitationally. It was suggested [38], [39] that if this hidden sector is strongly interacting, gaugino condensates $<\lambda\lambda>$ could form and give rise to spontaneous supersymmetry breaking. An intuitive way to describe this low-energy phenomenon is through a non-perturbative (recall $ReS = 1/g^2$) superpotential of the form $W(S) = e^{3S/2b_H}$, where $b_H$ is the $\beta$-function coefficient of the hidden gauge group $H$. This mechanism seems attractive, since the exponential factor could in principle generate the required small supersymmetry breakdown. However, it was soon realized that this scenario as discussed in ref.[38],[39] had several problems. 1) There is no stable (non-trivial) minimum for the dilaton field $ReS$. 2) It is not clear what dynamics could fix the size of the compactified variety $ReT$ since $W(S)$ does not depend on $T$. Furthermore, from the discussion in previous sections, it is also clear that a potential like $W(S) = e^{3S/2b_H}$ cannot possibly be an effective superpotential from super-string theory. Indeed, such a superpotential is invariant under duality, whereas we learned that any superpotential should transform as a weight $-3$ modular form!

It has been recently realized [25], [40] that a gaugino condensation mechanism for supersymmetry breaking can be naturally formulated in a manner consistent with duality invariance. Furthermore, this provides a solution to the second problem above, i.e. the dynamical determination of the compactification scale $ReT$. Let us consider for simplicity the case of a condensing gauge group without chiral matter fields. In this case one has $b'_H = -b_H$ (see eq. (11)) and $f_H = S - \frac{b_H}{8\pi^2}log(\eta^2(T))$. Now, the relevant effective non-perturbative superpotential
can be estimated to be [25]

$$W(S, T) = e^{3f_H(16\pi^2)/2b_H} = \frac{e^{3S(16\pi^2)/2b_H}}{\eta^6(T)}.$$  \hfill (16)

This superpotential has now two interesting properties. First, it explicitly depends on the $T$ field. The existence of such a $T$ dependence of the gaugino condensation superpotential was already remarked in refs. [36], [41] but in that case the large $T$ limit formula for $f$, eq.(9), was used and the corresponding scalar potential was unstable [41]. The second interesting property is that now $W(S, T)$ transforms as a modular form of weight $-3$ (recall that $\eta^2(T)$ has modular weight one), as required by duality invariance. Owing to the presence of the $\eta(T)$ function the scalar potential is now well behaved. If one only keeps the fields $S$ and $T$ and assumes a general non-perturbative superpotential of the form $W(S, T) = \Omega(S)/\eta^6(T)$, the obtained scalar potential is [25]

$$V = \frac{1}{S_RW^2R^3[\eta(T)]^4}\{[S_R^2\Omega_S - \Omega_S]^2 + 3\Omega_S^2(\frac{T_R^2}{4\pi^2})|\hat{G}_2|^2 - 1\}.$$  \hfill (17)

where $S_R = 2ReS, T_R = 2ReT$ and $\Omega_S = \partial\Omega/\partial S$. Here the function $\hat{G}_2$ is the weight-2 Eisenstein function which admits the expansion $\hat{G}_2 \simeq \frac{2\pi}{R} + \frac{\pi^2}{3} - 8\pi^2e^{-2\pi T} - ...$. From the form of the potential one can already draw some general conclusions. In the decompactification limit $T_R \to \infty$ (and its dual $T_R \to 0$), the potential diverges ($V \to \infty$) since the product $T_R^3[\eta(T)]^4$ vanishes exponentially at both points, and $\hat{G}_2 \to \pi^2/3$ as $T_R \to \infty$. Thus, if gaugino condensation takes place, the theory is forced to be compactified. The origin of the divergence for $T_R \to \infty$ has a clear physical meaning [42]: for large radius $R$ an infinite number of Kaluza-Klein excitations (with masses $\sim n^2/R^2$, $n \in \mathbb{Z}$) become massless and contribute to the effective coupling constants, which in turn diverge. Because of duality, the same arguments apply for $T_R \to 0$ replacing Kaluza-Klein excitations by winding modes. Another interesting property of the potential is its periodicity in $ImT$ (direct consequence of $T \to T + i$ invariance). The minimum of such a potential is at $ImT = \text{integer}$, leading to purely real soft SUSY-breaking terms in the effective theory. This might be interesting in trying to understand the CP-violating properties [43] of this effective theory.
In the above scheme, supersymmetry is spontaneously broken with $m_{3/2} = |\Omega|/(S_{R}^{1/2}T_{R}^{3/2}|\eta|^6)$. The size of supersymmetry breaking is essentially determined by the value of $|\Omega(S)|$. It is not yet clear what dynamics (such as multiple gaugino condensation or other mechanism) could lead to the required hierarchically small SUSY-breaking and it is not clear either what could make the cosmological constant vanish. Concerning the problem of determining the vev of the dilaton field $S$, the possibility of having a new kind of duality symmetry for the $S$ field is intriguing [44]. This new $S$ – duality will relate strong to weak coupling ($S \rightarrow 1/S$) and contain a new discrete Peccei-Quinn-type symmetry ($S \rightarrow S+i$). If such a symmetry (or some generalization) was present in the effective low-energy theory one would expect background values $S_{R}^{-1} \sim g^2 \sim 1$, not far away from the self-dual point. This is not very different from the estimations of the values of the gauge coupling constants at the unification scale (notice that $g_{GUT}$ is of order $\sim 0.7$ if one extrapolates the measured low-energy values). In fact this would provide an explanation for why indeed the measured coupling constants are so close to one and not to any other, e.g. much smaller, value. It is anyway not unreasonable to think that, independently of the dynamics which eventually fix the value of $<S>$, the $T$-duality of the effective action will force the theory to be compactified. Notice also that once the $T$ field gets a vev, duality is generically spontaneously broken [25].

In the above discussion I have considered the simplest possibility of a matterless condensing gauge group and focused only on the generic fields $S,T$. The more general case with matter fields [45] and several orbifold moduli $T_j$ can also be considered arriving at similar qualitative properties. A further complication arises in some cases if the dilaton field $ReS$ gets mixed at one loop with the $T_j$ fields in the Kahler potential [37]. Except for some limiting cases (the $Z_3$ and $Z_7$ orbifolds) the above simplified discussion gives the correct qualitative answer.

More generally [42] one can simply consider a generic non-perturbative superpotential of the form $W = \Omega(S)H(T)/\eta(T)^6$ in which $H$ is some modular invariant function (up to constant phases). This type of non-perturbative superpotentials could arise in more complicated situations (not necessarily linked to the gaugino condensation mechanism) involving some unknown non-perturbative dynamics. If $H$ depends only on $T$, one can see that either $|H| = constant$ or else
there will necessarily be singularities for finite values of the field $T$. It is not clear what kind of physical situations could lead to these singularities, but one can find particular expressions for $H(T)$ (polynomials of the known absolutely modular invariant function $j(T)$) which give rise to a vanishing cosmological constant [42]. On the other hand, if there are matter fields coupling to the condensing gauge group in the gaugino condensation mechanism, $H$ may explicitly depend on those matter fields.

5. Soft SUSY-breaking terms and duality invariance

It is well known that when supersymmetry is broken in a “hidden” sector of the theory, the physical world feels the breaking of SUSY through the existence of explicit SUSY-breaking soft terms (gaugino masses $M_\alpha$, scalar masses $m^2$, trilinear scalar couplings $A_j$, etc. [46]). In general the form of those soft terms is expected to depend on the way in which SUSY-breaking takes place and on the particular 4-D string considered. If the idea of low-energy supersymmetry is correct, the soft terms should eventually be measured at accelerators. It would thus be very important to find theoretical constraints on these terms coming from generic string properties like duality in order to check the theory. Even if one were unable to obtain model-independent predictions for those quantities, knowing the relation between given classes of 4-D strings and the resulting soft terms would perhaps allow us to rule out (or rule in !) large classes of models.

It turns out that in some simple situations something can be said about the soft SUSY-breaking terms in a relatively model-independent way, at least within the class of orbifold-like 4-D strings [20]. Consider, to start with, the case of gaugino masses and take again the simplified model with just $S, T$ as moduli fields. Let us assume that supersymmetry breaking takes place in this singlet $S, T$ sector, as in the scenarios discussed in the previous section. The gaugino masses $M_\alpha$ are then given by

$$M_\alpha = f_S G_{S,S}^{-1} h_S + f_T G_{T,T}^{-1} h_T ,$$

where $h_S = G_S e^{G/2}$ is the $S$ auxiliary field and $h_T$ is the corresponding $T$ auxiliary field. Using eq. (10), one can easily obtain an expression for each
gaugino mass of the form

\[ M_\alpha = k_\alpha M_0(S, T) + b'_\alpha M'(S, T), \quad (19) \]

where \( M_0, M' \) are gauge-group-independent quantities which depend on the details of supersymmetry breaking. The gauge-group dependence comes only through \( k_\alpha \) and \( b'_\alpha \). Consider now the phenomenologically interesting case with gauge group \( SU(3) \times SU(2) \times U(1) \). One can easily write down combinations of gaugino masses in which the dependence on \( M_0, M' \) drops out. A particularly interesting one is the following

\[
\frac{(M_1 + M_2 - \frac{k_1 k_4}{k_2} M_3)}{\left(\frac{k_2}{k_1} M_1 - M_2\right)} = \frac{B'}{A'} \equiv \gamma, \quad (20)
\]

where \( A', B', \) and \( \gamma \) were defined in section 3. (Notice that here \( \gamma \) is given by a ratio of combinations of \( b'\)'s; we are not necessarily imposing the constraint in eq. (15) which is required in order to explain the experimental results for \( \sin^2 \theta_W \) and \( \alpha_s \) in terms of string threshold corrections. We are not assuming a large \( ReT \) value either.) From eq. (20) one can obtain the general constraint amongst gaugino masses (assuming the standard \( k_\alpha \)'s) \[20\]

\[
M_1 \left( \frac{3}{5} \gamma - 1 \right) - M_2 (\gamma + 1) + \frac{8}{3} M_3 = 0. \quad (21)
\]

This equation applies at the unification scale and is authomatically satisfied by the standard “minimal” GUT constraints \( M_3 = M_2 = 3/5 M_1 \) but allows for more general possibilities depending on the model dependent parameter \( \gamma \). Thus if one wants to understand the measured values of \( \sin^2 \theta_W \) and \( \alpha_s \) in terms of string threshold corrections one must have, as we discussed in section 3, \( 2.2 \leq \gamma \leq 4.0 \) (one has \( \gamma = 25/7 \) if one further imposes joining of coupling constants at the minimal model value \( M_X \sim 10^{16} \text{GeV} \)). In models in which \( b'_\alpha = - b_\alpha \) (as in the case of \( Z_2 \times Z_2 \) orbifolds) one has \( \gamma = B'/A' = B/A \) and hence one can compute \( \gamma \) by simply knowing the low energy field content. Thus, for example, in the models of ref. [33] and [30] in which one adds particular extra fields in order to obtain adequate coupling unification, one can explicitely compute [20] the values of \( \gamma \).
A first lesson to be learned from the above discussion is that, in string models, there can be non-universal contributions to soft terms and, in particular, to gaugino masses. I believe that eq. (21) provides us with a useful parametrization for departures from gaugino mass universality, since a measurement of $\gamma$ gives us useful information about the underlying string models. Of course, we have considered above a case with only one relevant modulus $T$ and neglected the effect of matter fields in supersymmetry breaking. More general situations in which the above arguments still hold are considered in ref [20].

One can also obtain some interesting results for the soft SUSY-breaking scalar masses $m^2_j$. Indeed, using the Kahler potential of eq. (3) and assuming some generic non-perturbative superpotential, one can compute [42] the corresponding scalar potential $V_\phi$ in the presence of matter fields $\phi_j$. Expanding $V_\phi$ to quadratic order in the matter fields one gets

$$V_\phi = V_0(S, T) + \sum_j V_j \phi_j \phi_j^* + \ldots,$$

(22)

where $V_0$ has a generic form as in eq. (17) and the sum runs over the matter fields. To first order on those fields, all the non-universal dependence of their masses comes through the modular weights $n_j$ in eq.(3). After rescaling the kinetic terms, one finds for the soft masses [20]

$$m^2_{\phi_j} = m_{3/2}^2 + V_0 + n_j m_0^2(S, T),$$

(23)

where $m_{3/2}$ is the gravitino mass, $V_0$ is given by eq. (17) (and is essentially the cosmological constant), and $m_0^2$ is some model dependent mass parameter. As we did in the case of gaugino masses one can take linear combinations and ratios to isolate the dependence on the modular weights. Of course, the situation becomes more complex if one considers the dependence on other $T_j$ moduli and if the matter fields themselves get involved in the process of symmetry breaking. Anyway, the moral of eqs. (23) and (21) is clear: in generic string models SUSY-breaking soft terms are not in general universal but vary from particle to particle. Since a certain degree of universality is required from phenomenology (e.g. suppression of flavour-changing neutral currents), this may also lead to the elimination of large classes of string models.
6. Final remarks

Four-dimensional strings constitute the most promising candidates for unified theories of all the interactions. Their low-energy limits are described by effective field theory Lagrangians, which are invariant under certain symmetries including gauge and $N = 1$ supergravity. Of special interest are the duality-type symmetries, which are intrinsically stringy in character and which seem to be present in all 4-D strings studied up to know. They are important because they are supposed to be respected even by non-perturbative interactions hence allowing us to shed some light on the non-perturbative dynamics which are supposed to break supersymmetry and fix the values of the vev’s of the moduli and dilaton in the theory. We have shown above how in a large class of models (Abelian $(0, 2)$ orbifolds) the duality symmetry plays an important role in understanding some phenomenologically interesting questions such as gauge-coupling unification, supersymmetry breaking, and the nature of soft SUSY-breaking terms. Some specific string models and/or particle content assumptions may be ruled out by using these duality arguments.

We think that it is important to keep advancing in determining the form of the effective low-energy Lagrangian from 4-D strings and, in particular, the generalized duality symmetries present in non-orbifold models. With this knowledge one should, in principle, be able to constrain in a substantial way the expected form of the effective Lagrangian. If the idea of low-energy supersymmetry is correct, future accelerators will detect and measure in detail the masses of the sparticles and hence important experimental information on the SUSY-breaking terms will be available. In the context of 4-D strings, these soft terms encode important information on the structure of the underlying theory. It will then be an important challenge to learn to read this important information from these experimental data and to draw the corresponding conclusions concerning the underlying string theory.

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