Avoidance of instability of a superluminal Gaussian light pulse via control of nonlinear coherence Kerr effect in a gain-assisted medium

Bakht Amin Bacha
Department of Physics, Hazara University, Pakistan

Fazal Ghafoor
Department of Physics, COMSATS Institute of Information Technology, Islamabad, Pakistan

We investigate nonlinear Kerr-induced coherence effect on a superluminal probing light pulse in a gain-assisted N-type 4-level atomic system via an intense monochromatic laser field. The dispersion exhibits a novel, interesting and useful two-paired double gain lines processes. The system displays lossless characteristics similar to [L. J. Wang, A. Kuzmich, A. Dogariu, Nature 406, 277 (2000)] but with advantages of multiple controllable anomalous regions, significantly enhanced superluminal behavior and relaxed temperature, states of matter regardless of its isotropic or anisotropic conditions. Unlike the instability in [A. M. Akulshin, S. Barreiro, and A. Lezama, Phys. Rev. Lett. 82, 4277 (1999)], the present system also overcomes the instable-limit by the Kerr-induced coherence effect in the system. Indeed, the coherence enhances the group velocity remarkably by at least ~30ms more than of an instable Kerr-free system with almost negligible distortion in the retrieved pulse.

The significant development of theoretical and experimental techniques for the control of light pulse propagation through resonance optical media had taken place in the last few decades. The control over the properties of spontaneous emission [1–4], stimulated absorption [5], dispersion [6] and group index of media [7] had led to the observation of some fascinating phenomena such as coherent population trapping, lasing without inversion [8, 9]. In most of these types studies, coherent fields were used to control the optical properties of the media [10]. In fact, their practical and theoretical aspects were also investigated for slow and even for stopped (stored) light pulse [11–15].

The superluminal light pulse propagation had also been remained under extensive studies among researchers. One of these studies was reported by Bigelow et al. via beating of a pump and probe field to generate coherent population pulsation in the medium [15]. Similarly, a negative group velocity was studied in a medium with a gain doublet [16], and in a transparency medium with inverted population [17]. The list of such interesting studies is quite long, however some of these can be found in Refs. [18–20]. Be specific, we concentrate on two pioneer and sophisticated experiments, and their complications. In one [27–30], Wang et al. observed superluminal behavior of a probing light pulse using the region of lossless anomalous dispersion between two closely spaced gain lines. However, due to limited capability of the experiment, the light velocity could not be increased more than c/310 (where c is the velocity of light in vacuum). In the other remarkable experiment, Akulshin and his co-workers defined a stability limit for the superluminal probing pulse constrained by the strength of the bi-chromatic laser field [31]. Recently, the incompatibility of a pulse width with a steep anomalous region created a pulse distortion and the other related problems [see for example [32–36]]. Consequently, the applied aspects of these studies were limited. For example, precession temporal cloaking [33], precession image measurement [38] and may be many other, were suffered from a limited group velocity of the pulse when media of negative group index were used [39]. Therefore, enhancing superluminality while avoiding the earlier studied complication, in general, is an important task.

The Kerr effect is a nonlinear and noise free phenomenon used by Hai and co-workers for nonlinear index of a three-level atomic medium inside an optical ring cavity. The greatly enhanced Kerr effect near atomic resonance condition led them to measure group index up to 7.010−6cm/w [40]. The enhanced nonlinear kerr effect was also investigated both theoretically and experimentally in different systems in different contexts [40–42]. The various aspects of the Kerr effect were used for quantum phase gate, optical solitons [43–46], quantum logic gate, quantum non-demolition, quantum teleportation, and nonlinear light control [47–49]. Dey and Agarwal used the coherence generated by the Kerr effect to slowdown light pulse through an Electromagnetically Induced Transparency (EIT) medium [50].

In this letter, we investigate Kerr-induced coherence effect on a superluminal weak probing pulse in a gain-assisted medium via a strong monochromatic laser field. Intriguingly, the coherence created by the Kerr effect in the proposed atomic system is significant for the propagating pulse. Consequently, the generated coherence overcomes the instable limit of our system unlike the system of the Ref. [31], where the limit was created by the incoherence effect of the strengthened intensity (power broadening) of the bi-chromatic pump field. Nevertheless, the group index of our system gets a direct proportionality with the square of the Kerr-field intensity caused by the Kerr-induced coherence effect. Consequently, the superluminality is enhanced even with a less losses (undistorted retrieved pulse) like those of Ref. [27].
but with the advantages of multiple controllable anomalous regions, relaxed temperature, states of matter, and their isotropy or anisotropy conditions. Obviously, undistorted retrieved pulse is normally necessary for practical applications and the present system exhibits the same characteristics when analyzed analytically.

Three-level atoms of a gaseous medium in a vapor cell were considered by Wang et al. in their experimental set-up. Two strong continuous-wave Raman pump light beams $E_1$ and $E_2$, of frequencies $\nu_1$ and $\nu_2$ (with difference $2\Delta = \nu_2 - \nu_1$), were allowed to interact with the Raman atoms set at temperature 30°C. The upper excited energy level was considered as $6\Sigma_{1/2} | F = 4, m = 4 \rangle | a \rangle$ while $6\Sigma_{1/2} | F = 4, m = -4 \rangle | d \rangle$ and $6\Sigma_{1/2} | F = 4, m = -2 \rangle | c \rangle$ were assumed as the two ground states of the atom. Also, a probe field is coupled with the $|c\rangle$ and $|a\rangle$. Both the fields are detuned from the transition frequency $\nu_{ad}$. $|a\rangle$ to $|d\rangle$, by a large average detuning $\Delta_0$. The optical susceptibility of the probe field was calculated as:

$$\chi_{\omega} = \frac{M_1}{\nu_p - \nu_1 + i\gamma} + \frac{M_2}{\nu_p - \nu_2 + i\gamma},$$  

where $M_{1,2} = N|\sigma_{ac}|^2|\Omega_{1,2}|^2/4\pi\epsilon_0\hbar^2 \Delta_0^3$. Using experimental data they obtained the group index $-310 \pm 5$, corresponding to the advanced time $-62 ns$.

We consider an N-type four-level atomic system driven by two pump fields and a probe field. Meanwhile, we couple a monochromatic laser field, which we call the Kerr field, due to its induced coherence effect on the dynamics of the system [see Fig. 1]. The two lower levels $|d\rangle$ and $|c\rangle$ are coupled with the upper level $|a\rangle$ by two coherent pump fields and a weak probed field of Rabi frequencies $\Omega_1$, $\Omega_2$, and $\Omega_p$, respectively. Furthermore, the coupling of the intense Kerr field is with the levels $|c\rangle$ and $|b\rangle$ having the Rabi frequency $\Omega_k$. Now to present the model and equations of motion, we proceed with the following interaction picture Hamiltonian in the dipole and rotating wave approximations:

$$H_1 = -\frac{\hbar}{2}(\Omega_1 e^{-i\Delta_1 t} + \Omega_2 e^{-i\Delta_2 t}) |a\rangle \langle d| + \frac{\hbar}{2} \Omega_k e^{-i\Delta_k t} |b\rangle \langle c| - \frac{\hbar}{2} \Omega_k e^{-i\delta t} |a\rangle \langle c| + H.c(2)$$

where $\nu_1 = \nu_{ad} \pm \Delta_1$, $\nu_2 = \nu_{ad} \pm \Delta_2$, $\nu_k = \nu_{bc} \pm \Delta_k$ and $\nu_p = \nu_{ac} \pm \delta$, while $\nu_1$, $\nu_2$, $\nu_p$, $\nu_k$, are the frequencies of the two pump fields, the probe field and the Kerr field, respectively. The detuning parameters appearing in the system are $\Delta_1 = \Delta_0 - \Delta$, and $\Delta_2 = \Delta_0 + \Delta$, where $\Delta = (\nu_2 - \nu_1)/2$ is the effective detuning, $\Delta_0 = (\Delta_1 + \Delta_2)/2$ is the average detuning. We use the following general form of density matrix equation:

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar} [\rho, H_1] + \Delta \rho$$  

for the equations of motion, where $\Delta \rho$ is the damping part of our system. The equations of motion for slowing varying amplitudes $\tilde{\rho}$ for different matrix elements are then evaluated using its transformation relation with the fast varying amplitudes We evaluated the steady state solution for $\tilde{\rho}_{ac}$ and obtained $\tilde{\rho}_{ac} = -i \alpha_{ac}(\beta_1 + \beta_2)$. The parameters $\beta_1$ and $\beta_2$ are listed in the appendix-B. The susceptibility, which is a response function to the applied fields, is obtained as

$$\chi = \frac{2N|\sigma_{ac}|^2\rho_{ac}}{\epsilon_0\hbar\Omega_p} - i3N\gamma\lambda^3/32\pi^3(\beta_1 + \beta_2),$$

where $N$, is the atomic number density, $\lambda = 2\pi c/\omega_{ac}$, is the wavelength, and $A = 4\sigma_{ac}^2\omega_{ac}^3/\epsilon_0\hbar^3 = 4\gamma$, is the Einstein coefficient. Consistently, our main results is reducible to the theoretical result of Wang et al for the dispersion and absorption of the probe light pulse. In this connection, if we assume $\gamma_{ad} = \gamma_{bd} = 0$, $\Omega_k = 0$, $\Delta_k = 0$, $\gamma_{ac} = \gamma_{cd} = \gamma$, and $\Delta_0 \approx \delta$ while $\Delta/\Delta_0, \gamma/\Delta_0 \ll 1$, we obtained $\chi = M_1/(\delta - \Delta_1 + \gamma) + M_2/(\delta - \Delta_2 + \gamma)$, where $M_{1,2} = N|\sigma_{ac}|^2|\Omega_{1,2}|^2/4\pi\epsilon_0\hbar^2 \Delta_0^3$. $|\sigma_{ac}|$ appears for the corresponding dipole moment. Further, if $\nu_{ac} \approx \nu_{ad}$, and $\Delta_{1,2} = \nu_{1,2} - \nu_{ad}$, while $\delta = \nu_p - \nu_{ac}$, the results obtained is similar to Eq. (1) as calculated by Wang et al.

Generally, the Kerr effect means a change in the refractive index of a material in response to an applied electric field. It is the electric field intensity dependent nonlinear effect on the response function of the medium. The corresponding refractive index is proportional to the square or higher order power of the field. Therefore, the change in the refractive index can be measured from $\Delta n_e = \lambda K E_0^2/\omega_{0}^4$, where $\lambda$ is the wavelength and $E_0$ is the amplitude of the electric field, while $K$ is the Kerr coefficient. Specifically, the Kerr effect in a medium appears due to intense laser light. If un-important higher order terms are neglected, then the Kerr effect is given by $n_e = n_0 + n_2 I$, where $n_e = \sqrt{T + \chi}$ and $T = n_1 + n_2 I$. One of our main task is to explore a mechanism in the system for an enhancement of superluminality. The intense Kerr field may achieve this task under favorable conditions for superluminal light. These favorable conditions includes: both low (high) temperatures regimes for homogenous (nonhomogeneous) gaseous, liquid, and solid state media. Nevertheless, unfavorable conditions are very large optical bandwidth and the saturation limit of the Kerr effect.

Now, a formalism for the contribution from the Kerr effect is developed by the expansion of $\chi$ in the power
The group indices for the three different cases can now be dealt with the third order susceptibility in response to the Kerr field, which is known as cross Kerr nonlinearity. The expression for \( N_g = n_r(\omega) + \omega \frac{\partial n_r(\omega)}{\partial \omega} \), while the group velocity dispersion \( D_v \) is written as \( D_v = \frac{\partial^2}{\partial \nu^2}(e^{-1}) = \frac{1}{c} \frac{\partial N_g}{\partial \omega} \). The complex wave-number \( k(\omega) \) can be expanded via Taylor series in terms of group index as

\[
k(\omega) = \frac{N_g(\omega)}{c} + \frac{1}{2}(\omega - \omega_0)^2 \frac{1}{c} \frac{\partial N_g}{\partial \omega}|_{\omega=\omega_0} + \ldots \quad (11)
\]

Incidently, the transit time of a pulse through a material medium is defined as \( T = N_g L/c = k_1 L \), while the spread of transit time takes the form of \( \Delta T = (L \partial N_g/c \partial \omega) \Delta \omega \), where \( \Delta \omega \) appears for the frequency bandwidth. No significant distortion of the pulse is there if it satisfies the condition \( \Delta T < \tau_0 \), where \( \tau_0 \) is the characteristic pulse width. However, if the condition \( \partial N_g / \partial \omega = 0 \) is satisfied, then there is a negligible spread in the transit-time distortion. Successful experiments with slow and fast light are due to negligible distortion in the output pulse when \( \partial N_g / \partial \omega = 0 \) while having a larger group index \( N_g \). Obviously, if a system satisfies the condition for higher order terms of \( k(\omega) \), i.e. \( k_2, k_3, k_4 \ldots = 0 \), then there is no distortion in the system at all. The pulse shape remains unchanged, while the advance time \( \tau_a = k_0 \left( \frac{L}{N_g} - \frac{1}{c} \right) \) and the phase shift are changed. The refractive index, \( n_r \), of the medium, \( \partial n_r / \partial \omega \) represents dispersion, while \( \partial N_g / \partial \omega \) gives the distortion of the output signal. Furthermore, the real part of the higher order terms \( k_j(\omega) \) [where \( j = 1, 2, 3, \ldots \)] defines the dispersion and phase distortion, while the imaginary part of \( k_j(\omega) \) represents the gain [absorption] and the amplitude distortion. Obviously, the information about the output pulse \( S_{out}(\omega) \) can be extracted from the input pulse \( S_{in}(\omega) \) via the transfer function \( H(\omega) \) of the dispersive medium, and is formulated as \( S_{out}(\omega) = H(\omega) S_{in}(\omega) \), where \( H(\omega) = e^{-ik(\omega)L} \). We choose the Gaussian input pulse of the form \( S_{in}(t) = \exp[-t^2/T_0^2] \exp[i(\omega_0 + \xi)t] \), where \( \xi \) is the upshifted frequency from empty cavity. The Fourier transforms of this function is then given by \( S_{in}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} S_{in}(t) e^{i\omega t} dt \). The above integral is worked out as \( S_{in}(\omega) = \frac{\tau_0}{\sqrt{2\pi}} \exp\left[ -\left(\omega - \omega_0 + \xi\right)^2 \tau_0^2/4 \right] \).

By virtue of the convolution theorem, the output \( S_{out}(t) \) can be written as \( S_{out}(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} S_{in}(\omega) H(\omega) e^{i\omega t} d\omega \). Integrating the function analytically, we get

\[
S_{out}(t) = \frac{\tau_0}{\sqrt{2\pi}} \frac{\sqrt{c^2 t_0^2 + 4c^2 t^2}}{2 \left(\pi c^2 \right)^{1/2} \sqrt{2 \pi}} \exp\left[ -\left(\frac{c^2 t_0^2}{4c^2} + i(t - \frac{n_0L}{c})\omega_0\right) \right] \times \exp[\frac{n_2^2}{4} + \frac{2c^2 n_1^2}{c}] 
\]

where \( n_1 = \frac{2c^2 n_1 + c^2 t_0^2}{c} \), \( n_2 = \frac{c^2 n_2 - 2c^2 n_0^2 + c^2 t_0^2}{c} \), \( n_1 = \partial N_g / \partial \omega \) and \( n_2 = \partial^2 N_g / \partial \omega^2 \). Supposing \( n_2 = 0 \). Choosing \( \nu_{ac} = 1000\gamma \), the gain doublet occurs at \( \delta = 30\gamma \).
of the probe field between the gain doublet is 

\[ n = \frac{\nu_c}{\omega} \pm \frac{1}{\omega} \]

and \[ \omega = 2\pi \nu_0 \] is the probe angular frequency. Furthermore, if the central frequency of the probe field between the gain doublet is \( \nu_c \), then \[ \nu_c = (1000 \pm 30) \gamma \]. The central angular frequency is \( \omega_0 = 2\pi \nu_c \). Therefore, our estimation leads to \( n_0 = 6.61247 \times 10^5 \), and \( n_{0,k} = -2.43 \times 10^7 \) and their corresponding dispersions appear as \( n_1 = \pm 3.34 \times 10^8 \) and \( n_2 = \pm 2.99 \times 10^7 \), while \( n_{1,k} = \pm 3.04 \times 10^8 \) and \( n_{2,k} = \pm 8.43 \times 10^7 \). The pulse width is given by \( \gamma_0 = 3.5 \mu s \), which is less than \( T_d \). Consequently, it corresponds to a negligible distortion meaning lossless characteristics at the anomalous regions in our atomic system.

Prior to interpreting our results we define necessary terminology. The field coupled with energy levels \( |c\rangle \) and \( |b\rangle \) is called a control field when there is no Kerr effect in the system. In this case we do not include the additional nonlinear term contributed by the Kerr effect. The distinction is made for the strengthened field. The N-type four-level atomic system is reduced for the strengthened field. The stronger is the coupling laser is intensive. In this system, the scientifically reasonable results are associated with analysis of the additional nonlinear term in the dynamics of the system. This is done deliberately for a sake of comparison, as there is always Kerr effect in the system if the coupling laser is intensive. In this system, the scientifically reasonable results are associated with analysis of the additional nonlinear term contributed by the Kerr effect for the strengthened field. The distinction is made for the usefulness of the Kerr effect in the enhancement of superluminal group velocity, a reasonable scientific approach. The N-type four-level atomic system is reduced to the famous scheme of Wang et al. when the Kerr field is set to zero. In this approximation, the results of our system agree with the said reference [see Fig. 2]. Next, in this approximated scheme if we further exclude one of two coherently driven pump fields, then the atomic system reduces to a single Raman gain scheme, where only one gain peak exists. Furthermore, in this approximated setup if we include the Kerr field with the fourth level, then this forms the N-type scheme with only one coherent pump field and exhibits a similar behavior, like the one discussed in Ref. 50.

The cross Kerr nonlinearity is deliberately introduced in the medium with a goal to induce maximum coherence via a large Kerr effect. The susceptibility displays two pairs of double-peak gain lines, a novel and interesting behavior of the system. The physics here is different than earlier approaches 22,30. The coupling of an additional Kerr field splits the ground energy level to a doublet. Subsequently, the photons added to the probe field by the two pump fields follow two different paths provided by the two dressed states. In this way one pair of the gain appears for each coherently driven pump field. The additional Kerr field modifies the gain feature, but the created anomalous regions remain almost lossless similar to the Ref. 27. Therefore, the induced Kerr effect which drastically increases the group index is still practical. The anomalous regions at the three position of dispersion is controllable with strength of the Kerr field. Intriguingly, these controllable regions may resolve the issues of the distortion caused by incompatibility of the width of probing pulse and the anomalous regions for the transparency. In Fig. 2 and Fig. 3 we show the plots of dispersion and gain spectrum under similar parameters for the Kerr free system (\( \chi \)) as well as for a Kerr effect (\( \chi^k \)) for condition of two photon resonances: \( \delta = \Delta_1 \) or \( \delta = \Delta_2 \). The slopes of the dispersions between the gain regions are anomalous for both \( \chi \) and \( \chi^k \) but steeper for the Kerr one. Correspondingly, negative group index for the Kerr effect is larger than that of the Kerr free case. This then corresponds to an enhancement in the group velocity of the superluminal light pulse for the Kerr effect, meaning advancement in time under the same set of parameters [Fig. 2(c) and Fig. 3(c)] respectively. Next, we display the behavior of the dispersion, gain, and their corresponding group index, and the advanced time with the probe field detuning when the Kerr field vanishes. Obviously, there are two
gain peaks in the spectrum at the two photon resonance points: $\delta = \Delta_1$ or $\delta = \Delta_2$. This behavior is in accord with the scheme of Wang et al. Consequently, the slope of the dispersion becomes steeply anomalous in these resonance points. The group index and the negative time delay are $-2.5 \times 10^7$ and $-2.5\,\text{ms}$, respectively [see Fig. 4(a-d) dash lines]. These time delays are always greater than the negative delay time of the Wang et al. experiment for both the Kerr effect and the Kerr free systems. Further, the group indices in between the gain lines are also negative but have enhanced profile for the Kerr field [see the inset Fig. 4(c-d)]. The group index without the Kerr field at the point $\delta = 40\gamma$ is $-500000$, and in the presence of Kerr effect is $-550000$. The time delays are then $-50\mu s$ and $-55\mu s$ for Kerr effect and without the Kerr effect, respectively. The advance time of the pulse is increased by $5\mu s$ with the Kerr effect. The group index varies with $\Omega_k/\gamma$ for $\delta = 30\gamma$ or $\delta = 50\gamma$, and is always more negative for Kerr effect [see Fig. 5]. Group index in the presence of the Kerr effect for $\Omega_3 = 16\gamma$ is $N_k^g = -3.10046 \times 10^8$, and in the absence of the Kerr effect it is $N_g = -3.47245 \times 10^8$ while having $\delta = 30\gamma$ [see Fig. 5(a-d)]. The group velocity for a Kerr free system is $v_g = c/N_g$, while it is $v_k^g = c/N_k^g$ when there is no Kerr effect in the system. The corresponding group velocities are $-68611.4\,\text{cm/s}$ and $-96.7598\,\text{cm/s}$ under similar parameters while having $\delta = 30\gamma$ and $\Omega_3 = 16\gamma$, respectively. Moreover, the negative group delay time is $-48\mu s$ when there is no Kerr effect in the system. However, in the presence of the nonlinear Kerr effect the negative time delay is $-30\mu s$ while having $\delta = 30\gamma$ and $\Omega_3 = 16\gamma$, respectively. Furthermore, the comparison of fractional change in the group index and in the advance time also confirms the dominant stable behavior for the Kerr effect in the system. The negative group index means sooner arrival relative to the velocity of the pulse in vacuum. The pulse travelling in a medium with a larger negative group index, in fact, arrives much sooner than pulse seeing the medium with relatively lesser negative group
index. Consequently, the superluminal Gaussian pulse leaves the medium for the Kerr effect sooner by 30 ms than the pulse leaving the medium when there is no Kerr effect in the system.

In Ref. 31 the incompatibility of power broadening of the drive bi-chromatic laser field with closely spaced ground hyperfine splitting resulted in instability in comparison with a situation when a monochromatic field was used as mentioned by Akulshin and his coworker in another Ref. 36. Furthermore, the group index with a Kerr effect has a directly proportionality with the drive field intensity presented for analysis of the experimental result while the instability limit appears from a Kerr free result for group index [see Ref. 31]. In principle, if the drive field intensity is large enough then a coherence effect must be induced by a Kerr effect as presented in this manuscript in comparison to our assumed Kerr free system. Consequently the induced coherence effect overcome the fixed incoherence power broadening of the bi-chromatic field. Correspondingly, if we do not include the Kerr effect in our estimation then the system suffers from the instability limit which is in fact scientifically not reasonable as shown in Fig. (5) [compare the Kerr-free and Kerr effect cases]. In addition, the incoherence mechanism may be more reduced if a medium with very slow ground state relaxation is considered. For example, a cell with buffer gas [18] or a cell with an anti-relaxation coating [19] may facilitates an experiment to overcome ground state relaxation.

Conceptually, enhancement in negative group index is not a necessary condition for a useful superluminal eect rather it requires undistort retrieved pulse, in addition. Earlier, physical interpretation of superluminality by virtual reshaping is now not always reasonable 29, 53, 54 with the explanation of amplification of the front edge with the relatively absorption of its tail. Here, in the pulse distortion measurement we kept the probe pulse bandwidth much smaller than gain lines separation and interaction time to avoid resonances with the Raman transitions frequencies for the probe pulse. Consequently, there is no amplification of the front edge of the pulse. This means that both the front edge and the tail would be amplified if the atoms of the medium amplify the probe pulse. However, this is not in accord with the earlier claims. Obviously, the superluminal light propagation arises due to the anomalous dispersion regions created in between the nearby Raman gain resonances of our system. In fact, if the gain becomes large, its effect appears as a compression of the pulse 29 [see Fig. 7]. The detail analysis reveals that the pulse distortion measurement fully agrees with the Wang et al. studies even with the second order perturbation limit. Evidently, in our system it is shown that pulse shape remains preserve over the very small region in between the gain lines for some specific value of the upshift frequency of a little change in the probe pulse width. The shifting of this upshift frequency (the probe field pulse width) results in compression (enhancement) of the retrieved Gaussian pulse. Unlikely, this behavior does not agree with the earlier interpretation and is more likely agree with Wang et al. presented results and interpretation. Satisfactorily, we are providing analytical results to the literature where this transparency behavior may be demonstrated in a laboratory and proceeded beyond the existing literature.

In conclusion, we induce quantum coherence effect in a gain-assisted N-type 4-level atomic system via nonlinear Kerr effect by the coupled intense monochromatic laser field. The coherence avoids the instability limit constrained by the earlier traditional approach to superluminality. The proposed scheme displays novel, amazing and useful behavior of two-pair gain lines processes. Consequently, a remarkable enhancement in the superluminal effect on the Gaussian pulse is seen. Quantitatively, under experimentally feasible parameters, the pulse advances by almost 30 ms more than that of the Kerr free system of the Wang et al. The proposed system is less like Wang et al but with the advantages of multiple controllable anomalous regions, significantly enhanced and stable superluminal behavior, and relaxed temperature, states of matter along with their isotropy, or anisotropy conditions, respectively. The control of multiple anomalous regions may insure an undistorted retrieved pulse due to their compatibility with the probe pulse width. The superluminal effect may also be more enhanced if inhomogeneous solid state medium with a large enough atomic density is selected. Incorporating these aspects in an experiment may modify some related current experimental technologies to help improve applied aspects of a superluminal light pulse in a medium.

I. APPENDIX

The parameters β1,2 in Eq. (4) and the parameters β3, T1,2, T3, K1,2 in Eq. (7) are listed bellow:

$$\beta_{1,2} = \frac{4|\Omega_{1,2}|^2[P_{1,2} + L_{1,2}]}{4(\gamma_{ac} - i\delta)(\gamma_{ab} - i(\delta - \Delta_k)) + \Omega_k^2},$$  (13)

where

$$P_{1,2} = \frac{2\gamma_{ad}(\gamma_{ab} - i(\delta - \Delta_k))}{(\gamma_{ca} + \gamma_{da})(\gamma_{ad}^2 + \Delta_k^2)},$$  (14)

and

$$L_{1,2} = \frac{4(\delta - \Delta_k + i\gamma_{ab})(\delta - \Delta_{1,2} - \Delta_k + i\gamma_{bd}) + \Omega_k^2}{(\gamma_{ad} + i\Delta_{1,2})R_{1,2}},$$  (15)

with

$$R_{1,2} = [4(\Delta_{1,2} - \delta - i\gamma_{de})(\Delta_{1,2} + \Delta_k - \delta - i\gamma_{bd}) - \Omega_k^2]$$  (16)

while

$$\beta_3 = \frac{2\gamma_{ad}(\gamma_{ad}^2 + \Delta_k^2)\Omega_k^2}{(\gamma_{ca} + \gamma_{da})(\gamma_{ac} - i\delta)(\gamma_{ad}^2 + \Delta_k^2)(\gamma_{ad}^2 + \Delta_k^2)},$$  (17)
\[ T_{1,2} = \frac{|\Omega_{1,2}|^2}{(\gamma_{ac} - i\delta)(\delta - \Delta_{1,2} + i\gamma_{dc})(\Delta_{1,2} - i\gamma_{ad})} \]  
\[ T_3 = (\gamma_{ac} - i\delta)(\gamma_{ab} - i(\delta - \Delta_k)), \]  
and
\[ K_{1,2} = \frac{\iota^i}{\Delta_{1,2} + \Delta_3 - i\gamma_{bd}} \frac{\iota^i}{\kappa_{x,2} - i\kappa_{x,2}} |\Omega_{1,2}|^2, \]  
while
\[ \alpha_{1,2} = T_3(\gamma_{ad} + i\Delta_{1,2})(\gamma_{dc} - i(\delta - \Delta_{1,2})), \]  
and
\[ x_{1,2} = [\gamma_{dc} - i(\delta - \Delta_{1,2})]^2(\gamma_{ad} + i\Delta_{1,2}) \times [\gamma_{bd} - i(\delta - \Delta_{1,2} - \Delta_k)]. \]  

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