Semi-Inclusive DIS: an explicit calculation in the Target Fragmentation Region†

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Abstract

I present a calculation of the one particle deep inelastic cross section in the target fragmentation region in $(\phi^3)_6$. The renormalized cross section gets a large logarithmic correction whose coefficient is precisely the scalar DGLAP kernel. The result is found to be consistent with an extended factorization hypothesis and with infrared power counting.

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1 INTRODUCTION

Semi-inclusive deep inelastic scattering has been successfully studied in the framework of perturbative QCD [1], at least in the case in which the transverse momentum of the produced hadron is of order of the hard scale $Q^2$.

In the last few years a new attention has been devoted to this process in the limit in which the transverse momentum, or equivalently the momentum transfer $t = -(p - p')^2$ between the incoming and outgoing hadron, is very small with respect to $Q^2$. In this limit the process is dominated by the target fragmentation mechanism and, for this reason, a new approach in terms of the so called fracture functions has been proposed [2], and developed [3, 4].

In this talk I present a calculation [5] of the semi-inclusive cross section in the target fragmentation region ($t \ll Q^2$) in $(\phi^3)_6$ model field theory. This model has revealed itself a nice laboratory to study strong interactions at short distances, since it is asymptotically free and it has a much milder structure of infrared singularities with respect to QCD [6, 7]. In fact there are no soft but only collinear singularities and so factorization becomes simpler to deal with [8].

2 DIS IN $(\phi^3)_6$

I will start recalling some results one gets for inclusive DIS. Let us consider the process $p + J(q) \to X$ where $J = \frac{1}{2} \phi^2$. We define as usual

$$Q^2 = -q^2, \quad x = \frac{Q^2}{2pq}. \quad (1)$$

The structure function can be defined as

$$W(x, Q^2) = \frac{Q^2}{2\pi} \int d^6ye^{iqa} <p|J(y)J(0)|p> \cdot \quad (2)$$

It is easy to calculate the parton-current cross section $w(x, Q^2)$ in dimensional regularization ($D = 6 - 2\epsilon$). At lowest order we get (see Fig. [6])
Figure 1: Lowest order contribution to the deep inelastic cross section

\[ w_0(x, Q^2) = \frac{Q^2}{2\pi} 2\pi\delta((p+q)^2) = \delta(1-x). \]  

(3)

The first order corrections are shown in Fig. 2. External self energies are not taken into account since we work at \( p^2 = 0 \). In order to take into account the renormalization of the operator \( J \) one has to multiply the total contribution by \( Z_J^{-2}(Q^2) \) where

\[ Z_J(Q^2) = 1 + \frac{5}{12} \frac{\lambda^2}{(4\pi)^3} \frac{1}{\epsilon} \left( \frac{4\pi\mu^2}{Q^2} \right)^\epsilon. \]  

(4)

Up to finite corrections we get

\[ w(x, Q^2) = \delta(1-x) + \frac{\lambda^2}{(4\pi)^3} P(x) \left( -\frac{1}{\epsilon} \right) \left( \frac{4\pi\mu^2}{Q^2} \right)^\epsilon \]  

(5)

where

\[ P(x) = x(1-x) - \frac{1}{12} \delta(1-x) \]  

(6)

is the DGLAP kernel for our model. The contribution to the structure function is obtained as a convolution with a bare parton density \( f_0(x) \)

\[ W(x, Q^2) = \int_x^1 \frac{du}{u} f_0(u) w(x/u, Q^2). \]  

(7)

The collinear divergence in \( w(x, Q^2) \) can be lumped as usual in a \( Q^2 \) dependent parton density by means of the equation

\[ f_0(x) = \int_x^1 \frac{du}{u} \left[ \delta(1-u) + \frac{\lambda^2}{(4\pi)^3} P(u) \frac{1}{\epsilon} \left( \frac{4\pi\mu^2}{Q^2} \right)^\epsilon \right] f(x/u, Q^2). \]  

(8)
Figure 2: One loop corrections to the deep inelastic cross section

The scale dependent parton density \( f(x, Q^2) \) obeys the DGLAP evolution equation

\[
Q^2 \frac{\partial}{\partial Q^2} f(x, Q^2) = \int_x^1 \frac{du}{u} P(u) f\left(\frac{x}{u}, Q^2\right). \tag{9}
\]

For the process \( J(q) \to p + X \) with \( q \) timelike a fragmentation function \( d(x, Q^2) \) can be defined in the same way and it obeys the same DGLAP evolution equation. At one loop level the timelike DGLAP kernel is the same as in the spacelike case, but this relation is broken at two loops \[7\].

3 SEMI-INCLUSIVE DIS

In the semi-inclusive case a new structure function can be defined as

\[
W(p, p', q) = \frac{Q^2}{2\pi} \sum_X \int d^6x e^{iqx} \langle p | J(x) | p' X > < X p' | J(0) | p > . \tag{10}
\]

We have calculated \[5\] the partonic cross section in the limit \( t \ll Q^2 \) at leading power, by keeping only divergent terms and possible \( \log Q^2/t \) contributions. As expected, the cross
The section is dominated by target fragmentation. The first diagram which give contribution is the one in Fig. 3.

![Diagram](image)

Figure 3: Leading order contribution to one particle deep inelastic cross section in the region $t \ll Q^2$

It gives

$$w_1(x, z, t, Q^2) = \frac{\lambda_0^2}{t^2} x \delta(1 - x - z)$$

where $\lambda_0$ is the bare coupling constant and

$$z = \frac{p'q}{pq}.$$  \hspace{1cm} (12)

It turns out that the relevant one loop corrections come from the diagrams in Fig. 4.

The other diagrams in fact either give finite contributions or are suppressed by powers of $t/Q^2$.

The details of the calculation are presented in Ref. [5]. Summing up all the contributions, multiplying by $Z_f^{-2}(Q^2)$, introducing the running coupling constant we finally get

$$w(x, z, t, Q^2) = \frac{\lambda^2(t)}{t^2} x \left( \delta(1 - x - z) + \frac{\lambda^2}{(4\pi)^3} \frac{1}{\epsilon} \left( \frac{4\pi\mu^2}{t} \right)^\epsilon \left( \frac{1}{6} \delta(1 - x - z) 
- \frac{1 - x - z}{(x + z)^2} - \frac{1 - x - z}{(1 - x)^2} \right) + \frac{1}{x} P \left( \frac{x}{1 - z} \right) \frac{\lambda^2}{(4\pi)^3} \log \frac{Q^2}{t} \right).$$  \hspace{1cm} (13)

The structure function is obtained as a convolution with the bare parton density and fragmentation function. By using eq. (8) and the corresponding definition for the
Figure 4: One loop leading contributions to the one particle deep inelastic cross section

fragmentation function we get

\[
W(x, z, t, Q^2) = \int_{x+z}^1 \frac{du}{u} \int_{u-x}^1 \frac{dv}{v^2} f(u, t) \frac{\lambda^2(t)}{t} \frac{v^2}{u^2} \left[ \delta \left( 1 - \frac{x}{u} \right) \delta \left( 1 - \frac{z}{uv} \right) + \frac{\lambda^2}{(4\pi)^3} P \left( \frac{x}{1 - z/uv} \right) \log \frac{Q^2}{t} \right] d(v, t)
\]

where again only leading log $Q^2/t$ terms have been considered and the integration limits
are derived using momentum conservation.

From eq. (14) it appears that the renormalized hard cross section gets a large log $Q^2/t$ correction whose coefficient is the scalar DGLAP kernel. Such correction, if not properly resummed, can spoil perturbative calculations in the region $t \ll Q^2$.

Eq. (14) shows a new singularity, which corresponds to the configuration in which $p'$ becomes parallel to $p$. When we integrate over $t$, in order to absorb such singularity, the introduction of a new phenomenological distribution, the fracture function \[2\] becomes necessary \[3\]. Eq. (14) can also be rewritten in the following form

$$W(x, z, t, Q^2) = \lambda^2(t) \int_x^{1-z} \frac{dr}{r} \int_{z+r}^1 \frac{du}{u(u-r)} \hat{P} \left( \frac{u}{t} \right) f(u, t) \left[ \delta \left( 1 - \frac{x}{r} \right) + \lambda^2 \left( \frac{4\pi}{3} \right)^3 P \left( \frac{x}{r} \right) \log \frac{Q^2}{t} \right] d \left( \frac{z}{u-r}, t \right)$$ (15)

where we have defined the A-P real scalar vertex $\hat{P}(x) = x(1-x)$. The function

$$E^{(1)}(x, Q^2/Q_0^2) = \delta(1-x) + \lambda^2 \left( \frac{4\pi}{3} \right)^3 P(x) \log \frac{Q^2}{Q_0^2}$$ (16)

appears to be the first order approximation of the evolution kernel $E(x, Q^2/Q_0^2)$ which resums the leading logarithmic series \[10\]. This fact suggests that an interpretation of eq. (15) can be given in terms of Jet Calculus \[10\].

4 FACTORIZATION IN TERMS OF CUT VER-TICES

Cut vertices are a generalization of matrix elements of local operators originally proposed by Mueller in Ref.\[11\]. They can be very useful to give an interpretation of the results obtained in the previous sections.

Let us go back to Sect.2 and set $p^2 < 0$ with $p = (p_+, 0, p_-)$. If we choose a frame in which $p_+ \gg p_-$ we can write for the parton-current cross section \[14\]

$$w(p, q) = \int \frac{du}{u} v(p^2, u) C(x/u, Q^2)$$ (17)
where \( v(p^2, x) \) is a spacelike cut vertex with \( C(x, Q^2) \) the corresponding coefficient function.

If we define
\[
v(x, \epsilon) = \delta(1 - x) + \frac{\lambda^2}{(4\pi)^3} P(x) \left( -\frac{1}{\epsilon} \right)
\] (18)
\[
C(x, Q^2) = \delta(1 - x) + \frac{\lambda^2}{(4\pi)^3} P(x) \log \frac{Q^2}{4\pi\mu^2}
\] (19)
we can write eq. (13) in the form
\[
w(x, Q^2) = \int_x^1 \frac{du}{u} v(u, \epsilon) C(x/u, Q^2).
\] (20)

Here \( v(x, \epsilon) \) is a spacelike cut vertex defined at \( p^2 = 0 \) whose mass divergence is regularized dimensionally.

A similar interpretation can be given of eq. (13). We define
\[
\bar{x} = \frac{x}{1 - z}
\] (21)
and
\[
v(\bar{x}, z, t, \epsilon) = \frac{\lambda^2(t)}{t^2} \left[ \delta(1 - \bar{x}) + \frac{\lambda^2}{(4\pi)^3} \frac{1}{\epsilon} \left( \frac{4\pi\mu^2}{t} \right)^\epsilon \left( \frac{1}{6} \delta(1 - \bar{x}) + \frac{(1 - z)^2 \bar{x}(1 - \bar{x})}{(1 - \bar{x} + z) \bar{x}(1 - z)} + \frac{P(\bar{x})}{(4\pi)^3} \log \frac{4\pi\mu^2}{t} \right) \right]
\] (22)
as a generalized cut vertex [9] which contains all the leading mass singularities of the cross section. We can write up to \( O(t/Q^2) \) corrections
\[
w(\bar{x}, z, t, Q^2, \epsilon) = \int_x^1 \frac{du}{u} v(u, z, t, \epsilon) C(\bar{x}/u, Q^2)
\] (23)
where the coefficient function is the same which occurs in inclusive DIS.

The validity of this factorization relies on the fact that diagrams with more than two legs connecting the soft to the hard part are suppressed by powers of \( t/Q^2 \) [5]. This is a result which can be generalized at all orders by using the ideas of Ref. [8, 12].
The large $Q^2$ limit of the semi-inclusive cross section can be studied by looking at the singularities in the limit $p^2, p'^2, t \to 0$. The strength of such singularities can be predicted by using infrared power counting [9]. Starting from a given diagram, its reduced form in the large $Q$ limit is constructed by simply contracting to a point all the lines whose momenta are not on shell. In $(\phi^3)_6$ the general leading diagrams in the large $Q^2$ limit for the process under study involve a jet subdiagram $J$, composed by on shell lines collinear to the incoming particle, from which the detected particle emerges in the forward direction and a hard subgraph $H$ in which momenta of order $Q$ circulate, which is connected to the jet by the minimum number of collinear lines. Additional lines connecting $J$ to $H$ as well as soft lines connecting them are suppressed by power counting. So one can say that in $(\phi^3)_6$ the leading diagrams are of the form depicted in Fig. 5 and this means that in this model eq. (23) holds at all orders [9].

5 SUMMARY

In this talk I have presented an explicit calculation of the one particle deep inelastic cross section in the target fragmentation region within $(\phi^3)_6$ model field theory. The
renormalized hard cross section gets a large $\log Q^2/t$ correction as expected in a two scale regime and the coefficient driving this logarithmic correction is precisely the scalar DGLAP kernel.

Furthermore the result obtained fits within an extended factorization hypothesis \[9\]. In fact the partonic semi-inclusive cross section factorizes into a convolution of a new object, a generalized cut vertex $v(p, p', \bar{x})$ \[9\], with four rather than two external legs, and a coefficient function $C(\bar{x}, Q^2)$ which is the same as the one of inclusive DIS. Infrared power counting applied to this process allows to say that this last result holds in $(\phi^3)_6$ at all orders.

References

[1] G. Altarelli, R.K. Ellis, G. Martinelli, S.Y. Pi, Nucl. Phys. B160 (1979) 301.
[2] L. Trentadue and G. Veneziano, Phys. Lett. B323 (1994) 201.
[3] D. Graudenz, Nucl. Phys. B432 (1994) 351.
[4] D. De Florian and R. Sassot, Phys. Rev. D56 (1997) 426.
[5] M. Grazzini, UPRF-97-09, hep-ph/9709312.
[6] J.C. Taylor, Phys. Lett. B73 (1978) 85; Y. Kazama and Y.P. Yao, Phys. Rev. Lett. 41 (1978) 611, Phys. Rev. D19 (1979) 3111; L. Baulieu, E.G. Floratos and C. Kounnas, Phys. Rev. D23 2464 (1981).
[7] T. Kubota, Nucl. Phys. B165 (1980) 277.
[8] J.Collins, D.E. Soper and G. Sterman in Perturbative QCD ed. by A.H. Mueller (1982) 1.
[9] M. Grazzini, L. Trentadue and G. Veneziano, hep-ph/9709452.
[10] K. Konishi, A. Ukawa and G. Veneziano, Nucl. Phys. B157 (1979) 45.
[11] A.H. Mueller, Phys. Rev. D18 (1978) 3705; Phys. Rep. 73 (1981) 237.

[12] G. Sterman, Phys. Rev. D17 (1978) 2773, 2789.