Large scale alignment anomalies of CMB anisotropies: a new test for residuals applied to WMAP 5yr maps

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Abstract. We analyze the alignment of the low multipoles (quadrupole and octupole) of various maps of the WMAP 5yr release: the CMB maps obtained with ILC and MCMC methods, the CMB map in the V band after foreground reduction, and, for comparison, the (not cleaned) V band map. We study how much this alignment is polluted by residuals on the Galactic region. Among the considered maps, the WMAP-ILC turns out to be the most clean map from the point of view of the proposed test. This result has been found studying the redistribution (due to the masking process) of each bin of the probability distribution functions of the alignment estimators. By construction, our method, feasible through Monte Carlo simulations, works for any possible mask adopted in the analysis of data from current and forthcoming CMB anisotropy experiments and it can only exclude that the considered map is clean.

Keywords: CMBR experiments, CMBR theory
1 Introduction

The anisotropy pattern of the cosmic microwave background (CMB), obtained by Wilkinson Microwave Anisotropy Probe (WMAP), probes cosmological models with unprecedented precision (see [1, 2] and references therein). Although WMAP data are largely consistent with the concordance Λ cold dark matter (ΛCDM) model, there are some interesting deviations from it, in particular on the largest angular scales. They can be divided in the following categories. 1) Lack of power at large scales. The angular correlation function is found to be uncorrelated (i.e. consistent with 0) for angles larger than 60 degrees. In [3, 4] it has been shown that the probability associated to this event is low as 0.15%. Still in this category we mention the surprisingly low amplitude of the quadrupole term of the angular power spectrum (APS), already found by Cosmic Background Explorer (COBE) [5, 6], and now confirmed by WMAP [1, 2]. 2) Unlikely alignments of low multipoles. An unlikely (for a statistically isotropic random field) alignment of the quadrupole and the octupole is described in reference [7–12]. Moreover, both quadrupole and octupole align with the CMB dipole [4]. Other unlikely alignments are described in [13–15]. 3) Hemispherical asymmetries. It is found that the power coming separately from the two hemispheres (defined by the ecliptic plane) is too asymmetric (especially at low ℓ) [16, 17]. 4) Cold Spot. In ref. [18] it is reported a detection of a non Gaussian behaviour in the southern hemisphere with a wavelet analysis technique (see also [19]).

It is still unknown whether these anomalies come from fundamental physics or whether they are the residual of some not perfectly removed astrophysical foreground or systematic effect [20–24]. As an example of the latter kind, in references [25, 26] it is presented a study about the impact of the dipole straylight contamination on the low amplitude of the quadrupole and on the low ℓ alignments for Planck\footnote{http://www.rssd.esa.int/planck} characteristics and capabilities. Many efforts have been dedicated to the development of methods aimed at the discrimination...
between spurious and cosmological effects [27–30]. The still open question about the origin of such anomalies has attracted a lot of interest in the last few years. In [31–41] there are some papers about the possible generation of the low ℓ anomalies in the context of some specific models of fundamental physics. In [42] it is claimed that no model of contamination that is statistically independent of the source of the primary CMB anisotropy, can explain this large-scale power deficit. In other words, if a contamination (not taken into account) is responsible for the lack of power it must have a correlation with the primary CMB anisotropy.

In the present paper we focus on the second of the aforementioned list of anomalies, i.e. on the unlikely alignments of low multipoles (quadrupole and octupole). We take into account various maps of the WMAP 5 year release: the ILC map, the MCMC map, the (not cleaned) V band map and the foreground reduced V band maps (see figure 1). The ILC (Internal Linear Combination) map, available at pixel size of 6.87 arcmin, has been obtained from a weighted linear combination of the five intensity maps at the various WMAP frequencies smoothed to 1 degree (FWHM) resolution, with weights chosen to maintain the CMB anisotropy signal while minimizing the Galactic foreground contribution in different 12 regions covering the whole sky. The WMAP team believes that it is suitable for analyses on angular scales greater than about 10 degrees. The MCMC (Monte Carlo Markov Chain) map is again smoothed to 1 degree resolution but with a pixel size of 54.97 arcmin. From the MCMC fit the WMAP team derived anisotropy maps for the Galactic foreground diffuse components and that for the CMB component, considered in this work. We exploit also the foreground reduced (i.e. the CMB anisotropy map derived with a subtraction of the foreground components using a Foreground Template Model) map in the V band, the WMAP frequency channel where the original level of foreground is minimum. For comparison, we consider also the map in the V band without any kind of foreground subtraction. These two last maps centred at 61 GHz have the original beam resolution of 19.8 arcmin and are provided at a pixel size of 6.87 arcmin. Clearly, since we will analyse the large scale properties of the sky, the fact that the above maps have been provided at different resolutions and pixel sizes do not have any significant impact for this work. All these products are publicly available at LAMBDA web site where further information can be found.

In this paper we address the impact of residuals (that are unavoidably present in the considered maps) on the estimators for alignments of low multipoles. Uncertainties can be potentially introduced in each step of data analysis needed to generate the CMB map. For example, it is known that it is difficult to perform an accurate component separation close to the Galactic plane. Therefore we propose a new consistency test that can be capable of detecting the effects of the residuals on the estimators for the alignments at low multipoles. The basic idea of this check is the comparison between the value of the estimator of the map under analysis with the values of the estimator of the simulated maps that are unlikely (or likely, depending on the context) at the same level of the considered map. The application of the proposed test to the not cleaned V band map, certainly not meaningful for a cosmological analysis, is performed in order to verify the capability of the method to identify spurious contributions in a known dirty case.

The problem of minimizing potential residuals in a CMB map is crucial for cosmological analyses. In [43] it is proposed a way to limit the impact of foreground contamination present in the all sky maps adopted for the low ℓ analysis of the alignments. This new method makes use of a power equalization filter [44] and it has been proposed for a better control of residual

2http://lambda.gsfc.nasa.gov/
foregrounds and therefore for a potentially more robust cosmological analysis. However, even if this (or some other) component separation method could work perfectly, residuals can still be present because of uncertainties from potential systematics and from other stages in the data analysis. In [45] the $a_{\ell m}$ (coefficients of the expansion over the basis of the Spherical Harmonics) of the CMB are obtained with a minimum variance method in case of incomplete sky coverage, non uniform noise and foreground contamination. Unfortunately, if the considered mask excludes more than about 10\% of the sky then the error bars of the $a_{\ell m}$ coefficients become too large.

In this paper we adopt a drastic approach and mask the Galactic region (with masks of various size$^3$). Since the Multipole Vectors expansion (that is the mathematical tool that we will use to define alignments [7, 8]) is doable only over the all sky (i.e. over the full set of the orthonormal Spherical Harmonics), we do not look for a new basis defined in the uncut region but we screen the information coming from the Galactic region, setting the value of the pixels falling into the area we do not want to consider, to a fiducial value (namely 0). In section 2 it is shown how to consistently perform the analysis and how to use the masking process in order to analyze the contamination present in the masked area (i.e. the proposed test).$^4$ Technically, we study the redistribution (due to the masking process) of each bin of the probability distribution functions of the alignment estimators. The used technique can indeed only exclude that the considered map is clean in some region but it cannot state that the map is clean in that region since a possible residual could be in principle compatible even with random realizations. By construction, our method works for any possible mask suited to exclude regions that are possibly affected by various kinds of contamination.

The paper is organized as follows: in section 2 we present the adopted methodology and describe the performed simulations and the proposed test, in section 3 we show the obtained results, and in section 4 we draw our conclusions.

2 Methodology

2.1 Multipole vectors

The alignment of multipoles can be defined using a new representation of CMB anisotropy maps where the $a_{\ell m}$ are replaced by vectors [7, 8]. In particular, each multipole order $\ell$ is represented by $\ell$ unit vectors and one amplitude $A$

$$a_{\ell m} \leftrightarrow A^{(\ell)}, \hat{u}_1, \cdots, \hat{u}_\ell. \quad (2.1)$$

Note that the number of independent objects is the same in the l.h.s and r.h.s. of equation (2.1): $2\ell + 1$ for $a_{\ell m}$ equals $3\ell$ (numbers of components of the vectors) +1 (given by $A^{(\ell)} - \ell$ (because there are $\ell$ constraints due to the normalization conditions of the vectors).

Equation (2.1) can be understood starting from this observation [8]: if $f$ is a solution of the Laplace equation

$$\nabla^2 f = 0, \quad (2.2)$$

$^3$In 2 cases out of 3, larger than 10\% of the sphere.

$^4$As it will be clearer in section 2, we do use the masking process as a tool to define a test for residuals and to infer conclusions about the all sky maps but we do not use the scientific analysis on the masked maps to infer conclusion about the all sky maps.
where $\nabla^2 = \partial_x^2 + \partial_y^2 + \partial_z^2$ in Cartesian coordinates, then it is possible to build a new solution $f'$ applying a directional derivative to $f$

$$\nabla_{\vec{u}} f \equiv \vec{u} \cdot \nabla f = f', \quad \nabla^2 f' = 0,$$

(2.3)

with the gradient $\nabla = (\partial_x, \partial_y, \partial_z)$. This happens because the two operators $\nabla^2$ and $\nabla_{\vec{u}}$ commute. Maxwell [46] repeated this observation $\ell$ times considering the $1/r$ potential as starting solution. Here $\vec{r} = (x, y, z)$ and $r = \sqrt{\vec{r} \cdot \vec{r}} = \sqrt{x^2 + y^2 + z^2}$. In this way, one obtains

$$f_\ell(x, y, z) = \nabla_{\vec{u}_1} \cdots \nabla_{\vec{u}_\ell} \nabla_{\vec{u}_1} \frac{1}{r}. \quad (2.4)$$

Observe the simple pattern that emerges as we apply the directional derivatives one at a time:

$$f_0 = \frac{1}{r},$$
$$f_1 = \frac{(-1)(\vec{u}_1 \cdot \vec{r})}{r^3},$$
$$f_2 = \frac{(3 \cdot 1)(\vec{u}_1 \cdot \vec{r})(\vec{u}_2 \cdot \vec{r}) + r^2(-\vec{u}_1 \cdot \vec{u}_2)}{r^5},$$
$$f_3 = \frac{(-5 \cdot 3 \cdot 1)(\vec{u}_1 \cdot \vec{r})(\vec{u}_2 \cdot \vec{r})(\vec{u}_3 \cdot \vec{r}) + r^2(...)}{r^7}.$$

The (...) stands for a polynomial which we do not write explicitly, being not relevant to the current purposes.

Moreover, writing $f_\ell$ in spherical coordinates once $r$ is set to 1, one finds the following property

$$\tilde{\nabla}^2 f_\ell(1, \theta, \phi) = \ell(\ell + 1)f_\ell(1, \theta, \phi),$$

(2.5)

where $\tilde{\nabla}^2$ is the angular Laplace operator defined as

$$\tilde{\nabla}^2 = -\left[\frac{1}{\sin \theta} \partial_\theta (\sin \theta \partial_\theta) + \frac{1}{\sin^2 \theta} \partial_\phi^2\right]. \quad (2.6)$$

In other words $f_\ell(1, \theta, \phi)$ is eigenfunction of the angular part of the Laplace operator with eigenvalue given by $\ell(\ell + 1)$. This is nothing but the definition of spherical harmonics $Y_{\ell,m}$ (e.g. see [47]). Therefore, for every $\ell$ we can write

$$A^{(\ell)} f_\ell(1, \theta, \phi) = \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi), \quad (2.7)$$

where the amplitude $A^{(\ell)}$ has been inserted because of normalization purposes. Equation (2.7) makes evident the association represented by equation (2.1). From equation (2.7) it is possible to write down the set of equations that has to be solved to pass from $a_{\ell m}$ to multipole vectors. In order to see that this set is solvable we count the equations and the unknowns involved in this set. From equation (2.7) we have $2\ell + 1$ equations (one equation for each independent $a_{\ell m}$) plus $\ell$ equations from the normality conditions of the vectors (i.e. $\vec{u}_i \cdot \vec{u}_i = 1$ where $i$ is in fact we would have $4\ell + 1$ equation because each $\ell$ different from 0 has a real and imaginary part. But considering that $a_{\ell m}$ with $m > 0$ are related to those with $m < 0$ through $a_{\ell m}^* = (-1)^m a_{\ell - m}$ we are left with $2\ell + 1$ equations (i.e. the expression must be real).
runs from 1 to $\ell$). Therefore the total number of independent equations is $3\ell + 1$. This is also the number of unknowns because we have 3 unknowns for each vector plus 1 given by the amplitude $A^{(\ell)}$. This shows that the set is solvable.

One of the advantage of Multipole Vectors representation is that from these unit vectors one can easily construct scalar quantities that are invariant under rotation. Note that is not equally easy to obtain scalar quantities directly from the $a_{\ell m}$ coefficients since they depend on the coordinate system. For a more detailed explanation of equation (2.1) and of the properties of that association see for example references \[7, 8, 13, 26\].

Unfortunately, an explicit analytical expression for the association given in equation (2.1) is possible only for $\ell = 1$. For $\ell \neq 1$ numerical methods are needed.\(^6\) The Copi et al.’s algorithm (which use is acknowledged here) for constructing multipole vectors from a standard spherical harmonic decomposition is described in \[7\] and the implementation of it is public available.\(^7\) Other methods exist but, as far as we know, their implementation is not public available on a standard platform (see for example \[8, 50\] where the problem of finding $\ell$ vectors is translated into the problem of finding the zeros of a polynomial of degree $2\ell$).

### 2.2 Alignment estimators

We focus on the alignment quadrupole-octupole. Therefore we consider the following estimators widely used in literature (e.g. \[7, 8, 10, 13, 51\]):

\[
S_{23} = \sum_{i=1}^{3} |\hat{q} \cdot \hat{o}_i|/3, \tag{2.8}
\]

\[
D_{23} = \sum_{i=1}^{3} |\hat{q} \cdot \hat{o}_i|/3. \tag{2.9}
\]

Here the symbol \(\hat{\cdot}\) stands for a vector with norm equal to 1. The “area vectors” are defined as

\[
\hat{q} = \hat{q}_{21} \times \hat{q}_{22}, \tag{2.10}
\]

\[
\hat{o}_1 = \hat{o}_{32} \times \hat{o}_{33}, \tag{2.11}
\]

\[
\hat{o}_2 = \hat{o}_{33} \times \hat{o}_{31}, \tag{2.12}
\]

\[
\hat{o}_3 = \hat{o}_{31} \times \hat{o}_{32}, \tag{2.13}
\]

where $\hat{q}_{2j}$ represent the two normalized multipole vectors ($j = 1, 2$) associated to the quadrupole, $\hat{o}_{3k}$ represent the three normalized multipole vectors ($k = 1, 2, 3$) associated to the octupole.

Notice that all the estimators belong to the interval $[0, 1]$ and contain absolute values in order to make them invariant under the reflection symmetry (see for example \[13, 26\]).

### 2.3 Description of the performed simulations and test for residuals

We have performed $3 \times 10^5$ Gaussian random extractions of ΛCDM skies. We have masked each extraction\(^8\) setting the pixels inside the mask to the fiducial value equal to 0. This allows us to screen the information present in the Galactic region and, at the same time, to have an

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\(^6\)Indeed, for $\ell = 2$ it is possible to obtain the multipole vectors computing the eigenvectors of a symmetric and traceless tensor representing the quadrupole, see \[48, 49\].

\(^7\)http://www.phys.cwru.edu/projects/mpvectors/

\(^8\)Hereafter we will use random extraction, random realization and random map as synonyms.
Figure 1. Mollweide projections of the considered maps. ILC map (upper left), MCMC map (upper right), foreground reduced V band map (lower left), V band map (lower right). The number of pixels in each map is $N_{\text{pix}} = 3145728$ except for the MCMC map where $N_{\text{pix}} = 49152$. A part the V band map that is taken into account as test case, all the other maps are only CMB maps obtained with different techniques. See also the text.

all-sky map (even if artificial) such that the multipole vectors decomposition is still doable.\(^9\) We have largely exploited the extended temperature mask available at LAMBDA web site that has a sky coverage of 73\% and other two masks whose percentage of sky coverage are around 83\% and 93\%.\(^10\) For each realization (masked and not masked) we have computed the estimators defined in subsection 2.2 passing from $a_{\ell m}$ to multipole vectors through the Copi et al.’s routine. This allows us to compute the probability distribution function (henceforth pdf) of the considered estimators in the all-sky case and in the masked cases. In all the cases (masked and unmasked) we have computed the value of the estimators for four maps: the ILC map, the MCMC map, the V band map and the foreground reduced V band.\(^11\) We do not use the masked pdf to directly extract scientific conclusions because this would not tell anything about the all sky map that is what we are interested in. What we do use is the masking process itself. Here it is a description of the test we propose. It consists of the following steps:

- consider the random maps whose all-sky estimator value belong to the same bin of the pdf as the all-sky value of the estimator of the map under analysis;

\(^9\)In this paper we will call masked maps, the maps that present the value 0 for the pixels that fall into the mask, even if, strictly speaking, they are still all-sky.

\(^10\)We use ”sky coverage” to mean ”observed sky” (that has not to be confused with masked area).

\(^11\)A part the V band map that is taken into account as test case, all the other maps are only CMB maps obtained with different techniques. That’s why we compare the same simulations with all these maps.
• mask these maps (both the set of random maps and the map under analysis that belong to the same bin) as described above;

• transform to the multipole vectors representation and compute the masked values of the estimator (both for the set of random maps and for the map under analysis that belong to the same bin);

• build a pdf of these masked values obtained from the random extractions (what we call redistribution);

• compare the masked value of the estimator for the map under analysis with the redistribution.

In other words, we compute how each bin of interest of the all-sky pdf (for the considered estimator, D23 and S23 in this work) is redistributed when a mask is applied. This allows us to analyze how consistent are the four considered maps with the random realizations whose D23 or S23 values belong to the same bin of the all-sky distributions. More explicitly, if the estimator of the masked map under analysis does not behave consistently with the other random masked maps then we conclude that some residual was present in the all-sky map affecting the value of the all-sky estimator and making it belonging to a “wrong” bin. This consistency check is the test we propose in this paper to look for the effects of residuals, that are potentially present in the Galactic region of the considered WMAP 5yr maps, on the alignment estimators. Of course, in the case of the not cleaned V band map we are sure that a non-CMB signal is present at low Galactic latitudes.

Note that the process adopted in our test introduces a sort of kernel representing the effect of the mask for each considered bin. The global pdf in the presence of a mask can be seen as the convolution of the all-sky pdf with this kernel.

The resolution that we have considered to perform this analysis is represented by the HEALPix parameter $N_{\text{side}}$ that we have set to 16.

The impact of the noise in our test is addressed at the end of the next section.

3 Results

In this section we present and discuss the obtained results. We have chosen the following convention, in order to display our results on the pdf: red color for the all-sky pdf, cobalt blue color for the masked pdf, green color for the difference between masked and unmasked pdf. The green plots give the effect that has been introduced by the considered treatment of the mask. More precisely they show the difference between masked and unmasked distribution (masked minus unmasked). Moreover we use the blue color for the pdf of the redistribution for the large mask, the light blue color for the pdf of the redistribution for the medium sized mask and the white color for the pdf of the redistribution for the small mask. Vertical lines show the values of the estimators derived from the considered maps: black vertical line is for ILC map, blue vertical line for the MCMC map, green vertical line for the foreground reduced V band, yellow line for V band map. The V band map has been taken into account just for comparison with the foreground reduced V band and for method validation.

\footnote{http://healpix.jpl.nasa.gov \footnote{For the reader who is not familiar with HEALPix convention, $N_{\text{side}} = 16$ corresponds to maps of 3072 pixels on the whole sky.}}
Figure 2. All-sky pdf for D23. Vertical lines represent the values for considered maps (of course in the all sky case). More precisely, black vertical line is for ILC map, blue vertical line for the MCMC map, green vertical line for the foreground reduced V band, yellow line for V band map. The panel presents the counts (y-axis) versus the statistic (x-axis). See also the text.

3.1 Quadrupole-Octupole alignment estimators

In figure 2 we show the all sky pdf for D23 with the corresponding values for the considered maps. In figure 3 we show the same estimator D23 for the three considered masks (left column of panels). We note that for all the considered maps, but not for the V band map (yellow line) where foreground subtraction is not applied, the D23 estimator\(^\text{14}\) assumes similar values when intermediate or large masks are applied, as the black, green, and blue lines tend to overlap (or at least to be closer) when the size of the Galactic mask increases. In the right column of panels of figure 3 there is the difference of the pdf’s between masked and all-sky case, that turns out to be stronger for larger masks. At the chosen binning of 0.01, such an effect appears weaker for the smaller mask.\(^\text{15}\) It is interesting to notice that the area to the right of the black line (i.e. ILC map) below the pdf is larger for larger masks. At the chosen binning of 0.01, such an effect appears weaker for the smaller mask.\(^\text{15}\) It is interesting to notice that the area to the right of the black line (i.e. ILC map) below the pdf is larger for larger masks. In particular, the probability to obtain a value smaller than the ILC-WMAP value is 98.36\% for the all-sky case, 97.96\% for smaller mask, 88.62\% for the medium mask, 87.37\% for the extended mask. This would seem to indicate that excluding/screening the information coming from the Galactic region (as described in section 2.3) makes the estimator no longer anomalous (for the map under analysis). Unfortunately this does not indicate uniquely that the anomaly that is present in the all-sky ILC-WMAP map is due to residuals that are present in the Galactic region. The fact that the masked ILC-WMAP map is no longer anomalous might also be an effect of the masking process itself (i.e. of the prior of setting to 0 the masked pixels).\(^\text{16}\)

To discriminate between the two possibilities we compute the redistribution of the 87\(^\text{th}\) bin (that is the all-sky bin where the D23 value for the ILC-WMAP map falls) caused by the presence of zeros of the masking process. We analyze then if the masked ILC-WMAP D23 value is consistent with this redistribution, i.e. with a masked Gaussian random realization.

\(^{14}\)The same is true also for S23, see the following and figure 6.

\(^{15}\)Increasing the number of realizations (for example to \(10^6\)) or the size of the bin (for example to 0.05) would show a similar shape as the other masked cases. We do not report here these plots but we dedicate a short appendix to this aspect.

\(^{16}\)Therefore, as aforementioned in footnote 4 and in subsection 2.3, we cannot extract scientific information about the all sky map from the masked analysis.
that exhibits a value for D23 that falls in same bin as the all-sky ILC-WMAP (this is the consistency check we propose). Of course, the same has been done for the other maps and for all the three considered masks. Figure 4 shows the “masking flux” of the bin 87 for the ILC-WMAP, of the bin 50 for the MCMC map, of the bin 96 for V band map and of the bin 26 for the foreground reduced V band map. Observing the fourth column of figure 4 we can see that the ILC-WMAP map is always consistent with a Gaussian random realization that shows the same all-sky D23 value. For what concerns the other maps, figure 4 shows that they are not consistent with a Gaussian random realization because first, second and third column of figure 4 present the D23 value for the other three maps (vertical lines) far from the peak of the distribution. We interpret this as evidence for the presence of residuals in the Galactic region in the all starting all-sky maps (except the aforementioned ILC-WMAP map).

In figure 5 we show the all sky pdf for S23 with the corresponding values for the considered maps. In figure 6 we show the same estimator S23 for the three considered masks. In particular, the probability to obtain a value smaller than the ILC-WMAP value is 98.90% for
Figure 4. Pdf of the redistribution of some specific bin of D23. In the first column we plot the redistribution due the masking process of the bin 26 for the foreground reduced V band map (green vertical line). In the second column we plot the redistribution due the masking process of the bin 96 for the V band map (yellow vertical line). In the third column we plot the redistribution due the masking process of the bin 50 for the MCMC map (blue vertical line). In the fourth column we plot the redistribution due the masking process of the bin 87 for the ILC map (black line). The first row is for the largest mask (73% of sky coverage), the second row for the middle size mask (83% of sky coverage) and the third row is for the smallest considered mask (93% of sky coverage). All the panels present the counts (y-axis) versus the statistic (x-axis). See also the text.

Figure 5. The same as figure 2 but for S23. Vertical lines represent the values for considered maps (of course in the all sky case). More precisely, black vertical line is for ILC map, blue vertical line for the MCMC map, green vertical line for the foreground reduced V band, yellow line for V band map. The panel presents the counts (y-axis) versus the statistic (x-axis). See also the text.

the all-sky case, 99.24% for smaller mask, 98.50% for the medium mask, 95.44% for the extended mask. As for the D23 estimator, this does not necessarily mean presence of residuals coming from the Galactic region. In figure 7 we show the redistribution for the bins where
Figure 6. The same as figure 3 but for S23. Left column of panels: pdf of S23 for the masked case. Black vertical line is for ILC map, blue vertical line for the MCMC map, green vertical line for the foreground reduced V band, yellow line for V band map. Right column of panels: difference of pdfs between masked and unmasked distributions. Percentage of sky coverage from top to bottom row of panels: 73%, 83% and 93%. All the panels present the counts (y-axis) versus the statistic (x-axis). See also the text.

The S23 values of the considered maps stand. In analogy with what has been obtained for the redistribution of the D23 bins, figure 7 shows that the ILC-WMAP map is the only map that is consistent with the corresponding pdf of the redistribution.\textsuperscript{17} Note that the pdfs of redistributions of figures 4 and 7 are more peaked as larger is the sky coverage.

The redistributions shown in figures 4 and 7 have well defined shapes. The case of V band map and D23 estimator is the most anomalous (see figure 2), thus the corresponding bin is less populated (282 points). In spite of this, its redistributions are pretty stable. In order to show this, we take the case of the large mask and compute in figure 8 the redistribution for all the available points (left panel, as in figure 4) and for half of the available points (right panel). Since the shape appears to be stable for this case, the most critical one, we conclude that the shown results are robust. In general, when the number of realization are few, an increase of the number of realizations and/or of the bin size of the histogram of the redistribution might be needed.

\textsuperscript{17}In fact the V band for the larger mask might be considered consistent, but the inconsistency become evident as soon as it is considered a less sized mask. See also section 4.
Figure 7. Pdf of the redistribution of some specific bin of S23. In the first column we plot the redistribution due the masking process of the bin 21 for the foreground reduced V band map (green vertical line). In the same second column we plot the redistribution due the masking process of the bin 53 for the V band map (yellow vertical line). In the third column we plot the redistribution due the masking process of the bin 48 for the MCMC map (blue vertical line). In the fourth column we plot the redistribution due the masking process of the bin 73 for the ILC map (black vertical line). The first row is for the largest mask (73% of the sky coverage), the second row for the middle size mask (83% of the sky coverage) and the third row is for the smallest considered mask (93% of the sky coverage). All the panels present the counts (y-axis) versus the statistic (x-axis). See also the text.

Figure 8. Convergency of our results. Redistribution of D23 estimator, for bin 96 for the large mask case (73% of sky coverage). Left panel is exactly the same as in figure 4, while right panel uses half of the realizations belonging to the same bin. The yellow vertical line represents the value of the estimator for the V band map. See also the text.

Regarding the comparison between our estimators for the V band map and the foreground reduced V band map, we note that the analysis of the pdf of the redistribution in both cases always indicates the presence of residuals in the Galactic region, except perhaps
Figure 9. Impact of the noise. Upper panels: difference of the pdf’s for the D23 estimator. Lower panels: difference of the pdf’s for the S23 estimator. Left panels: difference of the pdf in the case with noise minus without noise in the all-sky case. Right panels: difference of the pdf in the case with noise minus without noise in the masked case (extended mask, i.e. 73% of sky coverage). All the panels present the counts (y-axis) versus the statistic (x-axis). See also the text.

for the case of the S23 estimator applied to the V band map as suggested by the largest mask case\textsuperscript{18} (see figure 7). Note that the power of our method depends also on the chosen estimator and on the size of the mask. For this reason, given the estimator, we recommend to apply our test exploiting various mask sizes. Regarding the power of the method, in the considered cases, we found that the width of the pdf of redistribution increases (although not too strongly) with the size of the mask. Note that for a map where we are aware of the presence of foregrounds, as the V band map, the test successfully detects spurious contamination.

We have also studied the impact of the noise in the pdf of S23 and D23 in the masked (73% of sky coverage) and all-sky case. We expect only a (very) weak impact of the noise for our analysis as intuitively suggested by the fact that at \( N_{\text{side}} = 16 \) the root mean square (RMS) of the signal (i.e. CMB) is \( \simeq 50 \mu K \) and the RMS of the noise is \( \simeq 2 \mu K \) (for example considering the V band map of WMAP). In figure 9 we show the effect of the noise on the pdf confirming the weak impact. These plots have been obtained with 300000 Gaussian random realizations. The CMB signal is extracted according to a \( \Lambda \)CDM model and the Gaussian random noise has been generated exploiting the non-uniform sensitivity of the V band map.

\textsuperscript{18}This exception is not so surprising since the nature of our test (note also that the CMB temperature anisotropy map in the V band is clearly not so affected by diffuse Galactic foregrounds when a large fraction of the sky at low Galactic latitudes is excluded).
4 Conclusions

We have studied how the alignment between quadrupole and octupole is polluted on the Galactic region.

We found that among the considered maps the WMAP-ILC is the most consistent with Gaussian random realizations. From the point of view of this test, we can say that WMAP-ILC is the most clean map (among the considered ones). This result has been found studying the redistribution (due to the masking process) of each bin of the probability distribution functions of the alignment estimators. Of course we cannot exclude that other tests might detect some residual contamination. The used technique can indeed only exclude that the considered map is clean in some region but it cannot state that the map is clean in that region since a possible residual could be in principle compatible with random realizations.

Since the WMAP-ILC passes this test for three Galactic masks, the unlikely alignment between Quadrupole and Octupole (probed by the two considered estimators) that is present in this map is confirmed.

Although the results presented here formally apply to the considered products, our study points out on the relevance of analyzing low multipole alignments with the possibility of testing sky regions potentially affected by systematics (of instrumental or astrophysical origin) and show how it is feasible through Monte Carlo simulations. By construction, our method works for any possible mask adopted in the analysis of data from current and forthcoming CMB anisotropy experiments, such as those expected by the Planck satellite.

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A Comparison of simulations with more realizations

In figure 10 we focus on the smallest mask case. This case for the difference of pdfs seems not to have a clear shape (see lower-right panels of figure 3 and 6). Therefore we increased the number of extractions in order to improve the statistics. In figure 10 we show the results for differences of the pdf’s between masked and unmasked distribution of D23 and S23 with 1000000 extractions with binning of 0.01 and 0.05. This demonstrates that even the smaller mask case exhibits qualitatively the same behavior of the other cases (even if weaker quantitatively). Anyway, we suggest to use larger number of realizations when small masks are exploited.
Figure 10. Impact of the number of realizations on the considered estimators for the smaller mask case (93% of sky coverage). All the panels are referring to the smaller mask case (93% of sky coverage). Upper panels: difference of the pdfs between masked and full sky of D23 for the masked case. Lower panels: difference of the pdfs between masked and full sky of S23 for the masked case. Left panels: 1000000 extractions with binning of 0.01. Right panel: 1000000 extractions and binning of 0.05. See lower-right panels of figures 3 and 6 for comparison with the corresponding pdf built with 300000 extractions. All the panels present the counts (y-axis) versus the statistic (x-axis). See also the text.

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