Neutrinos: Theoretical Aspects

Eligio Lisi
Istituto Nazionale di Fisica Nucleare (INFN), Sezione di Bari, Bari, Italy
Dipartimento Interataeneo di Fisica “Michelangelo Merlin,” U. di Bari, Bari, Italy
Via Amendola 173, 70126 Bari, Italy
E-mail: eligio.lisi@ba.infn.it

Abstract. Neutrino physics is a vibrant field of research, ignited by the experimental discoveries of neutrino masses and mixings, and by the quest to understanding their theoretical implications. We shall cover some theoretical and phenomenological aspects of this field, stemming from both recent and time-honoured research areas. In particular, we shall focus on (1) the status of the standard three-neutrino framework, (2) the nearby theory landscape, (3) neutrinos as a bridge to flavor physics and (4) neutrinos as a bridge to high energy and mass scales.

1. Prologue
The presentation of this contribution at TAUP 2019 in Toyama [1] started by showing the picture of a nice red bridge in the Toyama Castle Park, taken from an angle where one can see only the nearby trees and the first steps of the bridge, but not its end or the farthest landscape. It was intended as a metaphor for neutrino masses and mixings as a bridge to new physics. In a sense, we have just taken the first steps along a very interesting road, but we cannot see yet what really lies beyond. In this situation, theory can provide a guidance towards possible promising directions, both nearby and far away. Given the explosion of interest in this field during the last two decades (see Fig. 1), a comprehensive review would be a prohibitive task. Only a few theoretical (and phenomenological) aspects will be covered herein, in an attempt to provide an overarching context for a number of related contributions presented at this conference.

Figure 1. Yearly number of preprints with #neutrino# in the title, from the InSPIRE database.
2. Where we are: $3\nu$, circa 2019

Beautiful flavor oscillation experiments [2] have established a solid three-neutrino ($3\nu$) paradigm for the interpretation of basically all the available neutrino data. Possible explanations of a few anomalous data via a fourth (sterile) neutrino at the eV scale are reviewed elsewhere [3].

The mass-mixing parameters defining the $3\nu$ framework are summarized in Fig. 2. Global data analyses [4, 5, 6] converge on a detailed $3\nu$ picture where five parameters are known with increasing precision, while five parameters are still unknown, although with interesting hints emerging for two of them. Figure 3 summarizes the results of a preliminary update of the global $3\nu$ analysis in [4], as presented at this Conference. In particular, we know the two squared mass differences $\Delta m^2$ and $\delta m^2$ and the three mixing parameters $\sin^2 \theta_{ij}$ within $1\sigma$ errors ranging from 1.3% (for $\Delta m^2$) to $\sim 5\%$ (for $\sin^2 \theta_{23}$). The convergence of different data on the known parameters is remarkable, and their accuracy will be improved by further oscillation searches.

The status of the five $3\nu$ unknowns is as follows. The sign of $\Delta m^2$, namely, the ordering of $\nu$ masses, can be probed via interference of $\pm \Delta m^2$-driven oscillations with $Q$-driven oscillations, where $Q$ is a quantity with known sign, e.g., the $\nu$ interaction energy in matter. Interesting hints in favor of $+\Delta m^2$ (normal ordering) have been emerging from recent data, up to an overall level $>3\sigma$. At the same time, nearly-maximal CP violation ($\sin \delta \sim -1$) is also being favored by $\nu_\mu \rightarrow \nu_e$ data in combination with others [2], up to almost $2\sigma$; see [4, 5, 6] for details. The absolute $\nu$ mass scale has been limited in the sub-eV range by the very first KATRIN results shown at this conference [7], in agreement with cosmological constraints, but with no positive detection yet. The angle $\theta_{23}$ still flips around $\pi/4$ with a slight—but not yet convincing—overall preference for the second octant. Last but not least, the profound question concerning the Dirac or Majorana nature of neutrinos [8] remains elusive, despite intense $0\nu\beta\beta$ searches [9].

![Figure 2](https://example.com/figure2.png)

**Figure 2.** Parameters (masses, mixings, phases) of the standard three-neutrino framework.

![Figure 3](https://example.com/figure3.png)

**Figure 3.** Status of $3\nu$ knowns and unknowns, circa 2019 (from a preliminary update of [4]).
3. The nearby theory landscape

The emergence of the $3\nu$ framework represents not only a great experimental achievement but also a success for the theory of neutrino oscillations, especially concerning propagation in matter, i.e., in a background of fermions — whose density may change in space and time. Matter effects continue to be an endless source of inspiration even in the “trivial” case of constant density, whose linear algebra has recently revealed structures that had not been noticed by mathematicians [10]. Interest remains sustained in high-density matter effects [11], that may also be time-varying in astrophysical sources such as gamma-ray-burst [12] or core-collapse supernovae (SN).

The physics of SN neutrinos [13, 14] is particularly challenging in terms of oscillations, since the neutrinos themselves, having a density comparable to electrons and nucleons for a few seconds, are a relevant part of the SN fermion background. Understanding the very subtle (nonlinear, collective, unstable) effects resulting from neutrino-neutrino interactions [15, 16] probably represents the most difficult open problems in $\nu$ flavor transition theory. Possible couplings between flavor changes and explosion dynamics, not yet implemented in the current SN simulation toolbox, would involve an even higher level of complexity. Future experimental detections of SN neutrino events will greatly help to test theoretical expectations for the SN neutrino flavor evolution, possibly in combination with associated electromagnetic and gravitational wave (GW) signals in a multi-messenger approach [17].

Within the standard model realm, it is being increasingly recognized that strong-interaction effects play an ubiquitous role in neutrino physics, and should be better understood theoretically, in order to match the needs of current and future neutrino research programs. For instance, theoretical progress in modeling hadronic and nuclear processes is crucial to improve upon: (i) estimates of atmospheric [18] and accelerator neutrino spectra; (ii) neutrino interaction cross sections in various target nuclei at accelerator energies [19, 20]; (iii) constraints on low-energy processes, e.g. the axial coupling $g_A$ in weak decays [21], form factors in coherent scattering [22, 23], and reactor spectral shapes from the summation method [24]; (iv) constraints on low-$x$ parton distribution functions, to refine the estimates of prompt backgrounds in ultra-high-energy cosmic neutrino searches. These and similar areas of theoretical research are converging towards a new inter-disciplinary field that might be named as Electroweak Nuclear Physics [25], with the goal of understanding the response of nuclei to neutrino and other electroweak probes, using improved nuclear models across a wide energy range.

Going beyond the standard model (BSM), one may consider perturbations or “stress tests” of the $3\nu$ paradigm in terms of either new neutrino states or new interactions, that may either alter the determination of known $3\nu$ parameters or be degenerate with hints of the unknown ones (e.g., via unitarity violations or extra mixing terms or phases). E.g., nonstandard four-fermion interactions (NSI, with parametric couplings $\epsilon G_F$) are being actively studied to test possible anomalous results in accelerator, reactor and atmospheric neutrino experiments [26, 27, 28]. Perturbation effects of additional (sterile) neutrino states, either light of heavy, also represent an important line of research that is reviewed elsewhere [3].

Going farther beyond, one may consider both extra states and interactions, e.g., in terms of right-handed neutrinos as a bridge (so-called “portal”) to weakly coupled BSM physics (so-called “dark” sector, e.g., dark matter) [29, 30, 31]. This is an effective way to describe neutrino-related BSM phenomena in the absence of a full-fledged BSM theory, that can provide further stress tests of the standard $3\nu$ scenario.

Summarizing, there is still a lot to learn in neutrino theory, both within the standard scenario (e.g., studying matter and self-interaction effects, or electroweak nuclear physics) and “slightly” beyond, (e.g., assuming parametrized deviations from the $3\nu$ framework, or effective BSM portals). Of course, one hopes to reach also more ambitious objectives, far beyond the current realm, as described in the following.
4. A bridge to flavor physics

The standard model lacks an organizing principle for flavor (i.e., for Yukawa couplings $Y$). In principle, the discovery of neutrino masses and mixing might have shed some light on the $Y$’s structure and on the fermion flavor puzzle. Unfortunately, no obvious pattern has emerged from the $3\nu$ paradigm, and we do not know if its parameters are “accidental” or suggestive of some symmetry (such as a possible $\mu$-$\tau$ flavor symmetry based on $\theta_{23} \sim \pi/4$). Theoretical ideas can thus range from “anarchical” models to more elaborated assumptions, invoking specific symmetries for leptons and possibly for quarks (or, alternatively, texture zeros [32]).

Inter-family flavor symmetries based on invariance under permutation groups (such as $A_4$, $S_4$, $A_5$) or on grand-unified theory (GUT) assumptions linking leptons and quarks, are often adopted to make sense of the observed values for the neutrino mixing angles; see Fig. 4 for a qualitative description. Various assumptions and models can, in principle, be discriminated by measuring more precisely the known $3\nu$ parameters, as well as by determining the unknown ones. A drawback of these models is that specific symmetry realizations often require large and carefully crafted scalar sectors (so-called flavons with their v.e.v.’s).

Recently, a novel and more constrained approach to $\nu$ model building has been proposed, in terms of the so-called modular forms [33]. The approach is inspired by string theory, where the $10D \rightarrow 4D$ reduction is obtained by compactifying three times two dimensions on a torus. For each of the three torii, the coordinate ratio $\tau$ (called modulus) is subject to transformations known as “modular symmetries” that do not change the physics. It turns out that, if one posits $Y = Y(\tau)$ and supersymmetry, with modest tuning one can reasonably explain the known mass-mixing parameters for leptons, and possibly for quarks [34]. This approach to the flavor problem has sparked a wide interest for its rather constrained features, that make it highly predictive and testable, in comparison with scenarios assuming ad hoc scalar fields and v.e.v.’s.

![Figure 4](image1.png)

**Figure 4.** Schematic view of symmetry assumptions in neutrino (and fermion) model building.

![Figure 5](image2.png)

**Figure 5.** Basics of the modular-invariance approach to model building.
5. A bridge to higher scales

In a recent paper [35], E. Witten has reviewed from a modern theoretical perspective the time-honoured idea that global symmetries should only be approximate. In particular, there are no strong reasons to think that lepton and baryon numbers ($B$ and $L$) should be exactly conserved, or that CP should be conserved a priori by strong interactions. The related symmetry breaking operators would lead to Majorana neutrinos, proton decay, and axions, respectively; see also Fig. 6. Although such observables may stem from different sources of new physics, the discovery of one of them (apart from becoming a landmark discovery in science) would lend credibility to the occurrence of the others, and it would support the overarching theoretical idea of nonconservation of global symmetries. These arguments are not weakened by the absence of BSM physics at the $O(10)$ TeV scale of the LHC, since rather different scales may be involved.

The Majorana mass term in Fig. 6 is suppressed by one power of the new mass scale $M$, as also emerging from the celebrated see-saw mechanism, where $m_\nu \sim Y^2 v^2 / M$ and $v$ is the electroweak scale. Higher-dimensional operators with stronger suppression may also be envisaged [36]. In any case, Majorana neutrinos may offer a great opportunity to relate $m_\nu$ to a much higher scale $M \gg v$, where one expects not only heavy neutrino partners $N$ but possibly the origin of new processes as proton decay (if the $B$-breaking scale is the same, as in GUTs).

In this context, the celebrated leptogenesis scenario remains a very interesting theoretical research line, with important phenomenological links between low and high mass scales [37, 38]. We remind that C-, CP- and L-violating out-of-equilibrium decays of heavy neutrinos in the early universe, at a temperature $T \sim M$, can generate a lepton asymmetry $\Delta L$, and eventually a baryon asymmetry $\Delta B$ via nonperturbative processes (sphalerons); see also Fig. 7. This process involves (non)equilibrium evolution in an expanding and cooling universe, and significant work is needed to assess the ranges of $M$ and $T$ that successfully lead to the observed ratio $\eta = n_B/n_\gamma$ in given models. From the high-scale viewpoint, it has been shown that typical models can work from $M \sim 10^{14}$ down to $M \sim 10^6$ GeV; with some tuning (e.g., with nearly mass-degenerate $N$’s) one can even get $M \sim v$ and make contact with current heavy $N$ searches. From the low-scale viewpoint it has been shown that, in a wide range $M \sim 10^6-10^{13}$ GeV, leptogenesis can proceed via the $3\nu$ Dirac phase ($\delta$) and/or the Majorana phases ($\alpha$, $\beta$) [37]. In all cases, assessing the Majorana nature of neutrinos in $0\nu\beta\beta$ decay remains a most important challenge.

![Figure 6](image1.png)

**Figure 6.** Implications of broken global symmetries: extra Lagrangian terms and observables.

![Figure 7](image2.png)

**Figure 7.** Leptogenesis: $N$ decay into charged leptons $L$ and scalars $H$ via CP-violating $Y$’s.
A recent twist in testing leptogenesis scenarios involves the physics of gravitational relics from the big bang. The generation of the baryon asymmetry $\eta$ presumably takes place at $t < 1s$, while the two (experimentally consistent) snapshots of $\eta$ are taken at later times, from observations related to big-bang nucleosynthesis [BBN, $t \sim O(10^2)$s] and cosmic microwave background [CMB, $t \sim O(10^4)$ yr]. We have directly observed CMB relics ($\gamma$'s) and we might be on the (long) path to detect BBN relics ($\nu$'s) [39, 40]. In the far future, we may also hope to observe the background of relic GW from $t < 1s$, whose power spectrum can, in principle, encode some information about heavy $N$ decays occurring in the same epoch; this may happen if the heavy $N$ physics at scale $M$ involves the generation of cosmic defects or phase transitions, leading to extra components in the primordial GW spectrum [42, 41]. In a sense, after witnessing the birth of multi-messenger astronomy, we may start dreaming about far-future observations of multi-messenger relics.

6. Epilogue
Neutrino masses and mixings are a bridge to new physics. We have just taken the first steps within the $3\nu$ paradigm, and we are now exploring knowns and unknowns in this framework, as well as its theoretical surroundings. We do not know what lies farther beyond, but we have many promising theoretical ideas to go forward, with different degrees of phenomenological testability. There may be a long and difficult path after crossing the first $3\nu$ bridge, but there are also bright prospects for discovering beautiful landscapes and new vistas in particle physics. Neutrinos will never cease to amaze us!

Acknowledgments
E.L. thanks the TAUP 2019 organizers for kind hospitality in Toyama, Japan. This work was partially supported by the INFN initiative TAsP and by the research grant number 2017W4HA7S “NAT-NET: Neutrino and Astroparticle Theory Network” under the program PRIN 2017 funded by the Italian Ministero dell’Istruzione, dell’Università e della Ricerca (MIUR).

References
[1] TAUP 2019 Indico Website: http://www-kam2.icrr.u-tokyo.ac.jp/indico/event/3/timetable
[2] H. Tanaka, these Proceedings.
[3] D. Caratelli, these Proceedings.
[4] F. Capozzi, E. Lisi, A. Marrone and A. Palazzo, Prog. Part. Nucl. Phys. 102, 48 (2018) [arXiv:1804.09678 [hep-ph]].
[5] I. Esteban, M. C. Gonzalez-Garcia, A. Hernandez-Cabezudo, M. Maltoni and T. Schwetz, JHEP 1901, 106 (2019) [arXiv:1811.05487 [hep-ph]].
[6] P. F. de Salas, D. V. Forero, C. A. Ternes, M. Tortola and J. W. F. Valle, Phys. Lett. B 782, 633 (2018) [arXiv:1708.01186 [hep-ph]].
[7] G. Drexlin, these Proceedings.
[8] F. Simkovic, these Proceedings.
[9] Y.-H. Kim, these Proceedings.
[10] P. B. Denton, S. J. Parke, T. Tao and X. Zhang, “Eigenvectors from Eigenvalues,” arXiv:1908.03795 [math.RA].
[11] Z.-z. Xing, these Proceedings.
[12] J. Carpio, these Proceedings.
[13] A. Harada, these Proceedings.
[14] K. Nakazato, these Proceedings.
[15] S. Abbar, these Proceedings.
[16] M. Zaizen, these Proceedings.
[17] T. Takiwaki, these Proceedings.
[18] M. Honda, these Proceedings.
[19] T. Yuan, these Proceedings.
[20] C. Wilkinson, these Proceedings.
[21] H. Ejiri, these Proceedings.
[22] A. Studenikin, these Proceedings.
[23] V. Sharma, these Proceedings.
[24] A. Marrone, these Proceedings.
[25] E. Lisi, talk at NuInt 2018, 12th International Workshop on Neutrino Interactions in the Few-GeV Region (L’Aquila, Italy, 2019), https://indico.cern.ch/event/703880.
[26] M. Tortola, these Proceedings.
[27] D. Muhammed, these Proceedings.
[28] S. Blot, these Proceedings.
[29] E. Peinado, these Proceedings.
[30] W. Marks, these Proceedings.
[31] S.-F. Ge, these Proceedings.
[32] H. Borghain, these Proceedings.
[33] F. Feruglio, “Are neutrino masses modular forms?,” arXiv:1706.08749 [hep-ph].
[34] M. Tanimoto, these Proceedings.
[35] E. Witten, “Symmetry and Emergence,” Nature Phys. 14, 116 (2018) [arXiv:1710.01791 [hep-th]].
[36] M. Hirsch, these Proceedings.
[37] G. Mangano, these Proceedings.
[38] A. Gi, these Proceedings.
[39] M. Spinrath, these Proceedings.
[40] H. Okui, these Proceedings.
[41] H. Murayama, these Proceedings.