Grade 2 students’ representation of mathematical ideas on multiplication through open approach and lesson study

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Abstract. The objectives of this research were 1) to analyze students’ representation of mathematical ideas 2) to study students’ learning ability in Open Approach and Lesson Study class by specifying criterion in the learning achievement test on the title "multiplication (1) that was not less than 75% of the total number of students pass 75% of full score. The target group of this research were 42 students of grade 2/1 of The Demonstration school Primary level (Mardindang) Khon Khaen University, Khon Kaen Province. This research was one short case study design. Students were tested one time after finishing learning Unit. The instruments used for data collection were 1) 6 Multiplication (1) learning lesson plans 2) learning achievement test of multiplication (1). Learning achievement results were analyzed by comparing with specified criterion as average and percentage. Students’ representation of mathematical ideas were analyzed from note recording, VDO recording, teaching recording, and students’ works. The results revealed that: 1) 95.23% of total number of students obtained score passing criterion 75% of full score. 2) Students showed 5 ways of the representation of mathematical ideas which were following 2.1) Material manipulation, students used wooden blocks to represent the ideas so that they could find the answer. They understood the meaning of group and element by grouping the amount of element. 2.2) Using pictures, Students drew a picture of blocks, squares or diagrams. They drew pictures as elements equally of each group to represent the way to find the answer. 2.3) Speaking, Students explained verbally how to count equally, how to add repeatedly and recite the multiplication table. 2.4) Writing, Students wrote the numbers as repeated addition. They wrote the amount which increased equally. They explained the amount of equal element of each group. Moreover, they wrote the mathematical sentence of addition and multiplication. It was noted that the multiplicand was amount of group and the multiple was elements of group. 2.5) Using real situation, Students shared their own real life experiences that was connected coherently with the mathematical problem given by the teacher. They also explained their ideas by using mathematical symbol of multiplication to show how to get the answer.

Keywords: Open Approach, Lesson Study, Mathematical Representation.

1. Introduction

New paradigm of learning management system in 21st century focus on the development of people and learning style which is emphasize about people to be wise, learning community, learning how to learn, learning collaboration and letting the learner construct knowledge by themselves [2]. This is consistent with Inprasitha research [4]. He claims that arranging mathematics class activities by using Open Approach, in order that students are able to learn mathematics with their potentiality simultaneously their self-determination and group determination. Open Approach emphasizes students
have good attitude with learning mathematics more than just plentiful of mathematics knowledge. Moreover, this method provides students opportunities to learn by themselves. They have chances to be free to think. So, there are lots of answers in one open-ended problem that the teacher provides. Definitely, Open-ended problem situation is the key point to specify learning activities in class. As well as, teaching materials are the important key to urge students’ attention. There are 4 steps of Open Approach method 1) Posing Open-ended Problem; this is the step that the teacher post open-ended problem or task in order to let them feel like this is their problem. 2) Student’s self-learning; this is the step that the teacher get the students solve the problem as a group. The teacher must encourage students to show their different representation ideas. 3) Whole Class discussion and comparison; this step, the students come to the front and share his/her ideas or groups’ ideas about the way to get the answer. 4) Summarization through connecting students’ Mathematical ideas emerged in the classroom; this step is about conceptual ideas summarization and connection. The teacher help them to expand their ideas so that they can discover their mathematical representation ideas. Learning activity in Mathematics class by using Open Approach is merged in the innovation of Lesson Study. It is a process to develop teaching profession. The teachers will work collaboratively to enhance and develop efficiently students' thinking process and lesson plan. A team works together to plan, observe, analyze and revise all lesson plans [5], [6], [14], [15], [16].

Mathematical Idea is an insight idea in students’ mind during thinking about problem solution. They must need a concrete instrument like wording or symbol to approach this kind of idea. Individual creates his/her meaningful idea to discover the mathematical ideas by the self, Insight learning is considered as cognitive perception or Mental models. It can be observed while the, students are busily working. Getting them to know the concept of Mathematical idea, the students must be provided an opportunity to listen and see through speaking, writing an explanation, or writing a symbol. It can be implied that the representation of Mathematical idea supported students to understand the meaning of Mathematical idea. Lesh [8] explained the meaning of the representation of Mathematical ideas that students have a way to communicate in Mathematics when they are provided an opportunity to show the representation of Mathematical concept with different ways. It is implied that students are able to connect the ideas and understand in Mathematics. Lesh specified 5 ways of the representation of Mathematical ideas as these following 1) Manipulative aids 2) Pictures / Diagram 3) Written symbols 4) Verbal symbols 5) Real-life context.

Understanding of multiplication and division are harder than addition and subtraction. Generally, the definition of multiplication and division are complicated for students. The representation of Mathematical idea of kids will showed us naturally how do the kids think mathematically and understand about multiplication. As above mentioned, the researcher is interested in studying the representation of mathematical idea and students learning ability on the title "Multiplication in Open Approach and Lesson Study class.

2. Research Objectives / Purpose of study

1. To analyze students’ representation of mathematical ideas on titled multiplication of grade 2 students of The Demonstration School Primary level (Mordindang) Khon Kaen University by using Open Approach and Lesson Study

2. To study students’ learning ability in Open Approach and Lesson study class by specifying criterion in learning achievement test on title “multiplication” that: not less than 75% of the total number of students pass 75% of full score.

3. Conceptual framework
4. Terms and Definitions

4.1. Personal relevance (PR)
Mathematical Representation means the way to convey ideas by using, mathematical patterns according to individual conception and apprehension so that let others perceive his/her ideas. There were 5 ways as following

1) Material manipulative Representation refers to learning materials manipulation. Students manipulate learning materials given by the teacher in order that they are faced a problem situation and explain the meaning of situation. Learning materials which are related to problem situation such as pictures or objects were provided for problem situation.

2) Pictures Representation means using pictures to convey or reflect ideas and understanding such as drawing instead of writing number.

3) Verbal Representation refers to expression of mathematical conception by speaking or interpreting the problem situation verbally. For example Students describe their ideas verbally. Students talk about what they have seen or what they have manipulated

4) Written Representations refers to expression of mathematical conception by writing. For example students write whatever to show their ideas, students write what they have seen or what they have manipulated on

5) Real-world situation Representation refers to the connection between Students' answer with real situation. Students may explain verbally or write to show their ideas for example, the amount of nine means the amount of 9 apples in a plate.

4.2. Multiplication means addition of equal amount
It is showed in two number which are the amount of a specified number and the amount of an equal specified number of each time. For instance, 2+2+2=3x2 or 5+5+5=3x5

5. Research Methodology
Methodology regarded qualitative research. The students’ representation on multiplication will be interpreted through participant observation, lesson study members’ reflection, studentas’ works and multiplication learning achievement test.

5.1 Target group
The target group was 42 students (21 male and 21 female students) in grade 2/1 of The Demonstration School Primary level (Mordindang) Khon Kaen university.

5.2 Research instruments

Research instruments included 1) six lesson plans of unit 9 on the title "multiplication" by using Open Approach and Lesson Study, one hour was spent for one lesson plan. 2) Learning achievement test of multiplication (1) which is consisted of 10 items for objective test and 3 items subjective test. Open Approach was four steps of running activities in classroom which were: 1) posing the open-ended problem 2) students’ self-learning in solving open-ended problem 3) students’ collaborating in discussion for solving problems together and 4) students’ collaborating in concluding learning outcome, and Lesson Study was the collaboration of the researcher and co-researcher which worked together in three important phases of Lesson Study: 1) collaborative lesson planning, 2) classroom observation and 3) lesson reflection, one hour was spent for one lesson plan.

6. Data collection

1. A Researcher and team developed lesson plans from lesson plans which were used in a previous semester.
2. A Researcher followed the lesson as planning
3. A researcher collected data from VDO recording, teaching recording, (1)
4. A researcher and team collaboratively discussed, reflected and analyzed the representation of Mathematical ideas of students.

7. Findings

The findings revealed that students showed 5 ways the representation of Mathematical idea on the title "multiplication (1) in Open Approach and Lesson Study class, which was based on Lesh's conception. [8]. These could be seen as provided in the table 1.

7.1. Manipulative Representation by using materials

Students manipulated learning materials to represent ideas so that they could find the answer. For example, students used blocks as a tool to connect ideas with problem situation. They moved equally blocks as the representative of the number of the group. In addition, students also divided equally amount of stars into group. Learning material played very important role in teaching management, because it helped, students to discover variety of the representation of Mathematical ideas. Thailand Basic Education Core Curricular B.E. 2551 stated that learning materials must assistant students to learn systematically mathematical concepts in each level. And the advantages of learning material help students to clarify an abstract idea to be a concrete idea. Lastly, suitable manipulative aid can help students develop thinking process and understanding in abstract.
Table 1. The learning result of understanding Mathematical conception of multiplication (1).

Figure 2. Manipulative Representation
### Activity | Mathematical Representation | Outcome |
|---|---|---|
| Manipulative aid | Pictures | Written symbols | Verbal symbols | Real-life context |
| Let’s party! | ✓ | ✓ | ✓ | ✓ | - understanding the meaning of group and element  
- learning how to find the amount of object |
| How many…? | ✓ | ✓ | ✓ | ✓ | - understanding the meaning of multiplication.  
- writing representation of multiplication by using equation sentence and interpret Mathematical sentence of multiplication |
| Amusement Park | ✓ | ✓ | ✓ | ✓ | - understanding the meaning of multiplication.  
- writing representation of multiplication by using equation sentence and interpret Mathematical sentence of multiplication |
| How can we group the apple? | ✓ | ✓ | ✓ | ✓ | - using blocks as amount of object  
- using blocks to represent mathematical sentence of multiplication |
| Should we add…? | ✓ | ✓ | ✓ | ✓ | - understanding the meaning of “equal” |
| “The star” | ✓ | ✓ | ✓ | ✓ | - Applying knowledge of multiplication in situation. |

#### 7.2 Pictures Representation

Students drew pictures to represent the way to find the amount of object and to represent mathematical sentence of multiplication (1). The students drew blocks and pictures as a symbol that they are familiar like as a circle. They drew equally the picture of object as group to represent the way to discover the answer. This connected to how students are able to write a multiplication sentence. The amount of group is multiplicand and the amount of element is multiplier (see as figure 3)
7.3 Verbal Representation
Students explained verbally the way they discover the answer which was coherently with problem situation by counting equally number, number repeated addition and multiplication table recitation.

7.4 Written Representation
Explanation by writing symbols and words were used as the students' idea representation. Students explained in terms of writing of the repeated addition symbol sentence, a writing of equal number increasing. Meanwhile others represented their idea by writing a multiplication sentence. They noted that the amount of group of object x the amount of element. The amount of group of object was multiplicand and the amount of element was multiplier.

Figure 5. Written Representation

7.5 Real world situation Representation

Students represented their ideas by using their real life experiences. They connected their familiar experiences to mathematical problems by explaining about multiplication sentence, multiplication product, addition, and sum. For instance, “There were 3 stables of horse. Each stable has 6 horse. How many horse are there?” They discovered the way to get the answer by using multiplication which was 3x6 = 18. 3 was the amount of stables (the amount of group). 6 was the amount of horse in each stable (the amount of elements).

Figure 6. Real world situation Representation
The result of students learning achievement found that 40 students which was as 95.23% of the total number of student obtained score passing criterion at or higher 75%. As showed in table 2

Table 2. A summary of students' learning achievement test result on the title “multiplication (1)"

| The amount of student | Test score | Average Test Score | The amount of students who pass 75% or higher 75% of full score |
|-----------------------|------------|--------------------|---------------------------------------------------------------|
| Total Point           | Score criterion at 75% | Highest | Lowest | Average Score | Percent of test score | The amount of student | Percent |
| 42                    | 30         | 22.5               | 30     | 19           | 28.5               | 95.00               | 40      | 95.23   |

Students not less than 75% of the total number of students pass 75% of full score. The research revealed that students’ ability of learning was higher than the specified criterion. There are 40 students (95.23%) who passed the test criterion at 75% of full score. This result is consistent with Inprasitha [5] [6]. He points out that the method that arranging learning activity by using Open Approach emphasize students’ thinking process. Mathematical Open-ended problem is posted as a question. All the lesson plans are written collaboratively from the teacher team in order to improve students thinking ability and develop effectively the lesson plan. The teacher team collaboratively design, observe, analyze and revise all lessons together. Students are encouraged to be a part of problem so that they can learn mathematics by themselves. Students learn meaningfully from aid manipulation especially authentic materials. Moreover, facilitating a whole class discussion is supported, too. Discussion time is the most important. During the discussion, students have an opportunity to compare the similar or different ideas of their friend [10]. Concept reflection supports students’ mathematical communication and learning process. This study discover 5 ways of the representation of Mathematical ideas, which are Manipulative representation, Picture representation, Verbal representation, Written representation, and Real-world situation representation. At all events, students convey various conceptual ideas and understanding by themselves.

8. Data collection

1. The research should get more teachers to help and observe class activities so that the researcher obtain comprehensively information.

2. For the next study, the researcher should study more about higher level of the representation of idea in open-ended problem situation because students' idea are the important key that lead to learning process, when they perceive mathematical concept, they can apply and be able to use the skills in any situation.

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