**TECHNICAL NOTE**

An investigation into the dependence of virtual observation point-based specific absorption rate calculation complexity on number of channels

Stephan Orzada¹,²,³ | Safi Akash¹ | Thomas M. Fiedler¹ | Fabian J. Kratzer¹ | Mark E. Ladd¹,²,⁴,⁵

¹Medical Physics in Radiology, German Cancer Research Center (DKFZ), Heidelberg, Germany
²Erwin L. Hahn Institute for MRI, University Duisburg-Essen, Essen, Germany
³Radiation Oncology, University Hospital Heidelberg, Heidelberg, Germany
⁴Faculty of Physics and Astronomy, Heidelberg University, Heidelberg, Germany
⁵Faculty of Medicine, Heidelberg University, Heidelberg, Germany

Correspondence
Stephan Orzada, German Cancer Research Center (DKFZ), Im Neuenheimer Feld 280, 69120 Heidelberg, Germany.
Email: stephan.orzada@dkfz.de

**Purpose:** This study aims to find a relation between the number of channels and the computational burden for specific absorption rate (SAR) calculation using virtual observation point-based SAR compression.

**Methods:** Eleven different arrays of rectangular loops covering a cylinder of fixed size around the head of an anatomically correct voxel model were simulated. The resulting Q-matrices were compressed with 2 different compression algorithms, with the overestimation fixed to a certain fraction of worst-case SAR, median SAR, or minimum SAR. The latter 2 were calculated from 1e6 normalized random excitation vectors.

**Results:** The number of virtual observation points increased with the number of channels to the power of 2.3–3.7, depending on the compression algorithm when holding the relative error fixed. Together with the increase in the size of the Q-matrices (and therefore the size of the virtual observation points), the total increase in computational burden with the number of channels was to the power of 4.3–5.7.

**Conclusion:** The computational cost emphasizes the need to use the best possible compression algorithms when moving to high channel counts.

**KEYWORDS**
computational burden, MRI, SAR, VOP compression

1 INTRODUCTION

The constant drive to higher main magnetic field strengths in MRI is accompanied by challenges in terms of excitation homogeneity and increased specific absorption rate (SAR).¹⁻³ This has accelerated the development of multi-channel transmit approaches to counter these effects. Some examples of these methods are RF shimming,⁴,⁵ kT-points,⁶ 2D spokes,⁷ 3D tailored RF pulses,⁸ transmit SENSE,⁹,¹⁰ and Time Interleaved Acquisition of Modes.¹¹ Whereas these methods differ in multiple aspects, they all rely on altering the amplitudes and phases of the signals in the transmit channels. By altering these, not only are the transmitted magnetic field and therefore...
the $B_1^+$ changed but the electric field is as well. Consequently, SAR hotspots change both their location and amplitude when these techniques are applied. In contrast to single-channel systems, where the SAR can be described by a function of input power, the situation is therefore more complicated in multi-channel transmit systems. In these systems, the electric field distribution is a function of a complex excitation vector containing the amplitudes and phases of all channels. Therefore, SAR calculation in multi-channel systems is a multidimensional problem. To stay below the SAR limits defined by the regulatory guidelines, both global and locally averaged SAR need to be calculated and supervised.

Because SAR cannot be measured directly in vivo, information on the SAR distribution in the body must be derived from numerical RF simulations in body models. From the simulation results, Q-matrices can be computed that provide the SAR value for a certain volume through a vector–matrix–vector multiplication with the excitation vector. These matrices can then be used for pulse design with SAR constraints as well as for online SAR supervision. Due to the way in which SAR averaging is performed, the number of Q-matrices from a simulation with an anatomical body model can be on the order of several millions. Thus, SAR calculation has a high computational cost, increasing the computational effort for online SAR supervision as well as transmit pulse calculation with SAR constraints.

Especially for online supervision, the SAR calculations must be fast because the results must arrive within a limited time. For the common bandwidth of 500 kHz used in MRI pulses, the sampling rate according to Nyquist must be at least 1 MHz (1 million samples per second). This means that the SAR has to be calculated for each microsecond by performing vector–matrix–vector multiplications with all necessary matrices.

To reduce the number of matrices necessary for the calculation, compression algorithms have been proposed in the literature that trade overestimation against the number of matrices used for SAR calculation. With these algorithms, the number of matrices can be reduced from several million down to several hundred.

As the transmit channel count of MRI systems increases, increased computational burden is to be expected when calculating SAR because the number of entries in the Q-matrices as well as the excitation vectors grow. Commercial systems are available with 8 or 16 transmit channels; custom-built systems with up to 32 channels have been presented. A simple calculation provides the number of floating-point operations required to calculate SAR for a single complex Q-matrix (or virtual observation point (VOP)) with $N$ channels as $8N^2 + 6N - 2$ (assuming 6 floating point operations per multiplication and 2 per addition). Yet, to the best of the authors’ knowledge, no investigation has been undertaken to determine how the number of VOPs behaves with an increasing number of channels. This would be important to estimate the actual increase in computational burden for online SAR supervision when increasing the channel count in the future.

In this work, we investigate the increase in the number of SAR matrices after compression for a fixed array geometry with increasing number of elements.

## METHODS

A total of 11 arrays tuned at the proton resonance frequency at 7 Tesla was simulated in CST Microwave Studio 2019 (Simulia, Dassault Systèmes SE, Vélizy-Villacoublay, France). Rectangular loops in 1 to 4 rows of 2 to 6 elements, resulting in array counts of 2, 3, 4, 6, 8, 10, 12, 15, 16, 20 and 24, were bent to conform to a cylinder with a height of 220 mm and a diameter of 260 mm around the head of a male human body model (“GUSTAV,” Simulia). The array configurations are displayed in Figure 1. Arrays with channel counts of 6 and above were arranged in 2 or more rows to distribute them equally across the circumferential dimensions because it has been shown that multi-row arrays allow for higher degrees of freedom; that is, they allow for more complex field patterns. Especially the work by Guérin et al. is of interest here as the ratio of wavelength to array diameter is similar. The loops were gapped, and the gaps were bridged with capacitors suitable to achieve a homogenous current distribution. Current sources were used to feed the loops to avoid the necessity for other decoupling methods. The feed power was normalized to 1 W total accepted power. Metal structures were modeled as perfect electric conductors; capacitors were modeled as lossless. To reduce the simulated volume, the body model with a resolution of 2.08 by 2.08 by 2.0 mm$^3$ was truncated at the shoulders. After truncation, it contained 14 different tissue types.

After simulation, the resulting Q-matrices for 10 g-averaged local SAR were evaluated by applying 1e6 random excitation vectors normalized to 1 W power to calculate the respective maximum local SAR ($SAR_{\text{max,local}}$). From these 1e6 results per coil array, the mean and the median were calculated. The worst-case SAR ($SAR_{\text{wc}}$) was calculated from the highest eigenvalue of all Q-matrices of each array. Because finding the minimum possible $SAR_{\text{max,local}}$ within the excitation space is nontrivial, the mean of the 1000 lowest values from each set of 1e6 $SAR_{\text{max,local}}$ values was calculated and will be labeled “min$_{0.1\%}$” throughout this document.
The 11 array models used in this study. The gaps in the loops were bridged with capacitors appropriate to achieve a homogenous current distribution along the loops. The size of the array was chosen to cover the whole head. The channel count goes from 2 to 24 channels.

Two different compression algorithms were used: the algorithm presented by Eichfelder and Gebhardt\textsuperscript{18} because it is the most widely used compression algorithm; and the algorithm presented by Orzada et al.\textsuperscript{20} because, to the best of the authors’ knowledge, it is currently the algorithm with the most effective compression. The algorithm by Orzada et al. was started with 16 times the targeted overestimation and a reduction factor of 0.5 to reach the targeted overestimation at the fifth iteration. The version of the algorithm used in this work is available on SourceForge (https://sourceforge.net/projects/enhanced-sar-compression/files/Enhanced\%20VOP\%20Compression\%20GPU\%20Target\%20Fast/).

Because it has been shown that using a fraction of the SAR\textsubscript{wc} can lead to very high relative overestimation,\textsuperscript{23,26} compression was not only performed with a fixed maximum allowed overestimation of 5\% of the respective SAR\textsubscript{wc} but also with 25\% of the respective median SAR as well as 100\% of the respective min\textsubscript{0.1\%} SAR.

The number of VOPs obtained for each number of transmit channels was evaluated. A linear regression as the simplest approach was performed using the “cfit”-function in MatLab 2020a (MathWorks, Natick, MA) on the log–log data to estimate the dependency of the number of VOPs on the number of channels, which effectively results in the fit function:

\[ N_{\text{VOPs}} = a(N_{\text{channel}})^b, \]

where \( a \) and \( b \) are free parameters.

3 | RESULTS

The simulations resulted in 2–4 million Q-matrices per coil arrangement. Four representative examples for the distribution of SAR\textsubscript{max,local} are shown in Figure 2 for 2, 6, 15, and 24 channels. Min\textsubscript{0.1\%}, median, mean, and SAR\textsubscript{wc} are marked for reference. Whereas for 2 channels the SAR values are almost evenly distributed between the minimum and the worst-case local SAR, for higher channel counts the values are clustered around the median. The SAR\textsubscript{wc} increases with the number of channels. This is even more visible in Figure 3A, which shows the respective min\textsubscript{0.1\%}, median, and SAR\textsubscript{wc} for the various channel counts. Whereas median and min\textsubscript{0.1\%} are quite similar for all arrays, the SAR\textsubscript{wc} steeply increases with the number of channels. When the overestimation is a fixed fraction of SAR\textsubscript{wc}, this leads to an increasing relative overestimation, as shown by the relative overestimation elevation factor in Figure 3B. This factor provides a measure of how an overestimation based on the SAR\textsubscript{wc} translates to the overestimation relative to median and min\textsubscript{0.1\%}. For 24 channels in this example, the relative overestimation for the lowest SAR values is \( \sim 27 \) times higher than for the SAR\textsubscript{wc}. This means that an overestimation of 10\% of the SAR\textsubscript{wc} corresponds to up to 270\% maximum relative overestimation.

Compression with the algorithm presented by Eichfelder and Gebhardt\textsuperscript{18} resulted in up to 30,860 VOPs for 24 channels and for an overestimation of 25\% of the median. The results are shown in a log–log representation.
FIGURE 2  Examples of the distribution of the maximum local SAR for 1e6 random excitations. The excitations were normalized to 1 W total input power. Whereas for 2 channels the values are almost evenly distributed between the minimum and the worst-case local SAR, for higher channel counts the values are clustered around the median. SAR, specific absorption rate; W, watt.

in Figure 4. The data are not well represented by the regression model when using a fixed fraction of the SARwc for overestimation with the respective array, as indicated by the adjusted $R^2$-value of only 0.73. Using a fixed fraction of median and min$_{0.1\%}$ showed a clear relationship of the number of VOPs proportional to the number of channels to the power of $\sim$3.7, albeit with a 95% confidence interval of ±0.49 and ±0.70, respectively.

Compression with the algorithm presented by Orzada et al.\textsuperscript{20} resulted in up to 904 VOPs for 24 channels and for an overestimation of 25% of the median. The results are shown in a log–log representation in Figure 5. Again, the data are not well represented by the regression model when using a fixed fraction of the SARwc for overestimation with the respective array, as indicated by the adjusted $R^2$-value of 0.87. Using a fixed fraction of median and min$_{0.1\%}$ showed a clear relationship of the number of VOPs proportional to the number of channels to the power of $\sim$2.3, with a 95% confidence interval of ±0.23 and ±0.33, respectively.

4 | DISCUSSION

Most of the possible SAR$_{\text{max,local}}$ values were clustered around the median, whereas the SARwc increased steeply with the number of elements in the array. The steep increase is unsurprising considering that the fixed input power of 1 W can be distributed over an ever-smaller volume when increasing the number of array elements due to the diminishing size of each element. In theory, the maximum achievable 10 g-averaged local SAR with 1 W
Figure 3 (A) Median, $\min_{0.1\%}$, and worst-case SAR are shown for all arrays. Whereas median and $\min_{0.1\%}$ are quite similar for all arrays, the worst-case SAR increases steeply with the number of channels. (B) The relative overestimation elevation factor gives a measure of how an overestimation based on the worst-case SAR translates to an overestimation relative to median and $\min_{0.1\%}$. For 24 channels in this example, the relative overestimation for the lowest SAR values is $\sim 27$ times higher than for the worst-case SAR. This means that an overestimation of 10% of the worst-case SAR corresponds to up to 270% maximum relative overestimation.

The results in this work demonstrate the importance of understanding that when compressing SAR matrices, one must consider the larger ratio between SAR$_{wc}$ and the median and the minimum possible SAR. It is obvious that more channels generally lead to a larger ratio, which in turn leads to a much-increased relative overestimation when using a fixed fraction of SAR$_{wc}$ for overestimation, as proposed in the original work on VOP compression by Eichfelder and Gebhardt.

Using a fixed overestimation of 5% of the respective SAR$_{wc}$ did not result in VOP counts that were well represented by the linear regression model. The results in Figure 3 show a steep increase in SAR$_{wc}$ from 12 to 24 channels, whereas the number of VOPs shows a decline from 12 to 24 channels. This might be explained by the fact that while the complexity of the SAR matrices increases, the average relative overestimation also increases, resulting in fewer VOPs.

When the actual median relative overestimation is considered by using a fraction of the respective median for maximum allowed overestimation, a clear trend is visible for both compression algorithms for an increasing number of VOPs with increasing number of channels. The same holds for considering the actual maximum relative overestimation by using a fraction of the respective $\min_{0.1\%}$ for maximum allowed overestimation.

As a sidenote, we also simulated an 8-channel loop array with the loops aligned in a single row because this is the most commonly used configuration (not included in the figures). This turned out to produce fewer VOPs than the $4 \times 2$ and even the $3 \times 2$ array. For example, the algorithm by Orzada et al.\cite{20} compressing with 25% of the respective median, resulted in VOP counts of 36, 136, and 69 VOPs, respectively. Again, in the light of the work of Guerin et al.\cite{24}, this is not unexpected.

Another point worth noting is the difference in the exponent for the 2 algorithms used in this work. The increase in the number of VOPs is steeper for Eichfelder and Gebhardt’s algorithm\cite{18} than it is for the algorithm presented by Orzada et al.\cite{20}. From the literature, it is known that the latter algorithm provides superior compression to the former. The results presented here show that this superiority is magnified with an increasing number of channels.
Although there is no good fit of the regression model to the data when using 5% of the worst-case SAR for the overestimation, using 25% of the median or 100% of min$_{0.1\%}$ will provide a good fit. The number of VOPs increases with the number of channels to approximately the power of 3.7. VOP, virtual observation point.

Although there is no good fit of the regression model to the data when using 5% of the worst-case SAR for the overestimation, using 25% of the median or 100% of min$_{0.1\%}$ will provide a good fit. The number of VOPs increases with the number of channels to approximately the power of 2.3.
Considering that the number of floating point operations for calculating SAR with a single Q-matrix with $N$ channels is $O(N^2)$, with the results obtained in this paper we predict the relation between floating point operations and number of channels to be $N_{\text{flop}} = O(N^{4.3})$ (Orzada et al.) to $N_{\text{flop}} = O(N^{5.7})$ (Eichfelder and Gebhardt) for the same relative error. This means that when going from 8 to 32 channels, the computational burden for SAR supervision rises by a factor of 390 to 2700. To give the reader an impression of the actual burden, let us consider the 200 VOPs that Jin et al. present as the maximum possible on their 8-channel system. With 1 million samples per second, this means 112 Gflops, which is manageable on GPUs. Having the same accuracy in a 32-channel system would lead to 43–301 Tflops, whereas modern GPUs are theoretically capable of ~40 Tflops.

The results are also important for parallel transmit pulse calculation when SAR constraints are used. Fiedler et al. have shown that the overestimation can lead to unintuitive results in arrays with high channel counts in which the VOP-based SAR is reduced in the optimization while the actual SAR increases. This indicates that optimization on a VOP set with a high overestimation does not necessarily result in optimal actual SAR; thus, low overestimation should be used in pulse calculation. Whereas increasing the number of channels provides more degrees of freedom, which can be used to reduce SAR, high overestimation can potentially diminish this advantage. It should also be emphasized that overestimation cannot be used as a safety factor to account for uncertainties between the model and the actual experimental configuration because the actual overestimation varies between 0 and a maximum absolute overestimation defined when performing the compression; in the case it is 0, there would be no safety factor.

The actual impact on the computational effort on transmit pulse calculation is not trivial to calculate because it strongly depends on the algorithm used and the relation between the computational effort for the transmit field optimization and the SAR calculation; however, the results in this work indicate that the increase in computational burden is higher than through the impact of larger matrices alone. In this work, we did not investigate the impact of the number of channels on SAR performance itself. In the light of other works, the performance is expected to increase with the number of channels; however, as we could show here it does so at a steeply increasing cost in computational burden for SAR calculation.

The large increase in computational burden indicates that for higher channel counts the best possible compression algorithms, ideally with postprocessing, should be used to keep the computational burden for online SAR supervision manageable and the error in RF pulse calculation low.

## 5 | CONCLUSION

An investigation into SAR calculation complexity based on VOP compression has been presented. It was shown that the number of VOPs increases with the number of channels to the power of 2.3 to 3.7 when keeping the relative overestimation constant, depending on the compression algorithm. Together with the increasing size of the Q-matrices themselves, the computational burden rises with the number of channels to the power of 4.3 to 5.7. This emphasizes the need for the best possible compression algorithms when moving to high channel counts.

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## ORCID

Stephan Orzada @ https://orcid.org/0000-0001-9784-4354
Thomas M. Fiedler @ https://orcid.org/0000-0002-1556-375X
Fabian J. Kratzer @ https://orcid.org/0000-0001-7454-1641

## REFERENCES

1. Hoult DI, Phil D. Sensitivity and power deposition in a high-field imaging experiment. J Magn Reson Imaging. 2000;12:46-67.
2. van de Moortele PF, Akgun C, Adriany G, et al. B(1) destructive interferences and spatial phase patterns at 7 T with a head transceiver array coil. Magn Reson Med 2005;54:1503–1518.
3. Erturk MA, Li X, van de Moortele PF, Uguribil K, Metzger GJ. Evolution of UHF body imaging in the human torso at 7T: technology, applications, and future directions. Top Magn Reson Imaging. 2019;28:101-124.
4. Collins CM, Liu W, Swift BJ, Smith MB. Combination of optimized transmit arrays and some receive array reconstruction methods can yield homogeneous images at very high frequencies. Magn Reson Med. 2005;54:1327-1332.
5. Metzger GJ, Snyder C, Akgun C, Vaughan T, Uguribil K, van de Moortele PF. Local B1+ shimming for prostate imaging with transceiver arrays at 7T based on subject-dependent transmit phase measurements. Magn Reson Med. 2008;59:396-409.
6. Cloos MA, Boulant N, Luong M, et al. kT-points: short three-dimensional tailored RF pulses for flip-angle homogenization over an extended volume. Magn Reson Med 2012;67:72–80.
7. Setsompop K, Alagappan V, Gagoski B, et al. Slice-selective RF pulses for in vivo B1+ inhomogeneity mitigation at 7 Tesla using parallel RF excitation with a 16-element coil. Magn Reson Med 2008;60:1422–1432.
8. Saekho S, Yip CY, Noll DC, Boada FE, Stenger VA. Fast-kz three-dimensional tailored radiofrequency pulse for reduced B1 inhomogeneity. Magn Reson Med. 2006;55:719-724.
9. Grissom W, Yip CY, Zhang Z, Stenger VA, Fessler JA, Noll DC. Spatial domain method for the design of RF pulses in multicoil parallel excitation. Magn Reson Med. 2006; 56:620-629.

10. Katscher U, Bornert P, Leussler C, van den Brink JS. Transmit SENSE. Magn Reson Med. 2003;49:144-150.

11. Orzada S, Maderwald S, Poser BA, Bitz AK, Quick HH, Ladd ME. RF excitation using time interleaved acquisition of modes (TIAMO) to address B1 inhomogeneity in high-field MRI. Magn Reson Med. 2010;64:327-333.

12. International Electrotechnical Commission (IEC). IEC 60601-2-33. Medical electrical equipment: Part 2-33: Particular requirements for the basic safety and essential performance. Revision 4.0. 2022.

13. Fiedler TM, Ladd ME, Bitz AK. SAR simulations & safety. Neuroimage. 2018;168:33-58.

14. Zhu Y, Alon L, Deniz CM, Brown R, Sodickson DK. System and SAR characterization in parallel RF transmission. Magn Reson Med. 2012;67:1367-1378.

15. Graesslin I, Vernickel P, Bornert P, et al. Comprehensive RF safety concept for parallel transmission MR. Magn Reson Med. 2014;74:589-598.

16. Gumbrecht R, Fontius U, Adolf H, et al. Online Local SAR Supervision for Transmit Arrays at 7T. In Proceedings of the 21st Annual Meeting of ISMRM, Salt Lake City, UT, 2013. p. 4420.

17. IEC/IEEE. International Electrotechnical Commission (IEC). IEC/IEEE 62704-1. Determining the peak spatial-average specific absorption rate (SAR) in the human body from wireless communications devices, 30 MHz to 6 GHz – part 1: general requirements for using the finite-difference time-domain (FDTD) method for SAR calculations. Revision 1.0.2017.

18. Eichfelder G, Gebhardt M. Local specific absorption rate control for parallel transmission by virtual observation points. Magn Reson Med. 2011;66:1468-1476.

19. Lee J, Gebhardt M, Wald LL, Adalsteinsson E. Local SAR in parallel transmission pulse design. Magn Reson Med. 2012;67:1566-1578.

20. Orzada S, Fiedler TM, Quick HH, Ladd ME. Local SAR compression algorithm with improved compression, speed, and flexibility. Magn Reson Med. 2021;86:561-568.

21. Orzada S, Solbach K, Gratz M, et al. A 32-channel parallel transmit system add-on for 7T MRI. PLoS One. 2019;14:e0222452.

22. Auerbach EJ, Delabarre L, Van de Moortele PF, et al. An integrated 32-channel transmit and 64-channel receive 7 Tesla MRI system. In Proceedings of the 25th Annual Meeting of ISMRM, Honolulu, HI, 2017. p. 1218.

23. Fiedler TM, Orzada S, Floser M, et al. Performance analysis of integrated RF microstrip transmit antenna arrays with high channel count for body imaging at 7 T. NMR Biomed. 2021;34:e4515.

24. Guerin B, Gebhardt M, Serano P, et al. Comparison of simulated parallel transmit body arrays at 3 T using excitation uniformity, global SAR, local SAR, and power efficiency metrics. Magn Reson Med. 2015;73:1137-1150.

25. Wu X, Tian J, Schmitter S, Vaughan JT, Ugurbil K, Van de Moortele PF. Distributing coil elements in three dimensions enhances parallel transmission multiband RF performance: a simulation study in the human brain at 7 Tesla. Magn Reson Med. 2016;75:2464-2472.

26. Orzada S, Fiedler TM, Bitz AK, Ladd ME, Quick HH. Local SAR compression with overestimation control to reduce maximum relative SAR overestimation and improve multi-channel RF array performance. MAGMA. 2021;34:153-163.

27. Jin J, Weber E, Destruel A, et al. An open 8-channel parallel transmission coil for static and dynamic 7T MRI of the knee and ankle joints at multiple postures. Magn Reson Med. 2018;79:1804-1816.

28. Orzada S, Fiedler TM, Quick HH, Ladd ME. Post-processing algorithms for specific absorption rate compression. Magn Reson Med. 2021;86:2853-2861.

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