Positive Covert Capacity of the MIMO AWGN Channels

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Abstract—Covert communication, i.e., communication with a low probability of detection (LPD), has attracted a huge body of work. Recent studies concluded that the covert coding rates of the discrete memoryless channels and the additive white Gaussian noise (AWGN) channels are diminishing with the blocklength, \( n \). Only \( O(\sqrt{n}) \) nats can be transmitted covertly and reliably over such channels. As a result, the covert capacity is zero. In this paper, we show that the square root law can be overcome if the number of transmitting antennas is scaled. Thus, the covert capacity of the multiple-input multiple-output (MIMO) AWGN channels is shown to converge to the capacity of the MIMO AWGN channels under certain conditions. As a result, a positive covert capacity can be achieved. In addition, we characterize the covert capacity and the covert degrees-of-freedom of the MIMO AWGN channels. Besides, we derive the order-optimal scaling laws for the number of covert nats when the covert coding rate is diminishing. Moreover, we provide the scaling laws for the well-conditioned, the unit-rank MIMO AWGN channels as well as when the channel state information of the illegitimate receiver is only known to have a bounded spectral norm. Lastly, secrecy and LPD conditions are jointly investigated.

Index Terms—LPD communication, covert communication, stealth communication, square root law, physical layer security, secrecy capacity, effective secrecy, degrees-of-freedom, unit-rank MIMO.

I. INTRODUCTION

WIRELESS communication is prone to eavesdropping and hence cryptographic techniques are utilized to achieve secure wireless communication. However, in many situations, it is required that the entire communication session remains undetectable. This is a fundamentally different problem from secure communication and it needs to be addressed at the physical layer.

In this paper, the problem of covert communication over the MIMO AWGN is studied. First, the problem of maximizing the covert coding rate is considered and the relation between the n-letter and the single-letter LPD constraint is derived. When the covert coding rate is diminishing with the blocklength, upper and lower bounds on the scaling are derived and are shown to coincide. Only \( O(\sqrt{n}) \) nats can be transmitted covertly and reliably over such channels. Then, the scaling laws for special MIMO channels of interest such as the well-conditioned, the unit-rank MIMO AWGN channels as well as when the channel state information (CSI) of the illegitimate receiver is only known to have a bounded spectral norm are deduced and are shown to be \( O(\sqrt{n}) \) nats. Next, we incorporate physical layer security (PLS) into the problem and consider it simultaneously with the LPD constraint. One may think that the LPD constraint automatically leads to strong secrecy. Interestingly, however, an illegitimate receiver can decode the received signal even if the existence of communication may not be detected. Therefore, we show that covert and secure communication needs to be addressed independently.

Next, when the covert coding rate is not diminishing, the covert capacity and the covert degrees-of-freedom (DoF) are characterized. Last, we show that the square root law of covert communication can be overcome even with unknown CSI of the illegitimate receiver. We show that the covert coding rate need not be diminishing, even under the LPD constraint, as the number of transmitting antennas increases, similar to the massive MIMO setting, hence the covert capacity approaches the channel capacity even without the knowledge of CSI of the illegitimate receiver. Note that make only mild assumptions on the CSI of the attacker, trying to detect the communication session. This is in contrast to the existing studies, many of which assume the presence of global CSI to achieve LPD.

Recently, there has been a growing interest in the notion of the low probability of detection (LPD) communication, referred to as covert communication. The LPD condition was introduced in \([1]\) where some basic limits of the secure and the covert communications over quasi-static channels are investigated when space-time coding over the multiple-input multiple-output (MIMO) antennas is utilized. Inspired by Steganography \([2]\), it has been established that the maximum number of nats that can be transmitted reliably with LPD is \( O(\sqrt{n}) \) nats for both discrete memoryless channels (DMC) \([3, 4]\) and the additive white Gaussian noise (AWGN) channels \([3, 5]\). Besides, the square root law still holds for the MIMO AWGN channels, when the CSI of the illegitimate receiver is only known to have a bounded spectral norm \([6]\). Therefore, the covert coding rate is diminishing with the blocklength. However, in more practical settings, when the illegitimate receiver has uncertainty about the channels’ conditions or the transmission time \([7, 8]\), the covert coding rate can be non-diminishing and hence a positive covert capacity can be achieved.

Moreover, a general assumption is that a secret is shared between the legitimate entities, while the illegitimate receiver tries to detect whether there is a communication or not \([3, 5]\). In \([10]\), the capacities of the DMC and AWGN channels are derived when a sufficiently long secret key is shared and the coding rate is proved to be non-diminishing when the transmitter has a knowledge of the interference signal.
The shared secret can be the codebook itself or a secret binary vector that is added only once with the codeword [3] similar to the one-time padding developed by Shannon. Later, it is shown that $O(\sqrt{n})$ nats can be transmitted reliably and covertly using $O(\sqrt{n})$ pre-shared nats [4]. Nonetheless, covert communication can be achieved even when there is no shared secret but this requires the illegitimate receiver’s channel to be noisier than the legitimate receiver’s channel [4, 7, 11].

More recently, the MIMO wireless communication is proposed to improve wireless security as well as to offer diversity and multiplexing gains. Recent attentions focus on the massive MIMO systems and incorporate beamforming techniques that are being integrated into the future cellular networks [12]. Massive MIMO does not only offer diversity or multiplexing gains but also plays an important role in the delay constrained applications, since the time diversity techniques cannot be exploited in low latency applications. Further, the massive MIMO antenna array achieves high gain and directivity that offer an LPD and high confidentiality against eavesdropping attacks [13]. Therefore, PLS benefits from the MIMO systems and beamforming techniques. Accordingly, an extensive work of the PLS established the secrecy capacity of the MIMO wiretap channels as in [14].

The contributions of this paper can be summarized as follows:

- We derive the exact order-optimal scaling law for the maximum number of covert nats that can be transmitted reliably over the MIMO AWGN channels and show that $O(\sqrt{n})$ nats can be transmitted covertly and reliably when the coding rate is diminishing. This suggests that the square-root law holds for both SISO [3–5] and MIMO channels.
- We derive the exact order-optimal scaling law for the maximum number of covert and secure nats that can be transmitted reliably over the MIMO AWGN channels and show that $O(\sqrt{n})$ nats can be transmitted covertly, securely and reliably when the coding rate is diminishing, which coincides with [4] and Model 1 in [11].
- We characterize the covert capacity and the covert DoF of the MIMO AWGN channels when the CSI of the illegitimate receiver is known. We show that the covert capacity can be positive as long as there exists a null-space of the CSI of the illegitimate receiver.
- We provide conditions under which the covert capacity can converge to the channel capacity (without the LPD constraint) as the number of transmitting antennas increases even with unknown CSI of the illegitimate receiver, i.e., the square root law of covert communication can be overcome. This result is parallel to the findings in [15], which deal with eavesdropping attacks.

Organization: The paper is organized as follows. The system model and the problem formulation are presented in Section II. In addition, the scaling laws of the maximum number of covert nats are provided in Section III. Besides, the scaling laws of the maximum number of covert and secure nats are provided in Section IV. In Section V the covert capacity and the covert DoF of the MIMO AWGN channels are characterized. Moreover, the asymptotic performance of covert communication with the number of transmitting antennas is developed in Section VI. Furthermore, we provide a numerical examples in Section VII and the paper is concluded in Section VIII.

Notations: In the rest of this paper, $[x]^+$ denotes $\max(0, x)$, $(\cdot)^T$ denotes the transpose, $(\cdot)^{\dagger}$ denotes the conjugate transpose, $I_N$ denotes the identity matrix of a size $N$, $\text{tr}(\cdot)$ denotes the matrix trace operator, $|A|$ denotes the determinant of a matrix $A$, and $\text{diag}(a_1, a_2, \ldots, a_m)$ stands for a diagonal matrix with diagonal elements $(a_1, a_2, \ldots, a_m)$. Meanwhile, denote by $A \geq B$ when the difference $A - B$ is positive semidefinite. The distribution of a circularly symmetric complex Gaussian random vector $X$ with a mean vector $\mu$ and a covariance matrix $Q$ is denoted by $CN(\mu, Q)$. Moreover, the expectation and the variance of a random vector $X$ are denoted by $E[X]$ and $\text{var}[X]$, respectively. The entropy of a discrete random variable $x$ is $H(X)$ and the differential entropy of a continuous random variable $x$ is $h(X)$, while the mutual information between two random variables $x$ and $y$ is denoted by $I(X; Y)$. Further, $\lim$ denotes the limit inferior, and $\| \cdot \|_p$ and $\| A \|_{op}$ are the $L_p$ norm and the operator (spectral) norm of a matrix $A$, i.e., the maximum eigenvalue of $A$, respectively. The standard order notation $f(n) = O(g(n))$ is adopted to denote an upper bound on a function $f(n)$ that is asymptotically tight, i.e., there exist constants $m$ and $n_0 > 0$ such that $0 \leq f(n) \leq m g(n)$ for all $n > n_0$.

![Fig. 1. Covert communication over MIMO AWGN channels.](image-url)
Moreover, $\mathbf{H}_b \in \mathbb{C}^{N_b \times N_a}$, denoted by Bob’s channel, and $\mathbf{H}_w \in \mathbb{C}^{N_w \times N_a}$, denoted by Willie’s channel, are the channels’ coefficient matrices of Alice-Bob’s and Alice-Willie’s channels, respectively. Throughout this paper, $\mathbf{H}_b$ and $\mathbf{H}_w$ are assumed to be deterministic. We start with assuming that both are known. Then, in Section VII we relax this assumption. Across $n$ channel uses, the transmitted signal vector is defined by $\mathbf{X}_n \triangleq \{ \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n \}$ and $\mathbf{Y}_n^b, \mathbf{Y}_n^w, \mathbf{Z}_n^b$, and $\mathbf{Z}_n^w$ are defined similarly.

The AWGN channel is an ergodic channel scenario for which the channel matrix is a deterministic constant and independent of the channel uses, i.e., time-invariant channels \[15\]. The capacity, $C$, denotes the largest possible reliable communication rate. The capacity is an asymptotic quantity, achieved as the blocklength, $n$, goes to infinity. For a finite blocklength, we define the maximal coding rate, $R(n, \epsilon)$, where the probability of error, $P_e$, does not exceed a certain value $\epsilon$ for a blocklength, $n$, where

$$C = \lim_{\epsilon \downarrow 0} \lim_{n \to \infty} R(n, \epsilon). \quad (2)$$

**Definition 1.** A $(2^{nR}, n, \epsilon, \delta_L, \delta_S)$ code consists of:

1. A uniformly distributed message set, $\mathcal{M} \triangleq [1 : 2^{nR}]$.
2. A randomized encoder $f : \mathcal{M} \to \mathbb{C}^n$, that assigns a codeword $\mathbf{x}_n(m)$ to each message $m \in \mathcal{M}$ under an average transmission power constraint $P$ on every codeword, $\mathbf{x}_n(m) \triangleq \{ \mathbf{x}_1(m), \mathbf{x}_2(m), \ldots, \mathbf{x}_n(m) \}$, as follows:

$$\sum_{i=1}^{n} |\mathbf{x}_i(m)|^2 \leq nP. \quad (3)$$

3. A decoder, $\phi : \mathbb{C}^n \to \mathcal{M} \cup \{\epsilon\}$, that assigns an estimate, $\hat{m} \in \mathcal{M}$, or an error message, $\epsilon$, to each received sequence, $\mathbf{y}_n$.

4. The encoder-decoder pair satisfies:
   a) An average probability of error constraint:
      $$P_e(n, C) \leq \epsilon, \text{ where } C \text{ is the codebook associated with the code and } P_e(n, C) = \mathbb{P} \{M \neq \hat{M} \}.$$
   b) An LPD constraint: $\delta_L \leq \delta_S$.
   c) A secrecy constraint: $\delta_s \leq \delta_S$.

In the following, the LPD constraint is discussed. It is worthwhile noting that this paper follows a similar approach that is adopted in \[3\], which studies the covert communication over the SISO AWGN channels. To detect the communication between Alice and Bob, Willie performs an optimal statistical hypothesis testing using the received observations, $\mathbf{Y}_n^b = \mathbf{y}_n^b$, to decide whether Alice is transmitting, the true hypothesis $\mathcal{H}_1$, where the probability distribution of Willie’s observation is denoted by $\mathbb{P}_{\mathbf{Y}_n^b}$, or not, the null hypothesis $\mathcal{H}_0$, where the probability distribution of Willie’s observation is denoted by $\mathbb{P}_{\mathbf{Z}_n^b}$. Alice’s objective is to guarantee that the sum of the probability of rejecting $\mathcal{H}_0$ when it is true, type I error, $\alpha$, and the probability of rejecting $\mathcal{H}_1$ when it is true, type II error, $\beta$, by Willie’s test is as inefficient as a blind test for which $\alpha + \beta = 1$ \[16\].

The optimal test employed by Willie minimizes the sum of the error probabilities over all possible tests. The sum of the error probabilities is given by \[17\]:

$$\alpha + \beta = 1 - \mathcal{V} (\mathbb{P}_{\mathbf{Y}_n^b} ; \mathbb{P}_{\mathbf{Z}_n^b}), \quad (4)$$

where $\mathcal{V} (\mathbb{P}_{\mathbf{Y}_n^b} ; \mathbb{P}_{\mathbf{Z}_n^b}) = \frac{1}{2} \Vert \mathbb{P}_{\mathbf{Y}_n^b} (\mathbf{y}_n^b) - \mathbb{P}_{\mathbf{Z}_n^b} (\mathbf{z}_n^b) \Vert_1$ is the variational distance between the two distributions $\mathbb{P}_{\mathbf{Y}_n^b}$ and $\mathbb{P}_{\mathbf{Z}_n^b}$, with $\mathbb{P}_{\mathbf{Y}_n^b} (\mathbf{y}_n^b)$ and $\mathbb{P}_{\mathbf{Z}_n^b} (\mathbf{z}_n^b)$ are the density functions of $\mathbb{P}_{\mathbf{Y}_n^b}$ and $\mathbb{P}_{\mathbf{Z}_n^b}$, respectively. Further, the variational distance is upper bounded by the KL divergence by Pinsker’s inequality \[5\, 18\] as:

$$\mathcal{V} (\mathbb{P}_{\mathbf{Y}_n^b} ; \mathbb{P}_{\mathbf{Z}_n^b}) \leq \frac{1}{2} \mathcal{D} (\mathbb{P}_{\mathbf{Y}_n^b} \| \mathbb{P}_{\mathbf{Z}_n^b}), \quad (5)$$

where

$$\mathcal{D} (\mathbb{P}_{\mathbf{Y}_n^b} \| \mathbb{P}_{\mathbf{Z}_n^b}) \triangleq \mathbb{E}_{\mathbb{P}_{\mathbf{Y}_n^b}} [\log \mathbb{P}_{\mathbf{Y}_n^b} (\mathbf{y}_n^b) - \log \mathbb{P}_{\mathbf{Z}_n^b} (\mathbf{z}_n^b)]. \quad (6)$$

Accordingly, to guarantee the LPD by Willie’s optimal detector, Alice chooses some $\delta_L$ to upper bound $\mathcal{D} (\mathbb{P}_{\mathbf{Y}_n^b} \| \mathbb{P}_{\mathbf{Z}_n^b})$ for the desired probability of detection. Hence, Alice can achieve the LPD goal by designing the signaling strategy, based on the amount of information available, subject to the LPD constraint.

**Definition 2.** Maximal covert coding rate:

$$R(n, \epsilon, \delta_L) \triangleq \sup \left\{ R : \exists a (2^{nR}, n, \epsilon, \delta_L) \text{ code} \right\}, \quad (7)$$

where a $(2^{nR}, n, \epsilon, \delta_L)$ code satisfies the following LPD constraint:

$$\mathcal{D} (\mathbb{P}_{\mathbf{Y}_n^b} \| \mathbb{P}_{\mathbf{Z}_n^b}) \leq 2 \delta_L^2, \delta_L \geq 0, \quad (8)$$

while ensuring that the illegitimate receiver’s sum of the detection error probabilities is lower bounded by $1 - \delta_L$.

Consequently, covertness can be achieved by bounding the sum of the error probabilities at Willie as $\alpha + \beta \geq 1 - \delta_L$, regardless of the operating point on Willie’s ROC curve for any $\alpha$ \[18\]. It is worth to mention that the KL divergence in the LPD constraint is more restrictive than the variational distance. In addition, the LPD constraint with $\mathcal{D} (\mathbb{P}_{\mathbf{Y}_n^b} \| \mathbb{P}_{\mathbf{Z}_n^b})$ is also stricter than $\mathcal{D} (\mathbb{P}_{\mathbf{Z}_n^b} \| \mathbb{P}_{\mathbf{Z}_n^b})$ and gives a lower covert rate \[19\]. Although the KL divergence does not constrain the ROC curve tightly, minimizing $\mathcal{D} (\mathbb{P}_{\mathbf{Y}_n^b} \| \mathbb{P}_{\mathbf{Z}_n^b})$ is a sufficient condition to guarantee that Willie’s test is ineffective \[16\]. Therefore, the more restrictive LPD metric in Equation \[8\] is adopted in this paper as the LPD metric, i.e., the input distribution employed by Alice to generate the codebook should satisfy Equation \[8\]. Hence, the asymptotic condition, $\lim_{n \to \infty} \delta_L = 0$, corresponds to the LPD notion.

1A randomized encoder is exploited in the encoding process to capture the physically degraded Willie’s channel for secure communication (see Remark 9).

2Interpreting the KL divergence in the exponents of the error probabilities, $\alpha$ and $\beta$, when discriminating two different distributions is not valid in this setting since the distribution $\mathbb{P}_{\mathbf{Z}_n^b}$ is not i.i.d. \[19\]

3The Gaussian distribution is not optimal for the LPD constraint with $\mathcal{D} (\mathbb{P}_{\mathbf{Z}_n^b} \| \mathbb{P}_{\mathbf{Z}_n^b})$ \[19\].

4The covert communication under different LPD metrics is investigated in \[18\].
Definition 3. The scaling, with the blocklength, of the maximum number of nats that can be transmitted covertly, while satisfying the average probability of error constraint and the average power constraint, is defined as follows \([3]\):

\[
L \triangleq \lim_{n \to \infty} \lim_{\epsilon \to 0} \sqrt{\frac{n}{2 \delta_L^2}} R(n, \epsilon, \delta_L). \tag{9}
\]

Next, we state the first problem considered in this paper.

**Problem 1.** (With a pre-shared secret) Characterize the exact scaling law, \(L\), of the maximum number of covert nats over the MIMO AWGN channels as defined in Equation (9), when the maximal covert coding rate is diminishing.

This problem is investigated thoroughly in Section [III] and results for special cases of interest such as the well-conditioned, the unit-rank MIMO AWGN channels and when the CSI of Willie is only known to have a bounded spectral norm are deduced. In the second half of the paper, Section [IV] we consider LPD, jointly with secrecy. To formally state the problem, we provide the following definition.

**Definition 4.** Maximal covert secrecy coding rate:

\[
R(n, \epsilon, \delta_L, \delta_S) \triangleq \sup \{ R : \exists a (2^nR, n, \epsilon, \delta_L, \delta_S) \text{ code} \}, \tag{10}
\]

where a \((2^nR, n, \epsilon, \delta_L, \delta_S)\) code satisfies one of the following secrecy constraints:

- Weak secrecy: \(\frac{1}{n} I(M; Y^n_w) \leq \delta_S \),
- Strong secrecy: \(I(M; Y^n_w) \leq \delta_S \), \(\tag{11}\)
- Effective secrecy: \(D(\mathbb{P}_M Y^n_w || \mathbb{P}_M \mathbb{P} Z^n_w) \leq \delta_S \),

where \(\mathbb{P}_M\) is the probability distribution of the message, \(M\), while \(\mathbb{P}_M Y^n_w\) is the joint probability distribution of the message and Willie’s observation and \(D(\mathbb{P}_M Y^n_w || \mathbb{P}_M \mathbb{P} Z^n_w) = I(M; Y^n_w) + D(\mathbb{P}_M Y^n_w || \mathbb{P}_M Z^n_w)\).

The asymptotic condition, \(\lim_{n \to \infty} S_s = 0\), corresponds to different secrecy notions that exist in the literature.

**Definition 5.** The scaling, with the blocklength, of the maximum number of nats that can be transmitted covertly and securely, while satisfying the average probability of error constraint and the average power constraint, is defined as follows:

\[
L_S \triangleq \lim_{n \to \infty} \lim_{\epsilon \to 0} \sqrt{\frac{n}{2 \delta_L^2}} R(n, \epsilon, \delta_L, \delta_S). \tag{12}
\]

**Problem 2.** (Without a pre-shared secret) Characterize the exact scaling law, \(L_S\), of the maximum number of covert and secure nats over the MIMO AWGN channels as defined in Equation (12), when the maximal covert secrecy coding rate is diminishing.

Similar to the AWGN channel capacity, the covert capacity can be defined in terms of the maximal covert coding rate as follows:

\[
C_L = \lim_{\epsilon \to 0} \lim_{n \to \infty} R(n, \epsilon, \delta_L). \tag{13}
\]

The DoF is a metric that provides a capacity approximation as the total transmission power approaches infinity. More specifically, for a system with transmission power, \(P\), the DoF is defined in terms of the sum capacity, \(C(P)\), as the pre-log of capacity at high signal-to-noise ratio \([21]\), namely,

\[
D = \lim_{P \to \infty} \frac{C(P)}{\frac{P}{2 \log P}}. \tag{14}
\]

Similarly, we define the covert DoF as follows:

\[
D_c = \lim_{P \to \infty} \frac{C_L(P)}{\frac{P}{2 \log P}}. \tag{15}
\]

The following problem characterizes the nonzero covert capacity and the covert DoF, which is studied in Section [V].

**Problem 3.** Characterize the covert capacity and the covert DoF of the MIMO AWGN channels when the CSI of Willie is known and there exists a null-space between Alice and Willie.

**Problem 4.** (With unknown CSI of Willie)

1) Can the square root law of covert communication be overcome if the number of transmitting antennas is scaled. In particular, under what conditions can we achieve, \(\lim_{N \to \infty} R_n(n, \epsilon, \delta_L) = \lim_{N \to \infty} R(n, \epsilon)\), for any given \(\delta_L \geq 0\)?
2) What is the lower bound on the number of transmitting antennas that satisfies a given probability of detection?

III. COVERT COMMUNICATION OVER THE MIMO AWGN CHANNELS

In this section, the communication system is computationally-secured using a pre-shared secret. Thus, covertness is investigated by analyzing the LPD constraint. The following theorem extends the result in [3] for the DMC to the MIMO AWGN channels with infinite input and output alphabets. In this theorem, we derived that the LPD constraint can be represented by the single-letter KL divergence instead of the \(n\)-letter KL divergence.

**Theorem 1.** The maximum number of nats that can be transmitted reliably over the MIMO AWGN channels, when the covert coding rate is diminishing, scales like \(L\) which is given by:

\[
L = \lim_{n \to \infty} \sqrt{\frac{n}{2 \delta_L^2}} \max_{Q \geq 0, \text{tr}(Q) \leq P} \log \left| \frac{1}{\sigma_b^2} H_b Q H_b^\dagger + I_N \right| \tag{16}
\]

subject to: \(D(\mathbb{P} Y_w || \mathbb{P} Z_w) \leq \frac{2 \delta_L^2}{n}\).

Moreover, the input distribution that maximizes the first-order approximation of the covert capacity, while minimizing \(D(\mathbb{P} Y_w || \mathbb{P} Z_w)\), is the zero-mean circularly symmetric complex Gaussian distribution with a covariance matrix \(Q\).

**Proof.** Appendix [A]

**Remark 1.** The analyses and results for the scaling laws in this paper are given when the rate under the LPD constraint is diminishing with the blocklength, e.g., the allocated power should be reduced to satisfy the constraint. This case is when there is no null-space between Alice and Willie. On the other hand, when there exists a null-space, the average power constraint can be satisfied with equality, i.e., maximum
transmission power is used, and the LPD condition is satisfied as well. In this case, the covert coding rate is not diminishing and $O(\eta)$ covert nats can be transmitted reliably to Bob. Therefore, the average power constraint of the input signal is meaningful in the MIMO AWGN channels formulation, although it is inactive in the SISO AWGN channels due to the LPD constraint.\footnote{The LPD constraint, for the SISO AWGN channels requires the average power to go to zero as the blocklength tends to infinity.}

Remark 2. When the computation of the covert information rate of Willie’s channel, $I(X; Y_w)$, where $I(X; Y_w) = \inf \{ t : \lim_{n \to \infty} \Pr \{ \frac{1}{n} I(X^n; Y_w^n) > t \} = 0 \}$, there exists at least one sequence of codebooks satisfies $\lim_{n \to \infty} V(\mathbf{P}_{Y_w^n} \| \mathbf{P}_{Z_w^n}) = 0$, and $D(\mathbf{P}_{Y_w^n} \| \mathbf{P}_{Z_w^n}) = 0$. Moreover, the covert information rate, $I(X; Y_w)$ can be interpreted as Willie’s channel resolution.\footnote{Proof. Appendix [B]}\footnote{Theorem 2 extends the result in [3] for the SISO AWGN channels to the MIMO AWGN channels and both results coincides when $\delta_a = \delta_w, \lambda_{a,i} = \lambda_{w,i}, \forall i$, $N_a = N_b = 1$ and using a real channel input. Then, the scaling is $L = 1/\sqrt{n}$.}

Using the singular value decomposition, let $H_w^\dagger H_w$ be decomposed as $U_w \Lambda_w U_w^\dagger$, where $U_w \in \mathbb{C}^{N_w \times N_w}$ is a unitary matrix and $\Lambda_w = \text{diag}(\Lambda_{w,1}, \Lambda_{w,2}, \ldots, \Lambda_{w,N_w}) \in \mathbb{R}^{N_w \times N_w}$ is a diagonal positive semi-definite matrix with $\lambda_{w,i} \geq 0$, $\forall i \in \{1, \ldots, N_w\}$ and $w \in \{b, w\}$. Further, let $\Lambda_w = U \Lambda_w U^\dagger$ be the rotated eigenvalue matrix of Willie’s channel, in the direction of Bob’s channel, with diagonal elements, $\lambda_{w,i} \geq 0$, $\forall i$, such that $U = U_b^\dagger U_w$. Hence, the KL divergence, $D = D(\mathbf{P}_{Y_w} \| \mathbf{P}_{Z_w})$, can be calculated as follows:

$$D = \mathbb{E}_{P_{Y_w}} [\log f_{Z_w}(Z_w) - \log f_{Y_w}(Y_w)] = \log |\Sigma_{Y_w}^{-1} \Sigma_w| + \mathbb{E}_{P_{Y_w}} \left[ Z_w^\dagger \Sigma_{w}^{-1} Z_w - Y_w^\dagger \Sigma_{Y_w}^{-1} Y_w \right] = \sum_{i=1}^{N_w} \frac{q_i \lambda_{w,i}}{\sigma_w^2} \log \left( \frac{q_i \lambda_{w,i}}{\sigma_w^2} + 1 \right),$$

(16)

where

$$\log |\Sigma_{Y_w}^{-1} \Sigma_w| = -\log \frac{1}{\sigma_w^2} H_{w} Q H_{w}^\dagger + I_{N_w}$$

$$= -\log \frac{1}{\sigma_w^2} \tilde{Q} \Lambda_{w} + I_{N_w}$$

$$= -\sum_{i=1}^{N_w} \log \left( \frac{q_i \lambda_{w,i}}{\sigma_w^2} + 1 \right),$$

(17)

$$\mathbb{E}_{P_{Y_w}} \left[ Z_w^\dagger \Sigma_{w}^{-1} Z_w - Y_w^\dagger \Sigma_{Y_w}^{-1} Y_w \right] = \mathbb{E}_{P_{Y_w}} \left[ \text{tr} (\Sigma_{w}^{-1} Z_w^\dagger Z_w) - \text{tr} (\Sigma_{Y_w}^{-1} Y_w^\dagger Y_w) \right] = \text{tr} (\Sigma_{Y_w}^{-1} \Sigma_w) - \text{tr} (\Sigma_{Y_w}^{-1} \Sigma_{Y_w}) = \text{tr} \left( \frac{1}{\sigma_w^2} H_{w} Q H_{w}^\dagger + I_{N_w} \right) - N_w = \text{tr} \left( \frac{1}{\sigma_w^2} H_{w} Q H_{w}^\dagger \right) = \sum_{i=1}^{N_w} \frac{q_i \lambda_{w,i}}{\sigma_w^2},$$

(18)

$$f_{Z_w}(z_w) = |\pi \Sigma_{w}^{-1}| \exp \left( -z_w^\dagger \Sigma_{w}^{-1} z_w \right),$$

(19)

while $\tilde{Q} = U_b^\dagger \tilde{Q} U_b$ is a diagonal matrix with diagonal elements $q_i$, such that $\text{tr}(\tilde{Q}) = \text{tr}(Q)$, and $N = \min \{N_a, N_b\}$. Let $c_i$ be the normalized KL divergence in the $i$th eigendirection, i.e.,

$$c_i = \frac{q_i \lambda_{w,i}}{\sigma_w^2} - \log \left( \frac{q_i \lambda_{w,i}}{\sigma_w^2} + 1 \right) / \mathcal{D}, \quad \forall i \in \{1, \ldots, N\},$$

(20)

where $q_i$ is the optimal power in the $i$th eigendirection, i.e., $c_i$ is fixed $\forall i$ and $\sum_{i=1}^{N} c_i = 1$. The following theorem extends the scaling law in [3] for the SISO AWGN channels to the MIMO AWGN channels.

Theorem 2. The scaling, with the blocklength, of the maximum number of nats that can be transmitted covertly and reliably over the MIMO AWGN channels, when the covert coding rate is diminishing, is $O(\sqrt{n})$ nats and is given by:

$$L = \sum_{i=1}^{N} \sqrt{2} c_i \sigma_w^2 \lambda_{w,i} / \sigma_b^2,$$

(21)

while ensuring that the illegitimate receiver’s sum of the detection error probabilities is lower bounded by $(1 - \delta_L)$.

Proof. Appendix [B]

Remark 3. Theorem 2 extends the result in [3] for the SISO AWGN channels to the MIMO AWGN channels and both results coincides when $\delta_a = \delta_w, \lambda_{a,i} = \lambda_{w,i}, \forall i$, $N_a = N_b = 1$ and using a real channel input. Then, the scaling is $L = 1/\sqrt{n}$.

Remark 4. It is clear that, in the computationally-secured covert communication, Alice can transmit $O(\sqrt{n})$ nats to Bob even if Bob’s channel is noisier than Willie’s channel, i.e., $\lambda_{w,i} < \lambda_{b,i}, \forall i$, while Willie’s sum of the detection error probabilities is still lower bounded by $(1 - \delta_L)$. On the contrary, securing the communication at the physical layer from being decoded cannot be achieved unless Bob’s channel is less noisy than Willie’s channel.

A. Scaling Laws of Specific MIMO AWGN Channels

1) Well-Conditioned MIMO AWGN Channels: In a rich scattering environment, the channel matrix is well-conditioned where the eigenvalues are approximately identical and hence, equal power allocation is optimal. The scaling in this case is given in the following corollary.

Corollary 1. The scaling, with the blocklength, of the maximum number of covert nats that can be transmitted reliably over the well-conditioned MIMO AWGN channels, when the covert coding rate is diminishing, is $O(\sqrt{n})$ nats and is given by:

$$L = \frac{\sqrt{2N} \sigma_w^2 \lambda_b}{\sigma_b^2},$$

(22)

while ensuring that the illegitimate receiver’s sum of the detection error probabilities is lower bounded by $(1 - \delta_L)$. 

2) Bounded Spectral Norm of Willie’s CSI: Further, when the CSI of Willie is unknown but lies in the set of matrices with a bounded spectral norm similar to \([\mathbb{H}_w]^{\dagger}\),
\[
S_w = \left\{ \mathbb{H}_w : \frac{\|\mathbb{H}_w \mathbb{H}_w^{\dagger}\|_{op}}{\sigma_w^2} \leq \hat{\lambda}_w \right\},
\]
where \(\|\mathbb{H}_w \mathbb{H}_w^{\dagger}\|_{op}\) represents the largest possible power gain of Willie’s channel, i.e., the worst-case Willie’s channel is isotropic. Hence, the set \(S_w\) incorporates all possible matrices \(\mathbb{H}_w\), such that \(\frac{\|\mathbb{H}_w \mathbb{H}_w^{\dagger}\|_{op}}{\sigma_w^2} \leq \hat{\lambda}_w\), which implies that Willie’s channel has limited capabilities such as low secrecy sensitivity or cannot approach a certain area around transmitter. It is worth noting that the secrecy capacity such a class of channels is the worst-case secrecy capacity and equals the secrecy capacity of the compound channel \([\mathbb{H}_w]^{\dagger}\).

Similarly, the covert capacity of such a class of channels is the worst-case covert capacity and equals the covert capacity of the compound channels \([\mathbb{H}_w]^{\dagger}\). The following two corollaries give bounds on the scaling of such a class of channels.

**Corollary 2.** The scaling, with the blocklength, of the maximum number of covert nats that can be transmitted reliably over the MIMO AWGN channels, when the CSI of the illegitimate receiver belongs to the class of channels with a bounded spectral norm and when the covert coding rate is diminishing, is \(O(\sqrt{N})\) nats and is bounded as follows:
\[
\sum_{i=1}^{N} \frac{2 \sqrt{c_i \lambda_{w,i}}}{\sigma_w^2 \lambda_{w,i}} \leq L \leq \sum_{i=1}^{N} \frac{\sqrt{2c_i \sigma_w^2 \lambda_{b,i}}}{\sigma_b^2 \lambda_{b,i}},
\]
while ensuring that the illegitimate receiver’s sum of the detection error probabilities is lower bounded by \((1 - \delta_L)\).

Moreover, when the legitimate receiver’s channel is well-conditioned, \(L\) is bounded as:
\[
\frac{\sqrt{2N \lambda_b}}{\sigma_b^2 \lambda_{w}} \leq L \leq \frac{\sqrt{2N \sigma_w^2 \lambda_b}}{\sigma_b^2 \lambda_{w}},
\]

3) Unit-Rank MIMO AWGN Channels: In this subsection, the unit-rank MIMO channels are analyzed using the physical modeling of the MIMO channels in \([13]\). Without loss of generality, the focus is on evenly-spaced uniform linear antenna arrays. Consider the line-of-sight (LoS) MIMO channels and define \(u_i(\Omega_t)\) as the unit spatial transmit signature in the directional cosine, \(\Omega_t \triangleq \cos \phi_t\), where \(\phi_t\) is the angle of departure of the LoS from the transmit antenna array.

In addition, the angle, \(\theta\), between any two different spatial transmit signatures is related to directional transmit cosines as follows:
\[
|\cos \theta| = \left| u_1^{\dagger}(\Omega_1) u_2(\Omega_2) \right| = \frac{\sin(\pi L_t \Omega)}{N_t \sin(\pi L_t \Omega / N_t)},
\]
where \(\Omega \triangleq \Omega_2 - \Omega_1\), \(L_t \triangleq N_t \Delta\) is the length of the transmit antenna array normalized with respect to the carrier wavelength and \(\Delta\) is the normalized transmit antenna separation. Moreover, the quantity \(|\cos \theta| = |f(\Omega)|\) is a periodic function with a period \(N_t / L_t\) and nulls at \(\Omega = k / L_t\), \(k = 1, \ldots, N_t - 1\).

Figure 2 depicts the periodic function \(|f(\Omega_2 - \Omega_1)|\) for a fixed transmit direction, \(\Omega_1\), different values of the number of transmitting antennas and the normalized array length.

Alice tries to find the optimal spatial transmit signature in the directional cosine, \(\Omega^*\), that maximizes the achievable covert rate. The directional cosine, \(\Omega^*\), can be characterized as follows:
\[
\Omega^* = \arccos \frac{\sum P \hat{\lambda}_w \log \left( \frac{P \hat{\lambda}_w}{\sigma_w^2} + 1 \right)}{2\delta^2},
\]
where \(\hat{\lambda}_b = \frac{\lambda_b}{\sigma_b^2} f(\Omega - \Omega_b)\) and \(\hat{\lambda}_w = \frac{\lambda_w}{\sigma_w^2} f(\Omega - \Omega_w)\) are the eigenvalues of Bob’s and Willie’s channels, respectively, after projecting on the directional cosine, \(\Omega\). Therefore, the following corollary gives the scaling for the unit-rank MIMO AWGN channels.

**Corollary 3.** The scaling, with the blocklength, of the maximum number of nats that can be transmitted covertly and reliably over the unit-rank MIMO AWGN channels, when the covert coding rate is diminishing, is \(O(\sqrt{N})\) nats and is given by:
\[
L = \frac{\sqrt{2N \sigma_b^2 \hat{\lambda}_b}}{\sigma_b^2 \hat{\lambda}_w} = \frac{\sqrt{2N \sigma_w^2 \hat{\lambda}_w}}{\sigma_b^2 \hat{\lambda}_w |f(\Omega^* - \Omega_b)|^2},
\]
where \(\xi_b\) and \(\xi_w\) are the LoS path attenuations of the legitimate and the illegitimate receivers’ channels, respectively, while ensuring that the illegitimate receiver’s sum of the detection error probabilities is lower bounded by \((1 - \delta_L)\).

**Remark 5.** The scaling law for the SISO AWGN channels, with isotropic transmit and receive antennas, is given by \(L = \frac{\sqrt{2N \sigma_b^2 \hat{\lambda}_b}}{\sigma_b^2 \hat{\lambda}_w} \xi_b\), which coincides with the result in \([13]\) when \(\sigma_b = \sigma_w\), \(\xi_b = \xi_w\) and the channel input is real.

**Corollary 4.** A positive covert rate can be achieved by transmitting in the spatial transmit signature in the null directional cosine of the illegitimate receiver’s channel, i.e.,
been achieved in the null direction of Willie while achieving a non-diminishing covert coding rate. Similarly, Alice can achieve a non-diminishing covert coding rate by utilizing the null-space precoding if a null-space between Alice and Willie exists.

Figure 3 shows an example of a transmit beam that is steered in the null direction of Willie while achieving a non-diminishing covert coding rate. Similarly, Alice can achieve a non-diminishing covert coding rate by utilizing the null-space precoding if a null-space between Alice and Willie exists.

IV. JOINT COVERT AND SECURE COMMUNICATION OVER THE MIMO AWGN CHANNELS

In this section, Alice transmits secure information reliably to Bob while tries to lower bound Willie’s sum of the detection error probabilities by \((1 - \delta_L)\). To investigate the scaling laws under this regime, the secrecy capacity should be considered first. The secrecy capacity is the maximum achievable rate such that \(R = R_c\) where \(R_c \leq \lim_{n \to \infty} \frac{1}{n} H(M | Y^n_w)\), denoted as the equivocation rate, is Willie’s uncertainty about the message, \(M\), given the channels’ outputs \(Y^n_w\), i.e., the secrecy level at Willie. Hence, the secrecy capacity is given by:

\[
C_S = \max_{(R,R) \in C_E} R, \tag{32}
\]

where \((R, R)\) is the rate–equivocation pair and \(C_E\) is the capacity-equivocation region. Moreover, the secrecy capacity of the MIMO wiretap channels is defined as follows [12]

\[
C_S(Q^*) = \max_{Q^* \succeq 0, \text{tr}(Q) \leq P} [C_b(Q) - C_w(Q)]^+, \tag{33}
\]

where \(C_u(Q) = \log \left| \frac{1}{\sigma^2_u} H_u(Q) H_u^* + I_u \right|\) and \(u \in \{b, w\}\). Hence, using a similar approach that is adopted in the previous section, the following corollary gives the scaling law for covert and secure communication over the MIMO AWGN channels.

**Corollary 5.** The maximum number of covert and secure nats that can be transmitted reliably over the MIMO AWGN channels scales like \(L_S\) which is given by:

\[
L_S = \lim_{n \to \infty} \sqrt{\frac{n}{2 \delta^2_L}} \max_{Q \succeq 0, \text{tr}(Q) \leq P} [C_b(Q) - C_w(Q)]^+, \tag{34}
\]

subject to:

\[
D(\mathbb{P}_{Y^n_w} \parallel \mathbb{P}_{Z^n_w}) \leq 2 \frac{\delta^2_L}{n}. \tag{35}
\]

Moreover, the input distribution that maximizes the first-order approximation of the covert secrecy capacity, while minimizing \(D(\mathbb{P}_{Y^n_w} \parallel \mathbb{P}_{Z^n_w})\), is the zero-mean circularly symmetric complex Gaussian distribution with a covariance matrix \(Q\).

**Theorem 3.** The scaling, with the blocklength, of the maximum number of covert and secure nats that can be transmitted reliably over the MIMO AWGN channels, when the covert secrecy rate is diminishing, is \(O(\sqrt{n})\) nats and is given by:

\[
L_S = \sum_{i=1}^{N} \sqrt{2 \varepsilon_i} \left[ \frac{\sigma^2_{w,i} \lambda_{b,i}}{\sigma^2_{w,i} \lambda_{w,i}} - 1 \right]^+, \tag{36}
\]

while ensuring that the illegitimate receiver’s sum of the detection error probabilities is lower bounded by \((1 - \delta_L)\).

**Proof.** Appendix C

**Remark 6.** Theorem 3 coincides with Theorem 2 in [11] for binary symmetric (BSC) channels where Alice can transmit \(O(\sqrt{n})\) covert and secure nats reliably to Bob without any pre-shared secret, namely, hidable and deniable nats. Also, it coincides with the results in [4] for DMC and AWGN channels. In contrast to covert communication, secure communication cannot be achieved unless Bob’s channel is less noisy than Willie’s channel, i.e., \(\lambda_{b,i} > \lambda_{w,i}\), \(\forall i\). As long as the channel to Willie is noisier than the channel to Bob, Alice exploits the common randomness with Bob to use an ensemble of the public codebook and transmits secure messages without being decoded by Willie [17]. To ensure this, the randomized encoder uses dummy messages and chooses the secure message uniformly with a randomization rate that is determined by Willie’s channel quality. Hence, Willie can be overwhelmed by the dummy messages and cannot decode but still has the ability to detect. Therefore, the LPD constraint can be added to guarantee that Willie cannot detect the transmission as well as cannot decode. On the other hand, the effective secrecy capacity is developed in [24] where Willie tries to detect whether the transmission is meaningful or not, namely, stealth and secure communication. The effective secrecy capacity is positive and is the same as the weak and the strong secrecy capacities if there is a distribution \(\mathbb{P}_{Z^n_w}\) such that \(\mathbb{P}_{Y^n_w} = \mathbb{P}_{Z^n_w}\).
A. Scaling Laws of Specific MIMO AWGN Channels

Corollary 6. The scaling, with the blocklength, of the maximum number of covert and secure nats that can be transmitted reliably over the well-conditioned MIMO AWGN channels, when the covert secrecy rate is diminishing, is $\mathcal{O}(\sqrt{n})$ nats and is given by:

$$L_S = \sqrt{2N} \left[ \frac{\sigma^2_{w} \lambda_b}{\sigma^2_{b} \lambda_w} - 1 \right]^{+}.$$  \hfill (36)

Corollary 7. The scaling, with the blocklength, of the maximum number of covert and secure nats that can be transmitted reliably over the MIMO AWGN channels, when the CSI of the illegitimate receiver belongs to the class of channels with a bounded spectral norm and when the covert secrecy rate is diminishing, is $\mathcal{O}(\sqrt{n})$ nats and is bounded as follows:

$$\sum_{i=1}^{N} \sqrt{2c_i} \left[ \frac{\sigma^2_{w} \lambda_{b,i}}{\sigma^2_{b} \lambda_{w}} - 1 \right]^{+} \leq L_S \leq \sum_{i=1}^{N} \sqrt{2c_i} \left[ \frac{\sigma^2_{w} \lambda_{b,i}}{\sigma^2_{b} \lambda_{w,i}} - 1 \right]^{+}. \hfill (37)$$

Moreover, when the legitimate receiver’s channel is well-conditioned, $L_S$ is bounded as: $$\sqrt{2N} \left\{ \frac{\lambda_b}{\sigma^2_{b} \lambda_w} - 1 \right\}^{+} \leq L_S \leq \sqrt{2N} \left\{ \frac{\sigma^2_{w} \lambda_{b}}{\sigma^2_{b} \lambda_{w}} - 1 \right\}^{+}. \hfill \text{(38)}$$

Corollary 8. The scaling, with the blocklength, of the maximum number of covert and secure nats that can be transmitted reliably over the unit-rank MIMO AWGN channels when $\Omega_b = \Omega_w + \frac{k}{\sigma^2_{w}}, \ k = 1, \ldots, N_a - 1$, is $\mathcal{O}(\sqrt{n})$ nats and is given by:

$$L_S = \sqrt{2} \left[ \frac{\sigma^2_{w} \lambda_b}{\sigma^2_{b} \lambda_w} - 1 \right]^{+} = \sqrt{2} \left[ \frac{\sigma^2_{w} \xi_{b,i} N_b |f(\Omega^* - \Omega_b)|^2}{\sigma^2_{b} \xi_{w} N_w |f(\Omega^* - \Omega_w)|^2} - 1 \right]^{+}. \hfill (39)$$

V. COVERT CAPACITY AND COVERT DOF OF THE MIMO AWGN CHANNELS

Since the covert coding rate over the MIMO AWGN channels is diminishing with the blocklength, a positive covert capacity can be achieved only when there exists a null-space between Alice and Willie. The following corollaries exploit this fact to deduce the covert capacity of the MIMO AWGN channels.

Corollary 9. The covert capacity of the MIMO AWGN channels can be positive and is given by: $C_L = \sum_{i=1}^{N} \log \left( \frac{\lambda_{b,i}}{\lambda^2_{w,i}} + 1 \right)$, where the optimal power allocation is a water-filling over the null-space directions given by:

$$q_i = \begin{cases} \left[ \frac{1}{\mu} \frac{\lambda^2_{b,i}}{\lambda^2_{w,i}} \right]^+, & \lambda_{w,i} = 0 \text{ and } \lambda_{b,i} \neq 0, \\ 0, & \text{otherwise} \end{cases} \hfill (39)$$

$\forall \ i \in \{1, \ldots, N\}$, and $\mu$ is chosen such that:

$$\sum_{i=1}^{N} q_i = P, \ \mu > 0,$$

$$\sum_{i=1}^{N} q_i < P, \ \mu = 0. \hfill (40)$$

Remark 7. Exploiting the null-space prevents Willie from both detecting and decoding. Hence, the covert communication using only the null-space is secure, i.e., the covert capacity equals the covert secrecy capacity in this case.

As aforementioned, the channels’ matrices are well-conditioned and equal power allocation is optimal. Accordingly, the covert capacity of the well-conditioned MIMO AWGN channels depends on the relation between the dimension of available null-space, $M = N_a - N_w$, between Alice and Willie, and the dimension of available space, $N$, between Alice and Bob. The covert capacity of the well-conditioned MIMO AWGN channels is given in the following corollary.

Corollary 10. The covert capacity of the well-conditioned MIMO AWGN channels can be positive and is given by:

$$C_L = \begin{cases} M \log \left( \frac{P M N_w^2}{M N_w^2 + 1} \right), & 0 < M < N, \\ N \log \left( \frac{P N N_w^2}{N N_w^2 + 1} \right), & M \geq N, \\ 0, & M = 0. \end{cases} \hfill (41)$$

Since the covert capacity of the well-conditioned MIMO AWGN channels can be characterized by Equation (41), the following two corollaries give the covert DoF for the MIMO AWGN channels.

Corollary 11. The covert DoF for the MIMO AWGN channels is given by:

$$D_c = \begin{cases} M, & 0 < M < N, \\ N, & M \geq N, \\ 0, & M = 0. \end{cases} \hfill (42)$$

Corollary 12. The covert DoF for the MIMO multiple access AWGN channels with one illegitimate receiver is given by:

$$D_c = \min \left\{ \sum_{k=1}^{K} [N_k - N_w]^+, \ N_b \right\}, \hfill (43)$$

where $K$ is the number of transmitters and $N_k$ is the number of antennas of the $k^{th}$ transmitter.

VI. CAN WE OVERCOME THE SQUARE ROOT LAW?

This section answers an interesting question and gives some important insights on the asymptotic performance of the covert communication with unknown CSI of Willie’s channel. In this regime, Alice transmits directly to Bob without steering its transmit beam to the null direction of Willie’s channel or any other direction. Hence, the covert communication with unknown CSI of Willie’s channel is diminishing with the blocklength. However, increasing the number of transmitting antennas, for a fixed blocklength, overcomes the square root law of the covert communication and hence, a positive covert capacity is achieved. As the number of transmitting antennas goes to infinity, namely, the massive MIMO limit, the covert coding rate converges to the maximal MIMO limit of the MIMO AWGN channels. The same concept applies to secure communication but the scope of this section is limited to covert communication.
A. Unit-Rank MIMO Channels

With unknown CSI of Willie’s channel, Alice transmits in the spatial transmit signature in the directional cosine of Bob’s channel. To give more insights utilizing the antenna array design, we consider the received pattern at Willie, which is given by \( |f(\Omega)| = |f(\Omega_b - \Omega_w)| \), in the following two cases:

- Case (1): Fixed normalized array length, \( L_a \), of the transmit antenna array:
  \[
  \lim_{N_a \to \infty} \left| \frac{\sin(\pi N_a \Omega)}{N_a \sin(\pi \Delta \Omega)} \right| = \frac{\sin(\pi L_a \Omega)}{\pi L_a \Omega}.
  \]  

  This means that as the number of transmitting antennas increases, the main lobe does not change and all other lobes decrease which increases the covert coding rate as long as Willie is not aligned to the main lobes which are centered at \( \phi_b \) and \( 2\pi - \phi_b \), respectively, with a beamwidth equals to \( 2/L_a \).

- Case (2): Fixed normalized antenna separation, \( \Delta \):
  \[
  \lim_{N_a \to \infty} \left| \frac{\sin(\pi N_a \Delta \Omega)}{N_a \sin(\pi \Delta \Omega)} \right| = 0.
  \]  

In this case, the width of the main lobe decreases and becomes very directive, pencil beam, as well as all other lobes decrease substantially. Thus, Willie can not receive the transmitted signal wherever Willie is not aligned to the spatial transmit direction of Bob, i.e., \( \Omega_b \neq \Omega_w + \frac{k}{L_a} \mod \frac{1}{\lambda} \), \( k = 1, \ldots, N_a - 1 \). Therefore, a positive covert capacity can be achieved with zero probability of detection due to the high beamforming capability.

Proposition 1. Without the knowledge of the CSI of the illegitimate receiver and for a finite blocklength, the LPD constraint of the unit-rank MIMO AWGN channels is satisfied \( \forall \delta_L \geq 0 \) as \( N_a \to \infty \), under the following condition:

1. Either a fixed normalized antenna separation, while the illegitimate receiver is not aligned to the spatial transmit direction of the legitimate receiver, i.e., \( \Omega_b \neq \Omega_w + \frac{k}{L_a} \mod \frac{1}{\lambda} \), \( k = 1, \ldots, N_a - 1 \).
2. Or a fixed normalized array length, while the illegitimate receiver is not aligned to the main lobes which are centered at \( \phi_b \) and \( 2\pi - \phi_b \), respectively, with a beamwidth equals to \( 2/L_a \).

Moreover, a lower bound on the number of transmitting antennas that satisfies a given probability of detection, \( \delta_L \), is given by:

\[
N_a \geq \frac{\xi^2 P N_w \sin^2(\pi N_a \Delta \Omega)}{\sigma_w^2 \sin^2(\pi \Delta \Omega)} W_{-1}\left(-e^{-\frac{2\pi^2 \xi^2}{\sigma_w^2}}-1\right)^{-1},
\]

where \( W_{-1} \) is a branch of the Lambert function.

Proof. Appendix [13]

Remark 8. This result suggests that even without the knowledge of Willie’s CSI, the maximal covert coding rate of the unit-rank MIMO AWGN channels is asymptotically not diminishing and converges to the maximal coding rate of the unit-rank MIMO AWGN channels. Hence, Alice can transmit \( O(n) \) nats covertly and reliably for a fixed blocklength under some conditions. Thus, Alice can asymptotically achieve the capacity of the unit-rank MIMO AWGN channels without being detected.

Corollary 13. Under the stated conditions, the covert capacity, \( C_L \), of the unit-rank MIMO AWGN channels converges to the capacity, \( C \), of the unit-rank MIMO AWGN channels, while the LPD condition is satisfied as \( N_a \to \infty \), i.e., \( \lim_{N_a \to \infty} C_L = \lim_{N_a \to \infty} C \) for any given \( \delta_L \geq 0 \).

B. Multi-Path MIMO Channels

Consider there are multiple reflected paths in addition to an LoS path where the \( i^{th} \) path has an attenuation of \( \xi_i \) and makes an angle of \( \phi_{r,i} \) with the transmit antenna array and an angle of \( \phi_{t,i} \) with the receive antenna array, \( \forall i \).

The channels matrix, \( H \), is given by [15] as follows:

\[
H = \sum_{i=1}^{P} \xi_{i} \sqrt{N_r N_t} \exp(-j 2\pi d_i^p) u_{r,i}^p(\Omega_{r,i}^p) u_{t,i}^p(\Omega_{t,i}^p),
\]

where \( \xi_i \) is the attenuation of the \( i^{th} \) path, \( d_i^p \) is the distance between the first transmit antenna and the first receive antenna along the \( i^{th} \) path normalized to the carrier wavelength, \( u_{r,i}(\Omega_{r,i}^p) \) and \( u_{t,i}(\Omega_{t,i}^p) \) are the unit spatial receive and transmit signatures in the directional receive and transmit cosines of the \( i^{th} \) path, \( \forall i \). In addition, the channels matrix is full rank if there exist at least \( N \) paths such that \( \Omega_{r,i}^p \neq \Omega_{r,j}^p \) \( \mod \frac{1}{\Delta \Omega} \), \( \forall i, j \) and \( i \neq j \), where \( u \in \{t, r\} \).

Moreover, the channels matrix is well-conditioned if there exist at least \( N \) paths such that the angular separation at the transmit array, \( \Omega_{r,i}^p = [\Omega_{r,i}^p - \Omega_{r,j}^p] \), \( \forall i, j \) and \( i \neq j \), and at the receive array, \( \Omega_{t,i}^p = [\Omega_{t,i}^p - \Omega_{t,j}^p] \), \( \forall i, j \) and \( i \neq j \), of each two paths are no less than \( 1/L_t \) and \( 1/L_r \), respectively, otherwise, paths are not resolvable.

The individual physical paths can be aggregated to the resolvable paths similar to the resolvable channel taps when modeling the multi-path fading channels [15]. Hence, the \( (i,j)^{th} \) channel gain in the angular domain consists of all paths whose transmit and receive directional cosines are within an angular window of width \( 1/L_t \) and \( 1/L_r \) around \( 1/L \) and \( k/L \), respectively. Therefore, the orthonormal basis of the transmitted signal space, \( C^N_r \), and the received signal space, \( C^N_t \), are given by:

\[
S_u = \left\{ u_u(0), u_u\left(\frac{1}{L_u}\right), \ldots, u_u\left(\frac{N_u - 1}{L_u}\right) \right\},
\]

where \( u \in \{t, r\} \).

Let \( U_t \) and \( U_r \) be unitary matrices whose columns are the orthonormal vectors in \( S_t \) and \( S_r \), respectively. Then, the angular domain representation of the channels matrix is given by:

\[
H^a = U_t^H H U_r = \sum_{i=1}^{P} \xi_{i} \sqrt{N_r N_t} \exp(-j 2\pi d_i^p) u_{r,i}^a(\Omega_{r,i}^p) u_{t,i}^a(\Omega_{t,i}^p),
\]

\[
= \sum_{i=1}^{P} \xi_{i} \sqrt{N_r N_t} \exp(-j 2\pi d_i^p) u_{r,i}(\Omega_{r,i}^p) u_{t,i}^a(\Omega_{t,i}^a),
\]

(48)
where \( \mathbf{u}_a^o(\Omega_{a,i}^o) = \mathbf{U}_a^t \mathbf{u}_p^o(\Omega_{p,i}^o) \) and \( \mathbf{u}_u^o(\Omega_{u,i}^o) = \mathbf{U}_u^t \mathbf{u}_r^o(\Omega_{r,i}^o), \forall i \), are the angular representation of the unit spatial transmit signature, \( \mathbf{u}_p^o(\Omega_{p,i}^o) \), in the directional cosine, \( \Omega_{p,i}^o \) and the unit spatial receive signature, \( \mathbf{u}_r^o(\Omega_{r,i}^o) \), in the directional cosine, \( \Omega_{r,i}^o \), of the \( i \)-th path, respectively. In addition, \( \mathbf{u}_a(\Omega_{a,i}) \) and \( \mathbf{u}_u(\Omega_{u,i}) \) are the orthonormal vectors in \( \mathcal{S}_a \) and \( \mathcal{S}_u \), respectively, where \( \xi_a \) is the attenuation of the \( i \)-th angular window, \( d_i \) is the distance between the first transmit antenna and the first receive antenna along the \( i \)-th angular window normalized to the carrier wavelength, and (a) follows by aggregating all paths, i.e., unresolved paths in the angular window, to contribute in the \( i \)-th path. In the directional cosine, \( \Omega_{p,i}^o \) and \( \Omega_{r,i}^o \), of the \( i \)-th path, respectively. In addition, \( \mathbf{u}_a(\Omega_{a,i}) \) and \( \mathbf{u}_u(\Omega_{u,i}) \) are the orthonormal vectors in \( \mathcal{S}_a \) and \( \mathcal{S}_u \), respectively, where \( \xi_a \) is the attenuation of the \( i \)-th angular window, \( d_i \) is the distance between the first transmit antenna and the first receive antenna along the \( i \)-th angular window normalized to the carrier wavelength, and (a) follows by aggregating all paths, i.e., unresolved paths in the angular domain, that contribute in the same basis vector. For the \( i \)-th path to contribute in the \( i \)-th transmit basis vector and in the \( k \)-th receive basis vector: 
\[
| \Omega_{a,i}^o - \frac{\pi}{2} | < \frac{\pi}{\xi_a}, \forall i \text{ and } l \in \{0, \ldots, N_u - 1 \}, \text{ where } u \in \{t, r \}.
\]

Therefore, the angular representations of Bob’s and Willie’s channels power gain in terms of the resolvable paths are given by:
\[
\mathbf{H}_u^o \mathbf{H}_u^o = \sum_{i=1}^{N_u} \lambda_{u,i} \mathbf{u}_{u,i}(\Omega_{u,i}) \mathbf{u}_{u,i}^t(\Omega_{u,i}), \tag{49}
\]
where \( u \in \{b, w \} \) and \( \lambda_{u,i} = \xi_{u,i}^2 N_u N_a \).

**Proposition 2.** Without the knowledge of the CSI of the illegitimate receiver and for a finite blocklength, the LPD constraint of the multi-path MIMO AWGN channels is satisfied \( \forall \delta_L \geq 0 \) as \( N_a \to \infty \), under the following condition:

1. Either a fixed normalized antenna separation, while the illegitimate receiver is not aligned to any of the spatial transmit directions of the legitimate receiver, i.e., \( \Omega_{b,i} \neq \Omega_{w,i} + \frac{k}{\xi_b} \text{ mod } \frac{\pi}{\xi_b} \), \( k = 1, \ldots, N_w - 1 \), \( \forall i \) and \( j \).
2. Or a fixed normalized array length, while the illegitimate receiver is not aligned to lobes which are centered at \( \phi_{b,i} \) and \( 2\pi - \phi_{b,i} \), \( \forall i \), respectively, with a beamwidth equals to \( 2/|\xi_a| \).

**Proof.** Appendix E.

**Remark 9.** It should be clear that the stated conditions in the previous proposition can not be easily satisfied. If so, Alice can asymptotically achieve the capacity of the multi-path MIMO AWGN channels without being detected.

**VII. NUMERICAL ILLUSTRATIONS**

In this section, we provide some numerical results to illustrate the performance of covert communication when the number of transmitting antennas scales. Using the first-order approximation of the coding rates, the achievable covert nats is given by:
\[
nB \log \left( 1 + \frac{\xi_b^2 N_a N_b}{B\sigma_b^2} \right), \tag{50}
\]
where \( B \) is the occupied bandwidth, \( q = \min \{ P, \tilde{P} \} \) and
\[
\tilde{P} = \frac{\sigma_w^2}{\xi_b^2 N_a N_w |f(\Omega)|^2} \left[ -W_{-1} \left( e^{-\frac{2\sigma_w^2}{\xi_b^2}} - 1 \right) \right], \tag{51}
\]
while the achievable non-covert nats is given by:
\[
nB \log \left( 1 + \frac{P \xi_b^2 N_a N_b}{B\sigma_b^2} \right). \tag{52}
\]

The simulation parameters are given as follows. The distance between Alice-Bob and Alice-Willie is \( d = 1 \) km. Given the path-loss exponent, \( \alpha = 4 \), and the path-loss constant, \( L = 10^{-2} \), the LoS attenuation is \( \xi_b^2 = \xi_w^2 = L d^{-\alpha} \). The noise power densities for both Bob and Willie are assumed to be equal and are given by \( \sigma_b^2 = \sigma_w^2 = -174 \text{ dBm/Hz} \). The maximum transmission power is \( P = 10 \text{ dBm} \), the occupied bandwidth is \( B = 180 \text{ kHz} \), \( \Omega_w = \pi/4 \), \( \Omega_b = \pi/2 \) and the probability of detection is \( \delta_L = 10^{-2} \).

**Fig. 4.** First-order approximation of covert nats and non-covert nats vs. the number of transmitting antennas.

**Fig. 5.** First-order approximation of covert nats and non-covert nats vs. number of channel uses.

Figures 4 and 5 compare the achievable covert and non-covert nats that can be transmitted over the MIMO AWGN channels using the first-order approximation. Clearly, in Fig. 4 the achievable covert nats converges to the non-covert nats, for a fixed blocklength, \( n = 10^4 \), as the number of transmitting antennas increases, for different number of Willie’s antennas, \( N_w = 1, 10, 50 \). The achievable covert nats are \( \mathcal{O}(n) \) instead of \( \mathcal{O}(\sqrt{n}) \). Although the number of antennas of Willie are increased from 1 to 50, the achievable covert nats are still \( \mathcal{O}(n) \) for a large number of transmitting antennas. In case
of a small number of transmitting antennas, $N_a < 10$, the achievable covert nats are $O(\sqrt{n})$.

For a fixed small number of transmitting antennas, $N_a = 10$, as in Fig. 5 the achievable covert nats are $O(\sqrt{n})$ for a different number of Willie’s antennas, $N_w = 10, 20, 50$. In contrary, the achievable covert nats are $O(n)$ for a fixed large number of transmitting antennas, $N_a = 100$.

VIII. Conclusion

The covert communication over the MIMO AWGN channels has been studied thoroughly in this paper. Asymptotically with the number of transmitting antennas, a positive covert capacity can be achieved and it can be equal to the capacity of the MIMO AWGN channels as long as the illegitimate receiver is not aligned to any spatial transmit direction of the legitimate receiver. In addition, a non-diminishing covert coding rate can be achieved by utilizing the null-space between the transmitter and the illegitimate receiver, and hence, the covert capacity can be positive and the covert DoF are identified. Besides, exploiting the null-space only achieves both covert and secure communications. On the other hand, the covert coding rate is diminishing with the blocklength if there is no null-space between the transmitter and the illegitimate receiver. In this case, the number of covert nats is $O(\sqrt{n})$ nats and the same result holds for the number of covert and secure nats. The future research could be directed towards investigating the covert and the secure communications for delay constrained applications, under different attack models, and exploiting artificial noise transmission to limit the illegitimate receiver’s detection capabilities.

APPENDIX A

PROOF OF THEOREM 1

Proof: **Converse:** Consider a given $(2^{nR}, n, \epsilon, \delta_L)$ covert code$^6$ that satisfies the LPD constraint, $D(\mathbb{P}_{Y^n_w} \parallel \mathbb{P}_{Z^n_w}) \leq 2\delta_L^2$, with an average error probability $P_e^{(n)}$ such that $\lim_{n \rightarrow \infty} P_e^{(n)} = 0$.

**Analysis:**

1) *Probability of error analysis:* Starting with Fano’s and data processing inequalities: Let $R = R(n, \epsilon, \delta_L)$,

$$nR = H(M)$$

$$= I(M; Y^n_w) + H(M | Y^n_w)$$

$$\leq I(M; Y^n_w) + n \epsilon_n$$

$$= h(Y^n_w) - h(Y^n_w | M) + n \epsilon_n$$

$$\leq h(Y^n_w) - h(Y^n_w | M, X^n) + n \epsilon_n$$

$$\leq h(Y^n_w) - h(Z^n_w | M, X^n) + n \epsilon_n$$

$$\leq h(Y^n_w) - h(Z^n_w) + n \epsilon_n$$

$$= h(Y^n_w) - n \log |\Sigma_b| + n \epsilon_n$$

$$\leq \sum_{i=1}^{n} h(Y_{b,i}) - n \log |\Sigma_b| + n \epsilon_n$$

$$\leq \sum_{i=1}^{n} \log |\pi (H_b Q H_b^† + \Sigma_b)| - n \log |\Sigma_b| + n \epsilon_n$$

$$= n \log |\pi (H_b Q H_b^† + \Sigma_b)| - n \log |\Sigma_b| + n \epsilon_n$$

$$= n \log \frac{1}{\sigma_b^2} H_b Q H_b^† + I_{N_b} + n \epsilon_n$$

$$\leq n C_L(Q) + n \epsilon_n,$$

where $n \epsilon_n = 1 + P_e^{(n)} n R(n, \epsilon, \delta_L)$, $\epsilon_n$ tends to zero as $n$ goes to infinity by the assumption that $\lim_{n \rightarrow \infty} P_e^{(n)} = 0$, $Q$ is a decreasing function of $n$, $H$ is stationary over $n$ channel uses, (a) follows since conditioning does not increase the entropy, (b) since translation does not change the entropy, (c) since the noise is independent of both the message and the transmitted codeword, (d) follows from the chain rule of entropy and removing conditioning, (e) follows since the maximum differential entropy of a continuous random vector is attained when the random vector has a zero-mean circularly symmetric complex Gaussian distribution with a covariance matrix, $Q$, that attains the maximum, and (l) by the definition of the Gaussian vector channels’ capacity, while the optimal input covariance matrix, $Q$, is chosen such that the LPD constraint is satisfied.

2) *Low probability of detection analysis:* Following a similar approach of [25] and [3]:

$$2 \delta_L^2 \geq D(\mathbb{P}_{Y^n_w} \parallel \mathbb{P}_{Z^n_w})$$

$$= -h(Y^n_w) + \mathbb{E}_{\mathbb{P}_{Y^n_w}} \left[ \log \frac{1}{f_{Z^n_w}^{(n)}(Z^n_w)} \right]$$

$$= -h(Y^n_w) + \sum_{i=1}^{n} \mathbb{E}_{\mathbb{P}_{Y^n_w,i}} \left[ \log \frac{1}{f_{Z^n_w}^{(n)}(Z^n_{w,i})} \right]$$

$$\geq -h(Y^n_w) + \sum_{i=1}^{n} \mathbb{E}_{\mathbb{P}_{Y^n_w,i}} \left[ \log \frac{1}{f_{Z^n_w}^{(n)}(Z^n_{w,i})} \right]$$

$$\geq \sum_{i=1}^{n} D(\mathbb{P}_{Y^n_{w,i}} \parallel \mathbb{P}_{Z^n_{w,i}})$$

$$\geq n D(\mathbb{P}_{Y^n_w} \parallel \mathbb{P}_{Z^n_w}),$$

where $\mathbb{P}_{Y^n_w} = \frac{1}{n} \sum_{i=1}^{n} \mathbb{P}_{Y^n_{w,i}}$, (a) follows from the chain
rule of entropy and removing conditioning, and (b) follows since the KL divergence, $D(P_{Y_w} \| P_{Z_w})$, is convex in $P_{Y_w}$. Consequently, the constraint, $D(P_{Y_w} \| P_{Z_w}) \leq 2\delta^2/n$, is satisfied.

Hence, the input distribution that maximizes the first-order approximation of the capacity in Equation (53), while minimizing $D(P_{Y_w} \| P_{Z_w})$, Equation (54), is the zero-mean circularly symmetric complex Gaussian distribution with a covariance matrix $Q$. 

**Achievability:** Generate a random codebook by fixing the input distribution, $f_X(x)$, that achieves the covert capacity as well as satisfies the constraint, $D(P_{Y_w} \| P_{Z_w}) \leq 2\delta^2/n$. Randomly generate i.i.d. $2^nR$ sequences such that $x^n(m)$, $m \in \mathcal{M}$, and hence, $f_X^n(x^n) = \prod_{i=1}^n f_X(x_i)$. Consequently, the output distribution of this code is i.i.d. such that $f_Y^n(y^n) = \prod_{i=1}^n f_Y(y_{b,i})$, where $Y_i \sim CN(0, \Sigma_{Y_i} = H_{b,i} Q H_{b,i}^\dagger + \Sigma_u)$ and $u \in \{b, w\}$. Therefore, $D(P_{Y_w} \| P_{Z_w}) = n D(P_{Y_w} \| P_{Z_w})$ and hence, the LPD constraint is satisfied. Moreover, the probability of decoding error, $P_e^{(n)}$, can be made arbitrarily small for some $\epsilon_n$ that tends to zero as $n$ goes to infinity as long as the codebook size is smaller than that achieves the covert capacity, $C_s(Q)$. Adapting the proof of the achievability of Theorem 1 in [3] for the DMC channels, to prove the remaining part of the achievability of the MIMO AWGN channels. Hence, the sequence of codes $\{R\}$ is achievable if the random sequence $\{\Pi(X^n; Y^n_b)\}$ converges to $n C_s(Q)$ in probability, i.e., the following equation is satisfied:

$$\lim_{n \to \infty} Pr \left\{ \| (X^n; Y^n_b) - n C_s(Q) \| \geq \epsilon \right\} = 0, \quad \epsilon > 0. \quad (55)$$

To complete the achievability proof, the expectation of $\Pi(X^n; Y^n_b)$ is evaluated first. Then, Chebyshev’s inequality is exploited. Let $\Pi = \Pi(X^n; Y^n_b)$,

$$ \Pi = \log \frac{f_X^n x^n \cdot f_Y^n(y^n)}{f_X^n x^n \cdot f_Y^n(y^n)} = \log \frac{f_X^n x^n \cdot f_Y^n(y^n)}{f_Y^n(y^n)} = \log \frac{\prod_{i=1}^n f_X(x_i, y_{b,i})}{\prod_{i=1}^n f_Y(y_{b,i})} = \sum_{i=1}^n \log |\Sigma_{Y_i} \Sigma_{\delta}^{-1}| $$

$$+ \sum_{i=1}^n y_{b,i}^\dagger \Sigma_{\delta}^{-1} y_{b,i} - (y_{b,i} - H_b x_i)^\dagger \Sigma_{\delta}^{-1} (y_{b,i} - H_b x_i),$$

where $f_X(x, y_{b,i}) = \exp(-y_{b,i}^\dagger H_b x_i + y_{b,i}) |\Sigma_{Y_i}^{-1}|^{-1} (y_{b,i} - H_b x_i)$. Then,

$$f_Y(y_{b,i}) = |\pi \Sigma_{Y_i}^{-1}|^{-1} \exp \left\{ -y_{b,i}^\dagger \Sigma_{\delta}^{-1} y_{b,i} \right\}. \quad (56)$$

and $f_Y(y_{b,i}) = |\pi \Sigma_{Y_i}^{-1}|^{-1} \exp \left\{ -y_{b,i}^\dagger \Sigma_{\delta}^{-1} y_{b,i} \right\}$. Then,

$$E[\Pi] = \sum_{i=1}^n \log |\Sigma_{Y_i} \Sigma_{\delta}^{-1}| $$

$$+ \sum_{i=1}^n \mathbb{E} \left\{ \left( \text{tr} \left( \Sigma_{Y_i}^{-1} y_{b,i} y_{b,i}^\dagger \right) - \text{tr} \left( \Sigma_{\delta}^{-1} z_{b,i} z_{b,i}^\dagger \right) \right) \right\}$$

$$= n \log |\Sigma_{Y_i} \Sigma_{\delta}^{-1}| $$

$$+ \sum_{i=1}^n \left( \text{tr} \left( \Sigma_{Y_i}^{-1} y_{b,i} y_{b,i}^\dagger \right) - \text{tr} \left( \Sigma_{\delta}^{-1} z_{b,i} z_{b,i}^\dagger \right) \right)$$

$$= n \log \left| \sum_{i=1}^n \left( \frac{1}{\sigma_b^2} H_b Q H_b^\dagger + I_{N_b} \right) = n C_s(Q), \quad (57) \right.$$ 

$$\var[\Pi] = \var \left\{ \sum_{i=1}^n \log |\Sigma_{Y_i} \Sigma_{\delta}^{-1}| + \sum_{i=1}^n V_i \right\}$$

$$= \sum_{i=1}^n \sum_{i=1}^n \mathbb{E} \left\{ \left( \text{tr} \left( \Sigma_{Y_i}^{-1} y_{b,i} y_{b,i}^\dagger \right) - \text{tr} \left( \Sigma_{\delta}^{-1} z_{b,i} z_{b,i}^\dagger \right) \right) \right\}$$

$$= \sum_{i=1}^n \sum_{i=1}^n \left( \frac{1}{\sigma_b^2} L \right) \right.$$ 

$$= \frac{1}{\sigma_b^2} \sum_{i=1}^n \sum_{i=1}^n \left( \frac{1}{\sigma_b^2} L \right) \right.$$ 

and (a) follows by matrix inversion lemma. Since $Q$ tends to 0 as $n$ goes to infinity, $x_i, \forall i$, goes to 0 and $\Sigma_{Y_i}$, goes to $\sigma_b^2 I_{N_b}$. Hence, $v_i, \forall i$, goes to zero and therefore, $\lim_{n \to \infty} \var\{\Pi(X^n; Y^n_b)\} = 0$ by the bounded convergence theorem. Consequently, using Chebyshev’s inequality: $\lim_{n \to \infty} Pr \{|(X^n; Y^n_b) - n C_s(Q)| \geq \epsilon\} \leq \frac{1}{\var\{\Pi(X^n; Y^n_b)\}} \leq 0.$

**Appendix B**

**Proof of Theorem 2**

Proof. **Converse:** Consider a given $(2^nR, n, \epsilon, \delta)$ covert code that satisfies the LPD constraint, $D(P_{Y_w} \| P_{Z_w}) \leq 2\delta^2/n$, with an average error probability $P_e^{(n)}$ such that $\lim_{n \to \infty} P_e^{(n)} = 0$. From the converse of Theorem 1, $nR(n, \epsilon, \delta) \leq n C(L)(Q) + n \epsilon_n$, and $D(P_{Y_w} \| P_{Z_w}) \leq 2\delta^2/n$. Hence, the KL divergence can be lower bounded as follows:

$$\frac{2\delta^2}{n} \geq D(P_{Y_w} \| P_{Z_w})$$

$$= \sum_{i=1}^N \left\{ q_i \lambda_{w,i}^{\dagger} \log \left( \frac{q_i \lambda_{w,i}^{\dagger}}{\sigma_w} + 1 \right) \right\} \quad (60)$$

$$\geq \sum_{i=1}^N \frac{q_i^2 \lambda_{w,i}^{\dagger}}{2 \sigma_w^2} \right.$$
where the second inequality follows from the logarithm fact that \( \log(x + 1) \geq x - \frac{x^2}{2}, \forall x > 0 \). Let \( \frac{q_i^2 \lambda_{w,i}^2}{2 \sigma_w^2} \leq \frac{2 c_i \delta_L^2}{n}, \forall i \). Consequently,

\[
q_i \leq \frac{2 \sigma_w^2}{\lambda_{w,i}} \sqrt{\frac{c_i}{\delta_L^2}} \delta_L^2 \sqrt{\frac{c_i}{\delta_L^2}} + \frac{1}{n}, \forall i. \tag{61}
\]

Therefore, an upper bound on the scaling, \( L \), can be obtained as follows:

\[
L = \lim_{\epsilon \downarrow 0} \lim_{n \to \infty} \sqrt{n} \frac{R(n, \epsilon, \delta_L)}{C_L(Q)} \leq \lim_{n \to \infty} \sqrt{n} \frac{C_L(Q)}{C_L(Q)} \leq \sum_{i=1}^{N} \frac{\sqrt{2 c_i \sigma_w^2 \lambda_{w,i}}}{\sigma_b^2 \lambda_{w,i}},
\]

where

\[
C_L(Q) = \log \left( \frac{1}{\sigma_b^2} \mathbb{H}_b Q \mathbf{H}_b^t + I_{N_b} \right) \leq \log \left( \frac{1}{\sigma_b^2} \tilde{Q} \Lambda_b + I_{N_b} \right) = \sum_{i=1}^{N} \log \left( \frac{q_i \lambda_{b,i}}{\sigma_b^2} + 1 \right) \geq \sqrt{\frac{2}{n} \sum_{i=1}^{N} q_i \lambda_{b,i}} \leq \sum_{i=1}^{N} \frac{\sqrt{2 c_i \sigma_w^2 \lambda_{w,i}}}{\sigma_b^2 \lambda_{w,i}}.
\]

(a) follows from the logarithm inequality and (b) holds by substituting an upper bound on \( q_i, \forall i \).

**Achievability:** The KL divergence can be upper bounded as follows:

\[
\mathcal{D}(\mathbb{P}_{\mathbf{Y}_w} \| \mathbb{P}_{\mathbf{Z}_w}) = \sum_{i=1}^{N} \left[ q_i \lambda_{w,i}^2 - \log \left( q_i \lambda_{w,i}^2 \sigma_w^2 + 1 \right) \right] = \sum_{i=1}^{N} \left[ q_i \lambda_{w,i}^2 - \left( \frac{2 q_i \lambda_{w,i}^2}{\sigma_w^2} + \frac{q_i \lambda_{w,i}^2}{\sigma_w^2} \right) \right] \geq \sum_{i=1}^{N} \frac{q_i^2 \lambda_{w,i}^2}{2 \sigma_w^2} + q_i \lambda_{w,i} \sigma_w^2 \leq \frac{2 \sigma_L^2}{n},
\]

where (a) follows from the logarithm inequality, \( \frac{1}{1 + x} \leq \log(1 + x), \forall x \geq 0 \). Let \( \frac{q_i^2 \lambda_{w,i}^2}{2 \sigma_w^2} + q_i \lambda_{w,i} \sigma_w^2 \leq \frac{2 c_i \delta_L^2}{n}, \forall i \), then,

\[
q_i \leq \frac{\sigma_w^2}{\lambda_{w,i} \sigma_w^2} \sqrt{\frac{4 n}{c_i \delta_L^2}} + 1, \forall i. \tag{65}
\]

Hence, choosing \( q_i = \frac{2 \sigma_w^2}{\lambda_{w,i} \sigma_w^2} \sqrt{\frac{c_i}{\delta_L^2}} \delta_L^2 \sqrt{\frac{c_i}{\delta_L^2}} + \frac{1}{n}, \forall i \), satisfies the LPD constraint. Therefore, from the achievability of Theorem 1 and using Equation (55), the following bound on the scaling, \( L \), is achievable.

\[
L \geq \lim_{n \to \infty} \frac{1}{2 n \delta_L^2} \mathbb{E} \left[ \left( \mathbb{X}_n^n; \mathbb{Y}_b^n \right) \right] \leq \lim_{n \to \infty} \sqrt{n} \frac{C_S(Q)}{C_L(Q)} \leq \sum_{i=1}^{N} \sqrt{2 c_i \lambda_{b,i} \sigma_b^2},
\]

where

\[
C_L(Q) = \log \left( \frac{1}{\sigma_b^2} \mathbb{H}_b Q \mathbf{H}_b^t + I_{N_b} \right) = \log \left( \frac{1}{\sigma_b^2} \tilde{Q} \Lambda_b + I_{N_b} \right) = \sum_{i=1}^{N} \log \left( \frac{q_i \lambda_{b,i}}{\sigma_b^2} + 1 \right) \geq \sum_{i=1}^{N} \left( \frac{2 q_i \lambda_{b,i}}{\sigma_b^2} + \frac{q_i \lambda_{b,i}}{\sigma_b^2} \right) \geq \sum_{i=1}^{N} \left( \frac{4 \sigma_b^2 \lambda_{b,i} \sqrt{c_i \delta_L^2}}{\sigma_b^2} + 2 \sqrt{n} \lambda_{w,i} \sigma_b^2 \right),
\]

(a) follows from the logarithm inequality and (b) holds by substituting the chosen value of \( q_i, \forall i \).

**Remark 10.** It is worth noting that the choice of the power allocation, \( q_i, \forall i \), in this proof, does not violate the average power constraint, \( P \), and also that the LPD constraint is active. This can be inferred since the covert rate is diminishing, i.e., there is no null-space between Alice and Willie, \( \lambda_{w,i} \neq 0, \forall i \).

---

**APPENDIX C**

**Proof of Theorem 3**

**Proof.** Converse: Similar to the converse of Theorem 2, an upper bound on the scaling is obtained as follows:

\[
L_S \triangleq \lim_{\epsilon \downarrow 0} \lim_{n \to \infty} \sqrt{n} \frac{R(n, \epsilon, \delta_L, \delta_S)}{C_S(Q)} \leq \lim_{n \to \infty} \sqrt{n} \frac{C_S(Q)}{C_L(Q)} \leq \sum_{i=1}^{N} \sqrt{2 c_i \lambda_{b,i} \sigma_b^2} + 1, \forall i. \tag{68}
\]
where
\[
C_S(Q) = \left[ \log \left( \frac{1}{\sigma_w^2} Q A_b + I_{N_a} \right) - \log \left( \frac{1}{\sigma_w^2} Q A_w + I_{N_a} \right) \right]^+ \\
= \sum_{i=1}^{N} \left[ \log \left( \frac{q_i \lambda_{b,i}}{\sigma_b^2} + 1 \right) - \log \left( \frac{q_i \lambda_{w,i}}{\sigma_w^2} + 1 \right) \right]^+ \\
(a) \leq \sum_{i=1}^{N} \left[ \frac{q_i \lambda_{b,i}}{\sigma_b^2} - \left( \frac{q_i \lambda_{w,i}}{\sigma_w^2} + 1 \right) \right]^+ \\
(b) \leq \sum_{i=1}^{N} \left[ 2 \left( \frac{\sigma_w^2}{\sigma_w^2} \right) \lambda_{w,i} \sqrt{c_i \delta_i^2 n} \left( 2 \sqrt{c_i \delta_i^2 n} + 2 \sqrt{c_i \delta_i^2 n} + 1 \right) \right]^+ \tag{69}
\]

(a) follows from the logarithm inequality and (b) holds by substituting the chosen value of \( q_i \), \( \forall i \).

**Achievability:** Using the achievability of Theorem 2 the following bound on the scaling law is achievable:
\[
L_S \geq \lim_{n \to \infty} \frac{1}{2 \sqrt{n} \delta_L} \mathbb{E} \left[ \left( X^n ; Y^n_b \right) - I(X^n ; Y^n_a) \right]^+ \\
= \lim_{n \to \infty} \frac{1}{2 \sqrt{n} \delta_L} C_S(Q) \tag{70}
\]

where
\[
C_S(Q) = \sum_{i=1}^{N} \left[ \log \left( \frac{q_i \lambda_{b,i}}{\sigma_b^2} + 1 \right) - \log \left( \frac{q_i \lambda_{w,i}}{\sigma_w^2} + 1 \right) \right]^+ \\
(a) \geq \sum_{i=1}^{N} \left( \frac{q_i \lambda_{b,i}}{\sigma_b^2} - \frac{q_i \lambda_{w,i}}{\sigma_w^2} \right) \tag{71}
\]

Using the maximum transmission power aimed at Bob’s spatial transmit direction, the LPD constraint in Equation (73) can be rewritten as:
\[
P \lambda_w [\Omega] - \log \left( \frac{P \lambda_w [\Omega]}{\sigma_w^2} \right) \leq \frac{2 \delta_L^2}{n}, \tag{74}
\]
where \( \Omega = \Omega_b - \Omega_w \). Letting \( N_a \) go to infinity and using Equations (44)-(45), the left side of the previous equation goes to zero, under the stated conditions, i.e., the LPD constraint is satisfied \( \forall \delta_L \geq 0 \).

Further, to estimate the number of transmitting antennas that satisfies a given probability of detection, \( \delta_L \), the maximum transmission power that satisfies the given probability of detection is given by solving Equation (74) as follows:
\[
\min \left\{ P, \xi_{w} \frac{\sigma_w^2}{\xi_{w} P N_w [\Omega]} \left[ \frac{1}{\sqrt{1 - (e^{-2 \frac{T}{\sigma_w^2}})} - 1} \right] \right\} \tag{75}
\]

Therefore,
\[
N_a \leq \xi_{w} \frac{\sigma_w^2}{\xi_{w} P N_w [\Omega]} \left[ \frac{1}{\sqrt{1 - (e^{-2 \frac{T}{\sigma_w^2}})} - 1} \right] \tag{76}
\]

where \( \left[ \frac{\sin(\pi N_a \Delta \Omega)}{N_a \sin(\pi \Delta \Omega)} \right] \). Hence,
\[
N_a \geq \frac{\sigma_w^2}{\xi_{w} P N_w [\sin(\pi N_a \Delta \Omega)]^2} \left[ \frac{1}{\sqrt{1 - (e^{-2 \frac{T}{\sigma_w^2}})} - 1} \right] \tag{77}
\]

Besides, using the first-order approximation of the channel capacity, consider the following problem:
\[
\lim_{N_a \to \infty} R(n, 0, \delta_L) = \lim_{N_a \to \infty} \max_{\Omega \geq 0} \frac{C_b(Q)}{\text{tr}(Q) \leq P} \tag{78}
\]
subject to: \( D \leq \frac{2 \delta_L^2}{n} \).

For Appendix D

**Proof of Proposition 1**

**Proof.** Since Alice transmits in the spatial transmit signature in the directional cosine of Bob’s channel, it suffices only to investigate the LPD constraint as \( N_a \) goes to infinity. The LPD constraint is given by:
\[
\log \left( \frac{1}{\sigma_w^2} Q H_b H_w + I_{N_a} \right) + \text{tr} \left( \frac{1}{\sigma_w^2} Q H_b H_w \right) \leq \frac{2 \delta_L^2}{n}. \tag{72}
\]

Using the spatial transmit direction of Willie, Equation (72) can be rewritten as:
\[
\log \left( \frac{1}{\sigma_w^2} Q u_w (\Omega_w) u_w^\dagger (\Omega_w) + I_{N_a} \right) + \text{tr} \left( \frac{1}{\sigma_w^2} Q u_w (\Omega_w) u_w^\dagger (\Omega_w) \right) \leq \frac{2 \delta_L^2}{n}. \tag{73}
\]

Using the maximum transmission power aimed at Bob’s spatial transmit direction, the LPD constraint in Equation (72) can be rewritten as:
\[
\log \left( \frac{1}{\sigma_w^2} Q H_b H_w + I_{N_a} \right) - \log \left( \frac{1}{\sigma_w^2} Q H_w H_w + I_{N_a} \right) \leq \frac{2 \delta_L^2}{n}. \tag{78}
\]

Using the maximum transmission power aimed at Bob’s spatial transmit direction, the LPD constraint in Equation (72) can be rewritten as:
\[
P \lambda_w [\Omega] - \log \left( \frac{P \lambda_w [\Omega]}{\sigma_w^2} \right) \leq \frac{2 \delta_L^2}{n}, \tag{74}
\]

where \( \Omega = \Omega_b - \Omega_w \). Letting \( N_a \) go to infinity and using Equations (44)-(45), the left side of the previous equation goes to zero, under the stated conditions, i.e., the LPD constraint is satisfied \( \forall \delta_L \geq 0 \).

Further, to estimate the number of transmitting antennas that satisfies a given probability of detection, \( \delta_L \), the maximum transmission power that satisfies the given probability of detection is given by solving Equation (74) as follows:
\[
\min \left\{ P, \xi_{w} \frac{\sigma_w^2}{\xi_{w} P N_w [\Omega]} \left[ \frac{1}{\sqrt{1 - (e^{-2 \frac{T}{\sigma_w^2}})} - 1} \right] \right\} \tag{75}
\]

Therefore,
\[
N_a \leq \xi_{w} \frac{\sigma_w^2}{\xi_{w} P N_w [\Omega]} \left[ \frac{1}{\sqrt{1 - (e^{-2 \frac{T}{\sigma_w^2}})} - 1} \right] \tag{76}
\]

where \( \left[ \frac{\sin(\pi N_a \Delta \Omega)}{N_a \sin(\pi \Delta \Omega)} \right] \). Hence,
\[
N_a \geq \frac{\sigma_w^2}{\xi_{w} P N_w [\sin(\pi N_a \Delta \Omega)]^2} \left[ \frac{1}{\sqrt{1 - (e^{-2 \frac{T}{\sigma_w^2}})} - 1} \right] \tag{77}
\]

Besides, using the first-order approximation of the channel capacity, consider the following problem:
\[
\lim_{N_a \to \infty} R(n, 0, \delta_L) = \lim_{N_a \to \infty} \max_{\Omega \geq 0} \frac{C_b(Q)}{\text{tr}(Q) \leq P} \tag{78}
\]
subject to: \( D \leq \frac{2 \delta_L^2}{n} \).

Where \( C_b(Q) = \log \left| \frac{1}{\sigma_b^2} H_b Q H_w + I_{N_a} \right| + D \) and \( D = \mathcal{D}(\mathbb{P} Y_w || \mathbb{P} Z_w) \). This problem is a convex optimization problem.\[a\]

Moreover, the Slater’s condition holds if \( Q \) is chosen to be zero except the first entry, \( q_1 \), is chosen to satisfy \( \frac{q_1 |H_w|}{\sigma_w^2} - \log \left( \frac{q_1 |H_w|}{\sigma_w^2} + 1 \right) \leq \frac{2 \delta_L^2}{n} \) where \( h_{w,1} \) is the first entry of \( H_w \). Hence, \( Q \) satisfies all constraints and the Slater’s condition holds. Therefore, there is no duality gap and Karush–Kuhn–Tucker (KKT) conditions are necessary and sufficient for optimality. Moreover, the Lagrangian function is given by:
\[
\mathcal{L}(Q, \mu, u) = \log \left| \frac{1}{\sigma_b^2} Q H_b H_w + I_{N_a} \right| - \mu (\text{tr}(Q) - P) \tag{78}
\]

\[a\]The KL divergence \( \mathcal{D}(\mathbb{P} Y_w || \mathbb{P} Z_w) \) is convex in \( \mathbb{P} Y_w \) and hence in \( Q \).
From the KKT conditions, the Lagrangian function is optimal in \( Q^* \), i.e., \( \forall Q, \mu^*, \eta^* ) |_{Q=Q^*} = 0 \). Hence,
\[
\begin{align*}
[ Q^* + \sigma_w^2 (H_b^H H_b)^{-1} ]^{-1} + \eta \left[ Q^* + \sigma_w^2 (H_w^H H_w)^{-1} \right]^{-1} \\
- \mu I_{N_u} - \frac{\eta}{\sigma_w^2} H_u^H H_u = 0.
\end{align*}
\]
(81)

Rewrite Equation (81) using the spatial transmit signatures as follows:
\[
\begin{align*}
\left[ Q^* + \frac{\sigma_w^2}{\lambda_b} \left( u_b(\Omega_b) u_b^H(\Omega_b) \right)^{-1} \right]^{-1} \\
+ \eta \left[ Q^* + \frac{\sigma_w^2}{\lambda_w} \left( u_w(\Omega_w) u_w^H(\Omega_w) \right)^{-1} \right]^{-1} \\
= \mu I_{N_u} + \frac{\eta \lambda_w}{\sigma_w^2} u_w(\Omega_w) u_w^H(\Omega_w).
\end{align*}
\]
(82)

With unknown CSI of Willie’s channel, Alice transmits in the spatial transmit signature in the directional cosine of Bob’s channel and hence,
\[
\left[ q^* + \frac{\sigma_w^2}{\lambda_b} \right]^{-1} + \eta \left[ q^* + \frac{\sigma_w^2}{\lambda_w} \left( f(\Omega^2) \right)^{-1} \right]^{-1} = \mu + \eta \lambda_w \frac{|f(\Omega)|^2}{\sigma_w^2}.
\]
(83)

Accordingly, using Equations (44)–(45), under the stated conditions, \( q^* + \frac{\sigma_w^2}{\lambda_w} \rightarrow \mu \) as \( N_u \) goes to infinity. Hence, the maximal covert coding rate of the unit-rank MIMO AWGN channels converges to the maximal coding rate of the unit-rank MIMO AWGN channels and LPD condition is satisfied, i.e., \( \lim_{N_u \to \infty} R(n,0,\delta_L) = \lim_{N_u \to \infty} R(n,0) \), for any given \( \delta_L \geq 0 \) and under the stated conditions.

**APPENDIX E**

**PROOF OF PROPOSITION 2**

**Proof.** Rewrite the LPD constraint using the angular domain representation,
\[
\begin{align*}
\log \left( \frac{1}{\sigma_w} \tilde{Q}^* U_b^H U_w H_w^H H_w U_w^H U_b + I_{N_u} \right) \\
+ \text{tr} \left( \frac{1}{\sigma_w} \tilde{Q}^* U_b^H U_w H_w^H H_w U_w^H U_b \right) \leq \frac{2 \delta_L^2}{n}.
\end{align*}
\]
(84)

where \( \tilde{Q}^* = U_b^H Q U_b, U_b \) and \( U_w \) are unitary transmit matrices whose columns are the orthonormal vectors in the angular domain of Bob’s and Willie’s channel, respectively. Hence, the LPD constraint can be decomposed using the angular representation of Bob’s channel as follows:
\[
\sum_{i=1}^{N_u} q_{i}^* \lambda_{w,i} - \log \left( \frac{q_{i}^* \lambda_{w,i}}{\sigma_w^2} + 1 \right) \leq \frac{2 \delta_L^2}{n},
\]
(85)

where
\[
\tilde{\lambda}_{w,i} = U_b^H U_w \sum_{j=1}^{N_u} \lambda_{w,j} u_{w,j}(\Omega_{w,j}) u_{w,j}^H(\Omega_{w,j}) U_w^H U_b
\]
\[
= \sum_{j=1}^{N_u} \lambda_{w,j} |f(\Omega_{w,j})|^2,
\]
and \( \Omega_{w,j} = \Omega_{b,j} - \Omega_{w,j} \).

Letting \( N_u \) go to infinity and using Equations (44)–(45), the left side of the previous equation goes to zero, under the stated conditions, i.e., the LPD constraint is satisfied \( \forall \delta_L \geq 0 \).

Using the angular domain representation, Equation (81) can be rewritten as follows:
\[
\begin{align*}
[ Q^* + \frac{\sigma_w^2}{\lambda_b} (H_b^H H_b)^{-1} ]^{-1} \\
+ \eta \left[ Q^* + \frac{\sigma_w^2}{\lambda_w} (H_w^H H_w)^{-1} \right]^{-1} \\
\times \mu I_{N_u} + \frac{\eta \lambda_w}{\sigma_w^2} U_w U_w^H H_w H_w^H U_w^H U_b.
\end{align*}
\]
(87)

Hence, Equation (87) can be decomposed in each angular window of Bob’s channel as follows, \( \forall i = \{1, \ldots, N_u \} \),
\[
\left[ q_{i}^* + \frac{\sigma_w^2}{\lambda_{b,i}} \right]^{-1} + \eta \left[ q_{i}^* + \frac{\sigma_w^2}{\lambda_{w,i}} \right]^{-1} = \mu + \eta \lambda_{w,i}.
\]
(88)

Again, Alice transmits in the spatial transmit signatures in the directional cosines of Bob’s channel and hence, using Equations (44)–(45), \( \forall i \), \( q_{i}^* + \frac{\sigma_w^2}{\lambda_{w,i}} \rightarrow \mu \), as \( N_u \) goes to infinity. Thus, the maximal covert coding rate of the multi-path MIMO AWGN channels converges to the maximal coding rate of the multi-path MIMO AWGN channels and LPD condition is satisfied, i.e., \( \lim_{N_u \to \infty} R(n,0,\delta_L) = \lim_{N_u \to \infty} R(n,0) \), for any given \( \delta_L \geq 0 \) and under the stated conditions.

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