Conformal or Confining

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Abstract

We present a lattice study of the phase transitions at zero and nonzero temperature for the SU(3) gauge theory with a varying number of flavours \( N_f \) in the fundamental representation of the gauge group. We show that all results are consistent with a lower edge of the conformal window between \( N_f = 8 \) and \( N_f = 6 \). A lower edge in this interval is in remarkable agreement with perturbation theory and recent large-\( N \) arguments.

Keywords: Non-Abelian gauge theories, QCD, conformal symmetry, conformal window

1. Introduction

Physicists are familiar with two paradigms for conformal symmetry loss in four-dimensional interacting quantum field theories: Quantum Electrodynamics (QED), which is infrared free, and Quantum Chromodynamics (QCD), which loses conformal symmetry in a highly non trivial way, leading to two manifestations of one single breaking phenomenon: asymptotic freedom and confinement

A richer dynamics can be realised in the presence of non-trivial, i.e., interacting, ultraviolet or infrared fixed points of the quantum theory, and may turn out to play a role in unifying the standard model of particle physics with gravity. The possibility that such a scenario is realised somewhere between the electroweak symmetry breaking scale and the Planck scale has recently attained strong theoretical and experimental appeal. The search for renormalization-group fixed points in candidate theories for particle physics and cosmology also shares a wide range of tools and aspects with the study of quantum critical phenomena in condensed matter systems, in three as well as lower spatial dimensions.

The simplest step beyond the QCD paradigm is the so-called conformal window: a family of zero-temperature theories in the phase diagram of non-Abelian gauge theories ranging over a number of massless flavours \( N_f^U < N_f < N_f^U \) in a given representation of the gauge group. The critical number of flavours \( N_f^c \) marks its lower edge, and \( N_f^U \) the upper edge where ultraviolet freedom is lost. The beta-functions \( \beta(g) \) of theories inside the conformal window have a zero at some coupling \( g = g^* \), where the theory has a non-trivial infrared fixed point (IRFP) and it is conformal; they are negative for \( 0 < g < g^* \), and zero at \( g = 0 \) (ultraviolet freedom). Above the conformal window, \( \beta(g) > 0 \) for \( g \gtrsim 0 \) implies infrared freedom opening up the interesting possibility of asymptotic safety, i.e., a non-trivial ultraviolet fixed point (UVFP)

Many questions related to non-trivial fixed points and the emergence of the conformal window — its location in the parameter space and accompanying signatures — are genuinely nonperturbative, and require nonperturbative strategies. Gauge/gravity duality can come to rescue whenever the gauge theory is conformal, as in the conformal window, or near-conformal and still deconfined, as in the high-temperature quark-gluon-plasma. This seems to be not true in the confining and asymptotically free phase of QCD that precedes the conformal window; the recently proposed solution for the scalar glueball correlator in the ‘t Hooft limit of large-\( N \) QCD tells that its momentum dependent logarithms cannot be reproduced by AdS/CFT realisations so far.

In this letter we constrain the lower edge of the conformal window for the SU(3) gauge theory with \( N_f \) flavours in the fundamental representation (i.e., many-flavour massless QCD) by means of a lattice study, a genuinely nonperturbative approach, tailored to the physics problem at hand. Knowing the location of the lower edge is one way to establish how far large-\( N \) predictions, or perturbation theory to a given loop order are from the complete theory. It is also essential in order to correctly identify the properties of many phenomenological models for particle physics with a composite spectrum.

2. The lower edge of the conformal window

Figure summarises the results of this work and explains its strategy: theories with \( N_f < N_f^c \) have spontaneously broken chiral symmetry below a critical temperature \( T_c > 0 \), above which it is restored. Inside the conformal window, theories have exact chiral symmetry and...
are deconfined at all temperatures. Hence, one way to
determine $N'_c$ is to follow the chiral symmetry breaking
pattern, with varying $N_f$ and temperature, for the theory
regularised on a Euclidean spacetime lattice. For $N_f < N'_c$, a
thermal chiral symmetry restoring phase transition occurs
at some $T_c = 1/(a(g^c_L N_f)) > 0$, with $a(g^c_L)$ the lattice
spacing at the critical lattice bare coupling $g^c_L$ on a lat-
tice volume $N^3 \times N_l$, with temporal extent $N_l \ll N_f$. For
$N_f \geq N'_c$, the lattice theory exhibits exact chiral symme-
try at all temperatures, i.e., any $N_f$, and for all values of
$g_L$ in the interval $0 \leq g_L \leq g^c_L$, with $g^c_L$ a sufficiently large
coupling where chiral symmetry will eventually be broken.
This is the bulk phase transition.

The red solid line in Fig. 1 is the line of such transitions
for varying $N_f$. In any given renormalisation scheme, it
manifests the fact that fermion screening is increasingly
effective for increasing $N_f$ inside the conformal window;
this $N_f$ dependence is a leading order effect separating
two phases with different underlying symmetries, different
in nature from any lattice artefact that could occur for
$N_f < N'_c$ inside a single chirally broken phase on coarse
lattices. At finite lattice spacing $a$, the bulk line can be
seen as the $N_f = 1/(a T) \rightarrow \infty$ limit of an $N_f$-finite family
of chiral phase transitions that exists for all $N_f > 0$. The
line therefore stops at $N'_c$, because no $T = 0$ chiral phase
transition occurs for $N_f < N'_c$.

With this paradigm, we have studied the $N_l$ depen-
dence of chiral phase transitions for $4 \leq N_f \leq 12$ in QCD.
The setup of the simulations is the one of [8]. Through-
out, we have not observed an anomalous behaviour that
could hint at consequences of the fourth root of the fermion
determinant for staggered lattice fermions. We have stud-
ied the chiral condensate $\langle \bar{\psi} \psi \rangle$, order parameter of spontane-
ous chiral symmetry breaking, the disconnected chiral
susceptibility $\chi_{\text{disc}}$, and the connected chiral cumu-
late $R_{\pi} = \chi_{\text{conn}} / \chi_{\pi} = (\partial \langle \bar{\psi} \psi \rangle / \partial m_{\text{valence}}) / \langle \langle \bar{\psi} \psi \rangle / m \rangle$. This
quantity is a powerful probe of chiral symmetry: in the
chirally broken phase, the pseudoscalar lowest-lying state
is a Goldstone boson and its vanishing mass in the chiral
limit, together with the non-degeneracy of chiral part-
ners, guarantees that $R_{\pi} \rightarrow 0$ in the chiral limit; in the
chirally restored phase, the degeneracy of the scalar and
pseudoscalar chiral partners implies that $R_{\pi} \rightarrow 1$ in the
chiral limit. We have also studied the Polyakov loop $L$
at finite temperature. Despite it not being a true order
parameter in the presence of fundamental fermions, it is
observed to typically retain the correct features related to
a deconfined or a confined phase. We used volumes with
$N_f < N_l$ and aspect ratio $N_f / N_l \geq 2$, as well as zero tem-
perature volumes with $N_f = 2 N_l$, at a fixed bare fermion
mass $am = 0.01$.

Remarkably, the sharp crossovers in Fig. 2 for $N_f = 12$
to $N_f = 8$ are $N_f$ independent. We do observe a reduced
crossover for $N_f = 7$, an almost closing gap, and signs
of hysteresis in the interval $\beta = [4.275, 4.3]$ right on the
bulk line of Fig. 1. We defer the fate of $N_f = 7$ for a
future study. We have also extrapolated the bulk line to
$N_f = 16$ obtaining $\beta_{c} \sim 1.2$, consistently with previous
studies [7] carried out with different lattice actions and
heavier fermions.

A genuinely different behaviour is observed for the $N_f = 4$
and $6$ theories, whose markedly $N_l$-dependent chiral symme-
try breaking crossovers for varying $N_f < N_l$ are reported
in Fig. 1. Already at $N_l = 6$, the crossover occurs at a
coupling weaker than the one predicted by the linear in
$\beta$ extrapolation of the bulk line. It moves to weaker cou-
pling for increasing $N_l$, consistently with a thermal nature
of the transition. At the same time, no zero-temperature
($N_f > N_l$) phase transition or analytic crossover is observed
to occur along the extrapolated bulk line, nor at weaker
coupling.

These results provide evidence that the lower edge of the
conformal window lies between $N_f = 8$ and $N_f = 6$, the
main result of this work. In what follows, we further scru-
tinise the $N_f = 8$ and $N_f = 6$ theories, discuss implications and
future directions.
than the bulk coupling. These signatures are recognised of corresponding mass effects at weaker coupling for fixed $\langle N_0 \rangle$ and fermion masses has suggested a first order nature of the occurrence of a chiral symmetry breaking bulk transition with those of symmetry for the $\psi \bar{\psi}$ condensate in the chiral limit and with the $\psi \bar{\psi}$ and their perfect over-

Figure 3 shows strikingly the $N_f = 8$ theory and the exotic phase

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Towards weaker coupling, Fig. 2 and Fig. 3 show a sequence of $N_f$-dependent reduced crossovers of the chir-
peaks. For a comparison, the \( N_f = 6 \) signals for \( N_f = 6 \) in Fig. 4 and \( N_f = 8 \) in Fig. 3 are equally sharp, but with genuinely different characteristics.

The thermal nature of the \( N_f = 6 \) and \( N_f = 4 \) crossovers is corroborated by the two-loop asymptotic scaling study in Fig. 5 similar in spirit to [12]. In particular, we compare the lattice data to the two-loop scaling curve \((g_E-2\ell)\) \[ R(g_E)N_t = \left( \frac{T_c}{\Lambda_E} \right)^{-1} = \text{const}, \] with \( R(g_E)(=\alpha \Lambda_E) \) the two-loop asymptotic scaling function and the E-scheme improved coupling [17] \[ g_E^{-2} = (1/3)/(1 - \langle P \rangle_{MC}/3), \] with \( \langle P \rangle_{MC} \) the Monte Carlo determined zero temperature plaquette at the critical coupling. The constant in equation (1) is fixed by the data at \( N_t = 12 \).

The leading-order lattice-distorted two-loop scaling \((g_E-2\ell LD)\) [18], which corrects for lattice artefacts due to the finiteness of \( a \),

\[ R_{LD}^{-1}(g_E) = R^{-1}(g_E) \left[ 1 - h \frac{R^2(g_E)}{R^2(g_E(N_t = 12))} \right] \] is in good agreement with the data for \( h \ll 1 \). A larger \( h \) is needed when the tadpole improved [19] (lattice bare) coupling is used in equation (3), \( h = 0.024 \) \((h = 0.0275)\) and \( h = 0.018 \) \((h = 0.022)\) for \( N_f = 6 \) and \( N_f = 4 \), respectively. The increase of \( h \) from \( N_f = 4 \) to \( N_f = 6 \) is consistent with the fact that the latter theory has a lower critical temperature, hence a larger critical lattice spacing and larger lattice artefacts, for the same \( N_t \). However, if contributions to asymptotic scaling beyond two loops are significant (more likely for \( N_f = 6 \) than \( N_f = 4 \)) the \( h \) parameter in the two-loop scaling will be affected by higher loop effects. A complete four-loop analysis in the \( \overline{MS} \) scheme is desirable, but we currently lack the full conversion of the lattice bare coupling to the \( \overline{MS} \) coupling for the given lattice action. In using the asymptotic scaling formula valid in the massless limit, we have assumed that the relative shift of the (pseudo) critical couplings due to the nonzero fermion mass with varying \( N_t \) is within present uncertainties. A more refined study is certainly worthwhile, following the strategy used for the precise determination of the QCD pseudocritical temperature [20].

5. Discussion

This study consistently suggests that the conformal window of QCD opens between \( N_f = 6 \) and \( N_f = 8 \) fundamental flavours. The devised strategy makes use of the underlying symmetries of the \( SU(3) \) theory and their breaking pattern with varying \( N_f \) and temperature, and it should be preliminary to the lattice investigation of other properties.

Consider the hadron spectrum; it is numerically challenging to distinguish between the spectrum just above and just below the lower edge, in the presence of a nonzero fermion mass and at finite volume. For example, the features of the \( N_f = 8 \) spectrum studied in [21] can be explained by an underlying exact chiral symmetry, in agreement with this work: the lowest-lying scalar meson is light because it needs to be degenerate with its chiral partner, the pseudoscalar, in the chiral limit. One should also observe that inside the conformal window residual finite volume effects behave differently from QCD, given that only a long-range spin-dependent Coulomb potential is present. In fact, the study of \( N_f = 12 \) [8] has shown that spin-1
channels can be more affected than spin-0 channels by the finite size of the box, leading to an enhancement of the $m_{\rho}/m_{\pi}$ finite volume ratio not dissimilar to $N_f=8$ in [21].

A lower edge of the QCD conformal window around $N_f = 7$ is not far from the prediction of two-loop perturbation theory, it is in remarkable agreement with the large-$N$ result $N_f/N = 5/2$, for $N = 3$ [22] and with four-loop perturbation theory [23]. It is also consistent with the perturbatively small value of the fermion mass anomalous dimension of the $N_f = 12$ theory [8, 15], because $N_f = 12$ is not close to the lower edge of the conformal window. A more accurate study of $N_f = 7$ at a smaller fermion mass should allow to determine if this theory is conformal or confining, while any form of preconformal behaviour, if it exists, is likely to be manifest in the $N_f = 6$ theory. The latter is not far from real world QCD and it remains an instructive playground for standard model extensions, including dark matter.

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