RELATIVISTIC CORRECTIONS TO THE SUNYAEV-ZELDOVICH EFFECT FOR CLUSTERS OF GALAXIES. III. POLARIZATION EFFECT

NAOKI ITOH
Department of Physics, Sophia University, 7-1 Kioi-cho, Chiyoda-ku, Tokyo 102-8554, Japan

Satoshi Nozawa
Josai Junior College for Women, 1-1 Keyakidai, Sakado-shi, Saitama 350-0295, Japan

AND

Yasuharu Kohyama
Fuji Research Institute Corporation, 2-3 Kanda-Nishiki-cho, Chiyoda-ku, Tokyo 101-8443, Japan

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ABSTRACT

We extend the formalism of relativistic thermal and kinematic Sunyaev-Zeldovich effects and include the polarization of the cosmic microwave background photons. We consider the situation of a cluster of galaxies moving with a velocity \( \beta \equiv v/c \) with respect to the cosmic microwave background radiation. In the present formalism, polarization of the scattered cosmic microwave background radiation caused by the proper motion of a cluster of galaxies is naturally derived as a special case of the kinematic Sunyaev-Zeldovich effect. The relativistic corrections are also included in a natural way. Our results are in complete agreement with the recent results of relativistic corrections obtained by Challinor, Ford, & Lasenby with an entirely different method, as well as the nonrelativistic limit obtained by Sunyaev & Zeldovich. The relativistic correction becomes significant in the Wien region.

Subject headings: cosmic microwave background — cosmology: theory — galaxies: clusters: general — polarization — radiation mechanisms: nonthermal — relativity

1. INTRODUCTION

The present authors (Itoh, Kohyama, & Nozawa 1998; Nozawa, Itoh, & Kohyama 1998) have recently given accurate relativistic corrections to the thermal and kinematic Sunyaev-Zeldovich effects (Zeldovich & Sunyaev 1969; Sunyaev & Zeldovich 1972, 1980a, 1980b, 1981). Their method was based on the kinetic equation for the photon distribution function using a relativistically covariant formalism (Berestetskii, Lifshitz, & Pitaevskii 1982; Buchler & Yueh 1976). By using the generalized Kompaneets equation (Kompaneets 1957; Weymann 1965), they have derived analytic expressions for the thermal and kinematic Sunyaev-Zeldovich effects as a power series of \( \theta_e = k_B T_e/mc^2 \) and \( \beta \), where \( T_e \) is the electron temperature and \( \beta \) is the peculiar velocity of the cluster divided by the velocity of light. It has been shown by several groups (Stebbins 1997; Challinor & Lasenby 1998; Itoh et al. 1998) that the results for the thermal Sunyaev-Zeldovich effect obtained by the power series expansion agree with the previous numerical calculations by Rephaeli (1995) and by Rephaeli & Yankovitch (1997), thereby proving the validity of their method. In particular, the convergence of the power series expansion has been carefully studied in the paper by Itoh et al. (1998), where analytic expressions up to \( O(\theta_e^2) \) have been derived and the results have been compared with those of the direct numerical integration of the Boltzmann equation. It has been shown that the power series expansion approximation is sufficiently accurate for the region \( k_B T_e \leq 15 \) keV by taking into account up to \( O(\theta_e^3) \) contributions.

In the paper by Nozawa et al. (1998), a formalism for the kinematic Sunyaev-Zeldovich effect for clusters of galaxies with a peculiar velocity \( \beta \) has been derived by applying the Lorentz boost to the standard formalism of the extended Kompaneets equation. With the power series expansion approximation in terms of the electron temperature \( \theta_e \) and the peculiar velocity \( \beta \), an analytic expression for the kinematic Sunyaev-Zeldovich effect, which includes the relativistic corrections of up to \( O(\theta_e^3) \) and \( O(\beta^3 \theta_e) \), has been derived. It has been found that the relativistic correction is significant. Similar works have been reported by Sazonov & Sunyaev (1998) and also by Challinor & Lasenby (1999). They are in essential agreement with the works of the present authors.

In the present paper we address ourselves to the problem of the polarization of the cosmic microwave background radiation (CMBR) caused by the proper motion of the cluster of galaxies. This problem has been studied by Sunyaev & Zeldovich (1980b) in the nonrelativistic limit. The polarization Sunyaev-Zeldovich effect enables the measurement of the tangential velocities of the clusters of galaxies, thus contributing to the measurement of the radial velocities complemented by the measurement of the velocities along the line of sight which is achieved by means of the observation of the kinematic Sunyaev-Zeldovich effect. There exist, however, competing polarization effects connected with the finite depth of the intracluster plasma. These effects have been discussed by Sunyaev & Zeldovich (1980b) and more recently by Sazonov & Sunyaev (1999). In the present paper we solve this problem by extending our already-established formalism to the case of polarized photons. With the present formalism the effect of the polarization of the CMBR can be derived on the same footing as the calculation of the thermal and kinematic Sunyaev-Zeldovich effects. We thereby accurately take into account the relativistic corrections.

After submitting the original manuscript of the present paper to the Astrophysical Journal, we became aware of...
similar works by Audit & Simmons (1998), as well as by Hansen & Lilje (1999). More recently, papers by Sazonov & Sunyaev (1999) and by Challinor, Ford, & Lasenby (1999) have been submitted on this similar subject (the polarization of the CMBR). We have greatly benefited from communication with these authors and have revised the present paper. In particular, A. Challinor has kindly pointed out an error in the original manuscript of the present paper. In the present revised manuscript, we have fully taken into account their comments and have corrected the error in the original manuscript. With the present formalism we have confirmed the recent result obtained by Sunyaev & Zeldovich (1980b).

The present paper is organized as follows: the Kompaneets equation for polarized photons will be derived in § 2. Results of the calculation will be presented in § 3. Concluding remarks will be given in § 4.

2. LORENTZ-BOOSTED KOMPANEETS EQUATION

In the present section we will extend the Kompaneets equation studied by Nozawa et al. (1998) to the case of polarized photons. We will consider a cluster of galaxies (CG) moving with a peculiar velocity with respect to the CMBR. We will formulate the kinetic equation for the photon distribution function using a relativistically covariant formalism (Berestetskii et al. 1982; Buchler & Yueh 1976). As a reference system, we choose the system that is fixed to the CMBR. The z-axis is fixed to the line connecting the CG and the observer. (We assume that the observer is fixed to the CMBR frame.) We fix the positive direction of the z-axis as the direction of the propagation of a photon from the cluster to the observer. In this reference system, the center of mass of CG is moving with a peculiar velocity \( \beta (\equiv v/c) \) with respect to the CMBR. For simplicity, we choose the direction of the velocity in the x-z plane, i.e., \( \beta = (\beta_z, 0, \beta_x) \).

In order to describe Compton scattering of polarized photons by unpolarized electrons, we derive the Stokes parameters using the polarization density matrix (Berestetskii et al. 1982). Here we emphasize the importance of using relativistically covariant formalism. Sunyaev & Zeldovich (1980b) have shown that the degree of the polarization of the cosmic microwave background photon caused by the proper motion of the cluster of galaxies is of order \( \beta^2 \). On the other hand, Nozawa et al. (1998) have derived relativistic corrections to the kinematical Sunyaev-Zeldovich effect which are of order \( \beta^2 \). Therefore, one has to be extremely careful to take into account all the relevant terms of order \( \beta^2 \) when calculating the polarization Sunyaev-Zeldovich effect. This is the reason why we adopt relativistically covariant formalism in this paper.

In the CMBR frame, the time evolution of the photon distribution \( n(\omega) \) is written as follows:

\[
\frac{\partial n(\omega)}{\partial t} = -2 \int \frac{d^3p}{(2\pi)^3} \int d^3p' d^3k' \left( \frac{e^2/4\pi^2}{2\omega}\right) \delta^4(p - k - p' - k') \\
\times \left[ X_{k+p-k'+p} n(\omega) [1 + n(\omega') f(E)] - X_{k' + p - k + p} n(\omega') [1 + n(\omega)] f(E') \right],
\]

where \( k, k' \) are the four-momenta of photons and \( p, p' \) are the four-momenta of electrons, respectively. In equation (2.1) the first term corresponds to the Compton scattering \( k + p \rightarrow k' + p' \). The explicit form is given as follows:

\[
X_{k+p-k'+p} = \bar{X}[1 + \eta \cdot \zeta^{(j)}],
\]

where

\[
\bar{X} = \frac{4}{m^2} \left[ \left( \frac{1}{x} - \frac{1}{y} \right)^2 + \left( \frac{1}{x} - \frac{1}{y} \right) + \frac{1}{4} \left( \frac{y}{x} + \frac{x}{y} \right) \right],
\]

\[
\zeta^{(j)} = \frac{1}{\bar{X}} \left\{ \left( \frac{1}{x} - \frac{1}{y} \right)^2 e^{(1)p} e^{(2)p'} - \left( \frac{1}{x} - \frac{1}{y} \right) e^{(1)p} e^{(2)k'} \right\},
\]

\[
\zeta^{(2)} = 0,
\]

\[
\zeta^{(3)} = \frac{1}{\bar{X}} \left\{ \left( \frac{1}{x} - \frac{1}{y} \right)^2 \left[ e^{(1)p} e^{(2)p'} - \frac{e^{(1)p} e^{(2)p'}}{\bar{X}} \right] \right\} + 2 \left( \frac{1}{x} - \frac{1}{y} \right) \left[ e^{(2)p} e^{(2)k'} + \frac{1}{\bar{X}} e^{(2)k'} \right],
\]

In equation (2.2), \( \eta = (\eta_1, \eta_2, \eta_3) \) is the Stokes parameter, which corresponds to the selection of the polarization of the photon of momentum \( k \) by the detector (Berestetskii et al. 1982). In equations (2.4)–(2.6), \( e^{(1)} \) and \( e^{(2)} \) are the polarization vectors for photons of the momentum \( k \). We choose the explicit forms as follows:

\[
k = (\omega, k), \quad k = (0, 0, \omega),
\]

\[
e^{(1)} = (0, e^{(1)}), \quad e^{(1)} = \frac{k \times k'}{|k \times k'|},
\]

\[
e^{(2)} = (0, e^{(2)}), \quad e^{(2)} = \frac{k \times (k' \times k)}{|k \times (k' \times k')|}.
\]

Similarly, the second term in equation (2.1) corresponds to the Compton scattering \( k' + p' \rightarrow k + p \). The explicit form is as follows:

\[
X_{k' + p' \rightarrow k + p} = \bar{X}[1 + \eta \cdot \zeta^{(j')}],
\]

where

\[
\zeta^{(j')} = \frac{1}{\bar{X}} \left\{ \left( \frac{1}{x} - \frac{1}{y} \right)^2 e^{(1)p'} e^{(2)p'} - \left( \frac{1}{x} - \frac{1}{y} \right) e^{(1)p'} e^{(2)k'} \right\},
\]

\[
\zeta^{(2)} = 0,
\]

\[
\zeta^{(3)} = \frac{1}{\bar{X}} \left\{ \left( \frac{1}{x} - \frac{1}{y} \right)^2 \left[ e^{(1)p'} e^{(2)p'} - \frac{e^{(1)p'} e^{(2)p'}}{\bar{X}} \right] \right\} + 2 \left( \frac{1}{x} - \frac{1}{y} \right) \left[ e^{(2)p'} e^{(2)k'} + \frac{1}{\bar{X}} e^{(2)k'} \right].
\]
Inserting equations (2.2) and (2.14) into equation (2.1), we obtain
\[
\frac{\partial n(\omega)}{\partial t} = -2 \int \frac{d^3 p}{(2\pi)^3} d^3 p' d^3 k W[n(\omega)[1 + n(\omega')]f(E)
- n(\omega)[1 + n(\omega')]f(E')]
\]
\[= -2 \int \frac{d^3 p}{(2\pi)^3} d^3 p' d^3 k W\eta \cdot [\xi(\omega)n(\omega)[1 + n(\omega')]f(E)
- \xi(\omega)n(\omega)[1 + n(\omega')]f(E')], \quad (2.18)
\]
where
\[
W = \frac{(e^2/4\pi)^3}{2\omega_0 EE'}
\]
is the transition probability corresponding to Compton scattering of an unpolarized photon by an unpolarized electron. In equation (2.18) the first term corresponds to the unpolarized case, which has been studied in Itoh et al. (1998) and in Nozawa et al. (1998). The second term corresponds to the polarized photon contribution.

Equation (2.18) is further simplified. With the help of the following relations,
\[
e^{(1)p'} = e^{(1)p},
\]
\[
e^{(3)p'} = e^{(3)p} - e^{(3)p'},
\]
it is straightforward to show that
\[
\xi \equiv \xi(\omega) = \xi(\omega').
\]
Therefore, we have
\[
\frac{\partial n(\omega)}{\partial t} = -2 \int \frac{d^3 p}{(2\pi)^3} d^3 p' d^3 k W[n(\omega)[1 + n(\omega')]f(E)
- n(\omega)[1 + n(\omega')]f(E')]
\]
\[= -2 \int \frac{d^3 p}{(2\pi)^3} d^3 p' d^3 k W\eta \cdot [\xi(\omega)n(\omega)[1 + n(\omega')]f(E)
- \xi(\omega)n(\omega)[1 + n(\omega')]f(E')].
\]
(2.23)

The vector \(\xi\) represents the Stokes parameter of the photon \(k\) which is polarized due to the Compton scattering (Berestetskii et al. 1982).

Next we transform the Stokes parameter \(\xi = (\xi_1, \xi_2, \xi_3)\) to the Stokes parameter \(\xi = (\xi_1, \xi_2, \xi_3)\) defined with respect to the coordinate system \(x'y'z'\) fixed to the CMBR. According to Landau & Lifshitz (1975), we obtain
\[
\xi_1 = \xi_1 \cos 2\left[-\left(\phi' + \frac{\pi}{2}\right)\right] - \xi_3 \sin 2\left[-\left(\phi' + \frac{\pi}{2}\right)\right],
\]
(2.24)
\[
\xi_2 = \xi_2,
\]
(2.25)
\[
\xi_3 = \xi_1 \sin 2\left[-\left(\phi' + \frac{\pi}{2}\right)\right] + \xi_3 \cos 2\left[-\left(\phi' + \frac{\pi}{2}\right)\right],
\]
(2.26)
where \(\phi'\) is the azimuthal angle of the vector \(k'\) with respect to the \(x'y'z'\) plane. Since \(\xi_2 = 0\), we obtain \(\xi_2 = 0\). Therefore, we have
\[
\begin{pmatrix}
\xi_1 \\
\xi_3
\end{pmatrix}
= \begin{pmatrix}
-\cos 2\phi' & -\sin 2\phi' \\
\sin 2\phi' & -\cos 2\phi'
\end{pmatrix}
\begin{pmatrix}
\xi_1 \\
\xi_3
\end{pmatrix}, \quad (2.27)
\]
where \(\xi_1 = \xi(\omega)\) and \(\xi_3 = \xi(\omega)\) are given by equations (2.15) and (2.17).

Thus, the time evolution of the Stokes parameter corresponding to the photon polarization in the CMBR frame is written as follows (Acquista & Anderson 1974; Nagirner 1994):
\[
\frac{\partial}{\partial t}\left\{\begin{pmatrix}
\xi_1 \\
\xi_3
\end{pmatrix}\right\} = -2 \int \frac{d^3 p}{(2\pi)^3} d^3 p' d^3 k W
\times \begin{pmatrix}
-\cos 2\phi' & -\sin 2\phi' \\
\sin 2\phi' & -\cos 2\phi'
\end{pmatrix}\begin{pmatrix}
\xi_1 \\
\xi_3
\end{pmatrix}
\times \{n(\omega)[1 + n(\omega')]f(E) - n(\omega)
\times [1 + n(\omega')]f(E')\}.
\]
(2.28)
The electron Fermi distribution functions in the initial and final states are defined in the CG frame. They are related to the electron Fermi distribution functions in the CMBR frame as follows (Landau & Lifshitz 1975):
\[
f(E) = f_C(E_C),
\]
(2.29)
\[
f(E') = f_C(E_C'),
\]
(2.30)
\[
E_C = \gamma(E - \beta \cdot p),
\]
(2.31)
\[
E_C' = \gamma(E' - \beta \cdot p'),
\]
(2.32)
\[
\gamma = \frac{1}{\sqrt{1 - \beta^2}}
\]
(2.33)
where subscript \(C\) denotes the CG frame. We caution the reader that the electron distribution function is anisotropic in the CMBR frame for \(\beta \neq 0\). By ignoring the degeneracy effects, we have the relativistic Maxwellian distribution for electrons with temperature \(T_e\) as follows:
\[
f_C(E_C) = [e^{(\mu_e - m)/k_B T_e} - 1]^{-1}
\]
\[
\approx e^{-(\mu_e - m)/k_B T_e},
\]
(2.34)
where \((\mu_e - m)\) is the nonrelativistic chemical potential of the electron measured in the CG frame. We now introduce the quantities
\[
\chi \equiv \frac{\omega}{k_B T_e},
\]
(2.35)
\[
\Delta \chi \equiv \frac{\omega' - \omega}{k_B T_e}.
\]
(2.36)
Substituting equations (2.29)–(2.36) into equation (2.28), we obtain
\[
\frac{\partial}{\partial t}\left\{\begin{pmatrix}
\xi_1 \\
\xi_3
\end{pmatrix}\right\} = -2 \int \frac{d^3 p}{(2\pi)^3} d^3 p' d^3 k W
\times \begin{pmatrix}
-\cos 2\phi' & -\sin 2\phi' \\
\sin 2\phi' & -\cos 2\phi'
\end{pmatrix}\begin{pmatrix}
\xi_1 \\
\xi_3
\end{pmatrix}
\times f_C(E_C)[1 + n(\omega')]n(\omega)
where $\hat{k}$ and $\hat{k}'$ are the unit vectors in the directions of $k$ and $k'$, respectively. Equation (2.37) is our basic equation.

3. RESULTS OF THE CALCULATION

We expand the right-hand side of equation (2.37) in powers of $\Delta x$ by assuming $\Delta x \ll 1$, as has been done by Nozawa et al. (1998):

$$\frac{\partial}{\partial t} \left[ n(\omega) \left( \frac{\xi_1}{\xi_3} \right) \right] = 2 \left[ \frac{\partial^2 n}{\partial x^2} I_{1,0} + n(1 + n) I_{1,1} \right] + 2 \left[ \frac{\partial^3 n}{\partial x^3} I_{2,0} + 2(1 + n) \frac{\partial n}{\partial x} I_{2,1} + n(1 + n) I_{2,2} \right] + 2 \left[ \frac{\partial^3 n}{\partial x^3} I_{3,0} + 3(1 + n) \frac{\partial^2 n}{\partial x^2} I_{3,1} + 3(1 + n) \frac{\partial n}{\partial x} I_{3,2} + n(1 + n) I_{3,3} \right] + \cdots + 2n \left[ (1 + n) J_0 + \frac{\partial n}{\partial x} J_1 + \frac{\partial^2 n}{\partial x^2} J_2 + \frac{\partial^3 n}{\partial x^3} J_3 + \cdots \right],$$

where

$$I_{k,l} = \frac{1}{(2\pi)^3} \int d^3 p \int d^3 p' \int d^3 k' W$$

$$\times \begin{pmatrix} \cos 2\phi' & -\sin 2\phi' \\ \sin 2\phi' & \cos 2\phi' \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_3 \end{pmatrix} \times f_c(E_c)(\Delta x) e^{x^2(k' - k)} \gamma(1 - \beta \cdot \hat{k}') \gamma,$$ (3.2)

$$J_k = -\frac{1}{(2\pi)^3} \int d^3 p \int d^3 p' \int d^3 k' W$$

$$\times \begin{pmatrix} -\cos 2\phi' & -\sin 2\phi' \\ \sin 2\phi' & -\cos 2\phi' \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_3 \end{pmatrix} \times f_c(E_c)(\Delta x)^3 (1 - e^{x^2(k' - k)}).$$ (3.3)

For most of the clusters of galaxies, $\beta \ll 1$ is realized. For example, $\beta \approx 1/300$ for a typical value of the peculiar velocity $v = 1,000$ km s$^{-1}$. Therefore, it should be sufficient to expand in powers of $\beta$ and retain up to $O(\beta^2)$ contributions. We assume the initial photon distribution of the CMBR to be Planckian with a temperature $T_0$:

$$n_0(X) = \frac{1}{e^X - 1},$$ (3.4)

where

$$X = \frac{\omega}{k_B T_0}.$$ (3.5)

Assuming $T_0/T_e \ll 1$, we obtain

$$P = \frac{1}{n_0(X)} \Delta \left[ n(X) \left( \frac{\xi_1}{\xi_3} \right) \right]$$

$$= \frac{yXe^X}{e^X - 1} \beta_2^2 \left( F_0 + \theta_x F_1 + \theta_x^* F_2 \right) \left( \begin{array}{c} 0 \\ -1 \end{array} \right),$$ (3.6)

$$\beta_x = \beta \sin \theta_y, \quad \theta_x = \frac{k_B T_e}{mc^2},$$ (3.7)

$$F_0 = \frac{1}{20} \bar{X},$$ (3.8)

$$F_1 = \frac{3}{10} \bar{X} - \frac{2}{5} \left( \bar{X}^2 + \frac{1}{2} \bar{S}^2 \right) + \frac{1}{20}(\bar{X}^3 + 2\bar{X} \bar{S}^2),$$ (3.9)

$$F_2 = \frac{3}{4} \bar{X} - \frac{21}{5} \left( \bar{X}^2 + \frac{1}{2} \bar{S}^2 \right) + \frac{867}{280}(\bar{X}^3 + 2\bar{X} \bar{S}^2)$$

$$+ \frac{4}{7} \left( \bar{X}^3 + \frac{11}{2} \bar{X}^2 \bar{S}^2 + \bar{S}^4 \right) + \frac{1}{35} \left( \bar{X}^5 + 13\bar{X}^3 \bar{S}^2 + \frac{17}{2} \bar{X} \bar{S}^4 \right),$$ (3.10)

$$\bar{X} \equiv X \coth \left( \frac{X}{2} \right),$$ (3.11)

$$\bar{S} \equiv \frac{X}{\sinh \left( X/2 \right)}.$$ (3.12)

where $\theta_y$ is the angle between the directions of the peculiar velocity ($\beta$) and the photon momentum ($k$), which is chosen as the $z$-direction. $N_e$ is the electron number density in the CG frame. The integral in equation (3.8) is over the photon path length in the cluster, and $\sigma_T$ is the Thomson cross section. Here, the following comments should be emphasized. In equation (3.6) the function $F_0 + \theta_x F_1 + \theta_x^* F_2$ is exactly same as the form given in Challinor et al. (1999). It is known that the function is positive for all $X$-values, therefore the minus sign in equation (3.6) guarantees the linear polarization perpendicular to the plane formed by the velocity vector of the cluster and the vector which connects the cluster and the observer.

Equations (3.6)–(3.14) are in complete agreement with the recent result obtained by Challinor et al. (1999) with an entirely different method. It should be emphasized that the effect of the polarization of the CMBR has been derived on the same footing as the calculation of the thermal and kinematic Sunyaev-Zeldovich effects by using the present formalism. It is also worthwhile to mention here that the polarization (equation [3.6]) is proportional to $\beta_x^2$, where $\beta_x$ is the transverse component of the peculiar velocity of CG. In the present case the polarization of the CMBR is nonzero,
because the electron distribution is anisotropic in the CMBR frame for the nonzero peculiar velocity. It should also be noted that we have neglected higher order relativistic corrections such as $O(\theta_e^2)$ terms in deriving equation (3.6). This is because the observation of the higher order term in the polarization of the CMBR caused by the motion of the CG will not be feasible in the near future. Therefore, it should be sufficient to neglect higher order relativistic corrections.

In the Rayleigh-Jeans limit $X \rightarrow 0$ with $\theta_e = 0$, equation (3.6) gives the polarization degree

$$P = \frac{1}{10} y \beta_x^2 \begin{pmatrix} 0 \\ -1 \end{pmatrix},$$

which agrees with the result of Sunyaev & Zeldovich (1980b). Similar results have been reported in more recent works (Audit & Simmons 1998; Hansen & Lilje 1999; Sazonov & Sunyaev 1999; Challinor et al. 1999). We define the polarization function

$$\Xi \equiv \frac{1}{y \beta_x^2} \frac{|\Delta[n(X)]\xi_3|}{n_0(X)} = \frac{Xe^X}{e^X - 1} \left(F_0 + \theta_e F_1 + \theta_e^2 F_2\right).$$

(3.16)

It is clear from Figure 1 that the value of the polarization function $\Xi$ is small for the Rayleigh-Jeans region, however, it quickly becomes large for the Wien region, where the coefficient is larger than 10 at $X \gtrsim 11$. Finally, we define the distribution of the spectral intensity of the polarized radiation as follows:

$$\Delta I_{pol} \equiv X^3 |\Delta[n(X)]\xi_3|$$

$$= y \beta_x^2 \frac{X^4e^X}{(e^X - 1)^2} \left(F_0 + \theta_e F_1 + \theta_e^2 F_2\right).$$

(3.17)

The graph of the function $\Delta I_{pol}/y$ is shown in Figure 2. It has been found that the function has a maximum value of $11 \times 10^{-6}$ at $X \approx 5$ for $k_B T_e = 10 \text{ keV}$, $\beta_x = 1/300$. It is clear that the relativistic correction becomes significant for large values of $X$, typically for $X > 5$.

4. CONCLUDING REMARKS

We have calculated the Stokes parameter corresponding to the polarization of the CMBR caused by the proper motion of a cluster of galaxies. The calculation is based on covariant formalism. With the present formalism the polarization of the CMBR is calculated as a natural extension of the kinematic Sunyaev-Zeldovich effect calculated by Nozawa et al. (1998). Thus, the relativistic corrections have been derived from first principles. We have confirmed the recent result of the relativistic corrections obtained by Challinor et al. (1999) with an entirely different method as well as the nonrelativistic limit obtained by Sunyaev & Zeldovich (1980b). The relativistic correction becomes significant for large values of $X$, $X > 5$. The distribution of the spectral intensity of the polarized radiation has also been calculated. The maximum value of $\Delta I_{pol}/y$ is $11 \times 10^{-6}$ at $X \approx 5$ for $k_B T_e = 10 \text{ keV}$, $\beta_x = 1/300$. At present, the observation of the polarization of the CMBR caused by the motion of a cluster of galaxies is not feasible. However, it is hoped that its observation becomes possible in the future. (See Sazonov & Sunyaev (1999) for the future projects.)

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