Scalar Tetraquark Currents with application to the QCD sum rule

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We study the light scalar mesons in the QCD sum rule. We construct both the diquark-antidiquark currents (qq)(q̄q) and the meson-meson currents (q̄qq)(q̄qq). We find that there are five independent currents for both cases, and derive the relations between them. For the meson-meson currents, five independent currents are formed by products of color singlet q̄q pairs or color octet pairs. However, they can be related to each other, and the relations are derived. We obtain the masses of the light scalar mesons which are consistent with the experiments.

Keywords: tetraquark; QCD sum rule.

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1. Introduction

Most of hadron states, including mesons and baryons, can be well classified by the quark content q̄q and qqq in the quark model. However, there are still many observed states, which can not be explained or difficult to be explained by just using q̄q and qqq. One important example is the pentaquark Θ+ After three years of intense study, the status of Θ+ is still controversial. The charm-strange mesons D_{sJ}(2317), D_{sJ}(2460) and the charmonium state X(3872), Y(4260) are also difficult to be explained by the conventional picture of q̄q in the quark model.

Besides them, the light scalar mesons with masses below 1 GeV have been discussed for thirty years, but are still controversial. By using the conventional picture of q̄q, the mass ordering are expected be \( m_\sigma < m_{a_0} < m_\kappa < m_{f_0} \). However, from the experiments, the mass ordering are \( m_\sigma < m_\kappa < m_{f_0} \sim m_{a_0} \) which can be explained well assuming that they were tetraquark states.
In this paper, we study the light scalar mesons in the QCD sum rule employing tetraquark currents for the interpolating fields. We construct both the diquark-antidiquark currents \((qq)(\bar{q}\bar{q})\) and the meson-meson currents \((\bar{q}q)(\bar{q}q)\). We find that there are five independent currents for each scalar tetraquark state, and derive the relations between them. We also find that there are five meson-meson currents where “mesons” inside are color singlets, and five ones where “mesons” inside are color octets. However, they can be related to each other, and the relations are obtained. The masses of the light scalar mesons are calculated in the QCD sum rule, which are consistent with the experiments.

2. Diquark-Antidiquark Currents

In this section, we construct the diquark-antidiquark currents \((qq)(\bar{q}\bar{q})\) for the state \(\sigma(600)\). The currents for other scalar mesons are similar. In order to make a scalar tetraquark current, the diquark and antidiquark fields should have the same color, spin and orbital symmetries. Therefore, they must have the same flavor symmetry, which is either antisymmetric \(\bar{3}_c \otimes 3_c\) or symmetric \(6_c \otimes \bar{6}_c\). In this paper we choose the antisymmetric one. The details about their flavor structure can be found in the reference [4].

Using the antisymmetric combination for diquark flavor structure, we arrive at the following five independent currents

\[
\begin{align*}
S_3^\sigma &= (u_a^T C \gamma_5 d_b)(\bar{u}_a \gamma_5 C d_b^T - \bar{u}_b \gamma_5 C d_a^T), \\
V_3^\sigma &= (u_a^T C \gamma_\mu \gamma_5 d_b)(\bar{u}_a \gamma^\mu \gamma_5 C d_b^T - \bar{u}_b \gamma^\mu \gamma_5 C d_a^T), \\
T_6^\sigma &= (u_a^T C \sigma_{\mu\nu} d_b)(\bar{u}_a \sigma^{\mu\nu} C d_b^T + \bar{u}_b \sigma^{\mu\nu} C d_a^T), \\
A_6^\sigma &= (u_a^T C \gamma_\mu d_b)(\bar{u}_a \gamma^\mu C d_b^T + \bar{u}_b \gamma^\mu C d_a^T), \\
P_3^\sigma &= (u_a^T C d_b)(\bar{u}_a C d_b^T - \bar{u}_b C d_a^T),
\end{align*}
\]  

where the sum over repeated indices \((\mu, \nu, \cdots)\) for Dirac, and \(a, b, \cdots\) for color indices) is taken. Either plus or minus sign in the second parentheses ensures that the diquarks form the antisymmetric combination in the flavor space. The currents \(S, V, T, A\) and \(P\) are constructed by scalar, vector, tensor, axial-vector, pseudoscalar diquark and antidiquark fields, respectively. The subscripts 3 and 6 show that the diquarks (antidiquark) are combined into the color representation \(\bar{3}_c\) and \(6_c\) (\(3_c\) or \(\bar{6}_c\)), respectively.
3. Meson-Meson Currents

In this section, we construct the meson-meson currents $(\bar{q}q)(\bar{q}q)$ for the state $\sigma(600)$. We find that there are five currents where “mesons” inside are color singlets

\[ S_1^\sigma = (\bar{u}_a u_a)(\bar{d}_b d_b) - (\bar{u}_a d_a)(\bar{d}_b u_b), \]
\[ V_1^\sigma = (\bar{u}_a \gamma_\mu u_a)(\bar{d}_b \gamma^\mu d_b) - (\bar{u}_a \gamma_\mu d_a)(\bar{d}_b \gamma^\mu u_b), \]
\[ T_1^\sigma = (\bar{u}_a \sigma_{\mu\nu} u_a)(\bar{d}_b \sigma^{\mu\nu} d_b) - (\bar{u}_a \sigma_{\mu\nu} d_a)(\bar{d}_b \sigma^{\mu\nu} u_b), \]
\[ A_1^\sigma = (\bar{u}_a \gamma_\mu \gamma_5 u_a)(\bar{d}_b \gamma^\mu \gamma_5 d_b) - (\bar{u}_a \gamma_\mu \gamma_5 d_a)(\bar{d}_b \gamma^\mu \gamma_5 u_b), \]
\[ P_1^\sigma = (\bar{u}_a \gamma_5 u_a)(\bar{d}_b \gamma_5 d_b) - (\bar{u}_a \gamma_5 d_a)(\bar{d}_b \gamma_5 u_b). \]

The minus sign ensures that the diquarks (anti-diquarks) form the antisymmetric combination in the flavor space. These five currents are independent, and can be related to the five diquark-antidiquark currents

\[ 8S_3^\sigma = -2S_1^\sigma + 2V_1^\sigma + T_1^\sigma - 2A_1^\sigma - 2P_1^\sigma, \]
\[ 2V_3^\sigma = 2S_1^\sigma - V_1^\sigma + A_1^\sigma - 2P_1^\sigma, \]
\[ 2T_6^\sigma = 6S_1^\sigma + T_1^\sigma + 6P_1^\sigma, \]
\[ 2A_6^\sigma = 2S_1^\sigma + V_1^\sigma - A_1^\sigma - 2P_1^\sigma, \]
\[ 8P_3^\sigma = -2S_1^\sigma + 2V_1^\sigma + T_1^\sigma + 2A_1^\sigma - 2P_1^\sigma. \]

We find the other five currents where “mesons” inside are color octets

\[ S_8 = (\bar{u}_a \lambda^{\alpha\beta}_{ab} u_b)(\bar{d}_c \lambda^{\alpha\beta}_{cd} d_d) - (\bar{u}_a \lambda^{\alpha\beta}_{ab} d_b)(\bar{d}_c \lambda^{\alpha\beta}_{cd} u_d), \]
\[ V_8 = (\bar{u}_a \gamma_\mu \lambda^{\alpha\beta}_{ab} u_b)(\bar{d}_c \gamma^{\mu\nu} \lambda^{\alpha\beta}_{cd} d_d) - (\bar{u}_a \gamma_\mu \lambda^{\alpha\beta}_{ab} d_b)(\bar{d}_c \gamma^{\mu\nu} \lambda^{\alpha\beta}_{cd} u_d), \]
\[ T_8 = (\bar{u}_a \sigma_{\mu\nu} \lambda^{\alpha\beta}_{ab} u_b)(\bar{d}_c \sigma^{\mu\nu\alpha\beta} \lambda^{\alpha\beta}_{cd} d_d) - (\bar{u}_a \sigma_{\mu\nu} \lambda^{\alpha\beta}_{ab} d_b)(\bar{d}_c \sigma^{\mu\nu\alpha\beta} \lambda^{\alpha\beta}_{cd} u_d), \]
\[ A_8 = (\bar{u}_a \gamma_\mu \gamma_5 \lambda^{\alpha\beta}_{ab} u_b)(\bar{d}_c \gamma^\mu \gamma_5 \lambda^{\alpha\beta}_{cd} d_d) - (\bar{u}_a \gamma_\mu \gamma_5 \lambda^{\alpha\beta}_{ab} d_b)(\bar{d}_c \gamma^\mu \gamma_5 \lambda^{\alpha\beta}_{cd} u_d), \]
\[ P_8 = (\bar{u}_a \gamma_5 \lambda^{\alpha\beta}_{ab} u_b)(\bar{d}_c \gamma_5 \lambda^{\alpha\beta}_{cd} d_d) - (\bar{u}_a \gamma_5 \lambda^{\alpha\beta}_{ab} d_b)(\bar{d}_c \gamma_5 \lambda^{\alpha\beta}_{cd} u_d). \]

They are also independent, and can be related to the five diquark-antidiquark currents, as well as to the five meson-meson currents $S_1^\sigma$, $V_1^\sigma$, $T_1^\sigma$, $A_1^\sigma$ and $P_1^\sigma$

\[ 12S_8^\sigma = -2S_1^\sigma + 6V_1^\sigma + 3T_1^\sigma - 6A_1^\sigma - 6P_1^\sigma, \]
\[ 3V_3^\sigma = 6S_1^\sigma - 5V_1^\sigma - 3A_1^\sigma - 6P_1^\sigma, \]
\[ 3T_6^\sigma = 18S_1^\sigma - 5T_1^\sigma + 18P_1^\sigma, \]
\[ 3A_6^\sigma = -6S_1^\sigma - 3V_1^\sigma - 5A_1^\sigma + 6P_1^\sigma, \]
\[ 12P_3^\sigma = 6S_1^\sigma - 6V_1^\sigma + 3T_1^\sigma + 6A_1^\sigma - 2P_1^\sigma. \]

4. QCD sum rule analysis

We have performed the QCD sum rule analysis for each single current and their linear combinations. We have performed the OPE calculation up to dimension eight.
which contains the four-quark condensates. We find that the results for single currents are not always reliable, while a good sum rule is achieved by a linear combination of $A_6^\sigma$ and $V_3^\sigma$:

$$\eta^\sigma = \cos \theta A_6^\sigma + \sin \theta V_3^\sigma,$$

where $\theta$ is the mixing angle. The best choice of the mixing angle turns out to be $\cot \theta = 1/\sqrt{2}$. The mixed currents for $\kappa$, $a_0$ and $f_0$ can be found in the similar way.

By using this mixed current $\eta^\sigma$, we studied Borel mass $M_B$ and threshold value $s_0$ dependences, which are quite stable. The convergence of the OPE is also good with the positivity of the spectral densities being maintained, and with sufficient pole contribution. Therefore, we have achieved a good QCD sum rule within the present calculation of OPE. We also considered the finite decay width by using the Gaussian distribution instead of the pole term in the phenomenological side, where the predicted masses do not change much as far as the Borel mass is within a reasonable range. Then we can still reproduce the experimental data.

We have also performed the QCD sum rule analysis with the conventional $\bar{q}q$ currents. Their masses are calculated to be around 1.2 GeV as in the previous work. This indicates that the tetraquark currents are more suitable for the description of the light scalar mesons than the conventional ones.

In summary, our QCD sum rule analysis supports a tetraquark structure for low-lying scalar mesons. We construct both the diquark-antidiquark currents and the meson-meson currents. We find that there are five independent currents in both constructions. However, currents in different constructions can be related to each other. Therefore, all the scalar tetraquark currents can be written as a combination of five meson-meson currents where “mesons” inside are color singlets. This conclusion can be extended to other tetraquark currents of different quantum numbers, as well as pentaquark currents.

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References

1. W. M. Yao et al. [Particle Data Group], J. Phys. G 33, 1 (2006).
2. R. L. Jaffe, Phys. Rev. D 15, 267 (1977); R. L. Jaffe, Phys. Rev. D 15, 281 (1977).
3. E. M. Aitala et al., Phys. Rev. Lett. 89, 121801 (2002); M. Ablikim et al., Phys. Lett. B 633, 681 (2006).
4. H. X. Chen, A. Hosaka and S. L. Zhu, Phys. Rev. D 76, 094025 (2007).
5. L. J. Reinders, S. Yazaki and H. R. Rubinstein, Nucl. Phys. B 196, 125 (1982).