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Optimization Algorithms for Improving the Performance of Permutation Trellis Codes

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Abstract

In this paper, soft-decision decoders of permutation trellis codes (PTC) with $M$-ary frequency shift-keying are designed using three optimization algorithms and presented in four different decoding schemes. The bit error rate (BER) performance of the schemes are presented in an additive white Gaussian noise (AWGN) and power-line communication (PLC) channel while the complexities of the schemes are also presented. Results show that the soft-decision decoders improve the BER performance of PTC by up to 3dB for an AWGN channel and 2dB in a PLC channel at a BER of $10^{-4}$.

Keywords: $M$-FSK, Permutation codes, Permutation trellis, Soft-decision decoding, Viterbi decoding algorithm.

1. Introduction

Permutation trellis code (PTC) reported in \cite{1, 2} illustrate the combination of permutation codes with convolutional codes of different code rates, constraint lengths and minimum free distance. PTCs are obtained by using different one-to-one mappings such that the Hamming distance between any two binary sequences and their corresponding permutation sequences are preserved, increased or decreased \cite{1, 3, 2}. The decoder can either be hard-decision or soft-decision \cite{4} using the Viterbi algorithm \cite{5}.

Permutation codes with $M$-ary Frequency Shift Keying ($M$-FSK) have been proven to combat narrow-band interference (NBI) and impulse noise (IN) in a PLC channel \cite{1, 2, 6}. While permutation codes assist with spreading the information across multiple frequencies, $M$-FSK enables non-coherent detection. Using the square law or envelope detection (ED), the detector selects the signal with the highest energy from a pair of $M$ quadrature correlators at each sampling period \cite{4}.

In order to decode PTC with a hard-decision method, the decoded permutation sequence is forwarded to the hard-decision Viterbi decoder in order to recover the transmitted message. However, the demodulated sequence may share the same branch metric with more than one path in the node of the trellis, leading to a decoding error. In addition, the hard-decision decoder compares the received sequence with all the paths at

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In this paper, optimization algorithms are presented as soft-decision decoders of PTC with $M$-FSK in order to improve the BER performance of the Viterbi hard-decision decoder. First, a novel soft-decision decoder is designed using the branch-and-bound (BB) algorithm. The output of the soft-decision (SD) decoder is then combined with the Viterbi decoder in 2 different schemes in order to improve the BER performance. In addition, the iterative permutation soft-decision decoder (PSDD) in [8] is implemented in the PTC decoder in order to forward a sequence with zero Hamming distance with one path in the trellis node. Results show that the additional step of converting the SD output to its binary equivalent before decoding with the Viterbi algorithm produces better BER performance for both decoders. The complexities of the decoders are also presented and in some cases, the SD decoder reduces the decoding complexity for large codebooks while simultaneously improving the BER performance.

In this paper, optimization algorithms are presented as soft-decision decoders PTC with $M$-FSK. First, the branch-and-bound (BB) algorithm is introduced as a novel SD decoder. In addition, the iterative PSDD in [8] is implemented as a PTC decoder. Results show that the additional step of converting the SD output to its binary equivalent before decoding with the Viterbi algorithm produces better BER performance for both decoders. The complexities of the decoders are also presented and in some cases, the SD decoder reduces the decoding complexity for large codebooks while simultaneously improving the BER performance.

2. System Model

2.1. Permutation Trellis Codes

Permutation trellis codes combine convolutional codes and permutation codes at the encoder and uses a modified Viterbi decoder to decode the demodulated signal. The convolutional encoder with constraint length $K$ first encodes the incoming message in Fig. 1 by shifting $k$ binary bits of information via a linear shift register consisting of $K$ stages. The output is a corresponding $n$ bit binary sequence, hence a code rate of $R_C = k/n$. Each $n$-tuple from the convolutional encoder is then mapped onto a permutation codeword $c = c_1c_2\ldots c_M$ of length $M$ such that $n \leq M$. Each codeword belongs to a code book $C$ which consists of $2^n$ unique codewords and $C$ is a subset of the set of all possible permutations consisting of $M!$ codewords. Therefore, the combined code rate is $R_p = k/M$.

The mapping is done such that the Hamming distance between each binary sequence and its respective permutation sequence is preserved, reduced or increased. An example of a distance increasing mapping (DIM) is $\{00 \leftrightarrow 231, 01 \leftrightarrow 213, 10 \leftrightarrow 132, 11 \leftrightarrow 123\}$ where $n = 2, M = 3$ and $\{000 \leftrightarrow 1234, 001 \leftrightarrow 1342, 010 \leftrightarrow 1423, 011 \leftrightarrow 3241, 100 \leftrightarrow 4132, 101 \leftrightarrow 2314, 110 \leftrightarrow 2431, 111 \leftrightarrow 2143\}$ where $n = 3, M = 4$. We refer the reader to...
other examples of DIMs, distance reducing mappings and distance preserving mappings in [1, 2, 3].

2.2. M-ary Frequency Shift Keying with PTC

Consider a set of $M$ orthogonal signals $s_1, s_2, \ldots, s_M$, each having one of the $M$ different frequency components and a low pass vector representation [4]

$$s_m = (0, \ldots, 0, \sqrt{2E_s}, 0, \ldots, 0)^T.$$

The notation $(\cdot)^T$ denotes the transpose operation and the symbol energy $E_s$ is in the $m$-th ($m = 1, 2, \ldots, M$) element denoting the $m$-th frequency. The permutation sequence in Fig. 1 is modulated using the $M$ orthogonal vectors such that each symbol in the sequence has a one-to-one mapping with an $M$-FSK vector $s_m$. Therefore, the $M$-FSK-modulated permutation sequence in Fig. 1 produces $S = [s_{ij}] \in \{0, 1\}^{M \times M}$, which consists of a permutation of all vectors $s_1, s_2, \ldots, s_M$. Each vector $s_m$ represents a column in $S$ and appears once in every $M$ block. Therefore, it is obvious the modulated permutation sequence utilizes all $M$ available frequencies.

Assuming an additive white Gaussian noise (AWGN) channel and that the phase $\phi$ is unknown at the receiver, each vector is non-coherently received at each time slot as

$$y = e^{j\phi}s_m + v_G,$$

where $\phi$ is a random variable and is distributed uniformly between 0 and $2\pi$ while $v_G$ is a complex-valued Gaussian distribution $CN(0, 2N_0)$. In the presence of IN such as in a PLC channel, each transmitted vector is received as

$$y = e^{j\phi}s_m + v_G + v_I\sqrt{p}.$$  

The noise vector $v_I$ models the IN as complex variables with Gaussian distribution $CN(0, \frac{2N_0}{A})$ and $p$ is a Poisson-distributed random sequence whose values are determined by the impulsive index $A$. NBI is also a common noise source in PLC. While IN may affect some or all frequencies in a time slot, NBI may affect one of the $M$ frequencies over a period of time as illustrated in [2]. Each received $M$-FSK vector $y$ at each time slot is demodulated by choosing the $s_m$ with the highest envelope

$$\hat{r} = \arg \max_{m \in M} |y^H \cdot s_m|.$$  

This produces an integer $\hat{r}$ which is equivalent to the likely transmitted $m$-th frequency.

2.3. Hard-Decision Decoder

The received $M$-FSK modulated permutation sequence $Y = [y_{ij}] \in C^{M \times M}$ is equivalent to the transmitted $S$ and consists of a permutation of column vectors $y_1, y_2, \ldots, y_M$. Demodulating each column in $Y$ using (3) produces a permutation of integers $\hat{r} = \hat{r}_1\hat{r}_2\ldots\hat{r}_M$. The branch metric in the Viterbi decoder is then modified by comparing and choosing the minimum Hamming distance $\min_{c_q \in C} d(\hat{r}, c_q)$ between $\hat{r}$ and $c_q$ for $1 \le q \le 2^n$ on each transition path of the trellis. We hence refer to this decoder as the PTC decoder.

The envelope detector in (3) is not optimal in scenarios such as in the PLC channel where large envelopes are detected at the demodulator as a result of IN and NBI [1]. A more adequate threshold detector proposed in [2] sets a threshold of $0.6\sqrt{E_m}$ on the
symbol energy $E_m$ of each received sample, such that samples above the threshold are
demodulated as one while values below the threshold are demodulated to zero. This
results in an $M \times M$ matrix which is then compared with all possible transmitted codeword
matrices and the branch metric is computed as

$$M - \left\lceil \sum_{1 \leq i,j \leq M} (s_{ij} \land y_{ij}) \right\rceil,$$

(4)

for each branch where the notation $\land$ is the binary AND operation. Then, the branch
with the minimum metric value is chosen. We hence refer to this decoder as the PTC
threshold decoder.

2.4. The Permutation Soft-Decision Decoder

Each codeword $c_q$ produces a matrix $S$ such that

$$\sum_{i=1}^{M} s_{ij} = \sum_{j=1}^{M} s_{ij} = 1,$$

(5)

for $i = 1, 2, \ldots, M, j = 1, 2, \ldots, M$. If $Y$ is considered as a soft input to the PSDD
then the PSDD decodes $\hat{Y} = [\hat{y}_{ij}] \in C^{M \times M}$ at the first iteration $g = 1$ by finding the cost $Z_1$ that minimizes

$$Z_1 = \sum_{i=1}^{M} \sum_{j=1}^{M} \hat{y}_{ij}s_{ij},$$

(6)

subject to (5) and $\hat{y}_{ij} = -|y_{ij}|$. The Hungarian Algorithm (HA) \cite{10} can find the matrix $S = [s_{ij}]$ that produces the minimum cost $Z_1$ with a corresponding permutation sequence

$$a_1 = \{(1, j_1), (2, j_2), \ldots, (M, j_M)\},$$

(7)

where $j_1, j_2, \ldots, j_M$ is a permutation of symbols $1, 2, \ldots, M$. If $a_1$ produces a codeword $c \notin C$, the Murty’s algorithm \cite{11} ranks $Z_2, Z_3, \ldots, Z_{2^n}$ to find a codeword $c \in C$. Using
the solution matrix from $Z_1$, $n - 1$ non-empty subsets of $Y$ form nodes $N_1, N_2, \ldots, N_{n-1},$
by partitioning $a_1$. Nodes in this case are defined as $N_1 = \{(1, j_1)\}, N_2 = \{(1, j_1), (2, j_2)\},$
\ldots, $N_{n-1} = \{(1, j_1), \ldots, (n-1, j_{n-1})\}$. Row-column pairs with the bar (\cdot) indicate row-
column elements to be replaced with $\infty$ while row-column elements without the bar are
removed from $Y$. The minimum cost is then solved for each node and the node with the
least cost forms the next assignment $a_2$.

The PSDD makes an optimal decision (OD) by using a brute force to find the codeword
c from all possible permutations with the highest cost

$$Z_{OD} = \arg \min_{c_q \in C} \sum_{i=1}^{M} \sum_{j=1}^{M} \hat{y}_{ij}s_{ij}^{(c_q)}, \quad \text{for } 1 \leq q \leq 2^n$$

(8)

where $s_{ij}^{(c_q)}$ is each element in the permutation matrix produced by a codeword $c$.

3. Proposed Soft-Decision Decoder and Schemes

3.1. Branch and Bound

In order to solve $Y$ using branch and bound (BB) \cite{7}, $Y$ is divided into $M + 1$ levels
$\epsilon_i = \epsilon_0, \epsilon_1, \ldots, \epsilon_M$ and each column is a node defined as $\delta_i = \delta_1, \delta_2, \ldots, \delta_M$. Then, BB
Figure 2: Tree-based method to decode permutation codes using branch and bound with $M = 4$.

[7] is used as a SD technique to solve an assignment problem using a state space tree shown in Fig. 2 with $M = 4$. At level 0, there exists an initial node containing all integers $\{c_1, c_2, \ldots, c_M\}$ in no specific order and none of the $M$ jobs is yet assigned. For simplicity, the initial node for $M = 4$ is 1234 and nodes 1/234, 2/134 up to 4/123 are obtained at $\epsilon_1$. One node $\hat{\delta}_{\epsilon_1}$ at $\epsilon_1$ from the $M$ branches is scheduled if

$$\hat{\delta}_{\epsilon_1} = \sum_{i=1}^{M} \min_{1 \leq j \leq M} (\hat{y}_{ij}). \quad (9)$$

The other nodes are pruned while the surviving node at $\epsilon_1$ satisfying (9) is divided into $M - \epsilon_1$ branches at $\epsilon_2$. The node to be kept at this level satisfies

$$\hat{\delta}_{\epsilon_2} = \hat{y}_{1\delta_2} + \sum_{i=2}^{M} \min_{1 \leq j \leq M, \hat{y}_{i\delta_2} = \infty} (\hat{y}_{ij}). \quad (10)$$

This process of pruning and branching is done until the last level $M$ or the $M - \epsilon_i$ level at which only one assignment is possible.

3.2. Schemes

As shown in Fig. 3, the PSDD uses BB or HA and Murty algorithms to decode $Y$, hence the dashed lines. The dotted lines around the Murty block indicates the Murty algorithm is only used when $c / \in C$ at HA. Otherwise, the Murty step is skipped.

3.2.1. Scheme 1

$Y$ is decoded using HA and Murty and the resulting permutation sequence is decoded using the non-binary Viterbi decoder. If HA produces a sequence $c \notin C$, Murty iterates until $c \in C$ or the specified maximum number of iterations is reached. Therefore, more iterations increase the probability of forwarding a permutation sequence with zero Hamming distance with at least one transition path in the trellis of the Viterbi decoder. For large code books, the PSDD decodes with low complexity and the Viterbi decoder searches the path in the trellis that is exactly equal to $c$.

3.2.2. Scheme 2

Here, the Viterbi decoder is used to decode the binary output from the permutation to binary code demapper. The PSDD produces a permutation code $c$ that is demapped to its binary equivalent. If the final iteration of the PSDD produces a codeword $c \notin C$, minimum distance decoding between $c$ and all codewords in $C$ is performed before demapping. This binary output is then decoded normally using the Viterbi decoder.
3.2.3. Schemes 3 & 4

In Scheme 3, the channel output \( Y \) is decoded with BB to solve for \( Z_1 \). The permutation sequence produced is then forwarded to the non-binary Viterbi decoder. In Scheme 4, the permutation sequence produced by BB is demapped to its binary equivalent and forwarded to the binary Viterbi decoder, similar to Scheme 2. BB uses a single iteration, hence only \( Z_1 \) is solved in Schemes 3 and 4.

In the case of the PLC channel, the soft-decision algorithms can be applied after the threshold detector which applies \( 0.6 \sqrt{E_m} \) on the each received symbol energy as described in [2]. As an example, consider a transmitted codeword 3214. The received permutation matrix \( Y_I \) affected by impulse noise at time \( t_4 \) can be represented as

\[
Y_I = \begin{bmatrix}
t_1 & t_2 & t_3 & t_4 \\
0 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 \\
1 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]
or

\[
Y_N = \begin{bmatrix}
t_1 & t_2 & t_3 & t_4 \\
1 & 1 & 1 & 1 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

when affected by NBI at \( f_1 \). \( Y_I \) and \( Y_N \) can be easily solved using the PSDD to produce the codeword 3214 as shown by the boxed values. The codeword can then be either demapped to its binary equivalent or decoded with the non-binary sequence \( c \) using the Viterbi algorithm.

4. Simulation Results and Complexity Analysis

We compare the simulated BER of the modified non-binary PTC decoder in [2] with BERs obtained from the proposed decoding schemes discussed in Section III. DIMs of PTC were simulated with three different codes of \( R_C = 1/2 \) mapped onto \( M = 3 \), \( R_C = 2/3 \) mapped onto \( M = 4 \) code book while a distance preserving mapping from \( R_C = 1/4 \) onto an \( M = 4 \) code book is also simulated. The first is generated from \( K = 3 \), minimum free distance \( d_{\text{free}} = 9 \) with an octal generator polynomial \((7 \ 5)\), the second is generated from \( K = 4 \), \( d_{\text{free}} = 6 \) with an octal generator polynomial \((1 \ 3 \ 0; 3 \ 2 \ 3)\) and the third is generated from \( R_C = 1/4 \), \( K = 6 \), \( d_{\text{free}} = 18 \) with an octal generator polynomial \((53 \ 67 \ 71 \ 75)\). The first and second code are then mapped onto distance increasing permutation code books of \( M = 3 \) and \( M = 4 \) respectively, while the third code is mapped onto a distance preserving permutation code book of \( M = 4 \). We define SNR per symbol \( E_s/N_0 \) as the ratio of the signal energy \( E_s \) to noise power spectral density \( N_0 \). Therefore, the SNR per bit \( E_b/N_0 \) of the system for bit energy \( E_b \) is defined as

\[
\frac{E_b}{N_0} = \frac{E_s}{N_0 \times R_p \times \log_2 M}.
\]
Results in Fig. 4 show tight BER performance between the modified PTC decoder and the soft-decision schemes. When $M$ is increased in Fig. 5, the BER coding gains obtainable from the Schemes 1 and 2 become more evident with up to 3dB at $g = 4$. Figs. 6 and 7 show significant BER improvement in AWGN and PLC channels respectively with Schemes 3 and 4 performing close to the OD of Schemes 1 and 2.

4.1. BER Performance and $\frac{2^n}{M!}$ ratio

The soft-decision decoders presented provide the advantage of always outputting a permutation of integers by solving an assignment problem. The soft-decision decoder’s probability of producing a codeword $c \in C$ can be described by the ratio $\frac{2^n}{M!}$. Hence, the performance of the schemes approaches the OD performance as $2^n$ increases. For example, in Figs. 6 and 7 which use a convolutional code of $R_C = 1/4$, $K = 6$, an octal generator polynomial (53 67 71 75) and $M = n$, the $2^M$ unique binary sequences are mapped onto a code book with $M = 4$. Since $\frac{2^4}{4!} = 0.667$, each scheme of the soft-decision decoders is more likely to choose a codeword $c \in C$. Fig. 7 uses the threshold detector discussed in Section II-C. The OD of Schemes 1 and 2 are not optimal at lower SNRs because of the $\{0, 1\}$ cost matrix produced by the threshold detector. Hence, multiple assignments can produce the same highest cost and the resulting codeword, which may not belong in $C$ is randomly chosen. For lower values of $2^n$ such as in Figs. 4 and 5 more iterations such as $g = 4$ are required to produce $c \in C$ compared to $g = 1$. Schemes 2 and 4 further improve the performance of the soft-decision because the demapping process chooses one of the $2^n$ possible binary sequences using Hamming distance decoding. Hence, the binary sequence forwarded to the Viterbi decoder follows a path in Viterbi’s trellis with zero Hamming distance with the branches of the decoded path in the trellis.

4.2. Complexity Analysis

The HA and Murty’s algorithm add a combined complexity of $O(M^3)$ and $O(M^3 \ln^2(M))$ for BB for a fixed $M$. For a state sequence of length $T$, the HA and Murty’s algorithm complexity becomes $O(TM^3)$ in Schemes 1 and 2. Similarly, a complexity of $O(TM^3 \ln^2(M))$ is derived for Schemes 3 and 4. These complexities in the PSDD of Schemes 1-4 replace the $O(TM^3)$ complexity required by the ED. Table ?? lists the complexity required by each decoding step in Fig. 4.
Table 1: Time Complexity of Decoders

|                | ED      | PSDD    | Demapping | Viterbi              |
|----------------|---------|---------|-----------|----------------------|
| PTC Decoding   | $O(TM^2)$| 0       | 0         | $O(T \cdot 4^{K-k})$ |
| Scheme 1       | 0       | $O(TM^3)$| 0         | $O(T \cdot 4^{K-k})$ |
| Scheme 2       | 0       | $O(TM^3)$| $O(TM!)$  | $O(T \cdot 4^{K-k})$ |
| Scheme 3       | 0       | $O(TM^3 \ln^2(M))$ | 0         | $O(T \cdot 4^{K-k})$ |
| Scheme 4       | 0       | $O(TM^3 \ln^2(M))$ | $O(TM!)$  | $O(T \cdot 4^{K-k})$ |

The Viterbi decoder’s complexity is given as $O(T\eta^2)$ for a state size of $\eta = 2^{K-k}$, hence a modified complexity of $O(T \cdot 4^{K-k})$ per $T$ block. Schemes 2 and 4 however require an additional complexity of $O(TM!)$ for demapping every $M$ input. In cases where the threshold detector is used, the SD decoder reduces the complexity of computing the trellis branch metric. This is because the classical decoder compares the demodulated matrix with all possible code matrices with a complexity of $O(M!)$ per codeword. This becomes impractical for large codebooks. However, the increase in the complexities of the PSDD and BB for larger codebooks are minimal in comparison.

5. Conclusion

The design of four decoding schemes which combine three optimization algorithms to improve the BER performance of the PTC decoder is presented. The complexities of the different components of the decoding schemes are also analyzed. Demapping the permutation sequence to its binary equivalent before decoding with the Viterbi decoder produces the best BER coding gain when compared to the PTC decoder. The BER coding gains obtained from the schemes increase as the ratio of the permutation code book length to all possible permutations increases. While the improved BER coding gain is at the expense of additional complexity, a reasonable trade-off can be considered when the permutation code book contains large codewords. While multi-tone $M$-FSK has been established to improve the data rate of the $M$-FSK scheme, future work will require techniques for increasing the data rate of permutation-coded $M$-FSK.
Figure 6: BER comparing the four PSDD decoding schemes with the PTC decoder for $R_C = 1/4$ and $M = 4$ in AWGN channel.

Figure 7: BER comparing the four PSDD decoding schemes with the PTC decoder for $R_C = 1/4$ and $M = 4$ in AWGN and IN channel with $A = 0.1$. 
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