Exploring Students Mental Computing Based On Number Sense At 7th Grade Junior High School In Ponorogo

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Abstract. The ability of number sense possessed by students will affect the performance of these students in determining the strategies used in solving a mathematical problem. Students who have a good number sense will show the mental characteristics of computing possessed. According to Markovits and Shower (1994), there are four levels of mental computation in the way students of solving numbers problems, namely: standard, transition, non-standard without reformulation, and non-standard with reformulation. Therefore this study aims to categorize how to answer students in solving numbers problems based on the ability of number sense possessed. This type of research is descriptive qualitative. Subjects selection use the purposive sampling method obtained three subjects that represent the number sense category owned by students. The results of this research indicate the students who have high number sense abilities tend to have computational mental with non-standard categories with reformulation while students who have medium and low number sense abilities tend to have standard category mental computation.

1. Introduction

Learning mathematics in schools is one of the important efforts to support fluency in carrying out daily activities. Mathematics learning in schools is not only related to calculations but also emphasizes how students can think logically, form characters, and be able to decide strategies that are efficient and in accordance with the problems faced [1]. The problem that appears in the mathematics learning process is the students usually solve the routine problems using simple algorithms, when the students are given a high level problem, the students can not to solve the problems [2]. In fact, most students show their little understanding to solve the problem about the use of written algorithms that tend to memorize work procedures, and the lack of basic facts mastery makes students unable to activate the strategies that can be used in arithmetic, so the impact of these is the students cannot be able to find new procedures that are more efficient to solve the complex problems in mathematics[3, 4, 5].
Lately, educational research is increasingly interested in the process of developing students' adaptive skills in arithmetic [5, 6]. Based on research facts about the low performance of students in written algorithms [7, 8] refers to the majority of students' mental computing at a standard level with completion procedures in accordance with what does the teacher teach in class. As we know in our daily activities, we are never separated from numbers. The understanding of numbers meaning refers to the ability to understand numbers and their operations system.

The mastery of meaningfulness of numbers and their operations is called number sense [9, 10]. This number sense is formed and developed from the results of the numbers exploration into different variations and it is connected without being limited to pure computing [8, 11]. This student's sense number can be seen in the reaction when solving a problem through four components, namely: (1) understanding the concept of numbers, operations, recognizing the magnitude of numbers and the relationship of numbers or magnitude numbers, (2) using various numbers of representations and their operations, (3) recognize the relative size of numbers, (4) decompose and reorder numbers, and (5) determine the results of calculations through different strategies [12]. The importance of mastering these numbers attracts many people to research about student number sense [2, 8, 9, 10, 13]. Even the earlier the primary education curriculum such as in Malaysia, America, and Taiwan first emphasized to build and develop the students' understanding of number concept to achieve the needed skills [9, 14, 15]. Circumstances, where students are able to provide a quick response without using specific procedural that exists, is the ability of students to master basic facts [15]. Through mastering, these basic facts can activate and train student number sense. The students who had a good number sense, if he or she was observed by their teacher, they would tend to show their computational mental characteristics such as how to approach mentality, planning and control, as well as flexibility and suitability makes sense that depends on the context and purpose [10].

Expression of calculations made by students shows that one's mental was solving a problem. This computational mental is an individual perspective in arithmetic as a tool to conceptualize learning as well as a form of active development of a thought. Mentally, that is applying a calculation strategy to a problem without using written calculations [11]. Mental computing is the center of attention in the education sector because it the facts, mental computing is active construction of students in learning, especially strategies in solving problems. This computational mental can be seen from the number sense possessed by each student. This computational mental can be categorized into four categories, namely: standard, transition, non-standard without reformulation and non-standard with reformulation [16].

Other study [5, 8, 17] which only examines how the computational process takes place, whereas if viewed from the mental ability of this computation starts from the number sense that is owned so that it will determine the mental computing, direction or strategy of students in processing and dealing with number problems. Thus, this study aimed to identify which problem could cause students low level of competence in calculations. Therefore the researcher wanted to explore the data that obtained from the level of number sense and it could categorize the computational mental result based on the characteristics. The importance of understanding the number sense abilities was possessed by students as a form of prerequisite for all the computational developments that they can getting learning mathematics especially on the problem of numbers [18]. Researchers feel that there is some truth in [9] which states that the teaching process that focuses on written calculations alone cannot help children develop meaningful understanding. That is the reason why NCTM highlights the need for emphasis on understanding the meaning of numbers in the Principles and Standards for School Mathematics [19].
2. Methodology

The purpose of this study was to describe the computing mental exploration result of seventh grade students in solving problems based on their number sense, from the results of this exploration could be categorized as mental computing. Therefore, this research was a explorative descriptive qualitative research. Data collection techniques used were written tests and interviews. The instrument in this study used a descriptive test that contains mathematical problems with the number sense component inside. In addition the researchers also conducted interviews to confirm the results of the students’ answers. This study used triangulation of methods to get valid data. This research was conducted at SMPN 1 Ponorogo. Subjects involved in this study were students of class VII D with a total of 30 students. Based on the results of the number sense ability test, subjects were grouped into three groups namely subjects that have low, medium, and high number sense abilities. Researchers used a purposive sampling technique to choose the subjects for interview, with the same direction, they are the researcher intentionally selecting the students based on their communication fluency accompanied by suggestions from mathematics teachers. Then selected three subjects or students for each number sense category, namely S1 was a subject with a low number sense category, S2 was a subject with a medium number sense category, and S3 is a subject with a high number sense category.

3. Result and Discussion

The results number sense ability test that have been given by students of class VII D in SMPN 1 Ponorogo obtained the highest score data that is 85 and the lowest score obtained is 40 therefore the conversion table can be formed using intervals to classify students' sense numbers with high, medium, and low categories. The following data can be seen in table 1 below:

| No | Student’s categories of number sense | Score  |
|----|------------------------------------|--------|
| 1. | Low                                | $40 \leq x < 55$ |
| 2. | Medium                             | $55 \leq x < 70$ |
| 3. | High                               | $70 \leq x \leq 85$ |

Based on the results in Table 1, three subjects will be chosen to represent the number sense category they have. Following are the details of each chosen research subject.

| No | Name | Score | Student’s categories of number sense |
|----|------|-------|-------------------------------------|
| 1. | AYW  | 40    | Low                                 |
| 2. | ZNG  | 63    | Medium                              |
| 3. | RAS  | 85    | High                                |

The written test that has been given gives the results as shown in the following worksheet:

![Figure 1](image1.jpg) Solving of S1 in addition problems

![Figure 2](image2.jpg) Solving of S1 in multiplication problems
The results of the S1 work in Figure 1 shows that the calculation strategy used by the student was adding two numbers by arranging numbers of 599 above 299. Researchers also conduct interviews to clarify the answers obtained by S1. S1 explains the initial steps of the calculation starting with $9 + 9 = 18$, writing 8 as a result and then storing 1 above the number 9 in front of it. After that, $9 + 9 = 18$ because it was still saving 1 then it became $18 + 1 = 19$ in the end only writing 9 by storing 1 again above the number 5, then adding $5 + 2 = 7$ because it also still saved 1 then the result would be 8 and ends with answer 899. The answer in Figure 1 that is worked out is the correct result. Based on the results of the interview, it showed that even in this settlement strategy S1 did not know what the meaning of storing the number 1, because S1 only followed and memorized the procedures that the teacher taught in class. The following is a sample interview with S1:

\[
\begin{align*}
R & : "What is the purpose of saving 1 that you do?"
S1 & : "Because of the result of $9 \times 9 = 18$ then only eight are written, the number one is stored above 9 (refers to the number 9)."
R & : "Why does it have to be stored above 9?"
S1 & : "I don't know, my teacher since elementary school taught it like that"
R & : "Maybe you know, what is the meaning of the number 1 that you saved?"
S1 & : "I don't know, miss"
\end{align*}
\]

The results of interview were the students did not know understand the meaning of storing the number 1, and even the students did not know that 1 here was a dozens stated. This showed that the importance of basic facts mastering about the value of place in the sum operation, in this case, the students were expected to not only apply the procedures of the teacher but could understand correctly how the purpose of the strategy was done. Similar to Figure 1 and Figure 2 shows the same stacking pattern, it's just different in the operation used, which were multiplication. In this case, also found students did not know the reason why the placement of numbers (numbers contained Figure 2) more indented one number than the numbers above. In this case it is strong that students only memorize the procedures performed by their teacher. Based on Figure 1 and Figure 2 of the settlement strategy it could be noted that S1 has not used a number sense about recognizing the magnitude of numbers, therefore, S1 is only focused on using a written algorithm in a collated manner, whereas students who have good sense numbers will definitely have different points of view in calculate [7, 16].

The completion of Figure 3 about which one of these numbers rows that has the largest and smallest values? The sequence of numbers had the same numerator value, which is $\frac{7}{8}$, $\frac{7}{9}$, $\frac{7}{10}$, and $\frac{7}{11}$ but it was only different for each denominator. To determine these numbers based on the written results and interviews S1 began to find the least common multiple (LCM) from the denominators namely 8, 9, 10.
and 11, then obtained the results of equating the denominator's value with the results of the least common multiple (LCM) which is 3960. Completion conducted by S1 was also still focused on memorizing procedures if S1 known correctly the basic concept of numbers will be easier to determine the largest and smallest numbers without used a written algorithm. In this case, the speed of students in determining the value was also a measure in terms of mental computing that is the skill in mastering the number of numbers.

In Figure 4 the strategy for solving multiplication of integers with decimal numbers used by S1 was to convert 0.8 to fractions $\frac{8}{10}$ first. In this strategy S1 known that 0.8 can be converted into fractions of $\frac{8}{10}$ to facilitate the calculation process. When traced with the interview, the researcher proposed an argument whether 0.8 can be changed to $\frac{4}{5}$ not $\frac{8}{10}$? S1 also immediately answered "it can not, because the value of 0.8 yes $\frac{8}{10}$ not same $\frac{4}{5}$. "because it assumes that $\frac{8}{10} \neq \frac{4}{5}$. Though both had the same magnitude. In this case it could be seen that S1 is able to answer the question correctly but when traced by an interview it known that S1 had not really understood the numbers.

Researchers concluded that the results of the work done by S1 had not mastered the basic facts about numbers and therefore the calculation process only focus on memorize procedure taught by the teacher so that the determination of the results was always done with a written algorithm. In addition, it appears that S1 had not use a number sense about recognize number quantities. The results of this ability test are also strengthened by the number sense that S1 was included in the low category and the completion process was only focused on the computation algorithm by stacking, while students who had good sense numbers would certainly had different points of view in calculating. In this case according to [16] the mental category of S1 included the standard computational category.

Furthermore, the completion by S2 in Figures 5 and 6 showed the strategy carried out as same the settlement strategy on subject 1 used the written algorithm with the stacking method. In this case, it was also seen that S2 also did not use the ability of number sense in solving addition and multiplication problems as well as in Figure 8. The researcher tried to trace the answers obtained by S2 in Figure 8 because S2 directly wrote the answers on a worksheet, then obtained information that
S2 used stacking method for counting $40 \times 0.8$ namely by placed 40 above 0.8. In contrast to S1 at this completion S1 has been able to use its number sensing capabilities by changing 0.8 to $\frac{8}{10}$ to facilitate calculations. But in Figure 7 different from that done by S1 in Figure 3 shows that S2 has been able to use a number sense about recognizing the number than S1 that uses the least common multiple (LCM) to determine the answer even though the reason expressed in the concept of the denominator was not quite right. This number sense ability was used to reduce the use of written algorithms with traditional procedures, with the use of this procedure, it hope that the students could be skilled in processing numbers. Following are the results of researchers' interviews with S2 related to completion in Figure 7:

R : "In the answer to this problem (the researcher points to S2 answer at number 3, Figure 11) you answer that the biggest fraction is and the small one is with the reason that if the number is large, the amount will decrease"

S2 : "yes"

R : "What exactly is fraction here?"

S2 : "(silent) mmmm ... yes, just a fraction miss"

In this case it was also found that S2 did not know the purpose of the sign that divides between the numerator and the denominator in the fraction. Based on the results of the settlement in Figure 5,6,7,8 in this case the researcher concluded that S2 used a computational mental, that tends to be in the standard category [16,17], because S2 was more likely to use traditional procedures in problem solving.

The results of the S3 work Figure 9 on the sum problem showed that the numbers in the sum were rearranged so that it can be easily calculated. It appears that S3 changed the number 599 to 600 – 1 which still had the same value as 599 and 299 was changed to 300 – 1. Based on the results of the interview also S3 stated that for this problem Figure 9 changing the numbers to a value that was closer to 600 and 300 made it easier to calculate quickly without having to write to be able to find the answer. This computational mental strategy supports calculations that are not based on written algorithms which this strategy are more efficient than the stacking methods carried out by S1 and S2.
Likewise with the solution in Figure 10 for the multiplication problem, appears that the strategy used by S3 broke $750 \times 56 = (700 + 50) \times (70 + 6)$. The strategy carried out by S3 on this multiplication is by partitioning strategy, namely by splitting the two operations in decomposition [8] which is $750 \times 56$ to $(700 + 50) \times (70 + 6)$. From the results of written tests and interviews also researchers can conclude that S3 has undergone a change of mind with increasing experience in working on problems [18].

Next, Figure 11 showed S3 be able to determine which fraction had the largest and smallest value without using a written algorithm. S3 only wrote the reason for how he did to determine the value. Based on the reasons stated S3 took the benchmark for $\frac{7}{8}$ approaching $\frac{8}{8}$ or 1 while for $\frac{7}{11}$ approaching $\frac{11}{11}$ or 1. The selection of the strategy used the number sense with benchmark 1, pegging all the fractions with the least difference results. To assuming that to approach 1 difference between $\frac{7}{8}$ and $\frac{8}{8}$ only $\frac{1}{8}$ closer to 1, the fractional value was greater, if the difference was greater then the fractional value is smaller. In addition, the completion of Figure 12 based on the results of interviews and written tests showed that S3 was able to use number sense about the number of numbers that for multiplication of $40 \times 0.8$ would produce a value of less than 40, by looking at the magnitude said that 0.8 is less than 1 so certainly the results of the multiplication would be less than 40 without having to do a different writing algorithm with the strategies used by S1 and S2. Therefore based on Figures 9, 10, 11 and 12 it can be concluded that based on the categorization of [16] S3 computational mental is the non-standard category with reformulation provided that to solve the problem using a strategy by changing the numbers in the problem to make it easier for the calculation process.

4. Conclusion
The ability of number sense possessed by students will affect the performance of these students in determining the strategies used in solving a mathematical problem. The students who have a good number sense will show their mental characteristics of computing possessed. This research needs to determine the extent of student’s numbers mastery to be able to adapt calculations without having the use of written algorithms. Based on the results of exploration, the researcher concluded that the students who have high number sense abilities tend to have computational mentalities with non-standard categories with reformulation, while the students who have medium and low number sense abilities tend to have standard categories of mental computation. From the results of the interviews it could also be known why many students tend to have standard mental computing abilities, they are (1) Lack of the student’s basic numbers facts mastery in the learning process that does not give students the freedom to express and construct their own calculation strategies. (2) The students are accustomed to use the settlement procedure that given by the teacher can be applied to solve computational problems without knowing why the method is used so that students just use the solution to be applied so the students will tend to memorize the given procedure from the teacher. The implications of this research is the mastery of numbers basic facts must be emphasized in mathematics learning to be able to hone the student’s number sense so that the students will have number sense skill in arithmetic strategies to solve higher problems.
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