Exact $\mathcal{O}(g^2\alpha_s)$ top decay width from general massive two-loop integrals

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Abstract

We calculate the $b$-dependent self-energy of the top quark at $\mathcal{O}(g^2\alpha_s)$ by using a general massive two-loop algorithm proposed in a previous article. From this we derive by unitarity the $\mathcal{O}(\alpha_s)$ radiative corrections to the decay width of the top quark, where all effects associated with the $b$ quark mass are included without resorting to a mass expansion. Our results agree with the analytical results available for the $\mathcal{O}(\alpha_s)$ correction to the top quark width.
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Precision measurements of the electroweak parameters are a powerful tool for testing the validity of the standard model and searching for new physics. The impressive accuracy attained experimentally, which is also expected to improve in the future, makes a complete two-loop analysis of the data necessary.

However, on the theoretical side, such a complete two-loop electroweak analysis is far from being a simple task. Apart from the proliferation of diagrams, a special kind of technical problem is encountered in electroweak two-loop calculations. These processes involve particles with different masses, and in general need to be evaluated at finite external momenta. It has been known already for some time that general massive two-loop Feynman diagrams, when evaluated at nonvanishing external momenta, lead to complicated and often unknown special functions. See, for instance, ref. [1] which relates the sunset two-point topology to the Lauricella function.

Recent progress in two-loop electroweak analyses was mainly attained by using mass or momentum expansion methods. We note that in some situations mass expansions turn out to converge well, such as the top mass expansion of the $\alpha_s$ corrections to the $Z \to b\bar{b}$ process [2], while in other cases subleading terms are known to be substantial [3]. In certain situations it was possible to recover the exact function starting from an expansion [4]. On the other hand, when the process under consideration
involves more than one ratio of masses, recovering the exact result from an expansion would be difficult, the functions involved being complicated.

In order to circumvent the use of a mass or momentum expansion, we proposed in ref. [5] a general framework for treating two-loop Feynman graphs by a combination of analytical and numerical methods. Given a Feynman diagram, its tensorial structure is first reduced analytically into a combination of ten special functions $h_i$, which are defined by fairly simple integral representations. The result of this analytical reduction is then integrated numerically.

A subset of the ten special functions $h_i$, namely $h_1$ and $h_2$, are sufficient for treating two-loop corrections in a theory involving only scalars and no derivative couplings. This is the case of radiative corrections of enhanced electroweak strength in the standard model, and in ref. [6] this method was used for deriving corrections of leading power in the Higgs mass. The resulting radiative corrections agree with independent calculations which use different techniques [7].

Going from a scalar theory to a full renormalizable theory with fermions and derivative couplings, such as the standard model, increases a lot the complexity of the problem. The analytical reduction of the tensor structure of a graph into $h_i$ functions along the lines of ref. [5] needs to be handled by a symbolic manipulation program such as FORM or Schoonschip, and results in rather lengthy expressions. Once the reduction is performed, the numerical integration methods are the same as those used in ref. [6], with the only difference that more computing power is needed to handle the number of $h_i$ functions which need to be integrated.

In ref. [8] we have shown how these methods can be used for calculating momentum derivatives of two-loop two-point functions around a finite, non-zero value of the external momentum. Such momentum derivatives are encountered for instance when evaluating wave function renormalization constants on-shell.

In this letter we show how this method can be used for calculating two-point functions of physical interest. We note that, due to the general nature of the tensor reduction algorithm given in ref. [5], the inclusion of more than two external legs proceeds in the same way. The difference is that the resulting expressions are more complicated than those stemming from two-point functions, and more computing power is needed for performing higher-dimensional numerical integration.

In figure 1 we show the two-loop Feynman graphs relevant for the $b$-dependent self-energy of the top quark at order $g^2\alpha_s$.

As for the counterterm structure, we only show in figure 1 the counterterm diagrams which are necessary for renormalizing the imaginary part of the self-energy. These are the on-shell one-loop QCD counterterms of the $b$-quark mass $\delta m_b$ and of the top wave function renormalization constant $\delta Z_t$:

$$
\delta m_b = 2\alpha_s N_c\pi^2 m_b \left\{ \frac{3}{\epsilon} + \frac{3}{2} \left[ \gamma + \log \pi + \log \left( \frac{m_b^2}{\mu^2} \right) \right] - 2 \right\} 
$$

(1)
Figure 1: The two-loop Feynman graphs which contribute to the $b$-mass dependent correction of $\mathcal{O}(\alpha_s g^2)$ to the top self-energy. Only the counterterm diagrams are shown which are needed for subtracting the infinities of the imaginary part of the self-energy, which gives the $\mathcal{O}(\alpha_s)$ correction to the $t \to W + b$ decay.

$$\delta Z_t = \alpha_s N_c \pi^2 \left[ \frac{2}{\epsilon} + \gamma + \log \pi + \log \left( \frac{m_t^2}{\mu^2} \right) + 2 \log \left( \frac{m_t^2}{m_g^2} \right) - 4 \right]$$

Here, $\mu$ is the 't Hooft mass scale, and $m_g$ is the gluon mass infrared regulator. $N_c = 4/3$ is the color factor.

The imaginary part of the two-loop self-energy on-shell is physical. Writing the top self-energy as $\Sigma(p \cdot \gamma) = \Sigma_1(p \cdot \gamma) + \gamma_5 \cdot \Sigma_\gamma(p \cdot \gamma)$, the top decay width is given by $\Gamma_t = 2 \cdot \text{Im} \Sigma_1(p \cdot \gamma = m_t)$. This can be compared to known analytical and numerical results for the $\mathcal{O}(\alpha_s)$ QCD corrections to the top decay width [9], and thus provides a nontrivial test of the two-loop algorithm.

The algebraic reduction of the tensorial structure of the graphs was done by implementing the reduction formulae of ref. [5] into a symbolic manipulation program. The resulting intermediary decomposition into $h_i$ functions is too lengthy to reproduce here. Instead, we give the final results obtained after numerical integration over the analytical decomposition.

Here we would like to discuss briefly the treatment of infrared divergence associ-
Figure 2: The real and the imaginary parts of $\Sigma_1(p \cdot \gamma = m_t)$ at two-loop (see text), given by the graphs of figure 1, as a function of the top mass and of the gluon infrared mass regulator. An overall factor $\alpha_s N_c G_F M_W^2 m_t / \sqrt{2}/(2\pi)^8$ is understood.

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ated with the massless gluon. To regularize the infrared singularities, we introduce a mass regulator for the gluon. We note that in higher-order QCD calculations introducing a gluon mass would affect the Slavnov-Taylor identities. In such a case, a different approach is needed to treat the infrared singular diagrams. One possibility is to try to extract analytically the IR singular part of the diagram and to treat this by dimensional regularization, while the remaining IR finite part of the diagram can be calculated by numerical integration. Whether or not this can always be done in a simple way is beyond the scope of this article. We also note that our approach is mainly designed to treat massive graphs, while other approaches are available for massless QCD calculations. However, this being an $O(g^2\alpha_s)$ correction, the use of a gluon mass regulator is legitimate in this case. The physical quantity $\text{Im} \Sigma_1(p \cdot \gamma = m_t)$, which is associated with the top quark width, is free of infrared singularities, and we checked numerically that indeed the result becomes independent of the gluon regulator when the mass regulator is much smaller than the $b$ mass.

In figure 2 we give numerical results for the ultraviolet finite part of the quantity $\Sigma_1(p \cdot \gamma = m_t)$ derived from figure 1; the inclusion of the counterterm contributions shown in figure 1 makes only the imaginary part (which is physical) of the self-energy finite, both UV and IR. We plot the results for a range of the top mass, and assume $G_F = 1.16637 \cdot 10^{-5}$ GeV$^{-2}$, $m_W = 80.41$ GeV, $m_b = 4.7$ GeV, and $\alpha_s(m_t) = .108$.

In table 1 we give numerical values for the $O(\alpha_s)$ correction to the top decay rate $t \to W + b$ obtained from the imaginary part of the self-energy. Therefore, the $O(\alpha_s)$ QCD correction is obtained as an inclusive quantity, integrated over the gluon spectrum. To check the size of the finite mass of the $b$ quark, we ran our programs in the vanishing $b$ mass limit. At tree level the finite $b$ mass amounts to a 3-4 MeV...
Table 1: The $O(\alpha_s)$ correction to the top decay $t \to W + b$ as obtained from the imaginary part of the two-loop top self-energy of figure 1, integrated numerically. We took $G_F = 1.16637 \times 10^{-5}$ GeV$^{-2}$, $m_W = 80.41$ GeV, $m_b = 4.7$ GeV, and $\alpha_s(m_t) = 0.108$.

correction to the width, and in the $O(\alpha_s)$ correction the effect is negligible. These results agree with existing calculations of $O(\alpha_s)$ corrections to $t \to W + b$ [7], which provides a good test for the two-loop tensor decomposition and numerical integration algorithm.

To conclude, we have shown that the general methods proposed in ref. [5] can be used to calculate physical radiative corrections. We treated at two-loop order the $b$-mass dependent self-energy of the top quark at $O(g^2\alpha_s)$. From the imaginary part of the self-energy we extract the $O(\alpha_s)$ corrections to the top decay process $t \to b+W$, and find agreement with existing results for this process. The calculation is performed while respecting the actual mass and momentum kinematics of the process, and without resorting to a mass or momentum expansion of the diagrams.

Aknowledgements

The work of A.G. was supported by the US Department of Energy. The work of Y.-P. Y. was supported partly by the US Department of Energy.

References

[1] F.A. Berends, M. Buza, M. Böhm and R. Scharf, Z. Phys. C63 (1994) 227.

[2] J. Fleischer, O.V. Tarasov, F. Jegerlehner, P. Raczka, Phys. Lett. B293 (1992) 437; R. Harlander, T. Seidensticker, M. Steinhauser, Phys. Lett. B426 (1998) 125; J. Fleischer, F. Jegerlehner, M. Tentyukov, O. Veretin, BI-TP-99-07 (1999), hep-ph/9904250.

[3] G. Degrassi, F. Feruglio, A. Vicini, S. Fanchiotti, P. Gambino, Phys. Lett. B350 (1995) 75; G. Degrassi, S. Fanchiotti, P. Gambino, Int. J. Mod. Phys. A10 (1995) 1337.

[4] T. van Ritbergen, R.G. Stuart, Phys. Rev. Lett. 82 (1999) 488.

[5] A. Ghinculov and Y.-P. Yao, Nucl. Phys. B516 (1998) 385.
[6] A. Ghinculov and J.J. van der Bij, Nucl. Phys. B436 (1995) 30; A. Ghinculov, Phys. Lett. B337 (1994) 137; (E) B346 (1995) 426; A. Ghinculov, Nucl. Phys. B455 (1995) 21.

[7] P.N. Maher, L. Durand, K. Riesselmann, Phys. Rev. D48 (1993) 1061; (E) D52 (1995) 553; L. Durand, B.A. Kniehl and K. Riesselmann, Phys. Rev. D51 (1995) 5007; Phys. Rev. Lett. 72 (1994) 2534; (E) Phys. Rev. Lett. 74 (1995) 1699; A. Frink, B.A. Kniehl, D. Kreimer, K. Riesselmann, Phys. Rev. D54 (1996) 4548.

[8] A. Ghinculov and Y.-P. Yao, hep-ph/9910423.

[9] M. Jezabek, J.H. Kuhn, Muenchen/Annecy/Hamburg 1992-93: e+ e- collisions at 500-GeV, vol. C, 249-253; Nucl. Phys. B314 (1989) 1; J. Liu, Y.-P. Yao, Int. J. Mod. Phys. A6 (1991) 4925; A. Denner, T. Sack, Nucl. Phys. B358 (1991) 46; G. Eilam, R.R. Mendel, R. Migneron, A. Soni, Phys. Rev. Lett. 66 (1991) 3105.