Dynamical Mechanism for Varying Light Velocity as a Solution to Cosmological Problems

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Abstract

A dynamical model for varying light velocity in cosmology is developed, based on the idea that there are two metrics in spacetime. One metric $g_{\mu\nu}$ describes the standard gravitational vacuum, and the other $\hat{g}_{\mu\nu} = g_{\mu\nu} + \beta \psi_\mu \psi_\nu$ describes the geometry through which matter fields propagate. Matter propagating causally with respect to $\hat{g}_{\mu\nu}$ can provide acausal contributions to the matter stress-energy tensor in the field equations for $g_{\mu\nu}$, which, as we explicitly demonstrate with perfect fluid and scalar field matter models, provides a mechanism for the solution of the horizon, flatness and magnetic monopole problems in an FRW universe. The field equations also provide a “graceful exit” to the inflationary epoch since below an energy scale (related to the mass of $\psi_\mu$) we recover exactly the standard FRW field equations.

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1 Introduction

The standard inflationary epoch scenario can explain several of the observed features of the universe such as the flatness and homogeneity of the present universe, as measured by the cosmic microwave background (CMB) measurements, the isotropy of the universe (horizon problem), and the lack of relic magnetic monopoles [1, 2, 3]. Much effort has gone into constructing viable models of inflation, notably using a scalar inflaton associated with a large vacuum energy (cosmological constant) in the early universe. To invoke the required e-folds of inflation, it is necessary to begin with an approximately pre-inflationary homogeneous universe, otherwise, not enough e-folds of inflation can be achieved to solve the horizon and flatness problems. The generic prediction of inflationary models is that $\Omega = 1$, where $\Omega = \rho/\rho_{\text{crit}}$.

However, because the observed baryonic matter in the universe is not sufficient for this, one is forced into a scenario where most of the matter in the universe is non-baryonic dark matter [3]. Recently, the cosmological constant $\Lambda$ has been replaced with a dynamical, time dependent and spatially inhomogeneous component whose equation of state differs from the standard matter, dark matter and radiation [4]. This new contribution to the cosmological energy density (called “quintessence” or Q-component) can be described by fundamental fields or macroscopic objects, such as light cosmic strings. Fits to recent data are superior to those using $\Lambda$ and cold dark matter.

In the following, we shall develop a dynamical model of the superluminary phase transition that can solve the horizon, flatness and magnetic monopole relic problems, and can furnish a prediction for the temperature fluctuations observed in the CMB. It provides a specific dynamical mechanism to explain the origin of the spontaneous symmetry breaking of local Lorentz and diffeomorphism invariance postulated in earlier publications [5, 6, 7]. This leads to a concrete model in which light effectively travels at a much larger speed in the very early universe and undergoes a phase transition to its standard speed at some critical time $t = t_{pt}$, when the local Lorentz and diffeomorphism symmetries are restored.

Albrecht, Magueijo and Barrow have also proposed models of varying light speed as possible solutions to the initial value problems in cosmology [8, 9]. The model proposed here is distinct in that we are not considering the possibility that what was heretofore considered a ‘constant’ of nature is time varying. Instead we begin by motivating the type of theory that can lead to the physical idea of a speed of light which is ‘dynamical’, and, after re-
stricting ourselves to a concrete realization of this, only then proceed to show that the standard cosmological problems can be solved in the assumption of homogeneity and isotropy. This is certainly closer in spirit to the philosophy that motivated the development of general relativity, and therefore we feel that this is a significant (albeit philosophical) step forward. On a more practical side, although we also expect to see preferred-frame effects locally, there are no non-dynamical fields in our diffeomorphism-invariant formulation and therefore these effects are the result of local dynamics rather than a global frame chosen at the outset. We therefore expect that our model will be less tightly constrained by experiment [10].

One of the important improvements achieved by the new scenario described here, is that the initial conditions of the universe are not as restrictive as those required by the inflationary model. Moreover, although we solve the field equations of the theory assuming that the universe is initially a Friedmann-Robertson-Walker (FRW) flat and homogeneous universe, we expect that the initial universe can be generalized to more complicated inhomogeneous models without losing the generic solutions to the cosmological problems.

2 Bimetrics and Field Equations

The idea here is to present a model that embodies the physical content of a “varying speed of light” in a diffeomorphism invariant manner—without introducing a global preferred reference frame into spacetime. To accomplish this, we note that the causal propagation of electromagnetic fields is determined from the spacetime metric that appears in Maxwell’s equations, and therefore changing the speed at which light propagates is accomplished by making (non-conformal) alterations to this metric. In order for this to have physical consequences we need to ensure that this is not the only metric in spacetime, so that we can therefore concretely talk about the speed of light as being different than the speed of propagation of other fields. (This differs from Brans-Dicke theory in a fundamental way: there the scalar field is essentially a conformal factor and the light cones derived in the “Einstein frame” and the “Jordan frame” are identical, whereas here we are working with two metrics with inequivalent causal structure.)

As a simple and functional model, we consider the introduction of a covector field $\psi_\mu$ which relates the “gravitational metric” $g_{\mu\nu}$ to the “matter
metric” \( \hat{g}_{\mu\nu} \) by
\[
\hat{g}_{\mu\nu} = g_{\mu\nu} + \beta \psi_\mu \psi_\nu, \tag{1}
\]
where \( \beta > 0 \) is a dimensionless constant. The class of models we consider is described by the action
\[
S_{\text{tot}} = S_{\text{gr}}[g] + S_\psi[\psi, g] + S_{\text{matter}}[\hat{g}, \text{matter fields}], \tag{2}
\]
where
\[
S_{\text{gr}}[g] = -\frac{1}{\kappa} \int dt d^3x \sqrt{-g}(R[g] - 2\Lambda), \tag{3}
\]
is the usual Einstein-Hilbert action, \( \kappa = 16\pi G/c^4 \) and \( \Lambda \) is the cosmological constant. Mindful of the issues involved in constructing well-behaved vector field actions \[11\] and noting that the structure (1) is not invariant under local \( U(1) \) transformations, we assume a Maxwell-Proca action for the covector field \((m = \mu c/\hbar \) has dimensions of an inverse length):
\[
S_\psi[\psi, g] = \frac{1}{\kappa} \int dt d^3x \sqrt{-g} \left( \frac{1}{4} B^2 + \frac{1}{2} m^2 \psi^2 \right), \tag{4}
\]
where \( B_{\mu\nu} := \partial_\mu \psi_\nu - \partial_\nu \psi_\mu \), \( B^2 := g^{\mu\nu} g^{\alpha\beta} B_{\mu\alpha} B_{\nu\beta} \) and \( \psi^2 := g^{\mu\nu} \psi_\mu \psi_\nu \). We assume that the matter field action is one of the standard forms, but constructed out of \( \hat{g}_{\mu\nu} \), and therefore the field equations guarantee that the conservation laws \( \nabla_\nu T_{\mu\nu}^{\text{matter}}[\hat{g}] = 0 \), where \( \nabla_\nu \) denotes the covariant derivative with respect to the \( \hat{g}_{\mu\nu} \) metric connection, and
\[
T_{\text{matter}}^{\mu\nu}[\hat{g}] = \frac{2}{\sqrt{-\hat{g}}} \hat{g}^{\mu\alpha} \hat{g}^{\nu\beta} \left( \frac{\delta S_{\text{matter}}[\hat{g}]}{\delta \hat{g}^{\alpha\beta}} \right), \tag{5}
\]
are satisfied.

Variation of (2) with respect to \( g_{\mu\nu} \) and \( \psi_\mu \) leads to the field equations:
\[
\sqrt{-g}(G^{\mu\nu}[g] - \Lambda g^{\mu\nu}) = \frac{1}{2} \sqrt{-g} T^{\mu\nu}[g, \psi] + \frac{\kappa}{2} \sqrt{-\hat{g}} T_{\text{matter}}^{\mu\nu}[\hat{g}], \tag{6}
\]
\[
\sqrt{-g} \left( -\nabla_\nu B^{\mu\nu} + m^2 \psi_\mu \right) = \beta \kappa \sqrt{-\hat{g}} T_{\text{matter}}^{\mu\nu}[\hat{g}] \psi_\nu, \tag{7}
\]
where \( \nabla_\nu \) denotes the covariant derivative formed from the \( g_{\mu\nu} \) metric connection, and
\[
T_{\mu\nu} = -B_{\mu\alpha} B^{\alpha}_\nu + \frac{1}{4} g_{\mu\nu} B^2 + m^2 \psi_\mu \psi_\nu - \frac{1}{2} g_{\mu\nu} m^2 \psi^2. \tag{8}
\]
It is a straightforward exercise to show that the field equations and matter conservation laws are consistent with the Bianchi identities. (Note that this type of “vector-tensor” theory is distinct from those considered, for example, in [2].)

Some comments on this construction are in order. Note that the gravitational metric fields and the covector $\psi_\mu$ propagate on the geometry described by $g_{\mu\nu}$, whereas all other matter fields will propagate on the geometry described by $\hat{g}_{\mu\nu}$. Thus if we consider the motion of a (non-gravitational) test particle, it is reasonable to assume that it is the geodesics of $\hat{g}_{\mu\nu}$ that are of physical interest. It is also very important to recognize that the energy conditions normally imposed on the matter stress-energy tensor no longer have the same implications for the gravitational field equations. To illustrate this, consider a vector field $v^\mu$ which is null with respect to the matter metric: $\hat{g}(v, v) = 0$. From (1) we find that $g(v, v) = -\beta(\psi_\mu v^\mu)^2 \leq 0$, and therefore $v^\mu$ may be spacelike or null with respect to $g_{\mu\nu}$ (which motivates the choice $\beta > 0$). This will manifest itself in Section 3 as a fluid that behaves in a perfectly causal way with respect to the matter metric $\hat{g}_{\mu\nu}$, but appears as an acausally, propagating fluid in (6).

The field equations (7) have the important property that $\psi_\mu = 0$ is always a solution regardless of the matter content of spacetime, in which case the conventional general relativity coupled to matter models are realized and there is no conflict with experiment. In regions of spacetime where $\psi_\mu$ is nonvanishing and $g(\psi, \psi) > 0$, we can restrict ourselves to frames that are aligned with the covector field: $\psi_\mu \to (1, 0, 0, 0)$, and we have reduced the gauge group of the orthonormal frames to $O(3)$. Thus we see that $\psi_\mu \neq 0$ plays the role of a vacuum condensate $\langle \psi_\mu \rangle_0$ that can be said to spontaneously ‘break’ local Lorentz invariance.

Note that the model that we have introduced here is a “metric theory of gravity” in the sense that all matter and non-gravitational fields respond to the metric $\hat{g}_{\mu\nu}$. The dynamics that determine $\hat{g}_{\mu\nu}$ involve the tensor $g_{\mu\nu}$ as well as the covector $\psi_\mu$ and therefore preferred frame effects are possible. However, since we expect that the vector field will essentially lead to repulsive effects (the presence of $\psi_\mu$ locally increases the speed of matter propagation, thereby effectively decreasing the gravitational coupling to the matter) and the magnitude of the vector field is dependent upon the local matter energy density, we expect that the effects of $\psi_\mu$ will be negligible in the present universe.

Most of what appears in this work could be reproduced using a scalar
field to define the matter metric as (for example) \( \hat{g}_{\mu\nu} = g_{\mu\nu} + \beta \nabla_\mu \phi \nabla_\nu \phi \). Although in some ways the scalar field driven mechanism is preferable, the model presented here is much cleaner conceptually as well as algebraically. We will return to the scalar field driven case in a later publication.

3 A Homogeneous and Isotropic Model

We now examine what effect this additional structure has on the standard cosmological scenario. Beginning with a one-parameter family of perfect fluid matter sources \( (p \propto \rho) \), we find a solution that demonstrates that while the matter energy density is greater than a threshold \( \rho_{pt} \) (to be identified later), the matter will experience an inflating universe regardless of the equation of state. These matter models do not provide enough e-folds to solve the Horizon problem unless \( p \approx -c^2 \rho \), which is precisely the relation one derives from a “slowly rolling” scalar field in inflationary scenarios. We show that the mechanism that we are proposing enhances even the simplest inflationary scalar field model to the point where no fine-tuning is necessary.

Assuming that all the fields in the theory are spatially homogeneous and isotropic leads us to the FRW form of the gravitational metric in comoving coordinates:

\[
g_{\mu\nu} dx^\mu dx^\nu = c^2 dt^2 - R^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right].
\]  

(9)

where we will be employing a dimensionless radial coordinate \( r \) and \( k = 0, \pm 1 \) for the flat, closed and hyperbolic spatial topologies. Note that this has fixed the time reparameterization invariance of the theory, and because the spacetime symmetries require that \( \psi_\mu = (c\psi_0(t), 0, 0, 0) \), the matter metric is given by

\[
\hat{g}_{\mu\nu} dx^\mu dx^\nu = c^2 dt^2[1 + \beta \psi_0^2(t)] - R^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right].
\]  

(10)

From these we also find that

\[
\sqrt{-\hat{g}} = (1 + \beta \psi_0^2(t))^{1/2} \sqrt{-g}.
\]  

(11)

As a simple matter model, we consider a perfect fluid

\[
T_{\text{matter}}^{\mu\nu} = (\rho + \frac{P}{c^2}) u^\mu u^\nu - p \hat{g}^{\mu\nu},
\]  

(12)
where the vector field is normalized as \( \hat{g}_{\mu\nu}u^\mu u^\nu = c^2 \), resulting in

\[
u^0 = 1/\sqrt{1 + \beta \psi_0^2(t)}.
\]

(13)

The matter conservation equations lead to

\[
\dot{\rho}(t) + 3\left(\rho(t) + \frac{p(t)}{c^2}\right)\left(\frac{\dot{R}(t)}{R(t)}\right) = 0,
\]

and the single nontrivial field equation derived from (7) is equivalent to

\[
\psi_0(t)\left[ m^2 \sqrt{1 + \beta \psi_0^2(t)} - \beta \kappa c^2 \rho(t) \right] = 0.
\]

(15)

We will examine the nontrivial solution

\[
\beta \psi_0^2(t) = \left(\frac{\rho(t)}{\rho_{pt}}\right)^2 - 1,
\]

(16)

which we will refer to as the “broken phase” in analogy with spontaneous symmetry breaking. We have identified the time at which \( \psi_0(t) \) vanishes as \( t_{pt} \) and for convenience defined

\[
\rho_{pt} := \frac{m^2}{(\beta \kappa c^2)}, \quad H_{pt} := \sqrt{\frac{c^2 m^2}{(6 \beta)}},
\]

(17)

the latter of which we will identify later as (approximately) the Hubble function evaluated at \( t_{pt} \). Since we expect that \( \rho(t) \) will decrease from the initial singularity, if we assume that the universe begins in the “broken phase”, then \( \psi_0(t) \) will decrease to zero. After this time it will be forced to vanish identically because the “broken phase” is unavailable to the system when \( \rho(t) < \rho_{pt} \), and so for \( t > t_{pt} \) we smoothly match the solution to that of a standard FRW cosmological model [13].

Using (16), the nontrivial Friedmann equation obtained from (8) becomes

\[
\frac{\dot{R}(t)^2}{R^2(t)} + \frac{k c^2}{R^2(t)} = \frac{1}{3} c^2 \Lambda + \frac{1}{2} H_{pt}^2 \left[ 1 + \left( \frac{\rho(t)}{\rho_{pt}} \right)^2 \right].
\]

(18)

Adopting the one-parameter family of equations of state (\( \omega > 0 \) is a fixed constant)

\[
p(t) = \frac{\omega}{3} c^2 \rho(t),
\]

(19)
we use \((14)\) to find that
\[
\rho(t) = \rho_{pt}\left(\frac{R_{pt}}{R(t)}\right)^\omega,
\]
where we also define \(R_{pt} := R(t_{pt})\). This is of the same form as the standard Friedmann equation with an effective cosmological constant given by \(\Lambda_{eff} = \Lambda + m^2/(4\beta)\), an effective energy density \(\rho_{eff} = \rho^2(t)/(2\rho_{pt})\), and an effective pressure \(p_{eff} = 5c^2\rho_{eff}/3\) that appears to violate the causal energy requirements. This is not unexpected, for the fluid will appear to violate causality in the “gravitational frame”, while being perfectly causal in the “matter frame”. Although it is relatively trivial to include it, we will only consider solutions with vanishing bare cosmological constant (\(\Lambda = 0\)) here.

The equation \((18)\) is difficult to solve in general, however, if \(\omega \geq 1\) and we require that
\[
H_{pt}^2R_{pt}^2/c^2 \gg 1,
\]
which is essentially the condition that the “size” of the universe at \(t_{pt}\) is much larger than the fundamental length scale of \(\psi_{\mu}\), then it is straightforward to show that the effect of \(k\) in the Friedmann equation is negligible during this phase. (This is also true for \(0 < \omega < 1\) after the length scale of the universe has had time to grow much larger than \(m^{-1}\).) In this approximation (or assuming that \(k = 0\)) we find the solution
\[
R(t) = R_{pt}\sinh^{1/\omega}[((\omega H_{pt}/\sqrt{2})(t - t_{pt}) + \arcsinh(1)].
\]
Identifying the initial singularity at time \(t = t_{init}\) by \(R(t_{init}) = 0\), we observe that the universe remains in this phase for a time \(t_{pt} - t_{init} = \sqrt{2}\arcsinh(1)/(\omega H_{pt})\). The horizon scale on this interval:
\[
d_H(t_{init}; t_{pt}) = \frac{c(t_{init} - t_{pt})}{\arcsinh(1)} \int_0^{\arcsinh(1)} \frac{dy}{\sinh^{1/\omega}(y)},
\]
is finite for \(\omega > 1\) (and, indeed, if \(\omega = 3\) or \(\omega = 4\) does not differ significantly from the usual radiation or matter dominated result: \(2c(t_{init} - t_{pt})\)) and diverges for \(0 < \omega \leq 1\).

The matter metric \((19)\) has \(\hat{g}_{00} = c^2(\rho(t)/\rho_{pt})^2\), and so \(\hat{g}_{\mu\nu}\) may be put into a comoving frame by introducing the coordinate \(d\tau = dt(\rho(t)/\rho_{pt}) = dt(R_{pt}/R(t))^\omega\). Requiring that \(\tau_{pt} = t_{pt}\), we find
\[
\tau = t_{pt} + \frac{\sqrt{2}}{\omega H_{pt}} \ln\left\{\frac{\tanh[((\omega H_{pt}/\sqrt{2})(t - t_{pt}) + \arcsinh(1))/2]}{\tanh(\arcsinh(1)/2)}\right\},
\]

from which it is clear that the finite coordinate time between the initial singularity and \( t_{pt} \) in the gravitational frame is mapped into the infinite coordinate interval \( \tau \in (-\infty, t_{pt}] \) in the matter frame. Using these results, we find

\[
R(\tau) = R_{pt} \sinh^{-1/\omega}(x), \quad \rho(\tau) = \rho_{pt} \sinh(x), \quad H(\tau) = \left( H_{pt}/\sqrt{2} \right) \coth(x),
\]

(25)

where for convenience we have defined

\[
x := (\omega H_{pt}/\sqrt{2})(\tau_{pt} - \tau) + \text{arcsinh}(1), \quad x \in [\text{arcsinh}(1), \infty).
\]

(26)

Note that near the \( R = 0 \) singularity \( (\tau \ll \tau_{pt}) \), we have the approximate form

\[
R(\tau) \approx 2^{1/\omega} R_{pt} \exp\left(H_{pt} \tau/\sqrt{2}\right),
\]

(27)

and we see inflationary behaviour [1] irrespective of the value of \( \omega \). These results may also be derived directly by re-writing (18) in terms of \( \tau \) to find

\[
\frac{\dot{R}^2(\tau)}{R^2(\tau)} + \frac{k c^2}{R^2_{pt}} \left( \frac{R(\tau)}{R_{pt}} \right)^{2(\omega - 1)} = \frac{1}{3} c^2 \Lambda \left( \frac{R(\tau)}{R_{pt}} \right)^{2\omega} + \frac{1}{2} H_{pt}^2 \left[ 1 + \left( \frac{R(\tau)}{R_{pt}} \right)^{2\omega} \right],
\]

(28)

and setting \( \Lambda = 0 = k \).

Evaluating the horizon scale on \((\tau, \tau_{pt})\) we find

\[
d_H(\tau; \tau_{pt}) = 2 c (\tau_{pt} - \tau) \left[ \frac{1}{2(x - \text{arcsinh}(1))} \right]_{\text{arcsinh}(1)}^{x} \sinh^{1/\omega}(y) dy,
\]

(29)

which diverges as \( \tau \to -\infty \) for any \( \omega > 0 \), and therefore it is possible to solve the horizon problem [14]. We have written it in this form to emphasize that this divergence is not solely due to the fact that the time interval is infinite in the limit; the part of (29) in square brackets diverges separately. Of course, the horizon scales (23) and (29) have different physical meanings. The first describes the proper size of regions that are causally connected via gravitational radiation at \( \tau_{pt} \), whereas the latter describes the size of the regions that are connected causally by the propagation of matter fields. Although it may be that solving the horizon problem in the matter sector is sufficient, if \( 0 < \omega \leq 1 \) then the gravitational horizon scale is also “inflated”. We should emphasize that although it is precisely this case that drives the conventional inflationary models, what we have here is more like an “enhanced inflation”–assuming that \( \psi_0 \neq 0 \) initially, any form of matter energy will cause the
universe to inflate. As we shall see, the inflationary effect of matter with
negative pressure is enhanced over conventional models of inflation.

It is perhaps slightly disconcerting that the universe as it appears in the
matter frame is infinitely old. Our gravitational model, however, is classical,
and we do not expect it to be accurate when matter energy densities become
greater than the Planck density \( \rho_P := c^5/(\hbar G^2) \approx 5.2 \times 10^{93} \text{g/cm}^3 \). From (23)
this occurs at a time \( \tau_{qg} \) defined by \( \sinh(x_{qg}) \approx \rho_P/\rho_{pt} \), and therefore between
\( \tau_{qg} \) and \( \tau_{pt} \) the radial scale of the universe increases by a factor
\[
(\rho_P/\rho_{pt})^{1/\omega} =: e^N,
\]
where \( N \geq 60 \) to solve the horizon problem \( \mathbb{H} \). Using this to write \( H_{pt}^{-1} \approx (8\pi/3)^{-1/2} t_P \exp(\omega N/2) \), where \( t_P := (G\hbar/c^5)^{1/2} \approx 5.4 \times 10^{-44} \text{sec} \) is the
Planck time, it is straightforward to determine the coordinate time that the
universe spends in this phase:
\[
\tau_{pt} - \tau_{qg} \approx A(N\omega)N H_{pt}^{-1} \approx A(N\omega)N t_P \sqrt{3/(8\pi)} \exp(\omega N/2),
\]
where \( A(N\omega) := \sqrt{2}(\text{arcsinh}(\exp(N\omega)) - \text{arcsinh}(1))/(N\omega) \in (1, \sqrt{2}) \). We
see, therefore, that the time spent in this phase of the universe is longer than
would be possible in conventional models by a scaling factor \( \approx N \).

As one would expect from (18) or (28), if \( \omega < 1 \) we have a solution to the
flatness problem that mimics that which is provided by inflation (although
admittedly, the status of the flatness problem is not completely clear \( \mathbb{F} \)).
Evaluating \( \Omega - 1 \) and comparing it to the value that obtains at \( t_{pt} \) we find:
\[
|\Omega - 1| = |\Omega - 1| t_{pt} \frac{2c^2\tanh^2(y)}{\sinh^2(\omega y)},
\]
where \( y = (\omega H_{pt}/\sqrt{2})(t - t_{pt}) + \text{arcsinh}(1) \). As \( t \to t_{init} \) (\( \tau \to \tau_{qg} \)) we find that
\( \Omega - 1 \) vanishes for \( \omega > 1 \) and diverges for \( \omega < 1 \), and therefore the extreme
fine-tuning that is necessary in non-inflationary models is avoided \( \mathbb{F} \). For
\( \omega < 1 \) models the solution to the magnetic monopole problem is identical to
that described by Guth \( \mathbb{G} \) and will not be repeated here.

It is noteworthy that models with \( \omega \approx 0 \) achieve a given inflation factor
in the shortest possible time. Although as \( \omega N \to 0 \) the time interval \( \mathbb{I} \)
becomes very small: \( \tau_{pt} - \tau_{qg} \to N(8\pi/3)^{-1/2} t_P \), it is not difficult to show
that the horizon scale \( d_H(\tau_{qg}; \tau_{pt}) \to 2c(\tau_{pt} - \tau_{qg})(\exp(N) - 1)/(2N) \). It is also
interesting to note that within the limits of our approximation, if the broken
phase ends near electroweak symmetry breaking: $1/(2H_{pt}) \approx 10^{-11} \text{sec}$, then we are led to $N \omega \approx 152$. If we assume that radiative energy dominates the universe back to $\tau_{qg}$ then we get $N \approx 38$ which is not quite sufficient to solve the horizon problem. Requiring that $N \approx 60$ would imply that the broken phase lasts well into the observable universe. Choosing instead the broken phase to end at $1/(2H_{pt}) \approx 10^{-35} \text{sec}$ (roughly corresponding to the temperature at which some GUT symmetry is broken spontaneously [4]), we find $N \omega \approx 42$, clearly requiring that $\omega < 1$.

To achieve this with a more realistic matter model, we adopt a scenario familiar from inflation, namely, we assume that the universe exits the quantum gravitational stage with the matter energy of the universe dominated by a Higgs field close to a “false vacuum”. Assuming the “slow roll” approximation where the kinetic energy of the scalar field is (at least initially) dominated by the potential energy, the effective pressure is negative and the effective equation of state has $\omega \approx 0$.

Working in the gravitational frame, the field equation for the scalar field

$$\ddot{\phi}(t) + 3H(t)\dot{\phi}(t) + c^2\delta V[\phi] = 0,$$

(33)

assuming that the acceleration term can be neglected and that $H(t) \approx H_\phi$, has the approximate solution

$$\phi(t) \approx \phi_0 \exp \left[ \frac{c^2m_\phi^2}{6H_\phi^2}(t - t_{eg}) \right].$$

(34)

For clarity we use the minimal Higgs potential (we have chosen $\phi$ to be dimensionless) $V[\phi] = \lambda(\phi^2 - m_\phi^2)^2/(2\lambda)/(4l_p^2)$, where $h m_\phi/c$ is the mass of the scalar field in the physical vacuum which we will assume corresponds to the GUT scale $\approx 5 \times 10^{14} \text{GeV}$, $\lambda$ is a dimensionless coupling constant and $l_p := (G\hbar/c^3)^{-1/2} \approx 1.3 \times 10^{-33} \text{cm}$ is the Planck length. To find (34) we have also made the approximation $\delta V[\phi] \approx -m_\phi^2/2$; note that (34) is valid provided that $H_\phi^2 \gg c^2m_\phi^2/18$.

The energy density of a homogeneous and isotropic scalar field $\rho = (\frac{1}{2} \dot{\phi}^2 + c^2V[\phi])/(\kappa c^4)$ enters into the Friedmann equations (18) proportional to $\rho^2(t)$, and using $\rho(t) \approx l_p^4m_\phi^4/(16\lambda\kappa c^2)$, we find that

$$\frac{\dot{R}^2(t)}{R^2(t)} \approx H_\phi^2 := \frac{1}{2}H_{pt}^2 \left[ 1 + \frac{\beta l_p^2m_\phi^4}{16\lambda m^2} \right]^2.$$
This should be compared to \( H_\phi^2 \approx c^2 P_m^4/\lambda (96\phi) \) for a scalar field in the usual inflationary scenario, leading to the condition \( \lambda \ll 3 P_m^2 \phi^2/16 \approx 2.8 \times 10^{-10} \) for the slow-roll approximation to hold, indicating that the scalar field must be very weakly coupled. This fine-tuning is easily avoided in our model. For example, if we assume that \( \lambda \approx 1 \) and \( m \approx m_\phi \), then \( H_\phi^2 \approx c^2 m_\phi^2/(12\beta) \), and we find that our approximation is good provided that \( \beta \ll 3/2 \).

Since \( d\tau/dt \approx \rho(t)/\rho_m \approx \beta P_m^4/\lambda m^2 \), we see that we have effectively scaled the speed of light by a constant factor \( c \rightarrow c \beta l_P^2 P_m^4/\lambda m^2 \), recovering the scenario originally introduced by Moffat [5]. (Note that for \( m \approx m_\phi \) this indicates that the speed of propagation of matter fields is enhanced by a factor \( \approx 10^{10} \beta/\lambda \) over that of gravitational fields, which for \( \lambda \approx 1 \approx \beta \) is considerably smaller than the value \( \approx 10^{30} \) assumed in [5, 7]. We shall see though that we can still get enough \( e \)-folds without undue fine-tuning.)

Choosing \( \tau_{qg} = t_{qg} \), in the matter frame we find

\[
\frac{\dot{R}^2(\tau)}{R^2(\tau)} \approx \left( \frac{16\lambda m^2}{\beta P_m^4} \right)^2 H_\phi^2 \rightarrow R(\tau) = R_{qg} \exp \left[ \frac{16\lambda m^2 H_\phi}{\beta P_m^4} (\tau - \tau_{qg}) \right],
\]

and for the scalar field

\[
\phi(\tau) \approx \phi_0 \exp \left[ \frac{8\lambda c^2 m^2}{3\beta P_m^4} (\tau - \tau_{qg}) \right].
\]

Defining the number of \( e \)-folds that the universe expands within the slow roll approximation (which ends at \( \tau_{sr} \)) as

\[
N_\phi := \frac{16\lambda m^2 H_\phi}{\beta P_m^4} (\tau_{sr} - \tau_{qg}),
\]

we obtain

\[
\phi(\tau_{sr}) \approx \phi_0 \exp[2\beta N_\phi],
\]

Choosing \( m \approx m_\phi \) this becomes \( \phi_0 \exp(2\beta N_\phi) \), and taking \( \beta \approx 10^{-3} \) and \( N_\phi \approx 100 \) we see that the scalar field has not evolved appreciably over the interval. The requirement that the approximation of the potential leading to (35) is valid throughout this period is

\[
\frac{4\lambda \phi_0^2}{P_m^4 m_\phi^4} \exp \left( \frac{N_\phi c^2 m_\phi^2}{6 H_\phi^2} \right) \ll 1,
\]

and we obtain

\[
\phi(\tau_{sr}) \approx \phi_0 \exp[2\beta N_\phi],
\]
which becomes $\phi_0^2 \ll l_p^2 m_{pl}^2/(4\lambda)$ with the above choices. Assuming that $\lambda \approx 1$ leads to $\phi_0 \lesssim 2 \times 10^{-5}$.

We should stress that the “slow roll” approximation and the simple potential model have been introduced in order to show how to solve the horizon problem in this model without undue fine-tuning. Since the role of the scalar field is to enhance the inflationary effect, choosing $1/(2H_{pt})$ anywhere prior to $\approx 0.01\sec$, we expect that there are many other models with sufficient e-folds. Clearly other scenarios (inflation at the electroweak scale [10], or possibly more than one scale [17]) are also possible.

### 4 Concluding Remarks

A bimetric theory of gravitation is proposed in which two metrics are associated with spacetime. The trajectories of test particles are geodesics of the matter metric $\hat{g}_{\mu\nu} = g_{\mu\nu} + \beta \psi_\mu \psi_\nu$, the null cones of which are contained within the light cones of the gravitational metric $g_{\mu\nu}$. Thus, the gravitational matter spacetime acts as a “digravitational” medium and determines the speeds of clocks associated with the matter. In the gravitational frame, we have the standard Einstein equations with matter that can appear to violate causality, for the matter seems to propagate faster than the speed of light as defined by the gravitational metric $g_{\mu\nu}$. But in the matter frame causality is not violated—matter and gravity propagate less than or at the speed of light as defined by the metric $\hat{g}_{\mu\nu}$.

The Einstein-matter field equations and the proposed Proca-like dynamical field equations for $\psi_\mu$ reduce to the equations of GR when the vector field $\psi_\mu = 0$. We explicitly derived an exact solution for an FRW universe with $k = \Lambda = 0$, based on an equation of state $p = c^2(\omega/3 - 1)\rho$. Prior to a time $t_{pt}$ (which is when the matter density drops below a threshold defined by the mass of $\psi_\mu$) we find that there are two possible solutions to the field equations. One solution associated with what we call the “broken phase” leads to an inflationary epoch, characterized by an expansion of the universe with enough inflation to solve the horizon problem, the relic magnetic monopole problem and the flatness problem. Following the phase transition at $t_{pt}$ is the “unbroken phase”: only the solution $\psi_\mu = 0$ is available to the system. Since we found that the effect is maximized for $p \approx -c^2\rho$, we also considered a scalar field model with a potential that can lead to inflation ($V \propto \phi^4$). We found that our mechanism removed the need for the extreme fine-tuning of
the coupling constant that is required by ordinary inflation.

It is possible to speculate that the phase transition at \( t = t_{pt} \) could occur late enough in the evolution of the universe (or in patches thereof) that measurable effects could be observed in the present. This is a possible scenario which we plan to investigate elsewhere. Finally, we can obtain predictions for galaxy seeds and CMB temperature fluctuations from the Maxwell-Proca field equations for \( \psi_\mu \). Inhomogeneous fluctuations of \( \psi_\mu \) could also be the source of the Q-component of energy [4].

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