Nonlinear buckling analysis of arch structure to initial defect sensitivity

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Abstract. Due to the more and more prominent nonlinear buckling problems of large-span arch structures in engineering applications, and the lack of special design methods of large-span arch structures in existing specifications, this paper establishes a simplified mechanical model for the stability of arch structures with initial geometric defects. The finite element model of arch structure is established by using finite element software, and the characteristic value buckling analysis and geometric nonlinear analysis are carried out to find out the buckling mode of arch structure, and the sensitivity of defects to arch structure is studied. The results show that the existence of defects will reduce the bearing capacity of the structure, and the increase of defects will change the deformation form of the structure.

1. Introduction

Arch structure is a widely used and important structure type in China. Due to its beautiful form and large spanning capacity, arch structure is widely used in Bridges, tunnels and other buildings\(^{[1-3]}\). Especially in recent decades, the large span space arch structure has a stage of development. The defects of arch structure, such as connection, material, fatigue, stress concentration and welding, will directly affect the safety of long-span structure.

Considering the second order effect of transverse stress and shear stress, cheng peng et al\(^{[4]}\). Put forward the general theory of plane instability of arc arch. Timoshenko S P et al\(^{[5]}\). Established the balance equation of curved bar and deduced the critical load formulas of symmetric and antisymmetric elastic instability of circular arc arch under radial uniformly distributed load under different boundary conditions. In this paper, the finite element model of arch structure is established by using finite element software, and the buckling mode of arch structure under load is studied.

2. Calculation model of arch structure

As shown in figure 1, the calculation parameters of the arch structure are: radius \(R = 150\text{m}\) of the arch structure, area \(A = 0.3\text{m}^2\) of the section, elastic modulus \(E = 10^7\) of the material, moment of inertia \(I = 1\text{m}^4\) of the section, and poisson's ratio 0.3.
3. Nonlinear buckling theory
Nonlinear buckling analysis is to solve the critical load by nonlinear analysis of load increment. Through Newton lapson (N-R) equilibrium iteration, the end solutions of each load increment reach equilibrium convergence. Figure 2 (a) shows the purely incremental solution. Figure 2 (b) describes the use of the (n-r) balanced iteration in a single-degree-of-freedom nonlinear analysis

① Gradually apply the load in an incremental form;
② Complete the balance iteration in each load increment to balance the incremental solution;
③ Solving the equilibrium equation:
\[
\begin{bmatrix} K_T \end{bmatrix} \{ \Delta u \} = \{ F \} - \{ F_{nr} \} 
\]
In the formula, \([K_T]\) -- tangent stiffness matrix, \(\{ \Delta u \}\) -- displacement increment, \(\{ F \}\) -- external load increment, \(\{ F_{nr} \}\) -- internal force vector;
④Iterate until \(\{ F \} - \{ F_{nr} \}\) is within bounds.

\[
L = \sqrt{\Delta u^2 + \lambda^2}
\]

The convergence process of arc length method is shown in figure 3.

Figure 2. Relationship between pure incremental approximation and newton-rapson approximation

The other arc length method is similar to n-r method. The arc length method introduces load factor \(\lambda(1(\lambda(1), and the equilibrium equation of force is:

\[
\begin{bmatrix} K_T \end{bmatrix} \{ \Delta u \} = \lambda \{ F_b \} - \{ F_{nr} \} 
\]

In order to solve the load factor, arc length radius \(L\) must be introduced:

\[
L = \sqrt{\Delta u^2 + \lambda^2}
\]
4. Buckling analysis of deep arch structures with initial defects

Table 1 shows the fifth buckling load and buckling mode shapes of the arch structure. FIG. 4 shows the nonlinear buckling analysis and final deformation results of the arch structure. FIG. 4 (a) shows the nonlinear buckling analysis of the arch structure and FIG. 4 (b) shows the final deformation results of the arch structure. If the introduced defect is 9% of first-order eigenvalue modal shape, buckling load is. The results show that the larger the defect, the smaller the ultimate load. FIG. 5 shows the nonlinear buckling analysis and final deformation results of the defective arch structure. FIG. 5 (a) shows the nonlinear buckling analysis of the defective arch structure, and FIG. 5 (b) shows the final deformation results of the defective arch.

Table 1. The buckling modes of the arch perfect structure

| Modal                  | The buckling load (KN) | Buckling modal shape |
|------------------------|------------------------|---------------------|
| The first order modal  | 8751.7                 | ![First order buckling modal](image1) |
| The second order modal | 19418                  | ![Second order buckling modal](image2) |
| The third order modal  | 34950                  | ![Third order buckling modal](image3) |
| The fourth order modal | 54321                  | ![Fourth order buckling modal](image4) |
| The fifth order modal  | 78509                  | ![Fifth order buckling modal](image5) |
(a) Improve nonlinear buckling analysis of arch  (b) Perfect the final deformation of arch structure

Figure 4. Perfect the nonlinear buckling analysis and final deformation of arch structure

(a) Nonlinear buckling analysis of defective arch

(b) Ultimate deformation of defective arch structure

Figure 5. Nonlinear buckling analysis and final deformation of defective arch

5. Conclusion

(1) With the increase of defects, the geometric nonlinearity of the curve becomes more and more obvious, the bearing capacity of the structure also decreases significantly, and the displacement response at the same load level also increases.

(2) The ultimate load of arch structure with initial defects is less than the first buckling load of eigenvalue analysis, and its initial deformation is positive and symmetric. As the load increases, the deformation of the structure will jump due to the influence of defects.

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