The Interplay of Rashba Spin-Orbit Interaction and Landau Level Broadening on a Two-Dimensional Electron Gas Under a Tilted Magnetic Field

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Abstract. A two-dimensional electron gas in a tilted magnetic field with Rashba spin-orbit interaction (RSOI) was studied. The RSOI is accredited to the asymmetry of the heterostructure where the two-dimensional electron gas is found. The effects of the disorder-attributed Landau level broadening and the RSOI on the spin splitting were identified by simulating the density of states which was assumed to take a Gaussian shape. Increased Landau level broadening obscures the spin splitting and increases the overlap between spin states resulting to stout Gaussian peaks. On the other hand, stronger RSOI amplifies the splitting and lessens the overlap between spin states of the Landau levels. The splitting, however, results to stouter peaks. The similarity in the RSOI and Landau level broadening effects can be explained by recognizing that the asymmetry of the heterostructure is in itself a form of structural disorder.

1. Introduction
The presence of disorder is not necessarily always undesirable. The plateaux in the integer quantum Hall effect, for example, will not appear apart from a substantial density of localized states in between the Landau levels [1]. In a two-dimensional electron gas (2DEG), disorder can be due to charged impurities or spatial inhomogeneities [2]. No matter how the disorder is introduced, for a 2DEG under a perpendicular magnetic field, the ideal δ-shape of the density of states transforms into a smooth broadened function. The broadening was evidenced by the smooth, instead of sharp, variation of the thermodynamic quantities versus magnetic field [3].

When an electron system is subject to a magnetic field, its spin degeneracy is lifted as described by the Zeeman effect. To capitalize on the electron spin without the direct application of magnetic fields, spin-orbit interaction (SOI) can be taken advantage of. The latter is relativistic in nature whereby a moving electron sees an electric field as a magnetic field. This gives way to the electron spin coupling with its orbit or motion resulting to a spin-split energy spectrum.

Transporting and transferring information through the electron spin rather than through its charge is the main goal of spintronics [4]. The endeavor entails characterizing the efficiency of the spin splitting first. With the disorder present in the material how will spin information be affected? This is the question that we will try to answer in this paper.
2. The Approach

A 2DEG is considered to lie in an \( x - y \) plane with an external magnetic field \( \mathbf{B} \) applied at an angle \( \theta \) with respect to the normal axis. Taking into account two spin interactions, viz. the Rashba and Zeeman splitting, the Hamiltonian of a single electron in the system can be written as

\[
H = \frac{\hbar^2 \vec{k}^2}{2m^*} + \alpha(\vec{\sigma} \times \vec{k}) \cdot \hat{z} - \vec{\mu}_B \cdot \vec{B},
\]

where the first term is the free particle energy, the second is the Rashba SOI, and the third is the Zeeman energy. The strength of the Rashba SOI, assumed to be constant for this present work, is indicated by the parameter \( \alpha \). Here \( m^* \) is the electron’s effective mass, \( \vec{\sigma} \) are the Pauli matrices, \( \vec{\mu}_B \) is the Bohr magneton and \( \vec{k} \) is the wave vector. The magnitude of the latter is determined by \( k = -i \nabla + \frac{e}{\hbar} \vec{A} \), where \( e \) is the electronic charge, \( \hbar \) is Planck’s constant over 2\( \pi \), and \( \vec{A} \) is the magnetic vector potential.

When \( \vec{B} \) is tilted with respect to the normal, (1) cannot be solved exactly \([5]\). Hence, some researchers resorted to using perturbation theory \([6]\), the continued fraction numerical method \([7]\), and a quasi-classical approach \([5]\). In this work, we take advantage of the level crossing. The latter develops with increasing magnetic field \([8]\) or Rashba SOI strength \([9]\). Because opposite spin states of adjacent Landau levels cross each other, we invoke that the crossing states are equally probable. This yields to an exact solution of

\[
E_n^\pm = n + \frac{1}{2} + \frac{\Upsilon}{2} \left( \sqrt{n} + \sqrt{n + 1} \right) \pm \frac{1}{2} \sqrt{\left[ \Upsilon (\sqrt{n + 1} - \sqrt{n}) + 2\Omega_z \right]^2 + 4\Omega_+ \Omega_-}. \tag{2}
\]

The eigenenergies in (2) are in terms of \( \hbar \omega_c = \hbar eB_z/m^* \). Here \( n \) is the Landau level index, \( \Upsilon = \alpha(\vec{\sigma} \times \vec{k})_z/\hbar \omega_c \) is the dimensionless Rashba energy, \( \Omega_j = \mu_B B_j/\hbar \omega_e \) is the \( j \)-th component of the rescaled Zeeman energy and \( \Omega_\pm = \Omega_x \pm i\Omega_y \). The detailed derivation can be found in \([10]\). The eigenenergies in (2) is valid only for the tilted case and not for the perpendicular case. This is because imposing equal probability between opposite spins of neighboring Landau levels will result to diverging coefficients if the tilt angle \( \theta = 0 \).

In simulating the disorder-broadened Landau levels, we employ a Gaussian-shaped density of states (DOS). This form was chosen as most experimental results are best fitted by this bell-shaped function. The disorder enters through the Gaussian width \( \Gamma \). The DOS can be written as

\[
\text{DOS}(E) = \frac{2eB}{\hbar} \sum_n \left( \frac{1}{2\pi} \right)^{1/2} \frac{1}{\Gamma} \exp \left[ -\frac{(E - E_n^\pm)^2}{2\Gamma^2} \right]. \tag{3}
\]

Each energy level is highly degenerate with the degeneracy proportional to \( B_z = B \cos(\theta) \). The shape of the DOS versus \( B \) will give a picture of how well separated the two spin states of each Landau level are.

3. Results

The effect of disorder in the DOS is shown as figure 1. Increasing \( \Gamma \) resulted to an expanded overlap between spin levels of the Landau levels. Depending on the amount of disorder the overlap can be very large that the spin states become indistinguishable (\( \Gamma = 1.0 \) meV). It can also be very small that the overlap between adjacent spin levels is negligible (\( \Gamma = 0.1 \) meV). In this case an increased disorder is not favorable in resolving the splitting. The disorder, in effect, obscures the spin split levels by trapping more electrons in the localized states that are found at the tails of the energy levels. The small \( \Gamma \) in figure 1 also shows the \( B \) point where the splitting starts to become pronounced (\( B \approx 0.25 \) T). Below this \( B \), no splitting is seen from the
DOS. Hence, in addition to minimizing the disorder, it is also necessary to apply the appropriate magnetic field intensity in order to obtain a well resolved spin splitting.

In figure 2, the effect of an increased Rashba SOI strength is displayed. The Landau levels show more pronounced splitting for larger \( \alpha \) as expected. This, however, is compensated by peaks of wider width. The latter is akin to the influence of increased disorder as depicted in figure 1. With the similarity of the effects of increased disorder and Rashba SOI strength, it can be concluded that the latter is truly a form of disorder. We can see here that Rashba SOI has a drawback. Although it is able to resolve spin splitting, it also means that more disorder is introduced into the system. This disorder is brought about by the inversion asymmetry of the confining potential.

4. Conclusion
The effect of disorder on a two-dimensional electron gas under a tilted magnetic field and with Rashba spin-orbit interaction is studied. The eigenvalues are obtained by invoking that opposite spin states of adjacent Landau levels are equally probable. This results to larger spin splitting for each Landau level. It was shown that increased disorder effectively expands the overlap between energy levels. This corresponds to more electrons occupying the localized states. Although increasing the Rashba interaction constant produces more spin splitting per Landau level, the energy levels also become stouter allowing more electrons to be found at the tails of the Landau levels. The similarity in the effects of increasing the disorder and increasing the Rashba interaction supports the fact that the latter can also be considered as a form of disorder.

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