Thermal conductance and noise of Majorana modes along interfaced $\nu = 5/2$ fractional quantum Hall states

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Identifying the topological order of the fractional quantum Hall state at filling $\nu = 5/2$ is an important step towards realizing non-Abelian Majorana modes in condensed matter physics. However, to unambiguously distinguish between various proposals for this order is a formidable challenge. Here, we present a detailed study of transport along interfacial edge segments of fractional quantum Hall states hosting non-Abelian Majorana modes. With an incoherent model approach, we compute, for edge segments based on Pfaffian, anti-Pfaffian, and particle-hole-Pfaffian topological orders, thermal conductances, voltage biased charge current noise, and delta-$T$ noise. We determine how the thermal equilibration of edge modes impacts these observables and identify the temperature scalings of transitions between regimes of differently quantized thermal conductances. In combination with recent experimental data, we use our results to estimate thermal and charge equilibration lengths in real devices. We also propose an experimental setup which permits measuring several transport observables for interfaced fractional quantum Hall edges in a single device. It can, e.g., be used to rule out edge reconstruction effects. In this context, we further point out some subtleties in two-terminal thermal conductance measurements and how to remedy them. Our findings are consistent with recent experimental results pointing towards a particle-hole-Pfaffian topological order at filling $\nu = 5/2$ in GaAs/AlGaAs, and provide further means to pin-point the edge structure at this filling and possibly also other exotic fractional quantum Hall states.

I. Introduction

Non-Abelian anyons are exotic excitations in two-dimensional condensed matter systems with no counterpart in particle physics [1]. A system with non-Abelian anyons has a ground state degeneracy, and interchanging – or “braiding” – the anyons can shift the system between different ground states. This shift depends only on the order of exchanges and information about the braiding process is stored globally in the ground state wavefunction. Besides signifying a strongly correlated phase of matter, non-Abelian braiding forms the basis for the appealing, but so-far speculative, idea of topologically protected quantum computations [2, 3].

A most promising candidate system to host non-Abelian anyons is the fractional quantum Hall (FQH) state [4, 5] at filling $\nu = 5/2$ [6–9]. The quantum state that is realized at this filling is however still not fully understood and poses a long-standing question in condensed matter physics (see Ref. [10] for a recent overview).

The purpose of this paper is to model FQH edge transport involving various candidate states at this filling, in light of recent experiments [11, 12] that have brought progress towards identifying the 5/2 state. To date, the three most prominent candidate states are the Pfaffian (Pf) [7], anti-Pfaffian (aPf) [13, 14], and particle-hole-Pfaffian (phPf) [15–18]. Whereas numerical calculations have seemed to favor the aPf state [19–25], tunneling experiments are more consistent with the aPf, or the Abelian SU(2)$_2$, 331, or 113 states [26–28].

Although all three non-Abelian candidates are (by construction) compatible with the observed Hall conductance $G_H/(e^2/h) = \nu = 5/2$, they have distinct bulk topological orders [29]. Via the bulk-boundary correspondence, this order is in turn manifested by different edge structures, i.e., the number, chirality, and type of channels (or “modes”) propagating around the FQH edge. As depicted in Fig. 1, the proposed edge structures of the Pf, aPf, and phPf states differ only in the second Landau level (2LL). This distinction can be quantified by the topological quantum number

$$\nu_Q \equiv c - \bar{c},$$

where $c$ and $\bar{c}$ are the total central charges for the chiral and anti-chiral sectors, respectively, in the underlying conformal field theory [30]. For Abelian FQH edges, $c$ ($\bar{c}$)
equals the number of “downstream” (“upstream”) channels (where the downstream direction is defined as that of the equilibrated charge flow). For more exotic edge structures, $c$ and $\bar{c}$ may take rational values, e.g., $c = 1/2$ for a single non-Abelian Majorana mode (MM). As can be seen in Fig. 1, $\nu_{Q}^{PF} = 7/2$, $\nu_{Q}^{phPF} = 3/2$, and $\nu_{Q}^{phPf} = 5/2$.

Quite remarkably, the abstract quantity (1) can be related to the experimentally accessible edge thermal conductance $G^Q$ according to the relation [31, 32]

$$G^Q = \nu_{Q} \kappa_{0} \bar{T},$$

in which $\kappa_{0} \bar{T} \equiv \pi^{2} k_{B}^{2} / (3h) \bar{T}$ is the quantum of heat (with $k_{B}$ and $h$ the Boltzmann and Planck constants, respectively, and $\bar{T}$ is the temperature). Two-terminal conductance measurements for QH edges, first demonstrated in Ref. [33], have subsequently been performed for many FQH states in both GaAs/AlGaAs [11, 34–37] and graphene [38–41]. In particular, Ref. [35] reported $G^Q / (\kappa_{0} \bar{T}) \approx 5/2$ at filling $\nu = 5/2$, which fits well with Eq. (1) for the phPf state (see Fig. 1). This value further rules out Abelian candidate states, since those are incompatible with half-integer $G^Q$. However, Eq. (2) is valid for the two-terminal conductance only when the heat transport is fully equilibrated, i.e., when edge channels exchange energy efficiently [41–46]. This equilibration can be quantified by a characteristic thermal equilibration length $\ell_{eq}^{C}$, so that the condition for Eq. (2) to hold reads $\ell_{eq}^{C} \ll L$, where $L$ is the edge length. Importantly, $\ell_{eq}^{C}$ is non-universal and depends on microscopic details such as inter-channel interactions, the edge disorder strength, and the temperature. It is worth pointing out that an analogous condition, $\ell_{eq}^{\nu} \ll L$, with $\ell_{eq}^{\nu}$ a characteristic charge equilibration length, is needed also for robust charge conductance quantization for FQH edges with counter-propagating modes ($c, \bar{c} \neq 0$). However, almost all FQH experiments to date indicate that this condition is normally well fulfilled (see Refs. [47, 48] for exceptions).

Figure 2. Effective structures of interfaces between non-Abelian candidate states and integer states $n = 1, 2, 3$. In this work, we focus on $n = 2, 3$ (indicated by the frame), which are the integers that expose the second Landau level. The interfaces with orange text below their name are those for which the directions of equilibrated charge and heat transport are opposite.

The interpretation of the experiment in Ref. [35] as revealing a thermally equilibrated phPf edge was therefore questioned, since $G^Q / (\kappa_{0} \bar{T}) \approx 5/2$ can be obtained also for a partially equilibrated aPf edge [49–53]. The same value can, under certain conditions, be obtained from models with random puddles of alternating non-Abelian orders [54–59], or from reentrant states due to Landau level mixing [60].

Further progress has been made recently in GaAs/AlGaAs devices where the $\nu = 5/2$ state is interfaced with integer QH states [11, 12]. This results in an effective $\nu = 5/2 - n$ edge, where $n = 1, 2, 3$ (see Fig. 2). The basic idea is that since all candidate states share two Abelian, integer edge channels (coming from two filled LLS), successive elimination of them exposes the remaining non-Abelian “$\nu = 1/2$ structure” for which the states differ. This elimination occurs either due to Anderson localization [61–63] or efficient equilibration of the edge states. For $n = 3$, a possible particle-hole symmetry of the 2LL can be tested. In Ref. [11], the thermal conductance for the $\nu = 5/2 - 2$ and $\nu = 5/2 - 3$ edges were both measured to $G^Q / (\kappa_{0} \bar{T}) \approx 1/2$. It was argued that this is only possible if the underlying $\nu = 5/2$ state is the phPf, thereby further strengthening the case for this state. It has also been proposed that charge conductance measurements of $5/2 - n$ interfaces could distinguish between non-Abelian candidate states [64].

For the same type of interfaced structures but in another device, the authors in Ref. [12] measured the excess noise for a current biased edge segment. Previously, it was proposed [53, 65–67] that such noise discloses important properties of the edge. More specifically, for full thermal and charge equilibration of any edge structure, the noise $S$ scales with $L$ in one of three possible classes: $S \simeq \exp (-L/\ell_{eq}^{C})$ for $\nu_{Q} > 0$, $S \simeq \sqrt{\ell_{eq}^{C}/L}$ for $\nu_{Q} = 0$, or $S \simeq \text{const.}$ for $\nu_{Q} < 0$ [66]. This classification holds under conditions where heat leakage into the QH bulk is negligible. If the heat leakage is efficient, the noise is strongly suppressed in $L/\ell_{eq}^{C}$ also for $\nu_{Q} = 0$ and $\nu_{Q} < 0$. By contrast, under conditions where the thermal equilibration between downstream and upstream modes (if present) is negligible, the noise scales as $S \simeq \text{const.}$, and absence of upstream modes implies identically $S = 0$. This noise classification is the result of the chiral nature

\[
\begin{align*}
\text{Pf-1} && \text{aPf-1} && \text{phPf-1} \\
(\text{ds} : \rightarrow, \nu_{Q} > 0) && (\text{ds} : \rightarrow, \nu_{Q} > 0) && (\text{ds} : \rightarrow, \nu_{Q} > 0) \\
\text{Pf-2} && \text{aPf-2} && \text{phPf-2} \\
(\text{ds} : \rightarrow, \nu_{Q} > 0) && (\text{ds} : \rightarrow, \nu_{Q} < 0) && (\text{ds} : \rightarrow, \nu_{Q} > 0) \\
\text{Pf-3} && \text{aPf-3} && \text{phPf-3} \\
(\text{ds} : \leftarrow, \nu_{Q} < 0) && (\text{ds} : \leftarrow, \nu_{Q} > 0) && (\text{ds} : \leftarrow, \nu_{Q} > 0)
\end{align*}
\]
of the edge: For equilibrated charge transport, the voltage drop across a biased edge segment occurs only close to one of the contacts (in the so-called hot spot) whereas partitioning of charges, i.e., excess noise, is dominantly produced close to the other contact (in the so-called noise spot), see Fig. 3. This partitioning is enhanced with respect to the equilibrium noise only in the presence of upstream modes that transport heat from the hot spot to the noise spot. In turn, this heat transport depends strongly on $\nu_Q$ which leads to the correspondence between $S$ and $\nu_Q$.

For an interfaced Pf edge, the classification above implies that only the $5/2-3$ interface can generate excess noise, since only that interfacing results in upstream modes. For the aPf, both $5/2-1$ and $5/2-2$ can result in noise whereas $5/2-3$ cannot. Finally, for the phPf, all three interfaces host counter-propagating modes and can therefore produce finite noise. However, since $\nu_Q > 0$ for the phPf interfaces [68], the noise is exponentially suppressed in $L/\ell_C^{eq}$ and finite noise should only emerge for poor thermal equilibration, i.e., either for small $L$ and/or large $\ell_C^{eq}$.

Indeed, Ref. [12] reported finite excess noise for both $5/2-2$ and $5/2-3$ interfaces for short $L$, but the noise weakened significantly for larger $L$ (see Fig. 11 below). At the same time, the two-terminal charge conductance of the interface, $G_{2T}$, was always measured to $G_{2T}/(e^2/h) = 1/2$, indicating a well established charge equilibration between downstream and upstream (if present) modes.

Taken together, the two experiments [11, 12] suggest that the $\nu = 5/2$ state in GaAs/AlGaAs is of phPf type. However, a detailed model of how the upstream mediated noise is generated for interfaced $\nu = 5/2$ edges remains lacking. Moreover, the above interpretation of the noise measurements hinges on the absence of edge recon-
stion [69–72]. This effect introduces non-topological pairs of counter-propagating modes which complement the edge structure from the bulk boundary correspondence. The addition of such modes and conditions with poor thermal equilibration can result in noise generation for any FQH edge, which complicates experimental interpretations.

In this work, we incorporate the qualitative noise and conductance analysis above in a comprehensive theoretical model which further permits a quantitative comparison with experimental data. To this end, we study transport along interfaces between non-Abelian and integer $n$ edges with the incoherent edge approach, recently developed in Refs. [44, 53, 65–67, 73]. We review the basics of this model in Sec. II. In Secs. III and Sec. IV, we use the model to compute the thermal conductance and the noise, respectively, for interfaced edge structures. We focus on $n = 2, 3$, since it is those integers that expose the 2LL structure, see Fig. 2. A summary of the calculations in these sections is given in Tab. I. In Sec. V, we analyze the temperature scalings of the thermal equilibration lengths. This scaling permits us to analyze the temperature dependence of the noise and thermal conductance. In Sec. VI, we compare our results to the experiments in Refs. [11, 12] and provide estimates on thermal and charge equilibration lengths. We then propose in Sec. VII a unified experimental setup allowing several independent experiments for probing FQH edge structures in a single device. We argue that this device is beneficial for ruling out edge reconstruction effects as well as possible sample-to-sample differences between separate devices probing noise and the thermal conductance. We summarize and conclude our work in Sec. VIII. A number of technical calculations are delegated to Appendices A–E.

II. MODEL OF EDGE TRANSPORT

A. Charge and energy transport

Our starting point is the generic edge segment with two attached contacts, depicted in Fig. 3. Charge transport along this segment is described by [44, 67]

$$\partial_x \vec{V}(x) = \mathcal{M}_V \vec{V}(x).$$

Here, $\vec{V}(x) \equiv [V_1(x), \ldots, V_N(x)]^T$ (with $[\ldots]^T$ denoting vector transpose) describes the local voltages of $N$ edge channels and the matrix

$$\mathcal{M}_V = \begin{pmatrix}
\sum_{n \neq 1} (I_{1,n}^C)^{-1} & (I_{2,1}^C)^{-1} & \ldots & (I_{N,1}^C)^{-1} \\
(I_{1,2}^C)^{-1} & \sum_{n \neq 2} (I_{2,n}^C)^{-1} & \ldots & (I_{N,2}^C)^{-1} \\
\vdots & \vdots & \ddots & \vdots \\
(I_{1,N}^C)^{-1} & (I_{2,N}^C)^{-1} & \ldots & \sum_{n \neq N} (I_{N,n}^C)^{-1}
\end{pmatrix} \chi_{1\nu_1}
\chi_{2\nu_2}
\chi_{N\nu_N}$$

(4)

describes couplings between edge channels in terms of the channel chiralities $\chi_i = \pm 1$ (with $+1$ and $-1$ corresponding to downstream and upstream directions, respectively), filling factor discontinuities $\nu_i$, and charge equilibration lengths $l_{ij}^C = l_{ij}^C$ between modes $i$ and $j$. The microscopic content affecting these lengths can be obtained within a chiral Luttinger liquid approach, see e.g., Refs. [39, 46, 51]. As follows, we always label the modes of all edge structures in Fig. 2 with $i = 1$ starting from the top. The local electric currents $\vec{I}(x) \equiv [I_1(x), \ldots, I_N(x)]^T$, corresponding to $\vec{V}(x)$, obey a similar equation

$$\partial_x \vec{I}(x) = \mathcal{M}_I \vec{I}(x), \quad \mathcal{M}_I = \mathcal{D}_I \mathcal{M}_V \mathcal{D}_I^{-1},$$

(5)

with the diagonal matrix $\mathcal{D}_I = \text{diag}(\chi_1\nu_1, \ldots, \chi_N\nu_N)$. This description of edge currents was presented also in Ref. [51] and can further be mapped onto the capacitive
currents as

\[ \text{The local temperatures are further related to local heat temperatures (squared), and the matrix in Sec. III.} \]

Similarly to the charge transport, edge energy transport is described by

\[ \partial_x \vec{T}^2(x) = M_T \vec{T}^2(x) + \delta \vec{V}(x), \]

where \( \vec{T}^2(x) = [T_1^2(x), \ldots, T_N^2(x)]^T \) are the local temperatures (squared), and the matrix

\[
M_T = \begin{pmatrix}
-\sum_{\ell=1}^{Q_1} (t_{1,\ell}^Q)^{-1} & \sum_{\ell=1}^{Q_2} (t_{1,\ell}^Q)^{-1} & \cdots & \sum_{\ell=1}^{Q_N} (t_{1,\ell}^Q)^{-1} \\
\sum_{\ell=1}^{Q_1} (t_{1,\ell}^Q)^{-1} & -\sum_{\ell=2}^{Q_2} (t_{1,\ell}^Q)^{-1} & \cdots & \sum_{\ell=2}^{Q_N} (t_{1,\ell}^Q)^{-1} \\
\vdots & \vdots & \ddots & \vdots \\
\sum_{\ell=1}^{Q_1} (t_{N,\ell}^Q)^{-1} & \sum_{\ell=2}^{Q_2} (t_{N,\ell}^Q)^{-1} & \cdots & -\sum_{\ell=1}^{Q_N} (t_{N,\ell}^Q)^{-1}
\end{pmatrix}
\]

in which \( t_{ij}^Q = t_{ji}^Q \) are the thermal equilibration lengths (see Appendix A for an example). The final term in Eq. (6) is the Joule heating contribution

\[
\delta \vec{V}(x) = \sum_{n=1}^{N} \frac{e^2}{\hbar \kappa_0} \left( \frac{[V_1(x) - V_n(x)]^2}{t_{1,n}^C \chi_1}, \ldots, \frac{[V_N(x) - V_n(x)]^2}{t_{N,n}^C \chi_N} \right)^T,
\]

and originates from the voltage drops between the edge channels. In the matrix \( M_T \), the thermal conductance of the edge modes are described by the numbers \( n_i \), which equal 1 for Abelian edge channels and \( 1/2 \) for Majorana edge channels. These numbers are related to Eq. (1) as

\[
c = \sum_{i:y_i=+1} n_i, \\
c = \sum_{i:y_i=-1} n_i, \\
\nu_0 = \sum_{i} \chi_i n_i.
\]

The local temperatures are further related to local heat currents as

\[ J_i(x) = \frac{n_i \kappa_0}{2} T_i^2(x). \]

The transport equations (3) and (6) must be supplemented by boundary conditions that depend on the setup. Boundary conditions are discussed in more detail in Sec. III.

**B. Charge current noise**

We now focus on the regime of efficient charge equilibration, which we denote as \( \delta \equiv L/\ell_{eq} \gg 1 \), where \( L \) is the edge length and \( \ell_{eq} \) is defined as the largest equilibration length in the set of counter-propagating channels. As mentioned in Sec. I, this condition is normally fulfilled on all FQH edges. Then, the excess charge current noise \( S \), due to a voltage bias, in any of the two contacts (equal due to charge conservation), see Fig. 3, is to good approximation given by [65, 66, 75]

\[
S \approx \frac{2 e^2}{\hbar \kappa_0} \nu (\nu_+ - \nu_-) \int_0^L dx e^{\frac{-2 \nu_+}{\nu_+ - \nu_-}} \Lambda(x).
\]

A detailed derivation this formula can be found in Ch. 3 of Ref. [76]. In Eq. (14), “~” should here, and below, be understood as “equal in the limit of very large \( \delta \)” Furthermore,

\[
\nu_+ = \sum_{i:y_i=+1} \nu_i, \\
\nu_- = \sum_{i:y_i=-1} \nu_i,
\]

are the total filling factor discontinuities of the downstream (+) and upstream (−) edge modes respectively. They satisfy the relations \( \nu_+ > \nu_- \) and \( \nu = \nu_+ - \nu_- \), where \( \nu \) is the effective filling factor of the edge structure. For a \( 5/2 - n \) interface, we simply have \( \nu = 5/2 - n \) for \( \delta \gg 1 \). The exponential suppression Eq. (14) follows from the chiral nature of the edge [65]. It implies that the noise generation is predominantly influenced by the region of size \( \sim \ell_{eq} \) close to the most upstream contact. We call this the noise spot.

The key quantity in Eq. (14) is the local noise kernel

\[
\Lambda(x) = \frac{S_{loc}[\delta V(x), T_+(x), T_-(x)]}{2 g_{loc}[\delta V(x), T_+(x), T_-(x)]},
\]

where \( S_{loc} \) and \( g_{loc} \) is the local dc noise and the (dimensionless) tunneling conductance, respectively. It is assumed that all downstream and upstream edge channels charge-equilibrate separately very efficiently, e.g., by emanating from the same (ideal) contact [77]. Importantly, both \( S_{loc} \) and \( g_{loc} \) depend on microscopic details of the edge such as inter-channel interactions, the edge disorder strength, the local voltage difference between the modes \( \delta V(x) \), and the thermal equilibration-induced effective temperatures \( T_k \) of downstream and upstream edge modes. Appendix B outlines how noise kernels are obtained from a bosonization approach.

Our procedure to find the charge and heat flows along an edge segment is as follows. We first solve Eqs. (3) and (6) with suitable boundary conditions. These will
depends on the type of setup. We then use these solutions to compute charge and thermal conductances, or insert them first into Eqs. (17) and then (14) to obtain the noise.

III. THERMAL CONDUCTANCE

A. Two-terminal thermal conductance

In this section, we compute the two-terminal thermal conductance for various $5/2-n$ (with $n = 2, 3$) interfaces by applying Eq. (6) to the setup in Fig. 4. To this end, we set the voltages in both contacts equal to zero: $V = \Delta V = 0$. The solutions to Eq. (3) are then trivial: $\vec{V}(x) = 0$. For the thermal transport, the boundary conditions for the top edge segment read

$$T_i(0) = \bar{T} + \Delta T, \quad \text{for} \quad \chi_i = +1, \quad (18a)$$

$$T_i(L) = \bar{T}, \quad \text{for} \quad \chi_i = -1, \quad (18b)$$

and for the bottom segment we have

$$T_i(0) = \bar{T}, \quad \text{for} \quad \chi_i = +1, \quad (19a)$$

$$T_i(L) = \bar{T} + \Delta T, \quad \text{for} \quad \chi_i = -1. \quad (19b)$$

We obtain the heat currents on the top and bottom edge segments by solving Eq. (6) for

$$J_{Q,\text{top}}^i = \sum_{\chi_i = +1} J_i(L) - \sum_{\chi_i = -1} J_i(L), \quad (20)$$

$$J_{Q,\text{bot}}^i = \sum_{\chi_i = +1} J_i(0) - \sum_{\chi_i = -1} J_i(0), \quad (21)$$

with $J_i(x)$ given in Eq. (12) [see also Appendix A.9 in Ref. [76] for details in solving Eq. (6)]. Next, by differentiating the total edge current with respect to the temperature difference $\Delta T$, we obtain the two-terminal thermal conductance

$$G_{2T}^Q \equiv \lim_{\Delta T \to 0} \left( \frac{d(J_{Q,\text{tot}}^i + J_{Q,\text{bot}}^i)}{d\Delta T} \right). \quad (22)$$

Let us start with the edge structures Pf-2, and aPf-3, which have only co-propagating modes. For both these interfaces, we readily find $G_{2T}^Q/(\kappa_0 \bar{T}) = 3/2$, independently of $L$ and $\ell_{Q,ij}$, which follows from the fact that the heat exchanged between co-propagating channels never backscatters, and such processes can therefore not affect $G_{2T}^Q$.

For the other four interfaces aPf-2, Pf-3, phPf-2, and phPf-3, we consider only pairwise thermal equilibration between the counter-propagating channels, see Fig. 2. Thermal equilibration between co-propagating channels can be ignored, as discussed in the previous paragraph. For example, for the Pf-3 interface, we consider only the thermal equilibration between channel pairs 1-2 and 1-3. As we will show, the thermal conductance then depends on the degrees of thermal equilibration $L/\ell_{Q,ij}$ between these pairs. To parameterize the equilibration, we introduce two dimensionless equilibration parameters $\alpha$ and $\beta$ as

$$\alpha = \frac{L}{\ell_{Q,1,2}} \quad \text{and} \quad \beta = \frac{L}{\ell_{Q,1,3}}, \quad (23)$$

for the phPf-2, Pf-3, and aPf-2 interfaces and

$$\alpha = \frac{L}{\ell_{Q,1,2}} \quad \text{and} \quad \beta = \frac{L}{\ell_{Q,2,3}}, \quad (24)$$

for the phPf-3 structure (note the slight difference in the definition of $\beta$, due to their different structures). We plot the thermal conductances as functions of $\alpha$ and $\beta$ in Fig. 5. For the phPf-2 interface, Fig. 5a shows that $G_{2T}^Q$ has a step-like behavior and transitions from $G_{2T}^Q/(\kappa_0 \bar{T}) = 1/2 \to 3/2$ with decreasing $\alpha$ (there is no $\beta$-parameter for this interface). For large $\alpha$ the thermal transport is effectively mediated by a single, “collective mode” with thermal quantum number $n_{\text{tot}} = 1 - 1/2 = 1/2$. For small $\alpha$, the two edge channels are essentially decoupled and heat transport occurs in both directions along the edge. The thermal quantum numbers of the channels then add up, $n_{\text{tot}} = 1 + 1/2 = 3/2$, in their contribution to $G_{2T}^Q$.

The thermal conductance of aPf-2, Pf-3 and phPf-3, is depicted in Fig. 5b. For these interfaces, we see that, depending on which pair of modes that equilibrates most efficiently, the thermal conductance approaches different limits. More specifically we have

$$\lim_{\beta \to \infty} G_{2T}^Q = \frac{1}{2} \kappa_0 \bar{T}, \quad \lim_{\beta \to 0} G_{2T}^Q = \frac{1}{2} \kappa_0 \bar{T},$$

$$\lim_{\beta \to \infty} G_{2T}^Q = \frac{3}{2} \kappa_0 \bar{T}, \quad \lim_{\beta \to 0} G_{2T}^Q = \frac{5}{2} \kappa_0 \bar{T}. \quad (25)$$
Similar to the phPf-2 interface, the maximum value of the conductance is obtained when there is no thermal equilibration and the contributions of all edge channels along the interface add up: \( n_{\text{tot}} = 1 + 1 + 1/2 = 5/2 \), as expected. The value \( G_{2T}^Q/(\kappa_0 T) = 1/2 \) is generated when all edge channels being fully equilibrated \((\alpha, \beta \to \infty)\) and \( n_{\text{tot}} = 1 - 1 + 1/2 = 1/2 \), in accordance with Eq. (2), up to corrections exponentially small in \( L \). Alternatively, \( G_{2T}^Q/(\kappa_0 T) = 1/2 \) is produced for \( \alpha \to \infty, \beta \to 0 \), where the edge structure becomes a decoupled MM and a pair of two strongly equilibrated bosons: \( n_{\text{tot}} = 1/2 + 0 \). The zero here corresponds to a diffusive correction which vanishes as \( \sim \alpha^{-1} \) (see, e.g., the Supplement Material of Ref. [66] for a detailed discussion). In contrast, \( G_{2T}^Q/(\kappa_0 T) = 3/2 \) is produced by two decoupled collective modes generating \( n_{\text{tot}} = 1/2 + 1 = 3/2 \). The limits in Eq. (25) are clearly idealized, and real devices have finite values of \( \alpha, \beta \). However to estimate the relative magnitude of \( \alpha \) and \( \beta \) requires detailed microscopic information about inter-channel energy exchange mechanisms, which generically depend on, e.g., spin, orbital, or valley degrees of freedom of the LLs. Incorporating these effects is a challenging problem which, however, lies beyond the scope of the present work.

We end this subsection by pointing out that the experimental value of \( G_{2T}^Q/(\kappa_0 T) = 1/2 \) for both 5/2 – 2 and 5/2 – 3 can, at least in principle, be generated from a state other than the phPf. Consider an edge structure similar to the aPf edge but with the MM direction reversed. Then, for full thermal equilibration \( G_{2T}^Q/(\kappa_0 T) = 1/2 \) is indeed produced for both \( n = 2 \) and \( n = 3 \) \((n_{\text{tot}} = 1 - 1 + 1/2 = 1/2 \) and \( n_{\text{tot}} = 1 - 1/2 = 1/2 \), respectively). The non-equilibrated limits for those interfaces are however \( G_{2T}^Q/(\kappa_0 T) = 5/2 \) and \( G_{2T}^Q/(\kappa_0 T) = 3/2 \), respectively, which stand in contrast to the phPf which has \( G_{2T}^Q/(\kappa_0 T) = 3/2 \) and \( G_{2T}^Q/(\kappa_0 T) = 5/2 \) in this limit.

### B. Thermal Hall conductance

Equation (22) describes the two-terminal thermal conductance which we have shown to significantly depend on the degree of thermal equilibration. Only for efficient thermal equilibration does \( G_{2T}^Q \) take the universal value as specified in Eq. (2). Note that in Eq. (22), the two edge current contributions are added (cf., Fig. 4). By instead subtracting the two edge currents, one can define a thermal Hall conductance as

\[
G_H^Q \equiv \lim_{\Delta T \to 0} \left( \frac{d(J_Q^{\text{top}} - J_Q^{\text{bot}})}{d\Delta T} \right). \tag{26}
\]

In the most general case, the top and bottom edge heat currents depend on all mutual equilibration lengths between all pairs of counter-propagating modes. However, within the model in Sec. II, we prove (see Appendix C for the proof) that for any edge structure, universality emerges

\[
G_H^Q/(\kappa_0 T) = c - \bar{c} = \nu_Q, \tag{27}
\]

provided the degrees of equilibration on the top and bottom edges are equal, even if they are poor. Such a situation can, e.g., be achieved by designing devices with equal top and bottom edge segment lengths. Indeed, such a setup was presented in Ref. [37], in which the thermal Hall conductance was measured at \( \nu = 2/3 \) and was found to be in good agreement with the expected value \( G_H^Q/(\kappa_0 T) \approx 0.82 \), indicating an incomplete thermal equilibration. In the present context, Figs. 1–2 suggests that a Hall type of thermal conductance measurement for interfaces 5/2 – n would give \( G_H^Q/(\kappa_0 T) = 7/2 - n, 3/2 - n, \) and \( 5/2 - n \) for the Pf, aPf, and phPf edges, respectively. A thermal Hall measurement therefore unambiguously

---

Figure 5. (a) Two-terminal thermal conductance \( G_{2T}^Q \) (in units of the thermal conductance quantum \( \kappa_0 T \)) for the phPf-2 interface as a function of the degree of pairwise thermal equilibration \( \alpha \) [see Eq. (23)]. (b) \( G_{2T}^Q \) as a function of the two equilibration parameters \( \alpha \) and \( \beta \) [see Eqs. (23)-(24)] for interfaces phPf-3, aPf-2 and Pf-3.
distinguishes between these three edge structures. This conclusion holds even in the absence of interfacing, since already \( n = 0 \) results in different \( G^Q_H \).

Let us end our treatment of the thermal conductance by emphasizing that for edge states with co-propagating channels, there is no significant advantage gained from a thermal Hall measurement. By contrast, for edges with counter-propagating channels, \( G^Q_H \) provides information directly related to the state’s topological order, i.e., \( \nu_Q \), independent of the thermal equilibration (which might be hard to accurately control experimentally). Finally, we point out that it is simple to check, that within our model, corresponding conclusions hold for a similar definition of the charge Hall conductance as well, i.e.,

\[
G_H \equiv \frac{I_{\text{top}} - I_{\text{bot}}}{\Delta V} = \nu \frac{e^2}{h},
\]

when top and bottom charge equilibrations are identical. Here, \( I_{\text{top/bot}} \) are defined in perfect analogy with Eqs. (20)-(21).

## IV. NOISE GENERATION ON INTERFACED EDGES

In this section, we compute the noise generated on a single interfac ed edge segment with two attached contacts. Such a setup has been realized in experiments, see, e.g., Refs. [12, 36, 75]. We are here interested in three cases: First, in Sec. IV A, we take the two contacts to have equal temperatures and impose a voltage bias. This is illustrated in Fig 6a. We then move on to the case with no applied voltage bias, but let the two contacts have different temperatures. Here, we first take the most upstream contact (the left one in the figure) to be the hot one, see Fig 6b, and compute in Sec. IV B the excess noise in the colder, downstream contact. We call this downstream delta-\( T \) noise. Finally, in Sec. IV C we consider the situation in which the hot contact lies most downstream and the noise is measured in the upstream, colder contact, see Fig 6c. We call this upstream delta-\( T \) noise.

### A. Voltage biased edge segment

For an applied voltage bias \( \Delta V \), the injected downstream charge current dissipates heat only close to one of the contacts, when the charge equilibration is efficient (the hot spot location is independent of the voltage bias direction [65]). We assume this in the following, which further implies that in the noise spot region, all charged edge modes equilibrate to the same electrochemical potential [65, 66]. We can then set \( \delta V(x \lesssim l_{\text{eq}}^C) \approx 0 \) [see Eq. (8)], with negligible corrections \( \sim \exp[-L/l_{\text{eq}}^C] \ll 1 \) [65]. We further assume that \( \epsilon \Delta V \gg k_B T \), so that in this subsection, we may set the base temperature \( T \) to zero. Our computed noise then amounts to the excess noise. Such excess noise is generated only if heat from the hot spot can propagate upstream (see Fig. 6a). This possibility depends in turn on the edge structure and on how well the edge channels thermally equilibrate. This feature is captured within our model in Eq. (14). As follows, we consider the two limits of either very efficient or very poor thermal equilibration.

#### 1. Efficient thermal equilibration

For efficient thermal equilibration, we start by solving Eq. (3) for the boundary conditions \( V_i(0) = \Delta V \) (we set \( V = 0 \) for \( \chi_i = +1 \) and \( V_i(L) = 0 \) for \( \chi_i = -1 \). This is done for all the charged edge channels. From the solution, we extract the voltage drops (8), which we then insert into Eq. (6), which is finally solved with the boundary conditions \( T_i(0) = T_i(L) = 0 \). We give examples of voltage and temperature profiles in Appendix D. The resulting temperature profiles are used in Eq. (14). This integral is dominated by the temperatures at the noise spot, i.e., \( T_i(x \lesssim l_{\text{eq}}^C) \). Moreover, due to the efficient thermal equilibration, the channel temperatures are similar in this region. Performing the integration in Eq. (14), these two features result in noise on Nyquist-Johnson (NJ) form [78, 79]

\[
S \approx \frac{2e^2}{\hbar} \nu \frac{\nu_+ - \nu_-}{\nu_+ + \nu_-} k_B T_{\text{ns}},
\]

in terms of an effective noise spot temperature \( T_{\text{ns}} \). This temperature depends on the voltage bias \( \Delta V \) and the way \( T_{\text{ns}} \) depends on \( L \) stands is in one-to-one correspondence.
Figure 7. (a) Excess (basis temperature $\bar{T} = 0$) noise $S^{\text{exc}}$ in units of $e^3 \Delta V/h$, where $\Delta V$ is the bias voltage. The noise is computed vs the charge equilibration parameter $\delta = L/\ell_c^2$ for parameters causing efficient thermal equilibration: $(\alpha, \beta) = \{(250, 250), (100, 100), (25, 25)\}$ [see Eqs. (23)-(24)], for solid, dashed, and dotted lines, respectively. (b) same as (a) but in the limit of absent thermal equilibration $\alpha, \beta \to 0$. (c) Excess noise in the presence of a temperature gradient $\Delta T$ in downstream (ds) or upstream (us) direction for very efficient thermal equilibration with $(\alpha, \beta) = (100, 100)$. (d) same as (c) but for absent thermal equilibration. For the phPf-2 interface, we have used $\delta = \delta_{23}$, see Fig. 8.

to the three possible cases of a thermally equilibrated edge: $\nu_Q > 0$, $\nu_Q < 0$, and $\nu_Q = 0$ [66]. This classification of the noise was described in detail in Sec. I.

We first analyze the noise generated on the Pf-2, and aPf-3 interfaces. For these, we have that the noise vanishes identically, $S = 0$, for any value of $\delta$ and $\Delta V$. This happens simply because there are no upstream modes present that can transport heat to the noise spot, and thus both $\nu_- = 0$ and $\Lambda(x) = 0$ (the latter equality holds because we set $\bar{T} = 0$).

The other four interfaces have $\bar{c} \neq 0$, but the aPf-2 and Pf-3 have $\nu_Q < 0$ whereas phPf-2 and phPf-3 have $\nu_Q > 0$. For these four interfaces, we plot the noise $S$ vs $\delta$ in Fig. 7a. We see that the noise approaches a constant value $S \approx \text{const.}$, for aPf-2 and Pf-3 but decays exponentially $S \approx \exp(-\delta)$ for phPf-2 and phPf-3. We also see that with increasing thermal equilibration (increasing $\alpha, \beta$) the noise is overall suppressed for $\nu_Q > 0$, because the heat that reaches the noise spot is suppressed even further. In contrast, when $\nu_Q < 0$, increasing thermal equilibration leads to enhanced noise since, in this case, there is more heat propagating upstream to the noise spot.

2. Absent thermal equilibration

For absent thermal equilibration, downstream and upstream edge modes are generally at very different temperatures at the noise spot. To estimate these temperatures, we follow the approach in Ref. [36] and model the hot spot as a point-like heat source with power [67]

$$P_0 = \frac{e^2 \Delta V^2 \nu_- (\nu_+ - \nu_-)}{2h} \frac{\nu_+}{\nu_-}. \quad (30)$$

We next assume that $P_0$ is equally divided among all edge channels. The temperature of the modes propagating upstream from the hot spot is thus approximated as

$$T_- \approx \sqrt{\frac{\bar{c}}{\bar{c} + c} \sqrt{\frac{2P_0}{\kappa_0}}}. \quad (31)$$
Since we have set the temperature in the contacts to $\bar{T} = 0$, the downstream modes will have zero temperature at the noise spot

$$T_+ \approx 0. \quad (32)$$

Within the approximations (31)-(32), we compute noise kernels $\Lambda(x)$ in terms of finite and zero temperature bosonized Green’s functions. Details of these calculations are given in Appendix B.

We present the thermally non-equilibrated noise in Fig. 7b. Similarly to the case of efficient thermal equilibration, the Pf-2, and aPf-3 do not generate any excess noise. For the other interfaces, the noises are essentially constant as functions of $\delta$, since the heat reaching the noise spot is now independent of this parameter. The relative magnitude between the interfaces follow from different noise spot temperatures due to differing pre-factors in Eqs. (14) and (31).

The case of the phPf-2 interface warrants here an extra discussion. The noise of this interface was a crucial ingredient in the interpretation of the experiment in Ref. [12].

Figure. 1 shows that this structure only has a single downstream charge mode. Hence, no partitioning would be possible and no excess noise could be generated. However, by recalling that on sufficiently small length-scales, $x \ll \xi_{\text{eq}}^C$, there are in fact counter-propagating integer modes close to both source and drain contacts. These modes are responsible both for Joule-heating and charge partitioning. To correctly describe the noise, we include these modes, see Fig. 8. This procedure introduces two charge equilibration lengths between each pair of counter-propagating modes, see Fig. 8. This procedure introduces two charge equilibration lengths between each pair of counter-propagating modes. These modes are responsible both for Joule-heating and charge partitioning. To correctly describe the noise, we include these modes, see Fig. 8. This procedure introduces two charge equilibration lengths between each pair of counter-propagating modes.

As our next step, we assume that the charge equilibration of the integer channels is much faster than their equilibration with the bosonic $1/2$ mode. In terms of our equilibration parameters, this amounts to taking $\delta_{12} \gg \delta_{23}$. The main contribution to the Joule-heating then comes from the equilibration of integer channels. This limit is consistent with a quantized expected charge conductance, since we have that

$$\lim_{\delta_{12} \to \infty, \delta_{23} \to 0} G_{2T} = \frac{1}{2} \frac{e^2}{h}. \quad (33)$$

which was indeed observed experimentally for the $5/2 - 2$ interface [12]. With this implementation, our model describes how the phPf-2 interface generates noise in the thermally non-equilibrated limit, see Fig. 7d. We note that this noise is similar in magnitude as the phPf-3, in accordance with experimental observations [12], see also the discussion in Sec. VI below. We further note that the outlined procedure bares a similarity with implementing edge-reconstruction [69–72]. Effects of edge-reconstruction are discussed in more detail in Sec. VII.

B. Downstream delta-T noise

Here, we consider the situation where the two contacts are at the same potential, but at different temperatures. Excess charge current noise generated only by a temperature gradient and in the absence of an average charge current, is known as “thermally activated shot noise” or “delta-T noise” [80]. This noise has attracted considerable attention lately (for recent work on delta-T noise for weak tunneling between FQH edges, see Refs. [81–84]). Moreover, delta-T noise is, in fact, at play in two-terminal thermal conductance measurements when the central contact is at zero potential (see Sec. VII below).

We first take the hot contact to lie upstream from the cold contact, see Fig. 6b. The boundary conditions are then given in Eq. (18). We are interested in the noise in the right, cold contact, which we call downstream delta-T noise (because the noise is measured downstream from the heat source). Note however that the noise spot lies close to the hot contact (given that the equilibrated charge flow is from left to right). The noise is given by a modification of Eq. (14), namely

$$S \simeq \frac{2e^2}{h \xi_{\text{eq}}^C} \nu_+ (\nu_+ - \nu_-) \int_0^L dx \, e^{-\frac{x}{\xi_{\text{eq}}^C}} \Lambda(x) + \frac{2e^2}{h} \frac{(\nu_+ - \nu_-)^2}{\nu_+} k_B (T + \Delta T). \quad (34)$$

Here, the second term takes into account thermal fluctuations coming from the hot, left contact [36]. The thermal noise coming from the cold, right contact is suppressed by a factor that is exponential in $\delta = L/\xi_{\text{eq}}^C \gg 1$ [65, 76]. This term can thus be safely neglected due to the efficient charge equilibration. In contrast to the voltage biased edge segment (where we assumed $e\Delta V \gg k_B T \approx 0$), we focus here on excess noise obtained by subtracting from Eq. (34) the equilibrium noise in the cold contact, i.e.,

$$S^{\text{excess}} = S - 2G_{2T} k_B T, \quad (35)$$

where $G_{2T} = e^2 (\nu_+ - \nu_-)/h$ is the two-terminal charge conductance.
1. Efficient thermal equilibration

For efficient thermal equilibration, we find that the first term in Eq. (34) for any edge structure, reduces to $2e^2(\nu_+ - \nu_-)k_B(T + \Delta T)/(\hbar \nu_+)$. Adding this to the second term, we obtain the downstream delta-$T$ noise as

$$S_{\text{ds}} \simeq 2\frac{e^2}{\hbar}(\nu_+ - \nu_-)k_B(\bar{T} + \Delta T)$$

$$= 2G_{2T}k_B(\bar{T} + \Delta T). \quad (36)$$

The corresponding downstream excess noise (35) reduces to

$$S_{\text{excess}}^{\text{ds}} \simeq 2G_{2T}k_B\Delta T. \quad (37)$$

Hence, the excess downstream delta-$T$ noise for strong thermal equilibration equals the excess NJ-like noise emanating from the hot contact. This result is independent of $\nu_2$. We also note that for $\Delta T = 0$, the entire edge segment and the contacts are in thermal equilibrium and $S_{\text{ds}} = 2G_{2T}k_B\bar{T} \Rightarrow S_{\text{excess}}^{\text{ds}} = 0$ as expected. In Fig. 7c, we plot, for phPf-3, phPf-2, Pf-3, and aPf-2, $S_{\text{excess}}^{\text{ds}}/(2G_{2T}k_B)$ vs $\delta$. We see that with increasing $\delta$ this ratio approaches unity as expected. For aPf-3 and Pf-2, where $\bar{c} = 0$, we have trivially $S_{\text{excess}}^{\text{ds}} = 2G_{2T}k_B\Delta T$ for all $\delta$.

2. Absent thermal equilibration

In case of absent thermal equilibration, edge modes can have very different temperatures at the noise spot. When computing the noise in Eq. (34) for this case, we find that the excess noise still takes a form similar to the NJ noise (37), namely

$$S_{\text{excess}} = 2\lambda_{\text{ds}}G_{2T}k_B\Delta T. \quad (38)$$

Here, $\lambda_{\text{ds}} \sim O(1)$ is a correction factor [36], which reflects the poor thermal equilibration. Its origin is the first term in Eq. (34): For poor thermal equilibration, this term does take the simple form $\sim k_B(T + \Delta T)$ that produced Eq. (36). Instead, we compute (see Appendix B) the noise kernels with distinct temperatures at the noise spot and insert these kernels in Eqs. (34)-(35). We then obtain Eq. (38) with the correction factors $\lambda_{\text{ds}}$.

In Tab. II, we give the values of $\lambda_{\text{ds}}$ for weak temperature bias $\Delta T \ll \bar{T}$ and strong bias $\Delta T \gg \bar{T}$ (see Appendix B2 for the derivation). Note that the values are obtained in the extreme limit of no thermal equilibration, while full equilibration amounts to $\lambda_{\text{ds}} = 1$ according to Eq. (37). Partial equilibration then corresponds to values larger than those in Tab. II, but still below 1. We use the strong bias values for the plots in Fig. 7d.

Table II. Correction factors for thermally non-equilibrated downstream (ds) excess delta-$T$ noise [see Eq. (38)] for small and large applied bias $\Delta T$.

| $\lambda_{\text{ds}}$ | phPf-3 | aPf-2 | phPf-2 |
|----------------------|--------|-------|--------|
| $\Delta T \ll \bar{T}$ | 3      | 5     | 4      |
| $\Delta T \gg \bar{T}$ | 0.865  | 0.774 | 0.899  |

The noise is measured in the cold, upstream contact and we call this noise upstream noise (since noise is measured upstream of the heat source). Note that also in this case, the noise spot is close to the left contact. The boundary conditions are now given in Eq. (19) and the noise reads

$$S \simeq \frac{2e^2}{\hbar\ell_{\text{eq}}}\nu_+(\nu_+ - \nu_-) \int_0^L dx e^{-\frac{2x}{\hbar\ell_{\text{eq}}}} \Lambda(x)$$

$$+ \frac{2e^2}{\hbar}\nu_+(\nu_+ - \nu_-)^2 k_B\bar{T}. \quad (39)$$

The difference between Eq. (34) and Eq. (39) lies only in the temperature entering the last term: now the thermal charge fluctuations from the hot, right contact are exponentially suppressed in $\delta = L/\ell_{\text{eq}}$. The definition of excess noise (35) holds also for the upstream delta-$T$ noise.

1. Efficient thermal equilibration

Whether the noise spot acquires a temperature larger than $\bar{T}$ depends, for efficient thermal equilibration, strongly on the thermal quantum number $\nu_Q$ in Eq. (1). For $\nu_Q > 0$, the upstream heat flow is exponentially small in $\delta$ and the resulting noise is to excellent approximation given as the NJ noise corresponding to the cold contact

$$S_{\text{us}} \simeq 2G_{2T}k_B\bar{T}. \quad (40)$$

This implies $S_{\text{excess}} = 0$ for all edges with $\nu_Q > 0$. For $\nu_Q \leq 0$, the situation is similar to the voltage biased and thermally equilibrated edge in Sec. IV A 1. The only difference is that instead of heat coming from dissipation in the hot spot, it comes from a heated contact. Hence, the classification presented in Ref. [66] (this classification was outlined in Sec. 1) carries over to thermally equilibrated, upstream delta-$T$ noise.

We plot the upstream, equilibrated delta-$T$ noise in Fig. 7c. The interfaces Pf-2, aPf-3, phPf-2, and phPf-3 all have $\nu_Q > 0$ (relative to their charge flows) and their upstream delta-$T$ noises therefore either vanish identically $S = 0$ (aPf-2 and Pf-3) or decay exponentially in $\delta$ (aPf-3 and Pf-2). Due to the single Majorana mode on all interfaces, none of them have $\nu_Q = 0$. For aPf-2 and Pf-3, $\nu_Q < 0$ and $S_{\text{excess}}$ reaches a constant value with increasing $\delta$.

C. Upstream delta-$T$ noise

We now analyze the situation where the hot contact lies downstream to the cold contact, as depicted in Fig. 6c.
2. Absent thermal equilibration

For very poor thermal equilibration, upstream excess delta-$T$ noise becomes possible in the presence of upstream modes, $\varepsilon \neq 0$. For $\varepsilon = 0$, we have that $S_{\text{us}} \simeq 2G_T\bar{T} \Rightarrow S_{\text{us}}^{\text{excess}} \simeq 0$. This is the case for Pf-2 and aPf-3 interfaces.

All other interfaces have $\varepsilon \neq 0$. We compute the noise kernels for those in Appendix B. The resulting noise can be written as

$$S_{\text{us}}^{\text{excess}} = 2\lambda_{\text{us}}G_Tk_B\Delta T,$$

with $\lambda_{\text{us}}$ given in Tab. III, for both weak, $\Delta T \ll \bar{T}$, and strong temperature bias $\Delta T \gg \bar{T}$. Since the thermally equilibrated $S_{\text{us}}^{\text{excess}}$ depends on $\nu_Q$, we do not expect a continuous limit for $\lambda_{\text{us}}$, as was the case for $S_{\text{ds}}^{\text{excess}}$. The strong bias values for $\lambda_{\text{us}}$ are used for the plots in Fig. 7d: The $\varepsilon \neq 0$ interfaces display constant noise vs $\delta$.

Table III. Correction factors for thermally non-equilibrated upstream (us) excess delta-$T$ noise [see Eq. (41)] for small and large applied bias $\Delta T$.

| $\lambda_{\text{us}}$ | phPf-3 | aPf-2 | phPf-2 |
|-----------------------|-------|-------|-------|
| $\Delta T \ll \bar{T}$ | 3     | 7     | 7     |
| $\Delta T \gg \bar{T}$ | 0.365 | 0.438 | 0.338 |

The results for the various types of noise computed in Sec. IV are summarized in Tab. I.

D. Downstream-upstream symmetry of phPf-3 for absent thermal equilibration

Examination of $\lambda_{\text{ds}}$ and $\lambda_{\text{us}}$ exposes a unique relation for the phPf-3 interface. For strong applied biases, we have for this interface that $\lambda_{\text{ds}} = 0.5 + \lambda_{\text{us}}$. The constant offset follows from Eqs. (34) and (39) together with the fact that for the phPf-3 the noise kernels for excess delta-$T$ noise in the down- and upstream bias configurations are identical, see Appendix B4. By computing the noise kernels for general $\Delta T/\bar{T}$, we find that the above downstream-upstream symmetry holds for all $\Delta T$ (as long as the thermal equilibration is poor). In Fig. 9, this result is visualized by overlapping solid and dashed lines for all $\Delta T$ at constant $\bar{T}$ only for the phPf-3 (blue). We present an experimental method to test this symmetry in Sec. VII.

V. TEMPERATURE DEPENDENCE OF THERMAL EQUILIBRATION LENGTHS

Here, we discuss the impact of the base temperature $\bar{T}$ on the thermal equilibration. The degree of this equilibration is determined by parameters on the form $L/\ell^Q_{\text{eq}}$. Thus, to investigate phenomena related to thermal equilibration, one may either tune $L$, which requires advanced devices (see, e.g., Ref. [36]), or tune $\ell^Q_{\text{eq}}$ through its temperature dependence. Hence, this dependence is an important edge characteristic, as it determines

i) The value of the two-terminal thermal conductance $G_T^Q$ for a given temperature $\bar{T}$ and edge length $L$ [41].

ii) The sharpness of transitions between plateaus of quantized two-terminal thermal conductances (see Sec. III A).

iii) The magnitude of the noise generated on a single edge segment (see Sec. III B).

In the remainder of this section, we therefore discuss the temperature dependence of various $\ell^Q_{\text{eq}}$ for our considered interfaces.

In Ref. [46], the authors argued that most thermal equilibration lengths scale as

$$\ell^Q_{\text{eq}} \sim \bar{T}^{-2},$$

due to tunneling of particles between the edge channels. The exponent in (42) comes from the fact that in most cases, the most relevant (in the renormalization group sense) tunneling operators $\mathcal{O}$ take the form

$$\mathcal{O} \sim e^{i\sum_j m_j\phi_j} \text{ or } \sim \psi^\dagger\sum_j m_j\phi_j.$$  

Here, $\psi$ is a MM, $\phi_j$ are bosons, and $m_j$ are real-valued numbers indicating the number and charges of particles that tunnel. When the edge is sufficiently close to a strong disorder fixed point [43, 85–87], the $m_j$ take on

Figure 9. Noise kernel $\Lambda$ dependencies on the base temperature $\bar{T}$ (here, we set $\bar{T} = 0.1$ for concreteness) and applied bias $\Delta T$, for poor thermal equilibration. The boxed areas correspond to regions in which the approximation for weak or strong biases (wb and sb, respectively) are valid, see Appendix B4. The downstream and upstream noise kernels are identical for all $\Delta T, \bar{T}$ for the phPf-3 interface.
integer values which results in an integer scaling dimension of $O$, denoted $\Delta$ [30], and the exponent $\alpha = 2(\Delta - 1)$ in (42) is thus also an integer. Away from such a point, however, the exponents are expected to take on smaller, non-universal values which depend on the inter-edge interaction strength [43].

An important exception from Eq. (42) is the equilibration length between a counter-propagating boson and Majorana mode. The most relevant operator coupling these modes is instead on the form [46]

$$ O \sim \partial_x \phi \psi i \partial_x \psi. $$

(44)

Essentially, (44) follows from the fact that it is impossible to construct tunneling operators between a counter-propagating boson $\phi$ and a MM $\psi$ (the point is that only their combination can create electrons), and the simplest operator exchanging energy between the modes is instead of density-density type. This feature leads to the temperature scaling

$$ \ell_{eq}^Q \sim \bar{T}^{-4}. $$

(45)

We present a detailed derivation of this result in Appendix A. Equation (45) suggests that, for edges with a counter-propagating boson and a MM, which is the case for phPf-2 and phPf-3 interfaces (see Fig. 2), there can be unusually sharp transitions between thermal conductance plateaus. They are important signatures in thermal conductance measurements. For example, the phPf-3 interface produces a thermal conductance transition from $G_{2T}^Q/(\kappa_0 \bar{T}) = 1/2$ to $G_{2T}^Q/(\kappa_0 \bar{T}) = 3/2$ with decreasing temperature $\bar{T}$. The scaling of this transition is of the sharper form (45), which might enhance the prospects of its experimental observation at not too low $\bar{T}$.

Temperature scalings of the thermal equilibration lengths enter the incoherent model (summarized in Sec. II), as [39]

$$ \ell_{i,j}^Q = \frac{a}{g_{i,j} \gamma_{i,j} (n_i - n_j)} = \frac{a \kappa_0 \bar{T}}{G_{i,j}^Q \gamma_{i,j} (n_i - n_j)}, \quad n_i \neq n_j $$

(46a)

$$ \ell_{i,j}^Q = \frac{a}{g_{i,j} \gamma_{i,j}} = \frac{a \kappa_0 \bar{T}}{G_{i,j}^Q \gamma_{i,j}}, \quad n_i = n_j $$

(46b)

Here, $n_i$ and $n_j$ are the central charges of the two modes, $\gamma_{i,j}$ is a parameter characterizing deviations from the Wiedemann-Franz law [39, 44, 51], $a$ is the typical length-scale for inter-channel heat exchange, and $G_{i,j}^Q/(\kappa_0 \bar{T}) \equiv g_{i,j}$ are the dimensionful and dimensionless thermal conductances of this exchange. The latter quantities can be computed within the chiral Luttinger liquid model (see e.g., Refs. [39, 46, 51]).

For the special case of the counter-propagating boson and MM, denoted $\phi_2$ and $\psi$, respectively, we have

$$ \ell_{\phi,\psi}^Q = \frac{a}{g(\nu_\phi - \nu_\psi) \gamma_{\phi,\psi}} = \frac{2a \kappa_0 \bar{T}}{G_{int}^Q} $$

(47)

with $\gamma_{\phi,\psi} = 1$ and

$$ G_{int}^Q = \frac{8k_B^2 \pi \gamma T^4 \kappa_0 \bar{T}}{35h^2 \nu_\phi \nu_\psi}, $$

(48)

as computed in Appendix A. Here, $\Gamma_0$ is the bare coupling strength between the channels, which we assume to be weak, and $\nu_\phi$, $\nu_\psi$ are the propagation velocities of $\phi$ and $\psi$, respectively. The parameter $b$ is an ultra-violet cutoff, with dimension of length.

Knowledge of the temperature scalings of $\ell_{i,j}^Q$, permits us to analyze $G_{2T}^Q$ vs $\bar{T}$, rather than $\alpha, \beta$ (as we did in Sec. III). As an instructive example, we show in Fig. 10 such a plot for the phPf-3 interface. The characteristic feature is the presence of plateaus with different quantized values of $G_{2T}^Q$. These plateaus are associated to different regimes of equilibrated and non-equilibrated edge modes. In this particular example, there are two inter-channel transitions, which we label $I$ and $II$. These transitions are related to thermal equilibration by electron tunneling between $\phi_1$ and $\psi \times \phi_2$, transition $I$, and by a density-density interaction between $\phi_2$ and the MM $\psi$; transition $II$. From the above analysis, we have different temperature scalings for these transitions, namely $\ell_{\phi,\psi}^Q \sim T^{-2}$ and $\ell_{\phi,\psi}^{II} \sim T^{-4}$. Figure 10 shows that for full thermal equilibration (high $\bar{T}$), $G_{2T}^Q/(k_B \bar{T}) = 1 - 1 + 1/2 = 1/2$ which transitions at lower $\bar{T}$ to the value for absent thermal equilibration: $G_{2T}^Q/(k_B \bar{T}) = 1 + 1 + 1/2 = 5/2$. However, an intermediate plateau emerges in a temperature regime when there is a separation of the two equilibration length scales $\ell_{\phi,\psi}^Q \ll \ell_{\phi,\psi}^{II}$. This plateau vanishes for $\ell_{\phi,\psi}^Q \approx \ell_{\phi,\psi}^{II}$ within the range of the transition temperature. A similar analysis can be performed for any edge structure, with its own specific set of plateaus and transitions. The set of these two features can be viewed as a “pin-code” of the FQH
edge structure [41].

In the above analysis, we have have neglected interference effects from scattering of bosonic plasmon waves on contact-edge interfaces [43, 88, 89]. This amounts to assuming $L \gg L_T \sim T^{-1}$, where $L_T$ is a characteristic thermal length scale. Including the interference effects can potentially generate additional plateaus [36, 43].

VI. COMPARISON TO EXPERIMENTS

In this section, we compare the results from our model to the recent experimental findings in Refs. [11, 12]. We start with the thermal conductance. In Ref. [11], the two-terminal thermal conductance at the interface between the $5/2$ state and integer states $n$ was observed to obey $G_{2T}^Q/(\kappa_0 T) \approx |5/2 - n|$. We see that this result is incompatible with both the Pf and aPf edge structures, since for Pf-2 and aPf-3, we have $G_{2T}^Q/(\kappa_0 T) = 3/2$, independently of the thermal equilibration (see Sec. III A). The measured conductances can further be compared with Fig. 5, where we note that the measured values are consistent with a phPf edge structure, if we assume an efficient thermal equilibration when interfaced with integer modes. Indeed, among the three non-Abelian candidates in Fig. 1, the phPf is the only one compatible with the expected values of $G_{2T}^Q$ for all interfaced structures.

We further note that with decreasing temperature, and thus decreasing degree of equilibration (see Sec. V), the thermal conductance should increase and saturate at $G_{2T}^Q/(\kappa_0 T) = 3/2$ and $5/2$ for the phPf-2 and phPf-3 edges, respectively. As mentioned in the end of Sec. III A the non-equilibrated limit is an important check for uniquely pin-pointing the edge structure. This could be tested in future experiments similar to that in Ref. [41].

We next move on to the noise. Ref. [12] reported measurements of excess noise at interfaces of $5/2 - n$ for an applied voltage bias. From our analysis in Sec. III B, we note that for both Pf-2, and aPf-3, which consist of only co-propagating modes over long length scales, no excess noise is expected for any degree of thermal equilibration. As finite, but with $L$ and $T$ decreasing, noise was found for both $5/2 - 2$ and $5/2 - 3$, the most compatible edge structure is, just as in Ref. [12], the phPf, this time assuming the thermal equilibration to be neither full nor absent but rather in an intermediate regime.

Our model allows a comparison on a quantitative level: We compare the measured noise data with our Figs. 7a-7b. Due to the the well-established charge conductance quantization in Ref. [12], we can safely assume efficient charge equilibration $\delta \gg 1$. From our calculations, we find that the slope of the noise vs bias voltage curve for the phPf-3 interface approximately evaluates to

$$0 \leq \partial_{\Delta V} S^{\text{exc}}_{\text{phPf-3}} \leq 0.113 \frac{e^3}{\hbar} \approx 0.70 \times 10^{-30} \frac{A^2}{\mu V \text{Hz}},$$

which lies within our estimate and is moreover compatible with interfaced phPf edges that are not fully thermal equilibrated.

We finally estimate the thermal equilibration lengths from noise data measured at lengths $L = \{28, 38, 48, 58\} \mu m$ in Ref. [12], see Fig. 11. For concreteness, we use the phPf-3 edge structure and compute, for a given set of $\alpha, \beta, \delta$, the temperature profiles for a given bias $\Delta V \gg T \approx 0$. These profiles are then used with Eq. (14) to obtain the noise $S(\alpha, \beta, \delta)$. In Fig. 11, we present $S(L)$-profiles for two sets of model parameters that reasonably reproduce the observed noise.

Figure 11. Length dependent noise $S(L)$ at the $5/2 - 3$ interface. The red squares and dashed, black line are measured noise data and a fit, respectively, from Ref. [12]. The data is obtained for interface lengths $L = \{28, 38, 48, 58\} \mu m$ and was normalized to $S(L = 28 \mu m)$. Blue and green triangles is our computed noise for a phPf-3 interface. For the sets 1 and 2, we used parameters $(\alpha, \beta, \delta) = (6, 10, 100)$ and $(\alpha, \beta, \delta) = (4, 8, 25)$ at $L = 28 \mu m$ respectively, to obtain the normalizing constants $S(L = 28 \mu m) = \{0.023, 0.026\} \Delta V \cdot 10^{-30} A^2/(\mu V \text{Hz})$ and thus $S(L)/S(28 \mu m)$. 

$$0 \leq \partial_{\Delta V} S^{\text{exc}}_{\text{phPf-2}} \leq 0.086 \frac{e^3}{\hbar} \approx 0.54 \times 10^{-30} \frac{A^2}{\mu V \text{Hz}},$$

$$\partial_{\Delta V} S^{\text{measured}} \approx 0.1 \times 10^{-30} \frac{A^2}{\mu V \text{Hz}},$$

i.e., it is of similar magnitude to the one for phPf-3. In comparison, the experimentally observed noise characteristics for both $5/2 - 2$ and $5/2 - 3$ were [12]

$$0 \leq \partial_{\Delta V} S^{\text{exc}}_{\text{phPf-2}} \leq 0.086 \frac{e^3}{\hbar} \approx 0.54 \times 10^{-30} \frac{A^2}{\mu V \text{Hz}},$$

$$\partial_{\Delta V} S^{\text{measured}} \approx 0.1 \times 10^{-30} \frac{A^2}{\mu V \text{Hz}},$$

between the two limits of absent and full thermal equilibration (see Appendix E for conversion between units).
length dependent noise. For set 1 (blue triangles), we have taken \((\alpha, \beta, \delta) = (6, 10, 100)\) and for set 2 (green triangles), we took \((\alpha, \beta, \delta) = (4, 8, 25)\). For the shortest length \(L = 28\mu m\), the values \(\delta = 100\) and \(\delta = 25\) correspond to \(\ell_{eq}^C \approx 0.28\mu m\) and \(\ell_{eq}^C \approx 1.12\mu m\), respectively. Both these charge equilibration lengths accurately produce \(G_{2T}/(e^2/h) = 1/2\), as observed experimentally.

For both parameter sets, we find that thermal equilibration lengths

\[
3\mu m \leq \{\ell_{12}^Q, \ell_{23}^Q\} \leq 7\mu m
\]

(52)

fits the data well.

VII. COMBINED CONDUCTANCE AND NOISE MEASUREMENTS IN A SINGLE DEVICE

In this section, we propose a device designed for measurements of both \(G_{2T}^Q\) and various types of noise in a single device, see Fig. 12. This device is beneficial for two main reasons: First, it can be used to rule out possible sample-to-sample differences between separate devices targeted towards noise and conductance measurements. Second, it can be used to exclude effects of edge reconstruction [69–72]. To do so is particularly important, since in its presence, and under conditions of poor thermal equilibration, any edge structure can produce excess upstream noise. The benefit with our device is that the two-terminal thermal conductance at poor equilibration gives access to the total number of edge channels (with MM’s counted as a “half-channel”). This number can be used to ascertain that in the noise measurements there is no upstream heat transport in non-topological, spurious upstream modes from edge reconstruction. In describing our device below, we will also point out some subtleties in \(G_{2T}^Q\) measurement schemes for states with counter-propagating edge channels, and how to remedy them.

In Fig. 12, four QH regions are connected to a central, floating contact, \(\Omega\), which can act as a hot reservoir when electrical currents impinge on it. We take the three regions with fillings \(\nu_1, \nu_2,\) and \(n\) as the source regions: the source contacts \(S_1, S_2\) and \(S_n\) inject currents \(I_1 = \nu_1 V_{1e}^2/h, I_2 = \nu_2 V_{2e}^2/h,\) and \(I_n = nV_e^2/h,\) respectively, and \(V_1, V_2,\) and \(V_n\) are the corresponding three bias voltages with respect to ground. The drain contacts \(D_1, D_2, D_i,\) and \(D_{in}\) are grounded, and the amplifiers \(A_1\) and \(A_i\) are floating. Note that the currents that emanate from \(\Omega\) and enter the amplifiers are emitted to grounded contacts. Thus, these currents do not propagate back into source contacts. The filling in the region \(\nu_{in}\) is here tuned to 5/2 (or to some other state of interest), and this region is interfaced with a region with integer filling \(n\).

The contact potential \(V_{\Omega}\) and the dissipated electrical

\[
V_{\Omega} = \frac{h}{e^2} \frac{I_1 + I_2 + I_3}{\nu_1 + \nu_2 + \max(\nu_{in}, n)}, \quad (53)
\]

\[
P = \frac{h}{2e^2} \left( \frac{I_1^2}{\nu_1} + \frac{I_2^2}{\nu_2} + \frac{I_n^2}{n} - \frac{(I_1 + I_2 + I_n)^2}{\nu_1 + \nu_2 + \max(\nu_{in}, n)} \right). \quad (54)
\]

Strictly speaking, Eq. (54) holds when the sourced edges have \(\nu_Q > 0\). If this is not the case, it is not evident that all dissipated power heats the central contact. Instead, some Joule heat can leak out back on the source edge via upstream modes, without heating the contact. This produces a correction to Eq. (54) which will influence how the thermal conductance is extracted from the experimental data. We refer to Ref. [36] for a detailed discussion on this issue. However, this possible complication can be avoided by tuning the fillings \(\nu_1\) and \(\nu_2\) to, e.g., integer values. Then, it is certain that Eq. (54) is the dissipated power in the contact. In the following, we assume that this is the case.

Two operational modes with the device are of particular interest for this paper:

1) \(P = 0\) and \(V_{\Omega} \neq 0\) and the noise in the amplifier
$A_i$ is measured vs $V_{i\Omega}$. This configuration realizes the setup in Fig. 6a, upon the identification $V_{i\Omega} = \Delta V$ and assuming $n < 5/2$. For $n > 5/2$, the charge flow direction along the interface is from $A_i$ to $\Omega$ but this flow can be reversed by swapping the magnetic field direction [12]. Since $A_i$ is floating, no net current is injected into $\Omega$ for $n > 5/2$. Note that the charge fluctuations emanating from the hot contact in Fig. 12 is split between four edges. This requires a minor modification of the downstream noise as compared to Secs. IV A-IV B. Specifically, the device geometry is taken into account by substituting in Eqs. (36), (37), (40), and (41)

$$G_{ST} \rightarrow G_{m}^*,
\frac{1}{G_{m}^*} = \frac{1}{G_m} + \sum_{k \neq m} \frac{1}{G_k}.
\qquad (55)$$

Here, $G_m$ is the equilibrated charge conductance of edge $m$, where $m = 1, (in - n)$ labels the two outgoing edges with amplifiers and $k = 1, 2, in, (in-n)$ labels all outgoing edges.

ii) $P \neq 0$ and $V_{i\Omega} = 0$. This produces the situations considered in Figs. 6b-6c. A measurement of the excess noise downstream of the central contact (see Fig. 6b) allows to determine its excess temperature $T_{i\Omega} = T + \Delta T$. In turn, access to this noise allows extraction of the two-terminal thermal conductance [33]. Let us mention that $V_{i\Omega} = 0$ is not a necessary condition when measuring the thermal conductance. However, finite $V_{i\Omega}$ leads to additional hot spots in the device (close to all drain contacts downstream of $\Omega$). In the presence of upstream heat flow, the generated heat at these points could potentially effect the heating of the central contact. Such complications are absent for $V_{i\Omega} = 0$. Ideally, the downstream noise for this purpose is probed on an edge with $\tilde{c} = 0$ (e.g., at contact $A_1$ with $\nu_1$ tuned to an integer). This choice avoids possible corrections to the downstream thermal noise, compare Eq. (36) and Eq. (38). In fact, it is very useful to compare the downstream noises in $A_1$ and $A_i$ to investigate the validity of the NJ relation [78, 79] for QH states with counter-propagating channels and poor thermal equilibration. To illustrate this, we consider $\nu_1$ having only downstream modes. The excess noise in $A_1$ is then obtained from Eq. (36) as

$$\frac{S_{ds}^{\text{excess},1}}{2G_1^*} = k_B \Delta T.
\qquad (56)$$

With the assumption of no upstream modes above, this expression holds independently of any equilibration. In contrast, the excess noise in $A_i$ reads

$$\frac{S_{ds}^{\text{excess},i}}{2G_{in-n}^*} = \lambda_{ds} k_B \Delta T,
\qquad (57)$$

where $\lambda_{ds}$ does depends on the thermal equilibration of the $\nu_{in-n}$ interface. Only for full thermal equilibration is $\lambda_{ds} = 1$.

The ratio between Eq. (56) and Eq. (57) allows extraction of $\lambda_{ds}$:

$$\lambda_{ds} = \left( \frac{S_{ds}^{\text{excess},i}}{2G_{in-n}^*} \right) \times \left( \frac{S_{ds}^{\text{excess},1}}{2G_1^*} \right)^{-1}.
\qquad (58)$$

We next move on to upstream noise measurements. By swapping $\nu_n$ and $n$, the noise in $A_i$ corresponds to upstream delta-$T$ noise as presented in Fig. 6c (again, the substitution (55) is needed). Similarly to the downstream delta-$T$ noise, the upstream delta-$T$ can be measured vs $\Delta T$ where the latter is obtained from Eq. (56).

Finally, we discuss measurements of the phPF-3 downstream-upstream symmetry of the delta-$T$ noise at poor thermal equilibration, as presented in Sec. IV D. To detect this symmetry, we propose to first measure the downstream delta-$T$ noise (34), but in contrast to the excess noise (35), one subtracts in this case the hot contact thermal noise, i.e.,

$$S_{\Delta T}^{\text{ds}} \equiv S - 2G_{in-n}^* k_B (T + \Delta T) = \frac{G_{in-n}^*}{2} \Lambda^{\text{ds}}.
\qquad (59)$$

This quantity can then be compared with the upstream delta-$T$ noise (39) for which the cold contact thermal noise is subtracted

$$S_{\Delta T}^{\text{us}} \equiv S - 2G_{in-n}^* k_B T = \frac{G_{in-n}^*}{2} \Lambda^{\text{us}}.
\qquad (60)$$

In going from Eqs. (34) and (39) to Eqs. (59) and (60), we have again used the conductance substitution (55). The downstream-upstream symmetry is now exposed in the ratio

$$\frac{S_{\Delta T}^{\text{ds}}}{S_{\Delta T}^{\text{us}}} = \frac{\Lambda^{\text{ds}}}{\Lambda^{\text{us}}}
\qquad (61)$$

which uniquely equals unity for the phPF-3 interface. Comparison of delta-$T$ noises for thermally non-equilibrated interfaces thus provides another experimental signature to distinguish between non-Abelian candidate states.

VIII. SUMMARY AND CONCLUSIONS

We have presented a comprehensive theoretical description of quantum transport along interfaces formed between integer ($n = 2$ and $n = 3$) QH states and non-Abelian Pf, aPf, and phPF candidates for the $\nu = 5/2$ state. Such interfaces isolate the “non-Abelian part” of the edge and was used in recent experiments [11, 12], to distinguish between different candidate theories.

For such interfaces and experiments, we have here in detail determined the impact of thermal equilibration on the edge thermal conductance as well as on excess noise on voltage or temperature biased edge segments (so-called delta-$T$ noise). In contrast to Abelian edges, non-Abelian $5/2 - n$ interfaces feature Majorana modes,
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Note: Some of the results in this paper were reported in the Master thesis Ref. [76].

Appendix A: Heat exchange between counter-propagating boson and Majorana modes

The phPf-2 edge structure, depicted in Fig. 2, consists of two modes: One bosonic mode $\phi$ and one Majorana mode $\psi$. This structure is valid for length scales $L \gg \ell_{eq}^C$, where the contribution of integer edge modes can be neglected. A low energy effective description of the two counter-propagating modes is then given by the free Lagrangian density

$$\mathcal{L}_0 = \frac{2}{4\pi} \partial_x \phi (\partial_t - v_\phi \partial_x) \partial_x \phi + i \psi (-\partial_t - v_\psi \partial_x) \psi,$$  \tag{A1}

where $v_\phi$ and $v_\psi$ are the mode velocities. Our goal is to compute the heat exchange between the two modes due to a coupling between them. The edge electron operator reads $\psi e^{2\phi}$, so we cannot introduce electron or quasiparticle tunneling operators to couple these modes. Instead, the simplest, most relevant (in the renormalization group sense) operator coupling the channels is given by the density-density operator $[46]$ $\mathcal{O} = \partial_x \phi \psi \partial_x \psi.$  \tag{A2}

We therefore add to $\mathcal{L}_0$ the point-like perturbation

$$\mathcal{L}_T = \Gamma_0 \delta(x) \partial_x \phi \psi \partial_x \psi,$$  \tag{A3}

where $\Gamma_0$ is the coupling constant, assumed to be weak. We thus seek the heat current and inter-mode thermal conductance induced by $\mathcal{L}_T$, treated as a weak perturbation.

To do so, we consider the unperturbed bosonic energy current, which is given in terms of the stress energy tensor of the upstream bosonic field $\mathcal{F}_\phi \equiv (\partial_x \phi)^2$, as $[32]$ $J_{Q,\phi}^{(0)}(d,t) = \hbar \frac{v_\phi^2}{2\pi} \mathcal{F}_\phi(\tilde{t}).$  \tag{A4}

Here, $\tilde{t} = t - \frac{d}{v_\phi}$ is a shifted time referring to the transport of energy a small distance $d$ away from the point $x = 0$.

which although charge neutral, may influence the charge current noise generated on the edge. A major finding is that non-Abelian interfaces with counter-propagating modes are highly sensitive to thermal equilibration. This feature produces a significant length and temperature dependence for the two-terminal thermal conductance. For the noise, the degree of thermal equilibration influences the noise magnitude, the length dependence of the noise, as well as the Nyquist-Johnson relation for charge current noise emanating from a heated contact. Our more quantitative results are summarized in Tab. I.

In contrast to the two-terminal thermal conductance and the noise, we have proved that for the thermal Hall conductance, as it was defined in Eq. (27), the thermal equilibration dependency drops out for a generic edge structure. provided the two involved edge segments have the same degree of thermal equilibration. Such experimental conditions were put forward in Ref. [37]. Our results suggest that a similar experiment performed at $\nu = 5/2$ provides a clear distinction between all candidate states, without any interfacing.

The authors of Refs. [11, 12], interpreted their noise and thermal conductance measurements as pointing towards the phPf as the realized state in GaAs/AlGaAs. On the qualitative level, our findings favor such an interpretation as well. We further computed the voltage biased noise for phPf-2 and phPf-3 interfaces, and our results are quantitatively consistent with the experimentally obtained values. Importantly, our calculations show that the measured noise magnitude is indeed consistent with a FQH edge that is not fully thermally equilibrated, which was a crucial feature for the interpretation in Ref. [12].

We would like to further emphasize that the interpretation in Ref. [12] favoring the phPf state is based on the absence of edge reconstruction. If this effect would be present, any edge can generate noise for poor thermal equilibration. This severely complicates the interpretation of the experimental data. Ideally, one would therefore like to confidently rule out such effects. To address this problem, we proposed a device to do precisely that, by allowing thermal conductance and noise measurements in the same sample. Combining such measurements permits an unambiguous determination of the edge structure. We also pointed out some potential issues (highlighted previously in Ref. [36]) with the standard two-terminal thermal conductance setup [33] for edges with counter-propagating modes. Here, we proposed concrete improvements of the setup in order to mitigate these issues.

While our work targeted the famous $\nu = 5/2$ state, we envision that it can be generalized for the purpose of pin-pointing edge structures of other exotic FQH states, such as the even-denominator states in graphene [90–96] or the state at $\nu = 12/5$ [97–99].
In the interaction picture, the average heat current in the presence of \( \mathcal{L}_T \), can be written as

\[
\langle J_{Q,\phi}(t) \rangle = \langle T e^{i H'(t) \tau} J_{Q,\phi}^{(0)}(\tilde{t}) e^{-i H'(t) \tau} \rangle, \tag{A5}
\]

where \( H' \) is the Hamiltonian corresponding to \( \mathcal{L}_T \), and \( T \) denotes time ordering. We next expand the time-evolution operators up to \( O(\Gamma_0^0) \). Collecting terms corresponding to the same order in \( \Gamma_0 \), the first and second order correction to the heat current become

\[
J_{Q,\phi}^{(1)}(t) = \frac{i}{\hbar} \int_{-\infty}^{t} dt' [H'(t'), J_{Q,\phi}^{(0)}(\tilde{t})] \tag{A6}
\]

and

\[
J_{Q,\phi}^{(2)}(t) = \frac{i^2}{\hbar^2} \int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt'' [H'(t''), [H'(t'), J_{Q,\phi}^{(0)}(\tilde{t})]], \tag{A7}
\]

respectively. The commutators in these expressions can be computed with operator product expansions with the stress energy tenor \([30]\). After some algebra, we find the corrections to the average energy current in terms of the modes Green’s functions as

\[
\langle J_{Q,\phi}^{(1)}(t) \rangle = 0 \tag{A8}
\]

and

\[
\langle J_{Q,\phi}^{(2)}(t) \rangle = \frac{G_{\phi}(\tau)}{\hbar} \int_{-\infty}^{\infty} (\partial_{\tau} G_{\phi}(\tau)) G_{\psi}(\tau) d\tau. \tag{A9}
\]

In Eq. (A9), the integrand is given in terms of the finite temperatures Green’s functions, \( G_{\phi}(\tau) \) and \( G_{\psi}(\tau) \) \([30]\), which read

\[
G_{\phi}(\tau) \equiv \langle \partial_{\tau} \phi(\tau,0) \partial_{\tau} \phi(0,0) \rangle = \left( \frac{\pi b k_B T_{\phi}}{\hbar v_{\phi}} \csc \left( \frac{\pi k_B T_{\phi}}{\hbar v_{\phi}} (b - i v_{\phi} \tau) \right) \right)^2, \tag{A10}
\]

with \( b \) the UV cutoff, and

\[
G_{\psi}(\tau) \equiv \langle \mathcal{F}_{\psi}(\tau,0) \mathcal{F}_{\psi}(0,0) \rangle = \left( \frac{\pi b k_B T_{\psi}}{\hbar v_{\psi}} \csc \left( \frac{\pi k_B T_{\psi}}{\hbar v_{\psi}} (b - i v_{\psi} \tau) \right) \right)^4 + \frac{c^2}{36} \left( \frac{\pi k_B T_{\psi}}{\hbar v_{\psi}} \right)^4, \tag{A11}
\]

where \( c = 1/2 \) and \( \mathcal{F}_{\psi} = \psi i \partial_{\tau} \psi \) for the MM \( \psi \).

For completeness, we give also the correlation function for a tunneling operator of the form \( \mathcal{O} \sim \psi e^{2i \phi_1 + \phi_2} \). From the statistical independence of the involved fields we obtain

\[
\langle \mathcal{O}(\tau) \mathcal{O}(0) \rangle \propto G_{\phi_1}(\tau) G_{\phi_2}(\tau) G_{\psi}(\tau), \tag{A12}
\]

with

\[
G_{\phi}(\tau) \equiv \langle e^{i v_{\phi} \phi(\tau,0)} e^{i v_{\phi} \phi(0,0)} \rangle = \left( \frac{\pi b k_B T_{\phi}}{\hbar v_{\phi}} \csc \left( \frac{\pi k_B T_{\phi}}{\hbar v_{\phi}} (b - i v_{\phi} \tau) \right) \right)^{1/\nu_{\phi}}, \tag{A13}
\]

in which \( \nu_{\phi} \in \{1/2,1\} \), and

\[
G_{\psi}(\tau) \equiv \langle \psi(\tau,0) \psi(0,0) \rangle = \left( \frac{\pi b k_B T_{\psi}}{\hbar v_{\psi}} \csc \left( \frac{\pi k_B T_{\psi}}{\hbar v_{\psi}} (b - i v_{\psi} \tau) \right) \right) . \tag{A14}
\]

Next, we insert the expressions for \( G_{\phi}(\tau) \) from Eq. (A10) and \( G_{\psi}(\tau) \) from Eq. (A11) into Eq. (A9) and shift variables \( \tau \to \tau + \frac{i k_B T_{\phi}}{v_{\phi}} - \frac{i k_B T_{\psi}}{v_{\psi}} \). Following Ref. [100], the integral boundaries are switched back for a properly introduced cut-off \( b \) that satisfies \( \hbar v_{\phi} \beta_{\phi} > b \). Setting the temperatures \( T_{\psi} = \tilde{T} + \frac{\Delta T}{2} \) and \( T_{\phi} = \tilde{T} - \frac{\Delta T}{2} \), gives to leading order in \( \Delta T \) the interaction induced heat current

\[
\langle J_{Q,\phi}(t) \rangle = \frac{8 b^2 k_B^4 \pi^3 T_{\psi}^2}{105 \hbar^4 v_{\psi}^4} \tilde{T}^5 \Delta T. \tag{A15}
\]

The corresponding interaction thermal conductance thus reads

\[
G_{\text{int}}^Q = \lim_{\Delta T \to 0} \left( \frac{d}{d\Delta T} \langle J_{Q,\phi}(t) \rangle \right) = \frac{8 b^2 k_B^4 \pi^3 T_{\psi}^2}{35 \hbar^4 v_{\psi}^4} \tilde{T}^4 \kappa_0 \tilde{T}. \tag{A16}
\]

As our final step, we consider an array of points, distanced with the length \( a \), with couplings on the form \( \langle A3 \rangle \) and take the continuum limit (see Refs. [39, 51, 65, 67]). This procedure relates the conductance (A16) to the thermal equilibrium length as

\[
\ell_Q^Q = \frac{a}{g_{\phi,\psi}^2(n_{\phi} - n_{\psi})} = \frac{2a}{g_{\gamma,\psi}^2 G_{\text{int}}^Q} = 2 \frac{a \kappa_0}{G_{\text{int}}^Q} \sim T^{-4}, \tag{A17}
\]

where is Eq. (47). For the interaction (A3), we have \( \gamma_{\phi,\psi} = 1 \).

**Appendix B: Computation of noise kernels**

Our approach to compute noise kernels \( \Lambda(x) = S_{\text{loc}}(x)/[2 g_{\text{loc}}(x)] \) follows that in Ref. [75]. The local noise \( S_{\text{loc}} \) and local tunneling conductance \( g_{\text{loc}} \) generically read

\[
S_{\text{loc}}(x) \approx 4 \int_{-\infty}^{\infty} \langle \mathcal{O}(\tau,0) \mathcal{O}(0,0) \rangle d\tau \tag{B1}
\]

and

\[
g_{\text{loc}}(x) \approx 2 \int_{-\infty}^{\infty} \tau \langle \mathcal{O}(\tau,0) \mathcal{O}(0,0) \rangle d\tau . \tag{B2}
\]
Here, we have assumed zero voltage difference between the edge channels, since at the noise spot, edge channels equilibrate to the same electrochemical potential. Furthermore, $O$ denotes the most relevant tunneling operator charged edge channels, and the corresponding correlation function can be expressed as a product of Green’s functions [see Eqs. (A13)-(A14)] as

$$\langle O(\tau, 0)O^\dagger(0, 0) \rangle = \frac{\Gamma_0^2}{(2\pi b)^2} \prod_k G_k(\tau, 0).$$  \hspace{1cm} (B3)

Here, $k \in \{\psi, \phi_i(1,2)\}$, $b$ is a short distance cut-off, and $\Gamma_0$ is the bare coupling amplitude. As follows, we compute $\Lambda(x)$ for various interfaces in the two limiting cases of efficient and absent thermal equilibration. We emphasize that all $x$-dependence in $\Lambda(x)$ enters in the modes temperature profiles $T_k(x)$.

1. Voltage biased charge current noise for absent thermal equilibration

a. phPf-3

For this interface, equilibrated charge transport is from right to left (see Fig. 2). The hot spot and noise spot are therefore interchanged in comparison to Fig. 6a. Heat from the hot spot is transported upstream by the bosonic 1/2 mode. Absence of thermal equilibration leads, with the procedure outlined in Sec. IV A 2, to the temperature $T_{\phi_1} = T_\psi = T_+ = 0$ and $T_{\phi_2} = T_- = \sqrt{\frac{2PE}{h\kappa_0}}$. Following the same approach as in the previous section, using Eqs. (A13) and (A14), we arrive at the noise kernel

$$\Lambda_{\text{phPf-3}} = \frac{12\zeta(3)}{\pi^2} k_B T_-,$$  \hspace{1cm} (B4)

where $\zeta(z)$ is the Riemann zeta-function.

b. aPf-2 and Pf-3

In treating the interfaces aPf-2 and Pf-3, we notice that they are constructed by the same set of modes but in opposite directions. They are therefore expected to generate the same charge current noise and thus to have the same form of $\Lambda(x)$, also in absence of thermal equilibration. With use of Eqs. (31)-(32), the modes’ temperatures are given by $T_{\phi_1} = 0$, $T_{\phi_2} = T_\psi = T_- = \sqrt{\frac{2PE}{h\kappa_0}}$, we arrive at

$$\Lambda_{\text{aPf-2}} = \Lambda_{\text{Pf-3}}^{\Delta V} \approx 1.604 k_B T_-.$$  \hspace{1cm} (B5)

c. phPf-2

For the phPf-2 interface, we take into account an additional pair of integer modes which are not thermally equilibrated (see the discussion in Sec. IV A 2 and Fig. 8). The charge transport equation (3) then takes the form

$$\partial_x \hat{V}(x) = \frac{\delta_{12}}{L} \begin{pmatrix} -\chi_1/\nu_1 & \chi_1/\nu_1 & 0 \\ \chi_2/\nu_2 & -\chi_2/\nu_2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} V_1(x) \\ V_2(x) \\ V_3(x) \end{pmatrix} + \frac{\delta_{23}}{L} \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\chi_2/\nu_2 & \chi_2/\nu_2 \\ 0 & \chi_3/\nu_3 & -\chi_3/\nu_3 \end{pmatrix} \begin{pmatrix} V_1(x) \\ V_2(x) \\ V_3(x) \end{pmatrix},$$

(B6)

with $\nu_1 = \nu_2 = 1$, $\nu_3 = 1/2$ and $\chi_1 = -\chi_2 = \chi_3 = 1$. The boundary conditions are

$$V_1(0) = \Delta V, \quad V_2(1) = 0 \quad \text{and} \quad V_3(0) = \Delta V.$$  \hspace{1cm} (B7)

To ensure a quantized charge conductance, we choose $\delta_{12} \gg \delta_{23}$. In general it is not possible to give accurate estimates on $\delta_{12}$ and $\delta_{23}$ without further experimental data. For our purpose it is sufficient to adjust them such that $G_{2T}/(e^2/h) = 1/2$ to good accuracy. Within this description, charge partitioning occurs mainly within the region $0 \lesssim x \lesssim \delta_{12}$. Furthermore, the Joule heating contribution is non-zero and leads to a total dissipated power of

$$P_0 = \frac{3}{\pi^2} \frac{e^2 V_0^2}{h\kappa_0} \int_0^{\delta_{12}} \left[ \delta_{12}(V_1 - V_2)^2 + \delta_{23}(V_2 - V_3)^2 \right] dx,$$  \hspace{1cm} (B8)

where we omitted the $x$-dependence of $V_i$ for notational ease. The process of interest for the noise characteristics on the phPf-2 edge is the charge tunneling between the bosonic channels of filling $\nu_1 = 1$ and the electronic mode comprised of $\nu_3 = 1/2$ and the MM $\psi$. Then, the noise kernel for absent thermal equilibration is the same as in the phPf-3 case with the temperature $T_- = \sqrt{\frac{2PE}{h\kappa_0}}$. Following the same steps as before, the noise is computed vs the charge equilibration length $\delta_{23}$, giving a noise of similar magnitude as for absent thermal equilibration on the phPf-3 edge for a ratio $3 \lesssim \delta_{12}/\delta_{23} \lesssim 5$. The deviation from the expected $G_{2T}$, is less than 0.5% if $\delta_{23} \gtrsim 8$ which is what we assume here.

2. Downstream delta-T noise for absent thermal equilibration

For this type of noise, the appropriate boundary conditions are given in Eq. (18). We consider the limiting cases of a weak bias (wb), $\Delta T \ll T$ and strong bias (sb), $\Delta T \gg T$ by expanding the expressions to first order in $\Delta T/T$ and setting $T = 0$, respectively. We refer to the corresponding noise kernels as $\Lambda^{(\text{wb})}(x)$ and $\Lambda^{(\text{sb})}(x)$, respectively. Following the same steps as in Appendix A, using a shift of variables such that the mode with the largest temperature, say $T_m$ and thus smallest $\beta_m \equiv \left[ \frac{\hbar V_0}{T_m + \Delta T} \right]^{-1}$ fulfills $\hbar \nu_m \beta_m > b$ and performing the integrals, we arrive at our desired expressions. In
case of weak applied bias $\bar{T} \gg \Delta T$, we find to first order in $\Delta T$

\begin{align}
\Lambda_{\text{phPf-3}}^{(\text{wb})}(x) &= 2k_B\bar{T} + k_B\Delta T, \\
\Lambda_{\text{alPf-2}}^{(\text{wb})}(x) &= 2k_B\bar{T} + \frac{k_B}{2}\Delta T, \\
\Lambda_{\text{phPf-2}}^{(\text{wb})}(x) &= 2k_B\bar{T} + \frac{6k_B}{5}\Delta T,
\end{align}

which reduce to the equilibrium NJ form for vanishing applied bias $\Delta T$ as expected. For large bias $\Delta T \gg \bar{T}$ we obtain instead

\begin{align}
\Lambda_{\text{phPf-3}}^{(\text{sb})}(x) &= \frac{12\zeta(3)}{\pi^2}k_B\Delta T, \\
\Lambda_{\text{alPf-2}}^{(\text{sb})}(x) &= \frac{9\zeta(3)}{\pi^2}k_B\Delta T, \\
\Lambda_{\text{phPf-2}}^{(\text{sb})}(x) &= \frac{17\pi^4}{60(\pi^2 \log(4) + 3\zeta(3))}k_B\Delta T.
\end{align}

3. Upstream delta-$T$ noise for absent thermal equilibration

For the upstream delta-$T$ noise, we use the boundary conditions from Eq. (19). The noise kernels are computed in a similar manner as for the downstream delta-$T$ noise. To first order in $\Delta T$ we find for weak applied biases $\Delta T \ll \bar{T}$

\begin{align}
\Lambda_{\text{phPf-3}}^{(\text{wb})}(x) &= 2k_B\bar{T} + k_B\Delta T, \\
\Lambda_{\text{alPf-2}}^{(\text{wb})}(x) &= 2k_B\bar{T} + \frac{3k_B}{2}\Delta T, \\
\Lambda_{\text{phPf-2}}^{(\text{wb})}(x) &= 2k_B\bar{T} + \frac{4k_B}{5}\Delta T,
\end{align}

which reduce to equilibrium noise for $\Delta T \rightarrow 0$. For strong bias $\Delta T \gg \bar{T}$, we find

\begin{align}
\Lambda_{\text{phPf-3}}^{(\text{sb})}(x) &= \frac{12\zeta(3)}{\pi^2}k_B\Delta T, \\
\Lambda_{\text{alPf-2}}^{(\text{sb})}(x) &= \left( \log(4) + \frac{3\zeta(3)}{\pi^2} \right) k_B\Delta T, \\
\Lambda_{\text{phPf-2}}^{(\text{sb})}(x) &= \frac{\pi^4}{60\zeta(3)}k_B\Delta T.
\end{align}

4. Downstream-upstream delta-$T$ noise symmetry of phPf-3 at poor thermal equilibration

As depicted in Fig. 9, the most striking feature is that the downstream and upstream noise kernels are uniquely equal for the phPf-3 interface (blue curves in Fig. 9). This equality is clearly manifest also in the asymptotic expressions: compare (B9a) and (B11a), respectively (B10a) and (B12a).

Mathematically, this “downstream-upstream symmetry” follows from the fact that the exponent of the downstream and upstream sectors in the product of edge mode Green’s functions [see Eq. (B3)] are equal. More specifically, we have for the phPf-3 interface that

\begin{align}
G_{\phi_1}(\tau, \bar{T} + \Delta T)G_{\phi_2}(\tau, \bar{T})G_{\psi}(\tau, \bar{T} + \Delta T)
\propto G_{\phi_1}(\tau, \bar{T})G_{\phi_2}(\tau, \bar{T} + \Delta T)G_{\psi}(\tau, \bar{T}).
\end{align}

Here, the first and second line correspond to downstream and upstream bias conditions, respectively. Moreover, the proportionality factor in Eq. (B13), which includes powers of mode velocities and temperatures, crucially drops out when dividing the noise with the conductance to obtain $\Lambda$, see Eq. (17). Relations similar to (B13) do not hold for any other edge structure in Fig. 2. Our proposal to test this symmetry is presented in the end of Sec. VII.

5. Some integrals and their computation

In computing the noise kernel for a voltage biased phPf-3 interface, we face integrals

\begin{align}
S_{\text{loc}}^{\text{int}} &\propto \int_{-\infty}^{\infty} \text{sech}^2(z) \, dz, \\
g_{\text{loc}}^{\text{int}} &\propto \int_{-\infty}^{\infty} \text{sech}^2(z) \, dz.
\end{align}

Integrals of this kind can be solved by using Mittag-Leffler’s theorem [101], which amounts to expanding the hyperbolic functions as

\begin{align}
\text{sech}^2(z) &= -\sum_{k=0}^{\infty} \left[ \frac{1}{(z-A)^2} + \frac{1}{(z+A)^2} \right],
\end{align}

where $A = i\frac{3}{2}(2k + 1)$. Similar expansions can be found for other hyperbolic functions, e.g., sech$(z)$. Inserting the series expansion back into Eqs. (B14)-(B15), exchanging the order of integration and summation, we arrive at

\begin{align}
S_{\text{loc}}^{\text{int}} &\propto \frac{\zeta(3)}{\pi^2} \text{ and } g_{\text{loc}}^{\text{int}} \propto \frac{\pi}{6}.
\end{align}

For other integrals, complex contour integration and the residue theorem are more useful. For example, in the case of delta-$T$ noise at phPf-2 we encounter an integral

\begin{align}
S_{\text{loc}}^{\text{int}} &\propto \int_{-\infty}^{\infty} \frac{1}{\cosh(z)^3(\pi + 2iz)^2} \, dz.
\end{align}

Substituting $z \rightarrow 2\pi t$ and manipulating the resulting expression leads to

\begin{align}
S_{\text{loc}}^{\text{int}} &\propto \frac{1}{8\pi} \int_{-\infty}^{\infty} \frac{1}{\cosh(2\pi t)^3} \left( \frac{1}{z} - it \right)^2 \, dt.
\end{align}

The right-hand-side can now be written as a derivative

\begin{align}
S_{\text{loc}}^{\text{int}} &\propto -\frac{1}{8\pi} \int_{-\infty}^{\infty} \frac{\partial^2}{\partial a^2} \left( \ln(a - it) \cosh(2\pi t)^3 \right) \bigg|_{a = 1/4} \, dt \\
&\equiv -\frac{1}{8\pi} \frac{\partial^2}{\partial a^2} J(a) \bigg|_{a = 1/4},
\end{align}

where
We next write the function \( J(\alpha) \) as an integral along the closed rectangular contour \( \gamma(z) \), defined by

\[
-R \to R \to R + i \to -R + i \to -R, \quad R \in \mathbb{R}^+, \quad (B21)
\]

in the complex \( z \)-plane as

\[
J(\alpha) = -\int_{\gamma(z)} \ln (\Gamma(a - iz)) \cosh(2\pi z)^3 \, dz
\]

\[
= -2\pi i \sum_i \text{Res} \left( \frac{\ln (\Gamma(a - iz))}{\cosh(2\pi z)^3}, z_i \right),
\]

where we used the residue theorem in the final step with the two third order poles \( z_1 = i/4 \) and \( z_2 = 3i/4 \) enclosed by \( \gamma(z) \). After some additional algebraic manipulations, we arrive at

\[
J(\alpha) = \frac{1}{8\pi^2} \left[ \psi^{(1)}(a + 3/4) - \psi^{(1)}(a + 1/4) \right]
\]

\[
+ \frac{1}{2} \ln \left( \Gamma(a + 3/4) \right) - \ln \left( \Gamma(a + 1/4) \right),
\]

where \( \psi^{(1)}(z) = \partial_z^2 \ln \Gamma(z) \) is the trigamma function. Combining Eqs. (B20) and (B23) gives

\[
\frac{1}{8\pi} \int_{-\infty}^{\infty} \frac{1}{\cosh(2\pi t)^3(a - it)^2} dt = \frac{17\pi}{480}
\]

The same method can be used to produce the following identity

\[
\int_{-\infty}^{\infty} \frac{1}{\cosh(2\pi t)^3(a - it)^n} dt = \frac{(-1)^{n-1}}{(n-1)!} \frac{d^n J(\alpha)}{d\alpha^n}, \quad n \in \mathbb{Z}^+.
\]

**Appendix C: Proof of the universality of \( G_N^Q \) for equal thermal equilibration on top and bottom edges**

Here, we prove Eq. (27), namely that the thermal Hall conductance (26) is universal as long as the two edges have the same degree of equilibration (even if it is poor).

We consider a generic edge structure with \( k \) downstream (ds) and \( N-k \) upstream (us) modes, and write the heat transport equation (6) (with \( \delta V = 0 \), since we assume no voltage bias) as

\[
\partial_x \vec{\theta}(x) = \mathcal{M}_T \vec{\theta}(x).
\]

(C1)

Here, we defined \( \vec{\theta}(x) = \vec{T}^2(x) \), and the matrix \( \mathcal{M}_T \) (7) satisfies the heat current conservation law

\[
\sum_j (\mathcal{M}_T)_{ij} = 0, \quad \forall i.
\]

(C2)

The general solution of Eq. (C1) for an edge segment with length \( L > 0 \) can be written as

\[
\vec{\theta}(L) = e^{LM_T} \vec{\theta}(0),
\]

which we express in ds and us blockform as

\[
\left( \begin{array}{c}
\vec{\theta}_{ds}(L) \\
\vec{\theta}_{us}(L)
\end{array} \right) = \left( \begin{array}{c|c}
A & B \\
\hline \\
C & D
\end{array} \right) \left( \begin{array}{c}
\vec{\theta}_{ds}(0) \\
\vec{\theta}_{us}(0)
\end{array} \right).
\]

(C4)

Here, the block matrices \( A \in \mathbb{R}^{k \times k}, B \in \mathbb{R}^{k \times (N-k)}, C \in \mathbb{R}^{(N-k) \times k}, \) and \( D \in \mathbb{R}^{(N-k) \times (N-k)} \). In terms of these matrices, the heat conservation law (C2) translates to

\[
\sum_j A_{ij} + \sum_j B_{ij} = 1, \quad \forall i,
\]

(C5)

\[
\sum_j C_{ij} + \sum_j D_{ij} = 1, \quad \forall i.
\]

(C6)

Next, we rearrange the terms in Eq. (C4) as

\[
\begin{pmatrix}
\vec{\theta}_{ds}(L) \\
\vec{\theta}_{us}(0)
\end{pmatrix} = \begin{pmatrix}
A - BD^{-1}C & BD^{-1} \\
-\bar{D}^{-1}C & \bar{D}^{-1}
\end{pmatrix}
\begin{pmatrix}
\vec{\theta}_{ds}(0) \\
\vec{\theta}_{us}(0)
\end{pmatrix}.
\]

(C7)

Let us now consider the top edge, for which the boundary conditions read \( \vec{\theta}_{ds}(0) = (\vec{T} + \Delta T)^2 \times (1, \ldots, 1)^T_k \) and \( \vec{\theta}_{us}(L) = \vec{T}^2 \times (1, \ldots, 1)^T_{N-k} \) (see Fig. 4). Plugging these quantitites into Eq. (C7), we write the top edge heat current (20) as

\[
\begin{align*}
J^{top}_Q &= \frac{k_0}{2} \sum_{i:x_i=+1} n_i \theta_{ds}^i(L) - \vec{T}^2 \sum_{i:x_i=-1} n_i \\
&= \frac{k_0}{2} \left( (\vec{T} + \Delta T)^2 \sum_{i:j:x_i=+1} n_i (A - BD^{-1}C)_{ij} \\
&+ \vec{T}^2 \sum_{i:j:x_i=-1} n_i (BD^{-1})_{ij} - \vec{T}^2 \sum_{i:x_i=-1} n_i \right).
\end{align*}
\]

(C8)

where in the second line we used Eq. (C7). For the bottom edge, we have reversed boundary conditions \( \vec{\theta}_{ds}(0) = \vec{T}^2 \times (1, \ldots, 1)^T_k \) and \( \vec{\theta}_{us}(L) = (\vec{T} + \Delta T)^2 \times (1, \ldots, 1)^T_{N-k} \). The bottom edge heat current (21) then reads

\[
\begin{align*}
J^{bot}_Q &= -\frac{k_0}{2} \left( \sum_{i:x_i=+1} n_i \theta_{ds}^i(L) - \vec{T}^2 \sum_{i:x_i=-1} n_i \\
&= \frac{k_0}{2} \left( \vec{T}^2 \sum_{i:j:x_i=-1} n_i (A - BD^{-1}C)_{ij} \\
&+ (\vec{T} + \Delta T)^2 \sum_{i:j:x_i=+1} n_i (BD^{-1})_{ij} - (\vec{T} + \Delta T)^2 \sum_{i:x_i=-1} n_i \right). \quad \text{(C9)}
\end{align*}
\]

The crucial next step is the assumption that the degrees of thermal equilibration on the top and bottom edges are identical. This translates to identical block matrices \( A, B, C, \) and \( D \) for the two edges. Then, and only then, can
we combine the two currents as
\[ J_{Q}^{\text{top}} - J_{Q}^{\text{bot}} = \frac{\kappa_0}{2} \left( (\bar{T} + \Delta T)^2 + \bar{T}^2 \right) \times \left( \sum_{i,j,\chi_i = +1} n_i (A - BD^{-1}C + BD^{-1})_{ij} - \sum_{i,j,\chi_i = -1} n_i \right). \]  
(C10)

Now, by using Eqs. (C5)-(C6), and identifying
\[ \sum_{i,j,\chi_i = +1} n_i = c, \]
(C11)
\[ \sum_{i,j,\chi_i = -1} n_i = \bar{c}, \]
(C12)

the dependence on \(A, B, C,\) and \(D\) cancels out, and we find that Eq. (C10) reduces to
\[ J_{Q}^{\text{top}} - J_{Q}^{\text{bot}} = \frac{\kappa_0}{2} \left( (\bar{T} + \Delta T)^2 + \bar{T}^2 \right) (c - \bar{c}). \]  
(C13)

Finally, inserting this expression into the definition of \(G_{ij}^Q,\) given in Eq. (26), gives our desired result (27). Our proof generalizes the theoretical analysis for \(\nu = 2/3\) in Ref. [37] to any edge structure.

**Appendix D: Computation of voltage and temperature profiles for the phPF-3 interface**

We compute the noise and thermal conductance of all considered edge structure by using voltage and temperature profiles of the edge channels within the incoherent tunneling model introduced in Sec. II. Below, we present in detail their derivation for the phPF-3 interface using boundary conditions for charge current as well as downstream delta-\(T\) noise. Other edge structures are treated in a perfectly analogous manner.

We begin by labelling the edge channels as 1, 2, 3 from top to bottom according to Fig. 2. Transport along the edge is characterized by the channel specific filling factors \(\nu_i,\) heat conductances \(n_i\) and chiralities \(\chi_i,\) given by

1: \(\nu_1 = 1\) \(\quad n_1 = 1\) \(\quad \chi_1 = +,\)
2: \(\nu_2 = 1/2\) \(\quad n_2 = 1\) \(\quad \chi_2 = -,\)
3: \(\nu_3 = 0\) \(\quad n_3 = 1/2\) \(\quad \chi_3 = +.\)

These values further specify the transport matrices \(\mathcal{M}_V\) and \(\mathcal{M}_T\) in Eqs. (4) and (7) as
\[ \mathcal{M}_T = \alpha \begin{pmatrix} -\chi_1 n_2 & \chi_1 n_2 & 0 \\ \chi_1 n_2 & -\chi_2 n_1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \beta \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\chi_2 n_3 & \chi_2 n_3 \\ 0 & \chi_3 n_2 & -\chi_3 n_2 \end{pmatrix} \]  
(D1)
and
\[ \mathcal{M}_V = \frac{1}{\ell_{\text{eq}}} \begin{pmatrix} -\chi_1 & \chi_1 & 0 \\ \chi_1 & -\chi_2 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \]  
(D2)

1. **Charge current noise, \(\Delta V \neq 0, \Delta T = 0\)**

Figure 13. Channel resolved voltage profiles \(V_1(\tilde{x})\) and \(V_2(\tilde{x})\) of the charge carrying channels at phPF-3 using \(\delta = 20.\)

For the charge current noise, the appropriate boundary conditions are
\[ T_i(0) = 0, \quad \text{for} \quad i \in \{1, 2, 3\} \quad (D3a) \]
\[ V_i(0) = \Delta V, \quad \text{for} \quad \chi_i = +1 \quad (D3b) \]
\[ V_i(1) = 0, \quad \text{for} \quad \chi_i = -1. \quad (D3c) \]

Here, we assumed that \(\epsilon \Delta V \gg k_B T\) so that we may set the base temperature \(\bar{T} \to 0\). The voltage profiles \(\bar{V}(\tilde{x})\) for the two charge carrying channels along the edge segment are obtained by solving Eq. (3). They are visualized in Fig. 13. Clearly the voltage drop occurs only on the right-hand-side of the edge segment. This is the hot spot. Note also that the voltage profiles are independent of the degree of thermal equilibration, since the processes leading to charge and thermal equilibration are considered to take place at different length scales. Knowledge of \(\bar{V}(\tilde{x})\) allows us to further compute the Joule-heating contribution in Eq. (8). For efficient thermal equilibration, we next solve Eq. (6) and obtain temperature profiles which depend on the pairwise degrees of thermal equilibration: \(\alpha = L/\ell_{\text{eq},12}, \beta = L/\ell_{\text{eq},23}\) [see Eq. (24)], as well as the charge equilibration parameter \(\delta = L/\ell_{\text{eq}}C\). Here, \(L\) denotes the edge segment length and we define here \(\tilde{x} = x/L\) as a rescaled coordinate along the segment. The channel resolved temperature profiles are visualized in Fig. 14(a) for two different sets of thermal equilibration lengths. We see that large \(\alpha, \beta\) (red curves) are needed to have all channels at similar temperatures at the noise spot (i.e., for \(\tilde{x} \ll 1\)) and thus to describe efficient thermal equilibration. This is contrasted by the blue curves for which \(\alpha\) is not large and the channels are at different temperatures at the noise spot. Absent thermal equilibration on the other hand corresponds to \(\ell_{\text{eq},ij} \to \infty\) and Eq. (6) simplifies to
\[ \partial_{\tilde{x}} \bar{T}^{2}(\tilde{x}) = \delta \bar{V}(\tilde{x}). \]  
(D4)

The channel resolved temperature profiles for this solution are given in Fig. 14(b).
2. Downstream delta-T noise, $\Delta V = 0$, $\Delta T \neq 0$

This type of noise corresponds to the boundary conditions

$$
V_i(0) = 0 = V_i(1), \quad \text{for } i \in \{1, 2, 3\} \quad (D5a)
$$
$$
T_i(0) = \bar{T} + \Delta T, \quad \text{for } \chi_i = + \quad (D5b)
$$
$$
T_i(1) = \bar{T}, \quad \text{for } \chi_i = - \quad (D5c)
$$

which leads to trivial voltage profiles $V_i(\tilde{x}) \equiv 0, \forall i$. As a consequence, the energy transport equation Eq. (6) simplifies to

$$
\partial_{\tilde{x}} \tilde{T}^2(\tilde{x}) = \mathcal{M}_T \tilde{T}^2(\tilde{x}). \quad (D6)
$$

For efficient thermal equilibration, this equation is solved to obtain temperature profiles in terms of $(\alpha, \beta)$. Here, we consider the strong temperature bias limit, $\Delta T \gg \bar{T}$, and consider the temperature profiles to leading order in $\Delta T / \bar{T}$. The resulting temperature profiles are shown in Fig. 14(c). In case of absent thermal equilibration, the energy transport equation reads

$$
\partial_{\tilde{x}} \tilde{T}^2(\tilde{x}) = 0 \quad (D7)
$$

Hence, the temperature profiles of the channels will be constant and follow immediately from the boundary conditions, see Fig. 14(d).
3. **Upstream delta-T noise, $\Delta V = 0$, $\Delta T \neq 0$**

For the upstream delta-T noise, we use the boundary conditions

\begin{align}
V_i(0) &= 0 = V_i(1), \quad \text{for } i \in \{1, 2, 3\} \quad (D8a) \\
T_i(0) &= \bar{T}, \quad \text{for } \chi_i = + \quad (D8b) \\
T_i(1) &= \bar{T} + \Delta T, \quad \text{for } \chi_i = -. \quad (D8c)
\end{align}

Hence, the voltage profiles are also trivial $V_i(\tilde{x}) \equiv 0, \forall i$ and we obtain the temperature profiles using Eq. (D6) for efficient and Eq. (D7) for absent thermal equilibration. The resulting temperature profiles are visualized in Fig. 14(e) and Fig. 14(f), respectively.

4. **Connection of temperature profiles to noise**

The phPF-3 interface has $\nu_Q > 0$ so that for full thermal equilibration, the heat transport is dominantly in the downstream direction [see Fig. 2]. Any upstream heat transport is exponentially suppressed in $L$ [60]. For the situation of voltage biased charge current noise, in the regime of efficient thermal equilibration, we find non-vanishing temperatures only in the region $\tilde{x} \approx 1$. This is seen from the red curves in Fig. 14(a). This characteristic region, called the hot spot, is a result of Joule-heating close to the right (most downstream) contact and the dominantly downstream thermal transport. We see that the effective temperature at the noise spot is very small: $T_{ns} \approx 0$, and we expect small noise as well $S \approx \exp(-\delta)$. Indeed, by inserting the temperature profiles in the noise kernel $\Lambda(x) = k_B T(\tilde{x})$ in Eq. (14) and integrating, we obtain exponentially suppressed noise $S \sim \exp(-\delta)$, plotted in Fig. 7(a).

For thermally equilibrated down- and upstream delta-T noise, the boundary conditions for an edge with $\nu_Q > 0$ imply equal and constant temperatures of all channels at the noise spot [see Fig. 14(c) and (e)]. This leads to a hot spot temperature $T_{ns} = \bar{T} + \Delta T$ and $T_{ns} = \bar{T}$ for down- and upstream delta-T noise respectively. The corresponding noise profiles are shown in Fig. 14(c). Absent thermal equilibration on the other hand leads to constant but different temperatures of the channels at the noise spot for the three cases. We explore the out-of-equilibrium situation in the noise spot by using the Green’s function method outlined in Sec. B 1 to compute noise kernels $\Lambda(\tilde{x})$ with the full temperature profiles in case of charge current noise. This leads to the noise plots in Fig. 7(b). For thermally non-equilibrated down- and upstream delta-T noise there is no additional source of heating along the edge. Following the same approach as for charge current noise in Sec. B 1, we find a modified NJ noise for down- and upstream delta-T noise, described by Eq. (38) and Eq. (41) respectively. Adjusting $\mathcal{M}_V$, $\mathcal{M}_T$ as well as the boundary conditions according to the transport properties of the involved channels, the temperature and noise profiles of the other interfaces in Fig. 7 are obtained following the same steps.

**Appendix E: Unit conversion without tears: from theorist to experimentalist units**

The voltage bias noise excess noise $S_{exc}^{exc}$ computed using our approach is expressed in units of $e^2 \Delta V/h$. Let us denote the noise in such units as $S_{AV}^{exc}$. To connect these units to more experimentally relevant units, we start by using the current-voltage relation for chiral edge transport

\[ I_0 = \nu e^2 \frac{\hbar}{h} \Delta V. \]  

(E1)

This relation holds for efficient charge equilibration, and is manifest experimentally by robust charge conductance quantization. By using this relation, we re-write the noise in terms of the source current $I_0$ with the notation

\[ S_{I_0}^{exc} = \frac{S_{AV}^{exc}}{\Delta V} \frac{I_0}{\nu} e^2. \]  

(E2)

For a voltage bias $\Delta V$ given in $\mu V$, the excess noise can then be written as

\[ S_{AV}^{exc} \approx 6.20492 \left( \frac{S_{AV}^{exc}}{\Delta V e^3/\hbar} \right) \cdot 10^{-30} \frac{A^2}{\mu V \text{ Hz}}. \]  

(E3)

Alternatively, for a bias current $I_0$ given in $nA$, we have

\[ S_{exc} \approx 1.602 \left( \frac{S_{AV}^{exc}}{\Delta V} \frac{I_0}{\nu} \right) \cdot 10^{-28} \frac{A^2}{n A \text{ Hz}}. \]  

(E4)

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