Commercial software engineering under the digital economy concept

L N Borisoglebskaya¹, E N Provotorova² and S M Sergeev³

¹ Orel State University, Engineering centre of digital environment technologies for integrated security: telecommunications, communications and energy efficiency, 95, Komsomolskaya st., 302026, Orel, Russian Federation
² Voronezh State Technical University, 14, Moskow ave., Voronezh, 394026, Russian Federation
³ Peter the Great St. Petersburg Polytechnic University, 29, Polytechnitcheskaya st., Saint-Petersburg, 199000, Russian Federation

E-mail: sergeev2@yandex.ru

Abstract. The counterparties’ economic relations have been mainly transferred to the online sector. This reflects the concept of digital interaction between manufacturers, intermediaries and consumers. At the software engineering stage, it is necessary to use one of the main formalisms - sales funnel, which has transformed into a conversion funnel under the concept of a digital economy. In the presented article, a mathematical model is developed that formalizes in detail all the stages of commercial interaction. The research results provide a set of software engineering algorithms for the expert systems designed for commercial services managers.

1. Introduction
The process of the commercial service must be investigated in dynamics. The CRM (Customer Relationship Management) systems are used to assess the economic indicators. The sales funnel had been one of the most convenient for many years. It is based that for business owner, a sales funnel is probably the most important marketing tool. In recent years, sales technology has changed significantly, primarily due to the growth of the online segment, e-commerce. Meanwhile, the structure of the sales funnel has changed. The most important difference is the ability to obtain real-time [1] conversion data during the transition between stages. In the theory of sales, a new term Conversion funnel has appeared. This concept is derived from the area of e-commerce, the sale of products or services over the internet. It refers, for example, to a conversion of a visitor into a buyer.

2. Problem statement
The tasks of forecasting, optimization of trading activities, expert systems require a strictly formalized description of the processes when customers pass through the stages of commercial activities [2]. A two-dimensional representation of the sales funnel, common in the literature, gives a purely schematic representation of the processes. Usually the movement from leads to cash is formally [3] described. Also, depending on the business segment, traditions and legislation, the stages may differ slightly, but the general principle remains. If we compare the main stages of interaction with consumers in the conditions of offline trade and e-commerce [4], we can reflect them in table 1.
Table 1. Stages of interaction with consumers

| Stage     | Sales funnel                  | Conversion funnel                                                                 |
|-----------|-------------------------------|-----------------------------------------------------------------------------------|
| Awareness | Prospecting. Unqualified leads| Landing page, targeted page, blog, posts                                           |
| Discovery | Qualifying leads, Initial meeting, define prospects needs | Content, marketing, inbound marketing, online ads.                               |
| Evaluation| Make an offer                  | SEO search engine optimization, SEM Search Engine Marketing viral campaigns, social media, IFO (Irresistible Free Offer) |
| Intent    | Negotiation, finalize proposal | Site registration, subscription options of supply                                  |
| Purchase  | Closing the deal               | System for making orders Online payment                                           |
| Loyalty   | Deliver the product            | Online order tracking, deliveries and invoicing.                                    |

With conversions, the conversion rate can be measured. This is the percentage of all visitors who have eventually converted. The success of marketing activities is measured using the conversion rate of online shops. It should be noted that the conversion funnel reflects primarily not the behavior of the audience (consumers), but the technology of the marketer's (or seller).

The task of this work is to develop a mathematical model [5] of conversion funnel processes in the conditions of market uncertainty. The result is an algorithm for predicting economic [6] indicators of commercial activity.

3. The main formalisms

In the area from \(x_1\) to \(x_2\) at some point in time \(t\), there can be written the following:

\[
W = -\frac{Q(x_2,t) - Q(x_1,t)}{x_2 - x_1}.
\]

Moving [6] on to the limit: 

\[
\lim_{x_2 \to x_1} W = -\left. \frac{\partial Q}{\partial x} \right|_{x_1}.
\]

In that \(W\) is seen as value proportional to bounce rate during the progression on the axis \(X\). Accordingly, the bigger \(W\) - the steeper conversion funnel narrows. At the entrance the \(-\left. \frac{\partial Q}{\partial x} \right|_{x_1}\Delta t\) clients goes through \(x_1\) at considering area for the time \(\Delta t\), and at the exit from \(x_2\) the \(-\left. \frac{\partial Q}{\partial x} \right|_{x_2}\Delta t\) clients goes out. Thus, total loss on the area from \(x_1\) to \(x_2\) for the time \(\Delta t\) equals:

\[
(W(x_1) - W(x_2))\Delta t = -\left. \frac{\partial Q}{\partial x} \right|_{x_1} \Delta t - \left( -\left. \frac{\partial Q}{\partial x} \right|_{x_2} \Delta t \right).
\]

Now, there is indicator \(g(x)\), (grade of service) which represents work effectiveness of commercial service on stage \(x\). In that the professional qualities of enterprise managers are considered. The function \(g(x)\) - is integral effectiveness indicator [7] of the organization inside according unit which is in contact with client on this stage. The higher \(g(x)\) - the more organized is work of the unit. Conversion rate is calculated via formula:

\[
g(x)^{-1}\Delta t(Q(x,t_2) - Q(x,t_1)) = g(x)^{-1} \Delta x\Delta Q.
\]

Moreover, the work of the commercial service can be influenced by the external market environment. For example, the withdrawal of some customers to competitors as a result of aggressive policies, changes in legislation (for example, opening a foreign market or vice versa sanctions), conditions (inflation, crisis, etc.). Also, random factors [8] influence the client’s decision (for example, a change of fashion, the refusal of a transaction due to too large time delays).
It is required to foresee possibility of accounting this influence. The functional dependency \( D(x,t) \) (Disbenefits) is necessary [9] to be introduced. This function shows the level of influence of market uncertainty on the bounce rate of customers over the past one stage. The total quantity of loss equals:

\[
\frac{\partial Q}{\partial x} \Delta t - \left( \frac{\partial Q}{\partial x} \right)_{x=x_2} \Delta t = g(x)^{-1} \Delta x \Delta Q - g(x)^{-1} D(x) \Delta x \Delta t.
\]

Remarking, the separation of stages along the axis \( x \) implies finding customers at each stage in the scope of one service activity. Meanwhile, all employees have the same authority, qualifications, use the same type of hardware and software. Inside of the stage number \( n \) value \( g(x) \) is constant and equal to \( g(x) = g_n \). Writing again the equation [10] there is one for the stage \( n \):

\[
\frac{\partial Q}{\partial t} \Delta t - \left( \frac{\partial Q}{\partial t} \right)_{t=t_1} \Delta t = g_n^{-1} \Delta x (\Delta Q - D(x,t) \Delta t) .
\]

Taking out \( \Delta t \) and as \( \Delta x = x_2 - x_1 \), the Lagrange theorem is applied. The result is differential [11] equation:

\[
g_n \frac{\partial^2 Q}{\partial t^2} + D(x,t) = \frac{\partial Q}{\partial t} .
\]

Now the type of dependencies will be \( g(x) \) and \( D(x,t) \) [12]. The methods of queuing theory are used to do that [13]. As noted above, at each stage managers have the same skill level. Denote their performance by \( \mu_n \), where \( n \) - the number of stage. There is introduced \( \lambda_n \) - customer flow density from the previous stage, \( m \) - quantity of managers working [14] with clients, \( D \) - maximum quantity of them waiting for transition to the next stage. Than to calculate \( g(x) \), the formula for determining the conversion rate is used in the following form:

\[
g_n = 1 - \frac{\lambda_n}{m! \mu_n} \sum_{k=0}^{m} \frac{\lambda_n^k}{k! \mu_n^k} + \frac{\lambda_n^{m+1}}{m! \mu_n^{m+1}} \left( 1 - \left( \frac{\lambda_n}{m \mu_n} \right)^D \right)^{-1}.
\]

For modeling \( D(x,t) \) parameter \( v(t) \) is used as an indicator of the intensity of withdrawal of potential consumers due to the uncertainty of the market situation [15]. Then, denoting \( \alpha_n = \lambda_n / \mu_n \) follows [16] the next equation:

\[
D(x,t) = 1 - \frac{v(t)}{\lambda} \left[ \sum_{k=0}^{m} \frac{\alpha_n^k}{m! \mu_n^k} \sum_{i=0}^{\infty} \frac{\alpha_n^i}{m! \mu_n^i} \prod_{k=1}^{i} (m + k \frac{v(t)}{\mu_n}) \right]^{-1} \frac{\alpha_n^m}{m!} \sum_{i=0}^{\infty} \frac{\alpha_n^i}{m! \mu_n^i} \prod_{k=1}^{i} (m + k \frac{v(t)}{\mu_n})
\]

As a result, a complete set of formulas for a mathematical model [10] of processes in conversion funnel is obtained.

4. Application of results
For practical results, it is important to obtain expressions for calculating the amount of profit [17] from the activity for a certain period of time. In addition, the management of an enterprise can influence the values \( g_n \) through organizational and personnel interventions, as well as the implementing of various
technologies. Accordingly, it is necessary to obtain recommendations [18] for achieving the maximum profit for the planned period by choosing the optimal values \( g_n \).

As an example of the use of the proposed method, consider the link of conversion funnel with the widely used logistic curve economists. There is denoted \( Q_0(t) \) as the number of clients at the initial stage, value \( \Omega \) - customer pool capacity (potential customers). Then according to Verhulst [19] equation:

\[
\frac{dQ_0(t)}{dt} = \theta Q_0(t) \left( 1 - \frac{Q_0(t)}{\Omega} \right),
\]

where parameter \( \theta \) is a rate of change of preferences. The exact solution to the equation is the S-logistic [20] function:

\[
Q_0(t) = \frac{\Omega Q_0^0 e^{\theta t}}{\Omega + Q_0^0 (e^{\theta t} - 1)}.
\]

Remarkably, the starting value differs from zero, since there was an initial pool of pre-orders for this product. The first derivative gives an estimate of the intensity of the change in the number of clients entering the conversion funnel. It is calculated this way:

\[
Q_0'(t) = \frac{\theta \Omega Q_0^0 e^{\theta t} (\Omega - Q_0^0)}{[\Omega + Q_0^0 (e^{\theta t} - 1)]}.
\]

Since in reality there is discreteness, the Feigenbaum logistic map used. Sales funnels are right for businesses that rely on a high degree of prospect interaction and engagement to make sales or close deals [21]. Their sales process may be long and complex or they may be selling a high-ticket item that requires a lot of consideration by the customer. Both B2B and B2C businesses use sales funnels.

Conversion tracking is used both by web masters and affiliate networks to review performance. The latter use conversions made in the affiliate program as a basis for their payments to publisher. Creating a conversion funnel helps teams identify sales and marketing activities that will help to build trust and increase customer engagement. Once created these tasks can be built into CRM tools, helping teams, manage leads, stay on task, and prioritize their follow up efforts.

5. Conclusion

The interpretation of work results can be considered as a tool for the commercial service manager. Using the set of restored function values \( g(x) \) it is possible to evaluate the results of activities at all stages in dynamics, and make decisions about the redistribution of the load. A precise analysis of all relevant phases of the funnel can provide information about possible problems. Usability tests can also make weaknesses in the ordering process known. To increase the likelihood of purchase, various optimization approaches can be applied. The recognition of customers is according to this study of great value and can motivate them to buy. Through this, trust signals such as high usability and performance of the website can increase the chance of a conversion. Another equally important result is the ability to predict both the volume of secondary demand and related terms. Thus, investment planning will be made on the basis of a weighted analysis, which will allow assessing risks and provides an additional justification for the possibility of attracting credit funds.

References

[1] Prokhorov V V, Sergeev S M and Part A A 2019 Solvability of hyperbolic systems with distributed parameters on the graph in the weak formulation Vestnik of Saint Petersburg University Applied Mathematics Computer Science Control Processes 14(1) 107–17

[2] Prokhorov V V 2015 Boundary control of a parabolic system with distributed parameters on a graph in the class of summable functions Automation and Remote Control 76(2) 318-22

[3] Sergeev S, Kirillova T and Krasyuk I 2019 Modelling of sustainable development of megacities under limited resources E3S Web of Conferences 91 05007

[4] Podvalny S L and Prokhorov V V 2015 The questions of controllability of a parabolic systems with distributed parameters on the graph International Conference "Stability and Control Processes" in Memory of V.I. Zubov (SCP) 117-9
5

[5] Iliashenko O, Krasnov S and Sergeev S 2018 Calculation of high-rise construction limitations for non-resident housing fund in megacities E3S Web of Conferences 03006

[6] Borisoglebskaya L N, Provotorova E N and Sergeev S M 2019 Promotion based on digital interaction algorithm IOP Conf. Ser.: Mater. Sci. Eng. 537 042032

[7] Kamachkin A M and Yevstafyeva V V 2000 Oscillations in a relay control system at an external disturbance Control Applications of Optimization 2000 Proceedings of the 11th IFAC Workshop 2 459-62

[8] Provotorov V V, Ryazhskikh V I and Gnilitskaya Yu A 2017 Unique weak solvability of a nonlinear initial boundary value problem with distributed parameters in a netlike region. Vestnik of Saint Peterburg University Applied Mathematics Computer Science Control Processes 13(3) 264-77

[9] Provotorov V V 2008 Eigenfunctions of the Sturm-Liouville problem on astar graph Sbornik: Mathematics 199(10) 1523-45

[10] Borisoglebskaya L N, Provotorov V V, Sergeev S M and Kosinov E S 2019 Mathematical aspects of optimal control transference processes in spatial networks IOP Conf. Ser.: Mater. Sci. Eng. 537 042025

[11] Alexandrova I and Zhabko A 2018 A new LKF approach to stability analysis of linear systems with uncertain delays Automatica 91 173-8

[12] Kiseleva E, Artemova E, Litvinenko I, Kirillova T, Tupchienko V and Wang Bing 2017 Implementation of Innovative Management in the Actions of the Business Enterprise International Journal of Applied Business and Economic Research 15(13) 231-42

[13] Kravets O J, Podvalny E S and Barkalov S A 2015 Quality assessment of a multistage process in the case of continuous response functions from resource influences Automation and Remote Control 76(3) 500-6

[14] Krasyuk I A, Bakharev V V, Kozlova N A and Mirzoeva D D 2017 Staffing in the sphere of trade: the main issues and prospects of solution Proceedings of 2017 IEEE 6th Forum Strategic Partnership of Universities and Enterprises of Hi-Tech Branches (Science Education Innovations) 6 48-50

[15] Provotorov V V 2015 Boundary control of a parabolic system with delay and distributed parameters on the graph International Conference "Stability and Control Processes" in Memory of V.I. Zubov (SCP) 126-8

[16] Aleksandrov A, Zhabko A and Hu G-D 2014 Delay-independent stability conditions for some classes of nonlinear systems IEEE Transactions on Automatic Control 59(8) 2209-14

[17] Podvalny S L, Provotorov V V and Podvalny E S 2017 The controllability of parabolic systems with delay and distributed parameters on the graph Procedia Computer Science 12 324-30

[18] Yanenko M 2016 Cost-Based Brand Management International Business Management 10(26) 5991-5

[19] Krasyuk I A and Medvedeva Y Y 2018 Resource support in business analytics of innovative development of trade and technological systems IEEE Conference on Data Science: Challenges of Digital Transformation 482-8

[20] Karelin V V 2010 Penalty functions in the control problem of an observation process Vestnik of Saint Petersburg University Series 10 Applied mathematics Computer science Control processes 4 109-14

[21] Kapustina I V, Kirillova T V, Ilyina O V, Razzhivin O A and Smelov P A 2017 Features of Economic Costs of Trading Enterprise: Theory and Practice International Journal of Applied Business and Economic Research 15(11) 1-10