Modeling of quasiparticle-induced excitations of a Josephson charge-phase qubit

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Abstract

We have analyzed quasiparticle transitions in an Al charge-phase qubit inducing a dynamic change of the qubit states. The time-averaged mixed state is related to the strong coupling of the qubit to an ensemble of non-equilibrium quasiparticles in the leads. Such quasiparticles tunnel stochastically on and off the island and can excite the qubit. Continuous monitoring of the qubit impedance at a frequency of 80 MHz shows the admixture of the excited state. We present a numerical description of these cyclic transitions and compare it with our experimental data.

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Superconducting circuits, based on small Josephson junctions and tunneling of single Cooper pairs, are promising candidates for qubits [1]. The qubit operation relies on the coherent superposition of the macroscopic quantum states of the qubit circuits, but the incoherent tunneling of a single unpaired electron, i.e. a quasiparticle (QP), changes instantly and stochastically the even-parity to an odd-parity charge state in the system [2, 3] and thus shifts the bias point of the qubit. Moreover, it halts the coherent tunneling of Cooper pairs in a random telegraphic way, being a fundamental source of decoherence [4, 5] and hence setting limits to the qubit operation time.

Several experimental techniques have been applied to observe the even-odd states of superconducting circuits, basically either focusing on dc measurements of the superconducting branch of single-charge transistors [6, 7, 8], electrometry of the charge of Cooper pair boxes [9, 10, 11, 12], or by rf techniques with inductively-coupled resonant circuits, i.e. by the rf reflectrometry method imaging the effective Josephson-inductance of the device [13]. This rf reflectrometry method has been applied recently to investigate the kinetics of quasiparticle tunneling [14, 15, 16] and QP trapping in Cooper pair boxes [17].

In this paper we focus on the problem of single QPs entering the qubit island. By continuous monitoring of the qubit impedance at a frequency of 80 MHz, we exploit the quasiparticle-induced dynamic change of the qubit states. These quasiparticle transitions induce a time-averaged mixed qubit state related to the strong coupling of the qubit to an ensemble of non-equilibrium QPs in the leads. In a previous paper we reported this effect and showed, that these transitions obey a selection rule [18]. Here we focus on the kinetics of the cyclic quasiparticle transitions in our circuit and present a numerical modeling of these transitions.

In our experiment, we have investigated a Josephson charge-phase qubit [19, 20, 21, 22]. This device can be considered as a Cooper-pair box of SQUID configuration, i.e. a superconducting loop interrupted by two small-capacitance Josephson-junctions with a mesoscopic island in between. This island is capacitively coupled to a gate electrode, see Fig. 1 (a). The quantum states \(|n, q\rangle\) of our qubit system are associated with different Bloch bands of a particle in the periodic (Josephson) potential [23]. Here \(n\) is the band number and \(q\) the quasicharge governed by the gate voltage \(V_G\), i.e. \(q = C_G V_G\), where \(C_G\) is the gate capacitance, see, e.g., Ref. [24]. The quantum states of the transistor also involve the phase coordinate \(\varphi_{dc}\) set by the external magnetic flux \(\Phi_{dc}\) applied to the SQUID loop. The qubit Josephson
inductance $L_J$ (being much larger than the inductance $L$ of the loop) is related to the local curvature of the corresponding energy surface $E_n(q, \varphi)$, i.e. $1/L_J(n, q, \varphi) \propto \partial^2 E_n(q, \varphi)/\partial \varphi^2$, where the integer values $n = 0$ and 1 correspond to the ground and excited state, respectively.

Our Josephson qubit is inductively coupled to an rf-driven tank circuit. The qubit eigenstates can be identified by means of the Josephson inductance of the transistor which is probed by small rf oscillations $I_{RF}$ in the loop with the resulting phase $\varphi(t) = \varphi_{dc} + a \sin(2\pi f t)$. Here, $a$ is proportional to the amplitude of the rf oscillations in the tank circuit induced by an rf driving current of frequency $f$, close to the bare resonance frequency $f_0 \approx 77$ MHz of the tank circuit. Due to the coupling to the qubit, the effective inductance $L_{eff}$ of the circuit is equal to $L_T - M^2 L_J^{-1}(n, q, \varphi)$, with the geometrical inductance $L_T \approx 150$ nH of the resonant circuit, with the mutual inductance $M \approx 3.8$ nH, and with $k \approx 0.4$ being the coupling coefficient between the superconducting qubit loop and the resonant circuit (for details, see Ref. [18]). The resulting shift $\Delta f$ of the resonant frequency is $\Delta f/f_0 \propto -1/L_J(q, \varphi)$.

The investigated sample has been fabricated in two steps: First, the tank circuit inductor was fabricated on the basis of Nb technology, including e-beam lithography, dry etching, anodization and chemical-mechanical polishing, see Refs. [22, 25] for details. In a second step, the qubit loop and the Bloch transistor were co-fabricated on the same chip by means of the two-angle Al shadow evaporation technique. In such an rf-SQUID configuration, the Bloch transistor is galvanically decoupled from the measuring circuit, which in general leads to a reduction in the density of non-equilibrium QPs which are able to enter the island. Besides this, no further precautions for the suppression of QP poisoning of the island - such as, for example, the engineering of a barrier-like gap profile having the island gap value greater than the electrode gap value [26, 27], or the implementation of normal-metal QP traps [7] in the outer electrodes - were taken. The critical currents of the individual junctions of the transistor were approx. 25 nA, with the corresponding value of 45 $\mu$eV for the average Josephson coupling energy $E_{J0}$. The charging energy $E_C$ of the transistor island has a value of 110 $\mu$eV, such that both energies $E_C$ and $E_{J0}$ are smaller than the value of the superconducting gap in Al films, $\Delta_{Al} \approx 210$ $\mu$eV. These values of $E_{J0}$ and $E_C$ were taken from a fitting of the shape of the ground state extracted from rf measurements with a finite amplitude of the Josephson phase oscillations, see Ref. [18] for details.
FIG. 1: (a) Sketch of the charge-phase qubit with inductive readout. The core element is a double Josephson junction with a capacitive gate coupled to its island, i.e. the Bloch transistor, embedded in a macroscopic superconducting loop. The loop is inductively coupled to an rf-driven tank circuit which itself is capacitively coupled to a cold preamplifier, see Ref. [18]. The inset shows a scanning electron micrograph of the transistor island, fabricated in the Al shadow evaporation technique. (b) Experimental gate modulation dependencies $\alpha(q)$ for an amplitude of rf oscillations $a = 0.28$ (top curve) and $a = 0.84$ (bottom curve). The range of gate voltage shown by dashed-line boxes corresponds to the mixing of ground and excited states.

In our experiment, which was carried out in a dilution refrigerator at a base temperature of 20 mK, we measure the phase angle $\alpha$ between the driving signal $I_{rf}$ and the rf voltage $V_{rf}$ on the tank circuit instead of the frequency detuning $\Delta f$. From the $\alpha$-dependence one can deduce the Josephson-inductance $L_J(q, \varphi)$ by applying the simple formula [22]:

$$\tan \alpha = k^2 Q \frac{L}{L_J(n, q, \varphi)}.$$  \hspace{1cm} (1)

The measurement of the phase shift, $\alpha$ as a function of $\varphi_{dc}$ and $q$ allows the curvature of the energy surface to be mapped, showing a periodical dependence of $\alpha$ both on $\varphi_{dc}$ with a period of $2\pi$ and $q \propto V_G$ with a $2e$-periodic gate modulation. In this paper, we focus on the measurements carried out at $\varphi_{dc} = \pi$ that correspond to the minimum qubit energy (in the degeneracy point $q = e$ equal to the difference of the Josephson coupling energies of the individual junctions). In this case, the effect of the quasiparticle induced transitions, discussed in the following, is strongest. These transitions manifest themselves in an overshooting behavior of the gate dependence curve measured at $\varphi_{dc} = \pi$. This peculiar
shape is formed by two types of arcs, see Fig. 1 (b). The obtuse arcs, for example, (between the degeneracy points at \( q = -e \) and \( q = e \)) are interrupted by the acute arcs centered at the degeneracy points, i.e. the phase angle \( \alpha \) starts to rise sharply and remains in a broad range around \( q = \pm e \) at a level that is even higher than that for \( q = 0 \) (indicated by the dotted lines in Fig. 1 (b)). When increasing the amplitude \( a \) of the rf oscillations of the Josephson phase, the Josephson-inductance, i.e. the second derivative of the energy surface, is probed over a larger region of \( \varphi \). Thus, the gate curve is averaged over states with different \( E_J(\varphi) = \sqrt{E_{J1}^2 + E_{J2}^2 + 2E_{J1}E_{J2}\cos \varphi} \) for fixed \( q \) with \( E_{J1,2} \) being the Josephson energies of the individual tunneling junctions. As a consequence, the overshooting is reduced due to the increased effective splitting between ground-state and excited state.

We explain this overshooting behavior by a statistical mixture of the different quantum states of the qubit ground and excited state that are characterized by an opposite curvature of the corresponding energy bands [19]. Here, at the degeneracy point (\( q = e \) and \( \varphi_{dc} = \pi \)), the states considered are the ground state \( A \) and the excited state \( B \), both with even parity of the island. We can neglect the contribution of the ground state \( C \) with odd parity, as the admixture of the values of \( \alpha(q) \) corresponding to this state with small negative curvature cannot yield the above-mentioned overshooting of the experimental data in the vicinity of \( q = e \). Likewise, we are of the opinion that the contribution of the odd state is small due to the presumably short lifetime of a QP in the island, see, e.g., Refs. [5, 17]. Besides, we do not consider the corresponding odd-parity excited state (not marked here), since its excitation energy (\( \approx 3E_C \)) is too great, i.e. much larger than the energy gap between the states \( A \) and \( B \).

Explaining this mixture effect, we rule out the thermal activation of the excited state \( B \) at the given base temperature (less than 100 mK) and also the excitation as a result of the Landau-Zener (LZ) tunneling due to the rf drive which leads to a periodic passing of the degeneracy point (at \( \varphi_{dc} = \pi \) and \( q = e \)) [18]. This assumption is based on the smooth dependence of the observed effect on the detuning \( x = |q - e|/e \), whereas the energy splitting near the degeneracy point strongly depends on the parameter \( x \). Instead, we explain the observed mixed state by a strong coupling of non-equilibrium QPs to the qubit system, which allows the transfer of energy to the qubit and thus an excitation of its upper state. This QP-assisted pumping of the qubit can be considered as a cycle in which an unpaired non-equilibrium QP tunnels onto the island while the qubit is in the ground state \( A \). As
FIG. 2: QP-pumping mechanism: In the left panel, an unpaired non-equilibrium QP tunnels from the outer electrode onto the island of the qubit in the ground state. This corresponds to the transition $A \rightarrow C$. In the favored process (i), the QP tunnels back with a rate $\gamma_{CB}$ into a lower energetic state of the outer electrode and, hence, transfers the energy $\delta E_{BA}$ to the qubit by exciting it to the upper state. The alternative process (ii), i.e. the tunneling of a QP back to the outer electrode (with a rate $\gamma_{CA}$ without the qubit being excited is shown in the panel at the lower right. Due to the somewhat larger density of states for lower energies, the rate $\gamma_{CB}$ is larger than $\gamma_{CA}$.

soon as the QP enters the transistor island, it changes the excess charge and induces in this way an instantaneous change of the working point (from the even-parity states $A$ or $B$ to the odd-parity state $C$ with corresponding rates $\gamma_{AC}$ and of the energy level splitting, see Fig. 2). Only when the QP tunnels out to a lower energetic state of the electrode with a tunneling rate $\gamma_{CB}$, a transfer of energy to the qubit system with a value $E_{CB} = E_C - E_B$ occurs, exciting it to the upper state $B$, whereas a QP tunneling back to the initial state of the electrode with an energy difference $\delta E_{CA} = E_C - E_A$ and corresponding rate $\gamma_{CA}$ does not induce any excitation of the qubit. The former process should prevail due to the greater density of the states that are close to the edge of the QP band in the energy spectrum. Of course, such non-equilibrium QPs should have a kinetic energy which is larger at least by the value of $\delta E_{BA}$ than the value of the superconductor energy gap, in order to transfer energy
to the qubit. This would mean, however, that a QP having a lower kinetic energy could enter the island as well, but in that case the QP should leave the island without exciting the qubit. We assume that QPs having an energy lower than $\delta E_{BA}$ above the energy gap are available in the outer electrodes and that their relaxation to the gap edge occurs both via interaction with the lattice of the electrodes and via the traveling onto the island and back into the electrodes with simultaneous excitation of the qubit. Unfortunately, we are unable to draw conclusions about absolute values of quasiparticle tunneling rates, because in our technique we observe only the averaged values of the phase shift $\alpha$.

The QP transitions are described by the tunneling Hamiltonian, $H_T$, where the Josephson coupling term describing the tunneling of Cooper pairs is naturally included in the Hamiltonian of the qubit. Applying the Fermi Golden Rule for calculating the transition rates of the QP tunneling onto (or off) the island, we obtain, for example, for the transition $A \rightarrow C$, i.e. when the qubit is initially in the ground state and a single quasiparticle tunnels into the island:

$$
\gamma_{AC}(E_p) = 2\pi \sum_k |\langle A, p | H_T | C, k \rangle|^2 \delta(E_p + \delta E_{AC} - E_k),
$$

(2)

where the system state is described by the state of the qubit ($A$, $B$ or $C$) and of the QP with the energy $E_{p,k}$, where $\xi_{p,k}$ is the kinetic energy of the QPs with momentum $p, k$ in the lead (or on the transistor) and $E_{p,k} = \sqrt{\Delta^2 + \xi_{p,k}^2}$ the corresponding energy. As we have discussed in Ref. [18], the matrix element $|\langle A, p | H_T | C, k \rangle|^2$ is responsible for the interference effect occurring at phase values $\varphi_{dc} = 2\pi n$, $n = 0, 1, \ldots$, when QPs tunnel onto the island both as an electron-like particle and as a hole-like particle with different phases for the different trajectories. For these values of the Josephson phase a destructive interference takes place having a suppressing effect on the cyclic qubit excitation. Hence, a selection rule for the QP transitions between the ground state and the excited state for certain flux bias values can be deduced [18].

In this paper we address the problem of the energy spectrum of non-equilibrium QPs. Therefore, for our analysis we focus on the results of the measurements at the special flux bias $\varphi_{dc} = \pi$ (yielding the smallest energy splitting $\delta E_{BA}$), where the selection rule gives a negligible contribution, which cannot be resolved due to the finite value of the amplitude $a$ of the Josephson phase oscillations, see for more details Ref. [18]. Therefore, the matrix element is nearly energy-independent for $\varphi_{dc} = \pi$ and, as a result, one can formulate the
rates for the incoming tunneling events of the quasiparticles, with $f_{\text{dist}}(E_p)$ corresponding to the filling factor of the non-equilibrium quasiparticles:

\[
\gamma_{AC} \propto \int_{\Delta}^{\infty} dE_p \frac{E_p}{\sqrt{E_p^2 - \Delta^2}} \frac{E_p + \delta E_{AC}}{(E_p + \delta E_{AC})^2 - \Delta^2} f_{\text{dist}}(E_p), \tag{3}
\]

\[
\gamma_{BC} \propto \int_{\Delta}^{\infty} dE_p \frac{E_p}{\sqrt{E_p^2 - \Delta^2}} \frac{E_p + \delta E_{BC}}{(E_p + \delta E_{BC})^2 - \Delta^2} f_{\text{dist}}(E_p). \tag{4}
\]

This distribution function $f_{\text{dist}}$ reflects the occupation number of the QPs for different energy. The first term in the integrand of formulas (3) and (4) corresponds to the density of states in the electrodes and the second term to the density of states in the transistor island. For the tunneling out of the transistor island, the $\Theta$-function appears because there are no states to tunnel for a quasiparticle with energy lower than the threshold energy $E_k - (\Delta + \delta E_{CA, CB})$,

\[
\gamma_{CA} \propto \int_{\Delta}^{\infty} dE_k \frac{E_k}{\sqrt{E_k^2 - \Delta^2}} \frac{E_k + \delta E_{CA}}{(E_k + \delta E_{CA})^2 - \Delta^2} f_{\text{dist}}(E_k) \Theta (E_k - (\Delta + \delta E_{CA})), \tag{5}
\]

\[
\gamma_{CB} \propto \int_{\Delta}^{\infty} dE_k \frac{E_k}{\sqrt{E_k^2 - \Delta^2}} \frac{E_k + \delta E_{CB}}{(E_k + \delta E_{CB})^2 - \Delta^2} f_{\text{dist}}(E_k) \Theta (E_k - (\Delta + \delta E_{CB})). \tag{6}
\]

We calculate the steady-state values of the probability weights $w_{A,B}(q)$ from a system of rate equations

\[
\dot{w}_A = \gamma_R w_B + \gamma_{CA} w_C - \gamma_{AC} w_A, \tag{7}
\]

\[
\dot{w}_B = -\gamma_R w_B - \gamma_{BC} w_C + \gamma_{CB} w_C, \tag{8}
\]

\[
\dot{w}_C = -\gamma_{CB} w_C - \gamma_{CA} w_C + \gamma_{BC} w_B + \gamma_{AC} w_A, \tag{9}
\]

which allows us to estimate the ratio $\frac{w_B}{w_A} \approx \frac{\gamma_{AC} \gamma_{CB}}{\gamma_{CA} \gamma_{BC}}$. Here, we assume that the relaxation due to quasiparticle tunneling is dominant, i.e. the rate of relaxation due to other mechanisms, $\gamma_R = \gamma_{BA}$, can be neglected. As we shall see below, the admixture of the excited state is rather strong, which would be impossible if $\gamma_R$ played an important role. Such relaxation also includes the effect of environmental degrees of freedom (e.g., flux and gate control lines, external magnetic field, background charge, etc.).

Moreover, the occupation number $w_C$ is negligible, as discussed above. The filling factor function $f_{\text{dist}}$ is not necessarily normalized, but the corresponding prefactors cancel out in the ratio $w_B/w_A$.
With the tunneling rates calculated, we are able to model the peculiar experimental gate modulation (Fig. 3 (a)) in terms of occupation numbers of the ground state \( w_A \) and the excited state \( w_B \). The experimental values of these quantities are extracted from Eq. (11), fitted subsequently to the measured \( \alpha(q) \)-dependence and assume a statistical mixture of the states \( A \) and \( B \)

\[
L_j^{-1}(q, \pi) \rightarrow w_A(q)L_j^{-1}(0, q, \pi) + w_B(q)L_j^{-1}(1, q, \pi)
\]

(10)

(whereby the weight factors are non-negative occupation numbers which obey the relation \( w_A(q) + w_B(q) = 1 \)).

Hence, we are able to reconstruct the occupation numbers \( w_{A,B}(q) \) of the ground state and of the excited state, see Fig. 3 (b). As a result, we find at \( q = \pm e \) an increase in the occupation of the upper state up to \( w_A(e) \approx 0.46 \), which remains rather constant in a broad range around this degeneracy point. This observed steady-state population mirrors the competition between the rates of quasiparticle tunneling transitions.

As a result of the fitting procedure to the experimental gate dependence \( \alpha(q) \), we can extract some information about the spectrum of the non-equilibrium QPs in the electrodes. First, we assumed a Gaussian-like energy distribution \( f_{\text{dist}} \) (shown by the dashed line in Fig. 3 (c)), but such an approach could not recover the correct shape of neither the gate curve in Fig. 3 (a) nor the occupation number dependencies in Fig. 3 (b). A reasonable correspondence to the experimental data was achieved when using a spectral function \( f_{\text{dist}} \propto \sqrt{1 - (E_p/2.06\Delta)^2} \) with a very sharp cut-off for calculating the ratio \( w_B/w_A \) from the steady-state rate equations for the tunneling rates. The cut-off energy of \( 2.06\Delta_{\text{Al}} \approx 430 \) \( \mu \)eV agrees roughly with the turning-points \( q = 0.58e \), resp. \( q = 1.42e \) (marked by arrows) of the experimental gate curve in Fig. 3 (a), from which we can estimate the maximum possible energy transfer \( dE \) to the qubit. From the weak-coupling approximation yielding a parabolic shape of the energy bands \( (E \approx E_C(q/e)^2) \) for \( E \ll E_C \) we obtain \( dE = (1.42^2 - 0.58^2)E_C \approx 190 \) \( \mu \)eV, being of the order of \( \Delta \). The total energy \( E_p \) of a quasiparticle with respect to the Fermi level is therefore about \( 2\Delta \). One may speculate about the origin of the non-equilibrium QPs having such an energy spectrum with an almost equal distribution up to a sharp cut-off energy. This sharp cut-off energy of approx. 430 \( \mu \)eV cannot be explained by the filtering of our signal lines by ThermoCoax cables located close to the mixing chamber. These cables have a cut-off frequency of about 1 GHz and the monotonically increased damping at higher
frequencies \([28]\). Thus, the source of the found QP distribution is still unknown. On the other hand, previous works studying non-equilibrium effects in superconductors and their applications \([29, 30]\) have discussed mechanisms of QP relaxation and pointed out that the relaxation of high-energy QPs is a two-step process. In the first step, the most efficient mechanism of the relaxation of hot QPs is the emission of phonons via inelastic electron-phonon scattering, which enable themselves to break Cooper-pairs due to their short phonon wavelength and create secondary QPs. This happens fast with the electron-scattering time \(\tau_E\) as the characteristic time scale, e.g. \(\tau_E \propto 10^{-8}\) s \([31]\) for Al at the critical temperature. This avalanche stops at an average decay energy of \(2\Delta\). At this energy, the multiplication of quasiparticles due to the breaking of Cooper-pairs is forbidden for energies lower than \(2\Delta\) \([32]\) because of energy conservation principle. Instead, QP-QP scattering leads to a band of QP energies lying between \(\Delta\) and \(2\Delta\). In the second step, the decay to thermal equilibrium happens on much longer time scales (the so-called phonon bottleneck). Here, the energy is continually exchanged between the phonon and the QP bath, i.e. the QPs recombine slowly to Cooper pairs under the emission of thermal (long-wave) phonons. Consequently, such a two-step relaxation mechanism might induce the existence of the deduced energy distribution, as detected in our experiment.

One might utilize this effect for an alternative qubit design which contains electrodes with a lower gap energy than that of aluminium, e.g. titanium, and an Al island. Such a design is similar to the well-known band-gap engineering \((\Delta < \Delta_{\text{island}})\) \([26, 27]\) ensuring immediate escape of a QP from the island. In the case of a sufficiently low energy gap with respect to the energy of the qubit biased in the optimum point \(\varphi = 0\) and \(q = e\), i.e. \(\Delta < E_{J1} + E_{J2}\), the probability of QP-assisted excitation of the qubit is drastically reduced due to the low population of QPs. On the other hand, the process of poisoning the island without qubit excitation is suppressed due to the selection rule \([18]\). In such a regime of the qubit operation the unwanted QP tunneling may be substantially reduced.

In conclusion, we have analyzed the effect of a statistical mixing of qubit states related to the energy transfer between non-equilibrium quasiparticles and the qubit system. In our numerical analysis we have applied a rate-equation model to the quasiparticle-induced transitions. Thus, we were able to predict the statistical mixing ratio of the qubit states and to model the experimentally observed qubit gate dependence as a result of the qubit pumping. Since our set-up only permitted a continuous readout of the, therefore, averaged qubit
state, we were unable to provide absolute numbers for the rate of quasiparticle tunneling. On the basis of our simulations we can, nevertheless, deduce the energy distribution of the non-equilibrium quasiparticles having a sharp cut-off with an energy of roughly $2\Delta$.

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FIG. 3: (a) Gate dependence $\alpha(q)$ for a flux bias $\varphi_{dc} = \pi$; symbols: experimental curve, solid curve: calculated gate dependence with best-fit spectrum; dotted curve shows the calculated $\alpha(q)$-curve for a Gaussian-like spectrum. (b) Occupation probabilities of the ground state $w_A$ and excited state $w_B$, yielding the observed gate dependence. Symbols: reconstructed occupation numbers from the experimental gate dependence in (a). The solid curve is calculated using the best-fit spectrum of the non-equilibrium QPs (solid curve in (c)). The dotted curve is calculated assuming a Gaussian spectrum of the QPs (dotted curve in (c)). (c) Energy distribution function of the non-equilibrium quasiparticles used in the calculation of the QP transition rates. Solid curve: best-fit spectrum, dotted curve: Gaussian-like spectrum of the quasiparticles.