Revisiting pseudo-Dirac neutrinos

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Abstract

We study the pseudo-Dirac mixing of left and right-handed neutrinos in the case where the Majorana masses $M_L$ and $M_R$ are small when compared with the Dirac mass, $M_D$. The light Majorana masses could be generated by a non-renormalizable operator reflecting effects of new physics at some high energy scale. In this context, we obtain a simple model independent
closed bound for $M_D$. A phenomenologically consistent scenario is achieved with $M_L, M_R \simeq 10^{-7}$ eV and $M_D \simeq 10^{-5} - 10^{-4}$ eV. This precludes the possibility of positive mass searches in the planned future experiments like GENIUS or in tritium decay experiments. If on the other hand, GENIUS does observe a positive signal for a Majorana mass $\geq 10^{-3}$ eV, then with very little fine tuning of neutrino parameters, the scale of new physics could be in the TeV range, but pseudo-Dirac scenario in that case is excluded. We briefly discuss the constraints from cosmology when a fraction of the dark matter is composed of nearly degenerate neutrinos.

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I. INTRODUCTION

Measurements of the atmospheric neutrino fluxes by the Super-Kamiokande experiment [1] and of the solar neutrino fluxes by several experiments [2] have given a compelling experimental evidence for neutrino masses, mixing and oscillations. The recent results of the SNO experiment [3] favour the existence of neutrino oscillation among active flavours involving $\nu_e$ from the Sun. Upon inclusion of the LSND result [4], a simultaneous explanation of both the solar and atmospheric results in terms of oscillations would require the existence of at least one sterile neutrino which can oscillate with any of the active flavours. There are many analyses in the literature where various possible active-sterile neutrino oscillation patterns have been studied [5].

In most analyses, the atmospheric anomaly points for its solution towards large angle $\nu_\mu \to \nu_\tau$ or $\nu_\mu \to \nu_s$ oscillations, where $\nu_s$ denotes a sterile neutrino. Results obtained by CHOOZ reactor based $\bar{\nu}_e$ disappearance experiment [6] and later by PaloVerde [7] severely constrain $\nu_\mu \to \nu_e$ oscillations for neutrino mass scales relevant for atmospheric neutrinos. This is also in agreement with the flat spectrum observed for the atmospheric $e$-like events. In addition, an analysis of the neutral current data disfavours large transitions involving $\nu_e$ at the atmospheric scale [8]. Recently, the Super-Kamiokande collaboration argued that the oscillations between active-sterile flavours is disfavoured at $3\sigma$ level [9]. It should be mentioned, however, that this conclusion may depend on how one analyses the data, and it has been claimed that a maximal $\nu_\mu \to \nu_s$ oscillation solution to the atmospheric neutrino problem is not yet ruled out [10]. Furthermore, it has been argued that the study of neutral current events at Super-Kamiokande, combined with the information obtained from future long baseline experiments, might not even be sufficient to decide between active-active and active-sterile oscillation solutions [11].

The possible role of the active-sterile oscillations in explaining the solar neutrino problem has recently got new light from the first SNO results on the charged current rates. The pre-SNO situation was such that active-sterile large mixing angle (LMA) as well as low mass
(LOW) solutions were disfavoured whereas small mixing angle (SMA), vacuum (VAC) and Just-So solutions were well allowed [12]. Upon inclusion of the preliminary SNO results, within the two flavour analysis, it appears that only the VAC solution gives a good fit to the data with best fit point as $\Delta m^2_\odot = 1.4 \cdot 10^{-10} \text{eV}^2$ and $\tan^2 \theta_\odot = 0.38$ [13]. Alternatively, magnetic moment solutions to the solar anomaly are also feasible. Such solutions equally involve large active-sterile oscillations and are currently not ruled out [14].

It may of course be that the solar neutrino oscillations follow in reality a more complicated pattern than an effective two flavour scenario. The SNO and future experiments, especially those which are sensitive to both charged and neutral currents (Borexino and KamLAND), are believed to provide a crucial test of the existence of oscillations to sterile neutrinos of any form. On the other hand, Barger et al. [15] have recently argued that due to the poorly known value of the $^{8}$B flux normalization, even the forthcoming SNO neutral current measurement might not be sufficient to determine the sterile neutrino content in the solar neutrino flux.

Thus, given our current understanding and analyses of the neutrino data, large active-sterile oscillations may play some role in solving the solar and atmospheric neutrino anomaly, though it seems to be less probable than active-active solutions. Furthermore, a combined analyses of the neutrino data including the LSND result favours a $2 + 2$ spectrum which involves the possibility of large active-sterile oscillations either in the solar or atmospheric sector [16].

All of the above solutions require neutrinos to posses a small but non-vanishing mass. From the theoretical point of view, the seesaw mechanism [17] offers the simplest and the most natural explanation for small neutrino masses. In this mechanism, one assumes the existence of a large Majorana mass scale ($M_R$) for the right-handed neutrino ($\nu_R$), $M_R \gg M_D$ and $M_L$. Here, $M_D$ is a Dirac mass and $M_L$ is a Majorana mass of the left-handed neutrino ($\nu_L$), both of which occur in a general Dirac-Majorana mass Lagrangian for $\nu_L$. Upon diagonalization, the seesaw mass Lagrangian leads to two Majorana neutrinos, one with a very small mass ($\sim M_D^2/M_R$) and another one with a large mass ($\sim M_R$). Therefore,
the sterile neutrino in this scheme decouples from the low energy world and cannot play any role in the oscillation phenomena under discussion.

If, on the other hand, one assumes $M_D \gg M_R, M_L$, the situation is quite different. The resulting mass eigenstates have eigenvalues very close to each other, and they have opposite CP parities. Hence they can form a pseudo-Dirac neutrino \[18\]. There have been numerous suggestions in the literature for pseudo-Dirac neutrinos as solution to the neutrino anomalies, where the observed flavour suppression is due to a maximal or near to maximal mixing between an active and a sterile neutrino \[19\].

A relevant question in the pseudo-Dirac scenario is to explain the unorthodoxy in the hierarchy: $M_D \gg M_R$ which is necessary for sterile neutrinos to be light. In the standard model (SM) the Majorana masses $M_L$ and $M_R$ are non-existing due to the conservation of lepton number. Hence the origin of these mass terms goes beyond the SM and there could be many sources. One possibility is that the masses may be provided at the SM level by non-renormalizable effective operators of the type $L^2\phi^2/M$ and $\nu_R^2\phi'^2/M'$. Here $L = (\nu_L, l_L)$ is an ordinary lepton doublet, $\phi$ and $\phi'$ are Higgs fields, and $M$ and $M'$ are high mass scales derived from some beyond-the-SM theory. The masses $M$ and $M'$ are not necessarily connected with the vacuum expectation values of the Higgs fields $\phi$ and $\phi'$, so it is possible that both $M_L$ and $M_R$ are much smaller than $M_D$. In any case, it is known that in a viable model $M_L$ should be suppressed so that $M_D \gg M_L$. This is required to avoid a contradiction with the accurately determined $\rho$-parameter. It is conceivable to assume that a similar suppression also happens for $M_R$.

A subsequent question is to understand the smallness of $M_D$. A light Dirac mass can be either (i) due to a small Yukawa coupling in the mass term $\bar{\nu}_R \nu_L \phi$ or (ii) just like in the case of $M_L$ or $M_R$, a light $M_D$ could be generated by a non-renormalizable higher dimensional term \[20\]. Another possibility is realized in models with large extra spatial dimensions. In such theories, the Yukawa coupling of the term $\bar{\nu}_R \nu_L \phi$ may be suppressed as the right-handed neutrino can be most of the time in the bulk outside our four-dimensional brane \[21\]. In the following, we assume a small $M_D$ relevant for a pseudo-Dirac mixing without
addressing to its origin. We examine the mixing of $\nu_L$ and $\nu_R$ when the Dirac mass term dominates over the Majorana mass terms, i.e. $M_D \gg M_L, M_R$, and discuss the experimental and theoretical bounds one can obtain for the mass parameters. This is illustrated for the case of the electron neutrino.

Our paper is organised as follows. In the next Section, we give the basic formalism for pseudo-Dirac mixing and by a simple exercise we show that the effective electron neutrino mass as probed by neutrinoless double beta decay experiments is exactly $M_L$. In Section III, we set bounds for the masses, $M_L$ and $M_D$ and derive a closed bound for $M_D$. We also discuss the constraints from cosmology when some fraction of the dark matter is composed of nearly degenerate neutrinos. Finally, in Section IV, we conclude by summarising the main results of this paper.

II. THE PSEUDO-DIRAC SCENARIO

Let us consider the $2 \times 2$ Dirac-Majorana mass matrix in the $(\nu_L, \nu_L^C)$ basis of the form

$$\mathcal{M} = \begin{pmatrix} M_L & M_D \\ M_D & M_R \end{pmatrix},$$

and assume $M_D \gg M_L, M_R$. The mixing angle which diagonalises $\mathcal{M}$ is easily derived to be

$$\tan 2\theta = \frac{2M_D}{M_R - M_L}.$$

We get a pseudo-Dirac neutrino pair with mass eigenvalues

$$m_{\pm} = \delta \pm \bar{M},$$

where $\bar{M} = (M_L + M_R)/2$ and $\delta = \sqrt{(M_L - M_R)^2 + 4M_D^2}/2 \approx M_D$. For a nonzero $M_D$ and $M_L = M_R$, this system corresponds to a maximal interlevel mixing of $\pi/4$ between the Majorana pair. If $M_D > 0$ is assumed, the neutrino mass-squared difference is

$$\Delta m^2 = m_+^2 - m_-^2 = 4M_D \cdot M_L.$$
If $M_L \neq M_R$, i.e. when the mixing is not maximal, one has

$$\Delta m^2 \simeq 2M_D(M_L + M_R).$$

In the case of $M_D \gg M_L, M_R$, which we are interested in here, the Majorana mass parameters $M_L$ and $M_R$ do not contribute substantially to the kinematical masses $m_\pm$. As a result, the standard mass measurements based on particle decays are not sensitive to them but only probe the Dirac mass parameter $M_D$. The parameters $M_L$ and $M_R$ can be tested in processes where they have a dynamical role. The most important process for studying $M_L$ is the neutrinoless double beta ($0\nu\beta\beta$) decay. One can easily see that the effective neutrino mass $M_{\text{eff}}$ measured in $0\nu\beta\beta$ decay experiments is actually $M_L$ and it is independent of $M_D$ and $M_R$. The mass eigenstates, $\nu^\pm_L$, can be written in terms of the interaction states, $\nu_L$ and $\nu_C^L$, as

$$\nu^\pm_L = N_\pm[2M_D\nu_L + (M_R - M_L \pm 2\delta)\nu_C^L],$$

where

$$N_\pm = [2(M_R - M_L)^2 + 8M_D^2 \pm 4(M_R - M_L)\delta]^{-1/2}$$

are normalization factors. In the limit $M_D \gg M_L$ and $M_R$,

$$\nu^\pm_L \simeq N_\pm[2M_D\nu_L + (M_R - M_L \pm 2M_D)\nu_C^L];$$

$$N_\pm \simeq \frac{1}{2\sqrt{2}M_D}[1 \mp \epsilon]; \quad \epsilon = \frac{M_R - M_L}{4M_D} \ll 1.$$ (7)

Therefore, the active neutrino component $\nu_L$ in the mass eigenstates is given by the amplitude

$$\langle \nu_L|\nu^\pm_L \rangle = 2M_DN_\pm \simeq \frac{1}{\sqrt{2}}(1 \mp \epsilon),$$

implying

$$\nu_L = \frac{1}{\sqrt{2}}[(1 - \epsilon)\nu^+_L + (1 + \epsilon)\nu^-_L].$$ (9)
The effective electron neutrino mass as measured by 0νββ decay experiments is then given to be

\[ M_{\text{eff}} = \cos^2 \theta \eta_+ m_+ + \sin^2 \theta \eta_- m_- = \frac{1}{2} [(1 - \epsilon)^2 \eta_+ m_+ + (1 + \epsilon)^2 \eta_- m_-] \approx M_L , \quad (10) \]

where \( \eta_\pm = \pm 1 \) are the Majorana phases of the mass eigenstates. It is easy to check that without the assumption \( M_D \gg M_L, M_R \) one ends up with the exact result \( M_{\text{eff}} = M_L \).

### III. BOUNDS FOR \( M_L \) AND \( M_D \)

**A lower bound for \( M_D \).** From (4) and (10) a general result follows:

\[ M_{\text{eff}} \approx \frac{\Delta m^2}{2\beta M_D} , \quad (11) \]

where \( \beta \equiv M_R/M_L + 1 > 1 \). The most stringent experimental upper bound published by the Heidelberg-Moscow experiment in [22] implies \( M_{\text{eff}} \leq M_{\text{eff}}^{\text{exp}} = 0.2 \text{ eV} \) (more recently the experiment has quoted the limit 0.34 eV at 90 \% C. L. [23]). Thus, for a given \( \Delta m^2 \), to be consistent with 0νββ decay results, the Dirac mass \( M_D \) must obey the bound

\[ M_D \gtrsim \frac{\Delta m^2}{2\beta M_{\text{eff}}^{\text{exp}}} . \quad (12) \]

**A bound for \( M_L \) from unitarity.** A Majorana mass \( M_L \) of the left-handed neutrino reflects physics beyond SM. In its presence the SM should be considered as an effective theory. It should be replaced by a more fundamental theory at some high energy scale \( M_X \), where new physics should enter, since otherwise the processes induced by the Majorana mass term would spoil the unitarity. One can find an upper limit for \( M_X \), for example, by studying the high energy behavior of the lepton number violating reactions \( \nu \nu \rightarrow WW \) or \( ZZ \), which can occur because of the Majorana mass term. The amplitudes of these reactions increase as proportional to the center of mass energy, leading to a breakdown of the effective theory at high energies. It was recently shown [24] that the most stringent bound for \( M_X \) is obtained by considering the following linear combination of the zeroth partial wave...
amplitudes: \( a_0(\frac{1}{2}(\nu_+\nu_+ - \nu_-\nu_-)) \rightarrow \frac{1}{\sqrt{3}}(W^+W^+ + Z^0Z^0) \), where \( \nu_\pm \) are helicity components of the mass eigenstate neutrino \( \nu \) and the final state bosons are longitudinally polarized. This amplitude to obey unitarity, i.e. \( |a_0| \leq 1/2 \), requires [24]

\[
M_X \leq \frac{4\pi \langle \phi \rangle^2}{\sqrt{3}M_L} ,
\]

where \( \langle \phi \rangle = 174 \text{ GeV} \) is the vev of the ordinary Higgs boson. It should be stressed that the Majorana mass \( M_L \) appears in this formula, not the kinematical mass of the neutrino. At high energies, where neutrinos are ultra-relativistic, the kinematical mass of the neutrino is irrelevant.

The condition (13) can be used to set an upper limit for the Majorana mass \( M_L \). If new physics starts to operate at the Planck scale \( M_{pl} \simeq 1.2 \cdot 10^{19} \text{ GeV} \), then

\[
M_L \leq \frac{4\pi \langle \phi \rangle^2}{\sqrt{3}M_{pl}} \simeq 2 \cdot 10^{-5} \text{ eV} .
\]

The smaller the scale of the new physics, the less stringent is the bound. The \( 0\nu\beta\beta \) decay to be visible in the planned GENIUS experiment [25], i.e. \( M_L \gtrsim 10^{-3} \text{ eV} \), would require \( M_X \lesssim 10^{17} \text{ GeV} \).

A closed bound for \( M_D \). As was originally pointed out by Weinberg [26], Majorana masses for the left-handed neutrinos can be generated by higher dimensional operators of the form

\[
\mathcal{L}_5 = \frac{f_{\alpha\beta}}{M_X}(L_{i\alpha}^T C^{-1} L_{j\beta} \phi_k \phi_l \epsilon_{ik} \epsilon_{jl}) ,
\]

where \( i, j, k, l \) are \( SU(2)_L \) indices, \( \alpha, \beta \) are flavour indices, and \( M_X \) is the scale of new physics. This operator breaks the lepton number explicitly, and after spontaneous symmetry breaking it leads to the following Majorana mass (neglecting flavour mixing):

\[
M_L = f \frac{\langle \phi \rangle^2}{M_X} ,
\]

where \( f \approx O(1) \) is a numerical factor. With \( M_X \lesssim M_{pl} \) this implies

\[
M_L \gtrsim f \frac{\langle \phi \rangle^2}{M_{pl}} \simeq 3 \cdot 10^{-6} \text{ eV} \cdot f .
\]
Therefore, in this scheme we have

\[ 3 \cdot 10^{-6} \text{ eV} \cdot f \lesssim M_L \lesssim 0.2 \text{ eV} , \tag{18} \]

where the upper bound is due to the $0\nu\beta\beta$ decay results.

By using (11) one can infer from (18) the following closed bound for possible values of the Dirac mass $M_D$:

\[ \frac{\Delta m^2}{0.4\beta \text{ eV}} \lesssim M_D \lesssim \frac{\Delta m^2}{6 \cdot 10^{-6} f\beta \text{ eV}} . \tag{19} \]

Let us now turn to experimental numbers involving the electron neutrino. According to the analysis done in [13] for the solar neutrino problem (that takes into account the recent results of SNO on the $\nu_e$ charged current rate), the best fit values for pure vacuum solution ($\nu_e \leftrightarrow \nu_s$) are with $\Delta m^2 = 1.4 \cdot 10^{-10} \text{ eV}^2$ and $\tan^2 \theta = 0.38$. This does not correspond to a maximal mixing which is the case in the pseudo-Dirac scenario. However, as can be seen from the analysis [13], maximal mixing with $\theta = \pi/4$ is not completely ruled out even though it is less favoured. To illustrate the situation we set $\Delta m^2 = 1.4 \cdot 10^{-10} \text{ eV}^2$ and $\beta = 2$ as reference values which corresponds to maximal active-sterile mixing in the case $M_L = M_R$. With these values, (19) gives the numerical range

\[ 1.8 \cdot 10^{-10} \text{ eV} \lesssim M_D \lesssim 2.4 \cdot 10^{-5} \text{ eV}/f . \tag{20} \]

Comparison with (18) shows that for the small $\Delta m^2$ of the vacuum solution, the pseudo-Dirac requirement $M_R, M_L \ll M_D$ leads to a consistent picture only when $M_D$ is in the upper end of this range. If we take $f = 0.1$, then a possible situation could be, e.g., $M_L, M_R \simeq 10^{-7}$ eV and $M_D \simeq 10^{-5} - 10^{-4}$ eV. In any case, one can conclude that if the solar neutrino deficit is due to a pure sterile mixing, $M_L$ is necessarily so small that the $0\nu\beta\beta$ decay would stay outside the range that the upcoming GENIUS experiment would be able to probe. On the other hand, the kinematical determination of the electron neutrino mass in tritium decay [27] would also be extremely difficult because of the smallness of $M_D$. Nonetheless, the analysis
does predict a nonzero mass value from both these processes and hence the associated scale of new physics §.

**TeV scale physics.** It follows from (11) and (16), together with the requirement $M_L \ll M_D$, that with any natural values of $f$, the energy scale $M_X$ must be fairly close to the Planck scale $M_{pl}$. This can be illustrated with the following example. If we wanted to have new physics close the weak scale, e.g. in the TeV scale, it follows from (16) and the experimental limit $M_L \leq M_{\text{eff}} = 0.2$ eV that $f < 10^{-11}$, and further, the requirement $M_L \ll M_D$ to be satisfied, one must have $f < 10^{-16}$. In fact, if $f$ is $O(0.1)$, the feasible range for new energy scale is $M_X \gtrsim 10^{-2} M_{pl}$. With such high values of $M_X$ there is no hope to observe $M_L$ and $M_D$ at least in near future, as already mentioned.

A larger Majorana mass $M_L$ from TeV-scale new physics could be obtained in models where there are suitable additional scalars. Within the context of nonrenormalizable theories, this is feasible if we consider a higher dimension operator other than the one suggested in (15). To illustrate this, we consider the simplest extension to the SM with an extra scalar doublet, $\phi'$. In order to avoid the induced flavour changing neutral currents, we impose a discrete $Z_2$ symmetry for the field $\phi'$. In this case, the lowest possible higher dimensional operator, which can generate a Majorana mass, is of the type

$$\mathcal{L}_7 = \frac{f'}{M_X^3} (L\phi\phi')^2.$$  \hspace{1cm} (21)

A Majorana mass is obtained when the scalars get a vev:

$$M_L = \frac{f'}{M_X^3} (\langle \phi \rangle \langle \phi' \rangle)^2.$$  \hspace{1cm} (22)

If we choose $\langle \phi \rangle / \langle \phi' \rangle \approx 10$, and then set $\langle \phi \rangle \approx 100$ GeV and $M_X \sim 10 - 100$ TeV, we must require $f' \leq 10^{-4} - 0.1$ in order to satisfy the current limit $M_L \leq M_{\text{eff}} = 0.2$ eV.

But also in this model the condition $M_L, M_R \ll M_D$ is hard to realize if $\Delta m^2$ is as small as $10^{-10}$ eV which corresponds to the vacuum oscillation solution for the solar neutrino

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§The scale can be extracted depending on the specific nature of a model for $M_D$. 

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problem. For \( M_X = 10 \text{ TeV} \) (\( M_X = 100 \text{ TeV} \)), \( f' \) must have unnaturally small values, \( f' < 10^{-9} \) (\( f' < 10^{-6} \)).

From this example one can conclude that in the pseudo-Dirac scenario the scale of new physics could be very high and that \( M_L \) and \( M_D \) are outside laboratory detection at present and also for any future realistic experiments. Naturally, it follows that, if for example GENIUS observes a nonzero signal for the 0νββ decay, pseudo-Dirac scenario is very unlikely.

**Cosmological constraints.** Here, we discuss the constraints assuming that the new physics arises from an operator of the type \( \mathcal{L}_5 \) and is consistent with pseudo-Dirac scenario. In the context of cosmology, neutrinos being neutral can be ideal candidates for the hot dark matter. In the non-relativistic limit, the energy density is \( \rho_\nu = \sum_i m_{\nu_i} N_\nu \), where \( N_\nu \) is the number density and \( m_{\nu_i} \) are the mass values. In the context of four neutrino flavours, it is expected that there is at least a pair of nearly degenerate neutrinos. It is possible that the splitting between such nearly degenerate pairs could correspond to the solar sector. It is conceivable that the dark matter is composed of some fraction of such degenerate or nearly degenerate neutrinos with the splitting to be \( \sim \sqrt{\Delta m^2_{\odot}} \approx 10^{-4} - 10^{-5} \text{ eV} \); this value of the mass splitting in our case will be close to the Dirac mass. Therefore, for such quasi-degenerate masses \( m_\nu \approx M_D \), we can relate to the cosmological parameters as \[ \sum_\alpha M_D \approx 94 \Omega_\nu \text{ eV} \] \[ (23) \]

where \( \Omega_\nu \) is the neutrino density compared with the critical density, and \( \alpha \) runs from 1 to \( n_f \), where \( n_f \) is the number of flavors in thermal equilibrium. Using (4) and (10), we can rewrite (23) as

\[ \Delta m^2 \approx 94 \Omega_\nu \beta \frac{M_{\text{eff}}}{n_f} \text{ eV}. \] 

\[ (24) \]

The present allowed range is \( 0.003 < \Omega_\nu < 0.1 \) \[ 29 \]. This yields the lower limit

\[ \frac{0.1 n_f \Delta m^2}{\beta \text{ eV}} \lesssim M_{\text{eff}}. \] 

\[ (25) \]
Comparing this with the lower limit for $M_{\text{eff}}$ in (17), which was obtained by requiring that the scale $M_X \leq M_{\text{pl}}$, one notices that the bound obtained from cosmology is more stringent only if

$$\Delta m^2 n_f \gtrsim 4.7 \cdot 10^{-5} f \beta \text{ eV}^2.$$  

(26)

This is not in accordance with the vacuum oscillation solution of the solar deficit problem which requires $\Delta m^2 \sim 10^{-10} \text{ eV}^2$. Therefore we conclude that in the limit of the dark matter being composed of some fraction of degenerate neutrinos, cosmology does not give more stringent bounds on $\Delta m^2$ than the oscillation results.

IV. SUMMARY

We have investigated a pseudo-Dirac mixing of left and right-handed neutrinos assuming that the Majorana masses $M_L$ and $M_R$ are small compared with the Dirac mass $M_D$. In this scenario there exist light sterile neutrinos, which may be necessary for explaining the solar and atmospheric neutrino anomalies together with the LSND results on neutrino oscillations. We assume that the Majorana mass $M_L$ is generated by a non-renormalizable operator reflecting effects of new physics at some high energy scale. A consistent scenario relevant for the pure $\nu_e \leftrightarrow \nu_s$ vacuum oscillation is achieved with $M_L, M_R \simeq 10^{-7} \text{ eV}$ and $M_D \simeq 10^{-5} - 10^{-4} \text{ eV}$. In this case, the preferred value for $M_D$ is pushed to its upper end which arises due to the pseudo-Dirac criterion ($M_D \gg M_L$). The mass $M_L$ is easily correlated to the bound for the effective Majorana mass as probed in neutrinoless double beta decay searches. Unfortunately, the planned future experiments for probing $M_{\text{eff}}$ are still (at least) a couple of orders above the required sensitivity. If on the other hand, future experiments do observe a positive signal for $M_{\text{eff}}$, then this will disfavour a pseudo-Dirac scenario. If the Majorana mass is to be generated by the simplest non-renormalizable operator (15), then such a positive effect would furthermore imply that the scale of new physics has to be at the GUT scale or otherwise neutrino parameters should be unnaturally fine tuned.
An interesting possibility is if the new physics is much below the Planck scale along with a nonzero signal for $0\nu\beta\beta$ decay. In this case, without much fine tuning of the neutrino parameter, the scale of new physics could be at the TeV range. This scenario, based on an operator of the form (21), invokes additional scalar doublets with a possible $Z_2$ symmetry. However, also in this case the pseudo-Dirac scenario were ruled out. We also show that in the limit of nearly degenerate neutrino as dark matter components, the corresponding bounds for the neutrino parameters are less stringent than the ones obtained due to oscillations. This is primarily due to the small mass squared difference required for the solar solution.

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REFERENCES

[1] Super-Kamiokande Collaboration, Y. Fukuda, Phys. Rev. Lett. 81, 1562 (1998); Phys. Lett. B467, 185 (1999).

[2] GALLEX Collaboration, W. Hampel et al., Phys. Lett. B447, 127 (1999); SAGE Collaboration, J. N. Abdurashitov et al. Phys. Rev. C60, 055801 (1999); Kamiokande Collaboration, Y. Fukuda et al., Phys. Rev. Lett. 77, 1683 (1996); Homestake Collaboration, B. T. Cleveland et al., Astrophys. J. 496, 505 (1998); GNO Collaboration, M. Altmann et al., Phys. Lett. B490, 16 (2000).

[3] SNO Collaboration, G.R. Ahmad et al., Phys. Rev. Lett. 87, 071301 (2001).

[4] G. Mills for the LSND Collaboration, Nucl. Phys. Proc. Suppl. 91, 198 (2001).

[5] M. C. Gonzalez-Garcia, P. C. de Holanda, C. Peña-Garay, J. W. F. Valle, Nucl. Phys. B573, 3 (2000); J. N. Bahcall, P. I. Krastev and A. Yu. Smirnov, Phys. Rev. D58, 096016 (1998); M. C. Gonzalez-Garcia and C. Peña-Garay, Nucl. Phys. Proc. Suppl. 91, 80 (2000); N. Fornengo, S. Pakvasa, T. J. Weiler and K. Whisnant, Phys. Rev. B580, 58 (2000); M. C. Gonzalez-Garcia, H. Nunokawa, O. L. Peres and J. W. F. Valle, Nucl. Phys. B543, 3 (1999); R. Foot, R. R. Volkas and O. Yasuda, Phys. Rev. D58, 013006 (1998); O. Yasuda, Phys. Rev. D58, 091301 (1998); C. Giunti, M. C. Gonzalez-Garcia, C. Peña-Garay, Phys. Rev. D62, 013005 (2000); M. C. Gonzalez-Garcia, C. Peña-Garay, Phys. Rev. D63, 073013 (2001); G. L. Fogli, E. Lisi, A. Marrone, Phys. Rev. D63, 053008 (2001); G. L. Fogli, E. Lisi and A. Marrone, Phys. Rev. D64, 093005 (2001); O. Yasuda, hep-ph/0006315; M. C. Gonzalez-Garcia, H. Nunokawa, O. L. Peres and J. W. F. Valle, Nucl. Phys. B543, 3 (1999); M. C. Gonzalez-Garcia, M. Maltoni, C. Peña-Garay, Phys. Rev. D64, 093001 (2001).

[6] The CHOOZ collaboration, M. Apollonio et al., Phys. Lett. B420, 397 (1998).

[7] F. Boehm et al. Phys. Rev. Lett 84, 3764 (2000).
[8] K. R. S. Balaji, G. Rajasekaran and S. Uma Sankar, Int. J. Mod. Phys. A16, 1417 (2001).

[9] Super-Kamiokande Collaboration, S. Fukuda et al., Phys. Rev. Lett. 85, 3999 (2000).

[10] R. Foot, Phys. Lett. B496, 169 (2000); N. Fornengo, M. C. Gonzalez-Garcia and J. W. F. Valle, Nucl. Phys. B580, 58 (2000).

[11] A. Geiser, Eur. Phys. J. C7, 437 (1999).

[12] J. N. Bahcall, P. I. Krastev and A. Yu. Smirnov, JHEP 0105, 015 (2001).

[13] P. I. Krastev and A. Yu. Smirnov, hep-ph/0108177 (2001); see also, J. N. Bahcall, M. C. Gonzalez-Garcia and C. Peña-Garay, JHEP 0108, 014 (2001); M. C. Gonzalez-Garcia, M. Maltoni, C. Peña-Garay, Phys. Rev. D64, 093001 (2001).

[14] E. Kh. Akhmedov and Joao Pulido, Phys. Lett. B485, 178 (2000) and references therein.

[15] V. Barger, D. Marfatia and K. Whisnant, hep-ph/0106207 (2001).

[16] S. M. Bilenky, C. Guinti, W. Grimus and T. Schwetz, Phys. Rev. D60, 073007 (1999); W. Grimus and T. Schwetz, hep-ph/0102252

[17] M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity, edited by P. van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam, 1979), p. 315; T. Yanagida in Proceedings of the Workshop on Unified Theory and Baryon Number in the Universe, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, Japan, 1979); R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).

[18] L. Wolfenstein, Nucl. Phys. B186, 147 (1981). For recent considerations, see e.g. Yosef Nir, JHEP 0006, 39 (2000); M. Lindner, T. Ohlsson and G. Seidl, hep-ph/0109264 and references therein.

[19] S. M. Bilenky and B. M. Pontecorvo, Yad. Fiz. 38, 415 (1983) [Sov. J. Nucl. Phys. 38, 248 (1983)]; A. Geiser, Phys. Lett. B444, 358 (1999); Makoto Kobayshi and C. S. Lim,
Phys. Rev. D64, 013003 (2001); M. Kobayashi, C. S. Lim and M. M. Nojiri, Phys. Rev. Lett. 67, 1685 (1991); H. Minakata and H. Nunokawa, Phys. Rev. D45, 3316 (1992); C. Giunti, C. W. Kim and U. W. Lee, Phys. Rev. D46, 3034 (1992); S. T. Petcov, Phys. Lett. B110, 245 (1982); W. Krolikowski, Acta. Phys. Polon. B31, 663 (2000); D. Chang and O. C. W. Kong, Phys. Lett. B477, 416 (2000); C. Liu and J. Song, Nucl. Phys. B598, 3 (2001); C. Liu, Mod. Phys. Lett. A16, 1699 (2001); J. P. Bowes and R. R. Volkas, J. Phys. G24, 1249 (1998).

[20] D. Chang and O. C. W. Kong in ref. [19].

[21] N. Arkani-Hamed, S. Dimopoulos, G. Dvali and J. March-Russell, hep-ph/9811448 (1998); G. Dvali and A. Y. Smirnov, Nucl. Phys. B563, 63 (1999).

[22] Heidelberg-Moscow Collaboration, L. Baudis et al., Phys. Rev. Lett. 83, 41 (1999).

[23] H. V. Klapdor-Kleingrothaus, Part. Nucl. Lett. 104, 20 (2001).

[24] F. Maltoni, J. M. Niczyporuk and S. Willenbrock, Phys. Rev. Lett. 86, 212 (2001).

[25] GENIUS Collaboration, H. V. Klapdor-Kleingrothaus et al., hep-ph/9910205 (1999).

[26] S. Weinberg, Phys. Rev. Lett. 43, 1566 (1979).

[27] J. Bonn et al., Nucl. Phys. Proc. Suppl. 91, 273 (2001).

[28] See, e.g., E. W. Kolb and M. S. Turner, The Early Universe (Addison-Wesley, Redwood City, USA, 1990).

[29] D. E. Groom et al., The Eur. Phys. J. C15, 1 (2000).