Holographic Principle, Cosmological Constant and Cyclic Cosmology

Claudio Corianò and Paul H. Frampton

*Dipartimento di Matematica e Fisica "Ennio De Giorgi",
Università del Salento and INFN-Lecce,
Via Arnesano, 73100 Lecce, Italy*

Abstract

The holographic principle provides a deep insight into quantum gravity and resolves the fine-tuning crisis concerning the cosmological constant. Holographic dark energy introduces new ultra-violet (UV) and infra-red (IR) cutoffs into quantum gravity which are necessarily strongly related. The equation of state for dark energy $\omega = p/\rho$ is discussed from the holographic point of view. The phantom option of $\omega < -1$ is resurrected, as in an earlier cyclic cosmology. Such a cyclic model can, however, equally use the cosmological constant with $\omega = -1$. 
1 Introduction

The 1998 discovery of accelerating cosmic expansion gave rise to a theoretical explanation by a small non-zero cosmological constant (CC). At first the magnitude of the CC seemed exceedingly small when compared with the Planck scale.

Five years earlier, ’t Hooft [1] had made the stunning suggestion that the number of degrees of freedom for gravity in (3+1) spacetime is the same as the number of degrees of freedom for quantum field theory in (2+1) spacetime. It did not take long [2] to try to connect these two observations by pointing out that the usual calculation of the cosmological constant in quantum field theory had not taken account of ’t Hooft’s drastic reduction in the number of gravitational degrees of freedom. The initial work did not lead to the correct equation of state \( \omega = p/\rho \) for the dark energy [3]. This led to an interesting modification [4] which provides the jumping off point here.

There are some puzzles remaining in this approach [5] but since it makes dramatic progress towards the magnitude of the CC it is worth asking whether it can gain some traction in handling the equally vexing questions surrounding cyclic cosmology which confronts the Tolman Entropy Conundrum (TEC). This will be discussed in the present paper.

As we shall discuss in the next section, the holographic principle dictates that we take care to choose cutoffs such that they do not allow states which lie inside their own Schwarzschild radius. This imposes strong constraints which dramatically modify how we approach calculations in quantum gravity.

2 IR and UV Cutoffs

The naive estimate of the cosmological constant from quantum field theory (QFT) uses the vacuum energy from the 0-point function and results in

\[
\Lambda_{QFT} \sim \int_{M_{Planck}} d^3k \sqrt{k^2 + m^2} \sim M_{Planck}^4
\]

so that, using the reduced Planck mass \( M_{Planck} \sim 10^{18} \text{GeV} \), one estimates

\[
\Lambda_{QFT} \sim 10^{72} \text{(GeV)}^4 \equiv 10^{108} \text{(MeV)}^4
\]

(2)

to be compared with the observational value

\[
\Lambda_{obs} \sim 10^{-12} \text{(eV)}^4
\]

(3)

displaying the 120 orders of magnitude discrepancy between theory, \( \Lambda_{QFT} \), and experiment, \( \Lambda_{obs} \).

It is fair to say that before the advent [1] of the holographic principle, this discrepancy was simply described as the largest error in theoretical physics and defied any explanation.

According to the holographic principle, however, the expression in Eq. (1) for \( \Lambda_{QFT} \) includes a significant overestimate of the number of degrees of freedom. The point is that the UV cutoff is really much
less than $M_{\text{Planck}}$. Let us denote this ultraviolet cutoff for gravity by $M_{\text{UV}}$ and the infra-red cutoff by $M_{\text{IR}} = L_{\text{IR}}^{-1}$ where $L_{\text{IR}}$ is the size of the system.

The UV cutoff in the gravitational sector in Eq. (1) must be reduced to $10^{-30} M_{\text{Planck}}$ if the calculation is to be consistent with observation. That such a dramatic reduction is feasible is testament to the power of the holographic principle. Of course, in the non-gravitational sector the UV cutoff for the standard model (SM) must be higher, at least a few TeV, to accommodate the LHC experiments.

This separation of the gravitational and SM sectors is crucial to the results in [2–4]. According to the holographic principle, the volume $L_{\text{IR}}^3$ occupied by the effective field theory describing gravity must satisfy that its entropy is less than that of a black hole of radius $L_{\text{IR}}$. This requires the inequality

$$L_{\text{IR}}^3 M_{\text{UV}}^3 < L_{\text{IR}}^2 M_{\text{Planck}}^2$$

which implies a scaling law

$$L_{\text{IR}} \propto \left( \frac{1}{M_{\text{UV}}^3} \right)$$.  

Even Eqs. (5) is insufficiently strong to avoid disallowed states whose Schwarzschild radius exceeds $L_{\text{IR}}$. To see this, consider the effective field theory at a temperature satisfying

$$M_{\text{IR}} \ll T < M_{\text{UV}}$$

In the volume $L_{\text{IR}}^3$ the thermal energy $E$ and entropy $S$ are given by $E = L_{\text{IR}}^3 T^4$ and $S = L_{\text{IR}}^3 T^3$ respectively. If we saturate the inequality of Eq.(4) we find a system with Schwarzschild radius $R_S$ given by

$$R_S = \frac{E}{M_{\text{Planck}}^2} = L_{\text{IR}}(L_{\text{IR}} M_{\text{Planck}})^{\frac{2}{3}} \gg L_{\text{IR}}$$

which confirms that Eq.(5) is insufficiently strong to exclude states whose Schwarzschild radius exceeds the size of the box. To exclude all states which lie within their Schwarzschild radius requires that one impose the stronger inequality

$$L_{\text{IR}}^3 M_{\text{UV}}^4 \leq L_{\text{IR}} M_{\text{Planck}}^2$$

which implies that

$$L_{\text{IR}} \propto \left( \frac{1}{M_{\text{UV}}^2} \right)$$.  

which is much stronger than Eq.(5). When Eq.(8) is saturated the maximum entropy $S_{\text{max}}$ falls short of the black hole entropy $S_{\text{BH}}$ by

$$S_{\text{max}} = S_{\text{BH}}^{\frac{2}{3}}$$

The significant difference between $S_{\text{max}}$ and $S_{\text{BH}}$ resides in states which are not describable by conventional quantum field theory. However, there remains a fatal flaw in the discussion so far as the
alert reader may have noticed. The point it that by using the constraint in Eq.\([9]\) the cosmological constant \(\Lambda\) develops a dependence on the FLRW scale factor \(a(t)\) of the form

\[
\Lambda \propto \left( \frac{1}{a(t)^3} \right) \tag{11}
\]

which uses the fact that \(L_{IR} \propto a(t)^{\frac{2}{3}}\). But the scaling of Eq.\([11]\) means that the dark energy and matter terms on the right-hand-side of the Friedmann expansion equation behave similarly and that therefore the dark energy has equation of state \(\omega_{DE} = p/\rho = 0\) corresponding to pressureless dust rather than \(\omega_{DE} < -\frac{1}{3}\) as necessary for accelerated expansion.

The holographic approach to dark energy thus appeared doomed until the appearance of paper \([4]\) which made an interesting proposal of how to proceed more successfully. The idea is to replace the radius of the visible universe by the future event horizon \(L_{IR} = R_h\) as the infrared cutoff given by

\[
R_h = a \int_t^\infty \frac{dt}{a} = a \int_a^\infty \frac{da}{Ha^2} \tag{12}
\]

This future event horizon is the boundary of the volume a fixed observer may eventually observe.

Writing

\[
\rho_{DE} = 3c^2 M_{\text{Planck}}^2 L_{IR}^{-2} \tag{13}
\]

and assuming dominance by dark energy in the Friedmann expansion equation

\[
H^2 = \frac{1}{3M_{\text{Planck}}^2} \rho_{DE} \tag{14}
\]

we find that

\[
R_h H = c \tag{15}
\]

and consistency requires with a new normalisation that

\[
\frac{1}{H} = \left( \frac{\alpha}{ca} \right) a^{1/c} \tag{16}
\]

which means that the equation of state \(\omega = p/\rho\) satisfies

\[
-3(1 + \omega) = -2(1 - \frac{2}{c}) \tag{17}
\]

which means that

\[
\omega = -\frac{1}{3} - \frac{2}{3c} \tag{18}
\]

in which \(c > 0\).

From Eq.\([18]\) we see that \(\omega < -\frac{1}{3}\) as required for accelerated expansion and that \(\omega = -1\) when \(c = 1\), corresponding to a cosmological constant.

Fits to the observational data tend to favour \(c \leq 1\) corresponding to \(\omega \leq -1\), although \(c > 1, \omega > -1\) cannot yet be excluded.
3 Cosmological Constant

Observationally the magnitude of the cosmological constant $\Lambda$ is approximately $\Lambda \sim +10^{-12}\text{eV}^4$ and its equation of state is $\omega = p/\rho \simeq -1$, quite closely.

From the previous section, setting $c = 1$, we have

$$\Lambda = 3M^2_{\text{Planck}}R_{-2}^{-2} \tag{19}$$

and using $M_{\text{Planck}} = 10^{27}\text{eV}$ and $R_{h}^{-1} = 10^{-33}\text{eV}$ this gives immediately a result $\Lambda = +10^{-12}\text{eV}^4$ consistent with observation. Compared to Eq. (11) in the Introduction we notice that the holographic principle has decreased the UV cutoff by 30 orders of magnitude and hence the CC, which goes like the UV cutoff to the fourth power, by 120 orders of magnitude. This provides vindication of the radical proposal in [1] about quantum gravity.

Let us discuss the equation of state $\omega$ given by Eq.(18) in the previous section. With $c = +1$ we find $\omega = -1$. Consistent with observational data, the parameter may instead be, for example, $c = 0.986 < 1$ which corresponds to $\omega = -1.01$ and is characteristic not of a cosmological constant but of phantom dark energy. This small-seeming change drastically changes the fate of the universe. Both $\omega = -1$ exactly and $\omega = -1.01$ will be interesting cases for our ensuing discussion about cyclic cosmology.

4 Cyclic Cosmology

There is an undeniable attraction to the idea of a cyclic universe which goes an infinite number of times through an

$$\text{expansion} \rightarrow \text{turnaround} \rightarrow \text{contraction} \rightarrow \text{bounce} \rightarrow \text{expansion} \ldots \tag{20}$$

process. In the earliest days of theoretical cosmology most of the leading theorists (De Sitter, Einstein, Friedmann, Lemaitre, Tolman) at some point favoured such a theory, primarily to avoid the initial singularity present in the Friedmann expansion equation. However, considerations of entropy and the second law of thermodynamics led Tolman [6, 7] in 1931 to a no-go theorem about cyclic cosmology, often called the Tolman Entropy Conundrum (TEC). Simply put, if the entropy continuously increases as required by the second law, each cycle becomes larger and longer. Correspondingly, in the past the cycles were smaller and shorter and therefore must have at some finite past time originated from an initial singularity.

Entropy of the universe enters our considerations not only because of the TEC but also because of the necessity of exceptionally low entropy at the beginning of the present expansion era. Why should the universe be in such a homogenous uniform state at the start? Cyclic cosmology should address also this second entropy issue which is not explained in, for example, inflationary theory.

What we have in mind is an infinitely cyclic theory with an infinite past. The infinite past raises
interesting mathematical issues which were addressed in the 2009 preprint \[8\]. The cyclic model we shall discuss has, at present, an infinite number of universes forming an infiniverse. This will remain the case for the infinite future. What is more subtle is the infinite past where according to \[8\] there are two possibilities: (A) there was always an infinite number of universes; (B) by using the set theory idea of *absence of precedent* it can begin, an infinite time in the past, with a finite number of universes, possibly only one.

Of course, what was unknown to Tolman and to all other theoretical cosmologists until the end of the twentieth century is the dark energy which drives the observed accelerated cosmological expansion. This provides alternatives to prior thinking, providing novel ways to get rid of the entropy of our universe, for example at the turnaround from expansion to contraction.

One important issue is to provide observational tests for a given model. In \[9\] it was shown, based on conservative and plausible assumptions, that to be sensitive to any effects of dark energy an experiment must be at least the size of a galaxy. It is therefore discomfiting to read a recent paper \[10\] looking for dark energy at the LHC. Although \[9\] emphasises \( \omega < -1 \) the argument therein applies equally to \( \omega = -1 \).

The equation of state \( \omega = p/\rho \), where \( p \) is pressure and \( \rho \) is density, plays an important role although not as important as first thought when emphasis was (mis)placed on the phantom possibility \( \omega < -1 \) which can lead to a big rip \[11\], a little rip \[12\] or one of its variants \[13–15\]. As we shall discuss, the proposals for a cyclic cosmology survive in the case of \( \omega = -1 \) which is the equation of state for the cosmological constant.

In the model of \[16\] the method of evading the TEC was based on the Come Back Empty (CBE) idea. The CBE hypothesis is that our contracting universe contains no matter, including no black holes, only entropy-free dark matter and hence contract adiabatically with zero entropy. The almost vanishing number of other universes which do contain matter and / or black holes will be failed universes because they will prematurely bounce after the turnaround from expansion to contraction.

To discuss cyclicity, the holographic model with \( c = 0.986, \omega = -1.01 \) from the previous section is closest to the original discussion of \[16\]. We first note that the time from the present time \( t_0 \) to the big rip at \( t = t_{\text{rip}} \) is \[17\]

\[
    t_{\text{rip}} - t_0 = \left( \frac{11Gy}{-\omega_{\text{DE}} - 1} \right) \approx 1.1 \text{ Ty} \tag{21}
\]

so that to one-digit accuracy we can say \( t_{\text{rip}} = t_T = 1.0 \text{ Ty} \) where \( t_T \) is the time of turnaround from expansion to contraction which is only a fraction of a second before \( t_{\text{rip}} \).

At \( t_T \) the universe divides into a very large number \( N \) of causally disconnected patches, almost all of which are empty of matter and of black holes. The vanishingly small number of causal patches which do contain quarks and leptons and black holes will necessarily fail to contract all the way to a normal bounce because the matter will proliferate and cause a premature bounce. The successful universes, of which ours is one, can contract to a bounce a fraction of a second before the would-be big bang.

The *total* entropy of the infiniverse always increases consistent with the second law of thermodynamics,
but at turnaround the entropy of our universe drops very close to zero and remains nearly vanishing until the bounce, thereby explaining why the entropy at the beginning of the next expansion era is very low.

After turnaround at \( t = t_T \), the scale factor deflates to \( \hat{a}(t) = f a(t_T) \) where \( f < 10^{-28} \propto N^{-\frac{1}{4}} \) and a fraction \( (1 - f)^3 \) of the entropy is jettisoned at turnaround. During the contraction from \( t = t_T \) to the bounce at \( t = t_B \) the reduced scale factor satisfies a Friedmann contraction formula

\[
\left( \frac{\dot{a}(t)}{a(t)} \right)^2 = \frac{8\pi G}{3} \left[ \frac{(\dot{\rho}_\Lambda)_0}{a(t)^3(\omega_\Lambda + 1)} + \frac{(\dot{\rho}_r)_0}{a(t)^4} \frac{\ddot{a}(t)}{\dot{a}(t)} \right] \tag{22}
\]

with

\[
\dot{\rho}_i(t) = \frac{(\rho_i)_0 f^{3(\omega_i + 1)}}{a(t)^{3(\omega_i + 1)}} = \frac{\dot{\rho}_i(t)}{a(t)^{3(\omega_i + 1)}} \tag{23}
\]

and \( \dot{\rho}_m = 0 \) because of the CBE hypothesis. The CBE assumption was critically examined in \[18\] where it was confirmed that after turnaround the universe contains at most one photon. As the contraction progresses, spatial flatness is rapidly approached as an attractor point of Eq.(22).

We must comment on the different case \( c = +1, \omega = -1 \) of the previous section which is the cosmological constant. At first sight this is very different because there is no would-be big rip. However, it is straightforward to show that a turnaround and bounce can occur in a similar way by employing a right-hand-side to the Friedmann expansion equation containing \( (\rho_{DE} - \rho_{DE}^2) \) as can be justified by higher dimensional brane models e.g. \[19, 20\].

An analysis in \[21\] showed that CBE is feasible for any \( \omega_\Lambda < -2 \), which includes all the values of interest, since it allows the number \( N \) of causal patches at turnaround to be sufficient to satisfy the CBE constraint. Another study \[22\] showed that scale invariant density perturbations in the radiation field are provided during contraction which re-enter the horizon after the bounce. Finally, the typical time elapse before the turnaround was estimated \[23\] by a new matching condition method which reassuringly resulted in a value of \( t_T \) consistent with that estimated in Eq.(21) above.

5 Discussion

As we have seen, the holographic principle strongly modifies previous calculations in quantum gravity made without the benefit of its knowledge. Most striking is the diminution of the magnitude of the cosmological constant by 120 orders of magnitude, thereby ending that mystery.

This requires a decrease of 30 orders of magnitude in the UV cutoff \( M_{UV} \) for the gravity sector. The IR cutoff scales like \( L_{IR} \propto M_{UV}^{-2} \) and the correct proportionality constant involves the future event horizon in order to obtain the correct equation of state \( \omega_{DE} \simeq -1 \). This choice for the IR cutoff is somewhat counterintuitive because it uses information from the future, or at least assumes that the accelerated expansion will continue.

With respect to cyclic cosmology, the holographic dark energy permits an equation of state \( \omega = -1 \) or
slightly more negative e.g. $\omega = -1.01$. The latter case was emphasised in [16]. However, the former case can equally underly an infinitely cyclic model by appealing to a brane term in the Friedmann expansion equation.

Acknowledgements

We wish to acknowledge INFN for support under Iniziativa Specifica QFT-HEP.

References

[1] G. 't Hooft,
   in *Salamfestschrift*.
   Conf.Proc. C930308 (1993) 284-296.
   arXiv:gr-qc/9310026.

[2] A.G. Cohen, D.B. Kaplan and A.E. Nelson,
   Phys. Rev. Lett. 82, 4917 (1999).
   arXiv:hep-th/9803132.

[3] S.D.H. Hsu,
   Phys. Lett. B594, 13 (2004).
   arXiv:hep-th/0403052.

[4] M. Li,
   Phys. Lett. B603, 1 (2004).
   arXiv:hep-th/0403127.

[5] J.-W. Lee, H.-C. Kim and J. Lee,
   J.Korean Phys.Soc. 74, 1 (2019).
   arXiv:1812.01993 [gr-qc].

[6] R.C. Tolman,
   Phys. Rev. 38, 1758 (1931).

[7] R.C. Tolman,
   *Relativity, Thermodynamics and Cosmology*.
   Oxford University Press (1934).

[8] P.H. Frampton,
   arXiv:0903.4309 [hep-th].
[9] P.H. Frampton,
Mod. Phys. Lett. A19, 801 (2004).
arXiv:hep-th/0302007

[10] M. Aaboud, et al., (ATLAS Collaboration),
JHEP. 1905:142 (2019).
arXiv:1903.01400[hep-ex]

[11] R. Caldwell,
Phys. Lett. B545, 23 (2002).
arXiv:astro-ph/9908168

[12] P.H. Frampton, K.J. Ludwick and R.J. Scherrer,
Phys. Rev. D84, 063003 (2011).
arXiv:1106.4996[astro-ph.CO]

[13] P.H. Frampton, K.J. Ludwick, S. Nojiri, S.D. Odintsov
and R.J. Sherrer,
Phys. Lett. B708, 204 (2012).
arXiv:1108.0067[hep-th]

[14] P.H. Frampton, K.J. Ludwick and R.J. Scherrer,
Phys. Rev. D85, 083001 (2012).
arXiv:1112.2964[astro-ph.CO].

[15] P.H. Frampton and K.J. Ludwick,
Mod. Phys. Lett. A28, 1350125 (2013).
arXiv:1304.5221[astro-ph.CO]

[16] L. Baum and P.H. Frampton,
Phys. Rev. Lett. 98, 071301 (2007).
arXiv:hep-th/0610213

[17] P.H. Frampton and T. Takahashi,
Astropart. Phys. 22, 307 (2004).
arXiv:astro-ph/0405333

[18] L. Baum and P.H. Frampton,
Mod. Phys. Lett. A23, 33 (2008).
arXiv:hep-th/0703162
[19] L. Randall and R. Sundrum,
Phys. Rev. Lett. **83**, 3370 (1999).
arXiv:hep-ph/9905221

[20] C. Csaki, M. Graesser, L. Randall and J. Terning,
Phys. Rev. **D62**, 045015 (2000).
arXiv:hep-ph/9911406

[21] L. Baum, P.H. Frampton and S. Matsuzaki,
JCAP **04**:032 (2008).
arXiv:0801.4420[hep-th].

[22] P.H. Frampton,
Mod. Phys. Lett. **A31**, 1650076 (2016).
arXiv:1508.01079[gr-qc].

[23] P.H. Frampton,
Mod. Phys. Lett. **A32**, 1750132 (2017).
arXiv:1611.05716[gr-qc].