Rare $B$ Decays with a HyperCP Particle of Spin One

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Abstract

In light of recent experimental information from the CLEO, BaBar, KTeV, and Belle collaborations, we investigate some consequences of the possibility that a light spin-one particle is responsible for the three $\Sigma^+ \rightarrow p\mu^+\mu^-$ events observed by the HyperCP experiment. In particular, allowing the new particle to have both vector and axial-vector couplings to ordinary fermions, we systematically study its contributions to various processes involving $b$-flavored mesons, including $B$-$\bar{B}$ mixing as well as leptonic, inclusive, and exclusive $B$ decays. Using the latest experimental data, we extract bounds on its couplings and subsequently estimate upper limits for the branching ratios of a number of $B$ decays with the new particle. This can serve to guide experimental searches for the particle in order to help confirm or refute its existence.

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I. INTRODUCTION

The detection of a new particle having a sub-GeV mass would likely hint at the presence of physics beyond the standard model. This possibility has been raised recently by the observation of three events for the rare decay mode $\Sigma^+ \rightarrow p\mu^+\mu^-$ with dimuon invariant masses narrowly clustered around 214.3 MeV by the HyperCP collaboration a few years ago [1]. Although these events can be accounted for within the standard model (SM) when long-distance contributions are properly included [2], the probability that the three events have the same dimuon mass in the SM is less than 1 percent. This makes it reasonable to speculate that a light neutral particle, $X$, is responsible for the observed dimuon-mass distribution via the decay chain $\Sigma^+ \rightarrow pX \rightarrow p\mu^+\mu^-$ [1].

The new-particle interpretation of the HyperCP result has been theoretically explored to some extent in the literature [3–11]. Various ideas that have been proposed include the possibility that $X$ is spinless or that it has spin one. In the spinless case, $X$ could be a sgoldstino in supersymmetric models [5] or a $CP$-odd Higgs boson in the next-to-minimal supersymmetric standard model (NMSSM) [7, 8]. In the case of $X$ being a spin-1 particle, one possible candidate is the gauge ($U$) boson of an extra U(1) gauge group in some extensions of the SM [11].

The presence of $X$ in $\Sigma^+ \rightarrow p\mu^+\mu^-$ implies that it also contributes to other $|\Delta S| = 1$ transitions, such as the kaon decays $K \rightarrow \pi\mu^+\mu^-$. In general, the contributions of $X$ to $|\Delta S| = 1$ processes fall into two types. The first one is induced by the flavor-changing (effective) couplings of $X$ to $ds$. In addition to these two-quark contributions, there are so-called four-quark contributions of $X$, which arise from the combined effects of the usual four-quark $|\Delta S| = 1$ operators in the SM and the flavor-conserving couplings of $X$ to quarks, as well as its interactions with the SM gauge fields [6]. Although the two-quark contributions are generally expected to dominate over the four-quark ones, in some models the parameter space may have regions where the two types of contributions are comparable in size and hence could interfere destructively [6, 7]. Accordingly, to explore the $X$ hypothesis in detail and compare its predictions with experimental results in a definite way, it is necessary to work under some model-dependent assumptions.

There are a number of experiments that have recently been performed or are still ongoing to test the $X$ hypothesis [12–16]. Their results have begun to restrict some of the proposed ideas on $X$ in the literature. In particular, as already mentioned, $X$ could be a light $CP$-odd Higgs boson in the NMSSM. In the specific NMSSM scenario considered in Ref. [7], $X$ does not couple to up-type quarks and has the same flavor-conserving coupling $l_d$ to all down-type quarks, implying that the four-quark contributions of $X$ to $|\Delta S| = 1$ decays are proportional to $l_d$ [7]. Recent searches for the radiative decays $\Upsilon(1S, 2S, 3S) \rightarrow \gamma X \rightarrow \gamma\mu^+\mu^-$ by the CLEO and BaBar collaborations [13] have come back negative and imposed sufficiently small upper-bounds on $l_d$ to make the four-quark contributions negligible compared to the two-quark ones. With only the two-quark contributions being present, the scalar part of the $sdX$ coupling is already constrained by $K \rightarrow \pi\mu\mu$ data to be negligibly small, whereas its pseudoscalar part can be probed by $K \rightarrow \pi\pi\mu\mu$ measurements [3, 4]. There are now preliminary results on the branching ratio $B$ of $K_L \rightarrow \pi^0\pi^0X \rightarrow \pi^0\pi^0\mu^+\mu^-$ reported by the KTeV and E391a collaborations [14, 15]. The KTeV preliminary measurement $B < 9.44 \times 10^{-11}$ at 90% C.L. [14] is the much more stringent of the two and has an upper bound almost 20 times smaller than the lower limit $B_{95} = 1.7 \times 10^{-9}$ predicted in Ref. [3] under the assumption that the $sdX$ pseudoscalar coupling, $g_P$, is purely real. However, there is a possibility that $g_P$ has an
imaginary part, and in the case where this coupling is mostly imaginary the predicted lower bound, $B_{lo}$, can be much smaller. More precisely, one can find that $B_{lo} < 7 \times 10^{-11}$, which evades the above bound from KTeV, if $|\text{Im} \, g_{P}| > 0.98 |g_{P}|$ and, moreover, $B_{lo} = 1.7 \times 10^{-9} |\epsilon_{K}|^2 \sim 8 \times 10^{-15}$ if $g_{P}$ is purely imaginary, $\epsilon_{K} \sim \mathcal{O}(0.002)$ being the usual CP-violation parameter in kaon mixing. If the KTeV preliminary result stands in their final report, then it will have imposed a significant constraint on $g_{P}$, restricting it to be almost purely imaginary, for the scenario in which $X$ has spin zero and its four-quark contributions to flavor-changing transitions are negligible. To place stronger restrictions on $g_{P}$, it is important to look for the decays of particles other than neutral kaons, such as $K^{\pm} \rightarrow \pi^{\pm} \pi^{0}X$ and $\Omega^{-} \rightarrow \Xi^{-}X$ [17].

Although the $X$ couplings in the $|\Delta S| = 1$ sector are in general independent of those in the $|\Delta B| = 1$ sector, there is also new information from the latter sector that seems compatible with the results of the $K_{L}$ measurements. Very recently the Belle collaboration has given a preliminary report on their search for a spinless $X$ in $B \rightarrow \rho \mu^{+}\mu^{-}$ and $B \rightarrow K^{*}\mu^{+}\mu^{-}$ with $m_{\mu\mu}$ values restricted within a small region around $m_{\mu\mu} = 214.3\text{MeV}$. They did not observe any event and provided stringent upper-bounds on the branching ratios of $B \rightarrow \rho X$ and $B \rightarrow K^{*}X$ [16].

Unlike the spinless case, the scenario in which $X$ has spin one is not yet as strongly challenged by experimental data, for it predicts that the lower limit of the branching ratio of $K_{L} \rightarrow \pi^{0}\pi^{0}X \rightarrow \pi^{0}\pi^{0}\mu^{+}\mu^{-}$ arising from the two-quark $dsX$ axial-vector coupling, taken to be real, is $2 \times 10^{-11}$ [3]. This prediction is well below the preliminary upper-bound of $9.44 \times 10^{-11}$ from KTeV [14] and could get lower in the presence of an imaginary part of the $dsX$ coupling. It is therefore interesting to explore the spin-1 case further, which we will do here.

In this paper we focus on the contributions of $X$ with spin 1 to a number of rare processes involving mesons containing the $b$ quark. We will not deal with specific models, but will instead adopt a model-independent approach, assuming that $X$ has flavor-changing two-quark couplings to down-type quarks only and that its four-quark contributions to flavor-changing transitions are negligible compared to the two-quark ones. Accordingly, since the $bdX$ and $bsX$ couplings generally are not related to the $sdX$ couplings, we further assume that the $b(d, s)X$ couplings each have both parity-even and parity-odd parts, but we leave the parity of $X$ unspecified. Specifically, we allow $X$ to have both vector and axial-vector couplings to $b(d, s)$. The more limited case of $X$ being an axial-vector boson with only parity-even couplings to $b(d, s)$ has been considered in Ref. [10]. Following earlier work [3], to be consistent with HyperCP observations we also assume that $X$ does not interact strongly and decays inside the detector with $B(X \rightarrow \mu^{+}\mu^{-}) = 1$. In exploring the effect of $X$ with spin 1 on $B$ transitions, we will incorporate the latest experimental information and obtain constraints on the flavor-changing couplings of $X$ in order to predict upper bounds on the rates of a number of rare decays. At this point it is worth pointing out that, since we let $X$ have vector couplings to $b(d, s)$, the transitions in which we are interested include $B$ decays into $X$ and a pseudoscalar meson, such as pion or kaon, which were not considered in Ref. [10]. As our numbers will show, most of the branching ratios of the decays we consider can be large enough to be detected in near-future $B$ experiments. This can serve to guide experimental searches for $X$ in order to help confirm or rule out the spin-1 case.

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1 We gratefully acknowledge D. Gorbunov for pointing this out to us.
II. INTERACTIONS AND AMPLITUDES

Assuming that \( X \) has spin one and does not carry electric or color charge, we can express the Lagrangian describing its effective couplings to a \( b \) quark and a light quark \( q = d \) or \( s \) as

\[
\mathcal{L}_{bqX} = -\bar{q} \gamma_{\mu} (g_{Vq} - g_{Aq} \gamma_5) b X^{\mu} + \text{H.c.} = -\bar{q} \gamma_{\mu} (g_{Lq} P_L + g_{Rq} P_R) b X^{\mu} + \text{H.c.},
\]

where \( g_{Vq} \) and \( g_{Aq} \) parametrize the vector and axial-vector couplings, respectively, \( g_{Lq,R} = g_{Vq} \pm g_{Aq} \), and \( P_{LR} = \frac{1}{2} (1 \mp \gamma_5) \). Generally, the constants \( g_{Vq,Aq} \) can be complex. In the following, we derive the contributions of these two-quark interactions of \( X \) to the amplitudes for several processes involving \( b \)-flavored mesons. As mentioned above, we follow here the scenario in which the four-quark flavor-changing contributions of \( X \) are negligible compared to the effects induced by \( \mathcal{L}_{bqX} \).

The first transition we will consider is \( B_q^0 \to B_{q'}^0 \) mixing, which is characterized by the physical mass-difference \( \Delta M_q \) between the heavy and light mass-eigenstates in the \( B_q^0-B_{q'}^0 \) system. This observable is related to the matrix element \( M_{12}^q \) for the mixing by \( \Delta M_q = 2 |M_{12}^q| \), where \( M_{12}^q = M_{12}^{q,SM} + M_{12}^{q,X} \) is obtained from the effective Hamiltonian \( \mathcal{H}_{bq \to bq} \) for the SM plus \( X \)-mediated contributions using \( 2m_{B_q} M_{12}^q = \langle B_q^0 | H_{bq \to bq} | B_{q'}^0 \rangle \) [18].

The SM part of \( M_{12}^{q,SM} \) is dominated by the top loop and given by [18]

\[
M_{12}^{q,SM} \approx \frac{G_F m_W^2}{12\pi^2} f_{B_q}^2 m_{B_q} \eta_B B_{B_q} (V_{tb} V_{tq}^{*})^2 S_0(m_t^2/m_W^2),
\]

where \( G_F \) is the usual Fermi constant, \( f_{B_q} \) is the \( B_q \) decay-constant, \( \eta_B \) contains QCD corrections, \( B_{B_q} \) is a bag parameter, \( V_{kl} \) are elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, and the loop function \( S_0(m_t^2/m_W^2) \approx 2.4 \). To determine the \( X \) contribution \( M_{12}^{q,X} \), we derive the effective Hamiltonian \( \mathcal{H}_{bq \to bq}^X \) from the amplitude for the tree-level transition \( bq \to X^* \to \overline{bq} \) calculated from \( \mathcal{L}_{bqX} \). Thus

\[
\mathcal{H}_{bq \to bq}^X = \frac{\bar{q} \gamma_{\mu} (g_{Lq} P_L + g_{Rq} P_R) b \bar{q} \gamma_{\mu} (g_{Lq} P_L + g_{Rq} P_R) b}{2 (m_X^2 - m_{B_q}^2)}
+ \frac{\left\{ \bar{q} \left[ (g_{Lq} m_q - g_{Rq} m_b) P_L + (g_{Rq} m_q - g_{Lq} m_b) P_R \right] b \right\}^2}{2 (m_X^2 - m_{B_q}^2) m_X^2},
\]

where we have used in the denominators the approximation \( p_X^2 = m_{B_q}^2 \) appropriate for the \( B_q \) rest-frame and included an overall factor of 1/2 to account for the products of two identical operators. In evaluating the matrix element of this Hamiltonian at energy scales \( \mu \sim m_b \), one needs to include the effect of QCD running from high energy scales which mixes different operators. The resulting contribution of \( X \) is

\[
M_{12}^{q,X} = \frac{f_{B_q}^2 m_{B_q}}{3 (m_X^2 - m_{B_q}^2)} \left[ (g_{Vq}^2 + g_{Aq}^2) P_{1}^{VLL} + \frac{g_{Vq}^2 (m_b - m_q)^2 + g_{Aq}^2 (m_b + m_q)^2}{m_X^2} P_{1}^{SLL} + \frac{g_{Vq}^2 (m_b - m_q)^2 - g_{Aq}^2 (m_b + m_q)^2}{m_X^2} P_{1}^{PLR} \right],
\]
where $P_1^{VLL} = \eta_1^{VLL} B_1^{VLL}$, $P_1^{SLL} = -\frac{5}{2} \eta_1^{SLL} R_{Bq} B_1^{SLL}$, and $P_j^{LR} = -\frac{1}{2} \eta_1^{LR} R_{Bq} B_1^{LR} + \frac{3}{4} \eta_2^{LR} R_{Bq} B_2^{LR}$, $j = 1, 2$ [19], with the $\eta$’s denoting QCD-correction factors, the $B$’s being bag parameters defined by the matrix elements $\langle B_0^q | q \gamma^\mu P_L b \bar{q} \gamma_\mu P_L b | B_0^q \rangle = \langle B_0^q | \bar{q} \gamma^\mu P_R b \bar{q} \gamma_\mu P_R b | B_0^q \rangle = \frac{2}{3} f_{B_q} m_{B_q} B_1^{VLL}$, $\langle B_0^q | q_P L b \bar{q} P_L b | B_0^q \rangle = \langle B_0^q | q_P R b \bar{q} P_R b | B_0^q \rangle = -\frac{5}{12} f_{B_q} m_{B_q}^2 R_{Bq} B_1^{SLL}$, $\langle B_0^q | \bar{q} \gamma^\mu P_L b \bar{q} \gamma_\mu P_R b | B_0^q \rangle = -\frac{1}{3} f_{B_q} m_{B_q}^2 R_{Bq} B_1^{LR}$, and $\langle B_0^q | q_P L b \bar{q} P_R b | B_0^q \rangle = \frac{1}{2} f_{B_q} m_{B_q}^2 R_{Bq} B_2^{LR}$, and $R_{Bq} = m_{B_q}^2 / (m_b + m_q)^2$.

Bounds on $g_{Vq}$ and $g_{Aq}$ can then be extracted from comparing the measured and SM values of $\Delta M_q$.

The second transition of interest is $B_0^q \to \mu^+ \mu^-$, which receives a contribution from $B_0^q \to X^* \to \mu^+ \mu^-$. To derive the amplitude for the latter, we need not only $L_{bqX}$, but also the Lagrangian describing $X \to \mu^+ \mu^-$. Allowing the $X$ interaction with $\mu$ to have both parity-even and -odd parts, we can write the latter Lagrangian as

$$L_{\mu X} = \bar{\mu} \gamma_\alpha (g_{V \mu} + g_{A \mu} \gamma_5) \mu X^\alpha,$$  

(5)

where $g_{V \mu}$ and $g_{A \mu}$ are coupling constants, which are real due to the hermiticity of $L_{\mu X}$. Using the matrix elements $\langle 0 | \bar{q} \gamma^\mu b | B_0^q \rangle = \langle 0 | \bar{q} b | B_0^q \rangle = 0$, $\langle 0 | \bar{q} \gamma^\mu \gamma_5 b | B_0^q (p) \rangle = -i f_{B_q} p^\mu$, and $\langle 0 | \bar{q} \gamma_5 b | B_0^q \rangle = i f_{B_q} m_{B_q}^2 / (m_b + m_q)$, we then arrive at

$$\mathcal{M}(\bar{B}_0^q \to X \to \mu^+ \mu^-) = -\frac{2i f_{B_q} g_{A \mu} g_{A \mu} m_\mu}{m_X^2} \bar{\mu} \gamma_\mu.$$

(6)

The resulting decay rate is

$$\Gamma(\bar{B}_0^q \to X \to \mu^+ \mu^-) = \frac{f_{B_q}^2 |g_{A \mu}|^2 m_\mu^2}{2 \pi m_X} \sqrt{m_{B_q}^2 - 4 m_\mu^2}.$$

(7)

This implies that we need, in addition, the value of $g_{A \mu}$, which can be estimated from the contribution of $L_{\mu X}$ in Eq. (5) at one-loop level to the anomalous magnetic moment of the muon, $a_\mu$. We will determine $g_{A \mu}$ in the next section. Before moving on to other transitions, we note that from $L_{\mu X}$ follows the decay rate

$$\Gamma(X \to \mu^+ \mu^-) = \frac{g_{V \mu}^2 m_X}{12 \pi} \left(1 + \frac{2 m_\mu^2}{m_X^2}\right) \sqrt{1 - \frac{4 m_\mu^2}{m_X^2}} + \frac{g_{A \mu}^2 m_X}{12 \pi} \left(1 - \frac{4 m_\mu^2}{m_X^2}\right)^{3/2}.$$

(8)

The next process that can provide constraints on $g_{V q}$ and $g_{A q}$ is the inclusive decay $b \to q \mu^+ \mu^-$, to which $b \to b X$ can contribute. From $L_{b q X}$ above, it is straightforward to arrive at the inclusive decay rate

$$\Gamma(b \to q X) = \frac{|p_X|}{8 \pi m_b^2 m_X^2} \left[|g_{V q}|^2 \left(m_b + m_q\right)^2 + 2 m_\mu^2 \left(m_b - m_q\right)^2 m_X^2 \right] \left[ \left(1 - \frac{4 m_\mu^2}{m_X^2}\right) \left(1 - \frac{4 m_\mu^2}{m_X^2}\right) \right]$$

(9)

where $p_X$ is the 3-momentum of $X$ in the rest frame of $b$. One may probe the $b \to b X$ contribution to $b \to q \mu^+ \mu^-$ by examining the measured partial rate of the latter for the smallest range available of the dimuon mass, $m_\mu$, that contains $m_\mu = m_X$.

We will also consider the exclusive decays $B \to M X$, which contribute to $B \to M \mu^+ \mu^-$, where $M$ is a pseudoscalar meson $P$, scalar meson $S$, vector meson $V$, or axial-vector meson $A$. To evaluate
their decay amplitudes, we need the $\bar{B} \to M$ matrix elements of the $b \to q$ operators in $\mathcal{L}_{bqX}$. The matrix elements relevant to $\bar{B} \to PX$ and $\bar{B} \to SX$ are

$$\kappa_P \langle P(P_P)|q\gamma^\mu b|\bar{B}(p_B)\rangle = \frac{m_B^2 - m_P^2}{k^2} k^\mu F_0^{BP} + \left[(p_B + P_P)^\mu - \frac{m_B^2 - m_P^2}{k^2} k^\mu\right] F_1^{BP}, \quad (10)$$

$$i\kappa_S \langle S(P_S)|\bar{q}\gamma^\mu \gamma_5 b|\bar{B}(p_B)\rangle = \frac{m_B^2 - m_S^2}{k^2} k^\mu F_0^{BS} + \left[(p_B + P_S)^\mu - \frac{m_B^2 - m_S^2}{k^2} k^\mu\right] F_1^{BS}, \quad (11)$$

and $\langle P|q\gamma^\mu \gamma_5 b|\bar{B}\rangle = \langle S|\bar{q}\gamma^\mu b|\bar{B}\rangle = 0$, where $k = p_B - p_{P,S}$; the factor $\kappa_P$ has a value of 1 for $P = \pi^-, K, D$ or $-\sqrt{2}$ for $P = \pi^0$, the values of $\kappa_S$ will be given in the next section, and the form factors $F_{0,1}^{BP,BS}$ each depend on $k^2$. For $\bar{B} \to V X$ and $\bar{B} \to A X$, we need

$$\kappa_V \langle V(p_V)|\bar{q}\gamma_\mu b|\bar{B}(p_B)\rangle = \frac{2V^{BV}}{m_B + m_V} \epsilon_{\mu\nu\sigma\tau} \varepsilon^{*\nu}_V \varepsilon^{*\tau}_B p^\mu_B p^\nu_V, \quad (12)$$

$$\kappa_V \langle V(p_V)|\bar{q}\gamma^\mu \gamma_5 b|\bar{B}(p_B)\rangle = 2iA_0^{BV} m_V \frac{\varepsilon^{*\mu}_V \cdot k}{k^2} k^\mu + iA_1^{BV} (m_B + m_V) \left(\varepsilon^{*\mu}_V - \frac{\varepsilon^{*\nu}_V \cdot k}{k^2} k^\mu\right)
- \frac{iA_2^{BV} \varepsilon^{*\nu}_V \cdot k}{m_B + m_V} \left(p^\mu_B + p^\mu_V - \frac{m_B^2 - m_V^2}{k^2} k^\mu\right), \quad (13)$$

$$\kappa_A \langle A(p_A)|\bar{q}\gamma^\mu b|\bar{B}(p_B)\rangle = -2iV^{BA}_1 m_A \frac{\varepsilon^{*\mu}_A \cdot k}{k^2} k^\mu + iV^{BA}_1 (m_B - m_A) \left(\varepsilon^{*\mu}_A - \frac{\varepsilon^{*\nu}_A \cdot k}{k^2} k^\mu\right)
+ \frac{iV^{BA}_2 \varepsilon^{*\nu}_A \cdot k}{m_B - m_A} \left(p^\mu_B + p^\mu_A - \frac{m_B^2 - m_A^2}{k^2} k^\mu\right), \quad (14)$$

$$\kappa_A \langle A(p_A)|\bar{q}\gamma_\mu \gamma_5 b|\bar{B}(p_B)\rangle = -2A^{BA}_1 \frac{\epsilon_{\mu\nu\sigma\tau}}{m_B - m_A} \varepsilon^{*\nu}_A \varepsilon^{*\tau}_B p^\mu_B p^\nu_A, \quad (15)$$

where $k = p_B - p_{V,A}$, the factor $\kappa_V$ has a magnitude of 1 for $V = \rho^-, K^*, \phi, D^*$ or $\sqrt{2}$ for $V = \rho^0, \omega$, the values of $\kappa_A$ will be given in the next section, and the form factors $V^{BV}, A^{BA}_{0,1,2}, V^{BA}_{0,1,2}$, and $A^{BA}$ are all functions of $k^2$. Since $X$ has spin 1, its polarization $\varepsilon^*_X$ and momentum $p^*_X$ satisfy the relation $\varepsilon^*_X \cdot p^*_X = 0$. The amplitudes for $\bar{B} \to PX$ and $\bar{B} \to SX$ are then

$$\mathcal{M}(\bar{B} \to PX) = \frac{2g_{Vq}}{\kappa_P} F_1^{BP} \varepsilon^*_X \cdot p_P, \quad (16)$$

$$\mathcal{M}(\bar{B} \to SX) = \frac{2i g_{Aq}}{\kappa_S} F_1^{BS} \varepsilon^*_X \cdot p_S, \quad (17)$$

leading to the decay rates

$$\Gamma(\bar{B} \to P(S)X) = \frac{|p^*_X|^3}{2\pi \kappa_{P(S)}^2 m^2_X} \left|g_{V(A)q} F_1^{BP(S)}\right|^2, \quad (18)$$

where $p^*_X$ is the 3-momentum of $X$ in the rest frame of $B$. For $\bar{B} \to V X$ and $\bar{B} \to A X$, the amplitudes are

$$\mathcal{M}(\bar{B} \to VX) = -\frac{ig_{Aq}}{\kappa_V} \left[A_1^{BV} (m_B + m_V) \varepsilon^*_V \cdot \varepsilon^*_X - \frac{2A_2^{BV} \varepsilon^*_V \cdot p_X \varepsilon^*_X \cdot p_V}{m_B + m_V}\right]
+ \frac{2g_{Vq} V^{BV}}{\kappa_V (m_B + m_V)} \epsilon_{\mu\nu\sigma\tau} \varepsilon^{*\nu}_V \varepsilon^{*\tau}_X p^\mu_V p^\nu_X, \quad (19)$$
\[ \mathcal{M}(\bar{B} \to AX) = -\frac{ig_{Vq}}{\kappa_A} \left[ V_{1BA}^{BA}(m_B - m_A) \varepsilon_A^* \varepsilon_X^* - \frac{2V_{2BA}^{BA} \varepsilon_A^* \varepsilon_X^* p_X \varepsilon_A^* p_A}{m_B - m_A} \right] + \frac{2g_{Aq}A_{BA}^{BA}}{\kappa_A (m_B - m_A)} \epsilon_{\mu\sigma\tau} \varepsilon_{A}^{\mu} \varepsilon_{X}^{\nu} \epsilon_{A}^{\sigma} p_{X}^{\tau}. \] (20)

The corresponding decay rates can be conveniently written as [20]

\[ \Gamma(B \to M'X) = \frac{|p_X|}{8\pi m_B^2} \left( |H_0^{M'}|^2 + |H_+^{M'}|^2 + |H_-^{M'}|^2 \right), \] (21)

where \( M' = V \) or \( A \), \( H_0^{M'} = -a_{M'}x_{M'} - b_{M'}(x_{M'}^2 - 1) \), and \( H_{\pm}^{M'} = a_{M'} \pm c_{M'} \sqrt{x_{M'}^2 - 1} \), with \( x_{M'} = (m_B^2 - m_{M'}^2 - m_X^2)/(2m_{M'}m_X) \).

\[ a_V = \frac{g_{Vq}A_{BA}^{BA}}{\kappa_V} (m_B + m_V), \quad b_V = -\frac{2g_{Vq}A_{BA}^{BA}m_Vm_X}{\kappa_V (m_B + m_V)}, \quad c_V = \frac{2g_{Vq}m_{V}m_{X}V_{BA}^{BA}}{\kappa_V (m_B + m_V)}, \] (22)

\[ a_A = \frac{g_{Vq}V_{1BA}^{BA}}{\kappa_A} (m_B - m_A), \quad b_A = -\frac{2g_{Vq}V_{2BA}^{BA}m_Am_X}{\kappa_A (m_B - m_A)}, \quad c_A = \frac{2g_{Aq}m_{A}m_{X}A_{BA}^{BA}}{\kappa_A (m_B - m_A)}. \] (23)

In the next section, we employ the expressions found above to extract constraints on the couplings \( g_{Vq} \) and \( g_{Aq} \) from currently available experimental information. We will subsequently use the results to predict upper bounds for the branching ratios of a number of \( B \) decays.

Before proceeding, we remark that we have not included in \( \mathcal{L}_{baX} \) in Eq. (1) the possibility of dipole operators of the form \( \bar{q} \sigma^{\mu\nu}(1 \pm \gamma_5)b \partial_\mu X_\nu \). They would contribute to the processes dealt with above, except for \( B_q \to \mu^+ \mu^- \). However, we generally expect the effects of these operators to be suppressed compared to those of \( \mathcal{L}_{baX} \) by a factor of order \( p_X/\Lambda \sim m_b/\Lambda \), with \( \Lambda \) being a heavy mass representing the new-physics scale, if their contributions all occur simultaneously.

### III. NUMERICAL ANALYSIS

#### A. Constraints from \( B_q \)-\( \bar{B}_q \) mixing

As discussed in the preceding section, the \( X \) contribution \( M_{12}^{q,X} \) to \( B_q \)-\( \bar{B}_q \) mixing is related to the observable \( \Delta M_q = 2 |M_{12}^q| \), where \( M_{12}^q = M_{12}^{q,SM} + M_{12}^{q,X} \). The experimental value \( \Delta M_q^{exp} \) can then be expressed in terms of the SM prediction \( \Delta M_q^{SM} \) as

\[ \Delta M_q^{exp} = \Delta M_q^{SM} |1 + \delta_q|, \quad \delta_q = \frac{M_{12}^{q,X}}{M_{12}^{q,SM}}, \] (24)

and so numerically they can lead to the allowed range of \( \delta_q \), from which we can extract the bounds on \( g_{Vq,Aq} \). Thus, with \( \Delta M_d^{exp} = (0.507 \pm 0.005) \text{ps}^{-1} \) [21] and \( \Delta M_d^{SM} = (0.560^{+0.007}_{-0.006}) \text{ps}^{-1} \) [22], using the approximation \( |1 + \delta_d| \approx 1 + \text{Re} \delta_d \), we can extract the 1\( \sigma \) range

\[ -0.22 < \text{Re} \delta_d < +0.03. \] (25)
Similarly, $\Delta M_s^{\text{exp}} = (17.77 \pm 0.12) \text{ps}^{-1}$ [21] and $\Delta M_s^{\text{SM}} = (17.6^{+1.7}_{-1.8}) \text{ps}^{-1}$ [22] translate into
\[ -0.09 < \Re \delta_s < 0.11 . \] (26)

To proceed, in addition to $m_X = 214.3 \text{MeV}$, we use $m_b = 4.4 \text{GeV}$, $P_1^{\text{VLL}} = 0.84$, $P_1^{\text{SLL}} = -1.47$, $P_1^{\text{LR}} = -1.62$, $P_2^{\text{LR}} = 2.46$ [19], CKM parameters from Ref. [22], $f_{B_d} = 190 \text{MeV}$, $f_{B_s} = 228 \text{MeV}$, $\eta_B = 0.551$, $B_{B_d} = 1.17$, and $B_{B_s} = 1.23$ [18, 22], as well as meson masses from Ref. [21]. Also, we will neglect $m_d$ and $m_s$ compared to $m_b$. It follows that for the ratio in Eq. (24)
\[
\Re \delta_d = \left\{ -4.4 \left[ (\Re g_{V_d})^2 - (\Im g_{V_d})^2 \right] - 8.2 (\Re g_{V_d})(\Im g_{V_d}) \\
+ 17 \left[ (\Re g_{Ad})^2 - (\Im g_{Ad})^2 \right] + 33 (\Re g_{Ad})(\Im g_{Ad}) \right\} \times 10^{12} ,
\]
\[
\Re \delta_s = \left\{ -2.5 \left[ (\Re g_{V_s})^2 - (\Im g_{V_s})^2 \right] + 0.2 (\Re g_{V_s})(\Im g_{V_s}) \\
+ 9.9 \left[ (\Re g_{As})^2 - (\Im g_{As})^2 \right] - 0.7 (\Re g_{As})(\Im g_{As}) \right\} \times 10^{11} .
\] (27)

Hence constraints on the couplings come from combining these formulas with Eqs. (25) and (26). If only $g_{Vq}$ or $g_{Aq}$ contributes at a time, the resulting constraints are
\[
-0.7 \times 10^{-14} < (\Re g_{V_d})^2 - (\Im g_{V_d})^2 + 1.9 (\Re g_{V_d})(\Im g_{V_d}) < 5.0 \times 10^{-14} ,
\]
\[
-1.3 \times 10^{-14} < (\Re g_{Ad})^2 - (\Im g_{Ad})^2 + 1.9 (\Re g_{Ad})(\Im g_{Ad}) < 0.2 \times 10^{-14} ,
\] (28)
\[
-4.4 \times 10^{-13} < (\Re g_{V_s})^2 - (\Im g_{V_s})^2 - 0.1 (\Re g_{V_s})(\Im g_{V_s}) < 3.6 \times 10^{-13} ,
\]
\[
-0.9 \times 10^{-13} < (\Re g_{As})^2 - (\Im g_{As})^2 - 0.1 (\Re g_{As})(\Im g_{As}) < 1.1 \times 10^{-13} .
\] (29)

If one assumes instead that $g_{Vq,Aq}$ are real, then from Eqs. (25)-(27) one can determine the allowed ranges of the couplings shown in Fig. 1.

![Constraints from $B_d^{-\bar{B}_d}$ mixing](image1)

![Constraints from $B_s^{-\bar{B}_s}$ mixing](image2)

FIG. 1: Parameter space of $g_{Vq}$ and $g_{Aq}$ subject to constraints from $B_q^{-\bar{B}_q}$ mixing, $q = d, s$, if $g_{Vq,Aq}$ are taken to be real.
B. Constraints from leptonic decays $B_q \rightarrow \mu^+\mu^-$

As the $B_q \rightarrow \mu^+\mu^-$ width in Eq. (7) indicates, to determine $g_{Aq}$ requires knowing the $X^\mu\mu$ coupling constant $g_{A\mu}$. Since $\mathcal{L}_{\mu X}$ in Eq. (5) generates the contribution of $X$ to the muon anomalous magnetic moment $a_\mu$, we may gain information on $g_{A\mu}$ from $a_\mu$. The $X$ contribution is calculated to be \cite{3, 23}

$$a^X_\mu = \frac{m^2_\mu}{4\pi^2 m^2_X} (g^2_{V\mu} f_V(r) + g^2_{A\mu} f_A(r)) = 1.1 \times 10^{-3} g^2_{V\mu} - 9.0 \times 10^{-3} g^2_{A\mu}, \quad (30)$$

where $r = m^2_\mu/m^2_X$,

$$f_V(r) = \int_0^1 dx \frac{x^2 - x^3}{1 - x + rx^2}, \quad f_A(r) = \int_0^1 dx \frac{-4x + 5x^2 - x^3 - 2rx^3}{1 - x + rx^2}. \quad (31)$$

Presently there is a discrepancy of 3.2$\sigma$ between the SM prediction for $a_\mu$ and its experimental value, $\Delta a_\mu = a_\mu^{\exp} - a_\mu^{\SM} = (29 \pm 9) \times 10^{-10}$ \cite{24}, with $a_\mu^{\exp} = (11659208 \pm 6) \times 10^{-10}$ \cite{21}. Consequently, since the $g_{V\mu}$ and $g_{A\mu}$ terms in $a^X_\mu$ are opposite in sign, we require that $0 < a^X_\mu < 3.8 \times 10^{-9}$, which corresponds to the allowed parameter space plotted in Fig. 2. Avoiding tiny regions where the two terms in Eq. (30) have to conspire subtly to satisfy the $a^X_\mu$ constraint, we then have

$$g^2_{V\mu} \lesssim 1 \times 10^{-5}, \quad g^2_{A\mu} \lesssim 1 \times 10^{-6}, \quad (32)$$

provided that $0 < 1.1 g^2_{V\mu} - 9.0 g^2_{A\mu} < 3.8 \times 10^{-6}$. We note that combining these requirements for $g_{V\mu}$ and $g_{A\mu}$ with Eq. (8) results in the width $\Gamma(X \rightarrow \mu^+\mu^-) \lesssim 1.8 \times 10^{-8}$ GeV.\footnote{It is worth mentioning here that in Ref. \cite{3} the number for $\Gamma(X_A \rightarrow \mu^+\mu^-)$ in their Eq. (18), corresponding to $g_{V\mu} = 0$ and $g^2_{A\mu} = 6.7 \times 10^{-8}$, is too large by a factor of 3.}

Assuming that $\bar{B}_{d,s} \rightarrow X^* \rightarrow \mu^+\mu^-$ saturates the latest measured bounds $\mathcal{B}(B_d \rightarrow \mu^+\mu^-) < 6.0 \times 10^{-9}$ and $\mathcal{B}(B_s \rightarrow \mu^+\mu^-) < 3.6 \times 10^{-8}$ \cite{25}, respectively, we use Eq. (7) with $g^2_{A\mu} = 1 \times 10^{-6}$ to extract

$$|g_{Ad}|^2 < 2.8 \times 10^{-14}, \quad |g_{Aq}|^2 < 1.2 \times 10^{-13}, \quad (33)$$

which are roughly comparable to the corresponding limits in Eq. (28) from $B_q$-$\bar{B}_q$ mixing.

FIG. 2: Parameter space of $g_{V\mu}$ and $g_{A\mu}$ subject to constraints from the muon anomalous magnetic moment.
C. Constraints from inclusive decay $b \to q\mu^+\mu^-$

Since there is still no experimental data on the inclusive $b \to d\mu^+\mu^-$, we consider only the $q = s$ case. Thus, employing Eq. (9) and the $B^0_d$ lifetime [21], we find

$$B(b \to sX) \approx \frac{\Gamma(b \to sX)}{\Gamma_{B^0_d}} = 8.55 \times 10^{13} (|g_{Vs}|^2 + |g_{As}|^2) .$$

To get constraints on $g_{Vs,As}$, it is best to examine the measured partial rate for the smallest $m_{\mu\mu}$ bin available which contains $m_{\mu\mu} = m_X$. The most recent data have been obtained by the BaBar and Belle collaborations [26, 27], the former giving the more restrictive

$$B(b \to s\ell^+\ell^-)_{m_{\ell\ell} \in [0.2\text{ GeV, } 1.0\text{ GeV}]} = (0.08 \pm 0.36^{+0.07}_{-0.04}) \times 10^{-6} ,$$

which is the average over $\ell = e$ and $\mu$. This data allows us to demand that the $X$ contribution be below its 90%-C.L. upper-bound. With $B(X \to \mu^+\mu^-) = 1$, it follows that

$$B(b \to sX) < 6.8 \times 10^{-7} ,$$

which in combination with Eq. (34) implies

$$|g_{Vs}|^2 + |g_{As}|^2 < 8.0 \times 10^{-21} .$$

D. Constraints from exclusive decays $B \to P\mu^+\mu^-$

It can be seen from Eq. (16) that only the vector coupling $g_{Vq}$ is relevant to the $B \to PX$ decay, not $g_{Aq}$. As mentioned earlier, the possibility of $X$ having vector couplings was not considered in Ref. [10], and therefore $B \to PX$ decays were not studied therein. Currently there is experimental information available on $B \to \pi\mu^+\mu^-$ and $B \to K\mu^+\mu^-$ that can be used to place constraints on $g_{Vq}$. For the form factors $F_1^{BP}$, since they are functions of $k^2 = (p_B - p_P)^2 = m_X^2 \ll m_B^2$, it is a good approximation to take their values at $k^2 = 0$. Thus, for $B \to (\pi, K)$ we adopt those listed in Table I. Using Eq. (18), we then obtain

$$B(B^+ \to \pi^+ X) = 1.06 \times 10^{13} |g_{Vd}|^2 , \quad B(B_d \to \pi^0 X) = 4.96 \times 10^{12} |g_{Vd}|^2 ,$$

$$B(B^+ \to K^+ X) \approx B(B_d \to K^0 X) = 1.85 \times 10^{13} |g_{Vd}|^2 .$$

Experimentally, at present there are only upper limits for $B(B \to \pi\mu^+\mu^-)$, namely [25, 28]

$$B(B^+ \to \pi^+\mu^+\mu^-) < 6.9 \times 10^{-8} , \quad B(B_d \to \pi^0\mu^+\mu^-) < 1.84 \times 10^{-7}$$

at 90% C.L. Assuming that the contributions of $B \to \pi X \to \pi\mu^+\mu^-$ saturate these bounds and using Eq. (38) along with $B(X \to \mu^+\mu^-) = 1$, we find from the more stringent of them

$$|g_{Vd}|^2 < 6.5 \times 10^{-21} .$$

For $B \to K\mu^+\mu^-$, there is data on the partial branching ratio that is pertinent to $B \to KX$. The latest measurement provides $B(B \to K\mu^+\mu^-)_{m_{\mu\mu} \leq 2\text{ GeV}} = (0.81^{+0.18}_{-0.16} \pm 0.05) \times 10^{-7}$ [29]. The corresponding SM prediction is consistent with this data [30] and has an uncertainty of about 30% [31].
TABLE I: Form factors relevant to $B \to PX$ [32].

|                  | $B_d \to \pi$ | $B_d \to \eta$ | $B_d \to \eta'$ | $B_s \to K$ | $B_d \to K$ | $B_c \to D_s^+$ | $B_c \to D_s^+$ |
|------------------|---------------|----------------|----------------|-------------|-------------|----------------|----------------|
| $F_{1BP}(0)$     | 0.26          | 0.23           | 0.19           | 0.30        | 0.36        | 0.22           | 0.16           |

In view of this, we can demand that $B(B \to KX \to K\mu^+\mu^-)$ be less than 40% of the central value of the measured result.\(^3\) Thus, with $B(X \to \mu^+\mu^-) = 1$, we have

$$B(B \to KX) < 3.2 \times 10^{-8}.$$ \(42\)

Comparing this limit with Eq. (39) results in

$$|g_{V's}|^2 < 1.7 \times 10^{-21},$$ \(43\)

which is stronger than the $g_{V's}$ bound inferred from Eq. (37). One can expect much better bounds on $g_{Vq}$ from future measurements of $B \to (\pi, K)\mu^+\mu^-$ with $m_{\mu\mu}$ values restricted within a small region around $m_{\mu\mu} = m_X$.

E. Constraints from exclusive decays $B \to V\mu^+\mu^-$

For $B \to VX$ decays, the values of the relevant form factors at $k^2 = 0$ are listed in Table II. Employing those for $B = B_d$ and $V = \rho, K^*$ in Eq. (21), we find

$$B(B_d \to \rho^0X) = 1.77 \times 10^{10}|g_{V'd}|^2 + 6.18 \times 10^{12}|g_{Ad}|^2,$$

$$B(B_d \to K^*0X) = 5.45 \times 10^{10}|g_{V's}|^2 + 1.79 \times 10^{13}|g_{As}|^2.$$ \(44\)

It is worth noting here that the dominance of the $g_{Aq}$ terms in the preceding formulas over the $g_{Vq}$ terms also occurs in other $B \to VX$ transitions and corresponds to the fact that in the decay rate, Eq. (21), the $g_{Aq}$ term $|H_0V|^2$ is significantly enhanced with respect to the $g_{Vq}$ term in $|H_+|^2 + |H_V|^2$. Currently there is no published measurement of $B(B \to \rho\mu^+\mu^-)$, but there are publicly available experimental data on $B(B \to K^*\mu^+\mu^-)$ for the $m_{\mu\mu}$ bin containing $m_{\mu\mu} = m_X$, the most precise being $B(B \to K^*\mu^+\mu^-)_{m_{\mu\mu} \leq 2\text{GeV}} = (1.46^{+0.40}_{-0.35} \pm 0.11) \times 10^{-7}$ [29]. The corresponding SM prediction

TABLE II: Form factors relevant to $B \to VX$ [34].

|                   | $B_d \to \rho$ | $B_d \to \omega$ | $B_s \to K^*$ | $B_d \to K^*$ | $B_s \to \phi$ | $B_c \to D_s^+$ | $B_c \to D_s^+$ |
|-------------------|---------------|-----------------|-------------|-------------|-------------|----------------|----------------|
| $V_{BV}(0)$       | 0.32          | 0.29            | 0.31        | 0.41        | 0.43        | 0.63           | 0.54           |
| $A_{BV1}(0)$      | 0.24          | 0.22            | 0.23        | 0.29        | 0.31        | 0.34           | 0.30           |
| $A_{BV2}(0)$      | 0.22          | 0.20            | 0.18        | 0.26        | 0.23        | 0.41           | 0.36           |

\(^3\) In estimating $B(B \to MX \to M\mu^+\mu^-)$, we neglect the interference in the $B \to M\mu^+\mu^-$ rate between the SM and $X$ contributions because $X$ is very narrow, having a width of $\Gamma_X \lesssim 10^{-8}\text{GeV}$, as found earlier.
agrees with this data [30] and has an uncertainty of about 30% [33]. This suggests requiring \( B(B \to K^*X \to K^*\mu^+\mu^-) \) to be less than 40% of the central value of the measured result. Thus, with \( B(X \to \mu^+\mu^-) = 1 \), we have

\[
B(B_d \to K^{*0}X) < 5.8 \times 10^{-8}.
\]  

(45)

In addition, very recently the Belle collaboration has provided a preliminary report on their search for \( X \) with spin 1 in \( B \to \rho X \to \rho \mu^+\mu^- \) and \( B \to K^*X \to K^*\mu^+\mu^- \). They did not observe any event and reported the preliminary bounds [16]

\[
B(B_d \to \rho^0 X, \rho^0 \to \pi^+\pi^- \text{ and } X \to \mu^+\mu^-) < 0.81 \times 10^{-8},
\]

\[
B(B_d \to K^{*0}X, K^{*0} \to K^+\pi^- \text{ and } X \to \mu^+\mu^-) < 1.53 \times 10^{-8}
\]

(46)

at 90% C.L. Since \( B(\rho^0 \to \pi^+\pi^-) \simeq 1 \) and \( B(K^{*0} \to K^+\pi^-) \simeq 2/3 \), these numbers translate into

\[
B(B_d \to \rho^0 X) < 0.81 \times 10^{-8}, \quad B(B_d \to K^{*0}X) < 2.3 \times 10^{-8},
\]

(47)

the second one being more restrictive than the constraint in Eq. (45). In the absence of more stringent limits, in the following we use these numbers inferred from the preliminary Belle results. Accordingly, applying the limits in Eq. (47) to Eq. (44) yields

\[
0.00286 |g_{Vd}|^2 + |g_{Ad}|^2 < 1.3 \times 10^{-21},
\]

(48)

\[
0.00304 |g_{Vs}|^2 + |g_{Ad}|^2 < 1.3 \times 10^{-21}.
\]

(49)

The \( g_{As} \) bound implied from the last equation can be seen to be stricter than that from Eq. (37).

From Eqs. (41), (43), (48), and (49), we can then extract the individual limits

\[
|g_{Vd}|^2 < 6.5 \times 10^{-21}, \quad |g_{Ad}|^2 < 1.3 \times 10^{-21},
\]

(50)

\[
|g_{Vs}|^2 < 1.7 \times 10^{-21}, \quad |g_{As}|^2 < 1.3 \times 10^{-21}.
\]

(51)

These bounds are clearly much stronger than those in Eqs. (28) and (33) derived from \( B^0_q \bar{B}_q^0 \) mixing and \( B_0^0 \to \mu^+\mu^- \), respectively. Also, combining Eqs. (41) and (48), we have plotted the allowed parameter space of \( g_{Vd} \) and \( g_{Ad} \) in Fig. 3(a) under the assumption that they are real. Similarly, Fig. 3(b) shows the \( g_{Vs}g_{As} \) region allowed by Eqs. (43) and (49).

F. Predictions for \( B \to MX \) decays, \( M = P, V, S, A \)

We can now use the results above to predict the upper limits for branching ratios of a number of additional \( B \)-decays involving \( X \). Specifically, we explore two-body decays of \( B_{d,s}^0 \) and \( B_{u,c}^0 \) into \( X \) and some of the lightest mesons \( M \). We deal with \( M = P, V, S, \) and \( A \) in turn.

The \( g_{Vd} \) bound in Eq. (50) leads directly to limits on the branching ratios of \( B_d^0 \to \pi^0 X, \ B_d^0 \to \eta(0)X, \ B_d^0 \to K^0X, \) and \( B_c \to D_d^+X \). Thus, from Eq. (38) follows

\[
B(B_d^0 \to \pi^0 X) < 3.2 \times 10^{-8}.
\]

(52)
FIG. 3: Parameter space of $g_{Vq}$ and $g_{Aq}$, taken to be real, subject to constraints on (a) $B \to \pi X$ (lightly shaded, yellow region), $B \to \rho X$ (medium shaded, green region), and both of them (heavily shaded, red region) and (b) $B \to K X$ (lightly shaded, yellow region), $B \to K^* X$ (medium shaded, green region), and both of them (heavily shaded, red region).

Furthermore, employing Eq. (18) and Table I, with $\kappa_\eta = \kappa_\eta' = \sqrt{2}$, one gets

$$\mathcal{B}(B_d^0 \to \eta X) < 2.4 \times 10^{-8}, \quad \mathcal{B}(B_d^0 \to \eta' X) < 1.6 \times 10^{-8},$$
$$\mathcal{B}(B_s^0 \to K^0 X) < 8.2 \times 10^{-8}, \quad \mathcal{B}(B_c \to D_d^+ X) < 1.7 \times 10^{-8}.$$  (53)

Similarly, the $g_{V_s}$ bound in Eq. (51) implies

$$\mathcal{B}(B_s^0 \to \eta X) < 1.2 \times 10^{-8}, \quad \mathcal{B}(B_s^0 \to \eta' X) < 1.7 \times 10^{-8},$$
$$\mathcal{B}(B_c \to D_s^+ X) < 2.3 \times 10^{-9}.$$  (54)

where the first two numbers have been calculated using $\kappa_\eta = \kappa_\eta' = 1$, $F_{B_d}(0) = -F_{B_d}(0) \sin \varphi$, and $F_{B_d}(0) = F_{B_d}(0) \cos \varphi$ [35], with $F_{B_d}(0)$ from Table I and $\varphi = 39.3^\circ$ [36].

The $g_{Vq}$ and $g_{Aq}$ bounds in Eqs. (50) and (51), together with Fig. 3, lead to upper limits for the branching ratios of several other $B \to V X$ decays. Thus, combining Eq. (21) with the relevant form-factors in Table II yields for $q = d$

$$\mathcal{B}(B^+ \to \rho^+ X) < 1.7 \times 10^{-8}, \quad \mathcal{B}(B_d^0 \to \omega X) < 7.0 \times 10^{-9},$$
$$\mathcal{B}(B_s^0 \to K^{*0} X) < 2.2 \times 10^{-8}, \quad \mathcal{B}(B_c \to D_d^{*+} X) < 5.0 \times 10^{-9}$$  (55)

and for $q = s$

$$\mathcal{B}(B_s^0 \to \phi X) < 3.9 \times 10^{-8}, \quad \mathcal{B}(B_c \to D_s^{*+} X) < 3.9 \times 10^{-9},$$  (56)

where $|\phi| \simeq |s\bar{s}|$ has been assumed.

In contrast to the $B \to P X$ case, $g_{Aq}$ is the only coupling relevant to $B \to S X$ decays, as Eq. (17) indicates. From the $g_{Aq}$ bounds found above, we can then estimate the branching ratios of some of these decays. Since the quark contents of many of the scalar mesons below
are within the reach of near-future B measurements [21]. Adopting the form-factor values for $F_{B_d a_0(1450)}^{K^+}(0) = 0.26$ and $F_{B_d K^0_s(1430)}^{K^+}(0) = 0.26$ [37], we use Eq. (18) with $\kappa_S = 1$ for $S = a_0^0(1450), K^0_s(1430)$ and $\kappa_S = -\sqrt{2}$ for $S = a_1^0(1450)$, as well as the $g_{Aq}$ limits in Eqs. (50) and (51), to obtain

$$B(B^+ \to a_1^0(1450)X) < 1.1 \times 10^{-8}, \quad B(B^+_d \to a_1^0(1450)X) < 5.1 \times 10^{-9},$$

$$B(B^+ \to K_0^{++}(1430)X) \simeq B(B_d^0 \to K_0^{*0}(1430)X) < 1.0 \times 10^{-8}. \quad (57)$$

Similarly to the $B \to VX$ case, both $g_{Vq,Aq}$ contribute to $B \to AX$, as Eq. (20) shows. We will consider the decays with the lightest axial-vector mesons $A = a_1(1260), b_1(1235), K_1(1270)$, and $K_1(1400)$. The latter two are mixtures of the $K_{1A}$ and $K_{1B}$ states [21], namely $K_1(1270) = K_{1A} \sin \theta + K_{1B} \cos \theta$ and $K_1(1400) = K_{1A} \cos \theta - K_{1B} \sin \theta$, with $\theta = 58^\circ$, $m_{K_{1A}} = 1.37$ GeV, and $m_{K_{1B}} = 1.31$ GeV [38]. Incorporating the bounds in Eqs. (50) and (51) into Eq. (21) with $\kappa_A = 1$ for $S = a_1^0, b_1^+, K_1$ and $\kappa_A = -\sqrt{2}$ for $S = a_1^0, b_1^+$, as well as the form factors listed in Table III, we arrive at

$$B(B^+ \to a_1^+(1260)X) \simeq 2B(B_d^0 \to a_1^0(1260)X) < 1.6 \times 10^{-8},$$

$$B(B^+ \to b_1^+(1235)X) \simeq 2B(B_d^0 \to b_1^0(1235)X) < 1.2 \times 10^{-7},$$

$$B(B^+ \to K_1^{*+}(1270)X) \simeq B(B_d^0 \to K_1^{*0}(1270)X) < 2.6 \times 10^{-8},$$

$$B(B^+ \to K_1^{*+}(1400)X) \simeq B(B_d^0 \to K_1^{*0}(1400)X) < 1.3 \times 10^{-8}. \quad (58)$$

Before ending this section, we would like to make a few more remarks regarding our results above. The branching ratios of $B^+ \to \rho^+X$, $B_d^0 \to \phi X$, $B_d^0 \to K^0_s(1430)X$, and $B \to K_1X$ were also estimated in Ref. [10] under the assumption that the vector couplings $g_{Vq,Aq} = 0$. Compared to their numbers, our $B^+ \to \rho^+X$ result above is of similar order, but our numbers for $B_d^0 \to \phi X$ and $B_d \to K^0_s(1430)X$ are smaller by almost two orders of magnitude. This is mostly due to the more recent data that we have used to extract the $g_{Aq}$ values. On the other hand, our results for $B \to K_1(1270)X, K_1(1400)X$ are larger than the corresponding numbers in Ref. [10] by up to two orders of magnitude. The main cause of this enhancement is the nonzero contributions of $g_{Vq}$ to their decay rates. As one can see in Eq. (21) for the $B \to AX$ rate, the $g_{Vq}$ term in $|H^A_0|^2$ is significantly greater than the $g_{Aq}$ term in $|H^A_1|^2 + |H^A_2|^2$. For the same reason, without $g_{Vd}$, the $B \to a_1X, b_1X$ branching ratios in Eq. (58) would be orders of magnitude smaller. Thus our inclusion of the vector couplings of $X$ has not only given rise to nonvanishing $B \to PX$ decays, but also helped make most of our predicted $B \to MX$ branching ratios as large as $10^{-8}$ to $10^{-7}$, which are within the reach of near-future $B$ measurements.

| Table III: Form factors relevant to $B \to AX$ [37]. |
|-----------------|-----------------|------------------|----------------|----------------|
|                 | $B_d \to a_1(1260)$ | $B_d \to b_1(1235)$ | $B_d \to K_{1A}$ | $B_d \to K_{1B}$ |
| $A^{BA}(0)$     | 0.25             | 0.10             | 0.26            | 0.11           |
| $V_1^{BA}(0)$   | 0.37             | 0.18             | 0.39            | 0.19           |
| $V_2^{BA}(0)$   | 0.18             | -0.03            | 0.17            | -0.05          |

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IV. CONCLUSIONS

Recent searches carried out by the CLEO, BaBar, E391a, KTeV, and Belle collaborations for the HyperCP particle, $X$, have so far come back negative. Furthermore, the new preliminary result from KTeV has led to significant experimental restrictions on the $sdX$ pseudoscalar coupling in the scenario where $X$ is a spinless particle and has negligible four-quark flavor-changing interactions. In contrast, the possibility that $X$ is a spin-1 particle is not well challenged by experiment yet. In this paper, we have investigated some of the consequences of this latter possibility. Specifically, taking a model-independent approach, we have allowed $X$ to have both vector and axial-vector couplings to ordinary fermions. Assuming that its four-quark flavor-changing contributions are not important compared to its two-quark $bqX$ interactions, we have systematically studied the contributions of $X$ to various processes involving $b$-flavored mesons, including $B_q\rightarrow B_q$ mixing, $B_q\rightarrow \mu^+\mu^-$, inclusive $b\rightarrow q\mu^+\mu^-$, and exclusive $B\rightarrow M\mu^+\mu^-$ decays, with $q = d, s$ and $M$ being a spinless or spin-1 meson. Using the latest experimental data, we have extracted bounds on the couplings of $X$ and subsequently predicted the branching ratios of a number of $B\rightarrow MX$ decays, where $M$ is a pseudoscalar, vector, scalar, or axial-vector meson. The presence of the vector couplings $g_{Vq}$ of $X$ has caused the decays with a pseudoscalar $M$ to occur and also greatly enhanced the branching ratios of the decays with an axial-vector $M$. The $B\rightarrow MX$ branching ratios that we have estimated can reach the $10^{-7}$ level, as in the cases of $B_s^0\rightarrow K^0X$ and $B^+\rightarrow b^+_1(1235)X$, which is comparable to the preliminary upper limits for the branching ratios of $B_d\rightarrow \rho^0X, K^{*0}X$ recently measured by Belle. Therefore, we expect that the $B$ decays that we have considered here can be probed by upcoming $B$ experiments, which may help confirm or rule out the new-particle interpretation of the HyperCP result.

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