Cosmology in a Globally SO(1,1) Symmetric Scalar-Tensor Gravity

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A cosmological model is formulated in the context of a scalar-tensor theory of gravity in which the entire cosmic background evolution is due to a SO(1,1) doublet evolving in Minkowski space-time, such that one component of the doublet is conformally coupled, and the other (with opposite relative sign) only minimally coupled to gravitation. An interplay between the energy density of radiation and that of the kinetic energy associated with the minimally coupled component (which are of opposite relative signs) results in a non-singular, ‘bouncing’ cosmological model with essentially no horizon, flatness and anisotropy problems. Quantum excitations of this component during the matter dominated pre-bounce phase generate a flat spectrum of adiabatic gaussian scalar perturbations on cosmological scales. No detectable primordial tensor modes are generated in this scenario. Consistency with the measured amplitude of linear density perturbations at recombination implies that the bounce took place at $z = O(10^{12})$. Consequently, all features of the standard cosmological model at lower redshifts remain unchanged. Dark energy is identified with the quartic self-coupling of the scalar field.

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I. INTRODUCTION

The standard cosmological model with an early inflationary scenario has clearly been a very successful paradigm that provides a compelling interpretation of essentially all current cosmic microwave background (CMB), large scale structure (LSS) measurements, and the agreement between Big Bang nucleosynthesis (BBN) predictions and light element abundances. It is remarkable that the cosmological model provides a very good fit to extensive observational data, that sample phenomena over a vast dynamical range, using less than a dozen free parameters.

However, the essence of dark energy (DE) and cold dark matter (CDM) – two key ingredients in the model that determine the background evolution, LSS formation history, and gravitational potential on galactic scales – remain elusive. Additionally, what is considered by many as the most pristine fingerprint of cosmic inflation [1-3] – a major underpinning of the standard cosmological model – B-mode polarization of the CMB [4-6] induced by primordial gravitational waves (PGW), has not been detected. The latter is admittedly a very challenging measurement in the presence of e.g., polarized Galactic dust, nonlinear density perturbations, and instrumental systematics. In light of these hurdles and the allowed broad window for the energy scale of inflation, it is not unlikely that this signal will never be measured at sufficient statistical significance. Yet, its non-detection will not rule out the inflationary paradigm, but would rather set an (arguably very weak) upper bound on the energy scale of inflation.

According to the standard cosmological model global evolution is driven by space expansion, namely the time-dependent Hubble scale provides the ‘clock’ for the evolving properties of radiation and matter, resulting in a sequence of cosmological epochs. This clock is only meaningful if other time scales, e.g., the Planck time, or characteristic Compton times, evolve differently, particularly if they are non-varying; thus, space expansion is a relative notion.

The main objective of the present work is to demonstrate the viability of an alternative, non-singular ‘bouncing’ cosmological model within a physical framework based on a SO(1,1) symmetric scalar-tensor theory of gravity. Spacetime, effectively Minkowski, is static in this model. Cosmic evolution is then achieved by evolving scalar fields, i.e. evolution of fundamental particle masses and the Planck mass, while their dimensionless ratios remain fixed to their standard values. The time-dependent scalar field which regulates the (dynamical) masses of particles starts infinitely large, monotonically decreases until it ‘bounces’, then grow again unboundedly. In other words, the Compton wavelength associated with fundamental particles starts infinitely small, increases until it peaks at the ‘bounce’, then decreases again. Described in terms of these length ‘units’ the universe is said to undergo a ‘contraction’ phase, followed by a ‘bounce’ and ‘expansion’.

We explore a wide range of the possible ramifications of the model (albeit not exhaustively) which a priori demotes the gravitational constant, particle masses, and all other dimensional constants, from their fundamental-physical-constants status, and replaces them with the conformally-coupled component of a single SO(1,1) dynamical doublet.

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A dynamical vacuum expectation value (VEV) for the Higgs field is naturally accommodated by a Weyl-symmetric theory (which is effectively the case for the proposed model except at very near the bounce) that essentially incorporates all the fundamental interactions into this framework [7]. In particular, the standard model (SM) of particle physics, where the Higgs VEV is a fixed constant, is just one convenient gauge choice. Applying a Weyl transformation to such field configurations that appropriately endows the Higgs VEV with exactly the same dynamics of the evolving scalar field in the background cosmological model guarantees that the dynamical particle masses are continuous everywhere, exactly as in the SM and GR. Thus, a static field configuration, e.g. a planet or a galaxy, transforms into a stationary one, while observables in a planet or a star are the same as in standard physics.

While the intriguing possibility that the cosmological redshift could be explained by means of time-dependent fundamental ‘constants’ is nearly as old as (what has become) the standard expanding space interpretation [8], it has been waived off by big bang proponents as soon as it was proposed [9]. This basic idea has re-emerged later within the framework of e.g., scalar-tensor theories of gravity [10-14] and in the context of Weyl-geometry, e.g. [15].

Temporal variation of the gravitational constant, $G$, is a key feature in scalar-tensor theories, of which Brans-Dicke (BD) theory is archetypical. Standard interpretation of observational constraints, e.g. [16], usually renders this theory equivalent to GR due mainly to the convention that all other (dimensional) fundamental quantities are constant. Our approach is fundamentally different as it is based on the premise that all fundamental length scales have exactly the same dynamics which is regulated by the conformally coupled component of the doublet. In other words, the often cited $\omega_{BD} > 40000$ [16] that essentially fixes the BD scalar field (i.e. Newton’s $G$) to a constant thereby reducing the theory to GR, does not apply to the the Bergmann-Wagoner type of theory [17, 18] studied here. In particular, our approach guarantees that dimensionless observables (in a sense that will be more clearly defined below) are by construction unchanged compared to their corresponding values in the standard cosmological model in the domain of the latter validity, i.e. down to the bounce.

Throughout, we adopt a mostly-positive signature for the spacetime metric $(-1,1,1,1)$. Our units convention is $\hbar = 1 = c$. We outline our theoretical approach in section 2, and the cosmological model is presented in section 3. In Section 4 we discuss and summarize our main results.

II. THEORETICAL FRAMEWORK

Consider the following scalar-tensor theory of the Bergmann-Wagoner type [17, 18]

$$I = \int \left[-\frac{1}{6}|\phi|^2 R - \phi^T \phi + \lambda |\phi|^4 + \mathcal{L}_M(|\phi|)\right] \sqrt{-g} d^4x,$$

(1)

which differs from the BD action in that $\phi$ appears not only in the curvature and kinetic terms but also in $\mathcal{L}_M$ and $\lambda |\phi|^4$. Eq. (1) is nearly Weyl-symmetric in a sense that will be made clear below. Here and throughout, Greek indices run over spacetime coordinates. Throughout this work $f_\mu \equiv \frac{\partial f}{\partial x^\mu}$ for any function $f$.

The doublet $\phi$ is conveniently parameterized by $\rho$ and $\xi$, i.e. $\phi^T = (\rho \cosh \xi, \rho \sinh \xi)$. Inner products are performed using the metric $G_{IJ} = \text{diag}(-1,-1)$ in field space (where capital Latin letters assume the values 1 & 2) and $|\phi|^2 = \phi^T \phi = \phi^T G_{IJ} \phi^I$. Employing $G_{IJ} = \text{diag}(-1,1)$ [or even $G_{IJ} = \text{diag}(-1,-1)$ for that matter] guarantees that gravitation is attractive rather than repulsive. The lagrangian density describing the non-pure-scalar field sector, $\mathcal{L}_M = \mathcal{L}_M(|\phi|)$, is assumed to depend on $|\phi|^2 = \phi^T \phi = \rho^2$ but not on $\xi$. Thus, the latter is a free field that only minimally couples to gravity in this theory with a kinetic term of the canonical sign. The fact that $\xi$ is massless simply reflects the assumed $SO(1,1)$ symmetry. In case that this symmetry is only approximate then $\xi$ is in general massive. For the rest of this work we assume that $SO(1,1)$ is an exact global symmetry of Eq. (1). In particular, $\xi$ and its perturbations cannot be removed by conformal transformations or by local gauge transformations.

The pre-factor $\frac{1}{6}$ in the curvature term in Eq. (1) guarantees that excluding the kinetic term associated with $\xi$ the action described by Eq. (1) is invariant under the Weyl transformation (or rather local field redefinition) $g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu}$, $\rho \rightarrow \rho/\Omega$ and $\mathcal{L}_M \rightarrow \mathcal{L}_M/\Omega^4$ where $\Omega(x)$ is an arbitrary function. The kinetic term associated with $\xi$ is only dynamically important near the bounce as we explain below. Therefore, the proposed cosmological model is essentially described by a Weyl-symmetric theory – or as put by a few what only looks as such, e.g. [19-22] – through the entire cosmic history except at very near the bounce. Clearly, the single degree of freedom $\Omega$ is insufficient to simultaneously gauge-fix both $\rho$ & $\xi$. Throughout this work any reference to Eq. (1) and the field equations derived from it as ‘Weyl-symmetric’ should be understood in the sense discussed above.

The dimensionless parameter $\lambda$ appearing in Eq. (1) is the same coupling parameter appearing in the Higgs potential in particle physics, i.e. $\lambda \approx 0.129$ as inferred from the Higgs mass. Whereas the SM of particle physics is normally considered completely unrelated to gravitation we make the point in [7] that the structure of Eq. (1) can accommodate a ‘conformalised’ version of the SM.
Comparing the term $\propto R$ in Eq. (1) to the corresponding term in the Einstein-Hilbert (EH) action, while fixing $\phi$, implies the latter must be of order the Planck mass. However, $\mathcal{L}_M = \mathcal{L}_M(|\phi|)$ and therefore all particle masses are proportional to $\rho$ and although it is dynamical, dimensionless ratios of particle masses and of particle masses to the Planck mass are fixed to their SM values. This is a special property of the action described by Eq. (1) that significantly distinguishes it from other scalar-tensor theories in general, and BD in particular.

Variation of Eq. (1) with respect to $g_{\mu\nu}$ and $\phi^T$, results in the generalized Einstein equations, and scalar field equation, respectively,

$$\frac{|\phi|^2 G^\nu_\mu}{3} = -T^\nu_{M,\mu} - \Theta^\nu_\mu - \lambda \delta^\nu_\mu |\phi|^4$$  

(2)

$$\frac{\phi R}{6} - \square \phi - 2\lambda |\phi|^2 \phi + \frac{\partial \mathcal{L}_M}{\partial \phi^T} = 0,$$

(3)

and the generalized energy momentum (non-) conservation then follows, e.g. [23, 24]

$$T^\nu_{M,\mu;\nu} = \mathcal{L}_M,\phi + \mathcal{L}_M,\phi^T \phi^\nu_\mu,$$

(4)

where semicolons stand for covariant derivatives. The effective energy-momentum tensor $\Theta^\nu_\mu$ associated with the scalar fields is

$$\Theta^\nu_\mu = \frac{1}{3} \delta^\nu_\mu (\phi^T \square \phi + \phi \square \phi^T - \phi^\rho_\mu \phi^\nu_\rho) + \frac{1}{3} (2 \phi^\rho_\mu \phi^\nu_\rho - 2 \phi^T \phi^\nu_\mu - \phi^\nu_\mu \phi^T).$$

(5)

Here and throughout, $f'^\nu_\mu \equiv (f_\mu)^\nu$, the covariant Laplacian is $\square f$, and $(T_M)_{\mu\nu} \equiv \frac{2}{\sqrt{-g}} \frac{\delta (\sqrt{-g} \mathcal{L}_M)}{\delta g^\mu_\nu}$ is the energy-momentum tensor. Eq. (4), which is not independent of (2) & (3), implies that energy-momentum (of matter alone) is generally not conserved, which is expected in the case that $\Lambda$, or particle masses are space-time-dependent. Combining Eq. (3) with the trace of Eq. (2), we obtain the following consistency relation

$$\phi^T \frac{\partial \mathcal{L}_M}{\partial \phi^T} + \phi \frac{\partial \mathcal{L}_M}{\partial \phi} + T_M = 0,$$

(6)

which is indeed consistent with a traceless energy-momentum tensor but only in case that the matter Lagrangian density is independent of the scalar field, indeed an often-made assumption (for example in general relativity or BD theory where all particle masses are fixed) that we relax in the present work. It will be argued in section 3.1 that gravitation in the framework described by Eq. (1) is sourced only by the potential term of the $\phi$-dependent matter Lagrangian, while the kinetic term is in certain gauges the ‘curvature’ term itself. Since $\mathcal{L}_M = \rho_M$ it immediately follows from Eq. (6) that

$$\rho_M \propto |\phi|^{1-3w}.$$  

(7)

Here, $w$ is equation of state (EOS), and $\lambda |\phi|^4$ has been absorbed in $\mathcal{L}_M$ with an effective EOS parameter $w = -1$. As expected, $\rho_M$ is a quartic potential in the case $w = -1$, is independent of $|\phi|$, i.e. of masses, in the case $w = 1/3$, and linear in masses in case of NR fermions, i.e. the $w = 0$. In the case of ‘stiff’ matter, $w_{\text{stiff}} = 1$, $\rho_M \propto |\phi|^{-2}$. We assume that no contribution to the perfect fluid, described by $\mathcal{L}_M$, is characterized by a sound speed $c_s > 1/\sqrt{3}$, i.e. $w > 1/3$.

As mentioned above, it has been argued that the Weyl-symmetric action, Eq. (1), is obtained in the case of a singlet field ($\xi = 0$) from the EH action by merely redefining the metric and scalar fields, that there are no conserved currents associated with the symmetry of Eq. (1), and that consequently this is a ‘sham’ or ‘fake’ Weyl-symmetry, e.g. [19-22]. Indeed, in the next section we illustrate this by going from the standard Friedmann-Robertson-Walker (FRW) action to Eq. (1) in the case of a singlet by redefining fields. However, by adding another degree of freedom that is effectively only minimally-coupled to gravity (more specifically a free scalar field) to form a doublet, the fields appearing in Eq. (1) cannot be generally redefined to recover the EH action; the two actions are clearly inequivalent in this case. Here we only point out that integrating both Eqs. (2) & (3) results in additional integration constants absent from GR. This will be addressed more concretely elsewhere [25].

### III. COSMOLOGICAL MODEL

In this section the background evolution, the evolution of linear perturbations, and a singularity-free early universe scenario are described. The longitudinal mode of the doublet, $\rho$, has essentially the same dynamics that the scale factor $a(\eta)$ has in standard cosmology (except near the bounce, where the dynamics of $\xi$ significantly alters that of $\rho$). The field $\xi$ and its perturbations are responsible for the bounce and the flat power spectrum of density perturbations, respectively.
A. Recasting FRW as a Scalar-Tensor Model and its Implications

To motivate the construction in the following sections of a cosmological model based on Eq. (1) we work out the equivalence between the FRW action and Eq. (1) in the homogeneous and isotropic case in this section.

The issue of scalar fields characterized by negative kinetic energies as in Eq. (1) is characteristic of either general relativity (GR) or other theories of gravitation ‘dressed’ with conformal symmetry, e.g. [26, 27]. This is usually interpreted as an indication for instability of the theory, e.g. [28-30], and was at the heart of an intense debate regarding the physical equivalence of theories in the Einstein frame and Jordan frame where the latter was considered by many as ‘unphysical’, in spite of the fact of being classically equivalent to the former, e.g. [31, 32], while others maintained that at least classically the theory with negative kinetic energy of the scalar field is stable. Indeed, the latter is the case with the classical FRW action, as we see below. Some even speculated that in a semi-classical context, when $\phi$ is quantized and the spacetime metric is treated as a classical field, stability may still be maintained, e.g. [33]. But, common lore holds it that if attempted to be quantized this ‘ghost’ field causes a catastrophic production of particles due to the unlimited phase space available for such processes to take place. We thus conjecture that $\phi$ is unquantized, i.e. that it is always found in its particle vacuum state (its energy states are unquantized) – a conjecture that could be better motivated by the following example. Again, we assume that $\phi$ of Eq. (1) is always classical and not only in the cosmological context [7].

Consider the EH action $I_{EH} = (2\kappa)^{-1} \int (\mathcal{R} - 2\Lambda + \tilde{\mathcal{L}}_M) \sqrt{-g} d^4x$ with a cosmological constant $\Lambda$ and a matter Lagrangian density $\tilde{\mathcal{L}}_M$. Here, $\kappa = 8\pi G$ and $\tilde{\mathcal{R}}$ is derived from the metric $\tilde{g}_{\mu\nu}$ in the usual way. The FRW action

$$I_{FRW} = (3/\kappa) \int [-a'^2 + Ka^2 - \Lambda a^4/3 + a^4\tilde{\mathcal{L}}_M(a)/6] \sqrt{-g} d^4x,$$

is obtained for the metric $\tilde{g}_{\mu\nu} = a^2 diag(-1, 1/Kr^2, r^2, r^2 \sin^2 \theta)$, and after the term proportional to the corresponding curvature scalar $\tilde{\mathcal{R}} = 6(a''/a + K)/a^2$ is integrated by parts. Here, $K$ is the spatial curvature, a prime denotes derivatives with respect to conformal time, and the time coordinate in the volume element $d^4x$ is conformal. One can readily verify that the Euler-Lagrange equation for the scale factor $a(\eta)$ that extremizes the action $I_{FRW}$ is indeed the Friedmann equation $\mathcal{H}^2 + K = a^2\rho_M + \lambda a^4/\kappa + \text{const.}/a^2$, where $\mathcal{H} \equiv a'/a$ is the conformal Hubble function, $\rho_M \equiv -\frac{1}{8\pi G} \tilde{\mathcal{L}}_M$, and it is assumed that $\tilde{\mathcal{L}}_M(a)$ is a power-law in $a$, as $\tilde{\rho}_M$ is a power-law in standard cosmology. Completing the derivation requires that the integration constant is related to the energy density of radiation $\rho_r = \text{const.}/a^4$.

Defining $\rho^2 \equiv 3a^2/\kappa$, the FRW action is reformulated as a Weyl-symmetric scalar-tensor action with a non-positive kinetic term, defined on a static background

$$I_{FRW} = \int \left( \frac{1}{6} R \rho^2 - \rho^2 - V(\rho) + \tilde{\mathcal{L}}_M(\rho) \right) \sqrt{-g} d^4x.$$

Here, $g_{\mu\nu}$ is the static metric related to the FRW metric via a Weyl transformation, $g_{\mu\nu} \equiv \tilde{g}_{\mu\nu}/a^2 = \text{diag}(-1, 1/Kr^2, r^2, r^2 \sin^2 \theta)$, the 4D infinitesimal volume element is, e.g. $\sqrt{-g}d^4x = \sinh^2 \chi \sin \theta d\eta d\chi d\theta d\varphi$, where $\chi$ is a ‘radial’ coordinate in the hyperbolic coordinate system (assuming $K < 0$ for concreteness), $V(\rho) \equiv \chi \rho^4$ is an effective potential, $\mathcal{L}_M = a^2\tilde{\mathcal{L}}_M$, $R = 6K$ and $\lambda \equiv \kappa\Lambda/9$. The standard FRW spacetime is indeed described by a single degree of freedom, the scalar field $a(\eta)$, which is here replaced by $\rho(\eta)$, and therefore ignores the crucial role played by $\xi(\eta)$ as we will see below. Also, in standard cosmology, it is assumed that $a(\eta)$ has no perturbations, but rather only $g_{\mu\nu}$ is perturbed. We follow suit and assume that $\delta \rho = 0$ throughout, compatible with the choice $\delta a = 0$ in standard cosmology. This guarantees that the two descriptions applied to the late time universe are in agreement.

We relax this assumption and describe its far reaching ramifications in a follow up work [25].

This example illustrates that a well-defined solution of the Einstein field equations is derived from an action which is essentially equivalent to Eq. (1), i.e. with a ‘wrong’ sign of the kinetic term. The wrong sign of $a^2$ appearing in Eq. (8) is never considered a problem (a part is of the metric field) and similarly the sign of $\rho^2$ should not alarm us, provided that $\rho$ is a genuinely classical field; same way that $a$ is a classical function in the standard formulation of the FRW solution so should be $\rho$. We emphasize that unlike in standard cosmology where $\phi_{\mu\nu}$ and $\tilde{\mathcal{R}}$ are dynamical, the latter being divergent at the initial singularity, the curvature scalar $R = 6K$ is fixed. A Weyl transformation applied to Eq. (1) affects both the metric and scalar field to the extent that the dynamics of the metric (scale factor in our case) could be fully replaced by that of the scalar field in the the FRW spacetime which is only described in terms of a single degree of freedom. In standard cosmology the kinetic term associated with the inflaton field appears with the canonical sign and thus could be quantized. In the present framework a similar procedure is followed; whereas $\rho$, the analog of the scale factor $a$, is unquantized, the other member of the doublet, $\xi$, effectively appears in the action with the canonical sign and is therefore amenable to quantization.
Unlike $\rho(\eta)$ which is characterized by a vanishing current [22], the minimally-coupled field $\xi$ does have a conserved ‘charge’ associated with it which plays a central role in the present cosmological model as is shown below. We express the view in this work that the scale factor $a$ in the GR-based formulation of the FRW cosmology is not more genuine or fundamental degree of freedom than the scalar field $\rho$. Indeed, the scale factor $a(\eta)$ is not an observable and it is conventionally fixed to unity at present $a(\eta_0) = 1$, but its logarithmic derivative, the conformal Hubble function $\mathcal{H} = a'/a$, is. In analogy, $Q \equiv \rho'/\rho$ is an observable in the present reformulation of the FRW model, and can be viewed as the rate of variation of particle masses and the Planck mass, i.e. $m'/m = O(10^{-18})$ sec$^{-1}$ at present, on a static background space.

**B. Evolution of the Cosmological Background**

According to the alternative scenario proposed here a single doublet does not only resolve the flatness and horizon puzzles, but also accounts for scale-free density perturbations. The topological relics problem (the ‘magnetic monopole problem’) does not arise in the first place; as we see below, the highest redshift attainable in our model is $z \gtrsim 10^{12}$ which is equivalent to $O(100)$ MeV in standard units.

The Einstein tensor components $G^\mu_\nu$, associated with the metric $g_{\mu\nu} = \text{diag}(-1, \frac{1}{1-K\tau^2}, r^2, r^2 \sin^2 \theta)$, in conformal rather than cosmic time coordinates, are $G^0_0 = -3K$ and $G^i_i = -K \delta^i_i$. In the following $f' \equiv \partial f/\partial \eta$ denotes the derivative of a function $f$ with respect to conformal time $\eta$. Here, $i, j$ indices stand for the spatial coordinates. The energy-momentum tensor of a perfect fluid is $(T_M^\mu_\nu = \rho_M \cdot \text{diag}(-1, w, w, w)$, and employing Eq. (7) in Eqs. (2) & (3) we obtain that

\begin{align}
Q^2 + K &= \frac{\rho_M}{\rho^2} - \xi^2 \\
2Q' + Q^2 + K &= -\frac{3w\rho_M}{\rho^2} + 3\xi^2, 
\end{align}

where $Q \equiv \rho'/\rho$ is the conformal Hubble-like function. In case that $\xi = 0$ these reduce to the FRW equations but with $H$ replaced by $Q$. This implies that in the radiation-dominated (RD) or matter-dominated (MD) evolutionary phases, $\rho \propto \eta^{2(1+3w)}$ is a monotonically increasing function of conformal time. This continue to be the case insofar $w > -1/3$. The evolution of the Rydberg ‘constant’ thus explains the observed cosmological redshift on this static spacetime. The ‘Friedmann equations’, Eqs. (10) & (11), are augmented with $\propto \xi^2$ terms, which come from $\Theta^2$, the effective energy-momentum tensor associated with the kinetic term of the scalar field (Eq. 5). These terms are comparable in size (but of opposite sign) to other contributions to the energy budget of the universe only near the bounce and are negligible at all other times as is shown below.

Whereas $\rho(\eta)$ essentially replaces the scale factor $a(\eta)$ of the FRW model, $\xi$ is a new field. As is already mentioned and as shown in this section and in 3.4, inclusion of a non-vanishing $\xi$ in our cosmological model serves two purposes. First, it could be used to avoid the initial singularity by explicitly violating the weak, strong, and null energy conditions. Second, it couples excitations of $\xi$ to scalar metric perturbations, i.e. to perturbations of the gravitational potential ultimately resulting in a flat power spectrum. This mechanism has no parallel in the tensor perturbations sector and consequently no primordial generation of B-mode polarization on cosmological is expected.

Now, to close our system of background equations, the evolution of $\xi$ is governed by (Eq. 3)

$$\xi'' + 2Q \xi' = 0,$$

i.e. $\xi' = c_\xi/\rho^2$, where $c_\xi$ is an integration constant. Plugging $\xi' = c_\xi/\rho^2$ into Eqs. (10) & (11), and comparing with Eq. (7), shows that the $\propto \xi^2$ terms effectively play the role of a ‘stiff matter’ contribution with $w_\xi = 1$. This contribution to the energy budget would have dominated the cosmic evolution at early epochs, i.e. in the small field limit $\rho/\rho_0 \ll 1$, was it positive. However, from Eq. (10) it is clear that $\rho_\xi \equiv \rho^2 \xi^2$ is negative. Assuming no bounce takes place during the MD era, but if it ever takes place it does so deep into the RD era, then only when $\rho_\xi$ becomes comparable to the energy density of radiation does a ‘bounce’ take place, after which $|\rho_\xi|$ drops faster than any other form of energy density considered in this model (we assume a theoretical upper limit of $w \leq 1$).

The proposed scenario is non-singular and the cosmic history comprises of contracting and expanding evolution phases. The former, ‘deflationary’ epoch, is nearly symmetric in this scenario to the post-bounce expanding phase. It starts with a vacuum-like energy-, followed by MD, RD, and a brief era dominated by a mixture of radiation and ‘stiff’ energy density, a bounce, and an expansion with these various pre-bounce eras occurring in reverse order.

Accounting for the vacuum-like, nonrelativistic (NR), radiation and effectively stiff energy densities, Eq. (10) implies that

$$Q^2 = \lambda \rho^2 + \lambda_p n_{NR}/\rho + \rho_\rho_\xi/\rho^2 + \rho_{\xi}\rho/\rho^4,$$
where $\rho_{i,*}$ is the energy density associated with the $i$’th species at $\eta_*$, $\rho(\eta_*) \equiv 1$, $n_{NR}$ is the number density of NR particles (assuming a single species for illustration purposes), and $\lambda_\nu$ is a dimensionless parameter. The full analytic integration of this equation is not very illuminating, and therefore we treat two interesting limits separately that will suffice for our purposes. The first limit is obtained by neglecting the vacuum-like and NR terms near the bounce. In this case, the Friedmann-like equation integrates to

$$\rho^2 = \rho_{r,*} \eta^2 - \rho_{r,*}/\rho_{r,*}$$

(14)

where $\rho$ attains its minimum at $\eta = 0$. The latter could be readily integrated to give the cosmic time around the bounce, $t \propto \int \rho(\eta) d\eta$, and is easily verified to be non-singular as well. In other words, the effective time coordinates of both massless and massive particles can be extended through the bounce, i.e. spacetime is geodesically-complete. The causal horizon is thus larger than naively thought thereby avoiding the horizon problem, as is generically the case in non-singular bouncing cosmological models, [34].

In the other extreme – where the background dynamics is dominated by the quartic potential $\rho = \left(\sqrt{\chi(\eta_c - \eta)}\right)^{-1}$, and $\rho$ scales according to its canonical dimension $\text{length}^{-1}$, i.e. $\propto \eta^{-1}$, not $\propto t^{-1}$. This again highlights the privileged role played by conformal time as compared to cosmic time, in contrast to the standard cosmological model where conformal time is only used for computational convenience, or as the natural time coordinate parameterizing null geodesics. The integration constant $\eta_c$ determines the lower and upper limit on the (conformal) time coordinate in the proposed nearly symmetric bouncing model, $\eta \in (-\eta_c, \eta_c)$.

One may argue that instead of working with Eq. (1) applied to a homogeneous and isotropic spacetime as done in the present section, we could have equally well used the EH action, Eq. (8), supplemented by $\mathcal{I}_\xi = (3/8) \int (a^2 \xi^2 \sqrt{-g} d^4 x$, in formulating the cosmological model described in the present section on an expanding background space. However, this ad hoc procedure – while being mathematically equivalent to the approach taken here – seems less natural to us than the $SO(1,1)$ symmetric model outlined here and formulated on a static background space.

The ‘flatness problem’ arises in the hot big bang model due to the monotonic expansion of space and the consequent faster dilution of the energy density of matter (either relativistic or NR) compared to the effective energy density dilution associated with curvature. It is thus hard to envisage how could space be nearly flat (as is indeed inferred from observations, e.g. [35]) if not for an enormous fine-tuning at the very early universe, or an early violent inflationary era. In the proposed bounce scenario the matter content of the universe has always existed, and in particular the present ratio of matter - curvature-energy densities has been exactly the same when $\rho$ in the pre-bounce was equal to its present (post-bounce) value. However, in the pre-bounce phase matter domination over curvature is actually an attractor point as the contracting universe starts essentially from $\rho \to \infty$. In other words, had the universe been curvature-dominated (CD) at present (as is naively expected in the standard expanding hot big bang model but with no inflation), i.e. at $\rho_0$, it must have been CD at $\rho_0$ at the mirror pre-bounce phase, but since $\rho_M \propto \rho$ while $\rho_K \propto \rho^2$ then a CD domination at $\rho_0$ in pre-bounce would amount to an extremely fine-tuned $\rho_M/\rho_K \to 0$ at $\rho \to \infty$ at the pre-bounce phase.

As is well-known, entropy produced in the pre-bounce era could be processed at the bounce to thermal radiation, implying in effect that $\rho_i$ might somewhat change between pre- and post-bounce but the expectation that the contracting and expanding phases nearly mirror each other is not significantly changed – not at any rate that might change the conclusion regarding the (un-) naturalness of CD era at the present.

The kind of argument employed here in explaining away the flatness problem can be reversed to show that anisotropy actually does enormously grow relative to the other energy species in the contraction universe. In terms of standard cosmology $\rho$ represents some (geometric) average scale factor and $\xi$ determines deviation from isotropy, such as in Bianchi-type cosmological models. In bouncing models this anisotropy is a measure of the variation of expansion or contraction rates between the principal axes of the homogeneous model, which change very fast near the bounce, the Belinskii-Khalatnikov-Lifshitz (BKL) instability, i.e. that anisotropy is a natural attractor in contracting cosmologies, and in order to avoid it an enormous fine-tuning of initial conditions has to be invoked, e.g. [36]. The analog of this would be the extreme variation over time of $\xi$ around the bounce. We will argue below that our model is free of these illnesses that generically afflict bouncing cosmological models. It should be emphasized that slightly anisotropic expansion or contraction could be mimicked by a stiff matter but the opposite is not true – an effective stiff EOS does not necessarily imply anisotropic evolution. For example, in the proposed model space is static and isotropic and the evolution is only in the scalar field, not space. An effective anisotropy, or in other words, an effective ‘stiff’ component ($w = 1$) could arise, e.g. from terms in the Lagrangian Eq. (1) which are $\propto \phi^{-2}$. However, Weyl symmetry that prohibits the appearance of any dimensional quantity at the action level, severely limits the existence of such a term. It could appear in the form, e.g. $\Delta \mathcal{L} = (\dot{\psi}^2 - \phi^2)^{-1}$, or higher-curvature terms of the form $\propto |\phi|^{-2} \epsilon_{\alpha\beta} \epsilon_{\gamma\delta} C^{\alpha\beta}_{\gamma\delta} C^{\gamma\delta}_{\alpha\beta}$. The second term vanishes in the case of conformally-flat metric $g_{\mu\nu}$, as the one employed here. Perhaps more important, in the same fashion that no stiff matter component seems to be required in standard cosmology to explain observations
we ignore such terms in the proposed model, and in general do not allow non-canonical negative powers of the scalar field to appear in the action for a lack of an otherwise good reason to allow such terms.

C. Linear Perturbation Theory

The standard cosmological model has successfully passed numerous tests and has been quite effective in explaining the formation and linear growth of density perturbations over the background spacetime, predicting the CMB acoustic peaks, polarization spectrum, and damping features on small scales. It also correctly describes the linear and nonlinear evolution phases of the LSS (on sufficiently large scales) and abundance of galaxy and galaxy cluster halos. Therefore, it would seem essential to establish equivalence of linear perturbation theory between our model and the standard cosmological model.

As in section 3.2, we assume an effective energy density \( \rho_M \) characterized by a (generally time-dependent) EOS \( w = w(\eta) \) that encapsulates NR and relativistic baryons, radiation, and a vacuum-like energy density. In the following, \( \varphi \) and \( \alpha \) are gravitational potentials appearing in the rescaled perturbed FRW line element \( ds^2 = -(1 + 2\alpha) d\eta^2 + (1 + 2\varphi) \gamma_{ij} dx^i dx^j \) where \( \gamma_{ij} = \text{diag}[1/(1 - K r^2), r^2, r^2 \sin^2 \theta] \). The fractional energy density and pressure perturbation (in energy density units) are \( \delta_\rho_M = \delta \rho_M/\rho_M \) and \( \delta P_M = \delta P_M/\rho_M \), respectively. The matter velocity is \( v \).

In the shear-free gauge \( \chi = 0 \), and in the case of vanishing curvature \( K \) and stress anisotropy, i.e. \( \varphi = -\alpha \), the Arnowitt-Deser-Misner (ADM) energy & momentum constraints, Raychaudhuri equation, \( \delta \xi \) equation, and the perturbed continuity & Euler equations (Eqs. 39, 40, 42, 43, 48 & 49 of [23]) reduce to

\[
Q \varphi' + \left( \frac{k^2}{3} + Q^2 + \xi'^2 \right) \varphi = \frac{1}{2} (Q^2 + \xi'^2) \delta \rho_M - \xi' \delta \xi' \\
\varphi' + Q \varphi = - \frac{3(1 + w)}{2} (Q^2 + \xi'^2) u + 3 \xi' \delta \xi \\
\varphi'' + 2Q \varphi' + \left[ 2Q' - 4\xi'^2 - \frac{k^2}{3} \right] \varphi = - \frac{(1 + 3w)}{2} (Q^2 + \xi'^2) \delta \rho_M + 4 \xi' \delta \xi' \\
\delta \xi'' + 2Q \delta \xi' + k^2 \delta \xi = - 4 \xi' \varphi' \\
\delta \rho_M' + (1 + w)(3 \varphi' + k^2 u) = 0 \\
u' + (1 - 3w) Qu + \frac{w'u}{1 + w} = - \varphi' + \frac{w \delta \rho_M}{1 + w},
\]

where \( u \equiv v/k \) and we used Eq. (10) to eliminate \( \rho_M \) and \( \delta \rho = 0 \) identically, as explained in section 3.1. These are exactly the linear perturbation equations over an FRW background if we make the replacement \( Q \rightarrow H \) and recalling the conclusion from section 3.2 that \( Q \) is exactly \( H \) except of very near the bounce, where perturbations are negligibly small anyway.

Vector and tensor perturbations, which are described by Eqs. (53)-(55) and (58) of [23] respectively, similarly satisfy the same equations they do in GR, provided that \( a \rightarrow \rho \), i.e. \( H \rightarrow Q \) and the sources, e.g. anisotropic stress \( \pi^{(t)} \), are correspondingly rescaled by a multiplicative factor \( \rho^4 \), e.g. \( \pi^{(t)} \rightarrow \pi^{(t)} \rho^4 \).

The full kinetic theory, pertaining to the theory described by Eq. (1) applied to homogeneous and isotropic backround cosmology, involving collisional photons and collisionless neutrinos, e.g. [23], where the corresponding perturbed energy-momentum tensor components are given in terms of integrals over the respective distribution functions \( f \), can be easily incorporated in our scheme with minor adjustments; neutrino masses are \( \propto \rho \), the dynamical Higgs VEV, and this has to be accounted for in the collisional Boltzmann equation.

The Newtonian limit of the gravitational interaction in the framework of Eq. (1) is obtained from Eqs. (15)-(20) by setting \( Q, K, w, \) and \( \xi' \) to zero. In particular, Eqs. (15), (19) & (20) are the relativistic Poisson, continuity, and Euler equations, respectively.

In standard cosmology, a gravitational potential consistent with Eqs. (15)-(20) in the RD era, i.e. \( w = 1/3 \), is \( \varphi = A(k) j_1(q)/q + B(k) y_1(q)/q \) where \( q \equiv \sqrt{x} \), with \( x \equiv k \eta \). Assuming the two modes have been generated at approximately equal amplitudes at some \( q \ll 1 \) during the inflationary phase then the mode that diverges at \( q = 0 \) is negligible at later times. Consequently, the initial condition \( B(k) = 0 \) is selected in standard cosmology and \( \varphi \) is essentially constant at \( q \rightarrow 0 \). In the present model the argument is different; since the model is extended through the bounce to \( \eta < 0 \) this diverging mode which is an odd function of \( \eta \) near the bounce is set to zero by the mere requirement that \( \varphi \) is a continuous function of time, at \( \eta = 0 \) in particular. Hence, the adiabatic initial condition merely follows from the very existence of a non-singular bounce and continuity. A similar argument applies to the tensor modes; the mode singular at \( \eta = 0 \) is an odd function of \( \eta \) and is consequently discontinuous at \( \eta = 0 \).
It has been recently proposed that the universe has a CPT ‘anti-universe’ counterpart [37]. The two universes according to this picture share a common topological singularity at $\eta = 0$. In contrast, as is shown in section 3.4 below, our model is characterized by a non-singular ‘bounce’. In [37] it is shown that the adiabatic initial conditions could naturally result from time-reversal symmetry which is inherent to their model, a conjecture that is obviously not satisfied by the perturbed FRW universe in the scenario proposed here (but is clearly satisfied at the background level of the present model). Thus, rather than conjecturing a global time-reversal symmetry we make a more modest and natural requirement from our model – pointwise continuity. This implies in particular that the integration constants multiplying perturbation modes which are divergent odd functions of $\eta$ must vanish.

D. Primordial Flat Spectrum and Bounce

Although inflation provides a mechanism for generating scalar and tensor perturbations which are characterized by nearly-flat power spectra, it is not a prediction of the inflationary scenario; it has been known for nearly a decade before the advent of inflation that at least the density perturbations are described by a nearly flat spectrum [38, 39], and primordial tensor modes have not been detected yet anyway. Other early universe scenarios, e.g. the varying speed of light cosmology [40, 41], the ekpyrotic [42] and new ekpyrotic [43] scenarios, the cyclic universe [44, 45], string gas cosmology [46], Anamorphic cosmology [47], and pseudo-conformal universe [48, 49], are capable of explaining the observed flat spectrum as well.

It is well known that flat spectra are equally well generated from fluctuations of scalar fields and tensor mode perturbations during the MD contraction phase in bouncing scenarios [50]. Specifically, the amplitudes of the power spectra are $O(H^2)$ (where $H$ is measured in Planck units) which during MD contraction would result in essentially vanishingly small amplitudes for both scalar and tensor perturbations provided that the proposed scenario is symmetric around the bounce or is nearly so. Thus, no primordial horizon-scale tensor modes are expected in the proposed scenario. Alternatively, here we consider the massless transversal perturbation of the scalar field, $\rho \delta \xi$ (accounting for its minimal coupling to scalar curvature perturbations) as a viable source for scale-invariant density perturbations.

Combining Eqs. (15) & (17) we obtain for an arbitrary $w$ the equation

$$\varphi'' + 3(1 + w)Q \varphi' + wk^2 \varphi = 3(1 - w)\xi \delta \varphi'. \quad (21)$$

Using Eq. (18) to express $\varphi'$ in terms of $\delta \xi$ and its derivatives, taking the time-derivative of Eq. (21), and employing the relation $\xi' \sim \rho^{-2}$, i.e. $\xi''/\xi' = -2Q$, and $Q = \frac{2}{1 + 3w/\eta}$ (by virtue of the Friedmann equation assuming the dynamics is dominated by a species characterized by $w = \text{constant}$, a very good assumption throughout the cosmic history except at very brief transitions between the various epochs), we obtain the following fourth-order equation for $\delta \xi$

$$\delta \xi_{xxxx} + \frac{6(3 + w)}{(1 + 3w)^{6}x} \delta \xi_{xxx} + \left[1 + w + \left(\frac{80}{(1 + 3w)^{2}} - 2\right) \frac{1}{x^{2}}\right] \delta \xi_{xx}$$

$$+ \left[\frac{24(3 + w)}{(1 + 3w)^{4}x^{8}} + \frac{2(7 + 5w)}{(1 + 3w)^{2}x^{3}}\right] \delta \xi_{x} + \left[\frac{32}{(1 + 3w)^{2}} - 2\right] \frac{1}{x^{2}} \delta \xi = 0, \quad (22)$$

where again $x \equiv k\eta$, and small $\xi^{2}$ terms have been neglected. The latter is a very good approximation except for the immediate vicinity of the bounce. We have not been able to analytically solve Eq. (22) for arbitrary $w$. Specializing to the case $w = 0$ results in

$$\delta \xi = \frac{1}{x^{3}} \left[c_{1} + c_{2}e^{ix}(1 - ix) + c_{3}e^{-ix}(1 + ix)\right] + \frac{c_{4}e^{ix}}{x^{4}} \left[e^{2ix}(1 - ix)x^{5}\text{ExpIntegralEi}(-ix)\right]$$

$$+ i \left(4e^{ix}(72 - 4x^{2} + x^{4}) + x^{5}(x - i)\text{ExpIntegralEi}(ix)\right). \quad (23)$$

As usual, we impose the Bunch-Davis condition in the limit $x \gg 1$, i.e. $\rho \delta \xi \to \frac{1}{\sqrt{2k}}e^{-ikx}$. Employing $\rho = \rho_{0}(\eta/\eta_{0})^{2}$ in the MD era then implies that $c_{3} = \frac{\sqrt{2k/\eta_{0}^{3}}}{\sqrt{2k\rho_{0}}} e^{-ikx}$ and all other integration constants vanish. Employing Eq. (12), $\xi' = c_{\xi}/\rho^{2}$, in Eq. (21) we obtain in the limit $x \ll 1$

$$\varphi'' + \frac{6\varphi'}{\eta} = \frac{C}{\eta^{8}}, \quad (24)$$

where $C \equiv (-9c_{\xi}/\eta_{0})/(\sqrt{2k^{3/2}\rho_{0}^{3}})$. Its solution is $\varphi = \tilde{c}_{1} + \tilde{c}_{2}\eta^{-5} + (C/6)\eta^{-6}$. In the contraction phase the fastest growing mode is the $\propto \eta^{-6}$ term (assuming all modes are generated at approximately similar amplitudes, and neglecting
the $\propto \eta^{-5}$ term that is discontinuous at $\eta = 0$) and thus $\varphi = -9c_\xi/(6ik\sqrt{2k}\rho^3)$, which implies that

$$
\Delta^2 = \frac{k^4|\varphi|^2}{2\pi^2} = \frac{9\lambda_\rho}{16\pi^2\rho_{DE}}
$$

is flat. Perturbations generated earlier on during the MD contracting phase are relatively smaller at the time of production than those which are generated latter since $\rho_\xi$ is much smaller than $\rho_{DE}$ at large $\rho$ values. However, it has a longer time to grow than the perturbations generated closer to the bounce. Overall, the perturbations observed today are equally contributed at all times during pre-bounce MD era.

It should be stressed that unlike the mechanism proposed in [50] which reflects the fact that any field fluctuates on an evolving background (and with rms fluctuation determined by the Hubble scale), the curvature perturbations discussed here are a direct result of the coupling between $\xi$ and $\varphi$ (and the amplitude of scalar perturbations is therefore $\propto \rho_\xi$). There is no tensorial analog in our scenario to this coupling, and naively applying the treatment given by [50] to tensor perturbations in the proposed scenario results in signals many orders of magnitude smaller than the theoretical floor level for detection [51].

Repeating the same procedure in the RD era ($w = 1/3$), and imposing the Bunch-Davis vacuum on $\rho\delta\xi$ in the $x \gg 1$ limit eliminates one of the four integration constants. In the other limit, $x \ll 1$, we obtain $\delta\xi \propto \eta^{-4}$. Since $\xi' = c_\xi\rho^2$ and since in the RD era $\rho = \rho_0\eta/\eta_0$, then neglecting the $w k^2\varphi$ term in Eq. (21) we obtain

$$
\varphi = \frac{\beta}{\eta^3} + c_1\frac{\eta}{\rho_0} + c_2
$$

in the $x \ll 1$ limit, where $c_1$, $c_2$ & $\beta$ are independent on $\eta$ but possibly depend on $k$. The constant $\beta$ is also proportional to $c_\xi$. Again, the mere requirement of continuity at all times, and at $\eta = 0$ in particular, implies that $\varphi = c_2(k)$, and consequently Eq. (25) is not modulated during the RD era.

Inflation generically predicts a slightly red power spectrum. The Harrison-zeldovich spectrum $\Delta^2$ in Eq. (25) is rolled out by recent observations at the $\sim 7\sigma$ confidence level [35] assuming the vanilla $\Lambda$CDM cosmological model and provides yet another evidence for the inflationary scenario. However, there have been raised some doubts concerning the robustness of this conclusion in light of the recently claimed tension in inference of Hubble constant from cosmological data and local universe measurements, e.g. [52] and references within. More specifically to our model, taking the results of [35] at face value the model proposed here would have to be modified, probably via introducing cosmological data and local universe measurements, e.g. [52] and references within. More specifically to our model, taking the results of [35] at face value the model proposed here would have to be modified, probably via introducing a cosmological scale that breaks the Weyl invariance of Eq. (1). This possibility will be explored elsewhere.

Assuming that the bounce took place well into the RD era, using $\frac{\rho_\varphi}{\rho_{DE}} = \left(\frac{\rho_\varphi}{\rho_{DE}}\right)_{\eta_0} (\frac{\eta}{\eta_0})^6 = (1 + z)^6$, that the redshift for radiation-matter equality is $z_{eq} \sim 3400$ [35], that the gravitational potential perturbations only grow during the MD pre-bounce (and are frozen at the corresponding expansion phase), and adopting the observationally inferred normalization on super-horizon scales $\Delta^2 = 2.4 \times 10^{-9}$ [35] in Eq. (25) we obtain the ratio $\left(\frac{\rho_\varphi}{\rho_{DE}}\right)_{\eta_0}$. In addition, assuming that the bounce took place at $z < 10^{13}$ (when protons only start becoming relativistic) we obtain the bounce redshift $z_b \sim 8.6 \times 10^{11}$ from the requirement that at the bounce the energy density of radiation and that of ‘stiff matter’ are momentarily equal, i.e. $|\rho_{\varphi}|(1+z_b)^6 = \rho_{\varphi}(1+z_b)^6$. Here we assumed that radiation consists of the CMB photons, three families of (essentially massless) background neutrinos, and relativistic electrons with $\Omega_e \sim \frac{m_e^2}{m_p^2} \Omega_b$. This estimate is robust; for protons to be fully relativistic at the bounce and contribute to $\rho_r$, i.e. $z_b > 10^{13}$, then their relative contribution to the relativistic energy density is $\approx 10^{-14}\frac{\Omega_e}{\Omega_b}$, i.e. relatively negligible, and the bounce indeed takes place at $z_b \lesssim 10^{12}$. Consequently, the well-established physics of recombination and BBN is unaffected by the relatively late bounce (compared to typical Planck scale bounce models). Since $\varphi$ is linearly related to $\delta\xi$ via e.g. Eq. (21), and assuming all perturbations are linear, the former automatically inherits the statistical gaussian properties of the latter; scalar perturbations are thus expected to be gaussian and adiabatic (the latter property has been discussed in section 3.3).

IV. SUMMARY

While the standard cosmological model has no doubt been very successful in phenomenologically interpreting a wide spectrum of observations, it is fair to say that it still lacks a microphysical explanation of several key features, primarily the nature of DM and DE. Direct spectral information on the CMB is unavailable (due to opacity) in the early radiation-dominated era ($z \gg 3000$). From the observed cosmic abundance of light elements, BBN at redshifts $O(10^3)$ could be indirectly probed. Earlier on, at $z = O(10^{12})$ and $z = O(10^{15})$ (energy scales of $O(200)$ MeV and $O(100)$ GeV, respectively), the quantum chromodynamics (QCD) and electroweak phase transitions had presumably
occurred, although their (indeed weak) signatures in the CMB and LSS have not been found. In addition, inflation, a cornerstone in the standard cosmological model, is clearly beyond the realm of well-established physics; its detection via the B-mode polarization it induces in the CMB could be achieved only if it took place at energy scales $\sim 10^{13}$ orders of magnitude larger than achievable at present. Although the inflationary scenario is very flexible it is also plagued with certain undesirable problems, such as the $\eta$-problem, trans-Planckian problem, and the ‘measure problem’ in the multiverse. The latter is unavoidable in the currently favorite ‘eternal inflation’ scenario.

Ideally, an alternative cosmological model that agrees well with the standard cosmological model at BBN energies and lower, i.e. $z < 10^{10}$, while still addressing the classical problems of the hot big bang model that inflation was designed to solve, and all this in the $\lesssim 1$ TeV range of energies, will definitely be an appealing alternative. This could be in principle achieved with a (relatively late) non-singular bounce that also removes the technically and conceptually undesirable initial (curvature) singularity problem of GR-based cosmological models. In order to achieve such a bounce within GR, or a (mostly) conformally-related theory, certain ‘energy conditions’ have to be violated. One specific realization of this program has been the focus of the present work.

Symmetries play a key role in our theories of fundamental interactions. For example, the SM of particle physics is based on a local $U(1) \times SU(2) \times SU(3)$ gauge group with quantizable gauge fields. In addition, our favorite theory of gravitation, GR, is diffeomorphism-invariant. In this work we entertained the possibility that in addition to diffeomorphism-invariance, the fundamental scalar-tensor theory of gravitation, Eq. (1), also satisfies a global $SO(1,1)$ symmetry; one doublet component is conformally coupled to gravity whereas the other is only minimally coupled. The cosmological model based on this alternative theory of gravitation has some appealing properties, only a few of them have been discussed in the present work.

We have shown here that the standard cosmic scale factor might be replaced by a conformally coupled component of an $SO(1,1)$ doublet. Its kinetic term is non-canonical in the sense that it appears with the ‘wrong sign’ in the action (thereby guaranteeing that gravitation is an attractive force), and is inherently classical, much like the scale factor of the standard cosmological model is. Ignoring the other component (as is effectively done in the standard cosmological model), which is a free quantizable field minimally coupled to gravitation, results in ignoring its possible perturbations as well, which are described by scale-invariant gaussian and adiabatic perturbations. In the standard cosmological model we then obtain scalar perturbations with these desired properties from the fluctuations of another scalar field – the inflaton. Perhaps even more important, the existence of this minimally coupled doublet component guarantees that cosmic history goes through a bounce rather than an initial singularity.

A tantalizing alternative scenario explored in this work starts with a vacuum-like- followed by MD and RD deflationary evolution phase which culminates at a ‘bounce’ when the (absolute value of the negative) energy density associated with the effective ‘stiff matter’ (provided by the kinetic term of the minimally coupled component) momentarily equals that of radiation. We reiterate that the various cosmological epochs according to this scenario only result from the universal evolution of masses, not space expansion – space is static. In the vacuum-like-dominated epoch the energy density of the universe is dominated by the quartic potential of the scalar field which is genuinely classical with no quantum fluctuations. Therefore, DE according to the present scenario is not zero-point energy but rather a manifestation of the self-coupling of the scalar field, with all other fields (e.g. Dirac, electromagnetic, etc.) playing a subdominant role in the dynamically evolving background at the DE era.

During the MD contraction epoch, gaussian adiabatic scalar perturbations characterized by a flat spectrum, which are sourced by the (quantum) fluctuating minimally coupled field, are generated. The observed power spectrum is efficiently produced during the entire MD deflationary epoch. As in the case of inflation, the observed gaussianity is explained by the correspondence between the (quantum) vacuum state of an essentially free scalar field and the ground state of an harmonic oscillator. Adiabatic ‘initial’ conditions, generically predicted by inflation, are instead a natural outcome of the very existence of a bounce in the present model instead of a big bang; the mere requirement of continuity then select the ‘adiabatic’ initial conditions. Normalizing primordial scalar perturbations by their observed value implies that the bounce took place at $z \sim O(10^{12})$, safely remote from standard BBN or any other lower-energy standard cosmological epochs.

The scalar field does not ‘slow-roll’ along its potential (as it does in the standard inflationary scenario) but rather its kinetic energy is comparable to its potential energy at all times. It is therefore free from the fine-tuning problem of the inflaton potential shape generically required in the standard cosmological model due to radiative corrections. From the present work perspective ‘slow-roll’ is an artifact of the standard units convention, in which all scalar fields (e.g. particle masses, the cosmological constant, the inflaton itself, etc.) are effectively set to constants.

Linear perturbations generated during the deflationary phase generically survive the bounce due to continuity of metric perturbations and do not undermine the underlying homogeneity and isotropy of the cosmological model in the post-bounce phase. Matter is not created in the (non-singular and adiabatic) cosmological scenario laid out here, nor is it destroyed. The cosmological scenario from the BBN era onward is exactly as in the standard cosmological model. In addition, the ‘anisotropy’ problem that generally plagues bouncing scenarios does not exist in our construction. Weyl symmetry and the consequent absence of any dimensional parameter in the action severely limit the possibility...
that such an effective term will be present in the action. Also, as the proposed model is intimately linked to the SM we prohibit the existence of inverse powers of the scalar field at the action level, barring the existence of any energy contribution characterized by $w > 1/3$. In addition, although the energy density of minimally coupled component of the doublet behaves effectively as a perfect fluid with $w = 1$ it does not cause any anisotropy problem simply because bouncing takes place exactly once the anisotropy-like density starts taking over the cosmic dynamics, and this only happens since this ‘stiff’ energy density is negative – otherwise bouncing would not have taken place.

While conformal time is both past- and future-bounded in this scenario, i.e. $\eta \in (-\eta_c, \eta_c)$, the (effective) cosmic time in not. There is no ‘horizon problem’ associated with the model – not for radiation, and not even for, e.g. light (but still massive) neutrinos. Specifically, the pre-bounce starts from very large (and in principle infinite) particle masses and therefore the causal horizon is much larger than would be naively expected from monotonically growing masses (that corresponds to expanding space), i.e. essentially $\eta_0 \ll \eta_c + \eta_0$ if $\eta_c \gg \eta_0$. Likewise, the ‘flatness problem’ afflicting the hot big bang scenario stems from the slower decay of the energy density associated with curvature as compared to that of matter in a monotonically expanding universe. In bouncing scenarios the situation is reversed in the pre-bounce phase; starting at initially large $\rho$ (particle masses) one typically expects to find that the energy density in the form of matter largely exceeds that of curvature at any finite $\rho$ value in the pre-bounce phase. Since this adiabatic model is very nearly symmetric in $\rho$ around the bounce (barring entropy processing effects at around the RD phases), one generally expects the universe to look spatially flat at any finite $\rho$ after the would-be singularity (actually a non-singular bounce). From this perspective flatness is an attractor-, rather than an unstable-point that requires fine-tuning. The ‘monopole’ and ‘relic defects’ problems do not arise (in the proposed scenario) because the bounce took place at $z = O(10^{12})$.

We believe that, in addition to addressing the cosmological horizon, flatness and cosmological relic problems, the framework proposed here provides important insight on the nature of DE, initial singularity, cosmological ‘expansion’, and the flatness of the matter power spectrum on cosmological scales. Even so, the work presented here is by no means exhaustive, and indeed many of its basic aspects will be further elucidated in future papers.

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[27] Jackiw, R., & Pi, S.-Y. 2015, PRD, 91, 067501
[28] Ohanian, H. C. 2015, arXiv:1502.00020
[29] Faraoni, V., Gunzig, E., & Nardone, P. 1999, Fundam. Cosmic Phys., 20, 121
[30] Faraoni, V. 2004, PRD, 70, 081501
[31] Faraoni, V., & Nadeau, S. 2007, PRD, 75, 023501
[32] Postma, M., & Volponi, M. 2014, PRD, 90, 103516
[33] Flanagan, É. É. 2004, Classical and Quantum Gravity, 21, 3817
[34] Ijjas, A., & Steinhardt, P. J. 2018, Classical and Quantum Gravity, 35, 135004
[35] Planck Collaboration, Aghanim, N., Akrami, Y., et al. 2018, arXiv:1807.06209
[36] Levy, A. M. 2017, PRD, 95, 023522
[37] Boyle, L., Finn, K., & Turok, N. 2018, PRL, 121, 251301
[38] Harrison, E. R. 1970, PRD, 1, 2726
[39] Zeldovich, Y. B. 1972, MNRAS, 160, 1P
[40] Moffat, J. W. 1993, International Journal of Modern Physics D, 2, 351
[41] Albrecht, A., & Magueijo, J. 1999, PRD, 59, 043516
[42] Khoury, J., Ovrut, B. A., Steinhardt, P. J., & Turok, N. 2001, PRD, 64, 123522
[43] Buchbinder, E. I., Khoury, J., & Ovrut, B. A. 2007, PRD, 76, 123503
[44] Steinhardt, P. J., & Turok, N. 2002, Science, 296, 1436
[45] Steinhardt, P. J., & Turok, N. 2002, PRD, 65, 126003
[46] Nayeri, A., Brandenberger, R. H., & Vafa, C. 2006, PRL, 97, 021302
[47] Piao, Y.-S. 2011, arXiv:1112.3737
[48] Rubakov, V. A. 2009, JCAP, 9, 030
[49] Hinterbichler, K., & Khoury, J. 2012, JCAP, 4, 023
[50] Wands, D. 1999, PRD, 60, 023507
[51] Knox, L., & Song, Y.-S. 2002, PRL, 89, 011303
[52] Di Valentino, E., Melchiorri, A., Fantaye, Y., & Heavens, A. 2018, PRD, 98, 063508