A four-dimensional chaotic system and its oscillator circuit design

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\textbf{Abstract} This paper reports a novel four-dimensional autonomous chaotic system, which has a canonical and many interesting complex structure. Furthermore, throughout the study of these dynamical behaviors of this chaotic system some basic dynamical properties, such as continuous spectrum, Lyapunov exponents, fractal dimensions, strange attractor and Poincaré mapping are investigated briefly. Moreover, an oscillator circuit is designed to simulation the new chaotic system.

\textbf{1. Introduction}

In the early 1960s, E. Lorenz made an important contribution to discover the chaos phenomenon. Since that time, much effort has gone into showing that Lorenz’s mathematical community \cite{1}. It is well known that chaotic-dynamic systems are capable of exhibiting a variety of behaviors. In 1976, Rössler found another three dimensional autonomous chaotic system. Furthermore \cite{2}, LÜ J and Chen G have investigated the relationship among these different chaotic systems and summarized a unified and generalized Lorenz-like system \cite{3-6}. In 2004, Liu C X et al. reported a new chaotic system which displays very sophisticated dynamical behaviors \cite{7-10}. Such systems may be generated by many kinds of experiments from various fields of science such as hydrodynamics, astrophysics, chemistry, clinical neuroscience electronics, etc.

In this paper we report a new four-dimensional chaotic system. Some basic dynamical properties, such as continuous spectrum, Lyapunov exponents, fractal dimensions, strange attractor and Poincaré mapping are studied and numerical simulation results are verified briefly \cite{11,12}. Finally, the Oscillator circuit of this chaotic dynamical system has been designed by Electronics Workbench (EWB).

\textbf{2. System description}

Consider the following dynamical equations that describe the four-dimensional autonomous chaotic system are shown:

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Here, $a = 8, b = 40, c = \frac{10}{3}, d = 1, e = 4, g = 4, h = 2, j = 5, k = 1$ are real constants. Lyapunov exponents are related to the expanding or contracting nature of different directions and measure the exponential rates of divergence or convergence of nearby trajectories in phase space. The dynamical system (1) can be characterized with its Lyapunov exponents using the Wolf method and is found to be

$$\begin{align*}
\lambda_1 &= 0.6555, \\
\lambda_2 &= 0.0045, \\
\lambda_3 &= -0.7113, \\
\lambda_4 &= -12.7835
\end{align*}$$

The Lyapunov dimension of this system is

$$d_L = j + \frac{1}{|\lambda_j|} \sum_{i=1}^{j} \lambda_i = 3 + \frac{(\lambda_1 + \lambda_2)}{|\lambda_4 + \lambda_3|} = 3 + \frac{0.6555 + 0.0045}{-12.7835 - 0.7113} \approx 3.0489$$

Therefore, the equilibrium $O(0,0,0,0)$ is an unstable saddle in this non-linear four-dimensional autonomous system.

Three-dimensional view of the chaotic strange attractor is shown in Figure 1.

Figure 1. The $x - y - z$ trajectories of the system (1)

Observations of numerical simulation of system (1) are shown in Figure 2 (a)-(c)
Figure 2. Phase portrait of chaotic system (1) in $x-w$, $y-w$, $z-w$ plane with $a = 8, b = 40, c = \frac{10}{3}, d = 1, e = 4, g = 4, h = 2, j = 5, k = 1$

The waveform of $w(t)$ in time domain is shown in Figure 3. Apparently, the waveform of $w(t)$ are non-periodic in this continuous-time four-dimensional autonomous system (1), it has shown some basic dynamical properties. The spectrum of this non-linear system (1) is also studied; its spectrum is continuous as shown in Figure 4.

Figure 3. $w(t)$ waveform of the system (1)  Figure 4. Spectrum of $|x|$ in the system (1)

Figure 5. Poincaré mapping of $x-y$ plane of the system (1)

It can be seen that the Poincaré mapping are these points of the confusion as shown in Figure 5.
The bifurcation diagram of $x$ with increasing $a$ is given in Figure 6, it shown richer and complex dynamical behaviors. According to the aforementioned analysis results, apparently, it is a new chaotic attractor evolved from system (1).

3. Circuitry confirmation of system (1)

Using the Electronics Workbench (EWB), an electronic circuit can be designed to realize the model of this chaotic system (1). First, we change the system (1) into the following form:

\[
\begin{align*}
\dot{u}_{c_1} &= \frac{R_1}{R_2 R_4 C_1} u_{c_3} - \frac{R_1}{R_1 R_4 R_5 C_1} u_{c_1} - \frac{R_3}{R_4 R_5 C_1} u_{c_4} \\
\dot{u}_{c_2} &= \frac{R_3}{R_4 R_1 C_2} u_{c_1} - \frac{R_3}{10 R_5 R_7 R_10 C_2} u_{c_1} u_{c_1} \\
\dot{u}_{c_3} &= \frac{R_{14}}{10 R_1 R_6 C_3} u_{c_1} u_{c_2} - \frac{R_{14}}{R_1 R_{15} R_4 C_3} u_{c_2} - \frac{R_{14}}{R_{16} R_7 R_2 C_3} u_{c_3} \\
\dot{u}_{c_4} &= -\frac{R_{20}}{10 R_1 R_2 C_4} u_{c_1} u_{c_4}
\end{align*}
\]

The four state variables $x, y, z$ and $w$ are respectively obtained from the terminal outputs of $u_{c_1}, u_{c_2}, u_{c_3}$ and $u_{c_4}$ in this electronic circuit. This design is simple, Operational amplifiers (LM741), analog multipliers (AD633), linear resistors and capacitors are employed to perform the required addition, subtraction and multiplication operations. The designed chaotic oscillator circuit is shown in Figure 7.

Resistance / KΩ: $R_1, R_2, R_3, R_{10}, R_{13}, R_{16}, R_{19} = 10; R_5, R_6, R_{11}, R_{12}, R_{14}, R_{17} = 1 , R_{20} = 2$ ; $R_{20} = 2; R_9, R_{18} = 4; R_3 = 8; R_{23} = 20; R_{15} = 30; R_7 = 40 ; R_4, R_{21}, R_{24} = 100$. Capacitance / μF: $C_1, C_2, C_3, C_4 = 1$.

Now, we have the following circuit simulation result respectively. The attractors can be observed from the oscilloscope and shown in Figure 8.
4. Conclusion

In this paper, we introduce a simple chaotic system. It is a fourth-order autonomous nonlinear system. The proposed system has been demonstrated not only via numerical simulations to exhibit hyperchaotic behavior, but also by designed circuit to observe attractors. It is expecting that the application of more chaotic system to engineering problems would benefit from this paper in the near future.
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