Study of anomalous gauge boson self-couplings and the role of spin-1 polarizations

Rafiqul Rahaman

Department of Physical Sciences
IISER Kolkata, India

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Outline

1. Introduction
2. Anomalous couplings
3. Spin-polarization observables
The Standard Model (SM) of particle physics is highly successful theory which describe the governing principle of elementary constituents of matter and their interactions.

However, the SM has issues within the theoretical framework (the hierarchy of mass, strong CP problem) and also the inability to address neutrino oscillation, dark matter, baryogenesis and many more.

New physics has been postulated to cure those, predicting new particles and symmetry.

Sadly, nothing found beyond the SM at the current reach of energy ($\mathcal{O}(10)$ TeV).
One could expect that the new physics scale is too heavy to be directly explored at the LHC at the current energy range.

They may leave footprints in the available energy range. They will modify the structure of SM interactions or bring some new interactions.

These can be modelled by effective filed theory (EFT) through higher dimensional operators,

\[ \mathcal{L}_{\text{eft}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{c_{i}^{(6)}}{\Lambda^2} \phi_i^{(6)} + \sum_i \frac{c_{i}^{(8)}}{\Lambda^4} \phi_i^{(8)} + \ldots . \]
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2 Anomalous couplings

3 Spin-polarization observables
Anomalous triple gauge boson couplings (aTGC)

Possible triple gauge boson self interactions in electro weak (EW) theory:

Present in the SM: $WWZ$, $WW\gamma$ ($WW\nu$),

Not present in the SM: $ZZZ$, $ZZ\gamma$, $Z\gamma\gamma$ ($Z\nu\nu$).
The operators contributing to $WWV$ couplings are

$$\mathcal{O}_{WWW} = \text{Tr}[W_{\mu\nu} W^{\nu\rho} W_{\rho}^\mu], \quad \mathcal{O}_{\tilde{W}WW} = \text{Tr}[\tilde{W}_{\mu\nu} W^{\nu\rho} W_{\rho}^\mu],$$

$$\mathcal{O}_W = (\mathcal{D}_\mu \Phi)^\dagger W^{\mu\nu} (\mathcal{D}_\nu \Phi), \quad \mathcal{O}_{\tilde{W}} = (\mathcal{D}_\mu \Phi)^\dagger \tilde{W}^{\mu\nu} (\mathcal{D}_\nu \Phi).$$

The operator for $ZVV$ couplings are.

$$\mathcal{O}_{BW} = i \Phi^\dagger B_{\mu\nu} W^{\mu\rho} \{ \mathcal{D}_\rho, \mathcal{D}_\nu \} \Phi,$$

$$\mathcal{O}_{WW} = i \Phi^\dagger W_{\mu\nu} W^{\mu\rho} \{ \mathcal{D}_\rho, \mathcal{D}_\nu \} \Phi,$$

$$\mathcal{O}_{BB} = i \Phi^\dagger B_{\mu\nu} B^{\mu\rho} \{ \mathcal{D}_\rho, \mathcal{D}_\nu \} \Phi,$$

$$\mathcal{O}_{\tilde{B}W} = i \Phi^\dagger \tilde{B}_{\mu\nu} W^{\mu\rho} \{ \mathcal{D}_\rho, \mathcal{D}_\nu \} \Phi.$$

$$\mathcal{D}_\mu = \partial_\mu + \frac{i}{2} g \tau^I W^{I}_\mu + \frac{i}{2} g' B_\mu$$

$$W_{\mu\nu} = \frac{i}{2} g \tau^I (\partial_\mu W^{I}_\nu - \partial_\nu W^{I}_\mu + g \epsilon_{IJK} W^{J}_\mu W^{K}_\nu),$$

$$B_{\mu\nu} = \frac{i}{2} g' (\partial_\mu B_\nu - \partial_\nu B_\mu), \quad \tilde{V}^{\mu\nu} = 1/2 \epsilon^{\mu\nu\rho\sigma} V_{\rho\sigma}$$

K. Hagiwara, Nucl. Phys. B282 (1987) 253–307

Gounaris et al. Phys. Rev. D 61, 073013 (2000)
\[ \phi_i(=1,2) = \begin{pmatrix} \phi_i^+ \\ \phi_i^0 \end{pmatrix}, \quad (H_1) = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} (\phi_1) \], \quad \tan \beta = \frac{v_2}{v_1}, \quad v = \sqrt{v_1^2 + v_2^2}

\[ H_1 = \begin{pmatrix} -iG^+ \\ \frac{1}{\sqrt{2}} (v + h + iG^0) \end{pmatrix}, \quad H_2 = \begin{pmatrix} \frac{1}{\sqrt{2}} (R + il) \end{pmatrix}, \quad \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = T^T \begin{pmatrix} h \\ R \\ I \end{pmatrix} \]
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1 Introduction

2 Anomalous couplings

3 Spin-polarization observables
Spin density matrix and polarization parameters

\[ \rho_{1/2}(\lambda, \lambda') = \frac{1}{2} [\mathbb{I}_{2 \times 2} + \vec{p} \cdot \vec{\sigma}] , \]

\[ \rho_1(\lambda, \lambda') = \frac{1}{3} \left[ \mathbb{I}_{3 \times 3} + \frac{3}{2} \vec{p} \cdot \vec{S} + \sqrt{\frac{3}{2}} T_{ij} (S_i S_j + S_j S_i) \right] \]
Spin density matrix and polarization parameters

\[ \rho_{1/2}(\lambda, \lambda') = \frac{1}{2} \left[ \mathbb{I}_{2 \times 2} + \vec{p} \cdot \vec{\sigma} \right], \]

\[ \rho_1(\lambda, \lambda') = \frac{1}{3} \left[ \mathbb{I}_{3 \times 3} + \frac{3}{2} \vec{p} \cdot \vec{S} + \sqrt{\frac{3}{2}} T_{ij} (S_i S_j + S_j S_i) \right], \]

\[ P_{1/2}(\lambda, \lambda') = \frac{1}{2} \begin{bmatrix} 1 + p_z & p_x - ip_y \\ p_x + ip_y & 1 - p_z \end{bmatrix}, \]

\[ P_1(\lambda, \lambda') = \begin{bmatrix} \frac{1}{3} + \frac{p_z}{2} + \frac{T_{zz}}{\sqrt{6}} & p_x - ip_y + \frac{T_{xz} - iT_{yz}}{2 \sqrt{2}} + \frac{T_{xy} - 2iT_{xy}}{\sqrt{6}} \\ p_x + ip_y + \frac{T_{xz} + iT_{yz}}{2 \sqrt{2}} & \frac{1}{3} - \frac{2T_{zz}}{\sqrt{6}} & \frac{p_x - ip_y}{2 \sqrt{2}} - \frac{T_{xz} - iT_{yz}}{\sqrt{3}} \end{bmatrix}. \]

F. Boudjema and R. K. Singh JHEP 0907, 028 (2009)
Angular distribution of decayed fermion (spin-1)

\[
\frac{1}{\sigma} \frac{d\sigma}{d\Omega_f} = \frac{2S + 1}{4\pi} \sum_{\lambda, \lambda'} P_1(\lambda, \lambda') \times \Gamma_1(\lambda, \lambda'),
\]

\[
= \frac{3}{8\pi} \left[ \left( \frac{2}{3} - (1 - 3\delta) \frac{T_{zz}}{\sqrt{6}} \right) + \alpha p_z \cos \theta + \sqrt{\frac{3}{2}} (1 - 3\delta) T_{zz} \cos^2 \theta \\
+ \left( \alpha p_x + 2 \sqrt{\frac{2}{3}} (1 - 3\delta) T_{xz} \cos \theta \right) \sin \theta \cos \phi \\
+ \left( \alpha p_y + 2 \sqrt{\frac{2}{3}} (1 - 3\delta) T_{yz} \cos \theta \right) \sin \theta \sin \phi \\
+ (1 - 3\delta) \left( \frac{T_{xx} - T_{yy}}{\sqrt{6}} \right) \sin^2 \theta \cos(2\phi) \\
+ \sqrt{\frac{2}{3}} (1 - 3\delta) T_{xy} \sin^2 \theta \sin(2\phi) \right]
\]

F. Boudjema and R. K. Singh JHEP 0907, 028 (2009)
Polarization parameters from asymmetry

\[ A_x = \frac{1}{\sigma} \left[ \int_{\theta=0}^{\pi} \int_{\phi=-\pi/2}^{\pi/2} \frac{d\sigma}{d\Omega_f} d\Omega_f - \int_{\theta=0}^{\pi} \int_{\phi=\pi/2}^{3\pi/2} \frac{d\sigma}{d\Omega_f} d\Omega_f \right] = \frac{3\alpha p_x}{4}, \]

\[ \equiv \frac{\sigma(\cos \phi > 0) - \sigma(\cos \phi < 0)}{\sigma(\cos \phi > 0) + \sigma(\cos \phi < 0)} \equiv \frac{\text{Green Region} - \text{Red Region}}{\text{Green Region} + \text{Red Region}}. \]
A polarization parameter from asymmetry is defined as

\[ A_y \equiv \frac{\sigma(\sin\phi > 0) - \sigma(\sin\phi < 0)}{\sigma(\sin\phi > 0) + \sigma(\sin\phi < 0)} = \frac{3\alpha p_y}{4}, \]

\[ A_z \equiv \frac{\sigma(\cos\theta > 0) - \sigma(\cos\theta < 0)}{\sigma(\cos\theta > 0) + \sigma(\cos\theta < 0)} = \frac{3\alpha p_z}{4}, \]

\[ A_{xy} \equiv \frac{\sigma(\sin 2\phi > 0) - \sigma(\sin 2\phi < 0)}{\sigma(\sin 2\phi > 0) + \sigma(\sin 2\phi < 0)} = \frac{2}{\pi} \sqrt{\frac{2}{3}} (1 - 3\delta) T_{xy}, \]

\[ A_{xz} \equiv \frac{\sigma(\cos\theta \cos\phi > 0) - \sigma(\cos\theta \cos\phi < 0)}{\sigma(\cos\theta \cos\phi > 0) + \sigma(\cos\theta \cos\phi < 0)} = \frac{2}{\pi} \sqrt{\frac{2}{3}} (1 - 3\delta) T_{xz}, \]

\[ A_{yz} \equiv \frac{\sigma(\cos\theta \sin\phi > 0) - \sigma(\cos\theta \sin\phi < 0)}{\sigma(\cos\theta \sin\phi > 0) + \sigma(\cos\theta \sin\phi < 0)} = \frac{2}{\pi} \sqrt{\frac{2}{3}} (1 - 3\delta) T_{yz}, \]

\[ A_{x^2-y^2} \equiv \frac{\sigma(\cos 2\phi > 0) - \sigma(\cos 2\phi < 0)}{\sigma(\cos 2\phi > 0) + \sigma(\cos 2\phi < 0)} = \frac{1}{\pi} \sqrt{\frac{2}{3}} (1 - 3\delta) (T_{xx} - T_{yy}), \]

\[ A_{zz} \equiv \frac{\sigma(\sin 3\theta > 0) - \sigma(\sin 3\theta < 0)}{\sigma(\sin 3\theta > 0) + \sigma(\sin 3\theta < 0)} = \frac{3}{8} \sqrt{\frac{3}{2}} (1 - 3\delta) T_{zz}. \]
$CP$-even and $CP$-odd distinction
\[ V = \frac{Z}{\gamma} \]

\[
\mathcal{L}_{ZVV} = \frac{e}{m_Z^2} \left[ - \left[ f_4^\gamma (\partial_\mu F^{\mu\beta}) + f_4^Z (\partial_\mu Z^{\mu\beta}) \right] Z_\alpha (\partial^\alpha Z_\beta) + \left[ f_5^\gamma (\partial_\sigma F_{\sigma\mu}) + f_5^Z (\partial_\sigma Z_{\sigma\mu}) \right] \tilde{Z}_{\mu\beta} Z_\beta - \left[ h_1^\gamma (\partial_\sigma F_{\sigma\mu}) + h_1^Z (\partial_\sigma Z_{\sigma\mu}) \right] Z_\beta F^{\mu\beta} - \left[ h_3^\gamma (\partial_\sigma F^{\sigma\rho}) + h_3^Z (\partial_\sigma Z^{\sigma\rho}) \right] Z^\alpha \tilde{F}_{\rho\alpha} \right].
\]

Gounaris et al. Phys. Rev. D 61, 073013 (2000)
Sensitivity: \( \mathcal{S}(\vec{f}) = \frac{|\mathcal{O}(\vec{f}) - \mathcal{O}(\vec{f} = 0)|}{\delta \mathcal{O}} \),

Figure: \( e^+ e^- \rightarrow ZZ \rightarrow l^+ l^- q\bar{q} \), \( \sqrt{s} = 500 \text{ GeV}, \mathcal{L} = 100 \text{ fb}^{-1} \)

Rahaman, Singh, *Eur. Phys. J.* C76 no. 10, (2016) 539
Constraining a TGC
$$\mathcal{L}_{\text{wwv}} = ig_{\text{wwv}} \left[ + \Delta g_1^{\gamma} (W_\mu^+ W^-_\mu - W_\mu^+ W^-_\mu) V^\nu \\
+ ig_4^{\gamma} W_\mu^+ W^-_\nu (\partial \mu V^\nu + \partial \nu V^\mu) \\
- ig_5^{\gamma} \epsilon_{\mu \nu \rho \sigma} (W_\mu^+ \partial_\rho W^-_\nu - \partial_\rho W_\mu^+ W^-_\nu) V_\sigma \\
+ \frac{\lambda^{\gamma}}{m_W^2} W_\mu^+ W^-_\nu V_\mu + \frac{\lambda^{\gamma}}{m_W^2} W_\mu^+ W^-_\nu \tilde{V}_\rho^\mu \\
+ \Delta \kappa^{\gamma} W_\mu^+ W^-_\nu V^{\mu \nu} + \kappa^{\gamma} W_\mu^+ W^-_\nu \tilde{V}^{\mu \nu} \right].$$

$$V = Z / \gamma$$
$e^+ e^- \rightarrow W^+ W^- \rightarrow l^- \bar{\nu}_l q \bar{q}'$

Multi parameter variations

$\sqrt{s} = 500$ GeV, $\mathcal{L} = 100$ fb$^{-1}$

Rahaman, Singh arXiv:1909.05496
$e^+ e^- \rightarrow W^+ W^- \rightarrow l^- \bar{\nu}_l q \bar{q}'$, $\sqrt{s} = 500$ GeV, $\mathcal{L} = 100$ fb$^{-1}$
Extraction of parameters
Figure: $pp \rightarrow 3l + \not{E}_T(W^\pm Z)$, $\sqrt{s} = 13$ TeV, $\mathcal{L} = 100$ fb$^{-1}$

\[
A_{\Delta \phi} + A_i^Z + A_i^W \quad \text{---} \quad \sigma_i + A_{\Delta \phi} + A_i^Z \quad \text{---} \quad \sigma_i \quad \text{---} \quad \sigma_i + A_{\Delta \phi} + A_i^Z + A_i^W
\]

\[
\sqrt{s} = 13 \text{ TeV} \quad \mathcal{L} = 100 \text{ fb}^{-1} \quad \chi^2 = 4
\]

$(\bullet) \equiv \text{SM}$

$(\star) \equiv \{\Delta g_1^Z, \lambda^Z, \Delta \kappa^Z, \tilde{\lambda}^Z, \tilde{\kappa}^Z\} = \{0.01, 0.01, 0.1, 0.01, 0.1\}$

Rahaman, Singh arXiv:1911.03111
The polarization observables help to distinguish between $CP$-even and $CP$-odd couplings.

They help to constrain anomalous couplings.

A suitable choice of beam polarization at $e^+ e^-$ collider improve limits on anomalous couplings.

Polarization observables are instrumental in extracting the values of anomalous couplings if a deviation from the SM is observed at the LHC.

Spin-spin correlations can be added to further improve the limits.
Thank you
Backup slides
Relation of form factors with $SU(2) \times U(1)$ operators

\[
\Delta g^Z_1 = c_W \frac{m_Z^2}{2\Lambda^2}, \\
g^V_4 = g^V_5 = \Delta g^\gamma_1 = 0, \\
\lambda^\gamma = \lambda^Z = \lambda^V = c_{WWW} \frac{3g^2 M^2_W}{2\Lambda^2}, \\
\tilde{\lambda}^\gamma = \tilde{\lambda}^Z = \tilde{\lambda}^V = c_{\tilde{WWW}} \frac{3g^2 M^2_W}{2\Lambda^2}, \\
\Delta \kappa^\gamma = (c_W + c_B) \frac{M^2_W}{2\Lambda^2}, \\
\Delta \kappa^Z = (c_W - c_B \tan^2 \theta_W) \frac{M^2_W}{2\Lambda^2}, \\
\tilde{\kappa}^\gamma = c_{\tilde{W}} \frac{M^2_W}{2\Lambda^2}, \\
\tilde{\kappa}^Z = -c_{\tilde{W}} \tan^2 \theta_W \frac{M^2_W}{2\Lambda^2}.
\]

\[
\Delta g^Z_1 = \Delta \kappa^Z + \tan^2 \theta_W \Delta \kappa^\gamma, \\
\tilde{\kappa}^Z + \tan^2 \theta_W \kappa^\gamma = 0.
\]
Relation of form factors with $SU(2) \times U(1)$ operators

\[ f_5^Z = 0, \]
\[ f_5^\gamma = \frac{v^2 m_Z^2}{4 c_w s_w} \frac{C_{BW}}{\Lambda^4}, \]
\[ f_4^Z = \frac{m_Z^2 v^2 \left(c_w^2 C_{WW} + 2 c_w s_w C_{BW} + 4 s_w^2 C_{BB}\right)}{2 c_w s_w \Lambda^4}, \]
\[ f_4^\gamma = -\frac{m_Z^2 v^2 \left(-c_w s_w C_{WW} + C_{BW} \left(c_w^2 - s_w^2\right) + 4 c_w s_w C_{BB}\right)}{4 c_w s_w \Lambda^4}, \]
\[ h_3^Z = \frac{v^2 m_Z^2}{4 c_w s_w} \frac{C_{BW}}{\Lambda^4}, \]
\[ h_4^Z = h_3^\gamma = h_4^\gamma = h_2^Z = h_2^\gamma = 0, \]
\[ h_1^Z = \frac{m_Z^2 v^2 \left(-c_w s_w C_{WW} + C_{BW} \left(c_w^2 - s_w^2\right) + 4 c_w s_w C_{BB}\right)}{4 c_w s_w \Lambda^4}, \]
\[ h_1^\gamma = -\frac{m_Z^2 v^2 \left(s_w^2 C_{WW} - 2 c_w s_w C_{BW} + 4 c_w^2 C_{BB}\right)}{4 c_w s_w \Lambda^4}, \]

\[ f_5^\gamma = h_3^Z \quad \text{and} \quad h_1^Z = -f_4^\gamma. \]
Production density matrix and polarization parameters

\[ \rho_1(\lambda, \lambda') = \frac{1}{3} \left[ I_{3 \times 3} + \frac{3}{2} \vec{p} \cdot \vec{S} + \sqrt{\frac{3}{2}} T_{ij} (S_i S_j + S_j S_i) \right], \] spin-1/2 case

\[ S_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_y = \frac{i}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \]

\[ \vec{p} = \{ p_x, p_y, p_z \}, \quad T = \begin{pmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yy} & T_{yz} & T_{zz} \end{pmatrix}, \quad \sum_i T_{ii} = 0. \]

\[ P(\lambda, \lambda') = \begin{pmatrix} \frac{1}{3} + \frac{p_z}{2} + \frac{T_{zz}}{\sqrt{6}} & \frac{p_x - ip_y}{2\sqrt{2}} + \frac{T_{xz} - iT_{yz}}{\sqrt{3}} & \frac{T_{xx} - T_{yy} - 2iT_{xy}}{\sqrt{6}} \\ \frac{p_x + ip_y}{2\sqrt{2}} + \frac{T_{xz} + iT_{yz}}{\sqrt{3}} & \frac{1}{3} - \frac{2T_{zz}}{\sqrt{6}} & \frac{p_x - ip_y}{2\sqrt{2}} + \frac{T_{xz} - iT_{yz}}{\sqrt{3}} \\ \frac{T_{xx} - T_{yy} + 2iT_{xy}}{\sqrt{6}} & \frac{p_x + ip_y}{2\sqrt{2}} - \frac{T_{xz} + iT_{yz}}{\sqrt{3}} & \frac{1}{3} - \frac{p_z}{2} + \frac{T_{zz}}{\sqrt{6}} \end{pmatrix}. \]

F. Boudjema and R. K. Singh JHEP 0907, 028 (2009)
The decay density matrix of a spin-1 particle

\[ \Gamma_1(\lambda, \lambda') = \]

\[
\begin{bmatrix}
\frac{1+\delta+(1-3\delta) \cos^2 \theta+2\alpha \cos \theta}{4} & \frac{\sin \theta(\alpha+(1-3\delta) \cos \theta)}{2\sqrt{2}} e^{i\phi} & (1-3\delta) \frac{(1-\cos^2 \theta)}{4} e^{i2\phi} \\
\frac{\sin \theta(\alpha+(1-3\delta) \cos \theta)}{2\sqrt{2}} e^{-i\phi} & \delta + (1-3\delta) \frac{\sin^2 \theta}{2} & \frac{\sin \theta(\alpha-(1-3\delta) \cos \theta)}{2\sqrt{2}} e^{i\phi} \\
(1-3\delta) \frac{(1-\cos^2 \theta)}{4} e^{-i2\phi} & \frac{\sin \theta(\alpha-(1-3\delta) \cos \theta)}{2\sqrt{2}} e^{-i\phi} & 1+\delta+(1-3\delta) \cos^2 \theta-2\alpha \cos \theta
\end{bmatrix}
\]

F. Boudjema and R. K. Singh JHEP 0907, 028 (2009)

- \( V_{f'f} : \gamma^\mu \left( C_L^f \frac{(1-\gamma_5)}{2} + C_R^f \frac{(1+\gamma_5)}{2} \right) \).
- \( \delta \to 0 \) for \( m_f \to 0 \) and \( \alpha \to \frac{(C_R^f)^2 - (C_L^f)^2}{(C_R^f)^2 + (C_L^f)^2} \).
Lab frame and rest frame momentum configuration

Lab frame

Rest frame
Sensitivity of observables to aTGC

Sensitivity of an observable $\mathcal{O}$ dependent on parameter $\vec{f}$ is defined as

$$\mathcal{S}(\mathcal{O}(\vec{f})) = \frac{|\mathcal{O}(\vec{f}) - \mathcal{O}(\vec{f} = 0)|}{\delta \mathcal{O}}$$

$$\delta \mathcal{O} = \sqrt{(\delta \mathcal{O}_{\text{stat.}})^2 + (\delta \mathcal{O}_{\text{sys.}})^2}$$

For Asymmetry the error is

$$\delta A = \sqrt{\frac{1 - A^2}{\mathcal{L} \sigma} + \epsilon_A^2},$$

$\mathcal{L}$ being the integrated luminosity.

For the cross section the error is

$$\delta \sigma = \sqrt{\frac{\sigma}{\mathcal{L}} + (\epsilon_\sigma \sigma)^2}.$$
Observables with parametric dependence

\[ \text{e}^+ \text{e}^- \rightarrow \text{ZZ} \]

| Observables | Linear terms | Quadratic terms |
|-------------|--------------|-----------------|
| \( \sigma \) | \( f_5^Z, f_5^\gamma \) | \( (f_4^\gamma)^2, (f_5^\gamma)^2, (f_4^Z)^2, (f_5^Z)^2, f_4^\gamma f_4^Z, f_5^\gamma f_5^Z \) |
| \( \sigma \times A_x \) | \( f_5^\gamma, f_5^Z \) | – |
| \( \sigma \times A_y \) | \( f_4^\gamma, f_4^Z \) | – |
| \( \sigma \times A_{xy} \) | \( f_4^Z, f_4^\gamma \) | \( f_4^Z f_5^\gamma, f_4^\gamma f_4^Z, f_5^\gamma f_5^Z \) |
| \( \sigma \times A_{x^2-y^2} \) | \( f_5^Z, f_5^\gamma \) | \( (f_4^\gamma)^2, (f_5^\gamma)^2, (f_4^Z)^2, (f_5^Z)^2, f_4^\gamma f_4^Z, f_5^\gamma f_5^Z \) |
| \( \sigma \times A_{zz} \) | \( f_5^Z, f_5^\gamma \) | \( (f_4^\gamma)^2, (f_5^\gamma)^2, (f_4^Z)^2, (f_5^Z)^2, f_4^\gamma f_4^Z, f_5^\gamma f_5^Z \) |
1. 1\textsuperscript{st} point in chain: $\mathbf{f}_1 \in \text{Uniform}[\mathbf{f}_{\text{min}}, \mathbf{f}_{\text{max}}]$ and calculate likelihood, $\mathcal{L}(\mathbf{f}_1)$.

2. Set Standard deviation $\delta \mathbf{f} = \frac{\mathbf{f}_{\text{max}} - \mathbf{f}_{\text{min}}}{h}$.

3. Generate 2\textsuperscript{nd} point in chain: $\mathbf{f}_2 \in \text{Gaus}[\mathbf{f}_1, \delta \mathbf{f}]$ and calculate $\mathcal{L}(\mathbf{f}_2)$.

4. Accept nearby points:
   
   DO IF $\text{Uniform}[0, 1] < \frac{\mathcal{L}(\mathbf{f}_2)}{\mathcal{L}(\mathbf{f}_1)}$

   $\mathbf{f}_1 = \mathbf{f}_2$.

   Write $w, \mathcal{L}(\mathbf{f}_2), \mathbf{f}_2, O_i(\mathbf{f}_2)$.

   $w = 1$.

   ELSE $w = w + 1$

   .....Continue.

5. GetDist chain: $w, -\log[\mathcal{L}(\mathbf{f})], \mathbf{f}, O_i(\mathbf{f})$

6. Obtain correlations, BCI, etc.
Observables dependant on aTGC in $WW$ production

Parameters | $\sigma$ | $\sigma \times A_x$ | $\sigma \times A_y$ | $\sigma \times A_z$ | $\sigma \times A_{xy}$ | $\sigma \times A_{xz}$ | $\sigma \times A_{yz}$ | $\sigma \times A_{x^2 - y^2}$
--- | --- | --- | --- | --- | --- | --- | --- | ---
$\Delta g_1^V$ | ✓ | ✓ | — | ✓ | — | ✓ | — | ✓
g_4^V | — | — | ✓ | — | ✓ | — | ✓ | —
g_5^V | ✓ | ✓ | — | ✓ | — | ✓ | — | ✓
$\lambda^V$ | ✓ | ✓ | — | ✓ | — | ✓ | — | ✓
$\bar{\lambda}^V$ | — | — | ✓ | — | ✓ | — | ✓ | —
$\Delta \kappa^V$ | ✓ | ✓ | — | ✓ | — | ✓ | — | ✓
$\bar{\kappa}^V$ | — | — | ✓ | — | ✓ | — | ✓ | —
$(\Delta g_1^V)^2$ | ✓ | ✓ | — | — | — | — | — | ✓
$(g_4^V)^2$ | ✓ | — | — | — | — | — | — | ✓
$(g_5^V)^2$ | ✓ | — | — | — | — | — | — | ✓
$(\lambda^V)^2$ | ✓ | ✓ | — | — | — | — | — | ✓
$(\bar{\lambda}^V)^2$ | ✓ | ✓ | — | — | — | — | — | ✓
$(\Delta \kappa^V)^2$ | ✓ | ✓ | — | — | — | — | — | ✓
$(\bar{\kappa}^V)^2$ | ✓ | ✓ | — | — | — | — | — | ✓
$\Delta g_{\alpha}^V \alpha^V$ | ✓ | ✓ | — | — | — | — | — | —

back to sensitivity on couplings
End of backup slides