Magnetic dipole moments of the hidden-charm pentaquark states: $P_c(4440)$, $P_c(4457)$ and $P_{cs}(4459)$

Ulaş Özdem$^1, *$

$^1$Health Services Vocational School of Higher Education, Istanbul Aydin University, Sefakoy-Kucukcekmece, 34295 Istanbul, Turkey

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In this work, we employ the light-cone QCD sum rule to calculate the magnetic dipole moments of the $P_c(4440)$, $P_c(4457)$ and $P_{cs}(4459)$ pentaquark states by considering them as the diquark-diquark-antiquark picture with quantum numbers $J^P = \frac{3}{2}^-$, $J^P = \frac{1}{2}^-$ and $J^P = \frac{1}{2}^-$, respectively. In the analyses, we use the diquark-diquark-antiquark form of interpolating currents, and photon distribution amplitudes to obtain the magnetic dipole moment of pentaquark states. Theoretical examinations on magnetic dipole moments of the hidden-charm pentaquark states, are essential as their results can help us better figure out their substructure and the dynamics of the QCD as the theory of the strong interaction. As a by product, we extract the electric quadrupole and magnetic octupole moments of the $P_c(4440)$ pentaquark. These values are non-zero but small and show a non-spherical charge distribution.

Keywords: Pentaquarks, magnetic moment, $P_c(4440)$, $P_c(4457)$, $P_{cs}(4459)$, diquark-diquark-antiquark picture

I. MOTIVATION

With the experimental discovery of the exotic X(3872) state, i.e., state that cannot be interpreted by the conventional meson or baryon picture, a new era began in high energy physics. Since the discovery of this state, numerous exotic states have been added to the family of particles that have been experimentally reported. Study of exotic particles that are composed of tetraquark, pentaquark or hybrid states is among the most attractive subjects of the hadron physics. Experimental data collected with various collaborations in recent years and theoretical developments obtained in different theoretical models form the field of rapidly growing exotic studies [1–10].

In 2019, LHCb collaboration reported three new narrow pentaquark states [11] as

\begin{align*}
P_c(4312) & : M = 4311.9 \pm 0.7^{+6.8}_{-6.6} \text{ MeV}, \quad \Gamma = 9.80 \pm 2.7^{+3.7}_{-4.5} \text{ MeV}, \\
P_c(4440) & : M = 4440.3 \pm 1.3^{+4.1}_{-4.7} \text{ MeV}, \quad \Gamma = 20.6 \pm 4.9^{+8.7}_{-10.1} \text{ MeV}, \\
P_c(4457) & : M = 4457.3 \pm 0.6^{+1.1}_{-1.7} \text{ MeV}, \quad \Gamma = 6.40 \pm 2.0^{+5.7}_{-1.9} \text{ MeV}.
\end{align*}

Very recently, the LHCb Collaboration reported a pentaquark state with strangeness, $P_{cs}(4459)$, in the invariant mass spectrum of $J/\psi \Lambda$ in the $\Xi_b^0 \rightarrow J/\psi \Lambda K^-$ decay [12]:

\begin{equation}
P_{cs}(4459) : M = 4458.8 \pm 2.7^{+4.7}_{-1.1} \text{ MeV}, \quad \Gamma = 17.3 \pm 6.5^{+8.0}_{-5.7} \text{ MeV}.
\end{equation}

But, spin and parity of the $P_{cs}(4459)$ state have not been determined yet. As regards to their decay products, one can easily conclude that these newly discovered four states consist of at least five quarks, $c\bar{c}uud$ or $c\bar{c}uds$, therefore they are perfect candidates of hidden-charm pentaquark states. After the experimental discovery several phenomenological models have been adopted to calculate the spectroscopic parameters, decays and production mechanisms of the pentaquarks, like the QCD sum rule, the meson-exchange model, the quark delocalization model, and so on [13–54]. However, the substructure of these states are not determined yet. In other words, in order to understand the internal structure and nature of these particles, different properties should be studied besides their decay channels and spectroscopic properties. For instance, investigating their electromagnetic form factors may provide important insights on this point.

Electromagnetic form factors or multi-pole moments of hadrons are important parameters in study of their electromagnetic structure, and also they can ensure important knowledge about the dynamics of the QCD at low energy.

*ulasozdem@aydin.edu.tr
region. Electromagnetic multi-pole moments, especially magnetic dipole moment, are also a crucial part in the calculation of $J/\psi$ photo-production cross sections, which can provide an independent analysis of the hidden-charm pentaquark states. Investigating electromagnetic features of exotic resonances is relatively new topic. However, in the literature there are a few studies where the electromagnetic properties of the hidden-charm pentaquark states are investigated [55–58]. In Ref. [55], the magnetic dipole moment of the hidden-charm pentaquark states have been extracted in the diquark-diquark-antiquark picture via the LCSR with quantum numbers in the different color-flavor structure. In Ref. [56], the magnetic dipole, electric quadrupole and magnetic octupole moments of the $P_c(4380)$ pentaquark with $J^P = \frac{3}{2}^-$ quantum numbers have been obtained in the diquark-diquark-antiquark and molecular pictures in the framework of the light-cone QCD sum rule (LCSR). In Ref. [57], they acquired the ground state of hidden-charm pentaquarks with $J^P = \frac{3}{2}^-$ quantum numbers and their associated magnetic dipole moments and electromagnetic couplings, of interest to pentaquark photoproduction experiments in the framework of the constituent quark model. In Ref. [58], the magnetic dipole moment of the $P_c(3121)$ pentaquark state have been extracted in the molecular and diquark-diquark-antiquark pictures via the LCSR with quantum numbers $J^P = \frac{1}{2}^-$ quantum numbers. In Ref. [58], they achieved the magnetic dipole moment of the $P_c(4312)$ pentaquark state in the $\Sigma_c \bar{D}$ molecular picture by means of the QCD sum rule (QCDSR) in the external weak electromagnetic field.

In this study, we briefly introduce the LCSR method used to extract the magnetic dipole moments of the $P_c$ pentaquark states. In order to obtain the magnetic dipole moment of the corresponding states with the QCD sum rule method, we begin by writing the correlation function suitable for the calculations. This correlation function is obtained in two representations which are called hadronic and QCD representations. The LCSR for the physical quantities are obtained from the matches of the coefficients of the same Lorentz structures achieved on both hadron and quark-gluon degrees of freedom. At the hadron level, we embedding a complete set of intermediate states into the correlation function to acquire the hadronic representation, and isolate the ground state pentaquark states, and get the results:

$$\Pi^{Had}(p,q) = \frac{\langle 0 \mid J_{P_c(2\to 3)} \mid P_c(2\to 3)(p,s) \rangle}{[p^2 - m_{P_c(2\to 3)}^2]} \langle P_c(2\to 3)(p,s) \mid J_{P_c(2\to 3)} \mid 0 \rangle + ..., \quad (4)$$

The rest of paper is structured as follows. In Sect. II, the method used for the calculations are described. In Sect. III, we carry out numerical analysis of the acquired LCSR for the magnetic dipole moment of pentaquark states. The explicit expressions of the magnetic moment of the $P_{c1}$ is presented in Appendix A.

II. MAGNETIC DIPOLE MOMENTS OF THE PENTAQUARK STATES VIA LCSR

In this section, we briefly introduce the LCSR method used to extract the magnetic dipole moments of the $P_{c1}$, $P_{c2}$ and $P_{cs}$ pentaquark states. In order to obtain the magnetic dipole moment of the corresponding states with the QCD sum rule method, we begin by writing the correlation function suitable for the calculations. This correlation function is obtained in two representations which are called hadronic and QCD representations. The LCSR for the physical quantities are obtained from the matches of the coefficients of the same Lorentz structures achieved on both representations with the help of the quark-duality ansatz.

A. Formalism of the $P_{c2}$ and $P_{cs}$ states

The current correlation function for the $P_{c2}$ and $P_{cs}$ pentaquark states is written as

$$\Pi(p,q) = i \int d^4x e^{ipx} \langle 0 \mid T \{ J_{P_{c2}(cs)}(x) \bar{J}_{P_{c2}(cs)}(0) \} \rangle \gamma, \quad (2)$$

where $J_{P_{c2}(cs)}(x)$ is the interpolating current of $P_{c2}$ or $P_{cs}$ pentaquark state. In the diquark-diquark-antiquark picture with quantum number $J^P = \frac{1}{2}^-$, it is written as

$$\begin{align*}
J_{P_{c2}}(x) &= e^{abcde} e^{bfg} [u_T^c(x) C \gamma_5 d_c(x) u_T^d(x) C \gamma_5 C \gamma^a C \gamma^b C \gamma^e(x)], \\
J_{P_{cs}}(x) &= e^{abcde} e^{bfg} [u_T^a(x) C \gamma_5 d_c(x) s_T^d(x) C \gamma_5 C \gamma^a C \gamma^b C \gamma^e(x)],
\end{align*} \quad (3)$$

where $C$ is the charge conjugation matrix; and $a$, $b$, .. are color indices.

As we mentioned at the beginning of this section, in LCSR studies we need to evaluate the correlation function at both hadron and quark-gluon degrees of freedom. At the hadron level, we embedding a complete set of intermediate pentaquark states into the correlation function to acquire the hadronic representation, and isolate the ground state pentaquark states, and get the results:
The matrix element $\langle P_{2c}(p, s) | P_{c2}(p + q, s) \rangle_\gamma$ entering Eq. (4) can be parameterized in terms of Lorentz invariant form factors as follows:

$$
\langle P_{2c}(p, s) | P_{c2}(p + q, s) \rangle_\gamma = \varepsilon^\mu \langle p, s | \left[ f_1(q^2) + f_2(q^2) \right] \gamma_\mu + f_2(q^2) \frac{(2p + q)_\mu}{2m_{P_{2c}(s)}} \rangle u(p + q, s),
$$

where $\varepsilon$ and $q$ are the polarization vector and momentum of the photon, respectively.

Substituting Eq. (5) in Eq. (4) for hadronic side we get

$$
\Pi^{Had}_{p,q} = \frac{\lambda^2_{P_{2c}(s)} m_{P_{2c}(s)}}{(p + q)^2 - m^2_{P_{2c}(s)}} \mu_{P_{2c}(s)} \frac{1}{p^2 - m^2_{P_{2c}(s)}}.
$$

We observe from Eq. (6) that the correlation function contains many structures, any of them can be selected in obtaining magnetic dipole moment of the $P_{c2(c)}$ pentaquark, and thus we decide on the structure $f\bar{q}$. As a result, the correlation function can be expressed with regards to the magnetic dipole moment of $P_{c2(c)}$ pentaquark as,

$$
\Pi^{Had}_{p,q} = \frac{\lambda^2_{P_{2c}(s)} m_{P_{2c}(s)}}{(p + q)^2 - m^2_{P_{2c}(s)}} \mu_{P_{2c}(s)} \frac{1}{p^2 - m^2_{P_{2c}(s)}}.
$$

At the QCD level, we contract the relevant quark fields in the correlation function with the help of the Wick’s theorem,

$$
\Pi^{QCD}_{c,2}(p, q) = i \varepsilon^{abc} \varepsilon^{a'b'c'} \varepsilon^{ade} \varepsilon^{a'd'e'} \varepsilon^{bfg} \varepsilon^{b'f'g'} \int d^4 x e^{ip \cdot x} \left\{ \right.
$$

$$
\left. Tr \left[ \gamma_5 S^e_{d'}(x) \gamma_5 \tilde{S}^{dd'}_{u}(x) \right] \left[ \gamma_{\mu} S^e_{c'}(x) \gamma_{\nu} \tilde{S}^{ff'}_{\bar{u}}(x) \right] \left[ \gamma_5 \gamma_{\mu} \gamma_{\nu} \gamma_5 \right] + \right.
$$

$$
- \left. Tr \left[ \gamma_5 S^e_{d'}(x) \gamma_5 \tilde{S}^{dd'}_{u}(x) \right] \left[ \gamma_{\mu} S^e_{c'}(x) \gamma_{\nu} \tilde{S}^{ff'}_{\bar{u}}(x) \right] \left[ \gamma_5 \gamma_{\mu} \gamma_{\nu} \gamma_5 \right] \right\} |0\rangle_\gamma.
$$

where

$$
\tilde{S}^{ij}_{e(q)}(x) = C S^{ij}_{e(q)}(x) C,
$$

with $S_{q(c)}(x)$ being the full light and charm quark propagators. The relevant propagators are given as [63]

$$
S_q(x) = i \frac{\not{x} - \not{q}}{2 \pi^2 x^4} - \frac{\not{q} q}{12} (1 - \frac{m^2_f}{4}) - \frac{q \not{q}}{192} m^2_q x^2 \left( 1 - \frac{m^2_f}{6} \right) - \frac{i g_s}{32 \pi^2 x^2} G^{\mu\nu}(x) \left[ \# \sigma_{\mu \nu} + \# \sigma_{\mu \nu} \right],
$$

$$
S_c(x) = \frac{m^2_c}{4 \pi^2} \left[ K_1 \left( \frac{m_c \sqrt{-x^2}}{\sqrt{-x^2}} \right) + i \left. K_2 \left( \frac{m_c \sqrt{-x^2}}{\sqrt{-x^2}} \right) \right] \frac{g_s m_c}{16 \pi^2} \int_0^1 dv G^{\mu\nu}(v x) \left[ \# \sigma_{\mu \nu} + \# \sigma_{\mu \nu} \right],
$$

$$
K_1 \left( \frac{m_c \sqrt{-x^2}}{\sqrt{-x^2}} \right) + 2 \sigma_{\mu \nu} K_0 \left( m_c \sqrt{-x^2} \right),
$$

with $K_1(x)$ and $K_0(x)$ being the modified Bessel functions.
where $K_i$ are modified the second kind Bessel functions and $G^\mu\nu$ is the gluon field strength tensor.

The QCD representation of the correlation function can be obtained with the help of photon distribution amplitudes (DAs) according to quark-gluon properties and after performing the Fourier transform to transfer the calculations to the momentum space.

As a final step, by applying the double Borel transform on the variables $-p^2$ and $(p + q)^2$ and choosing the coefficients of the same Lorentz structures in both QCD and hadronic representations and matching them employing the quark-hadron duality approach, we obtain the desired LCSR for magnetic dipole moments of $P_{c2}$ and $P_{cs}$ states:

\[
\mu_{P_{c2}} \lambda_{P_{c2}}^2 m_{P_{c2}} = e^{\frac{-p^2}{2\Lambda^2}} \Delta_1^{QCD},
\]

\[
\mu_{P_{cs}} \lambda_{P_{cs}}^2 m_{P_{cs}} = e^{\frac{-p^2}{2\Lambda^2}} \Delta_2^{QCD}.
\]

The $\Delta_1^{QCD}$ and $\Delta_2^{QCD}$ functions are quite lengthy, therefore the explicit expression of this function are not presented here.

\section*{B. Formalism of the $P_{c1}$ state}

In this subsection we derive the LCSR for the magnetic dipole moment of the $P_{c1}$ pentaquark state. For this purpose, we consider following correlation function,

\[
\Pi_{\mu\nu}(p, q) = i \int d^4xe^{ipx}\langle 0| T\{J_{\mu}^{P_{c1}}(x)\bar{J}_{\nu}^{P_{c1}}(0)\}|0\rangle_{\gamma},
\]

where $J_{\mu}^{P_{c1}}$ is the interpolating current of $P_{c1}$ pentaquark with $J^P = \frac{3}{2}^-$ quantum numbers. In the diquark-diquark-antiquark picture, it is given as

\[
J_{\mu}^{P_{c1}}(x) = e^{abc}v_{d}^{a}v_{b}^{b}\varepsilon^{efg} [u_{d}^{T}(x)C_{\gamma}^{\Delta_1}d_{e}(x)u_{f}^{T}(x)C_{\gamma}^{\Delta_2}C_{\gamma}^{x}(x)],
\]

The hadronic side of the correlation function is written as,

\[
\Pi_{\mu\nu}^{Had}(p, q) = \langle 0| J_{\mu}^{P_{c1}}(0) | P_{c1}(p) \rangle_{\gamma} \langle P_{c1}(p) | P_{c1}(p + q) \rangle_{\gamma} \langle (p + q) | J_{\nu}^{P_{c1}}(0) | 0\rangle_{\gamma} + \ldots
\]

The matrix element of the interpolating current between the vacuum and the $P_{c1}$ pentaquark is defined as

\[
\langle 0| J_{\mu}^{P_{c1}}(0) | P_{c1}(p, s) \rangle = \lambda_{P_{c1}} u_{\mu}(p, s),
\]

where $\lambda_{P_{c1}}$ is the residue $P_{c1}$ pentaquark and $u_{\mu}(p, s)$ is the Rarita-Schwinger spinor.

The transition matrix element $\langle P_{c1}(p) | P_{c1}(p + q) \rangle_{\gamma}$ entering Eq. (19) can be parameterized in terms of four Lorentz invariant form factors as follows [64–67]:

\[
\langle P_{c1}(p) | P_{c1}(p + q) \rangle_{\gamma} = -\epsilon_{\mu\nu}(p) \left[ F_1(q^2)g_{\mu\nu} - \frac{1}{2m_{P_{c1}}} \left[ F_2(q^2)g_{\mu\nu} + F_4(q^2) \frac{q_{\mu}q_{\nu}}{2m_{P_{c1}}} \right] \phi \right] + F_3(q^2) \frac{1}{2m_{P_{c1}}} \phi \left[ G_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{2m_{P_{c1}}} \right] \phi
\]

In principle, we can obtain the final expression of the hadronic side of the correlation function using the above equations, but we encounter two difficulties: not all Lorentz structures are independent and the correlation function also includes contributions of spin-1/2 and these undesirable contributions must be eliminated. To remove undesirable contributions coming from the spin-1/2 particles and obtain only independent structures in the correlation function, we apply the ordering for Dirac matrices as $\gamma_{\mu}^{\text{R}}\gamma_{\nu}^{\text{L}}$ and remove terms with $\gamma_{\nu}$ at the beginning, $\gamma_{\mu}$ at the end and those proportional to $p_{\mu}$ and $p_{\nu}$ [68]. As a result, using Eqs. (17)-(21) the hadronic side take the form,

\[
\Pi_{\mu\nu}^{Had}(p, q) = \frac{\lambda_{P_{c1}}^2}{[(p + q)^2 - m_{P_{c1}}^2]^{2m_{P_{c1}}^2}} \left[ g_{\mu\nu} \phi \phi F_1(q^2) - m_{P_{c1}} g_{\mu\nu} \phi \phi F_2(q^2) - \frac{F_3(q^2)}{4m_{P_{c1}}} q_{\mu}q_{\nu} \phi \phi - \frac{F_4(q^2)}{4m_{P_{c1}}} c_{\mu}p_{\nu} \phi \phi + \text{other independent structures} \right].
\]
The final form of the hadronic side in terms of the selected structures in momentum space is:

$$\Pi^\text{Had}_{\mu \nu}(p, q) = \Pi^\text{Had}_1 g_{\mu \nu} \hat{p} \hat{q} + \Pi^\text{Had}_2 g_{\mu \nu} \hat{p} \hat{q} + \ldots,$$

(23)

where $\Pi^\text{Had}_1$ and $\Pi^\text{Had}_2$ are functions of the form factors $F_1(q^2)$ and $F_2(q^2)$, respectively; and other independent structures are represented by dots.

The magnetic, $G_M(q^2)$, form factor is defined in terms of the form factors $F_i(q^2)$ in the following way [64–67]:

$$G_M(q^2) = (F_1(q^2) + F_2(q^2))(1 + \frac{4}{5} \tau) - \frac{2}{5}[F_3(q^2) + F_4(q^2)]\tau(1 + \tau),$$

(24)

where $\tau = -\frac{q^2}{4m_{P_{c1}}^2}$. At $q^2 = 0$, the magnetic dipole moment is obtained in terms of the functions $F_i(0)$ and $F_2(0)$ as:

$$G_M(0) = F_1(0) + F_2(0).$$

(25)

The magnetic dipole moment, $(\mu_{P_{c1}})$, is defined in the following way:

$$\mu_{P_{c1}} = \frac{e}{2m_{P_{c1}}} G_M(0).$$

(26)

The next step is to calculate the correlation function in Eq. (17) in terms of quark-gluon parameters. When we apply the same procedures as in the previous subsection, we get the following result:

$$\Pi^{QCD}_{\mu \nu}(p, q) = i \varepsilon^{abc} \bar{c}^a d^b c^c \varepsilon^{ade} \bar{e} d^c \epsilon^{e f g} d^f \epsilon^{g h} \int d^4 x e^{-ip \cdot x} \langle 0 | \left\{ Tr \left[ \gamma_5 S^d_{\mu}(x) \gamma_5 \tilde{S}^{dd'}(x) \right] Tr \left[ \gamma_\mu S^g_{\nu}(x) \gamma_\nu \tilde{S}^{g g'}(x) \right] \right. \left( \tilde{S}_{c}^{e c'}(-x) \right) \left\} \right| 0 \rangle \gamma. $$

(27)

As a result, the QCD side of the correlation function in terms of the selected structures is obtained as

$$\Pi^{QCD}_{\mu \nu}(p, q) = \Pi^1_{\mu \nu} g_{\mu \nu} \hat{p} \hat{q} + \Pi^2_{\mu \nu} g_{\mu \nu} \hat{p} \hat{q} + \ldots$$

(28)

The following processes are applied as described in the previous subsection and magnetic dipole moment results are obtained in LCSR. The QCD and hadronic representations of the correlation function are then matched employing quark-duality assumption. By equating the coefficients of the structures $g_{\mu \nu} \hat{p} \hat{q}$ and $g_{\mu \nu} \hat{p} \hat{q}$, respectively for the $F_1$ and $F_2$ we obtain sum rules for these two form factors. As a result, we get,

$$\Pi^{Had}_{\mu \nu}(p, q) = \Pi^{QCD}_{\mu \nu}(p, q).$$

(29)

The explicit expressions of the LCSR for the $F_1$ and $F_2$ are presented in the Appendix A. We are now ready to move on to numerical analysis.

### III. NUMERICAL ANALYSIS AND DISCUSSIONS

This section is devoted to the numerical computations for the magnetic dipole moments of the $P_{c1}$, $P_{c2}$ and $P_{cs}$ pentaquark states. We use $m_u = m_d = 0$, $m_s = 96^{+3}_{-0} \text{MeV}$, $m_c = 1.275 \pm 0.02 \text{GeV}$ [69], $m_{P_{c1}} = 4440.3 \pm 1.3^{+1.1}_{-0.7} \text{MeV}$, $m_{P_{c2}} = 4457.3 \pm 0.6^{+1.1}_{-1.0} \text{MeV}$ [11], $m_{P_{cs}} = 4458.8 \pm 2.7^{+4.7}_{-1.0} \text{MeV}$ [12], $f_{\gamma} = -0.0039 \text{ GeV}^2$ [70], $\langle \bar{u}u \rangle = \langle dd \rangle = (-0.24 \pm 0.01)^3 \text{GeV}^3$, $\langle \bar{s}s \rangle = 0.8 \langle \bar{u}u \rangle \text{ GeV}^3$ [71], $m_0^2 = 0.8 \pm 0.1 \text{GeV}^2$ [71], $\langle g_5^2 G^2 \rangle = 0.88 \text{ GeV}^4$ [72], $\lambda_{P_{c1}} = (1.44 \pm 0.23) \times 10^{-5} \text{ GeV}^6$, $\lambda_{P_{c2}} = (3.02 \pm 0.48) \times 10^{-5} \text{ GeV}^6$ [28] and $\lambda_{P_{cs}} = (1.86 \pm 0.31) \times 10^{-3} \text{ GeV}^6$ [52]. Another set of main input parameters are the photon wavefunctions of different twists, entering the DAs. These wavefunctions are given in Ref. [70].

Except the above mentioned input parameters, the estimations for the magnetic dipole moments of pentaquark states depend on two auxiliary parameters: Borel mass parameter $M^2$ and continuum threshold $s_0$. According to the philosophy of the method used, the observables under examination should be weakly dependent on the variations of these auxiliary parameters. The continuum threshold is considered to be the point where the excited states and continuum begin to contribute to the correlation function. The upper and lower bound of the Borel parameter is
decided by demanding that both the contributions of the higher states and continuum are adequately suppressed and the contributions coming from higher dimensional terms are small. Our numerical analysis leads to the conclusion that these requirements are fulfilled in the regions shown below for the considered pentaquark states.

\begin{align*}
22.0 \text{ GeV}^2 &\leq s_0 \leq 24.0 \text{ GeV}^2 \text{ for } P_{c1} \text{ and } P_{c2} \text{ states}, \\
23.0 \text{ GeV}^2 &\leq s_0 \leq 25.0 \text{ GeV}^2 \text{ for } P_{cs} \text{ state}, \\
5.0 \text{ GeV}^2 &\leq M^2 \leq 7.0 \text{ GeV}^2 \text{ for } P_{c1} \text{ and } P_{c2} \text{ states}, \\
5.5 \text{ GeV}^2 &\leq M^2 \leq 7.5 \text{ GeV}^2 \text{ for } P_{cs} \text{ state}.
\end{align*}

By having the values of all input parameters, we can start carrying out numerical computations. In Fig. 1, we depict the dependencies of the magnetic dipole moments on \( M^2 \) and \( s_0 \). As is seen, the deviation of the results in connection with the \( s_0 \) is remarkable however there is much less dependence of the physical observables under consideration on the \( M^2 \) in its working window. Our final results for the magnetic dipole moments are

\begin{align}
\mu_{P_{c1}} &= 1.62^{+0.65}_{-0.57} \mu_N, \\
\mu_{P_{c2}} &= 0.88^{+0.32}_{-0.25} \mu_N, \\
\mu_{P_{cs}} &= 0.34^{+0.13}_{-0.11} \mu_N,
\end{align}

where \( \mu_N \) is the nucleon magneton. Our results cover errors originated from the uncertainty of the determinations of auxiliary parameters \((M^2 \text{ and } s_0)\) and other input parameters used in the analyses. Since the masses of the \( P_{c2} \) and \( P_{cs} \) states are close to each other, one can expect the magnetic dipole moment results to be close to each other. However, it is worth noting that the quark configuration for the \( P_{c2} \) and \( P_{cs} \) pentaquark states assigned as \([ud][uc]\bar{c}(1, 1, 1/2)\) and \([ud][sc]\bar{c}(0, 0, 1/2)\), respectively. Therefore, the results obtained for these two pentaquark states are different from each other. We observe that the \( P_c(4312) \) can be assigned to be the \([ud][uc]\bar{c}(0, 0, 1/2)\) pentaquark state with the spin-parity \( J^P = \frac{1}{2}^- \). It can be seen that the quark configurations of the \( P_{cs} \) and \( P_c(4312) \) particles are similar and therefore the magnetic dipole moment results of these particles can be expected to be close to each other at the SU (3) symmetry breaking limit. In Ref. [59], the magnetic dipole moment of \( P_c(4312) \) in the diquark-diquark-antiquark picture is extracted as \( \mu_{P_c} = 0.40 \pm 0.15 \mu_N \). As one can see from this estimation, the numerical values for the magnetic dipole moment of \( P_c \) and \( P_{cs} \) states acquired in the present study are close to each other and we see a reasonable SU(3) flavor violation which is roughly \( \%15 \). Since there are no theoretical results and experimental data for the results we obtained, we cannot compare our results. Comparing the results obtained using different theoretical models with our results can give an idea about the consistency of our predictions.

As a by product, we also obtain the electric quadrupole \( (Q_{P_{c1}}) \) and magnetic octupole \( (O_{P_{c1}}) \) moments of the \( P_{c1} \) pentaquark as

\begin{align}
Q_{P_{c1}} &= (2.0_{-0.5}^{+0.6}) \times 10^{-2} \text{ fm}^2, \\
O_{P_{c1}} &= (2.4_{-0.6}^{+0.7}) \times 10^{-4} \text{ fm}^3.
\end{align}

We see that nonzero but small value for the electric quadrupole and magnetic octupole moments of \( P_c(4440) \) pentaquark showing a non-spherical charge distribution.

In summary, the observation of new pentaquark states such as \( P_c(4312), P_c(4440), P_c(4457) \) and \( P_{cs}(4459) \), ensures a new platform to investigate the exotic states in QCD. There are different interpretations of their inner structures and quantum numbers, and these should be shedded light on with further research. In the present work, stimulated by the observation of the hidden-charm pentaquark states we have achieved the magnetic dipole moments of the \( P_c(4440), P_c(4457) \) and \( P_{cs}(4459) \) by considering them as diquark-diquark-antiquark picture with quantum numbers \( J^P = \frac{3}{2}^-, \frac{1}{2}^- \) and \( \frac{1}{2}^- \) by means of the light-cone QCD sum rule, respectively. As a by product, the electric quadrupole and magnetic octupole moments of the \( P_c(4440) \) pentaquark have also extracted. Our predictions for electromagnetic multi-pole moments of pentaquark states may be checked via different theoretical models such as Lattice QCD, chiral perturbation theory etc. Any experimental measurements of the electromagnetic multi-pole moments of the hidden-charm pentaquark states and comparison of the obtained results with the predictions of this study may provide as helpful knowledge on the internal structure of the these states as well as the non-perturbative nature of the QCD. We hope to extend our work with a detailed analysis of the electromagnetic multi-pole moments of the \( P_c(4440), P_c(4457) \) and \( P_{cs}(4459) \) hidden-charm pentaquark states by considering them as molecular picture.
FIG. 1: Variations of the magnetic dipole moments of $\mu_{Pc_1}$, $\mu_{Pc_2}$ and $\mu_{Pcs}$ with $M^2$ and $s_0$. 
Appendix A: Explicit forms of the $F_1$ and $F_2$ functions:

In this appendix, we give the explicit expressions for the $F_1$ and $F_2$ functions:

\[
F_1 = \frac{m_{\tilde{\chi}^0 H}}{\sqrt{\lambda}} \left\{ \frac{(g^2 G^2)}{108716359680 \pi^2} \right\} \begin{align*}
&20 \pi^2 f_{3\gamma} \left(4 \left(32 e_d + 41 e_u\right) I[0, 4, 4, 0] + 3 \left(9 e_d + 20 e_u\right) I[0, 4, 5, 0] \right) I_2[\gamma] \\
&+ (72 e_d + 720 e_u - 702 e_c) I[0, 5, 2, 1] - (324 e_d + 2184 e_u - 2673 e_c) I[0, 5, 2, 2] + (492 e_d + 2456 e_u - 3511 e_c) \\
&\times I[0, 5, 2, 4] - (300 e_d + 1240 e_u - 1811 e_c) I[0, 5, 2, 4] + (60 e_d + 248 e_u - 271 e_c) I[0, 5, 2, 5] + (216 e_d + 2160 e_u \\
&- 2106) I[0, 5, 3, 1] + (684 e_d + 4728 e_u - 5693 e_c) I[0, 5, 3, 2] - (624 e_d + 3424 e_u - 4483 e_c) I[0, 5, 3, 3] + (156 e_d \\
&+ 856 e_u - 893 e_c) I[0, 5, 3, 4] + (216 e_d + 2160 e_u - 2106 e_c) I[0, 5, 4, 1] - (396 e_d + 2904 e_u - 3375 e_c) \\
&\times I[0, 5, 4, 2] + (132 e_d + 968 e_u - 973 e_c) I[0, 5, 4, 3] - (72 e_d + 720 e_u - 702 e_c) I[0, 5, 5, 1] + (36 e_d + 360 e_u \\
&- 351 e_c) I[0, 5, 5, 2] \\
&\end{align*}
- \frac{m_c \langle \bar{q} q \rangle}{3774873\pi^2} \left\{ 192 e_c \left(I[0, 5, 2, 2] - 2I[0, 5, 2, 3] + I[0, 5, 2, 4] - 2I[0, 5, 3, 2] + 2I[0, 5, 3, 3] + I[0, 5, 4, 2] \right) \\
&- 40 \pi^2 f_{3\gamma} (e_d + 10 e_u) I_2[\gamma] I[0, 4, 4, 0] + 3(e_d + 14 e_u) I_4[S] I[0, 5, 4, 0] \\
&+ \frac{f_{3\gamma}}{3019898880 \pi^5} (13 e_d + 58 e_u) I_2[\gamma] I[0, 6, 5, 0] \\
&- \frac{e_c}{880803840 \pi^7} \left\{ 4 I[0, 7, 2, 3] - 13 I[0, 7, 2, 4] + 15 I[0, 7, 2, 5] - 7 I[0, 7, 2, 6] + I[0, 7, 2, 7] - 12 I[0, 7, 3, 3] \\
&+ 27 I[0, 7, 3, 4] - 18 I[0, 7, 3, 5] + 3 I[0, 7, 3, 6] + 12 I[0, 7, 4, 3] + 3 I[0, 7, 4, 5] - 4 I[0, 7, 5, 3] + I[0, 7, 5, 4] \right\} \right\}
\tag{35}
\]

\[
F_2 = \frac{m_{\tilde{\chi}^0 W}}{\sqrt{\lambda}} \left\{ \frac{(g^2 G^2)}{108716359680 \pi^2} \right\} \begin{align*}
&20 f_{3\gamma} \pi^2 \left\{ -4 \left(52 e_d - 19 e_u\right) I_2[\mathcal{A}] - 9 \left(32 e_d + 41 e_u\right) I_2[\mathcal{V}] + 4 \left(3 e_d + 2 e_u\right) \\
&\times I_6[\tilde{\zeta}\tilde{\zeta}] \right\} I[0, 4, 4, 0] + 3 \left\{ 8 \left(-34 e_d + e_u\right) I_2[\mathcal{A}] + 9 \left(9 e_d + 220 e_u\right) I_2[\mathcal{V}] \right\} I[0, 4, 5, 0] \\
&- \left\{ -72 e_d + 702 e_u - 720 e_c \right\} \\
&\times I[0, 5, 2, 1] + (324 e_d - 2673 e_u + 2184 e_c) I[0, 5, 2, 2] + (-492 e_d + 3511 e_c - 2456 e_u) I[0, 5, 2, 3] + (300 e_d \\
&- 1811 e_c + 1240 e_u) I[0, 5, 2, 4] + (-60 e_d + 271 e_c - 248 e_u) I[0, 5, 2, 5] + (216 e_d - 2106 e_c + 2160 e_u) I[0, 5, 3, 1] \\
&+ (-684 e_d + 5697 e_c - 4728 e_u) I[0, 5, 3, 2] + (624 e_d - 4484 e_c + 3424 e_u) I[0, 5, 3, 3] + (-156 e_d + 893 e_c - 856 e_u) \\
&\times I[0, 5, 3, 4] + (-216 e_d + 2106 e_c - 2160 e_u) I[0, 5, 4, 1] + (396 e_d - 3375 e_c + 2904 e_u) I[0, 5, 4, 2] + (-132 e_d \\
&+ 973 e_c - 968 e_u) I[0, 5, 4, 3] + (72 e_d - 702 e_u + 720 e_c) I[0, 5, 5, 1] + (-36 e_d + 351 e_c - 360 e_u) I[0, 5, 5, 2] \\
&\right\} \\
&- \frac{m_c \langle \bar{q} q \rangle}{125829120 \pi^8} \left\{ -120 \left(e_d + 10 e_u\right) f_{3\gamma} \pi^2 I_2[\mathcal{V}] I[0, 4, 4, 0] + (3 e_d + 14 e_u) I_4[S] + 2 e_d I_4[T_1] I[0, 5, 4, 0] \\
&+ 64 e_c \left(I[0, 5, 2, 2] - 2I[0, 5, 2, 3] + I[0, 5, 2, 4] - 2I[0, 5, 3, 2] + 2I[0, 5, 3, 3] + I[0, 5, 4, 2] \right) \\
&+ \frac{f_{3\gamma}}{3019898880 \pi^5} (13 e_d + 58 e_u) I_2[\gamma] I[0, 6, 5, 0] \\
&- \frac{e_c}{880803840 \pi^7} \left\{ 4 I[0, 7, 2, 3] - 13 I[0, 7, 2, 4] + 15 I[0, 7, 2, 5] - 7 I[0, 7, 2, 6] + I[0, 7, 2, 7] - 12 I[0, 7, 3, 3] \\
&+ 27 I[0, 7, 3, 4] - 18 I[0, 7, 3, 5] + 3 I[0, 7, 3, 6] + 12 I[0, 7, 4, 3] - 15 I[0, 7, 4, 4] + 3 I[0, 7, 4, 5] - 4 I[0, 7, 5, 3] \\
&+ I[0, 7, 5, 4] \right\} \right\}
\tag{36}
\]
We should also point out that in the above expressions, for simplicity we have only given the terms that give important contributions to the numerical values of the magnetic moments, and have not presented many higher dimensional contributions, although they have been considered in the numerical calculations.

The functions $I[n,m,l,k]$, $I_1[A]$, $I_2[A]$, $I_3[A]$, $I_4[A]$, $I_5[A]$, and $I_6[A]$ are defined as:

$$I[n,m,l,k] = \int_{4m^2}^{s_0} ds \int_{0}^{1} dt \int_{0}^{1} dw \ e^{-s^2/M^2} \ s^n (s - 4m^2)^m t^l w^k,$$

$$I_1[A] = \int D\alpha, \int_{0}^{1} dv \ A(\alpha,\alpha,\alpha) \delta(\alpha + \bar{\alpha} - u_0),$$

$$I_2[A] = \int D\alpha, \int_{0}^{1} dv \ A(\alpha,\alpha,\alpha) \delta'(\alpha + \bar{\alpha} - u_0),$$

$$I_3[A] = \int D\alpha, \int_{0}^{1} dv \ A(\alpha,\alpha,\alpha) \delta(\alpha + \bar{\alpha} - u_0),$$

$$I_4[A] = \int D\alpha, \int_{0}^{1} dv \ A(\alpha,\alpha,\alpha) \delta'(\alpha + \bar{\alpha} - u_0),$$

$$I_5[A] = \int_{0}^{1} du \ A(u) \delta'(u - u_0),$$

$$I_6[A] = \int_{0}^{1} du \ A(u),$$

where $A$ represents the corresponding photon distribution amplitudes.

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