Taming Deeply Virtual Compton Scattering

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We study recent Deeply Virtual Compton scattering (DVCS) data within a dual parameterization of the Generalized Parton Distributions (GPDs). This parameterization allows to quantify the maximum amount of information, that can be extracted from DVCS data, in a “quintessence” function. We present a “zero step” model for the latter solely based on the forward quark density, providing a parameter free prediction for the imaginary part of the DVCS amplitude. It is shown that the bulk effect of the ep → eγp beam helicity cross section difference can be understood within such a model.

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The generalized parton distributions (GPDs) [1] (see Refs. [2,3,4,5] for recent reviews) describe the response of the target hadron to the well-defined QCD quark-gluon operator of the type : \( \bar{\psi}(0) P e^{i g \int_z^x A_\nu(z) \psi(z) } \), which is defined on the light-cone, i.e. \( z^2 = 0 \). The hard exclusive processes provide us with a set of new fundamental probes of the hadronic structure. The hadron image seen by such non-local probes is encoded in the dependence of GPDs on its variables. The central quantities that contain invaluable information on the nucleon structure are momentum transfer (t) dependent quark densities. These densities can be obtained as the \( x \to 0 \) limit of the nucleon GPDs \( H(x, \xi, t) \) and \( E(x, \xi, t) \):

\[
q(x, t) = \lim_{\xi \to 0} H(x, \xi, t), \quad e(x, t) = \lim_{\xi \to 0} E(x, \xi, t), \quad (1)
\]

where \( x \pm \xi \) correspond to the quark longitudinal momentum fractions. The first density \( q(x, t) \) (at \( t = 0 \) it reduces to the usual quark distribution measured in DIS) is related to the distribution of quarks and anti-quarks in the longitudinal momentum and transverse plane of the nucleon, thus providing the 3D image of quarks and anti-quarks in the nucleon [7]. The new function \( e(x, t) \) is crucial for extraction of the angular momentum carried by quarks in the nucleon [7].

Although the quark densities \( q(x, t) \) and \( e(x, t) \) are defined as the simple limit of GPDs [1], one can not perform this limit measuring observables for hard exclusive processes. The reason is that the leading order amplitude of hard exclusive reactions (we restrict ourselves to DVCS and discuss only the GPD \( H(x, \xi, t) \), the discussion for \( E(x, \xi, t) \) is analogous) is expressed as:

\[
A(\xi, t) = \int_{-1}^{1} dx \ H(x, \xi, t) \left( \frac{1}{\xi - x - i0} - \frac{1}{\xi + x - i0} \right). \quad (2)
\]

The amplitude is given by the convolution integral in which the dependence of the GPDs on the variable \( x \) is “integrated out”. One cannot completely restore the GPD \( H(x, \xi, t) \) from (2), hence one is not able to perform a “complete imaging” of the target hadron from the knowledge of the amplitude and cannot perform the limit (1) to obtain the key quark densities \( q(x, t) \) and \( e(x, t) \).

The aim of this Letter is to estimate the contribution of the quark densities \( q(x, t) \) to DVCS observables. This is to be considered as a “zero step” to extract the GPD information from the data, as it allows us to quantify the deviations in terms of genuine non-forward parts of the GPDs. For our analysis we employ the dual parameterization of GPDs suggested in Ref. [8].

The dual parameterization is based on a representation of parton distributions as an infinite series of \( t \)-channel exchanges [11], allowing to express the GPD \( H \) in terms of a set of functions \( Q_2(\xi, x, t) \) \( (\nu = 0, 1, 2, \ldots) \), for details see [8]. We call the functions \( Q_2(\xi, x, t) \) forward-like because at leading order (LO), the scale dependence of the functions \( Q_2(\xi, x, t) \) is given by the standard DGLAP evolution equation, so that these functions behave as usual parton distributions under QCD evolution. Furthermore, the function \( Q_0(x, t) \) is related to the forward \( t \)-dependent quark densities \( q(x, t) \) (which reduces at \( t = 0 \) to the parton densities \( q(x) \) measured in DIS):

\[
Q_0(x, t) = [q + \bar{q}] (x, t) - \frac{x}{2} \int_x^1 \frac{dz}{z^2} \ [q + \bar{q}] (z, t). \quad (3)
\]

The expansion of the GPD \( H(x, \xi, t) \) around \( \xi = 0 \) with \( x \) fixed to the order \( \xi^2 \) involves only a finite number of functions \( Q_{2\nu}(x, t) \) with \( \mu \leq \nu \). One can then express the amplitude (2) in terms of forward-like functions as [8, 9]:

\[
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Im \( A(\xi, t) \) = \( 2 \int \frac{dx}{x} N(x, t) \frac{1}{\sqrt{\frac{2\pi}{\xi} - x^2 - 1}} \),
\( t > 0 \).
\( 1 \quad 1 \quad 2 \)

Re \( A(\xi, t) \) = \( 2 \int \frac{dx}{x} N(x, t) \left[ \frac{1}{\sqrt{1 - \frac{2\pi}{\xi} + x^2}} + \frac{1}{\sqrt{1 + \frac{2\pi}{\xi} + x^2}} - \frac{2}{\sqrt{1 + x^2}} \right] \)
\( t > 0 \).
\( 2 \quad 1 \quad 1 \)

Here \( D(t) \) is the D-form factor (FF) :
\[
D(t) = \sum_{n=1}^{\infty} d_n(t) = \frac{1}{2} \int_{-1}^{1} \frac{D(z, t)}{1 - z} \, dz,
\]
where \( D(z, t) \) is the D-term \( 12 \). One can check \( 9 \) that the amplitude given by Eqs. (4,5) automatically satisfies a dispersion relation with the subtraction constant given by the D-FF, as it should be on general grounds \( 13,14 \).

The function \( N(x, t) \) in Eqs. (4,5) is defined as :
\[
N(x, t) = \sum_{n=0}^{\infty} x^{2n} Q_{2n}(x, t).
\]

The information contained in the LO amplitude is in one-to-one correspondence with the knowledge of the function \( N(x, t) \) and the D-FF \( D(t) \), because Eq. (4) can be inverted \( 9 \) in terms of the amplitude. This inversion corresponds to the Abel transform tomography \( 15 \), and the corresponding inversion equation has the form \( 9 \) :
\[
N(x, t) = \frac{1}{\pi} \frac{x(1 - x^2)}{(1 + x^2)^{3/2}} \int_{1/x}^{1} \frac{d\xi}{\xi^{3/2}} \frac{1}{\sqrt{\xi - \frac{2\pi}{1 + x^2}}} \times \left\{ \frac{1}{2} \text{Im} A(\xi, t) - \xi \frac{d}{d\xi} \text{Im} A(\xi, t) \right\} .
\]

This equation implies that the function \( N(x, t) \) contains the maximal information about GPDs that is possible to obtain from the observables. Therefore this function can be called the GPD-quintessence function \( 1 \). Another important feature of the expressions \( 13 \) for the amplitude is that one can easily single out the contributions to the amplitude coming from the forward parton densities. Indeed, the first term in the sum \( 7 \) is given by the function \( Q_0 \) which is related to the \( (t\text{-dependent}) \) parton densities by Eq. \( 3 \). The big advantage of the dual parameterization is that one can clearly separate the contribution of the \( (t\text{-dependent}) \) parton densities from genuine non-forward effects encoded in the functions \( Q_2, Q_4, \ldots \). The authors of Ref. \( 10 \) developed an approach that also allows to separate the contribution of forward quark densities to observables. Calculations of DVCS observables in the dual parameterization were presented in \( 17 \), however in this paper a “predefined” model for forward-like functions \( Q_{2n} \) has been used and the “zero step” separation was not discussed.

In the following we study the separation between effects of forward quark densities and genuine non-forward contributions. In order to make this separation (“zero step”) as clean as possible we choose to analyze recent JLab/Hall A data \( 18 \) on the beam helicity \((\text{in})\text{dependent} cross sections in the } e^+ p \rightarrow e^+ p + \gamma \text{ process, as well as recent beam spin asymmetry data measured by the CLAS collaboration } 20 \). We make such choice because the beam helicity dependent cross sections and beam asymmetries are directly proportional to the imaginary part of the DVCS amplitude. This, in principle, gives the possibility to apply directly the Abel tomography formula \( 3 \), and requires a measurement at several values of \( x_B \). Furthermore, we choose the data with \( Q^2 > 2 \text{ GeV}^2 \) and \( -t < 0.3 \text{ GeV}^2 \). In this kinematical region one can safely neglect contributions of the nucleon GPDs \( E, H \) and \( \tilde{E} \) relative to the contribution of \( H \), also one keeps the contribution of higher twists negligible. This allows us to make direct conclusions about contribution of the quark densities \( q(x, t) \). The region of small \( t \) makes our results insensitive to the modelling of the \( t \)-dependence of the quark densities.

In order to make this “zero step” comparison of the data with the contribution of forward quark densities we have to make assumptions about the \( t \)-dependence of the quark densities \( q(x, t) \). As the model for this dependence we use the Regge motivated Ansatz : \( q(x, t) = q(x) \cdot x^{-\alpha' t} \), where \( q(x) \) the forward quark distribution, and the slope parameter \( \alpha' = 1.105 \text{ GeV}^{-2} \) was fixed from the form factor sum rule in Ref. \( 19 \).

In Fig. \( 11 \) we compare our predictions for the polarized cross section difference for different beam helici-
polarized cross section difference $(d^t\sigma_+ - d^t\sigma_-)/2$ for different beam helicities. The black bands are JLab/Hall A data. Dashed (blue) curves: double distribution parameterization with profile parameter $b = 1$; solid (red) curves: dual parameterization including only the forward function $Q_0$.

The JLab/Hall A Coll. also published results for unpolarized cross sections that are shown in Fig. 2. When comparing the unpolarized cross sections with the model independent Bethe-Heitler result, one sees that the latter dominates the cross section at $\Phi = 0$, where it makes up for about 85% of the cross section, at $-t = 0.23$ GeV$^2$. However, at $\Phi = 180$ deg it is more than a factor 2 below the data. Although both double distribution and dual parameterization models can explain part of the difference with the data, no single model is able to simultaneously describe the cross section at $\Phi = 0$ and $\Phi = 180$ deg, in line with the finding of [22]. In particular, within the dual parameterization in twist-2 approximation, adjusting the one subtraction constant $D(t)$ does not allow to describe this $\Phi$ dependence, as is also shown on Fig. 2.

It was also checked that when adding an estimate for the non-forward function $Q_2$ no agreement can be found either over the whole $\Phi$ range. Although the region around $\Phi = 180$ deg yields zero for the beam helicity asymmetry, it is very worthwhile to cross-check this puzzle further.

In Fig. 3 we show recent exclusive measurements of $e^- p \rightarrow e^- p\gamma$ beam helicity asymmetries from JLab/CLAS [20] and JLab/Hall A [18]. We note that for two middle bins in Fig. 3 where both experiments

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2 We checked that in the $t$ range shown, the dependence on $\alpha'$ is much smaller than the difference between the curves.
FIG. 3: Azimuthal angular dependence of the \( e^- p \rightarrow e^- p \gamma \) beam helicity asymmetry \( (d^e \sigma_+ - d^e \sigma_-)/(d^e \sigma_+ + d^e \sigma_-) \) for different kinematics. Black bands in the two middle panels are data points from JLab/Hall A [13]. Solid circles are data points from JLab/CLAS [20]. Curve conventions as in Fig. 2.

have overlapping kinematics, the data from both experiments are consistent with each other. When comparing both parameterizations with the data in Fig. 3 we note that the double distribution model (dashed curves) yields asymmetries that lie systematically above the data. The dual parameterization model based on the forward function \( Q_0 \) (solid curves) yields a good first ("zero step") description of the data, given that no parameter was adjusted here. There is a slight tendency for the asymmetries to be overestimated within the dual parameterization which is merely a reflection of the underestimate of the unpolarized cross sections as seen in Fig. 2.

The success of our "zero step" exercise shows that the contribution of the forward quark densities \( q(x,t) \) at small \( t \) to the \( e^- p \rightarrow e^- p \gamma \) polarized cross section difference yields the bulk effect. Deviations of this prediction from the data can be fitted by introducing forward-like functions \( Q_2, Q_4, \ldots \) which describe the genuine non-forward effects in GPDs. The dominance of the forward quark densities in the imaginary part of the DVCS amplitude is an important observation. It implies that precise measurements of DVCS observables in a wider kinematical region would allow us to extract the \( t \)-dependent quark densities \( q(x,t) \) and \( c(x,t) \). The latter can be accessed by measurements on the neutron.

In summary, we studied recently obtained high precision DVCS data in the valence region within a dual parameterization of the GPDs. This parameterization allows to extract a quintessence function which contains the maximal amount of information that can be extracted from DVCS observables. We established a "zero step" model for this function solely based on the forward quark density, which provides a parameter free prediction for the imaginary part of the DVCS amplitude. It was shown that the bulk effect of the \( ep \rightarrow ep \gamma \) beam helicity cross section difference can be understood within such a model. By systematically studying deviations between the data and such model will allow to reveal the non-forward effects to this quintessence function.

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