Abstract: This paper presents a framework for the design and analysis of an \(L_1\) adaptive controller with a switching reference system. The use of a switching reference system allows the desired behavior to be scheduled across the operating envelope, which is often required in aerospace applications. The analysis uses a switched reference system that assumes perfect knowledge of uncertainties and uses a corresponding non-adaptive controller. Provided that this switched reference system is stable, it is shown that the closed-loop system with unknown parameters and disturbances and the \(L_1\) adaptive controller can behave arbitrarily close to this reference system. Simulations of the short period dynamics of a transport class aircraft during the approach phase illustrate the theoretical results.

Keywords: Adaptive control, Control System Analysis, Flight Control, Switching System.

1. INTRODUCTION

In many aerospace applications, a local linear approximation of the plant is sufficient for local control. However, over the entire flight envelope, the dynamics can differ significantly. For example, at high speeds, with more air flowing over the control surfaces, there can be significantly more control authority than at low speeds. More modification of the natural dynamics of the vehicle is then possible, so the control objectives can be more ambitious. Sometimes the overall behavior of the system can vary throughout the envelope from stable to unstable. Typically, once control laws are designed in the different local regions of the flight envelope, they are then scheduled throughout. There are many examples of gain scheduling.

The design for the high-alpha research vehicle by Davidson et al. (1998) uses gain scheduling. In Gangsaaas et al. (2008) gain scheduling is used for control of a business jet, and the F-35 control law uses a dynamic inversion controller based on scheduled linear models, per Harris and Stanford (2018). Applying gain scheduling enables the use of analysis techniques and design criteria intended for linear plants while allowing different dynamics to be set at different locations within the flight envelope.

In recent years, researchers have begun investigating adaptive control designs for piecewise linear (gain scheduled) systems. In Sang and Tao (2012), piecewise linear reference models are used in the model reference adaptive control (MRAC) framework with projection based adaptation laws. The stability condition is given based on a dwell time argument. The authors of Yuan et al. (2016) sought to extend the results of Sang and Tao (2012) by proposing new stability criteria allowing for the Lyapunov matrix to be time varying. However, the adaptive laws and switching laws are coupled in this approach. Both of these results are only applicable to single-input systems. The work in Kersting and Buss (2017) provides the first results for multi-input MRAC for piecewise affine systems.

In the past decade, \(L_1\) adaptive control has been developed (see Hovakimyan and Cao (2010)) and applied to numerous flight control applications, including NASA’s AirSTAR in Gregory et al. (2010) and Calspan’s variable-stability Learjet in Puig-Navarro et al. (2019). However, in these flight applications, the control law was either not scheduled or the scheduling was done in an ad-hoc fashion, lacking a rigorous mathematical analysis and relying on extensive numerical simulations for stability and performance verification. This paper provides a method for analyzing an \(L_1\) adaptive controller where the desired dynamics are changing throughout the flight envelope. Note that these changes could occur due to known scheduling parameters, e.g. fuel state, or due to online model identification.

2. MOTIVATING EXAMPLES

Before presenting the main results, we present a few motivating examples. \(L_1\) adaptive control has been shown to compensate for uncertainties and disturbances quite well (see e.g. Ackerman et al. (2016); Gregory et al. (2010)). However, sometimes changes in the system response are also important.

Changes in Airspeed

In Ackerman et al. (2019), during flight testing of an \(L_1\) adaptive control law, pilots noted that they could not feel the typical change in stick force associated with the vehicle slowing down, causing pilots to spend more time
looking at the instrument gauges. As noted by the authors, “the lack of cuing through stick sensitivity is a direct (expected) result of the adaptive augmentation providing automatic compensation for the change in stick sensitivity and providing a consistent aircraft response despite the deviation in airspeed from the design condition.”

Changes in Inertia

Another common scenario where dynamics are expected to change is when the inertia changes, such as when payload is dropped or fuel stored on the wings of a vehicle is burned. For a given speed, Puig-Navarro et al. (2019) shows that the roll mode time constant changes by a factor of roughly 2.5 as fuel is burned. If the natural dynamics provide an acceptable response, it may not be worth spending the vehicle’s finite control power to modify the dynamics back to a fixed reference system. This would reduce the available control power for disturbance rejection and command tracking while making it more challenging to achieve desired stability margins.

System Identification

In Heim et al. (2018), a new paradigm in aircraft design and testing is suggested. The central idea is to use state-of-the-art system identification techniques to develop mathematical models of the aircraft onboard and in real time during flight. The control algorithms in this framework are adjusted based on the identified models.

The present work aims to add to the $L_1$ adaptive control literature an approach for handling switched reference systems in order to achieve expected (desired) changes in the system dynamics.

Throughout this paper, we use $\| \cdot \|$ to denote either the Euclidean norm of a vector or the induced 2-norm of a matrix. $\mathbb{R}^n$ denotes the $n$-dimensional real vector space. $I$ denotes an identity matrix of appropriate dimensions.

For symmetric matrices $P$ and $Q$, $P > Q$ means $P - Q$ is positive definite. For a function of time $x: t \to \mathbb{R}^n$, its Laplace transform is denoted by $x(s) = \mathcal{L}[x(t)]$, and its $L_\infty$ norm is defined as $\|x\|_{L_\infty} = \max_{t \geq 0} \|x(t)\|_2$.

3. $L_1$ ADAPTIVE CONTROL FOR SWITCHING REFERENCE SYSTEMS

3.1 Problem Formulation

Consider the family of multi-input multi-output LTI systems whose state-space matrices are given by

$$\{(A_i, B_i, C_i) : i \in I\},$$

where $I$ denotes the index set. Let $\mathcal{P} = \{p : [0, \infty) \to \mathcal{I}\}$ denote the family of piecewise constant switching signals. For a given switching signal $p \in \mathcal{P}$, define the following switched linear system subjected to time-varying parametric uncertainty and disturbances:

$$\dot{x}(t) = A_p x(t) + B_p (\omega_p u(t) + \hat{\theta}_p(t) x(t) + \sigma_p(t)),
\quad y(t) = C_p x(t),
\quad x(0) = x_0,$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^m$ is the system input, and $y(t) \in \mathbb{R}^m$ is the regulated system output. $A_p, B_p, C_p \in \mathbb{R}^{n \times n}, B_p \in \mathbb{R}^{n \times m}$, and $C_p \in \mathbb{R}^{m \times n}$ are the system matrices. $\omega_p \in \Omega \subset \mathbb{R}^{m \times m},$ $\theta_p(t) \in \Theta \subset \mathbb{R}^{n \times m}$, and $\sigma_p(t) \in \Delta \subset \mathbb{R}^m$ are unknown system parameters. Given a switching signal $p$, we assume that the sequence of finite switching time is $t_0, t_1, \ldots, t_l, \ldots$ with $t_0 = 0$.

Assumption 1. The sets $\mathcal{A}, \mathcal{B}, \mathcal{C}, \Omega, \Theta, \Delta$ are compact, convex polytopes. Without loss of generality, the sets $\Theta$ and $\Delta$ are assumed to contain 0. $\Omega$ is assumed to be diagonally dominant, and without loss of generality, it is assumed to contain $\mathbb{I}$. Define

$$D_\theta = \max_{\theta \in \Theta} \|\theta\|, \quad D_\sigma = \max_{\sigma \in \Delta} \|\sigma\|, \quad D_\omega = \max_{\omega \in \Omega} |\omega - I|.$$ 

It is further assumed that $\sigma_i(t)$ and $\theta_i(t)$ are continuous and have (unknown) bounded derivatives for all $i \in I$, i.e.

$$|\dot{\sigma}_i(t)| \leq d_\sigma, \quad |\dot{\theta}_i(t)| \leq d_\theta, \quad \forall i \in I.$$ (3)

Assumption 2. The switching signal $p$ has a dwell time, $\tau_d > 0$, i.e. the switching times $t_1, t_2, \ldots$ satisfy the inequality $t_{k+1} - t_k \geq \tau_d$ for all $k$. However, the results derived for dwell-time switching also hold for the more general case of average-dwell-time switching. For the details of (average) dwell-time switching, see Liberman (2003).

To analyze the performance of the adaptive system that will be presented in Section 3.2, we define a (non-adaptive) reference system that contains perfect knowledge of the parameters:

$$\dot{x}_{\text{ref}}(t) = A_p x_{\text{ref}}(t) + B_p (\omega_p u_{\text{ref}}(t) + \hat{\theta}_p(t) x_{\text{ref}}(t) + \sigma_p(t)),
\quad x_{\text{ref}}(0) = x_0,$$

$$u_{\text{ref}}(s) = -\frac{D_0(s)}{s} \mu_{\text{ref}}(s),$$

where $\mu_{\text{ref}}(t) = \omega_p u_{\text{ref}}(t) + \hat{\theta}_p(t) x_{\text{ref}}(t) + \sigma_p(t) - k_p r(t)$. The last equation in (4) is equivalent to

$$u_{\text{ref}} = -\omega_p^{-1} C_p x_{\text{ref}}.$$

where $\xi_{\text{ref}}(t) = \hat{\theta}_p(t) x_{\text{ref}}(t) + \sigma_p(t) - k_p r(t)$ with $k_p$ being a feedforward gain for reference tracking, and the (time-invariant) mapping $C_i$ has the transfer function form of

$$C_i(s) = \omega_i(s I + D_0(s) \omega_i)^{-1} D_0(s),$$

which denotes a low-pass filter with the DC gain equal to an identity matrix, i.e. $C_i(0) = \mathbb{I}$. From the first equation in (4) and (5), one can see that the reference control input tries to cancel the uncertainties within the bandwidth of the filter $C_p$. This reference system provides the target performance of the $L_1$ adaptive controller.

Letting $(A_f, B_f, C_f, D_f)$ be a minimal realization of $D_0(s)$ with $n_f$ states, the reference system dynamics can be written in state-space form:

\footnote{We use the input-output mapping form instead of a transfer function form in (5) since the mapping is time-varying due to the existence of switching.}
Lemma 4. If there exists symmetric matrices $\bar{P}_s$ provides a sufficient condition, which we assume to be a switching signal with dwell time $\tau_d$. The following lemma follows the switched reference system with quantifiable transient and steady-state performance bounds.

### 3.2 $\mathcal{L}_1$ Adaptive Control Architecture

For the switched system in (2), we define the state predictor as

$$\hat{x}(t) = A_p \hat{x}(t) + B_p \left( \hat{\omega}(t)u(t) + \hat{\theta}(t)\tau(t) + \hat{\sigma}(t) \right),$$

where $\hat{x}$ is the state of the predictor, and $\hat{\omega}$, $\hat{\theta}$, and $\hat{\sigma}$ are the parameter estimates governed by the adaptation laws:

$$\dot{\hat{\theta}}(t) = \Gamma \text{Proj} \left( \hat{\theta}(t), -x(t)\bar{x}(t) P_p B_p \right), \quad \hat{\theta}(0) = 0,$$

$$\dot{\hat{\sigma}}(t) = \Gamma \text{Proj} \left( \hat{\sigma}(t), -B_p^T P_p \bar{x}(t) \right), \quad \hat{\sigma}(0) = 0,$$

with $P_p$ being the (1,1) block of the $\bar{P}_p(t)$ in (11) (under the same partition as (7)), and $\bar{x}(t) = \bar{x} - x(t)$ being the prediction error.

The control law is defined as

$$u(t) = -\frac{D_0(s)}{s} \mu(s),$$

where $\mu(s)$ is the Laplace transform of $\mu(t) = \hat{\omega}(t)u(t) + \hat{\theta}(t)\tau(t) - k_p r(t)$, respectively, $D_0(s)$ is a proper stable transfer function, $k_p$ is a reference scaling gain, and $r(t)$ is a bounded, piecewise continuous reference signal. For autonomous systems, the reference scaling gain is often the inverse of the $p$th system's DC gain, i.e. $k_p = -(C_p A_p^{-1} B_p)^{-1}$, but for human-controlled applications, this could be some other scaling or shaping of the human input. Note that the control input will always be continuous due to the presence of the filter.

### 3.3 Analysis of the $\mathcal{L}_1$ Controller

Let $\hat{\omega}_p(t) = \hat{\omega}(t) - \omega_p$, $\hat{\theta}_p(t) = \hat{\theta}(t) - \theta_p$, and $\hat{\sigma}_p(t) = \hat{\sigma}(t) - \sigma_p(t)$. Defining

$$\tilde{\eta}_p(t) = \hat{\omega}_p(t)u(t) + \hat{\theta}_p(t)\tau(t) + \hat{\sigma}_p(t),$$

the prediction error dynamics can be formed from (2) and (12) as:

$$\dot{\tilde{x}}(t) = A_p \tilde{x}(t) + B_p \tilde{\eta}_p(t), \quad \tilde{x}(0) = 0.$$  

Lemma 7. The prediction error $\tilde{x}(t)$ is uniformly bounded,

$$||\tilde{x}(t)||_{\infty} \leq \sqrt{\frac{\beta_2}{\Gamma}},$$

for all $p \in \mathcal{P}$, where

$$\beta_2 = 4(D_p^2 + D_p^2 + D_p^2) + 4\lambda^{-1}(D_0 d_0 + D_a d_a).$$

**Proof.** Consider the Lyapunov function during the $i$-th time interval, $t \in [t_i, t_{i+1})$,

$$V_i(t) = \tilde{x}^T(t) P_i \tilde{x}(t) + \frac{1}{\Gamma} \left( \text{tr} (\hat{\theta}_p^T(t) \hat{\theta}_p(t)) + \text{tr} (\hat{\omega}_p^T(t) \hat{\omega}_p(t)) + \hat{\sigma}_p^2(t) \right)$$

Differentiating along trajectories of the prediction error dynamics (16) and substituting the adaptive laws (13),
The reference error dynamics can be compactly written as
\[
\begin{bmatrix}
\dot{\tilde{e}}(t) \\
\dot{\tilde{x}}_f(t)
\end{bmatrix} = \begin{bmatrix}
\bar{J}_p & \bar{J}_p \\
0 & \bar{J}_p
\end{bmatrix} \begin{bmatrix}
\tilde{e}(t) \\
\tilde{x}_f(t)
\end{bmatrix} + \bar{J}_p \bar{G}_p \tilde{e}(t),
\]
\[
|e_u(t)| = |C_L| \begin{bmatrix}
\dot{\tilde{e}}(t) \\
\dot{\tilde{x}}_f(t)
\end{bmatrix} - D_J B_J^T \tilde{e}(t),
\]
where \(\tilde{x}_f(t) = [x_f^T(t), x_f^T(t)]^T\).

**Theorem 9.** Consider the closed-loop adaptive system with the \(L_1\) controller defined via (12)–(14) and the closed-loop reference system (7) (or (4)). Suppose that there exist \(\bar{P}_i(\omega) (i \in \mathcal{I})\) and some constants \(\lambda > 0\) and \(\mu \geq 1\) such that the inequalities in (9) hold for all \((\theta, \omega) \in \Theta \times \Omega\), and the dwell time satisfies (10). Then, there exist positive constants \(\kappa_1\) and \(\kappa_2\) such that
\[
\|
\begin{bmatrix}
\tilde{x}(t) \\
\tilde{x}_f(t)
\end{bmatrix}
\|_\infty \leq \kappa_1 \begin{bmatrix}
\tilde{e} & \tilde{x}_f
\end{bmatrix} \|_\infty \leq \kappa_2 \begin{bmatrix}
\tilde{e} & \tilde{x}_f
\end{bmatrix} \|_\infty.
\]

**Proof.** Partitioning \(\bar{P}_i\) of (9) along the same partition as \(\bar{A}_i\) in (7), we have
\[
\bar{P}_i = \begin{bmatrix}
P_i & R_i \\
R_i & S_i
\end{bmatrix}.
\]

Further defining \(Q_i \triangleq S_i - R_i^T P_i^{-1} R_i\), and considering (9), one obtains
\[
Q_i \geq 0, \quad \forall i \in \mathcal{I},
\]
\[
\bar{F}_i^T Q_i + Q_i \bar{F}_i \leq -\lambda Q_i, \quad \forall i \in \mathcal{I},
\]
\[
Q_i \leq \mu Q_j, \quad \forall i, j \in \mathcal{I}.
\]

Let \(V_i(t) = e^T(t) \bar{P}_i e(t) + \nu \bar{x}_f^T(t) Q_i \bar{x}_f(t)\) on the time interval \([t_i, t_{i+1})\), where the scalar \(\nu > 0\) satisfies
\[
-\lambda \nu \bar{P}_i + \frac{1}{\nu \lambda^2} \bar{P}_i \bar{H}_i Q_i^{-1} \bar{H}_i^T \bar{P}_i < 0,
\]
with \(a \in (0, a^*)\). Such \(\nu\) always exists since \(\bar{P}_i > 0\).

Differentiating \(V_i(t)\) along the system trajectories, we have
\[
\begin{aligned}
\dot{V}_i(t) &= \nu \bar{x}_f^T(t) \left( \bar{P}_i \bar{Q}_i + Q_i \bar{F}_i \right) \bar{x}_f(t) + e^T(t) \left( \bar{A}_i^T \bar{P}_i + \bar{P}_i \bar{A}_i \right) e(t) + 2 \nu \bar{x}_f^T(t) Q_i \bar{x}_f(t) + 2 \nu \bar{x}_f^T(t) Q_i \bar{x}_f(t) \\
&\leq \begin{bmatrix}
\bar{x}_f^T(t) \\
\tilde{x}_f^T(t)
\end{bmatrix} \begin{bmatrix}
-\bar{P}_i \bar{H}_i, & -\nu \lambda Q_i, & \nu \bar{Q}_i \bar{G}_i, & 0
\end{bmatrix} \begin{bmatrix}
\bar{e}(t) \\
\tilde{x}_f(t)
\end{bmatrix} \\
&\leq -\nu \lambda V_i(t) + g \| \tilde{x}(t) \|^2, \quad \forall t \in [t_i, t_{i+1})
\end{aligned}
\]
where the last inequality follows from square completion, and the scalar \(g\) is given by
\[
g \triangleq \left\| \begin{bmatrix}
J^T \bar{P}_i & \nu \bar{G}_i^T Q_i
\end{bmatrix} \begin{bmatrix}
-\lambda \bar{P}_i & \bar{P}_i \bar{H}_i \\
\bar{H}_i^T \bar{P}_i & -\nu \lambda Q_i
\end{bmatrix}^{-1} \begin{bmatrix}
\bar{P}_i \bar{J}_i \\
\nu \bar{Q}_i \bar{G}_i
\end{bmatrix} \right\|.
\]

Integrating the last inequality in (25) and applying the bound on \(\tilde{x}\) from (17), we have
\[ V_i(t) \leq V_i(t) e^{-(1-a)\lambda(t-t_i)} + \int_{t_i}^{t} e^{-(1-a)\lambda(t-t)} g \| \dot{x}(\tau) \|^2 d\tau \]
\[ \leq V_i(t) e^{-(1-a)\lambda(t-t_i)} + \int_{t_i}^{t} e^{-(1-a)\lambda(t-t)} g \frac{\beta_\varepsilon}{\Gamma} d\tau \]
\[ \leq V_i(t) e^{-(1-a)\lambda(t-t_i)} + \frac{g}{(1-a)\lambda} \beta_\varepsilon \left( 1 - e^{-(1-a)\lambda(t-t_i)} \right) \]
\[ \leq \mu V_{i-1}(t) e^{-(1-a)\lambda(t-t_i)} + \frac{g}{(1-a)\lambda} \beta_\varepsilon \left( 1 - e^{-(1-a)\lambda(t-t_i)} \right), \]  
\[ (26) \]

for any \( t \in [t_i, t_{i+1}) \), where the last inequality follows from \( V_i(t) \leq \mu V_{i-1}(t) \) implied by (9), (24), and the definition of the Lyapunov functions. It follows that if at some switching time \( t_i \),
\[ V_{i-1}(t_i) \leq \frac{g}{(1-a)\lambda} \frac{1 - \mu^{1/a}}{1 - \mu^{-\xi}} \beta_\varepsilon \Gamma, \]  
\[ (27) \]
then by applying the dwell time constraint (10) and substituting (27) into (26), we have
\[ V_i(t_{i+1}) \leq \frac{g}{(1-a)\lambda} \frac{1 - \mu^{1/a}}{1 - \mu^{-\xi}} \beta_\varepsilon \Gamma. \]

Thus if the condition in (27) holds for some \( t_i \), it will hold for all \( t_j \) with \( j \geq i \). Since \( \dot{e}(0) = 0 \) and \( x_{f_2}(0) = 0 \), the inequality in (27) is satisfied at \( t = 0 \), and thus at every switching time \( t_i \) (\( t = 0, 1, \cdots \)). Applying the bound in (27) to (26), we obtain a uniform bound for the reference tracking error over the time interval \([t_i, t_{i+1})\) as
\[ \left\| \frac{e(t)}{\sqrt{\nu \dot{x}_{f_2}}} \right\|^2 \leq V_i(t) \leq \frac{\mu g}{(1-a)\lambda \mu^{1/a}} \frac{1 - \mu^{-\xi}}{1 - \mu^{-\xi}} \beta_\varepsilon \Gamma. \]  
\[ (28) \]
This holds for all \( i \in \mathbb{I} \), and thus the bound is uniform for all \( t \). We further notice that
\[ e_n(t) = C_\dot{e}(t) + L_p \dot{x}_{f_2}(t) - D_f B_\dot{p} \dot{x}(t). \]  
\[ (29) \]
From (28), (29) and the bound on \( \dot{x}(t) \) in (17), we can extract appropriate constants \( \kappa_1 \) and \( \kappa_2 \) such that (23) holds. The proof is complete. \( \square \)

To summarize, if the reference system is stable (meets the conditions of Assumption 2), Theorem 9 guarantees that the states of the adaptive system follow those of the reference system with a bound proportional to \( \| \dot{x} \|_{\infty} \). From Lemma 7, the bound on \( \| \dot{x} \|_{\infty} \) is proportional to the inverse of the square root of \( \Gamma \). Thus, by increasing the adaptation gain \( \Gamma \), under the adaptive control law in (12), (13), (14), the adaptive system can be made arbitrarily close to the reference system, which is stable.

4. SIMULATION RESULTS

In the following example, the short period dynamics of a transport class aircraft \(^2\) are considered during the approach phase. As noted in Section 2, pilots desire to sense changes in control effectiveness as the air speed varies, which motivates the usage of varying desired dynamics.

The flaps and gear are deployed at 162 knots, and the approach speed is taken to be 137 knots. A different model
\(^2\) Based on NASA’s Transport Class Model Aircraft Simulation https://software.nasa.gov/software/LAR-18322-1.
5. CONCLUSION

A framework for explicitly handling changes in the desired system dynamics during the design of an $L_1$ adaptive controller has been presented. This work was motivated by the common practice within the aerospace community of gain scheduling control laws. It provides a means to mathematically verify the stability and robustness of a design despite changes in the desired system response. The proposed method was validated by the simulation of the short period dynamics of a transport class aircraft during the approach phase.

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Appendix A. PROJECTION OPERATOR

Consider the following smooth convex function:

\[ f(\theta) \triangleq \frac{(\epsilon_\theta + 1)\theta^T\theta - \theta_{max}^2}{\epsilon_\theta \theta_{max}} \]

with \( \theta_{max} \) being the norm bound imposed on the vector \( \theta \), and \( \epsilon_\theta > 0 \) being a free parameter determining the projection tolerance. The projection operator is defined as

\[ \text{Proj}(\theta, y) \triangleq \begin{cases} 
  y - \frac{\theta\theta^Tyf(\theta)}{||\theta||^2} & \text{if } f(\theta) \geq 0 \text{ and } \theta^Ty > 0 \\
  y & \text{otherwise} 
\end{cases} \]

For more details on the projection operator and its properties, see Hovakimyan and Cao (2010).

Appendix B. SIMULATION PARAMETERS

\[
\begin{align*}
A_{162} &= \begin{bmatrix} -0.5301 & 0.9273 \\ -0.9106 & -0.6871 \end{bmatrix} & B_{162} &= \begin{bmatrix} -0.0009 \\ -0.0168 \end{bmatrix} \\
A_{157} &= \begin{bmatrix} -0.5272 & 0.9289 \\ -0.8557 & -0.6580 \end{bmatrix} & B_{157} &= \begin{bmatrix} -0.0008 \\ -0.0154 \end{bmatrix} \\
A_{152} &= \begin{bmatrix} -0.5201 & 0.9305 \\ -0.7229 & -0.6279 \end{bmatrix} & B_{152} &= \begin{bmatrix} -0.0008 \\ -0.0141 \end{bmatrix} \\
A_{147} &= \begin{bmatrix} -0.5168 & 0.9322 \\ -0.6618 & -0.5960 \end{bmatrix} & B_{147} &= \begin{bmatrix} -0.0007 \\ -0.0132 \end{bmatrix} \\
A_{142} &= \begin{bmatrix} -0.5171 & 0.9339 \\ -0.6669 & -0.5637 \end{bmatrix} & B_{142} &= \begin{bmatrix} -0.0007 \\ -0.0123 \end{bmatrix} \\
A_{137} &= \begin{bmatrix} -0.5147 & 0.9357 \\ -0.6219 & -0.5309 \end{bmatrix} & B_{137} &= \begin{bmatrix} -0.0006 \\ -0.0115 \end{bmatrix}
\end{align*}
\]