Thought experiment discriminating special relativity from preferred frame theories

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\section*{Abstract}

In relativistic theories the effect of nonconservation of simultaneity can be separated from that of the Lorentz-Fitzgerald contraction. Since with absolute synchronization simultaneity is conserved, we show that a simple kinematic test may discriminate absolute from Einstein synchronization, settling the century-long debate on the conventionality of the one-way speed of light. An immediate consequence is that Einstein’s postulate of a universal light speed can actually be tested and the unique physical meaning of special relativity (SR) is restored. Only an experiment can corroborate SR or identify the preferred frame of reference.

\section{1. Introduction}

In formulating special relativity (SR) theory, Einstein [1] postulated a universal constant speed of light $c$, together with his clock synchronization procedure. Except for experts, many physicists believe that the speed of light measured in experiments refers to the one-way speed [2], while what is actually being measured is the round-trip value $3c$, in accordance with Einstein synchronization procedure. Remarks on the conventional character of Einstein synchronization procedure were made early by Poincaré [4] and were reinforced in the works of Reichenbach [5] and, later, Grünbaum [6]. The lack of solution for the problem of synchronization, pointed out by Reichenbach [5] with the argument that the standard Einstein synchronization involves circular reasoning, implies that a nonstandard synchronization convention can be adopted, with unequal values of the speed of light in opposite directions. This argument has led to the ‘conventionalist thesis’, widely discussed by physicists and epistemologists, asserting that Einstein’s procedure is merely a convention, implying that the one-way speed $c$ cannot be measured in principle.

There is a vast literature in favor or against the conventionality of synchronization [7–9]. Literature has then witnessed the renaissance of relativistic theories that assume the existence of a preferred frame $\Sigma$ where space and light speed are isotropic while, in an inertial frame $S$ moving with velocity $v$ with respect to $\Sigma$, the one-way speed of light is $c = c(\Sigma)$, that is, no longer isotropic. Thus, in the last decades specialists have focused their attention to preferred frame theories (PFT) that use coordinate transformations (denoted by TT or IST [10]) with the same rod-contraction and clock-retardation as the Lorentz transformations (LT), but differ from LT by an arbitrary synchronization parameter [9–13], generally denoted by the symbol $\varepsilon$ [5, 9]. According to Mansouri and Sexl [9], by their very construction these preferred frame theories interpret all the known experiments that support SR and, thus, are physically equivalent to SR.

For many physicists, the question about whether Einstein’s postulates can be proved to be true is a philosophical matter rather than a scientific one. Furthermore, differences between PFT theories and SR do not necessarily imply that a universal physical constant, such as $c$, is, can be verified experimentally. Nevertheless, some epistemologists [14] claim that a theory is physically meaningless unless its basic postulates can be tested.

Then, within their view, it is crucial for the standing of the theory that the physical equivalence between PFT and SR can be verified and tested, at least in principle [15].

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Within the scenario of relativistic theories, in section 2 we discuss the possibility that SR and PFT can be discriminated by means of a dynamic or kinematic test. In section 3, we describe a kinematic test involving the length of a moving rod. The contracted length, measured in an inertial frame $S$, is different for PFT and SR. The difference leads, indirectly, to an observable effect that can be measured. Consequently, in principle, our kinematic test consents to identify the preferred frame, if it exists, and measure the one-way speed of light. The conclusions (section 4) point out that, at least in principle, the Lorentz transformations (based on Einstein synchronization) are not equivalent to and cannot be arbitrarily replaced by coordinate transformations (such as the TT) based on absolute synchronization. Section 5 (Appendix) corroborates the assumptions of the previous sections.

As the conventionalist thesis is disproved, we can no longer assume the equivalence between special relativity and preferred frame theories. Since Einstein’s postulate of a universal constant speed of light can be verified experimentally, it follows that SR maintains its unique physical meaning and can be tested against theories that assume the existence of an identifiable preferred frame.

2. Absolute versus relative simultaneity

Let us consider an inertial frame $\Sigma (X, Y, Z, T)$ where space is isotropic and the one-way speed of light is $c$. As shown in figure 1, the laboratory frame $S(x, y, z, t)$ is moving with velocity $W$ in the $X$ direction with respect to $\Sigma$. The following transformations from $\Sigma$ to $S$ apply [9] for the relativistic theories under consideration. For SR, with the Lorentz transformations (LT) based on Einstein synchronization, we have

$$x = \gamma_w (X - WT); \quad y = Y; \quad z = Z; \quad t = \gamma_w \left( T - \frac{WX}{c^2} \right),$$  \hspace{1cm} (1)

while, for PFT with the Tangherlini transformations (TT) [10] based on absolute synchronization,

$$x = \gamma_w (X - WT); \quad y = Y; \quad z = Z; \quad t = \gamma_w^{-1} T.$$  \hspace{1cm} (2)

Transformations (1) and (2) foresee the same clock-retardation for moving clocks ($X = WT$ in (1)) and rod-contraction by the Lorentz-Fitzgerald factor $\gamma_w$. The only difference between (1) and (2) is the clock synchronization procedure adopted:

For the LT, Einstein synchronization introduces nonconservation of simultaneity and $t = \tau(T, X)$. For the TT we have absolute synchronization that preserves simultaneity and $t = \tau(T)$. According to the conventionalist thesis, the transformations (1) and (2) are physically equivalent and, thus, they always foresee the same observable results and no experiment can lead to identify the preferred frame $\Sigma$. The fact that the speed of light can be $c$ or $c = c(W)$, is viewed as a consequence of the conventional different choice of synchronization. In
order to discriminate SR from PFT, attempts have been made to find an internal synchronization procedure not equivalent to that of Einstein. However, as remarked by Ohanian [8], there are no satisfactory alternative internal synchronizations that can be used as a substitute for Einstein procedure.

Let us consider a rod AB, stationary in the inertial frame S moving with uniform speed $W'$ relatively to frame $\Sigma$, as shown in figure 1. If the rod has rest length $L_0$ and the middle point M coincides with the origin $O_S$ of $\Sigma$ at $T = 0$, the equation of motion of the end point B in frame $\Sigma$ is,

$$X_B = X_{AB} + W'T = \frac{L_0}{2\gamma_w'} + W'T,$$

(3)

where $\gamma_w'$ is the Lorentz-Fitzgerald length contraction factor. A similar equation holds for the end point A. The description of the kinematical behavior of the rod in the laboratory frame S within either SR or PFT is the following.

2.1. Special Relativity

According to SR, the equation of motion of the end point B in frame S is,

$$x_B = x_{AB} + vt = \frac{L_0}{2\gamma} + vt,$$

(4)

showing that the rest length of the moving rod is contracted by the factor $\gamma$. Equation (4) can be derived from (3) by means of the LT (1).

According to (4), $x_B(t = 0) = (L_0/2)/\gamma$. By means of (3) and the LT (1), $x_B(T = 0) = \gamma_w'(L_0/2)/\gamma_w$. Then, we have, $\delta x_B = x_B(t = 0) - x_B(T = 0) = (L_0/2)(\gamma_w^{-1} - \gamma_w'^{-1})$, which represents the length change due to nonconservation of simultaneity. In fact, from the LT (1) the corresponding time delay is $\delta t = \gamma_w' WX/c^2$, which implies a rod displacement by $\delta x_B = v\delta t = \gamma_w' vWX/c^2 = \gamma_w' \gamma_w^{-1}(L_0/2)vWX/c^2$, for $X = X(0) = \gamma_w^{-1}(L_0/2)$.

Using the velocity composition relation, $W' = (v + W)/(1 + vW/c^2)$, we find $\gamma_w' \gamma_w^{-1} = \gamma(1 + vW/c^2)^{-1}$ and, $\delta x_B = (L_0/2)(\gamma_w^{-1} - \gamma_w'^{-1}) = \gamma_w' \gamma_w^{-1}(L_0/2)vWX/c^2 \simeq (L_0/2)vWX/c^2$, as expected. This result for $\delta x_B$ indicates that the Lorentz transformations contain two effects that modify the length of the moving rod when passing from the description in frame $\Sigma$ ($L_S = L_0/\gamma_w$) to that in frame S ($L_S = L_0/\gamma$). Considering that a meter stick at rest in S is contracted by the factor $\gamma_w$ in the direction of motion, the first effect, corresponding to a pure Lorentz-Fitzgerald contraction, shrinks the rod rest length to $\gamma_w' L_0/\gamma_w'$ in frame S. The result $\gamma_w' L_0/\gamma_w'$, for the purely contracted length, stands for the length of the moving rod when simultaneity between frames $\Sigma$ and S is conserved. The second effect is the one due to the introduction of nonconservation of simultaneity in SR by means of the Lorentz time transformation, which enhances the length $\gamma_w' L_0/\gamma_w'$. The purely contracted rod by $\delta x = x(t = 0) - x(T = 0) = L_0/\gamma - \gamma_w' L_0/\gamma_w' = \gamma_w' \gamma_w^{-1}L_0 vWX/c^2$. The two effects are combined in the LT, in order to provide in frame S at $t = 0$ the resulting length foreseen by SR,

$$x_B - x_A = L_S = \gamma_w' \frac{L_0}{\gamma_w} + \delta x = \frac{L_0}{\gamma},$$

in agreement with (4).

2.2. Preferred Frame Theory

According to PFT, where nonconservation of simultaneity does not apply, with the help of (3), the TT (2), and the relation $v_w = \gamma_w'(W' - W)$ for the velocity, the equation of motion of point B in S is,

$$x_B = x_{AB} + v_w t = \frac{\gamma_w L_0}{2\gamma_w'} + v_w t.$$

(5)

Equation (5) indicates that in frame S at $t = 0$ the moving rod is contracted by the factor $\gamma_w'/\gamma_w$. Thus, for PFT the moving rod in frame S is shorter than that foreseen by SR, the difference being due to nonconservation of simultaneity.

In (5) the speed $v_w$ represents the velocity of point B measured with clocks absolutely synchronized in S in agreement with the TT of PFT. Thus, $v_w$ differs from the SR velocity $v$ of (4). Considering that the preferred frame is unknown and absolute synchronization in S is not available and inviable, we are unable to directly measure $v_w$.

Then, it is convenient to express $v_w$ in terms of the measurable quantity $v$ and write (5) as,

$$x_B = \frac{L_0}{2\gamma(1 + vWX/c^2)} + \frac{v}{1 + vWX/c^2}t.$$

(6)

Result (6) coincides with (4) when $W = 0$, i.e., when $\Sigma$ and S coincide. In general, when $W \neq 0$, the two results are not equal and the contracted length of the moving rod differs, depending on the theory (SR or PFT).
2.3. Kinematics of a rotating rod according to SR and PFT

Let us consider the ring of radius $R$ of figure 1, stationary in frame $S$. As observed from frame $\Sigma$, the circular ring looks like an ellipse because the horizontal radius is contracted by the factor $\gamma_x$ and becomes the semiminor axis. However, meter sticks stationary in $S$ are equally contracted in the direction of motion and, therefore, for an observer of frame $S$, the measurable shape of the ring is still circular for both SR and PFT. Let another twin rod $AB$, with the same rest length $L_0$, be rotating with angular velocity $\omega$ with its two ends $A$ and $B$ sliding on the ring and keeping in contact with it, as shown in figure 1. We assume that the clocks in the inertial frame $S$ are properly synchronized and that the coordinates of an end point of the rod, located at the angular position $\phi$ on the circumference, can be described by the functions, $x = R \sin \phi$ and $y = -R \cos \phi$. With $\phi = \omega t + \alpha$, for point B we have,

$$x_B = R \sin(\omega t + \alpha); \quad y_B = -R \cos(\omega t + \alpha). \quad (7)$$

A similar expression holds for the end point A with $\phi = \omega t - \alpha$. It should be remarked that, even though for SR the angular velocity $\omega = \omega_0$ is uniform everywhere on the circumference, it is not uniform for PFT (when referring to the absolute quantities of the TT, the subscript $a$ will be added to them). For simplicity, we assume that, at the time $t = 0$, the rotating rod is straight and parallel to the $x$ axis, being $y_B = y_A = y_M = -R \cos \alpha$, as in figure 1.

Neglecting dynamical effects (such as the one due to centrifugal forces that tend to bend the rod. Ideally, the rod should be massless and perfectly rigid), the resulting dimensions and shape of the moving rod are of kinematical origin and due, in part, to the pure Lorentz-Fitzgerald contraction in the direction of motion. Although, at $t = 0$, the contracted length of the two rods (in rectilinear and rotational motion, respectively) is essentially the same in the $x$ direction, the elementary segments of the rotating rod possess in the $y$ direction and, therefore, the thickness of these segments is contracted also in the $y$ direction, changing slightly the rod shape. Using the LT and the TT, the resulting shape change of a rotating rod has been calculated in the appendix (see also, [16]) and is found to be inconsequential within our approximations. Considering that the shape change does not modify significantly our discussion and does not alter our conclusions, we disregard it in this section, assuming for simplicity that, for both SR and PFT, the rotating rod is straight in frame $S$.

When the time $t$ (and $\omega t$) is small, $\sin(\omega t + \alpha) = \sin \alpha \cos \omega t + \sin \omega t \cos \alpha \asymp \sin \alpha + \omega t \cos \alpha$, as first order in $\omega t$. Then, using the subscript $+$ for motion in the positive $x$ direction, with $\alpha = \alpha_+$, expression $x_B$ in (7) becomes,

$$x_B = R \sin \alpha_+ + (\omega R \cos \alpha_+) t = \frac{L_0}{2\gamma_+} + v_+ t, \quad (8)$$

where $v_+ = \omega R \cos \alpha_+$ and $\gamma_+$ the corresponding length contraction factor. For short times $t$ (and $\omega t$ small), equation (8) can be used to represent formally both (4) and (5).

Consequently, within our approximations, at the time $t = 0$ the rotating rod may be thought of being instantaneously at rest with and coinciding with the twin rod in rectilinear motion with the same speed component $v_+$ in the $x$ direction.

3. Testing nonconservation of simultaneity

Equation (8) may be applied to describe the rod behavior within SR or, separately and independently, within PFT. Assuming that the rod is essentially straight in frame $S$, regardless of the theory (SR or PFT)—at some specific time (e.g., $t = 0$) the rotating rod will be parallel to the $x$ axis and all the elements of the rod will share the same velocity component $v_+$ in the positive $x$ direction.

Thus, with the end-points $A$ and $B$ constrained to be sliding on the ring and denoting by $H_+$ the distance $|y_M| = OM$ of the middle point $M$ of the rod from the center $O$ of the circle, for the rectangular triangle $OMB$ we may write in frame $S$, $R^2 = H_+^2 + x_M^2$. Then, with the help of (8),

$$H_+ = (R^2 - x_M^2)^{1/2} = \left(R^2 - \frac{L_0^2}{4\gamma_+^2}\right)^{1/2} = (R^2 - R^2 \sin^2 \alpha_+)^{1/2} = R \cos \alpha_+, \quad (9)$$

3.1. Special Relativity

Referring to figure 1 and equation (4), according to SR we have $v_+ = v_1, \gamma_+ = \gamma, \alpha_+ = \alpha$, and $L_S = L_{SR} = x_{0B} - x_{0A} = 2R \sin \alpha = L_0/\gamma$. Then, (9) becomes,

$$H_+ = H = \left(R^2 - \frac{L_0^2}{4\gamma^2}\right)^{1/2} = (R^2 - R^2 \sin^2 \alpha)^{1/2} = R \cos \alpha, \quad (10)$$

independent of $W$. 

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3.2. Preferred Frame Theory

As illustrated in figure 2, it is apparent that, for PFT, the length \( L_S = L_a \) of the rod AB is smaller than \( L_{SR} \). Therefore, for PFT, the distance \( H_{PFT} = OM \), of the middle point M of the rod AB from the center O of the ring, is greater than the corresponding distance foreseen by SR for the rod of length \( L_{SR} \).

\[ H_{PFT} = \sqrt{R^2 + \left( \frac{L_0}{2\gamma^2} \cos \alpha \right)^2} \]

\[ \text{Figure 2. According to PFT, the rotating rod AB has the contracted length } L_a = L_0/\gamma (1 + vW/c^2) = 2R \sin \alpha_a \text{. According to SR, the length of the contracted rod (represented by the dotted line of length } L_{SR} \text{) is given instead by } L_{SR} = L_0/\gamma > L_a \text{. Therefore, for PFT, the distance } H_{PFT} = OM \text{, of the middle point M of the rod AB from the center O of the ring, is greater than the corresponding distance foreseen by SR for the rod of length } L_{SR} \text{.}

The SR result (10) may be derived from the PFT result (12) by setting \( W = 0 \) in (12). The distance \( H \) for the rod in the lower \( (\varphi = \omega t = 0) \) and upper \( (\varphi = \omega t = \pi \text{, or any other value}) \) part of the ring is the same for SR. Instead, for PFT, in the upper part \( (\varphi = \pi \text{, where } v \text{ changes to } -v) \), the rod length is greater than in the lower part because its absolute velocity is reduced to \( \approx -v \). Then, with \( v \cdot W = vW \cos \varphi \), for PFT we may write (12) as,

\[ H(\varphi) = H_{PFT}(\varphi) \approx H + L_0/W \sin \alpha \cos \varphi, \]

indicating that in the lower part \( (\varphi = 0) \) of the ring, \( H_{PFT} > H \), while in the upper part \( (\varphi = \pi) \), \( H_{PFT} < H \). When the rod is perpendicular to the direction of \( W \text{(e.g., when } \varphi = \pi/2 \text{, the result is essentially the same as that of SR.)}

The measurement of \( H_+ \) and \( H_- \) at different angular positions of the rod middle point M with respect to the fixed center O of the ring, can be accomplished in S with precision optical devices (equivalent to a high precision meter stick) by determining the relative position of M when it passes by.

For this measurement, there is no need of clock synchronization. Then, since \( v \cdot W = vW \cos \varphi \) and \( V \) are known and measurements may provide the maximum relative change

\[ \delta H = H(0) - H(\pi) = H_{PFT}+ - H_{PFT}− = L_0/W \frac{vW \sin \alpha}{c^2}, \]

the absolute velocity component \( W \) can be estimated from (14).

What marks the difference between SR and PFT is the change of the rod length \( (L_{SR} \text{ or } L_a) \), due to nonconservation of simultaneity, and the related value of \( H(\varphi) \). Due to the effect of nonconservation of simultaneity introduced in SR through Einstein synchronization, we find that isotropy (of both the light speed \( c \)
and space) is maintained in frame S. As a consequence of space isotropy, the distance OM, $H(\phi) = H$ in (10), is independent of $\phi$ for SR and $\delta H = 0$. Instead, for PFT, space is anisotropic in S and the variation, $\delta H = 0$, of (14) becomes observable. It follows that, by measuring $H(\phi)$ and $\delta H$, we are testing space isotropy and, indirectly, the effect of nonconservation of simultaneity.

4. Conclusions

The conventionalist scenario implies that the Lorentz transformations can be substituted by the Tangherlini transformations, a substitution hardly acceptable by physicists who have been exploiting for decades the symmetry properties of the Lorentz group in applications to several branches of modern physics, such as elementary particle physics, astrophysics, and quantum physics.

Overall, the polemic on conventionalism is an expression of the wider dispute about relative versus absolute time (or instantaneous). It is unlikely that historically long debates such as these can be settled through mere theoretical arguments. However, the controversy about the conventionality of the one-way speed of light and whether or not Einstein’s second postulate can be tested, reaches a turning point when we show in section 3 that the effect of nonconservation of simultaneity can be observed.

Only experimentally we can tell if standard SR is valid or not, and Einstein synchronization is corroborated if, for the test of section 3, space isotropy is confirmed. The mere existence of this test invalidates the thesis of conventionalism by showing, as required by epistemologists [14], that the basic postulates of the theory can be verified experimentally. Thus, considering that, in principle, the one-way speed is observable, the Lorentz transformations cannot be equivalently substituted by transformations based on absolute synchronization.

For the purpose of our paper, it is sufficient to consider our test as a thought experiment. Still, given the remarkable advance of technology, we believe it likely that physicists will be able to develop an experimental setup that can measure $\delta H(\phi)$ with sufficient precision to corroborate (or not) the theory. The outcome of the test, if realized, will have a significant impact on modern physics by either corroborating the postulate of a universal speed of light or identifying the preferred frame of reference.

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Appendix: Change of shape of the rotating rod

Due to the pure Lorentz-Fitzgerald contraction, the size of an object in motion suffers a contraction in the direction of motion. The transformation of coordinates from an inertial frame $\sigma(x,t)$ to a frame $\sigma'(x',t')$ in relative motion with velocity $\mathbf{u}$ is given by [17],

$$x' = x + \frac{\gamma_u - 1}{\beta^2}(\beta \cdot x)\beta - \gamma_u \beta ct, \quad (15)$$

where, relative to the reference frame $\sigma$, $\beta = \mathbf{u}/c$, and $\gamma_u$ is the contraction factor. Relation (15) represents a pure boost (Lorentz transformation without rotation) from $\sigma$ to $\sigma'$. For SR the time transformation from $\sigma$ to $\sigma'$ is, $t' = \gamma_u(t - x/c\beta)$. If frame $\sigma$ coincides with the preferred frame $\Sigma$ and the time transformation $t' = T/\gamma_u$ is used, the transformation (15) is valid for PFT.

We wish to determine the shape of the rotating rod in the laboratory frame S according to SR and to PFT. Generally speaking, if the rod is straight and parallel to the x axis when the rod is at rest, according to (15) its shape is modified when in motion because of the Lorentz-Fitzgerald contraction acting on the elements of the rod. Considering that the rod is rotating and translating as perceived by an observer stationary in frame $\sigma$, each rod element of length $\delta \xi \zeta$ possesses different velocity components $\dot{u}_\xi(\zeta)$ and $u_\zeta(\zeta)$ depending on the position $\zeta$ of the element with respect to the rod middle point M. Therefore, according to equation (15), the orientation and distortion of each element $\delta \xi \zeta$ of the rod is different and function of $\zeta$. In seeking the change of shape of the rod, we assume that at $t = 0$ and simultaneously in S, each rod element $\delta \xi \zeta$ is connected with their contiguous elements. Then, the resulting shape of the moving rod is determined by the resulting effect of different orientations of the elements that constitute the whole rod, as a continuous object.
A.1. Preferred Frame Theory: shape of the rotating rod in frames $\Sigma$ and $S$

According to PFT, simultaneity is conserved in every inertial frame. Therefore, in order to find the rod shape at $t = 0$ in $S$, we may first conveniently determine the change of the moving rod shape working in the preferred frame $\Sigma = \sigma$ at $T = 0$, where (15) is valid.

Then, we may find the rod shape in $S$ by transforming from $\Sigma$ to $S$ using the Tangherlini Transformations (2) and neglecting terms that lead to higher order corrections to result (14).

With $x' = x_0'$, $x' = X$ and $t = T$, (15) represents the transformation between the preferred frame $\Sigma$ and the frame $S'$ instantaneously at rest with an element of the rod with velocity components $U_x$ and $U_y$. Then, at $T = 0$, transformation (15) becomes

$$
x'_x = X + \frac{\gamma_u - 1}{U^2} U_x^2 X + \frac{\gamma_u - 1}{U^2} U_x U_y Y
$$
$$
y'_y = Y + \frac{\gamma_u - 1}{U^2} U_x^2 Y + \frac{\gamma_u - 1}{U^2} U_y U_x X.
$$

(16)

From (16) it appears that the axes of the frame $S'$ are not parallel to the axes of frame $\Sigma$ because of the Lorentz-Fitzgerald contraction. The velocity components $U_x$ and $U_y$ of an element of the translating and rotating rod depend on the position of the element with respect to the rod middle point $M$. For an element of length $d\zeta$, positioned at the distance $\zeta$ from point $M$ located at $X_M = 0$ on the $X$ axis at $T = 0$, we have, $U_x = W + v$ and $U_y(\zeta) = 2(\gamma/L_0)V \sin \alpha$ being $U_0 = 0$ for point $M$ at $\zeta = 0$ and $U_y \approx V \sin \alpha$ for the end point $B$ at $\zeta = L_0/2$.

Assuming that the rod is straight and parallel to the $X$ axis at $t = 0$, its shape is modified according to transformations (16) when in motion, because in the inertial frame $S'$ instantaneously at rest with the moving rod element of length $d\zeta$, the element is now aligned along the $x'_x$ axis, which is not parallel to the $X$ axis. With the origins of $\Sigma$ and $S'$ coincident with the rod element at $T = 0$, transformations (16) shows that the $x'_x$ axis of frame $S'$, defined by the equation $\eta(\zeta)$, turns out to be tilted as a function of $\zeta$ relative to the $X$ direction. For the chosen element, with $\eta' = 0$ in the second equation of (16), $\tan \eta(\zeta)$ is given by,

$$
\tan \eta(\zeta) = \frac{dY}{dX} = \frac{dY}{d\zeta} \approx -\frac{\gamma_u - 1}{U^2} U_x U_y \approx -\frac{\gamma_u - 1}{U^2} U_x \frac{V \sin \alpha}{L_0/2}.
$$

(17)

Consequently, due to the Lorentz-Fitzgerald contraction, the end point $B$ is displaced with respect to the middle point $M$ by the amount $\Delta_x = Y_M(0) - Y_0(0)$ given by,

$$
\Delta_a \approx -\int_0^{L_0/2} \tan \eta(\zeta)d\zeta \approx \frac{1}{2} \frac{\gamma_u - 1}{U^2} L_0 \frac{V}{2} U_x V \sin \alpha \approx \frac{L_0}{4} \frac{U_x V}{c^2} \sin \alpha.
$$

(18)

where in (18) we have used the relation, $\gamma_u - 1 \approx (1/2)U^2/c^2$. The same result (18) applies for the other end point of the rod $A$. Then, because of the resulting curved rod shape shown in figure 3, when both points $A$ and $B$ are aligned along the $X$ direction at $T = 0$, equation (18) indicates that (with $U_x \approx W + v$) point $M$ is located above points $A$ and $B$ by the amount,

$$
\Delta_a \approx \frac{1}{4} \frac{L_0 (W + v) V}{c^2} \sin \alpha \approx \frac{L_0}{8} \frac{W V}{c^2} \sin \alpha.
$$

(19)

For PFT simultaneity is conserved and, since the transformation of $Y$ from $\Sigma$ to $S$ is $y' = Y$, we conclude that, for our curved rod, at $t = 0$ in frame $S$, point $M$ is located above points $A$ and $B$ by the amount $\Delta_a$. The result (14) of section 3 has been derived in frame $S$ assuming that the rod keeps a rectilinear shape while translating and rotating. If, instead, its shape is changed, the correction $\Delta_a$ given by (19) must be taken into account, and the result (13) is modified to,

$$
H_{\text{PFT}}(\varphi) = H + \left(1 - \frac{1}{4}\right) \frac{L_0}{2} \frac{W V}{c^2} \sin \alpha \cos(\varphi),
$$

(20)

showing that the effect of the bending of the rod, due to the Lorentz-Fitzgerald contraction, compensates in part the discrepancy with the prediction of SR.

If we do not take into account the dependence on $\zeta$ of $U_y = U_y(\zeta)$ in (17) and (18), the correction in (20) becomes $(1 - 1/2)$, indicating that, in bending the rod, the weight of the pure Lorentz-Fitzgerald contraction is $1/2$ that of SR, the latter including the effect of nonconservation of simultaneity.
A.2. Special Relativity: shape of the rotating rod

The results obtained for PFT correspond to a rod shape evaluated simultaneously in both frames $\Sigma$ and $S$. If, for SR, we follow the same procedure applied above for PFT, complications arise because in transforming from $S$ to $\Sigma$ we need to take into account nonconservation of simultaneity.

In fact, the rod shape has been evaluated simultaneously in $\Sigma$ by applying transformation (16) to rod elements spatially separated, since the rod is an extended object. Because of the time dependence on space of the Lorentz transformations, for SR the mentioned rod shape is no longer simultaneous in the laboratory frame $S$ where the observable is measured. Nevertheless, as a short-cut to avoid complicated calculations, we may derive the shape change proceeding as follows.

A.3. Rod shape in $S$

According to SR, the preferred frame is not identifiable and does not exist. Then, the behavior of the rotating rod can be derived in frame $S$ without the need of any reference to the “preferred” frame $\Sigma$, and we may take equation (15) to stand for the relativistic coordinate transformation from the inertial frame $S$ to the generic frame $S'$ in relative motion with respect to $S$.

Applying (15) to the case of our rotating rod, for an observer of frame $S$, an element of the rod (located at the position $\zeta$ from the rod middle point $M$) has the velocity components, $u_x = V \cos \alpha = v$, $u_y(\zeta) \simeq 2(\zeta/L_0)V \sin \alpha$, being $u_y = 0$ for point $M$ at $\zeta = 0$ and $u_y \simeq V \sin \alpha$ the velocity component of point $B$ at $\zeta \simeq L_0/2$. In line with the approach used above for PFT, we may set $y' = 0$ for each element of the rod from point $M$, $\zeta = 0$, to point $B$, $\zeta \simeq L_0/2$ and find the resulting change of the rod shape calculating the corresponding $y$ deviation. Expression (17) for the angle $\eta(\zeta)$, evaluated in $S$, becomes now,

$$
\tan \eta(\zeta) \simeq -\frac{\gamma_\nu - 1}{u^2} - \frac{1}{2} \frac{u_x u_y(\zeta)}{c^2} \propto \frac{v^2}{c^2},
$$

which indicates that, for SR, the changes are of the order of $v^2/c^2$ and, as expected, independent of $W$.

The observables of our thought experiment are related to rod shape variations of the order of $v^2/c^2$ and $vW/c^2$, as foreseen by SR and PFT, respectively. The variations of the order $v^2/c^2$, which do not depend on $\varphi$, change the value of $H$ in (10) but do not alter the observable $\delta H$ in (14). Then, as far as measuring $\delta H$ is concerned, it is irrelevant whether, for SR, the rotating rod keeps straight or not. For the purpose of identifying the preferred frame and determining the absolute velocity $W$ by measuring $\delta H$, the relevant term is the one proportional to $(vW/c^2) \cos(\varphi)$ appearing in (13) and (20). If the test is performed but the experimental precision is such that only variations greater than $v^2/c^2$ can be detected, it would be impossible to check the predictions of SR about the shape of the rotating rod. Instead, in case that $W \gg v$, the variations of the order $vW/c^2$ predicted by PFT could be easily detected, if they exist. In practice, for a known precision of the
experimental set-up, even if \( W \) is not detected, the experiment can be used to establish an upper limit on the value of \( W \).

Our analysis shows that, in relativistic theories, the effect of the pure Lorentz-Fitzgerald contraction can be separated from that due to nonconservation of simultaneity. In the laboratory frame \( S \), the shape of the rotating rod foreseen by PFT, where nonconservation of simultaneity is absent, differs from that foreseen by SR.

As shown in figure 3, although for PFT the rod shape is curved, the rod is being bent by a lesser amount than that implying the equivalence with SR, where nonconservation of simultaneity applies. The main difference between (17) and (21) is due to the coupling term \( VW/c^2 \), present in (17) and (19), indicating that for PFT the rod shape depends on the absolute velocity \( W \). Instead, for SR, the coupling term is absent in frame \( S \) because, while passing from frame \( \Sigma \) to frame \( S \), it has been eliminated by virtue of the symmetry properties of the Lorentz transformations, which include nonconservation of simultaneity.

Wrapping up, the difference between a non-identifiable preferred frame \( \Sigma^* \) and an identifiable preferred frame \( \Sigma \) may be understood this way:

For our rotating rod, in order to attain a hypothetical non-identifiable PFT, a curved rod shape—similar to that of figure 3—must be ‘ad hoc’ assumed in frame \( \Sigma^* \). If the shape of the rotating rod is straight in frame \( S \), according to SR the transformed shape in frame \( \Sigma^* \) does possess a curvature pronounced enough as to provide in (10) the observable value \( H = |y_{\Sigma^*}| = |y_{\Sigma^*}| = 0 \) foreseen by SR. Thus, the related hypothetical non-identifiable Preferred Frame Theory, which assumes such a curved rod shape, is physically equivalent to SR (actually, the same theory with a change in the time transformation) and the arbitrary preferred frame \( \Sigma^* \) is not identifiable, in line with the conventionalist thesis.

Nevertheless, the required pronounced curved shape, which gives the same result of SR, is consequence of both the Lorentz-Fitzgerald contraction and nonconservation of simultaneity, the latter being a feature that is not present in a theory based on absolute synchronization. Accordingly, in a realistic PFT, where nonconservation of simultaneity is absent, the curvature of the rod, due to the pure Lorentz-Fitzgerald contraction only, is not very pronounced, as evidenced by result (19) derived above. Hence, in agreement with (20), the observable \( H_{\text{PFT}}(\varphi) \) differs from \( H \), and, thus, the corresponding preferred frame \( \Sigma \) is identifiable.

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