MSSM Anatomy of the Polarization Puzzle in $B \to \phi K^*$ Decays

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We analyze the $B \to \phi K^*$ polarization puzzle in the Minimal Supersymmetric Standard Model (MSSM) including the neutral Higgs boson (NHB) contributions. To calculate the non-factorizable contributions to hadronic matrix elements of operators, we have used the QCD factorization framework to the $\alpha_s$ order. It is shown that the recent experimental results of the polarization fractions in $B \to \phi K^*$ decays, which are difficult to be explained in SM, could be explained in MSSM if there are flavor non-diagonal squark mass matrix elements of 2nd and 3rd generations, which also satisfy all relevant constraints from known experiments ($B \to X_s \gamma$, $B_s \to \mu^+\mu^-$, $B \to X_s \mu^+\mu^-$, $B \to X_s g$, $\Delta M_s$, etc.). We have shown in details that the experimental results can be accommodated with the flavor non-diagonal mass insertion of chirality RL, RL+LR, RR, or LL+RR when the NHB contributions as well as $\mathcal{O}(\alpha_s)$ corrections of hadronic matrix elements of operators are included. However the branching ratios for the decay are smaller than the experimental measurements.

I. INTRODUCTION

The recent experimental results for polarization fractions in $B \to \phi K^*$ are \[1,2,3\]

$$|A_0|^2 = 0.52 \pm 0.08 \pm 0.03 \quad \text{Belle}$$
$$= 0.46 \pm 0.12 \pm 0.03 \quad \text{BaBar}$$

$$|A_\perp|^2 = 0.19 \pm 0.08 \pm 0.02 \quad \text{Belle}$$

(1)

for the mode $\phi K^{*+}$, and

$$|A_0|^2 = 0.45 \pm 0.05 \pm 0.02 \quad \text{Belle}$$
$$= 0.52 \pm 0.05 \pm 0.02 \quad \text{BaBar}$$

$$|A_\perp|^2 = 0.30 \pm 0.06 \pm 0.02 \quad \text{Belle}$$
$$= 0.22 \pm 0.05 \pm 0.02 \quad \text{BaBar}$$

(2)
The results deviate significantly from the SM prediction

\[ |A_0|^2 \sim 1 - O(1/m_b), \]

based on the naive counting rules which follow from a helicity argument. This significant deviation is referred to as a puzzle or anomaly in the literature. It has attracted many interests in searching for possible theoretical explanation in SM and new models beyond SM.

The naïve counting rules are obtained with the naive factorization in calculating hadronic matrix elements. It may be possible to explain the data if including the \( \alpha_s \) corrections to hadronic matrix elements in SM. It is shown that one can obtain \( |A_0|^2 \sim 0.5 \) due to the annihilation enhancement from the integral containing end-point singularity in QCDF approach. However, it is at the issue that makes the approach less-predictable. Li and Mishima point out that annihilation contributions are not enough to make \( |A_0|^2 \sim 0.5 \) in PQCD factorization approach. The effects from the final state interaction (FSI) have been studied in refs. One can get \( |A_0|^2 \sim 0.5 \), but \( |A_0|^2 (B \to \rho K^*) < |A_0|^2 (B \to \phi K^*) \) which does not agree with the measurements. Moreover, it has been shown in ref. that such FSI effects would lead to \( |A_0|^2 : |A_\parallel|^2 : |A_\perp|^2 = 0.43 : 0.54 : 0.03 \) which clearly contradicts the data. Therefore, one may draw the conclusion that it is difficult to explain the data within the SM.

A lot of works have been done to investigate polarizations of \( B \to \phi K^* \) in models beyond SM. A model with right currents can give \( |A_0|^2 \sim 0.5 \) but simultaneously leads to \( |A_\parallel|^2 \ll |A_\perp|^2 \) which is not in agreement with the data. It is also shown that the RL or LR+RL insertion in MSSM can lead to \( |A_0|^2 \sim 0.5 \) due to the \( C_{8g} \) enhancement, compared with that in SM. However, wrong formulas for the \( \alpha_s \) order hadronic matrix elements of the chromomagnetic dipole operator \( Q_{8g} \) in the case of transverse polarization are used in Ref. As shown in refs. the \( \alpha_s \) order hadronic matrix elements of \( Q_{8g} \) for transverse polarizations are very small. Moreover, the neutral Higgs boson (NHB) contributions are not considered in the work. Yang et al. show that the R-parity violating SUSY might explain the puzzle. A model-independent analysis for contributions of new operators, i.e., the operators beyond the operator basis in SM, has been carried out in ref. Recently, an analysis of polarizations in the model with scalar interaction of tree-level flavor changing neutral current (e.g., the model III two Higgs doublet model) has also been performed. In this paper, we shall perform a detailed analysis of polarizations in \( B \to \phi K^* \) as well as the decay rates in MSSM including neutral Higgs boson contributions and the \( \alpha_s \) corrections of hadronic matrix elements.

For the \( b \to s \) transition, besides the SM contribution, there are mainly two new contributions arising from the strong penguins and neutral Higgs boson (NHB) penguins with the gluino and squark propagating in the loop in MSSM. The former is not important because the Wilson coefficients of QCD penguin operators in MSSM are not changed significantly, compared with those in SM. Although \( C_{8g} \) can get a significant enhancement, the hadronic matrix elements of \( Q_{8g} \) in the case of transverse polarization are very small. The latter induces scalar operators as well as tensor operators due to renormalization. As well known, the effects of these new operators to leptonic \( B_s \) decays are significant, and their effects to some hadronic B decays are also important. For \( B \to VV \) decays, it is expected that the hadronic matrix elements of scalar and tensor operators can enhance transverse polarization fractions. Moreover, although the effects of the primed counterparts of the usual operators are
suppressed by $m_s/m_b$ and consequently negligible in SM, their effects in MSSM can be significant because they have the opposite chirality and the flavor non-diagonal squark mass matrix elements are free parameters which are only subjective to constraints from experiments. In particular, as discussed in ref. [13], the primed counterparts of the usual operators have contributions to longitudinal and transverse polarizations different from those of usual operators and consequently could enhance the transverse polarization fractions. The relevant Wilson coefficients at the $m_W$ scale have been calculated by using the vertex mixing method in Ref. [27] and the mass insertion approximation (MIA) method in ref. [26]. In this paper we shall use the results given in ref. [26]. For the hadronic matrix elements of operators relevant to the decays $B \to VV$, we shall use the BBNS’s approach (QCDF) to calculate the $\alpha_s$ order corrections to the naive factorization results.

We show that polarization fractions of the decays can agree with experimental data within 1σ deviation in MSSM with the parameter space satisfying all the constraints from $B_s - B_s$ mixing, $B \to X_s\gamma$, $B \to X_{s}g$, $B \to X_s\mu^+\mu^-$ and $B_s \to \mu^+\mu^-$. In particular, the puzzle for polarization in $B \to \phi K^*$ can be explained, while not in contradiction to the measurements of other two vector final states, in quite a large region of parameter space because we have included the contributions of the primed counterparts of usual operators and NHB induced operators in MSSM with the $\alpha_s$ corrections of their hadronic matrix elements included.

The paper is organized as follows. In Sec. II, we give the effective Hamiltonian responsible for the $b \to s$ transition in MSSM. In Sec. III, we present the decay amplitudes. In particular, the hadronic matrix elements of NHB induced operators to the $\alpha_s$ order are calculated. The Sec. IV is devoted to numerical results. We draw conclusions and discussions in Sec. V.

II. EFFECTIVE HAMILTONIAN

The effective Hamiltonian for $b \to s$ transition can be expressed as

\[
\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p=u, c} V_{pb} V_{ps}^* \left( C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3, \ldots, 16} [C_i Q_i + C_i' Q_i'] \right) + C_{77} Q_{77} + C_{8g} Q_{8g} + C_{77} Q_{77} + C_{8g} Q_{8g} \right) + h.c.
\]

(5)

Here $Q_i$ are quark and gluon operators and are given by *

\[
Q_1^p = (\bar{s} \gamma_\mu p)_{V-A} (\bar{p} \gamma_\mu b)_{V-A}, \quad Q_2^p = (\bar{s} \gamma_\mu p)_{V-A} (\bar{p} \gamma_\mu b)_{V-A},
\]

\[
Q_{3(5)} = (\bar{s} \gamma_\mu b)_{V-A} \sum_q (\bar{q} \gamma_\mu q)_{V-(+)^A}, \quad Q_{4(6)} = (\bar{s} \gamma_\mu b)_{V-A} \sum_q (\bar{q} \gamma_\mu q)_{V-(+)^A},
\]

\[
Q_{7(9)} = \frac{3}{2} (\bar{s} \gamma_\mu b)_{V-A} \sum_q e_q (\bar{q} \gamma_\mu q)_{V-(+)^A}, \quad Q_{8(10)} = \frac{3}{2} (\bar{s} \gamma_\mu b)_{V-A} \sum_q e_q (\bar{q} \gamma_\mu q)_{V-(+)^A},
\]

\[
Q_{11(13)} = (\bar{s} b)_{S+P} \sum_q \frac{m_q}{m_b} (\bar{q} q)_{S-(+)^P}, \quad Q_{12(14)} = (\bar{s} b)_{S+P} \sum_q \frac{m_q}{m_b} (\bar{q} q)_{S-(+)^P},
\]

\[
Q_{15} = \bar{s} \gamma_\mu (1 + \gamma_5) b \sum_q \frac{m_q}{m_b} \bar{q} \sigma_\mu (1 + \gamma_5) q,
\]

* For the operators in SM we use the conventions in Ref. [28] where $Q_1$ and $Q_2$ are exchanged each other with respect to the convention in most of papers.
\[ Q_{16} = \bar{s}_i \sigma^{\mu\nu} (1 + \gamma_5) b_j \sum_q \frac{m_q}{m_b} \bar{q}_j \sigma_{\mu\nu} (1 + \gamma_5) q_i, \]
\[ Q_{7\gamma} = \frac{e}{8\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu} F_{\mu\nu} (1 + \gamma_5) b_\beta, \]
\[ Q_{8g} = \frac{g_s}{8\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu} G^a_{\mu\nu} \frac{\lambda_\alpha \beta}{2} (1 + \gamma_5) b_\beta, \]

where \((\bar{q}_1 q_2)_{V, A} = \bar{q}_1 \gamma^\nu (1 + \gamma_5) q_2, (\bar{q}_1 q_2)_{S, P} = \bar{q}_1 (1 + \gamma_5) q_2 \), \(p = u, c, q = u, d, s, c, b\), and \(e_q\) is the electric charge number of \(q\) quark, \(\lambda_a\) is the color SU(3) Gell-Mann matrix, \(\alpha\) and \(\beta\) are color indices, and \(F_{\mu\nu}\) (\(G_{\mu\nu}\)) are the photon (gluon) field strength. The primed operators, the counterpart of the unprimed operators, are obtained by replacing the chirality in the corresponding unprimed operators with opposite ones.

For the processes we are interested in this paper, the Wilson coefficients should be run down to the scale of \(O(m_b)\). \(C_1 - C_{10}\) are expanded to \(O(\alpha_s)\) and NLO renormalization group equations (RGEs) should be used. However for the \(C_{8g}\) and \(C_{7\gamma}\), LO results should be sufficient. The details of the running of these Wilson coefficients can be found in Ref. [29]. The one loop anomalous dimension matrices of the NHB induced operators can be found in Refs. [26, 30].

There is the mixing of the new operators induced by NHBs with the operators in SM. The leading order anomalous dimensions have been given in Refs. [31, 32]. The mixing of NHB induced operators with the chromo-magnetic operator can enhance the Wilson coefficient \(C_{8g}\) significantly [26, 32]. Because at present no NLO Wilson coefficients \(C_i^{(t)}\), \(i = 11, \ldots, 16\), are available, we use the LO running of them in this paper.

### III. THE DECAY AMPLITUDE AND POLARIZATION

We use the BBNS approach [7, 28] to calculate the hadronic matrix elements of operators. In the BBNS approach, the hadronic matrix element of an operator in the heavy quark limit can be written as

\[ \langle V_1 V_2 | Q | B \rangle = \langle V_1 V_2 | Q | B \rangle_f \left[ 1 + \sum r_n \alpha_s^n \right], \]

where \(\langle V_1 V_2 | Q | B \rangle_f\) indicates the naive factorization result. The second term in the square bracket indicates higher order \(\alpha_s\) corrections to the matrix elements [28]. We calculate the hadronic matrix elements to the \(\alpha_s\) order in this paper. In order to see explicitly the effects of new operators in the MSSM, we divide the decay amplitude into three parts. The first one, \(H_o\), has the same form as that in SM, the second, \(H_o'\), is for primed counterparts of the SM operators, and the third, \(H_n\), is new which comes from the contributions of Higgs penguin induced operators. That is, we can write the decay amplitude for \(B \to VV\) as

\[ A(B \to V_1 V_2) = \frac{G_F}{\sqrt{2}} H^\lambda \]
\[ H^\lambda = H_0^\lambda + H_o'^\lambda + H_n^\lambda. \]

The helicity amplitudes can be obtained by set \(\lambda = 0, +1, -1\) in above expressions, respectively.

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\footnote{Strictly speaking, the sum over \(q\) in expressions of \(Q_i\) \((i = 11, \ldots, 16)\) should be separated into two parts: one is for \(q = u, c\), i.e., upper type quarks, the other for \(q = d, s, b\), i.e., down type quarks, because the couplings of upper type quarks to NHBs are different from those of down type quarks. In the case of large \(\tan \beta\) the former is suppressed by \(\tan^{-1} \beta\) with respect to the latter and consequently can be neglected. Hereafter we use, e.g., \(C_{11}^{(t)}\) to denote the Wilson coefficient of the operator \(Q_{11} = (\bar{s} b)_{S, P} \frac{\Delta b}{3 m_b} (\bar{c} c)_{S, P}. \)}
A. Helicity amplitude

Let $e_i^{(λ)}$ (i=1, 2) be the polarization vector of vector meson $V_i$, $λ_1 = λ_2 = λ$ in $B \rightarrow V_1 V_2$ due to the angular momentum conservation. The helicity amplitudes $H^λ (λ = 0, +1, -1)$ of $B \rightarrow K^* 0$ in MSSM are given by

$$H^λ = H^λ_0 + H^λ_1,$$

$$H^λ_0 = -λt \left[ a_3^λ + a_4^λ + a_5^λ - \frac{1}{2} (a_7^λ + a_8^λ + a_{10}^λ) \right] A_{(V-A)}(λ),$$

$$H^λ_1 = H^λ_0 (C_i \rightarrow C'_i, A_{(V-A)}(λ) \rightarrow A_{(V+A)}(λ)),$$

$$H^λ_n = -λt \left[ \frac{1}{8} (a_{14}^λ + r_1 a_{14}^λ) + (a_{15}^λ - r_1 a_{16}^λ) + \frac{1}{2} (a_{16}^λ + r_1 a_{16}^λ) \right] A_{T(1+γ_5)}(λ) - \frac{1}{2} (a_{14}^λ + r_2 a_{15}^λ).$$

(12)

where

$$a_i^λ = a_i^λ (C_i \rightarrow C'_i),$$

$$r_1 = \frac{A_{T(1+γ_5)}}{A_{(V-A)}}, \quad r_2 = \frac{A_{(V-A)}}{A_{(V+A)}},$$

$$A_{T(1+γ_5)}(λ) = \langle φ(e_2^{(λ)}, q)|\bar{s}σ_{μν}(1 ± γ_5)s|0⟩ (K^*(e_1^{(λ)}, p_{K^*})|\bar{s}σ_{μν}(1 ± γ_5)b|B(p_B)),$$

$$A_{(V±A)}(λ) = \langle φ(e_2^{(λ)}, q)|\bar{s}γ_μ(1 ± γ_5)s|0⟩ (K^*(e_1^{(λ)}, p_{K^*})|\bar{s}γ_μ(1 ± γ_5)b|B(p_B)).$$

(13)

In eq. (10) the coefficients $a^λ_i \ (i = 3, 4, 5, 7, 9, 10)$ have been given in ref. [6, 17, 18, 20]. Because there are contradicting results on penguin insertion contributions, especially the $C_{9y}$ effect to transversely polarized amplitudes, we revisit this part and confirm the results in ref. [6, 18]. We calculate coefficients in eq. (12) and results are

$$a_{12}^λ = \frac{m_s}{m_b} C_{12} + \frac{C_{11}}{N_c} + \frac{αs CF C_{11}}{4π N_c} P^λ_{11},$$

$$a_{14}^λ = \frac{m_s}{m_b} C_{14} + \frac{C_{13} + αs CF (V_{13} + H_{13}^λ)}{4π N_c} + \frac{αs CF C_{13}}{4π N_c} P^λ_{14},$$

$$a_{15}^λ = \frac{m_s}{m_b} C_{15} + \frac{C_{16} + αs CF (V_{14} + H_{14}^λ)}{4π N_c} + \frac{αs CF C_{16}}{4π N_c} P^λ_{15},$$

$$a_{16}^λ = \frac{m_s}{m_b} C_{16} + \frac{C_{15} + αs CF (V_{15} + H_{15}^λ)}{4π N_c} + \frac{αs CF C_{15}}{4π N_c} P^λ_{16}.$$

(14)

where

$$P^λ_{11} = \left[ \frac{m_s}{m_b} \frac{4}{3} \ln \frac{m_b}{μ} - G_φ^λ(0) \right] + \frac{4}{3} \ln \frac{m_b}{μ} - G_φ^λ(1) \right] r_2,$$

$$P^λ_{13} = -8 \left[ -2 \ln \frac{m_b}{μ} G_φ^λ - GF_φ^λ(1) \right] c_{13}^λ,$$

$$P^λ_{15} = P^λ_{13} + 4G_φ^λ,$$

$$P^λ_{16} = P^λ_{13},$$

$$c_{13}^λ = \begin{cases} r & \text{for } λ = 0, \\ \frac{1}{8} \frac{f_s}{f_0} \frac{m_s}{m_b} & \text{for } λ = ±1. \end{cases}$$

(15)

In eq. (15), we have defined

$$r = \frac{A_{(V-A)}(λ = 0)}{A_{T(1+γ_5)}(λ = 0)}$$

$$G_φ^λ(s) = \int_0^1 dx G(s - iε, x) \Phi_φ^λ_1 (x), \quad G(s, x) = -4 \int_0^1 dt (1 - t) \ln [s - t(1 - t)x]$$

$$G_φ^λ(s) = \int_0^1 dx \frac{Φ_φ^λ_2 (x)}{x} G(s - iε, x), \quad GF_φ^λ(s) = \int_0^1 dt \ln [s - t(1 - t)]$$

(17)
with $\bar{x} = 1 - x$. Here the distribution amplitudes of $\phi$ meson are given by

$$
\Phi_0^0 = \Phi_0^0 = \phi_0^0, \quad \Phi_\parallel^\pm = \left( g_\perp^{(v)} \pm \frac{g_\perp^{(a)}}{4} \right), \quad \Phi_\perp^\pm = \bar{x} \left[ g_\perp^{(v)} - \frac{\bar{\Phi}}{\bar{x}} - \frac{1}{4} \left( \frac{g_\perp^{(a)}}{\bar{x}} + g_\perp^{(a)} \right) \right],
$$

(18)

where $\phi_\parallel^0$, $g_\perp^{(v)}$, $g_\perp^{(a)}$ and

$$
\Phi = \int_0^\bar{x} dy (\phi_\parallel(y) - g_\perp^{(v)}(y))
$$

(19)

are defined in ref. [34]. Using the Wandzura-Wilczek approximation [33][34], one has

$$
G_\phi = 0.
$$

(20)

And numerically $GF_\phi^\pm(1)$ is smaller than $GF_\phi^0(1)$ by about a factor of two and $GF_\phi^-(1) = 0$. Thus, the penguin contract contributions of $Q_{13,15,16}$ to transverse amplitudes are smaller than those to the longitudinal amplitude. However, the penguin contract contribution of $Q_{11}$ to transverse amplitudes can be larger than that to the longitudinal amplitude, as $G_\phi^+(1)$ is larger than $G_\phi^0(1)$.

In eq. (14), $V^\lambda$ and $H_{K^*}\phi$ are vertex and hard-spectator scattering contributions respectively and numerically not important.

The amplitudes in transversal basis [35] for $\bar{B} \to VV$ are related to the helicity amplitudes by

$$
A_0 = H^0, \quad A_\parallel = \frac{H^- + H^+}{\sqrt{2}}, \quad A_\perp = \frac{H^+ - H^-}{\sqrt{2}}.
$$

(21)

And the longitudinal polarization is defined by

$$
f_L = \frac{|A_0|^2}{|A_0|^2 + |A_\parallel|^2 + |A_\perp|^2}.
$$

(22)

**B. Form factors for $B \to \phi K^*$**

Using the identity

$$
\sigma^{\mu\nu}\gamma_5 = -\frac{i}{2}\epsilon^{\mu\nu\rho\sigma}\sigma_{\rho\sigma},
$$

(23)

we have

$$
\langle \phi | \bar{s}\sigma^{\mu\nu}(1 \pm \gamma_5)s|0\rangle = (g^{\mu\rho}g^{\nu\sigma} \mp \frac{i}{2}\epsilon^{\mu\nu\rho\sigma})(\phi | \bar{s}\sigma_{\rho\sigma}s|0),
$$

$$
\langle K^*(e^{(\lambda)}, p_{K^*}) | \bar{s}\sigma^{\mu\nu}(1 \pm \gamma_5)b|B(p_B)\rangle = (g^{\mu\rho}g^{\nu\sigma} \mp \frac{i}{2}\epsilon^{\mu\nu\rho\sigma})(K^*(e^{(\lambda)}, p_{K^*}) | \bar{s}\sigma_{\rho\sigma}b|B(p_B)).
$$

(24)

Defining

$$
\langle K^*(e^{(\lambda)}, p_{K^*}) | \bar{s}\sigma_{\mu\nu}b|B(p_B)\rangle = -i\epsilon_{\mu\nu\rho\sigma}\epsilon^{\sigma\rho\nu}(p_B^0 C_1(s) + p_{K^*}^0 C_2(s)),
$$

(25)

where $s = q^2$ and $q = p_B - p_{K^*}$, one has

$$
\langle K^*(e^{(\lambda)}, p_{K^*}) | \bar{s}\sigma_{\mu\nu}q_\nu b|B(p_B)\rangle = i\epsilon^{\mu\nu\rho\sigma}\epsilon_{\rho\sigma\nu}(p_B p_{K^*} C_1 + C_2).
$$

(26)

Comparing with the usual definition, one has $(C_1 + C_2) = 2T_1$. From eqs. (24) and (25), it is easy to obtain

$$
\langle K^*(e^{(\lambda)}, p_{K^*}) | \bar{s}\sigma^{\mu\nu}q_\nu(1 \pm \gamma_5)b|B(p_B)\rangle = i\epsilon^{\mu\nu\rho\sigma}\epsilon_{\rho\sigma\nu}(p_B p_{K^*} C_1 + C_2) \pm \frac{1}{2}\epsilon^{\mu\nu}\left[ (m_B^2 - m_{K^*}^2) C_+ + s C_- \right]
$$

$$
\mp \frac{1}{2}\epsilon^\mu \cdot p_B (p^\mu C_+ + q^\mu C_-),
$$

(27)
where $C_\pm = C_1 \pm C_2$, $s = q^2$, and $p = p_B + p_{K^*}$. From eq. [27], it follows that there are only two independent form factors in the matrix element of the tensor operator between pseudo-scalar and vector meson states. That is, we need not introduce three form factors in the matrix element, as done in the usual definition in ref. [43]. Comparing with the usual definition of the same matrix element, one has

$$C_+ = 2 T_1, \quad C_- = 2 T_3, \quad T_2 = \frac{1}{2} \left( C_+ + \frac{s}{m_B^2 - m_{K^*}^2} C_- \right). \quad (28)$$

Define

$$\langle 0| \bar{s} \sigma^{\mu \nu} s| \phi(e^{(\lambda)}, q) \rangle = f_\phi \left( e^{(\lambda)_\mu} q^\nu - e^{(\lambda)_\nu} q^\mu \right), \quad (29)$$

we have the naive factorization amplitude of tensor operators as follows.

$$\mathcal{A}_{T(1 \pm \gamma_5)}(\lambda) \equiv \langle \phi(e^{(\lambda)}, q)| \bar{s} \sigma_{\mu \nu} (1 \pm \gamma_5) s| 0 \rangle \langle K^*(e_1^{(1)}), p_{K^*}| \bar{s} \sigma^{\mu \nu} (1 \pm \gamma_5) b| B(p_B) \rangle = 4 f_\phi \left\{ i \epsilon_{\mu \nu \rho \sigma} e_2^\rho e_1^\sigma p_B^\mu p_{K^*}^\nu \cdot 2 T_1 \pm \left( e_1^\nu \cdot e_2^\nu \right) (m_B^2 - m_{K^*}^2) T_2 \right\} \mp \left( e_1^\nu \cdot p_B \right) \left( e_2^\nu \cdot p_B \right) \left( 2 T_2 \pm \frac{2 q^2}{m_B^2 - m_{K^*}^2} T_3 \right), \quad (30)$$

where the superscript $(\lambda)$ has been suppressed in the right hand of eq. [30]. Therefore, it follows that

$$\mathcal{A}_{T(1 \pm \gamma_5)}(\lambda = 0) = \mp 2 \frac{f_\phi}{m_\phi m_{K^*}} \left\{ (m_B^2 - m_\phi^2 - m_{K^*}^2) T_2 - 4 m_B^2 p_c^2 \left( T_2 + \frac{m_\phi^2}{m_B^2 - m_{K^*}^2} T_3 \right) \right\},$$

$$\mathcal{A}_{T(1 \pm \gamma_5)}(\lambda = +1) = \mp 4 \frac{f_\phi}{m_\phi} \left\{ (m_B^2 - m_{K^*}^2) T_2 \mp 2 m_B p_c T_1 \right\},$$

$$\mathcal{A}_{T(1 \pm \gamma_5)}(\lambda = -1) = \mp 4 \frac{f_\phi}{m_\phi} \left\{ (m_B^2 - m_{K^*}^2) T_2 \pm 2 m_B p_c T_1 \right\}, \quad (31)$$

with $p_c$ is the center mass momentum in the $B$ rest frame.

The decay constants and the form factors of vector and pseudoscalar mesons are defined as usual [43]:

$$\langle 0| \bar{s} \gamma_\mu (1 \pm \gamma_5) s| \phi(q, e^{(\lambda)}) \rangle = i f_\phi m_\phi e^{(\lambda)_\mu}, \quad (32)$$

$$\langle K^*(p_{K^*}, e^{(\lambda)})| \bar{s} \gamma_\mu (1 \mp \gamma_5) b| B(p_B) \rangle = \epsilon^{\mu \nu \rho \sigma} e_2^\rho e_1^\sigma p_{p_B}^\mu p_{p_{K^*}}^\nu \frac{2 V(s)}{m_B + m_{K^*}} \mp i \epsilon^{(\lambda)_\mu} (m_B + m_{K^*}) A_1(s)$$

$$\pm i p_\mu e^{(\lambda)_\mu} \cdot p_B \frac{A_2(s)}{m_B + m_{K^*}},$$

$$\pm i q e^{(\lambda)_\mu} \cdot p_B \frac{2 m_{K^*}}{s} (A_3(s) - A_0(s)), \quad (33)$$

where $p = p_B + p_{K^*}$ and $q = p_B - p_{K^*}$. The above equations lead to

$$\mathcal{A}_{V \pm A}(\lambda) \equiv \langle \phi(q, e^{(\lambda)})| \bar{s} \gamma_\mu (1 \pm \gamma_5) s| 0 \rangle \langle K^*(p_{K^*}, e_1^{(1)})| \bar{s} \gamma_\mu (1 \mp \gamma_5) b| B(p_B) \rangle = f_\phi m_\phi \left[ i \epsilon^{\mu \nu \rho \sigma} e_2^\rho e_1^\sigma \frac{2 V(s)}{m_B + m_{K^*}} \mp i \epsilon^{(\lambda)_\mu} (m_B + m_{K^*}) A_1(m_B^2) \right.$$

$$\left. \mp i e_1^{(\lambda)_\mu} \cdot p_B \cdot e_2^{(\lambda)_\mu} \cdot p_B \frac{A_2(m_B^2)}{m_B + m_{K^*}} \right], \quad (34)$$

From eq. [34], we have

$$\mathcal{A}_{V \pm A}(\lambda = 0) = \mp \frac{f_\phi}{2 m_{K^*}} \left\{ (m_B^2 - m_{K^*}^2) (m_B + m_{K^*}) A_1 - 4 m_B^2 p_c^2 \frac{A_2}{m_B + m_{K^*}} \right\},$$

$$\mathcal{A}_{V \pm A}(\lambda = +1) = \mp f_\phi m_\phi \left\{ (m_B + m_{K^*}) A_1 \mp 2 m_B p_c \frac{V}{m_B + m_{K^*}} \right\},$$

$$\mathcal{A}_{V \pm A}(\lambda = -1) = \mp f_\phi m_\phi \left\{ (m_B + m_{K^*}) A_1 \pm 2 m_B p_c \frac{V}{m_B + m_{K^*}} \right\} \quad (35)$$
Comparing eq. (31) and eq. (35), one has
\[ A_{V+A} \sim \frac{m_\phi}{m_B} A_T(1+\gamma_5). \] (36)
That is, the contributions of tensor operator are enhanced by a factor of \( m_B/m_\phi \), compared with those of vector-axial vector operators. Therefore, the contributions of NHB are sizable although there is a suppression factor \( m_s/m_b \) in eq. (21).

IV. NUMERICAL RESULTS

A. Constraints from experiments

We impose two important constraints from \( B \to X_s\gamma \) and \( B_s \to \mu^+\mu^- \). Considering the theoretical uncertainties, we take \( 2.0 \times 10^{-4} < \text{Br}(B \to X_s\gamma) < 4.5 \times 10^{-4} \), as generally adopted in the literature. Phenomenologically, \( \text{Br}(B \to X_s\gamma) \) directly constrains \( |C_{71}(m_b)|^2 + |C'_{71}(m_b)|^2 \) at the leading order. Due to the strong enhancement factor \( m_\phi/m_b \) associated with single \( \delta^{LR(RR)}_{23} \) insertion term in \( C_{71}(m_b) \), \( \delta^{LR(RR)}_{23} \) \( (\sim 10^{-2}) \) are more severely constrained than \( \delta^{LL(RR)}_{33} \). However, if the left-right mixing of scalar bottom quark \( \delta^{LR}_{33} \) is large \( (\sim 0.5) \), \( \delta^{LL(RR)}_{33} \) is constrained to be order of \( 10^{-2} \) since the double insertion term \( \delta^{LL(RR)}_{23} \delta^{LR(RR)}_{33} \) is also enhanced by \( m_\phi/m_b \). The branching ratio \( B_s \to \mu^+\mu^- \) in MSSM is given as \( 24 \)
\[ \text{Br}(B_s \to \mu^+\mu^-) = \frac{G_F^2 \alpha^2_{em}}{64\pi^3 m_B^3 r_B f_B^2} |\lambda|^2 \sqrt{1 - 4\tilde{m}^2(1 - 4\tilde{m}^2)} |C_{Q_1}(m_b) - C'_{Q_1}(m_b)|^2 + |C_{Q_2}(m_b) - C'_{Q_2}(m_b) + 2\tilde{m}(C_{10}(m_b) - C'_{10}(m_b))|^2 \] (37)
where \( \tilde{m} = m_\phi/m_B \). In the moderate and large tan \( \beta \) case the term proportional to \( (C_{10} - C'_{10}) \) in Eq. (37) can be neglected. The new CDF experimental upper bound of \( \text{Br}(B_s \to \mu^+\mu^-) \) is \( 1.5 \times 10^{-7} \) \( 40 \) at 90\% confidence level. We have the constraint
\[ \sqrt{|C_{Q_1}(m_w) - C'_{Q_1}(m_w)|^2 + |C_{Q_2}(m_w) - C'_{Q_2}(m_w)|^2} \lesssim 1.2 \] (38)
Because the bound constrains \( |C_{Q_1} - C'_{Q_1}| \) \((i=1, 2)\), \( ^{\dagger} \) we can have values of \( |C_{Q_1}| \) and \( |C'_{Q_1}| \) larger than those in constrained MSSM (CMSSM) with universal boundary conditions at the high scale and scenarios of the extended minimal flavor violation in MSSM \( 23 \) in which \( |C'_{Q_1}| \) is much smaller than \( |C_{Q_1}| \). Just like the constraint from \( \text{Br}(B \to X_s\gamma) \), \( \delta^{LR(RR)}_{23} \) is also constrained to be order of \( 10^{-2} \) by \( \text{Br}(B_s \to \mu^+\mu^-) \), if \( \delta^{LR}_{33} \) is order of 0.5. At the same time we require that predicted \( \text{Br}(B \to X_s\mu^+\mu^-) \) falls within 1 \( \sigma \) experimental bounds, which gives no new limits on parameters once the updated CDF bound of \( \text{Br}(B_s \to \mu^+\mu^-) \) is imposed. It is shown recently that with the old CDF bound \( \text{Br}(B_s \to \mu^+\mu^-) \) upper bound, \( 2.6 \times 10^{-5} \) \( 42 \), the present experimental limit \( R_K \ (R_K = \text{Br}(B \to K\mu^+\mu^-) / \text{Br}(B \to K\mu^+\mu^-)) \leq 1.2 \) puts constraints on \( C_{Q_{1,2}} \) which are similar to ones from \( \text{Br}(B_s \to \mu^+\mu^-) \) \( 32 \) and Higgs penguin contributions (i.e., the terms relevant to \( C_{Q_{1,2}}^{(i)} \) to \( \text{Br}(B \to X_s\mu^+\mu^-) \) is order of 10\% or less \( 32 \) \( 41 \). We obtained smaller contributions by calculations with the updated CDF bound.

We also impose the current experimental lower bound \( \Delta M_s > 14.4 \text{ps}^{-1} \) \( 43 \). The correlation between \( S_{\phi_K} \) and \( \Delta M_s \) has been extensively discussed in the literature, in particular, in the fourth paper of ref. \( 23 \). So in this paper

\( ^{\dagger} \) \( C^{(i)}_{Q_{1,2}} \) are the Wilson coefficients of the operators \( Q^{(i)}_{1,2} \) which are Higgs penguin induced in leptonic and semileptonic B decays and their definition can be found in Ref. \( 43 \). By substituting the quark-Higgs vertex for the lepton-Higgs vertex, it is straightforward to obtain Wilson coefficients relevant to hadronic B decays.
we just analyze the constraints on parameters from the lower bound. Because $\delta^{LR(RL)}_{23}$ is constrained to be order of $10^{-2}$ by $\text{Br}(B \to X_s \gamma)$, their contribution to $\Delta M_s$ is small. The dominant contribution to $\Delta M_s$ comes from $\delta^{LL(RR)}_{23}$ insertion with both constructive and destructive effects compared with the SM contribution. Too large a destructive effect is ruled out, because SM prediction, $\Delta M_s^{SM} = 17.3^{+1.5}_{-0.7}$, is only slightly above the present experiment lower bound. However $\delta^{LL(RR)}_{23}$ are constrained to be order of $10^{-2}$ by the combined experimental measurement of $\text{Br}(B \to X_s \gamma)$ and upper bound of $\text{Br}(B_s \to \mu^+ \mu^-)$. Their effects to $\Delta M_s$ are limited. And we have checked, the effects are negligibly small with only one kind of chirality, LL or RR, while $\Delta M_s$ can be enhanced to $25\text{ps}^{-1}$ with both kinds of chirality, LL and RR, however it is not strongly correlated with $S_{\phi K_S}$ and $S_{\eta' K_S}$ provided that the other experimental constraints, in particular, those from $\text{Br}(B \to X_s \gamma)$ and upper bound of $\text{Br}(B_s \to \mu^+ \mu^-)$, have been imposed.

As pointed out in Sec. II, due to the gluino-sbottom loop diagram contribution and the mixing of NHB induced operators with the chromomagnetic dipole operator, the Wilson coefficients $C^{(l)}_{8g}$ can be large, which might lead to a too large $\text{Br}(B \to X_s g)$. So we need to impose the constraint from experimental upper bound $\text{Br}(B \to X_s g) < 9\%$ [46]. A numerical analysis for $C'_{8g}=0$ has been performed in Ref. [32]. We carry out a similar analysis by setting both $C_{8g}$ and $C'_{8g}$ non-zero.

### B. Numerical results

In the numerical calculations, we employ the latest Light-Cone Sum Rules results [34] for the form factors of $B \to K^*$, other parameters can be found in ref. [29].

Before moving to numerical results, we discuss some unique features of $B \to VV$ process. The contributions of non-primed operators to the helicity amplitude $H_+$ are much smaller than those to $H_-$, while the contributions of primed operators to the helicity amplitude $H_-$ are much smaller than those to $H_+$, because of the helicity flip of quarks and anti-quarks coming from non-primed or primed operators when they consist of a vector meson with some definite helicity. That is, in the transverse basis, $A_0$ and $A_\parallel$ are proportional to $C - C'$, while $A_\perp$ is proportional to $C + C'$. Therefore, we have $|A_\parallel/A_\perp| \simeq |(C - C')/(C + C')|$

In numerical analysis we fix $m_\tilde{g} = m_q = 500\text{GeV}$, $\tan \beta = 10$ and $\delta^{LR}_{23} = 0.4$. We vary the NHB masses in the ranges of $91\text{GeV} \leq m_h \leq 135\text{GeV}$, $91\text{GeV} \leq m_H \leq 200\text{GeV}$ with $m_h < m_H$ and $200\text{GeV} \leq m_A \leq 250\text{GeV}$ for the fixed mixing angle $\alpha = 0.6, \pi/2$ of the CP even NHBs and scan $\delta^{dAB}_{23}$ in the range $|\delta^{dAB}_{23}| \leq 0.06$ for $A=B$ and 0.01 for $A \neq B$ ($A = L, R$).

Numerical results for the correlation between longitudinal polarization $f_L$ and branching ratio $\text{Br}(B \to \phi K^*)$ are shown in Figs. 8[4][1] where the correlation between $f_L$ of $B \to \phi K^*$ and the indirect CP asymmetry $S_{B \to \phi K}$ is also given. Fig. 1[4][1][2][4] are the results of insertions of $\delta^{dRL}_{23}$, both $\delta^{dL}_{23}$ and $\delta^{dRL}_{23}$, $\delta^{dRR}_{23}$, both $\delta^{dLL}_{23}$ and $\delta^{dRR}_{23}$, respectively. In all four cases, $f_L$ can be dragged as low as 0.5, but the $\text{Br}(B \to \phi K^*)$ is smaller than the experimental measurement when $f_L \sim 0.5$. On the other hand, there are some parameter regions with $f_L$ as low as 0.5 and $S_{B \to \phi K}$ near 0.4, which is consistent with the present experimental measurements. In the case of new physics contributions from LR, RL insertions as shown in Fig. 1[4][1][2] the only large effects come from the SUSY contributions of the chromo-magnetic dipole operator $Q_{8g}$ (and/or $Q'_{8g}$) since the Wilson coefficient $C^{\text{new}}_{8g}(m_h)$ can be significantly larger than $C_{8g}^{SM}(m_h)$. Because $Q_{8g}$ does not contribute to $h = \pm 1$ amplitude, only the longitudinal amplitude can be largely modified. The experimental measurement of $f_L \sim 0.5$ requires that the magnitude of longitudinal amplitude in MSSM must be smaller than that in SM. Therefore, $\text{Br}(B \to \phi K^*)$ in MSSM decreases, compared with SM, when
FIG. 1: The correlations between $f_L$ and $\text{Br}(B \rightarrow \phi K^*)$, $S_{B \rightarrow \phi K^s}$ with $\delta^{\text{LR}}_{23}$ insertion. The Br is in unit of $10^{-5}$.

$\phi K^*$

$\text{Br} \phi K^*$

$S_{B \rightarrow \phi K^s}$

$\delta^{\text{LR}}_{23}$

$\delta^{\text{RL}}_{23}$

$\sim 0.5$, as can be seen from Fig. 1.

In the case of LL, RR insertions, the Wilson coefficient $C_{8g}(m_W')$ could be largely modified and $C_{11}(?)$ and $C_{13} (?)$ could be large. Running from the $m_W$ scale, $C_{13}(C_{13}')$ can induce sizable $C_{8g} (C_{8g}')$ at the $m_b$ scale, which effect we have discussed above. Running from a large electro-weak scale to $m_b$, $C_{13}(C_{13}')$ can also induce large $C_{13-16} (C_{13-16}')$.

However, the updated CDF bound of $B_s \rightarrow \mu^+ \mu^-$ has imposed a stringent constraint on $C_{11,13}$, which leads to that the Wilson coefficients of NHB induced operators are small and $C_{8g}$ can be largely modified only for small and moderate $\tan \beta$. The penguin insertions of operators $Q_{13-16}^{(t)}$ as well as $Q_{11,12}^{(t)}$ have been calculated and given in eqs.(13) and (14), but numerically $Q_{13-16}^{(t)}$ contributions to the magnitude of $h = \pm 1$ amplitude are small compared with the magnitude of $h = 0$ amplitude, due to small $GF^{t}_0$ (1) function in eq.(14). $Q_{13-16}^{(t)}$ can contribute through tree-level to $a_{14-16}$ in eq.(14) and $Q_{11,12}^{(t)}$ also do. However, $C_{11,13}$ are not large enough to enhance the transverse amplitudes sizably due to the constraint from $B_s \rightarrow \mu^+ \mu^-$, as pointed above. So even though $A_{\perp}$ has the structure of $C + C'$, which is different from the $A_{0,\parallel}$ amplitudes, it is still impossible to fine-tune the magnitude of $A_{\perp}$ to the level of $|A_0|$. Therefore, the overall contribution of LL, RR insertions are very similar to LR, RL insertions, as we see from Fig.s 1-4.

The numerical results are obtained for $m_{\tilde{g}} = m_{\tilde{q}} = 500$ GeV. For smaller gluino and squark masses, the Wilson coefficient $C_{8g}^{(t)}$ becomes larger, which could have larger effect on the $b$ to $s$ transitions. However, the effect is indeed limited due to the constraint from $B \rightarrow X_s g$. For fixed $m_{\tilde{g}}$, the Wilson coefficient $C_{8g}^{(t)}$ is not sensitive to the variation of the mass of squark in the range about from 100 GeV to 1.5 TeV. Therefore, the numerical results are not sensitive to the squark mass and would have a sizable change when the gluino mass decreases. When the gluino and squark
masses approach to infinity (indeed, the several TeV is big enough), SUSY effects drop, i.e., one reaches the decoupling limit.

Before concluding, we will comment on two channels, $B \to K^{*}\gamma$ and $B \to K^{*}l^{+}l^{-}$, which share the same $B \to K^{*}$ form factors as $B \to K^{*}\phi$. They have already been calculated within QCD factorization in ref. [50, 51] and new physics effects have been discussed in ref. [52]. For $B \to K^{*}\gamma$ channel, the SM prediction is about 2 times larger than the experimental measurement. A way out to reduce the theoretical prediction of $\text{Br}(B \to K^{*}\gamma)$ is to decrease the transverse form factors associated with $B \to K^{*}$. Then the magnitude of transverse amplitude of $B \to K^{*}\phi$ will be decreased as well, and the polarization problem becomes even worse within the SM. We carry out an analysis of the correlations between $\text{Br}(B \to K^{*}\gamma)$ and the polarization of $B \to K^{*}\phi$ within the new physics framework as we discussed above. We find that both $\text{Br}(B \to K^{*}\gamma)$ and $f_{L}$ can be accommodated within 1σ limits only in the case of both LR and RL insertions as shown in Fig. 5a. However, in all the cases, the predicted $\text{Br}(B \to K^{*}\phi)$ is still small when $f_{L}$ approaches 0.5. This situation can be relaxed to some extent in all the cases of insertions when we consider the $B \to K^{*}$ form factors $\xi_{\parallel}$ and $\xi_{\perp}$, as defined in [51], with 50% uncertainties. As an example, our results of the correlations between $f_{L}$ and $\text{Br}(B \to K^{*}\phi)$ are given in Fig. 5b in the case of both LL and RR insertions. At the same time, $\text{Br}(B \to K^{*}\gamma)$ and $f_{L}$ can be accommodated within 1σ limits in all the cases of insertions. The situation of $B \to K^{*}l^{+}l^{-}$ is more inconclusive due to the branching ratio measurement by BaBar and Belle with large uncertainties, and theoretically it has been discussed in ref. [51, 52].

V. CONCLUSIONS AND DISCUSSIONS

In summary we have analyzed the $B \to \phi K^{*}$ polarization puzzle in MSSM. The hadronic matrix elements of the new operators in MSSM for the decays have been calculated in the QCDF approach up to the $\alpha_{s}$ order. Using the Wilson coefficients in ref. [26] and hadronic matrix elements obtained, we have calculated the polarization fractions and branching ratios for the decays $B \to \phi K^{*}$. It is shown that in the reasonable region of parameter space where the constraints from $B_{s} - \bar{B}_{s}$ mixing, $B \to X_{s}\gamma$, $B \to X_{s}q\bar{q}$, $B \to X_{s}\mu^{+}\mu^{-}$ and $B_{s} \to \mu^{+}\mu^{-}$ are satisfied, the polarization fractions of the decays can agree with experimental data within 1σ deviation. In particular, the puzzle for polarization in $B \to \phi K^{*}$ can be explained, while not in contradiction to the measurements of other two vector final states, in a large region of parameter space because we have included the contributions of the primed operators.
and new operators including the $\alpha_s$ corrections of hadronic matrix elements of them in MSSM. However, the branching ratio is smaller than the measurements when the longitudinal fraction $f_L$ is near 0.5. We may not worry about it too much at present due to the large uncertainty in calculating hadronic matrix elements of operators.

It is necessary to make a theoretical prediction in SM as precise as we can in order to give a firm ground for signaling new physics. The twist-3 and weak annihilation contributions to $B \to \phi K_S$ in SM have been calculated in Ref. [25] using the method in Ref. [47] by which there is not any phenomenological parameter introduced. The numerical results show that the annihilation contributions to the decay rates are negligible, the twist-3 contributions are also very small, smaller than one percent. We expect that the conclusion would qualitatively remain for $B \to \phi K^*$ in MSSM, so that we have neglected the annihilation contributions in numerical calculations.

In conclusion, we have shown that the recent experimental measurements on the polarization fractions in $B \to \phi K^*$, which is difficult to be explained in SM, can be explained in MSSM if there are flavor non-diagonal squark mass matrix elements of second and third generations whose size satisfies all relevant constraints from known experiments ($B \to X_s\gamma, B_s \to \mu^+\mu^-, B \to X_s\mu^+\mu^-, B \to X_sg, \Delta M_s$, etc.). Therefore, if the present polarization puzzle persists in the future, it will be a signal for new physics beyond the SM and MSSM will be a possible candidate of new physics.
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Appendix. Vertex and hard scattering contributions

In the Appendix we give the explicit expressions of vertex corrections and hard scattering contributions at the $\alpha_s$ order for NHB induced operators which are not given in the content.

The hard spectator contributions up to the leading twist in Eq. (14) are given as following:

$$H_{13,14,15}^0 = 0,$$

$$H_{13}^\pm = 4H_{14}^\pm, \quad H_{15}^\pm = -12H_{14}^\pm,$$

$$H_{14}^\pm = \frac{4\pi^2}{N_c}(1 \mp 1) \int \frac{d\xi}{\xi} \Phi_B(\xi) \int_0^1 \frac{du}{u} \phi_{1\perp}^\pm(u) \int_0^1 \frac{dv}{v} \phi_{K^*\perp}^\pm(v),$$

where $\phi_{1\perp}$ is defined in ref. [3] and normalized as

$$\int_0^1 du \phi_{1\perp}(u) = 1.$$

The vertex corrections up to the leading twist in Eq. (14) are as follows.

$$V_{13,14,15}^0 = 0$$

$$V_{14}^\pm = -\frac{1}{6}[12 \ln \frac{m_b}{\mu} + \int_0^1 du g(u) \phi_{1\perp}(u)] + the \ regularization \ scheme \ dependent \ constant,$$

$$g(x) = 3 \left( \frac{1 - 2x}{1 - x} \ln x - i\pi \right),$$

where we have used that $\phi_{1\perp}(u, \bar{u})$ is symmetric with respect to $u, \bar{u}$. Omitting regularization scheme dependent constants, we have

$$V_{13}^\pm = 4V_{14}^\pm, \quad V_{15}^\pm = -12V_{14}^\pm.$$

We have verified that the $\mu$ dependance of $a_{14}^\perp = -\frac{1}{8}a_{14}^\perp + a_{15}^\perp + \frac{1}{2}a_{16}^\perp$ in eq.(11) has been cancelled up to the order of $\alpha_s$. That is, $\frac{d}{d\ln \mu} a_{14}^\perp = O(\alpha_s^2)$.

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