Spin Polarisability of the Nucleon
in the Heavy Baryon Effective Field Theory

*K.B. Vijaya Kumar, Yong-Liang Ma and Yue-Liang Wu

Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080, China

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Abstract

We have constructed a heavy baryon effective field theory with photon as an external field in accordance with the symmetry requirements similar to the heavy quark effective field theory. By treating the heavy baryon and anti-baryon equally on the same footing in the effective field theory, we have calculated the spin polarisabilities $\gamma_i$, $i = 1 \ldots 4$ of the nucleon at third order and at fourth-order of the spin-dependent Compton scattering. At leading order (LO), our results agree with the corresponding results of the heavy baryon chiral perturbation theory, at the next-to-leading order(NLO) the results show a large correction to the ones in the heavy baryon chiral perturbation theory due to baryon-antibaryon coupling terms. The low energy theorem is satisfied both at LO and at NLO. The contributions arising from the heavy baryon-antibaryon vertex were found to be significant and the results of the polarisabilities obtained from our theory is much closer to the experimental data.
I. INTRODUCTION

The Compton-scattering process is an important tool to probe the nucleon structure by measurements of various polarisabilities. For unpolarised proton the experimental amplitude is well determined and is in good agreement with the results of heavy-baryon chiral perturbation theory (HBCHPT). But, with regard to the scattering from polarised targets, it is less satisfactory. At present, there exists no direct measurements of the polarisabilities of polarised Compton scattering. For the spin dependent pieces the scattering amplitude in the Breit frame is:

\[ T = \epsilon^\mu \Theta_{\mu \nu} \epsilon^\nu \]

\[ = i\vec{\sigma} \cdot (\hat{\epsilon}' \times \hat{\epsilon}) A_3(\omega, \theta) + i\vec{\sigma} \cdot (\hat{k} \times \hat{k}') \hat{\epsilon}' \cdot \hat{\epsilon} A_4(\omega, \theta) \]

\[ + [i\vec{\sigma} \cdot (\hat{\epsilon}' \times \hat{k}') \hat{\epsilon}' \cdot \hat{k} - i\vec{\sigma} \cdot (\hat{\epsilon} \times \hat{k}) \hat{\epsilon} \cdot \hat{k}'] A_5(\omega, \theta) \]

\[ + [i\vec{\sigma} \cdot (\hat{\epsilon}' \times \hat{k}) \hat{\epsilon} \cdot \hat{k}' - i\vec{\sigma} \cdot (\hat{\epsilon} \times \hat{k}') \hat{\epsilon}' \cdot \hat{k}] A_6(\omega, \theta) \]

\[ + i\vec{\sigma} \cdot (\hat{k} \times \hat{k}') \hat{\epsilon}' \cdot \hat{k} \hat{\epsilon} \cdot \hat{k} A_7(\omega, \theta) + \text{spin independent terms} \]  

(1.1)

where \( \omega \) is the photon energy and \( k \) is the incoming photon momentum, \( \epsilon \) and \( \epsilon' \) are the incoming and outgoing photon polarization directions respectively and the hats indicate unit vectors. By crossing symmetry the functions \( A_i \) are odd in \( \omega \). The leading pieces in the expansion are governed by low-energy theorems[1], and the next order terms contain the spin polarisabilities \( \gamma_i \)

\[ A_3(\omega, \theta) = \frac{e^2 \omega}{2m_B} [Q(Q + 2\kappa) - (Q + \kappa)^2 \cos \theta] + 4\pi \omega^3 (\gamma_1 + \gamma_5 \cos \theta) \]

\[ - \frac{e^2 Q(Q + 2\kappa) \omega^3}{8m_B^4} + O(\omega^5) \]

\[ A_4(\omega, \theta) = -\frac{e^2 \omega}{2m_B^2} (Q + \kappa)^2 + 4\pi \omega^3 \gamma_2 + O(\omega^5) \]

\[ A_5(\omega, \theta) = \frac{e^2 \omega}{2m_B^2} (Q + \kappa)^2 + 4\pi \omega^3 \gamma_4 + O(\omega^5) \]

\[ A_6(\omega, \theta) = -\frac{e^2 \omega}{2m_B^2} Q(Q + \kappa) + 4\pi \omega^3 \gamma_3 + O(\omega^5) \]

\[ A_7(\omega, \theta) = O(\omega^5) \]  

(1.2)

where the charge of the nucleon is \( Q = (1 + \tau_3)/2 \) (\( \tau_3 \) is the third component of the isospin) and its anomalous magnetic moment is \( \kappa = ((\kappa_s + \kappa_v)\tau_3)/2 \). Only four of the polarisabilities are independent due to the relation \( \gamma_5 + \gamma_2 + 2\gamma_4 = 0 \).
At present there are only estimates of the polarisabilities. The best estimate exists for forward scattering where only $A_3$ contributes. The quantity $4\pi f_2(0)$ is defined as $dA_3(\omega, 0)/d\omega$ at $\omega = 0$, and depends only on $\kappa^2$ according to the low energy theorem (LET). The relevant polarisability is $\gamma_0 = \gamma_1 + \gamma_5$, which is related via a dispersion relation to measurements at energies above the threshold for pion photo production, $\omega_0$:

$$\gamma_0 = \frac{1}{4\pi^2} \int_{\omega_0}^{\infty} d\omega \frac{\sigma_-(\omega) - \sigma_+(\omega)}{\omega^3}$$

(1.3)

where $\sigma_\pm$ are the parallel and antiparallel cross-sections for photo absorption and the related sum rule for the model-independent piece, due to Gerasimov, Drell and Hearn[2], has the same form except that $1/\omega$ replaces $1/\omega^3$.

Very recently, the various measurements have been made with MAMI at Mainz[3], for photon energies between $200 \sim 800\,MeV$; the range will be extended downward to 140 MeV, and a future experiment at Bonn will extend it upwards to 3 GeV. The MAMI data does not currently go low enough in energy to give a reliable result for the spin polarisability $\gamma_0$. However electroproduction data have also been used to extract this quantity; Sandorfi et al.[4] find $\gamma_0^p = -1.3 \times 10^{-4}\,fm^4$ and $\gamma_0^n = -0.4 \times 10^{-4}\,fm^4$, while a more recent analysis of Drechsel et al.[5] gives a rather smaller value of $\gamma_0^p = -0.6 \times 10^{-4}\,fm^4$. (We shall use units of $10^{-4}\,fm^4$ for polarisabilities from now on). The spin polarisability has been calculated in the framework of HBCHPT: at lowest (third) order in the expansion $\gamma_0 = \alpha_{em} g_\Lambda^2/(24\pi^2 f_\pi^2 m_\pi^2) = 4.51$ which diverges as $1/m_\pi^2$ in the chiral limit. Here, the entire contribution comes from $\pi N$ loops. Being a spin dependent quantity, the spin polarisability should receive a sizable contribution from the delta resonance. The effect of the $\Delta$ enters in counter-terms at fifth order in standard HBCHPT, and has been estimated to be so large as to change the sign[6]. The calculation has also been done in an extension of HBCHPT with an explicit $\Delta$ by Hemmert et al.[7]. They find that the principal effect is from the $\Delta$ pole, which contributes $-2.4$ with the effect of $\pi\Delta$ loops being small $-0.2$. Also, one other combination of the polarisabilities, the backward scattering, $\gamma_\pi = \gamma_1 - \gamma_5$ has been estimated from the low-energy data for Compton scattering from the proton by Tonnison et al.[8]. The backward scattering is dominated by the anomalous $\pi^0$ exchange graph, which vanishes for the forward scattering, but at third order and at fourth order there are also pion loop contributions. The experimental value is $\gamma_\pi = -27.1$ with experimental and theoretical errors of about 10% each. The HBCHPT results of Hemmert et al. is $-36.7$ of which $-43.5$ is the anomalous
contribution, 4.6 is the \( \pi N \) piece and 2.2 comes from including the \( \Delta \).

The fourth order contribution to all the four polarisabilities has been worked by several groups in the framework of HBCHPT \[9\[10\[11\[12\]. At \( O(p^4) \) there are no seagulls and since the NLO pieces of the \( A_i(\omega) \) are of the fourth chiral order and are odd in \( \omega \), they will have expansions of the form \( e^2 \omega (am_{\pi} + b\frac{m^2_{\pi}}{m_{\pi}} + ...) \). These non-analytic powers of \( m^2_{\pi} \) cannot be present in the basic couplings in the Lagrangian, but can only be generated from loops. In the work of Gellas et al.\[9\] they have not included the one-particle reducible graphs (Fig.2g in our paper) in their definition of the polarisabilities. With the addition of that contribution their results agree with the results of ref.\[10\[11\]. The polarisability of interest, the forward spin polarisability to order \( O(p^4) \) turns out to be \( \gamma_0 = 4.5 - (6.9 + 1.5\tau_3) \), where the first term in the above expression is the contribution from the leading order. The NLO contributions are large and hence call the convergence of the expansion into question. As has been argued by Bernard et.al \[13\] the bad reason for the convergence is entirely related to the contributions from the Born terms. It has been argued that the calculations of the Born graphs to the fourth order is not sufficient to obtain convergence and hence is necessary to take into account the two-loop corrections which appears at the fifth order\[14\].

It is to be noted that HBCHPT\[15\] is an effective theory constructed by using the idea of heavy quark effective theory (HQET)\[16\]. In HBCHPT, baryon is considered as extremely heavy and only baryon momenta relative to the rest mass will count which is very small. The heavy source (baryon) is surrounded by a cloud of almost massless particles which is exactly the idea used in HQET. In HQET, after decoupling the "quark fields" and "antiquark fields" only one of them is treated independently. Strictly speaking, in quantum field theory, particle and antiparticle decouple completely only in the infinite heavy quark mass limit \( m_Q \to \infty \). To consider the finite quark mass correction, it is necessary to include the contribution from the components of the antiquark fields. For that, one can simply extend the usual HQET\[16\] to a heavy quark effective field theory (HQEFT) with keeping both effective quark and antiquark fields. This was first pointed out by one of us \[17\], where a new formulation of heavy quark effective Lagrangian was derived from the full QCD. Its form permits an expansion in powers of the heavy quark momentum characterizing its off-shellness divided by its mass. A detailed comparison between HQEFT and HQET is provided in the reference \[18\]. It is important to note that at the leading order the HQEFT is same as HQET, the difference arises from the sub-leading terms which is proportional to the inverse of heavy
quark mass $m_Q$. The reason is that in the construction of HQET, the particle and anti-particle are independently treated based on the assumption that the particle number and anti-particle number are conserved separately in the effective theory. Such an assumption is valid only in the infinite mass limit. Hence, quark-anti-quark coupled terms that correspond to the pair creation and annihilation interaction terms in full QCD were inappropriately dropped away in the usual HQET. Those terms have been shown in HQEFT of QCD to be suppressed by $1/m_Q$ and they become vanishing in the infinite mass limit. Thus, the usual HQET based on the assumption that the quark and anti-quark number conservation in the effective Lagrangian is an incomplete theory for evaluating the subleading order corrections. Unlike the derivation of HQET from QCD by making the assumption of particle and anti-particle conservation in the effective theory, the HQEFT was derived from QCD by treating all the contributions of the field components, i.e. large and small, particle and anti-particle in the effective Lagrangian, so that the resulting effective lagrangian forms the basis for a complete effective field theory of heavy quarks. A more systematic construction and detailed interpretation of the HQEFT is recently presented in [19]. The HQEFT has been used to study heavy quark systems [20, 21, 22, 23, 24, 25, 26]. Considering the shortcomings of the HQET and the rationality of HQEFT from the view point of quantum field theory, it is more reasonable to construct the chiral theory of baryons in the frame work of HQEFT. In a word, the HQET is not a complete theory from the view point of quantum field theory, so that the HBCHPT which was constructed by using the idea of HQET is also not complete. To construct a complete heavy baryon effective field theory (HBEFT), a similar idea of HQEFT should be used by treating heavy baryon and antibaryon fields equally on the same footing.

The paper is organised as follows. In Sec.II, we will construct the heavy baryon effective field theory (HBEFT) that contains both effective baryon and antibaryon fields. We construct $\mathcal{L}^{(1)}_{\pi N}$ and $\mathcal{L}^{(2)}_{\pi N}$ with photon as an external field. The constructed Lagrangian has all the symmetries of the HBCHPT. The Feynman rules from the leading order and next to leading order Lagrangian required in the study of Compton scattering are derived. In Sec.III, we calculate the spin polarisabilities at the lowest order and at NLO. In Sec.IV, we discuss the complete results of the fourth order calculations and the important conclusions. Appendix A gives the pertinent Feynman rules from $\mathcal{L}^{(1)}_{\pi N}$ and $\mathcal{L}^{(2)}_{\pi N}$. Appendix B gives the full amplitude for the diagrams of Fig.2 and Fig.3.
II. CONSTRUCTION OF THE EFFECTIVE LAGRANGIAN $\mathcal{L}_{\pi N}^{(1)}$ AND $\mathcal{L}_{\pi N}^{(2)}$

Here, we briefly describe the new formulation of the will be constructed HBEFT that contains both effective baryon and anti-baryon fields. In chiral perturbation theory, the transformation properties of baryon and meson fields under chiral symmetry $SU(2)_L \times SU(2)_R$ are

$$U(x) \rightarrow g_L U(x) g_R^\dagger, \quad g_L \times g_R \in SU(2)_L \times SU(2)_R$$

$$H(x) \rightarrow G(x) H(x), \quad G(x) \in SU(2)_{\text{local}}$$

where $H(x)$ is baryon matrix

$$H(x) = \begin{pmatrix} p \\ n \end{pmatrix}$$

$G(x)$ is determined by

$$\xi(x) \rightarrow g_L \xi(x) G^\dagger(x) = G(x) \xi(x) g_R^\dagger$$

and is hidden local symmetry. It can be verified that $SU(2)_{\text{local}} = SU(2)_V$ when $g_L = g_R$. In the following, we select

$$U(x) = \xi^2(x) = e^{\frac{2iU(x)}{f_\pi}}$$

and $f_\pi$ is the pion decay constant. The covariant derivative appearing in the kinetic energy term of the baryon can be written as

$$D_\mu H(x) = \partial_\mu H(x) - iV_\mu H(x)$$

$$V_\mu(x) = \frac{i}{2} [\xi(x) \partial_\mu \xi^\dagger(x) + \xi^\dagger(x) \partial_\mu \xi(x)]$$

under chiral transformation, it transforms as

$$D_\mu H(x) \rightarrow G(x) D_\mu H(x)$$

Besides the quantities introduced above, we have,

$$A_\mu = \frac{i}{2} [\xi(x) \partial_\mu \xi^\dagger(x) - \xi^\dagger(x) \partial_\mu \xi(x)]$$

under chiral transformation it transforms as

$$A_\mu \rightarrow G(x) A_\mu G^\dagger(x)$$
Then the lowest order Lagrangian is

$$L = \text{Tr}[\hat{H}(i\bar{\psi} - m_B)H] + 2D\text{Tr}[\hat{H}\gamma_5H]$$  \hspace{1cm} (2.11)

where \(m_B\) is the baryon mass matrix and the coupling constant \(D\) can be determined phenomenologically.

We expand the above Lagrangian in terms of the inverse of the heavy baryon mass similar to the HQEFT. We are making use of the some of the conclusions given in [17, 18, 19].

Baryon fields can be decomposed into baryon and anti-baryon which correspond to the positive and negative solutions respectively of the Dirac equation. Using the relation

$$H = \left[1 + \left(1 - \frac{i\bar{\psi} \cdot D + m_B}{2m_B}\right)^{-1}\frac{i\bar{\psi} \cdot D}{2m_B}\right] \hat{H}_v$$

$$\hat{H} = \hat{H}_v \left[1 + \frac{-i\bar{\psi} \cdot D}{2m_B} \left(1 - \frac{-i\bar{\psi} \cdot \bar{D} + m_B}{2m_B}\right)^{-1}\right]$$  \hspace{1cm} (2.12)

integrating out the small components of baryon and anti-baryon field in (2.11) we get,

$$L_v = \text{Tr}[\hat{H}_v(i\bar{\psi} \cdot D - m_B)\hat{H}_v] + \frac{1}{2m_B}\text{Tr}[\hat{H}_v i\bar{\psi}(1 - \frac{i\bar{\psi} \cdot D + m_B}{2m_B})^{-1}iD_\perp \hat{H}_v]$$

$$+ \frac{1}{2m_B}\text{Tr}[\hat{H}_v iD_\perp (1 - \frac{-i\bar{\psi} \cdot \bar{D} + m_B}{2m_B})^{-1}i\bar{\psi} \cdot D - m_B\hat{H}_v]$$

$$+ \frac{1}{4m_H^2}\text{Tr}[\hat{H}_v (-i\bar{\psi} \cdot \bar{D}) \left(1 - \frac{-i\bar{\psi} \cdot \bar{D} + m_B}{2m_B}\right)^{-1}\frac{i\bar{\psi} \cdot D + m_B}{2m_B} \left(1 - \frac{-i\bar{\psi} \cdot \bar{D} + m_B}{2m_B}\right)^{-1}i\bar{\psi} \cdot D - m_B\hat{H}_v]$$

$$+ 2D\text{Tr}\left\{\hat{H}_v \gamma_\mu \gamma_5 A_\mu \hat{H}_v + \hat{H}_v \gamma_\mu \gamma_5 A_\mu \left(1 - \frac{i\bar{\psi} \cdot D + m_B}{2m_B}\right)^{-1}\frac{i\bar{\psi} \cdot D + m_B}{2m_B}\hat{H}_v\right\}$$

$$+ \hat{H}_v \frac{-i\bar{\psi} \cdot \bar{D}}{2m_B} \left(1 - \frac{-i\bar{\psi} \cdot \bar{D} + m_B}{2m_B}\right)^{-1} \gamma_\mu \gamma_5 A_\mu \hat{H}_v$$

$$+ \frac{\hat{H}_v \frac{-i\bar{\psi} \cdot \bar{D}}{2m_B}}{2m_B} \left(1 - \frac{-i\bar{\psi} \cdot \bar{D} + m_B}{2m_B}\right)^{-1} \gamma_\mu \gamma_5 A_\mu \left(1 - \frac{i\bar{\psi} \cdot D + m_B}{2m_B}\right)^{-1}\frac{i\bar{\psi} \cdot D + m_B}{2m_B}\hat{H}_v$$  \hspace{1cm} (2.13)

where

$$\hat{H}_v = \hat{H}_v^{(+)} + \hat{H}_v^{(-)}$$  \hspace{1cm} (2.14)

$$\hat{H}_v^{(\pm)} = \frac{1 \pm \bar{\psi} \cdot H^{(\pm)}}{2}$$  \hspace{1cm} (2.15)

$$\bar{\psi} = D_{\parallel} + D_{\perp}$$  \hspace{1cm} (2.16)

$$D_{\parallel} = \bar{\psi} \cdot D, \hspace{0.5cm} D_{\perp} = \bar{\psi} - \bar{\psi} \cdot D$$  \hspace{1cm} (2.17)

and \(H^{(\pm)}\) are the solutions of the Dirac equations corresponding to the positive and negative energy respectively. For any operator \(O\), the operator \(\hat{O}\) is defined by \(\int \kappa \hat{O} \varphi \equiv - \int \kappa O \varphi\).
Introducing new field variables $H_v$ and $\hat{H}_v$ with the definition, We can rewrite the above Lagrangian as,

$$A_{\mu \parallel} = v_{\mu} \cdot A, \quad A_{\mu \perp} = A_{\mu} - v_{\mu} \cdot A$$ (2.18)

The Lagrangian can be written in terms of the baryon baryon $(++)$, baryon anti-baryon $(+-)$, anti-baryon baryon $(-+)$ and anti-baryon anti-baryon ($--$) explicitly as,

$$\mathcal{L}_v = \mathcal{L}_v^{(++)} + \mathcal{L}_v^{(-+)} + \mathcal{L}_v^{(+ -)} + \mathcal{L}_v^{(- -)} + \mathcal{L}_{A,v}^{(++)} + \mathcal{L}_{A,v}^{(-+)} + \mathcal{L}_{A,v}^{(+ -)} + \mathcal{L}_{A,v}^{(- -)}$$ (2.19)

where

$$\mathcal{L}_v^{(++)} = \text{Tr}[\bar{H}_v (i\hat{\mathcal{D}}_v - m_B) \hat{H}_v]$$ (2.20)

$$\mathcal{L}_v^{(+-)} = \frac{1}{2m_B} \text{Tr}[\bar{H}_v (i\hat{\mathcal{D}}_v)(1 - \frac{-i\psi \cdot \vec{D} + m_B}{2m_B})^{-1}(i\hat{\mathcal{D}}_v - m_B) \hat{H}_v]$$

$$\mathcal{L}_{A,v}^{(++)} = 2D \text{Tr}\left\{ \bar{H}_v \gamma_{\mu \gamma 5} A_{\mu \parallel} \hat{H}_v + \bar{H}_v \gamma_{\mu \gamma 5} A_{\mu \perp} \left(1 - \frac{i\psi \cdot \vec{D} + m_B}{2m_B}\right)^{-1} \frac{i\hat{\mathcal{D}}_v}{2m_B} \hat{H}_v \right\}$$

$$\mathcal{L}_{A,v}^{(-+)} = 2D \text{Tr}\left\{ \bar{H}_v \gamma_{\mu \gamma 5} A_{\mu \parallel} \hat{H}_v + \bar{H}_v \gamma_{\mu \gamma 5} A_{\mu \perp} \left(1 - \frac{i\psi \cdot \vec{D} + m_B}{2m_B}\right)^{-1} \frac{i\hat{\mathcal{D}}_v}{2m_B} \hat{H}_v \right\}$$

$$\mathcal{L}_{A,v}^{(+ -)} = 2D \text{Tr}\left\{ \bar{H}_v \gamma_{\mu \gamma 5} A_{\mu \parallel} \hat{H}_v + \bar{H}_v \gamma_{\mu \gamma 5} A_{\mu \perp} \left(1 - \frac{i\psi \cdot \vec{D} + m_B}{2m_B}\right)^{-1} \frac{i\hat{\mathcal{D}}_v}{2m_B} \hat{H}_v \right\}$$

$$\mathcal{L}_{A,v}^{(- -)} = 2D \text{Tr}\left\{ \bar{H}_v \gamma_{\mu \gamma 5} A_{\mu \parallel} \hat{H}_v + \bar{H}_v \gamma_{\mu \gamma 5} A_{\mu \perp} \left(1 - \frac{i\psi \cdot \vec{D} + m_B}{2m_B}\right)^{-1} \frac{i\hat{\mathcal{D}}_v}{2m_B} \hat{H}_v \right\}$$

with

$$i\hat{\mathcal{D}}_v = i\psi \cdot D + \frac{1}{2m_B} i\hat{\mathcal{D}}_v (1 - \frac{-i\psi \cdot \vec{D} + m_B}{2m_B})^{-1} i\hat{\mathcal{D}}_v$$ (2.22)

To make $1/m_B$ expansion, it is useful to remove the large mass term in the Lagrangian. Introducing new field variables $H_v$ and $\hat{H}_v$ with the definition, We can rewrite the above Lagrangian as

$$H_v = e^{i\int m_B v \cdot x} \hat{H}_v, \quad \bar{H}_v = \bar{H}_v e^{-i\int m_B v \cdot x}$$ (2.23)

We can rewrite the above Lagrangian as,

$$\mathcal{L}_v^{(\pm \pm)} = \text{Tr}[\bar{H}_v i\hat{\mathcal{D}}_v \hat{H}_v]$$ (2.24)
\[ L^{(\pm)}_v = \frac{1}{2m_B} \text{Tr}[\bar{H}_v(-i\gamma_5\vec{D}_\perp)(1 - \frac{-i\psi_v \cdot \vec{D}}{2m_B})^{-1}e^{-2i\phi m_B v \cdot x}(i\vec{D}_\perp)H_v] \]

\[ L^{(\pm)}_{A,v} = 2D\text{Tr}\{H_v \gamma_5 H_v \}
+ \bar{H}_v A_{\perp} \gamma_5 \left(1 - \frac{-i\vec{D}}{2m_B}\right)^{-1} \frac{i\vec{D}}{2m_B} H_v + \bar{H}_v \frac{-i\vec{D}}{2m_B} \left(1 - \frac{-i\vec{D}}{2m_B}\right)^{-1} A_{\perp} \gamma_5 \}
\]

\[ L^{(\pm)}_{A,v} = 2D \text{Tr}\{H_v A_{\perp} \gamma_5 e^{-2\phi m_B v \cdot x} H_v \}
+ \bar{H}_v A_{\perp} \gamma_5 \left(1 - \frac{-i\vec{D}}{2m_B}\right)^{-1} e^{2i\phi m_B v \cdot x} \frac{i\vec{D}}{2m_B} H_v + \bar{H}_v \frac{-i\vec{D}}{2m_B} \left(1 - \frac{-i\vec{D}}{2m_B}\right)^{-1} \frac{i\vec{D}}{2m_B} e^{-2i\phi m_B v \cdot x} H_v \} \] (2.25)

with

\[ i\vec{D}_\perp = i\phi v \cdot D + \frac{1}{2m_B} i\vec{D}_\perp (1 - \frac{-i\phi v \cdot D}{2m_B})^{-1} i\vec{D}_\perp \] (2.26)

The factor \( e^{\pm 2i\phi m_B v \cdot x} \) arises from the opposite momentum shift for the effective heavy baryon and anti-baryon fields. Introducing the electromagnetic field

\[ D_\mu H_v(x) = \partial_\mu H_v(x) - iV_\mu H_v(x) \] (2.27)

\[ V_\mu(x) = \frac{i}{2} [\xi(x)(\partial_\mu - ieQ \mathcal{A}_\mu)\xi^\dagger(x) + \xi^\dagger(x)(\partial_\mu - ieQ \mathcal{A}_\mu)\xi(x)] \]

\[ A_\mu(x) = \frac{i}{2} [\xi(x)(\partial_\mu - ieQ \mathcal{A}_\mu)\xi^\dagger(x) - \xi^\dagger(x)(\partial_\mu - ieQ \mathcal{A}_\mu)\xi(x)] \] (2.28)

where \( Q = (1 + \tau_3)/2 \) and \( \tau_3 \) is the third component of the isospin. In the Coulomb gauge

\[ A_0 = 0, \quad v \cdot A = 0 \] (2.29)

it should be noticed that the polarization direction introduced in this paper differs from the convention used in \[^{15}\].

**A. The Leading Order Lagrangian (\( L^{(1)}_{\pi N} \))**

The lowest order Lagrangian (\( L^{(1)}_{\pi N} \))(where the superscript \( (1) \) denotes the low energy dimension (number of derivatives and/or quark mass terms)) can be decomposed into

\[ L^{(\pm)}_v = \text{Tr}[\bar{\pi}/(i\phi v \cdot D)H_v] \]
\[
\mathcal{L}_{A,v}^{(\pm \mp)} = 2D \text{Tr}\{\bar{H}_v A_\perp \gamma_5 H_v\}
\]
\[
\mathcal{L}_{A,v}^{(\pm \mp)} = 2D \text{Tr}\{\bar{H}_v A_\parallel \gamma_5 e^{-2im_B v \cdot x} H_v\}
\] (2.30)

with \( S_\mu^v \) the covariant spin operator \[\text{[15]}\]
\[
S_\mu^v = \frac{i}{2} \gamma_5 \sigma_\mu v = -\frac{1}{2} \gamma_5 (\gamma_\mu \not{v} - v\gamma_\mu), \quad S_\mu^v = \gamma_0 S_\mu^v \gamma_0
\] (2.31)

with
\[
\sigma_\mu^\nu = \frac{i}{2} [\gamma_\mu, \gamma_\nu]
\] (2.32)

The Lagrangian can be expressed as
\[
\mathcal{L}_v^{(\pm \mp)} = \text{Tr}[\bar{H}_v^{(+)} (iv \cdot D) H_v^{(+)}] - \text{Tr}[\bar{H}_v^{(-)} (iv \cdot D) H_v^{(-)}]
\] (2.33)
\[
\mathcal{L}_{A,v}^{(\pm \mp)} = 4D \text{Tr}\{\bar{H}_v^{(+)} S_\nu A_\perp H_v^{(+)}\} - 4D \text{Tr}\{\bar{H}_v^{(-)} S_\nu A_\perp H_v^{(-)}\}
\] (2.34)
\[
\mathcal{L}_{A,v}^{(\pm \mp)} = 2D \text{Tr}\{\bar{H}_v^{(+)} \gamma_5 v \cdot A_\parallel e^{2im_B v \cdot x} H_v^{(+)}\} - 2D \text{Tr}\{\bar{H}_v^{(-)} \gamma_5 v \cdot A_\parallel e^{-2im_B v \cdot x} H_v^{(+)}\}
\] (2.35)

From the above Lagrangian, we can extract the Feynman rules (Appendix A) the relations between the constants in our convention and those given in \[\text{[15]}\] are
\[
f_\pi = 2F
\] (2.36)
\[
D = -\frac{1}{2} g_A
\] (2.37)

B. The Next to Leading Order Lagrangian (\(\mathcal{L}_{\pi N}^{(2)}\))

In this section we will write down the effective (\(\mathcal{L}_{\pi N}^{(2)}\)) Lagrangian. The terms which stem from the \(1/m_B\) expansion of the relativistic \(\pi N\) Lagrangian are
\[
\mathcal{L}_{v \pi N1/m_B}^{(\pm \mp)} = -\frac{1}{2m_B} \text{Tr}[\bar{H}_v \not{D}_\perp \not{D}_\perp H_v]
\]
\[
\mathcal{L}_{v \pi N1/m_B}^{(\pm \mp)} = \frac{1}{2m_B} \text{Tr}[\bar{H}_v (-i \not{D}_\perp) e^{-2i \beta m_B v \cdot x} (i \not{D}_\perp) H_v]
\]
\[
\mathcal{L}_{A,v,1/m_B}^{(\pm \mp)} = \frac{2D}{2m_B} \text{Tr}\{\bar{H}_v \gamma_5 i \not{D}_\perp H_v + \bar{H}_v (-i \not{D}_\perp) A_\parallel \gamma_5 H_v\}
\]
\[
\mathcal{L}_{A,v,1/m_B}^{(\pm \mp)} = \frac{2D}{2m_B} \text{Tr}\{\bar{H}_v \gamma_5 e^{2i \beta m_B v \cdot x} i \not{D}_\perp H_v + \bar{H}_v (-i \not{D}_\perp) A_\parallel \gamma_5 e^{-2i \beta m_B v \cdot x} H_v\}
\] (2.38)

The other terms involving the low energy constants (LECs) come from the most general relativistic Lagrangian at \(O(p^2)\) after translation into the heavy mass formalism. There are
constants which appear in the field $\chi$, and is related to explicit chiral symmetry breaking.

These are,

$$\mathcal{L}^{(2)}_{A\pi N} = c'_1 \bar{H} H \text{Tr}[\chi^+] + c'_2 \text{Tr}[\bar{H} \chi^+ H] + c'_3 \bar{H} \sigma^{\mu\nu} H \text{Tr}[f^+_{\mu\nu}] + c'_4 \text{Tr}[\bar{H} \sigma^{\mu\nu} f^+_{\mu\nu} H]$$

$$+ c'_5 \text{Tr}[\bar{H} (\gamma_\mu, \gamma_\nu) A_\mu A_\nu H] + c'_6 \text{Tr}[\bar{H} (\gamma_\mu, \gamma_\nu) A_\mu A_\nu H]$$

(2.39)

where $c'_i$ are the LECs, $\chi^+ = \xi^+ \chi_1^+ + \xi\chi^1$, $\gamma = 2BM(M)$ is quark mass matrix and $f^{+}_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu)(\xi Q\xi^T + \xi^T Q\xi)$ where $\gamma^{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu)$ is the canonical photon field strength tensor.

Integrating out the small component of the field, we get the $O(p^2)$ Lagrangian

$$\mathcal{L}^{(2)}_{A\pi N} = c'_1 \bar{H} v \hat{H} v \text{Tr}[\chi^+] + c'_2 \text{Tr}[\bar{H} v \chi^+ \hat{H} v] + c'_3 \bar{H} v (\partial A - \partial_\mu A_\mu) \hat{H} v \text{Tr}[e(\xi Q\xi^T + \xi^T Q\xi)]$$

$$+ c'_4 \text{Tr}[\bar{H} v (\partial A - \partial_\mu A_\mu) e(\xi Q\xi^T + \xi^T Q\xi) \hat{H} v]$$

$$+ c'_5 \text{Tr}[\bar{H} v A \cdot A \hat{H} v] + c'_6 \text{Tr}[\bar{H} v A \cdot A \hat{H} v]$$

(2.40)

again expressing the Lagrangian in terms of baryon and anti-baryon fields we have,

$$\mathcal{L}^{(2)(\pm, \pm)}_{A\pi N} = c'_1 \bar{H} v \hat{H} v \text{Tr}[\chi^+] + c'_2 \text{Tr}[\bar{H} v \chi^+ \hat{H} v] + c'_3 \bar{H} v (\partial A - \partial_\mu A_\mu) \hat{H} v \text{Tr}[e(\xi Q\xi^T + \xi^T Q\xi)]$$

$$+ c'_4 \text{Tr}[\bar{H} v (\partial A - \partial_\mu A_\mu) e(\xi Q\xi^T + \xi^T Q\xi) \hat{H} v]$$

$$+ c'_5 \text{Tr}[\bar{H} v A \cdot A \hat{H} v] + c'_6 \text{Tr}[\bar{H} v A \cdot A \hat{H} v]$$

(2.41)

$$\mathcal{L}^{(2)(\pm, \pm)}_{A\pi N} = c'_1 \bar{H} v \hat{H} v \text{Tr}[\chi^+] + c'_2 \text{Tr}[\bar{H} v \chi^+ \hat{H} v] + c'_3 \bar{H} v (\partial A - \partial_\mu A_\mu) \hat{H} v \text{Tr}[e(\xi Q\xi^T + \xi^T Q\xi)]$$

$$+ c'_4 \text{Tr}[\bar{H} v (\partial A - \partial_\mu A_\mu) e(\xi Q\xi^T + \xi^T Q\xi) \hat{H} v]$$

$$+ c'_5 \text{Tr}[\bar{H} v A \cdot A \hat{H} v] + c'_6 \text{Tr}[\bar{H} v A \cdot A \hat{H} v]$$

(2.42)

Following the definition [13], the Lagrangian can be rewritten as

$$\mathcal{L}^{(2)(\pm, \pm)}_{A\pi N} = c'_1 \bar{H} v H v \text{Tr}[\chi^+] + c'_2 \text{Tr}[\bar{H} v \chi^+ H v] + c'_3 \bar{H} v (\partial A - \partial_\mu A_\mu) H v \text{Tr}[e(\xi Q\xi^T + \xi^T Q\xi)]$$

$$+ c'_4 \text{Tr}[\bar{H} v (\partial A - \partial_\mu A_\mu) e(\xi Q\xi^T + \xi^T Q\xi) H v]$$

$$+ c'_5 \text{Tr}[\bar{H} v A \cdot A H v] + c'_6 \text{Tr}[\bar{H} v A \cdot A H v]$$

(2.43)

$$\mathcal{L}^{(2)(\pm, \pm)}_{A\pi N} = c'_1 \bar{H} v (\partial A) e^{-2i\gamma_{2mB^2}x} H v \text{Tr}[e(\xi Q\xi^T + \xi^T Q\xi)]$$

$$+ c'_4 \text{Tr}[\bar{H} v (\partial A) e(\xi Q\xi^T + \xi^T Q\xi) e^{-2i\gamma_{2mB^2}x} H v]$$

$$+ c'_5 \text{Tr}[\bar{H} v (A) e^{-2i\gamma_{2mB^2}x} H v]$$

(2.44)

The $O(p^2)$ Lagrangian without baryon-antibaryon mixing is

$$\mathcal{L}^{(\pm)}_{v\pi N 1/mB} = \frac{1}{2m_B} \text{Tr}[\partial_\mu \partial_\mu H v]$$
\[ \mathcal{L}_{A,v,1/m_B}^{(\pm \pm)} = \frac{2D}{2m_B} \text{Tr} \left\{ \vec{H}_v \vec{A}_\perp \gamma_5 i \vec{D} \cdot H_v + \vec{H}_v(-i \vec{D} \cdot \gamma_5) \right\} \]

\[ \mathcal{L}_{A,v,1/m_B}^{(2)(\pm, \mp)} = c_1' \vec{H}_v H_v \text{Tr}[\chi^+] + c_2' \text{Tr}[\vec{H}_v \chi^+ H_v] + c_3' \vec{H}_v(\vec{\partial}_\perp \vec{A}_\perp - \partial_\mu \vec{A}_\perp) H_v \text{Tr}[e(\xi Q \xi^\dagger + \xi^\dagger Q \xi)] + c_4' \text{Tr}[\vec{H}_v(\vec{\partial}_\perp \vec{A}_\perp - \partial_\mu \vec{A}_\perp) e(\xi Q \xi^\dagger + \xi^\dagger Q \xi) H_v]
\]

\[ + c_5' \text{Tr}[\vec{H}_v \cdot \vec{A} H_v] + c_6' \text{Tr}[\vec{H}_v(\vec{A} \cdot \vec{A}_\perp) H_v] \] (2.45)

The O\((p^2)\) Lagrangian with baryon-antibaryon mixing is

\[ \mathcal{L}_{v \pi N1/m_B}^{(\pm \pm)} = \frac{1}{2m_B} \text{Tr}[\vec{H}_v(-i \vec{D} \cdot \gamma_5) e^{-2i\kappa_m B v \cdot x}(i \gamma_v \cdot D) H_v] \]

\[ \mathcal{L}_{A,v,1/m_B}^{(\pm \pm)} = \frac{2D}{2m_B} \text{Tr} \left\{ \vec{H}_v \vec{A}_\perp \gamma_5 e^{2i\kappa_m B v \cdot x} i \vec{D} \cdot H_v + \vec{H}_v(-i \vec{D} \cdot \gamma_5) \right\} \]

\[ \mathcal{L}_{A,v,1/m_B}^{(2)(\pm, \mp)} = c_3' \vec{H}_v(\vec{\partial}_\parallel \vec{A}_\perp) e^{-2i\kappa_m B v \cdot x} H_v \text{Tr}[e(\xi Q \xi^\dagger + \xi^\dagger Q \xi)] + c_4' \text{Tr}[\vec{H}_v(\vec{\partial}_\parallel \vec{A}_\perp) e(\xi Q \xi^\dagger + \xi^\dagger Q \xi) e^{-2i\kappa_m B v \cdot x} H_v] \] (2.46)

In the above equations the Dirac matrices can be expressed as a combination of the \(S^\mu_v\), four velocity \(v_\mu\) and \(\gamma_5\). It is to be noted that the relations between the constants in our convention and that of [15] are

\[ c_3' = \frac{1}{2} \left[ \frac{i\kappa_s}{4m_B} - \frac{i\kappa_v}{4m_B} \right] \]

\[ c_4' = \frac{i\kappa_v}{4m_B} \] (2.47) (2.48)

where \(\kappa_s\) and \(\kappa_v\) are the scalar and vector anomalous magnetic moments.

### III. SPIN POLARISABILITY OF THE NUCLEON IN THE FRAME WORK OF HBEFT

In this section, we use the Lagrangian constructed in previous section to calculate the polarisabilities of the nucleon.

To calculate the spin-dependent scattering amplitude, we work in the gauge \(A_0 = 0\) \((v \cdot A = 0)\), or in the language of HBCHPT, \(\epsilon \cdot v = 0\), where \(v^\mu = (1, 0, 0, 0)\) is the unit vector which defines the nucleon rest frame. Here, there is no lowest-order coupling of a photon to a nucleon; the coupling comes in only at second order. The Feynman vertex consists of two pieces, one proportional to the charge current and one to the magnetic moment.

\[ \frac{ie}{2m_B} \{Q \epsilon \cdot (p_1 + p_2) + 2(Q + \kappa)[S \cdot \epsilon, S \cdot Q]\} \] (3.1)
At leading order\[7\], the diagrams in Fig. 1 should be considered. At the leading order LET is satisfied by the combination of the Born and seagull diagrams. The calculated polarisabilities agrees with those of ref.\[7\].

FIG. 1: Diagrams which contribute to spin-dependent Compton Scattering in the $\epsilon \cdot v = 0$ gauge at LO.

At NLO, the diagrams which contribute to spin dependent forward Compton scattering are given in Fig.2 and Fig.3 respectively. Fig.2 are the diagrams which arise from the $O(p^2)$Lagrangian without baryon anti baryon mixing (eq.2.45).The amplitude for the diagrams 2a-2h are listed in appendix B. At this order there can be no seagulls \[10\]. In Fig. 2

FIG. 2: Diagrams which contribute to spin-dependent Compton Scattering in the $\epsilon \cdot v = 0$ gauge at NLO. These are the diagrams from the Lagrangian without baryon anti baryon mixing (crossed diagrams are not shown).

the solid dots are vertices from $\mathcal{L}_{\pi N}^{(2)}$. The amplitudes for the loop diagrams are listed in Appendix B.

Without considering the resonance contributions, $A_i(\omega, \theta)$ can be divided into the following structure

$$A_i(\omega, \theta) = A_i(\omega, \theta)^{\text{Born}} + A_i(\omega, \theta)^{\text{HBChPT}} + A_i(\omega, \theta)^{\text{antibaryon}} \quad (3.2)$$
The first two terms in the above expression have been worked out in the framework of HBCHPT by number of authors. Our results of $\gamma_i$ naturally agrees with those of ref. [10][11]:

$$\gamma_1^{HBChPT} = \frac{\alpha_{em} g_A^2}{24\pi^2 F^2 m_\pi^2} \left[ 1 - \frac{\pi m_\pi}{8m_B} (8 + 5 \tau_3) \right]$$

$$\gamma_2^{HBChPT} = \frac{\alpha_{em} g_A^2}{48\pi^2 F^2 m_\pi^2} \left[ 1 - \frac{\pi m_\pi}{4m_B} (8 + \kappa_v + 3(1 + \kappa_v) \tau_3) \right]$$

$$\gamma_3^{HBChPT} = \frac{\alpha_{em} g_A^2}{96\pi^2 F^2 m_\pi^2} \left[ 1 - \frac{\pi m_\pi}{4m_B} (6 + \tau_3) \right]$$

$$\gamma_4^{HBChPT} = \frac{\alpha_{em} g_A^2}{96\pi^2 F^2 m_\pi^2} \left[ -1 + \frac{\pi m_\pi}{4m_B} (15 + 4\kappa_v + 4(1 + \kappa_v) \tau_3) \right]$$

$$\gamma_5^{HBChPT} = \frac{\alpha_{em} g_A^2}{24\pi^2 F^2 m_\pi^2} \left[ -\frac{\pi m_\pi}{8m_B} (7 + 3\kappa_v + (1 + \kappa_v) \tau_3) \right]$$

$$\gamma_0^{HBChPT} = \frac{\alpha_{em} g_A^2}{24\pi^2 F^2 m_\pi^2} \left[ 1 - \frac{\pi m_\pi}{8m_B} (15 + 3\kappa_v + (6 + \kappa_v) \tau_3) \right]$$

$$\gamma_1^{HBChPT} = \frac{\alpha_{em} g_A^2}{24\pi^2 F^2 m_\pi^2} \left[ 1 - \frac{\pi m_\pi}{8m_B} (1 - 3\kappa_v + (4 - \kappa_v) \tau_3) \right]$$

where $F$ is the pion decay constant. To get the above relations, the relations $\gamma_0 = \gamma_1 - (\gamma_2 + 2\gamma_4) \cos \theta$ and $\gamma_5 + \gamma_2 + 2\gamma_4 = 0$ were used.

At NLO, the diagrams which contribute to the scattering amplitude from the baryon and anti-baryon vertex are given in Fig.3. These are the diagrams which arise from the $O(p^2)$ Lagrangian with baryon-anti baryon mixing (eq.2.46). They have no analog in HBCHPT.

Fig3. The diagrams arising from baryon antibaryon vertex which contribute to spin dependent forward Compton scattering at NLO. The solid dots are vertices from $\mathcal{L}_{\pi N}^{(2)}$ (crossed diagrams are not shown)

In Fig.3 $\pm$ denotes baryon and anti-baryon vertex respectively.

The contribution to forward spin polarisability from the diagrams of Fig.3 is,

$$T^{O(p^3),\text{anti–baryon}} = i \frac{\omega^3 e^2 D^2}{3\pi m_B m_\pi f_\pi^2} \{2\kappa_s + 1\} \vec{\sigma} \cdot (\vec{e}' \times \vec{e})(t_3) + O(\omega^5)$$

(3.4)

+spin independent terms.
At this stage we would wish to make some comments. The LET is satisfied in our theory as the diagrams of Fig.2 which are same as that of HBCHPT satisfy the LET. There is no contribution from the diagrams of Fig.3 to the lowest order in $\omega$ and hence the LET is intact which is a non-trivial check to our theory. But, the diagrams of Fig.3 contribute to forward spin polarisability (see Eq.(3.4)). It should be noted that higher order terms in $\omega$ are model dependent quantities. Also, the baryon anti-baryon vertex gives contribution only to $A_3(\omega, \theta)$ and further, the relation $\gamma_3 + \gamma_2 + 2\gamma_4 = 0$ is still exact, which is another good check to consistency of our theory. The explicit contributions from the individual diagrams of Fig.3 are given in appendix B.

We have

$$A_3^{O(p^4), \text{anti-baryon}}(\omega, \theta) = \frac{\omega^3 e^2 D^2}{3\pi m_B m_\pi f_\pi^2} \{2\kappa_s + 1\}(t_3) + O(\omega^5)$$

and

$$= \frac{\omega^3 e^2 g^2_A}{48\pi m_B m_\pi F^2} \{2\kappa_s + 1\}(t_3) + O(\omega^5)$$

we get

$$\gamma_1^{O(p^4), \text{anti-baryon}}(\omega, \theta) = \alpha_{em} g^2_A F^2 \{2\kappa_s + 1\}(t_3)$$

Using eqs. (3.3) to (3.6), we get,

$$\gamma_1 = \frac{\alpha_{em} g^2_A}{24\pi^2 F^2 m_\pi^2} \{1 - \frac{\pi m_\pi}{8m_B} [8 + (1 - 8\kappa_s)\tau_3]\}$$

$$\gamma_0 = \frac{\alpha_{em} g^2_A}{24\pi^2 F^2 m_\pi^2} \{1 - \frac{\pi m_\pi}{8m_B} [15 + 3\kappa_v + (2 + 7\kappa_s)\tau_3]\}$$

$$\gamma_\pi = \frac{\alpha_{em} g^2_A}{24\pi^2 F^2 m_\pi^2} \{1 - \frac{\pi m_\pi}{8m_B} [1 - 3\kappa_v - 9\kappa_s\tau_3]\}$$

(3.7)

Using the physical values of the parameter $\kappa_v = 3.71$, $\kappa_s = -0.12$, we get

$$\gamma_1 = 4.5 - (2.0 + 0.17\tau_3)$$

$$\gamma_0 = 4.5 - (6.6 + 0.29\tau_3)$$

$$\gamma_\pi = 4.5 - (-2.6 + 0.27\tau_3)$$

(3.8)

Below, we present our numerical results in Table.1.

|       | $\gamma^p_0$ | $\gamma^p_\pi$ | $\gamma^n_\pi$ |
|-------|--------------|----------------|---------------|
| PRESENT | -2.4         | -35.8          | -34.3         |
| HBCHPT[10][11] | -3.9          | -36.6          | -34.4         |
| Expt   | -0.86 ± 0.13[29] | -38.7 ± 1.8[29] | -27.1 ± 3.6[30] |
Table 1. Numerical results of the real Compton scattering in comparison with the results of HBCHPT and the extracted results from the expt (in unit $10^{-4} fm^4$).

From the above table, the following conclusions can be drawn: The amplitude of the forward spin polarisability $\gamma_0$ of proton is smaller than the corresponding HBCHPT result, and closer to the experimental data. For the backward scattering $\gamma_\pi$, both calculations lead to similar results which are consistent with the experimental data within the errors\textsuperscript{30}. Since spin polarisabilities of neutron are sensitive to the resonance\textsuperscript{15, 32} and there is no direct experimental data yet, we shall not consider them in this note. To further compare our theoretical framework with HBCHPT, it is required to extend the calculations to the virtual Compton scattering which is under way\textsuperscript{33}.

IV. SUMMARY AND CONCLUSIONS

In this paper, we have constructed a complete HBEFT ($\mathcal{L}_{\pi N}^{(1)}$ and $\mathcal{L}_{\pi N}^{(2)}$) by considering the anti-baryon contributions through the idea of HQEFT with photon as an external degree of freedom. The pure baryon sector of the derived Lagrangian is same as the usual HBCHPT Lagrangian. In addition the Lagrangian of HBEFT also consists of the antibaryon contribution explicitly.

The calculated spin polarisability at order $O(p^3)$, agree with the corresponding results of the HBCHPT since from the chiral power counting, the vertex of $O(p^3)$ loop diagrams stems from the insertions of $\mathcal{L}_{\pi N}^{(1)}$ Lagrangian. This agrees with the conclusion given in\textsuperscript{18, 19} that the HQEFT is same as the usual HQET at the leading order. But, at the NLO the loop diagrams involve one insertion from $\mathcal{L}_{\pi N}^{(2)}$. The calculation shows that in the framework of HBEFT the result of $\gamma_1$, is different from the corresponding calculation of the HBCHPT. The anti-baryon terms reduces the magnitude of the spin polarisability of proton at NLO in comparison to the results of the HBCHPT.

At NLO, the leading order terms in the $\omega$ expansion satisfy the low energy theorems and the relation $\gamma_5 + \gamma_2 + 2\gamma_4 = 0$. Both of the above results give very good checks to our theory. Hence, the HBEFT, obtained by using the idea of the HQEFT is more complete than that of HBCHPT. If we ignore the diagrams of (Fig.3) (arising from baryon anti-baryon vertex), our results agree with the results of HBCHPT\textsuperscript{10, 11}. At NLO, the diagrams of Fig.3 do not contribute at the leading order in $\omega$ and hence low energy theorem are satisfied by
the leading order terms in $\omega$ of Fig.2. But, the diagrams of Fig.3 do contribute to spin polarisabilities.

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APPENDIX A: FEYNMAN RULES

The Lagrangian in the Sec.II denotes the baryon and anti-baryon explicitly. Below we write down the Feynman rules in terms of the baryon fields. To get the baryon or anti-baryon contributions, one just need to multiply the velocity projection operator $(1 + \hat{\beta})/2$ or $(1 - \hat{\beta})/2$ on both sides. For completeness we list the Feynman rules needed to calculate the tree and loop diagrams.

We use the following notations:

- $p$: Momentum of a nucleon in heavy mass formulation.
- $q$: Momentum of an external pion.
- $k$: Momentum of an external photon.
- $\epsilon$: Photon polarisation vector.
- $\hat{x}$: Co-ordinate operator.

Pion isospin indices are $a$ and $b$. $v_\mu$ is the nucleon four-velocity and $s_\mu$ is the covariant spin operator. Parameters $Q, D, f_\pi ...$ are meant to be taken in the chiral limit.

(1). The vertices from $\mathcal{L}^{(1)}_{\pi N}$ Lagrangian eqs. (2.33) – (2.35).
baryon propagator: $\frac{i\not{p}}{v_p}\delta_{ij}$

1 pion (q out): $\frac{-4i\not{q}}{f_\pi} [\not{S}_\pi \cdot q(t_a)_{ij}]$

1 pion, 1 photon: $\frac{4ieD\cdot\not{k}}{f_\pi} \not{S}_\pi \cdot \epsilon^{ab}(t_b)_{ij}$

2. The vertices from the $L^{(2)}_{\pi N}$ Lagrangian are.

Baryon propagator:

$$\frac{-i}{2m_B} [p^2 - (v \cdot p)^2] \delta_{ij} - 4iB[c'_1(m_u + m_d)\delta_{ij} + c'_2M_{ij}]$$

$$+ \frac{-i}{m_B} [v \cdot pS_v \cdot p\gamma_5\delta_{ij}] [\frac{1}{2}e^{2im_Bv\cdot\hat{x}} + \frac{1}{2}e^{-2im_Bv\cdot\hat{x}}]$$

1 Photon (k out):

$$\frac{-i}{2m_B} [(p + p') \cdot \epsilon - 2[S_v \cdot k, S_v \cdot \epsilon)]Q_{ij}$$

$$+ 4e[S_v \cdot k, S_v \cdot \epsilon] (c'_3 + c'_4Q)_{ij}$$

$$+ ieS_v \cdot \gamma_5 \{4i\epsilon \cdot k[c'_3\delta_{ij} + c'_4Q]_{ij} - \frac{1}{m_B}v \cdot pQ_{ij}\}$$

$$\times [\frac{1}{2}e^{2im_Bv\cdot\hat{x}} + \frac{1}{2}e^{-2im_Bv\cdot\hat{x}}]$$

It should be noticed that coordinate operator $\hat{x}$ only acts on the baryon and antibaryon moments.

APPENDIX B: FULL AMPLITUDE

1. The full amplitude for the diagrams 2a-2h (including the cross ones). These are the same diagrams which arise at NLO in HBCHPT. (10)

The notation $t_i$ is used for the tensor structures which multiply the amplitudes $A_i$ (10).

$$T_a = \frac{g^2e^2}{4m_Bf^2_\pi} \left[ \left( m_\pi^2 - \omega^2(1 + \cos\theta) \right) \frac{\partial J_0(\omega, m_\pi^2)}{\partial\omega} - 2\omega J_0(\omega, m_\pi^2) \right] t_3 - (\omega \to -\omega)$$

$$T_b = -\frac{g^2e^2}{2m_Bf^2_\pi} \left[ \frac{2m_\pi^2}{\omega} \left( J'_0(\omega, m_\pi^2) - J'_0(0, m_\pi^2) \right) t_3 + (1 + \cos\theta) \frac{\partial J_2(\omega, m_\pi^2)}{\partial\omega} \right] t_3$$

$$-\omega J'_2(\omega, m_\pi^2)t_5 + \omega(t_5 - 2(1 - \cos\theta)t_3) \int_0^1 dx J'_2(x\omega, m_\pi^2) \right] - (\omega \to -\omega)$$

$$T_c = -\frac{g^2e^2}{2m_Bf^2_\pi} \tau_3\omega J_0(\omega, m_\pi^2)t_3 - (\omega \to -\omega)$$

$$T_d = \frac{g^2e^2}{m_Bf^2_\pi} \tau_3\omega \int_0^1 dx J'_2(x\omega, m_\pi^2) t_3 - (\omega \to -\omega)$$
The diagrams 2i-2k does not contribute to spin dependent Compton scattering and hence

The J contributions to the Amplitudes from Fig.3 (including the crossed ones)

\[ T_e = \frac{g^2 e^2}{2m_B f^2_\pi} (1 - \tau_3) \frac{1}{\omega} \left( J_2(\omega, m^2_\pi) - J_2(0, m^2_\pi) \right) t_3 - (\omega \to -\omega) \]

\[ T_f = \frac{g^2 e^2}{4m_B f^2_\pi} \omega (2(\mu_\nu - \mu_s \tau_3)(t_3 \cos \theta - t_4) + (1 - \tau_3) t_6) \]
\[ \times \int_0^1 dx (1 - 2x) J'_2(x\omega, m^2_\pi) - (\omega \to -\omega) \]

\[ T_g = -\frac{g^2 e^2}{4m_B f^2_\pi} \omega (2(\mu_\nu + \mu_s \tau_3)(t_3 \cos \theta + t_4 - t_5) + (1 + \tau_3) t_6) \]
\[ \times \int_0^1 dx J'_2(x\omega, m^2_\pi) - (\omega \to -\omega) \]

\[ T_h = -\frac{g^2 e^2}{m_B f^2_\pi} \omega^2 \int_0^1 dy \int_0^{1-x} dx \left[ (7x - 1)(t_6 - t_5) + 7(1 - x - y) t_4 \right] \frac{\partial J''_2(\omega, m^2_\pi - xy\omega)}{\partial \omega} \]
\[ + (2V(x, y, \theta)(x t_6 - x t_5 + (1 - x - y) t_4) \right] \]
\[ \left. - y(1 - x - y) \omega^4 V(x, y, \theta) t_7 \frac{\partial J''_2(\omega, m^2_\pi - xy\omega)}{\partial \omega} \right] - (\omega \to -\omega) \] (B1)

where \( \omega = (1 - x - y)\omega \),

\[ J_0(\omega, m^2_\pi) = \frac{1}{d + 1} \left( (m^2_\pi - \omega^2) J_2(\omega, m^2_\pi) - \omega m^2_\pi \Delta_\pi \right). \] (B2)

The diagrams 2i-2k does not contribute to spin dependent Compton scattering and hence are not listed.

The \( J_0(\omega, m^2_\pi) \), \( J_2(\omega, m^2_\pi) \) and \( \Delta_\pi \) are defined in [15], prime denotes differentiation with respect to \( m^2_\pi \), and

\[ V(x, y, \theta) = (2xy - x +1) \cos \theta - x(1 - x) - y(1 - y). \] (B3)

2. Contributions to the Amplitudes from Fig.3 (including the crossed ones)

\[ T_{3a} = -2i\omega C \int_0^1 dx [S_v \cdot \epsilon', S_v \cdot \epsilon] \{ 8i [c'_3(t_3)_{ti} + c'_4 Q_{ti}] - \frac{1}{m_B} Q_{ti} \} \]
\[ \times [J'_2(x\omega, m^2_\pi) + J'_2(-x\omega, m^2_\pi)] \]

\[ T_{3b} = \frac{i\omega C}{2} [S_v \cdot \epsilon', S_v \cdot \epsilon] \{ 8[2ic'_3(t_3)_{ti} - ic'_4(1 - t_3)_{ti}] + \frac{1}{m_B}(1 - t_3)_{ti} \} \]
\[ \times [J_0(\omega, m^2_\pi) + J_0(-\omega, m^2_\pi)] \]

\[ T_{3c} = -i\omega C \int_0^1 dx [S_v \cdot \epsilon', S_v \cdot \epsilon] \{ 8[2ic'_3(t_3)_{ti} - ic'_4(1 - t_3)_{ti}] + \frac{1}{m_B}(1 - t_3)_{ti} \} \]

\[ \times [J_0(\omega, m^2_\pi) + J_0(-\omega, m^2_\pi)] \]
\begin{align*}
&T_3 d = i \omega C [S_v \cdot \epsilon', S_v \cdot \epsilon] \{ 8i [c_3'(t_3)_{ii} + c_4'Q_{ii}] - \frac{1}{m_B} Q_{ii} \} \{ J_0(\omega, m^2_\pi) + J_0(-\omega, m^2_\pi) \} \tag{B4} \\
\text{where} \\
&\quad C = \frac{8ie^2D^2}{f^2_\pi} \tag{B5}
\end{align*}

The other diagrams (3e-3h) do not contribute to the spin dependent Compton scattering.

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