On the Horava-Lifshitz-like extensions of supersymmetric theories

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Abstract

Within the superfield approach, we formulate two different extensions of the Wess-Zumino model and super-QED with Horava-Lifshitz-like additive terms, discuss their quantum properties and calculate lower contributions to the effective action. In the case of the gauge theory, the one-loop effective potential turns out to be gauge independent.

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I. INTRODUCTION

The possibility of Lorentz symmetry breaking is being intensively discussed presently. Following many studies, see e.g. [1], the Lorentz symmetry breaking can be treated as a natural ingredient of extensions of the standard model. At the same time, supersymmetry is known to be a crucial symmetry of the quantum world due to many remarkable properties of the supersymmetric field theories. In fact, it provides an improvement of the renormalization behaviour in supersymmetric theories due to the well-known mutual cancellation of bosonic and fermionic contributions. Furthermore, the superfield formalism allows a very compact treatment of supersymmetric field theory models (for a general review on supersymmetry see e.g. [2, 3]). Therefore, a natural problem consists in formulating Lorentz-breaking supersymmetric field theories.

There are essentially two ways to do it up to now. The first is the deformation of the supersymmetry algebra through a Lorentz symmetry breaking implemented by a constant tensor into the anticommutation relations of the supersymmetry generators (known as the Kostelecky-Berger approach [4]). This approach has been applied to the study of supersymmetric field theories in [5], where such models have been studied at the tree level, and in [6], where some studies of the perturbative aspects of these theories have been carried out. This approach is rather universal and can be applied to a wide class of theories, allowing to obtain their CPT-even Lorentz-breaking extensions. The second way to construct Lorentz-breaking theories consists in the introduction of an extra superfield, with some of its components depending on a constant vector (or tensor), as it has been done in [7]. We note that in this second method a supersymmetric extension of the Carroll-Field-Jackiw term has been successfully constructed.

None of these methods allow, up to now, to study the supersymmetric extension of the Horava-Lifshitz-like (HL-like) theories characterized by a strong asymmetry between time and space coordinates [8]. The reason for that is the presence of different orders in time and space derivatives, in the classical HL-like theories; such a difference apparently cannot be introduced within the two already mentioned approaches. As it was already noted by some of us in an earlier paper [6], a natural way of introducing a Lorentz breaking in a superfield model consists in inserting "by hand" extra spatial derivatives in the classical action. Some studies in this direction were performed in [9]. Following this line, in this paper, we add to the Wess-Zumino and gauge theory actions extra terms involving higher spatial derivatives of the superfields.

The structure of the paper is as follows. In the section 2, we consider the HL-like extensions for the Wess-Zumino model, with higher derivatives added either to the general or to the chiral
Lagrangians and calculate the one-loop low-energy effective actions. In the section 3, we formulate the HL-like extension of the supersymmetric QED, and in the summary we discuss our results.

II. THE HL-LIKE EXTENSIONS OF THE WESS-ZUMINO MODEL

Let us treat the simplest supersymmetric theory, that is, the Wess-Zumino model. Its action in superfields is

$$S = \int d^8 z \bar{\Phi} \Phi + \int d^6 z (\frac{m}{2} \Phi^2 + \frac{\lambda}{3!} \Phi^3) + h.c. \quad (1)$$

As usual (see e.g. [2]), the supersymmetry transformations are $\delta \Phi = i(\epsilon^a Q_\alpha + \bar{\epsilon}^{\dot{a}} \bar{Q}_{\dot{a}}) \Phi$, with $Q_\alpha$, $\bar{Q}_{\dot{a}}$ being the supersymmetry generators satisfying the relations

$$\{Q_\alpha, \bar{Q}_{\dot{a}}\} = 2i \sigma^m_{\alpha \dot{a}} \partial_m; \quad \{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{a}}, \bar{Q}_{\dot{b}}\} = 0; \quad [Q_\alpha, \partial_m] = 0. \quad (2)$$

In the most used symmetric representation they look like

$$\bar{Q}_{\dot{a}} = i(\partial_{\theta^{\dot{a}}} - i \theta^a (\sigma^m)_{\alpha \dot{a}} \partial_m), \quad Q_\alpha = i(\partial_{\bar{\theta}^\alpha} + i \bar{\theta}^{\dot{a}} (\bar{\sigma}^m)_{\dot{a} \alpha} \partial_m). \quad (3)$$

Let us try to implement a Horava-Lifshitz (HL)-like extension of this theory.

A. HL-like extension of the general Lagrangian

Since the term involving the integral over chiral (antichiral) subspace projected into components does not involve derivatives of any fields, let us, as a first attempt, modify only the general (Kählerian) Lagrangian. To do it, we add a new term with higher derivatives (but only spatial, to avoid the arising of ghost states). So, the new action compatible with the supersymmetry transformations is

$$S = \int d^8 z \Phi (1 + \rho \Delta^{z-1}) \bar{\Phi} + (\int d^6 z W(\Phi) + h.c.). \quad (4)$$

Here $\rho$ is some constant with a negative dimension, and $z \geq 2$ is a critical exponent. In the most studied case, $W(\Phi) = \frac{m}{2} \Phi^2 + \frac{\lambda}{3!} \Phi^3$.

To get some insights, let us write this action in components defined through the projections

$$\phi(x) = \Phi(z)|;$$
$$\psi_\alpha(x) = D_\alpha \Phi(z)|;$$
$$F(x) = \frac{1}{4} D^2 \Phi(z)|. \quad (5)$$
The superpotential term does not differ from the usual, and we have the following component action:

\[ S = \int d^4x \left[ \bar{\phi} \Box (1 + \rho \Delta z^{-1})\phi - i\bar{\psi}^\alpha \sigma^m_{\dot{\alpha} \alpha} \partial_m (1 + \rho \Delta z^{-1}) \psi^\alpha + \bar{F} (1 + \rho \Delta z^{-1}) F - \frac{m}{2} (\phi F + \frac{1}{2} \psi^\alpha \psi_\alpha + h.c.) \right]. \]  

(6)

Following the general methodology of study of HL theories (and using the arguments of the renormalization group flow [8]), we can consider the “ultraviolet” limit where we take into account only highest order spatial derivatives. In this limit the action above goes into

\[ S_{UV} = \int d^4x \left[ \bar{\phi} (\partial_0^2 - \rho \Delta z) \phi - i\bar{\psi}^\alpha (\sigma_0^{\dot{\alpha}} \partial_0 + \rho \sigma^i_{\dot{\alpha} \alpha} \partial_i \Delta z^{-1}) \psi^\alpha + \rho \bar{F} \Delta z^{-1} F - \frac{m}{2} (\phi F + \frac{1}{2} \psi^\alpha \psi_\alpha + h.c.) \right]. \]  

(7)

Here, first, we omitted the “crossed” terms involving both time and space derivatives (like \( \bar{\phi} \partial_0^2 \Delta z^{-1} \phi \)) whose existence does not participate in the usual definition of the HL-like action. We suggest that we can rule out such terms under some conditions. Second, we see that for the scalar field \( \phi \) the critical exponent is equal to \( z \) as we wanted, but for the spinor one it is \( 2z - 1 \) which differs from \( z \) unless \( z = 1 \), and for the auxiliary one we have a completely static dynamics (that is, the critical exponent in some sense tends to infinity). As a result, we conclude that the requirement of a unique critical exponent (different from 1) in this case is inconsistent with supersymmetry, and our supersymmetric theory is essentially different from the "simple” HL-like theories.

The superfield description of the theory (4) is rather interesting. The propagators are

\[ < \Phi(z_1) \bar{\Phi}(z_2) > = \frac{1 + \rho \Delta z^{-1}}{\Box (1 + \rho \Delta z^{-1})^2 - m^2} \delta^8(z_1 - z_2); \]

\[ < \Phi(z_1) \Phi(z_2) > = < \bar{\Phi}(z_1) \bar{\Phi}(z_2) >^* = -\frac{m}{\Box (1 + \rho \Delta z^{-1})^2 - m^2} (\frac{D^2}{4 \Box}) \delta^8(z_1 - z_2), \]  

(8)

with \( D^2, \bar{D}^2 \) factors associated with the vertices by the same rules as in the usual Wess-Zumino model.

We can calculate the superficial degree of divergence and the effective potential for the theory (4). In this case the effective dimensions of time and space derivatives are the same, that is, 1, since there is no ways to have structures like \( \partial_0^2 - \Delta^2 \). The effective dimensions of spinor derivatives are \( 1/2 \) again. So, we have the dimensions of propagators: \(-2z\) for \( < \Phi \bar{\Phi} >\), and \(1 - 4z\) for \( < \Phi \Phi >\), \( < \bar{\Phi} \bar{\Phi} >\) which for \( z = 1 \) reproduces the usual case. One has a factor 2 for any vertex (due to \( D^2, \bar{D}^2 \) factors), but factor 1 if the vertex has an external line attached, totaling \( 2V - E \) with \( V \)
and $E$ the numbers of vertices and external lines, respectively. All loop integrations contribute $4L$, but this number is reduced by $2L$ due to the contraction of any loop into a point, by the rule $\delta_{12} D^2 D^2 \delta_{12} = 16 \delta_{12}$. Totally we have

$$\omega = 2 - E - 2(z - 1)P - (2z - 1)P_c,$$

where $P$ is the number of all propagators, and $P_c$ – of only $\langle \Phi \Phi \rangle$, $\langle \bar{\Phi} \bar{\Phi} \rangle$ ones. This $\omega$ reproduces the usual result for $z = 1$ where we have only a wave function renormalization (see e.g. [2]). If $z \geq 2$, the theory is finite.

The effective potential can be obtained through the following standard summation of supergraphs, cf. [13]

![Fig.1](image)

The external lines are the alternating $\Psi = -W'' = m + \lambda \Phi$ (accompanied by $-\frac{D^2}{4}$) and the corresponding $\bar{\Psi}$ (accompanied by $-\frac{D^2}{4}$). So, we have the modified Feynman rules:

$$\langle \Phi(z_1)\Phi(z_2) \rangle = \frac{1}{\Box(1 + \rho \Delta z^{-1})} \delta^8(z_1 - z_2);$$

$$\langle \Phi(z_1)\Phi(z_2) \rangle = \langle \bar{\Phi}(z_1)\bar{\Phi}(z_2) \rangle^* = 0.$$

So, similarly to [13], we have the following sum:

$$K^{(1)} = \int d^8z \sum_{n=1}^{\infty} \frac{1}{2n} \left[ \Psi \bar{\Psi} \frac{D^2 D^2}{16 \Box^2 (1 + \rho \Delta z^{-1})^2} \right]^n \delta^8(z - z')|_{z=z'}.$$

Here the $\frac{1}{2n}$ is a symmetry factor of the corresponding Feynman diagram. We can use the property of the projecting operator: $(\frac{D^2}{16 \Box})^n = \frac{D^2}{16 \Box}$ together with the rule for contracting of a loop into a point: $\frac{D^2}{16 \Box} \delta^4(\theta - \theta')|_{\theta = \theta'} = 1$. So, after the Fourier transform we have

$$K^{(1)} = -\int d^4\theta \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \sum_{n=1}^{\infty} \frac{1}{2n} \left[ -\Psi \bar{\Psi} \frac{1}{k^2 (1 + \rho (-k^2 z^{-1})^2)} \right]^n.$$

We use the sum:

$$\sum_{n=1}^{\infty} \frac{a^n}{n} = -\ln(1 - a),$$

so, after the Wick rotation and replacement $\rho \rightarrow \rho(-1)^{z^{-1}}$ we have

$$K^{(1)} = -\frac{i}{2} \int d^4\theta \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \ln \left[ 1 + \frac{\Psi \bar{\Psi}}{k^2 (1 + \rho (-k^2 z^{-1})^2)} \right].$$
It is finite for \( z > 1 \) and logarithmically diverges at \( z = 1 \) as it happens in the usual Wess-Zumino model. It is more simple to study \( \frac{dK^{(1)}}{d(\Psi \bar{\Psi})} \), to have a purely algebraic expression. Integrating then over \( k_0 \) (and the corresponding Feynman parameter) by usual procedures of the quantum field theory, we have

\[
\frac{dK^{(1)}}{d(\Psi \bar{\Psi})} = \frac{1}{2} \int \frac{d^3 \vec{k}}{(2\pi)^3 k^2} \frac{1}{(\vec{k}^2)^{1/2}} \left( \vec{k}^2 (1 + \rho \vec{k}^{2z-2}) + \Psi \bar{\Psi} + \sqrt{(1 + \rho \vec{k}^{2z-2})(\vec{k}^4 + \Psi \bar{\Psi} \vec{k}^2 + \rho \vec{k}^{2z+2})} \right)^{-1}. \tag{15}
\]

This expression is evidently finite. Unfortunately, it can be calculated only approximately, by disregarding the subleading powers of \( \vec{k} \) and using the approximation \( \sqrt{1 + x} \simeq 1 + x/2 \). So, we get

\[
\frac{dK^{(1)}}{d(\Psi \bar{\Psi})} \simeq \frac{1}{2} \int \frac{d^3 \vec{k}}{(2\pi)^3} \frac{1}{(\vec{k}^2)^{1/2}} \frac{1}{2 \rho \vec{k}^{2z} + \frac{3}{2} \Psi \bar{\Psi}}. \tag{16}
\]

This integral can be found explicitly:

\[
\frac{dK^{(1)}}{d(\Psi \bar{\Psi})} = \frac{1}{12\pi \Psi \bar{\Psi}} \csc(\pi/z) \left( \frac{4\rho}{3\Psi \bar{\Psi}} \right)^{-1/z}. \tag{17}
\]

Therefore, the one-loop Kählerian effective potential is

\[
K^{(1)} = \frac{1}{12\pi} \csc(\frac{\pi}{z}) \left( \frac{4\rho}{3\Psi \bar{\Psi}} \right)^{-1/z} (\Psi \bar{\Psi})^{1/z}. \tag{18}
\]

This is an approximate result for the effective potential. It is finite at \( z > 1 \) and diverges at \( z = 1 \) as it must be. Indeed, for \( z \to 1 \) we can write \( z = 1 + \epsilon \). Since \( \csc(\frac{\pi}{1+\epsilon}) \simeq \frac{1}{\pi \epsilon} \), so, we have

\[
K^{(1)} \simeq \frac{\Psi \bar{\Psi}}{16\pi^2 \rho \epsilon} - \frac{\Psi \bar{\Psi}}{16\pi^2 \rho} \ln \frac{3\Psi \bar{\Psi}}{4\rho \mu^2}, \tag{19}
\]

which agrees with the behavior of the usual WZ model [11]. The divergence can be cancelled by a simple wave function renormalization.

**B. HL-like extension of the chiral Lagrangian**

An alternative HL-like extension of the Wess-Zumino model would be obtained by adding the HL-like term to the chiral Lagrangian, so, our action will be

\[
S = \int d^8 z \Phi \bar{\Phi} + \left[ \int d^6 z \left( \frac{1}{2} \Phi(m + a(-\Delta)^2)\Phi + \frac{\lambda}{3!} \Phi^3 \right) + h.c. \right]. \tag{20}
\]
The corresponding superfield propagators are

\[ < \Phi(z_1)\Phi(z_2) > = < \bar{\Phi}(z_1)\bar{\Phi}(z_2) >^* = \frac{m + a(-\Delta)^z}{\Box - (m + a(-\Delta)^z)^2} \frac{D^2}{4} \delta^8(z_1 - z_2); \]

\[ < \Phi(z_1)\bar{\Phi}(z_2) > = \frac{1}{\Box - (m + a(-\Delta)^z)^2} \delta^8(z_1 - z_2). \tag{21} \]

We note that at \( a = 0 \) these propagators reduce to the standard ones of the Wess-Zumino model.

There are extra \( D^2, \bar{D}^2 \) factors associated with the vertices just as in the previous case.

We can calculate the one-loop Kählerian effective potential. As usual [13], the corresponding Feynman supergraphs will represent themselves as rings with alternating chiral and antichiral legs, with any chiral leg accompanied by the factor \(-\frac{D^2}{4}\), and the antichiral one – by the factor \(-\frac{\bar{D}^2}{4}\).

All propagators are \( < \phi\bar{\phi} > \) (with \( \phi, \bar{\phi} \) being quantum fields), so, we can use again the Fig. 1.

Since the factor \((-\Delta)^z\) acts only in the internal propagators, it is natural to carry out the Fourier transform and include this factor to the definition of the external field; the propagator will be simply \( \frac{1}{k^2} \). So, we have the contribution in the form

\[ \Gamma^{(1)} = \sum_{n=1}^{\infty} \frac{1}{2n} \int d^4\theta \int \frac{d^4k}{(2\pi)^4} \left[ (\Psi + a\tilde{k}^{2z})(\bar{\Psi} + a\tilde{k}^{2z}) \frac{D^2\bar{D}^2}{16k^4} \right]^n \delta(\theta - \theta')|_{\theta=\theta'}, \tag{22} \]

where \( \Psi = m + \lambda\Phi \). Then, we use the fact that \( (\frac{D^2\bar{D}^2}{16})^n = (-k^2)^{n-1} \frac{D^2\bar{D}^2}{16} \), contract the loop into a point by the rule \( (\frac{D^2\bar{D}^2}{16})^n \delta(\theta - \theta')|_{\theta=\theta'} = 1 \) and use the identity (13).

As a result, after the Wick rotation we obtain

\[ K^{(1)} = -\frac{1}{2} \int \frac{dk_{0E}d^3\tilde{k}}{(2\pi)^4} \frac{1}{k^2_{0E} + \tilde{k}^2} \ln \left( 1 + \frac{(\Psi + a\tilde{k}^{2z})(\bar{\Psi} + a\tilde{k}^{2z})}{k^2_{0E} + \tilde{k}^2} \right). \tag{23} \]

The Euclidean momentum square \( k^2 = k^2_{0E} + \tilde{k}^2 \) emerges due to the presence of the \( \Box \) because of the usual structure of the spinor supercovariant derivatives. This integral can be calculated. Indeed, we can rewrite it as

\[ K^{(1)} = -\frac{1}{2} \int \frac{dk_{0E}d^3\tilde{k}}{(2\pi)^4} \frac{1}{k^2_{0E} + \tilde{k}^2} \ln \left( k^2_{0E} + \tilde{k}^2 + (\Psi + a\tilde{k}^{2z})(\bar{\Psi} + a\tilde{k}^{2z}) \right). \tag{24} \]

We can apply the same approximation as in the previous case, that is, disregard the subleading degrees of momenta, so, we get

\[ K^{(1)} \simeq -\frac{1}{2} \int \frac{dk_{0E}d^3\tilde{k}}{(2\pi)^4} \frac{1}{k^2_{0E} + \tilde{k}^2} \ln \left( k^2_{0E} + \tilde{k}^2 + \Psi\bar{\Psi} + a^2\tilde{k}^{4z} \right). \tag{25} \]

We can obtain

\[ \frac{dK^{(1)}}{d(\Psi\bar{\Psi})} \simeq -\frac{1}{2} \int \frac{dk_{0E}d^3\tilde{k}}{(2\pi)^4} \frac{1}{k^2_{0E} + \tilde{k}^2} \frac{1}{k^2_{0E} + \tilde{k}^2 + \Psi\bar{\Psi} + a^2\tilde{k}^{4z}}. \tag{26} \]
We integrate over $k_0$ and $\vec{k}$ as in the previous case. Finally, we get

$$K^{(1)} = -\frac{1}{8\pi a^{1/2}} \csc\left(\frac{\pi}{2z}\right)(\Psi \bar{\Psi})^{1/2z}.$$  (27)

We confirm that this effective potential is ultraviolet finite if $z \geq 1/2$, which agrees with the usual Wess-Zumino model. Treating the chiral effective potential, it is easy to find that the first non-trivial contribution to it arises only at the two-loop order just as in the usual Wess-Zumino model [14, 15].

III. THE HL-LIKE EXTENSION OF THE SUPERGAUGE THEORY

Now, let us try to introduce the gauge theories within this approach. It is well known (cf. [2]) that the free action of the Abelian gauge theory is

$$S_{SYM} = \frac{1}{64} \int d^6 z W^\alpha W_\alpha,$$  (28)

where

$$W_\alpha = -\bar{D}^2 D_\alpha V,$$  (29)

and the $V(z)$ is a real scalar superfield. We can rewrite the action as

$$S = -\frac{1}{2} \int d^8 z V \left(-\frac{1}{8} D^\alpha \bar{D}^2 D_\alpha\right) V.$$  (30)

The action (28) is invariant under gauge transformations

$$\delta V = i(\Lambda - \bar{\Lambda}),$$  (31)

with $\Lambda$ chiral and $\bar{\Lambda}$ antichiral. It is well known that $(-\frac{1}{8} D^\alpha \bar{D}^2 D_\alpha) = \Pi_{1/2}$ is a transverse projector. To perform the Horava-Lifshitz-like extension of this theory in a way compatible with the superfield structure and gauge covariance, we must maintain this projector. So, we deal as in the first example with the Wess-Zumino model (4), that is, introduce the HL-like operator $1 + \rho \Delta^{z-1}$ (so, at $\rho = 0$ the usual structure is recovered). Our action takes the form

$$S = -\frac{1}{2} \int d^8 z V \left(-\frac{1}{8} D^\alpha \bar{D}^2 D_\alpha\right)(1 + \rho \Delta^{z-1}) V.$$  (32)

From the component viewpoint, it looks like

$$S = -\frac{1}{4} \int d^4 x F_{\mu\nu} (1 + \rho \Delta^{z-1}) F^{\mu\nu} + \ldots.$$  (33)
Indeed, this action involves two time derivatives and $2z$ spatial derivatives, together again with some “crossed” terms.

We should as usual fix the gauge. It is natural to promote the following extension of the gauge-fixing action:

$$S_{GF} = \frac{1}{2\xi} \int d^8z V \Pi_0 \Box (1 + \rho \Delta^{z-1}) V,$$  \hspace{1cm} (34)

where $\Pi_0 = \{D^2, D^2\}_{16,16}$ is a longitudinal projector (so, $\Pi_{1/2} + \Pi_0 = 1$, and $\Pi_{1/2} \Pi_0 = 0$). As a result, we have the propagator

$$< V(z_1) V(z_2) > = -\frac{1}{\Box (1 + \rho \Delta^{z-1}) (\Pi_{1/2} + \xi \Pi_0)} \delta^8(z_1 - z_2).$$ \hspace{1cm} (35)

Now, we can implement the coupling of this field with chiral matter. Just as in [16], we can show that there must be only one possible term up to the multiplicative constant factor, that is,

$$S_m = \int d^8 z \Phi \bar{\Phi} e^{\gamma V} \Phi,$$ \hspace{1cm} (36)

with no mass or self-coupling terms unless we consider a set of chiral superfields, with the invariant mass matrix $m_{ij}$ and coupling tensor $\lambda_{ijk}$. So, in this case we cannot deform the action of the matter in a HL-like way.

Again as in [16], we can have two types of supergraphs. The first one involves only gauge propagators and quartic vertices in an explicit form:

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig2.png}
\caption{Fig. 2}
\end{figure}

We can perform the sum of these supergraphs:

$$K^{(1)}_a = \int d^8 z_1 \sum_{n=1}^\infty \frac{(-1)^n}{2n} (g^2 \Phi \bar{\Phi} \frac{1}{\Box (1 + \rho \Delta^{z-1}) (\Pi_{1/2} + \xi \Pi_0)})^n \delta_{12}|_{\theta_1=\theta_2},$$ \hspace{1cm} (37)

where $\frac{1}{n}$ is a symmetry factor. These diagrams do not involve the triple vertices which will be considered shortly.

By using the properties of the projection operators, we can write

$$K^{(1)}_a = \int d^8 z_1 \sum_{n=1}^\infty \frac{(-1)^n}{2n} (g^2 \Phi \bar{\Phi} \frac{1}{\Box (1 + \rho \Delta^{z-1}) (\Pi_{1/2} + \xi \Pi_0)})^n (\Pi_{1/2} + \xi^n \Pi_0) \delta_{12}|_{\theta_1=\theta_2},$$ \hspace{1cm} (38)
Since $\frac{D^2 D^2}{y_9} \delta_{12} = 1$, we have $\Box \Pi_0 \delta_{12} |_{\theta_1 = \theta_2} = 2$, and $\Box \Pi_{1/2} \delta_{12} |_{\theta_1 = \theta_2} = -2$. Thus, we have

$$K_{\alpha}^{(1)} = \int d^8 z \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \left[ \frac{1}{\Box (1 + \rho \Delta \bar{z}^{-1})} \right] \left[ g^2 \Phi \bar{\Phi} \right] (1 - \alpha^n) \delta^4 (x_1 - x_2) |_{x_1 = x_2}. \quad (39)$$

By carrying out the Fourier transform $\Box \rightarrow -k^2$ we arrive at

$$K_{\alpha}^{(1)} = -\int d^8 z \int \frac{d^4 k}{(2\pi)^4} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \frac{1}{k^2} \left( \frac{g^2 \Phi \bar{\Phi}}{k^2 (1 + \rho (-k^2) z^{-1})} \right)^n (1 - \alpha^n). \quad (40)$$

Then, by using the expansion (13), we have

$$K_{\alpha}^{(1)} = \int d^8 z \int \frac{d^4 k}{(2\pi)^4} \int \frac{1}{k^2} \left[ \ln \left( 1 + \frac{g^2 \Phi \bar{\Phi}}{k^2 (1 + \rho (-k^2) z^{-1})} \right) - \ln \left( 1 + \frac{\alpha g^2 \Phi \bar{\Phi}}{k^2 (1 + \rho (-k^2) z^{-1})} \right) \right]. \quad (41)$$

Notice that at $\alpha = 0$ (Landau gauge), the second term in this expression vanishes.

The second type of diagrams involves the triple vertices as well. We should first introduce a "dressed" propagator

$$\ldots = \ldots + \ldots \ldots + \ldots$$

\textit{Fig.3}

In this propagator, the summation over all quartic vertices is performed. As a result, this "dressed" propagator is equal to

$$<VV >_D = <VV > (1 + g^2 \Phi \bar{\Phi} <VV > + (g^2 \Phi \bar{\Phi} <VV >)^2 + \ldots) =$$

$$= -\sum_{n=0}^{\infty} (g^2 \Phi \bar{\Phi})^n \frac{1}{\Box (1 + \rho \Delta \bar{z}^{-1})^{n+1}} \Pi_{1/2} + \alpha \Pi_0 )^{n+1} \delta^4 (z_1 - z_2). \quad (42)$$

By summing up, we arrive at

$$<VV >_D = - \left( \frac{1}{\Box (1 + \rho \Delta \bar{z}^{-1}) + g^2 \Phi \bar{\Phi} \Pi_{1/2} + \frac{\alpha}{\Box (1 + \rho \Delta \bar{z}^{-1}) + \alpha g^2 \Phi \bar{\Phi} \Pi_0 } \right) \delta^4 (z_1 - z_2). \quad (43)$$

To proceed, we should sum over diagrams representing themselves as cycles of all possible number of links. Such diagrams look like

$$\ldots$$

\textit{Fig.4}

The complete contribution of all these cycles gives

$$K_{\beta}^{(1)} = \int d^8 z \sum_{n=1}^{\infty} \frac{1}{2n} (g^2 \Phi \bar{\Phi} (\langle \phi \bar{\phi} \rangle + \langle \bar{\phi} \phi \rangle) <VV >_D )^{n} \delta_{12} |_{\theta_1 = \theta_2}, \quad (44)$$
or, as it is the same,

\[ K^{(1)}_b = \int d^8 z_1 \sum_{n=1}^{\infty} \frac{1}{2\pi^2} (g^2 \Phi \bar{\Phi} \Pi_0 < VV >_D) \delta_{12} |_{\bar{\theta}_1 = \theta_2}. \] (45)

By noting that

\[ \Pi_0 < VV >_D = -\frac{\alpha}{\Box(1 + \rho \Delta z^{-1})} + \alpha g^2 \Phi \bar{\Phi} \Pi_0 \delta_{12}, \] (46)

we can rewrite the expression above as

\[ K^{(1)}_b = \int d^8 z_1 \sum_{n=1}^{\infty} \frac{(-1)^n}{2n} \frac{\alpha \Pi_0 \delta_{12} |_{\bar{\theta}_1 = \theta_2}}{\Box(1 + \rho \Delta z^{-1}) + \alpha g^2 \Phi \bar{\Phi}} \rho \delta_{12} |_{\bar{\theta}_1 = \theta_2}. \] (47)

Since \( \Box \Pi_0 \delta_{12} |_{\bar{\theta}_1 = \theta_2} = 2 \), we have

\[ K^{(1)}_b = \int d^8 z_1 \sum_{n=1}^{\infty} \frac{1}{n} \frac{\alpha \Pi_0 \delta_{12} |_{\bar{\theta}_1 = \theta_2}}{\Box(1 + \rho \Delta z^{-1}) + \alpha g^2 \Phi \bar{\Phi}} \rho \delta_{12} |_{x_1 = x_2}. \] (48)

By carrying out the Fourier transform and the summation as above, we arrive at

\[ K^{(1)}_b = \int d^8 z \int \frac{d^3 k dk_0 E}{(2\pi)^4} \frac{1}{k_E^2} \ln \left[ 1 + \frac{\alpha g^2 \Phi \bar{\Phi}}{k_E^2 (1 + \rho (-\bar{k}^2)^{z-1})} \right]. \] (49)

By adding this contribution to \( K^{(1)}_a \) (41), we see that the \( \alpha \) dependent contribution vanishes, and the total one-loop Kählerian effective potential is gauge independent, being, after relabeling \( \rho (-1)^z \rightarrow \rho \) equal to

\[ K^{(1)} = \int d^8 z \int \frac{d^3 k dk_0 E}{(2\pi)^4} \frac{1}{k_E^2} \ln \left[ 1 + \frac{g^2 \Phi \bar{\Phi}}{k_E^2 (1 + \rho (-\bar{k}^2)^{z-1})} \right]. \] (50)

It is clear that at \( \rho = 0 \), the one-loop Kählerian effective potential in the usual supersymmetric gauge theory [13] is re-obtained.

Again, we integrate with use of the same approximations as in the previous cases. We arrive at

\[ K^{(1)} = \frac{1}{16\pi^2} \csc \left( \frac{\pi}{z} \right) \left( \frac{g^2 \Phi \bar{\Phi}}{2\rho} \right)^{1/z}. \] (51)

This is our final (unfortunately approximate) result. Again, it is finite if \( z > 1 \) and diverges at \( z = 1 \). The same observations made after (18) can be applied.

**IV. SUMMARY**

We have formulated some Horava-Lifshitz-like extensions of the most used superfield theories, that is, the Wess-Zumino model and supersymmetric QED. It turns out that the critical exponent in these theories is not well-defined, however, we can still discuss some of their peculiarities.
We showed that, first, all strength of the superfield approach can be successfully applied to this class of theories, second, the renormalization behaviour of these theories is improved in comparison with their non-supersymmetric analogues, third, we found that, in general, the effective potential may be expressed as integrals over the spatial part of the loop momenta. However, these integrals cannot be expressed in terms of elementary functions. This led us to introduce some approximations to obtain a closed form.

It is clear that, in principle, this class of theories is not more complicated than the usual Lorentz-invariant supersymmetric field theories. In this respect, we should recall the statement that the theories we discussed flow in the infrared to $z = 1$ (or $z = 1/2$), that is, to the usual WZ model, and in the ultraviolet to a nontrivial $z$.

**Acknowledgements.** This work was partially supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) and FAPESP. The work by A. Yu. P. has been supported by the CNPq project No. 303438/2012-6.

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