Cosmic String in Scalar-Tensor Gravities

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Abstract

We consider a local cosmic string described by the Abelian-Higgs model in the framework of scalar-tensor gravities. We find the metric of the cosmic string in the weak-field approximation. The propagation of particles and light is analysed in this background. This analysis shows that the (unperturbed) cosmic string in scalar-tensor theory presents some analogous features to the wiggly cosmic string in General Relativity.

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1 Introduction

The scalar-tensor theories of gravity proposed by Bergmann [1], Wagoner [2] and Nordtverdt [3] - generalizing the original Brans-Dicke [4] theory -
have been considerably revived in the last years. Indeed, the existence of a
scalar field as a spin-0 component of the gravitational interaction seems to
be a quite natural prediction of unification models such as supergravity or
superstrings [5]. Apart from the fact that scalar-tensor theories may provide
a solution for the problem of terminating inflation [6, 7], these theories by
themselves have direct implications for cosmology and for experimental tests
of the gravitational interaction: One expects that in the Early Universe the
coupling to matter of the scalar component of the gravitational interaction
would be of the same order of the coupling to matter of the long-range
tensor component although in the present time the observable total coupling
strength of scalars ($\alpha^2$) is generically small [8]. Besides, any gravitational
phenomena will be affected by the variation of the gravitational “constant”
$G_{eff} \sim \tilde{\phi}^{-1}$. So, it seems worthwhile to analyse the behaviour of matters in
the framework of scalar-tensor theories, specially those which originated in
the early Universe such as topological defects. In this context, some authors
[9] have considered the solutions of cosmic string and domain walls in Brans-
Dicke theory.

The aim of this paper is to study the modifications on the metric of a local
cosmic string in more general scalar-tensor gravities. These modifications are
induced by the coupling of a scalar field to the tensor field in the gravitational
lagrangean. For simplicity, we will consider a class of scalar-tensor theories
where the potential $V(\tilde{\phi})$ (or as in Wagoner’s notation, the potential $\lambda(\tilde{\phi})$
[2]) is vanishing.
The action describing these theories is (in Jordan-Fierz frame)

\[ S = \frac{1}{16\pi} \int d^4x \sqrt{\tilde{g}} \left[ \tilde{\phi} \tilde{R} - \frac{\omega(\tilde{\phi})}{\tilde{\phi}} \tilde{g}_{\mu\nu} \partial_\mu \tilde{\phi} \partial_\nu \tilde{\phi} \right] + S_m[\Psi_m, \tilde{g}_{\mu\nu}], \]  

(1)

where \( \tilde{g}_{\mu\nu} \) is the physical metric in this frame, \( \tilde{R} \) is the curvature scalar associated to it and \( S_m \) denotes the action of the general matter fields \( \Psi_m \).

These theories are metric, which means that matter couples minimally to \( \tilde{g}_{\mu\nu} \) and not to \( \tilde{\phi} \). For many reasons it is more convenient to work in the so-called Einstein (conformal) frame, in which the kinematic terms of tensor and scalar fields do not mix

\[ S = \frac{1}{16\pi G} \int d^4x \sqrt{g} \left[ R - 2g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right] + S_m[\Psi_m, A^2(\phi)g_{\mu\nu}] \]  

(2)

where \( g_{\mu\nu} \) is the (unphysical) metric tensor in Einstein frame, \( R \) is the curvature scalar associated to it and \( A(\phi) \) is an arbitrary function of the scalar field. Action (2) is obtained from (1) by a conformal transformation in the physical metric

\[ \tilde{g}_{\mu\nu} = A^2(\phi)g_{\mu\nu} \]  

(3)

and by a redefinition of the quantities

\[ GA^2(\phi) = \tilde{\phi}^{-1}, \]

\[ \alpha^2 \equiv \left( \frac{\partial \ln A(\phi)}{\partial \tilde{\phi}} \right)^2 = [2\omega(\tilde{\phi}) + 3]^{-1}. \]

It is important to remark that \( \alpha(\phi) \) is the field-dependent coupling strength between matter and scalar fields. In the particular case of Brans-Dicke theory, \( A(\phi) \) has the following dependence on \( \phi \) : \( A(\phi) = e^{2\alpha \phi} \), with \( \alpha(\phi) = \alpha = \)
\[ (\omega + 3)^{-1/2} = \text{constant.} \] In the Einstein frame, the field equations are written as the following

\[
R_{\mu \nu} = 2 \partial_\mu \phi \partial_\nu \phi + 8 \pi G (T_{\mu \nu} - \frac{1}{2} g_{\mu \nu} T),
\]
\[
\Box_g \phi = -4 \pi G \alpha(\phi) T. \tag{4}
\]

We note that the first of the above equations can also be written in terms of the Einstein tensor \( G_{\mu \nu} \)

\[
G_{\mu \nu} = 2 \partial_\mu \phi \partial_\nu \phi - g_{\mu \nu} g^{\alpha \beta} \partial_\alpha \phi \partial_\beta \phi + 8 \pi G T_{\mu \nu}.
\]

The energy-momentum tensor is defined as usual

\[
T_{\mu \nu} = \frac{2}{\sqrt{g}} \frac{\delta S_m[A^2(\phi)g_{\mu \nu}]}{\delta g^{\mu \nu}},
\]

but in the Einstein frame it is no longer conserved \( \nabla_\nu T^\nu_\mu = \alpha(\phi) T \nabla_\mu \phi \). It is clear from transformation (3) that we can related quantities from both frames such that \( \tilde{T}^{\mu \nu} = A^{-6} T^{\mu \nu} \) and \( \tilde{T}_\nu^\mu = A^{-4} T_\nu^\mu \).

In what follows we will search for a regular solution of an isolated static straight cosmic string in the scalar-tensor gravity described above. Hence, the cosmic string arises from the action of the Abelian-Higgs model where a charged scalar Higgs field \( \Phi \) minimally couples to the \( U(1) \) gauge field \( A_\mu \)

\[
S_m = \int d^4 x \sqrt{-g} \left[ \frac{1}{2} D_\mu \Phi D^\mu \Phi^* - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} - V(|\Phi|) \right], \tag{5}
\]

with \( D_\mu \equiv \partial + ie A_\mu \), \( F_{\mu \nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \) and the Higgs potential \( V(|\Phi|) = \lambda (|\Phi|^2 - \eta^2)^2 \). \( e, \lambda \) and \( \eta \) are positive constants, \( \eta \) being the characteristic energy scale of the symmetry breaking (eg, for typical GUT strings, \( \eta \sim 10^{16} \text{GeV} \).)
We confine our attention to the static configurations of vortex type about the $z$-axis. In cylindrical coordinate system $(t, z, \rho, \varphi)$ such that $\rho \geq 0$ and $0 \leq \varphi < 2\pi$, we impose the following form for the Higgs $\Phi$ and the gauge $A_\mu$ fields

$$\Phi \equiv R(\phi) e^{i\varphi} \quad \text{and} \quad A_\mu \equiv \frac{1}{e} [P(\rho) - 1] \delta_\mu^r,$$

(6)

where $R$, $P$ are functions of $\rho$ only. Moreover, we require that these functions are regular everywhere and that they satisfy the usual boundary conditions for vortex solutions

$$R(0) = 0 \quad \text{and} \quad P(0) = 1,$$

$$\lim_{\rho \to \infty} R(\rho) = \eta \quad \text{and} \quad \lim_{\rho \to \infty} P(\rho) = 0,$$

(7)

In General Relativity, a metric for a cosmic string described by the action (5) above has been already found in the asymptotic limit by Garfinkle [11] and exactly by Linet [12] provided the particular relation $e^2 = 8\lambda^2$ between the constants $e$ and $\lambda$ is satisfied. The question is whether one can find a solution for the cosmic string in the framework of the scalar-tensor gravity. The answer for this question seems obviously negative. However, we can consider the weak-field approximation for this solution in the same way as Vilenkin [13] in the framework of General Relativity. In fact, the weak-field approximation breaks down at large distances from the cosmic string. Therefore, we assume that at large distances the $\phi$ dependence on the right-hand side of the first of Einstein eqs. (4) must dominate over the $T^\mu_\nu$ term.

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1 With winding number $n = 1$. 
So, we will neglect the energy-momentum tensor in Einstein eqs. (4) and find the vacuum solution as an asymptotic behaviour of $g_{\mu\nu}$ and $\phi$ and, then, match this vacuum metric with the metric of the cosmic string in the weak-field approximation.

The plan of this work is as follows. In section 2, we find the exact vacuum metric for the Einstein eqs. (4). In section 3, we find the metric of the cosmic string in the weak-field approximation and analyse under which conditions it can be matched to the vacuum metric. We then analyse the propagation of particles and light in the linearized metric of the cosmic string. Our main result is that particles and light propagate in this background in a similar way as they propagate in the background of a wiggly cosmic string in General Relativity.

2 The vacuum metric in scalar-tensor: an exact solution

In this section we will find the exact static vacuum metric which is solution to the Einstein eqs. (4) in scalar-tensor theories. This metric is supposed to match the weak-field solution of the cosmic string, so it seems natural to impose the same symmetries to this vacuum spacetime as those of the string. $^2$

$^2$As shown by Laguna-Castillo and Matzner [14], the $T^\mu_\nu$ components vanish far from the cosmic string in General Relativity. Gundlach and Ortiz showed that this feature remains valid for a cosmic string in Brans-Dicke theory. One must expect that this assumption can be applied to general scalar-tensor theories.
We write the following metric

$$ds^2 = g_1(\rho)dt^2 - g_2(\rho)dz^2 - d\rho^2 - g_3(\rho)d\varphi^2,$$

where $g_1, g_2, g_3$ are functions of $\rho$ only, and $(t, z, \rho, \varphi)$ are cylindrical coordinates such that $\rho \geq 0$ and $0 \leq \varphi < 2\pi$. Defining $u \equiv (g_1 g_2 g_3)^{1/2}$, the Einstein eqs. (4) in vacuum can be written as

$$R^i_i = \frac{1}{2u} \left[ u \frac{g_i'}{g_1} \right]' = 0, \quad (i = t, z, \varphi) \quad (9)$$

$$G^\rho_\rho = -\frac{1}{4} \left[ \frac{g_1' g_2'}{g_1 g_2} + \frac{g_1 g_3'}{g_1 g_3} + \frac{g_2 g_3'}{g_2 g_3} \right] = -(\phi')^2,$$

$$\frac{1}{u} \frac{d}{d\rho} (u\phi') = 0, \quad (11)$$

where the prime means derivative with respect to $\rho$. Besides, since $T^\mu_\nu = 0$, the following expression is also valid

$$\sum_i R^i_i = \frac{u''}{u} = 0 \quad (i = t, z, \varphi). \quad (12)$$

From (12) it follows that $u$ is a linear function of $\rho$ ($u \sim B\rho$). This result enables us to solve eqs. (9) and (11) to find

$$g_i = k_i^{(0)} \left( \frac{\rho}{\rho_0} \right)^{k_i} \quad \text{and} \quad \phi = \phi_0 + \kappa \ln(\rho/\rho_0), \quad (13)$$

respectively. $B, k_i^{(0)}, k_i$ and $\kappa$ are constants to be determined later. Combining the solution for $u$ with solutions (13), we obtain the following relations between the constants

$$[k_1^{(0)} k_2^{(0)} k_3^{(0)}]^{1/2} = B,$$

$$k_1 + k_2 + k_3 = 2, \quad \text{and}$$
\[ k_1k_2 + k_1k_3 + k_2k_3 = 4\kappa^2. \]

Moreover, if we suppose the boost invariance of the string along the z-axis (i.e., \( g_1 = g_2 \)) the above relations are simplified and we finally find

\[ k_1^{(0)}[k_3^{(0)}]^{1/2} = B, \]

\[ k_3 = 2 - 2k_1 \quad \text{and} \quad \kappa^2 = k_1(1 - \frac{3}{4}k_1). \quad (14) \]

The constant \( k_1^{(0)} \) can always be absorbed by a redefinition of \( t \) and \( z \). Then, we obtain the final form for the vacuum metric

\[ ds^2 = \left( \frac{\rho}{\rho_0} \right)^{k_1} \left( dt^2 - dz^2 \right) - d\rho^2 - \left( \frac{\rho}{\rho_0} \right)^{2-2k_1} B^2 d\phi^2, \quad (15) \]

in which \( k_1 \) (and consequently \( k_3 \)), \( B \) and \( \kappa \) will be fully determined after the introduction of matter fields.

The Ricci tensor in the vacuum metric (15) is regular (vanishes everywhere) if and only if \( \phi = \phi_0 = \text{const.} \) (i.e, \( \kappa = 0 \)). This should not be surprising in views of the structure of Einstein eqs. (9-11), in particular eq. (10). Besides, \( \kappa = 0 \) implies that the only allowed values for \( k_1 \) are \( k_1 = 4/3 \) and \( k_1 = 0 \). This latter value corresponds to the conical metric and only in this case \( B^2 = k_3^{(0)} \) can be interpreted as the deficit angle. As we will see in the next section, the values \( k_1 = 4/3 \) and \( k_1 = 0 \) are precisely the necessary conditions required to match metric (15) to the metric of the cosmic string in the weak-field approximation. Finally, we note that metric (15) can also be written in Taub-Kasner form [13] after a suitable coordinate transformation.
3 Cosmic string solution in scalar-tensor gravity: the weak-field approximation

We start by writing down the full Einsteins equations

\[ R^t_t = R^z_z = \frac{1}{2u} \left[ \frac{g_1'}{g_1} \right]' = \frac{8\pi G(T^t_t - \frac{1}{2}T)}{} \]  

(16)

\[ R^\rho_\rho = \frac{1}{2} \left[ 2 \left( \frac{g_1''}{g_1} \right) - \left( \frac{g_1'}{g_1} \right)^2 + \left( \frac{g_3''}{g_3} \right) - \frac{1}{2} \left( \frac{g_3'}{g_3} \right)^2 \right] = -2(\phi')^2 + 8\pi G(T^\rho_\rho - \frac{1}{2}T) \]  

(17)

\[ R^\varphi_\varphi = \frac{1}{2u} \left[ \frac{g_3'}{g_3} \right]' = 8\pi G(T^\varphi_\varphi - \frac{1}{2}T) \]  

(18)

\[ \frac{1}{u} [u\phi']' = 4\pi G\alpha(\phi)T. \]  

(19)

where the non-vanishing components of the energy-momentum tensor are

\[ T^t_t = T^z_z = \frac{1}{2}A^2(\phi) \left[ (R')^2 + g_3^{-1}R^2P^2 + \frac{1}{e^2}g_3^{-1}A^{-2}(P')^2 + 2\lambda A^2(R^2 - \eta^2)^2 \right], \]  

\[ T^\rho_\rho = -\frac{1}{2}A^2(\phi) \left[ (R')^2 - g_3^{-1}R^2P^2 + \frac{1}{e^2}g_3^{-1}A^{-2}(P')^2 - 2\lambda A^2(R^2 - \eta^2)^2 \right], \]  

\[ T^\varphi_\varphi = \frac{1}{2}A^2(\phi) \left[ (R')^2 - g_3^{-1}R^2P^2 - \frac{1}{e^2}g_3^{-1}A^{-2}(P')^2 + 2\lambda A^2(R^2 - \eta^2)^2 \right]. \]  

(20)

The matter fields equations can be written as (where form (6) for a vortex type solution is imposed)

\[ R'' + R' \left[ \frac{g_1'}{g_1} + \frac{1}{2} \frac{g_3'}{g_3} - 2\alpha(\phi)\phi' \right] - R[g_3^{-1}P^2 + 4\lambda A^2(R^2 - \eta^2)] = 0, \]  

(21)

\[ P'' + P' \left[ \frac{g_1'}{g_1} + \frac{1}{2} \frac{g_3'}{g_3} - 4\alpha(\phi)\phi' \right] - e^2A^2R^2P^2 = 0. \]  

(22)
It is impossible to find an analytical solution for eqs. (16-22). So, we will consider the solution of the cosmic string in the weak-field approximation. In fact, this approximation can only be justified if we consider the scalar field $\phi$ as a small perturbation on the gravitational field of the cosmic string. Thus, it may be expanded in terms of a small parameter $\epsilon$ about the values $\phi = \phi_0$ and $g_{\mu\nu} = \eta_{\mu\nu}$

$$
\phi = \phi_0 + \epsilon \phi^{(1)}
$$

$$
g_{\mu\nu} = \eta_{\mu\nu} + \epsilon h_{\mu\nu}
$$

$$
A(\phi) = A(\phi_0) + \epsilon A'(\phi_0)\phi^{(1)}
$$

$$
T^\mu_\nu = T^{(0)}_\mu_\nu + \epsilon T^{(1)}_\mu_\nu
$$

The term $(\phi')^2$ is neglected in the process of linearization of the Einstein eqs. (16-19). Moreover, in this approximation the $T^{(0)}_{(0)\nu}$ term is obtained from (20) by a limit process $\lambda \to \infty$ (see Linet [12]). Therefore, it tends to the Dirac distribution on the surface $(t, z) = \text{constant}$. In this way, the linearized equations reduce to those of General Relativity [13], except that in our case the terms $T^{(0)}_{(0)\nu}$ and $h_{\mu\nu}$ carry a conformal factor $A^2(\phi)$

$$
\nabla^2 h_{\mu\nu} = 16\pi G (T^{(\mu)}_{(0)\nu} - \frac{1}{2}\eta_{\mu\nu}T^{(0)}_\nu),
$$

in the harmonic coordinates $(h^{\mu}_\nu - \frac{1}{2}\delta^\mu_\nu h)_\nu = 0$. Besides, equation (19) for the scalar field is also linearized

$$
\nabla^2 \phi^{(1)} = 4\pi G\alpha(\phi_0)T_{(0)}.
$$

The solution for this equation is evident

$$
\phi^{(1)} = 4G\mu A^2(\phi_0)\alpha(\phi_0)\ln(\rho/\rho_0).
$$
This solution matches with the vacuum solution iff

\[ \kappa_{\text{lin}} = 4G\mu\alpha(\phi_0)A^2(\phi_0). \]  \hfill (25)

\( \kappa \) from (14) we see that the only allowed values for \( k_1 \) are \( k_1 = 0 + O(G^2\mu^2) \) and \( k_1 = 4/3 + O(G^2\mu^2) \). The result which is physically meaningful is \( k_1 = 0 \) plus negligible corrections of order \( (G\mu)^2 \); the other value corresponds to a non physical metric \([11, 13]\).

The solution for eq. (23) differs from that found by Vilenkin \([13]\) by a conformal factor \( A^2(\phi) \). However, the procedure is the same as in his paper. After a coordinate transformation to bring the metric to the cylindrical system, we have

\[
\begin{align*}
\text{ds}^2 &= A^2(\phi_0)[1 + 8GA^2(\phi_0)\mu\alpha^2(\phi_0)\ln(\rho/\rho_0)] [dt^2 - dz^2 - d\rho^2 \\
&\quad - [1 - 8GA^2(\phi_0)\mu] \left( \frac{\rho}{\rho_0} \right)^2 d\varphi^2],
\end{align*}
\] \hfill (26)

in which \( \rho \geq 0 \) and \( 0 \leq \varphi < 2\pi \). Therefore, metric (27) represents an isolated scalar-tensor cosmic string so long as the weak-field approximation is valid.

### 3.1 Propagation of particles and light

Concerning the light deflection in metric (27), it is easy to note that the angular separation of double image \( \delta \varphi \) is given by \( \delta \varphi = 8\pi GA^2(\phi_0)\mu = 8\pi \tilde{G}_0\mu \) (recall that \( GA^2(\phi_0) = \tilde{G}_0 \) is the effective Newtonian constant) and remains unchanged in the scalar-tensor gravity. For GUT strings, \( \delta \varphi \sim 10^{-5} \) rad. It means that, from the point of vue of this effect, it is impossible to distinguish a scalar-tensor cosmic string from a General Relativity one.
The second remark is related to the fact that the scalar-tensor cosmic string exerts a force on a non-relativistic test particle of mass $m$  
\[ f^\rho = -4mGA^2(\phi_0)\mu\alpha^2(\phi_0)\frac{1}{\rho}, \quad (27) \]
and it is always attractive. This force seems to be negligible at present time (recall that $GA^2(\phi_0)\mu = \tilde{G}_0\mu \sim 10^{-6}$ for GUT strings, and $\alpha^2(\phi_0) < 10^{-3}$ [8]).

Let us consider the deflection of particles moving past the string. First of all, we rewrite metric (27) in terms of conformally Minkowskian coordinates

\[ ds^2 = (1 + h_{00})(dt^2 - dx^2 - dy^2) \quad (28) \]

where $h_{00} = 8GA^2(\phi_0)\mu\alpha^2(\phi_0)\ln[(x^2 + y^2)^{1/2}]$ and, for simplicity, we consider $dz = 0$. The factor $A^2(\phi_0)$ was absorbed by a rescaling. Metric (29) has a missing wedge of angular width $\Delta = 8\pi\tilde{G}_0\mu$. The linearized geodesic equations in this metric have the form

\[ 2\ddot{x} = -(1 - \dot{x}^2 - \dot{y}^2)\partial_x h_{00} \]
\[ 2\ddot{y} = -(1 - \dot{x}^2 - \dot{y}^2)\partial_y h_{00} \quad (29) \]

where dots refer to derivative with respect to $t$. Now, if we choose the $x$-axis along which the particles are flowing with initial velocity $v_s$, the second of eqs. (30) can be integrated over the unperturbed trajectory $x = v_st$ and $y = y_0$. After the particles pass the string, their velocity gains a small $y$-component which can be computed in linear order in $\tilde{G}_0\mu$. For astrophysical applications, it may be interesting to express the relative velocity developed by the particles after the string has passed between them. To transform to the
frame in which the string has a velocity $v_s$ we make a Lorentz transformation \[\eqref{eq:16}\]. So, the final result is

$$u = 8\pi\tilde{G}_0\mu v_s\gamma + \frac{4\pi\tilde{G}_0\mu\alpha^2(\phi_0)}{v_s\gamma}$$

(30)

where $\gamma = (1 - v_s^2)^{-1/2}$. The first term in (31) is equivalent to the expression of the relative velocity of particles flowing past a straight cosmic string in General Relativity \[\eqref{eq:17}\] and it expresses the impulse velocity due to the conical deficit angle. The second term is due to the attractive force exerted by the scalar-tensor cosmic string. Let us compare the magnitudes of these two terms. String simulations show that $v_s \sim 0.15$ in the matter era \[\eqref{eq:18}\]. It means that the first term is about 45 times larger than the second one (recall that $\alpha^2(\phi_0) < 10^{-3}$ \[\eqref{eq:8}\]). At this point, it may be interesting to compare this effect in the scalar-tensor gravity with the relative velocity of particles in the background of a wiggly cosmic string in General Relativity \[\eqref{eq:19}\]. The fact that the wiggles produce an attractive force on the particles leads to similar correction to the usual expression \[\eqref{eq:3}\] for the relative velocity. However, one important difference is that in this case the term induced by the wiggles’ force is ten times larger than the usual term in the matter era \[\eqref{eq:19}\].

Conclusions

In this paper we studied the gravitational properties of a straight cosmic string in scalar-tensor gravities in the weak-field approximation. We showed

\[\text{By “usual” we mean the term produced by the unperturbed cosmic string in General Relativity.}\]
that the propagation of particles and light in this background presents some analogy with the propagation of particles and light in the background of a wiggly cosmic string in General Relativity. But in the latter case the term of the relative velocity of particles moving past the string induced by the force produced by wiggles dominates over the usual term in the matter era. While in the former case the usual term dominates over the term produced by the force induced by the scalar-tensor gravity in the matter era. However, this situation may change in the early Universe because the field-dependent coupling strength between matter and the scalar field $\alpha^2(\phi)$ is expected to be of the same order of the coupling to matter of the tensor component of gravity. Finally, it may be worthwhile to point out that we are comparing the wiggly string in General Relativity to the scalar-tensor string at purely formal level. One should keep in mind that the formation of wakes in these two backgrounds has different physical origin: in the wiggly case, it is induced by the perturbations along the string (the wiggles) and in our case, it is induced by the scalar-tensor gravity.

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