Modeling of peat tractor semi-trailer motion

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Abstract. Wheeled tractor vehicles are used to transport peat from open cast mines to storage stockpiles or processing facilities. This paper analyzes dynamic stability and control of an articulated vehicle. A mathematical model of linear dynamics is demonstrated for deformed supporting base and a two-tier transport unit consisting of an AWD tractor and a two-axle trailer. The functions obtained from analysis of maneuverability of the transport unit may be used in studies of operational and technical properties of the vehicles as calculation formulas to determine the trajectories of its components when transporting peat materials in peat mines. This analytic approach provided understanding of the propulsion system of the tractor vehicle and grounds for subsequent computer modeling of the process.

1. Introduction

Wheeled tractor vehicles are used to transport peat from open cast mines to storage stockpiles or processing facilities. The tractor transport unit is a combination of a power unit (tractor) and a transport unit (semi-truck) [1]. Transportation operations are related to process operation, usually they involve transfer of material without changes in its state.

Efficiency of movement of the tractor transport unit in peat extraction is facilitated by its cross-country performance and steerability. One of the most important operational properties of a tractor transport unit that influences its productivity is its steerability [2]. Characteristics of vehicle steerability on wet peat deposit significantly depend on its design, dimensions and weight of elements, the nature of their linkage, axle controlling laws, tire characteristics, modes and conditions of movement [3].

Energy property of a tractor transport unit is its ability to develop certain engine power and tractor pulling capacity. Tractor dynamics is related to mechanical modeling, as well as to mathematical description and analysis of the tractor systems [4, 5].

The general efficiency of a tractor transport unit operation depends on the efficiency of operation of its components. The process of substantiation of a quantitative composition of the tractor transport unit shall include the main characteristics of both the tractor and the semi-truck as separate test objects [3].

The subject of this paper is analysis of motion of the tractor transport unit used in peat extraction and consisting of an agricultural tractor and a semi-truck. The tractor dynamic models are important tools in research and development in the mining industry [6].
The objective of the work is to model the movement of the tractor transport unit in operational conditions in order to select the design parameters corresponding to the condition and modes of operation on weak soils.

2. Unit Description

The tractor transport unit is a mechanical linkage consisting of articulated transport elements. The principal kinematic and dynamic properties of the unit depend on physical phenomena taking place during its movement, which are in return determined by its structure. During consolidation of the tractor transport unit, its energy property does not have a fundamental importance in determining the quantitative composition of machinery in the unit, selecting their operational (in particular, speed-related) modes, etc. The main factor is flotation.

The flotation defines the ability of movement on weak soils without excessive immersion of wheels, when the possibility of movement depends on a ratio between the sliding surface resistance forces and the propulsive forces.

The main criteria in evaluation of unit's flotation is \[ \text{the trailing weight (adhesion weight) of the tractor } m_{tw} (G_{tw}) \] – a part of the tractor's weight that creates normal loads onto the tractor's driving wheels. The main factors influencing the traction between the tractor's wheels and the surface of peat deposit are:

- physical and mechanical properties of the soil (deposit type, shear strength, moisture content, etc.)
- weight per driving wheels – trailing weight \( G_{tw} \), characterized by a normal reaction of soil surface \( R_s \) for the driving wheels of the tractor;
- design of the undercarriage, wheels type and size (diameter, rim width, tire pressure, etc.)

For known values of the wheel traction coefficient on weak soil \( \mu \), and the total movement resistance coefficient \( \psi \) of the peat deposit, the condition of passability for a tractor transport unit with a semi-truck, a total weight of \( M_{st} \) and the trailing weight of the tractor of \( M_{tw} \) is their conformance with the inequality \[ \frac{M_{tw}}{M_{st}} \geq \frac{\psi}{\mu}. \]

For tractors on pneumatic tires, the traction coefficient between the tractor wheels and a weak soil is \( \mu = 0.35 \ldots 0.55 \). For wheeled 4WD tractors, the operating weight is their trailing weight \( G_{tw} \).

For agricultural general purpose wheeled tractor MTZ-1523 with wheels doubled and the body space of \( V_{tb} = 8 \text{ m}^3 \), the condition of passability is \( 0.67 > 0.4 \). The front wheels of the tractor are provided with 420/70R24 tires, the hind wheels – with 520/70R38 tires. Dimension of flotation tires of the semi-truck is 24.0/50-22.5.

Wheeled 4WD tractor is considered appropriate for peat extraction where the soil foundation is wet and soft, and tractor passability poses a serious problem.

3. System Description

When studying the movement modes of the unit, one may make a number of assumption that allow simplifying the movement equation and obtaining the formulas and criteria that determine the essential main parameters of maneuverability of the tractor transport unit with a constant hook load. Maneuverability analysis is performed at small speeds, without taking into account the dynamic effects, that is, from the point of view of kinematics of motion. The tractor transport unit is modeled as consisting of two articulated solid bodies that are moving in the horizontal plane at a constant travel speed [8].

At the first stage of solving the problem in the condition of small speeds and side accelerations, the influence of tire elasticity (lateral skid) on the trajectory of the tractor transport unit may be neglected. The movement of the tractor transport unit is assumed flat, parallel to a motionless supporting plane Q.
without account for vertical movement of its parts. Motion of each segment of the unit is described by
the motion of a figure that represents a projection of the segment onto the supporting plane, which has
an OXY coordinate system rigidly fixed onto it. The tractor's and semi-truck's weight forces $G_{tr}$ and $G_{st}$ are applied to the centers of gravity of those segments.

Simplification of the actual dynamic systems shall be based upon three main principles: research
objectives, operating modes and dynamic properties of the studied system [9].

It is possible to obtain a mathematical model of a wheeled machine by traditional methods using
Lagrange equation of the second kind on the basis of the analytical model. In order to analyze dynamic
parameters of the tractor under consideration, its design features, and to study the problem of
interaction between the wheeled tractor and a semi-truck, a diagram is proposed (Figure 1), where the
tractor and the semi-truck are idealized as two absolutely rigid bodies with their own centers of
gravity.

The bodies in the unit are articulated with single-degree joint and are installed on elastic
foundations, flexible in lateral, longitudinal and horizontal angular directions and characterized with
stiffness coefficients of $C_t$, $C_{st}$ [10, 11]. Its dynamics is described with a model that represents the
movements of the semi-truck and the tractor deviating them from the course [8]. According to Figure
1, the local coordinate systems $(x_1, y_1)$ and $(x_2, y_2)$ are centered at the centers of gravity of the tractor
and the semi-truck respectively [12].

When composing the motion equations for the system in question, the following assumptions may
be made [10]:
- the wheeled tractor in the unit with the semi-truck is moving steadily, without descents or
  ascensions, there are no translational vibrations;
- the tractor is symmetric with respect of its longitudinal plane;
- the translational speed of the unit is constant;
- the wheels are involved in continuous motion without lateral skidding;
- tire stiffness of both tractor and its semi-truck are constant;
- the left and right doubled tires on the axles of the tractor and semi-truck may be merged into
  the equivalent tires;
- the angulation and the gliding angle of the tires are small, the motion equations are linear.
- no aerodynamic forces are influencing the tractor transport unit.

Assumption of a steady motion of the system without translational fluctuations is common in
studying stability of motion and steerability of vehicles [13].

With these assumption, the position of the tractor transport unit with respect to the introduced
coordinate system is determined by a course angle of the leading component and coordinates of the
main trajectories of the points in the articulated component, which are functionally dependent on the
position of the tractor’s steerable wheels and the motion speed of the entire unit.
Stability of motion and steerability of the tractor transport unit may be improved by determining its rational design and operational parameters. Analysis of this problem should employ mathematical modeling of main motions, allowing evaluating a degree of influence that various parameters have over stability and steerability. The mathematical model of the unit shall be as close to the actual unit as possible, taking into account the interactions between its components and those between the unit and the supporting surface.

In this model, lateral forces in the left and right wheels of both the tractor and the semi-truck were deemed equal and summed up together [14, 15], as the influence of the lateral load transmission onto the total turning force is small for small angles of steering wheel rotation at the unit's speeds (<1.5 m/s).

The mathematical description of the motion of the tractor transport unit motion may be represented with Lagrange’s equation of the second kind [10, 11, 13]

\[
\frac{d}{dt} \frac{\partial T}{\partial q_i} - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = Q_i \quad i = 1 \ldots n,
\]

where \( n \) is the number of degrees of freedom of the system; \( T \) is the system's kinetic energy; \( U \) is the system's potential energy; \( q_i \) is the \( i \)-th joint variable; \( Q \) is a generalized force along the \( i \)-th joint variable.

The analysis uses a linear approach at small wheel slip angles. Let us apply the following coordinate relations reflected in Figure 1.

\[
x = x_1 = x_2;
\]

\[
y_2 = y_1 - h_1 \beta - a_2 \Theta,
\]

where \( h_1 \) is a distance between the tractor’s center of gravity and the coupling point; \( \beta \) is the sliding angle of the semi-truck; \( a_2 \) is a distance between the semi-truck’s center of gravity and the coupling point; \( \Theta \) is the coupling point’s sliding angle.
The kinetic energy of the system [11] \( T \) may be expressed as

\[
T = 0.5 m_1(\dot{x}^2 + \dot{y}_1^2) + 0.5m_2(\dot{x}^2 + \dot{y}_2^2) + 0.5I_1\dot{\beta}^2 + 0.5I_2\dot{\theta}^2 =
\]

\[
= 0.5(m_1 + m_2)(\dot{x}^2 + \dot{y}_1^2) + 0.5m_2(h_1^2\dot{\beta}^2 - 2h_1\dot{y}_1\dot{\beta})
\]

\[
+ 0.5m_2(a_2^2\dot{\theta}^2 - 2\dot{y}_1a_2\dot{\theta} + 2h_1\dot{\beta}a_2\dot{\theta}) + 0.5I_1\dot{\beta}^2 + 0.5I_2\dot{\theta}^2,
\]

where \( m_1 \) and \( m_2 \) are weights of the tractor and the semi-truck, respectively; \( I_1 \) and \( I_2 \) are the moments of inertia of the tractor and the semi-truck [12].

The mathematical description of the motion of the tractor transport unit has been made with an analytic method of engineering mechanics, namely, with Lagrange's equation of the second kind from the calculation diagram (Figure 1).

An expression for Lagrange's conditions

\[
\frac{d}{dt} \frac{\partial T}{\partial \dot{x}} = (m_1 + m_2)\ddot{x};
\]

\[
\frac{d}{dt} \frac{\partial T}{\partial \dot{y}} = (m_1 + m_2)\ddot{y}_1 - m_2h_1\dot{\beta} - m_2a_2\dot{\theta};
\]

\[
\frac{d}{dt} \frac{\partial T}{\partial \dot{\beta}} = m_2h_1(h_1\dot{\beta} + \ddot{y}_1) + m_2h_1a_2\dot{\theta} + l\dot{\beta};
\]

\[
\frac{d}{dt} \frac{\partial T}{\partial \dot{\theta}} = m_2a_2(a_2\dot{\theta} + \ddot{y}_1 + h_1\dot{\beta} + l\dot{\beta};
\]

\[
\frac{\partial T}{\partial q_i} = 0 \quad i = 1 \ldots n;
\]

\[
\frac{\partial T}{\partial U} = 0 \quad i = 1 \ldots n.
\]

Using the virtual work, one may evaluate the generalized forces, related to the joint variables [16]. The virtual work with accounts for the generalized forces

\[
\Delta W = (-F_{y1}\delta_1 - F_{y3}\varphi - F_{y4}\varphi)\Delta x + F_{y1}\Delta(y_1 + a_1\beta) + F_{y2}\Delta(b_1\beta) + F_{y3}\Delta(y_1 - h_1\beta - l_1\theta) + F_{y4}\Delta(y_1 - h_1\beta - l_3\theta).
\]

where \( \delta_1 \) is the steerable wheels' turning angle; \( \Delta \) is an angulation between the tractor and the semi-truck; \( F_{yki} \) is a lateral force created at the \( i \)-th tire on the axle \( k \) [12].

Generalized forces \( Q \) for each position

\[
Q_x = -F_{y1}\delta_1 - F_{y3}\varphi - F_{y3}\varphi;
\]

\[
Q_y = F_{y1} + F_{y2} + F_{y3} + F_{y3};
\]

\[
Q_\beta = a_1F_{y1} - b_1\beta - h_1F_{y3} - h_1F_{y3};
\]

\[
Q_\theta = -l_1F_{y31} - l_3F_{y32}.
\]

To express in the local coordinates with accounts for the speed of the tractor transport unit, let us apply the following correlations

\[
\dot{x} = u - v_1\beta;
\]

\[
\dot{y} = v_1 + u\beta;
\]

\[
\dot{\beta} = r_1;
\]

\[
\theta = \beta + \varphi.
\]
Thus, the forces on tires of the tractor and the semi-truck in local coordinates are [11, 6, 17] (Figure 1)

\[
F_{y1} = -\frac{1}{u} C_1 (v_1 + a_1 r_1) + C_1  \delta_1;
\]
\[
F_{y2} = -\frac{1}{u} C_2 (v_1 - b_1 r_1);
\]
\[
F_{y31} = -\frac{1}{u} C_{31} (v_1 - h_1 r_1 - l_2 \dot{\theta}) + C_{31} \varphi;
\]
\[
F_{y32} = -\frac{1}{u} C_{32} (v_1 - h_1 r_1 - l_3 \dot{\theta}) + C_{32} \varphi.
\]

where \( v_1 \) is a lateral speed in the tractor's center of gravity; \( r_1 \) is a yawing velocity of the tractor.

The motion equations may be expressed using only the local coordinates. System state vector

\[
x = [v_1 \ r_1 \ \dot{\varphi} \ \varphi]^T.
\]

The first equation that describes the longitudinal dynamics is not taken into account, as the traveling speed \( u \) is assumed constant. The following shortcuts were used [16, 18]

\[
C = C_1 + C_2;
\]
\[
C_t = C_{31} + C_{32};
\]
\[
c_{s1} = a_1 c_1 - b_1 c_2;
\]
\[
c_{q1}^2 = a_1^2 c_1 + b_1^2 c_2.
\]

where \( C \) is a total stiffness of the tractor in turns; \( C_{1,2} \) is the stiffness on axles during turns; \( C_t \) is a total stiffness of the semi-truck during turns; \( s_1 \) is a distance along the longitudinal axis of the vehicle from its center of gravity to the non-slip point of the turn; \( q_1 \) is a length corresponding to the medium moment in turn.

Motion equations of a tractor transport unit [16, 11]

\[
(m_1 + m_2) ( \ddot{v}_1 + u r_1 ) - \{ m_2 (h_1 + a_2) \} \dot{r}_1 - m_2 a_2 \ddot{\varphi} = -\frac{1}{u} \left[ (C + C_t) v_1 + \{ C_{s1} - C_{31} (h_1 + l_2) - C_{32} (h_1 + l_3) \} r_1 - (C_{31} l_2 + C_{32} l_3) \ddot{\varphi} - C_t u \varphi \right] + C_1 \delta_1 - h m_2 ( v_1 + u r_1 ) + \{ l_1 + m_2 h_1 (h_1 + a_2) \} \dot{r}_1 + m_2 h_1 a_2 \ddot{\varphi} = -\frac{1}{u} \left[ (C_{s1} - C_t h_1) v_1 + \{ C_{q1}^2 + C_{31} h_1 (h_1 + l_2) + C_{32} h_1 (h_1 + l_3) \} r_1 + \right. \\
+ (C_{31} h_1 l_2 + C_{32} h_1 l_3) \ddot{\varphi} + C_t h_1 u \varphi \right] + C_{k1} \delta_1 - m_2 a_2 ( \dot{v}_1 + u r_1 ) + \{ l_2 + m_2 a_2 (h_1 + a_2) \} \dot{r}_1 + \\
\left. + (l_2 + m_2 a_2^2) \ddot{\varphi} = -\frac{1}{u} \left[ (C_{31} l_2 + C_{32} l_3) v_1 + \{ C_{31} l_2 (h_1 + l_2) + C_{32} l_3 (h_1 + l_3) \} r_1 + \{ C_{31} l_2^2 + 
\right. \\
\left. + C_{32} l_3^2 \ddot{\varphi} + C_{31} l_2 + C_{32} l_3 u \varphi \right],
\]

where \( v_1 \) is a lateral speed in the tractor's center of gravity; \( r_1 \) is the yawing velocity of the tractor (\( r_1 = \dot{\beta} \)); \( \varphi \) is a relative angulation between the tractor and the semi-truck. The turning angle of the semi-truck is defined as \( \theta \) when \( \theta = \beta + \varphi \).

4. Conclusion

The mathematical model describing motion of a tractor with a semi-truck allows calculating current acceleration values from the values of forces and moments acting unto a peat transport unit. Passability conditions of the tractor transport unit were also studied in order to evaluate the parameters of the tractor's and semi-truck's tires. The functions obtained from analysis of maneuverability of the transport unit may be used in studies of operational and technical properties of the vehicles as calculation formulas to determine the trajectories of its components when transporting peat materials.
in peat mines. This analytic approach provided understanding of the propulsion system of the tractor vehicle and grounds for subsequent computer modeling of the process.

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