Abstract

Motivated by the possible large direct $CP$ asymmetry of $\bar{B}_d^0 \rightarrow D^+ D^-$ decay measured by Belle collaboration, we investigate double charm $B_{u,d}$ and $B_s$ decays in the minimal supersymmetric standard model with R-parity violation. We derive the bounds on relevant R-parity violating couplings from the current experimental data, which show quite consistent measurements among relative collaborations. Using the constrained parameter spaces, we explore R-parity violating effects on other observables in these decays, which have not been measured or have not been well measured yet. We find that the R-parity violating effects on the mixing-induced $CP$ asymmetries of $\bar{B}_d^0 \rightarrow D^{(*)+} D^{(*)-}$ and $\bar{B}_s^0 \rightarrow D_s^{(*)+} D_s^{(*)-}$ decays could be very large, nevertheless the R-parity violating effects on the direct $CP$ asymmetries could not be large enough to explain the large direct $CP$ violation of $\bar{B}_d^0 \rightarrow D^+ D^-$ from Belle. Our results could be used to probe R-parity violating effects and will correlate with searches for direct R-parity violating signals in future experiments.

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1 Introduction

Double charm decays of $B_{u,d}$ and $B_s$ provide us with a rich field to study $CP$ violation and final-state interactions as well as to extract information of Cabibbo-Kobayashi-Maskawa (CKM) elements. $CP$ asymmetries (CPAs) in these decays play important roles in testing the Standard Model (SM) as well as exploring new physics (NP) \cite{1, 2}.

Double charm decays, $\bar{B}_d^0 \to D^{(*)}+D^{(*)}$, $B_u^- \to D^{(*)0}D^{(*)}$ and $\bar{B}_s^0 \to D_s^{(*)}+D^{(*)}$, are dominated by color-allowed tree $b \to c\bar{c}d$ transition, but involve small penguin pollution from $b \to u\bar{u}d$ transition carrying a different weak phase. The latter contributions lead to direct CP As, which are very small (about the order of $10^{-2}$) in the SM. If penguin corrections are neglected, the SM predictions for the direct CPAs would be zero. It is interesting to note that both BABAR and Belle have measured the direct CPA in $B_d^0 \to D^+D^-$ decay

$$C(B_d^0, \bar{B}_d^0 \to D^+D^-) = \begin{cases} -0.91 \pm 0.23 \pm 0.06 \text{ (Belle \cite{3})}, \\ -0.07 \pm 0.23 \pm 0.03 \text{ (BABAR \cite{4})} \end{cases}$$

respectively. One would find the difference between the two measurements is

$$\Delta C = 0.84 \pm 0.32,$$  \hspace{1cm} (2)

i.e., the difference is as large as $2.7\sigma$. So far, such a large direct CPA has not been observed in the other measurements of $\bar{B}_d^0 \to D^{(*)}+D^{(*)}$, $B_u^- \to D^{(*)0}D^{(*)}$ and $\bar{B}_s^0 \to D_s^{(*)}+D^{(*)}$ decays \cite{4, 5, 6, 7, 8, 9, 10, 11}, which involve the same quark level weak decays. If the large CP violation in $\bar{B}_d^0 \to D^+D^-$ from Belle is true, it would establish the presence of NP. At present one cannot conclude the presence of NP in those decays. Equivalently one also cannot take the CPAs are in agreement with the SM expectations. Recently the large direct CPA in $\bar{B}_d^0 \to D^+D^-$ has been investigated with possible NP scenarios, such as unparticle interaction \cite{12} and the NP effects in electroweak penguin sector \cite{13, 14}, and so on.

In this paper, we would like to investigate $B_{u,d}$ and $B_s$ double charm decays systematically in the minimal supersymmetric standard model (MSSM) \cite{15, 16} with R-parity violation \cite{17, 18}. In the literature, the possible appearance of the R-parity violating (RPV) couplings \cite{17, 18}, which violate the lepton and/or baryon number conservations, has gained full attention in searching for supersymmetry \cite{19, 20}. The effects of supersymmetry with R-parity violation in $B$ meson decays have been extensively investigated, for instance in Refs. \cite{21, 22, 23, 24}. In our work, twenty-four double charm decays $\bar{B}_d^0 \to D^{(*)}+D^{(*)}$, $B_u^- \to D^{(*)0}D^{(*)}$ and $\bar{B}_s^0 \to D^{(*)}+D^{(*)}$,
\(D_s^{(*)} D_{(s)}^{(*)-}\) are studied in the RPV MSSM. For simplicity we employ naive factorization for the hadronic dynamics, which is expected to be reliable for the color-allowed amplitudes, which are dominant contributions in those double charm decays.

The color-allowed tree level dominated decays of \(b \to c\bar{c}d\), i.e. \(\bar{B}_d^0 \to D_s^{(*)+} D_{(s)}^{(*)-}\), \(B_u^- \to D_{(s)}^{(*)0} D_{(s)}^{(*)-}\) and \(\bar{B}_s^0 \to D_{(s)}^{(*)+} D_{(s)}^{(*)-}\), involve the same set of RPV coupling constants. For these processes, besides the CPA in \(\bar{B}_d^0 \to D_s^{(*)+} D_{(s)}^{(*)-}\), a few other observables in \(B_{u,d} \to D_s^{(*)+} D_{(s)}^{(*)-}\) have been already measured by BABAR and Belle collaborations [25, 26]. To derive constraints on the relevant RPV couplings, we will choose a set of data from the aforementioned measurements which have quite high consistency between the measurements of BABAR and Belle. Then, using the constrained RPV coupling parameter spaces, we predict the RPV effects on the other observables in \(B_{u,d} \to D_s^{(*)+} D_{(s)}^{(*)-}\) decays, whose measurements from BABAR and Belle are not compatible within 2σ error range, and/or which have not been measured yet. One of our goals is to see how large the direct CPA of \(\bar{B}_d^0 \to D_s^{(*)+} D_{(s)}^{(*)-}\) can be within the constrained parameter spaces. We find that the lower limit of \(C(B_{(s)}^0, B_{(s)}^0 \to D_s^{(*)+} D_{(s)}^{(*)-})\) could be just slightly decreased by the RPV couplings, and the RPV effects on this quantity are not large enough to explain the large direct \(CP\) violation from Belle, although the mixing-induced CPAs of \(\bar{B}_d^0 \to D_s^{(*)+} D_{(s)}^{(*)-}\) are very sensitive to the RPV couplings.

Decays \(\bar{B}_d^0 \to D_{(s)}^{(*)+} D_{(s)}^{(*)-}\), \(B_u^- \to D_{(s)}^{(*)0} D_{(s)}^{(*)-}\) and \(\bar{B}_s^0 \to D_{(s)}^{(*)+} D_{(s)}^{(*)-}\) are governed by the \(b \to c\bar{c}s\) transition at the quark level, and also involve the same set of RPV coupling constants. They have similar properties to \(\bar{B}_d^0 \to D_{(s)}^{(*)+} D_{(s)}^{(*)-}\), \(B_u^- \to D_{(s)}^{(*)0} D_{(s)}^{(*)-}\) and \(\bar{B}_s^0 \to D_{(s)}^{(*)+} D_{(s)}^{(*)-}\) decays, nevertheless the penguin effects are less Cabibbo-suppressed. For these decays, most branching ratios and one longitudinal polarization have been measured [27, 28, 30, 31, 32, 33, 34]. We will take the same strategy as the one for \(b \to c\bar{c}d\) decays to constrain relevant RPV couplings and estimate RPV effects in these decays. We find that RPV couplings could significantly affect the CPAs of these decays, and could flip their signs.

Our paper is organized as follows: In Sec. 2, we briefly introduce the theoretical framework for the double charm \(B_{u,d}\) and \(B_s\) decays in the RPV MSSM, and we tabulate all the theoretical input parameters. In Sec. 3, we deal with the numerical results and our discussions. At first, we give the SM predictions with full uncertainties of the input parameters. Then, we derive the constrained parameter spaces which satisfy all the experimental data with high consistency between different collaborations. Finally, we predict the RPV effects on other quantities, which
have not been measured or have not been well measured yet. Section 4 contains our summary.

2 Theoretical Framework

2.1 Decay amplitudes in the SM

In the SM, the low energy effective Hamiltonian for $\Delta B = 1$ transition at a scale $\mu$ is given by

$$\mathcal{H}_{\text{eff}}^{\text{SM}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left\{ C_1 Q_p^B + C_2 Q_p^B + \sum_{i=3}^{10} C_i Q_{i\gamma} + C_{7\gamma} Q_{7\gamma} + C_{8\gamma} Q_{8\gamma} \right\} + h.c., \quad (3)$$

here $\lambda_p = V_{pb} V_{pq}^*$ for $b \to q$ transition ($p \in \{u,c\}, q \in \{d,s\}$). The detailed definition of the effective Hamiltonian can be found in [35].

It is empirically observed that naive factorization [25] still works reasonably well in the color-allowed double charm $B_{u,d}$ and $B_s$ decay processes. We will describe the $B \to D^{(*)} D_q^{(*)}$ decay amplitudes within the naive factorization approximation in this paper. Under the naive factorization approximation, the factorized matrix elements are given by

$$A_{[BD^{(*)},D_q^{(*)}]} \equiv \left\langle D^{(*)}_q | \bar{q} \gamma^\mu (1 - \gamma_5) c | 0 \right\rangle \left\langle D^{(*)} | \bar{c} \gamma\mu (1 - \gamma_5) b | B \right\rangle. \quad (4)$$

Decay constants and form factors [36, 37] are usually defined as

$$\langle D_q(p_{D_q}) | \bar{q} \gamma^\mu \gamma_5 c | 0 \rangle = -if_{D_q} p_{D_q}^\mu, \quad (5)$$

$$\langle D_q^*(p_{D_q}^*) | \bar{q} \gamma^\mu c | 0 \rangle = f_{D_q}^* p_{D_q}^\mu, \quad (6)$$

$$\langle D(p_D) | \bar{c} \gamma\mu b | B(p_B) \rangle = \left[ (p_B + p_D)_\mu - \frac{m_B^2 - m_D^2}{q^2} q_\mu \right] F_1(q^2) + \frac{m_B^2 - m_D^2}{q^2} q_\mu F_0(q^2), \quad (7)$$

$$\langle D^*(p_D^*, \varepsilon^*) | \bar{c} \gamma\mu b | B(p_B) \rangle = \frac{2V(q^2)}{m_B + m_D^*} \epsilon_{\mu\nu\alpha\beta} \varepsilon^{\nu\alpha} p_{D^*}^\beta, \quad (8)$$

$$\langle D^*(p_D^*, \varepsilon^*) | \bar{c} \gamma_\mu \gamma_5 b | B(p_B) \rangle = i \left[ \varepsilon^\mu (m_B + m_{D^*}) A_1(q^2) - (p_B + p_{D^*})_\mu (\varepsilon^* \cdot p_B) \frac{A_2(q^2)}{m_B + m_{D^*}} \right]$$

$$-iq_\mu (\varepsilon^* \cdot p_B) \frac{2m_{D^*}}{q^2} [A_3(q^2) - A_0(q^2)], \quad (9)$$

with $q = p_B - p_{D^*}$. Then we can express $A_{[BD^{(*)},D_q^{(*)}]}$ in terms of decay constants and form
factors as follows

\[ A_{[BD^{(*)},D_q^{(*)}]} = \begin{cases} 
if_{D_q}(m_B^2 - m_D^2)F_0(m_{D_q}^2) & (D\bar{D}_q), \\
2f_{D_q}m_B|p_c|F_1(m_{D_q}^2) & (D\bar{D}_q^*), \\
-2f_{D_q}m_B|p_c|A_0(m_{D_q}^2) & (D^*\bar{D}_q), \\
-if_{D_q}^*m_{D_q}^* \left( (\varepsilon_D^{*} \cdot \varepsilon_D^{*}) (m_B + m_D^*) A_1(m_{D_q}^2) \\
- (\varepsilon_D^{*} \cdot p_D^*) (\varepsilon_D^{*} \cdot p_D^*) \frac{2A_2(m_{D_q}^2)}{m_B + m_D^*} \\
+ i\epsilon_{\mu \nu \alpha \beta} \varepsilon_D^\mu \varepsilon_D^{*\nu} p_{D_q}^\alpha p_{D_q}^{*\beta} \frac{2V(m_{D_q}^2)}{m_B + m_D^*} \right) & (D^*\bar{D}_q^*). 
\end{cases} \tag{10} \]

Decays \( B \to D^{(*)} D_q^{(*)} \) may occur through both tree level and loop induced (penguin) quark diagrams, and the SM decay amplitudes within the naive factorization are given as

\[ \mathcal{M}^{SM}(B \to D^{(*)} D_q^{(*)}) = \frac{G_F}{\sqrt{2}} \left( \lambda_c a^c_1 + \sum_{p = u, c} \lambda_p [a^p_4 + a^p_{10} + \xi (a^p_6 + a^p_8)] \right) A_{[BD^{(*)},D_q^{(*)}]} \tag{11} \]

where the coefficients \( a^p_i = \left( C_i + \frac{C_{i+1}}{N_c} \right) + P^p_i \) with the upper (lower) sign applied when \( i \) is odd (even), and \( P^p_i \) account for penguin contractions. The factorization parameter \( \xi \) in Eq. (11) arises from the transformation of \( (V - A)(V + A) \) currents into \( (V - A)(V - A) \) ones for the penguin operators \( Q_5, \ldots, Q_8 \), and it depends on properties of the final-state mesons

\[ \xi = \begin{cases} 
\frac{2m_{D_q}^2}{m_c + m_q} & (D\bar{D}_q), \\
0 & (D\bar{D}_q^*), \\
-\frac{2m_{D_q}^2}{m_c + m_q} & (D^*\bar{D}_q), \\
0 & (D^*\bar{D}_q^*). 
\end{cases} \tag{12} \]

For the penguin contractions, we will consider not only QCD and electroweak penguin operator contributions but also contributions from the electromagnetic and chromomagnetic dipole operators. \( P^p_i \) are given as follows

\[ P^p_1 = 0, \]
\[ P^p_4 = \frac{\alpha_s}{9\pi} \left\{ C_1 \left[ \frac{10}{9} - G_{D_q^{(*)}}(m_p) \right] - 2F_1 C^e_{8g} \right\}, \]
\[ P^p_6 = \frac{\alpha_s}{9\pi} \left\{ C_1 \left[ \frac{10}{9} - G_{D_q^{(*)}}(m_p) \right] - 2F_2 C^e_{8g} \right\}, \]
\[ P^p_8 = \frac{\alpha_e}{9\pi N_c} \left\{ (C_1 + N_c C_2) \left[ \frac{10}{9} - G_{D_q^{(*)}}(m_p) \right] - 3F_2 C^e_{7\gamma} \right\}, \]
\[ P^p_{10} = \frac{\alpha_e}{9\pi N_c} \left\{ (C_1 + N_c C_2) \left[ \frac{10}{9} - G_{D_q^{(*)}}(m_p) \right] - 3F_1 C^e_{7\gamma} \right\}, \tag{13} \]
where the penguin loop-integral function $G_{D_q}(m_p)$ is given by

$$G_{D_q}(m_p) = \int_0^1 duG(m_p,k)\Phi_{D_q}(u),$$

$$G(m_p,k) = -4\int_0^1 dx(1-x)\ln\left[\frac{m_p^2-k^2x(1-x)}{m_b^2}\right] - ie,$$

with the penguin momentum transfer $k^2 = m_c^2 + \bar{u}(m_b^2 - m_c^2 - m_{M_2}^2) + \bar{u}^2 m_{M_2}^2$, where $\bar{u} \equiv 1 - u$. In the function $G_{D_q}(m_p)$, we have used a $D_q^*$ meson-emitting distribution amplitude $\Phi_{D_q^*}(u) = 6u(1-u)[1+a_{D_q^*}(1-2u)]$, instead of keeping $k^2$ as a free parameter as usual. The constants $F_1$ and $F_2$ in Eq. (13) are defined by

$$F_1 = \begin{cases} 
\int_0^1 du\Phi_{D_q}(u)\frac{m_b^2}{k^2}(um_b + \frac{2um_{D_q}}{m_b-m_c}\epsilon_2\cdot p_1 - um_c) & (DD_q), \\
\int_0^1 du\Phi_{D_q^*}(u)\frac{m_b^2}{k^2}(um_b + \frac{2um_{D_q^*}}{m_b+m_c}\epsilon_2\cdot p_1 + um_c) & (D^*D_q), \\
\int_0^1 du\Phi_{D_q}(u)\frac{m_b^2}{k^2}(\bar{u}m_b - m_c) & (DD_q^*), \\
\int_0^1 du\Phi_{D_q^*}(u)\frac{m_b^2}{k^2}(\bar{u}m_b + m_c) & (D^*D_q^*), \\
\end{cases}$$

$$F_2 = \begin{cases} 
\int_0^1 du\Phi_{D_q}(u)\frac{m_b^2}{k^2}[\bar{u}(m_b - m_c) + m_c] & (DD_q), \\
0 & (DD_q^*), \\
\int_0^1 du\Phi_{D_q}(u)\frac{m_b^2}{k^2}[\bar{u}(m_b + m_c) - m_c] & (D^*D_q), \\
0 & (D^*D_q^*), \\
\end{cases}$$

where $\epsilon_{2L}^*, p_1 \approx (m_b^2 - m_{M_2}^2 - m_c^2)/(2m_{M_2}^2)$ and $\epsilon_{2T}^*, p_1 = 0$ for $B \rightarrow D^*D_q^*$ decays.

2.2 Decay amplitudes of the RPV contributions

In the RPV MSSM, in terms of the RPV superpotential [17], we can obtain the relative RPV effective Hamiltonian for $B \rightarrow D^{(*)} D_q^{(*)}$ decays as following

$$\mathcal{H}^{\text{RPV}}_{\text{eff}} = \sum_n \chi^{\mu n}_i\chi^{\mu n}_{j\ell} \eta^{-4/30} \left[ -(\bar{d}_k\gamma^\mu P_R u_j)\bar{u}_i\gamma_\mu P_R d_i)_1 + (\bar{d}_k\gamma^\mu P_R u_j)\bar{u}_i\gamma_\mu P_R d_i)_8 \right]$$

$$+ \sum_i \chi^{\mu n}_{i\ell} \chi^{\mu n}_{j\ell} \eta^{-8/30} (\bar{d}_k P_L u_j)\bar{u}_i P_R d_i)_1 + \text{h.c.},$$

where $P_L = \frac{1-\gamma_5}{2}$, $P_R = \frac{1+\gamma_5}{2}$, $\eta = \frac{\alpha_s(m_f)}{\alpha_s(m_b)}$ and $\beta_0 = 11 - \frac{2}{3} n_f$. The subscripts 1 and 8 of the currents represent the currents in the color singlet and octet, respectively. The coefficients $\eta^{-4/30}$ and $\eta^{-8/30}$ are due to the running from sfermion mass scale $m_{f}$ (assumed as 100 GeV) down to $m_b$ scale. Since it is usually assumed in phenomenology for numerical display that only one sfermion contributes at one time, we neglect the mixing between the operators when we use the renormalization group equation (RGE) to run $\mathcal{H}^{\text{RPV}}_{\text{eff}}$ down to the low scale.
The decay amplitudes of RPV contributions to $B \to D^{(*)} D_{q}^{(*)}$ are given by

\[
\mathcal{M}^{\text{RPV}}(B \to D^{(*)} D_{q}^{(*)}) = \Lambda' \left( -1 + \frac{1}{N_{C}} \right) \left\langle D_{q}^{(*)} \right| \bar{q} \gamma_{\mu}(1 + \gamma_{5})c \left| 0 \right\rangle \left\langle D^{(*)} \right| \bar{c} \gamma_{\mu}(1 + \gamma_{5})b \left| B \right\rangle \\
+ 2 \Lambda' \left\langle D_{q}^{(*)} \right| \bar{q}(1 - \gamma_{5})c \left| 0 \right\rangle \left\langle D^{(*)} \right| \bar{c}(1 + \gamma_{5})b \left| B \right\rangle,
\]

(19)

where $\Lambda'' \equiv \eta^{-4/3} \frac{\lambda_{23}^{f} \lambda_{23}^{b} \lambda_{24}^{f} \lambda_{24}^{b}}{8m_{d}^{2}} \left( \frac{\lambda_{23}^{f} \lambda_{23}^{b} \lambda_{24}^{f} \lambda_{24}^{b}}{8m_{d}^{2}} \right)$ and $\Lambda' \equiv \eta^{-8/3} \sum_{i} \frac{\lambda_{12}^{f} \lambda_{12}^{b} \lambda_{24}^{f} \lambda_{24}^{b}}{8m_{d}^{2}} \left( \frac{\lambda_{12}^{f} \lambda_{12}^{b} \lambda_{24}^{f} \lambda_{24}^{b}}{8m_{d}^{2}} \right)$ for $q = d$ ($q = s$).

$A'_{[BD^{*},D_{q}^{*}]}$ is defined by

\[
A'_{[BD^{*},D_{q}^{*}]} \equiv \frac{i f_{D^{*}} m_{D_{q}^{*}}}{2} \left[ (\varepsilon_{D}^{*} \cdot \varepsilon_{D_{q}^{*}}) (m_{B} + m_{D^{*}}) A_{1}(m_{D_{q}^{*}}^{2}) - (\varepsilon_{D}^{*} \cdot p_{D^{*}}) (\varepsilon_{D_{q}^{*}}^{*} \cdot p_{D^{*}}) \frac{2 A_{2}(m_{D_{q}^{*}}^{2})}{m_{B} + m_{D^{*}}} - i \epsilon_{\mu \nu \alpha \beta} \varepsilon_{D}^{* \mu} \varepsilon_{D_{q}^{*} \nu} p_{D^{*}}^{\alpha} p_{D^{*}}^{\beta} \frac{2 V(m_{D_{q}^{*}}^{2})}{m_{B} + m_{D^{*}}} \right].
\]

(21)

### 2.3 Observables to be investigated

We can get the total decay amplitudes in the RPV MSSM as

\[
\mathcal{M}(B \to D^{(*)} D_{q}^{(*)}) = \mathcal{M}^{\text{SM}}(B \to D^{(*)} D_{q}^{(*)}) + \mathcal{M}^{\text{RPV}}(B \to D^{(*)} D_{q}^{(*)}).
\]

(22)

The branching ratio $\mathcal{B}$ reads as

\[
\mathcal{B}(B \to D^{(*)} D_{q}^{(*)}) = \frac{\tau_{B}|p_{c}|}{8\pi m_{B}^{2}} \left| \mathcal{M}(B \to D^{(*)} D_{q}^{(*)}) \right|^{2},
\]

(23)

where $\tau_{B}$ is the $B$ lifetime, $|p_{c}|$ is the center of mass momentum in the center of mass frame of $B$ meson. In $B \to D^{*} D_{q}^{*}$ decays, the two vector mesons have the same helicity, therefore three different polarization states, one longitudinal and two transverse, are possible. We define the corresponding amplitudes as $\mathcal{M}_{0,\pm}$ in the helicity basis and $\mathcal{M}_{L,\parallel,\perp}$ in the transversity basis, which are related by $\mathcal{M}_{L} = \mathcal{M}_{0}$ and $\mathcal{M}_{\parallel,\perp} = \frac{M_{L,\parallel,\perp}}{\sqrt{2}}$. Then we have

\[
\left| \mathcal{M}(B \to D^{*} D_{q}^{*}) \right|^{2} = |\mathcal{M}_{0}|^{2} + |\mathcal{M}_{\pm}|^{2} + |\mathcal{M}_{-}|^{2} = |\mathcal{M}_{L}|^{2} + |\mathcal{M}_{\parallel}|^{2} + |\mathcal{M}_{\perp}|^{2}.
\]

(24)

The longitudinal polarization fraction $f_{L}$ and transverse polarization fraction $f_{\perp}$ are defined by

\[
f_{L,\perp}(B \to D^{*} D_{q}^{*}) = \frac{\Gamma_{L,\perp}}{\Gamma} = \frac{|\mathcal{M}_{L,\perp}|^{2}}{|\mathcal{M}_{L}|^{2} + |\mathcal{M}_{\parallel}|^{2} + |\mathcal{M}_{\perp}|^{2}}.
\]

(25)
In charged $B$ meson decays, where mixing effects are absent, the only possible source of CPAs is
\[
A_{CP}^{k, \text{dir}} \equiv \frac{|M_k(B^- \to \bar{f})/M_k(B^+ \to f)|^2 - 1}{|M_k(B^- \to \bar{f})/M_k(B^+ \to f)|^2 + 1},
\]
and $k = L, \|, \perp$ for $B^- \to D^+D_s^*$ decays and $k = L$ for $B_u^- \to DD_q, DD_s^*, D^*D_q$ decays. Then for $B_u^- \to D^*D_q^*$ decays, we have
\[
A_{CP}^{+, \text{dir}}(B \to D^*D_q^*) = \frac{A_{CP}^{||, \text{dir}} |M||^2 + A_{CP}^{L, \text{dir}} |M_L|^2}{|M||^2 + |M_L|^2}.
\]

For CPAs of neutral $B_q$ meson decays, there is an additional complication due to $B_q^0 - \bar{B}_q^0$ mixing. There are four cases that one encounters for neutral $B_q$ decays, as discussed in Refs.\[38, 39, 40, 41\].

(i) $B_q^0 \to f, \bar{B}_q^0 \to \bar{f}$, where $f$ or $\bar{f}$ is not a common final state of $B_q^0$ and $\bar{B}_q^0$, for example $B_q^0 \to D^+D_s^-$. 

(ii) $B_q^0 \to (f = \bar{f}) \leftrightarrow \bar{B}_q^0$ with $f^{\text{CP}} = \pm f$, involving final states which are CP eigenstates, i.e., decays such as $B_q^0 \to D^+D^-, B_q^0 \to D_s^+D_s^-$. 

(iii) $B_q^0 \to (f = \bar{f}) \leftrightarrow \bar{B}_q^0$ with $f^{\text{CP}} \neq \pm f$, involving final states which are not CP eigenstates. They include decays such as $B_q^0 \to (VV)^0$, as the $VV$ states are not CP eigenstates. 

(iv) $B_q^0 \to (f \& \bar{f}) \leftrightarrow \bar{B}_q^0$ with $f^{\text{CP}} \neq f$, i.e., both $f$ and $\bar{f}$ are common final states of $B_q^0$ and $\bar{B}_q^0$, but they are not CP eigenstates. Decays $B_q^0(\bar{B}_q^0) \to D^{*-}D^+, D^-D^{*+}$ and $B_q^0(\bar{B}_q^0) \to D_s^{-}-D_s^+, D_s^-D_s^{*+}$ belong to this case.

CPAs of neutral $B$ decays in case (i) are similar to CPAs of the charged $B$ decays, and there are only direct CPAs $A_{CP}^{\text{dir}}$ since no mixing is involved for these decays. For cases (ii) and (iii), their CPAs would involve $B_q^0 - \bar{B}_q^0$ mixing. The time-dependent asymmetries can be conveniently expressed as
\[
A_f^k(t) = S_f^k \sin(\Delta mt) - C_f^k \cos(\Delta mt),
\]
\[
S_f^k \equiv \frac{2 \text{Im}(\lambda_k)}{1 + |\lambda_k|^2}, \quad C_f^k \equiv \frac{1 - |\lambda_k|^2}{1 + |\lambda_k|^2},
\]
where $\lambda_k = \frac{4 M_k(B_q^0 \to f)}{p M_k(B_q^0 \to f)}$. In addition, $S_f^+$ and $C_f^+$ can be obtained from the similar relation given in Eq. (27).
Case (iv) also involves mixing but requires additional formulae. Here one studies the four time-dependent decay widths for $B^0_q(t) \rightarrow f$, $\bar{B}^0_q(t) \rightarrow \bar{f}$, $B^0_q(t) \rightarrow \bar{f}$ and $\bar{B}^0_q(t) \rightarrow f$ [38, 39, 40, 41]. These time-dependent widths can be expressed by four basic matrix elements [40]

$$g = \langle f | \mathcal{H}_{\text{eff}} | B^0_q \rangle, \quad h = \langle f | \mathcal{H}_{\text{eff}} | \bar{B}^0_q \rangle,$$

$$\bar{g} = \langle \bar{f} | \mathcal{H}_{\text{eff}} | \bar{B}^0_q \rangle, \quad \bar{h} = \langle \bar{f} | \mathcal{H}_{\text{eff}} | B^0_q \rangle,$$

which determine the decay matrix elements of $B^0_q \rightarrow f, \bar{f}$ and of $\bar{B}^0_q \rightarrow f, \bar{f}$ at $t = 0$. We will study the following quantities

$$S^k_f = \frac{2 \text{Im}(\lambda'_k)}{1 + |\lambda'_k|^2}, \quad C^k_f = \frac{1 - |\lambda'_k|^2}{1 + |\lambda'_k|^2},$$

$$S^k_{\bar{f}} = \frac{2 \text{Im}(\lambda''_k)}{1 + |\lambda''_k|^2}, \quad C^k_{\bar{f}} = \frac{1 - |\lambda''_k|^2}{1 + |\lambda''_k|^2},$$

with $\lambda'_k = (q/p)(h/g)$ and $\lambda''_k = (q/p)(\bar{g}/\bar{h})$. The signatures of $CP$ violation are $\Gamma(B^0_q(t) \rightarrow \bar{f}) \neq \Gamma(B^0_q(t) \rightarrow f)$ and $\Gamma(\bar{B}^0_q(t) \rightarrow f) \neq \Gamma(B^0_q(t) \rightarrow \bar{f})$, which means that $C_f \neq -C_{\bar{f}}$ and/or $S_f \neq -S_{\bar{f}}$.

### 2.4 Input parameters

Theoretical input parameters are collected in Table [1]. In our numerical results, we will use the input parameters which are varied randomly within $1\sigma$ range.

We have several remarks on the input parameters:

- **CKM matrix elements:** The weak phase $\gamma$ is well constrained in the SM, however, with the presence of R-parity violation, this constraint may be relaxed. We will not take $\gamma$ within the SM range, but vary it randomly in the range of 0 to $\pi$ to obtain conservative limits on RPV couplings.

- **Decay constants:** The decay constants of $D^*_q$ mesons have not been directly measured in experiments so far. In the heavy-quark limit ($m_c \rightarrow \infty$), spin symmetry predicts that $f_{D^*_q} = f_{D_q}$, and most theoretical predictions indicate that symmetry-breaking corrections enhance the ratio $f_{D^*_q}/f_{D_q}$ by $10\% - 20\%$ [45, 46]. Hence, we take $f_{D^*_q} = (1.1 - 1.2)f_{D_q}$ as our input values.

- **Distribution amplitudes:** The distribution amplitudes of $D^{(*)}_q$ mesons are less constrained, and we use the shape parameter $a_{D^{(*)}} = 0.7 \pm 0.2$ and $a_{D^{(*)}} = 0.3 \pm 0.2$. 


When we study the RPV effects, we consider only one RPV coupling for the form factors involving $W$. We obtain Wilson coefficients in terms of the expressions in [35].

Table 1: Summary of theoretical input parameters and $\pm 1\sigma$ error ranges for sensitive parameters used in our numerical calculations.

| Parameter | Value [GeV] |
|-----------|-------------|
| $m_{B_u}$ | 5.279 |
| $m_{B_d}$ | 5.280 |
| $m_{B_s}$ | 5.366 |
| $M_{D^0}$ | 1.865 |
| $M_{D^+}$ | 1.870 |
| $M_{D_s^+}$ | 1.969 |
| $M_{D^{*+}}$ | 2.007 |
| $M_{D_s^{*+}}$ | 2.010 |
| $m_{B_s}(2\text{ GeV})$ | 0.095 ± 0.025 |
| $m_{B_d}(2\text{ GeV})$ | 0.0015 ± 0.0030 |
| $\tau_{B_u}$ | 1.638 ± 0.011 |
| $\tau_{B_d}$ | 1.530 ± 0.009 |
| $\tau_{B_s}$ | 1.425 ± 0.041 |
| $|V_{ud}|$ | 0.97430 ± 0.00019 |
| $|V_{us}|$ | 0.22521 ± 0.00083 |
| $|V_{ub}|$ | 0.00344 ± 0.00022 |
| $|V_{cd}|$ | 0.22508 ± 0.00084 |
| $|V_{cs}|$ | 0.97350 ± 0.00021 |
| $|V_{cb}|$ | 0.04045 ± 0.00106 |
| $|V_{td}|$ | 0.00841 ± 0.00035 |
| $|V_{ts}|$ | 0.03972 ± 0.00015 |
| $|V_{tb}|$ | 0.999176 ± 0.00003 |
| $\alpha$ | 90.7 ± 4.5 $^\circ$ |
| $\beta$ | 21.7 ± 1.0 $^\circ$ |
| $\gamma$ | 67.6 ± 2.8 $^\circ$ |
| $f_D$ | 0.201 ± 0.003 ± 0.017 |
| $f_{D_s}$ | 0.249 ± 0.003 ± 0.016 |

- **Form factors:** For the form factors involving $B \rightarrow D^{(*)}$ transitions, we take expressions which include perturbative QCD corrections induced by hard gluon vertex corrections of $b \rightarrow c$ transitions and power corrections in orders of $1/m_{b,c}$ [37, 47]. As for Isgur-Wise function $\xi(\omega)$, we use the fit result $\xi(\omega) = 1 - 1.22(\omega - 1) + 0.85(\omega - 1)^2$ from Ref. [48].

- **Wilson coefficients:** We obtain Wilson coefficients in terms of the expressions in [35].

- **RPV couplings:** When we study the RPV effects, we consider only one RPV coupling product contributes at one time, neglecting the interferences between different RPV coupling products, but keeping their interferences with the SM amplitude. We assume the masses of sfermion are 100 GeV. For other values of the sfermion masses, the bounds on the couplings in this paper can be easily obtained by scaling them by factor $f^2 \equiv (m_{f}/100\text{ GeV})^2$.

### 3 Numerical results and discussions

In this section we summarize our numerical results and analysis in the exclusive color-allowed $b \rightarrow c\bar{c}q$ decays. First, we will show our estimates in the SM with full theoretical uncertainties of sensitive parameters. Then, we will investigate the RPV effects in the decays. We will constrain
relevant RPV couplings only from quite highly consistent experimental data and show the RPV MSSM predictions for the other observables, which have not been measured yet or have less consistency among different collaborations.

3.1 Exclusive color-allowed $b \to c\bar{c}d$ decays

Decays $\bar{B}_d^0 \to D^{(*)+} D^{(*)-}$, $B_u^- \to D^{(*)0} D^{(*)-}$ and $\bar{B}_s^0 \to D_s^{(*)+} D_s^{(*)-}$ are dominated by the color-allowed $b \to c\bar{c}d$ tree diagram, but involve small penguin pollution from the $b \to u\bar{u}d$ transition carrying a different weak phase. These decays involve the same set of RPV coupling constants $\lambda_{232}'\lambda_{212}'$ and $\lambda_{233}'\lambda_{121}'$ at tree level due to squark and slepton exchanges, respectively. For $\bar{B}_d^0 \to D^{(*)+} D^{(*)-}$ and $B_u^- \to D^{(*)0} D^{(*)-}$ processes, a few observables have been measured by BABAR and Belle collaborations. The latest experimental data and their weight averages are summarized in Table 2. We can see almost all physical quantities have been consistently measured between BABAR and Belle, and only $B(\bar{B}_d^0 \to D^{+} D^{-}, D^{*\pm} D^{\mp})$, $C(\bar{B}_d^0 \to D^{+} D^{-})$ and $C(\bar{B}_d^0, \bar{B}_d^0 \to D^{+} D^{*-})$ have low consistency between BABAR and Belle.

Our SM estimates predicted within the theoretical uncertainties of input parameters are given in the second columns of Table 3 and Table 4. Theoretical predictions for the branching ratios and the polarization fractions are given in Table 3. CPA predictions are given in Table 4. All the branching ratios are above $10^{-4}$ order. The direct CPAs are expected to be quite small. All mixing-induced CPAs of $\bar{B}_d^0$ decays are very large (about $-0.7$). There is an obvious signature of the mixing-induced $CP$ violations in $\bar{B}_s^0 \to D_s^{*+} D_s^{-}$, $D_s^{*+} D_s^{-}$ decays since $S(B_s^0, \bar{B}_s^0 \to D_s^{*+} D_s^{-}) \neq -S(B_s^0, \bar{B}_s^0 \to D_s^{+} D_s^{-})$, which are consistent with the experimental measurements. In addition, for $\bar{B}_d^0 \to D^{+} D^{-}$, $B_u^- \to D^{*0} D^{*-}$ and $\bar{B}_s^0 \to D_s^{*+} D_s^{-}$ decays, the longitudinal and transverse polarization fractions can be precisely predicted, and are about $\sim 0.5$ and $\sim 0.1$, respectively. Comparing present experimental data in Table 2 with the SM predictions in Table 3 and Table 4, we can find that all measured quantities agree with the SM expectations within the error ranges except $C(\bar{B}_d^0, \bar{B}_d^0 \to D^{+} D^{-})$ from Belle.

We now turn to explore the RPV effects in $\bar{B}_d^0 \to D^{(*)+} D^{(*)-}$, $B_u^- \to D^{(*)0} D^{(*)-}$ and $\bar{B}_s^0 \to D_s^{(*)+} D_s^{(*)-}$ decays. The most conservative existing experimental bounds are used in our analysis. We choose the averaged data, which have highly consistent measurements between BABAR and Belle (defined as a scale factor $S \leq 1$), and varied randomly within $2\sigma$ ranges to constrain the RPV effects. The current experimental data and theoretical input parameters are not yet precise enough to set absolute bounds on the relative RPV couplings. We obtain the
Table 2: Experimental data for $\bar{B}_d^0 \to D^{(*)+} D^{(*)-}$ and $B_u^- \to D^{(*)0} D^{(*)-}$ decays from BABAR and Belle. The branching ratios ($B$) are in units of $10^{-4}$. The scale factor $S$ is defined in introduction part of Ref. [42], and $S > 1$ often indicates that the measurements are inconsistent.

| Observable                      | BABAR          | Belle          | Average | $S$ |
|---------------------------------|----------------|----------------|---------|-----|
| $B(B_d^0 \to D^+ D^-)$          | $2.8 \pm 0.4 \pm 0.5$ [9] | $1.97 \pm 0.20 \pm 0.20$ [3] | $2.1 \pm 0.3$ | 1.2 |
| $B(B_s^0 \to D^{*-} D^*)$       | $5.7 \pm 0.7 \pm 0.7$ [9] | $11.7 \pm 2.6 \pm 2.6$ [26] | $6.1 \pm 1.5$ | 1.6 |
| $B(B_d^0 \to D^{**} D^{*-})$    | $8.1 \pm 0.6 \pm 1.0$ [9] | $8.1 \pm 0.8 \pm 1.1$ [7] | $8.1 \pm 0.9$ | ≤ 1.0 |
| $B(B_u^- \to D^0 D^-)$          | $3.8 \pm 0.6 \pm 0.5$ [9] | $3.85 \pm 0.31 \pm 0.38$ [10] | $3.8 \pm 0.4$ | ≤ 1.0 |
| $B(B_u^- \to D^{0*} D^-)$       | $6.3 \pm 1.4 \pm 1.0$ [9] | | | |
| $B(B_u^- \to D^0 D^{*-})$       | $3.6 \pm 0.5 \pm 0.4$ [9] | $4.57 \pm 0.71 \pm 0.56$ [11] | $3.9 \pm 0.5$ | ≤ 1.0 |
| $B(B_u^- \to D^{0*} D^*)$       | $8.1 \pm 1.2 \pm 1.2$ [9] | | | |
| $C(B_d^0 \to D^+ D^-)$          | $-0.07 \pm 0.23 \pm 0.03$ [4] | $-0.91 \pm 0.23 \pm 0.06$ [3] | $-0.48 \pm 0.42$ | 2.5 |
| $C(B_s^0, B_s^0 \to D^{**} D^-)$ | $0.08 \pm 0.17 \pm 0.04$ [4] | $-0.37 \pm 0.22 \pm 0.06$ [5] | $-0.09 \pm 0.22$ | 1.6 |
| $C(B_d^0, B_d^0 \to D^+ D^{**})$ | $0.00 \pm 0.17 \pm 0.03$ [4] | $0.23 \pm 0.25 \pm 0.06$ [5] | $0.07 \pm 0.14$ | ≤ 1.0 |
| $C^+(B_d^0 \to D^{**} D^*)$     | $0.00 \pm 0.12 \pm 0.02$ [4] | $-0.15 \pm 0.13 \pm 0.04$ [8] | $-0.07 \pm 0.09$ | ≤ 1.0 |
| $A^{\text{dir}}_{CP}(B_u^- \to D^0 D^-)$ | $-0.13 \pm 0.14 \pm 0.02$ [9] | $0.00 \pm 0.08 \pm 0.02$ [10] | $-0.03 \pm 0.07$ | ≤ 1.0 |
| $A^{\text{dir}}_{CP}(B_u^- \to D^{0*} D^-)$ | $0.13 \pm 0.18 \pm 0.04$ [9] | | | |
| $A^{\text{dir}}_{CP}(B_u^- \to D^0 D^{*-})$ | $-0.06 \pm 0.13 \pm 0.02$ [9] | $0.15 \pm 0.15 \pm 0.05$ [11] | $0.03 \pm 0.10$ | ≤ 1.0 |
| $A^{\text{dir}}_{CP}(B_u^- \to D^{0*} D^*)$ | $-0.15 \pm 0.11 \pm 0.02$ [9] | | | |
| $S(B_d^0 \to D^+ D^-)$          | $-0.63 \pm 0.36 \pm 0.05$ [4] | $-1.13 \pm 0.37 \pm 0.09$ [3] | $-0.87 \pm 0.26$ | ≤ 1.0 |
| $S(B_s^0, B_s^0 \to D^{**} D^-)$ | $-0.62 \pm 0.21 \pm 0.03$ [4] | $-0.55 \pm 0.39 \pm 0.12$ [5] | $-0.61 \pm 0.19$ | ≤ 1.0 |
| $S(B_d^0, B_d^0 \to D^+ D^{**})$ | $-0.73 \pm 0.23 \pm 0.05$ [4] | $-0.96 \pm 0.43 \pm 0.12$ [5] | $-0.78 \pm 0.21$ | ≤ 1.0 |
| $S^+(B_d^0 \to D^{**} D^*)$     | $-0.76 \pm 0.16 \pm 0.04$ [4] | $-0.96 \pm 0.25 \pm 0.12$ [8] | $-0.81 \pm 0.14$ | ≤ 1.0 |
| $f_L(B_d^0 \to D^{**} D^*)$     | $0.158 \pm 0.028 \pm 0.006$ [4] | $0.125 \pm 0.043 \pm 0.023$ [8] | $0.150 \pm 0.025$ | ≤ 1.0 |
| $f_L(B_d^0 \to D^{**} D^*)$     | $0.57 \pm 0.08 \pm 0.02$ [7] | | | |

allowed scattering spaces of the RPV couplings $\lambda^{rs}_{232} \lambda^{r'}_{121}$ and $\lambda^{rs}_{123} \lambda^{r'}_{211}$ as displayed in Fig. 1. These survived parameter spaces are not in conflict with the above mentioned highly consistent experimental data in $\bar{B}_d^0 \to D^{(*)+} D^{(*)-}$ and $B_u^- \to D^{(*)0} D^{(*)-}$ decays.

The squark exchange coupling $\lambda^{rs}_{232} \lambda^{r'}_{121}$ contributes to all twelve $\bar{B}_d^0 \to D^{(*)+} D^{(*)-}$, $B_u^- \to D^{(*)0} D^{(*)-}$ and $\bar{B}_s \to D_s^{(*)+} D^{(*)-}$ decay modes. The allowed space of $\lambda^{rs}_{232} \lambda^{r'}_{121}$ is shown in the left plot of Fig. 1. Its magnitude $|\lambda^{rs}_{232} \lambda^{r'}_{121}|$ and its RPV weak phase $\phi_{\text{RPV}}$ have been constrained significantly. We obtain $|\lambda^{rs}_{232} \lambda^{r'}_{121}| \in [0.14, 1.62] \times 10^{-3}$ and $\phi_{\text{RPV}} \in [-75^\circ, 84^\circ]$. The right plot of Fig. 1 displays the allowed space of the RPV couplings $\lambda^{rs}_{123} \lambda^{r'}_{211}$ due to slepton exchanges, which
Table 3: Theoretical predictions for CP-averaged branching ratios (in units of 10^{-4}) and ratios of polarization (in units of 10^{-2}) in exclusive color-allowed $b \rightarrow c\bar{c}d$ decays. The second column gives the SM predictions with the theoretical uncertainties of input parameters. The last two columns are the RPV MSSM predictions with different RPV couplings considering the input parameter uncertainties and experimental errors.

| Observable                  | SM          | MSSM w/ $\lambda_{232}^{\prime}$ | MSSM w/ $\lambda_{123}^{\prime}$ |
|-----------------------------|-------------|-----------------------------------|-----------------------------------|
| $\mathcal{B}(\bar{B}_d^0 \rightarrow D^+D^-)$ | [2.35, 4.15] | [2.77, 3.80]                      | [2.77, 4.39]                      |
| $\mathcal{B}(\bar{B}_d^0 \rightarrow D^{*+}D^-)$ | [2.27, 3.96] | [2.87, 4.22]                      | [2.30, 4.59]                      |
| $\mathcal{B}(\bar{B}_d^0 \rightarrow D^+D^{*-})$ | [2.56, 5.04] | [3.31, 4.65]                      |                                  |
| $\mathcal{B}(\bar{B}_d^0 \rightarrow D^{*+}D^{*-})$ | [4.84, 8.95] | [6.18, 8.70]                      | [5.21, 8.84]                      |
| $\mathcal{B}(\bar{B}_d^0 \rightarrow D^+D^{*-})$ | [6.21, 12.22] | [7.05, 8.90]                      |                                  |
| $\mathcal{B}(B_u^- \rightarrow D^0D^-)$ | [2.53, 4.43] | [3.00, 4.04]                      | [3.00, 4.68]                      |
| $\mathcal{B}(B_u^- \rightarrow D^{*0}D^-)$ | [2.42, 4.27] | [3.07, 4.53]                      | [2.25, 4.75]                      |
| $\mathcal{B}(B_u^- \rightarrow D^0D^{*-})$ | [2.73, 5.42] | [3.55, 4.96]                      |                                  |
| $\mathcal{B}(B_u^- \rightarrow D^{*0}D^{*-})$ | [6.61, 13.10] | [7.54, 10.67]                     |                                  |
| $\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^+D^-)$ | [2.33, 4.20] | [2.76, 3.81]                      | [2.77, 4.48]                      |
| $\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^{*+}D^-)$ | [2.24, 4.00] | [2.81, 4.25]                      | [2.08, 4.42]                      |
| $\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^+D^{*-})$ | [2.54, 5.03] | [3.27, 4.69]                      |                                  |
| $\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^{*+}D^{*-})$ | [6.15, 12.10] | [6.92, 10.04]                     |                                  |
| $f_L(B_d^0 \rightarrow D^{*+}D^{-})$ | [52.40, 52.97] | [50.35, 52.53]                    |                                  |
| $f_L(B_u^- \rightarrow D^{*0}D^{*-})$ | [52.43, 53.02] | [50.37, 52.56]                    |                                  |
| $f_L(B_s^0 \rightarrow D_s^{*+}D^{-})$ | [52.56, 53.16] | [50.60, 52.70]                    |                                  |
| $f_L(B_s^0 \rightarrow D_s^{*0}D^{*-})$ | [8.82, 9.51]  | [10.00, 12.94]                    |                                  |
| $f_L(B_u^- \rightarrow D^{*0}D^{*-})$ | [8.84, 9.53]  | [10.03, 12.97]                    |                                  |
| $f_L(B_s^0 \rightarrow D_s^{*+}D^{*-})$ | [8.35, 9.05]  | [9.50, 12.34]                     |                                  |

contributes only to six decay modes $\bar{B}_d^0 \rightarrow D^{(*)+}D^-$, $B_u^- \rightarrow D^{(*)0}D^-$ and $\bar{B}_s^0 \rightarrow D_s^{(*)+}D^-$. The magnitudes $|\lambda_{123}^{\prime}\lambda_{21}^{\prime}\lambda_{21}|$ have been limited within $|\lambda_{123}^{\prime}\lambda_{21}| \leq 1.28 \times 10^{-3}$, and the corresponding RPV weak phase $\phi_{\text{RPV}}$ for the range $|\lambda_{123}^{\prime}\lambda_{21}^{\prime}| \leq 0.4 \times 10^{-3}$ is not constrained so much, however, the RPV weak phase for $|\lambda_{123}^{\prime}\lambda_{21}| \in [0.4, 1.3] \times 10^{-3}$ is very narrow.

Using the constrained parameter spaces shown in Fig. 1, one may predict the RPV effects
Table 4: Theoretical predictions for CPAs (in units of 10^{-2}) in exclusive color-allowed b \to c \bar{c} d decays.

| Observable | SM | MSSM w/ $\lambda'_{232}\lambda''_{212}$ | MSSM w/ $\lambda'^*_{123}\lambda''_{121}$ |
|------------|----|--------------------------------------|--------------------------------------|
| $S(B_d^0, B_d^0 \to D^+ D^-)$ | $[-78.00, -71.67]$ | $[-97.52, -52.66]$ | $[-99.83, -35.16]$ |
| $S(B_d^0, B_d^0 \to D^{*+} D^-)$ | $[-70.40, -64.55]$ | $[-81.77, -32.17]$ | $[-98.01, -55.02]$ |
| $S(B_d^0, B_d^0 \to D^+ D^{*-})$ | $[-72.17, -66.83]$ | $[-83.29, -36.15]$ | $[-98.30, -57.69]$ |
| $S^+(B_d^0, B_d^0 \to D^{*+} D^{*-})$ | $[-72.73, -67.77]$ | $[-95.18, -53.01]$ | |
| $C(B_d^0, B_d^0 \to D^+ D^-)$ | $[-6.03, -3.87]$ | $[-7.61, 0.92]$ | $[-11.05, 2.59]$ |
| $C(B_d^0, B_d^0 \to D^{*+} D^-)$ | $[3.36, 13.83]$ | $[1.38, 14.75]$ | $[-13.35, 21.30]$ |
| $C(B_d^0, B_d^0 \to D^+ D^{*-})$ | $[-14.44, -3.53]$ | $[-14.83, -1.45]$ | $[-21.00, 13.28]$ |
| $C^+(B_d^0, B_d^0 \to D^{*+} D^{*-})$ | $[-1.35, -1.03]$ | $[-1.83, 0.20]$ | |
| $\mathcal{A}_{CP}^{dir}(B_u^- \to D^0 D^-)$ | $[3.87, 6.03]$ | $[-0.92, 7.61]$ | $[-2.59, 11.05]$ |
| $\mathcal{A}_{CP}^{dir}(B_u^- \to D^{*0} D^-)$ | $[-1.15, -0.45]$ | $[-1.10, 0.29]$ | $[-1.99, 0.23]$ |
| $\mathcal{A}_{CP}^{dir}(B_u^- \to D^0 D^{*-})$ | $[1.03, 1.35]$ | $[-0.40, 1.42]$ | |
| $\mathcal{A}_{CP}^{+, dir}(B_u^- \to D^{*0} D^{*-})$ | $[1.03, 1.35]$ | $[-0.20, 1.83]$ | |
| $\mathcal{A}_{CP}^{dir}(B_s^0 \to D^+_s D^-)$ | $[3.87, 6.03]$ | $[-0.92, 7.61]$ | $[-2.59, 11.05]$ |
| $\mathcal{A}_{CP}^{dir}(B_s^0 \to D^{*-} D^-)$ | $[-1.15, -0.45]$ | $[-1.10, 0.29]$ | $[-1.99, 0.23]$ |
| $\mathcal{A}_{CP}^{dir}(B_s^0 \to D^+_s D^{*-})$ | $[1.03, 1.35]$ | $[-0.40, 1.42]$ | |
| $\mathcal{A}_{CP}^{+, dir}(B_s^0 \to D^{*-} D^{*-})$ | $[1.03, 1.35]$ | $[-0.20, 1.83]$ | |

Figure 1: Allowed parameter spaces for relevant RPV couplings constrained by $\bar{B}_d^0 \to D^{(*)+} D^{(*)-}$ and $B_u^- \to D^{(*)0} D^{(*)-}$, where $\phi_{RPV}$ denotes the RPV weak phase.
on the other quantities which have not been measured yet or have less consistent measurements between BABAR and Belle. With the expressions for $B, C, S, A_{\text{dir}}^\text{CP}$, $f_L$ and $f_\perp$ at hand, we perform a scan on the input parameters and the constrained RPV couplings. Then we obtain the RPV MSSM predictions with different RPV coupling, whose numerical results are summarized in the last two columns of Table 3 and Table 4.

The contributions of $\lambda'^{\alpha\beta}_{232}, \lambda''^{\alpha}_{212}$ due to squark exchange are summarized in the third columns of Table 3 and Table 4. In Table 3, comparing with the SM predictions, we find $\lambda'^{\alpha\beta}_{232}, \lambda''^{\alpha}_{212}$ coupling could not affect all branching ratios much. Three $f_L(B \to D^*D^*)$ and three $f_\perp(B \to D^*D^*)$ are slightly decreased and increased by $\lambda''^{\alpha}_{232}, \lambda''^{\alpha}_{212}$ coupling, respectively, and their allowed ranges are scarcely magnified by this coupling. As given in Table 4, the $\lambda'^{\alpha\beta}_{232}, \lambda''^{\alpha}_{212}$ contributions could greatly enlarge the ranges of four $S(B_d^0, \bar{B}_d^0 \to D^{(*)+}D^{(*)-})$. The effects of $\lambda'^{\alpha\beta}_{232}, \lambda''^{\alpha}_{212}$ coupling could extend a little bit the allowed regions of four $C(B_d^0, \bar{B}_d^0 \to D^{(*)+}D^{(*)-})$ and eight $A_{\text{dir}}^\text{CP}(B^- \to D^{(*)0}D^{(*)-}, B^0_s \to D^{(*)0}D^{(*)-})$, too. But this squark exchange coupling cannot explain the large $C(B_d^0, \bar{B}_d^0 \to D^+D^-)$ from Belle. The predictions including slepton exchange couplings $\lambda'^{\alpha\beta}_{232}, \lambda''^{\alpha}_{21}$ are listed in the last columns of Table 3 and Table 4. The $\lambda'^{\alpha\beta}_{232}, \lambda''^{\alpha}_{21}$ couplings do not give very big effects on the relevant branching ratios, but could significantly magnify the ranges of $S(B_d^0, \bar{B}_d^0 \to D^{(*)+}D^-)$ from their SM predictions as well as extend the ranges of $C(B_d^0, \bar{B}_d^0 \to D^{(*)+}D^-)$ and $A_{\text{dir}}^\text{CP}(B^- \to D^{(*)0}D^-, \bar{B}_s^0 \to D^{(*)+}D^-)$. The lower limits of $C(B_d^0, \bar{B}_d^0 \to D^{(*)+}D^-)$ could be reduced by these slepton exchange couplings, too, but slepton exchange coupling effects are still not large enough to explain the large $C(B_d^0, \bar{B}_d^0 \to D^+D^-)$ from Belle.

It is worth noting that our investigation of the color-allowed $b \to c\bar{c}d$ decays was motivated by the large direct CPA of $B_d^0 \to D^+D^-$ reported by Belle, which has not been confirmed by BABAR and contradicted the SM prediction. Relative RPV couplings, constrained by all consistent measurements in $\bar{B}_d^0 \to D^{(*)+}D^{(*)-}$ and $B^- \to D^{(*)0}D^{(*)-}$ systems, could slightly enlarge the range of $C(B_d^0, \bar{B}_d^0 \to D^+D^-)$. Our RPV MSSM prediction for this observable is coincident with the BABAR measurement, but still cannot explain the Belle measurement. The unparticle interaction has positive effects on $C(B_d^0, \bar{B}_d^0 \to D^+D^-)$ as obtained in Ref. 12, in which the author used only experimental constraints of $B(\bar{B}_d^0 \to D^+D^-)$. Note also that very large value of $C(B_d^0, \bar{B}_d^0 \to D^+D^-)$ could be obtained by unparticle interaction, however, with the sign opposite to the Belle measurement.

For each RPV coupling product, we can present correlations of physical quantities within the
coupling contributions to $\bar{B}_d^0 \rightarrow D^{(*)+} D^{(*)-}$, $B^- \rightarrow D^{(*)0} D^{(*)-}$ and $\bar{B}_s^0 \rightarrow D_s^{(*)+} D_s^{(*)-}$ decays are very similar to each other. So we will take an example for a few observables of $\bar{B}_d^0 \rightarrow D^{(*)+} D^{(*)-}$ decays to illustrate RPV coupling effects. Effects of RPV couplings $\lambda^{\prime\prime*}_{232} \lambda^{\prime\prime}_{212}$ and $\lambda^{\prime\prime*}_{123} \lambda^{\prime\prime}_{121}$ on observables of $\bar{B}_d^0 \rightarrow D^{(*)+} D^{(*)-}$ decays are shown in Fig. 2 and Fig. 3 respectively.

In Fig. 2 we plot $B$, $f_L$, $C$ and $S$ as functions of $\lambda^{\prime\prime*}_{232} \lambda^{\prime\prime}_{212}$. Three-dimensional scatter plot Fig. 2 (a) shows $B(\bar{B}_d^0 \rightarrow D^{(*)+} D^{(*)-})$ correlated with $|\lambda^{\prime\prime*}_{232} \lambda^{\prime\prime}_{212}|$ and its phase $\phi_{\text{RPV}}$. We also give projections to three perpendicular planes, where the $|\lambda^{\prime\prime*}_{232} \lambda^{\prime\prime}_{212}|-\phi_{\text{RPV}}$ plane displays the constrained regions of $\lambda^{\prime\prime*}_{232} \lambda^{\prime\prime}_{212}$, as the left plot of Fig. 1. It is shown that $B(\bar{B}_d^0 \rightarrow D^{(*)+} D^{(*)-})$ is little decreasing with $|\lambda^{\prime\prime*}_{232} \lambda^{\prime\prime}_{212}|$ on the $B-|\lambda^{\prime\prime*}_{232} \lambda^{\prime\prime}_{212}|$ plane. From the $B-\phi_{\text{RPV}}$ plane, we see that $B(\bar{B}_d^0 \rightarrow D^{(*)+} D^{(*)-})$ is not sensitive to $\phi_{\text{RPV}}$. All other branching ratios have similar trends with $\lambda^{\prime\prime*}_{232} \lambda^{\prime\prime}_{212}$ coupling. From Fig. 2 (b-d), we can see $f_L(\bar{B}_d^0 \rightarrow D^{(*)+} D^{(*)-})$ and $C(\bar{B}_d^0, B_d^0 \rightarrow D^{(*)+} D^{(*)-})$ are not very sensitive with $|\lambda^{\prime\prime*}_{232} \lambda^{\prime\prime}_{212}|$ and $\phi_{\text{RPV}}$. RPV coupling $\lambda^{\prime\prime*}_{232} \lambda^{\prime\prime}_{212}$ contributions to $S(\bar{B}_d^0, B_d^0 \rightarrow D^{(*)+} D^{(*)-})$ are also very similar to each other. So we take an example for $S(\bar{B}_d^0, B_d^0 \rightarrow D^{(*)+} D^{(*)-})$ shown in Fig. 2 (e) to illustrate the RPV coupling effects. $S(\bar{B}_d^0, B_d^0 \rightarrow D^{(*)+} D^{(*)-})$ is decreasing (increasing) with $|\lambda^{\prime\prime*}_{232} \lambda^{\prime\prime}_{212}|$ when $\phi_{\text{RPV}} > 0$ ($\phi_{\text{RPV}} < 0$), and it is decreasing with $\phi_{\text{RPV}}$. $S(\bar{B}_d^0, B_d^0 \rightarrow D^{(*)+} D^{(*)-})$ have totally different trends to $S(\bar{B}_d^0, B_d^0 \rightarrow D^{(*)+} D^{(*)-})$ with $|\lambda^{\prime\prime*}_{232} \lambda^{\prime\prime}_{212}|$ and $\phi_{\text{RPV}}$, and we show only the squark exchange effects on $S(\bar{B}_d^0, B_d^0 \rightarrow D^{(*)+} D^{(*)-})$ in Fig. 2 (f).

Fig. 3 gives the effects of the slepton exchange couplings $\lambda^{\prime\prime*}_{232} \lambda^{\prime\prime}_{212}$ in $\bar{B}_d^0 \rightarrow D^{(*)+} D^{(*)-}$ decays. As displayed in Fig. 3 (a), $B(\bar{B}_d^0 \rightarrow D^{(*)+} D^{(*)-})$ is not very sensitive with $|\lambda^{\prime\prime*}_{232} \lambda^{\prime\prime}_{212}|$ and has only small allowed values when $|\phi_{\text{RPV}}|$ is small. Fig. 3 (b) shows that $B(\bar{B}_d^0 \rightarrow D^{(*)+} D^{(*)-})$ is decreasing with $|\lambda^{\prime\prime*}_{232} \lambda^{\prime\prime}_{212}|$ and is weakly sensitive to $|\phi_{\text{RPV}}|$. Fig. 3 (c) exhibits the slepton exchange coupling effects on $C(\bar{B}_d^0, B_d^0 \rightarrow D^{(*)+} D^{(*)-})$, which is decreasing with $|\lambda^{\prime\prime*}_{232} \lambda^{\prime\prime}_{212}|$ and has little sensitivity to $\phi_{\text{RPV}}$. Slepton exchange couplings have great effects on $C(\bar{B}_d^0, B_d^0 \rightarrow D^{(*)+} D^{(*)-})$ and $S(\bar{B}_d^0, B_d^0 \rightarrow D^{(*)+} D^{(*)-})$, and they have quite complex variational trends to $|\lambda^{\prime\prime*}_{232} \lambda^{\prime\prime}_{212}|$ and $|\phi_{\text{RPV}}|$. The effects of $\lambda^{\prime\prime*}_{232} \lambda^{\prime\prime}_{212}$ couplings on $C(\bar{B}_d^0, B_d^0 \rightarrow D^{(*)+} D^{(*)-})$ and $S(\bar{B}_d^0, B_d^0 \rightarrow D^{(*)+} D^{(*)-})$ are shown in Fig. 3 (d-f). $C(\bar{B}_d^0, B_d^0 \rightarrow D^{(*)+} D^{(*)-})$ has entirely different trends from $C(\bar{B}_d^0, B_d^0 \rightarrow D^{(*)+} D^{(*)-})$. $S(\bar{B}_d^0, B_d^0 \rightarrow D^{(*)+} D^{(*)-})$ has a similar trends as $S(\bar{B}_d^0, B_d^0 \rightarrow D^{(*)+} D^{(*)-})$. 

16
Figure 2: Effects of RPV coupling $\lambda_{232}^{\prime}\lambda_{212}^{\prime\prime}$ in $\bar{B}_d^0 \rightarrow D^{(*)+} D^{(*)-}$ decays, where $|\lambda_{232}^{\prime}\lambda_{212}^{\prime\prime}|$ is in units of $10^{-3}$, and $B$ in units of $10^{-4}$.

Figure 3: Effects of RPV coupling $\lambda_{123}^{\prime}\lambda_{121}^{\prime\prime}$ in $\bar{B}_d^0 \rightarrow D^{(*)+} D^{(*)-}$ decays, where $|\lambda_{123}^{\prime}\lambda_{121}^{\prime\prime}|$ is in units of $10^{-3}$, and $B$ in units of $10^{-4}$.
3.2 Exclusive color-allowed $b \to c\bar{c}s$ decays

Exclusive color-allowed $b \to c\bar{c}s$ tree decays include $\bar{B}^0_d \to D^{(*)-} D_s^{(*)-}$, $B_u^- \to D^{(*)0} D_s^{(*)-}$ and $\bar{B}^0_s \to D_s^{(*)-} D_s^{(*)-}$ decay modes. Almost all branching ratios and one longitudinal polarization have been measured by Belle [40], BABAR [27, 28, 29], CLEO [30, 31, 32, 33], and ARGUS [34] collaborations. Their averaged values from Particle Data Group [42] are listed as follows

$$
\mathcal{B}(\bar{B}^0_d \to D^+ D_s^-) = (7.4 \pm 0.7) \times 10^{-3}, \quad \mathcal{B}(\bar{B}^0_d \to D^{*-} D_s^-) = (8.3 \pm 1.1) \times 10^{-3}, \\
\mathcal{B}(\bar{B}^0_d \to D^+ D_s^{*-}) = (7.6 \pm 1.6) \times 10^{-3}, \quad \mathcal{B}(\bar{B}^0_d \to D^{*-} D_s^{*-}) = (17.9 \pm 1.4) \times 10^{-3}, \\
\mathcal{B}(B_u^- \to D^0 D_s^-) = (10.3 \pm 1.7) \times 10^{-3}, \quad \mathcal{B}(B_u^- \to D^{*0} D_s^-) = (8.4 \pm 1.7) \times 10^{-3}, \\
\mathcal{B}(B_u^- \to D^0 D_s^{*-}) = (7.8 \pm 1.6) \times 10^{-3}, \quad \mathcal{B}(B_u^- \to D^{*0} D_s^{*-}) = (17.5 \pm 2.3) \times 10^{-3}, \\
\mathcal{B}(\bar{B}^0_s \to D_s^+ D_s^-) = (11 \pm 4) \times 10^{-3}, \quad \mathcal{B}(\bar{B}^0_s \to D_s^+ D_s^{*-}) < 121 \times 10^{-3}, \\
\mathcal{B}(\bar{B}^0_s \to D_s^{*+} D_s^{*-}) < 257 \times 10^{-3}, \quad f_L(\bar{B}^0_d \to D^{*+} D_s^{*-}) = 0.52 \pm 0.05. \quad (33)
$$

The SM predictions, in which the full theoretical uncertainties of input parameters are considered, are given in the second columns of Table 5 and Table 6. Theoretical predictions for the branching ratios and the polarization fractions are given in Table 5. Predicted CPAs are also given in Table 6. Compared with the experimental data, only the SM predictions of $\mathcal{B}(\bar{B}^0_d \to D^{*+} D_s^{*-}, B_u^- \to D^{*0} D_s^{*-})$ are slightly larger than the corresponding experimental data given in Eq. (33), and all the other branching ratios are consistent with the data within $1\sigma$ error level. For the color-allowed $b \to c\bar{c}s$ decays the penguin effects are doubly Cabibbo-suppressed and, therefore, play a significantly less pronounced role in CPAs. These CPAs have not been measured yet. We obtain that all CPAs are expected to be very small (about $10^{-3}$ or $10^{-4}$ order) in the SM except $\mathcal{S}(B_s^0, \bar{B}^0_s \to D_s^{*+} D_s^{*-})$ and $\mathcal{C}(B_s^0, \bar{B}^0_s \to D_s^{*+} D_s^-, D_s^+ D_s^{*-})$. There is no obvious signature of $CP$ violation in $B_s \to D_s^{*\pm} D_s^\mp$ decays since $\mathcal{C}(B_s^0, \bar{B}^0_s \to D_s^{*+} D_s^-) \approx -\mathcal{C}(B_s^0, \bar{B}^0_s \to D_s^+ D_s^{*-})$ and $\mathcal{S}(B_s^0, \bar{B}^0_s \to D_s^{*+} D_s^-) \approx -\mathcal{S}(B_s^0, \bar{B}^0_s \to D_s^+ D_s^{*-})$.

There are two RPV coupling products, $\lambda_{231}'' \lambda_{221}''$ and $\lambda_{231}'' \lambda_{222}''$, contributing to these exclusive $b \to c\bar{c}s$ decay modes at tree level. We use the experimental data listed in Eq. (33) to constrain the RPV coupling products, and the allowed spaces are shown in Fig. 4. The coupling $\lambda_{231}'' \lambda_{221}''$ due to squark exchange contributes to all twelve relative decay modes. The allowed space of $\lambda_{231}'' \lambda_{221}''$ is shown in the left plot of Fig. 4. The slepton exchange couplings $\lambda_{233}'' \lambda_{222}''$ contribute to six $\bar{B}^0_d \to D^{(*)+} D_s^-$, $B_u^- \to D^{(*)0} D_s^-$ and $\bar{B}^0_s \to D_s^{(*)-} D_s^-$ decays, and the constrained space is displayed in the right plot of Fig. 4. From Fig. 4 we find both moduli of RPV couplings have
Table 5: Theoretical predictions for $CP$ averaged $\mathcal{B}$ (in units of $10^{-4}$) and polarization fractions (in units of $10^{-2}$) of exclusive color-allowed $b \to c\bar{c}s$ decays in the SM and the RPV MSSM.

| Observable | SM | MSSM w/ $\lambda''_{231}\lambda''_{221}$ | MSSM w/ $\lambda''_{123}\lambda'_{122}$ |
|-----------|----|--------------------------------|--------------------------------|
| $\mathcal{B}(B^0_d \to D^+D_s^-)$ | [6.70, 10.65] | [6.38, 7.59] | [6.42, 8.80] |
| $\mathcal{B}(B^0_d \to D^{*+}D_s^-)$ | [6.70, 10.45] | [6.47, 9.49] | [6.16, 9.30] |
| $\mathcal{B}(B^0_d \to D^+D_s^{*-})$ | [7.32, 13.22] | [6.90, 10.29] | |
| $\mathcal{B}(B^0_d \to D^{*+}D_s^{*-})$ | [19.27, 34.42] | [18.59, 20.70] | |
| $\mathcal{B}(B^-_u \to D^0D_s^-)$ | [7.21, 11.43] | [6.90, 8.12] | [6.90, 9.49] |
| $\mathcal{B}(B^-_u \to D^{*0}D_s^-)$ | [7.17, 11.24] | [6.95, 10.17] | [6.64, 9.96] |
| $\mathcal{B}(B^-_u \to D^0D_s^{*-})$ | [7.89, 14.27] | [7.43, 11.00] | |
| $\mathcal{B}(B^-_u \to D^{*0}D_s^{*-})$ | [20.57, 37.06] | [19.99, 22.10] | |
| $\mathcal{B}(\bar{B}^0_s \to D^+_sD_s^-)$ | [6.55, 10.72] | [6.36, 7.71] | [6.23, 9.05] |
| $\mathcal{B}(\bar{B}^0_s \to D^{*+}D_s^-)$ | [6.46, 10.44] | [6.51, 9.46] | [6.02, 9.26] |
| $\mathcal{B}(\bar{B}^0_s \to D^+_sD_s^{*-})$ | [7.08, 12.97] | [7.00, 10.40] | |
| $\mathcal{B}(\bar{B}^0_s \to D^{*+}D_s^{*-})$ | [18.64, 33.83] | [18.48, 20.93] | |
| $f_L(B^0_d \to D^{*+}D_s^-)$ | [50.25, 50.91] | [48.46, 51.13] | |
| $f_L(B^-_u \to D^{*0}D_s^-)$ | [50.28, 50.94] | [48.49, 51.16] | |
| $f_L(\bar{B}^0_s \to D^{*+}D_s^-)$ | [50.40, 51.10] | [48.71, 51.30] | |
| $f_{\perp}(B^0_d \to D^{*+}D_s^-)$ | [8.85, 9.55] | [8.15, 12.85] | |
| $f_{\perp}(B^-_u \to D^{*0}D_s^-)$ | [8.87, 9.57] | [8.17, 12.88] | |
| $f_{\perp}(\bar{B}^0_s \to D^{*+}D_s^-)$ | [8.38, 9.07] | [7.71, 12.23] | |

been limited as $|\lambda''_{231}\lambda''_{221}| < 8.05 \times 10^{-3}$ and $|\lambda''_{123}\lambda'_{122}| < 5.05 \times 10^{-3}$. Their RPV weak phases are not constrained much when their magnitudes are less than about $1 \times 10^{-3}$.

Next, using the constrained parameter spaces shown in Fig. 4 we are going to predict RPV effects on the observables which have not been measured yet. We summarize RPV MSSM predictions with two separate RPV coupling contributions in the last two columns of Table 5 and Table 6.

The contributions of $\lambda''_{231}\lambda''_{221}$ coupling due to squark exchange are summarized in the third columns of Table 5 and Table 6. In Table 5, we find that the ranges of all branching ratios
Table 6: Theoretical predictions for CPAs (in units of $10^{-2}$) of exclusive color-allowed $b \to c\bar{c}s$ decays in the SM and the RPV MSSM.

| Observable | SM | MSSM w/ $\lambda_{231}^{\prime\prime} \lambda_{221}^{\prime\prime}$ | MSSM w/ $\lambda_{233}^{\prime}\lambda_{422}^{\prime}$ |
|------------|----|-------------------------------------------------|-----------------------------------|
| $\mathcal{A}_{\text{CP}}(B_d^0 \to D^+ D^-_s)$ | $[-0.34, -0.22]$ | $[-3.06, 2.58]$ | $[-8.42, 7.94]$ |
| $\mathcal{A}_{\text{CP}}(B_d^0 \to D^{*+} D^-_s)$ | $[0.03, 0.06]$ | $[-0.32, 0.36]$ | $[-0.98, 1.07]$ |
| $\mathcal{A}_{\text{CP}}^{\text{dir}}(B_d^0 \to D^+ D^-_s)$ | $[-0.07, -0.06]$ | $[-0.51, 0.44]$ |
| $\mathcal{A}_{\text{CP}}^{\text{dir}}(B_d^0 \to D^{*+} D^-_s)$ | $[-0.07, -0.06]$ | $[-0.69, 0.56]$ |
| $\mathcal{A}_{\text{CP}}(B_u^- \to D^0 D^-_s)$ | $[-0.34, -0.22]$ | $[-3.06, 2.58]$ | $[-8.42, 7.94]$ |
| $\mathcal{A}_{\text{CP}}(B_u^- \to D^{*0} D^-_s)$ | $[0.03, 0.06]$ | $[-0.32, 0.36]$ | $[-0.98, 1.07]$ |
| $\mathcal{A}_{\text{CP}}^{\text{dir}}(B_u^- \to D^0 D^-_s)$ | $[-0.07, -0.06]$ | $[-0.51, 0.58]$ |
| $\mathcal{A}_{\text{CP}}^{\text{dir}}(B_u^- \to D^{*0} D^-_s)$ | $[-0.07, -0.06]$ | $[-0.69, 0.56]$ |
| $S(B_s^0, \bar{B}_s^0 \to D^+_s D^-_s)$ | $[0.40, 0.61]$ | $[-59.67, 61.80]$ | $[-99.84, 99.79]$ |
| $S(B_s^0, \bar{B}_s^0 \to D^{*+} D^-_s)$ | $[1.33, 2.16]$ | $[-46.48, 47.65]$ | $[-56.04, 59.14]$ |
| $S(B_s^0, \bar{B}_s^0 \to D^+_s D^-_s)$ | $[-2.20, -1.31]$ | $[-49.26, 45.22]$ | $[-58.96, 56.60]$ |
| $S^+(B_s^0, \bar{B}_s^0 \to D^{*+} D^-_s)$ | $[-31.81, -29.15]$ | $[-54.41, 55.49]$ |
| $C(B_s^0, \bar{B}_s^0 \to D^+_s D^-_s)$ | $[0.22, 0.34]$ | $[-2.58, 3.06]$ | $[-7.94, 8.42]$ |
| $C(B_s^0, \bar{B}_s^0 \to D^{*+} D^-_s)$ | $[2.77, 13.39]$ | $[3.15, 10.13]$ | $[0.79, 28.22]$ |
| $C(B_s^0, \bar{B}_s^0 \to D^+_s D^-_s)$ | $[-13.36, -2.76]$ | $[-10.08, -3.14]$ | $[-29.14, -0.54]$ |
| $C^+(B_s^0, \bar{B}_s^0 \to D^{*+} D^-_s)$ | $[0.06, 0.07]$ | $[-0.56, 0.69]$ |

Figure 4: Allowed parameter spaces for relevant RPV coupling products constrained by the measurements of exclusive color-allowed $b \to c\bar{c}s$ decays listed in Eq. (33).
are shrunk by $\lambda'_{231} \lambda''_{221}$ coupling and the experimental constraints. Especially, $\lambda'_{231} \lambda''_{221}$ coupling effects could reduce the range of $\mathcal{B}(B_s^0 \rightarrow D_s^{(*)} D_{s}^{(*)})$. However, the allowed ranges of three $f_\perp(B_s \rightarrow D_{s}^{(*)} D_{s}^{(*)})$ and three $f_{\perp}(B_s \rightarrow D_{s}^{(*)} D_{s}^{(*)})$ are enlarged by $\lambda'_{231} \lambda''_{221}$ coupling. In Table 6 we can see $\lambda'_{231} \lambda''_{221}$ coupling does not affect $\mathcal{C}(B_s^0 \rightarrow D_s^{(*)} D_{s}^{(*)})$ much.

Meanwhile, RPV coupling effects could remarkably enlarge the allowed ranges of the other direct CPAs (about 10 times). Unfortunately, they are still too small to be measured at presently available experiments. It is interesting to note that mixing-induced CPAs of $B_s$ decays are greatly affected by $\lambda'_{231} \lambda''_{221}$ coupling. For an example, $|S(B_s^0 \rightarrow D_s^{(*)} D_{s}^{(*)})|$ could be increased to $\sim 50\%$ and their signs could be changed by the squark exchange coupling.

The contributions of $\lambda'_{231} \lambda''_{122}$ due to the slepton exchanges are listed in the last columns of Table 5 and Table 6. From Table 5 we find the ranges of all branching ratios are shrunk by $\lambda'_{231} \lambda''_{122}$ coupling and the experimental constraints. The last columns of Table 6 show that $\lambda'_{231} \lambda''_{122}$ coupling could enlarge the ranges of all the CPAs. Particularly, $\lambda'_{231} \lambda''_{122}$ coupling could change the predicted $S(B_s^0, \bar{B}_s^0 \rightarrow D_s^{(*)} D_0)$ significantly from quite narrow SM ranges to $[-0.6, 0.6]$ or $[-1, 1]$. The upper limits of $|\mathcal{C}(B_s^0, \bar{B}_s^0 \rightarrow D_s^{(*)} D_0)|$ are increased a lot by $\lambda'_{231} \lambda''_{122}$ couplings.

Since RPV contributions to physical observables are also very similar in $B_d^0 \rightarrow D_s^{(*)} D_s^{(*)}$, $B_u^0 \rightarrow D_s^{(*)} D_s^{(*)}$ and $\bar{B}_s^0 \rightarrow D_s^{(*)} D_s^{(*)}$ systems, we show only a few observables of $B_s$ decays as examples. Fig. 5 and Fig. 6 show the variational trends in some observables with the $\lambda'_{231} \lambda''_{221}$ and $\lambda'_{231} \lambda''_{122}$ couplings, respectively.

First, we will elucidate the information implied in Fig. 5. From Fig. 5 (a), we find $\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^{(*)} D_s^{(*)})$ is not changed much by $|\lambda'_{231} \lambda''_{221}|$, and could have only small value when $\phi_{\text{RPV}}$ is not too large. As shown in Fig. 5 (b), $f_\perp(\bar{B}_s^0 \rightarrow D_s^{(*)} D_s^{(*)})$ is increasing with $|\lambda'_{231} \lambda''_{221}|$ and also could have small value when $\phi_{\text{RPV}}$ is small. From Fig. 5 (c-d), we find $|S(B_s^0, \bar{B}_s^0 \rightarrow D_s^{(*)} D_s^{(*)})|$ are all rapidly increasing with $|\lambda'_{231} \lambda''_{221}|$ and could be very large at the large values of $|\lambda'_{231} \lambda''_{221}|$ and $|\phi_{\text{RPV}}|$, furthermore, the RPV weak phase has opposite effects between $|S(B_s^0, \bar{B}_s^0 \rightarrow D_s^{(*)} D_s^{(*)})|$ and $|S(B_s^0, \bar{B}_s^0 \rightarrow D_s^{(*)} D_s^{(*)})|$. $\lambda'_{231} \lambda''_{221}$ coupling effects on $S(B_s^0, \bar{B}_s^0 \rightarrow D_s^{(*)} D_s^{(*)})$ are similar as ones on $S(B_s^0, \bar{B}_s^0 \rightarrow D_s^{(*)} D_s^{(*)})$ ($S(B_s^0, \bar{B}_s^0 \rightarrow D_s^{(*)} D_s^{(*)})$) in the future will strongly constrain the magnitude and RPV weak phase of $\lambda'_{231} \lambda''_{221}$ coupling, and then other mixing-induced CPAs will be more accurately predicted as indicated by Fig. 5 (c-d). As shown in Fig. 5 (e), $\mathcal{C}(B_s^0, \bar{B}_s^0 \rightarrow D_s^{(*)} D_s^{(*)})$ has similar trends as $S(B_s^0, \bar{B}_s^0 \rightarrow D_s^{(*)} D_s^{(*)})$ with $|\lambda'_{231} \lambda''_{221}|$ and $\phi_{\text{RPV}}$, 21
Figure 5: Effects of RPV coupling $\lambda'_{231}^n \lambda''_{221}$ in $\bar{B}_s^0 \to D_s^{*(+)} D_s^{(*)-}$ decays, where $|\lambda'_{231}^n \lambda''_{221}|$ is in units of $10^{-3}$, $\mathcal{B}$ in units of $10^{-4}$, and $f_T$ denotes the transverse polarization fraction $f_T$.

Figure 6: Effects of $\lambda'_{i23}^n \lambda'_{i22}$ in $\bar{B}_s^0 \to D_s^{*(+)} D_s^{(*)-}$ decays, where $|\lambda'_{i23}^n \lambda'_{i22}|$ are in units of $10^{-3}$, and $\mathcal{B}$ in units of $10^{-4}$.
however, $\lambda''_{23\lambda''_{221}}$ coupling effects on the former are much smaller than the effects on the latter. $\mathcal{A}_{CP}^{\text{dir}}(B_{u,d} \rightarrow DD_s^-, D^*D_s^-, D^*D_s^-)$ and $-\mathcal{A}_{CP}^{\text{dir}}(B_{u,d} \rightarrow DD_s^-)$ have the same variational trends with $\lambda''_{23\lambda''_{221}}$ as $\mathcal{C}(B_s^0, B_s^0 \rightarrow D_s^+D_s^-)$ has. $\mathcal{C}(B_s^0, B_s^0 \rightarrow D_s^+D_s^-, D_s^+D_s^-)$ are not affected much by $\lambda''_{23\lambda''_{221}}$ coupling, and we show $\mathcal{C}(B_s^0, B_s^0 \rightarrow D_s^{++}D_s^-)$ in Fig. 6 (c) as an example.

Fig. 6 illustrates $\lambda'_{i32\lambda'_{222}}$ contributions to the CPAs of $\bar{B}_s^0 \rightarrow D_s^{(*)+}D_s^-$. As displayed in Fig. 6 (a-b), $\mathcal{S}(B_s^0, \bar{B}_s^0 \rightarrow D_s^{(*)+}D_s^-)$ are very sensitive to $\lambda'_{i32\lambda'_{222}}$ couplings. $|\mathcal{S}(B_s^0, \bar{B}_s^0 \rightarrow D_s^{(*)+}D_s^-)|$ are strongly increasing with $|\lambda'_{i32\lambda'_{222}}|$, and they could reach extremum at $|\phi_{\text{RPV}}| \approx 120^\circ$. The $\lambda'_{i32\lambda'_{222}}$ coupling effects on $\mathcal{S}(B_s^0, \bar{B}_s^0 \rightarrow D_s^{(*)+}D_s^-)$ are same as the ones on $\mathcal{S}(B_s^0, \bar{B}_s^0 \rightarrow D_s^{(*)+}D_s^-)$. Fig. 6 (c) shows that $\mathcal{C}(B_s^0, \bar{B}_s^0 \rightarrow D_s^{(*)+}D_s^-)$ are also very sensitive to $|\lambda'_{i32\lambda'_{222}}|$ and $\phi_{\text{RPV}}$, but it is still too small to be measured in near future. In addition, $\mathcal{A}_{CP}^{\text{dir}}(B_{u,d} \rightarrow D_s^{(*)+}D_s^-)$ are affected much by $\lambda'_{i32\lambda'_{222}}$ couplings, and just RPV MSSM predictions of these quantities are very small. $\lambda'_{i32\lambda'_{222}}$ couplings could have similar impacts on $\mathcal{C}(B_s^0, \bar{B}_s^0 \rightarrow D_s^{(*)+}D_s^-)$ and $-\mathcal{C}(B_s^0, \bar{B}_s^0 \rightarrow D_s^{(*)+}D_s^-)$. We give these coupling effects on $\mathcal{C}(B_s^0, \bar{B}_s^0 \rightarrow D_s^{(*)+}D_s^-)$ in Fig. 6 (d), which shows $\mathcal{C}(B_s^0, \bar{B}_s^0 \rightarrow D_s^{(*)+}D_s^-)$ is increasing with $|\lambda'_{i32\lambda'_{222}}|$, and could have large value at large $|\phi_{\text{RPV}}|$. 

4 Summary

We have studied the twenty-four double charm decays $\bar{B}_d^0 \rightarrow D_s^{(*)+}D_s^-$, $B_u^- \rightarrow D_s^{(*)0}D_s^-$ and $\bar{B}_s^0 \rightarrow D_s^{(*)+}D_s^-$ in the RPV MSSM. We have treated these decays in the naive factorization and removed the known $k^2$ ambiguities in the penguin contributions via $b \rightarrow qg^*(\gamma^*) \rightarrow qc\bar{c}$ by calculating its hard kernel $b \rightarrow c + D_q$. Considering the theoretical uncertainties and the experimental error-bars, we have obtained fairly constrained parameter spaces of RPV couplings from the present experimental data, which have quite highly consistent measurements among the relative collaborations. Furthermore, using the constrained RPV coupling parameter spaces, we have predicted the RPV effects on the branching ratios, the CPAs and the polarization fractions, which have not been measured or have not been well measured yet.

The investigation of exclusive color-allowed $b \rightarrow c\bar{c}d$ decays is motivated by the large direct CPA $\mathcal{C}(B_d^0, \bar{B}_d^0 \rightarrow D^+D^-)$ reported by Belle, which has not been confirmed by BABAR yet and contradicted the SM prediction. Using the most conservative experimental bounds from $\bar{B}_d^0 \rightarrow D^{(*)+}D^{(*)-}$ and $B_u^- \rightarrow D_s^{(*)0}D_s^{(*)-}$ systems (choose only twelve highly consistent measurements between BABAR and Belle), we have first obtained quite strong constraints on the involved
RPV couplings $\lambda_{232}^{\nu_\tau} \lambda_{212}^{\nu_l}$ and $\lambda_{233}^{\nu_\tau} \lambda_{121}^{L}$ from $b \to c \bar{d}$ transition, due to squark exchange and slepton exchanges, respectively. Then, using the constrained RPV coupling parameter spaces, we have predicted the RPV effects on $C(B_d^0, \bar{B}_d^0 \to D^{+}D^{-})$ and other observables, which have less consistent measurements or have not been measured yet. We have found that the lower limit of $C(B_d^0, \bar{B}_d^0 \to D^{(*)+}D^{-})$ could be slightly reduced by the RPV couplings. Our RPV MSSM prediction of $C(B_d^0, \bar{B}_d^0 \to D^{+}D^{-})$ is consistent with BABAR measurement within $1\sigma$ error level, but cannot explain the corresponding Belle experimental data within $3\sigma$ level.

We have also found that the contributions of $\lambda_{232}^{\nu_\tau} \lambda_{212}^{\nu_l}$ and $\lambda_{233}^{\nu_\tau} \lambda_{121}^{L}$ cannot affect the relevant branching ratios much. $\lambda_{232}^{\nu_\tau} \lambda_{212}^{\nu_l}$ or $\lambda_{233}^{\nu_\tau} \lambda_{121}^{L}$ contributions could greatly enlarge the ranges of the relevant mixing-induced CPAs $S(B_d^0, \bar{B}_d^0 \to D^{(*)+}D^{(*)-})$ from their SM predictions, and these quantities are very sensitive to the moduli and weak phases of $\lambda_{232}^{\nu_\tau} \lambda_{212}^{\nu_l}$ and $\lambda_{233}^{\nu_\tau} \lambda_{121}^{L}$. So more accurate measurements of $S(B_d^0, \bar{B}_d^0 \to D^{(*)+}D^{(*)-})$ in the future will much more strongly constrain these RPV couplings, and then mixing-induced CPAs can be more accurately predicted as well. Effects of $\lambda_{232}^{\nu_\tau} \lambda_{212}^{\nu_l}$ coupling could slightly extend the allowed regions of four $C(B_d^0, \bar{B}_d^0 \to D^{(*)+}D^{(*)-})$ and eight $A_{\text{dir}}^{\nu_\tau}(B_u^0 \to D^{(*)0}D^{(*)-}, B_s^0 \to D^{(*)+}D^{(*)-})$. $C(B_d^0, \bar{B}_d^0 \to D^{(*)+}D^{-})$ are also sensitive to the slepton exchange couplings $\lambda_{123}^{\nu_\tau} \lambda_{212}^{L}$, and their signs could be changed by these couplings. Additionally, three $f_L(B(s) \to D^{(*)}_s D^*)$ and three $f_\perp(B(s) \to D^{(*)}_s D^*)$ are decreased and increased by $\lambda_{232}^{\nu_\tau} \lambda_{212}^{\nu_l}$ coupling, respectively, and their allowed ranges are magnified by these couplings.

For $\bar{B}_d^0 \to D^{(*)+}D_s^{(*)-}$, $B_s^- \to D^{(*)0}_s D^{(*)-}_s$ and $\bar{B}_s^0 \to D^{(*)+}_s D_s^{(*)-}$ decays, RPV couplings $\lambda_{231}^{\nu_\tau} \lambda_{221}^{\nu_l}$ and $\lambda_{233}^{\nu_\tau} \lambda_{122}^{L}$ contribute to these decay modes. We have found $\lambda_{231}^{\nu_\tau} \lambda_{221}^{\nu_l}$ coupling effects could apparently suppress the upper limit of $B(B_s^0 \to D_s^{(*)+}D_s^{(*)-})$, and could slightly enlarge the allowed ranges of three $f_L(B(s) \to D^{(*)}_s D^*_s)$ and three $f_\perp(B(s) \to D^{(*)}_s D^*_s)$, nevertheless these quantities are not very sensitive to the changes of $|\lambda_{231}^{\nu_\tau} \lambda_{221}^{\nu_l}|$ and $\phi_{\text{RPV}}$. $C(\bar{B}_s^0 \to D_s^{(*)+}D_s^{(*)-}, D_s^{(*)+}D_s^{(*)-})$ are not evidently affected by the squark exchange $\lambda_{231}^{\nu_\tau} \lambda_{221}^{\nu_l}$ coupling, and their upper limits are increased a lot by the slepton exchange $\lambda_{233}^{\nu_\tau} \lambda_{122}^{L}$ coupling. RPV couplings $\lambda_{231}^{\nu_\tau} \lambda_{221}^{\nu_l}$ and $\lambda_{233}^{\nu_\tau} \lambda_{122}^{L}$ could greatly enlarge all other CP asymmetries, which are also very sensitive to the relevant RPV couplings. However, the direct CPAs are still too small to be measured soon. We could explore RPV MSSM effects from the mixing-induced CPAs of $B_s$ decays.

With the large amount of B decay data from BABAR and Belle, especially from LHCb in the near future, measurements of previously known observables will become more precise and many unobserved observables will be also measured. From the comparison of our predictions
in Figs. 2, 3 and Figs. 5, 6 with near future experiments, one will obtain more stringent bounds on the product combinations of the RPV couplings. On the other hand, the RPV MSSM predictions of other decays will become more precise by the more stringent bounds on the RPV couplings. The results in this paper could be useful for probing RPV MSSM effects, and will correlate with searches for direct supersymmetry signals at future experiments, for example, the LHC.

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