Searching for neutrino emissions from multi-frequency sources

YU-LING CHANG 1, BRUNO ARSIOLI 2,3, WENLIAN LI 1, DONGLIAN XU 1,4,5, AND LIANG CHEN 6

1 Tsung-Dao Lee Institute, Shanghai Jiao Tong University, Shanghai 201210, P.R.China
2 University of Trieste and INFN, via Valerio 2, I-34127 Trieste, Italy.
3 ICRANet, P.zza della Repubblica 10, I-65122, Pescara, Italy
4 INPAC and School of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, P.R.China
5 Shanghai Laboratory for Particle Physics and Cosmology, Key Laboratory for Particle Physics and Cosmology(MOE), Shanghai 200240, P.R.China
6 Key Laboratory for Research in Galaxies and Cosmology, Shanghai Astronomical Observatory, Chinese Academy of Sciences, 80 Nandan Road, Shanghai 200030, P.R.China

ABSTRACT

Pinpointing the neutrino sources is crucial to unveil the mystery of high-energy cosmic rays. The search for neutrino-source candidates from coincident neutrino-photon signatures and electromagnetic objects with peculiar flaring behaviors have the potential to increase our chances of finding neutrino emitters. In this paper, we first study the temporal correlations of astrophysical flares with neutrinos, considering a few hundreds of multi-frequency sources from ALMA, WISE, Swift, and Fermi in the containment regions of IceCube high-energy alerts. Furthermore, the spatial correlations between blazars and neutrinos are investigated using the subset of 10-year IceCube track-like neutrinos with around 250 thousand events. In the second test, we account for 2700 blazars with different types of flaring phases in addition to sole position. No significant neutrino emissions were found from our analyses. Our results indicate an interesting trend showing the infrared flaring stages of WISE blazars might be correlated with arrival times of the neutrino alerts. Possible overflow of neutrinos associated with two of our blazar sub-samples are also illustrated. One is characterized by a significant flaring lag in infrared with respect to γ-rays, like seen for TXS0506+056, and the other is characterized by highly simultaneous infrared and γ-ray flares. These phenomena suggest the need to improve current multi-frequency light-curve catalogs to pair with the advent of more sensitive neutrino observatories.

Keywords: High energy astrophysics (739) — Neutrino astronomy (1100) — Active galactic nuclei (16) — Blazars (164) — Light curves (918)

1. INTRODUCTION

The origin of high-energy cosmic rays is one of the most important open questions for more than a century. Neutrinos are ideal messengers for tracking the origin of cosmic rays as they are undeflected when traveling through space. Since IceCube reported the detection of high-energy astrophysical neutrinos (IceCube Collaboration 2013; Aartsen et al. 2015, 2016; Abbasi et al. 2021), identifying the sources of those neutrino events is one of the most pressing challenges in modern astrophysics. With no significant anisotropy found from the diffuse flux of astrophysical neutrinos, a substantial fraction of the observed neutrino flux is expected to be of extragalactic origin. Blazars, a peculiar type of Active Galactic Nuclei (AGNs) and the most energetic sources of continuous non-thermal radiation in the Universe (Padovani et al. 2017), have long been suggested to be one of the most promising extragalactic astrophysical counterparts for neutrinos (for a recent review, see Giovannini & Padovani (2021)). Even though they are extremely rare compared to other types of AGNs, their distinctive features, such as superluminal motion and rapid variability, makes them of particular interest and importance. Neutrinos are expected to be produced via charged pions decay generated from the interaction of protons accelerated in the jet or the core of the blazars with ambient low-energy photons. The charged pions are accompanied by neutral pions that decay into very-
high-energy (VHE) γ-rays; therefore, γ-ray photons are
direct companions for neutrinos. A relationship between
the two messengers is expected if they are produced from
hadronic processes at the same site.

The first hint of a statistical connection between high
synchrotron peaked blazars (HSPs/HBLs) and neu-
trinos was reported by Padovani et al. (2016). They
suggested a chance probability of association between
2FHL\(^1\) HBLs with IceCube events to be \(\sim 0.4 - 1.3\%\)
depending on \(\gamma\)-ray fluxes. Applying the “energetic test”
presented in Padovani & Resconi (2014), Padovani et al.
(2016) further reported \(\sim 5\) probable HBLs counter-
parts for IceCube neutrinos. Other searches for neu-
trino counterparts with samples of \(\gamma\)-ray blazars found
no evidence of \(\gamma\)-ray emission associated with IceCube
neutrino events (Brown et al. 2015; Palladino & Vissani
2017; Krauss et al. 2018).

The importance of multi-frequency data to single out
the most likely candidates for neutrino events has been
highlighted in Padovani et al. (2016). Righi et al.
(2019b) and Franckowiak et al. (2020) analyzed the
\textit{Fermi}-detected blazars located within neutrino contain-
ment regions to search for additional neutrino-blazar
candidates and to reason out the jet’s composition and
emission processes with a multi-frequency approach.
Some of those potential neutrino blazars show tem-
poral coincidence between \(\gamma\)-ray flares and neutrino
events, while there is no compelling evidence to con-
clude the relationship between the \(\gamma\)-ray photons and
the IceCube neutrinos. Luo & Zhang (2020) further
suggested that the multi-frequency selected sample of
blazars, 5BZCat\(^2\), is not significantly correlated with
the list of IceCube alerts, and basically no association
between \(\gamma\)-ray fluxes of \textit{Fermi} Monitored sources\(^3\)
and neutrino flares are found. Later, Giommi et al.
(2020) reported a 3.23\(\sigma\) correlation excess with \(\gamma\)-ray
HBLs and IBLs\(^4\) in the vicinity of IceCube high-energy
track-like events. They identified probable \(\gamma\)-ray blazar
counterparts for IceCube neutrinos using an innovative
tool, the VOU-Blazars (Chang et al. 2020), designed to
find blazar/AGN candidates based on multi-frequency
data obtained with Virtual Observatory\(^5\) methods. The
VOU-Blazar tool can deal with relatively large areas of the
sky at once.

The production of VHE \(\gamma\)-ray photons is believed to
be inevitable during the photo-hadronic process from
which neutrinos are produced. Moreover, the \(\gamma\)-rays are
thought to be absorbed within the source or in further
interactions with the extragalactic background light via
photon-photon annihilation (Franckowiak et al. 2020),
cascading down to lower energy. The source environ-
ment might be optically thick to GeV \(\gamma\)-rays, causing
no apparent connection between \textit{Fermi} emissions and
neutrinos. Indeed, most of the neutrino activities has
no \(\gamma\)-ray flare companion, like the 2014-2015 neutrino
flares reported by IceCube Collaboration et al. (2018a).
Given that the pionic \(\gamma\)-ray photons may cascade down
to X-ray band in blazar jets with strong photons fields,
a stacking analysis of \textit{Swift} BASS\(^6\) objects (Goswami
et al. 2021) and a time-dependent search using the posi-
tion of X-ray selected blazars from 5BZCat (Sharma &
O’Sullivan 2021) were proposed.

Recently, Plavin et al. (2020) and Plavin et al. (2021)
found that the position of radio-bright blazars is sta-

tistically coincident with arrival directions of neutrino

\(^1\) 2FHL: Second \textit{Fermi}-LAT Catalog of High-Energy Sources, Ack-

\(^2\) The 5th edition of the Roma-BZCAT Massaro et al. (2015)

\(^3\) https://fermi.gsfc.nasa.gov/ssc/data/access/lat/mlc_lc/

\(^4\) According to the peak frequency of synchrotron radiation \((\nu_{\text{peak}})\),
blazars are divided into high-(HSP/HBL: \(\nu_{\text{peak}} \geq 10^{15}\) Hz),
intermediate- (ISP/IBL: \(10^{14}\) Hz < \(\nu_{\text{peak}} < 10^{15}\) Hz), and low-
(LSP/LBL: HSP/HBL: \(\nu_{\text{peak}} < 10^{15}\) Hz) peaked sources respec-
tively(Abdo et al. 2010a)

\(^5\) http://www.ivoa.net

\(^6\) BASS: The BAT AGN Spectroscopic Survey, Baumgartner et al.

\(^7\) http://astrogeo.org/rfc/

\(^8\) https://sites.astro.caltech.edu/ovroblazars/
In all stacking analyses, blazars’ contributions are found to be less than \(\sim 15-27\%\), depending on the assumed neutrino spectral shape. Constraints obtained from other methods, such as high-energy multiplets and auto-correlation, or predicted applying theoretical blazar hadronic models are all consistent with the stacking limits (Padovani et al. 2015, 2016; Murase et al. 2014, 2018; Yuan et al. 2020; Bartos et al. 2021). All these results suggest that blazars might not be the dominant sources of the IceCube diffuse astrophysical neutrino flux. Since these limits only allow to constrain the average neutrino emission of steady sources, it is not absolutely surprised that individual sources can outshine these limits over a shorter period, for example, during their flaring stages (Huber 2019).

In Sep. 2017, the \(\gamma\)-ray flaring stage of a bright blazar, TXS0506+056, was found in spatial and temporal coincidence with a 290 TeV neutrino alert IceCube-170922A at 3\(\sigma\) significance (IceCube Collaboration et al. 2018b). This neutrino event additionally was accompanied by the strong flares of TXS0506+056 across the electromagnetic spectrum. A dedicated search for archival neutrino flares from the same direction with 9.5 years of IceCube data (IceCube Collaboration et al. 2018a) found a 3.5\(\sigma\) excess of \(\sim 13.5\) neutrinos in 2014-2015. The association of 2017 alerts and 2014-2015 flares with TXS0506+056 pinpoints the blazar as the first promising extragalactic high-energy neutrino source and triggers a considerable interest in the nature of the electromagnetic counterparts of astrophysical neutrinos.

The case of TXS0506+056 also demonstrates that the observed coincident activities of neutrinos and photons would greatly increase the probabilities of identifying the counterparts of IceCube events (IceCube Collaboration et al. 2018b). The highly variable characteristics of blazars, with flux that could increase at least a factor of two in a day, make them play an indispensable role in finding astrophysical counterparts of high energy neutrinos. Murase et al. (2018) argue that neutrino emitting blazars can be dominated by the flares in the standard lepton scenario for their gamma-ray emission. Oikonomou et al. (2019) and Stathopoulos et al. (2021) estimated the neutrino emissions associated to \(\gamma\)-ray and X-ray flaring periods of 12 Fermi bright blazars (for which simultaneous observations exist) and another 66 blazars (observed more than 50 times with the Swift X-ray Telescope - XRT, Giommi et al. 2021), respectively. Those works predicted the highest rates of muon neutrinos to be \(\sim 1.2 - 3\text{yr}^{-1}\), concerning the X-ray flares of MrK421 and the \(\gamma\)-ray flares of AO0235+164 and OJ287.

Considering the multi-frequency activity of TXS0506+056 around the arrival time of IceCube-170922A, the strongest flaring period in the low-energy band (radio and infrared) and in the high-energy band (\(\gamma\)-rays) are not simultaneous. There is a significant lag of \(\sim 300\) days between the \(\gamma\)-ray and radio/infrared flares. In addition, PKS1502+106 and J0242+1101 also show long-lasting 15GHz radio flares in coincidence with IceCube-190730A (Franckowiak et al. 2020) and Antares flares (Illuminati & Plavin 2021), respectively. Both blazars show a significant lag of radio to \(\gamma\)-ray flaring stages. In general, lags in different energy bands might arise from changes in the source’s environment, causing different emission zones to shine in different energies. Taking the most plausible neutrino blazar, TXS0506+056, for example, many studies have shown difficulties of reconciling both neutrino activities (IceCube170922A and 2014-2015 flares) and multi-frequency spectral energy distribution (SED) through a single emission model (Murase et al. 2018; Reimer et al. 2019; Rodrigues et al. 2019; Petropoulou et al. 2020). Even though a single-zone lepto-hadronic model with a subdominant hadronic component assuming sufficient photons from the external field, and with right energy might be able to describe the high-energy alert from this source (Ansoldi et al. 2018; Keivani et al. 2018; Cerruti et al. 2019; Gao et al. 2019; Righi et al. 2019a), no single-zone scenario can explain the high neutrino flux from 2014-2015 flares and -at the same time- satisfy the constraints from the simultaneous SED. The neutrino flares, surprisingly, were not accompanied by any photon flare given all the observed information (IceCube Collaboration et al. 2018a), indicating that not all the neutrino emission is necessarily correlated to electromagnetic activities, especially in \(\gamma\)-ray (Halzen et al. 2019; Kun et al. 2021).

Here we propose a series of analyses to search for neutrino emission from flaring sources motivated by i) the ideal conditions for neutrino production during blazar flares (Oikonomou et al. 2019; Murase et al. 2018), ii) the remarkable characters of TXS0506+056, iii) the similarity of multi-frequency flaring behaviors between the TXS0506+056 and two additional/potential neutrino blazars: PKS1502+105 and J0242+1101. Our works consist of two parts. Firstly, we concentrate on the multi-frequency sources inside the containment regions of IceCube alerts. The probable radio-neutrino correlation among radio-bright AGNs reported in Plavin et al. (2020) as well as the electromagnetic cascade of VHE \(\gamma\)-rays, inspire us to investigate the correlation between the flaring phases in various wavebands and the arrival time of IceCube alerts. The second part of our analy-
ses will focus on three main blazar samples, analyzing their light curves in two different bands: low frequency (infrared) and high frequency (γ-ray). We aim to study the correlation with neutrinos among blazars with different types of multi-frequency activities. By investigating and selecting blazars with multi-frequency flaring behavior similar to TXS0506+056, we might be able to identify promising neutrino counterparts. No previous studies have looked for neutrino emission from blazars considering their multi-frequency light curves to investigate correlations when accounting for different flaring states.

The paper is structured as follows: in section 2 we introduce the neutrino data, the multi-frequency catalogs, and the blazar samples used in this work. In section 3, we investigate the correlations between the electromagnetic flaring periods of multi-frequency sources and the arrival times of the neutrino alerts. In section 4, we probe the possible neutrino emissions associated with blazars’ multi-frequency activities. We then discuss and summarize our results in section 5 and 6.

2. MULTI-MESSENGER SAMPLES AND DATA

2.1. IceCube neutrinos

IceCube has been announcing high-energy neutrino alerts since the spring of 2016, bringing about a total of 67 real-time alerts up to the end of May 2021, with a relatively high probabilities of those events being of astrophysical origin. Those reported events passed the selection criteria of the real-time alert system (Aartsen et al. 2017b) and generally have energy \( \gtrsim 100 \) TeV. Another 35 archival events from 2010 to 2016 fulfill the same criteria before the operation of the real-time alert system, summing up a total of 102 events considered in our analyses. The lists of real-time and archival alerts/events are taken from the Gamma-ray Coordinates Network (GCN) / Astrophysical Multimessenger Observatory Network (AMON) Notices and IceCube website\(^9\).

Apart from high-energy neutrino alerts and archival events which would have qualified as real-time alerts, IceCube published a sample of track-like neutrino events collecting 10 years data (from 2008 to 2018, IceCube Collaboration et al. 2021) that was assembled for neutrino point-source searches and used in the IceCube’s 10-year time-integrated point-source analysis (Aartsen et al. 2020). This sample covers a broader energy range than the high-energy alert list, and contains neutrino events with \( E < 100 \) TeV. To study only events with a higher probability of being of astrophysical origin and are well reconstructed, we cut the events with reconstructed energy at \( \gtrsim 60 \) TeV, which is the same energy cut applied in Padovani et al. (2016), and with angular uncertainty smaller than 5 degrees. The cuts result in 250,821 well-reconstructed high-energy track events studied in this paper.

2.2. Blazar samples

Three blazars samples are used in our studies.

The 3HSP: The 3HSP catalog is a multi-frequency selected sample of 2011 HSP and HSP candidates (Chang et al. 2019). The 3HSP catalog is currently the most extensive and complete HSP catalog\(^10\) and an ideal sample to study the statistical properties (such as completeness, evolution, etc.) of blazars.

The 5BZcat: We also selected blazars from the 5BZCAT catalog (Massaro et al. 2015), which consists of 3561 robust blazars, all confirmed via optical spectroscopy. Even though the 5BZCat is a compilation of blazars found by many different methods and thus not a complete sample, it is the largest catalog of confirmed blazars with optical spectral observations.

The WIBRaLS: The last samples we use are the WISE Wright et al. (Wide-field Infrared Survey Explorer, 2010) Blazar-like radio-loud sources, named WIBRaLS and WIBRaLS2 Catalogs\(^11\) (D’Abrusco et al. 2014, 2019). Those catalogs contain a total of \( \sim 12415 \) blazar candidates and are the largest samples of their kind to date. The WIBRaLS catalogs are samples of infrared selected radio-loud blazar candidates with WISE mid-infrared colors similar to that of confirmed γ-ray blazars.

In addition, there are 18 Fermi 4LAC-associated blazars (Lott et al. 2020) within the containment regions of the IceCube alerts that are not cataloged in 3HSP-DR2, 5BZCat, or WIBRaLS. Among those 18 sources, 11 are related to alerts which have relatively large angular uncertainty and were not considered in our analyses. In total, we have collected a meta-blazar sample with 15424 blazars and blazar candidates from three extensive blazar catalogs and extra blazars reported in 4LAC.

Considering the completeness and accessibility of multi-wavelength light curves, here we focus on objects

---

\(^9\) https://gcn.gsfc.nasa.gov/amon_hese_events.html,
https://gcn.gsfc.nasa.gov/amon_ebe_events.html,
https://gcn.gsfc.nasa.gov/amon_icecube_gold_bronze_events.html,
https://icecube.wisc.edu/science/data/TXS0506_alerts

\(^10\) A second and more complete version of the 3HSP catalog, the 3HSP-DR2, will be released soon and contains 2081 HSPs and HSP candidates.

\(^11\) In this paper, we use WIBRaLS to represent both WIBRaLS and WIBRaLS2 catalogs.
with both Fermi 4FGL-DR2 (Ballet et al. 2020) and WISE\textsuperscript{12} multi-epoch data, to study the flaring behavior in γ-rays and infrared. Among the meta-blazar sample, only 2700 sources have both γ-ray and infrared light curves available. We call these 2700 blazars the “Fermi-Infrared Blazar Sample (FIBS)”. In search for potential neutrino blazars, the correlation between radio and γ-ray flares is also of interest. Especially, the promising neutrino blazar TXS0506+056 shows a significant lag between the radio and γ-ray flaring stages. However, there is currently no other public and long-term monitored radio data available. The data from ALMA Calibration Catalog (ACC, in the millimeter band, Bonato et al. 2019) is the best option for us, even though the data do not cover the full time range of Fermi light curves and sometimes are triggered by high stage in other wavebands. In our meta-blazar sample, there are 504 sources with both Fermi and ACC multi-epoch data available.

2.2.1. Potential IceCube neutrino blazars

The IceCube Collaboration has been searching for neutrino excess from several lists of sources (Abbasi et al. 2011; Aartsen et al. 2013, 2014, 2017c, 2019, 2020). Among those sources, there are eight blazars for which the correlation with astrophysical neutrinos has the significance of the order of p-value $\leq 0.05$ for at least one of the IceCube all-sky point-source searches. Moreover, 101 out of 2700 FIBS sources are located within the containment region of 102 IceCube high-energy alerts (up to May 2021, Section 2.1). Here we slightly enlarged the 90% containment region of the alerts by a factor of 1.1, taking into account that 10% of the candidates expected to be outside of the 90% containment area (Giommi et al. 2020). We note that we did not consider the possible systematic position uncertainty for the alerts in our analyses. The blazar, TXS0506+056, is known for having a relatively high significance in IceCube point-source analysis and by its direct association with a track event IceCube-170922A and the neutrino excess in 2014–2015 (IceCube Collaboration et al. 2018b,a; Padovani et al. 2018). There are 108 sources either in the alert containment regions or associated with a weak neutrino excess signal (at 95% significance level). We call these 108 blazars potential neutrino sources and will thoroughly discuss the multi-frequency behaviors of these sources in section 4.3.2.

2.3. Multi-frequency data

Here we describe the multi-frequency data used in this paper, from millimeter radio up to γ-ray.

**Millimeter Radio:** The millimeter multi-epochs data were obtained from the ACC (Bonato et al. 2019), which is an astronomical-measurement database of calibration sources that are mostly bright quasars observed in seven different bands (ranging from 84 GHz to 950 GHz) and in different epochs (between May 2011 and July 2018). We used the band 3 data (84–116 GHz) to describe most of our sources. Only for 16 out of 504 sources in our meta blazar sample, we had to consider millimeter data other than band 3, with preference to band 4 (125–163 GHz), band 6 (211–275 GHz), and band 7 (275–373 GHz). All the ACC light curves in our analyses have at least three observations in the same band, with different time intervals varying from days to years. For those observations with time separation smaller than 15 days, we combined them and took their average value.

**Infrared:** This paper used 4.6 μm WISE W2 infrared light curves from AllWISE Multiepoch photometry (AllWISEMEP) dataset and NEOWISE (Near-Earth Object WISE, Mainzer et al. 2014) data release\textsuperscript{13}, with observing time ranging from January 2010 to December 2020. The WISE light curves have a large time interval (of several hundred days), and the observations are usually centered in 1–2 days along several months. Thus, we combined the infrared data with time separation smaller than 15 days, averaging the signal and removing the outliers. After the combination, the mean interval is roughly 180 days. Moreover, there is a break between 55600 to 56500 MJD which does affect the identification of relevant flaring periods, resulting from the gap between the AllWISEMEP and NEOWISE surveys. We manually added artificial points 180 days after the beginning and before the end of the break to remedy the data gap. The flux and error of the artificial points are based in the average infrared flux for each source.

**X-ray:** We consider the 3 keV multi-epoch observation data from Swift XRT, covering December 2005 to October 2020. The data is based on blazars frequently observed by Swift and was made available in Giommi et al. (2019) and Giommi et al. (2021). Similar to the data preprocessing in millimeter and infrared, the X-ray observations with time separation smaller than 15 days were combined and averaged.

**γ-ray:** The γ-ray data is retrieved from the aperture photometric light curves of Fermi-LAT 4FGL-DR2 catalog\textsuperscript{14}. Those aperture light curves are binned evenly

\textsuperscript{12} DOI of AllWISE source catalog: 10.26131/IRSA1

\textsuperscript{13} DOI of AllWISEMEP and NEOWISE datasets: 10.26131/IRSA134 and 10.26131/IRSA124

\textsuperscript{14} https://fermi.gsfc.nasa.gov/ssc/data/access/lat/10yr_catalog/ap_lcs.php
with 30 days intervals since June of 2008. By using 10-year-average photon indexes in the 4FGL-DR2 catalog, we converted the photon fluxes of the aperture light curves from 0.1-200 GeV to 0.8-200 GeV energy fluxes in units of erg cm$^{-2}$ s$^{-1}$, focusing on the high energy band to avoid the contamination from nearby sources due to the large point spread function at lower energy.

2.3.1. Multi-Frequency sources in the IceCube alert regions

In addition to blazars in our FIBS sample, sources in the vicinity of IceCube alert regions are of interest in this paper. We selected multi-frequency sources located in low declinations and outside the galactic plane (with similar location distribution concerning most of the IceCube alerts, $\delta < |40^\circ|$ and $b > |10^\circ|$) from above catalogs to study time-space correlations between the sources’ flaring activity and the IceCube alerts. This study is described in section 3. Only millimeter and X-ray sources with more than five detections at different epochs are considered. While for infrared sources, we took only those with a counterpart in WIBRaLS catalogs and with 1.4 GHz flux $\geq 100$ mJy (to reduce a large number of infrared emitters). In the 4FGL-DR2 catalog, when the variability index is larger than 18.48, the $\gamma$-ray variability is considered to be very likely, with a 99% level of confidence. Therefore, to select only sources with a high probability of being variable, we removed cases with the variability index smaller than 18.48. All the above criteria combined lead to a selection of 445 ACC, 3179 WIBRaLS, 876 XRT, and 992 4FGL sources, with 26, 165, 46, and 48 of them in the containment region of IceCube alerts, respectively.

A number of analyses are performed throughout the paper, and there are combinations of neutrino datasets and source samples that are built with multiple selection criteria. In Table 6 we summarize all the analyses titles and their corresponding neutrino datasets and source samples.

2.4. Quantification of variability and flares

The variability of a source can be quantified with the “Normalized Excess Variance” ($\sigma_{\text{NXS}}$) defined as:

$$\sigma_{\text{NXS}} = \frac{S^2 - \langle \sigma_i^2 \rangle}{\langle F_i \rangle^2}, \quad (1)$$

where $F_i$ and $\sigma_i$ represent the flux density and its associated error in time bin $i$, and $S$ is the flux standard deviation concerning the entire light curve.

Additionally, the Fermi-LAT team applied a likelihood ratio test to evaluate the variability of a $\gamma$-ray source. The column “Variability Index” in the 4FGL-DR2 catalog represents the sum of the log-likelihood difference between the flux fitted in each time interval and the average flux over the entire catalog interval. A value greater than 18.48 (over 12 time intervals) indicates a 1% chance of being a steady source.

To define and identify the flaring period of a source, we could adopt the criterion presented in Abdo et al. (2010b) to define the “bright state” for every observation in a light curve as:

$$F_i - \sigma_i > \langle F_i \rangle + 1.5 \times S \quad (2)$$

where $F_i$ and $\sigma_i$ represent the flux (density) and error of each detection of the light curve, while $\langle F_i \rangle$ and $S$ are the mean and standard deviation of the light curve.

The average flux $\langle F_i \rangle$ sometimes could be twice as bright as the quiescent state, especially for a highly variable source flaring in most of the observing period. Thus, we estimated the mean flux value when the source is in the quiescent stage (named quiescent flux) by averaging the faintest 30% detections in the light curve after removing no-detection time bins and faint/problematic outliers. Those outliers include detections with low signal-to-noise ratio or with low statistics.

A flaring state can also be objectively identified through the Bayesian Blocks Algorithm, aiming to find the optimal segmentation of the data in the observation interval (Scargle et al. 2013). In this work, we use Bayesian Blocks astropy implementation to detect statistically significant variations in multi-frequency light curves, and Equation 2 is applied to recognize the flaring period for comparison.

3. FLARES AND SOURCES WITHIN ICECUBE ALERT REGIONS

3.1. Time-dependent analysis

To study if the bright state epochs are consistent with the arrival time of the IceCube alerts, we follow the methods in Plavin et al. (2020) and perform a time-dependent analysis for multi-frequency sources inside the IceCube alert regions. Given that the majority of the IceCube alerts are close to the Celestial equator and to avoid the complicated Galactic plane region, we selected the alerts with galactic latitude $b > |10^\circ|$ and declination $\delta < |40^\circ|$. There are 26, 165, 46, and 48 ACC, bright WIBRaLS, XRT, and 4FGL variable sources, respectively, within the contaminant regions of 80 IceCube alerts with $b > |10^\circ|$ and $\delta < |40^\circ|$ (See section 2.3 for source selection). An observable $R(t = 0)$ was defined as the ratio of the average flux density within the $\Delta T$

15 https://docs.astropy.org/en/stable/api/astropy.stats.bayesian_blocks.html
time window and outside this time window.

\[
R(t = 0) = \frac{\text{average flux density within } \Delta T}{\text{average flux density outside } \Delta T} \tag{3}
\]

Where \( t = 0 \) means that the observing times of the multi-frequency sources were not shifted manually and \( \Delta T \) is the time window around the neutrino detected time. Higher \( R(t) \) values imply flares tend to center within the \( \Delta T \).

The \( \Delta T \) is determined by Monte Carlo simulation with the smallest p-value from several arbitrary trial values for 4FGL and WIBRaLS sources. For each \( \Delta T \) trial value, we scrambled the R.A. of the IceCube alerts 10,000 times using \( R(t = 0) \) as the test statistic, and the p-value is the probability that shifted alert positions yield a higher test statistic. According to the results from simulation (See Figure 1), we selected \( \Delta T = 630 \) days for bright WIBRaLS and \( \Delta T = 22 \) days for Fermi 4LAC-DR2 variable sources. We also ran the simulation with a \( \Delta T = 450 \) days, and the results is very similar to the ones with best-fit time windows of 630 days. For ACC and XRT sources, we use the medium time interval of roughly 30 days, given that the light curve intervals for the two samples are not equally binned. The multi-frequency fluxes are normalized by the quiescent flux (see section 2.4) of each source to avoid bias from different sources, and the normalized fluxes are defined as “flare levels”, which are the ratio between the flux of each detection and the quiescent flux.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{p_value.png}
\caption{P-values with respect to different \( \Delta T \) (time window) trials. The upper panel is for WIBRaLS sources, and the lower panel is for Fermi 4LAC-DR2 variable sources.}
\end{figure}

To test the correlation between electromagnetic flaring times and neutrino alert times, the observing times of multi-frequency sources are shifted with a time parameter \( t \), while the neutrino arrival time and time window remain the same. The correlation could be demonstrated once the highest flux ratio \( R(t) \) is centered at the original observing time \( t = 0 \). The observing times were shifted between \(-3600 < t < 3600 \) days, roughly spanning 10 years before and after the real observation, with shifted bin roughly equal to half of the time window \( \Delta T \). We note that the shifted bin of infrared data is 360 days, not half of the \( \Delta T \), given that the simulated time window for WIBRaLS sources is too large. IceCube has observed neutrino data and published alerts for around 10 years, including “alert-like” archival neutrino events before the real-time alert system was established. By choosing to shift the light curves from 10 years before the actual observing date to 10 years after that, the latest multi-frequency observation would be shifted through the time window of the first IceCube alert.

The maximum \( R(t) \) value might be at the center \(( t = 0 \), without shifting the observing time) by chance. We used Monte Carlo simulation to estimate the probability of flares coinciding with the alert arrival time. The control samples were then built by randomly selecting the same number of sources as the experimental sample from those outside the alert region, and assigning the “alert time” for each selected source. We repeat all the steps above with the control samples and obtained scrambled \( R(t) \), simulating the ratio of average flux (density) inside and outside the time window \( \Delta T \) with random sources and alert times for 10,000 times. The scrambled \( R(t) \) will be used to estimate the significance of the correlation between electromagnetic flares and IceCube alerts.

\subsection{3.2. Correlations with IceCube alerts}

The time-dependent results of temporal correlation between neutrino alerts and multi-frequency flares are shown from Figure 2 to 5. To investigate the variation of the correlation concerning the strength of the flares, we tested the relationships for several flare levels cuts (the ratio of observed flux values to the quiescent flux of the entire light curve). We selected the flare level values of 1.5 and 2.5 as our flare-criteria, simply because they are roughly the mean and 1\( \sigma \) upper limit of the multi-frequency light curves’ variability.

The X-axes are the delay of the multi-frequency flaring stages corresponding to the arrival times of the alerts. Time lags equal to zero means that the light curves are not shifted, and the flaring times of the multi-frequency sources are related to the neutrino alerts. On the other hand, a positive time lag value suggests that the multi-frequency flares occur ahead of neutrino alerts and vice
versa. The Y-axes R(t) represent the ratio between flux (density) averaged within and outside the time window ΔT (See section 3.1 for more details).

Figure 2. The ratio of ACC flux density averaged over a 30-day window to the average flux density outside it. The data average and 1σ statistical error are calculated with the observed experimental data, while the trials 1σ statistical error are obtained from random trials of the control samples.

The R(t) does not center at any time lag value in Figure 2. We do not find any evidence suggesting a correlation between the millimeter flares and neutrino alerts, but it should be noted that the millimeter light curves are not equally binned concerning some of the detections taken from the target of opportunity observations.

According to Figure 3, the maximum R(t) value (apart from the peak at around 3000 days) is at the time lag t = 0 for strong flares with flare level ≥ 1.5 or ≥ 2.5, implying that the strong infrared flares might be correlated with the neutrino arrival time. However, the correlation is not significant as the highest R(t) is just above the 1σ statistical trial error. We then removed the two promising neutrino blazars, TXS0506+056 and PKS1502+106, to evaluate if the possible association is driven by these two blazars. Without the two neutrino blazars, we could still tell the R(t) peak from the figure, suggesting that the possible trend is not driven by them. Other faint blazars might also play a role in the association between infrared flares and neutrino alerts. We note that the R(t) peak at t ~ 3000 days is too far away from the neutrino arrival time and thus we do not consider it to be associated with the neutrino events.

There are R(t) peaks around 2000 and 3000 days before the neutrino arrival time with time window ΔT = 30 days in Figure 4. However, the fluctuation in the figures is large, and the peaks do not stand out in the lower panel with a 200-day time window, which is the mean time interval for our XRT data. Our results suggest that the correlation between X-ray flares and neutrino events is not obvious, and probably the results are affected by the poor time coverage and uneven time interval of the

Figure 3. The ratio of WIBRaLS flux density averaged over a 630-day window to the average flux density outside it. Up: all WIBRaLS sources. Down: TXS0506+056 and PKS1502+106 are removed from the sample. Definition of legends are the same as Figure 2.

Figure 4. The ratio of XRT flux averaged over a time window to the average flux outside it. Up: a 30-day time window. Down: a 200-day time window. Definition of legends are the same as Figure 2.
Swift XRT data. A further investigation with dedicated X-ray time coverage might be fruitful in the future.

Figure 5. The ratio of 4FGL flux averaged over a 22-day window to the average flux outside it. Up: all Fermi variable sources. Down: TXS0506+056 and PKS1502+106 are removed from the sample. Definition of legends are the same as Figure 2.

Figure 5 indicates that the γ-ray flaring periods might be several hundred days delayed and roughly 1500 days ahead of the neutrino alerts. We have no evidence that the peak at \( t = -1500 \) days is related to neutrino emissions, but the possible correlation at \( t \sim 300 \) days might be more likely connected to a hadronic process. However, the results are dominated by the two most probable neutrino blazars, TXS0506+056 and PKS1502+106. After removing these two sources from the examined samples, there is no clear signal anymore.

3.3. Significance of the correlation between multi-frequency flares and the neutrino alerts

We statistically estimated the significance level of the correlations by calculating the probability that the simulated sources and alert times from the control samples which will lead to a higher \( R(t) \) values by chance. The chance probability is illustrated as pre-trial p-value in Figure 6 and 7. We have performed multiple tests with numbers of shifted time bins on the same neutrino sample, thus correcting our p-values is necessary. Trial factors could be estimated using family-wise error rate (FWER), \( 1 - (1 - p)^N \), with the assumption that the shifted time bins are independent. \( N \) is the number of the time bins in each test, and \( p \) is the pre-trial p-values. Given that our results are far from significant, the p-values shown in this section are the pre-trial ones.

Figure 6. Pre-trial p-values of the ratio of averaged flux over a 630-day window to that outside it for bright WIBRaLS sample.

We could tell from Figure 6 that the correlation between infrared flaring times and neutrino arrival times is not significant, with the smallest pre-trial p-value close to neutrino arrival time \( (t = 0) \) is \( \lesssim 0.1 \).

Figure 7. Pre-trial p-values of the ratio of averaged flux inside a 30-day window to that outside it for 4FGL variable sources.

Figure 7 shows that the smallest pre-trial p-value is approaching 0.004; however, the association might not be related to neutrino emissions given that the correlated flaring stage is around three years before the arrival of the neutrino alerts. The second smallest pre-trial p-value (\( \sim 0.01 \)), which is located at \( t \sim 300 \) days, might give us clues about the correlation between γ-ray flares and neutrino alerts, while it is not very significant. The trial factors for both the pre-trial p-values for WIBRaLS and 4FGL samples are very large, with 21 and 241 independent shifted bins in the tests, respectively. This renders the results much more insignificant.
The ratios of the number of bright detections (flare levels \(\geq 1.5\) or 2.5) within the time window (\(\Delta T\)) to that outside it (\(N(t)\)) are shown in Figure 8. The majority of both infrared and \(\gamma\)-ray flares seem to gather around 500--1000 days before the arrival of neutrinos, and some Fermi 4FGL-DR2 flares occur \(\sim 500\) days after the alerts as well. The number of bright infrared detections within the time window drops rapidly right after the neutrino alert time, causing the \(R(t)\) values in the last section to decrease quickly after \(t = 0\). The results are consistent with the Figures in section 3.2. It should be mentioned again that the \(\gamma\)-ray flares here are dominated by the two potential neutrino blazars TXS0506+056 and PKS1502+106 and compare the high stage in both infrared and \(\gamma\)-ray light curves for the selected blazar samples and identify sources with multifrequency activity similar to that of TXS0506+056.

Figure 8. The ratio of the number of flare detections inside the time window to that outside it. Up: a 630-day time window with WIBRaLS sources. Down: a 22-day time window with 4FGL variable sources. Definition of legends are the same as Figure 2.

4. BLAZARS WITH NON-SIMULTANEOUS MULTI-FREQUENCY FLARES

To find more potential neutrino or hadronic sources not only within the IceCube public alerts, we carefully analyzed the flaring phases of 3HSP, 5BZCat, and WIBRaLS blazars in various bands. The first plausible extragalactic neutrino blazar, TXS0506+056, has low-frequency flares in radio and infrared not simultaneous with high-energy flares in \(\gamma\)-ray, with a significant lag of flaring stages in low-frequency. Figure 9 shows the infrared and \(\gamma\)-ray light curves of TXS0506+056. Inspiring by this time lag in TXS0506+056, we try to recognize simultaneous infrared and \(\gamma\)-ray flares. The upper panel is the infrared light curve from WISE multi-epoch data, and the middle panel is the Fermi \(\gamma\)-ray light curve analyzed with Fermi Science Tool. The lower panel represents the flaring periods in infrared (blue thick lines) and \(\gamma\)-ray (red thick lines) as well as the simultaneous flaring stages (green lines). Black lines represent the arrival time of ICeCube-170922A.

Figure 9. Light curves of TXS0506+056 showing non-simultaneous infrared and \(\gamma\)-ray flares. The upper panel is the infrared light curve from WISE multi-epoch data, and the middle panel is the Fermi \(\gamma\)-ray light curve analyzed with Fermi Science Tool. The lower panel represents the flaring periods in infrared (blue thick lines) and \(\gamma\)-ray (red thick lines) as well as the simultaneous flaring stages (green lines). Black lines represent the arrival time of ICeCube-170922A.

4.1. Selection Criteria

All the analysis in this section will focus on the 2700 Fermi-Infrared blazar sample (FIIBS) with both Fermi and WISE light curves available, see section 2.2 and 2.3 for more details about the data reduction. Bayesian Blocks Algorithm (Scargle et al. 2013) were used to identify the flaring periods in the light curves (Section 2.4) with more than two observations at different time epochs. We chose the prior which makes the algorithm sensitive to variations that are significant at 99% confidence level (a false alarm probability 0.01), identifying only strong and clear flaring episodes and avoiding tentative flares. If no flares are recognized by the algorithm for a given light curve, we lower the confidence level to 95%. We keep those blocks with \(F_{\text{blocks}} > F_{\text{quiescent}} + 1.3 \times S\) as flaring phases, where \(F_{\text{blocks}}\) is the average flux density in a block, and \(F_{\text{quiescent}}\) is the quiescent flux density (the average value concerning the 30% faintest points, see section 2.4 for more details). \(S\) is the standard deviation associated with the entire light curve. Considering that we are comparing the average flux for multiple detections in a block, the flaring block phases are selected with 1.3 \(\times S\) instead of 1.5 \(\times S\) as in Abdo et al. (2010b) (Eq. 2). Sometimes for the infrared light curves, the \(F_{\text{quiescent}}\) is much lower than the mean value, and we require \(F_{\text{blocks}} > F_{\text{average}} + S\) to be considered as a flaring state.
Once the bright phases in both the high-energy and low-energy light curves of each blazar are identified, the "multi-frequency epoch" of a blazar could be divided into three flaring stages: high energy flaring, low energy flaring, and simultaneous flaring stages. The high energy flaring period is defined as the time-interval with brighter flux in the high energy light curve but relatively low flux (quiet stage) in low energy; and the low energy flaring period is defined vice versa. The simultaneous flaring stage refers to time intervals with both high-energy and low-energy flux in the high stage. We define sources with no simultaneous flaring stage as "Non-simultaneous Flaring Sources" (NFS). On the contrary, sources with simultaneous flaring stages longer than half of the whole infrared and γ-ray flaring time are considered as "Correlated Flaring Sources" (CFS). An example of the infrared and γ-ray light curves for sources in CFS control group are illustrated in Figure 10.

![Example of CFS light curves with J0739+0137](image)

**Figure 10.** Example of CFS light curves with J0739+0137. The upper panel is the infrared light curve from WISE multi-epoch data, and the middle panel is the γ-ray aperture light curve from Fermi-LAT. The lower panel represents the flaring periods in infrared (blue thick lines) and γ-ray (red thick lines) as well as the simultaneous flaring stages (green lines).

We then removed those without strong and clear flares and those with ambiguous and fluctuated light curves. Sources within the containment regions of IceCube alerts or with relatively high significance (of $p \leq 0.05$) in IceCube point source searches (potential neutrino sources, see section 2.2.1) are not dropped out. In Fermi 4FGL-DR2 catalog, sources with variability index less than 18.48 are treated as not variable. For sources with Fermi variability index $\leq 18$, we exclude the ones with $\sigma_{NXS} < 0.0001$ (See section 2.4) or with $F_{\text{block max}} / F_{\text{block min}} < 1.7$. The ratio 1.7 is roughly the median value of Fermi not variable sources, with variability index $< 18$. Those sources are considered to have no significant variability, and there are around one-third of Fermi not variable sources that were removed. Next, if the ratio of standard deviation between the non-flaring stage and the whole observing time of the sources is less than 0.9, it is regarded as ambiguous sources with extremely fluctuating light curves. The standard deviation ratio value of 0.9 is the third quartile of whole FIBS sources. We removed a total of 886 FIBS sources in this step. Sources with no flaring state identified in both frequencies are not considered in this analysis, including 18 potential neutrino sources. In the end, only 1353 FIBS sources remained to be investigated in the next step.

In the last step, we begin to select those blazars with low energy flares that are not simultaneous with high energy flares, like TXS0506+056. Once there is no simultaneous flaring period between low and high frequency flares, we directly selected those with long radio/infrared flares that happen after rapid high-energy Fermi flares and low-energy flaring lag smaller than 500 days. If the simultaneous flaring periods in a source only consists of a small fraction (usually not occupied by more than one-fourth of the successive flaring period in two light curves), the source is still considered as good candidates and are picked when relatively long low-frequency flares follow short high-frequency flares. On the contrary, we remove those cases where the majority of the multi-frequency flaring periods are simultaneous and complicated (i.e., when there are multiple γ-ray flares contained inside a single infrared flare) because this multi-frequency behavior is not the one we see for the TXS0506+056.

The above criteria led to the selection of 171 TXS-like sources from the FIBS sources. To check whether those 171 selected sources are more probable neutrino emitters concerning the remained ones, we took NFS and CFS samples as control groups. We note that some sources without simultaneous flares in two multi-frequency light curves have already been selected as TXS-like sources, and they are not included in the control groups. Sources with extremely ambiguous flares are removed from the control groups as well. Our control groups, in the end, contain 409 NFS and 62 CFS among FIBS sources. Further studies and analyses of these 171 selected sources, as well as control groups, will be discussed in the next section and in section 4.4.

Our selection steps are summarized as follows:

1. Collect the meta-blazar sample from 3HSP, 5BZ-Cat, and WIBRaLS catalogs.
2. Cut the meta-blazar sample: only select cases within the WISE infrared and Fermi 4LAC catalogs. That defines the Fermi-Infrared Blazar sample (FIBS), containing 2700 sources.
3. Identify the optimal segmentation and flares with Bayesian Blocks Algorithm and data reduction.
4. Remove sources with weak or ambiguous variability.

5. Select non-simultaneous multi-frequency flaring sources, like TXS0506+056.

As stated in section 2.2, we are also interested in blazars with non-simultaneous radio and γ-ray flaring stages. Following the above selection steps, we selected a total of 21 sources with millimeter and γ-ray flaring behaviors similar to that of TXS0506+056 from the 504 ACC-Fermi sub-sample. Given that the ACC multi-epoch data is much more sparse than the WISE and Fermi light curves, it is meaningless to find highly overlapped millimeter and γ-ray flares or the orphan flares. Therefore, there are no control groups for the millimeter-selected sample. We note that the selection concerning the ACC data suffers from a heavy incompleteness, and the 21 sources selected here are just meant to compare the results from millimeter with those from infrared.

4.2. Dedicated γ-ray light-curve analysis

In the last section, we applied Fermi aperture photometric light curves to select sources with multi-frequency flaring activities like TXS0506+056; however, the aperture light curves are not suitable to do detailed scientific analysis. The aperture light curves were used just to screen a large amount of FIBS sources in the first place, so further confirmation and filtering of those selected sources are required. We then performed dedicated γ-ray light curve analysis with Fermi Science Tool (for all the selected sources) to make sure our selection was robust. The time range of the dedicated Fermi analysis is from MJD 54862 to 59257 with a 30-day time bin, similar to the one used for the aperture photometric γ-ray light curves. The energy range used for the dedicated analysis covers from 900 MeV to 500 GeV, focusing on a slightly higher energy range when compared to the aperture light curves. A higher energy band was chosen mainly to reduce the computation load, and also it helps to avoid contamination at lower energy. We scanned for three different pivot energies (1, 2.5, and 5 GeV) for every selected source and time bin and selected the pivot energy that leads to the lowest uncertainty associated with the normalization (N_0), and photon index (Γ). For time bins where the test statistic (TS) is very similar for all pivot energies, we kept the one with a higher signal-to-noise ratio. For time bins where the likelihood analysis did not converge, we tried an extra run with pivot energy of 2 GeV.

Detection with TS \( \leq 5 \), Flux_{err} \( \geq \) Flux, or Photon Index(Γ) > 5.5 are considered as upper limits, and the limits of energy fluxes are set to 3 \( \times 10^{-14} \) erg s\(^{-1}\) cm\(^{-2}\). We assumed this value to represent an average upper limit level for our sample since most of the detections deemed as limits have energy fluxes smaller than 3\( \times 10^{-14} \). We reevaluated the flaring stages of our selected TXS-like sources from FIBS and ACC-Fermi sub-samples with these new dedicated analysis light curves one by one, removing problematic selections and the sources with ambiguous flaring stages. This procedure results in 32 (and 11) sources that remained from the infrared (and millimeter) selected lists. The source 3HSPJ064850.5-694522 was added to the selected sample with a slightly different criteria. This sources would be selected regarding an additional weak flare in γ-ray identified by the Bayesian Blocks Algorithm when setting the flux upper limits to 10\(^{-14} \) erg s\(^{-1}\) cm\(^{-2}\). We keep this source in our selected TXS-like sample to avoid missing any possible neutrino emitters.

A large number of sources removed might be a consequence of the large flux fluctuations associated with Fermi aperture light curves (especially for faint sources, where low photometric fluxes are usually associated with non-detection in the dedicated analysis, ending up flagged as flux upper limits). Contamination of the photons from nearby sources in the aperture light curves at low energy and the gap between MJD 55600 to 56500 in WISE multi-epoch data, additionally, would cause bogus selections. We note that some of the γ-ray flares from the dedicated light-curve analysis sometimes are associated with sources that produced only 1-2 detection(s) in the entire light curve. However, given that we have scanned the pivot energy with several values, carefully removed the problematic detections, and objectively and systematically identified the flaring phases with Bayesian Blocks Algorithm, the detected variation could be considered as robust. The final number of sources in each selected list and control group are shown in Table 1, and all the light curves of selected TXS-like sources, as well as the Bayesian Blocks results, are illustrated in the Appendix.

4.3. Sanity checks

Before performing the analysis with selected sources and control groups, we did a few tests to investigate the overall multi-frequency correlation of FIBS sources and to study the simultaneity of infrared and γ-ray flares of those potential neutrino sources.

4.3.1. Overall multi-frequency correlations of blazars

The first sanity check is to study the average time lag of the infrared flares with respect to the γ-ray ones for all the FIBS sources. Cross-correlation between two multi-frequency light curves could be estimated with Discrete Correlation Function (DCF). The time lag for
each source is retrieved from the time bin with maximum cross-correlation value. Here, the time lag is also evaluated with the fitting of DCF cross-correlated matrix, and we took the Gaussian mean as a representation of the time lag of the two light curves.

![Figure 11](image.png)

**Figure 11.** Correlation between infrared and *Fermi* light curves. The time lags of flaring periods in infrared regarding to γ-ray are estimated with the Discrete Correlation Function. The blue and orange bars represent the time lag corresponding to the maximum cross-correlation and corresponding to the Gaussian fitted mean of the cross-correlated matrix between two light curves, respectively.

Figure 11 illustrates the distribution of time lag of flaring phases in infrared regarding the γ-ray activity for 2700 FIBS sources. According to the figure, the time lag obtained directly with the time bin with maximum cross-correlation value substantially center at bin 0-30 days, while that obtained with the Gaussian fitting are more scattered but still gather around the zero. On average, there is no significant delay in infrared flares compared with γ-ray flares for most blazars. Blazars like TXS0506+056, with an infrared time lag of ~300 days, represent only a tiny fraction of the total FIBS sources, suggesting that they are not typical.

4.3.2. Sources related to IceCube neutrinos

Next, we would like to test whether the high-energy and low-energy flares of the potential neutrino sources are more likely to be simultaneous or “orphan”. The number and fraction of potential neutrino sources among the selected lists and control groups (see section 4.1 and 4.2) are also presented in Table 1. The total number of sources in each list is represented with a bracket. Compared with the control groups, the fraction of potential neutrino sources for TXS-like sources (see Figure 9) selected from both FIBS and ACC-\textit{Fermi} sub-samples are higher. The relatively high fraction for selected TXS-like sources from the infrared-γ-ray sub-sample indicates that sources with multi-frequency light curve behaviors similar to TXS0506+056 are more likely to be related to neutrinos, while the statistic is low given the small number of sources selected. The high fraction for sources selected from ACC-\textit{Fermi} sub-sample, on the contrary, is tentative and bias given that the multi-epoch data from the ACC catalog is far from complete (see section 4.1 for more detail).

| Source Type          | Selected TXS-like | CFS   | NFS   |
|----------------------|-------------------|-------|-------|
| Infrared-γ-ray (FIBS)| 4(32)             | 4(62) | 34(409) |
| Millimeter-γ-ray (ACC-\textit{Fermi})| 3(12) | —     | —     |

**Table 1.** Number and fraction of potential neutrino sources in our selected lists and control groups. The values inside the brackets represent the total number of sources in each selected list and control group. Example light curves of TXS-like sources and CFS samples are illustrated in Figure 9 and 10, and detailed selections of those groups of sources are described in Section 4.1.

Figure 12 shows the histogram of the ratio of the simultaneous flaring period (overlap flaring time) of FIBS (sub-)samples. For the following tests along this section, we separate the potential neutrino sources into “IceCube warm spot” and “IceCube region”, for sources with ≥95% significance level in IceCube’s point-source searches and within the containment regions of IceCube alerts (section 2.2.1). The flaring times are defined with the Bayesian Blocks Algorithm, and details of how we defined the flaring stage in a light curve are written in Section 4.1. Only sources with flares in both WISE and \textit{Fermi} light curves are plotted in the histogram. We note that the definition of “IceCube region” sources here are a bit different than the definition in section 2.2.1. Here we assumed the 90% IceCube containment region is a rectangle instead of an approximate ellipse. The number of infrared and γ-ray flaring blazars among FIBS, “IceCube warm spot” sources, and “IceCube region” sources are 2239, 8, and 88, respectively.

From Figure 12, it is clear that the distribution of the overlap flaring time ratio for “IceCube warm spot” sources is substantially different from that of typical FIBS sources and “IceCube region” sources, suggesting that for those with a weak neutrino signal in IceCube’s previous point-source analyses tend to have simultaneous flares in infrared and γ-ray.

The significance levels are estimated using Monte Carlo simulation. For sources within the 90% rectan-
Figure 12. Distribution of the simultaneous flaring period (overlap flaring time) ratio in infrared and γ-ray. The blue bars indicate the FIBS sources with both flaring in infrared and γ-ray, and the orange and green bars are the “IceCube warm spot” and “IceCube region” sources, respectively. See text for more details.

Figure 13. The ratio of orphan flares versus simultaneous multi-frequency flares with observed and simulated data. The stars represent the averaged values of the observed data, while the circles mean those from the simulations. The red, green, and blue points are “IceCube warm spot” sources, “IceCube region” sources, with a distance smaller than 1°, and “IceCube region” sources with a distance larger than 1°.

Table 2. Pre-trial p-values for three sub-groups of potential neutrino sources.

| Sources                                | p-value |
|----------------------------------------|---------|
| IceCube warm-spot sources              | 0.038   |
| Sources in IceCube region with a distance ≤ 1° | 0.218   |
| Sources in IceCube region with a distance > 1° | 0.466   |

According to Figure 13 and Table 2, sources with weak neutrino excess tend to have overlap and simultaneous flaring period in low and high frequency, at a 96.2% significance level, consistent with the histogram in Figure 12. Besides, the pre-trial p-value is smaller for those sources closer to the IceCube alerts, but still with a low significance level and the value larger than 0.2. The three groups of neutrino-related blazars in this section are supposed to be highly independent, with only one source, TXS0506+056, in both “IceCube warm spot” and “IceCube region” sub-samples. Therefore the post-trial p-values could be calculated with the FWER, which is given by 1 – (1 – p)^3, and the significance level will be lower after the trial correction.

In the last test, we estimate the time lag between the IceCube alerts and the multi-frequency flaring stages with FIBS sample within the IceCube alert region. Only the time of infrared and γ-ray flares closest to the arrival of the alerts are considered here. Figure 14 shows the distribution of the time lag of the closest multi-frequency flares. The majority of the closest infrared flaring stage seems to occur before the arrival of the alerts and around 500 days after the alert time. The closest γ-ray flaring stage tends to gather at ~ −1000 – 500 days of the neutrino arrival time with preference at around 500 – 1000 days before the arrival of the neutrinos. Zooming closer to the alert center, for sources with distance smaller than 1° to the alert position, we found that the distribution of closest γ-ray flares is approximately the same as when using all sources. Moreover, the closest infrared flares tend to move a bit more to the center, at ~ −500 – 500
days of the alert arrival time. The result is consistent with Figure 8, where the majority of the infrared and γ-ray flares - for sources within the IceCube containment region - happen around 500 to 1000 days before the neutrino alerts.

Figure 14. Distribution of the time lag of multi-frequency flares closest to the IceCube alert arrival time.

4.4. Correlations with IceCube 10-year track-like neutrinos

The main purpose of section 4 is to study the possible neutrino emissions from blazars based on their multi-frequency light curves. Since the most probable neutrino blazars TXS0506+056 has a significant flaring lag in infrared with respect to γ-ray (see Figure 9), we speculate that the neutrino blazars’ low-frequency and high-frequency flaring stage might not be simultaneous.

In section 4.1 and 4.2, we have built three groups of sources selected from the FIBS sources, that is, one trial group with 32 selected sources with TXS-like multi-frequency flaring behavior and two control groups according to their multi-frequency flaring stages: sources with no simultaneous infrared and γ-ray flares and sources with highly simultaneous flares (NFS and CFS, see also Figure 10). Eleven sources with ACC and Fermi flaring activities similar to that of TXS0506+056 were also selected for comparison. To test our hypothesis, we made use of IceCube 10 years of public data for point-source searches (IceCube Collaboration et al. 2021) and studied the spatial correlation between the track-like neutrino events and the four groups of blazars. We calculated the number of neutrino events that have at least one blazar located inside their uncertainty regions (\(N_{\text{observed}}\)) then used Monte Carlo simulation to estimate how many neutrino events are around our groups of blazars by chance. The simulation was iterated \(10^5\) times with all blazars’ positions randomly scrambled in R.A, and the expected number of neutrinos close to our blazars by chance (\(N_{\text{expected}}\)) is the average of those simulated number of events nearby. By subtracting \(N_{\text{expected}}\) from \(N_{\text{observed}}\), we got the excess number of neutrinos associated with our groups of blazars. The excesses of neutrino counts represent the fact that we observed more neutrino around our test blazars’ positions than expected. The p-values are estimated based on whether the number of neutrinos around our blazar samples is higher than what is expected to arise by chance.

Figure 15. Excess number of neutrinos around the position of four groups of blazars w.r.t. the reconstructed energy. Filled regions show the 1σ statistical errors from random trials of simulation.

Table 6 shows the measured and expected number of neutrino (\(N_{\text{observed}}\) and \(N_{\text{expected}}\)) around four groups of blazars. The excess number of neutrinos around the position of four groups of blazars as well as the corresponding p-values with respect to the reconstructed energy of 10-year IceCube events are illustrated in Figure 15 and 16. The figures suggest that, at energy bins between \(~ 60\) to \(~ 150\) TeV, generally there is a small neutrino overflow in the vicinity of blazars with non-
simultaneous infrared and γ-ray flares (those similar to TXS0506+056), with an average overflow of $\sim 10 - 15$ neutrinos. The significance levels of this overflow is not very high, with a pre-trial p-value around $0.13 - 0.27$. We also determined statistical significance using a right-tailed p-value, which describes the probability of randomly obtaining the number of neutrino events from simulation that is greater than the real observed value ($N_{\text{observed}}$). The right-tailed p-value for the neutrino overflow around selected TXS-like blazars is around $0.13 - 0.27$ as well, with the lowest value of 0.127 that occurs at 150 TeV. On the contrary, there is no neutrino excess for those selected sources at higher energy bins.

The excess count of neutrinos around the CFS control group is gradually increasing with the reconstructed energy of the neutrino events. This increment indicates that there might be a correlation between blazars with highly simultaneous infrared and γ-ray flares and extremely high-energy neutrinos. The significance of this possible trend is not high enough, with a right-tailed p-value of 0.2. As we tested our hypothesis for four independent groups of samples, the original insignificant overflows reported here are even less significant after considering the trial factors with the FWER formula. We note that the excess number of neutrinos fluctuates around zero for the NFS sample and for sources with millimeter and γ-ray flaring activities similar to TXS0506+056. This implies that the number of neutrinos observed around those blazars is consistent with the number of neutrinos that are randomly nearby only by chance. Even though the highest overflow occurs at $\sim 500$ TeV for NFS sample, it is regarded as random fluctuation given the extremely large trial errors.

According to our selection criteria, those selected sources with TXS-like flaring behaviors do have related infrared and γ-ray flares with time lag < 500 days, even though they are not highly simultaneous. Thus, we joined those 32 selected sources with the CFS sample to see if the neutrino excess around those blazars with “Related” flares in Figure 15 would be more significant. We did further tests for blazars with only infrared or γ-ray flares, but without both flares in multi-frequency light curves. Figure 17 shows the same analysis as described in the previous paragraph, but with three different groups of blazars: “Related” flares, only flaring in infrared, and only flaring in γ-ray. Apparently, there is no excess of neutrino events around these three groups of blazars.

5. DISCUSSION

5.1. Result 1: temporal correlation between infrared flares and IceCube alerts

According to our analyses, there might exist a weak trend that the high-energy neutrino alerts might be temporarily correlated with infrared flares (section 3.2). However, the correlation is not statistically significant. From a population perspective, it suggests that the relation between IceCube high-energy alerts and IR blazar flares is weak. Even though we have considered the emission from the whole sample of blazars within the containment regions of the alerts, there is no statistically significant neutrino signal.

On the other hand, we found no clear correlation between γ-ray flaring stages and the neutrino arrival times, which is consistent with the results of Franckowiak et al. (2020), Righi et al. (2019b), and references therein. The

**Figure 16.** Pre-trial p-values of the excess number of neutrino around the position of four groups of blazars w.r.t. the reconstructed energy.

**Figure 17.** Excess number of neutrinos around the position of three groups of blazars w.r.t. the reconstructed energy. Filled regions show the 1σ statistical errors from random trials of simulation.
“direct” relation between \( \gamma \)-ray and neutrino becomes ambiguous due to the electromagnetic cascade, which is related to the optical depth of the cascade and pionic photons. Even though the protons and electrons are co-accelerated or interact with the same photon field, we might not be able to detect related high-energy photon flux and neutrino flux. Not to mention that the Fermi detected \( \gamma \)-ray emissions could originate from a different region than the neutrino production locus (See Section 5.3).

5.2. **Result 2: spatial correlation between CFS and very-high-energy track-like neutrino events**

Our results suggest a possible trend that blazars with highly simultaneous infrared and \( \gamma \)-ray flares (CFS sample, Figure 10) might be correlated with very-high-energy \( \gamma \)-ray flaring period. Considering the steep spectral index of atmospheric neutrinos -compared to astrophysical neutrinos- the higher the energy of a neutrino event indicates a higher probability of that event being of astrophysical origin. Our results indicate that blazars with the \( \gamma \)-ray flaring stages correlated with the infrared ones are more likely to be astrophysical neutrino sources, with higher significance in IceCube searches than other blazars.

The results imply that a typical single-zone Synchrotron self-Compton (SSC) model might be able to explain the emission for those CFS blazars and suggest that neutrinos might be emitted from the same region as infrared and \( \gamma \)-ray emissions in the inner jet. This trend is consistent with prediction in Padovani et al. (2015) and Murase (2017), and references therein. They suggest that blazars might dominate the astrophysical neutrino flux at around PeV, or even at higher energies of \( 10 - 100 \) PeV, if the particles (primary electrons and protons) are co-accelerated in the jet. If so, we would expect a correlation among neutrinos, infrared flares, and \( \gamma \)-ray flares, with the assumption of an efficient cascade of TeV photons down to the GeV energy range, which is detectable by Fermi-LAT. The optical depth of the electromagnetic cascade depends on the photon field that drives the photo-hadronic process. In this case, the photon field is being produced via the inverse Compton scattering of the jets’ low-frequency synchrotron emission.

5.3. **Result 3: spatial correlation between TXS-like sources and track-like neutrino events at \( \sim 60 - 150 \) TeV**

We have also shown a small overflow of \( \sim 10 \) track-like neutrinos with energy between \( \sim 60 \) to \( 150 \) TeV around blazars with infrared and \( \gamma \)-ray flaring behaviors like TXS0506+056 (section 4.4). Table 1 additionally suggests that those TXS-like blazars contain more “potential neutrino sources” than the others and might be related to IceCube neutrino events. From DCF test (section 4.3.1), it is known that blazars with an infrared time lag of \( \sim 300 \) days (like TXS0506+056) are not common. This indicates that those TXS-like blazars might also contribute to the observed diffuse neutrino flux, especially at the energy of a few tens to hundreds of TeV.

For those TXS-like blazars (see Figure 9), the infrared flares significantly lag to the \( \gamma \)-ray flares imply that their low-frequency synchrotron emission might not come from the same site as where inverse Compton high-frequency photons are produced. In other words, the \( \gamma \)-ray emission might originate from a region closer to the central supermassive black hole, compared to the synchrotron radiation. The external Compton (EC) radiation might dominate the high-energy peaks in the SED of those blazars (instead of SSC radiation), and the external photon field may be highly relevant for the particles in their inner jet. One of the explanations for this significant lag is that when turbulence in the jet cause shock waves that propagate along with the jet, high-frequency emissions occur upstream of the jet before the low-energy synchrotron photons become transparent (Boula et al. 2018; Max-Moerbeck et al. 2014). Alternatively, the temporal evolution of emitting particles would also lead to the lag. Sahakyan & Giommi (2021) explained the lag of optical/UV flares regarding X-ray/\( \gamma \)-ray for a transient blazar, 4FGL J1544.3-0649, with the SSC model considering the acceleration and cooling of electrons and synchrotron photons. In their scenario, the injection of freshly accelerated electrons leads to a bright state in the high-energy band. We could not exclude the possibility that accelerated protons are also injected into the emission zone. Either explanation, as well as the impact of the external photon fields, might support the hypothesis that neutrino would be produced efficiently in those blazars, given the newly injected accelerated particles, sufficient particles acceleration by shock waves, or enhanced photon fields feeding photo-hadronic process. Especially, many studies have shown the difficulty to explain both neutrino and electromagnetic emissions of TXS0506+056 with a single-zone model (for a review of the neutrino emission
from TXS0506+056, see Cerruti (2020) and reference therein).

An optimistic scenario for the production of neutrinos is the existence of an external photon field. The remarkable point is that TXS0506+056 is found to be a “masquerading BL Lac object”, intrinsically an FSRQ, with a hidden broad-line region (BLR) and a radiatively efficient and geometrically thin disc accretion flow (Padovani et al. 2019). This hidden BLR might act as an external field for TXS0506+056 to increase the efficiency of the photo-hadronic process. Indeed, Padovani et al. (2021) suggests that the fraction of masquerading BL Lacs in their sample (47 Fermi IBLs and HBLs in the vicinity of IceCube high-energy track-like neutrinos, Giommi et al. (2020)) is > 24% and possibly as high as 80%. Another potential neutrino blazars, MG3 J225517+2409, reported to be (weakly) associated with ANTARES and IceCube neutrinos (The ANTARES Collaboration 2019), is also a masquerading BL Lac (Padovani et al. 2021). For other “intrinsic BL Lac objects”, the existence of a sheath of the jet in spine-sheath model (Tavecchio & Ghisellini 2015) or the complex and relatively broad-band photon spectrum produced from radiatively inefficient accretion discs (Righi et al. 2019b) might be able to act as a target field for neutrino production. Furthermore, the SEDs of a complete sample of 104 radio-bright blazars (all with 37 GHz flux density higher than 1 Jy) are found to be better described by an EC model with a dominant infrared external photon field that can originate from dust torus emission or molecular clouds in spine-sheath geometry (Arsioli & Chang 2018). Those radio-bright blazars are all with VLBI 8 GHz flux density ≥ 150 mJy and thus are also in the samples in Plavin et al. (2021), which shows correlation with the neutrinos. We note that Ros et al. (2020) found signs of a spine-sheath structure in the jet of TXS0506+056.

If those TXS-like blazars are supposed to be related to IceCube neutrinos, the natural question that arises is then: where do neutrinos come from? Under the assumption that high-energy and low-energy photons might not originate from the same site, we speculate that neutrinos might be emitted from (i) the same place as high-energy radiation (ii) the same place as low-energy synchrotron emission (iii) other places related to neither low nor high-frequency emissions. For the first possibility, Xue et al. (2019) proposed a scenario in which neutrinos might be produced from $p\gamma$ process and possibly $pp$ process in the central region with ambient gas cloud and external photon field from the BLR where the inverse Compton X-ray and $\gamma$-ray are radiated. On the other hand, the low-energy synchrotron radiation might originate from the outer region where external photons from BLR or accretion disc is negligible. The second possibility could be explained by the scenario proposed by Plavin et al. (2021) in which neutrinos are emitted from the parsec region in blazar jets where X-ray SSC photons interact with accelerated protons. GeV $\gamma$-rays (probably dominated by external Compton radiation) are supposed to be emitted from a different region than the neutrino and synchrotron radiation. While Neronov & Semikoz (2021) proposed an alternative scenario suggesting that the link between radio synchrotron and neutrinos is expected in proton-proton interaction. Both neutrinos and synchrotron emitting electrons from charged pion decays during the interaction between high-energy protons and circum-nuclear medium in the central region of blazars. To explain the third possibility, a collimated neutron beam is assumed to be produced from interaction between CR nuclei and synchrotron photons in the jet (Zhang et al. 2020; Murase et al. 2018). The neutron beam escaping from the blazar emission zone could further interact with the external photon field and produce additional neutrinos further away.

5.4. Upper limits of neutrino flux from TXS-like blazars

![Figure 18. Comparison between the neutrino flux upper limits from our selected TXS-like blazars and IceCube 9.5-year astrophysical neutrino flux.](image)

We could estimate the potential neutrino flux from our selected TXS-like blazars given the slight overflow of $\sim 60 - 150$ TeV track-like neutrinos around them. Since the excess is not significant (Figure 15), the flux we estimated here is an upper limit. The expected number of astrophysical neutrinos detected during $\Delta T$ at...
the neutrino flux is powered by low-luminosity blazars. In their hypothesis, resolved high-luminosity BL Lacs or FSRQs can only contribute to a limiting fraction of the observed astrophysical neutrino flux.

Alternatively, we could not exclude the possibility that the majority of blazars are inefficient neutrino emitters at the sub-TeV and sub-PeV range. Probably much fainter than blazars, other sources might also contribute to the IceCube astrophysical neutrino flux. While Stein (2019) constrained the contribution of the Tidal Disruption Events (TDE) to be $\leq 27\%$, Bartos et al. (2021) suggested that probably more than 50% of IceCube astrophysical flux might come from either AGN or TDE at 90% confidence level. Furthermore, at least a fraction of 10% of the astrophysical neutrinos might come from sources other than AGNs and TDEs with 80% probability (Bartos et al. 2021). Those neutrinos might originate from unresolved objects or truly diffuse processes, which might also dominate the diffuse $\gamma$-ray background below 100 GeV. On the contrary, blazars are supposed to be more relevant to account for neutrinos with higher energy $\gtrsim 10 - 100$ PeV (as discussed in Section 5.2), which constitute only an extremely small fraction ($\lesssim 0.5\%$) of the 10-year track-like neutrinos.

The incompleteness of our multi-frequency and multi-epoch data might also bias our results, especially the large blank from 55600 to 56600 in NEOWISE and AllWISE data. This gap in all the light curves of our blazars might lead to a number of sources with TXS-like multi-frequency behaviors not being properly selected because some of the infrared flaring stages fell exactly into the “blank period”, causing a lower significance level with insufficient sources. Apart from the gap and large time bin of the WISE data, the ACC and Swift XRT data used in this paper sometimes are taken from target of opportunity observations and are not equally binned. Unfortunately, there are no other complete and equally-binned light curves available. In the future, with better sensitivity of next-generation neutrino observatories and a more complete multi-frequency coverage for the light curves, we could further confirm or refute the hypothesis that those blazars are indeed efficient neutrino emitters.

6. CONCLUSIONS

We have performed a series of analyses to search for potential neutrino emissions from multi-frequency catalogs (ACC, WISE, Swift XRT, and Fermi 4FGL-DR2) and various blazar samples (3HSP, 5BZCat, WIBRaLS, and 4LAC), investigating possible correlations with IceCube alerts and 10-year track-like events. The associations between IceCube neutrinos and astrophysical

\[ N_{\nu} = \Delta T \times \int_{E_{\nu, \text{min}}}^{E_{\nu, \text{max}}} A_{\text{eff}} \phi_{\nu}(E_{\nu}, \delta) dE_{\nu} \]

where $\phi_{\nu}(E_{\nu})$ and $A_{\text{eff}}(E_{\nu}, \delta)$ are the differential neutrino flux and the effective area, respectively (Aartsen et al. 2020). We took the $A_{\text{eff}}$ from IceCube Collaboration et al. (2021) with two energy bins ($4.8 < \log_{10}(E/\text{GeV}) < 5$ and $5 < \log_{10}(E/\text{GeV}) < 5.2$) and averaged the effective areas over the declinations, assuming our selected sources are uniformly distributed on the sky. $N$ is set to 10 as we found an overflow of $\sim 10$ track-like neutrinos around those TXS-like blazars. The obtained upper limits of the differential neutrino flux in the two energy bins are $\phi_{\nu}(100 \text{ TeV})$ equal to $1.459 \times 10^{-18}$ and $1.646 \times 10^{-18}$ $\text{cm}^{-2} \text{s}^{-1}$. Assuming neutrino spectral index of $-2$, we evaluated the contribution of our selected TXS-like blazars to the 9.5-year astrophysical muon neutrino flux from Abbasi et al. (2021). Figure 18 indicates that those TXS-like blazars contribute with $\lesssim 10\%$ of the diffuse astrophysical neutrinos. These results do not conflict with previous stacking limits. A scenario where blazars with TXS-like multi-frequency activity dominates the blazar contribution to the IceCube astrophysical neutrino flux (at energy range of 60 – 150 TeV) still holds.

5.5. Possible reasons for the absence of significant correlations

Even though we have shown some tiny overflows of neutrino emissions from blazars with certain multi-frequency flaring phases, additional analyses is required to improve and refine our findings. Neither the overflow of track-like neutrinos around TXS-like or CFS blazars nor the correlation between infrared flares and IceCube alerts is statistically significant. Similarly, the trend showing that the sources probably with IceCube signals ($p$-value $\leq 0.05$ from the point-source analyses) tend to have simultaneous infrared and $\gamma$-ray flaring stages is not significant to a high level. The low significance could be caused by the limited ability of current neutrino observatories to detect weak neutrino signals with astrophysical origin. We need more sensitive detectors with improved pointing capability and larger volume to further detect those weak neutrino sources. It is known that the diffuse astrophysical neutrino flux detected by IceCube is the aggregate of enormous (probably at least $O(50)$, Brown et al. (2015)) faint neutrino emitters. Additionally, Palladino et al. (2019) suggested that unresolved BL Lacs with large baryonic loading might be the only sources that dominate the IceCube astrophysical neutrinos with stacking limits accounted for, assuming
sources are thoroughly discussed in the paper by examining the multi-frequency flaring stages of sources within the containment regions of IceCube alerts and blazars with different types of multi-frequency activities. A time-dependent analysis to investigate the coincidence between the bright stages of multi-frequency sources and the arrival times of the neutrino alerts suggests a possible correlated trend between the infrared flares and the IceCube alerts. The cross-matching between blazars with various infrared-\(\gamma\)-ray flaring behaviors and 10-year track-like neutrino events shows a small overflow of neutrinos around the blazars with flaring phases highly similar to TXS0506+056 (with a significant lag in infrared) or highly simultaneous (CFS sample). In a nutshell, we have shown that considering the infrared and \(\gamma\)-ray flaring behaviors, the CFS sample and the TXS-like sources might be the most likely neutrino-source candidates among blazars. Our results are consistent with the prediction that blazars might dominate the astrophysical neutrino flux at PeV or higher energies, and consistent with the current limits on the blazars’ contribution to the IceCube astrophysical neutrinos. Moreover, the possible neutrino emitting sites from the TXS-like blazars are discussed in detail, accounting for several models from the literature.

An unstable jet or changing accretion rate would lead to the formation of a jet blob, which expands during propagating outwards along the jets of blazars (Chen & Zhang 2021). The radio outburst might result from this inflating blob region, caused by long-term expansion when the synchrotron radiations transitioning from optically thick to thin. This expansion effect does not have to lead to an outburst at higher frequency (e.g., \(\gamma\)-ray), while the charged particles accelerated inside this large-scale blob may account for high energy neutrino emissions. The (plausible) statistical correlation between the radio flaring phases and the arrival of neutrinos (Plavin et al. 2020) might be a clue to this scenario (and maybe also include our infrared result, see Figure 3). Furthermore, the small perturbations in this blob would bring about flares at higher frequencies with a shorter and non-simultaneous time scale, considering the radio outburst related to the inflating blob. This is not in conflict with our results (Figure 15). A model considering a larger scale of accelerated sites inspired by the statistic correlation found in radio (and/or infrared) and by the multi-frequency activities of plausible neutrino blazars would be of importance to understand the neutrino emitting mechanism of those blazars. Given the observed correlation discussed along this work is not significant, our results need additional studies with more complete multi-frequency light curves and more sensitive neutrino detectors. In the future, with the next generation of neutrino observatories, we expect to better investigate the neutrino signatures from those sources, hopefully unveiling neutrino/hadronic processes taking place in blazars.

ACKNOWLEDGMENTS

We thank Ke Fang, Segev BenZvi, Weikang Lin, Neng-Hui Liao, and Ting-Gui Wang for helpful comments and discussions. YLC, DLX, and WLL acknowledge the National Natural Science Foundation of China (NSFC) grant (No. 12175137) on “Exploring the Extreme Universe with Neutrinos and multi-messengers” and acknowledge the Double First Class start-up fund provided by Shanghai Jiao Tong University. YLC and BA thank the support from China Postdoctoral Science Foundation Funded Project (BR4260008). LC thanks the support of NSFC grant (No. 12173066). We acknowledge the Centro de Computação John David Rogers (CCJDR) at IFIGW Unicamp, Campinas-Brazil, for granting access to the Feynman and Planck Clusters and the IcraNet - IT for the granted access to Joshua Computer Cluster (IT), on which the parallel computation of hundreds of \(\gamma\)-ray light curves were analyzed with the Fermi Science Tools.

Facilities: ALMA, WISE, Swift(XRT), Fermi(LAT)
### Searching for neutrino emissions (Section 3 and 4)

| Analyses                                      | Neutrino datasets | Main catalogs | Source samples and criteria |
|-----------------------------------------------|-------------------|---------------|-----------------------------|
| Temporal correlations with multi-frequency sources | IceCube alerts    | WIBRaLS       | $b > |10^\circ|$, $\delta < |40^\circ|$, and $F_{100\,\text{GHz}} \geq 100$ mJy |
|                                               |                   | *Fermi* 4FGL-DR2 | $b > |10^\circ|$, $\delta < |40^\circ|$, and Variability $\geq 18.48$ |
|                                               |                   | *Swift* XRT    | $b > |10^\circ|$, $\delta < |40^\circ|$, and # of detection $\geq 5$ |
|                                               |                   | *ACC*          | $b > |10^\circ|$, $\delta < |40^\circ|$, and # of detection $\geq 5$ |
| Spatial correlations with blazars with various flaring phases | 10 yrs track events | FIBS sources$^a$ | TXS-like sources from FIBS$^b$ |
|                                               |                   | FIBS sources   | CFS sample$^b$               |
|                                               |                   | FIBS sources   | NFS sample$^b$                |
|                                               |                   | *ACC-Fermi* blazars | TXS-like sources from *ACC-Fermi* sample$^b$ |

### Properties of potential neutrino blazars (Section 4.3.2)

| Analyses                                      | Neutrino datasets | Main catalogs | Source samples and criteria |
|-----------------------------------------------|-------------------|---------------|-----------------------------|
| Simultaneity of flaring stages of potential neutrino blazars$^d$ | IceCube alerts & IceCube warm spots$^c$ | FIBS sources | flaring in both infrared and $\gamma$-ray |
| Temporal correlations between the neutrino alerts and blazars | IceCube alerts | FIBS sources | with infrared/$\gamma$-ray flares and in IceCube alert regions |

### General correlations of multi-frequency light curves (Section 4.3.1)

| Analyses                                      | Main catalogs | Source samples and criteria |
|-----------------------------------------------|---------------|----------------------------|
| Temporal correlations of blazars in infrared and $\gamma$-ray | FIBS sources | all FIBS sources |

Table 3. Analyses as well as neutrino datasets and selection criteria of source samples applied throughout the paper. Detailed descriptions for neutrino datasets and main source catalogs are written in section 2.

$^a$ *Fermi*-Infrared Blazar Sample.

$^b$ There are four groups of blazars tested with Icecube 10-year track events. One experimental group of selected TXS-like sources (Figure 9) from FIBS, two control groups of CFS (Figure 10) and NFS sub-samples, and a comparison group of selected TXS-like sources from *ACC-Fermi* sub-sample. See section 4.1 and 4.2 for selection details.

$^c$ Sources with p-value $\leq 0.05$ in IceCube previous point source analyses.

$^d$ Sources within 90% containment regions of IceCube alerts or IceCube warm-spot sources.
### APPENDIX

#### A. TABLES OF SELECTED SOURCES AND CFS SOURCES

| Name        | 4FGL        | Name        | 4FGL        |
|-------------|-------------|-------------|-------------|
| J0112+3208  | 4FGLJ0112.8+3208 | J1246−2547  | 4FGLJ1246.7−2548 |
| J0143−3200  | 4FGLJ0143.5−3156 | J1248+5128  | 4FGLJ1248.7+5127 |
| J0509+0541  | 4FGLJ0509.4+0542 | 5BZQJ1256−0547 | 4FGLJ1256.1−0547 |
| J0646−3903  | 4FGLJ0646.7−3913 | J1258−1800  | 4FGLJ1258.6−1759 |
| 3HSPJ064850.5+694522 | 4FGLJ0648.4−6941 | 3HSPJ125848.0−04475 | 4FGLJ1258.7−0452 |
| J0701−4634  | 4FGLJ0701.5−4634 | 3HSPJ131146.0+395317 | 4FGLJ1311.8+3954 |
| J0729+6129  | 4FGLJ0728.5+6128 | J1318−1235  | 4FGLJ1318.7−1234 |
| J0753−5445  | 4FGLJ0733.5−5445 | J1354−1041  | 4FGLJ1354.8−1041 |
| J0808−0751  | 4FGLJ0808.2−0751 | 3HSPJ150644.5+081400 | 4FGLJ1506.6+0813 |
| J0816+5739  | 4FGLJ0816.3+5739 | J1546+1817  | 4FGLJ1546.5+1816 |
| J0916+3854  | 4FGLJ0916.7+3856 | J1616+4632  | 4FGLJ1616.6+4630 |
| J0921+2335  | 4FGLJ0921.7+2336 | J1716+6836  | 4FGLJ1716.1+6836 |
| J0930+3503  | 4FGLJ0930.7+3502 | 5BZQJ1753+2848 | 4FGLJ1753.7+2847 |
| J0944+6135  | 4FGLJ0943.7+6137 | J1806+6949  | 4FGLJ1806.8+6949 |
| 5BZQJ1145.5+6954 | 4FGLJ1145.7−6949 | J1844+1614  | 4FGLJ1845.0+1613 |
| 5BZBJ1149+6243 | 4FGLJ1149.2+6246 | J2142−0437  | 4FGLJ2142.7−0437 |

*Table 4.* Selected sources with infrared and γ-ray light curves similar to that of TXS0506+056.
| Name                  | 4FGL       | Name                  | 4FGL       |
|-----------------------|------------|-----------------------|------------|
| 3HSP J003552.6+595004 | 4FGLJ0035.9+5950 | J0958+4725           | 4FGLJ0958.0+4728 |
| J0050−5727            | 4FGLJ0050.0−5736 | J1037−2823           | 4FGLJ1037.7−2822 |
| 3HSP J005116.6−624204 | 4FGLJ0051.2−6242 | J1337−1257           | 4FGLJ1337.6−1257 |
| J0056−2117            | 4FGLJ0056.4−2118 | J1351+0031           | 4FGLJ1351.0+0029 |
| J0113−3551            | 4FGLJ0113.1−3553 | J1422+3223           | 4FGLJ1422.3+3223 |
| J0113+4948            | 4FGLJ0113.4+4948 | J1424+0434           | 4FGLJ1424.2+0433 |
| 3HSP J011546.1+251953 | 4FGLJ0115.8+2519 | J1436+2321           | 4FGLJ1436.9+2321 |
| J0210+5101            | 4FGLJ0210.7+5101 | J1443+2501           | 4FGLJ1443.9+2501 |
| J0231−4746            | 4FGLJ0231.2−4745 | J1457+4248           | 4FGLJ1457.3−4246 |
| J0231+1322            | 4FGLJ0231.8+1322 | J1459+7140           | 4FGLJ1459.0+7140 |
| J0239+0416            | 4FGLJ0239.7+0415 | J154824.38+145702.8  | 4FGLJ1548.3+1456 |
| J0243+7120            | 4FGLJ0243.4+7119 | J1551−1755           | 4FGLJ1550.8−1750 |
| J0253−5441            | 4FGLJ0253.2−5441 | J1617−7717           | 4FGLJ1617.9−7718 |
| 3HSP J033559.6−284619 | 4FGLJ0338.9−2848 | 3HSP J172818.6+501310 | 4FGLJ1728.3+5013 |
| 5BZQ J03354−1616      | 4FGLJ0354.7−1617 | J1743−0350           | 4FGLJ1744.2−0353 |
| J0359+5057            | 4FGLJ0359.6+5057 | 3HSP J181335.0+314417 | 4FGLJ1813.5+3144 |
| J0449+6332            | 4FGLJ0449.2+6329 | J1825−5230           | 4FGLJ1825.1−5231 |
| J0607+6720            | 4FGLJ0608.0+6721 | 5BZQ J1833.0+2103    | 4FGLJ1833.6−2103 |
| 5BZU J0621−2515       | 4FGLJ0620.5−2512 | 5BZBJ J1925.0−1018   | 4FGLJ1925.1−1019 |
| 5BZU J0622+3326       | 4FGLJ0622.9+3326 | J2000−1748           | 4FGLJ2000.9−1748 |
| J0644−6712            | 4FGLJ0644.4−6712 | J2022−4513           | 4FGLJ2022.3−4513 |
| J071304.54+573810.2   | 4FGLJ0713.0+5738 | 3HSP J213151.5−251558 | 4FGLJ2131.7−2515 |
| 5BZQ J0733+0456       | 4FGLJ0733.8+0455 | J2134−0153           | 4FGLJ2134.2−0154 |
| J0739+0137            | 4FGLJ0739.2+0137 | 5BZQ J2148+0657      | 4FGLJ2148.6+0652 |
| J0750+7909            | 4FGLJ0751.0+7908 | J2212+0646           | 4FGLJ2212.8+0647 |
| 5BZQ J0751+3313       | 4FGLJ0752.2+3313 | J2237−3921           | 4FGLJ2237.0−3921 |
| 3HSP J075936.1+132117 | 4FGLJ0759.6+1321 | 3HSP J224017.1−524113 | 4FGLJ2240.3−5241 |
| 3HSP J085409.9+440830 | 4FGLJ0854.3+4408 | 5BZQ J2249+2107      | 4FGLJ2249.9+2106 |
| 5BZQ J0912+4126       | 4FGLJ0912.2+4127 | J2311+3425           | 4FGLJ2311.0+3425 |
| J0923+4125            | 4FGLJ0923.5+4125 | 5BZQ J2323−0617      | 4FGLJ2323.6−0617 |
| J0930−8534            | 4FGLJ0931.2−8533 | J2336−4115           | 4FGLJ2336.6−4115 |

**Table 5.** Sources with highly simultaneous infrared and γ-ray flaring light curves.
B. MULTI-FREQUENCY LIGHT CURVES FOR SELECTED SOURCES.

Figure 19. Light curves of selected TXS-like sources. The upper panel is the infrared light curve from WISE multi-epoch data, and the middle panel is the Fermi $\gamma$-ray light curve analyzed with Fermi Science Tool. The lower panel represents the flaring periods in infrared (blue thick lines) and $\gamma$-ray (red thick lines) as well as the simultaneous flaring stages (green lines). Black lines represent the arrival time of the neutrino alerts.
Figure 19 continued. Light curves of selected TXS-like sources. The upper panel is the infrared light curve from WISE multi-epoch data, and the middle panel is the Fermi γ-ray light curve analyzed with Fermi Science Tool. The lower panel represents the flaring periods in infrared (blue thick lines) and γ-ray (red thick lines) as well as the simultaneous flaring stages (green lines). Black lines represent the arrival time of the neutrino alerts.
Figure 19 continued. Light curves of selected TXS-like sources. The upper panel is the infrared light curve from WISE multi-epoch data, and the middle panel is the Fermi γ-ray light curve analyzed with Fermi Science Tool. The lower panel represents the flaring periods in infrared (blue thick lines) and γ-ray (red thick lines) as well as the simultaneous flaring stages (green lines). Black lines represent the arrival time of the neutrino alerts.
Figure 19 continued. Light curves of selected TXS-like sources. The upper panel is the infrared light curve from WISE multi-epoch data, and the middle panel is the Fermi γ-ray light curve analyzed with Fermi Science Tool. The lower panel represents the flaring periods in infrared (blue thick lines) and γ-ray (red thick lines) as well as the simultaneous flaring stages (green lines). Black lines represent the arrival time of the neutrino alerts.
## C. TABLE OF NEUTRINO NUMBERS AROUND THE 4 GROUPS OF BLAZARS

| Log10(E/GeV) | Selected TXS-like | CFS | NFS | ACC selected |
|-------------|------------------|-----|-----|--------------|
| 4.8 | 243 | 595 | 3084 | 158 |
| $N_{\text{measured}}$ | 232 | 610 | 3071 | 156 |
| $N_{\text{expected}}$ | 15 | 25 | 60 | 13 |
| 4.9 | 223 | 551 | 2862 | 146 |
| $N_{\text{measured}}$ | 214 | 570 | 2859 | 145 |
| $N_{\text{expected}}$ | 14 | 24 | 58 | 12 |
| 5.0 | 209 | 505 | 2629 | 135 |
| $N_{\text{measured}}$ | 194 | 526 | 2627 | 133 |
| $N_{\text{expected}}$ | 14 | 23 | 55 | 11 |
| 5.1 | 188 | 463 | 2370 | 122 |
| $N_{\text{measured}}$ | 173 | 480 | 2379 | 119 |
| $N_{\text{expected}}$ | 13 | 22 | 52 | 11 |
| 5.2 | 159 | 416 | 2106 | 114 |
| $N_{\text{measured}}$ | 153 | 430 | 2116 | 103 |
| $N_{\text{expected}}$ | 12 | 21 | 48 | 10 |
| 5.3 | 135 | 366 | 1855 | 94 |
| $N_{\text{measured}}$ | 132 | 376 | 1855 | 88 |
| $N_{\text{expected}}$ | 12 | 20 | 45 | 9 |
| 5.4 | 108 | 304 | 1560 | 75 |
| $N_{\text{measured}}$ | 113 | 319 | 1568 | 75 |
| $N_{\text{expected}}$ | 11 | 18 | 41 | 9 |
| 5.5 | 80 | 252 | 1261 | 62 |
| $N_{\text{measured}}$ | 92 | 258 | 1260 | 59 |
| $N_{\text{expected}}$ | 10 | 16 | 36 | 8 |
| 5.6 | 59 | 207 | 936 | 38 |
| $N_{\text{measured}}$ | 70 | 202 | 942 | 41 |
| $N_{\text{expected}}$ | 8 | 14 | 30 | 6 |
| 5.7 | 46 | 131 | 642 | 26 |
| $N_{\text{measured}}$ | 48 | 136 | 629 | 28 |
| $N_{\text{expected}}$ | 7 | 12 | 25 | 5 |
| 5.8 | 25 | 82 | 407 | 18 |
| $N_{\text{measured}}$ | 31 | 89 | 406 | 18 |
| $N_{\text{expected}}$ | 6 | 9 | 19 | 4 |
| 5.9 | 12 | 62 | 261 | 15 |
| $N_{\text{measured}}$ | 20 | 56 | 257 | 11 |
| $N_{\text{expected}}$ | 5 | 7 | 15 | 3 |
| 6.0 | 7 | 39 | 171 | 7 |
| $N_{\text{measured}}$ | 12 | 34 | 161 | 7 |
| $N_{\text{expected}}$ | 3 | 6 | 12 | 3 |

Table 6. Number of neutrino events around blazars in our selected lists and control groups. $N_{\text{measured}}$ means the real number of neutrino, $N_{\text{expected}}$ means the average number from the simulation with blazars RA randomly scrambled 10000 times, and $1\sigma$ is the statistical error at 68% significance level of the simulation.
REFERENCES

Aartsen, M. G., Abbasi, R., Abdou, Y., et al. 2013, ApJ, 779, 132, doi: 10.1088/0004-637X/779/2/132
Aartsen, M. G., Ackermann, M., Adams, J., et al. 2014, ApJ, 796, 109, doi: 10.1088/0004-637X/796/2/109
—. 2015, PhRvD, 91, 022001, doi: 10.1103/PhysRevD.91.022001
Aartsen, M. G., Abraham, K., Ackermann, M., et al. 2016, ApJ, 833, 3, doi: 10.3847/0004-637X/833/1/3
—. 2017a, ApJ, 835, 45, doi: 10.3847/1538-4357/835/1/45
Aartsen, M. G., Ackermann, M., Adams, J., et al. 2017b, Astroparticle Physics, 92, 30, doi: 10.1016/j.astropartphys.2017.05.002
Aartsen, M. G., Abraham, K., Ackermann, M., et al. 2017c, ApJ, 835, 151, doi: 10.3847/1538-4357/835/2/151
Aartsen, M. G., Ackermann, M., Adams, J., et al. 2019, MNRAS, 483, L12, doi: 10.1093/mnrasl/sty210
Chang, Y. L., Arsioli, B., Giommi, P., Padovani, P., & Brandt, C. H. 2019, A&A, 632, A77, doi: 10.1051/0004-6361/201834526
Chang, Y. L., Brandt, C. H., & Giommi, P. 2020, Astronomy and Computing, 30, 100350, doi: 10.1016/j.ascom.2019.100350
Chen, L., & Zhang, B. 2021, ApJ, 906, 105, doi: 10.3847/1538-4357/abc42d
D’Abrusco, R., Massaro, F., Paggi, A., et al. 2014, ApJS, 215, 14, doi: 10.1088/0067-0049/215/1/14
D’Abrusco, R., Álvarez Crespo, N., Massaro, F., et al. 2019, ApJS, 242, 4, doi: 10.3847/1538-4365/ab16f4
Francowkiak, A., Barrapp, S., Paliya, V., et al. 2020, ApJ, 893, 162, doi: 10.3847/1538-4357/ab8307
Gao, S., Fedyinitch, A., Winter, W., & Pohl, M. 2019, Nature Astronomy, 3, 88, doi: 10.1038/s41550-018-0610-1
Giommi, P., Glauch, T., Padovani, P., et al. 2020, MNRAS, 497, 865, doi: 10.1093/mnrasl/staa2082
Giommi, P., & Padovani, P. 2021, Universe, 7, 492, doi: 10.3390/universe7120492
Giommi, P., Brandt, C. H., Barres de Almeida, U., et al. 2019, A&A, 631, A116, doi: 10.1051/0004-6361/201935646
Giommi, P., Perri, M., Capalbi, M., et al. 2021, MNRAS, 507, 5690, doi: 10.1093/mnras/stab2425
Goswami, S., Privon, G. C., Santander, M., & IceCube Collaboration. 2021, Journal of Instrumentation, 16, C09013, doi: 10.1088/1748-0221/16/09/C09013
Halzen, F., Kheirandish, A., Weisgarber, T., & Wakely, S. P. 2019, ApJL, 874, L9, doi: 10.3847/2041-8213/ab0d27
Hovatta, T., Lindfors, E., Kiehlmann, S., et al. 2021, A&A, 650, A83, doi: 10.1051/0004-6361/202039481
Huber, M. 2019, in International Cosmic Ray Conference, Vol. 36, 36th International Cosmic Ray Conference (ICRC2019), 916. https://arxiv.org/abs/1908.08458
IceCube Collaboration. 2013, Science, 342, 1242856, doi: 10.1126/science.1242856
IceCube Collaboration, Aartsen, M. G., Ackermann, M., et al. 2018a, Science, 361, 147, doi: 10.1126/science.aat2890
—. 2018b, Science, 361, eaat1378, doi: 10.1126/science.aat1378
