ABSTRACT: In the decoding of the Bose Chaudhuri-Hocquenghem (BCH) codes, the most complex block is the Chien search block. In the decoding process of the BCH codes, error correction is performed bit by bit, hence they require parallel implementation. The area required to implement the Chien search parallel is more, hence a strength reduced parallel architecture for the Chien search is presented. In this paper, the syndrome computation is done using conventional method, the inversion-less Berlekamp Massey Algorithm is used for the solving the key equations.

KEYWORDS: BCH Decoder, parallel Chien search architecture

I. INTRODUCTION

The algebraic codes which are widely used and most powerful are the BCH and RS codes [3]. The powerful error correcting codes which are used more often in modern communication systems like wireless communication, optical communication, computer networks, magnetic recording systems, various storage devices are the BCH codes. Error pattern of size t or less can be designed in the BCH codes. It is frequently convenient to define error correcting codes in terms of the generator polynomials \( G(x) \). For the BCH code capable of correcting \( t \) errors, generator polynomial is taken as the LCM of the minimal polynomials.

\[
G(x) = \text{LCM} (\Phi_1, \Phi_2, \Phi_3, \Phi_4, \ldots\ldots, \Phi_{2t-1})
\]  

The binary BCH code \((n, k, t)\) exists for integer values \( m \geq 3 \), \( t \leq 2m-1 \), with there properties as given below:

- \( n = 2^m - 1 \) length of code word
- \( k \geq n - mt \) number of information bits
- \( d_{\text{min}} \geq 2t+1 \) minimum hamming distance.
- \( t \) error correcting capability.

On the implementation of the parallel Chien search in area efficient manner is focused in this paper. Primitive binary BCH codes are considered in this paper.
II. BCH DECODER ARCHITECTURE

In Fig.1, the BCH decoder block diagram is shown. For a \((n,k,t)\) BCH code in which, \(c(x)\) represents the transmitted code polynomial, \(r(x)\) the code polynomial that is received at the decoder end and \(e(x)\) represents the code polynomial where the error has occurred.

A. Syndrome Computation

The first step in the decoding of the BCH codes is to calculate \(2t\) syndromes \(S_j\) by evaluating the received code polynomial \(r(x)\) at. The code polynomial that is received can be represented by:

\[
r(x) = (c(x) + e(x)) \tag{2}
\]

The calculation of the \(2t\) syndromes is given by

\[
S_j = \sum_{i=0}^{t-1} r(x)^i 1 \leq j \leq 2t \tag{3}
\]

Where is the root of the primitive polynomial. We can also define syndrome polynomial

\[
S_j = \sum_{i=0}^{t-1} S_i x^i \quad 1 \leq j \leq 2t \tag{4}
\]

B. Key Equation Solver

The coefficient of the error locator polynomial in the decoding of the BCH codes are calculated in this stage. The input to this stage are the syndromes that are generated in the first stage of the decoding of the BCH codes. The error locator polynomial is given as: \(\sigma(x) = \sigma_0 + \sigma_1 x + \cdots + \sigma_t x^t\). The error locator polynomial is related with the syndromes generated in the first stage by the following relationship:

\[
\sum_{i=0}^{t} S_{t+i-j} \sigma_j \tag{5}
\]

The co-efficient of the error locator polynomial can be calculated by making use different decoding algorithms. In this paper the error locator polynomial co-efficient are found by the Inversion-less Berlekamp Massey Algorithm to solve the key equation presented in the [1] is used in this paper. The output of this block is the error locator polynomial \(\sigma(x)\).

C. Chien Search

In decoding process, the error locator polynomial \(\sigma(x)\) is acquired by performing the syndrome estimation and then solving the key equation. The Chien search block comprehensively analyzes whether a root of \(\Lambda(x)\) is for \(i=0, 1, \ldots, n-1\); i.e., it checks whether yields zero or not for the following equation:
sig(\alpha^i) = \sum_{j=0}^{n-1} \sigma_j \alpha^{i j} \quad (6)

The above equation gives the straight forward execution of the Chien search block. The routine Chien search circuit is as shown in Fig. 2.

Fig. 2. Conventional Chien Search

It produces an error-vector e in manner that, if is the root, then the (n-i)th component en-i=1; otherwise en-i=0 for all 0≤i≤n-1. In the conventional Chien search circuit only single error location is checked for on clock cycle, therefore it requires n clock cycles to complete the search process. The parallel architecture for the Chien search [5] can be used in order to replace this traditional Chien search circuit as shown in Fig. 3.

Moreover, without any hardware complexity, the long critical path in the parallel design can be adequately shortened as discussed in [6], [9]. The parallelization of the Chien circuit reduces the n clock cycles required for computation to n/p clock cycles with the parallel factor p. The hardware requirement for the parallel process also increases linearly with p. Thus the efficient decoder complexity basically relies upon the design of proficient Parallel Chien Search circuit.

Fig. 3. Parallel Architecture for Chien Search

III. PROPOSED ARCHITECTURE

The given (n,k,t) BCH code is built on GF (2^m). The parallel Chien search circuit is more area consuming. The strength reduced parallel Chien search process is as given in Fig. 4.
The (15,7,2) BCH code is built on GF($2^4$). The proposed Chien search circuit for the BCH decoder is as given in the block diagram shown in Fig. 4.

**A. Chien Search Block**

The Chien search circuit and error correction block is the biggest area and timing consumption. The signals Zs and Zb is the output from control unit to indicate zero syndrome. If Zs = 1 then no error occurs in the received polynomial code due to $S(x)=0$. If Zb =1 then only one error occurs in the received polynomial code. We should use multiplexer to select original bit or correction bit. To reduce the critical path, pipelined register must be added to the selector of the output. It can reduce the critical path significantly.

To obtain the optimal design, the architecture of the constant finite field multiplier has to be optimized by reducing the number of XOR gate. In table I, we can see that the coefficients of constant FFM have the same pattern between the equation and each other. By implementing this circuit, the count of XOR gates can be minimized, by reducing the same pattern coefficients. To replace FFM, block Ci and block bo is used as shown in Fig. 5.

To find the roots of polynomials, each element in GF($2^4$) has to be evaluated. Since there are 15 elements, we need signals $C_0$-$C_{14}$ to form Ci block. The assignment is determined based on coefficient pattern of FFM. The input of bocan be transformed to perform an optimized computation by manipulating its bits. As we know, the error locator polynomial is defined as:

$$b(x)= x^2+b1x+b0$$  \hspace{1cm} (7)
so the value of \(b_0 + x^2\) can be computed in parallel computation by transforming the bits of \(b_0\). If \(x\) is the element of \(\text{GF}(2^4)\) and \(\alpha = \alpha_0, \alpha_1, \ldots, \alpha_{14}\) then \(x^2 = \alpha^{21} = \alpha_0, \alpha_1, \ldots, \alpha_{14}, \alpha_1, \alpha_3, \ldots, \alpha_{13}\).

### Table I

| FFM Type | GF Polynomial Equation |
|----------|------------------------|
| Pow(2)   | \(a_3 x^3 + (a_1+a_2) x^2 + a_2 x + (a_0+a_2)\) |
| Pow(4)   | \(a_3 x^4 + (a_2+a_3)x^2 + (a_1+a_3)x + (a_0+a_1+a_2+a_3)\) |
| Const    | \(a_0^3 + a_2 x^2 + a_1 x + a_0\) |
| Const    | \(a_1^3 + a_0^3\) |
| Const    | \(a_2^3 + (a_0+a_3)x^2 + (a_1+a_3)x + (a_0+a_3)\) |
| Const    | \(a_3^3 + (a_0+a_2)x^2 + (a_1+a_3)x + (a_0+a_2)\) |
| Const    | \(a_4^3 + (a_0+a_2)x^2 + (a_0+a_2+a_3)x + (a_0+a_1+a_3)\) |
| Const    | \(a_5^3 + (a_1+a_3)x^2 + (a_0+a_2+a_3)x + (a_0+a_2+a_3)\) |
| Const    | \(a_6^3 + (a_0+a_2+a_3)x^2 + (a_0+a_2+a_3)x \) + (a_1+a_3) |
| Const    | \(a_7^3 + (a_0+a_1+a_2+a_3)x^2 + (a_0+a_1+a_2+a_3)x + (a_0+a_2+a_3)\) |
| Const    | \(a_8^3 + (a_0+a_1+a_2+a_3)x^2 + (a_0+a_1+a_2+a_3)x + (a_0+a_2+a_3)\) |
| Const    | \(a_9^3 + (a_0+a_1+a_2+a_3)x^2 + (a_0+a_1+a_2+a_3)x + (a_0+a_2+a_3)\) |
| Const    | \(a_10 + (a_1+a_2+a_3)\) |
| Const    | \(a_11 + (a_1+a_2+a_3)\) |
| Const    | \(a_12 + (a_0+a_1+a_2+a_3)\) |
| Const    | \(a_13 + (a_0+a_1+a_2+a_3)\) |
| Const    | \(a_14 + (a_0+a_1+a_2+a_3)\) |
B. Control Unit

To perform our decoder system, we employ a simple control unit block that can support error correction. If there is no error in \( r(X) \), then the value of \( S_1 \) and \( S_3 \) is zero. If only one error occurs, then \( b_0 \) of error locator polynomial is zero. The design of the control unit is as appeared if Fig. 6.

![Fig. 6. Control Unit Block Diagram](image)

IV. RESULTS

The design of the BCH decoder architectures described in the previous sections namely conventional, strength reduced architecture are expected to be simulated in Cadence NC Verilog simulator tool. In order to compare the complexity overhead and power consumptions, the two variants of the BCH decoder described in the previous in the paper are expected to be simulated using Cadence Encounter RTL compiler tool.

The power and area parameters of the BCH decoder design is given in the table II.

| Area(um²)  | Power(nW)    |
|------------|--------------|
| Design in [8] | 14051        | 477932.501 |
| Proposed    | 6961         | 151867.464 |

V. CONCLUSION

This paper presents the strength reduced parallel Chien search architecture which uses different set of constant power FFM over GF(2\(^m\)). The proposed design has 3 clock latency since it consists of 3 pipelined stage. The optimized Chien search block consumes less area and power contrasted to the traditional Chien search BCH decoder. Hence the design presented in this paper can be used in modern communication systems.

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