Recent Observation of Short Range Nucleon Correlations in Nuclei and their Implications for the Structure of Nuclei and Neutron Stars

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Abstract
Novel processes probing the decay of nucleus after removal of a nucleon with momentum larger than Fermi momentum by hard probes finally proved unambiguously the evidence for long sought presence of short-range correlations (SRCs) in nuclei. In combination with the analysis of large $Q^2$, $A(e,e')X$ processes at $x > 1$ they allow us to conclude that (i) practically all nucleons with momenta $\geq 300$ MeV/c belong to SRCs, consisting mostly of two nucleons, ii) probability of such SRCs in medium and heavy nuclei is $\sim 25\%$, iii) a fast removal of such nucleon practically always leads to emission of correlated nucleon with approximately opposite momentum, iv) proton removal from two-nucleon SRCs in 90% of cases is accompanied by a removal of a neutron and only in 10% by a removal of another proton. We explain that observed absolute probabilities and the isospin structure of two nucleon SRCs confirm the
important role that tensor forces play in internucleon interactions. We find also that the presence of SRCs requires modifications of the Landau Fermi liquid approach to highly asymmetric nuclear matter and leads to a significantly faster cooling of cold neutron stars with neutrino cooling operational even for $N_p/N_n \leq 0.1$. The effect is even stronger for the hyperon stars. Theoretical challenges raised by the discovered dominance of nucleon degrees of freedom in SRCs and important role of the spontaneously broken chiral symmetry in quantum chromodynamics (QCD) in resolving them are considered. We also outline directions for future theoretical and experimental studies of the physics relevant for SRCs.

1 Introduction

1.1 Definition of short range correlations and key questions in their studies

It was understood many decades ago that stability of heavy nuclei and saturation of the nuclear density requires interplay between nucleon-nucleon attraction at intermediate distances $\leq 1.5$ fm and significantly stronger repulsion which sets in at distances $\leq 0.5$ fm. A strong compensation between attractive and repulsive potentials leads to the binding energy per nucleon which is much smaller than both average kinetic and potential energies (see e.g. Ref. [1]).

Presence of the strong short range repulsion and intermediate range attraction between nearby nucleons generates nucleons with momenta substantially larger than Fermi momentum characteristic for the given nucleus. Therefore the first implication of the presence of short-range interaction in nuclei is the presence of high momentum component in the nuclear ground state wave function. Since the main contribution to the high momentum component comes from spatial configurations where distances between two nucleons are significantly smaller than the average internucleon distances it is natural to refer to all these configurations as short-range correlations (SRCs). Often in the literature one separates these correlations into medium and short-distance correlations. We will not follow this tradition since such correlations are manifested in a similar way in the high momentum component of the nuclear wave function. Presence of high momentum component in the nuclear ground state wave function has been demonstrated in theoretical calculations of wave functions of light nuclei and infinite nuclear matter based on the nonrelativistic nuclear theory (see e.g. Refs. [2] and [3]). In these and similar calculations SRCs play an important role in the microscopic structure of nuclei with
more than 50% of kinetic energy originating from SRC\cite{footnote_1}. Understanding the dynamics relevant to SRC is important also for building a realistic equation of state for dense nuclear matter such as neutron stars where typical internucleon distances in the core are close to those encountered in SRCs $\sim 0.5 - 1.5$ fm.

In this respect the key questions in studying the dynamics of SRC’s are

- How large are the probabilities of SRCs in nuclei?
- What is the isotopic structure of SRCs?
- Are there significant three nucleon SRCs?
- How significant are non-nucleonic degrees of freedom in the SRC?
- What is kinematical range of applicability of the concept of SRC in QCD?
- What is the impact of SRCs on the dynamics of compact stars: neutron stars, hyperon stars etc?

### 1.2 SRC observables and strategy of their studies

Identifying the processes in which one can unambiguously probe SRCs including their microscopic properties is one of the goals of the present review. Nucleon momentum distributions in nuclei as well as nuclear spectral and decay functions represent the set of observables which elucidate the different aspects of the dynamics of SRCs, in particular: (i) the nature of SRCs as high density fluctuations of nuclear matter, (ii) dynamical correlation between initial momentum of struck nucleon and energy of residual nuclear system associated with a removal of a nucleon from two and three nucleon SRCs in the nucleus, and (iii) the isospin content of SRCs. These questions are discussed in details in Sec.2.

First we discuss the conditions under which it is possible to probe SRCs by suppressing contributions associated with long range (low momentum) processes in nuclei.

For many years SRCs were considered as an important but elusive feature of the nuclear structure. Referring to SRCs as elusive was due to lack of low energy processes which are dominated by the high momentum component of nuclear wave

\footnote{In a number of approaches such as Landau-Migdal Fermi liquid approach, mean field shell models and effective chiral theory approach, which aimed at describing low energy effects in nuclei the SRC are hidden in the parameters of the effective potential describing quasiparticles, with very little energy carried by nucleons with momenta above the Fermi momentum.}
function\footnote{2} It was shown in Refs. \[5\] and \[6\] that the fundamental problem was the use of the processes with energy-momentum scales comparable to that of the SRCs and that situation should drastically improve for high energy processes in which one can select kinematics corresponding to an energy and momentum transfer scale much larger than the scale characteristic for SRCs:

\[ q_0 \gg V_{NN}, \quad |\vec{q}| \gg 2k_F. \]  

This condition is well satisfied in high energy projectile-nucleus quasirelastic and inelastic interactions. As a result, generic lepton/hadron-nucleus processes could be treated as instantaneous as compared to the nucleon motion within SRC\[5\]–\[7\]. In such processes energy and momentum transferred to one of the nucleons in SRC significantly exceed relevant energies and momenta in SRC, leading to an effective release of nucleon spectators from the SRC. Formally this process is described by the decay function of the nucleus which will be defined below. Processes associated with a release of spectators produce significant correlation properties of the nuclear decay function, which can be used for identification of SRCs.

Possibility of instantaneous removal of nucleon from nucleus in high energy processes greatly simplifies identification of the observables that are sensitive to the high momentum component of nuclear ground state wave function. Therefore unambiguous identification of short range nucleon correlations in a nucleus requires an effective use of the resolution power of high momentum transfer processes.

The theoretical challenge of using high energy and momentum transfer reactions is that projectile and some of the final state particles move with relativistic velocities making it impossible to apply directly non-relativistic approaches for description of the nuclear wave functions as well as the scattering process itself. High energy projectile moving along the z-direction probes the light-cone (LC) slice of nuclear wave function near the hyperplane \( t - z = \text{const} \) (Fig.1) - the LC wave function of the nucleus: \( \psi_A(\alpha_1, k_1, t, \ldots \alpha_i, k_i, t, \ldots \alpha_A, k_A, t) \), where

\[ \alpha_i = A \left( \frac{E_i - p_{i,z}}{E_A - p_{A,z}} \right), \]  

\footnote{2}{It was argued in Ref. \[4\] that high momentum component of nuclear wave function could not be observed even in principle due to inability to separate the influence of measuring process from the measured quantity. The argument was based on implicit assumption of the proximity of scales characterizing measuring process and structure of SRCs. Such arguments were valid for intermediate energy processes in which cases energy and momentum scales characteristic for the measuring process and for the SRCs are comparable.}
are the light-cone fractions (scaled by A) of the nucleus momentum carried by constituent nucleons \( \left( \sum \alpha_i = A \right) \). Here, \( (E_i, p_{iz}) \) and \( (E_A, p_{Az}) \) are the energy and longitudinal momentum of constituent nucleons and target nucleus respectively. Due to the invariance of \( \alpha_i \) with respect to the Lorentz boosts in \( z \) direction, in the nucleus rest frame \( \alpha_i = A \left( \frac{E_i - p_{iz}}{M_A} \right) \), where \( E_i \) and \( p_{iz} \) now are the lab energy and \( z \) component of bound nucleon in the nucleus with mass \( M_A \).

![Figure 1: Fast projectile interacting with nucleus selects a light-cone slice of the wave function.](image)

The important feature of nuclear light cone wave function is that there exists a simple connection between LC and nonrelativistic wave functions of a nucleus:

\[
\psi_{nr}(\vec{k}_1, ..., \vec{k}_i, ..., \vec{k}_A) = (m_N)^{-\frac{A}{2}} \psi_{LC}(\alpha_1 = 1 + \frac{k_{1,z}}{m_N}, k_{1,t}, ..., \alpha_i = 1 + \frac{k_{i,z}}{m_N}, k_{i,t}, ..., \alpha_A = 1 + \frac{k_{A,z}}{m_N}, k_{A,t}),
\]

(3)

at \( k_i \ll m_N \) for \( i = 1, ..., A \). Therefore the knowledge of the nuclear LC wave function allows us to study the rest frame nonrelativistic nuclear wave function as well. However for large nucleon momenta in the nucleus the correspondence between nonrelativistic and light cone wave functions becomes more complex especially for the case of SRC of more than two nucleons.
1.3 Short survey of recent progress in SRC studies

For a long time the only class of high energy processes which was systematically studied experimentally and which appeared to be dominated by a projectile scattering off the SRC was production of fast backward nucleons and pions from nuclei in reactions (see Ref. [6] and references therein):

\[ \gamma(\nu,\pi,p) + A \rightarrow \text{fast backward } p(\pi) + X. \]  \hspace{1cm} (4)

It was demonstrated back in 1977 [5] that the data for the reaction (4) for \( \gamma + ^{12}\text{C} \) scattering at high energies (\( E_\gamma \geq 2 \text{ GeV} \)) can be described as due to the decay of SRC after the inelastic interaction of photon with one of the nucleons of two-nucleon SRC. The wave function of the SRC was found to be proportional to the deuteron wave function for \( 300 \leq k \leq 800 \text{ MeV}/c \) with a proportionality coefficient \( a_2(^{12}\text{C}) = 4 \div 5 \). The similarity of emission spectra for interaction of different projectiles with lightest (\(^2\text{H},^4\text{He}\)) and heavy (\(^{18}\text{F}\)) nuclei in kinematics in which scattering off the low momentum nucleons could not contribute, as well as several other regularities have been naturally explained based on the few-nucleon correlation model [6]. However these inclusive processes did not allow to reconstruct a complete final state of the reaction and therefore to perform quantitative investigation of the SRC structure of nuclei. Rather direct confirmation of the significant role of SRC has been obtained in the processes of neutrino (antineutrino) scattering off nuclei in the observation of correlation between momenta of backward nucleon and forward muon [8, 9] which was predicted in Ref. [5]. There was also an evidence for universality of SRCs coming from the comparison [10, 11] of \( A(e,e'X) \) cross sections for different nuclei at \( x \geq 1 \) and \( Q^2 \geq 1 \text{ GeV}^2 \) measured in several different experiments performed in the 1980’s.

A new qualitative and quantitative progress in determination of the structure of SRC have been achieved recently on the basis of two new theoretical ideas: (i) that the presence of two energy-momentum scales: high energy scale for the probe and lower energy scale for nuclear phenomena, justifies the application of the closure approximation. For \( A(e,e'X) \) reaction at \( x > 1 \) and \( Q^2 > 1.5 \text{ GeV}^2 \) closure application can be applied up to the final state interaction effects within SRC which are practically independent of \( A \). The latter observation leads to the scaling of the ratios of cross sections of different nuclei (see Sec.3); (ii) observation that hard exclusive processes, in which a nucleon from SRC is removed instantaneously, probe a new observable: the nuclear decay function \( D_A(k_1,k_2,E_R) \) [5, 6, 10, 12], which represents a probability of emission from the nucleus of a nucleon with momentum \( k_2 \) after the removal of a fast nucleon with momentum \( k_1 \) and leading to a residual...
nuclear state with recoil energy $E_R$ (see Sec. 2 for details). Although in general $D_A$ is a very complicated function of its variables, in the case of the removal of a nucleon from two-nucleon SRC its form is rather simple: the second nucleon of the SRC is released with momentum $\vec{k}_2 \sim -\vec{k}_1$. In this case emission of nucleons with $\vec{k}_2$ which are spatially far from the SRC from which the nucleon with $\vec{k}_1$ is removed does not contribute to $D_A$. Decay functions can be evaluated based on nuclear models which include SRCs in a rather wide kinematical region.

For more than a decade there were practically no new experimental data in this field. In the last three years a qualitative progress was reached as the new $A(e, e')X$ experiments have been performed at Jefferson Lab[13, 14] while studies of processes sensitive to the properties of the decay function in kinematics dominated by SRC were performed first at BNL[15]—[17] and then at JLab[18, 19].

Combined analysis of these data indicates that a) probability of short range correlations in carbon is $\sim 20\%$, b) these correlations are predominantly consist of two nucleons, c) probability for two protons to belong to a SRC as compared to that of a proton and neutron is very small $\sim 1/20$. These observations are in line with the concept of tensor forces dominating at intermediate to short distances in $I = 0$, and $S = 1$ channels of NN interaction (see e.g. discussion in Refs. [20]–[27]. All these observations together demonstrate strong potential of high energy processes for addressing long standing issues of the short-range nuclear structure.

In this review we outline theoretical expectations of realistic nuclear models that describe SRCs, summarize new information about properties of SRCs obtained recently in high energy processes, discuss compatibility of the ideas of nuclear theory with basics of QCD, outline implications of the discovery of SRCs for the theory of neutron stars and outline directions for the future studies both in theory and in high energy nuclear experiments. In particular we emphasize studies of isotopic structure of SRCs relevant to the study of internucleon forces in the region of the nuclear core, three-nucleon correlations, $\Delta$-isobar admixture, isospin effects etc.

## 2 The status of high momentum component of nuclear wave function in nonrelativistic theory

In this section we discuss the manifestations of the high momentum component of nuclear wave function in the properties of nucleon momentum distribution, $n(k)$, spectral function, $S(k, E)$ as well as decay function $D(k_1, k_2, E_r)$ (to be defined below). In particular, we will demonstrate how one can identify the signatures of two-
and three-nucleon SRCs in these quantities. We will also discuss nuclear reactions in which the above functions can be measured. Our consideration in this section is restricted to the nonrelativistic theory, though a number of arguments indicate that many SRC related properties we discuss will reveal themselves in a similar way in the relativistic theory.

2.1 Momentum Distribution

The nucleon momentum distribution \( n_A(k) \) is given by the modulus square of the ground state nuclear wave function integrated over all nucleon momenta except one,

\[
n(k) = \sum_{i=1}^{A} \int \psi_A^2(k_1, k_2, k_i, \ldots k_A) \delta^3(k - k_i) \delta^3(\sum_{j=1}^{A} k_j) \prod_{l=1}^{A} d^3 k_l.
\]  

(5)

Properties of \( n(k) \) at high momentum, \( k \gg k_{Fermi} \) follow directly from the Schrödinger equation in the momentum space. In general for given two-nucleon interaction potential, \( V \), the ground state wave function, \( \psi_A \) satisfies the equation:

\[
(E_B - \frac{k^2}{2m} - \sum_{i=2}^{A} T_i)\psi_A = \sum_{i=2}^{A} \int V(k - k'_i)\psi_A(k, k'_i, \ldots k_j, \ldots k_A) \frac{d^3 k'_i}{(2\pi)^3} \\
+ \sum_{i=2}^{A} \int V(k'_i - k_i)\psi_A(k, k'_i, \ldots k_j, \ldots, k_A) \frac{d^3 k'_i}{(2\pi)^3},
\]

(6)

where \( E_B \) is nuclear binding energy and \( T_i \) are kinetic energies of nucleon-spectators and \( V(k) = \int V(r)e^{-i(kr)}d^3r \) is the \( NN \) potential in the momentum space.

2.1.1 Theorem on high momentum tail of nuclear ground state wave function

Based on Eq.(6) it can be proven that if the potential decreases at large \( k \), like \( V(k) \sim \frac{1}{k^n} \) and \( n > 1 \) then the \( k \) dependence of the wave function for \( k^2/2mN \gg |E_B| \) is calculable in terms of the potential \( V \) as follows:

\[
\psi_A \sim \frac{V_{NN}(k)}{k^2} f(k_3, \ldots k_A),
\]

(7)

where \( f(k_3, \ldots k_A) \) is a smooth function of spectator nucleon’s momenta with \( k_2 \sim -k \). To prove the theorem it is sufficient to show that all higher order iterations will decrease faster with \( k \) and therefore preserve the form of Eq.(7). In the case of nucleon-nucleon potentials which contain both repulsive core and medium range attraction the accuracy of this equation maybe worse, however numerical studies described below are consistent with Eq.(7).
2.1.2 Nuclear wave function and short range correlations

In nonrelativistic nuclear theory interaction potential is constructed as a sum of potentials involving two, three, and higher number of nucleons. All realistic \( NN \) potentials deduced from fitting the NN scattering phase shifts at large \( k \) have the property that the potential of two-nucleon interaction, \( V_{NN} \), decreases significantly slower with an increase of \( k \) than triple and higher order nucleon potentials. (Such a behavior arises naturally if many body interaction results from the iteration of two-nucleon interactions). As a result, at large \( k \) limit the contribution of the pair nucleon potential \( V_{NN}(k) \) in which two nucleons have large relative momentum \( k \) will dominate. This will justify the use of Eq.(7) for calculation of asymptotic form of the ground state nuclear wave function at large \( k \). Consequently, using Eq.(5) one arrives at the asymptotic form of momentum distribution function:

\[
 n(k) \sim \left( \frac{V_{NN}(k)}{k^2} \right)^2.
\] (8)

The above relation can be improved by taking into account the center of mass motion of the NN pair. However the latter effect decreases with increase of \( k \).

The direct consequence of Eq.(8) is the dominance of two nucleons SRCs in the high momentum part of \( n_A(k) \). This results in the similarity of the shapes of \( n_A(k) \) for different nuclei at \( k > 300 \) MeV/c. This similarity is clearly seen in Fig.2 where momentum distributions for \( ^2H, ^3He, ^4He \) as well as \( ^{16}O \) from Ref. [24] are compared.

The ratios of nucleon momentum distributions are given in Fig.3 and Fig.4. It is spectacular that while the absolute magnitude of momentum distributions drop by three orders of magnitude in \( 0.3 < k < 1 \) GeV/c range, the ratios \( \frac{n_A}{n_d} \) for a given nucleus does not change significantly. It appears, that the non-uniformity of \( \frac{n_A}{n_d} \) ratios in Fig.4 is related to the difference between NN potentials for spin 0 and 1 states and to the center of mass motion of NN pair in nucleus \( A \). This can be seen in Fig.4, where the ratios \( \frac{n_A}{n_3He} \) and \( \frac{n_A}{n_4He} \) show significantly weaker momentum dependence at \( k > 400 \) MeV/c.

Other calculations of nuclear momentum distribution, \( n(k) \), within nonrelativistic nuclear theory with realistic NN potentials are also consistent with the dominance of two-nucleon correlations for momenta above 300 MeV/c. Moreover numerical studies [22, 25, 26] confirm that the dominant contribution in the momentum range \( 350 \leq k \leq 700 \) MeV/c is due to the tensor forces which dominate in NN channel with isospin 0 and spin 1. Independent evidence for the dominance of \( pn \) correlations comes from the study of two nucleon momentum distributions in nuclei [23, 27].
Note that quantity often used for the analysis of data is the partially integrated momentum distribution:

\[ n_n(k_z) = \int n(k) d^2 k_\perp. \quad (9) \]

One can see from Figs.3,4 that the scaling is even more pronounced for these quantities.

It is worth mentioning that although \( n_A(k) \) is the simplest function which can be constructed from the ground state nuclear wave function, it cannot be observed directly in any processes. The spectral function is the simplest quantity which is related to the cross sections of physical processes namely to \( A(e,e'N)X \) and \( A(e,e')X \) processes\(^3\). At the same time as we will see below the \( A(e,e')X \) cross section at large \( Q^2 \) is expressed through the light-cone nuclear density matrix which illustrates the nontrivial relation between light-cone and nonrelativistic quantities.

### 2.2 Spectral Function

The asymptotic behavior of \( n_A(k) \) for \( k \to \infty \), according to Eq.\(^8\), contributes only to the range of \( \alpha < 2 \) for the light-cone nucleon density \( \rho_A(\alpha, k_t) \). This result follows

\(^3\) In principle, one could infer \( n_A(k) \) from the sum rule \( n_A(k) = \int dE R S(k, E_R) \).
Figure 3: Curves describe momentum dependence of the ratio, \( \frac{n_A(k)}{n_d(k)} \). Points represent calculated \( k_z \) dependence of the ratio \( \frac{n_A(k_z)}{n_d(k_z)} \).

from the account of the energy-momentum conservation for the whole process which is contained in the spectral function.

Reaching the region of \( \alpha > 2 \) is possible only if at least three nucleons are involved in the process. This does not necessarily require specific three body forces - an iteration of two nucleon interactions is in principle sufficient. However in a wide range of the nucleon momenta, \( k \) above the Fermi momentum and for recoil nuclear energies close to \( \frac{k^2}{2m_N} \), the two nucleon SRC approximation with the motion of NN pair in the mean field of nucleus taken into account provides a good approximation for both nonrelativistic and light-cone description of nuclei. This approximation is effective in the calculation of the nuclear spectral function:

\[
S_A(p_i, E_R) = |\langle \phi_{A-1} | \delta (H_{A-1} - E_m) a(k) | \psi_A \rangle|^2,
\]

which represents a product of the probability of finding a nucleon in the nucleus with initial momentum \( p_i \), and the probability that after instantaneous removal of this nucleon the residual system will have recoil energy \( E_R \). Note that in the traditional definition one separates the recoil energy \( E_R \) into the sum of two terms - \( E_m \), the excitation energy of the \( A - 1 \) system in its center of mass, and the kinetic energy of the center of mass itself - \( \frac{p_i^2}{2m_{A-1}} \). Although such definition is very convenient for
Figure 4: Curves correspond to the momentum dependence of ratio, \( \frac{n_A(k)}{n_{^{3}He}(k)} \) and \( \frac{n_A(k)}{n_{^{4}He}(k)} \). Points are the calculated \( k_z \) dependence of the ratio \( \frac{n_A(k_z)}{n_{^{3}He}(k_z)} \) and \( \frac{n_A(k_z)}{n_{^{4}He}(k_z)} \).

The case of nucleon removal from nuclear shells, it gives a less transparent pattern of properties of the spectral function in the case of removal of nucleons from SRCs. Indeed, removal of the nucleon from say a two nucleon SRC leads to nearly universal \( E_R \) distribution for the spectral functions for different nuclei, while \( E_m \)-distributions strongly depend on \( A \) especially for light nuclei.

Within the plane wave impulse approximation the spectral function defined above is related to the differential cross section of \( A(e, e'N)X \) reaction as follows:

\[
\frac{d\sigma}{d\Omega_e dE_e d^3p_f dE_R} = \frac{j_N}{j_A} \sigma_{eN} \cdot S(p_t, E_R).
\]  

(11)

Here \( j_N \) is the flux calculated for moving bound nucleon with momentum \( p_t \), and \( \sigma_{eN} \) represents the cross section of electron- “bound-nucleon” scattering. In the limit of \( Q^2 \geq 1 \text{ GeV}^2 \gg p_t^2 \) the above form of factorization can be used also for more realistic case in which final state interactions (FSI) are taken into account. However in this case the spectral function is modified to \( S^{DWIA}(p_f, p_t, E_R) \) which now contains the factors that account for FSI as well as modification of the flux factor in the rescattering part of the spectral function. This approximation is usually referred to
as distorted wave impulse approximation\[1\]

In the case of spectral function SRCs manifest themselves clearly in the properties of the recoil nuclear system when a fast nucleon is removed from the nucleus. In this case the second nucleon is effectively removed from the nucleus as the potential between this and the struck nucleon is destroyed instantaneously. This mechanism of breaking of correlations we will refer hereafter as type 2N-I SRC mechanism of breaking SRC (Fig. 5a). Since momenta of the nucleons in NN correlation in average are equal and opposite, one finds for the average recoil energy of the residual system

\[<E_R> = \frac{p_i^2}{2m_N}, \quad (12)\]

as the \(A - 2\) system is essentially not perturbed during removal of the nucleon from 2N SRC. Eq. (12) agrees well with numerical studies of spectral functions for \(A = 3\) and infinite matter [28]. Moreover the distribution over \(E_R\) calculated in Refs. [29–31] is well described by the model which takes into account the motion of the \(NN\) pair in the mean field [28, 32] (see Fig. 6).

In many cases spectral functions are modeled based on type 2N-I SRCs modified only by taking into account the mean field momentum distribution of the center of mass motion of SRC in the nucleus (see e.g. Refs. [10, 28, 32]). Such approximations describe well the existing data and agree reasonably well with numerical calculations based on two-nucleon potentials only (see e.g. Ref. [31]). Note however, that these calculations did not include contribution of three particle three hole excitations so a good agreement of the models of Ref. [28] and [31] may reflect deficiency of both models.

If the nucleon with small momentum is removed, the residual system is predominantly in one of the lowest \(A - 1\) nucleon states. The contribution of SRCs into large recoil energy range is strongly suppressed in this case as compared to the expectation based on the total probabilities of SRCs. In the simplest case of \(A = 3\) system to observe suppression of the large recoil energies for the case of removal of a nucleon with momentum \(p_n \sim 0\) (Fig. 5b) one needs to take into account the difference between ”off-energy shell” and ”on-energy-shell” t-matrices of \(NN\) scattering. For further discussion see Sec. 2.3.

Note that mere observation of the correlation given by Eq. (12) in \(A(e,e^'N)X\) reactions will not allow conclude unambiguously that the spectral function, \(S(p_i, E_R)\) is sensitive to the SRC. In general, Eq. (12) is satisfied for any reaction dominated by

\[\text{It is possible also to perform unfactorized calculation of FSI in which case the interpretation of the scattering cross section through the spectral function is not possible. However at the kinematics in which FSI is a correction, effects due to unfactorization are insignificant.}\]
two–nucleon processes with two nucleons not necessarily belonging to a SRC (for example contribution due to meson exchange currents). However, if an additional kinematic conditions such as Eq. (1) are satisfied allowing to suppress long range two-nucleon processes, one could use the relation Eq. (12) to check the dominance of the SRCs.

In this respect it is worth mentioning the recent measurement [33] of three-body break up reaction of $^3$He in kinematics satisfying condition of Eq. (1). These experiment observed clear correlation consistent with Eq. (12) (Fig. 7).

The comparison of calculations based on DWIA [36, 37] with the data [33] demonstrates that a substantial contribution from final state reinteraction not only preserves the pattern of the correlation of Eq. (12) but also reinforces it (Fig. 7). This indicates a rather new phenomenon, that in high energy kinematics sensitive to SRC, FSI is dominated by single rescattering of struck nucleon with a spectator nucleon in SRC. As a result this rescattering does not destroy the correlation property of the spectral function.

Three nucleon (3N) SRCs also contribute to the spectral function. In Fig. 8 we consider two scenarios for 3N SRCs that can be evolved from 2N correlations with an increase of the c.m. momentum of 2N-SRCs. The one, which we refer to as type 3N-I SRC, (Fig. 8a) corresponds to the situation in which initial momentum of struck nucleon (typically $p_i \geq 600$ MeV/c) is shared by two spectator nucleons with invariant mass close to $2m_N$.

In reality integral over the recoil momentum should give slightly larger 2N recoil mass, $m_{23} > 2m_N$.

![Figure 5: Interaction of virtual photon with three nucleon system in configurations in which two of the nucleons are in SRC.](image)
energies. Thus effects of type 3N-I SRCs should be manifested in the strength of the spectral function at $p_i \geq 600$ MeV/c and recoil energies which (similar to the 2N SRCs) have universal, $A$-independent values:

$$\langle E_R \rangle \sim \frac{p_i^2}{4m_N}.$$  

(13)

It follows from Eq.(13) that the value of $\langle E_R \rangle$ in this case is approximately equal to one half of the recoil energy that characterizes type 2N-I correlations (Eq.(12)). These configurations give dominant contribution to $A(e,e')X$ cross section at $2 < x < 3$ and at large $Q^2$.

Another type of 3N SRCs (Fig.8b) (referred as type 3N-II SRCs) can originate from 2N SRCs in situations in which the center of mass momentum of NN correlation becomes comparable with relative momentum of nucleons in the NN correlation with momenta of all three nucleons considerably exceeding Fermi momentum. This corresponds to an average recoil energy:

$$\langle E_R \rangle \sim \frac{p_i^2}{m_N},$$  

(14)
Figure 7: The dependence of the differential cross section on the missing energy, for $^3$He three-body break up reactions at different values of initial nucleon momenta. Dotted, dashed and solid curves corresponds to PWIA, PWIA + single rescattering and PWIA + single + double rescatterings. Data are from Ref.33. Arrows define the correlation according to Eq.(12). Similar description of the data is achieved in Refs.34 and 35.

which are roughly twice as large as recoil energies characteristic to type 2N-I SRCS (Eq.(12)). Type 3N-II SRCS in general are more rare than type 3N-I SRCS, since they correspond to much larger recoil energy of residual nucleus, though it may be easier to observe them experimentally (See Sec.8.4).

As it follows from Eqs.(13,14) 3N SRCS generate correlations between $p_i$ and $E_R$, below and above the average recoil energy values characteristic to type 2N-I SRCS (Eq.(12). However since the correlation observed for type 2N-I SRCS in Eq.(12) has rather broad $E_R$ distribution it fully overlaps with $p_i - < E_R >$ correlations followed from Eqs.(13,14) up to rather large nucleon momenta. This situation indicates the limited capability of the spectral function to reveal details of three nucleon SRCS. One needs to study decay products of the residual nucleus to observe such configurations.

Thus we conclude, that even though 2N and 3N SRCS lead to a distinctive structure in the recoil energy distribution of the spectral function, the dominance of type
2N-I correlations overshadows the $p_i - \langle E_R \rangle$ correlation expected from 3N SRCs. The latter means that $p_i$ and $E_R$ variables are not enough to isolate 2N and 3N SRCs. However the important observation is that all correlation relations of Eqs. (12, 13, 14) have a local character and at large $p_i$ they should be manifested through the approximate universality of the spectral function distributions over $E_R$ for different nuclei.

It is worth noting that situation is different for the relativistic case for which use of the light-cone momentum fraction $\alpha_i$ (Eq. (2)) allows to separate 2N and 3N SRCs. Indeed choosing $\alpha_i > 2$, (see Ref. [10]) will significantly enhance the contribution of 3N SRCs in the spectral function.

### 2.3 Decay Function

Going one step further beyond the spectral function one can ask a question how the recoil energy $E_R$ is shared between the decay products of the residual nucleus. One can introduce the nonrelativistic decay function of the nucleus as follows [10]

$$D_A(k_1, k_2, E_R) = \left| \langle \phi_{A-1} a^\dagger(k_2) \delta(H_{A-1} - (E_R - T_{A-1})) a(k_1) | \psi_A \rangle \right|^2, \quad (15)$$

which is the probability that after a nucleon with momentum $k_1$ is instantaneously removed from the nucleus the residual A-1 nucleon system will have excitation energy $E_m = E_R - T_{A-1}$ and contain a nucleon with momentum $k_2$. This function can be extracted from the differential cross section of double-coincidence experiments in which knocked-out fast nucleon (with momentum $\vec{p}_f$) is detected in coincidence with a slow nucleon (with momentum $\vec{p}_r$) which is produced in the recoil kinematics.
In this case within PWIA differential cross section is expressed through the decay function as follows:

\[
\frac{d\sigma}{dE_e' d\Omega_e' d^3p_f d^3p_r} = \frac{j_N}{j_A} \sigma_{e,N}(p_f, p_i, Q^2) \cdot D_A(p_i, p_r, E_r).
\] (16)

Comparing Eq. (11) and (16) one observes that:

\[
S_A^e(p_i, E_R) = \int D_A(p_i, p_r, E_R) d^3p_r,
\] (17)

where \(S_A^e\) represents the part of the complete spectral function of Eq. (10) corresponding to the case of the break-up of \(A - 1\) residual nucleus with at least one nucleon in the continuum state.

Eq. (15) represents the lowest order nuclear decay function in which only one recoiling nucleon with large momentum is detected from the residual nucleus. In principle one can consider decay function containing more than one fast nucleons at recoil kinematics. This will require an introduction of additional factor in the r.h.s. of Eq. (15) due to presence of several nucleons in the recoil system. Note that in analogy with spectral function, the decay function defined above can be generalized to \(D_{A}^{DWIA}\) for the case distorted wave impulse approximation in which final state interactions are taken into account and factorization of nucleon electromagnetic current is justified.

To investigate the basic features of the decay function related to the SRC properties of nuclear ground state wave function, we analyze \(D_A(p_i, \vec{p}_r, E_m)\) function in the impulse approximation limit in which we neglect the final state interaction of the struck nucleon with spectator nucleons in the reaction. In this case the decay function can be represented as follows:

\[
D_A(p_i, \vec{p}_r, E_m) = \frac{1}{2s_A + 1} \sum_{s_A, s_f, s_r} \sum_{A-2} \int_{3}^{A-1} d^3p_i \cdot \delta(E_R - T_r - E_{A-2} - |\epsilon_A| - T_{A-2})
\times \left| \int d^3p_{r,A-2} \Psi_{\vec{p}_r,A-2}(p_3, ..., p_A) \cdot \Psi_A(p_i, p_r, p_3, ..., p_A) \right|^2,
\] (18)

where \(\Psi_{\vec{p}_r,A-2}\) represents the wave function of recoil nucleon and spectator \((A-2)\) system and \(p_{r,A-1}\) is the relative momentum of the recoil nucleon with respect to the c.m. of the \((A-2)\) system. The sum \(\sum_{A-2}\) accounts for the different configurations of the \((A-2)\) system. Within impulse approximation, the angular dependence of the decay function is defined by the relative angle between \(\vec{p}_i\) and \(\vec{p}_r\).
It follows from Eq. (18), that in order to a residual nucleus to decay into the state that contains a fast nucleon with momentum $p_r$, it should have a sufficient recoil energy $E_R > \frac{p_r^2}{2m_N} + |\epsilon_A|$. This energy should be transferred to the residual nucleus during the process of removal of the struck nucleon to compensate the binding energy of the struck nucleon with other nucleons. SRCs provide an effective mechanism of such energy transfer before the removal of struck nucleon. In this case an average energy transferred to residual (A-1) system is $\sim \frac{p_i^2}{2m_N} + |\epsilon_A|$.

The above discussed dynamics creates an additional suppression factor for kinematic situation in which recoil fast nucleons are produced in the situation in which the initial momentum of struck nucleon $p_i \approx 0$ (see also discussion in Sec.7.4 of Ref. [6]). Note that within the generalized eikonal model of $D_{DWIA}$, the recoil energy of the residual system is provided by the final state interaction of struck nucleon with nucleons of the residual system [36, 37].

Similar suppression exists for $A \geq 4$ nuclei for the kinematics in which a removal of a fast nucleon from one SRC is accompanied by an emission of a fast nucleon from another SRC separated by distances exceeding average internucleon distances in the nucleus ($\geq 1.5 fm$). Such decay is additionally suppressed by the short-range nature of NN interaction.

In Refs. [23] and [27] the nuclear double momentum distribution was considered. In the kinematics in which momentum of a proton $k_1 \gg k_F$ this quantity shows strong correlation with presence of a neutron with momentum $k_2 = -k_1$ reflecting presence of SRC and dominance of $pn$ correlations. Contribution of uncorrelated (A-2) nucleons in this kinematics is small since the factor (A-2) in the normalization of the double momentum distribution does not compensate (if $A$ is not very large) the small probability of 2N SRC per nucleon. However away from this kinematics uncorrelated contribution which is enhanced by a factor (A-2) becomes increasingly more important and difference from the decay function in which only correlated pairs contribute becomes large. As a result, it is difficult to use the double momentum distribution to calculate effects of c.m. motion of 2N SRC in the mean field of nucleus and dependence of the pp/pn ratio on this motion.

In following, we focus on kinematics, in which the removed nucleon momentum is large and therefore the above discussed suppression does not arise in the decay of the residual nucleus containing at least one fast nucleon.

Overall, one expects decay function to exhibit much stronger sensitivity to the SRC structure of nucleus than the spectral function. For example, for situation of Fig.5a, because of interaction within SRC is local as compared to the average

\footnote{Final state interaction of the struck nucleon which is not contained in the definition of the decay function in Eq. (18) can transfer this energy only after the removal of the struck nucleon.}
scale of internucleon distances it is natural to expect that after one nucleon of 2N SRC is removed the second one will be produced on the mass shell with momentum approximately equal and opposite to the one it had in the correlation: \( \vec{p}_r \approx -\vec{p}_i \).

Such correlations are clearly seen in Fig. 9 where the dependence of the decay function strength is given as a function of the relative angle of initial and recoil nucleons for different values of cuts imposed on initial nucleon momenta.

Such pattern resulting from the breaking of SRC by instantaneous removal of one of the correlated nucleons by energetic projectile was suggested in Ref. [5] as a spectator mechanism for production of nucleons in the reaction of Eq. (4). This pattern was experimentally confirmed in high momentum transfer triple coincidence \( A(p, 2pN)X \) experiment [15, 16] in which clear correlation between \( p_i \) and \( p_r \) was observed.

Already this example demonstrates that moving from spectral to decay function

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7 If SRCs are located at the center of the nucleus the escaping nucleon may rescatter in the nuclear medium leading to distortions of the spectrum which in kinematics of Eq. (1) can be taken into account in eikonal approximation (see e.g. Ref. [36]).
Figure 10: Dependence of the decay function on residual nuclei energy, $E_R$ and relative angle of struck proton and recoil nucleon, $\Theta_r$. Figure (a) neutron is recoiling against proton, (b) proton is recoiling against proton. Initial momentum of struck nucleon as well as recoil nucleon momenta is restricted to $p_i, p_r \geq 400$ MeV/c. 

provides an additional tool for probing SRCs, such as correlation between initial and recoil nucleon momenta.

Another advantage of the decay function is the possibility to isolate three-nucleon SRCs and to probe their dynamics. Fig. 10 shows the dependence of the decay function on the angle between initial, $\vec{p}_i$, and recoil, $\vec{p}_r$ nucleon momenta, and the recoil energy, $E_R$ for $p_i, p_r \geq 400$ MeV/c. This figure shows kinematical domains where it is possible to separate 2N and 3N correlations by varying energy of the recoil system. In the calculation presented above the minimal recoil energy for type 2N-I SRCs (Fig. 5a) will be $\sim \frac{p_{n,min}^2}{2m_N} \approx 80$ MeV, while for type 3N-I SRCs (Fig. 8) the minimum of recoil energies is twice as large. The upper left side of the figure demonstrates how type 2N-I SRCs evolve to a type 3N-I SRC with the third nucleon also recoiling against the removed nucleon. One can also see from the figure that with an increase of recoil energy type 3N-II correlations start to dominate. The important signature in this case is the relative angle of recoil nucleon emission being close to
$120^0$ which is characteristic for type 3N-II SRCs. The lower right part of the figure shows also different realization of 3N-I SRCs in which both struck and recoiled nucleons share/balance the momentum of the third nucleon which has roughly twice the momentum of $p_i$ or $p_r$.

![Graph](image)

Figure 11: Recoil energy dependence of the ratio of the decay functions when a proton in $^3$He is struck and the proton on one case and the neutron in other is produced in the decay. Both initial momenta of struck and recoil nucleons are set to be larger than 400 MeV/c. Also, the relative angle between initial and recoil nucleon momenta is restricted to $180^0 \geq \theta_r \geq 170^0$.

Figs.10a and 10b present the decay functions for proton removal with production of either proton or neutron in the decay. Comparison of these two cases shows (see upper left part of the graph) that in type 2N-I SRCs the strength of $pn$ correlation is larger than the strength of $pp$ correlation by factor of ten. This feature reflects the dominance of tensor interaction in $S = 1, T = 0$ channel of NN interaction at short distances and was confirmed experimentally, both for hadron- and electron-induced triple coincidence reactions on carbon [17, 18]. Interesting consequence of the onset of 3N SRCs is that these two rates become practically equal once recoil energy increases. More detailed view of relative strengths of $pp$ and $pn$ decay functions is given in Fig.11. The increase of the ratio of $pp$ to $pn$ strengths with an increase of the recoil energy represents an unambiguous indication of the dominance of type
3N-I SRC effects.

As we mentioned before, the notion of the decay function can be extended to the situations in which more than two nucleons are detected in the products of the decay of the residual nucleus. One such extension is the study of two recoil nucleons without detecting the struck nucleon. In this situation nucleons with approximately equal momenta will be emitted predominantly at large relative angles to minimize the momentum of the struck nucleon.

Concluding this chapter we would like to emphasize that experimental possibility of measuring nuclear decay function in high momentum transfer triple coincidence $A(e, e'N_f, N_r)X$ or $A(h, h'N_f, N_r)X$ reactions opens up completely new perspectives in studying the dynamics of 2N and 3N SRCs.

3 Scaling of the ratios of cross sections of $A(e, e')X$ reactions at $x > 1$.

3.1 Introduction

Here we consider $A(e, e')X$ reactions at kinematics:

$$x = A \frac{Q^2}{2m_A q} > 1,$$

which have been measured recently at JLab\[13, 14\]. They complement and improve the previous measurements which were performed at SLAC in the 80’s (see Ref. [11] and references therein).

Before presenting a more formal discussion we review an intuitive picture of the reaction. It follows essentially from the definition of $x$ that its magnitude cannot be larger than the number of nucleons in a given nuclei. This can be seen from the definition of the produced mass in the reaction

$$W^2 = Q^2(-1 + \frac{m_T}{xm_N}) + m_T^2 \geq m_T^2,$$

which leads to $x \leq m_T/m_N$. In the impulse approximation the process is described as an absorption of the virtual photon by a nucleon which had a momentum opposite to the direction of virtual photon momentum.

The kinematics of the process resembles that of the deep inelastic scattering off massive partons, and therefore in the limit of large $Q^2$ we expect that $x = \alpha_i$, where $\alpha_i$ is the light-cone momentum fraction of the nucleus carried by the
Figure 12: The $x$ Dependence of $|p_{min}|$ for different values of $Q^2$, with recoil energy given by the two nucleon approximation.

initial nucleon (defined in Eq.(2)) which is struck by virtual photon in quasielastic scattering. Therefore the relation $x = \alpha_i$ indicates that for $x \geq j$ at least $j$-nucleons should be involved in the process. Note here that inequality $x \leq j$ for the scattering off nucleus consisting of $j$ nucleons is valid for all $Q^2$. To see how the relation $x = \alpha_i$ emerges in the discussed process and to estimate the deviation from this relation for finite $Q^2$, it is convenient to introduce four-momentum of the struck nucleon $p_i^\mu = p_A^\mu - p_R^\mu$ where $p_A^\mu$ and $p_R^\mu$ are four-momenta of target and residual nuclei, and $m_i^2 = p_{i\mu}p_i^{\mu}$. In the impulse approximation the requirement that the produced nucleon is on-mass-shell leads to the relation

\[(q + p_A - p_R)^2 = m_N^2. \quad (21)\]

Using Eq.(21) and definition of $x$ one finds

\[x = \frac{\alpha - \frac{m_N^2 - m_i^2}{2m_Nq_0}}{1 + \frac{2p_i^{\mu}}{q_0 + q_3}}, \quad (22)\]

where $q_3 = |\mathbf{q}|$. It follows from Eq.(22) that in the limit $\mathbf{q} \gg p_i$, with $x$ being kept constant, $x = \alpha_i + O(1/Q^2)$.  

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One can see from Fig.12 that for discussed \( Q^2 \) the minimum momentum of the struck nucleon calculated in impulse approximation, (with recoil energy estimated based on type 2N-I SRC picture of high momentum component of nuclear wave function) increases with \( x > 1 \) and becomes significantly larger than \( k_F \sim 250 \text{ MeV/c} \) for \( x = 1.5 \) at \( Q^2 \geq 1.5 \text{ GeV}^2 \). Therefore with the gradual increase of \( x \) virtual photon should first probe the most abundant high momentum configuration which is 2N SRC and then with an increase of \( x \) above two, only high momentum nucleons whose removal is associated with a recoil energy smaller than the characteristic recoil energy for the interaction with two nucleon correlation (Eq.(12)). This can be achieved if struck nucleon momentum is balanced by momenta of two nucleons that exceed \( k_F \).

This picture corresponds to the type 3N-I SRCs (see Fig.8 and discussion in Sec.2.2) with average recoil energy defined according to Eq.(13). Hence we expect that the natural mechanism of quasielastic processes at large \( Q^2 \geq 1 \text{ GeV}^2 \) and \( x > 1 \) is the scattering off 2, 3,... nucleon SRCs which have universal (A-independent) properties.

Since the nucleus is a dilute system, in kinematics in which for example the scattering off \( j \times N \) SRC is possible, the scattering off \( (j + 1) \times N \) SRC should be a small correction. Therefore for two and three nucleon correlation kinematics, for \( Q^2 \) range where quasielastic scattering gives dominant contribution to the cross section\(^9\) we expect \([6, 10]\) inclusive cross section ratios to scale as follows:

\[
\frac{2}{A} \frac{\sigma(eA \rightarrow e'X)}{\sigma(e 2H \rightarrow e'X)} \bigg|_{2 > x \geq 1.5} = a_2(A), \quad \text{and} \quad \frac{3}{A} \frac{\sigma(eA \rightarrow e'X)}{\sigma(e "A = 3" \rightarrow e'X)} \bigg|_{3 > x \geq 2} = a_3(A),
\]

where it is assumed that the ratios are corrected for difference of the electron-proton and electron-neutron cross sections.

The most recent data from Jefferson Lab\([14]\) which confirm the prediction of Eq.(23) are shown in Fig.13.

The quantities, \( a_2(A) \) and \( a_3(A) \) represent the excess of per nucleon probabilities of finding 2N and 3N SRCs in nucleus, as compared to the deuteron and A=3 nucleus respectively. The fact that SRCs represent high density fluctuation of the nuclear matter and constitute only a small part of nuclear wave function allows us to calculate the \( A \) dependence of \( a_2 \) and \( a_3 \) through the nuclear matter density function evaluated within mean-field approximation. Indeed, the fluctuation character of SRCs allows

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\(^8\)Note that for intermediate values of \( Q^2 \sim 2 - 3 \text{ GeV}^2 \) and \( x \leq 2.5 \) the average value of \( \alpha_i \) for the struck nucleon could still be less than two.

\(^9\)For \( Q^2 \geq 6 \text{ GeV}^2 \) contribution from inelastic scattering becomes significant leading to an increase of the typical momenta of nucleons which dominate at given \( x \). This leads to an increase of the ratios with an increase of \( Q^2 \).\([11]\).
Figure 13: The $x$ dependence of the ratios of inclusive cross sections. Dashed curves indicate the scaling expectations from 2N- and 3N- SRCs.

us to justify the estimate\[6\]:

$$a_j \propto \int \rho_A(r)^j d^3r \approx \int \rho_A^{\text{mf}}(1 + j \frac{\rho_A^{\text{SRC}}}{\rho_A^{\text{mf}}}) d^3r.$$ (24)

Above we expressed the nuclear matter density function ($\rho_A(r)$) through the sum of mean field ($\rho_A^{\text{mf}}$) and SRC ($\rho_A^{\text{SRC}}$) density functions ($\rho_A = \rho_A^{\text{mf}} + \rho_A^{\text{SRC}}$). Using the fact that for nuclei with $A \geq 12$ the contribution to the normalization due to SRCs is much smaller than unity ($\int \rho_A^{\text{SRC}}(r) d^3r \ll 1$), for not very large $j \ll A$ the second term of the integrand in right hand part of Eq.(24) can be neglected. As a result one can estimate the $A$ dependence of $a_j$ using mean-field nuclear density function, $\rho_A^{\text{mf}}(r)$. Fig.14 compares the prediction of Eq.(24), using the Skyrme-Hartree-Fock model\[38\] for $\rho_A^{\text{mf}}(r)$, with the experimental data of $a_2(A)\[14,11\]$ and $a_3(A)\[13\]$. It is interesting that above estimates of $a_2$ work even for lightest

\[10\]Note that in Ref.\[11\] the values of $a_2(A)$ were extracted from the data assuming similar momentum dependences for $pn$, $nn$ and $pp$ momentum distributions in SRC region. Since recent studies\[17,18\] demonstrated that $pp$ and $nn$ SRCs are significantly suppressed as compared to that of $pn$, the estimates of $a_2$ in Ref.\[11\] should be reevaluated for nuclei with large excess of neutrons, such as $^{197}$Au. Thus we do not include in Fig.14 the values of $a_2(197)$ estimated in Ref.\[11\].
nucleus such as $^3He$. However in the $^4He$ case the estimate clearly fails for $a_3$ since one cannot use the mean field approximation for estimating correlation of three out of four nucleons.

![Graph](image)

Figure 14: The $A$ dependence of $a_2$ and $a_3$ calculated based on Eq.(24) and compared with the data from Refs.11,13,14. Both calculations and data are normalized to the corresponding values of $a_2(C)$ and $a_3(c)$.

It is worth mentioning that $A$ dependence of $a_2(A)$ obtained above is significantly slower than the $A$-dependence one would infer from the experimentally measured number of quasideuteron pairs in the nucleus, $\frac{L_{NZ}}{A}$, as it is determined from nuclear photoabsorption reaction at $E_\gamma \sim 100MeV$. Here $L$ is the Levinger factor. Also, the absolute value of $a_2(C)$ which follows from the value of Levinger factor for Carbon is by factor of two smaller than the one which follows from high energy data.

The extracted values of $a_2(A)$ allow us to estimate the absolute magnitude of high momentum component of nuclear wave function using information about the high momentum component of the deuteron wave function. For realistic deuteron wave functions the probability per nucleon to have momentum above 300 MeV/c is about $4 \div 5\%$. Hence the above estimates of $a_2(A)$ corresponds to the probability to find a nucleon with momentum $k \geq 300 MeV/c$, say in the iron, of $\sim 20 \div 25\%$. 

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3.2 Space-time structure of high $Q^2$ and $x > 1$ quasielastic scattering

For a more formal analysis of the process it is convenient to start with consideration of the expression for the cross section as a Fourier transform of the commutator of electromagnetic currents $J_\mu$ between wave functions of the nucleus in its rest frame:

$$2m_Aq_3\sigma^{(r)} = \int e^{iqy} \langle A | [J_\mu(y), J_\lambda(0)] | A \rangle \epsilon^{(r)}_\mu \epsilon^{(r)}_\lambda d^4y.$$  \hspace{1cm} (25)

where $q_3 = \sqrt{Q^2 + Q^4/4m^2x^2}$, and $\epsilon^{(r)}$ is the polarization vector of the virtual photon. Strong oscillations in the exponential lead to the condition that in the discussed kinematic range

$$y_+ \sim \frac{1}{q_-} < 1 \text{ fm}, \quad y_- \sim \frac{1}{q_+} < 0.2 \text{ fm},$$  \hspace{1cm} (26)

where we introduced the LC variables $y_\pm = y_0 \pm y_z$. Also it follows from causality (i.e. from condition that commutator of electromagnetic currents is 0 for space-like intervals) that $y_i^2 \leq 1/Q^2$. To reach this we use an approximation in which nucleons are point-like and nucleon structure is accounted for in terms of form factors. Account of meson currents leads to a nonlocality of electromagnetic current at $Q^2 = 0$ and restricts the size of the probed region to the radius of a nucleon. Challenging question is how this nonlocality depends on $Q^2$. The analysis of quark models of a nucleon shows that a selection of $Q^2 \approx \text{few GeV}^2$ squeezes an effective size of the nucleon \cite{39}. It is quite difficult to observe this phenomenon directly because of an expansion of small size configurations \cite{40} after it was produced in a hard subprocess.

3.3 SRCs and Final State Interaction

From the general considerations of Sec.3.2 one observes that the interaction between the knock out nucleon and residual system which may contribute to the total cross section in the kinematics of Eq.(19) is dominated by distances less than about 1 fm which corresponds to the interaction within the SRC and therefore it is canceled in the ratios like Eq.(23).

This conclusion can be reinforced by considering the rescattering diagram (Fig.15) and treating it as a Feynman diagram. In this case one can calculate the virtuality of struck nucleon at the intermediate state (before the FSI blob). We find \cite{11} that if the momentum of struck nucleon $\mathbf{p}_1$ is significantly different from the momentum, $\mathbf{p}_i = \mathbf{p}_f - \mathbf{q}$, corresponding to the initial momentum in impulse approximation, then
the virtuality of struck nucleon becomes large. Indeed on can estimate the virtuality of struck nucleon as follows:

\[ \Delta M^2 \equiv m^2 - (p_1 + q)^2 \approx m^2 + Q^2 (1 - \frac{1}{x}) - \tilde{m}^2, \]  

(27)

where \( \tilde{m}^2 = p_1^2 = (p_A - p'_{A-1})^2 \) and \( p'_{A-1} \) is the four momentum of the recoil nucleus at the intermediate state (see Fig.15). For small momenta of the initial nucleon, \( p_1 \approx 0 \) \( (m^2 - \tilde{m}^2 \sim 0) \) the virtuality grows linearly with \( Q^2 \) at fixed \( x \neq 1 \). It grows also with \( x \) moving away from \( x = 1 \). Thus for kinematics of Eq.(19) the rescattering amplitudes of Fig.15 for a struck nucleon with small initial momenta are suppressed due to large virtuality of the struck nucleon in the intermediate state. For example, for \( x = 1.5 \) and \( Q^2 = 2 \text{ GeV}^2 \) virtuality is \( \sim 1 \text{ GeV}^2 \).

To understand what physical phenomena cause this suppression, it is convenient to represent the FSI amplitude of Fig.15 within noncovariant theory in which time-ordering is explicitly present which allows to consider space-time evolution of the process.\(^{11}\) In this case the FSI amplitude can be represented as follows:

\[ A^{FSI,\mu}(eA \rightarrow eX) \sim \int d^3p_1 \psi_A(p_1) J_{em}^\mu(p_1, q) \frac{1}{\Delta E + i\epsilon} t_N(p_1 + q, p_f), \]  

(28)

where \( J_{em}^\mu \) is the electromagnetic current, \( t_N \) represents the rescattering amplitude of struck nucleon and \( \Delta E \) is the energy difference between intermediate and initial states:

\[ \Delta E = -q_0 - M_A + \sqrt{m^2 + (q + p_1)^2} + \sqrt{M_{A-1}^2 + p_{1}^2}. \]  

(29)

\(^{11}\)Note that relativistic effects in this case can be included within light-cone non-covariant theory.
Within this representation we can estimate the characteristic distances that struck nucleon propagates as:

\[ r \approx \frac{v}{\Delta E}, \quad (30) \]

where \( v \) is the velocity of the struck nucleon in the intermediate state. Averaging momentum \( p_1 \) over the \( p_1 \leq p_F \) region and using realistic nucleon momentum distribution one obtains the characteristic length, for \( x > 1 \) kinematics in which struck nucleon with small initial momentum propagates before it rescatters with nucleons from the residual nucleus. For \(^{27}\text{Al}\) target the \( Q^2 \) dependence of \( r \) for different values of \( x \) are presented in Fig. 16.

![Figure 16: Characteristic distance from \( \gamma^*N \) interaction point which struck nucleon (having \( p_1 \leq p_{\text{Fermi}} \)) can propagate before reinteraction that contributes to the total cross section of \( \gamma^*A \) scattering.](image)

This estimate demonstrates that FSI in kinematics of Eq. [10] overwhelmingly takes place at distances which are within SRC. Additionally, since the rescattering amplitude, \( t_N \) for \( p_1 \sim 0 \) is highly off-energy shell (this is equivalent to large virtuality of interacting nucleon in the covariant formalism) it is strongly suppressed as compared to the on-shell amplitude.
3.4 FSI in SRC and Generalized Eikonal Approximation

The distances relevant for FSI can be investigated in more detail by considering FSI in the eikonal approximation \[42, 44, 43, 46, 45, 34, 47, 48\].

We now introduce an additional restriction to the kinematics of Eq. (19) to ensure that the momentum of knocked-out nucleon, as it emerges from SRC, is large enough \((p_f \geq 500 \text{ MeV}/c)\) relative to all nucleons, including those in the SRC, so that the eikonal approximation can be applied for evaluation of final state reinteractions.\[12\]

Hence we require that

\[
W_{NN} - 2m_N = \sqrt{4m_N^2 + \frac{Q^2(2 - x)}{x}} - 2m_N \geq 60 \text{ MeV.} \tag{31}
\]

It is worth noting that to satisfy unitarity one should include both elastic and inelastic rescattering of knocked-out nucleon with spectator nucleons in the nucleus. This is technically very difficult to realize within eikonal approximation at finite energies (see e.g. Ref. \[41\]). However the unitarity condition will be automatically fulfilled if we calculate the inclusive \(A(e,e')X\) cross section through the imaginary part of the amplitude of virtual Compton scattering off the nuclei at forward direction as in Fig.17.

\[\text{Figure 17: Imaginary part of } \gamma^* A \text{ forward Compton scattering amplitude defines the nuclear structure function of inclusive } A(e,e')X \text{ scattering.}\]

In this case the nuclear matrix of inclusive scattering can be expressed through the forward Compton scattering amplitude as follows:

\[\text{For simplicity we discuss here } x < 2 \text{ kinematics where scattering off two nucleon correlation dominates, though our consideration can be easily extended to the case of scattering off } N > 2 \text{ SRC's.}\]

\[\text{If one would try to apply the Glauber theory for description of the cross section of inclusive processes by including only elastic reinteractions, the unitarity would be violated resulting in a strong overestimate of the role of the FSI.}\]
\[ W_A^{\mu
u}(Q^2, x) = \frac{1}{2\pi M_A} \text{Im} A^{\gamma^* A \rightarrow \gamma^* A}(P'_A = P_A). \] (32)

Let us now estimate the imaginary part of the forward Compton scattering amplitude within virtual nucleon approximation, in which the electromagnetic interaction takes place off the virtual nucleon and the motion of which is described by nonrelativistic nuclear wave function.

**Impulse approximation:** We first evaluate the impulse approximation part of the forward compton scattering amplitude at high \( Q^2 \) and \( x > 1 \). One can calculate contribution of the diagram of Fig.17a by applying effective Feynman diagrammatic rules for high energy electro-nuclear reactions (see e.g. Ref. [43]) and then performing nonrelativistic reduction procedure, to relate \((\text{nucleus}, A) \rightarrow (A \times \text{nucleons})\) transition to the nonrelativistic nuclear wave function. We obtain:

\[
\text{Im} A_0^{\gamma^* A \rightarrow \gamma^* A}(P'_A = P_A) = -A \sum_{\text{Em}} \sum_{s_1, s'_1, s_f} \int \Psi^*_A(p_1, s_1; p_2, \ldots; p_A) \times \\
J_{eN}^{\mu\nu}(p_f, s_f, p_1, s_1) \frac{1}{p_f^2 - m^2_N + i\epsilon} J_{en}^{\nu}(p_f, s_f, p_1, s'_1) \Psi_A(p_1, s'_1; p_2; \ldots; p_A) \times \\
d^3 p_i \prod_{j=2}^A d^3 p_j \delta^3 \left( \sum_{i=1}^A p_i \right),
\] (33)

where \( \sum_{\text{Em}} \) accounts for the sum and integration over the excitation energy of recoil nuclear system.

To see how the process at high \( Q^2 \) and \( x > 1 \) evolves in the space and time we transform Eq.(33) to the coordinate representation. To simplify following discussions we consider the kinematic limit:

\[ q_0 \gg E_m, \quad x \gg \frac{p_{12}^2}{2m^2_N}, \] (34)

in which case

\[ p_f^2 - m^2_N + i\epsilon = (p_1 + q)^2 - m^2_N + i\epsilon = 2q(p_{iz} - p_{1z} + i\epsilon), \] (35)

where

\[ p_{iz} = \frac{2q_0(m_N - E_R) - Q^2}{2q}, \] (36)

and \( E_R \) represents the total kinetic energy of the recoil system as it was defined in Sec.2.
Using
\[ \Psi_A(p_1, p_2, \ldots, p_A) = \frac{1}{((2\pi)^{3/2})^A} \int \Psi_A(r_1, r_2, \ldots, r_A) \prod_{i=1}^A e^{-ip_i r_i} d^3 r_i \delta^3(\sum r_i - r_A), \] (37)
and
\[ \frac{1}{p_{iz} - p_{iz} + i\epsilon} = -i \int \Theta(z_0) e^{i(p_{iz} - p_{iz})} dz_0, \] (38)
we can rewrite Eq.(33) in the coordinate space representation as:
\[ \mathcal{I} m A^{\gamma^* A \to \gamma^ A}(P'_A - P^A) = \frac{A}{2q} \sum_{E_m \text{ spins}} \sum J_{\mu N}^\mu(Q^2) J_{\nu N}^\nu(Q^2) \times \]
\[ \sum_{E_m} \int \Psi_A^\dagger(b_1, z_1; r_2; \ldots; r_A) \Theta(z_1 - z'_1) e^{ip_{iz}(z_1 - z'_1)} \Psi_A(b_1, z'_1; r_2; \ldots; r_A) \times \]
\[ dz_1 dz'_1 d^2 b_1 \prod_{j=2}^A d^3 r_j. \] (39)

The above expression allows a rather transparent interpretation of the space-time evolution of the inclusive process. First, the \( \Theta \) function enforces the condition that in longitudinal direction absorption of virtual photon happens before its emission. Secondly, one can see that at large \( p_{iz} \), the longitudinal distance which knocked-out nucleon propagates is proportional to \( \sim 1/p_{iz} \). Thus one observes that starting at \( p_{iz} \geq 300 \text{ MeV/c} \) the struck nucleon propagates the distances of \( \leq 1 \text{ Fm} \). This estimate is in accordance with that of Sec.3.2.

**Final State Interactions:** We now discuss the diagrams of Fig.17b representing final state interactions. In the \( W_{NN} \gg 2m_N \) limit the imaginary part of the sum of all rescattering diagrams in Fig.17b cancels out due to the closure condition for the intermediate states. However it follows from Eq.(31) that for \( x > 1 \) the mass of the produced two nucleon system, \( W_{NN} \), is close to \( 2m_N \) and condition for application of closure is not satisfied. Thus in this situation the explicit evaluation of the rescattering part of the Compton scattering amplitude is required.

We evaluate rescattering diagrams within the Generalized Eikonal approximation (GEA) \[ \text{[44, 43, 45]} \] which uses the set of effective Feynman diagram rules to calculate a given \( n^{th} \)-order rescattering of the struck-nucleon off the spectator nucleons in nuclei.

Within GEA nuclear Compton scattering amplitude corresponding to the \( n \)-fold rescattering of energetic knocked-out nucleon off the spectator nucleons can be ex-
pressed as follows:

\[ \text{Im} A_n^{\gamma^*A\rightarrow\gamma^A}(P'_A = P_A) = A \cdot \text{Im} \sum_{E_m} \sum_{\text{spans perm}} \int \Psi_A^i(p_1; ...; p_t; ...; p_{t+n-1}; ... P_A) \]

\[ \frac{J_{eN}^{i,t}(p_f, s_f, p_1, s_1)}{2q(p_{iz} - p_{isz} + \Delta + i\epsilon)} \prod_{j=1}^n \xi(s_j) f_{NN}(p_{1,j}^+, -p_{1,j-1}^+) \, d^3p_{1,j} \]

\[ J_{eN}^{i}(p_f, s_f, p_{1}, s_1) \Psi_A(p_{1}; ...; p'_i; ...; p'_{t+n-1}; ... P_A) d^3p_1 \prod_{k=1}^A d^3p_k \delta^3 \left( \sum_{i=1}^A \delta_i \right). \quad (40) \]

where \( \Delta \sim \Delta_j \approx \frac{2q}{q} E_m \), \( p_{t+j} = p_{t+j} - (p_{1,j} - p_{1,j-1}) \) and \( \xi = \frac{\sqrt{s_j(s_j - 4m_N^2)}}{2qm_N} \), where \( s_j \) is the c.m. invariant energy of two nucleons for the \( j \)th rescattering.

We transform the above equation to coordinate representation estimating all propagators at their pole values. Again considering the kinematical limit of Eq.(34) and using coordinate representations of Eqs.(37,38), after several straightforward steps we arrive at:

\[ \text{Im} A_n^{\gamma^*A\rightarrow\gamma^A}(P'_A - P_A) = A \frac{2q}{2q} \sum_{E_m} \sum_{\text{spans perm}} J_{eN}^u(Q^2)J_{eN}^v(Q^2) \sum_{l_1 \neq l_2 \ldots \neq l_n = 2} \]

\[ \int \Psi_A^i(b_1, z_1; r_2; ...; r_{l_1}; ...; r_{l_n}; ... r_A) \prod_{i=1}^n \Theta(z_1 - z_{l_i}) \text{Im} \Gamma_{NN}(b_i - b_{l_i}) \Theta(z_{l_i} - z'_i) \]

\[ e^{ip_x(z_1 - z'_1)} \Psi_A^i(b_1, z'_1; r_2; ...; r_{l_1}; ...; r_{l_n}; ... r_A) dz_1 dz'_1 d^3r_j. \quad (41) \]

where

\[ \text{Im} \Gamma_{NN}(b) = -\frac{1}{2} \int \text{Im} f_{NN}(k_{\perp}) e^{-ik_{\perp}b} \frac{q^2 k_{\perp}}{(2\pi)^2}. \quad (42) \]

Eq.(41) shows clearly how the scattering develops in the coordinate space. First of all the reinteractions take place over the longitudinal distances confined between the points of absorption \( (z_1) \) and emission \( (z'_1) \) of the virtual photon, this is enforced by the product of two \( \Theta \) functions: \( \Theta(z_1 - z_{l_1})\Theta(z_{l_1} - z'_1) \). Secondly, due to the exponential factor \( e^{ip_x(z_1 - z'_1)} \), the overall longitudinal distance is limited by the longitudinal momentum probed in the reaction - in the same way as it was for the case of impulse approximation. That is \( z_1 - z'_1 \sim \frac{1}{p_{iz}} \).

According to this result starting at \( |p_{iz}| \geq 300 \text{ MeV/c} \) the rescatterings will be confined at longitudinal distances of 1 Fm. In such case the first rescattering will take place predominantly with a nucleon in SRC. Contributions from double and higher
order rescatterings become numerically negligible since it is rather improbable to find two and more nucleons at longitudinal distances $\leq 1$ Fm from the struck nucleon. The latter observation reinforces the expectation that the higher order rescatterings play negligible role in Eq.(23) as soon as eikonal regime is established for final state reinteraction processes at $x > 1$ inclusive kinematics.

Large value of FSI and the violation of Eq.(23) were suggested in Ref. [49] in which dominant FSI effects have been generated at $x \geq 1$ due to electron scattering off almost stationary nucleon within a nucleus. However within eikonal approximation such rescattering corresponds to the non-pole contribution in the rescattering amplitude, such as Eq.(40), which has only real part once the unitarity of eikonal amplitude is restored (see above) and therefore does not contribute to the inclusive cross section.

3.5 FSI in near threshold kinematics

The above discussed picture of FSI for 2N SRC kinematics gradually changes with an increase of $x$ if the kinematic threshold is approached. For example if we consider the $x \rightarrow 2$ limit for the scattering off the deuteron for fixed $Q^2$, the final state mass of NN system is close to that of the deuteron. In this case, similar to the case of deuteron form factor one gets comparable contributions from small relative momenta in the initial and final states. In the nonrelativistic calculations [50] for $Q^2 \sim few \, GeV^2$ this contribution was found to be important for $W - m_D \leq 50 MeV$. For heavier nuclei this effect for $x \sim 2$ is washed out by the motion of the nucleon pair in the mean field and by the contributions of three nucleon correlations.

3.6 Measuring light cone momentum distribution of nucleons in nuclei

The above discussions allow us to conclude that for kinematics of Eq.(19) when additional condition for FSI being in the eikonal regime, (Eq.(31) is satisfied, the FSI is predominantly confined within SRC and as a result it should mostly cancel out in the ratios of Eq.(23).

3.6.1 The $\alpha$ distribution and FSI

Next question which we would like to address is whether in addition to observing the onset of SRC at $x > 1$ kinematics through the ratio of Eq.(23) one can extract information about SRC which is less affected by the FSI.
For this we discuss several important advantages we gain by using light-cone momentum fraction of interacting nucleon, $\alpha_i$, as it is defined in Eq. (2).

Using $\alpha_i$ we can rewrite the denominator of knocked-out nucleon’s propagator, which enters in Eq. (33), in the following form:

$$p^2_f - m^2_N + i\epsilon = mq_+ \left( \alpha_i - \frac{Q^2}{mq_+} + \frac{q_-}{m_Nq_+}(M_A - p_{R+}) + \frac{m_i^2 - m_N^2}{q_+m_N} + i\epsilon \right), \quad (43)$$

where we use energy and momentum conservation: $p^\mu_f = q^\mu + \rho^\mu_A - \rho^\mu_R$ and define $q_\pm = q_0 \pm q$ and $m_i^2 = (P_A - P_R)^2$. In Eq. (43) $p_{R+}$ represents the "+" component of the recoil nucleus four-momentum and it is a light-cone analog of the recoil energy $E_R$ (discussed in Sec. 2) which characterizes the total kinetic energy of recoil system in the nuclear lab frame.14

It is instructive to compare Eqs. (35,36) with Eq. (43). It follows from Eqs. (35,36) that the fast nucleon propagator depends on $E_R$ in large momentum transfer limit $q_0 \sim q$. As a result it cannot be factored out of the sum over the recoil system’s excitations.

However situation is quite different for representation based on the light-cone variables. It follows from Eq. (43) that in the limit, $Q^2 \to \infty, x = \text{const}$,

$$q_+ \gg q_- \quad \text{and} \quad q_+m_N \gg (m_i^2 - m_N^2).$$

As a result, the denominator is practically independent of the excitation energy of residual nucleus, $p_{R+}$. Consequently, $\alpha_i$ is factored out from the sum - $\sum_{p_{R+}}$ (which replaces $\sum E_R$ in Eq. (43) in the light cone representation) with $\alpha_i \approx \frac{Q^2}{m_Nq_+}$, which depends only on $x$ and $Q^2$.

The light cone factorization considerably simplifies the expression for the inclusive cross section. Using the correspondence relation in nonrelativistic limit between nonrelativistic and light-cone wave functions of the nucleus (see Eq. (3)), and above discussed factorization of the knocked-out nucleon’s propagator out of the integral of Eq. (33) for forward Compton scattering amplitude in impulse approximation one obtains:

$$\text{Im} A_{\gamma A}^{*\gamma A}(P_A = P_A) = \frac{\pi A}{q_+} W_{N}^\mu\nu(Q^2, x, \alpha_i) \cdot \rho_A(\alpha_i), \quad (44)$$

where $W_{N}^\mu\nu \sim J_{N}^{\mu\dagger} J_{N}^\nu$ represents the electromagnetic tensor of nucleonic currents and $\rho_A(\alpha)$ represents light-cone density matrix of nucleus integrated over the transverse

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14In light cone representation the sum over $E_R$ in the closure relation for $n_A(k)$ is replaced by the sum over $p_{R+}$ in the closure relation for $\rho_A^\dagger(\alpha, p_t)$. 

36
momentum of the nucleon which is defined as:

\[
\rho_A(\alpha) = \sum_{PR+\text{ spins}} \sum_{A} \int |\Psi_A(\alpha_1, p_{1t}; \alpha_2, p_{2t}; ... \alpha_A, p_{At})|^2 \times \\
\delta(\alpha - \alpha_1) \prod_{j=1}^{A} \frac{d\alpha_j}{\alpha_j} d^2 p_{jt} \delta(\sum_k \alpha_k - A) \delta(\sum_k p_{kt}). \tag{45}
\]

According to this result, within the impulse approximation, in high \(Q^2\) and \(x \geq 1\) kinematics, the inclusive \(eA\) scattering probes the light-cone density matrix, \(\rho_A(\alpha)\) of the nucleus.

We are in position now to analyze how the final state interaction alters this factorization. The key point here is that in eikonal approximation the propagator of fast rescattering nucleon is approximately independent of the excitation energy of the recoil system.

This can be seen from Eq.(40) in which the denominator of the fast nucleon propagator can be rewritten in the following form

\[
p_{iz0} - p_{iz} + \Delta + i\epsilon = m[\alpha_1 - \alpha_i + \frac{q_0 - q_3}{q_3 m} E_R + i\epsilon] \approx m[\alpha_1 - \alpha_i - \frac{Q^2 E_R}{2q_3^2 m} + i\epsilon], \tag{46}
\]

where \(\alpha_1 = \frac{p_{iz0}}{E_R} \) and \(E_R = m + E_{A-1} - M_A\). It follows from Eqs.(46,40) that in the limit \(\frac{Q^2 E_R}{2q_3^2 m} \ll 1\) the rescattering part of the amplitude is independent of the excitation energy of recoil nucleus. Hence using relation between nonrelativistic and light-cone nuclear wave functions (Eq.(3)) we obtain:

\[
\mathcal{I} m A_{(1)}^\gamma A \rightarrow \gamma A(P_A = P_A) = A \cdot \mathcal{I} m \sum_{PR+\text{ spins}} \sum_{i=2}^{A} \int \\
\Psi_A^\dagger(\alpha_1, p_{1t}; \alpha_2, p_{2t}; ... \alpha_i, p_{it}; ... \alpha_A, p_{At}) J_{en}^l(\alpha_f, p_{ft}, s_f, \alpha_1, p_{1t}, s_1) \\
\sqrt{s_{NN}(s_{NN} - 4m_N^2)} \frac{f_{NN}(p_{1t} - p_{1t})}{(\alpha_1 - \alpha_i + i\epsilon)(\alpha'_1 - \alpha_i + i\epsilon)} J_{en}^l(\alpha'_1, p'_{1t}, s_f, \alpha', p'_{1t}, s'_1) \\
\Psi_A(\alpha'_1, p'_{1t}; \alpha_2, p_{2t}; ... \alpha', p'_{it}; \alpha_A, p_{At}) d\alpha_1, d^2 p_{bt} \frac{d\alpha'_1, d^2 p'_{1t}}{(2\pi)^3} \\
\prod_{j=2}^{A} \frac{d\alpha_j}{\alpha_j} d^2 p_{jt} \delta(\sum_k \alpha_k - A) \delta^2(\sum_k p_{kt}). \tag{47}
\]

The imaginary part of the above expression is estimated through the imaginary part of \(NN\) scattering amplitude, \(f_{NN}\) and the pole values of the propagators. The latter
yields the conservation of light-cone momentum fraction of the struck nucleon, i.e. \( \alpha_i = \alpha_1 = \alpha'_1 \). If factorization of electromagnetic current is assumed then the overall effect of FSI can be represented through the correction part to the light cone density matrix\(^{15}\) in such a way that expression (44) will still describe the inclusive cross section replaced with modified density matrix:

\[
\mathcal{I}m A^{\gamma^* A \rightarrow \gamma^* A}(P'_A = P_A) = \frac{\pi A}{m q_+} W_{N}^{\mu, \nu}(Q^2, x, \alpha) \cdot \rho^{DWIA}_A(\alpha_i)
\]

where

\[
\rho^{DWIA}_A(\alpha) = \rho_A(\alpha) + \Delta \rho_A(\alpha)
\]

with

\[
\Delta \rho(\alpha) = -\frac{1}{4} \sum_{p_{R,s},\text{spins}} \sum_{l=2}^A \int \Psi_A^\dagger(\alpha_1, p_{1t}; \alpha_2, p_{2t}; ..; \alpha_l, p_{lt}; ..; \alpha_A, p_{At}) \times \\
\mathcal{I}m f_{NN}(p'_{1,\bot} - p_{1,\bot}) \Psi_A(\alpha_1, p'_{1t}; \alpha_2, p_{2t}; ..; \alpha', p'_{lt};..; \alpha_A, p_{At}) \times \\
d^2 p_{lt} \frac{d^2 p'_{1t}}{(2\pi)^2} \prod_{j=2}^A \frac{d\alpha_j}{\alpha_j} d^2 p_{jt} \delta(\sum_{k=1}^A \alpha_k - A) \delta^2(\sum_k p_{kt}).
\]

In this case the inclusive cross section can be represented as follows:

\[
\frac{d\sigma^{eA}}{dE_e d\Omega_{el}} = K \bar{\sigma}_{eN} \rho^{DWIA}_A(\alpha_i),
\]

where \( K \) contains kinematic factors. Eq.(51) leads again to the scaling prediction in the form

\[
\frac{d\sigma^{eA}}{dE_e d\Omega_{el}} = \bar{\rho}^{DWIA}_A(\alpha_i).
\]

Thus in high energy limit the dependence of the cross section on the nucleus structure is reduced to the dependence on modified LC nuclear density matrix which is a function of one variable, \( \alpha_i \). This result leads to a prediction of scaling law which is an analogue of the Bjorken scaling in deep inelastic scattering for the case of elastic scattering off nucleonic constituents in nucleus.

The structure of the rescattering term is similar to that in the deuteron break up at same \( x \) and \( Q^2 \). This allows us to estimate \( \Delta \rho(\alpha)/\rho(\alpha) \leq 2\% \) in the region of sufficiently large \( W_{NN} \) for which eikonal approximation is applicable. A detailed numerical study of this effect will be presented elsewhere.

\(^{15}\) This approximation is generally referred as Distorted wave impulse approximation
3.7 Light-cone scaling of the ratios

We demonstrated above that in kinematics of Eq. (19) FSI is predominantly acting within the SRC and therefore cancels out in the ratios of the cross sections at fixed $x$, and $Q^2$. However it follows from the discussion in section 3.1 that the same $x$ corresponds to different struck nucleon momenta for different $Q^2$. Since in the large $Q^2$ limit $x \to \alpha_i$ and cross section does not depend on the other three components of nucleon momentum (or equivalently $p_R$) it is natural to consider ratios for the same $\alpha_i$ (see Eq. (52)). In the region of $x < 2$ where scattering off two nucleon correlation is allowed we observe that the dominant contribution comes from configurations with recoil energy similar to that in the deuteron (see Sec. 2.2). The motion of the pair leads to smearing of the recoil energy distribution. However the maximum of the distribution over $\tilde{m}^2$ is close to the value of $\tilde{m}^2$ for the deuteron for the same $\alpha_i$. Moreover we checked in Ref. [11] that variation of $\tilde{m}^2$ around the deuteron value leads to a small variation of $\alpha_i$. Accordingly, one can integrate over the $p_{R,+}$, and $p_{i,t}$ leading to conclusion that for $x < 2$,

$$\frac{2\sigma_{eA}(x,Q^2)}{A\sigma_{e^2H}(x,Q^2)} = \frac{\rho_A(\alpha_{2N})}{\rho_{2H}(\alpha_{2N})},$$

(53)

where

$$\alpha_{2N} = 2 - \frac{q_- + 2m}{2m} \left(1 + \sqrt{\frac{W^2 - 4m^2}{W}}\right)$$

(54)

is the minimal value of $\alpha_i$ for the scattering off the deuteron for kinematics of Eq. (19). Here $W^2 = 4m^2 + Q^2(2/x - 1)$ is the invariant mass squared for the electron scattering off the deuteron in the kinematics of Eq. (19). Obviously, the relation of Eq. (53) will be violated at large $Q^2$ due to the contribution of inelastic processes (see Fig. 7 in Ref [11]). This relation predicts that even though the ratios of cross sections plotted as a function of $x$ somewhat change with $Q^2$, they should yield a much better scaling if plotted as a function of $\alpha_{2N}$. Indeed the SLAC data are consistent with this prediction, (see Fig.[18]). One can see from the figure that the data cover the region $\alpha \leq 1.55$ corresponding to the internal nucleon momenta $\leq 0.6$ GeV/c. For larger $Q^2$ and $x$ sufficiently close to 2 corresponding to the minimal values of $\alpha_{2N}$ being close to two, a new pattern may be expected. This is related to the observation of Ref. [6] that 3N SRCs appear to become important for $\alpha \geq 1.6$. Therefore when this kinematics is reached one expects that the increase of the ratio and onset of the new plateau should start below $x = 2$.

One can also define typical $\alpha_{3N}$ for scattering off a three nucleon correlation. It is rather insensitive in a wide $Q^2$ and $x$ range to the value of the recoil mass of the
two nucleon system. Hence it will be interesting to check at which values of \( x \) the scaling of the ratios as a function of \( \alpha_{3N} \) will set in for \( x > 2 \). At the same time it is worth emphasizing that for the region of \( \alpha_{3N} < 2 \) and \( x > 2 \) the cross section is not related directly to the LC density matrix but to the integral of the spectral function for approximately constant \( \alpha_{3N} \) over a restricted range of the recoil masses. These masses will exclude values typical for two nucleon correlations and therefore will not allow a closure approximation. A comparison of the \( x > 2 \) and \( 1 < x < 2 \) data for \( \alpha_{2N} = \alpha_{3N} \) would provide a unique information about relative importance of different contributions to the spectral function for \( \alpha_i \sim 1.5 \div 1.7 \). Such experiments are planned in JLab though the current plans for 6 GeV do not cover sufficiently high values of \( Q^2 \).  

Figure 18: The \( x \) and \( \alpha_{2n} \) dependence of the ratio \( R = \frac{2 \sigma_5}{\sigma_6} \) for different values of \( Q^2 = 1.2 \div 2.9 \) GeV\(^2\).  

4 Breaking SRCs in Hard Semi-Exclusive Reactions

We explained in Sec.2 that a removal of a nucleon from type 2N-I SRC leads to an emission of a nucleon in the direction opposite to the direction of initial momentum of struck nucleon. Therefore if projectile removes say a proton from 2N SRC the decay function mechanism leads to an emission of a neutron with momentum distribution
proportional to the square of the deuteron wave function at high momenta. This mechanism suggested in Ref. [5] allowed to explain the shape of the spectrum of fast nucleons emitted backward in reactions of Eq.(4) and led to the value of \( a_2(C) \approx 4/5 \) which is very close to the one obtained from \( A(e, e')X \) reactions at \( Q^2 \geq 1.5 \text{ GeV}^2 \) and \( x > 1 \) described in Sec.3.

A limitation of the reactions listed in Eq.(4) is that due to their inclusive nature it is impossible to fix the momentum of the nucleon (nucleons) which was removed in the process of producing fast backward nucleon. It can be partially alleviated by studying fast backward production in deep inelastic scattering in which case predominance of the scattering from forward moving nucleons leads to a reduction of average Bjorken \( x \) proportional to the light cone fraction of this nucleon[5]. This effect is observed in several neutrino bubble chamber experiments, (see e.g. Refs. [8] and [9]), although the production of fast backward nucleons due to hadronic reinteractions leads to a reduction of this effect.

Much more stringent tests of the structure of SRC can be performed in coincidence experiments in which a nucleon is removed from the nucleus with known momentum, and the second nucleon from the decay of the SRC is detected \[10, 51\]. An example of such process is the reaction, \( p + A \rightarrow p + p + (A - 1)^* \), in which both protons are detected at large c.m. angles \[16\].

For the purposes of studies of SRC it is sufficient to reach the range of large energy momentum transfers corresponding to \( E_p \geq 5 \text{ GeV} \) in which case color transparency effects appear to be small. Detection of two forward protons allows to determine with high precision the light-cone fraction of the interacting nucleon, \( \alpha_i \), and therefore to measure the light-cone density matrix of the nucleus. The analysis of Ref. [54] of EVA data\[15\] on \( \alpha_i \) distribution found that the data agree well with calculation which includes SRC with the strength determined from \( A(e, e')X \) data at \( Q^2 \geq 1.5 \text{ GeV}^2 \) and \( x > 1 \).

Since the cross section of elementary \( pp \rightarrow pp \) reaction decreases very rapidly with an increase of the energy:

\[
\frac{d\sigma_{pp \rightarrow pp}}{d\theta_{c.m.}} \propto s_{pp}^{-10},
\]

for \( \theta_{c.m.} \sim 90^o \), the scattering preferentially occurs off the forward moving proton which have \( \alpha_i < 1 \) since in this case \( s_{\pi}^{i} \approx \alpha_i s_{pp} < s_{pp} \).

If a forward moving proton \( (\alpha_i) \) belonging to a type 2N-I SRC is removed from the nucleus, correlated nucleon from the decay of SRC should be emitted backward.

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\[16\]Initially, the interest in experimental study of this reaction was prompted by suggestion of Refs. [53] and [52] that these processes can be used for studies of color transparency phenomenon.
Indeed, for the case of 2N SRC the light-cone momentum fraction for the correlated spectator is: $\alpha_s \approx 2 - \alpha_i > 1$. Expressing $\alpha_s$ through the lab frame energy $E_s$ and z-component of momentum $p_s$: $\alpha_s = \frac{E_s - p_{sz}}{m_N}$, one observes that condition $\alpha_s > 1$ indeed corresponds to a production of nucleon in the backward hemisphere ($p_{sz} < 0$). Based on this observation it was predicted\cite{10, 51} that in the process of $p + A \rightarrow p + p + (A - 1)^*$ there should be a strong correlation between knock-out of the fast forward proton and emission of a fast backward nucleons, mostly neutrons.

The BNL experiment E850 (EVA)\cite{16} was supplemented with neutron detectors which covered a large fraction of the backward angles. The experiment discovered \cite{15, 16} that in the process of $p + ^{12}C \rightarrow p + p + n + X$, a removal of a proton with momentum $|p_i| > k_F = 220 \text{ MeV/c}$ (the Fermi momentum for carbon) produces strong back-to-back directional correlation between $p_i$ and momentum of the neutron, $p_n$ (Fig.19).

![Figure 19: The correlation between $p_n$ and its direction $\gamma$ relative to $\vec{p}_i$. Data labeled by 94 and 98 are from Refs.15 and 16 respectively. The momenta on the labels are the beam momenta. The dotted vertical line corresponds to $k_F = 220 \text{ MeV/c.}]


The experiment[16] extracted the following quantity:

\[ F = \frac{\text{Number of (p,ppn) events } (p_i, p_n > k_F)}{\text{Number of (p,pp) events } (p_i > k_F)}, \]  

(55)

which represents the measure of correlation of backward neutrons with initial momentum of struck proton. For initial momentum range of 250 – 550 MeV/c experiment extracted \( F = 49 \pm 13\% \), which means about half of the events with \(|p_i| > k_F\) had directionally correlated neutrons with \(|p_n| > k_F\).

Further analysis of these data was performed in Ref. [17], which observed that in high momentum transfer kinematics, above mentioned ratio can be related to the quantity \( P_{pn/pX} \) which represents the relative probability of finding \( pn\)-correlation in the "pX" configuration that contains a proton with \( p_i > k_F \). Such relation reads:

\[ P_{pn/pX} = \frac{F}{T_n R}, \]  

(56)

where \( T_n \) accounts for the attenuation of the neutron in the nucleus and

\[ R \equiv \frac{\alpha_{i_{\max}}^{\alpha_{n_{\max}}} \rho_{i_{\max}}^{\rho_{n_{max}}} \int \int \int \int D_{pn}(\alpha_i, p_{ti}, \alpha_n, p_{nt}, P_{R+}) \frac{d\alpha}{d\alpha} d^2 p_t \frac{d\alpha}{d\alpha} d^2 p_{tn} dP_{R+}}{\alpha_{i_{\min}}^{\alpha_{n_{\min}}} \rho_{i_{\min}}^{\rho_{n_{min}}} \int \int \int \int S_{pn}((\alpha_i, p_{ti}, P_{R+}) \frac{d\alpha}{d\alpha} d^2 p_t dP_{R+}). \]  

(57)

Here \( D_{pn} \) is light-cone generalization of the decay function discussed in Sec.2, corresponding to the situation in which removal of a proton with light-cone momentum \( \alpha_i, p_{ti} \) from the nucleus was followed by an emission of neutron with momentum \( (\alpha_n, p_{nt}) \). The recoil energy is described by \( P_{R+} \) (see discussion in Sec.3). The spectral function \( S_{pn} \) in Eq.(57) represents the part of the spectral function \( S_p \) related to \( pn\)-correlation only and it is related to the decay function by the relation analogous to Eq.(17).

In Ref. [17] decay function was modeled based on 2N SRC model which includes the motion of the center of mass of the correlation in the nucleus mean field. Within this approximation the decay function is represented through the convolution of two density matrices representing relative \( (\rho_{SRC}) \) and center of mass \( (\rho_{cm}) \) momentum distributions as follows:

\[ D_{pn} = \rho_{SRC}^{\rho_{SRC}}(\alpha_{rel}, \bar{p}_{rel}, \rho_{cm}^{\rho_{cm}}(\alpha_{c.m.}, \bar{p}_{c.m.}) \delta \left( P_{R+} - m^2 + p_{tn}^2 \frac{m \alpha_n}{m(A - \alpha_{c.m.})} - \frac{M_{A-2}^2 + p_{c.m.}^2}{m(A - \alpha_{c.m.})} \right), \]  

(58)

43
where $\alpha_{rel} = \frac{\alpha_i - \alpha_n}{\alpha_{c.m.}}$, $p_{t,rel} = p_{ti} - \frac{\alpha}{\alpha_{c.m.}} p_{tn}$, $\alpha_{c.m.} = \alpha_i + \alpha_n$, and $p_{t,c.m.} = p_t + p_{tn}$.

Within 2N-SRC model [6], $\rho_{SRC}^{pn}$ is related to the LC density matrix of the deuteron as:

$$\rho_{SRC}(\alpha, p_t) = a_{pn}(A) \frac{\Psi_D^2(k)}{2 - \alpha} \sqrt{m^2 + k^2},$$

(59)

where $\Psi_D(k)$ is the deuteron wave function, and for $0 < \alpha < 2$

$$k = \sqrt{\frac{m^2 + p_t^2}{\alpha(2 - \alpha)} - m^2}.$$  

(60)

The parameter $a_{pn}(A)$ is the probability (relative to the deuteron) of having a $pn$ SRC pair in nucleus $A$, which is analogous to $a_2$ quantity discussed in Sec.3 but only for proton-neutron correlation.

The c.m. motion of the SRC relative to the $(A - 2)$ spectator system is described by a Gaussian ansatz similar to Ref. [32] with $\sigma$ being a parameter. This distribution can be expressed through the LC momentum of the c.m. of the SRC as follows:

$$\rho_{c.m.}(\alpha, p_t) = 2m \left(\frac{1}{2\pi\sigma^2}\right)^{\frac{3}{2}} e^{-\frac{m^2(2 - \alpha)^2 + p_t^2}{2\sigma^2}}.$$  

(61)

It is normalized as $\int \rho_{c.m.}(\alpha, p_t) \frac{d\alpha}{\alpha} dp_t = 1$. The parameter, $\sigma$ describing the width of the momentum distribution of the c.m. of 2N SRC was determined experimentally: $\sigma = 143 \pm 17$ MeV/c. It was found to be in excellent agreement with the value calculated in Ref. [32] for the carbon spectral function within the 2N-SRC model - $\sigma = 139$ MeV/c.

It was demonstrated that above approximation for the decay function describes major characteristics of the $A(p, 2pn)X$ data [17, 54]. Based on this model of decay function and constraining the integration region of Eq.(57) by kinematical cuts of experiment:

- struck proton: $0.6 < \alpha_i < 1.1$; $p_i > p_{min}^{\text{min}} = 0.275$ MeV/c
- recoil neutron: $0.9 < \alpha_n < 1.4$; $p_{min}^{\text{min}} < p_n < 0.55$
- $72^0 < \theta_n < 132^0$,

(62)

for which the recalculated value of correlation strength was $F = 0.43^{+0.11}_{-0.07}$ it was possible to estimate:

$$P_{pn/pX} = 0.92^{+0.08}_{-0.18}.$$  

(63)
The physical interpretation of this result is that the removal of a proton from the nucleus with initial momentum $275^{-550} \text{ MeV/c}$ is $92^{+8}_{-18}\%$ of the time is accompanied by the emission of a correlated neutron that carries momentum roughly equal and opposite to the initial proton momentum. Using this result, and assuming dominance of 2N SRCs in the high momentum component of the nuclear wave function at $250 < p_i < 550 \text{ MeV/c}$, it is possible to estimate an upper limit for the ratio of absolute probabilities of $pp$ and $pn$ SRCs as\cite{17}:

$$\frac{P_{pp}}{P_{pn}} \leq \frac{1}{2} (1 - \frac{P_{pn/pX}}{P_{pp/X}}) = 0.04^{+0.09}_{-0.04}. \quad (64)$$

This result indicates that probabilities of $pp$ or $nn$ SRCs in carbon are at least by factor of six smaller than that of $pn$ SRCs. This provided the first estimate of the isospin structure of 2N SRCs in nuclei and may have important implication for modeling the equation of state of asymmetric nuclear matter.

Further studies were performed very recently at JLab \cite{18, 19} for electro-nuclear reactions in which case $^{12}\text{C}(e,e'p)X$, $^{12}\text{C}(e,e'pp)X$, and $^{12}\text{C}(e,e'pn)X$ scatterings at $Q^2 = 2 \text{ GeV}^2$ and $x_B = 1.2$ were studied in the range $300 \leq p_i \leq 600 \text{ GeV/c}$. Since for the chosen kinematics the $\Delta$-production is suppressed and $p_i > k_{Fermi}$, the type 2N-I SRC has been predicted to give the dominant contribution to the cross section. Experiment also observed a recoiling partner, proton/neutron, from SRC produced back-to-back to the momentum $p_i$. When interpreted within the same model as used for the $A(p,ppn)X$ reaction it was found that only in $(9.5 \pm 2)\%$ of the $^{12}\text{C}(e,e'p)X$ events, a recoiling nucleons were protons. The ratio of the recoil neutron and recoil proton production cross section was found to be $8.2 \pm 2.2$.

If one considers the ratio of probabilities of emission from the $pp$ pair with respect to $pn$ pair the identity of the protons leads to a factor of two larger emission probability - in the $pp$ case one of the protons is always has a momentum in the forward direction while in the case of the $pn$ correlation this probability is 50%. Therefore for the ratio of the probabilities of $pn$ and $pp$ pairs one finds, $16.4 \pm 4.4$. This has to be corrected for the feeding of $pp$ events from the $pn$ events due to the charge exchange which yield the following final result for ratios of $pp$ to $pn$ SRC probabilities:

$$\frac{P_{pm}}{P_{pp}} = 18.0 \pm 5.0. \quad (65)$$

corresponding to

$$\frac{P_{pp}}{P_{pn}} = 0.056 \pm 0.018, \quad (66)$$

45
The pp/pn ratio was found to be practically constant in the whole studied momentum range: $300 \leq p_N \leq 600 \text{MeV}/c$. Also the sum of the absolute $pp$ and $pn$ probabilities was found to be close to one, indicating that practically all nucleons in this momentum range (with accuracy of the order 10%) belong to two nucleon SRC. The ratio found in Eq. (66) is in a good agreement with the one in Eq. (64). This is certainly not trivial since the mechanism of the reactions in two cases are very different (for example a virtual photon could couple to the exchange currents), and also in the proton projectile case a forward moving proton was struck, while in the electron experiment virtual photon was absorbed by a backward moving proton. In addition the invariant transferred momentum for $(p,2p)$ reaction was $-t \sim 5 \text{GeV}^2$ exceeding by far $Q^2 \sim 2 \text{GeV}^2$ for $(e,e'p)$ process. The observed ratio appears to be somewhat smaller than a naive expectation of a factor of $\frac{1}{9}$ based on the pion exchange.

Note also that in the kinematics of JLab experiment invariant mass of the two nucleon system is rather small, while transverse momenta are significant. As a result the final state rescatterings between two outgoing nucleons of SRC are rather large. However our studies indicate that this does not affect the ratios and overall probabilities discussed above.

To summarize, the study of hard semi-exclusive correlation processes demonstrate high discovery power in probing different aspects of SRCs. Already first experiments\cite{17}–\cite{19} confirm the SRC origin of high momentum nucleons in nuclei. For carbon case SRCs appear to dominate starting at momenta which are close to the Fermi momentum. Most of the SRCs are due to $pn$ correlations. Qualitatively these expectations are consistent with an expectations of potential models of the nucleus. For more quantitative comparisons it would be necessary to perform calculations of the decay functions of nuclei. One of the most important aspects of these studies is that it opens up a new venue in probing the isospin structure of SRCs. Such possibilities are especially important for studies of structure of asymmetric nuclear matter at high densities such as neutron stars.

5 Short-range nucleon correlations and neutrino emission by neutron stars

In this section we will discuss an example which demonstrates how our understanding of SRCs can be used in studies of the properties of cold dense nuclear matter. We will concentrate on the discussion of equilibrium issues of neutron star and related neutrino luminosity.
5.1 Introduction

A normal neutron star is bound by gravitational interactions. Global characteristics of neutron stars follow from the equations for the hydrostatic equilibrium in the general relativity, see Refs. [55, 56]. A neutron star can be divided into several layers: the crust, the outer and the inner cores. The outer core extends up to the densities \( \rho \sim (2 - 3) \rho_0 \), where \( \rho_0 \approx 0.16 \text{nucleon/fm}^3 \) is the nuclear matter density. The inner core extends to the center of the neutron star where densities can be significantly larger \( \sim 5 - 10 \rho_0 \) and may contain muons, hyperons, and exotic matter. Due to inverse \( \beta \) decay, the nuclear matter dissolves into a uniform liquid composed of neutrons at the density \( \sim 1/2 \rho_0 \), with

\[
x = N_p/N_n \sim 5 \div 10%,
\]

admixture of protons and equal admixture of electrons and tiny admixture of muons, see Refs. [57, 58, 59]. In the inner core the value of proton fraction is probably larger: \( \sim 10 \div 13% \) [60]. The most efficient neutrino cooling reactions are due to direct URCA processes involving neutron \( \beta \) decay:

\[
n \to p + e + \bar{\nu}_e,
\]

and \( \beta \) capture in

\[
e + p \to n + \nu_e.
\]

Thus it is worth to analyze how internucleon interactions influence thermally excited direct URCA processes within cold neutron stars. Standard cooling scenario assumes that direct URCA processes can occur in the inner core only [61].

In the ideal gas approximation the zero temperature neutron star is described as the system of degenerate neutron, proton and electron gases with the ratio of proton and neutron densities, \( x \ll 0.1 \). For any positive neutron density the Pauli blocking in the electron and proton sectors guarantees stability of a neutron star to the neutron \( \beta \)-decay cf. Refs. [62] and [63]. The number densities of protons and electrons are equal to ensure electrical neutrality of the star, so \( k_F(e) = k_F(p) \). The neutron Fermi momentum is significantly larger than the proton Fermi momentum because of the larger number of the neutrons:

\[
x^{1/3} k_F(n) = k_F(p).
\]

The internucleon interaction produces nucleons with momenta above Fermi surfaces, cf. Eqs. (71, 72). To guarantee conservation of the electric and the baryon charges nucleon occupation numbers below the corresponding Fermi surfaces - \( f_i(k, T = \)
0) should be smaller than unity especially for protons. The nonrelativistic Schrödinger equation with realistic nucleon-nucleon interactions gives occupation numbers for protons with zero momenta $\approx 70\%$ for the nuclear matter density. Even a larger depletion of occupation numbers is found for protons with momenta near the Fermi surface [65].

The Landau Fermi liquid approach [66] in which momentum distribution of quasiparticles coincides with the Fermi distribution for the ideal gas of fermions is effective starting approximation for describing strongly interacting liquid. It has been explained by A.B.Migdal that nucleon distribution at zero temperature should exhibit the Migdal jump at $k = k_F$ which justifies applicability of the Fermi step distribution at zero temperature. The value of the Migdal jump is equal to the renormalization factor $Z < 1$ of the single-particle Green’s function in the nuclear matter. The condition $Z < 1$ follows from the probability conservation [67, 68] and implies that occupation numbers for nucleons with momenta $k < k_F$ are below one. In the limit of small proton concentration Fermi surface nearly disappears since proton neighborhood is predominantly strongly interacting with neutron medium. So the height of the Migdal jump for the proton distribution should decrease $\propto x$ for $x \to 0$. (Decrease of the Migdal jump due to a large probability of SRC has been discussed a long time ago for the liquid $^3$He in Ref.[69]. ) Thus for a highly asymmetric mixture of protons and neutrons the interaction tends to extend proton momenta well beyond $k_F(p)$.

We show that for the temperatures $T \ll 1$ MeV the presence of the high momentum proton tail leads to a different value and temperature dependence of URCA processes for $x \geq 1/8$, cf. Eq. 82 as compared to that in Refs. [70, 72, 71] where the Fermi momentum distribution for quasiparticles was used. As the consequence of the presence of the high momentum proton tail the neutrino luminosity due to direct URCA processes differs from zero even for $x < 1/8$ i.e. in the region forbidden in the ideal gas approximation for quasiparticles by the Pauli blocking and the momentum conservation.

The electron gas within neutron star is ultrarelativistic. So the Coulomb parameter $e^2/v \ll 1$. Here $e$ is the electric charge of electron and $v = p/E \approx c$ is its velocity. Thus approximation of the free electron gas is justified. The Coulomb interaction between protons with momenta $k \geq k_F(p)$ and electrons produces electrons with momenta above the electron Fermi surface, although with a very small probability cf. Eq. 73. So the occupation probability for electrons: $f_e(k_e \leq k_F(e), T = 0)$ is slightly less than one.

Thus the interaction produces holes in all Fermi seas removing the absolute Pauli blocking for the direct neutron, muon, hyperon $\beta$-decays. We show however that the
account of the Pauli blocking in the electron sector ensures stability of a neutron to the direct $\beta$ decay in the outer core of a neutron star. Condition of stability may be violated in the inner core where however use of nucleon degrees of freedom is questionable.

If hyperon stars exist (for the review of this subject and references see Ref. [73]), neutrino luminosity due to direct $\beta$- decay may appear significantly larger than for a neutron star.

5.2 The role of the interaction

High momentum nucleon component of the wave function of a neutron star follows directly from the Schrödinger equation in the limit $k \gg k_F$ where $k_F$ is Fermi momentum. The derivation of the formulae is similar to that in [6].

At the leading order in $(k_F^2/k^2)$ the occupation numbers for protons and neutrons with momenta above Fermi surface are:

$$f_n(k, T = 0) \approx (\rho_n)^2 \left( \frac{V_{nn}(k)}{k^2/m_N} \right)^2 + 2x \left( \frac{V_{pn}(k)}{k^2/m_N} \right)^2,$$

(71)

and

$$f_p(k, T = 0) \approx (\rho_n)^2 \left( x^2 \left( \frac{V_{pp}(k)}{k^2/m_N} \right)^2 + 2x \left( \frac{V_{pn}(k)}{k^2/m_N} \right)^2 \right).$$

(72)

Here $\rho_i$ is the density of constituent $i$. The factor $V_{NN}(k)$ describes the high momentum tail of the potential of the $NN$ interaction. The factor 2 in the above formulae accounts for the number of spin states. In the first term, this factor is canceled due to the identity of nucleons within the pair. In the derivation of the formulae for the probability of SRCs we used the approximation of nucleon density uniform in coordinate space to describe the uncorrelated part of the wave function. Thus, the value of the high momentum tail depends strongly on the nucleon density in the core of a neutron star. Since $k_F(p)$ is significantly smaller than $k_F(n)$, the probability to find a proton with $k \geq k_F(p)$ for a neutron density close to the nuclear density should be significantly larger than in nuclei where $x \approx 1$. Note also that the analysis of the recent data on SRCs in the symmetric nuclear matter found a significant $\sim 20\%$ probability of nucleons above the Fermi surface in nuclei which is predominantly due to $I = 0$ SRCs[17, 19].

The Coulomb interaction between protons from SRCs and electrons produces electrons with momenta above the electron Fermi surface. Such electrons are ultrarelativistic so Feynman diagrams approach should be used to evaluate the high
momentum electron component rather than the nonrelativistic Schrödinger equation. We find for the high momentum electron component approximate expression:

\[ f_e(k_e \geq k_F(e), T = 0) \approx \frac{1}{2} \int \frac{d^3 k_p}{(2\pi)^3} f_p(k_p) \theta(k_p - k_F(p)) \rho_e. \]  

(73)

\[ \cdot (1 - f_p(k_p, T = 0)) \left( \frac{k_e + \frac{3}{4} k_F(e)}{\sqrt{k_e} \cdot \sqrt{\frac{3}{4} k_F(e)}} \left( \frac{V_{\text{Coulomb}}(k)}{k_e - k_e^2/2m_N - \frac{3}{4} k_F(e)} \right)^2 \right). \]

The factor \( 1 - f_p(k_p, T = 0) \) is the number of proton holes which prevent Pauli blocking for the proton after interaction with the electron. Effectively, Eq.\((73)\) gives the probability for triple (e-p-n) short range correlations. This equation can be simplified for applications by using average quantities:

\[ f_e(k_e \geq k_F(e), T = 0) \approx \frac{1}{2} P_{pn} \langle H \rangle \cdot \rho_e \left( \frac{k_e + \frac{3}{4} k_F(e)}{\sqrt{k_e} \cdot \sqrt{\frac{3}{4} k_F(e)}} \right) \left( \frac{V_{\text{Coulomb}}(k)}{k_e - k_e^2/2m_N - \frac{3}{4} k_F(e)} \right)^2. \]  

(74)

Here \( P_{pn} \) is the probability of pair nucleon correlation and \( \langle H \rangle \approx P_{pn} \).

The factor

\[ \frac{1}{2} \left( \sqrt{k^2 + m_e^2} + \sqrt{k_e^2 + m_e^2} \right) \left( \frac{k^2 + m_e^2}{(k^2 + m_e^2)^{1/4}} \right) \left( \frac{k_e^2 + m_e^2 > 1/4}{(k_e^2 + m_e^2)^{1/4}} \right) \]

follows from the Lorentz transformation of the electron e.m. current, conveniently calculable from the Feynman diagrams. Here \( < k_e^2 > \) is the average value of the square of electron momentum within the electron Fermi sea.

5.3 Impact of SRC on the direct and modified URCA processes at small temperatures

In the Landau Fermi liquid approach at finite temperature, \( T \) the direct URCA process Eqs.\(68\) and \(69\) is allowed by the energy-momentum conservation law if the proton concentration exceeds \( x = 1/8 \) [71]. The restriction on the proton concentration follows from the necessity to guarantee the momentum triangle:

\[ k_F(p) + k_F(e) \geq k_F(n), \]  

(75)

in the absorption of electrons by the protons.

50
If proton concentration is below threshold or direct URCA process is suppressed due to nucleon superfluidity neutrino cooling proceeds through the less rapid modified URCA processes:

\[ n + (n, p) \rightarrow p + (n, p) + e + \bar{\nu}_e, \]  

and

\[ e + p + (n, p) \rightarrow n + (n, p) + \nu_e, \]  

in which additional nucleon enables momentum conservation.

The neutrino luminosity resulting from the direct and modified URCA processes, \( \epsilon_{URCA} \), was evaluated in Ref.\[71\] for \( x \geq 1/8 \) where the Fermi distribution:

\[ f_{i,bare}(k, T) = \frac{1}{1 + \exp \frac{E_i - \mu_i}{k_F}}, \]  

(78)

describes the Pauli blocking factors \( 1 - f_e(k, T) \) and \( 1 - f_p(k, T) \) in the final state. After integration over the phase volume of the decay products it was found:

\[ \epsilon_{URCA} = c(kT)^6 \theta(k_F(e) + k_F(p) - k_F(n)). \]  

(79)

Here \( c(x \geq 0.1) \) has been calculated in terms of the square of the electroweak coupling constant relevant for low energy weak interactions and the phase volume factors.

In the case of realistic NN interactions significant fraction of protons has momenta above the proton Fermi momentum. So Eq.(75) is satisfied for the proton large momentum tail even for \( x \) smaller than 0.1. For the sake of illustrative estimate we substitute in the probability of neutron \( \beta \)-decay the Pauli blocking factor \( 1 - f_p(k, T) \), by the actual distribution of protons within the core of a neutron star. We account for the probability of additional neutron from \((p,n)\) correlation by the additional factor \( P_{pn} \).

To simplify the discussion we will ignore here tiny probability for electron holes at zero temperature and parameterize neutrino luminosity as

\[ \epsilon_{URCA} = c(kT)^6 R, \]  

(80)

where \( R \) accounts for the role of SRC in neutrino luminosity at small temperatures. We find

\[ R \approx \kappa_{pn}^2 \left[ \int (1 - f_p(k_p, T)) \theta(k_F(p) - k(p)) + 
\right.

\[ + f(k_p, T) \theta(k_p - k_F(p)) \theta(k_F(e) + k_F(n)) d^3k_p/(2\pi)^3 \bigg]. \]

\[ \times \left[ \int (1 - f_{p,bare}(k_p, T)) \theta(k_F(e) + k_F(p) - k_F(n)) d^3k_p/(2\pi)^3 \bigg]^{-1}. \]  

(81)
Here $f_p(k_p, T)$ is the occupation number of protons accounting the interaction and $f_{p,\text{bare}}(T, k)$ is the Fermi distribution function over proton momenta at nonzero temperature. The factor $\kappa_{pn}$ is the overlapping integral between a component of the wave function of the neutron star containing pair nucleon correlation and the mean field wave function of the star. For the numerical estimate we use approximation: $\kappa_{pn}^2 = P_{pn}$. For a rough estimate we neglect the first term in the numerator of the above formulae and put $T = 0$ in the second term. Using for the estimate $V_{NN}(k) \propto 1/k^2$ for $k \gg k_F$ and Eq. (72) to evaluate large $k$ behavior of $f_p \propto (1/k)^8$ we obtain:

$$R \approx \left(\frac{P_{pn}^2/5}{(m_N kT)^{3/2}}\right)\rho_n,$$

where $P_{pn}$ is the the probability for a proton to have momentum $k \geq k_F(p)$. For the illustration, we numerically evaluate the enhancement factor $R$ for neutron density close to $\rho_0$, $x = \rho_p/\rho_n = 0.1$, and $P_{pn} = 0.1$. So,

$$R \approx 0.16P_{pn}(MeV/kT)^{3/2}. \quad (83)$$

The enhancement is significant for $kT \ll 1\text{MeV}$. Remember that after one year a neutron star cools to the temperatures $T \leq 0.01 \text{ MeV}$.

Neutrino luminosity due to direct URCA processes decreases with decrease of $x$ but differs from zero even for the popular option: $x \leq 0.1$. So investigation of the neutrino luminosity of the neutron stars may help to narrow down the range of the allowed values of the $x$ ratio.

### 5.4 $\beta$ stability of neutron within the outer core of zero temperature neutron star

Normal neutron star is bound by gravity. Gravity does not forbid decays of constituents of the star if energy and momentum are conserved in the decay (the equivalence principle).

Constraints due to the energy-momentum conservation law and the Pauli blocking in the electron sector work in the opposite directions. Indeed, the maximal momentum of an electron from $\beta$-decay of a neutron with momentum $k_n$ is $\approx 1.19\text{MeV}/(1 - k_n/m_p)$. Hence, an electron produced in the neutron $\beta$ decay may fill the electron hole with momentum $k \approx 1\text{MeV}/c$ only. The dominant process which may lead to the formation of electron holes is the elastic interaction of an energetic proton with electrons within the free electron gas. Energy-momentum conservation is fulfilled in the case of nonrelativistic nucleons if electron in the Fermi sea kicked
out by proton has minimal energy in the range:

$$E_{\text{hole}}(k) = \frac{(p - k_f + k)^2 - p^2}{2m_N + k_f}. \quad (84)$$

Here $p$ is the proton momentum and $k_f$ is the electron momentum in the final state. Scattered electron has energy $k_f \geq E_F(e)$, so it is legitimate to neglect by the electron mass. Hence, the minimal energy of the hole (when electron and proton momenta are antiparallel in the initial state) is

$$E_{\text{hole}} \sim \left(\frac{1}{2} \div \frac{1}{3}\right)E_F(e), \quad (85)$$

for the proton momenta around $p = 0.4 \div 0.5 GeV/c$ typical for SRC and decreases with increase of $p$. Evident mismatch between energies of produced electron holes and electrons in the neutron decay guarantees that an electron from $\beta$-decay of a neutron can not fill an electron hole.

In the case of ultrarelativistic nucleon gas (inner core of a star?) energy-momentum conservation does not restrict energies of electron holes produced in (e-p) interaction:

$$E_{\text{hole}}(k) = \sqrt{m_N^2 + (p + k - k_f)^2} - \sqrt{m_N^2 + p^2} + \sqrt{m_e^2 + k_f^2} \quad (86)$$

In the limit $p/m_n \to \infty$ we obtain expression for minimal energy of hole:

$$E_{\text{hole}}(k) = \sqrt{m_e^2 + k_f^2} - k_f + k \approx k + m_e^2/k_f. \quad (87)$$

However in this regime use of nucleon degrees of freedom would be questionable. We will not discuss further in this paper interesting question on the possible $\beta$ instability of neutron within the inner core of star.

Direct $\beta$ decay of muon produces electrons with momenta up to $m_\mu/2$ which are not far from the electron Fermi momentum. So evaluation of Pauli blocking for muon, hyperon $\beta$-decays requires model building.

It follows from above discussion that the reduction of the difference between neutron and proton momentum distributions influences collective modes. The most significant effect would be the tendency to suppress the superfluidity of protons (superconductivity) due to the deformation of the proton Fermi surface because of an increase of the fraction of protons having momenta above the Fermi surface. Existence of SRC will not strongly influence the possible superfluidity of neutrons. Note that superfluidity of neutrons will further suppress neutron $\beta$ decay due to formation of neutron Cooper pairs near the Fermi surface.

Electrons and neutrinos in the $\beta$ decays of hyperons, muons, are vastly more energetic than in neutron decay. Hence, if hyperon or muon stars exist, they should decay significantly more rapidly than the neutron stars and produce larger neutrino flux.
6 QCD and nuclear physics

6.1 Introduction

Spontaneously broken chiral symmetry plays a critical role in the nuclear structure. Small mass of the pion (due to small mass of u and d quarks) ensures presence of a large distance scale in nuclei — small density of nuclear matter as compared to the density within the nucleon. At the same time since pion is a pseudogoldstone meson of QCD, low energy theorems for interaction of pions with nucleons ensure that probability of low momentum pionic degrees of freedom in nuclei is small. The spontaneously broken chiral symmetry is responsible practically for all of the nucleon mass, leading to a barrier of $\approx 1$ GeV between nonperturbative and perturbative vacua\textsuperscript{17}.

However the chiral symmetry breaking alone is not sufficient to answer questions concerning microscopic structure of SRCs such as the role of the mesonic and baryonic components in these configurations, as well as account of the relativistic motion of the nucleons in SRCs. In fact studies of nuclear structure within chiral perturbation theory introduce a perturbation series over the pion interaction with subtraction terms arising from the integration over the high momenta characteristic for SRC. As a result these approaches deal with effective wave functions of nuclei in which high momenta due to SRCs are absent (hidden in the subtractive terms).

To address the properties of nuclei at high resolution one needs to take into account several other fundamental properties of QCD namely the decrease of the coupling constant at small space-time intervals (asymptotic freedom) and proportionality of strong interaction strength to the region occupied by color - the color screening phenomenon. These features of QCD lead to a number of new phenomena in nuclei unexpected in pre QCD approaches. This includes transparency of nuclear matter for propagation of fast spatially small quark-gluon wave packets, increase of the radius of the region occupied by color within a bound nucleon - a precursor of color conductivity. Evidence for such phenomena in moderate energy processes is discussed below. Search for color opacity phenomenon at ultrahigh energies is one of the goals of LHC.

We discussed in Secs.3 and 4 that hard processes indicate the dominance of nucleon degrees of freedom in SRCs, even for higher densities present in nuclei for which nonnucleonic effects are expected to be enhanced as compared to average nuclear configurations. Within meson nucleon theories interaction grows with a

\textsuperscript{17}This property is absent in many bag models which are often used to evaluate properties of nuclear matter and to derive equation of state.
decrease of the distance due to the presence of the Landau pole,

\[ g^2(t) = \frac{g_0^2}{1 - bg_0^2 \ln(t/t_0)}, \]  

(88)

leading to very strong interactions within the SRC. Here \( g(t) \) is running coupling constant, \( t \) is a virtuality and \( b > 0 \). Therefore the dominance of nucleonic degrees of freedom does not seem natural within these models. No such phenomenon is expected in QCD due to the phenomenon of asymptotic freedom.

Overall QCD dynamics is strongly different from the expectations of preQCD field theory models: approximate Bjorken scaling for the cross sections of hard processes follows from asymptotic freedom \((b < 0 \text{ in Eq.}(88))\) and color screening phenomenon. Such scaling is absent in preQCD models due to increase of the invariant charge with virtuality i.e. due to the Landau ghost pole in the running coupling constant. Moreover, color fields are screened within spatially small wave packets of quarks and gluons leading to a decrease of interaction of these quark-gluon packets with ordinary hadrons. All these features of QCD have been observed in the numerous hard QCD and electroweak phenomena. Experimentally approximate Bjorken scaling for lepton-nucleon(nucleus) scattering sets in at \( Q^2 \geq 1 \text{ GeV}^2 \).

To summarize: nonrelativistic nuclear models are phenomenological approaches in which short-distance effects enter as a boundary condition and absorbed in the parameters of the models. Hence it is not surprising that expectations of these models for hard nuclear phenomena differ strongly from the QCD expectations. The difference is especially large for phenomena in which one deals with the coupling to the meson exchange currents at large virtualities.

6.2 Implications of hard phenomena for the structure of nonnucleonic configurations in nuclei

Nucleons are composite particles, and therefore internucleon interaction should lead to a deformation of the bound nucleon wave function. A certain modification of the bound nucleon is manifested in the difference between quark-gluon distributions within a nucleon and a nucleus (usually referred as EMC effect [74, 75, 76]). This conclusion follows from the combined application of the exact baryon charge and momentum sum rules[10]. Observation of bound nucleon structure function modifications in the nuclear medium rises question of how these modifications can be represented in terms of non-nucleonic degrees of freedom in nuclei and what is the probability of such components in the nuclear wave function.
PreQCD models predict enhancement of meson currents in nuclei and significant non-static meson fields in nuclei, onset of the meson condensate regime at densities comparable to the average nuclear density, $\rho_0$. Such a hypothesis leads also to the expectation of enhancement of the antiquark ratio $R_{\bar{q}} = \frac{\bar{q}_A(x, Q^2)}{\bar{q}_N(x, Q^2)}$ for $x \leq 0.1$ of $\geq 10\%$ for $A \geq 40$ while the data find $R_{\bar{q}}(0.05 \leq x \leq 0.1) < 1$ with the typical error bars of $1\%$. The enhancement of $R_{\bar{q}}$ originates in these models from pion virtualities, $p_{\pi}^2 \sim 1$ GeV$^2$, which are far from the region where concept of meson exchanges can be justified: ($|p_{\pi}^2| \leq \text{few } m_{\pi}^2$).

Presence of $\Delta$-isobars in the nuclear wave function on the level of few $\%$ is not excluded experimentally. In fact presence of $\Delta$-isobars in $A = 3$ nuclei is necessary to satisfy the exact QCD Bjorken sum rule [77].

QCD suggests possibility of direct role of color field in nuclear structure. Color screening phenomenon means that smaller is the size of the quark-gluon wave packet smaller is its interaction with a hadron. As a result quark-gluon configurations of smaller than average size are suppressed within the bound nucleon [78]. This can be demonstrated by applying variational principle which requires suppression of configurations with minimal attraction to increase binding energy. The strength of deformation, $\gamma$, is expected to increase with an increase of the momentum of nucleon, $k$, approximately as $\gamma \propto k^2$ (for simplicity we omit here a term related to the energy of the residual system due to which $\gamma$ is linear in virtuality of the interacting nucleon). As a result the deformations should increase with an increase of nuclear density. An analogous effect of induced polarization of two interacting atoms is well known in the atomic physics where such deformation is found to be different for directions along and perpendicular to the axis between the atoms. Similarly the deformation of parton distributions in the bound nucleon may depend both on the absolute value of the nucleon momentum, $k$, and its direction with respect to the momentum transfer $q$.

To summarize: the EMC effect unambiguously demonstrates presence of non-nucleonic degrees of freedom in nuclei. Description of the effect as deformation of bound nucleon is dual to the presence of baryonic non-nucleonic degrees of freedom within hadronic basis description of the modification effects. In the color screening mechanism, described above, these baryonic components are highly coherent with the nucleon components and their absolute probability is rather small. This is consistent with the experimental constrains on the admixture of such components coming from high energy data[10]. In Sec.8 we discuss some of these restrictions and how further measurements of cross sections of the reactions: $e + A \rightarrow e' + N_{\text{backward}} + \pi + X$ could improve the limits or lead to discovery of these components.
6.3 QCD and meson currents

One of the intriguing observations of low energy nuclear physics is that nucleon - nucleon potentials decrease rather weakly with momentum transfer. Within one boson exchange models (OBEP) such behavior requires very hard (nearly point-like) meson - nucleon form factors. Such a weak dependence of meson - nucleon form factors on $t$ seems to be difficult to reconcile with the observations of quark structure of mesons and nucleons at high energies as well as with the $t$-dependence of the exclusive pion electroproduction by the longitudinally polarized photon which is dominated by the pion pole contribution.

If one applies the OBEP model to the calculation of antiquark content of the nucleon at $Q^2 \sim \text{few GeV}^2$ one finds an inconsistency: hard form factors generate too many antiquarks at $x \geq 0.1$. For example, if one uses an exponential parameterization of the $\pi NN$ form factor, $F_{\pi NN}(t) = \exp(\lambda t)$, one finds $\lambda \geq 1 \text{ GeV}^{-2}$. Note that approaches like OBEP ignore the loop diagrams which are unavoidable in quantum field theories and which lead to the Landau pole in the running coupling constant at momentum transfer in the vicinity of $-t \approx 1 \text{ GeV}^2$. Still even in this form mesonic models do not match even qualitatively to the QCD pattern of the decrease of meson fields as a function of momentum transfer as a consequence of the asymptotic freedom and color screening phenomenon. Another shortcoming of the models is the increase of the antiquark density with nuclear density which contradicts the data, see discussion in section 6.2.

The listed paradoxes originate from the contribution of pion fields with momenta $\geq 0.5 \text{ GeV}/c$ in which case internucleon distances (distance between pion and a nucleon) become substantially smaller than the nucleon size of $\sim 0.6 \text{ fm}$ where geometrically one can hardly think of the emission of pion. This is related to the observation that the pion exchange can be separated from the other contributions only in the vicinity of $t = m^2_N$. At the same time an exchange by the meson quantum numbers in $t$-channel at small distances does not require physical presence of the exchanged mesons. For example, an exchange of quarks between two nucleons (see Fig. 20) can also provide an exchange of same meson quantum numbers in $t$-channel (see e.g. Refs. [79, 80]). A quark exchange does not lead to the change of the number of antiquarks in the intermediate state. This will remove contradiction with the measurements of the $A$-dependence of antiquark distributions, and may have relatively weak $t$ dependence in the discussed $t$-range. Hence a possible solution of the paradox

\[^{18}\text{We use here nucleon radius as given by the axial form factor since this radius does not contain contribution of the soft pion fields. Classically one can fit a pion between two nucleons only if } r_{NN} \geq 2r_N + 2r_\pi \geq 2 \text{ fm}.\]
maybe the matching of the interaction potential of nearly static pion exchange for small $t$ with the quark interchange at larger $t$.

\[ M = \pi^+, \rho^+, \ldots \]

**Meson Exchange**

**Quark interchange**

Figure 20: Diagrams corresponding to meson and quark exchange mediated NN interactions.

There is another constraint on the model of NN interaction coming from high energy behavior of the amplitudes that should match the Regge pole behavior. The Reggezation of the meson exchange leads to a strong modification of the energy dependence of corresponding amplitudes ($A$), for example from $A \propto s$ for a $\rho$-meson exchange to $A \propto s^{1/2}$ for the $\rho$-meson Regge trajectory at $t = 0$. For the pion case the difference is very small for $t=0$, but becomes large with increase of $-t$. It is interesting that the discussed change of energy dependence arises in the dual Veneziano type models as a result of strong cancellations between s-channel resonances. An early onset of the Regge type behavior at low $t$ limit for two body processes is known as Dolen-Horn duality.

Another set of phenomena characteristic for QCD which is absent in meson theories is the fluctuation of interaction strength of the hadrons. In particular, in meson models it is hardly possible to generate the characteristic QCD phenomenon of color transparency - suppression of interaction strength with nuclear media of those configurations in hadrons that have constituents occupying a volume much smaller than the volume of average hadronic configuration\(^{19}\). The color transparency phenomenon is observed in variety of experiments at collision energies ranging from hundred GeV\(^{82}\) to few GeV\(^{83}\). In particular, in the coherent process of dijet production by

\(^{19}\)The dual description of the color screening phenomenon is to represent spatially small quark-gluon configuration as a coherent superposition of hadrons\(^{81}\).
pions off the nuclei: $\pi^+ A \to two \text{jets} + A$ the observed ratio of the yields from platinum and carbon targets\cite{82} is at least by factor of eight larger than the ratio expected from preQCD estimates.

The same property of QCD is responsible for validity of QCD factorization theorem\cite{84} for exclusive processes which is confirmed by data on diffractive electroproduction of vector mesons at collider energies\cite{85}.

To summarize, the concept of the pion exchange currents which is popular in low energy nuclear physics for processes with low momentum transfer is qualitatively consistent with QCD. At the same time preQCD models predict that contribution of meson currents should increase with an increase of virtuality which contradicts to QCD prediction in which meson currents should decrease with an increase of the virtuality. The theoretical approaches in description of SRCs should be consistent with this basic property of QCD.

7 Light-cone description of high energy processes involving nucleons and nuclei.

We discussed in introduction and in Sec.3 that large $Q^2$ inclusive $A(e,e')X$ reactions probe the light-cone (LC) density matrix and ultimately LC wave function of the nucleus.

Account of relativistic effects should be done in accordance with basic properties of QCD. One of the theoretical challenges for the relativistic quantum field theory is to separate bound state wave function from the background of vacuum fluctuations which are always present within a field theory.

This can be easily done for LC wave functions of bound state in kinematical domain close to one in which nucleon motion is nonrelativistic. Thus a question arises about the possibility of an approximation in which motion of nucleons is treated relativistically while no additional degrees of freedom is included - the LC mechanics of nuclei. To estimate the relative role of different degrees of freedom in the nuclear wave function we use the experimental data on NN interaction and the idea (implemented in QCD string models) that inelasticities in hadron-hadron collisions are related to the production of resonances. Within this picture, in the $I = 0$ channel nucleon degrees of freedom should dominate up to $k^2/m_N \sim (m_{N^*} - m_N) \approx 600$ MeV. In the channel with isospin $T = 1$ inelasticities may appear important at a lower energy scale: $k^2/m_N \sim m_\Delta - m_N \approx 300$ MeV.

These estimates indicate that up to the very large momenta on nuclear scale, $k \leq 800$ MeV/c for the deuteron and $k \leq 550$ MeV/c for heavier nuclei an approx-
imation in which only nucleonic degrees of freedom are accounted for the nuclear wave function is a legitimate approximation.

7.1 Light cone quantum mechanics of nuclei

As it was shown above for rather wide range of internal momenta in the nucleus the inelasticities in NN interaction is very small which can be considered as a small parameter in the problem. Due to the presence of such small parameter it makes sense to consider two nucleon approximation for LC wave function of the deuteron\[7\] for bound nucleon momenta up to 800 MeV/c. Key result in considering two-nucleon system in the light-cone is the existence of a relationship between nonrelativistic (NR) and LC equations for two-nucleon wave functions. If nonrelativistic potential describes the phase shifts, the same is true for its LC analog. Hence there exists a simple approximate relation between LC and NR two nucleon waves functions and NN potentials. The proof (rather lengthy) is based on reconstruction of properties of NN potential from the Lorentz invariance of on-mass-shell NN amplitudes\[10, 86\]. One finds that the form of the LC potential which enters in the LC equation for scattering amplitude (Fig.21) is strongly constrained by the the angular momentum conservation.

After introducing irreducible part \( V \) (which does not contain two nucleon intermediate state) through the lengthy algebra we obtain relativistic equation for deuteron wave function and related equation for two nucleon system with mass \( M \) \[10, 86\] in the following form:

\[
(4(m^2 + p^2) - M_{2N}^2)\psi = \int V(p, p')(1/(2\pi)^3)d^3p'/\sqrt{p'^2 + m^2}\psi(p'),
\]

where \( p \) and \( p' \) are three dimensional "internal" momenta of two nucleon system.
which are related to the LC variables as follows:

\[ \alpha = 1 + \frac{p^3}{\sqrt{m^2 + p^2}} \text{ and } k_t = p_t. \]  

(90)

The relation between kernel \( V \) as given by Feynman diagram and potential \( U \) which enters in the nonrelativistic description of the NN interaction is given by:

\[ U(p_1, p_3) = \frac{V(p_1, p_3)}{\sqrt{2E_1 \cdot 2E_2 \cdot 2E_3f \cdot 2E_4f}}. \]  

(91)

This is the same relation that relates relativistic Feynman diagrams for QED to the Coulomb potential. Therefore

\[ \frac{[4(m^2 + p^2) - M^2]}{\sqrt{m^2 + p^2}} = \psi = \int U(p, p')d^3p'/(2\pi)^3\psi(p'). \]  

(92)

This equation has a unique solution provided the self consistency requirement is imposed that the equation of Fig.21 generates rotationally invariant NN scattering amplitude which satisfies angular momentum conservation. As a result the deuteron wave function in the two nucleon approximation depends only on a single variable, \( k \), which is defined as follows:

\[ k = \sqrt{\frac{m_N^2 + p_t^2}{\alpha(2 - \alpha)} - m_N^2}, \]  

(93)

and relates in a straightforward way to the nonrelativistic deuteron wave function:

\[ \Psi_{d}^{LC}(\alpha, p_t) = \Psi_{d}^{NR}(k)(m_N^2 + k^2)^{\frac{3}{4}}. \]  

(94)

To summarize, for two body system in two nucleon approximation the biggest difference between nonrelativistic or virtual nucleon approximation and LC is in the definition of “internal” momentum of two nucleons. For NR case the “internal” momentum corresponds to the relative momentum of two nucleons in the lab, while in LC it is defined according to Eq.(93). This results in a qualitatively different relation between the wave function and the scattering amplitude for large nucleon momenta.

### 7.2 Master equation for LC wave function of a many nucleon system

Similarly we can deduce many body equation for LC dynamics in terms of irreducible amplitudes of internucleon interactions[86]:

\[ \left[ \langle \sum_{i=A}^{i=A} \epsilon_i \rangle^2 - M_A^2 \right] \psi_A(p_j) = \int V(p_i, p'_i)\delta\left(\sum_{i=A}^{i=A} p_i \right) \frac{d^3p_i}{2\epsilon_i(2\pi)^3} \langle \sum_{i=A}^{i=A} \epsilon_i \rangle \psi_A, \]  

(95)
where $\epsilon_i = \sqrt{p_i^2 + m^2}$ and $M_A$ is the invariant mass of the eigenstate.

In the case of ground state for which binding energy is small master equation for LC wave function obtains the same form as that given by relativistic theory for Schrödinger wave function in c.m. in which particle production is neglected:

$$\sum \epsilon_i - M_A |\psi_A(p_i) = \int \frac{d^3 p_i}{(2\pi)^3} \delta(\sum p_i) \frac{d^3 p_i}{2(2\pi)^3} \epsilon_i \psi_A(p_i).$$  \hspace{1cm} (96)

Master equation with potential $V$ reproducing Lorentz invariance of on-mass shell amplitude allows to account for the angular momentum conservation and to satisfy the requirement of separability. Account of the angular momentum conservation for on-shell amplitudes within LC mechanics requires special many body forces described in [86]. Another pattern how potential $V$ can be chosen consistent with the properties of on-mass-shell amplitudes is to explore similarity of LC equations to the center mass equation of relativistic noncovariant perturbation theory in which antinucleon production is neglected.

Having master equation and fitting potentials to describe on mass shell amplitudes it should be feasible to calculate LC wave functions, spectral and decay functions. It is worth mentioning that due to factorization of LC momentum fraction $\alpha$ and recoil energy $p_{R+}$ (see Sec.3) the calculation of LC density matrix which enters in description of large $Q^2$ inclusive $A(e,e'X)$ processes does not require performing a more challenging calculation of the spectral function. The latter is the case in the nonrelativistic approach. Calculation of LC spectral function could be simplified by the angular condition which implies that the LC spectral function is a function of two variables only and by the sum rule relating LC density matrix to the spectral function.

At present, for state of art analysis of many phenomena in high momentum transfer reactions nonrelativistic wave functions, spectral functions and decay functions calculated within nonrelativistic theory of nuclei can be used as basis for building LC density matrix as well as LC spectral and decay functions [17].

8 Directions for the future studies

The progress in studies of SRCs described above and challenging problems of QCD as well as understanding the implication of SRCs in the dynamics of cold dense nuclear matter calls for a systematic studies of reactions described above as well as including series of new hard processes.

Here we briefly outline some possible directions for experimental research both for electron and hadron facilities.
8.1 Inclusive $A(e, e')X$ reactions

8.1.1 Probing 2N and 3N SRCs

We explained in Sec.3 that high $Q^2$ inclusive $A(e, e')X$ reactions at $x > 1$ directly probe SRC probabilities. Further progress in this direction will be the study of $A(e, e')X$ reactions at much broader range of $x$ and $Q^2$. In particular for $x \sim 3$ it is highly desirable to reach at least $Q^2 \sim 6 \div 8$ GeV$^2$. The transition to DIS regime at $Q^2 \geq 10$ GeV$^2$ is very interesting. One expects that with an increase of $Q^2$ the ratios at fixed $x \geq 1$ should increase since DIS scattering at given $x$ probes LC fractions $\alpha \sim x + 0.5$ [6]. For example the carbon/deuteron ratio at $x=1$ should increase from $\sim 0.4$ to $\sim 5$. Such regime will corresponds to deep inelastic scattering off superfat quarks in nuclei with $x \geq 1$ [10, 87].

The leading twist contribution is expected to dominate at $Q^2 \geq 12$ GeV$^2$ for $x=1$ and somewhat larger $Q^2$ for higher $x$ (the interplay between leading twist and higher twist contributions (quasielastic scattering) depends on relative importance of the 2N and 3N correlations). The $x$ dependence of $F_{2A}(x \geq 1, Q^2)$, $A \geq 4$ in the scaling limit is expected in the few nucleon correlation model [6, 10] to be $\propto \exp(-bx)$ with $b \sim 8 \div 9$, leading to a large increase of the $F_{2A}/F_{2D}$ ratio between $x = 1$ and $x = 1.5$, see Fig. 22. Experimental attempts to observe such ”superfat” quarks were inconclusive: the BCDMS collaboration [88] has observed a very small $x > 1$ tail ($b \sim 16$), while the CCFR collaboration [89] observed a tail consistent with the presence of very significant SRCs ($b \sim 8$). A possible explanation for the inconsistencies is that the resolution in $x$ at $x > 1$ of the high-energy muon and neutrino experiments is relatively poor, causing great difficulties in measuring $F_{2A}$ which strongly varies with $x$. The energy resolution, intensity and energy of Jefferson Lab at 11GeV may allow one to study the inset of the scaling regime and thereby confirm the existence of superfat quarks.

8.1.2 Study of isotopic structure of SRCs and the nuclear core in $A(e, e')X$ reactions

If $pn$ correlations dominate in high $Q^2$ $A(e, e')X$ reactions at $x > 1$ one expects that,

$$\frac{\sigma_{3H}(1 < x < 2, Q^2)}{\sigma_{3He}(1 < x < 2, Q^2)} \approx 1, \quad \text{and} \quad \frac{\sigma_{40Ca}(1 < x < 2, Q^2)}{\sigma_{48Ca}(1 < x < 2, Q^2)} \approx \frac{N(48Ca)}{N(40Ca)} = 1.4. \quad (97)$$

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Figure 22: Ratio of the per nucleon structure functions of carbon and deuteron for different $Q^2$ in the few nucleon and two nucleon correlation approximations including both quasielastic and inelastic contributions. Curves correspond to $Q^2 = 3, 5, 7, 10, 15$ and 20 GeV$^2$ values.

As we discussed in the text inclusive $A(e,e')X$ reactions at $x > 2$ and production of fast backward nucleons in semi-exclusive reactions presently are the only sources of information about 3N SRC.

The $A(e,e')X$ reactions at $x > 2$ allow one also to probe the isospin structure of 3N SRCs. In this process virtual photon is absorbed by a nucleon of 3N SRC with large momentum, $p_i$, which is balanced by two nucleons with a relative momentum much smaller than momentum characteristic of 2N SRCs.

If 3N SRC emerges in the nuclear wave function predominantly through the iteration of two nucleon interactions one expects that $ppn$ and $nnp$ correlations to have about the same strength considerably exceeding the strengths of $ppp$ and $nnn$ correlations. The dominant contribution to the $x > 2$ region comes from the configurations in which recoiling mass is close to minimal. Our numerical studies with the
realistic spectral function for $A = 3$ nucleon system suggest that in this kinematics it is by factor $2 \div 3$ more likely (depending on the nucleon momentum) for a recoil pair to be in $I = 0$, rather than $I = 1$, state. As a result the ratio

$$ R_3 = \frac{\sigma_{3H}(2 < x < 3, Q^2)}{\sigma_{3He}(2 < x < 3, Q^2)} \approx \frac{\sigma_{el}(en)(Q^2)}{\sigma_{el}(ep)(Q^2)} \ll 1, $$

(98)

where $\sigma_{el}(eN)$ is the cross section of elastic electron-nucleon scattering in the kinematics of the $A(e,e')X$ reaction. This should be compared with the expectation of $R_3 \sim 1$ for the scattering off two nucleon correlations. Similarly a strong increase of the cross section with the number of neutrons can be expected for scattering off different isotopes like calcium.

### 8.2 Neutrino processes off nuclei

Unique advantage of neutrino (antineutrino) initiated processes is in the feasibility to probe density of antiquarks within a nucleon and nucleus. Therefore investigation of the correlation between $y$ dependence of processes: $\nu + A \rightarrow \mu + \text{backward proton} + X$ and momentum of the backward proton will help to unambiguously establish dependence of meson currents on virtuality and nuclear density.

### 8.3 Spectral functions

#### 8.3.1 Probing 2N and 3N correlations

Further studies of spectral functions at large initial momenta of removed nucleon ($p_i$) are necessary in kinematics in which rescattering effects are minimized. Generally this is the case when produced nucleon has a small transverse momentum and large energy relative to the residual system. In this kinematics one can probe both 2N and 3N correlations. To enhance the contribution from 3N SRCs one can look for the ratio of cross sections of $^3\text{He}(e,e'p)X$, and $^3\text{He}(e,e'n)X$ reactions in quasielastic kinematics in which the recoil invariant mass is sufficiently small to suppress the contributions of 2N correlations (such study will be complementary to the study of $A(e,e')X$ processes at $x > 2$ discussed above).

#### 8.3.2 High excitation energies and possible presence of nonnucleonic components in nuclei

The current meson exchange based models of NN interactions involve $N\Delta$ and $\Delta\Delta$ intermediate states. These states lead to a high momentum component of the nuclear
wave function through the $N\Delta$ and $\Delta\Delta$ correlations. The current calculations of spectral functions with realistic NN interactions do not treat $\Delta$-isobar degrees of freedom explicitly. On the other hand, if a nucleon is removed at large $Q^2$ from a $\Delta N$ correlation, the typical excitation energy will be of the order of $m_\Delta - m_N \sim 300$ MeV. High values of recoil nucleus excitation energy will be characteristic also for scattering off more exotic configurations like six-quarks that are close together - a kneaded quark state. In order to observe the evidence for nonnucleon components like $\Delta$-isobars in $N\Delta$ and $\Delta\Delta$ SRCs one needs large $Q^2 \geq 2/3\text{GeV}^2$ to destroy instantaneously these correlations and to suppress the contribution from two step charge exchange processes. Note that studying $x$ dependence of these processes may help to estimate two step processes as they should be practically the same at $x = 1$ and away from the quasielastic kinematics.

One can also study a complementary process of knock out of a $\Delta$-isobar, preferably $\Delta^{++}$ which cannot be produced in the scattering off a single nucleon in the processes like $e + N \to e + \Delta$ or $p + p \to p + \Delta$. Similar to the processes discussed above we would need high enough energies of the produced $\Delta^{++}$ to suppress the charge exchange contribution.

Note that in this case too the study of $x$ dependence of $\Delta^{++}$ production rate relative to the nucleon rate will allow to separate mechanism of scattering off the preexisting $\Delta$-isobar like configurations from those events associated with $\Delta$ production due to charge exchange rescattering of nucleons.

### 8.3.3 Spin structure of $2N$ correlations

It is important to measure directly the ratio of the S- and D- wave contributions in $pn$ correlations in the momentum range where D-wave dominates. This is possible in the scattering off the polarized deuteron in the reaction $e + ^2 \vec{H} \to e + p + n$ if one chooses parallel kinematics to minimize rescattering effects. In the case of tensor polarized deuteron the $T_{20}$ asymmetry is expressed through the ratio of the D- and S- wave momentum distributions, $w(k)/u(k)$. This reaction also provides a unique way to study relativistic effects which are predicted within light-cone approach to be strongly sensitive to the angle between recoil nucleon momentum, $\vec{p}_r$ and the reaction axis ($\vec{q}$)\cite{4,10}. Similar investigations are possible using vector polarized deuteron target and studying polarization of the interacting nucleon which is also expressed through the $w(k)/u(k)$ ratio \cite{91}. Such a measurement was performed in Ref. 92 using the reaction $\vec{e} + ^2 \vec{H} \to e + p + n$ at $Q^2 = .21$ GeV$^2$ for the recoil nucleon momenta $\vec{p}_s < 350$ MeV/c. It would be important to extend these measurements to much higher $Q^2$ and sufficiently large $W$ where dynamics of final state rescatterings
is simplified and can be described within GEA. In this case it would be possible to cover much larger range of recoil nucleon momenta for a range of angles between \( \vec{q} \) and \( \vec{p} \), for which the FSI is small and can be reliably calculated.

### 8.4 Decay functions

#### 8.4.1 Tests of factorization, mapping pp, pn correlations

The first studies of decay function described in the review suggest several directions for further theoretical investigations. First it is important to find kinematics in which final state interaction is minimal.

Secondly, to identify kinematic conditions for which factorization of the cross section into a product of decay function and elementary electron-bound-nucleon cross section is justified. Such studies are desirable to perform for both electron scattering using range of \( Q^2 > 1 \text{ GeV}^2 \) and for high momentum transfer (anti)proton-nucleus scattering.

Ultimately studies along these lines will allow us to study both pp and pn correlations for larger range of correlated nucleons momenta in which case one expects central forces in NN potential to became dominant or comparable with respect to the tensor forces. The onset of this regime could be identified by an increase of the pp/pn ratios with an increase of initial momentum of the nucleons in SRC.

It is also important to establish minimal momenta of the struck nucleon for which correlation mechanism is still operational, namely how close it is to the Fermi momentum. Remember that for carbon (\( k_F(C) \sim 220 \text{ MeV}/c \)) the correlation is clearly seen at \( k \geq 300 \text{ MeV}/c \) while it may be setting in at somewhat smaller \( k \).

#### 8.4.2 Looking for SRC involving \( \Delta \)-isobars

As we mentioned above the \( \Delta N \) like correlations may be present in nuclei. They may be manifested in the decay of correlations when a nucleon correlated with \( \Delta \)-isobar is removed. Hence one needs to look for production of backward isobars in electron and proton scattering in high momentum transfer kinematics we discussed.

Although the yield of \( \Delta \)'s comparable to that of nucleons is clearly excluded by near saturation of decay function by pn and pp SRCs, a yield on the level of \( \sim 10\% \) is possible. It is worth noting that this is a scale expected in quark exchange models of NN interaction\[6\]. In addition, there exist data on inclusive production of \( \Delta \)'s in \( e^- \text{ "air" } \) scattering at \( E_e = 5 \text{ GeV} \) and at large \( \alpha \geq 1 \) which allowed to estimate the ratio of \( \Delta^{++} \) and proton yields for same \( \alpha \) of the order 5% \[93\].
latter corresponds to the order of 10% high momentum component per nucleon in the nuclear ground state wave function due to $\Delta$-isobars.

### 8.4.3 Probing 3N correlations

The structure of three nucleon SRCs can be explored in the processes like $p + A \rightarrow p' + p_1 + N_2 + N_3 + (A - 3)^*$ in which proton $p_1$ is produced in large angle center of mass $pp$ scattering and two nucleons $N_1$ and $N_2$ represent recoiling particles, or in analogous processes in $\bar{p}A$ scattering using the PANDA detector [94]. Similar to the case of $A(p, 2pn)X$ processes studied in Ref.[15, 16] the knock-out proton ($p_1$) in this case will have preferentially $\alpha_i < 1$, leading to recoiling nucleons with $\alpha_{2,3} > 1$ emitted backward. Since such reaction selects $\langle k_{ti} \rangle \sim 0$ this will correspond to the production of "$N_2$" and "$N_3$" nucleons with back-to-back transverse momenta. Such configurations will be significantly enhanced due to 3N SRCs.

It is worth emphasizing here that this reaction would allow to check the role of three-proton SRCs which cannot be easily generated from two nucleon SRCs, and which via isospin invariance are connected to three-neutron correlations. The latter are important for understanding the equation of state of neutron stars where the role of $nnn$ SRCs is expected to be enhanced due to much higher densities involved.

### 8.5 Theoretical studies

At several points in the text we explained that high energy processes require understanding of LC structure of nuclear wave functions as well as spectral and decay functions. A simple relation between nonrelativistic (or virtual nucleon) and light-cone approximations exists only for the two nucleon system. Already for the case of the motion of NN pair in a mean field of the nucleus LC and virtual nucleon approximations yield significantly different results due to different treatment of the recoil system in these approximations.

It is important to develop further the relativistic approaches to confront them with experimental data. In the case of LC approximation the most pressing task is to solve the light-cone three-nucleon bound state problem. The relevant equations which to a very high accuracy satisfy constrains imposed by rotational invariance of on-shell NN amplitudes have been derived in Ref.[86]. Solving numerically these equations would allow both to make predictions for various experiments discussed in the review and to address a delicate issue of matching LC and nonrelativistic spectral functions for three body systems.

In describing SRCs at very large internal momenta it is important also to develop theoretical approaches that describe NN interaction at very small separations. In this
respect it is important that recent approaches to derive nucleon-nucleon interaction based on Chiral QCD Lagrangian which are justified for small nucleon and pion momenta to be matched with theoretical approaches that account for asymptotic freedom and color screening of strong interaction.

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