On existence of the $\sigma(600)$
Its physical implications and related problems

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Abstract. We make a re-analysis of I=0 $\pi\pi$ scattering phase shift $\delta_0^0$ through a new method of $S$-matrix parametrization (IA; interfering amplitude method), and show a result suggesting strongly for the existence of $\sigma$-particle – long-sought Chiral partner of $\pi$-meson. Furthermore, through the phenomenological analyses of typical production processes of the $2\pi$-system, the $pp$-central collision and the $J/\Psi \rightarrow \omega \pi\pi$ decay, by applying an intuitive formula as sum of Breit-Wigner amplitudes, (VMW; variant mass and width method), the other evidences for the $\sigma$-existence are given.

The validity of the methods used in the above analyses is investigated, using a simple field theoretical model, from the general viewpoint of unitarity and the applicability of final state interaction (FSI-) theorem, especially in relation to the “universality” argument. It is shown that the IA and VMW are obtained as the physical state representations of scattering and production amplitudes, respectively. The VMW is shown to be an effective method to obtain the resonance properties from production processes, which generally have the unknown strong-phases. The conventional analyses based on the “universality” seem to be powerless for this purpose.  

Introduction

The light iso-singlet scalar $\sigma$-meson is a very important particle which is predicted to exist as a chiral partner of the Nambu-Goldstone $\pi$-meson, corresponding to the dynamical breaking of chiral symmetry existing in the massless limit of QCD [1], with mass $\approx 2m_q$ ($m_q$ being the constituent quark mass). This $\sigma$ gives quarks constituent masses, and in this sense $\sigma$ may be called “Higgs particle in QCD.” The possible existence of the $\sigma$-particle has been suggested from various viewpoints both theoretically and phenomenologically. In particular the importance of $\sigma$ in relation with the $D\chi$SB has been argued extensively by Refs. [2,3].

However, the existence of $\sigma$ as a resonant particle has not yet been generally accepted. A major reason for this is due to the analyses of $\pi\pi$ phase-shift obtained

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1) This is a brief review of essential points given in the four contributions in parallel sessions; (i) “re-analysis of $\pi\pi/K\pi$ phase shift...” by T. Ishida, (ii) “existence of $\sigma/\kappa$ particle...” by M.Y. Ishida, (iii) “$\sigma$ particle in production...” by K. Takamatsu, and (iv) “relation between scattering and...” by M.Y Ishida. These are referred as [psa], [lsm], [prd], and [rel], respectively.
from CERN-Münich experiment [4] in 1974, in which the $I=0$ $\pi\pi$ $S$-wave phase shift $\delta^0_0$ up to $m_{\pi\pi} = 1300$ MeV turned out to be only 270°. After subtracting a rapid contribution of the resonance $f_0(980)(180°)$, there remains only 90°, being insufficient for $\sigma$ around $m = 2m_{\eta} = 500 \sim 600$ MeV. Most analyses thus far made on it have yielded conclusions against the existence of $\sigma$ [5]. As a result the light $\sigma$-particle had been disappeared from the list of PDG since the 1976 edition [6].

On the other hand, in the recent $pp$-central collision experiment, a huge event concentration in $I=0$ $S$-wave $\pi\pi$-channel is seen [7] in the region of $m_{\pi\pi}$ around 500~600 MeV, which is too large to be explained as a simple “background” and seems strongly suggest the existence of $\sigma$. Actually it is shown that the characteristic shape of $\pi^0\pi^0$ effective mass spectra below 1 GeV is able to be explained [8] by the two, $\sigma$- and $f_0$-, Breit-Wigner resonant amplitudes interferring mutually.

However the claim, $\sigma$-existence in the production processes, had been criticized from the so-called “universality” argument [9]: “Unitarity requires a resonance that
decays to $\pi\pi$, for example, has to couple in the same way to this final state whether produced in $\pi\pi$ scattering or centrally in $pp \rightarrow pp(\pi\pi)$. Thus claims of a narrow $\sigma(500)$ in the GAMS results cannot be correct as no such state is seen in $\pi\pi$ scattering.

**Phenomenological Observation of $\sigma$-particle**

On the contrary, we have recently made a re-analysis \cite{10,11} of the $\pi\pi$ phase shift and found a strong evidence for existence of $\sigma$-particle\cite{psa}. The reasons which led us to a different result from the conventional one \cite{5}, even with the use of the same data of phase shifts, are twofold: On one hand technically, we have applied a new method of Interferring Breit-Wigner Amplitude (IA-method) for the analyses, where the $\mathcal{T}$-matrix (instead of $\mathcal{K}$-matrix in the conventional treatment) for multiple resonance case is directly represented by the respective Breit-Wigner amplitudes in conformity with unitarity, thus parametrizing the phase shifts directly in terms of physical quantities, such as masses and coupling constants of the relevant resonant particles. On the other hand physically, we have introduced a “negative background phase” $\delta_{BG}$ of hard core type, $\delta_{BG}(s) = -|p_1| r_c$, whose possible origin is discussed later. The results of our re-analyses are given in Fig. 1(a), while in Table 1 I compare the essential points and the results of our analysis with those of the conventional one. In our analysis the introduction of repulsive $\delta_{BG}$ with $r_c \sim 3\text{GeV}^{-1}$ (0.60fm, about the structural size of pion) plays a crucial role for the existence of $\sigma(600)$. The sum of the large attractive $\delta_{\sigma(600)}$, contribution due to $\sigma(600)$, and the large repulsive $\delta_{BG}$ gives a small positive phase shift, which was treated, in the conventional analysis, \cite{5} as a background (or broad $\epsilon(900)$) contribution $[\delta_{BG}^{\text{pos}}]$. Note that the fit with $r_c=0$ in our analyses corresponds to the conventional analyses without the repulsive $\delta_{BG}$ thus far made. In this case the

\footnote{Independently the other several groups have performed the re-analyses of the phase shift, leading also to positive conclusion \cite{12–15} for $\sigma$-existence. Reflecting this situation the $\sigma$-particle has been revived in the list of latest edition of PDG, \cite{16} after missing for twenty years, with somewhat a hesitating label “$f_0(400 \sim 1200)$ or $\sigma$”.

**TABLE 1.** Comparison between the fit with $r_c \neq 0$ and with $r_c = 0$ in our PSA. The latter corresponds to the conventional analyses thus far made.

|            | $r_c \neq 0 (\chi^2/N_f = 23.6/30)$ | $r_c = 0 (\chi^2/N_f = 163.4/31)$ |
|------------|----------------------------------|----------------------------------|
| $\sigma$   |                                  |                                  |
| $m_{\sigma}$ | $585 \pm 20 (535 \sim 675)$        | $920$                            |
| $\Gamma_{\sigma}$ | $385 \pm 70$                        | $660$                            |
| $\sqrt{s_{\text{pole}}/\text{MeV}}$ | $(602 \pm 26) - i(196 \pm 27)$    | $970-1320$                       |
| $r_c$      | $3.03 \pm 0.35 \text{GeV}^{-1}$    |                                 |
|            | $(0.60 \pm 0.07 \text{fm})$        |                                 |
mass and width of “$\sigma$” becomes large, and the “$\sigma$”-Breit-Wigner formula can be regarded as an effective range formula describing a positive background phase. The corresponding pole position is close to that of $\epsilon(900)$ in Ref. [5]. In this case the bump-like structure around $500 \sim 600$ MeV in $\delta_0^0$ cannot be reproduced, as shown in Fig. 1(a), and gives $\chi^2 = 163.4$, worse by 140 than the best fit. This seems strongly to suggest the $\sigma$-existence.

We also present[prd] possible evidences for the existence of the $\sigma$ particle as an intermediate state of the $\pi\pi$ system in production processes, by analyzing the data obtained through the $pp$ central collision experiment by GAMS [7,8] and the data in the $J/\psi \to \omega\pi\pi$ decay reported by DM2 collaboration [17]. As is seen in Fig. 1(b) and (c), in each process there is a huge concentration of events in the $\pi\pi$ effective mass spectrum below 1 GeV, which is able to be simply explained as the $\sigma$-resonance. In the analyses we apply the Variant Mass and Width(V MW)-method, where the production amplitude is represented by a sum of the $\sigma$ and $f_0$ Breit-Wigner amplitudes with relative phase factors

$$\frac{r_\sigma e^{i\theta_\sigma}}{m_\sigma^2 - s - i\sqrt{s}\Gamma_\sigma(s)} + \frac{r_f e^{i\theta_f}}{m_f^2 - s - i\sqrt{s}\Gamma_f(s)},$$

It is notable that, with this rather simple formalism, not only the effective mass distribution, but also the characteristic angular distribution are excellently reproduced. It seems to us that $\sigma$-particle is observed as a huge event concentration of $S$-wave states in these production processes. The obtained mass and width of $\sigma$, with those of $f_0$ being fixed, are given in Table.2.

**Relation between Scattering and Production Amplitudes**

In treating the $\pi\pi$-scattering and production amplitudes, there are two general problems to be taken into account: The scattering amplitude $T$ must satisfy the unitarity $T - T^\dagger = 2i\rho T^\dagger$, and the production amplitude $F$ must have, in case that the initial state has no strong phase, the same phase as $T$: $T \propto e^{i\delta} \rightarrow F \propto e^{i\delta}$ (FSI; Final-State-Interaction theorem [19]). Conventionally, the more restrictive relation between $F$ and $T$ is required on the basis of the “universality,” [5,9]

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3) It was named [18] historically after the following reason. The mass and width of “a” resonant particle, which is misinterpreted as one resonance, instead of actual two overlapping resonances, are observed variantly depending upon the respective processes.

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**Table 2.** Observed mass and width of $\sigma$-particle in production processes.

| Process                        | Mass (MeV) | Width (MeV) |
|--------------------------------|------------|-------------|
| $pp$-central (GAMS NA12/2)     | $590\pm10$ | $710\pm30$ |
| $J/\psi$ decay (DM2)           | $480(\pm5_{st})$ | $325(\pm10_{st})$ |
\[ \mathcal{F} = \alpha(s) \mathcal{T} \]  

(2)

with a slowly varying real function \( \alpha(s) \) of \( s \). The criticism on our results of phenomenological analysis on the production process using the VMW-method was raised [9] along this line, as was mentioned. I have already shown, through the reanalysis of the \( \pi \pi \)-scattering, that there exists surely the \( \sigma \)-pole in \( \mathcal{T} \). Accordingly a main reason of the above criticism loses its reason. However, there has been remained a problem in the VMW-method applied to \( \mathcal{F} \); whether it is consistent to the FSI-theorem, or not.

In the following I re-examine the relation between \( \mathcal{F} \) and \( \mathcal{T} \) concretely, by using a simple model [20,21]. In the NJL-type model as a low energy effective theory of QCD, (and in the linear sigma model, \( L\sigma M \), obtained as its local limit), or in the constituent quark model, the pion \( \pi \) and the resonant particles such as \( \sigma(600) \) or \( f_0(980) \) are the color-singlet \( q \bar{q} \)-bound states and are treated equally. These "intrinsic quark dynamics states," denoted as \( \bar{\pi}, \bar{\sigma}, \bar{f} \) are stable particles with zero widths and appear from the beginning. Actually these particles have structures and interact with one another (and a production channel "\( P \)") through the residual strong interaction:

\[
\mathcal{L}^{\text{int}}_{\text{scatt}} = \sum \bar{g}_\alpha \bar{\alpha} \pi \pi + \bar{g}_{2\pi}(\pi)^4 \quad \left( \mathcal{L}^{\text{prod}}_{\text{int}} = \sum \bar{\xi}_\alpha \bar{\alpha} \"P \" + \bar{\xi}_{2\pi} \pi \pi \"P \" \right). 
\]

(3)

Due to this, these bare states change [12] into the physical states, denoted as \( \pi(= \bar{\pi}), \sigma \) and \( f \) with finite widths. In the following we consider only the virtual two-\( \pi \) meson effects for the resonant \( \sigma \) and \( f \) particles.

There are following 3-ways representing scattering amplitudes, corresponding to the three sorts of the basic states for describing the resonant particles:

1. **Intrinsic quark-dynamics states (bare states) representation**
   In the bases of zero-width bare states, denoted as \( |\bar{\alpha}\rangle \), the \( \pi \pi \)-scattering amplitude \( \mathcal{T} \) is represented in terms of the \( \pi \pi \)-coupling constants \( \bar{g}_\alpha \) and the propagator matrix \( \bar{\Delta} \) as

   \[ \mathcal{T} = \bar{g}_\alpha \bar{\Delta}_{\alpha\beta} \bar{g}_\beta; \quad \bar{\Delta}^{-1}_{\alpha\beta} = (\bar{M}^2 - s - i\bar{G})_{\alpha\beta}. \]

   (4)

   The real and imaginary parts of the squared mass matrix take the non-diagonal forms, which mean the bare states have indefinite masses and life times. The imaginary part of the inverse propagator is \( G_{\alpha\beta} = \bar{g}_\alpha \rho \bar{g}_\beta \left( \rho = \sqrt{1 - 4m^2_\pi^2/s}/16\pi \text{ being the state density} \right) \), then our \( \mathcal{T} \) is easily shown to satisfy the unitarity.

2. **\( K \)-matrix" states representation**
   The real part of the \( \bar{\Delta}^{-1} \) is symmetric and can be diagonalized by an orthogonal transformation: It transforms the bare states \( |\bar{\alpha}\rangle \) into the "\( K \)-matrix" states as \( |\bar{\alpha}\rangle \equiv |\bar{\alpha}\rangle o_{\bar{\alpha}\bar{\alpha}} \). Correspondingly the \( \mathcal{T} \) is represented by
\[ T = \tilde{g}_\alpha \tilde{\Delta}_{\alpha\beta} \tilde{g}_\beta; \quad \tilde{\Delta}_{\alpha\beta}^{-1} = (\Delta_{\kappa^{-1}} - i\tilde{G})_{\alpha\beta}, \Delta_{\kappa^{-1}} = (\tilde{m}_\alpha^2 - s)\delta_{\alpha\beta}, \tilde{G}_{\alpha\beta} = \tilde{g}_\alpha \rho \tilde{g}_\beta, \] (5)

where the coupling constant \( \tilde{g}_\alpha = (\tilde{g}_\alpha \rho \tilde{g}_\alpha) \) is real. These states have definite masses, but indefinite life times. The propagator \( \tilde{\Delta} \) is able to be expressed in the form representing concretely the repetition of the \( \pi\pi \)-loop, as

\[ \tilde{\Delta} = (1 - i\Delta_{\kappa}\tilde{G})^{-1}\Delta_{\kappa} = \Delta_{\kappa} + i\tilde{\Delta}\Delta_{\kappa}. \] (6)

Then the \( T \) takes the same form\(^4\) as the \( K \)-matrix in potential theory:

\[ T = K + i\bar{T}\rho K = K(1 - i\rho K)^{-1}; \quad \bar{K} = \bar{g}_\alpha \Delta_{\kappa\alpha\beta} \bar{g}_\beta = \bar{g}_\alpha (\bar{m}_\alpha^2 - s)^{-1} \bar{g}_\alpha. \] (7)

3. **Physical resonant states representation,**

The imaginary part of \( \tilde{\Delta}^{-1} \) in the \( K \)-matrix state representation, which was remained in a non-diagonal form, can be diagonalized by a complex orthogonal \([20,22]\) matrix \( u \), satisfying \( uu = 1 \). It transforms \( |\tilde{\alpha} \rangle \) into the unstable physical states as \( |\alpha \rangle \equiv |\tilde{\alpha} \rangle u_{\tilde{\alpha} \alpha} \). It is to be noted that the transformation is not unitary and \( \langle \alpha | \neq (|\alpha \rangle)^\dagger \). Correspondingly the \( T \)-matrix is represented by

\[ T = F_{\alpha} \Delta_{\alpha\beta} F_{\beta} = \sum_{\alpha} F_{\alpha}(\lambda_{\alpha} - s)^{-1} F_{\alpha}; \quad F_{\alpha}(\equiv \bar{g}_\alpha u_{\tilde{\alpha} \alpha}) \] (8)

where the \( \lambda_{\alpha} \) is the physical squared mass of the \( \alpha \)-state, and the \( F_{\alpha} \) are the physical coupling constants, which are generally complex. The physical state has a definite mass and life time, and be observed as a resonant particle directly in experiments.

In the following I show how the formulas in the IA and VMW methods satisfying FSI theorem\([rel]\) are derived effectively in the physical state representation. We start from the “\( K \)-matrix” states, which are able to be identified with the bare states \( |\tilde{\alpha} \rangle (\equiv |\tilde{\alpha} \rangle) \) without loss of essential points, since the reality of the coupling constant is unchanged through the orthogonal transformation. The real part of the mass correction generally does not have sharp \( s \)-dependence, and the \( \tilde{g} \) is almost \( s \)-independent, except for the threshold region. Then, in the two( \( \bar{\sigma}, \bar{f} \) ) resonance-dominative case, the scattering amplitude \( T \) is given by Eq.(7) as

\[ T = K / (1 - i\rho K); \quad \bar{K} = \bar{g}_\sigma^2 / (\bar{m}_\sigma^2 - s) + \bar{g}_f^2 / (\bar{m}_f^2 - s). \] (9)

The production amplitude \( \bar{F} \) is obtained, by replacing the one of respective scattering-coupling-constant \( \tilde{g} \) with the production coupling-constant \( \bar{\xi} \), as

\[ \bar{F} = \bar{P} / (1 - i\rho K); \quad \bar{P} = \xi_\sigma \bar{g}_\sigma / (\bar{m}_\sigma^2 - s) + \xi_f \bar{g}_f / (\bar{m}_f^2 - s). \] (10)

\(^4\) From the viewpoint of the present field-theoretical model, this “\( K \)-matrix,” Eq.(7), has a physical meaning as the propagators of bare particles with infinitesimal imaginary widths, \( \bar{m}_\alpha^2 \rightarrow \bar{m}_\alpha^2 - i\epsilon \), while the original \( K \)-matrix in potential theory is purely real and has no direct meaning.
The FSI-theorem is automatically satisfied since both $\mathcal{K}$ and $\mathcal{P}$ can be treated as real and the phases of $\mathcal{T}$ and $\mathcal{F}$ come from the common factor $(1 - i\rho\mathcal{K})^{-1}$.

In the physical state representation the $\mathcal{T}$ is given by

$$
\mathcal{T} = \frac{F^2_\sigma}{\lambda_\sigma - s} + \frac{F^2_f}{\lambda_f - s} = \frac{g^2_\sigma}{\lambda_\sigma - s} + \frac{g^2_f}{\lambda_f - s} + 2i\rho \frac{g^2_\sigma}{\lambda_\sigma - s} \frac{g^2_f}{\lambda_f - s}.
$$

This is just the form of scattering amplitude, applied in IA-method[psa]. The $\lambda_\alpha(\alpha = f, \sigma)$ is identified to $M^2_\alpha - ipg^2_\alpha$ appearing in usual Breit-Wigner formula. Thus we define the physical mass $M_\alpha$ and the real physical coupling $g_\alpha (g^2_\alpha \equiv -\text{Im} \frac{\lambda_\alpha}{\rho}).$

Similarly in the physical state representation the $\mathcal{F}$ is given by

$$
\mathcal{F} = \frac{r_\sigma e^{i\theta_\sigma}}{\lambda_\sigma - s} + \frac{r_f e^{i\theta_f}}{\lambda_f - s}.
$$

The $r_\alpha$ and $\theta_\alpha$ are expressed in terms of $\bar{g}_\alpha$, $\bar{\xi}_\alpha$, and $\lambda_\alpha$ and shown to be almost $s$-independent except for the threshold region. Thus the Eq.(12) has the same form as Eq.(1) applied in VMW-method. However, we must note on the following: In the VMW-method essentially the three new parameters, $r_\sigma$, $r_f$ and the relative phase $\theta(\equiv \theta_\sigma - \theta_f)$, independent of the scattering process, characterize the relevant production processes. Presently they are represented by the two production coupling constants, $\bar{\xi}_\sigma$ and $\bar{\xi}_f$. Thus, among the three parameters in VMW-method there exists one constraint due to the FSI-theorem. The corresponding considerations in the case with the non-resonant background phase are also given in [rel].

Here it should be noted that the FSI-theorem is only applicable to the case of the initial state having no strong phase. This type of initial strong phases generally exists in all processes under the effect of strong interactions, which is effectively able to be introduced in the VMW-method by substitution of $\bar{r}_\alpha \rightarrow \bar{r}_\alpha e^{i\theta^\text{strong}_\alpha}$. However, we have few knowledge on the initial phases, and we are forced to treat the parameters in VMW-method as being effectively free.

The way of our analyses of scattering and production processes are compared with that of the conventional analyses based on the “Universality” argument pictorially in Fig. 2. The $\pi\pi$-scattering is largely affected by the effect of the non-resonant repulsive background, and the $\mathcal{T}$ cannot be described only by usual Breit-Wigner amplitudes with a non-derivative coupling. The spectrum of $\mathcal{T}$ shows a very wide peak around the $\sqrt{s} \simeq 850$ MeV, at which the phase $\delta_0$ passes through 90 degrees, and then falls down rapidly. In contrast the spectra of $\mathcal{F}$ in the $pp$-central collision and the $J/\Psi \rightarrow \omega\pi\pi$-decay have peaks at around $\sqrt{s} = m_\sigma(500 \sim 600\text{MeV})$. In the conventional way, with the universality relation $\mathcal{F} = \alpha \mathcal{T}$, the $\mathcal{T}$ is first analyzed and the phase shift $\delta$ around $\sqrt{s} = m_\sigma$ is interpreted as due to, instead of $\sigma$-contribution, the background. Then $\mathcal{F}$ is analyzed with the $\alpha(s)$ arbitrarily chosen with the polynomial form, $\alpha(s) = \sum \alpha_n s^n$. In the most simple case with $\alpha = \text{const}$, the universality relation implies that $\bar{\xi}_\sigma = \alpha \bar{g}_\sigma$, $\bar{\xi}_f = \alpha \bar{g}_f$ and $\bar{\xi}_{2\pi} = \alpha \bar{g}_{2\pi}$, that is,
all the production couplings to be proportional to the corresponding $\pi\pi$-couplings, and the spectra of $F$ and $T$ becomes the same. Actually they are different and the difference is fitted by the $\alpha_n$. The masses and widths of resonances are determined only from the $\pi\pi$-scattering, and the analyses of $F$ on any production process become nothing but the determination of the $\alpha_n$ for respective processes, which have no direct physical meaning. Thus all the production experiments lose their values in seeking for new resonances. On the other hand, in the VMW-method, only the physically meaningful parameters are introduced. The $\xi_\omega$, $\xi_\sigma$, and $\xi_{2\pi}(s)$ are independent parameters of the $\pi\pi$-scattering, and the difference between the spectra of $F$ and $T$ is explained intuitively by supposing the relations among the coupling constants such as $\xi_\omega/\xi_\sigma \gg \xi_{2\pi}/\xi_{2\pi}$, that is, the ratio of background effects to the $\sigma$-effects are weaker in the production processes than in the scattering process. Thus in this case the large low-energy peak structure in $|F|^2$ shows directly the $\sigma$-existence. In this situation the properties of $\sigma$ can be obtained more precisely in production processes than in scattering processes. Here I should like to note that this difference between two methods may reflect their basic standpoints: In the
“universality” argument only the stable (pion) state consists in the complete set of meson states, while the $\bar{\sigma}$ and $\bar{f}$, in addition to pion, are necessary as bases of the complete set in VMW-method.

Physics connected with $\sigma$-existence

Origin of repulsive core In our phase shift analyses the repulsive $\delta_{BG}$ of hard core type introduced phenomenologically plays an essential role. This type of $\delta_{BG}$ was also reported historically in the $\alpha\alpha$-scattering [23] and in the $NN$-scattering [24], whose origin may be related to the Fermi-statistics property of respective constituent particles.

The repulsive core in the $\pi\pi$- and $K\pi$-scatterings seems to have a strong connection to the $\lambda\phi^4$-interaction in $L\sigma M$: It represents a contact zero-range interaction and is strongly repulsive, and has plausible properties as the origin of repulsive core. Moreover, the magnitudes of core radii are shown to be explained qualitatively by this term[lsm]. In NJL-type model the composite pion field $\phi(x)$ is defined in the "local limit" from the constituent quark field $q(x)$ by $\phi(x) = \bar{q}(x)i\gamma_5\tau q(x)$, and the $\lambda\phi^4$-interaction is obtained from the quark box-diagram. The repulsiveness of the $\lambda\phi^4$-interaction, $\lambda > 0$, is explained by the loop factor -1 due to the Fermi-statistics property of constituent quarks.

$q\bar{q}$-meson spectra and “Chiralons” Taking the $SU(3)$ flavor symmetry into account, it is now natural to expect the existence of a scalar meson nonet. We also analyze[psa] the $I=1/2$ $K\pi$-scattering phase shift from a similar standpoint to the $\pi\pi$ system, and actually show [26] that its behavior is consistent with the existence of an $I=1/2$ scalar meson, the $\kappa(900)$ meson. The $\sigma(600)$ and $\kappa(900)$, and the observed resonances $a_0(980)$ and $f_0(980)$, are shown[lsm] to have almost plausible properties as the members of the $\sigma$-nonet in the $SU(3)L\sigma M$ and the $SU(3)L\sigma M$ with $\rho$- and $a_1$-nonets. The $\sigma$-nonet forms with the pseudoscalar $\pi$-nonet the linear representation of chiral symmetry. This result implies that the chiral symmetry plays the stronger role than ever thought in understanding the strong interaction, especially not only the low energy theorems derived through the non-linear realization, but also the spectroscopy and reactions related with all the mesons with masses below and around $\sim 1$ GeV through the linear realization.

Here I should like to mention that the $\sigma$-nonet is to be discriminated from the $^3P_0$ scalar nonet: There are well-known two contrasting views on $q\bar{q}$-meson spectra. One is based on LS-coupling scheme of non-relativistic quark model. In boosted LS-coupling scheme [27] the covariant $q\bar{q}$-wave function(WF) is obtained by boosting the non-relativistic space and spin WF’s, $f(r) \otimes \chi_i\bar{\chi}^j$ separately into $f(\tilde{r}_\mu) \otimes U^\beta_\alpha$. The Bargmann-Wigner function, $U^\beta_\alpha$, is represented by a direct product, $U^\beta_\alpha = u_\alpha(v)\bar{u}^\beta(v)$, of free Dirac spinors of quark and anti-quark with the same velocity $v_\mu$ as the meson. The $U$ includes the pseudoscalar and vector component, but no scalar since $\langle \bar{v}(v)u(v) \rangle = 0$. The space-time WF $f$ is a function of the $\tilde{r}_\mu \equiv (\delta_{\mu\nu} + v_\mu v_\nu)r_\nu$ ($r_\mu$ and $v_\mu$ being the relative space-time coordinate and the velocity of meson, respectively).
The other view is based on chiral symmetry and has a covariant framework from the beginning. In the NJL model the local $\sigma^i(x)$ field is defined by $\sigma^i(x) \equiv \bar{\psi}(x) \lambda^i \psi(x)$, missing the freedom of relative space-time coordinate $r_\mu$ which is required to describe orbital excitations. The local $\sigma^i(x)$ is only describable, so to speak, $L=0$ states. Being based on a similar consideration the axial $a_1$-nonet as a partner of $\rho$-nonet is also expected to exist besides of the $^3P_1$-nonet.

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