**L₁ adaptive control of a shape memory alloy actuated flexible beam**

Bongani Malinga and Gregory D. Buckner*

*Department of Mechanical and Aerospace Engineering, North Carolina State University, Campus Box 7910, Raleigh, NC 27695, USA*

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This paper details the synthesis of an \( L_1 \) adaptive controller for a shape memory alloy (SMA) actuated flexible beam. The controller manipulates applied voltage, which alters SMA tendon temperature to track reference bending angles. Simulated and experimental results show that the \( L_1 \) adaptive controller provides precise tracking of the reference trajectories and effectively compensates for the nonlinear hysteretic relationship between SMA Joule heating and bending angle without explicitly modelling these characteristics. A simulation model whose results closely resemble the experimental performance results is presented. As a first step towards the development of \( L_1 \) adaptive control implementation guidelines, a complete description of the \( L_1 \) control parameters and their correlation to tracking performance is presented.

**Keywords:** control systems; adaptive control; artificial neural networks; shape memory alloys; nonlinear systems; optimization; grid search

1. Introduction

In recent years, there has been an increased focus on the use of shape memory alloys (SMAs) in medical devices, including coronary stents (Kleinstreuer, Li, Basciano, Seelecke, & Farber, 2008) catheter guide wires (Haga, Mineta, Makishi, Matsunaga, & Esashi, 2010), miniature forceps (Nakamura, Matsui, Saito, & Yoshimoto, 1995), eyeglass frames (Wu and Schetky, 2000), and sensing applications (Stoeckel, 1991). These materials possess the unique ability to ‘memorize’ shapes through thermally induced phase transitions (Kohl, 2004), making them potentially attractive for micro-actuation applications. Their material composition (e.g. Cu–Zn, Cu–Zn–Al, Cu–Al–Ni, Ni–Ti, Ni–Ti–Fe, Fe–Pt) largely determines critical mechanical properties including ductility, corrosion, and ‘memory’ properties of the alloy. Three SMA characteristics are associated with crystal reorientation during stress and temperature-induced phase changes: the pseudoelastic effect, the one-way effect, and the two-way effect (Kohl, 2004; Van Humbeck, Chandrasekaran, & Delaey, 1991). The pseudoelastic effect exhibits a reversible response to deformation by temperature and applied stress. The one-way effect is characterized by shape change upon loading, but no reorientation upon cooling. Once deformed, the ‘memorized’ shape is recovered when heated above the transition temperature. The two-way effect refers to shape change upon cooling and heating without external loading.

Real-time control of SMA actuated devices is complicated by the highly nonlinear, hysteretic constitutive relationships between material stress, strain, and temperature. Numerous models have been developed to describe the hysteretic behaviour of SMAs; all can be broadly classified as being either physical or empirical in nature. Empirical models date back to Tanaka and Nagaki (1982), with continued development by Liang and Rogers (1990), Brinson (1993) and others. While these models are computationally efficient and adaptable to experimental data, they rely on conditional statements to account for hysteresis in the forward and reverse direction, and tend to be difficult to implement in real-time control algorithms. Physical models have been developed by Achenbach and Müller (1985), Seelecke and Müller (2003) and others. These models are based on statistical thermodynamics and phase transformation probabilities. While single crystal models are computationally efficient, their accuracy is frequently inadequate for control applications. These model-based control design approaches are prone to model errors, which could result from parametric uncertainty and unmodelled dynamics.

Traditional proportional-integral (PI) controllers have been applied to SMA tendon control, but their dependence on integral action to compensate for system hysteresis presents a major drawback. A reduced proportional gain is required to reduce overshoot caused by rapid changes in setpoint. The result is limited bandwidth and limited disturbance rejection characteristics.

This paper introduces an \( L_1 \) adaptive controller (Cao and Hovakimyan, 2006) for an SMA-actuated plant: a flexible beam deflected by an offset SMA tendon. The controller manipulates applied voltage, which causes heating in the SMA tendon and bending in the flexible
beam; the SMA’s two-way effect is used to track reference bending angles. Unlike traditional PI and model-based adaptive control schemes, $L_1$ adaptive control effectively compensates for hysteresis by considering it a general system uncertainty; it is robust and computationally efficient, enabling real-time implementation. $L_1$ adaptive control offers higher bandwidth, enhanced robustness, and improved disturbance rejection without the need for exhaustive tuning. Combining $L_1$ adaptive control with the hysteretic recurrent neural network (HRNN) model and the grid search algorithm provides a systematic framework for adaptive controller design, applicable to a wide range of dynamic systems. The remainder of the paper is organized as follows: Section 2 introduces the plant, model, and controller synthesis, along with computational and experimental methods. Simulated and experimental results are presented in Section 3, with concluding remarks presented in Sections 4 and 5.

2. Methods

2.1. Experimental methods: plant

The plant consists of a flexible beam actuated by a single SMA tendon, Figure 1. The SMA tendon is offset from the neutral axis of the beam by a fixed distance $a$. As the tendon contracts, the actuation force creates a moment about the beam, causing it to bend to an angle $\theta$. While fully maneuverable actuators (like those employed in robotic catheters (Moallem, 2003) require antagonistic actuation, the single-tendon actuator considered here effectively addresses the nonlinearities encountered in SMA actuation. The methods used here can be extended to other single-tendon applications or designs that require antagonistic actuation for increased maneuverability and bandwidth. A complete description of the SMA-actuated flexible beam is presented in (Veeramani, Buckner, Owen, Cook, & Bolotin, 2008).

An experimental realization of this SMA-actuated plant is shown in Figure 2. The central flexible beam (0.5 mm diameter super-elastic Nitinol) features nine equally-spaced plastic collets that maintain a 1 mm offset between the neutral axis and the SMA tendon (0.127 mm diameter Flexinol, Dynalloy, Inc. Tustin, CA). Actuation commands are sent via USB communication from the host PC to a microcontroller, which implements pulse-width modulation to regulate Joule heating and temperature-induced strain in the SMA tendon. Real-time bending is calculated from 3D base and tip measurements (trakSTAR 3D Magnetic Tracking System, Ascension Technology Corporation, Burlington, VT) using constant-curvature deflection relationships (Veeramani et al., 2008).

2.2. Simulation methods: plant model

The plant dynamics are simulated using a linear, first-order temperature model cascaded with a single-input, single-output HRNN, as shown in Figure 3. The SMA wire thermal response to the applied control voltage is modelled as an empirical first-order system:

$$G(s) = \frac{1}{\tau s + 1},$$

where $\tau$ is the experimentally determined Joule heating time constant.

A two-phase HRNN is used to map the highly nonlinear and hysteretic relationship between tendon temperature and bending angle. This unique neural network, illustrated in Figure 4, utilizes weighted recurrent neurons, each composed of conjoined sigmoid activation functions, to capture the directional dependencies typical of hysteric
smart materials. For a complete description of the HRNN and its application to SMA modelling, see (Veeramani, Crews, & Buckner, 2009). At time step $q$, the output of each activation function is

$$f_i(\hat{T}(q)) = \frac{1 - f_i(\hat{T}(q - 1))}{1 + \exp((T_{F,i} - \hat{T}(q))\chi_i)} + \frac{f_i(\hat{T}(q - 1))}{1 + \exp((T_{R,i} - \hat{T}(q))\chi_i)},$$

where $\hat{T}(q)$ refers to the estimated SMA tendon temperature at the $q$th time step. The activation functions range between 0 and 1, where 0 refers to an inactive neuron and 1 refers to an active neuron. The parameter $\chi_i$ controls how quickly the output switches between the two values, with a high value ($\chi_i \gg 1$) leading to step changes. A previously inactive neuron becomes active at the forward transition temperature $T_{F,i}$. A previously active neuron becomes inactive at the reverse transition temperature $T_{R,i}$.

The network output

$$\hat{\theta}(q) = \sum_{i=1}^{N} w_i^2 f_i(\hat{T}(q))$$

is a weighted combination of these activation functions. The network weights $w_i$ are subject to the constraint:

$$\sum_{i=1}^{N} w_i^2 = 1$$

and are optimized through network training.

The HRNN was trained using open loop data from the physical plant. The Levenberg–Marquardt algorithm (Marquardt, 1963) was used to solve the nonlinear optimization problem governed by the cost function:

$$P = \sum_{q=1}^{Q} e(q)^2,$$

where:

$$e(q) = \theta_m(q) - \hat{\theta}(q)$$

represents the error between an experimentally measured bending angle $\theta_m(q)$ and the HRNN-predicted angle $\hat{\theta}(q)$.

The optimization algorithm was implemented using MATLAB’s `lsqnonlin` command (The MathWorks, Inc.,...
Natick, MA) using a Jacobian defined by
\[
J_{q,1} = \frac{\partial e(q)}{\partial w_i} = -2w_i f_i. \tag{7}
\]

Input voltage and bending angle data from a previous investigation (Veeramani et al., 2009) was used for HRNN training using the same plant inputs of Figure 3. SMA tendon temperature was calculated using Equation (1), with the \( L_1 \) controller providing the input voltage. The HRNN was then trained to optimize the network weights as outlined in Equations (4)–(7); weights were then imported into the simulation environment for predicting the bending angle based on the calculated SMA tendon temperature.

### 2.3. \( L_1 \) adaptive control

The \( L_1 \) adaptive control objective is to manipulate the input voltage \( u(t) \) so that the measured bending angle \( y(t) \) tracks the reference bending angle \( r(t) \) as shown in Figure 5.

One fundamental difference between \( L_1 \) control and traditional model reference adaptive control (MRAC) is the inclusion of a low-pass filter that allows the decoupling of control and adaptation. The filter allows for higher adaptation rates while keeping the control signal within the bandwidth of the system actuator. Key features of \( L_1 \) adaptive control include enhanced robustness and transient performance coupled with rapid adaptation, without introducing or enforcing persistence of excitation, without any gain scheduling of controller parameters, and without resorting to high-gain feedback. These features can be achieved by explicitly building the robustness specification into the problem formulation, effectively decoupling adaptation from robustness and increasing the speed of adaptation, subject only to hardware limitations.

#### 2.3.1. \( L_1 \) control problem formulation

Consider an SISO system that can be represented by the linear model:
\[
y(s) = A(s)(u(s) + d(s)), \tag{8}
\]
where \( u(s) \) is the system input, \( y(s) \) is the system output, and \( A(s) \) is a strictly proper unknown transfer function. \( d(s) \) represents the time-varying nonlinear uncertainties and disturbances, denoted by \( d(t) = f(t, y(t)) \), and \( f : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R} \) is an unknown map, subject to the following assumptions.

**ASSUMPTION 1** Lipschitz continuity: There exist constants \( L > 0 \) and \( L_0 > 0 \), possibly arbitrarily large, such that the following inequalities hold uniformly in \( t \):
\[
|f(t, y_1) - f(t, y_2)| \leq L|y_1 - y_2|, \forall t \geq 0,
\]
\[
|f(t, y)| \leq L|y| + L_0.
\]

**ASSUMPTION 2** Uniform boundedness of the rate of variation of uncertainties: There exist constants \( L_1 > 0 \), \( L_2 > 0 \) and \( L_3 > 0 \), possibly arbitrarily large, such that for all \( t \geq 0 \):
\[
|\dot{d}(t)| \leq L_1 |\dot{y}(t)| + L_2 |y(t)| + L_3.
\]

The control objective is to design an adaptive output feedback controller \( u(t) \) such that the system output \( y(t) \) tracks the given bounded piecewise-continuous reference input \( r(t) \) following a desired reference model \( M(s) \). For implementation and demonstration simplicity and to streamline parameter selection, in this paper we consider a first-order reference model:
\[
M(s) = \frac{m}{s + m}, \quad m > 0. \tag{9}
\]

#### 2.3.2. \( L_1 \) control architecture

The system output can be expressed in terms of \( M(s) \):
\[
y(s) = M(s)(u(s) + \sigma(s)), \tag{10}
\]
\[
u(s) = C(s)(r(s) - \sigma(s)), \tag{11}
\]

where uncertainties associated with \( A(s) \) and \( d(s) \) are lumped into \( \sigma(s) \), which is given by
\[
\sigma(s) = \frac{(A(s) - M(s))u(s) + A(s)d(s)}{M(s)}. \tag{12}
\]

The design of the \( L_1 \) adaptive controller proceeds by considering a strictly proper low-pass filter \( C(s) \), with \( C(0) = 1 \), such that
\[
H(s) = \frac{A(s)M(s)}{C(s)A(s) + (1 - C(s))M(s)}. \tag{13}
\]
is stable, and the following $L_1$ norm condition holds

$$G(s)L < 1,$$  \hspace{1cm} (14)

where $G(s) \triangleq H(s)(1 - C(s))$, $L$ is the maximum of a compact set that represents time-varying unknown parameters, bounded by $\max|\sigma(s)|$. The $L_1$ norm is defined as

$$G(s)L_1 = \int_0^\infty |g(t)|dt,$$ \hspace{1cm} (15)

where $g(t)$ is the inverse Laplace transform of $G(s)$ and represents the impulse response of the system.

Reference model/output predictor:

We consider the following output predictor

$$\dot{\hat{y}}(t) = -m\hat{y}(t) + m(u(t) + \hat{\sigma}(t)), \quad \hat{y}(0) = 0,$$ \hspace{1cm} (16)

where $\hat{\sigma}(t)$ is the adaptive estimate, governed by the adaptation laws described in the next section.

Adaptation law:

The adaptation of $\hat{\sigma}(t)$ is defined to be:

$$\dot{\hat{\sigma}} = \Gamma \text{Proj}(\hat{\sigma}(t), -\hat{\tilde{y}}(t)), \quad \hat{\sigma}(0) = 0,$$ \hspace{1cm} (17)

where $\text{Proj}$ is the projection operator, $\Gamma \in \mathbb{R}^+$ is the adaptation rate, $\hat{\tilde{y}}(t) \triangleq \hat{y}(t) - y(t)$ and the projection is performed with the following bound

$$|\hat{\tilde{y}}(t)| \leq \Delta, \quad \forall t \geq 0.$$ \hspace{1cm} (18)

Control law:

The control signal is generated according to the following law

$$u(s) = C(s)(r(s) - \hat{\sigma}(s)),$$ \hspace{1cm} (19)

where $C(s)$ is the low-pass filter. In the interest of simplicity, the low-pass filter is parameterized as a first-order transfer function:

$$C(s) = \frac{\omega}{s + \omega}, \quad \omega > 0$$ \hspace{1cm} (20)

assuming zero initialization.

Figure 6 illustrates a closed-loop system with $L_1$ adaptive control consisting of Equations (16)–(20) subject to the $L_1$-norm condition in Equation (14).

The ideal control law provides the desired system response by effectively accounting for uncertainties. Thus, the reference system in Equation (16) has a different response as compared to the ideal one. It compensates only for the uncertainties within the bandwidth of $C(s)$, which can be selected to be compatible with the control channel specifications.

2.3.3. Closed-loop system stability

Considering the closed-loop system described by Equations (10)–(12), the choice of $M(s)$ and $C(s)$ can be restricted such that $H(s)$ is stable and Equation (14) holds. The condition in Equation (14) restricts the class of systems $A(s)$ in Equation (8) that can be stabilized by the $L_1$ controller architecture.

Letting

$$A(s) = \frac{A_n(s)}{A_d(s)}, \quad C(s) = \frac{C_n(s)}{C_d(s)}, \quad M(s) = \frac{M_n(s)}{M_d(s)},$$ \hspace{1cm} (21)

it follows from Equation (13) that

$$H(s) = \frac{C_d(s)M_n(s)A_n(s)}{C_n(s)M_d(s)A_n(s) + M_n(s)A_d(s)(C_d(s) - C_n(s))}.$$ \hspace{1cm} (22)

A strictly proper $C(s)$ implies that the order of $C_d(s)$ is smaller than that of $A_n(s)$, the transfer function $H(s)$ is strictly proper.

Using Equations (10)–(12) and (20) it can be shown that

$$y(s) = H(s)(C(s)r(s) + (1 - C(s))d(s)) = H(s)C(s)r(s) + G(s)d(s).$$ \hspace{1cm} (23)

Since $H(s)$ is strictly proper and stable, $G(s)$ is also strictly proper and stable and therefore

$$y(s)_{L_\infty} \leq H(s)C(s)L_{L_\infty}r(s)_{L_\infty} + G(s)L_1(Ly(s)_{L_\infty} + L_0).$$ \hspace{1cm} (24)
Thus,
\[ y(s)\mathcal{L}_\infty \leq \frac{H(s)C(s)L_rL_s + G(s)L_eL_0}{1 - G(s)L_eL}. \] (25)

Using the condition in Equation (14), one can write,
\[ y(s)\mathcal{L}_\infty \leq \rho, \quad \rho = \frac{H(s)C(s)L_rL_s + G(s)L_eL_0}{1 - G(s)L_eL} < \infty. \] (26)

Hence, \[ y(s)\mathcal{L}_\infty \] is bounded. It follows that if \( C(s) \) and \( M(s) \) verify the condition in Equation (14), the closed-loop reference system in Equations (10)–(12) is stable.

To determine the classes of systems that can satisfy Equation (14) via the choice of \( M(s) \) and \( C(s) \), we consider the first-order \( M(s) \) and \( C(s) \) as specified in Equations (9) and (20). It follows from Equations (9) and (20) that
\[ H(s) = \frac{m(s + \omega)A_r(s)}{\omega(s + m)A_n(s) + msA_d(s)}. \] (27)

Next, consider stabilization of \( A(s) \) by a PI controller, say, of the following structure
\[ PI(s) = \left( \frac{\omega}{m} \right) \frac{(s + m)}{s}, \] (28)

where \( m \) and \( \omega \) are the same as in (9) and (20). The open loop transfer function of the cascaded \( A(s) \) with the PI controller will be,
\[ H_{PI}(s) = \left( \frac{\omega}{m} \right) \frac{(s + m)}{s} A(s) \] (29)
resulting in the following closed-loop system:
\[ H_{PI}(s) = \frac{\omega(s + m)A_r(s)}{\omega(s + m)A_n(s) + msA_d(s)}. \] (30)

Hence, the stability of \( H(s) \) (27) is equivalent to that of Equation (30), and the problem can be reduced to identifying the class of \( A(s) \) that can be stabilized by a PI controller. It also permits the use of root locus methods for checking the stability of \( H(s) \) via the open loop transfer function \( H_{PI}(s) \).

The stability equivalence of the \( L_1 \) controller with appropriate \( M(s) \) and \( C(s) \) to that of a PI controller means that any system that can be stabilized by a PI controller belongs to the class of systems that can satisfy Equation (14) via the choice of \( M(s) \) and \( C(s) \). Noting that SMA systems have been successfully controlled and stabilized using PI controllers (Hannen, Crews, & Buckner, 2012), the choice of \( M(s) \) and \( C(s) \) in this paper therefore ensures the stability of the \( L_1 \) controller-based closed loop system.

2.4. \( L_1 \) control parameter optimization: grid search

The user-specified parameters of the \( L_1 \) controller include the low-pass filter \( C(s) \), the reference model \( M(s) \), and the adaptation rate \( \Gamma \). Intuitively, \( C(s) \) should be chosen such that its bandwidth does not exceed that of the actuator. In practice however, maximum controller bandwidth does not necessarily translate to optimal tracking performance. The adaptation rate \( \Gamma \) is essentially the gain of the adaptive estimator, and since the control signal is low-pass filtered, relatively large gain values can be used. For real-time controller implementation on a computer, \( \Gamma \) is limited by the stability of the numerical integration method, which is determined largely by available computational capabilities. As discussed in (Michini and How, 2009), the choice of \( M(s) \) is not so straightforward for achieving the desired specifications. Given these characteristic limitations, no intuitive method has been established to find combinations of \( C(s) \), \( M(s) \) and \( \Gamma \) that yield optimal tracking performance for specific application. This brings about the need to employ optimization techniques to determine the best \( L_1 \) control parameter set.

Generally, optimization combines the desired performance metrics into a cost function that can be minimized in an effort to find the best combination of \( C(s) \), \( M(s) \) and \( \Gamma \). The optimization process calculates a cost function and searches for the parameter set that minimizes the cost function subject to the relevant constraints (like system stability, available bandwidth, etc.). Using the parameterization of \( C(s) \) and \( M(s) \) given in Equations (9) and (20), a multi-dimensional grid search can be used to find \( \omega \), \( m \) and \( \Gamma \) that yield the least mean square error (MSE) for a given reference trajectory. The MSE is defined as
\[ MSE = \frac{1}{n} \sum_{i=1}^{n} (\theta_i - \theta_m)^2, \] (31)

where \( \theta_i \) is the reference bending angle and \( \theta_m \) is the measured or simulated bending angle. Figure 7 shows the general diagram of the optimization process.

In addition to minimizing the cost function, two constraints must be considered. First, filter parameter \( \omega \) is constrained between \( \omega = 0 \) (where the filter acts as a pure integrator) and \( \omega = 50 \) (where the actuator bandwidth becomes limiting). The second constraint addresses stability of the closed-loop system. One simple way to implement this constraint is to augment the cost function with a term that is arbitrarily large if the system is unstable. Instability, in this case, refers to a closed-loop system response that either results in a simulation singularity or diverges from the desired profile as the simulation time increases.

Optimization was conducted using a grid search technique (Ensor and Glynn, 1997), which finds the minimum of a multivariable function over \( \Lambda \subseteq \mathbb{R}^d \), the parameter space over which we wish to optimize. Let \( \alpha : \Lambda \rightarrow \mathbb{R} \) be a real-valued function that, for each \( \lambda \in \Lambda \), measures the performance of the system. The goal, then, is to minimize \( \alpha \) over \( \Lambda \). To numerically optimize \( \alpha \) over \( \Lambda \), we approximate \( \Lambda \) by some finite set of \( m \) points \( \lambda_m = \ldots \).
\{\lambda_1, \ldots, \lambda_m\} \subseteq \Lambda \text{ and then compute } \alpha \text{ over } \Lambda_m. \text{ The minimum of } \alpha \text{ over } \Lambda_m \text{ is then taken as the minimum of } \alpha \text{ over } \Lambda. \text{ Since } \Lambda_m \text{ is frequently taken to be a discrete grid (when } \Lambda \text{ is a hyper-rectangle), we refer to this approach as a ‘grid search’ for the minimum. In the bending actuator control application, } \alpha \text{ is the } \text{MSE (31) between the reference angle and the simulated angle, evaluated for a single run at each } \Lambda_m. \text{ Each grid dimension has a range of values which are divided into a set of equal-value intervals. The multidimensional grid has a centroid, which locates the optimum point. The search involves multiple passes and in each pass, the method finds the node with the lowest MSE. This node becomes the new centroid and builds a smaller grid around it. Successive passes result in the multidimensional grid shrinking as the centroid keeps moving towards the optimum point. The cost as a function of the three parameters can be visualized using a contour plot, making the process more intuitive to the designer.}

2.5. Implementation environment

The $L_1$ adaptive controller and plant model were simulated using Simulink (The MathWorks, Inc., Natick, MA). For real-time implementation, the $L_1$ controller was programmed in Visual Studio C++ (Microsoft Corporation, Redmond, WA) and set to run at 60 Hz, a sample rate that was governed by the capability of the TrakSTAR position sensor.

3. Results

3.1. Simulation results

Transient step response data obtained from the experimental setup, shown in Figure 2, were used to identify the SMA Joule heating time constant: $\tau = 0.25$ s. The reader is referred to (Crews, Smith, Pender, Hannen, & Buckner, 2012; Hannen et al., 2012) for more details on the experimental determination of related SMA temperature response model parameters.

The HRNN weights were optimized using 1071 equally spaced neurons. Figure 8 shows the training and testing data, originally published in Hannen et al. (2012), along with the trained HRNN prediction. The optimal solution resulted in a training cost of $2.8 \times 10^{-4}$ and a testing cost of $2.7 \times 10^{-4}$, reduced from initial values of 0.47.

The optimized HRNN weights and transition temperatures were implemented in the $L_1$ simulation environment to represent the constitutive relationship between SMA temperature and bending angle. The grid search algorithm identified the $L_1$ control parameter set $(m, \omega, \Gamma)$ that minimized the error cost function for a sinusoidal reference (amplitude 20 degrees, frequency 0.1 Hz).
Figure 9. Optimization results of the $L_1$ control parameters showing (a) the 3D optimization search space and (b) the 2D visualization of the $\omega$ and $m$ grid progressions.

Table 1. Simulation and experimental $L_1$ parameters.

| Parameter | Description | Optimization range | Simulation values | Experimental values |
|-----------|-------------|---------------------|-------------------|---------------------|
| $m$       | Reference model | [1–50]              | 1.94              | 1.94                |
| $\omega$  | Low-pass filter | [1–50]              | [16–50]           | 45                  |
| $\Gamma$  | Adaptive gain  | [100–1000]          | 500               | 500                 |

Figure 9 shows a 3D representation of the grid search’s convergence to the optimal solution.

Table 1 summarizes the resulting $L_1$ controller parameters. It shows the parameter ranges used in the grid search and the values that were subsequently used in the simulations and experimental implementation. With the reference model $m$ set to the grid search optimum of 1.94, simulation results revealed that tracking performance is relatively insensitive to low pass filter parameters $\omega$ in the range of 16–50. This can be seen in the flatness of the surface along the $\omega$ axis at around $m = 1.94$ as illustrated in Figure 9(a). This result helps define the minimum low-pass filter bandwidth for the $L_1$ controller.

Controller performance was simulated using a 0.1 Hz sinusoidal reference, with results shown in Figure 10. Figure 10(a) shows that the plant response $y(t)$ tracks the reference trajectory $r(t)$ reasonably well, but exhibits a lag of approximately 0.55 s due to the SMA’s relatively slow heat transfer dynamics. While this phase lag could be significantly reduced using feedforward control, the tracking performance is otherwise quite good. Figure 10(b) shows the tracking error and Figure 10(c) shows the actuator control voltage $u(t)$.

3.2. Experimental results

Using the simulation-optimized $L_1$ control parameters in the experimental system resulted in reasonable tracking performance for a 0.1 Hz sinusoidal reference bending angle. In order to achieve experimental tracking performance that was comparable to the simulation results, $\Gamma$ and $m$ were set to the simulation-optimized values (500 and 1.94, respectively) while $\omega$ had to be at least 45. The controller parameters that resulted in the best experimental tracking performance are presented in Table 1, and corresponding experimental tracking results for a 0.10 Hz sine wave are presented in Figure 11. Similar results for a triangle wave reference are presented in Figure 12. There were no significant changes in the system tracking performance for $\omega$ values greater than 45 (within the range that worked well for simulation: 16–50).

4. Discussion

Generally, a very strong correlation was observed between the dynamic behaviour of the simulated plant and the experimental SMA-actuated flexible beam, suggesting that the plant model used in simulation effectively captured the nonlinear characteristics of the flexible beam plant. The experimental tracking performance substantiates the viability of the $L_1$ adaptive control scheme to effectively control a highly nonlinear hysteretic system.

Although there is still no intuitive way to select the optimal set of $L_1$ controller parameters, a major a step towards a formal guideline for parameter selection and practical implementation was achieved. The following observations from the grid search parameter selection lay the foundation for the development of $L_1$ adaptive controller guidelines and considerations for both simulation and implementation.
The reference model, \( M(s) \): The reference model defines the desired system behaviour but, unlike those used in MRAC (Astrom and Wittenmark, 1994; Ioannou and Kokotovic, 1982; Krstic, Kanellakopoulos, & Kokotovic, 1995), it serves as a state estimator. The simulated grid search optimization and experimental performance results suggest that, assuming that the reference model \( M(s) \) itself is BIBO stable, the choice of the reference model alone cannot cause system instability. Thus, for transient performance enhancement the choice of \( M(s) \) can be trivialized to choosing a stable reference model whose response time is at least within the bandwidth of the expected reference trajectories.

Adaptive gain, \( \Gamma \): Low gains result in slow adaptation and very poor tracking, while high gains yield rapid adaptation. In general, the simulated and experimental evaluations show that by increasing the adaptation gain \( \Gamma \), the \( L_1 \) adaptive controller tracking performance improves. Thus, it can be concluded that an arbitrarily high adaptation gain can be set, reducing the parameter selection problem to the selection of the reference model \( M(s) \) and the low pass filter \( C(s) \) such that the system has the desired response. However, in almost all real-time implementations that use discretized algorithms running at fixed sample rates, the trade-off between numerical accuracy and fixed sampling interval almost always results in the loss of precision during integration. Therefore, finding the optimal compromise between accurate integration, a feasible sampling time, and numerical stability is of paramount importance. Numerical instability was observed in simulations where the adaptation gain was set excessively high (\( \Gamma > 2000 \)).

Low pass filter, \( C(s) \): While the low-pass filter bandwidth is upper-bounded by the actuator bandwidth, and no intuitive lower bound exists, the lower bound was experimentally determined. The simulation results showed slower system response as the filter bandwidth was reduced. The ideal control signal is the one that leads to the desired system response by compensating for the uncertainties exactly. In practice, however, only the uncertainties within the bandwidth of \( C(s) \), are cancelled, thus, we want the bandwidth of the low-pass filter to be as high as possible but also making sure it is lower than the actuator limit. The need for higher low-pass filter bandwidth in experiment is attributed to fact that the experimental setup, there is an inherent filter between the digital controller output and the analog electronics that supply the SMA wire with the joule heating current. In addition, there are additional filter and delay dynamics associated with the sensor information being fed back into the digital closed

Figure 10. Simulated \( L_1 \) controller tracking results for a 0.1 Hz sinusoidal reference trajectory showing (a) bending angle, (b) tracking error and (c) control input.
Figure 11. Experimental $L_1$ control tracking results for a 0.1 Hz sinusoidal reference trajectory showing (a) bending angle, (b) tracking error and (c) controller generated plant input.

Figure 12. Experimental $L_1$ control tracking results for a 0.1 Hz ramp reference trajectory showing (a) bending angle, (b) tracking error and (c) controller generated plant input.
loop controller. In experimental implementation, in order to realize a certain actuator bandwidth response, the digital filter bandwidth needs to be set much higher so that it compensates for the total effective feedback path delays and response times. Thus, in order to get similar results that the simulation model shows, it is conceivable that the digital filter in the experimental setup needs to have higher bandwidth.

5. Conclusions and future work
This paper demonstrates the implementation of the $L_1$ adaptive control law to regulate the bending angle of a flexible beam actuated by a single SMA tendon. The results demonstrate controller feasibility, showing fast response times and precise tracking of a variety of reference trajectories with the controller effectively compensating for hysteresis in the SMA tendon behaviour. Control parameter guidelines and limitations associated with experimental implementation of the output feedback $L_1$ controller realistic control applications are presented. The proposed parameterization of the $L_1$ parameters $C(s)$ and $M(s)$ enables the implementation of a manageable grid-search optimization, providing the control designer with an intuitive method of linking performance metrics to the selection of the $L_1$ parameters. This design process represents a step in the direction of more readily applying $L_1$ adaptive control to real-world control systems and taking advantage of its potential benefits.

Although SMA-based actuators have been successfully implemented in a number of applications, limitations associated with nonlinear and hysteretic behaviour have presented challenges to the development of robust, high performance, high bandwidth controllers that can be implemented in real-time. While high-performance control can be achieved using high fidelity models that are computationally expensive, such controllers can be difficult to implement in real-time and may have significant bandwidth limitations. This has motivated research into the development of simpler control strategies that can effectively compensate for nonlinearities and hysteresis while providing high performance over a broad frequency range. The performance results presented here suggest an advancement in this direction: a relatively simple but effective adaptive control architecture for systems that exhibit nonlinear and hysteretic behaviour.

A fundamental challenge of $L_1$ adaptive control theory involves optimizing the design of the bandwidth-limited filter, which defines the trade-off between performance and robustness. The design process requires specifying the filter order, parameterizing the filter, and ensuring system stability with a bounded $L_1$ norm (a non-convex constrained optimization problem). While this paper presents preliminary design guidelines with the assumed forms of $C(s)$ and $M(s)$, $L_1$-controller filter design is still an open problem. Another challenge involves specification of the adaptive gain. While increasing this gain can improve the performance bounds and stability margins of the $L_1$ adaptive controller, it is critical to ensure that high gain implementations do not cause numerical instabilities and that the CPU has enough computational power to robustly execute the real-time integration algorithms.

Future work includes exploring the potential benefits that higher order forms of $C(s)$ and $M(s)$ might provide, while preserving the parameterization simplicity presented in this paper. Additional future work will focus on extending the $L_1$ controller to bidirectional SMA systems. An antagonistic pair of SMA tendons can increase actuator bandwidth, limited in the single tendon case by slow tendon cooling. This would involve system model reformulation to incorporate the dynamics introduced by the two parallel SMA tendons and improving the actuator response time.

Using the $L_1$ controller as an adaptive augmentation to an existing base controller is an attractive research topic. This control architecture represents a hybrid system that is more likely to be accepted industry-wide and would provide quantifiable benefits without the additional verification and validation efforts required for certifying new control schemes. Anticipated implementation challenges include analysis of the $L_1$ adaptive controller performance bounds in the presence of input saturation. This might require appropriate modification of the state predictor to remove the effect of the control deficiency from the adaptation process.

Disclosure statement
No potential conflict of interest was reported by the authors.

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