A Review on Strut and Tie Methods

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Abstract. Strut and tie method (STM) provides simplified solutions to complicated design problems. It was an evolutionary discovery which lead to an improved understanding of shear and flexural deformations of the structures. STM is applicable to the regions of the structure where beam theory is not valid. Numerous studies were conducted in this field, and various modifications are being attached to the conventional STM. Indeterminate models replaced determinate models, and nonlinear analysis replaced linear elastic analysis. New ideas like automatic generation of models based on optimisation techniques, simplified inelastic analysis using secant stiffness method and 3-dimensional analysis were also introduced. This paper summarises some of these studies and discusses their merits and limitations. Some suggestions to simplify these complicated methods are also included.

1. Introduction
Strut and tie method is a consistent tool used for the analysis and design of structural concrete for several decades. Truss analogy suggested by Ritter and Mӧrsch [1,2] was the initial step of the evolution of the STM. Later, so many researchers have studied the possibilities of STM and developed it into a consistent tool. Determinate strut and tie models for several structural elements were developed. Linear elastic analysis was used for the analysis of these models. Hence, the models were able to predict only the elastic behaviour of the structures. One of the limitations of this method was the uncertainty in developing the models for complex loading and geometrical cases. New ideas like nonlinear STM, Topology optimised STM, Grid STM, Secant stiffness method, Compatibility STM were developed to overcome these limitations.

This paper includes a summary of the evolution of STM. Several new STMs are discussed, and their merits and limitations are highlighted.

2. Evolution of STM
The idea of considering cracked reinforced concrete (RC) beam as truss consisting of inclined concrete struts and steel ties was first presented by Ritter [1] in 1899. This truss analogy was based on the observation that cracks develop at an inclination of approximately 45° to the longitudinal axis of the reinforced concrete beams under shear stress. Later Mörsch [2] suggested that a continuous diagonal compression stress field reflects the behaviour of a beam better than that by prismatic strut elements. Mörsch used principal stress trajectories for proportioning the tensile reinforcement and also explained the ability of truss models to duplicate the behaviour of beams with bent up longitudinal bars.
The behaviour of reinforced concrete under shear remained a taxing issue for the researchers and engineers for several years. Since the application of Bernoulli hypothesis is limited to B regions that are away from the discontinuities, the disturbed regions (D regions) were designed using thumb rules or based on the previous experiences instead of sectional analysis. Figure 1 shows some examples of B and D regions. Later it was noticed that the loads at these disturbed regions are transferred to support or adjacent B region by some mechanism. This observation pointed out that STM can be effectively used for the analysis of D regions. These studies initiated the growth of strut and tie method. The evolution of STM is explained in the following sections.

![Figure 1. Examples of B and D regions.](image1)

2.1. Conventional strut and tie method

During '90s, STM was evolved and established as a consistent tool for the analysis and design of structures with both Bernoulli regions and disturbed regions and was recognised in many of the standards around the world [3-6]. Some of the classical works in STM was reported during this period [7-10]. These works explained the analysis and design procedure of several RC members with loading and geometrical complications like beam-column joints, corbels, deep beams and shear walls. It was recommended to use the elastic stress distribution or load path methods to develop determinate models in STM. In all these works, the capacity of the model was assessed based on the plasticity theorem which states that the stress field satisfying force equilibrium and boundary conditions and does not cause yielding at any point gives a lower bound solution for the capacity of elastic-perfectly plastic material. Since concrete is not such a material, it can endure only limited plastic deformations. Hence, one cannot choose any arbitrary strut and tie model for a structure but have to select the best model which has maximum stiffness and minimum weight. After the formulation of the proper strut and tie model for a structure or structural element, linear elastic analysis was conducted for obtaining the internal forces of each member. These member forces were checked with the corresponding member strength calculated [3-7]. Intersection points of these members termed as nodes were also checked with its strength. A conventional STM model for RC beam is shown in figure 1 (a). The required steel for carrying the tensile forces in tie elements of the model was designed and detailed.

![Figure 2. Different strut and tie models (a) Conventional (b) Model by Yun [13] (c) GSTM model[20].](image2)
2.2. Nonlinear STM

Basically, conventional STM uses force-based failure criteria and does not consider the inelastic deformations. The ability of determinate models in reflecting the exact load transfer mechanism of structures in certain complex cases was questionable. These limitations lead to the development of nonlinear STM and the use of indeterminate strut and tie models [11-16]. Yun [13] used nonlinear techniques in different stages of the proposed design algorithm. The model for an RC beam developed using the method is shown in figure 2 (b). Steel struts and concrete ties were positioned additionally at the locations of concrete struts and steel ties. The method was able to predict the load-deformation in the elastic and inelastic range of the structure precisely and properly design the complicated RC structures. Another important work in nonlinear STM was conducted by To et al. [16]. A specialised computer program was developed for the sectional analysis in this method. Nonlinear properties of concrete and steel were considered in the finite element analysis used in this method. To et al. [16] expanded the study from monotonic analysis to cyclic and dynamic analysis of complicated RC members. These nonlinear STMs were able to predict the behaviour of complex structures with increased precision. But the complexities and prolonged processing time made these methods impracticable for the generous use among the engineers.

2.3. Topology optimised STM

Another limitation of the conventional STM was the uncertainty in the selection of the model for each structural element. Various models were recommended in the classical works and codes for some of the important structural elements. But those were suitable only for certain geometrical and loading conditions which belonged to the domain studied up to then. Complexities like openings and different loading cases lacked proper strut and tie models and design recommendations. To overcome this constraint, topology optimisation methods [17-19] were adopted. Ali et al. [17] considered a ground structure as the model of the structure initially. Ground structure consisted of numerous truss elements which would be then optimised to the appropriate model (figure 3). Optimisation was based on minimum volume, and two factors (practicality factor and stress redistribution factor) were introduced to control the iterative procedure. Complex structural elements like deep beam with web opening, dapped end beam, beam-column joints and shear wall with openings were analysed using the method.

Another major work in this field was by Liang et al. [18] which was a performance-based optimisation procedure. It involved complicated iterative procedures comprising strain energy calculations and optimisation of weight of the model. Later the work was extended from the analysis of complex RC structural elements to prestressed concrete elements [19]. Optimisation methods were accurate and applicable to complex structures but were complicated and time-consuming. The whole iterative procedure must be repeated for each load cases to obtain the precise model for each load case.

2.4. Grid STM

A novel idea for obtaining a strut and tie model which can be used for different load cases of a structure was presented by Yun et al. [20]. It was termed as 2-Dimensional Grid Strut and Tie Method (GSTM). The structure was divided into several grids to construct the model (figure 2 (c)). Size of grid elements was governed by geometric compatibility. Strength of the members was optimised in the method, and a
computer program was developed for all the iterative procedures. The method appeared to be a valid
and improved analysis and design tool except for the need of the special program for carrying out all the
graphical decision, optimisation and numerical calculations.

2.5. Secant stiffness based STM
As a simplified method for the inelastic analysis of strut and tie model, Park et al. [21] proposed secant
stiffness method which was originally used by Park et al. [22] for the earthquake design of RC
structures. Portions of the structure which would undergo inelastic deformations are identified first.
Secant stiffness is allocated for the element at these regions and elastic stiffness for the elements where
only elastic deformation is anticipated. Hence basically, the method depends on the design strategy
adopted by the engineer. The method was able to predict the load-deformation behaviour of structures
using the linear analysis of the strut and tie model. The concept of replacing nonlinear analysis with
linear analysis using secant stiffness is explained in figure 4. This simplified method was not applicable
in complex cases having buckling and fracture failure modes, because of the difficulties in attaining the
accurate models.

![Comparison of nonlinear analysis and linear analysis using secant stiffness](image)

**Figure 4.** Comparison of nonlinear analysis and linear analysis using secant stiffness [21].

2.6. Compatibility STM
Yet another way for using strut and tie model was proposed by Scott et al. [23] namely compatibility
STM. It was a nonlinear analysis method which considered both the reinforcement and concrete in each
element of the model. Concrete and steel areas were constrained together as one element and assigned
separate stiffness for the same. Truss action and arch action were combined together to form the
compatibility strut and tie model (figure 5). Inelastic response of each element was attained using the
nonlinear constitutive relationship of cracked concrete. This method was not capable of handling
complex problems due to the complications in selecting suitable strut and tie models.

![Compatibility STM modelling](image)

**Figure 5.** Compatibility STM modelling [23] (a) Arch action (b) Truss action (c) Compatibility model.
2.7. Evaluation and modification of existing codal recommendations

There were studies conducted to check the sensitivity of the STM recommendations in various standards. Tuchscherer et al. [24] compared the recommendations in ACI 318-11, AASHTO LRFD and CEB-FIP [25-27]. The work concluded that CEB-FIB recommendation gives more realistic results and the other two standards are overly conservative in the deep beam design. Some modifications regarding the recommendations on the calculation of the strut strength and nodal strength were also suggested.

2.8. Latest works in STM

As the latest works in this field, two more studies are discussed in this paper. The extended work of Yun et al. [28] in the field of GSTM is the first one. The work proposed a 3-dimensional GSTM for the design of concrete structures. The method was assuring more accuracy and understanding of structural behaviour. The complexities and processing time was also increased for this extension work. Generous use of this method among engineers remains impractical. Another new trend was presented by Meléndez et al. [29]. The work proposed a refined 3-dimensional strut and tie model for four-pile caps. The major innovative modification was the usage of optimisation technique to arrive at the precise inclination of the strut elements. For simplicity, the determinate model was used, and shear and flexural failure criteria were considered simultaneously, unlike existing strut and tie models of pile caps. The work was purely limited to four-pile cap without shear reinforcement.

3. Discussions

All of the STMs explained in this paper have improved accuracy than the conventional STM, but are complicated and inappropriate for general use among engineers. Some of the modifications that may be used to simplify the methods are suggested below.

i. Geometrical compatibility of the GSTM can be achieved in a simplified way if grids are developed by considering all the main reinforcement in the structure along with a portion of concrete bonded to it as the elements of grids.

ii. Consider the diagonal concrete portion inside each grid as the strut elements of the model.

iii. Using linear elastic analysis of the model created using the above modifications may be enough for a satisfactory prediction of the load path and failure pattern.

iv. For the design purpose, the initial model may be developed from the minimum reinforcement throughout the structure.

4. Conclusions

Strut and tie method has evolved from a simple truss analogy to a complicated nonlinear analysis tool. Several modifications are being introduced to the conventional STM. Improvements like the generation of strut and tie models using optimisation techniques and the ability to predict the nonlinear load-deformation behaviour were achieved by the numerous works that are discussed in this paper. But since now, none of these methods has reached to the practising engineers as a generous analysis and design tool. This is due to the complications of the iterative procedures included, tedious calculations and unavailability of specialised computer programs. Some modifications that can be applied to simplify these methods are also discussed.

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