Observation of the Little-Parks Oscillations in a System of Asymmetric Superconducting Rings

A. A. Burlakov, V. L. Gurtovoi, S. V. Dubonos, A. V. Nikulov, and V. A. Tulin
Institute of Microelectronics Technology and High Purity Materials,
Russian Academy of Sciences, 142432 Chernogolovka, Moscow District, RUSSIA.

Little-Parks oscillations are observed in a system of 110 series-connected aluminum rings 2 \( \mu m \) in diameter with the use of measuring currents from 10 \( nA \) to 1 \( \mu A \). The measurements show that the amplitude and character of the oscillations are independent of the relation between the measuring current and the amplitude of the persistent current. By using asymmetric rings, it is demonstrated that the persistent current has clockwise or contra-clockwise direction. This means that the total current in one of the semi-rings may be directed against the electric field at measurement of the Little-Parks oscillations. The measurements at zero and low measuring current have revealed that the persistent current, like the conventional circulating current, causes a potential difference on the semi-rings with different cross sections in spite of the absence of the Faraday’s voltage.

The Little-Parks experiment [1], equally with the works [2, 3], is among the first observations of the quantization effects in superconductors, which follow from the Ginzburg-Landau theory [3] and the fluxoid quantization postulated by London [4] for explaining the Meissner effect. According to the second equation of the Ginzburg-Landau theory [3] and the fluxoid quantization effects in superconductors, which follow from the energy density of the superconducting state increases because of non-zero velocity

\[ j_c = \frac{2e}{m} |\Psi|^2 (\hbar \nabla \varphi - 2eA) = 2e |\Psi|^2 v \]  

(1)

and the requirement that the complex pair wave function \( \Psi = |\Psi| \exp(i\varphi) \) must be single-valued \( \oint dl \nabla \varphi = 2\pi m \) the superconducting current density \( j_s \) and the pair velocity \( v \) along a closed contour \( l \),

\[ \oint dl v = \frac{2e}{m} (n\Phi_0 - \Phi) \]  

(2)

cannot be equal to zero if the pair density \( |\Psi|^2 \) along the whole contour \( l \) is nonzero and if the magnetic flux within \( l \) is not divisible by the flux quantum \( \Phi_0 = \pi \hbar / e \), i.e. if \( \oint dl A = \Phi \neq n\Phi_0 \). The energy density of the superconducting state increases because of non-zero velocity

\[ f_{GL} = (\alpha + \frac{mv^2}{2}) |\Psi|^2 + \frac{\beta}{2} |\Psi|^4 \]  

(3)

and reduces the superconducting transition temperature \( T_c(v) \) corresponding to \( \alpha + mv^2/2 = \alpha_0[T/T_c(v = 0) - 1] + mv^2/2 = 0 \) [5]. The idea of the Little-Parks experiment [1] is based on the dependence of the critical temperature \( T_c(v) = T_c(v = 0)[1 - mv^2/2\alpha_0] = T_c(v = 0)[1 - (\xi^2(0)/\lambda^2)(n - \Phi/\Phi_0)^2] \) of a thin-walled (the wall thickness is smaller than the penetration depth of magnetic field: \( w \ll \lambda_L \)) superconducting cylinder, whose radius \( r \) is comparable with the correlation length \( \xi(0) = \hbar^2/2m\alpha_0 \). The paper by Little and Parks [1] was entitled “Observation of Quantum Periodicity in the Transition Temperature of a Superconducting Cylinder.” But in reality the authors observed not the periodicity of the transition temperature \( T_c(\Phi/\Phi_0) \) but the periodicity of the resistance \( R(\Phi/\Phi_0) \) of the cylinder at \( T \approx T_c \), where \( R(T) \) is non-zero, \( R(T) > 0 \), but smaller than the resistance in the normal state, \( R(T) < R_n \). This means that the Little-Parks effect is of a fluctuation nature, because, without fluctuations, the superconducting pair density is nonzero, i.e., \( |\Psi|^2 > 0 \), only in the superconducting state at \( T < T_c \) where \( R(T) = 0 \). Fluctuations in superconductors became the object of intensive research [6] several years after the publication by Little and Parks [1]. Therefore, in [1], and also in [5] (presumably, by tradition), a purely empiric relationship between the periodicities of resistance and critical temperature was employed: \( \Delta R(\Phi/\Phi_0) \approx (dR/d(T - T_c))\Delta T_c(\Phi/\Phi_0) \)

The normal state at \( T < T_c \) and the superconducting state at \( T > T_c \) have a finite probability \( P(\Psi) \propto \exp(-F_{GL}/k_BT) \) [5,7] because of the thermal fluctuations. Therefore, the thermodynamic average density of superconducting pairs \( |\Psi|^2 \neq 0 \) and the circulating persistent current \( I_p = s2e|\Psi|^2v \neq 0 \) is observed at \( \Phi \neq n\Phi_0 \) (when the pair velocity \( v \neq 0 \) according to (2)) in the crit-

![FIG. 1: Structure used for the observation of the Little-Parks resistance oscillations \( V(\Phi/\Phi_0) = V(\Phi/\Phi_0)/I_{ext} \).](image-url)
The discrepancy should be especially significant, if the persistent current $I_p$, as the conventional circulating current, has a direction, because, in this case, at $I_{ext} < 2I_p$, the total current in one of the semi-rings $I_w = I_{ext}/2 - I_p$ (or $I_n = I_{ext}/2 - I_p$) will be directed against the electric field $E = -\nabla V$ (Fig. 1). Because of a possibility of such the paradoxical situation the measurement of the Little-Parks oscillations with the $I_{ext}$ value as small as possible may have fundamental importance.

To measure the oscillations $\Delta V(\Phi/\Phi_0)$ at small $I_{ext}$ values, we used a system consisting of a great number of aluminum rings (Fig. 2). In order to prove that the persistent current, like the conventional current, has a direction, we used the system of asymmetric rings. The nanostructure shown in Fig. 2 was fabricated by the lift-off method by depositing a thin aluminum film of thickness $d = 20 \text{ nm}$ on a Si substrate. The lithography was performed using a JEOL-840A scanning electron microscope, which was transformed into a laboratory electron lithograph by the NANOMAKER program package. The structure consists of 110 series-connected asymmetric rings with the same inner diameter $2r = 1.9 \mu m$ and the semi-ring widths $w_n \approx 0.4 \mu m$ and $w_w \approx 0.2 \mu m$ (Fig. 2). The cross sections of the semi-rings are $s_n = w_n d \approx 0.008 \mu m^2$ and $s_w = w_w d \approx 0.004 \mu m^2$.

With the London penetration depth in aluminum being $\lambda_L(T) \approx 0.05 \mu m(1 - T/T_c)^{-1/2}$, these cross sections correspond to a weak screening $s_n < s_w < \lambda_L^2(T)$ at $T > 0.7T_c$. The distance between the rings and the width of the strips connecting them is $\approx 0.5 \mu m$. The midpoint of the resistive transition corresponds to $T_c \approx 1.36 K$, the width of the transition is $\Delta T_c(0.1 + 0.9R_n) \approx 0.04 K$, and the maximum slope is $dR/d(T - T_c) \approx 30000 \Omega/K$. The resistance of the structure in the normal state is $R_n \approx 970 \Omega$ the resistance per square is $R_n \approx 1.4 \Omega/\square$, and the resistance ratio is $R(300 K)/R(4.2 K) \approx 1.7$.

The measurements were performed by the four-probe method at a voltage $V - V$ contact pads, and, for the current, the $I_{ext} - I_{ext}$ contact pads were used. The four additional contact pads, which allowed the measurements on parts of the system, were not used in the experiment under discussion.

![FIG. 2: System of 110 series-connected asymmetric aluminum rings with a diameter of 2 \( \mu m \) and half-ring widths of 0.2 \( \mu m \) and 0.4 \( \mu m \). The voltage was measured between the \( V - V \) contact pads, and, for the current, the \( I_{ext} - I_{ext} \) contact pads were used. The four additional contact pads, which allowed the measurements on parts of the system, were not used in the experiment under discussion.](image)

![FIG. 3: Little-Parks oscillations measured with different values of the current: $I_{ext} = (1) 100$, (2) 200, (3) 300, (4) 400, (5) 500, and (6) 600 nA, at temperature $T = (1) 1.3530$, (2, 3) 1.3528, (4) 1.3515, (5) 1.3507, (6) 1.3503 K corresponding to the lower part $R(T) \approx 0.2R_n$ of the resistive transition (See Fig. 6).](image)
resistive transition occurs for $T$ showed that a noticeable displacement $\Delta R$ shifts the resistive transition and causes a resistance variation from $1\mu A$ to $2\mu A$ was used to measure the resistance versus magnetic field $R(B)$ (the Little-Parks oscillations) and temperature $R(T)$. The signal was used to obtain the dependences of the dc voltage on magnetic field $V_{dc}(B)$. The voltage was measured across the potential contact pads $V$ (Fig. 2) by an instrumental amplifier with a gain of 1000 and an input-normalized noise level of 20 $nV$ in the frequency band from 0 to 1 Hz. Then, the signal was supplied to an SR560 preamplifier (Stanford Research Systems), which was used for additional amplification and formation of the required signal frequency band by low-pass and high-pass filters. The temperature was measured by a calibrated resistance thermometer ($R(300K) = 1.5 k\Omega$) with a measuring current of 0.1 $\mu A$. The amplified voltage taken from the sample and the signals that were proportional to the current passing through the sample, to the magnetic field, and to the temperature were simultaneously digitized by a 16-digit A/D converter with eight differential inputs.

To estimate the persistent current in the critical region $T \approx T_c$, we used the fact that not only the persistent current $I_p$, but also the measuring current $I_{ext}$ shifts the resistive transition and causes a resistance variation $\Delta R$. The measurements of $R(T)$ performed by us with measuring currents $I_{ext}$ from 20 $nA$ to 2 $mA$ showed that a noticeable displacement $\Delta T_c(I_{ext})$ of the resistive transition occurs for $I_{ext} > 100 nA$ (at $I_{ext} = 200 nA$, $\Delta T_c(I_{ext}) \approx 0.001 K$). In accordance with the $T_c$ change the resistance increases with increasing $I_{ext}$ when $I_{ext} > 100 nA$ (Fig. 3). At $I_{ext} = 300 nA$, this increase is close to the amplitude of the $\Delta R(\Phi/\Phi_0)$ oscillations observed in the experiment (Fig. 3). This suggests that the amplitude $I_{p,A}$ of the persistent current oscillations is no smaller than 100 $nA$ at this temperature, corresponding to the lower part of the resistive transition, see Fig. 6. This $I_{p,A}$ value is approximately an order of magnitude smaller than the value obtained from magnetic susceptibility measurements on a similar ring (at $T \approx T_c$, $I_{p,A} \approx 1 \mu A$) [11] and is closer to the theoretical value predicted in [10]. For comparison, according to the results of magnetic susceptibility measurements [11] and the measurements of the critical current oscillations [12,13] in similar aluminum rings the amplitude of the persistent current should be $I_{p,A} \approx 100 \mu A(1 - T/T_c)$ in the superconducting state, i.e. at $T < T_c$.

Our measurements showed that the $\Delta R(\Phi/\Phi_0)$ oscillations are qualitatively independent of the ratio $I_{ext}/I_{p,A}$ of the measuring $I_{ext}$ and the persistent current $I_{p,A} \approx 100 nA$ values (Fig. 3). An increase in the $\Delta R(\Phi/\Phi_0)$ oscillation amplitude from $\Delta R_A \approx 50 \Omega$ at $I_{ext} = 100 nA$ to $\Delta R_A \approx 90 \Omega$ at $I_{ext} = 600 nA$ (Fig. 3) is caused by the difference in the slopes of the resistive transition $dR/d(T - T_c)$ at $R(T - T_c) \approx 0.2 R_n$ and at $R(T - T_c) \approx 0.5 R_n$ (Fig. 6) and corresponds to approximately equal values of the critical temperature oscillation amplitude $\Delta T_{c,A}/T_c \approx 0.0025$ observed in the experiment is close to the $\Delta T_c/T_c = (\xi^2(0)/r^2)(n - \Phi/\Phi_0)^2$ oscillation amplitude obtained from the theory [5]: $\Delta T_{c,A}/T_c = (\xi^2(0)/r^2)(1/4) \approx 0.004$ at the ring radius $r \approx 1 \mu m$ and the correlation length $\xi(0) \approx 130 nm$ (which was determined for similar aluminum films in [14]). Despite the fact that we performed the measurements with a system of rings rather than with one ring, the value observed for $\Delta T_{c,A}/T_c$ agrees well with the value estimated in the

FIG. 4: Little-Parks oscillations measured for the opposite directions of the measuring current: $I_{ext} = 50, 10, -10, and -50 nA$ at $T = 1.353 K$. The increase in the amplitude of the oscillations and the displacement of the extrema at $I_{ext} = 10 nA$ are related to the presence of the voltage $V_p(\Phi/\Phi_0)$ at $I_{ext} = 0$ (see Fig. 5). The subtraction of this voltage $[V(\Phi/\Phi_0) - V_p(\Phi/\Phi_0)]/I_{ext}$ leads to the Little-Parks universal dependence at $I_{ext} = 10 nA$ (curve $R \times 10 nA$).

FIG. 5: Voltage oscillations $V(\Phi/\Phi_0)$, measured for different values and directions of the direct current: $I_{ext} = 3, 1, 0, -1, -3 nA$ at $T = 1.358 K$. A constant or sine current generated by a Keithley 6221 precision source was supplied to the current contact pads $I_{ext} - I_{ext}$ (Fig. 2). The constant current from 1 $nA$ to 2 $\mu A$ was used to measure the resistance versus magnetic field $R(B)$ (the Little-Parks oscillations) and temperature $R(T)$. The chemical current was used to obtain the dependencies of the dc voltage on magnetic field $V_{dc}(B)$. The noise level of 20 $nV$ in the frequency band from 0 to 1 Hz. Then, the signal was supplied to an SR560 preamplifier (Stanford Research Systems), which was used for additional amplification and formation of the required signal frequency band by low-pass and high-pass filters. The temperature was measured by a calibrated resistance thermometer ($R(300K) = 1.5 k\Omega$) with a measuring current of 0.1 $\mu A$. The amplified voltage taken from the sample and the signals that were proportional to the current passing through the sample, to the magnetic field, and to the temperature were simultaneously digitized by a 16-digit A/D converter with eight differential inputs.

To estimate the persistent current in the critical region $T \approx T_c$, we used the fact that not only the persistent current $I_p$, but also the measuring current $I_{ext}$ shifts the resistive transition and causes a resistance variation $\Delta R$. The measurements of $R(T)$ performed by us with measuring currents $I_{ext}$ from 20 $nA$ to 2 $mA$ showed that a noticeable displacement $\Delta T_c(I_{ext})$ of the resistive transition occurs for $I_{ext} > 100 nA$ (at $I_{ext} = 200 nA$, $\Delta T_c(I_{ext}) \approx 0.001 K$). In accordance with the $T_c$ change the resistance increases with increasing $I_{ext}$ when $I_{ext} > 100 nA$ (Fig. 3). At $I_{ext} = 300 nA$, this increase is close to the amplitude of the $\Delta R(\Phi/\Phi_0)$ oscillations observed in the experiment (Fig. 3). This suggests that the amplitude $I_{p,A}$ of the persistent current oscillations is no smaller than 100 $nA$ at this temperature, corresponding to the lower part of the resistive transition, see Fig. 6. This $I_{p,A}$ value is approximately an order of magnitude smaller than the value obtained from magnetic susceptibility measurements on a similar ring (at $T \approx T_c$, $I_{p,A} \approx 1 \mu A$) [11] and is closer to the theoretical value predicted in [10]. For comparison, according to the results of magnetic susceptibility measurements [11] and the measurements of the critical current oscillations [12,13] in similar aluminum rings the amplitude of the persistent current should be $I_{p,A} \approx 100 \mu A(1 - T/T_c)$ in the superconducting state, i.e. at $T < T_c$.
approximation of the lowest permitted state, when the
n − Φ/Φ0 value varies from −1/2 to 1/2.

When \( I_{\text{ext}} = 50 \div 300 \, nA \), the amplitude \( \Delta R_A \) varies
insignificantly (Fig. 3), and the measured voltage oscillations
can be described by the general formula
\[
V(\Phi/\Phi_0) = R(\Phi/\Phi_0)I_{\text{ext}} = R_0I_{\text{ext}} + \Delta R(\Phi/\Phi_0)I_{\text{ext}}
\]
where \( R_0 \) is the resistance of the ring without the Little-Parks oscillations
taken into account. This resistance monotonically
varies insignificantly (Fig. 3), and the measured voltage oscillations
in a wide frequency range and that it nonmonotonically
depends on the frequency \( f \) at a given average temperature.
In [15], it was shown that the \( V_A \)
value does not depend on the frequency \( f \) at a given temperature.
Efficiency decreases to 0.2\( R_n \) independently
of the ring number in the system and its resistance \( R_n \)
[13]. For our system of 110 rings with \( R_n \approx 970 \, \Omega \), we
observed the voltage \( V_{dc}(\Phi/\Phi_0) \) oscillations with an amplitude
reaching \( V_{A,max} \approx 0.0045 \, V \), and \( R_{re} \approx 180 \, \Omega \approx 0.2 R_n \)
at the low temperature \( T \approx 1.20 \, K \). Near the beginning
of the transition to the normal state, the rectification efficiency decreases to \( R_{re} \approx 20 \, \Omega \approx 0.02 R_n \), and the decrease continues as the temperature increases
further (Fig. 6). Above \( T \approx 1.350 \, K \) corresponding to
max \( V_{p,A} \approx 0.6 \, \mu V \) the quantity \( R_{re} \) decreases simul-
taneously with \( V_{p,A} \) (Fig. 6), and, on the average, \( V_{p,A}/R_{re} \approx 50 \, nA \). This value estimates the noise amplitude in our system: \( 0_{\text{noise}} \approx 50 \, nA \). Note that the effective noise power equal to
\( (R_n/110)I_{\text{ext}}^2/\text{noise} \approx 2 \times 10^{-14} \, W \)
per one ring, corresponds to the wide frequency spectrum
within which the rectification effect is observed. For comparison,
the total equilibrium power of the Nyquist noise
\( W_{N_q} = k_B T \Delta \omega \approx (k_B T)^2/\hbar \approx 3 \times 10^{-12} \, W \) at \( T = 1.3 \, K \).
in the whole frequency band from 0 to the quantum limit $k_B T / h \approx 1.8 \times 10^{11} \text{ Hz}$ is two orders of magnitude greater. This estimate, as well as the decrease in $V_{p,A}$ to zero in the lower part of the resistive transition (Fig. 6), testifies to a low noise level in our system and suggests that a system of asymmetric superconducting rings can be used as a detector of this kind of noise [16].

Thus, the use of the system with a great number of rings allowed us to observe the Little-Parks oscillations with a measuring current $I_{ext}$ much smaller than the amplitude of the persistent current $I_{p,A}$. This result suggests that the Little-Parks oscillations $R(\Phi/\Phi_0)$ should be observed in the limit $I_{ext} \to 0$, i.e. under equilibrium, which confirms the assumption used in the explanation of this effect [5]. The use of asymmetric rings allowed us to show that the persistent current has a direction and that this direction periodically changes as the magnetic field varies. This means that, when $I_{ext}/2 < I_{p,A}$, the constant component of the total current in one of the semi-rings is directed against the dc electric field $E = -\nabla V$, because the persistent current, unlike the conventional circulating current, is observed in the absence of the Faraday electromotive force, i.e. at $d\Phi/dt = 0$. To reveal the cause and nature of this paradox, additional investigations are necessary.

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