Shot Noise as a probe for the pairing symmetry of Iron pnictide superconductors

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One of the outstanding problems in Iron-pnictide research is the unambiguous detection of its pairing symmetry. The most probable candidates are the two-band $s_{++}$ and sign reversed $s_{\pm}$ wave pairing. In this work the Andreev conductance and shot noise are used as a probe for the pairing symmetry of Iron pnictide superconductors. Clear differences emerge in both the zero bias differential conductance and the shot noise in the tunneling limit for the two cases enabling an effective distinction between the two.

I. INTRODUCTION

Andreev conductance and shot noise across a Metal-Superconductor\textsuperscript{1} or Ferromagnet-Superconductor\textsuperscript{2} have been subjects of extensive research in the past two-three decades. The main purpose of research in such setups is to probe their applications in tasks ranging from detection of pairing symmetry of superconductors\textsuperscript{3} to quantum information processing\textsuperscript{4}. In this respect while conductance calculations have been used extensively to probe the pairing symmetry, there is no record of the use of shot noise in such tasks. Shot noise has been used to measure the unit of transferred charge in fractional quantum hall experiments, in distinguishing particles from waves and as entanglement detector too\textsuperscript{4}. In contrast, probably for the first time, in this manuscript shot noise will be used to detect pairing symmetry of an Iron pnictide superconductor.

The aim of this work is to propose differential conductance and shot noise as a possible discriminator between the two possible $s_{++}$ and $s_{\pm}$ pairing symmetries of Iron based superconductors\textsuperscript{5}. Experimental tests like the half-flux quantum\textsuperscript{6} have utilized Josephson coupling, between an Iron superconductor and a s-wave superconductor, and have managed to zero in on the $s_{\pm}$ pairing but doubts remain\textsuperscript{7}. The spontaneous magnetic flux measured can identify the sign-reversed pairing symmetry($s_{\pm}$) in Josephson junction with Iron-based superconductor. In a recent work, the feasibility of tuning the coupling between two bands of the Iron superconductor was discussed so as to discriminate between the two possible pairing symmetries\textsuperscript{7,8}. The Josephson coupling changes from adding constructively for $s_{++}$ case to canceling destructively for $s_{\pm}$ case due to the $\pi$ phase shift. Thus due to phase sensitivity of Josephson junctions, there is almost complete cancellation of supercurrents from sign-reversed pairing symmetry in Iron pnictide Josephson junctions\textsuperscript{7}. We will also exploit this property in Iron superconductors to discriminate between the two pairing symmetries via the differential conductance and shot noise.

Two tunneling channels in Iron pnictide based junctions are due to the multiband nature of the Iron-superconducting electrode. This gives rise to complicated interference depending on the underlying pairing symmetry\textsuperscript{8}. We show it is the interference of waves reflected from different pairing symmetries of Iron pnictide superconductor junctions which helps in distinguishing between them. The layout of the paper is as follows: in the next section we briefly discuss the competing pairing symmetries in Iron superconductors and how they arise, next we discuss the first of our chosen settings namely a Normal Metal-Insulator-Normal Metal-Insulator-Iron pnictide junction focussing on the wavefunctions, boundary conditions and expressions for differential conductance and shot noise. After this we discuss the second setting a Ferromagnet-Insulator-Normal Metal-Insulator-Iron pnictide junction. This is followed by a discussion on the results for both the settings. We finally conclude with a note on experimental realization of our chosen settings.

II. THEORY OF ELECTRON AND HOLE POCKETS IN IRON SUPERCONDUCTORS

The kinetic energy term of an Iron pnictide superconductor can be derived using a tight binding model\textsuperscript{10}:

\[ H_{\text{Kinetic}} = \begin{pmatrix} \epsilon_x & \epsilon_{xy} \\ \epsilon_{yx} & \epsilon_y - \mu \end{pmatrix}, \]

where \( \epsilon_x = -2t_1 \cos(k_x a) - 2t_2 \cos(k_y a) - 4t_3 \cos(k_x a) \cos(k_y a), \) \( \epsilon_y = -2t_2 \cos(k_x a) - 2t_1 \cos(k_y a) - 4t_3 \cos(k_x a) \cos(k_y a), \) \( \epsilon_{xy} = -4t_4 \sin(k_x a) \sin(k_y a) \) and \( \mu \) denotes the chemical potential with \( a \) being the lattice constant. For the parameters \( t_1 = -1, t_2 = 1.3, t_3 = t_4 = 0.85 \) and \( \mu = 0.45 \) the FeAs (Iron pnictide) band structure is plotted in Fig. 1. The Fermi surfaces obtained by diagonalizing \( H_{\text{Kinetic}} \) are plotted in the unfolded Brillouin zone, it has two electron pockets (or, electron bands) centered at \((0, \pm \pi)\) and \((\pm \pi, 0)\) and two hole pockets (or, hole bands) centered at \((0, 0)\) and \((\pi, \pi)\). If the Iron pnictide superconductor lies on the \( x \) = \( y \) plane, an incident electron at the metal-superconductor interface with small \( p_x \) is transmitted through the electron and hole Fermi surface pockets. In this work we follow the assumption in Ref. \textsuperscript{12} and consider the Andreev reflection problem as envisaged with a Fermi surface consisting of just one hole and one electron pocket. The problem can be generalized to the four pocket Fermi surface shown in Fig. 1 as in Ref. \textsuperscript{10} \textsuperscript{13}. Another important point to note from Fig. 1 is the translation in-variance in the \( y \) direction. The full Hamiltonian of the Iron pnictide superconductor is then a sum of the Kinetic energy term and pairing potential and can be written as:

\[ H = H_{\text{Kinetic}} + V_{\text{pairing}} = \begin{pmatrix} H_{\text{Kinetic}}(k) & \Delta(k) \\ \Delta^*(k) & H_{\text{Kinetic}}(k) \end{pmatrix}. \]
The superconducting gap $\Delta(k)$ assumes two different values for the gap $\Delta_e$ and gap $\Delta_h$ in the electron and hole Fermi surfaces. In this work, we concentrate on two alternative scenarios for the pairing symmetry of Iron pnictide superconductor\cite{19} the two band s-wave case $s_{++}$ in which $\Delta_e$ and $\Delta_h$ have same sign and contrast it with the two band $s_{\pm}$-wave case for which $\Delta_e$ and $\Delta_h$ take on opposite signs.

III. METAL-INSULATOR-METAL-INSULATOR-IRON PNICTIDE SUPERCONDUCTOR JUNCTION

In Fig. 2, we show the first of our chosen settings to detect the pairing symmetry of Iron pnictide superconductor. The normal metal $N_1$ is at bias voltage $V$ with respect to the metal $N_2$ and Iron pnictide superconductor which are both grounded.

A. Hamiltonian

The Hamiltonian of Iron pnictide superconductor from Eq. 2 is given as below, with $\varepsilon_{k,1}$ and $\varepsilon_{k,2}$ the two electronic energy bands from Eq. 2, while $-\varepsilon_{k,1}$ and $-\varepsilon_{k,2}$ are the two hole energy bands with $H\Psi = E\Psi$, where

$$H = \begin{bmatrix}
    \varepsilon_{k,1} + U(x) & \Delta_1(k)\Theta(x) & \alpha\delta(x) & 0 \\
    \Delta_1^*(k)\Theta^*(x) & \varepsilon_{k,1} + U(x) & 0 & -\alpha\delta(x) \\
    \alpha\delta(x) & 0 & \varepsilon_{k,2} + U(x) & \Delta_2(k)\Theta(x) \\
    0 & -\alpha\delta(x) & \Delta_2^*(k)\Theta^*(x) & \varepsilon_{k,2} + U(x)
\end{bmatrix},$$

and $\alpha$ is the interband coupling strength between the two bands in Iron pnictide superconductor and $E$ defines the energy of the states. The two bands couple through the interface scattering as long as $\alpha \neq 0$\cite{11}.

B. Wavefunctions and Boundary Conditions

The wavefunctions in metal $N_1$ and $N_2$ are $\Psi_{N_1}$ and $\Psi_{N_2}$. The $N_1$/O/N$_2$/I/Iron-pnictide junction has insulators at $x = -a$ and $x = 0$, the two insulators are described by $\delta$-function potentials:

$$U(x) = U_1\delta(x + a) + U_2\delta(x)$$

with $U_1$ and $U_2$ being the barrier strengths. The Iron based superconductor possesses two superconducting gaps $\Delta_{1,2}$ in both the bands $1$ and $2$\cite{11}. The superconducting phases of the gaps are $\phi_1$ and $\phi_2$. The $s_{\pm}$ pairing model has unequal gaps ($\Delta_1 \neq \Delta_2$) with phases of opposite signs, i.e., $\phi_1 - \phi_2 = \pi$, while $s_{++}$ pairing model has unequal gaps ($\Delta_1 \neq \Delta_2$) but with same sign, i.e., $\phi_1 = \phi_2$.

Similar to the Iron pnictide junction, we consider the metals $N_1$ and $N_2$ to have two distinct bands with the band energies as was also done in Ref. \cite{11}, $\varepsilon_{k,1} = (h^2/2m)(k_F - \pi)^2 - E_F$ and $\varepsilon_{k,2} = (h^2/2m)(k_F - \pi)^2 + E_F$ as in Fig. 1. Further, we assume the hole and electron Fermi surfaces to be circular and of same size although in actuality they aren’t exactly circular. In the Andreev approximation ($E_F \gg \Delta_1, \Delta_2, \bar{E}$), however the additional phase shift in the first band makes no difference to the results at all and therefore in the subsequent calculation we neglect this additional phase shift.

From Eq. 3, the wave functions in the three regions when an electron is incident from the left in band 1 is-

$$\Psi_{N_1}(x) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{ik_Fx} + b_1 e^{-ik_Fx} + a_1 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} e^{ik_Fx} + b_2 e^{-ik_Fx} + a_2 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} e^{ik_Fx}, \text{ for } x < -a,$$
using which all the scattering amplitudes can be determined. In Eqs.(7-10) \( \hat{1} \) is the 2 \( \times \) 2 unit matrix and \( \text{diag} \) and \( \text{off diag} \) denote diagonal and off-diagonal \( 4 \times 4 \) matrices in which these unit matrices are embedded[11]. At this point we also introduce two dimensionless parameters characterizing the system, namely the barrier strength \( z_0 = 2mU_1/k_F \), \( i = 1,2 \) and the interband coupling strength \( \alpha = 2m\alpha/k_F \). From the scattering amplitude \( a_i, b_i, i = 1,2 \) we get the Andreev and normal reflection probabilities as \( \Lambda_{\sigma} = |a_\sigma|^2 \), \( B_{\sigma} = |b_\sigma|^2 \) where \( \sigma = 1,2 \). This procedure of solving the boundary conditions (Eqs. 7-10) is repeated for an electron incident in band 2 of metal \( N_1 \).

C. Conductance and shot noise in \( N_1/\text{I}/N_2/\text{Ip} \) junction

The well known BTK [14] approach to calculate the differential conductance in Normal metal-Superconductor junctions was previously extended to normal-metal-Iron pnictide superconductor junction in Ref. [11]. In this paper, we extend it to address both differential conductance and differential shot noise in both normal metal/insulator/normal metal/insulator/Iron pnictide superconductor as well as Ferromagnet/insulator/normal metal/insulator/Iron pnictide superconductor junction as a means to detect the pairing symmetry of Iron pnictide superconductor. To calculate the currents in the normal metals one has to sum the contributions of electron incident from both bands. The net charge current induced by a voltage drop \( eV \) across the junction \( I_2 \) for electron incident in band \( \Lambda = 1,2 \) is -

\[
I_2 = 2N(0)eV_F \sum_{\sigma=1,2} \int_{-\infty}^{+\infty} \left( 1 - B_{\sigma}(E) \right) [h_{\sigma}(E - eV) - h_{\sigma}(E)] + A_{\sigma}(E) [h_{\sigma}(E) - h_{\sigma}(E + eV)] dE. \tag{11}
\]

The incoming electrons from Iron pnictide superconductor have Fermi distribution \( h_{\sigma}(E) \), while incoming electrons from Normal metal \( N_1 \) have distribution \( h_{\sigma}(E - eV) \). In Eq. (11) \( \Lambda \) is the cross sectional area of the interface, \( v_F \) the Fermi velocity, \( N(0) \) is the density of states at the Fermi energy \( E_F \) and subscript \( \sigma \) in the scattering probabilities describes whether the reflection is from band 1 or band 2 of Iron-based superconductor. After determining the scattering probabilities we calculate the differential conductance from Eq. (11) as-

\[
G(E) \propto \sum_{\sigma=1,2} \int_{-\infty}^{+\infty} \frac{\partial h_{\sigma}(E - eV)}{\partial E} [1 + A_{\sigma}(E) - B_{\sigma}(E)] dE, \tag{12}
\]

where \( \Lambda \) denotes incoming electron from band \( \Lambda = 1,2 \). At temperature \( T = 0 \), Fermi function is a Heaviside theta function.
Thus, we have: 
\[ \frac{\partial G(E-eV)}{\partial E} = \delta(E-eV). \]

The normalized differential conductance of the system at temperature \( T = 0 \) is then [14,16]:
\[
G_{\lambda}(eV) \approx \frac{dI}{dV}_{NM} = \sum_{\sigma=1,2} [1 + A_{\sigma}(eV) - B_{\sigma}(eV)] / T_{NM}.
\]
(13)

where \( T_{NM} \) is the tunneling conductance in the normal state with iron pnictide replaced by a normal metal. The differential conductance for two band (iron based) superconductor thus is given as:
\[
G(eV)/G_0 = \frac{1}{2T_{NM}} \sum_{\lambda=1,2} G_{\lambda}(eV),
\]
(14)

where \( G_0 = \frac{2e^2}{h} \). \( G_{\lambda}(eV) = 1 + A_1(eV) + A_2(eV) - B_1(eV) - B_2(eV) \) for incoming electron in band \( \lambda \) and \( T_{NM} \) is the transmission probability of a Normal metal-Insulator-Normal metal-Insulator-Normal metal junction.

Next, we calculate the shot noise for our junction. Shot noise is defined as the temporal fluctuation in electric current in non-equilibrium(transport) across a system. Unlike thermal noise which vanishes at zero temperature shot noise exists even at zero temperature. This is a consequence of the discreteness of charge. The general result for shot noise power [18] \( P_{11} \) (the double subscript 11 refers to the fact that shot noise is current-current correlation in normal metal) across a normal metal/superconductor junction is:
\[
P_{11} = \frac{2e^2}{\hbar} \sum_{k,l=1,2,x,y,T,\delta,c,\delta,h} W_{k,\delta,c,\delta}(1x,E) f_{\delta}(E) [1 - f_{\delta}(E)],
\]
(15)

where the parameter \( W_{k,\delta,c,\delta}(1x,E) = \delta_{1\delta} \delta_{1\delta} \delta_{1\delta} \delta_{1\delta} - s^{T \delta}(E) s^{T \delta}(E) \) contains all the information about the scattering process, \( s^{T \delta}(E) \) represents the scattering amplitude for a particle of type \( \gamma \) incident from contact \( k \) which is transmitted to contact 1 as a particle of type \( x \) and \( f_{\delta}(E) \) is the Fermi function for particle of type \( \gamma \) in reservoir \( k \). It should be noted that normal metal is contact 1 while superconductor is contact 2. Here \( sgn(x) = +1 \) for \( x = e \), i.e. electron and \( sgn(x) = -1 \) for \( x = h \), i.e., hole. Because of Andreev reflection an electron incident in contact 1 can result in either an electron or a hole leaving contact 1 or 2. We can further simplify the shot noise expression by separating the electron-electron (or, hole-hole) correlations identified as \( P_{11}^{AA} \) and electron-hole (or, hole-electron) correlations as \( P_{11}^{2B} \). Thus, \( P_{11} = P_{11}^{AA} + P_{11}^{2B} \), where \( P_{11}^{AA} = \langle \Delta I_{11} \Delta I_{11} \rangle + \langle \Delta I_{11} \Delta I_{11} \rangle \) and \( P_{11}^{2B} = \langle \Delta I_{11} \Delta I_{11} \rangle + \langle \Delta I_{11} \Delta I_{11} \rangle \). Further \( P_{11}^{AA}, P_{11}^{2B} \) from Eq. (15) can be written as [18]
\[
P_{11}^{AA} = \frac{2e^2}{\hbar} \int sgn(x) sgn(y) dE W_{k,\delta,c,\delta}(1x,E)
\]
\[
T^{T \delta}_{11}(E) W_{k,\delta,c,\delta}(1x,E) f_{\delta}(E) [1 - f_{\delta}(E)] dE,
\]
(16)

in Eqs. (16-17) the scattering probabilities are related to scattering amplitudes, i.e., \( T^{T \delta}_{11}(E) = |s^{T \delta}(E)|^2 \) if \( x = e \) then \( x = h \). Further, at zero temperature, the term \( f_{\delta}(E) [1 - f_{\delta}(E)] \) vanishes, and only the second term in \( P_{11}^{AA} \) and \( P_{11}^{2B} \) remains. After some algebra, the shot noise power can be written as-
\[
P_{11} = \frac{2e^2}{\hbar} \int dE \{ T^{T \delta}_{11}(E) T^{hh}_{11}(E) + T^{eh}_{11}(E) T^{bh}_{11}(E) \}
\]
\[
+ T^{T \delta}_{11}(E) T^{eh}_{11}(E) + T^{eh}_{11}(E) T^{bh}_{11}(E) \}
\]
\[
\cdot \left[ 1 - T^{hh}_{11}(E) \right] + 2 T^{T \delta}_{11}(E) T^{hh}_{11}(E) \}
\]
(17)

Now \( T^{T \delta}_{11}(E) \) is the normal reflection probability \( B(E) \) while \( T^{eh}_{11}(E) \) is the Andreev reflection probability \( A(E) \). Therefore Eq. (18) can be written in terms of \( A \) and \( B \) as-
\[
P_{11} = \frac{2e^2}{\hbar} \int dE \{ A(E) (1-A(E)) + B(E) (1-B(E)) \}
\]
\[
+ 2A(E) B(E) \}
\]
(19)

Eq. (19) is the expression for shot noise power in a Metal-Superconductor junction. In a \( N_1//N_2//I \) iron pnictide superconductor junction due to multi-band structure of iron pnictide superconductor, an incident electron from band \( \lambda = 1 \) or 2 can result in reflection of an electron and hole in bands 1 and 2. Shot noise power can then be defined as \( P_{11} = P_{11}(1)+P_{11}(2) \), where \( P_{11}(\lambda) \) is shot noise power for incident electron from band \( \lambda = 1 \) or 2. Shot noise power derived for N/S junction, Eq. (19), above can be extended to \( N_1//N_2//I \) iron pnictide superconductor junction as follows:
\[
P_{11}(\lambda) = \frac{4e^2}{\hbar} \int dE \sum_{\sigma=1,2} A_{\sigma}(E) [1-A_{\sigma}(E)] + B_{\sigma}(E)
\]
\[
+ 2A_{\sigma}(E) B_{\sigma}(E) \}
\]
(20)

with \( \lambda = 1(2) \) and \( P_{11}(\lambda) = (1/e) \int S_{\lambda} dE \) where \( S_{\lambda} \) being the differential shot noise for incident electron from band \( \lambda \). Differential shot noise for \( N_1//N_2//I \) iron pnictide superconductor junction [4,20] is thus:
\[
S/S_0 = \frac{1}{2} \sum_{\lambda=1,2} S_{\lambda},
\]
(21)
of differential shot noise to differential conductance as
\[ F = \frac{\Delta}{\Sigma} \frac{\Delta}{\Sigma} \] (22-24). Next we study the differential conductance and shot noise in a Ferromagnet-Insulator-Normal Metal-Insulator-Iron pnictide superconductor junction.

\[ \psi_{FM}(x) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} (e^{ik_2x} + b_1 e^{-ik_2x}) + a_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{ik_3x} + b_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} e^{-ik_3x} + a_2 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} e^{ik_4x}, \text{ for } x < -a, \] (22)

\[ \psi_{NM}(x) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} (t_1 e^{ik_2x} + g_2 e^{-ik_2x}) + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} (h_2 e^{ik_2x} + f_1 e^{-ik_2x}) + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} (t_2 e^{ik_2x} + g_1 e^{-ik_2x}) + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} (h_1 e^{ik_2x} + f_2 e^{-ik_2x}), \] (23)

for \(-a < x < 0\), and

\[ \psi_{IP}(x) = c_1 \begin{pmatrix} u_1 \\ v_1 e^{-i\phi_1} \\ 0 \end{pmatrix} e^{ik_3x} + d_1 \begin{pmatrix} v_1 e^{-i\phi_1} \\ u_1 \\ 0 \end{pmatrix} e^{-ik_3x} + c_2 \begin{pmatrix} 0 \\ u_2 \\ v_2 e^{-i\phi_1} \end{pmatrix} e^{ik_4x} + d_2 \begin{pmatrix} 0 \\ 0 \\ u_2 \end{pmatrix} e^{-ik_4x}, \text{ for } x > -a, \] (24)

Similar to Eq. (22-24), we can write wavefunction resulting from electron/der incident in band 2 too. The possible reflection amplitudes are \(b_1\) — normal reflection in band 1, \(b_2\) — normal reflection in band 2, \(a_1\) — Andreev reflection in band 1, \(a_2\) — Andreev reflection in band 2. The electron/hole wave vectors\([21]\) are \(k_{e(h)} = \sqrt{2m(E_F + E + i\sigma h\phi_1)}, \) with \(s = 1\) for spin up and \(s = -1\) for spin down electron/hole. In the

IV. **FERROMAGNET-INSULATOR-METAL-INSULATOR-IRON PNICTIDE SUPERCONDUCTOR JUNCTION**

The Ferromagnet-Insulator-Metal-Insulator-Iron pnictide superconductor setting is shown in Fig. 3 with wave functions: \(\psi_{FM}(x)\), \(\psi_{NM}(x)\) and \(\psi_{IP}(x)\) for the ferromagnet, normal metal and Iron pnictide segments. For a spin up electron incident at the interface from left in band 1, the resulting wavefunctions in various segments are:

\[ \psi_{FM}(x) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} (e^{ik_2x} + b_1 e^{-ik_2x}) + a_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{ik_3x} + b_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} e^{-ik_3x} + a_2 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} e^{ik_4x}, \text{ for } x < -a, \] (22)

\[ \psi_{NM}(x) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} (t_1 e^{ik_2x} + g_2 e^{-ik_2x}) + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} (h_2 e^{ik_2x} + f_1 e^{-ik_2x}) + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} (t_2 e^{ik_2x} + g_1 e^{-ik_2x}) + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} (h_1 e^{ik_2x} + f_2 e^{-ik_2x}), \] (23)

for \(-a < x < 0\), and

\[ \psi_{IP}(x) = c_1 \begin{pmatrix} u_1 \\ v_1 e^{-i\phi_1} \\ 0 \end{pmatrix} e^{ik_3x} + d_1 \begin{pmatrix} v_1 e^{-i\phi_1} \\ u_1 \\ 0 \end{pmatrix} e^{-ik_3x} + c_2 \begin{pmatrix} 0 \\ u_2 \\ v_2 e^{-i\phi_1} \end{pmatrix} e^{ik_4x} + d_2 \begin{pmatrix} 0 \\ 0 \\ u_2 \end{pmatrix} e^{-ik_4x}, \text{ for } x > -a, \] (24)

Similar to Eq. (22-24), we can write wavefunction resulting from electron incident in band 2 too. The possible reflection amplitudes are \(b_1\) — normal reflection in band 1, \(b_2\) — normal reflection in band 2, \(a_1\) — Andreev reflection in band 1, \(a_2\) — Andreev reflection in band 2. The electron/hole wave vectors\([21]\) are \(k_{e(h)} = \sqrt{2m(E_F + E + i\sigma h\phi_1)}, \) with \(s = 1\) for spin up and \(s = -1\) for spin down electron/hole. In the

FM/I/NM/I/Iron-pnictide junction of Fig. 3, the ferromagnet has magnetization defined as \(h(x) = h_0 \Theta(x + a)\), where \(\Theta\) is the Heaviside step function\([22]\) and \(t_{\sigma}, f_{\sigma}, g_{\sigma}, h_{\sigma}\) are the transmission amplitudes in band \(\sigma\), wherein \(\sigma = 1, 2\) and \(u_i, v_i\) with \(i = 1, 2\) are the usual coherence factors defined as before with superconducting gap \(\Delta\). The boundary conditions at the interfaces are:
From Eqs. (25-28), all the scattering amplitudes can be determined when spin up/down electron is incident in band 1 or 2. From this we get the scattering probabilities as $B_1 = |b_1|^2$, $A_1 = (k_{b_1}/k_{c_1})|a_1|^2$, $B_2 = |b_2|^2$, $A_2 = (k_{b_2}/k_{c_2})|a_2|^2$ for spin up electron incident in band 1. Similarly, one can also determine the probabilities: $B_1$, $A_1$ and $B_2$, $A_2$ for electron incident in band 2.

### A. Conductance and shot noise for FM/I/NM/I/Ip junction

The differential conductance for two-band (Iron-based) superconductor normalized [11] by $G_0 = 2e^2/h$ within the BTK formalism for FM/I/NM/I/Iron pnictide is given as:

$$G(eV)/G_0 = \frac{1}{2TFM} \sum_{\lambda=1,2} G_\lambda(eV).$$  \tag{29}$$

where $G_\lambda = (2e^2/h)G_\lambda(eV) = 1 + \frac{A_1(eV) + A_2(eV) - B_1(eV) - B_2(eV)}{2}$ for incoming spin up electron in band $\lambda$ and $TFM$ being the transmission probability through a FM/I/NM/I/Iron pnictide junction. For FM/I/NM/I/Iron pnictide junction the differential shot noise as before can be calculated by generalizing of the Andreev shot noise across a Normal metal-Superconductor junction[20] as follows:

$$S/S_0 = \frac{1}{2} \sum_{\lambda=1,2} S_\lambda,$$  \tag{30}$$

where $S_\lambda = A_1(eV)(1 - A_1(eV)) + B_1(eV)(1 - B_1(eV)) + 2A_1(eV)B_1(eV) + A_2(eV)(1 - A_2(eV)) + B_2(eV)(1 - B_2(eV)) + 2A_2(eV)B_2(eV)$ and $S_0 = (4e^2/h)$, with $S_\lambda$ being the differential shot noise for spin up electron incident in band $\lambda$. The differential Fano factor is defined as ratio of differential shot noise to differential conductance, i.e., $F = (\sum_{\lambda=1,2} S_\lambda)/(\sum_{\lambda=1,2} G_\lambda)$.

### V. RESULTS AND DISCUSSION

In this section, for $s_{++}$ and $s_{\pm}$ pairing in Iron pnictide superconductor, we calculate the differential conductance, differential shot noise and differential Fano factor for the superconducting gap ratio $\beta = \Delta_2/\Delta_1$ as 1.5, first for normal metal/insulator/normal metal/insulator/Iron pnictide junction and then for ferromagnet/insulator/normal metal/insulator/Iron pnictide junction.

#### A. $N_1/I/N_2/I$ Iron-pnictide

![Figure 4: Plot of the differential Conductance in $N_1/I/N_2/I$ Iron-pnictide junction for $s_{++}$ pairing(solid) and $s_{\pm}$ pairing(dashed) vs bias voltage $eV/\Delta_1$ with $\alpha = 1$, $\beta = \Delta_2/\Delta_1 = 1.5$, $k_F a = \pi/2$ with (a) intermediate barrier strengths: $z_1 = z_2 = 1.0$ and (b) tunnel barriers: $z_1 = z_2 = 2.0$.](image)

As a first application of our model, we plot the differential conductance for a $N_1/I/N_2/I$ Iron-pnictide junction vs the bias voltage to illustrate the influence of barrier strengths and also focus on the zero bias limit for both $s_{++}$ and $s_{\pm}$ pairing symmetries. In the zero bias limit ($eV \to 0$) we see a peak for $s_{++}$ whereas a dip is seen for $s_{++}$ pairing as shown in Fig. 4(a) for intermediate barrier strengths. Unlike the zero bias limit, near the band gap edges $\Delta_1$, $s_{++}$ pairing shows a peak while $s_{\pm}$ pairing shows a dip. The differential conductance peaks become more prominent near the band gap edges in case of tunnel barriers, see Fig. 4(b) for $s_{++}$ pairing symmetry but for $s_{\pm}$ pairing symmetry a dip is seen near the band gap edges. With an increase in $z$, i.e., for tunnel barriers there is no peak in the zero bias limit for $s_{\pm}$ pairing.

1. Differential shot noise and differential Fano factor

Several interesting features are found in Fig. 5 where we plot the differential shot noise with respect to barrier strength for different values of interband coupling strength $\alpha$. When interband coupling strength is large($\alpha = 2.0$), with increase of barrier strength ($z \to \text{Large}$) the differential shot noise for $s_{\pm}$ pairing tends to zero whereas the differential shot noise tends to a finite non-zero value for $s_{++}$ pairing as in Fig. 5(a), for bias voltage tuned to the superconducting gap for band 1, i.e., $\Delta_1$. In contrast when interband coupling strength is low($\alpha = 0.3$)
Figure 5: Plot of the differential shot noise in $N_1/I/N_2/I$ Iron-pnictide junction for $s^{++}$ pairing(solid) and $s^\pm$ pairing(dashed) vs barrier strength $z$ ($z_1 = z_2 = z$) with $k_F a = \pi/2$, $\beta = \Delta_2/\Delta_1 = 1.5$ and $eV = \Delta_1$ for (a) $\alpha = 2.0$ and (b) $\alpha = 0.3$.

Figure 6: Plot of the differential Fano factor in $N_1/I/N_2/I$ Iron-pnictide junction for $s^{++}$ pairing(solid) and $s^\pm$ pairing(dashed) vs bias voltage $eV/\Delta_1$ with $\beta = \Delta_2/\Delta_1 = 1.5$, $\alpha = 1$ and $k_F a = \pi/2$ for (a) intermediate barrier strengths: $z_1 = z_2 = 1.0$ and (b) tunnel barriers: $z_1 = z_2 = 2.0$.

Figure 7: Plot of the differential Conductance in $FM/I/NM/I/Iron$-pnictide junction for $s^{++}$ pairing(solid) and $s^\pm$ pairing(dashed) vs bias voltage $eV/\Delta_1$ with $\beta = \Delta_2/\Delta_1 = 1.5$, $k_F a = \pi/2$, $\alpha = 1$ and $h_0 = E_F = 0.9$ for (a) $z_1 = z_2 = 1.0$ and (b) $z_1 = z_2 = 2.0$.

In Fig. 7 we plot the differential conductance for a FM/I/NM/I/Iron-pnictide(lp) junction. First we notice the remarkable similarity between Fig. 4(b) and Fig. 7(b) which is suggestive of the fact that for tunnel barriers magnetization in ferromagnet ceases to play much of a role. For intermediate barrier strength, differential conductance ($G_{s^\pm}$) in the zero bias limit shows a small dip in Fig. 7(a) compared to the $N_1/I/N_2/I/lp$ junction of Fig. 4(a). Differential conductance for $s^\pm$ pairing first increases then decreases with increase of bias voltage in Fig. 7(a) unlike differential conductance for $s^{++}$ pairing which decreases with increase in bias voltage in Fig. 4(a) for $eV < \Delta_1$. $G_{s^{++}}$ has almost similar behaviour as the case of $N_1/I/N_2/I/lp$ junction. The differential conductance is reduced to half for FM/I/NM/I/lp junction (see Fig. 7(b)) as compared to $N_1/I/N_2/I/lp$ junction in Fig. 4(b). The difference between $s^{++}$ pairing and $s^\pm$ pairing is that while for $s^{++}$ there are peaks at $eV \sim \Delta_1$ and $\Delta_2$, for $s^\pm$ the peaks at $eV \sim \Delta_2$ are absent in both $N_1/I/N_2/I/lp$ as well as FM/I/NM/I/lp junction.
**VI. EXPERIMENTAL REALIZATION AND CONCLUSION**

Part of the difficulty in determining pairing symmetry of iron-based superconductors is that different experiments seem to show different results in different doping regimes and in different compounds\[13\]. In certain samples, a small non zero resistance has been observed below \( T_c \) due to the presence of inter-growth defect\[25\] that may affect the experimental results in Josephson junctions. Real measurements are often influenced by thermal noise, which smears the shape of the current near the critical current\[26\]. We can avoid these difficulties by calculating the shot noise in the tunnel limit, i.e., at \( z \to \)Large, where differential shot noise vanishes for \( s_\pm \) pairing but is finite for \( s_{++} \) pairing when the interband coupling strength(\( \alpha \)) is large and shows opposite behavior when \( \alpha \) is small. This is the unique silver bullet, regardless of whether we use normal metal or ferromagnet, the differential shot noise in the tunneling limit vanishes for \( s_\pm \) pairing, while it is finite for \( s_{++} \) pairing in the strong coupling regime.

**VII. APPENDIX**

In this section, we give the analytic expressions for differential conductance and differential shot noise for \( Ni/I/N2/I \) iron pnictide junction. The expressions are valid for both \( s_{++} \) pairing and \( s_\pm \) pairing symmetries. We will use the gauge \( \phi_1 = 0 \), and make explicit use of the internal phase shift by writing \( e^{i(\theta_1 - \theta_2)} \equiv \delta = +1 \) for \( s_{++} \) pairing, while \( \delta = -1 \) for \( s_\pm \) pairing, respectively.

**A. Differential conductance**

The analytic expressions for differential conductance in units of \( \frac{2e^2}{h} \) for \( s_{++} \) and \( s_\pm \) pairing in Metal/Insulator/Metal/Insulator/Iron pnictide superconductor for the intermediate barrier strength \( z \to 1 \), \( k_F a = \pi/2 \) and \( \alpha = 1 \) are given as:

\[
G_{s_{++}} = 10\{2(-39E_d^2 + 6\sqrt{-E_d^2 + E_d^2 + 2\sqrt{-4E_d^3 + 9E_d^2} + 9} - (17 + 4\sqrt{1 - E_d^2}\sqrt{9 - 4E_d^2}) - \sqrt{1 - E_d^2}\sqrt{9 - 4E_d^2} - E_d^2(2049 + 224\sqrt{1 - E_d^2}\sqrt{9 - 4E_d^2}))} \}
\]

\[
G_{s_\pm} = \frac{1}{4}\{2079 + 452E^2 + 624E_d^2\}
\]
\[ G_{s_+} = -10\{ -153 + 2(-9E_d^2 - 30\sqrt{-E_d^4 + E_d^2} + 10 \sqrt{-4E_d^4 + 9E_d^2 + 18\sqrt{1 - E_d^4}\sqrt{9 - 4E_d^2}} \} / (2097) + 452E_d^2 + E_d^2(255 - 624\sqrt{1 - E_d^4}\sqrt{9 - 4E_d^2}) - 224\sqrt{1 - E_d^4}\sqrt{9 - 4E_d^2} \),

where \( E_d = eV / \Delta_1 \). Now for the special case of zero bias limit \( (eV \to 0) \), differential conductance for \( s_{++} \) pairing is \( G_{s_{++}} = 0.657 \) and for \( s_{\pm} \) pairing is \( G_{s_{\pm}} = 2.0 \), which is exactly same as in the Fig. 4(a) for \( N_1/l/N_2/l \) Iron pnictide superconductor.

### B. Differential shot Noise

The analytic expressions for differential shot noise in units of \( s_{h}^{k_{F}a} \) for \( s_{++} \) and \( s_{\pm} \) for \( N_1/l/N_2/l \) Iron pnictide superconductor with bias voltage \( eV = \Delta_1 \) and \( \alpha = 0.3 \) are given as:

\[ S_{s_{++}} = 1800(1 + (1/z^2)) \{ 512676/z^{14} + 507996/z^{12} + 572280/z^{10} + 361546/z^8 + 230221/z^6 + 108040/z^4 + 37405/z^2 + 9000 \} / \{ 83286/z^8 + 19782/z^6 + 51511/z^4 + 3420/z^2 + 10405 \}^2, \]

with \( \beta = \Delta_2 / \Delta_1 = 1.5 \) and \( k_Fa = \pi / 2 \). We get the values for differential shot noise in the tunneling limit at \( z \to \infty \) for \( s_{++} \) pairing is \( S_{s_{++}} = 0.006459 \) and for \( s_{\pm} \) pairing is \( S_{s_{\pm}} = 0.1496 \), which is seen in the Fig. 5(b) for \( N_1/l/N_2/l \) Iron pnictide superconductor.

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