Boosted Top Quark Signals for Heavy Vector Boson Excitations in a Universal Extra Dimension Model

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ABSTRACT

In view of the fact that the $n = 1$ Kaluza-Klein (KK) modes in a model with a Universal Extra Dimension (UED), could mimic supersymmetry signatures at the LHC, it is necessary to look for the $n = 2$ KK modes, which have no analogues in supersymmetry. We discuss the possibility of searching for heavy $n = 2$ vector boson resonances – especially the $g_2$ – through their decays to a highly-boosted top quark-antiquark pair using recently-developed top-jet tagging techniques in the hadronic channel. It is shown that $t\bar{t}$ signals from the $n = 2$ gluon resonance are as efficient a discovery mode at the LHC as dilepton channels from the $\gamma_2$ and $Z_2$ resonances.

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Extra spatial dimensions, introduced rather tentatively in physics during the period 1914 – 1926 [1, 2, 3], had been more or less relegated to the category of arcana till the late 1990’s, when they were re-introduced by Arkani-Hamed, Dimopoulos and Dvali [4, 5] as a possible solution to the insidious hierarchy problem that plagues the Standard Model (SM) and many of its extensions. The subsequent decade saw an outburst of creativity in the field of model building using this concept. Much of that effort was driven by the hope that having extra dimensions of different shapes and sizes, and locating the SM particles in different subspaces of these, could provide the long missing solutions to many of the puzzles inherent in the standard electroweak model. Today, a dozen years after the original proposal [4], some of the early euphoria has worn off. A sober appraisal will show that now there exist many offshoots of the original proposal, each having its own strengths as well as its own drawbacks. One of the most attractive of these suggestions is the model with a single Universal Extra Dimension (UED-5) [6], which can be said to provide a viable new physics alternative with a minimum of new assumptions. As this paper is devoted to considering some signatures of this UED-5 model, a short introduction to its main features seems appropriate.

The UED-5 model is rather close to the primitive model of Kaluza and Klein [7], in that it envisages a single extra compact dimension $x^4$ of space, and allows all the fields of the SM to propagate in this extra dimension, as well as in the canonical four dimensions of Minkowski space. It differs, however, from standard Kaluza-Klein (KK) theory in two crucial aspects:

1. Though the metric tensor contains off-diagonal elements which constitute a four-vector, no attempt is made to identify this with any of the gauge bosons of the SM. This sidesteps an important stumbling block of the KK theory and enables the extra dimension to be much larger than the Planck length. Interactions due to these off-diagonal elements will be completely negligible for elementary particle physics at laboratory energies and need not be considered further.

2. The extra fifth dimension $x^4$ is compactified, with the topology, not of a circle $S(1)$, as in the KK theory, but of a circle folded about one of its diameters, i.e. an ‘orbifold’ $S(1)/Z_2$. It is well known [8] that a fifth dimension in the form of a manifold – in this case a simple circle $S(1)$ – after compactification, cannot give rise to chiral fermions in four dimensions, whereas a $S(1)/Z_2$ orbifold can. Thus, the UED model can accommodate the chiral quarks and leptons of the Standard electroweak theory, which the primitive KK theory could not.

These two changes are enough to permit the construction of a model having heavy KK excitations of every SM particle. After compactification, at tree-level, the particle masses $M_n$ of the $n$-th order KK excitation are given by

$$M_n^2 = M_0^2 + n^2 R^{-2}$$  \hspace{1cm} (1)
where $M_0$ is the mass in five dimensions and $R$ is the radius of compactification of the extra dimension. These KK masses are, of course, determined by the momentum in the fifth direction, which is necessarily discretised as $p_4^{(n)} = nR^{-1}$ by the periodic boundary condition. The integer $n$ is referred to as the KK number. The $n = 0$ modes (i.e. those which never go into the fifth dimension) are identified with the SM particles.

The presence of large numbers of KK excitations leads to dramatic consequences when we consider the running of the SM gauge coupling constants $g_1$, $g_2$ and $g_3$. Since the beta functions change every time a KK threshold $R^{-1}, 2R^{-1}, 3R^{-1}, \ldots$ is crossed, the running over large energy ranges resembles a power-law behaviour rather than the usual logarithmic running \cite{9}. It can be shown that this leads to a meeting (within experimental error) of the three coupling constants at a scale of $\Lambda \simeq 20R^{-1}$, i.e. after crossing about 20 KK thresholds. Accordingly, the minimal UED-5 model should be cut off at the scale $20R^{-1}$, or earlier, where one would expect a unified gauge group to take over. Even if we do not believe in grand unification and put down the triple meeting of the coupling constants to be a coincidence, the theory must definitely be cut off around 40 thresholds, since at such energies the U(1) coupling $g_1$ develops a Landau pole. Note that with a typical experimentally allowed value of $R^{-1} \approx 500$ GeV, the unification point is $\Lambda \approx 10$ TeV, which is rather beyond the kinematic range of the CERN LHC, but far below the traditional grand unification scale. Though serious model building has not really been done in this context, it is obvious that, with such a low unification scale, any grand unified gauge group must include some discrete symmetry to prevent ultra-fast proton decay. However, in the present work we are concerned with the theory well below the cutoff $\Lambda$, which includes lepton and baryon number conservation exactly as in the SM, and hence, such issues are not of primary concern.

Thus, there are two unknown parameters in the theory. One is $\Lambda R$, for which the most reasonable value is around 20, but which can be taken as low as 5 (i.e. just beyond the LHC accessible range\cite{3}) and as high as 40 (the Landau pole in $g_1$). The other free parameter is the size parameter $R^{-1}$, which controls the masses of the KK excitations. Experimental constraints arising from the radiative decay $B \to X_s\gamma$ tell us \cite{10} that the value of $R^{-1}$ must definitely be above 300 GeV, and is likely to be above 600 GeV if the Higgs boson turns out to be as light as is hinted at by electroweak precision data \cite{11}. There is no experimental upper bound, of course, but if $R^{-1} > 1400$ GeV, then the UED-5 model is unable, at the 90% confidence level, to explain the dark matter density extracted from cosmic microwave background data \cite{12}. Taking all this into account, our choice of range is

\[ 400 \text{ GeV} < R^{-1} < 1400 \text{ GeV} \quad \text{and} \quad 5 < \Lambda R < 20 \]

\footnote{This reflects the most conservative view that one should not extrapolate the theory beyond the kinematically accessible experimental limit.}
which stretches the experimental and phenomenological limits within reason, but not to absolute extremes.

The $S^{(1)}/Z_2$ orbifold possesses local translation invariance, as a result of which the momentum in the fifth direction $x^4$ is conserved at the tree level. Noting that $p^{(i)}_4 = n_i R^{-1}$ for the $i$-th incoming particle entering into a reaction, momentum conservation $\sum_i p^{(i)}_4 = 0$ also implies the conservation of KK number, i.e. $\sum_i n_i = 0$. However, the translation invariance is clearly broken globally at the boundaries of the orbifold $S^{(1)}/Z_2$, i.e. at the two ends of the diameter about which folding has taken place. Thus, quantum corrections can break the translation invariance \cite{13} if they involve virtual states which wind around the extra dimension, and are, therefore, sensitive to the size of the compact dimension. Moreover, such propagators receive contributions from the boundary conditions at the so-called orbifold fixed points. This has two immediate consequences at the one-loop level. The first is to provide additional degeneracy-lifting contributions\cite{6} to the masses $M_n$, apart from the $M_0$. These have been calculated by Cheng et al \cite{13} and the corresponding spectrum has recently been automated in a CalcHEP framework by Datta et al \cite{14}. It is important to note that while the overall splitting more or less scales as $R^{-1}$, it also grows logarithmically with $\Lambda R$ – which is not unexpected since all the loop momenta have to be cut off at $\Lambda$.

The second – and more far-reaching – consequence of the breaking of translation invariance is that it is now possible to have non-conservation of momentum in the fifth direction, which implies non-conservation of KK number, i.e. it is possible to have $\sum_i n_i \neq 0$. Nevertheless, there still exists a $Z_2$ symmetry corresponding to interchange of the two ends of the folded circle, and this enforces conservation of a multiplicative quantum number $\eta = (-)^n$, which we call KK parity. All the SM particles have $n = 0$ and hence possess even KK parity $\eta = +1$; all the $n = 1$ excitations have odd KK parity $\eta = -1$; all the $n = 2$ modes again have even KK parity $\eta = +1$, and so on. If we focus on the $n = 1$ states, we note that they can neither be produced singly from SM particles nor decay individually into SM particles as that would led to non-conservation of KK parity. This is completely analogous to the way in which supersymmetric particle production and decay is controlled by the conservation of $R$ parity.

The analogy with supersymmetry goes further, indeed — for conservation of KK parity implies that the lightest of the $n = 1$ states, the Lightest KK Particle (LKP), must be absolutely stable, just as the Lightest Supersymmetric Particle (LSP) is. Like the LSP, the LKP is also an excellent candidate for the dark matter component of the Universe \cite{12} — in fact, the prediction of such a candidate is a strong à posteriori motivation for the UED-5

\footnote{Though these radiative corrections do increase the mass splitting between $n = 1$ modes of different fields, the splitting ($< 30\%$) is not very large since it is, after all, a perturbative effect.}

3
model. Further, once we accept the fact that the LKP will be stable, the conservation of KK parity also ensures that it will interact only weakly with matter, by exchanging heavy $n = 1$ excitations, in the same way as the LSP exchanges only heavy supersymmetric particles, and neutrinos exchange heavy $W$ and $Z$ bosons. Because of these weak interactions with matter, the LKP will escape detection in terrestrial experiments and its appearance can only be inferred from missing transverse energy (MET) and momentum.

At collider experiments, therefore, the UED-5 model bears a strikingly close resemblance to supersymmetry, for in both cases the SM particles have heavy undiscovered partners, of which the lightest will escape detection and be observable only as MET. This has lead to the UED-5 model being dubbed ‘bosonic supersymmetry’ [15]. Most of the production and decay processes which occur in supersymmetry have their counterparts in the UED-5 model, as a result of which it is difficult, considering collider signals alone, to distinguish between supersymmetry and the UED-5 model. Some attempts to do this may be found in Ref. [16]. These methods work well enough for a minimal SUGRA-inspired mass spectrum, but if we take the most general supersymmetric mass spectrum, it will be practically impossible to tell the difference between the two models using kinematic variables alone. The fact that the heavy partners in a UED-5 model carry the same spin whereas in a SUSY framework they have spin differing by a half unit, has also been proposed as a discriminator between the two models [17]. However, the biggest difference between UED-5 and supersymmetric models lies in the fact that the UED-5 model predicts a whole tower of KK partners of each SM particle, whereas in any $N = 1$ supersymmetric model, there is just one set of supersymmetric partners. Thus, additional discovery of one or more of the $n = 2$ KK modes of SM particles would be a ‘smoking gun’ signal of the UED-5 model. As the $n = 2$ modes (for example) have KK parity $\eta = (-)^2 = +1$, they may be produced as resonances in the collision of SM particles. Moreover, because of the mass relation $M_2 \approx 2R^{-1} \approx 2M_1$, the energy required to produce a pair of $n = 1$ KK modes is roughly the same as that required to excite a single $n = 2$ KK mode resonance. This means that if we can produce a pair of $n = 1$ KK modes and see their cascade decays to the LKP – which is the signal that mimics the signals for supersymmetry – then, kinematically, we should also be able to produce $n = 2$ resonances. The combination of a supersymmetry-like signal with the existence of such a resonance would be a very strong signal, indeed, for a UED-5, and, therefore, it is necessary to consider the $n = 2$ KK mode resonances at the LHC in all seriousness.

In order to produce $n = 2$ KK modes singly, we require to use their coupling to a pair of SM particles, i.e. to $n = 0$ modes. Such a vertex, which violates KK number but not KK parity, cannot be obtained at the tree level (i.e. with local operators) in the UED-5 model. However, as mentioned above such vertices are generated by one-loop diagrams of the form shown in Figure [1], which have $n = 1$ modes running in the loop and where every vertex is
KK number-conserving, but one (or three) of the propagators violate KK number by crossing an orbifold boundary somewhere. At these boundaries the reflection symmetry in the fifth dimension ensures that the momentum $p_4$ – and hence the KK number – flips sign. The relevant vertices at the LHC would be of the form $q \bar{q} - V_2$, where $V = \gamma, W, Z$ or $g$. These are readily available in the literature \[14\] and could lead to direct $s$-channel processes of the form $q \bar{q} \rightarrow V_2 \rightarrow f \bar{f}$, where $f$ is either a quark or a lepton. A similar class of vertex can be obtained by replacing the quarks in the above diagrams by leptons. However, for the purposes of this work, we neglect $g-g-g_2$ vertices, since these are removed, to leading order, by a re-diagonalisation procedure which is required so that the massless gluon states do not contain an admixture of massive $g_2$ states \[18\].

Figure 1: One-loop diagrams giving rise to vertices of the form $q \bar{q} - V_2$ in the UED-5 model. Here $q$ stands for any quark, and $V$ stands for any vector boson. If $V_2 = g_2$, the diagrams on the fourth row are absent.

Granted the possibility of having vertices of the form $q \bar{q} - V_2$, it follows that a $V_2$ boson can be produced on-shell at the LHC, kinematics permitting, through any of the following processes:

\[
\begin{align*}
    u + \bar{u} &\rightarrow \gamma_2, \ Z^0_2, \ g_2 \\
    d + \bar{d} &\rightarrow \gamma_2, \ Z^0_2, \ g_2 \\
    u + \bar{d} &\rightarrow W^+_2 \\
    \bar{u} + d &\rightarrow W^-_2
\end{align*}
\]

\(^7\)There could be, in principle, higher dimensional operators for this vertex, which would be suppressed by a suitable high energy scale. The investigation of such effects, though interesting in itself, is postponed to a future work.
where the parton density function (PDF) of $u$ is understood to include the flux of $c$ while the PDF of $d$ includes those of both $s$ and $b$. Now, the same vertices, together with their leptonic counterparts, can be responsible for the decay processes

$$
\begin{align*}
\gamma_2, \ Z_2^0, \ g_2 & \to \ \nu + \bar{\nu}, \ \ell^+ + \bar{\ell}^- , \ q + \bar{q}, \ b + \bar{b}, \ t + \bar{t}
W_2^+ & \to \ \nu + \ell^+, \ q + \bar{q}', \ t + \bar{b}, \\
W_2^- & \to \ \bar{\nu} + \ell^-, \ \bar{q}' + \bar{q}, \ b + \bar{t},
\end{align*}
$$

where $\nu, \ell$ denote neutrinos and charged leptons, respectively, of any of the three generations, and $q, q'$ denote quarks of the first two generations. Of the final states which can be easily tagged at the LHC, the dilepton signals have already been studied in Refs. [19] and [20], while the dijet signals arising from light quarks will have an overwhelmingly large QCD background. At LHC energies, traditional $b$-tagging techniques, which rely on the reconstruction of displaced vertices, do not work well when the parent $b$-quark (or antiquark) is highly boosted and the decay products collimated into a narrow cone, especially when the $b$-jet has $p_T > 300$ GeV [21]. However, when it comes to $t\bar{t}$ final states, it may be possible to detect resonances in them, provided we can efficiently reconstruct these massive quarks from their decay fragments. Tagging heavy quarks through jet sub-structure is a technique which has been recently been put to good use in new physics studies at the LHC [22], and, for top quarks, can be implemented easily enough using the software FastJet [23]. Thus, in this work, we study $\gamma_2, \ Z_2^0$ and $g_2$ resonances in the process $p + p \to t + \bar{t}$. We note that for typical $V_2$ masses in the range of a TeV or thereabouts, even the heavy top quarks will be highly boosted, and hence the traditional tag of an isolated lepton coming from their semileptonic decays will become inefficient [24]. The high boost can be easily understood since the $p_T$ of the top quarks will show a ‘Jacobian peak’ around half of the mass of the heavy resonance, subject to minor smearing effects due to the overall boost in the laboratory frame. Thus, we must turn to the hadronic decays of top quarks, which will lead to a pair of jets having three sub-jets when studied at higher resolution.

The rate of production of $V_2$ resonances – in particular, of $g_2$ resonances – will depend on their couplings to light quarks in the proton. Of these, the most important $q\bar{q}g_2$ vertices can be parametrised in the form [14]

$$
-ig_\mu \left( g_{Lq} \frac{1 - \gamma_5}{2} + g_{Rq} \frac{1 + \gamma_5}{2} \right)
$$

where $q = u, d$. In the limit where these quarks are massless and the masses of all the $n = 1$ fields are degenerate, one can write down these form factors $g_{Lq}$ and $g_{Rq}$ in closed form. These approximate formulae, given in Ref. [14], are not reproduced here in the interests of brevity. However, in Figure 2 we show their variation with the cutoff parameter $\Lambda R$. It may be noted that the dependence on the cutoff $\Lambda R$ arises from two different sources,
viz. the divergent terms in the one-loop diagrams, as well as through the running coupling constants in the theory, which we evaluate at the resonance scale of $Q \approx 2R^{-1}$. This induces, in addition to the logarithmic dependence on $\Lambda R$, a weaker logarithmic dependence on the compactification scale $R^{-1}$, which is illustrated by the thickness of the bands in Figure 2. These correspond to variation of $R^{-1}$ between 400–1400 GeV, the range of interest in this work.

Figure 2: Couplings of the $g_2$ to (a) $u$ quarks and (b) $d$ quarks, as a function of the cutoff parameter $\Lambda R$. The width of the curves, indicated by hatching, indicates the weaker variation induced by changing $R^{-1}$, which acts through the changed running of $\alpha_s$. It is worth noting that the $g_2$ couplings to quark pairs are considerably smaller than the corresponding couplings of a normal gluon.

The lesson which is implicit in Figure 2 is the fact that the couplings of the $g_2$ to light quarks are rather small, when compared with the strong coupling constant $g_s$ which is about 0.25 at these energies, and the electromagnetic coupling constant, which is about 0.3. This is not really a surprise, though, since we have established that the $q \bar{q} - g_2$ couplings occur at the one-loop level and hence will always be suppressed by a factor of $(16\pi^2)^{-1}$. The $g_2$ resonance will be rather small, therefore – in fact, it will not be much larger than a heavy Higgs boson resonance. To distinguish it above the background will, then, require a rather fine-grained search, as is prescribed in the case of Higgs boson searches.

We now take up the case of detecting resonances in the $pp \to t\bar{t}$ cross section. We have already mentioned that top quark pairs can be produced in large numbers through the resonant processes

\[
\begin{align*}
  u + \bar{u} &\to \gamma_2, \ Z_2^0, \ g_2 \to t + t \\
  d + \bar{d} &\to \gamma_2, \ Z_2^0, \ g_2 \to t + \bar{t}
\end{align*}
\]
where $u$ includes the (small) contribution of $c$ quarks in the proton, and $d$ includes the (likewise small) contributions of $s$ and $b$ quarks in the proton. Even though all these processes arise at the same order in perturbation theory, the dominant contribution is still expected to be from the intermediate $g_2$ state, which is, after all, strongly-interacting. The cross section for $t\bar{t}$ production at the LHC through such processes is shown in Figure 3.

![Figure 3: Illustrating (a) the summed resonant cross section for $t\bar{t}$ production at the LHC through $V_2 = \gamma_2, Z_2$ or $g_2$, as a function of $R^{-1}$. Solid (dotted) lines in the lower half correspond to $\Lambda R = 20$ (5) and the horizontal dotted line indicates the projected luminosity reach of the 7 TeV run at the LHC [25]. The QCD background is indicated by broken lines in the upper half. In (b), the partial contributions of $\gamma_2, Z_2$ and $g_2$ are shown, for $\sqrt{s} = 7$ TeV, where again solid (dotted) lines correspond to $\Lambda R = 20$ (5). The dominance of the $g_2$ resonance is obvious.

On the left side of Figure 3, we show the variation in the cross section for resonant $t\bar{t}$ production as a function of the size parameter $R^{-1}$ of the UED-5 model. It may be noted that this is the total cross section sans kinematic cuts and efficiency factors, calculated using the software CalcHEP [26]. Solid (dotted) lines correspond to the choices $\Lambda R = 20$ (5), forming a band within which we expect the intermediate values of $\Lambda R$ to lie. There are two such bands, one for $\sqrt{s} = 7$ TeV and one for $\sqrt{s} = 14$ TeV, as indicated on the figure itself. It is somehow gratifying that the larger signal corresponds to the more popular choice $\Lambda R = 20$. A horizontal dotted line at 1 fb indicates the luminosity reach of the 7 TeV run at the LHC [25]. It is clear that this run will not be able to probe values of $R^{-1}$ much above 600 GeV, and hence, though providing a stronger limit than the present Tevatron limit, will hardly improve on the low energy bound from radiative $B$ decay. However, in the 14 TeV run, assuming an integrated luminosity of 100 fb$^{-1}$, the graph indicates that we should be able to probe the UED-5 model all the way to $R^{-1} = 1.5$ TeV. Near the top of
this plot, two horizontal broken lines indicate the NLO SM background, which is enormous, viz. 82.9 (144) pb at LO(NLO) for 7 TeV and 406 (968) pb at LO(NLO) for 14 TeV, using MSTW-08 structure functions [27]. This background will be diminished somewhat in the case of a UED-5, since the running of the strong coupling constant $\alpha_s$ is faster in a UED-5 than in the SM. The nevertheless huge size of this background should not deter us from proceeding with this analysis, since the resonant cross section will be concentrated in a small bin of $t\bar{t}$ invariant mass, which is expected to lie at a value where the continuum QCD background falls off to small values. The viability of such signals will be demonstrated shortly, but this serves to drive home the point that kinematic reconstruction of the final state invariant mass plays a pivotal role in this analysis, and that this is feasible for highly-boosted top quarks only in the purely hadronic channels.

On the right side, marked (b), of Figure 3 we plot the relative contribution to the resonant $t\bar{t}$ cross section from intermediate $\gamma_2$, $Z_2$ and $g_2$ states, again, as a function of $R^{-1}$. These are drawn assuming that $\sqrt{s} = 7$ TeV; for $\sqrt{s} = 14$ TeV, the qualitative features will remain the same, but there will be some numerical changes due to unequal variation in the corresponding one-loop couplings. However, these are not so important. What is significant is the fact that the $g_2$ contribution dominates the resonant part of the cross section. This is clearly due to the fact that the $g_2$ cross section, though occurring at one loop like the others, still has a factor of $\alpha_s$ where the $\gamma_2$ and $Z_2$ have a factor of $\alpha$. Even at LHC energies, this difference is substantial, and if we add to it the various colour factors, the dominance of the $g_2$ contribution is quite understandable. Of course, for the same set of reasons, the $g_2$ resonance will not be as sharp as the others, but this will not matter at the LHC, where the invariant mass binning would be crude enough to render all three peaks indistinguishable. The $g_2$ dominance encourages us to look for the resonance in purely hadronic channels, complementing earlier work [20] in the dileptonic channels.

In order to study resonances in the $t\bar{t}$ cross section, therefore, it is first necessary to reconstruct the kinematics of top quarks at the LHC. A top quark (or antiquark) decays into a real $b$ quark and a real $W$ boson, which has a hadronic branching ratio of 67.6%. Thus, the dominant decay of the top quark will be to three jets, of which one is a $b$-jet. If the mother top quark is at rest, or is moving at low speeds, these three jets will come out widely separated in direction. At low energies, e.g. at the Tevatron, the principal tag for top quarks is, not the hadronic decay mode, but the rarer cases when the $W$ decays leptonically, and a hard isolated lepton can be used to trigger the top quark decays within a mass of similar-looking events. However, if the top quark is highly boosted — as is likely to happen at the LHC (e.g. a 1 TeV top quark will have a boost parameter $\beta$ of about 0.98) — then all its

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8 More details are given in the context of Figure 6.
decay products would tend to be collinear and the lepton will appear to be part of a jet, rather than a hard decay product in its own right. It has been suggested, therefore, that one should turn to the dominant hadronic channels for identification of decaying top quarks. Of course, the three jets arising from a top quark decay will also tend to be collinear and hence are likely to be close enough for the usual jet identification algorithms to clump them into a single jet. A smaller angular resolution, however, will reveal, in most cases, three sub-jets within the single jet, and this kind of sub-structure can be used quite efficiently, to tag top quark jets.

![Figure 4](image.png)

**Figure 4:** Illustrating the kinematics in the $\phi$-$\eta$ plane of hadronic final states in $t\bar{t}$ decay for $t$ and $\bar{t}$ with 'low $p_T$' $\sim$ 100 GeV (extreme left), $t$ and $\bar{t}$ with 'high $p_T$' $\sim$ 1 TeV (centre), and 'high $p_T$ (blown up)' (extreme right) i.e. the region in the little box enclosing the cluster (central box) plotted with high angular resolution. The size of the plotted points is indicative of the energy deposit expected in the calorimeter.

In Figure 4 we show a scatter plot in the plane of pseudorapidity $\eta$ and azimuthal angle $\phi$ representing some typical hadronic decays of a $t\bar{t}$ final state. The states in question have been generated using the well-known Monte Carlo event generator PYTHIA, which simulates the decay and subsequent hadronisation of the $t$ and $\bar{t}$ partons. In the box on the left, marked 'low $p_T$', each little blob indicates a final state hadron (mostly pions) arising as the end product of the $t\bar{t}$ decay. The plot ranges over the full acceptance range of the LHC hadron calorimeter, viz. $-3.5 \leq \eta \leq +3.5$ and $0 \leq \phi < 2\pi$. The size of the blob is an indicator of the energy of the hadron and hence, the tiny dots correspond mostly to soft pions etc.. In this plot, one can identify six jets simply by inspection and we may expect any reasonable clustering algorithm to do the same. Of course, this alone does not identify a $t\bar{t}$ event. The full identification will require the following further steps:

1. Two of the jets should be tagged as $b$-jets. If not, the event is rejected.

9The radius of the blob varies logarithmically with the energy, on an arbitrarily-chosen scale. This is a fair simulation of a typical calorimeter response to a hadronic shower.
2. Of the remaining four jets, labelled (say) 1, 2, 3 and 4, one pairing, i.e. \((12)(34)\) or \((13)(24)\) or \((14)(23)\) should have both pair invariant masses in the neighbourhood of the \(W\)-boson mass, i.e. between 70 – 90 GeV. If not, the event is rejected. We can now reconstruct the kinematics of the two \(W\) bosons arising in \(t\bar{t}\) decay.

3. Now, of the two \(W\)'s, say, \(W_1\) and \(W_2\) and the two \(b\)-jets, say, \(b_1\) and \(b_2\), one pairing \((W_1 b_1)(W_2 b_2)\) or \((W_1 b_2)(W_2 b_1)\) should have both pair invariant masses in the neighbourhood of the \(t\)-quark mass, i.e. between 165 – 190 GeV. If not, the event is rejected. We can now reconstruct the kinematics of the \(t\) and the \(\bar{t}\).

4. Finally, we reconstruct the invariant mass of the \(t\bar{t}\) system. This can either be done using the previous results, or simply by adding up all the final state momenta for the selected events and squaring – this has the advantage of including all the soft pions etc., which may have been left out of a clustering algorithm. We then check if the invariant mass of the entire six-jet system has a peak at some high value or not.

The situation changes if the \(t\) and the \(\bar{t}\) are highly boosted. This is illustrated in the central box, marked ‘high \(p_T\)’ in Figure 4. As before, final state hadrons in a PYTHIA simulation have been indicated by dark blobs of size varying according to the expected energy deposit. In this case, apart from a few scattered soft hadronic objects, we see that the main hadronic event consists of just two hard transverse jets (note the low value of \(\eta\) for both clusters), and this is also what a simple-minded jet clustering algorithm will tell us. We thus have a pair of \(t\) jets, which would normally be lost against the enormous QCD dijet background. However – and this is where ‘top-tagging’ techniques come into play – if we make a much more high resolution plot of one of the jet clusters, as shown in the box on the right, marked ‘high \(p_T\) (blown up)’, then the story changes again. The box on the extreme right is a high-resolution magnification of the tiny box enclosing the cluster near the centre of the middle box (marked ‘high \(p_T\)’). Once again, if we neglect soft hadrons, this shows a sub-structure with three sub-jets clearly identifiable by inspection alone. Thus, if we can pass the putative \(t\) jets through a suitably-tuned sub-clustering algorithm, which identifies three sub-jets as illustrated, and then subject the entire event to a filtering process similar to the one described above, we should be able to identify \(t\bar{t}\) events arising from a resonance, even when the \(t\) and the \(\bar{t}\) are heavily boosted.

Having established the rationale for a top-tagging study of dijet events, we now describe the exact process adopted in order to implement this. Cross sections for the generation of \(V_2\) resonances and their decay into \(t\bar{t}\) pairs has been done using a CALCHEP-based program [14]. The events from these have been interfaced with PYTHIA [28] to produce a bunch of hadronic final states, similar to those shown in Figure 4 These are then analysed using the program FASTJET [23], which is also freely available on the Internet. The working of
the CalcHEP and Pythia part of our analysis follows standard procedures and is hardly worth discussing here. However, as FastJet is a relatively new entrant to the field of collider studies, a brief review of its working seems appropriate. Here we closely follow the discussion of Ref. [29], mentioning our specific choices of parameters as and when the occasion arises.

The first step in the algorithm used by the FastJet program is to cluster the hadronic final states into ‘jets’ of angular width $R$ ($= 0.8$ in our analysis). This is done by using the Cambridge-Aachen (CA) algorithm\(^{10}\), i.e. to start with all the four-vectors corresponding to the hadronic final states, and then merge the pair having the smallest value of $\Delta R \equiv \sqrt{\Delta \eta^2 + \Delta \phi^2}$ into a single four vector. The process is repeated until there are no four-vectors with $\Delta R < R$. At this state, all the surviving four-vectors may be identified as jets. We thus identify a multi-jet event. Of course, all information on the original set of four-vectors is stored.

The next step is to consider one ‘jet’ at a time and to ‘decluster’ it, using the following algorithm. The clustering process mentioned above is reversed, starting from the last two four-vectors to be merged. If the final transverse momentum $p_T^{(J)}$ is decomposed as $p_T^{(J)} = p_T + p_T'$ we calculate the fractions $p_T/p_T^{(J)}$ and $p_T'/p_T^{(J)}$. If one of these ratios comes out to be less than a previously-chosen minimum value $\delta_p$ ($= 0.05$ in our analysis), the corresponding four-vector is discarded as not being identifiable as a sub-jet. The other four-vector, which must then have a large $p_T$ ratio, is then subjected to the same declustering process, i.e. it is split into the two four-vectors from which it was created by the original clustering process. The process then iterates. In every case, the ratio is created w.r.t. $p_T^{(J)}$, the total transverse momentum of the jet.

The declustering procedure is terminated if one of the following situations is encountered:

1. Both the declustered four-vectors have $p_T/p_T^{(J)} > \delta_p$ ($= 0.05$ in our analysis).
2. Both the declustered four-vectors have $p_T/p_T^{(J)} < \delta_p$ ($= 0.05$ in our analysis).
3. The objects are too close, i.e. $|\delta \eta| + |\delta \phi| < \delta_r$ ($= 0.1$ in our analysis).
4. There is only one ‘calorimeter cell’ left. For the LHC, it is convenient to take a calorimeter cell of 0.1 in both $\eta$ and $\phi$. Thus, this condition usually corresponds closely to the previous one.

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\(^{10}\)This uses the simplest of three possible measures of the angular distance between four-vectors, the other two being known as the ‘$k_T$-algorithm’ and the ‘anti-$k_T$ algorithm’. Though these other measures are more sophisticated than the simpleminded CA measure, a detailed study undertaken by the CMS Collaboration shows that in practice, all three measures lead to similar results.
If the procedure terminates because of condition 1 above, we consider the jet to have two sub-jets. If the procedure terminates due to any of the conditions 2, 3 or 4, we consider the original jet to be irreducible, i.e. having no sub-jet structure.

In case the original jet is found to have two sub-jets, each sub-jet is, in turn treated to the same declustering procedure to check for further sub-jets, and the process iterates. The procedure terminates when all the sub-jets are found to be irreducible by the criteria described above. If we are tagging the original jet for a $t$-quark origin, then we select only such cases where there are three distinct sub-jets. As $b$-tagging is not efficient for high-$p_T$ jets, we require only that 

1. one pair of sub-jets should have invariant mass peaking near the $W$-pole, i.e. between 65 – 95 GeV, and
2. the invariant mass of all three sub-jets should peak near the $t$-quark pole, i.e. between 145 – 205 GeV.

Once an event clears these criteria, we define a $W$ helicity angle as the angle $\theta_h$, in the rest frame of the reconstructed $W$ boson, between the 3-momentum vector corresponding to one of the $W$-decay sub-jets and the vector corresponding to the overall $t$ (or $\bar{t}$) jet’s 3-momentum. To qualify as a $t$ (or $\bar{t}$) jet, the helicity angle should satisfy $\cos \theta_h < 0.7$. This last criterion is very helpful in preventing QCD jets from being mistagged as $t$ jets, the reason being that for such QCD jets, the distribution in $\cos \theta_h$ diverges as $(1 - \cos \theta_h)^{-1}$, and hence is strongly peaked around $\cos \theta_h \approx 1$. On the other hand, for $t$-quark jets, the distribution in $\cos \theta_h$ is essentially flat, and hence the signal will be affected only marginally by the cut $\cos \theta_h < 0.7$. A sketch of the distributions in $\cos \theta_h$ for a $t$-quark jet, a quark jet and a gluon jet may be found in Ref. [29].

In Figure 5 we plot (solid line) the efficiency fraction obtained by us, as a function of the transverse momentum $p_T$, for identification of a $t(\bar{t})$ jet out of a genuine sample of $t(\bar{t})$ jets, together with a plot (broken line) of the fraction of QCD dijets that would be mistagged as $t(\bar{t})$ jets. The top-tagging efficiency, shown by the solid line, clearly peaks at around 30–40% for $p_T$ in the range 0.6–1.5 TeV, after which it falls faster, but still remains more than 10%. It is easy to understand the low efficiency for low values of $p_T$, for in that case, the $t$-quark will mostly decay into isolated jets, and our initial selection of only dijet events will exclude most of the genuine $t$-quark decays. Moreover, some of the $t$-jets will be fat jets, wider than our acceptance criterion of $R < 0.8$, in which case the jet momentum will not carry all of the information about the momentum of the parent $t$-quark momentum, resulting in rejection by the invariant mass criteria described above. Again, when the $p_T$ is very high, i.e. in the neighborhood of 1 TeV, the efficiency falls again, because the highly-boosted decay products are so sharply collimated that the sub-jets merge and lose their individual identity to an extent that the declustering algorithm fails. For the mistagging fraction, shown by the broken line which rarely exceeds 1%, the same arguments can be applied. Low-$p_T$ QCD jets are less efficient, but the difference is not too large.
jets will tend to spread out and form either multiple jets or fat jets, and hence the number mistaken for $t$-quark jets will be less. At high-$p_T$, again, what substructure does arise from random fluctuations, will be lost in the sharp collimation of all hadronic tracks. It is, in a way, advantageous, that the tagging efficiency and the mistagging probability curves have similar shapes. For this means that when the signal is low, the number of mistagged events will also be low, and when the latter is larger, the signal is more healthy.

It is important to note that our choice of fixed tolerance parameters $\delta_p$ and $\delta_r$ is somewhat looser than that those chosen by Kaplan et al [29] in their pioneering discussion of top-tagging techniques. As a result, we have a somewhat larger acceptance for $t$ ($\bar{t}$) jets with substructure at the cost of a greater acceptance for QCD dijets as well. However, the actual differences are small, and when combined with the invariant mass and other criteria, lead to very similar results. We feel, therefore, that it is reasonable to continue the analysis with fixed tolerance parameters.

The actual event selection will be as follows: we consider all generated events having two hard jets of $p_T > 500$ GeV and no other identifiable activity. If both hard jets can be tagged, using the procedure described above, as having a $t(\bar{t})$ origin (this will include a substantial number of mistagged QCD jets, since the QCD dijet cross section is very large), we then construct the invariant mass $M(t\bar{t})$ of this pair, expecting $V_2$ resonances to show up as bumps in the invariant mass distribution. A plot of the expected distribution is shown in Figure 6 for three different values of the size parameter $R^{-1}$. For this graph, and indeed, for the rest

\[ \text{Efficiency Fraction} \]

Figure 5: Illustrating the efficiency fractions obtained in our implementation of the FastJet algorithm. The solid line indicates the fraction of $t$-quark jets which are identified as such, while the broken line indicates the fraction of light quark or gluon jets which are mistagged as $t$-jets.
of this paper, we set $\Lambda = 20$ and consider only the 14 TeV run of the LHC.

The shaded histograms in Figure 6 correspond to values of $R^{-1} = (a) 500$ GeV, (b) 800 GeV and (c) 1 TeV respectively. Of the un-shaded histograms, the solid line corresponds to the Gaussian $1\sigma$ fluctuation in the SM background, which has been calculated by adding the $t\bar{t}$ contribution to the mistagged contribution from QCD dijets. It must be noted that once the $R^{-1}$ threshold is crossed, the QCD beta function will receive contributions from $n = 1$ KK excitations and hence, the running of the coupling constant $\alpha_s$ will be different from what we would get in the SM alone. The broken line shows what would have been obtained for the fluctuation in the background if this effect had not been taken into consideration. Obviously, deviations in the two histograms will arise only for $M(t\bar{t}) > R^{-1}$.

![Figure 6](image_url)  

**Figure 6:** Invariant mass distribution for reconstructed $t\bar{t}$ pairs at the 14 TeV run of the LHC, assuming an integrated luminosity of 100 fb$^{-1}$, for $R^{-1} = (a) 500$ GeV, (b) 800 GeV and (c) 1 TeV. The shaded histogram represents the signal $(S)$ and the solid un-shaded histogram represents the Gaussian fluctuation in the SM background (continuum $t\bar{t}$ pairs, as well as mistagged dijets) $(\sqrt{B})$. The broken line shows the fluctuation in the SM background if the (faster) UED-5 running of the strong coupling constant $\alpha_s$ is ignored.

The background histograms in Figure 6 are cut off on the left at $M(t\bar{t}) < 1$ TeV because of the cut $p_T > 500$ GeV imposed on the triggered jets. The background is largest where the identification efficiency and mistagging probabilities are largest, i.e. for $M(t\bar{t})$ between 1.0 and 1.5 TeV, and exhibits a monotonic decrease as $M(t\bar{t})$ grows larger. This decrease is caused by a combination of $s$-channel suppression, falling PDFs, and decreased efficiency for very high $p_T$ jets. It allows us to discern a diminished signal even for larger values of $R^{-1}$, such as 1 TeV, where the $g_2$ resonance lies well over 2 TeV. In order to calculate the signal

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13 These two contributions are quite comparable in magnitude, and hence it is essential to consider both together.
significance, we calculate a $\chi^2$ for the deviation from the continuum UED-5 prediction. This is given by the formula

$$\chi^2 = \sum_{(i)} \frac{S_i}{\sqrt{B_i}}$$

where $S_i$ and $B_i$ refer to numbers of events in the $i$-th bin from the resonant signal ($S$) and the continuum background ($B$) respectively, and the sum $(i)$ runs only over bins where $S_i/\sqrt{B_i} \geq 3$. This value of $\chi^2$ is then compared with the corresponding number predicted for Gaussian random fluctuations in the same number of bins at a given confidence level (CL).

Taking this as a general procedure, and noting that $\chi^2 \propto \sqrt{L}$, where $L$ is the integrated luminosity, we can obtain significance figures in terms of CL for the entire range of $R^{-1}$ values, for any luminosity estimate.

In Figure 7 we show, using the above procedure, the luminosity reach of the LHC, running at $\sqrt{s} = 14$ TeV, in order to get a significance of 90% (dotted line), 95% (dashes) and 99% (solid line). It is clear that even with the not-unreasonable estimate of 50 fb$^{-1}$ for the luminosity, we could obtain at least a 95% CL signal all the way up to about $R^{-1} = 850$ GeV, which is close to the upper bound derived from the dark matter constraint. If $R^{-1}$ happens to be smaller, in the neighbourhood of 600 GeV, then even in the early runs at 14 TeV, we may expect a 2$\sigma$ resonance peak in the $t\bar{t}$ cross section. Combined with missing energy signals (such as a trilepton plus jets plus missing energy), this would then be an unambiguous signal for the existence of a UED. Values of $R^{-1}$ above a TeV would be accessible only if the
much talked-about luminosity upgrade [31] does happen, and hence, at the moment, may be considered as being only a remote possibility.

In view of the rather promising results presented above, it is our belief that the reconstruction of $g_2$ resonances from $t\bar{t}$ final states could be the best method of detecting $n = 2$ states in a UED-5 theory. The luminosity reach in this channel is fully comparable with that predicted in the dilepton channel by Datta et al. [20], and may even be marginally better. We now turn to the issue of associated production of $V_2$ resonances at the LHC. This can arise as a result of several processes, such as $p + p \rightarrow q + V_2$, $p + p \rightarrow g + V_2$, and $p + p \rightarrow q + g_2$ where $V_2$ is any one of the vector bosons $\gamma_2$, $Z_2$, $W_{\pm}^2$ or $g_2$ and $q_2$ may be an SU(2) singlet or doublet quark. In turn, the heavier $g_2$ or $q_2$ may decay through cascades to the lighter $\gamma_2$, $Z_2$ and $W_{\pm}^2$. We may denote these associated production processes, in general, as $p + p \rightarrow V_2 + X$, where $X$ is inclusive of leptons, jets and missing energy. Cross sections for these can be readily calculated using the CALCHEP routines mentioned before. In Figure 8 we plot the cross sections, at the 14 TeV LHC, as a ratio with the resonant $g_2$ production cross section, taking only the decay of $\gamma_2$, $Z_2$ and $g_2$ to $t\bar{t}$ pairs and the decay of $W_2$ to $t\bar{b}$ or $\bar{t}b$ pairs (i.e. the production cross section of $p + p \rightarrow V_2 + X$ are convoluted with the branching ratios of $\gamma_2$, $Z_2$, $g_2 \rightarrow t + \bar{t}$ or $W_2 \rightarrow t + \bar{b}$ as the case may be).

![Figure 8](image)

Figure 8: Showing the ratio of $t\bar{t}$ cross sections for associated $V_2$ production as a fraction of the resonant $G_2$ production cross section shown in Figure 3. The solid line illustrates the sum of both resonant and associated cross sections for $g_2$, whereas the others are just the associated cross sections.

The horizontal dotted line in Figure 8 represents the resonant $g_2$ cross section (for $\Lambda R = 20$) for $p + p \rightarrow t + \bar{t}$ as a ratio with itself, i.e. unity. The solid line represents its ratio with the
sum of the associated and resonant production cross sections, i.e. we plot

$$\frac{\sigma(pp \to g_2X) + \sigma(pp \to g_2)}{\sigma(pp \to g_2)}$$

as a function of the mass parameter $R^{-1}$. It may be seen that an enhancement in the $g_2$ signal varying from a factor of about a half to a quarter may be achieved by adding on the associated production. However, this turns out not to be the best policy, for the following reason. If the signal in question is $p + p \to t + \bar{t} + X$, then we must also consider the background for this. There are a host of processes which can contribute to this, including QCD processes such as three-jet production, with mistagging as $t$-jets. These are likely to increase the background by a considerably larger factor than the enhancement factor we may get for a signal, and would lead, therefore, to a reduced significance and lower discovery limits than what we have already obtained. We advocate, therefore, that the search should concentrate on the clean signal when there are just two hard $t$-jets and nothing else in the final state, and do not pursue the issue of associated production in the $g_2 + X$ channel any further.

The dashes in Figure 8 indicate the cross section for associated production of the electroweak $\gamma_2$ and $Z_2$ in $p + p$ collisions, once again, as a ratio with the $g_2$ resonant cross section. These cross sections are quite large, varying from about 30\% - 45\%, and certainly much larger than the resonant cross sections for $\gamma_2$ and $Z_2$, as shown in Figure 8. This phenomenon has already been noted and used skilfully in Ref. [20] where the $\gamma_2$ and $Z_2$ have been studied in the context of their purely leptonic decays. In the present case, it is not very meaningful to consider the $t\bar{t}$ decays of the $\gamma_2$ and $Z_2$, since these will come associated with several possibilities $X$, and would again be swamped by the large background to a $t\bar{t}X$ final state. Finally, the dot-dashed line near the bottom of Figure 8 shows the cross section for $p + p \to W^{\pm}_2 + X$, again, as a ratio with the resonant $g_2$ cross section. This cross section is smaller and in any case, the only hadronic channel in which we can even think of searching for this resonance is $W_2 \to t\bar{b}$ or $\bar{t}b$. However, at these energies, we have seen that $b$-tagging is not possible, so that the $b$-jets will be indistinguishable from a light quark jet. Accordingly the selection of events will depend on the tagging of a single $t$ quark (or antiquark). When we compare this with the $t\bar{t}$ case, and consider the tagging/mistagging efficiencies, we see that the signal will be increased roughly three times due to the use of one efficiency factor rather than two, but the background will increase by two orders of magnitude since only one mistagging probability will be required instead of two. Accordingly, any $W_2$ resonance will be completely lost against the mistagged dijet background. It may be more useful to look, therefore, for the decay $W_2 \to \ell + \nu$, but this will be hampered by its very low branching ratio. Moreover, a recent study [32] seems to indicate destructive interference between the $W_2$ contribution to $\ell\nu$ and its SM counterpart. All in all, associated $V_2$ production is not nearly as viable a signal
as the resonant $g_2$, unless we look in the purely dilepton channel with a high luminosity of 100 fb$^{-1}$ or more.

To conclude, therefore, we have studied $tt$ production through a resonant $g_2$ in the simplest model with a universal extra dimension. Using recently-developed techniques of top quark tagging for jets with high transverse momentum, we show that it is possible to isolate $g_2$ resonances and obtain an observable signal for much of the parameter space of the model which is interesting in the context of a dark matter candidate. This will be possible at the 14 TeV run of the LHC, as the 7 TeV run will not collect enough luminosity for the signal to be discernible over the background. For very large values of the size parameter $R^{-1}$, however, it may not be possible to identify $g_2$ resonances with the luminosity actually available at the LHC. We have also studied associated production of $V_2$ bosons and shown that this is not very useful if we are triggering on top quark final states. Our work complements and extends the search for $\gamma_2, Z_2$ resonances explored in the 2005 work of Datta et al [20]. As $tt$ final states will be one of the primary channels of interest at the LHC, one may hope that even in the early days of the 14 TeV run, we would be able to start accessing the parameter space of a UED model in a simple and efficient way.

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