When it Comes to Spintronics, There May be Some Room in the Middle

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ABSTRACT

Spintronic devices that utilize the spin degree of freedom of a charge carrier to store, process or transmit information, may be better performers than their traditional electronic counterparts if special properties of “spin” are exploited in the design. I review here the example of single spin logic which follows this principle. It can reduce power dissipation by several orders of magnitude and is the progenitor of today’s spin based quantum logic gates. These devices are far easier to realize than “quantum computers”, while much more likely to exhibit the advantages touted for spintronics than mere spin-based analogs of conventional transistors.

INTRODUCTION

Authors of spintronic papers frequently introduce their work with the cliché that spin based devices will be superior to their electronic counterparts in power and speed. This dogmatic belief is now being increasingly questioned [1, 2, 3]. Spintronic analogs of classical field effect [4], or bipolar junction transistors [5, 6] are not likely to offer much advantage unless revolutionary advancements are made in spintronic materials. Recently, we showed that spin-based transistors actually lose to traditional transistors in almost every respect [2, 3]. This then begs the question as to where, if anywhere at all, spin devices will have an advantage. To answer this question, one needs to first establish why spin based devices could have an advantage in the first place.

In the context of classical switching devices, “spin” has a fundamental advantage. Most charge based digital devices are switched by moving charges in space. In the case of a normal field effect transistor, charge is moved into the channel to turn the device ON, and then out of the channel to turn the device OFF. A finite amount of work is done to physically move charges and this is ultimately dissipated as heat. In the case of bipolar junction transistors, the dissipation is associated with motion of charges into and out of the base of the transistor. Suffice it to say then that in order to reduce dissipation, one must eliminate physical motion of charges.

Imagine now a scenario where the spin orientation of a single electron becomes a bistable quantity. This is achieved by placing an electron in a quantum dot and applying a weak magnetic field so that the Hamiltonian describing this electron becomes

\[ H = \left( \vec{p} - e\vec{A} \right)^2 /2m^* - (g/2)\mu_B \vec{B} \cdot \vec{\sigma} \]  

where \( \vec{B} \) is the magnetic field and \( \vec{A} \) is the associated vector potential. If the magnetic field is directed along the z-direction (\( \vec{B} = B\hat{z} \)), then diagonalization of the above Hamiltonian immediately yields two mutually orthogonal eigenspinors \( |1, 0 \rangle \) and \( |0, 1 \rangle \) which are states with their spin quantization axes directed parallel and anti-parallel to the z-directed magnetic field. Thus, the spin quantization axis (or spin polarization) becomes a binary variable – “down” (parallel to the field), or “up” (anti-parallel to the field). These two states can encode logic bits 0 and 1, respectively.

We could switch from logic bit 0 to 1, or vice versa, by simply flipping spin without physically moving charges. Therefore, we expect the switching to be accomplished with minimal energy dissipation. The dissipation will be of the order of the energy separation between the two non-degenerate states (\( = |g\mu_B B| \)). If we assume realistic parameters such as \( |g| = 20 \) ¹ and \( B = 1 \) Tesla, then the energy dissipation is about 1 meV. If the switching delay is about 1 µsec (spin flip processes in quantum dots are rather inefficient), then the power dissipated in a bit flip is \( 1.6 \times 10^{-16} \) Watts. With current technology, we can expect to produce about \( 10^{11} \) dots/cm², so that the worst case energy dissipation per unit area – the

¹For bulk InAs, \( |g| = 15 \), but the g-factor in quantum dots can be engineered somewhat by changing the shape and size [7]
quantity device engineers are concerned with – is a paltry 16 $\mu$W/cm$^2$ with a device density of $10^{11}$/cm$^2$. This is several orders of magnitude better than the projection of the International Technology Roadmap for Semiconductors for the year 2010 [9]. The energy advantage accrues from exploiting a basic feature of spin: we can flip spin without moving charges. This saves an enormous amount of energy.

**Unwanted bit flips and error rates:** How stable are single electron spins in quantum dots? Unwanted spin flip processes are strongly suppressed in these systems [9,10]. Calculations for typical GaAs quantum dots have shown that the spin flip time is several hundreds of $\mu$s. This quantity was measured in a single electron GaAs quantum dot and was found to exceed 50 $\mu$s in a magnetic field of 7.5 Tesla and 14 Tesla, with no indication of a magnetic field dependence [11]. Thus, we expect unwanted bit flips to occur at low enough rates that conventional error correction schemes can easily handle them.

**Reading and writing of spin:** In order to controllably orient and detect spin polarization of a single electron in a quantum dot (i.e. read and write a bit), we have to design and fabricate the structure appropriately. The quantum dots are delineated electrostatically in a penta-layered structure consisting of a ferromagnet-insulator-semiconductor-insulator-ferromagnet combination as shown in Fig. (a). A wrap-around split Schottky gate delineates the boundary of the quantum dot in the semiconducting layer. This gate also allows the “write” operation. The ferromagnetic layers too serve multiple purposes. First, they apply a weak magnetic field on the electron in the semiconductor dot and thus define the spin quantization axis, while making spin polarization a bistable quantity. Second, they aid in performing the “reading” and “writing” operations.

The materials are so chosen that the conduction band energy diagram at equilibrium (in the direction normal to the heterointerfaces) is as shown in Fig. (b). “Writing” spin, or aligning its polarization either parallel or anti-parallel to a magnetic field, is achieved as follows. At first, the lowest subband edge in the semiconductor is above the Fermi level in the metallic ferromagnets, so that the quantum dot is completely depleted of any electron. In order to write a spin, a positive potential is applied to the wrap-around gate which decreases the confinement and effectively makes the semiconductor dot larger, thereby pulling the subband edge below the Fermi level in the ferromagnet. A single electron now tunnels into the semiconductor from a ferromagnet. This electron’s spin is that of the majority carriers in the ferromagnet. If the ferromagnet is a half metal with 100% spin polarization, and is magnetized in a known direction, then we know what the spin polarization of this electron is. Hence, we have “written” a bit.

What if this polarization was not the desired polarization and we had intended to write the opposite polarization? Then, we have to flip this bit. For this purpose, we apply a differential potential between the two Schottky gates to induce a Rashba interaction in the dot. The total spin-splitting energy in the semiconductor layer is now [12]

$$\Delta_s = 2\sqrt{\left(\frac{g\mu_B B}{2}\right)^2 + \frac{E_y^2}{m^*e^4} W_x W_x^f(W_x, W_x')},$$

(2)

where the function $f(W_x, W_x')$ is given by

$$f(W_x, W_x') = \cos^2\left(\frac{\pi W_x}{2W_x^f}\right) \cos^2\left(\frac{\pi W_x'}{2W_x'}\right) F(W_x, W_x'),$$

$$F(W_x, W_x') = \frac{1}{((W_x/W_x^f)^2 - 1)^2 ((W_x'/W_x')^2 - 1)^2},$$

(3)

where $E_y$ is the electric field due to the differential potential between the Schottky gates, $W_x$ is the spatial width of the wavefunction in the lower spin state, and $W_x'$ is the spatial width in the upper spin state (they are different because the potential barriers confining the electron are of finite height).

By adjusting $E_y$, we can make the total spin splitting energy in the chosen dot resonant with a global ac magnetic field with frequency $\omega$ ($\Delta_s = \hbar\omega$). We hold this resonance for a time duration $\tau$ such that $(2/\hbar)\mu_B B_{ac}\tau = 1$, where $B_{ac}$ is the amplitude...
of the ac magnetic field. This flips the spin and allows us to write the desired bit.

The Rashba interaction that occurs during the gating operation can cause spin relaxation in a quantum dot. Therefore, the writing operation must be completed quickly. In other words, we need $\tau$ to be much less than 1 $\mu$s. If $\tau = 0.1$ $\mu$s, then $B_{ac} = 3.6$ gauss, which is very reasonable.

How much energy is dissipated during the gating operation? It is of the order of $CV^2$, where $C$ is the dot capacitance and $V$ is the voltage applied to the dot. If we assume that $E_y = 10^5$ V/cm and the dot dimension is 10 nm, then $V = 100$ mV. If $C = 1$ aF, then the energy is $10^{-20}$ Joules. If the gating operation is completed in 0.1 $\mu$s, then the power dissipated is $10^{-13}$ Watts, which is still very small.

“Reading” a spin, or ascertaining its polarization, is more difficult than writing spin. Single spin reading has been demonstrated with magnetic resonance force microscopy [13], but this is difficult and slow. For electrical detection, we can use the technique of ref. [14] (which is by no means unique and variations exist).

**SINGLE SPIN LOGIC CIRCUITS**

The idea of using an entity like spin to encode information and perform computation is probably not very new. However, to my knowledge, no concrete scheme for actually doing this existed before 1994. In 1994, two of my colleagues and I presented the first design for a universal NAND gate using three interacting single electron spins in three quantum dots [15]. The NAND gate is universal; therefore, any combinational or sequential circuit can be realized with it. Below, I repeat the basic idea.

Consider a linear chain of three electrons in three quantum dots. Each electron is placed in a magnetic field because of the ferromagnetic contacts around it. As mentioned before, the spin polarization is a bistable quantity aligned either parallel or anti-parallel to the field. We assume that only nearest neighbor electrons interact via exchange since their wavefunctions overlap. We will also assume that the exchange energy (defined as the energy difference between the triplet and singlet states of two neighboring interacting electrons) is larger than the Zeeman splitting energy. In this case, the ground state of the system has anti-ferromagnetic ordering where nearest neighbors have opposite spin. In fact, the ground state of the array looks like as in Fig. 2(a).

Let us now regard the two peripheral spins as the two “inputs” to the logic gate and the central spin as the “output”. Since the downspin state ($\downarrow$) represents logic 0, and the upspin state ($\uparrow$) is logic 1, we find that when the two inputs are 0, the output is automatically 1 because of the anti-ferromagnetic ordering. That is encouraging since it is one of the four entries in the truth table of a NAND gate.

We now have to realize the other three entries in the truth table. If we change the two inputs to 1 from 0, then this takes the system to an excited state. We then let the system relax. In order to maintain the anti-ferromagnetic ordering, either the central spin will flip down, or else the two peripheral spins will flip down to their original state after the writing operation is complete. The former requires a single spin flip, while the latter requires two spin flips. However, the former process will take the system to a local metastable state, while the latter takes it to the global ground state.

Whether the metastable state is reached, or the global ground state is reached depends on the energy landscape. If the metastable state is reached, then we will achieve the configuration shown in Fig. 2(b). This is the desired configuration because when the two inputs are 1, the output is 0. This is yet another entry in the truth table of a NAND gate.

The metastable state, left to itself, must ultimately decay to the global ground state. But if the energy barrier between the two states is large enough, this process may take very long. If it is much longer than the inverse of the input data rate, we can ignore
it.

How do we guarantee that the metastable state is reached and not the global ground state? We can never do it with 100% certainty, but we can do it with a high degree of certainty if we make the two-spin-flip process much more inefficient than the single-spin-flip process. Since selection rules are rather stringent in quantum dots, this is a very likely scenario.

Finally, what happens if one input is 1 and the other 0? This situation seemingly causes a tie, but the weak magnetic field present in every dot causes a Zeeman interaction which resolves this situation in favor of the central dot having a down-spin configuration. This situation is shown in Figs. 2(c) and 2(d). Note that these conform to the other two entries in the truth table of a NAND gate.

Finally, we have realized the entire truth table:

| Input 1 | Input 2 | Output |
|---------|---------|--------|
| 0       | 0       | 1      |
| 1       | 1       | 0      |
| 0       | 1       | 0      |
| 1       | 0       | 0      |

In 1995, Molotkov and Nazin verified the entire truth table using a fully quantum mechanical exact many body calculation [16]. Further calculations of this system have been carried out by Bychkov and co-workers [17].

The all-important issue of unidirectionality: There is, however, a serious problem with this type of logic gates which may not be apparent to the untrained. There is no isolation between the input and output of this type of logic gate since the exchange interaction is bidirectional. It does not distinguish between which spin is the input bit and which is the output. This makes it impossible for logic signal to flow unidirectionally from an input stage to an output stage and not the other way around. We have discussed this at length in various publications [15, 18, 19, 20] since it is vital. In 1994, when we first proposed these logic gates [15], we proposed to enforce unidirectionality by progressively increasing the distance between quantum dots, so that there is spatial symmetry breaking. In 1996, we revisited this idea and explored imposing unidirectional flow of signal in time, rather than in space, by using clocking [18]. This is actually commonplace in bucket brigade shift registers realized with charge coupled devices, where a push clock and a drop clock are used to steer a charge packet unidirectionally from one device to the next. At that time, we realized that a single phase clock cannot impose the required unidirectionality. Later, we found that a 3-phase clock is required to do this job [20]. The reader is referred to [15, 18, 19, 20, 21] for a discussion of this important topic.

The clocking circuit however introduces additional dissipation. The energy dissipated in a clock cycle, in a single clocking line, is about $C V^2$, where $C$ is the capacitance of the clocking line and $V$ is the voltage swing in the line. If we assume $C = 1 \mu F$, $V_c = 100 \text{ mV}$, then the energy dissipated is $10^{-17}$ Joules. The clock frequency is about 1 MHz (of the same order as the spin flip rate), so that the power dissipation in a clock line is of the order of $10^{-11}$ Watts. This is still a small quantity.

**ADIABATIC/REVERSIBLE GATES**

So far, we have discussed a spintronic logic family that dissipates very little energy. But can we design logic gates that dissipate no energy at all? It is well known that such gates must be logically reversible [22], i.e., we should be able to infer the input unambiguously from the output state. In 1996, we presented such a gate which is a quantum adiabatic inverter [23]. Just two exchange coupled spins in a weak magnetic field make an inverter, which is logically reversible since the input can always be inferred from the output (because they are simply logic complements of each other). Therefore, the inverter could be switched adiabatically without dissipating any energy at all.

In ref. [23], we presented the adiabatic inverter. Its dynamics is governed by the time-dependent Schrödinger equation and the gate is both logically and physically reversible, dissipating no energy at all. We showed that once a fresh input is applied to switch the input (or align the spin in the input dot to the desired orientation) and $J$ is the exchange splitting energy, which is the energy difference between the triplet and singlet states of the two electrons. Note that neither the energy $h_A$, nor $J$ is dissipated in the switching process. We could switch the device arbitrarily fast by applying arbitrarily large $h_A$, but we also showed in ref. [24], that in order for the inverter to switch perfectly and behave correctly, we need $h_A = 2 J$, which makes $t_d = h / (8 \sqrt{2} J)$. Even if $J = 1 \text{ meV}$, $t_d$ is fractions of a picosecond, which is reasonably fast. Since the switching delay is less than 1 ps and the spin coherence time in a quantum dot can easily exceed 100 μs, this gate is suitable for...
fault tolerant quantum computing. Later, a similar idea was discussed by Openov and Bychkov [24].

Double quantum dot systems are routinely fabricated and their spin states have been both measured and controlled [24]. Therefore, adiabatic inverters currently exist.

**Toffoli-Fredkin gate:** The adiabatic inverter however is not a universal adiabatic gate. The universal adiabatic gate is the Toffoli Fredkin gate which has three inputs \(A, B, C\) and three outputs \(A', B', C'\). The input-output relationships are \(A' = A, B' = B\) and \(C' = C \oplus A \bullet B\), where \(\oplus\) is the EXCLUSIVE-OR operation and \(\bullet\) is the AND operation. That means \(C\) toggles iff \(A\) and \(B\) are both logic 1; otherwise, nothing happens. We can realize this gate with three spins in a magnetic field with nearest neighbor exchange interaction (exactly the same configuration as the NAND gate described earlier). However, there is only one difference with the NAND gate. This time, we will make the Zeeman interaction (due to the magnetic field) stronger than the exchange interaction, i.e. \(g \mu_B B > J\).

\[
\Delta_{11} C > \Delta_{10} C = \Delta_{01} C > \Delta_{00} C
\]

(5)

These situations are shown in the energy diagrams in Fig. 3.

To implement the dynamics of the gate, the whole system is pulsed with a global ac magnetic field of amplitude \(B_{ac}\) whose frequency is resonant with \(\Delta_{11}\). The pulse duration is \(h/(2 \mu_B B_{ac})\). Therefore, \(C\) will toggle iff \(A\) and \(B\) are both in logic state 1. Otherwise, nothing will happen. This is the standard technique for realizing a Toffoli-Fredkin gate (see ref. [25] for a similar idea).

**QUANTUM COMPUTING WITH SPINS**

The idea of using spins in quantum dots to encode qubits was already latent in ref. [23]. Subsequently, a series of proposals followed, articulating the use of spins in different systems to encode qubits and implement universal quantum gates [24, 25, 26, 27].

The advantage of using spin, as opposed to charge, as hosts for qubits is that spin coherence is more robust than charge coherence. This field is currently an arena of active research and some progress is expected in near future, although practical quantum computers with the capability to entangle and maneuver several qubits may be decades away.

**CONCLUSION**

In this short review, I have presented a genre of low power devices where switching is accomplished by...
flipping spin rather than physically moving charges. The ultimate embodiment of low power circuits are quantum computers which do not dissipate any power at all. These are, of course, at the top of the hierarchy, but also the most challenging and difficult to realize. At the bottom of the hierarchy are the spin based analogs of conventional field effect and bipolar transistors, which do not promise significant advancements at this time. In the middle is the single spin (classical) logic family which is less difficult to realize than quantum computers and more promising than mere spintronic analogs of transistors. There may be plenty of room in the middle for exciting research.

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