Theoretical Study of Spin-Torque Oscillator with Perpendicularly Magnetized Free Layer

Tomohiro Taniguchi, Hiroko Arai, Hitoshi Kubota, and Hiroshi Imamura*
Spintronics Research Center, AIST, Tsukuba, Ibaraki 305-8568, Japan

Abstract—The magnetization dynamics of spin torque oscillator (STO) consisting of a perpendicularly magnetized free layer and an in-plane magnetized pinned layer was studied by solving the Landau-Lifshitz-Gilbert equation. We derived the analytical formula of the relation between the current and the oscillation frequency of the STO by analyzing the energy balance between the work done by the spin torque and the energy dissipation due to the damping. We also found that the field-like torque breaks the energy balance, and change the oscillation frequency.

Index Terms—spintronics, spin torque oscillator, perpendicularly magnetized free layer, the LLG equation

I. INTRODUCTION

Spin torque oscillator (STO) has attracted much attention due to its potential uses for a microwave generator and a recording head of a high density hard disk drive. The self-oscillation of the STO was first discovered in an in-plane magnetized giant-magnetoresistive (GMR) system [1]. After that, the self-oscillation of the STO has been observed not only in GMR systems [2]-[6] but also in magnetic tunnel junctions (MTJs) [7]-[11]. The different types of STO have been proposed recently; for example, a point-contact geometry with a confined magnetic domain wall [12]-[14] which enables us to control the frequency from a few GHz to a hundred GHz. Recently, Kubota et al. experimentally developed the MgO-based MTJ consisting of a perpendicularly magnetized free layer and an in-plane magnetized pinned layer [15],[16]. They also studied the self-oscillation of this type of MTJ, and observed a large power (∼ 0.5 μW) with a narrow linewidth (∼ 50 MHz) [17]. These results are great advances in realizing the STO device. However, the relation between the current and the oscillation frequency still remains unclear. Since a precise control of the oscillation frequency of the STO by the current is necessary for the application, it is important to clarify the relation between the current and the oscillation frequency.

In this paper, we derived the theoretical formula of the relation between the current and the oscillation frequency of the STO consisting of the perpendicularly magnetized free layer and the in-plane magnetized pinned layer. The derivation is based on the analysis of the energy balance between the work done by the spin torque and the energy dissipation due to the damping. We found that the oscillation frequency monotonically decreases with increasing the current by keeping the magnetization in one hemisphere of the free layer. The validity of the analytical solution was confirmed by numerical simulations. We also found that the field-like torque breaks the energy balance, and change the oscillation frequency. The shift direction of the frequency, high or low, is determined by the sign of the field-like torque.

This paper is organized as follows. In Sec. II, the current dependence of the oscillation frequency is derived by solving the Landau-Lifshitz-Gilbert (LLG) equation. In Sec. III, the effect of the field-like torque on the oscillation behaviour is investigated. Section IV is devoted to the conclusions.

II. LLG STUDY OF SPIN TORQUE OSCILLATION

The system we consider is schematically shown in Fig. 1. We denote the unit vectors pointing in the directions of the magnetization of the free and the pinned layers as \( \mathbf{m} = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta) \) and \( \mathbf{p} \), respectively. The \( x \)-axis is parallel to \( \mathbf{p} \) while the \( z \)-axis is normal to the film plane. The variable \( \theta \) of \( \mathbf{m} \) is the tilted angle from the \( z \)-axis while \( \varphi \) is the rotation angle from the \( x \)-axis. The current \( I \) flows along the \( z \)-axis, where the positive current corresponds to the electron flow from the free layer to the pinned layer.

We assume that the magnetization dynamics is well described by the following LLG equation:

\[
\frac{d\mathbf{m}}{dt} = -\gamma \mathbf{m} \times \mathbf{H} - \gamma \mathbf{H}_s \times (\mathbf{p} \times \mathbf{m}) + \alpha \mathbf{m} \times \frac{d\mathbf{m}}{dt}. \tag{1}
\]

The gyromagnetic ratio and the Gilbert damping constant are denoted as \( \gamma \) and \( \alpha \), respectively. The magnetic field is defined by \( \mathbf{H} = -\partial E/\partial (M \mathbf{m}) \), where the energy density \( E \) is

\[
E = -M H_{\text{appl}} \cos \theta - \frac{M (H_K - 4 \pi M)}{2} \cos^2 \theta. \tag{2}
\]

Here, \( M, H_{\text{appl}} \), and \( H_K \) are the saturation magnetization, the applied field along the \( z \)-axis, and the crystalline anisotropy field along the \( z \)-axis, respectively. Because we are interested in the perpendicularly magnetized system, the crystalline anisotropy field, \( H_K \), should be larger than the demagnetization field, \( 4 \pi M \). Since the LLG equation conserves the

*Corresponding author. Email address: h-imamura@aist.go.jp
norm of the magnetization, the magnetization dynamics can be described by a trajectory on a unit sphere. The equilibrium states of the free layer correspond to \( \mathbf{m} = \pm \mathbf{e}_z \). In following, the initial state is taken to be the north pole, i.e., \( \mathbf{m} = \mathbf{e}_z \). It should be noted that a plane normal to the \( z \)-axis, in which \( \theta \) is constant, corresponds to the constant energy surface.

The spin torque strength, \( H_s \) in Eq. (1), is [18]-[20]

\[
H_s = \frac{\hbar n I}{2\epsilon(1 + \lambda m_x)MSd}, \tag{3}
\]

where \( S \) and \( d \) are the cross section area and the thickness of the free layer. Two dimensionless parameters, \( \eta \) and \( \lambda \) \((-1 < \lambda < 1)\), determine the magnitude of the spin polarization and the angle dependence of the spin torque, respectively. Although the relation among \( \eta, \lambda \), and the material parameters depends on the theoretical models [20]-[22], the form of Eq. (3) is applicable to both GMR system and MTJs. In particular, the angle dependence of the spin torque characterized by \( \lambda \) is a key to induce the self-oscillation in this system.

Figure 2 (a) shows the steady state precession of the magnetization in the free layer obtained by numerically solving Eq. (1). The values of the parameters are \( M = 1313 \text{ emu/c.c.} \), \( H_K = 17.9 \text{ kOe} \), \( H_{\text{appl}} = 1.0 \text{ kOe} \), \( S = \pi \times 50 \times 50 \text{ nm}^2 \), \( d = 2.0 \text{ nm} \), \( \gamma = 17.32 \text{ MHz/Oe} \), \( \alpha = 0.005 \), \( \eta = 0.33 \), and \( \lambda = 0.38 \), respectively [17]. The self-oscillation was observed for the current \( I \geq 1.2 \text{ mA} \). Although the spin torque breaks the axial symmetry of the free layer along the \( z \)-axis, the magnetization precesses around the \( z \)-axis with an almost constant tilted angle. The tilted angle from the \( z \)-axis increases with increasing the current; however, the magnetization stays in the north hemisphere \((\theta < \pi/2)\). The dots in Fig. 2 (b) show the dependence of the oscillation frequency on the current. As shown, the oscillation frequency monotonically decreases with increasing the current magnitude.

Let us analytically derive the relation between the current and the oscillation frequency. Since the self-oscillation occurs due to the energy supply into the free layer by the spin torque, the energy balance between the spin torque and the damping should be investigated. By using the LLG equation, the time derivative of the energy density \( E \) is given by \( dE/dt = \mathcal{W}_s + \mathcal{W}_\alpha \), where the work done by spin torque, \( \mathcal{W}_s \), and the energy dissipation due to the damping, \( \mathcal{W}_\alpha \), are respectively given by

\[
\mathcal{W}_s = \frac{\gamma M H_s}{1 + \alpha^2} [\mathbf{p} \cdot \mathbf{H} - (\mathbf{m} \cdot \mathbf{p}) (\mathbf{m} \cdot \mathbf{H}) - \alpha \mathbf{p} \cdot (\mathbf{m} \times \mathbf{H})], \tag{4}
\]

\[
\mathcal{W}_\alpha = -\frac{\alpha \gamma M}{1 + \alpha^2} [\mathbf{H}^2 - (\mathbf{m} \cdot \mathbf{H})^2]. \tag{5}
\]

By assuming a steady precession around the \( z \)-axis with a constant tilted angle \( \theta \), the time averages of \( \mathcal{W}_s \) and \( \mathcal{W}_\alpha \) over one precession period are, respectively, given by

\[
\mathcal{W}_s = \frac{\gamma M \hbar n I}{1 + \alpha^2 2e\lambda MSd} \left( \frac{1}{\sqrt{1 - \lambda^2 \sin^2 \theta}} - 1 \right) \times [H_{\text{appl}} + (H_K - 4\pi M \cos \theta) \cos \theta], \tag{6}
\]

\[
\mathcal{W}_\alpha = -\frac{\alpha \gamma M}{1 + \alpha^2} [H_{\text{appl}} + (H_K - 4\pi M \cos \theta)]^2 \sin^2 \theta. \tag{7}
\]

The corresponding oscillation frequency is given by

\[
f(\theta) = \frac{\gamma}{2\pi} [H_{\text{appl}} + (H_K - 4\pi M \cos \theta)]. \tag{9}
\]

Equations (8) and (9) are the main results in this section. The solid line in Fig. 2 (b) shows the current dependence of the oscillation frequency obtained by Eqs. (8) and (9), where the good agreement with the numerical results confirms the validity of the analytical solution. The critical current for the self-oscillation, \( I_c \), is given by

\[
I_c = \lim_{\theta \to 0} f(\theta), \tag{10}
\]

The value of \( I_c \) estimated by using the above parameters is 1.2 mA, showing a good agreement with the numerical simulation shown in Fig. 2 (a). The sign of \( I_c \) depends on that of \( \lambda \), and the self-oscillation occurs only for the positive (negative) current for the positive (negative) \( \lambda \). This is because a finite energy is supplied to the free layer for \( \lambda \neq 0 \), i.e., \( \mathcal{W}_s > 0 \). In the case of \( \lambda = 0 \), the average of the work done by the spin torque is zero, and thus, the self-oscillation does not occur.

It should be noted that \( f(\theta) \to \infty \) in the limit of \( \theta \to \pi/2 \). This means the magnetization cannot cross over the \( xy \)-plane, and stays in the north hemisphere \((\theta < \pi/2)\). The reason is as follows. The average of the work done by spin torque becomes zero in the \( xy \)-plane \((\theta = \pi/2)\) because the direction of the spin torque is parallel to the constant energy surface. On the other hand, the energy dissipation due to the damping is finite in the presence of the applied field [21]. Then, \( dE/dt(\theta = \pi/2) = -\alpha \gamma M H^2_{\text{appl}}/(1 + \alpha^2) < 0 \), which means the damping moves the magnetization to the north pole. Thus, the magnetization cannot cross over the \( xy \)-plane. The controllable range of the oscillation frequency by the current is \( f(\theta = 0) - f(\theta = \pi/2) = \gamma (H_K - 4\pi M)/(2\pi) \), which is independent of the magnitude of the applied field.
Since the spin torque breaks the axial symmetry of the free layer along the \( z \)-axis, the assumption that the tilted angle is constant used above is, in a precise sense, not valid, and the \( z \)-component of the magnetization oscillates around a certain value. Then, the magnetization can reach the \( xy \)-plane and stops its dynamics when a large current is applied. However, the value of such current is more than 15 mA for our parameter values, which is much larger than the maximum of the experimentally available current. Thus, the above formulas work well in the experimentally conventional current region.

Contrary to the system considered here, the oscillation behaviour of an MTJ with an in-plane magnetized free layer and a perpendicularly magnetized pinned layer has been widely investigated [23]-[26]. The differences of the two systems are as follows. First, the oscillation frequency decreases with increasing the current in our system while it increases in the latter system. Second, the oscillation frequency in our system in the large current limit becomes independent of the \( z \)-component of the magnetization while it is dominated by \( m_z = \cos \theta \) in the latter system. The reasons are as follows. In our system, by increasing the current, the magnetization moves away from the \( z \)-axis due to which the effect of the anisotropy field on the oscillation frequency decreases, and the frequency tends to \( \gamma H_{\text{appl}}/(2\pi) \), which is independent of the anisotropy. On the other hand, in the latter system, the magnetization moves to the out-of-plane direction, due to which the oscillation frequency is strongly affected by the anisotropy (demiagnetization field).

The macrospin model developed above reproduces the experimental results with the free layer of 2 nm thick [17], for example the current-frequency relation, quantitatively. Although only the zero-temperature dynamics is considered in this paper, the macrospin LLG simulation at a finite temperature also reproduces other properties, such as the power spectrum and the macrospin LLG simulation at a finite temperature also reproduces the experimental results with the free layer of 2 nm thick [17], for example the current-frequency relation, quantitatively. Although only the zero-temperature dynamics is considered in this paper, the macrospin LLG simulation at a finite temperature also reproduces other properties, such as the power spectrum and the linewidth, well. However, when the free layer thickness further decreases, an inhomogeneous magnetization due to the roughness at the MgO interfaces may affects the magnetization dynamics: for example, a broadening of the linewidth.

### III. Effect of Field-like Torque

The field-like torque arises from the spin transfer from the conduction electrons to the local magnetizations, as is the spin torque. When the momentum average of the transverse spin of the conduction electrons relaxes in the free layer very fast, only the spin torque acts on the free layer [19]. On the other hand, when the cancellation of the transverse spin is insufficient, the field-like torque appears. The field-like torque added to the right hand side of Eq. (1) is

\[
T_{\text{FLT}} = -\beta \gamma H_0 \mathbf{m} \times \mathbf{p},
\]

where the dimensionless parameter \( \beta \) characterizes the ratio between the magnitudes of the spin torque and the field-like torque. The value and the sign of \( \beta \) depend on the system parameters such as the band structure, the thickness, the impurity density, and/or the surface roughness [22],[27]-[29]. The magnitude of the field-like torque in MTJ is much larger than that in GMR system [30],[31] because the band selection during the tunneling leads to an insufficient cancellation of the transverse spin by the momentum average.

It should be noted that the effective energy density,

\[
E_{\text{eff}} = E - \beta M \frac{\hbar I}{2e\lambda MSd} \log (1 + \lambda m_x),
\]

satisfying \( -\gamma \mathbf{m} \times \mathbf{H} + T_{\text{FLT}} = -\gamma \mathbf{m} \times [-\partial E_{\text{eff}}/(\lambda \mathbf{m})] \), can be introduced to describe the field-like torque. The time derivative of the effective energy, \( E_{\text{eff}} \), can be obtained by replacing the magnetic field, \( \mathbf{H} \), in Eqs. (4) and (5) with the effective field \( -\partial E_{\text{eff}}/\partial (\lambda \mathbf{m}) = \mathbf{H} + \beta \mathbf{H}_p \). Then, the average of \( \frac{dE_{\text{eff}}}{dt} \) over one precession period around the \( z \)-axis consists of Eq. (6), (7), and the following two terms:

\[
\overline{W}_s = \frac{\beta \gamma M}{1 + \alpha^2} \left( \frac{\hbar I}{2e\lambda MSd} \right)^2 \frac{1 + \lambda^2 \cos^2 \theta}{(1 - \lambda^2 \sin^2 \theta)^{3/2}} - 1,
\]

\[
\overline{W}_v = -\frac{\alpha \gamma M}{1 + \alpha^2} \left( \frac{2\lambda \cos \theta}{1 - \lambda^2 \sin^2 \theta} \right) \frac{1}{(1 - \lambda^2 \sin^2 \theta)^{3/2}} - 1 \times [H_{\text{appl}} + (H_K - 4\pi M \cos \theta) \cos \theta].
\]

The constant energy surface of \( E_{\text{eff}} \) shifts from the \( xy \)-plane due to a finite \( |\beta| (\approx 1) \), leading to an inaccuracy of the calculation of the time average with the constant tilted angle assumption. Thus, Eqs. (13) and (14) are quantitatively valid for only \( |\beta| \ll 1 \). However, predictions from Eqs. (13) and (14) qualitatively show good agreements with the numerical simulations, as shown below.

For positive \( \beta \), \( \overline{W}_s \) is also positive, and is finite at \( \theta = \pi/2 \). Thus, \( \frac{dE_{\text{eff}}}{dt}(\theta = \pi/2) \) can be positive for a sufficiently large current. This means, the magnetization can cross over the \( xy \)-plane, and move to the south semiphere \( (\theta > \pi/2) \). On the other hand, for negative \( \beta \), \( \overline{W}_s \) is also negative. Thus, the energy supply by the spin torque is suppressed compared to...
the case of $\beta = 0$. Then, a relatively large current is required to induce the self-oscillation with a certain oscillation frequency. Also, the magnetization cannot cross over the $xy$-plane.

We confirmed these expectations by the numerical simulations. Figures 3 (a), (b) and (c) show the trajectories of the magnetization dynamics with $\beta = 0$, 0.5, and −0.5 respectively, while the time evolutions of $m_z$ are shown in Fig. 3(d). The current value is 2.0 mA. The current dependences of the oscillation frequency are summarized in Fig. 4.

On the other hand, in the case of $\beta = −0.5 < 0$, the magnetization stays near the north pole compared to the case of $\beta = 0$, because the energy supply by the spin torque is suppressed by the field-like torque. The zero frequency in Fig. 4 indicates the increase of the critical current of the self-oscillation. Compared to the case of $\beta = 0$, the oscillation frequency shifts to the high frequency region because the magnetization stays near the north pole.

IV. Conclusions

In conclusion, we derived the theoretical formula of the relation between the current and the oscillation frequency of STO consisting of the perpendicularly magnetized free layer and the in-plane magnetized pinned layer. The derivation is based on the analysis of the energy balance between the work done by the spin torque and the energy dissipation due to the damping. The validity of the analytical solution was confirmed by numerical simulation. We also found that the field-like torque breaks the energy balance, and changes the oscillation frequency. The shift direction of the frequency, high or low, depends on the sign of the field-like torque ($\beta$).

ACKNOWLEDGMENT

The authors would like to acknowledge T. Yorozu, H. Maehara, A. Emura, M. Konoto, A. Fukushima, S. Yuasa, K. Ando, S. Okamoto, N. Kikuchi, O. Kitakami, T. Shimatsu, K. Kudo, H. Suto, T. Nagasawa, R. Sato, and K. Mizushima.

REFERENCES

[1] S. I. Kiselev, J. C. Sankey, I. N. Krivorotov, N. C. Emley, R. J. Schoelkopf, R. A. Buhrman, and D. C. Ralph, *Nature*, vol.425, pp.380-383, 2003.
[2] W. H. Rippard, M. R. Pufall, S. Kaka, S. E. Russek, and T. J. Silva, *Phys. Rev. Lett.*, vol.92, pp.027201, 2004.
[3] J. C. Sankey, I. N. Krivorotov, S. I. Kiselev, P. M. Baraganca, N. C. Emley, R. A. Buhrman, and D. C. Ralph, *Phys. Rev. B*, vol.72, pp.224427, 2005.
[4] I. N. Krivorotov, N. C. Emley, R. A. Buhrman, and D. C. Ralph, *Phys. Rev. B*, vol.77, pp.054440, 2008.
[5] W. H. Rippard, M. R. Pufall, M. L. Schneider, K. Garello, and S. E. Russek *J. Appl. Phys.*, vol.103, pp.053914, 2008.
[6] J. Sinha, M. Hayashi, Y. K. Takahashi, T. Taniguchi, M. Draperko, S. Mitani, and K. Hono, *Appl. Phys. Lett.*, vol.99, pp.162508, 2011.
[7] A. V. Nazarov, H. M. Olson, H. Cho, K. Nikolaev, Z. Gao, S. Stokes, and B. B. Pant, *Appl. Phys. Lett.*, vol.88, pp.162504, 2006.
[8] A. M. Deac, A. Fukushima, H. Kubota, H. Maehara, Y. Suzuki, S. Yuasa, Y. Nagamine K. Tsunekawa, D. D. Djayaprawira and N. Watanabe, *Nat. Phys.*, vol.4, pp.803-809, 2008.
[9] K. Kudo, T. Nagasawa, R. Sato, and K. Mizushima, *Appl. Phys. Lett.*, vol.93, pp.022507, 2009.
[10] T. Devolder, L. Bianchini, J V Kim, P. Crozat, C. Chappert, S. Cornelissen, M. O. Beeck, and L. Lagaie, *J. Appl. Phys.*, vol.106, pp.103921, 2009.
[11] H. Suto, T. Nagasawa, K. Kudo, K. Mizushima, and R. Sato, *Appl. Phys. Express*, vol.4, pp.013003, 2011.
[12] K. Matsuishi, J. Sato, and H. Imamura, *IEEE Trans. Magn.*, vol.45, pp.3422-3425, 2009.
[13] T. Taniguchi and H. Imamura, *J. Phys. Conf. Ser.*, vol.292, pp.012007, 2011.
[14] H. Arai, H. Tsukahara, and H. Imamura, *Appl. Phys. Lett.*, vol.101, pp.092405, 2012.
[15] S. Yakata, H. Kubota, Y. Suzuki, K. Yakushiji, A. Fukushima, S. Yuasa, and K. Ando, *J. Appl. Phys.*, vol.105, pp.07D131, 2009.
[16] H. Kubota, S. Ishitabashi, T. Saruya, T. Noraki, A. Fukushima, K. Yakushiji, K. Ando, Y. Suzuki, and S. Yuasa *Appl. Phys. Express*, vol.111, pp.07C723, 2012.
[17] H. Kubota, presented at 12th Joint Magnetism and Magnetic Materials/International Magnetics Conference, 2013.
[18] J. C. Słonczewski, *Phys. Rev. B*, vol.39, pp.6995, 1989.
[19] J. C. Słonczewski, *J. Magn. Magn. Mater.*, vol.159, pp.L1-L7, 1996.
[20] J. C. Słonczewski, *J. Magn. Magn. Mater.*, vol.247, pp.324-338, 2002.
[21] In the absence of the applied field, $\delta \left[ H_{appl} / (H_K - 4\pi M) \right]$.
[22] J. C. Słonczewski, *J. Magn. Magn. Mater.*, vol.159, pp.L1-L7, 1996.
[23] In this case, $I(\theta)$ given by Eq. satisfies $I_{c} = I(\theta = 0) > I(\theta = \pi/2)$. These results mean that, above the critical current, the magnetization immediately moves from the initial state to the $xy$-plane, and stops its motion. Thus, the self-oscillation does not occur.