Polarization from Aligned Atoms as a Diagnostic of Circumstellar, Active Galactic Nuclei, and Interstellar Magnetic Fields. II.

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Received 2006 October 3; accepted 2006 November 6

ABSTRACT

We show that atomic alignment presents a reliable way to study the topology of astrophysical magnetic fields. The effect of atomic alignment arises from modulation of the relative population of the sublevels of the atomic ground state pumped by anisotropic radiation flux. As such aligned atoms precess in the external magnetic field, this affects the properties of the polarized radiation arising from both scattering and absorption by the atoms. As a result, the polarizations of emission and absorption lines depend on the three-dimensional (3D) geometry of the magnetic field as well as the direction and anisotropy of incident radiation. We consider a subset of astrophysically important atoms with hyperfine structure. For emission lines, we obtain the dependencies of the direction of linear polarization on the directions of the magnetic field and the incident pumping radiation. For the absorption lines we establish when the polarization is perpendicular and parallel to the magnetic field. For both emission and absorption lines we find the dependence on the degree of polarization on the 3D geometry of the magnetic field. We claim that atomic alignment provides a unique tool for studying magnetic fields in circumstellar regions, active galactic nuclei (AGNs), and the interplanetary and interstellar media. This tool allows one to study the 3D topology of magnetic fields and establish other important astrophysical parameters. We consider polarization arising from both atoms in the steady state and also those undergoing individual scattering of photons. We demonstrate the utility of atomic alignment for studies of astrophysical magnetic fields by considering a case of sodium alignment in a comet wake.

Subject headings: atomic processes — ISM: atoms — magnetic fields — polarization

Online material: color figures

1. INTRODUCTION

Magnetic fields play essential roles in many astrophysical circumstances. Yet despite its importance, our knowledge about astrophysical magnetic fields is very limited. Exploring new tools for measuring magnetic fields is thus extremely important.

In our previous paper (Yan & Lazarian 2006, hereafter Paper I), we discussed how the alignment of fine-structure atoms and ions can be used to detect the three-dimensional (3D) orientation of the magnetic field in a diffuse medium. As in Paper I, for the sake of simplicity, we term the alignment of both atoms and ions "atomic alignment." In this paper, we discuss atomic alignment within hyperfine structures. In fact, historically, optical pumping of atoms with hyperfine-structure atoms, e.g., alkali atoms, He, and Hg (Happer 1972), were first studied in the laboratory in conjunction with early maser research. This effect was noticed and made use of for the interstellar case by Varshalovich (1968). In a subsequent paper by Varshalovich (1971), it was first pointed out that the dependence of atomic alignment on the direction of the magnetic field can be used to detect a magnetic field in space. However, the study did not provide either detailed treatment of the effect or quantitative predictions.

Atomic alignment has also been addressed by solar researchers. The research into emission-line polarimetry resulted in an important change of the views on the solar chromosphere (see Landi Degl'Innocenti 1983, 1984, 1998; Stenflo & Keller 1997; Trujillo Bueno & Landi Degl'Innocenti 1997; Trujillo Bueno 1999; Trujillo Bueno et al, 2002; Manso Sainz & Trujillo Bueno 2003). However, they dealt with the emission of atoms in a very different setting. Similar to Paper I, below we concentrate on the "weak" field regime, in which it is the atoms at ground level that are repopulated due to magnetic precession, while the Hanle effect that the aforementioned works deal with is negligible. The closest to our study is the regime discussed in the work by Landolfi & Landi Degl'Innocenti (1986), who considered an idealized two-level fine-structure atom in a very restricted geometry of observations, namely, when the magnetic field is along the line of sight and perpendicular to the incident light. As we discussed in Paper I, which dealt with fine-structure atoms, these restrictions did not allow one to use this study to predict the directions of astrophysical magnetic fields from polarimetric observations.

In this paper we consider atomic alignment for atoms with hyperfine structure and provide quantitative predictions for both absorption and emission lines. To demonstrate the processes of alignment, we perform calculations for astrophysically important species.

Studies of magnetic field topology using atomic alignment are complementary to the studies of magnetic fields using aligned dust. Similar to the case of interstellar dust, the rapid precession of atoms in a magnetic field makes the direction of polarization sensitive to the direction of the underlying magnetic field (see Lazarian [2003] for a review). However, as the precession of magnetic moments of atoms is much faster than the precession of magnetic moments of grains, atoms can reflect much more rapid variations of the magnetic field. More importantly, alignable atoms and ions can reflect magnetic fields in the environments where either the properties of dust

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change or the dust cannot survive. This opens wide avenues for the magnetic field research in circumstellar regions, the interstellar medium, the interplanetary medium, the intracluster medium, active galactic nuclei (AGNs), etc. In addition, the polarization caused by atomic alignment is sensitive to the 3D direction of magnetic fields. This information is not available by any other techniques that are available for studies of magnetic fields in diffuse gas.

In what follows we formulate the conditions for atomic alignment of hyperfine species in $x_2$ and present our formalism for the treatment of atomic alignment and optical pumping in $x_3$. In $x_4$ we use Na $i$ and K $i$ to discuss the details of practical calculations of polarization of emission lines arising from the atomic alignment of species with hyperfine splitting. A nonequilibrium case is considered for Na $i$ in $x_5$, and it is shown how alignment and polarization changes with the number of scattering events. The results are then applied to turbulent comet wake in $x_6$. We show how the change of magnetic field direction influences the polarization of scattered Na D lines from the wake, which can be used for ground-based studies of interplanetary turbulence. In $x_7$ we consider the alignment of neutral hydrogen, N $\nu$, P $\nu$, and the resulting polarizations of Ly$\alpha$. In addition, we also discuss their implications for H $i$ 21 cm and N $\nu$ hyperfine radio lines. More complicated atomic species are considered in $x_8$, where we show that absorption from N $i$ atoms is polarized, even for unresolved hyperfine multiplets. We show how hyperfine structure changes its alignment compared to S $ii$, which has the same electron configuration, but without nuclear spin. In $x_9$ we discuss how the average along the line of sight affects the results. The discussion and the summary are provided in, respectively, $x_10$ and 11.

2. CONDITIONS FOR ATOMIC ALIGNMENT IN THE PRESENCE OF HYPERFINE STRUCTURE

We discussed atomic alignment in Paper I. Atomic alignment is caused by the anisotropic deposition of angular momentum from photons. As illustrated by the toy model shown in the left panel of Figure 1, absorption from $M_F = 0$ in the ground level is impossible, owing to conservation of angular momentum. The differential absorptions for a realistic example of Na $i$ are provided later in $x_4.1$. As a result, atoms scattering the radiation from a light beam are aligned in terms of their angular momentum. To have the alignment of the ground state, the atom should have nonzero ($\geq 1$) total angular momentum on its ground state to enable various projections of atomic angular momentum. A selected number of examples are given in Table 1.

For atoms with nuclear spin, hyperfine structure must be taken into account. It is the total angular momentum $J + I = F$, the vector summation of electron angular momentum $J$ and nuclear spin $I$, that should be considered. Alkali atoms are thus alignable with nuclear spins added. It is true that direct interaction between the nucleus and the radiation field is negligible. However, for resonant lines the hyperfine interactions cause substantial precession of electron angular momentum $J$ about the total angular momentum $F$ before spontaneous emission. Therefore, total angular momentum should be considered, and the $FM_F$ basis must be adopted (Walkup et al. 1982). For alkali-like atoms, hyperfine structure should be invoked to allow enough degrees of freedom to harbor the alignment and to induce the corresponding polarization.

In order for atoms to be aligned, the collisional rate should not be too high. In fact, as disalignment of the ground state requires spin exchange (or flips), it is less efficient than one can naively imagine. The calculations by Hawkins (1955) show that to disalign sodium one requires more than 10 collisions with electrons, and experimental data by Kastler (1957) support this. This reduced sensitivity of aligned atoms to disorienting collisions makes the effect important for various astrophysical environments.

A magnetic field would mix up different $M$ states. However, it is clear that the randomization in this situation is not complete, and the residual alignment and resulting polarizations reflect the magnetic field direction. Magnetic mixing happens if the angular
momentum precession rate is higher than the rate of the excitation of atoms from the ground state, which is true for many astrophysical conditions.

3. PHYSICS OF ATOMIC ALIGNMENT

We consider a realistic atomic system, which can have multiple upper levels $J_u$ and $F_u$ and lower levels $F_l$. When such an atomic system interacts with resonant radiation, there will be a photoexcitation followed by spontaneous emission. We describe the atomic occupation and radiation field by irreducible density matrices $\rho_{F_l}^k$ and $J_0^k$ (see Appendix B). Unlike fine-structure levels, the hyperfine separation $\nu_{F_l,F_{l}'} = [E(F_u) - E(F_{u}')]/h$ is comparable to the natural line width $A$, and therefore, the coherence between hyperfine levels must be taken into account on the upper level. The density matrix of the upper state is thus $\rho_{F_l}^k(F_u, F_{u}')$. There is no coherence on the ground state, and its occupation can thus be characterized by $\rho_{F_l}^k(F_u)$. These density matrices are determined by the balance of the three processes: absorption (at a rate $\sim B_{ul}J_0^k$), emission (at a rate $\sim A$), and magnetic precession (at a rate $\sim 2\pi g \nu_{J}$) among the sublevels of the state. The statistical equilibrium equations for hyperfine transitions can be extrapolated from the case of fine transitions (Paper I; see also Landi Degl’Innocenti & Landolfi 2004). The formalism for hyperfine transitions can be obtained by replacing $J$ and $M_J$ with $F$ and $M_F$ and taking into account those additional factors in equation (A2),

$$\rho_{F_l}^k(F_u, F_{u}') + 2\pi i \nu_{F_l,F_{l}'} \rho_{F_l}^k(F_u, F_{u}') + 2\pi i \nu_{F,F_{l}'} \rho_{F_l}^k(F_u, F_{u}') = -A(J_u - J_l) \rho_{F_l}^k(F_u, F_{u}') + [J_l] \sum_{F_{l}'} \left( \delta_{kk'} \rho_{F_l}^k J_0^k + r_{kk'} B_{ul} J_0^{k'} \right) \rho_{F_l,F_{l}'}^k(F_{u}', F_{u}'),$$

$$\rho_{F_l}^k(F_i) + 2\pi i \nu_{F_l} g q \rho_{F_l}^k(F_i) = \sum_{J_u,F_{l}'} \rho_{J_u,F_{l}'} A(J_u - J_l) \rho_{F_l}^k(F_{u}', F_{u}') - \sum_{J_u,F_{l}'} \left( \delta_{kk'} B_{ul} J_0^k + s_{kk'} B_{ul} J_0^{k'} \right) \rho_{F_l,F_{l}'}^k(F_{u}', F_{u}'),$$

where

$$p_k = [F_i](-1)^{F_{l'}+F_{l'}+k+1} \left\{ \begin{array}{ll} F_l & F_{l'} \\ F_u & F_{u}' \end{array} \right\} \left\{ \begin{array}{ll} k & 1 \\ 1 & 1 \end{array} \right\} \left\{ \begin{array}{ll} J_u & J_l \\ F_l & F_{l'} \end{array} \right\} \left\{ \begin{array}{ll} F_l & F_{l'} \\ F_u & F_{u}' \end{array} \right\}.$$
In equations (3)–(5), the matrices with parentheses are 3j symbols, and the matrices with braces represent the 6j or 9j symbols, depending on the size of the matrix. They are also known as Wigner coefficients (see Cowan 1981: Appendix C in Paper I). Throughout this paper, we define \[ j \equiv 2j + 1 \], which means \[ [F, F'] = (2F + 1)(2F' + 1) \]. The evolution of the upper state \([\rho_0^k(F_u, F_u')\)] is represented by equation (1), and the ground state \([\rho_0^k(F_g, F_g')\)] is described by equation (2). The second terms on the left-hand side of equations (1) and (2) represent mixing by a magnetic field, where \( g_u \) and \( g_l \) are the Landé factors for the upper and ground level, respectively. For the upper level, the mixing \( \sim \nu_l g_l \rho_0^k(F_u, F_u') \) is much slower than the emission \( \sim A \rho_0^k(F_u, F_u') \) and is thus negligible as we consider a regime in which the magnetic field is much weaker than the Hanle field\(^3\). The third term on the left-hand side of equation (1) gives a measure of coherence of two hyperfine levels. It is easy to see if \( \nu_F, F > A \), which is the Einstein emission coefficient, then the coherence component of the density matrix \([\rho_0^k(F_u, F_u')\)] would be zero. The two terms on the right-hand side of equations (1) and (2) are due to spontaneous emissions and the excitations from the ground level. Transitions to all upper states are taken into account by summing over \( J_u \) and \( F_u \) in equation (2). Vice versa, for upper level transitions to all ground sublevels \((F_l)\) are summed up in equation (1). The excitation is proportional to

\[
\mathcal{J}_u^0 = \int d\nu \, \frac{\nu^2}{\nu_w^2} \xi(\nu - \nu_0) \int \frac{d\Omega}{4\pi} \sum_{i=0}^3 \mathcal{J}_u^i(i, \Omega)S_i(\nu, \Omega),
\]

which is the radiation tensor of the incoming light averaged over the whole solid angle and line profile \( \xi(\nu - \nu_0) \), and \( S_i = [I, Q, U, V] \) represent Stokes parameters. The unit radiation tensors \( \mathcal{J}_u^i(i, \Omega) \) are given by

\[
\mathcal{J}_u^0(i, \Omega) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathcal{J}_u^1(i, \Omega) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 - 1.5 \sin^2 \theta \\ -3/2 \sin^2 \theta \\ 0 \end{pmatrix}, \quad \mathcal{J}_u^2(i, \Omega) = \sqrt{3} e^{i 2 \theta} \begin{pmatrix} \sin^2 \theta/4 \\ -2(1 + \cos^2 \theta)/4 \\ i \cos \theta/2 \end{pmatrix}, \quad \mathcal{J}_u^3(i, \Omega) = \sqrt{3} e^{i \theta/2} \begin{pmatrix} \mp \sin \theta/2 \\ 0 \\ -i \sin \theta/2 \end{pmatrix}.
\]

For an unpolarized point source from \((\theta_r, \phi_r)\), the radiation tensor is then

\[
\mathcal{J}_u^0 = I_s, \quad \mathcal{J}_u^1 = \frac{W_a}{2 \sqrt{2} W} (2 - 3 \sin^2 \theta_r) I_s, \quad \mathcal{J}_u^2 = \frac{W_a}{4W} \sin^2 \theta_r e^{i 2 \phi_r} I_s, \quad \mathcal{J}_u^3 = \mp \sqrt{3} \frac{W_a}{4W} \sin \theta_r e^{i 2 \phi_r} I_s,
\]

where \( W \) is the dilution factor of the radiation field, which can be divided into anisotropic part \( W_a \) and isotropic part \( W_i \) (Bommier & Sahal-Brechot 1978), and \( I_s \) is the solid-angle averaged intensity. In the case of a point source, \( W_i = 0 \). If \( W_i \neq 0 \), the degree of alignment and polarization will be reduced. The solid-angle averaged intensity for a blackbody radiation source is

\[
I_s = \frac{W_a}{e^{h\nu/k_0T} - 1}.
\]

Since we are interested in the regime in which for the ground level the magnetic mixing is much faster than the optical pumping, \( \nu_l \gg \gamma_u^{-1} = B \rho_0^k \) for the ground state, and magnetic coherence does not exist either. Thus, there are only components \( \rho_0^k(F_l) \) for the ground level. Taking into account this simplification, we obtain steady state solutions by setting the first terms of equations (1) and (2) on the left-hand side to zeros,

\[
\sum_{F_u F_u'} \frac{1}{1 + 2 \pi i \nu F_u F_u'} A_{F_u F_u'} \rho_0^k(J_u, J_l) \left( \delta_{k k'} \rho_0^k(B_{lu} \tilde{J}_u^0 + r_{kk'} B_{lu} \tilde{J}_u^k) \rho_0^k(F_l) \right) - \sum_{J_{k'} F_u} \left( \delta_{kk'} B_{lu} \rho_0^0 J_u^0 + s_{kk'} B_{lu} \rho_0^0 F_l \right) \rho_0^k(F_l) = 0,
\]

\[
\rho_0^k(F_u, F_u') = \frac{B_{lu}}{A + 2 \pi i \nu F_u F_u'} |J_l| \sum_{F_{k'}} \left( \delta_{k k'} \rho_0^0 \tilde{J}_u^0 + r_{kk'} \rho_0^k \right) \rho_0^0 (F_l).
\]

It can be proved from equation (10) that \( \rho_0^k \propto \tilde{J}_u^0 \). Equation (10) represents a set of linear equations. Considering the equation with \( k = 0 \), it only includes \( \rho_0^{0,2} \) due to the triangular rule of the 3j symbol in the coefficient \( r_{kk'} \). For \( \rho_0^0 \), the coefficient is \( \propto \tilde{J}_u^0 \), as the coefficient \( \rho_0^k \) is zero. Therefore, \( \rho_0^0 \propto -\rho_0^0 \propto \tilde{J}_u^0 \). As a result, the dipole component of the density matrix changes its sign at the Van Vleck angle, as we show later. This is a generic feature of atomic alignment independent of their specific structures of atomic levels. The corresponding

\(^3\) For the Hanle effect to be dominant, magnetic splitting ought to be comparable to the energy width of the excited level.
emission coefficient can be extrapolated from that for fine-structure atoms (see Landi Degl’Innocenti 1982) by replacing \((L, S, J, M)\) with \((J, I, F, M_F)\),

\[
e_{ij}(\nu, \Omega) = \frac{\hbar \nu_0}{4\pi} A n \xi (\nu - \nu_0) [J_0] \sum_{K \neq I, F \neq F_i} F_i \sqrt{\frac{3}{2}} [F_u, F_u'] (-1)^{F_u + F_i + 1} \left\{ \begin{array}{ccc} J_u & J_i & K \\ F_i & F_i & I \end{array} \right\} \left\{ \begin{array}{cc} J_u & K \\ F_u & I \end{array} \right\} \rho_{Q}^{K} (F_u, F_u') \mathcal{J}_{Q}^{K}(i, \Omega),
\]

where \(n\) is the total number density of the atoms. This is the expression if we can resolve the hyperfine components \(F_i = 1\) and \(F_i = 2\). For the D1 lines of alkali atomic species, polarization will be zero otherwise. The corresponding emissivities in the unresolved case can be found in Landolfi & Landi Degl’Innocenti (1985). Since the separations among the hyperfine levels on the upper state are much smaller, the absorption coefficients in this case can be obtained by making the analogy with the emissivities of the unresolved case,

\[
\eta_{ij}(\nu, \Omega) = \frac{\hbar \nu_0}{4\pi} B n \xi (\nu - \nu_0) [J_0] \sum_{K \neq I, F \neq F_i} F_i \sqrt{\frac{3}{2}} (-1)^{I - J_i - F_i + F_i} \left\{ \begin{array}{ccc} J_i & J_i & K \\ F_i & F_i & I \end{array} \right\} \left\{ \begin{array}{cc} 1 & K \\ 1 & I \end{array} \right\} \rho_{Q}^{K} (F_i, F_i) \mathcal{J}_{Q}^{K}(i, \Omega).
\]

For the optically thin case, the linear polarization degree and the positional angle are

\[
p = \sqrt{\frac{\Omega^2 + U^2}{I}} = \sqrt{\frac{\epsilon_2^2 + \epsilon_1^2}{\epsilon_0}} \quad \text{and} \quad \chi = (1/2) \tan^{-1} (U/\Omega) = (1/2) \tan^{-1}(\epsilon_2/\epsilon_1)
\]

(see Fig. 1); the polarization produced by absorption through optical depth \(\tau = \eta_0 d\) is

\[
\frac{\Omega}{I \tau} = \frac{-\eta_1 d \epsilon_0}{(1 - \eta_0 d) \epsilon_0 d \eta_0} \approx -\frac{\eta}{\eta_0}, \quad U = 0.
\]

The 6\(j\) symbol in equation (13)

\[
\left\{ \begin{array}{ccc} J_i & J_i & K \\ F_i & F_i & I \end{array} \right\} = 0
\]

for \(K = 2\) and \(J_i < 1\). This suggests that absorption is unpolarized for atoms with \(J_i < 1\). Alkali atoms can therefore only produce polarized emissions.

4. ALIGNMENT OF \(\text{Na}^{+}\) AND \(\text{K}^{+}\)

4.1. \(D1\) and \(D2\) lines of \(\text{Na}^{+}\)

The geometry of the radiation system is illustrated by Figure 2. The origin of this frame is defined as the location of the atomic cloud. The line of sight defines the \(z\)-axis, and together with the direction of radiation, they specify the \(x-z\) plane. The \(x-y\) plane is thus the plane of the sky. In this frame, the incident radiation is coming from \((\theta_0, 0)\), and the magnetic field is in the direction \((\phi_B, \phi_B)\).

The magnetic field is chosen as the quantization axis \((e^z)\) for the atoms. Alignment shall be treated in the frame \(\mathcal{L} \equiv (e^x, e^y, e^z)\) (see Appendix D for how these two frames are related). In this “theoretical” frame, the line of sight is in the \((\theta, \pi)\) direction (i.e., the \(x-z\) plane is defined by the magnetic field and the line of sight; see Fig. 2, right), and the radiation source is directed along \((\theta_r, \phi_r)\).

The ground state of \(\text{Na}^{+}\) is \(2S_{1/2}\), and the first excited states \(2^2S_{1/2}\) and \(2^2P_{3/2}\) correspond to D1 and D2 lines, respectively. The nuclear spin of \(\text{Na}^{+}\) is \(I = 3/2\), its total angular momentum can thus be \(F = I \pm J = 1, 2\) (see Fig. 3, left). According to the selection rule of the 3\(j\) symbol in equation (B1), the irreducible density tensor of the ground state \(\rho_{ij}^{K} (F_i = 1)\) has components with \(k = 1 - 1, 1 + 1 = 0, 1, 2\), and \(\rho_{ij}^{K} (F_i = 2)\) has components with \(k = 2 - 2, 2 - 1, \ldots, 2 + 2 = 0, 1, 2, 3, 4\). For unpolarized pumping light, we only need to consider even components. For the upper level \(\rho_{ij}^{K} (F_u)\), according to equation (12), only the components with \(k = K \leq 2\) need to be counted. For the Na D2 line, the hyperfine splittings of upper level \(2^2P_{3/2}\) are comparable to their natural width: \(\omega_{21} = 2.67\gamma\), \(\omega_{21} = 5.2\gamma\), and \(\omega_{22} = 7.7\gamma\). Thus, the interference between levels, measured by the factor \(1/(1 + 2\pi iv_{F_u} F_u/A)\) in equations (10) and (11), must be taken into account on the upper level. On ground level, there is no magnetic coherence term, namely, \(\rho_{ij}^{K = 0} = 0\). Owing to the triangle rule of 3\(j\) symbols (see eqs. [10], [4], and [5]), only \(\rho_{ij}^{K = 0} \rho_{ij}^{K = 0}\) components appear, and they are determined by the polar angle \(\theta_r\) of the radiation (eq. [8]; Fig. 2). As a result, \(\rho_{ij}^{K = 0}\) are real quantities, and they are independent of the azimuthal angle \(\phi_r\) (see eq. [10]). Physically, this results from fast propagation around the magnetic field. Their dependence on polar angle is shown in Fig. 3. To calculate the alignment for the multiplet, all transitions should be counted according to their probabilities, even if one is interested in only one particular line. This is because they all affect the ground populations and, therefore, the degree of alignment and polarization. This means that in practical calculations, a summation should be taken over all the hyperfine sublevels \(F_u\) of both the upper levels \(J_u = 1/2, 3/2\) in equation (10). The key is the coefficients \(p_{k}, r_{k k},\) and \(s_{k k}\), which are determined by the hyperfine structure of
an atomic species. By inserting the values of $J_u, F_u, J_l, F_l, k, k', K, \text{ and } J_u^g$ (eq. [8]) into equations (3)–(5), we get the coefficients as given by Table 2.

We see from Table 2 that the coefficient $s_{kk'} \equiv 0$, and this is actually true for all the alkali species. Since $s_{kk'}$ represents the differential excitation from the ground level (see eq. [2]), this means that alkali species are not aligned by the same mechanism4 (so-called depopulation pumping), as illustrated by the toy model (§ 2). Instead, they are aligned through another mechanism, repopulation pumping. Atoms are repopulated as a result of spontaneous decay from a polarized upper level (see Happer 1972). The upper level becomes polarized because of differential absorption rates to the levels (given by $r_{kk'}$; see eq. [1]). For instance, $r_{20} = -0.1042$ for the absorption from $F_l = 1$ to $F_u = 1$. As a result, the density component of the upper level $F_u = 1, \rho_0^b(1,0) < 0$, indicating that atoms are accumulated in the sublevel $F_l = 1, 0$ according to the definition of the irreducible tensor $\rho_0^b = [\rho(1, 1) - 2\rho(1, 0) + \rho(1, -1)]$ (see Appendix B).

With the coefficients $p_b, r_{kk'}$, and $s_{kk'}$ known, one can then easily attain the coefficient matrix of equation (10). For Na, there are in total five linear equations for $\rho_0^b(F_l = 1)$ and $\rho_0^b(F_l = 2)$. By solving them, we obtain

$$
\begin{pmatrix}
\rho_0^b(F_l = 2) \\
\rho_0^b(F_l = 1) \\
\rho_0^b(F_l = 2)
\end{pmatrix}
= \begin{pmatrix}
34,492 - 5511 \cos 2\theta_r + 100 \cos 4\theta_r \\
2327 + 7289 \cos 2\theta_r - 157.4 \cos 4\theta_r - 23.4 \cos 6\theta_r \\
1606 + 5823 \cos 2\theta_r - 435.1 \cos 4\theta_r - 4.5 \cos 6\theta_r \\
39,31 + 75.31 \cos 2\theta_r + 35.56 \cos 4\theta_r + 15.94 \cos 6\theta_r
\end{pmatrix} \theta_0^a,
$$

(16)

4 In fact, the absorptions from alkali species are not polarized for the same reason.

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**Fig. 2.**—Left: Typical astrophysical environment where atomic alignment can happen. A pumping source deposits angular momentum to atoms in the direction of radiation and causes differential occupations on their ground states. In a magnetized medium where the Larmor precession rate $\nu_L$ is larger than the photon arrival rate $\nu_p$, however, atoms are realigned with respect to the magnetic field. Atomic alignment is then determined by $\theta_r$, the angle between the magnetic field and the pumping source. The polarization of the scattered line also depends on the direction of the line of sight, $\theta$ and $\phi$ (or $\phi_\perp$, defined afterward). Center: Geometry of the observational frame. In this frame, the line of sight is the $z$-axis, together with the incident light, they specify the $x$-$y$ plane. The magnetic field is in the $(\theta_b, \phi_b)$ direction. Right: Transformation to the “theoretical frame,” where the magnetic field defines the $z'$-axis. This can be done by two successive rotations specified by Euler angles $(\phi_b, \theta_b)$ (see Appendix D for details). The first rotation is from the $xyz$-coordinate system to the $x'y'z'$-coordinate system by an angle $\phi_b$ about the $z$-axis, the second is from the $x'y'z'$-coordinate system to the $x''y''z''$-coordinate system by an angle $\theta$ about the $y'$-axis. Atomic alignment and transitions are treated in the “theoretical” frame, where the line of sight is in the $(\theta, \psi)$ direction, and the incident radiation is in the $(\theta_b, \phi_b)$ direction. [See the electronic edition of the Journal for a color version of this figure.]

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**Fig. 3.**—Left: Schematic of Na and K hyperfine levels. Middle: Normalized density tensor components $\rho_0^b/\rho_0^b(F_l = 1)$ for the ground state of Na. Right: Normalized density tensor components for the ground state of K. The difference of Na and K alignments is due to different degrees of coherence on the excited state (see text). [See the electronic edition of the Journal for a color version of this figure.]
where we have defined $\phi^0_\parallel = \phi^0_\parallel (F_1 = 1)/(26,416 - 4549 \cos 2 \theta_r - 163.4 \cos 4 \theta_r + 24.5 \cos 6 \theta_r)$. The results are demonstrated in Figure 3. The triangle dependence is caused by the precession of the atoms around the magnetic field. As we see, the alignment changes sign at the Van Vleck angle $\theta_r = 54.7^\circ$, which is the same as the case for atoms with only fine structures (Paper I). As explained in § 3 and in Paper I, this is a generic feature of atomic alignment determined by the geometric relation of the pumping source and the magnetic field.

Scattering from such aligned atoms causes polarization in both D lines. Put equation (16) into equations (11) and (12) and combine with equation (7), and we obtain the expression for emission coefficients after some tedious calculations. For the D2 line and $F_r = 1$,  

$$e_0 = \frac{3\sqrt{2}}{8\pi} AI_n\nabla^0_\parallel(v - \nu_0)(62,830 - 10,084 \cos 2 \theta_r - 412 \cos 4 \theta_r + 56 \cos 6 \theta_r + (0.8 \cos 8 \theta_r - 8.7 \cos 6 \theta_r + 30.3 \cos 4 \theta_r + 2236.6 \cos 2 \theta_r + 776) \cos 2 \theta_r \cos 2 \phi_r(395 - 513.4 \cos 2 \theta_r + 107.4 \cos 4 \theta_r + 12.3 \cos 6 \theta_r - 1.3 \cos 8 \theta_r) \sin^2 \theta]  
+ \cos \phi_r[104.3 + 1.95 \cos 2 \theta_r - 212.19 \cos 4 \theta_r - 1.78 \cos 6 \theta_r + 2.98 \cos 8 \theta_r - 0.18 \cos 10 \theta_r  
+ 8.5(\cos 2 \theta_r - 1.4 \cos 4 \theta_r + 0.1 \cos 6 \theta_r + 48.5) \sin 2 \theta \sin 2 \theta_r)]  
+ (34.0 \cos 2 \theta_r - 47.6 \cos 4 \theta_r + 2.8 \cos 6 \theta_r + 1649.9) \sin \phi_r \sin \theta \sin 2 \theta_r].  
(17)$$

For $F_r = 2$,  

$$e_0 = \frac{3\sqrt{2}}{8\pi} AI_n\nabla^0_\parallel(v - \nu_0)(136,670 - 28,893 \cos 2 \theta_r + 899 \cos 4 \theta_r - 2 \cos 6 \theta_r + \cos^2 \theta(20729 \cos 2 \theta_r - 1118 \cos 4 \theta_r - 19 \cos 6 \theta_r - 2 \cos 8 \theta_r + 6057) + 5769.4 - 6278.5 \cos 2 \theta_r + 656.2 \cos 4 \theta_r - 27.4 \cos 6 \theta_r + 0.3 \cos 8 \theta_r + 0.1 \cos 10 \theta_r  
+ (5090 - 591 \cos 2 \theta_r + 6 \cos 4 \theta_r) \sin 2 \theta \sin 2 \theta_r)]  
+ (10,097 - 1183.0 \cos 2 \theta_r + 12.4 \cos 4 \theta_r  
- 2675.4 \cos 2 \theta_r - 6549.4) \cos \phi_r(10,097 - 1183.0 \cos 2 \theta_r + 12.4 \cos 4 \theta_r  
- \cos 6 \theta_r) \sin 2 \theta \sin 2 \theta_r + 6057)  
+ 20729 \cos 2 \theta_r - 19.1 \cos 8 \theta_r + 0.1 \cos 10 \theta_r  
+ 0.3 \cos 8 \theta_r + 0.1 \cos 10 \theta_r  
+ (5090 - 591 \cos 2 \theta_r + 6 \cos 4 \theta_r) \sin 2 \theta \sin 2 \theta_r)]  
+ (10,097 - 1183.0 \cos 2 \theta_r + 12.4 \cos 4 \theta_r  
- \cos 6 \theta_r) \sin 2 \theta \sin 2 \theta_r + 6057)  
+ 20729 \cos 2 \theta_r - 19.1 \cos 8 \theta_r + 0.1 \cos 10 \theta_r  
+ 0.3 \cos 8 \theta_r + 0.1 \cos 10 \theta_r  
+ (5090 - 591 \cos 2 \theta_r + 6 \cos 4 \theta_r) \sin 2 \theta \sin 2 \theta_r)]  
+ (10,097 - 1183.0 \cos 2 \theta_r + 12.4 \cos 4 \theta_r  
- \cos 6 \theta_r) \sin 2 \theta \sin 2 \theta_r + 6057)  
+ 20729 \cos 2 \theta_r - 19.1 \cos 8 \theta_r + 0.1 \cos 10 \theta_r  
+ 0.3 \cos 8 \theta_r + 0.1 \cos 10 \theta_r  
+ (5090 - 591 \cos 2 \theta_r + 6 \cos 4 \theta_r) \sin 2 \theta \sin 2 \theta_r)]  
+ (10,097 - 1183.0 \cos 2 \theta_r + 12.4 \cos 4 \theta_r  
- \cos 6 \theta_r) \sin 2 \theta \sin 2 \theta_r + 6057).  
(18)$$
Fig. 4.—Contour graphs of polarization signals of Na D2 emission line for (left, right) Q/I and (middle) U/I. Polarization depends on three angles: $\theta_0$, $\theta$, and $\phi_0$ (Fig. 2). Here $\phi_0$ is fixed to either $\pi/2$ or 0. At $\phi_0 = 0$, $U = 0$. The Stokes parameter $Q$ represents the linear polarization along $e_1$ minus the linear polarization along $e_2$, and $U$ refers to the polarization along $(e_1 + e_2)/\sqrt{2}$ minus the linear polarization along $- (e_1 + e_2)/\sqrt{2}$ (see Fig. 1, right). [See the electronic edition of the Journal for a color version of this figure.]

For the D1 line, when $F_1 = 1$,

\[
\epsilon_0 = \frac{3\sqrt{3} \lambda^2}{8\pi} A_l n_0^0 Q(\nu - \nu_0) \left\{ 36,142 - 6350 \cos 2\theta_r - 285 \cos 4\theta_r + 35 \cos 6\theta_r - \cos 2\theta(40 - 2 \cos 6\theta_r + 76 \cos 4\theta_r + 26 \cos 2\theta_r) - \cos 2\phi_0(588 - 702 \cos 2\theta_r + 126 \cos 4\theta_r - 11 \cos 6\theta_r) \sin^2 \theta \right. \\
- \cos \phi_r(268 - 1.19 \cos 2\theta_r - 267 \cos 4\theta_r + 6 \cos 6\theta_r + 537.0 - 5.1 \cos 2\theta_r + 2.7 \cos 4\theta_r - 0.3 \cos 6\theta_r) \sin \theta \sin 2\theta_r \left. \right\},
\]

\[
\epsilon_1 = -\frac{3\sqrt{3} \lambda^2}{8\pi} A_l n_0^0 Q(\nu - \nu_0) \left\{ 130.89 \cos 2\theta_r - 153.45 \cos 4\theta_r + 5.26 \cos 6\theta_r + 1.82 \cos 8\theta_r - 2050.2 \sin^2 \theta \right. \\
+ \cos 2\phi_0(40 - 293.92 - 0.20 \cos 8\theta_r + 5.66 \cos 6\theta_r - 62.56 \cos 4\theta_r + 251.03 \cos 2\theta_r) + 1053.08 \cos 2\theta_r \\
- 187.674 \cos 4\theta_r + 16.975 \cos 6\theta_r - 0.606 \cos 8\theta_r - 881.772 \right. - \cos \phi_r(1073.92 - 10.216 \cos 2\theta_r + 5.472 \cos 4\theta_r \\
- 0.673 \cos 6\theta_r) \sin 2\theta_r + 1969.6 \right. \left.,
\]

\[
\epsilon_2 = -\frac{3\sqrt{3} \lambda^2}{8\pi} A_l n_0^0 Q(\nu - \nu_0) \left\{ 1404.1 \cos \theta_r(\cos 2\theta_r - 0.1782 \cos 4\theta_r + 0.0161 \cos 6\theta_r - 0.0006 \cos 8\theta_r - 0.8373) \sin 2\phi_0 \\
(20.4 \cos 2\theta_r - 10.9 \cos 4\theta_r + 1.3 \cos 6\theta_r - 2147.8) \sin \phi_r \sin \theta \sin 2\theta_r \right\}.
\]  

For $F_2 = 2$,

\[
\epsilon_0 = \frac{3\sqrt{3} \lambda^2}{8\pi} A_l n_0^0 Q(\nu - \nu_0) \left\{ 64,488 - 10,148 \cos 2\theta_r + 337 \cos 4\theta_r - 10 \cos 6\theta_r - \cos 2\theta(0.11 \cos 8\theta_r + 2.63 \cos 6\theta_r \\
- 66.73 \cos 4\theta_r + 65.45 \cos 2\theta_r - 40.301) \right. - \cos 2\phi_0[(-587.8 + 702 \cos 2\theta_r - 125.2 \cos 4\theta_r + 11 \cos 6\theta_r) \sin^2 \theta] \\
+ \cos \phi_r(268 - 1.19 \cos 2\theta_r - 267.1 \cos 4\theta_r + 1.1 \cos 6\theta_r - 0.7 \cos 8\theta_r + 0.1 \cos 10\theta_r + \sin 2\theta_r \sin 2\theta_r) \right. \left. \sin 2\theta_r \right\},
\]

\[
\epsilon_1 = -\epsilon_2(F_1 = 1), \quad \epsilon_2 = -\epsilon_2(F_1 = 1) \quad (19)
\]

For the unresolved D2 line, the result can be obtained by the summation of the two hyperfine components $F_1 = 1$ and $F_1 = 2$. The angular dependence of the polarization comes from both $\rho_Q^{F_1}(F_n, F_n')$, the density matrix of the upper level, and $J_Q^{F_1}(i, \Omega)$ (see eq. [12]). While the former reflects the direction of incident radiation seen in the theoretical frame, the latter is an observational effect that is solely determined by the line of sight in the theoretical frame. Figure 4 is a contour graph showing the dependence of polarization on $\theta_0$ and $\theta$ (see Fig. 2) with fixed $\phi_0 = 0^\circ$, 90°. Along the $\theta_0$ axis, the principle harmonic is of order 2 dependence, which is as expected from equations (17) and (18). At $\phi_0 = 90^\circ$, Q/I and U/I are shifted with respect to each other in $\theta$ by a quarter of their period 180°. At $\phi_0 = 0^\circ$ the U = 0 and Q/I is distorted, as the phase dependence of $\theta_0$ and $\theta$ is entangled. Since $U = 0$, the polarization always lies in the single plane formed by the radiation source, the magnetic field, and the observer, just as expected from the symmetry of the system (see Fig. 2). Figures 5 and 6 are the corresponding plots for degree of polarization $p$ and the positional angle of polarization $\chi$ (see Fig. 1, right) calculated according to equation (14).

Figure 7 is the polarization diagram (or Hanle diagram) of the Na D2 emission line. Solid lines are the contour of equal $\phi_0$, while dash-dotted lines are contours of equal $\theta_0$. For any pair of $\theta_0$, $\phi_0$, the polarization diagram gives the polarization $Q/I$, $U/I$. The actual diagram is 3D, and its shape depends on the perspective we observe (the angle $\theta$; see Fig. 2). We present here its projection at four directions $\theta = 0^\circ$, 30°, 60°, and 90°. For $\theta > 90^\circ$, the contour of the diagram $\phi_0$ at $\theta$ is the same as that for $180^\circ - \phi_0$ at $180^\circ - \theta$; the contour of the diagram of $\theta_0$ is the same as that for $180^\circ - \theta_0$ at $180^\circ - \theta$. Thus, at $\theta = 0^\circ$, the polarization is symmetric about $\theta_0 = 90^\circ$, i.e., the contour of $\theta_0$ coincides with that of $180^\circ - \theta_0$. At $\theta = 90^\circ$, the polarization is symmetric about $\theta_0 = 90^\circ$. In other cases, the equal-value lines are always increasing clockwise with antisymmetry of $U$ about 90°. At $\theta_0 = 0^\circ$, 180°, the lines degenerated to a point in every diagram (marked by “**” as expected. At $\phi_0 = 0^\circ$, 180°, $U = 0$ and the direction of polarization traces the magnetic field in the pictorial
Fig. 5.—Polarizations of Na D2 emission lines and their dependence on \((\theta_r, \theta)\) at \(\phi_r = 90^\circ\). Top panels give the degree of polarization of emission line; bottom panels show the positional angle of polarization measured from the plane parallel to the magnetic field (see Fig. 1, right). [See the electronic edition of the Journal for a color version of this figure.]

Fig. 6.—Degree of polarizations of Na D2 emission lines and their dependence on \((\theta_r, \theta)\) at \(\phi_r = 0^\circ\). The positional angle of polarization is 0, as the Stokes parameter \(U = 0\) (see Fig. 4 and the text). [See the electronic edition of the Journal for a color version of this figure.]
plane with 90° uncertainty. The lines at \( \phi_r = 180° \) are antisymmetric with respect to \( \theta_r = 90° \) with those of \( \phi_r = 0° \). These symmetric features are generic and independent of specific species, as they are solely determined by the scattering geometry. The polarizations of the two hyperfine components (\( F_1 = 1, 2 \); see Fig. 3, left) of the D1 line are reversals of each other (see eq. [19] and [20]). The polarization of the D1 line is thus dependent on the ratio of its line width and the separation of the two hyperfine components \( \sim 1 \) km s\(^{-1}\). As an illustration, the left panel of Figure 8 shows how the polarization of the D1 line with a fixed radiation and magnetic geometry changes with the line width.

**Fig. 7.**—Polarization diagrams (or Hanle diagrams) of Na D2 emission lines observed at different angles \( \theta \). In the diagram, solid lines represent contours of equal \( \phi_r \), while dash-dotted lines refer to contours of equal \( \theta_r \) (see Fig. 2). The numbers in the text box mark the values of \( \phi_r \), while the numbers without boxes give the values of \( \theta_r \). Note that when \( \theta_r = 0° \) and 180°, the corresponding polarizations degenerate to one point in each diagram (marked by diamonds). [See the electronic edition of the Journal for a color version of this figure.]

**Fig. 8.**—Left: Polarization of Na D1 line at \( \theta_r = 90° \), \( \theta = 90° \), \( \phi_r = 0° \). As we see, the polarization degree and the profile of the D1 line depends on the line width. The Stokes parameters of the two hyperfine components (\( F_1 = 1, 2 \); see Fig. 3, left) are reversals of each other. Thus, if the line width is much wider than their separation (\( \sim 1 \) km s\(^{-1}\)), their polarizations cancel each other out. Center and right: Contour graphs of the emissivity ratio of Na D doublet; \( \phi_r \) is fixed to either \( \pi/2 \) or 0. [See the electronic edition of the Journal for a color version of this figure.]
Measurements of the polarization degree of both D lines can constrain up to four parameters, from which we can extract both magnetic field \((B/C18)\) direction and information about the radiation source \((B/C14)\). In the future, in situations in which we can resolve the hyperfine components of D lines, we can cross-check and make the detection of the magnetic field even more accurate.

The intensity of the scattered light is also modulated by magnetic realignment. For comparison, we plot the D2/D1 line ratios of their intensities (see Fig. 8, center and right; Fig. 9). The corresponding line ratios without magnetic realignment are equal to those values at \(\theta_r = 0^\circ\), where the magnetic field is parallel to the incident radiation.

4.2. Results for \(K_i\)

\(K_i\) has the same electron configuration and nuclear spin as \(Na_i\) does (see Fig. 3, left). The only difference between them is the coherence between hyperfine sublevels on the excited state \(2P_{3/2}; \omega_{10} = 0.51 \gamma, \omega_{21} = 1.5 \gamma, \) and \(\omega_{12} = 3.5 \gamma\). However, the result for \(K_i\) differs from that of \(Na_i\) by only \(\leq 5\%\). Here we only provide the density tensors of the ground state,

\[
\begin{bmatrix}
\rho_0^1(F_i = 1) \\ \rho_0^2(F_i = 2)
\end{bmatrix}
= 
\begin{bmatrix}
0.9 \cos^{10} \theta_r + 8.3 \cos^8 \theta_r - 44.89 \cos^6 \theta_r - 135.8 \cos^4 \theta_r + 535.8 \cos^2 \theta_r - 161.9, \\
-2.5 \cos^6 \theta_r + 15.5 \cos^4 \theta_r + 67.0 \cos^2 \theta_r - 758.4 \cos^2 \theta_r + 1337.0, \\
-0.01 \cos^{12} \theta_r + 0.16 \cos^{10} \theta_r + 6.08 \cos^8 \theta_r + 2.54 \cos^6 \theta_r - 218.68 \cos^4 \theta_r + 502.59 \cos^2 \theta_r - 143.33.
\end{bmatrix}
/ \begin{bmatrix}
-1.0 \cos^{10} \theta_r - 4.2 \cos^8 \theta_r + 58.9 \cos^6 \theta_r - 57.1 \cos^4 \theta_r - 438.0 \cos^2 \theta_r + 1026.9.
\end{bmatrix}
\]

\(\rho_0^1(F_i = 2) = -7.4893 \cos^8 \theta_r + 91.6742 \cos^6 \theta_r - 46.534 \cos^4 \theta_r + 1.5742 \cos^2 \theta_r + 1.3429\)

\(2.8 \cos^8 \theta_r - 53.3 \cos^6 \theta_r + 422.2 \cos^4 \theta_r + 1320.0 \cos^2 \theta_r - 5521.9.\)
There are a number of parameters that determine the polarization: direction of the magnetic field \((\theta_B, \phi_B)\), direction of the radiation field \((\theta_0, \phi_0)\), and percentage of radiation anisotropy \(W_c/W\). For those cases in which the radiation source is known, \(\phi_0\) can be easily obtained. If we know the distances of the source and atomic cloud, we can also determine \(\theta_0\) and \(W_c/W\). Thus, we have two to four unknown numbers depending on the specific situation. The list of observables we have are degree and direction of polarization and the line intensity ratio of the doublet. If the hyperfine separation of the sodium doublet (~20 mÅ) is resolved, then the four line components are available. In this case one has enough observables to constrain the 3D direction of the magnetic field and the environment in situ.

4.3. Comparison with Earlier Works

As we mentioned in § 1, the optical pumping of atoms has been studied in the laboratory in conjunction with early maser research (see Hawkins 1955; Happer 1972). For instance, magnetic mixing of the occupations of sodium atoms was extensively studied by Hawkins (1955). Their quantitative results, however, are incorrect because of the classical approach they adopted for the radiation field. As pointed out in Paper I, because conservation and transfer of angular momentum is the essence of the problem, it is necessary to quantize the radiation field, as the atomic states are quantized. Hawkins (1955) claims that the alignment is zero at \(\theta_r = 90^\circ\) \((\theta_r\) is the angle between the magnetic field and the pumping light). Our results show that the alignment is zero at the Van Vleck angle 54.7°. This is a generic feature for optical pumping of any atom, regardless of their structures (see also Paper I).

5. TIME-DEPENDENT ALIGNMENT

In this paper we mostly deal with alignment in equilibrium states. Astrophysical environments may present us with the cases in which there are not enough scattering events to reach equilibrium. This can happen if either the mean free path of the atom is comparable to or larger than the system dimension, so that atoms are leaving the system before acquiring the steady state alignment, or the rate of randomizing collisions is comparable to the optical pumping rate. In this case, a collisional term should be added to the statistical equilibrium equations (see Appendix C). Below we study the alignment of Na I under a given number of scattering events.

The calculations are straightforward. Initially, when no pumping has occurred, the ground state occupation is isotropic. There is only \(\rho_0^0\) in this case. Since the energy splitting between the two hyperfine sublevels is negligible, atoms are distributed according to their level statistical weight,

\[
\begin{pmatrix}
\rho_0^0(F_1 = 1) \\
\rho_0^0(F_1 = 2) \\
\rho_0^0(F_1 = 1) \\
\rho_0^0(F_1 = 2)
\end{pmatrix} \propto \begin{pmatrix}
\sqrt{3} \\
\sqrt{3} \\
0 \\
0
\end{pmatrix}.
\]

(23)

We simply need to find the explicit expression for the scattering matrix and then multiply the initial density matrix by it as many times as the number of scattering events. For such a purpose, we need to go back to the statistical equilibrium equations (1) and (2). The first step, the stimulated emission is described by the second term on the right-hand side of equation (1). There are two routes then for the excited atoms, either they spontaneously decay (first term on the right-hand side of the same equation) to the ground level or precess under the magnetic field produced by nuclear spin (the third term on the right-hand side). The probability of spontaneous emission out of the two routes is thus easily obtained by multiplying the first term on the left-hand side of equation (2) by \(A/(A + 2\pi \tau_{r, F})\). It turns out that the scattering matrix is the same as the first term in equation (10) apart from the Einstein coefficient \(B\), which should be removed, as we are concerned about the probability rather than rates. The effect of the magnetic field is only to remove the magnetic coherence as explained earlier. We assume for the incident radiation the same intensity at the resonant frequencies of D lines. Then averaging over both D lines, we obtain the scattering matrix

\[
\begin{pmatrix}
1.09 & 0.497 & 0.0340 \cos^2 \theta_r - 0.0113 & 0.052 - 0.156 \cos^2 \theta_r & 0 \\
0.497 & 1.35 & 0.00878 - 0.0264 \cos^2 \theta_r & 0.121 \cos^2 \theta_r - 0.0403 & 0 \\
0.279 \cos^2 \theta_r - 0.0929 & 0.115 \cos^2 \theta_r - 0.0383 & 0.190 \cos^2 \theta_r - 0.225 & 0.105 \cos^2 \theta_r + 0.206 & 0.0219 - 0.0656 \cos^2 \theta_r \\
0.0738 \cos^2 \theta_r - 0.0246 & 0.269 \cos^2 \theta_r - 0.0896 & 0.0879 - 0.0315 \cos^2 \theta_r & 0.0602 \cos^2 \theta_r + 0.699 & 0.136 \cos^2 \theta_r - 0.0453 \\
0 & 0 & 0.0845 - 0.254 \cos^2 \theta_r & 0.294 \cos^2 \theta_r - 0.0981 & 0.165 \cos^2 \theta_r + 0.22
\end{pmatrix}
\]

(24)

For a given number of scattering events, the density matrix of the ground level can be obtained by multiplying equation (23) by the scattering matrix (eq. [24]) as many times as the number of scattering events. The results are shown in Figure 10. We see after about five scattering events that the system approaches its equilibrium. The scattering timescale is \(B_{\text{eff}}/2\pi \approx 5 \times 10^{-10} [R(R_{\odot})/r(\text{pc})]^2\). In contrast, the collisional transition rate is \(\sim 10^{-14} n_e \text{ cm}^2\) (Happer 1972). In this sense, collisions can be neglected if \(r \ll 1 \text{ pc}\) for a solar-type star. In the case in which collisions are not negligible, the effect of collisions would be to limit the number of scattering events.

We also obtained the polarization of emission and absorption from atoms having scattered only one photon. The shape of polarization curves for all the lines are very similar to the equilibrium cases. In fact, the curves for positional angle of polarization are the same for all

5 Note that in this paper we discuss a regime for which the external magnetic field is not strong enough to affect the upper level population.
the cases. In this sense, direction of polarization is a more robust measure for the detection of the magnetic field. Only the amplitude of the degree of polarization is decreased from the equilibrium case by \(\sim 10\%\).

6. EXAMPLE: SYNTHETIC OBSERVATION OF COMET WAKE

As an illustration, we discuss here a synthetic observation of a comet’s wake. Although the abundance of sodium in comets is very low, its high efficiency of scattering sunlight makes it a good tracer (Thomas 1992). It was suggested by Cremonese & Fulle (1999) that there are two categories of sodium tails. Apart from the diffuse sodium tail superimposed on a dust tail, there is also a third narrow tail composed of only neutral sodium and well separated from dust and ion tails. This neutral sodium tail is characterized by fast-moving atoms from a source inside the nuclear region and accelerated by radiation pressure through resonant D line scattering. While for the diffuse tail, sodium is considered to be released in situ by dust; it is less clear for the second case. Possibly, the fast narrow tail may also originate from the rapidly fragmenting dust in the inner coma (Cremonese et al. 2002).

The gaseous sodium atoms in the comet tail acquire not only momentum but also angular momentum from the solar radiation, i.e., they are aligned. Distant from comets, the Sun can be considered a point source. As shown in Figure 11, the geometry of the scattering is well defined, i.e., the scattering angle \(\theta_0\) is known. The polarization of the sodium emission thus provides exclusive information of the magnetic field in the comet wake. Embedded in solar wind, the magnetic field is turbulent in a comet wake. We take a data cube (Fig. 12) from magnetohydrodynamic (MHD) simulations of a comet wake. Depending on its direction, the embedded magnetic field alters the degree of alignment and, thus, polarization of the light scattered by the aligned atoms. Therefore, fluctuations in the linear polarization...
are expected from such a turbulent field (see Figs. 13 and 14). The calculation is done for the equilibrium case. Otherwise, the result for the degree of polarization will be slightly different ($\leq 10\%$) depending on the number of scattering events experienced by atoms as presented in § 5. The direction of polarization, nevertheless, should be the same as in the equilibrium cases. Except from polarization, intensity can also be used as a diagnostic. Note that the result also depends on the inclination with the plane of the sky $\alpha = 90^\circ - \theta_0$. Figure 14 shows that the patterns are completely different at $\alpha = 45^\circ$ from those with no inclination (Fig. 13). By comparing observations with our synthetic data, we can determine whether the magnetic field exists and its direction. For interplanetary studies, one can investigate not only spatial but also temporal variations of magnetic fields. Since alignment happens at a timescale $\tau_B$, magnetic field variations on this timescale will be reflected. This can allow a cost-effective way of studying interplanetary magnetic turbulence at different scales.

7. ALIGNMENT OF H I, P V, and N V

7.1. Aligned Atomic Hydrogen

Hydrogen has a structure similar to that of sodium. The nuclear spin of hydrogen is $I = 1/2$. The total angular momentum of the ground state can be $F_I = 1/2 \pm 1/2 = 0, 1$ (see Fig. 15, left). Only the sublevel of $F_I = 1$ can accommodate alignment. The hyperfine
splittings \( \nu_{\text{P},P} \) of \( nP_{1/2} \) and \( nP_{3/2} \) are smaller than their natural line width \( \gamma \), \( \nu_{21}(nP_{3/2}) = 0.229\gamma \) and \( \nu_{10}(nP_{1/2}) = 0.258\gamma \). Thus, coherence on both levels should be taken into account. We obtain from equation (10),

\[
\begin{bmatrix}
\rho_0^0(F_1 = 1) \\
\rho_0^2(F_1 = 1)
\end{bmatrix} = \begin{bmatrix}
(1.557 \cos^2 \theta + 3.114 \cos^2 \theta - 23.665) & 0.11008 - 0.33023 \cos^2 \theta \\
0 & 0
\end{bmatrix} \rho_0,
\]

where \( \rho_0^0 = \rho_0^0(F_1 = 0) / (\cos^4 \theta + 1.730 \cos^2 \theta - 13.652) \). Insert the ground density matrix \( \rho_0^{0,2}(F_i) \) into equations (12) and (13), and we obtain their emissivities. For the D2 line,

\[
\epsilon_0 = \frac{3\sqrt{3} \lambda^2}{8\pi} A n_0^0 \zeta(\nu - \nu_0) (\cos^2 \theta (1.092 \cos^6 \theta + 1.762 \cos^4 \theta - 17.148 \cos^2 \theta + 5.481) + \cos 2\phi \sin^2 \theta (6.131 \cos^2 \theta - 0.3446 \cos^4 \theta)) - 54.412 + 12.611 \cos^2 \theta + 2.9112 \cos^4 \theta - 3.641 \cos^6 \theta + \cos \phi \left(0.1190 \cos 2\theta + 2.2114 \cos 4\theta - 0.1190 \cos 6\theta, -0.010 \cos 8\theta - 2.2014 + 0.3207 (\cos^4 \theta + 1.970 \cos^2 \theta - 15.026) \sin 2\sin 2\theta - (5.422 + 0.3641 \cos^4 \theta) \sin^2 \theta \right),
\]

\[
\epsilon_1 = \frac{3\sqrt{3} \lambda^2}{8\pi} AL n_0^0 \zeta(\nu - \nu_0) (1.0923 \cos^6 \theta - 1.7621 \cos^4 \theta + 17.1512 \cos^2 \theta - 5.4808 - 0.6414 \cos \phi (\cos^4 \theta + 1.970 \cos^2 \theta - 15.026) \sin 2\sin 2\theta + 1.9023 \cos^2 \theta (\cos^6 \theta + 6.930 \cos^4 \theta - 15.702 \cos^2 \theta + 5.018) + \cos 2\phi \left(0.3641 \cos^4 \theta, 0.3446 \cos^4 \theta - 6.1312 \cos^2 \theta + 5.4225 + \cos^2 \theta (0.3641 \cos^6 \theta + 0.3446 \cos^4 \theta - 0.3641 \cos^6 \theta - 6.1312 \cos^2 \theta + 5.4225) \right)),
\]

\[
\epsilon_2 = \frac{3\sqrt{3} \lambda^2}{8\pi} AL n_0^0 \zeta(\nu - \nu_0) (3.4651 \sin 2\phi \cos \theta (0.2101 \cos^6 \theta + 0.1989 \cos^4 \theta - 3.5388 \cos^2 \theta + 3.1297) - 1.2829 \sin \phi (\cos^4 \theta + 1.970 \cos^2 \theta - 15.026) \sin \theta \sin 2\theta),
\]

as we see, \( H \) and \( N \) are less aligned than \( P \) and \( N \). This is due to the high degree overlap of the hyperfine levels on the upper state of \( H \). [See the electronic edition of the Journal for a color version of this figure.]
Here $\phi$ is fixed to either $\pi/2$ or $0$. At $\phi = 0, U = 0$. [See the electronic edition of the Journal for a color version of this figure.]

For the D1 line,

$$\epsilon_0 = \frac{3\sqrt{3} I_c^2}{8\pi} \alpha n_0 \beta (\nu - \nu_0)(1.749 \cos^4 \theta_r + 3.447 \cos^2 \theta_r - 26.292), \quad \epsilon_1 = 0, \quad \epsilon_2 = 0. \quad (28)$$

The polarization of Ly$\alpha$ lines are given in Figure 16. We see that compared to Na I and K I, the Ly$\alpha$ line is much more polarized. In general, the more substates the atom has, the less its polarizability is, as polarized radiation is mostly from those atoms with the largest axial angular momentum, which constitute a small percentage in atoms with more sublevels.

The alignment on the ground state also causes the change in optical depth $\tau_{21}$ of the H i 21 cm line, which is a transition between the hyperfine sublevels $F_i = 0, 1$ (see Fig. 15, left),

$$\eta_{21} = \frac{3}{8\pi} A m J^2 \xi(\nu - \nu_0) n \left[ J_0^0 \rho_0^0(F_i = 0) - \sum K J_0^K \rho_0^K(F_i = 1)/\sqrt{3} \right]$$

$$= \frac{3}{8\pi} A m J^2 \xi(\nu - \nu_0) n \left[ J_0^0 \rho_0^0(F_i = 0) - (1 - 1.5 \sin^2 \theta) \rho_0^2(F_i = 1)/\sqrt{2} \right]/\sqrt{3}/4 \sin^2 \theta \rho_0^2(F_i = 1). \quad (29)$$

For comparison, we plot in Figure 17 the ratio of the optical depth with alignment taken into account $\tau_{\text{real}}$ and that without alignment counted $\tau_0$,

$$\frac{\tau_{\text{real}}}{\tau_0} = \frac{\left[ J_0^0 \rho_0^0(F_i = 0) - \sum K J_0^K \rho_0^K(F_i = 1)/\sqrt{3} \right]}{\left[ J_0^0 \rho_0^0(F_i = 0) - J_0^0 \rho_0^0(F_i = 1)/\sqrt{3} \right]} \sim \frac{[-J_0^2 \rho_0^2(F_i = 1)/\sqrt{3}]}{[J_0^0 \rho_0^0(F_i = 0) - J_0^0 \rho_0^0(F_i = 1)/\sqrt{3}]. \quad (30)$$

For the D1 line,
Since the two hyperfine levels are almost evenly populated in the equilibrium case, the slight change among the populations can make a substantial influence on the transmission of 21 cm in the medium (see Fig. 17).

This may be related to the tiny-scale atomic structures (TSAS) observed in different phases of interstellar gas (see Heiles 1997). Besides, as we see from Figure 17, the optical depth can become negative in some cases, indicating the amplification of the 21 cm radiation or maser (see Varshalovich 1967). These effects should be included for H I studies, e.g., Lyα clouds (Akerman et al. 2005) and local bubbles (Redfield & Linsky 2004). Moreover, polarization can occur due to the alignment. The absorption coefficients for the radiation or maser (see Varshalovich 1967). These effects should be included for H I radiation field (see Van de Hulst [1950] for its expression). Thus, if there is an isotropic component in the radiation field, the results will be reduced by a factor of $W_r/W$. Note that if photons are scattered multiple times before reaching the atoms, the anisotropy of the radiation field will be diminished and so would these effects. Detailed study will be provided elsewhere.

7.2. Case of P v

The overlap of hyperfine structure of upper levels reduces the alignment. In fact, P v has the same electron configuration and nuclear spin (see Fig. 15, left). P v is more aligned, as it does not have the overlap on the upper level,

$$
\begin{bmatrix}
\rho_0^1(F_1 = 1) \\
\rho_0^2(F_1 = 1)
\end{bmatrix} = \rho_0^1 \left[ \cos^4 \theta_r + 2 \cos^2 \theta_r - 19 \right],
$$

where $\rho_0^1 = \rho_0(F = 0)/(1.876 \cos^4 \theta_r + 0.289 \cos^2 \theta_r - 10.825)$. The right panel of Figure 15 gives the comparison of the density tensor components of H I and P v.

7.3. Case of N v

N v is also an alkali atom. The nuclear spin of nitrogen is I = 1. The total angular momentum of the ground state thus can be $F = (1 \pm 1/2) = 3/2, 1/2$ (see Fig. 15, middle). Therefore the ground state has $(2 \times 3/2 + 1) + (2 \times 1/2 + 1) = 6$ sublevels in total, which enables alignment. For N v, the hyperfine splitting is much larger than the natural width of the excited state. The smallest hyperfine splitting is about 5.67 γ; their influence is thus marginal. We obtain its density matrix as follows,

$$
\begin{bmatrix}
\rho_0^1(F_1 = 3/2) \\
\rho_0^2(F_1 = 3/2)
\end{bmatrix} = \rho_0 \left( \frac{13.136 \cos^2 \theta_r - 240.06}{20.90 - 62.701 \cos^2 \theta_r} \right),
$$

where $\rho_0^1 = \rho_0(F = 1/2)/(16.62 \cos^4 \theta_r - 1.8 \cos^2 \theta_r - 167.90)$.

The hyperfine transition between the two sublevels on the ground state ($\lambda = 70.72$ mm) has been shown to be a good tracer of hot rarefied astrophysical plasmas (Sunyaev & Docenko 2007). Like H I 21 cm, the alignment alters the optical depth (according to eq. [29]), which one must take into account when analyzing the N v lines. Furthermore, polarization appears as a result of the alignment (see Fig. 18).

8. MORE COMPLEX ATOMIC SPECIES: N i

Unlike alkali atoms, neutral nitrogen is alignable within its fine structure. The ground state of N i is $4S^2_1/2$, and the excited state is $4P^2_{1/2,3/2,5/2}$. The ground state can therefore have four magnetic sublevels with $M = \pm 1/2, 3/2$. Thus, hyperfine structure is not a prerequisite for alignment. However, the alignment and resulting polarizations will be miscalculated if we do not include the hyperfine

---

Fig. 18.—Change of optical depth of N v 70.7 mm due to alignment. Left: Ratio of optical depth with alignment taken into account ($\tau_{\text{real}}$) and without alignment counted ($\tau_0$). Right: Ratio of density of effective absorbers to the total density for polarized radiation $Q$ of N v 70.7 mm (see eq. [29]). Note that without alignment $\rho_0^1$, polarization and this quantity would be zero. [See the electronic edition of the Journal for a color version of this figure.]
structure. For resonant lines, hyperfine interactions cause substantial precession of electron angular momentum \( J \) about total angular momentum \( F \) before spontaneous decay. Thus, total angular momentum \( F \) should be considered, and the \( FM_F \) base must be adopted (Walkup et al. 1982).

Nitrogen has a nuclear spin \( I = 1 \). Its ground level is thus split into three hyperfine sublevels \( F_i = J_i - 1, J_i, J_i + 1 = 1/2, 3/2, 5/2 \). The density tensor \( \rho(F_i = 5/2) \) has three components with \( k = 0, 2, 4 \); \( \rho(F_i = 3/2) \) has two components with \( k = 0, 2 \); sublevel \( F_i = 1/2 \) is not alignable, and we only need to consider \( \rho_0^0 \). Solving equation (10), we obtain

\[
\rho_0^{0,2}(F_i = 3/2) = \rho_0^0 \left( \begin{array}{c}
3 \cos^8 \theta_r + 4 \cos^6 \theta_r - 552 \cos^4 \theta_r - 872 \cos^2 \theta_r + 12325 \\
2 \cos^8 \theta_r - 72.6 \cos^6 \theta_r - 235.04 \cos^4 \theta_r + 1887.5 \cos^2 \theta_r - 600.4
\end{array} \right),
\]

\[(33)\]

\[
\rho_0^{0,2}(F_i = 5/2) = \rho_0^0 \left( \begin{array}{c}
10 \cos^8 \theta_r + 25 \cos^6 \theta_r - 765 \cos^4 \theta_r - 1017 \cos^2 \theta_r + 15087 \\
6 \cos^8 \theta_r - 99.2 \cos^6 \theta_r - 358.5 \cos^4 \theta_r + 4089.6 \cos^2 \theta_r - 1322.3
\end{array} \right),
\]

\[(34)\]

where \( \rho_0^0 = \rho_0^0(F_i = 1/2)/(0.2 \cos^0 \theta_r - 2 \cos^0 \theta_r - 8.6 \cos^0 \theta_r - 217.5 \cos^0 \theta_r - 727.6 \cos^2 \theta_r + 8733.1) \).

Since \( \text{N}\) has a \( J_i = 3/2 > 1 \), absorption from \( \text{N}\) can be polarized, unlike alkali species (see § 4). For the optically thin case, the polarization produced by absorption through optical depth \( \tau = \eta_0 d \) is (see eq. [15])

\[
\frac{Q}{I\tau} \simeq \frac{\eta_1}{\eta_0} = \frac{1.5 \sin^2 \theta w_{2,J_{2,J_2}}^2 \Sigma_F(-1)^{F_i-J_i} \Upsilon(F_i,2) \rho_0^2(F_i)}{\Sigma_F(-1)^{F_i-J_i} \sqrt{2} \Upsilon(F_i,0) \rho_0^2(F_i) + \Upsilon(F_i,2) \rho_0^2(F_i)(1 - 1.5 \sin^2 \theta) w_{2,J_2}^2},
\]

\[(35)\]

where

\[
\Upsilon = \left\{ \begin{array}{ccc}
J_i & J_i & K \\
F_i & F_i & I
\end{array} \right\}
\]

and \( I_0 \) is the intensity of the background source. We neglect emission here. For a generic case, in which the background source (e.g., a QSO) is polarized and optical depth is finite, we can obtain in the first-order approximation

\[
I = (I_0 + Q_0) e^{-\tau(1+\eta_1/\eta_0)} + (I_0 - Q_0) e^{-\tau(1-\eta_1/\eta_0)}, \quad Q = (I_0 + Q_0) e^{-\tau(1+\eta_1/\eta_0)} - (I_0 - Q_0) e^{-\tau(1-\eta_1/\eta_0)}, \quad U = U_0 e^{-\tau}, \quad V = V_0 e^{-\tau},
\]

\[(36)\]

in which \( I_0, Q_0, U_0, \) and \( V_0 \) are the Stokes parameters of the background source. The polarizations of the absorption to \( J_a = 1/2 \) is illustrated in Figure 19. Note that if the incident light is polarized in a direction different from alignment, then \( \text{circular polarization} \) can be generated due to dephasing, although it is a second-order effect. Consider a background source with a nonzero Stokes parameter \( U_0 \) shining on atoms aligned in the Q-direction.\(^7\) The polarization will be precessing around the direction of alignment and generate a \( V \) component representing a circular polarization

\[
\frac{V}{I\tau} \simeq \frac{\kappa Q}{\eta_1} \frac{I_0}{U_0} = \frac{\psi_1 \eta_0}{\eta_1} \frac{I_0}{U_0},
\]

\[(37)\]

where \( \kappa \) is the dispersion coefficient associated with the real part of the refractive index, whose imaginary part corresponds to the absorption coefficient \( \eta_1 \), \( \psi_1 \) is the dispersion profile, and \( \xi_1 \) is the absorption profile.

\[\text{We did the calculation assuming that hyperfine splitting is at least 3 times the natural line width, and therefore, the interference term is negligible for the excited state.}\]

\[\text{To remind our readers, the Stokes parameter } Q \text{ represents the linear polarization along } e_1 \text{ minus the linear polarization along } e_2; U \text{ refers to the polarization along } (e_1 + e_2)/\sqrt{2} \text{ minus the linear polarization along } (-e_1 + e_2)/\sqrt{2} \text{ (see Fig. 1, right).}\]
Optical depth also varies with the alignment (Fig. 20). The generic expression of the line ratio of a multiplet is given by

\[
\tau = \frac{\Sigma F_i(-1)^{F_i-F_{i0}}\left[\sqrt{2}Y(F_i, 0)\rho_0(F_i) + Y(F_i, 2)\rho_0^2(F_i)(1 - 1.5 \sin^2 \theta)\right]^{1/2}}{\Sigma F_i(-1)^{F_i-F_{i0}}\left[\sqrt{2}Y(F_i, 0)\rho_0(F_i) + Y(F_i, 2)\rho_0^2(F_i)(1 - 1.5 \sin^2 \theta)\right]^{1/2}}
\]

(38)

Similar to the cases without hyperfine structure (Paper I), absorption is determined by only two angles, \(\theta_i\) and \(\theta\). Among them, \(\theta_i\) determines the ground-state alignment, and \(\theta\) dependence occurs from the direction of observation. We also see the Van Vleck effect in Figure 19. Specifically, the polarization is either parallel or perpendicular to the magnetic field in the plane of the sky; the switch happens at the Van Vleck angle \(\theta_v = 54.7^\circ\).

In Figures 19 and 20 we plot together the polarizations and optical depth ratios for N \(\text{i}\) and S \(\text{ii}\), the result for which is taken from Paper I. As we know, N \(\text{i}\) and S \(\text{ii}\) have exactly the same term. The difference between them arises from the hyperfine structure of N \(\text{i}\). In other words, if we do not take into account the hyperfine structure of N \(\text{i}\), it would be polarized exactly the same way as S \(\text{ii}\). This can be tested observationally. As we explained above, it is usually true that the more complex the structure is, the less polarized the line is. It can also be interpreted by the nature of hyperfine interactions. Hyperfine interactions cause the precession of electron angular momentum in the field generated by the nuclear spin. Thus, similar to the case of magnetic mixing, the polarization is reduced.

9. DILUTION ALONG THE LINE OF SIGHT

For absorption lines, there are atoms far from any pumping source along the line of sight. These atoms are not aligned, and we need to take into account the averaging along the line of sight. Different components of the atomic density matrix are modulated by pumping. For atoms far from a source, their dipole component of density matrix \(\rho_0^2\) is zero. The zero-order term \(\rho_0^2\), representing total occupation of a level, however, changes with alignment by only \(\pm 10\%\). As a first-order approximation, we thus can ignore the variation of \(\rho_0^2\) due to the pumping and adopt a step function for \(\rho_0^2\):

\[
\rho_0^2(r) = \begin{cases} 
\rho_0^2(\text{aligned}), & \text{for } r < r_c, \\
0, & \text{for } r > r_c.
\end{cases}
\]

(39)

where \(r_c\) is the distance from a pumping source where the collisional transition rate becomes equal to the optical pumping rate. In this case, we only need to multiply \(\rho_0^2\) in equations (29), (35), and (38) by \(N_a/N_{\text{tot}}\), the ratio of alignable column density to the total column density along the line of sight. Accordingly, the contours (Figs. 17–20) do not change apart from their amplitudes, because the dependencies on \(\theta_i\) and \(\theta\) only appear in the terms containing \(\rho_0^2\). By overlapping the polarization contour maps and the line ratio contour map, we attain the angles \(\theta_i\) and \(\theta\). Then insert these values into either equation (35) or equation (38), and we get the ratio \(N_a/N_{\text{tot}}\). More precise results can be obtained by making iterations. From this ratio, we also learn the conditions in the vicinity of the pumping source. Combining different atomic species, we can make a tomography of the magnetic field as the ratio \(N_a/N_{\text{tot}}\) varies with each atomic species. As an example, let us consider the alignment of H \(\text{ii}\) by an O-type star (outside a H \(\text{ii}\) region), for which we know the collisional transition rate is \(C_{10}/n_{\text{H}} = 3.3 \times 10^{10} \text{ cm}^3 \text{ s}^{-1}\). By equating it with the Ly\(\alpha\) pumping rate \(B_0^2\), we can get \(r_c \simeq 15 \text{ pc}\) in the cold neutral medium where we adopt \(n_{\text{H}} = 30 \text{ cm}^3 \text{ s}^{-1}\). If the star is 100 pc away from us, then the dilution factor along the line of sight would be \(r_c \simeq 15 \text{ pc} \times n_{\text{H}}/30 \text{ cm}^3 \text{ s}^{-1} \approx 6\), where \(r_s\) is the size of Strömgren sphere.

It is also possible that there are multiple independent pumping sources along the line of sight. These situations, however, can be easily identified for diffuse interstellar medium.
10. DISCUSSION

10.1. Hyperfine Splittings

We have considered alignment of atoms with nuclear spin in this paper. For these atoms, it is the total angular momentum $F = J + I$ that should be considered. The prerequisite for alignment in this case is $F > 1/2$. There are two categories. Atoms like alkali atoms, Al, Cu, etc., would not be alignable without hyperfine structure, since their electron angular momentum in the ground state is $J < 1$. Another category of atoms is alignable within fine structure, e.g., N, Cl, etc. However, calculations of the alignment and polarization would render erroneous results if one does not take into account the hyperfine structure of the species, because for resonant lines, the hyperfine interaction time scale is shorter than that of resonant scattering.

The first category of atoms above, i.e., with $J < 1$, cannot produce any polarization in absorptions, even when the ground state is aligned within the hyperfine structure. As we explained in § 8, polarization only reduces because of hyperfine interaction. For alkali atoms, the absorption is not polarized in the frame of fine structure. Taking into account hyperfine structure does not make a difference in this case regarding the absorptions. However, the polarization of emission will be affected by the alignment. We note that although the ground level alignment is not a prerequisite for polarization of emission for every line, it does affect the polarization of emission.

We discussed a few examples of elements from our list in Table 1. They were chosen on the basis of astrophysical importance as we see it. For instance, Na D lines are very pronounced and easy to measure. They are also important for studies of comet wakes, as we discuss in the paper. More calculations should be done in the future in relation to particular astrophysical objects under study.

10.2. Polarization of Absorption Lines

We studied polarization of absorption resulting from alignment of atoms with nuclear spin and thus with hyperfine structure. The degree of polarization is reduced compared to atomic species of the same electron configurations but without hyperfine structure. The direction of polarization, however, has the same pattern, namely, either parallel or perpendicular to the magnetic field on the plane of the sky (Paper I). The switch between the two cases happens at the Van Vleck angle $\theta_v = 54.7^\circ$. In fact, this should be applicable to all absorption lines (including molecular lines) regardless of their different structures as long as the following conditions are satisfied. First, pumping light and background light are unpolarized. Second, the system is in the magnetic realignment regime, namely, magnetic precession is faster than the photon excitation rate.

This fact is very useful in practice. It means that even if we do not have an exact prediction and precise measurement of the degree of polarization of the absorption lines, we can still have a two-dimensional (2D) mapping of the magnetic field on the plane of the sky (the angle $\phi$ in Fig. 2, right) within an accuracy of $90^\circ$ once we observe their direction of polarizations. In this sense, it has some similarity with the Goldreich-Kylafis effect (Goldreich & Kylafis 1982), although it deals with radio emission lines.

For absorption lines, there is inevitably dilution along the line of sight, which adds another dependence on the ratio of alignable column density to total column density $N_a/N_\text{tot}$. For different species, this ratio is different. The ratio should be close to 1 for highly ionized species that only exist near radiation sources. The same is true for the absorption from the metastable state (see Paper I). Combining different species (with different $N_a/N_\text{tot}$), it is possible to acquire a tomography of the magnetic field in situ. To extend the technique, we will present elsewhere calculations for more atoms with metastable states.

An additional effect that we consider briefly in § 6 is the generation of circularly polarized light when linear polarized light passes through aligned atoms. This is a new interesting effect, the implications of which we intend to explore elsewhere.

10.3. Polarization of Emission Lines

In Paper I we dealt with absorption lines of species with fine structure. This paper deals with absorption and emission of both the atom species with hyperfine structure and hyperfine plus fine structure.

The work on emission of atoms with hyperfine structure can be traced back in time. Studies of alignment of neutral sodium in the laboratory was pioneered more that half a century ago by Brossel et al. (1952), Hawkins (1955), and Kastler (1957). These experiments revealed that sodium atoms can be efficiently aligned in laboratory conditions if atomic beams are subjected to anisotropic resonance pumping light and background light are unpolarized. Second, the system is in the magnetic realignment regime, namely, magnetic precession is faster than the photon excitation rate.

This fact is very useful in practice. It means that even if we do not have an exact prediction and precise measurement of the degree of polarization of the absorption lines, we can still have a two-dimensional (2D) mapping of the magnetic field on the plane of the sky (the angle $\phi$ in Fig. 2, right) within an accuracy of $90^\circ$ once we observe their direction of polarizations. In this sense, it has some similarity with the Goldreich-Kylafis effect (Goldreich & Kylafis 1982), although it deals with radio emission lines.

For absorption lines, there is inevitably dilution along the line of sight, which adds another dependence on the ratio of alignable column density to total column density $N_a/N_\text{tot}$. For different species, this ratio is different. The ratio should be close to 1 for highly ionized species that only exist near radiation sources. The same is true for the absorption from the metastable state (see Paper I). Combining different species (with different $N_a/N_\text{tot}$), it is possible to acquire a tomography of the magnetic field in situ. To extend the technique, we will present elsewhere calculations for more atoms with metastable states.

An additional effect that we consider briefly in § 6 is the generation of circularly polarized light when linear polarized light passes through aligned atoms. This is a new interesting effect, the implications of which we intend to explore elsewhere.

The disadvantage of emission lines compared to absorption lines is that the direction of the polarization of emission lines has a complex dependence on the direction of the magnetic field and the illusion light. Therefore, the use of emission lines is more advantageous when combined with other measurements.

The change of the optical depth is another important consequence of atomic alignment. Such an effect can be important for $H_\perp$ as was first discussed by Varshalovich (1967). However, the actual calculations that take into account the magnetic realignment in ubiquitous astrophysical magnetic fields are done, as far as we know, only in this paper. It might happen that the variations of the optical depth caused by alignment can be related to the TSAS observed in different phases of interstellar gas (see Heiles 1997). Similar effects may be present for Ly$\alpha$ clouds and other objects.
10.4. Implications

Atomic alignment opens a new channel of information about the physical properties of the various media, including environments of circumstellar regions, AGNs, and the interstellar medium. In particular, the topology of magnetic fields that are so important for these environments can be revealed. What is unique for this new window is the possibility of obtaining information about 3D directions of magnetic fields. By combining different emission and absorption lines, it seems possible to restore the entire structure of the region, which would include full 3D information, including the information about the position of illuminating stars.

We have done calculations for a number of representative atomic species (see Table 1). These emissions and/or absorptions from these atomic species are important lines seen in different astrophysical environments. Indeed, there are many more atomic lines that can be studied the same way as we did here. The particular choice of atoms to use depends on both the instruments available and the object to be studied. For instance, Na D lines have been observed in the interplanetary medium, including comet tails and Jupiter’s moon Io. Many lines from alignable species have been observed in QSOs and AGNs (see Verner et al. 1994), and they can be used to study the magnetic field in situ. Needless to say, this technique can be used in any interstellar medium near an emitting source, H ii region, circumstellar disk, supernova, etc., for intergalactic gas, Ly line, H i 21 cm, N v, and other radio lines, one should also be aware of the fact that alignment also changes the optical depth and the emissivity of the medium. With our quantitative predictions of the alignment, one can also attain the information of the magnetic field in the medium.

A detailed discussion of alignment of atoms in different conditions corresponding to circumstellar regions (see Dinerstein et al. 2006), AGNs (see Kriss 2006), Ly line clouds (see Akerman et al. 2005), interplanetary space (see Cremonese et al. 2002), the local bubble (see Redfield & Linsky 2004), etc., will be given elsewhere. Note that for interplanetary studies, one can investigate not only spatial but also temporal variations of magnetic fields. This can allow a cost-effective way of studying interplanetary magnetic turbulence at different scales.

Taken together, this paper and Paper I provide examples of treating both aligned absorbing and emitting species. For the sake of simplicity, we have not considered a few effects that can affect observations. For instance, we did not consider effects of the finite telescope resolution on the emission from the regions that have finite curvature of the magnetic field or a random magnetic field component. Such effects are well known and described in the existing polarimetric literature (see Hildebrand et al. 2000). For absorption, however, the effect of the telescope finite diagram is negligible if we study the absorption from a point source. In addition, we considered emission from an optically thin medium, which justifies our neglect of the radiative transfer.

In the present paper we considered the alignment of Na i atoms subjected to a limited number of scattering events. However, our approach to describing time-dependent alignment is general and can be easily applied to other species. This may be particularly important for studies of transient phenomena using our technique.

As the resolution and sensitivity of telescopes increases, atomic alignment will be able to probe the finer structure of astrophysical magnetic fields, including those in the halo of accretion disks, stellar winds, etc. Space-based polarimetry should provide a wide variety of species with which to study magnetic fields.

11. SUMMARY

In this paper we calculated the alignment of various atomic species having hyperfine structure and quantified the effect of magnetic fields on alignment. As a result, we obtained linear polarization that is expected for both scattering and absorption. We have shown the following.

1. Atomic alignment of atoms and ions with hyperfine structure of levels happens as a result of interaction of the species with an anisotropic flow of photons.
2. Atomic alignment affects the polarization state of the scattered photons as well as the polarization state of the absorbed photons. The degree of polarization is influenced by mixing caused by Larmor precession of atoms in an external magnetic field. This allows a new way of studying magnetic field direction in a diffuse medium using polarimetry.
3. The degree of polarization depends on the species under study. Atoms with more levels exhibit, in general, a smaller degree of alignment. More importantly, it depends on the angle between the direction of the pumping light and the observational direction with respect to the magnetic field embedded in the medium.
4. The direction of polarization depends on the direction of the anisotropic radiation and observation with respect to the magnetic field acting on an atom, if the polarization of scattered light is considered.
5. The direction of polarization is either parallel or perpendicular to the magnetic field, if the polarization of absorbed light is studied. The switch between the two options happens at the Van Vleck angle between the direction of the magnetic field and the pumping radiation.
6. The intensity ratio of scattered lines or absorption lines is also influenced by magnetic realignment and therefore also carries the information about the direction of the magnetic field.
7. Absorption and emission of species along the line of sight away from the pumping sources interferes with the detected signal, e.g., influence on the degree of the measured polarization and the ratio of absorption lines. This effect, however, can be accounted for iteratively.
8. If the light incident on the aligned atoms is linearly polarized, as this is a typical case of QSOs, circular polarization is created in the transmitted light.
9. Atomic alignment is an effect that is present for a variety of species and for different terms of the same atom. Combining the polarization information as well as using line intensity ratio data allows the improvement of precision in the mapping of magnetic fields and gives insight into the environments of the aligned atoms.
10. A steady state alignment is achieved after many scattering events. For a limited number of the scattering events, the alignment depends on this number, i.e., it is “time dependent.” While the direction of polarization is the same for both cases, the degree of polarization increases with the number of scattering until the steady state alignment is reached.
11. Time variations of the magnetic field in interplanetary plasma should result in time variations of the degree of polarization, thus providing a tool for interplanetary turbulence studies.
APPENDIX A

RADIATIVE TRANSITIONS

For spontaneous emission from a hyperfine state \( J' I' F'M' \) to another hyperfine state \( J'' I'' F'M'' \), the transition probability per unit time is

\[
a = \frac{64 \pi^4 e^2 a_0^2 \nu^3}{3 \hbar c^3} \sum_q \left| \langle JIFM'| V_q | J'F'M' \rangle \right|^2,
\]

where \( V_q^j = r \cdot e_q^j \) is the projection of dipole moment along the basis vector \( e_q \) of the radiation, \( e_\pm = (\mp \hat{x} - \hat{y})/\sqrt{2}, e_0 = \hat{z} \).

For hyperfine lines, in the case of weak interaction in which neighboring fine levels do not interact, the electrical dipole matrix element for transition from a hyperfine sublevel \( F', M'_F \) of the upper level \( J' \) to \( F, M_F \) of lower level \( J \) is given by

\[
R_{F'M'}^F = \langle JIFM_F | V_q | J'F'M'_F \rangle = (-1)^{F' + M_F - 1} \left( \begin{array}{cc} F & 1 \\ -M_F & q \\ M'_F & \end{array} \right) \langle JIF | V_q | J'F' \rangle
\]

\[
= (-1)^{F' + M_F - 1} \sqrt{|F, F'|} \left( \begin{array}{cc} F & 1 \\ -M_F & q \\ M'_F & \end{array} \right) \left( \begin{array}{cc} J_l & I \\ F' & 1 \\ J' & \end{array} \right) \langle J | V_q | J' \rangle,
\]

where \( M_F \) is the quantum number corresponding to the projection of the total angular momentum \( F \) and \( I \) corresponds to the nuclei spin. The matrix with braces represents the \( 6j \) or \( 9j \) symbol, depending on the order of the matrix. When more than two angular momentum vectors are coupled, there is more than one way to add them up and form the same resultant. The \( 6j \) (or \( 9j \)) symbol appears in this case as a recoupling coefficient describing transformations between different coupling schemes.

APPENDIX B

IRREDUCIBLE DENSITY MATRIX

We adopt the irreducible tensorial formalism for performing the calculations (see also Paper I). The relation between the irreducible tensor and the standard density matrix of atoms is

\[
\rho_{QQ}^\ell(F, F') = \sum_{M_0} (-1)^{F-M}(2K+1)^{1/2} \left( \begin{array}{cc} F & K \\ -M & Q \\ M' & \end{array} \right) \langle FM|\rho|F'M' \rangle.
\]

For photons, their generic expression of the irreducible spherical tensor is

\[
J_{QQ}^\ell = \sum_{qq} (-1)^{1+q}[3(2K+1)]^{1/2} \left( \begin{array}{cc} 1 & 1 \\ q & -q' \\ -Q & \end{array} \right) J_{qq}.
\]

APPENDIX C

EFFECT OF COLLISIONS

Collisions can cause transitions among the hyperfine sublevels and reduce the ground-state alignment. In the regime in which collisions are not negligible, the equilibrium equation for the ground-state equation (eq. [2]) should be modified to include the collisions. For the ground hyperfine level,

\[
\dot{\rho}_0^F(F_1^0) = \sum_{F'_1, l_1, k_1} p_k[j_l](C(F'_1 \rightarrow F_1^0)\rho_k^F(F'_1) + \sum_{J_aJ_bF_aF_b} p_a[j_b][A(J_a \rightarrow J_l)\rho_k^F(F_a,F_b)] \right.

\left. - \sum_{F'_1, l_1, k_1} \delta_{k_k}C(F_1^0 \rightarrow F'_1) + D^k + \sum_{J_a, k_1} (\delta_{k_k}B_{l_k}(\gamma_b^0 + s_{k_k}B_{l_k}) \right)

\left. \rho_0^F(F_1^0). \right)
\]
For other hyperfine levels on the ground state,

\[ \rho_0^k(k) = - \sum_{F'_{l-w},l} C(F_l \rightarrow F'_{l}) \rho_0^k(F_l) + \sum_{F'_{l-w},l} p_k(J_l) C(F_l \rightarrow F_{l}) \rho_0^k(F_l) + \sum_{J_{u-w},u} p_k(J_u) A(J_u \rightarrow J_l) \rho_0^k(F_{u},F'_{u}) \]

\[ - \sum_{F'_{l-w},l} \delta_{l,k} C(F_l \rightarrow F'_{l}) + D^k + \sum_{J_{l-w},l} (\delta_{l,k} B_{l}^{J_0} + s_{ll} B_{l}^{J_0} J_0^2) \rho_0^k(F_l) + \sum_{F'_{l-w},l} [J_l] p_k C(F_l \rightarrow F_{l}) \rho_0^k(F_{l}), \] (C2)

where \( C \) is the collisional transition rate among the hyperfine levels on the ground state, \( D^k \) is the depolarizing rate due to elastic collisions (see Landi Degl’Innocenti & Landolfi 2004), and \( E \) and \( E' \) are the energies of hyperfine levels \( F_l \) and \( F'_{l} \), respectively.

APPENDIX D

FROM OBSERVATIONAL FRAME TO MAGNETIC FRAME

In real observations, the line of sight is fixed, and the direction of the magnetic field is unknown. Thus, a transformation is needed from the observational frame to the theoretical frame in which the magnetic field is the quantization axis. This can be done by two Euler rotations, as illustrated in Figure 2. In the original observational coordinate \((x'yz')\) system, the direction of the radiation is defined as the \( z' \)-axis, and the direction of the magnetic field is characterized by polar angles \( \theta_B \) and \( \phi_B \). First, we rotate the whole system by an angle \( \phi_B \) about the \( z' \)-axis, so as to form the second coordinate system \( x'y'z' \). The second rotation is from the \( z'(z'') \)-axis to the \( z' \)-axis by an angle \( \theta_B \). Mathematically, the two rotations can be fulfilled by multiplying rotation matrices

\[
\begin{bmatrix}
\cos \theta_B & 0 & -\sin \theta_B \\
0 & 1 & 0 \\
\sin \theta_B & 0 & \cos \theta_B
\end{bmatrix}
\begin{bmatrix}
\cos \phi_B & \sin \phi_B & 0 \\
-\sin \phi_B & \cos \phi_B & 0 \\
0 & 0 & 1
\end{bmatrix}
= \begin{bmatrix}
\cos \theta_B \cos \phi_B & \cos \theta_B \sin \phi_B & -\sin \theta_B \\
-\sin \phi_B & \cos \phi_B & 0 \\
\sin \theta_B \cos \phi_B & \sin \theta_B \sin \phi_B & \cos \theta_B
\end{bmatrix}. \] (D1)

REFERENCES

Akerman, C. J., Ellison, S. L., Pettini, M., & Steidel, C. C. 2005, A&A, 440, 499
Bonnier, V., & Sahal-Brechot, S. 1978, A&A, 69, 57
Brossel, J., Kastler, A., & Winter, J. 1952, J. de Phys. Radium, 13, 668
Cowan, R. D. 1981, The Theory of Atomic Structure and Spectra (Berkeley: Univ. California Press)
Cremonese, G., & Fulle, M. 1999, Earth Moon Planets, 79, 209
Cremonese, G., Huebner, W. F., Rauer, H., & Boice, D. C. 2002, Adv. Space Res., 29, 1187
Dinerstein, H. L., Sterling, N. C., & Bowers, C. W. 2006, in ASP Conf. Ser. 348, Astrophysics in the Far Ultraviolet. ed. G. Sonneborn, H. Moos, & B-G Andersson (San Francisco: ASP), 328
Goldreich, P., & Kylafis, N. D. 1982, ApJ, 253, 606
Happer, W. 1972, Rev. Mod. Phys., 44, 169
Hawkins, W. B. 1955, Phys. Rev., 98, 478
Heiles, C. 1997, ApJ, 481, 193
Hildebrand, R. H., Davidson, J. A., Dotson, J. L., Dowell, C. D., Novak, G., & Vaillancourt, J. E. 2004, PASP, 112, 1215
Kastler, A. 1957, J. Opt. Soc. Am., 47, 460
Kris, G. A. 2006, in ASP Conf. Ser. 348, Astrophysics in the Far Ultraviolet. ed. G. Sonneborn, H. Moos, & B-G Andersson (San Francisco: ASP), 499
Landi Degl’Innocenti, E. 1982, Sol. Phys., 79, 291
———. 1983, Sol. Phys., 85, 3
———. 1984, Sol. Phys., 91, 1
———. 1998, Nature, 392, 256
Landi Degl’Innocenti, E., & Landolfi, M. 2004, Polarization in Spectral Lines (Dordrecht: Kluwer)
Landolfi, M., & Landi Degl’Innocenti, E. 1985, Sol. Phys., 98, 53
———. 1986, A&A, 167, 200
Lazarian, A. 2003, J. Quant. Spectrosc. Radiat. Transfer, 79, 881
Manso Sainz, R., & Trujillo Bueno, J. 2003, in ASP Conf. Ser. 307, Solar Polarization, ed. J. Trujillo-Bueno & J. Sanchez Almeida (San Francisco: ASP), 251
Redfield, S., & Linsky, J. L. 2004, ApJ, 602, 776
Stenflo, J. O., & Keller, C. U. 1997, A&A, 321, 927
Sunyaev, R., & Dolenc, A. 2007, Astron. Lett., 33, 67
Thomas, N. 1992, Suvey Geophys., 13, 91
Trujillo Bueno, J. 1999, in Solar Polarization, ed. K. N. Nagendra & J. O. Stenflo (Dordrecht: Kluwer), 73
Trujillo Bueno, J., & Landi Degl’Innocenti, E. 1997, ApJ, 482, 183
Trujillo Bueno, J., Landi Degl’Innocenti, E., Collados, M., Merenda, L., & Manso Sainz, R. 2002, Nature, 415, 403
Van de Hulst, H. C. 1950, Bull. Astron. Inst. Netherlands, 11, 135
Varshalovich, D. A. 1987, Soviet Phys.-JETP, 52, 242
———. 1968, Astrofizika, 4, 519
———. 1971, Soviet Phys.-Uspekhi, 13, 429
Verner, D. A., Barthel, P. D., & Tyler, D. 1994, A&AS, 108, 287
Walkup, R., Migdall, A. L., & Pritchard, D. E. 1982, Phys. Rev. A, 25, 3114
Yan, H., & Lazarian, A. 2006, ApJ, 653, 1292 (Paper I)