Optical Tomographic Image Reconstruction Based on Gradient Tree Calculation Method

Jinlan Guan

Department of Basic Courses, Guangdong AIB Polytechnic College, Guangzhou, 510507

*Corresponding author e-mail: guanjinlan@gdaib.edu.cn

Abstract. Optical coherence tomography is a new imaging method, which is widely used in many fields. This article introduces an iterative image reconstruction algorithm based on gradient trees. It also discusses image reconstruction methods containing void-like regions. It is proved that the image reconstruction based on the transportation model can overcome the shortcomings of the diffusion equation, and it can accurately reconstruct the optical tomographic image.

Keywords: Optical Coherence Tomography, Gradient, Image Reconstruction

1. Introduction

In recent years, laser technology has developed rapidly, and a new tomographic imaging technology - Optical coherence tomography (OCT) has also been developed. This technology combines semiconductor and ultra-fast laser technology, ultra-sensitive detection, fine mine automatic control and computer image processing, and other technologies into a whole. This is a brand new tomography technology.

2. Basic principles

2.1. Basic principles of OCT

The time it takes for a light pulse to be emitted back at different depths of the target is different. By measuring the time delay of the light pulse emitted from the target, the structure image in the depth direction of the target can be obtained. The process of obtaining structural images is image reconstruction [1].

2.2. Boltzmann transport equation

When photons propagate in a chaotic medium, two physical phenomena mainly occur: absorption and scattering. The purpose of optical tomography is to solve the distribution of absorption and scattering coefficients in a given medium. As we all know, light is an electromagnetic wave and its propagation law in tissues can be strictly described by Maxwells equation. This equation expresses that the relationship between electric waves and magnetic waves in time and space is the wave form of light. In optical tomography, the propagation of photons in the scattering medium obeys the Boltzmann transport
model, referred to as the transport equation. Most of the current reconstruction algorithms are based on the approximate form of Boltzmann's equation-diffusion equation. However, this approximate form is only used in the case of large scattering and dry absorption. Therefore, this paper presents an optical tomographic image reconstruction algorithm based on the radiation transfer equation and applies it to the image reconstruction of the region with holes [2].

3. Iterative image reconstruction method based on gradient

3.1. Gradient based iterative image reconstruction method (Gradient based iterative image reconstruction, GIIR)

Alexander DKlose uses the joint difference method in the gradient calculation of optical tomographic image reconstruction, but the derivation algorithm for optical parameters given by him has limitations. The algorithm can only achieve the calculation of the derivative of the optical parameters of the boundary point and cannot achieve the optical parameter of the internal point. The calculation of the parameter derivative will cause the image reconstruction to fail. Based on the joint difference algorithm, this paper studies the derivation method for the optical parameters of the internal points and proposes a strategy for deriving the optical parameters of the internal points based on the gradient tree, that is, the gradient of a node is from the root node to the node. The sum of all path derivatives. Direct gradient calculation is a recursive process. When there are many grids, the calculation speed is very slow and almost impossible to achieve. Therefore, an approximate gradient calculation strategy is proposed in the specific implementation to take a subset of branches to achieve gradient calculation, which greatly reduces the computational complexity. The application of the proposed reconstruction method based on the gradient tree can effectively realize the calculation of the derivative of the optical parameters of the internal points, while the approximate calculation method can effectively reduce the complexity of the gradient calculation, increase the calculation speed, and obtain good image reconstruction quality, which further proves the proposal of this paper. The effectiveness of the reconstruction algorithm [3].

3.2. The idea of joint difference algorithm

The OT imaging problem can be regarded as a nonlinear objective function optimization problem. The ultimate goal of OT image reconstruction is to solve the optical parameter distribution vector $\mu$ and minimize the objective function value. The gradient optimization algorithm is usually used to achieve the reconstruction of optical tomographic images through the optimization operation of the objective function. Most current reconstruction methods are iterative image reconstruction methods based on gradients. In recent years, this method has been used in OT reconstruction based on diffusion equation; at the same time, Klose and Hielscher used it in OT reconstruction based on radiation transfer equation. In image reconstruction, the steepest descent method, the conjugate gradient method, etc. are usually used to calculate a local optimization algorithm that only needs to calculate a gradient to achieve the iterative update of optical parameters. The joint difference algorithm does not need to explicitly calculate the Jacobian matrix, and does not need to invert it repeatedly, and the calculation is relatively simple [4].

The main challenge of the GIIR method is to find an effective method to calculate the objective function $\nabla \Phi$ with respect to the gradient of the optical parameter $\mu$, a method to directly calculate the approximate derivative of the finite difference method, namely:

$$
\frac{d\Phi}{d\mu} = \frac{\Phi(\mu + \Delta\mu) - \Phi(\mu)}{\Delta\mu}
$$

(1)

In this method, if the vector $\mu$ contains $n$ unknown optical parameters, $(n+1)$ forward calculations are required to obtain the gradient, so a large amount of calculation is required. Here, the joint difference method is used for gradient calculation, which can quickly and effectively calculate $\nabla \mu \Phi$. The joint difference method is a method that uses the intermediate data obtained in the forward calculation and the chain rule to realize the gradient calculation. It only uses one forward calculation. The gradient can
be obtained, saving calculation time. In this paper, this idea is used in OT reconstruction to calculate the gradient of the objective function with respect to the optical parameters $\nabla_{\mu} \phi$.

The joint difference method is used in optical tomography to realize the direct gradient calculation of the objective function to the optical parameters. In order to apply this method, the objective function must first be decomposed into a series of basic differentiable function stages along the reverse direction of the forward model iterative calculation to systematically apply the chain rule to each basic factor stage to obtain the gradient value [5].

3.3. Calculation method of internal optical parameter gradient

First, the objective function is decomposed into the form of Z sub-functions $F^{(Z)}$ (assuming that the convergent forward solution is obtained after Z-step forward iterative operation):

$$\varphi[F(\mu)] = \varphi[F^{(Z)}(F^{(Z-1)}(F^{(Z-2)}(\ldots(F^{2}(F^{1}(\mu))\ldots(\mu))))\mu)]$$ (2)

The sub-function $F^{(Z)}$ is defined by the iterative algorithm calculated by the forward model, and the intermediate result can be obtained in the Z-th iteration: $\phi_{k,i,j}^{(Z)}(k \epsilon \{1, k\}, i, j \epsilon \{1, N\})$. The length of the solution vector is $L = K \times N \times N$. $\phi_{k,i,j}^{(Z)}$ represents the value when the direction of the boundary position (i, j) is k in the last Z-th iteration.

$$F(\mu) = \varphi[f_1(\mu), f_2(\mu), \ldots, f_p(\mu)]$$ (3)

Analyzing the formula (2), we can see that it is essentially a form of a multivariate function about $\mu$.

Then the derivative of $F$ with respect to $\mu$ can be obtained by the total differentiation of $F$ with respect to $\mu$, namely:

$$\frac{dF}{d\mu} = \frac{\partial F}{\partial f_1} \frac{df_1}{d\mu} + \frac{\partial F}{\partial f_2} \frac{df_2}{d\mu} + \ldots + \frac{\partial F}{\partial f_p} \frac{df_p}{d\mu}$$ (4)

$F_1$, $f_2$, $\ldots$, $f_P$ are all multivariate functions of $\mu$. This relationship can be described visually with a tree structure (ie, gradient tree). For example, $\phi_{k,i,j}^{(Z)}$ is derived from $(\mu)i-1,j-1$. According to formula (4), the derivative is the sum of all path derivatives from the root node to the child node $\phi_{k,i-1,j-1}^{(Z)}$. Obviously, the derivation of $\phi_{k,i,j}^{(Z)}$ to $(\mu)i-1,j-1$ is the sum of the two branch derivatives in the tree structure. Then the number of branches required to derive $\phi_{k,i,j}^{(Z)}$ for any $(\mu)i,j$ is the sum of all the path derivatives from the root node to the child node $\phi_{k,i,j}^{(Z)}$ [6].

It can be seen from the above that the subscript of $\phi^{(Z)}$ is $(k,a_i,a_j)$, the subscript of $\mu$ is $(b_i,b_j)$, let $m = a_i-b_i$, $n = a_j-b_j$, then $\phi_{k,a_i,a_j}^{(Z)}$ is $(\mu)b_i,b_j$. The total number of branches to be derived is $C_{m+n}^m$.

Theoretically, the recursive program can complete the calculation of the derivative, but when $m$ and $n$ are relatively large, the amount of calculation of the recursive program is very huge. In order to improve the calculation speed, this paper adopts an approximate gradient calculation method.

In order to reduce the amount of calculation. It is proposed to take a subset of the $C_{m+n}^m$ branches to calculate the gradient. First, construct an initial binary sequence containing $(m+n)$ bits, the first $m$ bits are 1, and the last $n$ bits are 0. Among them, 1 represents the left branch, and 0 represents the right branch. Through a series of transposition operations on this sequence, a subset of branches can be found. The following transposition strategies are mainly used in the experiment:

(1) Perform right cyclic shift on the initial sequence until the last bit is 1, and get a branch set $S_1\{A_{i}=1,2,..\}$, where $r$ represents the number of elements in the set [7].

(2) For the initial sequence, swap each bit from the 2nd to the $m$th bit with the $(m+1)$th to $(m+n)$ bit to obtain the second branch set $S_2\{B_{i}=1,2,..,s\}$, $s$ represents the number of elements in the set.

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(3) Perform a reverse order operation on the elements in the set $S_1$. After selecting the reverse order, it is different from before the reverse order, and the elements that do not exist in $S_2$ form the third set.

### 4. Experiment and discussion

According to the obtained gradient, the conjugate gradient method is used to determine the update direction of the optical parameters. The experimental model is two $21 \times 21$ square grids with a grid spacing of 0.1cm. The reconstruction model given in Figure 1 is a reconstruction model for irregular areas. The black area in the upper left corner is the low-scattering area, and the scattering coefficient is $8 \text{ cm}^{-1}$, the white area in the lower right corner is the high scattering area, the scattering coefficient is $12 \text{ cm}^{-1}$, and the background scattering coefficient is $10 \text{ cm}^{-1}$. This experiment is aimed at the reconstruction of the scattering coefficient [8]. The absorption coefficient of the entire model is 0.01 cm$^{-1}$.

![Figure 1. Experimental model](image)

The light source is placed in the middle of each side, and the detector is placed on the second to 20 grid points of the other three sides (except the side where the light source is located). The spacing between the detectors is 0.1cm, which constitutes a total of $4 \times 3 \times 19$ detector pairs. Figure 1 shows the position of the light source and the detector when the light source is at the bottom. The arrow indicates the position of the light source, and the small black rectangle indicates the position of the detector.

The number of branches taken during the experiment is about 80% of the total number. Figure 2 shows the reconstruction results of the experimental model. Among them (a) is the reconstruction result after 5 iterations; (b) is the reconstruction result after 30 iterations. It can be seen from the experimental results that although the approximate gradient calculation method is used, a better reconstructed image can be obtained after only a few iterations, indicating that the approximate gradient calculation is effective [9].

![Figure 2. Reconstructed image of the model](image)

The following introduces two measures to measure the effect of image reconstruction, and further analyze the experimental data.
The objective function value. In GIIR reconstruction, the ultimate goal is to minimize the value of the objective function, so the change in the value of the objective function in the iterative process can reflect how close the reconstruction process is to the target.

Normalized root mean squared error (NRM S), which is a quantity that can directly reflect the quality of image reconstruction, and is defined as follows:

\[
\text{NRMS} = \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} (\mu_{ij}^0 - \mu)
\]

Where \( \mu_{ij}^0 \) is the value of the original image at \((i, j)\), is the reconstructed image value generated in each iteration, \( \mu \) is the average value of the original image, and \( N \) is the number of pixels in each row (column) of the reconstructed image. Figure 3 is the objective function of the experimental model, and Figure 4 is the NRM S curve [10].

![Figure 3. Objective function curve](image)

![Figure 4. NRMS curve](image)

It can be seen from the curve in the figure that as the number of iterations increases, the value of the objective function and NRMS become smaller and smaller, and the decline of the objective function fluctuates, while the NRM S has basically a linear downward trend, confirming the reconstruction Has been developing in a good direction.

5. Conclusion
This paper discusses the method of image reconstruction based on gradient tree with hollow regions under the transport model. It is proved that the image reconstruction based on the transport model can overcome the drawbacks of the reconstruction of the diffusion equation in the non-scattering region. This can reduce the complexity of gradient calculation and speed up the image reconstruction process. It can also accurately reconstruct optical tomographic images. However, the verification of the algorithm in this article is only a simulation model. In the future, we will continue to improve the reconstruction algorithm and explore reconstruction methods under complex backgrounds.
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