Spatially Constrained Fuzzy c-Means Clustering Algorithm for Image Segmentation

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Abstract. The fuzzy c-means (FCM) clustering is an unsupervised clustering method, which has been widely used in image segmentation. In this paper, a spatially constrained fuzzy c-means clustering algorithm for image segmentation is proposed to overcome the sensitivity of the FCM clustering algorithm to noises and other imaging artifacts. Firstly, the local prior probabilities of pixel classification are defined according to the fuzzy membership function values of neighbouring pixels, and then those local prior probabilities are incorporated into the objective function of the standard FCM. Thus, the local spatial information embedded in the image is incorporated into the FCM algorithm. Experimental results on the synthetic and real images are given to demonstrate the robustness and validity of the proposed algorithm.

1. Introduction

As an important part of image processing, image segmentation is considered to be one of the key and most difficult techniques in computer vision [1]. The goal of image segmentation is to divide an image into several disjoint regions, which are homogeneous with respect to some characteristics such as gray level, color, texture, etc. Fuzzy c-means (FCM) clustering method [2] has been widely used in image segmentation and its success chiefly attributes to the introduction of fuzziness for the belongingness of each image pixel. This allows the clustering procedure to retain more original image information than the crisp or hard segmentation methods. However, the standard FCM clustering method is sensitive to noise and other imaging artifacts, since it does not consider any spatial information in an image.

To solve this problem, a variety of studies incorporating local spatial information into the original FCM clustering method have been proposed [3-10]. In some methods, the spatial context was embedded in the objective function of FCM clustering. Pham [6] applied a spatial penalty on the membership functions. The penalty term leads to an iterative algorithm that allows the estimation of spatially smooth membership functions. A FCM_S algorithm, proposed by Ahmed [7], modified the FCM objective function to compensate the intensity inhomogeneity and allow the labeling of a pixel to be influenced by the labels in its immediate neighborhood. However, this method is very time-consuming to compute the neighborhood terms in each iteration step. To address this problem, Chen and Zhang [8] applied the mean-filter image and the median-filter image to simplify the neighborhood term of FCM_S, and proposed the FCM_S1 and FCM_S2, respectively. The extra mean-filtered image and median-filtered image introduced by these two algorithms can be computed in advance, so the computational complexity can be considerably reduced. Cai [9] proposed a generalized fuzzy c-means algorithm for fast and robust image segmentation. Krindis and Chatzis [10] proposed a fuzzy local
information c-means (FLICM) clustering algorithm. In this algorithm, a fuzzy local factor was introduced for solving the problem of noise sensitive and image detail lost. However, the objective function of FLICM is formulated without setting any parameter that controls the effect of this factor. Feng and Chen [11] used the spatial context constraint on the objective function of FCM, which is referred as GFCM. The spatial constraint based on MRF theory is defined as refusable level. The main drawback of the GFCM algorithm is that the hard maximum membership classification is necessary to compute the prior probabilities during the iteration.

Inspired by the ideas of GFCM algorithm, a novel spatially constrained fuzzy c-means (SCFCM) for image segmentation is proposed in this paper. At first, the local prior probabilities of pixel classification are defined according to the fuzzy membership function values of neighboring pixels, then those local prior probabilities are incorporated into the objective function of the standard FCM. Thus, the local spatial information embedded in the image is incorporated into the segmentation process. Therefore, the proposed method can effectively reduce the impact of noises and other imaging artifacts.

The rest of paper is organized as follows. Section 2 reviews the FCM clustering method briefly. The new image segmentation method is presented in Section 3. The experimental comparisons are presented in Section 4. Finally, conclusions are given in Section 5.

2. Fuzzy c-means clustering

The FCM clustering was first proposed by Dunn [12] and further improved by Bezdek [2]. Mathematically, the standard FCM clustering partitions an image $X$ into $c$ clusters by minimizing the objective function

$$J_{FCM} = \sum_{k=1}^{c} \sum_{i=1}^{n} u_{ik}^m \| x_i - v_k \|^2$$

where $X = \{x_1, x_2, \ldots, x_n\}$ denotes an image with $n$ pixels, $v_k$ is the prototype of the center of cluster $k$ and $u_{ik}$ is the fuzzy membership function of the pixel $i$ belonging to the cluster $k$. The fuzzy partition matrix $U = [u_{ik}]$ satisfies

$$U = \left\{ u_{ik} \in [0,1] \mid \sum_{k=1}^{c} u_{ik} = 1, \forall i; 0 < \sum_{i=1}^{n} u_{ik} < n, \forall k \right\}$$

The parameter $m$ control the fuzziness of the membership function.

Using the Lagrange multiplier method, the objective function $J_{FCM}$ can be minimized iteratively. the update equations of membership functions $u_{ik}$ and the cluster centers $v_k$ are given as follows:

$$u_{ik} = \frac{\left( \| x_i - v_k \| \right)^{2/(m-1)}}{\sum_{j=1}^{c} \left( \| x_i - v_j \| \right)^{2/(m-1)}}$$

$$v_k = \frac{\sum_{i=1}^{n} u_{ik}^m x_i}{\sum_{i=1}^{n} u_{ik}^m}$$

Apparently, the FCM algorithm does not use any spatial information in an image. i.e., the segmentation process is solely based on the histogram of image. Therefore, the FCM clustering method is sensitive to noises and imaging artifacts.

3. Proposed Algorithm
In this work, the local prior probabilities of each pixel are defined. Then those probabilities are incorporated into the FCM objective function, and a new spatially constrained fuzzy c-means (SCFCM) algorithm for image segmentation is presented.

3.1 Definition of the local prior probability
Given the membership function \( u_{ik} \) of FCM algorithm, the local prior probability is defined as

\[
\pi_{ik} = \frac{\left( \sum_{j \in N_i} w_j u_{jk} \right)^b}{\sum_{k=1}^c \left( \sum_{j \in N_i} w_j u_{jk} \right)^b}
\]  

(5)

Where \( b \) is the constant strength factor, \( w_j \) is a weight coefficient depending on their distance from the central pixel. The influence of the neighbouring pixels is controlled by \( w_j \), and it can be obtained by

\[
w_j = \exp \left( -\frac{L_{ij}^2}{2d^2} \right)
\]  

(6)

Where \( L_{ij} \) is the spatial Euclidean distance between pixels \( i \) and \( j \), and \( d = (s-1)/4 \), \( s \) is the size of the neighborhood window. The strength of \( w_j \) decreases as the distance between pixel \( i \) and \( j \) increases.

Comparing to GFCM algorithm, those prior probabilities can be computed directly from the membership functions of neighborhood pixels, and the hard maximum membership segmentation is unnecessary during the iteration. Furthermore, the spatial Euclidean distance between pixels has been incorporated into those prior probabilities. Therefore, the local prior probability defined in this paper has better characteristics, compared with GFCM algorithm.

3.2 SCFCM algorithm
By incorporating the local prior probability \( \pi_{ik} \) into the standard FCM objective function, the objective function of the proposed algorithm is formulated as

\[
J_{SCFCM} = \sum_{k=1}^c \sum_{i=1}^n (1 - \pi_{ik}) u_{ik}^m \| x_i - v_k \|^2
\]  

(7)

Similar to the standard FCM clustering method, the \( J_{SCFCM} \) can be minimized iteratively. The membership functions \( u_{ik} \) and the cluster centers \( v_k \) are updated as follows:

\[
v_k = \frac{\sum_{i=1}^n u_{ik}^m (1 - \pi_{ik}) x_i}{\sum_{i=1}^n u_{ik}^m (1 - \pi_{ik})}
\]  

(8)

\[
u_{ik} = \left[ \frac{(1 - \pi_{ik}) \| x_i - v_k \|^2}{\sum_{k=1}^c \left( (1 - \pi_{ik}) \| x_i - v_k \|^2 \right)^{-1/m}} \right]^{-1/m}
\]  

(9)

Thus, the SCFCM algorithm is given as follows:
Step 1: Set the parameter \( m \), and the number of the clusters \( c \).
Step 2: Initialize the membership functions randomly.
Step 3: Calculate the local prior probabilities using (5)-(6).
Step 4: Update the cluster center $v_k$ using (8).

Step 5: Update the membership functions $u_{ik}$ using (9).

Step 6: In case of convergence, exit; Otherwise, go to Step 3.

4. Experimental Results

To demonstrate the robustness and validity of the SCFCM method, we compare the performance of our proposed SCFCM algorithm with the FCM, FCM_S1, FLICM and GF CM algorithms. The experiment will be done on the synthetic and real images corrupted by noises respectively. We set the fuzzy parameter $m = 2$, and $b = 1.8$ for the proposed algorithm. The size of neighbourhood window is $3 \times 3$.

In the first experiment, a synthetic test image with $128 \times 128$ pixels, as shown in Fig. 1(a), is used to compare the performance of the proposed algorithm with others. The image contains four classes with intensity values 0, 70, 140 and 210. We compare the performance of above five algorithms on the synthetic test image, which is divided into 4 classes. The test image is corrupted by “Gaussian”, “salt & pepper” and hybrid “Gaussian” and “salt & pepper” noises, respectively, and the segmentation results are shown in Table 1 and Fig. 1. Table 1 gives the average segmentation accuracy (SA) of the five algorithms on three different noisy images, where SA is defined as the sum of the correctly classified pixels divided by the sum of the total number of pixels [8]. Fig. 1 shows the results on synthetic test image corrupted by “Gaussian” noises. we can see that FCM, FCM_S1, FLICM and GFCM are, respectively, influenced by the noises more or less., which indicates that these algorithms lack enough robustness to noises, while the proposed SCFCM algorithm removes almost all the added noises. Compared to the FCM, FCM_S1, FLICM and GFCM algorithm, we can see that the proposed SCFCM algorithm achieves much better segmentation.

![Table 1: SA% of five algorithms on synthetic test image.](image)

| Noise type   | FCM | FCM_S1 | FLICM | GFCM | SCFCM |
|--------------|-----|--------|-------|------|-------|
| Gaussian     | 98.21 | 99.08  | 98.53 | 98.79 | 99.98 |
| Salt & Pepper| 95.81 | 95.38  | 95.92 | 96.47 | 98.33 |
| Hybrid noise | 94.53 | 95.68  | 95.58 | 95.71 | 98.55 |

![Fig. 1: Segmentation results of five algorithms on synthetic test image.](image)

(a) Original image. (b) Noise image. (c) FCM. (d) FCM_S1. (e) FLICM. (f) GFCM. (g) SCFCM.
Fig. 2 presents a comparison of segmentation results on a real image with $308 \times 242$ pixels as shown in Fig. 2(a). Fig. 2(b) is the original image corrupted by hybrid “Gaussian” and “salt & pepper” noises. Fig. 2(c)-(g) present the results applying the five algorithms on Fig. 2(b), respectively. We use $c = 2$ in the experiment. Visually, FCM, FCM_S1, FLICM and GFCM are respectively affected by the noises to different extents, while the proposed SCFCM algorithm achieves an outstanding segmentation. From Fig. 2(c)-(g), It can be concluded that the SCFCM algorithm is less sensitive to noises than other algorithms.

5. Conclusions
In this paper, we have presented a novel spatially constrained fuzzy c-means clustering algorithm for image segmentation. Firstly, the local prior probabilities of pixels are defined according to the fuzzy membership function values of neighbouring pixels, then those local prior probabilities are incorporated into the objective function of the standard FCM. By this way, the local spatial information embedded in the image is incorporated into the FCM clustering algorithm. Experimental results on the synthetic and real images have demonstrated the robustness and validity of the proposed algorithm.

Acknowledgments
This work was supported by the National Natural Science Foundation of China under grant no. 51674200 and The Youth Foundation of Xi'an Shiyou University under grant no. 2013BS021.

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