Chapter 5
Quarantine and Isolation

The intervention-game framework introduced in Chap. 4 included provisions for vaccination and intermediate defense measures (IDM), which were defined as voluntary strategies decided upon by individuals. This Chapter introduces another framework for interventions that are forcefully imposed upon individuals, regardless of their intentions. These include quarantine and isolation. We show a theoretical
framework based on the intervention-game concept and discuss how the introduction of quarantine and isolation impacts the final epidemic size (FES) and average social payoff (ASP).

5.1 Social Background; Quarantine or Isolation?

In general, vaccination and IDM can be regarded as more or less spontaneous, simply because they are self-costly and self-beneficial. The point of how one’s decision impacts others is less addressed in the literature. Although commitment to those provisions helps to establish a herd immunity, the primary effect, i.e., benefit, comes to the person who commits. However, if the vigor of a spreading disease becomes critical, as we are currently experiencing with COVID-19, public authorities and governments cannot help but to interfere to save their own population as individual-based provisions are no longer sufficient. Thus, provisions no longer focus on an individual, but on the mass of individuals in the population. For this purpose, authorities sometimes resort to enforcing compulsory provisions.

There are two ideas: quarantine and isolation. Hereafter, we use “quarantine” to refer to a state that is more compulsory than isolation. In quarantine, the authorities force people who are not symptomatic but might be infected to be segregated. By contrast, isolation is only applied to people who are symptomatic. The word “quarantine” originally corresponded to a period of 40 days,\(^1\) which is the length of time that arriving ships suspected of black-plague infection were constrained from intercourse in Mediterranean ports in the fourteenth century. The terms quarantine and isolation have been heavily highlighted in the media in the wake of COVID-19, and would be familiar to lay people. Although the removal of even a small number of infected individuals from the general population is beneficial from the standpoint of public health, it impinges upon individual rights and freedom; thus, in democratic countries, it is believed that applying quarantine on a large scale without due cause would not be possible. However, COVID-19 has altered this traditional picture. Western democratic countries in Europe and the USA have introduced quite strong intervention provisions, even including legal penalties like lock-down to dam the voracious momentum of COVID-19. The only successful example of a democracy not employing such a strong intervention but still managing to control the epidemic is Japan, where the government basically “asks” people and industry to introduce “self-isolation,” rather than compulsory quarantine. One reason for their success is that Japanese people’s compliancy causes them to interpret the government’s “asking” as “demands” or even “decrees.”

The mechanism for quarantine and isolation to control the spread of infectious diseases is quite simple, as it forcefully reduces possible contacts between

\(^1\)“Quarantine” is a loan-word from Italian, where it means “segregation”; this in turn comes from the Latin word “quadrāgintā” meaning “40”.
susceptible (S) and infected (I) individuals. Before modern medicine and medical science were established, such a social provision of forcefully segregating infected people was the only workable way to cope with dreadful communicable diseases; this is shown, for instance, by biblical passages that refer to the ostracism of lepers.

This chapter concerns another intervention game in which both quarantine and isolation co-exist alongside pre-emptive vaccination. Thus, intervention provisions are threefold: vaccination, quarantine, and isolation. The key point of our theoretical framework is that vaccination is introduced as an individual’s voluntary strategy and is therefore affected by a decision-making process backed by the evolutionary game theory, while both quarantine and isolation are defined as globally imposed provisions that are irrespective of an individual’s intention.

A fundamental question coming to mind is whether the simultaneous introduction of these two different interventions affects the social dynamics observed in terms of FES, vaccination coverage (VC), or ASP. Another question is whether basic public-health measures, such as isolation (i.e., the removal of only symptomatic individuals from the general population) and quarantine (i.e., the removal of individuals who have had contact with an infectious individual but are not displaying symptoms) are likely to control the spread of the disease. For cases in which these public-health measures are good enough, the next likely question is whether both isolation and quarantine should be used or whether one of the two can be sufficient. For example, even in retrospect, it is not clear whether isolation or quarantine had the greater impact in stopping the spread of SARS, or whether both control measures were essential.\(^2\) Still another interesting question is whether the introduction of such non-voluntary provisions as quarantine and isolation cooperatively bolster the suppressing effect of committing to a voluntary vaccination.\(^3\)

We will answer these questions by establishing a new model for the intervention game.

5.2 Model Structure

5.2.1 Formulation of the SVEIR Model\(^4\)

We use the SEIR model as a baseline for the current intervention-game mode. The compartment E implies “infected but not infectious”; thus, those individuals in E can

\(^2\)There have been several studies on quarantine and isolation in the specific case of SARS. For example:

Gumel (2004), pp. 2223–2232.

\(^3\)There have been several pioneering studies on this point. One of those suggested that the combined use of quarantine and isolation policies with an imperfect vaccination strategy can lead to complete elimination of disease, even when a low efficacy level of the universal strategy is considered.

Safi and Gumel (2011), pp. 3044–3070.

\(^4\)Alam et al. (2020), p. 033502.
be picked up and transferred to the compartment J, meaning quarantine, in accordance with the “precautionary principle” to preserve public welfare. By contrast, to represent isolation, those who are at I can be transferred to J, meaning isolation. Hence, compartment J is defined as a haunt that implies both quarantine and isolation.

In practice, we rely upon the basic concepts of the widely used SVEIR (Susceptible → Vaccinated → Exposed → Infected → Recovered) epidemic model, because, as with what we discussed in the previous chapters, we presume that pre-emptive “vaccination” is a voluntary provision. To adjust to the specific situation, our new predictive scheme (depicted in Fig. 5.1) includes an additional compartment “J” (as mentioned above) to represent quarantine and isolation. Our proposed model also considers the population to be infinite and ideally well-mixed without having any spatial structure, in keeping with the mean-field approximation (MFA).

To this end, the epidemic model contains compartments: susceptible (S), vaccinated (V), exposed (E), infected (I), recovered (R), and one additional compartment (J), meaning a joint compartment which serves as a universal compartment when quarantine and isolation policies are taken simultaneously or separately.
A susceptible person is an uninfected individual who can be infected through active contact with an infectious person. An exposed person, initially introduced as susceptible or vaccinated, later on encounters an infectious person and gets infected but usually remains asymptomatic; by contrast, an infected person is symptomatic. A recovered person is someone who gets immunized from the disease, and we assume can no longer be infected. A quarantined person is infected but asymptomatic and is forcefully removed from contact with the mass of people (thus, they are transferred from E to J). On the other hand, an isolated person is definitely both infected and infectious and is removed from contact with the general population by being admitted to a hospital or any rehabilitation center, as represented by transferring from I to J. The most important feature of the present model is that both quarantine and isolation are assigned to the same compartment J, despite the original meaning of those two different provisions being preserved. When quarantine is taken as the only control policy, J serves as a single compartment called quarantine (Q). Similarly, when solely isolation is considered, J behaves like a single compartment for isolation (Is). Our theoretical model assumes that susceptible (S) and vaccinated (V) individuals get exposed (E) with an infection-spreading rate $\beta$ [day$^{-1}$ person$^{-1}$] before becoming infectious. When the joint policy is imposed, a fraction of exposed ($E$) individuals who were already becoming symptomatic are immediately transferred to the infected (I) state with progression rate $\alpha$, and the remaining asymptomatic exposed individuals are then quarantined and thereby forcefully moved to J at a constant quarantine rate, $q_1$. On the other hand, a certain fraction of infected individuals is isolated and consequently transferred to J with a constant isolation rate, $q_2$, while the rest of the infected individuals gradually reach the recovery state R with a disease-recovery rate $\gamma$ [day$^{-1}$]. Furthermore, the people in J can also recover with a revised recovery rate, $\delta$ [day$^{-1}$].

Our mathematical model substantively shows the interplay between voluntary vaccination (active provision) and the imposed quarantine-isolation (passive provision) policy during an epidemic outbreak. As we have presumed so far, vaccination is costly when introduced. Since we presume that pre-emptive vaccination is imperfect and voluntary, we would therefore like to address the possibility of vaccine failure. In particular, a vaccine may not work equally well for all vaccinators (i.e., the fraction of vaccinators for which the vaccine yields an immune response is termed the effectiveness of vaccination, denoted by $e$ (0 ≤ $e$ ≤ 1), and kept fixed for repeated seasons). Another key property is the VC, expressed as a positive-valued variable $x$ (0 ≤ $x$ ≤ 1), which evolves over repeated seasons in accordance with the predictions of evolutionary game theory, as explained later. Consequently, we can classify all pre-emptive vaccinators into two categories: immune individuals, who receive perfect immunity with a probability $e$, and non-immune individuals, who fail to obtain vaccine-induced immunity with a complementary probability of $1 - e$. Throughout this study, the ratio of the infection-spreading rate, $\beta$, and the disease-recovery rate, $\gamma$, is treated as the basic reproduction number, $R_0$ (= $\beta/\gamma$).

The evolution of individuals through the SVEIR model with quarantine, isolation, and joint policy in a single season is modeled by the following dynamical equations:
\[
\frac{dS(x,t)}{dt} = -\beta S(x,t)I(x,t), \tag{5.1a}
\]
\[
\frac{dV(x,t)}{dt} = -\beta(V(x,t) - eV(x,0))I(x,t), \tag{5.1b}
\]
\[
\frac{dE(x,t)}{dt} = \beta S(x,t)I(x,t) + \beta(V(x,t) - eV(x,0))I(x,t) - \alpha E(x,t)
- q_1E(x,t), \tag{5.1c}
\]
\[
\frac{dI(x,t)}{dt} = \alpha E(x,t) - q_2I(x,t) - \gamma I(x,t), \tag{5.1d}
\]
\[
\frac{dJ(x,t)}{dt} = q_1E(x,t) + q_2I(x,t) - \delta J(x,t), \tag{5.1e}
\]
\[
\frac{dR(x,t)}{dt} = \gamma I(x,t) + \delta J(x,t). \tag{5.1f}
\]

For simplicity, the entire population is kept at fixed as:
\[
S(t) + V(t) + E(t) + I(t) + J(t) + R(t) = 1. \tag{5.2}
\]

The initial conditions for the system of Eqs. (5.1)–(5.2) take the form:
\[
S(0) \geq 0, V(0) \geq 0, E(0) \geq 0, I(0) \geq 0, J(0) \geq 0, R(0) \geq 0. \tag{5.3}
\]

As we previously assumed, committing to vaccination is costly and being infected also entails an illness cost that is greater than the vaccination cost. In the current evolutionary framework, we can classify individuals into four states in terms of their health condition and cost burden: (i) a healthy vaccinator (HV) who pays only the vaccination cost and remains healthy in an epidemic season; (ii) an infected vaccinator (IV) who commits vaccination but unfortunately becomes infected and must therefore bear the vaccination cost as well as the infection cost; (iii) a successful free rider (SFR), who does not incur any cost and fortunately can survive without being infected; and (iv) a failed free rider (FFR), who relies utterly upon herd immunity and tries to free ride without taking any protective provisions but eventually becomes infected, thereby bearing the infection cost itself.

Presuming a positive VC, x, we can observe the level of each compartment when \( t \) (local timescale; meaning the time in a single season) becomes infinitely large \( (t \to \infty) \). Thus, we obtain:
\[
HV(x, \infty) = V(x, \infty), \tag{5.4a}
\]
\[
SFR(x, \infty) = S(x, \infty). \tag{5.4b}
\]

Meanwhile, we are not able to solve the set of Eqs. (5.1a, 5.1b, 5.1c, 5.1d, 5.1e, 5.1f, 5.2 and 5.3) in a fully analytical way; thus, we introduce the concept of flux
“φ_{A \rightarrow B}” indicating the total number of individuals passing through state (compartment) A to B. In practice, we define three flux terms: φ_{V \rightarrow I}(x, \infty), φ_{S \rightarrow I}(x, \infty), and φ_{E \rightarrow I}(x, \infty), to express the transferring flux from vaccinated to infected, susceptible to infected, and exposed to infected, respectively. We can calculate the fraction of infected vaccinators as the conditional expectation of vaccinated individuals who get exposed and are subsequently infectious. The details of this mechanism can be illustrated as a product of the conditional probability of vaccinated individuals who are exposed. The fraction of those exposed individuals who eventually become infectious is given by

\[
IV(x, \infty) = \frac{\left( \int_0^\infty \varphi_{V \rightarrow E}(x,t)dt \right)}{\int_0^\infty (\varphi_{S \rightarrow E}(x,t) + \varphi_{V \rightarrow E}(x,t))dt} * \left( \int_0^\infty \varphi_{E \rightarrow I}(x,t)dt \right).
\]

(5.4c)

In a similar fashion, the fraction of non-vaccinators who are infected (i.e., the fraction of failed free riders) can be obtained through

\[
FFR(x, \infty) = \frac{\left( \int_0^\infty \varphi_{S \rightarrow E}(x,t)dt \right)}{\int_0^\infty (\varphi_{S \rightarrow E}(x,t) + \varphi_{V \rightarrow E}(x,t))dt} * \left( \int_0^\infty \varphi_{E \rightarrow I}(x,t)dt \right).
\]

(5.4d)

Finally, the corresponding fractions of the four types of individuals existing in the population are encapsulated in Table 5.1.

### Payoff Structure

5.2.2 Payoff Structure

We should follow the Table 3.1, because there is only a single costly strategy, which is vaccination defined as an active provision. We recall the relative cost of vaccination, \( C_r = C_v/C_i \) \((0 \leq C_r \leq 1)\), where the costs for vaccination and infection are denoted by \( C_v \) and \( C_i \), respectively. As we discussed in the previous sub-section, at the end of a single season, we can use the game-theoretic approach to classify all individuals who initially chose either vaccination or free riding (defection) into the four classes depending upon their final health status and whether they are healthy or infected at the equilibrium point. Therefore, we depict the present payoff structure in Table 5.2, just as in Table 3.1.

#### Table 5.1 Fractions of four types of individuals

| Strategy/State               | Healthy       | Infected      |
|------------------------------|---------------|---------------|
| Vaccinated (Cooperator; V)   | HV(x, \infty) | IV(x, \infty) |
| Non-vaccinated; Defector (Free Rider; FR) | SFR(x, \infty) | FFR(x, \infty) |
An individual’s decision about vaccinating depends utterly upon the trade-off between the prevention costs and the perceived risks involved. Thus, their decision might evolve with global-scale time (repeating seasons) based on the epidemic incidence observed in the population. Therefore, we can formulate the overall expected payoff by means of the average social payoff $\langle \pi \rangle$, the average payoff of vaccinated individuals $\langle \pi_V \rangle$, and that of non-vaccinated (defective) individuals $\langle \pi_{NV} \rangle$, which are given below:

$$\langle \pi \rangle = -C_r HV(x, \infty) - (C_r + 1)IV(x, \infty) - FFR(x, \infty),$$

$$\langle \pi_V \rangle = (-C_r HV(x, \infty) - (C_r + 1)IV(x, \infty))/x,$$

$$\langle \pi_{NV} \rangle = (-FFR(x, \infty))/(1 - x).$$

### 5.2.3 Strategy Updating and Global Dynamics

In the present model, we only apply individual-based risk assessment (IB-RA), introduced in Sect. 3.2.2.\(^5\) As described in Eq. (3.25), there are eight state-transition-probability functions,

$$P(HV \leftarrow SFR) = \frac{1}{1 + \exp[-(-C_r)/\kappa]}, \quad P(HV \leftarrow FFR) = \frac{1}{1 + \exp[-(-C_r)/\kappa]}$$

$$P(IV \leftarrow SFR) = \frac{1}{1 + \exp[-(-C_r-1)/\kappa]}, \quad P(IV \leftarrow FFR) = \frac{1}{1 + \exp[-(-C_r-1)/\kappa]}$$

$$P(SFR \leftarrow HV) = \frac{1}{1 + \exp[-(-C_r)/\kappa]}, \quad P(SFR \leftarrow IV) = \frac{1}{1 + \exp[-(-C_r)/\kappa]}$$

$$P(FFR \leftarrow HV) = \frac{1}{1 + \exp[-(-C_r-1)/\kappa]}, \quad P(FFR \leftarrow IV) = \frac{1}{1 + \exp[-(-C_r-1)/\kappa]}$$

Throughout which we presume a noise parameter in the Fermi function, $\kappa$, of 0.1.

At the end of each epidemic season, everyone can update their strategy depending upon the last season’s payoff. Hence, with increasing or decreasing VC, $x$ is inevitable. Here, the independent variable, $t$, indicates the global timescale, which in other words means the number of elapsed seasons. Since we consider IB-RA strategy-updating rule for decision making in each subsequent season, the dynamic equation following this particular rule can be expressed as follows:

\(^5\)Needless to say, you can apply SB-RA instead of IB-RA.

**Table 5.2** Estimated payoff structure at the end of each epidemic season

| Strategy/State | Healthy | Infected |
|---------------|---------|----------|
| Vaccinated (V) | $-C_r$  | $-C_r - 1$ |
| Non-vaccinated (NV) | 0      | -1       |
\[
\frac{dx}{dt} = HV(x, \infty)SFR(x, \infty)(P(SFR \leftarrow HV) - P(HV \leftarrow SFR)) \\
+ HV(x, \infty) FFR(x, \infty)(P(FFR \leftarrow HV) - P(HV \leftarrow FFR)) \\
+ SFR(x, \infty) IV(x, \infty)(P(SFR \leftarrow IV) - P(IV \leftarrow SFR)) \\
+ FFR(x, \infty) \ast IV(x, \infty)(P(FFR \leftarrow IV) - P(IV \leftarrow FFR)).
\] (5.6)

5.3 Result and Discussion

5.3.1 Local Dynamics in a Single Season

Let us confirm what happens in a single season to show how well quarantine and isolation policies manage to suppress the spread of disease. The result here is not influenced by evolutionary game theory, which implies a fixed VC of 0.46 at the beginning of a season. We presume the set of parameters to be \( \beta = 0.833, \gamma = \frac{1}{3}, \alpha = 0.25, \delta = 0.33, \) and \( e = 0.5, \) where the value of \( \beta \) comes from the basic reproduction number, \( R_0 = 2.5, \) considered for the case of a seasonal-influenza-like flu.\(^6\) Another two crucially important parameters, \( q_1 \) and \( q_2, \) which are the pick-up rates for quarantine and isolation policies, respectively, have been introduced here to quantify the levels of quarantine and isolation policies. If both parameters take non-zero positive values, then both quarantine and isolation policies co-exist and we call this the “joint policy.” Figure 5.2 compares the no-policy, quarantine-policy, isolation policy, and joint-policy cases by displaying the fraction of people in each compartment at the equilibrium state. Note that the compartment denoted “isolated” refers to “J,” which contains individuals not only resulting from isolation, but also from quarantine policies. In all four cases, a monotonically increasing tendency is observed in the fraction of recovered individuals, which is triggered mostly by the monotonic decrement of the fraction of susceptible individuals; this is quite conceivable. Moreover, a decreasing tendency is also observed in the vaccinated fraction, which eventually reaches a stable equilibrium. This phenomenon can be justified by the fact that vaccines do not bring perfect immunity \( (e = 0.5 \) is presumed); thus, the population initially labeled “V” monotonically decreases as some of them are infected (transferring from V to E). This is also conceivable, since it confirms that the model works fairly closely to how it was designed. Figure 5.3 highlights the fraction of infection for four respective scenarios. Obviously, the promptness of an infection dying out due to the joint policy justifies its outperformance among the four scenarios. From the quantitative viewpoint of an infection spreading, quarantine and isolation policies are next implemented.

\(^6\)Fukuda et al. (2014), pp. 1–9.
We observe that the quarantine policy outperforms the isolation policy, while the joint policy (i.e., adopting both at the same time) performs understandably well in keeping the infection size smaller.

5.3.2 Social Equilibrium from Global Dynamics

In Fig. 5.4, varying the basic reproduction number $R_0$ and progression rate $\alpha$, we present a set of two-dimensional full-phase heat maps to quantify the FES under a wide range of parametric conditions used for the quarantine-isolation policy. Each 2D heat map displays FES along with vaccination cost, $C_r$, ranging from 0 to 1 along the x-axis, as well as the vaccine effectiveness $e$, ranging 0 to 1 along the y-axis. For the no-policy, joint-policy, isolation policy, and quarantine-policy cases (considered in a clockwise direction), we depict the fractions of individuals existing in each compartment at local equilibrium. The highest fraction of exposed individuals is observed in the case of the no-policy condition. The exposed fraction dies out much earlier when either joint or quarantine policy is imposed. In all four cases, the baseline values of the parameters are considered as $\beta = 0.8333$, $\gamma = 0.3333$, $\alpha = 0.25$, $\delta = 0.33$, and $e = 0.5$. For simplicity, we choose, $q_1 = q_2 = 0$ for the no policy condition, $q_1 = 0.1$, $q_2 = 0.1$ for the joint policy condition, $q_1 = 0.2$, $q_2 = 0$ for the quarantine policy condition, and $q_1 = 0.0$, $q_2 = 0.2$ for the isolation policy condition.

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a comprehensive understanding, we introduce two control parameters, namely, \( q_1 \) and \( q_2 \), which represent the contributions of quarantine and isolation policies, respectively. Additionally, we include six blocks labeled a, b, c, d, e, and f, each having nine panels to explain the comparative impact of quarantine and isolation policies besides pre-emptive vaccination at varying combinations of basic reproduction number \( R_0 \) in the row-wise direction and the infection-progression rate \( \alpha \) in the column-wise direction. For the sake of discussion, we presume two different values of \( R_0 = \{4.5, 2.0\} \) and three different \( \alpha = \{0.10, 0.25, 0.40\} \) to represent all evolutionary outcomes. Being shifted from the standard value of the basic reproduction number \( R_0 = 2.5 \), which came from the case of the flu in many previous studies and was presumed in Figs. 5.2 and 5.3 we investigate two extreme cases: \( R_0 = 4.5 \) (i.e., the infection-spreading rate \( \beta \) is higher and the recovery rate \( \gamma \) is lower than usual in flu cases) and \( R_0 = 2.0 \) (i.e., the infection-spreading rate \( \beta \) is lower and the recovery rate \( \gamma \) is a little bit higher). In a nutshell, \( R_0 = 4.5 \) is considered to be a worse situation than the standard flu, whereas \( R_0 = 2.0 \) can be treated as more moderate. Moreover, implementing a pair of control parameters \( q_i = \{0.0, 0.1, 0.2\} \) for \( i = 1, 2 \) in two different directions, we can classify nine panels

![Fig. 5.3](image-url)
Fig. 5.4 Final epidemic size (FES) observed under varying vaccine cost and efficacy. First, we distinguish the entire parameter space into six blocks labeled a, b, c, d, e, and f based on two different basic reproduction numbers ($R_0 = 4.5, 2.0$) in a row – wise direction and three different progression rates ($\alpha = 0.10, 0.25, 0.40$) in a column – wise direction. Also, while applying control policies, we use two different pick – up rates; $q_1(=0.0, 0.1, 0.2)$ for a quarantine policy and $q_2(=0.0, 0.1, 0.2)$ for an isolation policy. For example, block a is solely designed for $R_0 = 4.5$ and $\alpha = 0.10$, which contains nine panels, each labeled with (4a – *), depending upon the values of...
quantitatively inside each of the six blocks. Therefore, on the basis of parameter settings, we can distinguish four different policy regions as follows: (i) no-policy ($q_1 = 0$, $q_2 = 0$); (ii) quarantine policy ($q_1 \neq 0$, $q_2 = 0$); (iii) isolation policy ($q_1 = 0$, $q_2 \neq 0$); and (iv) joint policy ($q_1 \neq 0$, $q_2 \neq 0$). Panels (4*-i) show the FES when no-policy is initiated by the governing authorities (the default situation). Accordingly, panels (4*-vii), (4*-iii), and (4*-ix), respectively, indicate the full-scale implementation of quarantine, isolation, and joint policies considered under varying conditions.

An in-depth analysis of the basic reproduction number and progression rate reveals a few interesting points that must be shared with readers here. First, if we presume that $R_0 = 4.5$ and $\alpha = 0.10$, we can see from Fig. 5.4 that the quarantine policy (panel (4a-vii)) more effectively works to suppress disease than the isolation policy (panel (4a-iii)), while both perform better than the default case (panel (4a-i)). The introduction of control policies at a certain level can sometimes push up the critical line (for instance, the white dotted line in panel (4a-ii)) between the controlled and endemic phases (see, for example, panels (4b-ii), (4c-viii), (4d-ii), (4f-iv)) when varying $R_0$ and $\alpha$ are presumed.

Interestingly, when $R_0 = 4.5$ and $\alpha$ are set to 0.25 and 0.40 (depicted in blocks c and e, respectively), no such pushing takes place as long as a pure-quarantine policy is imposed (i.e., fixing $q_2 = 0$). Keeping the parameter settings unchanged for $R_0$ and $\alpha$, we observe that the pure-isolation policy reduces FES compared to the pure-quarantine policy (see panel (4c-iii) vs. (4c-vii) and panel (4e-iii) vs. (4e-vii)). In both cases, the isolation policy offers better disease attenuation, as measured in terms of the existing size of the epidemic, than the quarantine policy, especially in the endemic regions below the critical line. This is a quite interesting finding. One possible reason behind this is that the higher progression rate (when $\alpha > 0.10$) might increase the flow of individuals from an exposed situation to an infected state, rather than shifting them to a quarantined state. This in return enables the government to capture infected and infectious people (I) efficiently through an isolation policy, which ensures public safety. Therefore, an isolation policy seems to work better than quarantine in this condition. In Fig. 5.4, panel (4c-iii) clearly suggests the supremacy of an isolation policy in producing an impressive result compared to a quarantine policy (4c-vii), thereby contributing as a dominant factor when a joint policy is taken (4c-ix). In other words, we can say that the increasing progression rate, $\alpha$, gradually diminishes the contribution of the quarantine policy. This might be important when establishing the strategy for a public-health policy.

Meanwhile, when a smaller basic reproduction number ($R_0 = 2.0$) is presumed, the promptness of the epidemic outbreak is weakened, and thus moderate policies are expected from the government. As we can see from panels (4b-*), a reasonable level of quarantine can bring back the epidemic-free situation (when $\alpha = 0.10$). Even

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Fig. 5.4 (continued) pick – up rates $q_1$ and $q_2$. The construction of the remaining blocks is analogous to the way block “a” has been designed. Here, the white dotted line indicates the critical line between the controlled and endemic phases.
when the progression rate ($\alpha$) is a little bit higher than our prior consideration (e.g., $\alpha = 0.25, 0.40$), a reasonable contribution from either of the policies is fair enough (panels (4d-vi), (4d-viii), (4f-vi), (4f-viii)) to ensure a pleasant situation in terms of epidemic size.

Figure 5.5 shows the VC under different parametric conditions and varying $R_0$ and $\alpha$, as presented in the same manner as Fig. 5.4. Below the critical line, there is no hope for individuals to be vaccinated due to the meager effectiveness and higher vaccination cost throughout that region. However, in the no-policy cases (e.g., panels (5a-i), (5c-i), (5e-i)), VC converges toward its maximum limit compared to other policy cases. This is because neither quarantine nor isolation policies encourage people to vaccinate as long as a reasonably higher effectiveness and lower cost are ensured. When a full-scale quarantine or isolation policy (panel (5*-ix)) is taken, VC immediately declines to zero, but the corresponding FES observed in Fig. 5.4 is not bad. Meanwhile, for $R_0 = 2.0$, it is observed that an intermediate quarantine policy determined by the pick-up rate $q_1 = 0.1$ is adequate to maintain a small FES (see panel (4b-iv)), as long as the progression rate $\alpha$ is set to 0.10 although VC is zero (see panel (5b-iv)). That means that the spread of the epidemic can be kept under control by only quarantining a limited number of exposed individuals. In this sense, quarantine can be a powerful provision to suppress the epidemic. We also observe that, in such a situation, an isolation policy is relatively less effective than quarantine. Therefore, results coming from the dual-policy cases (see panels (4b-v), (4b-ix)) are mostly influenced by the quarantine policy. On the other hand, the incredible dominance with complete remission of the disease triggered by either quarantine or isolation policies is no longer observed when the progression rate, $\alpha$, is set to 0.25 to 0.40 (see panels (4d-*)) and (4f-*)).

Although it sounds trivial, implementing two control policies (quarantine and isolation) at full scale is the most efficient way to suppress the spread of a disease. Therefore, apart from pre-emptive vaccination, the public-health authority must prepare sufficient resources and support, as well as formulate a set of comprehensive quarantine-isolation policies to address this pressing issue. At times when a quarantine-isolation policy works very well, the spread of a disease can be controlled even under a low or zero vaccination rate. Looking at the panels ((4a-ix), (4c-ix), (4e-ix)) for FES and the corresponding panels ((5a-ix), (5c-ix), (5e-ix)) for VC, we can justify what we mentioned earlier. It also presents a positive impression of how these two types of preventive measures (pre- and post-provisions, or active provision and passive provisions using our earlier terminology) complement each other as a means of containing or preventing infectious diseases.

The importance of introducing a quarantine-isolation policy can be well understood from the phase diagrams drawn for ASP in Fig. 5.6. Although the holistic tendency observed at ASP is quite analogous to FES, it shows subtle differences above the critical line. The question then arises: why is this color-gradient change observed in the ASP heat map but not in the FES heat map? This might be an interesting phenomenon and the exact reasons must be noted. These reasons are twofold: a subtle change in FES can create an enlarged difference in ASP, and a fractional difference in vaccines contributes to producing a better social payoff.
Fig. 5.5 Vaccination coverage, VC, observed under varying vaccine cost and efficacy. First, we distinguish the entire parameter space into six blocks labeled a, b, c, d, e, and f based on two different basic reproduction numbers ($R_0 = 4.5, 2.0$) in the row-wise direction and three different progression rates ($\alpha = 0.1, 0.25, 0.40$) in the column-wise direction. Also, while applying control policies, we use two different pick-up rates, namely $q_1(=0, 0.1, 0.2)$ for quarantine policy and $q_2(=0, 0.1, 0.2)$ for isolation policy. For example, block a is solely designed
5.3.3 Public-Based (Passive) Provision: Quarantine and Isolation vs. Individual-Based (Active) Provision: Vaccination

In this sub-section, we consider variation in both the quarantine and isolation rates (i.e., $q_1$ and $q_2$) in two different directions. For this purpose, we choose several parametric values for vaccine effectiveness ($e = 0.2, 0.4, 0.6, 0.8, 1.0$) and the relative cost of vaccination ($C_r = 0.2, 0.4, 0.6, 0.8$). With the aforementioned parametric settings, we thus prepare Figs. 5.7, 5.8, and 5.9, which depict the heat maps for FES, VC, and ASP, respectively. Each heat map, both x-axis for quarantine rate; $q_1$, and y-axis for isolation rate; $q_2$, range 0 to 0.7.

In Fig. 5.7, a broken line colored white along 45° shows the state where equal contributions come from the two control-policy provisions enforced by authorities. Essentially, pre-emptive vaccination is not trustworthy for common people as long as its reliability is meager (e.g., $e = 0.4$), regardless of what relative cost of vaccination is presumed. In fact, under such conditions, they cannot help but to rely upon the quarantine-isolation policy to protect themselves against threats of infection, as depicted by dotted triangles in Fig. 5.7. The observed FES is quite high because of poor VC (see Fig. 5.8). Yet, when the effectiveness is reasonably high, a certain fraction of individuals prefer to take pre-emptive provisions (see Fig. 5.8), because they can protect themselves by pre-emptive vaccination without relying on the government policy. Intuitively, when we go downwards along the 45° line, we can see a monotonic increase tendency in FES. Except for panels ((7-a), (7-f), (7-k), (7-p), (7-g), (7-l), (7-q)), this monotonic increase takes place until it reaches a ring-shaped colored region observed in Fig. 5.7. Only panel (7-b) shows a slight deviation around the dotted triangular region. For this particular setting of the effectiveness ($e = 0.4$) and the relative cost of vaccination ($C_r = 0.2$), the epidemic size is still controllable (marked with a curved triangle), even though the isolation rate is quite meager. This phenomenon can be fully justified by the existence of a tiny fraction of vaccinators around the same parametric region observed in panel (8-b). Beneath this ring-shaped region (down the 45° line), a monotonic decreasing tendency is found where both pick-up rates $q_1$ and $q_2$ are relatively low compared to other regions. Complete reliance upon government policies always creates a negative impression among individuals, leading them to put their faith solely in pre-emptive vaccination.

Along the white broken line in the upward direction, both quarantine and isolation policies contribute equally to suppressing the spread of the disease. In
Fig. 5.6 Average social payoff (ASP) observed at varying vaccine costs and efficacies. We distinguish the entire parameter space into six blocks labeled a, b, c, d, e, and f based on two different basic reproduction numbers ($R_0 = 4.5, 2.0$) in the row-wise direction and three different progression rates ($\alpha = 0.10, 0.25, 0.40$) in the column-wise direction. Also, while applying control policies, we use two different pick-up rates, namely $q_1(=0, 0.1, 0.2)$ for quarantine policy and $q_2(=0, 0.1, 0.2)$ for isolation policy. For example, block a is solely designed...
that direction, pick-up rates $q_1$ and $q_2$ are symmetric, and their increasing tendency ensures a smaller epidemic size.

Interestingly, the level of FES seems quite analogous between the two regions (below and above the ring marked with two red dots along the 45° line), which indicates that individual-based pre-emptive provisions (vaccination) and public-

Fig. 5.7 Final epidemic size controlled by quarantine rate and isolation rate. The heat maps are drawn for constant values of vaccine efficacy ($e = 0.2, 0.4, 0.6, 0.8, 1.0$) and relative cost ($C_r = 0.2, 0.4, 0.6, 0.8$). This figure contains twenty panels (labeled (7-*)), each of which is generated by a particular combination of vaccine efficacy and cost values. The black dotted triangle indicates the severity of the epidemic existing in regions where both quarantine and isolation rates are relatively small. The white dotted straight line shows equal contributions coming from isolation and quarantine, whereas the red dots show where a phase transition takes place under certain parametric conditions.

Fig. 5.6 (continued) for $R_0 = 4.5$ and $\alpha = 0.10$, which contains nine panels, each labeled with (5a – +), depending upon the values of pick-up rates $q_1$ and $q_2$. The constructions of the remaining blocks are analogous to the way block a has been designed. At a glance we can see that the average payoff is higher for most cases in which $R_0 = 2.0$ (see blocks b, d, f) than those in which $R_0 = 4.5$ (see blocks a, c, e)
based late provisions (quarantine/isolation) exert an equal influence. Therefore, we can separate the phase diagrams (heat maps) into two major regions based on the instinct contributions coming from either of the two types of provisions. To justify our hypothesis through 2D heat maps, let us consider the example of panel (7-i), where the solid orange line shows the contribution of pre-emptive vaccination and the dotted red line indicates the contribution brought about by the quarantine-isolation policy. Thus, the contributory characteristics shown by the pre-emptive provision and control policies are immensely beneficial for each individual living in society. We can also verify our hypothesis through Fig. 5.9, which provides us with a more general idea of how a dual contribution driven by two types of provisions can significantly enhance the ASP. One obvious occurrence with vaccination is that, when reasonably reliable vaccines are offered at a cheap price, they strongly attract people despite the fact that fully relying on a public-based government policy can contribute significantly to producing a better social payoff (see, for example, panels (8-c), (8-d), (8-h), (8-i), (8-n), where the social payoff is radically improved).
5.3.4 Passive Provision Rather Compensates the Shadow by Active Provision Than Mutually Competing

In this chapter, we considered pre-emptive vaccination and forced control policies, two major protection approaches against pandemics and severe epidemics. We then systematically studied the positive impacts triggered by a quarantine-isolation policy while modeling the mathematical epidemiology aided by the theory of vaccination games. The central theme of this study has been to answer the questions of when, why, and under what conditions do authorities have the strongest justification for implementing quarantine and isolation policies to prevent infectious diseases. Our proposed model deeply elaborates the delicate impact of these control policies through the SVEIJR epidemic model, as well as their appropriateness under a wide range of parametric conditions to suppress the spread of disease among individuals who primarily took vaccination as a pre-emptive provision. However,
due to the transient effect of vaccination and the widespread occurrence of epidemic breakout, there is an emergent need to seek more sustainable approaches to protect the global community from viral diseases. This study emphasized the importance of using quarantine or isolation policies to prevent the spread of infection. Particularly at times when the majority of people prefer not take vaccination due to its meager efficacy or higher cost, these control policies can somehow offer some degree of reassurance. Through numerical simulation, it is obvious that adopting a quarantine-isolation policy can also relax any bad situation in the hopeless region. We also showed which policy performs best under what conditions. Our theoretical analysis suggests that a joint policy should be taken when the disease-spreading rate is higher. Nonetheless, the government should pay more attention to securing a healthy state in society by taking either of the control policies. The important question remains: how should the pick-up rates be designed? We justified this fact by varying the pick-up rate parameters $q_1$ (designed for quarantine policy) and $q_2$ (dedicated to isolation policy) in two different directions while generating phase diagrams for FES, VC, and ASP throughout this study. In other words, the relative contributions coming from either pre-emptive vaccination or late-control policies are equally important for maintaining the FES at a controllable state.

Using the MFA technique and the IB-RA strategy-updating rule, our model can successfully address the importance of dual provision techniques to the audience which has never been used before. In this study, we have deliberately skipped all visual datasets using the strategy-based risk-assessment (SB-RA) update rule introduced in Chap. 3. Basically the same general tendency was observed even when SB-RA was applied instead of IB-RA, other than suggesting some difference arising inherently from the substantial difference between IB-RA and SB-RA, as discussed in Chap. 4 (Sect. 4.3).

5.3.5 Comprehensive Discussion

This chapter has demonstrated that the mathematical-epidemiological model is quite useful for depicting various situations to adjust the respective circumstances brought about by an epidemic and the social provisions followed to contain it. Quarantine and isolation, which have been introduced for COVID-19 since no protective medical provisions like vaccines or antiviral treatments are available, can be embedded into the baseline ODE model. Note that those two provisions are treated as independent of an individual’s decision. That is why the focal compartments I and J do not connect with the framework of the evolutionary game. Hence, it can be said that the flexibility of that part of the mathematical-epidemiological model can be preserved.
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