Implications of chiral symmetry for scalar isoscalar channel and multi-nucleon forces

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Chiral perturbation theory has to be substantially modified for the problem of nuclear forces. Loop effects dominate over tree level effects. The dominant behaviours in the scalar isoscalar channel and multi-nucleon forces are obtained in the chiral limit and for a small explicit breaking.

In this Letter we point out that chiral symmetry has some clear model independent implications for the scalar isoscalar (referred to as the $\sigma$) channel of the nucleon nucleon interactions and for multinucleon forces. We obtain dominant effects of all pion exchanges between nucleons. It has been emphasised by Weinberg that the implications of chiral symmetry to the nucleon potential is in a different class from the usual low energy theorems involving pions with each other or scattering off a heavy nucleon because there is no such thing as a soft nucleon. When the effective Lagrangian is used it appears that there is no interesting prediction for an expansion in powers of momentum transfer. With the chiral transformation of the nucleon field defined as in the non-linear realisation (below) $\nabla^2 N'$ is a chiral invariant. Therefore two and multi-nucleon interactions of arbitrary strength are allowed by symmetry. This is in contrast to the scattering of pions with each other or with other chiral matter where an expansion in powers of momenta is sought in low energy theorems and chiral perturbation theory.

We argue that the implication of spontaneously broken chiral symmetry for the nuclear forces is elsewhere. Consider the spin zero isospin one channel of the nucleon nucleon scattering in the chiral limit. There is a characteristic dipole-dipole type of long range interaction as a consequence of the massless pion exchanged. With an explicit breaking of chiral symmetry, the dominant effect is to change the long range interaction to the Yukawa interaction. In the same way it is more appropriate to isolate the dominant (qualitative and quantitative) effect in the $\sigma$ channel in the chiral limit and for a small explicit breaking. We do this here. The implications of chiral symmetry for the nuclear forces has attracted considerable attention recently. In contrast, we point out that chiral perturbation theory has to be modified substantially for the NN interactions in the $\sigma$ channel as the loop effects dominate over the tree level effects. We also emphasise the crucial role played by explicit chiral symmetry breaking and the pion nucleon $\sigma$ term.

Our starting point is that not only the transverse but also the longitudinal susceptibility diverges in a ferromagnet. The implication is that the $\sigma$ propagator has an unusual though universal logarithmic infrared singularity. To see this in the linear $\sigma$ model, we need an infinite summation of graphs, an example of which is the $N \to \infty$ approximation. (This only refers to the infrared behaviour and does not rule out a pole at some $q^2 \neq 0$.)

$\sigma$ does not appear as a fundamental field in the non-linear $\sigma$-model but it is the chiral partner of the pions and therefore has a physical meaning as a composite field. Its correlation refers to the longitudinal susceptibility in a ferromagnet which is as important as the transverse susceptibility. It is easier to understand the logarithmic infrared singularity here. In a $O(N)$ non-linear $\sigma$-model we have $\sigma^2 + \bar{\pi}^2 = f^2$, so that $\sigma' = -\frac{\bar{\pi}^2}{2f} + \frac{\bar{\pi}^4}{8f^3} + \cdots$ where $\sigma' = \sigma - \langle 0 | \sigma | 0 \rangle$. Using the massless propagator for $\bar{\pi}$, we get Fig. (1a),

$$< \sigma'(-q) \sigma'(q) > \sim ln (\Lambda^2/q^2), \ q \to 0$$

where $\Lambda$ is the microscopic scale (or a cutoff) in the theory. We will refer to this as the logarithmic enhancement in the $\sigma$ channel. Application of the renormalization group shows that the leading infrared behaviour Eqn. is exact. The pion loop (Fig. 1a) plays a special role because it is the only infrared singular graph. Thus $\sigma'$ behaves like a composite: $\bar{\pi}^2$ of two pions in the infrared.

Chiral perturbation theory would indirectly include the effects of the infrared behaviour of the $\sigma'$ as an one loop effect. Indeed the $ln$ effect Eqn. appeared first when the soft pion amplitudes were corrected for unitarity. We emphasise that it survives as the dominant effect to all orders. Note that chiral perturbation theory orders the graphs according the powers of momenta and does not count the logarithms. As the logarithm is an important now, this phenomenon should be kept track of explicitly.
We now consider the dominant effect of all pion exchanges between nucleons. It is possible to handle this at low momentum transfers because the amplitude for emission of any number of soft (virtual) pions from a nucleon is given by the soft pion theorems. Consider two pion exchanges (Fig.1b) to be specific. In the chiral limit, the \( \pi N \) scattering amplitude can be expressed as a sum of two contributions. i) Through the gradient coupling of the pion, this triangle contribution could effectively produce a \( \sigma \) channel. ii) There is an effective Lagrangian approach. i) There is an effective interaction from the nucleon in the low momentum limit. This gives the same S-matrix elements. Therefore only the \( \sigma' \) channel is \( [2] \).

To extract this contribution we compute the coupling to the composite \( \pi^2 \) operator (Fig.2a). As a consequence of the gradient coupling of the pion, this triangle contribution vanishes linearly as \( q \to 0 \). This means the \( \sigma' \) decouples from the nucleon in the low momentum limit. This is a general result in the chiral limit as seen below using the effective Lagrangian approach. i) There is an effective local vertex, referred to as the \( \rho \) vertex, (Fig.2b) with the pions in spin = 1, isospin = 1 channel. This is irrelevant for the \( \sigma \) channel.

\[
L_N = \frac{\bar{N} \gamma^\mu \partial_\mu N}{g^2} - g\bar{N}(\sigma + i\gamma_5 \tau.\vec{\pi})N
\]  
(2)

It appears that \( \bar{N}N \) is a source for \( \sigma' \). If so its exchange would give an unusual long ranged nuclear force, Eqn[1] in the chiral limit, and would be a model independent consequence of the chiral symmetry. By a redefinition of fields \( L_N \) can be rewritten as [2]

\[
L'_N = \frac{\bar{N'}(i\gamma^\mu \partial_\mu - g(\sigma^2 + \vec{\pi}^2)^{1/2})N'}{g^2} \\
+ \frac{\bar{N'} \gamma^\mu (1 + \frac{\vec{\pi}^2}{4f^2})^{-1}(\gamma_5 \frac{\bar{\tau}\partial_\mu \pi - \bar{\tau} \times \partial_\mu \pi}{2f})N'}{g^2} \]  
(3)

This gives the same S-matrix elements. Therefore only the chiral invariant combination \( (\sigma^2 + \vec{\pi}^2)^{1/2} \) couples to the nucleons in the limit of vanishing momenta. The other couplings are the derivative coupling of (an odd number of) pions to the axial current (\( \gamma_5 \) vertex) and to even number of pions in the vector-isovector channel (\( \rho \) vertex). The \( SU(2)_L \times SU(2)_R \) singlet

\[
(\sigma^2 + \vec{\pi}^2)^{1/2} - f \sim \sigma' + \vec{\pi}^2/(2f) + \ldots \]  
(4)

does not have infrared divergent correlation functions. For instance, in the non-linear \( \sigma \) model this degree of freedom is frozen. Also in the linear \( \sigma \) model the infrared behaviour of the \( \sigma' \) is the same as that of \( -\vec{\pi}^2 \). Indeed the underlying reason for the identification of \( \sigma' \) with \( -\vec{\pi}^2 \) in the infrared is that the chiral invariant combination occurring in Eqn (4) does not have any infrared divergence while the individual terms do have logarithmic divergences (in four dimensions). Further, this chiral invariant field is expected to be massive. In QCD we may expect the chiral invariant field to have a mass of \( O(1 \text{ GeV}) \) and its exchange would generate scalar isoscalar forces with a very short range. It is not an explanation for the intermediate range attraction. Thus in the chiral limit there are no long range interactions in the \( \sigma \) channel of the nucleon- nucleon interactions, even though the \( \sigma' \)-propagator is infrared divergent. The local four (and multi-) nucleon terms permitted by chiral symmetry may be regarded as mediated by exchanges of such very massive chiral singlet fields. They are small not because they are forbidden by chiral symmetry but because they are suppressed by inverse powers of the heavy masses. Our general derivation explains the cancellations due to chiral symmetry in one loop calculation observed in Ref. [1].

We now argue that the small explicit breaking of chiral symmetry is crucial for the intermediate range attraction. In the linear \( \sigma \)-model the effects of the current quark mass \( m_q \) are mimicked by the term \( fm_q^2 \sigma \) (with \( m_q^2 \sim O(m_N) \)). It induces a non-zero pion mass \( m^2_\pi \sim O(m_q) \). In addition we include a small current mass \( \Sigma_N \bar{N}N \) with \( \Sigma_N = O(m_q) \) for the nucleons, as allowed by the transformation properties. This is not double counting as we are considering direct effects of the quark mass on two distinct sectors of the effective theory, the mesons and the baryons. It has been pointed out that canonical PCAC relation is not valid in the presence of the \( \Sigma_N \) term. This is not a problem as we are not using PCAC for calculating the amplitudes. We are explicitly putting in the information that the divergence of the axial vector current acts not only as an interpolating field for the pion but also has a non-zero matrix element between baryons of opposite chirality.

Making the Dyson-Weinberg transformation for the combination \( (\sigma + g^{-1}\Sigma_N) + i\gamma_5 \tau.\vec{\pi}N' \) we get the Yukawa interaction,

\[
\sim \bar{N}'N'((\sigma^2 + \vec{\pi}^2) + 2g^{-1}\Sigma_N \sigma + g^{-2}\Sigma_N^2)^{1/2} \]  
(5)

Now there is a direct coupling of \( \bar{N}'N' \) to \( \sigma' \) in addition to the coupling to the chiral invariant combination \( (\sigma^2 + \vec{\pi}^2)^{1/2} \)
This induces a scalar isoscalar force of range much larger than the microscopic scale. This is also seen in the non-linear $\sigma$ model where the two chiral symmetry breaking terms are represented by $[18]$, 
\[
L'_{N} = -\frac{1}{2}m_{\pi}^{2} \frac{\pi^{2}}{1 + \pi^{2}/(4f^{2})} (1 - \frac{\Sigma N N'}{f^{2}m_{\pi}^{2}})
\]  
(6)
as allowed by the chiral transformation properties. The second term, referred to as the $\Sigma$ vertex (Fig. 2c), is the well known pion nucleon $\Sigma$ term, important for the pion nucleon scattering length. It shows that there is a direct coupling of the nucleon to $\pi^{2}$ (equivalently to $\sigma'$), once the chiral symmetry is explicitly broken. This induces the intermediate range attraction. When chiral symmetry is explicitly broken, $\sigma'$ propagator is no longer infrared singular. The leading behaviour for small $\rho$ is simply obtained by using a massive propagator for $\pi$ of chiral symmetry. An expansion in powers of $\rho$ gives identical results as with the NN scattering. We focus on the three nucleon forces here. In the chiral limit only Fig.3a involving one $\rho$-vertex and two $\pi$ vertices survives and gives rise to a long range interaction. This contributes only for nn and ppn systems (related to the vanishing coupling $\rho^{0}\pi^{0}\pi^{0}$).

In the non-relativistic limit where the energy transfer is zero, each of the three vertices is suppressed by a power of the momentum transfer. With an explicit breaking of chiral symmetry, there are contributions from one $\Sigma$ and two $\pi$ vertices (Fig.3b) and from three $\Sigma$ vertices (Fig.3c). We estimate the three contributions as $q^{3}/m_{\pi}^{2}$, $\Sigma N q^{2}/m_{\pi}^{2}$ and $\Sigma_{\rho}/m_{\pi}^{2}$ respectively. In a theory like QCD we expect $\Sigma_{N} \sim m_{\pi}^{2} \sim m_{\rho}$. Hence for $q^{2} << m_{\pi}^{2}$ Fig 3c is leading order in $q^{2}$ but quadratic in $\Sigma_{N}$ while Fig 3b is of order $\Sigma_{N}$ but suppressed by one power of $q^{2}/m_{\pi}^{2}$. We emphasise that these contributions are unavoidable and computable consequences of chiral symmetry. All other contributions are suppressed by the heavy masses. Analogues of Fig 3c are also present for multinucleon forces and they are all of order $\Sigma_{N}$.

In the linear $\sigma$ model, $\sigma$ appears as a fundamental field, and an exchange of a dressed $\sigma'$ would appear as a tree diagram in the skeleton expansion. This is in contrast to the non-linear $\sigma$ model where it would appear as a loop effect. As the $\sigma$ channel plays a crucial role for nuclear forces it is more appropriate to use the formalism of the linear $\sigma$ model. On the other hand it would appear that the consequences of the low energy theorems are reflected better in the non-linear $\sigma$ model. The formalism of $1/N$ expansion [12] [14] provides a unified formalism combining the best of both.

In this Letter we have argued that because of the chiral symmetry it is possible to analyse exhaustively the dominant effects of pion exchanges between nucleli in a model independent way. The $\sigma$ ‘particle’ that effectively mediates the scalar isoscalar channel has a model independent and unusual logarithmic infrared behaviour. However in the chiral limit this decouples from the nucleon at low momenta. Consequently there are no long range interactions in this channel. Thus the intermediate range attraction is much weakened because of chiral symmetry and it is the ‘small’ explicit breaking which governs it. Also the $\sigma'$ is equivalent to $\pi^{2}$: in the infrared in a model independent way. As a result the intermediate range attraction is dominated by a very simple process: there is an effective $NN$ : $\pi^{2}$ : vertex whose strength is fixed by the $\pi N$ elastic scattering, and is theoretically of the order of the quark mass. The two pion exchange between two nucleons induced via this effective vertex...

\[FIG. 3. \text{Dominant contributions to the three body forces comes from a. one '}\rho\text{vertex and two '}\pi\text{vertices b. one '}\Sigma\text{vertex' and two '}\pi\text{vertices c. three '}\Sigma\text{vertices'}.\]
summarises the dominant effect of all kinds of virtual exchanges between the nucleons. Thus naive perturbation theory considerations turn out to be correct for not so naive reasons. Also there are small and computable multinucleon forces of intermediate range. Here we have considered only the dominant contributions for $q << m_{\pi}$. It is possible to systematically compute corrections of $O(q^2)$ and higher by a modification of chiral perturbation theory and thereby extend the range of validity to $q \sim m_{\pi}$. Nevertheless very accurate data for NN and Nd scattering in the range of few MeV exists, for which the calculations here are directly relevant.

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[1] S. Weinberg, Phys. Lett. B251, 288, (1990); Nucl. Phys. B363, 3, (1991).
[2] S. Weinberg, Phys. Rev. Lett. 18, 188, (1967).
[3] S. Weinberg, Phys. Rev. Lett. 17, 616, (1966).
[4] J. Gasser, H. Leutwyler, Phys. Repts. 87, 77, (1982); U. Meissner, Rep. Prog. Phys. B56, 903 (1993); H. Leutwyler, Recent Aspects of Quantum Fields, ed. H. Mitter and M. Gausterer, Lecture Notes in Physics 396 (1991); A. Pich, Rep. Prog. Phys. B58, 563 (1995); G. Ecker, Chiral Perturbation Theory, UWThPh-1996-34, June 1996 hep-ph/9608226.
[5] C. Celenza, A. Pantazi and C. M. Shakin, Phys. Rev. C46, 2213, (1992).
[6] Birse, Phys. Rev. C49, 2212, (1994).
[7] deRocha and M. R. Robilotta, Phys. Rev. C49, 1818, (1994).
[8] J. L. Friar and S. A. Coon, Phys. Rev. C49, 1272, (1994).
[9] Park T-S, Min and Rho M., Phys. Repts. B233, 341, (1993).
[10] C. Ordonez and V. Van Kolck, Phys. Lett. B291, 459, (1992); C. Ordonez and L. Ray and V. Van Kolck, Phys. Rev. Lett. 72, 1982 (1994); Phys. Rev. C53, 2086 (1996).
[11] J.-L. Ballot, M. L. Robilotta and C. A. da Rocha, nucl-th/9611.
[12] J. Delorme, G. Chanfray and M. Erickson, Nucl. Phys. A603, 239 (1996).
[13] D. B. Kaplan, M. J. Savage and M. B. Wise, nucl-th/9605002 (1996).
[14] E. Brezin, D. J. Wallace and K. G. Wilson, Phys. Rev. B7, 232 (1973); D. J. Wallace and R. K. P. Zia, Phys. Rev. B12, 5340 (1975); E. Brezin and J. Zinn-Justin, Phys. Rev. Lett. 36 (1976) 691.
[15] Ramesh Anishetty, Rahul Basu, N. D. Hari Dass and H. S. Sharatchandra, imsc-94/52, hep-th/9502003.
[16] Ramesh Anishetty, N. D. Hari Dass and H. S. Sharatchandra, under preparation.
[17] H. Lehmann, Phys. Lett. B41, 529 (1972).
[18] B. W. Lynn, Nucl. Phys. B402, 281 (1993).