Spatial Solitons
in Media with Delayed-Response Optical Nonlinearities

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PACS. 0340K - Waves and wave propagation: general mathematical aspects.
PACS. 4265J - Beam trapping, self focusing, thermal blooming, and related effects

Abstract

Near-soliton scanning light-beam propagation in media with both delayed-response Kerr-type and thermal nonlinearities is analyzed. The delayed-response part of the Kerr nonlinearity is shown to be competitive as compared to the thermal nonlinearity, and relevant contributions to a distortion of the soliton form and phase can be mutually compensated. This quasi-soliton beam propagation regime keeps properties of the inclined self-trapped channel.

1. Introduction

A competition between the cubic and thermal medium nonlinearities under different laser pulse durations seems to be firstly considered in [1]. Recently we noted [2] some special features of the soliton planar (two-dimensional) light beam propagation in a nonlinear medium with both the prompt Kerr-type nonlinearity and the inherently delayed-response thermal one. Under condition of scanning a powerful light beam, the Kerr-type nonlinearity was shown to be able to become dominant, while the thermal one being a perturbation exerting an influence upon the form and phase of a Kerr soliton. In this situation the planar beam keeps the auto-waveguide properties.

In this communication, we pay attention to some effects which occur when taking account of the material response of both simultaneously existing nonlinearities. Fast scanning light beam is the case where the dynamic cubic nonlinearity can be appreciable. It is shown that under certain condition the
delayed-response part of the Kerr-type nonlinearity and the thermal nonlinearity can be mutually cancelled.
2. Formulation of the problem

A system of equations governing the planar light beam propagation, allowing for both above nonlinearities and under the assumption of slight absorption, reads [3, 4]

\[ E_Z + \frac{1}{u} E_t + \frac{i}{2k} E_{XX} = -i \frac{k}{n_0}(\Delta n^{(K)} + \Delta n^{(T)}) E, \quad \Delta n^{(T)} = n_T T, \]

\[ \tau(\Delta n^{(K)})_t + \Delta n^{(K)} = n_2 |E|^2, \quad T_t = \chi T_{XX} + \frac{cn_0 \tilde{\gamma}}{2\pi \rho c_p} |E|^2. \]  

(1)

Here \( E \) is the beam field, \( T \) is the medium temperature measured with respect to the equilibrium one \( T_0 \), \( \tilde{\gamma} \) is the linear absorption coefficient, \( \chi \) is the temperature conductivity coefficient (the heat conductivity coefficient \( \sigma = c_p \rho \chi \)), \( c_p \) is the specific heat capacity at constant pressure, \( \rho \) is the medium density, \( n_0 \) is the linear refraction coefficient, \( n_2 \) is the Kerr constant, \( n = n_0 + \Delta n, \Delta n = \Delta n^{(K)} + \Delta n^{(T)} \) characterizes the contributions of the Kerr-type nonlinearity \( (K) \) and the thermal one \( (T) \), respectively, \( k_0 \) is the wave number, \( k = n_0 k_0 \), \( \tau \) is the relaxation time of the Kerr-type nonlinearity, \( X \) is the lateral coordinate directed along the interface, \( Z \) is the longitudinal coordinate directed into the bulk of the nonlinear medium, \( u \) is the group velocity in the first approximation of the dispersion theory, \( t \) is the running time. There is no \( Y \) dependence according to the two-dimensional model.

Let the action of a powerful radiation on the nonlinear medium be realized via translational scanning along the \( X \) axis of the planar beam, the incidence direction being normal to the interface and coincident with the \( Z \) axis. Also, let \( v \) be the scanning velocity and \( D \) be a width of the beam. Assume that the field \( E \) and the temperature \( T \) are dependent only on the moving frame coordinates \( X' = X - v(t - (Z/u)) \) and \( Z' = Z \), being independent on time explicitly, i.e., the stationarity condition holds in the moving reference system. In this system we have

\[ E_{Z'} + \frac{i}{2k} E_{X'X'} = -i \frac{k}{n_0}(\Delta n^{(K)} + \Delta n^{(T)}) E, \quad \Delta n^{(T)} = n_T T, \]

\[ \Delta n^{(K)} \approx n_2 |E|^2 + \tau v n_2 (|E|^2)_{X'}, \quad -v T_{X'} = \frac{cn_0 \tilde{\gamma}}{8\pi \rho c_p} |E|^2. \]  

(2)
The expression for $\Delta n^{(k)}$ is obtained from an approximate solution of the corresponding equation (1) under the assumption of smallness for $\tau \ll 1$. Note that the small term containing the derivative of $|E|^2$ can have another origin related to the Raman process. Besides, the second order derivative in temperature conductivity equation (1) may be neglected when $(\chi/vD) \ll 1$, which permits to represent $T$ as $T \sim \int_{-\infty}^{X} |E|^2 dX'$. Eventually, using (2), we obtain a resulting equation for $E$ representable in a convenient dimensionless form:

$$iA_z - A_{xx} - 2|A|^2A = \alpha(|A|^2)_xA \pm \beta A \int_{-\infty}^{x} |A|^2 dx'.$$

(3)

Here the following dimensionless quantities are introduced:

$$x = \frac{X'}{D}, \quad z = \frac{Z'}{2kD^2}, \quad A = \frac{1}{4} \left( \frac{c n_0 g}{\pi \sigma T_0} \right)^{1/2} E,$$

$$g = 16 \pi k^3 \sigma T_0 \left( \frac{D}{n_0} \right)^2 \frac{n_2}{c}, \quad \alpha = 32\pi k^3 \sigma T_0 \frac{D n_2 \tau}{c n_0^2 g} v, \quad \beta = \frac{\mu \delta}{g},$$

(4)

$$\delta = 2kD\hat{\gamma} v^{-1}, \quad \mu = 2(kD)^2 \frac{T}{n_0} |n_T|.$$

The signs ± for the last term in (3) correspond to the defocusing or focusing effect of the thermal nonlinearity, respectively.

The coefficients $\alpha$ and $\beta$ are seen from the derivation of the system (1) to be small ($\alpha \ll 1$ and $\beta \ll 1$), which allows Eq. (3) to be considered as a perturbed nonlinear Schrödinger equation (NSE). In the following, we shall conceive that the soliton relevant to the non-perturbative NSE is formed at the interface. Our task is to describe a behavior of the soliton under the action of the perturbation (3). For definiteness, we restrict ourselves to a consideration of the defocusing thermal nonlinearity.

3. Results.

In the adiabatic approximation where a distortion of the soliton form is neglected, we seek a solution of Eq. (3) as

$$A_s(x, z) = 2\eta(z) \exp(-i\theta(z)) \text{sech} y,$$
where \( y = 2\eta(x - \kappa(z)) \), \( \theta = [\xi(z)/\eta(z)]y + \Delta(z) \) and \( z \)-evolution of the parameters \( \xi, \eta, \kappa \) and \( \Delta \) is given by the formulae [3]:

\[
\xi_z = \frac{1}{2} \Re \int_{-\infty}^{\infty} R \tanh \operatorname{sech} e^{i\theta} dy, \quad \eta_z = -\frac{1}{2} \Im \int_{-\infty}^{\infty} R \operatorname{sech} e^{i\theta} dy,
\]

\[
\kappa_z - 4\xi = \frac{1}{4\eta^2} \Re \int_{-\infty}^{\infty} y R \operatorname{sech} e^{i\theta} dy,
\]

\[
\Delta_z - 2\xi\kappa_z + 4(\xi^2 - \eta^2) = \frac{1}{2\eta} \Re \int_{-\infty}^{\infty} R(1 - y \tanh) \operatorname{sech} e^{i\theta} dy.
\]

Here the perturbation is given by

\[
R = \alpha(|A|^2)_x A + \beta A \int_{-\infty}^{x} |A|^2 dx'.
\]

Solving the equations (5) we find

\[
\xi = \xi_0 + \frac{4}{15} \gamma \eta_0^2 z, \quad \eta = \eta_0, \quad \kappa = 4\xi_0 z + \frac{8}{15} \gamma \eta_0^2 z^2 + \kappa_0,
\]

\[
\Delta = 4(\xi_0^2 + \eta_0^2 - \frac{1}{2} \beta \eta) z + \frac{16}{15} \gamma \xi_0^2 \eta_0^2 z^2 + \frac{4}{3} \left( \frac{4}{15} \gamma \eta_0^2 \right)^2 + \Delta_0,
\]

where

\[
\gamma = 16\alpha \eta_0^2 - 5\beta.
\]

The soliton amplitude \( 2\eta \) is seen to be independent on the perturbation while an angle of inclination of the self-trapped channel to the \( z \) axis (related to the parameter \( \xi \)) varies, an increase or decrease of this angle being determined by the sign of a certain combination \( \gamma \) (8) of the parameters \( \alpha \) and \( \beta \). The function \( \kappa(z) \) characterizes a soliton centre location and depends on the perturbation also by means of \( \gamma \). The soliton phase \( \theta(z) \) acquires a nonlinear dependence on \( z \), the latter being completely determined by perturbations nonlocal in \( x \).

Finding corrections to the soliton form enables a conclusion to be drawn in relation to keeping or non-keeping a soliton localization under a perturbation. In the framework of the first-order approximation where a solution of Eq. (3) is written as \( A = A_s + A_1 \), the correction \( A_1 \) is evaluated by the formula [7, 2]

\[
A_1 = -\frac{1}{\pi} \left( \int_{-\infty}^{\infty} d\lambda \frac{\tilde{b}(\lambda)}{(\lambda - \lambda_1)(\lambda - \lambda_1)} e^{-2i\lambda x} (\lambda - \xi - i\eta \tanh)^2 \right)
\]
\[ + \eta^2 e^{-2i\theta} \operatorname{sech}^2 y \int_{-\infty}^{\infty} d\lambda \frac{b(\lambda)}{(\lambda - \lambda_1)(\lambda - \lambda_1)} e^{2i\lambda x} \]

where \( \lambda_1 = \xi + i\eta \), the bar implies complex conjugation, and \( b(\lambda) \) depends on the perturbation (6) (see [2]):

\[
b(z, \lambda) = \frac{i\pi}{30} (\lambda - \xi) \left[ 12\alpha + \frac{\gamma}{(\lambda - \lambda_1)(\lambda - \lambda_1)} \right] e^{i(\Delta - 2\lambda \kappa)} \operatorname{sech} \frac{\pi}{2} \frac{\lambda - \xi}{\eta}.
\]

Restricting ourselves to finding asymptotics, we obtain

\[
x \to +\infty : \quad A_1 \to e^{-y - i\theta} \left[ \frac{16}{15} \alpha \eta^2 + \frac{1}{15} \gamma y(y - 2) \right],
\]

\[
x \to -\infty : \quad A_1 \to e^{y - i\theta} \left[ \frac{16}{15} \alpha \eta^2 - \frac{1}{15} \gamma y(y - 2) \right].
\]

Thus, the soliton localization keeps in the presence of the perturbation (6) though a slight disturbance of the initial soliton \( x \)-symmetry is observed.

4. Discussion

To illustrate the plausibility of the idea proposed, let us adduce some typical parameters of available nonlinear condensed media and optical beams (order-of-magnitude estimates):

\[
n_0 \sim 1, \quad n_2 \sim 10^{-13} \ldots 10^{-12} \text{CGSE}, \quad \left| \frac{\partial n}{\partial T} \right| \sim 10^{-4} \ldots 10^{-3} \text{K}^{-1},
\]

\[
\tau \sim 10^{-12} \ldots 10^{-11} \text{s}, \quad D \sim 10^{-1} \text{cm}, \quad v \sim 10^8 \text{cm/s},
\]

\[
\rho c_p \sim 10^7 \text{erg/cm}^3\text{K}, \quad \tilde{\gamma} \sim 10^{-4} \text{cm}^{-1}, \quad k \sim 10^5 \text{cm}^{-1}.
\]

The magnitudes of the coefficients \( \alpha \) and \( \beta \) calculated with the above parameters confirm our conjecture concerning the smallness of \( \alpha \) and \( \beta \). Thus, the pertinent experimental conditions seem to be quite realizable, including a corresponding scanning velocity value. The last is achievable by means of up-to-date deflection techniques.

The perturbation (6) is described by two terms each of them reflects nonlocal character of the delayed-response nonlinearity. The non-locality of the perturbation determined by the inertial Kerr nonlinearity is of differential
type while the non-locality due to the thermal nonlinearity is associated with
the integral type. In the moving reference frame the above non-locality is
actually similar to the spatial dispersion influencing the soliton stability [8].

If the coefficients $\alpha$ and $\beta$ are of different signs, the action of both per-
turbations leads to a summation effect. On the other hand, if $\alpha$ and $\beta$ are of
the same sign, the nonlinear perturbations able to be compensated partially
or completely. This is seen from relevant corrections (7). A complete com-
pensation occurs at $\gamma = 0$. Let us write down $\alpha$ and $\beta$ isolating the explicit
dependence on the scanning velocity: $\alpha = \alpha^* v$ and $\beta = \beta^* v^{-1}$ (expressions
for $\alpha^*$ and $\beta^*$ can be readily obtained from (4)). Then we get from the
condition $\gamma = 0$:

$$v^2 = \frac{5}{16} \frac{\beta^*}{\alpha^* n^2}.$$  (9)

Hence, a feasibility of nonlinear distortion compensation is due to differ-
ence in $v$ dependence of both nonlinearities. Namely, the influence of the
Kerr-type nonlinearity increases with increasing $v$, while that of the thermal
nonlinearity decreases. Note that the relation (9) resembles the resonance
condition for the oscillating contour.

Thus, a compensation of distortions caused by the delayed-response part
of the cubic nonlinearity and defocusing thermal nonlinearities leads, in the
adiabatic approximation, to the ”pure” soliton having the envelope of sech-
type and the phase linear in $z$. In this case, there remains a linear correction
due to the thermal nonlinearity. It is worth to point out that one can also
use tilted pulses [9] which resemble translationally scanned beams.

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The authors are grateful to one of the referees for constructive suggestions.
This work was supported in part by the Fundamental Research Foundation
of Belarus under Contract No. Φ-17/205.
References

[1] Apanasevich P.A., Afanasiev A.A., and Urbanovich A.I., *Kvant. Elektron.*, 2 (1975) 2423.

[2] Doktorov E.V., Prokopenya I.N, and Vlasov R.A., *Phys. Lett. A*, 157 (1991) 181.

[3] Akhmanov S.A., Vorontsov M.A., et al., *Izv. VUZ. Radiofiz.*, 23 (1980) 3.

[4] Karamsin Yu.A., Sukhorukov A.P., and Trofimov V.A., *Mathematical Modelling in Nonlinear Optics* (Moscow State Univ., Moscow) 1989 (in Russian).

[5] Akhmanov S.A., Visloukh V.A., and Chirkin A.S., *Optics of Femtosecond Laser Pulses* (Nauka, Moscow) 1988 (in Russian).

[6] Florianczyk M. and Gagnon L., *Phys. Rev.*, A41 (1990) 4478.

[7] Karpman V.I. and Maslov E.M., *Sov. Phys. JETP*, 46 (1977) 537.

[8] Turitsyn S.K. *Teor. Mat. Phys.*, 64 (1985) 226.

[9] Janszky J. and Corrady G., *Appl. Phys.*, B33 (1984) 79.