Helium halo nuclei from low-momentum interactions

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Abstract. We present ground-state energies of helium halo nuclei based on chiral low-momentum interactions, using the hyperspherical-harmonics method for \( ^6\)He and coupled-cluster theory for \( ^8\)He, with correct asymptotics for the extended halo structure.

1 Motivation

The physics of strong interactions gives rise to new structures in neutron-rich nuclei. One prominent example are the helium halo nuclei, \( ^6\)He and \( ^8\)He, with two or four loosely-bound neutrons forming an extended halo around the \( ^4\)He core. \( ^6\)He is the lightest halo nucleus and the lightest Borromean system in nature. Recently, a combination of nuclear and atomic physics techniques enabled a new era of precision measurements of the ground-state energies (masses) and charge radii of \( ^6\)He \textsuperscript{12} and \( ^8\)He \textsuperscript{22}. Their reproduction poses extraordinary challenges for nuclear theory that will advance \textit{ab-initio} methods and our understanding of nuclear forces.

The existing \textit{ab-initio} calculations with traditional nucleon-nucleon (NN) and three-nucleon (3N) potentials are based on the Green’s Function Monte Carlo (GFMC) method \textsuperscript{5} and the No-Core Shell Model (NCSM) \textsuperscript{6}. In addition, there are larger-scale NCSM results but restricted to NN interactions \textsuperscript{7} and Fermionic Molecular Dynamics studies based on a unitary-correlated NN interaction plus a two-body potential introduced to mimic 3N effects \textsuperscript{8}.

One of the central advances in nuclear theory has been the development of effective field theory (EFT) and the renormalization group (RG) to nuclear forces. While light nuclei have been investigated using the NCSM \textsuperscript{9}, there are no results for helium halo nuclei based on chiral NN and 3N interactions. This is due to the challenges of the loosely-bound halo and the extended structure of the wave function. In this paper, we present results of an effort to study helium halo nuclei based on chiral EFT. These combine the RG evolution to low-momentum interactions with the \textit{ab-initio} hyperspherical-harmonics method for \( ^6\)He and coupled-cluster theory for \( ^8\)He. Our work goes beyond the previous investigation \textsuperscript{10} of the helium isotopes by studying the cutoff variation, as a tool to probe the effects of many-body forces, and we present first results based on the exact hyperspherical-harmonics expansion for \( ^6\)He, which is more difficult to describe in coupled-cluster theory due to its open-shell nature \textsuperscript{10}.

2 Effective field theory and the renormalization group for nuclear forces

Nuclear interactions depend on a resolution scale, which we denote by a generic momentum cutoff \( \Lambda \), and the Hamiltonian is always given by an effective theory for NN and corresponding...
many-nucleon interactions \[ H(\Lambda) = T + V_{NN}(\Lambda) + V_{3N}(\Lambda) + V_{4N}(\Lambda) + \ldots \quad (1) \]

For most nuclei, the typical momenta are of order of the pion mass, \( Q \sim m_\pi \), and therefore pion exchanges are included explicitly in nuclear forces. In chiral EFT \[11\], nuclear interactions are organized in a systematic expansion in powers of \( Q/\Lambda_b \), where \( \Lambda_b \) denotes the breakdown scale, roughly \( \Lambda_b \sim m_\rho \). At a given order, this includes contributions from one- or multi-pion exchanges and from contact interactions, with short-range couplings that depend on the resolution scale \( \Lambda \) and for each \( \Lambda \) are fit to data. Chiral EFT enables a direct connection to the underlying theory of Quantum Chromodynamics (QCD) through full lattice QCD simulations \[14\]. This can constrain long-range pion-nucleon couplings, the pion-mass dependence of nuclear forces, and has the potential to access experimentally difficult observables, such as three-neutron properties.

In Fig. 1, we show chiral EFT interactions at N^5LO of Entem and Machleidt \[15\] (EM with \( \Lambda = 500 \) and 600 MeV) and of Epelbaum et al. \[16\] (EGM with \( \Lambda = 450-600 \) MeV and a spectral-function cutoff in the irreducible two-pion exchange \( \Lambda_{\text{SF}} = 500-700 \) MeV). These accurately reproduce low-energy NN scattering. Using the renormalization group (RG) \[13\] we can change the resolution scale in chiral EFT interactions and evolve N^5LO potentials to low-momentum interactions \( V_{\text{low } k} \) with lower cutoffs. The RG preserves long-range pion exchanges and includes subleading contact interactions, so that NN scattering observables and deuteron properties are reproduced \[18\]. In the lower part of Fig. 1, we show the universality of \( V_{\text{low } k} \) by evolving all seven N^5LO potentials to a lower cutoff \( \Lambda = 2.0 \) fm^{-1}, and that the RG

**Fig. 1.** Diagonal (left) and off-diagonal (right) momentum-space matrix elements of different chiral EFT interactions at N^3LO \[15,16\] in the \(^1S_0\) channel (upper figures) and after RG evolution to low-momentum interactions \( V_{\text{low } k} \) (lower figures) for a smooth regulator with \( \Lambda = 2.0 \) fm^{-1} and \( n_{\text{exp}} = 4 \).
Fig. 2. Ground-state energy $E_0$ of $^6$He versus oscillator parameter $\hbar \omega$ for different SRG-evolved interactions with $\lambda = 3.0, 2.0, 1.5$ and $1.0 \text{ fm}^{-1}$. The NCSM results clearly show improved convergence with the maximum number of oscillator quanta $N_{\text{max}}$ for lower cutoffs. Since 3N interactions are neglected, the different NN calculations converge to different ground-state energies. For details see Ref. [22].

Evolution weakens the off-diagonal coupling between low and high momenta. This decoupling can also be achieved using similarity renormalization group (SRG) transformations towards band-diagonal [19] or block-diagonal [20] interactions in momentum space.

Changing the cutoff leaves observables unchanged by construction, but shifts contributions between the interaction strengths and the sums over intermediate states in loop integrals. The evolution of chiral EFT interactions to lower cutoffs is beneficial, because these shifts can weaken or largely eliminate sources of nonperturbative behavior such as strong short-range repulsion and short-range tensor forces [21]. Lower resolution needs smaller bases in many-body calculations, leading to improved convergence in nuclear structure applications. This is demonstrated by the very promising convergence for $N_{\text{max}} \sim 10$ in NCSM calculations with SRG interactions [22], shown in Fig. 2.

Chiral EFT interactions become more accurate with higher orders and the RG cutoff variation provides an estimate of the theoretical uncertainties due to neglected many-body interactions in $H(\Lambda)$ and due to an incomplete many-body treatment (see also Refs. [23,24]). For example, when three-nucleon (3N) interactions are neglected, we have found a universal correlation between the $^3$H and $^4$He binding energies [25], empirically known as “Tjon-line”.

3 Challenges for ab-initio calculations of halo nuclei

The extended structure and the asymptotic behavior of the wave function are theoretically challenging for halo nuclei, and to date there are no results based on chiral NN and 3N interactions. However, advances in ab-initio methods have the potential to overcome this challenge.

The hyperspherical-harmonics (HH) method has recently been extended to studies of $^6$He based on simple phenomenological NN potentials [26,27] using a powerful antisymmetrization algorithm [28]. The HH wave function is expanded in a translationally-invariant Jacobi basis and has the correct asymptotic behavior. In addition, the method is capable to handle nonlocal potentials, by expanding the interaction in harmonic-oscillator matrix elements [29,30].

Coupled-cluster (CC) theory [31] is a powerful method for nuclei for which a closed-shell reference state provides a good starting point [32,33]. The CC wave function is developed in a single-particle basis, which can be a Hartree-Fock or Gamow-Hartree-Fock basis [10] with correct asymptotic behavior. Combined with rapid convergence for low-momentum interactions,
Table 1. Ground-state energy $E_0$ of $^4\text{He}$ based on the \textit{ab-initio} FY, HH and CC approaches and the two-nucleon $V_{\text{low }k}$ interaction evolved from the EM 500 MeV chiral N$^3$LO potential \cite{15} for a cutoff $\Lambda = 2.0 \text{ fm}^{-1}$ using a smooth $n_{\text{exp}} = 4$ regulator.

| Method | $E_0(^4\text{He})$ [MeV] |
|--------|-----------------|
| Faddeev-Yakubovsky (FY) \cite{11} | $-28.65(5)$ |
| Hyperspherical harmonics (HH) | $-28.65(2)$ |
| CCSD (CC with singles and doubles) | $-28.44$ |
| $\Lambda$-CCSD(T) (CC with triples corrections \cite{31,42}) | $-28.63$ |

CC theory has pushed the limits of accurate calculations to medium-mass nuclei and set new benchmarks for $^{16}\text{O}$ and $^{40}\text{Ca}$ \cite{34}.

Three-nucleon interactions are a frontier in the physics of nuclei \cite{35} and including their contributions in \textit{ab-initio} calculations of neutron-rich and heavier nuclei presents a central challenge. For the helium isotopes, 3N interactions are crucial for binding energies and radii, for the evolution of nuclear structure with isospin, and for spin-orbit effects (see for example Refs. \cite{10,25,36}). In chiral EFT without explicit Deltas, 3N interactions start at N$^2$LO \cite{37,38} and typically constitute $\sim 10\%$ of the NN potential energy \cite{23}. Their contributions are given diagrammatically by

$$
\begin{array}{c}
\pi \otimes \pi \otimes \pi \\
\scriptstyle c_1, c_3, c_4 \\
\scriptstyle c_D \\
\scriptstyle c_E
\end{array}
$$

The long-range two-pion-exchange part is determined by the couplings $c_1, c_3, c_4$, which have been constrained in the $\pi N$ and NN system, and the remaining D- and E-term couplings are usually fitted to the $^3\text{H}$ binding energy and another observable in $A \geq 3$. The leading chiral 3N interaction generally improves the agreement of theory with experiment in light nuclei \cite{9}. At the next order, N$^3$LO, there are no new parameters in chiral EFT for many-body forces.

Since chiral EFT is a complete low-momentum basis, we have constructed 3N interactions $V_{3N}(A)$, corresponding to RG-evolved interactions, by fitting the leading D- and E-term couplings to the $^3\text{H}$ binding energy and the $^4\text{He}$ binding energy \cite{25} or radius \cite{89} for a range of cutoffs. By constraining the 3N interaction with the $^4\text{He}$ radius, we have found an improved cut-off dependence of nuclear matter and empirical saturation within theoretical uncertainties \cite{89}. For lower cutoffs, low-momentum 3N interactions become perturbative in light nuclei \cite{25}, and the first CC results with 3N forces show that low-momentum 3N interactions are accurately treated as effective 0-, 1- and 2-body terms, and that residual 3N forces can be neglected \cite{40}. This is very promising for developing tractable approximations to handle many-body interactions in \textit{ab-initio} approaches.

4 Results for helium halo nuclei based on chiral low-momentum interactions

In this Section, we present results for ground-state energies of helium nuclei based on chiral low-momentum NN interactions. These combine the RG evolution to low-momentum interactions and the resulting improved convergence with the \textit{ab-initio} hyperspherical-harmonics method for $^6\text{He}$ and coupled-cluster theory for $^8\text{He}$. Work towards including 3N interactions is in progress.

We first benchmark the HH and CC methods against the exact Faddeev-Yakubovsky (FY) $^4\text{He}$ ground-state energy in Table \cite{11} Our results are based on the low-momentum NN interaction $V_{\text{low }k}$ evolved from the EM 500 MeV chiral N$^3$LO potential \cite{15} for a cutoff $\Lambda = 2.0 \text{ fm}^{-1}$ using a smooth $n_{\text{exp}} = 4$ regulator. The (variational) HH ground-state energy agrees very well with the exact FY result, and the CC energy also agrees with the FY result after triples corrections are included. At the CCSD level (CC theory with singles and doubles excitations), only about
Ground-state energies and the CC matrix elements \[30\]. For the CC
4
He as a function of the grand-angular momentum \(K_{\text{max}}\) and the (CCSD level) coupled-cluster results for \(^{8}\)He as a function of the number of oscillator shells \(N_{\text{max}} = \max(2n + l)\).

\[\begin{array}{cccccc}
\text{A [fm}^{-1}\text{]} & E_0^{(4}\text{He)} & E_0^{(6}\text{He)} [E_0(K_{\text{max}} = 14)] & E_0^{(8}\text{He)} & \Lambda-\text{CCSD(T)} [\text{CCSD}] \\
1.8 & -29.30(2) & -30.28(3) [-30.13] & -31.21 [-30.33] \\
2.0 & -28.65(2) & -29.35(13) [-29.13] & -29.84 [-28.72] \\
2.4 & -27.40(2) & -27.62(19) [-26.91] & -27.54 [-25.88] \\
\hline
\text{experiment} & -28.296 & -29.268 \text{[4]} & -31.395 \text{[4]} \\
\end{array}\]

Table 2. Ground-state energies \(E_0\) in MeV of \(^{4}\)He (HH converged), \(^{6}\)He (HH extrapolated and for the largest \(K_{\text{max}} = 14\) space) and \(^{8}\)He (A-CCSD(T) and CCSD level) based on Fig. 3. For comparison we also give the experimental ground-state energies.

200 keV of correlation energy is missed. We have found similar agreement for all other cutoffs studied in this work.

In Fig. 3 we show the convergence as a function of basis size for the HH \(^{4}\)He and \(^{6}\)He ground-state energies and the CC \(^{8}\)He ground-state energy based on low-momentum NN interactions \(V_{\text{low} k}\) for a range of cutoffs \(A = 1.8, 2.0\) and \(2.4 \text{ fm}^{-1}\). The HH convergence as a function of the grand-angular momentum \(K_{\text{max}}\) is excellent for the \(^{4}\)He energies for all cutoffs studied, whereas the \(^{6}\)He energies are not completely converged, but as expected, we find improved convergence for lower cutoffs. The HH results practically do not show a dependence on the oscillator parameter \(\hbar\omega\) introduced by the expansion of nonlocal potentials in harmonic-oscillator matrix elements \[30\]. For the CC \(^{8}\)He energies, we obtain a good convergence with the number of oscillator shells \(N_{\text{max}} = \max(2n + l)\) using the spherical CC code \[43\]. The CCSD results are obtained in a Hartree-Fock basis and exhibit a small \(\hbar\omega\) dependence, which decreases with \(N_{\text{max}}\).

Our results for the ground-state energies of \(^{4}\)He, \(^{6}\)He and \(^{8}\)He are summarized in Table 2. For \(^{6}\)He, the largest \(K_{\text{max}} = 14\) space in the HH calculations includes three million basis states. The resulting matrices are dense and larger spaces are hard to achieve using the HH methods, but we are working towards accomplishing this and fully-converged \(^{6}\)He results. At present, we make an extrapolation assuming an exponential Ansatz \(E(K_{\text{max}}) = E^{\infty} + \alpha e^{-\beta K_{\text{max}}},\) with fit parameters \(\alpha, \beta\). The results of this extrapolation are listed in Table 2. Our procedure to obtain \(E^{\infty}\) is based on an extrapolation of the \(K_{\text{max}} = 8 – 14\) (last four) points. The first two \(K_{\text{max}} = 2 – 4\) points are omitted as they contain too few basis states to be considered as a reasonable expansion of the wave function. As an estimate of the error, we take double
the difference between the extrapolation of the $K_{\text{max}} = 8 - 14$ (last four) and $K_{\text{max}} = 6 - 14$ (last five) points. We adopted this procedure, since it was robust when applied to $^4\text{He}$. For $^4\text{He}$, besides neglecting the first two points as explained, we also omitted the $K_{\text{max}} = 12$ and $K_{\text{max}} = 14$ results, to simulate a not fully converged energy. By extrapolating the $K_{\text{max}} = 4 - 10$ (last four) points we obtained $E^\infty$ values that agree with $E(K_{\text{max}} = 14)$ within the theoretical uncertainty. For $^8\text{He}$, we show CC ground-state energies at the CCSD level, which typically accounts for 90% of the correlation energy [31] and at the $\Lambda$-CCSD(T) level [42]. The energies in Table 2 correspond to the $N_{\text{max}} = 14$ results, where a satisfactory convergence is reached.

The cutoff variation of the energies in Table 2 is significantly larger than the theoretical uncertainties due to an incomplete many-body treatment (the extrapolation errors or neglected quadrupole and higher corrections). Therefore, the cutoff variation is almost entirely due to neglected many-body interactions in the Hamiltonian $H(A)$ [23,25]. Our results highlight that 3N interactions are crucial for ground-state energies, and it is encouraging that the experimental energies are within the cutoff variation, so within the effects expected from many-body forces. Even for cutoffs around 2.0 fm$^{-1}$, where the NN-only results are reasonably close to the experimental energies for $^4\text{He}$ and $^6\text{He}$, the $^8\text{He}$ ground-state energy is underbound. Although low-momentum 3N interactions are overall repulsive in nuclear [24] and neutron matter [45], the same two-pion-exchange $c_i$-terms are attractive in $^4\text{He}$ for these cutoffs [23,25] and the N$^2$LO chiral 3N interaction provides an attractive contribution to spin-orbit splittings [9].

### 5 Outlook

This is an exciting era, with a coherent effort to understand and predict nuclear systems based on EFT and RG interactions, where 3N forces are a frontier, and with major advances in ab-initio methods for nuclear structure. On the experimental side, a highlight is set by the recent precision measurements of the charge radii and the first direct mass measurements of $^6\text{He}$ [12] and $^8\text{He}$ [34]. However, due to the extended halo structure, there are no results for the helium halo nuclei based on chiral NN and 3N interactions.

We have presented results for the ground-state energies of helium halo nuclei based on chiral low-momentum NN interactions. This combines the RG evolution to low-momentum interactions with the ab-initio HH method for $^8\text{He}$ and CC theory for $^6\text{He}$. These approaches overcome the challenges of the extended halo structure and have the correct asymptotic behavior of the wave function. The HH and CC methods have been validated against the exact FY result for $^4\text{He}$. For $^6\text{He}$ and $^8\text{He}$, our results highlight the importance of 3N interactions. For all studied cutoffs, the NN-only results underbind $^8\text{He}$ (see also Ref. [10]). Therefore, the helium isotopes probe 3N effects beyond the overall repulsion in infinite nuclear and neutron matter. Work is in progress towards including chiral 3N interactions in HH and CC approaches.

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