An outstanding challenge in large-scale quantum platforms is to simultaneously achieve strong interactions, giving rise to the most interesting behaviours, and local addressing, which can probe them. In the context of correlated phases, local addressing allows one to directly probe the nature of the system’s order. At the same time, such addressing allows the study of quantum information spreading and operator growth in out-of-equilibrium scenarios. Here we introduce a technique that enables the measurement of local correlation functions, down to single-site resolution, despite access to only global controls. Our approach leverages the intrinsic disorder present in a solid-state spin ensemble to dephase the non-local components of the correlation function. Utilizing this toolset, we measure both the spin and energy transport in nuclear spin chains. By tuning the interaction Hamiltonian via Floquet engineering, we investigate the cross-over between ballistic and diffusive hydrodynamics. Interestingly, in certain parameter regimes, we observe the coexistence of diffusive spin transport with ballistic energy transport, a hallmark of interacting integrable systems.

The complex dynamics of isolated quantum many-body systems are often amenable to a simple yet powerful description given by classical hydrodynamics. However, characterizing the nature of these hydrodynamical descriptions and how they emerge from microscopic quantum dynamics remains an area of active pursuit. Recently, this pursuit has seen tremendous advances owing to the development of large-scale quantum simulation platforms ranging from ultracold atoms and superconducting circuits, to solid-state spin systems.

To control and probe many-body dynamics in such systems, one typically requires a combination of strong interactions and local manipulation. In the majority of platforms, these two features are in tension: strong interactions arise when the constituent degrees of freedom are closely spaced, which in turn challenges the ability to perform local measurements. The tension is particularly acute in solid-state platforms where electronic and nuclear spins can exhibit strong interactions only when spaced at nanometre length scales. Here, we demonstrate that disorder, often times unavoidable in solids and long-considered detrimental for quantum coherence and transport, can be a powerful source of local control. First, by dephasing a homogenous state using the disorder, we demonstrate the preparation of states whose polarization on different sites is uncorrelated. Second, we show that single-site, spin–spin correlation functions can be directly measured using spin echo. The intuition behind our approach is the following—owing to the lack of spatial correlations, non-local components of the correlation function are averaged out, leaving only a sum of autocorrelations. Applying our technique in the context of nuclear magnetic resonance, we demonstrate the direct observation and characterization of nanoscale spin and energy transport, without the need for magnetic field gradients, subdiffraction techniques or multiple spin species.

Our experiments are performed on $S = 1/2$ $^{19}F$ nuclear spins within a single crystal of fluorapatite. The nuclear spins effectively form quasi-one-dimensional (1D) chains, since the interchain couplings are -40 times weaker than the intrachain couplings (Fig. 1a). We place...
our sample in a 7 T magnetic field along the [001] axis, which leads to a strong Zeeman splitting that reduces the dipolar interaction between \(^{19}\text{F}\) spins to its secular form,

\[
H_{\text{FF}} = \sum_{jk} \frac{1}{2r_{jk}} \left( 2s_j^x s_k^x - s_j^y s_k^y - s_j^z s_k^z \right),
\]

where \(r_{jk}\) is the distance between sites \(j\) and \(k\) (measured in units of the lattice constant). The presence of \(^{31}\text{P}\) nuclear spin-1/2’s leads to additional Ising interactions, \(H_{\text{FP}} = \sum_{jk} \frac{1}{2I_{jk}} s_j^z s_k^z\), where \(I_{jk}\) is the spin operator of \(^{31}\text{P}\) and \(J_{jk}\) includes the angular dependence of the dipole–dipole coupling (Methods). Crucially, the \(^{31}\text{P}\) nuclear spins are randomly polarized at room temperature and their interaction strength is significantly weaker than both \(H_{\text{FF}}\) and \(H_{\text{FP}}\); to this end, \(I_{jk}\) can be approximated as a scalar random variable, which effectively plays the role of a static, on-site disorder field for the \(^{19}\text{F}\) spins:

\[
H_{\text{dis}} = \sum_j w_j s_j^z,
\]

where \(w_j\) is drawn from a Gaussian distribution with an estimated width of 6 krads \(^{-1}\) (Methods and Extended Data Fig. 1).

To probe the infinite-temperature transport of spin and energy in our system, one must measure autocorrelation functions of the form \(\text{Tr}(S_j(t)S_k(0))\). To do so, we begin by evolving a weakly polarized thermal state \(\rho_0 \propto \sum_i S_i\) into a target initial state \(\rho \propto \sum_i c_i S_i\). Next, we evolve this initial state under a desired Hamiltonian \(H\) for a time \(t\), yielding \(\rho(t) = e^{-iHt}\rho e^{iHt}\). Finally, we measure a tunable observable, \(O_m\); in practice, via radiofrequency pulses, this observable is mapped onto the magnetization along the \(x\) axis, \(M = \sum s_i^x\), which we directly read out via an inductive measurement. The resulting signal is equivalent to the infinite-temperature correlation function, \(\text{Tr}(O_{m}(t)O_{m}(0))\). Clearly, if \(O_{p}\) and \(O_{m}\) are translationally invariant, the measured signal contains non-local correlations between all pairs of spins, for example \(\sum_{jk} \text{Tr}(S_j^x(t)S_k^x(0))\).

To access local correlation functions, such as the spin survival probability \(P_{\text{sur}}\), we prepare initial states and measure observables such that the spin-polarization at different sites is uncorrelated and averages to zero. An exemplary goal is to prepare and measure the random Zeeman state given by \(O_p = \sum_j \alpha_j S_j^z\), where \(\alpha_j\) are independent and identically distributed random variables with zero average. This would immediately enable the measurement of sum of single-site autocorrelations since

\[
\sum_{jk} \alpha_j \alpha_k \text{Tr}(S_j^z(t)S_k^z(0)) = \sum_{jk} \text{Tr}(S_j^z(t)S_k^z(0)),
\]

which is proportional to single-site autocorrelations \(\text{Tr}(S_j^z(t)S_k^z(0))\) for a translationally invariant Hamiltonian or a disordered Hamiltonian with translationally invariant statistics.

Let us now describe our disorder-based experimental protocol for preparing \(O_p\) (Fig. 1c). First, we rotate the thermal polarization to the \(x\) axis, initializing a state proportional to \(|1 + \epsilon \sum_j S_j^x\rangle\). Then, we evolve under \(H_{\text{dis}}\) for a time \(t\), such that the excess magnetization of each spin is oriented along a random direction in the xy plane. To ensure that the time evolution during \(t\) is generated only by \(H_{\text{dis}}\), we utilize concatenated WERHJA sequences (ref. 42) to dynamically decouple \(H_{\text{FF}}\). Next, we employ phase cycling to project the random polarization of each spin onto the \(y\) axis. A final radiofrequency pulse returns the polarization along \(z\), and we obtain \(O_p = \sum_j \alpha_j S_j^z\), with \(\alpha_j = \sin(w_j t)\) (for additional details see the Supplementary Information Section III). A similar strategy can be used to enable a measurement of \(O_m = \sum_j \alpha_j S_j^x\). In particular, just before the final inductive measurement of \(M\), we refocus the random state back to a uniform magnetization by applying the disorder field again. The above single-site autocorrelation function can be used to detect spin transport. An analogous approach can be used to detect autocorrelations of two-site observables, such as the local energy density. We first use the Jeener–Broekaert pulse pair \(\pi/4\) to create a homogeneous two-body correlated initial state proportional to \(|1 + \epsilon \sum (S_j^x S_k^x + S_j^y S_k^y)\rangle\) (here we assume nearest-neighbour coupling for representation simplicity, but the results also hold with \(1/r^4\) long-range coupling; for additional details see the Supplementary Information Section III). Evolution under the disordered field and phase cycling yields the random double-quantum (DQ) state with \(O_p = \sum_j \alpha_j' (S_j^x S_{j+1}^x - S_j^y S_{j+1}^y)\) where \(\alpha_j' = \sin(w_j t + w_{j+1} t)\) and

\[
\text{Tr}(S_j^x(t)S_k^x(0)) = \sum_{jk} \text{Tr}(S_j^x(t)S_k^x(0)),
\]

which is proportional to single-site autocorrelations \(\text{Tr}(S_j^x(t)S_k^x(0))\) for a translationally invariant Hamiltonian or a disordered Hamiltonian with translationally invariant statistics.
for large $\tau$. An additional $\pi/2$ pulse naturally realizes $O_p = \sum a'_i(S_i^{z_1} S_i^{z_2})$. We note that linear combinations of these two initial states allow us to reconstruct all of the subsequent operators we will consider. We can carefully characterize the initial state preparation, focusing on two properties: (i) demonstrating that $\sum a_i = 0$ and $\sum a'_i = 0$. For the first property, we measure $\langle \mathcal{L}(f, \theta, \gamma) \rangle = \text{Tr}(U_f(\phi, \theta, \gamma)O_p U_f^\dagger(\phi, \theta, \gamma)O_p)$ for various $\{\phi, \theta, \gamma\}$, where $U_f = \otimes_i e^{-i\gamma_i S_i^y} e^{-i\phi_i S_i^z}$. From $\langle \mathcal{L}(f, \theta, \gamma) \rangle = \sum a_i a'_i$, of the random observable up to a rotationally invariant component, $O_p = \sum d_i \mathcal{L}_i$, where $d_i$ are independent random variables satisfying $\sum d_i = 0$. For the second property, we measure the overlap of a random state $O_p$ with its corresponding homogeneous state. As depicted in Fig. 2c, the overlap quickly decays to zero as a function of the preparation time, indicating that, for sufficient

\[ \langle a'_i a'_j \rangle \propto \delta_{ij} \text{ for large } \tau. \]

\[ O_p = \sum a'_i(S_i^{z_1} S_i^{z_2}) \]

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but becomes interacting (case 2). Finally, the addition of a weak on-site random field, \( h \), causes the model to generically become non-integrable (case 3). We note that the long-range nature of the dipolar interaction \( H \) generically non-integrable for all of the above cases. However, our hope is that signatures of integrability will be present in the dynamics at short times; as we will see below, this is indeed borne out by the data.

These three different universality classes can be distinguished by the dynamical exponent, \( z \), associated with their spin and energy transport. Crucially, \( z \) can be directly measured via the power-law decay of the autocorrelation function \( -1/t^z \), with \( z = 1 \) corresponding to ballistic motion and \( z = 2 \) corresponding to diffusion.

Let us begin with case 1. We tune \( \{ u, v, h \} = \{0.5, 0, 0\} \) and measure the spin–spin autocorrelation function (Fig. 3c, green) and the energy autocorrelation function (Fig. 3c, blue). Both exhibit late-time power laws consistent with \( z = 1 \), in agreement with the expectation that quasiparticles propagate ballistically in a non-interacting, integrable model.

For case 2, we tune our system to \( \{ u, v, h \} = \{-0.15, 0.3, 0\} \) and find that spin transports diffusively while energy transports ballistically (Fig. 3d). This phenomenon is due to the existence of stable spinless quasiparticles and is a central feature of infinite-temperature transport in the so-called XXZ model. Finally, for case 3, we set \( \{ u, v, h \} = \{-0.15, 0.3, 0.23\} \) and observe that both spin and energy transport diffusively (Fig. 3e), consistent with a generic non-integrable model. The maximum time explored here is limited by the interchain coupling, which, albeit being 40 times smaller than intrachain coupling, becomes non-negligible at \( \nu t = 50 \) and breaks the quasi-1D approximation.

Two remarks are in order. First, the energy transport data in case 2 exhibit a weak deviation from ballistic transport at the longest times explored in the experiment (inset, Fig. 4d). To understand the origin of this deviation, we numerically compute the energy autocorrelation function in a 1D spin chain using density matrix truncation, with and without long-range couplings. The agreement between our experiment and numerics in the former case suggests that the observed deviation results from the weak breaking of integrability associated with the long-range intrachain couplings, which is eight times weaker than the nearest-neighbour interaction. Second, by tuning the disorder strength during the evolution, we can controllably break integrability and access the non-integrable regime on the experimental timescale.

In Fig. 4, we measure the energy and spin transport as we tune \( h \) from 0 to 0.3 (Methods and Extended Data Fig. 2). We extract \( z \) using different time windows of the autocorrelation function, starting at \( t_{\text{final}} = 7.7/\nu \) and ending at a variable \( t_{\text{end}} \). For the spin transport (Fig. 4a), after an initial transient, all of the models exhibit \( z = 2 \) at intermediate times. At the latest times, the interchain couplings begin to play a role, causing a decrease in \( z \). Meanwhile, for the energy transport at \( h = 0 \), \( z \) remains close to its initial ballistic value for all times. However, for \( h = 0.3 \), the system reaches a diffusive exponent \( z = 2 \) at intermediate times before exhibiting a weak decrease (possibly owing to interchain couplings).

In summary, our results introduce a new method to probe local spin and energy transport in solid-state spin ensembles. Our technique leverages the intrinsic disorder in such systems and requires only collective control. Our local probe enables exploration of quantum many-body phenomena unavailable in homogeneous systems, such as the ballistic and diffusive hydrodynamics demonstrated here, or subdiffusion near the many-body localization transition and the emergence of superdiffusion with long-range interactions. In addition to two-point correlation functions, which were the focus of the present work, our protocols can naturally be generalized to four-point, out-of-time-ordered correlations, and thus used to probe many-body quantum information scrambling. Beyond quantum simulation, transport measurements provide rich information of the system, and therefore can also boost quantum sensing applications in material and biological science. Finally, we point out that a general static
inhomogeneous field can similarly induce the dephasing process and thus create random states, broadening the application of the present scheme to quantum platforms where disorder is not naturally present, including cold atoms, trapped ions and superconducting circuits.

**Online content**

Any methods, additional references, Nature Portfolio reporting summaries, source data, extended data, supplementary information, acknowledgments, peer review information; details of author contributions and competing interests; and statements of data and code availability are available at https://doi.org/10.1038/s41567-023-02024-4.

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Methods

Experimental system

The sample in the experiment is a single crystal of fluorapatite with formula Ca$_5$(PO$_4$)$_3$F. The most abundant isotopes of F and P have 1/2 nuclear spin, while the most abundant isotopes of Ca and O have zero nuclear spin. Fluorapatite is a hexagonal mineral with space group $P6_3/m$, where the $^{19}$F spin 1/2 nuclei form linear chains along the c axis. Each fluorine spin in the chain is surrounded by three equidistant $^{31}$P spins. The sample we used is a cut from a natural crystal of $^{31}$P nuclei. The sample in the experiment is a single crystal of fluorapatite with the crystallographic axis parallel to the external magnetic field $B$ and the magnetic field z axis. We align the crystal is oriented with its c axis along the corresponding direction and with the strength given by the $^{19}$F–$^{31}$P interaction. Summing up the contribution from all $^{31}$P gives the total strength of the disorder field. In Extended Data Fig. 1, we include the 45 closest $^{31}$P and observe a smooth enough distribution of the field strength. The distribution is perfectly fitted by a sum of four Gaussian distributions with the same width, the centres at $±\frac{1}{2}^{19}F$ and $±\frac{3}{2}^{19}F$ and the height ratio of 3:3:1:1. Interestingly, even a single Gaussian function can still capture the distribution reasonably well, so we simply use a Gaussian distribution for simplicity in our numerical simulation. This can be qualitatively justified by comparing the computed $\tau$ decay profiles of the Gaussian distribution and the true distribution (Extended Data Fig. 1).

Ab initio calculation of disordered field

The disordered magnetic fields on the $^{19}$F originates from the $^{19}$F–$^{31}$P interaction and the random orientation of $^{31}$P. This picture allows us to directly calculate the distribution of the disordered field strength. In particular, we compute the interaction strength between $^{19}$F and its several closest neighbouring $^{31}$P based on their relative position and gyromagnetic ratio. Then we assume each $^{31}$P points along the $\pm z$ or $\pm y$ direction with the same probability, which effectively applies a magnetic field on $^{19}$F along the corresponding direction and with the strength given by the $^{19}$F–$^{31}$P interaction. Summing up the contribution from all $^{31}$P gives the total strength of the disorder field. In Extended Data Fig. 1, we include the 45 closest $^{31}$P and observe a smooth enough distribution of the field strength. The distribution is perfectly fitted by a sum of four Gaussian distributions with the same width, the centres at $±\frac{1}{2}^{19}F$ and $±\frac{3}{2}^{19}F$ and the height ratio of 3:3:1:1. Interestingly, even a single Gaussian function can still capture the distribution reasonably well, so we simply use a Gaussian distribution for simplicity in our numerical simulation. This can be qualitatively justified by comparing the computed $\tau$ decay profiles of the Gaussian distribution and the true distribution (Extended Data Fig. 1).

Data for transport with disorder

In Fig. 4 we show the dynamical exponent for various disorder field strengths. We present the source data—autocorrelation as a function of time here—in Extended Data Fig. 2.

Data availability

Source data are provided with this paper. All other data that support the plots within this paper and other findings of this study are available from the corresponding author upon reasonable request.

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Author contributions
P.P. designed and performed the experiment with assistance from P.C. B.Y. and N.Y.Y. performed the numerical and analytical calculations. P.C. supervised the project. All authors worked on the interpretation of the data and contributed to writing the manuscript.

Competing interests
The authors declare no competing interests.

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**Extended Data Fig. 1** Ab initio calculation of disordered field and decoherence profile. Disordered on-site field generated by $^{31}$P. a. Numerical calculation of distribution of the on-site field strength. The four-Gaussian fit gives a standard deviation of 2.217(2) krad/s for each Gaussian peak. The single-Gaussian fit gives a standard deviation of 6.05(6) krad/s. b. Left axis: Decoherence profile generated by the calculated distribution of on-site field and the single-peak Gaussian approximation. Right axis: Statistical correlation between the random amplitudes of local observables on two closest $^{19}$F. As the coherence approaches zero, the statistical correlation also vanishes.
Extended Data Fig. 2 | Raw data for transport with disorder. Spin (a) and energy (b) autocorrelation for various disorder field strength $h$. Data are presented as mean values +/- SD from readout noise (for additional details see the Supplementary Information).