A multi-AUV cooperative navigation method

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Abstract. Cooperative navigation is one of the key methods for multiple autonomous underwater vehicles (AUVs) to obtain accurate positions when performing tasks underwater. In the realistic state-space model of the multi-AUV cooperative navigation system, where the system noise does not satisfy the additivity, it is necessary to augment the dimension of the state variables before nonlinear filtering. Aiming at the problem that the error of traditional algorithms increases linearly with the dimension of state-space, a cooperative navigation method based on Augmented Embedded Cubature Kalman filter (AECKF) algorithm is proposed. The experiment results show that the AECKF cooperative navigation algorithm has better positioning accuracy and stability than the traditional algorithm.

Keywords: Autonomous Underwater Vehicle (AUV), cooperative navigation, state augmentation, Augmented Embedded Cubature Kalman filter (AECKF)

1. Introduction

In recent years, multi-AUV cooperative navigation has developed as a new type of underwater navigation technology, which has received extensive attention from countries all over the world[1]. To estimate and correct the cooperative navigation parameters of AUV, many different filtering algorithms are used based on different state-space models[2]. Using higher precision sensors or a more realistic motion model can improve navigation accuracy. However, the former will increase costs and the latter may cause the system noise to not meet the additivity.

Based on the above analysis, this paper proposes a multi-AUV cooperative navigation method based on the AECKF algorithm. Firstly, we add the system noise to the state variable to augment its dimension to solve the problem that the system noise cannot be added. Then, a multi-AUV cooperative navigation method based on the AECKF algorithm is proposed, which overcomes the shortcoming that the positioning error of traditional algorithms increases linearly with the dimension of state-space. The simulation results show that the positioning accuracy and stability of the proposed multi-AUV cooperative navigation method are better than the traditional algorithm.

2. Related Works

Among the Multi-AUV cooperative navigation methods, the algorithm based on Bayesian estimation is the widely used. Reference[3] uses the Cross Entropy algorithm combined with the Extended
Kalman filter (EKF) to effectively reduce the observation error of the following AUV. Reference[4] shows that Unscented Kalman filter (UKF) has a better approximation effect than EKF for nonlinear systems, but the sigma points used for approximation have negative values. For this reason, the Cubature Kalman filter (CKF) algorithm is proposed. However, the inherent defects make its filtering error increase linearly with the augmentation of state dimension[5]. In reference[6], the Embedded Cubature Kalman filter (ECKF) algorithm is proposed to achieve higher accuracy of filtering estimation and has better high-order expansibility.

Comprehensively considering the problems that the noise of the system does not satisfy the additivity and the increase of positioning error of the traditional algorithms after the augmentation of the state dimension during the multi-AUV cooperative navigation process, this paper proposes a multi-AUV cooperative navigation method based on the AECKF algorithm. In terms of verification methods, in addition to the simulation to verify the navigation and positioning accuracy of the algorithm.

### 3. Multi-AUV cooperative navigation state-space model

In actual underwater cooperative positioning, the research on the AUV positioning problem can be directly mapped to the two-dimensional space. The state equation of single AUV is as follow:

\[
\begin{align*}
    x_{k+1} &= x_k + \Delta t \cdot V_k \cos \phi_k \\
    y_{k+1} &= y_k + \Delta t \cdot V_k \sin \phi_k \\
    \theta_{k+1} &= \theta_k + \Delta t \cdot \omega_k
\end{align*}
\]

(1)

Among them, \(x_k\) and \(y_k\) are the AUV's dead reckoning positioning information; \(\Delta t\) is the dead reckoning cycle; \(V_k\) is the forward speed; \(\phi_k\) represents the heading angle, and \(\omega_k\) represents the angular velocity, where \(V_k\) and \(\phi_k\) are interfered by zero-mean and independent Gaussian white noise.

The relative distance measurement information between the pilot and following AUVs is \(Z_k\). Then the coordinate position relationship can be obtained:

\[
Z_k = \sqrt{(x_k^M - x_k^s)^2 + (y_k^M - y_k^s)^2} + v_k
\]

(2)

Where \(x_k^M\), \(y_k^M\), and \(x_k^s\), \(y_k^s\) are the east and north positions of the pilot AUV and the following AUV at \(t_k\), respectively, and \(v_k\) is the Gaussian white noise.

Define \(x_k = [x_k, y_k, \phi_k]^T\) as the state variable of the following AUV, \(w_k = [w_k^y, w_k^\theta]^T\) as the process noise vector, and \(v_k = [v_k, v_k^\theta]^T\) as the measured noise vector. Then we can get the following equation:

\[
Q_k = E[w_k w_k^T], R_k = E[v_k v_k^T]
\]

(3)

Where \(Q_k\) and \(R_k\) are the assumed known system process noise covariance matrix and the measured noise covariance matrix, respectively.

### 4. State augmentation and Embedded Cubature Kalman filter

The framework of the multi-AUV cooperative navigation method based on the AECKF algorithm is shown in Figure 1.
4.1. Embedded Cubature Kalman Filter (ECKF)
For \( n \)-dimensional state variables, use \( N = 2^n + 1 \) cubature points to achieve the numerical approximation of the integral:

\[
I_N(f) = \sum_{i=1}^{N} \tilde{w}_i \tilde{f}(\tilde{\xi}_i)
\]  

(4)

In equation (4), the corresponding cubature points and weight are as follow:

\[
\tilde{\xi}_i^E = \begin{cases} 
[0], & i = 1 \\
\sqrt{2}[\delta], & i = 2, \ldots, N
\end{cases}, \quad \tilde{w}_i^E = \begin{cases} 
1 - \frac{1}{2\sigma_i^2}, & i = 1 \\
\frac{1}{2\sigma_i^2}, & i = 2, \ldots, N
\end{cases}
\]  

(5)

Where \([\delta] = [s_1, \sigma_1^2, s_2, \sigma_2^2, \cdots, s_N, \sigma_N^2]^T \) is the cubature point set, and \([\delta]_i \) is the \( i \)-th point in the point set.

4.2. Algorithm steps of multi-AUV cooperative navigation based on AECKF
Consider the nonlinear system established in the previous section:

\[
\begin{align*}
X_{k+1} &= f(X_k, u_k, w_k) \\
Z_{k+1} &= h(X_{k+1}) + v_k
\end{align*}
\]  

(6)

Where \( X_k \in \mathbb{R}^n \) is the \( n \)-dimensional system state variable, \( w_k \) is the \( n_w \)-dimensional system process noise, and \( v_k \) is the \( n_z \)-dimensional system observation noise.

**Step1 State prediction**
Replacing the cubature points set and weights of CKF with the equation (5), the Cholesky decomposition of the error covariance is performed as follow:

\[
P_{k|k} = S_{k|k} S_{k|k}^T
\]  

(7)

augment the dimension of state variables and \( S_{k|k}^a \):

\[
\hat{X}_{k|k}^a = \begin{bmatrix} \hat{X}_{k|k}^T & 0_{n_u} & 0_{n_u} & \sqrt{Q_{k}} & 0 \\
\end{bmatrix}
\]

(8)

Calculate the cubature points \((i = 1, 2, \cdots, N, N = 2(n + n_u + n_z)):\)

\[
X_{i,k|k} = S_{k|k}^a \tilde{\xi}_i^E + \hat{X}_{k|k}^a
\]  

(9)

Propagate cubature points through system function:

\[
X_{i,k+1|k}^* = f(X_{i,k+1|k}^*, X_{i,k|k}^w)
\]  

(10)

Calculate the state prediction value at time \( t_{k+1} \):

\[
\hat{X}_{k+1|k} = \frac{1}{N} \sum_{i=1}^{N} X_{i,k+1|k}^*
\]  

(11)

Calculate the prediction value of state error covariance at time \( t_{k+1} \):

\[
P_{k+1|k} = \frac{1}{N} \sum_{i=1}^{N} X_{i,k+1|k}^* X_{i,k+1|k}^{*T} - \hat{X}_{k+1|k} \hat{X}_{k+1|k}^T
\]  

(12)

**Step2 Measurement prediction**
Propagate the cubature points through the measurement function:

\[
Z_{i,k+1|k} = f(X_{i,k+1|k}^*, X_{i,k+1|k}^{vr})
\]  

(13)

Calculate the prediction value of measurement at time \( t_{k+1} \):

\[
\hat{Z}_{k+1|k} = \frac{1}{N} \sum_{i=1}^{N} Z_{i,k+1|k}
\]  

(14)
Calculate the autocorrelation and the cross-correlation covariance matrix:

\[
P_{Z\!\!Z, k+1|k} = \frac{1}{N} \sum_{i=1}^{N} Z_{i,k+1|k} Z_{i,k+1|k}^T - \hat{\mathbf{Z}}_{k+1|k} \hat{\mathbf{Z}}_{k+1|k}^T, \quad P_{X\!\!Z, k+1|k} = \sum_{i=1}^{N} \mathbf{a}_i \mathbf{X}_{i,k+1|k} Z_{i,k+1|k}^T - \hat{\mathbf{X}}_{k+1|k} \hat{\mathbf{Z}}_{k+1|k}^T
\]

(15)

**Step 3 State update**

Calculate the filter gain at time \( t_{k+1} \):

\[
K_{k+1} = P_{X\!\!Z, k+1|k} P_{Z\!\!Z, k+1|k}^{-1}
\]

(16)

Calculate the state estimate value at time \( t_{k+1} \):

\[
\hat{\mathbf{X}}_{k+1|k+1} = \hat{\mathbf{X}}_{k+1|k} + K_{k+1} (\mathbf{Z}_{k+1|k} - \hat{\mathbf{Z}}_{k+1|k})
\]

(17)

Calculate the estimate value of the state error covariance at time \( t_{k+1} \):

\[
P_{k+1|k+1} = P_{k+1|k} - K_{k+1} P_{Z\!\!Z, k+1|k} K_{k+1}^T
\]

(18)

5. Simulation comparison and performance evaluation

5.1. Simulation environment construction

On a two-dimensional plane, a multi-AUV cooperative navigation system based on a single pilot boat is established. The speed of AUVs are maintained at \( 2 m/s \). The system status update cycle is \( \Delta t = 10 s \), and the carrier's underwater navigation time is \( T = 2500 s \). Set the variance of the process noises to \( \sigma_v^2 = (0.1 m/s)^2 \) and \( \sigma_\omega^2 = (1' / s)^2 \) and the variance of the measured noise to \( \sigma_r^2 = (1 m)^2 \). The initial value of the state estimate after taking the augmented dimension is as follow:

\[
\hat{\mathbf{X}}_{0|0} = [500 \quad 500 \quad \pi / 4 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0.01 \quad 0 \quad 0]^T, \quad P_{0|0} = \text{diag} [1 \quad 1 \quad 0.01 \quad 0 \quad 0]
\]

(19)

5.2. Simulation results and analysis

Figure 2 is the comparison of the positioning accuracy based on AECKF and other cooperative navigation algorithms. As shown, the AECKF algorithm can effectively reduce the errors of the other algorithms during the co-location process in the case of the high dimensionality of the system.

![The follow AUV trajectory comparison chart](image)

**Figure 2.** Different filter algorithms track and real track.
Figure 3. Positioning errors of different algorithms.

Figure 3 shows the comparison of the positioning error curves obtained by the four algorithms. It can be seen that under the simulation conditions, the maximum positioning error of the AECKF filter algorithm is less than 5m and the fluctuation range of the error curve is about 4m, indicating that it has better accuracy and robustness. The maximum error distance and error curve fluctuation degree of CKF, UKF, and EKF algorithms increase sequentially, which is difficult to maintain higher accuracy and stability compared with the AECKF algorithm.

Table 1. Performance comparison of different algorithms for cooperative navigation.

| Algorithm | x-axis position /m | y-axis position/m | Heading angle/rad | Distance error/m |
|-----------|-------------------|------------------|-------------------|------------------|
| EKF       | 22.494            | 7.688            | 2.343             | 23.772           |
| UKF       | 2.531             | 2.777            | 0.747             | 3.753            |
| CKF       | 2.009             | 2.250            | 0.612             | 3.016            |
| AECKF     | 1.106             | 1.538            | 0.307             | 1.894            |

Table 1 lists the co-location performance indicators of four different algorithms. From the statistical data in Table 1, it can be seen that the AECKF algorithm proposed in this paper has higher positioning accuracy than other cooperative navigation methods. Compared with EKF, UKF, and CKF, the multi-AUV cooperative navigation algorithm based on AECKF improves the positioning accuracy by 92.03%, 49.53%, and 37.20%, respectively. This is mainly due to the comprehensive consideration of the increased dimension of the system after the augmentation of the state. For the adverse effects of filtering accuracy, the embedded cubature criterion is used to overcome this shortcoming, thereby obtaining higher positioning accuracy.

6. Conclusions
This paper proposes an Augmented Eckf algorithm, which adds process noise and measurement noise to state variables and uses the ECKF algorithm to suppress the positioning estimation error that increases with the increase of the state-space dimension. The simulation results show that this algorithm can effectively overcome the shortcomings of reduced or even divergent filtering accuracy of the CKF and UKF algorithms when the system state dimension is relatively high. The accuracy and stability of the AECKF algorithm is significantly better than many other traditional nonlinear filtering
algorithms. But while improving the accuracy and stability of the navigation, it also increases the running time of the algorithm to a certain extent.

Acknowledgments
This work is partly supported by Major Scientific and technological innovation project of Shandong Province of China (2020CXGC010705), China Postdoctoral Science Foundation funded project (2020M672123), Post-doc Creative Funding in Shandong Province (244312), and Post-doc Funding in Weihai, science and technology development plan project of Weihai.

References
[1] Xu B, Li S, Razzaqi A A, et al. Cooperative Localization in Harsh Underwater Environment based on the MC-ANFIS[J]. IEEE Access, 2019, PP(99):1-1.
[2] Xu Bo, Li Shengxin, Jin Kunming, Wang Lianzhao. Multi-autonomous underwater vehicle cooperative positioning method based on radial basis function neural network-assisted Cubature Kalman filter[J]. Acta Armamentarius, 2019, 40(10): 2119-2128.
[3] Zhang L, Li Y, Liu L, et al. Cooperative Navigation Based on Cross Entropy: Dual Leaders[J]. IEEE Access, 2019, 7:1-1.
[4] Allotta B, Chisci L, Costanzi R, et al. A comparison between EKF-based and UKF-based navigation algorithms for AUVs localization[C]// Oceans. IEEE, 2015:1-5.
[5] Jiang L, Cai B, Tao T, et al. A CKF based GNSS/INS train integrated positioning method[C]// International Conference on Mechatronics & Automation. IEEE, 2010.
[6] Liu Hua, Miao Chen. Square root embedded cubature Kalman particle filter algorithm[J]. Journal of Nanjing University of Science and Technology: Natural Science Edition, 2015(4):471-476.