Phenomenology of neutrinoless double beta decay

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Abstract. Neutrinoless double beta decay \(0^{\nu}\beta\beta\) violates lepton number by two units, a positive observation therefore necessarily implies physics beyond the standard model. Here, three possible contributions to \(0^{\nu}\beta\beta\) decay are briefly reviewed: (a) The mass mechanism and its connection to neutrino oscillations; (b) Left-right symmetric models and the lower limit on the right-handed \(W\) boson mass; and (c) R-parity violating supersymmetry. In addition, the recently published “extended black box” theorem is briefly discussed. Combined with data from oscillation experiments this theorem provides proof that the \(0^{\nu}\beta\beta\) decay amplitude must receive a non-zero contribution from the mass mechanism, if neutrinos are indeed Majorana particles.

1. Introduction
Since the discovery of neutrino oscillations [1] most papers on neutrinoless double beta decay \((0^{\nu}\beta\beta)\) have exclusively concentrated on its implications for Majorana neutrino masses. However, as is well-known, any model beyond the standard model of particle physics, which allows for lepton number violation, potentially contributes to \(0^{\nu}\beta\beta\) decay. Thus, the basic physics of \(0^{\nu}\beta\beta\) decay can be summarized as:

\[
\left[T_{1/2}^{0^{\nu}\beta\beta}\right]^{-1} = \left(\sum_{i} \langle \epsilon_{i}\rangle M_{\epsilon_{i}}\right)^{2} F^{0^{\nu}\beta\beta}. \tag{1}
\]

The factor \(\langle \epsilon_{i}\rangle\) contains some (unknown, but lepton number violating) particle physics parameters. To determine the numerical value of \(\langle \epsilon_{i}\rangle\) input from both, experiment and theoretical nuclear physics, is needed. Experiments limit (or measure) \(T_{1/2}^{0^{\nu}\beta\beta}\), for a discussion of various different experiments see, for example [2]. \(M_{\epsilon_{i}}\) in eq. (1) stands for a nuclear structure matrix element. Different particle physics contributions to \(0^{\nu}\beta\beta\) decay depend on different matrix elements. No definite consensus about the value and, most importantly, the error of nuclear matrix elements exist up to now. For a thorough discussion see [3]. Finally, \(F^{0^{\nu}\beta\beta}\) is a leptonic phase space integral, its value can be calculated quite precisely [4].

This talk concentrates exclusively on particle physics aspects of \(0^{\nu}\beta\beta\) decay. The classic “black box” [5] theorem and its recently published “extended” version [6] are briefly discussed, before reviewing constraints on left-right symmetric models and supersymmetry with R-parity violation derived from a lower limit on the \(0^{\nu}\beta\beta\) decay half-live. Last but not least, expectations for the mass mechanism of \(0^{\nu}\beta\beta\) decay in light of neutrino oscillation data are discussed. It is curious to note, that combining the “extended black box” with oscillation data [7] already today demonstrates that there must be a non-zero contribution from the mass mechanism to the \(0^{\nu}\beta\beta\) decay amplitude, if neutrinos are indeed Majorana particles.
2. $0\nu\beta\beta$ decay and the Black Box

From the experimental point of view lepton number violation in $0\nu\beta\beta$ decay is observed through the appearance of two electrons in the final state with no missing energy. Many different, possible mechanisms have been discussed in the literature. Interestingly, however, one can show [5] that independent of which contribution to $0\nu\beta\beta$ decay is the dominant one, neutrinos are guaranteed to have a non-zero Majorana mass, if $0\nu\beta\beta$ decay is observed. The proof of this “black box” theorem [5] essentially follows from the observation that any effective low-energy $\Delta L \neq 0$ operator inducing $0\nu\beta\beta$ decay will contribute also - possibly at the same order in perturbation theory, for sure in some higher order - to the $\langle \nu_e - \nu_\mu \rangle$ entry of the Majorana neutrino mass matrix ($M_{ee}^{\nu}$). A perfect cancellation of all different contributions to $M_{ee}^{\nu}$ would then require a special symmetry and the proof of the black box theorem is completed by showing that no such symmetry can exist [7] in any gauge model containing the standard model charged current interaction.

This well-known theorem has recently been extended to the case of three generations of neutrinos and arbitrary lepton number and lepton flavour violating processes [6]. Combined with data from oscillation experiments this “extended” black box theorem can be used to show that $M_{ee}^{\nu} \neq 0$. The proof involves two steps. In the first step it is shown that any effective operator generating lepton number violating processes of the form $\Phi_k \rightarrow \Phi_m l\bar{l}/\beta$, where $\Phi_k$ and $\Phi_m$ stand symbolically for any set of SM particles with $L = 0$, necessarily generates a non-zero $M_{\alpha\beta}^{ee}$ entry in the Majorana neutrino mass matrix in higher order of perturbation theory. As for the original black box, one can show that there is no possible symmetry allowing for a perfect cancellation of different contributions to this entry. In the second step, then all allowed neutrino mass matrices with $M_{ee}^{\nu} \equiv 0$ are constructed. It is then easy to show that none of the possible five structures is consistent with oscillation data. One can thus conclude that $M_{ee}^{\nu} \neq 0$ is guaranteed for Majorana neutrinos [6] already today.

The above theorem(s) do not state which mechanism of $0\nu\beta\beta$ decay is the dominant one. Two instructive examples, in which the mass mechanism might indeed not be the dominant contribution to $0\beta\beta$ decay, are therefore discussed next.

2.1. Left-right symmetry

For $0\nu\beta\beta$ decay, with its typical low energy scale of a few MeV, all calculations can be done with the effective Hamiltonian [4]

$$\mathcal{H}_{W}^{CC} = \frac{G_F}{\sqrt{2}} \left\{ J_{\mu L}^{\dagger} J_{\mu L} - \kappa J_{\mu R}^{\dagger} J_{\mu R} + \eta J_{\mu L}^{\dagger} J_{\mu R} - \lambda J_{\mu R}^{\dagger} J_{\mu L} \right\}. \tag{2}$$

Here, $J_{\mu L/a}^{\dagger} = \bar{\nu}_\mu d_a$ and $J_{\mu L} = \bar{\nu}_\mu \nu_\mu$ are the hadronic and leptonic charged currents, $L/R$ stands for $P_{L/R} = \frac{1}{2}(1 \pm \gamma_5)$. $G_F$ is the Fermi constant, $\kappa \simeq \eta \simeq \tan \zeta$, i.e. the mixing angle between the $W_L$ and $W_R$ bosons, and $\lambda \simeq (m_{W_L}/m_{W_R})^2$.

The Hamiltonian of eq. (2) gives rise to the diagrams in fig. [4]. The graphs on the left and the middle represent so-called “long-range” contributions. The graph to the left is due to a product of two $j_{\mu L}$ and corresponds to the mass mechanism of $0\nu\beta\beta$ decay, proportional to $\langle m_\nu \rangle = \sum_i U_{ei}^2 m_\nu$. The graph in the middle is proportional to $\langle \lambda \rangle = \lambda \sum_i U_{ei} V_{ei}$. The graph to the right is proportional to $\langle \xi \rangle = \frac{\lambda^2 + \eta^2 - 2\lambda \eta \left( M_{mix}^2 + M_{\nu}^2 \right)}{\langle m_N \rangle} \langle m_N \rangle$. Here, $\langle \frac{m_\nu}{m_{N}} \rangle = \sum_j V_{ej}^2 \langle m_{N} \rangle$. [4]

Formally, the long-range contribution in LR models are suppressed only by one power of $\lambda/\eta$, compared to the short-range contribution, which is quadratic in $\lambda/\eta$. Many calculations therefore have taken into account only the long-range LR contributions. However, as first pointed out by Mohapatra [9] and confirmed by a detailed calculation of the relevant nuclear matrix elements [8], the short-range contribution can be much more important then the long-range one. This at
first sight contradictory statement can be easily understood. In left-right symmetric models the mixing between the active, left (and light) neutrinos with the heavy, sterile ones can be estimated “á la seesaw” to be very roughly of the order \( \sum \) mixing between the active, left (and light) neutrinos with the heavy, sterile ones can be estimated first sight contradictive statement can be easily understood. In left-right symmetric models the mixing between the active, left (and light) neutrinos with the heavy, sterile ones can be estimated “á la seesaw” to be very roughly of the order \( \sum \). Then, with a limit of \( \langle \lambda \rangle \lesssim 8 \times 10^{-7} \) one gets \( m_{W_R} \gtrsim 1.1 m_{W_L} (\frac{m_D}{1\text{TeV}})^{1/4} (\frac{M_\nu}{1\text{TeV}})^{-1/4} \). In the short range contribution, although some cancellation of terms in \( \langle m_N \rangle \) might occur, no such strong supression is expected. From \( \mathcal{N} \) and assuming a limit on the \( ^{76}\text{Ge} \) half-live of \( T_{1/2}^{0\nu\beta\beta} \gtrsim 1.2 \times 10^{25} \text{ys} \) a limit of

\[
m_{W_R} \gtrsim 1.3 (\frac{\langle m_N \rangle}{1\text{TeV}})^{-1/4} \text{ TeV}
\]

can then be derived. Note that the limit disappears as \( \langle m_N \rangle \) goes to infinity, as it should. Note also that the uncertainty in this limit due to the uncertainty in the nuclear matrix element calculation scales only as \( \Delta m_{W_R} \sim (\Delta \mathcal{M})^{-1/4} \) and thus is quite insensitive to the details of the nuclear model.

2.2. \( \mathcal{R} \)-parity violation

In the standard model lepton number is conserved, because there is (a) no right-handed neutrino and (b) only one Higgs doublet with \( L = 0 \). In supersymmetric models, on the other hand, if one does not assume lepton number conservation a priori, one can write down the following (trilinear) lepton number violating terms

\[
\mathcal{L}_{\mathcal{R}P} = - \lambda'_{ijk} \left[ (\bar{u}_L \tilde{d}_R)^j \cdot \left( e^c_{R} \right)_{i} (\tilde{d}_R)^k + (\bar{e}_L \tilde{\nu}_L)^j_i (d_R)^k \cdot \left( \tilde{u}^*_L \right)_{i} - (\bar{d}_L \tilde{d}_L)^j (d_R)^k \cdot \left( \tilde{e}^*_L \right)_{i} \right] + h.c.
\]

Here, the tilde indicates the scalar superpartners of the usual quarks and leptons. A product of two of the terms in eq. \( \lambda' \), together with an MSSM neutralino and/or gluino interaction lead to \( 0\nu\beta\beta \) decay diagrams without any virtual neutrinos being exchanged, as first pointed out in \( \mathcal{H} \). A dedicated calculation of all diagrams \( \mathcal{H} \), together with a limit of \( T_{1/2}^{0\nu\beta\beta} \gtrsim 1.2 \times 10^{25} \text{ys} \) for \( ^{76}\text{Ge} \) leads to

\[
\lambda_{111} \leq 3.2 \times 10^{-4} (\frac{m_T}{100\text{GeV}})^2 (\frac{m_T}{100\text{GeV}})^{1/2}.
\]

It is interesting to note, that such a small value of \( \lambda_{111} \) generates at 1-loop level an entry in the Majorana neutrino mass matrix of \( M^\nu_{ee} \simeq 10^{-6} \text{eV} \).
3. Neutrino oscillations and $0\nu\beta\beta$ decay

If the mass mechanism is dominant, the $0\nu\beta\beta$ decay half-life is proportional to the (square of the) $(\nu_e - \nu_\mu)$ element of the Majorana neutrino mass matrix. For three generations of light neutrinos, this so-called “effective Majorana” mass can be expressed as:

$$M_{\nu e}^\nu \equiv \langle m_\nu \rangle = c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 e^{i\alpha} m_2 + s_{13}^2 e^{i\beta} m_3$$  \hspace{1cm} (6)

Eq. (6) contains a priori seven unknowns: Three mass eigenstates, two angles and two phases.

With the help of data from neutrino oscillation experiments, one can trade two mass eigenstates for the observed $\Delta m_{\text{Atm}}^2$ and $\Delta m_\odot^2$ and relate the two angles to the solar ($\theta_\odot$) and reactor angle ($\theta_R$). For the case of normal hierarchy, $m_{\nu_1} \leq m_{\nu_2} \leq m_{\nu_3}$, eq. (6) can then be written as

$$\langle m_\nu \rangle = c_\odot^2 c_R^2 m_{\nu_1} + s_\odot^2 c_R e^{i\alpha} \sqrt{m_{\nu_1}^2 + \Delta m_\odot^2 + s_R^2 e^{i\beta} \sqrt{m_{\nu_1}^2 + \Delta m_\odot^2 + \Delta m_{\text{Atm}}^2}},$$  \hspace{1cm} (7)

while for the case of inverse hierarchy, $m_{\nu_3} \leq m_{\nu_1} \leq m_{\nu_2}$, it is given by

$$\langle m_\nu \rangle = c_\odot^2 c_R^2 \sqrt{m_{\nu_3}^2 - \Delta m_\odot^2 + \Delta m_{\text{Atm}}^2} + s_\odot^2 c_R e^{i\alpha} \sqrt{m_{\nu_3}^2 + \Delta m_{\text{Atm}}^2 + s_R^2 e^{i\beta} m_{\nu_3}}$$  \hspace{1cm} (8)

**Figure 2.** Allowed range of $\langle m_\nu \rangle$ as a function of the lightest neutrino mass eigenvalue. To the left normal hierarchy, to the right inverse hierarchy. To calculate the allowed range of $\langle m_\nu \rangle$ the 3 $\sigma$ c.l. intervals on the oscillation parameters have been used [13], except for the case of normal hierarchy, for which 3 different cases for the upper limit on $s_R^2$ are shown. These are $s_R^2 \leq 0.04$ (light blue), $s_R^2 \leq 0.025$ (medium blue), $s_R^2 \leq 0.005$ (darker blue).

Fig. 2 shows the resulting allowed range of $\langle m_\nu \rangle$ for both, normal and inverse hierarchy, taking into account the latest results from a global fit to all neutrino oscillation data [13]. The lower limit on $\langle m_\nu \rangle$, which appears in the case of inverse hierarchy, can be understood trivially. For $m_{\nu_3} = 0$ and $\alpha = \pi$ eq. (8) reads approximately

$$\langle m_\nu \rangle \simeq c_\odot^2 (c_\odot^2 - s_\odot^2) \sqrt{\Delta m_{\text{Atm}}^2}.$$  \hspace{1cm} (9)

Thus, as soon as data tells us that $s_\odot^2 \leq \frac{1}{3}$, exact cancellation is no longer a possibility. This statement remains true for any finite $m_{\nu_3}$, simply because $s_R^2 < \cos(2\theta_\odot)$ is guaranteed by data nowadays. Fig. 2 shows how this lower limit evolves with future data from neutrino oscillation experiments. A possible future smaller upper bound on $s_R^2$ would make it easier for $0\nu\beta\beta$ decay experiments to rule out inverse hierarchy.
There is no such simple quantitative lower limit for the case of normal hierarchy. Fig. (2), to the left, aims at demonstrating this point. If \( m_{\nu_1} \equiv 0 \), a lower limit appears if

\[
s^2_R \leq \frac{\sqrt{\Delta m^2_{\odot} s^2_{\odot}}}{\sqrt{\Delta m^2_{\odot} + \Delta m^2_{\text{Atm}}} + \sqrt{\Delta m^2_{\odot} s^2_{\odot}}} \sim 0.034
\]

However, from this superficial look at the data at the point \( m_{\nu_1} = \tan^2 \theta_{\odot} m_{\nu_2} \) exact cancellation yielding \( \langle m_\nu \rangle \equiv 0 \) seems possible. However, this is equivalent to saying \( M_{ee} = 0 \) and it is exactly this possibility which is ruled out by the “extended black box” theorem [6].

**Figure 4.** Summary of experimental data on the absolute neutrino mass scale and the half-life of \( ^{76}\text{Ge} \, 0\nu\beta\beta \) decay. For discussion see text.

In fig. (4) finally a summary of the current status of various experimental attempts on measuring/limiting the absolute scale of neutrino masses is given. The light and darker blue areas are allowed for the \( 0\nu\beta\beta \) decay half live of \( ^{76}\text{Ge} \) for normal and inverse hierarchy, calculated with matrix elements from [14]. Note, that matrix elements from [15] lead to slightly larger half-lives, see also the discussion in [3]. The green area labeled “Mainz & Troitsk” shows the latest upper limits derived from endpoint measurements in \(^3\text{H} \) decay [16, 17]. The bar labeled
“KATRIN” represents the expected sensitivity of the next generation $^3$H experiment KATRIN [18]. Note, that KATRIN claims a final sensitivity of $m_{\nu_e} \sim 0.2$ eV ($@ 90 \%$ c.l.) or a 5 $\sigma$ discovery threshold of $m_{\nu_e} \sim 0.35$ eV. Various limits on the absolute neutrino mass scale from cosmology have been published recently, derived from CMB data combined with information from large scale structure surveys. For three generations of neutrinos numbers ranging from $\sum_i m_{\nu_i} \sim 0.4 - 2.0$ eV, depending on input and bias, have been published. For a detailed discussion see, for example, the review [19]. The horizontal gray band indicates the range of the finite $T_{1/2}^{0\nu\beta\beta}$ claimed by some members of the Heidelberg-Moscow experiment [20]. Note that this result is highly controversial, see for example the discussion by Barabash in [2]. The vertical red lines indicate the sensitivity of two future Ge experiments. GERDA [21] is currently in phase I, phase II is funded. In the future Majorana [22] and/or GERDA phase III can test the range allowed by inverse hierarchy.

4. Conclusions
Lower limits on the $0\nu\beta\beta$ decay half live can be used to constrain various particle physics parameters. However, from the point of view of particle physics it would be interesting to determine the dominant contribution to $0\nu\beta\beta$ decay. Very little work has been done in this direction. Angular correlations between the electrons [4] or a comparative study of $0\nu\beta^-\beta^-$ and $0\nu\beta^+/EC$ decay [23] might be able to disentangle left-left and left-right-handed combinations of currents (of the long range type). However, other contributions to $0\nu\beta\beta$ decay possibly exist and ultimately it might be that only a combination of various different pieces of experimental data will provide the correct and final answer.

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