1. Introduction

In the last years the Polyakov-loop-extended Nambu–Jona-Lasinio (PNJL) model was widely used in the study of thermodynamics and the phase diagram of hot and dense matter. Results of this research are expected to play an important role in our understanding of the evolution of the early universe and physics of heavy ion collisions at relativistic energies. This improved field theoretical model is fundamental for interpreting the lattice QCD data and extrapolating into regions not yet accessible for lattice simulations.
An attractive property of the PNJL model is the synthesis of the Polyakov loop dynamics with the Nambu–Jona-Lasinio model, combining the two principal nonperturbative features of low-energy QCD: confinement and spontaneous chiral symmetry breaking. A particular feature of this model is that one can uniquely determine the coupling between the chiral condensate, which is an order parameter of the chiral phase transition when $m_q \rightarrow 0$, and the Polyakov loop, which is the order parameter for the deconfinement phase transition in the limit $m_q \rightarrow \infty$. The model is remarkably successful in reproducing lattice data on the QCD thermodynamics.

However, the choice of the parameter set as well as regularization of integrals, as noted by several authors, is a nontrivial question. As it is well known, the order of the phase transition in the $(T, \mu)$ plane is sensitive to the parameter choice. It was already noted that one can choose different sets of parameters which allow for a first order phase transition, giving a reasonable fit to physics observables in the vacuum but predicting different physical scenarios at finite temperature $T$ and chemical potential $\mu$. In addition, recently new lattice data for the pure gluon QCD sector defining the effective Polyakov loop potential have been obtained which differ noticeably from the old data.

In this paper we investigate how the input information from lattice QCD and the used forms of the effective potential influence general properties of thermodynamics at finite temperature $T$ and baryon chemical potential $\mu$. After introduction, in Sect. 2 we consider the polynomial and logarithmic parameterizations of the Polyakov loop effective potential for the new and old pure gluon lattice data within the two-flavor PNJL model. Independent of the temperature, the model parameters defined by properties of quarks and mesons are presented in Sect. 3. Comparative study of the thermodynamics and phase structure, their dependence on the lattice input and used parametrization are considered in Sect. 4 at finite $T$ and $\mu$. The last Section summarizes the obtained results.

1.1. The Nambu–Jona-Lasinio model with Polyakov-loop

The deconfinement in the pure $SU(N_c)$ gauge theory can be simulated by introducing an effective potential for a complex Polyakov loop field. The PNJL Lagrangian employed in this work is

$$\mathcal{L}_{\text{PNJL}} = \bar{q} (i\gamma_{\mu} D^\mu - \hat{m}_0) q + G \left[ (\bar{q}q)^2 + (\bar{q}i\gamma_5 \bar{\tau}q)^2 \right] - \mathcal{U} (\Phi[A], \bar{\Phi}[A]; T).$$

Here, a local chirally symmetric scalar-pseudoscalar four-point interaction of quark fields $q, \bar{q}$ is introduced with an effective coupling strength $G$, $\bar{\tau}$ is the vector of Pauli matrices in flavor space, $\hat{m}_0$ is the diagonal matrix of the 2-flavor current quark masses, $\hat{m}_0 = \text{diag}(m_u^0, m_d^0)$, $m_u^0 = m_d^0 = m_0$.

The quark fields are coupled to the gauge field $A^\mu$ through the covariant derivative $D^\mu = \partial^\mu - iA^\mu$. The gauge coupling $g$ is conveniently absorbed in the definition $A^\mu(x) = g A_\mu^\mu \frac{\lambda_0}{2}$ where $A_\mu^\mu$ is the $SU(3)$ gauge field and $\lambda_0$ is the Gell-Mann ma-
traces. The gauge field is taken in the Polyakov gauge
\[ A^\mu = \delta_0^\mu A^0 = -i \delta_4^\mu A_4. \]
The field \( \Phi \) is determined by the trace of the Polyakov loop \( L(\bar{x}) \) and its conjugate \( \Phi[A] = \frac{1}{N_c} \text{Tr}_c L(\bar{x}) \), \( \Phi[A] = \frac{1}{N_c} \text{Tr}_c L^\dagger(\bar{x}) \), where \( L(\bar{x}) = \mathcal{P} \exp \left[ i \int_0^\beta d\tau A_4(\bar{x}, \tau) \right] \), \( \beta = 1/T \) being the inverse temperature. In the absence of quarks, we have \( \Phi = \bar{\Phi} \) and the Polyakov loop servers as an order parameter for deconfinement.

The gauge sector of the Lagrangian density (1) is described by an effective potential \( U(\Phi[A], \bar{\Phi}[A]; T) \). The effective potential must satisfy the \( Z(3) \) center symmetry. In accordance with the underlying \( Z(3) \) symmetry, one can choose the following general polynomial form:
\[ U(\Phi, \bar{\Phi}; T) = -b_2(T) \sigma^2 - b_3(T) \sigma^3 + b_4(T) \sigma^4, \]
where \( \sigma = \Phi \Phi + \bar{\Phi}\bar{\Phi} \) and \( b_2(T) = a_0 + a_1 \left( \frac{T_0}{T} \right) + a_2 \left( \frac{T_0}{T} \right)^2 + a_3 \left( \frac{T_0}{T} \right)^3 \). The \( Z(3) \) symmetry leads to some freedom in the choice of the effective potential form. Along with the simplest polynomial form, Eq. (2), there exists an expression with a logarithm in place of the higher order polynomial terms in \( \bar{\Phi}, \Phi \). In the logarithmic form the potential is
\[ U(\Phi, \bar{\Phi}; T) = -b_2(T) \sigma^2 - b_3(T) \sigma^3 + b_4(T) \sigma^4, \]
The pressure of a pure-gauge system is given by \( p = -U \).

Pure-gluon lattice data from Ref. [22] are traditionally used to find the parameter set for both forms of the effective potential [33]. In contrast, our work is based on new gluon lattice data [21] looking for a new potential to fit the lattice pressure. In finding the potential parameters the following conditions should be satisfied: \( \Phi \to 1 \) and \( p/T^4 \to 1.75 \), when \( T \to \infty \). As immediately follows from these conditions, \( a_0 = 3.51 \) for the logarithmic potential and the constraint \( 1.75 = a_0/2 + b_3/3 - b_4/4 \) for the polynomial potential. Minimizing \( U(\Phi, \bar{\Phi}; T) \) with respect to variation of \( \Phi \) and taking into account that \( \Phi = \bar{\Phi} \) at \( \mu = 0 \), we can find parameters using the method of least mean squared deviations. Thus, for the critical temperature \( T_0 = 270 \text{ MeV} \) the following parameter sets were obtained (see Table 1 and Table 2).

In Fig. 1, old and new lattice gluon data are compared together with the results of their approximations. As is seen the lattice results differ by about 10% at \( T/T_0 \geq 2 \). New data are plotted by circles but the density of measured points is so high that
the results look like a shaded band. Both polynomial and logarithmic forms are in nice agreement with the data and it is hard to distinguish them from each other.

In general, the parameter \( T_0 \) depends on the number of active flavors and the chemical potential \( \mu \). In the pure gauge sector \( T_0 = 0.27 \) GeV was used \(^{21}\). The effective potential for both sets of parameters at \( T = 0, 0.2, 0.32, 0.54 \) GeV is shown in Fig. 2. Both sets describe quite satisfactorily the Polyakov loop as a function of

Table 1. Parameters of the effective potential \( \mathcal{U}[A] \) with the polynomial form.

|      | \( a_0 \) | \( a_1 \) | \( a_2 \) | \( a_3 \) | \( a_4 \) |
|------|----------|----------|----------|----------|----------|
| old data \(^{22}\) | 6.75 | -1.95 | 2.625 | -7.44 | 0.75 | 7.5 |
| new data \(^{21}\) | 6.47 | -4.62 | 7.95 | -9.09 | 1.03 | 7.32 |

Table 2. Parameters of the effective potential \( \mathcal{U}(\Phi, \bar{\Phi}; T) \) in the logarithmic form.

|      | \( \tilde{a}_0 \) | \( \tilde{a}_1 \) | \( \tilde{a}_2 \) | \( \tilde{b}_3 \) |
|------|-----------------|-----------------|-----------------|-----------------|
| old data \(^{22}\) | 3.51 | -2.47 | 15.2 | -1.75 |
| new data \(^{21}\) | 3.51 | -5.121 | 20.99 | -2.09 |

Fig. 1. Scaled pressure in the pure gauge sector as function of scaled temperature. The old\(^{22}\) and new\(^{21}\) lattice data are plotted by circles and triangles, respectively. Solid lines correspond to polynomial form of potential and dashed lines correspond to the logarithm form.
Fig. 2. The Polyakov loop effective potential $U$ as a function of $\Phi$ for various values of temperature for old (top) and new (bottom) sets of parameters. Left panel corresponds to the polynomial form and right panel corresponds to the logarithm form of the potential.

temperature. In accordance with the $Z(3)$ center symmetry, the following properties of the effective potential $U(\Phi, \bar{\Phi}; T)$ are seen. At low temperature $U(\Phi, \bar{\Phi}; T)$ has a single minimum at $\Phi = 0$ (a confinement phase); the effective potential is getting flat for the critical temperature $T = T_0$ and above critical temperature (a deconfinement phase) a second minimum arises at nonzero $\Phi$, as a consequence of $Z(3)$ symmetry breaking; in the $T \to \infty$ limit, $\Phi \to 1$ (see Fig. 2). One should note that after the second minimum the logarithmic potential $\Phi$ increases faster than the polynomial one forming a more distinct minimum. With the introduction of quarks the critical temperature goes down. The range of applicability of this model is $T \sim < 2.5 T_c$ since at higher temperature transverse gluons start to contribute significantly.

The Polyakov loop is compared with the lattice results in Fig. 3. In reasonable agreement of both forms with the lattice data, the new parameter set predicts slightly lower values of the field $\Phi$ because the pressure for new data is also below the old one.

2. Quarks and light mesons in the PNJL model

The grand potential density for the PNJL ($N_f = 2$) model in the mean-field approximation is given by the following equation.
Fig. 3. Temperature dependence of the Polyakov loop $\Phi$ for polynomial (2) (left) and logarithmic (4) (right) forms. Lattice data are from $^{25}$ Solid and dashed lines correspond to new and old parameter sets, respectively.

$$\Omega(\Phi, \bar{\Phi}, m, T, \mu) = U(\Phi, \bar{\Phi}; T) + G\langle \bar{q}q \rangle^2 + \Omega_q,$$

where the quark term is

$$\Omega_q = -2N_c N_f \int \frac{d^3p}{(2\pi)^3} E_p - 2N_f T \int \frac{d^3p}{(2\pi)^3} \left[ \ln N^+_\Phi(E_p) + \ln N^-_\Phi(E_p) \right],$$

and the functions are

$$N^+_\Phi(E_p) = \left[ 1 + 3\left( \Phi + \Phi e^{-\beta E_p^+} \right) e^{-\beta E_p^+} + e^{-3\beta E_p^+} \right],$$

$$N^-_\Phi(E_p) = \left[ 1 + 3\left( \bar{\Phi} + \bar{\Phi} e^{-\beta E_p^-} \right) e^{-\beta E_p^-} + e^{-3\beta E_p^-} \right],$$

where $E_p = \sqrt{p^2 + m^2}$ is the quasiparticle energy of the quark; $E_p^\pm = E_p \mp \mu$, the upper sign applying for fermions and the lower sign for antiparticles.

Since NJL-type models are nonrenormalizable, it is necessary to introduce a regularization, e.g., by a cutoff $\Lambda$ in the momentum integration. Following $^{9}$, we use in this study the three-dimensional momentum cutoff $\Lambda$ for vacuum terms and extend this integration to infinity for the matter contributions given by the second term of Eq. 7. A comprehensive study of the differences between the two regularization procedures (with and without cutoff on the quark momentum states at finite temperature) was performed in $^{14}$.

In the mean-field approximation, we can obtain the constituent quark mass $m$ from the condition that the thermodynamic potential $^{6}$ will have a minimum with respect to variation of this parameter, $\partial \Omega/\partial m = 0$. This condition is equivalent to the gap equation $^{9,26}$

$$m = m_0 - 2G \langle \bar{q}q \rangle,$$
where the quark condensate is defined as \( \langle \bar{q}q \rangle = \frac{\partial \Omega}{\partial m_0} \). For the mass gap equation we get

\[ m = m_0 + 4G N_c N_f \int_{\Lambda} \frac{d^3p}{(2\pi)^3} \frac{m}{E_p} \left[ 1 - f^+ - f^- \right] \tag{11} \]

with the modified Fermi-Dirac distribution functions for fermions and antifermions

\[ f^+ = \left[ \left( \Phi + 2\Phi e^{-\beta E_p} \right) e^{-\beta E_p} + e^{-3\beta E_p} \right] / N_\Phi^+ (E_p) , \tag{12} \]

\[ f^- = \left[ \left( \bar{\Phi} + 2\Phi e^{-\beta E_p} \right) e^{-\beta E_p} + e^{-3\beta E_p} \right] / N_\Phi^- (E_p) . \tag{13} \]

Moreover, for PNJL calculations we should find the values of \( \Phi \) and \( \bar{\Phi} \) by minimizing \( \Omega \) with respect to \( \Phi \) and \( \bar{\Phi} \) at given \( T \) and \( \mu \). One should note that if \( \Phi \to 1 \), the expressions Eqs. (12), (13) reduce to the standard NJL model.

For a self-consistent description of the particle spectrum in the mean-field approximation, the meson correlations have to be taken into consideration. These correlations are related to the polarization operator of constituent fields. For scalar and pseudoscalar particles the polarization operators are represented by loop integrals

\[ \Pi^{PP}_{ab}(P^2) = \int \frac{d^4p}{(2\pi)^4} \text{Tr} \left[ i\gamma_5 \tau^a S(p + P)i\gamma_5 \tau^b S(p) \right] , \tag{14} \]

\[ \Pi^{SS}_{ab}(P^2) = \int \frac{d^4p}{(2\pi)^4} \text{Tr} \left[ S(p + P)S(p) \right] , \tag{15} \]

where \( S(p) \) is the quark propagator and the operation \( \text{Tr} \) is taken over Dirac, flavor and color indices of quark fields.

From the point of view of the polarization operators, the pseudoscalar (\( \pi \)) and scalar (\( \sigma \)) meson masses can be defined by the condition that for \( P^2 = M_\pi^2 (M_\sigma^2) \) the corresponding polarization operator \( \Pi^{PP}(M_\pi^2) (\Pi^{SS}(M_\sigma^2)) \) leads to a bound state pole in the corresponding meson correlation function. For mesons at rest (\( P = 0 \)) in the medium, these conditions correspond to the equations

\[ 1 + 16G N_c N_f \int \frac{d^3p}{(2\pi)^3} \frac{E_p}{M_\pi^2 - 4E_p^2} \left( 1 - f^+ - f^- \right) = 0, \tag{16} \]

\[ 1 + 16G N_c N_f \int \frac{d^3p}{(2\pi)^3} \frac{1}{E_p} \left( \frac{E_p^2 - m_\sigma^2}{M_\sigma^2 - 4E_p^2} \right) \left( 1 - f^+ - f^- \right) = 0. \tag{17} \]

In order to solve Eqs. (10), (16) and (17), a set of model parameters has to be determined: the above-mentioned cutoff parameter \( \Lambda \), the current quark mass \( m_0 \) (in the chiral limit \( m_0 = 0 \)) and the coupling constant \( G \). These parameters are fixed at \( T = 0 \) to reproduce physical quantities: the pion mass \( M_\pi = 0.139 \) GeV, the pion decay constant \( F_\pi = 0.092 \) GeV and the quark condensate \( \langle \bar{q}q \rangle^{1/3} = -250 \) MeV. The obtained parameters are the same as in those obtained earlier in our papers and are shown in Table 3. Meson masses obtained as solutions of the
Table 3. The set of model parameters reproducing observable quantities (in brackets) and the chiral condensate \( \langle \bar{q}q \rangle^{1/3} = -250 \) MeV.

| \( m_0 \) [MeV] | \( \Lambda \) [GeV] | \( G \) [GeV]^{-2} | \( F_\pi \) [GeV] | \( M_\pi \) [GeV] |
|-----------------|-----------------|----------------|-------------|----------------|
| 5.5             | 0.639           | 5.227          | (0.092)     | (0.139)        |

gap-equation (10) and Eqs. (16), (17) at nonzero \( T \) are presented in Fig. 4 for two parameter sets.

Fig. 4. Temperature dependence of the masses \( m_q \), \( M_\pi \) and \( M_\sigma \) masses at \( \mu = 0 \) GeV for polynomial (left panel) and logarithmic (right panel) forms of potential. The PNJL results for new and old parameter sets are given by the solid and dashed lines, respectively. The Mott temperatures for both parameter sets are plotted by vertical lines.

The temperature modification of the quasiparticle properties is clearly seen in this figure. Up to the Mott temperature \( T_{Mott} \), defined as \( M_\pi(T_{Mott}) = 2m_q(T_{Mott}) \), the \( \sigma \) mass practically follows the behavior of \( 2m_q(T) \) with a drop toward the pion mass signaling partial chiral symmetry restoration. At \( T > T_{Mott} \) the masses of chiral partners become equal to each other, \( M_\sigma \approx M_\pi \), and then both masses monotonically increase with temperature. Below the Mott temperature, the pion mass remains practically constant. It justifies that \( T_{Mott} \) is a little bit lower for the new parameterization than for the old one when the polynomial form is used, while both values of \( T_{Mott} \) coincide in the case of logarithmic parametrization.

3. Thermodynamics of the PNJL models

The thermodynamics of particles is described in terms of the grand canonical ensemble which is related to the Hamiltonian \( H \) as follows:

\[
e^{-\beta V\Omega} = \text{Tr} \, e^{-\beta(H-\mu N)},
\]

(18)
where \( N \) is the particle number operator, \( \mu \) is the quark chemical potential and the operator \( \text{Tr} \) is taken over momenta as well as color, flavor and Dirac indices. If \( \Omega \) is known, the basic thermodynamic quantities - the pressure \( p \), the energy density \( \varepsilon \), the entropy density \( s \), the density of quark number \( n \) and the specific heat \( c_v \) - can be defined as follows:

\[
p = -\frac{\Omega}{V}, \tag{19}
\]

\[
s = - \left( \frac{\partial \Omega}{\partial T} \right)_\mu, \tag{20}
\]

\[
\varepsilon = -p + Ts + \mu n, \tag{21}
\]

\[
n = - \left( \frac{\partial \Omega}{\partial \mu} \right)_T, \tag{22}
\]

\[
c_v = T \left( \frac{\partial s}{\partial T} \right)_\mu. \tag{23}
\]

The thermodynamic potential in equilibrium corresponds to a global minimum
with respect to variations of the order parameter
\[
\frac{\partial \Omega(T, \mu, m)}{\partial m} = 0, \quad \frac{\partial^2 \Omega(T, \mu, m)}{\partial m^2} \geq 0.
\] (24)

All these relations (19), (20), (21), (22), (23) describe the thermodynamics of the system. For the considered models the thermodynamic potentials are defined from Eq. (6). From this equation we can read off the vacuum part
\[
\Omega_{\text{vac}} = \frac{(m - m_0)^2}{4G} - 2N_c N_f \int \frac{d^3 p}{(2\pi)^3} E_p.
\] (25)

This quantity does not vanish as \( T \to 0 \) and \( \mu \to 0 \). Therefore, in order to obtain the physical thermodynamical potential which corresponds to vanishing pressure and energy density at \( (T, \mu) = (0, 0) \), one has to renormalize the thermodynamic potential by subtracting its vacuum expression (25). This corresponds to the following definition of the physical pressure
\[
\frac{p}{T^4} = \frac{p(T, \mu, m) - p(0, 0, m)}{T^4}.
\] (26)

With increasing temperature the pressure has to reach the Stefan-Boltzmann limit \(^\text{14}\) which in the chiral limit for the PNJL model is given as
\[
\frac{p_{SB}}{T^4} = (N_c^2 - 1) \frac{\pi^2}{45} + N_c N_f \frac{7\pi^2}{180} \simeq 4.053,
\] (27)
where the first and second terms correspond to gluons and quarks, respectively.

If the regularization \( \Lambda = 0.639 \) is used, the \( T \)-behaviour of the thermodynamic quantities considered is roughly the same while their absolute values are noticeably lower, being far from the Stefan-Boltzmann limit \(^\text{14}\).

![Graph](image-url)

Fig. 6. Comparison of the scaled quark number density as a function of temperature at \( \mu = 0.8 \) and 0.6 GeV. Lattice data points are from Ref. 31.
Effects of model parameters in thermodynamics of the PNJL model

Within the PNJL model with \( \Lambda \to \infty \) (we can use \( \Lambda \to \infty \) because most of the integrals in the PNJL are convergent) the reduced pressure and energy density exhibit reasonable behavior consistent with the recent lattice QCD results for the vanishing chemical potential \(^3\) (see Fig. 5) keeping in mind that the \( m_{PS}/m_V \) ratio in lattice calculations is still far from that for physical masses \( m_{PS}/m_V \sim 0.2 \). As another example of thermodynamic characteristics, the temperature dependence of the reduced quark number density \( n_q/T^3 \) (see Eq. (22)) is presented in Fig. 6. Model results are in good agreement with the lattice data for both values of the chemical potential considered. The logarithmic approximation of the effective potential \( \mathcal{U} \) seems to describe lattice data better than the polynomial one.

### 3.1. Phase diagram and the CEP

Within NJL-like models there are several characteristic temperatures. The parameter \( T_0 \) entering into the effective potential \(^4\) of the PNJL model has been noted above. For \( \pi \)-mesons, the Mott temperature \( T_{\text{Mott}} \) is provided by the condition \( M_\pi(T_{\text{Mott}}) = 2m_q(T_{\text{Mott}}) \). Above \( T_{\text{Mott}} \) the pion dissociates into a quark and antiquark and does not exist as a bound state. Similarly the \( \sigma \) meson dissociation temperature \( T_\sigma^d \) is given by the equation \( M_\sigma(T_\sigma^d) = 2M_\pi(T_\sigma^d) \). Other characteristics of phase transitions are the pseudo-critical temperature for the chiral crossover \( T_\chi \), defined by the maximum of \( \partial \langle \bar{q}q \rangle /\partial T \), and the pseudo-critical temperature for the crossover deconfinement transition \( T_p \) that can be found from the maximum of \( \partial \Phi /\partial T \). Their difference is less than 0.013 but it increases with decreasing \( T_0 \). The third quantity is \( T_c \) assumed to equal \( T_\chi \). But in the PNJL model it is higher than the lattice result \( T_c \sim 192 \text{ GeV} \). Thus, it was suggested to define \( T_c \) as an average of two transition temperatures \( T_\chi \) and \( T_p \).

All these quantities obtained at \( \mu = 0 \) are presented in Table 4.

| Polytropic form of potential | \( T_\chi \) | \( T_p \) | \( T_c \) | \( T_{\text{Mott}} \) | \( T_\sigma^d \) |
|-------------------------------|-------------|-------------|-------------|----------------|-------------|
| Polynomial form of potential  | new         | 0.2455      | 0.2335      | 0.2395         | 0.259        | 0.247       |
|                               | old         | 0.2575      | 0.2485      | 0.253          | 0.27         | 0.257       |
| Logarithmic form of potential | new         | 0.2305      | 0.2295      | 0.23           | 0.264        | 0.2645      |
|                               | old         | 0.2345      | 0.2335      | 0.234          | 0.2645       | 0.252       |

To define the crossover transition line, the chiral condensate \( \langle \bar{q}q \rangle \) and the Polyakov loop \( \Phi \) were used as the order parameters. As shown in Fig. 7 these quantities are the temperature-dependent functions and demonstrate a quick change near the transition line which essentially depends on the chemical potential \( \mu \). The position of this line is defined by local maximum of \( d \langle \bar{q}q \rangle /dT = 0 \) and \( d\Phi/dT = 0 \). To find the first order transition line, it is convenient to introduce the baryon number susceptibility \( \chi_q = \left. \frac{dn_q}{d\mu} \right|_{T=\text{const}} \). The first order phase transition ends just at point...
where $\chi_q$ has a pronounced maximum and this point is called critical endpoint (CEP) where the phase transition of the second order. At $T \geq T_{CEP}$ the baryon number susceptibility has a sharp rise and it can be considered as the presence of an ideal gas of weakly interacting quarks.

![Temperature dependence of the chiral condensate and Polyakov loop potential with the new set of parameters within the PNJL model. Solid, dashed and dot-dashed lines are calculated for $\mu = 0, \mu = \mu_{CEP}$ and $\mu > \mu_{CEP}$, respectively.](image)

The behavior of the baryon number susceptibility $\chi_q$ as a function of the chemical potential for three different temperatures around the CEP is presented in Fig. 8. For $T < T_{CEP}, \mu > \mu_{CEP}$ we have a phase transition of the first order with clear discontinuity; for $T = T_{CEP}$ the susceptibility $\chi_q$ diverges at $\mu = \mu_{CEP}$; for $T > T_{CEP}$ the discontinuity at the transition line disappears and we observe crossover type of the phase transition. The polynomial and logarithmic approximations of the Polyakov loop predict very similar results. Similar behavior exhibits also the specific heat $c_v$.

As is seen from Fig. 8 both models show the CEP at the temperature $T_{CEP} = T_\chi$ below which the chiral phase transition is of the first order. At this point ($T_{CEP}, \mu_{CEP}$) the phase transition changes from the first order to crossover type. At this point the second order transition is present.

The phase diagram in the ($T, \mu$) plane is presented in Fig. 9. Within the PNJL model the positions of the critical endpoints ($T_{CEP}, \mu_{CEP}$) are (0.118,0.3166), (0.11,0.3192) for the logarithmic form and (0.10,0.3175),(0.09,0.322) for the polynomial form, where the first pair of numbers correspond to the new data set and the second one is for the old data set (in GeV). As was noted in Ref. 14, critical properties of observables are significantly influenced by the chosen parameter set and regularization procedure. As follows from Fig. 9 the substitution of the new basic lattice data with using the polynomial and the logarithmic forms for the $U$ approximation influences more significantly the chemical potential of the critical endpoint $\mu_{CEP}$ rather than its temperature $T_{CEP}$.
Effects of model parameters in thermodynamics of the PNJL model

4. Summary and conclusions

We have considered the PNJL (\(N_c = 3, N_f = 2\)) model and investigated its phase structure at finite \(T\) and \(\mu\). Two different sets of parameters based on the old and new lattice data for the pure gluon sector were used and for each of these sets the two different parameterizations of the effective potential \(U(\Phi, \bar{\Phi}, T)\) - polynomial and logarithmic - were applied. The thermodynamics for all the developed versions of the PNJL model was studied and compared with the available lattice data. Consideration of different thermodynamic observables like pressure and energy density, their \(T\) and \(\mu\) behavior as well as the quark number density serves as an important probe of the model. We found that in spite of a noticeable disagreement between the old and new original lattice data, the effective gluon potentials \(U\) are quite close to each other and a larger difference is due to the form of their approximation: the
The logarithmic form predicts a more distinct and narrower minimum at high $T$.

The model qualitatively reproduces both $\pi$ and $\sigma$ meson properties in hot, dense quark matter and the rich and complicated phase structure of this medium providing information on the order of phase transitions and the position of critical points. This information depends stronger on the form of the effective potential rather than on the used lattice data set.

Unfortunately, the position of the calculated CEP in the $(T, \mu)$ plane is still far from the predictions of lattice QCD and empirical analysis. Further elaboration of the presented model is needed. In particular, the inclusion of entanglement interactions between quark and gauge degrees of freedom in addition to the covariant derivative in the original PNJL model and the incorporation of explicit diquark degrees of freedom are of great interest. Both modifications reproduce lattice data at $\mu \geq 0$ better than the original PNJL model and influence the position and the nature of the critical endpoint in the $(T, \mu)$ phase diagram. Moreover, with the use of the logarithmic form of the effective potential, these models result in the appearance of new phases.

One should note that we are restricted to the case without diquark correlations and thus possible color superconducting phases at low $T$ and high $\mu$ are ignored. It is attractive also to include into consideration the color superconducting phases and nonlocality of the interaction as well as effects beyond the meanfield.

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