Update of the NNLO PDFs in the 3-, 4-, and 5-flavour schemes

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We report on an update of the next-to-next-to-leading order (NNLO) ABKM09 parton distributions functions. They are obtained with the use of the combined HERA collider Run I inclusive deep-inelastic scattering (DIS) data and the partial NNLO corrections to the heavy quark electro-production taken into account. The value of the strong coupling constant $\alpha_{s}^{\text{NNLO}}(M_Z) = 0.1147(12)$ is obtained. The standard candle cross sections for the Tevatron collider and the LHC estimated with the updated PDFs are provided.

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The parton distribution functions (PDFs) are an essential ingredient for hadron collider phenomenology. With increasing accuracy of the data and expansion of their kinematics, particularly due to the start-up of the LHC, further validation of the PDFs is required. This implies both employment of new, more accurate data sets and theoretical improvement in the data interpretation. These two issues are usually related since higher accuracy of the data often demands to improve the theoretical accuracy as well. Particularly precise PDFs are necessary for the calculation of the standard candle cross sections, as for Higgs-, top quark-, and $W/Z$ hadro-production. To provide a theoretical accuracy of $O(1\%)$ for those processes one has to take into account the next-to-next-to-leading order (NNLO) QCD corrections. Therefore the PDFs extracted in the NNLO QCD approximation are required.

In the following we report an update of the NNLO PDF set of Ref. [1], which are extracted from the data sets including the deep-inelastic-scattering (DIS) data obtained by the HERA collider experiments. The latter give an important constraint on the PDF kinematics relevant for the interpretation of the LHC data to be collected in the first run. The inclusive DIS data obtained by the H1 and ZEUS collaborations in the Run I of the HERA collider [2] were recently merged into one combined data set [3]. This set replaces the separate H1 and ZEUS data sets used in the analysis of Ref. [1]. In order to illuminate the trend of the new data set with respect to the previous version of the fit we first perform a model independent analysis of the combined HERA data on the inclusive structure function $F_2$. To obtain the values of $F_2$ we correct the cross section values of Ref. [3] for the contribution due to the longitudinal structure function $F_L$, which is calculated making use of the NNLO PDFs of Ref. [1]. The data points obtained are separated by bins in the Bjorken variable $x$. For each bin a second-order polynomial in $\ln(Q^2/3 \text{ GeV}^2)$, where $Q^2$ denotes the momentum transferred, is fitted to the data. In such a way we obtain the constant term, slope, and curvature of $F_2$ with respect to $\ln(Q^2)$, depending on $x$. To minimize the bias in the determination of the $F_2$-slope, which often serves as an important indicator of the QCD dynamics, we use only the data in the range of $2 \text{ GeV}^2 < Q^2 < 100 \text{ GeV}^2$ for this fit. With the systematic error

![Figure 1](image.png)

**Figure 1**: The slope on $\ln(Q^2)$ (left panel) and the constant term (right panel) obtained in the model-independent analysis of the combined HERA Run I data for the inclusive structure function $F_2$ in comparison to the predictions of ABKM09 fit [1]. The impact of the correction on the contribution of longitudinal structure function $F_L$ employed to extract the values of $F_2$ from the data on the cross sections is given by the stars.
correlations taken into account the value of $\chi^2/NDP$ obtained is $153/134 = 1.14$. Thus the fluctuations in data are somewhat bigger than the uncertainties quoted in Ref. [3]. On the other hand, in the variant of the fit with the systematic errors combined with the statistical ones in quadrature the value of $\chi^2/NDP = 123/134 = 0.9$ is obtained. Therefore in this case the fluctuations in the data are overestimated. The $x$-dependence of the slope and the constant term in the case of the model-independent fit to the $e^+ p$ data sample are compared to the NNLO predictions based on the ABKM09 PDFs in Fig. 1. The $F_2$-slope is found in reasonable agreement to the predictions, while the constant term in general overshoots it. This happens since the combined HERA data go higher than the separate data of Ref. [2] in general. Particularly this is the case for the H1 experiment. However, at small $x$ the trend is opposite, therefore the new data call for a change in the

$$ q_i(x, \mu_0) = \exp \left[ a \ln x (1 + \beta \ln x) (1 + \gamma_1 x + \gamma_2 x^2 + \gamma_3 x^3) \right] (1 - x)^b $$  \hspace{1cm} (1)

at the boundary factorization scale $\mu_0$. The high-$x$ asymptotics of Eq. (1) is defined by the parameter $\beta$ and its low-$x$ asymptotics – by two parameters, $a$ and $\beta$. If $\beta = 0$, at low $x$ Eq. (1) reproduces the general Regge-motivated ansatz. The parameter $\beta$ describes possible deviations off this ansatz, like in Fig. 1. For the valence quark distributions such a parameter is irrelevant. We tried to fit it only for the gluon and sea distributions. The strange quark sea at small $x$ is poorly constrained by the existing data and is not sensitive to such a modification of the low-$x$ asymptotics. For the gluon distribution the parameter $\beta$ is also comparable to zero within uncertainties. However for the

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{The pulls for the low-$x$ part of the combined HERA data used in the updated version of our QCD fit.}
\end{figure}
non-strange quark sea a small positive value of $\beta$ is preferred by the data, in agreement with the trend given in Fig. 1.

A correct interpretation of the inclusive DIS data at small $x$ implies account of the high-order QCD corrections, which are particularly large at this kinematic. In the fit of Ref. [1] we employ the NNLO corrections to the DIS light parton contribution and the NLO corrections to the heavy-quark DIS production, which are considered in the 3-flavor factorization scheme of $O(\alpha_s^2)$ for the neutral current and of $O(\alpha_s)$ for the charged current. The NNLO corrections to the heavy-quark DIS production are only partially known Ref. [4–6]. In particular, for the case of neutral current they are calculated for the heavy-quark threshold production kinematics using the soft-gluon resummation technique up to terms of $O(\beta_h)$ [8], where $\beta_h$ is the heavy-quark velocity. The threshold contributions are particularly big at small $x$ and $Q^2$ [7] and brings the QCD predictions in better agreement with the existing HERA data on the charm semi-inclusive structure functions [8].

In the updated version of our QCD fit based on the combined HERA data we employ the $O(\alpha_s^2)$ threshold corrections of Ref. [8] in combination with a new, more flexible, PDF shape of Eq. (1). The pulls obtained in this fit for the most accurate part of the HERA data with $Q^2 < 300$ GeV$^2$ are given in Fig. 3. The cut of $Q^2 > 2.5$ GeV$^2$ is also used in our fit and applied to the data in Fig. 3. The value of $\chi^2/NDP$ for this part of the combined HERA data set is 365/202 = 1.8, which is comparable to the value obtained in our model independent fit. In general the data do somewhat overshoot the fit. This is statistically admissible, once we take into account the error correlations. Since the systematic errors in the data are significant, in this case the coherent shift of the data points by the value of the systematic uncertainty does not lead to a big penalty on the value of $\chi^2$. However, this signals a tension at the level of 1σ between the combined HERA data and the fixed target DIS data by NMC collaboration, which spread down to $x \sim 0.01$.

The gluon and non-strange sea distributions at small $x$ obtained in different variants of our fit are compared in Fig. 3. In the updated version the sea distribution at small $x$ moves upward as compared to the one of Ref. [1] due to the impact of the update in the HERA data. The gluon distribution at small $x$ is correspondingly lower than that of Ref. [1], in particular, due to the mo-
momentum conservation imposed on the PDFs in the fit. The NNLO corrections to the heavy-quark electro-production also lead to a suppression of the gluon distribution at small $x$. Nonetheless, despite this decrease, the gluon distribution remains positive down to the relevant factorization scale of $\sim 2$ GeV$^2$. The value of the strong coupling constant $\alpha_s$ is fitted simultaneously with the PDFs. For the value of $\alpha_s(M_Z^2)$ and in the 5-flavor scheme we obtain 0.1147(12). This is by 1σ bigger than the value of 0.1135(14), which was obtained in the analysis of Ref. [1], mainly due to the impact of the update in the HERA data.

The cross sections of the standard candle processes at the energies of the Tevatron collider and at LHC calculated with the use of updated PDFs are given in Table 1. The same cross sections obtained for the previous version of our PDFs [1] are also given in Table 1 for comparison. In both cases we use the PDFs in the 5-flavor scheme, which are derived from the 3-flavor ones (cf. also Ref. [9]) using the matching conditions of Ref. [10]. The $W^-, Z,$ and Higgs cross sections are calculated in the NNLO QCD approximation [11, 12]. The $t\bar{t}$- cross sections are calculated in the NLO approximation with a partial account of NNLO corrections [13]. The impact of the PDF update on the $W$ and $Z$ production cross sections is significant, particularly at Tevatron. This difference can be traced back to the upward shift of the quark distributions at small $x$, due to shift of the HERA data. The Higgs and $t\bar{t}$- production cross sections are quite sensitive to the value of $\alpha_s$. Therefore the variation of these cross sections is defined by the balance in $\alpha_s$ and the gluon distribution changes.

|       | $W^\pm$ (nb) | $Z$ (nb) | $t\bar{t}$ (pb) ($M_t = 173$ GeV) | $H$ (pb) ($M_H = 150$ GeV) |
|-------|--------------|----------|---------------------------------|-----------------|
| Tevatron |              |          |                                 |                 |
| ABKM09 (updt.) | 26.8 ± 0.3   | 7.88 ± 0.07 | 6.81 ± 0.12                    | 0.35 ± 0.02     |
| ABKM09        | 26.2 ± 0.3   | 7.73 ± 0.08 | 7.00 ± 0.18                    | 0.36 ± 0.03     |
| LHC (7 TeV)   |              |          |                                 |                 |
| ABKM09 (updt.) | 100.9 ± 1.3  | 29.3 ± 0.4 | 132.7 ± 6.0                    | 9.4 ± 0.2       |
| ABKM09        | 98.8 ± 1.5   | 28.6 ± 0.5 | 133.8 ± 7.7                    | 8.8 ± 0.3       |

Table 1: Selected standard candle and other hard process cross sections for the Tevatron collider and the LHC calculated employing our updated NNLO PDFs in comparison to those of ABKM09 PDFs, Ref. [1].

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