Numerical Studies on the Dynamic Responses of Levitated High-temperature Superconductor with a Strongly Coupled Thermo-electromagnetic Model

To cite this article: C Q Ye et al 2018 J. Phys.: Conf. Ser. 1054 012087

View the article online for updates and enhancements.
Numerical Studies on the Dynamic Responses of Levitated High-temperature Superconductor with a Strongly Coupled Thermo-electromagnetic Model

C Q Ye¹, T Y Gong¹,², G T Ma¹, W J Yang¹ and Z Y Yang¹

¹ State Key Laboratory of Traction Power, Southwest Jiaotong University, Chengdu, China, 610031
² College of Electrical Engineering, Southwest Jiaotong University, Chengdu, China, 610031
E-mail: changing_ye@foxmail.com

Abstract. In this paper, we present a 2-D numerical model based on the H-formulation and nonlinear E-J relationship to study the dynamic responses of a bulk high-temperature superconductor (HTSC) levitated above two kinds of permanent magnetic guideway (PMG). Being different from the existing related models and results of this subject, the thermal effect was brought into account by a strongly coupled model with electromagnetic and thermal. The levitation forces were calculated by finite element method and then the vertical motion of the levitated HTSC subject to external disturbance is characterized by a second-order dynamic equation which couples the electromagnetic model via the levitation force. We studied the thermal influence on the vertical force and dynamic characteristics of the HTSC levitated above two kinds of PMGs. The obtained results prove that this strongly coupled model can better simulate the dynamic response of HTSC levitation system.

1. Introduction

With the distinct merits of self-stable levitation, contactless, less friction and energy-saving, the high temperature superconductor (HTSC), such as bulk REBCO superconductors and stacked tapes of REBCO coated conductor materials, levitated above a permanent magnet (PM), is promising in the industrial applications of rail transit, magnetic bearing, and flywheel energy storage system, etc [1]-[2].

To explore the dynamic stability of HTSC levitated above PM, experiments and numerical simulations have been conducted, including the cylinder and cuboid bulks excited by PM, known as rotational superconducting magnetic bearing (SMB) and linear SMB. The nonlinear dynamic phenomena of quasi-periodic, periodic double, chaotic, bifurcation, levitation drift behaviors were discovered in rotational SMB and discussed in approximate analytical models. Comparing with the rotational SMB, published papers on the dynamics of linear SMB mostly devoted to the effects of field-cooling height, working height, off-center, derailment operation, load-weight and its respective distribution on the initial frequency, dynamic stiffness and dumping parameters of system, including that the levitator composed of multiple YBaCuO bulks [3]-[5]. The abovementioned achievements regarding the vibration, to some extent, lead to better understanding the dynamics of linear SMB. However, the thermal effect induced by vibration of magnet or flux creep and motion, should be accounted in dynamic behaviors. Hence, this paper is devoted to make a primary discussion on how the thermal influence affect on the levitation force and vertical dynamics.
This paper is organized as following, Part II introduces the linear SMB and establish the 2-D model that couples the electromagnetic and thermal fields bidirectionally, to investigate the dynamic responses of a bulk HTSC levitated. Part III presents the coupled simulation of dynamics and conclusion is provided in Part IV.

2. Theoretical modeling

![Figure 1](image)

Figure 1. (a) Two-dimensional geometrical configuration of the linear superconducting magnetic bearing (SMB) composed of a bulk HTSC of YBaCuO levitated above a permanent magnetic guideway (PMG) with translational symmetry along the invisible z-axis, being axis-symmetrical in term of the y-axis of the so-called Cartesian system. (b) Two common kinds of PMG, PMG1 and PMG2 (unit: mm).

The 2-D reduced structure of the linear SMB addressed in this paper, consisting of a bulk YBaCuO and a permanent magnetic guideway (PMG) that extends infinitely in the translational direction, is shown in Fig. 1, where PMG1 is a common type and PMG2 is the Halbach-derived type that concentrates most the magnetic energy to the upper side. All PMGs are made of NdFeB permanent magnets that have remanent flux density $B_r = 1$ T. The magnetic field of PMG is calculated using a 2-D magnetostatic finite element model by the mf(magnetic fields) interface in Comsol[6]. The applied bulk HTSC in this work is a melt-textured YBaCuO block. The electromagnetic governing equation for the bulk HTSC is based on the H-formulation[7]. We only consider an unidirectional coupling between the PMG model and the bulk HTSC model, through applying the superposition of external field generated by PMG and self-field on the outer boundaries of HTSC model. This hypothesis is reliable because the magnetic field generated by the supercurrent in bulk HTSC excited by the PMG does not significantly exceed the coercive field of PMs. Additionally, we calculated the magnetic field of bulk HTSC through the Biot-Savart law. Last but not the least, this model has been validated by experiments, comparing with other models by different groups[8]–[9].

2.1 Thermo-Electromagnetic modeling

The bulk HTSC is modeling by the H-formulation with the nonlinear power law of E-J relationship, the multiphysics finite element software Comsol has been successfully applied in superconductor modeling in 2D and 3D application. In this part, we will present the fundamental modeling strategy. According to Faraday’s law and neglecting the displacement current, we obtain the governing partial differential equation (PDE) of HTSC in the $H$-formulation of general form,

\[ \mu \frac{\partial H}{\partial t} + \nabla \times E = 0. \]  (1)
where the electrical field \( \mathbf{E} = \rho \mathbf{J} = \rho \nabla \times \mathbf{H} \). To satisfy the condition \( \nabla \cdot \mathbf{H} = 0 \) in finite element method discretization of the 2nd-order differential operator \( \nabla \times \nabla \times \) and avoid both the ill-conditioned matrices and slowly iteration convergence in nodal finite element method, the edge elements are applied in Comsol. Considering the translational symmetry of linear SMB, the induced current and electrical field in HTSC only have the \( z \)-components, \( J_z \) and \( E_z \), therefore the magnetic and electric fields can be represented by \( \mathbf{H} = [H_x, H_y, 0] \) and \( \mathbf{E} = [0,0,E_z] \) in \( xy \)-plane. Consequently, the governing electromagnetic equation (1) can be rewritten as

\[
\mu \frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial y} = 0, \tag{2}
\]

\[
\mu \frac{\partial H_y}{\partial t} - \frac{\partial E_z}{\partial x} = 0. \tag{3}
\]

And the current density along the translational direction can be defined through the simplified equation (1)

\[
J_z = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}. \tag{4}
\]

Equation (2) and (3) can be solved by PDE module in Comsol. The nonlinear constitute relationship in HTSC is defined by the \( E-J \) power law,

\[
\rho(J_z) = \frac{E_z}{J_c} \left| \frac{J_z}{J_c} \right|^{n-1}, \tag{5}
\]

usually, \( J_c \) is the critical current depended on the magnetic and thermal fields when \( E_c \) equals to 1 \( \mu \)V/m, and \( n \) is a parameter according to material used. To account the effects of magnetic and thermal fields on the critical current density \( J_c(B,T) \), we apply the empirical equation[4],

\[
J_c(B,T) = J_{c0} \left( \frac{B_0}{B_0 + B_x^2 + B_y^2} \right) \frac{T_c-T}{T_c-T_0^\circ}, \tag{6}
\]

where \( B_0 \) is the characteristic magnetic field based on the bulk HTSC properties, \( B_x \) and \( B_y \) represent the magnetic field components along the \( x \)-axis and \( y \)-axis directions respectively in Fig. 1, the temperature \( T \) is defined through the thermal model, and the parameters \( T_0 \) and \( T_c \), depending on material thermal characteristic, respectively, represent the initial and critical temperatures of bulk HTSC.

**Table 1.** Parameters of bulk HTSC of YBaCuO used in calculations

| Parameter                        | Value                      |
|----------------------------------|----------------------------|
| Bulk width                       | 24 mm                      |
| Bulk thickness                   | 10 mm                      |
| Critical temperature, \( T_c \)  | 92 K [9]                   |
| Initial temperature, \( T_0 \)  | 77 K                       |
| Critical current density at operating temperature/zero field, \( J_{c0} \) | \( 7.5 \times 10^7 \) A/m² [8] |
| Critical electrical field, \( E_c \) | 10⁻⁴ V/m [7]               |
| Reference value of the magnetic induction, \( B_0 \) | 0.37 T [7]                     |
| \( n \)-exponent at 77 K         | 21 [7]                     |
| Thermal conductivity, \( k \)    | 6 W/K·m⁻¹ [10]             |
| Heat capacity per unit volume, \( c \) | \( 73.75 \ T^2 + 5599.78 \ T + 87669.311 \) J/(m³·K) |
| Heat transfer coefficient, \( h \) | 400 Wm⁻²K⁻¹ [4]           |

We set the thermal model based on the classical governing equation for \( T \),

\[
k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) - c \frac{\partial T}{\partial t} = -Q, \tag{7}
\]
where the thermal conductivity $k$ and heat capacity per unit volume $c$ herein are independent of the spatial coordinates, and the source term $Q$, known as the Joule heat arise from the conversion of electromagnetic energy, can be calculated through

$$Q = J_x E_z.$$  

(8)

We apply the convective boundary condition

$$k \frac{\partial T}{\partial n} + h(T - T_0) = 0,$$  

(9)

on the bulk HTSC, which means the thermal flux exchanges between the bulk HTSC and liquid nitrogen, where $h$ is the convective heat transfer coefficient. We summarized the values of parameters in Table 1.

2.2 Dynamics model

Once we obtain the induced supercurrent in each domain at any time instant, the levitation force of bulk superconductor is solved through Lorentz’s force equation

$$F_y = \int_0^S J_z B_x dxdy = \sum_{i=1}^{N} j_{ie}^e B_x^e \Delta S^e$$  

(10)

where the superscript $e$ indicates an element in the mesh of bulk HTSC, $S$ is the cross-sectional area of bulk, $N$ are the number elements of a bulk. $B_i$ is the combination of both the magnetic field of PMG and all rest elements.

Figure 2. The levitation force acted on the bulk HTSC with the descending velocities at 1 mm/s, from 60 mm to 10 mm above the center of (a) PMG1 and (b) PMG2, when the thermal effect is taking into account and not.

We refer to the linear SMB schematically drawn in Fig. 1 to establish the electromagnetic model and dynamic equation, in which the actual 3-D problem was simplified to be a 2-D one, recalling the practical case that the PM rails usually extends infinitely in the translational direction and are fixed on the ground or an experiment platform. Generally, according to Newton’s second motion law, the motion of HTSC in the vertical direction can be expressed by,

$$m \ddot{y} + F_y - mg = f_y$$  

(10)

where $m$ is the mass of levitator (not always equals to the mass of HTSC) and equals to 20 kg in this work, $y$ represents the vertical displacement of levitator, therefore, $\ddot{y}$ represents the acceleration of levitator in the vertical direction, $g$ is the gravitational acceleration (normally 9.8 N/kg), $F_y$ is the vertical magnetic force acting in the bulk HTSC and can be solved by Lorentz’s law, $f_y$ is the
3. Results and discussions

Figure 3. Dynamic responses of the bulk HTSC levitated above the center of PMG1 vary with time, (a) vertical displacement around the minimal height 10 mm, (b) vertical electromagnetic force acted on the bulk HTSC, when the thermal effect is taking into account and not.

The bulk HTSC is a melt-textured YBaCuO block that has 24 mm in width and 10 mm in thickness. To appraise the thermal effect on the dynamic responses of bulk HTSC levitated above the two kinds of PMGs, numerical simulation were carried out by starting the vertical movement at a position of 60 mm above the center of PMG where the external applied field is almost negligible at this height, and then descending to the position of 10 mm above the PMG, after the relaxation of 60 seconds, and finally vibrating for 10 seconds excited by the difference between the gravitational force and levitation force. It is noted that the descending speed of bulk HTSC is 1 mm/s to create a quasi-static state, therefore, the displacement current can be neglected. Furthermore, we set the relaxation time for 60 seconds to reduce the levitation height and oscillation center drifts during vibration that obviously displayed in Fig. 6 of Ref.[4].

exciting force acting on the levitator. It is worth noting that the air resistance is such tiny that can be neglected, regarding the tiny volume of triple-seeded bulk HTSC in this work.
Figure 4. Dynamic responses of the bulk HTSC levitated above the center of PMG2 vary with time, (a) vertical displacement around the minimal height 10 mm, (b) vertical electromagnetic force acted on the bulk HTSC, when the thermal effect is taking into account and not.

The levitation forces acted on the bulk HTSC as a function of time from the initial height 60 mm to the minimal height 10 mm, are portrayed in Fig. 2, when the thermal effect is taking into account and not. Specially, the levitation force generated by PMG1 is almost the twice of that generated by PMG2, because the magnetic field above the center of PMG1 is much stronger than that of PMG2. Generally speaking, it is safe to conclude that, for both cases of PMG1 and PMG2, the levitation forces are almost the same, whether the thermal effect is taking into account or not, although the levitation force without accounting the thermal effect at the minimal working height is little higher than the case considering the thermal effect.

Figures 3 and 4 display the responses of displacement and levitation force of bulk HTSC levitated above the center of PMG1 and PMG2 experienced oscillations between the corresponding balance position, after relaxation for 60 seconds at the position of 10 mm, where the initial velocity is 0 mm/s. To portray the dynamic behaviors clearly, the insets in each figure show the oscillations of displacement and vertical force from 111 to 112 s and 118 to 120 s.

Roughly speaking, both for PMG1 and PMG2, we can observed the invisible downward drift of the oscillation center both in the dynamics responses of displacement and levitation centers, whether the
thermal effect is taking into account or not. Comparing with the obvious downward drift of the oscillation center during the vibrations[3]-[4], the relaxation of bulk HTSC is necessary and usually applied in our experiments[5]. In the initial vibration phase, from 110 to 114 s, the oscillation behaviors, including the oscillation amplitudes, periods and phase angles, manifest almost synchronous for the cases considering thermal effect and not. As the oscillation went on from 114 to 120 s, however, the oscillation behaviors of the cases of PMG1 and PMG2, including the oscillation amplitudes and frequencies, display increasingly nonsynchronous, comparing with the models with and without thermal effects. Obviously, the oscillation amplitudes of vertical displacement and force decrease quicker, in the case of accounting the thermal effect, than that case without accounting the thermal effect, both in PMG1 and PMG2.

Specially, comparing with PMG1 in Fig. 3 and PMG2 in Fig. 4, it can be observed that the phases of PMG2 with and without thermal effects are almost corresponding from 110 to 120 s, however, the phases of PMG2 with and without thermal effects are not corresponding in the last half of oscillations. As a matter of fact, the dynamic equation of bulk HTSC levitated above the PMGs are nonlinear, as shown in Eq. (10). Furthermore, the vertical magnetic force between the bulk HTSC and PMG solved by Lorentz equation, depends not only on the displacement, velocity, time, motion condition, and motion history of bulk HTSC, but also on the magnetic field generated by PMG and physical parameters of HTSC and PMG(Fig. 5 of Ref. [3]). As a result, the superconducting levitation system demonstrates hysteretic characteristic and its stiffness is not a constant during the free-vibration process(see Fig. 5 of Ref. [3]). Consequently, the damping frequency and period time are not constants during the vibration(It also demonstrated in Figs. 6 and 7 of Ref. [4]). Concisely, the PMGs with different arrangement of PMs shape the magnetic field distribution, and affect the phases of vibration in Fig. 3 and Fig. 4 eventually. It can be safely concluded that the thermal effect influences the stability of SMB system and the thermal effect in SMB can be employed as a damper of practical application.

4. Conclusions

In this paper, a 2-D coupled thermo-electromagnetic modeling method for the bulk HTSC levitated above a normal and a Halbach-derived PMGs implemented in Comsol was introduced and employed with the dynamic model to study the influence of thermal effect on dynamic responses of levitated bulk HTSC. It is concluded from the numerical results that the bulk HTSC levitated above both kinds of PMG converges more quickly, when we apply the strongly coupled thermo-electromagnetic model which can better simulate the dynamic response of SMB, than that model without taking account the thermal effect. On that ground, an eddy current damper can improve the dynamic system stability. Further work should be done for better evaluate the thermal effect on the practical case of SMB.

Acknowledgements

The authors would like deeply to thank the anonymous reviewers and editors, whose critical comments considerably improved the quality of this paper. This work was supported in part by the National Natural Science Foundation of China under Grants 51475389, 51722706 and 51707164, in part by China Postdoctoral Science Foundation under Grant 2017M623055, and in part by the Sichuan Youth Science&Technology Foundation under Grant 2016JQ0003.

References

[1] Wang J, Wang S, Zeng Y, et al. 2002 Physica C 809-814 378–381
[2] Werfel F N, Floegel-Delor U, Rothfeld R, Riedel T, Goebel B, Wippich D and Schirrmeister P 2012 Supercond. Sci. Technol. 25 014007
[3] Gou X F, Zheng X J and Zhou Y H 2007 IEEE Trans. Appl. Supercond. 17(3) 3795–3802
[4] Alloui L, Ben-Alia K, Bouillault F, Mimoune S M, Bernard L and Leveque J 2013 Physica C 487(4) 1-10
[5] Jiang D, Ma G, Xu Y, Deng Z, Wang S and Wang J 2013 *IEEE Trans. Appl. Supercond.* 23(3) 3600404
[6] https://www.comsol.com/
[7] Sass F, Sotelo G G, Junior R D A and Sirois F 2015 *Supercond. Sci. Technol.* 28(12) 125012
[8] Queval L et al. 2016 *Trans. Appl. Supercond.* 26(3) 3601905
[9] Ye C Q, Ma G T and Wang J S 2016 *IEEE Trans. Appl. Supercond.* 26(8) 3603309
[10] Poole C P, Farach H A, Creswick R J and Prozorov R 2007 *Superconductivity* (Amsterdam: Elsevier)
[11] Roy F, Dutoit B, Grilli F and Sirois F 2008 *IEEE Trans. Appl. Supercond.* 18(1) 29–35