Baryogenesis at the End of Hybrid Inflation

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The baryon asymmetry of the universe may originate in the phase transition at the end of hybrid inflation, provided that the reheat temperature is low enough. I show that if the field that triggers the end of inflation is the electroweak Higgs field and CP is violated, the transition leads to baryon asymmetry even if no preheating or non-thermal symmetry restoration takes place. I estimate the strength of this effect and the constraints it imposes on the inflationary model.

PRESENTED AT

COSMO-01
Rovaniemi, Finland,
August 29 – September 4, 2001
1 Introduction

In order to explain the baryon asymmetry of the universe, a theory must satisfy three conditions [1]: It must (obviously) violate baryon number, but it must also violate the C and CP symmetries, and at the same time the baryon-number violating interaction must be out of thermal equilibrium. As the CP violation present in the standard model of particle physics is not strong enough, any proposal must include some extra source of CP violation, but apart from that, the standard model seems to have all the requires properties [2]. At high temperatures, the baryon number is violated by sphaleron processes [3], and the electroweak phase transition gives rise to the necessary non-equilibrium state.

Unfortunately, creating the baryon asymmetry is not enough: It must also survive until the present day, and therefore the baryon number violation must stop before the fields equilibrate. In the standard scenario of electroweak baryogenesis, a strongly first-order phase transition is needed to avoid this baryon washout. More precisely, the jump of the Higgs expectation value should be of the same order as the critical temperature [4], and numerical simulations [5] have shown that this is not the case for any realistic Higgs mass.

This problem can, however, be avoided if the energy scale of inflation is well below the electroweak scale [6, 7], because then the equilibrium temperature of the universe never exceeds the electroweak critical temperature, no sphaleron processes occur and no baryon washout takes place. The problem, then, is how to create the baryon asymmetry in the first place.

In Refs. [6, 7], it was argued that this is possible in hybrid models of inflation, if the inflaton field resonates with the electroweak Higgs field. This preheating heats up the long-wavelength modes of the Higgs and gauge fields, and leads to baryon number violation, which stops as soon as the fields start to approach thermal equilibrium. For this mechanism to work, both the preheating and the baryon number violation must be extremely efficient, but numerical simulations [8] seem to indicate that it is very difficult to generate enough baryons.

On the other hand, it was recently shown in Ref. [9] that even without any preheating at all, electroweak-scale hybrid inflation generally leads to baryogenesis, as the breakdown of the SU(2) gauge invariance leads to a non-zero Higgs winding number, which decays into baryons. In this talk, I will review this argument and estimate the amount of baryon asymmetry generated by this mechanism.

2 Hybrid inflation

In models of hybrid inflation [10], the inflaton field $\sigma$ is coupled to another scalar field $\phi$, which becomes unstable at a certain critical value of $\sigma_c$. This causes the slow roll conditions to break down, and inflation ends. The simplest realization of this idea is the
potential
\[ V(\sigma, \phi) = \frac{1}{2} m^2_\sigma \sigma^2 + g^2 \sigma^2 \phi^\dagger \phi - |m^2_\phi| \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 + \frac{m^4_\phi}{4\lambda} \] (1)

In the following, we will assume that \( \phi \) is the electroweak Higgs field. The amplitude of the CMB fluctuations forces \( m_\sigma \) to be extremely small, around \( 10^{-10} \text{ eV} \), and for our purposes, we can treat it as zero.

During inflation, the inflaton \( \sigma \) has a large value, \( \sigma \gg \sigma_c = \frac{m_\phi}{g} \). Therefore the effective mass term of the Higgs field,
\[ m^2_\phi(\sigma) = -|m^2_\phi| + g^2 \sigma^2 = g^2 (\sigma^2 - \sigma_c^2), \] (2)
is positive and the electroweak SU(2) symmetry is restored. The inflaton \( \sigma \) slowly rolls down the potential, and when it reaches \( \sigma_c \), \( m^2_\phi(\sigma) \) becomes negative, implying that the \( \phi \) field becomes unstable and the symmetry gets spontaneously broken.

In order to avoid the baryon washout, the final reheat temperature \( T_{rh} \) must be far enough below the electroweak critical temperature \([4]\), \( T_{rh} \lesssim 150 \text{ GeV} \), and this constrains the potential at \( \sigma = \sigma_c \) to be below the corresponding critical energy density,
\[ V(\sigma_c, 0) \approx \rho(T_{rh}) \approx \frac{\pi^2}{30} g_* T_{rh}^4 \lesssim 10^{10} \text{ GeV}^4, \] (3)
where \( g_* \approx 100 \) is the effective number of degrees of freedom at the electroweak scale. This implies that the Hubble rate is around \( 10^{-5} \text{ eV} \), well below any interesting energy scale, and therefore we can ignore the expansion of the universe altogether.

It is actually rather natural to identify the field \( \phi \) with the electroweak Higgs field. Indeed, if it were merely a real scalar field, the symmetry breakdown would lead to formation of domain walls, with disastrous consequences. The breakdown of any global continuous symmetry would imply the existence of Goldstone bosons, which have not been observed, and therefore the broken symmetry must be a gauge invariance. Then, if we want to avoid the formation of monopoles or cosmic strings, the simplest choice is that \( \phi \) is an SU(2) doublet, which means that it is the electroweak Higgs field.

This has the consequence that it fixes the couplings \( \lambda \) and \( m^2_\phi \), if we assume that the Higgs mass is \( m_H \approx 115 \text{ GeV} \):
\[ \lambda \approx 0.11, \quad m^2_\phi = \frac{m^2_H}{2} \approx -6.6 \times 10^3 \text{ GeV}^2. \] (4)
This implies that
\[ V_0 \equiv V(\sigma_c, 0) = V(0, 0) = \frac{m^4_H}{16\lambda} \approx 10^8 \text{ GeV}^4, \] (5)
which satisfies the bound in Eq. (3) easily. The final reheat temperature is
\[ T_{rh} \approx \left( \frac{15}{8\pi^2 g_* \lambda} \right)^{1/4} m_H \approx 42 \text{ GeV}. \] (6)
In this simplest realization, the only remaining free parameter is $g$. We shall assume that $g \sim O(1)$, even though this means that radiative corrections arising from Higgs loops are going to cause problems. These problems are alleviated in inverted hybrid models [1].

3 Baryogenesis

When the inflaton field $\sigma$ reaches its critical value $\sigma_c$, the Higgs field becomes unstable and the SU(2) gauge invariance breaks spontaneously. Some aspects of this process have recently been discussed in Refs. [11, 12]. For our purposes, it is enough to note that symmetry-breaking phase transitions typically lead to formation of topological defects via the well-known Kibble-Zurek mechanism [13, 14, 15].

Of course, the electroweak theory does not contain any genuine topological defects. The zeroth, first and second homotopy groups of the vacuum manifold are trivial, and therefore no domain walls, strings or monopoles are formed. However, as the vacuum manifold is a three-sphere, it has a non-trivial third homotopy group, whose elements are given by the integer-valued Higgs winding number $N_H$ [16]

$$N_H = -\frac{1}{24\pi^2} \int d^3x \epsilon^{ijk} \text{Tr} \left( \partial_i \Phi^\dagger \partial_j \Phi \partial_k \Phi^\dagger \Phi \right).$$ (7)

Here $\Phi$ is a unitary matrix $\Phi = \hat{\phi}^4 + i\sigma^i \hat{\phi}^i$ with $\phi = |\phi| \left( \hat{\phi}^2 + i\hat{\phi}^1, \hat{\phi}^4 - i\hat{\phi}^3 \right)$.

If the SU(2) symmetry were global, the Higgs winding number would label physically distinct vacua of the theory. In the gauge theory, however, there is for every Higgs field configuration a corresponding gauge field configuration that compensates exactly for the gradient energy, and in such a configuration the Chern-Simons number

$$N_{\text{CS}} = \frac{g^2}{16\pi^2} \int d^3x \epsilon^{ijk} \text{Tr} \left( F_{ij} A_k + \frac{2}{3} ig A_i A_j A_k \right)$$ (8)

is equal to the Higgs winding number, $N_H = N_{\text{CS}}$. Large gauge transformations can change both $N_{\text{CS}}$ and $N_H$, and therefore a vacuum with equal but non-zero Higgs winding and Chern-Simons numbers is always merely a gauge transform of the vacuum with $N_H = N_{\text{CS}} = 0$, and therefore physically indistinguishable from it.

Or, more precisely, it would be if there were no fermions. An anomaly relates the Chern-Simons number to the baryon and lepton numbers, and in particular,

$$\Delta N_B = 3\Delta N_{\text{CS}}.$$ (9)

This means that if an instanton or a sphaleron transition moves the system from one Chern-Simons vacuum to another, the baryon number changes as well. This is the basic mechanism of electroweak baryogenesis. Starting from a state with $N_B = 0$, some non-equilibrium mechanism changes the Chern-Simons number, and when the universe finally
settles down in that vacuum, all other physical observables are identical to the original vacuum, but the baryon and lepton numbers have changed.

In the standard scenario [2], the baryon number violation takes place on the bubble walls during a first-order electroweak phase transition, but lattice simulations show that in the standard model, the transition is not of first order. In a modified scenario, preheating after electroweak-scale inflation leads to non-thermal symmetry restoration, which allows the Chern-Simons number to change [6, 7]. However, that scenario is very sensitive to the non-equilibrium dynamics during and after preheating, and numerical simulations indicate that the baryon number violation may not be as strong as was hoped [8].

However, the Chern-Simons number can also change if it simply happens to differ from the Higgs winding number [16, 17], and this does not need high temperatures or symmetry restoration. The fields simply relax to the vacuum configuration with \( N_H = N_{CS} \), and this can happen in two different ways: Either the Higgs unwinds as in the global theory, in which case \( N_H \) changes, or the Chern-Simons number of the gauge field changes. In the latter case, baryons are created. The presence of CP violation biases this process [16, 17], so that even if the Higgs winding number averages to zero and only has spatial fluctuations, the regions with, say, positive \( N_H \) are more likely to form baryons than those with negative \( N_H \) are to form antibaryons. This leads to baryon asymmetry even if the mechanism which forms the Higgs winding is symmetric.

Figure 1: The Kibble mechanism in a one-dimensional complex field theory. After the transition, the Higgs field (arrows) is correlated only inside domains of size \( \xi \), and this leads to the formation of Higgs windings.

In our case [9], these spatial variations in the Higgs winding number are generated by the Kibble-Zurek mechanism [13, 14]. At the end of inflation, the universe is cold, and we can ignore the gauge field [15, 16]. In the gauge where \( \vec{A} = 0 \), the direction of the Higgs field has a well defined meaning, and in the phase transition spatial domains will form inside each of which the direction will be roughly constant, but between which it is totally uncorrelated. However, the Higgs field must smoothly interpolate between these domains, and as illustrated by the one-dimensional example in Fig. 1, this generally leads to non-zero Higgs winding number. Because the only relevant length scale for this process is the domain size \( \xi \), we can estimate that the typical Higgs winding density after the
transition is
\[ n_H = N_H / V \sim \hat{\xi}^{-3}. \] (10)
Positive and negative values of \( n_H \) are obviously just as likely, and therefore Eq. (10) should be interpreted as the rms value
\[ n_{H}^{\text{rms}} = \langle n_H^2 \rangle^{1/2}. \]

Assuming that each positive Higgs winding has a certain probability \( p_+ = p_0 + \epsilon \) to decay by changing \( NCS \) and thereby creating baryons, and correspondingly that the decay of a negative winding to antibaryons has a lower probability \( p_- = p_0 - \epsilon \), we obtain the estimate
\[ n_B \approx 3\epsilon n_H \sim \epsilon \hat{\xi}^{-3}, \] (11)
where \( \epsilon \) depends on the strength of the CP violation.

When the fields finally thermalize, they reach the reheat temperature in Eq. (8), which corresponds to the entropy density
\[ s = (2\pi^2/45)g_s T_{\text{rh}}^3 \sim 2.1m_H^3. \]
In order to explain the observed baryon asymmetry \( n_B / s \sim 3 \times 10^{-10} \), we therefore need
\[ n_H \gtrsim 10^{-9}\epsilon^{-1}m_H^3. \] (12)

4 Estimating \( n_H \)

Although defect formation is usually discussed in the context of thermal phase transitions [13, 14, 15], the same principles apply to our case. The role of the temperature is played by the inflaton field, which gives the Higgs field a time-dependent mass term (2). Near \( \sigma_c \), we can approximate
\[ m_\phi^2(t) = g^2(\sigma(t)^2 - \sigma_c^2) \approx -2g^2\sigma_c|\dot{\sigma}|t + O(t^2). \] (13)

The correlation length of the Higgs field is given by the inverse mass
\[ \xi(t) \approx \left(2g^2\sigma_c|\dot{\sigma}||t|\right)^{-1/2}, \] (14)
which shows that the correlation length diverges at the critical point. In the absence of any thermal fluctuations, this is equal to the relaxation time of the field, \( \tau(t) = \xi(t) \), and so the dynamics of the field also gets slower. Because of this critical slowing down, a point is eventually reached when the field cannot respond to the change of the mass, and therefore, instead of actually diverging, the correlation length reaches a maximum value \( \hat{\xi} \), which determines the size of the correlated domains and thereby also the Higgs winding density (10).

The value of \( \xi \) is approximately given by the correlation length at the time when \( |t| \) is equal to the relaxation time, which gives
\[ \hat{\xi} = \xi \left(t = -\hat{\xi}\right) \approx \left(2g^2\sigma_c|\dot{\sigma}|\right)^{-1/3}, \] (15)
or
\[ n_H \sim 2g^2\sigma_c|\dot{\sigma}| \approx g m_H |\dot{\sigma}|. \tag{16} \]

If we use the slow roll condition
\[ \dot{\sigma} = -\frac{V'(\sigma)}{3H} \tag{17} \]

for \( \sigma \), we find that \( \dot{\sigma} \) is far too low, and therefore we must assume that before \( \sigma_c \), the potential becomes steeper. This may happen because of radiative corrections, which become much more important when the effective mass of the Higgs field is low, or it can even be the case at tree level in more complicated models \[9\]. If we simply require that the inflation has not ended before \( \sigma_c \), we have
\[ |\dot{\sigma}| \lesssim V_0^{-1/2} \approx \frac{m_h^2}{4\sqrt{\lambda}}, \tag{18} \]

which implies
\[ n_H \lesssim \frac{g}{4\sqrt{\lambda}} m^3_H \approx g m^3_H. \tag{19} \]

Combining this with Eq. (12) gives the requirement
\[ \epsilon g \gtrsim 10^{-9}, \tag{20} \]

which means that the coupling \( g \) and the CP violation \( \epsilon \) do not have to be particularly strong for this mechanism to be able to explain the observed baryon asymmetry of the universe.

5 Conclusions

In this talk, I have discussed the possibility of baryogenesis at the end of electroweak-scale hybrid inflation. I have shown that the Higgs winding number generated by the Kibble-Zurek mechanism leads to baryon asymmetry, provided that CP is violated. There are still certain questions that need to be addressed before a concrete estimate of the resulting baryon density can be obtained: We need to establish the constant of proportionality in Eq. (10), and the dependence of \( \epsilon \) in Eq. (11) on the CP violating couplings in the Lagrangian. Furthermore, we do not yet have a natural model of electroweak-scale inflation that would have all the necessary properties.

Nevertheless, the estimates I have presented here indicate that in a simple toy model, it is possible to achieve high enough baryon densities to explain the observed baryon asymmetry of the universe, without having to assume that either the CP violation or the coupling between the Higgs and the inflaton is particularly strong.
ACKNOWLEDGEMENTS

I would like to thank Ed Copeland, David Lyth and Mark Trodden for collaboration on this topic.

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