Polygram Stars: Resonant Tidal Excitation of Fundamental Oscillation Modes in Asynchronous Stellar Coalescence

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Abstract

The prevalence of binary stars at close separations implies that many of these systems will interact or merge during the binary’s lifetime. This paper presents hydrodynamic simulations of the scenario of binary coalescence through unstable mass transfer, which drives the pair to closer separations. When the donor star does not rotate synchronously with respect to the orbit, dynamical tidal waves are excited in its envelope. We show that resonance crossings with high azimuthal order \((m \sim 3 \text{ to } 6)\) fundamental modes induce a visible “polygram” distortion to the star. As the binary orbit tightens, the system sweeps through resonance with modes of decreasing azimuthal order, which are selectively excited. We compare our hydrodynamic simulations to predictions from linear theory of resonant-mode excitation. The linear theory provides an estimate of mode amplitudes to within a factor of two, even as the oscillations become quite nonlinear as the stars coalesce. We estimate that a wave with 10% radial amplitude generates approximately 1% photometric variability; this may be detectable if such a binary coalescence is caught in action by future photometric all-sky surveys.

Key words: binaries: close – methods: numerical – stars: oscillations

Supporting material: animation

1. Introduction

Approximately half of all stars reside in binary or multiple systems (e.g., Duchêne & Kraus 2013; Moe & Di Stefano 2017). Many of these systems have sufficiently small separations that stellar evolution or dynamical interactions within a multiple-star system can drive them into direct interaction (De Marco & Izzard 2017). These interactions can lead to mass exchange, orbital transformation, or even the complete merger of the binary pair.

In this paper, we study a particular scenario of such hydrodynamic binary interaction: unstable mass transfer from a Roche-lobe filling donor star onto a more compact, but less massive, accretor star. This mass transfer is unstable in that the exchange of orbital angular momentum via mass removal from the donor star drives the binary system to increasing degrees of Roche lobe overflow and escalating mass transfer rate. This scenario inevitably leads to a common envelope phase in which the donor engulfs the accretor within its extended envelope (Paczynski 1976). From this point, the binary may go on to merge into a single object, or to eject the donor’s envelope, leaving behind a transformed binary system (Iben & Livio 1993; Taam & Ricker 2010; Ivanova et al. 2013).

Many systems are thought to have evolutionary histories such that when they reach the point of Roche lobe overflow, the donor star’s envelope is likely to be in synchronous rotation with the orbital motion (Ogilvie 2014). In this case, the gravity of the accretor star raises a static tidal distortion of the donor star’s envelope in the corotating frame of the binary. The dynamics of these synchronous scenarios have been analyzed recently by MacLeod et al. (2018a, 2018b). Deviation from synchronized rotation can lead to work being done as the binary potential raises dynamical waves in the gaseous envelope. Wave motions, in turn, lead to dissipation, which tends to bring the system back toward the quasi-equilibrium configuration.

However, there are several cases in which we might imagine that the synchronous configuration is never achieved. One such case, known as the Darwin tidal instability (Darwin 1879), is when there is insufficient angular momentum in the binary orbit to pull the donor’s envelope into synchronous rotation (e.g., Counselman 1973; Hut 1980, 1981). Tidal dissipation then leads to orbital decay toward the onset of mass exchange without ever reaching a synchronous state (e.g., Lai et al. 1994; Eggleton & Kiseleva-Eggleton 2001; Weinberg et al. 2017; Sun et al. 2018). This scenario tends to arise when the binary mass ratio is far from unity, or when the moment of inertia of the donor’s envelope is relatively large. Another situation in which synchronous rotation might not be achieved is when the mechanism that assembles the interacting binary allows insufficient time for synchronization to act. One example is in massive-star systems, where the star’s evolutionary timescale can be rapid compared to the tidal dissipation timescale (Savonije & Papaloizou 1983).

We study the excitation of dynamical tides following the assembly of an asynchronous binary at the Roche limit, in which the donor star is beginning to lose mass. Our simulated system contains an isentropic giant-star donor interacting with a companion (“accretor”) of one-tenth the donor’s mass. As the orbit shrinks, the tidal forcing frequency, set by the difference between the orbital frequency and envelope rotation of the donor, passes through integer resonances with the frequencies of fundamental, acoustic modes of the star. Waves of successive azimuthal order (first high \(m\) then lower \(m\)) are preferentially excited as the system passes through these resonances. This finding is in contrast to binaries at wider separations, which predominantly exhibit quadrupolar mode...
excitations by the tidal potential (e.g., Cowling 1941; Lai 1994).

In Section 2, we describe our simulated model and numerical approach. In Section 3, we discuss the criteria for resonance between tidal forcing and oscillatory modes of the star. In Section 4 we analyze the amplitudes of resonantly excited tidal waves that arise, and compare these results to predictions from linear theory. In Section 5, we discuss the implications and potential observability of these oscillations, and we conclude in Section 6.

2. Numerical Model and Methods

We report on the results of gas dynamics simulations performed using the Athena++ hydrodynamics code\(^5\) and based on the approach developed by MacLeod et al. (2018a, 2018b). Our simulated system involves a donor star of mass \(M_1 = 1\) and radius \(R_1 = 1\). Together with a choice of gravitational constant \(G = 1\), these choices define our system of units. The donor star interacts with a lower-mass accretor of mass \(M_2 = 0.1\). Both the core of \(M_1\) and the entirety of \(M_2\) are treated as softened point masses. We perform our calculation in the frame of the orbiting donor star, in spherical-polar coordinates. We adopt an adiabatic equation of state in our gas dynamical model. We further solve for mode frequencies under the Cowling approximation, which entails neglecting the oscillations perturbation on the star’s gravity and is equivalent to the static self-gravity applied in our hydrodynamic model.

In general, stellar oscillatory modes may be restored by pressure (\(p\)-modes), gravity (\(g\)-modes), or both (fundamental or \(f\)-modes). Only \(f\) and \(p\) modes are supported within an isentropic star, such as our stellar model. The fundamental modes lack radial nodes, and we observe that these modes are most strongly excited in our simulated systems.

We note that the key assumptions of the model, a polytropic stellar structure, a zero-radial-displacement inner boundary at 0.3 \(R_1\), and a static self-gravitational potential, all affect the frequencies and occurrence of various modes as compared to more realistic stellar models. In the Appendix we compare to a more detailed stellar-evolution model to show that normalized mode frequencies and overlap integrals are not strongly affected by our simplifications.

3. Resonant Tidal Excitation During Binary Coalescence

Here we discuss the conditions for resonance between the tidal forcing field and oscillation modes in the decaying binary. We demonstrate that such a binary crosses resonance with a number of high azimuthal order modes and trace these resonantly excited modes in our hydrodynamic model binary system.

3.1. Resonance Condition

The characteristics of fundamental oscillation modes of different degree, \(l\), and azimuthal order \(m = \pm l\), within our stellar model are given in Table 1. We show modes with \(l \geq 2\) because physically there is no \(l = 1\) fundamental mode. The mode frequencies, \(\omega_m\), depend only on \(l\) (and not \(m\)) because our model is non-rotating (in general, rotation introduces frequency splittings for modes of different \(m\)). Because of the equatorial symmetry of the binary gravitational field, excited modes tend to have order of \(m = \pm l\), therefore we focus on

\[ S_f(l) = \frac{1}{2} \int_{\Omega} v_l^2 d\Omega, \]

where \(v_l \left( R_1, \theta, \phi \right) \) is the radial velocity of fluid relative to the donor’s core as a function of \(\theta\) and \(\phi\) at the donor’s original radius, \(r = R_1\). The power spectrum is computed as

\[ S_f(l) = \frac{1}{2} \int_{\Omega} v_l^2 d\Omega, \]

which implies a normalization such that the sum of \(S_f\) over \(l\) is equal to the integral of the square of \(v_l\), divided by its angular area, \(\int v_l^2 d\Omega\). See Wieczorek et al. (2018) for a full description of the implementation.
Table 1
Properties of \( l = m \) Fundamental Modes of Our Non-rotating Stellar Model
Calculated with GYRE, in Units of \( G = M_1 = R_1 = 1 \)

| \( l \) | \( \omega_\alpha \) | \( Q_\alpha \) | \( M_\alpha \) | \( C_\alpha \) |
|--------|----------------|----------------|----------------|----------------|
| 2      | 1.918          | 0.273          | 3.39 \times 10^{-3} | 0.4575        |
| 3      | 2.197          | 0.259          | 1.86 \times 10^{-3} | 0.3117        |
| 4      | 2.426          | 0.247          | 1.20 \times 10^{-3} | 0.2379        |
| 5      | 2.630          | 0.237          | 8.36 \times 10^{-4}  | 0.1926        |
| 6      | 2.818          | 0.229          | 6.14 \times 10^{-4}  | 0.1618        |

Note. Here \( \omega_\alpha \) is the mode frequency, \( Q_\alpha \) is the overlap integral between the tidal potential and the mode (Equation 8), and \( M_\alpha \) is the mode mass (Equation 9). The coefficient \( C_\alpha \) pertains to the perturbation of mode frequencies by rotation, and is described by Equation (14).

3.2. Orbital Decay and Resonance Crossing

Mass removal from the donor star leads to runaway orbital decay in our simulated binary system. As mass is pulled from the donor and expelled from the vicinity of the \( L_2 \) Lagrange point, it is imparted angular momentum at the expense of the binary orbit. As the orbit tightens, the rate of mass and angular momentum transfer increases because the donor increasingly overflows its Roche lobe. This leads to increasingly rapid orbital decay due to mass stripping from the donor and ejection out of the binary system (MacLeod et al. 2018a, 2018b).

As the orbit tightens, the orbital frequency changes, roughly as \( \Omega_{\text{orb}} \propto a^{-3/2} \) (despite some mass loss and the extended donor star structure; see Figure 16 of MacLeod et al. 2018a). The constantly changing orbital frequency implies that the forcing frequency of waves within the donor’s envelope is always evolving. It may, therefore, sweep through resonances with particular modes.

Figure 1 examines the decaying binary orbit in our simulation and its implications for mode resonance crossing. The left panels show binary separation and orbital frequency as a function of time. The simulation begins at approximately the analytic Roche limit of 1.73 donor radii (Eggleton 1983). The binary separation then decays, initially over many tens of donor dynamical times. Eventually, the orbital decay rate increases dramatically, and the accretor plunges within the envelope of the donor (MacLeod et al. 2018a).

In the right panel of Figure 1, we examine the terms of the resonance condition, Equation (3). We plot the forcing frequency, \( \Omega_{\text{orb}} - \Omega_{\text{spin}} \), equivalent to \( \Omega_{\text{orb}} \) in this restricted case where \( \Omega_{\text{spin}} = 0 \). With horizontal lines we plot the eigenfrequencies of modes divided by their azimuthal order, \( \omega_\alpha/m \).

As the binary separation shrinks, the forcing frequency crosses resonances with different modes, beginning with higher azimuthal order modes at larger separations and progressing toward low-order modes at small separations. At the initial separation of 1.73 donor radii, the system is near resonance with the \( l, m = 6 \) mode. By the time the binary separation shrinks to just outside the donor’s original radius, the nearest resonance crossing is with the \( l, m = 2 \) mode.

3.3. Mode Excitation During Coalescence

Figure 2 shows a time series of properties of the envelope of the donor star in the binary system. The large panels show gas density in a slice through the orbital midplane. The lower panels show radial velocity on a spherical surface (lower left) at \( r = R_1 \) and the spherical-harmonic power spectrum, \( S_{fl}(l) \), Equation (2), of this velocity field (lower right).

The gas density slices of Figure 2 reveal strongly excited waves within the donor star’s envelope. The wave nearest the accretor can be seen to break and overlap toward the accretor. These waves can be traced to the \( r = R_1 \) radial velocity field by alternating regions of positive and negative radial velocity, measured relative to the donor’s core in the orbiting system. The power spectrum plot reveals that much of the power in the velocity field is carried by particular degrees, \( l \), of the spherical-harmonic decomposition.

Figure 3 shows a three-dimensional rendering of a density iso-surface of the first snapshot shown in Figure 2. The three-dimensional structure reveals that mode amplitudes are largest in the binary equatorial plane and vanish near the donor star’s pole. Figure 3 also highlights the three-dimensional structure of the mass transfer stream, which is tidally compressed as it approaches the accretor, and the subsequent, breaking wave that trails the stream.

A comparison of different panels of Figure 2 reveals that the number of peaks of the excited waves changes as the binary coalesces. While in the upper left panel (\( a = 1.69 \)) there are six wave peaks (\( l, m = 6 \)), as the binary comes together we trace the appearance of five, then four, then three, giving the equatorial structure of the star a polygon shape. Waves also increase in amplitude as the binary separation shrinks: both in height relative to the donor radius, and in magnitude of peak velocity. Turning to the power spectrum for a more quantitative description, we see that the spherical-harmonic degree with the highest power also shifts, starting with higher orders and moving lower as the binary separation decreases.

Figure 4 highlights the decomposition of the \( r = R_1 \) velocity field into its spherical-harmonic components. We observe a shift in \( S_{fl}(l) \) as a function of binary separation. The most excited mode transitions from \( l = 6 \) down to \( l = 2 \) by the time that the accretor is engulfed within the donor’s envelope and a common envelope phase has begun (the \( l = 3 \) and \( l = 2 \) mode amplitudes peak at separations less than the donor’s original radius).
Resonance crossings are indicated by vertical dashed lines in Figure 4 (the binary separations that satisfy the resonance condition of Equation (3)). Interestingly, peak power within a given spherical-harmonic order does not tend to occur at the separation of resonance crossing, but instead lies at somewhat smaller separation. For the lower-order modes this offset is particularly pronounced. Although modes are resonantly excited principally at the resonance crossing locations, the mode pattern itself takes time to set up and propagate around the donor star. The apparent polygram shape of the stellar equator is thus maximized not at the resonance crossing itself, but lagging, and therefore at tighter orbital separations.

4. Mode Amplitudes in Decaying Binary System

Resonance crossing excites fundamental oscillation modes of a range of azimuthal orders during the binary’s trend toward merger. We now contextualize these findings from our simulation model by comparison with the predictions of linear theory.

4.1. Predictions from Linear Theory

We can predict the amount of energy transfer to a mode during a resonance crossing using linear hydrodynamics. We follow a formalism introduced by Lai (1994). This general approach has since been applied to numerous problems involving white dwarf binaries (Fuller & Lai 2011, 2012a, 2012b, 2013, 2014; Burkart et al. 2013; Vick et al. 2017), neutron star binaries (Reisenegger & Goldreich 1994; Ho & Lai 1999; Lai & Wu 2006; Weinberg et al. 2013; Yu & Weinberg 2017a, 2017b; Xu & Lai 2017), stellar binaries (Lai 1997; Fuller & Lai 2012c; Fuller 2017; Hambleton et al. 2018; Sun et al. 2018; Vick & Lai 2018), and planet-star systems (Ogilvie & Lin 2004; Lai 2012; Weinberg et al. 2012, 2017; Essick & Weinberg 2016; Vick et al. 2019; Vick & Lai 2018; Wu 2018).

For comparison with the numerical model, we take $\Omega_{\text{spin}} = 0$ in the following expressions. The gravitational potential produced by $M_2$ can be expanded in a spherical-harmonic basis as

$$U(r, t) = -GM_2 \sum_{ml} \frac{W_{lm}}{D(t)^{l+1}} e^{-im\Phi(t)} Y_{lm}(\theta, \phi),$$

where $r = (r, \theta, \phi)$ is the position vector in spherical coordinates with respect to the center of mass of the donor star, $M_2$, $D(t)$ is the distance between the two stars, $\Phi(t) = \int dt \Omega_{\text{orb}}$ is the orbital true anomaly and

$$W_{lm} = (-1)^{l+m}/2 \left[ \frac{4\pi}{2l+1} \right]^{1/2} \left[ \frac{2}{\sqrt{l+m}} \right]^{1/2}.$$
To leading order in the binary separation $a$, the energy transfer to the $l = m$ mode is given by

$$\Delta E_{\text{es}} = \frac{GM^2}{R_1} \left( \frac{4\pi^2}{3l^3} \right) \left( \frac{M_1}{M} \right) \times \left( \frac{a}{\dot{a}P_{\text{orb}}} \right) \frac{1}{l} \frac{W_0^2 \Omega_0}{a_t} \left( \frac{R_1}{a_t} \right)^{2l-1},$$

where $a/\dot{a}P_{\text{orb}}$ is evaluated at the resonance separation $a_t = (GM)^{1/3} (\omega_0 / l)^{2/3}$ (see Equation (3) for $l = m$), and

$$Q_\alpha \equiv \int d^3x \rho \psi_\alpha^* \cdot \nabla (r^l \psi_m),$$

which is tabulated in Table 1. In Equation (8), $\psi_\alpha$ is normalized such that $\int d^3x \rho \psi_\alpha^* \cdot \psi_\alpha = 1$, and $Q_\alpha$ is in units $G = M_1 = R_1 = 1$ (identical to the hydrodynamic model code units).

Figure 2. Slices through the simulation orbital plane at increasing time (and decreasing binary separation). The large panels in each time show density in a slice through the binary equator. The lower panels show radial velocity on a spherical surface, $r = R_1$, and the power spectrum of this velocity field, $S_{\psi l}(l)$, as defined by Equation (2). Large-amplitude modes of different azimuthal order are clearly excited in these snapshots. As the binary separation decreases and the system sweeps through various resonances, the most excited mode transitions from higher to lower azimuthal order. The 20 s animation runs from time $t - t_1 = -75.97$ to 2.63 and from binary separations of $a = 1.72$ to 0.61. The animation shows the accelerating tightening of the binary orbit. We can observe times where resonance locks the phase of oscillatory modes with the orbit, as well as times of transition between the excitations of different modes. (An animation of this figure is available.)
The amount of energy transferred to the mode dictates the size of the surface velocity perturbation \( \delta v(R_1, \theta, \phi) \). For comparison with simulations, we are interested in the radial component of the velocity \( \delta v_r(R_1, \theta, \phi) = \delta v_r(R_1) Y_{lm}(\theta, \phi) \). To relate the energy transfer and radial velocity perturbation at \( r = R_1 \), we define the mode mass

\[
M_\alpha = \int d^3x \rho \left| \frac{\xi_\alpha}{\xi_\alpha(R_0)} \right|^2 ,
\]

where \( \xi_\alpha(R_0) \) is the radial component of the Lagrangian displacement vector at the surface. The mode masses for fundamental modes of our model star are tabulated in Table 1. Note that \( |\delta v_r(R_0)| = \omega_\alpha |\xi_\alpha(R_0)| \), and we have

\[
|\delta v_r(R_0)| = \left( \frac{\Delta E_\alpha}{M_\alpha} \right)^{1/2} .
\]

We can find a rms velocity at the stellar equator by fixing \( \theta = \pi/2 \) and computing

\[
\delta v_{\text{rms}}(R_0) = \left[ \frac{1}{2\pi} \int d\phi |\delta v_r(R_0)|^2 |Y_{lm}(\pi/2, \phi)|^2 \right]^{1/2} = \frac{1}{2l} \sqrt{\frac{(2l+1)}{4\pi}} \left( \frac{\Delta E_\alpha}{M_\alpha} \right)^{1/2} .
\]

These expressions contain a complete prediction of the mode velocities at the stellar surface given a mode, \( \alpha \), and knowledge of the mode profile and its properties.

### 4.2. Comparison to Model System

In this section, we employ the result of Section 4.1 with input from our hydrodynamic model to directly compare the oscillatory surface velocities predicted by linear theory to those obtained in the simulation. As a first ingredient, Table 1 displays the calculated mode properties \( Q_\alpha \) and \( M_\alpha \) for the stellar model described in Section 2 and employed in our hydrodynamic simulation.

#### 4.2.1. Orbital Decay and Resonance Crossings

In our model binary system, orbital decay arises primarily from mass removal from the donor by the gravitational force from the accretor. As discussed in Section 3, this leads to runaway orbital tightening. This orbital decay allows the system to pass through resonances and determines the total energy that may be transferred to a given mode.

Figure 5 examines how the ongoing orbital decay drives to mode excitation. The orbit first decays slowly at large separations (large \( a/d_0 \)), then more quickly as the binary separation decreases. We note that this overall trend is due to the mass transfer and ejection from the binary, not the interaction of the modes with the orbit. However, features of slower and more rapid orbital decay can be seen in the decay timescale of Figure 5, which will be discussed further in Section 5.3. Figure 5 also plots the binary orbital period and the resonance crossing time, Equation (6). We compare these properties of the binary orbit to the timescale for a mode pattern to propagate around the whole star following excitation, \( 2\pi/l/\omega_\alpha \). Resonance crossing separations are indicated with vertical lines. We note that at resonance crossing separations, \( 2\pi l/\omega_\alpha = P_{\text{orb}} \).

At large separations, the resonance crossing timescale, \( \Delta t_\alpha \), is intermediate between the shorter orbital period and the longer orbital decay rate. Furthermore, \( \Delta t_\alpha \) is longer than the mode pattern timescale (as indicated by horizontal lines of \( 2\pi l/\omega_\alpha \)). This hierarchy of timescales is in accordance with the assumptions that lead to Equation (7). Because resonance crossing is long compared to the time that the mode takes to fully establish, we observe a fully developed mode pattern at larger separations in Figure 2. As the separation decreases, however, \( \Delta t_\alpha \) becomes similar to the orbital period. This implies a rapid sweep through resonance compared to the time for the mode to be fully excited about the star.
Comparison of Predicted to Observed Oscillation Amplitudes

Together, these properties completely specify the input for the linear theory’s predictions of the oscillatory velocity field. We use the rms velocity on the stellar equator (in the plane of orbital motion, where the tides have maximum amplitude) as our point of comparison between hydrodynamic model and linear theory in Figure 6. The equatorial rms velocity is specified by Equation (11) in the linear case. In the hydrodynamic model, we compute the rms velocity of all zones in the shell with $r = R_1$ within $\pm \pi/32$ of the equator.

Figure 6 shows that the linear theory captures both the qualitative trends and the approximate amplitude of the rms velocity field. The prediction is particularly accurate for the $l = m = 5$ modes, while somewhat overpredicting the rms velocity of the lower-order modes. This may relate to the comparison of the resonance crossing time to the orbital period. For the higher-order modes $P_{\text{orb}} < \Delta t_f$ and the mode pattern is able to fully establish around the star. Orbital decay is so rapid through the lower-order resonance crossings that $P_{\text{orb}} < \Delta t_f$, and the mode pattern is only able to partially establish around the star’s equator. Additionally, we note that the mode amplitudes are quite extreme by the time the lower-order modes are excited. In Figure 2, clear features of shocks and mode interactions with the circumbinary material are present. The fact that mode amplitudes have clearly become nonlinear may thus also play a role in the inexact prediction of the linear theory for these cases. The lower values of simulated $\delta v_{\text{rms}}(R_1)$ may therefore reflect either the partial establishment of modes given the rapid orbital decay or nonlinear effects as the mode amplitudes grow and perhaps saturate at order unity amplitudes.

In closing, we note that while modes with higher azimuthal order than $m = 2$ have been rarely considered in cases of tidal excitation, the linear model nonetheless encodes much of the key behavior when applied to these higher-order modes.

5. Discussion

Here we explore some extensions of our basic findings, including the influence of a rotating donor star, coupling between oscillatory modes and the orbital evolution, dissipation of mode energies into the donor star gas, and the potential detectability of resonantly excited modes.

5.1. Influence of Donor-star Rotation

For simplicity, we have so far considered a non-rotating donor star. However, the qualitative results of our study extend to the generalized scenario of a rotating donor with two key modifications (e.g., Lai 1997). First, the apparent forcing frequency in the frame of the stellar fluid changes. Second, rotation modifies the mode frequencies themselves.

For a donor star in solid body rotation with frequency $\Omega_{\text{spin}}$, the condition for resonance is

$$m\Omega_{\text{orb}} = \sigma_\alpha,$$

where $\sigma_\alpha$ is the mode frequency in the inertial frame. Equation (12) is equivalent to $m(\Omega_{\text{orb}} - \Omega_{\text{spin}}) = \omega_\text{rot}$, with $\omega_\text{rot}$ referring to the mode frequency in the rotating frame. In the case of synchronized rotation, $\Omega_{\text{orb}} = \Omega_{\text{spin}}$, the forcing has no apparent time variability for the stellar fluid.

Next we compute the rotational modification of the mode frequencies. When the unperturbed mode frequency, $\omega_\alpha^{(0)}$, is much greater than $\Omega_{\text{spin}}$, we can treat the effect of rotation as a perturbation. The frequency in the inertial frame is then

$$\sigma_\alpha = \omega_\alpha^{(0)} + m\Omega_{\text{spin}}(1 - C_\alpha),$$

where $C_\alpha$ is the degree of synchronization.
where

\[ mC_\alpha \Omega_{\text{spin}} = i \int d^3x \, \rho \xi^*_{\alpha} \cdot (\Omega_{\text{spin}} \times \xi_{\alpha}), \]  

which is normalized such that \( \int d^3x \, \rho \xi^*_{\alpha} \cdot \xi_{\alpha} = 1 \). The \( C_{\alpha} \) for \( l = m \) \( f \)-modes up to \( l = 6 \) of the stellar model are shown in Table 1.

In Figure 7, we examine two example cases of an asynchronously rotating donor star. The upper panel shows the separation of \( l = m \) fundamental mode resonance crossings as a function of donor spin, where \( \Omega_{\text{spin}} \) is oriented along the same axis as \( \Omega_{\text{orb}} \). As the donor star rotates faster, the azimuthal order of the mode excited at a given separation increases. The lower panels show realizations of this prediction for a donor’s spinning with 20% of the initial orbital frequency (\( \Omega_{\text{spin}} = 0.092 (GM/R)^{1/2} \); left panel) exhibits primarily an \( l, m = 8 \) oscillation; a donor star rotating with 40% of the initial orbital frequency (\( \Omega_{\text{spin}} = 0.184 (GM/R)^{1/2} \); right panel) has a higher-order wave excited with \( l, m = 12 \). These models may be compared to the non-spinning donor, which initially exhibits an \( l, m = 6 \) oscillation.

\begin{align*}
\text{Figure 7. Influence of donor rotation on resonantly excited oscillations. The upper panel shows how resonance crossings shift in separation with donor star rotation. For donors that partially corotate with the orbital motion, higher azimuthal order modes are excited at a given separation as rotational frequency increases. Two examples are shown in the lower panels: a donor star rotating at 20% the initial orbital frequency (\( \Omega_{\text{spin}} = 0.092 (GM/R)^{1/2} \); left panel) exhibits primarily a \( l, m = 8 \) oscillation; a donor star rotating with 40% the initial orbital frequency (\( \Omega_{\text{spin}} = 0.184 (GM/R)^{1/2} \); right panel) has a higher-order wave excited with \( l, m = 12 \). These models may be compared to the non-spinning donor, which initially exhibits an \( l, m = 6 \) oscillation.}
\end{align*}

5.2. Mode Dissipation

Oscillatory modes reach large amplitudes in our models, and transition between different modes as the orbit decays. Here we briefly examine dissipation associated with the evolving wave pattern. In our particular scenario, the small mode energies and angular momenta computed in Section 5.3 imply that even if the modes were entirely dissipative, they would not be expected to significantly disrupt or spin up the donor’s envelope.

\begin{align*}
\text{Figure 8 examines frames from the time of transition between an } l, m = 6 \text{ mode dominating to an } l, m = 5 \text{ mode dominating. Each row in Figure 8 shows slices through the orbital plane of one snapshot, with columns showing gas Mach number, specific entropy, and angular velocity (with all velocities measured relative to the donor core). Contours trace 1 dex intervals of density between } \rho = 10^{-5} \text{ and } \rho = 10^{-1}.}
\end{align*}
The left panels of Figure 8, in which the gas Mach number is plotted, show that oscillatory motions are supersonic, \( M > 1 \), near the donor-star limb. Mode pattern velocities are highest here, and the sound speed is lowest at the stellar limb. Supersonic motion implies that wave motions will steepen into shocks, particularly where waves crest and break. The influence of these shocks can be traced by examination of the gas specific entropy. The donor’s unperturbed envelope structure has constant specific entropy, and shocks raise the entropy of gas in surface layers along the limb of the donor. This effect is most obvious in the dramatically breaking wave seen in the lower right of the donor star in Figure 8.

Finally, the shock-heated boundary layer that forms around the donor is pulled from its initial asynchronous state, \( \Omega = v_\phi / R = 0 \), toward corotation with the orbit, \( \Omega_{\text{orb}} \approx 0.4 \). Because the higher-shock-heated boundary layer material would extend to large scale height in the absence of the accretor’s gravity, it dramatically overflows the donor’s Roche lobe and is pulled toward the accretor then expelled from the binary system. Therefore, the fact that the system is already Roche lobe overflowing implies that perturbed material is removed from the donor rather than accumulating in a steady-state surface layer. A secondary consequence is that gas that experiences dissipative evolution that pulls it toward corotation is lost from the donor, implying that the donor’s average spin rate does not evolve significantly as the binary tightens (as can, for example, be observed in the right panels of Figure 8).

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### 5.3. Mode-orbit Coupling

When the orbit sweeps through a resonance, an amount of energy \( \Delta E_\alpha \), Equation (7), is transferred from the orbit to the mode, effectively coupling the orbital and oscillatory evolutions. Figure 5 clearly shows peaks and troughs in the orbital decay rate, along with a general trend toward shorter decay time with decreasing binary separation. For example, the feature at separation between 1.6 and 1.5 donor radii represents a factor of approximately 5 slowdown in orbital decay rate. In what follows, we explore the origins of these features.

Figure 9 compares the total orbital energy change during resonance crossings to the energy transfer to the mode predicted by the linear model. The former is computed by multiplying the orbital decay rate, as shown in Figure 5, by the resonance crossing time, \( \Delta t_\phi \). We observe that the total model orbital decay is always significantly larger than that which can be attributed to the modes. This conclusion implies that, in our situation, rapid non-conservative mass transfer instead drives the bulk of the orbital decay (as discussed in MacLeod et al. 2018a, 2018b) and that tidal exchange with oscillatory motions in the stellar envelope has a negligible effect on the orbit.

However, upon examination of the model snapshots, the explanation for the modulation of the orbital decay rate seen in Figure 5 becomes more clear. Because the binary separation is small, and the mode amplitudes become large, the accretor begins interacting strongly with the perturbed stellar density distribution. Figure 10 examines the torque density on the orbit from donor-star material in the orbital plane. Green-colored material accelerates the binary’s orbital motion (positive torque density), while pink-colored material decelerates the orbit (negative torque density). Imbalance of the otherwise nearly symmetric distribution of positively and negatively torquing material generates a net torque that drives the orbital evolution (for more context, see Section 4.3.3 of MacLeod et al. 2018a).

![Figure 9](image)

**Figure 9.** Fractional orbital decay generated during resonance crossings. We compute the total orbital energy change during a given resonance crossing time, \( \Delta t_\phi \) in our numerical model as \( \hat{E}_{\text{orb}} \), and compare it to the deposition of energy into oscillatory motions, \( \Delta E_\alpha \). This comparison shows that the energy deposited into modes accounts for only a small fraction of the orbital evolution. It therefore cannot explain the large-amplitude modulations of the orbital decay rate observed in Figure 5.

The frames of Figure 10 show similar times to those of Figure 8, in which the model transitions from an \( l = m = 6 \) polygram to an \( l = m = 5 \) polygram shape. In Figure 10, we label the wave peaks (a) and (b), and mark their merged peak in the final panel as (a/b). This transition in dominant mode is
accompanied by dissipation, as discussed in the previous section, and results in transient torques on the accretor’s orbital motion. While approaching wave peak (a) the excess positive torque from (a)’s mass distribution accelerates the orbital motion and slows the orbital decay rate (upper two panels of Figure 10, $a = 1.58$ and $a = 1.55$). In the final frame of Figure 10, wave peak (a) passes the location of the accretor (equivalently, the $y' = 0$ axis). Mode peaks (a) and (b) combine. The excess negative torque from (a/b)’s mass distribution enhances the orbital decay rate.

5.4. Comments on Potential for Observability

Stellar coalescence events occur in the Galaxy at a rate of order 0.5 per yr (Kochanek et al. 2014). Brighter events involving more massive stars occur at lower specific rates but are observable in other Local Group galaxies, raising predicted detection rates to tens per year with contemporary surveys (e.g., Adams et al. 2018). The fraction that will be sufficiently asynchronous to develop signatures of resonantly excited dynamical tides depends on the details of tidal dissipation and is, therefore, considerably more uncertain.

Two scenarios should preferentially lead to conditions that generate asynchronous rotation and polygram tidal excitation. In cases of extreme mass ratio between the donor and accretor stars there may not be sufficient angular momentum for the binary to reach a synchronous rotation state (Darwin 1879). This case is of particular relevance to mergers involving giant stars and compact objects or other stellar remnants—which often have masses much less than their progenitors (one such scenario is a neutron star-giant star interaction, e.g., Levesque et al. 2014). Massive stars in binaries may evolve on timescales more rapid than their tidal dissipation timescales, allowing for asynchronicity during mass exchange even when the binary is not Darwin unstable (Savonije & Papaloizou 1983). Measuring the fraction of stellar coalescence events exhibiting these polygram tidal signatures is thus one avenue toward constraining tidal dissipation models. In the remainder of this section, we discuss several potentially observable consequences of resonantly excited modes.

Although the waves in our model coalescing binary systems have very high amplitudes, our ability to unambiguously detect their presence is not guaranteed. Photometric variability will be a consequence of the non-radial oscillations excited in our model systems. We might expect this signature to be particularly observable in situations where the binary is not vigorously losing mass, such that the donor-star surface still remains observable. While non-radial stellar oscillations are routinely observed and used for astroseismic analyses, the typical modes are of low azimuthal order. Higher-order modes suffer increasing geometric cancellation when the distant star is observed as a point source. We consider two possible avenues for photometric detection—direct detection of higher azimuthal order oscillations and modification of ellipsoidal variations.

The photometric amplitude at wavelength $\lambda$ of an isolated oscillatory mode can be approximated as

$$A_\lambda \approx \text{abs}[\varepsilon b_\lambda^l(2 + l)(1 - l)],$$

where $\varepsilon$ is the wave’s radial displacement amplitude $8R/R = \varepsilon \lambda_{\text{in}}(\theta, \phi)$, $b_\lambda^l$ is a stellar disk averaging factor, and $(2 + l)(1 - l)$ is the geometric influence on the observed flux. This expression is reasonable for $l \gtrsim 6$, where the geometric modulation of stellar flux is dominant over temperature or surface gravity modulations (Daszyńska-Daszkiewicz et al. 2002). We also adopt the approximation of a system viewed edge-on. For the full expression, see Daszyńska-Daszkiewicz et al. (2002, Equation 17). The disk averaging factor decreases with increasing $l$, but is approximately compensated by $(2 + l)(1 - l)$. We arrive, very roughly, at $A_\lambda \approx 0.1\varepsilon$, for even $l$, in the range $l = 6$ to $l = 12$ (with factor of a few variation depending on $\lambda$). Odd $l$s suffer greater disk averaging and are a factor of nearly 10 further suppressed (see Daszyńska-Daszkiewicz et al. 2002, Figure 2).

Given the relatively large fractional amplitudes, $\varepsilon$, of the tidally driven waves that arise in our models, photometric detection of similar waves in real stars would be possible, with $A_\lambda \approx 0.01$ for an even $l$ mode with radial displacement $\varepsilon = 0.1$. For a resonant mode, the frequency is set by the resonance condition. The associated photometric variation imprint on the light curve of a binary as a higher frequency modulation with frequency $m \Omega_{\text{orb}}$. Space-based facilities like Kepler and TESS can achieve better than milli-magnitude precision for bright stars, implying the ability to measure or constrain an oscillatory modes with amplitude $\varepsilon \gtrsim 0.01$.

Were high-order oscillations themselves unobservable, it might still be possible to constrain a departure from the expectation of a system exhibiting an equilibrium tide. In a synchronous system, the projection of the Roche contours of equipotential result in “ellipsoidal” variations in the light curve due to the projection of the star’s $l$, $m = 2$ distortion. Because the bulk of oscillatory power in our models is not in the quadrupole distortion, we expect deviations from the baseline prediction for a synchronously rotating system exhibiting an equilibrium tidal distortion. At a given orbital period, we expect less amplitude in the quadrupole mode when the binary is asynchronous compared to synchronous. If the other parameters of the binary system were otherwise constrained (masses, separation, stellar radii—as is, for example,
possible in an eclipsing binary with radial velocity data), it might be plausible to confirm such a deviation.

Other possible avenues for detectability include spectral signatures of atmospheric disturbance by waves, either in the form of line profiles modulated by the surface radial velocity variations (e.g., Arras et al. 2012), or emission associated with shock-heated gas. Typical velocities would be expected to be on the order of $\delta v_{\text{rms}}$, as shown in Figure 6, and therefore may be a large fraction of the stellar escape velocity, creating very broad lines. However, atmospheric signatures may only be visible in the scenario of a non-mass-transferring binary because mass exchange would complicate gas kinematics and line profiles.

Finally, we note that the nonlinear modulation of the orbital decay rate, noted in Section 5.3, might be traceable in binary systems that are seen as eclipsing binaries prior to coalescence. A challenge in the application of this method would be an accurate determination of the binary orbital period over a relatively short duration. Nonlinear mode-orbit interactions occur over a timescale less than or similar to an orbital period. This implies that such a measurement would require constraining the orbit of a single cycle, which is generally much less accurate than if a longer time baseline can be accrued and requires high-cadence observations.

A known binary that may serve as a test case for some of these ideas is V1309 Sco, which underwent an outburst in 2008 (Mason et al. 2010) but was serendipitously monitored in the years prior and was shown to be an eclipsing binary with decreasing orbital period (Tylenda et al. 2011). As such, this system presents the strongest known evidence for a stellar-coalescence-driven outburst. Constraints from the eclipsing light curve suggest a relatively asymmetric mass ratio $q \approx 0.1$ (Zhu et al. 2016), and it has therefore been suggested that the system was driven into merger by asynchronous donor rotation and the Darwin tidal instability (Nandez et al. 2014). The pre-outburst light curve, from OGLE, is strongly affected by mass loss from the binary (Pejcha 2014; Pejcha et al. 2017). However, Tylenda et al. (2011) stated that the typical photometric precision is 0.01 mag, implying that a search for oscillations at integer multiples of the orbital frequency with amplitude $\epsilon \gtrsim 0.1$ is possible for this source.

6. Conclusion

We analyze models of binary coalescence in which the donor star is asynchronously rotating and show that high azimuthal order oscillatory modes of the stellar envelope are resonantly excited, making the equatorial structure of the star effectively a polygram. A few key findings of our study are as follows:

1. As the orbital separation shrinks in the binary’s trend toward coalescence, the system sweeps through resonance with different oscillatory modes. At the largest separations, high-order (high $m$) modes are excited, and the resonant-mode order decreases as the orbit tightens (Figures 1 and 2).

2. In a rapidly decaying binary system, the peak-excitation of a mode lags the crossing of the maximal resonance separation (Figure 4), but the oscillatory amplitude predicted by linear theory is otherwise accurate to within a factor of two even as the modes attain quite large amplitudes (Figure 6).

3. Stellar rotation affects mode excitation, as explored in Section 5.1. At a given binary separation higher-order modes are excited as the stellar rotation rate increases (see Figure 7).

4. Directly observing the photometric imprint of high azimuthal order modes is possible but challenging. We estimate that the photometric amplitude of even $l$ modes is $A_\lambda \approx 0.1\epsilon$ for high-order modes. Therefore, milli-magnitude level photometry could constrain the presence of any modes with fractional amplitude greater than 1%, $\epsilon \gtrsim 0.01$. Modes in resonance would exhibit a frequency that is an integer multiple of the orbital frequency.

We have considered a restricted scenario of a donor star with an isentropic envelope transferring mass and coalescing with a lower-mass companion. However, the qualitative features of our results should be applicable to a wide range of astrophysical systems. Non-isentropic structures, such as those of stars with radiative envelopes, also support g-modes, which may be resonantly excited (e.g., Lai 1994; Fuller & Lai 2011, 2012; Vick & Lai 2018). Our initial testing via simulation models indicates that high azimuthal order modes may be selectively excited in these cases as well, and the linear model described in section 4.1 provides a framework for exploring this more fully. In addition, when considering red giant stars, fundamental modes excited in a star’s envelope could, in principle, couple and mix with $g$-modes of similar frequency in the star’s radiative cores, a topic that may be interesting to explore in future work (e.g., Dziembowski et al. 2001). Other astrophysical scenarios of possible interest and applicability for our findings include close-in planetary perturbers such as hot Jupiters and the plunge toward merger in double neutron star systems (For example, see Figure 24 of Baiotti & Rezzolla 2017 and Stergioulas et al. 2004).

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Software: Athena++, Stone et al. (in preparation) http://princetonuniversity.github.io/athena, Astropy Collaboration et al. 2013), GYRE (Townsend & Teitler 2013; Townsend et al. 2018), shtools (Wieczorek et al. 2018).
The magnitude of the Lagrangian displacement, of the star occurs at $r = 0.3 R_1$. The right boundary of the convective envelope is at $r = 0.3 R_1$ and $m(r) = 0.43 M_1$, where $m(r)$ is the enclosed mass within radius $r$, similar to the simple stellar model in the hydrodynamic simulations, which has the effect of preventing mode propagation in the unmodeled regions (see Section 2).

Figure 11 shows that in the MESA model, $g$-modes propagate in the radiative interior, where the Brunt Väisälä buoyancy frequency is real, where $N = \sqrt{\frac{1}{\rho} \frac{d\rho}{dp} - \frac{\rho}{\Gamma_1 \rho}}$, in which $\Gamma_1$ is the first adiabatic index. Because $N$ becomes imaginary in the convective exterior, only fundamental and pressure modes propagate; $g$-modes do not propagate in this region. Pressure and fundamental modes, whose dispersion relation is governed by the Lamb frequency, $S_\ell^2 = \ell (\ell + 1) c_s^2 / r^2$, where $c_s$ is the local sound speed, propagate throughout the star. The right panel of Figure 11 shows the normalized Lagrangian mode amplitudes for the $l = 2$ through $l = 6$ fundamental modes of the MESA model. These modes retain non-zero amplitudes in the radiative inner region, $r < 0.3 R_1$, but they are fractionally small compared to the surface amplitudes. This is especially true for the higher azimuthal orders, which have vanishingly small amplitudes at small radii. The fact that fundamental mode amplitudes are maximized near the stellar surface validates our simplifying approach of removing the stellar core from our hydrodynamic models.

For comparison, the polytrope model and the stellar model adopted here to the properties of a more realistic giant star model generated with the MESA stellar evolution code (Paxton et al. 2011, 2013, 2015, 2018). This model adopts $M_1 = 4 M_\odot$ and is evolved until $R_1 = 35 R_\odot$. At this point, the radiative core extends to $r = 0.3 R_1$ and $m(r) = 0.43 M_1$, where $m(r)$ is the enclosed mass within radius $r$, similar to the simple stellar model in the hydrodynamic simulations, which has the effect of preventing mode propagation in the unmodeled regions (see Section 2).

Table 2 lists the mode properties. These were calculated with GYRE using a boundary condition of zero-radial displacement at $r = 0.2 R_1$ (interior to the convective envelope but outside of the helium core), and without the Cowling approximation.

Figure 11 shows that in the MESA model, $g$-modes propagate in the radiative interior, where the Brunt Väisälä buoyancy frequency is real, where $N = \sqrt{\frac{1}{\rho} \frac{d\rho}{dp} - \frac{\rho}{\Gamma_1 \rho}}$, in which $\Gamma_1$ is the first adiabatic index. Because $N$ becomes imaginary in the convective exterior, only fundamental and pressure modes propagate; $g$-modes do not propagate in this region. Pressure and fundamental modes, whose dispersion relation is governed by the Lamb frequency, $S_\ell^2 = \ell (\ell + 1) c_s^2 / r^2$, where $c_s$ is the local sound speed, propagate throughout the star. The right panel of Figure 11 shows the normalized Lagrangian mode amplitudes for the $l = 2$ through $l = 6$ fundamental modes of the MESA model. These modes retain non-zero amplitudes in the radiative inner region, $r < 0.3 R_1$, but they are fractionally small compared to the surface amplitudes. This is especially true for the higher azimuthal orders, which have vanishingly small amplitudes at small radii. The fact that fundamental mode amplitudes are maximized near the stellar surface validates our simplifying approach of removing the stellar core from the hydrodynamic models. The $l = 2$ mode has a fractional Lagrangian displacement of approximately 10% at $0.3 R_1$ and is thus most strongly affected by our simplification of excising the radiative interior of the stellar model in the hydrodynamic simulations, which has the effect of preventing mode propagation in the unmodeled regions (see Section 2).

The mode properties of the MESA model are in good agreement with those of the simple stellar model used in the paper (see Table 1). In particular, the mode frequencies of the MESA model differ from the simplified model by at most 7% (for the $l = 2$ mode). The most significant difference between the models is that the overlap integral, $Q_\alpha$, is larger for the MESA model, particularly for lower degree modes. This implies somewhat stronger coupling between modes and the tidal field in the MESA model than in our simplified model.

Together these properties indicate that the simplified, polytropic stellar model (described in Section 2 and used throughout this paper) compares quite favorably to a more realistic stellar model with otherwise similar properties. We therefore expect that the qualitative feature of resonant tidal
excitation of high azimuthal order modes and polygram stellar distortions will be robust against changes in the particulars of a given star’s structure.

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