Analysis Methods for the Practical Application of Fracture Mechanics

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Abstract. Advances in numerical methods have enabled detailed fracture mechanics calculations to be employed in practice. However, more often simplified analysis methods are applied as these often enable adequate margins to be demonstrated. This paper describes detailed and approximate methods for low temperature fracture and creep fracture assessments. Comparisons of the two approaches are given demonstrating that high accuracy can be obtained with simplified methods even for complex applications.

1. Introduction
When a crack is found or postulated in a component, it is necessary to estimate relevant fracture mechanics parameters in order to determine if the component is fit-for-service [1]. When the material response is elastic, estimation of the stress intensity factor, the linear elastic fracture mechanics parameter, is straightforward and comparison with the corresponding fracture toughness enables an assessment of fitness-for-service to be made. For elastic-plastic or creep material response, non-linear fracture mechanics is required and estimation of the parameters controlling fracture at the crack tip is more complex. However, methods for estimating such parameters from detailed finite element analyses have been developed so that non-linear fracture mechanics methods can now be applied in practice [2, 3].

The detailed finite element analysis methods and the associated materials data requirements are expensive and time consuming to use. Therefore, simplified approximate methods have been developed for practical application of inelastic fracture mechanics to defect assessment of plant components [4, 5]. Often, these approximate methods are sufficient to demonstrate adequate margins on limiting defect size or limiting load.

This paper first sets out the background to the fracture mechanics methods. Then the detailed finite element approach and the simplified approaches for estimating the corresponding fracture mechanics parameters are presented. Finally, axi-symmetric and three-dimensional (3-D) finite element (FE) analysis results are presented to illustrate some aspects of component response and to compare the finite element and simplified approaches. Both elastic-plastic and creep fracture approaches are addressed and attention is focussed on combined mechanical and thermal loading.

2. Fracture Mechanics Methods
There are three general classes of fracture mechanics methods. The first class is energy methods as pioneered by Griffiths for brittle fracture but subsequently applied more widely [6]. The second class is the use of parameters which describe the stress and strain (or strain rate) near the crack tip and
which are therefore assumed to control fracture [5, 7, 8]. Finally, micromechanical models of brittle,
ductile or creep fracture can be used with detailed stress analysis to make a direct prediction of crack
tip response [9, 10].

This section concentrates on the second class, the use of parameters which describe conditions in
the region of the crack tip. First, Section 2.1 describes single-parameter fracture mechanics, the most
commonly applied approach. Then Section 2.2 briefly describes two-parameter methods.

2.1. Single Parameter Fracture Mechanics

For a linear elastic material under Mode I loading, the stress tensor \( \sigma \) in polar co-ordinates \((r, \theta)\)
centred at the crack tip may be written as

\[
\sigma_{ij} = \frac{K}{(2\pi)^{1/2}} g_{ij}(\theta) + T \delta_{ij} + O(r^{1/2})
\]

as \( r \to 0 \). Here, \( g_{ij} \) are angular functions of \( \theta \), \( \delta_{ij} \) is Kronecker’s delta and \( K \) is the stress intensity
factor; the second-order \( T \) stress term can be regarded as the stress parallel to the crack flanks and is
discussed further in Section 2.2. This second-order term is generally neglected and elastic fracture is
described by the single parameter \( K \) which controls both the stress and strain fields near the crack tip.

For non-linear elastic materials in which strain, \( \varepsilon \), is related to stress, \( \sigma \), by

\[
\varepsilon = \alpha \varepsilon_Y (\sigma / \sigma_Y)^n
\]

where \( \alpha, \varepsilon_Y, \sigma_Y \) and \( n \) are constants, the stress field near the crack tip as \( r \to 0 \) may be written as

\[
\frac{\sigma_{ij}}{\sigma_Y} = \left( \frac{J}{\alpha \sigma_Y \varepsilon_Y I_0 r} \right)^{1/(n+1)} \tilde{\sigma}_{ij}(\theta, n) + Q \tilde{\sigma}_{ij}, \quad |\theta| < \frac{\pi}{2}
\]

where \( I_n \) and \( \tilde{\sigma}_{ij} \) are functions of \( n \) as determined by the asymptotic analyses of Hutchinson [7], Rice
and Rosengren [8]. Again, if the second-order term \( Q \) is neglected, fracture is controlled by the single
parameter \( J \).

The analogy between widespread plasticity and steady state creep [5] means that if creep strain
rate, \( \dot{\varepsilon}^c \), is related to stress by

\[
\dot{\varepsilon}^c = \dot{\varepsilon}_0 (\sigma / \sigma_0)^n
\]

where \( \dot{\varepsilon}_0, \sigma_0 \) and \( n \) are constants, the first-order term in the stress field at the crack tip is given in a similar
manner to eqn (3) by

\[
\frac{\sigma_{ij}}{\sigma_0} = \left( \frac{C^*}{\sigma_0 \dot{\varepsilon}_0 I_0 r} \right)^{1/(n+1)} \tilde{\sigma}_{ij}(\theta, n)
\]

where \( C^* \) is the steady state creep parameter. On initial loading, the crack tip fields are given by eqn
(1), for elastic response, or eqn (3) for elastic-plastic response. Subsequently, there is a transient
period during which stress redistribution occurs until the steady state conditions of eqn (5) are reached.
During this transient period, the stress and strain rate fields are described by eqn (5) but with
\( C^* \) replaced by the transient parameter \( C(t) \) where \( C(t) \geq C^* \).
Either detailed numerical methods or simplified methods are used to evaluate the single parameters 
K, J, C* and C(t) as functions of the applied loadings, material properties and crack size and shape. 
These are then related to corresponding material properties, which are obtained from test specimens 
that control fracture or crack growth. For linear material response, K is directly proportional to the 
applied load and independent of the material properties and there are many solutions available for 
evaluation of K [1]. Evaluation of J, C* and C(t) is more complex and methods for estimating these 
parameters are set out in Sections 3 and 4.

2.2. Two Parameter Fracture Mechanics
It has been found that material fracture response is generally not simply defined by the first-order 
parameters discussed in Section 2.1. Instead fracture toughness and creep crack growth have been 
found to be influenced by the nature of the loading, crack size and shape. These influences may be 
conveniently described in terms of the second-order parameters T and Q in eqns (1, 3). Conservative 
fracture mechanics assessments may be made by neglecting these second-order effects and obtaining 
materials data from deeply cracked bend specimens in which T, Q>0 tending to minimise the fracture 
resistance, or maximise the creep crack growth rates. These conditions are referred to as “high 
constraint”. For so-called low constraint conditions, such as shallow cracks and tension loading, for 
which T and Q are significantly negative and reduce the crack tip stress fields from the high constraint 
fields, the fracture resistance can be significantly higher. Although these second-order effects are not 
considered in detail in this paper, they are briefly discussed here.

Although T is an elastic parameter, it has been argued that it can be used to quantify crack tip 
constraint under elastic-plastic conditions [11]. As T can be evaluated by elastic analysis, it is 
relatively straightforward to determine compared to Q. Therefore, there are a number of solutions 
available in the literature. A compendium of these solutions has been compiled in [12] for two-
dimensional test specimen geometries, some axisymmetric geometries and for surface cracked plates 
under tension and bending. A more comprehensive collation for 1-D cracks has been produced by Fett 
[13]. This includes tabulations of weight functions for a number of cases, enabling the T-stress to be 
evaluated for non-linear thermal or residual stresses as well as uniform tension or linear bending stress 
distributions.

Unlike eqn (1) for elastic response, eqn (3) is not a strict asymptotic analysis of the higher-order 
terms for the non-linear problem. Instead, the hydrostatic term, Q, is an approximation to the 
collective behaviour of a number of higher-order terms in the forward sector,|θ| < π/2 , ahead of the 

3. Finite Element Approaches
Finite element analysis may be used to aid fracture mechanics assessments in a variety of ways. These 
include:
• Linear elastic analysis of the uncracked structure to determine the stresses at the position where a 
  crack is to be assessed, for input to a weight function solution to calculate K or T for a range of 
  crack sizes.
• Linear elastic analysis of the cracked structure to determine K or T for the crack size of interest.
• Inelastic analysis of the cracked structure to determine J, C(t) or Q for the loading, material and 
  crack size of interest.

This last use of finite element analysis is examined in more detail here for evaluation of J or C(t). 
It can be seen from eqns (3, 5) that in principle these parameters can be determined from the amplitude 
of the stress fields at the crack tip. However, this requires material behaviour to be described by the 
simple forms of eqns (2, 4) and requires very fine finite element meshes at the crack tip to capture the
asymptotic behaviour. Therefore, more generally, the crack tip parameters are obtained from contour integrals. For example, Rice [17] showed that a path-independent integral could be used to define J so that it was not necessary to determine the crack tip fields with high accuracy very close to the crack tip. However, this early definition of J did not allow for thermal or residual stresses in the structure and was limited to proportional loading. Extensions to determine J under more general loading conditions have been developed [2, 18] and to demonstrate that the resulting stress fields are consistent with eqn (3). Thus, it is now possible to perform detailed finite element analysis to quantify the crack tip stress field as a function of the loading applied to the structure. This is illustrated in Section 5.

For time-dependent creep calculations, finite element values C(t) can be extracted as a function of time t from a contour integral defined by

\[ C(t) = \int_{\Gamma} \mathbf{W} \, d\mathbf{y} - T_i \left( \frac{\partial \mathbf{u}_i}{\partial x} \right) \, ds \]  \hspace{1cm} (6)

where \( T_i \) are components of the traction vector, \( \mathbf{u}_i \) are the displacement rate vector components, and \( \mathbf{W} \) is strain energy rate density. At small times, a transient creep condition occurs, during which the stress redistributes within the cracked body. Under this condition, the C(t)-integral results exhibit significant path dependence and according to the definition in eqn (6), C(t) values should be extracted close to the crack tip. Therefore, care is needed in selecting a suitably refined mesh at the crack tip and highly refined meshes have been used to obtain the results presented later in Section 5. At longer times, when the stress has re-distributed within the cracked body, a steady state condition is achieved, where path-independence occurs and eqn (6) can be replaced by:

\[ C^* = \int_{\Gamma} \mathbf{W} \, d\mathbf{y} - T_i \left( \frac{\partial \mathbf{u}_i}{\partial x} \right) \, ds \]  \hspace{1cm} (7)

which defines the steady state stress field in eqn (5). This integral is path independent and therefore does not need to be evaluated close to the crack tip.

4. Simplified Methods

While the detailed numerical methods outlined in Section 3 can be followed, the results are dependent on the particular loading, crack size and material properties input to the finite element analysis. Thus, where it is required to determine margins on loadings or limiting defect sizes the methods are time consuming and expensive to use. Therefore, simplified methods which can provide more rapid assessments are attractive. Here, simplified methods for estimating J and C(t) for combined mechanical and thermal stresses are presented. Comparisons between these simplified methods and application of the detailed numerical approaches of Section 3 are then given in Section 5.

For the general loading case considered here is to necessary to quantify the effects of combined mechanical and thermal loading applied at time \( t=0 \) on the crack tip stress field as characterized by the elastic-plastic J (Section 4.1). Then, it is necessary to characterise the subsequent variations of C(t) with time \( t(>0) \) under transient creep conditions (Section 4.2).

4.1 Estimation of J

Reference stress methods [4] were developed a number of years ago to estimate, under mechanical loading only, the elastic-plastic value of J for a cracked component, denoted \( J^p \), from

\[ \frac{J^p}{J^p_e} = \frac{E \varepsilon^p_{ref}}{\sigma^p_{ref}} + \frac{1}{2} \frac{L^2}{E \varepsilon^p_{ref}} \sigma^p_{ref} \]  \hspace{1cm} (8)
where the reference stress, $\sigma_{\text{ref}}^p$, and the parameter $L_r$ are defined by the plastic limit load of the cracked component of interest, $P_L$:

$$L_r = \frac{\sigma_{\text{ref}}^p}{\sigma_y} = \frac{P}{P_L} \quad (9)$$

In eqn (8), the reference strain $\varepsilon_{\text{ref}}^p$ is defined as the true strain at the reference stress, determined from true stress-strain data of the material, and $J_e^p$ denotes the elastically calculated value of J due to primary stress, defined by the stress intensity factor $K$ due to primary stress, $K^p$:

$$J_e^p = \left(\frac{K^p}{E'}\right)^2$$

where $E' = E / (1 - \nu^2)$ for plane strain, $E' = E$ for plane stress; and $E$ and $\nu$ denote Young’s modulus and Poisson’s ratio, respectively.

When the cracked component is subject to combined primary and secondary stress, the elastic $J$ for combined primary and secondary stresses, $J_{e+ss}$ can be obtained by linear superposition:

$$J_{e+ss} = \left(\frac{K^p + K^s}{E'}\right)^2$$

where $K^s$ is the stress intensity factor for the secondary stresses. If eqn (11) was used to replace $J_e^p$ in eqn (8), the resulting estimate of J could significantly overestimate the actual J, due to mechanical stress relief of the secondary stresses. Therefore, a multiplying factor $V$ has been introduced [19] to estimate $J$ due to combined primary and secondary stresses, $J_{e+ss}$, from

$$J_{e+ss} = \frac{\left(\frac{K^p + V \cdot K^s}{E'}\right)^2}{\left[\frac{E \varepsilon_{\text{ref}}^p}{\sigma_{\text{ref}}^p} + \frac{1}{2} L_r^2 \frac{\sigma_{\text{ref}}^p}{E \varepsilon_{\text{ref}}^p}\right]}$$

It can be seen that the value of $V=1$ indicates a full effect of the secondary stress on the estimated J, whereas $V=0$ corresponds to the secondary stress being completely relaxed and not contributing to the elastic-plastic value of J. Methods for estimating $V$ as a function of the magnitudes of the primary and secondary loadings are given in [19].

The above elastic-plastic approach has been presented in terms of an estimate of J. An alternative and fully equivalent approach [1] is to present the analysis in terms of a Failure Assessment Diagram (FAD) [20]. Such approaches are not described in detail here but it is worth noting that these are very convenient methods for practical fracture mechanics assessment.

4.2 Estimation of $C(t)$

Reference stress methods [4] estimate the steady state creep parameter from

$$C^* = \sigma_{\text{ref}}^p (a) \varepsilon_{\text{ref}}^c [\sigma_{\text{ref}}^p (a), \varepsilon_{\text{ref}}^c] (K^p / \sigma_{\text{ref}}^p)^2$$

(13)
where $\sigma^p_{ref}$ is the reference stress for the primary loading as in Section 4.1 and $\dot{\varepsilon}_{ref}^c$ is the creep strain rate at the current reference stress and creep strain, $\varepsilon_{ref}^c$. Eqn (13) is analogous to eqn (8) apart from a small-scale yielding term in eqn (8).

For combined primary and secondary loading, if the initial response on loading is elastic, an initial reference stress, $\sigma_{ref}^0$, can be defined by

$$\sigma_{ref}^0 = \sigma^p_{ref} (K^p + K^s)/K^p$$

(14)

The reference stress may also be defined from the stress resultants for combined loading using an equation similar to eqn (9). This is not discussed in detail here but is particularly suitable for loadings with significant stresses not in the plane of the defect. The combined reference stress of eqn (14) may relax due to both creep straining and crack growth and the rate of change is given by

$$\dot{\sigma}_{ref}^p + \frac{K^s}{(K^p + K^s)} \left[ \frac{\partial K^p}{\partial (a/w)} - \frac{\partial K^s}{\partial (a/w)} \right] + \frac{\partial \sigma^p_{ref}}{\partial a/w} = \frac{\dot{a}}{w} + \frac{\dot{\varepsilon}_{ref}^c}{Z\sigma_{ref}} = 0$$

(15)

for a crack of depth $a$ in section width $w$, where $Z$ is the elastic follow-up factor and the creep strain rate is calculated at the current combined reference stress. Evaluation of the elastic follow-up factor is discussed later in Section 5.

Equations (14, 15) enable the initial reference stress and its relaxation to be determined. Omitting algebraic details, the resulting value of the transient crack tip characterising parameter, generalised to the case of combined primary and secondary loading involving stress relaxation due to both creep and crack growth and plasticity on initial loading is

$$C(t)/C^* = \left( \frac{\sigma_{ref}}{\sigma^p_{ref}} \right) \left[ \frac{\varepsilon_{ref}^c / \varepsilon_{ref}^c}{(\varepsilon_{ref}^c / \varepsilon_{ref}^c)^{n+1} - (\sigma_{ref}^0 / \varepsilon_{ref}^c)^{n+1}} \right]$$

(16)

where $\dot{\varepsilon}_{ref}^c$ and $\dot{\varepsilon}_{ref,p}^c$ are the creep strain rates at $\sigma_{ref}$ and $\sigma^p_{ref}$, respectively, $\varepsilon_{ref}$ is the total strain at $\sigma_{ref}$. $C$ refers to the value evaluated for the primary loading only, and $\varepsilon_{ref}^c$ is the total elastic-plastic strain corresponding to the initial value of the total reference stress $\sigma_{ref}^0$. For pure primary loading, it is straightforward to evaluate the strain terms in eqn (16) as the reference stress is well defined from the limit load expression of eqn (9). For more general loading, the initial strain term may be obtained from an estimate of the initial value of $J$, $J_0$, using the method discussed in Section 4.1.

5. Results

In this section the approaches of Sections 3 and 4 are applied to a cylindrical geometry subjected to a variety of combined mechanical and thermal stresses. The geometry and loadings are first presented in Section 5.1. Results for $J$ are presented in Section 5.2 and results for $C(t)$ in Section 5.3.

5.1 Geometry and Loading

Finite element analyses have been performed for circumferentially cracked cylinders under combined mechanical and thermal loading. The pipe has mean radius, $r$, and thickness, $t$, with $r/t=10$. Circumferential through-wall and part-through surface cracks are considered, as shown in Fig. 1. The circumferential through-wall crack is characterized by the relative crack length, $\theta/\pi$, where $\theta$ denotes the half crack angle (Fig. 1b). The circumferential part-through surface cracks have a rectangular shape, i.e., constant depth, characterized by the relative crack depth, $a/t$. 

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The mechanical loading was applied as an axial tension $N$. Three different types of temperature variation were applied to generate thermal stresses. The first, termed a “radial temperature gradient”, has a temperature gradient through the pipe wall, as shown in Fig. 2a, and produces through-wall bending stresses. The second type has a gradient along the longitudinal direction to produce global bending stresses, as shown in Fig. 2b, and is termed an “axial temperature gradient”. The final type has a temperature gradient across the cross-section, as depicted in Fig. 2c. This type of thermal loading produces global bending stresses, and is termed a “sectional temperature gradient”. The magnitude of the temperature difference, $\Delta T_{\text{max}}$, was adjusted to produce different values of the thermal stress intensity factor.

Plastic strains were described by eqn (2) with $n=5$ or $10$. Creep strain rates were defined by eqn (4), again with $n=5$ or $10$. For elastic-plastic-creep analyses, the value of $n$ was taken to be the same for both plasticity and creep.

5.2 Results for elastic-plastic response

Rather than presenting results for $J$, it is more convenient for the combined loading cases examined to present the results in terms of the parameter $V$ in eqn (12). Combining eqns (8, 12) leads to

\[
V = \frac{J_{p}^{s}}{J_{e}^{s}} \left( \frac{J_{p}^{s+}}{J_{p}} - 1 \right)
\]

(17)

where $J_{e}^{s}$ is the elastic value of $J$ when only the secondary stress is applied. In this case, the elastic-plastic $J$, $J^{s}$, can be written in terms of an initial value $V_{o}$ of $V$ as

\[
J^{s} = \left( V_{o} \cdot K^{s} \right)^{2} / E^{s}
\]

(18)

Then eqn (17) becomes

\[
\frac{V}{V_{o}} = \sqrt{J^{s}} \left( \frac{J_{p}^{s+}}{J_{p}} - 1 \right)
\]

(19)

The finite element results for the various $J$ estimates in eqn (19) have been used to express $V/V_{o}$ for the three types of thermal loading as functions of the level of axial stress and some results are shown in Figs 3-5. More comprehensive results which show similar trends are given in [21]. It is noteworthy that the results, when presented in the form of Figs 3-5, are insensitive to crack size and the type of thermal loading. Note that the normalising load, $N_{\text{OR}}$, in these figures is slightly different from the limit load in eqn (9) and has been chosen to ensure that eqn (8) is accurate for axial load acting alone. Thus, the comparisons in Figs 3-5 address the influence of combined loading on $J$. Also shown in these figures are simplified estimates of $V/V_{o}$ given in the R6 defect assessment procedure [20]. It can be seen that these are generally conservative. An alternative less conservative estimate [21]

\[
\frac{V}{V_{o}} = \left[ \frac{E_{\text{ref}}^{p}}{\sigma_{\text{ref}}^{p}} + \frac{1}{2} L_{i}^{2} \frac{\sigma_{\text{ref}}^{p}}{E_{\text{ref}}^{p}} \right]^{-1/2}
\]

(20)

is also shown in Figs 3-5. This is similar to the equation used to estimate $J$ in eqn (8) and it can be seen that this enables accurate but not overly conservative estimates of $V$, and hence $J$ (see eqn (19))
for combined primary and secondary loading. The results in Figs 3-5 have been presented for different magnitudes of the secondary loading as expressed by the ratio

\[ \beta = \frac{K^r L_r}{K^p} \]  

(21)

with Fig 3 having a relatively small secondary stress, Fig 4 corresponding to a large radial temperature gradient and Fig 5 corresponding to a modest sectional temperature gradient. It is apparent that the results are similar in all cases when normalised as presented in the figures, confirming that approximate estimation methods can be used.

5.3 Results for creep response
Following the initial loading which gave the results for J reported in Section 5.2, the finite element analyses were continued with creep relaxation of the secondary stresses occurring. The resulting values of C(t) evaluated according to eqn (6) fell rapidly with time with the rate of relaxation dependent on the magnitude of both the primary and secondary loadings, as quantified by \( L_r, \beta \) of eqns (9, 21). The rate of relaxation is governed by the elastic follow-up factor Z in eqn (15). From observation of the finite element results [22], it was found that this could be represented by a time normalised by a redistribution time dependent on the magnitudes of the primary and secondary loadings. The normalised time is:

\[ \tau^{p+s} = \frac{t}{t_{\text{red}}} \]  

(22)

where the redistribution time is defined by the ratio of the initial value of J for the combined loading to the steady state value of \( C^* \) for the mechanical loading:

\[ t_{\text{red}}^{p+s} = \frac{J_0^{p+s}}{C^*} \quad \text{for} \quad J_0^{p+s} < 10J_0^p \]  

(23)

At higher values of secondary loading the redistribution time was found to saturate so that eqn (23) was replaced by

\[ t_{\text{red}}^{p+s} = \frac{10J_0^p}{C^*} \quad \text{for} \quad J_0^{p+s} \geq 10J_0^p \]  

(24)

Some finite element results are shown in Figs 6-8 for selected cases with initial elastic response; more comprehensive results including plasticity on initial loading show similar trends and are given in [22]. It can be seen that the results when normalised by the time of eqn (22) are insensitive to the magnitudes of the primary and secondary loadings, although there is some sensitivity to the magnitude of the sectional temperature gradient, as shown on Fig 8. Also shown on Figs 6-8 is a simplified estimate of eqn (16) from [22]:

\[ \frac{C(t)}{C^*} = \frac{(1 + \tau^{p+s})^{4.5}}{(1 + \tau^{p+s})^{4.5} - \phi} \quad \text{with} \quad \phi = 1 - \frac{\alpha \varepsilon_y \sigma^n_0}{\varepsilon_0 \sigma^n_0} \frac{C^*}{J_0^{p+s}} \]  

(25)

where \( \phi = 1 \) corresponds to elastic response on initial loading, and \( \phi \rightarrow 1 \) corresponds to widespread plasticity on initial loading so that the steady state creep field is established at the crack tip; note that \( \phi \) depends on the magnitude of the thermal stress for elastic-plastic response but the dependence is weak. It can be seen that this simplified estimate is accurate for the axial and radial temperature gradients but generally a little conservative for the sectional temperature gradient, with increasing conservatism at higher secondary stresses. As for the case of elastic-plastic response, this confirms
that simplified estimates sufficient for practical applications can be made for elastic-plastic-creep crack tip response.

6. Concluding Remarks
This paper has first presented some background to fracture mechanics methods and their practical application by finite element and simplified methods. Then, results from finite element solutions for elastic-plastic J and creep C(t) for circumferentially cracked pipes under combined mechanical and thermal loads have been presented. The finite element results, when presented in normalised form, have found to be not particularly sensitive to the crack geometry (length and depth), the loading mode (axial tension and bending), the material properties and the thermal loading type. This suggests that simplified methods for estimating this ratio can be used in practical defect assessments and such simplified estimates have been presented.

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Figure 1. (a) Pipes under axial tension N, and (b) circumferential through-wall and part-through surface cracks. Directions for contour integral calculations are also shown.
Figure 2. Thermal loading: (a) radial temperature gradient, (b) axial temperature gradient, (c) sectional temperature gradient.

Figure 3  Elastic-plastic results for a small axial temperature gradient ($\beta = K^*L_r/ K_p = 0.5$).

Figure 4  Elastic-plastic results for a large radial temperature gradient ($\beta = K^*L_r/ K_p = 5.0$).
Figure 5  Elastic-plastic results for a modest sectional temperature gradient ($\beta = K^*L_r / K^p = 1.0$).

Figure 6  Creep results for the axial temperature gradient with a modest mechanical load, analysed elastically.

Figure 7  Creep results for the radial temperature gradient with a small mechanical load, analysed elastically.

Figure 8  Creep results for the sectional temperature gradient with a modest mechanical load, analysed elastically.