Screening Length in 2 + 1-dimensional Abelian Chern-Simons Theories

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Abstract

In this paper, we systematically study the question of screening length in Abelian Chern-Simons theories. In the Abelian Higgs theory, where there are two massive poles in the gauge propagator at the tree level, we show that the coefficient of one of them becomes negligible at high temperature and that the screening length is dominantly determined by the parity violating part of the self-energy. In this theory, static magnetic fields are screened. In the fermion theory, on the other hand, the parity conserving part of the self-energy determines the screening length and static magnetic fields are not screened. Several other interesting features are also discussed.

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1 Introduction

The properties of a charged plasma, at finite temperature, have been studied extensively in the past in 3 + 1 dimensions \[1, 2, 3, 4\] and it is known that there are several interesting features that emerge in thermal QED. For example, it is known that the photon becomes massive at finite temperature, much like a particle moving in a medium. Furthermore, since thermal amplitudes are, in general, non-analytic at the origin in the energy-momentum plane \[5, 6\], the mass of the photon that manifests in different processes is distinct. For example, the screening length between two static charges is related to the electric mass of the photon, \( m_{el} \), while the length associated with plasma oscillations due to a sudden excitation of the plasma is related to the plasmon mass, \( m_{pl} \), and the two masses are quite distinct. In 3 + 1 dimensional QED, for example, the electric mass is defined as

\[
\lim_{\vec{p} \to 0} \Pi^{00}(p^0 = 0, \vec{p}) = m_{el}^2, \tag{1}
\]

where \( \Pi^{\mu\nu} \) denotes the photon self-energy and the potential between two static charges separated by a distance \( R = |\vec{R}| \) is obtained to be

\[
\sim \int \frac{d^3p}{(2\pi)^3} D_{00}(p^0 = 0, \vec{p}) e^{i\vec{p} \cdot \vec{R}} = \int \frac{d^3p}{(2\pi)^3} \frac{e^{i\vec{p} \cdot \vec{R}}}{\vec{p}^2 + m_{el}^2} = \frac{e^{-m_{el}R}}{4\pi R}. \tag{2}
\]

This shows that the screening length and the electric mass are inversely related.

In 2 + 1 dimensional QED, if we naively carry over the definition of the electric mass as in (1) (as well as the propagator), then, although the potential between two static charges would not have the form of a Yukawa potential as in (2), it can be determined to be

\[
\sim \int \frac{d^2p}{(2\pi)^2} D_{00}(p^0 = 0, \vec{p}) e^{i\vec{p} \cdot \vec{R}} = \int \frac{d^2p}{(2\pi)^2} \frac{e^{i\vec{p} \cdot \vec{R}}}{\vec{p}^2 + m_{el}^2} = \frac{1}{2\pi} K_0(m_{el}R). \tag{3}
\]

Here \( K_0(m_{el}R) \) is the Bessel function with the asymptotic behavior (for large \( z \))

\[
K_0(z) \to \sqrt{\frac{\pi}{2z}} e^{-z}, \tag{4}
\]

so that once again, we see that the screening length and the electric mass are inversely related.

In 2 + 1 dimensions, however, we can also add a parity violating Chern-Simons term to the gauge Lagrangian \[\] and we know that, in some theories, such a term can be generated through quantum corrections even if it is not present at the tree level \[\]. In such a case, we expect that \( \Pi^{00} \) alone cannot determine the electric mass which can, in principle, depend on the Chern-Simons coefficient. Furthermore, in a 2 + 1 dimensional Abelian Higgs model with a Chern-Simons term \[\], it is known that even the tree level gauge boson propagator has two distinct poles. This raises the interesting question, namely, whether there is a unique screening length in such theories. In this paper, we study this question in detail in various Abelian Chern-Simons theories, which leads to some interesting results. We note here that the question of screening length, in a Yang-Mills-Chern-Simons theory interacting with fermions, has been discussed in the past \[\] and we compare our results with these.
2 Abelian Higgs model with a Chern-Simons term

Let us start with an Abelian gauge field, $A_\mu$, in 2+1 dimensions with both a Maxwell and a Chern-Simons term interacting with a charged scalar field with a symmetry breaking quartic potential $[9, 10, 11, 12]$,

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{\kappa}{2} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda + (D_\mu \Phi)^* (D^\mu \Phi) - \frac{\lambda}{4} (\Phi^* \Phi - v^2)^2$$  \hspace{1cm} (5)

where $\kappa$ represents the Chern-Simons coefficient.

In the spontaneously broken phase, where $\Phi$ has a nonzero vacuum expectation value, $\langle \Phi \rangle = v$, we expand the scalar field as $\Phi = v + \frac{1}{\sqrt{2}} (\sigma + i\chi)$ to obtain

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{\kappa}{2} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda + \frac{m^2}{2} A_\mu A^\mu + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - m \partial^\mu \chi A_\mu - e (\sigma \partial^\mu \chi - \chi \partial^\mu \sigma) A_\mu + \frac{e^2}{2} (\sigma^2 + \chi^2 + 2\sqrt{2}v\sigma) A_\mu A^\mu - \frac{\lambda}{16} (\sigma^2 + \chi^2 + 2\sqrt{2}v\sigma)^2,$$  \hspace{1cm} (6)

where we have defined

$$m = \sqrt{2} ev.$$  \hspace{1cm} (7)

We can add to this a gauge fixing Lagrangian as well as the corresponding ghost Lagrangian. Let us note here that in the $R_\xi$ gauge the $A_\mu - \chi$ mixing term disappears and all the fields have nontrivial mass parameters

$$m^2 = 2e^2v^2, \quad m_\sigma^2 = \lambda v^2, \quad m_\chi^2 = m_\xi^2 = \xi m^2,$$  \hspace{1cm} (8)

where $\xi$ is the gauge fixing parameter.

At zero temperature, the tree level gauge propagator, in this case, has the form

$$D^{(0)}_{\mu\nu}(p) = -\frac{1}{(p^2 - m_+^2)(p^2 - m_-^2)} \left[ \eta_{\mu\nu} (p^2 - m_+^2) - p_\mu p_\nu \frac{(1 - \xi)(p^2 - m_+^2) + \xi \kappa^2}{p^2 - \xi m_+^2} + i \kappa \epsilon_{\mu\nu\lambda} p^\lambda \right],$$  \hspace{1cm} (9)

where the superscript “0” denotes the tree level propagator and

$$m_\pm^2 = \frac{\kappa^2 + 2m^2 \pm (\kappa^4 + 4m^2 \kappa^2)^{1/2}}{2}.$$  \hspace{1cm} (10)

The two distinct poles of the propagator, alluded to in the introduction, are manifest in this case.

For the purpose of studying the gauge boson self-energy, it is much more convenient for us to work in the unitary gauge, $\chi = 0$, where the number of relevant Feynman graphs is much smaller. At zero temperature, the tree level gauge and the scalar propagators, in the unitary gauge, have the forms

$$D^{(0)}_{\mu\nu}(p) = -\frac{1}{(p^2 - m_+^2)(p^2 - m_-^2)} \left[ \eta_{\mu\nu} (p^2 - m_+^2) - p_\mu p_\nu \frac{p^2 - m_-^2 - \kappa^2}{m^2} + i \kappa \epsilon_{\mu\nu\lambda} p^\lambda \right],$$

$$D^{(0)}_{\sigma}(p) = \frac{1}{p^2 - m_\sigma^2}.$$  \hspace{1cm} (11)
where, again, the two poles in the gauge boson propagator are manifest.

We would like to study, systematically, the question of the screening length in this theory at finite temperature before turning to the fermion theory later. To make the problem precise, let us note that we will work in the imaginary time formalism \[14, 15, 16\] where the tree level propagators take the forms

\[
D^{(0)}_{\mu\nu}(p) = \frac{1}{(p^2 + m^2)} \left[ \delta_{\mu\nu} (p^2 + m^2) + p_\mu p_\nu \frac{p^2 + m^2 + \kappa^2}{m^2} - \kappa \epsilon_{\mu\nu\lambda} p_\lambda \right],
\]

\[
D^{(0)}_\sigma(p) = \frac{1}{p^2 + m^2},
\]

with \(p^0 = \frac{2n\pi}{\beta}, \beta = \frac{1}{T}\) and the Boltzmann constant \(k = 1\). Let \(u_\mu\) denote the velocity of the heat bath with \(u_\mu u_\mu = 1\). In the rest frame of the heat bath, \(u_\mu = (1, 0, 0, 0)\). Let us also define \[3, 16\] (all of our discussion is in the imaginary time formalism and, therefore, in Euclidean space.)

\[
\tilde{u}_\mu = u_\mu - \frac{u \cdot p}{p^2} p_\mu, \quad \tilde{p}_\mu = p_\mu - (u \cdot p) u_\mu, \quad \tilde{\delta}_{\mu\nu} = \delta_{\mu\nu} - u_\mu u_\nu,
\]

which satisfy

\[
p \cdot \tilde{u} = 0 = u \cdot \tilde{p} = u_\mu \tilde{\delta}_{\mu\nu}.
\]

With these structures, let us define

\[
P_{\mu\nu} = \tilde{\delta}_{\mu\nu} - \frac{\tilde{p}_\mu \tilde{p}_\nu}{\tilde{p}^2}, \quad Q_{\mu\nu} = \frac{p^2}{\tilde{p}^2} \tilde{u}_\mu \tilde{u}_\nu.
\]

It can be easily checked that

\[
p_\mu P_{\mu\nu} = 0 = p_\mu Q_{\mu\nu} = P_{\mu\nu} Q_{\nu\lambda},
\]

\[
P_{\mu\nu} P_{\nu\lambda} = P_{\mu\lambda}, \quad Q_{\mu\nu} Q_{\nu\lambda} = Q_{\mu\lambda},
\]

\[
P_{\mu\nu} + Q_{\mu\nu} = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}.
\]

The self-energy for the gauge boson, at finite temperature, can now be parameterized, to all orders, in the unitary gauge as

\[
\Pi_{\mu\nu}(p) = P_{\mu\nu} (\Pi_1 + \Pi_3) + Q_{\mu\nu} (\Pi_2 + \Pi_3) + \frac{p_\mu p_\nu}{p^2} \Pi_3 + \epsilon_{\mu\nu\lambda} p_\lambda \Pi_{\text{odd}}.
\]

Adding the tree level term, the complete two point function has the form

\[
\Gamma_{\mu\nu}(p) = P_{\mu\nu} (p^2 + m^2 + \Pi_1 + \Pi_3) + Q_{\mu\nu} (p^2 + m^2 + \Pi_2 + \Pi_3) + \frac{p_\mu p_\nu}{p^2} (m^2 + \Pi_3) + \epsilon_{\mu\nu\lambda} p_\lambda (\kappa + \Pi_{\text{odd}}),
\]
where we have defined
\[ M_1^2 = m^2 + \Pi_1 + \Pi_3, \quad M_2^2 = m^2 + \Pi_2 + \Pi_3. \] (19)

The complete propagator, which is the inverse of the complete two point function, can now be determined to be
\[
D_{\mu\nu}(p) = \frac{1}{(p^2 + M_2^2)(M_2^2 - p^2)} \left[ P_{\mu\nu}(p^2 + M_2^2) + Q_{\mu\nu}(p^2 + M_2^2) - (\kappa + \Pi_{\text{odd}}) \epsilon_{\mu\nu\lambda} p_{\lambda} \right] \\
+ \frac{1}{p^2} \frac{1}{m^2 + \Pi_3},
\] (20)

where we have defined
\[ M_2^2 = \frac{(\kappa + \Pi_{\text{odd}})^2 + M_1^2 + M_2^2 \pm ((M_1^2 - M_2^2)^2 + 2(M_1^2 + M_2^2)(\kappa + \Pi_{\text{odd}})^2 + (\kappa + \Pi_{\text{odd}})^4)^{1/2}}{2}. \] (21)

There are several things to note from this structure. First of all, the propagator continues to have two distinct poles. Second, the poles of the propagator correspond to the mass scales \( M_{\pm} \) which involve the Chern-Simons term (with radiative corrections) non-trivially so that it is not possible to identify the electric mass with \( \Pi_{00} \) as in (1). Finally, let us note that we can rewrite the propagator also as
\[
D_{\mu\nu} = \frac{P_{\mu\nu}}{M_1^2 - M_2^2} \left[ \frac{M_2^2 - M_1^2}{p^2 + M_1^2} - \frac{M_1^2 - M_2^2}{p^2 + M_2^2} \right] + \frac{Q_{\mu\nu}}{M_2^2 - M_1^2} \left[ \frac{M_2^2 - M_1^2}{p^2 + M_1^2} - \frac{M_1^2 - M_2^2}{p^2 + M_2^2} \right] \\
+ \frac{\kappa + \Pi_{\text{odd}}}{M_1^2 - M_2^2} \epsilon_{\mu\nu\lambda} p_{\lambda} \left[ \frac{1}{p^2 + M_1^2} - \frac{1}{p^2 + M_2^2} \right] + \frac{p_\mu p_\nu}{p^2} \frac{1}{m^2 + \Pi_3}.
\] (22)

Each tensor structure now has a sum of two simple poles and the problem of the uniqueness of a screening length is now clear. In fact, depending on the parameters of the theory, we note from eq. (22) that a screening potential can even become an anti-screening potential. Of course, this has been a general analysis so far and only an actual calculation can determine what really happens.

Before presenting the actual calculations, let us note here that although our discussion has so far been within the context of the Abelian Chern-Simons Higgs system, the same general features arise in the \( 2 + 1 \) dimensional QED with a Chern-Simons term, as we will discuss in section 4. Interestingly, although the answer to the uniqueness of the screening length is similar in the two theories, the mechanisms responsible for this are quite different, as we will see.

### 3 The calculations

In the unitary gauge, there are only two diagrams which contribute to the photon self-energy (see figure 1). Of the two diagrams, it is only the rising sun diagram which contributes to the parity violating part of the photon self-energy which has already been calculated in [12]. Therefore, we will concentrate only on the parity conserving part of these diagrams at finite temperature. Furthermore, the tadpole diagram is independent of the external momentum and consequently gives an analytic contribution – it has the same value both in the static as well as the long wave limits. The only non-analyticity may possibly arise from the rising sun diagram.
Before evaluating the individual diagrams, let us note that our interest lies in calculating the form factors $\Pi_1$, $\Pi_2$, and $\Pi_3$ ($\Pi_{\text{odd}}$ has already been calculated in [12]). From the parameterization of the self-energy in (17), we note that these can be determined from the self-energy as $(i, j = 1, 2)$

$$\Pi_1 = -\frac{p_0^2}{p_0^2 - \vec{p}^2} \Pi_{00} + \delta_{ij} \Pi_{ij} - \frac{p_0^2 - 2\vec{p}^2}{p_0^2 - \vec{p}^2} \frac{p_i p_j}{p^2} \Pi_{ij},$$

$$\Pi_2 = \frac{p_0^2 + \vec{p}^2}{p_0^2 - \vec{p}^2} (-\Pi_{00} + \frac{p_i p_j}{\vec{p}^2} \Pi_{ij}),$$

$$\Pi_3 = \frac{p_0^2}{p_0^2 - \vec{p}^2} \Pi_{00} - \frac{\vec{p}^2}{p_0^2 - \vec{p}^2} \frac{p_i p_j}{\vec{p}^2} \Pi_{ij}. \quad (23)$$

Therefore, rather than calculating the self-energy, it is simpler to calculate $\Pi_{00}$, $\delta_{ij} \Pi_{ij}$ and $\frac{p_i p_j}{p^2} \Pi_{ij}$ from which the three quantities of interest can be determined. We note that, in the static limit ($p_0 = 0$), eq. (23) leads to

$$\Pi_1^{(\text{static})} = \delta_{ij} \Pi_{ij}^{(\text{static})} - 2 \frac{p_i p_j}{\vec{p}^2} \Pi_{ij}^{(\text{static})},$$

$$\Pi_2^{(\text{static})} = \Pi_{00}^{(\text{static})} - \frac{p_i p_j}{\vec{p}^2} \Pi_{ij}^{(\text{static})},$$

$$\Pi_3^{(\text{static})} = \frac{p_i p_j}{\vec{p}^2} \Pi_{ij}^{(\text{static})}, \quad (24)$$

while, in the long wave limit ($\vec{p} = 0$), we obtain

$$\Pi_1^{(\text{long wave})} = -\Pi_{00}^{(\text{long wave})} + \delta_{ij} \Pi_{ij}^{(\text{long wave})} - \frac{p_i p_j}{\vec{p}^2} \Pi_{ij}^{(\text{long wave})},$$

$$\Pi_2^{(\text{long wave})} = -\Pi_{00}^{(\text{long wave})} + \frac{p_i p_j}{\vec{p}^2} \Pi_{ij}^{(\text{long wave})},$$

$$\Pi_3^{(\text{long wave})} = \Pi_{00}^{(\text{long wave})}. \quad (25)$$

Note that $\frac{p_i p_j}{\vec{p}^2} \Pi_{ij}|_{\text{long wave}}$ is well behaved.

The calculation of the tadpole diagram is straightforward.
\[ \Pi_{\mu\nu}^{(\text{tadpole})} = \frac{1}{2} (2e^2 \delta_{\mu\nu})^\beta \sum_n \int \frac{d^2 k}{(2\pi)^2} \frac{1}{k_0^2 + \omega_\sigma^2} = e^2 \delta_{\mu\nu} \frac{\beta}{(2\pi)^2} \sum_n \int \frac{d^2 k}{(2\pi)^2} \frac{1}{k^2 + \frac{1}{(2\pi)^2}}. \]  

where we have identified \( k_0 = \frac{2\pi n}{3} \) and \( \omega_\sigma = (\vec{k}^2 + m_\sigma^2)^{1/2} \). The sum over the Matsubara frequencies can be evaluated using

\[ \sum_n f(n) = -\pi \text{Res} f(z) \cot \pi z, \]  

where the residues are calculated at the poles of the function \( f(z) \). Using this (as well as the periodic properties of the trigonometric functions), we obtain

\[ \Pi_{\mu\nu}^{(\text{tadpole})} = e^2 \delta_{\mu\nu} \int \frac{d^2 k}{(2\pi)^2} \frac{1}{\omega_\sigma} \coth \left( \frac{\beta \omega_\sigma}{2} \right) = e^2 \delta_{\mu\nu} \frac{1}{2} \int \frac{d^2 k}{(2\pi)^2} \frac{1}{\omega_\sigma} \left( 1 + \frac{2}{e^{\beta \omega_\sigma} - 1} \right). \]  

The finite temperature contribution of the tadpole, therefore, follows to be

\[ \Pi_{\mu\nu}^{(\text{tadpole})}(T) = e^2 \delta_{\mu\nu} \int \frac{d^2 k}{(2\pi)^2} \frac{1}{\omega_\sigma} e^{\beta \omega_\sigma} = -\delta_{\mu\nu} - \frac{e^2}{2\pi \beta} \ln(1 - e^{-\beta m_\sigma}). \]  

As noted earlier, this diagram is independent of the external momentum and, therefore, gives an analytic contribution. In fact, from the definition in (23), we see that independent of the static or the long wave limit, this diagram, eq. (29), leads to (we will ignore the superscript \( T \) remembering all along that our interest is in the temperature dependent part)

\[ \Pi_1^{(\text{tadpole})} = 0 = \Pi_2^{(\text{tadpole})}, \quad \Pi_3^{(\text{tadpole})} = -\frac{e^2}{2\pi \beta} \ln(1 - e^{-\beta m_\sigma}). \]  

In fact, although the integrals, that we are interested in, can be evaluated in closed form, for simplicity, let us consider the high temperature limit, where we assume \( T \gg m_i \) and yet is small compared with the critical temperature where symmetry may be restored (such a regime exists). In this limit, we have

\[ \Pi_1^{(\text{tadpole})} = 0 = \Pi_2^{(\text{tadpole})}, \quad \Pi_3^{(\text{tadpole})} \sim -\frac{e^2}{2\pi \beta} \ln \beta m_\sigma. \]  

The rising sun diagram, on the other hand, does depend on the external momentum and can, in principle, lead to a non-analytic contribution for the parity conserving part of the self-energy

\[ \Pi_{\mu\nu}^{(\text{rising sun})} = -(2em)^2 \frac{1}{\beta} \sum_n \int \frac{d^2 k}{(2\pi)^2} \frac{\delta_{\mu\nu} (k_0^2 + \omega^2)}{((k_0 + p_0)^2 + \omega_\sigma^2)(k_0^2 + \omega_\perp^2)(k_0^2 + \omega_\parallel^2)} \]  

where we have defined

\[ \omega = (\vec{k}^2 + m^2)^{1/2}, \quad \omega_\sigma = ((\vec{k} + \vec{p})^2 + m_\sigma^2)^{1/2}, \quad \omega_\perp = (\vec{k}^2 + m_\perp^2)^{1/2}. \]  

Let us note here that the integrand in (22) involves propagators with distinct masses. In such a case, it has been argued in (17) that the amplitude will be analytic at the origin in the energy-momentum plane. More recently, it has been recognized that the non-analyticity arises in the self-energy only if in some limits the integrand develops double (or higher order) poles (18). When there are distinct masses in the propagators, however, such a possibility cannot arise and we will not expect a non-analyticity in the lowest order terms. Nevertheless, let us evaluate the integral separately both in the static limit as well as the long wave limit to understand this further.

7
3.1 Static Limit

In this case, we set \( p_0 = 0 \) and evaluate the amplitude in (32) as \( \vec{p} \to 0 \). For \( \Pi_0^{(\text{rising sun})} \), we obtain

\[
\Pi_0^{(\text{rising sun})} = -4e^2m^2 \frac{1}{\beta} \sum_n \int \frac{d^2k}{(2\pi)^2} \frac{1}{(k_0^2 + \omega^2 + \kappa^2)} - k_0 \frac{2}{m^2}
\]

\[
= -4e^2m^2 \frac{\beta^3}{(2\pi)^4} \sum_n \int \frac{d^2k}{(2\pi)^2} \frac{(n^2 + (\frac{\beta \omega}{2\pi})^2)}{(n^2 + (\frac{\beta \omega}{2\pi})^2)(n^2 + (\frac{\beta \omega}{2\pi})^2)}.
\]

It is worth noting here that the integrand has only simple poles owing to the fact that the masses inside the loop are distinct. The sum can be evaluated, as before, using eq. (27). Separating the temperature dependent part, we obtain the value of this in the high temperature limit as \( \vec{p} \to 0 \) to be

\[
\lim_{\vec{p} \to 0} \Pi_0^{(\text{rising sun})(T)} = \frac{e^2}{2\pi\beta} \left[ 2 + \frac{4m^2(m^2 - m^2_\sigma)}{(m^2_\sigma - m^2_\gamma)(m^2_\gamma - m^2_\sigma)} \ln \frac{m_\sigma}{m_\gamma} + \frac{4m^2(m^2 - m^2_\gamma)}{(m^2_\sigma - m^2_\gamma)(m^2_\gamma - m^2_\sigma)} \ln \frac{m_\gamma}{m_\sigma} \right].
\]

Here, we have used the high temperature limits of the integrals

\[
\int \frac{d^2k}{(2\pi)^2} \frac{1}{\omega e^{\beta \omega} - 1} \to -\frac{1}{2\pi\beta} \ln \beta m + O(\beta^0),
\]

\[
\int \frac{d^2k}{(2\pi)^2} \frac{\omega}{e^{\beta \omega} - 1} \to -\frac{2\zeta(3)}{2\pi^3 \beta^3} + \frac{m^2}{4\pi^3 \beta^3} + O(\beta^0).
\]

The other projections can also be calculated, in the static limit, in a similar manner. Without going into details, let us simply note the high temperature limits of these quantities.

\[
\lim_{\vec{p} \to 0} \delta_{ij} \Pi^{(\text{rising sun})(T)} \quad \rightarrow \quad -\frac{e^2}{2\pi\beta} \left[ -4 \ln(\beta m_\sigma) - \frac{4(2m^4 - 3m^2m^2_\mu - m^2_\gamma m^2 + m^4_\gamma)}{(m^2_\sigma - m^2_\mu)(m^2_\gamma - m^2_\sigma)} \ln \frac{m_\sigma}{m_\mu} 
\]

\[
+ \frac{4(2m^4 - 3m^2m^2_\mu - m^2_\sigma m^2 + m^4_\sigma)}{(m^2_\sigma - m^2_\mu)(m^2_\gamma - m^2_\sigma)} \ln \frac{m_\sigma}{m_\mu} + 2 \right],
\]

\[
\lim_{\vec{p} \to 0} \frac{p_i p_j}{p^2} \Pi^{(\text{rising sun})(T)} = \lim_{\vec{p} \to 0} \frac{1}{2} \delta_{ij} \Pi^{(\text{rising sun})(T)}.
\]

The second of the above relations is, in fact, required by the Ward identity of the theory and, consequently, our calculation is consistent with the requirements of gauge invariance (BRS invariance). It now follows, from eqs. (24), (33) and (37), that in the static limit, as \( \vec{p} \to 0 \) (we drop the superscript \( T \)),

\[
\lim_{\vec{p} \to 0} \Pi^{(\text{rising sun})}_1 = 0,
\]

and that the leading high temperature behavior of the other two form factors is given by

\[
\lim_{\vec{p} \to 0} \Pi^{(\text{rising sun})}_2 = -2 \left( \frac{e^2}{2\pi\beta} \ln \beta m_\sigma \right) + O\left(\frac{1}{\beta}\right),
\]

\[
\lim_{\vec{p} \to 0} \Pi^{(\text{rising sun})}_3 = 2 \left( \frac{e^2}{2\pi\beta} \ln \beta m_\sigma \right) + O\left(\frac{1}{\beta}\right).
\]
Adding the contribution from the tadpole diagram, eq. (21), we see that, in the static limit, as \( \vec{p} \to 0 \),
\[
\Pi_1^{(\text{static})} = \Pi_1^{(\text{tadpole})} + \Pi_1^{(\text{rising sun})} = 0,
\]
and that the leading high temperature behavior of the other two form factors are given by
\[
\Pi_2^{(\text{static})} = \Pi_2^{(\text{tadpole})} + \Pi_2^{(\text{rising sun})} = -2 \left( \frac{e^2}{2\pi\beta} \ln(\beta m_\sigma) \right),
\]
\[
\Pi_3^{(\text{static})} = \Pi_3^{(\text{tadpole})} + \Pi_3^{(\text{rising sun})} = \left( \frac{e^2}{2\pi\beta} \ln(\beta m_\sigma) \right).
\]

3.2 Long Wave Limit

The form factors can also be calculated in a completely analogous manner in the long wave limit, where we set \( \vec{p} = 0 \) and look at the amplitudes in the limit \( p_0 \to 0 \). Such a limit can be taken only after the sum over the Matsubara frequencies have been carried out and the external energies have been analytically continued to Minkowski space. Let us indicate how this is done only in the case of \( \Pi_0 \).

\[
\Pi_0^{(\text{rising sun})} = -4e^2m_0^2 \sum_n \frac{1}{\beta} \int \frac{d^2k}{(2\pi)^2} \frac{(k_0^2 + \omega^2) + \frac{k^2(k^2 + \omega^2 + \kappa^2)}{m^2}}{((k_0 + p_0)^2 + \omega_\sigma^2)(k_0^2 + \omega_\sigma^2)(k_0^2 + \omega^2)},
\]

where \( p_0 = \frac{2\pi \tau}{\beta} \) and (since \( \vec{p} = 0 \)) we have now defined \( \omega_\sigma^2 = k^2 + m_\sigma^2 \). The sum can be evaluated using eq. (27) as well as using the periodicity of trigonometric functions. If we now analytically continue \( p_0 \) to Minkowski space and look at the limit \( p_0 \to 0 \), then, the leading term is identical to the leading term in the static limit. Therefore, the high temperature limit leads to

\[
\lim_{p_0 \to 0} \Pi_0^{(\text{rising sun})(T)} \to \frac{e^2}{2\pi\beta} \left[ 2 + \frac{4m^2(m^2 - m_\sigma^2)}{(m_\sigma^2 - m_\tau^2)(m^2 - m_\tau^2)} \ln \frac{m_\sigma}{m_-} - \frac{4m^2(m^2 - m_\tau^2)}{(m_\sigma^2 - m_\tau^2)(m^2 - m_\tau^2)} \ln \frac{m_+}{m_-} \right].
\]

As we had alluded to earlier, the presence of distinct masses in the propagator regulates the non-analyticity as a result of which the lowest order term, in the long wave limit, is the same as in the static limit \([17]\). This is also reflected in the other calculations and yields

\[
\lim_{p_0 \to 0} \delta_{ij} \Pi_{ij}^{(\text{rising sun})(T)} \to -\frac{e^2}{2\pi\beta} \left[ -4 \ln(\beta m_\sigma) - \frac{4(2m^4 - 3m_\sigma^2m_\tau^2 - m_\tau^2)}{(m_\sigma^2 - m_\tau^2)(m_\tau^2 + m_\tau^2)} \ln \frac{m_\sigma}{m_+} + \frac{4(2m^4 - 3m_\sigma^2m_\tau^2 - m_\tau^2)}{(m_\sigma^2 - m_\tau^2)(m_\tau^2 + m_\tau^2)} \ln \frac{m_\sigma}{m_-} + 2 \right],
\]

\[
\lim_{p_0 \to 0} \frac{p_i p_j}{p^2} \Pi_{ij}^{(\text{rising sun})(T)} \to \lim_{p_0 \to 0} \frac{1}{2} \delta_{ij} \Pi_{ij}^{(\text{rising sun})(T)}.
\]

However, even though the lowest order terms in the integrand are the same in the two limits, the form factors are not. As can be seen from eq. (23), in the long wave limit, we obtain (suppressing the superscript \( T \)) the leading high temperature behaviors to be
\[
\begin{align*}
\lim_{p_0 \to 0} \Pi_1^{\text{(rising sun)}} &= 2 \left( \frac{e^2}{2\pi\beta} \ln(\beta m_\sigma) \right) + O(\frac{1}{\beta}), \\
\lim_{p_0 \to 0} \Pi_2^{\text{(rising sun)}} &= 2 \left( \frac{e^2}{2\pi\beta} \ln(\beta m_\sigma) \right) + O(\frac{1}{\beta}), \\
\lim_{p_0 \to 0} \Pi_3^{\text{(rising sun)}} &= O\left( \frac{1}{\beta} \right). 
\end{align*}
\]

Adding the contribution from the tadpole diagram, eq. (31), the complete form factors, in the long wave limit, have the leading high temperature behaviors, as \( p_0 \to 0 \),

\[
\begin{align*}
\Pi_1^{\text{(long wave)}} &= 2 \left( \frac{e^2}{2\pi\beta} \ln(\beta m_\sigma) \right), \\
\Pi_2^{\text{(long wave)}} &= 2 \left( \frac{e^2}{2\pi\beta} \ln(\beta m_\sigma) \right), \\
\Pi_3^{\text{(long wave)}} &= - \left( \frac{e^2}{2\pi\beta} \ln(\beta m_\sigma) \right). 
\end{align*}
\]

## 4 Discussion of results

Our calculations are completely consistent with the known results about loop diagrams with distinct masses in that the photon self-energy is analytic in the lowest order \([17, 18]\). However, the form factors are different in the static as well as the long wave limits. Beyond the lowest order terms, however, we do not expect the distinct masses in the propagators to lead to analytic results. This is already evident in the parity violating part of the photon self-energy coming from the rising sun diagram, where it is known that the leading high temperature behavior of the radiative correction to the Chern-Simons coefficient is different in the static and the long wave limits \([12]\).

\[
\begin{align*}
\lim_{\vec{p} \to 0} \Pi_1^{\text{(static) \ odd}} &= 4 \kappa m^2 F(m_+, m_-, m_\sigma) \left( \frac{e^2}{2\pi\beta} \right), \\
\lim_{p_0 \to 0} \Pi_3^{\text{(long wave) \ odd}} &= \frac{4 \kappa m^2}{(m_\sigma^2 - m_+^2)(m_\sigma^2 - m_-^2)} \left( \frac{e^2}{2\pi\beta} \ln(\beta m_\sigma) \right), 
\end{align*}
\]

where

\[
F(m_+, m_-, m_\sigma) = \frac{m_+^2}{(m_\sigma^2 - m_+^2)^2(m_+^2 - m_-^2)} - \frac{m_-^2}{(m_\sigma^2 - m_-^2)^2(m_+^2 - m_-^2)} + \frac{1}{2(m_\sigma^2 - m_+^2)(m_\sigma^2 - m_-^2)}.
\]

To understand the question of the screening length, let us tabulate all the results that we know so far. Thus, we see, from table 1, that, at high temperature, the contribution of \( \Pi_{\text{odd}} \) to \( M^2 \) is dominant and that \( M^2 \) is negligible by comparison. Thus, for example, in the static limit, we have

\[(\text{high } T)\]


| Parameter | Static limit | Long wave Limit |
|-----------|--------------|-----------------|
| $M_1^2$   | $\frac{e^2}{2\pi\beta} \ln \beta m_\sigma$ | $\frac{e^2}{2\pi\beta} \ln \beta m_\sigma$ |
| $M_2^2$   | $-\frac{e^2}{2\pi\beta} \ln \beta m_\sigma$ | $\frac{4\kappa m^2}{(m_+^2-m_-^2)(m_+^2-m_-^2)} \ln \beta m_\sigma$ |
| $\Pi_{\text{odd}}$ | $4\kappa m^2 F(m_+,m_-,m_\sigma) \left(\frac{e^2}{2\pi\beta}\right)$ | $\frac{16\kappa^2 m^4}{(m_+^2-m_-^2)^2} \left(\frac{e^2}{2\pi\beta}\ln \beta m_\sigma\right)^2$ |
| $M_+^2 \approx \Pi_{\text{odd}}^2$ | $16\kappa^2 m^4 F^2(m_+,m_-,m_\sigma) \left(\frac{e^2}{2\pi\beta}\right)^2$ | $\mathcal{O}(1)$ |
| $M_2^2$   | $(\ln \beta m_\sigma)^2$ | $(\ln \beta m_\sigma)^2$ |

Table 1: Summary of results

\[
D^{(\text{static})}_{00} = Q_{00} \left[ \frac{M_1^2 - M_1^2}{M_+^2 - M_2^2} \frac{1}{\bar{p}^2 + M_+^2} + \frac{M_2^2 - M_1^2}{M_+^2 - M_2^2} \frac{1}{\bar{p}^2 + M_-^2} \right] \\
\approx \frac{1}{\bar{p}^2 + M_+^2} + \frac{M_2^2}{M_+^2} \frac{1}{\bar{p}^2} \\
= \frac{1}{\bar{p}^2 + M_+^2} + \frac{\pi \beta \ln(\beta m_\sigma)}{8e^2 \kappa^2 m^4 F^2(m_+,m_-,m_\sigma)} \frac{1}{\bar{p}^2} \quad (49) \\
\approx \frac{1}{\bar{p}^2 + M_+^2}. \quad (50)
\]

Namely, even though the propagator, $D_{00}$, has two poles, the coefficient of the massless pole is negligible at high temperature. Consequently, the propagator effectively has a single pole and the screening length is related to $M_+$ which is determined by $\Pi_{\text{odd}}$. It is also worth noting that the same massive pole corresponds to the dominant term in the transverse part of $D_{ij}$ as well.

Although our discussion so far has been within the context of the Abelian Chern-Simons Higgs theory, a similar behavior is also manifest in the 2 + 1 dimensional QED with a Chern-Simons term (where there is no symmetry breaking). Let us note that in this theory, the tree level propagator in a general covariant gauge has the form (in Euclidean space)

\[
D^{(0)}_{\mu\nu} = \frac{1}{p^2 + \kappa^2} \left[(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}) - \kappa \epsilon_{\mu\nu\lambda} \frac{p_\lambda}{p^2}\right] + \xi \frac{p_\mu p_\nu}{(p^2)^2}, \quad (51)
\]

indicating a single massive pole. However, the complete two point function, at finite temperature, can be parameterized as (Conventionally, one identifies $\Pi_1 = \Pi_T$ and $\Pi_2 = \Pi_L$. However, we will follow the notation of the earlier section for consistency.)

\[
\Gamma_{\mu\nu} = P_{\mu\nu} (p^2 + \Pi_1) + Q_{\mu\nu} (p^2 + \Pi_2) + \epsilon_{\mu\nu\lambda} p_\lambda (\kappa + \Pi_{\text{odd}}) + \frac{1}{\xi} p_\mu p_\nu, \quad (52)
\]

leading to the complete propagator of the form

\[
D_{\mu\nu} = \frac{1}{(p^2 + M_+^2)(p^2 + M_-^2)} \left[P_{\mu\nu} (p^2 + \Pi_2) + Q_{\mu\nu} (p^2 + \Pi_1) - (\kappa + \Pi_{\text{odd}}) \epsilon_{\mu\nu\lambda} p_\lambda\right] + \frac{\xi p_\mu p_\nu}{(p^2)^2}. \quad (53)
\]
Here, \( M_\pm \) are the same as in (21) with \( m = 0 = \Pi_3 \). We see that even though the tree level propagator has a single pole, radiative corrections can generate two distinct poles in the propagator, much like the Abelian Chern-Simons Higgs system.

In the case of fermions, the masses inside the loop are identical (to the fermion mass \( M_f \)). Therefore, we expect that the amplitudes will be non-analytic \([19]\). For simplicity, we only list the leading behavior of various quantities in the static limit, without giving any technical details, which are quite standard. The radiative correction to the Chern-Simons term has already been calculated for this theory \([19]\) and we have, in the static limit, (at high \( T \))

\[
\Pi_{\text{odd}} = \frac{e^2}{8\pi} \beta M_f. \tag{54}
\]

Without giving details, we note that, in the static limit, the parity conserving part of the self-energy yields

\[
\lim_{\vec{p}\to 0} \Pi_1 = \lim_{\vec{p}\to 0} \delta_{ij} \Pi_{ij} = 2e^2 \int \frac{d^2k}{(2\pi)^2} \frac{1}{\omega_k} \frac{\partial}{\partial \omega_k} \left( \frac{(\omega_k^2 - m^2)n_F(\omega_k)}{\omega_k} \right) = 0, \tag{55}
\]

where \( n_F \) denotes the fermion distribution function and the leading high temperature behavior

\[
\lim_{\vec{p}\to 0} \Pi_2 = \lim_{\vec{p}\to 0} \Pi_{00} = -\frac{4e^2}{\pi^2} + O(\beta). \tag{56}
\]

Thus, we see that, in contrast to the Abelian Higgs model which we have studied in detail in the earlier sections, here the contribution of the parity violating part is negligible, at high temperature, compared with the parity conserving part, namely, the roles of the parity conserving and the parity violating parts appear to be reversed. In this case, it is easy to calculate

\[
M_1^2 = \Pi_1 = 0 = M_2^2,
\]

\[
M_\pm^2 = (\kappa + \Pi_{\text{odd}})^2 + \Pi_2 \approx \frac{e^2}{\pi^2} + O(\beta). \tag{57}
\]

As a result, in this case, we have

\[
D_{\text{00}}^{(\text{static})} = Q_{00} \left[ \frac{M_2^2 - M_1^2}{M_2^2 - M_1^2} \frac{1}{\vec{p}^2 + M_2^2} - \frac{M_2^2 - M_1^2}{M_2^2 - M_1^2} \frac{1}{\vec{p}^2 + M_2^2} \right] = \frac{1}{\vec{p}^2 + M_2^2}. \tag{58}
\]

Once again, we see that, even though \textit{a priori} we would have expected the existence of two poles, there is only one pole and that the screening length is determined by \( M_\pm \), which does not have any leading contribution from the parity violating form factor. This is quite different from the Abelian Higgs model where it is the parity violating part of the form factor that dominantly determines the screening length. We also note here that, since \( M_1 = 0 = M_- \) (and this is true to all orders as we will argue shortly), the propagator in the static limit, has the behavior to all orders (in the Landau gauge)

\[
D_{\mu\nu}^{(\text{static})} = P_{\mu\nu} \frac{1}{\vec{p}^2 + M_2^2} + Q_{\mu\nu} \frac{1}{\vec{p}^2 + M_2^2} - \frac{(\kappa + \Pi_{\text{odd}})}{\vec{p}^2 + M_2^2} \epsilon_{\mu\nu\lambda\rho} \lambda \tag{59}
\]

The presence of the massless pole in the space-like components of the propagator has already been observed in the non-Abelian theory \([13]\) and implies that static magnetic fields will not be screened in this theory. This is reminiscent of the vanishing magnetic mass in QED in \( 3 + 1 \)
dimensions and, in the present theory, this arises because \( \Pi_1 = 0 \). (Namely, in this theory, the uniqueness of the screening length and the absence of screening of static magnetic fields are directly related.) Let us argue next that this holds true to all orders in perturbation theory. Let us note that, gauge invariance (in QED) implies that \( p_\mu \Pi_{\mu \nu} = 0 \), which in the static limit gives

\[
p_k \Pi_{kj} = 0,
\]

or,

\[
\Pi_{ij} + p_k \frac{\partial \Pi_{kj}}{\partial p_i} = 0.
\]

(60)

Assuming that the self-energy is analytic in the external momentum, \( \vec{p} \), (we note that, at finite temperature, amplitudes are non-analytic in the external energy and momentum, but in the static limit, they are analytic in the external momentum unless there are infrared divergences), this implies upon using the symmetry properties of the amplitude that the parity conserving part of \( \Pi_{ij}(0, \vec{p}) \sim \mathcal{O}(p^2) \) so that \( \Pi_1(0, 0) = 0 \). As we have mentioned, this formal argument may be invalid when infrared divergences are present. While we have not carried out any higher order calculation to verify this, we do not expect infrared divergence to be a problem in the Abelian theory (the infrared divergence is much more severe in the non-Abelian theory). We would like to emphasize here that long range correlations of static magnetic fields are well known in 3 + 1 dimensional QED \[\square\]. The interesting feature here is that the photon field in 2 + 1 dimensional theory is massive at the tree level because of the Chern-Simons term and nonetheless a massless pole develops at the loop level. Another interesting feature to note is that had we started with a pure Chern-Simons theory \[20\] (without the Maxwell term) interacting with a fermion, the complete propagator would have the form (in the static limit in the Landau gauge)

\[
D^\text{(static)}_{\mu \nu} = \frac{1}{(\kappa + \Pi_{\text{odd}})^2 p^2 + \Pi_1 \Pi_2} [P_{\mu \nu} \Pi_2 + Q_{\mu \nu} \Pi_1 - (\kappa + \Pi_{\text{odd}}) \epsilon_{\mu \nu \lambda \rho} p_\lambda].
\]

(61)

Since, \( \Pi_1 = 0 \), it follows that, in this case, there will be no “00” component of the gauge propagator. As a result, in this theory, static electric charges will not feel any force (which is, of course, true at the tree level, but continues to hold at all loops), in addition to static magnetic fields not being screened.

Finally, it is worth mentioning that even in the Abelian-Chern-Simons-Higgs theory of section 2, it is easy to show using the Ward identities that \( \Pi_1 = 0 \) to all orders in the static limit (assuming no infrared divergence). This, however, does not result in a massless propagator (unlike in the fermion theory) for the space-like indices. This difference in the behavior of the two theories, namely, the fact that in the fermion theory, static magnetic fields are not screened while they are in the Abelian Higgs theory has a simple physical explanation. In the fermion theory, there are no magnetic charges which can screen the magnetic fields \[\square\], while the Abelian Higgs theory has vortex solutions which can achieve this.

In conclusion, we have systematically studied the question of screening length in 2 + 1 dimensional Abelian theories with a Chern-Simons term. We have shown that even though the starting gauge propagator, in the Abelian Higgs theory has two poles, at finite temperature, the coefficient of one of the poles becomes negligible leading to a unique screening length that is related to the parity violating part of the amplitude. In contrast, in a fermion theory, the parity violating part is negligible and the screening length is determined by the parity conserving part of the amplitude. In addition, we have pointed out various other interesting features that arise in these theories.
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