Analytic design methodology for flexible wing parametrization and spanwise load estimation

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Abstract. Presented is an analytic study of aerodynamic loading along a high aspect ratio wing, typical for airliners, under passive aeroelastic deformation. Such a highly non-planar, curved wing is feasible using the latest composite material technology, and is expected to enhance airliners aerodynamic performance at transonic cruise conditions, given it allows lower induced drag, similar to a wingletted wing. Also, gust and manoeuvring loads can be partially alleviated by the structure elasticity, hence the concept is of great interest for the airliner industry. A major problem to designing highly flexible wings is their constantly shifting shape and, as a result, their hardly predictable, non-constant lift force generated. This paper provides an attempt to precisely estimate load distribution along the statically inflected wing at different stages of its deformation, using mathematical parametrization of the curved wing shape to relate its geometry to aerodynamic loading and structural stiffness in a closed form function. This is achieved using the geometric dependency of local angles of attack on local wing curvatures, therefore load distribution is a function of the parametrized deformed wing shape. Spanwise load distributions in turn can be coupled with structural stiffness distributions using cantilever beam theory.

1. Introduction

Despite interest in biologically inspired morphing wings began as early as flight itself, only a handful of researchers before the onset of the 21-st century ever attempted to investigate its potential use for commercial aviation, due to the late implementation of flexible composite materials in primary structures such as the wing box. Introduction of recent airliners with composite wings revived interest in hyper flexibility, and some research took place immediately after the Boeing “787” first flight. The benefits of drooped and inflected wings for transonic commercial aviation have been studied both in theory and experimentally by a number of researchers. One of the most comprehensive studies on aeroelastic wing shape optimization was published in 2013 by Nguyen et al. [1], where two curved configurations have been considered: a bio-inspired wing that mimics a seagull with drooped wings (downward deflection) and an upward inflected configuration that offers less drag benefits, but which is much more feasible within available aircraft manufacturing capabilities. Results have shown that the upward inflected wing could achieve up to 3.5% decrease in aerodynamic drag, which is significant. Khosravi and Zingg [2], showed that potential range increases between 2.6% and 4.9% could be achieved using non-planar curved wing configurations combined with optimized airfoil leading and trailing edges. Reliability of numerical optimizations carried out in the above and other studies depends
closely on the methods used for aerodynamic loading estimation on the non-planar inflected wing, which itself is still a subject of intense research.

Given aerodynamic loading on a lifting surface is directly impacted by its geometrical shape, any attempt for load estimation must start with a proper shape parametrization, i.e. choosing design variables that control the wing shape, which can be subsequently related to local parameters such as local angles of attack and hence, local forces can be obtained. While wing plan form and front view are limited by predominantly straight lines, requiring only few parameters to describe their geometry, airfoil shapes required much greater parametrization effort. Techniques have been developed since the 1930’s with the development by NACA of a comprehensive set of standard airfoil shapes, designated by a 4-digit code which parameters can be entered into equations to precisely generate the cross-section of the airfoil and empirically calculate its properties [3]. Popular methods nowadays include evolutionary algorithms based on parametric curves that cover a wide range of existing airfoils, such as Bezier curves, Parsec and Sobieczky methods [4], as well as various types of b-splines [5].

With the inevitable advent of flexible and morphing wings, development of a future framework for aerodynamic shape optimization methodologies will require a full 3D wing geometry parametrization, where both frontal and plan views are dominated by curvilinear shapes, greatly extending the degrees of freedom and posing a particular challenge for load estimation. Some computational solutions already exist, using free form deformation volumes, such as the performed by Koo and Zingg numerical parametrization and three-dimensional wing shape optimization based on a Newton-Krylov approach [6]. CAD platforms such as “Blender” [7], have been applied for a parametric shape deformation with great potential applications in aerodynamic shape optimization [8]. However, the required large number of discrete points required to constraint the full wing geometry implies significant computational resources, hence developing analytic methodologies remains a priority, and is the subject of this conference paper.

In this paper, an attempt is made to quantify the interaction between the wing structural stiffness, its aerodynamic loading and the statically deformed wing geometry. For this end, identified are local geometric parameters of the deflected wing, and their dependencies on the local angles of attack, hence on local aerodynamic loadings. Local bending-torsional stiffness of the wing structure is then coupled with local forces using cantilever beam theory. Analytic parametrization of the statically deformed wing shape is performed using exponential functions of the wing frontal projection, which reflects bending deflections. Bending-torsional stiffness is then the driver for obtaining twist displacements, which themselves adjust local angles of attack, hence closing the cycle. Bending-torsional behavior of swept back wings, which tip sections are known to experience a negative twist, is well studied and there are multiple methodologies that relate bending-twist displacements to the sweep angle. However, there are very few works that focus on the frontal projection of the wing, as it didn’t shift significantly for “rigid” metallic wings, remaining nearly straight. Given, a hyper-flexible wing would change in shape in an entirely different fashion, acquiring a highly non-planar curvilinear shape, the urgency is for a novel parametrization technique, where the starting point is the frontal projection of the deformed wing.

2. Exponential parametrization of the flexible wing geometry

The deformed wing shape is parametrized with the goal of quantifying its aerostructural behavior, allowing a definition of an optimal stiffness distribution S(z), one that produces an optimal deformed wing shape Y(z). The statically inflected wing converges to a curvilinear shape that can be best parametrized using an exponential function to describe its frontal projection. An example of such parametrization for the Common Research Model (CRM) is given in figure 1, where Y(z) = i z curve describes the front view of the deformed wing. The exponential base i > 1 defines the degree of inflection. Examples in figure 1 include a moderately inflected static cruise configuration (g=1) with i=1.005, and highly loaded maneuvering configurations with greater values of ‘i’. The slope of the deflected sections is equal to the first derivative Y’(z) = i z ln(i). Then, structural behavior of the deformed wing can be analyzed using cantilever beam theory, where the inflected shape is a function of both local structural stiffness S(z) and local aerodynamic force F(z). Local upward (y-coordinate)
displacements can be estimated using Hooke’s law: \( Y(z) = \frac{F(z)}{S(z)} \). Therefore, the required structural stiffness distribution to produce the wing shape \( Y(z) \) is equal to: \( S(z) = \frac{F(z)}{Y(z)} \). Or, using the exponential parametrization \( Y(z) = i^z \):

\[
S(z) = \frac{F(z)}{i^z} \quad (1)
\]

As shown in figure 1 below, ‘i’ is the design variable that controls the frontal shape of the inflected wing at different stages of its deformation depending on the g-load:

**Figure 1.** Exponential parametrization of CRM wing spanwise displacements. Examples of different wing loading configurations and their respective parametrization functions.

### 3. Derivation of a closed-form force-structure-geometry relationship

In equation (1), given that an optimal wing shape \( Y_{\text{opt}}(z) = i_{\text{opt}} z \) at a particular flight regime is usually known either from CFD or analytic aerodynamics, the key to obtaining the optimal stiffness function \( S(z) \) lies in precisely calculating the distribution of aerodynamic loading \( F(z) \) along the deformed wing. The structural stiffness distribution \( S(z) \) includes two components:

- Bending stiffness \( S_1(z) = EI(z) \) about the X-axis of the bending moment. It affects the magnitude of vertical displacements \( y(z) \) and the deflection slope \( y'(z) \), as well as local curvatures \( y''(z) \).
- Torsional stiffness \( S_2(z) = J E(z) \) about the Z-axis of the torque moment. It impacts local angles of twist, and hence directly defines local angles of attack and therefore local loadings \( f(z) \). Given the studied CRM wing is swept back, bending deflections cause a negative twist of the wing, hence the torsional and bending stiffnesses are interrelated: \( S_2(z) = S_1(S_1(z)) \).

As tip sections of the highly non-planar shape of the inflected wing alters the local flow field in a much similar way as curved winglets do, load distribution patterns are similar, and the methodology developed for calculating load distribution along curved winglets, using local angles of attack distribution [9] can be applied to estimate loading on the flexible part of the wing. From [9], the local angle of attack \( \alpha(z) \) depends on the local cant angle \( \psi(z) \) and the aircraft angle of attack as follows:
\[ \alpha(z) = \frac{\psi(z)}{\pi/2} \alpha_{a/c} + \alpha_{\text{twist}}(z) \]  

(2)

where:

- \( \alpha_{\text{twist}}(z) \) accounts for angle of attack increments due to torsional deformation, hence it is function of the local bending-torsional stiffness \( S_z(z) \);
- \( \psi(z) \) is the local cant angle, complementary to the slope angle of the parametrization curve \( Y(z) \) tangent line, and therefore it can be found by derivation of the parametrization function \( Y(z) \) as follows:

\[ \psi(z) = \frac{\pi}{2} - \arctan[Y'(z)] \]

(3)

By substituting equation 3 in equation 2, the angle of attack distribution along the inflected wing can be calculated for any wing shape \( Y(z) = i' \), at any aircraft angle of attack, \( \alpha \) as follows:

\[ \alpha(z) = \frac{\pi}{2} - \frac{\arctan[i' \cdot \ln(i)]}{\pi/2} \alpha_{a/c} + \alpha_{\text{twist}}(z) \]

(4)

Taking equation 4 into account and given a known distribution of airfoil sections, it is easy to obtain a closed form load distribution using the lift equation:

\[ F(z) = C_L(z) \cdot S(z) \cdot \frac{\rho v^2}{2} \]

(5)

where local lift coefficient \( C_L(z) \) depends linearly on the local angle of attack. For example, a standard supercritical airfoil with a lift slope \( \approx 0.1 \) and a zero-alpha lift coefficient of \( \approx 0.45 \), has a linear part: \( C_L(z) \approx 0.1\alpha(z) + 0.45 \). Local (elementary) surface is equal to the local chord length: \( S(z) = c(z) \cdot \Delta z \), where \( \Delta z = 1 \) is a unit span length. Substituting in equation 5, load distribution is equal to:

\[ F(z) = [0.1\alpha(z) + 0.45] \cdot c(z) \cdot \frac{\rho v^2}{2} \]

(6)

Substituting from equation 4 the local angle of attack distribution along deformed wing span, equation 6 becomes:

\[ F(z) = \left[ \frac{\pi}{2} - \frac{\arctan[i' \cdot \ln(i)]}{\pi/2} \alpha_{a/c} + \alpha_{\text{twist}}(z) + 0.45 \right] \cdot c(z) \cdot \frac{\rho v^2}{2} \]

(7)

Equation 7 is a closed form load distribution along an inflected wing, where all the ingredients are predetermined: \( i, c(z) \) and \( \alpha_{a/c} \) are purely geometric values. Dynamic pressure \( (\rho v^2)/2 \) is given at any flight speed and altitude. \( \alpha_{\text{twist}}(z) \) can be calculated if the torsional stiffness distribution \( S_z(z) \), the aerodynamic and rigidity centers are known, which is usually the case at higher levels of design maturity of the wing geometry and structure. Determination of these values is beyond the scope of the concept exploration level, studied in this paper.
4. Discussion
Presented is a brief analytic study to correlate the tightly coupled aspects of a flexible wing design: aerodynamic, structural and geometrical. This was achieved through mathematical parametrization of the statically inflected wing shape, then using parametrized functions to calculate local values of aerodynamic loading, which are dependent on local angles of attack. With a known aerodynamic loading distribution, cantilever beam theory can be effectively exploited to determine the required distribution of bending and torsional stiffnesses along the wing span, therefore reducing the computational time and resources required for higher fidelity optimization. Future computational studies are required to validate this methodology, as well as to study the dynamic behavior of flexible wings. Therefore, fluid-structure interaction (FSI) experiments will determine to what curvilinear shape a wing with a predetermined anisotropic stiffness will converge to.

Practical realization of this wing concept is possible through the use of anisotropic composite materials with controllable stiffness distribution in different axes. Hence, a realistic composite material data and lamination parameters are needed to get the pre-calculated bending-torsional stiffness distribution, that in turn will produce the desired optimal curvilinear wing at each flight regime and loading conditions.

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