Searching for New Physics with $B$-Decay Fake Triple Products

Alakabha Datta $^a$, Murugeswaran Duraisamy $^a,^2$ and David London $^b,^3$

\textit{a}: Department of Physics and Astronomy, 108 Lewis Hall, University of Mississippi, Oxford, MS 38677-1848, USA
\textit{b}: Physique des Particules, Université de Montréal, C.P. 6128, succ. centre-ville, Montréal, QC, Canada H3C 3J7

Abstract

In pure-penguin $\bar{b} \to \bar{s} \ B \to V_1 V_2$ decays ($V_{1,2}$ are vector mesons), $f_T/f_L \simeq 1$ has been observed ($f_T$ ($f_L$) is the polarization fraction of transverse (longitudinal) decays). Explanations of this unexpectedly large result have been given within the standard model (SM) and with new physics (NP). In this paper, we show that these two explanations can be partially distinguished through the triple products (TP’s) in these transitions. In particular, the SM predicts one of the two fake, CP-conserving TP’s to be small ($|A^{(2)}_T| \leq 9\%$), while NP often gives larger values for $|A^{(2)}_T|$. We discuss the implications of the measurements of both fake TP’s in $B \to \phi K^*$ – the present data prefer a SM explanation of $f_T/f_L$ – and provide the SM predictions for $B^0_s \to \phi \phi$. 

\begin{itemize}
  \item $^1$datta@phy.olemiss.edu
  \item $^2$duraism@phy.olemiss.edu
  \item $^3$london@lps.umontreal.ca
\end{itemize}
Over the past 10-15 years, a great deal of effort has been put into measuring CP violation in the $B$ system. The great majority of these measurements have been of direct and indirect CP asymmetries in $B$ decays. As always, the goal is to find a discrepancy with the predictions of the standard model (SM). To date, the measurements are generally in agreement with the SM. However, there are some small hints of disagreements in some $\bar{b}\to s$ decays.

Some time ago, it was pointed out that there is another signal of CP violation in $B\to V_1V_2$ ($V_{1,2}$ are vector mesons) – a triple product (TP) \footnote{1} \footnote{2}. In the rest frame of the $B$, the TP takes the form $\bar{q}\cdot(\vec{\epsilon}_1\times\vec{\epsilon}_2)$, where $\vec{q}$ is the difference of the momenta of the final vector mesons, and $\vec{\epsilon}_1$ and $\vec{\epsilon}_2$ are the polarizations of $V_1$ and $V_2$. The TP is odd under both parity and time reversal, and thus constitutes a potential signal of CP violation.

The most general Lorentz-covariant amplitude for the decay $B(p)\to V_1(k_1,\vec{\epsilon}_1) + V_2(k_2,\vec{\epsilon}_2)$ is given by \footnote{1} \footnote{2}

$$M = a\vec{\epsilon}_1^*\cdot\vec{\epsilon}_2^* + \frac{b}{m_B^2}(p\cdot\vec{\epsilon}_1^*)(p\cdot\vec{\epsilon}_2^*) + \frac{c}{m_B^2}\epsilon_{\mu\nu\rho\sigma}p^\mu q^\nu\vec{\epsilon}_1^*\vec{\epsilon}_2^*\epsilon_\rho^*\epsilon_\sigma^*,$$

where $q \equiv k_1 - k_2$. The quantities $a$, $b$ and $c$ are complex and will in general contain both CP-conserving strong phases and CP-violating weak phases. In $B\to V_1V_2$ decays, the final state can have total spin 0, 1 or 2, which correspond to the $V_1$ and $V_2$ having relative orbital angular momentum $l = 0$ (s wave), $l = 1$ (p wave), or $l = 2$ (d wave), respectively. The $a$ and $b$ terms correspond to combinations of the parity-even $s$- and $d$-wave amplitudes, while the $c$ term corresponds to the parity-odd $p$-wave amplitude.

In order to obtain experimental information about the TP, one uses the linear polarization basis. Here, one decomposes the decay amplitude into components in which the polarizations of the final-state vector mesons are either longitudinal ($A_0$), or transverse to their directions of motion and parallel ($A_\parallel$) or perpendicular ($A_\perp$) to one another. $A_0$, $A_\parallel$, $A_\perp$ are related to $a$, $b$ and $c$ of Eq. (\footnote{1} via \footnote{2})

$$A_\parallel = \sqrt{2}a, \quad A_0 = -ax - \frac{m_1m_2}{m_B^2}b(x^2 - 1), \quad A_\perp = 2\sqrt{2}\frac{m_1m_2}{m_B^2}\sqrt{x^2 - 1},$$

where $x = k_1 \cdot k_2/(m_1m_2)$ ($m_1$ and $m_2$ are the masses of $V_1$ and $V_2$, respectively.). Now, in the rest frame of the $B$, the $c$ term of Eq. (\footnote{1} is proportional to the TP $\vec{q}\cdot(\vec{\epsilon}_1\times\vec{\epsilon}_2)$. Thus, there are two TP terms in $|M|^2$, proportional to $\text{Im}(ca^*)$ and $\text{Im}(ba^*)$ \footnote{2}. Equivalently, from the above equation, the two TP’s are proportional to linear combinations of $\text{Im}(A_\perp A_\parallel^*)$ and $\text{Im}(A_\parallel A_\parallel^*)$. (Note that this is to be expected – the TP is parity-odd, and one can generate such an effect through the interference of either the $l$-even s-wave or $d$-wave state (i.e. $A_0$ or $A_\parallel$) with the $l$-odd $p$-wave state ($A_\perp$).

Assuming that $V_{1,2}$ both decay into pseudoscalars, i.e. $V_1 \to P_1P_1'$, $V_2 \to P_2P_2'$,
the angular distribution of $B \to V_1V_2$ is then given by [3, 4]

$$
\frac{d\Gamma}{d\cos\theta_1 d\cos\theta_2 d\phi} = N \left( |A_0|^2 \cos^2\theta_1 \cos^2\theta_2 + \frac{|A_\perp|^2}{2} \sin^2\theta_1 \sin^2\theta_2 \sin^2\phi + \frac{|A_\parallel|^2}{2} \sin^2\theta_1 \sin^2\theta_2 \cos^2\phi + \frac{\text{Re}(A_0 A_0^*)}{2\sqrt{2}} \sin 2\theta_1 \sin 2\theta_2 \cos \phi - \frac{\text{Im}(A_\perp A_0^*)}{2\sqrt{2}} \sin 2\theta_1 \sin 2\theta_2 \sin \phi - \frac{\text{Im}(A_\parallel A_0^*)}{2} \sin^2\theta_1 \sin^2\theta_2 \sin 2\phi \right),
$$

(3)

where $\theta_1 (\theta_2)$ is the angle between the directions of motion of the $P_1 (P_2)$ in the $V_1 (V_2)$ rest frame and the $V_1 (V_2)$ in the $B$ rest frame, and $\phi$ is the angle between the normals to the planes defined by $P_1 P_2'$ and $P_2 P_1'$ in the $B$ rest frame. (For other decays of the $V_1$ and $V_2$ (e.g. into $e^+e^-$, $P\gamma$ or three pseudoscalars), one will obtain a different angular distribution, see Refs. [3, 4, 5].) The key point is that, by performing a full angular analysis of $B \to V_1V_2$, one can obtain $\text{Im}(A_\perp A_0^*)$ and $\text{Im}(A_\parallel A_0^*)$, i.e. both TP’s, from Eq. (3) above.

Now, above we indicated that TP’s are a signal of CP violation. This is not quite accurate. As already noted, in general the $A_i$ ($i = 0, \parallel, \perp$) possess both weak (CP-odd) and strong (CP-even) phases. Thus, $\text{Im}(A_\perp A_0^*)$ and $\text{Im}(A_\parallel A_0^*)$ can both be nonzero even if the weak phases vanish. In order to obtain a true signal of CP violation, one has to compare the $B$ and $\bar{B}$ decays. The amplitude for $\bar{B}(p) \to \bar{V}_1(k_1, \varepsilon_1) + \bar{V}_2(k_2, \varepsilon_2)$ can be obtained by operating on Eq. (1) with CP.

This yields

$$
\bar{M} = \bar{a} \varepsilon_1^* \cdot \varepsilon_2^* + \frac{\bar{b}}{m_B^2} (p \cdot \varepsilon_1^*)(p \cdot \varepsilon_2^*) - i \frac{\bar{c}}{m_B^2} \epsilon_{\mu\nu\rho\sigma} p^\mu q^\nu \varepsilon_1^{\rho\sigma},
$$

(4)
in which $\bar{a}$, $\bar{b}$ and $\bar{c}$ are equal to $a$, $b$ and $c$, respectively, except that the weak phases are of opposite sign. Thus, the above equation can be obtained from Eq. (1) by changing $a \to \bar{a}$, $b \to \bar{b}$ and $c \to -\bar{c}$. Similarly, the angular distribution of this decay is the same as that in Eq. (3), with $A_0 \to \bar{A}_0$, $A_\parallel \to \bar{A}_\parallel$ and $A_\perp \to -\bar{A}_\perp$, in which the $\bar{A}_i$ are obtained from the $A_i$ by changing the sign of the weak phases.

The point is that the TP’s for the $\bar{B}$ decay are $-\text{Im}(\bar{A}_\perp \bar{A}_0^*)$ and $-\text{Im}(\bar{A}_\parallel \bar{A}_0^*)$. The true (CP-violating) TP’s are then given by $\frac{1}{2}[\text{Im}(A_\perp A_0^*) - \text{Im}(\bar{A}_\perp \bar{A}_0^*)]$ and $\frac{1}{2}[\text{Im}(A_\parallel A_0^*) - \text{Im}(\bar{A}_\parallel \bar{A}_0^*)]$. But there are also fake (CP-conserving) TP’s, due only to strong phases of the the $A_i$’s. These are given by $\frac{1}{2}[\text{Im}(A_\perp A_0^*) + \text{Im}(\bar{A}_\perp \bar{A}_0^*)]$ and $\frac{1}{2}[\text{Im}(A_\parallel A_0^*) + \text{Im}(\bar{A}_\parallel \bar{A}_0^*)]$. For the fake TP’s, it is necessary to distinguish $B$ and $\bar{B}$, i.e. untagged samples contain no fake TP’s [6].

\footnote{Another way to see this is to note that when one acts on the $B$ decay amplitude [Eq. (1)] with CP, there is a factor $(-1)^l$ for each term, where $l$ is the relative angular momentum of the two vector mesons. Since the $c$-term corresponds to a state with $l = 1$, the $\bar{c}$ term, which is related to $\bar{A}_\perp$, has an additional factor of $-1$ associated with it. As a consequence, the TP’s in the angular distribution, which are proportional to $\bar{A}_\perp$, also have a factor of $-1$.}
In order to illustrate characteristics of the true and fake TP’s, suppose that there are two amplitudes $A_1$ and $A_2$ contributing to a given decay, and that the TP is proportional to $\text{Im}(A_1 A_2^*)$. It is straightforward to show that

\begin{align*}
TP_{\text{true}} &\propto \sin \phi \cos \delta, \\
TP_{\text{fake}} &\propto \cos \phi \sin \delta,
\end{align*}

where $\phi$ and $\delta$ are, respectively, the relative weak and strong phases between $A_1$ and $A_2$. As is clear from these expressions, the true TP requires a nonzero $\phi$ and is relatively insensitive to $\delta$. That is, as with any genuine CP-violating effect, the interference of two amplitudes with a relative weak phase is required. On the other hand, the fake TP requires only a nonzero strong-phase difference $\delta$, and can be nonzero even if the weak-phase difference $\phi$ vanishes. Since the linear polarization amplitudes will, in general, have different strong phases, this will lead to nonzero fake TP’s for all decays.

For the two TP’s of Eq. (3), we define

\begin{align*}
A^{(1)}_T &\equiv \frac{\text{Im}(A_{\perp} A_0^*)}{A_0^2 + A_{\parallel}^2 + A_{\perp}^2}, \\
A^{(2)}_T &\equiv \frac{\text{Im}(A_{\perp} A_0^*)}{A_0^2 + A_{\parallel}^2 + A_{\perp}^2}.
\end{align*}

The corresponding quantities for the charge-conjugate process, $\bar{A}^{(1)}_T$ and $\bar{A}^{(2)}_T$, are defined similarly, but with a multiplicative minus sign. Consider now $B \to V_1 V_2$ decays in which the final vector mesons are light: $m_{1,2} \ll m_B$. In Ref. [2] it was shown that, in the SM within factorization,

$$\frac{|A_{\parallel}|}{|A_0|} = O\left(\frac{m_{1,2}}{m_B}\right).$$

That is, the transverse amplitudes are naively expected to be much smaller than the longitudinal amplitude. This implies that, in general, $|A^{(2)}_T| \ll |A^{(1)}_T|$.

One also expects that, in $B \to V_1 V_2$, the fraction of transverse decays, $f_T$, is much less than the fraction of longitudinal decays, $f_L$. However, it was observed that these two fractions are roughly equal in the decay $B \to \phi K^*$: $f_T/f_L \simeq 1$ [7]. There are two possible explanations of this. The first is that the SM is still valid, but one must go beyond the minimal version. One scenario is that nonfactorizable QCD-factorization penguin-annihilation effects are important [8]. A second scenario involves nonperturbative rescattering [9, 10]. Alternatively, one can explain the $f_T/f_L$ measurement by introducing physics beyond the SM. Suppose there is a new-physics (NP) contribution to the $\bar{b} \to \bar{s}ss$ quark-level amplitude. If the NP operator has the structure $(1 - \gamma_5) \otimes (1 - \gamma_5)$ or $\sigma(1 - \gamma_5) \otimes \sigma(1 - \gamma_5)$ (denoted $ST_{LL}$ below), or $(1 + \gamma_5) \otimes (1 + \gamma_5)$ or $\sigma(1 + \gamma_5) \otimes \sigma(1 + \gamma_5)$ ($ST_{RR}$), this will contribute dominantly to $f_T$ in $B \to \phi K^*$ and not to $f_L$ [11, 12]. One can therefore reproduce the measured value of $f_T/f_L$ if the NP amplitude has the right size. In this paper, we do not
Table 1: Longitudinal polarization fraction $f_L$ for various $B \to V_1 V_2$ decays, taken from Ref. [13].

| Decay         | Final State | $f_L$         |
|---------------|-------------|---------------|
| $B \to \phi K^*$ [7] | $\phi K^{*0}$ | $0.480 \pm 0.030$ |
|               | $\phi K^{*+}$ | $0.50 \pm 0.05$ |
| $B \to \rho K^*$ [14] | $\rho^0 K^{*0}$ | $0.57 \pm 0.12$ |
|               | $\rho^+ K^{*0}$ | $0.48 \pm 0.08$ |
| $B \to K^* K^*$ [15] | $K^{*0} K^{*0}$ | $0.80 \pm 0.12$ |
|               | $K^{*+} \bar{K}^{*0}$ | $0.75 \pm 0.16$ |
| $B \to \rho \rho$ [16] | $\rho^0 \rho^0$ | $0.75 \pm 0.12$ |
|               | $\rho^+ \rho^-$ | $0.978 \pm 0.025$ |

Assess the advantages and disadvantages of the two explanations. Rather, our aim is to propose a way of distinguishing them.

$B \to V_1 V_2$ decays can be separated into four types. These include $\bar{b} \to \bar{s}$ transitions: (i) pure penguin (e.g. $B \to \phi K^*$), (ii) tree and penguin contributions (e.g. $B \to \rho K^*$), and $\bar{b} \to \bar{d}$ transitions: (i) pure penguin (e.g. $B \to K^* K^*$), (ii) tree and penguin contributions (e.g. $B \to \rho \rho$). The polarizations have been measured for the decays in parentheses (and others [13]). The results are shown in Table 1.

As noted above, there is an effect in the $\bar{b} \to \bar{s}$ penguin amplitude which leads to $f_T/f_L \approx 1$. There is a similar, though weaker, effect in the $\bar{b} \to \bar{d}$ penguin amplitude giving $f_T/f_L \approx 1/3$. The data suggest that the tree amplitude(s) reproduce the naive expectations, i.e. the transverse amplitudes are much smaller than the longitudinal amplitude. Thus, in $B_0^+ \to \rho^+ \rho^-$ and $B^+ \to \rho^+ \rho^0$, which are $\bar{b} \to \bar{d}$ decays with both tree and penguin contributions, we have $f_T/f_L \approx 0$. This is because the color-allowed tree amplitude is the dominant contribution. (In $B^0_2 \to \rho^0 \rho^0$, the color-suppressed tree amplitude is smaller, and the contribution of the $\bar{b} \to \bar{d}$ penguin amplitude leads to $f_T/f_L \approx 1/3$.) And in the $\bar{b} \to \bar{s}$ decay with both tree and penguin contributions, the tree amplitude, though nonzero, is subdominant. This gives a value for $f_T/f_L$ which is slightly smaller than that for the pure penguin $\bar{b} \to \bar{s}$ decay. The upshot of all of this is that there are three classes of decays in which the transverse polarizations are reasonably large. Therefore, for these decays, we have $|A_T^{(2)}| \approx |A_T^{(1)}|$, contrary to our naive expectation.

However, there is more, and this is the main point of this paper. It is also possible to express the polarization amplitudes using the helicity formalism. Here, the transverse amplitudes are written as

$$A_\parallel = \frac{1}{\sqrt{2}}(A_+ + A_-),$$
A_\perp = \frac{1}{\sqrt{2}}(A_+ - A_-) . \quad (8)

The key observation is the following. Due to the fact that the weak interactions are left-handed, i.e. the couplings are $V - A$, the helicity amplitudes obey the hierarchy

$$\left| \frac{A_+}{A_-} \right| = \frac{\Lambda_{QCD}}{m_b} . \quad (9)$$

Thus, in the heavy-quark limit, $A_+$ is negligible compared to $A_-$, so that $A_\parallel = -A_\perp$. But in this case, $A_T^{(2)}$, which is proportional to $\text{Im}(A_\perp A^*_\parallel)$, vanishes. This means that if the large $f_T/f_L$ observed in several $B \to V_1 V_2$ decays is due to the SM, $A_T^{(2)} = 0$ should be found. On the other hand, suppose that the large $f_T/f_L$ is due to NP. If the new interactions have a different weak phase from the SM, they can be detected using the true TP’s (of $A_T^{(1)}$ or $A_T^{(2)}$). However, the NP could have the same weak phase as the SM, so that the true TP’s vanish [Eq. (5)]. It may therefore not be ideal to concentrate on the true TP’s – it also may be useful to measure the fake TP constructed from $A_T^{(2)}$ and $\bar{A}_T^{(2)}$. As we will see below, it is possible to partially distinguish the SM from NP through the measurement of the fake $A_T^{(2)}$ TP.

Of course, there are corrections to the prediction that $A_T^{(2)} = 0$, since the heavy-quark limit is just an approximation. Below, we take these corrections into account, and estimate $A_T^{(2)}$ for the pure-penguin $\bar{b} \to \bar{s}$ decays $B \to \phi K^*$ and $B^0_s \to \phi \phi$. We also comment on the size of $A_T^{(2)}$ for other $\bar{b} \to \bar{s}$ and $\bar{b} \to \bar{d}$ transitions.

We take $A_\lambda = |A_\lambda|e^{i\delta_\lambda}$ ($\lambda = 0, \perp, \parallel$), and define $r_T \equiv |A_+/A_-|$. $A_T^{(2)}$ is then given by

$$A_T^{(2)} = \frac{r_T f_T}{1 + r_T^2} \sin (\delta_+ - \delta_-) , \quad (10)$$

where the polarization fractions are

$$f_i = \frac{|A_i|^2}{|A_0|^2 + |A_\perp|^2 + |A_\parallel|^2} , \quad i = 0, \perp, \parallel , \quad (11)$$

with $f_T = f_\perp + f_\parallel$. In $\bar{b} \to \bar{s}$ transitions, all contributions to the decay are proportional to the Cabibbo-Kobayashi-Maskawa (CKM) factors $V_{tb}^* V_{ts}$, $V_{cb}^* V_{cs}$, or $V_{ub}^* V_{us}$. The term $V_{cb}^* V_{cs}$ can be eliminated in terms of the other two using the unitarity of the CKM matrix. Furthermore, although $V_{ub}^* V_{us}$ has a large weak phase, its magnitude is greatly suppressed relative to that of $V_{tb}^* V_{ts}$. In pure-penguin $\bar{b} \to \bar{s}$ decays, the $V_{ub}^* V_{us}$ term is negligible, to a good approximation. That is, there is effectively only one weak amplitude (i.e. $\delta_+ - \delta_- \equiv \pi$ is purely a strong phase), and so all CP-violating effects are tiny. Thus, $A_T^{(2)} = -\bar{A}_T^{(2)}$ and so $A_T^{(2)}$ is by itself a fake TP.

In order to estimate the size of $A_T^{(2)}$, we proceed as follows. First, within QCD factorization [18], $r_T$ is expected to be about 4%. When the penguin-annihilation
amplitude is added, $r_T$ is increased to lie in the range 5%-15%. Second, it is straightforward to show that
\[
\frac{[(1 - r_T^2)^2 + 4r_T^2 \sin^2 (\delta_+ - \delta_-)]^{1/2}}{1 + r_T^2 + 2r_T \cos (\delta_+ - \delta_-)} = \sqrt{\frac{f_\perp}{f_\parallel}} .
\] (12)

Given the experimental values for $f_\perp$ and $f_\parallel$, the above equation provides a constraint on $r_T$ and the phase $(\delta_+ - \delta_-)$.

We begin with $B \to \phi K^*$. If desired, one can avoid tagging altogether by considering charged-$B$ decays, or by using self-tagging decays of the $K^{*0}$ in $B^0 \to \phi K^{*0}$. The estimate for $A_T^{(2)}$ is found using Eq. (10). $r_T$ is varied in the range (0.05, 0.15), and the phases $\delta_\pm$ in the range (0, 2$\pi$). The constraint of Eq. (12) is imposed using the measured polarization fractions $f_\perp = 0.241 \pm 0.029$ and $f_\parallel = 1 - f_\perp - f_\perp = 0.279 \pm 0.042$ [13]. The experimental numbers are varied within their $\pm 1\sigma$ errors. The result is shown in Fig. 1. There we see that $|A_T^{(2)}| \leq 9\%$ is predicted.

This prediction can be compared with the experimental result. $A_T^{(2)}$ has not been explicitly measured, but its value can be deduced using other measurements. The relevant $B_d \to \phi K^{*0}$ polarization observables are shown in Table 2. Here, the relative phases between $A_{\perp,\parallel}$ and $A_0$, denoted $\phi_\perp$ and $\phi_\parallel$, are defined to be
\[
\phi_i = \arg \frac{A_i}{A_0} - \pi \, \text{sign}(\arg \frac{A_i}{A_0}) , \quad i = \perp, \parallel .
\] (13)

We follow the convention of Ref. [19] for the polarization fractions, and of Ref. [17] for the phases, defining
\[
f_L^Q \equiv f_L (1 + Q A_{CP}^0) , \quad f_\perp^Q \equiv f_\perp (1 + Q A_{CP}^1) ,
\]
\[
\phi_h^Q = \phi_h + Q \Delta \phi_h , \quad h = |\parallel, \perp .
\] (14)
Polarization fractions

|                |       |
|----------------|-------|
| $f_L$          | 0.480 ± 0.030 |
| $f_\perp$      | 0.241 ± 0.029 |

Phases

|                |       |
|----------------|-------|
| $\phi_\parallel$ (rad) | 2.40±0.14 |
| $\phi_\perp$ (rad)     | 2.39 ± 0.13 |
| $\Delta \phi_\parallel$ (rad) | 0.11 ± 0.13 |
| $\Delta \phi_\perp$ (rad)    | 0.08 ± 0.13 |

CP asymmetries

|                |       |
|----------------|-------|
| $A_{CP}^\parallel$ | 0.04 ± 0.06 |
| $A_{CP}^\perp$      | −0.11 ± 0.12 |

Table 2: $B_d \to \phi K^*$ polarization observables [13].

Here, $Q = 1$ ($-1$) for $\bar{B}^0$ ($B^0$). Using the numbers above we can calculate $A^{(2)}_T$:

$$A^{(2)}_T = \frac{1}{2} (A_{T,B}^{(2)} - \bar{A}_{T,B}^{(2)}) = 0.002 ± 0.049.$$ (15)

The measured value of $A^{(2)}_T$ is therefore in agreement with the SM prediction. Indeed, it is consistent with zero. What does this say about the NP explanations of the large observed value of $f_T/f_L$? In the heavy-quark limit, $A_+ = 0$ in the $ST_{LL}$ scenario, so that $A_\parallel = -A_\perp$ (as in the SM) and $A^{(2)}_T = 0$. Similarly, $ST_{RR}$ predicts that $A_- = 0$, so that $A_\parallel = A_\perp$ and $A^{(2)}_T = 0$. Thus, the result that $A^{(2)}_T \simeq 0$ is consistent with $ST_{LL}$. It also appears to be consistent with $ST_{RR}$. However, the SM and $ST_{RR}$ make very different predictions for $A_+$ and $A_-$. Since both the SM and $ST_{RR}$ operators are present in this NP scenario – the value of $f_L$ confirms the importance of the SM contribution – the predicted value of $A^{(2)}_T$ is nonzero. Thus, the measurement of $A^{(2)}_T \simeq 0$ rules out $ST_{RR}$, or at least strongly constrains it. Furthermore, in real model calculations (e.g. in the two-Higgs-doublet model [20]), in general both $ST_{LL}$ and $ST_{RR}$ operators appear, so that neither $A_+$ nor $A_-$ is zero, and $A^{(2)}_T \neq 0$. As above, such NP scenarios are generally ruled out. (Note that even if the NP operators have new weak phases, this will not significantly affect the fake $A^{(2)}_T$ TP, see Eq. (5). The one exception is if the new phase is near $\frac{\pi}{2}$ or $\frac{3\pi}{2}$. In this case, the fake TP is small, and the NP must be detected through a true TP, which is maximal.) We therefore see that the measurement of the fake $A^{(2)}_T$ TP allows us to partially differentiate the SM from the NP explanations of $f_T/f_L$. The present $B \to \phi K^*$ data suggest that the SM is preferred over NP.

We now turn to $B_s \to \phi \phi$. In this case, tagging is necessary to distinguish the $B_s^0$ and $\bar{B}_s^0$ decays. Furthermore, $B_s^0-\bar{B}_s^0$ mixing must be taken into account. Within the SM, in which the weak phase of the mixing is negligible, the TP terms of Eq. (3) are modified as follows [21]:

$$\text{Im}(A_\perp A_{0,\parallel}^* - e^{-i\tau} \left( \text{Re}(A_\perp A_{0,\parallel}^*) \cos \Delta m t - \text{Im}(A_\perp A_{0,\parallel}^*) \sin \Delta m t \right),$$ (16)

where we have set the mixing phase to 0. As before, we use Eq. (10) to estimate $A^{(2)}_T$, taking $r_T$ and $\delta_\pm$ in the ranges $(0.05, 0.15)$ and $(0, 2\pi)$, respectively. The CDF
data for the polarization observables for this decay are [22]

\[
\begin{align*}
  f_L &= 0.348 \pm 0.041 \text{(stat)} \pm 0.021 \text{(syst)}, \\
  f_{\|} &= 0.287 \pm 0.043 \text{(stat)} \pm 0.011 \text{(syst)}, \\
  f_{\perp} &= 0.365 \pm 0.044 \text{(stat)} \pm 0.027 \text{(syst)}, \\
  f_T &= 0.652 \pm 0.041 \text{(stat)} \pm 0.021 \text{(syst)}. \\
\end{align*}
\]

These are used to impose the constraint of Eq. (12) (the experimental numbers are varied within ±1σ). The result is shown in Fig. 2. The prediction is that $|A_T^{(2)}| \leq 10\%$.

It is interesting to compare the decays $B \rightarrow \phi K^*$ and $B_s \rightarrow \phi \phi$. These are the same in the flavor SU(3) limit. However, at present there are significant differences. For example, the polarization fraction $f_L$ differs between the two decays by more than 3σ. (Still, SU(3) breaking, which can have an effect of \~25%, could account for this.) Due to the fact that $f_L$ is relatively small in $B_s \rightarrow \phi \phi$, a sizeable fraction of the $(\delta_+ - \delta_-)$ space is not allowed in Fig. 2. If we ignore the differences between the two decays and use the allowed values of $(\delta_+ - \delta_-)$ from $B_s \rightarrow \phi \phi$ as an input for $B_d \rightarrow \phi K^{*0}$, we see that $A_T^{(2)}$ is predicted to be very small in this decay (see Fig. 1).

As noted above, $\bar{b} \rightarrow \bar{s}$ decay amplitudes can be written in terms of $V_{tb}^*V_{ts}$ and $V_{ub}^*V_{us}$; and in pure-penguin $\bar{b} \rightarrow \bar{s}$ decays, the $V_{ub}^*V_{us}$ term is negligible. However, this approximation is not valid for $\bar{b} \rightarrow \bar{s}$ transitions in which there is a tree contribution, such as $B \rightarrow \rho K^*$. In such decays, since $V_{ub}^*V_{us}$ has a large weak phase, the $A_T^{(i)}$’s are no longer purely fake TP’s. Thus, in order to estimate the TP’s, one also has to compute the $A_T^{(i)}$’s for the charge-conjugate decays. Now, while it is still true that the fake $A_T^{(2)}$ TP vanishes in the heavy-quark limit, calculating corrections to this due to the finite $b$-quark mass is much more complicated. Because there is
more than one amplitude, the TP’s depend on additional variables (magnitudes of diagrams, weak and strong phases), and there are not enough experimental observations to constrain the parameters. For this reason we cannot provide a reliable estimate of the fake $A_T^{(2)}$ in this case.

Pure-penguin $\bar{b} \to \bar{d}$ decays are similar in this respect. The penguin diagram proportional to $V_{ub}^* V_{ud}$ is not negligible, so there are two amplitudes contributing to the decay. As such, the TP’s depend on more parameters than in pure-penguin $\bar{b} \to \bar{s}$ decays, and so the corrections to the heavy-quark limit result cannot be estimated reliably.

Finally, for pure-penguin $\bar{b} \to \bar{s}$ decays, we can estimate the $A_T^{(1)}$ fake TP. Defining $r_0 \equiv |A_0/A_-|$, $A_T^{(1)}$ is given by

$$A_T^{(1)} \equiv \frac{1}{\sqrt{2}} r_0 f_L [ r_T \sin (\delta_+ - \delta_0) - \sin (\delta_- - \delta_0) ] . \quad (18)$$

$r_0$ can be fixed from

$$r_0^2(1 + r_T^2) = \frac{f_T}{f_L} . \quad (19)$$

For $B \to \phi K^*$, we vary $r_T$ in the range $(0.05, 0.15)$, all phases $\delta_\lambda (\lambda = 0, \pm)$ in the range $(0, 2\pi)$, and all polarization fractions within $\pm 1\sigma$. This gives $r_0$ in the range $(0.95, 1.1)$. $r_T$ and $(\delta_+ - \delta_-)$ are further constrained by Eq. (10). The result for $A_T^{(1)}$ is shown in Fig. 3. $|A_T^{(1)}| \leq 40\%$ is predicted.

This prediction can be compared with the experimental result, which is deduced from other measurements as before. We find

$$A_T^{(1)} = -0.23 \pm 0.03 , \quad (20)$$
in agreement with the SM.

For $B_s \rightarrow \phi \phi$, we use the same procedure as above. $r_0$ is found to lie in the range (1.25, 1.47). The prediction for $A_T^{(1)}$ is shown in Fig. 4. We find $|A_T^{(1)}| \leq 40\%$.

In summary: the angular distribution of $B \rightarrow V_1V_2$ ($V_{1,2}$ are vector mesons) contains triple products (TP’s), odd under parity and time reversal. There are two TP’s, denoted $A_T^{(1)} \sim \text{Im}(A_\parallel A_0^\ast)$ and $A_T^{(2)} \sim \text{Im}(A_\perp A_0^\ast)$, where the $A_i$ ($i = 0, ||, \perp$) are the polarization amplitudes. There are TP’s in $B$ decays as well: $\bar{A}_T^{(i)}$ ($i = 1, 2$). They are equal to $-A_T^{(i)}$, with the weak phases having the opposite sign. There are two categories of TP’s: (i) real TP’s: $\frac{1}{2}[A_T^{(i)} + \bar{A}_T^{(i)}]$, and (ii) fake TP’s: $\frac{1}{2}[A_T^{(i)} - \bar{A}_T^{(i)}]$. Real TP’s are CP-violating, and are nonzero only if the decay has two contributing amplitudes with a relative weak phase. Fake TP’s are CP-conserving, and can be generated by strong phases alone. For fake TP’s, it is necessary to distinguish $B$ and $\bar{B}$, so that tagging is needed, possibly by using self-tagging decays.

In the heavy-quark limit, the standard model (SM) predicts that $A_\parallel = -A_\perp$, so that $A_T^{(2)} = 0$. We have computed the finite-mass corrections to this for the pure-penguin $\bar{b} \rightarrow \bar{s}$ decays $B \rightarrow \phi K^*$ and $B_0^0 \rightarrow \phi \phi$. These are especially interesting because, to a good approximation, they have only one weak amplitude. As a consequence, all CP-violating effects essentially vanish. In particular, these decays have only fake TP’s. For $B \rightarrow \phi K^*$, we find that the SM predicts $|A_T^{(2)}| \leq 9\%$, consistent with the measured value of 0.002 $\pm$ 0.049.

There is a further consequence of this measurement. In $B \rightarrow \phi K^*$, it is expected that $f_T \ll f_L$, where $f_T$ and $f_L$ are the fractions of transverse and longitudinal decays, respectively. However, $f_T/f_L \approx 1$ is found. Explanations of this result have been given within the SM and with new physics (NP). Interestingly, the NP scenarios often predict large values for $|A_T^{(2)}|$, and are thus ruled out, or at least strongly constrained, by the current measurement of $A_T^{(2)}$. Thus, the measurement of the
fake $A_T^{(2)}$ TP allows us to partially differentiate the SM from the NP explanations of $f_T/f_L$.

We find that the SM predicts $|A_T^{(2)}| \leq 10\%$ for $B^0_s \to \phi\phi$. We have also estimated $A_T^{(1)}$ within the SM, with the result that $|A_T^{(1)}| \leq 40\%$ for both decays. For $B \to \phi K^*$, this is consistent with the measured value of $-0.23 \pm 0.03$.

Acknowledgments: We are extremely grateful to Mirco Dorigo, Marco Rescigno and Anna Maria Zanetti for asking the questions which led to this study. Thanks also to Marco Rescigno for pointing out an error in an earlier version of this paper. This work was financially supported by the US-Egypt Joint Board on Scientific and Technological Co-operation award (Project ID: 1855) administered by the US Department of Agriculture (AD, MD), and by NSERC of Canada (DL).

References

[1] G. Valencia, Phys. Rev. D 39, 3339 (1989).

[2] A. Datta and D. London, Int. J. Mod. Phys. A 19, 2505 (2004) [arXiv:hep-ph/0303159].

[3] A. S. Dighe, I. Dunietz, H. J. Lipkin and J. L. Rosner, Phys. Lett. B 369, 144 (1996) [arXiv:hep-ph/9511363]; B. Tseng and C. W. Chiang, [arXiv:hep-ph/9905338].

[4] N. Sinha and R. Sinha, Phys. Rev. Lett. 80, 3706 (1998) [arXiv:hep-ph/9712502]; C. W. Chiang and L. Wolfenstein, Phys. Rev. D 61, 074031 (2000) [arXiv:hep-ph/9911338].

[5] G. Kramer, W. F. Palmer, Phys. Rev. D45, 193-216 (1992), Phys. Lett. B279, 181-188 (1992), Phys. Rev. D46, 2969-2975 (1992); G. Kramer, T. Mannel and W. F. Palmer, Z. Phys. C 55, 497 (1992); G. Kramer, W. F. Palmer and H. Simma, Nucl. Phys. B 428, 77 (1994) [arXiv:hep-ph/9402227]; A. N. Kamal and C. W. Luo, Phys. Lett. B 388 (1996) 633; D. Atwood and A. Soni, Phys. Rev. Lett. 81, 3324 (1998) [arXiv:hep-ph/9804393], Phys. Rev. D 59, 013007 (1999) [arXiv:hep-ph/9805212].

[6] R. Fleischer and I. Dunietz, Phys. Rev. D 55, 259 (1997) [arXiv:hep-ph/9605220].

[7] B. Aubert et al. [The BABAR Collaboration], Phys. Rev. D 78, 092008 (2008) [arXiv:0808.3586 [hep-ex]]; K. F. Chen et al. [BELLE Collaboration], Phys. Rev. Lett. 94, 221804 (2005) [arXiv:hep-ex/0503013].

[8] A. L. Kagan, Phys. Lett. B 601, 151 (2004) [arXiv:hep-ph/0405134].
For example, see P. Colangelo, F. De Fazio and T. N. Pham, Phys. Lett. B 597, 291 (2004) [arXiv:hep-ph/0406162]; M. Ladisa, V. Laporta, G. Nardulli and P. Santorelli, Phys. Rev. D 70, 114025 (2004) [arXiv:hep-ph/0409286]; H. Y. Cheng, C. K. Chua and A. Soni, Phys. Rev. D 71, 014030 (2005) [arXiv:hep-ph/0409317].

C. W. Bauer, D. Pirjol, I. Z. Rothstein and I. W. Stewart, Phys. Rev. D 70, 054015 (2004) [arXiv:hep-ph/0401188]. Long-distance “charming-penguin” effects from charm intermediate states were discussed earlier in M. Ciuchini, E. Franco, G. Martinelli and L. Silvestrini, Nucl. Phys. B 501, 271 (1997) [arXiv:hep-ph/9703353]; M. Ciuchini, E. Franco, G. Martinelli, M. Pierini and L. Silvestrini, Phys. Lett. B 515, 33 (2001) [arXiv:hep-ph/0104126].

P. K. Das and K. C. Yang, Phys. Rev. D 71, 094002 (2005) [arXiv:hep-ph/0412313]; S. Nandi and A. Kundu, J. Phys. G 32, 835 (2006) [arXiv:hep-ph/0510245]; C. H. Chen and C. Q. Geng, Phys. Rev. D 71, 115004 (2005) [arXiv:hep-ph/0504145]; C. S. Huang, P. Ko, X. H. Wu and Y. D. Yang, Phys. Rev. D 73, 034026 (2006) [arXiv:hep-ph/0511129]; A. Datta, M. Imbeault and D. London, Phys. Lett. B 671, 256 (2009) [arXiv:0811.2957 [hep-ph]].

S. Baek, A. Datta, P. Hamel, O. F. Hernandez and D. London, Phys. Rev. D 72, 094008 (2005) [arXiv:hep-ph/0508149];

E. Barberio et al. [Heavy Flavor Averaging Group], [arXiv:0808.1297 [hep-ex]].

B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 97, 201801 (2006) [arXiv:hep-ex/0607057].

B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 100, 081801 (2008) [arXiv:0708.2248 [hep-ex]].

B. Aubert et al. [Babar Collaboration], Phys. Rev. D 76, 052007 (2007) [arXiv:0705.2157 [hep-ex]]; A. Somov et al. [Belle Collaboration], Phys. Rev. Lett. 96, 171801 (2006) [arXiv:hep-ex/0601024].

For a discussion, see M. Beneke, J. Rohrer and D. Yang, Nucl. Phys. B 774, 64 (2007) [arXiv:hep-ph/0612290].

M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Rev. Lett. 83, 1914 (1999), [hep-ph/9905312]; Nucl. Phys. B591, 313 (2000), [hep-ph/0006124].

B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 93, 231804 (2004) [arXiv:hep-ex/0408017].

A. Datta, M. Imbeault and D. London, Phys. Lett. B 671, 256 (2009) [arXiv:0811.2957 [hep-ph]].
[21] This formula appears in a number of places. It can be derived from the analysis in I. Dunietz, Phys. Rev. D 52, 3048 (1995) [arXiv:hep-ph/9501287].

[22] M. Rescigno, on behalf of the CDF collaboration, [arXiv:1101.1485[hep-ex]].