Automatic selection of a subset size at vector fields construction

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Abstract. An algorithm for selection of the size of a correlation kernel at displacement vector field construction by the method of digital image correlation has been proposed. The algorithm has been tested on simulated and experimental optical images having different texture. The influence of the correlation kernel size and image texture on noise immunity at determining displacements has been studied. It is shown that the proposed algorithm allows to find this size providing the minimum error when determination of displacements and estimation of deformation.

1. Introduction

Optical method for estimation of deformation based on digital image correlation (DIC), consists of two main stages: 1) construction of a displacement vector field, 2) the subsequent calculation of strain components [1]. Most of the research in the area of development of algorithms for constructing displacement vectors are aimed to improve the accuracy and increase noise immunity in determining displacements [2, 3], or increase in speed.

In modern DIC-systems, before loading a speckle pattern is formed on a surface of tested material with a help of paint covering [1] that improves contrast and provides a reliable determination of displacements. In doing so, the shape and size of a speckle can significantly affect the accuracy and noise immunity of displacement measure.

In the method of optical flow determination the subset size (correlation area (SA)) usually corresponds to small fragments of the image which is caused, first of all, the need to reduce computational costs when construction of a complete displacement field, as well as providing a sufficiently high density of displacement vector fields [4, 5, 6]. The presence of noise in the image, deformation of the observed objects, as well as the small subset size causes an appearance of errors in the displacement estimation.

Partially, the problem of their presence can be solved by post-correction of the displacement vector field, such as a spatial filter [7], smoothing [8] and so on. Another way to solve the problem of their influence is to increase the subset size [9, 10]. This size must be sufficient to compare a few portions of images and minimize the noise influences, on the other hand to lead to the minimum smoothing of displacement vector fields.

The subject of this research is the problem of the automatic sizing of the subset in modern DIC-systems containing enough unique and identifiable characteristics (objects in the image) to ensure reliable and accurate determination of displacements.
2. Algorithm of a subset size selection

As indicated above, one of the main stages of the DIC method – is the displacement vector field construction. The algorithm for determining the displacements based on establishing consistency between the areas of two images by calculating the cross-correlation function (CCF) and its extremum seeking. [11] Finding the maximum CCF within the scanning zone is performed line by line by one pixel. The size of the scanning area (sa) and the step of vector construction (Figure 1, a) is initially set by the operator. There are following notation in the figure 1: n - the size of the subset side within which the correlation coefficient is calculated; sa - the size of the scanning area side; step - step of construction of the vectors; \( I_e, J_e \) - the coordinates of the upper left corner of the image area.

![Diagram](image1.png)

**Figure 1.** Illustration of determination displacement: a) principle of the construction of displacement vector fields; b) mapping example of the autocorrelation function for the three values \( n \): 1) \( n = 8 \), 2) \( n = 64 \), 3) \( n = 512 \).

The algorithm for determining the subset size for a set of numbers \( n \) in the range of \( 0 < n < \min \{ w/4; h/4 \} \), where \( w \) and \( h \) – width and height of an image, \( \min \) - the operator selecting the minimum values of the arguments (image parameters) is presented bellow in 5 steps:

1. Values of the autocorrelation function are calculated in the horizontal and vertical directions. To calculate the normalized correlation coefficient is used with zero average [4]

\[
ZNCC = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2\sum(y-\bar{y})^2}}
\]  (1)
where \( X, Y \) – brightness of comparable areas of the image,
\[ \bar{X} = \frac{1}{n} \sum_{t=1}^{n} X_t, \quad \bar{Y} = \frac{1}{n} \sum_{t=1}^{n} Y_t \] – the average values of brightness of the same areas.

2. For calculated values of the autocorrelation function in the horizontal and vertical directions for the parameter \( n \) varying in the range of \( 2 < n < 64 \), the following parameters are calculated:

\( FS \) – width of the autocorrelation function (ZNCC) at the level of 0.5 pixels (fig. 1, b);
\( N \) – the number of low-contrast areas of the image.
\( P \) – the number of autocorrelation function peaks that exceed the level of 0.5.

3. Steps 1-2 are repeated for any \( n \).

4. The minimum value of \( n \) is selected such for that the following conditions are fulfilled in order to select the subset size:
   - the number of peaks \( P \) should be equal to 1;
   - the number of areas \( N \) in the image, for which the condition of low contrast is performed (see above step 2) must be equal to zero.

5. Then, starting since the value \( n \) selected at the 4th step, one is chosen for which the parameter \( FS \) is less or equal than that calculated for the next value of \( n \). The resulting value is accepted as the required subset size.

3. Methods of testing

In the general statement generation of series model images consists of two stages: 1) formation of the image of a model surface; 2) forming a series of images of the model surface based on the increment of deformation. In total 6 series of images were examined, three of which were the model (simulated).

- Model of multi-layer image [12].
- Model of speckle (painted surface).
- For the third series the spot size was quadrupled.

The following three series were formed of the experimentally obtained images of loaded materials:

- Aluminum alloy A2024-with sprayed speckle.
- The carbon-carbon composite with a sprayed speckle [13].
- Aluminum alloy A2024 without sprayed speckle [14].

As the result, six series of images were generated, each of which includes 6 frames imitating specimen tension with strain increment of 1% at the final elongation of 5%. Further these series will be named "Series 1", "Series 2" and so on.

The error of deformation estimation was defined by an arithmetic average step-by-step absolute difference of strain fields specified by the model and calculated \( \epsilon_{xx} \):

\[ \delta \epsilon_{xx} = \frac{1}{n} \sum_{i=0}^{N} \left| \epsilon_{xx_{model}} - \epsilon_{xx_{actual}} \right| \]

4. Calculation results and discussion

The maximum values of \( FS, N, P \) at low values of \( n \) (Figure 2) explains the small amount of contrast objects (irregularities) in the image which are within the subset area. Therefore:

- number of the area pairs in the images with a correlation coefficient close to unity will be sufficiently large, too (parameter \( P \));
- the number of low-contrast areas is also high (parameter \( N \));
- a plurality of peaks with an amplitude greater than 0.5, resulting in a higher final value of the parameter \( FS \) at the autocorrelation function distribution will be illustrated.

The subsequent decline of the parameters \( FS, N, P \), due to increasing \( n \), is the consequence of the increasing the number within the subset area of objects (irregularities) of the image (e.g., speckle spots). Accordingly, the number of pairs of image fragments having a correlation coefficient close to 1 (parameter \( P \)), tends to unity. The number of low-contrast areas will tend to zero (parameter \( N \)); the
number of peaks at the distribution of the autocorrelation functions tends to unity, whereas the peak width should have a minimum value (parameter $FS$). In figure 2 observe various gradient of the characteristic $FS, N, P$ decline: for series 1, 3, 6, there is a gentle decline; for series 2, 4, 5 – sharper. This is due to the degree of image contrast and the number of objects (irregularities) in the image (e.g., speckle size spots, relief on the material surface, and so on.).

For images with sufficient small contrast of the texture (series 1, 6), and also for images having a relatively large area of low contrast (objects) (series 3) the value of $n$ must be greater to reach minimum values of the $FS, N, P$. For this reason, the character of the parameters with increasing $n$ is smoother. Conversely, for sharper contrast images (series 4 and 5), and images with a small size of speckle spots (series 2) even for relatively small values $n$ these parameters reach their minimum. As a result, a fairly sharp decline of $FS, N, P$ is visible in graphs. In particular, the parameter of the subset size increases from 10 to 48 (table. 1) when the size of speckle spots rises by 4 times on the surface of the colored model (series 2 and 3) which is associated with the low-contrast area increasing (speckle spots). Series 6, as well as series 3 are characterized by a large value of the subset area and the gradual decline in the number of low-contrast areas (Figure 2 c, h), which is expected, since the surface of the specimen of series 6 is not colored. At the same time for the specimen coated with speckle (series 4), model with colored surface (series 2) and images with a contrast surface (series 1 and 5) there is a sharp decline in the calculated parameters that indicates the higher noise-immunity of the algorithm of subset area determination. Thus, there is a clear relationship between the degree of image contrast, and the nature of the texture of the material surface, and the change in the parameters $FS, N, P$ with increasing value $n$.

Analysis of the graphs (Figure 2) allows to select the optimal size of the parameter $n$ (table. 1) according to the proposed algorithm. Calculations of dependence of strain measuring error (chapter 3) on the value $n$ were carried out to confirm the optimal choice of SA. On their basis, dimensions of SA were determined with the smallest measure strain error (table. 1). It is clear from the table that these dimensions of SA coincide to that values at the minimum error $\delta \varepsilon_{xx}$ or exceed them.

![Figure 2](image)

**Figure 2.** Dependence of parameters $FS, N, P$ of the pad size of correlation $n$: a) model of multi-layer image; b) aluminum alloy A2024 without sprayed speckle.

Moreover, if SA is greater than the value corresponding to the minimum error $\delta \varepsilon_{xx}$ it is able to get even smaller error for strain determination compared to that with using a smaller SA. This is due to the fact that reducing SA gives rise to a large number of false peaks of the autocorrelation function (Figure 1, b).

From the study it is following that the algorithm makes it possible to conduct successfully image processing as with the model, and with the experimental ones. More sloping graphs of dependence of parameters $FS, N, P$ on SA correspond to the model (series 3) and experimental (series 6) images with little contrast. The rest of the series as model images so and experimental ones with a more contrast texture have cool-changing dependences. Thus, based on the obtained data (Figure 2 and Table. 1) and its analysis it can be concluded that the proposed algorithm can be used effectively in practice.
Table 1. The results of parameter calculation when choosing SA.

| Image model | According to the algorithm parameters | At the minimum error δe_{e2} |
|-------------|----------------------------------------|------------------------------|
|             | P  | N  | FS | Summ |                              |
| 1           | 16 | 16 | 16 | 16   | 14                           |
| 2           | 10 | 10 | 10 | 10   | 10                           |
| 3           | 26 | 48 | 48 | 48   | 46                           |
| 4           | 14 | 12 | 16 | 16   | 12                           |
| 5           | 12 | 12 | 12 | 12   | 10                           |
| 6           | 22 | 52 | 52 | 52   | 42                           |

5. Conclusions
An algorithm to automatically select the subset size for the problem of constructing vector fields in the evaluation of deformation by the digital image correlation method. The algorithm functions were studied by simulated and experimental data. According to the results of the algorithm for the six series of images with different textures the subset providing a minimum error when estimation of deformation, was determined.

The comparison of revealed dimensions as a result of applying the algorithm to the values of SA when the error of deformation determining reaches to a minimum value has been conducted. The values of SA to which the minimum error corresponds have been found. The comparison of values obtained as a result of the algorithm applying and values which correspond to the minimum error shows a slight deviation of the first to the next higher.

Thus, the developed algorithm is effective to measure deformation of the materials having different relief as for surface images with a sprayed speckle so and for low contrast images (non colored).

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