Time history analysis method for the combined action of extreme fluctuating wind and wave on the maximum double cantilever structure of rigid frame bridge considering the influence of flow velocity

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Abstract: Taking the maximum double cantilever structure of rigid frame bridge in construction period as the research object, the structural dynamic response analysis under the combined action of extreme wind and wave is carried out, considering the influence of flow velocity. An analysis method of short-term pulsation effect of a structure is proposed considering the influence of various adverse factors under extreme wind and wave conditions, which provides a reference for structural safety assessment under extreme conditions; The dynamic response results of the maximum double cantilever structure of rigid frame bridge show that the wave-flow effect under extreme wind and wave conditions will increase the structural response during the construction period, resulting in adverse effects.

1. Introduction

For offshore structures, wind and wave are two important actions, which are related to the safety of the structure and the loss of life and property. In the design, the wind and wave loads are considered separately, and the values are taken according to the corresponding design reference period in the codes[1-2]. However, in fact, the wind and wave actions are interdependent and associated with each other, and they are not independent of each other. Especially when the wind speed is large, the wave will be higher. At this time, if the values are still taken according to the codes, the combined effect may be underestimated, which is unfavorable to structural analysis. On the other hand, when some structures such as continuous rigid frame bridge are constructed to the maximum double cantilever state, only the foundation is constrained by bedrock, pile foundation and pile cap above bedrock are impacted by waves, and superstructure is subjected to wind load, so the structural safety needs to be evaluated. Based on the above considerations, this paper takes the maximum double cantilever structure of a continuous rigid frame bridge as the research object, and proposes an analysis method for the combined effect of short-term extreme wind and wave considering the influence of water flow.
Based on the existing theory of wave force[1-3], this paper deduces the time-varying equations of wave-flow action on single pile and pile cap considering the influence of flow velocity, and considers the piles group effect for piles group foundation. Three most unfavorable analysis cases are worked out for the maximum double cantilever structure of rigid frame bridge, the corresponding wind-wave excitation time histories are generated, and the structural dynamic response analysis under the combined action of short-term extreme wind and wave is carried out. The analysis method provides a method for the structural response analysis under extreme wind and wave, and the analysis results provide a reference for the structural safety assessment under extreme wind and wave.

2. Wave force on pile foundation considering flow velocity

When it comes to the calculation of wave force, the accompanied water flow influence during the wave propagation is often ignored. In order to reflect the real effect of wave on the structure more truly, this paper deduces the time-varying calculation equations of horizontal wave-flow action on pile foundation under the influence of flow velocity, so as to be combined with fluctuating wind to analyze the dynamic response of corresponding structure.

2.1. Time varying horizontal wave-flow forces on pile foundation

2.1.1. Time varying horizontal wave-flow force on single pile. Since the circular column of pile foundation below the pile cap meets the small-scale pile condition, Morison Equation and Linear Wave Theory[4] can be applied to calculate the total horizontal wave-flow force on a pile of length \( z_1 - z_2 \) as follows:

\[
F_p(t) = \int_{z_1}^{z_2} df(z,t)
 = \int_{z_1}^{z_2} df_2(z,t) + \int_{z_1}^{z_2} df_1(z,t)
 = \text{Drag Force}, F_{dp}(t) + \text{Inertial Force}, F_{fp}(t)
 = F_{D1p}(t) + F_{D2p}(t) + F_{D3p}(t)
\]  

Where, \( F_{D1p}(t) \) includes the direct effect of flow velocity \( u_c \) and the indirect effect of wave number \( k \) (= \( 2\pi / L \)).

Where, \( F_{D1p}(t) \) is a part of drag force of wave without the direct influence of \( u_c \), and the equation is as follows:

\[
F_{D1p}(t) = F_{D1p,\text{max}} \cdot \cos(kx - \omega t) \cdot \cos(kx - \omega t)
 = F_{D1p,\text{max}} \cdot \frac{1}{16} \cdot \frac{\gamma D_h^2}{sh2k^2d}(2k(z_2 - z_1) + sh2kz_2 - sh2kz_1)
\]  

Where, \( C_D \) is the drag force coefficient, and the value is taken from figure 8.4.1 of the code[1], \( \gamma \) (unit: kN/m³) is the volumetric weight of water, \( D_p \) (unit: m) is the diameter of the pile, \( H \) (unit: m) is the design wave height, \( d \) (unit: m) is the water depth.

\( F_{D2p}(t) \) is the linear term directly affected by \( u_c \), and the equation is as follows:

\[
F_{D2p}(t) = F_{D2p,\text{max}} \cdot \cos(kx - \omega t)
 = F_{D2p,\text{max}} \cdot \frac{1}{2g} \cdot \frac{\gamma D_h H}{shkd}(shkz_2 - shkz_1) \omega_u c
\]  

Where, \( \omega_u = \omega - ku_c \) (unit: rad/Hz) is the relative circular frequency of wave.

\( F_{D3p} \) does not change with time \( t \), it is the quadratic term directly affected by \( u_c \), and the equation is as follows:
\[ F_{D3}^p = \pm C_D \frac{\gamma D_b}{2g} \cdot (z_2 - z_1)u_c^2 \]
\[ = C_D \frac{\gamma D_b}{2g} (z_2 - z_1)u_c^2 \cdot \frac{\cos(kx - \omega t)}{|\cos(kx - \omega t)|} \]

Its “+” and “-” change with the sign of \( \cos(kx - \omega t) \).

In \( F^p(t) \), the flow velocity \( u_c \) has no direct effect, and the equation is as follows:
\[ F^p(t) = F^p_{\text{max}} \sin(kx - \omega t) \]
\[ F^p_{\text{max}} = C_M \frac{\gamma \pi D_b^2 H}{8} \cdot \frac{shkz_2 - shkz_1}{chkd} \]

Where, \( C_M \) is the mass force coefficient (or known as the inertial force coefficient), and the value is taken from figure 8.4.1 of the code[1].

2.1.2. Time varying horizontal wave-flow force on pile cap.

The pile cap can be regarded as a large-scale pile, and the horizontal wave-flow force acting on it is determined according to the following equation:
\[ F^c(t) = F^c_{\text{max}} \sin(kx - \omega t) \]
\[ F^c_{\text{max}} = C'_M \frac{\gamma \pi D_c^2 H}{4} \cdot \frac{shkz_2 - shkz_1}{chkd} \]

Where, the coefficient \( C'_M \) is related to wave number \( k \) and flow velocity \( u_c \), which can be determined according to figure 8.4.4 of the code[1], \( D_c \) (unit: m) is the equivalent diameter of pile cap.

2.2. Influence analysis of flow velocity \( u_c \).

Taking a section of single pile as the analysis object, when the flow velocity is 0, 1, 2 or 3 m/s, the time histories of horizontal wave-flow force on the single pile are shown in figure 1.

![Figure 1. Time history curves of horizontal wave-flow force of single pile at various flow velocities.](image)

It can be seen from the figure that with the increase of velocity \( u_c \), the horizontal wave-flow force of single pile increases multiply, which indicates that the flow velocity has a decisive influence on the horizontal mechanical performance of single pile, which must be considered in the horizontal mechanical performance analysis of single pile. When the velocity \( u_c \) is 0 m/s, the horizontal force of wave on the single pile is continuous, but when the velocity \( u_c > 0 \) m/s, the magnitude or direction of wave-flow force of single pile changes suddenly at some moments, the dynamic response of the structure is worthy of further study.

Taking a round-ended pile cap as the analysis object, under the same wave effect, when the flow velocity is 0, 1, 2 or 3 m/s, the time histories of horizontal wave-flow force on the pile cap are shown in figure 2.
It can be seen from the figure that with the increase of velocity $u_c$, the amplitude of horizontal wave-flow force of pile cap increases slightly, which indicates that the flow velocity has little influence on the structure with larger relative diameter such as pile cap.

2.3. Piles group effect
When the center distance $l$ of pile foundation is less than 4 times of pile diameter $D$, piles group effect shall be considered according to table 8.3.5 of code[1]. The total horizontal wave-flow force on the whole piles group is as

$$\sum F(t) = \sum_{i} \sum_{j} K_{ij} F_{ij}(t)$$  \hspace{1cm} (7)

Where, $K_{ij}$ is the piles group coefficient, which is determined by the ratio of $l/D$ of piles along the wave direction.

The layout of a pile foundation is shown in figure 3, and the coordinate origin is set at point O. According to the ratio of $l/D$, and considering the shielding effect and solid effect of the piles group[5], the piles group coefficient $K_{ij}$ of the pile foundation can be obtained, as seen in table 1. When the wave direction and the long side direction of the piles group are in a certain angle, the linear interpolation can be carried out.

**Table 1. Piles group effect coefficient $K_{ij}$.**

| $i$ | 1   | 2   | 3   | 4   | 5   | 6   |
|-----|-----|-----|-----|-----|-----|-----|
| 4   | 0.857(0.850) | 0.857(1.245) | 0.857(1.245) | 0.857(1.245) | 0.857(0.850) | \   |
| 3   | 1.286(0.850) | 1.286(1.245) | 1.286(1.245) | 1.286(1.245) | 1.286(1.245) | 1.286(0.850) |
| 2   | 1.286(0.850) | 1.286(1.245) | 1.286(1.245) | 1.286(1.245) | 1.286(1.245) | 1.286(0.850) |
| 1   | 0.857(0.850) | 0.857(1.245) | 0.857(1.245) | 0.857(1.245) | 0.857(0.850) | \   |

* The values outside the brackets are that the wave direction is parallel to the long side of the piles group, and the values inside the brackets are that the wave direction is perpendicular to the long side of the piles group.
Time history curves of the total wave-flow force of the pile foundation considering the piles group effect or not are shown in figure 4. It can be seen that it is necessary to consider the piles group effect in the piles group, otherwise, the extreme value of the total wave-flow force will be about 8% lower, which will make the structural force prediction insufficient.

Figure 4. Time history curves of horizontal wave-flow force of piles group.

3. Dynamic response analysis of the combined action of extreme wind and wave on the maximum double cantilever state of rigid frame bridge during construction

When the construction of a rigid frame bridge reaches the maximum double cantilever state (in figure 5), the left and right cantilevers are respectively 90m long. Other parts of the structure may have greater dynamic response under the combined action of extreme wind and wave, except the underpart of pile foundation, which is driven into the bedrock to be constrained, so it is necessary to conduct corresponding analysis.

Figure 5. Analysis model of the maximum double cantilever structure of a rigid frame bridge.

3.1. Analysis cases

In order to inspect the dynamic response of the structure under various extreme conditions, the dynamic response analysis of the structure under three combined wind and wave cases is carried out as follows:

1. Case 1: Transverse wind + longitudinal wind;
2. Case 2: Transverse wind + longitudinal wind + transverse wave-flow, the angle between wave-flow and transverse main girder is 0°;
Case 3: Transverse wind + longitudinal wind + longitudinal wave-flow, the angle between wave-flow and transverse main girder is 90°.

3.2. Wind-wave excitation time history

3.2.1. Time history of fluctuating wind speed.

During the construction period of the rigid frame bridge, the basic wind speed at 10m high with a return period of 20 years is 38.8m/s, and the time histories of fluctuating wind speed of a section of main girder and pier at 10 m high are simulated by using the method of WAWS, as seen in figure 6.

![Figure 6. Time history curves of fluctuating wind speed.](image)

3.2.2. Time history of horizontal wave-flow force.

The design wave is considered to be related to the design wind speed. Using the method in references[6-7], the correlation coefficient $\rho_{XY}$ between wave height and period is 0.6, and the correlation coefficient $\rho_{XZ}$ between wave height and wind speed is 0.6. The design wave height and design period are 7.38m and 6.794s respectively under the condition of extreme wind wave combination therefore. The water is 28.225m deep, and the flow velocity $u_c$ is 2.377m/s. According to equations (1) ~ (6), the time histories of horizontal wave-flow force on pile (1,1) and pile cap (in figure 3) are shown in figure 7. Each piles group coefficient shall be considered according to the corresponding working cases.

![Figure 7. Time history curves of horizontal wave-flow force on pile foundation.](image)

3.3. Structural dynamic response of wind-wave excitation

Through the short-term (150s) dynamic response analysis of the maximum double cantilever structure in construction period of rigid frame bridge under the combined action of extreme wind and wave, the following results can be obtained:
3.3.1. Displacement.
The response time histories of maximum displacements of the structure in three directions are shown in figure 8.

![Figure 8. Maximum response displacements of the structure in three directions.](image1)

It can be seen from the figure that the maximum response displacements of the maximum double cantilever structure of the rigid frame bridge occur at the cantilever end of the structure, in which the vertical displacement and transverse displacement are larger, which are mainly caused by the vertical and transverse fluctuating wind. When the extreme wind and wave act together, the transverse wave-flow action almost has no effect, but the longitudinal wave-flow action increases the dynamic response displacement of the structure.

3.3.2. Internal force.
The response time histories of maximum bending moments of the structure in three directions are shown in figure 9.

![Figure 9. Maximum response bending moments of the structure in three directions.](image2)
It can be seen from the figure that the maximum response bending moments of the maximum double cantilever structure of the rigid frame bridge occur at the bottom of the pile cap of the structure, in which the bending moments in two directions are larger, which are mainly caused by the vertical and transverse fluctuating wind. When the extreme wind and wave act together, for the longitudinal bending moment, the transverse wave-flow has an enhancement effect, as for the transverse bending moment, it is mainly caused by vertical wind, and the wave-flow actions in two directions have little influence.

4. Conclusions
In this paper, the dynamic response analysis of the structure under the combined action of extreme wind and wave is carried out for the maximum double cantilever state of rigid frame bridge in construction stage, considering the influence of wind-wave correlation, flow velocity and piles group effect, and the response results of displacement and internal force of the structure are obtained. On the one hand, an analysis method of short-term pulsation effect of a structure is proposed considering various adverse factors under extreme wind and wave conditions, which provides a reference for structural safety assessment under extreme conditions. On the other hand, the dynamic response results of the maximum double cantilever structure of the rigid frame bridge show that the wave-flow action under extreme wind and wave conditions will increase the structural response during the construction period, thus causing adverse effects, which deserves special attention.

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