THE BEAMING PATTERN OF EXTERNAL COMPTON EMISSION FROM RELATIVISTIC OUTFLOWS:
THE CASE OF ANISOTROPIC DISTRIBUTION OF ELECTRONS

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ABSTRACT
The beaming pattern of radiation emitted by a relativistically moving source, such as jets in microquasars, active galactic nuclei, and gamma-ray bursts, is a key issue for understanding acceleration and radiation processes in these objects. In this paper, we introduce a formalism based on a solution of the photon transfer equation to study the beaming patterns for emission produced by electrons accelerated in the jet and the upscattering photons of low-energy radiation fields of external origin (the so-called external Compton scenario). The formalism allows us to treat non-stationary, non-homogeneous, and anisotropic distributions of electrons, but assuming homogeneous/isotropic and non-variable target photon fields. We demonstrate the non-negligible impact of the anisotropy in the electron distribution on angular and spectral characteristics of the EC radiation.

Key words: galaxies: active – gamma rays: galaxies – radiation mechanisms: non-thermal – relativistic processes – scattering

1. INTRODUCTION
The inverse Compton scattering (ICS) of relativistic electrons is one of the major radiation mechanisms in high-energy astrophysics. Since the Compton cooling time of electrons decreases linearly with energy, this channel becomes particularly prolific in the gamma-ray band. The universal presence of dense radiation fields makes ICS an effective gamma-ray production mechanism in various astronomical environments, in particular in sources containing relativistic outflows–microquasars, active galactic nuclei (AGNs), gamma-ray bursts (GRBs), etc. The dramatically enhanced fluxes, the shift of the spectral energy distribution towards higher energies, and the reduction of characteristic timescales are distinct features of the Doppler-boosted radiation produced in a relativistically moving source with a Doppler factor \( D \gg 1 \). In relativistic outflows, ICS can be realized, depending on the origin of target photon fields, in two different scenarios.

In the so-called synchrotron self-Compton (SSC) model, the synchrotron photons of relativistic electrons constitute the main target for Compton scattering of the same electrons. In its simplified homogeneous one-zone version, the SSC model assumes that a spherical source (a “blob”) filled with an isotropic electron population and a random magnetic field moves toward the observer with a constant Doppler factor \( D \). The beaming pattern for the emission of a relativistically moving source derived by Lind & Blandford (1985) can be directly applied to the synchrotron and SSC components of radiation.

In the second scenario, the seed photon field for ICS is dominated by external radiation, i.e., by low-energy photons produced outside the moving source of relativistic electrons. In this scenario, the so-called external Compton (EC) model, the beaming pattern of the inverse Compton radiation differs significantly from the beaming pattern of the synchrotron and SSC components (Dermer 1995).

The characteristics of EC radiation of a relativistically moving source can be calculated using two different approaches: (1) first transforming the external radiation photon field to the jet frame, calculating the characteristics of the high-energy photon (the outcome of ICS) in the same frame and finally transforming the latter back to the observer frame and (2) transforming the electron distribution from the jet frame to the observer frame, and then calculating the spectrum of IC photons directly in the observer frame.

Using the first approach, Dermer (1995) derived the EC beaming pattern in the Thomson limit assuming a power-law distribution of electrons. Georganopoulos et al. (2001), using the second approach, extended this result to the general case of Compton scattering, including the Klein–Nishina regime. In both treatments, the distribution of relativistic electrons has been assumed to be isotropic and homogeneous. In the zeroth approximation, this could be a reasonable assumption, and thus can, in principle, be applied to the interpretation of gamma-ray observations of many blazers. However, in some other cases, one cannot exclude significant deviations of distributions of electrons from homogeneous and isotropic realizations. The non-stationary treatment of the problem is another issue that has not yet been addressed.

In this paper, we develop a new approach that provides a strict formalism for the treatment of the beaming pattern for EC radiation. Namely, we solve the photon transfer equation, which allows us to treat the beaming pattern in a concise way, and, more importantly, extend the formalism to a more general (non-stationary and anisotropic) case of electron distributions, but assuming isotropic, homogeneous, and isotropic distribution of the seed (target) photon fields. To demonstrate the potential of the proposed formalism, we examine how anisotropy in the electron distribution affects the energy spectrum and the angular distribution of EC radiation.

2. THE BEAMING OF EXTERNAL COMPTON EMISSION
2.1. The Photon Transfer
In two close points located on the line of sight, the specific intensity (the spectral radiance) \( I \) and emissivity \( j \) are related as (see, e.g., Rybicki & Lightman 1979)

\[ I(s) - I(s - ds) = j \, ds, \]
where \( s \) is the distance along the line of sight. It is assumed that one can neglect the scattering and absorption of radiation during its propagation. It is convenient to consider \( I \) and \( j \) as functions of the radius-vector \( r \), the time \( t \), and the wave-vector of the photon \( k \), i.e., \( I \equiv I(k, r, t) \), and \( J \equiv J(k, r, t) \). For simplicity, hereafter we will use the system of units in which the speed of light \( c = 1 \) and the Planck constant \( \hbar = 1 \). Also, instead of \( I \) and \( J \), we will use the distribution function of photons \( g = I/e^3 \) and the source function \( Q = J/e^3 \), where \( \epsilon = |k| \) is the photon energy. The function \( g(k, r, t) d^3k d^3r \) describes the number of photons in the volume element \( d^3k d^3r \) of the phase space at the moment \( t \), while \( Q(k, r, t) d^3k d^3r \) is the number of photons in the momentum interval \( d^3k \), emitted during the time interval \( (t, t + dt) \) from the volume \( d^3r \) of the source located at the point \( r \).

With these new notations, Equation (1) can be written in the form

\[
g(k, r, t) - g(k, r - nds, t - ds) = Q(k, r, t) ds,
\]

where \( n = k/|k| \) is a unit vector along the photon momentum. Using Equation (2), the following relation can be derived as

\[
g(k, r, t) = \int_0^\infty ds \ Q(k, r - ns, t - s),
\]

which expresses \( g \) through \( Q \). For calculations of the flux, one should integrate \( g \) over the directions of the vector \( n \). This determines the total number of photons of a given energy which reach the point \( r \) at the moment \( t \),

\[
\tilde{g}(k, r, t) = \int g(k, r, t) d\Omega_n.
\]

For integration over \( d\Omega_n \), it is convenient to write Equation (3) in the form

\[
g(k, r, t) = \int_0^\infty ds \ \int d^3r_0 \ Q(k, r_0, t - s) \ \delta(r - r_0 - ns),
\]

where \( \delta(r - r_0 - ns) \) is the three-dimensional \( \delta \)-function, and \( k_r = \epsilon(r - r_0)/|r - r_0| \). After a simple integration of Equation (5) over \( d^3r_0 \), we apparently would return to Equation (3). In the integrand of Equation (5), the vector \( n \) enters only in the argument of the \( \delta \)-function; therefore, to determine \( \tilde{g}, \) one should calculate the integral

\[
X = \int \delta(r - r_0) d\Omega_n.
\]

Using the equation

\[
\delta(r - r_1) = \frac{1}{r_2^2 \sin \theta} \delta(r - r_1) \delta(\theta - \theta_1) \delta(\phi - \phi_1),
\]

where \( r, \theta, \phi \) \((r_1, \theta_1, \phi_1)\) are the spherical coordinates of the point \( r \) \((r_1)\), we find

\[
X = \frac{\delta(|r - r_0| - |s|)}{|r - r_0|^2}.
\]

Then the function \( \tilde{g} \) becomes

\[
\tilde{g}(k, r, t) = \int_0^\infty ds \ \int d^3r_0 \ \frac{Q(k, r_0, t - s)}{|r - r_0|^2} \ \times \ \delta(|r - r_0| - s),
\]

and the integration over \( s \) results in

\[
\tilde{g}(k, r, t) = \int d^3r_0 \ \frac{Q(k, r_0, t - |r - r_0|)}{|r - r_0|^2}.
\]

This equation has a simple physical meaning and can be also obtained from heuristic considerations. From the definition of the function \( Q \), it follows that

\[
\frac{Q d^3r_0}{|r - r_0|^2} \ \epsilon^2 d\epsilon
\]

is the number of photons in the unit volume at the point \( r \) and in the energy interval \( d\epsilon \), emitted by a point-like source located at the point \( r_0 \). Note that due to the delay, the number of photons at the instant \( t \) is determined by the source function at the moment \( t - |r - r_0| \). In the case of an extended source, to calculate the number of photons, one should use the sum of Equation (11) type expressions. The result will be the quantity \( \tilde{g} \ \epsilon^2 \ d\epsilon \). Replacing the sum by the integral, we arrive at Equation (10).

Equation (10) is correct for any \( r \) and \( t \). In order to calculate the photon distribution at a distance that significantly exceeds the characteristic size of the source \( R_0 \), let us define the origin of coordinates somewhere inside the source. Then, for \( r \gg R_0 \), one can set \( n_r = r/r \) and \( |r - r_0| = |r - (n_r)R_0| \). Neglecting \((n_r)R_0\) compared to \( r \) in the denominator, we obtain

\[
\tilde{g}(k, r, t) = \frac{1}{r^2} \ \int d^3r_0 \ Q(k, r_0, t - r + (n_r)R_0),
\]

where \( k = kn_r \). This means that all photons detected by a distant observer travel essentially in the direction of \( r \). Therefore, below we will not distinguish between the vectors \( n = k/\epsilon \) and \( n_r = r/r \), i.e., \( n_r = n \). On the other hand, we should note that in general one cannot neglect the last term in the argument \( t - r + (n_r)R_0 \). The possibility of such an approximation is determined not by the relation of \( r \) to \((n_r)R_0\), but by the speed of variation of the function \( Q \) during the time \((n_r)R_0\).

### 2.2. The beaming pattern

Let us consider the collision of ultrarelativistic electrons and soft photons. It is convenient to describe the ensemble of electrons by the distribution function in the phase space \( f(p, r, t) \). By definition, the quantity \( f_s(p, r, t) d^3p d^3r \) is the number of electrons located at the moment of time \( t \) in the phase volume element \( d^3p d^3r \). Let us define \( n_{ph}(\epsilon_{ph}) \) as the number of photons of energy \( \epsilon_{ph} \) within the interval \( d\epsilon_{ph} \) in the unit volume. We assume that the distribution of these target photons is homogeneous, isotropic, and stationary. Then, the high-energy photons appearing due to ICS are described by the source function

\[
Q(k, r, t) = \int w(p, \epsilon_{ph}, k) f_s(p, r, t) n_{ph}(\epsilon_{ph}) d^3p d\epsilon_{ph},
\]

where \( w(p, \epsilon_{ph}, k) \) is the probability of scattering averaged over the directions of the soft photons. According to Equation (12), the corresponding expression for \( \tilde{g} \) is given by

\[
\tilde{g}(k, r, t) = \frac{1}{r^2} \ \int d^3r_0 \ d\epsilon_{ph} \ w(p, \epsilon_{ph}, k) \ \times \ f_s(p, r_0, t - r + n_r) n_{ph}(\epsilon_{ph}).
\]
All quantities in Equation (14) are relevant to a reference system \( K \) (the observer system). Let us assume that the accelerated electrons belong to a blob that moves with a relativistic speed \( V \sim 1 \) and Lorentz factor \( \Gamma = 1/\sqrt{1 - V^2} \). In order to determine the impact of the blob’s bulk motion on the energy distribution of secondary photons, we introduce a co-moving coordinate system \( K' \), and define \( f'(p', r', t') \) as the distribution function of electrons in the blob, i.e., in the \( K' \) system. Below all quantities in the \( K' \) system will be indicated by the prime symbol. Note that the distribution function in the phase space is a relativistic invariant (Landau & Lifshitz 1975; Rybicki & Lightman (1979)), i.e.,

\[
f(p, r, t) = f'(p', r', t'),
\]

where the “primed” and “unprimed” variables are connected via Lorentz transformations.

If we assume (without a loss of generality) that the source is moving along the \( z \)-axis, then Equation (15) can be written in the form

\[
f(p, x, y, z, t) = f'(p', x, y, \Gamma(z - V t), \Gamma(t - V z)),
\]

where

\[
p'_x = p_x, \quad p'_y = p_y, \quad p'_z = \Gamma(p_z - V E_e).
\]

In Equation (14), the function \( f \) is given at an instant delayed in time. Replacing \( r \) by \( t - r + n r_0 \) in Equation (16), one finds

\[
f(p, x_0, y_0, z_0, r + n_x x_0 + n_y y_0 + n_z z_0) = f'(p', x_0, y_0, \Gamma(z_0 - V t + n_x x_0 + n_y y_0 + n_z z_0),
\]

\[
\Gamma(t + n_x x_0 + n_y y_0 + n_z z_0 - V z_0)),
\]

where \( n_x, n_y, n_z \) are the components of the unit vector \( n \). For simplicity, we have set the retarded time as \( \tau \equiv t - r \).

Let us introduce the new variables of integration in a way that the space arguments of the function \( f' \) can be written in the form \((x'_0, y'_0, z'_0)\). Apparently, for this we should set

\[
x'_0 = x_0, \quad y'_0 = y_0, \quad z'_0 = \Gamma(z_0 - V (t + n_x x_0 + n_y y_0 + n_z z_0)).
\]

As it follows from Equation (19), \( d^3 z'_0 = \Gamma(1 - V n_z) d^3 z_0 \); therefore the volume element is transformed according to

\[
d^3r_0 = \frac{d^3r'_0}{\Gamma(1 - V n_z)} = \mathcal{D} d^3r'_0,
\]

where

\[
\mathcal{D} = \frac{1}{\Gamma(1 - V n_z)} = \frac{1}{\Gamma(1 - n V)}
\]

is the Doppler factor. Note that the transformation given by Equation (20) differs from the standard Lorentz transformation. At \( \mathcal{D} > 1 \), we have an increase (but not a contraction, as in the case of Lorentz transformation) of the volume by a factor of \( \mathcal{D} \).

The time-argument in the right part of Equation (18) with respect to the new variables \((x'_0, y'_0, z'_0)\) becomes

\[
\mathcal{D} \cdot (t + n_x x'_0 + n_y y'_0 + \Gamma(n_z - V) z'_0).
\]

Let us introduce a unit vector along \( k' \). Using Equation (17), we obtain the following well known expressions for the aberration of light:

\[
n'_x = \mathcal{D} n_x, \quad n'_y = \mathcal{D} n_y, \quad n'_z = \mathcal{D} \Gamma(n_z - V).
\]

Then the time argument in Equation (18) can be written as

\[
\mathcal{D} \tau + (n' r'_0).
\]

As a result, we find that the function \( \tilde{g} \) in the system \( K \) is expressed via \( f' \) in the system \( K' \) as

\[
\tilde{g}(k, r, t) = \frac{\mathcal{D}}{r^2} \int d^3r'_0 \mathcal{D} d^3r' \epsilon_p w(p, \epsilon_p, k) \times f'(p', r', \mathcal{D} \tau + n'_r r'_0) n'_p(\epsilon_p).
\]

Remarkably, the function \( f' \) is not constrained by any condition; the distribution of electrons in the comoving frame can be non-stationary, non-homogeneous, and anisotropic. We should note that, in the case of a homogeneous, isotropic, and stationary target photon field, the homogeneity of electrons becomes irrelevant, i.e., Equation (25) does not depend on the spatial distribution of electrons.

Equation (25) can be significantly simplified after the following approximations. Let us assume that the distribution function of electrons in the comoving frame \( K' \) is stationary and isotropic, i.e., \( f' = f'(E'_e, r'_0) \), where \( E'_e \) is the energy in the \( K' \) frame. The energy \( E_e \) in the \( K \) frame and \( E'_e \) are related as \( E'_e = E_e \Gamma(1 - v_e V) \), where \( v_e \) is the electron speed. Furthermore, we propose that the Lorentz factor of electrons significantly exceeds the Lorentz factor of the bulk motion, \( \gamma > \Gamma \), and take into account that the up-scattered photon moves practically in the direction of the electron (the accuracy of this approximation is of the order of \( 1/\gamma \)). Therefore, \( v_e \approx n \) and, consequently, \( E'_e \approx E_e / \mathcal{D} \).

Integration of \( \tilde{W} \) over all directions gives

\[
\tilde{W}(E_e, \epsilon_p, \epsilon) = \epsilon^2 \int w(p, \epsilon_p, k) d\Omega_e = \frac{8 \pi r_e^2}{E_e \eta} \left[ 2q \ln q + (1 - q) \right.
\]

\[
\left. \times \left( 1 + 2q + \frac{\eta^2 q^2}{2(1 + \eta q)} \right) \right],
\]

where

\[
\eta = \frac{4 \epsilon_p E_e}{m^2}, \quad q = \frac{\epsilon}{\eta (E_e - \epsilon)}.
\]

For the given values of \( \epsilon_p \) and \( E_e \), the maximum energy of the upscattered photon is

\[
\epsilon_{\text{max}} = \frac{E_e}{1 + 1/\eta}.
\]

Equation (26) is typically obtained by integration over the directions of the momentum of the upscattered photon (Jones 1968; Blumenthal & Gould 1970; Aharonian & Atoyan 1981). However, the argument that depends on the angle enters in the integrand as \( (k \cdot p) \), therefore, there is no difference over the directions of which vectors, \( k \) or \( p \), the integration is performed. The above formulae are applicable under the conditions

\[
\epsilon_p \ll E_e, \quad m \ll E_e
\]

Let us denote by \( \hat{\tilde{W}} \) the upscattered photon energy spectrum per electron:

\[
\hat{\tilde{W}}(E_e, \epsilon) = \int \tilde{W}(E_e, \epsilon_p, \epsilon) n_p(\epsilon_p) d\epsilon_p.
\]
The above simplifications allow us to write Equation (25) in a more familiar and convenient form. Let us introduce two new quantities, the observed energy flux, \( F_e = \epsilon^3 \tilde{g} \), and the total number of electrons in the comoving frame per energy and per solid angle,

\[ N'_e(E'_e) = E'^2_e \int f'(E'_e, r'_0) d^3 r'_0. \]  

In Equation (25), at integration over the electron momenta \((p = E_e)\), a quantity \( E'^2_e \int f'(E'_e, r'_0) d^3 r'_0 \) appears that is apparently equal to \( D^2 N'_e(E'_e) \). Therefore, the observed flux is

\[ F_e = \frac{e D^3}{r^2} \int N'_e \left( \frac{E_e}{D} \right) \tilde{W}(E_e, \epsilon) dE_e. \]  

Further simplification of the structure of Equation (32) is possible in the Thomson limit \((\eta \ll 1)\). At \( \eta \ll 1 \), in Equation (26) one can ignore the last term in the parentheses. Note that the maximum energy of upscattered photons is \( \epsilon_{\text{max}} = E_e \eta \ll E_e \), therefore, \( q = \epsilon/(\eta E_e) \). Then, as it follows from Equations (26) and (30), the quantity \( \tilde{W} \) becomes a function of a single argument, \( \epsilon/E^2_e \). Writing \( \tilde{W}(E_e, \epsilon) = \Phi(\epsilon/E^2_e) \), and performing an integration over \( E'_e = E_e/D \), we obtain

\[ F_e = \frac{D^3}{r^2} \int N'_e \Phi \left( \frac{\epsilon}{D^2 E^2_e} \right) dE'_e. \]  

This implies that if the EC flux of a source at rest is described by some function \( S(\epsilon) \), the relativistic bulk motion of the source results in

\[ F_e = D^4 S(\epsilon/D^2). \]  

In a log–log plot, the function \( F_e \) is obtained from \( S \) by moving the latter up by a factor of \( \log_{10}(D^4) \), and shifting it to the right by a factor of \( \log_{10}(D^2) \). The total intensity is enhanced by a factor of \( D^6 \):

\[ \int_0^\infty F_e d\epsilon = D^6 \int_0^\infty S(\epsilon) d\epsilon. \]  

Integrating over all angles, we obtain the luminosity detected by the observer (the apparent luminosity):

\[ L = \frac{1}{5} (16 \Gamma^4 - 12 \Gamma^2 + 1) L_0, \]  

where \( L_0 \) is the luminosity of the source at rest (the intrinsic luminosity).

The above results are obtained for an arbitrary distribution of relativistic electrons. For electrons with a power-law distribution, \( N'_e(E'_e) \propto E'^{-\gamma}_e \), in the Thomson limit we arrive at the well-known result for the beaming pattern, \( F_{\epsilon_b} \propto D^{3+\gamma} \) (Dermer 1995).

### 3. Non-Isotropy

In this section, we study the impact of a possible anisotropy of the electron distribution inside the moving source on the angular and energy distributions of EC radiation. Non-isotropic distribution of electrons can occur for different reasons. In particular, anisotropic particle distributions are expected within different acceleration scenarios, including the particle acceleration by relativistic shocks (see, e.g., Dempsey & Duffy 2007), by the converter mechanism (Derishev et al. 2003), or due to magnetic reconnection (Cerutti et al. 2012).

We will use the general Equation (25) assuming that the source function (distribution of electrons) in the comoving system is not isotropic, but stationary. Let us introduce anisotropy (in \( K' \)) in the form

\[ f'(p', r') = \Psi(n'_e) f'(E'_e, r'), \]  

where \( n'_e = p'/|p'| \). The calculations for anisotropy in a general form are quite complex; therefore, to demonstrate, we will use an empirical approach, namely, we will adopt the specific form of elongated ellipsoid of revolution described by the function

\[ \Psi(n'_e) = \frac{\sin \alpha}{\alpha \sqrt{1 - (s n'_e)^2 \sin^2 \alpha}}. \]  

Here \( s' \) denotes a constant unit vector, which could have an arbitrary direction, and the parameter \( \alpha \) is confined within the interval \( 0 < \alpha < \pi/2 \). The function \( \Psi \) is normalized by the condition

\[ \int \Psi(n'_e) d\Omega'_{n'_e} = 1. \]  

The angular distribution given by Equation (38) is azimuthally symmetric relative to \( s' \). If the axis \( z \) is directed along \( s' \), the function \( \Psi(n'_e) \) can be considered as an equation of the surface of the ellipsoid of revolution written in spherical coordinates. The semi-major axis of the ellipsoid is directed along \( s' \) and is equal to \( a_z = (\tan \alpha/4\pi \alpha) \); the another two semi-axes are \( a_\perp = (\sin \alpha/4\pi \alpha) \).

It is convenient to introduce the asymmetry parameter \( \lambda \),

\[ \lambda = \frac{a_z}{a_\perp} - 1 = \frac{1}{\cos \alpha} - 1. \]  

In the limit of \( \lambda \rightarrow 0 \), the ellipsoid degenerates into a sphere, i.e., the angular distribution becomes isotropic (\( \Psi|_{\lambda=0} = 1 \)). At \( \lambda \gg 1 \), the angular distribution is strongly extended in the directions of \( s' \) and \(-s'\). We allow \( \lambda \) to be function of \( E'_e \), i.e., the anisotropy can be energy-dependent.

In the case of anisotropic angular distribution, the integration of Equation (26) over directions of the electron momentum results in

\[ W_{\text{anis}} = \epsilon^2 \int \Psi(n'_e) w(p, \epsilon_{\text{ph}}, k) d\Omega_e. \]  

Since the function \( w \) is different from zero when the directions of vectors \( p \) and \( k \) practically coincide, the argument of the function \( \Psi \) in Equation (41) can be replaced by \( n'_e \), and \( \Psi(n') \) can be taken out under the sign of integral. This gives

\[ W_{\text{anis}} \approx \Psi(n') W(E_e, \epsilon_{\text{ph}}, \epsilon), \]  

where \( n' \) should be expressed via \( n \) in the argument of \( \Psi \) according to relations in Equation (23). Therefore, for the anisotropic angular distribution of electrons, the observed flux is

\[ F_e = \frac{e D^3}{r^2} \int \Psi(n'_e) \Phi \left( \frac{\epsilon}{D^2 E^2_e} \right) \tilde{W}(E_e, \epsilon) dE_e. \]  

It is interesting to compare this equation with Equation (32). In the Thomson regime of scattering, Equation (43) is simplified,

\[ F_e = \frac{D^3}{r^2} \int \Psi(n'_e) \Phi \left( \frac{\epsilon}{D^2 E^2_e} \right) dE'_e. \]
Figure 1. Intensity of the EC radiation detected by an observer as a function of the angle between the line of sight and the direction of the relativistically moving source \( \theta \). Calculations are performed for four different combinations of the jet’s Lorentz factor \( \Gamma \) and the asymmetry parameter \( \lambda \) characterizing the angular distribution of electrons. The curves correspond to different directions of the axis of symmetry of the angular distribution of electrons in the comoving system \( \theta' \): 0° (curve 1), 30° (curve 2), 45° (curve 3), 60° (curve 4), and 90° (curve 5). For comparison the intensity corresponding to the isotropically distributed electrons is also shown (dotted curves).

In the case of an energy-independent anisotropy (\( \lambda = \text{const} \)), for the total (integrated over energy) intensity of radiation in the given direction, we have

\[
\int F_\epsilon \, d\epsilon \propto \Psi(n') D^6. \tag{45}
\]

To illustrate our results, we fix the orientations of the axes in the following way. As before, the axis \( z \) is directed along the velocity \( V \); the axis \( y \) we choose from the condition that the vector \( s' \) becomes parallel to the \((y, z)\) plane. Then, the components of the vector \( s' \) can be written in the following form:

\[
s' = (0, \sin \theta' \cos \theta', \cos \theta'). \tag{46}
\]

Here the angle \( \theta' \) determines the direction of the axis of symmetry of the angular distribution of electrons in the comoving frame.

To simplify the analysis, we assume that the radiation is detected in the plane \((y, z)\), and introduce polar coordinates in this plane. Then \((s' \cdot n') = \cos(\theta' - \theta'_n)\), where \( \theta' \) varies in the interval from \(-\pi\) to \(\pi\) (\( \theta'_n = 0 \) for the points on the axis \( z \)). Expressing in this equation \( \theta' \) through the viewing angle \( \theta \) in the system \( K \), we obtain

\[
(s' \cdot n') = D (\sin \theta \sin' \theta_n + \Gamma (\cos \theta - V) \cos' \theta_n). \tag{47}
\]

This expression is to be substituted into Equation (38).

In Figure 1, we show the dependence of the intensity of the total EC radiation (the flux integrated over the photon energies) on \( \theta \)—the angle between the line of sight and the direction of the jet. The calculations are performed for different combinations of the bulk motion Lorentz factor \( \Gamma \) and the asymmetry parameter \( \lambda \). The curves shown in these figures correspond to five different directions of the axis of symmetry of the electrons angular distribution described by the angle \( \theta' \). For comparison, we also show the intensity for isotropically distributed electrons. One can see that except for \( \theta'_n = 0 \) and \( \theta'_n = \pi/2 \), the maximum of the observed radiation appears not at \( \theta = 0 \), as in the isotropic case. Instead, it is shifted to larger angles as the level of anisotropy increases. For the chosen parameter, the shift can be as large as several degrees. This implies that in the case of anisotropic distributions of electrons in the source, we should be able to see misaligned jets as it has been indicated by Derishev et al. (2007).

In a more general approach, the \( \lambda \)-parameter can depend on the electron energy. This could be realized, for example, in the case of diffusive shock acceleration of electrons when the low-energy particles can be effectively isotropized in the downstream region due to pitch-angle scattering, whereas higher-energy particles radiate away their energy before being fully isotropized (see, e.g., Derishev et al. 2007). To demonstrate this effect, let us assume a simple energy dependence of the asymmetry parameter, \( \lambda = \lambda' \gamma'^p \), and consider a distribution of electrons in the standard “power-law with an exponential cutoff” form:

\[
N'_e(\gamma', n') = A \Psi(n') \gamma'^{-p} e^{-\gamma' / \gamma'_0} \tag{48}
\]

where \( A = \text{const} \). Here instead of the electron energy \( E'_e \), we use its Lorentz factor, \( \gamma' = E'_e / m_e \). From the condition of
normalization in Equation (39), it follows that \( \int N'_*(\gamma', n') d\Omega' \) does not depend on the level of anisotropy.

In Figure 2, we show the spectral energy distribution of EC radiation assuming that a relativistic jet propagates through the 2.7 K microwave background radiation. This might be relevant to the inverse Compton X-ray emission of extended jets of AGNs which could (still) be relativistic on kiloparsec scales (see, e.g., Sambruna et al. 2002). The radiation spectra are calculated for the viewing angle \( \theta = 0 \), thus \( D \approx 2 \Gamma \), and for the following combination of model parameters: \( p = 2, \lambda' = 0.1, p_1 = 1/2, \) and \( \gamma'_0 = 10^4 \). Since \( \eta \sim 4kT \gamma'_0/m \approx 2 \times 10^{-5} \ll 1 \), the Compton scattering proceeds in the Thomson limit. One can see from Figure 2 that if the photon energy is expressed in units of \( D^2 \epsilon_* = 4kT \gamma'_0^2/D^2 \), the shape of the energy spectrum of EC radiation does not depend on the jet's Doppler factor. On the other hand, it depends on the angle \( \theta'_0 \). The apparent reason is the dependence of the electron energy distribution on \( \theta'_0 \). For example, for \( \theta = 0 \) and \( \lambda'_0 \gamma^{p_1} \gg 1 \), the spectrum given by Equation (48) for \( \theta'_0 = 0 \) can be written in the form

\[
N'_*(\gamma', n') = \frac{\lambda' A}{2 \pi^2} \gamma'^{-p+p_1} e^{-\gamma'/\gamma'_0}, \quad (49)
\]

For \( \theta'_0 = \pi/2 \), Equation (48) differs from the spectrum corresponding to the isotropic distribution of electrons by a constant factor of \( 2/\pi \).

4. SUMMARY

In various astrophysical objects, such as microquasars, AGNs, and GRBs, the observed fluxes of radiation emerge from relativistically moving jets. The Doppler boosting caused by this motion can significantly enhance (by orders of magnitude!) the emitted absolute flux and shift the spectrum toward higher energies. Therefore, the beaming pattern of radiation is a key issue for proper understanding of acceleration and radiation processes in these objects.

In this paper, we derived the energy distribution of the EC radiation by solving the photon transfer equation for an optically thin source in a rather general case. It is described by Equation (25), which allows non-stationary and non-isotropic distribution of electrons in the frame of a relativistically moving source. Equation (25) does not specify the energy distribution of electrons either, but requires isotropic, homogeneous, and non-variable fields of seed photons for ICS. The latter condition makes the solution independent of the spatial distribution of electrons. For a power-law energy distribution of isotropically distributed electrons, Equation (25) is reduced to previously derived results (Dermer 1995; Georganopoulos et al. 2001).

The anisotropic distribution of electrons in a relativistically moving source can be realized in some acceleration scenarios, therefore, it is of a special practical interest. The formalism developed in this paper has been used to study the impact of the electron anisotropy on the EC emission. The calculations show that the anisotropy of emitting particles can significantly modify the beaming pattern. Most notably, the emission peak can be significantly shifted relative to the line of sight. This implies that, due to the anisotropic distribution of electrons in the source, modestly misaligned jets may become detectable. In contrast, while the energy of strongly anisotropic distribution of electrons in a source at rest can be radiated away from the observer, the relativistic motion of the source would make the radiation detectable, even in the case of most unfavorable anisotropy of electrons.

Generally, the electron anisotropy is expected to be energy-dependent. In this case, the anisotropy could result in harder spectra of EC emission compared to the isotropic distribution of electrons. The effects related to the anisotropic distribution of electrons in general, and in the context of the EC scenario, in particular, are quite strong. They cannot be ignored when interpreting the high energy emission from highly relativistic jets in AGNs and GRBs.

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