Methods and applications of gravity resonance spectroscopy within the qBounce experiment

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Abstract. We give a short overview of the gravity resonance spectroscopy methods used recently by the qBounce experiment. It has been demonstrated that mechanically induced transitions between bound neutron states in the gravitational field can be observed. Applications of this new method test Newton’s gravity law, search for extra dimensions of space-time, and study the neutrality of the neutron.

1. Introduction
A new method allows to study resonant transitions between discrete energy-states of ultra-cold neutrons (UCNs) bound in the gravity potential of the Earth [1]. It has the potential to shed new light on a broad range of questions. For example, the method provides a new way of looking on the limits of Newton’s gravitational law and is sensitive to hypothetical fifth forces in the micrometer regime. New mechanisms like axion-like couplings within this length regime can be detected. Furthermore, it allows to probe neutrons neutrality.

Ultra-cold neutrons with mass $m_n$ are placed on top of a flat neutron mirror. This works when their energy $E_n$ is less than the Fermi potential $V_{\text{Fermi}}$ of the mirror. The neutron, subject to the gravity field, forms hence bound states with discrete eigen energies $E_n$ with $n > 0$ which are in the pico-eV regime. The first observation of bound neutrons in the gravity field were published in 2002 [2]. A review about gravity tests with neutrons can be found in [3, 4]. A speciality of the solution of the Schrödinger equation is that the energy levels are non-equidistant. Each pair of states has thus a unique energy difference $\Delta E_{pq} = \hbar (\omega_q - \omega_p)$. Quantum mechanical transitions with a characteristic energy exchange between an externally driven modulator and these energy levels are observed on resonance. The technique presents a novelty in the sense that the quantum mechanically transition is driven by an oscillating mirror and is not a consequence of a direct coupling of an electromagnetic charge or electromagnetic moment to an external field. This is an additional approach to the high sensitivity method used in atomic clocks [5], nuclear magnetic resonance [6] and atom interferometry [7].

2. Rabi’s method
Transitions between quantum states can be realized by a coupling to a modulator using Rabi’s method [8]. Several ways are possible: by applying varying magnetic fields and by mechanical
vibrations, which modify the border condition periodically. The latter has been recently realized [1]. A different proposal can be found in [9, 10].

A typical Rabi-like setup can be seen in Figure 1(a). Neutrons fly through the three-part setup with the steps: preparation, excitation and analysis. The first part projects the state onto the ground state as higher states are suppressed by being removed from the neutron scatterer on top. The second region induces the transition by vibrating the mirror with a tuneable frequency and vibration amplitude. The third region, identical to the first, projects the state again onto the ground state. A detector at the end captures the neutron flux. This flux will be reduced if in region two a transition from the ground state to an excited state takes place.

Transitions in region two between two states \( |p\rangle \) and \( |q\rangle \) can be studied by treating the states as a spin-1/2 system. This is valid if the frequency of the other transitions is sufficiently different from \( \omega_{pq} \).

The time dependence of the states takes the following form:

\[
|\Psi(t)\rangle = C_p(t) e^{-i\omega_p t}|p\rangle + C_q(t) e^{-i\omega_q t}|q\rangle,
\]

with time dependent coefficients \( C_{p,q} \) which can be obtained by changing into the rotating frame of reference:

\[
C_p(t) = \tilde{C}_p(t) e^{-i\delta\omega t}, C_q(t) = \tilde{C}_q(t) e^{+i\delta\omega t},
\]

where \( \delta\omega = \omega - \omega_{pq} \) is the detuning from the resonance frequency \( \omega_{pq} \). This results in a coupled differential equation for the time dependent coefficients \( \tilde{C}_n \):

\[
\frac{d}{dt} \begin{pmatrix} \tilde{C}_p(t) \\ \tilde{C}_q(t) \end{pmatrix} = \frac{i}{2} \begin{pmatrix} \delta\omega & \Omega_R \\ -\delta\omega & -\Omega_R \end{pmatrix} \begin{pmatrix} \tilde{C}_p(t) \\ \tilde{C}_q(t) \end{pmatrix},
\]

with \( \Omega_R \) being the Rabi frequency, a measure for the coupling to the vibrating mirror.

When initially in the state \( p \), the probability of finding the neutron in state \( q \) after a time \( t \) is given by:

\[
|\Psi(t)|^2 = \left( \frac{\Omega_R}{\Omega_R^t} \right)^2 \sin^2 \frac{\Omega_R^t t}{2},
\]
where $\Omega_R = \sqrt{\Omega_R^2 + \delta\omega^2}$ incorporates the detuning.

Let us now consider the experimentally realized setup shown in Figure 1(b). In comparison to the three-part setup, one region performs all three tasks at the same time. For that purpose, a second mirror with a rough surface is placed on top of the first mirror which modifies the problem. The orthonormal eigen states, previously Airy Ai and Bi functions, are now a superposition of Airy Ai and Bi functions and the eigen energies are increased. The rough mirror removes higher, unwanted states as described by [11]. Such an additional loss mechanism modifies the system and is different for each state as it is proportional to the overlap of the wave function with the additional absorbing layer. Each state has thus an additional damping term $\gamma_n$ which enters the equation:

$$
\frac{d}{dt} \left( \begin{array}{c} \tilde{C}_p(t) \\ \tilde{C}_q(t) \end{array} \right) = i \left( \begin{array}{cc} (\delta\omega + i\gamma_p) & \Omega_R \\ -\delta\omega + i\gamma_q & \Omega_R \end{array} \right) \left( \begin{array}{c} \tilde{C}_p(t) \\ \tilde{C}_q(t) \end{array} \right).$$

(5)

The solution of this differential equation is:

$$
\left( \begin{array}{c} \tilde{C}_p(t) \\ \tilde{C}_q(t) \end{array} \right) = e^{-(\gamma_p+\gamma_q)t/4} \left( \begin{array}{cc} \Omega_R & i\gamma_q \Omega_R^2 & \Omega_R & i\gamma_p \Omega_R^2 \\ \Omega_R & \gamma_q \Omega_R^2 & -\Omega_R & \gamma_p \Omega_R^2 \\ \Omega_R & \gamma_q \Omega_R^2 & -\Omega_R & \gamma_p \Omega_R^2 \\ \Omega_R & \gamma_q \Omega_R^2 & -\Omega_R & \gamma_p \Omega_R^2 \end{array} \right) \left( \begin{array}{c} C_p(0) \\ C_q(0) \end{array} \right),$$

(6)

with $\Omega'_R = \sqrt{-((\gamma_q - \gamma_p)/2 + i\delta\omega)^2 + \Omega_R^2}$ now including the damping. Even though the structure of the solution is more complex the same behavior as in the three-part setup is recovered.

Without vibration the solution simplifies to

$$C_n(t) = e^{-\gamma_n t/2} C_n(0).$$

(7)

The recorded flux at the detector is proportional to the norm of the state:

$$|\Psi|^2 = |\tilde{C}_p|^2 + |\tilde{C}_q|^2.$$  

(8)

2.1. Experimental Realization

The experimentally feasible version, shown in Figure 1(b), combines the three regions into one and has been used within the qBounce experiment performed at the fundamental physics instrument PF2 at the Institut Laue-Langevin in 2009 [1] and 2010. The setup consists of a neutron mirror on bottom and a neutron scatterer on top. They are vibrating with a tuneable frequency and desirable amplitude. The vibration of the setup is induced by piezo actuators, which allow vibrations in the range 100 – 800 Hz and a vibration amplitude of up to 5 mm/s. The neutron mirrors were made of BK7 glass with a Fermi potential of about $V_{\text{Fermi}} \approx 100$ neV, which is several orders higher than the eigen energy of the neutron. The bottom mirror is polished and has a roughness less than 2 nm, whereas the top mirror has a surface roughness within the micrometer regime to provide the loss mechanism described in previous section. The velocity selection of the UCNs was performed by a collimation system in front of the setup. Only neutrons following a certain classical flight path in the gravity field are accepted into the setup resulting in an allowed velocity spectrum in the direction along the mirrors. This ensured that neutrons with a horizontal velocity $5.7 \leq v_h \leq 9.5$ m/s enters the setup resulting in a mean interaction time of $\tau \approx 20$ ms. To avoid a shift of states energy due to the magnetic moment of the neutron, magnetic shielding was provided by one layer of $\mu$-metal around the vacuum chamber. This was sufficient to reduce the ambient magnetic field inside our setup by more than one order.
3. Ramsey’s method of separated oscillating fields

The sensitivity of a Rabi-setup is inverse to the interaction time which is fixed by the length of the region two in Figure 1(a). The experimental limit lies in the scalability of this vibrating mirror setup. With the same length of the vibrating regions the interaction time can be increased by applying Ramsey’s method of separated oscillating fields [12]. Another advantage of the Ramsey-like setup is the self focusing. This means that a velocity distribution of the neutrons will not deteriorate the effect as strongly as for a Rabi-like setup. Less demanding requirements allow a greater neutron flux.

In the Ramsey-like setup, shown in Figure 2, interaction takes place in two smaller regions instead of in one longer. They are separated by a distance, where the wave function can propagate freely. If a $\pi/2$-pulse is applied in the second region, it creates a coherent superposition between the states $|p\rangle$ and $|q\rangle$. Region four vibrates in phase with region two. The reversal of the transition in region four depends on the relative phase $\phi_{pq}$ between the states $|p\rangle$ and $|q\rangle$ obtained in the intermediate region. The neutron flux is hence dependent on this relative phase. For further details see [13].

4. Applications

The presented methods allow for a search for deviations from Newton’s gravity law at short distances [14, 15]. A hypothetical energy shift is detectable via a change of the resonance frequency $\omega_{pq}$. A generic parametrisation of such a deviation between two masses $m_i$ and $m_j$ is accounted for by adding a Yukawa-like term [16]

$$V(r) = -G\frac{m_im_j}{r} \left(1 - \alpha e^{-r/\lambda}\right),$$

(9)

where $\alpha$ characterizes the strength of the deviation compared to Newton’s gravity law and $\lambda$ defines an interaction length. There are many theoretical predictions giving rise to such a deviation [16]. Large extra dimensions of space-time introduced by string theory which are compactified at length scales $\lambda$ would modify the exponent of the Newton’s inverse squared law [17]. The observed smallness of the cosmological constant and dark energy could be explained by Supersymmetric large extra dimensions [18, 19] or by a scalar chameleon field [20].

Possible axion-like couplings between matter and spin would also affect neutrons. Depending on their spin orientation a signal resolvable by such an experiment would be obtained.
Comparing the resonance frequency $\omega_{pq}$ for different spin orientations allows to detect an additional potential of the form:

$$V(\vec{r}) = \hbar g_s g_p \frac{\vec{\sigma} \cdot \vec{n}}{8\pi mc} \left( \frac{1}{\lambda r} + \frac{1}{r^2} \right) e^{-r\lambda}, \quad (10)$$

where $g_s g_p$ denotes the pseudo-scalar coupling constants. The geometry of the studied matter, ie. the neutron mirrors is noted by $\vec{n}$ and $\vec{\sigma}$ indicates the neutrons spin. Axion-like particles are possible candidates for dark matter [17] in the so-called axion window.

As has been shown in [21] such a setup can be used to provide new limits for the neutrons charge. Applying an electric field of $6 \times 10^6$ V/m in the intermediate region modifies the energy of the states and leads to a change in the relative phase $\phi_{pq}$ of the states $|p\rangle$ and $|q\rangle$. Current limits on the neutron charge are based on the deflection of neutrons within a present electric field [22].

To visualize the methods presented here fly-through setups have been used. However they can been realized as well in storage mode. All the applied steps are then not different parts in space but sequences performed at different times. This removes the constraint to choose neutrons with a narrow velocity distribution for Rabi like setups. It would allow a significant increase of the interaction time [23].

Gravity experiments with neutrons are motivated mainly due to the fact that in contrast to atoms the electrical polarizability of neutrons is extremely low [25]. Neutrons are not disturbed by short range electric forces such as van der Waals or Casimir forces. Together with its neutrality one holds the key for a sensitivity many orders of magnitude below the strength of electromagnetism.

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