The D-dimensional non-relativistic particle in the Scarf Trigonometry plus Non-Central Rosen-Morse Potentials

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Abstract. The D-Dimensional Non-Relativistic Particle Properties in the Scarf Trigonometry plus Non-Central Rosen-Morse Potentials was investigated using an analytical method. The bound state energy is given approximately in the closed form. The approximate wave function for arbitrary l-state in D-dimensions are expressed in the form of generalised Jacobi Polynomials. The energy spectra of the particle are increased when the dimensions are higher. The relationship between the orbital number in each dimension is recursive. The special case in 3 dimensions is given to the ground state.

1. Introduction

The solution of Schrodinger equations for some potentials is very essential. It provides the information of the quantum system. Some researchers investigated the approximate solution of the shape invariant potentials. This potential has been investigated using several techniques. For example, Coulomb potential using Laplace transformation \cite{1}, Morse potential using series expansion method \cite{2}, Modified Poschl-Teller, Hulten, Scarf hyperbolic Manning-Rosen plus Scarf potentials using NU method \cite{3-7}, q-deformed Rosen-Morse Potential using Supersymmetry Quantum Mechanics \cite{8}. The solution of shape-invariant potentials in D-dimensions only in radial part \cite{1-6}. Deta \cite{9} try to solve the angular part with D-dimensional Poschl-Teller potential. The other D-Dimensional shape invariant potentials has not been solved yet. The exact solution of Schrodinger Equation can be solved exactly only for \(l = 0\) in 1 dimension. For the other value of \(l\) and \(D\) (\(l \neq 0\) and \(D > 1\)), the Schrodinger equation might be solved approximately with appropriate approximation scheme. One of the suitable approximation schemes is proposed by Greene and Aldrich \cite{10,11}.

Recently, Deta \cite{9} solved the combination between radial plus angular potential in D-dimensions. He solved Scarf hyperbolic plus non-central Poschl-Teller potential. Suparmi \cite{12} solved four dimensional extended hyperbolic Scarf I plus three dimensional separable trigonometric noncentral potentials. However, other combinations of radial and angular potential in D-dimension have not been studied yet. One of the combinations that still has not been studied is Scarf Trigonometry plus Non-Central Rosen-Morse potentials.

In this research, we will try to solve the Schrodinger equation in D-Dimensions for Scarf Trigonometry plus Non-Central Rosen-Morse potential using Nikivorof-Uvarov method. The N-U method developed by Nikiforov-Uvarov \cite{13}. This method based on solving a second order linear differential equation. The equation is reduced to a generalized equation of hypergeometric type differential equation by a suitable change of variable.
The Scarf potential describes the periodically arranged particles like a crystal [5,6,14,15]. The application of this potential is a crystal model in the solid state physics [5,6]. On the other hand, the Rosen-Morse potential describes the quark-gluon dynamics of Quantum Chromodynamics [16]. The solution of this potential remains in the radial part [16-18]. The combination of these potential in D-Dimensions with the centrifugal term is separable potential. Hence, this system can be solved by the separation of variable method. The solution of radial part of Schrodinger equation is one solution. Nevertheless, the solution of angular part of Schrodinger equation is D-1 solutions.

2. The Nikiforov-Uvarov Method
The Schrodinger equation in D-dimensions of any shape invariant potential can be reduced into hypergeometric-type of a differential equation using a suitable variable transformation [19-21]. The hypergeometric-type of differential equation in Nikiforov-Uvarov method is:

$$\frac{\partial^2 \Psi(s)}{\partial s^2} + \frac{\tau(s)}{\sigma(s)} \frac{\partial \Psi(s)}{\partial s} + \frac{s(s)}{\sigma^2(s)} \Psi(s) = 0$$

(1)

where $\sigma(s)$ and $\bar{\sigma}(s)$ are polynomials, mostly in the second order, and $\tau(s)$ is the first order polynomial. Equation (1) can be solved using separation of variable, that is:

$$\Psi = \phi(s)y(s)$$

(2)

By inserting equation (2) into equation (1), we get the hypergeometric type equation, that is:

$$\sigma \frac{\partial^2 y}{\partial s^2} + \tau \frac{\partial y}{\partial s} + \lambda y = 0$$

(3)

Where $\phi(s)$ is a logarithmic derivative. The solution of $\phi(s)$ is obtained from condition:

$$\frac{\phi'}{\phi} = \frac{\sigma}{\tau}$$

(4)

while the function $\pi(s)$ and the parameter $\lambda$ are defined as:

$$\pi = \left(\frac{\sigma - \bar{\tau}}{2}\right) \pm \sqrt{\left(\frac{\sigma - \bar{\tau}}{2}\right)^2 - \sigma + k \sigma}$$

$$\lambda = k + \pi^2$$

(5)

(6)

The value of $k$ in equation (5) can be found from the condition that the expression under the square root of equation (5) must be a square of a polynomial. This polynomial is mostly first-degree polynomial, therefore the discriminate of the quadratic expression is zero. A new eigenvalue of equation (3) is:

$$\lambda = \lambda_n = -n\pi - \frac{n(n-1)}{2} \sigma^n; \quad n = 0, 1, 2, \ldots$$

(7)

where

$$\tau = \bar{\tau} + 2\pi$$

(8)

The new bound state energy obtained using the equation (6) and (7). To generate the bound state energy and the corresponding eigenfunction, the condition that $\tau' < 0$ is required. The solution of the second part of the wave function, $y_n(s)$, is connected to Rodrigues relation [20]:

$$y_n(z) = \frac{C_n}{\rho(z)} \int d^z \left\{ \sigma^z(z) \rho(z) \right\}$$

(9)

where $C_n$ is normalization constant, and the weight function $\rho(s)$ must satisfies the condition:

$$\frac{\partial \sigma \rho}{\partial s} = \tau(s) \rho(s)$$

(10)

The wave function of the system obtained from equation (4), (9) and (10).
3. Scarf Trigonometry plus Non-Central Rosen-Morse Potentials

The Scarf Trigonometry potential can be expressed as:

\[
V = \frac{\hbar^2}{2m} \left\{ \frac{b^2 + a(a - 1)}{\sin^2 r} - \frac{2b \left( a - \frac{1}{2} \right) \cos r}{\sin^2 r} \right\}
\]  
(11)

The Non Central Rosen-Morse potential can be expressed as:

\[
V = \frac{\hbar^2}{2mr^2} \left\{ \frac{\nu(\nu + 1)}{\sin^2 \Omega_{\nu}} - 2q \cot \Omega_{\nu} \right\}
\]  
(12)

The Schrödinger Equation in D-Dimensions for Scarf Hyperbolic plus Non-Central Pocshl-Teller Potential can be expressed as:

\[
-\frac{\hbar^2}{2m} \nabla^2 \Psi(r, \Omega) + \frac{\hbar^2}{2m} \left\{ \frac{b^2 + a(a - 1)}{\sin^2 r} - \frac{2b \left( a - \frac{1}{2} \right) \cos r}{\sin^2 r} \right\} \Psi(r, \Omega) + \frac{\hbar^2}{2mr^2} \left\{ \frac{\nu(\nu + 1)}{\sin^2 \Omega_{\nu}} - 2q \cot \Omega_{\nu} \right\} \Psi(r, \Omega) = E \Psi(r, \Omega)
\]  
(13)

where

\[
\nabla^2 = \frac{1}{r^{D-1}} \frac{\partial}{\partial r} \left( r^{D-1} \frac{\partial}{\partial r} \right) - \Lambda_{D,2}^2 (\Omega_{\nu})
\]  
(14)

with \( \Lambda_{D,2}^2 (\Omega_{\nu}) \) is hyperspherical harmonics, that is, D-Dimensional angular momentum operator. For \( 2 \leq k \leq D - 1 \), we have:

\[
\Lambda_{D,2}^2 (\Omega_{\nu}) = L_{k}^2
\]

\[
= \sum_{i=1}^{k-1} L_{i}^2 = -\frac{1}{\sin^2 \theta_{k}} \frac{\partial}{\partial \theta_{k}} \left( \sin^{k-1} \theta_{k} \frac{\partial}{\partial \theta_{k}} \right) + \frac{L_{k-1}^2}{\sin^2 \theta_{k}}
\]  
(15)

and for \( k = 1 \), we have:

\[
L_{1}^2 = -\frac{\partial^2}{\partial \theta_{1}^2}
\]  
(16)

Substitute (14), (15), (16), using variable substitution, and separation variable, that is,

\[
\varepsilon^2 = -\frac{2m}{\hbar^2} E
\]

\[
\Psi(r, \Omega_{\nu} = \theta_{1}, \theta_{2}, \ldots \theta_{D-1}) = \Psi(r)\Phi(\varepsilon = \varphi)H(\theta_{1}, \ldots \theta_{D-1})
\]  
(18)
we have the radial, polar, and azimuth parts of Schrodinger equation are:

\[
\frac{1}{r^{d-1}} \frac{\partial}{\partial r} \left( r^{d-1} \frac{\partial}{\partial r} \right) \Psi(r) - \left\{ b^2 + a(a-1) \frac{\sin^2 r}{\sin^2 r} - \frac{2b(a-1/2)\cos r}{\sin^2 r} + \epsilon^2 \right\} \Psi(r) = \frac{\lambda^2}{r^2} \Psi(r)
\]

(19)

\[
\frac{1}{\sin^{d-2} \theta} \frac{\partial}{\partial \theta} \left( \sin^{d-2} \theta \frac{\partial}{\partial \theta} \right) H - \frac{l_m (l_m + k - 2)}{\sin^2 \theta} H + l_i (l_i + k - 1) H - \left\{ \frac{\nu(\nu+1)}{\sin^2 \Omega_b} - 2q \cot \Omega_b \right\} H = 0
\]

(20)

for \( k = 2, 3, 4, \ldots, D - 1 \).

\[
-\frac{1}{\Phi} \frac{d^2}{d\theta^2} \Phi = L_i^2
\]

(21)

3.1. Solution of azimuth part of Schrodinger equation in D-dimensions

The azimuth part of Schrodinger equation in D-Dimensions (equation 20) is the ordinary differential equation. The unnormalized solution of azimuth part of Schrodinger equation in D-Dimensions (21) is:

\[
\Phi = Ae^{im\phi}
\]

(22)

where A is normalization constant.

3.2. Solution of radial part of Schrodinger equation in D-dimensions

The radial part of Schrodinger equation in D-Dimensions (Eq. 19) is hypergeometric differential equation. Equation (19) can be solved using some approximation and variable substitution, that is,

\[
\Psi(r) = r^{\frac{\nu}{2}} R(r)
\]

(23)

\[
r \approx \sin r
\]

(24)

\[
\cos r = s
\]

(25)

Substitute equation (23), (24), and (25) to equation (19), we have:

\[
\frac{d^2}{ds^2} R(s) - \frac{s}{(1-s^2)} \frac{d}{ds} R(s) - \frac{b^2 + a(a-1) - 2b(a-1/2)s}{\sin^2 s} + \frac{\epsilon^2 (1-s^2) + \lambda^2 + (D-1)(D-3)}{(1-s^2)^2} R(r) = 0
\]

(26)
By comparing equation (1), (26), and using eigenvalue of Nikivorof-Uvarov method, we obtain the energy spectra of Scarf Trigonometry plus Non-Central Rosen-Morse potentials in D-Dimensions is:

\[
E = \frac{\hbar^2}{2m} \left( p + n + \frac{l}{2} \right)^2
\]

where

\[
p = \sqrt{\left[ b^2 + \left( a - \frac{1}{2} \right)^2 + \lambda^2 + \frac{(D-1)(D-3)}{4} \right]} - \sqrt{\left[ b^2 + \left( a - \frac{1}{2} \right)^2 + \lambda^2 + \frac{(D-1)(D-3)}{4} \right] - \sqrt{\left[ b^2 - \left( a - \frac{1}{2} \right)^2 + \lambda^* + \frac{(D-1)(D-3)}{4} \right]}}
\]

and \( A_{D,1} \) is centrifugal term which depends on the eigenvalue of polar part of Schrodinger equation in D-Dimensions.

By using eigenfunction of NU method, we obtain:

\[
R_n = B_n \left( 1 - s^2 \right)^{\nu_{\alpha,\beta}/2} \left( 1 + s \right)^{\nu_{\alpha,\beta}/2p} \frac{\nu_{\alpha,\beta}}{2p} P_n^{(\alpha,\beta)}(s)
\]

where \( P_n^{(\alpha,\beta)}(s) \) is Jacobi Polynomial, that is,

\[
P_n^{(\alpha,\beta)}(s) = \frac{(-1)^n}{2^n n!} (1-s)^{-\alpha} (1+s)^{-\beta}
\]

\[
\frac{d^n}{dz^n} \left\{ (1-s)^{\alpha} (1+s)^{\beta} \right\}
\]

and \( B_n \) is normalization constant.

Finally, from equation (23), (25), and (29), we have the radial wave function of Schrodinger equation for Scarf Hyperbolic plus Non-Central Poshl-Teller Potential in D-Dimensions is:

\[
\Psi(r) = B_n^\frac{\nu_{\alpha,\beta}}{2p} B_s \left( \sin r \right)^{\nu_{\alpha,\beta}/2} \left( 1 + \cos r \right)^{\nu_{\alpha,\beta}/2p} \frac{\nu_{\alpha,\beta}}{2p} P_n^{(\alpha,\beta)}(\cos r)
\]
The effect of extra dimensions on bound state energy and radial wave function can be visualized as

![Figure 1](image)

**Figure 1.** Radial Wave Function on Ground State for Scarf Trigonometry Potential \((a = 2\) and \(b = 2\)) in (a) 3D, (b) 4D, and (c) 5D.

From Figure 1, we have the effect of extra dimensions in radial wave function is increase the amplitude of radial wave function. It was found to agree with previous work [6]. Based on superstring theory [22], the number of dimensions in the universe restricted to 10-spatial dimensions and 1-time dimension. If the amount of spatial dimension more than 10, the universe unstable and collapse. Thus, the maximum value of D is limited to 10 spatial dimensions.

### 3.3. Solution of polar part of Schrodinger equation in D-dimensions

According to the value of \(k\), there are two equations at polar part of Schrodinger equation in D-Dimensions. First, for \(k = 2, 3, 4, \ldots, D - 1\), we solve equation (20) using variable substitution, that is,

\[
A_k = l_k \left( l_k + k - 1 \right) \tag{32a}
\]

\[
A_{k-1} = l_{k-1} \left( l_{k-1} + k - 2 \right) \tag{32b}
\]

\[
H = \frac{Q}{\left( \sin \theta_k \right)^{\frac{24}{s}}} \tag{33a}
\]

\[
s = \cot \theta_k \tag{33b}
\]
Substitute equation (32a), (32b), (33a), (33b) to (21a), we have
\[
\frac{d^2 Q}{ds^2} + \frac{2s}{(1 + s^2)} \frac{dQ}{ds} = \left\{ \left[ \frac{1}{4} \frac{1}{4} + A_{k+1} + \nu (\nu - 1) \right] \left(1 + s^2\right) \right\} - \frac{-2qs - \left( \frac{k}{2} \right)^2 - A_k}{\left(1 + s^2\right)^2} Q = 0
\] (34)

By comparing equation (1), (34), and using eigenvalue of Nikivorof-Uvarov method, we obtain the eigenvalue of Scarf Hyperbolic plus Non-Central Pocshl-Teller potential in D-Dimensions:
\[
A_k = \left( \sqrt{\frac{1}{4} \frac{1}{4} + \frac{1}{4} + A_{k+1} + \nu (\nu - 1) + n + \frac{1}{2}} \right)^2
\]
\[
- \frac{q^2}{\left( \sqrt{\frac{1}{4} \frac{1}{4} + \frac{1}{4} + A_{k+1} + \nu (\nu - 1) + n + \frac{1}{2}} \right)^2 - \left( \frac{k}{2} \right)^2} \right)^2
\] (35)

By using eigenfunction of NU method, we obtain:
\[
H \left( \cot \theta_k \right) = C_n \left( 1 + \cot^2 \theta_k \right)^{\frac{3}{2}} e^{-\frac{2}{\nu} \arctan(\cot \theta_k)}
\]

\[
\frac{d^n}{d \cot \theta_k^n} \left\{ \left( 1 + \cot^2 \theta_k \right)^{n - \frac{1}{2}} e^{\nu \arctan(\cot \theta_k)} \right\}
\]
with
\[
p = \sqrt{\left( \frac{k}{2} \right)^2 + A_k} + 4q^2
\] (37)

Note that the value of \( k = 2, 3, 4, \ldots, D - 1 \), depend on the number of dimensions.

4. Special cases for the ground and first excited state in 3 dimensions

Special cases for 3 Dimensional problems for Scarf Hyperbolic plus Non-Central Pocshl-Teller Potential, that is:
\[
\Psi (r, \Omega) = \Psi (r) \Phi (\Omega)
\] (38)

with solution of radial part of 3 dimensional Schrodinger equation:
\[
\Psi (r) = B_n r^{-s} \left( \sin r \right)^{\nu - \frac{s}{2}} \left( 1 + \cos r \right)^{\frac{b(\alpha - 1)}{2p}} \left( 1 - \cos r \right)^{-\frac{b(\alpha - 1)}{2p}} P_n^{(\alpha, \beta)} \cos r
\] (39)
\[ p_n^{(\alpha, \beta)}(\cos r) = \frac{(-1)^n}{2^n n!} (1 - \cos r)^\alpha (1 + \cos r)^\beta \]  
(40)

\[ \frac{d^n}{d(\cos r)^n}\left\{ (1 - \cos r)^\alpha (1 + \cos r)^\beta \left( \sin^2 r \right)^n \right\} \]

\[ \alpha_r = -p_r + \frac{b(a - \frac{1}{2})}{p_r} \]  
(41a)

\[ \beta_r = -p_r - \frac{b(a - \frac{1}{2})}{p_r} \]  
(41b)

\[ p_r = \sqrt{\frac{\left\{ b^2 + (a - \frac{1}{2})^2 + A_2 \right\} - \sqrt{\left\{ b + (a - \frac{1}{2})^2 + A_2 \right\}} + \sqrt{\left\{ b - (a - \frac{1}{2})^2 + A_2 \right\}}}{2}} \]  
(42)

From equation (39) to (42), we can make a visualization of radial wave function using Matlab, that is:

**Figure 2.** Ground State Radial Wave Function on Ground State for Scarf Trigonometry Potential \((a = 2\) and \(b = 2\)) in 3 Dimensions.

**Figure 3.** Ground State Radial Wave Function on Ground State for Scarf Trigonometry plus Non-Central Rosen-Morse Potential \((a = 2, b = 2, v = 2,\) and \(q = 2\)) in 3 Dimensions.

From Figure 2 and 3, we have the effect of each potential parameter. The effect of parameter Non-Central Rosen-Morse will not appear if the parameters of Scarf trigonometry are zero. It causes the radial wave function vanishes. If the Scarf trigonometry parameters are not zero, the Non-Central Poschl-Teller parameters duplicate the radial wave function as shown in Figure 3.
Solution of Angular part divided into 2 parts, that is:

\[ Y_{l}^{m}(\Omega_{l} = \theta_{1}, \theta_{2}) = \Phi(\theta_{1} = \phi)H(\theta_{2}) \]  

(43)

Solution of azimuth part equation:

\[ \Phi = Ae^{imp} \]  

(44)

\[ A = \frac{1}{\sqrt{2\pi}} \]  

(45)

Solution of polar part equation:

\[ H(\cot \theta_{2}) = C_{\nu_{l}} \left(1 + \cot^{2} \theta_{2}\right)^{\nu_{l}^{2} + \frac{1}{2} - \nu_{l}} e^{-\frac{2\pi}{\nu_{l}} \tan(\cot \theta_{2})} \]  

(46)

\[ \frac{d^{n}}{d \cot \theta_{2}^{n}} \left(1 + \cot^{2} \theta_{2}\right)^{\nu_{l} - \nu_{l}^{2}} e^{\nu_{l}^{2}} \]  

with

\[ p_{\nu_{l}} = \sqrt{\left[\frac{1}{4} + A_{2}\right] + \sqrt{\left[\frac{1}{4} + A_{2}\right]^{2} + 4q^{2}}} \]  

(47)

\[ A_{2} = \left(\sqrt{A_{1} + \nu(\nu - 1) + \frac{1}{2}} + \frac{q^{2}}{\left(\sqrt{A_{1} + \nu(\nu - 1) + \frac{1}{2}}\right)^{2} - \frac{1}{4}}\right) \]  

(48)

Using substitution of all equation (46) to (48), we get unnormalized solution of 3 dimensional Schrodinger equations for Scarf hyperbolic plus non central Poschl-Teller potential, for example:

**Table 1. Visualization of angular part wave function for Non-Central Rosen Morse-Teller Potential.**

| \( H(\cos \theta)_{n,m,q} \) | 3 Dimensions | 2 Dimensions Projection |
|-----------------------------|--------------|------------------------|
| \( H(\cos \theta_{2})_{0000} \) | ![Visualization of angular part wave function for Non-Central Rosen Morse-Teller Potential.](image) | ![Visualization of angular part wave function for Non-Central Rosen Morse-Teller Potential.](image) |
| $H(\cos \theta)_{n,m,q}$ | 3 Dimensions | 2 Dimensions Projection |
|--------------------------|--------------|-------------------------|
| $H(\cos \theta)_{0033}$ | ![3D Plot](image1) | ![2D Projection](image2) |
| $H(\cos \theta)_{0100}$ | ![3D Plot](image3) | ![2D Projection](image4) |
| $H(\cos \theta)_{0133}$ | ![3D Plot](image5) | ![2D Projection](image6) |
| $H(\cos \theta)_{1000}$ | ![3D Plot](image7) | ![2D Projection](image8) |
The effect of polar potential parameters in 3-dimensions of Non-Central Rosen-Morse Potential on the orbital (subshell) particle can be described in Table 1. Without any polar potential ($v$ and $q$ are zero), the orbital electron will be changed into orbitals in spherical harmonics or hydrogen-like atom. These phenomena are figured in the number 1, 3, and 7, which correspond to an orbital in spherical harmonics in the same parameters [23].

5. Conclusion
In this paper, we present the solutions of Schrodinger equation in D-dimension for Scarf Trigonometry plus non-central Rosen-Morse Potentials within the framework of an approximation to the centrifugal and high dimension term. The bound state energy is obtained in D-dimensions using the Nikiforov-Uvarov method, and it was found to agree with previous works [6, 18]. The corresponding wave function of the Scarf trigonometry plus non-central Rosen-Morse potentials was obtained in terms of the Jacobi polynomials. The example of bound state energy and wave function in 3 dimensions presented in the condition of the ground state. The existence of arbitrary dimensions increases the amplitude of wave function.

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