QED processes in peripheral kinematics at polarized photon-photon and photon-electron colliders

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Abstract

The calibration QED process cross sections for experiments on planned electron-photon and photon-photon colliders for detecting the small angles scattered particles are calculated. These processes describe the creation of two jets moving sufficiently close to the beam axis directions. The jets containing two and three particles including charged leptons, photons and pseudoscalar mesons are considered explicitly. Considering the pair production subprocesses we take into account both bremsstrahlung and double photon mechanisms. The obtained results are suitable for further numerical calculations.

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Introduction

QED processes of the type $2 \rightarrow 3, 4, 5, 6$ at colliders of high energies have attracted both theoretical and experimental attention during the last four decades. Accelerators with high-energy colliding $e^+e^-$, $\gamma e, \gamma\gamma$ and $\mu^+\mu^-$ beams are now widely used or designed to study fundamental interactions \[1\]. Some processes of quantum electrodynamics (QED) might play an important role at these colliders, especially those inelastic processes whose cross section does not drop with increasing energy. The planned colliders will be able to work with polarized particles, so these QED processes are required to be described in more detail, including the calculation of cross sections with definite helicities of the initial particles – leptons (l=e or $\mu$) and photons $\gamma$. These reactions have the form of a two-jet process with the exchange of a virtual photon $\gamma^*$ in the t-channel (see Fig.1).

A lot of attention to the calculation of helicity amplitudes of QED processes at high energy colliders was paid in the literature (see \[2\] and references therein). Keeping in mind the physical programs at planned $\gamma\gamma$ and lepton $\gamma$ colliders, a precise knowledge of a set of calibration and monitoring processes is needed. The calibration processes are the QED processes with sufficiently large cross sections and clear signatures for detection. Rather a rich physics can be investigated in peripheral processes such as heavy leptons and mesons (scalar and pseudoscalar) creation, where the relevant QED monitoring processes must be measured.

Let us remind the general features of peripheral processes namely the important fact of their nondecreasing cross sections in the limit of high total energies $\sqrt{s}$ in the center of mass frame of the initial particles. The possibility of measuring the jets containing two or three particles can be relevant. This is a motivation of our paper.

It is organized in the following a way.

In Section 1, the kinematics of peripheral processes is briefly described.

In Section 2, the impact factors describing the conversion of initial photon, to the pair of charged particles (fermions or spinless mesons) are calculated.

In Sections 3, 4, and 5 a similar calculation is made for the initial polarized electron and photon, in particular subprocesses such as the single and the double Compton process, and the processes of pair creation are considered.

As well as the helicity amplitudes for subprocesses of type $2 \rightarrow 3$ have in general compli-
FIG. 1: The processes $\gamma\gamma, \gamma l \ (l = e,\mu)$ with the exchange of a virtual photon $\gamma^*$ in the t-channel.

FIG. 2: The scheme of collision of initial beams with detection of two jets moving in the cones within the angles $\theta$.

cated form we do not put explicit expressions for the corresponding cross sections indicating only the strategy to obtain it.

A. Kinematics

Throughout the paper it is implied that the energy fractions of jet component are positive quantities of the order of unity by magnitude (the sum of energy fractions of each jet is unity) and the values of transversal to the beam direction component of their 3-momenta are much larger compared to their rest masses. So we neglect the mass of jet particles.

The corresponding amplitudes include a large amount of Feynman diagrams (FD). Fortunately, in the high-energy limit the number of essential FD contributing the "leading" approximation greatly reduces. The method used permits one to estimate the uncertainty caused by "nonleading" contributions which have following magnitudes of the order

$$\frac{m^2}{s_1}, \frac{s_1}{s}, \frac{s_2}{s}, \frac{\alpha}{\pi} \ln \frac{s}{m^2}$$  \tag{1}
where $s_{1,2}$ is the jet invariant mass squares compared with the terms of order unity. The last term in (1) is caused by the absence of radiative corrections in our analysis. The angles $\theta_i$ of particle emission to the corresponding projectile direction of motion is assumed to be of the order (see Fig.2)

$$\frac{m_i}{\sqrt{s}} \ll \theta_i \sim \frac{\sqrt{s_i}}{\sqrt{s}} \ll 1,$$

(2)

where $m_i$ is the typical mass of the jet particle.

In this approach we can consider initial particles (having the 4-momenta $p_1, p_2$) as massless and use the Sudakov parameterization of 4-momenta of any particle of the problem

$$q_i = \alpha_i p_2 + \beta_i p_1 + q_{i\perp},$$

(3)

$$q_{i\perp} p_{1,2} = 0, \quad q_{i\perp}^2 = -\vec{q}_i^2 < 0.\)  

The Sudakov parameters $\beta_i$ are the quantities of order of unity for the momenta of the particles belonging to the jet1 and obeying the conservation law $\sum_{jet1} \beta_i = 1$, whereas the components of the jet1 particle momenta along the four momentum $p_2$ are small positive numbers which can be determined from the on mass shell conditions of the jet1 particles

$$q_i^2 = s \alpha_i \beta_i - \vec{q}_i^2 = 0, \quad \alpha_i = \frac{\vec{q}_i^2}{(s \beta_i)} \ll 1.$$

The same is valid for the 4-momenta of the particles belonging to the jet2, namely, $\alpha_j \sim 1, \sum_{jet2} \alpha_j = 1, \beta_j = \vec{q}_j^2/(s \alpha_j) \ll 1.$

Among the large amount of Feynman diagrams (FD), describing the process in the lowest (Born) order of perturbation theory (PT) (tree approximation), only ones survive (i.e., give

FIG. 3: Feynman diagrams describing a) the subprocess $\gamma^* e^- \to \gamma \gamma e^-$ and pair production $\gamma^* e \to e\bar{a}a$ subprocess by the bremsstrahlung b) and double photon c) mechanisms.
a contribution to the cross section which do not decrease with increasing $s$) which have a photonic t-channel one-particle state.

It is known \cite{3} that the matrix elements of the peripheral processes have a factorized form and the cross section can be written in terms of the so-called impact factors, each of which describe the subprocess of interaction of the internal virtual photon with one of the initial particles to produce a jet moving in the direction close to this projectile momentum. So the problem can be formulated in terms of computation of impact factors. For processes with initial photons with definite state of polarization described in terms of Stoke’s parameters we construct the relevant chiral matrices from bilinear combinations of chiral amplitudes. The last step consists in the construction of differential cross sections.

The matrix element, which corresponds to the main (”leading”) contribution, to the cross section, has the form

$$M = iJ_1^\mu \frac{g_{\mu\nu}}{q^2} J_2^\nu,$$

(4)

where $J_1^\mu$ and $J_2^\nu$ are the currents of the upper (associated with jet1) and lower blocks of the relevant Feynman diagram, respectively, and $g_{\mu\nu}$ is the metric tensor. The current $J_1^\mu$ describes the scattering of an incoming particle of momentum $p_1$ with a virtual photon and subsequent transition to the first jet (similar for $J_2^\nu$). Matrix elements (4) can be written in the form (see the appendices in \cite{3})

$$M = 2i \frac{s}{q^2} I_1 I_2,$$

(5)

$$I_1 = \frac{1}{s} J_1^\mu p_2^\mu, \quad I_2 = \frac{1}{s} J_2^\nu p_1^\nu.$$ Really, it follows from the Gribov representation of the metric tensor,

$$g^{\mu\nu} = \frac{2}{s} (p_2^\mu p_1^\nu + p_2^\nu p_1^\mu) + g_\perp^{\mu\nu} \approx \frac{2}{s} p_2^\mu p_1^\nu.$$ (6)

Invariant mass squares of jets can also be expressed in terms of the Sudakov parameters of the exchanged photon

$$q = \alpha p_2 + \beta p_1 + q_\perp, \quad (q + p_1)^2 = s_1 = -\vec{q}^2 + s\alpha, $$

$$(-q + p_2)^2 = s_2 = -\vec{q}^2 - s\beta, \quad q^2 = s\alpha\beta - \vec{q}^2 \approx -\vec{q}^2.$$ (7)

Here and below we mean by the symbol $\approx$ the equation with neglect the terms which do not contribute in the limit $s \to \infty$. 

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The singularity of the matrix element (5) at $\vec{q} = 0$ is fictitious (excluding the elastic scattering). Really one can see that it cancels due to the current conservation

$$q_\mu J_{\mu}^1 \approx (\alpha p_2 + q_\perp) J_{\mu}^1 = 0, \quad p_{2\mu} J_1^{\mu} = \frac{s}{s\alpha} \vec{q} \vec{J}_1, \quad (8)$$

$$q_\nu J_{\nu}^2 \approx (\beta p_1 + q_\perp) J_{\nu}^2 = 0, \quad p_{1\nu} J_2^{\nu} = \frac{s}{s\beta} \vec{q} \vec{J}_2. \quad (9)$$

We arrive at the modified form of the matrix element of peripheral process

$$M(a(p_1, \eta_1) + b(p_2, \eta_2)) \rightarrow jet_{1\lambda_1} + jet_{2\lambda_2} = i(4\pi\alpha) \frac{n_1 + n_2}{s} \frac{2s}{q^2} \eta_1^{\mu_1} \eta_2^{\mu_2}, \quad (10)$$

where $\eta_i$ describe the polarization states of the projectile $i = a, b$; $\lambda_i$ describes the polarization states of participants of its initiated jet. The numbers of QED vertices in the upper and lower blocks of FD (see Fig.1) are denoted by $n_{1,2}$.

We give here two alternative forms for the matrix elements $m_{1,2}$ of the subprocesses $\gamma^*(q) + a(p_1, \eta_1) \rightarrow jet_{1(\lambda_1)}$ and $\gamma^*(q) + b(p_2, \eta_2) \rightarrow jet_{2(\lambda_2)}$

$$m_{1\lambda_1}^{\eta_1} = \frac{\vec{q} \vec{J}_{1\lambda_1}}{s_1 + q^2}, \quad (11)$$

$$m_{1\lambda_1}^{\eta_1} = \frac{1}{s} p_{2\mu} J_{1\lambda_1}^{\eta_\mu}, \quad (12)$$

and the similar expressions for the lower block. We use the second representation (12). The form (11) can be used as a check of validity of gauge invariance, namely turning the matrix elements to zero in the limit $\vec{q} \rightarrow 0$.

A remarkable feature of the peripheral processes – is their differential cross sections do not depend on the total center of mass energy $\sqrt{s}$. To see this property, let us first rearrange the phase volume $d\Phi$ of the final two-jet kinematics state to a more convenient form

$$d\Phi = (2\pi)^4 \delta^4(p_1 + p_2 - \sum_i p_i^{(1)} - \sum_j p_j^{(2)}) dF^{(1)} dF^{(2)} = (2\pi)^4 d^4q \delta^4(1) \delta^4(2) dF^{(1)} dF^{(2)}, \quad (13)$$

$$\delta^4(1) = \delta^4(p_1 + q - \sum_i p_i^{(1)}), \quad \delta^4(2) = \delta^4(p_2 - q - \sum_j p_j^{(2)}),$$

$$dF_{(1,2)} = \prod_i \frac{d^3 p_i^{(1,2)}}{2\pi^2 \varepsilon_i^{(1,2)} (2\pi)^3}.$$

Using Sudakov’s parameterization for the transferred 4-momentum $q$ phase volume

$$d^4q = \frac{s}{2} d\alpha d\beta d^2 q_\perp = \frac{1}{2s} ds_1 ds_2 d^2 q_\perp, \quad (14)$$
with \( s_{1,2} \) the invariant mass squares of the jets, we put the phase volume in the factorized form

\[
d\Phi = \frac{(2\pi)^4}{2s} d^2q_\perp ds_1 dF^{(1)}(\delta_{(1)}) ds_2 dF^{(2)}(\delta_{(2)}).
\] (15)

Using the modified form of the matrix element and the phase volume for the peripheral process cross section in the case of polarized initial particles (photons or electrons), we have

\[
d\sigma^{\eta_1\eta_2} = \alpha^{n_1+n_2} \pi^2 (4\pi)^2 d^2q_\perp \Phi^{\eta_1}_1(\vec{q}) \Phi^{\eta_2}_2(\vec{q}),
\] (16)

with the impact factors \( \Phi^{\eta}_i(\vec{q}) \) in the form

\[
\Phi^{\eta}_i(\vec{q}) = \int ds_i \sum_{\lambda_j} |m^{\eta}_{i\lambda_j}|^2 dF^1_{\delta(i)} , \quad i = 1, 2.
\] (17)

The matrix elements with the definite chiral states of all particles \( m^{\eta}_{i(\lambda)} \), where the subscript \( \lambda \) denotes the set of chiral parameters of the final state, are calculated and listed below.

In the case of initial polarized photons the description in terms of Stoye’s parameters \( \xi_1, \xi_2, \xi_3 \), \( \xi_1^2 + \xi_2^2 + \xi_3^2 \leq 1 \) is commonly used. The matrix element squared in r.h.s. (17) must be replaced by [4]

\[
T_\gamma = \text{Sp}(\mathcal{M}\rho) = \frac{1}{2} \text{Sp} \left( \begin{array}{cc} m^{++} & m^{+-} \\ m^{-+} & m^{--} \end{array} \right) \begin{pmatrix} 1 + \xi_2 & i \xi_1 - \xi_3 \\ -i \xi_1 - \xi_3 & 1 - \xi_2 \end{pmatrix},
\] (18)

with the spin matrix \( \mathcal{M} \) elements

\[
m^{++} = \sum_{\lambda} |m^{++}_{(\lambda)}|^2, \quad m^{+-} = \sum_{\lambda} m^{++}_{(\lambda)}(m^{+-}_{(\lambda)})^*,
\] (19)

\[
m^{-+} = \sum_{\lambda} |m^{-+}_{(\lambda)}|^2, \quad m^{--} = (m^{+-})^*.
\]

We choose \( \lambda = +1 \) for the initial fermion

\[
T_e = \sum_{\lambda} |m^{+}_{\lambda}|^2.
\] (20)

The cross sections \( d\sigma_{n_1,n_2} \) of the process of type 2 \( \rightarrow n_1 + n_2 \) with production of two jets

\[
a(p_1, \eta_1) + b(p_2, \eta_2) \rightarrow a_1(r_1, \lambda_1) + \cdots + a_{n_1}(r_{n_1}, \lambda_{n_1}) + b_1(q_1, \sigma_1) + \cdots + b_{n_2}(q_{n_2}, \sigma_{n_2}),
\] (21)
where energy fractions \( x_1, \ldots, x_n, \sum x_i = 1 \) and transversal components of momenta \( \vec{r}_1, \ldots, \vec{r}_n; \sum \vec{r}_i = \vec{q} \) of jet \( a \) and similar quantities \( y_i, \vec{q}_i, \sum y_i = 1, \sum \vec{q}_i = -\vec{q} \) for the other jet \( b \), have the form

\[
d\sigma_{22} = \frac{\alpha^4}{2^2\pi^4} T_2^{(1)} T_2^{(2)} \frac{d^2q}{(q^2)^2} d^2r_1 d^2q_1 \frac{dx_1 dy_1}{x_1 x_2 y_1 y_2},
\]

\[
d\sigma_{23} = \frac{\alpha^5}{2^4\pi^6} T_2^{(1)} T_3^{(2)} \frac{d^2q}{(q^2)^2} d^2r_1 d^2q_1 d^2q_2 \frac{dx_1 dy_1 dy_2}{x_1 x_2 y_1 y_2 y_3},
\]

\[
d\sigma_{33} = \frac{\alpha^6}{2^6\pi^8} T_3^{(1)} T_3^{(2)} \frac{d^2q}{(q^2)^2} d^2q_1 d^2q_2 d^2r_1 d^2r_2 \frac{dx_1 dx_2 dy_1 dy_2}{x_1 x_2 x_3 y_1 y_2 y_3}.
\]

**B. Subprocesses** \( \gamma^*\gamma \rightarrow e^+e^-, \pi^+\pi^- \)

Let us consider first the contribution to the photon impact factor from the lepton pair production subprocess

\[
\gamma(k_1, \eta) + \gamma^*(q) \rightarrow e^-(q_-, \lambda) + e^+(q_+, -\lambda).
\]

The matrix element of the subprocess has the form (we suppress the factor \( 4\pi\alpha \))

\[
m_{1\alpha}^{\mu} = -\bar{u}_\lambda(q_-) \left[ \varepsilon^\eta \hat{q}_- - \frac{k_1}{\kappa_{1-}} \gamma^\mu + \gamma^\mu \hat{q}_+ + \frac{k_1}{\kappa_{1+}} \varepsilon^\eta \right] v_\lambda(q_+) , \quad \bar{u}_\lambda = \bar{u}_{\omega_-\lambda} , \quad v_\lambda = \omega_-\lambda v .
\]

We imply all the particles to be massless. A definite chiral state initial photon polarization vector has the form

\[
\varepsilon_1^\lambda = N_1 [\hat{q}_- \hat{k}_1 \omega_- - \hat{k}_1 \hat{q}_- \omega_+],
\]

where

\[
N_1^2 = \frac{2}{s_1 \kappa_+ \kappa_-} , \quad s_1 = 2q_+ q_- , \quad \kappa_{1\pm} = 2k_1 q_{\pm} .
\]

Chiral amplitudes \( m_{1\lambda}^{\eta} = (1/s) m_{1\lambda}^{\mu} p_{2\mu} \) have the form

\[
m_{1+}^+ = -\frac{N_1}{s} \bar{u}_q \hat{q}_+ \hat{q}_2 \omega_+ v , \quad m_{1-}^+ = -\frac{N_1}{s} \bar{u}_p \hat{q}_2 \hat{q}_- \omega_- v ,
\]

\[
m_{1-}^- = -\frac{N_1}{s} \bar{u}_q \hat{q}_+ \hat{q}_2 \omega_- v , \quad m_{1+}^- = -\frac{N_1}{s} \bar{u}_p \hat{q}_2 \hat{q}_- \omega_+ v .
\]
The elements of the spin-matrix $\mathcal{M}$ in the case of lepton pair production are,

$$m_{e^+ e^-}^{++} = m_{e^+ e^-}^{-} = \frac{2q^2}{q^2_+ q^2_-} x_+ x_- (x_+ + x_-^2),$$

$$m_{e^+ e^-}^+ = (m_{e^+ e^-}^{-})^* = -\frac{4q^2}{q^2_+ q^2_-} (x_+ x_-)^2 e^{2i\theta},$$

$x_\pm$ are the energy fractions carried out by pair components, $x_+ + x_- = 1$ and $\theta$ is the angle between two Euclidean vectors $\vec{q} = \vec{q}_- + \vec{q}_+$ and $\vec{Q} = x_+ \vec{q}_- - x_- \vec{q}_+$.

In the case of charged pion pair production

$$\gamma(p_1, e_1^\mu) + \gamma^*(q) \rightarrow \pi^+(q_+) + \pi^-(q_-)$$

we have

$$m_1^\mu = \frac{1}{s} \varepsilon_1^\mu p_2^\nu m_\nu = \frac{1}{p_1 q_-} \varepsilon_1^\mu q_- + \frac{1}{p_1 q_+} \varepsilon_1^\mu q_+ - \frac{2}{s} (\varepsilon_1^\mu p_2).$$

Using the photon polarization vector written as

$$\varepsilon_1^\mu = N_1[(q_+ p_1) q_- - (q_- p_1) q_+ + i\eta \varepsilon_{\alpha\beta\gamma\delta} q^\alpha q^\beta q^\gamma q_1^\delta],$$

we obtain the chiral amplitude of the pion pair production process (we define $(p_1 p_2 q_- q_+) = \varepsilon_{\alpha\beta\gamma\delta} p_1^\alpha p_2^\beta q_+^\gamma q_-^\delta = (s/2)[q_- q_+]$)

$$m_1^\mu = -N_1[\vec{Q} q + i\eta[\vec{Q}, \vec{q}]_z] = -N_1[\vec{q}] \vec{Q} e^{i\eta\theta}, \quad \theta = \vec{q} \vec{Q}.\quad (33)$$

where we imply the $z$ axis direction along the photon 3-vector and use the relation $[\vec{q}^-, \vec{q}^+]_z = [\vec{Q}, \vec{q}]_z$. For the pion chiral matrix we have

$$m_{\pi^+ \pi^-}^{++} = m_{\pi^+ \pi^-}^{-} = \frac{2q^2}{q^2_+ q^2_-} (x_+ x_-)^2, \quad (34)$$

$$m_{\pi^+ \pi^-}^+ = (m_{\pi^+ \pi^-}^{-})^* = \frac{2q^2}{q^2_+ q^2_-} (x_+ x_-)^2 e^{2i\theta}. \quad (35)$$

For the two-pair production process

$$\gamma_1(p_1, \vec{q}_1) + \gamma_2(p_2, \vec{q}_2) \rightarrow a(q_-) + \bar{a}(q_+) + b(p_-) + \bar{b}(p_+),$$

$$q_\pm = \alpha \pm p_2 + x \pm p_1 + q_{\perp}; \quad p_\pm = y \pm p_2 + \beta \pm p_1 + p_{\perp},$$

the differential cross section (assuming that the pair $a\bar{a}$ moves along the photon 1 direction and the pair $b\bar{b}$ moves along the photon 2 direction) has the form (22) with

$$T^{(1)} = \frac{q^2}{q^2_+ q^2_-} (x_+ x_-)^2 [1 - \xi_3 \cos(2\theta) + \xi_1 \sin(2\theta)], \quad \text{for } \pi^+, \pi^-, \quad (36)$$

$$T^{(1)} = \frac{q^2}{q^2_+ q^2_-} (x_+ x_-) \{x^2_+ + x^2_- + 2x_+ x_- [\xi_3 \cos(2\theta) + \xi_1 \sin(2\theta)]\}, \quad \text{for } e^+, e^- \quad (37)$$
and the similar expression for $T^{(2)}$. We remind that the formulae obtained are valid at a large, compared to masses of particles, transverse component of jet particles

$$q_-^2 \sim q_+^2 \sim p_+^2 \sim p_-^2 \gg m^2, \quad \vec{q} = \vec{q} - \vec{q}_-; \quad \vec{p}_+ = -\vec{q} - \vec{p}_-,$$

(38)

and finite energy fractions $x_\pm \sim y_\pm \sim 1$, which corresponds to the emission angles of jet particles $\theta_i = |\vec{q}_i|/(x_i\varepsilon) \gg m/\varepsilon$ that are considerably larger than the mass to energy ratio.

C. Subprocesses $\gamma^*\gamma \rightarrow e^+e^-\gamma, \pi^+\pi^-\gamma$

Here and below for subprocesses of type $2 \rightarrow 3$ we restrict ourselves to calculating the chiral amplitudes and checking their gauge invariance properties.

The subprocess

$$\gamma(k, \lambda) + \gamma^*(q) \rightarrow e^+(q_+, -\lambda_-) + e^-(q_-, \lambda_-) + \gamma(k_1, \lambda_1),$$

is described by 6 FD. A standard calculation of chiral amplitudes $m^\lambda_{\lambda_1\lambda_-}$ leads to

$$m^+_{++} = \frac{s_1 NN_1}{s} \hat{u}(q_-) \hat{q}_+ \hat{p}_2 \omega_+ v(q_+) = (m^-_-)^*,$$

$$m^+_{+-} = \frac{s_1 NN_1}{s} \hat{u}(q_-) \hat{p}_2 \hat{q}_- \omega_- v(q_+) = (m^-_+)^*,$$

$$m^-_{++} = \frac{NN_1}{s} \hat{u}(q_-) A^+_{++} \omega_+ v(q_+) = (m^+_-)^*,$$

$$m^-_{+-} = \frac{NN_1}{s} \hat{u}(q_-) A^+_{++} \omega_- v(q_+) = (m^+_-)^*,$$

(39)

with $A^+_{-+}(k, k_1) = A^+_{++}(-k_1, -k)$

$$N^2 = \frac{2}{s_1 \kappa_- \kappa_+}, \quad N^2_1 = \frac{2}{s_1 \kappa_{1+} \kappa_{1-}}, \quad s_1 = 2q_+ q_-, \quad \kappa_\pm = 2k q_\pm, \quad \kappa_{1\pm} = 2k_1 q_\pm$$

(40)

and a rather cumbersome expression for $A^+_{++}$

$$A^+_{++} = \frac{s_1}{(q_+ - q)^2} \hat{k} \hat{q}_+ \hat{k}_1 (-\hat{q}_+ + \hat{q}) \hat{p}_2 - \hat{q}_+ (\hat{q}_- - \hat{k}) \hat{p}_2 (\hat{q}_+ + \hat{k}_1) \hat{q}_- - \frac{s_1}{(q_+ - q)^2} \hat{p}_2 (\hat{q}_- - \hat{q}) \hat{k} \hat{q}_- \hat{k}_1.$$  

(41)

Substituting

$$\hat{p}_2 \approx \frac{1}{\alpha}(\hat{q} - \hat{q}_\perp) = \frac{s}{s\alpha} [\hat{q}_+ + \hat{k}_1 + (\hat{q}_- - \hat{k}) - \hat{q}_\perp],$$

1 In paper [6] formula 37 contains a misprint in the sign of $\xi_3^{(1,2)}$. 

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in the second term of r.h.s. (41) we have

\[ A^+ = -\alpha s_1 \kappa_1 \left[ \frac{x_+}{(q_+ - q)^2} + \frac{1}{s_1 \alpha} \dot{k} - s_1 \kappa_- \left[ \frac{x_-}{(q_- - q)^2} + \frac{1}{s_1 \alpha} \dot{k}_1 \right] \right. \]

\[ + \frac{s_1}{(q_+ - q)^2} \dot{k} q_+ \dot{k}_1 \dot{p}_2 + \frac{s_1}{(q_- - q)^2} \dot{p}_2 q_1 \dot{k}_1 + \frac{s_1}{s_1 \alpha} \dot{q}_+ (\dot{q}_- - \dot{k}) \dot{q}_- (\dot{q}_+ + \dot{k}_1) \dot{q}_- \]  

(42)

with

\[ (q_+ - q)^2 = -q^2 + 2q q_\pm - s \alpha x_\pm, \quad s \alpha = \frac{k^2_{\perp}}{x_1} + \frac{q^2_{\perp}}{x_-} + \frac{q^2_{\perp}}{x_+}, \]

(43)

\[ x_1 + x_- + x_+ = 1, \quad \kappa_\pm = \frac{q^2_{\perp}}{x_\pm}, \quad \kappa_1 = \frac{1}{x_1 x_\pm} (x_1 q_\pm - x_\pm k_1)^2. \]

A gauge property (the chiral amplitudes must vanish as \( \vec{q} \to 0 \)) can be seen explicitly.

A further procedure of constructing the chiral matrix is straightforward and can be performed in terms of simple traces. We will not touch it here.

Consider the subprocess

\[ \gamma(k, \lambda) + \gamma^*(q) \to \pi^+(q_+) + \pi^-(q_-) + \gamma(k_1, \lambda_1). \]

There are 12 FD describing rather a cumbersome expression for the matrix element. It can be considerably simplified when using the modified expressions for the photon polarization vectors in the form (8)

\[ \varepsilon^\lambda_\mu(k) = \frac{N}{2} S p \gamma_\mu \dot{q}_- \dot{q}_+ \dot{k}_1 \omega_\lambda, \quad \varepsilon^\lambda_1(k_1) = \frac{N_1}{2} S p \gamma_\mu \dot{q}_- \dot{q}_+ \dot{k}_1 \omega_\lambda \]

(44)

with the same expressions for \( N, N_1 \) as in the case of the \( \gamma \gamma^* \to e^+ e^- \gamma \) subprocess. Polarization vectors chosen in such a form satisfy Lorenz condition \( \varepsilon(k) k = 0, \varepsilon(k_1) k_1 = 0 \) and gauge condition \( \varepsilon(k) q_\pm = \varepsilon(k_1) q_\pm = 0 \).

The matrix element has the (we lost the Bose symmetry at this stage) form

\[ m^\lambda_\lambda = \frac{1}{s_2} \rho^\mu_\nu \varepsilon^\nu(k) \varepsilon^\mu(k_1) O_{\rho \mu \sigma} \]

\[ = \frac{4 x_-}{(q_- - q)^2} \left[ \frac{\varepsilon_1 q_+ (\varepsilon q)}{\kappa_1} - \frac{\varepsilon q_1 (\varepsilon q_+)}{\kappa_+} \right] + \frac{4 (\varepsilon q_1 p_2) (\varepsilon q_1)}{s \kappa_1} - \frac{4 (\varepsilon_1 q_1 p_2) (\varepsilon q_1)}{s \kappa_+} \]

(45)

where we imply \( \varepsilon = \varepsilon^\lambda_\lambda, \varepsilon_1 = \varepsilon^\lambda_1 \) and \( x_\pm = 2 p_2 q_\pm / s, \quad x_1 = 2 p_2 k_1 / s \) where \( x_1 + x_- + x_1 = 1 \).

For \( \lambda_1 = \lambda \) we have

\[ m^\lambda_\lambda = s_1 N N_1 [A_1 + i \lambda B_1], \quad A_1 = -\vec{Q} \vec{q}, \quad B_1 = [\vec{Q} \vec{q}]_z. \]

(46)
For the case of opposite chiralities we have

\[ m^\lambda_{\lambda} = s_1N N_1 [A + i\lambda B], \]

\[ A = -\vec{Q}\vec{q} + \frac{1}{2x_1 x_- x_+} [\vec{Q}^2 \vec{k}_1^2 - \vec{q}^2 (x_1\vec{q}_+ - x_+\vec{k}_1)^2 - \vec{q}^2 (x_1\vec{q}_- - x_-\vec{k}_1)^2] \times \left( \frac{x_+}{(q_+ - q)^2} - \frac{x_-}{(q_+ - q)^2} \right), \]

\[ B = \left( \frac{x_+}{(q_+ - q)^2} + \frac{x_-}{(q_+ - q)^2} \right) (s\alpha [\vec{q}_-\vec{q}_+]_z - s\alpha_-[\vec{q}\vec{q}_+]_z + s\alpha_+ [\vec{q}\vec{q}_-]_z + 2[\vec{q}_-\vec{q}_+]_z - [\vec{Q}\vec{q}]_z, \]

\[ s\alpha_\pm = \frac{\vec{q}_\pm^2}{x_\pm}, \quad s\alpha = \frac{\vec{k}_1^2}{x_1} + s\alpha_+ + s\alpha_. \]

We can see that the Bose-symmetry is restored.

**D. Subprocesses** \( e\gamma^* \to e\gamma; e + \gamma + \gamma \)

Consider first the Compton subprocess \(^2\)

\[ \gamma^*(q) + e(p, \lambda_1) \to \gamma(k, \lambda) + e(p', \lambda_1). \]

For the chiral matrix elements we have (we chose \( \lambda_1 = +1 \))

\[ m^\lambda_\lambda = N \frac{\bar{u}(p')[-\hat{p}\omega_\lambda(\hat{p}' + \vec{k})\hat{p}_2 - \hat{p}_2(\hat{p}_1 - \vec{k})\omega_{-\lambda}]\omega_+ u(p)}, \]

\[ m^\lambda_+ = -N \frac{\bar{u}(p')\hat{p}\hat{q}\omega_2 u(p)}, \quad m^\lambda_- = -N \frac{\bar{u}(p')\hat{p}_2\hat{q}\omega_+ u(p)}. \]

The sum of modulo square of the matrix elements is

\[ T_e = \sum_\lambda |m^\lambda_\lambda|^2 = 2 \frac{\vec{q}_\pm^2}{\kappa \kappa'} [1 + (1 - x)^2], \]

with

\[ \kappa = 2kp = \frac{\vec{k}_1^2}{x}, \quad \kappa' = 2kp' = \frac{1}{x(1 - x)} (\hat{p}'x - \vec{k}(1 - x))^2, \]

and \( x = 2kp_2/2p_1p_2, 1 - x \) are the energy fractions of photon and electron in the final state.

Consider now the double Compton subprocess (see Fig. 3a)

\[ e(p, \eta) + \gamma^*(q) \to e(p', \eta) + \gamma(k_1, \lambda_1) + \gamma(k_2, \lambda_2). \]

\(^2\) The case of real initial photons was considered in paper \([7]\).
The chiral matrix elements \( m_{A_1A_2} \) are

\[
m_{A_+} = (m_{A-})^* = -\frac{s_1 N_1 N_2}{s} \bar{u}(p') \hat{p} \hat{p}_2 \omega_+ u(p); \quad (52)
\]

\[
m_{A-} = (m_{A+})^* = -\frac{s_1 N_1 N_2}{s} \bar{u}(p') \hat{p}_2 \hat{p}' \omega_+ u(p);
\]

\[
m_{A_+} = (m_{A-})^* = \frac{N_1 N_2}{s} \bar{u}(p') A_{A+}^+ \omega_+ u(p);
\]

\[
m_{A_-} = (m_{A+})^* = \frac{N_1 N_2}{s} \bar{u}(p') A_{A-}^+ \omega_+ u(p),
\]

with \( A_{A+}^+(k_1, k_2) = A_{A-}^-(k_2, k_1) \) and

\[
A_{A-}^+(k_1, k_2) = \frac{s_1}{(p' - q)^2} \hat{p}_2 (\hat{p}' - \hat{q}) \hat{k}_1 \hat{k}_2 \hat{\hat{p}} + \hat{p} (\hat{p}' + \hat{k}_1) \hat{\hat{p}}_2 (\hat{p} - \hat{k}_2) \hat{p}' + \frac{s_1}{(p + q)^2} \hat{k}_1 \hat{\hat{p}}_2 (\hat{p} + \hat{q}) \hat{p}_2,
\]

with

\[
s_1 = 2pp', \quad N_i^2 = \frac{2}{s_1 \kappa_i \kappa_i'}, \quad \kappa_i = 2pk_i, \quad \kappa_i' = 2p'k_i. \quad (54)
\]

To see the gauge invariance property of two last amplitudes, we make a substitution \( p_2 = (q - q_\perp) / \alpha_q \) in the second term of r.h.s. and arrive at the form

\[
A_{A-}^+(k_1, k_2) = ss_1 \kappa_i' \left( \frac{x'}{(p' - q)^2} + \frac{1}{s \alpha_q} \right) \hat{k}_2 + ss_1 \kappa_2 \left( \frac{1}{(p + q)^2} - \frac{1}{s \alpha_q} \right) \hat{k}_1 + \frac{s_1}{(p + q)^2} \hat{k}_1 \hat{\hat{p}}_2 \hat{q}_\perp \hat{p}_2 - \frac{s_1}{(p - q)^2} \hat{p}_2 \hat{q}_\perp \hat{k}_1 \hat{\hat{p}}_2 - \hat{p} (\hat{p}' + \hat{k}_1) \hat{\hat{q}}_\perp (\hat{p} - \hat{k}_2) \hat{p}' \frac{s}{s \alpha_q}. \quad (55)
\]

We can verify that this expression turns to zero at \( q = 0 \). Really, we can use

\[
(p' - q)^2 = -\vec{q}^2 + 2\vec{p}' \vec{q} - sx' \alpha_q, \quad (p + q)^2 = -\vec{q}^2 + s \alpha_q,
\]

\[
\alpha_q = \alpha' + \alpha_1 + \alpha_2, \quad x' + x_1 + x_2 = 1, \quad s \alpha' = \frac{(s \vec{p}')^2}{x'}, \quad s \alpha_i = \frac{\vec{k}_i^2}{x_i},
\]

\[
\kappa_i = s \alpha_i, \quad \kappa_i' = \frac{1}{x_i/x_i'} (\vec{k}_i x_i' - \vec{p}' x_i)^2.
\]

A further strategy is similar to the one mentioned above (45).

E. Subprocesses \( e\gamma^* \to e\pi^+\pi^-, e\mu^+\mu^- \)

The matrix element of the pion pair production subprocess

\[
e(p, \eta) + \gamma^*(q) \to \pi^+(q_+) + \pi^-(q_--) + e(p', \eta)
\]
can be written in the form
\[ m^\eta = \bar{u}(p') [\hat{B} + \hat{D}] \omega_\eta u(p), \] (57)

where bremsstrahlung mechanism contribution is (see Fig.3b)
\[ \hat{B} = \frac{1}{q_1^2} \left[ B \hat{q}_1 + \frac{1}{s(p + q)^2} \hat{q}_1 \hat{p}_2 - \frac{1}{s(p' - q)^2} \hat{p}_2 \hat{q}_1 \right], \quad q_1 = q_+ + q_- \quad q_2 = p' - p_1; \quad (58) \]
\[ \hat{D} = \frac{1}{q_2^2} \left[ D(2\hat{q}_- + \hat{q}_2) - 2 \frac{x_+}{(q - q_-)^2} \hat{q}_2 + \frac{2(\hat{q}_2^2 - 2\hat{q}_-^2)}{s(q - q_-)^2} \hat{p}_2 \right]. \]

For the squares of modulo of the chiral amplitudes, which enter in (23,24), we have
\[ T^{(\pi)}_3 = |m^+|^2 = Sp(\hat{p}'(\hat{B} + \hat{D}) \hat{p}(\hat{B} + \hat{D}) \omega_+) \], (59)
with $B$ and $D$ specified below [62].

For the subprocess of the muon pair production
\[ e(p, \eta) + \gamma^*(q) \to \mu^+(q_+) + \mu^-(q_-) + e(p', \eta). \] (60)
The bremsstrahlung and two-photon mechanisms must be taken into account (see Fig.3b,c)
\[ m^+_\lambda = \frac{1}{q_1^2} \bar{u}(p') B_{\mu} \omega_+ u(p) \times \bar{u}(q_-) \gamma^\mu \omega_\lambda v(q_+) + \frac{1}{q_2^2} \bar{u}(p') \gamma_\nu \omega_+ u(p) \bar{u}(q_-) D_{\nu} \omega_\lambda v(q_+), \] (61)
with double photon mechanism contribution (not considered in paper [2])
\[ D_{\nu} = D\gamma_\nu + \frac{1}{s(q - q_+)^2} \gamma_\nu \hat{q}_2 - \frac{1}{s(q - q_-)^2} \hat{p}_2 \hat{q}_\nu \]
and bremsstrahlung mechanism one
\[ B_{\mu} = B \gamma_\mu - \frac{1}{s(p' - q)^2} \hat{p}_2 \hat{q}_\gamma_\mu + \frac{1}{s(p + q)^2} \gamma_\mu \hat{q}_2, \]
with
\[ B = \frac{x'}{(p' - q)^2} + \frac{1}{(p + q)^2}, \quad D = \frac{x_-}{(q - q_-)^2} - \frac{x_+}{(q - q_+)^2}, \]
\[ x_\pm = \frac{2p_2 q_\pm}{s}, \quad x' = \frac{2p_2 p'}{s}, \quad x_+ + x_- + x' = 1. \]
To perform the conversion in the Lorentz indices $\mu, \nu$ in (66), one can use the projection operators. For the case of equal chiralities $\eta = \lambda = +1$ we choose the projection operator as
\[ P_+ = \frac{\bar{u}(p) \hat{q}_+ \omega_+ u(q_-)}{\bar{u}(p) \hat{q}_+ \omega_+ u(q_-)}. \] (62)
Inserting it and using the relation $\omega_+ u(p)\bar{u}(p) = \omega_+ \hat{p}$, we obtain

$$m_+^\pm = \frac{-2}{\bar{u}(p)\hat{q}_+ \omega_+ u(q_-)} \bar{u}(p') \left[ \left( \frac{D}{q_2^2} + \frac{B}{q_1^2} \right) \hat{q}_+ \hat{p}\hat{q}_- \hat{p}_2 \quad + \frac{\hat{p}_2 \hat{q}_+ \hat{q}_- \hat{p}}{s} \left( \frac{1}{q_2^2(q - q_-)^2} - \frac{1}{q_1^2(p + q)^2} \right) \right] \omega_+ v(q_+)$$

$$= \frac{-2}{\bar{u}(p)\hat{q}_+ \omega_+ u(q_-)} \bar{u}(p') A_+^\pm \omega_+ v(q_+). \quad (63)$$

For the case of opposite chiralities $\eta = -\lambda = +1$ we use the projection operator

$$P_- = \frac{\bar{u}(p)\omega_- u(q_-)}{\bar{u}(p)\omega_- u(q_-)}. \quad (64)$$

Similar calculations lead to the result

$$m_-^\pm = \frac{2}{\bar{u}(p)\omega_- u(q_-)} \bar{u}(p') \left[ \left( \frac{D}{q_2^2} + \frac{B}{q_1^2} \right) 2(pq_-) + \frac{\hat{p}_2 \hat{q}_+ \hat{q}_- \hat{p}}{s} \left( \frac{1}{q_2^2(q - q_-)^2} + \frac{1}{q_1^2(p - q_-)^2} \right) \right] \omega_+ v(q_+)$$

$$- \frac{\hat{p}_2 \hat{q}_+ \hat{q}_- \hat{p}}{s} \left( \frac{1}{q_2^2(q - q_-)^2} + \frac{1}{q_1^2(p - q_-)^2} \right) \omega_+ v(q_+)$$

$$= \frac{2}{\bar{u}(p)\omega_- u(q_-)} \bar{u}(p') A^-_+ \omega_+ v(q_+). \quad (65)$$

The property of $A_+^+, A_-^+$ tending to zero as $|\hat{q}| \to 0$ is explicitly seen from (71, 72).

For the sum of squares of chiral amplitudes entering (23,24), one has

$$T^{(\mu)}_3 = \sum |m_\lambda|^2 = \frac{1}{(pq_+)(q_-)} Sp(\hat{p}'A_+ q_+ \tilde{A}_+^+ \omega_+) + \frac{2}{pq_-} Sp(\hat{p}'A_+ q_+ \tilde{A}_+^+ \omega_+). \quad (66)$$

A further strategy is straightforward.

**F. Subprocess $e\gamma^* \to e\bar{e}$**

Kinematics of subprocess is defined as

$$e(p, l_p) + \gamma^*(q) \to e(p_1, l_1) + e(p_2, l_2) + \bar{e}(p_+, t),$$

with $l_i, t = \pm$ the chiralities of initial and final fermions. Without loss of generality we can put below $l_p = +$. For the sum on chiral states of the modulo square of relevant matrix element we obtain:

$$\sum |M_{l_i l_+ 2l_2}|^2 = 2[|M_{++-}^+|^2 + |M_{+-+}^+|^2 + |M_{-++}^+|^2]. \quad (67)$$
Eight Feynman diagrams are relevant which form 4 gauge-invariant sets of amplitudes:

\[
M^+_{ii} = (4\pi\alpha)^{3/2} \left(-\frac{1}{s_1}\right) (\delta_{t_1}+\delta_{t_2}) \left[ \bar{u}^{t_2}(p_2)\gamma^\lambda u^t(p_+) \bar{u}^{t_1} A_{\lambda} u^t(p) + \bar{u}^{t_1}(p_1)\gamma^\sigma \bar{u}^+(p) \bar{u}^{t_2} B_{\sigma} u^t(p_+) \right] + \delta_{t_2} \left[ \bar{u}^{t_1}(p_1)\gamma_\eta u^t(p_+) \bar{u}^{t_2}(p_2) D_\eta u^t(p) + \bar{u}^{t_2}(p_2)\gamma_\delta \bar{u}^+(p) \bar{u}^{t_1}(p_1) C_\delta u^t(p_+) \right].
\]

(68)

Applying projection operators to provide the conversion on vector indices we have

\[
|M^+_{++}|^2 = \frac{(4\pi\alpha)^3}{2s_1^n p_{pp_+}} \left[ \frac{1}{p_2 p_{++}} \right] \frac{1}{2p_1 p_{++}} \frac{1}{4} \left[ Sp\hat{p}_1 m^{(1)}_{++} \hat{p}_+ (m^{(1)}_{++})^+ \right] + \frac{1}{p_+ p_{1}} \frac{1}{4} \left[ Sp\hat{p}_2 m^{(2)}_{++} \hat{p}_+ (m^{(2)}_{++})^+ \right] + \frac{1}{p_{1} p_+ p_{++}} \frac{1}{4} \left[ Sp\hat{p}_1 m^{(1)}_{++} \hat{p}_+ (m^{(1)}_{++})^+ \hat{p}_2 \hat{p}_+ \right];
\]

(69)

\[
|M^+_{+-}|^2 = \frac{(4\pi\alpha)^3}{2s_1^n p_{pp_2}} \frac{1}{4} \left[ Sp\hat{p}_1 m_{+-} \hat{p}_+ (m_{+-})^+ \right],
\]

\[
|M^+_{-+}|^2 = \frac{(4\pi\alpha)^3}{2s_1^n p_{pp_1}} \frac{1}{4} \left[ Sp\hat{p}_2 m_{-+} \hat{p}_+ (m_{-+})^+ \right];
\]

with

\[
m_{++} = \gamma_\sigma \hat{p}_p B_\sigma + A_\lambda \hat{p}_p B_\gamma,
\]

\[
m_{+-} = \gamma_\sigma \hat{p}_p C_\sigma + D_\eta \hat{p}_p \gamma_\eta,
\]

\[
m^{(1)}_{++} = A_\lambda \hat{p}_p B_\gamma + \gamma_\sigma \hat{p}_p B_\sigma,
\]

\[
m^{(2)}_{+-} = \gamma_\sigma \hat{p}_p C_\sigma + D_\eta \hat{p}_p \gamma_\eta,
\]

\[
A_\lambda = \frac{\hat{q}_+ (\hat{p}_+ - \hat{q}_-)}{(p_+ - q)^2} + \frac{\gamma_\gamma (\hat{p}_+ - \hat{q}_-) \hat{q}_+}{(q - p_+)^2},
\]

\[
B_\sigma = \frac{\hat{q}_+ (\hat{p}_+ - \hat{q}_-) \gamma_\gamma}{(p_+ - q)^2} + \frac{\gamma_\gamma (\hat{p}_+ - \hat{q}_-) \hat{q}_+}{(q - p_+)^2},
\]

\[
C_\sigma = \frac{\hat{q}_+ (\hat{p}_+ - \hat{q}_-) \gamma_\sigma}{(p_+ - q)^2} + \frac{\gamma_\gamma (\hat{p}_+ - \hat{q}_-) \hat{q}_+}{(q - p_+)^2},
\]

\[
D_\eta = \frac{\hat{q}_+ (\hat{p}_+ - \hat{q}_-) \gamma_\eta}{(p_+ - q)^2} + \frac{\gamma_\eta (\hat{p}_+ - \hat{q}_-) \hat{q}_+}{(q - p_+)^2}.
\]

(70)

(71)

**Conclusion**

In our paper [6], we wrote down the explicit expressions for the spin matrix elements \( M_{ij} \) for subprocesses of the type \( 2 \to 2 \), which are reviewed here. For the subprocesses of the type \( 2 \to 3 \), we formulated the algorithm of the calculation of spin matrix elements. We considered all possibilities of pair creation in the mentioned subprocesses as they were not completely considered in recent a work [2]. The gauge condition \( M_{ij}(q) \to 0 \) for \( |q| \to 0 \) is explicitly fulfilled in all cases. The subprocesses with the pions in the final state were also considered in the paper for the first time.

Radiative corrections to chiral amplitude was calculated only for some subprocesses of type \( 2 \to 2 [9] \).

The magnitude of the cross sections (22-24) is of the order \( \alpha^n/\mu^2 \gg \alpha^n/s \), \( n = 4, 5, 6 \) where \( \mu^2 = \max(s_1, s_2) \) is large enough to be measured, and does not depend on \( s \). The
strategy of calculation of cross section, using the helicity amplitudes of subprocesses $2 \rightarrow 3$, is described above and can be implemented to numerical programs which take into account details of experiments.

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