Rational Galaxy Structure and its Disturbance

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Abstract: Why is there little dust in elliptical galaxies? Here is a promising answer. Firstly, galaxies are rational. Rationality means that the density distribution of stars is proportional. Secondly, galaxy arms are linearly-shaped and irrational. The presence of arms is the disturbance to the rational disks and bars. Therefore, any disturbance to rational structure produces cosmic dust.

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1. Is Rational Gravity the Origin of Galaxies?

Scientists have fully proved that there exist only four forces among particles: electromagnetic, weak nuclear, strong nuclear, and gravitational. The nuclear forces are short-ranged while the electromagnetic force is long-ranged. Each of the three has two contradictory aspects of attracting and repelling and has generally no net effect in the macroscopic world due to offsetting effect. Gravitational force, however, has no contradictory aspects. Gravity has the only effect of attraction and can not offset itself. Because of this, the true origin of natural structure may be the gravitational force.

The origin of natural structure can not be other forces. Modern science has fully proved that independent system of microscopic particles combined by electromagnetic force or nuclear forces inevitably moves towards chaotic state rather than orderly one. This is the well known principle of entropy increase. If there were no gravitational force then the whole universe would be simply uniform gas without structure. However, there exist in the macroscopic world such orderly structures large as galaxies and small as solar system and even human beings. Therefore, varied kinds of macro-world structures
may result from the struggling of the gravitational force against the electromagnetic and nuclear forces.

Unfortunately, gravity is very very weak. For example, it is $10^{-40}$ times weaker than the electricity for protons. Only the Earth, Moon, Sun and so on present gravity. However, we can use man-made telescopes to take images of large-scale material systems such as galaxies. We can analyze the images.

The usually familiar gravity refers to the force which exerts between Earth and Moon or between Sun and Earth. These examples of gravity are the interaction between two bodies. As for the behavior of gravity interacted on many bodies, the solar system can not be the example. However, each galaxy is composed of billions of stars, which demonstrates the gravitational interaction among many bodies. Galaxy images show that each galaxy has a center. Star density at the center is the highest. From the center outward, the density is smaller and smaller and presents a regular pattern, known as galaxy structure. The principle behind the formation of galaxy structures is believed to be the gravitational interaction in many-body systems.

Then, what is the behavior of gravitational interaction in many-body systems? I pioneered the study on galaxy structures [1-3] and the study suggests that stars in any galaxy are controlled by a very simple orderly force involving many-bodies: proportion. Because solar system is just a point at the Milky Way galaxy, the proportion force reduces to Newtonian gravity between two bodies. Proportion means that the distribution of matters in the universe is orderly. For example, there are four giants standing in array. Their heights are A, B, C, D, respectively. According to the common view, the four giants can have any heights and can stand at any position. That is why current scientific theories can not explain the origin of natural structures. They can not provide any basic principle to resolve the motion of the simplest gravitational systems (interactional free three-bodies). However, galaxies are orderly. The orderly force requires that the distribution of heights is in proportion. That means that $A$ divided by $C$ is equal to $B$ divided by $D$. If there are nine giants standing in array, then the ratios of heights from neighboring two rows are constant (proportion rows). Similarly, the ratios of heights from neighboring two columns are constant (proportion columns). In this way are galaxies created.

The above-said rows and columns are all straight (proportion lines). But each galaxy is a regional distribution of matters in the universe. Therefore, the proportion lines of each galaxy are curved but the rows and columns still cross at each other vertically and they form the net of orthogonal curves. In general, a distribution of similar bodies is called the rational structure if its density varies proportionally along some particular net of orthogonal curves. Therefore, independent galaxies are all rational structures. The force which leads to the rational structure is called the rational force, i.e., the proportion force which is the demonstration of gravity at large-scale and many-body system.
2. Two Examples of Rational Structure

1. Logarithmic Density of Galaxy Structure. Now we start the scientific and mathematical investigation into galaxy structures. A galaxy is a distribution of stars. But we cannot see individual stars on a galaxy image. A galaxy image is the distribution of star densities. Therefore, we use a mathematical function to describe a distribution of densities. Because spiral galaxies are planar, we use a function of two variables, $x, y$, to describe the stellar distribution of a spiral galaxy:

$$\rho(x, y)$$  \hspace{1cm} (1)

where $x, y$ is the rectangular Cartesian coordinates on the spiral galaxy plane. The coordinate origin is the galaxy center. Therefore, $\rho(0, 0)$ is the stellar density at the galaxy center. We want to study the ratio of the density $\rho_2$ to the density $\rho_1$ at two positions 2 and 1 respectively: $\rho_2/\rho_1$. In fact, the logarithm of the ratio divided by the distance $s$ between the two positions is approximately the directional derivative of the logarithmic density ($f(x, y) = \ln \rho(x, y)$) along the direction of the two positions: $(\ln(\rho_2/\rho_1))/s \approx \partial f/\partial s$. There is no systematic mathematical theory on ratios. Therefore, we from now on, study the logarithmic function $f(x, y)$ instead of the density function $\rho(x, y)$:

$$f(x, y) = \ln \rho(x, y).$$  \hspace{1cm} (2)

2. Description of a Net of Orthogonal Curves. The following equations

$$x = x(\lambda, \mu), \ y = y(\lambda, \mu)$$  \hspace{1cm} (3)

tell us how to describe a net of curves by employing mathematics. Given two functions, $x(\lambda, \mu)$, $y(\lambda, \mu)$, you have the transformation between the curvilinear coordinates $(\lambda, \mu)$ and the rectangular Cartesian coordinates $(x, y)$. It describe a net of curves. Letting the second parameter $\mu$ be a constant, you have a curve (called a row curve, i.e., the proportion row defined in the following). That is, the above formula is a curve with its parameter being $\lambda$. For the different values of the constant $\mu$, you have a set of “parallel” rows. Similarly, you have a set of “parallel” columns of parameter $\mu$. However, The row curves and the column curves are not necessarily orthogonal to each other. The following equation is the necessary and sufficient condition for the net of curves to be orthogonal:

$$\frac{\partial x}{\partial \lambda} \frac{\partial x}{\partial \mu} + \frac{\partial y}{\partial \lambda} \frac{\partial y}{\partial \mu} = 0.$$  \hspace{1cm} (4)

To study the rational structures described in the following, we need more knowledge of the description of row and column curves. The arc length of the row curve is $s$ whose differential is $ds = \sqrt{x^2_\lambda + y^2_\lambda} d\lambda = P d\lambda$ where $P$ is the arc derivative of the row curve. The arc length of the column curve is $t$ and $Q$ is its arc derivative:

$$P = s'_\lambda = \sqrt{x^2_\lambda + y^2_\lambda}, \ Q = t'_\mu = \sqrt{x^2_\mu + y^2_\mu}. \hspace{1cm} (5)$$

3. The Condition of Rational Structure. The formulas (3) and (4) are the general description of a set of orthogonal curves, and the formula (1) is the general description
of a distribution of densities (a structure). This paper talks about rational structure. A distribution of densities is called the rational structure if its density varies proportionally along some particular net of orthogonal curves. That is, you walk along a curve from the net and the ratio of the density on your left side to the immediate density on your right side is constant along the curve. However, the constant ratio of this curve is generally different from the constant ratios of the other curves.

We have shown that a logarithmic ratio of two densities divided by the distance between the two positions is approximately the directional derivative of the logarithmic density along the direction of the distance. Therefore, we always study the logarithmic function \( f(x,y) \). If we know the two partial derivatives \( \frac{\partial f}{\partial x} \) and \( \frac{\partial f}{\partial y} \) then the structure \( f(x,y) \) is found. The partial derivatives of \( f(x,y) \) are the directional derivatives along the straight directions of the rectangular Cartesian coordinate lines. However, we are interested in the net of orthogonal curves and what we look for is the directional derivatives along the tangent directions of the curvilinear rows and columns. These are denoted \( u(\lambda,\mu) \) and \( v(\lambda,\mu) \) respectively. The condition of rational structure is that \( u \) depends only on \( \lambda \) while \( v \) depends only on \( \mu \):

\[
u = u(\lambda), \ v = v(\mu).
\] (6)

What a simple condition for the solution of rational structure.

Now we prove the condition. Assume you walk along a row curve. The logarithmic ratio of the density on your left side to the immediate density on your right side is approximately the directional derivative of \( f(x,y) \) along the column direction. That is, the logarithmic ratio is approximately the directional derivative \( v(\lambda,\mu) \). Because \( v(\lambda,\mu) \) is constant along the row curve (rational), \( v(\lambda,\mu) \) is independent of \( \lambda \): \( v = v(\mu) \). Similarly, we can prove that \( u(\lambda,\mu) = u(\lambda) \).

4. The Equation of Rational Structure. It is known that, given an arbitrary function, we can have its two partial derivatives. However, given two functions, we may not find the third function whose partial derivatives are the given functions. For the given two functions to be some derivatives, a condition must be satisfied. The condition is the Green’s theorem. In the case of derivatives along orthogonal curves, the Green’s theorem is the following:

\[
\frac{\partial}{\partial \mu}(u(\lambda,\mu)P) - \frac{\partial}{\partial \lambda}(v(\lambda,\mu)Q) = 0
\] (7)

In the case of rational structure, directional derivatives are the functions of the single variables, \( \lambda \) and \( \mu \) respectively. Therefore, the Green’s theorem turns out to be the following which is called the rational structure equation:

\[
u(\lambda)P_{\mu} - v(\mu)Q_{\lambda} = 0
\] (8)

This equation determines rational structure. To find a rational structure, generally we are, first of all, given a net of orthogonal curves. Accordingly the arc derivatives of both the rows and columns, \( P(\lambda,\mu) \) and \( Q(\lambda,\mu) \), are known. Therefore, the remaining functions, \( u(\lambda) \) and \( v(\mu) \), are the only unknowns. Because the rational structure equation
involves no derivative of the unknowns, the equation is not a differential equation at all. It is an algebraic equation and what we need to do is to add two factors (the two unknowns) to the derivatives of \( P \) and \( Q \) so that the rational structure equation holds: factorization. How simple is rational structure.

5. The First Example: Spiral Galaxy Disk Structure. Images of spiral galaxies taken with infrared light show that each spiral galaxy is mainly a disk (the circularly symmetric disk with respect to the galaxy center, i.e., the disk center) and the disk light density decreases exponentially outwards along the radial direction from the center. There are other minor or weak structures in spiral galaxies. Spiral galaxies gain their name by the fact that they present more or less spiral structures, known as arms.

The first example of rational structure is the spiral galaxy disk and is determined by the following net of equiangular spirals (or called logarithmic spirals, see Fig. 1):

\[
\begin{align*}
x &= e^{d_1 \lambda + d_2 \mu} \cos(d_3 \lambda + d_4 \mu), \\
y &= e^{d_1 \lambda + d_2 \mu} \sin(d_3 \lambda + d_4 \mu)
\end{align*}
\] (9)

where \( d_1(>0), d_2(>0), d_3(<0), d_4(>0) \) are real constants and we choose \( d_3 = -d_1d_2/d_4 \) so that the curves are orthogonal. The polar angle and polar distance of the point \((x, y)\) are \( r = e^{d_1\lambda + d_2\mu}, \theta = d_3\lambda + d_4\mu \). The coordinate lines are spiral-shaped. The corresponding arc-length derivatives \( P, Q \) are

\[
\begin{align*}
P(\lambda, \mu) &= s'_\lambda = (d_1/d_4)\sqrt{d_2^2 + d_2^4}e^{d_1\lambda + d_2\mu}, \\
Q(\lambda, \mu) &= t'_\mu = \sqrt{d_2^2 + d_4^4}e^{d_1\lambda + d_2\mu}.
\end{align*}
\] (10)

The rational structure equation does help factor out the required directional derivatives for our spiral galaxy disks,

\[
u(\lambda) = d_5d_4, \quad v(\mu) = d_5d_2
\] (11)

where \( d_5 \) is another constant. So far, we did not specify the variance domain \( S \) on \((\lambda, \mu)\) coordinate plane on which the coordinate system is defined. There are many such domains, and the spiral curves are equiangular spirals as shown in [2].
The spiral disk density along all orthogonal coordinate lines can be found by performing path integrations of the formulas, \( df = u ds = u(\lambda) P d\lambda, \ df = v(\mu) Q d\mu, \) along the row curves, \( \mu = \text{constant}, \) and the column curves, \( \lambda = \text{constant}, \) respectively. Finally, we have the logarithmic function \( f(x, y) \) corresponding to the spiral-shaped coordinate system. The density distribution \( \rho(x, y) \) represents spiral galaxy disks (we choose \( d_5 < 0, \) because light density \( \rho \to 0 \) when \( r \to +\infty \)),

\[
\begin{align*}
  f_d &= d_5 \sqrt{\frac{d_2}{d_3^2} + \frac{d_4}{d_5^2} e^{d_1 \lambda + d_2 \mu}}, \\
  \rho_d &= d_0 \exp(d_5 \sqrt{\frac{d_2}{d_3^2} + \frac{d_4}{d_5^2} e^{d_1 \lambda + d_2 \mu}})
\end{align*}
\]

where \( d_0 \) is the light density (star density) at the galaxy center. Note that we use the letter \( d \) as well as the subscript \( d \) for disk parameters and formulas. Similar notations are used for bar parameters and formulas. We can see the disk light pattern by displaying \( \rho_d(x, y). \)

Because the polar distance is \( r = \exp(d_1 \lambda + d_2 \mu), \) the galaxy disk light distribution is circularly symmetric,

\[
\rho_d = d_0 e^{f_d} = d_0 e^{(d_5 \sqrt{\frac{d_2}{d_3^2} + \frac{d_4}{d_5^2}}) r}.
\]

Since galaxy light density \( \propto \rho, \) we have recovered the known exponential law of spiral galaxy disk (the exponential disk).

In fact, galaxy structures depend only on the geometric curves, not the choice of coordinate parameters. Therefore, the following is the general expression for equiangular spirals:

\[
\begin{align*}
  x &= e^{d_1 f(\lambda) + d_2 g(\mu)} \cos(d_3 f(\lambda) + d_4 g(\mu)), \\
  y &= e^{d_1 f(\lambda) + d_2 g(\mu)} \sin(d_3 f(\lambda) + d_4 g(\mu))
\end{align*}
\]

where \( f(\lambda), g(\mu) \) are arbitrary functions. All these expressions give the same equiangular spirals and generate the same exponential disks, independent of the choice of coordinate parameters.

6. The Second Example: Dual Handle Structure. Now we study galactic bar model. A bar pattern is composed of two or three dual handle structures which are generally aligned with each other (spiral galaxy NGC 1365 is not the case). The dual handle structure is determined by the following orthogonal curves of confocal ellipses and hyperbolas:

\[
\begin{align*}
  x &= e^\sigma \cos \tau, \ y = \sqrt{e^{2\sigma} + b_1^2} \sin \tau, \\
  -\infty < \sigma < +\infty, \ 0 \leq \tau < 2\pi.
\end{align*}
\]

where \( b_1(> 0) \) is a constant. The orthogonal coordinate system no longer shares the coordinate lines with the polar coordinate system. The coordinate lines are confocal ellipses and hyperbolas (Fig.2). The distance between the two foci is \( 2b_1 \) which measures the distance between the two handles. The eccentric anomaly of the ellipses is \( \tau. \) The
inverse coordinate transformation of the formulas is easily found,

\[ p(x, y) = e^\sigma = \sqrt{(r^2 - b_1^2 + \sqrt{(r^2 - b_1^2)^2 + 4b_2^2x^2})/2}, \]

\[ \cos \tau = xe^{-\sigma} = x/p(x, y) \]

where \( r^2 = x^2 + y^2 \). We find the density of the dual handle structure,

\[ f_b(x, y) = (b_2/3)(p^2(x, y) + b_1^2x^2/p^2(x, y))^{3/2}, \]

\[ \rho_b = b_0 \exp(f_b(x, y)) \]

where \( b_0 \) is the dual handle density at the galaxy center. We need to choose \( b_2 < 0 \) so that \( f_b < 0 \) and \( \rho_b \rightarrow 0 \) when \( r \rightarrow +\infty \). We can see that \( b_0 \) corresponds to the central dual handle strength and \( b_1 \) corresponds to the dual handle length while \( b_2 \) measures the density slope off the dual handle. If we display the dual handle structure as a curved surface in 3-dimensional space then we can see that the surface is camelback-shaped with two humps (i.e., handles).

Fig. 2 The orthogonal curves of confocal ellipses and hyperbolas. The distance between the two foci, \( F \) and \( F' \), is \( 2b_1 \) which measures the distance between the two handles.

In fact, galaxy structures depend only on the geometric curves, not the choice of coordinate parameters.

3. The Origin of Spiral Galaxies

1. Proposition 1: Rational Structures are at Most Bilaterally Symmetric. This is a mathematical proposition: any net of orthogonal curves is either circularly symmetric with respect to the center point, or bilaterally symmetric. I spent three years from 2001 to 2004 to look for a net of orthogonal curves whose shape has odd symmetry [1]. That is, I wanted to find a rational structure which resembled a two-arm spiral pattern like the spiral galaxy M51. The three-year study indicates that a net of orthogonal curves is generally connected to some complex analytical function. From my experience, I do not find any complex analytical function whose graph of the real or imaginary part has odd symmetry. Therefore, I have the proposition that rational structures are at most
bilaterally symmetric. This is left for verification by able people. Surprisingly, galaxy structures happen in the same way as indicated in the following.

2. **Coincidence 1: Galaxy Patterns (except Arms) are at Most Bilaterally Symmetric.** Amazingly, any component of any galaxy pattern (except the arm pattern) is either circularly symmetric, or bilaterally symmetric. And my academic papers [2-3] show that, except the arm structure, any component of any galaxy structure (such as exponential disks, galactic bars, even the whole elliptical galaxy) is a rational structure. That is, any galaxy image (ignoring the arm) can be fitted identically to rational structures. The following will present more and more cases of coincidences. Therefore, they may not be coincidences at all. They may point to cosmic truth.

3. **Coincidence 2: Dust and Irrationality.** It is the observational fact that spiral galaxies are full of dust while elliptical galaxies have little dust. Why does this happen? However, people have not found the appropriate answer since galaxies were discovered more than 80 years ago.

Arm structure is neither circularly symmetric with respect to the center point, nor bilaterally symmetric. Therefore, arms are not rational structures. Arm pattern tends to be odd symmetry with respect to the center. Arm patterns exist only in spiral galaxies and they are weak compared with the main disk structure of spiral galaxies. Therefore, the presence of arm structure is the disturbance to the rational structure. Because arm patterns exist only in spiral galaxies and only spiral galaxies present dust, I come to the critical answer to the above question: any disturbance to rational structure produces cosmic dust.

The disturbing waves try to achieve the minimal disturbance and, as a result, they follow the proportion rows or columns of the rational structures.

4. **Coincidence 3: Exponential Disk is Correlated with Equiangular Spiral.** The exponential disk of any spiral galaxy is a rational structure which is circularly symmetric about the galaxy center. It is surprising that its proportion curves are the equiangular spirals which are the curves represented by normal spiral galaxy arms.

5. **Coincidence 4: The Only Rational Structure which is not Circularly Symmetric is Dual Handle Structure.** I have given the definition of rational structure. However, given an arbitrary net of orthogonal curves, we are not always possible to arrange a distribution of stellar density on the net to form a rational structure. In fact, there are only a few types of very simple orthogonal curves which correspond to rational structures. Rational structures are usually circularly symmetric about the center points. The rational structure we can find which is not circularly symmetric, is the bilaterally symmetric structure, namely, dual-handle structure.

We have two types of rational structures: exponential disks and dual-handle structures. Adding the two structures together leads to the barred pattern as we expected. It is amazing that only two kinds of spiral galaxies are observed in the universe. One kind of spirals are the normal spiral galaxies while the other kind are the barred spiral ones. What is more surprising is that some barred galaxies do show a set of symmetric enhancements at the ends of the stellar bar, called the ansae or the “handles” of the bar
[4] (see upper-left panel of Fig. 3). This indicates that a bar itself is nothing but a set of several pairs of ansae (handles). That is, bars are the superposition of several aligned or misaligned dual-handle structures. If the outer dual-handle structure is far more away from the galaxy center then it demonstrates the pattern of ansae or “handles” of the bar.

6. Coincidence 5: There are Barred Spiral Galaxies which Present Two Nonparallel Bars.

Fig. 3 Upper-left panel is galaxy NGC 2983 [4]. Upper-right is the infrared image of galaxy NGC 1365. Lower-left is the infrared image of NGC 1300. Lower-right is NGC 1365 (ultraviolet image: European Southern Observatory).

The main structure of spiral galaxies is the exponential disk. When the dual-handle structure (i.e., sub-bar) is near the galaxy center, the superposition of the dual-handles to the bright disk center presents a bar shape. This precisely explains the origin of galaxy bars. A galaxy bar is usually composed of two or more sets of aligned or misaligned dual-handle structures. Surprisingly, there are barred spiral galaxies which present two nonparallel bars (see upper-right panel of Fig. 3).

7. Coincidence 6: Bar Structure is So Weak in the Outer Areas of Spiral Galaxies that it is Ignored. Compared with the exponential disk, the bar is observationally weak structure. That is, bar structure is so weak in the outer areas of spiral galaxies that it is ignored. It is very surprising that the theoretical calculation of dual-handle structure shows that it is weak when compared with the disk (see Fig. 4).

We know that the disk density of spiral galaxies decreases outwards exponentially, which is the numerical result obtained over 80 years since the discovery of galaxies in the universe. Spiral galaxy disks are thus called exponential disks. We add the dual-handle structure to the exponential disk for them to be the model of barred spiral galaxies. If the density of dual-handle structure were comparable to or stronger than the exponential disk in the far distances from the galaxy center then our model would fail. That would suggest that the main structure of spiral galaxies were not the exponential disk, a result inconsistent with astronomical observation. The mathematical result is that the density distribution of dual-handle structure decreases outwards cubic-exponentially as shown below.
For large $r$, the first formula in (16) reduces to $r$, and the first formula in (17) approaches $(b_2/3)r^3$. Because $b_2 < 0$, we come to the conclusion that the density distribution of dual-handle structure decreases outwardly cubic-exponentially. Far away from the galaxy center, the dual-handle structure is negligible when compared with the disk. Mathematical result is consistent to observation.

8. Coincidence 7: The Arms of Barred Galaxies Spin around the Bar and are No Longer the Equiangular Spirals. With simple mathematical calculation we know that spiral-shaped proportion curves exist in dual-handle structure. However, they are not equiangular because they surround the central line of the dual-handles (recall that the spirals in exponential disks are equiangular and surround the center point). Two proportion curves which are oddly symmetric about the center point in dual-handle structure make approximately elliptical shape and its long axis must be parallel to the central line of the dual-handles. Surprisingly, astronomical observations show that arms of barred spiral galaxies do surround the middle lines of their bars, and they are not equiangular, and the two arms make approximately elliptical shapes with the long axes being parallel to the bar middle lines (see Fig.3).

9. Coincidence 8: Circular and Elliptical Rings. We have already known that exponential disks have circular proportion curves (one family of polar curves). Observationally, some normal spiral galaxies do have closed arms which are circular, called rings. Dual-handle structures also have closed proportion curves which are ellipses whose long axes must be parallel to the central lines of the dual-handles. Observationally, some barred spiral galaxies do have closed rings which are ellipses and the long axes are parallel to the galaxy bars.

10. Coincidence 9: Fitting Bar Images with Dual-handle Structures. In fact, we can directly use dual-handle structures to fit into the bar images of spiral galaxies. From the barred spiral galaxy images we subtract the superposition of two or three sets of dual-handle structures and see if the resulting images are exponential disks plus the weak arm structures. The results (Fig.4) are very good: the resulting images are exponential disks plus the weak arm structures.

11. Coincidence 10: Elliptical Galaxies are Completely Rational Structures. I have proved that elliptical galaxies are completely rational structures in three-dimensions [2,3]. The only proportion surfaces of elliptical galaxies are the intersecting nets of orthogonal spheres, where disturbing waves are difficult to form and spread. On the other hand, spiral galaxies are two-dimensional and their proportion curves are all open spirals where disturbance waves are easy to form and spread. Astronomical observations do show that arms do not exist in elliptical galaxies.

12. Coincidence 11: Rational Galaxy Structures are Determined by Elementary Functions. It is amazing that rational galaxy structures are determined by the most elementary functions. Spiral galaxy disks and bars are determined by the complex exponential function $g = \exp w$ where $g = x + iy$ and $w = \lambda + i\mu$ (see the formula (9)). Elliptical galaxy structure is determined by the complex reciprocal function $g = 1/w$ (see [3]). In fact, there are a few elementary functions which correspond to rational structures, and
the nature chooses the most elementary functions to be its design of the universe.

13. Future Work and Coincidence 12: Galaxy Nuclear Rings. There are lots of future theoretical and applied works to be done. One of them is the search of the condition under which the summation of two rational structures is still rational. Let $\rho_1$ and $\rho_2$ be the two rational structures. Their logarithmic densities are $f_1$ and $f_2$ respectively. The logarithm of the summation is $f$. It is straightforward to show that

$$\nabla f = (\rho_1 \nabla f_1 + \rho_2 \nabla f_2)/(\rho_1 + \rho_2).$$

That is, the gradient of summation is the summation of weighted gradients. Because galaxy disks take their maximum values at around the galaxy center and dual handle structures take the values away from the center, our above conclusions are still true. Furthermore, the proportion curves of the summation structure explain why there are nuclear rings or arms for some barred galaxies. An example is the galaxy NGC 4314.

Conclusion

1. The well-known fact can not be ignored that gravity is very very weak. For example, it is $10^{-40}$ times weaker than the electricity between protons. Therefore, humans in the foreseeing future can not design a physical precision experiment which can resolve the $10^{-40}$ strength of the earth’s gravitational field. That means we have not had a full understanding of gravity. But scientists assume they had it and applied the preliminary results of Newton and Einstein to the whole universe. This resembles the situation of cycles and epicycles in the old geocentric model.

2. It is a fact that the results of Newton and Einstein deal with the motion of two-bodies. When applied to the free motion of many-bodies, the theories give chaotic results. However, the universe has orderly motion. Whenever a problem involves free many bodies, Newton and Einstein theories have no power. For example, the Bode law of planetary distribution in the solar system has not been explained.
3. Newton and Einstein theories have no power for the explanation of natural structures. Galaxy structure is the simplest one in the observational world. Every one with common sense must suggest that there exists a law on galaxy structure. Newton and Einstein theories can not provide such law because they are the theories of two-bodies. The law is very possibly the rationality explained in my article.

4. The mainstream model of the universe (the Big Bang theory) which is based on Newton and Einstein theories, is being declined. A new article [6] describes: Nearly every month new observations arise that pose further challenges to the $\Lambda$CDM paradigm: Correlations in galaxy structures [7]; absence of baryon acoustic oscillations in galaxy-galaxy correlations [8]; galaxies formed already when the universe was 4 to 5 billion years old [9]; dwarf satellites that swarm our own galaxy just like its stars [10]. Observational data [11, 12] strongly suggest a paradigm shift for cosmology.

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