Contemporary earth crust strain rate tensor computed across Pamir region based on triplets of GPS velocities

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Abstract. We present detailed pictures of contemporary earth crust strain rate tensor values for territories of Pamir and its surroundings computed by triplets of GPS observations’ points. We characterize directions and intensity of contemporary tectonic activity at some fault zones.

1. Introduction
Pamir is a mountain system in Central Asia; it is a northward-directed ledge of Alpine-Himalayan orogen belt. The north and north-west edges of Pamir conjunct with South-West Tien-Shan. To the west of Pamir there is Tajik Depression – lowland with several small folding structures stretching in submeridional direction. To the east from Pamir there lies Tarim Plate – flat highland constrained between Tien-Shan from the north and Kun-Lun from the south (figure 1).

Pamir and neighbor mountain systems arose as the result of collision Hindustan continent with Eurasia [1]. At present day Pamir keeps moving relative to its surrounding structures and studying earth crust deformations caused by this movement is of a big interest. Investigation of Pamir’s contemporary tectonics has been going for last several decades involving various methods and approaches. Since last 20 years using high precision GPS observations made possible estimating velocities of individual point on earth (in further text we use the common term “GPS site” to refer a point where such GPS observations were done). Present research is based on the fullest set of GPS observations for years 2007-2011 in Tajikistan and years 1998-2012 in Chinese Pamir and South Tien-Shan. All this data was processed using the GAMIT/GLOBK software suite in a united computation procedure. This fact guarantees absence of errors that could be caused by mixing GPS velocity estimates taken from different sources.

Another research was done earlier using the same source dataset and covering area of Pamir [2]. In that research contemporary earth crust strain rate was calculated using a smoothing method [3] that produced a general picture of spatial distribution of strain rate tensor values over Pamir, Tien-Shan and neighbor territories. One of conclusions from [2] was that generally Pamir moves: westward relative to Tajik Depression (collides with it), north-westwards relative to South-West Tien-Stan (collides and slips) and westwards relative to Tarim Plate (goes away from it).

In this research we try to calculate a detailed distribution of contemporary earth surface’s strain rate tensor values across Pamir and neighbor areas to localize deformation’s intensity and types: horizontal shortening (thrusts, collision), horizontal extension (normal faults, divergence) or horizontal shift (strike-slip faults).
2. Method

The method we use is based on calculating earth surface points’ velocity gradient tensor. Mathematical model is the same as in researches [2] and [3]; the difference of method in this work is that instead of spatial smoothing we compute the velocity gradient tensor by small subsets of the source velocity catalogue. For each small area of earth surface we pick a set of three (rarely four) GPS sites surrounding that area more or less evenly. Then we compute the value of velocity gradient tensor by that subset. This method was already tried for some areas of Tien-Shan in [6]; it maximized the level of detail but gives erratic results in case if any of selected GPS sites has significant error of its velocity estimation.

The calculation was performed using the SUR_GPS_STRAINS software suite [7]

2.1. Mathematical model

Let us set the reference system for the study area. Its reference point will be the point where we calculate the value of earth crust strain rate tensor. The directions of the axes will be the same as in the reference system of the input data (GPS sites’ velocities). Then the motion of a given point of the earth surface (in particular, of \( k \)-th GPS site with coordinates \( \Delta P_{(k)} \)) can be described by an equation that is based on a linear term of the Taylor series expansion for the function of point’s velocity versus point’s radius-vector \( \dot{r} = F(\dot{r}) \):

\[
U_{(k)} = F(\Delta P_{(k)}) = T + \frac{dF}{dr} \Delta P_{(k)} + E_{(k)} = T + G\Delta P_{(k)} + E_{(k)}
\]  

or, in scalar form:

\[
u_{x(k)} = t_x + g_{xk} \Delta P_x + g_{y} \Delta P_y + e_{x(k)}
\]

\[
u_{y(k)} = t_y + g_{yk} \Delta P_x + g_{yy} \Delta P_y + e_{y(k)}
\]
where: \( U(k) \) is the vector of the GPS site’s velocity; \( T \) is the vector of the reference point velocity; \( G \) is the velocity gradient tensor at the reference point; \( \Delta P(k) \) is the radius vector of the GPS site; \( E(k) \) is the remainder term in the Taylor series expansion; \( i, j \) are indices of tensor components, each of them may refer to coordinate \( x \) or coordinate \( y \); \( k \) is the sequential number of the GPS site in the source data catalogue (fig. 2):

\[
U(k) = [u_{x(k)}, u_{y(k)}]; \quad T = [t_x, t_y]; \quad G = \begin{bmatrix} g_{xx} & g_{xy} \\ g_{yx} & g_{yy} \end{bmatrix}; \quad \Delta P(k) = \begin{bmatrix} \Delta P_{x(k)} \\ \Delta P_{y(k)} \end{bmatrix}; \quad E(k) = \begin{bmatrix} e_{x(k)} \\ e_{y(k)} \end{bmatrix}
\]

Figure 2. An example of location of GPS sites used in computing the velocity gradient tensor. The black lines show radius-vectors of GPS sites (\( \Delta P \)). The blue arrows denote velocities of surface points (not to scale; \( U \) for GPS sites and \( T \) for the coordinate origin point).

In equation (1) \( U(k) \) and \( \Delta P(k) \) are determined from the input velocities’ catalogue, while \( T \) and \( G \) are unknown. By combining unknowns of equations (3) and (4) in a vector, these equations can be rewritten as:

\[
\begin{bmatrix} u_{x(k)} \\ u_{y(k)} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta P_{x(k)} \\ \Delta P_{y(k)} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} t_x & t_y \\ g_{xx} & g_{xy} \\ g_{yx} & g_{yy} \end{bmatrix}^{\top} + \begin{bmatrix} e_{x(k)} \\ e_{y(k)} \end{bmatrix}
\]

Then by vertically concatenating equations from \( N \) GPS sites, by denoting the matrix containing \( \Delta P_{i(k)} \) values as \( A \) and the vector with unknowns as \( \bar{T} \) we get:

\[
\bar{u} = A\bar{T} + \bar{e},
\]

is a system of linear algebraic equations; it’s a classic linear regression problem and we used a standard least mean square (LMS) solution to minimize the components of vector \( \bar{e} \).
Because for every GPS site we know also estimates of root-mean-square errors $\sigma_{u_x(k)}$ for components of its velocity $u_x(k)$ we use weighted LMS to obtain the vector of the parameters by minimizing the quadratic form $J = \bar{e}^T \cdot W \cdot \bar{e}$, where $W$ is the weighting matrix:

$$W = \text{diag}\left(\sigma_{u_x(1)}^{-2}, \sigma_{u_y(1)}^{-2}, \sigma_{u_x(2)}^{-2}, \sigma_{u_y(2)}^{-2}, \ldots, \sigma_{u_x(N)}^{-2}, \sigma_{u_y(N)}^{-2}\right)$$

Obviously system of equations (5) has always 6 unknowns and $2N$ equations ($N$ is the number of GPS sites we use in the computation). Thus, if $N = 3$ we get system of 6 equations with 6 unknowns, having only one solution that turns the misfit vector $\bar{e}$ into zero (with the exception of the case if all $N$ GPS sites lie on a straight line). If $N > 3$ then system (5) is overdetermined and resulting value of tensor $G$ will depend on components of weighting matrix $W$ to minimize misfits for GPS sites with finer estimates of their velocity.

Note that in all cases we consider here the value of tensor $G$ depends neither on where we put reference point of the coordinate system (at what point we calculate $G$) nor on what GPS reference frame (ITRF) was used in computation of source velocity catalogue. Consider analogy between (1) and the problem of approximation several data points with a linear function $Y = B + K \cdot X$. Only $B$ (corresponding to $T$ in (1)) depends on where we put the reference point for any of axes $X$ and $Y$; it will not affect $K$ (corresponding to $G$ in (1)). After calculating the velocity gradient tensor $G$ we take its symmetric component that characterizes horizontal strain rate of the earth crust. Then this strain rate tensor is diagonalized and plotted on a geographic map at the point where it was calculated.

3. Results and analysis

All source data used (velocities of GPS sites) and all results (values of earth crust strain rate tensor) are presented at figure 3. This figure generally shows proportion between strain rate magnitudes at different areas of the research region. Three areas of the most intensive deformations marked with thick black rectangular frames are analyzed in the text below.

**Figure 3.** Broad picture of the resulting contemporary earth crust strain rate tensor distribution. Light brown arrows show source data – estimates of GPS sites’ velocities; ellipses on their tips show uncertainties of these estimates. Eigenvalues and directions of eigenvectors of diagonalized strain rate tensor are shown with blue (positive = extension) and red (negative = shortening) arrows. Black lines connect representation of tensor value with GPS sites used to calculate that value. Black arrows show scale of GPS sites’ velocities and strain rate tensor eigenvalues.
3.1. North Pamir

Figure 4 presents the resulting earth crust strain rate tensor values. It’s clearly seen that deformations are most intense in a narrow strip-shaped area of Pamir where it touches Tien-Shan. Traditionally the term “Main Pamir Thrust” is associated with this strip. Some researches describe it as a single fault; others (like [5]) show it as a band of separated fault fragments. Results of strain rate tensor’s calculation show just that GPS sites KRK4, DAT4, T334, T504 and IKZ4 are located to the north (Tien-Shan side) of this stripe while the site ALD4 is more to the south (Pamir side). The BAB5 site does probably lie inside this strip between two parallel faults but much more deformation rate separates it from Pamir than from Tien-Shan. There is also significant shortening to the south from the ALD4 site (triangle T285-ALD4-T390) that can be associated to the fault depicted along the crest of Transalai ridge. Considering this fault to be a part of Main Pamir Thrust can be a subject for (mostly formal) discussions. In general according to GPS data Main Pamir Thrust undergoes shortening across it also with some right-shift component. We can neither finely constrain the location of Main Pamir Thrust nor thoroughly characterize deformations across it because spatial density of GPS sites is not high enough.

![Figure 4. Contemporary earth crust strain rate tensor values across North Pamir. Green lines show faults according to [4]. Purple lines show active faults according to [5].](image)

Another widely known zone of modern tectonic activity at North Pamir is Karakul (Sarez-Karakul) graben. It splits the bottom of Karakul Lake submeridionally and continues further in south-south-east direction towards Lake Sarez. Figure 4 shows it not as a single fault line but rather as a system of normal faults. GPS data show significant divergence between sides of this graben only in Lake Karakul area between GPS sites T390 and T470 on its east side and site T380 on the west side.
In other areas of North, Central and South Pamir the calculated eigenvalues of contemporary earth crust strain rate tensor do not exceed 20-50 nanostrain per year; in general eigenvectors show shortening along South-North direction and extension along West-East direction.

3.2. East Pamir

Figure 5 presents the resulting earth crust strain rate tensor values. We can distinguish two zones with strong deformations of different kind.

On the north part of plotted area there is a latitudinally oblong area of submeridional shortening. The zone stretches from Main Pamir Trust on the west into a stripe at the north border of Tarim Plate where it conjunts with Tien-Shan. This zone clearly marks that not only Pamir but also Tarim Plate is now colliding with Tien-Shan.

Figure 5. Contemporary earth crust strain rate tensor values across East Pamir. Green lines show faults according to [4]. Purple lines show active faults according to [5].
The second zone matches another widely known tectonic feature – East Pamir Pull-apart – characterized as extension across the set of grabens and normal faults with submeridional strike. In fault model [4] this feature is represented with two main normal faults: the longer Kongur-Shan fault situated mostly to the north of 38° and the shorter Tashkorgan fault to the south of 38°. Latitudinal extension across northern part of Kongur-Shan fault is clearly shown by the values of strain rate tensors calculated between GPS sites MUJI and I090 to the west of the fault and sites WUPA and GAZE to the east. Investigation of deformations in the southern part is made difficult by the fact that there are no GPS sites close to the fault from both east and west sides. The only GPS site is QIAE and its velocity estimation has a very large error making all the strain rate results gained from it unreliable (they are marked unreliable on figure 5). The GPS site TASC is located very close to Tashkorgan fault. If it is actually on the eastern side of the fault then there is unlikely any contemporary tectonic activity on Tashkorgan fault. If the position of the fault shown on figure 5 is wrong and the TASC site is at the western side then we can not estimate current deformations at the fault because of lack of GPS sites nearby on eastern side.

3.3. West Pamir
In the research [2] following conclusions about deformations in Pamir, Tien- Shan and Tajik depression conjunction zone were made based on results of smoothed strain rate distribution: the part of Tajik Depression lying near West Pamir undergoes latitudinal shortening while on the northern border of Tajik Depression where it touches Tien-Shan’s Gissar ridge there is a latitudinal oblong band showing right lateral shear deformations associated with South-Gissar right lateral strike-slip fault from the global scheme of Pamir structural faults.

![Figure 6. Contemporary earth crust strain rate tensor values across West Pamir and Tajik depression. Green lines show local faults according to [1]. Purple lines show active faults according to [5].](image-url)
Figure 6 shows the strain rate tensor values calculated by triplets of GPS in this area plotted over more detailed map of faults from [1]. There are several thrust faults of submeridional strike in the southern considered part of Tajik Depression and there is significant latitudinal shortening across them. On the north these thrust faults end at a system of sublatitudinal-strike faults located to the south from Gissar ridge. We do not have enough of good GPS velocities to get any reliable detailed distribution of strain rate tensor. The key GPS site on the west of this area – T030 – lacks precision of its velocity estimate and the second site T120 has a velocity value that differ from neighbor GPS sites too much to be trustworthy. The results from these two GPS sites are marked as unreliable on figure 6. We computed strain rate tensor values for the larger areas using more distant GPS sites T075, T084, T150, T165 and T200. These values actually match with right lateral shift deformations on sublatitudinal-strike faults.

4. Conclusion
We calculated a detailed distribution of contemporary earth crust strain rate tensor for North, East and West Pamir and nearby areas of neighbor tectonic formations. As the result we distinguish areas with more contemporary tectonic activity and characterize their kinds of deformation:
- Main Pamir Thrust – cross-directional shortening with moderate right lateral shift;
- Karakul graben – moderate cross-directional extension;
- East Pamir Pull-apart – cross-directional extension of its northern part;
- central and eastern parts of Tajik Depression – latitudinal shortening;
- conjunction between Tajik Depression and Gissar ridge – probably right lateral shift.

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