Learning Task Specifications from Demonstrations via the Principle of Maximum Causal Entropy

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Abstract: In many settings (e.g., robotics) demonstrations provide a natural way to specify sub-tasks; however, most methods for learning from demonstrations either do not provide guarantees that the artifacts learned for the sub-tasks can be safely composed and/or do not explicitly capture history dependencies. Motivated by this deficit, recent works have proposed specializing to task specifications, a class of Boolean non-Markovian rewards which admit well-defined composition and explicitly handle historical dependencies. This work continues this line of research by adapting maximum causal entropy inverse reinforcement learning to estimate the posteriori probability of a specification given a multi-set of demonstrations. The key algorithmic insight is to leverage the extensive literature and tooling on reduced ordered binary decision diagrams to efficiently encode a time unrolled Markov Decision Process.

Keywords: Learning from demonstrations, Specification Mining, Maximum Causal Entropy, Inverse Reinforcement Learning, Binary Decision Diagrams

1 Introduction

As the ubiquity of autonomous robotic agents has increased, so has the need to easily convey what task a robotic agent should perform. In many settings, episodic demonstrations provide a natural way to specify a task. In such a setting, one traditionally models the demonstrator as operating within a dynamical system whose transition relation only depends on the current state and action (called the Markov condition). However, even if the dynamics are Markovian, many tasks are naturally modeled in history dependent (non-Markovian) terms. For example, “if the robot enters a blue tile, then it must touch a brown tile before touching a yellow tile”. Unfortunately, most methods for learning from demonstrations, e.g. standard inverse reinforcement learning [5], either do not provide guarantees that the artifacts learned for the sub-tasks can be safely composed and/or do not explicitly capture history dependencies.

Motivated by this deficit, recent works have proposed specializing to task specifications, a class of Boolean non-Markovian rewards which admit well-defined composition and explicitly handle historical dependencies [1][2]. A particularly promising direction has been to adapt maximum entropy inverse reinforcement learning [3] to task specifications, enabling a form of robust specification inference, even in the presence unlabeled demonstration errors [2]. However, while powerful, the principle of maximum entropy is limited to a setting where the dynamics are deterministic or agents that use open-loop policies [3].

This work continues this line of research by instead using the principle of maximum causal entropy, which generalizes the principle of maximum entropy to general stochastic decision processes [4]. While a conceptually straightforward extension, a naïve application of maximum causal entropy inverse reinforcement learning to non-Markovian rewards results in an algorithm with run-time exponential in the episode length, a phenomenon sometimes known as the curse of history. The key algorithmic insight in this paper is to leverage the extensive literature and tooling on reduced ordered binary decision diagrams to efficiently encode a time unrolled Markov decision process.

Related Work. Our work is intimately related to Maximum Entropy Inverse Reinforcement Learning. In Inverse Reinforcement Learning (IRL) [5] the demonstrator, operating in a stochastic environment,
is assumed to attempt to (approximately) optimize some unknown reward function over the trajectories. In particular, one traditionally assumes a trajectory’s reward is the sum of state rewards of the trajectory. This formalism offers a succinct mechanism to encode and generalize the goals of the demonstrator to new and unseen environments.

In the IRL framework, the problem of learning from demonstrations can then be cast as a Bayesian inference problem [6] to predict the most probable reward function. To make this inference procedure well-defined and robust to demonstration/modeling noise, Maximum Entropy [3] and Maximum Causal Entropy [4] IRL appeal to the principles of maximum entropy [7] and maximum causal entropy respectively [4]. This results in a likelihood over the demonstrations which is no more committed to any particular behavior than what is required for matching the empirically observed reward expectation. While this approach was initially limited to learning a linear combination of feature vectors, IRL has been successfully adapted to arbitrary function approximators such as Gaussian processes [8] and neural networks [9]. As stated in the introduction, while powerful, traditional IRL provides no principled mechanism for composing the resulting rewards.

To address this deficit, composition using soft optimality has recently received a fair amount of attention; however, the compositions are limited to either strict disjunction (do X or Y) [10] [11] or conjunction (do X and Y) [12]. Further, because soft optimality only bounds the deviation from simultaneously optimizing both rewards, optimizing the composition does not preclude violating safety constraints embedded in the rewards (e.g., do not enter the lava).

Recently, work has been done on inferring Linear Temporal Logic (LTL) by finding the specification that minimizes the expected number of violations by an optimal agent compared to the expected number of violations by an agent applying actions uniformly at random [1]. The computation of the optimal agent’s expected violations is done via dynamic programming on the explicit product of the deterministic Rabin automaton [13] of the specification and the state dynamics. A fundamental drawback of this procedure is that due to the curse of history, it incurs a heavy run-time cost, even on simple two state and two action Markov Decision Processes.

The closest work to ours, the inspiration for this paper, is the recent work on adapting maximum entropy inverse reinforcement learning to learn task specifications [2]; however, due to its use of the principle of maximum entropy rather than maximum causal entropy, this work is limited to settings where the dynamics are deterministic or agents with open-loop policies [3].

Finally, this work makes heavy use of reduced ordered binary decision diagrams [14], which while traditionally used for computer aided design, e.g. circuit verification, have also been used to efficiently implement value iteration in symbolic Markov decision processes [15].

Contributions: The primary contributions of this work are two fold. First, we leverage the principle of maximum causal entropy to provide a posteriori probability of a specification given a set of demonstrations. This formulation removes the deterministic and/or open-loop restriction imposed by prior work based on the principle of maximum entropy. Second, in an attempt to mitigate the curse of history, we propose using a reduced ordered binary decision diagram to encode the time unrolled Markov decision process that the soft-bellman backup is defined over.

Outline: In Sec 2, we define task specifications, probabilistic automata (Markov Decision Processes without rewards), Markov Decision Processes, and introduce Maximum Causal Entropy Inverse Reinforcement Learning. In Sec 3, we start by introducing the problem of specification inference from demonstrations and reduce the problem of assigning a posteriori probability to a specification to Maximum Causal Entropy Inverse Reinforcement Learning [4]. The remainder of the section adapts the initially naïve soft bellman backup formulation to work on reduced ordered decision diagrams. Finally, in Sec 4, we illustrate how to utilize the extensive literature on reduced ordered binary decision diagrams to implement the adapted soft bellman backup.

2 Background

We seek to learn task specifications from demonstrations provided by a teacher who executes a sequence of actions that probabilistically change the system state. For simplicity, we assume that the set of actions and states are finite and fully observed and that all demonstrations are a fixed length, \( \tau \in \mathbb{N} \). Formally, we begin by modeling the underlying dynamics as a probabilistic automaton.
Definition 1 A probabilistic automaton (PA) is a tuple $M = (S, s_0, A, \delta)$, where $S$ is the finite set of states, $s_0 \in S$ is the initial state, $A$ is the finite set of actions, and $\delta : S \times A \times S \rightarrow [0, 1]$ specifies the transition probability of going from $s$ to $s'$ given action $a$, i.e., $\delta(s, a, s') = \Pr(s' | s, a)$ and $\sum_{s' \in S} \Pr(s' | s, a) = 1$ for all states $s$.

A sequence of state/action pairs is called a trace. That is, for a trace, $\vec{x}$, of length $\tau \in \mathbb{N}$, $\vec{x}$ is an element of $(S \times A)^\tau$.

or alternatively a trajectory or behavior.

Note that probabilistic automata are equivalently characterized as $1/2$-player games where each round has the agent choose an action and then the environment samples a state transition outcome. In fact, this alternative characterization is implicitly encoded in the directed bipartite graph used to visualize probabilistic automata (see Fig 1b). In this language, we refer to the nodes where the agent makes a decision as a decision node and the nodes where the environment samples an outcome as a chance node.

Figure 1: Example dynamics and corresponding probabilistic automata.

Definition 2 A Markov Decision Process (MDP) is a probabilistic automaton endowed with a reward map from states to reals, $R : S \rightarrow \mathbb{R}$. This reward mapping is lifted to traces via,

$$R(\vec{x}) \stackrel{\text{def}}{=} \sum_{s, a \in \vec{x}} R(s).$$  \hspace{1cm} (1)

Given a MDP, the goal of an agent is to maximize the expected trace reward. Formally, we model an agent as acting according to a policy.

Definition 3 A policy, $\pi$, is a state indexed distribution over actions, $\Pr(a | s) = \pi(a | s)$.

In this language, the agent’s goal is equivalent to finding a policy which maximizes the expected trace reward. We shall refer to a trace generated by such an agent as a demonstration.

One of the reasons that MDPs have proven to be such a fruitful modeling formalism is the way rewards can succinctly encode complicated tasks, particularly when defined over state features, $f : S \rightarrow \mathbb{R}^n_+$. For example, $R$ is commonly taken to be a linear combination of feature weights, $R(s) = \theta \cdot f(s)$ for some $\theta \in \mathbb{R}^n$.

Inverse Reinforcement Learning. Given the incredible expressivity of MDPs, one naturally wonders about the inverse problem. Namely, given a set of demonstrations, find the reward that best “explains” the agent’s behavior, where by “explain” one typically means that under the conjectured reward, the agent’s behavior was approximately optimal. Notice however, that many undesirable rewards satisfy this property. For example, consider the reward $s \mapsto 0$, in which every demonstration is optimal.

In this paper, we shall choose to resolve this ambiguity by appealing to the principle of maximum causal entropy [4], which provides a likelihood distribution over demonstrations that matches any observed feature moments while (i) encoding the least amount of bias in the distribution (ii) ensuring that the agent’s predicted policy does not depend on the future. Concretely, as proven in [4], when an
agent is attempting to maximize the sum of feature state rewards, \( \sum_{t=1}^{T} \theta \cdot f(s_t) \), the principle of maximum causal entropy prescribes the following policy:

\[
Q_\theta(a_t, s_t) = \mathbb{E}[V_\lambda(s_{t+1}) \mid s_t, a_t] + \theta \cdot f(s_t)
\]

\[
V_\theta(s_t) = \ln \sum_{a_t} Q_\theta(a_t, s_t) = \text{softmax}_{a_t} Q_\theta(a_t, s_t)
\]

\[
\ln (\pi_\theta(a_t \mid s_t)) = Q_\theta(a_t, s_t) - V_\theta(s_t).
\]

Assuming that each demonstration is given i.i.d., what remains then is to find \( \theta \) that maximizes the product of the likelihoods of each demonstration. Finally observe that in the special case of scalar state features, \( f : S \to \mathbb{R}_{\geq 0} \), the maximum causal entropy policy (2) becomes increasingly optimal as \( \theta \in \mathbb{R} \) increases. In this setting, we shall refer to \( \theta \) as the agent’s rationality.

**Non-Markovian Rewards.** So far in our formalization, all trace rewards have been defined with respect to a Markovian (i.e., state based) reward map, \( R : S \to \mathbb{R} \); however, in practice, many interesting tasks are necessarily history dependent, e.g., touch a red tile and then a blue tile. A common trick within the reinforcement learning literature is to simply change the MDP and add the necessary history to the state so that the reward is then made Markovian.

However, in the case of inverse reinforcement learning, by definition, one does not know what the reward is. Therefore, one cannot assume to a priori know what history suffices. Further, for general rewards, including the entire history can result in an exponential blow up in the state space representation, i.e., the curse of history. Explicitly, we shall refer to the process of adding all history to a probabilistic automaton’s (or MDP’s) state as unrolling.

**Definition 4** For \( \tau \in \mathbb{N} \), a PA, \( M' = (S', s'_0, A', \delta') \), is called a \( \tau \)-unrolling of \( M = (S, s_0, A, \delta) \) if \( S' = \bigcup_{i=1}^{\tau} S^i \), \( s'_0 = s_0 \), \( A' = A \), and \( \delta'((s_0, \ldots, s_n)) = \delta(s_n) \).

Further, if \( R : S' \to \mathbb{R} \) is a non-Markovian reward over \( \tau \) length traces, then we endow the corresponding \( \tau \)-unrolled PA with the now Markovian Reward,

\[
R'(\{(s_0, \ldots, s_n)\}) = \begin{cases} R((s_1, \ldots, s_n)) & \text{if } n = \tau \\ 0 & \text{otherwise} \end{cases}
\]

Crucially, note that by construction, any trace (or policy) in the unrolled dynamics yields the same expected reward when interpreted in the original dynamics.

Finally, before continue, we briefly remark that in the 1½ player game formulation, the underlying probabilistic automata bipartite graph now forms a tree. Given the non-Markovian reward \( R \), this tree can be thought of as a decision tree of depth \( \tau \), where the interior decision and chance nodes act as before, and the leaves at depth \( \tau \), corresponding to \( \tau \)-length traces, are annotated with the correspond trace’s reward. We shall refer to these nodes as end nodes and denote by \( T[M] \) the decision tree corresponding to a \( \tau \)-unrolled MDP (see Fig 2a).

**Task Specifications.** As stated in the introduction, this work focuses on a subclass of non-Markovian trace rewards, referred to here as task specification rewards, whose additional structure enable composition and helps mitigate the curse of history.

**Definition 5** A trace reward is called a Task Specification Reward (TSR) if

\[
R(\bar{x}) \in \{0, \theta\}
\]

for some \( \theta > 0 \). The set of non-zero reward traces is called the task specification\(^a\),

\[
\varphi_R(\bar{x}) = \{ \bar{x} \in S^\tau \mid R(\bar{x}) \neq 0 \}.
\]

Further, we define true \( \overset{a}{\equiv} \) \( S^\tau \), \( \neg \varphi \overset{a}{\equiv} \) \( \text{true} \setminus \varphi \), and false \( \overset{a}{\equiv} \) \( \neg \text{true} \). A collection of specifications, \( \Phi \), is called a concept class.

\(^a\)or equivalently a finite trace property.
In practice, it is often conceptually easier to define a task specification first and then define a TSR as,

\[ R_{\varphi}(\xi) \overset{\text{def}}{=} \theta \cdot 1[\xi \in \varphi] \overset{\text{def}}{=} \begin{cases} 1 & \text{if } \xi \in \varphi \\ 0 & \text{otherwise} \end{cases}, \]

for some rationality coefficient \( \theta > 0 \). Finally, note that specifications themselves may be defined via a large range of formalisms, ranging from explicit set constructions to accepting automata over an abstract feature alphabet.

### 3 Specification Inference from Demonstrations

The primary task in this paper is to find the specification that best explains the behavior of an agent. We follow the authors in [2] and define our formal problem statement as:

**Definition 6 (Specification Inference from Demonstrations)** The specification inference from demonstrations problem is a tuple \((M, X, \Phi)\) where \(M = (S, s_0, A, \delta)\) is a probabilistic automaton, \(X\) is a (multi-)set of \(\tau\)-length traces drawn from an unknown distribution induced by a teacher attempting to demonstrate (satisfy) some unknown task specification within \(M\), and \(\Phi\) a concept class of specifications.

A solution to \((M, X, \Phi)\) is:

\[ \varphi^* \in \arg \max_{\varphi \in \Phi} \Pr(\varphi \mid M, X) \tag{7} \]

where \(\Pr(\varphi \mid M, X)\) denotes the probability that the teacher demonstrated \(\varphi\) given the observed traces, \(X\), and the dynamics, \(M\).

As discussed in the background section, up to a rationality coefficient, fixing a specification uniquely defines a family of TSRs. In the sequel, we propose approximately solving specification inference from demonstration problems using the following high-level algorithm.

1. Fix a prior distribution over specifications: \(\Pr(\varphi \mid M)\).
2. Sample a specification \(\varphi\) from the prior distribution.
3. Estimate the demonstration likelihood, \(\Pr(X \mid M, \varphi)\).
4. Return the specification with the highest posterior probability.

In particular, this paper focuses on estimating the demonstration likelihood given a particular specification \(\varphi\) using maximum causal entropy IRL.

**Naïve Reduction to IRL.** To start, recall that fixing a specification induces a TSR, \(R_{\varphi}(\xi) = \theta \cdot 1[\xi \in \varphi]\), for some rationality coefficient \(\theta\). Next, observe that using the demonstrations, we can estimate the empirical probability of the agent satisfying the task specification, \(p_{\varphi} = \mathbb{E}[1[\xi \in \varphi]] \). Thus, letting \(R'_{\varphi}\) denote the unrolled reward, we have an IRL instance with state feature, \(f(x) = \frac{1}{\theta} R'_{\varphi}(x)\), where the expected value of \(f\) is fixed to \(p_{\varphi}\). Substituting into (2) yields:

\[ Q_\theta(a_t, s_t) = \mathbb{E}_{P_{s_{t+1} \mid s_t, a_t}} [V_\lambda(s_{t+1} \mid s_t, a_t)] \]

\[ V_\theta(s_t) = \begin{cases} \theta \cdot R_\lambda(s_{t:T}) & \text{if } t = \tau \\ \text{softmax}_{a_t} Q_\theta(a_t, s_t) & \text{otherwise} \end{cases} \tag{8} \]

Note that because the unrolled MDP is always a tree, one could apply a naïve dynamic programming scheme over \(T[M]\) starting at the \(t = \tau\) leaves to compute \(Q_\theta\) and \(V_\theta\) (and thus \(\pi_\theta\)). Namely, in \(T[M]\), the chance nodes, which correspond to state/action pairs, are responsible for computing \(Q\) values and the decision nodes, which correspond to states waiting for an action to be applied, are responsible for computing \(V\) values. For decision nodes this is done by taking the softmax of the values of the child nodes. Similarly, for chance nodes, this is done by taking a weighted average of the child nodes, where the weights correspond to the probability of a given transition.

\[1\text{ Using the empirical satisfaction frequency, the accuracy of } p_{\varphi} \text{ could be estimated using a Chernoff bound.} \]
Further note that (i) the above dynamic programming scheme can be trivially modified to compute the average satisfaction probability of the maximum causal entropy policy and (ii) as the rationality coefficient $\theta$ increases, the probability of the agent satisfying the specification increases. Thus, by performing a binary search over $\theta$, one can find a maximum causal entropy policy consistent with the observed satisfaction probability. Finally, the likelihood of each demonstration can be computed by traversing the path in $T[M]$ corresponding to the trace and multiplying the corresponding policy and transition probabilities.

Thus, letting $\epsilon > 0$ denote the tolerance for $\theta$ and $|T[M]|$ denote the number of nodes in $T[M]$, the total run-time to compute the likelihood of the demonstrations using this naïve scheme is $O((\log(1/\epsilon) + |X|) \cdot |T[M]|)$.

**Reduced Ordered Decision Diagrams.** Of course, the problem with this naïve approach is that explicitly encoding the unrolled tree results in an exponential blow-up in the state space. A key insight in this paper is that by specializing to TSRs, one can leverage the extensive literature on reduced ordered decision diagrams, defined momentarily, to succinctly encode the unrolled dynamics in a manner that admits efficient dynamic programming.

First, recall that (action, outcome) pairs are ordered in time, and thus cannot in general be rearranged without changing the semantics of $T[M]$. We refer to a decision tree with a fixed variable order as an **ordered decision tree**, or more generally, an **ordered decision diagram**. Second, observe that, often, (i) ordered decisions are inconsequential (ii) can be interchanged and achieve the same result. For example, (i) if the task is to reach a goal, then after the goal is reached, all decisions are inconsequential (ii) if the goal is to move north-east within a gridworld, then first moving north and then east is equivalent to first moving east and then moving north.

These last two observations suggest applying the following two semantic preserving transformations: (i) Eliminate nodes whose children are isomorphic, i.e., inconsequential decisions (ii) Combine all isomorphic sub-graphs i.e., equivalent decisions. We refer to the limit of applying these two operations as a **reduced ordered decision diagram**. Further, the reduced variant of $T[M]$ shall be denoted, $\hat{T}[M]$.

As Fig 2b illustrates, reduced decision diagrams can be much smaller than their corresponding decision tree. For instance, since there are only two types end node rewards, the resulting reduced decision diagram has at most two end nodes. Exactly how much smaller the reduced diagram will be depends on many details including the candidate specification and the structure of the probabilistic automaton.

**Remark 3.1** Note that because the decisions are still ordered, the eliminated nodes can be recovered by seeing which nodes are skipped over.

Computationally, two problems yet remain. First is the question of how to construct $\hat{T}[M]$ without first constructing $T[M]$. Second is the question of how to adapt the naïve dynamic programming scheme to this compressed structure. In particular, because many interior nodes have been eliminated, and thus one must take care when applying (8). We shall start by addressing the latter problem.
Recall that in the variable ordering, nodes alternate between decision and chance nodes (i.e., agent and environment decisions), and thus alternate between taking a softmax and expectations of child values in (8). Next, recall that if a node is skipped in \( \hat{T}[M] \), then it must have been inconsequential and thus the resulting reward was independent of the decision made at that node. Thus, in \( T[M] \), the arguments to the softmax and expectations would have all had the same value. Letting \( \alpha \) denote the value of an eliminated node’s children yields the following identities.

\[
\text{softmax}(\alpha, \ldots, \alpha) = \ln(|A|) + \alpha \quad \text{and} \quad E[\alpha] = \alpha \tag{9}
\]

Of course, it could also be the case that a sequence of nodes is skipped in \( \hat{T}[M] \). Using (9), one can compute the change in value, \( \Delta \), that the eliminating a sequence \( n \) decision nodes and \( m \) chance nodes would have applied in \( T[M] \):

\[
\Delta(n, \alpha) = \ln(|A|^n) + \alpha = n \ln(|A|) + \alpha \tag{10}
\]

Further, letting \( \beta \) denote the maximum number of outcomes when applying an action, then each node operation takes at most, \( O(|A| + \beta) \) time. It follows then that the run-time of this dynamic programming scheme is \( O(|T[M]| \cdot (|A| + \beta)) \), where by denote the number of nodes in \( \hat{T}[M] \).

### 4 Reduction to Ordered Binary Decision Diagrams

In the next section, we illustrate how one can leverage the extensive literature and engineering in reduced ordered binary decision diagrams (ROBDD) to efficiently encode \( \hat{T}[M] \), while in practice avoiding the construction of \( T[M] \). We begin with the definition of an ROBDD.

**Definition 7** A reduced ordered binary decision diagram (ROBDD), is a representation of a Boolean predicate \( h(x_1, x_2, \ldots, x_n) \) as a reduced ordered decision diagram, where each decision corresponds to testing a bit \( x_i \in \{0, 1\} \).

One of the biggest benefits of the ROBDD representation of a Boolean function is the ability to build ROBDDs from a Boolean combinations of other ROBDDs. Namely, given two ROBDDs with \( n \) and \( m \) nodes respectively, it is well known that the conjunction or disjunction of the ROBDDs has at most \( n \cdot m \) nodes. Thus, in practice, if the combined ROBDD’s remain relatively small, Boolean combinations remain efficient to compute. Thus, in practice, one often does not construct the full binary decision tree.

Noting that elements of \( A \) can always be encoded as \( \log(|A|) \) length bit-strings, to make \( \hat{T}[M] \) a ROBDD, all that remains is to encode the original MDP’s transition function \( \delta \), and thus the environment’s decisions, as a Boolean function. Such an encoding can be achieved by modeling the environment’s decision as being the result of a sequence of fair coin flips. Formally, let \( R \in \{0, 1\}^q \) denote the random variable representing the result of flipping \( q \in \mathbb{N} \) fair coins, and let \( \hat{\delta} : S \times A \times \{0, 1\}^q \rightarrow S \) be a mapping from state, action, and coin flip outcome to the next state such that:

\[
\delta(s, a, s') \approx \Pr \left( \hat{\delta}(s, a, R) = s' \right) \tag{11}
\]

Given this transformation, \( \hat{T}[M] \) can be viewed as a Boolean function over an alternating sequence of action bit strings and coin flip outcomes determining if the task specification is satisfied. We denote the ROBDD encoding of \( \hat{T}[M] \) as \( \hat{T}[M] \). For example, in our gridworld example (Fig 1a), if elements of \( s \) are interpreted as pairs in \( \mathbb{R}^2 \), the right and down actions are interpreted as the unit vectors \((1, 0)\) and \((0, 1)\), and \( R \in \{0, 1\}^3 \) is interpreted as an unsigned int between 0 and 7, then,

\[
\hat{\delta}(s, a, R) = \begin{cases} 
s & \text{if } \max_i[(s + a)_i] > 1 \\
(0, 1) & \text{else if } R = 0 \\
 s + a & \text{otherwise} \end{cases} \tag{12}
\]

\(^2\)For a detailed explanation on how to systematically derive such an encoding, we refer the reader to [16].
One more (conceptual) change must be made to the dynamic programming scheme over $\hat{T}[M]$ given in the previous section. Namely first, note that in $\hat{T}[M]$, decision and chance nodes from $T[M]$ are now encoded as sequences of decision and chance nodes. For example, if $a \in A$ is encoded by $4$-length bit sequence $b_1 b_2 b_3 b_4$, then four decision’s are made by the agent before selecting an action. Notice however that one can still interpret decision and chance nodes as applying softmax and $E$ resp. In particular, recall that by definition,

$$\text{softmax}(\text{softmax}(\alpha_1, \alpha_2), \text{softmax}(\alpha_3, \alpha_4)) \overset{\text{def}}{=} \ln(e^{\ln(e^{\alpha_1}+e^{\alpha_2})} + e^{\ln(e^{\alpha_3}+e^{\alpha_4})})$$

$$= \ln\left(\sum_{i=1}^{4} e^{\alpha_i}\right) \overset{\text{def}}{=} \text{softmax}(\alpha_1, \ldots, \alpha_4),$$

and thus the semantics of the sequence decision nodes is equivalent to the decision node in $T[M]$. Similarly, recall that the coin flip are fair, and thus expectations are computed via $\text{avg}((\alpha_1, \ldots, \alpha_n) = \frac{1}{n}(\sum_{i=1}^{n} \alpha_i)$. Therefore, averaging over two sequential coin flips yields,

$$\text{avg}(\text{avg}(\alpha_1, \alpha_2), \text{avg}(\alpha_3, \alpha_4)) = \frac{1}{2} \left(\frac{1}{2}(\alpha_1 + \alpha_2) + \frac{1}{2}(\alpha_3 + \alpha_4)\right)$$

$$= \frac{1}{4} \sum_{i=1}^{4} \alpha_i = \text{avg}(\alpha_1, \ldots, \alpha_4),$$

which by assumption (11), is the same as applying $E$ on the original chance node. Finally, note that skipping over decisions needs to be adjusted slightly to account for sequences of decisions. Namely, recall that via (10), the corresponding change in value, $\Delta$, is a function of initial value, $\alpha$, and the number of agent actions skipped, i.e., $|A|^n$ for $n$ skipped decision nodes. Thus, in the ROBDD, since each decision node has two actions, skipping $k$ decision bits corresponds to skipping $2^k$ actions. Thus, if $k$ decision bits are skipped over in the ROBDD, the change in value, $\Delta$, becomes,

$$\Delta(k, \alpha) = \alpha + k \ln(s).$$

Further, note that $k$ can be computed in constant time while traversing the ROBDD. Thus, since the branching factor is always two, the run-time complexity of the dynamic programming scheme is linear in the size of $\hat{T}[M]$.

5 Conclusion and Future Work

Motivated by the problem of learning task specification from demonstrations, we have adapting the principle of maximum causal entropy to provide a posterior probability to a candidate task specification given a multi-set of demonstrations. Further, to exploit the structure of task specifications, we proposed an algorithm that computes this likelihood by first encoding the unrolled Markov Decision Process as a reduced ordered binary decision diagram (ROBDD). As illustrated on a few toy examples, ROBDDs are often much smaller than the unrolled Markov Decision Process and thus could enable efficient computation of maximum causal entropy likelihoods, at least for well behaved dynamics and specifications.

Nevertheless, three major questions remain unaddressed by this work. First is the question of how to select which specifications to compute likelihoods for. Second is how to reuse ROBDDs across specifications, particular specifications which are compositions of simpler specifications. Third, is the question of the real life savings of these optimizations, e.g., a set of benchmarks illustrating empirical run-times for various domains and specifications.

Finally, additional future work includes extending the formalism to infinite horizon specifications, continuous dynamics, and characterizing the optimal set of teacher demonstrations.

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