Serially concatenated extended CPM scheme based on TDRSS

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Abstract. Tracking and Data Relay Satellite System (TDRSS) can meet the needs of Low Earth Orbit (LEO) satellite real-time transmission, but it also has large Doppler shift, transmission delay and channel fading. Continuous Phase Modulation (CPM), with excellent properties of both bandwidth and energy efficient, are very suitable for nonlinear and/or fading channels such as satellite channel due to the constant envelope nature. In this paper, the CPE in the SCCPM scheme is replaced by the designed ECE to construct a new scheme called SC-ECPM scheme, in order to enhance the robustness of the signal. Compared with previous SCCPM scheme, the proposed system shows an improvement concerning the convergence threshold and bit-error-rate (BER).

1. Introduction

Tracking and Data Relay Satellite System (TDRSS) is a novel space-based tracking, telemetry and command system[1], which is a satellite communication method which using Geostationary Earth Orbit (GEO) relay satellite as a repeater to transmit the signals of Low Earth Orbit (LEO) satellites and other aircraft. The power and bandwidth resources of satellite communication are very limited. In order to meet the demand of higher bandwidth, the working frequency of satellite communication has developed from the original L/S/X band to the current Ku/Ka band. At the same time, it also brings a series of challenges, such as large transmission delay and channel fading.

Continuous Phase Modulation (CPM) has shown a great prospective in the wireless and mobile communication due to excellent properties of both bandwidth and energy efficient[2]-[5]. In consequence of the constant envelope nature of CPM, these schemes are very suitable for nonlinear and/or fading channels such as satellite channels because non-linear transmitter power amplifiers can be used for such systems. Much work on coding for CPM has been done to improve its energy efficiency[6]-[10].

Combining coding with CPM to achieve coding gain is of practical interest. The usual approach has been to combine an external binary convolutional channel encoder (CE) with an M-ary CPM scheme via a binary-to-M-ary mapper. It has been shown that a CPM scheme with modulation index \( h=K/P \) can be modeled by a CPE followed by a MM, where CPE is a linear encoder over the ring of integers modulo \( P \)[11]. Rimoldi propose a more natural way to encode CPM, namely use a CE over the ring of integers modulo \( P \)[12]. Now the CE and the CPE are over the same algebra allows the state of CPE to be fed back and no mapper is needed. Hereafter the combination of the CE and the CPE which are over the same algebra is called the extended channel encoder (ECE). A similar approach to encoding
CPM has been taken by using a systematic ring convolutional encoder combined with a CPE that is in feedback-free form has been considered[13]. In the two approaches, only rate-1/2 coded CPM was considered.

In this paper, rate-1 convolutional encoders over the ring of integers modulo M are combined with M-ary CPM schemes with modulation index \( h = l/M \). More specifically, we consider modulo-2 ECE with rate-1 and modulation index \( h = l/2 \). The serially concatenated CPM (SCCPM), i.e., coded and interleaved CPM with iterative decoding was proposed in[14]. The designed ECE were used to replace the CPE in SCCPM schemes, we call the new schemes SC-ECPM[15]. In order to compare with previous SC-CPM schemes, the extrinsic information transfer (EXIT) chart is given to predict the convergence threshold.

This paper is organized as follows. In order to establish notation, we describe the CPM signal following in Section 2. Section 3 introduces the ECE and the SC-ECPM scheme. In Section 4, the SC-ECPM scheme is compared with the previous SCCPM scheme. The EXIT chart used to predict the convergence threshold and the BER performance is also given. Numerical results and conclusion are presented in Section 5. It is shown that compared to the SCCPM schemes, up to 0.5dB coding gains can be achieved in the SC-ECPM schemes.

2. Decomposed model of CPM
A CPM modulator is a finite-state machine that generates a continuous-phase, constant-envelope waveform which depends on the input symbols and internal states, the CPM signal can be described as

\[
s(t,u) = \sqrt{\frac{2E_s}{T}} \cos\left[ 2\pi ft + \psi(t,u) + \varphi_0 \right]
\]

whose phase

\[
\psi(t,u) = 4\pi h \sum_{k=-\infty}^{\infty} u_k q(t-kT)
\]

depends on the input information symbols \( u_k \in \{0,1,2,\ldots,(M-1)\} \), where \( M=2m \) is the size of the input alphabet. \( E_s, T, q(t), f, \varphi \) denote symbol energy, symbol interval, phase pulse, carrier frequency and initial phase respectively. For a given modulation index \( h = K/P \) (\( K \) and \( P \) are relatively prime positive integers), we always use tilted-phase[5], which is given by

\[
\psi(\tau+nT,u) = 2\pi h \left[ \sum_{k=0}^{L-1} u_k \mod P \right] + 4\pi h \sum_{k=0}^{L-1} u_{k+n} q(\tau+kT) + W(\tau)
\]

where \( W(\tau) \) is a data-independent function, \( L \) is the number of intervals under the varying part of the phase pulse \( q(t) \).

A CPM scheme can be decomposed into the cascade of a Continuous Phase Encoder (CPE) and a Memoryless Modulator (MM)[11] as in figure 1. Such the decomposition has two advantages. Firstly, we can study the “encoding” operation independently from the modulation. Secondly, we can model the cascade of the isolate MM, the waveform channel and the demodulator as a discrete memoryless channel (DMC).

The output

\[
c_n = [v_n, u_{n-L+1}, \ldots, u_n]
\]

from the CPE is fed to the MM, which produces one of \( P^M \) signals. Where

\[
v_n = \sum_{i=0}^{N-1} u_i \mod P
\]

denotes the accumulated value of the previous data symbols which can take only \( P \) values.
3. The ECE and SC-ECPM scheme

In this Section, we describe structure of the 2-ary ECE with rate-1 and the SC-ECPM scheme using the ECE.

3.1 The ECE

The CPM schemes with modulation index $h=K/P$ can be decomposed into a CPE followed by a MM, where the CPE is a linear encoder over the ring of integers modulo $P$. If we use a CE over the ring of integers modulo $P$, the CE and the CPE are over the same algebra. So they can naturally combine with each other and no mapper is needed, the combination is again a convolutional encoder called ECE. In this paper, the ECE concatenated with MM is called extended CPM (ECPM). More specially, the CE over ring of integers modulo $M$ are combined with $M$-ary CPE whose modulation indexes are of the form $h=1/M$. In this paper, the ECE with $M=2$ were mainly considered.

The structure of ECE changes as the CE or the CPE varying. The $v$ delays of the CE and the $L$ delays of the CPE determine the overall number of states of the ECE. The tap coefficients of the CE and the feedback coefficients from the CPE to the CE determine the output of the CE. And the rate of the ECE also depends on the rate of the CE.

Consider a rate-1 CE over $Z_2$ combined with the CPE whose modulation index is $h=1/2$. The generator polynomials (feedback polynomials $G_b$; feedforward polynomials $G_f$) of the CE are given in octal numbers. The feedback coefficients (terminal coefficients $F_t$, initiative coefficients $F_i$) from the CPE to the CE are also given in octal numbers. The structure of the ECE is seen in figure 2. The parameters of the ECE can be described by table 1.
### Table 1. Parameters of ECE

| \( M \) | \( v \) | \( L \) | Rate | Generator polynomials | Feedback coefficients |
|---|---|---|---|---|---|
| 2 | 3 | 3 | 1 | \((G_b; G_f)_8\) | \((F_b; F_f)_8\) |

#### 3.2 The SC-ECPM Scheme

The designed ECE were used into SCCPM scheme as the substitutes of the CPE, the new scheme can be depicted as figure 3.

![Block diagram of the SC-ECPM scheme. The \( \Pi \) denotes the interleaver.](image)

In this scheme, an outer convolutional encoder connected to the ECE using an interleaver to form a serially concatenated convolutional encoder which is known as SCC[16]. The iterative receiver operates between the ECE soft-input-soft-output (SISO) decoder and convolutional code (CC) SISO decoder can get very good performance.

### 4. Performance of the SC-ECPM scheme

In this Section, the EXIT chart is used to predict the convergence threshold for SC-ECPM schemes. The convergence threshold is the lowest signal noise ratio (SNR) at which the BER can be made arbitrarily small by increasing the interleaver size and the number of iterations. With it, system performance can be predicted.

### Table 2. Parameters of ECE

| \( M \) | \( v \) | \( L \) | Rate | Generator polynomials | Feedback coefficients |
|---|---|---|---|---|---|
| 2 | 1 | 2 | 1 | \((G_b; G_f)_8\) | \((F_b; F_f)_8\) |

The ECE were designed according to parameters shown in table 2. Then the designed ECE is serially concatenated with the outer convolutional encoders to construct a SC-ECPM scheme, and the raised cosine (RC) frequency pulse is considered. In this paper, a peculiar code is given as the outer code in the SC-ECPM scheme, which is a recursive systematic convolutional (RSC) code. The generator polynomials of the code given in octal numbers are of the form \((n, n)\), such as \((7,7)\) code and \((3,3)\) code.
Figure 4. EXIT chart for SC-ECPM and SCCPM schemes

The EXIT chart is shown in figure 4. The solid thin line is the trajectory of the rate-1/2 (7,7) code. The solid heavy line denotes the decoding trajectory of the ECPM with SNR \((E_b/N_0)\) 0.5dB, dashed heavy line CPM with SNR 3 dB. \(E_b\) is energy per information bit. CPM is binary 2RC with \(h=1/2\).

The convergence threshold of SCCPM scheme is 3dB, while the convergence threshold of the SC-ECPM scheme is 0.5dB. Compared to the SCCPM scheme, the SC-ECPM scheme improves the convergence threshold of the system up to 2.5dB.

In SCCPM schemes, the (7,5) code is always used as outer code rather than the (7,7) code. The EXIT chart of SCCPM scheme with (7,5) code is given in figure 5.

Figure 5. EXIT chart for SCCPM scheme.

The solid heavy line denotes the decoding trajectory of the CPM with SNR \((E_b/N_0)\) 1dB, dashed heavy line binary CC (7,5). The dashed thin line is the trajectory of the CPM with SNR 2 dB. \(E_b\) is energy per information bit. CPM is binary 2RC with \(h=1/2\).

We can observe that the convergence threshold of this system is 1dB. So we can say that, compared to the SCCPM scheme with (7,5) code, the SC-ECPM scheme with (7,7) code also convergences earlier, it improves the convergence threshold of the system up to 0.5dB.

To make the performance improvement of the system more convincive, the comparison of BER performance between the SCCPM scheme with (7,5) code and the SC-ECPM scheme is given in figure 6.
Figure 6. Bit error rate for SCCPM and SC-ECPM schemes at different SNR ($E_b/N_0$). CPM is binary 2RC with $h=1/2$. The size of interleaver is 2048 bits and the number of iterations is 50. Simulations are over the additive white Gaussian noise (AWGN) channel.

As the figure 6 showing, the BER curve of the SC-ECPM scheme with (7,7) code is more steep than the other two. The SC-ECPM scheme features a coding gain of roughly 0.3 dB over the SCCPM scheme with (7,5) code at BER=$10^{-4}$. Compared to the SCCPM scheme with (7,7) code, the coding gain is up to 2.7 dB.

5. Numerical results and conclusion

5.1 Numerical Results

Some other SC-ECPM schemes with different ECE also show an improvement on the convergence threshold and BER performance compared with the SCCPM scheme with (7,5) code, the coding gains of the systems are partly listed in table 3.

Simulations have been performed over AWGN channel and the RC frequency pulses are used in all systems.

Table 3. The SC-ECPM schemes with different ECE and coding gains

| $M$ | $v$ | $L$ | Rate | Generator polynomials | Feedback coefficients | Coding gain |
|-----|-----|-----|------|-----------------------|-----------------------|-------------|
| 2   | 1   | 2   | 1    | (2;1)                 | (2,2)                 | 0.5         |
|     |     |     |      | (2;1)                 | (2,3)                 | 0.2         |
|     |     |     |      | (2;1)                 | (3,1)                 | 0.5         |
|     |     |     |      | (2;3)                 | (2,2)                 | 0.2         |
|     |     |     |      | (3;1)                 | (1,2)                 | 0.5         |
|     |     |     |      | (3;3)                 | (1,1)                 | 0.2         |
| 2   | 2   | 2   | 1    | (4;5)                 | (5,2)                 | 0.2         |
|     |     |     |      | (6;7)                 | (5,1)                 | 0.2         |
|     |     |     |      | (7;5)                 | (1,2)                 | 0.2         |
|     |     |     |      | (7;7)                 | (4,1)                 | 0.2         |

5.2 Conclusion

In this letter, the 2-ary ECE are mainly considered, which are the combination of the CE with rate-1 and the 2-ary CPE with modulation index $h=1/2$. The SC-ECPM schemes in which the CPE are
replaced by the designed ECE show an improvement on convergence threshold and BER performance. In some SC-ECPM schemes, the coding gain is up to 0.5dB.

It can be seen from the results that, the SC-ECPM scheme has a significant improvement in coding gain and can effectively enhance the robustness of signals in satellite communication.

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