First Search for the $K_L \to \pi^0 \gamma$ Decay

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(Dated: June 29, 2020)

We report the first search for the $K_L \to \pi^0 \gamma$ decay, which is forbidden by Lorentz invariance, using the data from 2016 to 2018 at the J-PARC KOTO experiment. With a single event sensitivity of $(7.1 \pm 0.3\text{stat.}\pm 1.6\text{syst.}) \times 10^{-7}$, no candidate event was observed in the signal region. The upper limit on the branching fraction was set to be $1.7 \times 10^{-7}$ at the 90% confidence level.

The $K_L \to \pi^0 \gamma$ decay is forbidden by the conservation of angular momentum. In the $K_L$ rest frame, the spin of a massless photon must be polarized along the decay axis, but the back-to-back configuration of two-body decays does not allow the parallel component of the orbital angular momentum. In the broader context, $K_L \to \pi^0 \gamma$ threatens Lorentz invariance and gauge invariance [1]. Such restrictions on $K_L \to \pi^0 \gamma$ provide the opportunity to search for new physics beyond the standard model (SM). In particular, as Ref. [2] suggests, similarly to experiments such as ones using optical resonators [3, 4], Lorentz invariance should be tested in short distances. Several scenarios predict a finite rate of the $K_L \to \pi^0 \gamma$ decay [1, 5].

Using charged kaons, the E949 experiment at BNL searched for the $K^+ \to \pi^+ \gamma$ decay and set the upper limit on the branching fraction to be $2.3 \times 10^{-9}$ [6] at the 90% confidence level (C.L.); no measurements have been made for neutral kaons.

The KOTO experiment is being carried out using 30 GeV Main Ring accelerator at J-PARC in Ibaraki, Japan. A $K_L$ beam was produced by protons hitting a gold target, and was transported into the KOTO detector at an angle of $16^\circ$ from the primary beam [7]. Photons in the beam were removed by a 35-mm-thick lead plate placed in the upstream, and charged particles were removed by a sweeping magnet. The solid angle of the neutral beam after collimation was 7.8 $\mu$sr, and the size was $8 \times 8\text{ cm}^2$ at 20 m downstream from the target. At the exit of the beam line, the peak of $K_L$ momentum was 1.4 GeV/c and the flux was $5 \times 10^7$ per $2 \times 10^{14}$ protons on target (POT), corresponding to 7 MHz with beam power of 50 kW from a measurement of $K_L \to 3\pi^0$ and $K_L \to 2\pi^0$ decays.

The primary purpose of the KOTO experiment is to study the CP-violating $K_L \to \pi^0 \nu \bar{\nu}$ decay, which is suppressed in the SM and the branching fraction, $\mathcal{B}(K_L \to \pi^0 \nu \bar{\nu})$, is predicted to be $(3.0 \pm 0.3) \times 10^{-11}$ [8]. The signature of $K_L \to \pi^0 \nu \bar{\nu}$ is $2\gamma + nothing$; hence the KOTO detector consisted of a fine-grained electromagnetic calorimeter and hermetic veto counters surrounding the decay volume. Thus, the apparatus is ideal when searching for the $K_L \to \pi^0 \gamma$ decay.

Figure 1 shows the sectional view of the KOTO detector, in which the $z$ axis is in the center of the beam line. The decay volume was kept in vacuum of $10^{-5}$ Pa to suppress interactions of neutrons in the beam with the atoms in the air. An electromagnetic calorimeter (CSI), consisting of 2716 50-cm-long undoped Cesium Iodide crystals, measured the energies and hit positions of incident photons from $K_L$ decays. The central and outer regions of CSI were made of $2.5 \times 2.5\text{ cm}^2$ and $5.0 \times 5.0\text{ cm}^2$ crystals in cross section, respectively [9, 10]. The crystals were stacked inside a 1.9-m-diameter cylinder, leaving a central $20 \times 20\text{ cm}^2$ hole for the beam path. The FB, MB, and IB [11] were lead-scintillator sandwich counters, hermetically covering the decay volume to veto extra particles from $K_L$ decays. The inner surfaces of IB, MB, and the beam hole of CSI were covered with plastic scintillators (their thicknesses were 5 mm, 10 mm, and 3 mm, respectively) to veto charged particles. Two layers of 3-mm-thick plastic scin-
illulation counters (CV) [12] were placed upstream of CSI to veto charged particles entering it. To veto charged particles escaping through the beam hole of CSI, three layers of wire chambers were placed downstream of CSI in the beam (newB-HCV). To veto photons passing through the beam hole, four sets of collar-shaped undoped Cesium Iodide counters (CC03, CC04, CC05, and CC06) were placed. To veto photons passing through the beam hole and staying in the beam, sixteen modules, each made of lead and aerogel (BHPV) [13], and four modules, each made of lead and acrylic plates (BHGC), were placed downstream of CSI. To veto particles going upstream, a counter made of undoped Cesium Iodide crystals (NCC) was placed inside of FB. The waveform of the signal from all of the detector components was recorded with either 125 MHz or 500 MHz digitizers. Details of the KOTO detector are available in Refs. [14, 15].

Data taken in the periods of from May to June 2016, from May to July 2017, and from January to February 2018 with the proton beam power of 42–50 kW, corresponding to $2.8 \times 10^{18}$ POT in total, was used in this analysis. The trigger required an energy deposit of $> 550$ MeV in CSI with no coincident signals in IB, MB, CC03, CC04, CC05, CC06, CV, and NCC. The online energy thresholds for the veto counters were set sufficiently higher than those used in the offline analysis to avoid acceptance loss. With the CSI, the number of clusters was calculated online. A cluster is defined as a collection of contiguous crystals with energies deposited larger than 22 MeV for the small and large crystals, respectively. The clusters were required to be outside of the region of the beam hole: $\max(|x|, |y|) > 150$ mm.

The decay vertex position of the $\pi^0$ along the beam, $z = z_{\text{vtx}}^\pi$, was first reconstructed assuming two of the three clusters were from the $\pi^0$, the decay position was along the $z$ axis, and the invariant mass, $M_{\gamma_1, \gamma_2}$, was equal to the nominal $\pi^0$ mass. There were three combinations; the one with the smallest absolute magnitude of two vertex displacement, $\Delta z_{\text{vtx}} = z_{\text{vtx}}^\pi - z_{\text{vtx}}^\gamma$, was selected, where $z_{\text{vtx}}^\pi$ was calculated by assuming the invariant mass of all the three $\gamma$’s equals the nominal mass of $K_L$. The four-momenta of the three $\gamma$'s were then reconstructed assuming that they are produced at $z_{\text{vtx}}^\gamma$.

The optimization of selection criteria (cuts) and estimation of acceptances were based on the Monte Carlo (MC) simulation using the GEANT4 package [18–20]. To reflect the real beam-related activities, the MC events were further overlaid with random trigger data taken during physics data collection.

To avoid bias we adopted a blind analysis technique; the signal region (SR) had been defined in the two-dimensional space of ($z_{\text{vtx}}^\pi, M_{\gamma_1, \gamma_2}$), and the selection criteria were determined using a data set with all the events in the SR removed. To gain the largest possible efficiency for $K_L \rightarrow \pi^0\gamma$ while suppressing the background contribution, the SR was defined to be $1500 \text{ mm} < z_{\text{vtx}}^\pi < 3500 \text{ mm}$ and $490 \text{ MeV} < M_{\gamma_1, \gamma_2} < 520 \text{ MeV}$. The side band regions were used as control regions (CRs). The region referred to as CR2, defined as $3500 \text{ mm} < z_{\text{vtx}}^\pi < 6200 \text{ mm}$ and $400 \text{ MeV} < M_{\gamma_1, \gamma_2} < 490 \text{ MeV}$, dominated by $K_L \rightarrow 2\pi^0$ decays, was used to calculate the $K_L$ yield.

The shape of the cluster in the $x$-$y$ plane was required to be consistent with the shape of electromagnetic shower from a single photon obtained by the MC simulation. The MC template of the nominal energy deposits and their standard deviations in crystals was prepared as a function of the incident angle of $\gamma$ and the observed cluster energy, and was used to compute a $\chi^2$ value (shape $\chi^2$). We required all the $\gamma$ cluster candidates to satisfy $\chi^2 < 5.0$. These requirements discriminated a hadronic cluster due to neutrons or a fusion cluster from photon overlaps in close proximity. To further remove clusters produced by neutrons, a neural network technique (NN) [21] was used to distinguish $\gamma$ clusters from neutron clusters based on the information on two-dimensional cluster shape, relative

FIG. 1. Sectional view of the KOTO detector. The $K_L$ beam, pointing in the $+z$ direction in the figure, was transported from the left side. The names without (with) underline were for neutral (charged) particles. The origin of $z$ axis is the upstream edge of FB.
energies of crystals, energy-weighted $xy$-position of the cluster, timings of the observed signals in crystals, and the $\gamma$'s incident angle. All the $\gamma$'s were required to have a likelihood of more than 0.8, which corresponds to 90% efficiency for $\gamma$'s and $\times 33$ reduction for neutrons.

To suppress other $K_L$ decays, we required no in-time signals in the veto counters above each threshold. In particular, we imposed stringent energy thresholds of 1 MeV in the three barrel counters (FB, MB, and IB) and NCC. After imposing all the veto cuts, the $K_L \rightarrow 2\pi^0$ decay was turned out to be the largest contribution of all the background sources. This decay mode could be a background if a photon with a small energy was undetected by the veto counters or two of the four clusters in CSI fused. To suppress this contribution, the two vertex displacement was required to satisfy $-100 \text{ mm} < \Delta z_{\text{vtx}} < 200 \text{ mm}$ as shown in Fig. 2a. Furthermore, the minimum photon energy of three $\gamma$'s ($E_{\gamma \text{min}}$) was required to be larger than 600 MeV, as shown in Fig. 2b. The $K_L$ momentum was calculated as a sum of momenta of $\gamma_0, \gamma_1$, and $\gamma_2$, and its transverse momentum and polar angle with respect to the $z$ axis were required to be less than 100 MeV/c and 4°, respectively ($K_L$ direction cuts). Figures 3a and 3b show the reconstructed mass ($M_{\pi^0\gamma}$) versus $\pi^0$ decay vertex ($z_{\text{vtx}}$) plots after imposing the cuts described above for the $K_L \rightarrow \pi^0\gamma$ decay and other $K_L$ decays generated by MC, respectively. A summary of the estimated acceptances for the $K_L \rightarrow \pi^0\gamma$, $K_L \rightarrow 2\pi^0$, and $K_L \rightarrow 3\pi^0$ decays at each step by the MC samples is shown in Table I. From the indices of 1 to 10, the accumulated acceptances are shown, whereas for 11 and 12, cuts of 1 to 8 are included.

Table II summarizes the expected number of background events in the SR. The contribution of the $K_L \rightarrow 2\pi^0$ events in the SR was estimated to be:

$$N_{\gamma \text{MC}}^{\text{SR}} \frac{p_{2\pi^0}}{N_{\gamma \text{CR2}}^{\text{CR2}}} \times \frac{N_{\text{CR2}}}{N_{\gamma \text{MC}}},$$

where $N_{\gamma \text{obs}}^{\text{CR2}} = 528$ is the number of the observed events in the CR2 region with an energy threshold for photons of $E_{\gamma \text{min}} > 300$ MeV, $N_{\gamma \text{MC}}^{\text{SR}}(N_{\gamma \text{MC}}^{\text{CR2}})$ is the number of events in the corresponding region with an energy threshold of $E_{\gamma \text{min}} > 600$ MeV (300 MeV) estimated by the MC simulation, and $p_{2\pi^0}^{\text{CR2}} = 97\%$ is the purity of the $K_L \rightarrow 2\pi^0$ decay in the CR2.

The number of $K_L \rightarrow 3\pi^0$ events in the SR was estimated based on another control region dominated by the $K_L \rightarrow 3\pi^0$ decays, $M_{\pi^0}^{\text{obs}} = (490 \text{ MeV})^2$, where $M_{\pi^0}^2 = (p_i + p_j)^2$, and $p_i$ ($i = 0, 1, 2$) are the four-vectors of the three $\gamma$'s [23].

Using the number of events in this region ($N_{\gamma \text{obs}}^{\text{CR}} = 108$ and $N_{\gamma \text{MC}}^{\text{SR}}(N_{\gamma \text{MC}}^{\text{CR}}) < 4.3 \times 10^{-3}$ at 68% C.L., we estimated the number of events in the SR to be less than 0.5 at the 68% C.L.

If the $K_L \rightarrow 2\gamma$ decay was coincident with an accidental hit in CSI, it could become a background. We generated the MC samples of the $K_L \rightarrow 2\gamma$ decay, corresponding to 18 times the experimental data, and no events satisfied the cuts.

Another type of background was the $\pi^0$ production by an interaction of beam halo neutrons in NCC, where two photons from the $\pi^0$ decay entered CSI with an additional accidental hit. We produced a MC sample and confirmed that the $\Delta z_{\text{vtx}}$ requirement removed all the events in the SR setting the upper limit as 0.02 at the 68% C.L.

The $K_L \rightarrow \pi^0\gamma\gamma$ decay mode could be a background to $K_L \rightarrow \pi^0\gamma$ if one of the $\gamma$ energies was soft in the laboratory frame. However, due to its small branching fraction of $B(K_L \rightarrow \pi^0\gamma\gamma) = 1.27 \times 10^{-6}$ [22], its contribution was negligible.

We also studied the contributions from $K_L$ decays with
TABLE I. Acceptances of the $K_L \rightarrow π^0γ$, $K_L \rightarrow 2π^0$, and $K_L \rightarrow 3π^0$ decays at each step of event selection.

| Index | Selection | $K_L \rightarrow π^0γ$ | $K_L \rightarrow 2π^0$ | $K_L \rightarrow 3π^0$ |
|-------|-----------|----------------------|----------------------|----------------------|
| 1     | $K_L$ decay | 9%                   | 9%                   | 9%                   |
| 2     | Geometry and trigger | $2.2 \times 10^{-3}$ | $3.1 \times 10^{-4}$ | $1.5 \times 10^{-5}$ |
| 3     | Shape $χ^2$ of clusters | $2.0 \times 10^{-3}$ | $2.4 \times 10^{-4}$ | $1.1 \times 10^{-5}$ |
| 4     | $xy$ position of clusters | $1.9 \times 10^{-3}$ | $2.2 \times 10^{-4}$ | $9.7 \times 10^{-7}$ |
| 5     | $K_L$ direction | $1.8 \times 10^{-3}$ | $2.0 \times 10^{-4}$ | $5.8 \times 10^{-7}$ |
| 6     | Veto | $7.1 \times 10^{-4}$ | $1.9 \times 10^{-5}$ | $1.5 \times 10^{-5}$ |
| 7     | Separation of $γ/n$ with NN | $5.1 \times 10^{-4}$ | $1.3 \times 10^{-5}$ | $8.2 \times 10^{-6}$ |
| 8     | $Δv_{τπ}$ | $4.9 \times 10^{-4}$ | $4.2 \times 10^{-5}$ | $9.1 \times 10^{-6}$ |
| 9     | $E_{min}^π > 300$ MeV | $2.4 \times 10^{-4}$ | $1.0 \times 10^{-5}$ | $4.4 \times 10^{-5}$ |
| 10    | $E_{min}^π > 600$ MeV | $5.0 \times 10^{-5}$ | $4.1 \times 10^{-5}$ | $4.7 \times 10^{-5}$ |
| 11    | CR2, $E_{min}^π > 300$ MeV | - | $8.9 \times 10^{-5}$ | $1.2 \times 10^{-10}$ |
| 12    | SR, $E_{min}^π > 600$ MeV | $(2.11 \pm 0.03) \times 10^{-5}$ | $(5.6 \pm 1.8) \times 10^{-10}$ | - |

* A probability that $K_L$ decay occurs in the decay volume.

TABLE II. Expected numbers of backgrounds in the signal region. The upper limits are at 68% C.L.

| Source | Number of events |
|--------|-----------------|
| $K_L \rightarrow 2π^0$ | $0.32 \pm 0.10$ |
| $K_L \rightarrow 3π^0$ | $< 0.5$ |
| $K_L \rightarrow 2γ$ | $< 0.06$ |
| Neutron | $< 0.02$ |
| $K_L \rightarrow π^0γγ$ | $0.02 \pm 0.002$ |
| Other $K_L$ decays | $< 0.04$ |
| Total | $0.34 \pm 0.10$ (1.0) |

* The total number of events and its uncertainty in the SR only include ones with central values, because all of the upper limits came from the limited statistics of MC samples. If the contributions of all the sources are assumed, the upper limit of the number of backgrounds in the SR is 1.0 at 68% C.L.

TABLE III. Summary of uncertainties in the single event sensitivity.

| Source | Uncertainty[%] |
|--------|----------------|
| Offline veto | 17 |
| Kinematic selection | 12 |
| Online veto | 6.4 |
| Online cluster counting | 1.8 |
| Shape $χ^2$ and $γ/n$ separation with NN | 1.5 |
| Geometrical | 1.5 |
| Clustering | 1.0 |
| Reconstruction | 0.3 |
| $B(K_L \rightarrow 2π^0)$ | 0.6 |
| Statistics for normalization | 4.4 |
| Total | 4.4_{stat} + 22_{syst} |

$K_L \rightarrow 3π^0$ decays in each region, respectively, and $B(K_L \rightarrow 2π^0) = 8.64 \times 10^{-4}$ and $B(K_L \rightarrow 3π^0) = 19.5\%$ are the branching fraction of the $K_L \rightarrow 2π^0$ and $K_L \rightarrow 3π^0$ decays, respectively [22]. The obtained $SES$ was $(7.1 \pm 0.3_{stat} \pm 1.6_{syst}) \times 10^{-8}$, where the first and second uncertainties are statistical and systematic, respectively.

The various sources of uncertainties on $SES$ are summarized in Table III. The largest contribution came from the discrepancies of the offline veto acceptances between data and MC. The systematic uncertainty of a given offline cut was calculated using a double ratio:

$$r = \frac{n_{data}/n_{MC}}{n_{data}/n_{MC}}$$

where $n_{data,MC}$ are the numbers of events after imposing all the vetoes, and $n_{data,MC}$ are the corresponding numbers when one of the vetoes was removed. The deviation of $r$ from 1 was the systematic uncertainty from the offline veto. The quadratic sum of all the vetoes was the total systematic uncertainty due to offline veto. The second largest effect came from the systematic uncertainty of the kinematic selection described before. Similarly to the offline veto cuts, we relaxed one of the kinematic cuts and compared the double ratio between data and MC. This uncertainty was mainly caused by...
of $K_L \rightarrow 2\pi^0$ was taken from the PDG value [22]. The total statistical and systematic uncertainties were 4.4% and 22%, respectively.

After determining the cuts described above, we unmasked the SR and observed no candidate events, as shown in Fig. 4. The discrepancy between the number of observed events and the MC simulation in the upper right region could be explained by the limited statistics of the simulated $K_L \rightarrow 3\pi^0$ decay sample. In fact, when we loosened the cut on the minimum photon energy ($E_{\gamma}^{\text{min}}$) from 600 MeV to 300 MeV, the contribution from the $K_L \rightarrow 3\pi^0$ and $K_L \rightarrow 2\pi^0$ decays by the MC simulation increased to $41 \pm 7$ and $47 \pm 1$, respectively, in which uncertainties were statistical only, and the sum of them was consistent with the observation of 96 events. Taking into account the systematic uncertainty of $S E S$ [24], the upper limit was set to $B(K_L \rightarrow \pi^0\gamma) < 1.7 \times 10^{-7}$ at the 90% C.L. This is the first experimental upper limit set on the $K_L \rightarrow \pi^0\gamma$ decay.

In conclusion, we searched for the $K_L \rightarrow \pi^0\gamma$ decay, which is forbidden by Lorentz invariance and gauge invariance, for the first time. From the data collected between 2016 and 2018, we observed no candidate events in the signal region. The first upper limit of the branching fraction of the $K_L \rightarrow \pi^0\gamma$ decay is $1.7 \times 10^{-7}$ at the 90% confidence level.

**ACKNOWLEDGEMENT**

We would like to express our gratitude to all members of the J-PARC Accelerator and Hadron Experimental Facility groups for their support. We also thank the KEK Computing Research Center for KEKCC and the National Institute of Information for SINET4. This work was supported by the Ministry of Education, Culture, Sports, Science, and
Technology (MEXT) of Japan and the Japan Society for the Promotion of Science (JSPS) under the MEXT KAKENHI Grant No. JP18071006, the JSPS KAKENHI Grants No. JP23224007, and No. JP16H06343, by the research fellowship program for postdoctoral scientists No. 17J02178, and through the Japan-U.S. Cooperative Research Program in High Energy Physics; the U.S. Department of Energy, Office of Science, Office of High Energy Physics, under Awards No. de-sc0006497, No. de-sc0007859, and No. de-sc0009798; the Ministry of Education and the Ministry of Science and Technology in Taiwan under Grants No. 104-2112-M-002-021, 105-2112-M-002-013 and 106-2112-M-002-016; and the National Research Foundation of Korea (2017R1A2B2011334 and 2019R1A2C1084552).

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