Electron-phonon scattering in quantum wires exposed to a normal magnetic field

Mher M. Aghasyan
Department of Physics, Yerevan State University, 375049 Yerevan, Armenia

Samvel M. Badalyan
Department of Radiophysics, Yerevan State University, 375049 Yerevan, Armenia

Garnett W. Bryant
National Institute of Standards and Technology, Gaithersburg, MD 20899 USA

Abstract

A theory for the relaxation rates of a test electron and electron temperature in quantum wires due to deformation, piezoelectric acoustical and polar optical phonon scattering is presented. We represent intra- and inter-subband relaxation rates as an average of rate kernels weighted by electron wave functions across a wire. We exploit these expressions to calculate phonon emission power for electron intra- and inter-subband transitions in quantum wires formed by a parabolic confining potential. In a magnetic field free case we have calculated the emission power of acoustical (deformation and piezoelectric interaction) and polar optical phonons as a function of the electron initial energy for different values of the confining potential strength. In quantum wires exposed to the quantizing magnetic field normal to the wire axis, we have calculated the polar optical phonon emission power as a function of the
electron initial energy and of the magnetic field.
I. INTRODUCTION

Semiconductor quantum wires attract considerable interest both for unraveling novel fundamental phenomena and for possible device applications. Rapid carrier relaxation is crucial for many of technological applications of these systems, therefore, understanding and characterizing carrier scattering in quantum wires are important for controlling carrier dynamics in thermalization, optical, and transport processes.

Theoretically, electron-phonon relaxation in quantum wires has been addressed in several works\textsuperscript{1–11}. Scattering by optical phonons has been investigated in rectangular wires\textsuperscript{1,5}. In cylindrical wires, a simple model with constant electron wave function inside the wire\textsuperscript{2}, an infinite and a finite well confining potential\textsuperscript{3,7} have been considered. Inter-subband scattering has been treated\textsuperscript{10}. Acoustical phonon relaxation has been studied in wires with parabolic (in one direction)\textsuperscript{6} and infinite well confining potentials\textsuperscript{8,9}. Optical phonon generation has been investigated due to electrophonon resonances\textsuperscript{11}. Although significant progress has been achieved, the problem still cannot be considered as solved.

In this work we present a theory for calculations of the relaxation rates of a test electron and electron temperature in quantum wires exposed to a normal magnetic field. We represent the intra- and inter-subband relaxation rates as an average of rate kernels weighted by the electron wave functions across the wire. Exploiting these expressions and the appropriate forms of the electron subband wave functions, we evaluate the relaxation rates in quantum wires under different environments. We discuss the scattering rates in quantum wires in zero and quantizing magnetic fields. In the magnetic field free case we present calculations of the electron scattering rates due to emission of deformation (DA) and piezoelectric (PA) acoustical, and polar optical (PO) phonons in quantum wires with a parabolic confining potential as a function of the electron initial energy for different values of the inter-subband separation. In the quantizing magnetic field applied normal to the wire axis we study electron scattering rate due to PO phonon emission as a function of electron initial energy and of the magnetic field.
II. RELAXATION OF A TEST ELECTRON

In quantum wires, particle motion is described by eigenfunctions $|\lambda\rangle \equiv |nlk\rangle = |nl\rangle |k\rangle$ which factor into subband functions $|nl\rangle = \chi_{nl}(R)$ ($R = (x, y)$) labeled by indices $n$ and $l$ corresponding to the lateral quantization across the wire and into plane waves $|k\rangle = e^{ikz}$ labeled by a wave vector $k$ corresponding to the free translation electron motion along the wire axes $z$. The single-particle energy is given by $\varepsilon_{nl}(k) = \varepsilon(k) + \varepsilon_{nl}$ where the kinetic energy is $\varepsilon(k) = \hbar^2 k^2 / 2m^*$ ($m^*$ is the electron effective mass) and $\varepsilon_{nl}$ is the subband energy.

The energy-loss power $Q$ for a test electron between subbands $n, l$ and $n', l'$ is defined as

$$Q^{\pm}_{nl \rightarrow n',l'}(\varepsilon(k)) = Q^+_{nl \rightarrow n',l'}(\varepsilon) - Q^-_{nl \rightarrow n',l'}(\varepsilon),$$

where $W_{nlk \rightarrow n'l'k'}^{\pm\eta}$ is the scattering probability at which one phonon of the mode $\eta$ with the wave vector $q = (q_z, q_\perp)$ and the frequency $\omega = \omega_\eta$ is emitted or absorbed by an electron, $f_T$ is the Fermi factor at crystal temperature $T$. The summation $(+)$ and $(-)$ over the final states $\varepsilon(k') < \varepsilon(k)$ and $\varepsilon(k') > \varepsilon(k)$ corresponds to the phonon emission and absorption processes, respectively. In the Born approximation using the explicit form of the transition probability $W_{nlk \rightarrow n'l'k'}^{\pm\eta}$, we represent the energy-loss power $Q$ in the following general form

$$Q^{\pm \eta}_{nl \rightarrow n',l'}(\varepsilon(k)) = Q_0^{\eta} \int d^2R \int d^2R' \chi_{nl'}^* (R') \chi_{nl} (R')$$

$$\times \chi_{n'l'} (R) \chi_{nl'} (R) K^{\pm \eta}_{nl \rightarrow n',l'}(\varepsilon(k), |R - R'|)$$

where $Q_0^{\eta}$ is the nominal power and $K^{\pm \eta}_{nl \rightarrow n',l'}$ is a rate kernel which depends on the type of electron-phonon interaction. By considering different interaction mechanisms, we obtain the rate kernels $K^{\pm \eta}_{nl \rightarrow n',l'}$ and the nominal powers $Q_0^{\eta}$. For $\eta = PO$ phonons: $Q_0^{PO} = h\omega_{PO}/\tau_{PO}$ and

$$K^{\pm PO}_{nl \rightarrow n',l'}(\varepsilon(k), |R - R'|) = \sqrt{\frac{\hbar\omega_{PO}}{2\sqrt{\varepsilon + \Delta_{nl,n'l'}} + \hbar\omega_{PO}}}$$

$$\times K_0 \left( q_+^\pm |R - R'| \right) \Psi^{\pm} (\varepsilon_{l,n}(k), \hbar\omega_{PO}).$$
For Υ = DA and Υ = PA phonons: $Q_0^\Upsilon = \hbar s p_{pO}/\tau_\Upsilon$ and

$$K_{nl\rightarrow n',l'}^\pm (\varepsilon_i |R - R'|) = \frac{\sqrt{2m s^2}}{2(s p_{pO})^{2+\sigma_\Upsilon}} \times$$

$$\int_{\omega^-_l}^{\omega^+_l} d\omega^{1+\sigma_\Upsilon} J_0 \left( q^\pm |R - R'| \right) \Psi^\pm (\varepsilon_{ln}(k); \hbar \omega),$$

with $\sigma_{DA} = 2$ and $\sigma_{PA} = 0$. Here the nominal scattering times are given by

$$\frac{1}{\tau_{pO}} = 2 \alpha_{pO} \omega_{pO}, \quad \frac{1}{\tau_{DA}} = \frac{\Xi^2 p_{pO}^3}{2\pi \hbar g s^2}, \quad \frac{1}{\tau_{PA}} = \frac{(e \beta)^2 p_{pO}}{2\pi \hbar g s^2}$$

where $\alpha_{pO}$ is the Fröhlich coupling, $\Xi$ and $e \beta$ are the deformation and piezoelectric potential constants, $g_0$ is the crystal mass density, $s$ the sound velocity, $\hbar p_{pO} = \sqrt{2m \hbar \omega_{pO}}$, and $\omega_{pO}$ the polar optical phonon frequency. In Eqs. (4) and (5) $\Delta_{n,n'} = \varepsilon_{l,n} - \varepsilon_{l',n'}$, $J_0$ is the Bessel function of the first kind and $K_0$ the modified Bessel function of the second kind, the function $\Psi^\pm$ is

$$\Psi^\pm (x, y) = \left( N_T(y) + \frac{1}{2} \pm \frac{1}{2} \right) \frac{1 - f_T (x + y)}{1 - f_T (x)}$$

where $N_T$ is the Bose factor. The phonon momenta are given by

$$q^\pm_z = \sqrt{\frac{2m}{\hbar^2}} \left| \sqrt{\varepsilon(k)} - r \sqrt{\varepsilon(k) + \Delta_{n,n'}} \mp \hbar \omega_{pO} \right|,$$

$$q^\pm_\perp = \left| \frac{\omega^2}{s^2} - \left( q^\pm_z \right)^2 \right|^{1/2}.$$
\[
\omega_1^\pm \approx \frac{1}{\hbar} \sqrt{\frac{\sqrt{(\epsilon(k) - r\sqrt{\epsilon(k) + \Delta_{n,l,n' l'}})} + \Delta_{n,l,n' l'}}{\sqrt{2ms^2}}} - r^{-1} \left(2\sqrt{\epsilon(k) + \Delta_{n,l,n' l'}}\right)^{-1}.
\]

Thus, Eqs. (1)–(3) with the rate kernels given by Eqs. (4) and (5) provide a new approach to calculate the energy-loss power due to PO, PA, and DA phonon scattering in quantum wires with an arbitrary cross section and under different environments.

### III. Electron Temperature Relaxation

If the distribution of hot electrons can be described by an electron temperature \(T_e > T\), we can determine the energy relaxation rate for the whole electron gas. In this case electron temperature relaxation between subbands \(n, l\) and \(n', l'\) can be described by the energy-loss power per electron which is given by

\[
\mathbb{Q}^{\pm}_{n,l \rightarrow n',l'}(T_e, T) = \frac{1}{N_1 L} \sum_{n,l,k} f_{T_e}(\epsilon) \sum_{n',l,k'} \bar{\hbar}\omega \times W^{\pm q\mp}_{n,l,k \rightarrow n',l'}[1 - f_{T_e}(\epsilon \mp \bar{\hbar}\omega)].
\]

Here \(N_1\) is the electron linear concentration. Direct calculations show that to obtain \(\mathbb{Q}^{\pm}_{n,l \rightarrow n',l'}\) one can use Eq. (3) but with the kernel \(K^{\pm \mp}_{n,l \rightarrow n',l'}\) replaced by the average rate kernel \(K^{\pm \mp}_{n,l \rightarrow n',l'}\).

For \(\Upsilon = \text{PO}\) phonons, we obtain

\[
\mathbb{K}^{\pm \text{PO}}_{n,l \rightarrow n',l'}(T_e, T; |R - R'|) = \frac{p_{\text{PO}}}{2\pi N_1} \int_0^\infty d\epsilon(k) \frac{\sqrt{\epsilon(k) + \Delta_{n,l,n' l'}}}{\sqrt{\epsilon(k)}} \Phi^{\pm}(\epsilon_{L,n}(k), \hbar\omega_{\text{PO}})
\]

\[
\times J_0 \left( q_{\mp}^\pm |R - R'| \right) \Phi^{\pm}(\epsilon_{L,n}(k); \hbar\omega).
\]

For \(\Upsilon = \text{DA}\) and \(\Upsilon = \text{PA}\) phonons

\[
\mathbb{K}^{\pm \Upsilon}_{n,l \rightarrow n',l'}(T_e, T; |R - R'|) = \frac{ms}{2\pi\hbar N_1(s_{p\text{PO}})^2 + \sigma_x} \int_0^\infty d\epsilon(k) \int_0^{\omega_{\pm}} \frac{\omega d\omega}{\sqrt{\epsilon(k) + \Delta_{n,l,n' l'}}} \Phi^{\pm}(\epsilon_{L,n}(k); \hbar\omega)
\]

\[
\times J_0 \left( q_{\mp}^\pm |R - R'| \right) \Phi^{\pm}(\epsilon_{L,n}(k); \hbar\omega).
\]

where \(\Phi\) is the following function.
\[
\Phi^\pm(x; y) = \left( N_T(y) + \frac{1}{2} \pm \frac{1}{2} \right) \\
\times f_{T_e}(x) (1 - f_{T_e}(x \mp y))
\]  

(15)

IV. SCATTERING IN THE ZERO MAGNETIC FIELD

In quantum wires with confining potential of cylindrical symmetry, the electron wave functions in the absence of the magnetic field in the plane perpendicular to the wire axis are represented in the form.

\[
\chi_{nl}(R) = \frac{1}{\sqrt{2\pi}} e^{il\phi} \chi_{nl}(R).
\]  

(16)

Substituting this into Eq. (3) and integrating over \(\varphi\), we represent the energy-loss power in the form

\[
Q^{\pm}_n \rightarrow n', l' (\varepsilon) = Q_0^T \int R dR \int R' dR' \chi^*_{n'l'}(R') \chi_{nl}(R) \\
\times \chi_{n'l'}(R) \left( K^{\pm}_n \rightarrow n', l' (\varepsilon(k); R, R') \right)_{cyl}
\]  

(17)

where the rate kernel factors into a product of two functions, each of them depends only on the modulus \(R\) or \(R'\). We find for PO interaction that the rate kernels \(\left( K^{\pm PO}_{n,l \rightarrow n', l'} \right)_{cyl} \) and \(\left( K^{\pm PO}_{n,l \rightarrow n', l'} \right)_{cyl} \) for a test electron and for electron temperature relaxation can be obtained by the replacement

\[
K_0 \left( q^+_z |R - R'| \right) \rightarrow K_{l-l'} \left( q^+_z R \right) I_{l-l'} \left( q^+_z R' \right)
\]

in Eqs. (4) and (13), respectively. Here \(I_l\) is the modified Bessel function of the first kind and \(R > R'\). To obtain the kernels \(\left( K^{\pm P,A,D,A}_{n,l \rightarrow n', l'} \right)_{cyl} \) and \(\left( K^{\pm P,A,D,A}_{n,l \rightarrow n', l'} \right)_{cyl} \), the replacement

\[
J_0 \left( q^+_z |R - R'| \right) \rightarrow J_{l-l'} \left( q^+_z R \right) J_{l-l'} \left( q^+_z R' \right)
\]

should be done in Eqs. (5) and (14), respectively.

For the parabolic confining potential \(V(R) = m^* \omega_0^2 R^2 / 2\), the electron wave functions are given by

\[\text{[Equation]}\]
\[ \chi_{nl}(R) = \sqrt{\frac{2n!}{(n+|l|)!}} e^{-\frac{r^2}{2a_0^2}} \left( \frac{r}{a_0} \right)^{|l|} L^{|l|}_n \left( \frac{r^2}{a_0^2} \right). \]

The subband energy \( \varepsilon_{n,l} = (2n + |l| + 1) \hbar \omega_0 \) where \( \omega_0 \) is confining potential strength, \( a_0 = \sqrt{\hbar/(m^* \omega_0)} \), \( L^{|l|}_n(x) \) gives the generalized Laguerre polynomial. Using these functions we have calculated PO, PA, and DA emission power for electron transitions between subbands \( n, l \) and \( n', l' \) with \( n, l = 0, 1 \) as a function of the electron initial energy for different values of \( \omega_0 \). In Figs. 1 and 2 we present the results of calculations. It is seen from Fig. 1 that the PO phonon emission rate diverges at \( \varepsilon = \Delta_{nl,n'l'} + \hbar \omega_{PO} \) due to transitions with the electron final states at the subband bottom where the 1D density of states has a square-root singularity. For energies far from \( \Delta_{nl,n'l'} + \hbar \omega_{PO} \) we find a weak dependence of the emission rate on \( \varepsilon(k) \) while there is a strong dependence on the subband separation \( \omega_0 \) and the quantum numbers \( n, l \). The intra-subband acoustical phonon emission rate has a peak at small energies (Fig. 2) while inter-subband emission is finite even at \( \varepsilon(k) = 0 \). The peak position decreases with an increase of \( a_0 \). We find that intra-subband DA phonon emission is weaker than PA phonon emission. This difference is less pronounced at inter-subband emission.

V. SCATTERING IN THE MAGNETIC FIELD NORMAL TO THE WIRE AXIS

When the magnetic field is applied perpendicular to the wire axis, the electron energy and wave functions in the parabolic confining potential \( V(x) = m \omega^2_x x^2 / 2 \) and \( V(y) = m \omega^2_y y^2 / 2 \) are given by

\[ \chi_{n,l}(\mathbf{R}) = \frac{\exp \left[ -\frac{(x-x_0)^2}{2a_x^2} \right] H_n \left( \frac{x-x_0}{a_x} \right) \exp \left[ -\frac{y^2}{2a_y^2} \right] H_l \left( \frac{y}{a_y} \right)}{\sqrt{2^n n! a_x \sqrt{\pi}}} \frac{\sqrt{2^{|l|} l! a_y \sqrt{\pi}}}{\sqrt{2^{2|l|} l! a_y \sqrt{\pi}}}, \]

\[ \varepsilon_{n,l}(k) = \varepsilon_B(k) + \hbar \Omega_x \left( n + \frac{1}{2} \right) + \hbar \omega_y \left( l + \frac{1}{2} \right), \varepsilon_B(k) = \frac{\hbar^2 k^2}{2m_B} \]

where \( a_x = \sqrt{\hbar/m^* \Omega_x} \), \( a_y = \sqrt{\hbar/m^* \omega_y} \), \( \Omega_x^2 = \omega_x^2 + \omega_B^2 \), \( m_B = m^* \Omega_x^2 / \omega_x^2 \), \( \omega_B = eB/m^*c \), and \( H_n \) gives the Hermite polynomials.
In this case to obtain the rate kernels $K^{\pm \Upsilon}$ we should multiply Eqs. (4), (5), (13), and (14) by a factor $\Omega_x/\omega_x$ and replace $\varepsilon(k)$ by $\varepsilon_B(k)$ in these equations. Below we will discuss only electron PO phonon scattering. Scattering by acoustical phonons in the normal magnetic field has been studied by Shik and Challis.\(^6\)

Substituting the kernel $K^{PO}$ and wave functions (18) in Eq. (3), we represent the PO phonon emission power in the form

$$Q^{PO}_{n,l \rightarrow n',l'} = Q^{PO}_0 \frac{\sqrt{\hbar \omega_{PO}}}{\sqrt{\varepsilon_B + \Delta_{n,l,n',l'} - \hbar \omega_{PO}} \omega_x} \Omega_x I_{n,l}^{n',l'}$$

where the form factors $I_{n,l}^{n',l'}$ for the most important intra-subband $00 \rightarrow 00$ and inter-subband $10 \rightarrow 00$ transitions are reduced to the following one-dimensional integrals

$$I_{00}^{00} = \int_0^\infty \frac{e^{-\xi} d\zeta}{(2\zeta + q_x^2 a_x^2)(2\zeta + q_y^2 a_y^2)},$$

$$I_{10}^{00} = I_{00}^{00} + \frac{2}{q_x^2 a_x^2 - q_y^2 a_y^2} \int_0^\infty (1 - \zeta) e^{-\xi} \sqrt{\frac{2\zeta + q_y^2 a_y^2}{2\zeta + q_x^2 a_x^2}} d\zeta$$

which we calculate numerically. The results of calculation are shown in Figs. 3-7. The diagrams of Figs. 3 and 4 represent the intra-subband PO phonon emission power dependencies on the electron initial energy and on the magnetic field, respectively, for several values of the confining potential strengths $\omega_x$ and $\omega_y$. It is seen from both figures that, as it was in the magnetic field free case, the PO phonon emission power diverges at the phonon emission threshold which is given in this case by $\varepsilon_B(k) = \hbar \omega_{PO}$. Because of the electron mass dependence on the magnetic field, the threshold values are trivially shifted to higher electron energies with an increase of the magnetic field. At energies far from the threshold, the effect of the magnetic field is weak. At inter-subband transitions, $\Delta_{10,00}$ differs from zero and depends on the magnetic field. In this case there is a threshold line given by $\varepsilon_B(k) = \hbar \omega_{PO} - \Delta_{10,00}$ and shown in Fig. 5. The electron threshold energy increases in the magnetic field from $\varepsilon_0 = \hbar(\omega_{PO} - \omega_x)$ at $B = 0$ up to the value $\varepsilon_1$ at $B = B_1$ ($B_1$ is determined from threshold conditions $\Omega_x = 2\omega_{PO}/3$). For magnetic fields larger than $B_1$ the
threshold energy decreases and vanishes at $B = B_{PO}$ ($B_{PO}$ is determined from the resonance $\Omega_x = \omega_{PO}$). According to this the emission power dependence on the initial energy has no divergence for $B = 21, 22$ and $25$ T (see Fig. 3). For magnetic fields larger but near $B_{PO}$, $Q^{PO}$ has a peak for small values of $\varepsilon$ while for magnetic fields far from $B_{PO}$, $Q^{PO}$ increases monotonically in $\varepsilon$. For a given value of $\varepsilon < \varepsilon_1$, there is an interval of magnetic fields where PO phonon emission is not possible (Fig. 3) while $Q^{PO}$ diverges at the edges of this interval (Fig. 7). This interval vanishes at $\varepsilon = \varepsilon_1$ so that at energies larger but not far from $\varepsilon_1$, $Q^{PO}$ as a function of the magnetic field has a peak at $B = B_1$ (Fig. 7). The second peak in the magnetic field dependence of the PO phonon emission power occurs at the resonance field $B_{PO}$ and corresponds to the vertical electron transitions with the phonon momentum $q_z = 0$.

In conclusion, we have presented a theory for the test carrier and carrier temperature relaxation rates in quantum wires. This theory has been exploited to calculate the PO, PA and DA phonon emission power for electron intra- and inter-subband transitions in quantum wires with the parabolic confining potential for different values of the potential strength. We have discussed the phonon emission power in quantum wires in the zero and quantizing magnetic field normal to the wire axis.

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FIGURES

FIG. 1. The PO phonon emission power versus the electron initial kinetic energy in zero magnetic field at the intra- and inter-subband electron transitions and for different values of the subband separation $\hbar \omega_0$.

FIG. 2. The PA and DA phonon emission power versus the electron initial kinetic energy in zero magnetic field at the intra- and inter-subband electron transitions and for different values of the subband separation $\hbar \omega_0$.

FIG. 3. The PO phonon emission power dependence on the electron initial kinetic energy in the quantizing magnetic field at the intra-subband electron transitions for various values of the subband separations $\hbar \omega_x$ and $\hbar \omega_y$.

FIG. 4. The PO phonon emission power dependence on the magnetic field at the intra-subband electron transitions for various values of the subband separations $\hbar \omega_x$ and $\hbar \omega_y$.

FIG. 5. The threshold line in the $(\varepsilon, B)$-plane which separates regions where PO phonon emission is and is not possible. $\varepsilon_0/\hbar \omega_{PO} = 0.84$, $\varepsilon_1/\hbar \omega_{PO} = 5.97$, $B_1 = 13.64$ T, $B_{PO} = 20.67$ T.

FIG. 6. The PO phonon emission power dependence on the electron initial kinetic energy at the inter-subband electron transitions for various values of the magnetic field.

FIG. 7. The PO phonon emission power dependence on the magnetic field at the inter-subband electron transitions for various values of electron initial kinetic energy.
PO-phonon emission power $Q_{PO}^P(\epsilon)/Q_0^P$

Electron initial energy $\epsilon(k)/(\hbar \omega_{PO})$

- $\omega_0 = 5.77$ meV, $n=0, l=0 \Rightarrow n'=0, l'=0$
- $\omega_0 = 1.44$ meV, $n=0, l=0 \Rightarrow n'=0, l'=0$
- $\omega_0 = 0.64$ meV, $n=0, l=0 \Rightarrow n'=0, l'=0$
- $\omega_0 = 5.77$ meV, $n=0, l=1 \Rightarrow n'=0, l'=0$
- $\omega_0 = 1.44$ meV, $n=0, l=1 \Rightarrow n'=0, l'=0$
- $\omega_0 = 0.64$ meV, $n=0, l=1 \Rightarrow n'=0, l'=0$
Electron initial energy [K]

Acoustic phonon emission power \([x 10^{-12} \text{ watt}]\)

PA, \(\omega_0 = 5.77 \text{ meV}, n=0, l=0 \Rightarrow n'=0, l'=0\)

DA, \(\omega_0 = 5.77 \text{ meV}, n=0, l=0 \Rightarrow n'=0, l'=0\)

PA, \(\omega_0 = 5.77 \text{ meV}, n=0, l=1 \Rightarrow n'=0, l'=0\)

DA, \(\omega_0 = 5.77 \text{ meV}, n=0, l=1 \Rightarrow n'=0, l'=0\)

PA, \(\omega_0 = 1.44 \text{ meV}, n=0, l=0 \Rightarrow n'=0, l'=0\)

DA, \(\omega_0 = 1.44 \text{ meV}, n=0, l=0 \Rightarrow n'=0, l'=0\)

DA, \(\omega_0 = 1.44 \text{ meV}, n=1, l=0 \Rightarrow n'=0, l'=1\)
Electron initial energy $\epsilon(k)/(\hbar\omega_{PO})$

PO-phonon emission power $Q_{PO}^{PO}(\epsilon) / Q_0$

$n'=0, l'=0 \Rightarrow n=0, l=0$

- $h\omega_x = 2h\omega_y = 5.77 \text{ meV}, B=2 \text{ T}$
- $h\omega_x = h\omega_y = 5.77 \text{ meV}, B=2 \text{ T}$
- $h\omega_x = h\omega_y = 2.885 \text{ meV}, B=2 \text{ T}$
- $h\omega_x = h\omega_y / 2 = 2.885 \text{ meV}, B=2 \text{ T}$
- $h\omega_x = 2h\omega_y = 5.77 \text{ meV}, B=5 \text{ T}$
- $h\omega_x = h\omega_y = 5.77 \text{ meV}, B=5 \text{ T}$

Electron initial energy $\epsilon(k)/(\hbar\omega_{PO})$
\( n'=0, l'=0 \Rightarrow n=0, l=0 \)

- \( h\omega_x = 2h\omega_y = 5.77 \text{ meV}, \varepsilon(k) = 3.3h\omega_{PO} \)
- \( h\omega_x = 2h\omega_y = 5.77 \text{ meV}, \varepsilon(k) = 4h\omega_{PO} \)
- \( h\omega_x = h\omega_y = 5.77 \text{ meV}, \varepsilon(k) = 3.3h\omega_{PO} \)
- \( h\omega_x = h\omega_y = 5.77 \text{ meV}, \varepsilon(k) = 4h\omega_{PO} \)
Phonon emission is not possible in this region.

Electron initial energy $\varepsilon(k)/(\hbar\omega_{PO})$

Magnetic field $B$ [T]

$n'=1$, $l'=0 \Rightarrow n=0$, $l=0$

$\hbar\omega_x=\hbar\omega_y=5.77$ meV

Threshold line

Phonon emission is not possible in this region.
\[ n' = 0, l' = 1 \Rightarrow n = 0, l = 0 \]

\[ h\omega_x = h\omega_y = 5.77 \text{ meV} \]

Electron initial energy \( \varepsilon(k)/(h\omega_{PO}) \)

- \( B = 0 \text{ T} \)
- \( B = 5 \text{ T} \)
- \( B = 13 \text{ T} \)
- \( B = 19 \text{ T} \)
- \( B = 20.5 \text{ T} \)
- \( B = 21 \text{ T} \)
- \( B = 22 \text{ T} \)
- \( B = 25 \text{ T} \)
\[ h\omega_x = h\omega_y = 5.77 \text{ meV} \]

\[ \epsilon = 10h\omega_{PO} \]

\[ \epsilon = 6h\omega_{PO} \]

\[ \epsilon = 5.97h\omega_{PO} \]

\[ \epsilon = 5h\omega_{PO} \]

\[ \epsilon = 4.5h\omega_{PO} \]

\[ \epsilon = 2.5h\omega_{PO} \]

\[ \epsilon = h\omega_{PO} \]

\[ \epsilon = 0 \]

Magnetic field \( B_0 \) [T]