Simulation Analysis of Six-axis Manipulator

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Abstract. The D-H parameter method is used to model the six-axis manipulator. Based on this model, the forward and inverse kinematics equations of the manipulator can be derived, so that the relationship of manipulator joint angle and the pose of manipulator can be obtained. Using the fifth-order polynomial interpolation method realizes the manipulator trajectory planning. In this paper, using the free Robotics Toolbox to simulate manipulator, the results show that the simulation of the manipulator is accurate and intuitive, which can provide theoretical basis and reference basis for the actual design and motion control of the manipulator.

1. Introduction

With the development of science and technology, the application of manipulator is more and more extensive. Manipulator was first used in manufacturing industry, but now it exists in aerospace, ocean exploration and even in daily life, where manipulator has had an irreplaceable position[1]. That kinematic analysis about how to determine the relationship between the rotation angle of each manipulator joint and the pose of the manipulator is necessary, so the rotation angle of the joint can be determined to make the end of the manipulator reach the specified position. And the known of trajectory planning of the end of the manipulator is also indispensable for the design and control of the manipulator.

The traditional physical entity design method of manipulator is not only time-consuming and laborious, but also difficult to achieve ideal results. With the continuous development of computer technology, the motion model of the manipulator can be designed by computer simulation technology, which is close to the real object and has high precision. Combined with the simulation that realized by Robotics Toolbox and the actual debugging, the ideal effect can be achieved. So the simulation can not only shorten the design cycle of manipulator but also save a lot of production costs.

2. Kinematics analysis of six-axis manipulator

2.1. Establishment of Manipulator model by D-H parameter method

The D-H parameter method that proposed by Denavit and Hartenberg in 1955, is a method to establish the coordinate system for each joint in the joint chain of a manipulator[2]. In this paper, a six-axis manipulator with rotating joints is adopted, as shown in Fig. 1(a).
The space coordinate system of this manipulator is established by D-H parameter representation, as shown in Fig.1(b). The parameters that determined by D-H parameter method are shown in Table 1.

### Table 1. D-H parameter table

| Manipulator joint | $\theta$  | d     | a    | $\alpha$ |
|-------------------|-----------|-------|------|----------|
| 1                 | $\theta_1$| 262   | 0    | $-90^\circ$ |
| 2                 | $\theta_2$| 0     | 250  | 0        |
| 3                 | $\theta_3$| 0     | 142.6| $90^\circ$ |
| 4                 | $\theta_4$| 263.16| 0    | $90^\circ$ |
| 5                 | $\theta_5$| 0     | 0    | $-90^\circ$ |
| 6                 | $\theta_6$| 193.9 | 0    | 0        |

2.2. **Forward kinematics analysis of manipulator**

According to the space coordinate system of the manipulator determined by D-H parameter method, the total transformation matrix of the manipulator can be obtained to determine the relationship between the joint variables of the manipulator and the end of manipulator\(^{[3]}\).

Taking the transformation from $\{O_0\}$ base coordinate system to the $\{O_1\}$ coordinate system as an example, the concrete transformation steps are as follows:

1. Rotate $\theta_1$ around the $z_0$ axis so that $x_0$ axis and $x_1$ axis are parallel.
2. Translating the distance of $d_1$ along the $z_0$ axis makes the $x_0$ axis and $x_1$ axis collinear.
3. Translating $a_1$ distance along $x_0$ axis makes the origin of $x_0$ axis coincide with that of $x_1$ axis.
4. Rotate the $z_0$ axis around the $x_1$ axis at an angle of $\alpha_1$, so that $z_0$ and $z_1$ are aligned.

At this time, the $\{O_0\}$ coordinate system and the $\{O_1\}$ coordinate system are completely identical, that is, the coordinate system is successfully transformed from one coordinate system to the next coordinate system.

Similarly, the transformation from $n+1$ coordinate system to $n+2$ coordinate system should be strictly in accordance with the above four orders.
According to the space coordinate system of the manipulator established in Figure 2, the D-H parameters in Table 1, the transformation matrixes at each joint can be obtained. Defining the transformation matrix is from \( \{O_o\} \) coordinate system to \( \{O_i\} \) coordinate system, \( A_1 \) transformation matrix is from \( \{O_1\} \) coordinate system to \( \{O_2\} \) coordinate system, by analogy, every joint transformation matrix as shown in equation (1) ~ (6).

\[
A_i = \begin{bmatrix}
\cos \theta_i & 0 & -\sin \theta_i & 0 \\
\sin \theta_i & 0 & \cos \theta_i & 0 \\
0 & -1 & 0 & d_i \\
0 & 0 & 0 & 1
\end{bmatrix} \quad i = 1, 2, 3, 4
\]

(1)

\[
A_2 = \begin{bmatrix}
\cos \theta_2 & -\sin \theta_2 & 0 & a_2 \cos \theta_2 \\
\sin \theta_2 & \cos \theta_2 & 0 & a_2 \sin \theta_2 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(2)

\[
A_3 = \begin{bmatrix}
\cos \theta_3 & 0 & -\sin \theta_3 & a_3 \cos \theta_3 \\
\sin \theta_3 & 0 & \cos \theta_3 & a_3 \sin \theta_3 \\
0 & -1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(3)

\[
A_4 = \begin{bmatrix}
\cos \theta_4 & 0 & -\sin \theta_4 & 0 \\
\sin \theta_4 & 0 & \cos \theta_4 & 0 \\
0 & -1 & 0 & d_i \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(4)

\[
A_5 = \begin{bmatrix}
\cos \theta_5 & 0 & -\sin \theta_5 & 0 \\
\sin \theta_5 & 0 & \cos \theta_5 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(5)

\[
A_6 = \begin{bmatrix}
\cos \theta_6 & -\sin \theta_6 & 0 & 0 \\
\sin \theta_6 & \cos \theta_6 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(6)

Based on the equation (1)~(6), we can obtain the total transformation matrix of this manipulator, which is expressed by desired posture as follows:

\[
A = A_1A_2A_3A_4A_5A_6 = \begin{bmatrix}
n_x & o_x & a_x & p_x \\
n_y & o_y & a_y & p_y \\
n_z & o_z & a_z & p_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(7)

The \( \mathbf{n} = [n_x, n_y, n_z, 0]^T \) is the direction vector on the x-axis of the \( \{O_1\} \) coordinate system relative to the \( \{O_o\} \) coordinate system, \( \mathbf{O} = [o_x, o_y, o_z, 0]^T \) is the direction vector on the y-axis of the \( \{O_1\} \) coordinate system relative to the \( \{O_o\} \) coordinate system, \( \mathbf{a} = [a_x, a_y, a_z, 0]^T \) is the direction vector on the z-axis of the \( \{O_1\} \) coordinate system relative to the \( \{O_o\} \) coordinate system.

2.3. Inverse Kinematics Analysis

If wanted the end of the manipulator moves to a specified point and has a specific attitude, the rotation angle of each joint can be calculated by inverse kinematics solution. From the beginning of \( A_1^{-1} \), multiply the each inverse transformation matrix by left, finally can get each joint angle \( \theta_i \) as \( \cos \theta_i \), \( \theta_i \) as \( \sin \theta_i \), by analogy, \( \theta_j \) as \( \sin(\theta_i + \theta_j) \), \( \theta_j \) as \( \cos(\theta_i + \theta_j) \).

Rotation angle of joint 1:

\[
\theta_1 = \arctan^{-1}\left[\frac{(-p_y + d_6 \cdot a_y)}{(-p_x + d_6 \cdot a_x)}\right]
\]

(8)

Rotation angle of joint 2:

\[
\theta_2 = \arctan^{-1}\left[\frac{\sqrt{\left(q_1^2 + q_2^2\right)/2} \cdot \left(-d_x c_y - a_y s_y\right) - q_3 \cdot \left(-d_y s_x + a_x c_x + a_z\right)}{\left(q_1^2 + q_2^2\right)/2} \cdot \left(-d_x c_y + a_y c_y + a_z\right) + q_3 \cdot \left(-d_y s_x - a_x c_x\right)}\right]
\]

(9)

Rotation angle of joint 3:

\[
\theta_3 = 2\arctan^{-1}\left[\frac{\left(-d_4 \pm (d_4^2 + a_3^2 - a_4 c_3 - d_4 s_3)^{1/2}\right)}{a_3 c_3 - d_4 s_3 + a_3}\right]
\]

(10)

Rotation angle of joint 4:

\[
\theta_4 = \arctan^{-1}\left[\frac{\left(-a_x \cdot s_1 - a_y \cdot c_1\right)}{-a_x \cdot c_1 + a_y \cdot s_1 + a_z \cdot s_{23} - a_z \cdot c_{23}}\right]
\]

(11)
Rotation angle of joint 5:
\[ \theta_5 = \arctan \left( \frac{a \cdot s_{23} c_4 - a \cdot (c_1 c_{23} c_4 + s_1 s_4) - a \cdot (s_1 c_2 c_4 - c_1 s_4)}{-a \cdot c_3 s - a \cdot s_3 s - a \cdot c_3} \right) \] (12)

Rotation angle of joint 6:
\[ \theta_6 = \arctan \left( \frac{[n \cdot s_{23} s_4 - n \cdot (c_1 c_{23} s_4 - s_1 c_4) - n \cdot (s_1 c_2 s_4 + c_1 c_4)]}{o \cdot s_{23} s_4 - o \cdot (c_1 c_{23} s_4 - s_1 c_4) - o \cdot (s_1 c_2 s_4 + c_1 c_4)} \right) \] (13)

In these equations, \( q_1 = c_1 \bullet (-d_4 s_{23} + a_5 c_{23} + a_2 c_2) \), \( q_2 = s_1 \bullet (-d_4 s_{23} + a_5 c_{23} + a_2 c_2) \), \( q_3 = -d_4 c_{23} - a_5 s_{23} - a_2 s_2 \).

2.4. Dynamic Simulation Analysis

According to the D-H parameters, the forward and inverse kinematics simulation can be implemented by functions that provided by the Robotics Toolbox. Using the link function to set the D-H parameter to model the arm, the result as shown in Fig. 2. Draging the horizontal bar of q1~q6 in the blue frame, and the joint of the mechanical arm will rotate accordingly.

Combining the actual parameters of the manipulator with the initial state of the actual manipulator, it can be seen that the D-H parameter method correctly models the six-axis manipulator. According to Table 1 and the value of each joint variable in the initial state of the arm \( \theta_1 = \theta_3 = \theta_4 = \theta_6 = 0^\circ, \theta_2 = 90^\circ, \theta_5 = -90^\circ \), the ikine function in the Robotics Toolbox in Matlab can be used to solve the inverse kinematics equation of the manipulator. Using this function can obtain the transformation matrix of the end of the manipulator relative base coordinate system:

\[
A = \begin{bmatrix}
1 & 0 & 0 & 263.16 \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 460.7 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\] (14)

Replacing the variables \( \theta_1 = \theta_3 = \theta_4 = \theta_6 = 0^\circ, \theta_2 = 90^\circ, \theta_5 = -90^\circ \) into equation (7), the calculated transformation matrix is consistent with equation (14). It can be shown the correctness of the positive kinematics equation of the manipulator is verified.

Substituting the initial state pose transformation matrix as equation(14) into the ikine function which is in the Robotics Toolbox of Matlab to solve the inverse kinematics equation, the six rotation angles are obtained, as \([0, -90^\circ, 0, 90^\circ, 0]\), which are the same as the result that obtained by equation(8)~(13), so it can prove the correctness of the kinematic equation which we derived.
3. Simulation analysis of trajectory planning

The third polynomial trajectory planning and the fifth-order polynomial trajectory planning are commonly used methods, but the trajectory is required to be smooth in the running process, and the fifth-order polynomial interpolation can solve the problem that the angular velocity change of the cubic polynomial interpolation is not smooth and the acceleration has a jump\(^5\). Finally, the trajectory of the fifth-order polynomial is selected.

3.1. Fifth-order polynomial trajectory planning

If the position, velocity and acceleration of the start and end points of the motion segment are known, the following five-degree polynomial can be used to plan the trajectory\(^6\):

\[
\begin{align*}
\theta(t) &= c_n + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 + c_5 t^5 \\
\dot{\theta}(t) &= c_1 + 2c_2 t + 3c_3 t^2 + 4c_4 t^3 + 5c_5 t^4 \\
\ddot{\theta}(t) &= 2c_2 + 6c_3 t + 12c_4 t^2 + 20c_5 t^3
\end{align*}
\] (15)

According to the equation (15), the coefficients of the fifth-order polynomial can be calculated by these six boundary conditions.

3.2. Trajectory planning simulation

The fifth-order polynomial method is used to simulate the six-axis manipulator trajectory. Setting the starting value of each joint variable of the manipulator is \(\text{init\_ang} = [0, -\pi/2, 0, 0, \pi/2, 0]\) and the ending value \(\text{targ\_ang} = [\pi/4, -\pi/3, \pi/5, \pi/2, -\pi/4, \pi/6]\), select 40 sampling points, the initial termination acceleration and velocity are 0, and the obtained trajectory is shown in Fig. 3.

Fig. 3(a) shows the motion trajectory of the end of the arm, in which the blue curve is the motion trajectory, and Fig. 3(b)~ (c) are the joint velocity and acceleration curves respectively, from this can
be seen that between at 40 sampling points, the acceleration is approximately sinusoidal, between 1 and 20 sampling points, the acceleration is greater than 0, the speed is increasing. Between the sampling points 20~40, the acceleration is less than 0, the speed decreases, and the speed changes smoothly, and the end of the arm moves smoothly.

4. Conclusion
The space coordinate system of the manipulator can be established by D-H parameter method, and the kinematics equation of the manipulator is derived by it. Using fifth-order polynomial method can realize the trajectory planning of the end of the manipulator. The Robotic Toolbox which users can download free online establishes the manipulator simulation model, and it can simulate the forward and inverse kinematics equations, and the trajectory planning. These correct simulation analyses of the manipulator can provide reference for the design of the manipulator and trajectory control, etc.

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