Inclusive semileptonic D decays and the heavy quark expansion

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Motivation: CKM
Unitarity Analysis

- UTA within the SM
  \[ \epsilon_K, \Delta m_d, \left| \frac{\Delta m_s}{\Delta m_d} \right|, \left| \frac{V_{ub}}{V_{cb}} \right] \]
  relying on theoretical calculations of hadronic matrix elements

K. Trabelsi
(CKMFitter)
Beauty 2009
Motivation: CKM Unitarity Analysis

- UTA within the SM
  \[ \epsilon_K, \Delta m_d, \left| \frac{\Delta m_s}{\Delta m_d} \right|, \frac{V_{ub}}{V_{cb}} \]

- relying on theoretical calculations of hadronic matrix elements

- Projected Super Flavour Factory sensitivity

  - \( V_{ub} \) (exclusive): 3–5%
  - \( V_{ub} \) (inclusive): 2–6%

T. Browder et al.
0710.3799
Status of $B \rightarrow X_u \ell \nu$

See talks by R. Kowalewski & T. Mannel

- Inclusive determination of $V_{ub}$ using OPE and HQE
- Expansion in $\alpha_s$ and $1/m_b$
- Present precision around 6–7%
- However 15% tension with UTA
- Dominant source of theoretical uncertainty due to shape-function modeling (kinematical phase-space cuts)
- A fully inclusive analysis would carry a tiny 2–3% theoretical error

Lange, Neubert and Paz [hep-ph/0504071]
Andersen and Gardi [hep-ph/0509360]
Gambino, Giordano, Ossola, Uraltsev [arXiv:0707.2493]
Aglietti, Di Lodovico, Ferrera, Ricciardi [arXiv:0711.0860]
Bauer, Ligeti and Luke [hep-ph/0107074]

Antonelli et al. 0907.5386

CLEO ($E_2$)
3.83 ± 0.45 + 0.32 - 0.33
BELLE sim. ann. ($m_\gamma$, $q^2$)
4.23 ± 0.45 + 0.30 - 0.45
BELLE ($E_4$)
4.64 ± 0.43 + 0.29 - 0.31
BABAR ($E_2$)
4.18 ± 0.24 + 0.29 - 0.31
BABAR ($E_4$, $q^2_{max}$)
4.28 ± 0.29 + 0.36 - 0.37
BELLE multivariate ($p_{T}$)
4.36 ± 0.26 + 0.21 - 0.23
BABAR ($m_{\gamma}$)
4.02 ± 0.19 + 0.27 - 0.29
BABAR ($m_{\gamma}q^2$)
4.32 ± 0.28 + 0.29 - 0.31
BABAR ($p_{T}$)
3.65 ± 0.24 + 0.25 - 0.27

Average +/- exp + theory - theory
4.20 ± 0.16 + 0.22 - 0.23

$\chi^2/\text{dof} = 12.3/8$ (CL = 14.00%)
Bosch, Lange, Neubert and Paz (BLNP)
Phys. Rev. D72, 073006, 2005

HAGF (Summer09)

$|V_{ub}| \times 10^{-3}$
Status of $B \rightarrow X_u \ell \nu$

- At $1/m_b^3$ leading spectator effects due to dimension 6 four quark operators (WA contributions)

- $16\pi^2$ phase space enhanced compared to LO & NLO contributions

- Affect both the total rate and spectra (expected to populate the $q^2/\text{lepton energy endpoint region}$)

- Cannot be extracted from inclusive $B \rightarrow X_c \ell\nu$ analysis

- Nor completely from comparing $B^+$ and $B^0$ decay modes

- Difficult to study non-perturbatively

Existing estimates spread between 3–10%
Inclusive Semileptonic Charm Decays

See talk by K. Ecklund

$D_q \rightarrow X \ell \nu$

Recently determined experimentally

$B(D^+ \rightarrow Xe\nu) = (16.13 \pm 0.20 \pm 0.33)\%$

$B(D^0 \rightarrow Xe\nu) = (6.46 \pm 0.17 \pm 0.13)\%$

Similar results for muons

Very recently results also for $D_s$ decays

$B(D_s \rightarrow Xe\nu) = (6.52 \pm 0.39 \pm 0.15)\%$

Including spectra

N. E. Adam et al. [CLEO] hep-ex/0604044

M. Ablikim et al. [BES] arXiv:0804.1454

Asner et al. [CLEO] 0912.4232
Ratio of $D_s$ and $D^0$ rates shows significant [17(6)\%] deviation from unity

$$\frac{\Gamma(D^+ \to Xe^+\nu)}{\Gamma(D^0 \to Xe^+\nu)} = 0.985(28),$$

$$\frac{\Gamma(D^+_s \to Xe^+\nu)}{\Gamma(D^0 \to Xe^+\nu)} = 0.828(57).$$

Signs of WA in $D_s$ decays?

How to disentangle from possible SU(3) violation?
SU(3) violation in Charm
(Two examples)

- **Hyperfine mass splitting**
  \[ \Delta_{Dq}^{hf} = \frac{3(m_{Dq}^2 - m_{Dq}^2)}{4} \]
  \[ \Delta_{D^+}^{hf} = 0.409(1)\text{GeV}^2, \quad \Delta_{D_0}^{hf} = 0.413(1)\text{GeV}^2, \quad \Delta_{D_s}^{hf} = 0.440(2)\text{GeV}^2. \]

- **SU(3) violation at 10%**

- **Decay constants**
  \[ f_{D_s} = 260(10)\text{MeV}, \quad f_D = 217(10)\text{MeV} \]

- **SU(3) violation at 20%**
Inclusive Semileptonic Charm Decays in OPE

- Treating charm quark mass as heavy, one can attempt an expansion in $\alpha_s(m_c)$, $\Lambda/m_c$
- Need to estimate local operator matrix elements between hadronic states
  - First appear at $1/m_c^2$ ← sources of SU(3) violation
- Heavy quark symmetry relates these estimates between the charm and beauty sectors
  - Quantitative translation (renormalization) not straightforward
- Alternative approach involves an educated sum over known exclusive modes

I. I. Bigi & N. G. Uraltsev, Phys. Lett. B 280 (1992)
Gronau & Rosner 0903.2287
Optical theorem

\[ \Gamma(HQ\bar{q}) = \frac{1}{2m_H} \langle HQ\bar{q} | T | HQ\bar{q} \rangle \]

\[ T = \text{Im} \ i \int d^4x T \{ \mathcal{H}_{eff}(x) \mathcal{H}_{eff}(0) \} \]

(Global) quark-hadron duality, HQE & OPE

Equations of motion

\[ \bar{c}c = \bar{c}bc + \frac{1}{2m_c^2} \left( \bar{c}(iD_\perp)^2c + \bar{c}\frac{g_s}{2}\sigma Gc \right) + O(1/m_c^3) \]

HQE parameters

\[ \mu_\pi^2 = -\frac{1}{2m_D} \langle D|\bar{c}(iD_\perp)^2c|D \rangle \]

\[ \mu_G^2 = \frac{1}{2m_D} \langle D|\bar{c}\frac{g_s}{2}\sigma Bc|D \rangle \]

Only applicable for the total rate
Analogously define current correlator whose imaginary part gives the hadronic tensor contributing to inclusive semileptonic spectra.

Again use HQE & OPE.

Requires local quark-hadron duality to hold.

Can be softened by instead computing spectral moments.

Any spectral cuts will reintroduce sensitivity to contributions beyond OPE.

Bigi et al. [hep-ph/9207214]
Manohar and Wise, [hep-ph/9308246]...
OPE for the rate & leptonic moments

Rate & leptonic energy moments in HQE & OPE

\[ \Gamma^{(n)} \equiv \int_0^{1-r} \frac{d\Gamma}{dx} x^n dx = \frac{G_F^2 m_c^5}{192 \pi^3} |V_{cs}|^2 \left[ f_0^{(n)}(r) + \frac{\alpha_s}{\pi} f_1^{(n)}(r) + \frac{\alpha_s^2}{\pi^2} f_2^{(n)}(r) + \frac{\mu^2}{m_c^2} f^{(n)}(r) + \frac{\mu_G^2}{m_c^2} f_G^{(n)}(r) \right. \\
\left. + \frac{\rho_{LS}^3}{m_c^3} f_{LS}^{(n)}(r) + \frac{\rho_D^3}{m_c^3} f_D^{(n)}(r) + \frac{32 \pi^2}{m_c^3} B_{WA}^{(n)s} \right], \]

- \( \alpha_s \) corrections known up to \( \alpha_s^2 \) for the total rate
  (\( \alpha_s^2 \beta_0 \) for the higher moments)

- \( 1/m_c \) corrections known up to \( 1/m_c^4 \) (all present analyses use \( 1/m_c^3 \))

- Cabibbo suppressed modes contribute to the total rate at the level of 5%, but their effect is highly suppressed in the normalized moments

A. Pak & A. Czarnecki
0803.0960,
K. Melnikov
0803.0951
V. Aquila et al.
hep-ph/0503083
Czarnecki & Jezabek
hep-ph/9402326
Gremm and Kapustin
hep-ph/9603448
Dassinger et al.
hep-ph/0611168
WA in OPE

- WA contributions to the rate can be related to matrix elements of dim=6 four quark operators
  \[ \langle H_Q q | O_{V-A}^q | H_{Qq} \rangle \equiv \langle H_Q q | \bar{Q} \gamma_\mu (1 - \gamma_5) q' \bar{q}' \gamma^\mu (1 - \gamma_5) Q | H_Q q \rangle \]
  \[ \langle H_Q q | O_{S-P}^q | H_{Qq} \rangle \equiv \langle H_Q q | \bar{Q} (1 - \gamma_5) q' \bar{q}' (1 - \gamma_5) Q | H_Q q \rangle \]

- In the SU(3) limit one distinguishes between isosinglet/triplet contributions - only the later can be estimated from the rate differences of $B^+$ and $B^0$

- Conventionally one parametrizes deviations from VSA: bag parameters
  \[ \langle D | O_{V-A} | D \rangle = f_D^2 m_D^2 B_1 \]
  \[ \langle D | O_{S-P} | D \rangle = f_D^2 m_D^2 B_2 \]

- Renormalization scale dependent, mix with the Darwin contributions at LO
  \[ \delta \Gamma \sim \left[ C_{WA} B_{WA}(\mu_{WA}) - \left( 8 \ln \frac{m_c^2}{m_{WA}^2} - \frac{77}{6} \right) \frac{\rho_D^3}{m_c^3} + \mathcal{O}(\alpha_s) \right] \]

- can be used to estimate WA contributions to the rate
Modeling WA in leptonic moments

- WA contributions to the current correlators vanish in the OPE - need to model
- Expected to populate the spectrum endpoint
- Develop a perturbative tail & non-perturbative smearing
- Possible phase-space suppression by hadronic thresholds
- Can be studied directly using exclusive channels ($D_s \rightarrow \omega l \nu$)
The WA interpretation of rate differences

Without resorting to quantitative OPE predictions, one can estimate WA from rate differences by equating the difference between $D^0$ and $D^0$ rates with the isotriplet component of WA.

\[
\begin{align*}
\Gamma_{WA}(D^0) & \propto \cos^2 \theta_c B_{WA}^s(D^0) + \sin^2 \theta_c B_{WA}^d(D^0), \\
\Gamma_{WA}(D^+) & \propto \cos^2 \theta_c B_{WA}^s(D^+) + \sin^2 \theta_c B_{WA}^d(D^+), \\
\Gamma_{WA}(D_s) & \propto \cos^2 \theta_c B_{WA}^s(D_s) + \sin^2 \theta_c B_{WA}^d(D_s),
\end{align*}
\]

by equating the difference between $D_s$ and $D^0$ rates with the isotriplet component of WA.

- Assumes SU(3) violating effects are sub-leading.
- Isosinglet component unconstrained.
Confronting OPE convergence in charm

In order to constrain WA fully, need to explicitly compute semileptonic rates and/or distribution moments - compare with exp.

Perturbative corrections known in the pole scheme

\[ \Gamma = \Gamma_0 \left[ 1 - 0.72 \alpha_s - 0.29 \alpha_s^2 \beta_0 - 0.60 \mu_G^2 - 0.20 \mu_\pi^2 + 0.42 \rho_D^3 + 0.38 \rho_{LS} + 80 B_{WA}^{(0)} \right], \]

\[ < E > = < E >_0 \left[ 1 - 0.03 \alpha_s - 0.03 \alpha_s^2 \beta_0 - 0.07 \mu_G^2 + 0.20 \mu_\pi^2 + 1.4 \rho_D^3 + 0.29 \rho_{LS} + 135 B_{WA}^{(1)} \right], \]

\[ < E^2 > = < E^2 >_0 \left[ 1 - 0.07 \alpha_s - 0.05 \alpha_s^2 \beta_0 - 0.14 \mu_G^2 + 0.52 \mu_\pi^2 + 3.5 \rho_D^3 + 0.66 \rho_{LS} + 204 B_{WA}^{(2)} \right], \]

\[ \sigma_E^2 = (\sigma_E^2)_0 \left[ 1 - 0.09 \alpha_s - 0.05 \alpha_s^2 \beta_0 - 0.14 \mu_G^2 + 1.7 \mu_\pi^2 + 9.4 \rho_D^3 + 1.4 \rho_{LS} + 641 B_{WA}^{(\sigma)} \right], \]

Renormalon (\( \Lambda/m_c \)) ambiguity of pole mass

- all moments affected (\( n \)-th scales as \( m_c^n \))

Better to use a short distance - threshold mass definition
Convergence of perturbative corrections

- Marginal in the pole scheme ($\alpha_s(m_c) \approx 0.35$)
  \[
  \frac{\Gamma}{\Gamma_0[m_c^{\text{pole}}]} = 1 - 0.269 \epsilon - 0.360 \epsilon^2_{\text{BLM}} + 0.069 \epsilon^2 + \ldots,
  \]

- Improves in short distance $m_c$ schemes
  \[
  \frac{\Gamma}{\Gamma_0[m_c^{1S}]} = 1 - 0.133 \epsilon - 0.006 \epsilon^2_{\text{BLM}} - 0.017 \epsilon^2.
  \]

- One can try to soften the strong dependence on the charm quark mass using information from inclusive B decays
  \[
  \frac{\Gamma}{\Gamma_0[m_b^{1S} - \Delta]} = 1 - 0.075 \epsilon - 0.013 \epsilon^2_{\text{BLM}} - 0.021 \epsilon^2, \quad (\Delta = m_b - m_c)
  \]

Ligeti et al.
1003.1351
Convergence of perturbative corrections

- In schemes with explicit IR cut-off, one needs to choose proper (low) IR scale (0.5–0.8 GeV)

- Need to translate OPE parameters as well (from global B fits)

- Perturbative and OPE corrections translated to kinetic scheme

\[
\begin{align*}
\Gamma_{\text{kin}} &= 1.2(3) \times 10^{-13} \text{GeV} \left\{ 1 + 0.23 \alpha_s + 0.18 \alpha_s^2 \beta_0 - 0.79 \mu_G^2 - 0.26 \mu_{\pi}^2 + 1.45 \rho_D^3 + 0.56 \rho_{LS}^3 + 120 \bar{B}_{WA}^{(0)} \right\}, \\
\langle E^1 \rangle_{\text{kin}} &= 0.415(21) \text{GeV} \left\{ 1 + 0.03 \alpha_s + 0.02 \alpha_s^2 \beta_0 - 0.09 \mu_G^2 + 0.26 \mu_{\pi}^2 + 2.7 \rho_D^3 + 0.44 \rho_{LS}^3 + 203 \bar{B}_{WA}^{(1)} \right\}, \\
\langle E^2 \rangle_{\text{kin}} &= 0.192(20) \text{GeV}^2 \left\{ 1 + 0.001 \alpha_s + 0.02 \alpha_s^2 \beta_0 - 0.18 \mu_G^2 + 0.68 \mu_{\pi}^2 + 6.6 \rho_D^3 + 0.99 \rho_{LS}^3 + 307 \bar{B}_{WA}^{(2)} \right\}, \\
\sigma_{E,\text{kin}}^2 &= 0.019(2) \text{GeV}^2 \left\{ 1 - 0.53 \alpha_s - 0.17 \alpha_s^2 \beta_0 - 0.18 \mu_G^2 + 2.2 \mu_{\pi}^2 + 17 \rho_D^3 + 2.1 \rho_{LS}^3 + 961 \bar{B}_{WA}^{(\sigma)} \right\},
\end{align*}
\]

- Rate uncertainty dominated by \( m_c \) & \( \mu_G \)

- Higher leptonic moments by \( \rho_D \)
Extraction of WA contributions

- Comparing theoretical expressions with experimental rates (in 1S scheme)
  - using OPE parameters and masses as extracted from global B decay fits
  - neglecting possible SU(3) violations
- Indication of a non-zero isosinglet WA contribution
  
  \[
  \begin{align*}
  a_0 &= 1.25 \pm 0.15, \\
  a_8 &= -0.20 \pm 0.12,
  \end{align*}
  \]
  
  Translates into \(O(1-2\%)\) effect in \(B \to X_u \ell \nu\) rate

\[
a_{0,8} = \frac{m_c^2 m_D f_D^2}{m_c^5} 16\pi^2 (D_{2,ns}^{8} - D_{1,ns}^{8}),
\]
Extraction of WA contributions

- Including information on the leptonic energy moments
- Different dependence of moments on the OPE parameters allows to possibly disentangle SU(3) violating effects from WA contributions
- Introduces dependence due to the modeling of the WA shape in the spectra
- Correlated WA determination from the rate and the moments
Extraction of WA contributions

- Including information on the leptonic energy moments
- Different dependence of moments on the OPE parameters allows to possibly disentangle SU(3) violating effects from WA contributions
- Introduces dependence due to the modeling of the WA shape in the spectra
- Correlated WA determination from the rate and the moments
  - Allowing for $O(20\%)$ SU(3) violation in OPE parameters
  - Largest uncertainty due to $\rho_D$ - linear (scale dependent) combination of $\rho_D$ and WA contributions determined precisely
  - For $\mu_{WA}\approx 1$GeV no clear indication of non-zero WA contributions

\[ B_{WA}^{s} = -0.0003(25) GeV^3 \]

- Translates into $O(2\%)$ uncertainty in $B \to X_u \ell \nu$ decay rate

Gambino & J.F.K
1004.0114
Conclusions

- Inclusive semileptonic charm decays can be used as a laboratory to test the OPE techniques used in the extraction of $|V_{ub}|$ and $|V_{cb}|$ from inclusive B decays.
  - Perturbative convergence seems to be surprisingly good.
- Use several observables to over-constrain the OPE parameter uncertainties and test OPE convergence.
- Indications that WA related uncertainties in inclusive $|V_{ub}|$ extraction smaller than previously expected [$O(1\%)$].
- More tests possible in the future with additional experimental inputs (experimentally determined leptonic energy and hadronic invariant mass moments) from Cleo and BESIII.
Backup Slides
Status of $B \rightarrow X_u \ell \nu$

See talks by R. Kowalewski & T. Mannel

- Experimental cuts on the leptonic energy and hadronic invariant mass to suppress dominant charm final state contributions

- Introduce theoretical sensitivity to effects beyond the OPE

- Modeled by s.c. shape-functions

- A fully inclusive analysis would carry a tiny 2-3\% theoretical error

P. Gambino, G. Ossola
hep-ph/0505091

$M_{X}^{cut} = 2.4 \text{ GeV}$

$\xi = \frac{2E_{\ell}^{cut}}{m_b} = 0.5$
Playing the experimentalist

One would want to compare completely inclusive leptonic energy moments in the rest-frame of the decaying hadron.

This is not what Cleo presently provide:
- do not compute the leptonic energy moments
- spectra given in the lab frame
- involve a lower $E_e = 0.2$ GeV cut
- do subtract the $D_s \rightarrow \tau \nu$ leptonic background

Asner et al. [CLEO]
0912.4232

See talk by K. Ecklund

[Graphs showing distributions with labels D^0 \rightarrow \pi^+ anything, D^+ \rightarrow \pi^+ anything, and D_s^+ \rightarrow \pi^+ anything]
Playing the experimentalist

One would want to compare completely inclusive leptonic energy moments in the rest-frame of the decaying hadron.

We try to compensate:

- Extrapolate the spectra down to $E_e=0$ using inclusive model shapes.
- Compute the leptonic energy moments from extrapolated spectra (in the lab frame).
- Boost the moments to the D frame by directional averaging.

\[
\langle E'_e \rangle = \gamma \langle E_e \rangle \quad \langle E'^2_e \rangle = \gamma^2 (1 + \beta^2/3) \langle E^2_e \rangle
\]

- D's produced in pairs at $E_{CM}=3774\,\text{MeV}$.
- $D_s$'s produced associated with $D_s^{*}$'s and through their decays.