Electroweak Corrections to the Top Quark Decay

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Abstract

We have calculated the one-loop electroweak corrections to the decay $t \rightarrow bW^+$, including the counterterm for the CKM matrix elements $V_{tb}$. Previous calculations used an incorrect $\delta V_{tb}$ that led to a gauge dependent amplitude. However, since the contribution stemming from $\delta V_{tb}$ is small, those calculations only underestimate the width by roughly one part in $10^5$.

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Due to its large mass, $m_t = 174.3 \pm 5.1$ GeV/$c^2$, the top quark, $t$, decays almost exclusively into a bottom quark, $b$, and a $W$-boson. This two-body channel, which is not available to the other quarks, makes the top quark singular. In fact, it is the only known quark where the weak decay takes place before the strong hadronization process. Hence, contrary to the other hadronic weak processes, one can calculate the width for the transition $t \to bW^+$ without being involved with the non-perturbative aspects of QCD. Because of this advantage this process will be a good testing ground for models beyond the Standard Model (SM). On the other hand, within the SM, the experimental measurement of the decay rate $\Gamma (t \to bW^+)$ gives a direct measurement of the $V_{tb}$ element of the Cabbibo-Kobayashi-Maskawa (CKM) matrix.

Recently, a similar result was obtained [9] applying the optical theorem to the two-body self-energy of the top quark. At order ($\alpha \alpha$), the contribution stems from the one-loop gluon correction to the weak vertex. This $O(\alpha^2)$ contribution was first evaluated by Jezabek and Kühn [5], and later confirmed by Denner and Sack (DS) [6] and Eilam et al. [7].

Presently, the direct observation of the top quark at the Tevatron [3] implies that $V_{tb}$ is known with a 30% error. The Particle Data Book [1] gives $V_{tb}$ with a smaller error, but using the CKM unitarity conditions. At the CERN LHC, with $10^7$ or $10^8$ top pairs per year, one expects to extract $V_{tb}$ with an error of the order 10%. It is then desirable to calculate the top width with a few percent precision.

At tree-level the $t \to bW^+$ width, $\Gamma_0$, is

$$\Gamma_0 = \frac{\alpha}{8 \sin^2 \theta_W} |V_{tb}|^2 \left[ \frac{(m_t^2 - (m_W + m_b)^2)^{1/2}}{2m_t} \right] \left[ \frac{(m_t^2 - (m_W - m_b)^2)^{1/2}}{2m_t} \right] \left[ \frac{m_t^2 + m_b^2}{2m_t^2} + \frac{(m_t^2 - m_b^2)^2}{2m_t^2 m_W^2} - \frac{m_W^2}{m_t^2} \right],$$

where $\alpha = 1/137.03599$ is the fine-structure constant and $\theta_W$ is the Weinberg angle ($\cos \theta_W = m_W/m_t$). The main correction to $\Gamma_0$ stems from the one-loop gluon correction to the weak vertex. This $O(\alpha \alpha_\pi)$ contribution was first evaluated by Gambino, Grassi and Madricardo (GGM) [12], and recently confirmed by Chetyrkin et al. [8].

Recently, a similar result was obtained [13] applying the optical theorem to the two-body self-energy of the top quark. At order ($\alpha \alpha_\pi$) there are two calculations. Czarnecki and Melnikov [10] evaluated the two-loop vertex diagram for $t \to bW^+$ using the $m_W^2 = 0$ approximation, while Chetyrkin et al. [8] expanded the imaginary part of the three-loop self-energy as a series in $q^2/m_t^2$. A recent approach to the same problem by Chinculov and Yao [11] uses a combination of analytical and numerical methods to evaluate the general massive two-loop Feynman diagrams. The electroweak corrections of order $\alpha^2$ were only evaluated in refs. [3] and [10]. However, as Gambino, Grassi and Madricardo (GGM) [12], have pointed out, in these papers the renormalization of $V_{tb}$ was done in such a way that the final result was gauge dependent. Recently we [13] have considered the renormalization of the CKM matrix, $V_{ij}$, in the generic linear $R_L$ gauge. We have confirmed that the DS [6] renormalization prescription leads to a gauge dependent amplitude and we have solved the problem introducing a condition to fix $\delta V_{ij}$, different from the one proposed by GGM [12]. Despite the fact that DS [6] have used a gauge dependent $\delta V_{ij}$, their numerical values for the $W$ partial decay widths are essentially correct. In fact, the $\delta V_{ij}$ contribution is negligible. This was confirmed by Kniehl et al. [14] using the GGM prescription. Clearly, it is in the top decay process that a wrong renormalization prescription leads to a gauge dependent amplitude and we have solved the problem introducing a condition to fix $\delta V_{ij}$, different from the one proposed by GGM [12].

We will compare our renormalization scheme [13] with the one proposed by GGM [12].

Denoting by $p$ and $q$ the four-momenta of the incoming top quark and the outgoing $W^+$, respectively, the tree level decay amplitude $T_0$ is:

$$T_0 = V_{tb} A_L,$$

with

$$A_L = \frac{g}{\sqrt{2}} \tilde{u}(p - q)/\varepsilon \gamma_L u(p),$$

where $\varepsilon^\mu$ is the polarization vector and, as usual, $\gamma_L = (1 - \gamma_5)/2$. The one-loop amplitude $T_1$ can be written in terms of four independent form factors, $F_L$, $F_R$, $G_L$ and $G_R$, each one associated with a given Lorentz structure.
for the spinors. $F_L$ is associated with $A_L$ and $F_R$ with $A_R$ which is given by eq. (3) replacing $\gamma_L$ by $\gamma_R$. Similarly, $G_L$ and $G_R$ are multiplied by $B_L$ and $B_R$, respectively, given by:

$$B_{L,R} = \frac{g}{\sqrt{2}} (p - q) \frac{\varepsilon \cdot p}{m_W} \gamma_{L,R}(p).$$

(4)

Besides the form factors, $T_1$ also depends on the counterterms. The final result is:

$$T_1 = A_L \left[ V_{tb} \left( F_L + \frac{\delta g}{g} + \frac{i}{2} \delta Z_W + \frac{1}{2} \delta Z_{it}^{L*} + \frac{1}{2} \delta Z_{tb}^{L*} \right) + \sum_{I \neq t} \frac{1}{2} \delta Z_{iI}^{L*} V_{ib} + \sum_{j \neq b} V_{ij} \frac{1}{2} \delta Z_{jb}^{L*} + \delta V_{tb} \right]$$

$$+ V_{tb} \left[ A_R F_R + B_L G_L + B_R G_R \right].$$

(5)

A detailed discussion of the counterterms can be found in our previous work [13] and so there is no need to repeat it here. In particular, we have shown [13] that one obtains a finite and gauge invariant $T_1$ with the $V_{tb}$ counterterm, $\delta V_{tb}$, given by:

$$\delta V_{tb} = -\frac{1}{2} \sum_{I \neq t} \delta Z_{iI}^{L*} V_{tb} - \frac{1}{2} \sum_{j \neq b} V_{ij} \delta Z_{jb}^{L*} - \frac{1}{2} V_{tb} \left[ \delta Z_{tt}^{L*} - \delta Z_{tt[1]}^{L} + \delta Z_{bb}^{L*} - \delta Z_{bb[1]}^{L} \right].$$

(6)

where $\delta Z_{iI}^{L*}$ and $\delta Z_{jj}^{L*}$ are the up and the down left-handed quark wave functions renormalization constants, respectively. A $\delta Z$ with the subscript [1] means that in its evaluation the CKM matrix was replaced by the identity matrix.

Let us stress that the only difference between our calculation and the previous ones [6, 7] is entirely due to a different choice of $\delta V_{tb}$. Unfortunately, the choice made by DS [6] is not physically acceptable. However, as we will see, $\delta V_{tb}$ gives a rather small contribution. Hence, the numerical result does not show any meaningful change. Perhaps, the best way to discuss the result is to define $\delta$ as:

$$\delta = \frac{2 \text{Re}[T_0 T_1^+]}{|T_0|^2}.$$

(7)

This, in turn, means that up to $\mathcal{O}(\alpha^2)$ the decay amplitude can be written as:

$$\Gamma = \Gamma_0 [1 + \delta].$$

(8)

In table [1] we show the different contributions to $\delta$ arising from the individual terms of eq. (6). In the calculations the program packages FeynArts [15], FeynCalc [16] and LoopTools [17] were used. Notice that, with our renormalization prescription for $\delta V_{tb}$, all contributions from the off-diagonal quark wave-functions renormalization constants are canceled and one simply needs to evaluate $\delta Z_{tt}^{L*}$ and $\delta Z_{bb}^{L*}$. They, together with the other counterterms give a large positive $\delta$ (23.66%) which is then reduced to 4.46% with the negative contribution of $F_L$ ($-18.75\%$) and $G_R$ ($-0.44\%$). The other form factors give negligible contributions. It is interesting to see the difference when we follow the CKM renormalization prescription given by GGM [12]. The calculation is slightly more complicated: the off-diagonal terms proportional to $\delta Z_{tt}^{L*}$ and $\delta Z_{jj}^{L*}$ have to be included; the diagonal terms $\delta Z_{tt}^{L*}$ and $\delta Z_{bb}^{L*}$ have to be calculated without the approximation of replacing the CKM matrix by the unit matrix; and finally one ought to add $\delta V_{tb}^{G}$ given by:

$$\delta V_{tb}^{G} = \frac{1}{2} \left[ \sum_{I} \delta Z_{iI}^{L,A} V_{tb} - \sum_{j} V_{ij} \delta Z_{jb}^{L,A} \right].$$

(9)
where the $\delta Z^{L,A}_{ij}$ are “special” anti-hermitian wave function renormalization constants fixed in terms of the quark self-energies at $q^2 = 0$, namely,

$$\delta Z^{L,A}_{ij} = \frac{m_i^2 + m_j^2}{m_i^2 - m_j^2} \left[ \Sigma^L_{ij}(0) + 2\Sigma^S_{ij}(0) \right]. \tag{10}$$

For the sake of completeness we have also listed in table 1 the numerical values of these additional contributions.

| Form Factors and Counterterms | Contributions to $\delta$ (%) |
|-------------------------------|-----------------------------|
| $F_L$                         | $-18.753$                   |
| $F_R$                         | $-2 \times 10^{-3}$        |
| $G_L$                         | $-8 \times 10^{-4}$        |
| $G_R$                         | $-0.445$                   |
| $\delta g$                   | $10.419$                   |
| $\frac{1}{2}\delta Z^L_{W[1]}$ | $3.193$                   |
| $\frac{1}{2}\delta Z^L_{tt[1]}$ | $5.220$                   |
| $\frac{1}{2}\delta Z^L_{bb[1]}$ | $4.831$                   |
| Total                         | $4.46$                      |

| $\frac{1}{2} \sum_{I \neq t} \delta Z^L_{tI} V_{tI}$ | $9 \times 10^{-6}$ |
| $\frac{1}{2} \sum_{I \neq t} V_{ti} \delta Z^L_{tI}$ | $-1.8 \times 10^{-3}$ |
| $\frac{1}{2} \delta Z^L_{tt} - \frac{1}{2} \delta Z^L_{tt[1]}$ | $-0.1 \times 10^{-3}$ |
| $\frac{1}{2} \delta Z^L_{tb} - \frac{1}{2} \delta Z^L_{bb[1]}$ | $-5.3 \times 10^{-3}$ |
| $\delta V^G_{tb}$                         | $6.4 \times 10^{-3}$ |
| Total                                      | $-0.8 \times 10^{-3}$ |

Table 1: Contributions to $\delta$ from the individual terms in eq.(5) evaluated at $m_t = 174.3 \text{ GeV}/c^2$ and $m_H = 114 \text{ GeV}/c^2$. The lower part lists the additional contributions needed if the GGM renormalization prescription is used.

They are all extremely small which means that $\delta$ is practically the same in both renormalizations schemes.

Certainly, the uncertainty introduced in the calculation by the error in the top quark mass is far more important. To illustrate this remark and to avoid the need to repeat this calculation in the future we have done it varying $m_t$ in the two-sigma interval around the present experimental mean value. We have found, that within this interval the value of $\delta$ can be very well reproduced by the linear fit:

$$\delta = \left[ 12.7715 - 0.0477 \frac{m_t}{\text{GeV}/c^2} \right] \times 10^{-2}. \tag{11}$$

Figure 1 shows the quality of this fit. Another parameter that enters the calculation is the Higgs mass $m_H$. In the results given in table 1 and in fig. 1 we have used rather arbitrarily $m_H = 114 \text{ GeV}/c^2$. As it is well known $\delta$ depends logarithmically on $m_H$. Again for $m_t = 174.3 \text{ GeV}/c^2$ and for $100 \text{ GeV}/c^2 \leq m_H \leq 400 \text{ GeV}/c^2$, $\delta$ could be fitted with the following expression:

$$\delta = \left[ 4.4457 + 0.1172 \ln \frac{m_H}{100 \text{ GeV}/c^2} \right] \times 10^{-2}. \tag{12}$$
In figure 2 we show the result and the fitted curve.

We would like to summarize our conclusions as follows:

i. Using our [13] prescription for the renormalization of the CKM matrix elements we have calculated the electroweak radiative corrections to the decay width $t \to bW^+$;

ii. For $m_t = 174.3 GeV/c^2$ and $m_H = 114 GeV/c^2$, the correction is $\delta = 4.46\%$. This increases the tree level value of $\Gamma$ from 1.4625 $GeV/c^2$ to 1.5277 $GeV/c^2$;

iii. We have checked that an alternative renormalization prescription advocated by GGM [12] gives a width that differs from ours by less than one part in $10^5$;

iv. The contribution to $\delta$ stemming from the $\delta V_{tb}$ counterterm is rather small. It is $7.2 \times 10^{-3}\%$ versus $6.4 \times 10^{-3}\%$ in the GGM [12] scheme, while the old DS [6] $\delta V_{tb}$ counterterm would have given $6.6 \times 10^{-3}\%$.

![Figure 1: $\delta$ as a function of the top mass.](image-url)
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Figure 2: $\delta$ as a function of the Higgs mass.
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