The Standard Model Partly Supersymmetric

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Abstract

We present a novel class of theories where supersymmetry is only preserved in a partial (non-isolated) sector. The supersymmetric sector consists of CFT bound-states that can coexist with fundamental states which do not respect supersymmetry. These theories arise from the 4D holographic interpretation of 5D theories in a slice of AdS where supersymmetry is broken on the UV boundary. In particular, we consider the Standard Model where only the Higgs sector (and possibly the top quark) is supersymmetric. The Higgs mass-parameter is then protected by supersymmetry, and consequently the electroweak scale is naturally smaller than the composite Higgs scale. This not only provides a solution to the hierarchy problem, but predicts a “little” hierarchy between the electroweak and new physics scale. Remarkably, the model only contains a single supersymmetric partner, the Higgsino (and possibly the stop), and as in the usual MSSM, predicts a light Higgs boson.
1 Introduction

In quantum field theories it is unnatural to impose symmetries that are only restricted to certain parts of the Lagrangian. This is because at the quantum level, interactions with sectors that are not symmetric will, in general, spoil the underlying symmetry of the symmetric sectors. In fact these quantum effects are generally divergent, which signals that one must include, from the beginning, all possible non-symmetric terms in the Lagrangian to absorb the infinities. Thus, in general, symmetries can only be maintained if the full theory respects them.

It is for this reason that in supersymmetric theories, which are advocated to solve the hierarchy problem, supersymmetry must be respected in all sectors of the theory. If there is a sector where supersymmetry is not realized below the scale $\Lambda$, and is coupled to another supersymmetric sector with coupling $g$, then a fermion-boson mass splitting of order $\frac{g^2}{4\pi}\Lambda$ will be induced in the supersymmetric sector. Therefore to maintain supersymmetry in the presence of a nonsupersymmetric sector, either $g$ must be very tiny (decoupling limit), or $\Lambda$ is small (supersymmetry is preserved in the full theory down to low energies).

In this work we will present theories which are an exception to the above general argument. These theories will consist of two sectors: a (super)symmetric sector that contains bound-states of a spontaneously broken conformal field theory (CFT), and a non-(super)symmetric sector which contains fundamental states. Using the AdS/CFT correspondence, we will show that the CFT bound-states decouple from the non-supersymmetric sector at energies above $1/L$, where $L$ is the size of the bound-states (the scale of the conformal symmetry breaking). Thus, the bound states are insensitive to (super)symmetry breaking effects at high energies.

This scenario allows us to have a non-supersymmetric sector coexisting with the supersymmetric sector, even though the coupling $g$ is of order one, and the scale $\Lambda$ is large. As an interesting application, we consider the Standard Model (SM) where only the Higgs sector is (approximately) supersymmetric. This enables one to have a prediction for the tree-level Higgs potential. For example, the quartic coupling is determined by supersymmetry to be $(g^2 + g^2)/8$, whereas in the minimal supersymmetric SM (MSSM), this leads to a light Higgs boson mass. Furthermore, the Higgs mass-parameter, that determines the electroweak scale, is protected by supersymmetry down to low-energies $\lesssim 1/L$. This mass can be induced at the quantum level by SM fields, giving

$$m_{EW} \sim \frac{g}{4\pi} \frac{1}{L} \ll \Lambda \sim M_P.$$  

Thus we see that these theories provide a rationale for having the electroweak scale naturally smaller than the composite Higgs scale $1/L$, which in turn can be much
smaller than the Planck scale. In fact, this “little hierarchy” between the electroweak scale and the scale of new physics (which in our scenario is $1/L$) is currently suggested by present collider experiments. This has also motivated other theoretical models with this property, such as TeV extra dimension models or the “little Higgs” models [2].

2 Partly (super)symmetric theories

To understand why (super)symmetric CFT bound-states are not sensitive to supersymmetry breaking effects at high energy scales, we will use the AdS/CFT correspondence, and consider the 5D anti de-Sitter (AdS$_5$) point of view. In this 5D dual picture the possibility of having partly (super)symmetric theories is very simple to understand.

Let us then start by considering a quantum field theory in a slice of 5D AdS $\mathbb{R}^3$ (Fig. 1). The 5D metric is given by

$$ds^2 = a^2(y)dx^2 + R^2 dy^2,$$

where $a(y) = e^{-kRy}$ is the warp factor, $1/k$ is the AdS curvature radius, and $\pi R$ is the proper length of the extra dimension $y$. This 5D theory has two 4D boundaries at $y = 0$ and $y = \pi$. Due to the warp factor, energy scales on the $y = \pi$ boundary are reduced by a factor $a(\pi)$ compared to those on the $y = 0$ boundary (this can be thought of as a redshift due to the warped space). We will suppose that the
full 5D theory is supersymmetric, and that supersymmetry is broken only on the $y = 0$ boundary at a scale of order the cutoff scale $\Lambda$ of the theory. Scalar fields localized on the $y = 0$ boundary will receive masses of order $\Lambda$. Similarly, bulk scalar fields will receive boundary masses of this order. However, scalar fields living on the $y = \pi$ boundary, will only know about the breaking of supersymmetry by loop effects of bulk fields. Consequently, the contribution to their masses will arise from bulk fields propagating from the $y = \pi$ boundary to the $y = 0$ boundary (where supersymmetry is broken) as shown in Fig. 1. This is a non-local effect which gives a finite contribution to the scalar masses. What is the order of magnitude of these corrections? A rough estimate of the energy involved in the virtual contribution to the scalar mass is given by $E \sim 1/\tau \sim ka(\pi)$, where $\tau$ is the time it takes to propagate from one boundary to the other (the conformal distance). After multiplying by the coupling of the bulk field to the $y = 0$ boundary, $g_0(E)$, and the $y = \pi$ boundary, $g_\pi(E)$, we obtain an estimate of the induced scalar mass

$$m^2 \sim \frac{g_0(\frac{1}{2})g_\pi(\frac{1}{2})}{16\pi^2} a^2(\pi)k^2.$$ (3)

Notice that due to the warp factor $a(\pi)$ this mass can be very small, and consequently fields on the $y = \pi$ boundary receive supersymmetry-breaking masses much smaller than the cutoff scale, $\Lambda$. Of course, this is expected since we know that energy scales on the $y = \pi$ boundary are redshifted with respect to those on the $y = 0$ boundary. More interestingly, it is the interpretation of this effect in the dual 4D theory to which we now turn to.

The above theory has a 4D interpretation based on the AdS/CFT correspondence [4]. This correspondence relates the 5D AdS theory to a strongly coupled 4D CFT with a large number of “colors” $N_c$. The 5D bulk fields at the $y = 0$ boundary, $\Phi(x)$, are identified as sources of CFT operators

$$\mathcal{L} = g \Phi(x)\mathcal{O}(x),$$ (4)

where the mass of $\Phi$ is related to the dimension of the operator $\mathcal{O}$. The boundary at $y = 0$ corresponds to an ultraviolet (UV) cutoff at $p = k$ in the 4D CFT [5], while the boundary at $y = \pi$ corresponds to an infrared (IR) cutoff at $p = ka(\pi)$ [6, 7]. Therefore, a slice of the bulk AdS space corresponds in 4D to a slice of CFT in momentum-space. Due to the term in Eq. (4), the CFT can generate a kinetic term for the sources $\Phi(x)$ which become dynamical, and these must then be included in the theory as extra “fundamental” fields. The IR breaking of the CFT theory introduces a mass gap of order $1/L = ka(\pi)$, where bound states, $M$ (“mesons” or “baryons”), are formed. For large $N_c$ [8], we know that the infinite number of bound-states are weakly coupled, and have a mass-spacing of order $ka(\pi)$.

The fundamental fields $\Phi$, and the CFT bound-states are not mass eigenstates, since, due to Eq. (4), they will mix with each other. However, after a rediagonaliza-
tion, one can obtain the mass eigenstates. In this basis we can map these eigenstates into the zero-mode, and Kaluza-Klein (KK) modes of the dual 5D theory. The question of whether the mass eigenstates are fundamental or CFT bound-states depends on the amount of mixing or where the fields are localized. The Kaluza-Klein states always have wave-functions which are peaked towards the boundary at $y = \pi$. Thus, they will have a small wave-function overlap with the fundamental fields $\Phi$, since by the AdS/CFT correspondence $\Phi$ is associated with the 4D field localized on the boundary at $y = 0$. Thus, to a good approximation, the KK-states always correspond to the CFT bound-states. On the other hand, fields living completely on the boundary at $y = \pi$ will correspond to pure CFT bound-states because they cannot mix with the fundamental fields $\Phi$.

By supersymmetrizing the theory, the scalar bound-states can be massless since the scalar masses are then protected by supersymmetry. However, if in the 5D AdS theory we break supersymmetry at the $y = 0$ boundary, then this corresponds in the 4D dual theory to breaking supersymmetry at the UV cutoff scale in the $\Phi$ sector. The CFT sector, however, is coupled to $\Phi$ via Eq. (4) and therefore it will also feel the breaking of supersymmetry. Whenever $\langle 0|\mathcal{O}|M \rangle \neq 0$, we have that $M$ and $\Phi$ mix. The states $M$ will then receive a (tree-level) correction to their masses that will not respect supersymmetry. For small mixing this is giving by the diagram of Fig. 2. This mixing is completely negligible when $M$ is the CFT bound-state dual to a field living on the boundary at $y = \pi$. In this case the dominant supersymmetry-breaking effect will arise at the one-loop level as shown in Fig. 3. Nevertheless, we learnt from the AdS$_5$ dual theory that these contributions are generated at scales $\lesssim 1/L$. Qualitatively, this can also be understood in the CFT picture. The bound-state $M$ is a CFT lump of size $L$ that decouples from fields of wavelength smaller than $L$. Conformal invariance protects the symmetries of the $M$ sector at short distances [$9$], and any (super)symmetry breaking effect is only induced at distances larger than $L$. We can determine more quantitatively this decoupling by using the AdS/CFT correspondence. For example, we can calculate the amplitude of the deep-inelastic scattering between a “probe” particle, $e$, and a CFT meson $M$, mediated by $\Phi$ which we take to be a photon (see Fig. 4)

$$eM \rightarrow eX ,$$  

(5)
Figure 4: Inelastic scattering of a fundamental field, $e$, with a CFT bound-state, $M$.

where by $X$ we refer to any combination of final states. In the 5D AdS picture, the photon is simultaneously coupled to the boundary at $y = \pi$ (where $M$ lives), and to the boundary at $y = 0$ (where non-CFT fields, like $e$, live). Therefore, the amplitude is proportional to the 5D photon propagator $G(p, y = 0, y' = \pi)$ calculated in Ref. [10] where $p$ is the 4D momentum. This propagator drops exponentially at momentum scales larger than $ka(\pi) = 1/L$,

$$G(p, y = 0, y' = \pi) \sim e^{-pL},$$

and shows that the CFT meson $M$ quickly decouples from the photon when $pL > 1$. Alternatively at high momenta, where the conformal symmetry is restored, the meson constituents are no longer localized particle states, and so become transparent to the short wavelength photon probe.

We must stress that this is not a general property of composite models. For example, a theory like technicolor where the strong dynamics are assumed to be similar to QCD will not show this decoupling. At high energies, the non-supersymmetric sector is not exponentially decoupled, since it can couple to the constituents of the bound-states. Therefore bound-states made of scalars (partners of the techniquarks in a supersymmetric technicolor sector) will receive large corrections to their masses.

### 3 The SM partly supersymmetric

Here we want to consider the possibility that only the Higgs sector of the SM is supersymmetric. Our motivation is to obtain a Higgs mass parameter that is insensitive to high-energy physics, and is induced at low energies at the quantum level. This will guarantee a partial decoupling between the scale of new physics and the electroweak scale that experiments, in particular LEP, are suggesting. From the previous section,
we know that this can be achieved by requiring the Higgs to be a CFT bound-state, while the SM fields are fundamental. This scenario is most simply realized if we start in the AdS picture, where a slice of AdS$_5$ is bounded by a UV-brane with $\Lambda \sim M_P$ (or Planck-brane), and an IR-brane with $\Lambda_{IR} = a(\pi)\Lambda \sim$ TeV (or TeV-brane). We will also assume that $k \sim M_P$, and then $L = 1/(a(\pi)k) \sim 1$/TeV. All the SM fields are assumed to reside in the bulk, except for the Higgs sector that will be localized on the TeV-brane. We will start with a supersymmetric bulk theory, and then we will break supersymmetry on the Planck-brane at the Planck scale. The supersymmetry breaking on the Planck-brane will be parametrized by the spurion superfield, $\eta = \theta^2 F$. Only bulk fields with Neumann boundary conditions can couple to the spurion superfield. Similar scenarios but with the breaking of supersymmetry on the TeV-brane have been previously considered in Refs. [11, 12, 13, 14]. Unlike the model that we will present here, these scenarios resemble the MSSM at low energies.

Consider first the gauge sector [11], where the supersymmetric action in superfield notation can be found in Ref. [13]. The 4D massless spectrum corresponds to an $N = 1$ vector multiplet

$$V = \{ A_\mu^a, \lambda^a, D^a \} .$$  \hspace{1cm} (7)

It has a wave-function that is flat, and couples with equal strength to the two branes. Therefore, by adding the extra term

$$\int d^2 \theta \frac{\eta}{\Lambda^2} \frac{1}{g_5^2} WW \delta(y) + h.c. ,$$  \hspace{1cm} (8)

the gaugino $\lambda^a$ will receive a huge mass $m_\lambda \sim F/\Lambda \sim M_P$ (see Appendix). Consequently, the gaugino effectively decouples, and the spectrum reduces to simply the gauge boson $A_\mu^a$, and the auxiliary field $D^a$ with the Lagrangian

$$\mathcal{L} = -\frac{1}{4g^2} F^{a\mu\nu} F_{a\mu\nu} + \frac{1}{2g^2} D^a D^a ,$$  \hspace{1cm} (9)

where $g$ is the 4D gauge coupling related to the 5D coupling by

$$\frac{1}{g^2} = \frac{\pi R}{g_5^2} .$$  \hspace{1cm} (10)

Thus, at the massless level supersymmetry is completely broken, and we just have the SM gauge bosons.

Similarly, matter fields (such as quarks and leptons) arise from 5D hypermultiplets. In the supersymmetric limit, the 4D massless spectrum is described by an $N = 1$ chiral multiplet [13]

$$Q = \{ \tilde{q}, q, F_Q \} .$$  \hspace{1cm} (11)
Their wave-functions depend on the 5D hypermultiplet mass, and for simplicity we will assume the value $M_{5D} = k/2$ ($c = 1/2$ in the notation of Ref. [11]) that corresponds to flat wave-functions [11, 13]. For this value of $M_{5D}$ the holographic interpretation is the same as that for the gauge sector (see Appendix). Other values of $M_{5D}$ will be considered for the top quark in Section 3.1. Supersymmetry breaking is induced by adding on the Planck-brane the interaction

$$- \int d^4 \theta \frac{\eta^\dagger \eta}{\Lambda^4} Q^\dagger Q k \delta(y),$$

which gives rise to squark and slepton masses of order $m_{\tilde{q}} \sim F/\Lambda \sim M_P$ (see Appendix). These scalar superpartners effectively decouple, and the massless spectrum solely consists of the fermions $q$ and auxiliary fields $F_Q$.

In this way all the supersymmetric partners of the SM fields have received Planck-scale masses, and the massless spectrum reduces to the usual SM. At the massive level, the first KK-states appear at the scale $1/L \sim \text{TeV}$. Unlike the massless states, the KK-spectrum is (approximately) supersymmetric. This is because the wave-functions of the KK-states are localized toward the TeV-brane and are less sensitive to the supersymmetry breaking on the Planck-brane.

The theory considered so far is the 5D dual theory of the SM coupled to a supersymmetric CFT theory. Next we will consider the Higgs sector. Since we want the Higgs sector to be supersymmetric, it must be confined to the TeV-brane. In the 4D dual, this corresponds to having a CFT composite Higgs. The supersymmetric partner of the Higgs, the Higgsino, generates gauge anomalies that must be cancelled. This leads us to consider the following three possible Higgs scenarios:

a) Two Higgs doublets: As in the MSSM, an extra Higgs (and Higgsino) is introduced to cancel the anomalies.

b) One Higgs doublet: The Higgsino anomalies are cancelled by an extra fermion that, as for the SM fermions, arises from a 5D hypermultiplet. Hence, the scalar partner of the extra fermion will obtain a Planck-sized mass.

c) Higgs as a slepton: No Higgsino is introduced. Instead, the tau (or other lepton) is assumed to be the supersymmetric partner of the Higgs. For this purpose the tau must live on the TeV-brane (contrary to the rest of the matter fields).

Let us now discuss in detail each of these possibilities.
a) Two Higgs doublet model

Suppose that the Higgs sector consists of two $N = 1$ 4D chiral multiplets, $H_1 = \{h_1, \tilde{h}_1, F_{H_1}\}$, and $H_2 = \{h_2, \tilde{h}_2, F_{H_2}\}$. The fact that the Higgs sector is doubled guarantees two things. First, anomalous contributions arising from the Higgsinos $\tilde{h}_i$ are automatically cancelled, and secondly, that quark and lepton masses can be simply obtained from the following superpotential on the TeV-brane

$$\int d^2\theta \left[ y_d H_1 Q D + y_u H_2 Q U + y_e H_1 L E \right].$$

(13)

Although the Higgs spectrum is supersymmetric, it directly couples to the SM which does not respect supersymmetry (since it is broken at the Planck scale). This gives rise to the following effective theory for the Higgs

$$\mathcal{L} = -|D_\mu h_i|^2 - i\bar{\tilde{h}}_i D\tilde{h}_i - D^a(h_1^\dagger T^a h_i) + y_d h_1 q d + y_e h_1 l e + y_u h_2 q u,$$

(14)

where $D_\mu$ is the covariant derivative, and $T^a$ are the generators of the gauge group. The complete effective Lagrangian below the TeV scale is given by Eqs. (9) and (14).

Eliminating the auxiliary field $D^a$, we obtain the tree-level Higgs potential

$$V = \frac{1}{2g^2} D^a D^a = \frac{g^2}{2} (h_1^\dagger T^a h_i)^2.$$

(15)

For the neutral component of the Higgs the potential becomes

$$V = \frac{1}{8} (g^2 + g'^2) (|h_1|^2 - |h_2|^2)^2,$$

(16)

where $g$ and $g'$ are the SU(2)$_L$ and U(1)$_Y$ couplings, respectively. We have obtained an interesting result. Although the Higgs mass is zero by supersymmetry, the Higgs potential has a quartic coupling that does not respect supersymmetry since it is generated from the $D$-term of the gauge sector. At the quantum level, radiative corrections will induce a mass term for the Higgs which will be naturally smaller than $1/L$. In section 3.1 we will calculate the one-loop effective potential. For now we will present the result of the Higgs mass induced by gauge loops given by the two-point contribution to the effective potential (first term of the sum of Eq. (31))

$$m_{h_i}^2 \simeq \left( \frac{0.14}{L} \right)^2,$$

(17)

where $1/L = a(\pi)k \sim$ TeV. This is a finite contribution since it comes from a non-local effect (see Fig. 1). As expected, the result (17) shows that the supersymmetry breaking effects in the Higgs sector are an order of magnitude below the scale $L^{-1}$, and are much smaller than the Planck scale. This gauge contribution is positive but, as we will show in Section 3.1, can be overcome by a negative contribution from the top-quark loop, in order to trigger electroweak symmetry breaking.
Higgsino masses: The $\mu$-problem

While radiative corrections induce a scalar Higgs mass, the Higgsinos remain massless. Of course, phenomenologically this is a problem, and we need to simultaneously consider possible mechanisms for generating Higgsino masses. In addition, the minimum of the Higgs potential depends critically on the value of the $B\mu$-term (the bilinear term $B\mu h_1 h_2$). This term is not generated by radiative corrections. This means that while $\langle h_2 \rangle \neq 0$, we have that $\langle h_1 \rangle = 0$, and the fermions in the down-sector remain massless. So, we will also need to generate a VEV for $h_1$.

The simplest possibility is to consider the following superpotential on the TeV-brane

$$\int d^2\theta \left[ \mu H_1 H_2 + \frac{\beta}{2\Lambda_{IR}}(H_1 H_2)^2 \right]. \quad (18)$$

The first term directly gives a mass to the Higgsino. By supersymmetry the Higgs scalar field will also receive a mass, since Eq. (18) leads to the Higgs potential

$$V = \left[ |\mu|^2 + \frac{\beta \mu^*}{\Lambda_{IR}}(h_1 h_2 + h.c.) \right](|h_1|^2 + |h_2|^2) + O \left( \frac{h_6^6}{\Lambda_{IR}^2} \right). \quad (19)$$

In order to have electroweak symmetry breaking, the value of $|\mu|^2$ cannot be larger than the negative one-loop top-quark contribution. This requires that $\mu \sim m_h \sim 0.1/L$ (if this value is smaller, then the Higgsino mass will be too small). The second term of Eq. (18) has been introduced to generate a linear term in $h_1$ (it plays the role of the $B\mu$-term in the MSSM). When $h_2$ gets a VEV, this linear term generates a VEV for $h_1$ of order $\langle h_1 \rangle \sim -\beta \langle h_2 \rangle^3/(\mu \Lambda_{IR}) \sim 0.1 \langle h_2 \rangle$.

Instead of introducing a $\mu$-term from the beginning in our theory, we can imagine generating it by the one-loop supersymmetry breaking effects. In this case the $\mu$-term will naturally be of order the electroweak scale. For example, we can consider a sector that has a field $X$ whose $F$-term squared is induced at the one-loop level. If this field couples to the Higgs in the following way

$$\int d^4\theta \frac{X^\dagger}{\Lambda_{IR}} H_1 H_2,$$  \quad (20)

then it will generate a $\mu$-term with $\mu = \langle F_X \rangle/\Lambda_{IR}$. Furthermore, $h_1$ does not need to get a VEV since fermion masses can be generated from the Kahler potential terms

$$\int d^4\theta \frac{X^\dagger}{\Lambda_{IR}^2}(H_2^\dagger Q D + H_2^\dagger LE). \quad (21)$$

The bottom quark Yukawa coupling will be given by $y_b \sim \langle F_X \rangle/\Lambda_{IR}^2 \sim 0.1$, where we have assumed that the Kahler-term coupling is of order one. For the other quarks and leptons in the down sector, these couplings will need to be hierarchically smaller.
It is also possible to introduce a singlet chiral supermultiplet, $S$, on the TeV-brane with a superpotential $SH_1H_2 + S^3$. A $\mu$-term can then be generated if $S$ gets a VEV that, parametrically, will be of the same order as the electroweak scale. This possibility deserves further analysis which will not be carried out here.

b) One Higgs doublet model

A natural solution to the $\mu$-problem exists if the Higgs arises from a 5D bulk hypermultiplet that consists of two $N = 1$ chiral multiplets, $H_{1,2}$, of opposite charges. If the boundary conditions for $H_2$ are taken to be Neumann, while those for $H_1$ are Dirichlet (as in the matter sector), then the 4D massless sector will correspond to a single 4D chiral multiplet. This theory will then be anomalous.

However, if $H_1$ is assumed to have Neumann boundary conditions on the Planck-brane but Dirichlet on the TeV-brane (and vice-versa for $H_2$), then the lowest lying state is a massive pair of 4D chiral multiplets. These twisted boundary conditions then lead to a theory that is not anomalous. The mass of these chiral multiplets can be written as a superpotential term, $\mu H_1H_2$, where the value of $\mu$ depends on the 5D hypermultiplet mass. In particular, assuming a 5D mass term $M_{5D} = k/2$, we find that

$$\mu \simeq \sqrt{\frac{2}{\pi k R}} ke^{-\pi k R}. \quad (22)$$

This mass term is analogous to the gaugino mass term obtained in Ref. [12]. Note that the $\mu$-term is naturally suppressed below the TeV-scale by the factor, $1/\sqrt{\pi k R}$. The Higgs wave-functions of the lowest lying modes become

$$H_2(y) \simeq \sqrt{2\pi k R} e^{-\pi k R} \frac{|y|}{\pi R} e^{\frac{2}{\pi k R} |y|}, \quad (23)$$

$$H_1(y) \simeq 2e^{-\pi k R} \sinh[k(|y| - \pi R)]e^{\frac{5}{2}k |y|}, \quad (24)$$

showing that $H_2$ is localized towards the TeV-brane, while $H_1$ is localized towards the Planck-brane. Hence, $H_1$ will be sensitive to the breaking of supersymmetry, and as in the matter sector, the scalar $h_1$ will receive a Planck mass.

Thus, the effective theory of the bulk Higgs with twisted boundary conditions below the TeV-scale consists of one chiral supermultiplet $H_2$, an extra fermion $\tilde{h}_1$, and the $F_{H_1}$ auxiliary field. The effective Lagrangian is given by

$$-\mathcal{L}_{eff} = \mu \tilde{h}_1 h_2 + \mu^2 |h_2|^2 + \frac{1}{8} (g^2 + g'^2) |h_2|^4, \quad (25)$$

\footnote{For $M_{5D} < k/2$ the two Higgsinos are localized towards the TeV-brane and the $\mu$-term becomes $\mathcal{O}(\text{TeV})$. For $M_{5D} > k/2$ one Higgsino becomes strongly localized towards the Planck-brane, while the other towards the TeV-brane. In this case the $\mu$-term is driven to exponentially small values, $\mu \sim e^{-(M_{5D}/(k-\frac{1}{2})\pi k R)}$.}
where the second (last) term in (25) is the $F_{H_1}$-term ($D$-term) contribution. In the 4D dual description, $h_2$ and $\tilde{h}_2$ are composite states of the CFT, while $\tilde{h}_1$ is a fundamental field that has been added to the CFT. The $\mu$-term can then be understood as arising from the marriage of the Higgsino $\tilde{h}_1$ with the fermion bound-state corresponding to $\tilde{h}_2$. This is exactly analogous to the dual interpretation of the gaugino mass in the warped MSSM [12].

c) Higgs as a slepton

Neither anomalies nor the $\mu$-problem will arise if the Higgs is considered to be the superpartner of the tau (or other lepton), and forms a 4D chiral multiplet, $L_3 = (h, \tau, F_{\tau})$, on the TeV-brane. This idea is not new, and dates back to the early days of supersymmetry [15]. The major obstacle in implementing this identification is that neutrino masses are large, and consequently, experimentally ruled out. This is because the gaugino induces the effective operator

$$\frac{g^2}{M_\lambda} \nu \nu hh,$$

where $M_\lambda$ is the gaugino mass. Thus, for $M_\lambda \sim \text{TeV}$, this operator generates a neutrino mass of order $(\langle h \rangle^2/M_\lambda) \sim 10 \text{ GeV}$.

However, in our warped model with only a partly supersymmetric spectrum, there is an approximate $U(1)_R$ symmetry which acts as a continuous lepton symmetry, and suppresses the neutrino masses. This symmetry is exact in the low-energy effective Lagrangian because there is no gaugino partner to the SM gauge boson, and the Kaluza-Klein gauginos have Dirac masses which are invariant under the $U(1)_R$. However, the $U(1)_R$ symmetry is broken at a high scale by the large gaugino mass $M_\lambda \sim M_P$, naturally giving small neutrino masses $m_\nu \sim 10^{-5} \text{ eV}$.

Having obtained acceptable neutrino masses, we can also generate the required fermion mass spectrum from operators only involving $L_3$. The couplings that generate masses for the down quarks and charged leptons can come from the superpotential (on the TeV-brane)

$$\int d^2 \theta \left( y_d^{(i)} L_3 Q_i D_i + y_e^{(i)} L_3 L_i E_i \right),$$

except for the third generation charged fermion in $L_3$, since due to the antisymmetry of the $SU(2)$ indices, $L_3 L_3 = 0$. In this case one must rely on higher-dimensional operators in the Kahler potential [16]. By holomorphy, there is no superpotential term which generates masses for the up quarks. Instead one must consider Kahler potential couplings like

$$\int d^4 \theta y_u^{(i)} \frac{X^i}{N_{IR}} L_3^\dagger Q_i U_i,$$
where $X$ is a spurion field that, as in the previous examples, has an $F$-term, $F_X$, on the TeV-brane. Notice, however, that one needs $F_X \lesssim 0.1 \Lambda_{IR}^2$, otherwise Kahler terms such as $X^\dagger X L_3^\dagger L_3$ give a large contribution to the Higgs mass-parameter, and lead to an electroweak scale which is too close to $\Lambda_{IR}$. Such a small value of $F_X$ can be problematic to generate the top quark mass from Eq. (28) unless the coefficient $y_u^{(3)}$ is large. In spite of this problem, we find this scenario very interesting because no extra matter is needed in order to have supersymmetry protect the Higgs mass.

3.1 One-loop effective potential and electroweak symmetry breaking

The Higgs fields will receive supersymmetry-breaking contributions from the bulk vector multiplets and bulk hypermultiplets containing the quarks and leptons as shown in Fig. 1. The simplest way to compute these contributions is to use the 5D AdS propagator in loop calculations. The general expressions for these propagators in a slice of AdS$_5$ were presented in Ref. [12]. Since the Higgs multiplet is located on the TeV-brane, we will be interested in the propagator expressions evaluated at the TeV boundary ($y = y' = \pi$). Following the notation of Ref. [12], we have that for bulk fields $\{V_\mu, \phi, e^{-2\sigma} \psi_{L,R}\}$ with masses $\hat{M}^2 = \{0, ak^2, c(c \pm 1)k^2\}$, the general expression for the propagator is given by

$$G(p) = -\frac{e^{\pi kR}}{k} \left[ J^P_\alpha(ip/k)Y_\alpha(ip\pi kR/k) - Y^P_\alpha(ip/k)J_\alpha(ip\pi kR/k) \right] ,$$

(29)

where $\alpha = \sqrt{(s/2)^2 + \hat{M}^2/k^2}$, $s = \{2, 4, 1\}$ and

$$J^i_\alpha(x) = \left( \frac{s}{2} - r_i \right) J_\alpha(x) + xJ_{\alpha-1}(x), \quad i = P,T.$$  

(30)

The functions $J_\alpha$ and $Y_\alpha$ are Bessel functions, and the values of $r_P(r_T)$ are determined by the boundary conditions on the Planck (TeV) brane. Only for fields with Neumann boundary conditions on the TeV-brane will the propagator Eq. (29) be nonzero.

3.1.1 Bulk gauge contributions

The gauge contributions to the Higgs potential arise from loops of gauge bosons, gauginos and $D$-terms. For the SU(2)$_L$ gauge sector the contribution to the effective
potential of the neutral component of the Higgs $h$ is given by

$$V_{\text{gauge}}(h) = 6 \sum_{n=1}^{\infty} \int_{0}^{\infty} \frac{dp}{8\pi^2} p^3 \frac{(-1)^{n+1}}{n} \left[ G_B^n(p) - G_F^n(p) \right] m_W^{2n}(h)$$

$$= 6 \int_{0}^{\infty} \frac{dp}{8\pi^2} p^3 \left[ \frac{1 + m_W^2(h)G_B(p)}{1 + m_W^2(h)G_F(p)} \right],$$

(31)

where $m_W^2(h) = \frac{g^2}{2} |h|^2/2$. The boson propagator is defined as $G_B(p) = \pi R G(p)$, where $G(p)$ is given by Eq. (29) with $\alpha = 1$, and $r_T = r_P = 0$, while for the fermion propagator we define $G_F(p) = e^{\pi k R} \pi R G(p)$, where $G(p)$ is obtained with $\alpha = 1$, $r_P = -\frac{1}{2} + \frac{ip F}{4 \Lambda^2}$, and $r_T = -\frac{1}{2}$. The breaking of supersymmetry is parametrized by $F$. We must stress that the effective potential is very insensitive to the actual value of $F$ since the contribution to the integral in Eq. (31) comes from the region $p^2 \approx 1/L^2 \ll F$. The gauge contribution generates a potential that monotonically increases with $h$.

### 3.1.2 Bulk matter contributions

Similarly, we can calculate the contribution from the bulk hypermultiplets to the Higgs effective potential. Unlike the gauge contributions, the loop diagrams now involve two bulk fermions. For the top quark the contribution to the effective potential is given by

$$V_{\text{top}}(h) = 6 \sum_{n=1}^{\infty} \int_{0}^{\infty} \frac{dp}{8\pi^2} p^{2n+3} \frac{(-1)^{n+1}}{n} \left[ G_B^{2n}(p) - G_F^{2n}(p) \right] m_t^{2n}(h)$$

$$= 6 \int_{0}^{\infty} \frac{dp}{8\pi^2} p^3 \left[ \frac{1 + m_t^2(h)G_B^2(p)}{1 + m_t^2(h)G_F^2(p)} \right].$$

(32)

where $m_t^2(h) = y_t^2 |h|^2$, with $y_t$ defined as the 4D top-quark Yukawa coupling. The scalar boson propagator is given by $G_B(p) = e^{-2\pi k R} G(p)/f_t^2$ where $f_t$ is the fermion zero-mode wave-function evaluated at $y = \pi$, and $G(p)$ is given by Eq. (29) with $\alpha = |c_t + \frac{1}{2}|$, $r_P = \frac{3}{2} - c_t + \frac{F^2}{2\Lambda^2}$ and $r_T = \frac{3}{2} - c_t$. Note that we are assuming an arbitrary 5D mass, $M_5 = c_t k$. For the fermion we instead have $G_F(p) = e^{\pi k R} G(p)/f_t^2$ where $\alpha = |c_t + \frac{1}{2}|$, and $r_P = r_T = -c_t$. The top quark contribution generates a potential that monotonically decreases with $h$, thus making it possible to break the electroweak symmetry.
Electroweak symmetry breaking and the physical Higgs mass

To study electroweak symmetry breaking (EWSB), we will restrict to a single Higgs doublet. The effective potential of the neutral component, $h$, is given by

$$V(h) = \mu^2|h|^2 + \frac{1}{8}(g^2 + g'^2)|h|^4 + V_{gauge}(h) + V_{top}(h).$$  \hbox{(33)}

This potential corresponds to the Higgs of model (b), and, for $\mu = 0$, to that of model (c). Although model (a) has another Higgs $h_1$, its effect on the breaking of electroweak symmetry is small since, as was argued earlier, $h_1$ obtains a VEV that is smaller than that of $h_2$. Note that we are only considering the one-loop contributions arising from the SU(2)$_L$ gauge sector Eq. (31) and the top-quark sector Eq. (32).

The potential Eq. (33) depends on two parameters, $\mu$ and $c_t$. For $\mu$ close to zero, we find that $c_t \gtrsim -0.5$ in order to have EWSB. In particular, for $c_t \approx -0.5$ we obtain the prediction $L^{-1} \approx 2$ TeV. In this case, however, we find a very light Higgs $m_{Higgs} \approx 95$ GeV. By increasing $c_t$, we increase the top-quark contribution to the effective potential. The Higgs mass can then be larger, but the scale $L^{-1}$ becomes closer to the electroweak scale. For example, when $c_t \approx -0.4$, we have $L^{-1} \approx 350$ GeV and $m_{Higgs} \approx 100$ GeV. If $\mu$ is nonzero, we have more freedom and a heavier Higgs can be obtained. For $\mu \approx 200$ GeV and $c_t \approx -0.4$ we find $L^{-1} \approx 1$ TeV and $m_{Higgs} \approx 105$ GeV. Larger values of $L^{-1}$ and the Higgs mass can be obtained, but this requires a precise tuning between $c_t$ and $\mu$. The fact that $c_t$ is preferred to be approximately $-0.5$ (instead of near 0.5 as assumed for the other quarks) implies that, in the 4D dual theory, the top quark is mostly a CFT bound-state instead of a fundamental state (see Appendix). The stop is then present in the low-energy spectrum with a mass of order TeV. All these results, however, are subject to some uncertainties which we will discuss next.

There are other one-loop corrections to the Higgs potential which we did not consider in Eq. (33). An important one-loop effect is the renormalization of the $D$-term in Eq. (15). The $D$-term is proportional to the gauge coupling, and therefore the renormalization of the $D$-term depends on the corrections of the gauge coupling. In a slice of AdS$_5$ the gauge coupling receives logarithmic corrections at the one-loop level due to the fundamental fields (zero modes) \hbox{[10]}. This makes the gauge couplings in Eq. (16) differ from the gauge couplings measured at low-energies by sizable logarithmic corrections. In the case where the fundamental sector consists of only the SM, these corrections reduce the Higgs quartic coupling by $\sim 10\%$. The origin of these corrections is easily understood in the 4D dual theory. By supersymmetry, the Higgs quartic coupling is proportional to the gauge coupling. However, in our scenario supersymmetry is broken in the gauge sector at high energies $F/\Lambda \sim M_P$. Hence the evolution from $M_P$ to TeV of the gauge couplings is different compared to that of the Higgs quartic coupling, because the two kinetic terms in \hbox{[10]} do not have

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the same renormalization. Since supersymmetry is only broken in the fundamental sector only these fields can contribute to this difference.

Other type of effects that can affect the electroweak breaking and the Higgs mass are due to boundary terms that can be present in the theory. For example, we can have a term like

\[- \int d^4x \int dy \frac{1}{4g_b^2} F^{a \mu \nu} F^{a \mu \nu} \delta (y - \pi). \tag{34}\]

Eq. (34) then modifies Eq. (10) to

\[ \frac{1}{g^2} = \pi R \frac{g_5^2}{g_6^2} + \frac{1}{g_b^2}. \tag{35}\]

Also the boundary conditions for the propagators are now modified due to the presence of the boundary kinetic term. Their effect can easily be incorporated into the propagators of Eq. (29) by taking

\[ r_T = \frac{g_5^2 p^2}{2g_b^2} e^{2\pi kR} \equiv \epsilon \frac{p^2}{k^2} e^{2\pi kR}. \tag{36}\]

For positive values of $\epsilon$ the gauge contributions to the effective potential become smaller. For example, when $\epsilon \simeq 1$, Eq. (17) changes to $m_{h_i}^2 \simeq (0.08/L)^2$ and the physical Higgs boson mass can increase by approximately 10 GeV. Thus we see that this type of boundary effect can be a substantial correction to the Higgs mass.

Also higher-dimensional operators can contribute to the Higgs mass. For example, if the $\mu$-term is nonzero, the operator of Eq. (38) gives a contribution to the Higgs quartic coupling. However, the coefficient of the operator of Eq. (38) cannot be very large, otherwise it will also give a very large correction to the electroweak observables, which we will comment on in the next section.

In summary, due to the above uncertainties, we cannot obtain a precise prediction for the scale $L$ as a function of the electroweak scale. Nevertheless, we can conclude that without any large tuning of the parameters, the scale of new physics, $L^{-1}$ lies an order of magnitude above the electroweak scale, and the physical Higgs mass is smaller than approximately 120 GeV.

### 3.2 Electroweak bounds

The success of the SM predictions places strong constraints on the scale of new physics. In our model the KK-states affect the SM relations between the electroweak observables. There are two kinds of effects [17]. The first effect arises from the exchange of the KK excitations of the $W$, $Z$ and $\gamma$, which induces extra contributions
to four-fermion interactions. However, these contributions are very model dependent, and can be zero for certain cases where the SM fermions do not couple to the KK-states [11]. Therefore, we will not consider them here. A second type of effect arises if the Higgs is on the boundary. In this case the SM gauge bosons will mix with the KK-states, and modify their masses [17]. These are the most important effects, and we will study them below. A similar analysis can also be found in Ref. [18].

In superfield notation the 4D Lagrangian of the SM gauge-boson KK-states is given by

$$\mathcal{L} = \int d^4 \theta \sum_n \left[ H_i^\dagger e^{-2g_5 f_0 V + f_n V^{(n)}} H_i \right]_{y=\pi} + M_n^2 V^{(n)^2},$$

(37)

where $f_0$ ($f_n$) is the wave-function of the massless mode $V$ (KK-state $V^{(n)}$), and $i$ labels the number of SM Higgs doublets. Integrating out $V^{(n)}$, by using their equation of motion, we obtain the effective 4D Lagrangian term

$$\mathcal{L}_{\text{eff}} = -\int d^4 \theta \sum_n \frac{g^2 f_n^2(\pi)}{M_n^2 f_0^2(\pi)} \left( H_i^\dagger e^{-2g V^a H_i} \right)^2,$$

(38)

where $g$ is given in Eq. (10). When the Higgs boson obtains a VEV, this operator will induce extra mass terms for the SM gauge bosons, $W_\mu$ and $Z_\mu$, namely

$$-\frac{1}{2} X m_Z^2 Z_\mu Z^\mu - \frac{1}{2} X \frac{m_W^4}{m_Z^2} W_\mu W^\mu,$$

(39)

where in analogy with the flat extra dimension case [19] we have defined

$$X = \sum_n \frac{m_Z^2 f_n^2(\pi)}{M_n^2 f_0^2(\pi)}.$$

(40)

The value of $X$ can easily be calculated from the 5D gauge boson propagator Eq. (29) by subtracting out the the zero-mode contribution. Thus, we obtain

$$X = \frac{m_Z^2}{f_0^2(\pi)} \left[ G(p=0) - \frac{f_0^2(\pi)}{p^2} \right] \simeq (4.1 m_Z L)^2.$$

(41)

By comparing with the flat extra dimension case [19], where $X \simeq (1.8 m_Z R_{\text{flat}})^2$, we obtain the following bound for the warped case

$$R_{\text{flat}}^{-1} \gtrsim 4 \text{ TeV} \quad \Rightarrow \quad L^{-1} \gtrsim 9 \text{ TeV}.$$  

(42)

Although the bound Eq. (42) is quite strong, we must keep in mind that there are inherent uncertainties in the above calculation. First, there can be brane kinetic terms (34). These can be taken into account by using Eq. (36) in the propagator part of Eq. (11). However, we find that for $\epsilon \sim 1$ the bound Eq. (42) is not affected very
much. Second and more importantly, we have used Eq. (10) to relate the coupling of the KK-states to the Higgs (the $g^2$ in front of Eq. (38)) with the gauge coupling measured at low-energy experiments. However, Eq. (10) receive one-loop corrections $\propto \ln(k/\text{TeV}) = \pi k R$[10] that are model dependent. This gives a sizable uncertainty to the bound in Eq. (42).

3.3 The TeV/$M_P$ hierarchy and radius stabilization

In our model the large hierarchy between the Planck scale and $1/L$ is explained by the warp factor $a(\pi) = e^{-\pi k R} \sim \text{TeV}/M_P$ as in the Randall-Sundrum model [3]. However, this requires a stabilization mechanism for the radion. Several mechanisms [20, 21] have been proposed in the past. The mechanism of Ref. [20] is not supersymmetric, while we find that those of Ref. [21] are not operative in our scenario.

Nevertheless, it has been recently pointed out [22] that the radius can be stabilized by quantum effects (Casimir energy) if certain fields (e.g. gauge bosons) are in the bulk. This is a very simple solution to the stabilization of the hierarchy that can also be operative in our scenario. Furthermore, it does not affect the results presented above.

4 Conclusion

We have presented a novel class of particle models in which different sectors of the theory have different scales of supersymmetry breaking. The model is based on a 5D theory compactified in a slice of AdS as shown in Fig. 1. It has a very interesting holographic interpretation: one sector is composed of CFT bound-states, while the other sector consists of fundamental fields. Even if the scale of supersymmetry breaking is large ($M_P$), the CFT sector is only sensitive to supersymmetry breaking effects of order $L^{-1}$ (TeV), where $L$ is the size of the bound-states.

We have applied our scenario to the SM in which only the Higgs sector is supersymmetric, and depending on the value of $c_t$, also the top quark. While the supersymmetric Higgs sector can either consist of one or two Higgs doublets, an interesting alternative involves identifying the Higgs as the partner of the tau lepton. The Higgs mass-parameter is protected by supersymmetry, which can be an order of magnitude smaller than $1/L$, and much smaller than $M_P$. Therefore this scenario not only solves the hierarchy problem but naturally explains the “little” hierarchy between the electroweak scale and the composite Higgs scale $1/L \sim \text{TeV}$ in a novel way. This is one of the main points of the paper.
The model also has several interesting predictions. First, the quartic Higgs coupling is fixed by supersymmetry, and as in the MSSM, leads to a light Higgs. The precise value of the Higgs mass is very model dependent (like in the MSSM!) but if no large tuning of parameters is imposed, we obtain $m_{\text{Higgs}} \lesssim 120$ GeV. Second, the only supersymmetric SM partner is the Higgsino (and possibly the stop), with a mass around the electroweak scale (unless we choose the tau to be the partner of the Higgs). Experimental searches for this Higgsino are very different from ordinary chargino searches due to the degeneracy of the charged and neutral Higgsino component [23]. Finally, at energies $L^{-1} \sim$ TeV, there are plenty of new CFT resonances (KK-states) that approximately respect supersymmetry. This leads to new and interesting possibilities for physics at the TeV scale.

Acknowledgements

The work of TG was supported in part by a DOE grant DE-FG02-94ER40823 at the University of Minnesota. The work of AP was supported in part by the MCyT and FEDER Research Project FPA2002-00748 and DURSI Research Project 2001-SGR-00188.
Appendix: Mass spectrum of bulk fields and its holographic interpretation

In this Appendix we will derive the 4D mass spectrum of the supersymmetric bulk fields (vector and hypermultiplet) when supersymmetry is broken on the Planck-brane by Eqs. (8) and (12). We will show that the 4D particle states split into two types: those which are sensitive to the Planck-brane are to be associated with fundamental fields in the 4D dual theory, while those which are sensitive to the TeV-brane are to be associated with CFT bound-states. Only the first type of states directly feel the breaking of supersymmetry.

If we consider the supersymmetry-breaking terms Eqs. (8) and (12) then the boundary mass terms for the gauginos and squarks are

\[ -\int d^4x \int dy \left[ \frac{F}{\sqrt{3} \Lambda^2} \lambda^a \lambda^a + \frac{F^2}{\Lambda^4} k|\tilde{q}|^2 \right] \delta(y). \]  \tag{43}

The 4D mass spectrum is obtained from the poles of the propagators of Eq. (29)

\[ J^T_\alpha(m \pi k R/k) Y^P_\alpha(m/k) = Y^T_\alpha(m \pi k R/k) J^P_\alpha(m/k). \]  \tag{44}

For the gauginos we have \( \alpha = 1, r_P = -\frac{1}{2} + \frac{m}{4k \Lambda} \) and \( r_T = -\frac{1}{2} \). For these values, the r.h.s. of Eq. (44) can be neglected for any value of the supersymmetry-breaking parameter \( F \), and the masses can be determined by the zeros of \( J^T_\alpha(m \pi k R/k) Y^P_\alpha(m/k) \).

This allows one to separate the solutions into two types. Those that depend on the boundary conditions at the TeV-brane, and those that depend on the boundary conditions at the Planck-brane

\[ J^T_1(m \pi k R/k) = 0 \rightarrow J_0(m \pi k R/k) = 0, \]  \tag{45}

\[ Y^P_1(m/k) = 0 \rightarrow m \simeq -\frac{F}{4 \Lambda^2 k} \left( \log \frac{m}{2k} + \gamma_E \right)^{-1}, \]  \tag{46}

where \( \gamma_E \) is the Euler-Mascheroni constant. Eq. (45) determines the KK spectrum and it is associated with the 4D CFT spectrum. Note that the KK spectrum does not depend on \( F \), and so the gauge boson and gaugino KK masses are the same. Eq. (46) corresponds to the “zero mode” which in the 4D dual picture is associated with a fundamental field (this solution is valid if \( m \lesssim k \)). This zero mode is the partner of the SM gauge boson which, as advertised, has a mass of order \( F/\Lambda \). When \( F \rightarrow 0 \), this gaugino becomes massless. If we actually take into account the r.h.s. of Eq. (44), then this corresponds to considering the mixing between the CFT bound-states and fundamental fields (Fig. 2). This mixing introduces a breaking of supersymmetry in the CFT sector.

\[ ^2 \text{More precisely, the fundamental states are identified with the poles of the 5D propagator evaluated at the Planck-brane} (y = y' = 0) \text{ in the limit of } R \rightarrow \infty. \]
For the squarks, the mass spectrum is determined by Eq. (44) with \( \alpha = |c + \frac{1}{2}| \), \( r_P = \frac{3}{2} - c + \frac{F^2}{2\Lambda^2} \) and \( r_T = \frac{3}{2} - c \), where \( c \) parametrizes the 5D hypermultiplet mass \( M_{5D} = ck \). The holographic interpretation of the mass spectrum depends on the values of \( c \). For \( c \geq \frac{1}{2} \), we have a situation similar to the gauge sector where the r.h.s. of Eq. (44) can be neglected and the masses are determined by

\[
J_{c+\frac{1}{2}}^T(me^{\pi kr}/k) = 0 \quad \rightarrow \quad J_{c-\frac{1}{2}}^T(me^{\pi kr}/k) = 0 ,
\]

\[
Y_P^{c+\frac{1}{2}}(m/k) = 0 \quad \rightarrow \quad m^2 \simeq (c - 1/2) \frac{F^2}{\Lambda^4}k^2 \quad (c > \frac{1}{2}) ,
\]

\[
\rightarrow \quad m^2 \simeq -\frac{F^2}{2\Lambda^4}k^2 \left( \log \frac{m}{2k} + \gamma_E \right)^{-1} \quad (c = \frac{1}{2}) . \]

(47)

The state whose mass is given by Eq. (48) is the one to be associated, in the 4D dual picture, to the fundamental state (the partner of the quark). Its mass is of order \( F/\Lambda \sim M_P \) (this is valid if \( m \lesssim k \)), showing that supersymmetry in the fundamental sector is broken at the scale \( M_P \). For \(-\frac{1}{2} < c < \frac{1}{2}\) the r.h.s. of Eq. (44) cannot be neglected independently of the value of \( F \), and the solutions to Eq. (44) cannot be separated into those that depend on the TeV-brane and those that depend on the Planck-brane. This makes it difficult to identify the fundamental and CFT states. This non-decoupling effect is due to the large mixing that exists between the sources \( \Phi \) and the CFT states. For \( c \leq -\frac{1}{2} \) the r.h.s. of Eq. (44) can be neglected again, and we find

\[
J_{c+\frac{1}{2}}^T(me^{\pi kr}/k) = 0 \quad \rightarrow \quad J_{c-\frac{1}{2}}^T(me^{\pi kr}/k) = 0 ,
\]

\[
Y_P^{c+\frac{1}{2}}(m/k) = 0 \quad \rightarrow \quad \text{No solution for } m \lesssim k .
\]

(49)

(50)

Thus, in this case there is no fundamental state whose mass is proportional to \( F \), which would become massless in the limit \( F \rightarrow 0 \). However, there is an extra massless mode that can be found by looking at the pole of the full propagator Eq. (29) in the limit that the Planck-brane decouples \( (k \rightarrow \infty \text{ with } e^{\pi kr}/k \text{ fixed}) \). This massless mode corresponds to a CFT bound-state. It picks a tree-level mass from its mixing with the fundamental field \( \Phi \) (this can be seen by not neglecting the r.h.s. of Eq. (44)). In the formal limit \( c \rightarrow -\infty \), the spectrum consists of a single supersymmetric field localized on the TeV-brane.

A similar analysis can be done by using the standard holographic correspondence to calculate the two-point functions \( \langle OO \rangle \) where \( O \) is the CFT operator of Eq. (44). In a theory with a TeV-brane \( \langle OO \rangle \) has poles corresponding to the CFT bound-states. One will find that the CFT spectrum is determined by Eqs. (45), (47) and (49). If the sources are dynamical, then their propagators are given by \( 1/(p^2 - \langle OO \rangle) \), and the mass spectrum is obtained from \( p^2 - \langle OO \rangle = 0 \).
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