A method for measuring tire characteristic using brake tester

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Abstract. Slip characteristic of a tire is essential for controlling braking, traction, or stability of a vehicle. In this paper, we propose a new method to measure longitudinal slip characteristic using a developed plate brake tester. The slip is calculated based on the plate displacement instead of the angular speed of the wheel. An algorithm for modelling the relationship between friction force and slip was made using Julien’s Theory. The algorithm was verified through a simulation. With this method, the longitudinal slip characteristic of a tire can be obtained quickly. The brake tester developed in this research is potential to be used as indoor test equipment for simulating a tire in various road conditions.

Keywords. Pacejka’s magic formula; anti-lock braking system (ABS); traction control; vehicle stability control; roller brake tester; vehicle dynamics; skid

1. Introduction

Indonesia is a nation with the second highest traffic accident rate in ASEAN [1]. More than 28,000 people die per year due to traffic accident [2, 3]. There are four factors causing a traffic accident: human, vehicle, environment, and time [4]. On a vehicle, braking system is an active safety system that prevents collision.

To ensure the performance of braking system of a vehicle, it is compulsory for a public transport and a commercial vehicle to be tested periodically on Ministry of Transport’s (MOT) facilities. The equipment used for testing a vehicle in the facilities, where a vehicle is tested statically, is usually a roller brake tester. It means the vehicle stays still above the tester during testing. A parameter representing the performance of braking system is braking efficiency [5, 6]. With this tester, the rolling resistance of the wheel is also known [7]. However, this kind of tester has a weakness. Braking force measurement is affected by diameter of the roller and distance between rollers [8, 9]. Hence, a vehicle may have different braking efficiency if it is tested on different brake testers with different roller diameters.

A roller brake tester measures the total force resulted from all components in the braking system. Therefore, it cannot test the performance of each component individually such as brake pad and disc. Methods and devices have been developed to test the brake pad and the disc using test benches [10–13]. Tire is an important part of a vehicle. It gives a traction force or braking force to the ground directly. Tire has important characteristic called slip [14]. Slip characteristic of a tire can be divided in-to two, i.e. lateral slip and longitudinal slip. Lateral slip characteristic has been used for vehicle stability control.
Meanwhile, longitudinal slip characteristic has been used for controlling braking, traction, or reducing fuel consumption [18–22]. Experimental methods and devices to measure longitudinal slip characteristic have been developed. Dynamometric platform was used to investigate friction force during braking [23, 24]. A device called dynamometric rim was also used to measure tire characteristics [25]. With this installed on the wheel rim, braking force and slip during braking from a certain speed until stop can be recorded. To record the traction slip characteristic, the fifth wheel was used [26]. On the fifth wheel, there are sensors measuring angular speed of the wheel and traction force resulted from drive wheel. When the vehicle is moving forward, slip and traction force is measured and recorded.

Although there are many methods used for measuring slip characteristic of a tire, it is difficult to be applied on MOT’s facilities. All methods explained above are drive test. Therefore, it takes a long time to get the result. Moreover, it needs more sensors installed on the vehicle.

In this paper, we report a development of a plate brake tester and an algorithm to measure the slip characteristic of a tire based on Julien’s Theory. The main contribution of this paper is an estimation of slip based on the plate displacement instead of the angular velocity of the wheel. First, the new concept of plate brake tester was developed. Second, equations to calculate slip and friction force based on the developed brake tester were derived. After that, an algorithm for tire characteristic modelling using Julien’s Theory was made. Finally, a MATLAB script made based on the algorithm was verified using tire data obtained from a reference.

2. Methods
The development concept of the plate brake tester is shown in figure 1. A braking test is carried out statically. The vehicle wheel is placed above a moving plate. The plate is moved when the wheel is in a braking condition. A force sensor is installed on the plate. Therefore, force measured by the sensor is the braking force. A braking efficiency can be calculated by dividing the maximum braking force with normal force.

With this device, it is possible to measure not only braking efficiency, but also tire characteristics. This can be done if braking force resulted from vehicle braking system is strong enough to make the wheel in a locked condition. In other words, the wheel does not rotate while the plate is moving. However, more sensors will be needed to measure parameters representing tire characteristics.

![Figure 1. The development concept of a plate brake tester. The plate is moved to the right when the wheel is braked. \( F_d \) is used to measure braking force.](image)

The experiment set-up design to do a tire test according to the concept in figure 1 is shown in figure 2, 3, 4, and 5. It requires two load cells and two LVDT (Linear Variable Differential Transformer). Load cell 1 is used to measure normal force while load cell 2 is used to measure friction force. LVDT 1 is used to measure the contact patch and LVDT 2 is used to measure plate displacement. The main plate
is moved manually from the input wheel. Power from input wheel is transmitted to the main plate through gear box, ball screw, secondary plate, and load cell 2.

**Figure 2.** The overall view of experiment set-up that can be used to measure tire characteristics. LVDT 1 and LVDT 2 are used to measure the contact patch length and the plate displacement, respectively. Load cell 1 is used to measure normal force.

Data needed to model tire characteristics based on Julien’s Theory are $\lambda$, tangential stiffness, contact patch length, coefficient of friction, normal force, and slip at maximum friction force. However, the data obtained from the experiment set up are the force measured by load cell 2 against plate displacement, normal force, and displacement measured by LVDT 1. Hence, a derivation of some equations to analyse the data is needed. Those equations are used to calculate friction force based on load cell 2 reading, contact patch length based on LVDT 1 reading, and slip based on the main plate displacement. After that, an algorithm of data analysis program is made based on equations which have derived previously and Julien’s Theory.

**Figure 3.** The different view of the experiment set-up design.
Figure 4. Detail assembly of ball screw, load cell 2, and plates. The main plate is moved through input wheel, gear box, ball screw, secondary plate, and load cell 2.

Figure 5. The bottom view of the plates showing the arrangement of the load cell 2.

2.1. Friction force
The based on both free body diagram and kinetic diagram shown in figure 6, the equation of friction force can be written as equation (1).

\[ F_f = F_d - F_l - m_p a_p \]  \hspace{1cm} (1)

where \( F_f \) is friction force between the tire tread and the main plate, \( F_d \) is force measured by load cell 2, \( F_l \) is friction force of the linear motion, \( m_p \) is mass of the main plate, and \( a_p \) is the acceleration of the main plate. If friction force of the linear motion is negligible and the velocity of the plate is constant, the force measured by load cell is equal to the friction force between the tire tread and the main plate.
Figure 6. Main plate motion analysis to calculate friction force ($F_g$). (a) Free Body Diagram. (b) Kinetic Diagram.

2.2. Length of the contact patch

Based on figure 7, the length of contact patch can be calculated with equation (2).

$$l_t = 2a = 2\sqrt{r_t^2 - (r_l - y)^2}$$  \hspace{1cm} (2)

where $l_t$ is a contact-patch length, $r_t$ is unloaded tire radius, $y$ is the difference between unloaded and loaded tire radius ($r_l$). The value of $y$ is measured by LVDT 1. When the load $W$ is applied to the wheel, the tire will deflect in vertical direction. Because of that, the wheel centre will move down at distance $y$ from the original position.

Figure 7. Wheel geometric when: (a) no vertical load; (b) under vertical load $W$. Centre height difference $y$ is used to calculate length of the contact patch ($l_t$).

2.3. Slip based on the plate displacement

The relationship between slip and plate displacement is shown in figure 8. Figure 8(a) shows the wheel condition when the plate is not moved yet. Figure 8(b) is the condition when the plate is moving at distance $x$. The wheel is locked. It means that the wheel does not rotate. In this research, the tire characteristic modelling is done only from a slip which equals to zero until a slip at maximum friction force. Therefore, it can be assumed that the total deformation happened along contact patch $l_t$ and $\lambda$ will be the same as the plate displacement $x$. The $\lambda$ is a part of the tire tread affected by deformation (stretching).
Figure 8. Phenomenon on tire tread: (a) Before the main plate moves; (b) When the plate is moved at a distance \( x \), the tire tread also deform as long as \( x \).

Figure 9. The consequence caused by deformation on tire tread when the plate is moved. (a) An angle \( \Delta \theta \) is produced because of deformation \( x \); (b) The effective radius becomes longer than the free rolling radius \( (r_e > r_0) \).

The effect of tire tread deformation toward distance travelled by the wheel is shown in figure 9. From figure 9(a) it can be seen that when the plate is moved, an angle \( \Delta \theta \) appears. This angle is also shown in figure 8(b). Thus, if the wheel rotates with an angle \( \theta \), it will travel like if it rotates with an angle \( \theta + \Delta \theta \). Therefore, the distance travelled by the wheel will be further compared to the wheel with no deformation \( \Delta \theta \). This effect also can be explained by figure (b). Deformation \( \Delta S \) makes the effective radius \( (r_e) \) longer than the free rolling radius \( (r_0) \). As a result, with the same angular velocity, the braked wheel will travel further than the non-braked wheel.

Slip can be defined as the percentage of distance difference travelled by a wheel. Its difference is calculated between braking and free rolling conditions. If this definition is used in the experiment setup design as in figure 2, slip can also be defined as percentage deformation to the initial length. Therefore, the relationship between slip and the plate displacement can be written as equation (3).

\[
i = \frac{\Delta S}{S_0} = \frac{x}{l_i + \lambda}
\]  

(3)

where \( i \) is slip, \( \Delta S \) is the total deformation, \( S_0 \) is the initial length, \( x \) is the plate displacement, \( l_i \) is the length of contact patch, \( \lambda \) is part of the tire tread in front of contact patch that stretches.

2.4. Velocity of the main plate

According to figure 2-5, the velocity of the main plate can be calculated with equation (4).

\[
V_p = n_{in} \cdot z_\theta \cdot z_b
\]  

(4)

where \( V_p \) is the main plate velocity, \( n_{in} \) is an angular velocity of the input wheel, \( z_\theta \) is a gear ratio of the gear box, \( z_b \) is a gear ratio of the ball screw.
2.5. The algorithm of data analysis program

The aim of data analysis is shown in figure 10. Data analysis is used to process input data to be output data. Input data are obtained from a tire test using the brake tester as in figure 2. Those inputs are friction force against plate displacement, length of contact patch, and normal force. Output data are friction coefficient, tangential stiffness, slip at maximum friction force, and length of $\lambda$. With those output data, tire characteristics can be modelled using Julien's Theory.

![Figure 10](image)

*Figure 10. The block diagram of data analysis purpose. Input is obtained from the brake tester. Output is used for tire characteristics modelling.*

There are some terms in Julien's Theory. These terms can be seen in figure 11 showing the relationship between friction force and the slip of a tire. Julien's Theory only models from point O to B. There are two regions between point O and B. First, point O to A is called the full adhesion region. Second, point A to B is called the combination of adhesion and the sliding region. In the adhesion region, there is no sliding in contact patch. Point A is called the critical point. Above point A, sliding region starts to takes place. Sliding region will spread along the contact patch if slip is increased. Point B is the peak value where all of contact patch is in sliding region for the first time.

The flow chart of data analysis program is shown in figure 12. All equations used are based on Julien's Theory [27]. Input data are taken from zero friction force until sliding friction force occurs. In other words, data needed are from the plate which is in rest until it moves at certain distance causing 100% slip. Therefore, the data obtained is from point O to C as shown in figure 11.

![Figure 11](image)

*Figure 11. Nomenclature used in Julien’s Theory [27]. Julien’s Theory only models from point O to B. From O to A is called the full adhesion region. From point A to B is called the adhesion and sliding region combination. Point A is critical point, while point B is peak point.*

First, the program will calculate the maximum friction force and the friction coefficient. The coefficient of friction is calculated with equation (5).

$$\mu_p = \frac{F_{gm}}{W}$$  \hspace{1cm} (5)
where $\mu_p$ is the peak friction coefficient, $F_{gm}$ is the maximum friction force (point B in figure 11), $W$ is normal force. $F_{gm}$ is obtained by finding the maximum friction force from input data.

The next step is setting the value of $\lambda$ equals to zero. With this initial value, the program will calculate slips, critical friction force, critical slip, tangential stiffness, friction force at the adhesion region, and friction force at the combination of the adhesion and sliding region. Slip ($s$), when the plate moves, is calculated based on the plate displacement according to equation (3). Critical friction force ($F_{gc}$) is calculated with equation (6). Critical slip ($i_c$) is obtained by finding slip ($s$) when critical friction force ($F_{gc}$) occurs. Tangential stiffness ($k_t$) is calculated with equation (7).

\begin{equation}
F_{gc} = \left( \frac{I_t}{2} + \lambda \right) \frac{F_{gm}}{I_t + \lambda}
\end{equation}

\begin{equation}
k_t = \frac{F_{gm}}{k_i I_c (I_t + \lambda)}
\end{equation}

Because the tangential stiffness has already known up to this step using equation (7), friction force from the initial slip until the slip at maximum friction force can be modelled. Friction force from zero slip up to critical slip (point O to A) is calculated using equation (8), while friction force from point A to B is calculated with equation (9).

\begin{equation}
F_{ga} = \left( \frac{k_t \cdot I_c^2}{2} + k_i I_c \lambda \right) I_s
\end{equation}

\begin{equation}
F_{gs} = F_{gm} - \left( F_{gm} - k_t I_c \lambda I_s \right)^2
\end{equation}

where $F_{ga}$ is friction forces in the adhesion region (point O to A), $i_s$ is the range of slip in the adhesion region, i.e. slip from zero until critical slip ($i_c$). $F_{gs}$ is friction forces in the combination of the adhesion and the sliding region (point A to B), while $i_s$ is the range of slip from critical slip ($i_c$) until slip at maximum friction force ($i_m$).

From calculations above, the maximum friction force (the last $F_{gs}$) will be below the peak value $F_{gm}$. This is because the value of $\lambda$ is set to zero. Therefore, there will be needs to calculate the error between the analysis result and the true value using equation 10.

\begin{equation}
dF = F_{gm} - F_{gs (end)}
\end{equation}

where $dF$ is the error of the analysis result, $F_{gs}$ is the maximum friction force, $F_{gs (end)}$ (end) is the last value of $F_{gs}$ (maximum friction force from analysis).

The program, then, iterates the value of $\lambda$ by adding it with increment of 1 mm. By increasing the value of $\lambda$, the error will be smaller. Iteration process will be stopped if the error is less than 0.1 Newton. If that condition is fulfilled, the program ends. The output data in the form of friction coefficient, $\lambda$, tangential stiffness, and the slip at maximum friction force can be displayed. With those output data, tire characteristics can be modelled using Julien’s Theory.

## 3. Results and discussion

A program made with MATLAB Script for analysing the input data based on figure 12 has been made. This program needs a verification to check its correctness. To do that, the plate displacement and friction force are simulated according to the brake tester design shown in figure 2. Those plate displacement and friction force data are used as an input of the program. After that, the results of the program are compared to the actual value obtained from a reference.
3.1. Simulation of plate displacement

A simulation of friction force and plate displacement is done based on tire characteristics data from a reference [28]. These tire data are as follow:

\[
\begin{align*}
\lambda &= 31.5 \text{ mm} \\
\lambda_t &= 282 \text{ mm} \\
\mu_p &= 0.888 \\
W &= 4000 \text{ N} \\
\end{align*}
\]

From the tire data above, the slip at maximum friction force \(i_m\) can be calculated with Julien’s Theory as in equation (11).

\[
(11) \quad i_m = \frac{P_{gm}}{k_t \lambda_t \lambda}
\]

The data of input wheel rotation \(n_{in}\), the ratio of the gear box \(z_g\), and the ratio of the ball screw \(z_b\) used in the experiment design as in figure 2 are:

\[
\begin{align*}
n_{in} &= 1 \text{ revolution/second} \\
z_g &= 1 : 60 \\
z_b &= 5 \text{ mm/revolution}
\end{align*}
\]

Using equation (4) and data above, the velocity of the plate is:

\[
V_p = \frac{1}{60} \times 5 \text{ mm/s} = 0.0833 \text{ mm/s}
\] (12)

The data of the friction force versus the slip can be calculated with equation (8) and (9). Then, those slip data are converted to plate displacement with equation (3). Plate displacement data can be converted to time-based data with equation (12). Hence, the data of friction force versus slip, plate displacement, and time can be plotted as shown in figure 13.

Slip, plate displacement, and time needed from zero to the maximum friction force are shown in figure 13. Peak point is achieved when the slip is around 9%. Plate displacement needed to reach its peak point is about 27 mm. It needs around 350 seconds to take data from the plate, which is in rest until moving at distance around 27 mm. There is enough time to observe the phenomenon of friction force using the experiment set up as in figure 2. Only friction force versus plate displacement is used as the input for the data analysis program based on the flow chart in figure 12.

3.2. Results of data analysis program

When the friction force versus plate displacement data in figure 13 are used as the input for the MATLAB Script based on figure 12, the results are the following tire characteristics:

\[
\begin{align*}
\lambda &= 31.5 \text{ mm} \\
\lambda_t &= 282 \text{ mm} \\
\mu_p &= 0.888 \\
\end{align*}
\]

k_t = 4610304.1 N/m²

i_m = 8.6871
Figure 12. Flow chart of data analysis program. The aim is to model tire characteristics from the friction force and the plate displacement data.
Figure 13. Plot of tire characteristics based on data known in the literature. Slip axis (i) is original plot of Julien’s Theory. Plate displacement axis (x) is calculated from the equation relating slip and x. Time (t) axis is calculated based on the set-up experiment design in figure 2. Only the friction force vs plate displacement axis (x) is used as the input for data analysis program.

The comparison between analysis result and real value is shown in table 1. It can be seen from table 1 that there is a little difference between the analysis result from the program and the true value. This is due to the influence of $\lambda$ toward the tangential stiffness ($k_t$) and the slip at peak point ($i_m$) according to equation (7) and (11). In the data analysis program, the value of $\lambda$ is iterated starting from zero. The iteration is stopped when the peak value difference between analysis result and true value is less than 0.1 Newton. Because the peak value of the data analysis program is never exactly the same as the true value, the value of $\lambda$ from analysis will slightly be smaller than its true value. Therefore, the value of $k_t$ is slightly smaller, and $i_m$ is slightly higher than the true value.

| Tire Characteristics | Analysis Result | True Value |
|----------------------|-----------------|------------|
| $\lambda$            | 31 mm           | 31.5 mm    |
| $\mu_p$              | 0.888           | 0.888      |
| $k_t$                | 4610300 N/m$^2$ | 4610304.1  |
| $i_m$                | 8.6871 %        | 8.6733 %   |

The effect of $\lambda$ variation on friction force versus slip curve is shown in figure 14. The program iterates the value of $\lambda$ from zero with 1 mm increment. When $\lambda=0$, the slip is longer than the true slip, and the peak value is lower than the true peak value. By increasing $\lambda$, the curve will get closer to the true curve. The figure shows the effect of $\lambda$ equals to 0, 10, and 20 mm on the curve. The $\lambda$ equals to 20 mm is the closest to the true curve. With $dF$ less than 0.1 Newton as shown in figure 12, the program has given the output $\lambda=31$ mm and $i_m=8.6871$ %. Those results are almost the same with the true value. It can be concluded that the data analysis program has been made is correct.
Figure 14. The effect of different value of $\lambda$ toward friction force versus slip curve. The data analysis program iterates the value of $\lambda$ from 0 to a value of $\lambda$ where the peak friction force of analysis is almost the same as the peak friction force of the input data.

The results of the data analysis program above have proven that the development of the plate brake tester as in figure 1 and 2 has several advantages. It has the ability to measure not only braking efficiency but also tire characteristics based on Julien’s Theory. Moreover, it takes a short time to test the tire characteristics because it only requires one stroke of plate movement. It is possible to test a tire in different road condition with changing the plate material e.g. dry or wet asphalt, concrete, etc.

However, there are several weaknesses. First, the data analysis method in this research is valid if the assumption is correct. It is assumed that the deformation of tire tread will be the same as the plate displacement as long as the friction force has not reached its peak value. Therefore, further research is needed to prove that assumption. Second, if the data analysis method is applied to the real vehicle brake tester, it needs a mechanism that isolates one wheel to the others. The aim of this isolation is to eliminate the influence of other wheels on the wheel being tested. Third, it may be difficult to ensure that the wheel is in a locked condition during testing.

4. Conclusions
A new concept of a plate brake tester has been developed. An algorithm to calculate slip characteristics of a tire based on the plate displacement using Julien’s Theory has been made as well. From a simulation, it was proved that the algorithm gave insignificant error in relation to the reference values. Hence, the developed brake tester has the ability not only to measure braking efficiency but also tire characteristics. Moreover, it takes a short time to test the tire because the plate only moves once to obtain tire characteristic data. However, if this developed brake tester is applied to Ministry of Transport (MOT) testing, it needs a mechanism isolating the wheel being tested form the other wheels. Furthermore, it may be difficult to ensure the wheel in a locked condition when the plate moves. It may also difficult to measure the length of contact patch accurately. Further experimental research is required to validate the algorithm. The new method discussed in this research is potential to be used as indoor test equipment. The tire characteristic in various road conditions can be simulated by adding the road condition above the plate e.g. dry or wet asphalt, dry or wet concrete, etc.

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