Deconfined quantum criticality and logarithmic violations of scaling from emergent gauge symmetry

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We demonstrate that the low-energy effective theory for a deconfined quantum critical point in $d = 2 + 1$ dimensions contains a leading order contribution given by the Faddeev-Skyrme model. The Faddeev-Skyrme term is shown to give rise to the crucial Maxwell term in the CP$^1$ field theory governing the deconfined quantum critical point. We derive the leading contribution to the spin stiffness near the quantum critical point and show that it exhibits a logarithmic correction to scaling of the same type as recently observed numerically in low-dimensional models of quantum spin systems featuring a quantum critical point separating an antiferromagnetically ordered state from a valence bond solid state. These corrections, appearing away from upper or lower critical dimensions, reflect an emergent gauge symmetry of low-dimensional antiferromagnetic quantum spin systems.

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Much of our understanding of phase transitions is based on the concept of spontaneous symmetry breaking [1], which provides a mechanism for the spontaneous generation of an ordered state as one or more parameters of a many-body system are varied. In Abelian systems the ordered state can be characterized by an order parameter. In the case of a $U(1)$ symmetry, the order parameter is nonzero when topological defects of the broken symmetry state in the original picture. The disorder parameter is nonzero when topological defects of the $U(1)$-theory (vortices) are condensed. The $U(1)$ symmetry of the dual theory is then spontaneously broken. The superfluid phase corresponds to the $U(1)$ symmetric state. The vortex condensation of the dual theory reflects the nontriviality of the first homotopy group of $U(1)$, namely $\pi_1(U(1)) = \mathbb{Z}$, the group of integers. This leads, for instance, to flux quantization in superconductors.

When the order field is composed of more elementary constituents, the disordered phase exhibits non-trivial features which do not follow from standard spontaneous symmetry breaking arguments. This is so, for instance, in two-dimensional quantum spin systems featuring a paramagnetic phase where the symmetries of the underlying lattice is broken. On a square lattice where $SU(2)$-invariant spin interactions compete, a valence-bond solid (VBS) state emerges in the paramagnetic phase [5]. An example of this is the $J-Q$ model [6], with a four-spin exchange around the plaquette of a square lattice in addition to the usual Heisenberg term,

$$H = J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - Q \sum_{\langle jkl \rangle} \left( \mathbf{S}_j \cdot \mathbf{S}_k - \frac{1}{4} \right) \left( \mathbf{S}_k \cdot \mathbf{S}_l - \frac{1}{4} \right).$$

When $J \gg Q$ the Heisenberg term dominates the physics and the ground state is antiferromagnetic (AF), while for $Q \gg J$ the four-spin term favors a VBS state. Numerical works [6–9] show that the $J-Q$ model has an emergent $U(1)$ symmetry. This had previously been predicted in models with the same phase structure by introducing a new paradigm for phase transitions [10], the so called deconfined quantum criticality (DQC) scenario. Introducing two different order parameters to describe a phase transition, one for the AF phase and another one for the VBS phase, it was argued that near the phase transition more fundamental building blocks, namely elementary excitations known as spinons, constitute both order parameters. In this scenario staggered Berry phases interfere destructively with the hedgehogs (magnetic monopoles in spin space), leading to spinon deconfinement at the phase transition [10]. This mechanism has been confirmed by large scale Monte Carlo (MC) simulations of an easy-plane antiferromagnet [11].

Previous analyses of spinon deconfinement considered a model for easy-plane antiferromagnets exhibiting a $U(1)$ symmetry [12]. The resulting theory is described by a CP$^1$ model, which due to the easy-plane anisotropy has a global $U(1)$ symmetry in addition to the local one [10, 12]. This property makes the model self-dual, and it was argued that this self-duality would imply a second-order quantum phase transition at zero temperature [10]. However, Monte Carlo (MC) simulations [11, 13] revealed a first-order phase transition, also found by a subsequent renormalization group (RG) analysis of the model [14]. Thus, one may ask if the same would also be found in the globally $SU(2)$ invariant case supposed to be described by an isotropic CP$^1$ model with a non-compact Maxwell term. MC simulations on such a model on a lattice have found a first-order phase transition [15]. This contradicts previous MC results [16] obtaining a second-order phase transition. Numerical work on the $J-Q$ model appears to show a second-order phase transition [6–7, 9, 17], see however Ref. 8.

Recently [9, 17], a feature of the $J-Q$ model which does not seem to follow from DQC, was observed. Namely, logarithmic violations of scaling in the zero-temperature spin stiffness and in the finite-temperature uniform susceptibility,
were found. In systems with continuous symmetries, this normally occurs either at the upper or lower critical dimensions. In Refs. [9] and [17], the systems considered are $2 + 1$-dimensional. Thus, the corresponding field theory prescribed by DQC would be neither at the upper nor the lower critical dimension. Moreover, no logarithms are expected to occur in the zero-temperature spin stiffness or in the finite-temperature uniform susceptibility, since both these quantities can be derived from a current correlation function. For this reason, it has been suggested [9] that the DQC scenario should be revised in order to accommodate this new aspect. A phenomenological theory at finite temperature involving a gas of free spinons has been proposed recently [18] to fit the logarithmic behavior of the simulations. A non-standard power behavior for the thermal gap at criticality was introduced to make a logarithm appear in the uniform susceptibility. Since the free spinon gas has the usual spectrum at zero temperature, no quantum critical logarithmic behavior can be derived in this way for the spin stiffness at zero temperature. Moreover, the origin of the anomalous scaling of the thermal gap was not addressed. In this paper, we show that the logarithmic violation of scaling in the $J - Q$ model is actually encoded in the DQC scenario and that this is a direct consequence of the emergent $U(1)$ gauge symmetry.

If quantum criticality in the $J - Q$ model follows from DQC, it should be governed by an effective lattice gauge theory where a staggered Berry phase suppresses its magnetic monopoles [10]. In the absence of this staggered Berry phase this lattice gauge theory is given by the $CP^1$ model with a compact Maxwell term [10].

$$S = -\frac{1}{g} \sum_{j,\mu,\alpha} \bar{z}_{aj}^\alpha e^{-iQ\mu} z_{aj+\mu} + \text{h.c.} \sim \frac{1}{e^2} \sum_{\mu,\nu,\lambda} \cos(\epsilon_{\mu\nu\lambda} A_\mu A_\nu A_\lambda),$$  \hspace{1cm} (2)

where $a = 1, 2$, $\Delta_\mu$ denotes the $\mu$-th component of the lattice gradient, and the complex scalar fields satisfy the local constraint $|z_{1j}|^2 +|z_{2j}|^2 = 1$. The action (2) has been studied numerically [19], and a second order phase transition was found. Also, the field theory of Eq. (2) has been studied via a renormalization group analysis [20]. The resulting flow diagram interpolates between the quantum critical points of the $O(3)$ and $O(4)$ non-linear $\sigma$ models, see Fig. 1. We thus expect the universality class of the phase transition of Eq. (2) to be $O(3)$, which obviously features a quantum critical point, in agreement with Ref. [19].

Within the DQC paradigm, the effect of the staggered Berry phase is essential. This can conveniently be accounted for by rewriting the Maxwell term in Eq. (2) in Villain form, $S_{\text{Maxwell}} = \frac{1}{2g} \sum_{j,\alpha} (\epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda) - 2m_{\mu j}^2$, and coupling the integer fields $n_{\mu j}$ to fixed, time-independent fields $\xi_j$ taking the values $0, 1, 2, 3$ on the dual lattice, as follows $i(\pi/2) \sum_j \xi_j \Delta_\mu n_{\mu j}$. This defines the Berry phase of a compact CP$^1$ model believed to describe the essential physics at deconfined quantum critical points [21]. A partial dualization of the model yields [21]

$$S_{\text{SI}} = \frac{1}{4} \sum_j \left[ e^2 \left( N_{\mu j} - \frac{1}{4} \Delta_\mu \xi_j \right)^2 + g(\epsilon_{\mu\nu\lambda} N_{\mu j})^2 \right] + S_n - 2\pi i \sum_j N_{\mu j} k_{\mu j}(n).$$  \hspace{1cm} (3)

Here, $S_n$ is the action of the nonlinear $\sigma$ model and $k_{\mu j}$ is the topological current. In the continuum limit, we have

$$k_{\mu j} \approx \frac{1}{4\pi} \epsilon_{\mu\nu\lambda} n \cdot (\partial_\nu n \times \partial_\lambda n).$$  \hspace{1cm} (4)

The lattice fields $N_{\mu j}$ are integer-valued and $\xi_j$ are fixed fields arising from the Berry phase in the original model having specific values on the dual lattice (details can be found in Refs. [21] and [22]). Using the Poisson formula to promote the integer fields $N_{\mu j}$ to real fields $B_{\mu j}$, performing the shift $B_{\mu j} \rightarrow B_{\mu j} + \Delta_\mu \xi_j / 4$, and integrating over $B_{\mu j}$, one obtains

$$\tilde{S}_{\text{SI}} = 2\pi^2 \sum_{i,j} D_{\mu\nu}(x_i - x_j)[k_{\mu j}(n) + m_{\mu j}][k_{\mu j}(n) + m_{\mu j}] + S_n - \frac{i\pi}{2} \sum_j m_{\mu j}^2 \Delta_\mu \xi_j.$$  \hspace{1cm} (5)

Here, $D_{\mu\nu}(x_i - x_j)$ satisfies the equation $[-g\Delta^2 + e^2] \delta_{\mu\nu} + 2\Delta_\mu \Delta_\nu]D_{\mu\nu}(x_i - x_j) = 2\delta_{\mu\nu} \delta_{ij}, m_{\mu j}$ are new integer-valued vector fields arising from the Poisson summation, and we have used $\Delta_\mu k_{\mu j}(n) = 0$. The first term in Eq. (5) can be approximated by

$$L_{\text{Skyrme}} = \frac{1}{2e^2} \left[ \epsilon_{\mu\nu\lambda} n \cdot (\partial_\nu n \times \partial_\lambda n) \right]^2,$$  \hspace{1cm} (6)

first introduced in Ref. [23]. The effective Lagrangian in the continuum limit is thus

$$L = \frac{1}{2g}(\partial_\mu n)^2 + L_{\text{Skyrme}} + \ldots.$$  \hspace{1cm} (7)
where the three-component direction field $\mathbf{n}$ satisfies the local constraint $\mathbf{n}^2 = 1$ and the ellipses denote other terms related to the Berry phase, which are being neglected in the above approximation. We return to this below. The model in Eq. (7) [24] is known to have a rich geometric and topological structure [25][27]. The emergent $U(1)$ symmetry of the Skyrme term follows from the compact $U(1)$ gauge group in the continuum arising as a subgroup of the $SU(2)$ gauge group [28]. To see this, consider the functional integral

$$Z = \int \mathcal{D}\mathbf{n} \mathcal{D}C_\mu \exp \left( -\frac{1}{4e^2} \int d^4x \mathbf{F}_{\mu\nu} \cdot \mathbf{F}_{\mu\nu} \right),$$

(8)

where $\mathbf{F}_{\mu\nu} = \partial_\mu \mathbf{W}_\nu - \partial_\nu \mathbf{W}_\mu - \mathbf{W}_\mu \times \mathbf{W}_\nu$ is an $SU(2)$ field strength with an adjoint non-Abelian gauge field of the form $\mathbf{W}_\mu = \mathbf{n} \times \partial_\mu \mathbf{n} + n C_\mu$, and $C_\mu$ is an Abelian gauge field. The Skyrme contribution Eq. (6) follows from integrating out $C_\mu$ in Eq. (5). This is reminiscent of the arguments of Ref. [29] where a four-dimensional version of Eq. (7) [30] is argued to be a low-energy description of $SU(2)$ Yang-Mills theory.

The model Eq. (7) has to be modified to account for the deconfinement of magnetic monopoles. As discussed in detail in Ref. [10], this interference mechanism suppresses the monopoles, which implies an effective model given by the non-compact $CP^1$ model with a Maxwell term. Such a model may be written on the form [27],

$$L = \frac{1}{2g} (\partial_\mu n)^2 + \frac{1}{2g} C_\mu^2 + \frac{1}{2e^2} \left( \epsilon_{\mu\nu\lambda} \partial_\nu C_\lambda + \epsilon_{\mu\nu\lambda} n \cdot (\partial_\nu n \times \partial_\lambda n) \right)^2.$$  

(9)

Setting $\mathbf{n} = z_α \sigma_{αβ} z_β$, we obtain the $CP^1$ realization of Eq. (9)

$$L = \frac{1}{g} (\partial_μ - iA_μ) z_α^2 + \frac{1}{2g} C_μ^2 + \frac{1}{2e^2} \left( \epsilon_{μνλ} \partial_ν C_λ + 2i \epsilon_{μνλ} z_α^2 \partial_ν z_α \right)^2.$$  

(10)

Classically, by enforcing the constraint $|z_1|^2 + |z_2|^2 = 1$, we have,

$$A_μ = (i/2)(z_2^* \partial_μ z_1 - z_1^* \partial_μ z_2).$$  

(11)

We can now perform the singular gauge transformation $A_μ \to A_μ - C_μ$, $z_α \to e^{i \frac{1}{2} \int d^4x C_μ(\hat{r})z_α}$, to obtain,

$$L = \frac{1}{g} (\partial_μ - iA_μ) z_α^2 + \frac{1}{2e^2} (\epsilon_{μνλ} \partial_ν A_λ)^2.$$  

(12)

This is precisely the standard DQC model [10]. Note that a similar decoupling does not hold in the case of the Ginzburg-Landau theory for two-component superconductors discussed in Ref. [27], since there the sum of the respective Cooper pair densities is not $CP^1$-constrained. Thus, we see that if we insist on the emergent character of $A_μ$, as expressed in Eq. (11), the Maxwell term in Eq. (12) is just the Skyrme term, which this is shown to be contained in Eq. (5).

The gauge transformation employed to decouple $C_μ$ and thus derive Eq. (12) was performed at the classical level, in which case the equation of motion for $A_μ$ yields $A_μ = (i/2)(z_2^* \partial_μ z_1 - z_1^* \partial_μ z_2)$. In some calculations involving higher order quantum fluctuations, it might be more appropriate to consider the Lagrangian [10], since quantum fluctuations give the gauge field $A_μ$ an independent dynamics. However, in most lowest order approximations, Eq. (12) is sufficiently accurate. Note that Eq. (11) does not follow from the equation of motion for $A_μ$ derived from the Lagrangian (12).

Next we calculate the spin stiffness, $\rho_σ$. The latter is obtained from the gauge invariant response to a twist associated to the spin current tensor $\mathbf{J}_μ = (\mathbf{n} \times \partial_μ \mathbf{n})/g$ [31]. Due to the constraint $\mathbf{n}^2 = 1$, we have,

$$\frac{g}{2} \frac{\partial}{\partial n_μ} = \frac{1}{2g} (\partial_μ \mathbf{n})^2$$  

(13)

which provides some insight into the meaning of $1/g$ as the bare stiffness. Note that when $d = 1 + 0$, $\mathbf{J}_μ$ is just the angular momentum of a particle having moment of inertia $1/g$ and constrained to move on the surface of the $S^2$ sphere. By generalizing the mechanics of a particle on a sphere to a field theory in $d = D + 1$ spacetime dimensions, the twist is realized by the response to an external constant triplet field $S_μ$, which amounts to making the replacement $\partial_μ \mathbf{n} \to \partial_μ \mathbf{n} + S_μ \times \mathbf{n}$. In order to facilitate the calculations, we can assume without loss of generality that $S_μ_3 = \delta_μ_3 S_3$. Thus, in the $CP^1$ representation $S_μ$ will couple to the third component of $\mathbf{J}_μ$, which is written in terms of spinon fields as $J_μ_3 = \frac{1}{g} (j_μ^{(1)} - j_μ^{(2)})$, where $j_μ^{(α)} = i(z_α^* \partial_μ z_β - z_β^* \partial_μ z_α) - 2A_μ z_α z_β$, with no sum over $α$. Variation of the Lagrangian Eq. (12) yields, $\frac{1}{g} j_μ^{(1)} + j_μ^{(2)} = \frac{1}{g} \partial_μ S_μ$, which allows us to write $J_μ_3 = \frac{2}{g} j_μ^{(1)} - \frac{1}{g} \partial_μ S_μ$. The spin stiffness is calculated by computing the response of $J_μ_3$ when it is coupled to an external source, $S_μ$. Differentiating the free energy functional with respect to the source, letting the source vanish at the end, yields the spin stiffness in the form,

$$\rho_σ = \frac{4}{g} (\langle |z_1|^2 |z_2|^2 \rangle - d) \int d^4x K_μ(\mathbf{x})$$  

(14)

where $K_μ(\mathbf{x}) = \langle J_μ_3(x) J_3(0) \rangle$ is the current correlator. The above formula is a generalization to the $O(3)$ case of the well-known formula for the superfluid density of a globally $U(1)$-invariant system. Interestingly, the $O(3)$ stiffness involves the correlator of a gauge-invariant $U(1)$ current within a framework associated to a global $SU(2)$ symmetry.

When only global symmetries are involved, conserved currents do not renormalize [32][33]. This fact elegantly provides a foundation for the scaling relation $ρ_σ \sim E_σ^{-D}$ for the superfluid stiffness in systems with a global $U(1)$ symmetry [34]. Indeed, current conservation implies that the current correlation function does not exhibit an anomalous dimension, whence the scaling behavior of the superfluid stiffness is simply determined by dimensional analysis. However, the non-renormalization theorem fails in gauge theories [35], such as the Lagrangian in Eq. (12). In the absence of the Maxwell, we have the standard $CP^1$ model, which is equivalent to the $O(3)$ nonlinear sigma model. In this case the non-renormalization
obtained here is fundamentally di-
separatrix of Fig. 1. Thus, the logarithmic violation of scaling
non-linear sigma model. The DQC regime lies on the critical

magnetic Mott insulating compound EtMe

testing scaling in experiments by measuring the spin suscepti-

...ganic model, and its primary role in the log-correction.

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