In 1916 Karl Schwarzschild obtained a spherically symmetric solution of Einstein field equations that we know as black hole solution. An interesting case is the time reversal solution in which gravitational collapse to a black hole occurs. This is known as the Schwarzschild white hole solution. In this framework a white hole is an object exploding from highly dense or singular state when it was originally well inside its black hole. Currently, this idea has been subject of great debate with respect to its physical meaning, existence and importance. For some authors the physical relevance of such objects is considered rather doubtful \cite{1}. However for some other researchers a current view of white holes bears a revision of the standard cosmological model (SCM) as it is done for example in \cite{2}. In this new view is examined the possibility that the big bang is a blast that happened inside of a black hole, followed by an expansion. In other words, the possibility that our universe has been originated due to the explosion of a white hole has already been considered in physics. As in general theory of relativity, some other objects that have similar properties to white holes have appeared in physics. For instance, objects that have the unusual property of reflecting all test particles due to a repulsive (anti-gravitational) effect on them have been considered in some treatments of Supersymmetry. These objects are called repulsons \cite{3}. Thus, in order to avoid misunderstanding, many researchers find convenient referring to white holes as compact objects which have the remarkable characteristic of having a repulsive naked singularity which reflects all test particles due to a repulsive (anti-gravitational) effect, being also this a better way to include all the possible configurations that could have these properties. In this letter we will refer to a white hole in the former sense.

The idea that our universe is a 4D space-time embedded in a higher dimensional manifold with large extra dimensions has been a topic of increased interest in several branches of physics, and in particular, in cosmology. This idea has generated a new kind of cosmological models that includes quintessential expansion. In particular theories on which is considered only one extra dimension have become quite popular in the scientific community. Among these theories are counted the braneworld scenarios, the space-time-matter (STM) \cite{4} and all noncompact Kaluza-Klein theories. In this letter we have particular interest on the ideas of the STM theory. A success of the STM theory is that all matter fields in 4D can be geometrically induced from a 5D apparent vacuum leading an effective 4D energy-momentum tensor \cite{5} and for this reason the theory is also called Induced-Matter theory. The assertion that all matter fields can be geometrically induced is hardly supported by the Campbell-Magaard theorem and their extensions \cite{6}. In the case of braneworld cosmologies, our universe is modeled by a brane embedded in a higher dimensional manifold called the bulk. Matter fields are confined to the brane and gravity can propagate along the bulk. In general braneworld models and STM have different physical motivation and interpretation. However, their equivalence has been recently shown by Ponce de Leon \cite{7}. In both theories 4D physics is obtained by evaluating the 5D metric at some fixed hypersurface, i.e. by establishing a foliation choosing a constant value of the fifth coordinate. The possibility of having a dynamical foliation in a consistent manner was explored by the present authors in \cite{8} and more formally established by J. Ponce de Leon in \cite{9}. In these papers by dynamical foliation the authors mean to take the fifth coordinate as a function of the cosmic time under

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the requirement of continuity of the metric, selecting in this way a dynamical 4D hypersurface. What makes important this new approach is that new physics can be derived from a new contribution of the fifth dimension in the induced matter on a time-varying 4D hypersurface. This modification is absent when we choose a constant foliation [9].

However, there are some other questions that have not even been analyzed in the mentioned approach. Is it possible to establish a dynamical foliation by choosing the fifth coordinate as a function of the rest of the spatial coordinates? On the other hand, as it is well known the fifth dimension is considered in the STM theory as a geometric source of the 4D physical matter fields. So that, as well as a dynamical foliation via a temporal dependence of the fifth coordinate allows to describe geometrically new sources of matter in 4D, is it possible to derive new physics by adding a spatial dependence in the fifth coordinate when implementing the foliation? Is there another mechanism to describe sources of matter by using foliations of the higher dimensional space-time? In this letter we present a manner to address the problem outlined in the last question. We propose a new formalism where, instead of implementing a dynamical foliation by taking a spatial dependence of the fifth coordinate including its time dependence, we consider another extra dimension, the sixth dimension, in such a way that we can implement two dynamical foliations in a sequential manner. The first one by choosing the fifth coordinate depending of the cosmic time, and the second one by choosing the sixth coordinate dependent of the 3D spatial coordinates. All of these choices preserve the continuity of the metric. In addition, the 6D metric must be Ricci-flat. This requirement is a natural extension of the vacuum condition used in the STM theory, in which 5D Ricci-flat metrics are used. In simple words, we are using the Campbell-Magaard theorem and their extensions for embedding a 5D Ricci-flat space-time in a 6D Ricci-flat space-time. The conditions of 6D Ricci-flatness and the continuity of the metric give us the foliation of the sixth coordinate. In other words, these conditions specify the sixth dimension as a function of the 3D spatial coordinates, in order to establish the foliation. This function, for a particular 6D metric, can be seen in 4D as a gravitational potential associated to a localized compact object that has the characteristics of a white hole. From a more general point of view, given a 6D Ricci-flat metric, this is a mechanism for inducing localized matter onto a time-varying 4D hypersurface by establishing a spatial foliation of a sixth coordinate. This is very relevant because we can describe matter at both, cosmological and astrophysical scales, in a cosmological model. Of course, this is not the first 6D gravitational model developed from a 6D vacuum. The STM theory was extended to more than five dimensions by Fukui [10]. In his model, Fukui obtained a simple vacuum cosmological solution in the context of a Space-Time-Matter-Charge theory of the universe [11]. This theory was proposed by Wesson in order to obtain a unified field theory of gravity and electromagnetism following the same line of his 5D STM theory [12]. On the other hand, a family of cosmological solutions to the Einstein field equations obtained from a (D + 4)-dimensional (D ≥ 2) vacuum, was studied in [13].

Inflationary theory of the universe provides a physical mechanism to generate primordial energy density fluctuations on cosmological scales [14]. However, it fails when we try to predict the spectrum of these fluctuations on more smaller (astrophysical) scales. Looking ahead for a novel cosmological approach that addresses this problem, we consider feasible to use the geometrical formalism described previously to study an asymptotic spatially flat FRW universe (on large scales) emerging from the explosion of a white hole obtained from a 6D vacuum state. We shall start our treatment considering a 6D vacuum state rather than the usual 5D one [15], to develop an effective 4D inflationary model of the universe where the expansion is affected by a local geometrical deformation induced by the mass M of a white hole induced in turn through the sixth dimension. Within this approach, the 4D early universe is viewed as a white hole that expands (the expansion is driven by the scalar field ϕ) on the effective (4D) spatially curved background metric, where the naked singularity is identified with the big bang singularity. Such a scenario should be feasible on very extreme conditions, where the mass of the white hole is of the order of the Planckian mass $M_p = 1.2 \times 10^{19}$ Gev, on which Grand Unification mass scale could take place. A similar approach (but without STM theory of gravity) was considered many years ago in [16]. Since we are aimed to describe an effective 4D cosmological scenario, the continuity condition applied to our 5D metric leaves a foliation on the fifth coordinate in such a way that its time dependence is related to the Hubble parameter, while the foliation on the sixth coordinate induces a 4D white hole gravitational potential. For simplicity, in this letter will be considered a de Sitter expansion, where the Hubble parameter $H$ is a constant. However, our formalism could be extended for whatever $H = H(t)$. Thus, in our model the fifth dimension is responsible for the 4D de Sitter expansion, which is physically driven by the inflaton field $\varphi$. From the physical point of view, the sixth dimension is responsible for the spatial curvature induced by the mass of the white hole (located at $R = 0$). In more general terms, within our approach the fifth dimension is physically related to the vacuum energy density which is the source of the effective 4D global inflationary expansion whereas the sixth one induces local gravitational sources.
II. EFFECTIVE 4D DYNAMICS FROM A 6D VACUUM STATE

In order to describe a 6D vacuum, we consider the 6D Riemann flat metric

\[ dS^2 = \psi^2 dN^2 - \psi^2 e^{2N} [dt^2 + r^2 d\Omega^2] - d\psi^2 - d\sigma^2 \]  

which defines a 6D vacuum state \( G_{\mu \nu} = 0 \) (\( a, b = 0, 1, 2, 3, 4, 5 \)). Since we are considering the 3D spatial space in spherical coordinates: \( \vec{r} = \vec{r}(r, \theta, \phi) \), here \( d\Omega^2 = d\theta^2 + \sin^2(\theta) d\phi^2 \). Furthermore, the coordinate \( N \) is dimensionless and the extra (space-like) coordinates \( \psi \) and \( \sigma \) are considered as noncompact. We define a physical vacuum state on the metric (1) through the action for a scalar field \( \varphi \), which is nonminimally coupled to gravity

\[ I = \int d^6x \left\{ \frac{(6)R}{16\pi G} + \frac{1}{2} g^{ab} \varphi, a \varphi, b - \frac{\xi}{2} (6)R \varphi^2 \right\} , \]  

where \((6)R = 0\) is the Ricci scalar and \( \xi \) gives the coupling of \( \varphi \) with gravity. Implementing the coordinate transformation \( N = Ht \) and \( R = r\psi = r/H \) on the frame \( U^\nu = (d\psi/dS) = 0 \) (considering \( H \) as a constant), followed by the foliation \( \psi = H^{-1} \) on the metric (1), we obtain the effective 5D metric

\[ (5) dS^2 = dt^2 - e^{2Ht} \left[ dR^2 + R^2 d\Omega^2 \right] - d\sigma^2. \]  

Unfortunately this metric is not Ricci-flat because \((5)R = 12H^2 \). However, it becomes Riemann flat in the limit \( H \to 0 \) i.e. \( R_{BCD}^{A}|_{H \to 0} = 0 \), describing in this limit a 5D vacuum given by

\[ G_{AB}|_{H \to 0} = 0, \quad (A, B = 0, 1, 2, 3, 4). \]  

Thus, keeping this fact in mind, now we consider on the metric (5) the following foliation on the sixth coordinate:

\[ d\sigma^2 = 2\Phi(R) \, dR^2. \]  

The effective 4D metric that results is

\[ (4) dS^2 = dt^2 - e^{2Ht} \left[ 1 + 2\Phi(R) \right] dR^2 + R^2 d\Omega^2, \]  

where \( t \) is the cosmic time, \( H = \dot{a}/a \) is the Hubble parameter for the scale factor \( a(t) = a_0 e^{Ht} \), with \( a_0 = a(t = 0) \). The Einstein equations for the effective 4D metric (4) are \( G_{\mu \nu} = -8\pi G \, T_{\mu \nu} \) (\( \mu, \nu = 0, 1, 2, 3 \)), where \( T_{\mu \nu} \) is represented by a perfect fluid: \( T_{\mu \nu} = (p + \rho) u_\mu u_\nu - g_{\mu \nu} p \), being \( p \) and \( \rho \) the pressure and the energy density on the effective 4D metric (5). The relevant components of the Einstein tensor are (the non-diagonal components are zero)

\[ G_{tt} = -\frac{3H^2 R^2 \left[ 1 + 4\Phi(1 + \Phi) \right] + 2e^{-2Ht} \left[ R^2 \frac{d\Phi}{dR} + \Phi(1 + 2\Phi) \right]}{R^2 \left[ 1 + 2\Phi(R) \right]^2}, \]  

\[ G_{RR} = \frac{2\Phi + 3H^2 R^2 e^{2Ht} \left[ 1 + 2\Phi \right]}{R^2}, \]  

\[ G_{\theta \theta} = \frac{R \left[ 3RH^2 e^{2Ht} \left[ 1 + 4\Phi(1 + \Phi) \right] \right]}{\left[ 1 + 2\Phi \right]^2}, \]  

\[ G_{\phi \phi} = \frac{\sin^2(\theta) R \left[ 3RH^2 e^{2Ht} \left[ 1 + 4\Phi(1 + \Phi) \right] + \frac{d\Phi}{R^2} \right]}{\left[ 1 + 2\Phi \right]^2}. \]

Now we are aimed to obtain the function \( \Phi(R) \). In order to make that, we are supposing that \( \Phi(R) \) is induced by a mass \( M \) located at \( R = 0 \) in absence of expansion (\( H = 0 \)), guaranteeing this way the flatness of (3). Once we have determined \( \Phi(R) \), the Hubble parameter \( H \) in (3) is not necessarily null. Thus we must take the equation

\[ G_{tt}|_{H \to 0} = -8\pi G \rho(R, t)|_{H \to 0}, \]  

where \( \rho(R, t)|_{H \to 0} = -3M/(4\pi R^3) \) is the gravitational energy density in absence of expansion. In this limit, we obtain the following differential equation for \( \Phi(R) \)

\[ -\frac{R^2 \frac{d\Phi}{dR}}{\left[ 1 + 2\Phi \right]^2} = 3GM + \frac{R\Phi}{\left[ 1 + 2\Phi \right]} . \]
The exact solution for this equation is

$$\Phi(R) = \frac{-3GM\ln(R/R_\ast)}{R + 6GM\ln(R/R_\ast)},$$ (12)

being $R_\ast$ the value of $R$ such that $\Phi(R_\ast) = 0$ and $G = M^{-2}_p$ the gravitational constant. Hence, the function $\Phi(R)$ describes the geometrical deformation of the metric induced from a 5D flat metric [the metric (3) with $H = 0$], by a mass $M$ located at $R = 0$. This function is $\Phi > 0$ (or $\Phi < 0$) for $R < R_\ast$ ($R > R_\ast$), respectively. Furthermore, $\Phi(R)|_{R \to \infty} \to 0$ and thereby the effective 4D metric (5) is (in their 3D ordinary spatial components) asymptotically flat. In this analysis we are considering the usual 4-velocities $u^a = (1,0,0,0)$. Thus, for the metric (5), the energy density and the components of the pressure are

$$\rho(R,t) = \frac{3}{8\pi G} \left[ H^2 - \frac{2GM}{R^3} e^{-2Ht} \right],$$ (13)

$$p_R(R,t) = -\frac{3}{8\pi G} \left[ H^2 - \frac{2GM}{R^3} e^{-2Ht\ln(R/R_\ast)} \right],$$ (14)

$$p_\theta(R,t) = -\frac{3}{8\pi G} \left[ H^2 - \frac{GM}{R^3} e^{-2Ht [1 - \ln(R/R_\ast)]} \right],$$ (15)

$$p_\phi(R,t) = \frac{3}{8\pi G} \left[ H^2 - \frac{GM}{R^3} e^{-2Ht [1 - \ln(R/R_\ast)]} \right].$$ (16)

Note that $p_\theta + p_\phi = 0$, so that the equation of state for a given $R$ is

$$\frac{p}{\rho} = -1 - \frac{3GM}{R^3} \left[ 1 - \ln(R/R_\ast) \right] e^{-2Ht},$$ (17)

being $p = p_R + p_\theta + p_\phi$. From the equation (17) we can see that at the end of inflation, when the number of e-folds is sufficiently large, the second term in (17) becomes negligible on cosmological scales [on the infrared (IR) sector], and thereby

$$p_{IR}^{(end)} \simeq -\rho_{IR}^{(end)},$$ (18)

which is the equation of state that describes inflationary cosmology.

The effective 4D action for the universe is

$$(4) \mathcal{I} = \int d^4x \left[ \frac{(4)R}{16\pi G} + \frac{1}{2} g^{\mu\nu} \phi^{,\mu} \phi^{,\nu} - \frac{\xi_1}{2} (4)R \bigg|_{\Phi=0} \phi^2 - \frac{\xi_2(R)}{2} (4)R \bigg|_{H=0} e^{-2Ht} \varphi^2 \right],$$ (19)

where $^{(4)}R = 12H^2 - 12GM/R^3 e^{-2Ht}$ is the effective 4D Ricci scalar for the effective 4D metric (5), $\xi_1$ gives the coupling of the scalar field $\phi(R,t)$ with gravity on the background induced by the foliation of the first extra dimension $\psi$ at $\Phi(R) = 0$ and $\xi_2$ gives the coupling of $\varphi$ with gravity, on the background induced by the foliation of the second extra dimension $\sigma$ at $H = 0$. The equation of motion for the field $\varphi$ on the metric (5) is

$$\ddot{\varphi} + 3H \dot{\varphi} + e^{-2Ht} \left[ \frac{1}{R^2(1 + 2\Phi(R))} \frac{\partial}{\partial R} \left[ R^2 \frac{\partial \varphi}{\partial R} \right] - \frac{1}{(1 + 2\Phi(R))^2} \frac{\partial \Phi}{\partial R} \frac{\partial \varphi}{\partial R} \right]$$

$$+ \frac{1}{R^2 \sin(\theta)} \frac{\partial}{\partial \theta} \left[ \sin(\theta) \frac{\partial \varphi}{\partial \theta} \right] + \frac{1}{R^2 \sin^2(\theta)} \frac{\partial^2 \varphi}{\partial \phi^2} + \left[ \xi_1 (4)R \bigg|_{\Phi=0} + \xi_2 (4)R \bigg|_{H=0} e^{-2Ht} \right] \varphi = 0.$$ (20)

Given the form of $\Phi$ according to (12), solving this equation is a hard nut to scratch. However, finding solutions in some limit approximations is easier.

### III. WEAK FIELD APPROXIMATION

In this section we study the weak field approximation for the equation (20). In that limit approximation the function $\Phi(R)$ can be written as

$$\Phi(R) \simeq -\frac{3GM}{R} + \left( \frac{6GM}{R} \right)^2,$$ (21)
being $M$ the mass of the compact object located at $R = 0$. Note that the function [21], as well as the exact one [12], goes to zero at $R \to \infty$. For $M > 0$ there is a stable equilibrium for test particles at $R_c = 12GM$ and exhibits a gravitational repulsion (antigravity) for $R < R_c$. Hence, this object has the properties of a white hole [3]. In order to obtain solutions of the equation (20) we propose $\varphi(\vec{R}, t) \sim \varphi_l(t) \varphi_R(R) \varphi_{\theta, \phi}(\theta, \phi)$. With this choice and using the equation (11), we obtain

\[
\begin{align*}
\varphi_l + 3H \varphi_l - 12H^2 \xi_1 \varphi_l &= -\alpha_l \varphi_l e^{-2Ht}, \\
\frac{\partial}{\partial R} \left[ R^2 \frac{\partial \varphi}{\partial R} \right] + 3GM(1 + 2\Phi) + R\Phi \frac{\partial \varphi}{\partial R} &= \varphi_R \left[ \frac{12GM}{R} \xi_2(R) - \alpha_l R^2 + \alpha_R \right] (1 + 2\Phi), \\
\sin(\theta) \frac{\partial}{\partial \theta} \left[ \sin(\theta) \frac{\partial \varphi_{\theta, \phi}}{\partial \theta} \right] + \frac{1}{\sin^2(\theta)} \frac{\partial^2 \varphi_{\theta, \phi}}{\partial \phi^2} &= \alpha_R \varphi_{\theta, \phi},
\end{align*}
\]

where $\alpha_l$ and $\alpha_R$ are separation constants.

### A. confined field

As a first step we shall study the initial state of the universe with purely gravitational energy density at $t = 0$. In this case the field $\varphi$ is confined to negative eigenvalues of energy, i.e. $\alpha_l = -|\alpha_l|$. The solution of the equation (23) for $\alpha_R = l(l + 1)$ is given by the spherical harmonics $Y_{l,m}(\theta, \phi)$

\[
\varphi_{\theta, \phi}(\theta, \phi) \sim \sum_{l,m} A_{lm} Y_{l,m}(\theta, \phi) = \sum_{l,m} A_{lm} \sqrt{(2l + 1)(l - m)!} P_l^m(\cos(\theta)),
\]

where $m = -l, -(l - 1), \ldots, 0, \ldots, (l - 1), l$ is a separation constant and $P_l^m(\cos(\theta))$ are the Legendre polynomials:

\[
P_l^m(\cos(\theta)) = \left[ (1 - \cos^2(\theta))^{m/2} \right] (\sin^2(\theta))^{l-m} (\sin^2(\theta))^{m} = (2l + 1) \frac{(2l)!}{2^l l!} \frac{d^l}{d\theta^l} ([1 - \cos^2(\theta)]^m).
\]

To complete the study of the spatial dependence of $\vec{\varphi}(\vec{R}, t)$ we must solve the equation (23). For solving this equation we propose

\[
\varphi_R(R) \sim e^{-\int f(R) dR} \varphi_H(R),
\]

with $f(R) = -[1/(2R^2)] [3GM(1 + 2\Phi(R)) + R\Phi(R)]$ and thus equation (23) can be replaced by the system

\[
\begin{align*}
\frac{d^2 \varphi_H}{dR^2} + \frac{2}{R} \frac{d \varphi_H}{dR} + \left\{ \left| \alpha_l \right| R_e - \frac{l(l + 1)}{R^2} \right\} \varphi_H(R) &= 0, \\
\frac{d \varphi_H}{dR} \left[ 6GM(1 + 2\Phi) + 2R\Phi \right] + \varphi_H \left\{ \frac{27G^2M^2}{4R^2} (1 + 2\Phi)^2 + \frac{3}{4} \Phi^2 \\
+ 9GM \Phi(1 + 2\Phi) + 3GM \frac{d \Phi}{dR} + \frac{R \frac{d \Phi}{dR}}{2} - 2 |\alpha_l| R^2 - 2l(R + 1) \\
- \frac{12GM \xi_2(R)}{R} (1 + 2\Phi) - \frac{6GM}{R} \Phi \right\} &= 0.
\end{align*}
\]

The solution for the equation (27) is

\[
\varphi_H(R) = A_{nl} \left( \frac{2R}{\lambda_p} \right)^l e^{- \left[ \frac{R}{2(l + 1) \lambda_p} \right] \left( 1 + \frac{16}{3} \frac{\lambda_p}{\lambda_p} \right)} L_{n+l}^{2l+1} \left[ \frac{2R}{\lambda_p} \right],
\]

where $L_{n+l}^{2l+1} \left[ \frac{2R}{\lambda_p} \right]$ are the associated Laguerre polynomials with $l = 0, 1, \ldots, n - 1$, for a given $n$, $A_{nl}$ is a normalization constant, and

\[
|\alpha_l| = G^{-1} = \lambda_p^{-2}, \quad R_e = 4n \lambda_p, \quad M \equiv M_n = \frac{n}{3\lambda_p},
\]

being $\lambda_p = M_p^{-1}$ the Planckian wavelength. Hence, the mass of the compact object $M_n = nM_p/3$ and the function $\Phi(R) \equiv \Phi_n(R) = - (3GM_n/R) + (6GM_n/R)^2$ are now quantized. The interesting of this result is that for any value of $n$ ($n$ can take any integer positive value), the eigenvalue $|\alpha_l| = G^{-1} = M_p^{-1}$ is the same. In other words $|\alpha_l|$ is
degnerated. Note that \( \varphi_H(R) \big|_{R \to 0, R \to \infty} \to 0 \), so that it is well defined along \( R \geq 0 \). On the other hand, the function \( f(R) \) now depends on \( n \): \( f_n(R) = -[1/(2R^2)] [3GM_n(1 + 2\Phi_n(R)) + R\Phi_n(R)] \). From equation (28) we obtain that the coupling \( \xi_2(R) \equiv \xi_2^{n,l}(R) \) is given by

\[
\xi_2^{n,l}(R) = \frac{1}{\varphi_H^2} \frac{d\varphi_H}{dR} \left[ \frac{R}{2} + \frac{R^2\Phi_n}{6GM_n(1 + 2\Phi_n)} \right] - \left[ \frac{R^3}{12GM_n(1 + 2\Phi_n)} \left( f_n^2 - \frac{df_n}{dR} \right) \right] + \frac{2\Phi_n R (|\alpha| |R|^2 - l(l + 1)) - 2R^2 f_n - R\Phi_n f_n}{12GM_n(1 + 2\Phi_n)} \frac{f_n R}{4},
\]

which depends on \( n \) and \( l \). Furthermore, the solution for the equation (35) is given respectively by (26) and (32). Note that the solution for \( \varphi(R) \) in the case where the field is confined, will be

\[
\varphi(R) = \varphi_{n,l}(R) \equiv \frac{M_p}{H} e^{-Ht} \left[ A_1 K_\nu \left( \frac{R}{H} \right) + A_2 I_\nu \left( \frac{R}{H} \right) e^{-Ht} \right],
\]

where \( K_\nu \) and \( I_\nu \) are the modified Bessel functions and \( \nu = (1/2) \sqrt{9 + 4\xi_1} \). Hence, the solution for \( \varphi(R, t) \) in the case where the field is confined, will be

\[
\varphi(R, t) = \varphi_{n,l}(R) \sum_{l=0}^{n-1} \sum_{m=-l}^{l} \varphi_{n,l}(R) \left[ A_{lm} Y_{l,m}(\theta, \phi) + A_{lm}^* Y_{l,m}^*(\theta, \phi) \right],
\]

with \( \varphi_{n,l}(R) \) and \( \varphi_{l}(t) \) given respectively by (26) and (32). Note that \( \varphi(R, t) \) is well defined for all \( R \geq 0 \).

### B. Dispersive case

Now we study the dispersive case \( (\alpha_1 = |\alpha| = k_R^2) \) for \( \varphi(R, t) \). In this case \( k_R \) is the wavenumber related to the coordinate \( R \). Proceeding in a similar manner that in the previous section for solving the equation (22), we propose

\[
\varphi(R) = \varphi_{k_R}(R) e^{-\int f_n(R) dR},
\]

such that \( \varphi_{k_R}(R) = \varphi_R[\Phi = 0] \). With this choice the expression (22) can be replaced by the equations

\[
\frac{d^2 \varphi_{k_R}}{dR^2} + \frac{2}{R} \frac{d\varphi_{k_R}}{dR} + \varphi_{k_R} \left[ k_R^2 - \frac{l(l + 1)}{R^2} \right] = 0,
\]

\[
\frac{d\varphi_{k_R}}{dR} \left[ 6GM_n(1 + 2\Phi_n) + 2R\Phi_n \right] + \varphi_{k_R} \left[ R^2 \left( f_n^2 - \frac{df_n}{dR} \right) - 2R f_n + 2\Phi_n \left( k_R^2 R^2 - l(l + 1) \right) \right] \left( 12GM_n \xi_2(R) \right) = 0.
\]

The solution for the equation (36) is

\[
\varphi_{k_R}(R) = \frac{A}{\sqrt{k_R}} J_{l+1/2}[k_R R],
\]

where \( J_{l+1/2} \) are the Bessel functions. On the other hand, from the equation (36) we obtain the coupling \( \xi_2^{n,l}(R) \)

\[
\xi_2^{n,l}(R) = \frac{R}{12GM_n(1 + 2\Phi_n)} \left\{ \frac{1}{\varphi_{k_R}} \frac{d\varphi_{k_R}}{dR} \left[ 6GM_n(1 + 2\Phi_n) + 2R\Phi_n \right] - \left[ R^2 \left( f_n^2 - \frac{df_n}{dR} \right) \right] \right\} - 2R f_n + 2\Phi_n \left[ k_R^2 R^2 - l(l + 1) \right] - 3GM_n(1 + 2\Phi_n) f_n - R\Phi_n f_n \}
\]

with \( f_n(R) = -[1/(2R^2)] [3GM_n(1 + 2\Phi_n(R)) + R\Phi_n(R)] \). The solution of the equation (22) for the dispersive case is

\[
\varphi(t) \sim \xi_{k_R}(t) = A e^{-3Ht/2} H^{(2)} \left[ \frac{k_R}{H} e^{-Ht} \right],
\]
being $A$ a constant, $\nu = (1/2)\sqrt{9 + 48\xi^2}$ and $H_{\nu}^{(2)}$ is the second kind Hankel function. Taking into account all the possible values of $k_R$, the complete solution for the field $\varphi(\vec{R}, t)$ can be written as

$$
\varphi(\vec{R}, t) = \frac{1}{(2\pi)^{3/2}} e^{-\left[\left(\frac{3\nu}{\lambda_p}\right)^2 (1 + 16\alpha \nu^3)\right]} \int d^3k_R \sum_{m=-l}^{l} \sum_{l=0}^{n-1} \left[ a_{k_R lm} Y_{l,m}(\theta, \phi) \varphi_{k_R}(R) \xi_{k_R}(t) \right] + a_{k_R lm}^* Y_{l,m}^*(\theta, \phi) \varphi_{k_R}^*(R) \xi_{k_R}^*(t),
$$

where $\varphi_{k_R}(R)$ and $\xi_{k_R}(t)$ are given respectively by the expressions (37) and (39). Notice that the exponential factor in (40) tends to 0 for $R/\lambda_p \to 0$. It solves all possible UV divergences of the $\varphi$-fluctuations in a natural manner.

C. Large scale asymptotic behavior

In general, the spatial homogeneity and isotropy of the universe on cosmological scales is an accepted fact. This property introduce new symmetries to the solution (40) allowing to explore the possibility to recover the well-known nearly invariant spectrum on super-Hubble scales, usually obtained from inflationary models. Thereby it results convenient to study the large scale asymptotic behavior of the solution (40). We can obtain this limit approximation by considering the conditions $R/\lambda_p > RH \gg 1$ and $k_F R \gg 1$ (the IR sector). Thus, taking into account that $\mathcal{J}_\mu[x] \mid_{x \gg 1} \simeq \sqrt{2/(\pi x)} \cos[x - \mu x^2 - 2/\pi]$, the expression (40) on the infrared (IR) sector becomes

$$
\varphi(\vec{R}, t) \mid_{IR} \simeq \frac{1}{(2\pi)^{3/2}} \sqrt{\frac{2}{\pi R}} \int d^3k_R \sum_{m=-l}^{l} \sum_{l=0}^{n-1} \left[ a_{k_R lm} Y_{l,m}(\theta, \phi) e^{i[k_R R - \frac{\nu}{2}]} \xi_{k_R}(t) \right] + a_{k_R lm}^* Y_{l,m}^*(\theta, \phi) e^{-i[k_R R - \frac{\nu}{2}]} \xi_{k_R}^*(t),
$$

On the other hand, it is important to emphasize that on cosmological scales we can consider $\Phi(R) \simeq 0$. Considering these limit conditions and transforming (41) to cartesian coordinates, we obtain

$$
\varphi(\vec{R}, t) \mid_{IR}^{(\Phi(R) = 0)} \simeq \frac{1}{(2\pi)^{3/2}} \int d^3k_R \left[ a_{k_R lm} e^{i\vec{k}_R \cdot \vec{R}} \xi_{k_R}(t) \right] + a_{k_R lm}^* \xi_{k_R}^*(t),
$$

being $d\vec{R}^2 = dx^2 + dy^2 + dz^2$ with $x = \tilde{R} \sin(\theta) \cos(\phi)$, $y = \tilde{R} \sin(\theta) \sin(\phi)$ and $z = \tilde{R} \cos(\phi)$. In the obtaining of (42) we have used the Rayleigh expansion $e^{ik_R \cdot \vec{R}} = 4\pi \sum_{l,m} i^l \mathcal{J}_l(k_R R) Y_{l,m}(\Omega_R) Y_{l,m}(\Omega_R)$ and the relations between annihilation-creation and creation-creation operators from spherical to cartesian representations $a_{k_R lm} = i^l \int d\Omega_R Y^*_l(\Omega_R) a_{k_R lm}$ and $a_{k_R lm}^* = (-i)^l \int d\Omega_R Y_{l,m}(\Omega_R) a_{k_R lm}^*$. Thus, the corresponding asymptotic expression for the modes on IR sector is

$$
\xi_{k_R}(t) \mid_{IR}^{(\Phi(R) = 0)} \simeq -\frac{i}{2} \left( \frac{1}{\pi R^2} \sqrt{\frac{2}{\pi}} \right) e^{Ht(\nu - 3/2)},
$$

Note that the expressions (42) and (43) are the same that we have obtained in previous papers [17]-[19]. In this manner, we recover cosmological solutions which agree with an isotropic and homogeneous universe in absence of the potential $\Phi(R)$ (and therefore of the compact object). As in [17]-[19], in what follows we shall use the cartesian coordinate representation in order to make explicit the asymptotic spatial homogeneity and isotropy on very large scales.

Now, we are able to calculate the effective 4D super-Hubble squared fluctuations, for the cosmological limit in cartesian coordinates $\langle \varphi^2 \rangle_{IR}^{(\Phi(R) = 0)}$, which are given by

$$
\langle \varphi^2 \rangle_{IR}^{(\Phi(R) = 0)} = \frac{1}{2\pi^2} \int_{0}^{k_H(t)} \frac{dk_R}{k_R^2} \left[ \xi_{k_R}(t) \xi_{k_R}^*(t) \right]_{IR}^{(\Phi(R) = 0)},
$$

where $k_H = \sqrt{12\xi + (9/4) H e^{Ht}}$ is the Hubble wavenumber on physical coordinates in a de Sitter expansion. Inserting (43) into (44) we obtain

$$
\langle \varphi^2 \rangle_{IR}^{(\Phi(R) = 0)} = \frac{2^{\nu - 3}}{\pi} \Gamma^2(\nu) H^{2\nu - 1} e^{-(3 - 2\nu)Ht} \int_{0}^{k_H} \frac{dk_R}{k_R} k_R^{3 - 2\nu}.
$$

(45)
Note that when the coupling parameter $\xi_1 \ll 1$, the index $\nu \simeq 3/2$ and thus the corresponding 3D power spectrum $P(k_R) \sim k_R^{3-2\nu}$ becomes nearly scale invariant. Performing the remaining integration, the expression (15) yields

$$\langle \varphi^2 \rangle_{k_R}^2 \sim 2^{2\nu-3} \frac{\Gamma^2(\nu)}{3 - 2\nu} \frac{1}{\pi^3} H^{2\nu} e^{-(3-2\nu)Ht} k_R^{3-2\nu} = \frac{2^{2\nu-3} H^2 \Gamma^2(\nu)}{\pi^3 (3 - 2\nu)} \left( 12 \xi_1 + \frac{9}{4} \right)^{\nu-3/2}. \quad (46)$$

The 3D power spectrum corresponds to the spectral index $n_s = 4 - \sqrt{9 + 48\xi_1}$. On the other hand, it is well known from observational evidences [20] that $n_s = 0.97 \pm 0.03$. Hence, we can establish the next range of values for the coupling parameter $0 \leq \xi_1 \leq 75.75 \times 10^{-4}$.

**D. Small Scale Approximation**

Once we have studied the asymptotic behavior on large scale of the scalar field [40], we are aimed to obtain its behavior on small scales $k_H R \ll 1$. Since the modes responsible for the posterior structure formation must be those that become classical on super-Hubble scales after inflation ends, in order to obtain the corresponding spectrum on small scales, we should consider those modes whose physical length scale is a little bigger than the horizon scale (the Hubble radius) at that time. Hence, for small scales, in presence of the compact object [and thus for $\Phi(R) \neq 0$], we must understand the today astrophysical scales ($\sim 100$ Mpc). In our conventions we will refer to this part of the spectrum as the small scale (SS) spectrum. Therefore, the effective small scale squared fluctuations in presence of $\Phi(R) \neq 0$ are given by

$$\langle \varphi^2 \rangle_{SS} = 2^{2\nu-3} \frac{\Gamma^2(\nu)}{3 - 2\nu} \frac{1}{\pi^3} H^{2\nu} e^{-(3-2\nu)Ht} e^{- \left[ \frac{1}{2} \left( \frac{\nu k R}{\pi^2} \right)^2 \left( 1 + \frac{16n_\lambda}{3m_\lambda} \right) \right]} k_R^{3-2\nu} (1 - \epsilon)^{3-2\nu}. \quad (47)$$

Comparing (46) with (48), we obtain

$$\langle \varphi^2 \rangle_{SS} = \frac{2^{2\nu-3} \Gamma^2(\nu)}{3 - 2\nu} \frac{1}{\pi^3} H^{2\nu} e^{-(3-2\nu)Ht} e \left[ \frac{1}{2} \left( \frac{\nu k R}{\pi^2} \right)^2 \left( 1 + \frac{16n_\lambda}{3m_\lambda} \right) \right] k_R^{3-2\nu} (1 - \epsilon)^{3-2\nu}. \quad (49)$$

However, since $\epsilon \ll 1$ then we can make the approximation $(1 - \epsilon)^{3-2\nu} \simeq 1$. Hence, the expression (49) becomes

$$\langle \varphi^2 \rangle_{SS} \approx e^{- \left[ \frac{1}{2} \left( \frac{\nu k R}{\pi^2} \right)^2 \left( 1 + \frac{16n_\lambda}{3m_\lambda} \right) \right]} \langle \varphi^2 \rangle_{IR} \quad (50)$$

Finally, we can write the power spectrum of $\langle \varphi^2 \rangle_{SS}$, making $k_R = 2\pi/R$ and $k_p = 2\pi/\lambda_p$

$$P(k_R) \langle \varphi^2 \rangle_{SS} \sim e^{- \left[ \frac{1}{2} \left( \frac{\nu k R}{\pi^2} \right)^2 \left( 1 + \frac{16n_\lambda}{3m_\lambda} \right) \right]} k_R^{3-2\nu}. \quad (51)$$

This result resembles the whole obtained by J. Einasto et. al[21], who obtained from observation that the spectrum of galaxies at present is determined by two fundamental spectral indices: $n_s \simeq 1$ (which is dominant on cosmological scales) and $n \simeq -1.9$ (which is dominant on astrophysical scales). Such a result is in agreement with our calculations. In the figure (1) we have plotted $\log_{10} \left[ \frac{\delta \langle \varphi^2 \rangle_{SS}}{\rho} \right]$, where

$$\frac{\delta \langle \varphi^2 \rangle_{SS}}{\rho} \simeq \frac{\xi_1}{2} \langle \varphi^2 \rangle_{IR} \langle \varphi^2 \rangle_{SS} \frac{\langle \varphi^2 \rangle_{SS}}{V(\langle \varphi \rangle)} \quad (52)$$

as a function of $x = k_R/k_p$. We have used $H = 1.7 \times 10^{-7}$ Mpc and $V(\langle \varphi \rangle) = \frac{8}{3} (4) \left( \frac{\langle \varphi^2 \rangle_{IR}}{\langle \varphi^2 \rangle_{SS}} \right) 10^{-20} M^4$, $\xi_1 = 75 \times 10^{-4}$ and $n = 10$ with $\nu = 1.518$ (i.e., for a spectral index $n_s = 0.964$). Note that the energy density fluctuations are scale dependent on scales of the order of $k_H/k_p > 10^{-2}$, which corresponds to $R < 10^{-3}H^{-1}$ during inflation.
IV. FINAL COMMENTS

In this letter using ideas of STM theory of gravity we have studied an effective 4D inflationary expansion of the universe that emerges from a 6D vacuum state. Under this approach, the expansion is affected by a geometrical deformation induced by the gravitational attraction of a white hole of mass $M_n = nM_p/3$. This deformation is described by the function $\Phi_n(R)$, which tends to zero on cosmological scales. However, on sub Hubble scales this function is determinant for the spectrum of the squared $\varphi$-fluctuations. An important result of our formalism is that the spectrum of energy density fluctuations is almost independent of the scale on cosmological scales, but this is not the case on current small (sub Hubble) scales. We suggest that it could be the origin of the current $k_R$-non invariant spectrum observed for the distribution of galaxies on astrophysical scales. A different, but in some sense emparented approach, was recently suggested in the literature[22]. For simplicity, our calculations were made in the weak field approximation of the expression (21) for $\Phi(R)$, so that they are valid on the infrared and blue sectors of the spectrum. However, these results could be extensive for strong interactions, using the exact expression (12) for $\Phi(R)$. Notice that in this letter we have neglected the last term in the action (19) to obtain the matter density spectrum. This term, through $\xi^2(R)$, could be responsible for the present day galaxy clusters correlations[21]. A more exhaustive study of this topic will be done in a forthcoming paper.

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FIG. 1: $\log_{10}\left[\frac{\delta \rho(k_R)}{\rho}\right]$ as a function of $x = k_R/k_p$. The energy density fluctuations are nearly scale invariant on large scales, but become scale dependent on the blue sector.