Deep learning-based topological optimization for representing a user-specified design area

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Abstract Presently, topology optimization requires multiple iterations to create an optimized structure for given conditions. Among the conditions for topology optimization, the design area is one of the most important for structural design. In this study, we propose a new deep learning model to generate an optimized structure for a given design domain and other boundary conditions without iteration. For this purpose, we used open-source topology optimization MATLAB code to generate a pair of optimized structures under various design conditions. The resolution of the optimized structure is 32 × 32 pixels, and the design conditions are design area, volume fraction, distribution of external forces, and load value. Our deep learning model is primarily composed of a convolutional neural network (CNN)-based encoder and decoder, trained with datasets generated with MATLAB code. In the encoder, we use batch normalization (BN) to increase the stability of the CNN model. In the decoder, we use SPADE (spatially adaptive denormalization) to reinforce the design area information. Comparing the performance of our proposed model with a CNN model that does not use BN and SPADE, values for mean absolute error (MAE), mean compliance error, and volume error with the optimized topology structure generated in MATLAB code were smaller, and the proposed model was able to represent the design area more precisely. The proposed method generates near-optimal structures reflecting the design area in less computational time, compared with the open-source topology optimization MATLAB code.

Keywords deep learning · machine learning · topology optimization · convolutional neural network

1 Introduction

In recent years, significant improvements in deep learning algorithms and computer hardware have made it possible to apply deep learning to medical imaging and speech recognition tasks. The most remarkable improvement in deep learning algorithms is the convolutional neural network (CNN), which is particularly well suited to image recognition. CNNs have achieved high performance in tasks such as making and synthesizing images, recognizing and classifying subjects, completion of perforated images, removing noise, and generating high resolution images from low resolution images.

Several studies show that deep learning could be applied not only to imaging and language applications, but also in mechanical fields, such as fluid simulation (Kim et al. 2019), and structure optimization (Yu et al. 2019). Such research demonstrates the ability of CNNs to reduce the cost of computational time.

Structural optimization is a method for optimally designing structures to maximize the target performance under imposed conditions, or design areas.

Structural optimization is categorized into three types: Size optimization (Pragert 1974; Svanberg 1982), shape optimization (Ding 1986; Haslinger and Mäkinen 2003), and topology optimization (Bendsøe and Kikuchi 1988; Bendsøe 1989). In size optimization, structure is optimized by changing only the length and the thickness, without altering the shape of the structure. In shape optimization, the outer shape of the structure is changed in addition to length and thickness. Compared with size optimization, shape optimization
provides greater freedom, and can produce optimized structures with higher performance than is achievable with size optimization. In topology optimization, structures are designed by considering the mass density distribution of the material as a design objective. Hence, the size, outer shape, and topology of the structure can be designed. Compared with size and shape optimization, it provides greater freedom in structural design, and can also design optimized structures with higher performance. However, topology optimization requires a large number of design parameters and many updates, which is associated with high computational cost.

When the design area is 2D, the subject of the topology optimization is the material density of each cell arranged in a grid, and displayed as a matrix. In topology optimization, optimizing the material density of all cells determines the shape of the structure, to achieve high strength and rigidity.

On the other hand, an image can be displayed as a matrix with luminance values arranged in grid pattern pixels. CNNs are good at extracting the shape of a subject from images, and making the images(matrix information). Therefore, it is possible that CNNs are also good at estimating a structure’s rigidity from the material density’s matrix, and also making the material density’s matrix. There have been several studies aiming to reduce the computational cost of topology optimization using CNNs.

CNN models were applied to topology optimization by Sosnovik et al. (2017), who reported success in reducing computational cost. The SIMP method (Bendsoe and Sigmund 1999), which is a conventional form of topology optimization, requires many updates to the material density distribution during structural optimization. While the optimization process is ongoing, the structure at that point (i.e. the structure before reaching its final optimized form) is defined as the intermediate structure. The difference between the pre- and post-update structure at a given step in the optimization process is defined as the gradient. The CNN model proposed by Sosnovik et al. takes the intermediate structure and gradient as inputs, and outputs the final optimized structure. However, since the inputs to Sosnovik’s CNN model do not include information on the design area or boundary conditions, these parameters cannot be directly reflected in the structure output from the model.

In 2018, Banga et al. used a CNN to reduce the computational cost of 3D topology optimization. However, as with Sosnovik’s approach, their model uses the intermediate structure and gradient as inputs to the CNN.

Since these two methods depend on the solution obtained from the SIMP method, they do not achieve topology optimization using a CNN alone. Rather, both methods use a CNN to reduce the number of material density updates required to complete optimization.

In 2018, Rawat and Shen proposed a method for obtaining topology-optimized structures (final material density distribution) without iteration. Their method is based on a generative adversarial network (GAN) (Goodfellow et al. 2014) model. The GAN model is composed of two deep learning (DL) models, and is trained by competition between the two models; a form of adversarial learning. In the GAN model proposed by Rawat and Shen, the DL models are both CNNs, hence the GAN model outputs a topology-optimized structure using CNNs only, without any input from the SIMP method. However, constraints such as volume fraction and boundary conditions, which are important for performing topology optimization, cannot be specified in this CNN model.

Conversely, with the DL model proposed by Yu et al. (2019), it is possible to specify a point where external force is applied, the external force’s direction, the structure’s volume constraint, and the position of the fixed point of the structure. Yu’s DL model is composed of a CNN model and a GAN model, and can output topology-optimized structures based on the above conditions. Not only does Yu’s CNN model achieve topology optimization without iteration, it allows specification of conditions for topology optimization.

As described above, some research exits into CNN-based topology optimization without iteration. However, to the best of our knowledge, no previous CNN models exist that support specification of the design area for topology optimization. All the studies mentioned above use a predetermined design area that cannot be changed. In other words, these CNN models can only output an optimized structure with a design area of the same shape and size. Design area is an important condition for performing topology optimization. Having a higher degree of freedom for the design area increases the versatility of the optimization method.

In this paper, we propose a CNN model that takes the distribution of external forces, value of external forces, volume fraction, and design area as inputs, and outputs the material density distribution without iteration. The output has the same rigidity as a structure optimized using conventional methods.

To output the optimized structure, our CNN model uses SPADE ResBlk (spatially adaptive denormalization residual block) (Park et al. 2019); a technique for generating images by applying a semantic segmentation mask to a CNN. Park et al. created a GAN model using SPADE. A semantic segmentation mask depicting silhouettes of the sky, sea, mountains, clouds, soil, trees, etc. in different colors is input to this GAN model via SPADE ResBlk, and an image of the actual scene based on the mask is output by the GAN model. SPADE ResBlk is described in detail in Section 2.2.

Design area and external force distribution, which are important conditions for topology optimization, can be regarded as image information. For example, if the design area is divided into a grid, and the area where materials are arranged is represented by black (1), while the area where materials are not arranged is represented by white (0), the de-
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Design area becomes an image composed of black and white. In the case of external distribution, if a luminance value is set in a place where an external force is applied, and the luminance values depend on the external load values, the external force distribution becomes a grayscale image. It is expected that using SPADE ResBlk, the mask (design area and external force distribution) expressed in image format can be passed to the CNN accurately. Using SPADE ResBlk makes it possible for the design area and external force distribution to be accurately represented in the optimized structure (optimal material density distribution) output by the CNN.

The most novel aspect of this research is the development of a CNN model that can support specification of design area information for topology optimization. Having compared and verified our model against the CNN model mentioned above (Yu et al. 2019), our CNN model was found to output a material density distribution that more accurately reflects the volume constraint, with greater rigidity. In addition, our model more accurately represents the design area, and outputs a material density distribution closer to the optimal structure obtained using conventional topology optimization (SIMP method). The results of this comparison are described in detail in Section 4.

2 Related research

2.1 The CNN model proposed by Yu et al. (2019)

The DL model for topology optimization proposed by Yu et al. (2019) is composed of a CNN model and a GAN model. The CNN model is able to accept specification of a point where external force is applied, the external force’s direction, the volume constraint of the structure, and the position of the fixed point of the structure; its output is a topology-optimized material density distribution of $32 \times 32$ pixels. The GAN model upscales the resolution of the material density distribution output by the CNN model from $128 \times 128$ pixels into $32 \times 32$ pixels. This paper focuses on the CNN model that outputs a material density distribution of $32 \times 32$ pixels. Their model consists of a CNN (encoder CNN) (Fig. 1) that extracts and compresses the features of the input information, and a CNN (decoder CNN) (Fig. 2) that expands the extracted features to $32 \times 32$ pixels.

The input information is: The single point where external force is applied, the external force’s direction, the volume constraint of the structure, and the position of the fixed point of the structure. The design area is fixed as a square of $32 \times 32$ pixels. The conditions are as follows.

- Volume fraction: 0.2–0.8
- Position of the fixed point
- Application point of single external force: Node 1 to 1089
- Direction of external force: 0° to 360°

Based on these input conditions, Yu’s CNN model outputs a structure (material density distribution) that maximizes rigidity. When generating datasets for training and evaluation using FEM (finite element method), they use MATLAB open-source topology optimization code (Andreassen et al. 2011). Yu et al. set a fixed displacement position and external force load position for the nodes, so both the fixed position information and the load external force information are a $33 \times 33$ matrix (whereas the material density distribution is a $32 \times 32$ matrix).

First, the position of the fixed point and load external force information, in the $33 \times 33$ matrix, are input to the CNN model.

In the encoder CNN, the input condition features are extracted using a convolutional layer and a max pooling layer, and a feature map of $4 \times 4$ pixels is generated. This $4 \times 4$ pixel feature map and volume fraction form the input to the decoder CNN. This decoder CNN gradually enlarges (upsampling, un-pooling) the input $4 \times 4$ pixel feature map, finally outputting a $32 \times 32$ material density distribution.

Both the encoder and decoder CNNs use a rectified linear unit (ReLU) function (Nair and Hinton 2010) as their activation function. However, in the output layer of the decoder CNN, a sigmoid activation function (Mitchell 1997) is used to output a real value of 0 to 1, representing the material density distribution of each element.

The MAE (mean absolute error) of the difference in material density distribution between the optimized structure obtained using a conventional topology optimization method and that obtained with Yu’s method is used as a loss function, and the CNN model is trained to minimize this MAE. In addition, an ADAM optimizer (Kingma and Ba 2014) is used as the learning algorithm.

2.2 SPADE (Park et al. 2019)

The CNN image generation literature includes several models that use a semantic segmentation mask (mask) as the input to a CNN, outputting an image based on the mask; for example, SIMS (Qi et al. 2018) and pix2pixHD (Wang et al. 2018). In 2019, Park et al. proposed a normalization method called SPADE as a method to represent the mask in the output image. The GAN model (GauGAN), created by Park et al., uses SPADE. By inputting a mask drawn by coloring the labels of sky, sea, clouds, mountains, forests, etc., GauGAN can generate images that closely approximate real-life scenery, based on the mask, as shown in Figure 3.

Figure 4 shows the flow in the SPADE layer. Here, the mask is first projected onto the convolution layer to extract a feature map. Then, the scale and bias of each pixel of the mask’s feature map are obtained through the convolution
layer. Mask, scale, and bias information is injected into the normalized feature map, which comes from inside the CNN model, and is normalized by performing batch normalization (BN) (Loffe and Szegedy 2015). This is the normalization method performed inside the SPADE layer.

We shall now describe the process in the SPADE layer with calculation formulas. Let $H, W$ be the number of pixels in the vertical and horizontal directions of the mask. Then, let the matrix indicating the mask be $\mathbf{m} \in \mathbb{L}^{H \times W}$. $\mathbb{L}$ is a set of integers, and each integer has label information such as mountains or sky. Assuming that the feature map in the $i$-layer of the CNN that generates the image is input to the SPADE layer. Let $\mathbf{h}^i$ be the feature map in the $i$-layer with batch size $N$, and let $C^i, H^i, W^i$ be the number of channels and pixels in the vertical and horizontal directions that $\mathbf{h}^i$
has. The value of each pixel in the feature map output by the SPADE layer is then given by equation (1).

\[ y_{i,c,x}^{\text{(post-BN)}} = \gamma_{i,c,x}(m) \left( h_{i,c,x}^{\text{pre-BN}} + \beta_{i,c,x}(m) \right) \quad (i\in \mathbb{N}, c\in \mathbb{C}, x\in W^n) \]  

(1)

where \( h_{i,c,x}^{\text{pre-BN}} \) are the values of the feature map input to the SPADE layer from the \( i \)-layer, and \( h_{i,c,x}^{\text{pre-BN}} \) represents the values before BN. \( \mu_{i,c}^{\text{(post-BN)}} \) and \( \sigma_{i,c}^{\text{(post-BN)}} \) are the mean and standard deviation values, used to normalize \( h_{i,c,x}^{\text{pre-BN}} \) for each channel in the BN process. \( \mu_{i,c}^{\text{(post-BN)}} \) and \( \sigma_{i,c}^{\text{(post-BN)}} \) are derived from equations (2) and (3).

\[ \mu_{i,c}^{\text{(post-BN)}} = \frac{1}{NHW_i} \sum_{n,y,x} h_{n,y,x}^{i} \]  

(2)

\[ \sigma_{i,c}^{\text{(post-BN)}} = \sqrt{\frac{1}{NHW_i} \sum_{n,y,x} \left( h_{n,y,x}^{i} - \mu_{i,c}^{\text{(post-BN)}} \right)^2} \]  

(3)

The values \( \gamma_{i,c,x}(m) \) and \( \beta_{i,c,x}(m) \) are learned parameters obtained by inputting masks to two convolution layers in the SPADE layer. The values \( \gamma_{i,c,x}(m) \) and \( \beta_{i,c,x}(m) \) represent scale and bias for each pixel. The values \( \gamma_{i,c,x}(m) \) and \( \beta_{i,c,x}(m) \) are used for scale and bias for the \( i \)-layer’s feature map, normalized by BN, and calculated as per equation (1). In this manner, the mask information is represented on the \( i \)-layer’s feature map at the SPADE layer.

Park et al. also created the SPADE ResNet Block (SPADE ResBlk) (Figure 5 (left)) by combining the SPADE layer with ResNet (He et al. 2016). The SPADE ResBlk is used multiple times during the up-sampling process (Figure 5 (right)).

In this study, we developed a CNN model that outputs the optimal material density distribution representing the specified design area, using a SPADE ResBlk that represents the mask information on the feature map. At this point, the masks input to the SPADE ResBlk are design area, volume fraction, and external forces.

3 Proposed method

3.1 User-specified conditions

In this study, the conditions for topology optimization solved by the proposed CNN model are as follows.

- Volume fraction
- Design area
- Distribution and load value of external forces

In the case of Yu’s topology optimization CNN model (Yu et al. 2019), the user can specify the volume fraction, position of the external force and direction, and position of the fixed point. However, they cannot specify the shape of the design area.

In this study, we constructed a CNN model supporting specification of not only the volume fraction, the location of each external force and each load value, but also the design area shape as a topology optimization condition. Our CNN model outputs an optimized topology structure of 32 \( \times \) 32 pixels (the material density distribution with the highest rigidity) under the specified conditions.

3.2 CNN model architecture

Figures 6 and 7 show the architecture of our CNN model, based on the encoder and decoder CNNs in Yu’s model. The first information input into the encoder CNN is the design area and external force conditions (external force location and load value) (Fig. 8). The shape of the design area is represented by a 32 \( \times \) 32 matrix. In other words, the shape of the design area is represented by a matrix where the design area is 1 and the material non-placeable area is 0. The external force condition is represented by two matrices of 32 \( \times \) 32. Each component of the first matrix represents the load value in the \( x \) direction of the external force applied to
Fig. 6 Architecture of our encoder CNN model

Fig. 7 Architecture of our decoder CNN model

Fig. 8 Example of the format of the design area and external force information input into the encoder CNN model

Each element (in FEM, each component represents the element force), and the second matrix represents the load value of the external force in the y direction.

In the encoder CNN, at the time of input and during the process of compressing information, the BN normalization...
process is performed multiple times. The information input into the encoder CNN is compressed and finally output as a $4 \times 4$ feature map. This feature map and volume fraction information are input into the decoder CNN. Specifically, the feature map output by the encoder CNN and a matrix in which all $4 \times 4$ components are volume fraction values are concatenated in the channel direction, and the processed $4 \times 4$ feature map is input to the decoder CNN.

The decoder CNN gradually expands the $4 \times 4$ input information (feature map), using SPADE ResBlk (Park et al. 2019) to reinforce the area information, before finally generating the optimal $32 \times 32$ material density distribution. The information (feature map) input into the decoder CNN includes optimization conditions (a matrix representing the design area, and a matrix representing the external force distribution, and volume fraction). However, some of the input information may be lost during feature map expansion. To counter this information loss, SPADE ResBlk is used not only to represent the area information, but also to reinforce the optimization conditions. First, a matrix representing the design area, and a matrix representing the external force distribution and volume fraction information are input into SPADE ResBlk; based on the resulting output, the feature maps of multiple layers in the decoder CNN are denormalized. The volume fraction information (scalar information) is combined with the matrix representing the design area information to form the SPADE ResBlk input. The volume fraction and the design area information are combined by multiplying the volume fraction value and the matrix representing the design area where each component is 0 or 1, as shown in Figure 9; the resulting matrix is input into the SPADE ResBlk. In other words, the matrix, including volume fraction and design area information, and two matrices representing the external force distribution are input into the SPADE ResBlk. The output layer of the decoder CNN uses a sigmoid activation function (Mitchell 1997) so that the material density (real value of 0 to 1) can be output.

4 Evaluation of the effectiveness of the proposed method

4.1 Conditions of the dataset for training and evaluating our CNN model

We set the following topology optimization conditions and used them to evaluate the effectiveness of the proposed method in this study.

- Volume fraction: 0.2–0.8 (same as Yu’s study (Yu et al. 2019))
- Design area: The design area is a square, with one circle inside. This circle represents the area where the material cannot be placed. The radius of the circle can be specified as smaller than 25% of the length of the square.

The center of the circle is located within the square design area of $32 \times 32$ pixels. To determine whether our model could output an optimized structure with a complex shape, we selected a curved circle instead of a polygon (e.g. a rectangle) as the shape of the area where the material cannot be placed.

- Fixed boundary condition: Fix the left edge of the design area
- External forces’ distribution and load value: External forces are located in the design area. The maximum number of external load points is 4, and the load values are selected from five patterns, $-2, -1, 0, 1, 2$ in x and y directions, respectively. The load value is discrete (integer) rather than continuous.

4.2 Method to generate the training, evaluation, and test dataset

Based on the conditions described in the previous section, we generated training, evaluation, and test datasets using the SIMP method (Bendsøe and Sigmund 1999), which, as previously noted, is a conventional topology optimization method. We used MATLAB open-source topology optimization code (Andreassen et al. 2011), in which topology optimization is performed using FEM based on the SIMP method.

When performing FEM, the structure or design space to be analyzed is divided into elements. The conditions of the force applied to each element are supplied, and the analysis is performed. There are two types of forces in FEM: Element forces and nodal forces. Element force refers to the force applied to the entire element (Figure 10 (left)), whereas nodal force refers to the force applied to a node. Nodes represent the points that comprise an element, and four nodes form a square element. The force applied to this node is called the nodal force (Figure 10 (right)). As shown in figure 10, when defining the force on one element, a single value is used to represent the element force. Conversely, when defining the nodal force on one element, four values are needed. From the above, the number of values required for setting the external force condition varies depending on the type of force, element force, and nodal force. In this case, the design area is composed of a matrix of $32 \times 32$ pixel elements, and the nodes that comprise this design area are represented by a $33 \times 33$ matrix.

In this study, the design conditions input into the CNN included external force conditions and design areas. When inputting an external force condition into the CNN as a nodal force, the input matrix will be a $33 \times 33$ matrix, which differs from the $32 \times 32$ matrix showing the design area information, and indicating the element information. Since it is difficult to input matrices of different sizes to the CNN simultaneously, it is inconvenient for the matrices containing the design area information and external force conditions to...
be different sizes. Therefore, in our study, the external force condition input into the CNN represents the element force defined for each element. In Andreassen’s MATLAB code (2011), when specifying the nodal force, which is external force information, we split the body force into four nodal forces with a quarter of the body force’s value, as shown in figure 10 (right).

Under these rules, we varied the volume fraction, the shape of the design area, and the external force conditions to create a total of 370,000 datasets. The entire collection of datasets was split into training, validation, and test datasets with an 8:1:1 ratio, such that 296,000 datasets were used for learning, 37,000 for verification, and 37,000 for test.

4.3 Training and evaluating the effectiveness of proposed method

Our CNN model was trained under the following learning conditions.

- Batch size: 512
- Loss function: MAE between the material density distribution obtained using the SIMP method and our CNN model
- Learning algorithm: ADAM
- Learning rate: 0.01

Our CNN model was trained using the above conditions. Figure 11 shows the value of MAE loss for each epoch during training. We evaluated the speed of calculating the material density distribution between our CNN model trained as above, and the SIMP method (Andreassen et al. 2011) (Table 1). The time required to calculate the optimal material density distribution was 0.07634 seconds per case using our model, and 0.4393 seconds per case for the SIMP model, using the same CPU. In other words, our trained CNN model demonstrated an 83% reduction in calculation time versus the conventional SIMP method.

Next, to compare the effects of SPADE ResBlk and BN, we trained the proposed model without BN, and Yu’s CNN model, which is equivalent to the proposed model without BN and SPADE ResBlk.

4.3.1 Results for validation datasets

Validation error results (minimum value of MAE for the validation datasets while training) for each model are as follows.

- Yu’s CNN model: 0.112 (115 epochs)
- Proposed CNN model (Without BN): 0.059 (37 epochs)
- Proposed CNN model: 0.055 (51 epochs)

The number in parentheses indicates the epoch number when the validation error reached the minimum value. The proposed method with SPADE ResBlk achieved a smaller MAE loss value than Yu’s CNN model, thus confirming the effectiveness of SPADE ResBlk. Comparing our proposed CNN model with and without BN, the MAE loss value was smaller with BN. Results for the validation datasets therefore show that the CNN model incorporating both SPADE ResBlk and BN was more effective.

4.3.2 Results for test datasets

In the section above, we compared the models’ performance using the validation datasets. To evaluate performance with the test datasets, we used Yu’s CNN model, our proposed CNN model (without BN), and our proposed CNN model...
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Table 2 Comparison of accuracy between Yu’s CNN model and our proposed CNN model (with or without BN)

|                      | Average error for mean compliance (higher 80%) | Average error for mean compliance (lower 20%) | Average error for volume fraction | Rate of the number following design area (threshold: 0.01) |
|----------------------|-----------------------------------------------|-----------------------------------------------|----------------------------------|--------------------------------------------------------|
| Yu’s model           | 18.7%                                         | 51624956%                                     | 10.28%                           | 68.2%                                                  |
| Proposed model       | 3.80%                                         | 424987%                                       | 9.91%                            | 98.0%                                                  |
| (without BN)         |                                               |                                               |                                  |                                                        |
| Proposed model       | 2.75%                                         | 103872%                                       | 9.70%                            | 99.6%                                                  |

Fig. 12 Optimized structures obtained with the various methods and the specified design areas

Fig. 13 Red circles show examples of structures with disconnection

when the validation error reached the minimum value. Figure 12 shows the design area, the respective structures output by each CNN model, and calculated using conventional topology optimization (SIMP method), based on the design area in the test datasets. The following three indices were used for evaluation.

- Average error for mean compliance with optimal material density distribution calculated from the SIMP method
- Average error between the specified volume fraction and the volume of the output material density distribution
- Ratio of output material density distribution that follows the specified design area

Table 2 presents the results comparing the three models described above against the three evaluation indices. From the result, the CNN model equipped with SPADE ResBlk and BN had the best performance for all three indices, confirming the validity of our proposed model.

In the following section, we shall compare each index in detail. We calculated and compared the mean compliance error between the material density distributions output by each CNN model and calculated using the SIMP method, respectively. Mean compliance is the index relating to rigidity, with low mean compliance indicating high rigidity. Note that each CNN model occasionally outputs a discontinuous material density distribution, as shown in figure 13. In this case, the discontinuous material density distribution has very low rigidity compared to the optimal material density distribution obtained using the SIMP method, and the mean compliance becomes very large. Therefore, when calculating the average error of the mean compliance, we adopted a method to evaluate what percentage of the average error is in the higher ranked dataset among all the test dataset, assuming that the material density distributions output by CNN which has smaller error is the higher ranked data.

Figure 14 shows the average error for each percentage of higher rank datasets.

![Fig. 14 Average error of the mean compliance of the optimized structures for each percentage of higher rank datasets](image-url)
the material density distribution of the top 80%, as a representative value. For Yu’s CNN model, the average error for the top 80% of results was 18.7%, compared to 3.80% for the proposed model without BN, and 2.75% for the proposed model with BN, demonstrating that the average error for mean compliance is reduced by using SPADE ResBlk and BN. Table 2 also shows the average error for the lower 20% of the datasets. Again, the proposed model has the smallest error; about 1/516 of the error of Yu’s model. These results indicate that our proposed model can reduce error for the rarely output discontinuous material density distribution. The effectiveness of our proposed model in outputting material density distributions of higher rigidity than Yu’s model is thus demonstrated.

Next, we compared the models in terms of volume fraction. We used the average volume fraction error between the value provided to the CNN model and the volume of the material density distribution output by the CNN model. The average error of the volume error of the volume fraction in all test datasets was 10.28% for Yu’s CNN model, 9.91% for the proposed model without BN, and 9.70% for the proposed model. Our model kept the average error within 10%. Based on this result, the proposed model clearly outputs a structure that more accurately reflects the volume fraction in topology optimization.

Finally, we evaluated whether the material density distribution output by each CNN model reflected the shape of specified design area. This evaluation is the most important element of this study. Ideally, the material density should be 0 in the area where the material cannot be placed (circular area in figure 12 design area). However, since the CNN only has a sigmoid output function, the material density distribution actually output from the CNN model is unlikely to be completely zero. Therefore, we decided that the condition of the design area would be satisfied if the maximum value of the material density in the circular area was below the threshold value. For example, with a threshold value of 0.01, if the maximum value of the material density in the circular area is 0.005, it is lower than the threshold value, and thus the design area condition is satisfied. Figure 15 summarizes the percentage of datasets where the design area condition was satisfied for each threshold value in the test datasets. It is apparent that our model achieved a higher ratio of results representing the design area for each threshold value than the other models. In fact, Yu’s model output more instances of material density distributions that did not satisfy the design area, as per the example shown in figure 16. Using the data obtained for a threshold value of 0.01 as a representative result, the ratio of the material density distribution representing the design area in each model is 68.2% for Yu’s model, 98.0% for the proposed model without BN, and 99.6% for the proposed model.

However, it has been confirmed that our proposed model occasionally outputs structures with a discontinuous material density distribution like Yu’s CNN model, as shown in figure 13. In this case, the error of the mean compliance becomes very large. This suggests that using the MAE of the material density distribution as the loss function is not sufficient to train the CNN model. Even if the CNN model is trained to minimize the MAE of the material density distribution, it cannot minimize the error between the mean compliance of the structure output by the CNN model and
the mean compliance of the training datasets’ structure. To overcome this problem, we propose the following approach, which we plan to investigate in future work.

- Add the mean compliance error to the loss function (setting loss function as the weighted sum of the mean compliance error and the MAE of the material density distribution).
- Make the GAN model incorporating our CNN model output more realistic continuous material density distributions.

5 Conclusion

We have proposed a new topology optimization method using a CNN, which is a key technology for DL, currently attracting much attention in the field of image generation. Our CNN model can support specification of important design areas, volume fractions, and external force distribution (load position and load value of external force) as design conditions for topology optimization. No previous CNN model has achieved such topology optimization.

In a previous study, Yu et al. (Yu et al. 2019) presented a CNN model for topology optimization, capable of outputting the optimized structure (optimal material density distribution) in a single calculation, and also able to accept specified volume fractions, external force conditions, and positions of fixed point. Our model differs from Yu’s in its application of BN (Loffe and Szegedy 2015) and SPADE ResBik (Park et al. 2019).

Evaluating the material density distribution output by the CNN model for the specified conditions of the shape of the design area, volume fraction, and external force distribution, our model was shown to be highly effective, as follows.

- Compared to Yu’s model, our CNN model can output a material density distribution closer to that achieved by the conventional SIMP method (Bendsøe and Sigmund 1999).
- Among the material distributions output by our CNN model, at an 80% material density distribution, the average error of the mean compliance with the material density distribution obtained by the SIMP method is kept to 2.75%. Therefore, our model can output a material density distribution almost identical to that of SIMP.
- 99.6% of all material density distributions output by our CNN model reflect the shape of the specified design area. In other words, our CNN model does not place material outside the specified design area.
- Computational time needed to obtain the optimal material density distribution using our model was reduced by 83%, compared with the SIMP method.

In the future, we would like to improve the accuracy of the material density distributions output by our CNN model by implementing the improvements described at the end of section 4. Additionally, in this paper, verification was performed for 2D design areas of 32 x 32 pixels, but we aim to extend our CNN model to support higher resolutions or 3D design areas.

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