Stochastic Strictly Contractive Peaceman-Rachford Splitting Method

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Abstract

In this paper, we propose a couple of new Stochastic Strictly Contractive Peaceman-Rachford Splitting Method (SS-PRSM), called Stochastic SCPRSM (SS-PRSM) and Stochastic Conjugate Gradient SCPRSM (SCG-PRSM) for large-scale optimization problems. The two types of Stochastic PRSM algorithms respectively incorporate stochastic variance reduced gradient (SVRG) and conjugate gradient method. Stochastic PRSM methods and most stochastic ADMM algorithms can only achieve a $O\left(\frac{1}{\sqrt{t}}\right)$ convergence rate on general convex problems, while our SS-PRSM has a $O\left(\frac{1}{t}\right)$ convergence rate in general convexity case which matches the convergence rate of the batch ADMM and SCPRSM algorithms. Besides our methods has faster convergence rate and lower memory cost. SCG-PRSM is the first to improve the performance by incorporating conjugate gradient and using the Armijo line search method. Experiments shows that the proposed algorithms are faster than stochastic and batch ADMM algorithms. The numerical experiments show SCG-PRSM achieve the state-of-the-art performance on our benchmark datasets.

1 Introduction

We consider the minimization problem with separable objective functions and a linear constraint which can be formulated as follows:

$$\min_{x \in \mathcal{X}, y \in \mathcal{Y}} \theta_1(x) + \theta_2(y) \quad \text{s.t.} \quad Ax + By = b,$$

where $\theta_1$ and $\theta_2$ are convex functions, $\mathcal{X}$, $\mathcal{Y}$ are nonempty convex sets, and $A$, $B$ are constant matrices, and $x$, $y$ are variables to be learned. The solution set of the problem is assumed to be nonempty.

To solve this convex minimization problem, the Alternating Direction Method of Multipliers (ADMM) [3, 5, 6] was proposed, which the rates of convergence is $O(1/t)$, where $t$ is the number of iterations. It is the foundation method of a group of splitting algorithms. Adopting the "batch" setting, the algorithm needs to visit all the samples in each iteration. With the increase of data set, it can quickly become computationally expensive. To alleviate this problem, the stochastic and online techniques, which visit only one sample or a mini-batch of samples each time, have recently become a popular research topic.

First, Wang and Banerjee [13] proposed the online ADMM. After that, Ouyang et al. [11] and Suzuki [12] gave three variants (STOC-ADMM, OPG-ADMM, RDA-ADMM) respectively. However,
with the decrease of their low per-iteration complexities, these stochastic algorithms all achieve a convergence rate of $O(1/\sqrt{t})$ for general convex problems. As it says to bridge the $O(1/\sqrt{t})$ vs $O(1/t)$ gap in the convergence rates for ADMM, Zhong and Kwok [15] proposed the SA-ADMM design for large-scale optimization problems with the same convergence rate of SA-ADMM, and does not need an extra memory to store the historic gradients on all samples decrease the storage cost.

The ADMM is just a special case of the Douglas-Rachford splitting method (DRSM). Meanwhile the Peaceman-Rachford Splitting Method (PRSM) which is similar to the DRSM has also been proposed to solve the splitting problem. Later, a modified version Strictly Contractive PRSM (SCPRSM) was developed by He et al. [8] They reduced the assumptions that PRSM required to ensure the convergence, and make SCPRSM converge under the same assumption with ADMM. As analyzed by He et al. [8] and Gu et al. [7], the group of PRSM algorithms often converge faster than ADMM. Recently a stochastic version of PRSM called Semi-Proximal-Based Strictly Contractive Peaceman-Rachford Splitting Method (Stochastic SPB-SPRSM) was designed by Na and Cho-Jui [10], but it only achieve the convergence rate of $O(1/\sqrt{t})$. From the convergence analysis of the above papers, it can be found that the proof of convergence rate of PRSM is much more difficult than ADMM.

Unlike the ADMM, there are few people doing the PRSM which has the better performance and there are little variants of the method. In this paper, we propose a couple of new variants of PRSM, called Scalable Stochastic Strictly Contractive Peaceman-Rachford Splitting Method (SS-PRSM) and Stochastic Conjugate Gradient Peaceman-Rachford Splitting Method (SCG-PRSM) for large-scale optimization problems. We get inspiration from the SCAS-ADMM to make the SS-PRSM algorithms. The main features of our algorithms are outlined as follows:

1. We combined the batch SPB-SCPRSM with the stochastic setting using the method of the stochastic variance reduced gradient (SVRG) to get the SS-PRSM. The computations of our algorithm for every update step are efficiently carried out.

2. SCG-PRSM is inspired by the conjugate gradient method. The Conjugate Gradient method only needs to use the first derivative information, and overcomes the shortcomings of the Gradient Descent method which may have slow convergence. Meanwhile the required storage capacity is small, and it has the advantage of high stability.

3. We show that the convergence rate of the SS-PRSM algorithm is $O(1/t)$ as same as the batch ADMM, SCPRSM and SA-ADMM in the general convexity case. And we show that the SCG-PRSM converge faster than the previous algorithms in several numerical experiments.

2 Background

In this section, we introduce several related algorithms: ADMM, SPRSM, SPB-SPRSM and Scalable Stochastic ADMM. And then we gave some notations.

2.1 Related Algorithms

First, we discuss the Alternating Direction Method of Multipliers. ADMM starts with the augmented Lagrangian of problem (1):

$$
L_\beta(x, y, \lambda) = \theta_1(x) + \theta_2(y) - \langle \lambda, Ax + By - b \rangle + \frac{\beta}{2}\|Ax + By - b\|^2
$$

where $\lambda \in \mathbb{R}^m$ is the vector of Lagrangian multipliers and $\beta > 0$ is a penalty parameter. The ADMM algorithm minimizes $L_\beta$ sequentially updating $x, y, \lambda$ (denoted $x_k, y_k, \lambda_k$) at each iteration, as shown below:

$$
\begin{align*}
  x_{k+1} &\leftarrow \arg \min_{x \in \chi} L_\beta(x, y_k; \lambda_k) \\
y_{k+1} &\leftarrow \arg \min_{y \in \gamma} L_\beta(x_{k+1}, y; \lambda_k) \\
\lambda_{k+1} &\leftarrow \lambda_k - \beta(Ax_{k+1} + By_{k+1} - b)
\end{align*}
$$

Peaceman-Rachford Splitting Method (PRSM) is another way to minimize the Lagrangian. As pointed out in Gabay & Mercer (1976), the only difference between PRSM and ADMM is the
addition of the intermediate step $\lambda_{k+1/2}$, which leads to a faster contraction speed in experiment whenever it is convergent. Unfortunately, PRSM becomes less robust—it converges under more restrictive assumptions than ADMM. To make PRSM convergent in more general cases, Strictly Contractive PRSM was proposed by He. The key idea is to employ an underdetermined relaxation factor $\alpha \in (0, 1)$ when updating the Lagrange multipliers $\lambda$. Recently, a new variant of PRSM called Semi-Proximal-Based SPRSM was published [7] and it could cover the SPRSM case. All work can make SPRSM easy to apply:

\[
\begin{align*}
\lambda_{k+1/2} &\leftarrow \lambda_k - \alpha \beta (Ax_{k+1} + By_k - b) \\
y_{k+1} &\leftarrow \arg \min_{y \in \mathcal{Y}} L_\beta (x_{k+1}, y; \lambda_{k+1/2}) + \frac{1}{\gamma} \|y - y_k\|_2^2 \\
\lambda_{k+1} &\leftarrow \lambda_{k+1/2} - \gamma \beta (Ax_{k+1} + By_{k+1} - b)
\end{align*}
\]

(3)

When the data set is large, solving (2.1) can be computationally expensive. To alleviate this problem, a family of stochastic algorithms were focused on. In the context of regularized risk minimization problem, the function $\theta_1(x)$ usually has the following structure:

\[
\theta_1(x) = \frac{1}{n} \sum_{i=1}^{n} f_i(x)
\]

(4)

where $x$ denotes the model parameter, $n$ is the number of training samples, and each $f_i(x)$ is the empirical loss caused by the $i$th sample. The function $\theta_2(y)$ is usually a regularization term.

In recent years, researchers have proposed several Stochastic algorithms, and proved their convergence rate. For Stochastic ADMM:

\[
\begin{align*}
x_{k+1} &\leftarrow \arg \min_{x \in X} \hat{L}_{\beta,k}(x, y_k; \lambda_k) \\
y_{k+1} &\leftarrow \arg \min_{y \in \mathcal{Y}} \hat{L}_{\beta,k}(x_{k+1}, y; \lambda_k) \\
\lambda_{k+1} &\leftarrow \lambda_k - \beta (Ax_{k+1} + By_{k+1} - b)
\end{align*}
\]

(5)

where

\[
\hat{L}_{\beta,k}(x, y; \lambda) = \theta_1(x_k) + \langle \theta_1'(x_k, \xi_{k+1}), x \rangle + \theta_2(y) - \langle \lambda, Ax + By - b \rangle + \frac{\beta}{2} \|Ax + By - b\|_2^2 + \frac{1}{2\eta_k} \|x - x_k\|_2^2.
\]

### 2.2 Notations

All notations are consistent throughout the paper and supplement material. We denote the objective function as $\theta(u) = \theta_1(x) + \theta_2(y)$; the constraint function as $r(w) = Ax + By - b$. For simplicity, we define the following vectors that will be used in this paper:

\[
u = \begin{pmatrix} x \\ y \\ \lambda \end{pmatrix}, w = \begin{pmatrix} y \\ \lambda \end{pmatrix}, u_t = \begin{pmatrix} \frac{1}{t} \sum_{k=1}^{t} x_k \\ \frac{1}{t} \sum_{k=1}^{t} y_k \\ \frac{1}{t} \sum_{k=1}^{t} \lambda_k \end{pmatrix}, w_t = \begin{pmatrix} \frac{1}{t} \sum_{k=1}^{t} x_{k+1} \\ \frac{1}{t} \sum_{k=1}^{t} y_{k+1} \\ \frac{1}{t} \sum_{k=1}^{t} \lambda_{k+1} \end{pmatrix},
\]

\[
F(w) = \begin{pmatrix} -A^T \lambda \\ -B^T \lambda \\ Ax + By - b \end{pmatrix}, P = \begin{pmatrix} S & 0 & T \\ 0 & I_n & \alpha \beta B (\alpha + \gamma) \beta I_m \end{pmatrix}, M = \begin{pmatrix} I_{n_2} & 0 \\ \alpha \beta B & (\alpha + \gamma) \beta I_m \end{pmatrix},
\]

\[
G = \begin{pmatrix} P & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & H \end{pmatrix} = \begin{pmatrix} S & 0 & T + \frac{\alpha + \gamma - \alpha \beta}{\alpha + \gamma} \beta B^T B - \frac{\alpha}{\alpha + \gamma} B^T \\ 0 & 0 & -\frac{\alpha}{\alpha + \gamma} B \\ 0 & 0 & \frac{1}{\alpha + \gamma} \beta I_m \end{pmatrix},
\]

\[
K = \begin{pmatrix} (1 - \alpha) \beta B^T B & (1 - \alpha) \beta B \\ (1 - \alpha) \beta B & 2 - \alpha - \gamma \beta I_m \end{pmatrix}, H = \frac{1}{\alpha + \gamma} \begin{pmatrix} (\alpha + \gamma - \alpha \gamma) \beta B^T B & -\alpha B \\ -\alpha B & -\alpha \beta I_m \end{pmatrix},
\]

For positive semidefinite matrix $G$, we also define $G$-norm of $x$ as $||x||_G = \sqrt{x^T G x}$.  


3 Scalable Stochastic PRSM

In this section, we present the details of our algorithms, which is scalable in terms of both convergence rate and storage cost. In the general convex problems, \( \theta_1(\cdot) \) is \( L \)-smooth and general convex but not necessarily to be strongly convex.

3.1 Stochastic Strictly Contractive Peaceman-Rachford Splitting Method (SS-PRSM)

For the updating of \( x \), we have the following function:

\[
x_{k+1} \leftarrow \arg\min_{x \in \chi} L_\beta(x, y_k; \lambda_k) + \frac{1}{2} \|x - x_k\|_S^2
\]

We focus on proposing different solution for updating \( x \). Our algorithm is inspired by the stochastic variance reduced gradient (SVRG) as same as Zhao et al. [14].

In machine learning, we often encounter the following optimization problem.

\[
\theta_1(x) = \frac{1}{n} \sum_{i=1}^{n} f_i(x)
\]

The basic approach is gradient descent. However, at each step, gradient descent requires evaluation of \( n \) derivatives. It is expensive. A popular modification is stochastic gradient descent (SGD): where at each iteration \( k \), we draw \( i_t \) randomly from \( \{1, \ldots, n\} \):

\[
x_{k+1} = x_k - \eta_k \nabla f_{i_t}(x_k)
\]

For SGD, due to the variance of random sampling, the convergence is slow. Johnson and Zhang [9] use the average gradient to reduce the variance:

\[
\tilde{\mu} = \frac{1}{n} \sum_{i=1}^{n} \nabla f_i(\bar{x})
\]

\[
x_{k+1} = x_k - \eta_k (\nabla f_{i_t}(x_k) - \nabla f_{i_t}(\bar{x}) + \tilde{\mu})
\]

Now, we design a new strategy to update \( x \) in the ADMM framwork. We make \( \theta_1(x) \) to be \( g(x) \):

\[
g(x) = L_\beta(x, y_k; \lambda_k) + \frac{1}{2} \|x - x_k\|_S^2
\]

and then the updating \( x \) can be exchange as:

\[
x_{k+1} = x_k - \eta_k (\nabla g_{i_t}(x_k) - \nabla g_{i_t}(\bar{x}) + \tilde{\mu})
\]

3.2 Stochastic Conjugate Gradient Peaceman-Rachford Splitting Method (SCG-PRSM)

In the numerical method, the conjugate gradient has a significant acceleration effect relative to the steepest descent method, and it can usually reach the optimal solution in the finite step. Here we introduce the idea of random conjugate gradient, and optimize the iterative process of \( x \). In order to avoid the problem that the search direction is no longer conjugate for large-scale problems, we use the recursive FR conjugate gradient based on Wolfe line search method [4].

Now, we design a new strategy to update \( x \).

\[
x_{k+1} = x_k + \alpha_k d_k,
\]

\[
d_{k+1} = -g_k + \beta_k d_k.
\]

where

\[
g_k = \nabla f_{i_t}(x_k) + A^T \beta_k + \rho A^T (A_k + B y_k - b) + S(x_k - x_{k-1}), \beta_k = \frac{g_{k+1}^T g_k}{g_k^T g_k}
\]

Wolfe criterion is given \( \rho \in (0, 0.5), \sigma \in (\rho, 1) \), let \( \alpha_k \) make the following inequalities holds.

\[
f(x_k + \alpha_k d_k) \leq f(x_k) + \rho \alpha_k g_k^T d_k,
\]

\[
\nabla f(x_k + \alpha_k d_k)^T d_k \geq \sigma g_k^T d_k.
\]
Algorithm 1 SS-PRSM

Initialize \((x_0, y_0, \lambda_0)\)

for \(k = 0, 1, 2, \ldots\) do
  Compute \(z_k = \frac{1}{n} \sum_{i=1}^{n} \nabla f_i(x_k)\);
  \(h_0 = bx_k\);
  \(a = h_0\);
  for \(m = 0, 1, 2, \ldots, M_k\) do
    Randomly select an \(i_m\) from \(\{1, 2, \ldots, n\}\);
    \(h_{m+1} = h_m - \eta_k (\nabla f_{i_m}(h_m) - \nabla f_{i_m}(h_0)) + z_k + A^T \beta_k + \rho A^T (Ah_m + By_k - b) + S(h_m - h_0)\);
    \(a = a + h_{m+1}\);
  end for
  \(x_{k+1} = \frac{a}{M_k}\);
  \(\lambda_{k+1} = \lambda_k - \alpha \beta(Ax_{k+1} + By_{k+1} - b)\);
  \(y_{k+1} = \arg\min_{y \in Y} L_{\beta}(x_{k+1}, y, \lambda_k)\);
  \(\lambda_{k+1} = \lambda_{k+1} - \gamma \beta(Ax_{k+1} + By_{k+1} - b)\).
end for

Algorithm 2 SCG-PRSM

Initialize \((x_0, y_0, \lambda_0)\)

for \(k = 0, 1, 2, \ldots, N\) do
  \(z_k = \frac{1}{n} \sum_{i=1}^{n} \nabla f_i(x_k)\);
  \(h_0 = bx_k\);
  \(a = h_0\);
  for \(m = 0, 1, 2, \ldots\) do
    Randomly select an \(i_m\) from \(\{1, 2, \ldots, n\}\);
    if \(\|g_{k,m}\| \leq \varepsilon\) then
      break;
    end if
    Compute \(\alpha_m\) using Armijo Method;
    \(h_{m+1} = h_m + \alpha_m d_m\);
    \(d_{m+1} = -g_{k,m} + \beta_m d_m\);
    \(a = a + h_{m+1}\);
  end for
  \(x_{k+1} = \frac{a}{m}\);
  \(\lambda_{k+1} = \lambda_k - \alpha \beta(Ax_{k+1} + By_{k+1} - b)\);
  \(y_{k+1} = \arg\min_{y \in Y} L_{\beta}(x_{k+1}, y, \lambda_k)\);
  \(\lambda_{k+1} = \lambda_{k+1} - \gamma \beta(Ax_{k+1} + By_{k+1} - b)\).
end for

4 Convergence Analysis

In this section, we will show that our SS-PRSM given in Algorithm 1 has a \(O(1/t)\) of convergence in terms of both the objective value and the feasibility violation, under the same assumptions of Scalable Stochastic ADMM:

\[
E \left[ \theta(\bar{u}_t) - \theta(\bar{u}^*) + \rho \|Ax_t + By_t - b\|^2 \right] = O\left(\frac{1}{t}\right),
\]

where \(\bar{u}^*\) and \(\bar{u}^*\) are the optimal solution.

All proofs in this section are provided in the supplement material, and we just list the important lemmas and a sketch of the proof here.

Assumptions. To prove the convergence results, we need the following assumptions:

1. \(\theta_1\) and \(\theta_2\) are convex functions.
Lemma 1. In Stochastic SS-PRSM (Algorithm 7), let the sequence $h_m$ be generate by iteration scheme. If we define

$$g_{m,k} = \nabla f_m(h_m) - A^T \lambda_k + \beta A^T (Ah_m + By_k - b) + S(h_m - h_0) + z_k + \nabla f_m(h_0)$$

$$= \nabla L_{i_m}(h_m) - \nabla L_{i_m}(h_0) + \nabla L(h_0) + S(h_m - h_0)$$

Then we have:

$$E(||g_{m,k}||^2) \leq 2C_L^2 + 4G_k^2 + 4C_S^2D^2.$$  

where $D$ is the bound of the domain of $x$, $C_L$ is the Lipschitz constant of the function $L(x)$ defined in ($\lambda$, and $G_k = ||\nabla L(x_k)||$, and $C_S = ||S||$. Let $N_k = C_L^2 + 2G_k^2 + 2C_S^2D^2$.

Lemma 2. Let the sequence $x_k$ be generate by iteration scheme. We have

$$\theta_1(x_{k+1}) - \theta_1(x) + \langle -A^T \lambda_{k+1}, x_{k+1} - x \rangle \leq \langle S(x_{k+1} - x_k), x - x_{k+1} \rangle + \eta_k N_k$$

$$+ \langle (1 - \alpha - \gamma) \beta A^T r_{k+1} + (1 - \alpha) \beta A^T B(y_k - y_{k+1}), x - x_{k+1} \rangle + \frac{D^2}{2\eta_k M_k}.$$  

Lemma 3. Let the sequence $y_k$ be generate by iteration scheme. We have

$$\theta_2(y_{k+1}) - \theta_2(y) + \langle -B^T \lambda_{k+1}, y_{k+1} - y \rangle \leq \langle (1 - \gamma) \beta B^T r_{k+1} + T(y_{k+1} - y_k), y - y_{k+1} \rangle.$$  

Lemma 4. Let the sequence $x_k, y_k, \omega_k$ be generate by iteration scheme. Then, we have

$$\theta_1(x_{k+1}) - \theta_1(x) + \theta_2(y_{k+1}) - \theta_2(y) + \langle \omega_{k+1} - \omega, F(\omega_{k+1}) \rangle \leq \langle (1 - \omega_{k+1}) \beta(\omega_{k+1} - \omega_k)$$

$$- (1 - \alpha - \gamma) \beta ||r_{k+1}||^2 + (1 - \alpha) \beta \langle r_{k+1}, B(y_k - y_{k+1}) \rangle + \frac{D^2}{2\eta_k M_k} + \eta_k N_k.$$  

Based on all the above lemmas, we can derive the convergence rate of the averaged iterates $\bar{u}$ by the following theorem:
We use the following algorithms as a comparison to solve the optimization problem (1):

1. BatchADMM: The original ADMM algorithm [3].
2. STOC-ADMM: The stochastic ADMM variant which has a convergence rate of $O\left(\frac{1}{\sqrt{T}}\right)$ for general convex problem [11].
3. SCAS-ADMM: The stochastic ADMM variant which has a convergence rate of $O\left(\frac{1}{T}\right)$ [14].
4. BatchSCPRSM: The original SCPRSM algorithm [7].

Experiments are performed on two simulated data sets (Table 1).

As talked about in SPRSM [3], the underdetermined relaxation factor $\alpha$ and $\gamma$ are not easily determined. Here, for simplicity, we take $\alpha = 0.4$, $\gamma = 1$ in all simulations and examples. Also, all the

| Data Set | #Samples | #Features | $\mu$ |
|----------|----------|----------|-------|
| NSL-KDD  | 125973   | 115      | 1e-2  |

**Theorem 1.** If $\theta_1, \theta_2$ are convex, then under Assumptions 1 and 2, the averaged iterates $\bar{w}_t = [\bar{x}_T^T, y_T^T, \bar{\lambda}_T^T]^T$ generated by Algorithm 1 satisfy \( \forall t \geq 1 \)

\[
E\left[ \theta(\bar{u}_t) - \theta(\bar{u}^*) + \rho \| Ax + B y - b \|^2 \right] = \frac{1}{t} \sum_{k=1}^{t} \left( \frac{D^2}{2\eta_k M_k} + \eta_k N_k \right) \\
+ \frac{1}{2t} \left( \| w_1 - w^* \|^2_G + \eta_1 \| y_1 - y_0 \|^2 + \rho_{\alpha, \gamma} \| r_1 \|^2 \right)
\]

To make our algorithm have a convergence rate, the $\frac{1}{t} \sum_{k=1}^{t} \left( \frac{D^2}{2\eta_k M_k} + \eta_k N_k \right)$ is bounded. We take the $\eta_k = \frac{1}{N_k(t + 1)^2}, M_k = N_k(t + 1)^2$. When $\delta > 1$, the $\left( \frac{D^2}{2\eta_k M_k} + \eta_k N_k \right)$ converges to a constant. So we have proved Equation 10.

**5 Experiments**

In this section, we will use the generalized lasso model to evaluate our algorithms. First, we conduct experiments on simulated datasets. We randomly generate three types of data for numerical experiments (the data generation algorithm reference [7]). Next, real datasets are adopted to evaluate our methods and other algorithms. They are a9a, abalone and NSL-KDD. Among them, NSL-KDD is a network intrusion detection data set. In the implementation, we always set $A = I_m, B = -I_m, b = 0, T = 0, S = I_m$, and then get the following format:

$$
\min_{x, y} \frac{1}{n} \sum_{i=1}^{n} f_i(x) + \mu \| y \|_1
\tag{11}
$$

$$
\min_x \left\{ \frac{1}{n} \sum_{i=1}^{n} \log(1 + \exp(-r_i(d_i^T x))) + \mu \| y \|_1 \right\}.
$$

We use the following algorithms as a comparison to solve the optimization problem (1):

1. BatchADMM: The original ADMM algorithm [3].
2. STOC-ADMM: The stochastic ADMM variant which has a convergence rate of $O\left(\frac{1}{\sqrt{T}}\right)$ for general convex problem [11].
3. SCAS-ADMM: The stochastic ADMM variant which has a convergence rate of $O\left(\frac{1}{T}\right)$ [14].
4. BatchSCPRSM: The original SCPRSM algorithm [7].

Experiments are performed on two simulated data sets (Table 1).
three algorithms need $\beta$ and we follow [8, 11] to set $\beta = 1$. We set $S = I_m$ and $T = 0$, which means we can add the proximal term in subproblem of $x$.

5.1 Numerical Simulated Experiments

Figure 1 shows the objective value obtained by the various algorithms versus iterations. Overall, SS-PRSM, SS-PRSM-BB (Barzilai-Borwein Step Size) and SCG-PRSM are the fastest and rapidly lead to a model with good generalization performance.

5.2 Real Data Experiments

Following the previous work [11], four widely used datasets are adopted to evaluate our methods. The hyper-parameter in (14) is set by using the same values in Ouyang et al. [11] (see Table 1).

Figure 4 shows the objective value obtained by the various algorithms versus iterations. Overall, SS-PRSM is the fastest and rapidly lead to a model with good generalization performance.

5.3 NSL-KDD Data Experiments

In recent years, machine learning methods have been widely used in abnormal flow detection, but with the computer network hardware and software updates, the abnormal flow of data increased dramatically, the original machine learning methods have a great impact. The ADMM framework has a good parallel performance to effectively cope with large data crises. For example, Seyed et al. [2] used the ADMM framework to achieve excellent results. Here we evaluate our algorithm on the NSL-KDD dataset (Table 2), illustrating its validity in the field of information security.

Figure 5.3 shows the accuracy obtained by the various algorithms versus iterations. Overall, SS-PRSM-BB can get a best accuracy.

6 Conclusion and future work

In this paper, we propose three new stochastic PRSM called SS-PRSM, SCG-PRSM and SS-PRSM-BB. We have proofed that the SS-PRSM can achieve the same convergence rate as the current best stochastic ADMM method on general convex problems. Furthermore, our three proposed stochastic methods have no extra memory cost compared with SCAS-ADMM and S-ADMM.

Empirical results on the general convex problems demonstrate our methods have better performance over batch and stochastic ADMM algorithms. In the future, we will apply the proposed algorithms to information security and compression sensing areas.

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