The simple interpretations of lepton anomalies in the lepton-specific inert two-Higgs-doublet model

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(Dated: December 12, 2018)

There exist about 3.7σ positive and 2.4σ negative deviations in the muon and electron anomalous magnetic moments \((g - 2)\). Also, some ratios for lepton universality in \(\tau\) decays have almost 2σ deviations from the Standard Model. In this paper, we propose a lepton-specific inert two-Higgs-doublet model. After imposing all the relevant theoretical and experimental constraints, we show that these lepton anomalies can be explained simultaneously in many parameter spaces with \(m_H > 200\) GeV and \(m_A (m_{H^±}) > 500\) GeV for appropriate Yukawa couplings between leptons and inert Higgs. The key point is that these Yukawa couplings for \(\mu\) and \(\tau/e\) have opposite sign.

PACS numbers:

Introduction — The Standard Model (SM) describes the elementary particles, as well as the fundamental interactions between them. In particular, such description is sensitive to the quantum corrections. For example, since Schwinger’s seminar calculation of the electron anomalous magnetic moment \(a_e = \alpha/2\pi\) [1], the charged lepton anomalous magnetic moments have become the powerful precision tests of Quantum Electrodynamics (QED), and subsequently the full SM. The muon anomalous magnetic moment \(g - 2\) has been a long-standing puzzle since the announcement by the E821 experiment in 2001 [2]. The experimental value has an approximate 3.7σ discrepancy from the SM prediction [2]

\[
\Delta a_{\mu} = a_{\mu}^{\exp} - a_{\mu}^{SM} = (274 \pm 73) \times 10^{-11}.
\]

Very recently, an improvement in the measured mass of atomic Csium used in conjunction with other known mass ratios and the Rydberg constant leads to the most precise value of the fine structure constant [3]. As a result, the experimental value of the electron \(g - 2\) has a 2.4σ deviation from the SM prediction [3]

\[
\Delta a_e = a_e^{\exp} - a_e^{SM} = (-87 \pm 36) \times 10^{-14},
\]

which has opposite sign from the muon \(g - 2\).

The Lepton Flavor Universality (LFU) in the \(\tau\) decays is an excellent way to probe new physics. The HFAG Collaboration reported three ratios from pure leptonic processes, and two ratios from semi-hadronic processes, \(\tau \to \pi/K\nu\) and \(\pi/K \to \mu\nu [4]

\[
\left(\frac{g_\tau}{g_\mu}\right) = 1.0011 \pm 0.0015, \quad \left(\frac{g_\tau}{g_e}\right) = 1.0029 \pm 0.0015,
\]

\[
\left(\frac{g_\pi}{g_e}\right) = 1.0018 \pm 0.0014, \quad \left(\frac{g_\pi}{g_\mu}\right) = 0.9963 \pm 0.0027,
\]

\[
\left(\frac{g_\tau}{g_\mu}\right)_K = 0.9858 \pm 0.0071,
\]

where the ratios of \(g_\tau/g_e\) and \(\left(\frac{g_\tau}{g_\mu}\right)_K\) have almost 2σ deviations from the SM.

Muon \(g - 2\) anomaly can be simply explained in the lepton-specific two-Higgs-doublet model (2HDM) and aligned 2HDM. However, the tree-level diagram mediated by the charged Higgs gives negative contribution to the decay \(\tau \to \mu \nu\) [4,11], which will raise the discrepancy in the LFU in \(\tau\) decays. In addition, these two types of 2HDM do not explain the muon and electron \(g - 2\) simultaneously since there is an opposite sign between them. Therefore, we shall propose a lepton-specific inert 2HDM to explain all three anomalies of muon and electron \(g - 2\) as well as LFU in \(\tau\) decay simultaneously. Although the muon and electron \(g - 2\) have been addressed simultaneously in a few recent papers [12–16], it seems to us that our model is simpler from the renormalized theory point of view.

Lepton-specific inert 2HDM — We introduce an inert Higgs doublet \(\Phi_2\) in the SM as well as a discrete \(Z_2\) symmetry under which \(\Phi_2\) is odd while all the SM particles are even. The scalar potential for the SM Higgs field \(\Phi_1\) and inert doublet \(\Phi_2\) is

\[
V = Y_1(\Phi_1^\dagger \Phi_1) + Y_2(\Phi_2^\dagger \Phi_2) + \frac{\lambda_1}{2}(\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{\lambda_5}{2}(\Phi_2^\dagger \Phi_2)^2 + h.c.,
\]

(4)

We focus on the CP-conserving case where all \(\lambda_i\) are real. The two complex scalar doublets can be written as

\[
\Phi_1 = \left(\begin{array}{c}
\frac{1}{\sqrt{2}}(v + h + iG_0) \\
\frac{1}{\sqrt{2}}(v + h + iA)
\end{array}\right), \quad \Phi_2 = \left(\begin{array}{c}
\frac{1}{\sqrt{2}}(H^+) \\
\frac{1}{\sqrt{2}}(H + iA)
\end{array}\right).
\]

(5)

The \(\Phi_1\) field has the vacuum expectation value (VEV) \(v = 246\) GeV, and the VEV of \(\Phi_2\) field is zero. \(Y_1\) is
fixed by the scalar potential minimization condition. The $H^+$ and $A$ are the mass eigenstates of the charged Higgs boson and CP-odd Higgs boson. Their masses are given as

$$m_{H^\pm}^2 = y_L^2 + \frac{\lambda_3}{2} v^2, \quad m_A^2 = m_{H^\pm}^2 + \frac{1}{2} (\lambda_4 - \lambda_5) v^2.$$  \hspace{1cm} (6)

The $h$ and $H$ have no mixing, and they are two mass eigenstates of the CP-even Higgses. In this paper, we take the light CP-even Higgs $h$ as the SM-like Higgs. Their masses are given as

$$m_h^2 = \lambda_1 v^2 \equiv (125 \text{ GeV})^2, \quad m_H^2 = m_A^2 + \lambda_3 v^2.$$  \hspace{1cm} (7)

The fermions obtain the mass terms from the Yukawa interactions with $\Phi_1$

$$-\mathcal{L} = y_u \overline{Q}L \Phi_1 u_R + y_t \overline{Q}L \Phi_1 d_R + y_L \overline{L}L \Phi_1 e_R + \text{h.c.},$$  \hspace{1cm} (8)

where $Q_L = (u_L, d_L)$, $L_L = (\nu_L, e_L)$, $\Phi_1 = i \tau_2 \Phi_1^\dagger$, and $y_u$, $y_d$ and $y_e$ are $3 \times 3$ matrices in family space. In addition, only in the lepton sector we introduce the $Z_2$ symmetry-breaking Yukawa interactions of $\Phi_2$,

$$-\mathcal{L} = \sqrt{2} \kappa_e \overline{L}L \Phi_2 e_R + \sqrt{2} \kappa_\mu \overline{L}L \Phi_2 \mu_R + \sqrt{2} \kappa_\tau \overline{L}L \Phi_2 \tau_R + \text{h.c.}. \hspace{1cm} (9)$$

Such the $Z_2$ symmetry-breaking effect only for the lepton sector can be realized in the high-dimensional brane world scenario, which will be studied elsewhere. From Eq. \ref{eq:6}, we can obtain the lepton Yukawa couplings of extra Higgses ($H$, $A$, and $H^\pm$). The neutral Higgses $A$ and $H$ have no couplings to $ZZ$, $WW$.

**Numerical results** – According to Eqs. \ref{eq:6} and \ref{eq:7}, the values of $\lambda_1$, $\lambda_3$ and $\lambda_4$ can be determined by $m_h$ ($=125$ GeV), $m_\mu$, $m_A$ and $m_{H^\pm}$. $\lambda_1$ controls the quartic couplings of extra Higgses, but does not affect the physics observables. So we simply take $\lambda_2 = \lambda_1$. Because the precision electroweak data favor small mass splitting between $m_A$ and $m_{H^\pm}$, we simply choose $m_A = m_{H^\pm}$. We employ the 2HDMC \ref{2HDMC} to implement the theoretical constraints from vacuum stability, unitarity and perturbativity, as well as the constraints of the oblique parameters ($S$, $T$, $U$). We scan over several key parameters in the following ranges

$$0.5 < \kappa_e < 1, \quad -0.25 < \kappa_\mu < 0, \quad 0 < \kappa_\tau < 0.01,$$

$$200 \text{ GeV} < m_H < 350 \text{ GeV},$$

$$500 \text{ GeV} < m_A = m_{H^\pm} < 700 \text{ GeV}. \hspace{1cm} (10)$$

In such ranges of $\kappa_e$, $\kappa_\mu$ and $\kappa_\tau$, the corresponding Yukawa couplings do not become non-perturbative. At the tree-level, the SM-like Higgs has the same couplings to the SM particles as the SM, and no exotic decay mode. The masses of extra Higgses are beyond the exclusion range of the searches for the neutral and charged Higgs at the LEP. Since the extra Higgses have no couplings to quarks due to $Z_2$ symmetry, we can safely neglect the limits from the observables of meson. The extra Higgs bosons are dominantly produced at the LHC via electroweak processes. We generate the Monte Carlo events using MG5\textunderscore{}aMC@NLO 2.4.3 \ref{MG5} with PYTHIA6 \ref{PYTHIA}, and adopt the constraints from all the analysis for the 13 TeV LHC in CheckMATE 2.0.7 \ref{CheckMATE}. The latest multi-lepton searches for electroweakino \ref{EWK} are further applied because of the dominated multi-lepton final states in our model.

In the model, the extra one-loop contributions to muon $g - 2$ is given as

$$\Delta a_\mu(1\text{loop}) = \frac{1}{2 \pi^2} \sum_i \kappa_\mu^2 r_i^2 F_i(r_i), \hspace{1cm} (11)$$

where $i = H$, $A$, $H^\pm$, $r_i = m_h^2/M_i^2$. For $r_i \ll 1$ we have

$$F_H(r) \approx -\ln r - 7/6, \quad F_A(r) \approx \ln r + 11/6, \quad F_{H^\pm}(r) \approx -1/6. \hspace{1cm} (12)$$

The contributions of the two-loop diagrams with a closed fermion loop are given by

$$\Delta a_\mu(2\text{loop}) = \frac{m_\mu}{8 \pi^2} \sum_{i, \ell} \kappa_\mu^2 \kappa_\ell^2 \frac{r_i r_\ell}{m_\ell} G_i(r_\ell), \hspace{1cm} (13)$$

where $i = H$, $A$, $\ell = \tau$, and $m_\ell$ and $Q_\ell$ are the mass and electric charge of the lepton $\ell$ in the loop. The functions $G_i(r)$ are given in Refs. \ref{G1}, \ref{G2}.

$$G_H(r) = \int_0^1 \frac{dx}{x(1-x) - r} \ln \frac{x(1-x)}{r}, \hspace{1cm} (14)$$

$$G_A(r) = \int_0^1 \frac{1}{x(1-x) - r} \ln \frac{x(1-x)}{r}. \hspace{1cm} (15)$$

We also consider the contributions of the two-loop diagrams with a closed charged Higgs loop, and find that their contributions are much smaller than the fermion loop. The calculations of $\Delta a_\mu$ are similar to $\Delta a_\tau$, but for the contributions of the two-loop diagrams, we include both $\mu$ loop and $\tau$ loop.

The HFAG Collaboration reported three ratios from pure leptonic processes, and two ratios from semi-hadronic processes, $\tau \rightarrow \pi/K\nu$ and $\pi/K \rightarrow \mu\nu$ \ref{HFAG}. In the model, we have the ratios

$$\left(\frac{g_\tau}{g_\mu}\right)^2 \equiv \frac{1}{1 + 2 \delta_\text{tree}}, \quad \left(\frac{g_\tau}{g_\mu}\right)^2 \equiv \frac{1}{1 + 2 \delta_\text{loop}}, \hspace{1cm} (16)$$

$$\left(\frac{g_\tau}{g_\mu}\right)^2 \equiv \frac{1}{1 + 2 \delta_\text{tree}}, \quad \left(\frac{g_\tau}{g_\mu}\right)^2 \equiv \frac{1}{1 + 2 \delta_\text{loop}}.$$
Here $\bar{\Gamma}$ denoting the partial width normalized to its SM value. $\delta_{\text{tree}}$ and $\delta_{\text{loop}}^{\tau/\mu}$ obtain corrections from the tree-level and one-loop diagrams mediated by the charged Higgs, respectively. They are given as \cite{9, 11}

$$
\delta_{\text{tree}} = \frac{v^4 \kappa_{\tau}^2 \kappa_{\mu}^2}{8 m_{H^\pm}^2} - \frac{v^2 m_{\mu}}{m_{H^\pm}^2 m_{\tau}} \kappa_{\tau} \kappa_{\mu} g(m_{\mu}^2/m_{\tau}^2) \frac{f(m_{\mu}^2/m_{\tau}^2)}{f(m_{\tau}^2/m_{\tau}^2)} \quad (17)
$$

$$
\delta_{\text{loop}}^{\tau/\mu} = \frac{1}{16 \pi^2} \kappa_{\tau/\mu}^2 \left[ 1 + \frac{1}{4} (H(x_A) + H(x_H)) \right] \quad (18)
$$

where $f(x) \equiv 1 - 8x + 8x^3 - x^4 - 12x^2 \ln(x)$, $g(x) \equiv 1 + 9x - 9x^2 + 6x(1+x) \ln(x)$, and $H(x_\phi) \equiv \ln(x_\phi)(1 + x_\phi)/(1 - x_\phi)$ with $x_\phi = m_\phi^2/m_{H^\pm}^2$.

The correlation matrix for the above five observables is

$$
\begin{pmatrix}
1 & +0.53 & -0.49 & +0.24 & +0.12 \\
+0.53 & 1 & +0.48 & +0.26 & +0.10 \\
-0.49 & +0.48 & 1 & +0.02 & -0.02 \\
+0.24 & +0.26 & +0.02 & 1 & +0.05 \\
+0.12 & +0.10 & -0.02 & +0.05 & 1
\end{pmatrix}
$$

We perform $\chi^2$ calculations for these five observables. The covariance matrix constructed from the data of Eqs. 3 and 19 has a vanishing eigenvalue, and the corresponding degree of freedom is removed in our calculation. In our discussions we require $\chi^2 < 9.72$, which corresponds to be within the 2\sigma range for four observables, and is smaller than the SM value, $\chi^2(\text{SM}) = 12.25$.

The measured values of the ratios of the leptonic Z decay branching fractions are given as \cite{28}

$$
\begin{align*}
\frac{\Gamma_{Z \to \mu^+\mu^-}}{\Gamma_{Z \to e^+e^-}} &= 1.0009 \pm 0.0028, \\
\frac{\Gamma_{Z \to \tau^+\tau^-}}{\Gamma_{Z \to e^+e^-}} &= 1.0019 \pm 0.0032,
\end{align*}
$$

with a correlation of +0.63. In the model, the width of $Z \to \tau^+\tau^-$ can have sizable deviation from the SM value due to the loop contributions of the extra Higgs bosons, because they strongly interact with charged leptons. The calculations of quantities in Eq. (21) are similar to Ref. 29.

After imposing the constraints of the theory and the oblique parameters, in Fig. 1 we show the surviving samples which are consistent with $\Delta a_{\mu}$ and $\Delta a_e$ at 2\sigma level. Both one-loop and two-loop diagrams give positive contributions to $\Delta a_{\mu}$. For $\Delta a_e$, the contributions of one-loop are positive and those of two-loop are negative. Only the contributions of two-loop can make $\Delta a_e$ to be within 2\sigma range. $\Delta a_{\mu}$ and $\Delta a_e$ respectively favor negative $\kappa_{\mu}$ and positive $\kappa_e$ for increasing $m_H$, and $m_H$ is required to be smaller than 320 GeV from $\Delta a_e$. A large mass splitting

FIG. 1: The samples within 2\sigma ranges of $\Delta a_{\mu}$ (left panel), $\Delta a_e$ (middle panel), and both $\Delta a_{\mu}$ and $\Delta a_e$ (right panel). All the samples satisfy the constraints of the theory and oblique parameters.

FIG. 2: The surviving samples fit the data of LFU in $\tau$ decay within the 2\sigma range. All the samples satisfy the constraints of the theory and oblique parameters.
between $m_A$ and $m_H$ can lead to sizable corrections to $\Delta a_\mu$ and $\Delta a_e$. Therefore, the right panel of Fig. 4 shows that $m_A$ is favored for increasing $m_H$, especially for a large $m_H$.

After imposing the constraints of the theory and the oblique parameters, we show the surviving samples with $\chi^2 < 9.72$ in Fig. 2. Such samples fit the data of LFU in $\tau$ decay within 2σ range. Because $\kappa_\mu$ is opposite in sign to $\kappa_\tau$, the second term of $\delta_{\text{tree}}$ in Eq. (17) is positive, which gives a well fit to $g_\tau/g_\tau$. Fig. 2 shows that $\chi^2$ can be as low as 7.4, which is much smaller than the SM value (12.25). The value of $\chi^2$ decreases with an increase of $-\kappa_\mu\kappa_\tau$ and increases with $m_{H^\pm}$.

In Fig. 3 we show the surviving samples after imposing the constraints of theory, the oblique parameters, $\Delta a_\mu$, $\Delta a_e$, the data of LFU in $\tau$ decay and $Z$ decay, and the direct searches at LHC. The model can give sizable corrections to $Z \to \tau^+\tau^-$ for large $\kappa_\tau$ and mass splitting between $m_A$ and $m_H$. Therefore, the region of small $m_H$ and large $\kappa_\tau$ is excluded by the data of LFU in $Z$ decay, as shown in the middle panel of Fig. 3. The left panel of Fig. 3 shows that the exclusion limits from the direct searches at LHC favor large $m_H$, $m_A$, and $m_{H^\pm}$. After imposing the theoretical constraint and relevant experimental constraints, the model can explain the anomalies of $\Delta a_\mu$, $\Delta a_e$ and LFU in the $\tau$ decay in many parameter space of 200 GeV < $m_H$ < 320 GeV, 500 GeV < $m_A$ = $m_{H^\pm}$ < 680 GeV, 0.006 < $\kappa_e$ < 0.01, -0.25 < $\kappa_\mu$ < -0.147, and 0.53 < $\kappa_\tau$ < 1.0.

Conclusion — We have proposed a lepton-specific inert 2HDM, where an inert Higgs doublet field with a discrete $Z_2$ symmetry is introduced to the SM. Considering all the current theoretical and experimental constraints, we showed that our model can provide a simple explanation for the anomalies of muon $g-2$, electron $g-2$, and LFU of the $\tau$ decays in many viable parameter spaces.

Acknowledgments — This work was supported by the Projects 11475238, 11575152, 11647601, and 11875062 supported by the National Natural Science Foundation of China, by the Natural Science Foundation of Shandong province (ZR2017MA004, ZR2017JL002), by the Key Research Program of Frontier Science, CAS, and by the ARC Centre of Excellence for Particle Physics at the Tera-scale under the grant CE110001004.

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