The Field-Tuned Superconductor-Insulator Transition with and without Current Bias

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(November 1, 2018)

The magnetic-field-tuned superconductor-insulator transition has been studied in ultrathin Beryllium films quench-condensed near 20 K. In the zero-current limit, a finite-size scaling analysis yields the scaling exponent product $\nu z = 1.35 \pm 0.10$ and a critical sheet resistance $R_c$ of about $1.2 R_Q$, with $R_Q = \hbar/4 e^2$. However, in the presence of dc bias currents that are smaller than the zero-field critical currents, $\nu z$ becomes $0.75 \pm 0.10$. This new set of exponents suggests that the field-tuned transitions with and without dc bias currents belong to different universality classes.

PACS numbers: 73.50.-h, 74.76.-w, 74.25.Dw, 74.40.+k

The superconductor-insulator (SI) transition in ultrathin films is believed to be a continuous quantum phase transition $\delta = 0$ occurring at $T = 0$ as the quantum ground state of the system is tuned by varying disorder, film thickness, magnetic field, or carrier concentration through a critical value. Models of the SI transition can be grouped into two categories: ones that rely on fluctuations in the phase and others that rely on fluctuations in the amplitude of the order parameter to drive the transition. For models in the first category, Cooper pairs are assumed to be present on the superconducting side as well as on the insulating side of the transition. Based on this "dirty-boson" model, Fisher et al.

\begin{equation}
R(\delta, T) \propto R_c f(\delta t),
\end{equation}

where $t(T) = T^{-1/\nu z}$, and $\delta$ is the deviation of a tuning parameter from its critical value. For field-tuned transitions, $\delta = |B - B_c|$ with $B$ and $B_c$ being the applied magnetic field and the critical field, respectively. The critical resistance $R_c$ is predicted to have a universal value of $R_Q = \hbar/4 e^2 = 6.5$ kΩ/□. Scaling arguments set a lower bound on the correlation length exponent $\nu \geq 1$ and give the value of the dynamical critical exponent $z = 1$. Experimentally, critical exponents consistent with these predictions have been found in scaling analyses of the field-tuned transitions in InO$_x$ and MoGe films, but disagrees with a recent report of $\nu z = 0.7$ for the field-tuned transition in ultrathin Bi/Ge films. For our Be films having a robust superconducting phase, $R_c$ appears to be near $1.2 R_Q$ in the field-tuned transition. This agrees well with results from the Bi/Ge films. We have also carried out for the first time studies of the field-tuned transition in the presence of a dc bias current. The applied dc bias current should exert a force of $\mathbf{j} \times \mathbf{B}$ on the vortices in the direction perpendicular to both the applied magnetic field $\mathbf{B}$ and the current density $\mathbf{j}$. We have observed that the scaling exponent product $\nu z$ becomes $0.75 \pm 0.10$ in the presence of dc bias currents, suggesting that the field-tuned transitions with and without dc bias current belong to different universality classes.

In Fig. 1, we show, with zero bias current, the temperature dependence of the sheet resistance, $R_D$, measured...
at various field values for one of our Be films with the magnetic field applied perpendicular to the plane of the film. These Be films were quench-condensed onto bare glass substrates which were held near 20 K during evaporations, under UHV conditions inside a dilution refrigerator. This in situ progressive evaporation setup allowed for systematic studies of the SI transition as film thickness was varied. We deposited each set of films in fine steps. We carefully monitored $R_\text{Q}$ during each evaporation step until a desirable value of $R_\text{Q}$ was reached. The films were very close to 10 Å in thickness, however, our quartz thickness monitor was not sensitive enough to pick up the small thickness increments after each step. Film resistance was measured in a standard four-terminal geometry using a PAR-124A lock-in amplifier operating at 27 Hz. The ac probe current was fixed at 1 nA. At the finite measuring temperatures in the vicinity of the field-tuned transition, the $I$-$V$ characteristics in this low-current regime were linear. More details regarding these Be films have been published elsewhere.

Quench-condensed Be films are chosen for this study because such films were found to be nearly amorphous. These Be films undergo a transition from insulating to superconducting when the normal state sheet resistance, $R_\text{N}$, is reduced below $\sim 10 \, k\Omega/\square$ with increasing film thickness. We have now studied the field-tuned SI transition in several films of $R_\text{N}$ between 5.6 and 12 kΩ/□. The superconducting transition temperature, $T_c$, of these films varied between 0.5 to 4 K in zero-field.

The film in Fig. 1, with $R_\text{N} = 10.7 \, k\Omega/\square$ at 15 K, is considered marginally superconducting, as we will discuss later. With increasing field, corresponding to curves from the bottom to the top in Fig. 1, this film was driven from superconducting to insulating, with a rather flat $R_\text{Q}$ vs. $T$ curve at a critical field of $B_c = 0.66$ T. The main part of Fig. 2(a) shows $R_\text{Q}$ vs. $B$ at various temperatures for the same film as shown in Fig. 1. In the vicinity of a quantum critical point, the resistance of a two-dimensional system is predicted to obey the scaling law in Eq. (1). Determining the critical exponents involves plotting the $R_\text{Q}$ vs. $B$ data at various temperatures according to the scaling law. The good crossing point, over one decade in temperature, in the $R_\text{Q}$ vs. $B$ plot in the main part of Fig. 2(a) identifies $B_c = 0.66$ T and $R_c = 4.4 \, k\Omega/\square$ for this film. We have used two methods to determine the scaling exponent $\nu z$. First, we can find $\nu z$ by evaluating the derivative of the $R_\text{Q}$ vs. $B$ curve at the critical field. In this case, we have the following scaling equation:

$$\frac{\partial R}{\partial B}|_{B_c} \propto R_c T^{-1/\nu z} f'(0).$$

A plot of $\frac{\partial R}{\partial B}|_{B_c}$ vs. $T^{-1}$ on a log-log scale, shown in the inset to Fig. 2(a), yields a straight line with a slope equal to $1/\nu z$, from which we determine $\nu z = 1.36 \pm 0.10$. Alternatively, we can plot $R/R_c$ vs. $|B - B_c|/t$ and treat $t(T)$ as an unknown variable. The values of $t(T)$ at various temperatures are determined by obtaining the best collapse of the data. Following this procedure presented in recent papers by N. Markovic et al., we obtain the temperature dependence of $t(T)$ which we plot on a log-log scale in the inset to Fig. 3(a). The collapse of the data is shown in the main part of Fig. 3(a) in a $R/R_c$ vs. $|B - B_c|/t$ plot. Figure 3(a) shows good collapse of the data over three orders of magnitude in $|B - B_c|/t$ and two orders of magnitude in $R/R_c$. The straight line in the inset to Fig. 3(a) shows a power-law fit, as expected by the scaling function, Eq. (1). The slope of the line in the inset gives $-1/\nu z$, from which we find $\nu z = 1.34 \pm 0.10$. The exponents obtained from the above two methods agree with each other, showing the consistency of the scaling analysis. The scaling exponents obtained in our Be films in the zero-bias limit appear to be in very good agreement with the predictions of theories based on the "dirty boson" model. Our result is also in very good agreement with renormalization group theories and Monte Carlo simulations.

Another important prediction of the bosonic model is the universal critical resistance $R_c = R_Q = h/4e^2 \approx 6.5 \, k\Omega/\square$ at the quantum critical point. This remains a controversial issue since only in the Bi/Ge sys-
spin-orbit coupling is strong, or negative, if the spin-orbit coupling is very weak.

Our Be films showed a zero-field SI transition as \( R_N \) was reduced below \( \sim 10 \, \text{kΩ/□} \) with increasing film thickness \([11]\). Films of \( R_N \) near \( 11 \, \text{kΩ/□} \) are considered marginally superconducting since they have near-zero critical field values. Further increasing film thickness, the critical field, \( B_c \), increases with decreasing \( R_N \), as shown in Fig. 4(a). In Fig. 4(b), we plot how \( R_c \) in the field-tuned transition varies with \( R_N \). For marginally superconducting films of \( R_N \) between 9 to 12 kΩ/□, \( R_c \) was significantly smaller than \( R_Q \). Nevertheless, we see clearly by comparing Fig. 4(a) with Fig. 4(b) that, as \( R_N \) was reduced by increasing film thickness, films with robust critical field values have a \( R_c \) of 7.1 \( \sim 8.0 \, \text{kΩ/□} \) in the field-tuned SI transition, which is about 1.2\( R_Q \). This result is in excellent agreement with results from the Bi/Ge system \([10]\). However, our results disagree with the suggestion \([10]\) that the magnetoresistance of the unpaired electrons caused the discrepancy between \( R_c \) and \( R_Q \). Since Be has the weakest spin-orbit coupling among metals, the magnetoresistance should be negative, leading to \( R_c < R_Q \) following the two-channel model. We note that we have shown in Fig. 1, Fig. 2(a), and Fig. 3(a) data measured on a marginally superconducting film of \( R_N = 10.7 \, \text{kΩ/□} \) at 15 K. Although the critical resistance of this film, \( R_c \sim 4.4 \, \text{kΩ/□} \), is significantly smaller than \( R_Q \), we have found that, for all the marginally and robust superconducting films of \( R_N \) ranging from 5.6 to 12 kΩ/□, the critical exponents are the same with \( \nu z = 1.35 \pm 0.10 \) in the zero-current limit.

Below, we describe results of the field-tuned SI transition in the presence of a dc bias current, \( I_{bias} \). For such studies, \( I_{bias} \) was varied between 125 nA and 2.5 µA and kept below the zero-field critical current (\( \sim 15 \, \text{µA} \)). At each fixed \( I_{bias} \), we used a magnetic field to tune a film from superconducting to insulating. We believe that joule heating was insignificant in our experiments based on the following arguments. First, we have performed extensive \( I-V \) measurements over the entire tem-

![Fig. 3](image-url)  
**FIG. 3.** Main figures show the scaling plots of \( R/R_c \) vs. \( |B - B_c|t \), with \( I_{bias} = 0 \) in (a) and \( I_{bias} = 2.5 \, \text{µA} \) in (b). Insets show the power-law relations between the parameter \( t \) and the temperature, which determine the values of \( \nu z \).

![Fig. 4](image-url)  
**FIG. 4.** Figures show (a) the critical field, \( B_c \), and (b) the critical resistance, \( R_c \), as functions of the normal-state sheet resistance, \( R_N \), measured at 15 K.
temperatures and magnetic field range of our experiments, with the time span for the $I$-$V$ sweeps ranging from ten minutes to one hour. The $I$-$V$ curves were completely reproducible without any observable hysteresis and independent of the sweep rate. Secondly, we can estimate the temperature increase, $\Delta T$, on the Be films due to joule heating. The thin Be films were deposited on glass microslide substrates of thickness 0.23 mm, which was attached to a copper sample holder by a very thin layer of grease. The glass substrate was the dominant source of heat resistance, with a thermal conductivity of $\sim 0.0003 \text{ W/Km}$ at 100 mK. For a typical film square of size 3x3 mm$^2$ and resistance 10 k$\Omega$, joule heating for $I_{\text{bias}} = 125 \text{ nA}$ is about 0.15 nW, resulting in $\Delta T \sim 0.012 \text{ mK}$ at 100 mK. For $I_{\text{bias}} = 2.5 \mu\text{A}$, $\Delta T \sim 5.0 \text{ mK}$ at 100 mK. We note that the heating power at 2.5 $\mu\text{A}$ was 400 times larger than the heating power at 125 nA. If joule heating were significant, the data obtained with $I_{\text{bias}} = 2.5 \mu\text{A}$ should show a flattening of the data in the low temperature region when compared to the data obtained with $I_{\text{bias}} = 125 \text{ nA}$. The fact that the scaling results at 125 nA and 2.5 $\mu\text{A}$ agree well suggests that heating was insignificant. This also argues against the existence of significant electron heating decoupled from the lattice.

We plot in Fig. 2(b) and Fig. 3(b) for $I_{\text{bias}} = 2.5 \mu\text{A}$, the results of scaling analyses based on the two previously described methods. In Table I, we list the parameters from scaling analyses of the field-tuned transition at various $I_{\text{bias}}$. Results from the data collapsing method are presented for $I_{\text{bias}}$ values of 250 nA and 2.5 $\mu\text{A}$, for which the amount of data taken was adequate for such analyses. It appears that $\nu_z \sim 0.75 \pm 0.10$, showing no systematic change with $I_{\text{bias}}$ in the range we have studied. Nevertheless, it is significantly smaller than the $\nu_z$ found in the zero-current limit. We can only speculate that the bias current could lead to a symmetry-breaking, resulting in different critical behavior at the transition. We note that experiments need to be carried out in other systems in order to find out whether this result is universal. In addition, experiments at low $I_{\text{bias}}$ values should be carried out to determine whether $\nu_z$ changes abruptly or gradually as $I_{\text{bias}}$ is increased from zero. Such experiments can probe the threshold $I_{\text{bias}}$ for the change in the scaling exponents, and have the potential of revealing whether the new scaling exponents are produced by certain nonlinear effects in the vortices under a bias current. On the other hand, we need to discuss other possible origins that might have lead to the new scaling exponents. For example, transport in marginally superconducting films, or, in the presence of a dc bias, could be dominated by narrow superconducting filaments, leading to a change in the dimensionality of the system. While it is difficult to determine to what extent film inhomogeneity affects our experiments, we would like to comment on the critical currents measured in our films. For the film shown in Fig. 2(b) and Fig. 3(b), the critical current in zero field was about $1.5 \times 10^{-5} \text{ A}$. This film was about 10 Å in thickness and 3 mm in width. Thus, the critical current density would be about $5 \times 10^6 \text{ A/m}^2$ if the current were uniformly distributed in the bulk of the film. On the other hand, if we assume that the current runs through a few filaments of 100 Å in total width, the critical current density would be $1.5 \times 10^{12} \text{ A/m}^2$. The critical currents of amorphous Be films have been measured by other groups, for example by Okamoto et al. Their Be films had a resistivity of $3.6 \times 10^{-6} \Omega\text{m}$. This value gives rise to $R_\text{Q} \sim 3.6 \text{ k}\Omega/\square$ for a film of thickness same as ours ($\approx 10 \text{ Å}$), meaning that their films were roughly 3 times less resistive than ours. They found that their films had critical current density of about $1 \times 10^8 \text{ A/m}^2$. Compared to their critical current density, the assumption in our films of conduction dominated by a few narrow filaments leads to an unreasonably large critical current density. In addition, we can compare our marginally superconducting films with robust superconducting films which are unlikely to be dominated by narrow filaments. As we have discussed, with $I_{\text{bias}} = 0$, these two types of films showed identical scaling exponents $\nu_z$. It is possible that film inhomogeneity has lead to a depressed $R_\text{Q}$ in marginally superconducting films, but has not changed the effective dimensionality of the system. Therefore, the change in the scaling exponents with a dc bias is unlikely due to a dimensional cross-over as a result of filamentary superconductivity.

In conclusion, the scaling exponents found in our Be films in the zero-biased, field-tuned SI transition agree very well with results from InO$_x$ and MoGe films, but disagrees with recent results from quench-condensed Bi/Ge films. For Be films having a robust superconducting phase, the critical sheet resistance in the field-tuned SI transition was about $1.2 R_\text{Q}$. Our field-tuned transition in the presence of a dc current reveals a set of new scaling exponents, suggesting that the field-tuned transitions with and without dc bias belong to different universality classes. We gratefully acknowledge S. Teitel and Y. Shapir for numerous useful discussions.

| $I_{\text{bias}}$ (nA) | $B_c$ (T) | $R_c$ (k$\Omega$/\square) | $Q$ (k$\Omega$/\square) | $\nu_z$ | $\nu_z$ |
|-------------------|--------|----------------|----------------|-------|-------|
| 125               | 0.33   | 15.5          | 9.36           | 0.81  | –     |
| 250               | 0.35   | 16.1          | 9.36           | 0.76  | 0.73  |
| 1000              | 0.36   | 15.5          | 9.36           | 0.73  | –     |
| 2500              | 0.38   | 15.0          | 9.36           | 0.77  | 0.75  |

$^*\nu_z$ obtained by the $\frac{\partial R}{\partial T_{\text{bias}}}$ method

$^\dagger\nu_z$ obtained by the data collapse method
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