Odderon Exchange from Elastic Scattering Differences between \textit{pp} and \textit{pp} Forward Scattering Measurements

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Regge poles, lead to terms of the form in the angular-momentum plane. The simplest examples, such as the Froissart-Martin bound showing that on very general grounds the total cross section is bounded by $\sigma_{\text{tot}} \sim \log^2 s$ as $s$ becomes asymptotically large.

Experimental discoveries in the 1970s showed that the $pp$ and $p\bar{p}$ total cross sections at the intersecting storage rings (ISR) do rise with energy [5] and can be parametrized with this functional form albeit, with a much smaller constant term than in the Froissart-Martin bound. The observed experimental rise of $\sigma_{\text{tot}}$ with energy has now been extended to much higher energies like $\log^2 s$ at the Tevatron, Large Hadron Collider (LHC), and with cosmic rays [6].

This behavior was understood in terms of Regge theory in which S-matrix elements for elastic scattering are based on the assumptions of Lorentz invariance, unitarity, and analyticity. In the high energy Regge limit, the scattering amplitude can be determined by singularities in the complex angular-momentum plane. The simplest examples, Regge poles, lead to terms of the form $\eta f(t)(s/s_0)^{\alpha(t)}$, where $t$ is the four-momentum transferred squared, $\eta$ the “signature” with value ±1, and $\alpha(t)$ the “trajectory” of the particular Regge pole. Positive signature poles give the same (positive) contribution to both $pp$ and $p\bar{p}$ scattering. Negative signature poles give opposite sign contributions to $pp$ and $p\bar{p}$ scattering. Using the optical theorem, each such Regge pole contributes a term proportional to $s^{\alpha(0)-1}$ to the total cross section. The largest contributor at very high energy, called the Pomeron, is the positive signature Regge pole whose $\alpha(0)$ is the largest. To explain the rising total cross section, the Pomeron should have $\alpha(0)$ just larger than one. A $\eta = -1$ Regge exchange with a similarly large $\alpha(0)$, called the odderon [7,8] was recognized as a possibility but was initially not well motivated theoretically and no clear evidence for it was found [9–11].

With the advent of quantum chromodynamics (QCD) as the theory of the strong interaction, the theoretical underpinnings evolved. The exchange of a family of colorless C-even states, beginning with a t-channel exchange of two gluons, was demonstrated to play the role of the Pomeron [12–15] with a predominantly imaginary amplitude near $|t| = 0$. QCD also firmly predicted the corresponding predominantly real odderon exchange of a family of colorless C-odd states, beginning with a $t$-channel exchange of three gluons, and $\alpha(0)$ near one [16–25]. However, the odderon remained elusive experimentally due to the dominating contribution by the Pomeron to total cross sections and small angle elastic scattering. The effect of the odderon should be felt most strongly when the dominant Pomeron amplitude becomes small compared to the odderon (e.g., near the so-called diffractive minimum in
and dip locations as a function of the differential cross sections measured at the bump and dip or bump in the D0 flattens out at LHC energies. Since there is no discernible decrease as a function of \( \sqrt{s} \) for ISR [9,34], \( SppS \) [35,36], Tevatron [28] and LHC [30–33] in the multi-TeV range would constitute a direct demonstration for the existence of the odderon.

The D0 Collaboration [27] measured the \( pp \) elastic differential cross section at \( \sqrt{s} = 1.96 \text{ TeV} \) [28]. The TOTEM Collaboration [29] at the CERN LHC measured the differential elastic \( pp \) cross sections at \( \sqrt{s} = 2.76 \) [30], 7 [31], 8 [32] and 13 [33] TeV. Figure 1 shows the TOTEM differential cross sections used in this study as functions of \( \sqrt{s} \). All \( pp \) cross sections show a common pattern of a diffractive minimum ("dip") followed by a secondary maximum ("bump") in \( da/dt \). Figure 2 shows the ratio \( R \) of the differential cross sections measured at the bump and dip locations as a function of \( \sqrt{s} \) for ISR [9,34], \( SppS \) [35,36], Tevatron [28] and LHC [30–33] \( pp \) and \( pp \) elastic cross section data. The \( pp \) data are fitted using the formula \( R = R_0 + a_0 \exp(b_0 \sqrt{s}) \). We note that the \( R \) of \( pp \) decreases as a function of \( \sqrt{s} \) in the ISR regime and flattens out at LHC energies. Since there is no discernible dip or bump in the D0 \( pp \) cross section, we estimate \( R \) by taking the maximum ratio of the measured \( da/dt \) values over the three neighboring bins centered on the evolution as function of \( \sqrt{s} \) of the bump and dip locations as predicted by the \( pp \) measurements. The D0 \( R = 1.0 \pm 0.2 \) value differs from the \( pp \) ratio by more than 3\( \sigma \) assuming that the flat \( R \) behavior of the \( pp \) cross section ratio at the LHC continues down to 2 TeV. The \( R \) values shown in Fig. 2 for \( pp \) scattering at the ISR [9] and the \( SppS \) [35,36] are similar to those of the D0 measurement.

Motivated by the features of the \( pp \) elastic \( da/dt \) measurements, we define a set of eight characteristic points as shown in Fig. 3(a). For each characteristic point, we identify the values of \( |t| \) and \( da/dt \) at the closest measured points to the characteristic point, thus avoiding the use of model-dependent fits. In cases where two adjacent points are of about equal value, the data bins are merged. This leads to a distribution of \( |t| \) and \( da/dt \) values as a function of \( \sqrt{s} \) for each characteristic point as shown in Figs. 3(b) and 3(c). The uncertainties correspond to half the bin size in \( |t| \) (comparable to the \( |t| \) resolution) and to the published uncertainties on the cross sections.

The values of \( |t| \) and \( da/dt \) as functions of \( \sqrt{s} \) for each characteristic point are fitted using the functional forms \( |t| = a \log(\sqrt{s}) + b \) and \( (da/dt) = c \sqrt{s} + d \) respectively. The parameter values are determined for each characteristic point separately and the same functional form describes the dependence for all characteristic points. The fact that the same forms can be used for all points is not obvious and might be related to general properties of elastic scattering [37]. The \( \chi^2 \) values for the majority of fits are close to 1 per degree of freedom (d.o.f.). The above forms were chosen for simplicity after it was checked that alternative forms providing adequate fits yielded similar extrapolated values within uncertainties.

The \( |t| \) and \( da/dt \) values for the characteristic points for \( pp \) interactions extrapolated to 1.96 TeV are displayed as open black circles in Fig. 1. The uncertainties on the extrapolated \( |t| \) and \( da/dt \) values are computed using a full treatment of the fit uncertainties, taking into account the fact that the systematic uncertainties of the different characteristic points are not correlated because they...

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**FIG. 1.** The TOTEM measured \( pp \) elastic cross sections as functions of \( |t| \) at 2.76, 7, 8, and 13 TeV (filled circles), and the extrapolation (discussed in the text) to 1.96 TeV (empty circles). The lines show the double exponential fits to the data points (see text).

**FIG. 2.** The ratio \( R \) of the cross sections at the bump and dip as a function of \( \sqrt{s} \) for \( pp \) and \( pp \). The \( pp \) data are fitted to the function noted in the legend.
are taken into account below as systematic conditions.

correspond to different detectors, data sets, and running conditions.

To compare the extrapolated pp elastic cross sections with the pp measurements, we fit the pp cross section with the function

$$h(t) = a_1 e^{-a_2|t|^2} + a_3 e^{-a_4|t|^2}$$  \hspace{1cm} (1)

to allow interpolation to the $t$ values of the D0 measurements in the range $0.50 \leq |t| \leq 0.96 \text{ GeV}^2$. The fit gives a $\chi^2$ of 0.63 per d.o.f. [38]. The first exponential in Eq. (1) describes the cross section up to the location of the dip, where it falls below the second exponential that describes the asymmetric bump and subsequent falloff. This functional form also provides a good fit for the measured pp cross sections at all energies as shown by the fitted functions in Fig. 1.

We evaluate the pp extrapolation uncertainty from Monte Carlo (MC) simulation in which the cross section values of the eight characteristic points are varied within their Gaussian uncertainties and new fits given by Eq. (1) are performed. Fits without a dip and bump position matching the extrapolated values within their uncertainties are rejected, and slope and intercept constraints are used to discard unphysical fits [39]. The MC simulation ensemble provides a Gaussian-distributed pp cross section at each $t$ value. However, the dip and bump matching requirement causes the mean of the pp cross section ensemble distribution to deviate from the best-fit cross section obtained above using Eq. (1) with the parameters of Ref. [38]. For the $\chi^2$ comparison with the D0 measurements below, we choose the mean value of the cross section ensemble at each $t$ value with its corresponding Gaussian variance.

We scale the pp extrapolated cross section so that the optical point (OP), $\frac{da}{dt}(t = 0)$, is the same as that for pp. The cross sections at the OP are expected to be equal if there are only C-even exchanges. Possible C-odd effects [37] are taken into account below as systematic uncertainties. Rescaling the OP for the extrapolated pp cross section would not itself constrain the behavior away from $t = 0$. However, as demonstrated in Refs. [40,41] the ratio of the pp and $p\bar{p}$ integrated elastic cross sections becomes one in the limit $\sqrt{s} \to \infty$. The parts of the elastic cross sections in the low $|t|$ Coulomb-nuclear interference region and in the high $|t|$ region above the exponentially falling diffractive cone that do differ for pp and $p\bar{p}$ scattering contribute negligibly to the total elastic cross sections. Thus, to excellent approximation, the integrated pp and $p\bar{p}$ elastic cross sections in the exponential diffractive region should be the same, implying that the logarithmic slopes should be the same. As this is the case within uncertainty for the pp and $p\bar{p}$ cross sections before the OP normalization, we constrain the scaling to preserve the measured logarithmic slopes. We assume that no $t$-dependent scaling beyond the diffractive cone ($|t| \geq 0.55$) is necessary.

To obtain the OP for pp at 1.96 TeV, we compute the total cross section by extrapolating the TOTEM measurements at 2.76, 7, 8, and 13 TeV. A fit using the functional form [42] for the $s$ dependence of the total cross section valid only in the range 1 to 13 TeV

$$\sigma_{\text{tot}} = b_1 \log^2(\sqrt{s}/1 \text{ TeV}) + b_2$$  \hspace{1cm} (2)

gives $\sigma_{\text{tot}}^{pp}(1.96 \text{ TeV}) = 82.7 \pm 3.1 \text{ mb}$ [43]. The extrapolated cross section is converted to a differential cross section $\frac{da}{dt} = 357 \pm 26 \text{ mb/GeV}^2$ at $t = 0$ using the optical theorem

$$\sigma_{\text{tot}}^2 = \frac{16\pi(h\rho)^2}{1 + \rho^2} \left(\frac{da}{dt}(t = 0)\right).$$  \hspace{1cm} (3)

We assume $\rho = 0.145$ based on the COMPETE extrapolation [44]. The D0 fit of $\frac{da}{dt}$ for $0.26 < |t| < 0.6 \text{ GeV}^2$ [28] to a single exponential is extrapolated to $t = 0$ to give the OP cross section of $341 \pm 49 \text{ mb/GeV}^2$. Thus the
TOTEM OP and extrapolated $d\sigma/dt$ values are rescaled by $0.954 \pm 0.071$ (consistent with the OP uncertainties), where this uncertainty is due to that on the TOTEM extrapolated OP. We do not claim that we have performed a measurement of $d\sigma/dt$ at the OP at $t = 0$ since this would require additional measurements of the elastic cross section closer to $t = 0$, but we require equal OPs simply to obtain a common and somewhat arbitrary normalization for the two data sets.

The assumption of the equality of the $pp$ and $p\bar{p}$ elastic cross sections at the OP could be modified if an odderon exists [8,16]. A reduction of the significance of a difference between $pp$ and $p\bar{p}$ cross sections would only occur if the $pp$ total cross section were larger than the $p\bar{p}$ total cross section at 1.96 TeV. This is the case only in maximal odderon scenarios [37], in which a 1.19 mb difference of the $pp$ and $p\bar{p}$ total cross sections at 1.96 TeV would correspond to a 2.9% effect for the OP. This is taken as an additional systematic uncertainty and added in quadrature to the quoted OP uncertainty estimated from the TOTEM total cross section fit. The effect of additional (Reggeon) exchanges [45–47], different methods for extrapolation to the OP, and potential differences in $\rho$ for $pp$ and $p\bar{p}$ scattering are negligible compared with the uncertainties in the experimental normalization. The comparison between the extrapolated and rescaled TOTEM $pp$ cross section at 1.96 TeV and the D0 $p\bar{p}$ measurement is shown in Fig. 4 over the interval $0.50 \leq |t| \leq 0.96$ GeV$^2$.

We perform a $\chi^2$ test to examine the probability for the D0 and TOTEM differential elastic cross sections to agree. The test compares the measured $p\bar{p}$ data points to the rescaled $pp$ data points shown in Fig. 4, normalized to the integral cross section of the $p\bar{p}$ measurement in the examined $|t|$ range, with their covariance matrices. The fully correlated OP normalization and logarithmic slope of the elastic cross section are added as separate terms to the $\chi^2$ sum. The correlations for the D0 measurements at different $t$ values are small, but the correlations between the eight TOTEM extrapolated data points are large due to the fit using Eq. (1), particularly for neighboring points. Given the constraints on the normalization and logarithmic slopes, the $\chi^2$ test with six degrees of freedom yields the $p$ value of 0.000 61, corresponding to a significance of 3.4$\sigma$.

We make a cross check of this result using an adaptation of the Kolmogorov-Smirnov test in which correlations in uncertainties are taken into account using simulated data sets [48,49]. This cross check, including the effect of the difference in the integrated cross section in the examined $|t|$ range via the Stouffer method [50], gives a $p$ value for the agreement of the $pp$ and $p\bar{p}$ cross sections that is equivalent to the $\chi^2$ test.

We interpret this difference in the $pp$ and $p\bar{p}$ elastic differential cross sections as evidence that two scattering amplitudes are present and that their relative sign differs for $pp$ and $p\bar{p}$ scattering. These two processes are even and odd under crossing (or C parity), respectively, and are identified as Pomeron and odderon exchanges. The dip in the elastic cross section is generally associated with the $t$ value where the Pomeron-dominated imaginary part of the amplitude vanishes. Therefore the odderon, believed to constitute a significant fraction of the real part of the amplitude, is expected to play a large role at the dip. In agreement with predictions [37,51], the $pp$ cross section exhibits a deeper dip and stays below the $p\bar{p}$ cross section at least until the bump region.

We combine the present analysis result with independent TOTEM odderon evidence based on the measurements of $\rho$ and $\sigma_{\text{tot}}$ for $pp$ interaction at different $\sqrt{s}$. These variables are sensitive to differences in $pp$ and $p\bar{p}$ scattering. The $\rho$ and $\sigma_{\text{tot}}$ results are incompatible with models with Pomeron exchange only and provide independent evidence of odderon exchange effects [26], based on observations in completely different $|t|$ domains and TOTEM data sets.

The significances of the different measurements are combined using the Stouffer method [50]. The $\chi^2$ for the total cross section measurements at 2.76, 7, 8, and 13 TeV is computed with respect to the predictions given from models without odderon exchange [44,51] including also model uncertainties when specified. The same is done separately for the TOTEM $\rho$ measurement at 13 TeV [52]. Unlike the models of Ref. [44], the model of Ref. [51] provides the predicted differential cross section without an odderon contribution, so we choose to use the $\chi^2$ comparison of the model cross section at 1.96 TeV with D0 data instead of the D0-TOTEM comparison [53].

When a partial combination of the TOTEM $\rho$ and total cross section measurements is done, the combined significance ranges between 3.4 and 4.6$\sigma$ for the different models. The full combination leads to total significances ranging
from 5.2 to 5.7σ for t-channel odderon exchange for all the
models of Refs. [44] and [51]. In particular, for the model
favored by COMPETE (RRPnfL2ν) [44], the TOTEM ρ
measurement at 13 TeV provides a 4.6σ significance [54],
leading to a total significance of 5.7σ for t-channel odderon
exchange when combined with the present result [55].

In conclusion, we have compared the D0 p̅p elastic cross
sections at 1.96 TeV and the TOTEM pp cross sections
extrapolated to 1.96 TeV from measurements at 2.76, 7, 8,
and 13 TeV using a model-independent method [56]. The
pp and p̅p cross sections differ with a significance of 3.4σ,
and this stand-alone comparison provides evidence that a
t-channel exchange of a colorless C-odd gluonic com-
pound, i.e., an odderon, is needed to describe elastic
scattering at high energies [37]. When combined with
the result of Ref. [26], the significance is in the range 5.2
to 5.7σ and thus constitutes the first experimental observation
of the odderon.

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[54] Reference [26] reports a 4.7\( \sigma \) difference between the favored model of Ref. [44] and the TOTEM measurements. This is reduced to 4.6\( \sigma \) when the model uncertainties are taken into account.

[55] Here, the freedom provided by the Stouffer method to use a subset (e.g., \( \rho \) and present analysis) of the significances to compute the final significance is used.

[56] The model dependence is limited to (a) the functions used for extrapolating the \( |t| \) and \( d\sigma/dt \) values for \( pp \), (b) the use of Eq. (1) for interpolating the extrapolated \( pp \) cross sections at 1.96 TeV to the D0 \( t \) values, and (c) the use of existing models to estimate the systematic uncertainty due to the range of possible odderon contributions to the OP difference between \( pp \) and \( p\bar{p} \).