STAR FORMATION LAWS AND THRESHOLDS FROM INTERSTELLAR MEDIUM STRUCTURE AND TURBULENCE

Florent Renaud, Katarina Kraljic, and Frédéric Bournaud
Laboratoire AIM Paris-Saclay, CEA/IRFU/SAP, Université Paris Diderot, F-91191 Gif-sur-Yvette Cedex, France; florent.renaud@cea.fr
Received 2012 September 18; accepted 2012 October 17; published 2012 November 2

ABSTRACT

We present an analytical model of the relation between the surface density of gas and star formation rate in galaxies and clouds, as a function of the presence of supersonic turbulence and the associated structure of the interstellar medium (ISM). The model predicts a power-law relation of index 3/2, flattened under the effects of stellar feedback at high densities or in very turbulent media, and a break at low surface densities when ISM turbulence becomes too weak to induce strong compression. This model explains the diversity of star formation laws and thresholds observed in nearby spirals and their resolved regions, the Small Magellanic Cloud, high-redshift disks and starbursting mergers, as well as Galactic molecular clouds. While other models have proposed interstellar dust content and molecule formation to be key ingredients to the observed variations of the star formation efficiency, we demonstrate instead that these variations can be explained by ISM turbulence and structure in various types of galaxies.

Key words: galaxies: ISM – hydrodynamics – methods: analytical – stars: formation

Online-only material: color figures

1. INTRODUCTION

Galactic-scale star formation (SF) laws are not universal. Galaxy mergers, when they experience a starburst phase, convert their gas into stars within a depletion time up to 10 times shorter than spiral galaxies (Daddi et al. 2010b; Genzel et al. 2010; Saintonge et al. 2012). Oppositely, SF in dwarf galaxies is less efficient than in spirals (Leroy et al. 2008; Bolatto et al. 2011). On smaller scales, molecular clouds may follow similar scaling laws but with shorter conversion timescales (e.g., Lada et al. 2010).

To explain these environment-dependent differences, some models have emphasized the role of interstellar medium (ISM) chemistry (metallicity and dust content) in triggering gas cooling and molecule formation. The transition from long to short gas depletion times may correspond to the critical column density needed to shield gas from the ambient ultraviolet radiation (Schaye 2004) and/or to efficiently convert H I into H2 (Krumholz et al. 2009), which strongly depends on the metallicity. This is supported by recent observations of the Small Magellanic Cloud (SMC), of lower metallicity than spirals, where the star formation rate (SFR) surface density hardly reaches the standard regime even at relatively high gas surface densities (Bolatto et al. 2011). However, such descriptions do not explain the very long gas depletion timescales in metal-rich and molecule-rich ellipticals (Saintonge et al. 2012). Furthermore, significant SF activity can be found in regions where the molecular fraction is low (Boissier et al. 2008). More generally, variations in the SF scaling laws about as strong in H2 as in total gas have been detected, suggesting that molecule formation alone does not drive these variations (Saintonge et al. 2012).

Nevertheless, other factors may explain the observed scaling laws. The gas surface density observed in an entire galaxy or a relatively large region is a global property that results from a very heterogeneous distribution of the volume density on small scales. The latter varies broadly from quasi-empty holes to dense clouds and cores, under the effect of the ISM turbulence, which is supersonic for a large fraction of the mass (e.g., Audit & Hennebelle 2010). Elmegreen (2002) has shown that the turbulent structure of the ISM can naturally explain the Kennicutt (1998) relation for spiral galaxies.

In this Letter, we develop an analytic model describing the variations and thresholds in SF laws depending on the supersonic nature of the turbulence and the resulting structure of the ISM in several types of galaxies.

2. THEORY

2.1. Analytical Formalism

We consider a region of surface S and thickness h, whose total mass M is distributed according to a mass-weighted probability density function (PDF) f. The volume of the region can be written as the sum of the volumes occupied by the gas at all possible densities ρ, i.e.,

$$hS = \int_0^\infty \frac{Mf(x)}{\rho} dx = \frac{M}{\rho} \int_0^\infty \frac{f(x)}{x} dx,$$

where $x = \rho/\bar{\rho}$ is a normalization of the local gas volume density ρ (i.e., for scales $\leq 1$ pc), related to gas surface density of the region $\Sigma = M/S$ through

$$\bar{\rho} = \frac{\Sigma}{h} \int_0^\infty \frac{f(x)}{x} dx.$$

The SFR surface density reads

$$\Sigma_{SFR} = \frac{1}{S} \int_0^\infty \frac{Mf(x)}{\rho_{SFR}} dx,$$

where $\rho_{SFR}$ is the local SFR volume density. Therefore, the general expression of the SFR surface density is

$$\Sigma_{SFR} = h \int_0^\infty \frac{f(x)x^{-1}\rho_{SFR}}{\int_0^\infty f(x)x^{-1} dx} dx.$$

See also Padoan & Nordlund (2011), Hennebelle & Chabrier (2011) and Federrath & Klessen (2012) for comparable approaches.
2.1.1 Log-normal PDF

Most of the ISM mass is supersonically turbulent, which generates a log-normal PDF (e.g., Vazquez-Semadeni 1994; Nordlund & Padoan 1999; Wada & Norman 2001):

\[ f_\sigma(x) = \frac{1}{x\sqrt{2\pi\sigma^2}} \exp \left[ -\frac{(\ln x - \langle \sigma \rangle)^2}{2\sigma^2} \right], \quad (5) \]

whose dimensionless width \( \sigma \) is related to the Mach number \( M \) (and thus to the gas velocity dispersion) through \( \sigma^2 \approx \ln(1 + 3M^2/4) \). This was primarily found for Isothermal gas, but models of non-isothermal ISM including gravity and stellar feedback have shown that deviations from the log-normal functional form are negligible, at least in isolated disk galaxies (Tasker & Tan 2009; Bournaud et al. 2011a).

2.1.2 Dynamical Star Formation

At small scale, cold enough gas becomes supersonically turbulent and hosts the shocks which trigger the process of SF. This typically happens above a certain volume density \( \rho_0 \); in nearby spirals for instance, the velocity dispersion of 6–10 km s\(^{-1}\) requires cooling below \( \sim 10^4 \) K for ISM turbulence to become supersonic. This only becomes possible above \( \rho_0 \approx 10^2 \) cm\(^{-3}\), at solar metallicity, according to calculations at galactic scale (Bournaud et al. 2010), and detailed ISM models including developed turbulence and radiative transfer which show \( M \approx (\rho/10^{-2} \text{cm}^{-3})^{1/2} \) (Audit & Hennebelle 2010, Figures 4 and 9). Above this density threshold, a fraction of the gas becomes gravitationally unstable and collapses, converting a constant fraction \( \epsilon = 0.01 \) of its mass into stars per free-fall time \( t_{\text{ff}} \), as supported by observations (Elmegreen 2002):

\[ \rho_{\text{SFR}} = \begin{cases} 0 & \text{if } \rho \leq \rho_0 \\ \epsilon \frac{\rho}{t_{\text{ff}}} & \text{else} \end{cases} \quad (6) \]

2.1.3 Regulation by Stellar Feedback

In dense cores, stellar feedback limits the conversion of the gas into stars, by heating, ionizing, and even ejecting the gaseous leftovers. This gas becomes available again for SF after a time \( t_s \). No more than \( \epsilon_s = 30\% \) of the gas mass can be consumed for SF per timescale \( t_s \) (Bontemps et al. 1996; Matzner & McKee 2000). This translates into a saturation in the local SF law:

\[ \rho_{\text{SFR}} = \begin{cases} 0 & \text{if } \rho \leq \rho_0 \\ \min \left( \epsilon_s \left( \frac{32G\rho}{3\pi} \right)^{3/2}, \rho_{\text{SFR}} \right) & \text{else} \end{cases} \quad (7) \]

The (re)formation of star-forming clouds is generally triggered by galactic-scale processes (e.g., Dobbs & Pringle 2009 for spirals; Teysier et al. 2010 for mergers; and Bournaud et al. 2007 at high redshift), therefore \( t_s \approx 100 \) Myr over a broad range of galaxy masses. For instance, this represents the interval between the compression by two spiral arms, or for giant clumps to collapse in high-redshift disks. In nuclear starbursts however, the dynamical timescale is much shorter (\( \sim 10 \) Myr) but strong stellar feedback from OB-type stars takes over and also regulates SF, by limiting the conversion of gas into stars to \( \epsilon_s \approx 0.3 \) (Murray et al. 2010) over the duration of the starburst event, i.e., \( t_s = 100 \) Myr here again (Di Matteo et al. 2008).

\[ \text{erfc} : \xi \mapsto (2/\sqrt{\pi}) \int_\xi^\infty \exp(-t^2) dt = \frac{1}{2} \text{erf}(\xi) \]

The thickness of the SMC is derived using Combes et al. (2002, Chapter 1) with a velocity dispersion of \( \sim 20 \) km s\(^{-1}\) and a rotation speed of 40 km s\(^{-1}\) at the H\(_\text{I}\) half-mass radius (1.5 kpc; Stanimirović et al. 2004).
Figure 1. Surface density of the star formation rate, computed from Equation (8), with $\epsilon = 0.01$. The dependences with the Mach number $\mathcal{M}$ (via the width $\sigma$ of the PDF (a)), the thickness $h$ (b), and the density threshold $\rho_0$ (c) are shown. Panel (d) shows the regulation of star formation due to the stellar feedback (solid lines, Equation (9)), compared to no regulation, as above (dashed lines). The black dashed line is the same in all panels.

(A color version of this figure is available in the online journal.)

Figure 2. Star formation rate surface density computed with parameters ($\mathcal{M}$, $h$ in pc, $\rho_0$ in cm$^{-3}$) representative of spiral galaxies (black) and of the Small Magellanic Cloud (SMC; red). The models are compared to observational data of the THINGS survey (black contours; Bigiel et al. 2008), other spiral galaxies (triangles; Kennicutt 1998), M51 regions (black dots; Kennicutt et al. 2007), and the SMC (red contours; Bolatto et al. 2011). The blue and green lines are alternative fits of lower quality (see the text).

(A color version of this figure is available in the online journal.)

that $\rho_0$ is independent of the metallicity (and thus has the same value as for spirals) would require adjusting the thickness to an unrealistic value (3 kpc) to match the data (green curve in Figure 2). Second, not accounting for the structure of the ISM ($\mathcal{M}$ and $h$) and only focusing on the role of the onset of supersonic turbulence through a high $\rho_0$ would lead to a too steep relation (blue line in Figure 2). Alternatively, if we would assume that the formation of H$_2$ triggers SF instead of the transition to supersonic turbulence, we could model this with an even higher threshold ($\rho_0 \approx 100$–300 cm$^{-3}$ for the formation of molecules at subsolar metallicity), and thus get a steeper slope failing to fit the observations. Hence, the observed SF law in the SMC is best explained by the onset of supersonic turbulence and the associated ISM structure.5

3.2. Application to Starbursting Mergers

When they experience a starburst phase, galaxy mergers often have part of their gas compressed in a nuclear disk, which we model by converting a mass fraction $m$ of the initial PDF into the form of a denser component (using a dimensionless parameter $\delta$; see Figure 3): $(1 - m)f_{\sigma_1}(x) + mf_{\sigma_2}(x / \exp \delta)$. Hydrodynamic simulations of mergers found such excess of high-density components compared to a log-normal PDF, even outside nuclear disks (Teyssier et al. 2010; Bournaud et al. 2011b), and PDFs from these works are well fitted by our arbitrary model using $\delta = 3$ and $m = 0.2$. With feedback,

5 In these regimes of low $\mathcal{M}$ and $\Sigma_{\text{SFR}}$ (spiral disks and SMC), the feedback has a very mild effect, not affecting the relations.
our model may slightly underpredict the deviation from the 3-slope toward the shallower slopes in all regimes. However, if, as we have proposed here, the local density threshold corresponds to the critical density needed to initiate collapse. In pressure equilibrium, $\rho_0$ would be the density for which tidal, centrifugal (shear), and Coriolis forces balance self-gravity. Although it depends on the galactocentric radius, this tidal density averaged over the galaxy is $\sim 10\,\text{cm}^{-3}$, with limited variations with galaxy type and redshift. Second, support by magnetic fields might stabilize molecular clouds, leading to higher thresholds and shallower slopes in all regimes.

Both these alternative interpretations would translate into mild variations of the density threshold with the galactic environment. However, if, as we have proposed here, the local density threshold corresponds to the onset of supersonic turbulence, high turbulent media (e.g., mergers and high-redshift disks) would have very low thresholds leading to breaks in the observed SF relation shifted to low surface brightness, close to the detection limits and thus difficult to detect. Preliminary observations of resolved regions of low-$z$ mergers (P.-E. Belles et al., in preparation; Boquien et al. 2011) started to probe this regime (see Figure 4): some regions are spiral-like (as expected since not all mergers are starbursting) but $\sim 20\%$ of them have significant $\Sigma_{\text{SFR}}$ for very low $\Sigma$, well below the break of spirals. Larger samples are needed to tell apart starbursting regions from spiral-like ones, and to confirm the trend suggested here. This could also be tested with resolved observations of $z = 2$ disks, to further probe the physical origin of the local density threshold.

5. SUMMARY AND CONCLUSION

In this Letter, we present an analytical formalism aimed at describing the relation between the surface density of gas and the surface density of SFR observed in several types of galaxies. The only two ingredients of the model are the gas density PDF (representing the turbulence-driven structure of the ISM) and a local SF law, with a threshold due to the onset of supersonic turbulence, plus regulation by stellar feedback. Our main findings are as follows.

1. When integrated over regions of galaxies or entire galaxies, the threshold in the local SF law translates into a break at low surface densities.
2. Above this break, the local SF law directly imprints the global SFR surface density, leading to a $3/2$ index power law, followed by a slope of unity at high surface density, when feedback takes over turbulence as the main regulation agent.

3. The slow nature of SF in nearby spirals with gigayear-long gas depletion timescales is mostly explained by turbulent regulation (i.e., divergent flows disrupting gas clouds; Elmegreen 2002). Feedback regulation becomes more prominent in denser and more turbulent systems (e.g., high-$z$ galaxies or Galactic molecular clouds) and produces the observed shallower relation for the entire disk population (average slope 1.1–1.3).

4. The different properties of the SMC can explain the onset of supersonic turbulence at higher surface densities than in spirals, and our model retrieves the observed longer depletion times without necessarily invoking molecule formation or shielding by dust.

5. In starbursting mergers, the strong interaction-induced turbulence results in high-efficiency SF, with a break shifted to low densities but tentatively observed in resolved galaxies.

By simply using the typical values of ISM properties from the literature for several galactic environments, our model naturally explains the observations, without fitting them or fine tuning the parameters.

The temperature of the gas shielded from ultraviolet radiation and in thermal equilibrium would drop when the density reaches $\approx 0.8 \, \text{cm}^{-3}$ (Audit & Hennebelle 2010), allowing ISM turbulence to become supersonic at these relatively low volume densities. In spiral galaxies, such low threshold would translate into an apparent break at $<1 \, M_\odot \, \text{pc}^{-2}$. Instead, according to models including not only ultraviolet radiative transfer but also turbulent compression, the gas slowly cools below $\sim 10^4$ K and enters the supersonic regime at about $10 \, \text{cm}^{-3}$ (Audit & Hennebelle 2010), which naturally explains the observed break at $\sim 10 \, M_\odot \, \text{pc}^{-2}$.

Our results emphasize that the supersonic turbulence plays a key role in triggering and regulating galactic-scale SF, along with feedback in dense and turbulent media, and that the

![Figure 4](https://example.com/figure4.png)

**Figure 4.** Comparison of our models, using physically motivated parameters, with observations of galaxies or regions of galaxies at low and high redshift ($z$).

(A color version of this figure is available in the online journal.)
resulting ISM structure can broadly explain the observed SF laws and thresholds.

We thank Pierre-Emmanuel Belles, Pierre-Alain Duc, and Elias Brinks for providing us with their data prior to publication and for comments, Avishai Dekel, Bruce Elmegreen, Mark Krumholz, and Amélie Saintonge for stimulating discussions, and the anonymous referee for a useful report. We acknowledge support from the EC through grant ERC-StG-257720 and the CosmoComp ITN.

REFERENCES

Audit, E., & Hennebelle, P. 2010, A&A, 511, A76
Besla, G., Kallivayalil, N., Hernquist, L., et al. 2012, MNRAS, 421, 2109
Bigiel, F., Leroy, A., Walter, F., et al. 2008, AJ, 136, 2846
Boissier, S., Gil de Paz, A., Boselli, A., et al. 2008, ApJ, 681, 244
Bolatto, A. D., Leroy, A. K., Jameson, K., et al. 2011, ApJ, 741, 12
Bolatto, A. D., Leroy, A. K., Rosolowsky, E., Walter, F., & Blitz, L. 2008, ApJ, 686, 948
Bontemps, S., Andre, P., Terebey, S., & Cabrit, S. 1996, A&A, 311, 858
Boquien, M., Lisenfeld, U., Duc, P.-A., et al. 2011, A&A, 533, A19
Bothwell, M. S., Kennicutt, R. C., & Lee, J. C. 2009, MNRAS, 400, 154
Bouché, N., Cresci, G., Davies, R., et al. 2007, ApJ, 671, 303
Bournaud, F., Chapon, D., Teyssier, R., et al. 2011a, ApJ, 730, 4
Bournaud, F., Elmegreen, B. G., & Elmegreen, D. M. 2007, ApJ, 670, 237
Bournaud, F., Elmegreen, B. G., Teyssier, R., Block, D. L., & Puerari, I. 2010, MNRAS, 409, 1088
Bournaud, F., Powell, L. C., Chapon, D., & Teyssier, R. 2011b, in IAU Symp. 271, Star Formation in Galaxy Mergers: ISM Turbulence, Dense Gas Excess, and Scaling Relations for Disks and Starbursts, ed. N. H. Brummell, A. S. Brun, M. S. Miesch, & Y. Ponty (Cambridge: Cambridge Univ. Press), 160
Combes, F., Boisse, P., Mazure, A., Blanchard, A., & Seymour, M. 2002, Galaxies and Cosmology (2nd ed.; New York: Springer)
Daddi, E., Bournaud, F., Walter, F., et al. 2010a, ApJ, 713, 686
Daddi, E., Elbaz, D., Walter, F., et al. 2010b, ApJ, 714, L118
Di Matteo, P., Bournaud, F., Marig, M., et al. 2008, A&A, 492, 31
Dobbs, C. L., & Pringle, J. E. 2009, MNRAS, 396, 1579
Downes, D., & Solomon, P. M. 1998, ApJ, 507, 615
Elmegreen, B. G. 2002, ApJ, 577, 206
Elmegreen, B. G., & Elmegreen, D. M. 2006, ApJ, 650, 644
Elmegreen, B. G., & Hunter, D. A. 2006, ApJ, 636, 712
Elmegreen, D. M., Kaufman, M., Brinks, E., Elmegreen, B. G., & Sundin, M. 1995, ApJ, 453, 100
Federrath, C., & Klessen, R. S. 2012, ApJ, in press (arXiv:1209.2856
Förster Schreiber, N. M., Genzel, R., Bouché, N., et al. 2009, ApJ, 706, 1364
Genzel, R., Tacconi, L. J., García-Carpio, J., et al. 2010, MNRAS, 407, 2091
Glover, S. C. O., & Clark, P. C. 2012, MNRAS, 421, 9
Heiderman, A., Evans, N. J., II, Allen, L. E., Huard, T., & Heyer, M. 2010, ApJ, 723, 1019
Hennebelle, P., & Chabrier, G. 2011, ApJ, 743, L29
Irwin, J. A. 1994, ApJ, 429, 618
Kennicutt, R. C. 1998, ApJ, 498, 541
Kennicutt, R. C., Jr., Calzetti, D., Walter, F., et al. 2007, ApJ, 671, 333
Krumholz, M. R., Dekel, A., & McKee, C. F. 2012, ApJ, 745, 69
Krumholz, M. R., McKee, C. F., & Tumlinson, J. 2009, ApJ, 699, 850
Krumholz, M. R., & Tan, J. C. 2007, ApJ, 654, 304
Krumholz, M. R., & Thompson, T. A. 2007, ApJ, 669, 289
Lada, C. J., Lombardi, M., & Alves, J. F. 2010, ApJ, 724, 687
Leroy, A. K., Walter, F., Brinks, E., et al. 2008, AJ, 136, 2782
Matzner, C. D., & McKee, C. F. 2000, ApJ, 545, 364
Murray, N., Quataert, E., & Thompson, T. A. 2010, ApJ, 709, 191
Nordlund, Å., & Padoan, P. 1999, in Proc. 2nd Guillermo Haro Conference on Interstellar Turbulence the Density PDFs of Supersonic Random Flows, ed. J. Franco & A. Carraminana (Cambridge: Cambridge Univ. Press), 218
Padoan, P., & Nordlund, Å. 2011, ApJ, 730, 40
Saintonge, A., Tacconi, L. J., Fabello, S., et al. 2012, ApJ, 758, 73
Schaye, J. 2004, ApJ, 609, 667
Stanimirović, S., Staveley-Smith, L., & Jones, P. A. 2004, ApJ, 604, 176
Tacconi, L. J., Genzel, R., Neri, R., et al. 2010, Nature, 463, 781
Tasker, E. J., & Tan, J. C. 2009, ApJ, 700, 358
Teyssier, R., Chapon, D., & Bournaud, F. 2010, ApJ, 720, L149
Vazquez-Semadeni, E. 1994, ApJ, 423, 681
Wada, K., & Norman, C. A. 2001, ApJ, 547, 172