SINR-Based Time Domain Synthesis Algorithm for Radar

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Abstract. For optimal radar waveform design and multi-target recognition in cognitive radar, a time-domain synthetic algorithm based on the signal to interference plus noise ratio (SINR) in the presence of clutter. Simulation results show that after adding phase information, the time-domain synthesis algorithm based on SINR can obtain more target information than the general water-filling algorithm.

1. Introduction
For radar multi-target transmit waveform design, Goodman.NA [1] proposes a water-filling algorithm based on mutual information (MI) and signal-to-interference ratio (SINR), then compares and analyzes the relationship between single target and multi-target detection probability under different criteria in 2009. In 2011, Fan Meimei [2] researches the waveform adaptive technology of multi-target recognition, and proposes an optimal waveform design method that uses the linear weighted sum of mutual information (LWS-MI) between each target and the observed signal as the target function to achieve the best performance for the small number of targets optimal waveform design [3]. In 2014, Jiu Bo reduces the uncertainty of the target information by maximizing the mutual information between the target echo and the extended target, and applies this method to the problem of target transmit waveform optimization, thereby increasing various target determinism [4, 5]. In 2011, Meng Huadong et al. propose that the phase modulation method be applied to the optimal waveform design of the radar to minimize the mean square spectrum difference between the optimal waveform and the designed waveform to minimize the Signal to clutter noise ratio (SCNR) loss, thereby making the overall detection performance has been improved accordingly [6]. In 2015, Yan Dong [7] proposes to design the optimal transmit waveform and considers the phase information and simulation analysis under the maximum SCNR condition to obtain more target information, finally performs error analysis. Kim, H.-S [8] proposes a multi-target classification algorithm, which analyzes the determined target and the random target through a large number of derivations, and finally judges that the proposed algorithm is better in view of the detection probability.

2. Problem Formulation
This paper studies the design of multi-target radar waveforms based on the signal-to-interference ratio in the presence of related clutter and noise. Based on Goodman [1] multi-target water-filling algorithm, we can know that the energy spectral density $\sigma_x^2(f)$ of each target is to set a certain weight for each target for the convenience of research. The target spectrum designed in the research process is the total
target spectrum of all targets. The sum of the weights set by all targets is 1, i.e. $\sum_{l=1}^{L} w_l = 1$. There $S_n(f)$ represents the noise energy spectral density and $S_c(f)$ represents the clutter energy spectral density. Through analysis and calculation, the target function based on SINR is finally determined as follows

$$\max_{T} \int_{0}^{T} \left| X(f) \right|^2 \sum_{l=1}^{L} w_l \sigma^2_{l}(f) \frac{S_n(f)}{S_c(f)} \, df$$

$$s.t. \int_{0}^{T} \left| X(f) \right|^2 \, df \leq E_t$$

(1)

According to experience, the target function is a concave function. According to the nature of the concave function, the Lagrange multiplier method is used to calculate. Assuming that the multiplier is $\lambda$, the final transmit waveform energy spectrum expression is shown as (2)

$$\left| X(f) \right|^2 = \frac{\left( \sum_{l=1}^{L} w_l \sigma^2_{l}(f) \cdot S_n(f) \right)^{1/2}}{S_c(f)} - S_n(f)$$

(2)

Because the energy spectrum is always positive, the expression of $\left| X(f) \right|^2$ shown as (3)

$$\left| X(f) \right|^2 = \max\left[ \frac{\left( \sum_{l=1}^{L} w_l \sigma^2_{l}(f) \cdot S_n(f) \right)^{1/2}}{S_c(f)}, 0 \right]$$

(3)

The final energy spectrum density of the optimal transmit waveform is as follows

$$\varepsilon_{opt}(f) = \max\left[ \frac{\left( \sum_{l=1}^{L} w_l \sigma^2_{l}(f) \cdot S_n(f) \right)^{1/2}}{S_c(f)}, 0 \right]$$

(4)

3. Time Domain Synthesis Algorithm

Based on the multi-target water-filling algorithm in the previous section, phase is added and a time domain synthesis algorithm is proposed. The target transmit waveform is a non-linear frequency modulation with an amplitude of 1. The target model is as follows

$$x(t) = e^{j\varphi(t)}, t \in [-T/2, T/2]$$

(5)

Where $\varphi(t)$ is time-domain dependent phase function. $T$ is the total duration time of the transmit waveform design, then

$$x(n) = e^{j\varphi_0}, e^{j\varphi_1}, \ldots, e^{j\varphi_{N-1}}$$

(6)

Time domain to frequency domain, $F_s, T_s, M$ and $N$ are the sampling frequency, the sampling interval, the number of sampling points and the number of samples for sampling frequency. It is assumed that the energy spectral density of the transmit waveform is $\varepsilon_{pm}(f)$ after phase modulation. After $N(M = N)$ point discrete sampling, the energy spectral density $\varepsilon_{pm}(k)$ is shown as

$$\varepsilon_{pm}(k) = \left| X(k) \right|^2$$

(7)
Adding phase, the time domain synthesized signal is \( |X(k)|^2, k = 0, 1, 2, \ldots, N - 1 \). Using discrete complex signals \( x(n) \) of length \( M \) to represent time-domain signals, \( n = 0, 1, 2, \ldots, M - 1 \), the N-point Discrete Fourier Transform (DFT) of \( x(n) \) is

\[
X(k) = \sum_{n=0}^{M-1} x(n)e^{-\frac{j2\pi kn}{N}}
\]

(8)

Based on the design of multi-target radar transmit waveforms, this paper combines the ideas of cyclic iteration and minimum mean square error to find the synthesized waveform of the optimal transmit waveform \( x(n) \). Assuming \( \varepsilon_{opt}(k) = D(k) \), \( k = 0, 1, 2, \ldots, N \), where \( D(k) = [D(1), D(2), D(3), \ldots, D(N-1)]^T \). Let the amplitude spectrum of \( X(k) \) and the optimal transmit signal \( D(k) \) as close as possible [7], assuming that \( \varepsilon_{opt}(k) = |X(k)|^2 \), where

\[
x(n) = F^{-1}\left(\varepsilon_{opt}(k)\right)^{1/2}
\]

(9)

Combined with the analysis of the previous content, we can know that the cost function is expressed by the minimum mean square error following as (10)

\[
H(\phi) = \sum_{k=0}^{N-1} \left[ D(k) - |X(k)|^2 \right]^2
\]

(10)

Assuming that the phase of \( X(k) = \theta(k) \), the expression of \( X(k) \) is shown in Equation 8, then the expression is \( X(k) = |X(k)|e^{j\theta(k)} \). Thus, the above formula (10) can be expressed as

\[
H(\phi) = \sum_{k=0}^{N-1} \left[ D(k)e^{j\theta(k)} - X(k) \right]^2
\]

(11)

\( W \) is a Discrete Fourier Transform (DFT) matrix of \( N \times M \). Using the principle of discrete Fourier transform

\[
X(k) = \sum_{n=0}^{N-1} x(n)W_n^k, k = 0, 1, 2, \ldots, N - 1
\]

(12)

Where \( (W_n)^k = e^{-\frac{j2\pi n}{N}} \).

Using the Fourier transform principle, the transformation matrix \( W \) proposed in this paper is

\[
W = \begin{bmatrix}
W_0^0 & W_0^1 & \cdots & W_0^{N-1}\\
W_1^0 & W_1^1 & \cdots & W_1^{N-1}\\
\vdots & \vdots & \ddots & \vdots \\
W_{N-1}^0 & W_{N-1}^1 & \cdots & W_{N-1}^{N-1}
\end{bmatrix}
\]

(13)

Therefore, from the above analysis, finally we get

\[
X = Wx
\]

(14)

Supposing \( q = [D(0)e^{j\theta_0}, D(1)e^{j\theta_0}, D(2)e^{j\theta_0}, \ldots, D(N-1)e^{j\theta_{N-1}}]^T \). After this conversion, the minimum mean square error can be expressed as

\[
H(\phi) = (q - Wx)^H(q - Wx)
\]

(15)

Where, the above formula \( H \) represents conjugate transpose. Using the linear minimum estimator in the mathematical estimation method to finally find the minimum estimator of the time-domain waveform \( x \) is
\[ \hat{x} = W^{H} q / N \tag{16} \]

The phase \( \theta(k) = [\theta(0), \theta(1), \theta(2), \ldots, \theta(N-1)] \) of the vector \( q \) is determined by \( X(k) \). The specific steps are: Initialization number of iterations \( l, \theta \); Calculate \( \hat{x}^{l+1} \); Update \( \hat{x}^{l+1} \); Calculated \( \tilde{X}^{l+1} \) and error \( \Delta \).

4. Simulation Results and Analysis

System bandwidth \( B \) is 10 MHz, system transmit energy \( E = 10^3 J \). The frequency \( f \) takes the value within \([0, B]\). Figure 1 shows the target spectrum of four known targets in the research process, the time-domain synthesis method proposed in this paper is used to further perform \( x(n) \) in time-domain synthesis. Figure 2 shows the distribution of clutter energy spectral density within the normalized frequency and the total target spectrum within the normalized frequency after multi-target weighting during the experiment.

![Figure 1. Multi-target spectrum](image1)

![Figure 2. Target spectrum and clutter spectrum based on mutual information clutter](image2)

This paper proposes a time-domain synthesis algorithm based on SINR under energy constraints. The optimal transmit waveform spectrum based on SINR is evenly distributed in all frequencies.

![Figure 3. Time-domain synthesis signal based on signal-to-interference ratio](image3)

Figure 3 is the phase information of the time-domain synthesized signal. It can be seen from Figure 3 that the amplitude of the transmitted waveform is constant at 1.
It can be seen from Figure 4 that as the number of iterations increases, in the case of $\gamma = 0.01$, it will become smaller and smaller, and eventually approaches 0.01. At the 17th iteration, the error value was 0.0099628, and the iteration was finally stopped; at this time, the optimal transmit waveform was closest to the time-domain synthesized signal.

5. Conclusion
Based on the traditional water-filling algorithm, this paper proposes LWS algorithm SINR-based. The optimal transmit waveform energy spectrum $e_{opt}(f)$ is obtained. Adding the idea of phase, by synthesizing constant-amplitude time-domain signals, a new method is used to study the design of the transmit waveform based on the LWS algorithm. Finally, through simulation verification, after multiple iterations, the designed transmit waveform is very close to the optimal transmit waveform. The algorithm proposed in this paper adds the idea of phase, and can obtain more target information through time-domain synthesis algorithm of the transmit waveform.

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7. References
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