Generation of custom acoustic harmonic bursts from spherical Helmholtz resonators using Q-switched Nd:YAG laser induced plasma

H. S. Ayoub1,2,3 · Ashraf F. El-Sherif2 · Diaa Ibrahim4 · M. Khairy ElTahlawy2 · Walid Gomaa2 · Y. S. Nada3 · Sana M. A. Maize3 · Y. H. Elbashar5

Received: 28 March 2021 / Accepted: 7 August 2021 / Published online: 14 August 2021 © The Author(s), under exclusive licence to Springer Science+Business Media, LLC, part of Springer Nature 2021

Abstract
We present a new method for generating a 4 to 25 ms high power acoustic harmonic bursts, reaching more than 110 dB Sound Pressure Level (SPL), from the spherical Helmholtz resonators. The method uses Q-switched Nd:YAG laser pulses (wavelength = 1064 nm, pulse width = 6 ns, and energy = 450 mJ) to induce plasma shocks inside an AISI 316L stainless steel cavity. The confined plasma shock produces an acoustic burst of temporal standing waves which are characterized by a wide harmonic bandwidth. The frequency response of the system depends on the geometry of the used Helmholtz resonator as well as the laser wavelength (with constant laser pulse duration and fluence). The experiments reveal the dependence of the odd/even harmonic on laser wavelength. This method is a prospective alternative for the dodecahedron loudspeakers, other sources in ISO and standard audio tests.

Keywords Acoustics harmonic bursts · Spherical Helmholtz resonators · Q-Switched Nd:YAG · Laser-induced plasma · AISI 316L stainless steel · Dodecahedron loudspeakers

1 Introduction
One of the challenges in the field of sound measurements and testing is to obtain an omnidirectional, short pulse width and high-power pulsed acoustic source (Papadakis and Stavroulakis 2019). However, the smaller is the pulse width, the harder is the problem, because of the limited frequency response and low sonic/subsonic yield of the acoustic source. For
example (Fahy 2000), the fluctuating fluid volume–mass audible sources (like the dynamic speakers, the pressurized gas nozzle injectors, the laser-induced breakdown (LIB) (Qin and Attenborough 2004; Oksanen and Hietanen 1994; Gómez Bolaños et al. 2013,2014; Hosoya et al. 2013; Gómez-Bolaños et al. 2015), the electric sparks, and the explosive charges) generate a high-intensity sound with a very poor impulse response and/or bad frequency selectivity. Also, the accelerating-fluctuating force on fluid audible sources, where the net volume acceleration equals zero (like the rotating blades, the rotating aerofoils, and vortex shedding), have a very bad response on the pulsed modulation. Finally, the fluctuating fluid shear stress audible sources, where the net fluid volume nor net force equal zero (like the jet flows, the vibrating spheres and the tuning forks), have a very long pulse width in spite of their good frequency-selectivity. In this work, we have combined the fluctuating fluid volume–mass and the fluctuating fluid shear stress audible sources into a new hybrid category. The idea behind this combination, is to make use of the high power and the short pulse-width LIB, to induce a Helmholtz resonance inside a hollow metallic sphere. The yielded acoustic spectrum of the high power impulse, is then customized according to the geometry of the resonator, the wall material characteristics and the laser beam parameters. The customized acoustic harmonic burst spectrum is very broad and independent of that produced by LIB, which is characterized by a random and uncontrollable high frequency bandwidth. Usually, Helmholtz resonators are used to create or to absorb a specific frequency. The reason behind this sharp resonance is due to the presence of reverberation air column inside the Helmholtz cavity neck, which acts as an air spring-dash pot resonance system (Fahy 2000). Therefore, the utilization of a Helmholtz resonator in order to create an omnidirectional acoustic source with even frequency response, seems contradictory. But it is easy to understand this exception given that the cavity is neck-less, which broadened the resonance curve, to comprise an infinite number of resonance frequencies inside the spherical cavity. The contribution of cavity thin wall vibrations, allows the generation of odd/even frequency response with an omnidirectional acoustic field granted by the spherical symmetry of the resonator. Unlike dodecahedron speakers (Kuttruff 2016), the proposed method merged between the tuned high power audible bursts and the short emission duration in one design, where the sound radiation omnidirectivity is pretty much the same. To maximize the acoustic output of the resonator, the pulsed laser beam is focused through a fine hole, on the internal wall of the spherical cavity (as shown in Fig. 1a), resulting in ablating the thin metallic layer and the generating a high pressure plasma plume (as shown in Fig. 1b).

Fig. 1 Illustration of the laser induced plasma Helmholtz resonator: a an imaginary section in the resonator showing the laser strike spot and the plasma plume shockwave b plasma plume formation
Hence, the wall of the resonator was selected to be made from a ferrous alloy, such as stainless steel, to withstand the successive laser ablative interaction (Zhang et al. 2019). The confined plasma plume induces a series of harmonic standing waves bursts, where the characterization of such acoustic output is the subject of this work. Since that the used Helmholtz resonator was manufactured in form of a spherical cavity of relatively thin metallic shell, the spherical harmonics reactions are expected to propagate through the walls. The photoacoustic/thermoelastic reactions (Ayoub et al. 2021) will be ignored, and our analysis will be focused on the through-air Laser-induced plasma (LIP) acoustics. The prospective applications of this method in physical testing and engineering that requires a spherical sound field are numerous (Rossing 2007). For example, in architectural acoustics, the impact sound level measurements, the airborne sound insulation, the reverberation time testing, the audio calibration, and the standard acoustic testing (where the most important are the ISO 3382, ISO 140, ISO 10140, ISO 16283, and ISO 717 test series). The key component in these tests, is the dodecahedron loudspeaker (shown in Fig. 2a) that is capable of delivering a Sound Pressure Level (SPL) up to 120 dB. This type of acoustic transducers is made from a polymeric hollow chassis in the form of a regular dodecahedron with 12 faces, where each one has a dynamic speaker. All speakers are connected together in phase, in order to produce an overlapped homogeneous sound field that simulates an acoustic point source (Arnela et al. 2018). As shown in Fig. 2b, the dodecahedron loudspeaker has a spherical sound field at the low frequencies (i.e. 100 Hz), but it appears to turn into the flower shape at the high frequencies (i.e. 3.15 kHz) (Quested et al. 2014), which is not the case for our method.

2 Theory

The kernel of this work is to transform the energy of the LIP (with a high-frequency acoustic response (Hosoya et al. 2013)) into a long-lasting harmonic acoustic pulse with a custom lower frequency spectrum. To achieve this goal, a simple derivation is used to build up a theoretical model, capable of characterizing the basic parameters of this process. So, we started with an intense laser-pulse striking a metallic surface in the x–y plane, then an induced ablative-reaction starts, and a plasma plume is formed. This particular interaction can be modeled

![Diagram showing a typical dodecahedron loudspeaker assembly and its sound field polar plot.](image-url)
through the one-dimensional heat equation, describing the heat flow through the z-direction (which representing the laser-beam propagation), the melting and solidification front, into the irradiated homogenous metal, which take the form (Stafe et al. 2010):

\[
\rho \left[ \frac{C_p}{\rho} + \Delta H_m \delta (T - T_m) \right] \left( \frac{\partial T}{\partial t} - v_{ab} \frac{\partial T}{\partial z} \right) - K \frac{\partial^2 T}{\partial z^2} = \alpha (1 - R) I e^{-\alpha z - f_{pl}^2 a_{pl} v_{pl} dt} \tag{1}
\]

where \( \rho \) is metal density, \( C_p \) is heat capacity of the metal at constant pressure, \( T \) is temperature, the melting \( \Delta H_m \) enthalpy at melting point \( T_m \), \( t \) is the time, \( v_{ab} \) is the ablation velocity, \( K \) is the thermal conductivity, \( \alpha \) is the absorption coefficient, \( R \) is the reflectivity of the surface, \( \alpha_{pl} \) is the plasma optical absorption coefficient, \( I \) is the laser intensity, \( v_{pl} \) is the hydrodynamic velocity of the plasma plume. The right hand-side term represents the laser beam as a heating source, over the time interval from \( t_1 \) to \( t_2 \) (representing the time span of the full width at half maximum (FWHM) of the laser pulse). Due to the intense ablative reaction, thermo-elastic stress waves are generated, with energy proportional to the square of the ablation velocity \( v_{ab} \) which is obtained from the Hertz–Knudsen equation using the expression (Stafe et al. 2010):

\[
v_{ab} = 0.32 \left( \frac{MP_s}{\rho K_B T_s} \right)^{1/2} \tag{2}
\]

where \( T_s \) is the metal surface temperature, \( P_s \) is the saturated vapor pressure above the metal surface, \( K_B \) is the Boltzmann constant and \( M \) is the metal atomic mass. The hydrodynamic velocity of the plasma plume \( v_{pl} \) is then given by the expression:

\[
v_{pl} = 0.82 \left( \frac{\gamma K_B T_s}{M} \right)^{1/2} \tag{3}
\]

where \( \gamma \) is the ratio between heat capacitances at constant pressure and volume \( \left( \frac{C_p}{C_v} \right) \). The sound intensity \( I_{pl} \) of the plasma plume shockwave at the Knudsen layer, is approximately the product of \( P_s \) times \( v_{pl} \) hence:

\[
I_{pl} = 8 \rho v_{pl}^2 \gamma^2 \left( \frac{K_B T_s}{M} \right)^{3/2} \tag{4}
\]

Equation 4 shows that \( I_{pl} \) is proportional to \( T_s^{1.5} \) under the assumption that the plasma formation threshold temperature is exceeded. Hence, \( T_s \) depends on the radiation energy density, the threshold laser fluence \( F_{th} \) of nanosecond plasma formation can be expressed as (Cabalin and Laserna 1998):

\[
F_{th} = L_v \left( \frac{\tau \rho K}{C_p} \right)^{1/3} \tag{5}
\]

where \( L_v \) is the latent heat of evaporation and \( \tau \) is the laser pulse duration. By solving Eqs. 4 and 5 together and by eliminating \( \rho \) to obtain the expression:

\[
I_{pl} = \frac{8 C_p v_{pl}^2 v_{ab}^2 F_{th}^2}{\tau K L_v^2} \left( \frac{K_B T_s}{M} \right)^{3/2} \tag{6}
\]
As the plasma plume expands and continues to cool down, the plasma velocity tends to drop quickly, and the shockwave is propagating through the cavity atmosphere at the speed of sound. The sound intensity is proportional to the square of the sound pressure $p$ so that (Sound Power Measurements 1992):

$$\frac{I}{I_0} = \frac{p^2}{p_o^2} \quad (7)$$

By choosing $I = I_{pl}$, $I_0 = 1 \times 10^{-12}$ W/m$^2$ which is a standard reference value and by choosing $p_o = 20 \times 10^{-6}$ Pa which represents the hearing threshold pressure, then the sound pressure in front of the plasma plume is given by the expression:

$$p_{pl} = 14.37 \frac{\gamma_{va} F_{th}}{L_v} \left( \frac{C_p}{\gamma K} \right)^{\frac{1}{2}} \left( \frac{K_B T_s}{M} \right)^{\frac{3}{4}} \quad (8)$$

Equation 8 shows that the sound pressure at the plasma plume front, does not depend on any geometrical factors related to the target. The SPL expressed in decibels is given by the expression:

$$SPL_{pl (dB)} = 20 \log_{10} \left( \frac{p_{pl}}{p_o} \right) = 20 \log_{10} \left( \frac{7.1865 \times 10^5 \gamma_{va} F_{th}}{L_v} \left( \frac{C_p}{\gamma K} \right)^{\frac{1}{2}} \left( \frac{K_B T_s}{M} \right)^{\frac{3}{4}} \right) \quad (9)$$

The shockwave induces a wide range of harmonic standing waves inside the Helmholtz cavity, each one has a specific intensity given by the expression:

$$I_{H_n} = 2 \rho_{air} C \pi^2 n^2 \delta_n^2 f_H^2 \quad (10)$$

where $\rho_{air}$ is air density, $C$ is the sound speed in air, $n$ is the harmonic multiplicity, $\delta_n$ is the amplitude of pressure variation and $f_H$ is the fundamental Helmholtz frequency, which is given by the expression (Greene et al. 2009; Wolfe 2000; Raichel 2006; Hussain et al. 2016; Kramida et al. 2019):

$$f_H = \frac{dC}{\pi} \sqrt{\frac{3}{8(L + 0.85d)D^3}} \quad (11)$$

where $D$ is the entrance hole diameter, is the internal cavity diameter and $L$ is the true length of the neck. Since $C = 340$ m/s and $L = 0$ for neckless Helmholtz cavities, therefore Eq. 11 can be simplified to:

$$f_H = 72.6 \sqrt{\frac{d}{D^3}} \quad (12)$$

Usually, Helmholtz cavities have a very strong resonance at $f_H$, with no higher-order harmonics due to the presence of the neck, controlling the acoustic impedance of the system. But in our case, where the cavity has no neck, it is possible to generate higher-order harmonics. Since the energy of the plasma shockwave is transformed into harmonic standing waves of different pressure amplitudes then:
Equation 13 is just an acceptable approximation to simplify the analysis. If a detailed analysis is required for decomposing the pressure amplitude, then a standard discrete Fourier transform should be applied and followed by Paseval’s inversion theorem. Applying Eqs. 6 and 12 into Eq. 13 we obtain the expression:

$$I_{pl} = \sum_{n=1}^{\infty} I_{H_n} = 2\rho_{air} C_\pi^2 f_H^2 \sum_{n=1}^{\infty} n^2 \delta_n^2$$  

(13)

Equation 14 shows that total pressure amplitude variation function $\sqrt{\sum_{n=1}^{\infty} n^2 \delta_n^2}$ at the plasma threshold, is proportional to $D^{1.5}$ and $d^{-0.5}$ which are a geometrical parameters, $F_{th}$ and $\tau^{-0.5}$ which are a radiation parameters and proportional to $v_{ab}$, $T_{th}^{0.75}$, $M^{-0.75}$, $L_v^{-1}$, $c_p^{0.5}$, $\gamma$ and $K^{-0.5}$, which are all a material characteristic-parameters. This result will help us in optimizing the suitable material that produces the maximum sound pressure in this particular application. For example, the best material should be made of a metal of low atomic mass and high melting-point. Since the other parameters are almost similar for most metals, especially the heat capacitance, it is recommended to select an alloy with a good mechanical and chemical properties, which withstands the extreme operation-condition under the high-power laser irradiation. One of the best materials for this application (Stainless Steels Grade Datasheets 2013) is the stainless steel which has a good oxidization resistance, a high melting point, and a good mechanical properties. One more advantage is that stainless steel alloys contain Ni, Cr, and Mo in addition to Fe (the major constituent), which are all transition elements from the same 3d group in the periodic table of elements. These elements are close in atomic mass so that $M$ in our calculation will be based on that of Fe. It should be mention that $F_{th}$ depends strongly on the laser wavelength $\lambda_L$. Therefore, $I_{pl}$ will depend proportionally on the laser fluence $F$ at this specific wavelength and pulse duration. In general, the plasma plume lasts for more than 20 ns in case of Q-switched LIP, but the acoustic pressure signal will last very much longer. The acoustic pulse duration depends basically on acoustic time constant of the Helmholtz resonator cavity $\tau_c$, where the impulse response of the cavity (defined as the ratio of the output to input sound intensity $I_{in}$) is given by the expression:

$$\frac{I_{out}}{I_{in}} = \left( \frac{p_{out}}{p_{pl}} \right)^2 = e^{-1/f_H \tau_c}$$  

(15)

Moreover, $\tau_c$ can be calculated from the basic expression:

$$\tau_c = \frac{Q}{\pi f_H}$$  

(16)

where $Q$ is the quality factor of the resonator. Eventually, the repetition rate of the laser pulses equals that of the acoustic bursts as long as their relaxation time is always smaller than the time between two successive laser pulses. One important issue that should be mentioned before ending our formulation, is the mode coupling. This phenomena controls the amplification of some harmonic mode over others. One of the principle causes of this phenomena, is the surface acoustic waves propagating through the spherical shell of the
Helmholtz cavity (Ayoub et al. 2021), at sixteen times the sound speed in air and causing complex breathing modes as shown in Fig. 3.

These elastic modes participate in the inhalation and the creation of Helmholtz harmonic pressure oscillations in front of the shockwave of plasma plume, by scattered reflection and interference. To minimize the effect of this phenomena on the acoustic harmonic yield of the resonator, it is recommended to avoid using cavities of thin shell thickness, in order to quench Rayleigh-Lambert and Tesseral waves. It is recommended also to control the laser fluence to moderate the generation of thermoelastic reaction inside the cavity wall. If the laser wavelength is changed at constant fluence, it is expected that mode coupling will take place, because of the dependency of wall reflectivity on laser wavelength. Usually, the metallic reflectivity drops at high laser fluence to 20% of its original value (Redmond and Lones 1952), causing more energy absorption, higher thermoelastic reaction, and consequently higher cavity vibration modes.

3 Experimental work

The aim of the experimental work, is to measure and characterize the effect of cavity diameter, laser wavelength and fluence on the temporal acoustic yield of the spherical Helmholtz resonators. Finally, to evaluate the performance of this method in comparison with dodecahedron speaker.

3.1 Experimental samples

According to Eq. 8, the resonator material is selected from Ashby diagrams to satisfy and maximize the acoustic pressure yield of the method. AISI 316L stainless steel was our selection that fulfill the parametric requirements previously discussed in Sect. 2. Table 1 summarizes the most important physical characteristics for the selected material.

To determine \( f_H \) for the samples, Eq. 12 are used to plot \( f_H \) as a function of \( d_H \), for two cavities of diameters 11 and 23 cm respectively (as shown in Fig. 4a). Then, \( d \) was selected to be of 3.5 mm for both cavities, leading to a fundamental resonance of 78 Hz and 31 Hz. If these frequencies are multiplied by the multiplicity \( n \), then we obtain the harmonic Helmholtz spectrum of both cavities as shown in Fig. 4b.

A sheets of 0.5 mm-thick of AISI 316L stainless steel were machined into hemispherical cups using high speed press stamping technique. A special molds where used to produce interlocking edges with a tolerance of ±0.01 mm. The hemispheres were then polished to 1 µm roughness and interlocked to form a hollow spheres of 11 and 23 cm inner diameter. Two holes were drilled in each sphere such that, the first hole was of 3.5 mm wide, located at the mid distance between a hemisphere’s pole and its edge to be used as beam entrance. The second hole was of 4 mm wide and located 1 cm from the edge of the

![Fig. 3 Illustration of cavity breathing modes as a result of an intense laser thermoelastic impulse](image)
### Table 1: Physical characteristics of AISI 316L stainless steel*

|              | \(C_p\) (kJ/kg.K) | \(\gamma\) (-) | \(\Delta H_m\) (kJ/kg) | \(L_v\) (kJ/kg.K) | \(K\) (W/m.K) | \(T_{\text{melting}}\) (K) | \(F_{\text{th} \, 1064\text{nm}}\) (J/cm\(^2\)) | \(F_{\text{th} \, 532\text{nm}}\) (J/cm\(^2\)) | \(F_{\text{th} \, 355\text{nm}}\) (J/cm\(^2\)) |
|--------------|-------------------|-----------------|-------------------------|--------------------|----------------|---------------------------|----------------------------------------|----------------------------------------|----------------------------------------|
| Stainless Steels Grade Datasheets (2013), Pichler et al. (2020) | 0.5              | 1.66            | 270                     | 6330               | 16             | 1381                      | 1.42                                   | 1.56                                   | 1.75                                   |
| Redmond and Lones (1952) |                   |                 |                         |                    |                |                           |                                        |                                        |                                        |
| Stainless Steels Grade Datasheets (2013), Pichler et al. (2020) |                   |                 |                          |                    |                |                           |                                        |                                        |                                        |
| Kim (1975) | Stainless Steels Grade Datasheets (2013) | Stainless Steels Grade Datasheets (2013) | Cabalin and Laserna (1998) | Cabalin and Laserna (1998) |

*69wt% Fe, 18wt% Cr, 10wt% Ni, 3wt% Mo (Cabalin and Laserna 1998), †Obtained by exponential regression of reference (Cabalin and Laserna 1998) data
other hemisphere, to be used as a mount for an optical fiber window. Both hemispheres were interlocked so that each hole is radially perpendicular to the other.

3.2 Experimental setup

An experimental setup was designed to measure the sound pressure and the LIB spectrum of a Nd:YAG laser generated plasma inside the Helmholtz cavities as shown in Fig. 5a and b. The major components of the setup are: Thorlabs SIR5-FC fast response fiber optic photodetector amplifier was used at first for beam characterization. Ocean optics USB4000 VIS–NIR spectrometer was used for spectral irradiance and LIP spectroscopy. The experimental setup shown in Fig. 5b is consisting of a Quantel Q-smart 450 Nd:YAG laser source (with second and third harmonic generation units) to induce a plasma shocks inside the Helmholtz cavities sphere mounted on x–y–z precision translation stage. Foam rubber ring of 10 mm-thick and 40 mm inner-diameter was used to mount the Helmholtz cavities. Pigtailed fiber and sockets for connecting optical devices to the spherical cavities. GRAS 46AG high precision flat response microphone connected to a 40 dB amplifier unit, used for sound pressure measurements. Acoustic insulation foam panels, used to build a $1.75 \times 2 \times 2$ m$^2$ full anechoic chamber around the resonators holder, of 21 dB noise level. The laser source was placed outside the chamber and the laser beam penetrates inside through a tinny quartz window in the foam.

![Fig. 4](image1) Effect of sphere diameter on Helmholtz resonance a as function of hole diameter b as function of multiplicity

![Fig. 5](image2) Measurement setup a illustration of the system b experimental setup
The laser divergence was controlled using \( \times 10 \), zoom beam expander/reducer, coated for Nd:YAG laser wavelengths. PixeLink PL-B781U CMOS camera synchronized with the laser source, to acquire the LIP plume peak optical emission profile. Gentec Solo 2, laser energy and power meter, used for laser energy measurements. Finally, TSI Quest SoundPro sound level meter was used to perform average SPL calibrations and measurements. We used an Agilent DSO-X 3052A digital storage oscilloscope/spectrum analyzer to acquire the time and frequency domain data (Table 2).

### 3.3 Experimental procedure

The experimental procedure was performed on 4 stages, each stage was repeated 3 times using different laser wavelengths. In some particular stages, the tests were repeated twice using different sphere diameter. The first stage was to characterize the laser pulses inside the resonator by connecting the fast response fiber optic photodetector amplifier to the Helmholtz cavity. In this stage, the SLP was measured using the microphone placed just near the entrance hole, to characterize the acoustic output signal. The second stage was to acquire the SPL inside the anechoic chamber and to plot the acoustic pressure field around the resonator with the microphone. In this stage the laser was kept outside the chamber, to exclude the laser head discharge noise from being measured. The third stage was to characterize the LIP spectral irradiance inside the cavity, by connecting the fiber optic spectrometer to the resonator. Finally, the fourth stage was to capture the peak optical emission profile of LIP plume on the outer surface of the cavity, since it was difficult to introduce the camera inside the closed sphere. Part of the data analysis was performed using Mikropack-SpecLine and DADiSP softwares.

### 4 Results and analysis

All of the acquired data during the experimental procedure was processed, plotted and summarized in the following order:

#### 4.1 Effect of resonator diameter on the acoustic output

##### 4.1.1 Impulse response

As shown in Fig. 6a, a train of pulses (with wavelength = 1064 nm, and pulse width = 6 ns) struck the inner surface of the 23 cm-diameter cavity, at 20 Hz repetition rate. In response, an analogue train of acoustic bursts, was received by the microphone 3 cm-apart from the cavity entrance hole. The bursts were stable, repetitive and identical in shape as shown in Fig. 6b. Noting that the laser fluence was about 5 J/cm², which is larger than \( F_{th} \), in order to guarantee the formation of LIP plume. The time constant of the decayed oscillations was in the order of 21 to 25 ms. A similar but different results

| Wavelength | 1064 nm | 532 nm | 355 nm |
|------------|---------|--------|--------|
| Energy per pulse (mJ) | 450 | 220 | 130 |
| Pulse duration (ns) | 6 | 5 | 5 |
were obtained with other laser wavelengths. When the diameter of the resonator was reduced to 11 cm, the time constant dropped to a less than 4 to 5 ms depending on the used laser wavelength. The delay time between optical impulse and the acoustic signal is less than 0.79 ms in most cases.

As shown in Fig. 7a, the impulse response of the larger cavity is characterized by a high pressure direct sound (DS) peak of pulse duration $\tau_{DS}$, followed by an initial time delay gap (ITDG) of 0.25 ms separating a first order reflection (FOR) and reverberant time (RT). To calculate the distance from the microphone to the cavity surface, we can multiply half the ITDG times the speed of sound to yield a reasonable span of approximately 4.25 cm. In this context, the RT could be of non-architecture origin, since that the measurement took place in anechoic chamber. The time interval separating the RT signal is almost 1 ms, representing a cavity flexure modes of 1 kHz. The complexity of the flexure peaks is due to modulation by cavity breathing modes. As shown in Fig. 7b, when the diameter of the cavity is reduced to 11 cm, the DS pressure peak and the FOR amplitude also decreased. The ITDG remains unchanged, but the RT has become of much shorter interval representing a cavity flexure modes of 2.5 kHz, causing the broadening DS profile.

Fig. 6. 20 Hz pulse train of a 1064 nm, 6 ns Nd:YAG laser irradiating the inner surface of a 23 cm diameter Helmholtz cavity at 5 J/cm$^2$ fluence b acoustic burst of standing Helmholtz pressure wave at the cavity entrance hole

Fig. 7 Impulse response of the Helmholtz cavities for a D = 23 cm b D = 11 cm
4.1.2 Frequency response

As shown in Fig. 8a and b, the acoustic spectral response for both cavities was obtained by applying fast Fourier transforms (FFT) on the impulse responses of Fig. 7a and b. Both spectral patterns showed an exponential SPL decay as proceeding toward the high-end band that can be approximated using the empirical expression:

$$\text{SPL}_{\text{approximated}} (\text{dB}) = \text{a} \cdot \text{e}^{\text{nbfH}} = \left(20 \log_{10} \frac{\bar{V}_m}{p_0} \frac{G}{S}\right) e^{72.6 \text{n} b \sqrt{\frac{2 \pi}{\nu}}}, \quad (17)$$

where a is the average SPL, represented as a function of the voltage signal $\bar{V}_m$, obtained by a microphone of pressure sensitivity S, followed by an amplifier of gain G, and b is the attenuation coefficient. The value of a is very broad and ranging from 53 to 83 dB, unlike the value of b, which equals approximately from $-3$ to $-4 \times 10^{-6}$ Hz$^{-1}$, over the whole wavelength range of the used laser. The peak pressure variation $\delta_{\text{max}}$ occurred at $n = 16$ for the large cavity and at $n = 29$ for the small cavity at laser wavelength of 1064 nm. Then, $\delta_{\text{max}}$ occurred at $n = 26$ for the large cavity and $n = 104$ for the small cavity at laser wavelength of 532 nm. And finally drops to $n = 23$ for the large cavity and $n = 33$ for the small cavity at laser wavelength of 355 nm. The peak pressure resonance frequency for the large cavity is one order of magnitude less than that of the small cavity. The reason behind the generation of higher frequency modes in case of the 532 nm laser, may due to low plasma self-absorption, which led to higher thermoelastic cavity modes. A summary of resonators temporal and spectral responses versus laser wavelength is inclusive in Table 3.

4.1.3 SPL field

As shown in Fig. 9a, the laser beam lost 4% of its initial intensity by reflection on the window’s surface, but the SPL generated by the photo-acoustic effect was negligible because of the low optical absorption coefficient of quartz. The sound pressure field around the Helmholtz cavities was homogenous, with peak average SPL in front of the resonator holes as shown in Fig. 9b. The acoustic yield of the large resonator seemed to be higher than the small one. The contour plot shows a quasi-spherical sound field around the cavities,

---

**Fig. 8** Measured (unsmoothed) and empirically modeled frequency response of Helmholtz cavities of a D = 23 cm b D = 11 cm
which means that part of the acoustic pressure is delivered through the solid cavity wall bounce. This result supports our interpretation of mode coupling and its effect in selecting higher-order resonance frequencies. The resultant average SPL field is compared to that of a semi-dodecahedron speaker. It was found also that changing the laser wavelength has no influence on the average SPL field distribution, but has a remarkable effect on the SPL magnitude.

### 4.2 Effect of laser wavelength

#### 4.2.1 Plasma plume shape

At fixed laser fluence, the plasma vapor temperature $T_p$ is the most effective parameter on which depends the pressure of the shockwave in front of the plasma plume. For plasma vapor of sonic speed $T_p \sim 0.67 T_s$ which means that irradiance of the plasma depends upon

\[ T_p \sim 0.67 T_s \]


the temperature of the Knudsen layer (Mościcki et al. 2011). The radius of the plasma plume depends on $T_S$, but since Mie scattering plays an important role in increasing the amount of absorbed radiation, it is expected that $T_p$ will be higher than the predicted as the wavelength decreases.

As shown in Fig. 10a, the plasma plume (induced by the principal Nd:YAG wavelength) has a flame-dome shape of 7 mm radius, with no apparent optical scattering seen in the direction of the imaging. A similar result was obtained with the second harmonic generation wavelength, but the plasma plume was of 6 mm radius regular dome shape, with apparent optical scattering seen at the direction of imaging as shown in Fig. 10b. Finally, a 4 mm radius dome shape plasma plume with intense apparent optical scattering was induced by the third harmonic generation laser wavelength as shown in Fig. 10c.

4.2.2 LIBS analysis

The spectral irradiance of the obtained plasmas (shown in Fig. 11), revealed that the thermal yield of the laser interaction with the stainless steel was based on the rotational transition of the ionized atomic constituents rather than the vibrational transitions of
the air molecules. The major contributing element was the ionized iron, then came the chromium and the nickel. It was obvious that the principal Nd:YAG laser line, produces the most intense plasma irradiance over the entire VIS/NIR spectrum. It has the strongest effect on ionizing the air atoms in the range from 550 to 650 nm. This could explain why the shape and the length of the plasma plume, induced by this laser wavelength, was the largest among other harmonics (Fig. 10a). Also, it was noticed that the second harmonic generation laser line has a stronger effect on exciting atmospheric atoms than the third harmonic generation line that eventually has a stronger ionization effect in the spectral range from 355 to 500 nm. This result came into agreement with the visual characterization of Fig. 10b and c, where the length of the plasma plume induced by the second harmonic generation laser line, as compared to that generated by the principal line of Fig. 10a and larger than than that of the third harmonic generation line. The reason behind this result may be referred to plasma self-absorption and Mie scattering (Mościcki et al. 2011) of the laser radiation by the plasma constituents.

To determine the plasma temperature $T_p$, which represents the speed of heavy ions, we must first estimate the electrons temperature $T_e$ by using the Boltzmann plot method (Gornushkin et al. 2010; Hussain et al. 2016), given that the intensity of a spectral line representing the transition between two energy levels is given by the expression:

$$I_s = \frac{hc g_k A_{ik} N_o}{4\pi \lambda_s u} e^{-E_k/K_BT_e}$$  \hfill (18)

where $h$ is Planck’s constant, $c$ is the velocity of light, $\lambda_s$ is the wavelength of optical transition between lower energy states i and higher energy state k, $g$ is the degeneracy of upper state, $A_{ik}$ is the transition probability, $E_k$ is the energy of the upper state, $U$ is the partition function and $N_o$ is electron density of the state. By modifying Eq. 18 to yield the expression:

$$\ln\left(\frac{\lambda_s I_s}{g_k A_{ik}}\right) = -\frac{E_k}{K_BT_e} + \ln\left(\frac{4\pi U}{hc N_o}\right)$$  \hfill (19)

By selecting four $\text{Fe}^+$ emission lines and plotting $\ln\left(\frac{\lambda_s I_s}{g_k A_{ik}}\right)$ versus $E_k$ to obtain the slope $-\frac{1}{K_BT_e}$ and calculate the plasma temperature, assuming that the term $\ln\left(\frac{4\pi U}{hc N_o}\right)$ is constant. This procedure was repeated for each of the three laser wavelengths for emission lines 363.14 nm, 370.55 nm, 387.85 and, 526.95 nm, corresponding to upper energies of 3.42 eV, 3.34 eV, 3.2 eV and 2.36 eV. The transition probabilities and the state degeneracies were obtained from standard LIBS data base of reference (Kramida et al. 2019). The difference between $T_e$ and $T_p$ is only a few thousand of Kelvins so that $T_e \sim T_p \sim 0.67 T_s$ approximately. Table 4 summarizes the temperature results obtained from the Boltzmann plots.

| Table 4 | Calculated plasma temperature for different laser wavelengths |
|---------|-------------------------------------------------------------|
|         | 1064 nm | 532 nm | 355 nm |
| $T_e$(K) | 81,000 | 74,000 | 71,000 |
| $T_s$(K) | 54,270 | 49,580 | 47,570 |
| SPL pl (dB) | 289 | 283 | 279 |
These results revealed that, under constant fluence, $F_{th}$ and $T_s$ were the most effective parameters in determining the magnitude of sound pressure variation in Eq. 14, ignoring the optical characteristics of the cavity surface and of the plasma itself. The sources of error in our calculations were the exclusion of Mie absorption and the assumption that the plasma gas is monochromatic.

### 4.2.3 Acoustic multiplicity versus laser wavelength.

As shown in Fig. 12, the smaller cavity frequency response for 1064 nm laser pulse, was of the highest magnitude among other wavelengths, followed by the response for 532 nm, in accordance with the previously obtained temperature results. It was noticed also that the frequency response of the cavity is very poor in the bass range, but very intense in the midrange for all laser wavelengths. At the high-end range, a fast $-3$ dB cutoff of 9.906 kHz was seen in the case of 355 nm laser pulse and at 18.72 kHz for the 1064 nm laser pulse. Only the frequency response of the cavity for the 532 nm laser pulse remained flat at the high-end region. A similar behavior was seen in the case of the larger cavity, but the highest responses occurred at lower frequencies within the same octave ranges. It is good to mention that the 1064 nm laser pulse tends to induce resonance frequencies of odd multiplicity rather than the other wavelengths that tend to induce resonance frequencies of even multiplicity. This result is illustrated in Table 5. The reason behind this behavior, was due to the intensive mode coupling phenomena between the Helmholtz cavity’s breathing modes and the acoustic pressure resonance inside it, as mentioned before in Sect. 2.

If a monochromatic acoustic pulse is required, then a neck should be add to the Helmholtz cavity, to tolerate the acoustic impedance of the resonator and boost the pressure resonance at a certain frequency. In this case, Eq. 11 should apply instead of Eq. 12.

![Fig. 12 Unsmoothed octave band response of Helmholtz cavity D=11 cm induced by principal, second and third harmonic wavelengths of Q-switched Nd:YAG at F=5 J/cm²](image)
5 Conclusion

A few milliseconds width, high SPL harmonic acoustic impulses in the range of 60 to 110 dB, has been generated from neckless Helmholtz resonators, by intra-cavity nano-second LIP shockwaves. It was found that the generated SPL depends on the laser wavelength, the cavity diameter, the laser ablation threshold fluence of the cavity material, the plasma temperature, and the plasma pressure. Also, it was found that the frequency response of the cavities depending on the cavities diameter and laser wavelength. Hollow spheres resonators of different diameters made of AISI 316L stainless steel were subject to Q-switched Nd:YAG LIP at the principal, second, and third harmonic generation wavelength. Under constant laser fluence of 5 J/cm², an odd/even acoustic harmonic multiplicity dependence on laser wavelength was revealed. The SPL contour plot around the resonators seemed to be quasi-spherical due to the contribution of cavity breathing modes in sound generation. The dependence of Helmholtz harmonics on mode coupling phenomena is believed to be governing the acoustic frequency response. The LIBS analysis of the plasma spectrum estimated a plasma temperature in the order of $10^5$ K and a plasma pressure in the order of $10^8$ Pa. The prospective application of this method is to append dodecahedron loudspeakers in some standard acoustic tests.

References

Arnela, M., Guasch, O., Sánchez-Martín, P., Camps, J., Alsina-Pagès, R.M., Martínez-Suquía, C.: Construction of an omnidirectional parametric loudspeaker consisting in a spherical distribution of ultrasound transducers. Sensors 18(12), 4317 (2018)
