DC motor simulation transfer function estimation: case study
Proteus Ver. 7

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Abstract: Computing technology development today has a significant influence on the design process of a product or system even including the learning process in the field concerned. The electronics and control field is a field that greatly helped by these developments. Developments of electronic products or systems become faster and more efficient with the availability of CAD (Computer-Aided Design) software. Some Electronic CAD software provides DC motor model, as Proteus 7.0. Availability of this model is beneficial in control system designing and learning but the availability of a DC motor models are often constrained because are not accompanied with adequate information about internal parameters of the dc motor model or dc motor modeling done in programming so it is not possible to derive dc motor transfer function based on internal parameters. This study successfully showed transfer function derivation of dc motor in Proteus CAD simulation with a black-box approach for linear systems.

1. Introduction
Understanding the character of a system is a prerequisite for the analysis of systems[1,7]. Often this knowledge is also needed in designing a new system composed of several well-known systems before. For systems that are known in detail we can derive system model from the system components mathematics model that has been developed previously. For a system with unknown subsystem, then the system identification can be done by observing the system's response to a particular input. This approach is often referred to as a black box approach [2,3].

Within time domain, the black-box approach includes applied several alternative signals into the system; some of the signals are step signal, ramp signal, and impulse signal.

Proposals to use the unit step system response for the system transfer function determination started by Kupmuller [3] and this method is commonly used in industry [4,9].

In this study, we use the system response of step signal input to estimate system transfer function using MatLab identification toolbox. The system in this study is a dc motor model in Proteus ECAD tools ver7. A comparison between estimated transfer functions with real transfer function is carried out to judge validity of estimated transfer function. The contribution of this study is that it allows performing dc motor modeling on ECAD tools without having to obtain prior internal parameters of the dc motor.

2. Method
2.1 DC motor modeling
For a dc motor open-loop dynamic equation is[7,8,9]:
\[
L \frac{di}{dt} + iR + k_b \sigma = V \\
J \frac{d\omega}{dt} = k_t i + T_d
\]

where \( L \) is armature inductance (h), \( R \) is armature resistance (ohm), \( V \) is input voltage (volt), \( I \) for armature current (ampere), \( k_b \) for back emf constants, and \( K_t \) reveals as torque constants.

2.2 DC motor model in ECAD Proteus ver 7.0

Proteus ver 7.0 has DC motor model. DC motor model accommodates dc motor with inertia, and loading while the dc motor model with encoder is a model that accommodates dc motor and load inertia and equipped with encoder position. This study used a model of dc motors. The schematic model for dc motors as shown in figure 1.

2.3 DC motor model used in this study

In this study, DC motor with encoder is used. The motor model parameters are shown in Table 1.

| Parameters                  | Nominal             |
|-----------------------------|---------------------|
| \( R_a \) (Armature Resistance) | 22 ohm              |
| \( L_a \) (Armatur Inductance) | 100 mH              |
| Nominal Voltage             | 5V                  |

The output of Proteus dc motor model is in the form of pulses. These pulses are generated by an internal rotary encoder. Pulse duty cycle and frequency can be set. Due to output is not in the voltage form, but in form of pulse, it is necessary to use a frequency-to-voltage converter circuit so that the block diagram of the system shown in Figure 2.
2.4 Frequency to Voltage Converter Circuit Design

In this F/V converter circuit, LM131 IC is used as active component. F/V converter output at maximal input is designed at 10 V, with a maximum of 10% ripple voltage at a speed of 100 rpm. According to table 1, the maximum frequency is:

\[ f_{\text{max}} = \frac{20}{\text{rev min}} \times \frac{900}{60} = 300 \text{ hz} \]

From this maximal frequency, we can derive a minimal time period (\( T_{\text{min}} \)) as:

\[ T_{\text{min}} = \frac{1}{f_{\text{max}}} = \frac{1}{300} = 3.3 \text{ ms} \]

Based on \( T_{\text{min}} \), we get \( T_{\text{pulse min}} \):

\[ T_{\text{pulse min}} = 0.5 T_{\text{min}} = 1.67 \text{ ms} \]

In this study, we use \( C_t = 0.33 \mu F \), so \( R_t \) becomes:

\[ R_t = \frac{2.664 ms}{1.1 x 0.33 \mu F} = 7.3 \Omega \]

\( R_d \) and \( C_d \) determination based on rule 5 \( R_d C_d << T_{\text{pulse min}} \) (Jacob, J., 1989). Because \( T_{\text{pulse min}} \) is 1.67 ms and \( R_d = 10 \Omega \), taken as reference, we can conclude to calculate \( C_d \) as:

\[ 5R_d C_d \leq (0.1)(1.67 ms) \]

\[ C_d \leq \frac{(0.1)(1.67 ms)}{(5)(10 \Omega)} = 0.33 \mu F \]

\[ V_{\text{ave}} = \frac{2V(1.1R_t C_t)R_L f_{\text{in}}}{R_x} \]

\( V_{\text{ave}} \) is used to determining \( R_s \) value. With \( V_{\text{ave}} = 10 \text{ V} \), \( R_s \) has value (Jacob, J., 1989):

\[ R_s = \frac{(2V)(1.1R_t C_t)R_L f_{\text{in}}}{V_{\text{ave}}} \]

\[ R_s = 14.8 \Omega \]

As part of F/V converter circuit, a filter is added. This filter is consists of \( R_L \) and \( C_F \), filter circuit has time constant, \( \tau \), at \( \tau = R_L C_F \). \( V_{\text{out}} \) equation becomes [5]:

In order to get ripple voltage at 10% then:

\[ \frac{V_{\text{out}}}{V_{pk}} = 90\% = e^{-t/T} \]

\[ \ln(0.9) = - \frac{t}{R_L C} \]

\[ C_F \geq 0.72 \mu F, \]

In this filter circuit, \( C_F = 1 \mu F \) is used.
3. Results and Discussion

3.1 Characteristic and Stability test of DC motor simulation and In Proteus

In order to obtain the characteristics of the system, the system is fed with several step voltages. These step voltages have magnitude at 14V and 15 V. These step voltage response graphs shown in Figure 3 and Figure 4.

Figure 3. Graph Response for 14 V step input

Figure 4. Graph Response for 15 V step input

Figure 4 to 5 visual examination, shows the system is an overdamped system, its means that system has a dumping ratio greater than 1. Also based on graphics examination, proteus simulation results give consistent results, the system has a rise time value and setting a relatively constant as shown in the following table 2.

| V_step, V | N_settling, rpm | t_r, s | t_s, s |
|-----------|-----------------|--------|--------|
| 14        | 940             | 5.2    | 5.5    |
| 15        | 1100            | 5.2    | 5.5    |
| 16        | 1080            | 5.2    | 5.6    |

3.2 Sampled data from the DC motor model step response.

In order to get step response data sample, motor dc model is fed with 15 V step input and then data is sampled in 0.25 second. Table 3 shown this sampling result.

| Time, s | N, rpm | No. | N, rpm | No. | N, rpm | No. | N, rpm | No. | N, rpm |
|---------|--------|-----|--------|-----|--------|-----|--------|-----|--------|
| 0       | 0      | 1   | 6.4900 | 2   | 8.8200 | 3   | 9.7100 | 4   | 9.99   |
| 0.25    | 0.6900 | 1.25| 6.8200 | 2.25| 8.9600 | 3.25| 9.7600 | 4.25| 9.99   |
| 0.50    | 1.5900 | 1.50| 7.1100 | 2.50| 9.0800 | 3.50| 9.7900 | 4.50| 9.99   |
| 0.75    | 2.3400 | 1.75| 7.3600 | 2.75| 9.2000 | 3.75| 9.8100 | 4.75| 10     |

Data in Table 3 is used for transfer function estimation. Transfer function estimation obtained using Matlab, through the identification tool application. The model used is a process model for a second-order system, without delay and overdamped. The transfer function estimation results are:

\[
\frac{0.676}{0.583s^2 + 2.629s + 1}
\]

where \( Wn \) equals to 1, \( K \) is 0.676, and Dumping ratio is 1.31.
3.3 Transfer function confirmation
Transfer function confirmation test is done by comparing the dc motor response to a step input its included response from estimated transfer function and response from dc motor model. From both step response graph, graphically its seen that the value of \( t_r \), \( t_s \) and settling consistency between the two graphics, as we can see in figure 5 and figure 6.

Table 4 shows the comparison of \( t_r \), \( t_s \) and settling time between estimated transfer function and dc motor when it is fed with 14V.

### Table 4. Comparison table when the system is feed with 14V step voltage

| Parameter            | Proteus DC motor model | Matlab Transfer function | Deviation |
|----------------------|------------------------|--------------------------|-----------|
| \( N_{setting}, \text{rpm} \) | 9.4                    | 8.6                      | -8.5%     |
| \( t_r, \text{s} \)   | 5.2                    | 5.2                      | 0%        |
| \( t_s, \text{s} \)   | 5.5                    | 5.7                      | 3.6%      |

Based on the value of \( t_r \) and \( t_s \) from the three tables above, it can be concluded that the resulting estimates MatLab function approach with the actual transfer function from a dc motor in proteus simulation deviation values for \( t_r \), \( t_s \) and \( N_{setting} \) quite small at less than 1.5% when it was feed with 15 v input.

Confirmation of the transfer function estimation by transfer functions are also performed by comparing the actual value of a point per point for the sample time 0.25 seconds when 14 v was used as input (Table 5).

### Table 5. Result when system is feed with 14V step voltage

| Time(s) | Simulation | Estimation | \( \sqrt{\text{Err}^2} \) (%) | Time(s) | Simulation | Estimation | \( \sqrt{\text{Err}^2} \) (%) |
|---------|------------|------------|-------------------------------|---------|------------|------------|-------------------------------|
| 0.25    | 0.497      | 0.36       | 27.57                         | 6       | 8.59       | 9.23       | 7.45                          |
| 0.5     | 1.48       | 1.06       | 28.38                         | 6.25    | 8.74       | 9.34       | 6.86                          |
| 0.75    | 2.02       | 1.82       | 9.90                          | 6.5     | 8.81       | 9.48       | 7.60                          |
| 1       | 2.96       | 2.55       | 13.85                         | 6.75    | 8.86       | 9.47       | 6.88                          |
| 1.25    | 3.65       | 3.23       | 11.51                         | 7       | 8.91       | 9.58       | 7.52                          |
| 1.5     | 4.26       | 3.86       | 9.39                          | 7.25    | 8.99       | 9.65       | 7.34                          |
| 1.75    | 4.64       | 4.05       | 12.72                         | 7.75    | 9.03       | 9.71       | 7.53                          |
The average error between real value and estimated value from table 7 above is 8.8%. It means that estimated transfer function close enough approach real transfer function.

4. Conclusion
Based on the description above, we can conclude that the dc motor model contained in Proteus 7 can be modeled mathematically using the black-box approach.

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