CPT invariance and neutrino physics

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Abstract

CPT invariance in neutrino physics has attracted attention after the revival of the hypothetical idea that neutrino and antineutrino might have nonequal masses \( m_\nu \neq m_\bar{\nu} \) when realizing neutrino oscillations as a new sensitive phenomenon to search for the violation of this fundamental symmetry. Moreover, the profound relation between the CPT and Lorentz symmetries turns the studies of CPT and Lorentz invariance violations into the one two-sided problem. We present a guide for non-experts through the literature on neutrino physics. The basic works are reviewed thoroughly while for the other papers only current results or discussion issues are quoted. The review covers, mostly, oscillations of neutrinos, resonant change of their flavors and cosmic neutrino physics to systematize possible evidences of CPT/Lorentz violation in this sector of the Standard Model.

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Introduction

This review involves more than two hundred publications that study implications of the CPT violation in various neutrino processes and analyze experimental data on the oscillations of solar, atmospheric and reactor neutrinos and antineutrinos, including the well-known LSND anomaly. However, for ten years past, among review papers and talks on neutrino oscillations (see Refs. [1] – [66]) solely Leung’s talk [1], published in 2000, was devoted in total to summarizing neutrino tests of general and special relativity while only a few ones touched on the CPT symmetry or treated the question in more details. Talks of Akhmedov [5] and of Kayser [6] at the ‘Neutrino-2002’ and the plenary talk of Gonzalez-Garcia [10] at the ‘ICHEP-2002’ contained brief remarks on \( m_\nu \neq m_\bar{\nu} \). Before the ‘Neutrino 2004’, three Mavromatos’ review works on this theme (talks [20, 21] and lectures [22]) were published. Almost at that time the analytical review by Bahcall, Gonzalez-Garcia and Peña-Garay [23] with a relevant section appeared. And at the ‘Neutrino-2004’ de Gouvêa included two sections on tests of Lorentz and CPT invariance in neutrino physics into his talk [38]. Since then, the authors of many survey works are discussing these problems.¹

Several remarks are due on the structure of the present paper and the notation used. By updating the 2004 version of this review the number of referred works was approximately doubled, so that the reference list covers now about ten years (up to January, 2009 deadline). The main body of the review is presented in five Sections while Introduction has a rather informative character; in Conclusion one makes quotations from the few published Solomonian-type outlooks we know.

Publications devoted to general theoretical status of CPT and Lorentz invariance are given in Sections 1 – 2. Concrete substantial issues of hypothetical CPT violation are distributed among eleven alphabetical Subsections A – K. Section 3 (Subsections A, B, C) devoted to general basic works: while Subsection A is of introductory character, Subsection B, C, and D describe, what type of CPT/LI-violating parameters may be introduced, in principle, into the lagrangian formalism for NO. Sections 4 (Subsections E, F, G) and 5 (Subsections H, I, J, K) give to the reader up-to-date information about experimental and observational bounds on these parameters in the framework of the theoretical approaches.

Note here that while the relevant phenomenological methods made used in Subsections B and C are a natural generalization of the Standard Model, the ‘unconventional’ ones discussed in Subsection D have typically problematic validity.

To compensate some ambiguity when choosing Subsections for reviewing a given work, we use cross references and footnotes. Numerous remarks beyond topics discussed, which we consider as a supplement information, are given as footnotes too.

We used the following abbreviations: CPT, NO, MSW, LI, EP, QD and MDR; these stand for, respectively, the CPT symmetry, neutrino oscillations, the Mikheyev–Smirnov–Wolfenstein resonance solution in medium, the Lorentz invariance, the equivalence principle, the quantum decoherence, and the modified dispersion relations (considered as relations between energy, \( E \), 3-momentum, \( p \), and mass, \( m \), of a particle).

1. The CPT theorem (quotations)

The theoretical basing of the Pauli–Lüders–Schwinger CPT theorem is not the goal of this review. So, in this Section we present the fundamental conclusions on CPT symmetry mostly by quotations from familiar monographs and collected volumes (see also Section 2).

• ... Unter sehr allgemeinen und wohlbegründeten Voraussetzungen, zu denen die fur die spezielle Relativitätstheorie charakteristische Lorentz-Invarianz gehört, gilt nämlich das sogenannte CPT-Theorem. Dieses sagt aus, dass aus diesen allgemeinen Voraussetzungen –

¹In this connection, see also section "CP and CPT violation" in very detailed and informative Web-site created by C. Giunti [67].
wir verweisen für Einzelheiten hier auf die Literatur 2 – die Invarianz der Theorie für die Zusammensetzung (Produkt) aller drei Operationen $C$, $P$ und $T$ (in irgend einer Reihenfolge) bereits folgt.

Dieses hat unter anderem zur Folge, dass die Massen von Teilchen und Antiteilchen (allgemeiner die Energiewerte eines Systems von Teilchen und die zu ihnen $C$-konjugierten Teilchen) einander gleich sein müssen. (W. Pauli [73], §1)

{... Very general and well-founded assumptions, including the requirement of Lorentz invariance in special relativity, imply the so-called CPT theorem. The theorem states that these general assumptions (for details see the literature 3) immediately imply the invariance of the theory relative to the combined action (product) of all three operations $C$, $P$, $T$ (in an arbitrary order). This in turn implies, among other things, that the masses of particles and antiparticles (in the general case – energy levels for two systems: of the particles and of their charge-conjugated particles) must be strictly identical. (W. Pauli [73], §1)}

• ... We assume further for the sake of simplicity the local character of the field equation, which means that all field quantities are spinors or tensors of finite rank and that the interaction part of the Lagrangian (or the Hamiltonian) contains only derivatives of finite order of these field quantities... (W. Pauli [70], §1)

• Unabhängig von Schwinger [69] kam Lüders [68] zu dem sehr nahverwandten Resultat, dass unter sehr weiten Voraussetzungen eine $P$ invariante Theorie, in welcher die normalen Vertauschungsrelationen bestehen, automatisch CT invariant ist. Die endgültige und allgemeine Formulierung des hier zuständigen Theorems aber stammt wiederum von Pauli [70] und lautet CTP Theorem: Eine bezüglich der eigentlichen Lorentzgruppe invariante Feldtheorie mit normalen Vertauschungsrelationen ist auch CTP invariant.

Der Fortschritt der neuen Fassung besteht darin, dass (natürlich vor der Entdeckung der Paritätsverletzung) nur die Invarianz bezüglich der eigentlichen Lorentzgruppe vorausgesetzt wird. Ausserdem wird das Theorem für beliebigen Spin bewiesen, während Lüders sich auf die wichtigsten Spinwerte 0, 1/2 und 1 beschränkt. (R. Jost [74], Abschn. 1, §3)

{Lüders, independently of Schwinger [69], obtained a very similar result [68], namely that under not very restricting assumptions, a $P$-invariant theory with normal commutation relations is automatically invariant under CT.

However, the final formulation of this theorem is again Pauli’s [70]; the CPT theorem states: a field theory with normal commutation relations, invariant under the Lorentz eigengroup, is also CPT-invariant.

The advantage of the new formulation is the fact that only invariance under Lorentz eigengroup is assumed (obviously, prior to the discovery of parity non-conservation). Furthermore, the theorem is proved for an arbitrary spin while Lüders only considered the more important spin values 0, 1/2 and 1. (R. Jost [74], Ch. 1, §3)}

• Normal commutation relations are defined as follows: tensor fields (belonging to one-valued representations of $L_+^\uparrow$) commute with themselves and with the spinor fields (belonging to two-valued representations of $L_+^\downarrow$) at space-like separation; spinor fields anticommute at space-like separation...

If we anticipate the results of the last chapter, where particles are introduced into a Wightman field theory, then the above results imply the law of the connection between spin and statistics: particles with integer spin obey Bose–Einstein statistics, particles with half integer spin obey

\[2\] Das CPT-Theorem wurde zuerst von G. Lüders [68] klar erkannt. – Ferner: J. Schwinger [69]. – W. Pauli [70]. – Für nicht lokale Theorien gab R. Jost [71] eine dem CPT-Theorem äquivalente Bedingung, die für lokale Theorien identisch erfüllt ist. – Weitere Anwendungen s. T. D. Lee, R. Oehme und C. N. Yang [72]. (Die Fußnote von Pauli [73], §1)

\[3\] In the first time, CPT-Theorem was realized by G. Lüders [68]. See, besides: J. Schwinger [69]; W. Pauli [70]. For nonlocal theories R. Jost [71] produced a condition equivalent to the CPT theorem that holds identically for local theories. For further applications see: T. D. Lee, R. Oehme and C. N. Yang [72]. (Pauli’s footnote [73], §1)
Let us next consider the restrictions imposed by the requirement that the theory be invariant under (Wigner) time inversion. An important theorem due to Pauli [70] and Lüders [68] (this discovery was essentially anticipated by Shell (1948) and by Schwinger [76]), and currently known as the TCP theorem, asserts that within the framework of relativistically invariant local field theories, assuming the usual connection between spin and statistics, invariance under time reversal is equivalent to invariance under \( U_P U_C \), i.e., the combined operation of charge conjugation \( U_C \) and space inversion \( U_P \). In a Lagrangian formulation, the TCP theorem is a result of the assumed invariance under proper Lorentz transformation of \( \mathcal{L} \) [Lagrangian density], the hermiticity of \( \mathcal{L} \), the locality of the theory, and the assumption that particles of integer spin (bosons) must obey Bose–Einstein statistics and those of half-integer spin (fermions) must obey Fermi–Dirac statistics, i.e., the particles obey the usual connection with statistics. ([77], Ch. 10, §2, p. 264)

...Hence it follows that the Lagrangian (14.16) from which we have demanded only that it must be Hermitian and invariant under the proper Lorentz transformations, is also invariant under PCT (CPT, TCP, and so on). This is the essence of the Lüders–Pauli CPT theorem (for further details see Pauli [70] and Gravert, Lüders and Rollnik [78])...

The requirement that the interaction is local has played an essential role in the above discussion. In the axiomatic formulation of quantum field theory, this requirement can be made less stringent. The proof of the CPT theorem in axiomatic approach has been given by Jost [75], Streater and Wightman [79], and Bogolyubov, Logunov, and Todorov [80]. In this approach, it is also assumed that the Lagrangian is written in the form of the normal product and there is a connection between spin and statistics: fields with integer spin commute with one another and with other fields, whereas fields with half-integer spin anticommute with one another but commute with integer-spin fields. ([81], Ch. II, §14)

The TCP-theorem is remarkable because a discrete symmetry is shown to exist in theories which, to begin with, are only assumed to be invariant under connected continuous groups... (R. Jost [75], Ch. V, §2)

... A very important consequence concerns the equality of masses and total lifetimes of particle and antiparticle, a result which is true irrespective of the particle conjugation non-invariance of the weak decay interactions... ([83], Ch. 3, §5)

2. Theoretical and experimental status of the CPT

The general principles of the quantum field theory that lie at the foundation of the CPT theorem and were formulated in the mid-20th century, connect any violation of the CPT invariance with far-reaching changes in such fundamental concepts of the theory as the causality principle (locality of the lagrangian) and the relation between spin and statistics (see, e.g., [84]). Hence a critical discussion of modern unconventional (and also Lorentz-non-invariant) theories involving CPT violation and their experimental testing are necessary elements in the progress of physics. Further theoretical scrutiny of the current status of the CPT and of the conditions of validity of the CPT theorem is no less important.

Does the Lorentz invariance (LI) still hold in the theory when the CPT symmetry breaks down, the way this occurs in models with unequal masses of particles and antiparticles \( m \neq m' \)? As follows from Greenberg’s paper [85], the answer is negative: the general Greenberg theorem states that the interacting fields that break the CPT symmetry inevitably break the LI as well. The CPT invariance here is necessary but not sufficient for the LI. Theories that break the CPT as a consequence of mass difference between particles and antiparticles must be non-local. Then Greenberg discusses what does the property of locality mean in quantum field theory.

The starting points of Greenberg’s work [85] are as follows. Quantum field theory is Lorentz-covariant on the mass shell if vacuum matrix elements of unordered products of the fields \( \phi(x_n) \) (Wightman functions \( W^{(n)} \) [79]) are covariant. The Lorentz covariance (in fact, the Poincaré
covariance) on the mass shell is assumed from the beginning. Quantum field theory is covariant off the mass shell if the vacuum matrix elements of time-ordered products of fields (τ functions) are covariant. For the LI to hold, quantum field theory must be covariant both on and off the mass shell.

Greenberg’s proof employs Jost’s axiomatic approach [71]. Jost formulated the fundamental theorem [71] stating that the necessary and sufficient condition of the CPT symmetry is that the so-called weak local commutativity holds at Jost points in the form

\[ W^{(n)}(x_1, x_2, \ldots, x_n) = W^{(n)}(x_n, x_{n-1}, \ldots, x_1), \]

(1)

\[ W^{(n)}(x_1, x_2, \ldots, x_n) = \langle 0|\phi(x_1)\phi(x_2)\cdots\phi(x_n)|0 \rangle. \]

(2)

Because the τ functions can be expressed in terms of the properly arranged sum of Wightman functions, it follows that the invariance condition is a constraint on \( W^{(n)} \) given by the relations (1), i.e., by the condition of weak local commutativity (WLC) [4]. This immediately implies the CPT symmetry. Consequently, any violation of the CPT invariance in any of Wightman functions signifies non-covariance of the corresponding τ function and, hence, breaking of the LI of the theory. Besides, there is no reason to deny the possibility of CPT violation in scattering and other physical processes even if particles and antiparticles have equal masses.

In Ref. [88], Greenberg gave a critical analysis of an attempt to justify the model with CPT violation caused by \( m \neq \bar{m} \) [89] by introducing free hybrid (‘homeotic’) fields that are, in the case of appropriate normalization, linear combinations of positive- and negative-frequency components of Dirac fields with the masses \( m \) and \( -m \). It was shown that even though such free fields could satisfy the Lorentz covariance condition on the mass shell, the interacting hybrid fields inevitably violate the Lorentz covariance in accordance with the Greenberg theorem [85]. The model proposed in Ref. [89] cannot serve as an example of a theory with CPT violation. When discussing the fundamental nature of the CPT symmetry in quantum field theory as compared to other discrete symmetries or their combinations, Greenberg emphasized that for LI to result in CPT, it is necessary and sufficient to have a certain weakened form of space-time commutativity (or anticommutativity), i.e., WLC [4]. This remark explains why free fields with \( m \neq \bar{m} \) can satisfy LI on the mass shell but at the same time violate the CPT symmetry.

Summarizing his investigation of the relation of the LI of a theory to the CPT, Greenberg [86] again returned to the question of what was lacking for the CPT symmetry to hold (in the presence of LI). A free or generalized free field can be Lorentz covariant but not obey CPT invariance if the particle and antiparticle masses are different [85]. What fails in that case is that WLC does not hold at Jost points... Note that although the fields in these examples transform covariantly their time-ordered products are not covariant. Thus if we require that time-ordered products be covariant as part of Lorentz covariance of a theory then, as shown in [85], free fields that violate CPT are not covariant. See [88] for a detailed analysis of hybrid Dirac fields (‘homeotic’ fields

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4As it is mentioned by Greenberg in his recent work [86], the Jost’s approach has two advantages over the lagrangian one: (1) it makes clear why CPT is fundamental but non of the individual C, P, and T symmetries is; (2) it gives a simple way to calculate CPT without calculating C, P and T separately. The goal of this pedagogical paper is to ‘de axiomatize’ CPT theorem considering a few items: the lagrangian CPT theorem; representation of the real and complex Lorentz groups; vacuum matrix elements of τ functions and analytic functions; enlargement of the domain of τ-functions analyticity; the general formula for CPT; CPT for the S matrix.

5This property was called WLC by Dyson who, as a further deduction from the ideas of Jost and Wightman, proved the well-known theorem [87]: a Wightman function will be analytic and one-valued at a real set of space-time points if and only if the fields possess a property of WLC at the same points. This statement assumes that CPT invariance is hold, while for the case when CPT symmetry is absent a similar but more complicated statement is proved.

6Note that only the normal spin–statistics relationship is possible in the axiomatic approach to quantum field theory as discussed here since selecting incorrect commutation relations for the field results in the field vanishing identically (see, e.g., Ref. [77], Ch. 17, §1).
The difference (see the review talk [93], the review [94] and talk [95]). It should be mentioned that the exceptional importance of testing the CPT experimentally was first realized in connection with the discovery of the violation of the P, C and CP invariance (see the review [97] and also [98]). The general constraints imposed by analyticity and discrete symmetries P, C, CP, TCP on the description of binary systems of neutral mesons of the type \( (K^0, \bar{K}^0) \) were obtained in the framework of quantum field theory in [99].

The current status of the CPT was also presented in recent reviews and talks. The monograph [101] as well as talks in a series of meetings on CPT/Lorentz symmetry [102]-[106] and in other conferences [107, 108] discuss theoretical sources and experimental limits of CPT violation. The talk [107] dealing with the classification of the effects of violation of all discrete symmetries also describes the relation between CPT invariance and hermiticity of lagrangian of the theory. The processes discussed are those that have not yet been studied experimentally. These are the circular polarization of \( \mu^0 \rightarrow \gamma \) and \( \eta^0 \rightarrow \gamma \) decays (and also the longitudinal polarization of muons in the decay \( \eta^0 \rightarrow \mu^+\mu^- \)) and circular polarization of photons in the decay of parapositronium [11]. It is emphasized that in contrast to the case of \( m \neq \bar{m} \) the above examples of CPT-odd polarizations can be formulated in a Lorentz-invariant manner.

Both the invited talk at the EXA’2005 conference and the plenary talk at the LEAP’05 conference [108] discuss CPT from the standpoint of violations of the basic underlying assumptions of the CPT theorem in models of quantum gravity. The possible ways of CPT violation are classified, and their phenomenology is described in terrestrial as well as astrophysical experiments. An attention is payed on disentangling genuine quantum-gravity induced CPT violation from ‘fake’ violation due to ordinary matter effects, particularly when CPT breaking of this type is connected with unitarity violations. Further discussion of the subject see, e.g., in Ref. [110].

Having finished the discussion of the general status of CPT we present also information on the violation of LI which is based on contents of the recent reviews [111]-[114].

The comprehensive up-to-date review on modern tests of Lorentz invariance [111] summarizes both theoretical frameworks and advances for new precision measurements in terrestrial experiments and astrophysical observations. The problems involved include the following issues:

\[ \Delta m \] which can be covariant only when they are non-interacting but even in the free case have time-ordered products that are not covariant. [86]

The information given above on the general theoretical status of the CPT problem is directly connected with the experimental tests of CPT conservation (in elementary particle physics [8] that is presented below and, especially, in neutrino physics).

It should be also noted that neither the monograph [101] nor the paper [109] used by the authors of Ref. [101] contain a theoretical justification of the validity of the relations for NO proposed in which \( m_\nu \neq m_\bar{\nu} \). What is hiding behind this fact is in all likelihood certain internal inconsistency in these relations.

The decays listed above conserve C parity while the magnitude of the PT-odd effect of \( sk \)-type correlation between the photon spin and momentum are controlled by the difference \( \beta = g^*h - gh^* \) where \( g \) and \( h \) are the coefficients in scalar and pseudoscalar terms of the effective lagrangian. Therefore an experimental observation of the effect would indicate that \( \beta \neq 0 \), i.e. that CPT is violated because lagrangian is non-hermitian.
defining Lorentz violation (QFT, modified Lorentz groups); kinematics vs dynamics; other symmetries (CPT, SUSY, Poincaré); diffeomorphism invariance and conservation of matter stress tensors; Lorentz violation and equivalence principle; Lorentz violation, causality and stable ground state; kinematic framework (modified dispersion relations, "doubly special" relativity (DSR) \[115\], non-systematic dispersion); dynamical framework (renormalizable CPT- and Lorentz-odd operators, non-commutative space-times, symmetry and Lorentz-odd operators, Lorentz violation with gravity); terrestrial constraints (penning traps, clock comparison, cavity experiments, torsion balances, neutral mesons, Doppler shift in Li, muon experiments, higgs sector); astrophysical constraints (time of flight (and DSR), birefringence, threshold \(\gamma\) reactions, threshold particle reactions in EFT (Čerenkov effect, GKZ cutoff etc.), threshold in DSR etc., synchrotron radiation), neutrino physics (oscillations, Čerenkov effect \[12\]); phase coherence of light; gravitational observations (gravitational waves, cosmology, post-newtonian corrections).

The principal goal of the review was to form a logical structure of various theoretical frameworks (with relevant types of experiments). While theoretical issues are present without details the list of references is voluminous (more than 280 works).

One should also mention the relevant problem of a varying speed of light (VSL) and its origins (see the review \[112\] of recent works on VSL theories): hard LI breaking, bimetric theories, locally Lorentz-invariant theories, color-dependent \(c\)-speed models, extra-dimension schemes, and field theories where VSL comes from vacuum polarization or CPT violation. Thereby, non-linear realization of the Lorentz group is connected with VSL theories and DSR, in particular.

As to LI and CPT violation in the framework of Standard Model Extension (SME) \[117, 106\] that is discussed below in Subsection B, all the information on experimental values (or limits) for proper parameters, available up to January 2010, are summarized in Ref. \[113\]. Ten comprehensive data tables include, first of all, such sectors of the minimal SME (with Lorentz-violating operators of mass dimension four or less) as the surrounding matter sector (protons, neutrons, and electrons) as well as the photon sector. The data are given also for charged leptons, neutrinos, mesons and for interaction sectors (electroweak, gluon, and gravity) as well as for non-minimal photon sector. Three special summary tables are composed by extracting from data tables the maximum attained sensitivities for the surrounding matter, photon, and gravity sectors.

Although the deadline on the reference list of our work is the end of 2008, it is instructive to mention here additionally the review of June, 2009 \[114\], where the authors discuss, in particular, such themes as the Effective Field Theory (EFT) approach, the SME with both renormalizable and non-renormalizable operators, the naturalness problem and higher dimension Lorentz-violating operators, as well as the DSR and the Very Special Relativity (VSR) scheme \[116\] corresponding to the breaking of space isotropy.\[13\]

Let us return now to CPT violation problems in neutrino physics. They are presented from the standpoint of both standard field-theory approaches (mostly considered in Subsections A, B, C, E, F, H, I) and more contemporary models \[14\] (reviewed in Subsections D, E (final paragraphs), F, G, J, K).

3. General consequences of hypothetical CPT and Lorentz invariance violation in neutrino physics

A. CPT and neutrino physics

The early history of studying the relation between possible CPT violation in neutrino physics

\[12\] See also in the beginning of Subsection B below, where Goldstone–Čerenkov effect is discussed when violating LI spontaneously.

\[13\] See also the second footnote in Subsection B; some information on DSR and VSR is given in the middle of Subsection D.

\[14\] Two schemes are conventionally mentioned in connection with the mechanisms that could produce the spontaneous CPT violation in string theories: the phenomenological \[117\] and one based on decoherence due to quantum gravity effects \[118\] (see also Refs. \[119, 120\] and review talks \[121, 122\]).
and neutrino oscillations (NO) covers about two decades. The first attempt was the paper by Bigi in 1982 [109]. Starting with the speculations in the literature on possible violation of Lorentz invariance outside the Standard Model that could be detectable in the lepton sector of the theory, Bigi investigated not the possibility of interpreting the NO data but a more general problem of expanding the range of phenomena and experiments whose analysis could promise sufficient progress in improving the sensitivity of results to CPT violation. It was pointed out that at least in principle the effects of CP and CPT nonconservation could be separated: the CPT conservation signifies the equality of probabilities \( P(\nu_\alpha \rightarrow \nu_\beta) = P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha) \) while the CP conservation results in the equality \( P(\nu_\alpha \rightarrow \nu_\beta) = P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha) \); the indices \( \alpha, \beta = e, \mu, \tau \). The subsequent description of NO for \( m_\nu \neq m_\bar{\nu} \) in the case of CPT violation was achieved by introducing a double set of parameters without writing out the lagrangian (see also the monograph [101]) and without introducing an explicit definition of the masses of \( \nu \) and \( \bar{\nu} \). Obviously, this description corresponds at the same time to the violation of the LI (see remarks on the discussion in the first part of Section 2) with all the consequences this implies for the theory. Therefore, neutrino oscillations models with \( m_\nu \neq m_\bar{\nu} \) mentioned in Subsection K are theoretically unfounded and in fact incorrect.

Note also that the discussion of the relation of the CP, T and CPT symmetries with NO made it quite clear for a long time [123] (see also the review [105]) that if the CPT is preserved, the effects of CP and T violation could only occur in experiments that would monitor an excess of neutrinos with the initial flavor. At the same time, the CPT violation may also manifest itself (in contrast to the CP- or T-non-invariance) in measuring the deficit of initial-flavor neutrinos.

Another important note concerns the type of neutrino mass and the lepton number conservation. Extending the Standard Model in subsequent Subsections B and C supposes Majorana (Dirac) masses for the neutrinos while in the general case the situation appears to be more complicated [124]. First of all, we do not know whether neutrino processes violate the conservation of the lepton number \( L \) and whether neutrinos are identical to their own antiparticles. In itself, introduction of the Dirac neutrino mass into the model keeps \( L \) conserved. Any non-conservation of \( L \) would imply the presence of Majorana mass terms that transform the neutrino into antineutrino. With CPT conserved and in the presence of Majorana mass, the mass eigenvalues are of Majorana type, i.e. the neutrino is its own antiparticle. If the CPT is conserved but the theory has no Majorana mass terms, then mass states are of Dirac type, \( L \) is conserved, and the neutrinoless double beta decay is forbidden. Authors of Ref. [124] discuss CPT violations using a simple example of a theory with a single neutrino \( \nu \) interacting with the electron, and its CPT-conjugate antiparticle \( \bar{\nu} \) coupled to the positron. They suppose that for a given spin direction, the \( \nu, \bar{\nu} \) mass matrix \( M_\nu \) has the form

\[
\begin{pmatrix}
\mu + \Delta & y^* \\
y & \mu - \Delta
\end{pmatrix},
\]

where the upper row corresponds to the neutrino and the lower one to the antineutrino. For stable neutrinos the matrix \( M_\nu \) is hermitian so that the parameters \( \mu \) and \( \Delta \) (Dirac masses) are real; \( \Delta \neq 0 \) denotes CPT violation and \( y \neq 0 \) (Majorana mass) denotes nonconservation of \( L \). An analysis shows that the mass eigenstates for \( \Delta \neq 0 \) cannot any more correspond to Majorana neutrinos. However, if \( y \neq 0 \) then there is mixing of \( \nu \) with \( \bar{\nu} \), the lepton number \( L \) is not conserved and the neutrinoless double beta decay is allowed. A further elaboration of the presented problem see in recent talk of Kayser [125].

B. Extension of the Standard Model: spontaneous violation of Lorentz invariance and CPT

Although concrete origins of LI violation in neutrino physics, which are conventionally thought to be related to cosmology and gravity, are not yet elaborated, authors of recent work [126] has been able to present a model-independent picture of their basic features. So, the assumption
by itself that general relativity is hold up to the Planck mass scale implies the spontaneousness of Lorentz violation. Indeed, violation of LI simultaneously breaks gauge symmetry of gravity, which can only be violated spontaneously; hence a proper Goldstone boson must be present. The Lorentz-violating vacuum in which the neutrino propagates may create 'static' effects in preferred ether frame due to the vacuum expectation value ("ghost condensation"), the basis of which is a modification of neutrino dispersion relation (similar to Eqs. (11) and (12) below in Subsection D), and 'dynamic' effects owing to coupling between the neutrino and the Goldstone boson ("gauged ghost condensation"). Static effects are described by effective hamiltonian \( h_{ij} = \pm \mu_{ij} + a_{ij}|p| + m_i^2/2|p| \), where \( \pm \mu \) is for left/right-handed neutrinos/antineutrinos, respectively, and violates CPT (\( i \neq j \) are flavor indices); the analogous hamiltonian \( h_{im} \) is used for dynamic Goldstone–Čerenkov effects. As it follows from kinematics, any neutrino has certain possibility to produce single (or several) goldstone quantum (quanta) of the corresponding mode, i.e. to change its original direction and energy.

As the central point of Ref. [120], dynamic emission of goldstones by neutrinos is specially presented below in the end of Subsection E, while static effects are discussed mostly in some referred works in Subsections D, E, F, G.

Regardless of paper [109] mentioned above but also in connection with searching for new more stringent constraints on the presence of Lorentz-non-invariant terms in the lagrangian of the Standard Model, some perturbation-theory approaches to description of CPT-odd effects were formulated. So as to construct a CPT-non-invariant generalization of the Standard Model in the framework of an effective low-energy theory, an approach [127] was developed for treating spontaneous CPT and LI violation in quantum field theory and in relativistic quantum mechanics. In this case the neutrino component of the lagrangian \( L_a \):

\[
L = \frac{1}{2} iL_a^{\gamma\mu} \overrightarrow{D}_\mu L_a - (a_L)_{\muab} L_a^{\gamma\mu} L_b + \frac{1}{2} i(c_L)_{\muab} L_a^{\gamma\mu} \overrightarrow{D}_\nu L_b;
\]

here \( \mu, \nu = 1, 2, 3, 4 \), \( a, b = \epsilon, \mu, \tau \); the first term is the kinetic term, the second and third terms correspond to LI violation, the term with \( (a_L) \) corresponds to CPT violation. When taking gravity into account \(^1\) this extension of the Standard Model was investigated in Ref. [128].

A detailed general analysis of a possible violation of the LI and CPT in the neutrino sector, not using the assumption of space isotropy \(^2\), was given in Refs. [141, 106]. The authors gave a clear scheme for estimating the sensitivity of various neutrino experiments relative to the value of three parameters \( -a_L \) and \( c_L \) included into the lagrangian (5), and the difference between

\(^1\) For a discussion of the problem of calculation of the NO phase in curved space-time see, e.g., work [129], part II, and also Refs. [130, 131] and papers cited therein.

\(^2\) The isotropy of space was tested relatively recently in measurements of the direction independence of the gravitational constant \( G \), in experiments with light propagation using the theory and practical methods of wave front inversion [132], as well as in experiments measuring the amplitude \( A(t) \) in the angular dependence \( 1 + A(t) \cos \theta \) of \( e^- \) emission in \( ^{90}\text{Sr} \), where \( \theta \) is the angle relative to the South–North axis. It was found that \( \Delta G/G \) does not exceed the level \( 10^{-10} \) (see, e.g., analysis in [133]) and that the speed of light in air and refraction index in glass are independent of direction, at least to within \( 5 \times 10^{-8} \) [134].

In view of connection to the Standard Model Extension, the most recent analysis of possible limits of the space anisotropy \( r \) is given in Ref. [135] (see also references therein). Some estimates are presented for two types of experiments with the transverse Doppler effect considered in terms of conventional special relativity theory and on the basis of its Finslerian generalization by the author. He concludes that from first-type experiments (soon after the discovery of the Mössbauer effect) aimed at searching for ether wind one obtains the boundary \( r < 5 \times 10^{-10} \), the present day limit could be made lower by at least three orders. As for future second-type measurements (with the effect of harmonic oscillation frequency modulation) the author expects that one would lower \( r \) down to \( \sim 10^{-14} \). (See also [136] and references therein.)

Another problem with anisotropy of our Universe stems from the so-called cosmololgical birefringence (see, e.g., [137]) which can results in a non-zero rotation polarization angle \( \Delta \varphi \) of cosmic microwave background (CMB). As summarized in the review talk of Ref. [138] (see also references therein and Ref. [139]), CMB data of the experiments WMAP and BOOMERANG give the best current constraints of \( \Delta \varphi \sim 100 \) mrad; the Planck Surveyor will improve sensitivity upon \( \Delta \varphi \sim 10^{-2} - 10^{-3} \) (i.e., 1–10 mrad). The updated result by April 21, 2008 [140] for testing CPT with CMB is \( \Delta \varphi = 2.6 \pm 1.9 \) deg at 68% CL.
the squared masses of neutrino eigenstates $\Delta m^2$, which determines the NO. It was shown that even in the framework of the simplest scheme (with nonzero element $c_L$ in the case of isotropic effective hamiltonian for the transitions $\nu_e \leftrightarrow \nu_e$, and with equal nonzero real elements $a_L$ in the case of preferred direction along the axis of revolution of the Earth for the transitions $\nu_e \leftrightarrow \nu_\mu$ and $\nu_e \leftrightarrow \nu_\tau$) it is still possible to reproduce the main features of the experimental behavior of the probabilities of the corresponding NO. The simplified model with two free parameters analyzed by the authors (instead of the usual four in the case of standard oscillations) in which $\Delta m^2 = 0$ and there is no mixing $\nu$ with $\bar{\nu}$ (‘bicycled model’), predicts, among other things, a considerable azimuthal dependence for the number of atmospheric neutrinos and a large decrease in the half-annual variation in the flux of solar neutrinos during some weeks before and after the equinox – an effect due to LI violation. The authors of [141] emphasized that the model serves to illustrate certain key effects caused by LI violation, and demonstrates how the presence of Lorentz non-invariance and CPT non-conservation on the scale $M_{Pl}$ can be identified using a certain signal in NO.

The results of analyzing the consequences of the Standard Model extension for neutrino physics [141] were summarized in a recent talk [106] (see also Refs. [142]), which gave an exhaustive description of the theoretical investigation of LI and CPT violation in NO. The work is based on conventional equations of motion for the Dirac and Majorana neutrinos, where matrices in the spinor space are written in a more general form:

\[
(i\Gamma^\nu_{AB}\partial_\nu - M_{AB})\nu_\nu = 0, \\
\Gamma^\nu_{AB} \equiv \gamma^\nu_\mu \delta_{AB} + d^{\mu\nu}_{AB}\gamma_5\gamma_\mu + e^\nu_{AB} + i f^{\nu}_{AB}\gamma_5 + g^{\nu}_{AB}\sigma_\mu/2. \\
M_{AB} \equiv m_{AB} + im_{5AB}\gamma_5 + a^\mu_{AB}\gamma_\mu + b^\mu_{AB}\gamma_5\gamma_\mu + H^{\mu\nu}_{AB}\sigma_\mu\sigma_\nu/2.
\]

Here, all neutrino fields (including the C-conjugate ones) are collected into a single spinor $\nu_A$, $A = 1, 2, ..., 2N$ where $N$ is the number of neutrino types; $\lambda, \mu, \nu = 1, 2, 3, 4$; $m$ and $m_5$ are the mass terms and the other coefficients in (5)–(6) correspond to LI violation, with $a, b, e, f, g$ determining CPT violation. If the coefficients of the type $g$ and $H$ are nonzero, a mixing of $\nu$ with $\bar{\nu}$ arises. In the framework of the scheme described here, the terms with LI violation are characterized by dimensionless combinations of $a^\mu L, b^\mu L, H^{\mu\nu}L$ and $e^{\mu\nu}L, d^{\mu\nu}L, g^{\mu\nu\sigma}L$ and can reproduce direction-dependent effects in oscillations.

Very recently another extensions of the Standard Model was studied [143], which are renormalizable in a more general framework of "weighted power counting" [144]. In this approach space and time have different weights, the theory does not contain right-handed neutrinos, nor other extra fields, and gives Majorana mass to the neutrinos after symmetry breaking. The author considers the simplest of minimally Lorentz-breaking schemes in detail; this model preserves CPT and space rotation invariance, violates LI explicitly at very high energies and restores it at low energies.

C. Perturbation-theory formalism for violation of Lorentz invariance, CPT and equivalence principle

Similarly to the approach in [127], outlined in Subsection B, Coleman and Glashow [145] developed the general formalism for introducing CPT and/or Lorentz non-invariant perturbative terms into the theory. The authors aimed at a concrete problem of testing special relativity in highly relativistic cosmic rays and NO [146]–[148] (see also [149]). Provided that the rotation invariance holds in a preferred reference frame (e.g., when one considers the rest frame of the cosmic background radiation) the renormalizable and gauge-invariant CPT-even LI-violating additional term in the Standard Model lagrangian results in the emergence of maximum attainable velocities (MAV) of particles which can be not equal to photon velocity $c_\gamma$. Each particle type is put in correspondence with not only the mass $m_a$ that characterizes it but also with the quantity MAV in vacuum denoted by $c_a$, so that $c_a \neq c_\gamma$. It is found that this assumption is sufficient...
for oscillations to appear even with massless neutrinos\textsuperscript{17} that are typically described in terms of the differences $\Delta c_{\nu} \equiv c_{\nu} - c_{\nu'}$ and angles of the corresponding mixing matrix for MAV eigenstates. In the most general form, the neutrino eigenstates are characterized in the ultrarelativistic case with given momentum $p$ by the following sum of three hermitian 3×3-matrices\textsuperscript{145,148}:

$$\hat{c}p + \hat{m}^2/2p + \hat{b}.$$  \hspace{1cm} (7)

Here $\hat{c}$ is the matrix of MAV values for the neutrino, $\hat{m}^2$ is the diagonal matrix of squared Majorana masses, $m^2 = mm^\dagger$, and $\hat{b}$ is the matrix related to the CPT non-invariant additional term $\nu_{\alpha}b_{\mu}b_{\nu}^\dagger$ in the lagrangian (the case of timelike $b_{\mu} \sim (b,0)$ for $b = 0$ is considered). The matrices $\hat{c}$ and $\hat{m}^2$ determine the energy eigenstates as MAV states in the high energy limit and as mass states in the low energy limit, respectively. In the case of NO of two flavors, the expression for the probability of diagonal transition on the baseline $L$ has the form

$$P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2 2\Theta \sin^2(LF/4).$$  \hspace{1cm} (8)

The generalized mixing angle $\Theta$ and the phase factor $F$ are written explicitly in terms of eight parameters – three mixing angles ($\theta_m, \theta_b,$ and $\theta_c$), three differences ($\Delta m^2, \Delta b, \Delta c$) corresponding to the matrices $\hat{m}^2, \hat{b},$ and $\hat{c},$ and two complex phases ($\eta$ and $\eta'$):

$$\Phi\sin2\Theta = |\Delta m^2 E^{-1}\sin2\theta_m + 2e^{i\eta}\Delta b\sin2\theta_{b} + 2e^{i\eta'}\Delta c E\sin2\theta_{c}|,$n$$\Phi\cos2\Theta = |\Delta m^2 E^{-1}\cos2\theta_m + 2e^{i\eta}\Delta b\cos2\theta_{b} + 2e^{i\eta'}\Delta c E\cos2\theta_{c}|. \hspace{1cm} (9)

Clearly, the type of possible violation of LI and CPT can be found from the essentially different dependences of the terms containing $\Delta m^2, \Delta b,$ and $\Delta c$ on $E$.

The phase of NO when the effect is due to the violation of equivalence principle (EP) of general relativity (first treated in\textsuperscript{150,151,152}) depends on $E$ in the same way as the term with $\Delta c$. One would expect that a corollary of EP violation in gravitation theories discussed in the literature\textsuperscript{18} will also be LI- and CPT-non-conserving. As mentioned in Ref.\textsuperscript{149}, the phenomenological equivalence of NO under EP or LI violation makes it possible to find directly the constraints on the parameters $\Delta c$ and $\theta_c$ from the range of values of $|\phi\Delta f|$ and $\sin2\theta_G$ obtained in the former case ($\phi$ is the dimensionless gravitation potential, $\Delta f$ characterizes the degree of EP violation, and $\theta_G$ is the corresponding mixing angle). In addition to referring to previous publications on the relation of NO to the effects of EP violation\textsuperscript{19}, the paper\textsuperscript{149} offers an important general statement that the experimental observation of NO in itself is insufficient for the decisive conclusion on nonzero mass of at least one of the neutrinos, since the oscillations may be caused by a very small violation of LI and/or EP.

D. ‘Non-standard’ violating mechanisms

‘Non-standard’ sources of LI or EP violation and novel NO mechanisms are usually connected with certain properties of the vacuum on the Planckian (or even considerably larger) scales. These aspects were treated in review talks\textsuperscript{20,21,22} and lectures\textsuperscript{22} that offered arguments in favor of the inherent sensitivity of the NO to CPT violation in comparison with experimental data

\textsuperscript{17}Massless neutrinos cannot oscillate if special relativity is unbroken. However, they can oscillate if different neutrinos travel at slightly different speeds in vacua.\textsuperscript{146}.

\textsuperscript{18}In Ref.\textsuperscript{153} (see also\textsuperscript{154}) NO are considered (even for mass-degenerate neutrinos) as caused by EP violation due to the string theory effects. They contribute to macroscopic gravity and itself caused by the massless scalar dilaton partner of the graviton\textsuperscript{155}. In connection to Refs.\textsuperscript{150} - \textsuperscript{153} see also Ref.\textsuperscript{130} (the next to last footnote in Subsection B).

\textsuperscript{19}See also the papers\textsuperscript{150,157,158} that discuss experiments carried out by the time of its publications on solar, accelerator (including the LSND experiment) and atmospheric neutrinos, correspondingly.

\textsuperscript{20}More recently, numerous mechanisms for LI and CPT violation have been pointed out once again in\textsuperscript{159}. 


involving other particles. The mechanism that could explain the loss of unitarity in quantum gravity [118, 119] – which would result in LI and CPT violation in one form or another – is so far illustrated only by a hypothetical though visually clear picture of the manifestation of the space-time structure (the "foam") at the quantum level; this is caused by appearance and disappearance of black holes and large metric fluctuations that are accompanied by formation of virtual horizons. In these talks the author uses an idea (see, e.g., [160]) that when a particle crosses such horizons, the information on its state may be partly lost. Correspondingly, pure state evolves into a mixed one, and it is suggested to consider the density matrices, instead of pure quantum-mechanical states. While, in conventional case, the connection of in and out states is described by the scattering matrix $S$,ting in space-time foamy situations, when unitarity may be lost, the notion of the $S$ matrix is replaced by that of the superscattering matrix, $\mathcal{S}$, introduced by Hawking, which is a linear, but non-invertible representation between in and out density matrices: $\rho_{\text{out}} = \mathcal{S} \rho_{\text{in}}$ where $\mathcal{S}$ is the irreversible matrix. Thereby, $\mathcal{S} \rho$ may not be defined as a product $S \rho S^\dagger$. All that implies the loss of the unitarity in an effective low-energy theory and the violation of CPT – in accordance with Wald’s theorem [163] which states that in the above case the CPT theorem is violated, at least in its strong form because the CPT operator is not well defined. In this connection the author of Ref. [21] discusses the problem of the relation between this scheme of CPT violation and LI (see [164], [165] and review [11]). A range of aspects of this problem is considered in the author’s different publications in which he discussed other possibilities too – e.g. the ill-defined definition of antiparticle [166], as well as the idea of direct violation of CPT [20] caused by the nonzero $\Lambda > 0$ term that accelerates the expansion of the Universe and results in the formation of a cosmological horizon [108].

In addition to a brief review of theoretical ideas concerning CPT violation at lengths of the order of $M_{\text{QG}}^{-1}$ that are characteristic of quantum gravity, the talk [21] and lectures [22] contained different issues of phenomenological testing CPT in various neutrino processes including astrophysical and cosmological manifestations. A review of a number of papers is also given in Refs. [21, 22] of the feasibility of the above picture. Author investigates whether it is possible to use the available data, including NO data, for the evaluation of parameters that characterize, first, the openness of the system that results in quantum decoherence (see Refs. [118, 119, 120]) according to the right-hand side of the Liouville equation,

$$\dot{\rho} - i[\rho, H] = \delta H \rho, \quad (10)$$

and, second, the distortion of standard dispersion relations (see Refs. [168, 169, 170]) via the addition of new terms (which are represented in the general case by a model-dependent function $F$),

$$E^2 = p^2 + m^2 + F(E, \vec{p}, M), \quad (11)$$

(here energy-scale factor $M$ stands for $M_{\text{QG}}$ or Planck mass $M_{\text{Pl}}$) that result in CPT and/or LI violation.

In the conventional approach when the preferred frame is at rest relative to the cosmic microwave background radiation, to change LI minimally with keeping energy-momentum conservation it is mentioned, Hawking has stated in his talk at the GR17 (17th Intern. Conf. on General Relativity and Gravitation, Dublin, July 2004) that information is not lost during formation and evaporation of black holes because in all likelihood the true (not the apparent) horizon is never formed. Hawking’s claims are discussed in the literature – by the author of [20] [21] also within the reference number six in his invited talk of Ref. [108] and, in details, in one of his recent theoretical works (see Section I of Ref. [161]) as well as in Hawking’s paper [162]: It is like burning an encyclopedia. Information is not lost, if one keeps the smoke and the ashes.

Below, the debate on the space-time foam rôle is considered as open.

22 Such issues as strong form and weak form of CPT invariance are discussed also in talks [108] and plenary talk [41].

23 Nonunitary evolution of a quantum system in which a pure state is transformed into a mixed one was discussed by Marinov [167] who used an equation of a type similar to (10).
must be only the boost invariance broken but the rotation symmetry is preserved while, in the reverse order case, the rotation invariance breaking entails the boost one too. Then, as assumed in the most of QG models, modified dispersion relations (MDR) come to the form:

\[ E^2 = p^2 + m^2 + f^{(1)}_{\alpha} p^2 |p|/M + f^{(2)}_{\alpha} p^2 (|p|/M)^2 + f^{(3)}_{\alpha} p^2 (|p|/M)^3 + \ldots ; \]

(12)

the parameters \( f^{(u)}_{\alpha} \) \((u = 1, 2, 3\ldots)\) are dimensionless and are labeled in accordance to particle species.

We notice here the theoretical problem of the presence in quantum field theory and in Standard Model, particularly, self-energy contributions to MDR in Eq. (11). As it has been shown in Ref. [171] (see also Refs. [172, 173] and the literature discussing these papers), the resulting contribution to MDR even with counterterms and renormalization procedure cannot preserve LI from violation at percent level without any suppression, unless the bare couplings of all the particles involved are strongly fine tuned. Hence the matter is also to search for mechanisms to maintain LI but not just to lower its breaking limits.

As for the quantum decoherence (QD), the general case of a phenomenological description of NO with two flavors treated as an open system was analyzed in detail in Ref. [174]. Dissipation effects in the right-hand side of (10) were treated in the approximation in which quantum gravity results in linear decoherence (with linear dependence on density matrix)\(^{24}\) they are simply parametrized by six real variables. These quantities are related via a number of inequalities that correspond to the property of ‘total positivity’ required to ensure that the density matrix, which describes the states of the extended system that includes not only neutrinos but also their microenvironments with a characteristic scale length, is positive. Three additional parameters are then introduced into the effective hamiltonian; they correspond to the interaction with the surrounding part of the system (for simplification, the authors kept only one of two which is additive to the conventional parameter \( \Delta m^2/2E \)).

The authors of Ref. [175] obtained and analyzed formulas for NO probability in the case of general dependence of decoherence effects on all parameters; they emphasized that these effects manifest themselves even with massless neutrinos and depend on the CP-odd phase which is present in the mixing matrix for the Majorana neutrino. In principle, this feature may serve to distinguish this case from that of the Dirac neutrino. Later, a study of simplified model for flavor oscillation was carried out [176], coming from Liouville decoherence and emphasizing attention to the cosmological constant (dark energy) as an origin of exponential quantum suppressing of low-energy observables. Invited talk [44] discusses the rôle of space-time foam already as the small possible contribution to \( \Delta m^2 \) in NO and speculates on the connection of QD with the dark energy, i.e., on involving QG-foam effects in the cosmological \( \Lambda \)-term origin. Then, elaborated ideas and methods of QG-induced QD in the string theory framework with underlying phenomenology was presented in invited talk [57].\(^{25}\) The decoherence evolution is considered with its characteristic feature of exponential time-dependent damping in NO probability both linear in \( t \), as in the conventional case, and quadratic in \( t \), as in cases of stochastically (random) fluctuating space-time foam [178, 161].

Besides considering several topics, above-quoted Ref. [21] discussed also non-linearly modified Lorentz transformations: both in connection with unitary non-equivalence of Fock’s flavor and mass spaces in the NO description in quantum field theory [179, 23] and in view of the natural requirement of invariant definition of the scale of Planck length/energy [181, 182]. Here, in a

\(^{24}\)This linear approximation may not comply with the complete theory [175] (see also Ref. [165]).

\(^{25}\)Contemporary methods and models for studying decoherence in particle physics, in general setting and, particularly, with its role in qualitatively new phenomena which imply discrete-symmetry violations, are described in review [177].

\(^{26}\)The low-energy limit of QG with \( \Lambda > 0 \) which must be invariant under deformed Poincaré symmetry [180] is discussed in the end of Ref. [41].
speculative scheme of "doubly special relativity" (DSR)\cite{115,24} the Lorentz group acts non-linearly on physical quantities, and new choice of group action leads to a new invariant energy scale (usually, Planck mass) as well as the invariant velocity $c$. In DSR models there is a dependence of the speed of light on the wavelength, and LI violation is really only 'apparent' effect when the usual linear Lorentz group action is violated.

While early DSR-type studies used broken LI, the most popular DSR schemes have 'deformed' LI, with no actual 'loss of symmetry' when in presence of a second observer-independent length scale $\lambda_{\text{DSR}} = M_{\text{Pl}}^{-1}$ all inertial frames remain equivalent, but modified Lorentz transformations appear as the transformation laws between the frames. In particular, MDR of the form $0 = 2[\cosh(\lambda E) - \cosh(\lambda m)] - p^2 \exp(\lambda E) \simeq E^2 - p^2 - m^2 - \lambda E p^2$ (here $\lambda M_{\text{Pl}} \equiv f^{(1)}$ in (12)) can hold for all inertial frame if a deformation of the boost transformations is $\lambda$-dependent\cite{183}.

Basing on exact form which is found for energy-momentum conservation law that is characteristic for deformed LI only, in contrast to the broken LI case when the usual form is valid, the authors managed to distinguish both scenarios by the sign of parameter $\lambda$\cite{183}: DSR requires $\lambda > 0$ as well as has typically small threshold anomalies and photon stability, the very properties that is absent in broken LI scheme where the possibility of positive $\lambda$ is not acceptable and is already excluded experimentally.

Although the DSR as a generalization of special relativity, but solely in momentum space, has the same number of generators (i.e., ten ones), it is not clear how to relate momentum to position while the notion of DSR space-time is still not developed; therefore now DSR is only a kinematic scheme\cite{114}.

As for the well-known problem whether flavor eigenstates or mass eigenstates emerge as real objects in a QFT-treatment of NO, that mentioned above in view of Ref.\cite{179}, the authors’ further studies of the existence of a Hilbert space for the flavor states showed that mixed neutrinos have proper MDR and that the corresponding non-linear realization of the Lorentz algebra is of the DSR-type\cite{182}.

Besides perturbative schemes of Kostelecký et al. and of Coleman and Glashow in Subsections B and C, respectively, another, more drastic approach to the problem of possible failure of LI was suggested by the authors of Ref.\cite{116}. In the Very Special Relativity (VSR) they substitute for the Poincaré group (as the exact symmetry of nature) by one of its certain subgroups which include space-time translations along with at least a 2-parameter subgroup of the Lorentz group (LG). Their subsequent work\cite{184} where VSR contains only a 4-parameter subgroup of LG, supposes a non-standard origin of lepton-number conserving neutrino masses without need for Yukawa couplings and see-saw models but with $2 \beta \nu$ processes forbidden as well as tritium $\beta$-decay end-point spectrum different from standard\cite{25}.

In contrast to the analysis of renormalizable Lorentz non-invariant terms in Ref.\cite{127} and Refs.\cite{145,146} that is described in Subsection C, the general discussion in Ref.\cite{185} of the possible LI violation on Planck scales that would affect NO was focused on studying nonrenormalizable effects that result in the energy dependence of oscillation length of the type $L_{\text{osc}} \propto E^{-n}$ with $n = 2$. The dependence with $n \neq -1$ is essential evidence of LI and CPT violation\cite{186}.

In the literature, there are the cases of this group with $n = 0$\cite{187,146,130,24} $n = 1$\cite{151,145}, and also $n = 2$\cite{189,190,191,30} and $n = -3$\cite{192,31}. Note that all possible terms in the effective

\begin{itemize}
  \item[25] In the next to last paragraph of Section 2 see also some issues of the DSR within the problems of review\cite{111}.
  \item[26] As for the tritium $\beta$-decay end-point spectrum in the VSR framework see discussion after the middle part of Subsection G.
  \item[27] Cases of energy-independent NO at $m_\nu = 0$ are also treated in Ref.\cite{158}; here, by analogy with solid state physics, the LI and CPT violations are introduced in the fermion vacuum of quantum field theory.
  \item[28] $L_{\text{osc}}$ is proportional to $M_{\text{QG}}^2/E^2$ if it is assumed that LI/EP is violated as a result of unequal values of MAV due to recoil effects in neutrino scattering by virtual $D$ branes\cite{169}.
  \item[29] $L_{\text{osc}}$ is proportional to $E^{-3}$ in the case considered of $m = 0$ in a $q$-deformed non-commutative theory\cite{193}. EP violation in effective Schwarzschild geometry modified by the hypothetical presence of the maximum acceleration $A_m = 2mc^3/h$ in the chosen gravity model corresponds to $L_{\text{osc}} \propto \Delta m^2/E^3$\cite{194} (see also Ref.\cite{131}).
\end{itemize}
action that are renormalizable and invariant under rotations correspond to \( n = 0, \pm 1 \).

The consequences of EP violation in noninertial reference frames were considered in Ref. [130] with \( n = 0 \); if \( \Delta m^2 = 0 \) and linear acceleration is zero, \( L_{osc}^{-1} \propto \omega \cos \beta \) where \( \omega \) is the angular velocity of the system (in the case of the Earth, \( \omega \sim 7 \times 10^{-5} \text{ rad/s} \) and \( \beta \) is the angle between the rotation axis and the momentum of the neutrino. It is emphasized the fact (already discussed in the literature) that the choice of metric affects the estimates of EP violation from NO data.

The authors of papers [192], in which the dependence on energy corresponds to \( n = -3 \), \( L_{osc} \propto \frac{E^3}{(\Delta m^2)^2} \), started with a speculation that the inertial and gravitational masses, \( m_i, m_g \), are two independent objects and the flavor oscillations carry the fluctuations \( \Delta E \Delta t \sim \hbar \) in a coherent manner: the inherent energy uncertainty for flavor states is related to the inverse of time period of NO. This general statement leads to quantum-induced violation of EP: \( m_g = (1 + f) m_i \), where \( f \) is non-zero for systems with no classical counterpart. Modification of standard commutation relation on Planck scales is also discussed in the last paper of Ref. [192]: \([x, p] = i\hbar (1 + L^2_{Pl}/p^2/\hbar^2)\) where \( L_{Pl} = \sqrt{\hbar G/c^3} \sim 10^{-33} \text{ cm} \).

One more origin of the spontaneous violation of LI invariance can arise in non-commutative field theories (see, e.g., [195]). In such a framework [196] and in the simple case, equal-time anticommutation relations have the form \( \{\psi(x), \psi^{\dagger}(y)\} = \mathcal{A}^{i\bar{j}}(x - y) \) \((i, j \text{ stands for flavor indexes})\) where \( \mathcal{A}^{i\bar{j}} = (\delta^i_j) \) is a constant matrix, \( \alpha \) is a deformation parameter in flavor space. In treating this deformation as the some kind of the extension of Standard Model [142], the massless-type MDR appear with non-trivial scale factor (so that \( \nu \) and \( \bar{\nu} \) of different flavors are degenerate in energy). This implies LI violation, leads to mass-independent energy difference for flavor states and to \( L_{osc} \propto |\alpha|EL \), while CPT symmetry remains intact.

4. Experimental and observational consequences of CPT and Lorentz invariance violation in neutrino physics

This Section contains the information on works dealing with those specific models of CPT, LI and EP violation in various neutrino processes where, as a rule, flavors are changed and for which the estimates of parameters that characterize the appropriate violation were obtained by comparing with measured data. The information on early works which discuss these subjects see in Section 5 of Ref. [59].

E. Perturbative violation of Lorentz invariance and CPT

In this Subsection we review papers that consider constraints on the parameters of perturbative violation of LI and CPT (with EP not violated) [52]; these parameters are predicted or expected on the basis of analyzing NO manifestations.

A comparison of the expressions (6)-(7) in Subsections B and C with the neutrino data of the 1990s showed that Lorentz non-invariant terms are found to be too small and do not significantly affect the interpretation of the available NO results (except for CPT-odd effects at very long baselines). At the same time, further investigation of oscillations of solar neutrinos and accelerator neutrinos at \( E \sim \text{TeV} \) and baseline \( L \sim 10^3 \text{ km} \) may detect LI violation when \( \Delta c \sim 10^{-25} \) [145, 148]. A recent analysis in Ref. [197] showed that a more stringent constraint than earlier ones may be obtained from the Super-Kamiokande (S-K) and MACRO experiments for atmospheric neutrinos at \( E \sim 100 \text{ GeV} \) and \( L \sim 10^4 \text{ km} \): \( \Delta c < 10^{-25} \).

The authors of Ref. [198] treated the cases of manifestation of CPT-odd effects caused by the interference of terms with \( \Delta m^2 \) and \( \Delta b \) in (9) when resonant amplification of NO amplitude becomes possible at \( \sin^2 2\Theta = 1 \), by analogy to the well-known MSW resonance when neutrinos pass through a sufficiently dense medium [34]. For instance, the resonance occurs in a medium with number density of electrons \( N_e \) for a simplified situation of \( \theta_m = \theta_b \equiv \theta \) and \( \eta = 0 \) when

\[^{32}\text{See also the end of Subsection I.}\]

\[^{33}\text{The phenomenon of the resonant flavor change of the MSW transition type was reported earlier in Refs. [150, 151, 152] and in other papers on EP violation (see, e.g., the references in Ref. [149]).}\]
the denominator of the generalized mixing angle $\Theta$,

$$\tan 2\Theta = \frac{(\Delta m^2 + 2E\Delta b)\sin 2\theta}{(\Delta m^2 + 2E\Delta b)\cos 2\theta - 2\sqrt{2}G_F E N_e},$$

vanishes for flavor index $\alpha = e$ in (8); here $G_F$ is the Fermi constant. The Ref. [198] argues that it is possible, in principle, to achieve estimates as low as $\Delta b \sim 5 \times 10^{-23}$ GeV, when analyzing the CPT violation in atmospheric neutrinos. And in neutrino factories, CPT violations could be detectable at the $3\sigma$ level for $\Delta b \approx (1 - 3) \times 10^{-23}$ GeV depending on the baseline length $L$ (at 29 GeV energy of stored muons).

At the same time, new analyses of the data on the absence of $\nu_{e\mu} \rightarrow \nu_\tau$ oscillations in the latest accelerator short-baseline experiments CHORUS and NOMAD are expected to furnish the limiting values at the levels of $\Delta b < 10^{-18}$ GeV and $\Delta c < 10^{-20}$ [141].

As for the value of $\Delta b$, a joint analysis [199] of the data on the solar neutrino and the expected sensitivity of the KamLAND reactor experiment gives the upper bound at the level $10^{-20} - 10^{-21}$ GeV. The result of mid-2004 global fitting [23] of solar and reactor data points to $\Delta b < 0.6(1.5) \times 10^{-20}$ GeV at $1\sigma (3\sigma)$, respectively.

More stringent restriction can be derived by taking into account that the neutrino and the charged leptons sectors of the theory are closely connected. So, the estimate $\bar{b}_i < 10^{-17}$ eV for spatial components of the quantity $\bar{b}$ (a conventionally chosen additive combination of $b$, and coefficients $d, H$, and $g$) in the sector of left-handed neutrinos was obtained for models with heavy right-handed Majorana neutrinos [200] on the basis of the available very strict limit on the axial term $\mathcal{E}b_\mu \gamma_\mu \gamma_5 \epsilon$ defined by the relation $\vert \bar{b}_i(electron) \vert \lesssim 10^{-19}$ eV. This estimate for $\bar{b}_i$ is weakened by four orders of magnitude ($\bar{b}_i \sim 10^{-4}\bar{b}_0$) by taking into account the motion of the solar system relative to the Galactic halo and that of the Earth around the Sun, so that a selection of a reference frame for $\bar{b}_\mu$ brings the obtained constraint down to the level $10^{-13}$ eV – still much more stringent than is anticipated for direct neutrino experiments [34].

To make the manifestations of the possible LI and CPT violation in oscillations accessible for realistic observations, the neutrino sector should be ‘shielded’ [202] from the sector of charged leptons. The authors of Ref. [202] connected the implementation of this idea with a unique operator $h^\mu_\beta(\nu_L)^\beta_\alpha \sigma_{\mu\beta}(\nu_L)_{\beta}$ that emerges in the light left-handed neutrino sector via a see-saw type mechanism through introduction of the appropriate LI violation for the heavy Majorana neutrino characterized by the constants $H_{\alpha\beta}$. This approach results in non-conservation of the lepton number $L (\Delta L = 2)$ while LI violation (with CPT conserved) valid for conventional neutrinos does not cover charged leptons via the radiative corrections in all orders of perturbation theory. The appropriate oscillations length is found to be independent of energy (as it is in the case of flavor transitions due to magnetic moment of the neutrino) and is dictated only by the constants $H_{\alpha\beta}$. A comparison of this approach with data (or estimates) provided the authors with the following constraints: $H_{\mu\tau} \lesssim 10^{-20}$ GeV (for atmospheric $\nu_\mu$), $H_{\mu\beta} \lesssim 10^{-22}$ GeV (for accelerator $\nu_\mu$ on a long baseline), $H_{\mu\beta} \lesssim 10^{-23}$ GeV (for neutrino factories), $H_{e\beta} \lesssim 10^{-19}$ GeV (for $\nu_e$ of the reactors CHOOZ and Palo Verde); the results of the KamLAND experiment with reactor neutrinos are described in this case at $H_{\mu\beta} \lesssim 7.2 \times 10^{-22}$ GeV.

Finally, the ratios of the expected numbers of $\nu_\mu$ and $\bar{v}_\mu$ events were estimated [203] in connection with new multi-kiloton magnetized iron calorimeter projects for studying atmospheric neutrinos in laboratories at Gran Sasso (Italy) and INO (India); the authors compared the results with predictions found in the CPT and LI violation scheme. The dependence of this ratio on $L$, $L/E$ and $LE$ obtained for a number of values of $\Delta b$ confirms the possibility of detecting these

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34The upper limit shown above for $\vert \bar{b}_i(electron) \vert$ was obtained in a precision experiment with torsion balance in which the probe body possessed certain residual magnetization caused by the spin dipole moment (due to the polarization of electrons). Later on, this experiment reduced the limit down to the level $10^{-20}$ eV [211]. Hence, CPT-violating constraint appears to be harder by a factor of ten.
violations for $\Delta b > 3 \times 10^{-23}$ GeV. These estimates are more stringent than was predicted for future neutrino factory projects.

Additional information on constraints of the quantities $\Delta c$ and $\Delta b$ in (9) that comes from NO data was presented in Ref. [105]. Similar results on constraining the parameters of possible violation of LI and CPT that come from experiments with atomic systems and muons were reported in the review talks of Ref. [204]. Constraints on LI violation parameters on Planck scales are also obtained by analyzing the high-energy parts of the cosmic rays spectrum; these constraints indicate that there is no effect of the Čerenkov radiation in vacuum for $p, e, \mu$ and $\nu$ [205].

As for using astrophysical and cosmological neutrinos to test LI violation via their Goldstone–Čerenkov emission (see Ref. [126] in the beginning of Subsection B), the authors evaluate three quantities – the emission rate, the neutrino energy loss rate, and the average deflection angle for single emission event. Comparing this value to SN1987A $\nu$-events data and CMB information on the energy accumulated in cosmological $\nu$ results in the following set of LI-violating bounds [126]:

$$
\mu_{SN} \lesssim 10^{-11}\text{GeV}(\frac{M}{10\text{ MeV}})^{3/2}(\frac{0.1\text{ eV}}{m}); \quad a_{SN} \lesssim 10^{-17}(\frac{M}{10\text{ MeV}})^{3/2};
$$

$$
\mu_{CMB} \lesssim 10^{-22}\text{GeV}(\frac{M}{1\text{ eV}})^{2}(\frac{0.1\text{ eV}}{m}); \quad a_{CMB} \lesssim 10^{-11}(\frac{M}{1\text{ MeV}})^{2};
$$

$$
\mu_{SN} \lesssim 10^{-15}\text{GeV}(\frac{M}{10\text{ MeV}})(\frac{10^{-3}\text{ eV}}{g})^{3/2}, \quad a_{SN} \lesssim 10^{-15}(\frac{M}{10\text{ MeV}})(\frac{10^{-3}\text{ eV}}{g})^{1/2}.
$$

Here, the first SN and CMB bounds are due to "ghost condensation" ('static' effects) while lower SN bounds are due to "gauged ghost condensation" ('dynamic' effect), $M$ is the scale of spontaneous Lorentz violation, $m$ is the neutrino mass, $g$ is the gauge coupling, parameters $\mu$ and $a$ correspond to dimension five and eight operators, respectively.

**F. Violation of Lorentz invariance and equivalence principle in terrestrial and cosmic neutrino physics**

In the review talk [11] that outlined the fundamentals of EP and LI violation in NO when neutrinos interact with the background gravitational field, the corresponding results of analysis of solar and atmospheric neutrino data available at the end of 1990s, as well as references to earlier works were presented. [36] The best constraints (the safest estimates) on the parameters $\Delta c$ and $|\phi \Delta f|$ for atmospheric neutrinos were [209] $6 \times 10^{-24}$ and $3 \times 10^{-24}$ at 90% CL, respectively, regardless of the values of mixing angles. The result for solar neutrinos was found to be at a similar level but was affected by the choice of assumptions. A detailed analysis of the available at that time atmospheric neutrino data with arbitrary values of the parameter $n$ in the energy dependence of the oscillation length, $L_{\text{osc}}^{-1} \propto E^n$, resulted in the constraint [209] $n = -0.9 \pm 0.4$ at 90% CL ($n = -1$ corresponds to ordinary oscillations of massive neutrinos). Still earlier results of experiments with atmospheric neutrinos failed to provide an opportunity to exclude any of the existing scenarios of EP violation in the spin-$J$ field exchange [210]: scalar with $J = 0$ and $n = -1$ (dilaton), vector with $J = 1$ and $n = 0$ (torsion in the Einstein–Cartan theory), tensor with $J = 2$ and $n = +1$ (graviton).

The authors of Ref. [211] give a description of global fitting of all solar neutrino data obtained before the publication of the results of the experiment SNO-2002 with solar neutrinos, and also provided information on previous interpretations of oscillation data, including those based on...

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35 In the framework of SME the goldstone emission mechanism is considered as secondary in the context of testing LI in NO and is disregarded (see, e.g., Ref. [206]).

36 In addition, see the talk [186], paper [207] and references therein, paper [130] mentioned earlier (in the second part of Subsection D), and also papers [208] where the gravitational interaction with the neutrino takes into account, besides the potential $\phi$, next-order terms in the post-newtonian approach, that describe new anisotropy effects. Violations of Einstein's EP are considered also in Ref. [135] (see last footnote in Subsection B), in connection with the cosmological birefringence (sizable rotation polarization angle via, in particular, the number density difference between neutrino and antineutrino).
The purpose of this work was to obtain a numerical comparison of possibilities of explaining experimental results in terms of different flavor-changing mechanisms for $\nu_e$. It was shown that in addition to the known large-mixing LMA(MSW) solution, oscillations can be explained at the same confidence level ($\gtrsim 60\%$) due to several mechanisms, in particular, by EP violation, by neutrino flavor changing through interaction of its magnetic moment with the external magnetic field, and by nonstandard neutrino interactions (NSNI) parametrized by two constants, one of which characterizes the contribution of flavor-changing interactions while the second determines the ordinary neutrino–medium interaction and plays a role similar to that of $\Delta m^2$ at the MSW resonance. It is also emphasized that experimental data do not warrant obtaining stringent constraints for the existence of solutions based on NSNI or on EP violation. The best description of solar NO reported in Ref. $[211]$ corresponds, in the case when they are caused by EP violation, to $|\phi\Delta f| \simeq 1.6 \times 10^{-24}$ and $\sin 2\theta_G = 1$ (vacuum-type solution); the MSW-type resonance solution requires $[215]$ the values of $|\phi\Delta f|$ that would be incompatible with the CCFR data.

The last analysis in Ref. $[216]$ of EP violation (within a simple model of the gravity going on physical mass basis) for solar $\bar{\nu}_e$ data from experiments Homestake, Sage, Gallex/GNO, S-K, and SNO (including reactor KamLAND results) gives the values for NO parameters near standard mass-flavor MSW solution and the conclusion, that the superior limit is $|\phi\Delta f| \lesssim 1.3 \times 10^{-20}$ (3$\sigma$) and the effect might take place for reactor antineutrinos.

A fitting of atmospheric neutrino data within EP violation or in the presence of NSNI provides very poor results (see talks $[217]$ and references therein); no interpretation of these data on the basis of pure NSNI mechanism is acceptable for 99% CL $[218]$, mostly due to the independence of this mechanism on energy. The subsequent fitting $[219]$ of atmospheric S-K data and the K2K experiment results showed that resorting to LI and CPT violation as an additional mechanism of NO hardly affects standard parameters. The restrictions (giving eightfold improvement on the results of Ref. $[209]$ (see the beginning of Subsection F) obtained at 90% CL in the $\nu_\mu - \nu_\tau$ sector are the following $[219]$: $|\Delta c| \lesssim 8.1 \times 10^{-25}$, $|\phi\Delta f| \lesssim 4.0 \times 10^{-25}$, $|\Delta b| \lesssim 3.2 \times 10^{-23}$ GeV, and $|\Delta\delta_0| \lesssim 4.0 \times 10^{-23}$ GeV – the last constraint is for NSNI with vector-type torsion field ($J = 1$) via CPT-even effects for $n = 0$. The corresponding 3$\sigma$ limits were also obtained and found to be greater by a factor of 1.5 – 2. $[39]$ The NSNI three-flavor interpretation of experimental results with solar and atmospheric neutrinos $[221]$ supports solar data description of Ref. $[211]$ (in the case when it is made more complicated due to mutual influences of these two sectors) while the inclusion of the KamLAND data disfavors NSNI hypothesis. Basing on upper limits of CPT/LI-violating coefficient differences $\Delta b$, which were obtained in Refs. $[198, 203]$ (see Subsection E) and in Ref. $[219]$, authors of a later three-flavor analytic treatment $[222]$ calculated bounds of the same order but for quite different exact combinations of the matrix $\hat{b}$ elements (see Eq. (7)) for the FNAL two-detector experiment NO$\nu$A at $E_{\nu_\mu} < 5$ GeV and for neutrino factories at $E_{\nu_\mu} > 15$ GeV, correspondingly.

The posterior work $[223]$ has shown, that these limits could be improved by over two orders of magnitude at high energy and high statistics with IceCube detector.

Using a subsample of 300 upward-throughgoing muon events in the MACRO detector, upper limits were established $[224]$ on LI- and EP-violating differences $\Delta c$ and $\Delta f$. As the subdominant

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$[37]$ See also talks $[212]$ and papers $[213, 214]$.

$[38]$ Many papers appeared recently in the literature, which interpret NO data in terms of NSNI mechanisms. Their phenomenological manifestations are typically characterized by that the effect is independent on neutrino energy, which inhere also in the contribution of the parameter $\Delta b$ in (9). It is seen from the discussion in Subsection C that this is also the case for the scalar version of EP violation. Hence, information on the limiting values of $\Delta b$ and $|\phi\Delta f|$ is likely to be extractable also from the data that yielded constraints on the parameters of NSNI.

$[39]$ At maximal mixing the results given are much stronger, e.g., $|\Delta c| \leq 2.0 \times 10^{-27}$; see also the $\nu_{\mu\tau} + K2K$ limits and sensitivities of the AMANDA-II, IceCube and MACRO detectors in Proc. of the first workshop EPNT06 $[220]$. 

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origins of atmospheric NO via $\nu_\mu \to \nu_\tau$ transition, these differences satisfy the following bounds at 90\% CL: $|\Delta c| < 2.5 \times 10^{-26}$ at maximal mixing, $|\Delta c| < 3 \times 10^{-25}$ at marginalization with respect to all the other parameters; as for EP violation, $|\phi \Delta f| \cong |\Delta c|/2$.

It can be expected also [225], that for a wide range of parameter values the estimates of EP violation there will be obtained in muon storage rings by recording changes of neutrino flavor by ordinary NO manifestations.

Some more formerly in the Ref. 226, a scheme was suggested in order to calculate $2\beta 0\nu$ decay rate with LI/EP violation. The main conclusions, when comparing the NO and the $2\beta 0\nu$ as probes of the violation in such scheme, were the following: (1) $2\beta 0\nu$ gives the most stringent bound on LI violation while NO cannot limit it at mixing angle $\theta_c \to 0$; (2) the dominant contribution to Lorentz violation comes from momentum-dependent $\Delta c$ term with the effective neutrino mass $\langle m \rangle$; (3) using $2\beta 0\nu$ data of the Heidelberg–Moscow experiment and known estimates for $\langle m \rangle$, the bound $\Delta c \lesssim 10^{-16} - 10^{-18}$ ($\theta_c = \theta_m = 0$) was obtained. (Similar independent calculation scheme for EP violation, as it was shown, points out that the LI bound can be directly translated to analogous EP bound for $|\phi \Delta f|$ when gravitational mixing vanishes while this region is constrained only by the $2\beta 0\nu$.

A detailed study of possible constraints on the EP- and LI-violating parameters in neutrino factories [227] led to a conclusion that measuring the T-odd probability difference $P(\nu_\alpha \to \nu_\beta) - P(\nu_\beta \to \nu_\alpha)$ provides the most sensitive evaluation of this violation. For $(\nu_e, \nu_\mu)$ and $(\nu_e, \nu_\tau)$ sectors, the limiting value $|\phi \Delta f| \lesssim 10^{-26}$ can be achieved [227] with a suitable baseline of several thousand kilometers, that is comparable to the maximum constraint in the $(\nu_\mu, \nu_\tau)$ sector obtained in Ref. 209 for atmospheric neutrinos ($\nu_{\text{atm}}$) using the S-K results.

In paper 228, a brief review of the literature is given on consequences of EP violation in various neutrino processes, including neutrino astrophysics and primary nucleosynthesis [10]. It was shown that the low probability of $\bar{\nu}_e \to \bar{\nu}_{\mu,\tau}$ transitions coming from supernova SN1987A data points to a very strong constraint on the corresponding parameters: for massless or mass-degenerate neutrinos $|\Delta f| \lesssim O(10^{-31})$ [11] and $\tan^2 \theta_G \ll 10^{-4}$. However, these constraints become invalid or weakened if the effect due to the mass is dominant. As it should be noted in this case, an analysis of consequences of EP violation in NO, which may be detected in future observations of super-high-energy neutrinos arriving from cosmologically remote active galactic nuclei, is likely to yield an even stronger constraint – at a level $|\Delta f| \sim 10^{-41}$ [230].

EP violation may be directly related to the formation of neutron stars, namely to the fact that pulsars at the moment of birth acquire considerable peculiar velocities. First evaluations of EP violation in resonant flavor transitions (for the maximum efficiency case of $J = 2$) that can sustain required velocities in the anisotropic ejection of neutrinos from a presupernova (provided there is a magnetic field $> 10^{15}$ Gauss) yielded the value $|\Delta f| \simeq 10^{-10} - 10^{-9}$ [231]. The translational and rotational motion of pulsars caused by directionality of neutrino ejection can be interpreted even at zero magnetic field [232] if resonance transitions are assumed to be caused by above mentioned anisotropy effects [208] in post-newtonian approach to gravitational neutrino interactions.

Finally, Ref. 233 attracted attention to the importance of simultaneous neutrino and optical monitoring of type-II presupernovas. The data on the time of recording and the characteristics of both signals, as well as the observation of the frequency difference in the atomic spectra on the surface of the star before and after the neutrino ejection pulse provide information both on the gravitational potential of the neutrino flux and the neutrino mass, and on the EP violation.

Constraints on the EP-violating parameters are also considered in Ref. 138 (see second footnote in Subsection B) and in Refs. 190, 191 (see two last footnotes in Subsection D). Possible violation of EP in some exotic cosmological models is mentioned in the end of Subsection G.

\[\text{footnote1: See also Ref. 229 about NO in wormhole-type objects.}\]

\[\text{footnote2: Evaluations of EP violation expressed in terms of constraints on the parameter } |\phi \Delta f| \text{ typically assume that the quantity } \phi = \text{ const } \sim 3 \times 10^{-5} \text{ is determined by the mass of the local galaxy supercluster.}\]
G. Manifestations of CPT and Lorentz invariance violation with ‘non-standard’ mechanisms in neutrino physics

This Subsection mostly deals with CPT/LI-violating neutrino processes involving flavor change owing to resonant effects and to ‘non-standard’ mechanisms (see Subsection D). The resonant effects originate in the energy level crossing of neutrinos (and, separately, antineutrinos), at low and high density regions of supernovas. The ‘non-standard’ mechanisms imply, in particular, quantum decoherence (QD) in the form of Eq. (10) in the linear formalism mentioned in the middle of Subsection D and MDR of Eq. (11) in Einstein’s gravity as well as in loop quantum gravity.

Several papers reported an analysis of NO data in two-flavor approximation based on the possible decoherence effect that is described by the right-hand side of (10) and parametrized by six variables (as mentioned in the second paragraph below Eq. (12)). Earlier attempts of explaining the deficit of solar neutrinos ($\nu_{\text{sol}}$), as well as atmospheric ($\nu_{\text{atm}}$) data, disregarded the requirement of ‘total positivity’ that relates these parameters to one another. Then stringent constraints for one of them, $\gamma = \gamma_0(E/\text{GeV})^k$ ($k = -1, 0, 1, 2$), were found in the simplest single-parameter case (in the limit when the neutrinos are weakly influenced by environments); this parameter characterizes the suppression of the conventional oscillations term with $\Delta m^2$ through additional factor $\exp(-2\gamma L)$ due to QD. Strong constraints for the parameter $\gamma_0$ were extracted from the $\nu_{\text{atm}}$ data at 90% CL for $k = 0, 2, -1$, equal to $3.5 \times 10^{-23} \text{ GeV}^{-1}$, $0.9 \times 10^{-27} \text{ GeV}^{-1}$, and $0.7 \times 10^{-21} \text{ GeV}$, respectively. A detailed analysis of a more realistic case of $k = -1$ on the basis of the result of S-K + K2K experiments in the channel $\nu_\mu \to \nu_\tau$ failed to detect evidence for QD effect; however, it equally not ruled out its presence at $\Delta m^2 = 0$.

As for QD as a subdominant effect, still more hard limits on parameter $\gamma_0$ were obtained very recently from a global fit to solar and KamLAND NO data. They are given at 95% CL for, respectively, $k = -2, -1, 0, +1, +2$ (here the neutrino energy $E$ is taken in GeV to have all bounds in the same GeV units): $\gamma_0 < (0.81 \times 10^{-28}, 0.78 \times 10^{-26}, 0.67 \times 10^{-24}, 0.58 \times 10^{-22}, 0.47 \times 10^{-20}) \text{ GeV}$. The last value at $k = +2$ contradicts to the model estimate and is worse than the atmospheric limit.

As it was found recently in Ref. the expected sensitivity limits of modern experiments CNGS (CERN-SPS, 400 GeV/c, to OPERA (Gran Sasso), $L \approx 730$ km) and T2K (Tokai JPARC, 40 GeV/c, to S-K, $L \approx 730$ km), which should be directly compared with above S-K and K2K results, are as follows, respectively for $k = 0, -1, +2$: CNGS $2 \times 10^{-22}, 9.7 \times 10^{-22}, 4.3 \times 10^{-26}, T2K - 2.4 \times 10^{-23}, 3.1 \times 10^{-23}, 1.7 \times 10^{-23}$.

Earlier fitting of reactor and short-baseline accelerator experiments (CHOOZ, CHORUS, E776, CCFR) established upper bounds on $\gamma_0$ for all values of $k$ at 99% CL; constraints in the $\nu_\mu \to \nu_\tau$ channel were found to be considerably weaker than those obtained from the $\nu_{\text{sol}}$ data. In the channel $\nu_\mu \to \nu_e$ they were, by the order of magnitude, $10^{-22} \text{ GeV}^2$, $5 \times 10^{-24} \text{ GeV}$, and $10^{-26} \text{ GeV}^{-1}$, respectively, for $k = -1, 0, 1, 2$ (it appears that the last two constraints will unlikely be improved using the $\nu_{\text{sol}}$ data); the limits in the channel $\nu_e \to \nu_\tau$ are such that the results are more stringent than those obtained from the $\nu_{\text{sol}}$ data only if $k = 2$: $\gamma_0 \lesssim 10^{-24} \text{ GeV}^{-1}$.

The same authors carried out a quantitative analysis of the potential uses of long-baseline

42These topics are presented in detail in lectures in the framework of a review on CPT violation in quantum gravity models.

43See also a pessimistic note of Ref. 237.

44See also a note on the equivalence between two models: averaged over experimental parameters model of NO and QD model, which is used, e.g. for analysing the S-K $\nu_{\text{atm}}$ data.

45See also Ref. 241.

46Above-mentioned as well as similar limits are discussed in Ref. 243 in view of studying QG-induced lepton flavor violation with AMANDA and IceCube detectors at the South pole.
accelerator experiments, K2K, MINOS, OPERA, and of a neutrino factory, in order to discriminate between ordinary NO and NO due to purely decoherence effects in ν_{atm} transitions ν_{μ} → ν_{τ}. Very recently estimated NO parameters for the MINOS experiment, as it was reported in Ref. [245], are in agreement with their current values, provided that observed ν_{μ} deficit and spectrum are due to ν_{μ} → ν_{τ} oscillations at L ≈ 730 km between two detectors. Best fits to energy spectra in the far detector disfavor the alternative QD scheme of Ref. [236] at the 5.7 σ level. The subsequent work [246] followed Ref. [244] extended the initial formalism of Ref. [174] to the three-flavor system in the framework of a general approach, which is independent of specifics of the model of QD interaction between neutrino and surroundings, and obtained explicit formulas for NO probabilities in this case. The authors of this work also studied the correspondence of the three-flavor analysis to the above-described two-flavor decoherence analysis for ν_{atm}. Two qualitative scenarios were investigated: (1) flavor change in NO due to QD only, and (2) joint effect of this mechanism and the conventional one. It was shown that with a simplifying assumption of diagonality of dissipation matrix in the right-hand side of (10), both versions of taking QD into account fail to comply with experimental data if the mixing in the channel ν_e → ν_μ or ν_e → ν_τ is included.

With a view to improve (by a factor of at least 10^{12}) the sensitivity for observing QD by future neutrino telescopes such as IceCube, the authors of Ref. [247] suggest making use of ultra-high energy ¯ν_e from cosmic neutron β decays. In this case after usual NO with initial neutrino flavor ratios 1 : 0 : 0 one obtains 0.56 : 0.24 : 0.20 while effects of the QD in all cases [248] as well as NO in the case of charged-pion→muon decays result in 1 : 6 : 3. As was shown in Ref. [249], the IceCube detector can strongly improve the current sensitivity to the ¯ν_e TeV-energy flux from cosmic-ray neutron decays in the massive star forming region Cygnus OB2 at L ≈ 1.7 kpc. In a parametrization of QD coefficients (similar to schemes of above-quoted works) in the form γ = γ_n(E/GeV)^n, their upper bounds at 90 (99) % CL are the following: γ_{-1} ≤ 1.0 × 10^{-34} (2.3 × 10^{-31}) GeV, γ_0 ≤ 3.2 × 10^{-36} (3.1 × 10^{-34}) GeV, γ_1 ≤ 1.6 × 10^{-40} (7.2 × 10^{-39}) GeV, γ_2 ≤ 2.0 × 10^{-44} (5.5 × 10^{-42}) GeV, γ_3 ≤ 3.0 × 10^{-47} (2.9 × 10^{-45}) GeV. [47]

We need to mention in this context that there is an extremely strong astrophysical constraint on the QD effect: γ_0 ≤ 10^{-40} GeV at k = 0 [251]. It is based on the published estimate of the upper limit of the probability for recording the fact of NO from data of the supernova SN1987A at L ≈ 50 Mpc (with taking into account all the NO processes): P(¯ν_e → ν_μ,τ) < 0.2 [252]. This constraint imposes a very considerable limitation on the expectation of observing the effect in other experiments, even though the data on NO from active galactic nuclei may amplify it by many orders of magnitude [251].

In the theoretical analysis of LI tests by Planck-scale-MDR of the type (12), a possible sensitivity to M_{Pl} suppressed effects is expected [253] in time delay experiments with very-high energy neutrinos from gamma ray bursts at observatories such as ANTARES. This case, with and without the emergence of a preferred inertial frame, is not affected by the delicate problem, discussed by the author, of a deformed law of energy-momentum conservation. The author of paper [254] argues that for cosmogenic neutrinos at very high energies of 10^{18} − 10^{21} eV the Planck-scale corrections to NO length, that are suppressed even by M_{Pl} factor in QG-MDR of Eq. (12)-type, may be detected with telescopes such as IceCube and ANITA.

In the paper [255] a review is presented on the theoretical foundation and motivation for studying CPT/LI violation with high-energy neutrinos originated from cosmically distant decays of the charged pions and the neutrons. The goal is to obtain signatures for identifying the violation effect by confronting observed neutrino flavor ratios R_q (q = ν_e,ν_μ,ν_τ) to predicted ones, which were calculated and displayed for two simplified illustrative scenarios of the energy

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[47] Previous upper bounds of the QD constants γ_0 and κ_n, which are obtained from simulating atmospheric NO at E ≥ 200 GeV for neutrino telescopes, such as ANTARES, were presented in Ref. [250].
behavior of $R_q(E)$ at $10 \text{ GeV} \lesssim E \lesssim 10^8 \text{ GeV}$ with MDR of the Eq.(12)-type when substituted $M_{Pl}$ instead of $M$. When considering Lorentz-violating case based on a general off-diagonal hamiltonian formalism with one real parameter explicitly proportional to $E^2$ or $E^3$, the initial set of $R_q$ for pionic origin is radically changed above the energy threshold of the effect at $E_{thr} \sim 1 \text{ TeV}$ or $\sim 10 \text{ PeV}$ (1 PeV = $10^{15}$ eV), respectively, while for neutronic origin it alters there to a smaller degree. In the case of CPT violation, a second approximation scheme is considered, with one nonzero QD parameter only, in which, for simplicity, only one $L$-dependent term $e^{L\tilde R}$ was left, after averaging to zero the sin and cos terms over large distances. Then, in a model with $\tilde \delta = (E/10\,000 \text{ GeV})^2/L$ all the $R_q(E)$ came to the same value, $\frac{1}{3}$, – gradually for pionic origin while dramatically for neutronic one. In conclusion, the signatures are discussed of QD effects on neutrino events observed in high-energy telescope, such as IceCube and KM3.

Because the first physical data from neutrino telescopes can be obtained even in the $\nu_{\text{atm}}$ exposition, authors of Ref. [250] proceeded to study probing LI but in the (12)-type scheme of MDR as in Ref. [255] at $u \geq 2$ (in particular, of DSR-type) and simulated the ANTARES sensitivity, as in Ref. [254]. In further discussion they used upper bounds on Lorentz-violating parameter estimated, when additional term in NO phase is proportional to $E^u$, in two neutrino mass cases: at $\Delta m^2 \neq 0$, $-2.9 \times 10^{-24}$, $u' = 1$; $2.9 \times 10^{-35} \text{ eV}^{-1}$, $u' = 2$; $6.9 \times 10^{-46} \text{ eV}^{-2}$, $u' = 3$; and at $\Delta m^2 = 0$, $-8.2 \times 10^{-25}$, $u' = 1$; $1.0 \times 10^{-35} \text{ eV}^{-1}$, $u' = 2$.

Meanwhile, recent consideration of MDR of the (12)-type and its influence on $R_q$ of high-energy neutrinos originated from distant cosmic sources shows [257] that even in this case the sensitivity of future IceCube and ANTARES telescopes will not ensure the identification of the CPT/LI-violating effects through variation in the averaged $R_q$ observed.

Possible tests of LI violation when they sensitive to Planck scale physics due to Eq. (12) depend on neutrino energy extension: in atmospheric $\nu$ experiments the PeV energies will be accessible while in gamma ray bursts (GRB) the values about exo-eV ($1 \text{ EeV} = 10^{18} \text{ eV}$) are expected. From this point of view, in Ref. [258] the problem of observing GRB neutrino events of three flavors with its time delays and energies is reconsidered in detail for the case of IceCube-type detectors.

Another source of CPT violation is the interaction of fermion spin with the spin connection of the external gravitational field in Einstein’s theory provided its sign is not reversed under CPT. The contributions of this interaction to the energy for Dirac neutrino and antineutrino are of opposite signs. So, MDR are appeared with helicity energy gap for $\nu$ and $\bar{\nu}$. This fact results in unequal number densities of $\nu$ and $\bar{\nu}$: in the early Universe – due to their scattering on primordial black holes [259] and in axially symmetric cosmological solutions [259, 260], in today’s epoch – via scattering on rotating black holes [259, 261]. In the general case of CPT/LI violation with Dirac and Majorana masses [264] and taking into consideration recent works on NO in gravitation field, authors of Refs. [262, 263] and [264] argue, that strong axial spin-gravity coupling results also in neutrino-antineutrino oscillations without flavor changes, which have no dependence on $m$ and $E$ [262, 263, 264]. The $\nu - \bar{\nu}$ asymmetry [263] and $\nu \leftrightarrow \bar{\nu}$ oscillations in the anisotropic phase of early Universe may lead to lepto- and baryogenesis [263, 266] (and, possibly, contribute to $r$-process nucleosynthesis in supernovae [263]); the $\nu - \bar{\nu}$ mixing influences the $2\beta0\nu$-decay rate [264] as the mass of the flavor state is modified.

Some papers, in which authors work within loop quantum gravity [267, 271], treat MDR of Eq. (11) too. Besides, the theory assumes the existence of an intermediate scale $L \gg L_{Pl}$ that separates the lengths $d \ll L$ at which the loop structure of space manifests itself, and the lengths $d \gtrsim L$ at which flat classical geometry is regained. Detailed investigation showed [190, 267] that in the framework of this approach the function $F$ in (11) for the neutrino in vacuum is in general parametrized by nine constants of different degrees of suppression (compared to unity) through a factor $(L_{Pl}/L)^{3T+2}$ ($T \geq 0$ is an additional phenomenological parameter that is possibly a function of energy), with the scale $L$ defined for two scenarios ($L \sim 1/E$ and $L \sim \text{const}$); the linear in $p$ additive term in $F$ includes the sign ‘±’ in accordance to the helicity. The authors
of Ref. [267] analyzed in detail the possibility of extracting information (with evaluating the restrictions on the parameter \(\Upsilon\) based on the \(\nu_{\text{atm}}\) data) on two characteristics of observable (in principle) effects of cosmic GRB as they are accompanied with powerful ejection of massive neutrinos with \(E \sim 10^5 - 10^{10}\) GeV: (1) by the signal delay time for various neutrinos in comparison with the light signal, found to be of the order of \((EL_{\text{Pl}})/c \approx 10^4\) s; (2) by the \(E\)-dependence, \(L_{\text{osc}}^{-1} \propto E^2L_{\text{Pl}}\), which differs from that discussed in Ref. [209] (see the beginning of Subsection F).

On the basis of the above formalism with working set of parametrization constants and by comparing theoretical results with NO data and with the spectrum of cosmic rays of extragalactic origin, the following problems were also considered: the energy dependence of NO length [269], constraints on the intermediate scale \((\mathcal{L} \gtrsim 10^{-18}\text{ eV}^{-1}) [269, 270]\) and the working constant [270], and a novel mechanism [271] for generating the primary cosmological asymmetry of the Universe originating from the density difference of neutrinos and antineutrinos caused by the above distinction in signs for the linear-in-momentum contribution to function \(F\) in Eq. (11).

According to some particular QG models (see, e.g., Refs. [170, 267]), the lower order in \(E_{\text{QG}}\) energy-dependent corrections of the general type to Lorentz-violating MDR of Eq. (11) may originate from the dimension \(d = 5\) operator while \(d = 4, 6\) operators are \(E\)-independent. The paper [268] is the first study to date in which the time delay, \(\Delta t\), for massive neutrinos from GRB is calculated with correctly accounting the time dependence of the Hubble constant due to matter effects as well as dark energy effects. The analysis for \(F \propto \pm E^2(E/E_{\text{QG}})\) was carried out for observable values in the planes \(\Delta t\) \textit{vs} \(E\) in the wide energy interval, \(\Delta t\) \textit{vs} \(m/E\) and \(E/E_{\text{QG}}\) for redshift \(z = 0.01 - 10\) and for three different cosmological models.

Very recently, after considering the stochastic space-time foam models, more generic than in Ref. [161], with small random fluctuations about flat Minkowski background, the authors came to conclusion [272], that the current NO data do not have the sensitivity to test such the models; only high-energy neutrinos from GRB can constitute sensitive probes of QD [247, 249].

For the deformation parameter in a non-commutative fields scheme with massless-type MDR (mentioned at the very end of Subsection D), the following bounds are obtained from LMA scenario at mixing angles \(\theta \sim \pi/4\) on the basis of atmospheric and solar neutrino results, correspondingly: \(|\alpha_{23}| < 10^{-22}, |\alpha_{12}| < 10^{-17}\) [196].

For exploring quantum gravity-induced violation of LI in Ref. [273], the MDR (11) with \(F \simeq \pm E^2(E/\xi M_{\text{Pl}})^{u}\) (\(\xi\) is scale parameter, \(u = 1, 2, 3, \ldots\)) were used. The authors consider, for \(u = 1\) and 2, the feasibility of detecting the effect, with the presence of background muon events from \(\nu_{\text{atm}}\), on the basis of estimating \(\Delta t\) between neutrino and low-energy photon signals from cosmic GRB while \(\Delta t\) of order of hours are expected for 100-TeV \(\nu\)'s at redshift \(z = 1\) and \(u = 1\). MDR of the similar type are considered in Ref. [274] to put bounds on LI-violating neutrino coefficient \(\tilde{\alpha}_2(\mu)\) and mass scale \(m_\mu\) by using relations \(E^2 = p^2 + 2m_\mu|p|\) and \(E^2 = p^2 + \tilde{\alpha}_2(\mu)|p|^3/\xi M_{\text{Pl}}\) (as the \(u = 1\) and \(u = 3\) cases of MDR (12) for \(M_{\text{QG}} \to M_{\text{Pl}}\) within the framework of the doubly special relativity (DSR) [48] for its substitution to formulæ for the widths of the \(\pi_\mu(\mu_2)\) decay and the neutron \(\beta\) decay. The authors give also examples of MDR in other interesting models.

When studying a generalized approach for deducing the equation of motion and the dispersion relation of a propagating fermionic particle, the author of Ref. [275] recovered the corresponding VSR formulæ of Ref. [184]. Then, the new LI-violating VSR-dynamics was applied to the tritium \(\beta\)-decay in order to compare the modified Curie-plot with the endpoint spectrum data as was pointed out in Ref. [184].

In subsequent work [276], with usual approach for obtaining LI-violating neutrino dynamics similar to VSR, new MDR are found via a specific transformation combined boosts and rotations with the aid of a light-like preferred vector, while the Lorentz algebra holds. Then an effective

\[\text{References:}\]
[111] at the end of Section 2 where the role of the DSR is presented in modern tests of LI.

\[\text{References:}\]
[275] at the end of Section 2 where the role of the DSR is presented in modern tests of LI.
neutrino mass effect in this preferential direction scenario also confronts the MDR parameters with data for tritium end-point spectrum.\(^{49}\) Analogous suggestion was made early in paper\(^ {179}\), with using MDR of DSR-type\(^ {182}\) which appears in a study of LI for mixed neutrinos (see mentioning these works in discussion on non-linear MDR in Subsection D below Eq. (12)).

As it has been suggested about ten years ago, space-time foamy structure in quantum gravity may imply LI breaking with MDR (see discussion around Eq. (11)) that leads to deviations of velocities (sub- or super-luminal) for massless neutral particles from speed of light. In this respect, the Ref.\(^ {278}\) gives the relevant limits on linear or quadratic Lorentz violation when observing energetic neutrinos from SN1987A in the form \(\delta v = (E/M_{QG})^l\), with \(l = 1, 2\). Based on statistically poor data sets from KamiokaII, IMB and Baksan detectors with event lists of neutrino energy and arrival time and using minimal dispersion method, the authors obtained the following results at \(95\%\) CL for sub-(super-)luminal cases, respectively: \(M_{QG1} > 2.7(2.5) \times 10^{10}\) GeV, \(M_{QG2} > 4.6(4.1) \times 10^4\) GeV. For the possible registration of future galactic supernova at 10 kpc in the S-K, Monte Carlo simulation gives rise: \(M_{QG1} > 2.2(4.2) \times 10^{11}\) GeV, \(M_{QG2} > 2.3(3.9) \times 10^5\) GeV. Potential sensitivity (in the case of future improvements) of the OPERA detector (long-baseline beam from CERN to Gran Sasso) as estimated in Ref.\(^ {278}\) exceeds the \(M_{QG2}\) limit: \(M_{QG2} \sim 7 \times 10^5\) GeV.

With the goal of using supernova neutrinos for testing CPT symmetry, authors of Ref.\(^ {279}\) consider feasible level-crossing patterns of the resonant flavor conversion in SN progenitors and estimate fluxes of three neutrino species \(\nu_e, \bar{\nu}_e, \nu_x\), as collective notation for \(\nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau\). These authors concentrate on difference between \(m_\nu\) and \(m_\bar{\nu}\) patterns (as well as on \(\theta_{13}\) and \(\bar{\theta}_{13}\) ones) and analyse how flux features of six working patterns can be discriminated during the observation of future galactic supernovas.

Finally, a separate problem discussed in the literature stems from attempts at describing the closeness of the observed neutrino masses and the energy scale of the dark energy, which determines the acceleration of the cosmological expansion of the Universe. The authors of Refs.\(^ {178, 131}\), extending and specifying the general line of their above-mentioned work\(^ {165}\) (besides a new interpretation of all NO data including LSND\(^ {54}\)) continue also the study of possible model approaches to interpret the cosmological \(\nu - \bar{\nu}\) asymmetry and to obtain a meaningful evaluation of the vacuum energy (the so-called dark energy or the cosmological term)\(^ {51}\) caused by mixing of neutrinos through QD. The starting point is the assumption\(^ {178, 131}\) that QD effects (caused by the interaction of the neutrino with the foam structure of the vacuum (see Subsection D)) contribute to the terms of hamiltonian in Eq. (10) for the evolution of the density matrix and result in the emergence of effective mass shifts, by analogy to the MSW effect in a medium.\(^ {52}\) Note that the application of the idea of a generic origin of the neutrino mass and the dark energy in one form or another is typical for a lot of works studying the possible interactions between neutrino (taking account also of varying mass) and dark energy. Different models with spontaneous and dynamical baryo- and lepto-genesis induce CPT and LI (and in some cases also EP) violation\(^ {281, 282, 283, 53}\) that can be tested in future NO experiments. Among earlier works in this field, we mention Ref.\(^ {285}\) where EP violation in a scheme of the cosmological ‘quintessence’ is considered.

\(^{49}\) Additional corrections to this spectrum via de Sitter symmetry breaking in VSR are discussed in Ref.\(^ {277}\).

\(^{50}\) Issues on LSND anomaly see in Section 5 (Subsections I, J and K).

\(^{51}\) The authors proceed from the non-equivalence of the flavor-state and of mass-state neutrino vacua; this results in non-trivial contribution to the cosmological term owing to the mixing effect itself\(^ {280}\).

\(^{52}\) In Ref.\(^ {178}\) a possible scenario is suggested in which, provided that only in antineutrino sector there is CPT-violating QD, due to charge conservation microscopic charge black hole–anti-black hole pairs in space-time foam can produce mostly \(e^+e^-\) (but rather not \(\mu^+\mu^-\)) pairs. Then, if, e.g., positrons are absorbed by event horizons during evaporation of anti-black holes, stochastically fluctuating electron density excess is originated.

\(^{53}\) For reviews of another analogous schemes see Ref.\(^ {138}\) (which discusses the cosmological birefringence in the end of second footnote in Subsection B), Refs.\(^ {282}\), and talk\(^ {65}\); in view of Refs.\(^ {283}\) see also references therein and the discussion in paper\(^ {284}\).
Completing the discussion of the general consequences which relate CPT with neutrino physics, we need to remind the reader that the CPT non-invariance of the theory results first of all in the independence of the CP- and T-violation effects.

5. Interpretations of neutrino oscillations based on CPT violation

In what follows we consider those various (different in principle) manifestations of CPT-odd effects that may originate NO in terrestrial experiments. We begin (see Subsection H) with publications in which no true (fundamental) violation of CPT invariance is assumed in the theory but in which oscillations are treated in the conventional (for the most part, CPT-non-symmetric) medium.

In Subsection I attempts are discussed to interpret NO not by deriving them from the fact that neutrinos are massive, but on the basis of LI and CPT violation in the theory (similarly to those we discussed in Section 3, including those originated from EP violation in gravity theories). Some hypothetical ways of explaining the LSND anomaly are given in Subsection J. Subsection K discusses papers aimed at obtaining constraints on the parameters of the fundamental CPT violation in the case when neutrino and antineutrino parameters are distinct (or at least $\mu_\nu \neq m_\nu$). We note here once more, that the authors of the papers with $m_\mu \neq m_\nu$ assume groundlessly that LI is conserved (see discussion of this problem in the beginning of Section 3).

H. False CPT-odd effects in matter

A number of papers [286]-[288] analyzed false CPT-odd effects caused by medium influence on NO while conserving the CPT invariance of the theory; these effects result, first of all, in nonzero asymmetry of the probabilities of diagonal transitions, $\Delta P_{\alpha\alpha}^\text{CPT} \equiv P_{\nu_{\alpha} \to \nu_{\alpha}} - P_{\bar{\nu}_{\alpha} \to \bar{\nu}_{\alpha}}$, i.e., the difference of survival probabilities of neutrino and antineutrino of a given flavor, and in similar asymmetries, $\Delta P_{\alpha\beta}^\text{CPT} \equiv P_{\nu_{\alpha} \to \nu_{\beta}} - P_{\bar{\nu}_{\beta} \to \bar{\nu}_{\alpha}}$, for non-diagonal transitions. For instance, in very-long-baseline experiments for terrestrial and atmospheric neutrinos, the dependence was demonstrated [287] of $\Delta P_{\mu\mu}^\text{CPT}$ on energy and angle $\theta_{13}$, i.e., on a small matrix element $U_{e3}$ which characterizes the connection between atmospheric and solar ranges of $\Delta m^2$ in NO. As it was shown there is the familiar resonance effect in atmospheric neutrinos that manifests itself clearly at $L \gtrsim 7000$ km through the interactions of $\nu$ and $\bar{\nu}$ in the earth’s mantle and crust; in this case the measurement of the CPT-odd asymmetry will provide an information on $\theta_{13}$ and on the sign of the corresponding difference of squared masses. In Ref. [288] a calculation of $\Delta P_{\alpha\alpha}^\text{CPT}$ for reactor-based oscillation experiments on long baselines from 730 to 3200 km was also given.

A detailed set of approximate analytical formulae for $\Delta P_{\alpha\alpha}^\text{CPT}$ and $\Delta P_{\alpha\beta}^\text{CPT}$ in a medium with an arbitrary density distribution was presented in works [289]. Particular cases of constant density and stepwise density distribution are also considered, the latter corresponding to NO in accelerator and reactor experiments at long baseline as well as at future neutrino factories. Estimates were obtained (numerically and on the basis of perturbative and low-energy approximations) for about a dozen of experiments – current and in preparation – for the indicated CPT-odd differences. Also shown graphically are the energy and baseline dependencies of the effect for three more efficient accelerator experiments, KamLAND, BNL NWG and NuMI, for which numerical values of $\Delta P_{ee}^\text{CPT}$ and $\Delta P_{\mu e}^\text{CPT}$ are $-0.033$, $0.032$ and $0.026$, respectively [289].

In Ref. [290] a general phenomenological approach to NO (in matter, in particular) is discussed when different damping effects are introduced to $\Delta P_{\alpha\beta}^\text{CPT}$. A three flavor analysis is presented to distinguish among damping signatures the decoherence-like effects in short- and long-baseline reactor and neutrino factory experiments. The information given may help to...
preserve from faking effects when one interprets, e.g., QD as a wave packet decoherence \[292\] or as NO to sterile neutrinos.

False CPT-odd effects are discussed also in Subsection J when explaining the LSND anomaly \[178\] \[41\] in connection with disentangling genuine CPT/LI violation due to QD from "fake" effects in matter \[238\] \[289\].

I. Neutrino oscillations due to violation of Lorentz invariance

The last part of the talk \[106\] cited in Subsection B contains, besides a discussion of application of the Extended Standard Model (SME) to neutrino physics, a description of qualitative features of the simplified two-parameter model (‘bicycle’ model \[141\]) and an analysis of its compatibility with the data on atmospheric and solar neutrinos. It is noted that it is currently difficult and would most likely be wrong to exclude the possibility of describing the observed oscillations due to LI and CPT violation instead of assigning mass to the neutrino. Therefore, to explain NO exclusively in the framework of CPT- and LI-violating models, detailed analyses of data of the concrete experiments are required.

Three articles of Kostelecký and Mewes \[141\] \[293\], in two of which the general formalism presented \[141\] for analysing violation of CPT and/or LI in the framework of SME, consider also possibilities of this scheme to interpret the LSND anomaly by the effects of unusual energy and directional dependence and \(\bar{\nu} - \nu\) mixing. The third paper \[293\] contains the results of studying LI violations in short-baseline NO in the framework of the formalism of the SME (see formulas (6) and (7)). Using general form of parametrization of these effects \[106\] \[141\] allowed to combine without contradiction the descriptions of the accelerator experiment LSND \[55\] and reactor experiments CHOOZ and KARMEN, due to both a nonstandard functional behavior of relevant terms on \(E\) and a dependence on the neutrino beam direction.

The two-flavor analysis of LSND data that covered a large number of parameters (the terms \(a^\mu L\) \(\bar{c}^\mu LE\) in (6) corresponds to taking into account 41 degrees of freedom, including the sidereal-time dependence on direction) yielded a nonzero quantity for a combination of coefficients that give the value of the LI violation. It is found to be \((3 \pm 1) \times 10^{-19} \) GeV, which is characteristic of effects at the Planck energy scale and is based on the measured probability over a sidereal day \(\langle P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \rangle \approx (0.26 \pm 0.08)\% \[293\].

Thereupon, the authors of Ref. \[293\] tried to use a five-parameter scheme of CPT/LI violation as a linear combination of generalized four-parameter bicycle model with the dependence of coefficients \((a_L)_{\mu\mu} = (a_L)_{ee}\) on preferred direction of \(a_L\) interaction and of original two-parameter bicycle model of Ref. \[141\] without such dependence. It was shown, however, that this scheme with \(m = 0\) contradicts atmospheric, long-baseline and reactor neutrino data in total.

Meanwhile, very recently a searches for sidereal effects in accelerator \(\nu_\mu\) data with the MINOS Near Detector were failed \[295\] as well as attempts to discover analogous periodic variations with respect to the Sun-centered frame in any previous terrestrial NO experiments. At expected suppression factor of \(M_W/M_{Pl} \sim 10^{-17}\) upper bounds on the CPT/LI violation coefficients in the SME analysis are found to be at the level of 0.01–1 \% \[295\] while the LSND data are consistent with no sidereal dependence \[296\].

Nevertheless, the further elaboration of SME-based models pointed out \[206\] that all the NO experiments, including solar, atmospheric and reactor data, are described by simplified effective CPT/LI-violating hamiltonian with three degrees of freedom, provided the high-energy pseudomass coefficient comes from one Lorentz-violating see-saw mechanism while the second see-saw in this ‘tandem’ model works at low energies to incorporate \(L/E\) dependence of the KamLAND and features of the LSND. The global NO data, adding also the LSND result, consistently answer the model at \(\bar{a} \equiv (a_L)_{\alpha\alpha} = -2.4 \times 10^{-10} \) GeV (here \(\alpha \equiv \alpha' = e, \mu, \tau\)), \(\bar{c} \equiv -\frac{1}{3}(c_L)_{ee} = 3.4 \times 10^{-17}\), \(\bar{m}^2/2 \equiv (m^2)_{\tau\tau}/2 = 5.2 \times 10^{-3} \) eV\(^2\) in the direction-independent

\[55\] In addition, attempts to explain the LSND anomaly are discussed in Subsections J and K.
approximation and without MSW matter effects within the Sun.  

Another three-flavor direction-independent approach but without CPT violation is explored in Ref. [298], where LI-odd effects are very small for experiments with solar and KamLAND neutrinos while atmospheric neutrinos are described, in fact, as in the two-flavor scheme. In order to accommodate the LSND data, only one relevant neutrino was coupled to LI violation provided that the non-trivial energy fine-tuning MDR are taken on.

**J. CPT non-invariant ‘ether’, quantum decoherence, and LSND anomaly**

Another idea tested for the interpretation of neutrino experiments was the CPT-non-invariant ‘ether’ [299] acting as a dense medium and creating interaction potential of opposite signs (and, correspondingly, different effective masses) for neutrino and antineutrino. As it was shown in Ref. [300], a two-flavor analysis for this model with introducing Lorentz-non-invariant effective operators cannot solve problems of the solar neutrino deficit and of the anomalous result of LSND. However, in principle, the increased number of fitting parameters and their non-standard dependence on energy provide a feasibility of describing NO data, including LSND.

In Subsection I we quoted the possibility of interpreting experiments on a short baseline only [293] via LI-violating effect in the Extended Standard Model. Such approach, assuming also [165] non-identical QD parameters for neutrino and antineutrino due to the strong CPT violation but with \( m_\nu = m_\bar{\nu} \), made it possible to successfully fit all the available NO results. The application of the three-flavor analysis, of simplifying assumptions on parametrization of the QD effect (which works in this particular model only in the antineutrino sector and is described by two quantities that are directly and inversely [57] proportional to \( E \)), and of the conventional NO mechanism with \( \Delta m^2 \) allows to explain [165, 178, 41] the LSND anomaly. Important points, which contribute to this explanation, are the suggestion [178, 41] that the observed neutrino mass differences are originated from a stochastic space-time foam and the appearing an exponential-suppression factor multiplying the oscillation term provided that one takes into account the difference in \( L \) for different experiments.

In contrast to above purely phenomenological model [165, 41] of quantum gravity-induced decoherence, in Ref. [301] authors presented also a complete, mathematically consistent, QD scheme of three-flavor mixing to describe all NO data (including LSND plus KamLAND spectral distortion results) assuming the same parameters for \( \nu \) and \( \bar{\nu} \). One of significant conclusions from successfully performed fitting, as it seems, is fairly unexpected interpretation of it: at obtained values of parameters it is more naturally to connect NO not with QD but rather with different possible origin – usual uncertainties in the (anti)neutrino beam energy that appear as "fake" effects [238]. Therewith, the fitting leads to the large QD effects with so non-naturally suppressed factors that may only be tested at higher energies [301].

Note in addition, that in the review talks [44, 57] (which followed above works) with attempts to explain all the NO data due to CPT-violating effects, the authors now exclude foam-induced QD as the sole cause of the fitting result obtained [44] and hope rather upon future experimental ‘solution’ of the LSND anomaly [57].

Nowadays summary of reconciling all the NO results on the base of dumping QD-type effects is presented in Ref. [302], the main goal of which is to suggest the consistent three-flavor scenario with the same suppression in neutrino and antineutrino sectors. The model assumes the fast energy dependence of the QD parameter \( \gamma \) which works only in the \( \nu_3 \)-mass state (and explains the LSND \( \bar{\nu}_e \rightarrow \bar{\nu}_\mu \) events) but does not affect \( \nu_1 - \nu_2 \) mixing while it can strongly influence supernova signals.

The LSND experiment and its result themselves deserve some additional remarks. We know

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56 A brief information is given in Ref. [297] on LI-violating models (bicycle and tandem) which may promote the joint explanation of all the NO data (including LSND).

57 The terms proportional to \( E^{-1} \) were attributed to usual matter effects.

58 To survey problems under discussion see also Section 13.2 of the White paper [42] and review [58].
(see, e.g., reviews \[34, 35\]) that the data on the deficit of solar and atmospheric neutrinos (confirmed in a number of experiments) were successfully explained in the scheme of three-flavor mixing; this approach operates only with two independent differences of squared masses $\Delta m_{ij}^2$ ($i, j = 1, 2, 3$). Therefore the indication in favor of the third value of $\Delta m^2$ that was obtained in the LSND experiment requires a modification of the scheme through incorporation of sterile neutrinos (i.e., by adding $i, j > 3$) or through a radical increase in the number of its free parameters in the case of CPT breaking (see the beginning of Subsection B).

The LSND experiment \[303\] searched for $\bar{\nu}_e$ events originating in decays of positive muons. These muons were produced in decays of stopped pions generated in interactions of protons from the LAMPF linac. An analysis of the data led to a conclusion \[303\] that a non-diagonal transition $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ at such third value $\Delta m^2_{\text{LSND}} \sim 1 \text{ eV}^2$ was present. This result was only obtained in a single experiment and was never confirmed by similar measurements at the KARMEN2 experiment \[304\] (see also the negative conclusions in Ref. \[305\] on a joint analysis of these experiments and of searching for oscillations $\nu_{\mu} \rightarrow \nu_e$ in the accelerator experiment NuTeV \[306\]).

The current accelerator experiment MiniBooNE (FNAL) \[307\] aims to test the LSND result. The refutation of this result would mean that there is no need to introduce sterile neutrinos \[59\] or a hypothetical inequality of the neutrino and antineutrino masses. As for MiniBooNE vs LSND, the very last status (prior to 2009) was presented in Ref. \[312\] in which a joint data analysis of these experiments with Bugey and KARMEN2 obtained a maximal compatibility of 3.94\% within two-neutrino approach \[60\]. However, any confirmation of the LSND anomaly may attract additional attention to using the simplest, even though theoretically unfounded, model for interpreting oscillations via $m_\nu \neq m_\bar{\nu}$ (see Subsection K).

K. Antineutrino- vs neutrino-oscillation parameters;

**LSND-anomaly models with $m_\bar{\nu} \neq m_\nu$**

As for the schemes with $m_\nu \neq m_\bar{\nu}$ (i.e., $m_\alpha \neq \bar{m}_\alpha$, where the mass-state index $\alpha = 1, 2, 3$), it was shown, for example, that in neutrino factory experiments sensitivity at the level of $|\bar{m}_3 - m_3| \lesssim 1.9 \times 10^{-4}$ eV can be achieved \[314\].

In Ref. \[315\] data fitting was carried out for the S-K + K2K experiments at $m_\nu \neq m_\bar{\nu}$ in the range of atmospheric squared mass difference. The first parameter in the $\Delta m^2_\nu$ vs $\Delta m^2_\bar{\nu}$ diagram proved to be constrained as in the case of CPT preserved while the second one, in contrast, showed the allowed values larger by about an order of magnitude.

NO experimental data were also analyzed in order to evaluate $\delta = \Delta m^2_\nu - \Delta m^2_\bar{\nu}$ \[316\]. The author cites the result $(-7.5 \times 10^{-3} \text{ eV}^2 < \delta < 5.5 \times 10^{-3} \text{ eV}^2)$ presented by the S-K collaboration at the conference ICHEP-2002. This result is based on studying the flux of atmospheric neutrinos and points to its dependence on the assumption that mixing is maximal and identical for $\nu$ and $\bar{\nu}$. However, a more detailed analysis of the most recent SNO data using the MSW mechanism inside the solar matter and the information on the deficit of reactor antineutrinos in the KamLAND experiment \[317\] yields a better constraint on $\delta$ for any pair of flavors and for the case when

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59 In the past, in Refs. \[308\] (see, also, review \[37\]) exotic scenarios were used to interpret LSND data in conjunction with all other NO results: with the fourth sterile neutrino and nonobligatory equality of $\nu$ and $\bar{\nu}$ parameters (with $m_\nu \neq m_\bar{\nu}$, in particular). It was shown in the CPT-violating case that while mass spectra of active + sterile neutrinos of the types $(3 + 1)$ and $(2 + 2)$ were possible for $\bar{\nu}$, only the $(3 + 1)$ scheme is valid for $\nu$; in CPT-conserving scenarios the $(3 + 2)$ models fit data better than $(3 + 1)$. Then, after the first results from the MiniBooNE experiment \[309\], which found no evidence for the LSND $\nu_\mu \rightarrow \nu_e$ signal, the three-flavor global NO fitting \[310, 311\] disfavored models with one and more than one sterile neutrino without CP-violation effects. The present status of active + sterile mixing models (up to 5-$\nu$ and 6-$\nu$ schemes) is also summarized in Ref. \[59\].

60 As multiple cycles of NO have not yet been found and assuming, as usual, only two mixing channels, the definitive conclusions on the compatibility among LSND, KARMEN2 and MiniBooNE data may essentially depend \[313\] on taking into account multichannel mixing, e.g., intermediate sterile-neutrino channels.
\( \Delta m^2_{\nu,\bar{\nu}} \) may have different signs: \( |\Delta m^2_{\nu} - \Delta m^2_{\bar{\nu}}| < 1.3 \times 10^{-3} \text{ eV}^2 \) (90\% CL) \textsuperscript{316}.  

As for the data on possible CPT non-conservation in the neutrino and antineutrino sectors, they are given in the review analysis \textsuperscript{23}, where this situation is represented graphically by comparing the allowed areas of NO parameters before and after the Neutrino-2004 conference (including the most recent KamLAND results presented to the conference \textsuperscript{318}). After the Neutrino-2004 the area for reactor \( \nu_e \) was shrunk like one for solar \( \nu_e \), confirming the CPT symmetry in \( \nu_e \) sector.  

Very recently in Ref. \textsuperscript{319} the authors investigated the sensitivities of future NO experiments for measuring \( \Delta m^2_{ij} \) and \( \theta_{ij} \) independently for neutrinos and antineutrinos. Expected sensitivities of neutrino factories to the atmospheric, (anti)neutrino parameters were also updated. Present and future bounds for solar, \( \nu \) and \( \bar{\nu} \) parameters, which expected at 3\( \sigma \) level in a dedicated \( \bar{\beta} \)-Beam facility, in combination with a SPMIN reactor experiment, are as follows (see below reduced variant of the complete Table 1 of Ref. \textsuperscript{319}).

| quantity | present bound | future (\( \beta B \) 750 km) |
|----------|---------------|---------------------------------|
| \( |\sin^2 \theta_{12} - \sin^2 \theta_{13}| \) | 0.3 | 0.14 |
| \( |\sin^2 \theta_{13} - \sin^2 \theta_{13}| \) | 0.3 | 5.7\times10^{-4} |
| \( |\sin^2 \theta_{23} - \sin^2 \theta_{23}| \) | 0.45 | 0.044 |
| \( |\Delta m^2_{31} - \Delta m^2_{31}| \) | 1.1\times10^{-4} \text{ eV}^2 | 2.2\times10^{-5} \text{ eV}^2 |
| \( |\Delta m^2_{31} - \Delta m^2_{31}| \) | 1\times10^{-2} \text{ eV}^2 | 3.3\times10^{-5} \text{ eV}^2 |

In connection with difficulties arisen from interpreting the entire body of experimental results on NO, even resorting to ‘marginal’ solutions with the sterile neutrinos (see, e. g., Refs \textsuperscript{315}, \textsuperscript{320} \textsuperscript{61}, an extended set of squared mass differences was used (via an independent value of \( \Delta m^2_{ij} \) for the antineutrino); to justify this usage, its origin is connected to the hypothetical CPT violation in the neutrino sector of the theory via \( m_{\nu} \neq m_{\bar{\nu}} \). It should be emphasize that all models involved \textsuperscript{326}, \textsuperscript{89} \textsuperscript{299} \textsuperscript{327} lack theoretical foundation: in fact, the difference between the neutrino and antineutrino masses is introduced into them ‘by hand’ since CPT violation is impossible in a Lorentz-invariant theory (see \textsuperscript{85} \textsuperscript{88}) \textsuperscript{62} as we stressed in Section 2.

Murayama and Yanagida \textsuperscript{326} were the first to try this approach to interpret experimental results on NO and supernova neutrino events (see also \textsuperscript{328}). By analyzing the energies of neutrino events in the Kamiokande and IMB experiments which originated from the SN1987A, these authors obtained arguments against preferred values of \( \Delta m^2_{\text{SND}} \approx 0.1 - 1 \text{ eV}^2 \). Then they proposed a scheme of neutrino and antineutrino masses that is compatible with all neutrino oscillations data and the LSND anomaly without adding the sterile neutrino. Furthermore, characteristics of above neutrino events do not contradict this scheme too. The LSND result is interpreted as a consequence of assuming large squared mass difference for the antineutrino, \( \Delta \bar{m}^2 \). The authors considered it essential to test for this assumption in the MiniBooNE experiment with an antineutrino beam.

The situation with interpretation of NO data on the basis of CPT non-invariant neutrino mass spectra is outlined in Ref. \textsuperscript{329} and in review talks \textsuperscript{330} \textsuperscript{311}. An analysis of all results, with the LSND experiment either taken or not taken into account, was carried out in the three-flavor approach.

It was shown that areas allowed for the LSND and for all other experiments on the \( \Delta m^2 \ vs \ \sin^2 2\theta \) plane do not overlap at the 3\( \sigma \) level while the values of these parameters that correspond to the best fit are practically identical in scenarios with and without CPT violation. The obtained level for rejecting the interpretation with CPT violation is 4.6\( \sigma \) \textsuperscript{311}.

\textsuperscript{61}The SNO experiments data eliminate the need to consider sterile neutrinos (\textsuperscript{321} and \textsuperscript{322}); for the subsequent estimates see paper \textsuperscript{323}, talk \textsuperscript{324} and review \textsuperscript{37}; see the situation after the MiniBooNE first result, e.g., in very recent works \textsuperscript{325}.

\textsuperscript{62}An obvious assumption is made of the usual connection between spin and statistics.
After the recent global fitting [329, 59] of all the data for NO except for the LSND result, the current values of mixing parameters were obtained separately for neutrinos and antineutrinos. The resultant picture of a possible scenario with CPT violation is illustrated at this page by Fig. 55 (four joined panels are due to absence of information about the relative ordering of neutrinos vs antineutrinos states), which is taken from Ref. [59]. It turns to be impossible to tell CPT-violating scenarios with normally ordered $\nu$ states plus inversely ordered $\bar{\nu}$ states (or vice versa) from the CPT-conserving one. The analysis shows also [329, 59] that the CPT-violating scenario of Ref. [327] fails to give a good description simultaneously of both the LSND result and the all-but-LSND NO data.

**Conclusion**

Contents of this article characterize the current state of the old problem of theoretical and experimental investigation of the hypothetical violation of the CPT and/or Lorentz invariance within the neutrino sector of the Standard Model. The information presented gives us evidence for constituting the neutrino oscillations as a novel area in which promising possibilities are opened for testing this symmetries. One should not forget, however, the status of the CPT invariance as one of the fundamental concepts underpinning the theory.

This is a proper place to quote the following outlook which is undoubtedly shared by most of the researchers: *Of course, whatever could be measured should be measured and whatever could be tested should be tested. There should be no reservations: such a fundamental symmetry as CPT should be tested. However one should keep in mind that unlike breaking of $C$, $P$, $T$, $CP$, $PT$, and $TC$, the breaking of CPT is non-compatible with the standard quantum field theory, the only basis for a self-consistent phenomenological description of any process, which we know up to now. Therefore the chances that CPT breaking would be discovered are vanishingly small.* [107]

Indeed, searches for CPT violation are unsuccessful yet (see Section 2), even in neutrino physics, as one discusses in Section 3 – 5.

No evidence for Lorentz violation was found too, in spite of plenty of theoretical studies and advances in the precision testing which are carried out over the last decade or two. Hence, it is natural to start a prudent outlook of the wise expert with the question: *...when have we tested enough? We currently have bounds on Lorentz violation strong enough that there is no way to*
put Lorentz violating operators of dimension $\leq 6$ coming solely from Planck scale physics into our field theories. It therefore seems hard to believe that Lorentz invariance could be to restrict the classes of quantum gravity theories/spacetime models we should consider. Without a positive signal of Lorentz violation, this is all that can reasonably be hoped for.\[111\]

On the other hand, from the standpoint of quantum gravity (QG), as the future theory to which theoreticians address as a physical origin of both CPT and Lorentz violation (LV), true adherents of QG believe that ...LV is not the only possible low energy QG signature. Nonetheless, it is encouraging that it was possible to gather such strong constraints on this phenomenology in only a few years. This should motivate researchers to further explore this possibility as well as to look even harder for new QG induced phenomena that will be amenable to observational tests. This will not be an easy task, but the data so far obtained prove that the Planck scale is not untestable after all.\[114\]

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