We study three-dimensional optical Anderson localization in medium with a percolating disorder, where the percolating clusters are filled by the light nanoemitters in the excited state. The peculiarity of situation is that in such materials the field clusters join a fractal radiating system where the light is both emitted and scattered by the inhomogeneity of clusters. Our numerical FDTD simulations found that in this nonlinear compound the number of localized field bunches drastically increases after the lasing starts. At longer times the bunches leave nonlinear percolating structure and are radiated from the medium. Monitoring the output of system allows to observe experimentally such dynamic localized bunches in three-dimensional setup.

**Introduction.** Disordered photonic materials can diffuse and localize light through random multiple scattering that leads to formation of the electromagnetic modes depending on the structural correlations, scattering strength, and dimensionality of the system [1], [2], [3], [4], [5], [6], [7]. The Anderson localization was predicted as a non-interacting linear interference effect [8]. However, in real systems the non-negligible interactions between light and medium can take place. Therefore important aspect of the optical localization is the interplay between nonlinear interactions and linear Anderson effect [6]. Nonlinear interactions appear in optics, due to nonlinear responses of a disordered medium that normally gives rise to indirect interactions between the photons through various mechanisms. In case of classical waves the localization can be interpreted as interference between the various amplitudes associated with the scattering paths of optical waves propagating among the diffusers. The study for Anderson transition in 3-D optical systems still has not been conclusive despite considerable efforts. The localization transition may be difficult to reach for the light waves due to various effects in dense disordered media required to achieve strong scattering [see [9] and references therein]. The experimental observation of optical Anderson localization [10] just below the Anderson transition in 3D disordered medium shows strong fluctuations of the wave function that lead to nontrivial length-scale dependence of the intensity distribution (multifractality).

Here we study the optical Anderson localization in 3D percolating disorder, where the percolating clusters are filled by the light nanoemitters in the excited state. The peculiarity of situation is that in such materials the field clusters join a fractal nonlinear radiating system where the light is both emitted and scattered by the inhomogeneity of clusters. Such system may be suggested as an extension to 3D optical case the localization released in one-dimensional waveguide in the presence of a controlled disorder [19]. In 3D disordered percolating systems the optical transport was observed [11] and also it is found that the random lasing assisted by nanoemitters incorporated into such a disordered structure can occur [12]. The system with spanning clusters produces a global percolation that results in qualitatively modification of its spatial properties. One can argues that already in a vicinity of the percolating phase transition the fractal dimension of such system $D_H \simeq 2.54$ considerably differs from the dimension of the embedded space $D = 3$ (multifractality). The crucial question is whether the optical Anderson localization can be achieved for a non-integer dimension case with a fractal (Hausdorff) dimension of $D_H < 3$, where the strong randomness for properties of the system is expected. In this paper we show that the dynamic field localization can arise in such 3D active fractal percolation system. To the best of our knowledge, the dynamic 3D Anderson localization assisted by fractal disordered inhomogeneity is discussed here for the first time.

**Basic equations.** The percolation normally refers to the leakage of fluid or gas through the porous materials. In this paper we consider that the percolating clusters are filled by gaseous light nanoemitters in the excited state. A typical incipient percolation cluster has a dendrite shape. From Fig. 1 (a) we observe that such a system consists of two essentially different areas. In the first region (in the vicinity of the entrance shown by arrow) there is a strong concentration of percolation clusters, which may lead to accumulation of the nonlinear field effects and the lasing. In the other part, the concentration of percolation clusters is small, so the disorder of medium is lower. We study the emission of electromagnetic energy from a cubical sample $(x, y, z) \in [0, l_0]$ in 3D medium with a percolating disorder, where the percolating clusters are filled by the light nanoemitters in the excited state. The output flux of energy can be written as $I = \int_{S} \mathbf{E} \cdot (\mathbf{K} - \mathbf{n}) dS = I_x + I_y + I_z$, where $\mathbf{K}$ is the Pointing vector, $\mathbf{n}$ is the normal unit vector to the surface $S$ of cube, and $I_{x,y,z}$ indicate the fluxes from two faces of the cube perpendicular to a particular direction. To find the emission from the system we solve numerically the following equations that couple the polarization density $P$, the electric field $E$, and occupations of the levels of emitters $N_i$ [14].
\[ \frac{\partial P}{\partial t} + \Delta \omega a \frac{P}{\partial t} + \omega^2 P = \frac{6\pi x_0 a^3}{\tau_{21} a^2} (N_1 - N_2) E, \]
\[ \frac{\partial N_{0,1}}{\partial t} = -\frac{1}{\tau_{10}} N_0 \pm \frac{N_{1,2}}{\tau_{10}(1)} (j E \cdot H) \]
\[ \frac{\partial N_{1,2}}{\partial t} = N_{1,2}(t) \frac{1}{\tau_{21}(1,2)} - \frac{N_{1,2}}{\tau_{10}(1)}, \]

where \( \Delta \omega a = \tau_{21}^{-1} + 2T_2^{-1} \), where \( T_2 \) is the mean time between dephasing events, \( \tau_{21} \) is the decay time from the second atomic level to the first one, and \( \omega a \) is the frequency of radiation. The electric and magnetic fields, \( E \) and \( H \), and the current \( j = \frac{\partial P}{\partial t} \) are found from the Maxwell equations, together with the equations for the densities \( N_i(t) \) of atoms residing in \( i \)-th level, see Ref. [15] and references therein. An external source excites emitters from the ground level \( (i = 0) \) to third level at a certain rate \( A_r \), which is proportional to the pumping intensity in experiments. After a short lifetime \( \tau_{32} \), the emitters transfer nonradiatively to the second level. The second level and the first level are the upper and the lower lasing levels, respectively. Emitters can decay from the upper to the lower level by both spontaneous and stimulated emission, and \( (j \cdot E)/\hbar \omega_0 \) is the stimulated radiation rate. Finally, emitters can decay nonradiatively from the first level back to the ground level. The lifetimes and energies of upper and lower lasing levels are \( \tau_{32}, E_2 \) and \( \tau_{10}, E_1 \), respectively. The individual frequency of radiation of each emitter is then \( \omega_0 = (E_2 - E_1)/\hbar \), and \( \hbar \) is Planck’s constant.

**Numerics.** In our calculations we considered the gain medium with parameters close \( GaN \) powder, similar to Ref. [15]. The lasing frequency \( \omega_0 \) is \( 2\pi \times 3 \times 10^{13} \text{ Hz} \), the lifetimes are \( \tau_{32} = 0.3 \text{ ps}, \tau_{10} = 1.6 \text{ ps}, \tau_{21} = 16.6 \text{ ps} \), and the dephasing time is \( T_2 = 0.0218 \text{ ps} \). The percolating cluster has been generated inside the cube of \( L_0 = 1 \text{ \mu m} \) edge having \( L^3 \) nodes, with \( L = 100 \) (see Fig. 1 (a)) that was sufficient to simulate the percolating structure [11]. Each node is supposed to indicate the position of many emitters, with the total concentration of emitters inside the percolation cluster being \( N = N_0 + N_1 + N_2 + N_3 = 3.3 \times 10^{24} \text{ m}^{-3} \). The initial (at \( t = 0 \)) values of densities \( N_0(0) = 0.001 N, N_1(0) = 0.002 N, N_2(0) = 0.002 N, \) and \( N_3(0) = 0.995 N \) are used. The dielectric permittivity of a host material is \( n = 2.2 \) that is close to the typical values for ceramics \( Lu_3Al_5O_{12}, SrTiO_3, ZrO_2 \), see review [13]. The result of simulations shown in Fig. 1 (b) obtained for cw pumping given by \( A_r = 10^7 \text{ s}^{-1} \). At this pumping the simulations show the formation a well-defined lasing for \( t > t_s \), and we refer \( t_s \) as the lasing start time. To simulate the noise in our system the initial seed for the electromagnetic field has been created with random phases at each node. In what follows it is used dimensionless time \( t \) renormalized as \( t \to t c/l_0 \), where \( c \) is the light velocity in vacuum.

Briefly, the **strategy** of our simulations consists in the following: (i) Calculating the geometry of the spanning percolating cluster, (ii) Calculating the photon field \( E \) generated by emitters incorporated in spanning cluster with the use of finite-difference time-domain (FDTD) technique [17], and (iii) Solution of nonlinear dynamic coupled equations for field, polarization \( P \) and the occupation numbers \( N_i \) for all the emitters in 3D system.

We use here the extended 3D percolating model that allows the empty percolating voids to have a small random-valued 3D shift from the reference grid. We use the percolating probability \( p = 0.4 \) that is slightly less of corresponding threshold 0.415 for such extended model. Fig. 1 (a) shows the typical spatial structure of the incipient percolating cluster in the cube \( l_0 \times l_0 \times l_0 \) corresponding to the numerical grid with \( L \times L \times L, L = 100 \) nodes. Fig. 1 (b) displays the dynamics of optical lasing generated by exited emitters in percolating medium with intensities \( I_{x,y,z} \) through the sides of 3D sample. The normalized field amplitudes \( E_y \) in the central intersection \( (x, y = L/2) \) of 3D percolating system at different times nearly of the initial time of laser generation \( (t = 4, 5, 6) \) are shown in Fig. 1 (c), (d), and (e). Fig. 1 (c) shows the fields distribution before the lasing starts. In this case the field amplitudes are small, one can observe only narrow point-like fields (indicated by arrow) generated by random emitters in percolating cluster. At larger time \( t = 5 \) (see Fig. 1 panel (d)) the laser genera-

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**FIG. 1.** (Color online.) (a) Typical spatial structure of the incipient percolating cluster near the percolation threshold in the cube \( l_0 \times l_0 \times l_0 \). (b) The dynamics of lasing (the critical lasing time is \( t_c = 5 \)), where the field fluxes \( I_{xyz} \) in the \( xyz \) -directions of the system are displayed; (c) The normalized fields in the central intersection \( xy \) of 3D system for various times closely to the critical time at: (c) \( t = 4 < t_c \), and (d) \( t = 5 = t_c \). In such situation only narrow point-like fields generated by random emitters are observed. For longer time at \( t = 6 > t_c \) (e) the field bunches having smooth localized shape and large amplitudes arise already beyond the emitter peaks. Hence the lasing in percolating disordered system will stimulate the localization of emitted field.
The field distribution for longer times. Panels (a) $t = 7$ and (b) $t = 8$ show the dynamic states (coupled to emitters) when the light bunches begin to migrate from the random emitter area. (That corresponds to lasing evolution in Fig. 1 (b) for the interval $t = 7$, 8 when in the system after emission the accumulation of energy occurs. Panels (c) $t = 9$ and (d) $t = 10$ show that for long time the optical bunches reach the output of system, and at $t = 10$ (d) the localized light is radiated to surrounding space.

Localization. In the above the properties of generated field bunches were analyzed in general. In what follows we study the dynamics and structure of localized fields confined in the light bunches. A field (quasi-spherical) bunch will be mentioned as a bounded 3D domain where field $E_i$ in every layer $i$ is a decreasing function $E_i > E_{i+1}$ ($i \in 0, 1, \ldots p - 1$) from center to periphery that leads to $E_0 > E_1 > \ldots > E_p > 0$. Such domain (bunch) have two parameters: radius $p$ and the field value in the center $E_0 = E_{max}$. [More detailed approaches in such 3D localization problem require too large amount of computer resources.]

From Fig. 4 one observes for time $t = t_D = 7$ the structure of some localized field with large amplitudes $E_{max}$ and characteristic radius $p$. The centers of such objects are established in various points of 3D medium. Fig. 4 displays the isosurface of normalized fields for such items. In general the bunches are situated in variety points of

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Finally it is especially interesting to study the dependence of number of optical bunches $N$ on the radius $p$ for various times $t = 4, 5, 6, 7, 8, 9, 10$. We observe that $N$ with large radius $p$ drastically increments after $t > 5$ when the lasing starts. At $t = t_D = 7$ and $t = 8$ it is generated the bunch with maximal characteristic radius $p = 7$.

3D system, see Fig. 3 due to disordering positions of radiated emitters and the fractal shape of the percolating clusters. As we observe from Fig. 4 all the bunches have well established localized shape.

It is interesting to study the dependence of number of bunches at different times $t$ as function of radius $p$. Fig. 5 shows such dependence for times $t = 4, 5, 6, 7, 8, 9, 10$. We observe from Fig. 5 that below the lasing threshold ($t \leq 5$, see Fig. 4(b)) there are a lot of fields with small radius $p$. Most of them correspond to narrow point-like fields generated by random light emitters (see Fig. 4(c)). However, as one can see from Fig. 5 after the lasing starts (since $t > 5$) the number of optical bunches with large radius $p$ drastically increments. Besides, Fig. 2 displays that such bunches (with large $p$) are situated mainly beyond the percolating area. Moreover, as Fig. 2 shows such fields have well-localized shape (Fig. 4), they longer are not pinned to the static emitters and they migrate to the output of system. That allows indicating such dynamic bunches as 3D zones of optical Anderson localization arisen by the active fractal percolating clusters.

Conclusion. We have shown that dynamic 3D optical Anderson localization can arise in medium with percolating disorder when the percolating clusters are filled by the light nanoemitters in the excited state. In such materials the field emitters in clusters create a fractal radiating system, where the field is not only emitted, but also scattered by the boundaries of clusters. Our 3D simulations have found that in such nonlinear compound the amount of localized field bunches drastically increases after the optical lasing starts. At longer times all the bunches leave nonlinear percolating structure and are radiated from the medium.
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