Optimization calculation method and weighing test of static friction coefficient of RPC spherical joint based on half space contact model

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Abstract. The application of new concrete materials in bridge engineering has always been a research direction of bridge scholars. Reactive powder concrete (RPC) has attracted many scholars’ attention in recent years due to its high strength, high toughness and high durability. But when it is applied to the spherical hinge in the bridge turning project, the result of static friction coefficient of RPC composite spherical hinge obtained by traditional calculation method is small. It may lead to the danger of overturning of the structure during the actual turning. For this reason, a more accurate method for calculating the static friction coefficient of this new type of spherical joint is needed. In this paper, we presented and verified a new stress distribution law and function expression of the friction surface of the spherical hinge, which was based on the contact model of half space body, combined with finite element analysis, weighing test and field measurement. This paper then proposes an optimized method for calculating the static friction coefficient of spherical joints. The results show that: The static friction coefficient of the spherical hinge obtained by the traditional calculation method is about 67% smaller than the actual static friction coefficient; The static friction coefficient of spherical hinge is 15.2% larger than that of the actual one, which is more accurate than that of the traditional method. If there is no test data, the static friction coefficient of this kind of RPC concrete composite spherical hinge is recommended to be between 0.03 and 0.06.

1. Introduction
RPC (reactive powder concrete) composite spherical joint is a new type of spherical joint used in the construction of bridge swivel. The spherical hinge is made of steel plate and reactive powder concrete. The shell is welded with steel plate first, and then the reactive powder concrete is filled in to form a composite structure. Compared with the traditional steel ball hinge, its volume is only 2 / 3 of that of the steel ball hinge under the same bearing capacity, and its bearing capacity can be as high as 45Mpa. Which greatly solves the problems that the traditional steel ball hinge volume is too large and the common concrete ball hinge is easy to crush.

RPC is composed of cement, quartz sand, quartz powder, silica fume, steel fiber and superplasticizer. It is a new type of cement-based composite material with ultra-high strength, high toughness, high durability and good volume stability[1]. Research and engineering practice interiorly and abroad show that reactive powder concrete has better cost performance than steel. In addition, the CO2 produced in the production process of reactive powder concrete is only 1 / 2 of that of ordinary concrete, which has good environmental protection performance[2]. When RPC is applied to rotary...
construction, it has many advantages, such as less steel, higher strength, stronger bearing capacity and smaller rotating moment.

2. Literature review

In recent years, many scholars have paid attention to the application of RPC materials in bridge engineering, but most of them are devoted to the study of the mechanical properties of RPC girder. Wenyu Ji[3] designed a softened truss model to effectively predict the shear failure model of T-shaped RPC girder, and then used 12 groups of T-shaped girder shear tests to verify. The results show that the shear span ratio, longitudinal reinforcement ratio and stirrup ratio have significant influence on the shear performance of beams. Xudong Shao[4] proposed a new type of RPC box girder bridge structure. This structure adopts the diaphragm structure with dense distribution, which improves the load distribution of the structure, as well as the shear performance of the box girder web and the stability of the flange plate. Then a wheel load test and a shear failure test were taken to verify the feasibility of the structure. Renyuan Du[5] carried out a vertical loading test of reactive powder concrete arch. He recorded the vertical deformation and section strain of the arch during the loading process, and compared them with the mechanical properties of the conventional concrete arch. The test results show that the cracking load, steel yield load and ultimate bearing capacity of RPC arch are significantly higher than those of conventional concrete arch, which can effectively reduce the self weight of the structure and improve the span capacity of the arch bridge. Zhe Wang[6] used a three-axis test to record compressive failure mode and crack development of concrete cylinder. The test was taken under three friction conditions: RPC cylinder directly contacts with metal indenter, is padded with two layers of polytetrafluoroethylene film, and is padded with three layers of polytetrafluoroethylene film coated with lubricating grease.

A large number of studies show that RPC material has excellent mechanical properties in the bridge structure, which can improve the bearing capacity of the structure, increase the stiffness, reduce the structural deformation, and has good crack resistance[7]. Based on the advantages of RPC materials, RPC composite spherical joints are more and more used in the bridge turning project, but no relevant theoretical or experimental research results have been found. As the most important structure in the bridge swivel project, the spherical hinge plays an important role in the success of the swivel. Among them, the vertical stress value of the contact surface of the upper and lower turntable is the main factor to control the stress state of the spherical hinge. If the calculation parameters on the contact surface are not estimated accurately, there may be insufficient traction and the bridge will not move. It is also possible that the rotating structure may overturn due to insufficient friction. Therefore, it is necessary to study the mechanical parameters of the new RPC spherical joint.

If there is no test basis, according to technical code for construction of highway bridge and culvert - JTG TF 50-2011, the static friction coefficient of ball joint is usually 0.1-0.12, and the dynamic friction coefficient is 0.06-0.09. This is based on the construction technology level test in the 1980s. Now, the unbalanced weighing test is often used to get the unbalanced moment and static friction moment of the ball joint, and calculate the static friction coefficient of the ball joint. The traditional static friction calculation model of spherical joints is relatively simple, and many basic assumptions are set in this calculation model. For example, the force arm of the friction force at the contact point is the distance from the ring to the central axis of the spherical hinge; the contact surface material between the spherical hinge and the base is smooth and isotropic[8]. Therefore, the calculated static friction force is smaller than the actual value. Yuxuan Wang[9] proposed a method to modify the friction force arm by the actual distance from the center of the micro ring to the center of the ball. The static friction coefficient of RPC composite spherical hinge calculated by this method is 0.009, which is quite different from the value 0.06 given by the spherical hinge manufacturer. Therefore, it is necessary to propose a more accurate static friction coefficient correction algorithm.
3. Methodology

3.1. Background

Relying on Yumenkou Yellow River Bridge West approach turning project of national highway. The swivel weight of pier 14# and pier 15# is 11020.2t and 8485.8t respectively. The swivel spherical hinge adopts RPC concrete composite spherical hinge system. The actual bearing capacity of pier 14# is 13000t, and that of pier 15# is 10000t. Taking pier 14# RPC concrete composite spherical hinge as an example. The spherical hinge is made of C120 grade RPC material. Its elastic modulus is $E=42\text{Mpa}$, Poisson's ratio is $\mu=0.2$, the radius of the upper spherical hinge is $R_1=4.8\text{m}$, the radius of the lower spherical hinge is $R_2=4.942\text{m}$, the plane diameter of the upper spherical hinge is $D_1=3\text{m}$, the plane diameter of the lower spherical hinge is $D_2=2.4\text{m}$, and the distance between the jack and the spherical hinge center is $L=4.35\text{m}$.

RPC concrete composite spherical hinge structure is composed of four parts, from the top to the bottom are: spherical hinge upper seat plate, spherical hinge upper plate, spherical hinge lower plate and spherical hinge lower seat plate. In order to leave enough construction operation space to facilitate construction, the distance between the upper and lower walls shall be considered as 0.7m. A friction pair composed of stainless-steel plate and ultra-high molecular weight polyethylene plate is arranged between the upper plate and the lower plate of the ball joint. The center of the spherical hinge is provided with a central rotating shaft filled with reactive powder concrete in the steel tube, and the detailed structure is shown in figure 1.

![Figure 1](image.png)

Figure 1. RPC concrete composite spherical hinge structure.

In this figure: 1-ball joint upper plate; 2-Ball joint lower plate; 3-upper seat plate; 4-lower seat plate; 5-central shaft; 6-connecting plate; 7-plug plate.

3.2. Unbalanced weighing test

There are two situations in the weighing test of spherical hinge, and different weighing calculation principles should be adopted. The first test scheme is shown in figure 2 when the friction distance of the spherical hinge is less than the unbalanced moment of the bridge structure. After the temporary consolidation between the rotary table and the bearing platform is relieved, the swivel inclines along the direction of the unbalanced moment because the unbalanced moment is greater than the friction moment of the spherical hinge. Assuming that the falling end is the B end of the main beam, the unbalanced load-bearing test shall be carried out at this time. As the end B of the structure falls, the reaction force at the end a of the swivel structure is zero, and the reaction force $P_2$ at the end B is measured. Then jacking is carried out at the B end to change the tilt direction of the swivel structure and the direction of the friction moment of the spherical hinge. Also ensure that the reaction force of end a is zero, and record the jack reading and percentage reading during turning. Finally, according to the load displacement curve, the force $P_3$ to overcome the maximum static friction moment is determined. At this time, the unbalanced moment $M_G$ of the swivel and the friction moment $M_Z$ of the spherical hinge are respectively:
When the unbalanced moment of the swivel is less than the friction moment of the spherical hinge, the test scheme is shown in figure 3: After the temporary consolidation between the turntable and the bearing platform is relieved, because the unbalanced moment of the swivel structure is less than the friction moment of the spherical hinge, the swivel structure will not tilt at this time. First, jack up the a-end of the main beam to make the structure rotate slightly clockwise in the plane, and then jack up the b-end of the main beam to make the structure rotate slightly counterclockwise in the plane. Record the jack reading and percentage reading during turning, and judge the force P4 and P5 when overcoming the maximum static friction moment according to the load displacement curve. At this time, the unbalanced moment \( M_G \) of the swivel and the friction moment \( M_Z \) of the spherical hinge are respectively:

\[
M_G = \frac{(P_4 + P_5) L_2}{2} \\
M_Z = \frac{(P_3 - P_2) L_2}{2}
\]

(a) Lifting of girder A end  \hspace{1cm} (b) Lifting of girder B end

Figure 2. Weighing principle of ball hinge friction moment less than rotating body unbalanced moment.

Figure 3. Principle of weighing when the friction moment of spherical hinge is greater than the unbalanced moment of rotating body.
After the friction moment of the ball joint is obtained through the weighing test, the static friction coefficient of the ball joint can be solved by the differential relationship between the friction on the friction surface of the ball joint and the friction moment of the ball joint. The instruments used in this weighing test are shown in figure 4-6.

The load displacement test results of the swivel bridge are listed in table 1, table 2 and figure 7, figure 8. It can be seen from the analysis of the figure that the unbalanced moment is less than the static friction.

The change of load-displacement under the condition of stage by stage equal amplitude loading shows that, When the load on the north side is greater than 18 MPa, the load-displacement curve changes at a turning point. When the load on the south side is greater than 8Mpa, the load displacement curve changes at a turning point. It can be concluded that when 18 MPa (P5 = 231.29t) and 8 MPa (P4 = 98.92t), the spherical hinge is in a critical state to overcome the static friction.

| Displacement | Oil gauge value (Mpa) | Jacking force (t) |
|--------------|----------------------|-------------------|
| 3            | 2                    | 17.95             |
| 7            | 4                    | 44.62             |
| 13           | 6                    | 71.29             |
| 19           | 8                    | 97.95             |
| 25           | 10                   | 124.62            |
| 30           | 12                   | 151.29            |
| 38           | 14                   | 177.95            |
| 42           | 16                   | 204.62            |
| 52           | 18                   | 231.29            |
| 60           | 20                   | 257.95            |
Table 2. Load-Displacement of South swivel beam.

| Displacement | Oil gauge value (Mpa) | Jacking force (t) |
|--------------|----------------------|-------------------|
| 0            | 0                    | 0                 |
| 4            | 2                    | 19.98             |
| 9            | 4                    | 46.29             |
| 15           | 6                    | 72.61             |
| 19           | 8                    | 98.93             |
| 34           | 10                   | 125.24            |
| 85           | 12                   | 151.56            |

3.3. Finite element analysis

Midas FEA, a large general finite element analysis software, is used to analyze the details of spherical hinge and bearing platform. All parts of the spherical hinge are solid elements. In order to ensure the calculation accuracy, the upper and lower bearing caps are added in the modeling process, which can reduce the stress deviation caused by the equivalent boundary conditions. The overall mesh division of the finite element model is shown in figures 9 to 11. At the interface of each part of the spherical hinge,
the coupling is carried out to ensure the consistency of mesh division. However, on the articulated contact surface of the upper and lower ball, in order to carry out contact analysis, the coupling is removed.

![Figure 9. Grid generation of rotating structure.](image)

Under the balanced load, the stress analysis of the upper and lower plates of the spherical hinge is shown in figure 12. From the result analysis, we can see that: The overall stress of the upper plate of RPC concrete spherical hinge is small, and most areas are between 10MPa and 20MPa. The maximum compressive stress of the first principal stress is 28.23mpa, which appears in the edge area of the ball hinge contact surface. The stress distribution trend on the contact surface is from the central axis to the edge of the contact sphere, and the stress gradually increases from 7MPa to 28.23mpa. There is a sudden change of stress at the central shaft and a local compressive stress of about 28Mpa, which is caused by the stress concentration caused by the spherical reaming hole.

![Figure 10. Grid division of RPC spherical hinge upper plate.](image)  ![Figure 11. Grid division of RPC spherical hinge lower plate.](image)

![Figure 12. Stress distribution of upper and lower plates of spherical hinge.](image)

The stress distribution curve along the arc length on the axial section of RPC concrete composite spherical hinge is extracted. As shown in figure 13, take 20 points along the arc length on the central axis section. See table 3 for the stress calculation results. Draw the distance stress distribution curve as shown in figure 14.

![Figure 13. Schematic diagram of contact point selection.](image)
Table 3. Load-Displacement of South swivel beam.

| Distance from the middle axis of the section (m) | Principal stress (Mpa) | Distance from the middle axis of the section (m) | Principal stress (Mpa) |
|-----------------------------------------------|------------------------|-----------------------------------------------|------------------------|
| -1.0158                                       | 28.2347                | 0.12                                          | 20.5467                |
| -0.9175                                       | 27.5617                | 0.2203                                         | 16.5332                |
| -0.8188                                       | 23.9263                | 0.3206                                         | 17.4914                |
| -0.7197                                       | 21.4658                | 0.4207                                         | 18.7486                |
| -0.6203                                       | 19.7721                | 0.5206                                         | 19.8133                |
| -0.5206                                       | 19.975                 | 0.6203                                         | 20.651                 |
| -0.4207                                       | 18.2893                | 0.7197                                         | 23.5763                |
| -0.3206                                       | 18.2023                | 0.8188                                         | 24.0224                |
| -0.2203                                       | 15.0327                | 0.9175                                         | 26.2788                |
| -0.12                                         | 18.8135                | 1.0158                                         | 28.6344                |

Figure 14. Stress distribution curve along arc length.

There is a sudden change of stress at two points ± 0.12m from the middle axis of the section. This is due to the stress concentration phenomenon caused by the opening of the central shaft of the spherical hinge. These two points should be ignored when drawing the stress distribution curve. It can be seen from the stress distribution curve that the stress distribution on the friction surface of the spherical hinge presents a certain regularity along the axis of the spherical surface, which is symmetrical with respect to the central axis. In order to accurately calculate the stress on the friction surface of the spherical hinge, it is necessary to analyse its distribution law and obtain the curve fitting equation. For this reason, a contact model of half space body is introduced.

3.4. Calculation theory of contact model of half space body

The theoretical calculation method of spherical hinge strength is based on the model established under the actual stress during the process of spherical hinge rotation. This method is based on the theory of contact stress calculation and derivation for solving space problems by elastic mechanics, and the contact part of horizontal rotating spherical hinge is locally abstracted as a half space body under the action of uniform stress. Then the three-dimensional principal stress of the complex stress state of the half space body is calculated, and the strength condition of the normal stress of the spherical hinge is obtained according to the material yield strength theory[10]. In this theory, the stress of the contact surface is quadratic along the axis of the sphere. On this basis, the calculation results of the theoretical model are compared with the settlement results of the finite element simulation to verify the
correctness of the finite element calculation results. Therefore, the friction moment of the friction surface of the spherical hinge can be calculated by fitting the quadratic curve of the finite element results.

Calculate the contact surface stress according to the contact stress method of half space body. The contact stress calculation diagram of half space body is shown in figure 15. According to the knowledge of solving the contact problem of half space body in elasticity, when the half space body contacts on the boundary, when Poisson's ratio $\nu=0.3$, the expression of the maximum contact force is

$$q = 0.388f \frac{FE^2 (R_1 + R_2)}{R_1^2 R_2^2}$$

(3)

Figure 15. Calculation diagram of contact stress in half space.

In formula, $E$ is the elastic modulus of the contact material; $F$ is the gravity of the spherical hinge superstructure; $R_1, R_2$ are the radius of the sphere of the lower spherical hinge and the upper spherical hinge respectively. For the structural characteristics of the spherical hinge of the rotary construction bridge, $R_1$ is negative and $R_2$ is positive.

The contact boundary of the two spherical joints is enlarged locally to investigate the structural characteristics and stress state of the contact part with the maximum contact stress. Because the contact surface of the spherical joint is curved and subjected to the uniform load of the spherical joint, the stress state of the contact surface is calculated according to the uniform load model of the half space body. The stress state is shown in figure 16.

Figure 16. Uniform load of half space body.

According to the method of solving space problems in elasticity[11], the derivation formula under uniform load is as follows:

$$\sigma_z = \frac{3z^3}{2\pi} \int_0^1 2\pi r q rdr \left( r^2 + z^2 \right)^{\frac{3}{2}} = -q \left[ 1 - z^3 (z^2 + a^3)^{\frac{3}{2}} \right]$$

(4)

$$\sigma_x = \sigma_y = -\frac{q}{2} \left[ (1 + 2\mu) + z^3 (z^2 + a^2)^{\frac{3}{2}} - 2(1 + \mu) z (z^2 + a^2)^{\frac{1}{2}} \right]$$

(5)
When the half space is below, \( a = 0 \), at this point, \( \sigma_Z = -q \). Poisson's ratio \( \nu = 0.2 \), By substituting this result into equation (6), \( z = 2a \), and then into equation (5), the contact pressure can be obtained. By comparing the contact pressure with the finite element method, the correctness of the contact theory of half space body can be verified. Based on the contact theory of half space body, the distribution of stress on the cross section of friction surface obeys the relation of one-dimensional quadratic curve. And the stress on the friction surface is the same at the same distance from the central shaft. Therefore, according to the stress results of finite element calculation, the stress on one cross section can be taken for curve fitting. The calculation formula of the stress on the arc of the cross section is obtained, and then the friction moment on the whole friction surface is obtained by integration.

4. Case study

4.1. Traditional calculation method

According to the weighing test results, the unbalanced moment of the longitudinal swivel of the project is less than the friction moment of the spherical hinge. Test results:

\[
P_1 = 98.92 \times 9.8 \text{kN} = 969.42 \text{kN}
\]

\[
P_2 = 231.29 \times 9.8 \text{kN} = 2266.64 \text{kN}
\]

Substituting equation (2) can obtain:

Unbalance moment of swivel:

\[
M_\alpha = \frac{P_2L - P_1L}{2} = \frac{2266.64 \times 4.35 - 969.42 \times 4.35}{2} = 2821.45 \text{kN} \cdot \text{m}
\]

Friction moment of RPC concrete composite spherical hinge:

\[
M_Z = \frac{P_2L + P_1L}{2} = \frac{2266.64 \times 4.35 + 969.42 \times 4.35}{2} = 7038.43 \text{kN} \cdot \text{m}
\]

Eccentricity:

\[
e = \frac{M_\alpha}{N} = \frac{2821.45}{13000 \times 9.8} = 0.022 \text{m}
\]

![Figure 17. Friction surface section of spherical hinge.](image)

Let the arc length and half angle of the spherical hinge friction surface be \( \alpha \), and the geometric calculation is as follows:

\[
\cos \alpha \approx 0.968
\]

\[
\sin \alpha = 0.25
\]
Substitute formula
\[ M_x = \int_0^3 dM_x = \int_0^3 R \cos \theta dF = 2 \mu_0 (1 - \cos^3 \alpha) \ \text{NR} \left( \frac{3 \sin^2 \alpha}{3 \times 0.0625} \right) \]
\[ M_x = \frac{2 \mu_0 (1 - \cos^3 \alpha) \ \text{NR}}{3 \sin^2 \alpha} \approx \frac{2 \mu_0 (1 - 0.9077) \ \text{NR}}{3 \times 0.0625} \]
\[ \mu_0 \approx \frac{1}{0.9842} \ \text{NR} \approx 0.01169 \]

Unbalanced moment of pier 14# RPC concrete composite spherical hinge longitudinal rotation \( M_G = 2821.45 \text{kN} \cdot \text{m} \), frictional resistance moment \( M_Z = 7038.43 \text{kN} \cdot \text{m} \), longitudinal eccentricity \( e = 0.022 \text{m} \). The longitudinal friction moment of RPC concrete composite spherical hinge is larger than the unbalanced moment of the rotating body, and there will be no longitudinal small rotation in the plane. The longitudinal static friction coefficient of RPC concrete composite spherical hinge is about 0.01169.

4.2. Modified calculation method based on contact theory of half space body

According to the calculation equation (3) of the half space calculation model, the maximum contact force is calculated by substituting the spherical joint data:

\[ q = 0.388 \left\{ \frac{FE^2 (R_1 + R_2)}{R_1^2 R_2^2} \right\}^{\frac{1}{3}} \]
\[ = 0.388 \times \left[ \frac{13 \times 9.8 \times (42 \times 10^3)^2 \times (4.8 + 4.83)}{4.8^2 \times 4.83^2} \right]^{\frac{1}{3}} \approx 1595.8476 \]
\[ \sigma_x = \sigma_y = -\frac{q}{2} \left[ (1 + 2 \times 0.2) + \frac{(2a)^3}{(2a)^2 + a^2} - \frac{2(1 + 0.2) \times 2a}{(2a^2 + a^2)^{\frac{3}{2}}} \right] \approx -0.01553q \]
\[ \sigma_x = \sigma_y = -0.01553q = 24.78 \text{Mpa} \]

Therefore, according to the contact model of half space body, the maximum contact stress of the spherical joint is 24.78Mpa.

According to the finite element analysis, the maximum compressive stress on the friction surface of RPC concrete composite spherical hinge is 28.2Mpa, and the maximum compressive stress on the friction surface of spherical hinge is 24.78Mpa calculated according to the contact model of half space body. The calculation result of the contact model of half space body is 12.2% lower than that of the finite element calculation. Within the allowable deviation range, it is proved that the contact surface stress calculated by the finite element calculation and the contact model of half space body is calculated. Roughly. Based on the contact theory of half space body, the distribution of stress on the cross section of friction surface obeys the relation of one-dimensional quadratic curve, and the stress is the same when the distance between the friction surface and the central rotating axis is the same.

Through the stress distribution point set extracted from table 3 and figure 14, curve fitting is carried out to obtain the stress distribution on the friction surface of the spherical hinge. The friction moment of the spherical hinge is calculated by the area division of the friction surface stress, and the static friction coefficient of the spherical hinge is obtained. The results are as follows:

Suppose the curve of distance-stress distribution is a one-dimensional quadratic regression equation:

\[ y = aX^2 + bX + c \]  

(7)

The linear equations are listed according to the multiplication of Lagrange number:
Where \( x \) and \( y \) are known parameters and \( a, b, c \) are unknown parameters. Because \( b = 0 \), the equations can be simplified to
\[
\begin{align*}
\sum a x^3 + c \sum x &= \sum xy \\
\sum a x^2 + \sum x + cn &= \sum y
\end{align*}
\] (9)

Taking in the data in table 3
\[
\sum x^3 = 3.2101, \sum x^2 = 4.0462, \sum x = 5.5743, \sum xy = 130.0972, \sum y = 190.7494
\]
Substituting the result into equation (9)
\[
a = 12.0651, c = 16.3258
\]
Substituting equation (7), the fitting equation of one variable quadratic regression is as follows:
\[
Y = 12.0651 X + 16.3258
\] (10)

The friction force on the micro element area of the friction surface is integrated and multiplied by the force arm. Then, by integrating the moment on the whole friction surface, the expression of friction moment can be obtained as follows:
\[
M_z = \int \int XFdx dz
= \int \int R \cdot \sin \alpha \cdot F \cdot R \cdot \cos \alpha \cdot Rd \theta \, d\alpha
\] (11)

Substituting equation (10) into the above equation, we can get:
\[
M_z = 4\pi R^3 \int_0^{0.25} 277.98 \sin^3 \alpha \cos \alpha + 16.3258 \sin \alpha \cos \alpha \, d\alpha = 25028.97
\]

Static friction coefficient of ball joint:
\[
\mu \approx \frac{1}{0.9842} \frac{M_z}{NR} = \frac{1}{0.9842} \frac{25027.97}{13000 \times 9.8 \times 4.8} \approx 0.0415
\]

4.3. Back calculation of actual traction
Actual starting traction during turning:
\[
T = 2995.2 \text{kN}.
\]

Back calculated static friction coefficient:
\[
\mu = \frac{3DT}{2NR} = \frac{3 \times 4.9 \times 2995.2}{2 \times 13000 \times 9.8 \times 4.8} = 0.036
\]

Through the above calculation, the calculation results of three static friction coefficients are compared, as shown in table 4 below.
Table 4. Comparison of static friction coefficient calculation results under different calculation theories.

| Computational theory | Traditional calculation method | Optimization calculation method | Actual traction reverse |
|----------------------|-------------------------------|---------------------------------|-------------------------|
| Coefficient of static friction $\mu$ | 0.01169 | 0.0415 | 0.036 |

5. Conclusions

In this paper, the theory of half space contact model and the method of finite element simulation are combined. The static friction coefficient of spherical hinge contact surface is optimized. At the same time, the calculation results are compared with the traditional weighing test results and the actual traction back calculation results. The following conclusions are drawn:

1. The theory of half space contact model shows that the stress on the friction surface of the spherical hinge is quadratic along the axis of the sphere. The maximum stress error of the contact surface calculated by this model and the finite element model is small, so the stress curve can be fitted by combining the finite element calculation results with the general equation of quadratic curve.

2. The opening of the spherical hinge shaft will lead to the stress concentration in the section of the accessories, and the change trend of the local stress is opposite to the overall trend of the spherical hinge. In the curve fitting calculation, the stress here should be ignored.

3. The static friction coefficient of the spherical hinge obtained by the traditional calculation method is 67.5% smaller than the actual static friction coefficient. The static friction coefficient of RPC concrete composite spherical hinge calculated by this method is small. As the basis of the calculation of the traction force of the bridge, there will be insufficient estimation of the traction force.

4. The static friction coefficient of the spherical hinge obtained by the stress curve fitting method is 15.2% larger than the actual static friction coefficient. Compared with the traditional calculation method, the calculation result of this method is more accurate, and the error between this method and the measured value is relatively small.

Based on the research results of this paper, it is suggested that the static friction coefficient of spherical hinge should be determined according to the weighing test results and theoretical calculation. For the composite spherical hinge of RPC concrete, the sliding plate of ultra-molecular weight polyethylene and stainless steel are used as friction pairs. When there is no test data, the static friction coefficient based on the contact stress fitting curve can be calculated accurately. On this basis, the value of static friction coefficient should be increased to ensure the safety and stability of the bridge. The recommended value is 0.03-0.06.
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References
[1] Yoo, D.Y., Shin, H.O., Yang, J.M., Yoon, Y.S. (2014) Material and bond properties of ultra-high performance fiber reinforced concrete with micro steel fibers. COMPOSITES PART B-ENGINEERING, 58: 122-133.
[2] Song, J.W., Liu, S.H. (2016) Properties of Reactive Powder Concrete and Its Application in Highway Bridge. ADVANCES IN MATERIALS SCIENCE AND ENGINEERING, 5460241.
[3] Ji, W., Li, W., An, M., Zhu, L. (2018) Shear Capacity of T-Section Girders Made of Reactive Powder Concrete. JOURNAL OF BRIDGE ENGINEERING, 23(7): 04018041.
[4] Shao, X., Pan, R., Zhan, H., Fan, W., Yang, Z., Lei, W. (2017) Experimental Verification of the Feasibility of a Novel Prestressed Reactive Powder Concrete Box-Girder Bridge Structure. JOURNAL OF BRIDGE ENGINEERING, 22.
[5] Du, Y., Chen, B. (2013) Experimental research on the ultimate load of capacity of reactive power concrete arches. Engineering Mechanics, 30: 42-48.
[6] Wang, Z., Wu, L. (2019) Triaxial test study of reactive powder concrete with different sizes under different friction reducing conditions. Journal of Zhejiang University (Engineering Science), 53: 40-50.
[7] Luo, K. (2018) Study of interface bonding performance between shape steel and Reactive Power Concrete (RPC) in steel reinforced RPC structures. Hunan University.
[8] Zhao, J. (2016) Study on calculation methods for static friction force in bridge swivelling erection. Shandong Communications Technology, 03: 91-95.
[9] Wang, Y. (2018) Study on Swivel Construction Monitoring and Study on the Parameters of the RPC Concrete Spherical Hinge. Lanzhou Jiaotong University.
[10] Xu, Z. (2013) A concise course in elasticity. HIGHER EDUCATION PRESS, Beijing.
[11] Sun, X., Fang, X., Guan, L. (2011) Mechanics of Materials (I). HIGHER EDUCATION PRESS, Beijing.
[12] Pan, W., Fan, J., Nie, J., Hu, J., Cui, J. (2016) Experimental Study on Tensile Behavior of Wet Joints in a Prefabricated Composite Deck System Composed of Orthotropic Steel Deck and Ultrathin Reactive-Powder Concrete Layer. JOURNAL OF BRIDGE ENGINEERING, 21(10): 04016064