On Nontrivial Solutions around a Marginal Solution in Cubic Superstring Field Theory

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Abstract

We construct tachyon vacuum and half-brane solutions, using an extension of $K\bar{B}c$ algebra, in the theory around a type of identity-based marginal solutions in modified cubic superstring field theory. With explicit computations, we find that their vacuum energies are the same as those of corresponding solutions around the original theory. It implies that the vacuum energy for the identity-based marginal solution vanishes although straightforward computation of it is subtle. We also evaluate the gauge invariant overlaps for those nontrivial solutions. The values for them are deformed according to the marginal solution in the same way as the case of bosonic string field theory.
1 Introduction

In a recent development in string field theory (SFT), so called $KBc$ algebra [1] is often used because of its algebraic simplicity in order to construct classical solutions and evaluate gauge invariants. In [2], the authors have investigated “$K'Bc$ algebra,” where the Kato-Ogawa BRST operator $Q_B$ and a string field $K = \{Q_B, B\}$ are replaced by $Q'$ and $K' = \{Q', B\}$, respectively, in $KBc$ algebra. Here, $Q'$ is the BRST operator in a theory around a class of identity-based marginal solutions [3, 4, 5] in bosonic open SFT. Using $K'Bc$ algebra, the tachyon vacuum solution on the deformed background was constructed as in [6] and vacuum energy and gauge invariant overlap are evaluated explicitly. It turned out that the value of vacuum energy does not change. But the expression of the gauge invariant overlap is deformed appropriately. In particular, for a closed tachyon vertex, a phase factor appears and it is the same value as that evaluated in [7] using a different method.

In this paper, we extend computations performed in [2] to the NS sector of modified cubic superstring field theory (SSFT) [8, 9, 10] without GSO projection. In the case of superstring, $KBc$ algebra with a string field $\gamma$ was applied to a construction of tachyon vacuum solution [11] and its variants [12, 13, 14, 15, 16]. It was further extended to “$GBKc\gamma$ algebra” by including a string field $G$, such as $G^2 = K$, made of superconformal generators and then half-brane solutions, whose vacuum energy is half of that of the tachyon vacuum solution, were constructed [17]. In this context, we explore $G'K'Bc\gamma$ algebra, where $Q_B$, $K$ and $G$ are replaced by $Q'$, $K' = \{Q', B\}$ and $G'$ (such as $G'^2 = K'$), respectively, corresponding to $K'Bc$ algebra in the bosonic case [2]. Here, $Q'$ is the BRST operator in a theory around a class of identity-based marginal solutions in modified cubic SSFT [11]. Around an identity-based marginal solution $\Psi_J$, which is constructed using a supercurrent algebra, we have an explicit expression of a deformed BRST operator $Q'$. Using $G'K'Bc\gamma$ algebra, we can construct various solutions to the equation of motion, $\hat{Q}'\Phi + \Phi^2 = 0$, in the NS sector without GSO projection. In particular, we focus on a tachyon vacuum solution and a half-brane solution in the marginally deformed background and we evaluate the vacuum energy and gauge invariant overlaps for them. The results are similar to the bosonic case: The vacuum energies are the same as those for corresponding solution in the original theory. The values of the gauge invariant overlap with a closed tachyon vertex change by a phase factor just as in [2]. We find that the vacuum energy and the gauge invariant overlap for the half-brane solution in the marginally deformed background are half of those for the tachyon vacuum solution, respectively. These results are consistent with our expectation that the vacuum energy for the identity-based solution $\Psi_J$ vanishes and it corresponds to a marginal deformation.

This paper is organized as follows. In the next section, we will develop $G'K'Bc\gamma$ algebra in a theory around a particular identity-based marginal solution. In §3 and §4, we will calculate vacuum energy and gauge invariant overlap for a tachyon vacuum solution and a half-brane solution. Then, we will give some concluding remarks in §5.

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1 In SSFT, other types of marginal solutions based on wedge states are also constructed [18, 19, 20, 21, 22], but identity-based solutions are suitable for our purpose.
In appendices B and C, we will show technical details. In appendix D, we will discuss the gauge invariant overlap with a closed tachyon vertex and a field redefinition induced by the identity-based marginal solution as in \[7\].

### 2 A version of $GKBc\gamma$ algebra and classical solutions in a marginally deformed background

Here, we give a version of “$GKBc\gamma$ algebra” developed in \[17\] in the theory around an identity-based marginal solution. Using this algebra, we can construct classical solutions including GSO(-) sector easily as in the original theory.

First of all, we consider a theory around a particular identity-based solution $\Psi_J$, which is given by

$$
\Psi_J = -V_L^a(F_a)I + \frac{1}{8}\Omega^{ab}C_L(F_aF_b)I,
$$

where $F_a(z)$ is some function such as $F_a(-1/z) = z^2F_a(z)$, $C_L$ denotes a half unit circle: $|z| = 1, \Re z \geq 0$ and $I$ is the identity state. In (2.1), the repeated indices, $a, b$, are contracted. The above $\Psi_J$ satisfies the equation of motion to modified cubic SSFT in the NS sector because we can show

$$
Q_B\Psi_J + \Psi_J^* \Psi_J = 0.
$$

Actually, $\Psi_J = e^{-\Phi_J}Q_Be^{\Phi_J}$ holds, where $\Phi_J = -\tilde{V}_L^a(F_a)I$ is an identity-based marginal solution to Berkovits’ WZW-type SSFT investigated in \[13\]. Here, we suppose that $J^a(z, \theta) = \psi^a(z) + \theta J^a(z)$ is a supercurrent associated with a Lie algebra, where $a$ is its index, in the matter sector. Its component fields satisfy following operator product expansions (OPE) \[19\]:

$$
\psi^a(y)\psi^b(z) \sim \frac{1}{y - z} \frac{1}{2} \Omega^{ab}, \quad J^a(y)\psi^b(z) \sim \frac{1}{y - z} f^{ab}_{\ c}\psi^c(z), \quad J^a(y)J^b(z) \sim \frac{1}{(y - z)^2} \frac{1}{2} \Omega^{ab} + \frac{1}{y - z} f^{ab}_{\ c}J^c(z),
$$

where constants $\Omega^{ab}, f^{ab}_{\ c}$ satisfy following relations

$$
\Omega^{ab} = \Omega^{ba}, \quad f^{ab}_{\ c}\Omega^{cd} + f^{ad}_{\ e}\Omega^{eb} = 0,
$$

$$
f^{ab}_{\ c} = -f^{ba}_{\ c}, \quad f^{ab}_{\ d}f^{cd}_{\ e} + f^{bc}_{\ d}f^{ad}_{\ e} + f^{ca}_{\ d}f^{bd}_{\ e} = 0.
$$

\[2\] See appendix A for a review and our conventions.
By re-expanding the NS action of modified cubic SSFT around the solution $\Psi_f$ (2.1), the BRST operator is deformed as

\[
Q' = Q_B + [\Psi_f, \cdot],
\]

\[
= Q_B - V^a(\mathcal{F}_a) + \frac{1}{8}\Omega^{ab}\mathcal{C}(\mathcal{F}_a\mathcal{F}_b),
\]

(2.8)

where $V^a(\mathcal{F}_a)$ and $\mathcal{C}(\mathcal{F}_a\mathcal{F}_b)$ are given by integrations along the whole unit circle, $|z| = 1$:

\[
V^a(f) \equiv \oint \frac{dz}{2\pi i} \frac{1}{\sqrt{2}}f(z)(cJ^a(z) + \gamma_3a(z)), \quad C(f) \equiv \oint \frac{dz}{2\pi i} f(z)c(z).
\]

(2.9)

The superconformal generators, $L'_n, G'_r$, corresponding to the above $Q'$ (2.8) are given by

\[
L'_n \equiv \{Q', b_n\} = L_n - \frac{1}{\sqrt{2}} \sum_{k \in \mathbb{Z}} F_{a,k} J_{n-k}^a + \frac{1}{8} \Omega^{ab} \sum_{k \in \mathbb{Z}} F_{a,n-k} F_{b,k},
\]

(2.10)

\[
G'_r \equiv [Q', \beta_r] = G_r - \frac{1}{\sqrt{2}} \sum_{k \in \mathbb{Z}} F_{a,k} \psi_{r-k}^a,
\]

(2.11)

where we define the coefficients as $F_{a,n} \equiv \oint \frac{d\sigma}{2\pi i} e^{i(n+1)\sigma} F_a(e^{i\sigma})$ and then $F_a(-1/z) = z^2 F_a(z)$ implies $F_{a,n} = (-1)^n F_{a,-n}$. Only the matter sectors of them are deformed and the central charge is not changed. Replacing $L_n, G_r$ in $K_1^L$ (A.9) and $\mathcal{G}_L$ (A.12) with $L'_n, G'_r$, respectively, we define $K'_I, \mathcal{G}'_L$ and string fields $K', G'$:

\[
K' = \pi K_1'^L I, \quad G' = \mathcal{G}'_L I \sigma_1
\]

(2.12)

as in (A.8), (A.11). Here, $\sigma_1$ in $G'$ is a Chan-Paton factor, which is involved to treat a theory without GSO projection. Because of $Q'|I) = 0$, we have

\[
K'_1 |I) = (Q'B_1 + B_1 Q')|I) = 0,
\]

(2.13)

\[
\mathcal{G}'|I) = \sqrt{\frac{\pi}{2}} (Q'\beta_{-\frac{1}{2}} - \beta_{-\frac{1}{2}} Q')|I) = 0,
\]

(2.14)

where $K'_1$ and $\mathcal{G}'$ are defined by replacing $L_n, G_r$ with $L'_n, G'_r$ in $K_1 = \{Q_B, B_1\}, \mathcal{G} = \sqrt{\frac{\pi}{2}}[Q_B, \beta_{-\frac{1}{2}}]$, respectively. The above relations are consistent with a deformed version of (A.24):

\[
\partial' \Phi \equiv K' \Phi - \Phi K' = \frac{\pi}{2} K_1' \Phi, \quad \delta' \Phi \equiv G' \Phi - (-)^{F(\Phi)} \Phi G' = (G' \sigma_1) \Phi,
\]

(2.15)

where $(-)^{F(\Phi)}$ is a sign factor due to worldsheet spinor. Attaching a Chan-Paton factor $\sigma_3$ and using a notation $Q' \equiv Q' \sigma_3$, we have a deformed version of $GKBc\gamma$ algebra reviewed in appendix A:

\[
G'^2 = K', \quad Q'B = K', \quad \hat{Q}'K' = 0, \quad \hat{Q}'G' = 0,
\]

(2.16)

\[
\hat{Q}'c = cK'c - \gamma^2 = cKc - \gamma^2,
\]

(2.17)
\[
BG' = G'B, \quad BK' = K'B, \quad G'K' = K'G', \\
\delta'c = 2i\gamma, \quad \delta'\gamma = -\frac{i}{2}\partial'c = -\frac{i}{2}\partial c, \quad \delta'G' = 2K', \quad \delta'K' = 0, \quad \delta'B = 0. 
\] (2.18)

We also note that
\[
\hat{Q}'\gamma = \hat{Q}\gamma = c\partial\gamma - \frac{1}{2}(\partial c)\gamma, \quad c\partial\gamma = -(\partial\gamma)c, \quad \gamma\partial c = - (\partial c)\gamma. 
\] (2.20)

In the following, we call the above relations among string fields as \(G'K'Bc\gamma\) algebra, which is the same form as \(GKBc\gamma\) algebra. We will use the above relations extensively in various calculations.

Let us consider the action \(S'[\Phi]\), which is obtained by re-expanding around \(\Psi J_3\sigma_3\), where \(\sigma_3\) is the Chan-Paton factor for GSO(+) string field (A.3) in the NS action without GSO projection (A.1). More explicitly, it is defined by
\[
S'[\Phi] = S[\Phi + \Psi J_3\sigma_3] - S[\Psi J_3\sigma_3] = \frac{1}{2}\langle \langle \Phi \hat{Q}'\Phi \rangle \rangle + \frac{1}{3}\langle \langle \Phi^3 \rangle \rangle. 
\] (2.21)

The equation of motion of \(S'[\Phi]\) is
\[
\hat{Q}'\Phi + \Phi^2 = 0. 
\] (2.22)

Using the method in [17] with \(G'K'Bc\gamma\) algebra instead of \(GKBc\gamma\) algebra, we can easily construct a class of solutions to (2.22):
\[
\Phi_f' = \sqrt{f'} \left( c \frac{K'B}{1-f'} + Bc^2 \right) \sqrt{f'} = \sqrt{f'} \left( c \frac{K'f'}{1-f'} Bc + \hat{Q}'(Bc) \right) \sqrt{f'}, 
\] (2.23)

where \(f'\) is a function of \(G'\), noting \(K' = G'^2\).

In the following sections, we consider two solutions, which correspond to \(f' = \frac{1}{1+K'}\) (a tachyon vacuum solution \(\Phi_T\)) and \(f' = \frac{1}{1+icG'}\) (a half-brane solution \(\Phi_H\)). More explicitly, they are given by
\[
\Phi_T = \frac{1}{\sqrt{1+K'}} \left( c + \hat{Q}'(Bc) \right) \frac{1}{\sqrt{1+K'}}, \quad \Phi_H = \frac{1}{\sqrt{1+icG'}} \left( -icG'Bc + \hat{Q}'(Bc) \right) \frac{1}{\sqrt{1+icG'}}. 
\] (2.24, 2.25)

### 3 Vacuum energy of the solutions on the marginally deformed background

In this section, we evaluate the vacuum energy, or the value of the action, for solutions \(\Phi_T\) (2.24) and \(\Phi_H\) (2.25) in the theory given by the marginally deformed action (2.21). In

\[\text{In this paper, we ignore the kernel of the picture changing operator with picture number } (-2), Y_{-2}, \text{ for simplicity. See [24] for a recent argument.}\]
general, for a solution $\Phi_f' (2.23)$ to the equation of motion (2.22), the value of the action is computed as

$$S'[\Phi_f'] = \frac{1}{6} \langle \langle \Phi_f' \hat{Q}' \Phi_f' \rangle \rangle = \frac{1}{6} \langle \langle \frac{c K' f'}{1 - f' B c f'} \hat{Q}' \left( \frac{c K' f'}{1 - f' B c} \right) f' \rangle \rangle, \tag{3.1}$$

using $G'K' B c\gamma$ algebra and a deformed version of (A.4) and (A.6):

$$\hat{Q}'(\Phi \Psi) = (\hat{Q}' \Phi) \Psi + (-1)^{\epsilon(\Phi)+F(\Phi)} \Phi (\hat{Q}' \Psi), \tag{3.2}$$

$$\langle \langle \hat{Q}'(\cdots) \rangle \rangle = 0, \tag{3.3}$$

where $(-)^{\epsilon(\Phi)}$ is a sign factor from Grassmannality of $\Phi$.

### 3.1 Tachyon vacuum solution

In the case of $f' = \frac{1}{1 + K'}$, namely, $\Phi_T (2.24)$, the expression of (3.1) is simplified as

$$S'[\Phi_T] = \frac{1}{6} \langle \langle \frac{1}{1 + K'} (c \partial c - \gamma^2) \frac{1}{1 + K'} \rangle \rangle = -\frac{1}{6} \langle \langle \gamma^2 \frac{1}{1 + K'} \rangle \rangle. \tag{3.4}$$

In the second equality, we have used the form of the picture changing operator $Y_{-2}$ defined in (A.7).

In order to define the inverse of a string field $I + K'$, which is denoted by $\frac{1}{1 + K'}$, we use the following expression:

$$\frac{1}{1 + K'} = \int_0^\infty dt e^{-t(1 + K')}. \tag{3.5}$$

Here, $K'$ can be rewritten as

$$K' = K - J + \frac{\pi}{2} CI, \tag{3.6}$$

$$J = \frac{\pi}{2} \int_{-\infty}^{\infty} dt f_a(t) \hat{U}_1 \tilde{J}^a (it) |0\rangle, \quad f_a(t) = \frac{F_a(tan(it + \frac{\pi}{4}))}{2\pi\sqrt{2} \cos^2(it + \frac{\pi}{4})}, \tag{3.7}$$

$$C = \int C_L \frac{dz}{2\pi i} (1 + z^2) \frac{\Omega_{ab}}{8} F_a(z) F_b(z) = \frac{\pi}{2} \int_{-\infty}^{\infty} dt \Omega_{ab} f_a(t) f_b(t). \tag{3.8}$$

($\tilde{J}^a(\tilde{z}) = (\cos \tilde{z})^{-2} J^a (\tan \tilde{z})$ is $J^a$ in the sliver frame.) Using the expansion formula (B.4) and the methods developed in [25], the order $J^N$ term of $e^{-tK+tJ}$ is computed as

$$e^{-tK+tJ} \bigg|_{O(J^N)} \quad = \int_0^1 dt_1 \int_0^{1-u_1} dt_2 \cdots \int_0^{1-u_1-u_2-\cdots-u_{N-1}} dt_N \int_0^\infty dt_1 f_{a_1}(t_1) \int_0^\infty dt_2 f_{a_2}(t_2) \cdots \int_0^\infty dt_N f_{a_N}(t_N)$$

$$= t^N \int_0^1 dt_1 \int_0^{1-u_1} dt_2 \cdots \int_0^{1-u_1-u_2-\cdots-u_{N-1}} dt_N \int_0^\infty dt_1 f_{a_1}(t_1) \int_0^\infty dt_2 f_{a_2}(t_2) \cdots \int_0^\infty dt_N f_{a_N}(t_N)$$

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Using the above, the integration in (3.9) can be rewritten as

\[ \times \frac{\pi N}{2N} \hat{U}_{t+1} \tilde{J}^{a_1} (it_1 + \frac{\pi}{4} y_1) \tilde{J}^{a_2} (it_2 + \frac{\pi}{4} y_2) \cdots \tilde{J}^{a_N} (it_N + \frac{\pi}{4} y_N) |0\rangle, \]  

(3.9)

where in the real part of the argument of \( \tilde{J}^{a_i} \), \( y_i \) (\( i = 1, 2, \ldots, N \)) are defined as

\[ y_1 = 2t \sum_{k=1}^{N} u_k - t, \quad \cdots, \quad y_i = 2t \sum_{k=i}^{N} u_k - t, \quad \cdots, \quad y_N = 2t u_N - t. \quad (3.10) \]

Hence, \( u_i \) can be expressed using \( y_i \) and the Jacobian is computed as

\[ \frac{\partial (u_1, \ldots, u_N)}{\partial (y_1, \ldots, y_N)} = 2^{-N} t^{-N}. \quad (3.12) \]

Using the above, the integration in (3.9) can be rewritten as

\[ e^{-tK + t\hat{J}}|_{O(J^N)} = \frac{\pi N}{4} \int_{-t}^{t} dy_1 \int_{-t}^{y_1} dy_2 \cdots \int_{-t}^{y_{N-1}} dy_N \int_{-\infty}^{\infty} dt_1 f_{a_1}(t_1) \int_{-\infty}^{\infty} dt_2 f_{a_2}(t_2) \cdots \int_{-\infty}^{\infty} dt_N f_{a_N}(t_N) \]

\[ \times \hat{U}_{t+1} \tilde{J}^{a_1} (it_1 + \frac{\pi}{4} y_1) \tilde{J}^{a_2} (it_2 + \frac{\pi}{4} y_2) \cdots \tilde{J}^{a_N} (it_N + \frac{\pi}{4} y_N) |0\rangle. \quad (3.13) \]

If we use an ordering symbol \( \mathbf{T} \) with respect to the real part of the arguments of \( \tilde{J}^{a_i} \) in the above integrations, we have

\[ e^{-tK + t\hat{J}}|_{O(J^N)} = \frac{1}{N!} \hat{U}_{t+1} \mathbf{T} \left( \frac{\pi}{4} \int_{-t}^{t} du \int_{-\infty}^{\infty} dt' f_a(t') \tilde{J}^a (it' + \frac{\pi}{4} u) \right)^N |0\rangle, \quad (3.14) \]

which implies the following expression of \( e^{-tK'} = e^{-t(K - \frac{\pi}{4} \hat{C} \hat{I})} \):

\[ e^{-tK'} = e^{-t\frac{\pi}{4} \hat{C} \hat{U}_{t+1} \mathbf{T} \exp \left( \frac{\pi}{4} \int_{-t}^{t} du \int_{-\infty}^{\infty} dt' f_a(t') \tilde{J}^a (it' + \frac{\pi}{4} u) \right) |0\rangle. \quad (3.15) \]

Therefore, (3.4) is computed as

\[ S'[\Phi_T] = -\frac{1}{6} \int_{0}^{\infty} dt \int_{0}^{\infty} ds e^{-t-s} \langle \gamma^2 e^{-tK'} ce^{-sK'} \rangle, \quad (3.16) \]

where the integrand can be rewritten as

\[ \langle \gamma^2 e^{-tK'} ce^{-sK'} \rangle \]

\[ = \frac{4}{\pi^2} \langle I | Y_{-2} \hat{U}_{t+s+1} \tilde{J}^2 (\frac{\pi}{4} (t + s)) \tilde{C} (\frac{\pi}{4} (s - t)) \]

\[ \times e^{-t(t+s)} \frac{\pi}{4} \mathbf{T} \exp \left( \frac{\pi}{4} \int_{-t-s}^{t+s} du \int_{-\infty}^{\infty} dt' f_a(t') \tilde{J}^a (it' + \frac{\pi}{4} u) \right) |0\rangle. \]
\[ \langle i M \rangle c (-i M) \tilde{c}(z) \sim \frac{i}{8} e^{4M}, \quad (M \to +\infty) \]

\[ \langle \delta'(\tilde{\gamma}(iM))\delta'(-iM)\tilde{\gamma}(x)\tilde{\gamma}(y) \rangle \sim -i \frac{4}{e^{4M}} \cos(x - y), \quad (M \to +\infty) \]

in the ghost sector, we have

\[ \langle \gamma^2 e^{-tK'} e^{sK'} \rangle = -\frac{(t + s)^2}{\pi^2} \lim_{M \to \infty} \langle \delta'(\tilde{\gamma}(iM))\delta'(-iM)\tilde{\gamma}(\frac{\pi(s - t)}{2(s + t)}) \rangle_{bc} \]

\[ \times e^{-(t+s)\tilde{x}c} \exp \left( \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} du \int_{-\infty}^{\infty} dt' f_a(t') \tilde{J}^a \left( \frac{2it'}{t + s} + u \right) \right)_{\text{mat}} \]

\[ = -\frac{(t + s)^2}{\pi^2} \lim_{M \to \infty} (-4ie^{-4M}) \frac{i}{8} e^{4M} \cdot 1 = -\frac{(t + s)^2}{2\pi^2}. \]

The minus sign comes from Grassmannality of \( \delta'(\gamma) \), which corresponds to \( \partial \xi e^{-2\phi} \) in the \( \xi\eta\phi \)-expression. In the first equality, a \( T \)-ordered exponential becomes a conventional exponential in the CFT correlator and in the second equality, we have used (3.17) in the matter sector, which is essential for \( J \)-independence.

Using this result, the value of the action (3.16) is

\[ S'(\Phi_T) = \int_0^\infty dt \int_0^\infty ds e^{-t-s} \frac{(t + s)^2}{12\pi^2} = \frac{1}{2\pi^2}, \]

which is the same as a D-brane tension. Namely, the vacuum energy of \( \Phi_T \) is the same as that of the tachyon vacuum solution [11, 13] in the original theory.

### 3.2 Half-brane solution

Let us consider the vacuum energy for \( \Phi_H \) (2.25). Namely in (3.1), we use

\[ f' = \frac{1}{1 + iG'} = \frac{1}{1 + \tilde{K}'} - i \frac{G'}{1 + \tilde{K}'}. \]

\[ \langle \delta'(\gamma(w))\delta'(\gamma(z))\gamma(y_1)\gamma(y_2) \rangle = -\frac{1}{(w - z)^3} (2wz + 2y_1y_2 - (w + z)(y_1 + y_2)), \]
Noting the structure of the Chan-Paton factor, we have
\[ S'[\Phi_H] = \frac{1}{6}(-A_1 + A_2), \quad (3.24) \]
\[ A_1 = (cG'Bc, \hat{Q}'(cG'Bc)'), \quad A_2 = (cG'BcG', \hat{Q}'(cG'Bc)G'), \quad (3.25) \]
where we have used the notation:
\[ (\Phi, \Psi)' \equiv \langle \langle \Phi \frac{1}{1 + K' \Psi} \frac{1}{1 + K} \rangle \rangle = \int_0^\infty dt \int_0^\infty ds e^{-t-s} \langle \langle \Phi e^{-tK'} \Psi e^{-sK'} \rangle \rangle. \quad (3.26) \]

Some relations in worldsheet supersymmetric transformation investigated in \[17\] hold in the following sense:
\[ \langle \langle G' \cdots \rangle \rangle = \frac{1}{2} \langle \langle \delta' \cdots \rangle \rangle, \quad \delta' \hat{Q}' = \hat{Q}' \delta', \quad \delta' \delta' = \partial \delta', \quad (3.27) \]
\[ \delta'(\Phi \Psi) = (\delta' \Phi) \Psi + (-1)^{F(\Phi)} \Phi (\delta' \Psi). \quad (3.28) \]

Using the above, \( A_1 \) and \( A_2 \) given in \[3.25\] are rewritten as
\[ A_1 = -(\gamma^2, cK')' + 5(\gamma^2, c\partial cB)' - 4(cB\gamma, \gamma K'c)' + 2(cB\gamma, \partial c)' + 2(\gamma, K'\gamma c)', \quad (3.29) \]
\[ A_2 = (B\gamma^2, K'c\partial c)' + 4(Bc\gamma K', c\gamma K')' + 2(Bc\partial \gamma, c\gamma K')' - 2(B\gamma\partial c, c\gamma K')' + (B\gamma \partial \gamma, c\partial c)' - (\gamma^2, K'c\partial c)' - (cB\gamma, \partial c)' - 2(cB\gamma, \partial^2 \gamma c)' + (cB\gamma, \gamma \partial^2 c)', \quad (3.30) \]
respectively. To evaluate each term explicitly, following formulas are useful:
\[ \langle \langle B e^{-rK'} e^{-sK'} \gamma e^{-tK'} e^{-uK'} \rangle \rangle = \frac{r T}{2 \pi^2} \cos \frac{\pi t}{T}, \quad (3.31) \]
\[ \langle \langle B e^{-rK'} \gamma e^{-sK'} e^{-tK'} e^{-uK'} \rangle \rangle = \frac{(r + s)T}{2 \pi^2} \cos \frac{\pi (s + t)}{T}, \quad (3.32) \]
\[ \langle \langle B e^{-rK'} \gamma e^{-sK'} \gamma e^{-tK'} e^{-uK'} \rangle \rangle = -\frac{(r + s + t)T}{2 \pi^2} \cos \frac{\pi s}{T}, \quad (3.33) \]
\( (T \equiv r + s + t + u) \), which are computed as \[3.21\]. We should note that these values are the same as undeformed ones, namely \( K' \rightarrow K \) thanks to the identity \[C.1\] in the deformed matter sector. Using these formulas, it turns out that
\[ A_1 = \frac{3}{\pi^2} - \frac{24}{\pi}, \quad A_2 = \frac{9}{2 \pi^2} - \frac{24}{\pi}, \quad S'[\Phi_H] = \frac{1}{4 \pi^2}. \quad (3.34) \]
Therefore, the vacuum energy of \( \Phi_H \) is a half of that of \( \Phi_T \), namely, \( S'[\Phi_H] = \frac{1}{2} S'[\Phi_T] \) from \[3.22\] and thus we call \( \Phi_H \) a “half-brane” solution as in \[17\].

### 4 Gauge invariant overlaps for the solutions

In this section, we evaluate gauge invariant overlaps for solutions \( \Phi_T \) \[2.24\] and \( \Phi_H \) \[2.25\] in the theory with the action \[2.21\]. Here, we define a gauge invariant \( \langle \Phi \rangle \rangle \) for a string...
field $\Phi$ after [17] as

$$\langle \langle \Phi \rangle \rangle_V = \frac{1}{2} \text{Tr}(\sigma_3 \langle I | \mathcal{V}(i) | \Phi \rangle),$$

where $\mathcal{V}(i)$ denotes a midpoint insertion of a closed string vertex operator in the NS-NS sector of the form $c \bar{c} \delta(\gamma) \delta(\bar{\gamma}) V_m(z, \bar{z})$ and $V_m(z, \bar{z})$ is a superconformal matter primary field with dimension $(1/2, 1/2)$. The above (4.1) is given by replacing $Y_{-2}$ with $\mathcal{V}(i)$ in (A.2). Because of the Chan-Paton factor $\sigma_3$, only the GSO(+) sector of a string field $\Phi$ gives nontrivial contribution.

In general, for a solution $\Phi_{f'}$ (2.23) to the equation of motion (2.22), the value of the above gauge invariant overlap is calculated as

$$\langle \langle \Phi_{f'} \rangle \rangle_V = \langle \langle \mathcal{C} K'_{f'} \rangle \rangle_V,$$

where we have used

$$\langle \langle \Phi \Psi \rangle \rangle_V = (-1)^{\epsilon(\Phi)+F(\Phi)+\epsilon(\Psi)+F(\Psi)} \langle \langle \Psi \Phi \rangle \rangle_V, \quad (4.3)$$

$$\langle \langle \mathcal{Q}'(\cdots) \rangle \rangle_V = 0. \quad (4.4)$$

The first equation implies the cyclic symmetry and the second equation comes from the gauge invariance.

### 4.1 Tachyon vacuum solution

In the case of $\Phi_T$ (2.24), that is, the case of $f' = \frac{1}{1+K'}$, the gauge invariant overlap (4.2) is simplified as

$$\langle \langle \Phi_{f' K'} \rangle \rangle_V = \langle \langle \mathcal{C} \frac{1}{1 + K'} \rangle \rangle_V = \int_0^\infty dt e^{-t} \langle \langle ce^{-t K'} \rangle \rangle_V. \quad (4.5)$$

Using relations

$$\frac{1}{2}(\mathcal{L}_0' - \mathcal{L}_0^t) c = -c, \quad \frac{1}{2}(\mathcal{L}_0' - \mathcal{L}_0^t) K' = K', \quad (4.6)$$

where $\mathcal{L}_0' = \{Q', B_0\}$ and $\mathcal{L}_0' - \mathcal{L}_0^t$ is a derivation with respect to the star product of string fields, we have

$$ce^{-t K'} = t^{1+\frac{1}{2}(\mathcal{L}_0' - \mathcal{L}_0^t)} (ce^{-K'}). \quad (4.7)$$

In addition, we have the equations

$$\langle I | \mathcal{V}(i) (B_0 - B_0^t) | 0 \rangle = 0, \quad \langle I | \mathcal{V}(i) (\mathcal{L}_0' - \mathcal{L}_0^t) | 0 \rangle = 0, \quad (4.8)$$
where the second equation is derived from the first equation (see \[20\] for example) and \([4.4]\). Therefore, \([4.7]\) and \([4.8]\) imply

\[
\langle \langle ce^{-tK'} \rangle \rangle_{\mathcal{V}} = t \langle \langle ce^{-K'} \rangle \rangle_{\mathcal{V}},
\]

(4.9)

and the integration with respect to \(t\) in \((4.5)\) can be performed explicitly:

\[
\langle \langle \Phi_{T} \rangle \rangle_{\mathcal{V}} = \int_{0}^{\infty} dt \ e^{-t} \langle \langle ce^{-K'} \rangle \rangle_{\mathcal{V}} = \langle \langle ce^{-K'} \rangle \rangle_{\mathcal{V}}.
\]

(4.10)

For computation of \(e^{-K'}\), we can apply \((3.13)\) as in the case of the evaluation of the action:

\[
\langle \langle \Phi_{T} \rangle \rangle_{\mathcal{V}} = \frac{2}{\pi} \langle (0|\hat{V}(i\infty)\hat{U}_{t} \vec{c}(\frac{\pi}{4})e^{-\frac{\pi}{2}C_{T}} \exp(\frac{\pi}{4} \int_{-1}^{1} du \int_{-\infty}^{\infty} dt' f_{a}(t')\tilde{J}^{a}(it' + \frac{\pi}{4}u))|0)\rangle
\]

\[
= \frac{e^{-\frac{\pi}{2}C_{T}}}{\pi} \left( \hat{V}(i\infty)\vec{c}(\frac{\pi}{2}) \exp \left( \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} du \int_{-\infty}^{\infty} dt' f_{a}(t')\tilde{J}^{a}(2it' + u) \right) \right). \tag{4.11}
\]

Furthermore, using the ghost structure of the closed string vertex

\[
\mathcal{V}(i) = c(i)c(-i)\delta(\gamma(i))\delta(\gamma(-i))\mathcal{V}_{m}(i, -i), \tag{4.12}
\]

the above is computed as

\[
\langle \langle \Phi_{T} \rangle \rangle_{\mathcal{V}} = \frac{e^{-\frac{\pi}{2}C_{T}}}{\pi} \lim_{M \to \infty} \left[ \langle \delta(\tilde{\gamma}(iM))\delta(\tilde{\gamma}(-iM)) \rangle_{\beta} \langle c(iM)c(-iM)\vec{c}(\frac{\pi}{2}) \rangle_{bc} \right]
\]

\[
\times \left\langle \exp \left( \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} du \int_{-\infty}^{\infty} dt' f_{a}(t')\tilde{J}^{a}(2it' + u) \right) \mathcal{V}_{m}(iM, -iM) \right\rangle_{\text{mat}}
\]

\[
= \frac{e^{-\frac{\pi}{2}C_{T}}}{\pi} \lim_{M \to \infty} \frac{e^{2M}}{4} \left\langle \exp \left( \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} du \int_{-\infty}^{\infty} dt' f_{a}(t')\tilde{J}^{a}(2it' + u) \right) \mathcal{V}_{m}(iM, -iM) \right\rangle_{\text{mat}}. \tag{4.13}
\]

Generally, this value depends on \(\tilde{J}^{a}\) and \(\mathcal{V}_{m}\) in the matter sector.

In order to perform further explicit calculations, let us consider the case of \((4.5)\) for a closed string vertex in the gauge invariant overlap and \(f_{a}\tilde{J}^{a} = f\tilde{J} = f\sqrt{2a} \partial \bar{X}^{a}\), i.e. \(J(z, \theta) = \psi^{a}(z) + \theta \sqrt{\frac{1}{2a}} \partial \bar{X}^{a}(z)\) as a supercurrent, for the identity-based marginal solution \(\Psi_{J}(2.1)\). We expand the exponential in \((4.13)\) as

\[
\langle \langle \Phi_{T} \rangle \rangle_{\mathcal{V}} = \frac{e^{-\frac{\pi}{2}C_{T}}}{\pi} \sum_{n=0}^{\infty} I_{n}^{(k_{o})},
\]

(4.14)

where \(I_{n}^{(k_{o})}\) is the \(n\)-th order term of \(\tilde{J}^{a}\):

\[
I_{n}^{(k_{o})} = \frac{1}{n!} \lim_{M \to \infty} \frac{e^{2M}}{4} \left\langle \left( \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} du \int_{-\infty}^{\infty} dt' f(t')\tilde{J}^{a}(2it' + u) \right)^{n} \mathcal{V}_{m}(iM, -iM) \right\rangle. \tag{4.15}
\]
In the case of the lowest order term, $I_0^{(k_0)}$ corresponds to the undeformed background and it is computed as
\[
I_0^{(k_0)} = \lim_{M \to \infty} \frac{e^{2M}}{4} \left\langle e^{\frac{i}{2}k_0 \tilde{X}^9(iM)} e^{-\frac{i}{2}k_0 \tilde{X}^9(-iM)} \right\rangle = \lim_{M \to \infty} \frac{e^{2M}}{4} \frac{1}{\sin(2iM)} = -\frac{i}{2}. \tag{4.16}
\]

To evaluate the other terms, following relation among CFT correlators, which is similar to (C.4), is useful:
\[
\left\langle \tilde{J}(\bar{z}) \tilde{J}(\bar{z}_1) \cdots \tilde{J}(\bar{z}_n) \tilde{V}_m(\bar{w}, \bar{\tilde{w}}) \right\rangle = \sum_{i=1}^n \frac{1}{\sin^2(\bar{z} - \bar{z}_i)} \left\langle \tilde{J}(\bar{z}) \cdots \tilde{J}(\bar{z}_{i-1}) \tilde{J}(\bar{z}_{i+1}) \cdots \tilde{J}(\bar{z}_n) \tilde{V}_m(\bar{w}, \bar{\tilde{w}}) \right\rangle + \frac{k_0 \sqrt{2\alpha'}}{2\cos \bar{z}} \left( \frac{\cos \bar{w}}{\sin(\bar{z} - \bar{w})} - \frac{\cos \bar{\tilde{w}}}{\sin(\bar{z} - \bar{\tilde{w}})} \right) \left\langle \tilde{J}(\bar{z}) \cdots \tilde{J}(\bar{z}_n) \tilde{V}_m(\bar{w}, \bar{\tilde{w}}) \right\rangle. \tag{4.17}
\]

Then, we have
\[
I_1^{(k_0)} = \lim_{M \to \infty} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} du \int_{-\infty}^{\infty} dt f(t) \frac{e^{2M}}{4} \left\langle i \sqrt{2\alpha'} \tilde{\partial} \tilde{X}^9(2it + u) e^{\frac{i}{2}k_0 \tilde{X}^9(iM)} e^{-\frac{i}{2}k_0 \tilde{X}^9(-iM)} \right\rangle = \lim_{M \to \infty} \int_{-\infty}^{\infty} dt f(t) I_0^{(k_0)} \equiv \tilde{I}_1^{(k_0)} I_1^{(k_0)}, \tag{4.18}
\]
for the first order and
\[
I_2^{(k_0)} = \frac{1}{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} du_1 \int_{-\infty}^{\infty} dt_1 f(t_1) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} du_2 \int_{-\infty}^{\infty} dt_2 f(t_2) \left[ \frac{1}{\sin^2(u_1 - u_2 + 2i(t_1 - t_2))} I_0^{(k_0)} \right] + \lim_{M \to \infty} \left\{ \frac{k_0 \sqrt{2\alpha'} \cos i M}{2 \cos(2it_1 + u_1)} \left( \frac{1}{\sin(2it_1 + u_1 - i M)} - \frac{1}{\sin(2it_1 + u_1 + i M)} \right) \right. \\
\left. \times \frac{e^{2M}}{4} \left\langle i \sqrt{2\alpha'} \tilde{\partial} \tilde{X}^9(2it_2 + u_2) e^{\frac{i}{2}k_0 \tilde{X}^9(iM)} e^{-\frac{i}{2}k_0 \tilde{X}^9(-iM)} \right\rangle \right\} = I_2 f_0^{(k_0)} + \frac{1}{2} \tilde{I}_1^{(k_0)} I_1^{(k_0)} = \left( I_2 + \frac{1}{2}(\tilde{I}_1^{(k_0)})^2 \right) I_0^{(k_0)} \tag{4.19}
\]
for the second order, where we have used (C.6) and
\[
I_2 = \frac{\pi^2}{2} \int_{-\infty}^{\infty} dt (f(t))^2 = \frac{\pi}{2} C. \tag{4.20}
\]
(3.8) with $\Omega = 2$, where $X^9(y)X^9(z) \sim -2\alpha'\log(y - z)$ in our convention, is used for the last equality.) In the same way, higher order terms can be computed and the results for even order terms and odd order terms are given by

\begin{align}
I_{2n}^{(k_9)} &= I_0^{(k_9)} \sum_{l=0}^{n} \frac{1}{(n - l)! (2l)!} I_2^{n - l}(\hat{i}_1^{(k_9)})^{2l}, \quad (4.21) \\
I_{2n-1}^{(k_9)} &= I_0^{(k_9)} \sum_{l=0}^{n-1} \frac{1}{(n - 1 - l)! (2l + 1)!} I_2^{n - 1 - l}(\hat{i}_1^{(k_9)})^{2l+1}, \quad (4.22)
\end{align}

respectively. Thus the gauge invariant overlap is calculated as

\begin{align}
\langle \langle \Phi_T \rangle \rangle V &= e^{-\frac{\pi c}{\pi}} \sum_{n=0}^{\infty} I_n^{(k_9)} = e^{-\frac{\pi c}{\pi}} e^{I_2 + I_1^{(k_9)}} I_0^{(k_9)} = \frac{1}{\pi} e^{\hat{i}_1^{(k_9)}} I_0^{(k_9)} = \frac{1}{2\pi i} e^{i\hat{i}_1^{(k_9)}}. \quad (4.23)
\end{align}

The exponent of the above can be rewritten as

\begin{align}
\hat{i}_1^{(k_9)} &= i\pi k_9 \sqrt{2\alpha'} \int_{-\infty}^{\infty} dt f(t) = i\pi k_9 \sqrt{\alpha'} \int_{C_L} \frac{dz}{2\pi i} F(z). \quad (4.24)
\end{align}

Therefore, the phase factor $e^{i\hat{i}_1^{(k_9)}}$, which is induced by a current $J = \frac{i}{\sqrt{2\alpha'}} \partial X^9$, is exactly the same as that in (D.9) obtained by a different method. It corresponds to the phase factor appeared in [2, 7] in the case of bosonic SFT.

### 4.2 Half-brane solution

In the case of $\Phi_H$ (2.25), or $f'$ given in (3.23), the gauge invariant overlap (4.2) is simplified as

\begin{align}
\langle \langle \Phi_H \rangle \rangle V &= -\langle \langle cG'BcG'e^{-tK'} \rangle \rangle V = -\int_{0}^{\infty} dt e^{-t} \langle \langle cG'BcG'e^{-tK'} \rangle \rangle V, \quad (4.25)
\end{align}

thanks to the structure of Chan-Paton factor. Using (4.6), (4.8) and

\begin{align}
\frac{1}{2}(L'_0 - L'_0^\dagger)B = B, & \quad \frac{1}{2}(L'_0 - L'_0^\dagger)G' = \frac{1}{2}G', \quad (4.26) \\
t^\dagger(L'_0 - L'_0^\dagger)(cG'BcG'e^{-K'}) = cG'BcG'e^{-tK'}, & \quad (4.27) \\
-cK'e^{-K'} = \frac{1}{2}(L'_0 - L'_0^\dagger)(ce^{-K'}) + ce^{-K'}, \quad (4.28)
\end{align}

which are a deformed version of relations in [17], the integration with respect to $t$ in (4.25) can be explicitly performed and (4.25) is simplified as

\begin{align}
\langle \langle \Phi_H \rangle \rangle V &= -\langle \langle cG'Be^{-K'} \rangle \rangle V = -\langle \langle cB(cK' + 2i\gamma G')e^{-K'} \rangle \rangle V \\
&= -\langle \langle (cK' + 2icB\gamma G')e^{-K'} \rangle \rangle V = \langle \langle (c + 2ic\gamma BG')e^{-K'} \rangle \rangle V. \quad (4.29)
\end{align}
Furthermore, noting

\[ \frac{1}{2}(B_0 - B'_0)K' = B, \quad \frac{1}{2}(B_0 - B'_0)B = 0, \quad \frac{1}{2}(B_0 - B'_0)c = 0, \quad \frac{1}{2}(B_0 - B'_0)\gamma = 0, \quad (4.30) \]

for a derivation \( B_0 - B'_0 \) with respect to the star product, the second term of the last expression in (4.29) is calculated as

\[
2i\langle\langle c\gamma' Be^{-K'} \rangle \rangle_V = 2i\langle\langle G' c\gamma Be^{-K'} \rangle \rangle_V = i\delta'(c\gamma' Be^{-K'})_V \\
= \langle\langle -2\gamma^2 BG'e^{-K'} + \frac{1}{2}\epsilon\partial\partial B e^{-K'} \rangle \rangle_V = \langle\langle (B_0 - B'_0) \left( \gamma^2 e^{-K'} - \frac{1}{4} cK' e^{-K'} \right) - \frac{1}{2} c e^{-K'} \rangle \rangle_V \\
= -\frac{1}{2} \langle\langle ce^{-K'} \rangle \rangle_V. \quad (4.31)
\]

Therefore, (4.29) can be rewritten as

\[
\langle\langle \Phi_H \rangle \rangle_V = \frac{1}{2} \langle\langle ce^{-K'} \rangle \rangle_V \\
= \frac{e^{-\frac{\pi}{2}c}}{2\pi} \langle\langle \tilde{V}(i\infty)\tilde{c}(\frac{\pi}{2}) \exp \left( \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} du \int_{-\infty}^{\infty} dt' f_a(t') \tilde{J}^a(2it' + u) \right) \rangle \rangle_V. \quad (4.32)
\]

This value is just a half of (4.11), namely, \( \langle\langle \Phi_H \rangle \rangle_V = \frac{1}{2} \langle\langle \Phi_T \rangle \rangle_V \). We notice that this relation itself does not depend on details of \( \tilde{J}^a \) and \( \tilde{V}_m \) in the matter sector.

5 Concluding remarks

We have considered a version of "\( GKBc\gamma \) algebra" in the theory around an identity-based marginal solution \( \Psi_J \), which is made of supercurrents associated with a Lie algebra in the matter sector, in modified cubic SSFT. Corresponding to a deformed BRST operator \( Q' \), string fields \( G \) and \( K \) are deformed to \( G' \) and \( K' \), respectively. Using these ingredients, we constructed a tachyon vacuum solution \( \Phi_T \) and a half-brane solution \( \Phi_H \) and evaluated the vacuum energy and gauge invariant overlap for them. The values of the vacuum energy for these solutions are exactly the same as those of the tachyon vacuum solution \( \Psi_T \) and the half-brane solution \( \Psi_H \) in the original theory, respectively. Namely, \( S'[\Phi_T] = S[\Psi_T] = 2S[\Psi_H] = 2S'[\Phi_H] \) holds, where we note the relation: \( S'[\Phi] = S[\Phi + \Psi_J\sigma_3] - S[\Psi_J\sigma_3] \) (2.21) in general. By introducing a parameter \( s \) in the weighting function: \( F_a(z) \to sF_a(z) \) and taking a differentiation of the action and integration from \( s = 0 \) to \( s = 1 \), we can show \( S[\Phi_T + \Psi_J\sigma_3] = S[\Psi_T] \) and \( S[\Phi_H + \Psi_J\sigma_3] = S[\Psi_H] \) in the same way as in [2]. Therefore, from consistency, the vacuum energy of \( \Psi_J \) vanishes: \( S[\Psi_J\sigma_3] = 0 \) although direct computation of \( S[\Psi_J\sigma_3] \) is difficult due to singular property of the identity state.

The values of the gauge invariant overlap for both of \( \Phi_T \) and \( \Phi_H \) change according to the marginal current in \( \Psi_J \). However, the relation between them: \( \langle\langle \Phi_H \rangle \rangle_V = \frac{1}{2} \langle\langle \Phi_T \rangle \rangle_V \) holds as in the case of the original theory. If we take a closed tachyon vertex for a Dirichlet
direction as the matter part of $V$ and $\partial X^9$ as a current $J$, a phase factor appears in the gauge invariant overlap and it is consistent with the effect caused by a field redefinition induced by $\Psi_J$.

These results in the above support the expectation that the identity-based solution $\Psi_J$ corresponds to a marginal deformation and they are an extension of results in [2] for bosonic SFT to modified cubic SSFT.

If we take a zero momentum graviton vertex $\psi^\mu \bar{\psi}^{\mu}$ as a matter part of closed string vertex and $J \sim \partial X^9$ in the gauge invariant overlap, the values for $\Phi_T$ and $\Phi_H$ are exactly the same as $\Psi_T$ and $\Psi_H$ in the undeformed background, respectively (i.e. $\langle \langle \Phi_T \rangle \rangle_V = 2\langle \langle \Psi_T \rangle \rangle_V = 2\langle \langle \Phi_H \rangle \rangle_V$), which can be proved in the same way as the evaluation of the action thanks to (C.1). These are reminiscent of the relation between the energy and the gauge invariant overlap in bosonic SFT [27].

In this paper, we considered the NS sector without GSO projection and as a closed string vertex for the gauge invariant overlap, we have only considered the NS-NS sector. It may be interesting to investigate gauge invariant overlaps for closed string vertices in the R-R sector.

Here, we have considered a theory only around an identity-based marginal solution in SSFT and we found that $G'K'Bc\gamma$ algebra has the same algebraic structure with undeformed $GKBc\gamma$ algebra. If we consider a theory around another type of identity-based universal solutions found in [28], we expect that the algebraic structure might be changed when a homotopy operator exists. Using such an algebra, vacuum energy and/or gauge invariant overlap might be evaluated directly.

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A brief review of $KBc$ algebra and its extension

Here, we summarize some results on a supersymmetric extension of $KBc$ algebra developed in [17] and we list our convention and notation in this paper. We consider string fields in the NS sector without GSO projection and therefore we introduce Chan-Paton factors, which are represented by $2 \times 2$ Pauli matrices: $\sigma_i$ ($i = 1, 2, 3$) (and the identity matrix implicitly). There are four sectors corresponding to Grassmann parity ($\epsilon$) and worldsheet spinor ($F$) and we assign Chan-Paton factors as in Table [1]. The NS action
Grassmann parity ($\epsilon$) | worldsheet spinor ($F$) | Chan-Paton factor
--- | --- | ---
even | even | 1
odd | even | $\sigma_3$
even | odd | $\sigma_2$
odd | odd | $\sigma_1$

Table 1: Assignment of the Chan-Paton factor

$S[\Psi]$ is written as

$$S[\Psi] = \frac{1}{2} \langle \langle \Psi \hat{Q} \Psi \rangle \rangle + \frac{1}{3} \langle \langle \Psi^3 \rangle \rangle, \quad (A.1)$$

where we omit a symbol for the star product among string fields and $\langle \langle \cdot \rangle \rangle$ includes a trace for $2 \times 2$ Chan-Paton matrices and a picture changing operator with picture number $(-2)$ denoted by $Y_{-2}$:

$$\langle \langle A \rangle \rangle \equiv \frac{1}{2} \text{Tr} (\sigma_3 \langle I | Y_{-2} A \rangle). \quad (A.2)$$

Here, $\langle I \rangle$ is the identity state and we denote $\hat{Q} \equiv Q_B \sigma_3$ in (A.1). The NS string field $\Psi$ in the action (A.1) can be expanded as

$$\Psi = \Psi_+ \sigma_3 + \Psi_- \sigma_2, \quad (A.3)$$

where $\Psi_+$ ($\Psi_-$) is in the GSO$(+)$ (GSO$(-)$) sector. In general, we have

$$\langle \langle \Phi \Psi \rangle \rangle = (-1)^{(\epsilon(\Phi)+F(\Phi))} (\sigma(\Psi)+F(\Psi)) \langle \langle \Psi \Phi \rangle \rangle, \quad (A.5)$$

$$\langle \langle \hat{Q}(\cdots) \rangle \rangle = 0. \quad (A.6)$$

In this paper, we define $Y_{-2}$ in (A.2) using two inverse picture changing operators:

$$Y_{-2} = Y(i) Y(-i), \quad Y(z) \equiv c(z) \delta'(\gamma(z)). \quad (A.7)$$

In the above convention, we define string fields $K, B, c$ as

$$K = \frac{\pi}{2} K^L_1 I, \quad B = \frac{\pi}{2} B^L_1 I \sigma_3, \quad c = \frac{1}{\pi} c(1) I \sigma_3 = \frac{2}{\pi} \hat{U}_1 \check{c}(0) |0\rangle \sigma_3. \quad (A.8)$$

In the above, $K^L_1, B^L_1, \hat{U}_1$ are defined as in [25]:

$$K^L_1 = \{Q_B, B^L_1 \}, \quad B^L_1 = \frac{1}{\pi} (B_0 + B^L_0), \quad \hat{U}_r = U^d_r U_r, \quad U_r = \left( \frac{2 \pi}{r} \right)^{\mathcal{L}_0} \quad (A.9)$$

$$B_1 = b_{-1} + b_1, \quad B_0 = b_0 + \sum_{k=1}^{\infty} \frac{2(-1)^{k+1}}{4k^2 - 1} b_{2k}, \quad \mathcal{L}_0 = \{Q_B, B_0 \}. \quad (A.10)$$
and $\tilde{c}(\tilde{z}) = (\cos \tilde{z})^2 c(\tan \tilde{z})$ is $c$-ghost in the sliver frame.

Similarly, we define $G, \gamma$ in the case of superstring as \cite{17}:

$$G = G_L I \sigma_1, \quad \gamma = \frac{1}{\sqrt{\pi}} \gamma(1) I \sigma_2 = \sqrt{\frac{2}{\pi}} \tilde{U}_1 \tilde{\gamma}(0) |0\rangle \sigma_2, \quad \text{(A.11)}$$

where $\tilde{\gamma}(\tilde{z}) = (\cos \tilde{z}) \gamma(\tan \tilde{z})$ is $\gamma$-ghost in the sliver frame and

$$G_L = \frac{1}{2} (G + G^*), \quad \text{(A.12)}$$

$$G = \oint \frac{dz}{2\pi i} \sqrt{\frac{\pi}{2}} \sqrt{1 + z^2} G(z) = \sqrt{\frac{\pi}{2}} \sum_{n=0}^{\infty} \left( \frac{1}{2} \right)_n G_{2n-\frac{1}{2}}, \quad \text{(A.13)}$$

$$G^* = \oint \frac{dz}{2\pi i} \sqrt{\frac{\pi}{2}} z \sqrt{1 + z^{-2}} G(z) = \sqrt{\frac{\pi}{2}} \sum_{n=0}^{\infty} \left( \frac{1}{2} \right)_n G_{\frac{1}{2} - 2n}. \quad \text{(A.14)}$$

Here, \( \binom{r}{s} \equiv \frac{\Gamma(r+1)}{\Gamma(s+1)\Gamma(r-s+1)} \) is a binomial coefficient and $G_r = [Q_B, \beta_r]$ is a superconformal generator. Actually, $G$ can be rewritten as

$$G = \sqrt{\frac{\pi}{2}} [Q_B, \tilde{\beta}_{-\frac{1}{2}}], \quad \text{(A.15)}$$

where $\tilde{\beta}_{-\frac{1}{2}}$ is a mode of $\tilde{\beta}(\tilde{z}) = (\cos \tilde{z})^{-\frac{3}{2}} \beta(\tan \tilde{z})$ in the sliver frame:

$$\tilde{\beta}_{-\frac{1}{2}} = \oint \frac{dz}{2\pi i} \sqrt{1 + z^2} \beta(z) = \sum_{n=0}^{\infty} \left( \frac{1}{2} \right)_n \beta_{2n-\frac{1}{2}}. \quad \text{(A.16)}$$

From explicit computation using mode expansions, we find that $G, B_0$ and $\tilde{\beta}_{-\frac{1}{2}}$ satisfy

$$\{G, B_0\} = \frac{1}{2} \sqrt{\frac{\pi}{2}} \tilde{\beta}_{-\frac{1}{2}}, \quad \{G, B_0^\dagger\} = -\frac{1}{2} \sqrt{\frac{\pi}{2}} \tilde{\beta}_{-\frac{1}{2}}. \quad \text{(A.17)}$$

Then, noting $[G, \mathcal{L}_0^\dagger] = \frac{1}{2} G, Q_B |I\rangle = 0$ and $B_0 |I\rangle = B_0^\dagger |I\rangle$, we have

$$G |I\rangle = 2^{c_{0}^{1} + 1} G |0\rangle = 0, \quad \text{(A.18)}$$

$$\tilde{\beta}_{-\frac{1}{2}} |I\rangle = \sqrt{\frac{2}{\pi}} \{G, B_0 - B_0^\dagger\} |I\rangle = 0. \quad \text{(A.19)}$$

Among the above string fields, i.e., $G, K, B, c$ and $\gamma$, we have following relations:

$$Bc + cB = 1, \quad G^2 = K, \quad B^2 = 0, \quad c^2 = 0, \quad \text{(A.20)}$$

$$\hat{Q} B = K, \quad \hat{Q} K = 0, \quad \hat{Q} G = 0, \quad \hat{Q} c = c K c - \gamma^2, \quad \text{(A.21)}$$

$$BG = GB, \quad BK = KB, \quad GK = KG, \quad B\gamma + \gamma B = 0, \quad c\gamma + \gamma c = 0. \quad \text{(A.22)}$$
Furthermore, worldsheet supersymmetry transformations of string fields are given by

\[
\delta c = 2i\gamma, \quad \delta \gamma = -i\partial c, \quad \delta G = 2K, \quad \delta K = 0, \quad \delta B = 0,
\]

(A.23)

where \( \partial \) and \( \delta \) are defined by

\[
\partial \Phi \equiv K\Phi - \Phi K = \pi K_1 \Phi, \quad \delta \Phi \equiv G\Phi - (-)^F(\Phi)G = (G\sigma_1)\Phi,
\]

(A.24)

which satisfy the relation \( \delta^2 = \partial \).

\[B\] An expansion formula

We consider a generalization of a well-known formula,

\[
\delta(e^X) = \int_0^1 d\alpha e^{(1-\alpha)X}(\delta X)e^{\alpha X}.
\]

(B.1)

Namely, by expanding as

\[
e^{X+\delta X} = e^X + \sum_{N=1}^{\infty} (e^{X+\delta X})_{O(\delta X)^N}
\]

(B.2)

for \([X, \delta X] \neq 0\) in general, we will find a similar expression as (B.1) for the order \((\delta X)^N\) term: \((e^{X+\delta X})_{O(\delta X)^N}\). By a straightforward expansion, we have

\[
(e^{X+\delta X})_{O(\delta X)^N} = \sum_{n=0}^{\infty} \frac{1}{n!}(X + \delta X)^n_{O(\delta X)^N}
\]

\[
= \sum_{n=N}^{\infty} \frac{1}{n!} \sum_{k_1=0}^{n-N} \sum_{k_2=0}^{n-N-k_1} \cdots \sum_{k_N=0}^{n-N-k_1-k_2-\cdots-k_{N-1}} X^{k_1}(\delta X)X^{k_2}(\delta X)\cdots X^{k_N}(\delta X)X^{n-N-k_1-k_2-\cdots-k_N}
\]

\[
= \sum_{k_0=0}^{\infty} \sum_{k_1=0}^{\infty} \cdots \sum_{k_N=0}^{\infty} \frac{1}{(k_0 + k_1 + \cdots + k_N + N)!} X^{k_0}(\delta X)X^{k_1}(\delta X)\cdots X^{k_{N-1}}(\delta X)X^{k_N}.
\]

(B.3)

Noting \(B(p, q) = \int_0^1 dt (1 - t)^{p-1}t^{q-1} = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}\) and comparing coefficients, we find that the above can be rewritten using integrals:

\[
(e^{X+\delta X})_{O(\delta X)^N} = \int_0^1 du_1 \int_0^{1-u_1} du_2 \cdots \int_0^{1-u_1-u_2-\cdots-u_{N-1}} du_N e^{(1-u_1-u_2-\cdots-u_N)X}(\delta X)e^{u_1X}(\delta X)e^{u_2X} \cdots (\delta X)e^{u_NX}.
\]

(B.4)
C \quad J\text{-independence in the matter sector}

In this section, we demonstrate a formula
\begin{equation}
    e^{-T\frac{\pi}{2} C} \left\langle \exp \left( \int_{-\frac{T}{2}}^{\frac{T}{2}} du \int_{-\infty}^{\infty} dt' f_n(t') \bar{J}^a \left( \frac{2it'}{T} + u \right) \right) \right\rangle_{\text{mat}} = 1 \tag{C.1}
\end{equation}
which is essential to evaluate the vacuum energy for \( \Phi_T \) (2.24) and \( \Phi_H \) (2.25). The above result was also used in \cite{2} in the context of bosonic SFT.

Let us define \( I_n \) as the order of \( J^a \) in the above \( \langle \cdots \rangle \), namely,
\begin{equation}
    I_n \equiv \frac{1}{n!} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} du_1 \int_{-\infty}^{\infty} dt_1 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} du_2 \int_{-\infty}^{\infty} dt_2 \int_{-\infty}^{\infty} dt_n f_n(t_1) \cdots f_n(t_n) \langle \bar{J}^a_1 \left( \frac{2it_1}{T} + u_1 \right) \cdots \bar{J}^a_n \left( \frac{2it_n}{T} + u_n \right) \rangle. \tag{C.2}
\end{equation}
To evaluate \( I_n \), we note the relation among CFT correlators:
\begin{equation}
    \langle J^a(z) J^{a_1}(z_1) \cdots J^{a_n}(z_n) \rangle
    = \sum_{i=1}^{n} \left[ \frac{1}{2} \Omega^{a_i}_{a_a} \langle J^{a_1}(z_1) \cdots J^{a_{i-1}}(z_{i-1}) J^{a_{i+1}}(z_{i+1}) \cdots J^{a_n}(z_n) \rangle + f^{a_i}_{a_a b} \langle J^b(z_i) J^{a_1}(z_1) \cdots J^{a_{i-1}}(z_{i-1}) J^{a_{i+1}}(z_{i+1}) \cdots J^{a_n}(z_n) \rangle \right], \tag{C.3}
\end{equation}
which is the Ward identity derived from the OPE (2.5). In terms of the sliver frame, we have
\begin{equation}
    \langle \bar{J}^a(\tilde{z}) \bar{J}^{a_1}(\tilde{z_1}) \cdots \bar{J}^{a_n}(\tilde{z_n}) \rangle
    = \sum_{i=1}^{n} \left[ \frac{1}{\sin^2(\tilde{z} - \tilde{z}_i)} \frac{1}{2} \Omega^{a_i}_{a_a} \langle \bar{J}^{a_1}(\tilde{z_1}) \cdots \bar{J}^{a_{i-1}}(\tilde{z}_{i-1}) \bar{J}^{a_{i+1}}(\tilde{z}_{i+1}) \cdots \bar{J}^{a_n}(\tilde{z_n}) \rangle + \frac{\cos \tilde{z}_i}{\cos \tilde{z} \sin(\tilde{z} - \tilde{z}_i)} f^{a_i}_{a_a b} \langle \bar{J}^b(\tilde{z}_i) \bar{J}^{a_1}(\tilde{z_1}) \cdots \bar{J}^{a_{i-1}}(\tilde{z}_{i-1}) \bar{J}^{a_{i+1}}(\tilde{z}_{i+1}) \cdots \bar{J}^{a_n}(\tilde{z_n}) \rangle \right]. \tag{C.4}
\end{equation}
Using the above identity and some relations in (2.6), (2.7), we have
\begin{equation}
    I_0 = 1, \quad I_1 = 0, \quad I_2 = \frac{\pi}{2} CT, \quad I_3 = 0, \tag{C.5}
\end{equation}
where \( C \) is defined in (3.8) and we have used \cite{2}:
\begin{equation}
    \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} du_1 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} du_2 \frac{1}{\pi} \frac{1}{\sin^2(u_1 - u_2 + 2i(t_1 - t_2))} = \delta(t_1 - t_2). \tag{C.6}
\end{equation}
\footnote{It can be obtained by calculating the contour integral \( \oint \frac{dz}{2\pi i} \epsilon_a(z) J^a(z) \) inserted in a correlation function of \( J^a(z_i) \)'s. (See \cite{29} for example.)}
In order to evaluate $I_n$ in general, we define

$$I_n^a(z) \equiv \frac{1}{n!} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} du_2 \int_{-\infty}^{\infty} dt_2 f_{a_2}(t_2) \cdots \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} du_n \int_{-\infty}^{\infty} dt_n f_{a_n}(t_n)$$

$$\times \left\langle \tilde{j}^a_1(z) \tilde{j}^{a_2}(\frac{2it_2}{T} + u_2) \cdots \tilde{j}^{a_n}(\frac{2it_n}{T} + u_n) \right\rangle, \quad (n \geq 1) \tag{C.7}$$

so that

$$I_n = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} du \int_{-\infty}^{\infty} dt f_{a_1}(t) I_n^a(\frac{2it}{T} + u). \tag{C.8}$$

Then, we have

$$I_1^a(z) = 0, \quad I_2^a(\frac{2it}{T} + u) = \frac{\pi}{4} T \Omega^{ab} f_b(t). \tag{C.9}$$

Using mathematical induction with relations in (C.4), (2.6) and (2.7), we can show

$$I_n^a(\frac{2it}{T} + u) = \frac{2n}{n} I_n^a(\frac{2it}{T} + u) I_{n-2} = \frac{1}{n} T \Omega^{ab} f_b(t) I_{n-2}. \tag{C.10}$$

This implies $I_n = 2n I_2 I_{n-2}$ and hence

$$I_{2m} = \frac{2m}{(2m)!!} I_2^m I_0 = \frac{1}{m!} I_2^m, \quad I_{2m+1} = 0, \quad (m = 0, 1, 2, 3, \ldots) \tag{C.11}$$

from (C.5). Using the above results, we have

$$\sum_{n=0}^{\infty} I_n = \sum_{m=0}^{\infty} \frac{1}{m!} I_2^m = \exp \left( \frac{\pi}{2} TC \right), \tag{C.12}$$

which is equivalent to (C.1).

### D On the gauge invariant overlap and field redefinition

In this section, we comment on a relation among the gauge invariant overlaps for string fields related by a field redefinition, induced by the solution $\Psi_J$ (2.1). In the following, we consider $u(1)$ supercurrent in the 9-th spatial direction: $J(z, \theta) = \psi^9(z) + \theta \frac{1}{\sqrt{2\alpha'}} \partial X^9(z)$ for the identity-based marginal solution $\Psi_J$ for simplicity. Then, the BRST operator $Q'$ (2.8) at the solution $\Psi_J$ (2.1) can be rewritten as a similarity transform from the conventional BRST operator $Q_B$ as [4]:

$$Q' = e^{\frac{\alpha'}{2\alpha'} X(F)} Q_B e^{-\frac{\alpha'}{2\alpha'} X(F)}, \tag{D.1}$$
where $X(F)$ is given by an integration along a unit circle:

$$
X(F) = \oint \frac{dz}{2\pi i} F(z)X^9(z). \tag{D.2}
$$

Therefore, in the NS action around $\Psi_J \eqref{2.21}$, a field redefinition:

$$
\Phi = e^{i2\sqrt{\alpha'}X(F)}\Phi' = e^{X_L(F)I} \Phi' \star e^{-i2\sqrt{\alpha'}X(F)I}, \tag{D.3}
$$

where

$$
X_L(F) = \int_{C_L} \frac{dz}{2\pi i} F(z)X^9(z), \tag{D.4}
$$

gives the undeformed NS action with respect to a string field $\Phi'$. In this sense, the solution $\Psi_J \eqref{2.1}$ induces a field redefinition \eqref{D.3}.

Let us consider the effect of \eqref{D.3} for a gauge invariant overlap. In order to do that, we take

$$
V_m(i, -i) = e^{\frac{i}{2}k_9X^9(i)}e^{-\frac{i}{2}k_9X^9(-i)} \tag{D.5}
$$
in \eqref{4.1}, where the on-shell condition: $(k_9)^2 = 2/\alpha'$ is satisfied, and it corresponds to a closed tachyon vertex for a Dirichlet direction. Furthermore, noting $F(-1/z) = z^2F(z)$, we expand $F(z)$ as

$$
F(z) = \sum_{m=1}^{\infty} f_m(z^{-m} + (-1)^{m+1}z^m)z^{-1}. \tag{D.6}
$$

Then, we have

$$
\langle I|\mathcal{V}(i)X(F) = -4\alpha'k_9 \sum_{n=1}^{\infty} \frac{(-1)^n f_{2n-1}}{2n-1} \langle I|\mathcal{V}(i) = 2\pi\alpha'k_9 \int_{C_L} \frac{dz}{2\pi i} F(z)\langle I|\mathcal{V}(i), \tag{D.7}
$$

because the $X^9$ sector of $\langle I|\mathcal{V}(i)$ is proportional to \[7, 30\]

$$
\langle 0| \exp \left( -\sum_{n=1}^{\infty} (-1)^n \frac{1}{2n}(\alpha_n^9)^2 - \sum_{n=1}^{\infty} \frac{2i\sqrt{2}\alpha'(-1)^n}{2n-1}k_9\alpha_{2n-1}^9 \right). \tag{D.8}
$$

Using \eqref{D.7}, the gauge invariant overlap for $\Phi \eqref{D.3}$ is evaluated as

$$
\langle \langle \Phi \rangle \rangle_{\mathcal{V}} = \frac{1}{2} \text{Tr}(\sigma_3\langle I|\mathcal{V}(i) e^{i2\sqrt{\alpha'}X(F)}|\Phi'\rangle)
= \exp \left( i\pi\alpha'k_9 \int_{C_L} \frac{dz}{2\pi i} \langle O| F(z) \langle \langle \Phi' \rangle \rangle_{\mathcal{V}}. \tag{D.9}
$$

Namely, the solution $\Psi_J \eqref{2.1}$ induces a phase factor in the above gauge invariant overlap. In the case of bosonic SFT, such an effect was derived in \[7\].
References

[1] Y. Okawa, “Comments on Schnabl’s analytic solution for tachyon condensation in Witten’s open string field theory,” JHEP 0604, 055 (2006) [hep-th/0603159].

[2] S. Inatomi, I. Kishimoto and T. Takahashi, “Tachyon Vacuum of Bosonic Open String Field Theory in Marginally Deformed Backgrounds,” arXiv:1209.4712 [hep-th].

[3] T. Takahashi and S. Tanimoto, “Wilson lines and classical solutions in cubic open string field theory,” Prog. Theor. Phys. 106, 863 (2001) [hep-th/0107046].

[4] T. Takahashi and S. Tanimoto, “Marginal and scalar solutions in cubic open string field theory,” JHEP 0203, 033 (2002) [hep-th/0202133].

[5] I. Kishimoto and T. Takahashi, “Marginal deformations and classical solutions in open superstring field theory,” JHEP 0511, 051 (2005) [hep-th/0506240].

[6] T. Erler and M. Schnabl, “A Simple Analytic Solution for Tachyon Condensation,” JHEP 0910, 066 (2009) [arXiv:0906.0979 [hep-th]].

[7] F. Katsumata, T. Takahashi and S. Zeze, “Marginal deformations and closed string couplings in open string field theory,” JHEP 0411, 050 (2004) [hep-th/0409249].

[8] C. R. Preitschopf, C. B. Thorn and S. A. Yost, “SUPERSTRING FIELD THEORY,” Nucl. Phys. B 337, 363 (1990).

[9] I. Y. Aref’eva, P. B. Medvedev and A. P. Zubarev, “NEW REPRESENTATION FOR STRING FIELD SOLVES THE CONSISTENCY PROBLEM FOR OPEN SUPERSTRING FIELD THEORY,” Nucl. Phys. B 341, 464 (1990).

[10] I. Y. Aref’eva, P. B. Medvedev and A. P. Zubarev, “BACKGROUND FORMALISM FOR SUPERSTRING FIELD THEORY,” Phys. Lett. B 240, 356 (1990).

[11] T. Erler, “Tachyon Vacuum in Cubic Superstring Field Theory,” JHEP 0801, 013 (2008) [arXiv:0707.4591 [hep-th]].

[12] I. Y. Aref’eva, A. S. Koshelev, D. M. Belov and P. B. Medvedev, “Tachyon condensation in cubic superstring field theory,” Nucl. Phys. B 638 (2002) 3 [hep-th/0011117].

[13] I. Y. Aref’eva, R. V. Gorbachev, D. A. Grigoryev, P. N. Khromov, M. V. Maltsev and P. B. Medvedev, “Pure Gauge Configurations and Tachyon Solutions to String Field Theories Equations of Motion,” JHEP 0905 (2009) 050 [arXiv:0901.4533 [hep-th]].

[14] R. V. Gorbachev, “New solution of the superstring equation of motion,” Theor. Math. Phys. 162, 90 (2010) [Teor. Mat. Fiz. 162, 106 (2010)].

[15] E. A. Arroyo, “Generating Erler-Schnabl-type Solution for Tachyon Vacuum in Cubic Superstring Field Theory,” J. Phys. A A 43, 445403 (2010) [arXiv:1004.3030 [hep-th]].
[16] E. Aldo Arroyo, “Multibrane solutions in cubic superstring field theory,” JHEP 1206, 157 (2012) [arXiv:1204.0213 [hep-th]].

[17] T. Erler, “Exotic Universal Solutions in Cubic Superstring Field Theory,” JHEP 1104, 107 (2011) [arXiv:1009.1865 [hep-th]].

[18] T. Erler, “Marginal Solutions for the Superstring,” JHEP 0707, 050 (2007) [arXiv:0704.0930 [hep-th]].

[19] Y. Okawa, “Analytic solutions for marginal deformations in open superstring field theory,” JHEP 0709, 084 (2007) [arXiv:0704.0936 [hep-th]].

[20] Y. Okawa, “Real analytic solutions for marginal deformations in open superstring field theory,” JHEP 0709, 082 (2007) [arXiv:0704.3612 [hep-th]].

[21] E. Fuchs and M. Kroyter, “Marginal deformation for the photon in superstring field theory,” JHEP 0711, 005 (2007) [arXiv:0706.0717 [hep-th]].

[22] M. Kiermaier and Y. Okawa, “General marginal deformations in open superstring field theory,” JHEP 0911, 042 (2009) [arXiv:0708.3394 [hep-th]].

[23] N. Mohammedi, “On bosonic and supersymmetric current algebras for nonsemisimple groups,” Phys. Lett. B 325 (1994) 371 [hep-th/9312182].

[24] M. Kohriki, T. Kugo and H. Kunitomo, “Gauge Fixing of Modified Cubic Open Superstring Field Theory,” Prog. Theor. Phys. 127, 243 (2012) [arXiv:1111.4912 [hep-th]].

[25] M. Schnabl, “Analytic solution for tachyon condensation in open string field theory,” Adv. Theor. Math. Phys. 10, 433 (2006) [hep-th/0511286].

[26] I. Kishimoto, “Comments on gauge invariant overlaps for marginal solutions in open string field theory,” Prog. Theor. Phys. 120, 875 (2008) [arXiv:0808.0355 [hep-th]].

[27] T. Baba and N. Ishibashi, “Energy from the gauge invariant observables,” [arXiv:1208.6206 [hep-th]].

[28] S. Inatomi, I. Kishimoto and T. Takahashi, “Homotopy Operators and Identity-Based Solutions in Cubic Superstring Field Theory,” JHEP 1110, 114 (2011) [arXiv:1109.2406 [hep-th]].

[29] P. H. Ginsparg, “Applied Conformal Field Theory,” [hep-th/9108028].

[30] T. Kawano, I. Kishimoto and T. Takahashi, “Gauge Invariant Overlaps for Classical Solutions in Open String Field Theory,” Nucl. Phys. B 803, 135 (2008) [arXiv:0804.1541 [hep-th]].