The pseudo-two-dimensional dynamics in a system of macroscopic rolling spheres

M. A. López-Castaño\textsuperscript{1,3}, J. F. González-Saavedra\textsuperscript{2,3}, A. Rodríguez-Rivas\textsuperscript{4}, E. Abad\textsuperscript{2,3}, S. B. Yuste\textsuperscript{1,3}, and F. Vega Reyes\textsuperscript{1,}\textsuperscript{*}

\textsuperscript{1}Departamento de Física, Universidad de Extremadura, 06071 Badajoz, Spain
\textsuperscript{2}Departamento de Física Aplicada, Centro Universitario de Mérida, Universidad de Extremadura, 06800 Mérida, Spain
\textsuperscript{3}Instituto de Computación Científica Avanzada (ICCAEx), Universidad de Extremadura, 06071 Badajoz, Spain
\textsuperscript{4}Departamento Matemática Aplicada II, Escuela Politécnica Superior, Virgen de África, 7, 41011, Sevilla, Spain

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We study in this work the dynamics of a collection of identical hollow spheres (ping-pong balls) that rest on a horizontal metallic grid. Fluidization is achieved by means of a turbulent air current coming from below. The upflow is adjusted so that the particles do not levitate over the grid, resulting in quasi-two-dimensional dynamics. We show that this kind of set-up facilitates experimental studies of a variety of phases in granular dynamics. We observe gas, liquid, glass and hexagonal crystal phases. Interestingly, the melting of the hexagonal crystal occurs in co-existence with a liquid phase, unlike in previous observations in two-dimensional phase transitions in granular matter, which followed more closely the predictions of equilibrium theory.

I. INTRODUCTION

The dynamics of macroscopic particle systems has attracted the interest of physicists and engineers since long ago \cite{1,2}. Its significance is partly due to the potential analogies with microscopic particle systems, and partly because the manipulation of granular materials finds widespread application in industry \cite{4}. Therefore, the understanding of what has been termed ‘granular dynamics’ is important both from the point of view of theory and applications. More specifically, there has been an increasing interest on two-dimensional granular systems over the last decades \cite{5,6}.

Granular media share important similarities with molecular matter (as already outlined by O. Reynolds in 1885 \cite{7}), but yet there are also significant differences and peculiarities. Convection and turbulence \cite{8,9}, jamming \cite{10}, Brownian motion \cite{11}, crystallization \cite{12,13,14}, and other phenomena well known in molecular matter have also been observed in granular matter, but they are usually more complex and they often exhibit peculiarities. Furthermore, some of the phenomenology reported in previous works is exclusive to granular matter, such as inelastic collapse \cite{15} and clustering instabilities \cite{16}.

In particular, the attention drawn by crystallization and ordering phenomena in 2D granular systems is partly due to the impact of 2D equilibrium theories in the field of condensed matter \cite{17,18}. The seminal work by Kosterlitz, Thouless, Halperin, Nelson, Young \cite{19,20,21} (subsequently extended by others \cite{22}) highlights the role of spatial dimension, as it predicts fundamental differences in the behavior of two-dimensional (2D) systems with respect to that of their three-dimensional counterparts. For instance, theoretical findings and experimental observations \cite{15,17,18} show that the crystal melting transition in 2D equilibrium systems is in general continuous and defect-mediated \cite{20}. The explanation to this 2D transition is usually referred to as the KTHNY theory \cite{19,21}. This emphasizes the interest of studying 2D granular systems. An additional advantage of such systems is that both the experimental measurements and the characterization of many properties of interest are often more straightforward than in 3D systems \cite{22}.

In order to induce granular matter thermalization, some kind of energy input is necessary, since energy is lost in macroscopic particle collisions \cite{23}. Depending on the type of driving, experimental work in 2D systems has relied mostly on air fluidization \cite{11,24,25} or shaking, either tangent \cite{26} or perpendicular to the plane to which the motion is constrained. Up to some exceptions \cite{27,28}, in most works the plane in which the particle motion takes place coincides with the horizontal plane; hence, tangent and perpendicular shaking are equivalent to horizontal and vertical shaking, respectively. Additionally, there are some interesting shaking experiments with no gravity \cite{29} (for which the horizontal direction is of course not defined). However, more recent work makes use of alternative methods of thermalization with the advantage that boundary friction with boundaries effects are not present, such as an AC electric field on charged particles \cite{30} or acoustic levitators \cite{31}.

For the purpose of studying phase transitions, horizontal shaking experiments have the drawback that field gradients are generated from the boundaries, thereby giving rise to inherently inhomogeneous systems \cite{32}, which renders the analysis of order transitions more difficult. In vertical shaking (quasi) 2D experiments, however, homogeneous dynamics can more easily be achieved. A variety of very interesting results have been obtained in vertical shaking experiments with spheres \cite{3,4,11,12,13,14,33,34,35,36}. In particular the existence of a liquid-to-crystal continuous transition mediated by the hexatic phase has been con-

\footnote{\texttt{fvega@unex.es}}
firmed, in agreement with the predictions of the KTHNY theory for equilibrium systems [21, 38].

In air tables, an appropriately adjusted air current flowing from below prevents levitation of the particles (thereby restraining the dynamics to a single plane), and also generates thermal-like motion via the stochastically fluctuating turbulent wakes that are caused by the interstitial air upflow [11]. Moreover, the dynamics is found to be homogeneous if the upflow is homogeneous as well [23]. In this way, horizontal dynamics is effectively achieved (i.e., no translational kinetic energy is stored in the vertical degree of freedom) for both plane (disks [24, 25]) and non-plane particles (spheres [11]).

At this stage, a comment on a subtle difference between air table experiments and vertical shaking experiments [6] is in order. In the latter, there is an intrinsic (non-measurable) fraction of the translational kinetic energy directly input in the vertical direction via mechanical collisions between the particles and the shaking boundaries [6]. However, in air tables the motion of spheres outside the horizontal plane is limited to sphere rolling, implying that there are no vertical displacements of the center of mass of the particles. For the sake of precision, we will make use of the term quasi-2D or pseudo-2D to refer to the dynamics of rolling spheres described in this work (as already explained, for an analogous but slightly different reason, vibrated thin layers are also referred to as quasi-2D systems [35]).

Furthermore, in quasi-2D vertical shaking experiments, low density phases are not observable in wide regions of the parameter space [39]. In contrast, we will see that set of spheres on a horizontal air table may undergo a variety of different phases, ranging from the low density granular gas to highly packed crystals. Additionally, repulsion forces between the spheres (of hydrodynamic origin [10]) are at play in our system, and this will be seen to influence the phase behavior, which is interesting to analyze. In this context, our set-up allows one to obtain an experimental characterization of a variety of different configurations in 2D granular dynamics.

This paper is organized as follows. In the next section, we describe the experimental system and the particle tracking method [11] we have used. In Section III we analyze the behavior of dynamical properties (distribution function, velocity autocorrelation and diffusion). Section IV discusses the ordering properties of the system and the emergence of phase transitions. In Section 5 we discuss the results and outline some open problems that could be studied with similar experimental set-ups.

II. DESCRIPTION OF THE SYSTEM

We perform experiments with a variable number $N$ of identical spherical particles. Specifically, our particles are ping-pong balls with diameter $\sigma = 4$ cm (ARTENGO® brand balls, made of ABS plastic with mass density 0.08 g cm$^{-3}$). The spheres rest on a metallic mesh (circular holes of 3 mm diameter arranged in a triangular lattice) and are enclosed by a circular wall made of polyactic acid (PLA). The diameter of this circular boundary is $D = 72.5$ cm and its height is $h \approx 4.5$ cm $> \sigma$. Thus, the total area of the system available to the spheres is $A = 0.413$ m$^2 = 328.65 \times \pi(\sigma/2)^2$, which means that up to $N_{\text{max}} = (\pi/\sqrt{12}) \times 328.65 \approx 298$ balls can fit in our system neglecting boundary effects (the $\pi/\sqrt{12} \approx 0.9069$ factor corresponds to the maximum packing fraction for disks in an infinite system [12]).

A state of the particulate system with stationary statistical properties is achieved by means of a vertical air flow in upward direction, as depicted in Figure 1. This upflow through the metallic grid generated with a fan, SODECA® HCT-71-6T-0.75/PL, and has stream velocities in the range $u_{\text{air}} = [2, 5.5]$ m/s. We have observed an approximately linear relationship between $u_{\text{air}}$ and fan power. An intermediate foam ($\sim 2$ cm thick) homogenizes the air current from the fan.

In order to assess the homogeneity of the flow throughout all the interstitial regions of the system, the air flow distribution over the grid was measured with a turbine digital anemometer plugged to a computer for the sake of data collection. We took measurements over a square grid of regularly spaced points on the table, and found local deviations of the air current of less than 10% with respect to the average $u_{\text{air}}$. The air current coming from the fan produces turbulent wakes as it flows past
the spheres [13]. We thus achieve an effectively two-dimensional particle dynamics, since the relevant particle motion is restrained to the grid plane (for more details on our particle fluidization mediated by turbulent air flow, see the Supplementary material [44]).

Summarizing, our experimental system has the following properties: 1) It is a many-particle system; 2) energy input (in absence of particles) can be measured and is found to be homogeneous; 3) motion is contained in a horizontal plane (the grid), and as a consequence gravity does not single out a predominant direction.

A series of experiments have been carried out by modifying the values of air flow intensity (2 m/s ≤ uair ≤ 5.5 m/s) and packing fraction φ = N(σ/D)², (0.03 ≤ φ ≤ 0.79). We have recorded a 99.92 s clip of each experiment with a high-speed camera [44] at 250 frames/s, or fps, (well below the maximum operational speed of our camera); i.e. the camera records a new image every ∆fps = 1/250 s. Particle positions are detected and tracked throughout the movies by means of a particle-tracking algorithm [41] that is applied to the digital images taken by the camera. More details on particle-tracking and experimental methods and as well as the measurement accuracy we achieved can be found in [44].

III. DYNAMICAL PROPERTIES

Air-fluidized granular systems have already been studied by other researchers. Of special relevance for us are the work by Oger et al. [24] (with disks) and particularly the work by Durian et al. [45] (with spheres). In this latter work, a series of experiments with ping-pong balls was carried out to characterize microscopic fluctuations, particle-particle and wall-particle forces [11, 40, 45], and jamming [46] among other aspects.

However, in contrast to previous works, here we focus on a complete description of the different phase transitions that can be observed in this kind of systems. A full exploration of the parameter space (φ, T) defined by the packing fraction and the granular temperature can be achieved by controlling the number of particles N and the air upflow velocity uair. We must also note that some of our results differ from those of previous works for closely related systems; in particular, in our experiments particles do not appear to be trapped in a harmonic potential, as opposed to the results in [11]. Furthermore, in contrast with the results obtained in [24] and [37], we find that granular temperature does not decrease monotonically with particle density [44]. These differences will be further discussed in the remainder of this paper.

A. Distribution function and velocity autocorrelation

In Figure 2(a) we show the distribution function f(c) of the rescaled velocity c = v/vth (with vth = (2T/m)½ being the thermal velocity and T ≡ (m/2)(v²) the granular temperature, and (⋅⋅⋅) denotes ensemble average). The results show a clear tendency to deviate from the Maxwellian distribution function (represented here by a solid line); this tendency being stronger the denser the system. As observed in previous experimental works on quasi-2D granular dynamics, as the tails deviate from the Maxwellian, they become exponential-like [48–50].

Moreover, it is interesting to note that this behavior was previously reported for constant particle density series with increasing granular temperature, but not for (approximately) constant temperature series, as displayed in Figure 2(a). We chose to compare systems with similar temperature in order to isolate the effects of modifying φ from the effects of changing the energy input-dissipation balance.

There is a certain difficulty in creating these constant temperature series, since the range of attainable granular temperatures can be very narrow depending on particle density.

Figure 2(b) shows the kurtosis K = ⟨v⁴⟩/⟨v²⟩² of the distribution function, which can be used to quantify the deviations from the Maxwellian distribution. As we can see, there is a strong overall tendency to deviate significantly from the Maxwellian at low temperatures and low densities (see Figure 2b). This probably signals the prevalence in this regime of friction effects due to the interaction between the irregular mesh surface and the balls, and as we will see later, can also be an indication of ordering processes.

The velocity autocorrelation function (VAF) reflects the memory effects in the fluid and is related to key transport properties. Within our experimental accuracy, it has been verified to depend only on time differences. We thus define this quantity as follows:

\[ \overline{A(\tau)} = \frac{\langle \vec{v}(t) \cdot \vec{v}(t + \tau) \rangle_{i,t}}{\langle (\vec{v}(t))^2 \rangle_{i,t}}, \]  

where subscripts i, t indicate averaging over particles and time.

Results are shown in Figure 3 where it can be readily noticed that there is a significant time interval during which autocorrelations are negative. We interpret this as a clear indication of particle effects due to non-contact distance interactions mediated by the circulating air, as opposed to the behavior for hard particles [51]. Moreover, the decay time to negative autocorrelations can be regarded as a measure of the typical collision time (in this context, “collision” should be understood as a particle entering a region where it can feel the repulsive forces as it approaches other neighboring particles). This col-
Figure 2. (a) Velocity distribution functions in logarithmic scale for a series taken at approximately constant granular temperature. The experimental data reveal that high density systems with exhibit more pronounced non-Maxwellian high-energy tails at $T = 0.76 \sigma^2 s^{-2}$. (b) Here we represent the kurtosis for constant density series vs. $T$.

Collision time has been found to decrease with increasing density.

In order to characterize this effect, Figure 2 (d) presents measurements of the velocity autocorrelation for a wide range of densities. The displayed results clearly indicate that non-contact interactions are in general more important at low densities. At very low densities the negative dip in the time behavior can be due to a single-particle effect (e.g. vortex shedding). Interestingly, the behavior is not monotonic, and in the very dense regime the dip becomes less pronounced again. This indicates that the interstitial hydrodynamic effects are more complex than expected. An example is the behavior of the autocorrelation function for $\phi = 0.365$ which remains non-negative up to $t \sim 1$ s, thereby resembling the behavior of hard particles; in contrast, at lower densities the negative values extend even up to $t \sim 2$ s, indicating that the particles are caged by their neighbors, but the corresponding plots exhibit a slight curvature due to the hydrodynamic effects.

**B. Diffusion**

An important characteristic of the experiment particles random motion is the mean square displacement (MSD) $\langle r^2 \rangle$. Most frequently, systems exhibit a power-law long-time behavior of the MSD, i.e., $\langle r^2 \rangle \sim D_\alpha t^\alpha$, where $\alpha$ is termed as diffusion exponent, whereas $D_\alpha$ is the so-called diffusion coefficient.

The drawback of only having at our disposal a limited number of trajectories can be alleviated through the standard procedure of constructing the time average of the mean square displacement (TAMSD) for each trajectory,

$$\langle r^2(t) \rangle = \frac{1}{t_m - t} \int_0^{t_m - t} d\tau \left| \vec{r}(\tau + t) - \vec{r}(\tau) \right|^2,$$

($t_m$ is the measurement time) and subsequently taking the mean over the ensemble of time averages for the individual trajectories. This yields the ensemble average of the time averaged mean square displacement (ETAMSD) $\langle r^2(t) \rangle$.

In this procedure it is assumed that both the MSD $\langle r^2 \rangle$ and the TAMSD $\langle r^2(t) \rangle$ follow the same power-law dependence $t^\alpha$, so that $\alpha$ can be accurately computed from a limited number of trajectories. However, this is not always the case. A well-known counterexample exhibiting non-equivalence between the TAMSD and the MSD as a result of weak ergodicity breaking is transport generated by the so-called CTRW model [52]. Fortunately, in our experimental system, there are no indications of such a behavior (for example, our VAFs are qualitatively different from those obtained from the CTRW model [54]).

In Figure 4 we show some representative ETAMSD curves obtained from three experiments with $N = 60$ ($\phi = 0.183$, $T = 0.422$), 120 ($\phi = 0.365$, $T = 0.618$) and 233 balls ($\phi = 0.709$, $T = 0.632$). Only data corresponding to trajectories longer than 40 s are considered. We have calculated linear fits of the ETAMSD in the time interval $2 < t < 16$ s (gray region in Figure 4). This choice is a trade-off ensuring that such an interval starts well after the end time of the ballistic regime, but is at the same time short enough to yield a sufficiently long time window $T_m - t$, so that statistical problems in the computation of the time average can be largely avoided.

In a further effort to obtain an improved estimate of $\alpha$, we have also plotted curves displaying the time dependence of the so-called mean logarithmic square displacement (MLSD) [53], which is the ensemble average of the log of the TAMSD, log $\langle r^2(t) \rangle$. A linear fit of this quantity as a function of log $t$ leads, in general, to better estimates for $\alpha$, provided that the localization error in the particles position remains small (as is the case in our experiments) [53].

All curves clearly exhibit an initial ballistic regime during which $\langle r^2(t) \rangle \sim t^2$. This holds up to times $\lesssim 0.1$ s.
The ballistic regime is always followed by a subdiffusive regime ($\alpha < 1.0$). For $\phi = 0.183$ and $\phi = 0.365$ one can spot an increase in the slope of the final part of the experimental curves, which could indicate the eventual onset of normal diffusion at even longer times, not covered by our experiment. This terminal increase in the slope has indeed been found to be a typical feature in granular dynamics experiments (e.g., in ref. [46], both a transient subdiffusive regime and a final normal diffusion regime were identified for a proper parameter choice). Nevertheless, one should bear in mind that the quality of the TAMSD deteriorates for larger values of $t$, since for such values the size $t_m - t$ of the time window over which the average is performed decreases. The offset of the MLSD and ETAMSD lines, quite noticeable for $\phi = 0.183$, is not completely unexpected [53]. In this case the fit of the MLSD and ETAMSD curves between $t = 2 \, \text{s}$ and $t = 16 \, \text{s}$ leads to $\alpha = 0.53$ and $\alpha = 0.70$, respectively. However, the case with $\phi = 0.365$ leads to $\alpha = 0.8$ and $\alpha = 0.9$, respectively, whereas the case with $\phi = 0.709$ leads to $\alpha = 0.1$ for the two curves. The noteworthy difference in the values of $\alpha$ for $\phi = 0.183$ turns out to be a persistent feature in our experiments, see Figure 5.

In Figure 5 we plot the values of $\alpha$ obtained by using the time interval $2 \, \text{s} < t < 16 \, \text{s}$ to fit the MLSD. As a reference, we also provide the values of $\alpha$ obtained from the ETAMSD computed for a number of experiments. As in Ref. [53], we have found that these $\alpha$-values are generally higher than those yielded by the MLSD (the difference is around one or, at most, two tenths); yet, they follow the same qualitative behavior.

The results in Figure 5 reveal a large variability of $\alpha$, probably due statistical limitations inherent to our experiments (number of trajectories and limited movie clips duration due to camera memory limitations). We do note however two main regimes, according to the behavior with respect to granular temperature. At low temperatures ($T \lesssim 0.7$), $\alpha$ is clearly increasing with $T$. In this regime $\alpha$ remains fairly small at the lowest measured granular temperatures; i.e., there is strong subdiffusion behavior. Interestingly, these are precisely the cases where the velocity distribution function deviates to a greater extent from a Maxwellian form (see Figure 2). At higher temperatures, we find a second diffusive regime

![Figure 3. Velocity autocorrelations. Panels (a), (b) & (c) display data series taken at constant packing fraction ($\phi = 0.183, \phi = 0.365 & \phi = 0.749$ respectively); (d) shows a data series taken at approximately constant granular temperature.](image-url)
densities this could be an indication of a crystallization process (crystals are typically colder than granular fluids for the same forcing conditions \[56\]). In Ref. \[55\] the cage size is identified as the value of \((\langle r^2 \rangle)^{1/2}\) for which its logarithmic derivative \(d \ln (\langle r^2 \rangle)^{1/2} / d \ln t\) attains a minimum. It is interesting to note that in those cases where the ballistic behavior changes to strong subdiffusion (small \(\alpha\)), the cage effect is so strong that the MSD is even seen to decrease during a short crossover regime. This effect can be clearly observed in the curves corresponding to \(\phi = 0.709\). We ascribe this behavior to the same transient viscoelastic forces that are responsible for the first dip exhibited by the VAF when \(\phi = 0.183\) and \(\phi = 0.709\) (cf. Figs. 3(a) and 3(c)).

On the contrary, the high temperature diffusive regime should correspond to regions of the phase space where the dynamics is dominated by a fluid phase \[11\]. In summary, strong indications of a rich phase behavior in this system emerge out of its diffusive properties. We will address this issue in more detail in the next section.

IV. STRUCTURAL PROPERTIES

We analyze in this section the structural properties of the system, based on the pair correlation function \(g(r)\) and the Voronoi tessellation \[57\]. As we will see, \(g(r)\) already yields clear indications of ordering transitions in the system. Voronoi tessellation –is a graphical representation that partitions space in cells enclosing only one particle, so that all the points inside a given cell are closer to the associated particle than to any other particle in the system \[57\]. This will confirm the expectations arising from the behavior of \(g(r)\), but also conveys additional information, thereby providing clear evidence for the onset of hexagonal order at high densities.

A. Radial distribution function

Following a standard procedure, we have computed the radial distribution function both from simulations and from our experimental data; taking into account that the system is 2D and has constant particle density \(\phi_0\), we employ the following formula:

\[
g(r) = \sum_{i,j>i}^{N} \frac{1}{2\phi_0(2\pi r_{ij})dr} \Pi(r_{ij} - r - dr/2),
\]

where \(\Pi(r_{ij} - r - dr/2) \equiv \Theta(r - r_{ij})\Theta(r + dr - r_{ij})\) is the rectangular pulse function \[58\] (\(\Theta\) being the Heaviside function \[59\]).

Measurements of the radial distribution function reveal interesting structural changes in the system, as already advanced in the previous sections. Results are displayed in Figure 6. As we can see, for \(\phi = 0.183\) (panel a) there is a liquid-like structure that is highly dependent on
Figure 6. Pair correlation function as processed from experimental 3D particle positions. Panels (a), (b) & (c) display data series taken at constant packing fraction ($\phi = 0.18, \phi = 0.365 \& \phi = 0.749$ respectively); (d) shows a data series taken at approximately constant granular temperature. A crystallization process is evident from the (a)-(c) panel series; clearly, complete crystallization is attained for $\phi = 0.749$ (see cf. panel (c)).

Notice that, in this case, the main peak appears at a distance clearly larger than $r = \sigma$ (we recall $\sigma$ is the particle diameter). At a higher density ($\phi = 0.365$, panel b) an analogous liquid-like structure emerges, but in this case it is very robust against temperature variations. At even high densities ($\phi = 0.749$, panel c), we can clearly see a series of sharp peaks, denoting positional ordering. Finally, in Figure 6 (d), the behavior of $g(r)$ for different densities is displayed in a series of curves at nearly constant temperature, where we can clearly see the transition from liquid-like to crystal-like $g(r)$ curves as the density is increased. It is worth pointing out that sharp secondary peaks already appear at densities as low as $\phi \sim 0.6$, which is an indication of a lattice parameter that is larger than the particles diameter. Furthermore, the first maximum splits into two, which implies the onset of a second maximum at $r = 2\sigma$, a typical situation before crystallization. Note that this behavior is reminiscent of that observed in early subcooled molecular liquids close to the glass transition \[60\]. The pair correlation function reveals the emergence of some kind of spatial correlations and translational symmetry, but it does not provide any information on the geometrical properties of this symmetry. For that purpose, the Voronoi diagrams we present in the next subsection are more adequate.

B. Phase changes

In order to detect emerging structural changes, we explored large regions of the parameter space (see \[44\] for a list of experiment data). Interestingly, the series with varying particle density at constant temperature shows a very rich and peculiar phase behavior. In order to visualize the varying degree of symmetry and the qualitative changes in related properties, we use both 2D spatial histograms and Voronoi tessellation diagrams \[57\], see Figures 7 and 8.
Figure 7. Set of 2D histograms for different system configurations. These histograms have been generated from the complete set of images in the movie clips, each grey dot representing a particle's instantaneous position. First row: each figure corresponds, from left to right, to increasing granular temperature at constant density; as the system heats up, first glassy behavior, and then transition to liquid are observed. Second and third rows: each figure corresponds, from left to right, to increasing density at constant temperature; a transition from liquid to crystal takes place, with phase coexistence. Parameters: (a) $\phi = 0.183, T = 0.158$; (b) $\phi = 0.183, T = 0.586$; (c) $\phi = 0.183, T = 0.743$; (d) $\phi = 0.274, T = 0.667$; (e) $\phi = 0.365, T = 0.618$; (f) $\phi = 0.457, T = 0.703$; (g) $\phi = 0.548, T = 0.696$; (h) $\phi = 0.639, T = 0.682$; (i) $\phi = 0.709, T = 0.632$. 
As already anticipated in Section II, each of the 2D histograms depicted in Figure 7 visualizes the positions of each particle (represented by white pixels) averaged over the time duration of each movie (~ 100 s), as is usual in previous studies on phase transitions. In contrast, Figure 8 represents Voronoi diagrams of instantaneous states of the same system. Both figures complement each other, i.e., the 2D spatial histograms tell us about the persistence in time of a given geometrical structure, whereas the Voronoi tessellation diagrams inform us about the specific geometry of that structure.

The evidence provided by Figures 7, 8 is rather compelling in spite of the fact that diagrams at very low densities (φ < 0.1), for which the dynamics is primarily driven by individual particles (particle-particle interactions are not frequent, and the dynamics is expected to be very similar to what has been previously reported for analogous single-particle systems [11]). In contrast, it is interesting to note that for somewhat larger densities 0.15 ≤ φ ≤ 0.25, we observe glassy-like states at sufficiently low temperatures, as shown in panels (a) of Figures 7, 8. Indeed, the histogram of time-averaged positions displayed in panel Figure 7 (a) reveals significant caging effects that are persistent in time; on the other hand, the corresponding Voronoi tessellation (Figure 8 a) shows a variety of cells with different coordination numbers, which signals the absence of a clearly dominant symmetry structure. At higher densities (0.25 ≤ φ ≤ 0.5), only a single phase is observed, which is seemingly disordered and isotropic, and can therefore be regarded as liquid-like [see panels (c)–(e)]. In panel (f), we have noted an even higher degree of disorder, with particles distributed in a more uniform fashion. This can be a signal of hyperuniformity [61]. Interestingly enough, upon further increase of the density (φ ≥ 0.5), we see the development of areas of hexagonal ordering coexisting with the fluid phase [see panel (g)]. The hexagonally ordered phase is seen to grow with increasing density – panel (h)– until it eventually extends over the whole system, see panel (i).

It is interesting to note that hexagonal ordering appears at much lower densities (φ ≈ 0.5 and higher) than in systems of hard particles or in soft disk models used in molecular dynamics to mimic active or passive particles, see e.g. [62]. This crystallization process into the hexagonal phase occurs in coexistence with the liquid phase (Figure 7 (g)) differs from an analogous transition in a confined monolayer of vertically vibrated spheres in that the latter system undergoes a melting transition of KTHNY type (i.e.; the melting transition for the vibrated layer is continuous and mediated by the successive unbinding of dislocation and disclination pairs, and it does not involve phase coexistence). Finally, it is also remarkable that for very cold systems there is no collective order collapse as it may happen in a vibrated monolayer of hard spheres. Instead, the particles in the gas phase gradually undergo strong caging effects, which results in the collective formation of disordered lattices.

In addition to Figures 7, 8 movie clips and experimental data of the different phases observed are available in the Supplementary material [44]. The complexity and richness of the phase transitions that we have observed is worth being studied in more detail. Such a study will be carried out in subsequent work.

V. CONCLUSIONS

In this work we have studied the pseudo-2D dynamics of a set of air-driven identical spheres which, excited by turbulent air, are rolling on a horizontal metallic grid.

The distribution function exhibits non-Maxwellian high energy tails, a feature also reported in previous works on quasi-2D granular dynamics. However, in contrast with the behavior of a monolayer of vertically vibrated particles, these non-Maxwellian tails seem to be more prominent at higher temperatures (Figures 2 a, b, c), although this trend is rather weak. In addition, we have described how the distribution function at nearly constant temperature behaves as a function of the particle density (Figures 2 d).

Velocity autocorrelations illustrate the relevance of hydrodynamic forces due to airflow-mediated particle interactions. The latter result in the onset large negative autocorrelations at comparatively short times (Figure 3). Direct observation confirms that particles initially approaching each other then experience an effective repulsion at short enough distances, which at very low densities (φ ≤ 0.15 ) prevents the formation of gas-like states; instead, a collection of Brownian-like particles is obtained in this limit (see Supplementary material experiments clips [44]). Clearly, the effect of repulsive forces in the dynamics of the system is more important at low densities, which is consistent with the observation that negative autocorrelations at short times are more pronounced in dilute systems (Figure 3 d, see also movies in Supplementary material [44]).

Two distinct diffusive regimes have been observed. The first regime, at low temperatures, is strongly increasing with granular temperature. At the lowest temperature values the system is strongly subdiffusive. Here, we can find the configurations with glassy or crystal configurations. At slightly higher temperatures, diffusion approaches normal diffusion. In the second regime, at high temperatures, we reach a plateau with slightly subdiffusive behavior. In this region we find systems in isotropic (gas or liquid) states.

The phase transitions observed in our system display a surprisingly rich and peculiar behavior, ranging from a collection of independent Brownian-like particles at low densities to glassy or liquid states at moderate densities, and to the onset of regular hexagonal lattices at higher densities. Most notably, we have described a very peculiar behavior in granular dynamics so far, consisting in the 2D hexagonal crystal melting in coexistence with a
Figure 8. Voronoi diagrams corresponding to the systems depicted in Figure 7. These diagrams confirm the glassy behavior in panels (a) and (b), the lack of order in the liquid-like systems, see panels (d), (e), (f), and the emergence (initially in coexistence with a liquid phase) of hexagonal ordering, see panels (g), (h). (i). (Irregular) polygons have been marked according to the following color code: blue for squares, green for pentagons, yellow for hexagons, dark red for heptagons, and light red for octagons. Parameters: a) $\phi = 0.183$, $T = 0.158$; (b) $\phi = 0.183$, $T = 0.586$; (c) $\phi = 0.183$, $T = 0.743$; (d) $\phi = 0.274$, $T = 0.667$; (e) $\phi = 0.365$, $T = 0.618$; (f) $\phi = 0.457$, $T = 0.703$; (g) $\phi = 0.548$, $T = 0.696$; (h) $\phi = 0.639$, $T = 0.682$; (i) $\phi = 0.709$, $T = 0.632$.

liquid phase. This result is fairly consistent and strikingly differs from previous observations in similar experiments on quasi-2D granular dynamics [21, 38].

In conclusion, while there is an extensive bibliography referring to engineering applications of air table systems [25], here we have used one such system for a more fundamental purpose, namely, to describe a variety of non-equilibrium 2D phase transitions and to identify the analogies with and departures from equilibrium theories and previous observations in granular dynamics experiments.

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