Order effects in dynamic semantics

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Abstract

In their target article, Wang and Busemeyer (2013) discuss question order effects in terms of incompatible projectors on a Hilbert space. In a similar vein, Blutner recently presented an orthoalgebraic query language essentially relying on dynamic update semantics. Here, I shall comment on some interesting analogies between the different variants of dynamic semantics and generalized quantum theory to illustrate other kinds of order effects in human cognition, such as belief revision, the resolution of anaphors, and default reasoning that result from the crucial non-commutativity of mental operations upon the belief state of a cognitive agent.

Keywords: Question order effects, belief revisions, anaphor resolution, default reasoning, generalized quantum theory, dynamic semantics

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1. Introduction

In their target article, Wang and Busemeyer (2013) discuss question order effects in terms of incompatible, i.e. non-commuting, projectors on a Hilbert space. In their model, a person’s belief state is expressed by a vector in a linear space that is equipped with a scalar product while the answers to yes/no-questions correspond to orthogonal subspaces in Hilbert space. A question is answered by projecting the current belief state vector either onto the question’s yes or no subspace. When answer subspaces to different questions do not coincide, the sequence of projections matters and questions are incompatible to each other.

In a similar vein, Blutner (2012) presented an (ortho-) algebraic approach for a query language that is not only able to explain question order effects but also allows the analysis of conditional questions of the form “If Mary reads this book, will she recommend it to Peter?” (Blutner, 2012). Also Blutner’s approach essentially relies upon a Hilbert space representation of belief states where questions induce a decorated partition into orthogonal answer subspaces. Yet, Blutner (2012) explicitly constructs his query language as a “version of update semantics” (Blutner, 1996; Veltman, 1996) where the “meaning of a sentence is not its truth condition but rather its impact on the hearer” (Kracht, 2002).

In my commentary on the target article of Wang and Busemeyer, I shall further elaborate the interesting analogies between the different variants of update semantics (Blutner, 1996; Veltman, 1996), dynamic semantics (Gärdenfors, 1988; beim Graben, 2006; Kracht, 2002), and dynamic logics (Groenendijk and Stokhof, 1991; Staudacher, 1987) on the one hand and generalized quantum theory (Atmanspacher et al., 2002), respective quantum dynamic logic (Baltag and Smets, 2011) on the other hand in order to illustrate some other kinds of order effects in human cognition, such as belief revision, the resolution of anaphors, and default reasoning that essentially result from the non-commutativity of mental operations upon a person’s belief states.
2. Generalized Quantum Theory

In generalized (or “weak”) quantum theory, [Atmanspacher et al. (2002)] consider a set \( X \) as a general state space and functions (morphisms) \( \text{Mor}(X) = \{ A | A : X \to X \} \), transforming a state \( x \in X \) into another state \( y \in X \) through

\[
y = A(x).
\]  

(1)

Particular functions from a subset \( \mathcal{A} \subseteq \text{Mor}(X) \) are called observables. Observables can be concatenated, i.e. iteratively invoked, such that \( (B \circ A)(x) = B(A(x)) = B(y) \), for all \( x \in X \). This observable product \( AB = A \circ B \) is associative: \( A(BC) = (AB)C \), but in general not commutative: \( AB \neq BA \). Only when \( AB = BA \), observables are called compatible, otherwise they are called incompatible.

[Atmanspacher et al. (2002)] supply a number of further axioms describing the properties of such observables and their impact upon the state space \( X \). One of these axioms introduces a neutral element \( 1 \), such that

\[
1 \circ A = A \circ 1 = A
\]

(2)

for all \( A \in \mathcal{A} \). Another axiom additionally introduces a zero observable \( 0 \in \mathcal{A} \) and a zero state \( o \in X \), such that

\[
\begin{align*}
0(x) &= o \quad (3) \\
A(o) &= o \quad (4) \\
0A &= A0 = 0 \quad (5)
\end{align*}
\]

for all \( x \in X \) and \( A \in \mathcal{A} \).

An important class of observables \( \mathcal{P} \subset \mathcal{A} \) are projectors which are idempotent

\[
A^2 = AA = A.
\]

(6)

Applying a projector \( A \) to a state \( x \in X \) yields another state \( y = A(x) = A^2(x) = A(A(x)) = A(y) \) that does not change under subsequent applications of \( A \) anymore. The projected state \( y = A(y) \) is hence an eigenstate of \( A \).
3. Classical Dynamic Semantics

Regarding the state space $X$ of generalized quantum theory as the set of epistemic states of a cognitive agent, yields an instantiation of dynamic update semantics \cite{Blutner1996, Gardenfors1988, Veltman1996} in the following way: Elements $x, y, z \in X$ are called epistemic states, or belief states while observables $A, B \in \mathcal{A}$ become interpreted as epistemic operators. By restricting observables only to commutative and idempotent operators, one obtains propositions. Their (commutative) composition can then be identified with logical conjunction

$$A \land B = AB = BA = B \land A.$$  \hfill (7)

An important notion in dynamic semantics is that of acceptance. A proposition $A \in \mathcal{P}$ is said to be accepted in state $x \in X$ (or $x$ is accepting $A$), if

$$A(x) = x.$$ \hfill (8)

That means, the state $x$ is an eigenstate of $A$. Because propositions are idempotent, the state $y = A(x)$ always accepts $A$. Thus, Eq. (8) receives a straightforward interpretation as information update.

Furthermore, logical consequence (or stability in \cite{Blutner1996}) is defined as follows: A proposition $B$ is called a logical consequence of a proposition $A$, if

$$B \land A = A \land B = A.$$ \hfill (9)

In this case, $y = A(x)$ entails $B(y) = B(A(x)) = A(x) = y$, such that $B$ is accepted whenever $A$ is accepted in an epistemic state (but not vice versa).

The given system can be equipped with other logical connectives such as negation ($\neg A$) or disjunction ($A \lor B$). \cite{Gardenfors1988} has proven that the resulting calculus is equivalent to intuitionist logics which can be further extended to classical propositional logics. Another important extension is Bayesian update semantics where states are interpreted as probability distributions $\rho$ over propositions. Then the impact of a proposition $A$ upon a belief state $\rho$ is expressed by Bayesian conditionalization

$$\rho_A(B) = \frac{\rho(B \land A)}{\rho(A)} =: \rho(B|A)$$ \hfill (10)
of the distribution with respect to $A$ (van Benthem et al., 2009; Gärdenfors, 1988; beim Graben, 2006).

4. Non-classical Dynamic Semantics

Classical dynamic semantics comprises epistemic operators that are commutative and idempotent propositions. Moreover, such systems are monotonic, as propositions which already have been accepted remain accepted during the updating of epistemic states. This follows from commutativity: Let $A$ be accepted in state $x$ (i.e. $A(x) = x$) and let $B(x) = y$, such that $B$ is learned during the updating from $x$ to $y$. Then $A(y) = A(B(x)) = (A \land B)(x) = (B \land A)(x) = B(A(x)) = B(x) = y$, saying that $A$ and $B$ are both accepted in the updated state $y$.

4.1. Belief revision

However, this account is not appropriate when belief states have to be revised by new evidence. Belief revision processes are in general not commutative and hence non-monotonic such that order effects become ubiquitous. Gärdenfors (1988) introduces a belief-revision operator as a mapping $*: \mathcal{P} \rightarrow \mathcal{A} \setminus \mathcal{P}$ assigning a revision $A^* \in \mathcal{A} \setminus \mathcal{P}$ to a proposition $A \in \mathcal{P}$. This revision dynamics has to obey several minimality axioms.

In order to illustrate this process, consider an agent in a belief state $x$ that accepts the proposition $A = “the moon consists of blue cheese”$ (beim Graben, 2006). Its revision is hence $A^* = “the moon does not consist of blue cheese”$. Another proposition might be $B = “the moon consists of stone$”. Since $x$ accepts $A$, the application of $B$, $B(x)$, leads to the zero state $o \in X$ of generalized quantum theory, that becomes now interpreted as the absurd state accepting every proposition (Gärdenfors 1988). Therefore also $BA = “the moon consists of blue cheese and of stone”$ is accepted in $o$. This state does not change under the revision $A^*$, hence $(A^*B)(x) = o$. On the other hand the product $BA^*$ applied to $x$ yields $B(y)$ where $y = A^*(x)$ accepts the revision of $A$. Therefore, $BA^*(x) \neq o$ because $BA^* = “the moon does not consist of blue cheese$, it rather consists of stone” can be consistently accepted. Thus
$A^*B \neq BA^*$, i.e. belief revisions and propositions do generally not commute and are hence incompatible to each other.

Belief revision in a probabilistic, Bayesian framework requires conditionalization with respect to a minimally altered probability distribution $\rho^*$ which involves several technical peculiarities such as epistemic entrenchment (Baltag and Smets, 2008; Gärdenfors, 1988; beim Graben, 2006). In the framework of quantum cognition, however, Bayesian conditionalization is replaced by the Lüders-Niestegge rule (Lüders, 1950; Niestegge, 2008)

$$\rho_A(B) = \frac{\rho(ABA)}{\rho(A)}$$

(11)

(see also Blutner (2009); Blutner et al. (2013)) resulting from the non-commutativity of Hilbert space projections (Atmanspacher et al., 2002), where $\rho(A) = |\langle \psi | A | \psi \rangle|^2$ gives the quantum probability in state vector $|\psi\rangle$. Therefore, belief revision seems to be a good candidate for quantum probability models in dynamic semantics (Engesser and Gabbay, 2002).

4.2. Anaphor resolution

Another important example for order effects in dynamic semantics is the resolution of anaphors. For this aim, Staudacher (1987) and Groenendijk and Stokhof (1991) have independently developed models of dynamic predicate logics, where quantifiers, such as “there exists an $x$” or “for all $x$”, and anaphors, e.g. pronouns, are described by epistemic operators acting upon model-theoretic valuations (see also Kracht (2002)).

As an instructive example we consider three propositions $A =$“John sat at the table”, $B =$“George came in”, and $C =$“he was wearing a hat”
Here, the pronoun “he” assumes conflicting interpretations for the compositions $CBA = “John sat at the table; George came in; he was wearing a hat”$ and $CAB = “George came in; John sat at the table; he was wearing a hat”$. In the first case, the pronoun “he” refers to “George”, while it refers to “John” in the second case. These anaphors therefore have to be described as non-commutative operators as well.

4.3. Default reasoning

Finally, Blutner (1996) and Veltman (1996) have observed that by relaxing the stability condition of logical consequence in (9), dynamic logics becomes non-monotonic. This allows the treatment of default operations, such as “may” or “normally”. Veltman (1996) presented a nice example for such an ordering effect in default reasoning: Let $A = “Somebody is knocking at the door”, B = “Maybe it’s John”, and C = “It’s Mary”. Then the composition $CBA = “Somebody is knocking at the door. Maybe it’s John. It’s Mary.”$ makes perfect sense, while $BCBA = “Somebody is knocking at the door. Maybe it’s John. It’s Mary. Maybe it’s John.” does not.

5. Conclusion

Classical dynamic semantics formalizes propositional logics in terms of commutative and idempotent epistemic operators that constitute a monotonic system of belief updating dynamics. By contrast, belief revision, the resolution of anaphors and non-monotonic reasoning in default logics require non-commutative operations.

In probabilistic dynamic semantics, updating is expressed by means of Bayesian conditionalization, whereas the description of belief revision processes requires rather peculiar mechanisms that could probably be more naturally expressed by means of quantum probability theory.

Other types of cognitive order effects such as anaphor resolution or default reasoning have been successfully described by extensions of dynamic semantics including predicate calculus or non-monotonicity. To my present knowledge, probabilistic generalizations of these models utilizing quantum
probability theory have not yet been devised. This might be a promising direction for future research.

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