Impurity Scattering in a Bose-Einstein Condensate at finite temperature.

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We consider the effects of finite temperature on the scattering of impurity atoms in a Bose-Einstein condensate, showing that the scattering rate is enhanced by the thermal atoms. Collisions can increase or decrease the impurity energy. Below the Landau velocity only the first process occurs, i.e., the collisions cool the condensate. Above the critical velocity the dissipative collisions prevail over the cooling ones for sufficiently low temperatures. These considerations are applied to a recent experiment.

Superfluidity is a phenomenon observed for the first time in $^4$He. It consists in the suppression of the viscosity of the superfluid below a critical velocity $v_L$, due to linear behavior of the dispersion curve $E(p)$ in the neighborhood of $p = 0$. Landau \cite{Landau} showed with kinematic arguments that $v_L = \min|E(p)/p|$. Superfluidity can be observed also in atomic Bose-Einstein condensates (BEC), Bogoliubov equations indicate that $v_L$ is equal to the local speed of sound. The first evidence of a critical velocity in a BEC was obtained by stirring the condensate with a laser beam \cite{Ishin}, but the observed critical velocity was much lower than the speed of sound, because of the presence of quantized vortices \cite{Haus97,Feder98}.

A recent experiment \cite{Dax02} demonstrated the superfluid suppression of scattering in agreement with the Landau criterion. This outcome was obtained by propagating microscopic impurities through the condensate. The impurities were created by a Raman transition from the $|F = 1, m_F = -1\rangle$ state to the untrapped $|F = 1, m_F = 0\rangle$ state. The velocity acquired by the untrapped atoms was due mainly to gravitational acceleration. The radial trapping frequency was varied to change the velocity of sound and the average impurity velocity. The number of the overall scattered atoms was evaluated for some values of the radial frequency. A suppression of the scattering was observed when the average velocity was near the Landau velocity at the center of the condensate. In Ref. \cite{Dax02} a zero-temperature theory is used to evaluate the scattering rate \cite{Ishin}.

In this work we first evaluate the scattering rate at finite temperature for an homogeneous system, demonstrating that the thermal cloud can enhance the scattering rate. This effect would provide an indirect measurement of the condensate temperature. We then evaluate the number of scattered atoms in a trapped system and show that the thermal atoms may have an observable effect upon the experimental data of Ref. \cite{Dax02}. Furthermore, we calculate the variation of temperature per impurity atom, showing that the impurity collisions cool the condensate for tight confinements.

The scattering rate at $T = 0$ can be evaluated using Fermi’s golden rule \cite{Ishin,Feder98}:

$$\Gamma = n_0 \left( \frac{2\hbar a}{M} \right)^2 \int dq d\Omega q^2 S(q) \times \delta \left( \frac{\hbar \vec{k} \cdot \vec{q}}{M} - \frac{\hbar q^2}{2M} - \omega_q^B \right),$$

(1)

where $n_0$ is the condensate density, $M$ the atomic mass, $\hbar$ the Planck constant, $a$ the scattering length for s wave collisions between the impurity atoms and the condensate atoms, and $S(q) = \omega_0^q/\omega_q^B$ the static structure factor of the condensate, with $\hbar \omega_q^0 = \hbar^2 q^2/2M$ and $\hbar \omega_q^B = \sqrt{\hbar \omega_q^0 (\hbar \omega_q^0 + 2\mu)}$ being the energies of a free particle and a Bogoliubov quasiparticle of momentum $q$, respectively. $\mu$ is the chemical potential.

In the scattering process an impurity atom changes its momentum and a phonon in the condensate is created. If the temperature is finite and the phononic modes are already populated we expect that the scattering is enhanced because of the bosonic stimulation, i.e.,

$$\Gamma_1(\beta) = n_0 \left( \frac{2\hbar a}{M} \right)^2 \int dq d\Omega q^2 S(q) \times \delta \left( \frac{\hbar \vec{k} \cdot \vec{q}}{M} - \frac{\hbar q^2}{2M} - \omega_q^B \right) e^{\beta \hbar \omega_q^B} e^{\beta \hbar \omega_q^B} - 1,$$

(2)

where $1/\beta \equiv T$ is the condensate temperature.

At finite temperatures there is another important channel that contributes to $\Gamma$: in a scattering process an impurity can change its momentum annihilating a phonon. In this case a thermal atom loses energy and increases the condensate population (this picture is exact for non-interacting atoms, but it is not completely appropriate for interacting ones), therefore the second channel is important because a single quantum state is macroscopically populated. We get the following additional contribute (cooling scattering rate)

$$\Gamma_2(\beta) = n_0 \left( \frac{2\hbar a}{M} \right)^2 \int dq d\Omega q^2 S(q) \times \delta \left( \frac{\hbar \vec{k} \cdot \vec{q}}{M} - \frac{\hbar q^2}{2M} + \omega_q^B \right) \frac{1}{e^{\beta \hbar \omega_q^B} - 1}.$$

(3)

There are two differences between $\Gamma_1$ and $\Gamma_2$. First, in the Dirac’s delta the sign of $\omega_q^B$ and $\vec{k}$ are changed, because a phonon is annihilated in the second channel (II), whereas it is created in the first one (I). The second difference
is a increasing monotonic function in \( \eta \), i.e., for \( \eta \) enhanced by a factor equal to the

demonstrate that \( Q \) the stimulated dissipative one, because \( \eta \) value of

condensate, furthermore for \( \eta > \eta_0 \), it grows upon decreasing the temperature. Therefore in an inhomogeneous trapped condensate the cooling scattering rate is always larger than \( \eta \) (spontaneous transition), because of the presence of the condensate. The maximum cooling rate is reached above the Landau critical velocity the collisions can only annihilate phonons, cooling the condensate. The maximum cooling rate is reached below the critical velocity. It vanishes for an impurity velocity approaching zero, because the required energy of the annihilated phonons increases. Above the critical velocity the cooling collisions can prevail over the dissipative ones for sufficiently high temperatures.

In Fig. 2, we have plotted \( Q(\beta) \) as a function of \( \beta \) for some values of \( \eta \). For a large \( \beta \), i.e. a small temperature, the stimulated scattering drops to zero and only the spontaneous scattering contributes to \( Q \). For \( \beta \rightarrow 0 \) \( Q \) remains finite. Therefore below the Landau critical velocity the collisions can only annihilate phonons, cooling the condensate. The maximum cooling rate is reached below the critical velocity. It vanishes for an impurity velocity approaching zero, because the required energy of the annihilated phonons increases. Above the critical velocity the cooling collisions can prevail over the dissipative ones for sufficiently high temperatures.

To refer to a realistic situation we apply Eqs. 3 and 4 to the data of Ref. 5. As reported, a variation of the
radial trapping frequency between 165 and 33 Hz implies a variation of the speed of sound from 1.1 to 0.55 cm/s. These values require that $N_{0\text{GP}} \sim 2.18 \times 10^{-43} \text{Jm}^3$, where $N_0$ is the number of condensed atoms and $q_{\text{GP}} = 4\pi \hbar^2 a_{0\text{GP}} / M$, $a_{0\text{GP}}$ being the s-wave scattering length of the condensate atoms. In the following we will use this value for $N_{0\text{GP}}$. The impurity atoms are created in an untrapped hyperfine level, therefore they are gravitationally accelerated. For experimental reasons they are produced with an initial axial velocity equal to $7 \text{mm/s}$. The number of scattered atoms is obtained by counting the impurity atoms in a region of the time-of-flight image below the unscattered impurity atoms, where the impurities that have lost kinetic energy are present. Since in this region there are Raman outcoupled thermal $m_F = 0$ atoms, the number of collided atoms in the counting region is obtained by subtracting the thermal background, determined by counting a similar sized region above the unscattered impurity atoms, where a few collision products are present. This background can be overvalued because some impurities, that have acquired energy by annihilating phonons, can be in the region above the unscattered atoms, where the presence of sole thermal atoms is supposed. In fact in Ref. [8] the authors conclude that, because of this overvaluation, the data of Ref. [5] are not affected by finite temperature. However, we show now that if the velocity is sufficiently larger than the sound velocity, the atoms that scatter annihilating phonons populate equally both regions.

If the scattering creates a phonon, the impurity loses the velocity $\vec{v}_i$, with $|\vec{v}_i| = v \cos \theta - c^2 / (v \cos \theta)$. Here $v$ is the impurity velocity and $\theta$ is the angle between that velocity and the phonon momentum. $\theta$ goes from 0 to $\arccos(c/v)$, therefore the lost velocity has a direction around the impurity one. Because of this directionality the dissipative scattering products move in the region of the time-of-flight image below the unscattered impurity atoms. If the scattering annihilates a phonon, the impurity increases the velocity by $\vec{v}_i$, with $|\vec{v}_i| = c^2 / (v \cos \theta) - v \cos \theta$, where $\theta$ ranges between $\arccos(c/v)$ and $\pi/2$. If $v/c \gg 1$ the acquired velocity is near orthogonal to the impurity velocity, i.e. the cooling scattering products are not confined within a narrow cone as the particles that collide creating phonons. Therefore we expect that the products of $(II)$ are equally distributed in both the measurement regions [8] and thus their contribution is negligible because of the background subtraction method. For example, with $v/c = 2$ and $v/c = 5$, $\theta$ ranges, respectively, between 60° and 90° and between 80° and 90° (the velocities are distributed near the surface of an open umbrella): furthermore its mean value approaches 90° upon increasing the temperature.

The problem is complicated both by the inhomogeneity of the condensate and the continuous variation of the impurity direction of motion during the free fall. Therefore to understand or to evaluate the exact contribution of the two collisions channels to the experimental data is not a trivial task. We have calculated only the overall collisional density for each channel.

If the initial axial velocity is small, an impurity produced at the point $(x, y, z)$ has the following probability to collide dissipatively [5]

$$p(x, y, z) = \int_z^{c(x, y)} dz_1 n(x, y, z_1) \sigma_1(\eta, c(x, y, z_1)), \quad (9)$$

where $\eta$ is determined by the local condensate density and the downward impurity velocity, $n$ is the condensate density and $c(x, y) = \sqrt{2\mu/m_0^2 - (\omega_0^2 x^2 + \omega_0^2 y^2)/\omega_0^2}$. $c(\vec{x})$ is $c_0 \sqrt{n(\vec{x})/n_0}$, $c_0$ being the speed of sound at the center of the condensate. Note that for $1/\beta = 0$ the $\sigma_1$ function depends only on $\eta$, as considered in Ref. [5].
As a first approximation, we have taken into account also the axial velocity using the total velocity, and not only the vertical component, to evaluate \( \eta \). We have also replaced \( dz \) with \( dz_1(v_1^2 + v_2^2)^{1/2}/v_2 \), where \( v_1 \) and \( v_2 \) are the axial and vertical velocities, respectively. We have supposed that the condensate is nearly homogeneous in the axial direction, so the variation of sound velocity and of \( c(x, y) \) in that direction is negligible.

The total number of dissipatively scattered atoms is 
\[
C_B(\bar{\eta}) = \int d\vec{r} n_I(\vec{r}) p(\vec{r}),
\]
where \( n_I(\vec{r}) \) is the impurity density and \( \bar{\eta} \equiv v_3/c_0 \), \( v_3 \equiv \sqrt{2g z_0} \) being the average vertical velocity and \( z_0 \) the Thomas-Fermi radius in the \( z \) direction. We can assume \( n_I \propto n \) \([5]\). In Fig. 3 we have reported (solid lines) \( C_B(\bar{\eta})/C_\infty \) as a function of \( \bar{\eta} \) and for some values of the temperature \( T \). \( C_\infty \) is the collisional density for high velocities and zero temperature. We have reported also the collisional density for the channel \( (II) \) (dashed line), calculated in a similar fashion. The dashed-dotted line refers to \((I)\) and is evaluated for \( T = 0 \) and \( v_3 = 0 \). We can see that the contribution of the axial velocity is important also for high \( \bar{\eta} \). This is true especially for \((II)\), because the scattering rate is higher for small velocities and, therefore, we have an important contribution at the beginning of the free fall. In particular, for high \( \bar{\eta} \) the horizontal velocity decreases the scattering rate, because it is higher than \( v_3 \). In fact, the cooling scattering rate has a maximum peak below \( v_3 \). For \( \bar{\eta} = 5 \) the temperature extracted from the time-of-flight pictures is about \( 100nK \) \([9]\). For this value the dissipative scattering rate is enhanced by about 30%.

For low \( \bar{\eta} \) the cooling scattering rate is higher than the dissipative one, suggesting that the overall effect of the collisions is the cooling of the condensate. We have calculated for each channel the average energy loss or gain of the trapped system per impurity atom. The temperature growth per impurity atom is 
\[
\Delta T = \Delta E/C_N(T),
\]
where \( \Delta E \) is the energy increase per impurity atom and \( C_N(T) \) is the heat capacity, which for \( N \) interacting trapped atoms is 
\[
C_N = \left( \frac{\partial E}{\partial T} \right)_N = Nk_B \left[ 12\zeta(4)/\zeta(3) + 1.27 t^2(5-8t^3)/(1-t^3)^{7/5} \right].
\]

In conclusion, we have considered the effects of finite temperature on the scattering of impurity atoms and we have shown that there are two important collision channels. In \((I)\) the collisions are dissipative and the impurities lose energy heating the condensate, in \((II)\) the impurities acquire energy cooling the condensate. Below the Landau velocity only \((II)\) has a non zero scattering rate. We have calculated the scattering rate for each channel with reference to the experimental configuration of Ref. \([5]\) and we have shown that the temperature may have a visible effect upon the experimental data. We have evaluated the energy loss or gain of the trapped atoms and demonstrated that the overall scattering process cools the condensate for tight traps and sufficiently high temperatures. This effect can become much more efficient using different experimental configurations \([11]\).

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