On the spin-isospin decomposition of nuclear symmetry energy

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The decomposition of nuclear symmetry energy into spin and isospin components is discussed to elucidate the underlying properties of the NN bare interaction. This investigation was carried out in the framework of the Brueckner-Hartree-Fock theory of asymmetric nuclear matter with consistent two and three body forces. It is shown the interplay among the various two body channels in terms of isospin singlet and triplet components as well as spin singlet and triplet ones. The broad range of baryon densities enables to study the effects of three body force moving from low to high densities.

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I. INTRODUCTION

Over the last two decades the nuclear symmetry energy has been one of most studied observables in nuclear physics for the important role it plays in the study of the spectroscopy of nuclei, heavy-ion collisions (HIC) and nuclear astrophysics (for a review see Ref.\textsuperscript{[1]}). The symmetry energy is the response of symmetric nuclear matter (SNM) to a small neutron-to-proton unbalance (we assume $\beta = (N - Z)/A > 0$) and it is the main property of asymmetric nuclear matter (ANM). In the framework of the Brueckner-Hartree-Fock (BHF) the energy per particle displays, as shown in Fig.1, the well known linear $\beta^2$-dependence within a broad range of nuclear-matter densities\textsuperscript{[2]}. The deviation due to the kinetic part is negligible. Such a behavior justifies the calculation of the symmetry energy as difference

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FIG. 1: Energy per particle in ANM vs. $\beta^2$ from BHF approximation with two different realistic two body forces: AV18 (left) and BONN B (right).

of binding energies between pure neutron matter (PNM) and symmetric nuclear matter. Whereas this estimate is validated by the empirical nuclear mass law, it is would not be so for neutron stars, whenever the $\beta^2$ linearity was lost. In that case the symmetry energy should be determined by a small isospin deviation from the PNM state. From the point of view of the nucleon-nucleon interaction, the $\beta^2$ linearity seems to be in conflict with the breakdown of rotational invariance in isospin space when moving from the bare interaction to the effective interaction $\mathcal{F}$. Considering the isospin shift of the single-particle potential $\Delta u_\tau = u_\tau(\beta) - u_\tau(0)$, it is easily proved that

$$\Delta u_n - \Delta u_p = \frac{1}{2} \rho [\mathcal{F}_{nn} + \mathcal{F}_{pp} - 2 \mathcal{F}_{np}] \beta.$$  \hspace{1cm} (1)

The difference $\mathcal{F}_\tau - \mathcal{F}_{\tau'}$ ($\tau \neq \tau'$) determines the Landau-Migdal parameter $\mathcal{F}_\tau'$. Despite $F'_n \neq F'_p$ in BHF approximation, the sum is almost constant at any value of symmetry parameter for $\rho = \rho_n + \rho_p$ constant. This is also true for any component of the $E_{sym}$ expansion in two body channels. In the next section we will discuss the individual spin-
isospin contributions to the symmetry potential energy which enter in the decomposition

\[ U_{\text{sym}} = \sum_{ST}(U_{\text{PNM}}^{ST} - U_{\text{SNM}}^{ST}), \]

(2)

where \( S \) is total spin, \( T \) is total isospin, and the \( z \)-projection of isospin \( T_z \) is dropped out according to the preceding discussion.

II. NUMERICAL RESULTS

The spin and isospin decomposition of the the symmetry energy potential has been calculated in the framework of the BHF approximation. Two versions two and three body force (2BF and 3BF) have been employed: Argonne V18 plus consistent meson-exchange 3BF and Bonn B plus consistent meson-exchange 3BF. In Fig.2 it is reported the isospin shift

\[ U^T(\rho, \beta) - U^T(\rho, 0) = \sum_S [U^{ST}(\rho, \beta) - U^{ST}(\rho, 0)] \]

(3)

with only two body force Bonn B. Due to the simple \( \beta^2 \) law, one can calculate the total

FIG. 2: Total potential energy per particle in ANM vs. \( \beta^2 \) (left) and isospin triplet contribution (right).

isospin contribution to the symmetry potential energy from the value of PNM, namely \( \beta = 1 \).
It is seen around the saturation density that the isospin-singlet term yields by far the largest contribution to the symmetry energy whereas the isospin-triplet is negligible. The reason is that at low density the G-matrix still keeps the rotational invariance in isospin space of the bare nucleon-nucleon interaction, so that the isospin-triplet contribution disappears ($G^1$ independent of the $z$-projection $T_z$) as it can be realized from Eq.1 expressed in term of total isospin

$$\Delta u_n - \Delta u_p = \frac{1}{2} \rho [G^1_{nn} + G^1_{pp} - 2G^1_{np} - 2G^0_{np}] \beta, \quad (4)$$

here

$$G^T_{\tau\tau'} = \sum_S G^{ST}_{\tau\tau'}, \quad (5)$$

and the effective interaction $G^T(T = 0, 1)$ is the G-matrix $G^T$ in the Brueckner theory. It amounts to say that the spin-singlet contribution is vanishing due to the generalized Pauli principle ($L + S + T = odd$), when restricting to only $L = 0$ angular momentum. The conclusion is that the spin-triplet two-body channel $^3S_1$ gives the largest contribution to the symmetry potential energy, being $L > 0$ channels much smaller. This result was already found in the earliest ab initio calculations of the symmetry energy [2, 7]. At supra-saturation density the isospin symmetry is violated so that the isospin-singlet starts to compete with isospin-triplet. The interplay between isospin-singlet and isospin-triplet is quite clearly illustrated in Fig.2. The decomposition of $T = 0$ and $T = 1$ potential energy per particle is reported in Table I for two densities. It is worthwhile noticing the rather good agreement between the two interactions adopted in the BHF calculations, giving comparable values for the symmetry energy as well as the the individual components, except for the $T = 0$ ones at the higher density. At saturation density the isospin-triplet components

| POT | $\rho (fm^{-3})$ | $U^{T=1}_{PNM}$ (MeV) | $U^{T=1}_{SNM}$ (MeV) | $U^{T=0}_{SNM}$ (MeV) | $E_{sym}$ (MeV) |
|-----|-----------------|----------------------|----------------------|----------------------|----------------|
| AV18 | 0.175           | -0.253               | -23.437              | -0.341               | -19.981        | 50.038        |
|      | 0.400           | 11.739               | -39.210              | 3.219                | -36.342        | 50.866        |
| BONN B | 0.175        | 0.295                | -23.414              | -0.136               | -20.438        | 50.470        |
|      | 0.400           | 10.105               | -39.626              | 2.948                | -37.170        | 50.470        |

TABLE I: Partial wave decomposition of the potential energy from BHF with 2BF.
slightly change going from SNM to PNM, leaving the isospin-singlet T=0 term to play the major role in determining the symmetry energy, as discussed before. At the higher density also the isospin-triplet T=1 to contribute to the enhancement of the symmetry energy. As already firmly established the 3BF is necessary to reproduce the saturation density of nuclear matter. At super-saturation density the 3BF becomes the dominant interaction. The simplest way to extend the BHF approximation is to replace the 3BF by a density dependent 2BF weighting the effect of the third particle by means of its correlation with the other two particles [8]. In coordinate space it can be written formally

\[ W(r_{12}) = \rho \int d^3r_3 V(r_1, r_2, r_3) g^2(r_{13}) g^2(r_{23}) \]  

where \( g(r) = 1 - \eta(r) \), \( \eta(r) \) being the defect function. The energy per particle in ANM is still a \( \beta^2 \) function for all densities considered, as shown in Fig.3. The 3BF contribution to the isospin shift of the single-particle potential \( \Delta u_\tau \) is given by

\[ \Delta u_n - \Delta u_p = \frac{1}{2} \rho [\tilde{W}_{nn} + \tilde{W}_{pp} - 2\tilde{W}_{np}] (n_n + n_p) \beta, \]
where
\[
\tilde{W}_{\tau\tau} = V_{\tau\tau\tau} g_{\tau\tau}^2 + V_{\tau\tau\tau'} g_{\tau\tau'}^2,
\]
\[
\tilde{W}_{\tau\tau'} = V_{\tau\tau'\tau} g_{\tau\tau'} g_{\tau\tau'} + V_{\tau\tau'\tau} g_{\tau\tau} g_{\tau'\tau'},
\]
and \(\tau \neq \tau'\). This contribution is weakly asymmetry dependent, the same as the 2BF term, so that the \(\beta^2\)-linearity is to be expected. The numerical results in fact confirm such a property, as shown in Fig.3, for both 3BF: meson-exchange 3BF consistent with Argonne V18 \[5\] and meson-exchange 3BF consistent with Bonn B \[6\]. Now the interplay between

FIG. 4: Total potential energy per particle in ANM vs. \(\beta^2\)(left), and isospin triplet contribution (right).

isospin singlet and triplet is displayed in Fig.4. It is dominated by the 3BF, whose strength is strongly increasing with density. The isospin-triplet contribution reaches 95% of the total symmetry potential energy at the highest considered density \(\rho = 0.6 \, fm^{-3}\).

The decomposition of isospin singlet and triplet in L partial waves is reported in Table II. In this case the agreement between the two 3BF adopted in the calculations is not so good as before, but the difference in the symmetry energy is about 5%. Since the 3BF strength is still small at the saturation density, the isospin-triplet is dominated by the isospin-singlet,
but at higher density it participates (with odd L) to increase the symmetry energy at the same footing as the isospin-singlet (with even L), as noticed in Fig.4. In conclusion the 3BF reinforces the spin-triplet component of the full interaction.

In the present note some properties of nuclear symmetry energy, calculated within the BHF approximation with two and three body forces, have been discussed in connection with symmetries of the interaction. The spin-isospin decomposition was performed to illustrate the interplay between different two body channels in terms of isospin singlet and triplet components as well as the spin singlet and triplet ones. This investigation is a preliminary step to calculate the nuclear symmetry energy beyond the mean field approximation, including medium polarization effects.

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