Branching Fraction Measurements of the SM Higgs with a Mass of 160 GeV at Future Linear $e^+e^-$ Colliders

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Abstract

Assuming an integrated luminosity of 500 fb\(^{-1}\) and a center-of-mass energy of 350 GeV, we examine the prospects for measuring branching fractions of a Standard Model-like Higgs boson with a mass of 160 GeV at the future linear \(e^+e^-\) collider TESLA when the Higgs is produced via the Higgsstrahlung mechanism, \(e^+e^- \rightarrow HZ\). We study in detail the precisions achievable for the branching fractions of the Higgs into \(WW^*\), \(ZZ^*\) and \(b\bar{b}\). However, the measurement of BF\((H \rightarrow \gamma\gamma)\) remains a great challenge. Combined with the expected error for the inclusive Higgsstrahlung production rate the uncertainty for the total width of the Higgs is estimated.

1 Introduction

Discovery and study of Higgs boson(s) will be of primary importance at a next linear \(e^+e^-\) collider. After discovery of a Higgs \((H)\), the task will be to determine as precisely as possible and in a model-independent manner its fundamental couplings and total width. In the Standard Model (SM) \([1]\), the mass range from \(\sim 100\) GeV to about 220 GeV is favored by comparing precision electroweak data with SM predictions \([2]\). For Higgs masses at 120 and 140 GeV detailed branching fraction measurement investigations can be found in e.g. refs.\([3]\), \([4]\), \([5]\). In this paper we assume a Higgs mass of 160 GeV. For such masses the Higgs decays mostly into \(WW^*\) whereas the \(b\bar{b}\) decay rate is strongly suppressed to few percent, opposite to the situation at smaller masses. Additionally, at \(M_H \sim 160\) GeV the background expected to contribute is more complicated since a mixture of \(WW\) (where both W’s are on-mass-shell) and \(WW^*\) (with one of the W’s being off-mass-shell) exists under the Higgs, and due to detector resolution effects the observed width of the Higgs boson is expected to be much larger than its natural total width. Therefore finite W width effects have to be accounted for to enable accurate results.

Future linear \(e^+e^-\) colliders operating in the 300 to 500 GeV center-of-mass energy region are ideal machines to investigate the Higgs sector in this mass regime: besides easily Higgs discovery, all major decay modes can be explored.

In order to access the actual capability of an \(e^+e^-\) collider to an analysis of the SM Higgs boson it is extremely appealing to apply recently developed tools, thanks to the effort of several groups. In particular, we include in this study

- the full matrix elements for 4-particle final states beyond the usual approximation of computing Higgsstrahlung production cross sections times branching fractions, using the program package CompHEP \([6]\). In this way,
all irreducible background and possible interferences between signal and background diagrams are accounted for. Total and partial widths of the Higgs boson corrected for QCD and QED loop contributions were implemented in CompHEP and cross-checked with the HDECAY program [7]. For parton shower and hadronization procedures as well as particle decays the PYTHIA/JETSET package [8] has been interfaced;

- initial state QED [9] and beamstrahlung for the TESLA linear collider option [10];
- a detector response [11], with detector parameters as designed in a series of workshops for the linear $e^+e^-$ collider Conceptual Design Report [12];
- all important reducible background reactions expected to contribute.

The cm energy $\sqrt{s}$ chosen for our study is 350 GeV avoiding thus $t\bar{t}$ pair production as background. Although $\sqrt{s}$ is not optimized for the best Higgs production rate, it is a compromise between largest machine luminosity and sufficient event production for the process

$$e^+e^- \rightarrow H(160)Z.$$

All the results presented are based on the high-luminosity option of the TESLA machine [13] with an instantaneous luminosity $L = 3 \cdot 10^{34}$ cm$^{-2}$ sec$^{-1}$ resulting to an accumulated luminosity of 500 fb$^{-1}$ within about two years of running.

2 SM Signals, Backgrounds and Branching Fractions

Within the SM, the Higgs boson with $M_H = 160$ GeV decays mostly to $WW^*$, in few percent of the time to $ZZ^*$ and $b\bar{b}$ and with a very small rate of about $0.5 \cdot 10^{-3}$ to two photons. We consider only these decay modes as all others are expected to be more difficult to measure. The aim of this study is to provide the statistical uncertainties of the branching fractions for the decay modes mentioned by measuring $\sigma(e^+e^- \rightarrow HZ) \cdot BF(H \rightarrow X) \cdot BF(Z \rightarrow Y)$, $\sqrt{S + B}/S$, where $S(B)$ is the number of Higgs (background) events observed in a small interval of the invariant mass $X$, centered around $M_H$. Branching fraction errors of the Higgs boson are then computed after convolution with the precision of the inclusive Higgs production cross section $\sigma(HZ)$ of 2.8% [4], [14]. Any Z boson branching fractions errors are neglected.
2.1 The Branching Fraction BF (H \rightarrow WW^*)

For M_H = 160 GeV, the Higgs decays mostly to WW^*, i.e. to a final state where one of the W’s is on-mass-shell and the other is off-mass-shell. Since we are interested to make a signal-to-background analysis as meaningful as possible we also consistently evaluate the background rates. Most of this background comes from e^+e^- \rightarrow WW^*Z and WWZ events and requires an accurate treatment of finite W width effects.

Therefore exact matrix elements are used to calculate by means of CompHEP the processes

\[ e^+e^- \rightarrow \bar{u}dWZ \]  (2)
\[ e^+e^- \rightarrow u\bar{d}WZ \]  (3)
and
\[ e^+e^- \rightarrow WWZ, \]  (4)

with proper matching below and above the W pair threshold, and to generate unweighted events. The final states \( \bar{c}sWZ \) and \( csWZ \) were taken into account by doubling the number of events for reactions (2) and (3), respectively. In order to avoid large not useful event samples the following cuts were applied to each final state particle during the generation stage:

- energy > 10 GeV;
- polar angle \( \Theta > 5^\circ \);
- the invariant mass of any particle pairing \( M_{ik} > 10 \) GeV.

Due to the large H \rightarrow WW^* branching fraction and the luminosity assumed, thousands of signal events are expected. In the following we restrict our analysis to only Z \rightarrow e^+e^- / \mu^+\mu^- and W \rightarrow q\bar{q} decays for reasons of simplicity. Therefore, the signature we have to search for consists of four jets accompanied by two opposite-charged leptons.

For each of the reactions (2) - (4) we apply a series of cuts to remove much of the reducible background and, if needed, further dedicated criteria are applied to suppress any remaining irreducible background.

In particular, the cuts we have adapted to select \( e^+e^- \rightarrow HZ \rightarrow WW^*Z \) events are

- the visible energy of the event, \( E_{vis} \), exceeds 200 GeV;
- the total transverse energy of the event is larger than 40 GeV;
• the total momentum along the beam direction is restricted to be within $\pm 120$ GeV;
• the number of tracks per event is larger than 20;
• out of all leptons found, at least one $e^+e^-$ or $\mu^+\mu^-$ pair has an invariant mass within $M_Z \pm 6$ GeV;
• for each jet we require
  - $E_{jet} > 10$ GeV ;
  - $|\cos \theta_{jet}| < 0.85$ ;
  - number of particles /jet $\geq 4$ ;
  - angle (jet$_i$, jet$_k$) $> 10^\circ$;
• select from all two-jet pairings with $70 \text{ GeV} < M(q\bar{q}) < 90 \text{ GeV}$ the one best compatible with $M_W$.

According to this procedure events were selected in which the Higgs recoils against the $Z \rightarrow l\bar{l}$ boson, with $H \rightarrow WW^*$ and $W \rightarrow q\bar{q}$ decays.

Reducible background events from

\begin{equation}
e^+e^- \rightarrow W^+W^- \rightarrow 4jets \tag{5}
\end{equation}

\begin{equation}
e^+e^- \rightarrow ZZ \rightarrow 4jets \tag{6}
\end{equation}

which might mimic our topology and fulfill the selection criteria are small. As an example, 150,000 events of reaction (6) expected for $500 \text{ fb}^{-1}$ were processed and no entry appeared in the final $WW^*$ mass distribution after all cuts.

Fig.1 shows the $WW^*$ mass spectrum for the surviving events. A clear Higgs signal is visible over a very small background which is mainly due to non-signal diagrams contributing in reactions (2) and (3). We estimate a statistical error of 2.5 % for $\sigma(HZ) \cdot BF(H \rightarrow WW^*) \cdot BF(Z \rightarrow l\bar{l})$. After convolution with the error of $\sigma(HZ)$ the precision for $BF(H \rightarrow WW^*)$ is found to be $\pm 3.5$ %. It is worth to mention that $e^+e^- \rightarrow HZ \rightarrow (ZZ^*) (l\bar{l}) \rightarrow 4 \text{ jets} + e^+e^- /\mu^+\mu^-$ events, which have the same signature and could contribute to the final $WW^*$ mass spectrum, were subtracted from the distribution in Fig.1.

2.2 The Branching Fraction BF ($H \rightarrow ZZ^*$)

Information for the $H \rightarrow ZZ^*$ branching fraction can be obtained either from the inclusive Higgs production cross section $\sigma(HZ)$ which is proportional to the ZZH

\footnote{The two leptons selected in the previous step were excluded from the jet finder.}
coupling-squared or from studies of $ZZ^*$ mass distributions. The latter spectra, obtained from the reaction

$$e^+e^- \rightarrow ZZ^*Z,$$  \hspace{1cm} (7)

contribute to four event topologies depending on the $Z$ decay modes selected:

- 6-jet events;
- 4-jet plus two opposite-charged lepton events;
- 2-jet plus two pairs of opposite-charged lepton events;
- three pairs of opposite-charged lepton events.

Figure 1: The combined $WW^{(*)}$ mass distribution after signal and background event selection.
Since large event rates are expected from the background reaction $e^+e^- \rightarrow \text{WWZ} \rightarrow 6 \text{ jets}$ and the signal process $e^+e^- \rightarrow \text{HZ} \rightarrow \text{WW}^* \text{Z} \rightarrow 6 \text{ jets}$, the 6-jet topology has been discarded from our study. Also the three lepton pair topology had been omitted due to its very small event rate.

We first consider the 2-jet 4-lepton topology. By means of CompHEP the following reactions

\[ e^+e^- \rightarrow d\bar{d}ZZ \]  
\[ e^+e^- \rightarrow u\bar{u}ZZ \]  
\[ e^+e^- \rightarrow ZZZ \]  

were generated and properly weighted to account for $s\bar{s}$, $c\bar{c}$ and $b\bar{b}$ contributions in reactions (8) and (9). The two Z bosons (and for reaction (10), two Z’s out of the three) are allowed to decay to electron respectively muon pairs. After event reconstruction we require besides some more general cuts

- only $e^+e^- / \mu^+\mu^-$, $e^+e^- / e^+e^-$ or $\mu^+\mu^- / \mu^+\mu^-$ lepton combinations in the final state;
- $|\cos \theta_l| < 0.95$ for each lepton;
- there exists at least one lepton pair
  - with $M_{ll} = M_Z \pm 6 \text{ GeV}$;
  - $E_L + E_{\bar{l}} > 125 \text{ GeV}$;
  - which recoils against the $(q\bar{q} \ l\bar{l})$ system with an invariant mass of less than 220 GeV;
- two and only two jets in the final state, after excluding the four leptons from the jet finder;
- for each jet we demand
  - $E_{jet} > 10 \text{ GeV}$;
  - $|\cos \theta_{jet}| > 0.9$,
  - number of particles/jet $\geq 4$;
  - angle $(\text{jet}_1, \text{jet}_2) > 20^\circ$.

The resulting $M(q\bar{q} \ l\bar{l})$ mass distribution is shown in Fig.2. Higgs boson production is clearly observed on very small background.

The 4-jet 2-lepton topology, our second candidate to search for $H \rightarrow \text{ZZ}^*$ decays, is expected to involve about 10 times more signal events due to the large
$Z \to q \bar{q}$ decay rate. This advantage is however to a great extent compensated by demanding stronger selection criteria to reduce the larger background expected. The procedure applied to select signal from background events requires

- $E_{\text{vis}} > 260$ GeV;
- the total transverse energy is restricted between 50 and 290 GeV;
- the total momentum along the beam line is within $\pm 80$ GeV;
- only $e^+e^-$ or $\mu^+\mu^-$ pairs are accepted;
- the number of jets, excluding the two leptons, is four;

Figure 2: The $ZZ^* \to q \bar{q} \ell\bar{\ell}$ mass distribution from the selected events of the 2-jet 4-lepton topology.
• for each jet we require
  \begin{itemize}
  \item $E_{\text{jet}} > 10$ GeV;
  \item $|\cos \theta_{\text{jet}}| > 0.9$;
  \item number of particles/jet $\geq 6$;
  \item $\angle (\text{jet}_i, \text{jet}_k) > 15^\circ$;
  \end{itemize}
• the two-jet invariant mass is within $M_Z \pm 10$ GeV;
• the dilepton invariant mass is within $M_Z \pm 6$ GeV;
• $E_\ell + E_{\bar{\ell}} > 125$ GeV;
• the mass recoiling against the Z (with $Z \rightarrow q \bar{q}$) does not exceed 200 GeV.

According to these criteria we try to select $e^+e^- \rightarrow \text{HZ} \rightarrow (ZZ^*) Z \rightarrow (l \bar{l} q \bar{q})(q \bar{q})$ events, where the Z boson against the Higgs decays hadronically to $q \bar{q}$.

In the final $(q \bar{q} \; l \bar{l})$ mass spectrum, shown in Fig.3, a convincing Higgs signal can be seen on some remaining background which peaks near 190 GeV due to surviving $e^+e^- \rightarrow ZZ^*(\rightarrow Z)Z$ events. The combined spectrum of Figs.2 and 3 yields for $\sqrt{S} + B/S = \Delta[\sigma(HZ) \cdot BF(H \rightarrow ZZ^*)] = 14.2\%$, which results to a statistical error of $\pm 14.5\%$ for BF $(H \rightarrow ZZ^*)$, after convolution with the uncertainty of $\sigma(HZ)$.

### 2.3 The Branching Fraction BF $(H \rightarrow b \bar{b})$

In contrast to the Higgs mass region below $\sim 130$ GeV with the dominant $H \rightarrow b \bar{b}$ decay mode, heavier Higgs bosons are predicted to have a small or negligible decay rate to $b \bar{b}$. For $M_H = 160$ GeV, the SM predicts a $3.8\%$ branching fraction which requires high luminosity for a precise measurement.

The topologies expected for $e^+e^- \rightarrow \text{HZ} \rightarrow (b \bar{b}) Z$ production consist either of 4-jet events involving two b-jets

$$e^+e^- \rightarrow b \bar{b} \ q \bar{q} \quad (11)$$

or two b-jet events accompanied by a pair of opposite-charged leptons

$$e^+e^- \rightarrow b \bar{b} \; e^+e^-/\mu^+\mu^- \quad (12)$$

Events with $Z \rightarrow \nu \bar{\nu}$ decays are excluded from the analysis since we demand visible Z decays.

Most of the background comes from the channels

$$e^+e^- \rightarrow q \bar{q} \; q \bar{q} \quad (13)$$
and

\[ e^+ e^- \rightarrow q\bar{q} \, e^+ e^- / \mu^+ \mu^- , \]  

with \( q = u, d, s \) and \( c \).

Our criteria to select events of reaction (11) are the following:

- four and only four jets occur in the final state;
- for each jet we require
  - \( E_{\text{jet}} > 12 \) GeV;
  - \( |\cos \theta_{\text{jet}}| < 0.85; \)
− angle (jet$_i$, jet$_k$) > 10$^\circ$;
− number of particles/jet $\geq$ 8;

• out of the four jets demanded two jets are tagged as b-jets; a jet is defined as a b-jet if
  − the number of charged particles with large impact parameter is $\geq$ 3 for one jet and $\geq$ 4 for the other;
  − a track is considered as a 'large impact parameter track' if its distance of closest approach to the primary vertex in the (r,phi) or the (r,z) projection is $\geq$ 3, in units of its error.

• the two remaining jets have an invariant mass within $M_Z \pm$10 GeV.

The resulting b-tagging efficiency turns out to be $\sim$ 65% while the probability for a light $q \bar{q}$ pair to be tagged as a $b \bar{b}$ pair is below 3%. For the surviving events the $b \bar{b}$ invariant mass spectrum is shown in Fig.4. Besides the dominating Z boson, the Higgs in its $b \bar{b}$ decay mode is clearly visible at 160 GeV. An estimate of the signal and background rates in the vicinity of $M_H$ results to an error of $\sigma(HZ) \cdot BF(H \rightarrow b \bar{b}) = \pm 16\%$.

Events of the 2-jet dilepton signal topology, produced about ten times less frequently, are selected by

• identifying an $e^+e^-$ or $\mu^+\mu^-$ pair with an invariant mass $M_Z \pm$6 GeV;

• two and only two jets, excluding the lepton pair from the jet finder, are tagged as b-jets, with properties as defined above.

The resulting $b \bar{b}$ mass distribution is shown in Fig.5 and, if combined with the spectrum of Fig.4, the statistical error for $BF(H \rightarrow b \bar{b})$ results to $\pm 12\%$.

2.4 The Branching Fraction BF ($H \rightarrow \gamma\gamma$)

The Standard Model predicts for $BF(H \rightarrow \gamma\gamma)$ with $M_H = 160$ GeV a rate of 0.54 $\cdot 10^{-3}$, so that only very large statistics experiments combined with an electromagnetic calorimeter of excellent resolution might have access to this quantity. Deviations of $BF(H \rightarrow \gamma\gamma)$ measurements from the SM expectations would be of great importance. In particular, by virtue of the fact that the coupling $H \rightarrow \gamma\gamma$ arises from charged loops, large deviations from the SM value due to new particles (e.g. a fourth generation, supersymmetry) are possible. Measuring $BF(H \rightarrow \gamma\gamma)$ at the next linear collider will be, regardless of the size of the deviation from the SM prediction, important to understand the nature of
the Higgs and provides hints of new physics that may be beyond the Standard Model.

In principle, similar arguments are also valid for Higgs decays like $H \to 2$ gluons or $H \to \gamma Z$. However, their measurability is expected to be difficult and requires dedicated studies in future. Here we consider the easily recognizable $H \to \gamma\gamma$ decay mode with the hope to observe the 2-photon Higgs decay despite the presence of large background.

For an accumulated luminosity of 500 $fb^{-1}$ we only expect 39 events from the Higgsstrahlung process

$$e^+e^- \to H(160)Z \to \gamma\gamma q\bar{q},$$

which have to be confronted with orders of magnitude larger irreducible back-
Figure 5: The $b\bar{b}$ mass distribution from the selected events of the 2-jet dilepton topology.

The signal and background diagrams contributing to these reactions are presented in Fig. 6.

Additional background comes from the reaction

$$e^+e^- \rightarrow \gamma Z \rightarrow \gamma \gamma q\bar{q}.$$  \hfill (16)

The signal and background diagrams contributing to these reactions are presented in Fig. 6.

Additional background comes from the reaction

$$e^+e^- \rightarrow \gamma Z \rightarrow \gamma q\bar{q} (\gamma),$$  \hfill (17)

where the photon in parentheses comes from initial state radiation and/or quark fragmentation tail and fluctuation effects. Since initial state photon radiation is already covered to a great extent by reaction (16) we have not combined both background contributions in the following. Hence, reaction (16) represents only some lower background limit.
In order to remove most of this background while retaining significant signal events in the $\gamma\gamma$ mass close to $M_H$ dedicated selection procedures are absolutely mandatory. After exploring a variety of requirements like e.g.

- total visible energy $> 240$ GeV;
- total transverse energy $> 30$ GeV;
- $p_T(\gamma_1) > p_T(\gamma_2) > 20$ GeV;
- $p_T(\gamma_1) + p_T(\gamma_2) > 64$ GeV;
- $|\cos \theta_{\gamma_1/2}| < 0.7$;
- two and only two jets with an invariant mass within $M_Z \pm 10$ GeV, after removal of the two selected photons from the jet finder algorithm;
- $|\cos \theta_{\text{jet}}| < 0.7$ for each jet,

we obtain the $\gamma\gamma$ invariant mass distribution as shown in Fig.6. As can be seen, the background in the Higgs region is overwhelming. The surviving Higgs event rate, indicated as cross-hatched in the figure, is so small that a reasonable BF ($H \rightarrow \gamma\gamma$) measurement remains challenging for future investigations. A multidimensional analysis based on a likelihood estimator which includes 16 variables had not improved the results obtained. Whether the addition of $H \rightarrow \gamma\gamma$ fusion events, with an about four times less production cross section, would significantly alter our conclusion remains to be studied in detail.

\[\text{Figure 6: SM diagrams contributing to reactions (15) and (16).}\]
Figure 7: The $\gamma \gamma$ mass spectrum from the surviving signal and background events of the reaction $e^+e^- \rightarrow \gamma \gamma \ Z \rightarrow \gamma \gamma \ q \bar{q}$ . The Higgs events are indicated cross-hatched.

3 The total Higgs width $\Gamma_{tot}(H)$

Besides the measurements of the Higgs branching fractions to fermions and bosons - precisely as possible and independent of any model or assumptions - the measurement of its total width is an important concern. The procedure for ascertaining this quantity is based on the precisions for $BF(H \rightarrow ZZ^*)$ of 14.5 % and the inclusive Higgsstrahlung cross section of 2.8 % . From that the error of the total Higgs width is calculated to $\Delta \Gamma_{tot}(H) = \pm 15 \%$ which is mainly governed by the uncertainty of $BF(H \rightarrow ZZ^*)$. 
It is worthwhile to remind that for $M_H$ of about 120 GeV the suggestion is to measure the $H \rightarrow \gamma \gamma$ partial width by means of a $\gamma \gamma$ collider and if combined with BF($H \rightarrow \gamma \gamma$) and BF($H \rightarrow b \bar{b}$) measurements from $e^+ e^-$ collisions, $\Gamma_{tot}(H)$ can be calculated. An analogous procedure for a 160 GeV Higgs seems to be prevented as long as an acceptable measurement fails for BF($H \rightarrow \gamma \gamma$); see sect.2.4. Fortunately, BF($H \rightarrow ZZ^*$) and $\sigma(HZ)$ measurements allow to set a rather precise value for $\Delta \Gamma_{tot}(H)$ and, if in addition the accurate value of BF($H \rightarrow WW^*$) can be combined with a precise measurement for the WWH coupling from e.g. the fusion reaction $e^+ e^- \rightarrow \nu \bar{\nu} \ H \rightarrow \nu \bar{\nu} \ WW^*$, a superior total Higgs width is feasible.

4 Conclusions

We have studied the prospects for measuring branching fractions for a SM-like Higgs boson with $M_H = 160$ GeV at the TESLA linear $e^+ e^-$ collider, assuming $\sqrt{s} = 350$ GeV and an integrated luminosity of 500 fb$^{-1}$. The resulting errors of these measurements convoluted with an expected uncertainty of 2.8 % for the total Higgsstrahlung cross section are summarized in Table 1.

| Branching fraction | Expected error |
|--------------------|----------------|
| BF ($H \rightarrow WW^*$) | ± 3.5 % |
| BF ($H \rightarrow ZZ^*$) | ± 14.5 % |
| BF ($H \rightarrow b \bar{b}$) | ± 12 % |

Table 1: Branching fraction errors expected for the SM Higgs boson with $M_H = 160$ GeV, at $\sqrt{s} = 350$ GeV and an integrated luminosity of 500 fb$^{-1}$.

The measurement of BF($H \rightarrow \gamma \gamma$) remains a great challenge since besides its very small SM value itself the background expected to contribute is orders of magnitude larger which renders a meaningful signal over background selection.

The procedure for ascertaining the total Higgs width and its error has been outlined, with the result of $\Delta \Gamma_{tot}(H) = ±15$ %. This estimate is based on uncertainties expected for BF($H \rightarrow ZZ^*$) of 14.5 % and the total Higgsstrahlung cross section of 2.8 %.

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1 The $H \rightarrow \gamma \gamma$ partial width itself stands out as an observable of considerably physical importance. Recent studies indicate that $\gamma \gamma$ collisions allow for a $\sim$5 % error determination of $\Gamma(H \rightarrow \gamma \gamma)$, for $M_H = 160$ GeV.
Finally, we would like to emphasize that the selection criteria proposed are rather simple and not yet optimized. For the future, we expect significant improvements if the precise detector behaviour is known, further topologies not yet involved in the analyses are taken into account and, once the Higgs mass is known, \( \sqrt{s} \) is optimized for running in the HZ measurement mode.

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