ABSTRACT: The essential purpose of the footbridges is carrying over pedestrians to a certain destination, ensuring their safety and comfort. Their design is not always an easy task, considering the huge variety of load impacts induced by human motion combined with the complexity of the structure itself. The natural frequencies of the contemporary footbridges belong to a lower range, close to the one of the excitation, following from the use of lightweight materials and extravagant constructive solutions (stress ribbon footbridges, suspension footbridges, long-span footbridges).

In the present article, an extensive study of an existing footbridge is performed. The dynamic analysis of the structure is conducted by employing a load model accounting for the human-human interaction.

KEY WORDS: Social force model, vertical accelerations, footbridge.

1 INTRODUCTION
The human-human interaction between participants in a crowd flow represents the process of adjusting their motion based on the perception of the surrounding circumstances. The current paper accounts for this phenomena by using the social force model [1]. By employing the model, the behaviour of every single pedestrian is defined and the respective temporal and spatial variables (velocities and positions) are acquired.

The present article employs determining parameters for the social force model.

2 SOCIAL FORCE MODEL
The social force model of a crowd represents an assembly of simulations of pedestrian motion [1]. The model output, pedestrian positions \( r_\alpha(t) \) and pedestrian velocities
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$v_\alpha(t)$, can be subsequently used as input data for dynamic analysis of pedestrian structures [2].

Social force itself is the variation in the pedestrian actual velocity. These forces represent the influence of the ambient circumstances on pedestrian’s behaviour. The overall force comprises three components:

- Driving force $-f^0_\alpha(v_\alpha)$
- Repulsive force due to pedestrian interaction $-f_{\alpha\beta}(r_\alpha, v_\alpha, r_\beta, v_\beta)$
- Repulsive force due to the presence of boundaries and obstacles $-f_{\alpha B}(r_\alpha)$

2.1 SAMPLING PEDESTRIAN FREQUENCIES AND STEPS

Based on the integration of the differential equations, the trajectories, as discrete position points, and speed of each pedestrian at every time step $t_s$ of the mathematical solution are obtained.

Next, an empirical relation, based on experimental data from laboratory trials on a treadmill, can be used [3]:

$$f_s(t) = 0.35v_{\text{tot}}^3(t) - 1.59v_{\text{tot}}^2(t) + 2.93v_{\text{tot}}(t),$$

where $v_{\text{tot}}$ is a total velocity, m/s.

The plausible velocity limits for this empirical formula (1) are adopted 0.23–2.2 m/s in conformity with [4]. Employing this relationship using the total instantaneous velocity directly taken from the integration of the differential equations, the sampled velocities, frequencies, coordinates and time instants for each step of the pedestrian on the bridge are derived (Figs. 1, 2) [2].

For the velocity and frequency depending on the particular circumstances, an averaging over a relevant time window is implemented aiming to avoid extraordinary values which are of main interest.

The sampling procedure is repeated for the period while a pedestrian is on the bridge based on averaged velocities. Namely, these averaged variables are used as governing for calculating the response of the structure.

Figure 1 manifests the detailed trajectory as a result of the mathematical solution for each time step (discontinuous blue line) of a random pedestrian being part of a crowd with density $d = 0.5$ ped./m$^2$. The red crosses applied on the trajectory represent the virtual steps performed by the pedestrian when walking on the bridge. The trajectory of a pedestrian participating in an unidirectional flow is quite fluent compared to the trajectory of pedestrian of bidirectional flow – the trajectory has many abrupt changes corresponding to the meeting of oppositely moving pedestrians.
Figure 2 displays the alteration of the velocities, frequencies and stride lengths in the course of time for unidirectional (left) and bidirectional flows (right). A high resemblance in respect to the form is noticeable between the plots of the velocity, frequency and stride length. It can be seen that for the case of the unidirectional flow, there is a good similarity between the instantaneous and averaging sampling. This is a rationale for the settled shape of the graphs in Figs. 2a, 2c, 2e as well as the graph of the trajectory (Fig. 1a) – they are smooth with no outstanding sectors. On the contrary, the graphs describing the bidirectional flow (Figs. 1b, 2b, 2d, 2f) may be recognized as immensely fluctuating due to the inconstant nature of the process. There are large discrepancies between the values of the parameters (velocities, frequencies and stride lengths) derived by instantaneous and averaged sampling which are most noticeable in Fig. 2f – first, there are considerable mismatches between the actual stride lengths and these evaluated using the relationship \( \ell_s(t) = \frac{v_{tot}(t)}{f_s(t)} \), second – between the instantaneous and averaged sampling. Namely, in order to avoid such abnormal variations – especially the step frequency has instant drops to 0.6 Hz – the averaged sampling is used further as leading for the evaluation of the structural response [2].

3 RANDOM FLOW

The random crowd flow is defined for the crowd densities concerned in the current codes of practice [5,6]. It is strictly unidirectional, since the pedestrians do not affect each other. The trajectories are defined as straight lines. The walking paths are con-
Fig. 2: Depiction of the total velocity, step frequency and stride length for an arbitrary pedestrian in a unidirectional (left) and bidirectional (right) crowd flow with $d = 0.5 \text{ ped./m}^2$ [2]
considered uniformly distributed along the width of the bridge deck. During the initial stage the pedestrians’ arrival times follow Poisson distribution and afterwards every pedestrian leaving the structure is replaced by a new one, hereby keeping the desired density of the flow. The step frequencies follow Gaussian distribution $N(\mu_{fs} = f_s, \sigma_{fs} = 0.175 \text{ Hz})$, where the mean $\mu_{fs}$ varies for the different crowd densities (Table 1). As opposed to the main premise in the current codes of practice for resonant condition, for the particular case of Eeklo footbridge the mean step frequencies are taken in compliance to [7]. The walking speeds $v_s$ are adopted as constant for each pedestrian in the crowd flow according to [8] (Table 1).

Table 1: Mean step frequencies according to [7] and constant velocities according to [8] for different crowd densities

| Crowd density $d$, ped./m$^2$ | Mean step frequency $\mu_{fs}$, Hz | Speed, $v_s$, m/s |
|-------------------------------|-----------------------------------|------------------|
| 15 pedestrians                | 2.00                              | 1.32             |
| 0.2                           | 1.93                              | 1.30             |
| 0.5                           | 1.81                              | 1.23             |
| 0.8                           | 1.69                              | 1.12             |
| 1.0                           | 1.61                              | 1.02             |
| 1.5                           | 1.41                              | 0.78             |

4 Response Prediction Including Human-Human Interaction

In the following, the slender Eeklo footbridge is considered as ongoing example (Fig. 3).

Fig. 3: Eeklo footbridge
Fig. 4: Plan (left) and side view (right) of the footbridge
4.1 MODAL ANALYSIS – EEKLO FOOTBRIDGE

The structural model of the footbridge is a continuous beam with three spans and overall length of 96 m (Fig. 4). The central span has length of 42 m, whereas the two side-ward spans – 27 m. The structure comprises two main I-section steel girders.

(a) Mode 1 - $f_1 = 1.03$ Hz

(b) Mode 2 - $f_2 = 1.70$ Hz, $\xi_2 = 1.94\%$

(c) Mode 3 - $f_3 = 3.00$ Hz, $\xi_3 = 0.19\%$

(d) Mode 4 - $f_4 = 3.30$ Hz, $\xi_4 = 1.45\%$

(e) Mode 5 - $f_5 = 3.43$ Hz, $\xi_5 = 2.97\%$

Fig. 5: Natural modes of vibrations
with height 1.2 m and axial distance between them 3.4 m, using the girders with their height as railings. T-shaped beams are implemented parallel to the main girders, spaced approximately through 0.85 m. There are secondary transverse beams of same shape implemented through 4.2 m in the midspan and through 4.5 m in the other two spans. The bridge deck is implemented by steel sheets of thickness 8 mm. The weight of the top structure is 440 kN.

Modal analysis is executed for 10 mode shapes. The results acquired by a finite element model are calibrated in such a way to coincide with the results from the in situ measurements [9]. Modes from 2-nd to 5-th are taken into account in the dynamic analysis of the structure (Fig. 5).

4.2 SINGLE-FOOT FORCE MODEL

In the literature sources, the following two methods are most generally applied to describe the loading: continuous loading function and single step function. For the purposes of the current study, the single-foot force model derived by Li [10] is used. The plot obtained through this model is a double-hump graph (Fig. 6). The first peak results from the heel strike and the second from the toe-off effect [11]. Increasing the pacing frequency makes the peaks more distinctive. That is the reason why the pacing frequency has the greatest influence on the force-time history, and hence on the dynamic effect of the human motion.

Fig. 6: Force-time function due to single steps for $f_s = 1.8 \text{ Hz}$, $f_s = 1.9 \text{ Hz}$ and $f_s = 2.0 \text{ Hz}$ according to [10]
Single-foot force model differentiates the human-induced loading as discrete loading for each step (Fig. 6). The formulation by Li [10] is as follows:

\[ F_e(t) = G \sum_{i=1}^{n} \alpha_i \sin\left(\frac{\pi i}{T_e} t\right), \]

where \( G \) is static component of the force, namely the weight of the pedestrian; \( \alpha_i \) – dynamic load factor of the \( i \)-th component; \( T_e \) – duration of the step; \( T_s = 1/f_s \) – period of stepping; \( n \) – number of harmonic components accounted for. The ratio between the walking period and the duration of the step is considered as constant during the moving process and appears to be \( T_s/T_e = 0.76 \) [12].

4.3 RESPONSE TO PEDESTRIAN FLOWS

For the purposes of the present computations the pedestrian flows are defined for all crowd densities considered in [5, 6]. The number of random flows is selected such that the vertical acceleration has converged - 500 random crowd flows are simulated. In addition, the calibrated sets of the social force model parameters [13] are used in the present section to describe realistic pedestrian flows on the Eeklo footbridge. Using the information acquired in Subsection 2.1, related to positions and step frequencies, and employing the classical uncoupled equations of motion for linear elastic system [14], the results in terms of vertical accelerations at the center of the midspan at the side of the railing are derived. Modes from 2 to 5 are incorporated in the calculations, assuming that the higher modes would have minor contribution to the overall response. The time step for the response calculations is \( t_s = 0.025 \text{ s} \).

Figure 7 shows the power spectral density (PSD) of the vertical accelerations for all relevant densities for unidirectional, bidirectional and random flows. No matter the crowd density or the type of the flow, the graphs show the most prominent peak for frequency 3.00 Hz which corresponds to the natural frequency of the first bending mode of the footbridge. The higher the crowd density, the more pronounced the peak, which shows the major importance of the third mode of vibrations. From the graph, it can be observed that there is a better accordance between results for the two flows defined by the social force model (especially for low densities up to \( d = 0.5 \text{ ped./m}^2 \)), than with the random flow.

- Random flow

For the case of the random flows, apart from the highest peak centered around the third natural frequency 3.00 Hz, two lower peaks are present at 1.70 Hz and 6.45 Hz. The first being the frequency of the second mode of vibration and the latter being close to the second harmonic component for the third mode.
Fig. 7: PSD of the vertical accelerations for unidirectional (solid), bidirectional (dotted) and random crowd (dashed) flows with densities according to the current codes of practice [5, 15]

- Social force model

For the case of the unidirectional flow, except for the major peak around 3.00 Hz, significant peaks are present around the frequencies of the second and fourth natural modes, respectively $f = 1.71$ Hz and $f = 3.30$ Hz – they are more pronounced for densities 0.5 ped./m², 0.8 ped./m², 1.0 ped./m². Especially for the case with
$d = 1.0$ ped./m$^2$, the three peaks are of one and the same order, showing that the mean step frequency is close to the frequency of the second mode $\mu_{fs} = 1.75$ Hz (Table 3) and simultaneously its second multiple is close to the fourth mode. A serious peak centered around $f = 3.00$ Hz is apparent for crowd density $d = 1.5$ ped./m$^2$. The vertical acceleration for this case acquire eminent values of 2.96 m/s$^2$ with 50% and 4.55 m/s$^2$ with 5% probability of exceedance, which is approximately five times higher than the response of the previous traffic density of $d = 1.0$ ped./m$^2$. A justification of these values may be detected in the distribution of step frequencies for this flow $\mu_{fs} = 1.58$ Hz, $\sigma_{fs} = 0.12$ Hz (Table 3). Consequently, this results in almost resonant response due to the second harmonic of the human-induced loads in Mode 3 and explains the sole and major peak in the PSD for density $d = 1.5$ ped./m$^2$. In general, Table 2 shows that as the pedestrian density increases, so does the structural response, with a major increase for $d = 1.5$ ped./m$^2$.

Similar to the response due to unidirectional and random flows, the results for bidirectional flows show a dominant excitation of the structure in the third mode. Again, less significant peaks are observed around the frequency of the second natural mode of vibration. The latter get larger for higher densities, as the mean step frequency $\mu_{fs}$ decreases toward the frequency of the second natural mode of vibration $f_{n,2}$ and for the ultimate density $d = 1.5$ ped./m$^2$, $\mu_{fs}$ is almost equal to $f_{n,2}$, namely $\mu_{fs} = 1.68$ Hz. On the whole, the results for bidirectional flows show a progressive increase with the crowd density, reaching maximum values of 1.48 m/s$^2$ with 50% and 2.02 m/s$^2$ with 5% probability of excess for the highest density of $d = 1.5$ ped./m$^2$.

Finally, the results for the vertical accelerations for the three types of flows, expressed with 50% and 5% probability of exceedance are summarized in Table 2.

| Crowd density $d$, ped./m$^2$ | Unidirectional flow | Bidirectional flow | Random flow |
|--------------------------------|---------------------|--------------------|-------------|
|                                | $a_{vert,50\%}$, m/s$^2$ | $a_{vert,5\%}$, m/s$^2$ | $a_{vert,50\%}$, m/s$^2$ | $a_{vert,5\%}$, m/s$^2$ |
| 15 ped.                        | 0.20                | 0.37               | 0.19        | 0.33        | 0.17        | 0.25        |
| 0.2                            | 0.33                | 0.47               | 0.41        | 0.48        | 0.32        | 0.47        |
| 0.5                            | 0.49                | 0.62               | 0.67        | 0.87        | 0.60        | 0.88        |
| 0.8                            | 0.57                | 0.69               | 0.94        | 1.27        | 1.04        | 1.43        |
| 1.0                            | 0.59                | 0.82               | 1.14        | 1.42        | 1.35        | 1.91        |
| 1.5                            | 2.96                | 4.55               | 1.48        | 2.02        | 1.81        | 2.58        |
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Accelerations derived for bidirectional flow exceed the ones for the unidirectional flow with some acceptable differences, except for \( d = 1.5 \) ped./m\(^2\). For this last particular case the accelerations for the unidirectional flow are approximately twice as high as the results for bidirectional flow due to the reasons stated above. On the other hand, a better correspondence in the results is observed between bidirectional and random flows. A gradual increase of the vertical acceleration with the crowd density is present for the three types of flows.

Table 3 together with Fig. 8 give the distribution of the step frequencies for the unidirectional and bidirectional flows. It can be observed that, as expected, the unidirectional flow is characterized by lower standard deviations than the bidirectional flow, which results into lower variability for the former and higher variability for the latter flow.

A simple comparison of the \( \mu_{fs} \) and \( \sigma_{fs} \) for the unidirectional and bidirectional flows (Fig. 8) shows almost identical values for \( \mu_{fs} \) for one and the same density and significant differences for the \( \sigma_{fs} \). The values of \( \sigma_{fs} \) for bidirectional flow are higher, which implies larger dispersion of the step frequencies and a possibility that more pedestrians from the flow will have pacing frequency equal or in close range to some of the natural frequencies. This is a simple explanation for the bigger responses acquired for bidirectional flow. The only exception for the case of \( d = 1.5 \) ped./m\(^2\) stems from the fact there is more considerable difference between the \( \mu_{fs} \) for the two flows with \( \mu_{fs,uni} > \mu_{fs,bi} \) and the higher \( \sigma_{fs} \) for unidirectional flow. From Fig. 8f, it can be seen that for unidirectional flow the number of pedestrians having step frequency equal or close to some of the natural frequencies or its multiples, and more specifically to the one of the third mode, is larger than for bidirectional flow more serious responses are evaluated for unidirectional flow for this density.

Figure 9 (left) resumes the results for the uni- and bidirectional flows and for the random crowd flow with 50 % probability of exceedance. The figure shows that for

| Crowd density \( d \), ped./m\(^2\) | Unidirectional flow | Bidirectional flow |
|-------------------------------------|---------------------|---------------------|
|                                     | \( \mu_{fs, uni} \), Hz | \( \sigma_{fs, uni} \), Hz | \( \mu_{fs, bi} \), Hz | \( \sigma_{fs, bi} \), Hz |
| 15 pedestrians                      | 1.89                | 0.14                | 1.87                | 0.18                |
| 0.2                                 | 1.88                | 0.11                | 1.86                | 0.19                |
| 0.5                                 | 1.85                | 0.07                | 1.84                | 0.22                |
| 0.8                                 | 1.81                | 0.07                | 1.81                | 0.26                |
| 1.0                                 | 1.75                | 0.06                | 1.78                | 0.29                |
| 1.5                                 | 1.58                | 0.12                | 1.68                | 0.34                |
Fig. 8: PDF of the sampled pedestrian frequencies for first (blue) and second (green) harmonic component for unidirectional crowd flows (dashed) and bidirectional crowd flows (continuous) with densities according to the current codes of practice [5, 6]
small densities up to $d = 0.5$ ped./m$^2$, there is convergence between the responses due to the three flows. For higher densities – $d = 0.8$ ped./m$^2$, $d = 1.0$ ped./m$^2$ – a more obvious resemblance is present between the results for bidirectional and random flows, whereas the values for the unidirectional flow are relatively smaller. Generally, it might be stated that, at least minimum comfort level according to [5,15] is insured for the pedestrians, except for the case of unidirectional flow with density $d = 1.5$ ped./m$^2$, as a resonant phenomena is quite probable.

Figure 9 (right) generalizes the vertical accelerations for the three types of flows (unidirectional, bidirectional and random) with 5% probability of exceedance. Again, a good agreement of the results for the lower densities is observable. For higher densities, the results are quite scattered. For the case of $d = 1.5$ ped./m$^2$ the dispersion of the results is most notable. At least minimum comfort level is assured for all traffic densities, traffic flow, except for the unidirectional flow for density of $d = 1.5$ ped./m$^2$.

5 CONCLUSIONS

The dynamic response of Eeklo footbridge was comprehensively studied for the cases of unidirectional, bidirectional and random flows. The unidirectional and bidirectional were defined by the popular social force model.

The data acquired from the social force model is translated into sequence of single-step forces and is applied on the Eeklo footbridge. The results of the dynamic response obtained through the social force model shows diverse results for the unidirectional and bidirectional flows. An interpretation for that is found in the
difference of the probability distributions of the step frequencies. The specific case of unidirectional crowd flow with density $d = 1.5 \text{ ped./m}^2$ attracts attention with the fact nearly resonance occurs due to the second harmonic component of the load - a phenomena denied as fictional in the literature sources. For all densities accounted for, a better correspondence between the results for bidirectional and random flows is present, rather than between unidirectional and bidirectional flows. The accelerations due to unidirectional and bidirectional flows differ almost two times in favour of the bidirectional flow.

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