Superconducting triplet spin valve

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We study the critical temperature $T_c$ of SFF trilayers (S is a singlet superconductor, F is a ferromagnetic metal), where the long-range triplet superconducting component is generated on noncollinear magnetizations of the F layers. We demonstrate that $T_c$ can be a nonmonotonic function of the angle $\alpha$ between the magnetizations of the two F layers. The minimum is achieved at an intermediate $\alpha$, lying between the parallel (F, $\alpha = 0$) and antiparallel (AP, $\alpha = \pi$) cases. This implies a possibility of a “triplet” spin-valve effect: at temperatures above the minimum $T_c^{P}$ but below $T_c^{AP}$ and $T_c^{MP}$, the system is superconducting only in the vicinity of the collinear orientations. At certain parameters, we predict a reentrant $T_c(\alpha)$ behavior. At the same time, considering only the P and AP orientations, we find that both the “standard” ($T_c^P < T_c^{AP}$) and “inverse” ($T_c^P > T_c^{AP}$) switching effects are possible depending on parameters of the system.

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In superconducting spin valves with the layer sequence F1/S/F2 the superconducting transition temperature $T_c$ of the system can be controlled by mutual alignment of magnetizations $M_{1,2}$ of the two ferromagnetic layers F1 and F2. Therefore, at a temperature $T$ fixed inside the range of $T_c$ variation, there is an opportunity for switching the superconductivity on and off by reversing the magnetization direction of the F1 or F2 layer. Model calculations have shown that the transition temperature $T_c^{AP}$ for the antiparallel ($M_1 \uparrow \downarrow M_2$) orientation of the F1 and F2 magnetizations should be higher than the transition temperature $T_c^P$ for the opposite case ($M_1 \uparrow \uparrow M_2$) [1, 2, 3]. The situation with this order of $T_c$’s (i.e., $T_c^{P} < T_c^{AP}$) is commonly referred to as the “standard” switching (see, e.g., [4]), and the switching in this case actually occurs at temperatures $T$ such that $T_c^{P} < T < T_c^{AP}$. The basic physical reason for the difference $\Delta T_c = T_c^{AP} - T_c^{P} > 0$ is partial compensation of the pair-breaking ferromagnetic exchange field, if the magnetizations of the F1 and F2 layers are aligned antiparallel.

Several experimental groups have published results on superconducting spin valves of the F1/S/F2 type [4, 5, 6, 7, 8, 10, 11, 12, 13]. The experimental results turned out to be controversial. Some studies of F1/S/F2 structures have shown the standard spin-valve effect [4, 5, 6, 7, 8] with the maximum shift $\Delta T_c \approx 41$ mK reported for the Ni/Nb/Ni trilayer in [7]. However, some experiments revealed the “inverse” spin-valve effect [9, 10, 11, 13] with $T_c^P > T_c^{AP}$ (i.e., $\Delta T_c < 0$). The most advanced calculations within the proximity effect theory, which take into account the triplet components of the superconducting pairing [14], demonstrate only the standard switching [3, 15] with $T_c$ monotonically increasing from the P to AP configuration [3]. Additional physical mechanisms like spin imbalance effect [9, 11] or magnetic domain structure [10, 12] should be recruited to explain the inverse spin-valve effect in the studied F1/S/F2-type structures.

A bit earlier an unconventional spin-valve-like S/F1/F2 structure was theoretically proposed in [16] to control the superconducting $T_c$ in the S layer by mutual alignment of the magnetizations of the two adjacent ferromagnetic layers F1 and F2. The authors of [16] argued that $T_c^P < T_c^{AP}$ in their system because of partial cancelation of the pair-breaking exchange fields just within the magnetic F1/F2 subsystem of the structure, thus predicting the standard switching as in the interleaved F1/S/F2 structure.

The S/F1/F2 structures are much less investigated experimentally [8, 17], and the experiments indicate the standard switching effect [16] with the maximal size of about 200 mK. In this Letter we study the critical temperature of a S/F1/F2 trilayer at arbitrary angle between the in-plane magnetizations of the ferromagnetic layers (see Fig. 1). We demonstrate that this structure allows not only the standard but also inverse spin-switching effect. Moreover, we show for the first time...
that the minimal critical temperature $T_c^{TV}$ of the structure is achieved at a noncollinear alignment of the magnetizations, when the long-range triplet component of the superconducting pairing is generated. Since $T_c^{TV}$ is lower than both $T_c^S$ and $T_c^{AP}$, this offers a possibility of a “triplet spin-valve effect” never reported before.

1. Model. We consider the S/F1/F2 structure in the dirty limit, which is described by the Usadel equations. Near $T_c$, the Usadel equations are linearized and contain only the anomalous Green function $\tilde{f}$:

$$D \frac{d^2 \tilde{f}}{dx^2} - \omega |\tilde{f} - \frac{i \text{sgn}\omega}{2} \left( \hat{\tau}_0 (\mathbf{h} \sigma), \tilde{f} \right) + \Delta \hat{\tau}_1 \hat{\sigma}_0 = 0.$$  (1)

Here, $\tilde{f}$ is a $4 \times 4$ matrix, $\hat{\tau}_i$ and $\hat{\sigma}_i$ are the Pauli matrices in the Nambu-Gor’kov and spin spaces, respectively, $D$ is the diffusion constant, and $\omega = \pi T_c (2n + 1)$ with integer $n$ is the Matsubara frequency. The exchange field in the middle F1 layer is along the $z$ direction, $\mathbf{h} = (0, 0, h)$, while the exchange field in the outer F2 layer is in the $yz$ plane: $\mathbf{h} = (0, h \sin \alpha, h \cos \alpha)$. The angle $\alpha$ changes between 0 (parallel configuration, P) and $\pi$ (antiparallel configuration, AP). The order parameter $\Delta$ is real-valued in the superconducting layer, while in the ferromagnetic layers it is zero. In general, the diffusion constant $D$ acquires a proper subscript, S or F, when Eq. (1) is applied to the superconducting or ferromagnetic layers, respectively. However, for simplicity we take them equal in this paper, because this assumption does not influence qualitative behavior of $T_c(\alpha)$.

The Green function $\tilde{f}$ can be expanded into the following components:

$$\tilde{f} = \hat{\tau}_1 (f_0 \hat{\sigma}_0 + f_3 \hat{\sigma}_3 + f_2 \hat{\sigma}_2),$$  (2)

where $f_0$ is the singlet component, $f_3$ is the triplet with zero projection on the $z$ axis, and $f_2$ is the triplet with $\pm 1$ projections on $z$ (the latter is present only if $\alpha \neq 0, \pi$). The singlet component is even in frequency (and real-valued), while the triplet ones are odd (and imaginary): $f_0(-\omega) = f_0(\omega)$, $f_3(-\omega) = -f_3(\omega)$, and $f_2(-\omega) = -f_2(\omega)$, which makes it sufficient to consider only positive Matsubara frequencies, $\omega > 0$.

As we show below, the problem of calculating $T_c$ can be reduced to an effective set of equations for the singlet component in the S layer: the set includes the self-consistency equation and the Usadel equation,

$$\Delta \ln \frac{T_c}{T_c} = 2\pi T_c \sum_{\omega > 0} \left( \frac{\Delta}{\omega} - f_0 \right),$$  (3)

$$\frac{D}{2} \frac{d^2 f_0}{dx^2} - \omega f_0 + \Delta = 0,$$  (4)

with the boundary conditions

$$\left. \frac{df_0}{dx} \right|_{x=-d_S} = 0, \quad -\xi \left. \frac{df_0}{dx} \right|_{x=0} = W f_0 \left|_{x=0} \right..$$  (5)

Here $T_{cS}$ and $\xi = \sqrt{D/2\pi T_{cS}}$ are the superconducting transition temperature and coherence length for an isolated S layer, and we assume that the S layer occupies the region $-d_S < x < 0$ (see Fig. 1). This is exactly the problem for which the multi-mode solution procedure (as well as the fundamental-solution method) was developed in [19] and then applied to F1/S/F2 spin valves in [3]. We only need to determine the explicit expression for $W$ in Eq. (5), solving the boundary problem for the S/F1/F2 structure.

2. Solution of the model. To simplify derivations, while keeping the essential physics, we consider the middle ferromagnetic layer F1 of arbitrary thickness ($0 < x < d_F$) but the outer ferromagnetic layer F2 being semi-infinite ($d_F < x < \infty$). The Usadel equation (1) generates the following characteristic wave vectors:

$$k_\omega = \sqrt{\frac{2\omega}{D}}, \quad k_h = \sqrt{\frac{h}{D}}, \quad \tilde{k}_h = \sqrt{k_h^2 + 2ik_h^2}. \quad (6)$$

Only $k_\omega$ appears in the solution for the S layer, while the F-layers’ solutions are described by $k_\omega$, $\tilde{k}_h$, and $k_h^*$. Since the exchange energy is usually larger than the superconducting energy scale, $h \gg T_c$, the $k_\omega$ mode in the ferromagnetic layers (arising at noncollinear magnetizations) represents the long-range triplet component [14], which plays the key role in the present study.

In the S layer the solution of Eq. (1) is:

$$\begin{pmatrix} f_0(x) \\ f_3(x) \\ f_2(x) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ A \end{pmatrix} \cosh \left( k_\omega (x + d_S) \right) \cosh \left( k_\omega ds \right). \quad (7)$$
The singlet component $f_0(x)$ in the S layer cannot be written explicitly, since it is self-consistently related to the (unknown) order parameter $\Delta(x)$ by Eqs. (6)-(9). Our strategy now is to obtain the effective boundary conditions for $f_0(x)$, eliminating all other components in the three layers.

In the middle F1 layer the solution of Eq. (11) reads:

\[
\begin{pmatrix}
  f_0(x) \\
  f_3(x) \\
  f_2(x)
\end{pmatrix} = C_1 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \cosh (k_\omega x) + S_1 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \sinh (k_\omega x) + \\
+ C_2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \cosh (k_h x) + C_3 \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \cosh (\tilde{k}_h x) + \\
+ S_2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \sinh (k_h x) + S_3 \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \sinh (\tilde{k}_h x). \tag{8}
\]

Finally, the solution in the semi-infinite, outer F2 layer is built only from descending modes:

\[
\begin{pmatrix}
  f_0(x) \\
  f_3(x) \\
  f_2(x)
\end{pmatrix} = E_1 \begin{pmatrix} 0 \\ -\sin \alpha \\ \cos \alpha \end{pmatrix} \exp (-k_\omega (x - d_F)) + \\
+ E_2 \begin{pmatrix} 1 \\ \cos \alpha \sin \alpha \end{pmatrix} \exp (-\tilde{k}_h (x - d_F)) + \\
+ E_3 \begin{pmatrix} -1 \\ \cos \alpha \sin \alpha \end{pmatrix} \exp (-\tilde{k}_h (x - d_F)). \tag{9}
\]

We will use the simplest, perfect-transparency boundary conditions at the S/F1 and F1/F2 interfaces (the case $\gamma = 1$ and $\gamma_B = 0$ in the notations of [20]):

\[
f_1|_{\text{left}} = f_1|_{\text{right}}, \quad \frac{df_1}{dx}|_{\text{left}} = \frac{df_1}{dx}|_{\text{right}}. \tag{10}
\]

Altogether there are 12 boundary conditions at the two interfaces (S/F1 and F1/F2). We are mainly interested in one of them, determining the derivative of the singlet component on the S side of the S/F1 interface ($x = 0$):

\[
\frac{df_0}{dx}|_{x=0} = 2 \text{Re}(\tilde{k}_h S_2). \tag{11}
\]

The remaining 11 boundary conditions form a system of 11 linear equations for 11 coefficients entering Eqs. (4-9). The solution of this system is nonzero due to $f_0(0)$ coming from Eq. (4) and entering the “right-hand side” of the system. Finding the $S_2$ coefficient [which is proportional to $f_0(0)$], we substitute it into Eq. (11) and thus explicitly find $W$ entering the effective boundary conditions [3].

3. Analysis of the solution. After reducing the problem to Eqs. (3-5), all the information about the two F layers is contained in the single real-valued function $W$. This function makes $f_0(x)$ bend at the S/F1 interface, hence the larger $W$, the stronger $T_c$ is suppressed.

The explicit expression for $W(\alpha)$ is very cumbersome and we do not write it here. However, certain analytical development (as well as complete numerical analysis) is possible. For the analytical consideration, we make an additional assumption of $T_c \ll h$, which implies $k_\omega \ll k_h$. For the collinear cases ($\alpha = 0$ and $\alpha = \pi$) we then find $W(0) = 2k_h \xi$ and

\[
W(0) - W(\pi) = 2k_h \xi \frac{\sqrt{2} \sin (2k_h d_F + \pi/4) - e^{-2k_h d_F}}{\sinh (2k_h d_F) + \cos (2k_h d_F)}, \tag{12}
\]

which oscillates as a function of $d_F$, changing its sign (see Fig. 2). Thus, as a result of interference in the middle F1 layer, we can either have the standard spin-switching effect with $T_c^{\text{P}} < T_c^{\text{AP}}$ [when the pair-breaking at the P alignment is stronger than at the AP alignment of magnetizations, i.e., at $W(0) - W(\pi) > 0$ as in the range $2k_h d_F < 3\pi/4$ in Fig. 2] or the inverse spin-switching effect with $T_c^{\text{P}} > T_c^{\text{AP}}$ [at $W(0) - W(\pi) < 0$ as in the range $3\pi/4 < 2k_h d_F < 7\pi/4$ in Fig. 2]. Note that the amplitude of the inverse effect is notably smaller compared with the standard one.
The analytical results obtained at $k_\omega \ll k_h$ are illustrated and extended by numerical calculations at arbitrary relation between $k_\omega$ and $k_h$. Figure 3 shows dependence of the transition temperature $T_c$ on the angle $\alpha$ between the magnetizations. We see that at small thicknesses $d_F$ of the middle ferromagnetic layer F1, the switching effect is standard, while at larger $d_F$ the effect is inverse ($T_c^P > T_c^{AP}$). Moreover, when the F1 layer thickness is around a half of the coherence length $\xi$, the minimal critical temperature $T_c^{\xi}$ at noncollinear orientations is significantly lower than both $T_c^P$ and $T_c^{AP}$ — this case corresponds to the triplet spin-valve effect. Note that depending on the parameters of the system, the minimum of $T_c(\alpha)$, predicted analytically, can shift to a close vicinity of either $\alpha = 0$ or $\alpha = \pi$, becoming shallower and indistinguishable.

Figure 4 demonstrates the possibility of reentrant $T_c(\alpha)$ dependence. In this situation the triplet spin-valve effect takes place even at $T = 0$.

### 4. Discussion

The physical interpretation of the triplet spin-valve effect can be given as follows: at the collinear configurations, both the singlet component $f_0$ and the zero-projection triplet component $f_3$ of the pairing function are short-ranged (with the characteristic penetration depth of the order of $k_h^{-1}$), so that at $k_h^{-1} \ll d_F$ the middle F1 layer plays a role of a shield separating the S layer from the ferromagnetic half-space F2. When the angle between magnetizations declines from the collinear configurations, the long-range triplet component opens up an additional channel for this “leakage”, hence $T_c$ is suppressed stronger.
from the boundary conditions which produce the linear system of equations for the coefficients entering Eqs. (17)-(19). We find that in the limit of \( d_F \gg k_{\omega}^{-1}, k_{\Lambda}^{-1} \), the amplitudes of the long-range triplet components near the S/F1 interface (which are given by \( C_1, S_1 \), and \( B \)) are suppressed by the factor \( e^{-k_{\omega}d_F - k_{\Lambda}d_F} \) which has a clear physical interpretation. The long-range components are generated from the short-range ones at the F1/F2 interface (i.e., at \( x = d_F \)), where electrons “feel” inhomogeneous magnetization. Therefore, the long-range contribution at the S/F1 interface is obtained as a result of a “wave” that goes from the S/F1 interface as a short-range component with the wave vector \( k_{\omega} \) and returns after reflection at the F1/F2 interface as a long-range component with the wave vector \( k_{\Lambda} \). At the same time, the self-consistency equation (3) that determines \( T_c \), contains only the singlet short-range component. Therefore, the influence of the long-range components on \( T_c \) is indirect: the long-range components influence \( T_c \) only through their influence on the singlet component. We find that while the difference between \( W \) (that encodes the information about the suppression of \( T_c \)) for the P and AP cases is suppressed as \( e^{-2k_{\omega}d_F} \) (the short-range components go from the S/F1 to F1/F2 interface and back), the changes in \( W \) due to noncollinear magnetizations contain the same exponential, \( e^{-2k_{\Lambda}d_F} \). Of course, the influence of the long-range triplet components is contained in prefactors but no long-ranged exponential (with \( k_{\omega} \) instead of \( k_{\Lambda} \)) appears in \( W \), because \( W \) still originates from the short-range components.

In conclusion, we have considered a mesoscopic S/F1/F2 structure composed of a superconducting layer S, a ferromagnetic layer of arbitrary thickness F1, and a ferromagnetic half-space F2. We have demonstrated that the structure exhibits different relations between the critical temperatures in the parallel and antiparallel configuration: both the standard (\( T_{cP}^P < T_{cP}^{AP} \)) and inverse (\( T_{cP}^P > T_{cP}^{AP} \)) switching can be realized depending on the system’s parameters. At the same time, our main result is that \( T_{cP}^P \) at noncollinear magnetizations is lower than both \( T_{cP}^{AP} \) and \( T_{cP}^{AP} \), which makes this system a triplet spin valve. Possible experimental observation of a nonmonotonic (like curve “3” in Fig. 3 or curves “1” and “3” in Fig. 4) or even reentrant (like curves “2” and “4” in Fig. 4) behavior of \( T_c(\alpha) \) could be a signature of existence of the long-range triplet superconducting correlations [14] in SF hybrid structures.

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