Local U(1) symmetry in Y(SO(5)) associated with Massless Thirring Model and its Bethe Ansatz

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Abstract

Keyword: Current algebra, \(U(1)\) gauge invariance, Yangian
PACS number: 03.50.-d 03.65.Fd
(I) Introduction

Recently, it has been proposed by S.C. Zhang et al. that the antiferromagnetic (AF) and superconducting (SC) phases of high-$T_c$ cuprates are unified by an approximated $SO(5)$ symmetry principle[1]. Considerable support for this proposal came from numerical investigations in models for high-$T_c$ materials. In particular, it was shown that the low-energy excitations can be classified in terms of an $SO(5)$ symmetry multiplet structure[2,3]. Subsequently, extended Hubbard models and a two-leg ladder model related to $SO(5)$ symmetry have been introduced and analyzed in detail[4,5,6]. On the other hand, Shelton and Sénchal[7] have studied the problem of two coupled 1D Tomonaga-Luttinger chains and concluded that approximate $SO(5)$ symmetry can emerge in low-energy limit of this model. It is well-known that the Luttinger liquid is connected with the massless Thirring model. It is worth to deal with massless Thirring model with $SO(5)$ symmetry.

This paper is organized as follows: In the section (II), the massless Thirring model with $SO(5)$ symmetry will be diagonalized and the Bethe ansatz wavefunction is constructed. In section (III), we shall give the current algebra realization of $Y(SO(5))$ in terms of q-deformed fermion current that give rise to the local $U(1)$-gauge transformation.

(II) The massless Thirring model with $SO(5)$ symmetry and its Bethe ansatz wavefunction

Let us consider the massless Thirring model constructed by the four-component fermion field operator $\psi(x) = [\psi_1(x), \psi_2(x), \psi_3(x), \psi_4(x)]^T$. The Hamiltonian takes the form:

\[
H = \int \left[ iv \sum_{i=1}^{4} C_i \psi_i^\dagger(x) \partial_x \psi_i(x) + g \sum_{i,j=1}^{4} C_{ij} n_i(x)n_j(x) \right] dx
\]

where $C_{ij} = C_{ji}$, $C_{ii} = 0$ and $n_i(x) = \psi_i^\dagger(x)\psi_i(x)$ ($i, j = 1, 2, 3, 4$) satisfy the anticommutation relations:

\[
[\psi_i^\dagger(x), \psi_j^\dagger(y)]_+ = 0 \quad (i, j = 1, 2, 3, 4) \quad (i, j = 1, 2, 3, 4) \]

For four-component fermion field operators $\psi(x) = [c_\sigma(x), d_\sigma^\dagger(x)]^T$ and forms the current algebra obeying $SO(5)[6]$. In momentum space, this Hamiltonian can be written as:

\[
H = \int [-v \sum_{i=1}^{4} (kC_i n_i(k))] dk
\]
that obviously is made up of the pairs, so this model may be applied to superconducting.

To diagonalize $H$, we introduce local $U(1)$ transformation:

$$
\Phi_i(x) = \exp[-i \sum_{k=1}^{4} \theta_{ik} \phi_k(x)] \psi_i(x)
$$

(6)

where $\phi_i(x) = \int_{-\infty}^{x} \psi_i^\dagger(y) \psi_i(y) dy$ and $\theta_{ik}$ are constants.

According to eq. (6), eq. (8) and eq. (9) by direct calculation, we obtain (no summation over the repeated $j$):

$$
\Phi_i(x)\Phi_j(y) = -\exp[i \theta_{ij}] \Phi_j(y)\Phi_i(x)
$$

(7)

$$
\Phi_i^\dagger(x)\Phi_j^\dagger(y) = -\exp[i \theta_{ij}] \Phi_j^\dagger(y)\Phi_i^\dagger(x)
$$

(8)

$$
\Phi_i(x)\Phi_j^\dagger(y) = -\exp[-i \theta_{ij}] \Phi_j^\dagger(y)\Phi_i(x) + \delta_{ij} \delta(x - y)
$$

(9)

This is a special case of Zamolodchikov-Faddeev algebra[15,16].

$$
\theta_{ii} = 0 \pmod{2\pi}
$$

(10)

$$
\theta_{ij} + \theta_{ji} = 0 \pmod{2\pi}
$$

(11)

Therefore, eq. (10) and eq. (11) are conditions given by the associativity of the special case of Zamolodchikov-Faddeev algebra. The meaning of eq. (11) is clear that the particle itself must still be fermion for the same "i-spin" states, however, eq. (11) show that the commutation relations between different "i-spin" states can be q-deformed and the q-deformation parameters should obey eq. (11) because of two-body interaction between different "i-spin" states.

Under the local $U(1)$ transformation eq. (6), the Hamiltonian eq. (4) can be diagonalized and find the physical constraint conditions for real $C_i$ and $C_{ij}$. The Heisenberg equation $i\partial_t \psi_i(x,t) = [\psi_i(x,t), H]$ reads:

$$
\partial_t \psi_i(x,t) = v C_i \partial_x \psi_i(x,t) - i2g \sum_{j=1}^{4} C_{ij} n_j(x,t) \psi_i(x,t)
$$

(12)
On account of the transformation eq.(6) and the Heisenberg equation eq.(12), we obtain:

\[ \frac{\partial}{\partial t} \Phi_i(x,t) - vC_i \frac{\partial}{\partial x} \Phi_i(x,t) = i \sum_{j=1}^{4} [v(C_i - C_j)\theta_{ij} - 2gC_{ij}]n_j(x,t)\psi_i(x,t) \exp[-i \sum_{k=1}^{4} \theta_{jk}\phi_k(x)] \]  

(13)

By choosing

\[ \theta_{ij} = \begin{cases} 2g & (C_i \neq C_j) \\ 0 & (C_i = C_j) \end{cases} \]  

(14)

and

\[ \theta_{ii} = 0 \]  

(15)

then \( \Phi_i(x,t) \) satisfy the free-field equation. The Hamiltonian becomes diagonalized (here we suppose: \( C_i \neq C_j \); if \( C_i = C_j \), the Hamiltonian can be diagonalized only when \( C_{ij} = 0 \):

\[ H' = iv \sum_{i=1}^{4} C_i \int \Phi_i^+(x)\frac{\partial}{\partial x} \Phi_i(x)dx \]  

(16)

The direct calculation shows that \( \Phi_i(x) \) and \( H' \) also satisfy the Heisenberg equation \( i\partial_t \Phi_i(x,t) = [\Phi_i(x,t),H'] \), so \( \Phi_i(x,t) \) are really dynamic variables regarding \( H' \).

Therefore, by using the local \( U(1) \) transformation eq.(6), the original Hamiltonian eq.(1) constructed by \( \psi_i(x) \) with anticommutation relations eq.(2)-eq.(4) has been transformed into the quadratic Hamiltonian eq.(16) in terms of the \( \Phi_i(x) \) obeying q-deformed relations eq.(7)-eq.(9). In the following, we shall show how the method in[8,9,10] works to find the Bethe ansatz wavefunction in a simple manner for \( SO(5) \) massless Thirring model.

Let us denote by \( |n_1,n_2,n_3,n_4> \) a eigenstate with \( n_i \) \( \Phi_i \)-particles \( (i=1,2,3,4) \), it can be expressed by

\[ |n_1,n_2,n_3,n_4> = \int \cdots \int dx_j \varphi(x_1,\ldots,x_M) \prod_{j_1=1}^{n_1} \Phi_i^+(x_{j_1}) \times \prod_{j_2=1}^{n_2} \Phi_2^+(x_{M_1+j_2}) \prod_{j_3=1}^{n_3} \Phi_3^+(x_{M_2+j_3}) \prod_{j_4=1}^{n_4} \Phi_4^+(x_{M_3+j_4}) | 0 > \]  

(17)

where \( M_i = n_1 + n_2 + \ldots + n_i \), \( M = M_4 \) and \( | 0 > \) is the vacuum defined by

\[ \psi_j(x) | 0 >= 0 \]  

(18)

or equivalently

\[ \Phi_j(x) | 0 >= 0 \]  

(19)

Substituting eq.(17) and eq.(16) into Schrödinger equation

\[ H' |n_1,n_2,n_3,n_4> = E_{n_1,n_2,n_3,n_4} | n_1,n_2,n_3,n_4 > \]  

(20)

it yields equation for \( \varphi(x_1,\ldots,x_M) \)

\[ iv \left( \sum_{i=1}^{4} C_i \sum_{j_1=1}^{n_i} \frac{\partial}{\partial x_{M_{i-1}+j_1}} \right) \varphi(x_1,\ldots,x_M) = E_{n_1,n_2,n_3,n_4} \varphi(x_1,\ldots,x_M) \]  

(21)
whose solution is:

\[ \varphi(x_1, \ldots, x_M) = A \exp(i \sum_{j=1}^{M} k_j x_j) \]

\[ E_{n_1, n_2, n_3, n_4} = -\nu(\sum_{i=1}^{4} C_i \sum_{j_i=1}^{n_i} k_{M_i+j_i}) \]  

(22)

where \( k_j \) and \( A \) are constants. Since the constant \( A \) is not essential, we shall omit it hereafter. The Bethe ansatz wavefunction \( \hat{\varphi}(x_1, \ldots, x_M) \) is defined by

\[ | n_1, n_2, n_3, n_4 > = \int \ldots \int d x_j \varphi(x_1, \ldots, x_M) \prod_{j=1}^{n_1} \psi^+_1(x_{j_1}) \]

\[ \times \prod_{j_2=1}^{n_2} \psi^+_2(x_{M_1+j_2}) \prod_{j_3=1}^{n_3} \psi^+_3(x_{M_2+j_3}) \prod_{j_4=1}^{n_4} \psi^+_4(x_{M_3+j_4}) | 0 > \]  

(23)

Substituting eq.(6) into eq.(17), by detail calculation, we have

\[ | n_1, n_2, n_3, n_4 > = \int \ldots \int d x_j \varphi(x_1, \ldots, x_M) \prod_{1 \leq p < q \leq 4} \prod_{j_p=1}^{n_p} \prod_{j_q=1}^{n_q} \exp[i \theta_{pq} \epsilon(x_{M_{p-1}+j_p} - x_{M_{q-1}+j_q})] \]

\[ \times \prod_{j_1=1}^{n_1} \psi^+_1(x_{j_1}) \prod_{j_2=1}^{n_2} \psi^+_2(x_{M_1+j_2}) \prod_{j_3=1}^{n_3} \psi^+_3(x_{M_2+j_3}) \prod_{j_4=1}^{n_4} \psi^+_4(x_{M_3+j_4}) | 0 > \]  

(24)

where \( \theta(x) = 0 \) \( (i f \ x < 0) \); 1 \( (i f \ x > 0) \) and \( \epsilon(x) = \theta(x) - \theta(-x) \) hereafter. Thus, the Bethe ansatz wavefunction \( \hat{\varphi}(x_1, \ldots, x_M) \) takes the form:

\[ \hat{\varphi}(x_1, \ldots, x_M) = \exp[i \sum_{j=1}^{M} k_j x_j] \prod_{1 \leq p < q \leq 4} \prod_{j_p=1}^{n_p} \prod_{j_q=1}^{n_q} [1 - itg\frac{\theta_{pq}}{2} \epsilon(x_{M_{p-1}+j_p} - x_{M_{q-1}+j_q})] \]  

(25)

which describes the many-body problem with \( \delta \)-interactions.

Suppose that \( M \) particles move in a region with the length \( L \). For an arbitrary \( x_j \) \( (M_{p-1} \leq j \leq M_p) \). Imposing the periodical boundary conditions (PBC), we have

\[ k_j L = -i \sum_{q \neq p}^{4} n_q l_{j} \ln \frac{1 - itg\theta_{pq}/2}{1 + itg\theta_{pq}/2} + 2l_j \pi \quad (l_j \ integer) \]  

(26)

i.e.

\[ k_j L = -\sum_{q \neq p}^{4} n_q \theta_{pq} + 2l_j \pi \quad (l_j \ integer) \]  

(27)
that is exactly the Bethe ansatz equation. Obviously, the local \( U(1) \) transformation eq.(6)

much helps the derivation of the Bethe ansatz condition for the massless Thirring model.

**III. Current realization of \( Y(SO(5)) \)**

The \( SO(5) \) algebra does have the current realization, however the fermionic construction is not unique. In parallel to the diagonalization of eq.(1) we shall show that the q-deformed operators \( \Phi_q(x) \) shown in eq.(8) also provides a realization of \( SO(5) \) algebra, henceforth Yangian associated with \( SO(5) \).

The original commutation relations of \( Y(g) \) were given by Drinfeld[17,18] in the form:

\[
[I_\lambda, I_\mu] = c_{\lambda \mu \nu} I_\nu \quad ; \quad [I_\lambda, J_\mu] = c_{\lambda \mu \nu} J_\nu
\]  
(28)

\[
[J_\lambda, [J_\mu, I_\nu]] - [J_\lambda, [J_\mu, J_\nu]] = \hbar^2 a_{\lambda \mu \alpha \beta \gamma} \{ I_\alpha, I_\beta, I_\gamma \}
\]  
(29)

\[
[[J_\lambda, J_\mu], [J_\sigma, J_\tau]] + [[J_\sigma, J_\tau], [I_\lambda, J_\mu]] = \hbar^2 (a_{\lambda \mu \alpha \beta \gamma} c_{\sigma \tau \nu} + a_{\sigma \tau \nu \alpha \beta} c_{\lambda \mu \gamma}) \{ I_\alpha, I_\beta, I_\gamma \}
\]  
(30)

where \( c_{\lambda \mu \nu} \) are structure constants of a simple Lie algebra \( g \), \( \hbar \) is a constant and

\[
a_{\lambda \mu \alpha \beta \gamma} = \frac{1}{4!} c_{\lambda \sigma \tau \gamma} c_{\mu \beta \tau \sigma} c_{\nu \rho \sigma \tau}
\]  
(31)

For Lie algebra \( SO(5) \), \( Y(SO(5)) \) is generated by antisymmetric generators \( \{ J_{ab}, J_{ab} \} \).

eq.(28) reads

\[
[J_{ab}, J_{cd}] = i (\delta_{bc} J_{ad} + \delta_{ad} J_{bc} - \delta_{ac} J_{bd} - \delta_{bd} J_{ac})
\]  
(32)

\[
[I_{ab}, J_{cd}] = i (\delta_{bc} J_{ad} + \delta_{ad} J_{bc} - \delta_{ac} J_{bd} - \delta_{bd} J_{ac})
\]  
(33)

\[
I_{ab} = -I_{ba}; \quad J_{ab} = -J_{ba}; \quad (a, b, c, d = 1, 2, 3, 4, 5)
\]

Not all of the relations in eq.(27)-eq.(30) are independent. After tedious calculation we can prove that there is only one independent relation:

\[
[J_{23}, J_{15}] = \frac{i}{24} \hbar^2 \left( \{ I_{13}, I_{42}, I_{45} \} + \{ I_{12}, I_{45}, I_{34} \} - \{ I_{14}, I_{42}, I_{35} \} - \{ I_{14}, I_{34}, I_{25} \} \right)
\]  
(34)

where \( J_{23} \) and \( J_{15} \) are the Cartan subset.

All the other relations other than eq.(28) can be generated on the basis of eq.(34) by using Jacobi identities together with eq.(32) and eq.(33). Therefore, for \( Y(SO(5)) \), eq.(28)-eq.(30) can be expressed with eq.(32)-eq.(34) in such a simple manner. This conclusion can also be verified by RTT relation independently through tremendous computation.

The generators of \( Y(SO(5)) \) can be realized by fermion current algebra as follows:

\[
I_{ab} = \int I_{ab}(x) dx \quad ; \quad I_{ab}(x) = -\frac{1}{2} \psi^+(x) \Gamma_{ab} \psi(x)
\]

\[
J_{ab} = T_{ab} + U J^0_{ab} \quad ; \quad T_{ab} = \int dx \psi^+(x) \Gamma_{ab} \partial_x \psi(x)
\]

\[
J^0_{ab} = \int \int dx dy \epsilon(x - y) I_{ac}(x) I_{cb}(y)
\]  
(35)

where \( \Gamma_{ab} = -i \Gamma^a \Gamma^b \), \( \Gamma^a \) are 4x4 Dirac matrices, \( U = \pm \frac{i}{\hbar} \hbar \) (\( h \) being arbitrary constant) and \( \psi(x) \) satisfies anticommutation relations eq.(2)-eq.(4). It can be checked that the set \( \{ I_{ab}, J_{ab} \} \) satisfies algebraic relations eq.(32)-eq.(34) of \( Y(SO(5)) \).
As given by Ref[6] if \( \psi(x) = [c_\sigma(x), d_\sigma(x)]^T \), then local generators \( I_{ab}(x) \) of Lie algebra \( \text{SO}(5) \) are expressed in terms of spin \( \vec{S}(x) = \frac{1}{2}(c^+(x)\vec{\sigma}c(x) + d^+(x)\vec{\sigma}d(x)) \), charge \( Q(x) = \frac{1}{2}(c^+(x)c(x) + d^+(x)d(x) - 2) \) and \( \vec{\pi}^+(x) = -\frac{1}{2}c^+(x)\vec{\sigma}\sigma_2d^+(x) \) with

\[
I_{ab}(x) = \begin{pmatrix}
\pi^+_1(x) + \pi_1(x) & 0 \\
\pi^+_2(x) + \pi_2(x) & -S_3(x) & 0 \\
\pi^+_3(x) + \pi_3(x) & S_2(x) & -S_1(x) \\
Q(x) & i(\pi_1(x) - \pi^+_1(x)) & i(\pi_2(x) - \pi^+_2(x)) & i(\pi_3(x) - \pi^+_3(x)) & 0
\end{pmatrix}
\] (36)

where the value of matrix elements on the upper right triangle are determined by antisymmetry, \( I_{ab}(x) = -I_{ba}(x) \).

Under the local \( U(1) \) transformation eq.(3), the four-component fermion field operators \( \psi(x) = [\psi_1(x), \psi_2(x), \psi_3(x), \psi_4(x)]^T \) is changed into q-deformed operator \( \Phi(x) = [\Phi_1(x), \Phi_2(x), \Phi_3(x), \Phi_4(x)]^T \). The generators of \( Y(\text{SO}(5)) \) is constructed by q-deformed fermionic current algebra as follows:

\[
\mathcal{T}_{ab} = \int T_{ab}(x)dx \quad ; \quad T_{ab}(x) = -\frac{1}{2}\Phi^+(x)\Gamma^{ab}\Phi(x)
\]

\[
\mathcal{J}_{ab} = \mathcal{T}_{ab} + UJ^0_{ab} \quad ; \quad J_{ab}(x) = \int dx\Phi^+(x)\Gamma^{ab}\partial_x\Phi(x)
\]

\[
J^0_{ab} = \int \int dx dy (x - y)J_{ac}(x)I_{cb}(y)
\] (37)

Substituting eq.(37) into eq.(32)-eq.(34), we can obtain the constraint conditions:

\[
\theta_{im} - \theta_{jm} = \theta_{in} - \theta_{jn} \quad (\text{mod} \ 2\pi)
\] (38)

where eq.(38) sets the condition making \( Y(\text{SO}(5)) \) constructed by q-deformed field operator \( \Phi(x) \), This indicates that the current realization of \( Y(\text{SO}(5)) \) is not unique. Careful calculation shows that there are three free parameters in \( \theta_{ij} \) under conditions eq.(4), eq.(11) and eq.(38). So there exists an additional freedom in \( Y(\text{SO}(5)) \). Substituting eq.(14) into eq.(38), we obtain

\[
\frac{C_{im}}{C_i - C_m} - \frac{C_{jm}}{C_j - C_m} = \frac{C_{in}}{C_i - C_n} - \frac{C_{jn}}{C_j - C_n} \quad (\text{mod} \ 2\pi)
\] (39)

\[
(i, j, m, n = 1, 2, 3, 4)
\]

In another words, \( H' \) is the Hamiltonian expressed by \( \Phi_i(x) \), under the transformation eq.(4), it becomes \( H \) where two-body interaction appears. So the physical meaning of \( U(1) \) transformation in \( Y(\text{SO}(5)) \) is connected with two body interaction in massless Thirring model with \( \text{SO}(5) \) symmetry.

From the above analysis we see that there is a local \( U(1) \) gauge-invariance in the construction of the current algebra realization for \( Y(\text{SO}(5)) \).
Under local $U(1)$ transformation eq.(3), eq.(36) is changed into

$$
\mathcal{T}_{ab}(x) = \begin{pmatrix}
0 & \bar{\pi}_1(x) + \pi_1(x) & 0 & 0 \\
\bar{\pi}_2(x) + \pi_2(x) & 0 & -\bar{S}_3(x) & 0 \\
\bar{\pi}_3(x) + \pi_3(x) & \bar{S}_3(x) & 0 & -\bar{S}_1(x) \\
\bar{Q}(x) & i(\bar{\pi}_1(x) - \pi_1^+(x)) & i(\bar{\pi}_2(x) - \pi_2^+(x)) & i(\bar{\pi}_3(x) - \pi_3^+(x)) & 0
\end{pmatrix}
$$

(40)

where the value of matrix elements on the upper right triangle are determined by anti-symmetry, $\mathcal{T}_{ab}(x) = -\mathcal{T}_{ba}(x)$, and

$$
\begin{pmatrix}
\bar{S}_1(x) \\
\bar{S}_2(x) \\
\bar{S}_3(x)
\end{pmatrix} = \begin{pmatrix}
\cos(\alpha + \beta \phi(x)) \cos(\alpha - \beta \phi(x)) & -\sin(\alpha + \beta \phi(x)) \cos(\alpha - \beta \phi(x)) & 0 \\
\sin(\alpha + \beta \phi(x)) \cos(\alpha - \beta \phi(x)) & \cos(\alpha + \beta \phi(x)) \cos(\alpha - \beta \phi(x)) & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
S_1(x) \\
S_2(x) \\
S_3(x)
\end{pmatrix}
$$

(41)

$$
\begin{pmatrix}
\pi_1^+(x) \\
\pi_2^+(x)
\end{pmatrix} = \exp(i\frac{\nu \phi(x)}{2}) \begin{pmatrix}
\cos(\frac{\alpha + \beta \phi(x)}{2}) & -\sin(\frac{\alpha + \beta \phi(x)}{2}) \\
\sin(\frac{\alpha + \beta \phi(x)}{2}) & \cos(\frac{\alpha + \beta \phi(x)}{2})
\end{pmatrix} \begin{pmatrix}
\pi_1^-(x) \\
\pi_2^-(x)
\end{pmatrix}
$$

(42)

$$
\pi_3^+(x) = \exp(i\frac{\nu \phi(x)}{2})[\cos(\frac{\alpha - \beta \phi(x)}{2})\pi_3^-(x) + \frac{1}{2}\Delta^+(x)\sin(\frac{\alpha - \beta \phi(x)}{2})]
$$

(43)

where $\phi(x) = \sum_{i=1}^{4} \phi_i(x)$, $\theta_{43} = \beta$, $\theta_{12} = \alpha$, $\theta_{13} - \theta_{42} = \nu$, SC order parameter $\Delta^+(x) = -ic^+(x)\sigma_2d^+(x)$ and AF order parameter $\bar{N}(x) = \frac{1}{2}(c^+(x)\bar{\sigma}c(x) - d^+(x)\bar{\sigma}d(x))$.

From the above analysis we see that under the $U(1)$ transformation eq.(3), eq.(36) still obey Yangian algebra as eq.(35) does if $\theta_{ij}$ satisfy the conditions eq.(10),eq.(11) and eq.(13). This indicates that there is a local $U(1)$ gauge-invariance in the construction of the current realization for $Y(SO(5))$. It turns out that after the transformation, there are local phase factors in the current realization of $Y(SO(5))$ (shown by eq.(11),eq.(42) and eq.(43), but eq.(37) still satisfy $Y(SO(5))$ Yangian algebraic relations. i.e., there is a local $U(1)$ gauge-invariance in such a current realization of $Y(SO(5))$.

We also find that the transformation eq.(3) can be used to diagonalize the massless Thirring model with $SO(5)$ symmetry that will help to understand the physical meaning of the introduced local $U(1)$ symmetry.

In another words, $H'$ is the Hamiltonian expressed by $\Phi_i(x)$, under the transformation eq.(3), it becomes $H$ where two body interaction appears. So the physical meaning of $U(1)$ transformation in the current realization of $Y(SO(5))$ connected with two body interaction in this physical models. Applying the transformation we find the local $U(1)$ gauge-invariance in $Y(SO(5))$ explicitly.

Noting that there are some non-trivial phase factors in the generators of Yangian, but they still satisfy the commutation relations of $Y(SO(5))$, i.e., there is a local $U(1)$ gauge-invariance in $Y(SO(5))$.

(IV).Conclusion and Acknowledge

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Using a local $U(1)$ transformation connecting the four-component fermionic field operator $\psi_i(x)$ with q-deformed one $\Phi_i(x)$, it is helpful to diagonalize massless Thirring model with $SO(5)$ symmetry. The Bethe ansatz wavefunction is obtained in a simple manner. It turns out that the current realization of $Y(SO(5))$ is not unique and exist a local $U(1)$ gauge transformation. This shows the existence of a local $U(1)$ symmetry in the current realization of $Y(SO(5))$. Correspondingly, the transformation leads to the local $U(1)$-gauge invariance for $Y(SO(5))$. The explicit forms of phase factors for $SO(5)$ has been shown.

The authors would like to thank Dr. Jing-Ling Chen helpful discussion. This work is in part supported by NSF of China.
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