Scaling of Acceleration Statistics in High Reynolds Number Turbulence

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The scaling of acceleration statistics in turbulence is examined by combining data from the literature with new data from well-resolved direct numerical simulations of isotropic turbulence, significantly extending the Reynolds number range. The acceleration variance at higher Reynolds numbers departs from previous predictions based on multifractal models, which characterize Lagrangian intermittency as an extension of Eulerian intermittency. The disagreement is even more prominent for higher-order moments of the acceleration. Instead, starting from a known exact relation, we relate the scaling of acceleration variance to that of Eulerian fourth-order velocity gradient and velocity increment statistics. This prediction is in excellent agreement with the variance data. Our Letter highlights the need for models that consider Lagrangian intermittency independent of the Eulerian counterpart.

Introduction.—The acceleration of a fluid element in a turbulent flow, given by the Lagrangian derivative of the velocity, resulting from the balance of forces acting on it, is arguably the simplest descriptor of its motion. This is directly reflected in the Navier-Stokes equations:

\[ \mathbf{a} = \mathbf{D} \mathbf{u}/\mathbf{D}t = -\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}, \]

where, \( \mathbf{u} \) is the divergence-free velocity (\( \nabla \cdot \mathbf{u} = 0 \)), \( p \) the kinematic pressure, \( \nu \) is the kinematic viscosity, and \( \mathbf{f} \) is a forcing term. Besides its fundamental role in the study of turbulence [1–4], understanding the statistics of acceleration is of paramount importance for a diverse range of applications constructed around stochastic modeling of transport phenomena in turbulence [5–8]. The application of Kolmogorov’s 1941 phenomenology implies that the variance (and higher-order moments) of any acceleration component \( a \) can be solely described by the mean-dissipation rate \( \langle \epsilon \rangle \) and \( \nu \) [9–11]:

\[ \langle a^2 \rangle = \frac{1}{3} \langle |a|^2 \rangle = a_0 \langle \epsilon \rangle^{3/2} \nu^{-1/2}, \]

where \( a_0 \) is thought to be a universal constant.

However, extensive numerical and experimental work has shown that \( a_0 \) increases with Reynolds number [12–20]. Thus, obtaining data on \( a_0 \) and modeling its \( R_a \) variation has been a topic of considerable interest. While several theoretical works have focused on acceleration statistics [21–24], the most notable procedure, whose validity should not be taken for granted, stems from the multifractal model [25–27], which quantifies acceleration intermittency (and, in general, the intermittency of other Lagrangian quantities) by adapting to the Lagrangian viewpoint the well-known Eulerian framework, based either on the energy dissipation rate [28] or velocity increments [29]. A key result from this consideration is that \( a_0 \sim R_a^2 \), \( \chi \approx 0.135 \), where \( R_a \) is the Taylor-scale Reynolds number. While data from direct numerical simulations (DNS) and experiments do not directly display this power law, it was nevertheless presumed to be asymptotically correct at very large \( R_a \), and an empirical interpolation formula [16,19],

\[ a_0 \approx \frac{c_1 R_d^\chi}{(1 + c_2/R_d^{\chi+1})^2}, \]

was suggested to fit the data, showing reasonable success [16]. An alternative scaling: \( a_0 \sim R_d^{1.25} \) was proposed by Hill [21], which was indistinguishable from Eq. (3) at low \( R_d \) [16,20]; we discuss the veracity of this proposal later.

In this Letter, we revisit the scaling of acceleration variance (and higher-order moments) by presenting new DNS data at higher \( R_d \). The new variance data agrees with previous lower \( R_d \) data, where the \( R_d \) range overlaps, but increasing deviations from Eq. (3) occur at higher \( R_d \). Results for high-order moments show even stronger...
deviations from previous predictions. Further analysis shows that the extension of Eulerian multifractal models to the Lagrangian viewpoint is the source of this discrepancy. We develop a statistical model that shows excellent agreement with variance data at high $R_\lambda$, and also provide an updated interpolation fit to include low $R_\lambda$ data.

Direct numerical simulations.—The DNS data used here correspond to the canonical setup of forced stationary isotropic turbulence in a periodic domain [30], allowing the use of highly accurate Fourier pseudospectral methods [31]. The novelty is that we have simultaneously achieved very high Reynolds number and the necessary grid resolution to accurately resolve the small scales [32,33]. The grid resolution is as high as $k_{\text{max}} \eta_K \approx 6$, which is substantially higher than $k_{\text{max}} \eta_K \approx 1-2$, used in previous acceleration studies [16,19,20]; $k_{\text{max}} = \sqrt{2N/3}$ is the maximum resolved wave number on a $N^3$ grid and $\eta_K = \nu^{3/4} \langle \epsilon \rangle^{-1/4}$ is the Kolmogorov length scale. This improved small-scale resolution is especially necessary for capturing higher-order statistics of acceleration, since acceleration is even more intermittent than spatial velocity gradients [12,19].

Acceleration variance.—Figure 1 shows the compilation of data from various sources including data from both DNS [19,20,38] and bias-corrected experiments [39]. We have also included DNS data obtained directly from Lagrangian trajectories of fluid particles [40-42], which give identical results for acceleration variance [43]. As evident, while Eq. (3) works for the previous range of $R_\lambda$, it does not fit the new data. In fact, a $R_\lambda^{0.25}$ scaling is more appropriate at higher $R_\lambda$, and as discussed later, the failure of multifractal models in fitting higher-order moments is even more conspicuous. To gain clarity on this point, it is useful to discuss the multifractal models first.

\[ \langle a^p \rangle \sim \langle \epsilon^{3p/4} \rangle \nu^{-p/4}, \]  

Alternatively,

\[ \langle a^p \rangle / \langle a^q \rangle \sim \langle \epsilon^{3p/4} \rangle / \langle \epsilon^{3q/4} \rangle, \]

where $a_K = \langle \epsilon \rangle^{3/4} \nu^{-1/4}$, i.e., acceleration based on Kolmogorov variables. Since Eulerian intermittency dictates that $\langle \epsilon^q \rangle \neq \langle \epsilon \rangle^q$ for any $q \neq 1$ [28,29], the key assumption in its extension to Lagrangian intermittency is that the $p$th moment of acceleration scales as the $(3p/4)$th moment of $\epsilon$ [25,27]. The scaling of $\langle \epsilon^q \rangle / \langle \epsilon \rangle^q$ can be obtained by several approaches, all leading to similar results. We briefly summarize a few approaches below, with additional details in the Supplemental Material [44].

The most direct approach is to use the multifractality of dissipation rate [25,45]. Within the multifractal framework, a scale-averaged dissipation $\epsilon_r$, over scale $r$, is assumed to be Hölder continuous: $\epsilon_r / \langle \epsilon \rangle \sim (r/L)^{q-1}$, where $\alpha$ is the local Hölder exponent, with a corresponding multifractal spectrum $F(\alpha)$ and $L$ is the large-scale length. Note, the 1D spectrum $f(\alpha)$ is more common in the literature [45], which is simply: $f(\alpha) = F(\alpha) - 2$. Now, $\epsilon_r$ reduces to the true dissipation for a viscous cutoff defined as $r \approx (\nu^3 / \langle \epsilon \rangle)^{1/4}$ or equivalently, $r/L \approx (R_\lambda)^{-3/4}$. Here, $R = u'L/\nu$, $u'$ being the large-scale velocity; we also use $R = R_\lambda^2$ and $\langle \epsilon \rangle \sim u'^3/L$ from dissipation anomaly [46].

The above framework leads to the result:

\[ \langle \epsilon^q \rangle / \langle \epsilon \rangle^q \sim R_\lambda^{\tau_q}, \quad \tau_q = \sup_{\alpha} \frac{6(q(1-\alpha) - 3 + F(\alpha))}{3 + \alpha}. \]

An approximation for $F(\alpha)$, such as the $p$-model [45,47], can be used to obtain $\tau_q$. The $p$th moment of acceleration can then be simply obtained as [48]

\[ \langle a^p \rangle / a_K^p \sim R_\lambda^{\xi_p}, \quad \text{with} \quad \xi_p = \tau_{3p/4}. \]

Instead of dissipation, one can also start by taking the velocity increment $\delta u_r$ over scale $r$ to be Hölder continuous: $\delta u_r / u' \sim (r/L)^h$, where $h$ is the local Hölder exponent.
and $D(h)$ is the corresponding multifractal spectrum. A scale-dependent dissipation rate $\epsilon_\tau$ can then be defined as $\epsilon_\tau \sim (\delta u_r)^3/r$, which reduces to the true dissipation for the viscous cutoff defined by the condition $\delta u_r r/\nu \approx 1$. This framework leads to the same result as in Eq. (6), corresponding to $\alpha = 3h$ and $F(\alpha) = D(h)$. A well-known approximation for $D(h)$ is given by the She-Lévêque model [49]. Finally, we can also use the Kolmogorov (1962) log-normal model [50], which gives $\tau_\epsilon = 3\mu(q-1)/4$, even though it is untenable for large $q$ [29]. Here, $\mu$ is the intermittency exponent, with experiments and DNS suggesting $\mu \approx 0.25$ [51,52].

The scaling of acceleration moments obtained from these three approaches and also from DNS data is listed in Table I, up to sixth-order. All approaches give essentially the same result for the acceleration variance, with the exponent of about 0.135 used in Eq. (3). However, the high-$R_\lambda$ DNS data clearly do not conform to any of the power laws shown in Table I. The results for normalized fourth- and sixth-order moments, also plotted in Fig. 2, clearly show that the power laws increasingly differ from multifractal predictions.

As noted earlier, the use of multifractals is primarily motivated by Eq. (4). To get better insight, in Fig. 3(a), we plot $a_0$ and $\langle \epsilon^{3/2} \rangle / \langle \epsilon \rangle^{3/2}$ versus $R_\lambda$. While the latter shows a clear $R_\lambda^{4/14}$ scaling as anticipated from multifractals (and also the log-normal model), the former shows a steeper scaling of $R_\lambda^{4/25}$. An even more general and direct test is presented in Fig. 3(b), by checking the validity of Eq. (4) for different $p$ values. The data clearly suggest that the acceleration intermittency, being stronger, cannot be described by extending the Eulerian intermittency of the dissipation rate. In fact, a similar observation has been made for Lagrangian velocity structure functions, where extensions of the $p$-model and the She-Lévêque model severely underpredict their intermittency (i.e., overpredict the inertial-range exponents) [53].

It is worth considering if one might describe the scaling of acceleration moments in terms of enstrophy $\Omega = |\omega|^2$ ($\omega$ being the vorticity), instead of dissipation. This change addresses the likelihood that acceleration is influenced more by transverse velocity gradients than by longitudinal ones [54,55]. In isotropic turbulence $\langle \Omega \rangle = \langle \epsilon \rangle/\nu$, but the higher moments differ, enstrophy being more

![TABLE I. Scaling exponents $\zeta$ for $R_\lambda$ scaling of acceleration moments $M_\alpha \propto R_\lambda^\zeta$, as predicted from intermittency models, compared with current DNS results (see Figs. 1 and 2).](image)

| $M_\alpha$ | $p$-model | She-Lévêque | K62 log-normal | DNS result |
|------------|-----------|-------------|---------------|------------|
| $\langle a^2 \rangle / a_K^2$ | 0.135 | 0.140 | 0.140 | 0.25 |
| $\langle a^4 \rangle / a_K^4$ | 0.943 | 1.00 | 1.13 | 1.60 |
| $\langle a^6 \rangle / a_K^6$ | 2.06 | 2.30 | 2.95 | 3.95 |

![FIG. 2. Normalized fourth-order (top) and sixth-order (bottom) moments of acceleration as a function of $R_\lambda$.](image)

![FIG. 3. (a) Scaling of $a_0$, $\langle \epsilon^{3/2} \rangle$, and $\langle \Omega^{3/2} \rangle$. For clarity, data for $\epsilon$ and $\Omega$ are shifted up by factors of 2 and 1.5, respectively. (b) Scaling of $p$th moments of acceleration normalized by moments of $\epsilon^{p/4}$ (filled symbols) and $\Omega^{p/4}$ (open symbols). For clarity, data for $p = 1$–5 are respectively shifted by factors of 1, 0.75, 0.5, 0.25, 0.12 for $\epsilon$ and 0.94, 1.01, 1.2, 1.35, 1.6 for $\Omega$.](image)
intermittent [32,56]. The resulting modification to Eq. (4) is
\[ \langle |a|^2 \rangle \sim \langle \Omega^3 \rangle^{2/4} \langle \nu^2 \rangle^{2/4}. \] However, as tested in Figs. 3(a) and (b), the differences arise only for large \( \rho \); even then, it is not sufficient to explain the stronger intermittency of acceleration (also see Supplemental Material [44]).

**Acceleration variance from fourth-order structure function.**—A statistical model for acceleration variance is now obtained using a methodology similar to that proposed by [21], but differing in some crucial aspects. From Eq. (1), acceleration variance can be obtained directly as [57]
\[ \langle |a|^2 \rangle = \langle |\nabla p|^2 \rangle + \nu^2 \langle |\nabla^2 u|^2 \rangle. \] (8)

The viscous contribution is known to be small and can be ignored [13]. An exact relation for variance of pressure gradient is also known [58,59]:
\[ \langle |\nabla p|^2 \rangle = \int_r r^{-3} [D_{1111}(r) + D_{\text{mnsn}}(r) - 6D_{11\rho\rho}(r)] dr, \] (9)

where the \( D \)s are the fourth-order longitudinal, transverse, and mixed structure functions, in order. The above results can be rewritten as [21]
\[ \langle |a|^2 \rangle \approx 4H_x \int_r r^{-3} D_{1111}(r) dr, \] (10)

where \( H_x \) is a constant defined by Eqs. (8), (9). At sufficiently high \( R_4 \) (\( \geq 200 \)), DNS data [13,20] confirm that \( H_x \approx 0.65 \) (also see Supplemental Material [44]). We can normalize both sides by Kolmogorov scales to write
\[ a_0 \approx \frac{4H_x}{3} \int_r (\frac{r}{\eta_K})^{-3} \frac{D_{1111}(r)}{u_k^4} \left( \frac{r}{\eta_K} \right). \] (11)

Assuming standard scaling regimes [29], we can write
\[ \frac{D_{1111}(r)}{u_k^4} = \begin{cases} \frac{K}{225} \left( \frac{r}{\eta_K} \right)^4 & r < \ell', \\ C_4 \left( \frac{r}{\eta_K} \right)^{\xi_4} & \ell' < r < L, \\ C & r > L, \end{cases} \] (12)

where \( F \) is the flatness of \( \partial u/\partial x \), \( \xi_4 \) is the inertial-range exponent, and \( C_4, C \) are constants that depend on \( R_4 \); \( \ell' \) is a crossover scale between the viscous and inertial range and is determined by matching the two regimes as
\[ \frac{F}{225} \left( \frac{\ell}{\eta_K} \right)^4 = C_4 \left( \frac{\ell}{\eta_K} \right)^{\xi_4}. \] (13)

Now, taking
\[ F \sim R_4^\alpha, \quad C_4 \sim R_4^\beta, \] (14)

we have

\[ \ell/\eta_K \sim R_4^{(\beta-\alpha)/(4-\xi_4)}. \] (15)

Finally, from piecewise integration of Eq. (11), it can be shown that (see Supplemental Material [44] for intermediate steps):
\[ a_0 \sim F(\ell/\eta_K)^2. \] (16)

Substituting the \( R_4 \) dependencies, we get
\[ a_0 \sim R_4^{(2\alpha-\alpha\xi_4+2\beta)/(4-\xi_4)}. \] (17)

The values of \( \alpha, \beta, \) and \( \xi_4 \) are in principle obtainable from Eulerian intermittency models. The exponent \( \alpha \) simply corresponds to \( \tau_2 \) in Eq. (6), since \( F \sim \langle \epsilon^2 \rangle/\langle \epsilon \rangle^2 \). Multifractal and log-normal models predict \( \alpha = \tau_2 \approx 0.38 \). The DNS data for \( F \) are shown in Fig. 4(a), giving \( \alpha \approx 0.387 \), in excellent agreement with the prediction, and also with previous experimental and DNS results in literature [18,20].

![FIG. 4.](image-url)

(a) Flatness of longitudinal velocity gradient as a function of \( R_4 \). (b) Fourth-order structure function compensated by its inertial-range scaling. The inset shows the variation of coefficient \( C_4 \) in Eq. (12) as a function of \( R_4 \).
On the other hand, intermittency models predict $\xi_4 \approx 1.28$ [49]. Our DNS shows $\xi_4 \approx 1.3$, which is well within statistical error bounds. Finally, the prediction for $\beta$ from multifractal model is $\beta \approx (4 - 3\xi_4)/2$, which reduces to $\beta \approx \mu/3$ for log-normal model; both predictions give $\beta \approx 0.08$ (also see Supplemental Material [44]). Figure 4(b) shows the normalized fourth-order structure function from our DNS data, using $\xi_4 \approx 1.3$. Note, as expected, the inertial-range increases with $R_j$. The inset of the bottom panel shows $C_4$, giving $\beta \approx 0.2$. This observed $\beta$ is substantially larger than 0.08 anticipated from multifractal and log-normal models.

The use of $\alpha = 0.387$, $\beta = 0.2$, and $\xi_4 = 1.3$ in Eq. (17) leads to

$$a_0 \sim R_j^\mu, \quad \chi = \xi_2 \approx 0.25,$$

which is in excellent agreement with the high-$R_j$ data shown in Figs. 1 and 3(a). The exponent 0.25 is virtually insensitive to a small variation in $\xi_4$, but is significantly impacted by the choice of $\beta = 0.2$ (instead of 0.08). Moreover, the use of $\beta \approx 0.08$ in Eq. (17) gives $a_0 \sim R_j^{0.15}$, which is essentially the same as the exponent 0.14 obtained earlier in Table I. This shows the robustness of piecewise integration leading to the result in Eq. (17) and also suggests that the discrepancy from multifractal prediction is due to the exponent $\beta$ (and hence the proportionality constant $C_4$). In this regard, the role of $\beta$ needs to be further explored, especially in relation to the inadequacy of Eq. (4).

We note that the exponent 0.25 was also suggested by Hill [21]. However, Hill arrived at this result by deriving that $a_0 \sim F_0^{0.70}$ and $F \sim R_j^{2.10}$ based on [60]; evidently, the current data do not agree with both of these results. It appears that the two errors fortuitously cancelled out each other to give the 0.25 exponent. Finally, we point out that the exponent 0.25 describes the data for $R_j \gtrsim 200$. To describe the data at lower $R_j$, an empirical interpolation formula in the spirit of Eq. (3) can be devised with $\chi = 0.25$. Least-square fit gives $c_1 \approx 0.89$, $c_2 \approx 40$ (also see Supplemental Material [44]).

Conclusions.—The moments of Lagrangian acceleration are known to deviate from classical K41 phenomenology due to intermittency. Attempts were made to quantify these deviations by extending the Eulerian multifractal models to the Lagrangian viewpoint and devising an ad hoc interpolation formula to agree with available data from DNS and experiments. The first contribution of this article is to present new, very well resolved DNS data on Lagrangian acceleration at higher $R_j$, and show that they disagree with the results from multifractal models, and the interpolation formula. The disagreement gets increasingly stronger with the moment order. As a second contribution, the article devises a statistical model that is able to correctly capture the scaling of acceleration variance. While this framework does not seem amenable for generalization to higher-order moments, our results show that the intermittency of Lagrangian quantities remains an open problem, even more compellingly than before.

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The forcing term has negligible contribution to acceleration moments. This is also reaffirmed by collapse of data in Fig. 1 from various sources that use different forcings.

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