We study analytically a black hole domain wall system in dilatonic gravity being the low-energy limit of the superstring theory. Using the C-metric construction we derive the metric for an infinitesimally thin domain wall intersecting dilaton black hole. The behavior of the domain wall in the spacetime of dilaton black hole was analyzed and it was revealed that the extreme dilaton black hole always expelled the domain wall. We elaborated the back reaction problem and concluded that topological kink solution smoothes out singularity of the considered topological defect. Finally we gave some comments concerning nucleation of dilaton black holes on a domain wall and compared this process with the creation of static dilaton black holes in the presence of the domain wall. We found that domain walls would rather prefer to nucleate small black holes on them, than large ones inside them.

1. INTRODUCTION

Motivated by the earlier investigations on uniqueness theorems for black holes (see Ref. [1] and references therein) Wheeler coined the metaphoric dictum black holes have no hair. Regardless of the specific details of the collapse or the structure and properties of the collapsing body a stationary black hole emerged in the resultant process which geometry was characterized by mass, charge and angular momentum. Nowadays there has been a considerable resurgence of mathematical works on black hole equilibrium states. The no-hair conjecture has been extended to the problem of nontrivial topology of some fields configurations. The considerations presented in [2] announced the existence of the Euclidean Einstein equations corresponding to a vortex sitting on the horizon of the black hole. In Refs. [3–5] numerical and analytical evidences for an Abelian-Higgs vortex acting as a long hair for Schwarzschild or Reissner-Nordström (RN) solution were established.

The superstring theories are the most promising candidates for a consistent quantum theory of gravity. Numerical studies of the low-energy string solutions revealed that Einstein-dilaton black holes in the presence of a Gauss-Bonnet type term were endowed with a nontrivial dilaton hair [6]. The extended moduli and dilaton hair connected with axions for Kerr-Newmann black hole background were studied in [7]. The dilaton black holes pierced by a thin vortex were intensively studied both numerically and analytically [8–11]. It was shown that the horizon of a charged dilaton black hole could support the long-range fields of the Nielsen-Olesen vortex which could be considered as black hole hair. It has been argued that the Euclidean dilaton black hole can support a vortex solution sitting at the horizon [8]. Allowing the dilaton black hole to approach extremality it was shown that the vortex was always expelled from the extreme dilaton black hole. In the thin string limit the metric of a conical dilaton black hole was obtained [9–11]. On their own topological defects arising during spontaneous symmetry breaking during phase transitions and their cosmological evolution play the very important role in our understanding of the cosmological evolution [12]. Topolog-
ical defects produced in the early stages of our universe could shed some light on the high energy phenomena which were beyond the range of our accelerators.

Recently domain walls were intensively studied due to the fact that our universe might be a brane or defect being immersed in some higher dimensional spacetime. The motivation for this fact comes from the unifications attempts such superstring theories or M-theory \[13\]. This idea enables us to solve the very intriguing phenomenological possibility of a resolution of the hierarchy problem.

In this paper we shall try to provide some continuity with our previous works \[8–10\] and consider the problem of another topological defect, a domain wall and dilaton black hole. We would like to find an equivalent of black hole string solution given by Aryal *et al.* \[14\] namely a domain wall dilaton black hole metric.

The paper is organized as follows. In Sec.II we derive an infinitesimal domain wall black hole metric in dilaton gravity. In Sec.III we analyze the fields equations of domain wall in the background of dilaton black hole and dilaton C-metric. We derived an analytic thin wall approximation useful in the back reaction problem. In Sec.IV we consider the problem of the expulsion of the domain wall by extremal dilaton black hole. We gave analytical arguments that expulsion always holds for this kind of black hole being the analog of the Meissner effect. The same situation takes place for Abelian-Higgs vortex and extremal dilaton black hole. In Sec.V we deal with the gravitational back reaction and conclude that topological kink solution smooths out the singularity of the domain wall. In Sec.VI we study the problem of a nucleation of dilaton black holes on a domain wall and the process of nucleation of static dilaton black hole pairs in the presence of a domain wall. Sec.VII concludes our results.

II. THE DILATON BLACK HOLE-WALL METRIC

In this section we will try to find an equivalent of a black hole string solution given by Aryal *et al.* in \[14\], i.e., an infinitesimally thin domain wall with a dilaton black hole being the static spherically symmetric solution in dilaton gravity. Dilaton gravity is the low-energy limit of the superstring theory. The action of this theory is given by \[13\]

\[
S = \int dx^4 \sqrt{-g} \left[ R - 2 (\nabla \phi)^2 - e^{-2a\phi} F^2 \right].
\]

The equations of motion derived from the variational principle may be written as follows:

\[
\nabla_{\mu} \left( e^{-2a\phi} F_{\mu\nu} \right) = 0,
\]

\[
\nabla_{\mu} \nabla_{\nu} \phi + \frac{a}{2} e^{-2a\phi} F^2 = 0,
\]

\[
G_{\mu\nu} = T_{\mu\nu}(\phi, F),
\]

where the energy-momentum tensor yields

\[
T_{\mu\nu}(\phi, F) = e^{-2a\phi} \left( 4 F_{\mu\rho} F_{\nu}^\rho - g_{\mu\nu} F^2 \right) - 2 g_{\mu\nu} (\nabla \phi)^2 + 4 \nabla_{\mu} \phi \nabla_{\nu} \phi.
\]

In the case of RN black hole-domain wall system the metric was derived in Ref. \[16\]. It constituted the extension of the results obtained in \[17,18\]. The key point in the above derivation will be the notion of the C-metric being an axially symmetric solution of Einstein gravity which represents two black holes uniformly accelerating apart. The
force for acceleration is provided by a conical excess between black holes or by a conical deficit (string) extending from each black hole to infinity. An external gravitational field can remove the nodal singularities \[19\]. If the black holes are electrically (magnetically) charged the cause of the acceleration is electric (magnetic) \[20\].

We begin with the generalization of the C-metric in dilaton gravity given by \[2\]:

\[
ds^2 = \frac{1}{A^2(x - y)^2} \left[ F(x) \left( G(y) dt^2 - \frac{dy^2}{G(y)} \right) + F(y) \left( \frac{dx^2}{G(x)} + G(x)d\phi^2 \right) \right],
\]

where we have denoted

\[
e^{-2a\phi} = \frac{F(y)}{F(x)} \quad F(\xi) = (1 + r_+ A\xi)^{2a^2 \xi^{1+a^2}}, \quad A_\phi = qx,
\]

\[
G(\xi) = \left[ 1 - \xi^2 (1 + r_+ A\xi) \right] \left[ 1 + r_- A\xi \right]^{1-a^2 \xi^{1+a^2}}.
\]

The metric \[3\] has two Killing vectors \( \partial/\partial t \) and \( \partial/\partial \phi \). The norm of the Killing vector \( \partial/\partial \phi \) vanishes at \( x = \xi_3 \) and \( x = \xi_4 \), this fact corresponds to the existence of the poles of the spheres surrounding the black holes. The axis \( x = \xi_3 \) points along the symmetry axis towards spatial infinity, while the axis \( x = \xi_4 \) points towards the other black hole.

For \( r_+ A > 2/3\sqrt{3} \) the function \( G(\xi) \) has four real roots which in ascending order are denoted by \( \xi_2, \xi_3, \xi_4 \) and we define \( \xi_1 = -\frac{1}{r_+ A} \). The surface \( y = \xi_1 \) is singular for \( a > 0 \). It is analogous to the singular surface (the inner horizon of the dilaton black hole). The surface \( y = \xi_2 \) is the black hole horizon, while \( y = \xi_3 \) is the acceleration horizon for an observer comoving with the black hole. These two surfaces are both Killing horizons for \( \partial/\partial t \). In the limit \( r_+ A \ll 1 \) and \( r_- A \ll 1 \) one obtains

\[
\begin{align*}
\xi_1 &= -\frac{1}{r_+ A} + O(A), \quad \xi_2 = -\frac{1}{r_+ A} + O(A), \\
\xi_3 &= -1 - \frac{r_+ A}{2} + O(A^2), \quad \xi_4 = 1 - \frac{r_+ A}{2} + O(A^2).
\end{align*}
\]

As was discussed in Ref. \[21\] in the ordinary C-metric is impossible to choose generally such a range of \( \phi \) that the metric \[3\] is regular at both \( x = \xi_3 \) and \( x = \xi_4 \). One can get rid of the nodal singularity at \( x = \xi_4 \) by choosing \( \phi \in [0, \frac{4\pi}{G(\xi_4)}] \), but then there is a positive deficit angle running the \( \xi_3 \) direction. It has the interpretation as a string with a positive mass per unit length \( \mu = 1 - \frac{|G(\xi_3)|}{G(\xi_4)} \) pulling the accelerating dilaton black holes away to infinity. On the other hand choosing \( \phi \in [0, \frac{4\pi}{G(\xi_3)}] \), means that one has a negative deficit angle along \( \xi_4 \) direction, interpreted as the black holes are being pushed apart by a rod of the negative mass per unit length equals to \( \mu = 1 - \frac{|G(\xi_4)|}{G(\xi_3)} \).

In order to construct the wall-dilaton black hole metric we shall follow the Israel procedure \[22\], i.e., the discontinuity of the extrinsic curvature is provided by the tension \( \sigma \) of a domain wall. Thus, it implies the following:

\[
[K_{ij}] = 4\pi G\sigma h_{ij},
\]

where \( h_{ij} \) is the metric induced on the wall. Having in mind \[16\], an appropriate umbilic surface can be found at \( x = 0 \). This has normal \( n = \frac{1}{A_y} dx \), and the induced metric is of the form

\[
ds^2 = \frac{1}{A^2 y^2} \left[ G(y) dt^2 - \frac{dy^2}{G(y)} + F(y) d\phi^2 \right].
\]

The extrinsic curvature in this case is \( K_{ij} = A h_{ij} \) and the Israel condition implies that the domain wall tension is equal to \( \sigma = A/2\pi G \).
One can choose the conical singularity to lie at $x = \xi_3$, on the side $x < 0$ of this surface. However the string will vanish from the spacetime if we take two copies of the side $x > 0$ and glue them together along $x = 0$. This construction is equivalent to determining $|x|$ for $x$ in the metric (3). The metric induced on the domain wall has an interesting form after introducing new radial and time coordinates of the form $r = -\frac{1}{A\eta}$ and $T = \frac{1}{A}$, namely it reduces to

$$ds^2 = -\left(1 - \frac{r_+}{r} - A^2 r^2\right) \left(1 - \frac{r}{r} - \frac{1}{1 + \alpha^2} \right) dT^2 + \frac{dr^2}{\left(1 - \frac{r_+}{r} - A^2 r^2\right) \left(1 - \frac{r}{r} - \frac{1}{1 + \alpha^2} \right)} + r^2 \left(1 - \frac{r}{r} - \frac{2\alpha^2}{1 + \alpha^2} \right) d\phi^2. \quad (11)$$

As in Ref. [16] we have chosen the conical singularity to be at $x = \xi_3$ on the side $x < 0$. The consequence of this is that if one takes two copies of the side $x > 0$ and glue them along $x = 0$, the string disappears from the spacetime. The charge of the black hole can be measured by integrating the flux on a sphere surrounding it. It implies

$$Q = 2\frac{1}{4\pi} \int dx d\phi F_{x\phi} = \frac{\Delta \phi}{2\pi} (A_\phi(x = \xi_4) - A_\phi(x = 0))$$

$$= \frac{2\xi_4}{qA^2 (r_+ A) (\xi_4 - \xi_3)(\xi_4 - \xi_2)(\xi_4 - \xi_1) \left(1 + \alpha^2\right)}$$

where $\Delta \phi = \frac{dr}{d\phi}$ is the period of $\phi$ coordinate.

As was pointed out in [16] the constructed domain wall contained two black holes at antipodal points of a spherical domain wall. The constructed black hole will neither swallow up the brane nor slide off of it [31]. The letter fact was revealed because of acting the elastic restoring force by means of which the brane acted on the dilaton black hole.

### III. Domain Wall Black Holes

In this section we shall describe behavior of the domain wall in the spacetime of dilaton black hole. A static, spherically symmetric solution of the equations of motion derived from the action $S$ is concerned it is determined by the metric of a charged dilaton black hole. The metric may be written as [27]

$$ds^2 = -\left(1 - \frac{2M}{r} \right) dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r} \right)} + r \left( r - \frac{Q^2}{M} \right) \left(d\vartheta^2 + \sin^2 \theta d\varphi^2\right), \quad (13)$$

where we define $r_+ = 2M$ and $r_- = \frac{Q^2}{M}$ which are related to the mass $M$ and charge $Q$ by the relation $Q^2 = \frac{r_+ r_-}{2} e^{2\phi_0}$. The charge of the dilaton black hole $Q$, couples to the field $F_{\alpha\beta}$. The dilaton field is given by $e^{2\phi} = (1 - \frac{r}{r_+}) e^{-2\phi_0}$, where $\phi_0$ is the dilaton’s value at $r \to \infty$. The event horizon is located at $r = r_+$. For $r = r_-$ is another singularity, one can however ignore it because $r_- < r_+$. The extremal black hole occurs when $r_- = r_+$, when $Q^2 = 2M^2 e^{2\phi_0}$.

We consider a general matter Lagrangian with real Higgs field and the symmetry breaking potential of the form as follows:

$$\mathcal{L}_{dw} = -\frac{1}{2} \nabla_\mu \varphi \nabla^\mu \varphi - U(\varphi). \quad (14)$$

The symmetry breaking potential $U(\varphi)$ has a discrete set of degenerate minima. The energy-momentum tensor for the domain wall yields
\[ T_{ij}(\varphi) = -\frac{1}{2} g_{ij} \nabla_m \varphi \nabla^m \varphi - U(\varphi) g_{ij} + \nabla_i \varphi \nabla_j \varphi. \] (15)

For the convenience we scale out parameters via transformation \( X = \varphi/\eta \) and \( \epsilon = 8\pi G\eta^2 \). The parameter \( \epsilon \) represents the gravitational strength and is connected with the gravitational interaction of the Higgs field. Defining \( V(X) = \frac{V(\varphi)}{V_F} \), where \( V_F = \lambda \eta^4 \) we arrive at the following expression:

\[ 8\pi G L_{dw} = -\frac{\epsilon}{w^2} \left[ w^2 \frac{\nabla \mu X \nabla^\mu X}{2} + V(X) \right], \] (16)

where \( w = \sqrt{8\pi G V_F} \) represents the inverse mass of the scalar after symmetry breaking, which also characterize the width of the wall defect within the theory under consideration. Having in mind (16) the equations for \( X \) field may be written as follows:

\[ \nabla_\mu \nabla^\mu X - \frac{\partial V}{\partial X} = 0, \] \hspace{1cm} (17)

where without loss of generality we have set \( w = 1 \) in order to fix our unit. In the background of the dilaton black hole spacetime the equation of motion for the scalar field \( X \) yields

\[ \frac{1}{r \left( r - \frac{Q^2}{M} \right)} \partial_r \left[ (r - \frac{Q^2}{M})(r - 2M) \partial_r X \right] + \frac{1}{r \left( r - \frac{Q^2}{M} \right)} \sin \theta \partial_\theta \left[ \sin \theta \partial_\theta X \right] = \frac{\partial V}{\partial X}. \] (18)

As in the case of the vortex and black hole \[2,3,9,10\] the fields were approximated as functions of \( \sqrt{g_{33}} \). Now, we guess the ansatz \( X(z) = X(r \cos \theta) \). Then, we can establish the following:

\[ \nabla_\mu \nabla^\mu X = X'' - \frac{2M z^2}{r^3} X'' + \frac{X'}{z} - \frac{2M}{r^2} X'. \] (19)

Taking into account the fact that outside the black hole horizon \( r \) is far more greater than \( M \), and assuming that the thickness of the wall is much less than the black hole horizon, i.e., \( M \gg 1 \) one can deduce that \( X \) is approaching the flat space solution. Thus in the thin wall approximation the thin wall can be painted on a dilaton black hole. In the case of Schwarzshild solution this fact was confirmed by the numerical calculations \[28\]. Preliminary numerical studies in the dilaton black hole case also confirmed this analytic results \[29\].

Now we proceed to the problem of painting the domain wall onto the dilaton C-metric. As we have expected our gravitating wall-black hole system will be described by the dilaton C-metric. By virtue of the new variables defined by

\[ r = -\frac{1}{Ay}, \quad T = \frac{t}{A}, \quad \theta = \int^x_3 \frac{dx}{\sqrt{G(x)}}, \] (20)

one can reach to the following metric:

\[
d s^2 = \frac{1}{(1 + Arx)^2} \left[ -F(x)H(r)dT^2 + \frac{F(x)}{H(r)} dr^2 + K^2(r) (d\theta^2 + G(x) d\phi^2) \right], \] (21)

where we have denoted

\[ H(r) = -\left( 1 - \frac{r_+}{r} - A^2 r^2 \right) \left( 1 - \frac{r_+}{r} \right)^{\frac{1-a^2}{1+a^2}}, \] \hspace{1cm} (22)

\[ K^2(r) = r^2 \left( 1 - \frac{r_+}{r} \right)^{\frac{2a^2}{1+a^2}}. \]
Now it is easily seen that the variable $x$ is basically $\cos \theta$ and as in Ref. [16] we guess $z = \frac{A}{x}$. Thus, after straightforward but tedious calculations we have:

$$\nabla^\mu \nabla^\nu X = X'' \left( zA - 1 \right)^2 \frac{G(x)}{F(y)} - \frac{A^2 z^2 G(y)}{F(x)}$$

\[ + X' A \left( zA - 1 \right) \left( (F(x)G(x))' y + (F(y)G(y))' x \right) - 2 \left( \frac{G(x)}{F(y)} + \frac{AG(y)}{F(x)} \right) - \frac{2zA(zA - 1)G(y)}{F(x)} \]  

(23)

The thin wall approximation [16] in the context of the C-metric means that $A | \xi_2 | \ll 1$, i.e., the black hole horizon radius has to be large. However for a self-gravitating domain wall there is a limit due to the wall formation. This limit is given by the size of the spontaneously compactified spacetime which corresponds to the acceleration horizon, then we will work having in mind the large regime of the accelerated horizon, i.e., $A | \xi_3 | \ll 1$. In this case the wall fields differ significantly from their vacuum because $z \sim 1$. On the other hand, the values of $y$ are bounded by the black hole and the acceleration horizon $1 < | \xi_3 | \leq y < | \xi_2 |$ and therefore $x \leq Ay \leq 1$. Thus we have

$$\Box X = X'' + O(A).$$  

(24)

We complete this section by the conclusion that the flat spacetime solution $X_0(z)$ is a good approximation to the solutions of the field equations in the dilaton C-metric spacetime.

IV. EXPELLING OF THE WALL BY THE EXTREMAL DILATON BLACK HOLE

In the previous section we argued the existence of the domain wall solution in the case of a large black hole masses. Now we shall consider the case of a very small extremal black hole sitting inside the domain wall. Inside the core of the wall the potential term is very small compared to the gradient terms, so we can neglect it. The solution for a wall in the absence of black hole in the region adjacent to its core is $X \simeq z - z_0$, we try the ansatz $X(r, \theta) = b(r) \cos \theta$ [16]. Applying the ansatz to Eq. (18) one gets

$$2\left( r - M - \frac{Q^2}{2M} \right) b' + \left( r - \frac{Q^2}{M} \right) (r - 2M) b'' - 2b = 0.$$  

(25)

The solution of Eq. (25) is provided by

$$X \approx \left( r - M - \frac{Q^2}{2M} \right) \cos \theta.$$  

(26)

One can observe that if we have to do with a non-extreme dilaton black hole, one has $X \neq 0$ on the horizon. For an external dilaton black hole for which $r_+ = r_- \text{ occurs}$, i.e., $Q = \sqrt{2}M$ we obtain that $b(r_+) = 0$ and $X(r_+) = 0$. This is in accord with the fact of the expulsion of a domain wall by a small extremal dilaton black hole sitting inside the domain wall. The same situation was revealed in the case of the extreme dilaton black hole and the Nielsen-Olesen vortex [9,10]. We gave analytical and numerical arguments that the vortex was always expelled from the considered black hole. In the case of an Abelian-Higgs vortex and extremal dilaton black hole system the analog of the Meissner effect was found.

After proving that the flux expulsion must take place for a sufficiently thick domain wall from the extremal dilaton black hole, we treat the case of a thin domain wall. As was shown in Ref. [30] the metric of the extreme dilaton black hole near horizon may be written in the form of Bertotti-Robinson metric, namely
We denote for simplicity $\Omega = A_0 \Omega = \Omega$ from the extremal dilaton black hole. Then using the fact that $\rho < 0$ on $[0, \frac{\pi}{2}]$ we deduce that

$$X_{,\theta \theta} < \frac{X(\theta) - X(\theta_0)}{\theta - \theta_0} < \frac{X(\theta)}{\theta - \beta} < \frac{X(\theta)}{\theta - \beta}.$$  

(30)

This enables us to write the following:

$$\frac{1}{M^2 - \Sigma^2} > (\theta - \beta) \cot \theta.$$  

(31)

The above relation must hold over the range of $\theta \in (\beta, \frac{\pi}{2})$ for the expulsion to occur. Since $\theta - \beta > 0$, $\cot \theta$ on this interval is greater than zero, then the relation (31) always holds and one gets the expulsion of the thin domain wall from the extremal dilaton black hole.

V. GRAVITATIONAL BACK REACTION

In order to study the gravitational back reaction problem, we shall consider the thick-wall dilaton black hole metric (9). We denote for simplicity $\Omega = A(x - y)$ and perform a linearized calculations in $\epsilon = 3A/2$ as in Ref. [14], writing $\Omega = \Omega_0 + A\Omega_1$ and so on. Near the core of the domain wall $\Omega_1/\Omega_0 = O(1)$ and tends to zero far away from it. Let us calculate

$$g^{xx}X_{,x}X_{,x} = \frac{A^2(x - y)^2 \left[ 1 - x^2 (1 + r_+ Ax) \right] (1 + r_- Ax) \frac{1}{1 + A^2}}{(1 + r_- Ay) \frac{2}{1 + A^2}} \frac{1}{A^2 y^2 (X')^2} \simeq (X'_0)^2,  

(32)

$$

$$g^{yy}X_{,y}X_{,y} = -\frac{A^2(x - y)^2 \left[ 1 - y^2 (1 + r_+ Ay) \right] (1 + r_- Ay) \frac{1}{1 + A^2}}{(1 + r_- Ax) \frac{2}{1 + A^2}} \frac{1}{2 y X'} \simeq O(A^2),  

(33)

$$

$$g^{xx}\phi_{,x}\phi_{,x} = \frac{A^2(x - y)^2 \left[ 1 - x^2 (1 + r_+ Ax) \right] (1 + r_- Ax) \frac{1}{1 + A^2}}{(1 + r_- Ay) \frac{2}{1 + A^2}} \frac{1}{A^2 y^2 (\phi')^2} \simeq (\phi'_0)^2,  

(34)

$$

$$g^{yy}\phi_{,y}\phi_{,y} = \frac{A^2(x - y)^2 \left[ 1 - y^2 (1 + r_+ Ay) \right] (1 + r_- Ay) \frac{1}{1 + A^2}}{(1 + r_- Ax) \frac{2}{1 + A^2}} \frac{1}{2 y \phi'} \simeq O(A^2).  

(35)
From now on, for simplicity, we set $a = 1$ in our considerations. Then, equations of motion (3) and (4) take the forms as follows:

$$\partial_x \left[ e^{-2\phi} \frac{F(x)F(y)}{\Omega^4} F_{\phi x} \right] + \partial_y \left[ e^{-2\phi} \frac{F(x)F(y)}{\Omega^4} F_{\phi y} \right] = 0,$$

(36)

$$\partial_x \left[ \frac{F(x)G(x)}{\Omega^4} \phi_{,x} \right] - \partial_y \left[ \frac{F(y)G(y)}{\Omega^4} \phi_{,y} \right] + \frac{1}{2} \sqrt{-g} e^{-2\phi} F^2 = 0,$$

(37)

As in Ref. [10] we shall assume that the first order perturbed solutions are determined by

$$\phi_1 = \phi_1(z), \quad A_{\mu}^{(1)} = g(z)A_{\mu}^{(0)}.$$  

(38)

Taking into account (24) and (37) we draw a conclusion that to the leading order in $A$ we get $\phi_1 = \text{const}$. Next from the relation (36) we have

$$-2\partial_x (\phi_1 q) + \partial_x^2 \left[ g(z)A_{\phi}^{(0)} \right] = 0.$$  

(39)

Then, one gets that $g(z) = \text{const}$. The gauge potential for the Maxwell fields is unaltered by the presence of the domain wall.

To the leading order in $A$ we have the following generalized Einstein equations:

$$R_{0}^0 = -2e^{-2\phi} q^2 \Omega^4 + \epsilon V(X),$$  

(40)

$$R_{x}^x = 3e^{-2\phi} q^2 \Omega^4 + 4(\phi')^2 + \epsilon V(X) + \epsilon (X_0')^2,$$  

(41)

$$R_{y}^y = -2e^{-2\phi} q^2 \Omega^4 + \epsilon V(X) + O(A),$$  

(42)

$$R_{\phi}^\phi = 6e^{-2\phi} q^2 \Omega^4 + \epsilon V(X),$$  

(43)

$$R_{xy} = -\frac{4z}{A_y q^2} (\phi')^2 - \frac{2\epsilon z}{A_y q^2} (X_0')^2.$$  

(44)

As was mentioned in [10] because of the fact that the variation of the extrinsic curvature due to the wall is carried by $\Omega$, one guesses that $F$ and $G$ will effectively take their background values. After lengthy calculations we find the following:

$$R_{0}^0 - R_{y}^y = \frac{1}{2F(x)F(y)} \left[ G(y)\Omega^2 F'(y)^2 + 4G(y)\Omega_{,yy} F(y)^2 \right],$$  

(45)

$$R_{\phi}^\phi - R_{x}^x = \frac{1}{2F(x)^2F(y)} \left[ -G(x)\Omega F'(x)^2 - 4G(x)F(x)^2 \Omega_{,xx} \right],$$  

(46)

$$R_{0}^0 - R_{\phi}^\phi = \frac{1}{2F(x)^2F(y)} \left[ F(x)\Omega^2 G''(x) + F(y)\Omega^2 G''(y) + 2G(y)F'(y)\Omega_{,y} \right. \left. - 2F(y)G'(y)\Omega_{,y} + 2G(x)F'(x)\Omega_{,x} - 2F(x)G'(x)\Omega_{,x} \right].$$  

(47)

The above relations suggest that $\Omega$ may be written as

$$\Omega = A(f - y),$$  

(48)

where $f_0 = |x|$. Inputting the ansatz for $\Omega$ and $X_0 = \tanh z$ one obtains
\[
R_\theta^0 - R_y^\gamma = \frac{1}{2F(x)F(y)} \left[ G(y)A^2(f - y)^2(r - A)^2 + 4G(y)A^2y(1 + A | z | )f_{yy}F^2(y) \right],
\]
\[
R_\phi^0 - R_x^\gamma = \frac{1}{2F^2(x)F(y)} \left[ -G(x)A^2(f - y)^2 + 4G(x)F^2(x)A^2y(1 - A | z | )f_{xx} \right] = -4\phi^2 - \frac{\epsilon}{\cosh(\frac{xy}{A})},
\]
\[
R_\theta^0 - R_\phi^0 = \frac{1}{2F(x)F(y)} \left[ -2F(x)A^2(f - y)^2(1 + 3r_+Ax) - 2F(y)A^2(f - y)^2(1 + 3r_+Ay) + 2G(y)A^3y(A | z | - 1)(f_{yy} - 1) + 2F(y)A^2y^2(2 + 3r_+Ay)(A | z | - 1)(f_{yy} - 1)
+ 2G(x)r_-A^3yf_{xx}(A | z | - 1) + 2F(x)A^2yx(2 + 3r_+Ax)f_{xx}(A | z | - 1) \right].
\]

Since \(f_{xx} = O(A)\) and \(f_{yy} = O(A^2)\) the generalized Einstein’s equations in dilaton gravity are satisfied to the leading order in \(A\).

Thus, we get the solution
\[
f = -\frac{y}{2} \left[ \frac{1}{6} \operatorname{tanh}^2 \left( \frac{x}{Ay} \right) + \frac{2}{3} \ln \cosh \left( \frac{x}{Ay} \right) \right] - 2A \int dx' dx'y' \left( \frac{d\phi}{dx'} \right)^2.
\]

As in the case of the domain wall black hole system in general relativity the topological kink solution smoothes out the shell-like singularity of the infinitesimal domain wall. The same situation takes place for topological vortex solutions which smooth out the delta function singularity in various kind of metrics.

VI. NUCLEATION OF DILATON BLACK HOLES ON AND IN THE PRESENCE OF DOMAIN WALLS

In this section we shall be concerned with the process of nucleation of dilaton black holes on the domain wall and black holes enclosed by a wall. Let us first consider the case of dilaton black hole on a domain wall. The probability of nucleation of a domain wall with a black hole on it is given by \(\exp[-(I - I_0)]\), where \(I_0\) is the Euclidean action of the initial configuration, while \(I\) is the Euclidean action of the final state with dilaton black hole on it. We assume the no-boundary conditions for the wave function of the considered universe. The considered exponent can be also viewed as the ratio of probabilities to nucleate a domain wall with or without black hole on it. In order to construct the wall black hole instanton one can use the on-shell equation
\[
I = -\frac{1}{4} \left( A_{ac} + A_{bh} \right),
\]
giving the action in terms of the appropriate area of the horizons.

One obtains the following:
\[
I = -\frac{\xi_4}{2\pi\sigma^2} \left| G'(\xi_4) \right| \left[ \frac{1}{(\xi_3 - \xi_4)\xi_3} \left( 1 + r_-A\xi_3 \right) \frac{\phi^2}{\xi_3^2} \right] + \frac{1}{(\xi_2 - \xi_1)\xi_2} \left( 1 + r_-A\xi_2 \right) \frac{\phi^2}{\xi_2^2},
\]
while the Euclidean action for a domain wall is given by
\[
I_0 = -\frac{1}{8\pi\sigma^2}.
\]
If one considers the case of \(a = 1\) and the limit \(r_+ \ll 1\) and \(r_-A \ll 1\) the expression simplifies and the resulting relation yields
\[ I = -\frac{1}{8\pi\sigma^2} \left[ 1 - 2\pi\sigma \left( 4M + \frac{Q^2}{M} \right) \right] + O(A^2). \] (56)

Hence,
\[ I - I_0 = (I - I_0)_{RN-dw} + \frac{Q^2}{4\sigma M}, \] (57)

where \((I - I_0)_{RN-dw}\) is the value of the exponent in the nucleation rate for the RN-domain wall system. In the extremal dilaton black hole case it yields
\[ (I - I_0)_{ext} = 1.5 \frac{M}{\sigma}. \] (58)

The black hole mass \(M\) is a parameter which can be varied independently of the domain wall tension. Therefore it can be arbitrary small.

In addition to the process of nucleation of dilaton black holes on the wall there exists the process of nucleation of black holes enclosed by a domain wall. In Ref. [24] it was found that domain walls could nucleate black holes at a finite distance from them. The double-sided nature of the domain wall caused that it enclosed a black hole on each side of it. The authors, among all, considered both charged and neutral black holes, while Ref. [26] was devoted to the pair creation of black holes in the presence of supergravity domain walls with broken and unbroken symmetry. Of course, the tantalizing question arises, which of these two processes is more likely to take place.

In order to build the instanton for static black hole nucleation one should first construct the Lorentzian section. To get a nonzero probability it ought to be required that a spatially closed universe has a finite three-volume (it caused that the total energy at the instant of nucleation vanished). This cut-and-paste procedure is depicted on Fig. 2 in Ref. [24]. For the reader’s convenience we quote this procedure, namely, one has two copies of dilaton black hole spacetime and join them along \(r = const\) timelike hypersurface at the location of the domain wall. Then, one identifies two external regions of the dilaton spacetime (this implies the \(R \times S^1 \times S^2\) topology of the spacetime). The resulting spacetime contains two domain walls and two domains containing dilaton black hole in each. In order to get the domain wall and dilaton black hole instanton one starts with the usual Riemannian section of dilaton black hole with mass \(M\), with topology \(S^2 \times R^2\) which will be cut along \(r = const\) and glue back to back. The outcome will be a baguette with \(S^2 \times S^2\) topology with a ridge at the domain wall. As in the Schwarzschild case one may check that if we take the hypersurface \(r = 3M\), it will be totally umbilic, its extrinsic curvature \(K_{ij}\) is proportional to induced metric \(h_{ij}\) on the domain wall and \([K_{ij}] = 4\pi\sigma h_{ij}\). In the end the Riemannian space has been joined to the Lorentzian spacetime depicting the creation from nothing of a closed spacetime containing two domain walls and two domains containing a dilaton black hole in each of them.

Now we shall proceed to the problem of creation of static magnetically charged dilaton black holes. The static black hole is the black hole which attractive gravitational energy exactly counterbalances the repulsive energy of the wall. We pointed out that the static limit domain wall are the wall for which \(\dot{r} = 0\), where the derivative was taken with respect to the proper time of the wall. The static domain wall lies at \(r_{st} = 3M\) [24]. The case of electrically charged dilaton black hole is quite analogous, except the fact that the electromagnetic charge must be imaginary on the Riemannian section [23]. The Euclidean action for the instanton will include an electromagnetic and dilaton contributions. Thus, we obtain
\[ I_{st} = I_{E_1} + I_{E_2} + I_{E_3} = -\frac{1}{2} \int_W d^3x \sqrt{h} + \int_M d^4x \sqrt{g} \frac{1}{16\pi} \left[ 2(\nabla \phi)^2 + e^{-2\phi}F^2 \right]. \] (59)

There are no boundary terms because the considered instantons are compact and without boundary. Calculating the first term in (59), due to the presence of the domain wall [24], gives

\[ I_{E_1} = 4\pi \sqrt{f(r)} \beta R(r)|_{r_{st}}, \] (60)

where \( R(r) = \sqrt{f(r) - \frac{Q^2}{M}} \) and \( f(r) \) is the \( g_{00} \) Euclidean dilaton black hole coefficient in \((t, r)\) coordinates. The action is evaluated at \( r_{st} \) and \( \beta = 8\pi M \) is the instanton period for the dilaton black hole. For the second and third term the integration over \( M \) covers both sides of the domain wall and yield

\[ I_{E_2} = \frac{\beta}{4} \left( \frac{Q^2}{M} - 2M \right) \ln \left( \frac{r - \frac{Q^2}{M}}{r} \right) - 2 \frac{Q^2}{r} \bigg|_{r_+ \to r_{st}}, \] (61)

while

\[ I_{E_3} = \frac{M}{Q^2} \ln \left( \frac{r - \frac{Q^2}{M}}{r} \right) \bigg|_{r_+ \to r_{st}}, \] (62)

where \( r_+ \) is the outer horizon of dilaton black hole.

In order to get the probability for the pair creation of static charged dilaton black hole in the presence of the domain wall we divide the amplitude for this process by the amplitude for the domain wall creation, which implies the following expression \( \exp(- (I_{st} - I_0)) \). In contrast to the process of nucleation of dilaton black holes on the domain wall, the nucleation in the presence of the domain wall is characterized by the fact that the mass of black hole is fixed [23] and equal to \( M = 1/6\sqrt{3\pi\sigma} \). Therefore it cannot be varied independently of the wall’s tension. For the extremal dilaton black hole \( P_{dil \text{ ext}} = \exp(- (I_{st} - I_0)) = -73/648\pi\sigma^2 \). If we take the probabilities for nucleation of Schwarzschild black hole \( P_{\text{Schw}} = \exp(-11/216\pi\sigma^2) \) and extremal RN black hole \( P_{\text{RN ext}} = \exp(-3/32\pi\sigma^2) \), one can see that the following takes place:

\[ P_{\text{Schw}} > P_{dil \text{ ext}} > P_{\text{RN ext}}. \] (63)

In the considered case the black hole of a certain (large) size can nucleate, therefore this process will be heavily suppressed. In the nucleation of black holes on the domain wall the black hole mass is a parameter which can be varied independently of \( \sigma \), and can be made arbitrarily small. Thus, the domain walls will prefer to nucleate small black holes on them rather than large ones.

**VII. CONCLUSIONS**

In our work we had studied the problem of dilaton black hole sitting on a domain wall. Applying the recently devised C-metric construction [17,18] we found the metric for an infinitesimally thin wall intersecting dilaton black hole. The behavior of the domain wall in the spacetime of the considered black hole and dilaton C-metric was studied. We derived the thin wall approximation useful in our further studies namely in a gravitational back reaction problem. Having in mind the behavior of the Abelian-Higgs vortex and extremal dilaton black hole, we analyzed the domain wall
extreme dilaton black hole system and gave analytic arguments that the extreme dilaton black hole always expelled
the domain wall. Thereby we have extended the phenomenon of the flux expulsion to the case of dilaton black hole
domain wall system. We also considered the gravitational back reaction problem concluding that the topological kink
solution smoothed the shell-like singularity of the infinitesimal domain wall. We studied the nucleation process of
dilaton black holes on the domain wall and compared it to the nucleation of black hole pairs in the presence of a
domain wall. In the last case the black hole of a certain, large size can be produced. Therefore it will be heavily
suppressed. But in the nucleation of black holes on the domain wall the black hole mass parameter can be varied
independently of the wall tension and can be made arbitrarily small. Then, domain walls will rather prefer to nucleate
small black holes on them, than large ones inside them.

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