Using a new analysis method to extract excited states in the scalar meson sector

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  - Search for $a_0$ including loops
TARGET: extract mass states from correlation matrix

→ for large time distances lowest state can be extracted, but . . .
  - how to extract if signal is only good for few time slices?
  - how to extract if signal is dominated by more than one states?

We are using: ensemble generated with 2+1 dynamical clover fermions and Iwasaki gauge action by the PACS-CS Collaboration [Aoki et.al. 2008]
Lattice $64 \times 32^3$ with $a \sim 0.09$ fm and
$\sim 500$ configurations at $M_\pi \sim 300$ MeV

We are using $a_0$ interpolators with $J^P = 0^+$ [Joshua Berlin’s Talk]
interpolators quark content: $\bar{d}u + \bar{s}s$
→ 6x6 correlation matrix with $q\bar{q}, di\bar{Q}d\bar{Q}, \pi\eta_s, 2K$

→ we will start with the $q\bar{q}$ correlator
The Scalar channel: $qar{q}$

Operator with $J^P = 0^+$: $\mathcal{O}(x) = \overline{d}(x)u(x)$

Correlator $C(t) = \langle \mathcal{O}(t)\mathcal{O}^\dagger(0) \rangle$ on a periodic lattice:

Standard technique to extract effective masses using asymptotic behavior:

$$C(t/a) = \sum_i A_i \cosh(E_i(t - T/2)) \quad \xrightarrow{t \gg 1, T \gg 1} A_0 \cosh(E_0(t - T/2)) + \ldots$$

$\rightarrow$ here dominated by two states
The Scalar channel: $q\bar{q}$

Operator with $J^P = 0^+$: $O(x) = \bar{d}(x)u(x)$

Effective mass on a periodic lattice:

$$C(t/a) = \frac{\cosh(E_{\text{eff}}(t - T/2))}{\cosh(E_{\text{eff}}(t + 1 - T/2))}$$

→ dominated by two states
The Scalar channel: $q\bar{q}$

here: lowest energy easy to identify (high statistics)
however: we are not interesting in lowest state (unphysical)

→ lattice artefact, backwards traveling pion

- higher state do not have a plateau
- can’t be extracted by a simple plateau average

→ what is with the correlator?
The Scalar channel: $q\bar{q}$

here: we simply fit the logarithm behavior for $t \in [2; 4]$ and $t \in [8; 12]$

Another possibility: fitting the correlator

$$f(t) = \sum_{n=0}^{N} A_n \cosh(E_n(t - T/2))$$

This is a non-linear minimization
AMIAS

- Standard least-squares algorithms: are numerically unstable, depend on the initial condition and fail for very low signal-to-noise ratio

⇒ to circumvent this we will use AMIAS

AMIAS (Athens Model Independent Analysis Scheme)

- relies on statistical concepts
- sample probability distributions of parameters
- also insensitive parameters are fully accounted and do not bias the results
- can access a large number of parameters by using Monte Carlo techniques

[Alexandrou et.al. arXiv:1411.6765],
[C. Papanicolas and E. Stiliaris, arXiv:1205.6505],
[E. Stiliaris and C. Papanicolas, AIP Conf. Proc. 904, 257 (2007)]
AMIAS: Basic Idea

- Using the $\chi^2$ of the fit function $f(t) = \sum_{n=0}^{N} A_n \cosh(E_n(t - T/2))$

$$\chi^2 = \sum_{k=1}^{N_t} \left( \frac{(C(t_k) - \sum_{n=0}^{\infty} A_n \cosh\{E_n(t_k - T/2)\})^2}{(\sigma_{t_k}/N)} \right).$$

- Using the central limit theorem
  $\Rightarrow$ each value assigned to the model parameters has a statistical weight proportional to

$$P(C(t_n); n) = e^{-\chi^2/2}$$

Model parameters
The probability for parameter $A_i, (E_i)$ to have a specific value $a_i$ is given by

$$\Pi(A_i = a_i) = \frac{\int_{b_i}^{c_i} dA_i \int_{-\infty}^{\infty} \prod_{j \neq i} dA_j, A_i e^{-\chi^2/2}}{\int_{-\infty}^{\infty} \left( \prod_j dA_j \right) A_i e^{-\chi^2/2}}.$$

$\Rightarrow$ Multi-dimensional integrals $\rightarrow$ Monte Carlo sampling with $P(C(t_n); n)$
Scalar channel: $q\bar{q}$

Results from AMIAS:

⇒ distribution of two energies and their corresponding amplitudes
- clean distinction
- need to set intervals for identifying them
  we use multiple tempering to explore the whole parameter region
Scalar channel: $q\bar{q}$

Results from AMIAS:

![Graph showing results from AMIAS with peaks E1, E2, A1, and A2.]

**lattice artefact:**
the $\pi$ is propagating backwards in time
generate a lattice artefact state with
$\rightarrow aE_{art} = a(m_{\eta_s} - m_\pi) \sim 0.2$

state around the $a_0$-region $\sim 0.6$:
in this region is the is $\pi + \eta_s$ and the 2 kaon channel
$\rightarrow$ we need to couple the meson-meson interpolators to $q\bar{q}$
Using in search for $a_0$ particle

**AMIAS:** can be also adapted for correlation matrices (“Search for $a_0$ interpolators $J^P = 0^+$”)

\[
\begin{align*}
(j = 0) : \quad & O^{q\bar{q}} = \sum_x \left( \bar{d}_x u_x \right) \\
(j = 1) : \quad & O^{K\bar{K}}_{\text{point}} = \sum_x \left( \bar{s}_x \gamma_5 u_x \right) \left( \bar{d}_x \gamma_5 s_x \right) \\
(j = 2) : \quad & O^{\eta\pi}_{\text{point}} = \sum_x \left( \bar{s}_x \gamma_5 s_x \right) \left( \bar{d}_x \gamma_5 u_x \right) \\
(j = 3) : \quad & O^{d_1 Q d_1 \bar{Q}} = \sum_x \epsilon_{abc} \left( \bar{s}_x, b C \gamma_5 \bar{d}_x, c \right) \epsilon_{ade} \left( u^T_{x,d} C \gamma_5 s_x, e \right) \\
(j = 4) : \quad & O^{K\bar{K}}_{2-\text{part}} = \sum_{x,y} \left( \bar{s}_x \gamma_5 u_x \right) \left( \bar{d}_y \gamma_5 s_y \right) \\
(j = 5) : \quad & O^{\eta\pi}_{2-\text{part}} = \sum_{x,y} \left( \bar{s}_x \gamma_5 s_x \right) \left( \bar{d}_y \gamma_5 u_y \right)
\end{align*}
\]

\[\Rightarrow 6 \times 6 \text{ correlation matrix:} \]
\[
C_{jk}(t) = \langle O_j(t) O^\dagger_k(0) \rangle_t \sim \sum_{n=0}^{\infty} \langle 0 | O_j(t) | n \rangle \langle n | O^\dagger_k(0) | 0 \rangle e^{-E_n t}.
\]

To analyze Correlation matrix:

**Generalized Eigenvalue Problem (GEVP) and AMIAS**

Cross check with GEVP by solving:

\[
[C(t)] v_n(t, t_0) = \lambda_n(t, t_0) [C(t_0)] v_n(t, t_0),
\]

on the otherside: using Amias by fitting every matrix element with:

\[
C_{jk}(t) = \sum_{n=0}^{\infty} A_j^{(n)} A_k^{*(n)} \cosh\{ -E_n (t - T/2) \}.
\]
Measurements: GEVP 5x5 without loops

- Lowest states coincidence with $\eta_s \pi$ and 2–kaon
- Diquark-antidiquark interpolating field ($j = 4$) mixed with excited states
- No $a_0$ candidate expected to be around the $\eta_s \pi$ and the 2–kaon states
Measurements: AMIAS 4x4 without loops

Using AMIAS for correlation matrix with 2–meson interpolators \((2 \times (\pi + \eta_s) \text{ and } 2 \times 2K)\)

- Energies correspondes to expectations from single channels
- No \(a_0\) candidate \(\rightarrow\) expected to be around the \(\eta_s \pi\) and the 2–kaon states
- Clear signal for energies and corresponding amplitudes
Measurements: AMIAS 4x4 without loops

Overlap of states with different interpolating fields?
→ using energies and corresponding amplitude to reconstruct correlation matrix
→ extract overlap from eigenvectors of the orthogonalized GEVP

→ agree with results of pure GEVP
→ needs to identify the amplitudes
→ can be difficult due to overlapping amplitudes/energies
Measurements: AMIAS 6x6 with loops *PRELIMINARY*

4 lowest states can be resolved (higher states can not be identify)

- strange quark loops introduces large noise (higher states very difficult to resolve)
- indication for an addtional state in the \(a_0\) region
  \[\rightarrow\] measurements are ongoing (*this are PRELIMINARY results*)
Conclusions

**AMIAS:**
- sampling of fit parameters by using $\chi^2$
- can be used for single channels and correlation matrices

avoid plateau fits

can extract states from channels which are dominated by more than one state
→ like it is done in case of $q\bar{q}$

**AMIAS in the search for $a_0$:**

**without loops:** resolve four lowest state

**with loops:** $a_0$ candidate

**Ongoing:**
- increasing statistics
- investigate AMIAS: for example:
  - excluding noisy matrix elements
  - identification of amplitudes and energies
Thanks