Terahertz Massive MIMO with Holographic Reconfigurable Intelligent Surfaces

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Abstract

We propose a holographic version of a reconfigurable intelligent surface (RIS) and investigate its application to terahertz (THz) massive multiple-input multiple-output systems. Capitalizing on the miniaturization of THz electronic components, RISs can be implemented by densely packing subwavelength unit cells, so as to realize continuous or quasi-continuous apertures and to enable holographic communications. In this paper, in particular, we derive the beam pattern of a holographic RIS. Our analysis reveals that the beam pattern of an ideal holographic RIS can be well approximated by that of an ultra-dense RIS, which has a more practical hardware architecture. In addition, we propose a closed-loop channel estimation (CE) scheme to effectively estimate the broadband channels that characterize THz massive MIMO systems aided by holographic RISs. The proposed CE scheme includes a downlink coarse CE stage and an uplink finer-grained CE stage. The uplink pilot signals are judiciously designed for obtaining good CE performance. Moreover, to reduce the pilot overhead, we introduce a compressive sensing-based CE algorithm, which exploits the dual sparsity of THz MIMO channels in both the angular and delay domain. Simulation results demonstrate the superiority of holographic RISs over the non-holographic ones, and the effectiveness of the proposed CE scheme.

Index Terms

Terahertz communications, reconfigurable intelligent surface, massive MIMO, holographic communications, compressive sensing (CS), channel estimation.

I. INTRODUCTION

Over the past few years, the demand for wireless data traffic has increased significantly due to the explosive growth of mobile devices and multimedia applications [1]–[3]. To accommodate...
these demands, the possible use of the terahertz (THz) band has attracted great interest from both industry and academia [1]–[7]. The THz band can provide more abundant bandwidth (from 0.1 THz to 10 THz), higher data rates (from tens of Gbps to several Tbps), and lower latency (of the order of micro-seconds [1]), as compared with the millimeter-wave (mmWave) band. The THz band is considered to be a promising candidate to enable beyond 5G and 6G communications.

In spite of the appealing advantages of the THz band, establishing a reliable transmission link at THz frequencies is a non-trivial task. This is because (i) there exist strong atmospheric attenuation and extremely high free-space loss in the THz band; and (ii) the line-of-sight (LoS) link is very sensitive to blockage effects in the THz band and thus the links are usually intermittent. These disadvantages may negatively affect the communication range and may severely degrade the service coverage of THz communication systems. The deployment of massive [5] or even ultra-massive [6], [7] multiple-input multiple-output (MIMO) systems in the context of THz communications may provide considerable beamforming gain in order to overcome the mentioned limitations, but it may result in an unaffordable power consumption and may put an overweight burden on the overall communication system design.

Recently, the emerging technology of reconfigurable intelligent surface (RIS) has been proposed and applied to wireless communications in order to enhance the communication performance [8]–[25]. Made of passive and metamaterial-based reconfigurable elements, RIS can manipulate both the phase and amplitude of the incident electromagnetic (EM) signals so as to reflect them towards the desired directions. More importantly, unlike other transmission technologies such as active relay [13], an RIS does not need power-hungry radio frequency chains (RFCs) and power amplifiers, which may be beneficial for developing green and cost-efficient communications. Although the application of RIS to the mmWave band has been investigated recently [18], [21]–[24], the utilization of RIS for THz communications is still at its infancy.

A. Prior Work

RIS-aided MIMO communications have attracted lots of research interest lately. The authors of [16] propose a joint active and passive beamforming scheme based on convex optimization to maximize the signal-to-interference-plus-noise ratio (SINR) at the receivers. A similar scenario is considered in [17], where the phases of the RIS that maximize the SINR are computed via the projected gradient ascent algorithm. In [18], broadband beamforming for RIS-aided hybrid mmWave MIMO systems is investigated. A geometric mean decomposition (GMD)-based beamforming scheme is applied to achieve better bit-error-rate (BER) performance than conventional singular value decomposition (SVD)-based beamforming.
The beamforming designs in [16]–[18] rely on the knowledge of global channel state information (CSI). Since no active elements are used in RISs, channel estimation (CE) is a challenging task and an essential prerequisite in RIS-aided MIMO systems. In [19] and [20], the authors propose a CE algorithm for RIS-aided systems, based on the least square (LS) estimator, for application to frequency-flat fading channels and frequency-selective fading channels, respectively. To further facilitate the CE task when large-size arrays are deployed, compressive sensing (CS)-based CE schemes are investigated in [21], [22], where the inherent sparsity of mmWave channels in the angular domain is exploited in order to reduce the pilot overhead. In [23] and [24], a new architecture of RIS for application to mmWave band is proposed, where a few active RFCs are available at the RIS in order to learn the channel in real time. Based on this architecture, deep learning (DL) techniques are adopted for CE and beamforming.

As far as the application of RISs to THz communications is concerned, the authors of [25] present an RIS-aided THz MIMO communication system for indoor applications. Joint CE and data rate maximization schemes are proposed in [25] based on both CS and DL techniques. In [26], the problem of RIS-assisted secure transmission in the THz band is investigated. The phase shifts at the RIS that maximize the secrecy rate are obtained based on convex optimization. Based on the current state of research, we evince that the design of RIS-aided THz communication systems is still at an early development stage.

Recently, the concept of holographic communication has been proposed as a new paradigm shift in MIMO [27]–[29] and RIS-aided [30] communications. One of the main features of holographic communications is the integration of very large numbers of tiny and inexpensive antennas or reconfigurable elements into a compact space in order to realize a holographic array with a spatially continuous aperture [27]–[30]. This holographic architecture is easier to realize in the THz band thanks to the miniaturization of THz electronic components. In [25], for example, the size of each graphene-based reflecting element is 200µm × 190µm at a carrier frequency of 0.22 THz that corresponds to a wavelength $\lambda \approx 1360\mu m$. Therefore, the reflecting elements can be spaced more densely than $\lambda/2$, as done in previous works [16]–[25], so as to form a spatially continuous surface [30], since the resulting surface is homogenizable [8]. This densely spaced or continuous implementation of RISs is referred to as holographic RIS. Channel modeling and data transmission schemes based on active holographic surfaces are investigated in [27] and [30], respectively. However, to the best of our knowledge, no current research works have tackled the physical layer transmission design of passive holographic communications, where nearly-passive holographic RISs with spatially continuous apertures are deployed.
B. Paper Contributions

In this work, we focus our attention on the analysis of holographic RISs for application to massive MIMO systems in the THz band. In particular, the main contributions of this paper can be summarized as follows:

- **We derive the beam pattern of an RIS made of discrete elements, and propose an angular-domain beamforming framework.** By applying Fourier analysis to the reflection coefficients of the elements at the RIS, we prove that the beam pattern of an RIS with discrete elements can be represented as a weighted integral of Dirichlet kernel functions. On this basis, we propose an angular-domain beamforming framework. The weighting factors in the beam pattern are designed and the corresponding reflection coefficients of the RIS are reconstructed via the Fourier transform of the obtained weighting factors.

- **We generalize the analysis and design to (continuous) holographic RISs.** Based on the proposed beamforming framework, we derive and obtain closed-form solutions for the beamforming design in two important cases, i.e., narrow beam steering (NBS) and spatial bandpass filtering (SBF), which play an important role in RIS-aided communication systems. We further extend these solutions to holographic RISs, in which the elements are closely spaced so as to yield a virtually continuous spatial aperture. The results reveal that the beam pattern of an ideal holographic RIS can be well approximated by an ultra-dense RIS, which results in a practical hardware architecture.

- **We propose a closed-loop CE scheme to effectively estimate the broadband channels of THz massive MIMO systems based on holographic RISs.** The proposed approach consists of downlink and uplink transmissions. In the downlink transmission, the holographic RIS uses SBF beamforming so that the users can coarsely estimate the range of LoS angles. In the subsequent uplink transmission, the users with similar LoS angles are scheduled into the same group, and the coarsely-estimated LoS angles are exploited to design the uplink pilot signals for the finer-grained uplink CE. To further reduce the uplink pilot overhead, a CS-based CE scheme is introduced, where the dual sparsity of THz MIMO channels in both the angular and delay domain is leveraged.

C. Notation

Column vectors and matrices are denoted by lower- and upper-case boldface letters, respectively. \((\cdot)^*, (\cdot)^T, (\cdot)^H\) and \((\cdot)^\dagger\) denote the conjugate, transpose, conjugate transpose and the pseudo-inverse, respectively. \(\mathbb{C}\) and \(\mathbb{Z}\) are the sets of complex-valued numbers and integers, respectively. \(a \propto b\) denotes \(a = Cb\) with \(C\) being a non-zero constant. \([\cdot]_i\) and \([\cdot]_{i,j}\) represent
the $i$-th element of a vector and the $i$-th row, $j$-th column element of a matrix, respectively. $[A]_I$ denotes the submatrix consisting of the columns of $A$ indexed by the set $I$. $\text{diag}(\cdot)$, $\|\cdot\|_F$, and $\otimes$ represent the diagonalization, Frobenius norm, and Kronecker product, respectively. $\text{vec}(\cdot)$ is the vectorization operation according to the columns of the matrix, and $\text{vec}^{-1}(\cdot)$ is the corresponding inverse operation. $|I|$ is the cardinality of the set $I$. $\Xi_N(x)$ is the $N$-order Dirichlet kernel function given by $\Xi_N(x) = \frac{\sin(Nx/2)}{N\sin(x/2)}$ for $x \neq 2k\pi$, and $\Xi_N(2k\pi) = (-1)^{k(N-1)}$, $k \in \mathbb{Z}$. The “sinc” function is defined as $\text{sinc}(x) = \frac{\sin x}{x}$ for $x \neq 0$, and $\text{sinc}(0) = 1$. $\delta(x)$ is the Dirac function. $F_N$ is an $N \times N$ normalized discrete Fourier transformation (DFT) matrix with the elements $[F_N]_{m,n} = \frac{1}{\sqrt{N}} e^{-j\frac{2\pi}{N}mn}$. $U(a,b)$ stands for the uniform distribution within $(a,b)$.

II. SYSTEM DESCRIPTION AND CHANNEL MODEL

In this section, we present the system model and the effective baseband channel model of the considered RIS-aided THz massive MIMO system over frequency-selective fading channels. We consider an RIS-aided THz massive MIMO system that operates in time division duplex (TDD) mode, as illustrated in Fig. 1. The base station (BS) and user-equipments (UEs) are equipped with half-wavelength spaced uniform planar arrays (UPAs) that consist of $M_B = M_{Bx} \times M_{By}$ and $M_U = M_{Ux} \times M_{Uy}$ antennas, respectively, where $M_{Bx}$ ($M_{Ux}$) and $M_{By}$ ($M_{Uy}$) is the number of antennas along the azimuth and elevation directions, respectively. An RIS of physical size $A_x \times A_y$ is deployed to enhance the effective coverage of the BS. As shown in Fig. 1(a), we assume that the LoS link between the BS and the UE is blocked by an obstacle or the human body. Thus, the UE communicates with the BS only via the RIS, which is regarded as a virtual LoS transmission in RIS-aided systems [22]. To reduce the power consumption and hardware cost, a hybrid analog-digital architecture is considered at the BS, i.e., there are only $N_{RF} \ll M_B$.

![Fig. 1. The model of an RIS-aided THz massive MIMO system. (a) A multi-antenna UE is served by the BS with the help of an RIS when the LoS path is blocked by possible obstacles. (b) The hardware architectures at the BS and the UE.](image)
RFCs at the BS, and each of them is connected to $M^B$ antennas through $M^B$ phase shifters. In addition, each UE employs analog beamforming, in which only one RFC is connected to $M^U$ antennas through $M^U$ phase shifters. An orthogonal frequency division multiplexing (OFDM) transmission scheme with $K$ subcarriers and sampling period $T_s$ is adopted. The cyclic prefix (CP) of length $N_{CP}T_s$ is added before each OFDM symbol to avoid inter-symbol interference. The center-carrier frequency is $f_c$ whose corresponding wavelength is $\lambda$.

Through an appropriate deployment of the RIS, we assume that there exists a LoS path between the BS and the RIS. The LoS angles between the BS and the RIS are assumed to be known in advance based on the location of the RIS [22]. In THz channels, the path loss of the non-LoS (NLoS) paths is known to be much larger than that of the LoS paths. Therefore, we neglect the NLoS paths in the channel between the BS and the RIS.

In the following text, we first introduce the physical channel model that is widely considered in previous works on RIS-aided MIMO systems [21]–[25]. Then, we introduce an effective baseband channel model by taking into account the beamforming design at the BS, the RIS and the UE.

a) Physical Channel Model with Discrete RISs: Under the assumptions that the RIS has $N$ reflecting elements and that narrowband signals are transmitted, the downlink flat fading channel $G \in \mathbb{C}^{N \times M^B}$ from the BS to the RIS can be modeled as

$$G = \alpha a_{RIS}(\psi_{RIS}) a_{BS}^H(\psi_{BS}),$$

(1)

where $\alpha$ is the channel coefficient, $\psi_{BS} = [\psi_{azi}^{BS}, \psi_{ele}^{BS}]^T$ and $\psi_{RIS} = [\psi_{azi}^{RIS}, \psi_{ele}^{RIS}]^T$ are the virtual LoS angle of departure (AoD) and virtual LoS angle of arrival (AoA) of the BS-RIS channel, respectively. $\psi_{BS}$ ($\psi_{RIS}$) includes both the azimuth part $\psi_{azi}^{BS}$ ($\psi_{azi}^{RIS}$) and elevation part $\psi_{ele}^{BS}$ ($\psi_{ele}^{RIS}$), which are assumed to be fixed and known as detailed in previous text. $a_{BS}(\psi_{BS}) \in \mathbb{C}^{M^B \times 1}$ and $a_{RIS}(\psi_{RIS}) \in \mathbb{C}^{N \times 1}$ denote the steering vectors at the BS and the RIS, respectively. $a_{BS}(\psi_{BS})$ is given by

$$a_{BS}(\psi_{BS}) = \left[1, ..., e^{-jd_{UPA}}(m_x \psi_{azi}^{BS} + m_y \psi_{ele}^{BS}), ..., e^{-jd_{UPA}}((M_x^B - 1) \psi_{azi}^{BS} + (M_y^B - 1) \psi_{ele}^{BS})\right]^T,$$

(2)

where $d_{UPA} = \lambda/2$ is the element spacing of the UPA at the BS, and $0 \leq m_x \leq (M_x^B - 1)$, $0 \leq m_y \leq (M_y^B - 1)$. $a_{RIS}(\psi_{RIS})$ can be written by using a similar notation and assumptions.

As for the RIS-UE channel $H \in \mathbb{C}^{M^U \times N}$, we consider a Rician fading channel model that consists of one LoS path and $L$ NLoS paths, as shown in Fig. 1(b). In particular, we have

$$H = \beta a_{UE}(\nu_{LoS}) a_{RIS}^H(\mu_{LoS}) + \frac{\beta}{\sqrt{LK_f}} \sum_{l=1}^{L} a_{UE}(\nu_l) a_{RIS}^H(\mu_l),$$

(3)
where $\beta$ is the channel coefficient, $K_f$ is the Rician factor that denotes the ratio of the energy between the LoS and NLoS channels, $\mu_{\text{LoS}}$ and $\nu_{\text{LoS}}$ are the virtual LoS AoD and virtual LoS AoA, respectively, $\mu_l$ and $\nu_l$ are the virtual NLoS AoD and virtual NLoS AoA of the $l$-th NLoS path. The steering vector at the UE $a_{\text{UE}}(\nu_l) \in \mathbb{C}^{M_{\text{U}} \times 1}$ can be formulated similar to (2).

b) Effective Baseband Channel Model: The effective baseband channel is defined as the inner product between the analog beamforming vector at the transceiver and the steering vector of the physical channels. Assuming that the $r$-th RFC ($1 \leq r \leq N_{\text{RF}}$) of the BS and the UE use the analog beamforming vectors $f_r \in \mathbb{C}^{M_{\text{B}} \times 1}$ and $w \in \mathbb{C}^{M_{\text{U}} \times 1}$, respectively, and that the RIS uses $\varphi \in \mathbb{C}^{N \times 1}$ as the reflection coefficients, the effective baseband channel $h^\text{eff}_r$ can be represented as

$$h^\text{eff}_r = w^H H \Phi G f_r$$

$$= \alpha g^\text{BS} (\psi_{\text{BS}}) \left[ \beta \tilde{g} (\mu_{\text{LoS}}, \psi_{\text{RIS}}) g_{\text{UE}} (\nu_{\text{LoS}}) + \frac{\beta}{\sqrt{LK_f}} \sum_{l=1}^{L} \tilde{g} (\mu_l, \psi_{\text{RIS}}) g_{\text{UE}} (\nu_l) \right], \quad (4)$$

where $\Phi = \text{diag}(\varphi) \in \mathbb{C}^{N \times N}$, and $g^\text{BS} (\psi_{\text{BS}}) = a_{\text{BS}}^H (\psi_{\text{BS}}) f_r$, $\tilde{g} (\mu, \psi_{\text{RIS}}) = a_{\text{RIS}}^H (\mu) \Phi a_{\text{RIS}} (\psi_{\text{RIS}})$, and $g_{\text{UE}} (\nu) = w^H a_{\text{UE}} (\nu)$ are the beam patterns of the BS, the RIS, and the UE, respectively.

We observe that the dimension of the effective baseband channel is agnostic to the dimensions of the arrays deployed in the system, and thus this model simplifies the channel representation.

The effective baseband channel model in (4) can be generalized to the case of frequency-selective channel. In a frequency-selective channel, in particular, the BS-RIS-UE effective baseband channel in the delay domain can be formulated as

$$h_r (\tau) = \underbrace{\alpha g^\text{BS} (\psi_{\text{BS}})}_{\text{BS to RIS}} \left[ h^\text{LoS} (\tau) + h^\text{NLoS} (\tau) \right], \quad (5)$$

where

$$h^\text{LoS} (\tau) = \beta \tilde{g} (\mu_{\text{LoS}}, \psi_{\text{RIS}}) g_{\text{UE}} (\nu_{\text{LoS}}) p (\tau - \tau_{\text{LoS}}), \quad (6)$$

and

$$h^\text{NLoS} (\tau) = \frac{\beta}{\sqrt{LK_f}} \sum_{l=1}^{L} \tilde{g} (\mu_l, \psi_{\text{RIS}}) g_{\text{UE}} (\nu_l) p (\tau - \tau_l). \quad (7)$$

In (6) and (7), $\tau_{\text{LoS}}$ and $\tau_l$ denote the delay offset of the LoS path and the $l$-th NLoS path of the RIS-UE channel, respectively, and $p(\tau)$ is the pulse shaping filter function.

It is worth noting that, in contrast with RISs made of discrete elements [16]–[25], a holographic RIS is modeled as an array with a spatially continuous aperture [30] (i.e., $N \to \infty$). The physical channels associated with a holographic RIS cannot be represented in terms of the finite-dimensional matrices in (1) and (3). Therefore, in this paper, we utilize the effective baseband channel model in (5)-(7) to describe the channels associated with a holographic RIS. The beam patterns in (6) and (7) are elaborated in detail in the next section.
Since we consider that the system operates in TDD mode, we exploit the channel reciprocity between the uplink and downlink transmissions. In particular, the uplink channels can be modeled based on the downlink channels reported in previous text. We omit the details of the uplink channel model for brevity.

III. BEAMFORMING DESIGN FOR HOLOGRAPHIC RISs

To complete the formulation of the effective baseband channel model in (5)-(7), we first derive and analyze the beam pattern of RISs based on discrete planar arrays (DPAs). Then, we extend the results to RISs with spatially continuous apertures, which are referred to as continuous metasurfaces (CMSs). In addition, we derive closed-form beamforming solutions in two important cases, i.e., narrow beam steering and spatial bandpass filtering, which both play an essential role in the proposed CE scheme for RIS-aided THz massive MIMO systems.

A. RISs Based on Discrete Planar Arrays

As shown in Fig. 2(a) and (b), a DPA-based RIS placed on the $x$-$y$ plan consists of numerous evenly-spaced reflecting elements. The number of elements along the $x$- and $y$-directions is $N_x \in \mathbb{Z}$ and $N_y \in \mathbb{Z}$, respectively. The distance between two adjacent elements is $d$, and we assume $d \leq \lambda/2$ due to the Nyquist sampling theorem. We define the total physical size of a DPA-based RIS as $A_x \times A_y$ with $A_x = N_x d$ and $A_y = N_y d$, and assume that $A_x$ and $A_y$ remain unchanged unless stated otherwise. Let $(x_m, y_n)$ be the coordinate of the $(m, n)$-th reflecting element, then we have

$$x_m = (m-1)d, \quad y_n = (n-1)d,$$

where $1 \leq m \leq N_x$, $1 \leq n \leq N_y$. Assume that a narrowband reference signal “1” impinges on the RIS with an azimuth AoA $\theta_{azi}^{\text{in}} \in [0, 2\pi)$ and an elevation AoA $\theta_{ele}^{\text{in}} \in [0, \pi/2]$ (defined in Fig. 2(a)). The phase-difference of the incident signal at the $(m, n)$-th element (compared to the center point $(0, 0)$) can be written as

$$a(x_m, y_n; \psi_{\text{in}}) = e^{j \frac{2\pi}{\lambda} \mathbf{p}^T \mathbf{r}} = e^{j \frac{2\pi}{\lambda} (x_m \cos \theta_{azi}^{\text{in}} \sin \theta_{ele}^{\text{in}} + y_n \sin \theta_{azi}^{\text{in}} \sin \theta_{ele}^{\text{in}})} = e^{j (x_m \psi_{azi}^{\text{in}} + y_n \psi_{ele}^{\text{in}})}.$$

![Fig. 2. Illustration of different types of RIS. (a) Critically-spaced RIS with $d = 0.5\lambda$; (b) ultra-dense RIS with $d < 0.5\lambda$, which has a spatial quasi-continuous aperture; (c) CMS with $d \to 0$ (ignoring the physical size of a single reflecting element), which has an ideal spatial continuous aperture.](image)
where \( \mathbf{p} = [x_n, y_m, 0]^T \) is the position vector of the \((m, n)\)-th element, \( \mathbf{r} = [\cos \theta_{\text{in}}^{\text{azi}} \sin \theta_{\text{in}}^{\text{ele}}, 
abla \sin \theta_{\text{in}}^{\text{azi}} \sin \theta_{\text{in}}^{\text{ele}} \cdot \cos \theta_{\text{in}}^{\text{ele}}]^T \) is the vector of the incident direction, and \( \psi_{\text{in}} = \begin{bmatrix} \psi_{\text{azi}}^{\text{in}} \\ \psi_{\text{ele}}^{\text{in}} \end{bmatrix} \) with \( \psi_{\text{azi}}^{\text{in}} = \frac{2\pi}{\lambda} \cos \theta_{\text{azi}}^{\text{in}}, \psi_{\text{ele}}^{\text{in}} = \frac{2\pi}{\lambda} \sin \theta_{\text{azi}}^{\text{in}} \sin \theta_{\text{ele}}^{\text{in}} \) is a 2-tuple variable representing the virtual AoA of the incident signal. Similarly, for the AoD denoted by \( \psi_{\text{azi}}^{\text{out}} \) and \( \psi_{\text{ele}}^{\text{out}} \), we define \( \psi_{\text{out}} = \begin{bmatrix} \psi_{\text{azi}}^{\text{out}} \\ \psi_{\text{ele}}^{\text{out}} \end{bmatrix} \) with \( \psi_{\text{azi}}^{\text{out}} = \frac{2\pi}{\lambda} \cos \theta_{\text{azi}}^{\text{out}}, \psi_{\text{ele}}^{\text{out}} = \frac{2\pi}{\lambda} \sin \theta_{\text{azi}}^{\text{out}} \sin \theta_{\text{ele}}^{\text{out}} \) as the virtual AoD. These definitions of virtual angles simplify the notations. The received signal along the direction of observation (i.e., reflection) \( \psi_{\text{out}} \) after the incident signal is reflected by the RIS is denoted by \( g(\psi_{\text{out}}, \psi_{\text{in}}) \), which is given by the superposition of the signals reflected by all the individual elements of the RIS

\[
g(\psi_{\text{out}}, \psi_{\text{in}}) = \sum_{n=1}^{N_x} \sum_{m=1}^{N_y} a^* (x_m, y_n; \psi_{\text{out}}) \Phi (m, n) a (x_m, y_n; \psi_{\text{in}})
\]

\[
= \sum_{n=1}^{N_x} \sum_{m=1}^{N_y} \Phi (m, n) e^{-j(x_m \Delta_x + y_n \Delta_y)},
\]

where \( \Delta_x = \psi_{\text{azi}}^{\text{in}} - \psi_{\text{azi}}^{\text{out}}, \Delta_y = \psi_{\text{ele}}^{\text{in}} - \psi_{\text{ele}}^{\text{out}} \), and \( \Phi (m, n) \in \mathbb{C} \) is the reflection coefficient of the \((m, n)\)-th element of the RIS, whose amplitude and phase are software-programmable via an RIS controller. We refer to \( g(\psi_{\text{out}}, \psi_{\text{in}}) \) as the beam pattern of the DPA-based RIS, which is consistent with the definition in [4]. The amplitude of \( g(\psi_{\text{out}}, \psi_{\text{in}}) \) can be used to evaluate the intensity of the signal along the direction \( \psi_{\text{out}} \) after reflection from the RIS. In general terms, \( \Phi (m, n) \) can be treated as a two-dimensional discrete signal in the spatial domain with spatial sampling period equal to \( d \) in both the azimuth and elevation directions. By using the discrete-time Fourier transform (DTFT), \( \Phi (m, n) \) can be represented as

\[
\Phi (m, n) = \left( \frac{d}{2\pi} \right)^2 \int_{\frac{\pi}{d}}^{\frac{\pi}{d}} \int_{\frac{\pi}{d}}^{\frac{\pi}{d}} \omega (k, l) e^{j(dm_k + dn_l)} dk dl,
\]

where

\[
\omega (k, l) = \sum_{m=1}^{N_x} \sum_{n=1}^{N_y} \Phi (m, n) e^{-j(dm_k + dn_l)}
\]

is a 2-dimensional periodic function whose period is \( \frac{2\pi}{d} \) with respect to both \( k \) and \( l \), and \( \int_{\frac{\pi}{d}}^{\frac{\pi}{d}} (\cdot) dl(\cdot) \) denotes the integral in an arbitrary interval of length \( \frac{2\pi}{d} \). Substituting (11) into (10), we obtain

\[
g(\psi_{\text{out}}, \psi_{\text{in}}) = \left( \frac{d}{2\pi} \right)^2 \sum_{m=1}^{N_x} \sum_{n=1}^{N_y} \int_{\frac{\pi}{d}}^{\frac{\pi}{d}} \int_{\frac{\pi}{d}}^{\frac{\pi}{d}} \omega (k, l) e^{j(dm_k + dn_l)} dk dl \cdot e^{-j(x_m \Delta_x + y_n \Delta_y)}
\]

\[1\] We assume that the virtual AoA \( \begin{bmatrix} \psi_{\text{azi}}^{\text{in}} \\ \psi_{\text{ele}}^{\text{in}} \end{bmatrix} \) is known and is treated as a constant in the beamforming design. This follows from the assumption that the LoS angles of the BS-RIS channel are fixed and known.
\[
\left(\frac{d}{2\pi}\right)^2 \int_{\frac{2\pi}{d}} \int_{\frac{2\pi}{d}} \omega'(k, l) \sum_{m=1}^{N_x} e^{j(dmk - x_m \Delta_x)} \sum_{n=1}^{N_y} e^{j(dnl - y_n \Delta_y)} dk dl
\]

\[
= \frac{A_x A_y}{4\pi^2} e^{j\frac{d-\Delta_x}{2} \Delta_x} e^{j\frac{d-\Delta_y}{2} \Delta_y} \times \int_{\frac{2\pi}{d}} \int_{\frac{2\pi}{d}} \omega'(k, l) \cdot \Xi_{N_x} [d (k - \Delta_x)] \Xi_{N_y} [d (l - \Delta_y)] dk dl,
\]

Equation (13) reveals that the beam pattern \( g(\psi_{out}, \psi_{in}) \) can be formulated as a weighted integral of Dirichlet kernel functions \([31]\) whose weighting factors are \( \omega'(k, l) \). Equations (11)-(13) shed light on the design of angular-domain beamforming for DPA-based RIS. In particular, the proposed design is based on two steps: (i) first, \( \omega'(k, l) \) in (13) is optimized in order to design \( g(\psi_{out}, \psi_{in}) \) that corresponds to the desired direction of observation; and (ii) then, the corresponding reflection coefficients \( \Phi(m, n) \) are reconstructed via (11). In the next sub-section, we further explain this procedure by providing the optimal beamforming for two specific cases.

B. Beamforming Design for DPA-Based RIS

In this sub-section, we describe the following two use cases for beamforming design based on the proposed approach.

a) Narrow beam steering. Given the desired beamforming direction \( \psi_{opt} = [\psi_{opt}^{azi}, \psi_{opt}^{ele}]^T \), the target of NBS is to maximize \( |g(\psi_{out}, \psi_{in})| \) for \( \psi_{out} = \psi_{opt} \), and to minimize (null) \( |g(\psi_{out}, \psi_{in})| \) for \( \psi_{out} \neq \psi_{opt} \). Since \( |\Xi_{N_x} [d (k - \Delta_x)]| \) and \( |\Xi_{N_y} [d (l - \Delta_y)]| \) attain their maximum if and only if \( k = \frac{2\pi u}{d} + \Delta_x \), \( u \in \mathbb{Z} \) and \( l = \frac{2\pi v}{d} + \Delta_y \), \( v \in \mathbb{Z} \), respectively, we can impose the following design for the angular-domain coefficients of NBS

\[
\omega_{NBS}(k, l; \psi_{opt}) \propto \sum_{u \in \mathbb{Z}} \delta(k - k_{opt, u}) \sum_{v \in \mathbb{Z}} \delta(l - l_{opt, v}),
\]

where \( k_{opt, u} = \frac{2\pi u}{d} + (\psi_{opt}^{azi} - \psi_{azi}^{opt}) \) and \( l_{opt, v} = \frac{2\pi v}{d} + (\psi_{opt}^{ele} - \psi_{ele}^{opt}) \). It can be readily verified that (15) ensures the required periodicity of \( \frac{2\pi}{d} \) with respect to both \( k \) and \( l \). The corresponding beam pattern for NBS can be obtained by substituting (15) into (13) and (14), which yields

\[
g_{NBS}(\psi_{out}, \psi_{in}; \psi_{opt}) \propto e^{j\frac{d-\Delta_x}{2} \Delta_x} e^{j\frac{d-\Delta_y}{2} \Delta_y} \Xi_{N_x} [d (k_{opt} - \Delta_x)] \Xi_{N_y} [d (l_{opt} - \Delta_y)],
\]

where \( k_{opt} \triangleq k_{opt, 0} \) and \( l_{opt} \triangleq l_{opt, 0} \), since in (13) we choose the integral intervals containing \( k_{opt, 0} \) and \( l_{opt, 0} \) for \( k \) and \( l \), respectively. From (11), the reflection coefficients for NBS can be formulated as follows

\[
\Phi_{NBS}(m, n; \psi_{opt}) \propto e^{j(dmk_{opt} + dnl_{opt})},
\]
Fig. 3. An example of beam pattern of NBS based on (16). $N_x = N_y = 64$, $d = \lambda/2$. The desired angle is $\frac{\lambda}{2\pi} \psi_{\text{opt}} = N_x = N_y = 64$, $d = \lambda/2$. The cut-off angles are $\frac{\lambda}{2\pi} \psi_{\min} = [0.6, -0.2]^T$ and $\frac{\lambda}{2\pi} \psi_{\max} = [0.2, 0.6]^T$.

Fig. 4. An example of beam pattern of SBF based on (23). $N_x = N_y = 64$, $d = \lambda/2$. The cut-off angles are $\frac{\lambda}{2\pi} \psi_{\min} = [0.6, -0.2]^T$ and $\frac{\lambda}{2\pi} \psi_{\max} = [0.2, 0.6]^T$.

b) Spatial bandpass filtering. As far as SBF is concerned, we aim to design $|g(\psi_{\text{out}}, \psi_{\text{in}})|$ to be quasi-constant for $\psi_{\text{azi}} \min \leq \psi_{\text{azi}} \text{out} \leq \psi_{\text{azi}} \max$ and $\psi_{\text{ele}} \min \leq \psi_{\text{ele}} \text{out} \leq \theta_{\text{ele}} \max$, and to be almost zero, i.e., $|g(\psi_{\text{out}}, \psi_{\text{in}})| \approx 0$ otherwise, where $\psi_{\min} = [\psi_{\text{azi}} \min, \psi_{\text{ele}} \min]^T$ and $\psi_{\max} = [\psi_{\text{azi}} \max, \psi_{\text{ele}} \max]^T$ are referred to as the cut-off angles. To clearly explain the design of SBF, we first decompose the values of $k$ in one period as follows

$$k_{\min} \leq k \leq k_{\max}, \quad (18)$$

$$a < k < k_{\min} \text{ or } k_{\max} < k \leq a + \frac{2\pi}{d}, \quad (19)$$

where $k_{\min} = \psi_{\text{azi}} \min - \psi_{\text{azi}} \text{in}$, $k_{\max} = \psi_{\text{azi}} \max - \psi_{\text{azi}} \text{in}$, and $a \in \mathbb{R}$ is an arbitrary number that satisfies the constraints $a < k_{\min}$ and $a + \frac{2\pi}{d} > k_{\max}$. Since we assume $d \leq \lambda/2$, the existence of $a$ is guaranteed. Similarly, the values of $l$ in one period can be decomposed as follows

$$l_{\min} \leq l \leq l_{\max}, \quad (20)$$

$$b < l < l_{\min} \text{ or } l_{\max} < l \leq b + \frac{2\pi}{d}, \quad (21)$$

where $l_{\min} = \psi_{\text{ele}} \min - \psi_{\text{ele}} \text{in}$, $l_{\max} = \psi_{\text{ele}} \max - \psi_{\text{ele}} \text{in}$, and $b \in \mathbb{R}$ is an arbitrary number that satisfies the constraints $b < l_{\min}$ and $b + \frac{2\pi}{d} > l_{\max}$.

The angular-domain coefficients for SBF (in one period of $k$ and $l$) can be designed as follows

$$\omega_{\text{SBF}}(k, l; \psi_{\min}, \psi_{\max}) \propto \begin{cases} e^{-j\frac{N_x+1}{2}dk}, & \text{for (18) and (20)} \\ e^{-j\frac{N_y+1}{2}dl}, & \text{for (19) and (21)} \\ 0, & \text{otherwise} \end{cases} \quad (22)$$

Substituting (22) into (13) and (14), and choosing the intervals of integration equal to $(a, a + \frac{2\pi}{d})$ for $k$ and equal to $(b, b + \frac{2\pi}{d})$ for $l$, we obtain the following beam pattern

$$g_{\text{SBF}}(\psi_{\text{out}}, \psi_{\text{in}}; \psi_{\min}, \psi_{\max}) \propto e^{j\frac{d-A_x}{2}\Delta_x} e^{j\frac{d-A_y}{2}\Delta_y}$$
\[
\int_{l_{\min}}^{l_{\max}} \int_{k_{\min}}^{k_{\max}} \Xi_N[x \{d(k - \Delta_x)\}] \Xi_N[y \{d(l - \Delta_y)\}] \, dk \, dl. \tag{23}
\]

The corresponding reflection coefficients can be calculated from (11), which yields
\[
\Phi_{\text{SBF}}(m,n; \psi_{\min}, \psi_{\max}) \propto \int_{l_{\min}}^{l_{\max}} \int_{k_{\min}}^{k_{\max}} e^{-j\frac{N_x+1}{2} dk} e^{-j\frac{N_y+1}{2} dl} e^{j(dmk+dlN)} \, dk \, dl
\]
\[
= e^{-j\Delta_x(m\psi_{\min} + n\psi_{\max})} \frac{e^{j\Delta_y(m\psi_{\min} + n\psi_{\max})} - e^{j\Delta_y(m\psi_{\min} + n\psi_{\max})}}{d\bar{m}} \frac{e^{j\Delta_y(m\psi_{\min} + n\psi_{\max})} - e^{j\Delta_y(m\psi_{\min} + n\psi_{\max})}}{d\bar{n}}, \tag{24}
\]

where \( \bar{m} = m - \frac{N_x+1}{2} \) and \( \bar{n} = n - \frac{N_y+1}{2} \).

To validate the proposed designs, we show two realizations of \(|g_{\text{NBS}}(\psi_{\text{out}}, \psi_{\text{in}}; \psi_{\text{opt}})|\) and \(|g_{\text{SBF}}(\psi_{\text{out}}, \psi_{\text{in}}; \psi_{\min}, \psi_{\max})|\) with normalized amplitudes in Fig. 3 and Fig. 4, respectively. It can be observed that the obtained beam patterns based on the proposed beamforming framework well fulfill the desired design. It is noteworthy that the reflection coefficients in (17) and (24) are given in closed-form and are physically realizable. By direct inspection of Fig. 3 and Fig. 4, we observe that (i) the NBS design criterion allows one to obtain a small focused region, which can be useful for beamforming applications; and (ii) the SBF design criterion allows one to obtain a wide focused region, which can be useful for broadcasting applications.

### C. Extension to Holographic RISs

As far as the design of DPA-based RISs is concerned, we have considered an element spacing equal to \( d = \lambda/2 \), which is a critical spacing based on the Nyquist sampling theorem. This assumption has been, implicitly or explicitly, adopted in previous research works [16]–[25]. However, the design and optimization of RISs based on elements spaced at the critical distance has some inherent drawbacks. In particular, (i) due to the periodicity and the non-negligible sidelobes of the Dirichlet kernel functions, a power leakage phenomenon [32] is usually observed, which may result in inter-beam interference; (ii) the energy received and reflected by the RIS highly depends on its effective reflection area [35]. More specifically, the use of critically-spaced RISs usually degrades the effective reflection area, which may result in a reduced energy efficiency.

Since the physical size of an RIS is limited by several practical factors, an effective solution to overcome the just mentioned drawbacks is to increase the number of reflecting elements and to reduce their spacing \( d < \lambda/2 \) while keeping \( A_x \) and \( A_y \) unchanged, as illustrated in Fig. 2(b). A DPA-based RIS whose elements have an element spacing \( d < \lambda/2 \) is referred to as ultra-dense RIS. It is worth mentioning that the analysis reported in the previous sub-section can be applied to critically-spaced and ultra-dense RISs by appropriately choosing the value of \( d \). In Figs. 5 and 6 to elucidate these aspects, we report the beam patterns of DPA-based...
RISs for different values of $d$ (only the beam patterns along the azimuth direction, i.e., with respect to $\psi_{\text{azi}}^\text{out}$, are reported for ease of illustration). It can be observed that the beam patterns of an ultra-dense RIS with $d = \{\lambda/4, \lambda/8\}$ are quite similar to those of a critically-spaced RIS ($d = \lambda/2$), but the power leakage is suppressed. By increasing the number of reflecting elements, more importantly, the effective reflection area of an ultra-dense RIS is expected to increase as compared with critically-spaced RIS, and thus a larger fraction of the energy of the incident EM signal can be steered towards the desired direction. This point is elaborated in detail in Section V, where the numerical results are presented.

The performance improvement from critically-spaced RISs to ultra-dense RISs naturally motivates us to ask: what is the performance of DPA-based RISs in the asymptotic regime $d \to 0$ and $N_x, N_y \to \infty$ (while keeping $N_x d = A_x$ and $N_y d = A_y$ fixed)? An RIS that is obtained by letting $d \to 0$ is referred to as CMS. In the following text, we show that the beam patterns and the design of the reflection coefficients of a CMS can be obtained by extending the analysis of DPA-based RISs.

Consider the CMS illustrated in Fig. 2(c), where each point $(x, y), x \in [0, A_x], y \in [0, A_y]$ is capable of manipulating the phase and amplitude of the incident signal. The reflection coefficients of a CMS, which are denoted by $\tilde{\Phi}(x, y)$, are continuously distributed within $[0, A_x] \times [0, A_y]$. Based on the properties of the Fourier transform, the DTFT in (11) and (12) tends to the continuous-time Fourier transformation (CTFT). In particular, we have

$$\tilde{\Phi}(x, y) \propto \lim_{d \to 0} \Phi(m, n) \big|_{dm=x, dn=y}, \ x \in [0, A_x], \ y \in [0, A_y].$$

(25)
The corresponding beam pattern of a CMS, \( \tilde{g}(\psi_{\text{out}}, \psi_{\text{in}}) \), can be calculated as

\[
\tilde{g}(\psi_{\text{out}}, \psi_{\text{in}}) \propto \lim_{d \to 0} g(\psi_{\text{out}}, \psi_{\text{in}}).
\]  

(26)

In this sub-section, the parameters related to CMSs are top-marked with \( \sim \) in order to distinguish them from DPA-based RISs. We note, in particular, that the constraints \( N_x d = A_x \) and \( N_y d = A_y \) need to be inherently enforced in (25) and (26). Based on (25) and (26), the next two corollaries report the beamforming design of NBS and SBF for application to CMSs.

**Corollary 1.** Consider a CMS based on the NBS-based beamforming design. The reflection coefficients and the beam pattern, which are denoted by \( \tilde{\Phi}_{\text{NBS}}(x, y; \psi_{\text{opt}}) \) and \( \tilde{g}_{\text{NBS}}(\psi_{\text{out}}, \psi_{\text{in}}; \psi_{\text{opt}}) \), respectively, can be formulated as

\[
\tilde{\Phi}_{\text{NBS}}(x, y; \psi_{\text{opt}}) \propto e^{j(xk_{\text{opt}} + yl_{\text{opt}})},
\]

(27)

\[
\tilde{g}_{\text{NBS}}(\psi_{\text{out}}, \psi_{\text{in}}; \psi_{\text{opt}}) \propto e^{-j\frac{A_x}{2} \Delta x} e^{-j\frac{A_y}{2} \Delta y} \sin\left[\frac{A_y}{2} (k_{\text{opt}} - \Delta x)\right] \sin\left[\frac{A_y}{2} (l_{\text{opt}} - \Delta y)\right].
\]

(28)

**Proof.** See Appendix A.

**Corollary 2.** Consider a CMS based on the SBF-based beamforming design. The reflection coefficients, \( \tilde{\Phi}_{\text{SBF}}(x, y; \psi_{\text{opt}}) \), can be formulated as

\[
\tilde{\Phi}_{\text{SBF}}(x, y; \psi_{\text{min}}, \psi_{\text{max}}) \propto e^{-j(\bar{x}\psi_{\text{azi}} - \bar{y}\psi_{\text{ele}})} e^{-j\frac{A_x}{2} \Delta x} e^{-j\frac{A_y}{2} \Delta y} \int_{k_{\text{min}}}^{k_{\text{max}}} \int_{l_{\text{min}}}^{l_{\text{max}}} \sin\left[\frac{A_x}{2} (k - \Delta x)\right] \sin\left[\frac{A_y}{2} (l - \Delta y)\right] dk dl.
\]

(29)

**Proof.** The proof of **Corollary 2** is similar to that of **Corollary 1** in Appendix A.

In Figs. 5 and 6, we report the beam patterns of a CMS and compare them with those of a critically-spaced RIS and an ultra-dense RIS. As far as CMSs are concerned, we observe that the Dirichlet kernel functions that characterize the beam patterns of critically-spaced and ultra-dense RISs are replaced by the “sinc” functions, which have relatively small sidelobes and no periodicity compared with Dirichlet kernel functions.

Although CMSs are not realizable in practice, since it is not possible to build surfaces with an infinite number of reflecting elements, Figs. 5 and 6 show that the beam pattern of a CMS can be well approximated by an ultra-dense RIS, which is a practical extension of conventional...
critically-spaced RISs, especially in the THz band. Based on the examples illustrated in Figs. 5 and 6, we observe that a spacing \( d \leq \lambda/4 \) makes the beam patterns of ultra-dense RISs and CMSs almost indistinguishable from each other. In this context, a holographic RIS can be defined as a CMS in theory and as an ultra-dense RIS in practice.

As illustrated in Fig. 5, the minimum width of the mainlobe of the NBS beam pattern is mainly determined by the physical size, i.e., \( A_x \) and \( A_y \), of the surface. We refer to this minimum width as spatial resolution. This parameter characterizes the ability of RISs of distinguishing different UEs that are closely located. Based on Fig. 5, we observe that the spatial resolution is the same for the three implementations, i.e., critically-spaced RISs, ultra-dense RISs, and CMSs, as long as their physical size is kept unchanged [28].

IV. CLOSED-LOOP CHANNEL ESTIMATION SCHEME

In this section, we investigate the CE problem for the holographic RIS described in the previous section and for application to THz massive MIMO systems, as illustrated in Fig. 1. The transmission frame structure is illustrated in Fig. 7. During the whole CE stage, short-length OFDM symbols, which are referred to as unique words (UWs) [34], are transmitted as pilot signals. Each UW consists of \( N_{CP} \) subcarriers in the frequency domain and has length (duration) equal to \( 2N_{CP}T_s \) (half of which constitutes the mainbody of \( N_{CP} \)-point OFDM symbol while the rest constitutes the \( N_{CP} \)-point CP) in the time domain, as elaborated in Fig. 7. For each RFC at the BS, the beamforming is assumed to be the NBS in order to obtain the high beamforming gain, and it is obtained by assuming that the LoS direction of the BS-RIS channel is known. In particular, we have

\[
\Phi_r^{BS} (m,n) = \Phi_{NBS}^{BS} (m,n; \psi_{BS}), \quad \forall r = 1,\ldots,N_{RF}.
\]

The specific expression of \( \Phi_{NBS}^{BS} \) is obtained from (17). It is worth noting that the constraint of constant modulus in the hybrid analog-digital architecture is implicitly satisfied when the NBS beamforming is considered. The signals of all the RFCs are added together and a single data stream is obtained. Thus, by dropping the index \( r \) in (5), we re-write it as

![Fig. 7. The transmission frame structure of the system.](image)
\[ h(\tau) = \alpha G_{BS} \left[ h^{LoS}(\tau) + h^{NLoS}(\tau) \right], \tag{32} \]

where \( G_{BS} = \Phi_{NBS}^{BS}(\psi_{BS}; \psi_{BS}) \) is the beam pattern corresponding to \( \Phi_{NBS}^{BS}(m, n; \psi_{BS}) \).

The proposed CS scheme consists of two phases, which are applied to the downlink and uplink transmissions. The two phases are described in the following two sub-sections, respectively.

### A. Downlink CE Stage

During the downlink CE stage, the UE coarsely estimates \( \mu_{LoS} \) and \( \nu_{LoS} \) by constraining their values within smaller ranges. First, the whole range of virtual AoD at the RIS is divided into \( G_x \) groups along the azimuth direction and into \( G_y \) groups along the elevation direction. The range of virtual azimuth-AoD in the \( g_x \)-th azimuth group, \( g_x = 1, \ldots, G_x \), is \([\psi_{\text{azi}}^{\text{min},g_x}, \psi_{\text{azi}}^{\text{max},g_x}]\) with

\[
\psi_{\text{azi}}^{\text{min},g_x} = \frac{2\pi}{\lambda} \left[ -1 + \frac{2(g_x - 1)}{G_x} \right], \quad \psi_{\text{azi}}^{\text{max},g_x} = \frac{2\pi}{\lambda} \left[ -1 + \frac{2g_x}{G_x} - \frac{\lambda}{A_x} \right], \tag{33}\]

where \( \frac{\lambda}{A_x} \) is the azimuth resolution of the RIS. Without loss of generality, a gap equal to \( \frac{\lambda}{A_x} \) between two adjacent groups is assumed. Similarly, the range of virtual elevation-AoD in the \( g_y \)-th elevation group, \( g_y = 1, \ldots, G_y \), is \([\psi_{\text{ele}}^{\text{min},g_y}, \psi_{\text{ele}}^{\text{max},g_y}]\), where

\[
\psi_{\text{ele}}^{\text{min},g_y} = \frac{2\pi}{\lambda} \left[ -1 + \frac{2(g_y - 1)}{G_y} \right], \quad \psi_{\text{ele}}^{\text{max},g_y} = \frac{2\pi}{\lambda} \left[ -1 + \frac{2g_y}{G_y} - \frac{\lambda}{A_y} \right]. \tag{34}\]

Then, we aim to find the group that contains the LoS AoD of the RIS-UE channel, \( \mu_{LoS} \). In particular, the downlink CE stage can be formulated as follows

\[
(\hat{g}_x, \hat{g}_y) = \left\{ (g_x, g_y) \mid \psi_{\text{azi}}^{\text{min},g_x} \leq \mu_{\text{LoS}} \leq \psi_{\text{azi}}^{\text{max},g_x} \text{ and } \psi_{\text{ele}}^{\text{min},g_y} \leq \mu_{\text{LoS}} \leq \psi_{\text{ele}}^{\text{max},g_y} \right\}. \tag{35}\]

To this end, for the \((g_x, g_y)\)-th group, the RIS uses the SBF beamforming with the cut-off angles given in (33) and (34), that is

\[
\Phi(x, y) = \Phi_{\text{SBF}}(x, y; \psi_{\text{azi}}^{\text{min},g_x}, \psi_{\text{azi}}^{\text{max},g_x}, \psi_{\text{ele}}^{\text{min},g_y}, \psi_{\text{ele}}^{\text{max},g_y}), \tag{36}\]

where \( \psi_{\text{azi}}^{\text{min},g_x} = \left[ \psi_{\text{azi}}^{\text{azi},g_x}, \psi_{\text{azi}}^{\text{azi},g_x} \right]^T \), \( \psi_{\text{azi}}^{\text{max},g_x} = \left[ \psi_{\text{azi}}^{\text{max},g_x}, \psi_{\text{azi}}^{\text{max},g_x} \right]^T \), \( \psi_{\text{ele}}^{\text{min},g_y} = \left[ \psi_{\text{ele}}^{\text{ele},g_y}, \psi_{\text{ele}}^{\text{ele},g_y} \right]^T \), and the specific expression of (36) is obtained from (24) (for the DPA-based RIS) or from (29) (for the CMS).

At the UE, given that the dimension of the antenna array at the UE is relatively small, the NBS beamforming towards \( M^U \) desired directions is employed to coarsely estimate \( \nu_{LoS} \). Specifically, for the \((n_x, n_y)\)-th desired direction, \( 1 \leq n_x \leq M_x^U, 1 \leq n_y \leq M_y^U \), the UE uses the beamforming design

\[
\Phi^{\text{UE}}(m, n) = \Phi_{\text{NBS}}^{\text{UE}}(m, n; \psi_{\text{opt},n_x,n_y}), \tag{37}\]

where \( \psi_{\text{opt},n_x,n_y} = \frac{2\pi}{\lambda} \left[ -1 + \frac{2(n_x - 1)}{M_x^U}, -1 + \frac{2(n_y - 1)}{M_y^U} \right]^T \).

In an OFDM-based system, the effective baseband channel in the delay domain (32) can be transformed to the frequency domain as follows.
Algorithm 1 The downlink CE stage

1: Determine $\psi_{BS}$, $\psi_{RIS}$, $G_x$ and $G_y$. 
2: Set the beamforming design for each RFC at the BS as in (31); 
3: for the $(g_x, g_y)$-th group at the RIS, $1 \leq g_x \leq G_x$, $1 \leq g_y \leq G_y$, do 
   4: Set the beamforming design at the RIS as in (36); 
   5: for the $(n_x, n_y)$-th group at the UE, $1 \leq n_x \leq M_x^U$, $1 \leq n_y \leq M_y^U$, do 
      6: Set the beamforming design at each UE as in (37); 
      7: The BS broadcasts one UW as pilot signals; 
      8: Each UE receives the pilot signals $y_{k,g_x,g_y,n_x,n_y}$ as in (39); 
   9: end for 
10: end for 
11: Each UE obtains its index of optimal groups, $(\hat{g}_x, \hat{g}_y, \hat{n}_x, \hat{n}_y)$, via (40);
we define the search space for the uplink CE stage as the considered RIS is $\lambda/A$ in order to determine the LoS angle with a finer resolution. Given that the spatial resolution of $\psi$ of SBF (as shown in Fig. 4), we can expect that Considering the energy focusing property of NBS (as shown in Fig. 3) and the bandpass property of SBF (as shown in Fig. 4), we can expect that $\nu_{\text{LoS}} \approx \psi_{\text{opt},\hat{n}_x,\hat{n}_y}$, $\psi_{\text{azi},\hat{g}_x,\hat{g}_y} \leq \mu_{\text{LoS}}^{\text{azi}} \leq \psi_{\text{max},\hat{g}_x,\hat{g}_y}^{\text{azi}}$, and $\psi_{\text{ele},\hat{g}_x,\hat{g}_y} \leq \mu_{\text{LoS}}^{\text{ele}} \leq \psi_{\text{max},\hat{g}_x,\hat{g}_y}^{\text{ele}}$. In other words, after the downlink CE stage, the range of possible values for $\mu_{\text{LoS}}$ is narrowed down to $\psi_{\text{azi},\hat{g}_x,\hat{g}_y}^{\text{azi}} - \psi_{\text{azi},\hat{g}_x,\hat{g}_y}^{\text{azi}} = \frac{2\pi}{\lambda} \left( \frac{2}{G_x} - \frac{\lambda}{A_x} \right)$ along the azimuth direction, and to $\psi_{\text{ele},\hat{g}_x,\hat{g}_y}^{\text{ele}} - \psi_{\text{ele},\hat{g}_x,\hat{g}_y}^{\text{ele}} = \frac{2\pi}{\lambda} \left( \frac{2}{G_y} - \frac{\lambda}{A_y} \right)$ along the elevation direction. This significantly reduces the search space of the uplink finer-grained CE discussed in the next subsection. The detailed procedure of the proposed downlink CE algorithms is summarized in Algorithm 1.

B. Uplink CE stage

After the downlink CE stage, each UE obtains the indices of the optimal groups $(\hat{g}_x, \hat{g}_y, \hat{n}_x, \hat{n}_y)$, and this information is fed back to the BS via the control links for UE scheduling. Specifically, the BS schedules the UEs having the same $(\hat{g}_x, \hat{g}_y)$ into the same uplink group. Each scheduled group performs the finer-grained uplink CE and the subsequent payload data transmission. In this sub-section, therefore, we consider a generic group as follows

$$G_{\hat{g}_x,\hat{g}_y} = \{ \text{All the UEs that have the same } (\hat{g}_x, \hat{g}_y) \text{ obtained from the downlink CE} \},$$

and we assume $|G_{\hat{g}_x,\hat{g}_y}| = N_{\text{UE}}$. During the uplink CE and payload data transmission stages, the beamforming design at the $u$-th UE (1 $\leq u \leq N_{\text{UE}}$) in $G_{\hat{g}_x,\hat{g}_y}$ is chosen as follows

$$\Phi_u^{\text{UE}}(m, n) = \Phi_u^{\text{NBS}}(m, n; \psi_{\text{opt},\hat{n}_x,\hat{n}_y})$$

where $\{\hat{n}_x, \hat{n}_y\}$ is the index of the optimal group obtained by the $u$-th UE during the downlink CE stage.

As mentioned in the previous sub-section, after the downlink CE stage, the LoS angle of the $u$-th UE in $G_{\hat{g}_x,\hat{g}_y}$, i.e., $\mu_{\text{LoS}}^{\text{azi}, u}$, is coarsely estimated to be $\psi_{\text{azi},\hat{g}_x,\hat{g}_y}^{\text{azi}} - \psi_{\text{azi},\hat{g}_x,\hat{g}_y}^{\text{azi}} = \frac{2\pi}{\lambda} \left( \frac{2}{G_x} - \frac{\lambda}{A_x} \right)$ along the azimuth direction, and to $\psi_{\text{ele},\hat{g}_x,\hat{g}_y}^{\text{ele}} - \psi_{\text{ele},\hat{g}_x,\hat{g}_y}^{\text{ele}} = \frac{2\pi}{\lambda} \left( \frac{2}{G_y} - \frac{\lambda}{A_y} \right)$ along the elevation direction. Hence, during the uplink CE stage, we only need to search within the range between $\psi_{\text{azi},\hat{g}_x,\hat{g}_y}^{\text{azi}}$ and $\psi_{\text{azi},\hat{g}_x,\hat{g}_y}^{\text{azi}}$ in order to determine the LoS angle with a finer resolution. Given that the spatial resolution of the considered RIS is $\lambda/A_x$ along the azimuth direction and $\lambda/A_y$ along the elevation direction, we define the search space for the uplink CE stage as

$$\begin{align*}
\zeta_{b_x}^{\text{azi}} &= \psi_{\text{azi},\hat{g}_x}^{\text{azi}} + \frac{2\pi (b_x - 1) \lambda}{A_x}, \quad 1 \leq b_x \leq B_x, \\
\zeta_{b_y}^{\text{azi}} &= \psi_{\text{azi},\hat{g}_x}^{\text{azi}} + \frac{2\pi (b_y - 1) \lambda}{A_y}, \quad 1 \leq b_y \leq B_y,
\end{align*}$$

Algorithm 1.
where $B_x = \frac{\Delta \psi_{\text{max},x} - \psi_{\text{min},x}}{\lambda/A_x} + 1 = \frac{2A_x}{\lambda A_x}$, $B_y = \frac{\Delta \psi_{\text{max},y} - \psi_{\text{min},y}}{\lambda/A_y} + 1 = \frac{2A_y}{\lambda A_y}$ ($B_x, B_y \in \mathbb{Z}$ without loss of generality). In particular, the target of the uplink finer-grained CE is to solve the following optimization problem

$$
\hat{(b_x, b_y)} = \arg \max_{(b_x, b_y)} \tilde{g}_{\text{NBS}} \left( \mathbf{\mu}_{\text{LoS}}^u, \psi_{\text{RIS}}; \zeta_u, b_y \right),
$$

where $\zeta_{b_x, b_y} = [\zeta_{b_x}, \zeta_{b_y}]^T$. Instead of executing an exhaustive beam scanning over the $B_x B_y$ directions in (43) and (44), during the $i$-th time slot of the uplink CE stage, the RIS employs the overlapped NBS beamforming towards all $B_x B_y$ directions with different random phases. This can be formulated as follows

$$
\Phi(x, y) = \frac{1}{\sqrt{B_x B_y}} \sum_{b_x = 1}^{B_x} \sum_{b_y = 1}^{B_y} e^{j\theta_{i,b_x,b_y}} \Phi_{\text{NBS}}(x, y; \zeta_{b_x, b_y}),
$$

where $\theta_{i,b_x,b_y} \sim U[0, 2\pi)$.

By using the beamforming designs in (42) and (46), and by capitalizing on the channel reciprocity between the downlink and uplink transmissions, the uplink effective baseband channel related to the $u$-th UE in the $i$-th time slot can be formulated as

$$
\hat{h}_{u,i}(\tau) = \hat{h}_{\text{LoS}, u,i}(\tau) + \hat{h}_{\text{NLoS}, u,i}(\tau),
$$

which consists of the LoS part and the NLoS part. The LoS part $\hat{h}_{\text{LoS}, u,i}(\tau)$ can be expressed as

$$
\hat{h}_{\text{LoS}, u,i}(\tau) = \tilde{g}_{\text{NBS}} \left( \mathbf{\mu}_{\text{LoS}}^u, \psi_{\text{RIS}}; \zeta_{b_x, b_y} \right) g_{\text{UE}} \left( \mathbf{\nu}_{\text{LoS}}^u, \psi_{\text{opt}}, \hat{\nu}_u, \hat{\nu}_y \right) p \left( \tau - \tau_{\text{LoS}}^u \right)
$$

$$
= w_i^T h_{\text{LoS}, u,i} p \left( \tau - \tau_{\text{LoS}}^u \right),
$$

where the variables top-marked as (⌣) denote the uplink versions of the channel parameters,

$$
w_i = \frac{1}{\sqrt{B_x B_y}} \left[ e^{j\theta_{1i,1}}, ..., e^{j\theta_{1i,b_y}}, e^{j\theta_{2i,1}}, ..., e^{j\theta_{2i,b_y}} \right]^T \in \mathbb{C}^{B_x B_y \times 1},
$$

$$
h_{\text{LoS}, u,i} = \tilde{g}_{\text{NBS}} \left( \mathbf{\mu}_{\text{LoS}}^u, \psi_{\text{RIS}}; \zeta_1, \tau_1 \right) \times
$$

$$
\left[ g_{\text{NBS}} \left( \mathbf{\mu}_{\text{LoS}}^u, \psi_{\text{RIS}}; \zeta_{1,1} \right), ..., g_{\text{NBS}} \left( \mathbf{\mu}_{\text{LoS}}^u, \psi_{\text{RIS}}; \zeta_{b_x,b_y} \right), ..., g_{\text{NBS}} \left( \mathbf{\mu}_{\text{LoS}}^u, \psi_{\text{RIS}}; \zeta_{B_x,B_y} \right) \right]^T \in \mathbb{C}^{B_x B_y \times 1}
$$

is the effective angular-domain LoS channel with finer angular resolution. The elements in $h_{\text{LoS}, u,i}$ show the relation between the LoS angle $\mathbf{\mu}_{\text{LoS}}^u$ and all the pre-defined codewords $\zeta_{b_x, b_y}$. The NLoS part $\hat{h}_{\text{NLoS}, u,i}(\tau)$ in (47) can be written as follows

$$
\hat{h}_{\text{NLoS}, u,i}(\tau) = w_i^T \sum_{l=1}^{L} h_{\text{NLoS}, u,i} p \left( \tau - \tau_{l}^u \right),
$$

where $h_{\text{NLoS}, u,i}$ is the effective angular-domain channel that corresponds to the $l$-th NLoS path. The formulation of $h_{\text{NLoS}, u,i}$ is similar to (49) and thus it is omitted for brevity.
Fig. 8. Examples of the received UW at the BS with DSCs, where $N_{CP} = 16$ and $N_{UE} = 4$. Three different DSC allocation schemes are illustrated. (a) Block allocation; (b) Uniform allocation; (c) Random allocation.

Similar to (38), the uplink channel in the delay domain (47) can be transformed to the frequency domain. In particular, the frequency-domain representation of the channel related to the $u$-th UE corresponding to the $n$-th DSC in the $i$-th time slot can be written as

$$
\bar{h}_{i,u,n} = \frac{1}{\sqrt{N_{CP}}} \sum_{d=0}^{N_{CP}-1} h_{i,u'} (d T_s) e^{-j \frac{2\pi}{N_{CP}} f_{u,n}'},
$$

where $f_{k_{u,n}}$ is the $k_{u,n}$-th column vector of the $N_{CP} \times N_{CP}$ DFT matrix $F_{N_{CP}}$, and

$$
H_{u}^{AdDd} = [h_1, ..., h_{N_{CP}}] \in \mathbb{C}^{B_x B_y \times N_{CP}}
$$

with $h_d = h_u^{LoS} \rho [(d-1) T_s - \tilde{\tau}_u^{LoS}] + \sum_{l=1}^{L} h_u^{NLoS} \rho [(d-1) T_s - \tilde{\tau}_l^{u}], \; d = 1, ..., N_{CP}$, being the effective angular-domain and delay-domain (AdDd) uplink channel to be estimated.

In order to simultaneously perform the uplink CE for all the UEs in $G_{\hat{g}_x, \hat{g}_y}$ and to avoid the interference of different UEs’ pilot signals at the BS, we consider the set of dedicated subcarriers (DSCs) for the $u$-th UE as follows

$$
\mathcal{K}_u = \{k_{u,n} | n = 1, ..., N_{used} \},
$$

where $N_{used} \triangleq N_{CP}/N_{UE}$ ($N_{used} \in \mathbb{Z}$ without loss of generality), $\mathcal{K}_u \subseteq \{1, ..., N_{CP}\}$, $|\mathcal{K}_u| = N_{used}$, and $\mathcal{K}_{u_1} \cap \mathcal{K}_{u_2} = \emptyset$ for $\forall u_1 \neq u_2$. During the uplink CE stage, the $u$-th UE transmits its own pilot signals by using only the $N_{used}$ subcarriers indexed by $\mathcal{K}_u$ out of the $N_{CP}$ available subcarriers. On the other hand, no signals are transmitted in other subcarriers. This makes easier to separate the pilot signals of different UEs at the BSs, since different DSCs are used. Three examples of the possible structure of an UW with DSCs are illustrated in Fig. 8.

We assume that each UE transmits the pilot signal $\sqrt{\frac{P_{UL}^{Tx}}{N_{used}}}$ over its DSCs during the uplink CE stage, where $P_{UL}^{Tx}$ is the total transmit power of the UE. The received pilot signal in the $i$-th time slot at the BS can be expressed as

$$
y_{i,u,n} = \sqrt{\frac{P_{UL}^{Tx}}{N_{used}}} \bar{h}_{i,u,n} + n_{i,u,n} = \sqrt{\frac{P_{UL}^{Tx}}{N_{used}}} w_i^T H_{u}^{AdDd} f_{k_{u,n}} + n_{i,u,n},
$$

with
Algorithm 2 The uplink CE stage

1: Determine $G_{gx,gy}$ for $N_{UE}$ UEs, and the DSC allocation scheme $K_u, \forall u \in \{1, ..., N_{UE}\}$;
2: Set the beamforming design at each UE as in (42);
3: for the $i$-th time slot, $1 \leq i \leq N_P$, do
4: Generate $w_i$ and accordingly set the beamforming design at the RIS as in (46);
5: All $N_{UE}$ UEs transmit the UWs with pilot signals by using the assigned DSCs;
6: The BS receives the pilot signals $y_{i,u}$ from each UE in (55);
7: end for
8: Compute $W, Y_u, \text{ and } F_u$ in (56), $\forall u \in \{1, ..., N_{UE}\}$;

% CS-based CE algorithm below. Take a certain $u$-th UE as an example.

9: Initialization: $j = 0$, $I = \text{empty set}$, $r = \text{vec}(Y_u), \hat{H}$ is an all-zero matrix of size $B_xB_y \times N_{CP}$, $N_{max}$ is the maximum number of iterations;
10: while $j < N_{max}$, do
11: $i^* = \text{arg max}_i \left| \left( F_u^T \otimes W \right) H r \right|_i$;
12: $I = I \cup \{i^*\}$;
13: $\hat{h}_{\text{temp}} = \left[ F_u^T \otimes W \right]_I^{+} \text{vec}(Y_u)$;
14: $r = \text{vec}(Y_u) - \left[ F_u^T \otimes W \right]_I \hat{h}_{\text{temp}}$;
15: $j = j + 1$;
16: end while
17: $\hat{h} = \text{vec}(\hat{H})$;
18: $[\hat{h}]_I = \hat{h}_{\text{temp}}$;
19: $\hat{H}_u^{\text{AdDd}} = \text{vec}^{-1}(\hat{h})$;

Output: The estimated channels $\hat{H}_u^{\text{AdDd}}$, $\forall u \in \{1, ..., N_{UE}\}$;

where $n_{i,u,n} \sim \mathcal{CN}(0, \sigma_n^2)$ is the AWGN. By collecting the received pilot signals of all the DSCs for the $u$-th UE $\{y_{i,u,n}\}_{n=1}^{N_{\text{used}}}$, we have

$$y_{i,u}^T = \sqrt{\frac{P_{UL}^{\text{UL}}}{N_{\text{used}}}} w_i^T H_u^{\text{AdDd}} F_u + n_{i,u}^T, \quad (55)$$

where $y_{i,u} = [y_{i,u,1}, ..., y_{i,u,N_{\text{used}}}]^T \in \mathbb{C}^{N_{\text{used}} \times 1}$, $n_{i,u} = [n_{i,u,1}, ..., n_{i,u,N_{\text{used}}}]^T \in \mathbb{C}^{N_{\text{used}} \times 1}$, and $F_u = [F_{N_{CP}}]_{K_u} \in \mathbb{C}^{N_{CP} \times N_{\text{used}}}$ is the partial DFT matrix. For $N_P$ successive time slots, we aggregate the channel observations of the $u$-th UE $\{y_{i,u}^T\}_{i=1}^{N_P}$ into the matrix $Y_u \in \mathbb{C}^{N_P \times N_{\text{used}}}$, which can be formulated as follows

$$Y_u = [y_{1,u}, ..., y_{N_P,u}]^T = WH_u^{\text{AdDd}} F_u + N_u, \quad (56)$$
where $\mathbf{W} = \sqrt{\frac{P_{UL}}{N_{used}}} [\mathbf{w}_1, \ldots, \mathbf{w}_{N_P}]^T \in \mathbb{C}^{N_P \times B_x B_y}$ and $\mathbf{N}_u = [\mathbf{n}_{1,u}, \ldots, \mathbf{n}_{N_P,u}]^T \in \mathbb{C}^{N_P \times N_{used}}$. The objective of the uplink CE stage is therefore, to estimate $\mathbf{H}_{u}^{AdDd}$ by exploiting the knowledge of $\mathbf{W}$, $\mathbf{F}_u$, and the noisy matrix $\mathbf{Y}_u$. Usually, we have $N_{P} < B_x B_y$ due to the limited channel coherence time, and $N_{used} < N_{CP}$ due to the allocation of different DSCs among multiple UEs. These constraints make (56) an under-determined system, which is not possible to solve by using traditional estimation techniques such as the LS estimator [19], [20]. Due to the strong LoS link between the UE and the RIS, the channel matrix $\mathbf{H}_{u}^{AdDd}$ is expected to be sparse, and, in particular, only the elements whose indices fulfill the conditions $\mu_{LoS}^u \approx \zeta_{b_x,b_y}$, $b_x \in \{1, \ldots, B_x\}$, $b_y \in \{1, \ldots, B_y\}$, and $\bar{\tau}_{LoS}^u \approx dT_s$, $d \in \{0,1,\ldots,N_{CP} - 1\}$ have a non-negligible absolute value. On the other hand, the other entries have a much smaller absolute value, which can be safely ignored. The dual sparsity of the channel matrix $\mathbf{H}_{u}^{AdDd}$ in both the angular and delay domain is a main feature of THz channels [25], which can be exploited to solve the CE problem. In particular, we use CS methods to estimate $\mathbf{H}_{u}^{AdDd}$ directly from the under-determined measurements in (56).

Based on these considerations, Algorithm 2 reports the details of the proposed uplink CE stage, which is based on the orthogonal matching pursuit (OMP) algorithm for estimating sparse channels. Once the estimated channel $\hat{\mathbf{H}}_{u}^{AdDd}$ is obtained, interpolation-based methods can be applied to reconstruct the channels in all $K$ subcarriers [20].

V. SIMULATION RESULTS

In this section, we present numerical results to evaluate the different types of RISs, and the performance of the proposed CE scheme.

A. Experimental Setting

We consider the system model as shown in Fig. 9. The BS and the RIS serve the active UEs distributed within a sector of radius $R$ and central angle $120^\circ$. The BS and the RIS are elevated to the height of $h_1$ and the UEs have the height of $h_2$. We assume that the normal directions of the arrays at the BS and RIS point towards each other, which yields $\psi_{BS} = \psi_{RIS} = [0,0]^T$. The simulation setup is detailed as follows unless stated otherwise: $R = 20$ m, $h_1 = 10$ m, $h_2 = 1.5$ m, $M_B^B = M_B^B = 64$, $N_{RF} = 4$, $M_U^B = M_U^B = 8$, $A_x = A_y = 0.2$ m, $f_c = 0.15$ THz ($\lambda \approx 0.2$ mm), $T_s = 2 \times 10^{-9}$ sec (bandwidth BW = $1/T_s = 500$ MHz), $N_{CP} = 64$, $K = 256$. A raised cosine filter $p(\tau)$ with roll-off factor 0.8 is employed. The noise power spectrum density at the receiver is $\sigma_{NSD}^2 = -174$ dBm/Hz. Thus, the power of the AWGN $\sigma_n^2$ is $\sigma_n^2 = \sigma_{NSD}^2 \times$ BW $\approx -87$ dBm. The number of iterations $N_{max}$ in Algorithm 2 is set to 20.

As for the downlink channel model in (5), we set $L = 1$, $K_f = 30$ dB. The angle $\mu_{LoS}$ is calculated according to the position of the UE, and the angles $\nu_{LoS}$, $\mu_l$ and $\nu_l$ are randomly
generated. The delay offsets $\tau_{\text{LoS}}$ and $\tau_l$ ($\tau_{\text{LoS}} < \tau_l$) follow a uniform distribution $U(0, (N_{\text{CP}} - 1)T_s)$. The channel coefficients $\alpha$ and $\beta$ are defined as follows [3], [35, Eq. (20)]

$$\alpha = e^{j\theta_\alpha} \sqrt{\frac{G_{\text{Tx}} S_{\text{eff}}}{4\pi R^2 A_{\text{abs}}(f_c, R)}},$$

$$\beta = e^{j\theta_\beta} \sqrt{\frac{G_{\text{RIS}} G_{\text{Rx}}}{A_{\text{abs}}(f_c, d_{\text{RIS-UE}})} \frac{\lambda}{4\pi d_{\text{RIS-UE}}}},$$

where $\theta_\alpha, \theta_\beta \sim U[0, 2\pi]$ are the phase shifts introduced by the channels, $S_{\text{eff}}$ is the effective reflection area of the RIS, $d_{\text{RIS-UE}}$ is the distance between the UE and the RIS, $G_{\text{Tx}}$, $G_{\text{RIS}}$, and $G_{\text{Rx}}$ are the array gains of the BS, the RIS, and the UE, respectively, and $A_{\text{abs}}(f_c, d_{\text{Tx-Rx}})$ is the attenuation caused by molecular absorption [1]–[3]. $A_{\text{abs}}(f_c, d_{\text{Tx-Rx}})$ is related to the carrier-frequency $f_c$ and the transmission distance $d_{\text{Tx-Rx}}$, and its specific values are obtained based on the recommendations of the International Telecommunications Union (ITU) [41]. The effective reflection area $S_{\text{eff}}$ of the RIS can be modeled as its whole physical area (aperture) [35]

$$S_{\text{eff}} = \begin{cases} A_x A_y, & \text{for CMS}, \\ \frac{A_x A_y}{d^2} S_{\text{ele}}, & \text{for DPA-based RIS with the element spacing } d, \end{cases}$$

where $S_{\text{ele}} \leq d^2$ is the physical size of a single reflection element in a DPA-based RIS. We assume $S_{\text{ele}} = 200\mu m \times 190\mu m$ as demonstrated in [25, Fig. 1]. The parameters of the uplink channel in (47)-(50) can be similarly modeled and thus are omitted for brevity.

### B. Numerical Results

We first evaluate the pilot overhead and computational complexity of the proposed closed-loop CE scheme. From Algorithm 1 and Algorithm 2, we evince that $G_x G_y M_x U M_y U$ and $N_P$ UWs are required for the downlink and uplink CE stages, respectively. Therefore, the total pilot overhead is $T_{\text{DL}} + T_{\text{UL}} = 2N_{\text{CP}} T_s (G_x G_y M_x U M_y U + N_P)$, according to Fig. 7. As for the uplink CS-based CE algorithm in Algorithm 2, we consider the total number of complex-valued multiplications to evaluate the computational complexity, as listed in [32, Table I]. Fig. 10 shows

$$G = \int_{\psi_{\text{out}} = 0}^{\pi/2} \int_{\phi_{\text{out}} = 0}^{\pi/2} |g(\psi_{\text{out}})|^2 \sin \phi_{\text{out}} d\phi_{\text{out}} d\psi_{\text{out}}$$

[35], where $g(\psi_{\text{out}})$ is the beam pattern of the array that is discussed in previous text.
Fig. 10. The trade-off between the total pilot overhead and computational complexity of the proposed closed-loop CE scheme, where $N_P = 40$ is considered. The required pilot overhead and computational complexity versus the number of groups $\{G_x, G_y\}$ when $N_P = 40$. It is observed that, by setting different groups $\{G_x, G_y\}$, the proposed closed-loop scheme provides a trade-off between the pilot overhead and the computational complexity. Two cases deserve further attention in Fig. 10: 1) The case of $G_x = G_y = 1$ refers to estimating the complete channels in the uplink (in an open-loop manner). In this case, the uplink channel matrix to be estimated in (52) has a total size of $40,000 \times 64$, which causes an unaffordable computational complexity and storage burden [40]; 2) the case of $G_x = 2A_x/\lambda = 200$ and $G_y = 2A_y/\lambda = 200$ refers to acquiring the complete CSI only in the downlink by exhaustive beam scanning. This case would suffer from a long $T_{DL}$ (about 0.6 sec as shown in Fig. 10), which may degrade the net spectral efficiency.

Fig. 11 shows the accuracy of the downlink CE stage by investigating the probability of grouping failure versus the total downlink transmission power at the BS. The indices obtained by Algorithm 1 are compared with the oracle LoS angles $\mu_{LoS}$ and $\nu_{LoS}$ to decide whether the downlink CE succeeds or not. Three different types of RISs, i.e., critically-spaced RIS ($d = \lambda/2$), ultra-dense RIS ($d < \lambda/2$) and CMS ($d \to 0$) are considered. We observe that the CMS provides the best performance among the three types of RISs. The performance of ultra-dense RISs is significantly better than traditional critically-spaced RISs, since ultra-dense RISs have larger effective reflection area and smaller sidelobes (i.e., higher array gain in the mainlobe) which bring a better receive signal-to-noise ratio (SNR). Fig. 11 also reveals that the performance of a practical ultra-dense RIS can well approach that of an ideal (but unrealistic) CMS. A practical $d = \lambda/8$ renders only a minor performance gap compared with the CMS, so it would be sufficient to treat the ultra-dense RIS as the holographic RIS. Moreover, it is observed that the downlink
CE performance improves as the number of groups \( \{ G_x, G_y \} \) increases. This is because with a larger \( \{ G_x, G_y \} \) the SBF beam pattern has a narrower passband, which enhances the amplitude of the beam pattern (i.e., array gain) in the passband and yields a better receive SNR.

Next, we investigate the performance of the uplink finer-grained CE stage that is obtained by using Algorithm 2, and compare it with some existing schemes. We set \( G_x = G_y = 10 \), i.e., \( B_x = B_y = 20 \). In Figs. 12-14, the normalized mean square error (NMSE) is adopted as the performance metric of interest, which is given by \( \mathbb{E} \left\{ \frac{\| \hat{H}_{AdDd} u - H_{AdDd} u \|_F^2}{\| H_{AdDd} u \|_F^2} \right\} \). Fig. 12 depicts the uplink NMSE performance versus the uplink pilot overhead \( N_P \) and the different DSC allocation schemes as illustrated in Fig. 8 for \( N_{UE} = 4 \). It can be observed that the random DSC allocation scheme achieves better CE performance, while the uniform allocation scheme fails to work properly. If the random DSC allocation scheme is considered, in addition, a sufficiently high CE accuracy can be ensured even with a low compression ratio \( (N_P N_{used})/(B_x B_y N_{CP}) \) in the range \( \{0.0063, 0.0187, 0.0313, 0.0437, 0.0563, 0.0688\} \) in Fig. 12. Due to its superior performance, the random DSC allocation strategy is adopted to obtain the rest of the results.

In Fig. 13, we plot the NMSE performance of the proposed CE scheme as a function of the number of simultaneously-served UEs \( N_{UE} \) for \( N_P = 40 \). As the benchmark, we adopt the simultaneous weighted OMP (SW-OMP) [38], [39] algorithm which estimates the channels in the frequency domain (rather than the delay domain) via well-determined measurements. When SW-OMP is considered, \( N_P \) UWs are equally divided into \( N_{UE} \) parts, each of which is dedicated for one UE. Therefore, only \( N_P/N_{UE} \) UWs are available for each UE to conduct CE. It can be observed that the proposed CE scheme outperforms the considered benchmark even if a larger number of UEs are served simultaneously. This is obtained because the proposed CE scheme exploits the dual sparsity of THz MIMO channels in both the angular and delay domain, while
the CE scheme based on SW-OMP only utilizes the sparsity in the angular domain.

Fig. 14 compares the NMSE performance of different CE schemes against the uplink transmission power $P_{UL}^{UL}$ for $N_{UE} = 4$. As a benchmark, we consider the LS estimator \cite{19, 20} with well-determined measurements in both the angular domain and delay domain, which requires a large number $N_P = B_x B_y N_{UE} = 1600$ of UWs as pilot signals. By leveraging the dual sparsity of THz MIMO channels in both the angular and delay domain, the proposed CS-based CE scheme outperforms the LS estimator even if a much smaller number of pilot signals ($N_P = \{40, 80\}$) is used. As a second benchmark scheme, we analyze the open-loop CE scheme that implements the uplink CE stage without using the downlink grouping. In such a case, the UEs do not have any prior information of the coarsely-estimated LoS angles and thus the NBS beamforming in (42) is unavailable. Instead, random pilot signals \cite{36, 37} are employed at the UEs to realize an omni-directional beam pattern. Compared with the NBS beamforming towards the coarsely-estimated LoS direction, the omni-directional beam pattern disperses the transmit energy towards several directions. This results in the poor CE performance, as demonstrated in Fig. 14.

In Fig. 15, we compare the uplink average spectral efficiency (ASE) performance that is achieved by different schemes. The ASE of the $u$-th UE is defined as

$$\text{ASE} = \left(1 - \frac{T_{DL} + T_{UL}}{T_{coh}}\right) E \left\{ \frac{1}{K} \sum_{k=1}^{K} \log_2 \left[ 1 + \frac{P_{UL}^{UL} |\hat{f}_{u,k}|^2}{(K\sigma_n^2)} \right] \right\} \text{[bit/sec/Hz]},$$

where $\hat{f}_{u,k}$ is the frequency-domain effective baseband channel of the $k$-th subcarrier by using the NBS beamforming towards the estimated LoS direction of the RIS. The NBS beamforming towards the oracle LoS direction is illustrated as an upper-bound (i.e., perfect CSI without uplink pilot overhead). We observe that the ASE obtained by the proposed CE scheme has good tightness.
Fig. 16. SBF beam patterns used in the proposed CE scheme with different quantization bits. The ultra-dense RIS with elements spacing $d = \lambda/4$ is considered.

with the upper-bound, while the ASE obtained by the well-determined LS estimator is worse due to the long time duration that is required for pilot transmission. It is also observed that the ASE improves as the element spacing $d$ decreases, and it can approach the performance of the ideal CMS with practical $d$ (e.g., $d = \lambda/8$), which further verify that the ultra-dense RIS is a good realization of the holographic RIS. Further, the ASE that is obtained in the absence of an RIS is illustrated. In this case, we assume that the UE communicates with the BS via the NLoS link with perfect CSI and no pilot overhead. We observe that the virtual LoS link provided by an RIS in THz massive MIMO systems significantly increases the ASE as compared with the case that does not use RIS.

At last, we investigate how the phase resolution of the reflecting element in the RIS affect the system performance. We only consider the practical DPA-based RISs. Suppose that each element in the RIS has $B$ phase quantization bit(s), which means that the phases of the reflection coefficients can only be drawn from a finite set $\mathcal{B} = \left\{ 2\pi \frac{1 - 2^{B-1}}{2^B}, 2\pi \frac{2 - 2^{B-1}}{2^B}, \ldots, 2\pi \frac{2^{B-2}}{2^B} \right\}$. For the proposed beamforming solutions (NBS and SBF), we quantize the obtained reflection coefficients $\Phi(m,n)$ from (17) or (24) as $\Phi^Q(m,n) = |\Phi(m,n)| e^{j\phi_{m,n}}$, where $\phi_{m,n} \in \mathcal{B}$ is the phase element in $\mathcal{B}$ that is closest to the phase of $\Phi(m,n)$. Then, the corresponding beam patterns are computed via (10) by replacing $\Phi(m,n)$ as $\Phi^Q(m,n)$. In Fig. 16, we present the examples of the SBF beam patterns with different quantization bits. We note that for $B = 1$ the quantization error is so severe that it degrades the beam patterns. However, when $B = \{2, 3\}$, the beam patterns after quantization distinctly exhibit the desired band-pass property, which play an effective role as compared with that without quantization ($B = \infty$). In Fig. 17, we evaluate the impact of quantization on the ASE performance. It shows that the ASE performance loss due to the
quantization error is tiny for the practical phase resolutions \(B = \{2, 3\}\) when non-holographic RIS \((d = \lambda/2)\) and holographic RIS \((d < \lambda/2)\) are considered. From these results, we gain insight for system design that a practical phase resolution (e.g., \(B = 3\)) of the reflecting elements can well realize the desired beamforming for both non-holographic RISs and holographic RISs.

VI. CONCLUSIONS

Motivated by the concept of holographic communications, we studied the physical layer transmission of holographic RISs in the THz band, where a large number of sub-wavelength reconfigurable elements are densely integrated into a compact space to approach a spatially continuous aperture. We derived the beam pattern of a holographic RIS and proposed two beamforming designs that are formulated in closed-form. Based on the proposed beamforming designs, we proposed a closed-loop broadband CE scheme for the RIS-aided THz massive MIMO systems. The proposed CE scheme encompasses a downlink grouping stage and an uplink finer-grained CE stage. In order to reduce the pilot overhead, a CS-based CE algorithm was proposed that exploits the dual sparsity of THz MIMO channels in both the angular and delay domain. Simulation results showed that holographic RISs are able to outperform traditional designs based on non-holographic RISs as well as communication schemes that do not use RISs.

APPENDIX A

PROOF OF Corollary 1

The result in (27) can be obtained by letting \(dm = x\) and \(dn = y\) in (17). As far as (28) is concerned, in particular, we have

\[
\tilde{g}_{\text{NBS}}(\psi_{\text{out}}, \psi_{\text{in}}, \psi_{\text{opt}}) \propto \lim_{d \to 0} g_{\text{NBS}}(\psi_{\text{out}}, \psi_{\text{in}}, \psi_{\text{opt}})
= \lim_{d \to 0} e^{j \frac{d \Delta x}{2} \Delta_x} e^{j \frac{d \Delta y}{2} \Delta_y} \Xi_{N_x} [d (k_{\text{opt}} - \Delta_x)] \Xi_{N_y} [d (l_{\text{opt}} - \Delta_y)]
= e^{-j \frac{\Delta x}{2} \Delta_x} e^{-j \frac{\Delta y}{2} \Delta_y} \lim_{d \to 0} \Xi_{N_x} [d (k_{\text{opt}} - \Delta_x)] \Xi_{N_y} [d (l_{\text{opt}} - \Delta_y)].
\]

(61)

Given the symmetry, we only need to prove \(\lim_{d \to 0} \Xi_{N_x} [d (k_{\text{opt}} - \Delta_x)] = \text{sinc} \left[ \frac{\Delta x}{2} (k_{\text{opt}} - \Delta_x) \right]\) in order to obtain (28). When \(k_{\text{opt}} \neq \Delta_x\), based on the definition of the function \(\Xi_{N_x}\), we have

\[
\lim_{d \to 0} \Xi_{N_x} [d (k_{\text{opt}} - \Delta_x)] = \lim_{d \to 0} \frac{\sin \left[ \frac{N_x d}{2} (k_{\text{opt}} - \Delta_x) \right]}{N_x \sin \left[ \frac{d}{2} (k_{\text{opt}} - \Delta_x) \right]}
= (a) \lim_{d \to 0} \frac{d \sin \left[ \frac{\Delta x}{2} (k_{\text{opt}} - \Delta_x) \right]}{\Delta_x \sin \left[ \frac{d}{2} (k_{\text{opt}} - \Delta_x) \right]}
= (b) \lim_{d \to 0} \frac{\sin \left[ \frac{\Delta x}{2} (k_{\text{opt}} - \Delta_x) \right]}{\frac{\Delta x}{2} (k_{\text{opt}} - \Delta_x) \cos \left[ \frac{d}{2} (k_{\text{opt}} - \Delta_x) \right]}
= \frac{\sin \left[ \frac{\Delta x}{2} (k_{\text{opt}} - \Delta_x) \right]}{\frac{\Delta x}{2} (k_{\text{opt}} - \Delta_x)},
\]
\[ = \text{sinc} \left( \frac{A_x}{2} \left( k_{\text{opt}} - \Delta_x \right) \right), \quad (62) \]

where the equality (a) follows from \( N_x d = A_x \), and the equality (b) is obtained by applying the De l’Hôpital rule with respect to \( d \). In addition, when \( k_{\text{opt}} = \Delta_x\), \( \lim_{d \to 0} \Xi_{N_x}(d \cdot 0) = 1 = \text{sinc} \left( \frac{A_x}{2} \cdot 0 \right) \).

Thus, we obtain (62). This completes the proof of Corollary 1.

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