Sudakov suppression of jets in QCD media

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We compute modifications to the jet spectrum in the presence of a dense medium. We show that in the large-$N_c$ approximation and at leading logarithmic accuracy the jet nuclear modification factor factorizes into a quenching factor associated to the total jet color charge and a Sudakov suppression factor which accounts for the energy loss of jet substructure fluctuations. This factor, called the jet collimator, implements the fact that subjets, that are not resolved by the medium, lose energy coherently as a single color charge, whereas resolved large angle fluctuations suffer more quenching. For comparison, we show that neglecting color coherence results in a stronger suppression of the jet nuclear modification factor.

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The properties of fully reconstructed jets in heavy-ion collisions [1–4] reveal the effects of notable final state interactions. They are currently actively investigated as probes of the underlying deconfined hot matter produced in these collisions. A remarkable observation is the strong suppression of the jet yield that persists over a large range of transverse momentum. In contrast, in-cone jet modifications tend to decrease in the same variable [5–7]. This challenges our understanding of the mechanisms underlying jet modifications in the presence of a QCD medium.

In this context, quenching typically refers to the energy loss of jets caused by elastic and inelastic interactions with the medium. The amount of energy, $\epsilon$, transported out of the jet cone is sampled by a probability distribution $P(\epsilon)$, called the quenching weight. Hence, the final-state spectrum reads

$$d\sigma_{\text{med}}/dp_T^2dy = \int_0^\infty d\epsilon P(\epsilon) \frac{d\sigma_{\text{vac}}(p_T + \epsilon)}{dp_T^2dy},$$

where $d\sigma_{\text{vac}}$ is the jet spectrum in vacuum. The quenching probability distribution is expected to depend on the medium properties, such as the jet quenching parameter $q$, which is a medium diffusion coefficient in transverse momentum space, and the medium length $L$ [8], but it should also be sensitive to the jet $p_T$ and cone size $R$.

The tools for analyzing jet quenching were developed to account for radiative and elastic energy loss of a single color charge propagating in the medium [9–14], see also recent reviews [15, 16] and references therein. However, owing to the QCD mass singularity, the jet-initiating parton tends to branch rapidly—in particular inside the medium. This leads one to question the single charge energy loss approximation for jet quenching at high-$p_T$.

Radiative energy loss of color-connected subjets was recently shown to be sensitive to interference effects between the emitters [17], see also [18–21]. On the other hand, most Monte Carlo implementations of jet quenching ignore interference effects and assume independent energy loss of all jet constituents. One may expect a substantial quantitative discrepancy between the two pictures.

In vacuum [22, 23], higher-order corrections to the fully inclusive jet spectrum are suppressed by powers of the coupling constant [30]. In the medium however, induced energy loss of the final-state particles causes a mismatch between real and virtual diagrams that is accompanied by logarithms of the available phase space. This takes place whenever the medium resolves the individual color.
charges created in hard splittings, namely at the time $t = t_d \sim (\hat{q}^2)^{-1/3}$, when the transverse distance between the two daughters, $r_{12} \sim \theta t$, becomes of order the medium correlation length $(\hat{q}^2)^{-1/2}$. This translates to a minimum characteristic angle $\theta_q \sim (\hat{q}L^3)^{-1/2}$ when $t_d = L$, so that, for $R < \theta_q$, the jet is not resolved. In this case, the energy loss probability distribution is not sensitive to the fluctuations of the jet substructure.

An example of a higher-order correction and the related phase space are depicted in Fig. 1. In the inset the thick lines represent hard, vacuum-like real emission while the thin lines represent multiple medium-induced emissions. The formation time of splitting is given by $t_t = 2p_x/m^2$, where $m^2 = z(1-z)p_T^2/\hat{q}^2$ is the invariant mass-squared of two-parton system. The real contribution will be affected by the combined quark and gluon energy loss while for the virtual contribution only the quark quenching plays a role. The mismatch appears when $t_t \sim (zp_T^2)^{-1} \ll t_d \ll L$, corresponding to vacuum splittings inside the medium. As a result the double logarithmic phase space for this correction, given by the area of the shaded area in Fig. 1, is enhanced by the jet scales

$$\tilde{\alpha} \int_{t_1 < t_2 < L} \frac{d\theta}{\theta} \int \frac{dz}{z} = \tilde{\alpha} \ln \frac{R}{\theta_c} \left( \ln \frac{p_T}{\omega_c} + \frac{2}{3} \ln \frac{R}{\theta_c} \right).$$

This calls for the resummation of higher-order contributions.

The main purpose of this Letter is to compute the effect of fluctuations of the jet substructure on the jet spectrum Eq. (1). As will become clear in the moment, it is convenient to directly consider the ratio of the jet spectrum in medium and the unmodified vacuum spectrum, known as the nuclear modification factor,

$$R_{\text{jet}} = \frac{\frac{d\sigma_{\text{med}}}{dp_T^2 dy}}{\frac{d\sigma_{\text{vac}}}{dp_T^2 dy}}.$$

If one assumes a steeply falling power spectrum with a constant power $n$, one can make the following approximation [24]

$$\frac{d\sigma_{\text{vac}}(p_T + \epsilon)}{dp_T^2 dy} \approx \frac{d\sigma_{\text{vac}}}{dp_T^2 dy} \exp\left(\frac{-n\epsilon}{p_T}\right),$$

which holds for small $\epsilon/p_T$ and large $n$. Corrections to the exponential spectrum for finite or momentum dependent index $n$ can be systematically calculated. Using Eq. (4), it becomes clear that the ratio (3) is related to the Laplace transform of the quenching weight, $\mathcal{P}(\nu) = \int_0^\infty d\epsilon \mathcal{P}(\epsilon) e^{-\nu \epsilon}$, as follows

$$R_{\text{jet}} = Q(p_T),$$

where $Q(p_T) \equiv \tilde{\mathcal{P}}(n/p_T)$.

Let us presently describe the effects of the fluctuating substructure on the total energy loss that a jet experiences, determining it from a perturbative expansion in what follows. At leading order in the strong coupling constant, $Q(p_T)$ is related to the quenching of a single quark [24], $Q^{(0)}(p_T) \equiv Q_q(p_T)$. In the approximation of independent soft medium-induced radiation the quenching factor, $Q_q(p_T) = \mathcal{P}_q(n/p_T)$, reads

$$Q_q(p_T) = \exp \left[ \int_0^\infty d\omega \frac{dI}{d\omega} \left( e^{-\omega p_T / \theta_q} - 1 \right) \right],$$

where the medium induced radiation spectrum is given by $\omega I/d\omega = 2\tilde{\alpha} \ln \cos(1 + i\sqrt{\omega/2\omega_c})$ [10, 11], $\omega_c \equiv \sqrt{qL^2/2}$. In Eq. (6), the coupling constant $\tilde{\alpha} \equiv \alpha_C p_T/\pi$ should be evaluated at the transverse scale $k_\perp(\omega) \equiv \sqrt{qL} = (2\tilde{\alpha} \omega)^{1/4}$ of the radiated gluons, but since the integral in Eq. (6) is dominated by $\omega \sim p_T/n$, to logarithmic accuracy, it can be taken to run as $\alpha_s((p_T \hat{q}/n)^{1/4})$.

When $\omega \sim p_T/n \ll \omega_c$, the radiation spectrum can be approximated by $\tilde{\alpha}\sqrt{\omega_c/2\omega}$, which yields the quenching factor $Q_q(p_T) = \exp\left[ -\sqrt{\pi \alpha_s^2 \omega_c / p_T} \right]$ [24]. But in general, the quenching factor depends on the jet radius. In the regime of interest, $Q_q \ll 1$ which corresponds to gluon frequencies, $p_T/n \lesssim \tilde{\alpha}\omega_c$, the radiated gluons undergo a democratic cascading process and eventually, lose all of their energy to the medium [25]. Therefore, their energy is uniformly distributed in angles and for small jet radii their contribution inside the jet cone can be neglected. On the other hand, small angle hard medium induced radiation $\omega \gtrsim \tilde{\alpha} \omega_c$ are rare, i.e. $O(\alpha_s)$ corrections, and thus can be treated as higher order contributions.

The characteristic angle that separates the two regimes is $\theta_q((\tilde{\alpha}^2 \omega_c) \sim \tilde{\alpha}^{-2} \omega_c \gg \theta_c$, where we have used that $\theta_q(\omega) \equiv k_\perp(\omega)/\omega = (\tilde{\alpha}^2 \omega_c)^{1/4}$. Hence, in what follows we shall work in the approximation $R \ll \tilde{\alpha}^{-2} \omega_c$. This allows for an angular separation between medium-induced gluon radiation and collinear enhanced vacuum splittings.

Throughout, we will work in the large-$N_c$ limit and consider only soft medium-induced gluons. We will also assume the dominance of quark jets, described by a hard spectrum with constant spectral index $n$.

For high-$p_T$ jets, the logarithmically enhanced contribution to the quenching at first order in the coupling constant, $Q^{(1)}(p_T)$, arises in the region where the formation time of the hard quark-gluon pair is shorter than the medium length, i.e. when the splitting forms promptly in the medium and propagates through approximately the whole length of the medium. It reads

$$Q^{(1)}(p_T) = \frac{\alpha_s}{\pi} \int_0^1 dz P_{qg}(z) \int_0^R \frac{d\theta}{\theta} \left[ Q_{qg}(p_T) - Q_q(p_T) \right],$$

for fixed coupling, where $P_{qg}(z) = C_F[1+(1-z)^2]/z$ is the quark-gluon Altarelli-Parisi splitting function, with $z$ the gluon momentum fraction, and $Q_{qg}(p_T) = \mathcal{P}_{qg}(n/p_T, \theta)$ is the quenching factor of the promptly produced quark-gluon pair with opening angle $\theta$ [17]. The above correction is negative since two protons are expected to suffer more energy loss than one, i.e., $Q_{qg} < Q_q$. In this expression, we have assumed $zp_T \gg \tilde{\alpha}^2 \omega_c$, which leaves
the argument of the splitting function unmodified. Indeed, when $p_T < \alpha^2 \omega_c$, then the minimum splitting angle $\theta_1(p_T)$ that follows from the constraint, $t_\ell \ll t_d$, satisfies $\theta_1(p_T) \sim \alpha^{-3/2} \theta_c$, which is assumed to be parametrically larger than $R$.

In the large-$N_c$ approximation, $Q_{qg}(p_T) = Q_q(p_T) Q_{\text{sing}}(p_T)$, where $Q_{\text{sing}}(p_T) \equiv P_{\text{sing}}(n/p_T, \theta)$ is the quenching factor of a quark-antiquark single-antenna. The single-antenna factor in the limit $t_\ell \ll t_d$ (see also [26]), which explicitly depends on the angle of the fluctuation, is found from $Q_{\text{sing}}(p_T) = Q_0^2(p_T, L) - 2 \int_0^{L} dt S_2(t) \gamma(p_T) Q_0^2(p_T, L - t)$, where we have restored the time-dependence in the single-parton quenching weights on the right-hand-side and $\gamma(p_T, t) = \int_0^\infty d\omega \frac{d\theta}{d\omega}(1 - e^{-n\omega/p_T})$ [17]. For further details on the interference spectrum, see [20, 27]. Finally, $S_2(t) = \exp(-\tilde{q} \theta^2 t^3/12)$ describes the color decoherence of the pair that is sensitive to the characteristic time scale $t_d \sim (\tilde{q} \theta^2)^{-1/3}$ [19]. At later times, interferences are suppressed and the gluon and quark radiate independently.

For the purpose of this work, it is sufficient to identify two limiting cases for $Q_{\text{sing}}(p_T)$ in order to extract the regimes of logarithmic enhancement. When $t_d \gg L$, which translates into the small angle region $\theta \ll \theta_c \equiv (\tilde{q} L^3/12)^{-1/2}$, we have $Q_{\text{sing}}(p_T) \simeq 1$ and when $t_d \ll L$, i.e. at large angles $\theta \gg \theta_c$, we have $Q_{\text{sing}}(p_T) \simeq Q_0^2(p_T)$.

We note that in the former situation $Q_{qg} \simeq Q_q$ and, as a result, Eq. (7) vanishes. Hence, only the region $\theta > \theta_c$ contributes to logarithmic accuracy. This conveys the main physics message, namely that the medium resolves only sufficiently wide jet fluctuations. When $t_\ell > t_d$, the medium resolves the quark-gluon fluctuation resulting in a hard medium-induced splitting that does not exhibit a double-logarithmic structure particular of vacuum splittings. Finally, vacuum splittings outside the medium cancel out in Eq. (7).

Let us now introduce a new object, that we call the collimator function, which encodes the quenching of wide angle jet fluctuations. It is defined as,

$$C(p_T) \equiv \frac{Q(p_T)}{Q_q(p_T)}$$

and represents the total quenching factor modulo the energy loss of the total color charge of the (quark) jet. The expansion of the collimator takes the form $C(p_T) = 1 + C^{(1)}(p_T) + \mathcal{O}(\alpha_2^3)$, where the first non-trivial correction appears at next-to-leading order. We focus on the leading logarithmic (LL) behavior, that is $z \ll 1$ and $\theta \gg \theta_\ell$. In this situation, one can approximate $Q_{\text{sing}}(p_T) \simeq Q_0^2(p_T)$, then Eq. (8) yields

$$C^{(1)}(p_T) \simeq 2\tilde{q} \int_{\theta_c}^{R} \frac{d\theta}{\tilde{q}/\theta^{1/3}} \int_{\theta_c}^{p_T} \frac{d\omega}{\omega} \left[ Q_0^2(p_T) - 1 \right].$$

Here we have denoted $\omega = z p_T$ and explicitly demanded that we must have $t_\ell < t_d < L$ to preserve the collinear logarithm. This condition implies that $\theta > \theta_\ell(\omega) = (\tilde{q}/\omega^3)^{1/4}$ or equivalently $\omega > (\tilde{q}/\theta_\ell^{1/3})$. Hence, the NLO result in the LL approximation becomes

$$\frac{C^{(1)}(p_T)}{2\tilde{q} [Q_0^2(p_T) - 1]} \simeq \ln \frac{R}{\theta_c} \left[ \ln \frac{p_T}{\omega_c} + \frac{2}{3} \ln \frac{R}{\theta_c} \right],$$

for $p_T > \omega_c$, cf. Eq. (2). Remarkably, in the relevant high-$p_T$ regime, we obtain a single-logarithmic contribution that scales with the jet momentum. In fact, this term arises from semi-hard radiation, $p_T > \omega > \omega_c$, and implies that the enhancement stems from splittings that appear early in the medium. A finite coherence angle $\theta_c$ moderates the enhancement, as will be demonstrated below.

The specific product of logarithms in Eq. (10) appears due to the fact that, at small angular scales, the jets are unresolved and lose energy coherently. Let us for the moment contrast this first-principle description [17] with a scenario wherein we treat the jet substructures as completely independent with respect to medium interactions. The logarithmic phase space is only limited by the condition that the splittings happen inside the medium, i.e. $t_\ell < L$, cf. the Lund diagram in Fig. 1. Instead of the phase-space relevant for (9), we now obtain a double-logarithmic enhancement,

$$C^{(1)}(p_T) \bigg|_{\text{incoh}} \simeq \frac{\tilde{\alpha}}{2} \left[ Q_0^2(p_T) - 1 \right] \ln^2 \left( \frac{p_T R^2 L}{q} \right).$$

This leads to a stronger energy-dependence and hence a stronger suppression than in Eq. (10).

Let us now come back to dealing with the generalization of Eq. (10) to all orders. This is an arduous task which demands tracing multiple emissions with various formation times. In order to limit the scope, we will pursue the resummation only within the LL approximation. At this precision, all formation times can be assumed to be arbitrarily small as it corresponds to the phase-space giving rise to the maximal logarithmic enhancement. Writing the full dependence of the collimator function as $C(p_T) \equiv C(z, p_T, R)$ and assuming small energy losses, and hence neglecting any modification of the splitting functions, we can iterate the procedure by replacing $Q_0^2(p_T) \to C_g(z, p_T, \theta) C_0((1 - z), p_T, \theta) Q_0^2(p_T)$, and $Q_g(p_T) \to C_g(1, p_T, \theta) Q_g(p_T)$, in the first and second terms of Eq. (7), respectively. This results in a nonlinear, coupled evolution equation for the resummed collimator function for quarks and gluons,

$$C_i(1, p_T, R) = 1 + \int_0^1 dz \int_{\theta_\ell}^{R} \frac{d\theta}{\theta^3} \frac{\alpha_s(k_{\perp})}{\pi} P_{gi}(z) \Theta(t_4 - t_\ell) \times \left[ C_g(z, p_T, \theta) C_i((1 - z), p_T, \theta) Q_0^2(p_T) - C_i(1, p_T, \theta) \right],$$

where $i = q, g$ and we have restored the full Altarelli-Parisi splitting functions (cf. $P_{gg}(z) = C_F(1 + (1 - z)^2/z(1 - z))$). Here we have explicitly assigned the collimator functions to the quark and gluon daughters of the splitting. Recall that it is implied that $C(z, p_T, \theta) = C(z, q_T, \theta)$.
form factor, obtains the exponentiation of Eq. (10) into the Sudakov asymptotic limit is recovered. On the other hand, when \( p_T \) is at high energy, therefore, the equation only has a fixed point of the equation. Hence, the expected large splitting into quarks is neglected since it does not contribute at large-\( N_c \) and does not contain IR divergences. At logarithmic accuracy one has again used the fact that \( \theta > \theta_c \) is not enough to avoid the large-angle jet fluctuations. Coherence effects play an important role in moderating the effect of the collimator, giving rise to a single logarithm of the jet energy in contrast with the double logs of the jet scale in the incoherent energy loss approximation. Our results demonstrate the sensitivity of inclusive jet observables to color coherence. It also opens the possibility to extend and refine studies of high-\( p_T \) jet resummations in the presence of medium effects. We expect that the effects of fluctuating energy loss will have an impact on other jet quenching observables, see e.g. [28, 29] for Monte Carlo studies.

Let us end with a final remark. One generally expects to recover the missing energy by opening up the jet cone and hence an increase of the nuclear modification factor. This should be the result of the interplay between the suppression of high-\( p_T \) jet fluctuations and the recapture of an increasing number of medium-induced soft particles within the jet cone. The physics of collimation produces an opposite trend, so for quantitative predictions it will also be important to implement the description of secondary medium-induced soft gluon emissions in order to improve the description of energy flow within and outside the jet cone.

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[30] Note that for small cone sizes, powers of $\alpha_s \ln R$ needs to be resummed.