Quark Matter and Nuclear Collisions*

A Brief History of
Strong Interaction Thermodynamics

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Abstract:
The past fifty years have seen the emergence of a new field of research in physics, the study of matter at extreme temperatures and densities. The theory of strong interactions, quantum chromodynamics (QCD), predicts that in this limit, matter will become a plasma of deconfined quarks and gluons – the medium which made up the early universe in the first 10 microseconds after the big bang. High energy nuclear collisions are expected to produce short-lived bubbles of such a medium in the laboratory. I survey the merger of statistical QCD and nuclear collision studies for the analysis of strongly interacting matter in theory and experiment.

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1 Denser than Nuclei: Neutron Stars, Big Bang, Heavy Ion Collisions

How dense can matter be - is there a limit? This question has been around for quite a while. A celebrated answer was triggered by Sir Walter Raleigh, who was wondering how one could best stack the cannon balls on his ships. He asked his expert on board, the English mathematician Thomas Harriot, who passed the question on to his German colleague Johannes Kepler. Kepler, in 1611, suggested that the ordered stack of hard balls, still used today by grocers to display oranges, is the greatest possible density for such objects; it filled the volume to a fraction of $\pi/\sqrt{18} \approx 0.74$. In 1831, Carl Friedrich Gauss proved that Kepler’s conjecture is in fact correct, for orderly packing. But if we simply pour balls into a container, we never reach such a density, since the randomly falling balls do not arrange themselves in such an orderly pattern. The maximum density of the disordered medium is much more difficult to calculate – in fact, that this density is always less than that of the Kepler pyramid was established only recently by the American mathematician Thomas C. Hales, using a computer-aided proof. And when the crate is filled, we can still increase the density a little by shaking the container for a while. Such random close packing leads to a density limit of 0.65 – about 10 % less dense than the orderly stacked pile.

![Figure 1: Stacking cannon balls](image)

The nuclei of heavy atoms are the densest matter found on Earth, and they are also the only strongly interacting matter here. Standard nuclear density is $0.16/fm^3$; with a nucleon radius of about 0.8 fm, that means about 35 % of the nuclear volume is filled with nucleons, 65 % remain empty. So the nucleons can still rattle around a little inside the nucleus. But they are “jammed”, confined to remain in a restricted local environment. And twice normal nuclear density comes already close to Kepler’s conjecture for the orderly packing limit.

Neutron stars are the result of stellar collapse under gravity, occurring when the nuclear fuel has burnt up and there is no further thermal pressure to balance the gravitational force. If the mass of the star is large enough, gravity will force electrons and protons to undergo inverse beta-decay ($p + e^- \rightarrow n$) and fuse to neutrons, so that now a combination of nucleon repulsion and Fermi pressure stops gravity from causing further compression.
This, however, means that the mass can also not be too large: otherwise gravity will win after all and turn the dead star into a black hole. The density of a neutron star increases in going from the crust to the core, and present estimates of core densities go up to five times normal nuclear density. That is well above all close packing limits, random as well as orderly, and so it must mean that the nucleons suffer considerable squeezing and/or overlap. Which raises the question of whether this allows them to remain as nucleons or whether the dense neutron star cores consists of deconfined quarks.

The expansion of the present universe, as attested by Hubble’s law as well as by the cosmic background radiation, allows us to extrapolate its density back to early times, close to its origin in a hot Big Bang. The relation between the age of the universe and its energy density is is given by

$$t = \frac{1}{H(t)} = \sqrt{\frac{3}{8\pi G \epsilon(t)}},$$

where $H(t)$ is the Hubble parameter, $G$ is the gravitation constant and $\epsilon(t)$ the cosmic energy density. Considering the instant of the big bang as starting point, we can determine the time needed to reach, on a cosmic level, the energy density $\epsilon \approx 0.5 \text{ GeV/fm}^3$, found inside a single nucleon. With the value $G = 6.708 \times 10^{-39} \text{ GeV}^{-2}$, it becomes

$$t_q \approx 10^{-5} \text{ s},$$

so that in its first ten microseconds, our universe was in a state for which the existence of individual hadrons is not really conceivable. Today we take that primordial medium to consist of deconfined quarks and define the time up to $t_q$ as the quark era. Only at the end of this era hadrons were formed, and there appeared for the first time the stage for today’s complex cosmic structure, the physical vacuum.

Given the density of a single heavy nucleus: if we collide two of them at high energy, they will either compress each other or overlap, and the resulting medium must be, at least for a short time, of considerably higher nucleon density, and hence also of rather high energy density. And indeed, the collision of two gold nuclei at a center of mass energy of 200 GeV turns this high energy density into the production of hundreds of secondary hadrons, see Fig. 2.

Figure 2: Particle production in the collision of two gold nuclei at $\sqrt{s} = 200$ GeV, from the STAR experiment at RHIC, Brookhaven National Laboratory
Tracing all these hadrons back to the interaction region of the two colliding nuclei, it is clear that the energy density there must have been very much higher than the 0.5 GeV/fm$^3$ inside a single nucleon. Whether it is meaningful to call this dense medium “matter” is another issue, subject of many studies, both theoretical and experimental.

2 All those Resonances: Hagedorn’s Legacy

In the middle of the last century, the ultimate constituents of matter seemed fairly clear: protons and neutrons giving the mass of nuclei, the almost massless electrons to assure electric neutrality. The short-range strong force, required to bind the nucleons, led Yukawa to introduce the pion as the boson mediating the interaction, and challenged experimentalists to look for it in cosmic rays as well to produce it in the laboratory. Proton-proton collisions indeed did that, but also much more: they opened a Pandora’s box containing, by now, an almost unlimited number of new strongly interacting elementary or not so elementary particles, hadrons – with more than a thousand entries in the Particle Data List, according to the latest count. New discrete, non-kinetic quantum numbers were needed: isospin, strangeness, charm and beauty, and for a given such quantum number, ever growing towers of resonances of different angular momenta and parities were observed.

This multitude of hadrons had two consequences which changed the course of physics. One was in its initial concept not really revolutionary, though it eventually provided us with the theory of strong interactions, quantum chromodynamics. The idea was simply that so many elementary particles must be compound states of fewer and simpler constituents, and such reductionism has been with us ever since the atoms of ancient Greece. We shall return to the results for strong interaction physics in the next section.

The second outcome was indeed totally new, and the basic idea came from Rolf Hagedorn [1,2]. He first asked whether the composition of resonances might not follow a self-similar pattern, so that a resonance of a given mass would consist of lighter resonances constructed according to the same composition law. In Hagedorn’s words, “fireballs consist of fireballs which consist o fireballs, and so on...”. A more poetic summary was already given around the middle of the nineteenth century by the English mathematician Augustus de Morgan (inspired by some lines of Jonathan Swift),

\begin{quote}
Great fleas have little fleas upon their backs to bite ’em.
And little fleas have lesser fleas, and so ad infinitum.
And the great fleas themselves, in turn, have greater fleas to go on,
While these again have greater still, and greater still, and so on.
\end{quote}

Hagedorn asked how many different resonances of a state of given mass $M$ could contain or could decay into, given self-similar composition, and he found the answer: exponentially many, $n(M) \sim \exp\{bM\}$. It is essentially a partition problem [3] – how many ways are there of dividing an integer into integers? Consider as an illustration the simplest case, ordered partitions. We then have
\[ 2 = 1 + 1, \quad 2 \to \rho(2) = 2^1 \]
\[ 3 = 1 + 1 + 1, \quad 1 + 2, \quad 2 + 1, \quad 3 \to \rho(3) = 2^2 \]
\[ 4 = 1 + 1 + 1 + 1, \quad 1 + 1 + 2, \quad 1 + 2 + 1, \quad 2 + 1 + 1, \quad 1 + 3, \quad 3 + 1, \quad 2 + 2, \quad 4 \to \rho(4) = 2^3 \]

and hence in general

\[ \rho(n) = 2^{n-1} = (1/2) \exp\{n \ln 2\} \]  

showing that the number of ordered partitions of an integer \( n \) grows exponentially with \( n \). And such a growth in fact also holds in the more general case of self-similar resonance decay or composition.

Hagedorn then went on to ask what effect such a resonance spectrum might have on the thermodynamics of strongly interacting matter, and found a very striking answer. Rather basic arguments \[4,5\] suggest that a medium of interacting elementary constituents (pions and nucleons) could be replaced by an ideal gas of all possible resonance states arising from the interaction. That leads to the grand partition function

\[ \ln Z(T) \sim \int dM \ n(m) \ \exp\{-M/T\} \]

which is seen to exist only if \( T \leq 1/b \). In other words, there is an ultimate temperature of strongly interacting matter, the Hagedorn temperature \( T_H = 1/b \). For a while, Hagedorn took it to be the upper limit of the temperature of all matter, just like there is a lower limit of \(-271^\circ \) C. But a few years later, Cabbibo and Parisi \[6\] pointed out that it was the power \( a \) of the prefactor in \( n(M) = M^a \exp\{bM\} \) which determined if \( \ln Z(T) \) itself or only derivatives above a certain order would diverge at \( T = T_H \). And such divergences are well known in statistical physics: they just correspond to critical behavior. So \( T_H \) only defined the end of the hadronic state of strongly interacting matter; for \( T > T_H \), there were no more hadrons, the matter had undergone a phase transition into a new state. Today then, Hagedorn’s limiting temperature is for us the first indication of a new, hotter and denser state of strongly interacting matter. Once hadron dynamics is correctly taken care of, hadronic thermodynamics defines its own limit, even without knowing anything about the quark infrastructure.

And Hagedorn’s vision of the self-similar origin of the critical behavior turned out to be fruitful far beyond expectation. Shortly afterwards, and quite independently, Fortuijn and Kasteleyn \[7,8\] showed that spontaneous symmetry breaking as the origin of critical behavior in spin systems could equivalently be replaced by the fusion of clusters of all sizes. And still a little later, also the dual resonance model \[9,10\] was found to result in an exponential growth of the number of states, derived from the assumption of a complete resonance determination of the scattering amplitude. This model subsequently became the progenitor of string theory, and even there the Hagedorn temperature is still alive and well today.
3 The Conjecture of Lucretius: Quark Confinement

The other outcome of the multitude of resonances was their origin as composite states of something more elementary. The different quantum numbers – spin, parity, charge, isospin, baryon number, and more – ruled out the possibility of simply additive subconstituents and led to the quark model with its non-Abelian composition laws. And this in turn provided the basis for what we now believe is the theory of strong interactions, quantum chromodynamics.

We had already mentioned that reductionism, the idea that the complexity of the visible world must arise from a few simple microscopic building blocks on an invisible level, was already introduced in ancient Greece. The perhaps most striking consequence in their logical derivation was put forward by the Roman philosopher Lucretius \[11\], who noted around 50 B. C.

*So there must be an ultimate limit to bodies, beyond perception by our senses. This limit is without parts, is the smallest possible thing. It can never exist by itself, but only as primordial part of a larger body, from which no force can tear it loose.*

I believe that this is the earliest reference to quark confinement; with QCD, Lucretius’ conjecture was finally, and for the first time, realized in the framework of physics. Also the other essential result of QCD, asymptotic freedom, was a novel feature, distinguishing it from all previous composition schemes. The theory had two basic pillars of support. The quark infrastructure now accounted for all the observed resonances as bound states of different quantum number combinations. And for short distance physics, asymptotic freedom allowed a perturbative approach, which provided a wealth of subsequently confirmed predictions. In spite of this immense success, three important areas of strong interaction physics remained out in the cold: hadronic collisions in the non-perturbative regime (which meant more than 95 % of all collision data), quantitative hadron spectra (calculate the masses of the mesons and the baryons), and the thermodynamics of strongly interacting matter in the region of the transition from hadronic to quark matter.

While high energy collisions in the non-perturbative regime still remain a largely unsolved realm of QCD, a new avenue was found for the calculation of hadron spectra and for the thermodynamics of strongly interacting matter. It had been shown that the partition function, normally written in the canonical form with \( \beta = 1/T \),

\[
Z(\beta, V) = \text{Tr} \exp\{-\beta \mathcal{H}\},
\]

could in field theory be reformulated as a Euclidean path integral \[12\],

\[
Z_E(\beta, V) = \int \mathcal{D}A \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp \{ - \int_V d^3x \int_0^\beta d\tau \mathcal{L}(A, \psi, \bar{\psi}) \},
\]

where \( \mathcal{H} \) and \( \mathcal{L} \) denote the QCD Hamiltonian and Lagrangian, respectively. In the reformulation, the trace over all particle states is replaced by path integrals over the gauge \( (A) \) and quark/antiquark \( (\psi/\bar{\psi}) \) fields. In this form, the integrations over space and temperature could be discretized, i.e., replaced by lattice sums \[13\]. If one now carried out the integrations over the quark spinor fields and changed variables from gauge fields to the
corresponding $SU(3)$ color matrices the structure of the resulting lattice partition function turned out to have the same form as that of a spin system on a lattice. And such spin systems could be addressed by numerical simulations on high performance computers [14].

4 Shift of Paradigm: The Computer Simulation of Lattice QCD

This not only provided a method to address strong interaction thermodynamics; that was just one application of a new way of doing physics calculations, replacing for certain classes of problems analytical mathematics (which was not able to solve them) by numerical computer simulation. It has been called “a betrayal of theoretical physics”, but, like many heresies, it opened new vistas and allowed in particular the quantitative study of many aspects of collective behavior.

In QCD, one used the exponential of the QCD action, $\exp\{-S(U, \beta)\}$, as weight to determine, essentially by throwing dice, equilibrium configurations for the state of the system. Given these, one could then “measure” on them the observables of interest: energy density, pressure, specific heat, and more. In a way, it was a realization of Boltzmann’s dream: one could now indeed generate for a many-body system all the possible configurations allowed by the overall dynamics, energy and volume constraints, assign them equal a priori weights, measure the value of the desired observable for each configuration, and then average.

The main drawback was that this method gave answers, but did not specify the underlying reasons for the answers. Thus it was found that the energy density, as function of the temperature, suddenly increased in a very narrow region from rather low hadronic values to much higher values, not far from the Stefan-Boltzmann limit expected for an ideal gas of colored constituents [15, 16]. So the sudden increase seemed to be the onset of deconfinement. In pure gauge theory, strong interaction thermodynamics had in fact been initiated [17–19] by measuring the expectation value $L(T)$ of the Polyakov,

$$L(T) \sim \exp\{-F_Q(T)/T\},$$

and showing that it acted as confinement/deconfinement order parameter, much like the magnetization did for spin systems. Here $F_Q(T)$ is the free energy needed to separate a static quark-antiquark pair infinitely far, and in the confinement regime, this diverged,

$$F_Q(T) = \lim_{r \to \infty} F_Q(r, T) \to \infty,$$

causing $L(T)$ to vanish there. In a deconfined medium, in contrast, gluon screening left $F_Q(T)$ and hence also $L(T)$ finite.

But in full QCD, quarks acted like an external field, leading to some alignment of the generalized spin $L(T)$ at all temperatures, just as the magnetization in the Ising model always remains finite for a finite external field. So how should one now specify deconfinement? In addition, there was the “other” transition, the restoration of chiral symmetry. At low temperatures, gluon dressing gave the quarks a constituent mass of hadronic scale; with increasing temperature, this dressing would melt, shifting the effective quark mass
back to its (almost vanishing) current quark value. Did this mass shift coincide with color deconfinement? In the chiral limit, for vanishing quark masses, the chiral condensate was indeed a *bona fide* order parameter, but what about deconfinement?

Today, we may have at least a partial answer. Consider QCD with two massless quark flavors. The chiral condensate identifies the chiral transition temperature $T_{ch}$. One way of defining deconfinement is to consider the transition at which the average baryon number per constituent flips from unity (in the hadronic phase, with nucleons) to 1/3, in the quark phase. This is evidently the point at which quarks are no longer bound to color neutral three-quark entities. And it is indeed observed in lattice studies, for higher moment fluctuations, that this point coincides with the chiral symmetry restoration temperature. So in what one could call an *ab initio* approach, we conclude that deconfinement and chiral symmetry restoration coincide, occurring (according to latest results) at a unique critical temperature of about $160 \pm 10$ MeV.

But what these calculations cannot tell us is what happens just beyond the transition. It is clear, and has been for twenty years, that the plasma of deconfined color is strongly interacting. The crucial measure, the so-called trace anomaly

$$\Delta(T) = \frac{\epsilon - 3P}{T^4}$$

comes from small values to reach a peak just above, but definitely above, the critical temperature. Beyond this peak, it decreases, but it reaches something like perturbative behavior only at very much higher temperatures, if at all.

So we can summarize our present state of knowledge in statistical QCD, as obtained from lattice studies, schematically as shown in Fig. 3. Approaching the transition temperature $T_c$, initially the energy density increases faster than the pressure, as a reflection of critical behavior. At a certain temperature $T_p > T_c$, the two roles are interchanged, since eventually the system tends towards the Stefan-Boltzmann limit $\epsilon/T^4 \rightarrow 3P/T^4$. The peak of the interaction measure is the result of the interchange.

![Figure 3](image.png)

Figure 3: Schematic view of energy density and pressure (a) and of the interaction measure (b) in the deconfined medium; in (a), SB denotes the Stefan-Boltzmann limit.

Here again we note both the advantages and the problems of the lattice simulation. We obtain beautifully the over-all pattern of the crucial thermodynamic observables, but we don’t really know the specific “microscopic” or “dynamic” origins of this behavior. On
the confined hadronic side, the model of an ideal resonance gas, as first proposed by Hagedorn and since then refined to become a very useful tool of analysis, provides us with a fairly detailed understanding of what is happening; we return to this in section 6. On the plasma side, there exist various models, but one probably has to admit that the origin of the observed behavior is not really understood.

A serious shortcoming of the computer simulation approach is its (present?) failure for systems of large baryon density. The origin of the difficulty seems purely technical: the weight-function used to generate equilibrium configurations is no longer positive definite at non-zero baryonic potential $\mu$. In other words, the integrals to be evaluated now have integrands fluctuating between positive and negative values. All that is known (or believed to be known) for the QCD phase diagram as function of temperature $T$ and baryonic potential $\mu$ is thus based on approximate lattice methods, trying extensions from vanishing to finite $\mu$, or on more or less phenomenological models.

My own favorite model assumes color deconfinement to set in when the density of constituents surpasses the hadronic scale, i.e., when each quark sees within its confinement range several other quarks, and chiral symmetry restoration when the temperature of the medium melts the gluon dressing of the constituent quarks. At $\mu = 0$, the two coincide, since the density increase is due to the temperature increase. For $T = 0$, that is not the case, and so one expects deconfinement there to lead to a plasma of massive quarks. The resulting phase diagram is schematically illustrated in Fig. 4.

![Figure 4: Schematic view of a possible QCD phase diagram](image)

It should be noted that the deconfined quarks could, at low temperature, bind to form bosonic colored diquarks, and these in turn could condense to form a color superconductor. Such a phase, as a special form within the quark plasma, is not included in Fig. 4.

5 The Little Bang: Making Matter in Collision

Given what we know about strong interaction thermodynamics, how can we apply it? Neutron stars are far away, and the Big Bang was long ago. Is there some way to create strongly interacting matter in terrestrial experiments? We had already mentioned heavy ion collisions, and in the course of the 1980’s, that possibility gained more and more interest.
The idea is quite straightforward. At the start of the program, beams of energetic nuclei were made to hit stationary nuclear targets. In the modern versions, however, at the LHC/CERN as well as at the RHIC/BNL, two beams of nuclei, moving in opposite directions, collide “head-on” in the detectors of the experiment. Through Lorentz contraction in the longitudinal direction, the incoming nuclei appear as pancakes to an observer in the laboratory. In the collision, they overlap and form a system of many quarks contained in a small disk, an excited colored medium. This highly compressed bubble subsequently expands, still retaining its colored nature, and through many interactions it could form a thermal system: a quark-gluon plasma. This colored plasma would then cool off and lead to the production of the hadrons detected in the laboratory.

From the very beginning, T. D. Lee was one of the main proponents of such a new research program, and he moreover explained the idea to the famous Chinese painter Li Keran, who in 1989 composed a picture of two fighting bulls, with the heading

![Image of two fighting bulls]

*Nuclei as heavy as bulls through collision generate new states of matter.*

Today the bulls also exist as a beautiful life-size sculpture, close to the campus of Tsinghua University in Beijing, as probably the only existing memorial to heavy ion collisions. The justification for a research program devoted to the empirical study of quark matter is quite different from most others in recent times. It is not the search for a well-defined and theoretically predicted entity, such as the Higgs’ boson, ultimate aim of several presently ongoing experiments at CERN and Fermilab. It is also not the purely exploratory study of the strong interaction in the last century: what happens if we collide two protons at ever higher energy? Instead, it is almost alchemistic in nature: can we find a way to make gold? Is it possible, through the collisions of two heavy nuclei at high enough energy, to make strongly interacting matter of high energy density and study its behavior in the laboratory?

The opinions on the feasibility of such an endeavor were mixed. Richard Feynman was as always ready with a concise judgement and said “if I throw my watch against the wall, I get a broken watch, not a new state of matter”. Actually, the problem had two sides. One was the aspect Feynman had addressed: is it possible to create through collisions something we would call matter? The other was the question of whether the experimentalists would be able to handle the large number of hadrons to be produced in such collisions. Both
were indeed serious, but the promise of finding a way to carry out an experimental study of the stuff that made up the primordial universe – that promise was enough to get a first experimental program going, in 1986, at Brookhaven and at CERN. To minimize the costs, both Labs used existing injectors, existing accelerators (BNL-AGS, CERN-SPS), existing detectors and, as someone pointed out, existing physicists not needing additional salaries: one big recycling project. Whatever, the second of the two questions mentioned above was indeed and resoundingly answered in the affirmative. At present collision energies, one single interaction of two heavy nuclei produces some thousands of new particles (see Fig. 2), and the detectors, the analysis programs and the experienced physicists can handle even that.

The first and conceptually more serious question is not yet answered as clearly, and in the physics community at large (in contrast to the heavy ion community), there are still adherents of Feynman’s point of view. It is clear that present collisions, with a center of mass energy of more than a million GeV for a central lead-lead interaction at the LHC, provide the highest energies ever reached on Earth, and the interactions produce thousands of new particles in a rather small volume, so that also the initial density of constituents is extremely high. The canonical estimate for the energy density produced in a central $A - A$ collision is given by the Bjorken form [20]

$$\epsilon = \frac{\bar{p}_0}{\pi R_A^2 \tau_0} \left( \frac{dN_h}{dy} \right)_A,$$

(11)

where $(dN_h/dy)_A$ is the number of hadrons produced in a unit of central rapidity, $\bar{p}_0$ the average energy per hadron, $R_A$ the nuclear radius and $\tau_0 \simeq 1$ fm the equilibration time needed to produce a thermal medium. For gold-gold collisions at RHIC (top energy) this leads to some 6 - 7 GeV/fm$^3$, and for lead-lead collisions at the LHC to 10 GeV/fm$^3$ or more. These are values exceeding the energy density within a single hadron by up to a factor twenty. So the early medium was indeed hot, and it seems possible to understand what is happening in this stage only in terms of quarks, gluons and their interactions. But does that allow us to speak of quark matter? What are the essential features of matter? And how can one show that the media produced in high energy nuclear collisions share these features, both in the early stages and in the later evolution?

6  The Abundance of the Species: Universal Hadrosynthesis

Multiparticle production in high energy collisions of strongly interacting particles has fascinated physicists for well over half a century. The little bang of such collisions produces with increasing energy an ever growing number of mesons and baryons of an ever growing number of species, and from the beginning, these large numbers were a challenge to describe the reactions by collective or statistical approaches. It was tempting to go even further and imagine that the collisions really produced droplets of strongly interacting matter, thus providing a means to access in the laboratory the thermodynamics of strong interaction physics.
The main features observed in high energy collisions are the multiplicity, i.e., the number of produced particles as function of the collision energy, the momentum spectra of the particles, and the relative abundances of the different species. And the detailed study of the abundances has led to what seems to me the perhaps most striking result of high energy strong interaction physics.

We have seen that high energy collisions produce an initial state of an energy density much higher than that of any hadronic medium. Leaving open the issue of whether or not this hot medium in equilibrium, it will expand, cool off and eventually hadronize. At this point, we have a medium of strongly interacting hadrons, and as already argued by Hagedorn, this system can be considered as an ideal gas of the possible resonances. Using Hagedorn’s self-similar resonance pattern, that had led to the prediction of the critical temperature $T_H$. But we can take a more pragmatic point of view and simply calculate the partition function of all the resonances listed in the Partial Data List [21], with all their known discrete quantum number states. This will require us to stop at some high mass, since resonances of masses above some 2 - 3 GeV are poorly known. But if the effective temperature of the medium is not too high, that should not matter too much; in any case, one can check what a mass cut does to the resulting description. The partition function of the medium is then, for vanishing baryon density, given by

$$\ln Z(T, \mu, V) = \frac{VT}{2\pi^2} \left[ \sum_{i}^{\text{mesons}} d_i m_i^2 K_2(m_i/T) + 2 \cosh(\mu/T) \sum_{i}^{\text{baryons}} d_i m_i^2 K_2(m_i/T) \right],$$  

where $d_i$ denotes the degeneracy of the resonance (spin, charge, etc.), and $\mu$ the baryonic chemical potential. Of course, charge and strangeness conservation has to be taken into account; we have here just suppressed it for the sake of simplicity.

Considered as a function of temperature, this partition function will never lead to critical behavior; it never diverges, for two reasons. The mass cut makes it a power series in temperature, always convergent. But even if we remove the mass cut, the empirical degeneracy factors $d_i$ are not exponential and therefore cannot compensate the $\exp\{-m_i/T\}$ contained in the Hankel functions $K_2(x)$. Nevertheless, the “shadow” of the transition makes its appearance, as it turned out, if we consider the abundances of different species $i$ and $j$. They are (at $\mu = 0$) given by

$$\frac{N_i}{N_j} \approx \left( \frac{d_i m_i}{d_j m_j} \right)^2 \frac{K_2(m_i/T_H)}{K_2(m_j/T_H)} \approx \left( \frac{d_i m_i}{d_j m_j} \right)^{3/2} \exp\{-(m_i - m_j)/T_H\}.$$  

And the curious thing, nature’s hint, was that apart from minor deviations, the observed abundances of all species in all high energy collisions, whether in $e^+ e^-$ annihilation, in $p-p$ or in nuclear collisions, always pointed to the same hadronization temperature $T_H \sim 170 \pm 10$ MeV of a quark-gluon plasma. Up to thirty different hadron species all agreed in their abundances that they were formed at a universal temperature of about 160 – 170 MeV. A recent summary compilation is shown in Fig. 5 [22, 24]. Just as the cosmic background radiation was left over from the Big Bang at the time of last scattering, at the end of the radiation era, at $T_{\text{rad}} \sim 3000^\circ$ K, so all observed hadrons were formed at what statistical QCD had obtained for the hadronization temperature $T_H \sim 170 \pm 10$ MeV of a quark-gluon plasma.
What does this tell us? Here opinions in the community differ. It is obviously tempting to consider the many partons of two colliding nuclei to interact and thermalize, and to have the resulting QGP then hadronize, just like vapor condenses below 100°C. In other words, the thermal nature of the hadronic state is taken as a result of a prior thermal quark-gluon state, which in turn was formed through thermalization of the (non-thermal) initial multiparton state. This leaves open the question of why $e^+e^-$ annihilation, in two-jet events just one hard quark-antiquark pair, and similarly $p-p$ interactions, lead to the same form of thermal behavior. I don’t believe that one should ignore such clear and pointed hints of nature. There must be a universal hadronization mechanism, operative whenever a quark and an antiquark reach their confinement horizon [25]; this can happen in jet cascades as well as in a cooling QGP.

7 Melting of Quarkonia: The QGP Temperature

In the search for probes of the early hot stages of heavy ion collisions, measuring the temperature of stellar cores can serve as an inspiration. This temperature is determined largely through a spectral analysis of the light the stars emit. Their interior is generally so hot that it is a plasma of electrons and nuclei, emitting a continuous spectrum of light, and the frequency of this light is proportional to the energy density of the inner core. In the cooler outer corona of the star, atoms can survive, and the passing light from the interior excites their electrons from the ground state to higher level orbits. The photons doing this are thereby removed from the continuous spectrum, and this shows up: there are absorption lines, whose position indicates the elements present in the corona, and whose strength measures the energy of the light from the interior. To take the simplest case: if the corona contains hydrogen atoms, then the frequencies needed to excite these to
the different excitation levels are candidates for absorption lines. In the case of relatively cool stars, the photons will not be energetic enough to do much except to bring the atoms into their lowest excited state. Sufficiently hot stars, on the other hand, will generally result in jumps to higher excitation states. So by looking which excited states are the target of the photons, we can tell what the temperature of the stellar core is.

A similar method can be applied to study the early interior of the medium produced in high energy nuclear collisions. Here one can observe the mass spectrum of dileptons, replacing the photon spectrum from stars. Ideally, this dilepton spectrum arises from the annihilation of quark-antiquarks pairs in the hot plasma; in practice, a number of competing sources come into play and have to be eliminated. This has so far made the identification of the thermal radiation from quark matter rather difficult. But on top of the smooth curve found for the dilepton mass distribution there are some very pronounced sharp peaks at well-defined positions (see Fig. 6): they are the signals of quarkonium production in nuclear collisions, and they can play the role which the atoms in the corona had in the stellar spectral analysis.

![Figure 6](image-url)

Figure 6: The dimuon spectrum in the bottomonium range, measured in $p - p$ collisions at the LHC for a collision energy $\sqrt{s} = 7$ TeV. The peaks correspond to the $\Upsilon(1S)$, $\Upsilon(2S)$ and $\Upsilon(3S)$ states, respectively.

Quarkonia are unusual hadrons, bound states of the heavy charm ($m_c \simeq 1.3$ GeV) and bottom quarks ($m_b \simeq 4.5$ GeV) and their antiquarks. They are very much smaller than normal hadrons ($r_{\Upsilon} \simeq 0.1$ fm), and their ground states are very tightly bound ($\Delta E_{\Upsilon} = 2M_B - M_{\Upsilon} > 1$ GeV). That’s why both charmonia and bottomonia can in principle survive deconfinement and exist as color-neutral hadrons in a colored quark-gluon plasma, up to some temperature. The different binding energies of the different quarkonia imply that they melt in a hot QGP at different temperatures, and the quarkonium spectrum can therefore serve as plasma thermometer [26–28].

That leaves theory with the charge of calculating the dissociation points of quarkonia in a hot deconfined medium, and experiment with the problem of determining the parameters (collision energy, energy density, temperature) at which the quarkonia disappear in the
dimuon spectrum emitted in nuclear collisions [29]. If both problems were solved, and if the numbers would match, we would have quantitative evidence for the production of a quark-gluon plasma in heavy ion collisions.

The calculation of the in-medium dissociation temperatures of quarkonia has been pursued for over twenty years, and we are still waiting for the final word. Initially, one made a model for the screened heavy quark potential in the QGP [27, 28]. When lattice data for the heavy quark interaction became available, that was employed in numerous studies to determine the potential [30]. It led to a still not resolved issue: should one use the free energy or the internal energy of the $Q\bar{Q}$ for the potential? At $T = 0$, the two coincide, but for finite $T$, the internal energy leads to a stronger binding. Arguments based on QED indicate that there the free energy is the relevant quantity [32–34]. But in the weak-coupling regime the binding structure is very different from the strong-coupling QCD world, since now the separation of the $Q$ and the $\bar{Q}$ requires both the work against the binding force and the energy to create the gluon dressing of the separated quarks. If both contribute to the binding, the $J/\psi$ will survive up to about $2 T_c$; the free energy alone lets it melt around $1.2 T_c$. – Another open issue is width of the quarkonium states, and related here, the imaginary part of the potential.

The solution will eventually come from direct lattice calculations, which are in fact in progress, thanks to the availability of ever more powerful computing facilities. They provide an integral transform of the desired in-medium quarkonium spectrum (the so-called correlator), and the inversion of this transform, given the precision of the calculations and probabilistic inversion technique (MEM: maximum entropy method), is not yet unambiguous. Present results seem to lie in the range of 1.5 - 2.0 $T_c$ for the dissociation of the $J/\psi(1S)$.

It should be emphasized that we need, in fact, the dissociation temperatures of all the quarkonium states, not just those for the ground states. The experimentally observed $J/\psi$ and $\Upsilon$ are only in part directly produced; almost half come from the decay of higher excited states. Since these states are very narrow, the decay occurs far outside the interaction region, and so the medium affects the passing excited states. As a result, one predicts sequential quarkonium suppression [35]: first the fractions from the excited states are suppressed, then eventually also the ground states. The result is a stepwise suppression pattern as function of the energy density or temperature, see Fig. 7.

On the experimental side, one also encounters a variety of complicating factors. Before the effect of any QGP on quarkonia can be determined, the role of more profane initial and final state effects has to be brought under control: shadowing/antishadowing, parton energy loss, nuclear and hadronic absorption, etc. The elimination of these began at the SPS by measuring $J/\psi$ suppression relative to the Drell-Yan dimuon background. Later, and at RHIC, this was replaced by combining $J/\psi p – p$ data with nuclear effects determined in $p/d – A$ collisions. At the LHC, this is also in progress. Nevertheless, one should keep in mind that all these schemes are in a sense “crutches”; the obvious and natural quantity to consider is the ratio of quarkonium production to that of open charm or bottom. In this ratio, most initial state effects will cancel out, and it addresses directly the question of whether the fraction of charm or bottom production going into the hidden sector is reduced or fully suppressed by the presence of the QGP [36].
Another interesting aspect has come up in the study of $J/\psi$ production. We have so far considered primary quarkonium production, i.e., a $c\bar{c}$ pair is produced in a specific nucleon-nucleon collision and subsequently, a certain small fraction combines to form a $J/\psi$, the remainder lead to open charm. If the medium would contain sufficiently many remaining unbound pairs, and if these would become part of the thermalized plasma, then at the hadronization point, a $c$ from one collision could statistically combine with a $\bar{c}$ from another collision to make a $J/\psi$ \cite{37,40}. Such secondary $J/\psi$ production would then result in an overall enhancement of the rate. Clearly, a necessary prerequisite is that there is a sufficiently large number of available charm/anticharm quarks, i.e., the nuclear collision energy has to be high enough. The prediction of such statistical combination is just the opposite of sequential suppression: while the latter has the production rate of hidden charm relative to open charm decrease or vanish in nuclear collisions, compared to $p-p$ interactions, statistical combination predicts an increase of this ratio.

A crucial input for the statistical combination mechanism is that indeed the mere statistical presence of a $c$ and $\bar{c}$ suffices to form a $J/\psi$. Additional dynamical conditions (sufficiently close approach, etc.), could strongly affect the predictions. On the other hand, if statistical combination is in effect, i.e., if the suppressed primary production is more than compensated by the statistical secondary process, then this would constitute evidence for a thermalization of even the heavy charm quarks. It would remove, however, the possibility of checking experimentally QCD predictions for charmonium dissociation.

This check would then have to be applied to bottomonium production, for which even at LHC energies statistical modifications seem excluded. And here on does indeed observe at $\sqrt{s} = 2.75$ TeV a suppression of the higher excited state relative to the $\Upsilon(1S)$ \cite{41}, as shown in Fig. 8. Perhaps a more detailed quantitative study of this suppression, both in amount and in onset thresholds, will allow a direct comparison to statistical QCD calculations.

## 8 Quenching of Jets: The QGP Density

Probing the density of the medium produced in the early stages of nuclear collisions is also a task which finds parallels in other fields of physics. A standard tool for this is the
The peaks in (a) correspond to the Υ(1S), Υ(2S) and Υ(3S), respectively [41].

Study of the attenuation of a flux of fast particles losing energy by scattering in the course of their passage through a medium.

In our case, we consider the production of jets in nucleus-nucleus collisions, consisting of one or more hadrons with very high momentum transverse to the collision axis. These jets are formed initially by a very energetic parton, quark or gluon, produced in the early hard collision stages and emitted in the transverse direction. Given a QGP formation time of about 1 fm, the nascent jet will pass through several fermi of hot deconfined matter before it escapes the interaction region and eventually hadronizes. How much energy it has lost when it finally emerges will tell us something about the density of the medium [42–45].

In particular, the density in a quark-gluon plasma is by an order of magnitude or more higher than that of a confined hadronic medium, and so the energy loss of a fast passing color charge is expected to be correspondingly higher as well. Let us consider this in more detail.

An electric charge, passing through matter containing other bound or unbound charges, loses energy by scattering. For charges of low incident energy $E$, the energy loss is largely due to ionization of the target matter. For sufficiently high energies, the incident charge scatters directly on the charges in matter and as a result radiates photons of average energy $\omega \sim E$. Per unit length of matter, the ‘radiative’ energy loss due to successive scatterings is thus proportional to the incident energy. This probabilistic picture of independent successive scatterings breaks down at very high incident energies [46–48]. The squared amplitude for $n$ scatterings now no longer factorizes into $n$ interactions; instead, there is destructive interference, which for a regular medium (crystal) leads to a complete cancellation of all photon emission except for the first and last of the $n$ photons. This Landau-Pomeranchuk-Migdal (LPM) effect greatly reduces the radiative energy loss. The medium produced in nuclear collisions is certainly not a regular crystal, so that here the cancellation becomes only partial.
Nevertheless, the rates become considerably modified. The incoherent radiative energy loss

$$\frac{-dE}{dz} \simeq \frac{3\alpha_s E}{\pi \lambda},$$

is through interference replaced by the coherent form

$$\frac{-dE}{dz} \simeq \frac{3\alpha_s}{\pi} \sqrt{\frac{\mu^2 E}{\lambda}}.$$

The former grows linearly with the energy of the passing charge and decreases as its mean free path increases, i.e., if there is less scattering. In the latter the screening mass \(\mu\) specifies in addition how much of the medium affects the jet and hence determines the amount of destructive interference. The crucial quantity is thus the transport coefficient \(\hat{q} = \mu^2/\lambda\); it specifies the overall energy loss of the jet, and it is much higher in a hot quark-gluon plasma than in a cool medium of confined constituents.

Estimates of the jet energy loss predicts for a QGP at \(T = 250\) MeV some 3 GeV/fm, while cold nuclear matter only gives about 0.2 GeV/fm [49] – more than an order of magnitude less. Such a difference in jet quenching has in fact been observed at the BNL Relativistic Heavy Ion Collider RHIC. In proton-proton collisions, a high transverse momentum particle is in general accompanied by another such particle flying in the opposite direction, to balance the overall transverse momentum ("back-to-back" jets). In \(d - Au\) collisions, the balancing jet has to traverse normal nuclear matter to "get out", and this is found to have rather little effect. In \(Au - Au\) interactions, on the other hand, the balancing jet has to pass through the produced hot QGP (if there is such a medium), and this should lead to strong suppression. In Fig. 9 it is seen that this is indeed observed – the “away-side” jet has essentially become fully quenched.

Figure 9: Azimuthal distribution of hadrons with transverse momenta above 5 GeV in \(p - p, d - Au\) and \(A - A\) collisions [50–52].

The interpretation of this result in terms of jet quenching is further supported by the observation that hard transverse photons are produced in nucleus-nucleus collisions at the rate predicted from proton-proton results scaled by the number of collisions; in contrast,
hard hadron production is reduced up to a factor five. The hadrons are quenched in the QGP, while the photons are not affected by a strongly interacting medium.

Numerous other features of jet production were studied at RHIC and described in terms of QCD-based models; for more details, see e.g. [53]. They all indicate that high energy nuclear collisions indeed produce a very dense and very strongly interacting medium. Hopefully the advent of further data from the LHC will eventually also lead to a quantitative comparison with \textit{ab initio} QCD calculations.

9 Horizons

The vast expanses of the universe, combined with its finite age, make it not so surprising that there are regions even today which can never have exchanged information with each other, which are causally not connected. Nevertheless the cosmic microwave background radiation observed from these regions is of the same universal temperature of some 2.7° Kelvin, up to better than one part in ten thousand. How could this arise, how can regions be in apparent equilibrium with each other, even though they cannot communicate? That is what cosmologists call their horizon problem, and they conclude that in classical Big Bang theory, there is no physical process which can make the temperature so uniform.

We want to close our analysis of strongly interacting matter by noting that such a horizon problem also occurs in the study of high energy collisions. It made its first appearance when it was found that the relative abundances of hadrons were essentially the same in $e^+e^-$ annihilation as in high energy nuclear collisions. In the latter, one can imagine some kind of thermalisation through multiple parton interactions, but in $e^+e^-$ annihilation both the number of partons and that of hadrons is so small that any kinetic equilibration is ruled out. Hagedorn concluded that the hadrons are simply “born in equilibrium”.

![Figure 10: The evolution of a high energy nucleus-nucleus collision](image)

The canonical view of the evolution of a high energy nuclear collision is illustrated in Fig. 10. The projectiles pass through each other, leaving behind an excited vacuum, which after a brief equilibration time $\tau_q$ forms a quark-gluon plasma. This subsequently, after a further time $\tau_h$, hadronises to form the observed secondaries. The evolution is generally assumed to be boost-invariant; this means that in each local rest frame, the same times $\tau_q$ and $\tau_h$ govern the development. As a result, the QGP bubble at mid-rapidity ($y = 0$) and that at some larger rapidity are not in causal contact; if the corresponding hadronisation
times are the same, it is not because the two systems “thermalised”, established a mutual equilibrium. They don’t even know of each other’s existence.

There seems to be only one way out of this dilemma: The hadron production is due to a local stochastic process. In other words, at each space-time point, hadronisation occurs through randomly choosing one out of the entire set of allowed states – allowed by the boundary conditions of the problem. The set of all the hadrons obtained in this way looks “thermal”, and it is indeed the same set as would have been obtained through kinematic thermalisation starting from a given non-equilibrium configuration. This implies a form of stochastic equivalence principle. Just as one cannot tell, by Einstein equivalence, if one is in a gravitational field or in a rocket experiencing the same acceleration, so one cannot tell if a given ensemble of states was obtained through kinematic equilibration or through throwing dice. Once it’s “thermal”, it no longer remembers how it got there.

A model of this type was proposed five years ago [25], based on the stochastic radiation seen by an accelerating observer [54]. The “Unruh” temperature obtained in this way is specified by the acceleration, which in turn is determined by the string tension. And such a scheme does in fact reproduce the universal hadrosynthesis pattern observed in high energy collisions [55]. Further work along these lines is in progress.

All in all, we can thus conclude that the little bang of nuclear collisions perhaps has more features in common with its big brother than we had bargained for. But it does leave us with many interesting problems still to be solved. For more details, see Ref. [56], where also references are given in a more systematic fashion; here I have concentrated on some indicative works only.

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