\textbf{Abstract}

We introduce conformal coupling of the Standard Model Higgs field to gravity and discuss the subsequent modification of $R^2$-inflation. The main observation is a lower temperature of reheating which happens mostly through scalaron decays into gluons due to the conformal (trace) anomaly. This modifies all predictions of the original $R^2$-inflation. To the next-to-leading order in slow roll parameters we calculate amplitudes and indices of scalar and tensor perturbations produced at inflation. The results are compared to the next-to-leading order predictions of $R^2$-inflation with minimally coupled Higgs field and of Higgs-inflation. We discuss additional features in gravity wave signal that may help to distinguish the proposed variant of $R^2$-inflation. Remarkably, the features are expected in the region available for study at future experiments like BBO and DECIGO. Finally, we check that (meta)stability of electroweak vacuum in the cosmological model is consistent with recent results of searches for the Higgs boson at LHC.

1 Introduction and Summary

The Starobinsky model of inflation [1] is the first, yet realistic example of new physics capable of solving major problems of the Hot Big Bang theory, see e.g. [2]. It exploits dynamics of gravity sector, modified by a quadratic in scalar curvature term added to the gravity action. The attractive feature of the model is that one and the same force–gravity–is responsible for both inflation and subsequent reheating of the early Universe. Such \textit{minimality} is of some interest, given the absence of any direct evidence of relevant new physics in laboratory and accelerator experiments.
In this paper we consider the Starobinsky inflation with matter sector described by the Standard Model of particle physics (SM) which scalar sector is slightly modified. Namely, we add a conformal coupling of the SM Higgs field to gravity. This term leaves intact the low energy phenomenology of the SM, but impacts on the history of the early Universe. Indeed, we found that with the Higgs boson becoming conformal at high energies, reheating of the Universe takes place later and occurs via gluon production due to the conformal (trace) anomaly. The idea of conformal anomaly being responsible for reheating was discussed in literature, e.g., [3, 4]. Here it is natural consequence of the conformal symmetry in our model.

Lower reheating temperature implies longer matter dominated stage between inflation and reheating. This modifies all predictions for power spectra of scalar and tensor perturbations generated at inflation. Likewise, this modifies predictions for gravity wave signals expected from nonlinear structure dynamics at post-inflationary stage. These are special signals in gravity waves given the long-lasting post-inflationary matter dominated stage. Remarkably, the signals fall in the region expected to be reached by proposed future experiments like BBO [5] and DECIGO [6] on searches for gravity waves. These signals have been proposed [7] as signatures of $R^2$-inflation. The same is true for our variant with conformal Higgs, where the features in gravity wave spectrum are expected at different frequencies, which allows to test the model. Finally, the non-minimal coupling provides with additional term in the Higgs effective potential, which becomes important at large scalar curvature. This can change the answer to the question: in which vacuum does the Higgs field fall in the expanding Universe, given the value of the Higgs self-coupling (or the Higgs boson mass)?

We address all these issues below. The model is presented in Sec. 2, and reheating is studied in Sec. 3. Predictions for amplitudes and spectral indices of scalar and tensor perturbations are obtained in Sec. 4. The calculations are performed to the next-to-leading order in slow roll parameters and the results are compared to similar predictions obtained there for the original Starobinsky model and for Higgs-inflation [8]. All the three models exhibit the same inflationary dynamics, so the only difference is in reheating temperature, which helps to distinguish the model predictions. In Sec. 5 we discuss the gravity wave signals expected in the model: one comes from inflation, others from nonlinear evolution of inhomogeneities at post-inflationary matter-dominated stage. Sec. 6 is devoted to analysis of stability of electroweak vacuum. There we estimate the lower bound on the Higgs boson mass, corresponding to the viable cosmological evolution in the model and find it to be consistent with recent results of LHC.
The Starobinsky model of inflation is described in the Jordan frame by the following action [1, 9],

\[ S = -\frac{M_P^2}{2} \int \sqrt{-g} \, d^4x \left( R - \frac{R^2}{6\mu^2} \right) + S_{\text{matter}} . \]  

(1)

Here the reduced Planck mass is \( M_P = M_{\text{Pl}}/\sqrt{8\pi} = 2.4 \times 10^{18} \text{ GeV} \) and \( S_{\text{matter}} \) in our case refers to the SM action. At large values of scalar curvature \( R \) model (1) allows for an inflationary stage in a slow roll regime: a scalar degree of freedom (dubbed scalaron), emerging when \( R^2 \)-term is added to the gravity action, plays inflaton. Scalaron quantum fluctuations evolving at inflationary stage freeze out with amplitude \( \sim \mu \) when exit the horizon. Later they give rise to matter perturbations, which amplitude is fixed by the global fit to cosmological data, consequently the value of parameter \( \mu \) equals [9]

\[ \mu = 1.3 \times 10^{-5} M_P = 3.1 \times 10^{13} \text{ GeV} . \]  

(2)

After inflation the Universe expansion is driven by oscillating massive scalaron field responsible for the effective matter-dominated post-inflationary stage. Later, scalaron oscillations decay into the SM Higgs bosons due to gravity interactions and a hot stage in the Universe starts with temperature [10]

\[ T_{\text{reh}}^{R^2} = 3.1 \times 10^9 \text{ GeV} . \]  

(3)

In this paper we introduce in model (1) conformal coupling of the SM Higgs field to gravity. Then the part of action with the Higgs doublet \( \mathcal{H} \) reads (we omit irrelevant for our study Yukawa terms):

\[ S_H = \int \sqrt{-g} \, d^4x \left( \frac{1}{6} R \mathcal{H}^\dagger \mathcal{H} + D^\mu \mathcal{H}^\dagger D_\mu \mathcal{H} - \frac{\lambda}{4} (\mathcal{H}^\dagger \mathcal{H} - v^2)^2 \right) . \]  

(4)

Note that a non-minimal coupling term is generally required by renormalizability of the model in the curved space-time, and the particular value of non-minimal coupling is stable with respect to perturbative quantum corrections. After the conformal (Weyl) transformation to the Einstein frame

\[ g_{\mu\nu} \rightarrow e^{\sqrt{2/3} \phi/M_P} g_{\mu\nu} \]  

(5)

action (1) takes form [11]

\[ S = \int \sqrt{-g} \, d^4x \left( -\frac{M_P^2}{2} R + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right) + \tilde{S}_{\text{matter}} , \]

3
Here $\tilde{S}_{\text{matter}}$ is the conformally transformed action of matter (SM fields). Any conformal non-invariance in the matter sector produces interaction between scalaron $\phi$ and SM particles.

3 Reheating via the conformal (gauge) anomaly

In our model (1), (4) the SM Higgs is conformal at large values of the field and hence it decouples from scalaron. Then the strongest relevant coupling between scalaron and SM fields comes from the conformal (gauge) anomaly which takes care of reheating in the model. Indeed, the conformal transformation (5) yields scalaron coupling to the trace of energy-momentum tensor of matter fields $T_\mu^\mu$:

$$S_{\text{int}} = \int \sqrt{-g} \, d^4x \frac{1}{\sqrt{6}} \frac{\phi}{M_P} T_\mu^\mu.$$

The relevant terms in (6) are due to conformal (gauge) anomaly, see e.g. [12]:

$$T_\mu^\mu = \frac{\beta(\alpha)}{4\alpha} (F_\mu^{\alpha \nu})^2, \quad \beta(\alpha) = \frac{b_\alpha \alpha^2}{2\pi}.$$

Here $F_\mu^{\alpha \nu}$ stand for gauge field tensors and $b_\alpha$ are the first coefficients of $\beta$-functions for corresponding gauge coupling constants $\alpha$; for the SM couplings of $U(1)_Y$, $SU(2)_W$ and $SU(3)_c$ gauge interactions these coefficients are $\frac{41}{6}$, $-\frac{19}{6}$ and $-7$, respectively.

As a result, the scalaron decay rate into the gauge bosons is

$$\Gamma_{\phi \rightarrow 2 \text{bosons}} = \frac{b_\alpha^2 \alpha^2 N_{\text{adj}}}{768 \pi^3} \frac{\mu^3}{M_P^2},$$

where $N_{\text{adj}} = 1, 3, 8$ for $U(1)_Y$, $SU(2)_W$ and $SU(3)_c$ gauge interactions, correspondingly. Values of $\alpha$ must be taken at the scale of $\mu/2$ and we obtain them by making use of the numerical code [13] operating with the SM 3-loop $\beta$-functions [14].

Scalaron decays mostly into gluons, which immediately rescatter producing all the SM particles\(^1\). The scalaron total decay rate $\Gamma_\phi$ in our model with conformal Higgs field (1), (4) is about 140 times lower than that in the model with the Higgs field minimally coupled to SM fields. Any conformal non-invariance in the matter sector produces interaction between scalaron $\phi$ and SM particles.

\(^1\)Amplitudes of scalaron direct decays to pair of SM massive fermions are suppressed by corresponding Yukawa coupling constants and ratio of the Higgs field $\sim H$, see Sec. 6, to the Planck scale. Rates of three and four body scalaron decays to SM particles are strongly suppressed by coupling constants and the phase space volume.
gravity (1). For completeness, let us note that if the Higgs non-minimal coupling to gravity parameterised as \( L_{\text{int}} = \xi R \mathcal{H}^\dagger \mathcal{H} \) we obtain for the total scalaron decay width:

\[
\Gamma_\phi = \frac{\mu^3}{192\pi M_P^2} \left[ \frac{\Sigma b_i^2 \alpha_i^2 N_{\text{adj}}^i}{4\pi^2} + 4(1 - 6\xi)^2 \right]
\]

(9)

where the sum in brackets is taken over the SM gauge groups. The decay to gauge fields dominates for \(|\xi - 1/6| < 0.007\).

We define the reheating temperature of the Universe after inflation as temperature at the moment of equality between the energy densities of scalaron condensate and relativistic matter \([10]\). Then numerically

\[
T_{\text{reh}} = 1.1 \times g_*^{-1/4}(T_{\text{reh}}) \sqrt{\Gamma_\phi M_P} = 1.4 \times 10^8 \text{ GeV},
\]

(10)

where effective number of degrees of freedom in the plasma of SM particles is \( g_* (T_{\text{reh}}) = 106.75 \). For all other values of non-minimal coupling constant \( \xi \) in front of the first term in Eq. (4) but 1/6 the reheating temperature is higher than (10). In particular, when \( \xi \) drops to zero the reheating temperature approaches \( 3.1 \times 10^9 \text{ GeV} \) \([10]\) as \( T_{\text{reh}} \propto (1 - 6\xi) \) provided Eq. (9).

4 Parameters of scalar and tensor perturbations produced at the inflationary stage

In both variants of \( R^2 \)-inflation (with ordinary and with conformal Higgs field) the scalaron potential at inflation is the same. Therefore, the only difference in predictions for parameters of scalar and tensor perturbations generated at inflation is due to different number of e-foldings because of different reheating temperature in the models (for detailed explanation see e.g. \([7]\)). Since parameters of perturbation power spectra depend on the reheating temperature very mildly (logarithmically) and the latter differs in the two models only by factor of 20, cf. Eqs. (3) and (10), they must be evaluated to the next-to-leading order in slow-roll parameters.

The procedure is described e.g. in Refs. \([15, 16]\). For the leading order estimates one usually exploits the number \( N_e \) of e-foldings passed after the perturbation of a given conformal moment \( k \) exited the horizon \([2, 17]\),

\[
N_e = \log \left( \frac{a(k)}{a_e} \right) \approx 53.27 - \frac{1}{3} \log \left( \frac{1.4 \times 10^8 \text{ GeV}}{T_{\text{reh}}} \right).
\]

(11)
Here $a(k)$ and $a_e$ refer to the scale factor at the moment of horizon exit and at the end of inflation, respectively. The value of $k$ is chosen to match the WMAP pivot scale $k/a_0 = 0.002$ Mpc$^{-1}$, where $a_0$ is the present scale factor.

For the next-to-leading order calculations a more convenient measure of the moment of horizon exit is [16]

$$\tilde{N}_e = \log \left( \frac{a(k) H(k)}{a_e H_e} \right) ,$$

(12)

where $H(k)$ and $H_e$ stand for the Hubble parameter at the exit and at the end of inflation. The latter is defined as the moment when the Universe stops to expand with acceleration, i.e. when $d^2 a/dt^2 = 0$. Then one obtains

$$\tilde{N}_e = 53.80 - \frac{1}{3} \log \left( \frac{1.4 \times 10^8 \text{GeV}}{T_{\text{reh}}} \right) .$$

(13)

Quantity (12) is related to the small slow roll parameters as follows [16]

$$\tilde{N}_e \approx -2 \sqrt{\pi} \int_{\phi_k}^{\phi_e} \frac{d\phi}{\sqrt{\epsilon(\phi)}} \left( 1 - \frac{1}{3} \epsilon(\phi) - \frac{1}{3} \eta(\phi) \right) ,$$

(14)

where $\phi_k$ and $\phi_e$ refer to the moment of horizon exit and the end of inflation, correspondingly. Introducing variable $\chi = \exp(\sqrt{2/3} \phi/M_P)$ we write down the slow roll parameters in $R^2$-model (see e.g. [2]):

$$\epsilon \equiv \frac{M_P^2}{2} \left( \frac{V'}{V} \right)^2 = \frac{4}{3} \left( \frac{1}{\chi - 1} \right)^2 ,$$

(15)

$$\eta \equiv M_P^2 \frac{V''}{V} = \frac{4}{3} \frac{2 - \chi}{(\chi - 1)^2} = \epsilon - \frac{2}{\sqrt{3}} \epsilon ,$$

(16)

$$\zeta^2 \equiv M_P^4 \frac{V'''V''}{V^2} = \frac{4}{3} \epsilon \left( 1 - \frac{3}{2} \sqrt{3} \epsilon \right) ,$$

(17)

where prime denotes derivative with respect to scalaron field $\phi$. Plugging (15), (16) into (14) and integrating then Eq. (14) one extracts slow roll parameters as functions of

$$N \equiv \frac{4}{3} \tilde{N}_e + \chi_e - 1 ,$$

namely:

$$\epsilon = \frac{4}{3} \frac{1}{N^2} + O \left( \frac{\log(N)}{N^3} \right) , \; \eta = -\frac{4}{3} \frac{1}{N} + \frac{4}{3} \frac{1}{N^2} + O \left( \frac{\log(N)}{N^3} \right) , \; \zeta = \frac{4}{3} \frac{1}{N} + O \left( \frac{\log(N)}{N^3} \right) .$$

(18)

Numerically, from the Friedman equation at the end of inflation, $\chi_e \approx 4.6$, and the relevant slow roll parameters are reasonably small, $\epsilon_e \approx 0.10$, $\eta_e \approx 0.27$, which justifies using of approximate relation (14) (see Ref. [16] for the exact relation).
Tilts of the power spectra of scalar and tensor perturbations, \(1-n_s\) and \(n_T\), and tensor-to-scalar ratio \(r\) to the next-to-leading order in the slow roll parameters are given by \([15, 16]\)

\[
1 - n_s = 6 \epsilon - 2 \eta - \frac{2}{3} \eta^2 + 0.374 \zeta^2 = \frac{8}{3} \frac{1}{N} + \frac{4.813}{N^2} + O\left(\frac{\log(N)}{N^3}\right),
\]

(19)

\[
r = 16 \epsilon = \frac{64}{3} \frac{1}{N^2} + O\left(\frac{\log(N)}{N^3}\right),
\]

(20)

\[
n_T = -2 \epsilon = -\frac{8}{3} \frac{1}{N^2} + O\left(\frac{\log(N)}{N^3}\right).
\]

(21)

Finally, substituting (10), (13) into (19)-(21) we obtain predictions for the cosmological parameters. They are presented in Table 1, together with predictions for two other inflationary models exhibiting the same inflaton potential at inflationary stage: \(R^2\)-model with the Higgs field minimally coupled to gravity (1) and the so-called Higgs-inflation \([8]\). For these latter two models we refine the results of Ref. \([7]\) for \(n_s\) derived there to the leading order in slow roll parameters. The (absolute) error is expected to be of about \(1/N^2 \simeq 10^{-3}\). Indeed, our estimates of \(n_s\) in these models deviate from those in Ref. \([7]\) by this amount.

For all the three models the interesting values of \(n_s\) and \(r\) are well inside the region preferred by combined analysis of present cosmological data \([18]\). Nominally, the (absolute) error of the next-to-leading approximation \(\log(N)/N^3 = 10^{-5}\) is enough to ensure that it is possible to distinguish all three models by measuring the fourth digit in the values \(n_s\) and \(r\), parameter \(n_T\) seems less promising. Provided the lower reheating temperature, predictions in our model deviate slightly stronger from those in the Higgs-inflation, as compared to the predictions in \(R^2\)-model with the Higgs field minimally coupled to gravity. Thus, future experiments, like CMBPol \([19]\) with accuracy of \(10^{-3}\) in \(r\) and \(0.0016\) in \(n_s\), have better chance to distinguish the Higgs-inflation from our variant of \(R^2\), if any signal would point at the right ballpark.

| Model                  | \(T_{\text{reh}}, \text{GeV}\) | \(n_s\)   | \(r\)         | \(n_T\)          |
|------------------------|-------------------------------|---------|-------------|-----------------|
| \(R^2\) with conformal Higgs | \(1.4 \times 10^8\)          | 0.9638  | 0.0038      | -0.00047        |
| \(R^2\)                | \(3.1 \times 10^9 [10]\)     | 0.9644  | 0.0036      | -0.00045        |
| Higgs-inflation        | \(6 \times 10^{13} [17]\)   | 0.9664  | 0.0032      | -0.00040        |

Table 1: Next-to-leading order predictions for spectral parameters.
5 Gravity wave signals from inflation and from scalaron clumps

In this model there are two sources of gravity waves: metric fluctuations at inflationary stage and scalaron clumps at late post-inflationary stage. Let us discuss them in order.

The first source is inherent in any inflationary model. In the model under discussion it gives rise to tensor perturbations (gravity waves) with almost flat power spectrum after inflation: a deviation from flatness is characterized by index $n_T$ presented in Table 1. The total power in tensor perturbations is about 0.4% of that in scalar perturbations, see values of parameter $r$ in Table 1. In the expanding Universe the perturbations, which length became smaller than horizon, start to evolve. Energy density of subhorizon tensor modes decreases with scale factor as radiation energy density, i.e. as $1/a^4$. Hence at post-inflationary stage when the Universe is dominated by oscillating scalaron with energy density scaling as $1/a^3$, the relative contribution of subhorizon gravity waves to total energy density drops as $1/a$ up to reheating, and later at radiation domination remains constant. Hence one expects a knee-like feature in the gravity wave spectrum to be observed at a frequency $f_\ast$ determined by the horizon size at reheating $H_{\text{reh}}$. The latter is related to the reheating temperature through the Friedman equation, so that

$$H_{\text{reh}} = \frac{\pi}{\sqrt{90}} \frac{g_*(T_{\text{reh}}) T_{\text{reh}}^2}{M_P}.$$  \hfill (22)

The relative contribution of tensor modes of conformal momenta

$$k > k_{\text{reh}} = \frac{H_{\text{reh}}}{a_{\text{reh}}}$$

are suppressed. At present $k_{\text{reh}}$ corresponds to the physical momentum

$$p_\ast = \frac{k_{\text{reh}}}{a_0} = \frac{a_{\text{reh}}}{a_0} H_{\text{reh}}.$$  \hfill (23)

By making use of entropy conservation we obtain

$$\frac{a_{\text{reh}}}{a_0} = \frac{T_0}{T_{\text{reh}}} \left( \frac{g_*(T_0)}{g_*(T_{\text{reh}})} \right)^{1/3}$$  \hfill (24)

with effective degrees of freedom at present $g_*(T_0) = 3.91$ and $g_*(T_{\text{reh}}) = 106.75$ [2]. Then substituting (24) and (22) into (23) we get for the present frequency, where the knee in gravity wave spectrum is expected, see Fig. 1,
Figure 1: Energy density in gravity waves (in units of the present day critical density) $\Omega_{gw}$ as a function of frequency and the projected sensitivities of next-generation gravitational wave detectors: LIGO [20], BBO [5], DECIGO [6]. The picture shows the gravity wave signal from inflation (solid line) and from structure evaporation at reheating (red star); results are presented for three particular values of the nonminimal coupling $\xi$.

The second source of gravity waves is scalaron inhomogeneities: subhorizon modes at intermediate matter dominated stage grow proportionally to scale factor and have enough time to enter nonlinear regime before reheating [7]. Then, gravity waves can be produced at formation of scalaron clumps, at their subsequent merging and at final evaporation, see e.g. [21, 22]. The highest amplitude, hence the best chance to be observed, is expected from the latter process. Evaporation is the scalaron decays into relativistic SM particles. It is out-of-equilibrium process yielding a non-vanishing transverse-traceless part of energy-momentum tensor feeding gravity waves. The typical frequency at production is about $H_{\text{reh}}$ [21], which gets redshifted and presently coincides with $f_\star$ (25). The signal amplitude does not depend on the reheating temperature [21]. The estimate in [22] gives for a relative contribution of

$$f_\star = \frac{p_\star}{2\pi} = 2.8 \text{ Hz} \left( \frac{T_{\text{reh}}}{1.4 \times 10^8 \text{ GeV}} \right).$$  

(25)
the gravity waves to the present total energy density $\Omega_{gw} \sim 4 \times 10^{-13} \varepsilon$, where $\varepsilon < 1$ is an efficiency factor which represents a measure of the asphericity of structure evaporation. The possible signal is shown in Fig. 1.

Similar signal is expected [7] in $R^2$-model with minimally coupled to gravity Higgs field. However, the typical frequency is by a factor $T_{reh}^{R^2}/T_{reh}$ higher. The same conclusion holds for position of the knee in gravity wave spectrum from inflation. One observes in Fig. 1 that those signals are either at the board of or out of reach of proposed experiments on searches for gravity waves. The signals in our model discussed above are right in the region available for investigation by future detectors like BBO [5] and DECIGO [6], see Fig. 1. This allows for independent test of our model: signatures in gravity waves pin down the value of reheating temperature.

6 Electroweak vacuum of the Higgs potential: lower limits on the Higgs boson mass

Since we have modified the Higgs sector by introducing conformal coupling to gravity, the stability of the electroweak (EW) vacuum and whether the Universe ends up in it, given the cosmological evolution suggested in this paper, have to be investigated. The object under study is the Higgs effective potential, which in the unitary gauge $\mathcal{H}^T = (0, (h + v)/\sqrt{2})$ at large $h \gg v = 246.2$ GeV reads

$$V(h) = \frac{\lambda(h)}{4} h^4 - \frac{1}{12} R h^2 .$$

Here $\lambda(h)$ solves the renormalization group equation when $h$ is replaced with renormalization energy scale $[23, 24]$. At large $h$ selfcoupling $\lambda(h)$ may become negative, hence EW vacuum is metastable. Coefficient 1/12 in front of the second term in (26) is renorminvariant (neglecting graviton loops) $[25]$. For homogeneous, isotropic and flat Universe

$$R = -12 H^2 - 6 \dot{H} ,$$

where dot denotes the time derivative. Thus, in the hot Universe: at radiation domination $R = 0$, at matter domination $R = -3 H^2 < 0$, at present (dark energy domination), $R < 0$. Hence, the conformal coupling only enlarges the area of stability of the EW (i.e. widens the range of the allowed Higgs boson mass related to the selfcoupling constant as $M_h \approx \sqrt{2 \lambda v}$).

Actually, metastability of the Higgs potential doesn’t mean the theory is invalid. The weaker condition to require is the lifetime of the EW vacuum (with respect to tunneling
and thermal decay) exceeds the Universe lifetime. For the usual case of minimally coupled Higgs the bound coming from tunneling is \( M_h \gtrsim 111 \text{ GeV} \) [26, 27] which is below the direct lower bound by ATLAS (115.5 GeV at 95% CL [28]). Since in our model the additional term \( R \mathcal{H}^\dagger \mathcal{H}/6 \) stabilizes the potential, the condition from tunneling is fulfilled.

The bound coming from the thermal decay depends on the reheating temperature [29, 27]. The numerical calculation (3-loop \( \beta \)-functions [14] and \( \mathcal{O}(\alpha_s^3), \mathcal{O}(\alpha \alpha_s) \) corrections in matching of pole and running masses included [13]) yields a lower limit which is even below that from tunneling:

\[
M_{\text{therm}} = \left[ 109.76 + \frac{M_t - 173.2 \text{ GeV}}{0.9 \text{ GeV}} \times 1.9 - \frac{\alpha_s - 0.1184}{0.0007} \times 0.14 \right] \text{ GeV} . \tag{28}
\]

Hence, given the direct limit from ATLAS, the problems with EW vacuum stability in our model are neither at hot stage of the Universe evolution, nor at present.

In our model the strongest lower limit on the Higgs boson mass comes from analysis of the Higgs field evolution from inflation to reheating. There at short periods of time \( \Delta t \sim 1/\mu \) the scalar curvature is positive because of oscillating scalaron (27),

\[
R \simeq -\frac{4}{3 \ell^2} (1 - 3 \cos (2\mu t)) . \tag{29}
\]

Then the Higgs potential (26) gets negative mass squared of order \( H^2 \), which may stimulate escape to large values of fields and hence to wrong vacuum at late times. Let us analyze this process.

Even if initially (classically) at origin, the Higgs field takes value \( h \sim H \) by the end of inflation, because of its quantum fluctuations, which get amplified when became superhorizon. This estimate may be refined, e.g. by making use of the stochastic approach, see e.g. [30, 31]. The comoving probability \( P_c(h, t) \) for the field to take value \( h \) at moment \( t \) obeys the Fokker-Planck equation

\[
\frac{\partial P_c}{\partial t} = \frac{\partial}{\partial h} \left[ \frac{H^3}{8 \pi^2} \frac{\partial P_c}{\partial h} + \frac{V'(h)}{3H} P_c \right] . \tag{30}
\]

One can integrate it over \( h \) and introducing the Higgs correlators

\[
\langle h^2 \rangle = \int h^2 P_c(h, t) \, dh , \quad \langle h V'(h) \rangle = \int h V'(h) \, P_c(h, t) \, dh ,
\]

cast it in the following form

\[
\frac{d}{dt} \langle h^2 \rangle = \frac{H^3}{4 \pi^2} - \frac{2}{3H} \langle h V'(h) \rangle . \tag{31}
\]
Neglecting the \(h\)-dependence of \(\lambda\) and treating perturbations as Gaussian quantities (\(\lambda\) is reasonably small) we simplify \(\langle hV'(h)\rangle \simeq 3\lambda\langle h^2\rangle^2 + 2H^2\langle h^2\rangle\) and finally arrive at

\[
\frac{d}{dt}\langle h^2 \rangle = \frac{H^3}{4\pi^2} - \frac{2\lambda}{H}\langle h^2 \rangle^2 - \frac{4H}{3}\langle h^2 \rangle.
\]  

(32)

Fluctuation \(\langle h^2 \rangle\) grows with time due to the first term and reaches maximum value when the r.h.s. of Eq. (32) is zero, that is

\[
\sqrt{\langle h^2 \rangle}_{\text{max}} = \frac{\sqrt{3}}{4\pi} H.
\]  

(33)

After inflation the Higgs field starts from (33) and evolves according to the classical equation of motion,

\[
\ddot{h} + 3H \dot{h} + \left(-\frac{1}{6} R + \lambda(h) h^2\right) h = 0,
\]  

(34)

where \(R(t)\) is given by Eq.(29). Given Eq.(29) the last term in parentheses in (34) is negligible at small \(t\), and the field falls from initial value (33) as \(h \propto t^{-1/3} \propto 1/\sqrt{a}\). However at some moment the potential starts to dominate (\(\lambda h^2 \propto t^{-2/3}\) falls slower than \(R \propto 2/9t^2\) and if the corresponding value of \(h\) is above the maximum of the effective potential (26) then it rolls down to wrong minimum (towards large \(h\)). Numerical solution with \(\lambda(h)\), evaluated by using 3-loop \(\beta\)-functions [14] and \(O(\alpha_s^2), O(\alpha\alpha_s)\) corrections in matching of pole and running masses [13], reveals that the Higgs field remains in small value region (and hence later evolves to the EW vacuum) provided the Higgs boson mass is above the following critical value

\[
M_{\text{crit}} = \left[126.2 + \frac{M_t - 173.2 \text{ GeV}}{0.9 \text{ GeV}} \times 1.55 - \frac{\alpha_s - 0.1184}{0.0007} \times 0.3\right] \text{ GeV}.
\]  

(35)

Uncertainties of estimate (35) are associated with errors in extraction of SM parameters \(M_t\) and \(\alpha_s\) from experimental data (about 2 GeV at 65% CL) and unknown higher-order QCD-corrections to the effective potential (26) (about 1 GeV) [13]. Given these numbers, we conclude that obtained limit (35) does not contradict to the recent observation of the SM Higgs-like signals at LHC

\[
\text{ATLAS [33]} : \quad M_h = 126.0 \pm 0.4\text{(stat.)} \pm 0.4\text{(syst.) GeV}, \quad (36)
\]

\[
\text{CMS [32]} : \quad M_h = 125.3 \pm 0.4\text{(stat.)} \pm 0.5\text{(syst.) GeV}. \quad (37)
\]

\(^2\)In fact the resulting lower limit on the Higgs boson mass (35) is (almost) insensitive to the numerical coefficient of order one in Eq.(33) in front of \(H\).
Hitherto we considered after inflation only classical evolution of the Higgs field, so a question about its quantum tunneling arises. We have addressed it adopting the usual instanton approach \[35\]. The initial state for tunneling is a classically evolving Higgs field. Its rate is of the order of the Universe expansion rate determined by the Hubble parameter. In the interesting case of our Universe the tunneling rate is (much) smaller than the expansion rate. Thus we treat the initial state \( h_{\text{in}} \) as a stationary state, which implies that at \( h_{\text{in}} \) the Higgs effective potential is quite flat, almost reaching the extremum (minimum), \( V'(h_{\text{in}}) \approx 0 \).

Then we approximate the Higgs effective potential by a polynomial of the fourth degree providing the same positions of the minimum \((h_{\text{in}}, V(h_{\text{in}}))\) and maximum \((h_{\text{max}}, V(h_{\text{max}}))\) as the Higgs effective potential at 3-loop level we used. So the height of the potential barrier remains the same and it’s width becomes smaller than for the real potential. We expect the tunneling rate for the approximate potential to be less than that in the real case.

We found the approximate instanton solution sewing together polynomial solution inside the new-phase bubble of radius \( R_b \) with the solution of the linear equation outside (neglecting the subdominant in this region Higgs selfcoupling) and it’s Euclidean action \( S_E \). The tunneling probability per unit time per unit volume is given by \[36\]

\[
\frac{\Gamma}{\cal V} = D \frac{{S_E}^2}{4 \pi^2} e^{-S_E},
\]

where dimensionful parameter \( D \) comes from the scalar determinant. It is determined by the size of the tunneling configuration, so we adopt \( D \sim R_b^{-4} \) as an order-of-magnitude estimate. Then we scanned over the Higgs mass with step 0.1 GeV starting from the central value of Eq. (35) and using the central values of top mass and \( \alpha_s \). For each Higgs mass value we calculated numerically the tunneling rate as a function of initial time \( t_{\text{in}} \) (or Hubble parameter \( H_{\text{in}} \)) referring to \( h_{\text{in}} \). The initial state \( h_{\text{in}} = h(t_{\text{in}}) \) was obtained by solving the classical equation of motion (34). Requiring for the tunneling rate in a horizon volume to be always (much) smaller than the Hubble rate we found the smallest critical mass of the Higgs boson to be\(^3\)

\[
M_{\text{crit}} = 126.6 \text{ GeV},
\]

that shifts the estimate (35) up by 0.4 GeV. With the SM Higgs boson mass above the estimate (39) the model is safe from tunneling to a wrong minima in the early Universe.

---

\(^3\)The result is almost insensitive to the numerical coefficient in the estimate \( D \sim R_b^{-4} \) used in (38). The tunneling rate changes with the Higgs mass mostly because of exponential factor: at critical mass (39) the value of \( S_E \) jumps by a factor of five.
We conclude, that being (minimally) extended with free scalar [34] or fermion [10] to serve as the dark matter, and with sterile neutrinos [10] to generate baryon asymmetry of the Universe and explain neutrino oscillations, the model we discussed becomes a phenomenologically complete, yet minimal, viable and testable model of particle physics.

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