Flow rate control for constant film thickness of hydrostatic bearing system

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Abstract. Restriction or constant flow is utilized to compensate recess pressures of a hydrostatic bearing which can be highest load capacity than any other type bearing, but not stiffness. Like to other type bearings supporting the worktable that is displaced continuously with changing load. This study proposes a control scheme with both feedbacks of flow rate and film thickness and measurements of temperature and pressure for constant thickness of hydrostatic film. The control algorithm of this scheme results in changing pump speed by using PID controller. As long as film thickness is controlled to be constant, worktable can be kept stable without steady-state displacement when load is changing. The steady-state accuracy in few sub-microns can be achieved by PID controller using the algorithm of this study, which have been verified by numerical simulation and measurement in experiments.

1. Introduction
A hydrostatic bearing can be heavier load capacity, lower friction coefficient, higher dynamic stability, and higher capacity to resist vibration than other type bearings. Also, the hydrostatic bearings have longer life than hard guideways, rolling bearings and hydro-dynamic bearings, moreover, zero torque starting ability and higher accuracy than rolling bearings. However, there are still innate advantages of hydrostatic bearings have not been developed. Thus, the hydrostatic bearings like other type bearings have the same drawbacks such as variations of the film thickness due to load changing or due to the variation of lubricant viscosity as temperature changing, and the tilt of the worktable or spindle tilt due to non-uniform load.

In past, many researchers have studied the principle, design data, and application of hydrostatic bearings, Rippel [1], Bassani [2, 3], and Rowe [4] summarized and integrated them into classical literatures. Also, Kang et al. [5-8] have inquired a series of investigations in static characteristics of open-type and closed-type bearings which regulate recess pressures by constant compensation or variable restrictor or self-compensation. Their summary is that compensation using membrane type restrictors can achieve higher stiffness of hydrostatic bearing than other type compensations. Moreover, infinite stiffness can be achieved at a certain pressure ratio when a specific compliance of active element is used, but negative stiffness occurs immediately when the compliance exceeds this specific value. Additionally, stiffness decreased rapidly as pressure ratio changing or the compliance deviate this specific value from a small value. In practice, this specific compliance is difficult to be obtained since the membrane must be designed too soft to sustain required pressure [9-11]. Several researches studied recess and pad shapes of hydrostatic bearing such as Sharma et al. [12], however,
the optimization of shape design has no remarkable effect to upgrade stiffness. Hence, film thickness control should be effective and easily realizable for upgrading stiffness.

Rare researches have addressed in film control for constant thickness till now. Early, Garg et al. [13] survey the design and compensation of hybrid and hydrostatic bearing. They didn’t mention active control of constant film thickness. Lately, few researches [14-16] conceived, however, they didn’t present control scheme by using the relationships among flow rate, recess pressure, viscosity, and film thickness. In their studies, only one feedback of flow rate and film thickness is used, so they required reference model of adaptive control or neural network to establish the compensation algorithm of flow rate and displacement. Thus their results show that the variation of film thickness is very large as load changing. Recently, Liu et al. [17] reviewed articles about hydrostatic bearings since 1990, they cited only one of servo control according to dynamic model of hydrostatic journal bearing but not for film thickness control [18].

Therefore, this paper will give the control scheme of constant film thickness for high accuracy and high stiffness of hydrostatic bearing. The control algorithm for flow rate of pump with both feedbacks of flow rate and film thickness and the measurements of temperature and pressure will be established. A preliminary experiment is performed by a PID controller to verify the feasibility of this algorithm. A worktable supported by closed-type hydrostatic bearing which has four recesses on both sides. In this experiment, identification for geometric coefficient of pad and calibration for level of worktable are involved.

2. Control algorithm of the compensation of flow rate

The scheme of flow rate control for constant thickness of hydrostatic bearing film is keeping initial film thickness constant as load is changing. The control algorithms are illustrated in the following. Initially, the film between bearing and worktable is subjected to the initial applied load, the flow rate through a recess is

$$Q_0 = \frac{P_0 h_0^3}{12\mu_0}$$  \hspace{1cm} (1)

where recess pressure $P_0$ induced by initial load $W_0$, film thickness $h_0$, average of dynamic viscosity $\mu_0$, determined from average of initial temperature, and $\gamma$ is dimensionless geometric coefficient of bearing pad and that can be obtained by the determination of equation (1) by substituting the measurements of $P_0$, $h_0$, $\mu_0$, and $Q_0$. In the initial condition, pump speed is $n_0$ that supply flow rate $Q_0 + L_0$ where $L_0$ is leakage under the discharge pressure ($P_0$).

When the load changes, a recess pressure changes from $P_0$ to $P_p$, if pump speed (n) is kept constant then the flow rate is $n_0q - L_p$, where $L_p$ is leakage for discharge pressure $P_p$, the film becomes $h_0 + \Delta h$. Thus, pump supply flow can be obtained by the determination of

$$Q'_p = n_0q - L_p = (Q_0 + L_0) - L_p = \frac{P_p(h_0 + \Delta h)^3}{12\mu} - \gamma = \frac{P_0 h_0^3}{12\mu_0} - \gamma + L_0 - L_p$$ \hspace{1cm} (2)

where $q$ is pump discharge per one revolution, $Q_0$ is net flow rate obtained by subtracting leakage $L_0$ from pump discharge and which can be measured by flow meters. If film thickness is kept constant as initial condition ($h_0$), the flow rate entering recess must be

$$Q_p = \frac{P_0 h_0^3}{12\mu}$$ \hspace{1cm} (3)

Thus, subtracting Eq. (2) by Eq. (3) gives the compensation of flow rate as follows:

$$\Delta Q = Q'_p + L_p - n_0q = \frac{\mu_0}{\mu} \left( \frac{P_p - P_0}{\mu - \mu_0} \right) + L_p - L_0$$ \hspace{1cm} (4)
where \( n_0q \) is the sum of \( Q_0 \) and \( L_0 \), the initial pump speed is set by \( n_0 = (Q_0 + L_0)q^{-1} \), \( n_0q \) and \( Q_0 \) can be obtained by measuring flow rates, \( L_0 \) is obtained by the determination of subtraction of both measurements, and \( L_p \) is obtained by the same manner.

For keeping the film a constant thickness as the initial one (\( h_0 \)), the required flow rate must be expressed by equation (3), and the actual flow rate can be obtained by measurement. Consequently, the compensation of flow rate \( \Delta Q \) is subtracting determined \( Q_p \) from measured \( Q \). This is traditional algorithm, which utilizes flow rate feedback only and has been presented in previous researches. A traditional model of constant film thickness control system is shown in figure 1, which uses servo motor to control pump speed for compensate flow rate, additional pressure and temperature feedback are used to determine expected flow rate (\( Q_p \)). This figure notates elements by R: recess, f: film, W: worktable, FS: flow meter, PS: pressure sensor, TS: thermometer and notates signal \( v \) for input voltage of motor driver.

![Figure 1](image1.png)  
**Figure 1.** System model of scheme 1 of pump speed control for constant film thickness with flow rate feedback only.

In this algorithm, an initial pump speed is regulated to obtained optimal initial flow rate \( Q_0 \) for maximizing static stiffness. When load is changing, the pump speed is compensated by \( \Delta n = \Delta Q q^{-1} \) for the compensation of flow rate. On the basis of flow rate feedback, actual film thickness cannot be constant accurately. Since the determination of flow rate in algorithm is not absolutely exact. This algorithm doesn’t use the feedback of film thicknes or worktable displacement, the steady-state error will be come from the measurement error and identification error.

For improving the accuracy of film thickness the control algorithm 1 is modified by adding film thickness feedback as shown by figure 2, which uses servo motor to control pump speed for compensate flow rate and pressure and temperature feedback also.

![Figure 2](image2.png)  
**Figure 2.** System model of scheme 2 of pump speed control for constant film thickness with both feedbacks of flow rate and film thickness.

In the second algorithm, the initial film thickness (\( h_0 \)) can be obtained by measuring the level difference between \( Q_0 \) supply and dry condition without supply when the worktable is subjected to
initial load. When load becomes from \( W_0 \) to \( W \), substituting measured recess pressure (\( P_r \)), viscosity (\( \mu \)), and film thickness \( h_0 + \Delta h \) to determine the flow rate entering a recess as shown by

\[
Q'_p = \frac{P_r (h_0 + \Delta h)^3}{12 \mu \gamma}
\]

(5)

However, the flow rate of constant thickness film should be

\[
Q_p = \frac{P_r h_0^3}{12 \mu \gamma}
\]

(6)

Substituting \( \gamma = 12 \mu Q_0 \left( P_0 h_0^3 \right)^{-1} \) into both above equations gives the variation of flow rate pass through the recess which is caused by the difference of film thickness (\( \Delta h \)) and can be expressed by

\[
\Delta Q = Q_p - Q'_p = \frac{P_r \mu_0}{P_0 \mu} \left[ 1 - \left( \frac{h_0 + \Delta h}{h_0} \right)^3 \right] Q_0
\]

(7)

From Eq. (5) to Eq. (7) gives a proof for constant film control by using the compensation of flow rate as expressed by Eq. (7). Additionally, leakage must be taken into consideration to modify the above equation of the compensation of flow rate by

\[
\Delta Q = \frac{P_r \mu_0}{P_0 \mu} \left[ 1 - \left( \frac{h_0 + \Delta h}{h_0} \right)^3 \right] Q_0 + L_p - L_0
\]

(8)

As shown in figure 2, using PID control rule for \( \Delta n \) which is transformed from \( \Delta Q = Q_p - Q'_p \) by \( \Delta n = \Delta Q q^{-1} \) and adding initial pump speed \( n_0 \) gives compensated pump speed \( n \).

3. Experimental verification

The experimental device is taken a photo as shown in figure 3, which is installed by eight variable-flow pump with their independent servo motors for a closed-type thrust bearing supporting a worktable with four recesses respectively on top and bottom, and additionally with two recesses on both sides with constant restrictors for pressure compensation.

![Figure 3](image-url)
stiffness is regulated to maximum by identification and the level is calibrated by secondary tuning of flow rates. Additionally, eight pressure sensors, eight flowmeters, and eight thermometers for each recess are installed. The measurement of temperatures is for determining average dynamic viscosity of lubricant passing through sill.

For each recess, independent measures for feedback into independent control algorithm give independent compensation of pump speed. The worktable displacement at one corner is feedback to both recesses on top and bottom of this bottom, which gives opposite changes of supply flow for both recesses. For example, top recess is required reducing flow rate and bottom recess is required flow rate increasing when the worktable is rising. 160 c.c/min is selected as initial flow rate to support only worktable, 85.4 um of the initial film thickness is measured. Comparison in variations of film thickness of control results from this initial state applying 20 kg per one time on the worktable till to 80 kg at the fourth time. If no control is used, the film decreases more than 22.5 um. Variations of film thickness for three conditions are shown in figure 5. When P control is used, the film decreases about 4.5 um. Moreover, film decrease less than 0.5 um when PI control is used.

As shown in this figure, P control cannot eliminate extra correction of system response but PI control can eliminate oscillation in transient state. However, integral compensation will accumulate error which induces oscillation in steady state. D control has the ability to stabilize error oscillation. Similarly for initial flow rate 160c.c/min, variations of film thickness by PID control can be seen in figure 6 when the load is applied from 20 to 80 kg. Individual parameters of P, I, and D compensations are selected by Ziegler-Nichols method, since the mathematical model of this study isn’t linear or simple. The simulation results reveal that the overshoot cannot be eliminated by all compensations but that can be abated by D compensation added in to PI control.
Figure 5. Comparison in variations of film thickness of control results from initial state to 80 kg adding for no control, P control and PI control.

Figure 6. Variations of film thickness from initial state to 80 kg adding for PID control.

Figure 7 is obtained from magnifying the thickness scale by 20 times, which shows clear comparison between both steady-state responses due to PI and PID controls. The variations of film thickness are shown by blue and red for PI and PID controller, respectively. This figure reveals that both controls result in same precision but more accuracy result can be obtained by PID control.

Figure 7. Comparison in large scale of variations of film thickness for PI and PID control.

4. Conclusions
This study presents the mathematical model of control scheme and algorithm of flow rate for constant film thickness of hydrostatic bearings. The algorithm is established by the relationship among recess pressure, flow rate, viscosity and film thickness. In deduce of this algorithm, constant thickness of film is verified theoretically. This algorithm is used in the same scheme, which includes both feedbacks of film thickness and flow rate but with P, PI and PID controllers to control pump speed, respectively. One of P, PI and PID controller are used, sub-micron of film thickness in steady-state precision can be achieved. Comparisons in steady-state errors when adding 20 Kg load step by step, P controllers has 1 micron error, and PI, PID controllers have 0.2 micron error, respectively. Additionally, PID controller can reduce steady-state error by compensating disturbance due to adding loads. Thus the PID
controller with the scheme and algorithm proposed by this article used in precise machine can be high precise and high accuracy.

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