On steam bubble patterns in superheated liquids

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Abstract. This paper presents the results of numerical simulation of steam bubble growth in a uniformly superheated liquid. The numerical calculation scheme takes into account both inertial and thermal effects. The main regularities of the process are discussed by the example of different superheat levels and different liquids: water, mercury and glycerin. Simple estimations of characteristic times of the inertial and transition stages are offered.

1. Introduction
Boiling is a process realized in a wide class of engineering problems. It can be initiated in various types of heat exchangers in order to perform effective cooling. Boiling can be also used in modern medical technologies, for example, the technology of minimally invasive laser surgery, where as a result of the laser radiation on the biological fluid its explosive boiling occurs, causing destructive effects on pathological formations [1]. An elementary boiling is the steam bubble growth in a superheated liquid; without detailed knowledge of its mechanism the correct description of the boiling process is impossible.

At present, in addition to many experimental studies, there is a great amount of theoretical works, which describe the growth of the steam bubble for cases of various liquids and at different conditions. However, most of them are either asymptotic or empirical in nature. This prevents from accurately predicting the dynamics of bubble growth in a wide range of regime parameters needed to effectively use heat mass processes. Obviously, it is not possible to solve the problem for a general case by analytical methods. Therefore, most researchers make significant assumptions that limit the commonality of the obtained solutions such as in classical works [2–5] and comparatively modern research works [6–8], where inertial and thermal stages of bubble growth are separately described. Some attempts of using both dynamic and thermal factors were made in [9–13]. However, the transitional stage of bubble growth is insufficiently described in time, although, it is extremely necessary in some cases. This paper is devoted to numerical modeling of steam bubble growth in a wide range of Jacob numbers and at all stages of the process, including the transition stage.

2. Solution procedure
The present work considers numerical modeling in the general statement including inertial and thermal effects. For this purpose, the modified Rayleigh-Plesset equation and the boundary thermal problem, which includes the non-stationary equation of thermal conductivity and the equations of mass balance, momentum and energy on the interface (which is a mobile boundary), are solved jointly. The equation of steam state and the Clapeyron-Clausius equation are recorded for the relationship between pressure...
and temperature. The following assumptions are used to set the problem: liquid is assumed to be incompressible; steam temperature and pressure in the bubble are homogeneous; steam in the bubble is stationary and saturated throughout the process; thermophysical properties of liquid and steam are constant throughout the process; heat generation due to viscous dissipation is not taken into account.

Calculations were performed in dimensionless variables (at their use the interphase boundary remains stationary as in the works on modeling gas bubble growth [14,15]) with the following characteristic values of time, speed and size, respectively: $t_0 = a_1\rho_l / \Delta p_l$, $w_0 = (\Delta p_l / \rho_l)^{1/2}$, $R_0 = a_1(\rho_l / \Delta p_l)^{3/2}$, where $a_1$ is the temperature conductivity of the liquid, $\rho_l$ is the density of the liquid, and $\Delta p_l$ is the initial pressure difference between the inner saturated vapor and the surrounding liquid.

For numerical modeling, the 4th order Runge-Kutta method and the implicit Crank-Nicholson method were used. The Brent method was used to find the pressure and temperature of steam set at each point in time by an implicit function.

3. Results and discussion

Numerical calculations of the steam bubble growth dynamics in water, mercury and glycerine have been carried out (in future we will denote them by the lower indices "w", "hg", "gl," respectively). The analysis of the dependences of the bubble radius, velocity and acceleration of the interphase boundary on time for different superheating: low ($< 1$ K), medium (2 – 5 K) and high ($> 5$ K) (according to classification [12]) at the atmospheric pressure, is presented.

3.1. At the low superheat level.

Let us consider the bubble growth at the low superheats, i.e. 0.8 K and 1 K (figure 1). The boiling processes can be conventionally divided into three stages: the inertial stage, the stage of rapid growth (transitional) and stable thermal growth stage. At the initial (inertial stage) there is a delay of growth caused by the fact that the initial pressure difference between the vapor and the surrounding liquid is almost balanced by the forces of surface tension and viscous friction (viscosity is noted to be negligible at this stage). During this stage the vapor pressure and temperature are kept constant without an apparent increase in the bubble radius. Then the effect of surface tension decreases, which leads to a rapid expansion of the bubble. This stage (transitional) is characterized by a significant increase in speed and a decrease in acceleration of the interface. The vapor pressure and temperature in the bubble also decreases rapidly. This leads to setting a significant temperature gradient on the boundary, which in turn leads to a heat supply from the surrounding liquid to the interface. After a certain time, the acceleration of the interface decreases to almost zero. This stage can be described as the end of the transition. After that, the stage of "stable" growth (thermal), caused exclusively by heat supply to the interphase, starts. The solution of this stage has a classical root dependence of the bubble radius on time characteristic for the thermal growth model [3–5].

All the above is shown in figure 1. Despite the fact that the obtained dependences for two different superheats are quite similar, there are certain features. In particular, the bubble growth rate at the transition stage is the larger, the greater is the initial overheating. This is explained by the larger initial pressure and temperature of the bubble at higher superheat. Let us notice, that time when the maximum speed is reached depend on the type of liquid and it is characterized by delay time $t_d \approx 6 R^*(2 \rho_l / 3 \Delta p_l)^{1/2}$ (where $R^*$ is the critical radius). So, for superheating by 1 K for mercury $t_{dhg} \approx 7.7$ ms, for glycerin $t_{dgl} \approx 0.13$ ms and for water $t_{dw} \approx 0.1$ ms. Such a significant difference is explained by a large higher surface tension coefficient ($\sigma_{hg} / \sigma_w \approx 6.9$, $\sigma_{hg} / \sigma_{gl} \approx 9.6$) and mercury density ($\rho_{hg} / \rho_w \approx 13.1$, $\rho_{hg} / \rho_{gl} \approx 10$) compared to surface tension and density of glycerin and water, respectively. The time of reaching the thermal stage is also different. This characteristic time can be written $t_s \approx \frac{12 a_j^2 \eta_l}{Pe \Delta p_l}$, where $Ja$ is the Jacob number, $Pe$ is the Peclet number, and $\eta_l$ is the dynamic viscosity of the liquid. From the latter ratio we can see that the time to reach the thermal growth stage

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depends on both dynamic and thermophysical factors (but first of all, on the Jacob number). For superheating by 1 K \( t_{thg} \approx 1.3 \text{ ms} \), \( t_{tw} \approx 5.5 \mu\text{s} \), \( t_{gl} \approx 3.6 \mu\text{s} \). It should be noted that for low level superheat the characteristic delay time is much larger than the characteristic transition stage time.

### 3.1.1. At the medium superheat level.

Now let us consider the bubble growth at the medium superheat from 2.5 to 5 K (figure 2). As superheating increases, the radius of the critical radius decreases. In this case, the system becomes more sensitive to changes in the bubble radius: the pressure equalizes faster than at low superheating, which in turn reduces the delay time \( t_d \). Hence, for a superheat by 2.5 K \( t_{thg} \approx 2 \text{ ms} \); \( t_{gl} \approx 34 \mu\text{s} \); \( t_{tw} \approx 25 \mu\text{s} \). Thus, the transition stage of the bubble growth ends faster than at the low superheats. However, the time to the thermal growth stage increases slightly: \( t_{thg} \approx 3.2 \text{ ms} \); \( t_{gl} \approx 8.7 \mu\text{s} \); \( t_{tw} \approx 13.5 \mu\text{s} \). These

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**Figure 1.** Time dependences of bubble radius, speed and acceleration of interface calculated at low superheats (1 K – solid blue line, 0.8 K – dotted black line) and for different liquids: water (upper row), mercury (middle row), and glycerin (lower row).
times seem to be comparable and they can be neglected in the case of bubble growth at large times. Thus, the traditional asymptotic solutions obtained in the frame of the thermal growth model [3-5, 7] assumingly work quite well at medium superheating.

3.1.2. At the high superheat level.
Let us consider the bubble growth at high overheats (over 5 K). As we can see in figure 3 the critical radius has even smaller size, and there is even faster pressure equalization in comparison with the previously considered cases. All this results in lower growth delay time and higher bubble growth rate. So, for superheating by 10 K the delay times \( t_{dhg} \approx 0.2 \) ms; \( t_{dgl} \approx 3.7 \) \( \mu \)s; \( t_{dw} \approx 2.6 \) \( \mu \)s. Other researchers use this fact to neglect delay and to apply the classical solutions of the thermal growth model. However, we will estimate the transition time and obtain: \( t_{trh} \approx 12.2 \) ms; \( t_{trw} \approx 48.3 \) \( \mu \)s; \( t_{trg} \approx 32 \) \( \mu \)s. We can see that in this case it significantly exceeds the delay time and can be compared with the observed time (sometimes even greater). Hence it is obviously incorrect to use the solutions obtained within the thermal model. The transition stage of growth must be taken into account!

Although the dependences of velocity and acceleration for high superheating are similar to those for the medium superheat level, they have large maximum values. We should additionally note an abnormal dependence of growth rate on time calculated for glycerin. The appearance of the additional local

Figure 2. Time dependences of bubble radius, speed and acceleration of interface calculated at the medium superheat by 2.5 K (solid blue line) and for different liquids: water (upper row), mercury (middle row), and glycerin (lower row).
maximum can be explained by the effect of viscosity. This is due to the fact that the forces caused by viscous stresses are of great importance at the stage of rapid bubble growth in the case of high-viscosity liquids. They do begin balancing the pressure drop together with the forces of inertia. As a result, an additional delay is created. This is the greater the higher the initial superheat, which is well illustrated in figure 3(h).

![Figure 3](image-url)

**Figure 3.** Time dependences of bubble radius, speed and acceleration of interface calculated at high superheats (10 K – solid blue line, 100 K – dotted black line) and for different liquids: water (upper row), mercury (middle row), glycerin (lower row).

**Conclusions**

This paper has numerically studied the dynamics of the steam bubble growth in the volume of a uniformly superheated liquid. The problem has been considered in a wide range of regime parameters of the process and at all its stages, including the transition. The numerical scheme, which allows modeling bubble growth taking into account both inertial and thermal factors, has been implemented. The bubble growth mechanism has been analyzed in detail. Characteristic times of dynamic and transition stages of growth are illustrated by the example of some liquids. Calculations have shown that for large superheats (with large Jacob numbers), the characteristic time of entry into the thermal growth stage may be unattainable at reasonable times, which means that the known solutions obtained in the framework of the thermal growth model are not applicable.
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References
[1] Chudnovskii V M, Levin A A, Yusupov V I, Guzev M A, Chernov A A 2020 International Journal of Heat and Mass Transfer 150 119286
[2] Lord Rayleigh 1917 The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science 34(200) 94–8
[3] Plesset M S and Zwick S A 1952 Journal of Applied Physics 23(1) 95–8
[4] Forster H K and Zuber N 1954 Journal of Applied Physics 25(4) 474–8
[5] Scriven L E 1959 Chemical Engineering Science 10 1–13
[6] Robinson A J, Judd R L 2004 International Journal of Heat and Mass Transfer 47(23) 5101–13
[7] Avdeev A A 2016 Bubble Systems 446
[8] Prosperetti A 2017 Vapor Bubbles Annual Review of Fluid Mechanics 49(1) 221–48
[9] Mikic B B, Rohsenow W M, Grith P 1970 International Journal of Heat and Mass Transfer 13(4) 657–66
[10] Miyatake O, Tanaka I, Lior N International Journal of Heat and Mass Transfer 40(7)
[11] Lesage F J, Siedel S, Cotton J S, Robinson A J 2014 Chemical Engineering Science 112 35–46
[12] Wang Q, Gu J, Li Z, Yao W 2017 Chemical Engineering Science 172 169–81
[13] Cai C, Liu H, Xi X, Jia M, Yin H 2018 International Journal of Heat and Mass Transfer 127 629–38
[14] Chernov A A, Kedrinsky V K, Pil’nik A A 2014 Physics of Fluids 26(11) 116602
[15] Chernov A A, Pil’nik A A, Davydov M N, Ermanyuk E V, Pakhomov M A 2018 International Journal of Heat and Mass Transfer 123 1101–08