A search of multiparticle correlations in 10.7 A GeV $^{197}$Au and 200 A GeV $^{32}$S interactions with emulsion nuclei by the Hurst method

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Abstract
An analysis of pseudorapidity correlations in 10.7 A GeV $^{197}$Au and 200 A GeV $^{32}$S interactions with emulsion nuclei by the normalized range method has been carried out. The evidence for events with large multiparticle correlations is presented. The most significant "correlation force" effect is apparent in the interactions of light emulsion nuclei (CNO group) and gold nuclei at an energy of 10.7 A GeV, corresponding to an absolute disintegration of the target nucleus $n_b + n_g \simeq 0$.

1 Introduction
A study of interactions of relativistic nuclei is related to a search for unusual nuclear matter conditions (as quark-gluon plasma (QGP), for example), which can be observed at high energies [1]. The behaviour of simple features of these processes, such as total cross sections, multiplicity and pseudorapidity distributions, etc. is rather precisely described by various models, even if they are based on the various assumptions about nuclear structure and dynamics of nuclear interactions. More detailed information, critical to a choice between existing models, can be deduced by investigating a fluctuation structure of secondary particle distributions.

Recent studies of pseudorapidity distributions have shown that some considerable deviations from average distribution can be observed in individual events. Certainly, these deviations can be initiated by the statistical factors, connected with finite number of particles in the given event. If the statistical explanation cannot describe these fluctuations, there are some dynamic mechanisms, which could produce peculiarities of pseudorapidity distributions.

In our previous work [2] we have proposed a normalized range method for analysing pseudorapidity correlations in multiparticle production processes. This method allows not only to discriminate dynamic correlations from statistical ones, but also to determine a "force" and "length" of these correlations.

In the present paper we apply this method to the analysis of experimental pseudorapidity distributions of secondary particles obtained in 10.7 A GeV $^{197}$Au and 200 A GeV $^{32}$S interactions with emulsion nuclei.

2 Analysis procedure
As result of high-energy interaction of two nuclei plenty of secondary particles is produced. According to the existing notions, secondary particles, which are "emitted" from "interaction volume", have pseudorapidities, corresponding to a central region
of pseudorapidity distribution. At borders of the distribution fragments (of the target-nucleus and projectile-nucleus) bring in considerable contribution. And so, in order to investigate of pseudorapidity correlations in distribution of particles from "interaction volume" we have chosen pseudorapidity interval $\Delta \eta = 4$ (for the $^{32}$S, 200 A GeV, $\eta_{\text{min}} = 1.0$, $\eta_{\text{max}} = 5.0$ and for the $^{197}$Au, 10.7 A GeV, $\eta_{\text{min}} = 0.3$, $\eta_{\text{max}} = 4.3$). This interval has been subdivided into $k$ parts with $\delta \eta = \Delta \eta / k$. By counting the number of particles in each subinterval we arrive at a sequence $n_i^\xi$. A pseudorapidity fluctuation, or the normalized relative deviation of an individual event from average pseudorapidity distribution is given by

$$\xi_i = \frac{n_i^\xi/n^e - n_i/n}{n^e/n_i - 1}$$

where $n_i^\xi$ is the number of particles in the i-th bin of an event with particles number $n^e$, and $n_i = \sum_n n_i^\xi$ is the total number of particles for all events in the i-th bin, and $n = \sum_n n^e$ is the total number of particles for all events.

For analysis we have selected events with the number of secondary particles (except fragments) greater than 90, because high statistics are necessary for a study of correlations by our method.

As described in our previous work [2] for an investigation of pseudorapidity correlations we analysed the normalized range $H(k') = R(k')/S(k')$ (where $R(k')$ and $S(k')$ are a "range" and a standard deviation, which is calculated by Eqs.(3)-(6), see below) versus the size of the pseudorapidity interval $d\eta = k' \delta \eta$, $(1 \leq k' < k)$ using a function

$$H(k') = (ak')^h$$

where $a$ and $h$ are two free parameters and $h$ is the correlation index (or Hurst index). If the signal $\xi_i$ represents white noise (a completely uncorrelated signal), then $h = 0.5$. If $h > 0.5$, long-range correlations are in a system [3, 4].

In our calculations we have used $k = 8192$. The choice of such large value of $k$ is not necessary (it is possible using lesser $k$ also). But the more $k$ the more accuracy of method is approachable.

So, for the sequence $\xi_i$, $1 \leq i \leq k$, quantities of $R(k)$ and $S(k)$ were calculated by following formulas:

$$S(k) = \left[ \frac{1}{k} \sum_{i=1}^{k} (\xi_i - <\xi>)^2 \right]^{1/2}$$

$$R(k) = \max_{1 \leq m \leq k} X(m, k) - \min_{1 \leq m \leq k} X(m, k)$$

where the quantity $X(m, k)$ characterizes the accumulated deviation from the average

$$<\xi> = \frac{1}{k} \sum_{i=1}^{k} \xi_i$$

It is possible to use also absolute deviation $\xi_i = n_i^\xi/n^e - n_i/n$. 

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for a certain interval $m \delta \eta$,

$$X(m, k) = \sum_{i=1}^{m} [\xi_i - \langle \xi \rangle], \quad 1 \leq i \leq m \leq k \quad (6)$$

Then the sequence $\xi_i$ has been subdivided into two parts: $\xi'_i$, $1 \leq i \leq k' = k/2$ and $\xi''_i$, $k' + 1 \leq i \leq k$, and the value of $H(k/2) = R(k/2)/S(k/2)$ was found for each of the two independent series. Similarly $\xi'_i$ and $\xi''_i$ have been subdivided further, followed by the calculation of $H(k/4) = R(k/4)/S(k/4)$. This subdivision and analysis procedure for newly obtained series of $\xi_i$-values is continued until $d \eta > 0.1$. $H$ corresponding to the same value of $k'$ have been averaged and drawn on a log-log scale as a function of $k'$ (Fig. 1).

![Figure 1: The correlation curve for (Au+Em)-interactions at 10.7 A GeV. Points are calculated values, lines are fits by (2)](image)

For our correlation analysis it is possible to use this dependence visually and determine the points belonging to the same line and their fit by (2). But more precisely and more clearly one can investigate the behaviour of the correlation index $h$ versus the width of pseudorapidity bin $d \eta$ with the help of a 3-point fitting procedure. Those three points were: the first has been subsequently conferred by obtained Hurst index and other two ones are neighboring points. For example, $h$ corresponding to $k' = 1024$ (ln($k'$) = 6.931, $d \eta = 0.5$) in the Fig.1 has been found, using (2) and with following a fitting procedure for points $k' = 512$ (ln($k'$) = 6.238), $k' = 1024$ (ln($k'$) = 6.931), $k = 2048$ (ln($k'$) = 7.625) from the Fig.1.

3 Data

The above analysis procedure has been applied to three samples of events
1. Experimental data of the $^{32}\text{S}$ 200 A GeV (264 events) and the $^{197}\text{Au}$ 10.7 A GeV (315 events) interactions with emulsion nuclei. NIKFI BR-2 stacks of nuclear emulsions, sizes of $(10 \times 20 \times 0.06)\text{ cm}^3$ have been exposed horizontally to the 10.7 A GeV $^{197}\text{Au}$ beam at the BNL/AGS and the 200 A GeV $^{32}\text{S}$ at the CERN/SPS. Details about scanning and measurements are given elsewhere [5],[6].

2. "Data" obtained by FRITIOF 7.02 model [7], of the $^{197}\text{Au}$ (10.7 A GeV, 315 "events") and $^{32}\text{S}$ (200 A GeV, 264 "events") interactions with emulsion nuclei.

3. A random process simulated by the HRNDM program, included in the programming package HBOOK [8]. Multiplicities and pseudorapidity distributions for this process are similar to the experimental data ($^{197}\text{Au} + \text{Em}$ at 10.7 A GeV, 315 "events" and $^{32}\text{S} + \text{Em}$ at 200 A GeV, 264 "events").

4 Results and discussion

It should be noted that the $\xi_i$ distributions for all data samples are similar (at least formally) to themselves. At the same time, the experimentally observed dependence of the correlation index on the size of the interval $d\eta$ is different from both HRNDM and FRITIOF events (see Fig. 2).

As it is seems from Fig.2 the most obvious difference is evident at large $d\eta$ for $(\text{Au}+\text{Em})$ interactions at 10.7 A GeV, where a significant increase of the correlation index is observed, thus indicating stronger correlation "force" [2].

A detailed analysis of individual events has shown that all experimental events can be conditionally divided into two classes. In Fig. 3 correlation curves for two events from two different classes, is presented.

Firstly, the events without a "kink" of the correlation curve correspond to random pseudorapidity distribution of secondary particles. In these events fluctuations are es-
sentially statistical, connected with the finite number of particles in the event. Secondly, the events “with kink” of the $ln H(k')$ versus $ln k'$ correlation curve, in which we do observe significant multiparticle correlations at $\delta \eta \geq 1$.

The event selection was made by the average correlation index

$$h_{ev} = \frac{1}{i_{max} - 1} \sum_{i=1}^{i_{max}} \frac{ln(H(k_i)) - ln(H(k_{i-1}))}{ln(k_i) - ln(k_{i-1})}, \quad (k_i = k/2^i)$$

which was determined by a power-law fit to the correlation curve $ln H(k')$ versus $ln k'$. If $h_{ev}$ was greater than 0.62, then the event was related to the group of events with kink. We can analyze the more correlated events by increasing the criterion of selection up to $h_{ev} \geq 0.7$ or $h_{ev} \geq 0.8$. In Fig.4 the curves of $ln H(k')$ versus $ln k'$ for 0.62 criterion are shown.

The behaviour of these without kink curves are similar to the FRITIOF events.

It is interesting to note that the relative number of correlated ($S+Em$)-events was considerably lower than for ($Au+Em$) interactions. Namely, we have 187 events with $h_{ev} > 0.62$ for gold and only 97 ones for sulfur. The ($S+Em$)-interactions with very strong correlations (with $h_{ev} > 0.8$) were absent at all. In the case of ($Au+Em$)-collisions the number of such events was 16.

To study possible distinctions in the mechanism of formation of a final condition for the two types of interactions (with kink and without kink of the correlation curve), we have analysed the behaviour of fragments of the projectile and the target nuclei. Thus, most difference is revealed for the $N_b$-distributions. $h$-particles are called a sum of $b$- and $g$-particles ($N_h = n_b + n_g$). According to the existing photoemulsion experiments criteria the $b$-particles are fragments of target nucleus with kinetic energy $E_{kin} < 26 AMeV$, $g$-particles are recoil protons with $26 < E_{kin} < 400 MeV$. As

\[\text{Figure 3: The correlation curve for two of events of (Au+Em)-interactions at 10.7 A GeV}\]
Figure 4: The correlation curve for two of events of (Au+Em)-interacions at 10.7 A GeV. The value of Hurst index $h$ versus a width of pseudorapidity bin $d\eta$ for the events with $h_{ev} \geq 0.62$ and the events with $h_{ev} < 0.62$ from experimental data of Au+Em 10.7 A GeV (left) and S+Em 200 A GeV (right).

it is seen from Fig. 5 a large part of events with kink, as opposed to the interactions without kink, proceed with a complete (or almost complete) disintegration of the target nucleus (peak in area $N_h = 0$).

Most of these kink events (events with very strong correlations) corresponds to such events (Fig. 6left).

The photoemulsion consist of Hydrogen(39.2%), nuclei of CNO-group(35.3%) and AgBr nuclei (25.5%). In order to know what interactions give main contribution in peak of $N_h = 0$ we have analysed $n_s$ distribution for events with $n_b = 0$ (Fig. 6right).

The first peak in this distribution at $n_s < 30$ corresponds to peripheral interactions of gold with photoemulsion nuclei and central interactions, $(p+Au)$. The second peak at $30 < n_s < 200$, is a result of central (since $n_s$ are large) interactions of light nuclei (CNO group) and nuclei of gold. The peak in the area of $n_s > 200$ corresponds to central collisions of $(Ag + Au)$ and $(Br + Au)$.

Hence, by analysing events with $n_s > 200$, we study interactions of heavy nuclei of Ag and Br with nuclei of gold. We expect that greatest correlations in such interactions. However, as it can be seen from Fig. 7left, the peak in area of $N_h = 0$, corresponding to the greatest values of a correlation index, has disappeared!

Therefore, the events which give the main contribution to the peak in the area of $N_h \sim 0$ and which have great kink of correlation curve, correspond to central interactions of Au and light nuclei of CNO group.

It is possible that there are several probable reasons for the existence of such large signs have coincided by accident. 

$^3$s-particles are secondary ones with $E_{kin} > 400$ AMeV.
correlations in multiparticle production processes. But, nevertheless, at present we know only one way when similar behaviour (to experimental one) of the correlation curve can be obtained. As result of interactions of two particles at high energy there is formed some large cluster (or several clusters), which then disintegrates into a plenty of secondary particles in the interval of $\delta \eta < 1$.

This process was simulated with the help of the programme "HRNDM", included into the software package of "HBOOK" [8]. We simulated stochastic events with pseudorapidity distributions similar to experimental data. We have designate: a total number of generated "particles" in an event as $n^c$, probability of production of cluster as $p$ and number of the clusters as $j$ ($0 \leq pj \leq 1$). Each simulated “particle” can be disintegrated (with probability $pj$) into $pn^c$-particles with pseudorapidity $\eta_l = \eta_m + f_1$, $1 \leq l \leq pn^c$, where $f_1$ is random function, which changes with uniform probability in region $[-RN, +RN]$, $\eta_m$ is pseudorapidity of the "parent particle".

By varying $p$, $j$ and $RN$, we can define curves most precisely reproducing the behaviour of our experimental data.

From Fig.7 (right) it is seen that the more dimension of the compound system, the more "kink" of the correlation curve will be displayed.

Moreover, such the large kink of correlation curve, which observed for experimental events, cannot be explained by two-particle disintegrations. So, the curve with $j=50$ and $p=0.02$ in Fig.7 (right) corresponds to process with maximal two-particle correlations (all secondary particles are produced as result of two-particles disintegrations). In this case the correlation index reached only 0.62. In order to obtain the values $h$ which observed for experimental data, it is necessary to assume that the compound system, which produces $\sim 40 \%$ (for Au+Em) and $\sim 24\%$ (for S+Em) of secondary particles, is formed.

It should be noted that for search of the most interesting effects as quark-gluon
plasma, for example, in CERN experiments as NA44, NA49, WA98 etc. symmetric collisions of heavy ion as Pb+Pb (or Au+Au) are used. If our results will be confirmed by other experimental data (for instance, in interactions of Au (or Pb) and nuclei of CNO group at other (more or less than 10.7 a GeV) energies) then it will be possible declare that probability of exhibition of QGP in asymmetric interaction is more than that in symmetric ones.

To clear this point it is possible by primitive model. Supposing that a nucleon is an elastic ball, inside which quarks are disposed, and nucleus is some system consisting the balls, which are glued together. At the moment of collision first balls, coming into contact, are squeezed by other balls, which push them from behind and in front. And so, a real energy of interaction (at this “region”) is significantly more than an energy per nucleon (some cumulative effect). A pressure (at this “region”) is enormous. Volumes of the “interacting balls” is getting significantly diminished (is squeezed). As result a nucleon ”envelope” can burst and quarks (from the nucleon) will be in quasifree state inside dense ”encirclement” of squeezed (but not bursting) neighbouring nucleons. This region can exist and be expanded till large nucleus are not split (when ”force of glue” will be less than ”breaking force of the wedged (smaller) nucleus”). If an interaction is less central (almost peripheral) one, then quarks region and time of its existence will be less, because of that the probability to chop off smaller ”scrap” is more than that to split nucleus to ”fifty-fifty”. In case of collision of symmetric nuclei the quarks region can appear also, but its dimensions will be still less, because dense ”encirclement”, which is capable to retain (inside itself) quarks, is absent (it is similar

\footnote{It should be noted that the process of nuclei interaction at high energies is more complicated one. Nevertheless, the model allow to realize why largest correlations should be exhibited in asymmetric collisions instead of symmetric ones}.
to peripheral interaction of asymmetric nuclei).

### 5 Conclusion

The analysis of pseudorapidity correlations in \(10.7\) GeV \(^{197}\text{Au}\) and \(200\) GeV \(^{32}\text{S}\) interactions with emulsion nuclei has been carried out by the normalized range method. This analysis has revealed events with large multiparticle correlations (with large kink of correlation curve). We assume that the main cause of this behaviour of the correlation curve is a formation of large compound system, which then disintegrates into a large number of secondary particles in the interval of \(\eta \pm \delta\eta\) (\(\delta\eta < 1\)). The most significant pseudorapidity correlations are observed in the central interactions of nuclei, which have considerable difference in volume (in nuclear weight, charge, etc.) at rather “low” energies (the nuclei of Au and CNO group at \(10.7\) GeV).

It would be interesting to see whether a similar effect is observed in interactions of other nuclei (for example, lead and nuclei of CNO group) and whether it depends on the interaction energy (if the effect will be increase at energy of collision more than \(10.7\) GeV).

### References

[1] I.M. Dremin, A.V. Leonidov Phys.Usp.53 1123-1149, 2011; arXiv:1006.4603v2 [nucl-th] 2010

[2] I.A. Lebedev, B.G. Shaikhatdenov J.Phys.G:Nucl.Part.Phys. 23 (1997) 637
[3] H.E. Hurst, R.P. Black, Y.M. Simaika (1965), Long-Term Storage: An Experimental Study (Constable, London)

[4] J. Feder "Fractals", Plenum Press, New York, 1988,

[5] M.I.Adamovich et al. Phys.Lett. B352 (1995) 1472

[6] M.I.Adamovich et al. Phys.Lett. B227 (1989) 285

[7] B.Nilsson-Almqvist, E.Stenlund, Comut.Phys.Commun. 43 (1987) 387,

[8] R.Brun, D.Lienart, CERN computer centre program library long write-up.