Neutrino Oscillations With Recently Measured Sterile-Active Neutrino Mixing Angle

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Abstract
This brief report is an extension of a prediction of neutrino oscillation with a sterile neutrino using parameters of the sterile neutrino mass and mixing angle recently extracted from experiment.

1 Introduction
Recently there was an investigation of how the neutrino transition probability $P(\nu_\mu \rightarrow \nu_e)$ is modified by the introduction of a sterile neutrino[1], motivated in part by recent experiments on neutrino oscillations[2], which suggested the existence of at least one sterile neutrino. The 3x3 U-matrix method of Sato and collaborators for three active neutrino oscillations[3,4] was extended to 4x4 with a sterile neutrino.

In the present brief report we use the formalism of Ref[1] with parameters for the sterile neutrino that have recently been extracted by a global fit to neutrino oscillation data[5].

2 $P(\nu_\mu \rightarrow \nu_e)$ Using a 4x4 U Matrix
Active neutrinos with flavors $\nu_e, \nu_\mu, \nu_\tau$ and a sterile neutrino $\nu_s$ are related to neutrinos with definite mass by

$$\nu_f = U \nu_m ,$$

where $U$ is a 4x4 matrix and $\nu_f, \nu_m$ are 4x1 column vectors, which is an extension of the 3x3 matrix used in Refs.[3,4].
As shown in Ref[1], the transition probability $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$, assuming $\delta_{CP} = 0$ giving $U_{ij}^* = U_{ij}$, is

$$\mathcal{P}(\nu_\mu \rightarrow \nu_e) = U_{12}^2 U_{21}^2 + U_{13}^2 U_{23}^2 + U_{14}^2 U_{24}^2 + 2U_{11} U_{21} U_{12} U_{22} \cos \delta L +$$

$$2(U_{11} U_{13} U_{23} + U_{12} U_{22} U_{13} U_{23}) \cos \Delta L +$$

$$2U_{14} U_{24} (U_{11} U_{21} + U_{12} U_{22} + U_{13} U_{23}) \cos \gamma L , \quad (2)$$

with neutrino mass differences $\delta m_{ij}^2 = m_i^2 - m_j^2$, $\delta = \delta m_{12}^2/2E$, $\Delta = \delta m_{13}^2/2E$, $\gamma = \delta m_{j4}^2/2E$ (j=1,2,3), where E is the energy and L the baseline. The neutrino mass differences are $\delta m_{12}^2 = 7.6 \times 10^{-5} (eV)^2$, $\delta m_{13}^2 = 2.4 \times 10^{-3} (eV)^2$, and from recent MiniBooNE analysis $\delta m_{j4}^2 = 0.9 (eV)^2$. Note that in Ref[1] there is a typo in the third line of Eq(2), with $U_{13} U_{22}$ rather than the correct $U_{13} U_{23}$.

Using $c_{12} = .83$, $s_{12} = .56$, $s_{23} = c_{23} = .7071$, and $s_{13} = .15$, (with $s_{ij}, c_{ij} = \sin \theta_{ij}, \cos \theta_{ij}$),

$$U_{11} = .822 c_\alpha$$
$$U_{12} = .554 c_\alpha - .821 s_\alpha^2$$
$$U_{13} = -.821 s_\alpha^2 c_\alpha - .554 s_\alpha^2 + .15 c_\alpha$$
$$U_{14} = .821 s_\alpha^2 + .554 s_\alpha c_\alpha + .15 s_\alpha$$

$$U_{21} = -.484 c_\alpha$$
$$U_{22} = .484 s_\alpha^2 + .527 c_\alpha$$
$$U_{23} = .699 c_\alpha - (-.484 s_\alpha c_\alpha + .527 s_\alpha)s_\alpha$$
$$U_{24} = -.484 s_\alpha c_\alpha^2 + .527 s_\alpha c_\alpha + .699 s_\alpha , \quad (3)$$

with $\alpha$ the sterile-active neutrino mixing angle, $s_\alpha, c_\alpha = \sin(\alpha), \cos(\alpha)$.

A recent analysis of neutrino oscillation data[5] found $\sin(\alpha) \simeq 0.1$. Using this, from Eqs(2,3) we obtain the results shown in the figure. The 3x3 results with only active neutrinos are somewhat different than those of Ref[6] as the current value of $s_{13} = .15$ from the Daya Bay experiment[7] was not known at that time.
Figure 1: The ordinate is $P(\nu_\mu \rightarrow \nu_e)$ for MINOS ($L=735$ km), MiniBooNE ($L=500$ m), T2K ($L=295$ km), and Double CHOOZ ($L=1.03$ km) using the 4x4 U matrix with $\delta m^2_{4j} = 0.9 (eV)^2$ and $\sin(\alpha) \simeq 0.1$. The dashed curves are for $\alpha = 0$ (3x3).
3 Conclusions

As shown in the figure, the neutrino oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$, is quite different for a model with four neutrinos, $\nu_e, \nu_\mu, \nu_\tau, \nu_s$ than with three active neutrinos. The effect of the sterile neutrino is largest for large values of $E/L$, as seen in the figure for MiniBooNE, which has a baseline of $L=0.5$ km. Although Double CHOOZ has a baseline of only 1.03 km, the energy is only about 3 MeV, so the effects of a sterile neutrino are smaller, but are still important for the extraction of neutrino parameters from neutrino oscillation data.

In the present work we have used the sterile-active neutrino mixing angle $\alpha$ ($\sin(\alpha) \simeq 0.1$), which is now known to be approximately correct for $\delta m^2_{41} = 0.9(eV)^2$, rather than the mixing angles used in Ref[1]. Therefore the present results are more reliable for comparison with future neutrino oscillation experiments, including measurements of CP and T-reversal violations, than the previous publication.

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