ABSTRACT

The classical 2D cosmological model of Callan, Giddings, Harvey and Strominger possesses a global symmetry that is responsible for decoupling of matter fields. The model is quantized on the basis of the extended phase space method to allow an exhaustive, algebraic analysis to find potential anomalies. Under a certain set of reasonable assumptions we show that neither the BRST symmetry of the theory nor the global symmetry suffers from an anomaly. From this we conclude that there is nothing to recognize the existence of black hole and therefore nothing to radiate in their cosmological model.
Inspired by string theory [1], much attention has recently paid to cosmology in two-dimensions [2]. In particular, Callan, Giddings, Harvey and Strominger (CGHS) [3] have proposed a two-dimensional (2D) model in which the classically exact solutions to field equations correspond to black holes. Their model therefore has been considered as a theoretical laboratory to study the essence of black hole physics.

The mass parameter in their black hole solution turns out to be arbitrary, and cannot be related to the parameters of the theory. So, we are lead to wonder whether there is some sort of degeneracy. One of our findings in this letter is that in fact the global symmetry found in ref. [6] is responsible for “degeneracy” of the black hole background and at the same time for decoupling of the matter. To see this, we start by writing down the classical action of CGHS [3]

\[ S = \int d^2\sigma \sqrt{-g} \left\{ \exp(-2\phi) \left[ R(g) + 4 g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi + 4 \mu^2 \right] - \frac{1}{2} \sum_{i=1}^{n} g^{\alpha\beta} \partial_\alpha f_i \partial_\beta f_i \right\}, \quad (\alpha, \beta = 0, 1), \]

where \( f \)'s stand for matter fields, \( \phi \) for the dilaton field, \( \mu^2 \) for a cosmological constant, and the curvature scalar for the metric \( g_{\alpha\beta} \) is denoted by \( R(g) \). In addition to 2D general coordinate transformations, the action is invariant under the global, non-linear transformation defined by [3]

\[ \phi \rightarrow \phi' = \phi - \frac{1}{2} \ln(1 + \Lambda \exp(2\phi)), \quad g_{\alpha\beta} \rightarrow g'_{\alpha\beta} = g_{\alpha\beta}(1 + \Lambda \exp(2\phi))^{-1}, \]

where \( \Lambda \) is a constant parameter. The conservation of the corresponding current, \( \partial_\alpha j^\alpha = \sqrt{-g} \, R(\hat{g}) = 0 \), expresses the flatness of \( \hat{g}_{\alpha\beta} \), where \( \hat{g}_{\alpha\beta} \equiv g_{\alpha\beta} \exp(-2\phi) \). (The importance of this current conservation in investigating the Hawking radiation in two dimensions has been emphasized in ref. [4], too.)

This global symmetry manifests itself in the arbitrariness of classical solutions. An easy way to see this is to work in the conformal gauge, \( g_{\alpha\beta} = \eta_{\alpha\beta} \exp(2\rho) \). One finds the equations of motion, \( \partial_\pm \exp(-2\phi) + \mu^2 \exp(2\rho - \phi) = 0 \), \( \partial_\pm \partial_\pm (\rho - \phi) = 0 \), along with two constraints (that are equivalent to \( \varphi_\pm (8) \) below in the conformal gauge). CGHS have

\[ ^{1}\text{See refs. [1, 3] for instance.} \]
found that the above set of equations admits a classical solution

\[
\frac{1}{2} \sum_{i=1}^{n} (\partial_+ f_i)^2 = -a \delta(\sigma^+ - \sigma_0^+) \, \, , \, \, \rho = \phi = \rho_{\text{BH}} \, ,
\]
\[
\exp(-2\rho_{\text{BH}}) = a(\sigma^+ - \sigma_0^+) \Theta(\sigma^+ - \sigma_0^+) - \mu^2 \sigma^+ \sigma^- \, ,
\] (3)

which describes the formation of a black hole of mass \(\mu a \sigma_0^+\) by an \(f\) shock wave traveling in the \(\sigma^-\)-direction. However, the solution (3) is not a unique solution for the given shock wave. One easily verifies that \(\exp(-2\rho) = \exp(-2\phi) = \Lambda + \exp(-2\rho_{\text{BH}})\) is a solution (with mass \(\mu(a \sigma_0^+ + \Lambda)\)) too, where \(\Lambda\) is an arbitrary constant. Since under the global transformation, \(\exp(-2\phi)\) and \(\exp(-2\rho)\) change to \(\Lambda + \exp(-2\phi)\) and \(\Lambda \exp(2(\phi - \rho) + \exp(-2\rho)\), respectively, it is clear that the arbitrariness of the black hole background originates from the global symmetry defined by (2).

The gravitational radius in the Schwarzschild solution in \(D = 4\) is arbitrary, too. It is in fact possible to find a similar symmetry there. Although this arbitrariness and the existence of a corresponding symmetry in both dimensions are closely related to the arbitrariness in choosing boundary conditions, there is a crucial difference in both dimensions: Thanks to the Weyl invariance of the matter coupling in \(D = 2\), the symmetry (2) is exact at the classical level even in the presence of the matter. That is, the matter does not feel the gravity mediated by the Weyl degree of freedom at the classical level, unless the matter coupling violates the Weyl invariance. (A test particle, for instance, would feel the black hole because its coupling violates the symmetry (2).)

At the quantum level, Weyl symmetry is anomalous in general \footnote{\textsuperscript{8}}. Therefore, the global symmetry (2) may be anomalous too, because it is closely related to the Weyl invariance. But the question of whether or not the global symmetry really remains intact in quantum theory certainly requires an independent investigation.

The subsequent sections are devoted to perform an exhaustive, algebraic analysis to find possible anomalies in the CGHS theory (1). We will use the method which we have developed in ref. \footnote{\textsuperscript{9}} on the basis of the extended phase space method of Batalin, Fradkin and Vikovisky (BFV) \footnote{\textsuperscript{10}}. The advantage of using this method is that the results so obtained are independent of the choice of gauge and regularization. This feature of the method is desirable, especially in a situation in which anomaly might cause “measurable”
effects, such as the Hawking radiation in the present case \(^2\). Throughout this paper we shall assume that space time singularities do not influence the invariance property such that we may perform our analysis without taking into account the presence of a black hole background explicitly.

We shall deal with classical, cohomological problems whose cohomologically non-trivial solutions exhibit potential anomalies. Under a certain set of reasonable assumptions, we will show that the global symmetry (2) as well as the BRST symmetry in the CGHS theory are free from an anomaly, to all orders in \(\hbar\). Therefore, the matter in their model does not feel the existence of a black hole at all, and there is nothing to radiate \(^3\).

To apply the BFV quantization method \([10]\), we have to go to the Hamiltonian formulation of the theory, and we use the parametrization of \(g_{\alpha \beta}\), according to Arnowitt, Deser, and Misner:

\[
g_{\alpha \beta} \equiv \begin{pmatrix} -\lambda^+\lambda^- & (\lambda^+ - \lambda^-)/2 \\ (\lambda^+ - \lambda^-)/2 & 1 \end{pmatrix} \exp 2\rho .
\]  

(4)

In terms of these new variables, the original action can be re-written as (we use the abbreviations \(\dot{f} = \partial_0 f = \partial_\tau f\), \(f' = \partial_1 f = \partial_\sigma f\))

\[
S = \int d^2\sigma \left\{ \frac{\psi'}{\lambda^+ + \lambda^-}\left[ 2(\lambda^+ - \lambda^-)(\dot{\rho} - \dot{\phi}) + 4\lambda^+\lambda^- (\rho' - \phi') + 2(\lambda^+\lambda^-)' \right] \\
+ \frac{\dot{\psi}}{\lambda^+ + \lambda^-}\left[ 4(\rho - \phi) + 2(\lambda^+ - \lambda^-)(\rho' - \phi') + 2(\lambda^+ - \lambda^-)' \right] \\
+ 2\mu^2 (\lambda^+ + \lambda^-) \exp 2(\rho - \phi) \\
+ \frac{1}{\lambda^+ + \lambda^-} \sum_{i=1}^n \left[ \dot{f}_i \dot{f}_i - (\lambda^+ - \lambda^-) f_i f_i' - (\lambda^+\lambda^-) f_i' f_i' \right] \right\} ,
\]

(5)

where \(\psi = \exp(-2\phi)\). The conjugate momenta to \(\lambda^\pm, \rho, \phi\) and \(f_i\) are respectively defined as

\[
\pi^\lambda_+ = 0 , \quad \pi^\lambda_- = 0 ,
\]

(6)

\[
\pi_\rho = (\lambda^+ + \lambda^-)^{-1}\left[ 2(\lambda^+ - \lambda^-)\psi' - 4\psi \right] ,
\]

\[
\pi_\phi = -\pi_\rho - 2(\lambda^+ + \lambda^-)^{-1}\psi\left[ -4(\dot{\rho} - \dot{\phi}) + 2(\lambda^+ - \lambda^-)(\rho' - \phi') + 2(\lambda^+ - \lambda^-)' \right] ,
\]

(7)

\(^2\)There is in fact some gauge-dependent observation in 2D cosmology \([11]\).

\(^3\)Our observation might be related to the recent result \([12]\) on the equivalence between \(c = 1\) conformal field theory and the Wess-Zumino-Witten model based on \(SL(2,R)/U(1)\).
\[ \pi_f^i = (\lambda^+ + \lambda^-)^{-1} \left[ 2 \dot{f}_i - (\lambda^+ - \lambda^-) f'_i \right]. \]

As can be seen from (6), \( \pi^\lambda_\pm \) are primary constraints, and generate the secondary constraints

\[ \varphi_\pm = -2\mu^2 \exp(2(\rho - \phi)) + (\partial_\sigma - Y_\pm)(\psi' \mp \frac{1}{2} \pi_\rho) + \frac{1}{4} \sum_{i=1}^n (\pi_{f_i}^i \pm f'_i)^2, \]

with \( Y_\pm \equiv (\rho - \phi)' \pm \frac{1}{4} \psi (\pi_\rho + \pi_\phi), \)

which satisfy under Poisson bracket the Virasoro algebra:

\[ \{ \varphi_\pm(\sigma), \varphi_\pm(\sigma') \}_PB = \mp(\varphi_\pm(\sigma)\partial_{\sigma'} - \varphi_\pm(\sigma')\partial_{\sigma}) \delta(\sigma - \sigma'), \]

\[ \{ \varphi_\pm(\sigma), \varphi_\mp(\sigma') \}_PB = 0. \]

Since there are four first-class constraints, the theory without the matter would have no physical degree of freedom \[13\].

According to BFV, we define the extended phase space by including to the classical phase space the ghost-auxiliary field sector

\[ (C^A, \overline{P}_A), (P^A, \overline{C}_A), (N^A, B_A), \]

where \( A(=\lambda^\pm, \pm) \) labels the first-class constraints given in (6) and (8) \[4\]. We assign 0 to the canonical dimension of \( \phi, \lambda^\pm \) and \( \rho \), and correspondingly +1 to \( \pi_\phi, \pi^\lambda_\pm \), and \( \pi_\rho \). The canonical dimensions of \( C^\pm_\lambda, \overline{P}_\pm \) and \( \overline{C}^\pm_\lambda \) are fixed only relative to that of \( C^\pm, c \equiv \dim(C^\pm): \)

\[ \dim(C^\pm_\lambda) = 1 + c, \quad \dim(\overline{P}_\pm) = 1 - c, \quad \dim(\overline{C}^\pm_\lambda) = -c. \]

The canonical dimensions of other fields are not needed for our purpose.

Given the first-class constraints with the corresponding algebra, we can construct a BRST charge

\[ Q = \int d\sigma [ C^\pm_\lambda \pi^\lambda_\pm + C^\mp_\lambda \pi^\lambda_\mp + C^+(\varphi_+ + \overline{P}_+ \partial_\sigma C^+) \\
+ C^-(\varphi_- - \overline{P}_- \partial_\sigma C^-) + B_A P^A ] , \quad (A = \lambda^\pm, \pm) \]

with gh\((Q) = 1, \dim(Q) = 1 + c),

\[ 4 \quad C^A \text{ and } P^A \text{ are the BFV ghost fields carrying one unite of ghost number, } \text{gh}(C^A) = \text{gh}(P^A) = 1, \]

while \( \text{gh}(\overline{C}_A) = \text{gh}(\overline{P}_A) = -1 \) for their canonical momenta, \( \overline{P}_A \) and \( \overline{C}_A \). The last canonical pairs in (10) are auxiliary fields and carry no ghost number.
which satisfies
\[ \{ Q , Q \}_{PB} = 0 . \] (13)

We would like to emphasize that the BRST charge (12) is given prior to gauge fixing. The gauge fixing appears in defining the total Hamiltonian \( H_T \). Since the canonical Hamiltonian vanishes in the present case, it is given by the BRST variation of gauge fermion \( \Psi \), \( H_T = \{ Q , \Psi \}_{PB} \), which immediately leads to
\[ \{ Q , H_T \}_{PB} = \frac{d}{dt} Q = 0 . \] (14)

In terms of the phase space variables, the charge \( W \), which generates the non-linear symmetry transformation (2), can be written as
\[ W = -\frac{1}{2} \int d\sigma (\pi_\phi + \pi_\rho) \psi^{-1} = -\int d\sigma (Y_+ - Y_-) , \] (15)
where \( Y_\pm \) are defined in (8), and its Poisson brackets with \( Q \) and \( H_T \) are
\[ \{ Q , W \}_{PB} = 0 , \] (16)
\[ \{ H_T , W \}_{PB} = 0 . \] (17)

Eqs. (13), (14), (16) and (17) are the basic bracket relations which exhibit the 2D general covariance and the global invariance at the classical level.

3 The “nilpotency of \( Q \)” expressed in eq. (13) means that the underlying constraints in the theory are first-class while eq. (14) expresses the consistency of the constraints with the dynamics of the system. Eqs. (16) and (17) exhibit the presence of the global symmetry. Note however that because of the relation, \( H_T = \{ Q , \Psi \}_{PB} \), eqs. (14) and (17) are consequences of (13) and (16). We therefore shall consider only (13) and (16) as the fundamental bracket relations.

At the quantum level, these quantities must be suitably regularized to become well-defined. An anomaly arises if the fundamental algebra between \( Q \) and \( W \) can not be maintained upon quantization, and the anomalous Schwinger terms, which exhibit an anomaly in the algebra, may be expanded in \( \bar{\hbar} \) as
\[ [Q , Q] \equiv i \sum_{n=2} h^n \Omega^{(n)} , \] \[ [Q , W] \equiv i \frac{1}{2} \sum_{n=2} h^n \Xi^{(n)} , \] (18)
where $[ , ]$ denotes super-commutator. Our basic assumption \cite{9,14} is that the super-commutation relations between $Q$ and $W$ obey: (i) the commutation law, (ii) the distribution law, (iii) the super-Jacobi identity, and (iv) $[A , B] = i\hbar \{A , B\}_{\text{PB}} + O(\hbar^2)$.

One easily observes that the outer commutators in the super-Jacobi identities for $Q$ and $W$,

$$[Q , [Q , Q ] ] = 0 , \quad 2 [Q , [Q , W ] ] + [W , [Q , Q ] ] = 0 ,$$

define a set of consistency conditions at each order in $\hbar$ in terms of Poisson brackets. In the lowest order, one finds that \cite{3} \cite{4}

$$\delta \Omega^{(2)} = 0 ,$$

$$\delta \Xi^{(2)} = \{W , \Omega^{(2)}\}_{\text{PB}} ,$$

where $\delta$ is the BRST transformation which is defined by $\delta A = -\{Q , A\}_{\text{PB}}$. The true anomalies $\Omega^{(2)}$ and $\Xi^{(2)}$ should be cohomologically non-trivial; if $\Omega^{(2)}$ and $\Xi^{(2)}$ are solutions, then $\Omega^{(2)} + \delta X$ and $\Xi^{(2)} + \{W , X\}_{\text{PB}} + \delta Y$ also solve (20) and (21), respectively. They can be then absorbed to order $\hbar^2$ into a re-definition of $Q$ and $W$, defined by $Q \rightarrow Q - (\hbar X/2)$ and $W \rightarrow W - (\hbar Y/2)$. Furthermore, if there is no non-trivial solution to (20) (this is the case in the CGHS theory as we shall see later), the non-trivial $\Xi^{(2)}$ is a non-trivial solution to the homogeneous part of (21):

$$\delta \Xi^{(2)}_h = 0 .$$

At this stage we would like to emphasize that the consistency conditions of $O(\hbar^2)$ are exactly the same as those of $O(\hbar^3)$ if there is no non-trivial solution at $O(\hbar^2)$. That is, the non-existence of the non-trivial solutions to the consistency conditions (20) and (22) is sufficient for the non-existence of those to the higher order consistency conditions.

Our main task is to solve the classical, algebraic problem defined by (20) and (21) (or (22) if there is no non-trivial solution to (20)). We seek solutions $\Omega^{(2)}$ and $\Xi^{(2)}$ in the form

$$\Omega^{(2)} = \int d\sigma \omega , \quad \Xi^{(2)}_h (\Xi^{(2)}_h) = \int d\sigma \xi (\xi_h) ,$$

where $\omega$ and $\xi (\xi_h)$ are polynomials of local operators with $\text{gh}(\omega) = 2$, $\text{gh}(\xi) = \text{gh}(\xi_h) = 1$, $\text{dim}(\omega) = 3 + 2c$, and $\text{dim}(\xi) = \text{dim}(\xi_h) = 2 + c$. According to the general structure
of the BFV formalism, the total phase space can be divided, with respect to the action of $\delta$, into two sectors:

$$S_1 \text{ consisting of } (f_i, \pi_f^i), (\rho, \pi_\rho), (\phi, \pi_\phi) \text{ and } (C^\pm, \overline{P}^\pm),$$

$$S_2 \text{ consisting of all the other fields}.$$ (24)

It is easy to see that on each sector the $\delta$ operation closes: $\delta_1^2 = \delta_2^2 = 0$, $\delta_1 \delta_2 + \delta_2 \delta_1 = 0$, where $\delta = \delta_1 + \delta_2$, and $\delta_1(\delta_2)$ acts on $S_1$ ($S_2$) variables only. The $S_2$-sector is BRST trivial because it is made of pairs $(U^a, V^a)$ with $\delta_2 U^a = \pm V^a$. As shown in ref. [9], there exists no non-trivial solution to (20) ((22)) if $\omega(\xi_h)$ contains the $S_2$-variables.

The linear independence of the generalized Virasoro constraints

$$\Phi_\pm \equiv -\delta \overline{P}_\pm = \varphi_\pm \mp 2\overline{P}_\pm C^\pm + \overline{P}_\pm C^\pm$$ (25)

and the fact that $\overline{P}_\pm$ is the only one which produces $\overline{P}_\pm$ under BRST transformation further implies that $\overline{P}_\pm$ can not be involved in the non-trivial part of $\omega$ and $\xi_h$. Therefore, $\omega$ and $\xi_h$ are functions of $f_i, \pi_f^i, \rho, \pi_\rho, \phi, \pi_\phi, C^\pm$ and their spatial derivatives only [5].

To proceed, we note that there is a time-independent reparametrization invariance (i.e., $\{\Phi_\pm, Q, W\}_\text{PB} = 0$). Its transformation can be generated by $\Phi_\pm$, and it is convenient to define covariant objects with respect to the transformations. The $Y_\pm$ given in eq. (8), for instance, are “gauge fields” transforming as

$$\delta_\pm Y_\pm = \pm u^\pm(\sigma)'' \pm [u^\pm(\sigma) Y_\pm]'$$ (26)

where $\delta_\pm \equiv -\{\Phi^u_\pm, \cdot\}_\text{PB}$ with $\Phi^u_\pm \equiv \int d\sigma u^\pm(\sigma) \Phi_\pm$. Then we may define the weight $w_\pm$ of a field $\chi$, according to

$$\delta_\pm \chi \equiv \pm u^\pm \chi' \pm w_\pm u^\pm \chi,$$ (27)

and the covariant derivatives by

$$D_\pm \equiv \partial_\sigma - Y_\pm w_\pm.$$ (28)

5We regard spatial total derivative terms as null.
Note that $\rho$ and $\phi$ do not transform covariantly and there is no bosonic field of weight zero. The only covariant quantities are:

\begin{align*}
Y_\pm & \quad \text{gauge fields} , \\
C^\pm & \quad \text{with } w_\pm = -1 , \\
F_{\pm}^i & \equiv \pi_j^i \pm f'_i \text{ with } w_\pm = 1 , \\
G_\pm & \equiv -2\mu^2 \exp 2(\rho - \phi) + (\partial_\sigma - Y_\pm)(\psi^\prime \mp \frac{1}{2}\pi_\rho) \text{ with } w_\pm = 2 ,
\end{align*}

and those which can be obtained by successive applications of the covariant derivatives thereon. The BRST transformations of these quantities are closed, as one can see from

\begin{align*}
\delta Y_\pm &= \pm (D_\pm C^\pm)' , \quad \delta C^\pm = \pm C^\pm C'^\prime , \\
\delta F_{\pm}^i &= \pm (C^\pm F_{\pm}^i)' , \quad \delta G_\pm = \pm C^\pm G'_\pm \pm 2C'^\prime G_\pm ,
\end{align*}

(30)

At this stage, we make our main assumption to reduce the complexity of our cohomological problem: We assume that the non-trivial parts of $\omega$ and $\xi_h$ are functions only of the quantities in (29) and their spatial derivatives and that they respect the discrete symmetry defined by $C^\pm \rightarrow C'^\mp , \overline{P}_\pm \rightarrow \overline{P}'_\pm , \partial_\sigma \rightarrow -\partial_\sigma$.

With this assumption in mind, we then group all the possible terms that may be present in $\omega$ and $\xi_h$:

\begin{align*}
g_{1(2)} & \quad \text{consisting of terms containing at least one } G_\pm (F_{\pm}^i) , \\
g_3 & \quad \text{stands for the rest}.
\end{align*}

(31)

It is clear that BRST transformations do not mix the terms of different groups.

We begin to solve (20). Since $\omega$ (dim($\omega$) = $3 + 2c$) must contain two $C$’s, each term of $g_1$ for $\omega$ has just one of $G_\pm$ (dim($G_\pm$) = 2). Exactly four terms that contain besides one of the gauge fields $Y_\pm$ come into the question. One can easily verify that they can never organize to a BRST invariant. As for the rest of $g_1$, one can write down four independent terms for $\omega$ that are consistent with our assumption. We find that all the BRST invariants are trivial, namely proportional to $\delta \left[ (\kappa_1 C^+ G_+ - \kappa_2 C^- G_+) - (+ \leftrightarrow -) \right]$, where $\kappa_{1,2}$ are arbitrary constants. The $g_2$-elements for $\omega$ can be further divided into two groups; the

\footnote{We have dropped $\overline{P}_\pm$ (with $w_\pm = 2$) from the list because it is not involved in $\omega$ and $\xi$ ($\xi_h$).}
one consisting of terms with one of $\sum_{i=1}^{n} F_i^+ F_i^\pm$, and the other one consisting of terms with one $\sum_{i=1}^{n} F_i^i F_i^\pm$. The BRST-transformation property of $\sum_{i=1}^{n} F_i^i F_i^\pm$ is the same as that of $G_{\pm}$, and this is the reason why the terms of the first group are absent in $\omega$. However, the second group of $g_2$ contains one non-trivial BRST invariant [14]:

$$\omega_2 = \kappa_3 \left[ (C^+ C^+ + C^- C^-) - (+ \leftrightarrow -) \right] \sum_{i=1}^{n} F_i^i F_i^\pm .$$  (32)

This matter-field dependent term $\omega_2$ is algebraically allowed as a $Q^2$-anomaly, but such matter-field dependent expressions have never appeared in the explicit calculations of anomalies. This may easily be understood if one employs Fujikawa’s method [15] to calculate anomalous terms. Therefore, we demand

$$\kappa_3 = 0 .$$  (33)

The third group, $g_3$, contains the Kato-Ogawa anomaly term [17],

$$\omega_{KO} = \kappa_{KO} \left[ (C^+ C^+ + C^- C^-) - (+ \leftrightarrow -) \right] ,$$  (34)

which however is trivial because of the identity

$$C^\pm' C^\pm'' = \mp \delta (C^\pm' Y_{\pm}) + (C^\pm C^\pm' Y_{\pm})' .$$  (35)

After similar algebraic calculations, one finds that no non-trivial BRST invariant in $g_3$ can be formed to become an independent part of $\omega$.

We finally would like to come to the solution of (21). Since $Q^2$ has turned out to be trivial, the true anomaly for the global non-linear symmetry corresponds to the non-trivial solution to the homogeneous equation (22). We have to perform algebraic calculations, similar to the previous ones but with ghost number and canonical dimension changed ($gh(\xi_h) = 1$, $\text{dim}(\xi_h) = 2 + c$). For $\xi_h$ there are two independent terms in $g_1$, consistent with our assumption. It is easy to find that they can not give any BRST invariant. Similarly, terms in $g_2$ with $\sum_{i=1}^{n} F_i^i F_i^\pm$ can not be present in $\xi_h$.

But, as the case for $\omega_2$, there is exactly one BRST invariant in $g_2$:

$$\xi_2 = \nu_3 (C^+ + C^-) \sum_{i=1}^{n} F_i^i F_i^\pm ,$$  (36)

---

7See ref. [10] for a method to calculate anomalous commutators from anomalous path-integral Jacobians.
where $\nu_3$ is an arbitrary constant. Terms in $g_3$ for $\xi_h$ must contain at least one of $Y_{\pm}$. Those with $(Y_+)^2$ or $(Y_-)^2$ can be simply excluded. The only possibility is

$$
\xi_3 = \nu_4 \left[ D_+ C^+ Y_- + D_- C^- Y_+ \right],
$$

$$
= \nu_4 \delta[(\rho - \phi) (Y_- - Y_+)] - \nu_4 \left\{ (\rho - \phi)(D_+ C^+ + D_- C^-) \right\}', \quad (37)
$$

which is trivial as the second equation indicates.

The same reason why $\omega_2 = 0$ (see (33)) can be applied to exclude that matter-field dependent term $\xi_2$ as an independent anomalous Schwinger term. We thus arrive at the conclusion that there is no non-trivial solutions to (20) and (22) and hence the BRST symmetry and the global symmetry (2) are exact to all orders in $\hbar$.

The general result on the trace anomaly of ref. [8] and the analysis on the relation between the trace anomaly and Hawking radiation [18] remain of course correct. But what we have found here implies that due to the very nature of the dilaton field the trace anomaly term can be absorbed into a re-definition of the various fields without violating the reparametrization invariance.

We thus have shown that the CGHS theory is free from the BRST anomaly and has an exact global invariance (2) that is responsible for decoupling of the matter. It is certainly worthwhile to collect our assumptions which have led to the conclusion: We have assumed that (I) the black hole background does not influence the invariance property of the theory so that we may perform our analysis on anomalies without taking into account its presence explicitly, (II) the commutators of $Q$ and $W$ satisfy the super-Jacobi identities (19), (III) the anomalous commutators can be expanded in $\hbar$, and (IV) the non-trivial solutions to the consistency conditions (20) and (22) involve only the quantities listed in (29).

The main reason why the BRST invariance in the CGHS model is intact is that the Kato-Ogawa anomaly term (34) (that corresponds to the central extension of the Virasoro algebra) is BRST trivial. That is, this term can be canceled by adding to the action a local counterterm of the form

$$
- \frac{\hbar}{2} \kappa_{KO} \int d^2 \sigma \left( N^+ Y_+ + N^- Y_- \right), \quad (38)
$$

which becomes

$$
- \frac{\hbar}{2} \kappa_{KO} \int d^2 \sigma \frac{1}{N^+ + N^-} \{ 2(N^+ - N^-)'(\dot{\rho} - \dot{\phi})
$$
\[ +2(N^+N^-)'(\rho' - \phi') - (N^+ - N^-)^2 ] , \quad (39) \]
in the standard gauge \((N^+ = \lambda^+, N^- = \lambda^-)\) \([14]\). The integrand is a total derivative in the conformal gauge \((N^+ = \lambda^+ = N^- = \lambda^- = 1)\), as one can easily see from (39). The contribution of this counterterm to the energy momentum tensor in the conformal gauge is
\[ \Delta T_{\pm\pm} = \frac{\hbar}{2} \kappa_{KO}' Y_{\pm}' = \hbar \kappa_{KO} \partial_{\pm}^2 (\rho - \phi) , \quad \Delta T_{\mp} = 0 , \quad (40) \]
which vanish on the black hole background (3). This should be contrasted to the case in string theory where any counterterm to cancel the term (34) necessarily contributes to the trace of the energy momentum tensor in a non-trivial manner \([8]\).

An independent verification of our result, that the BRST symmetry is anomaly-free and it is possible to redefine the charge \(W\) so as to be conserved and BRST invariant, by an explicit computation would be of course desirable. (There is some indication \([11, 19]\).) In this context it may be worth-mentioning that the case at hand is quite similar to that of the ghost-number current anomaly in string theory \([20]\) because at subcritical dimensions it is possible to redefine the charge associated with the ghost number conservation so as to commute with the BRST charge as well as the Hamiltonian \([20]\).

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