Special Right Jacobson Radicals for Right Near-rings

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Abstract. In this paper three more right Jacobson-type radicals, $J_{\nu}^r$, are introduced for near-rings which generalize the Jacobson radical of rings, $\nu \in \{0, 1, 2\}$. It is proved that $J_{\nu}^r$ is a special radical in the class of all near-rings. Unlike the known right Jacobson semisimple near-rings, a $J_{\nu}^r$-semisimple near-ring $R$ with DCC on right ideals is a direct sum of minimal right ideals which are right $R$-groups of type-$\nu$, $\nu \in \{0, 1, 2\}$. Moreover, a finite right $g_2$-primitive near-ring $R$ with $eRe$ a non-ring is a near-ring of matrices over a near-field (which is isomorphic to $eRe$), where $e$ is a right $g_2$-primitive idempotent in $R$.

1. Introduction

Special radicals for near-rings are introduced in [1] by G. L. Booth and N. J. Groenewald using equiprime near-rings. Among the known left Jacobson-type radicals, $J_3$, $J_{3(0)}$ are the only special radicals in the class of zero-symmetric near-rings and in the class of all near-rings respectively.

Srinivasa Rao and Siva Prasad [6, 7] introduced and studied $J^r_{\nu}$, the right Jacobson radical type-$\nu$, $\nu \in \{0, 1, 2\}$. In [9, 10] Srinivasa Rao and Siva Prasad along with T. Srinivas showed that $J^r_{\nu}$ is a Kurosh-Amitsur radical in the Fuchs variety $\mathcal{F}$ of all near-rings $R$ in which the constant part $R_c$ of $R$ is an ideal of $R$, $\nu \in \{0, 1, 2\}$. But $J^r_{\nu}$ is not s-hereditary in the class of all zero-symmetric near-rings and hence it is not an ideal-hereditary radical in that class, $\nu \in \{0, 1, 2\}$.

Also in [5][11] Srinivasa Rao and Siva Prasad (along with T. Srinivas) intro-
duced and studied the right Jacobson type of radical $J^R_{\nu(e)}$, $\nu \in \{1, 2\}$ ($J^R_{0(e)}$) and showed that it is a Kurosh-Amitsur radical in the class of all near-rings and is an ideal hereditary Kurosh-Amitsur radical in the class of all zero-symmetric near-rings. Moreover, they are special radicals in the class of all near-rings.

In this paper we introduce three more right Jacobson radicals, $J^R_{g_\nu}$, $\nu \in \{0, 1, 2\}$. We show that they are special radicals in the class of all near-rings. So, in the class of all near-rings, they are Kurosh-Amitsur radicals, their semisimple classes are hereditary and radicals classes are c-hereditary. Unlike the known right Jacobson semisimple near-rings, a $J^R_{g_0}$-semisimple near-ring $R$ with DCC on right ideals is a direct sum of right ideals which are right $R$-groups of type-$g_0$, $\nu \in \{0, 1, 2\}$. A finite right $g_2$-primitive near-ring $R$ with $eRe$ a non-ring is a near-ring of matrices over a near-field (which is isomorphic to $eRe$), where $e$ is a right $g_2$-primitive idempotent in $R$.

Near-rings considered are right near-rings (not necessarily zero-symmetric) and $R$ is a near-ring. Now we present some definitions and results of [6] and [7].

A group $(G, +)$ is called a right $R$-group if there is a mapping $(g, r) \rightarrow gr$ of $G \times R$ into $G$ such that (1) $(g + h)r = gr + hr$, (2) $g(rs) = (gr)s$ for all $g, h \in G$ and $r, s \in R$. A subgroup (normal subgroup) $H$ of a right $R$-group $G$ is called an $R$-subgroup (ideal) of $G$ if $hr \in H$ for all $h \in H$ and $r \in R$.

Let $G$ be a right $R$-group. An element $g \in G$ is called a generator of $G$ if $gR = G$ and $g(r + s) = gr + gs$ for all $r, s \in R$. $G$ is said to be monogenic if $G$ has a generator. $G$ is said to be simple if $G \neq \{0\}$ and $G$, and $\{0\}$ are the only ideals of $G$.

A monogenic right $R$-group $G$ is said to be a right $R$-group of type-$0$ if $G$ is simple.

A right $R$-group $G$ of type-$0$ is said to be of type-$1$ if $G$ has exactly two $R$-subgroups, namely $\{0\}$ and $G$.

A right $R$-group $G$ of type-$0$ is said to be of type-$2$ if $gR = G$ for all $0 \neq g \in G$.

Note that a right $R$-group of type-$2$ is of type-$1$ and a right $R$-group of type-$1$ is of type-$0$.

Let $\nu \in \{0, 1, 2\}$. A right modular right ideal $K$ of $R$ is called right $\nu$-modular if $R/K$ is a right $R$-group of type-$\nu$.

An ideal $P$ of $R$ is called right $\nu$-primitive if $P$ is the largest ideal of $R$ contained in a right $\nu$-modular right ideal of $R$. $R$ is called a right $\nu$-primitive near-ring if $\{0\}$ is a right $\nu$-primitive ideal of $R$.

Now we present some definitions of [11] and [5].

Let $G$ be a right $R$-group of type-$\nu$, $\nu \in \{0, 1, 2\}$. Suppose that $G0 = \{0\}$ for $\nu = 0$ and $P$ is the largest ideal of $R$ contained in $(0 : G) = \{r \in R \mid Gr = \{0\}\}$. Then $G$ is said to be a right $R$-group of type-$\nu(e)$ if $0 \neq g \in G, r_1, r_2 \in R$ and $gxr_1 = gxr_2$ for all $x \in R$ implies $r_1 - r_2 \in P$.

A right modular right ideal $K$ of $R$ is called right $\nu(e)$-modular if $R/K$ is a right $R$-group of type-$\nu(e)$.

Let $G$ be a right $R$-group of type-$\nu(e)$. Then $(0 : G)$ is an ideal of $R$ and is called a right $\nu(e)$-primitive ideal of $R$.  

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A near-ring $R$ is called right $\nu(e)$-primitive if $\{0\}$ is a right $\nu(e)$-primitive ideal of $R$.

A near-ring $R$ is called an equiprime near-ring [2] if $0 \neq a \in R$, $x, y \in R$ and $arx = ary$ for all $r \in R$, implies $x = y$. An ideal $I$ of $R$ is called equiprime if $R/I$ is an equiprime near-ring. Moreover, an equiprime near-ring is zero-symmetric.

It is known that a near-ring $R$ is equiprime if and only if ([2])

1. $x, y \in R$ and $xRy = \{0\}$ implies $x = 0$ or $y = 0$.
2. If $\{0\} \neq I$ is an invariant subnear-ring of $R$, $x, y \in R$ and $ax = ay$ for all $a \in I$ implies $x = y$.

In [1], G. L. Booth and N. J. Groenewald defined special radicals for near-rings. A class $E$ consisting of equiprime near-rings is called a special class if it is hereditary and closed under left invariant essential extensions. If $\mathcal{R}$ is the upper radical in the class of all near-rings determined by a special class of near-rings, then $\mathcal{R}$ is called a special radical. A class of near-rings $\mathcal{E}$ is said to satisfy condition $F_1$ if $J < I < R$ and $I$ is left invariant in $R$ and $I/J \in \mathcal{E}$ implies $J < R$. We need the following theorem:

**Theorem 1.1.** ([12]) Let $\mathcal{E}$ be a class of zero-symmetric near-rings. If $\mathcal{E}$ is regular, closed under essential left invariant extensions and satisfies condition ($F_1$), then $\mathcal{R} := \{\mathcal{E} : \mathcal{E}\}$ is a c-hereditary radical class in the variety of all near-rings, $\mathcal{SR} = \mathcal{E}$ and $\mathcal{SR}$ is hereditary. So, $\mathcal{R}(R) = \cap \{I < R \mid R/I \in \mathcal{E}\}$ for any near-ring $R$.

### 2. Right Jacobson Radicals of Type-$g_0$

Let $G$ be a right $R$-group and $T$ be a subset of $G$. Then $(0 : T) := \{r \in R \mid t r = 0$ for all $t \in T\}$. By Proposition 3.7 of [11], if $G$ is a right $R$-group of type-$0$ and $G0 = \{0\}$, then there is a largest ideal of $R$ contained in $(0 : G)$. Moreover, by Proposition 3.1 of [5], if $G$ is a right $R$-group of type-$\nu$, then $G0 = \{0\}$, $\nu \in \{1, 2\}$.

**Definition 2.1.** Let $\nu \in \{0, 1, 2\}$. Let $G$ be right $R$-group of type-$\nu$ and $G0 = \{0\}$ for $\nu = 0$, and $T$ be the set of all generators of the right $R$-group $G$. Then $G$ is said to be a right $R$-group of type-$g_0$ if $(0 : T) = P$, where $P$ is the largest ideal of $R$ contained in $(0 : G)$.

We present an example of a right $R$-group of type-$g_0$ which is not of type-$g_1$.

**Example 2.2.** Let $(G, +)$ be a finite non-abelian simple group. Since $\{0\}$ is the maximal normal subgroup of $(G, +)$, $\{0\}$ is the maximal right ideal of $M_0(G)$ and hence $M_0(G)$ is a right $M_0(G)$-group of type-$0$. This example was considered in [7] and it was shown that $M_0(G)$ is not a right $M_0(G)$-group of type-$1$. Each $0 \neq h \in G$ give rise to the inner automorphism $t_h$ of $G$ defined by $t_h(x) = h + x - h$ for all $x \in G$. Clearly, a generator of the right $M_0(G)$-group $M_0(G)$ is an automorphism of $(G, +)$. Let $T$ be the set of all automorphisms of $G$. Suppose that for some $t \in M_0(G)$ and $0 \neq h \in G$, $t_h t = 0$. Now $0 = (t^{-1}h)t_h = (t_h)^{-1}t_h t = t$. Therefore $\{0\} = (0 : t_h) = (0 : T)$. Since the largest ideal of $M_0(G)$ contained in $(0 : M_0(G))$ is $\{0\}$, $M_0(G)$ is a right $M_0(G)$-group of type-$g_0$ but not of type-$g_1$.

Now we present an example of a right $R$-group of type-$g_1$ which is not of type-$g_2$. 

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Example 2.3. Let \((G, +)\) be a finite cyclic group of prime order \(p\), where \(p \neq 2\). Since \(\{0\}\) is the only proper subgroup of \(G\), \(\{0\}\) is the only proper right \(M_0(G)\)-subgroup of \(M_0(G)\). Therefore, \(M_0(G)\) is a right \(M_0(G)\)-group of type-1. Clearly, \(M_0(G)\) is not a right \(M_0(G)\)-group of type-2, as \(M_0(G)\) is not a near-field. This example was considered in [7]. A generator of the right \(M_0(G)\)-group \(M_0(G)\) an is automorphism \((G, +)\). We know that \(G\) has \(p - 1\) automorphisms. Let \(T\) be the set of all these automorphisms. Suppose that for some \(s \in M_0(G)\) and \(t \in T\), \(ts = 0\). Now \(0 = (t^{-1})ts = s\). So \(\{0\} = (0 : t) = (0 : T)\). Since the largest ideal of \(M_0(G)\) contained in \(0 : M_0(G)\) is \(\{0\}\), \(M_0(G)\) is a right \(M_0(G)\)-group of type-\(g_1\) but not of type-\(g_2\).

The following are examples of right \(R\)-groups of type-\(g_2\).

Example 2.4. Let \(R\) be a near-field. Then \(R\) is a right \(R\)-group of type-2. Clearly, \(R\) is also a right \(R\)-group of type-\(g_2\).

Example 2.5. Let \((R, +)\) be a group and let \(K\) be a subgroup of \((R, +)\) of index 2. The trivial multiplication on \((R, +)\) determined by \(R \setminus K\) given by \(a.b = a\) if \(b \in R \setminus K\) and 0 if \(b \in K\). Now \((R, +, \cdot)\) is a near-ring. It is clear that \(K\) is a maximal (right) ideal of \(R\). Let \(a \in R \setminus K\). Now \(R = K \cup a + K\). It can be easily verified that \(a + K\) is the generator of the right \(R\)-group \(R/K\). So \(R/K\) is a right \(R\)-group of type-2 and \((0 : a + K) = (0 : R/K)\) is the largest ideal of \(R\) contained in \((0 : R/K)\). Hence \(R/K\) is a right \(R\)-group of type-\(g_2\).

Now we introduce some notions related to the right \(R\)-groups of type-\(g_ν\).

Definition 2.6. Let \(ν \in \{0, 1, 2\}\) and \(K\) be a right modular right ideal of \(R\). Then \(K\) is said to be right \(g_ν\)-modular right ideal of \(R\) if \(R/K\) is a right \(R\)-group of type-\(g_ν\).

Definition 2.7. Let \(ν \in \{0, 1, 2\}\). An ideal \(P\) of \(R\) is called a right \(g_ν\)-primitive ideal of \(R\) if \(P\) is the largest ideal of \(R\) contained in \((0 : G) := \{r \in R | Gr = \{0\}\}\) for some right \(R\)-group \(G\) of type-\(g_ν\).

Definition 2.8. Let \(ν \in \{0, 1, 2\}\). A near-ring \(R\) is called a right \(g_ν\)-primitive near-ring if \(\{0\}\) is a right \(g_ν\)-primitive ideal of \(R\).

Definition 2.9. Let \(ν \in \{0, 1, 2\}\). The intersection of all right \(g_ν\)-primitive ideals of \(R\) is called the right Jacobson radical of \(R\) of type-\(g_ν\), and is denoted by \(J_{g_ν}(R)\). If \(R\) has no right \(g_ν\)-primitive ideals, then \(J_{g_ν}(R)\) is defined to be \(R\).

Note that if \(R\) is a ring then \(J_{g_ν}(R) = J(R)\), where \(J\) is the Jacobson radical of \(R\).

By Proposition 3.1 of [11], for a right \(R\)-group \(G\), \(G_0 = \{0\}\) if and only if \(GR_\infty = \{0\}\). Since for a right \(R\)-group \(G\) of type-\(g_ν\), \(G_0 = \{0\}\), \(R_\infty\) is contained in \((0 : g)\) for every generator \(g\) of \(G\). So \(R_\infty \subseteq P\) for every right \(g_ν\)-primitive ideal \(P\) of \(R\). Hence a right \(g_ν\)-primitive ideal \(P\) of \(R\) is invariant. This shows that a right \(g_ν\)-primitive near-ring is zero-symmetric.
Proposition 2.10. Let \( \nu \in \{0,1,2\} \). An ideal \( P \) of \( R \) is a right \( g_\nu \)-primitive ideal of \( R \) if and only if \( P \) is the largest ideal of \( R \) contained in a right \( g_\nu \)-modular right ideal of \( R \).

Proof. Let \( P \) be a right \( g_\nu \)-primitive ideal of \( R \). There is a right \( R \)-group \( G \) of type-\( g_\nu \) such that \( P \) is the largest ideal of \( R \) contained in \( (0 : G) \). Let \( g_0 \) be a generator of the right \( R \)-group \( G \). The mapping \( r \to g_0 r \) is a right \( R \)-homomorphism of \( R \) on to \( G \) with kernel \( K := (0 : g_0) \). So \( R/K \) is right \( R \)-isomorphic to \( G \) (as right \( R \)-groups). Now \( K \) is a right \( g_\nu \)-modular right ideal of \( R \) and \( P \) is contained in \( K \). Let \( Q \) be the largest ideal of \( R \) contained in \( K \). Now \( GQ = \{0\} \), that is, \( Q \subseteq (0 : G) \) as \( RQ \subseteq Q \), \( Q \) being invariant ideal of \( R \). Since \( P \) is the largest ideal of \( R \) contained in \( (0 : G) \), \( Q \subseteq P \). Now \( P \subseteq Q \) as \( Q \) is the largest ideal of \( R \) contained in \( K \). Therefore \( P = Q \), that is, \( P \) is the largest ideal of \( R \) contained in \( K \). On the other hand suppose that \( P \) is the largest ideal of \( R \) contained in a right \( g_\nu \)-modular right ideal \( K \) of \( R \). Now \( G := R/K \) is a right \( R \)-group of type-\( g_\nu \). We have \((0 : G) = (0 : R/K) = (K : R) \) and \( RP \subseteq P \) as \( P \) is an invariant ideal of \( R \). So \( P \subseteq (K : R) \). Let \( T \) be the largest ideal of \( R \) contained in \((K : R) = \{r \in R \mid Rr \subseteq K\} \). Since \( P \) is an invariant ideal of \( R \), and \( P \subseteq T \), \( T \) is an invariant ideal of \( R \). So \( RT \subseteq T \). Let \( K \) be right modular by \( e \). Now \( r - er \in K \) for all \( r \in R \). We have \( t - et \in K \) for all \( t \in T \). Since \( RT \subseteq T, T \subseteq K \). Since \( P \) is the largest ideal of \( R \) contained in \( K \), \( T \subseteq P \). So \( T = P \). Now \( P \) is the largest ideal of \( R \) contained in \((K : R) \) and hence \( P \) is a right \( g_\nu \)-primitive ideal of \( R \). \( \square \)

Proposition 2.11. Let \( \nu \in \{0,1,2\} \). An ideal \( P \) of \( R \) is a right \( g_\nu \)-primitive ideal of \( R \) if and only if \( R/P \) is a right \( g_\nu \)-primitive near-ring.

Proof. Let \( \nu \in \{0,1,2\} \) and \( P \) be an ideal of \( R \). Suppose that \( P \) is a right \( g_\nu \)-primitive ideal of \( R \). So, we get a right \( g_\nu \)-modular right ideal \( M \) of \( R \) such that \( P \) is the largest ideal of \( R \) contained in \( M \). Now \( M/P \) is a right \( g_\nu \)-modular right ideal of \( R/P \). Since \( P \) is the largest ideal of \( R \) contained in \( M \), the zero ideal of \( R/P \) is the largest ideal of \( R/P \) contained in \( M/P \). Therefore, \( R/P \) is a right \( g_\nu \)-primitive near-ring. Suppose now that \( R/P \) is a right \( g_\nu \)-primitive near-ring. So, we get a right \( g_\nu \)-modular right ideal \( M/P \) of \( R/P \) such that the zero ideal of \( R/P \) is the largest ideal of \( R/P \) contained in \( M/P \). Clearly, \( M \) is a right \( g_\nu \)-modular right ideal of \( R \). Since the zero ideal of \( R/P \) is the largest ideal of \( R/P \) contained in \( M/P \), \( P \) is the largest ideal of \( R \) contained in \( M \). Therefore, \( P \) is a right \( g_\nu \)-primitive ideal of \( R \). \( \square \)

Proposition 2.12. \( J^*_{g_\nu} \) is the H"ochhne radical determined by the class of all right \( g_\nu \)-primitive near-rings, \( \nu \in \{0,1,2\} \).

Theorem 2.13. Let \( G \) be a right \( R \)-group of type-\( g_\nu \) and \( S \) be an invariant subnear-ring (and right ideal for \( \nu = 0 \)) of \( R \) with \( GS \neq \{0\} \). Then \( G \) is a right \( S \)-group of type-\( g_\nu \), \( \nu \in \{0,1,2\} \).
Proof. If $G$ is a right $R$-group of type-0 and $S$ is an invariant subnear-ring and right ideal of $R$ with $GS \neq \{0\}$, then under the restriction of $G$ to $S$, by Theorem 3.2 of [9], $G$ is a right $S$-group type-0. Also if $G$ be a right $R$-group of type-$\nu$ and $S$ is an invariant subnear-ring of $R$ with $GS \neq \{0\}$, then under the restriction of $G$ to $S$, by Theorems 3.1 and 3.2 of [10], $G$ is a right $S$-group type-$\nu$, where $\nu \in \{1, 2\}$. Therefore $G$ is a right $S$-group of type-$\nu$, $\nu \in \{0, 1, 2\}$. Let $A$ be the set of generators of the right $R$-group $G$ and $P$ be the largest ideal of $R$ contained in $(0 : G)_R := \{r \in R \mid Gr = \{0\}\}$. A generator of the right $R$-group $G$ is also a generator of the right $S$-group $G$. From the proof of Theorem 3.10 of [9] (and Theorems 3.9 and 3.10 of [10] for $\nu \in \{1, 2\}$) as the extension of $G$ from $S$ to $R$ coincides with the action of $G$ on $R$, it follows that a generator of the right $S$-group $G$ is also a generator of the right $R$-group $G$. So $A$ is the set of generators of the right $S$-group $G$. We have $P = (0 : A) = \{r \in R \mid ar = 0 \text{ for all } a \in A\}$. Now $P \cap S = (0 : A) \cap S = \{s \in S \mid As = \{0\}\}$. Let $Q$ be the largest ideal of $S$ contained in $(0 : G)_S := \{s \in S \mid Gs = \{0\}\} = (0 : G) \cap S$. Clearly $P \cap S \subseteq (0 : G)_S$. By the definition of $Q$, $P \cap S \subseteq Q$. Since $AQ = \{0\}$, $Q \subseteq P$. So $Q \subseteq P \cap S$. Therefore $Q = P \cap S$. Hence $G$ is a right $S$-group of type-$g_{\nu}$.\hfill $\square$

Proposition 2.14. A right $R$-group of type-$g_{\nu}$ is an $R$-group of type-$\nu(e)$, $\nu \in \{0, 1, 2\}$.

Proof. Let $G$ be a right $R$-group of type-$g_{\nu}$, $\nu \in \{0, 1, 2\}$. So $G$ is a right $R$-group of type-$\nu$. In view of Remark 3.9 of [11] $G$ is a right $R$-group of type-$\nu(e)$ if $r, s \in R$ and $gr = gs$ for all $g \in G$, then $r - s \in P$ where $P$ is the largest ideal of $R$ contained in $(0 : G)$ := \{r \in R \mid Gr = \{0\}\}. Let gr = gs for all $g \in G$, $r, s, \in R$ and $P$ be the largest ideal of $R$ contained in $(0 : G)$. Let $A$ be the set of all generators of the right $R$-group $G$. Now $ar = as$ for all $a \in A$. Since each $a \in A$ is distributive, $a(r - s) = 0$ for all $a \in A$. Therefore $r - s \in P$ as $P = (0 : A)$. Hence $G$ is a right $R$-group of type-$\nu(e)$.

Remark 2.15. If $G$ is a right $R$-group of type-$\nu(e)$, then by Proposition 3.12 of [11], $(0 : G)$ := \{r \in R \mid Gr = \{0\}\} is an ideal of $R$. Also, by Theorem 3.24 of [11], a right $g_{\nu}$-primitive near-ring is an equiprime near-ring.

Definition 2.16. Let $G$ be a right $R$-group of type-$g_{\nu}$, $\nu \in \{0, 1, 2\}$. Then $G$ is called faithful if $(0 : G) = \{0\}$.

Theorem 2.17. Let $G$ be a faithful right $S$-group of type-$g_{\nu}$ and $S$ be an essential left invariant ideal of $R$. Then $G$ is a faithful right $R$-group of type-$g_{\nu}$, $\nu \in \{0, 1, 2\}$.

Proof. Let $h_0$ be a generator of the right $S$-group $G$. From the proof of Theorem 3.10 of [9], for $h \in H, r \in R$ the operation defined by $hr := h_0(sr)$ if $h = h_0s, s \in S$, makes $G$ a right $R$-group and is an extension the action of $G$ on $S$ to $R$. Moreover, Theorem 3.10 of [9] and Theorems 3.9 and 3.10 of [10], $G$ is a right $R$-group of type-$\nu$, for $\nu \in \{1, 2\}$. Since $G$ is a right $R$-group of type-$\nu(e)$, by Theorem 3.33 of [11] and Theorem of [5], $G$ is a faithful $R$-group of type-$\nu$. Let $A$ be the set of
all generators of the right $S$-group $G$. Now $(0 : G)_S := \{ s \in S \mid Gs = \{0\} \} = \{0\}$. We have $\{0\} = (0 : A)_S := \{ s \in S \mid As = \{0\} \}$. Since $G$ is a faithful right $R$-group, $(0 : G)_R := \{ r \in R \mid Gr = \{0\} \} = \{0\}$. From the proof of Theorem 3.10 of [9], it can be easily seen that a generator of the right $S$-group $G$ is also a generator of the right $R$-group $G$. So $A$ is the set of generators of the right $R$-group $G$. Suppose that $r \in (0 : A)$. Now $Ar = \{0\}$. So $\{0\} = (Ar)S = A(rS)$ and hence $rS = \{0\}$ as $rS \subseteq S$. Since $S$ is an ideal, $KS = \{0\}$ and $S$ is a prime near-ring, we have $K = \{0\}$, where $K$ is the ideal of $R$ generated by $r$. Therefore $r = 0$ and hence $(0 : A)_R = \{0\}$. So $G$ is a faithful right $R$-group of type-$g_\nu$.

From the above theorem we have:

**Theorem 2.18.** The class of all right $g_\nu$-primitive near-rings is closed under essential left invariant extensions, $\nu \in \{0, 1, 2\}$.

In view of Theorem 1.1, we have the following:

**Theorem 2.19.** Let $\nu \in \{0, 1, 2\}$. Let $E$ be the class of all right $g_\nu$-primitive near-rings and $UE$ be the upper radical class determined by $E$. Then $UE$ is a c-hereditary Kurosh-Amitsur radical class in the variety of all near-rings with hereditary semisimple class $\mathcal{S}UE = E$. So, $J_{g_\nu}^r$ is a Kurosh-Amitsur radical in the class of all near-rings and for any ideal $I$ of $R$, $J_{g_\nu}^r(I) \subseteq J_{g_\nu}^r(R) \cap I$ with equality, if $I$ is left invariant.

**Theorem 2.20.** $J_{g_\nu}^r$ is an ideal-hereditary Kurosh-Amitsur radical in the class of all zero-symmetric near-rings.

**Theorem 2.21.** $J_{g_\nu}^r$ is a special radical in the class of all near-rings.

3. Examples

In this section we present some examples of near-rings $R$ and their right $R$-groups to show that the present right Jacobson radicals are distinct from the known right Jacobson radicals of near-rings. Now we present an example of a right $R$-group of type-$\nu(e)$ which is not of type-$g_\nu$, $\nu \in \{0, 1, 2\}$.

**Proposition 3.1.** If $G$ be a finite group and $G$ has a subgroup of index two, then $M_0(G)$ is a right $2(e)$-primitive near-ring.

**Proof.** Let $G$ be a finite group and $H$ be a subgroup of $G$ of index two. So $H$ is a normal subgroup of $G$. Let $R = M_0(G)$. Then $R/K$ is a right $R$-group of type-$2(e)$, where $K = (H : G) = \{ r \in R \mid r(g) \in H, \text{ for all } g \in G \}$. To show this we consider the two distinct cosets $H$ and $H + a$ of $H$ in $G$. Now $G = H \cup H + a, H$ and $H + a$ are disjoint sets. $K$ is a right ideal of $R$ which is right modular by the identity element of $R$. So $R/K$ is a monogenic right $R$-group. Now we show that $R/K$ is a right $R$-group of type-2. Let $0 \neq r + K \in R/K$. $(r + K)R = R/K$ if and only if
there is an \( s \in R \) such that \((r + K)s = 1 + K\), that is, \(1 - rs \in K\). Let \( P_1 = \{x \in G \mid r(x) + H\} \) and \( P_2 = \{x \in G \mid r(x) \in H + a\} \). Let \( b \in P_2 \) and \( r(b) = h' + a, h' \in H\). Define \( s : G \to G \) by \( s(g) = b, \) if \( g \in H + a\), and \(0, \) if \( g \in H\). We have \( s \in R\). For \( y \in H\), \( (1 - rs)(y) = y - r(s(y)) = y - r(0) = y \in H\) and for \( z = h + a \in H + a\), \((1 - rs)(z) = z - r(s(z)) = z - r(b) = (h + a) - (h' + a) = h - h' \in H\). Therefore, \(1 - rs \in (H : G) = K\) and hence \( R/K \) is a right \(R\)-group of type-2. Since \( R\) is simple, \(\{0\}\) is the largest ideal of \(R\) contained in \((0 : R/K) = (K : R) = \{t \in R \mid Rt \subseteq K\}\). Let \( u, v \in R \) and \((t + K)u = (t + K)v \) for all \( t + K \in R/K\). Now \( tu - tv \in K\), for all \( t \in R\). Suppose that \( g \in G\) and \( u(g) \neq v(g)\). We can choose a \( t \in R \) such that \((tu)(g) - (tv)(g) \in H + a\), a contradiction to the fact that \( tu - tv \in K\). Therefore, \( u = v\) and hence \( R/K \) is a right \(R\)-group of type-2(e). Since \( R\) is simple, it is a right 2(e)-primitive near-ring.

**Example 3.2.** Let \( G \) be the non-abelian group of order 6. Let \( N \) be the subgroup of \( G \) of order 3. By Proposition 3.1, \( M_0(G)/(N : G) \) is a right \( M_0(G)\)-group of type-2(e) and \( M_0(G) \) is a right 2(e)-primitive near-ring. Since \( N \) is the maximal (normal) subgroup of \( G\), \((N : G)\) is the only proper (maximal) right ideal of \( M_0(G)\). So a right \( M_0(G)\)-group of type-0 is \( M_0(G)\)-isomorphic to \( M_0(G)/(N : G)\). Therefore, if \( f + (N : G) \) is a generator of the right \( M_0(G)\)-group \( M_0(G)/(N : G)\), then \((0 : f + (N : G)) = (N : G) \neq \{0\}\). Note that as \( M_0(G) \) is a simple near-ring, \(\{0\}\) is the largest ideal of \( M_0(G) \) contained in \((0 : M_0(G)/(N : G))\). Hence \( M_0(G)/(N : G) \) is not a right \( M_0(G)\)-group of type-\( g_\nu\), \( \nu \in \{0, 1, 2\}\).

Now we present another example to show that there are right \( R\)-groups of type-\( \nu(e)\) which are not of type-\( g_\nu\). The following example was considered in [3] and [11].

**Example 3.3.** Consider \( G := Z_8\), the group of integers under addition modulo 8. Now \( T : G \to G \) defined by \( T(g) = 5g \) for all \( g \in G \) is an automorphism of \( G\). \( T\) fixes 0, 2, 4, 6 and maps 1 to 5, 5 to 1, 3 to 7 and 7 to 3. Now \( A := \{I, T\} \) is an automorphism group of \( G\) and \( \{2\}, \{4\}, \{6\}, \{1, 5\} \) and \{3, 7\} are the orbits. Let \( R \) be the centralizer near-ring \( M_4(G)\), the near-ring of all self maps of \(G\) which fix 0 and commute with \( T\). An element of \( R\) is completely determined by its action on \( \{1, 2, 3, 4, 6\}\). Note that for \( f \in R\) we have \( f(2), f(4), f(6)\) are arbitrary in \(2G\) and \( f(1), f(3)\) are arbitrary in \( G\). In [3] shown that \( I := (0 : 2G) = \{f \in R \mid f(h) = 0\), for all \( h \in 2G\}\) is the only non-trivial ideal of \( R\). Let \( K := (2G : G) = \{t \in R \mid t(G) \subseteq 2G\} \neq R\). Let \( t_0 \) be the identity element in \( R\). Now \( t_0 + K \) is a generator of the right \(R\)-group \( R/K\). Let \( h \in R - K\). We show now that \((h + K)R = R/K\). Since \( h \notin K\), there is an \( a \in G - 2G\) such that \( b := b(a) \notin 2G\). We construct an element \( s \in R\) such that \( s(1) = s(3) = a, \) so that \( s(5) = s(7) = a + 4, \) and \( s = 0 \) on \( 2G\). Since \( s\) maps \( G - 2G\) to \( G - 2G\), we get that \( t_0 - hs \in K\) and hence \((h + K)s = t_0 + K\). So \((h + K)R = R/K\). Therefore, \( R/K\) is a right \(R\)-group of type-\( \nu\). Moreover, \((R/K)I \neq \{K\}\). Therefore, \(\{0\}\) is the largest ideal of \( R\) contained in \((K : R)\) and hence \( J^\nu_\nu(R) = \{0\}\). Consider \( s_1, s_1 \in R\), where \( s(1) = 1 \) and \( 0 \) on \( G - \{1, 5\}\) and
$s_2(1) = 5$ and 0 on $G - \{1,5\}$. Clearly $(h + K)s_1 = (h + K)s_2$ for all $h \in R$ as $h(1) - h(5) = 2G$ for all $h \in R$. But $s_1 - s_2 \not\in \{0\} \Rightarrow R/K$ is not a right $R$-group of type-$\nu(\epsilon)$.

**Proposition 3.4.** Let $R$ be the near-ring considered in the Example 3.3 and let $K$ be a right ideal of $R$. Then $H_1 := \{f(g) \mid f \in K, g \in G \} \subseteq G$ and $H_2 := \{f(g) \mid f \in K, g \in 2G \} \subseteq 2G$ are (normal) subgroups of $G$ and $2G$ respectively.

**Proof.** We show that $H_1$ is a subgroup of $G$. Since $0 \in H_1$, $H_1$ is non-empty. Let $h_1, h_2 \in H_1$. We get $f_1, f_2 \in K$ and $g_1, g_2 \in G$ such that $h_1 = f_1(g_1)$ and $h_2 = f_2(g_2)$. Clearly, $-h_1 = (-f_1)(g_1) \in H_1$ as $-f_1 \in K$. Suppose that one of the $g_i$ is in $G - 2G$. With out loss of generality, suppose that $g_1 \in G - 2G$. We get $f_3 \in R$ such that $f_3(g_1) = g_2$. Now $f_1 - f_2f_3 \in K$ and $h_1 - h_2 = (f_1 - f_2f_3)(g_1) \in H_1$. Assume now that $g_1, g_2 \in 2G$. So, $h_1, h_2 \in 2G$. If $g_3 = 0$, then $h_1 - h_2 = -h_2 \in H_1$. Suppose that $g_1 \neq 0$. So, we get $f_4 \in R$ such that $f_4(g_1) = g_2$. Now $f_1 - f_2f_4 \in K$ and $h_1 - h_2 = (f_1 - f_2f_4)(g_1) \in H_1$. Therefore, $H_1$ is a subgroup of $G$. Similarly, we get that $H_2$ is a subgroup of $2G$.

**Proposition 3.5.** Let $R, K, H_1$ and $H_2$ be as defined in Proposition 3.4. If $H_1 = G$ and $H_2 = 2G$, then $K = R$.

**Proof.** Suppose that $H_1 = G$ and $H_2 = 2G$. We have 1, 3 $\in H_1$. So, for $i \in \{1, 3\}$, we get $f_i \in K$ such that $f_i(g_i) = i$, where $g_i \in \{1, 3, 5, 7\} = G - 2G$. For $i = 1, 3$ we also get $m_i \in R$ such that $m_i(i) = g_i$, so that $m_i(i + 4) = g_i + 4$ and $m_i = 0$ on $G - \{i, i + 4\}$. Now $m_i \in K, i = 1, 3$. Clearly, $f_1m_1 + f_3m_3$ fixes all the elements of $G - 2G$ and maps all the elements of $2G$ to 0. We have 2, 4, 6 $\in H_2 = 2G = \{0, 2, 4, 6\}$. For $i = 2, 4, 6$ we get $f_i \in K$ such that $f_i(g_i) = i, g_i \in 2G$. So, for $i = 2, 4, 6$ we get $m_i \in R$ such that $m_i(i) = g_i$ and $m_i = 0$ on $G - \{i\}$. Now $f_im_i \in K, i = 2, 4, 6$. $f_2m_2 + f_4m_4 + f_6m_6$ fixes all the elements of $2G$ and maps all the elements of $G - 2G$ to 0. Therefore, the identity map I of $G$ can be expressed as $I = f_1m_1 + f_2m_2 + f_3m_3 + f_4m_4 + f_6m_6 \in K$. Hence, $K = R$.

**Proposition 3.6.** Let $R, K, H_1$ and $H_2$ be as defined in Proposition 3.4. If $K$ is a maximal right ideal of $R$, then $K = (2G : G) = \{f \in R \mid f(G) \subseteq 2G\}$ or $(4G : 2G) = \{f \in R \mid f(2G) \subseteq 4G\}$.

**Proof.** Suppose that $K$ is a maximal right ideal of $R$. Clearly, if $H$ and $T$ are (normal) subgroups of $G$ and $2G$ respectively, then $(H : G) = \{f \in R \mid f(G) \subseteq H\}$ and $(T : 2G) = \{f \in R \mid f(2G) \subseteq T\}$ are right ideals of $R$. Now $2G$ and $4G$ are the maximal (normal) subgroups of $G$ and $2G$ respectively. We have $K \subseteq (H_1 : G)$ and $K \subseteq (H_2 : 2G)$. Since $K$ is a maximal right ideal of $R$, by Proposition 3.5, either $H_1 \neq G$ or $H_2 \neq 2G$.

Case (i) Suppose that $H_2 \neq 2G$. Since $K$ is a maximal right ideal of $R$ and $K \subseteq (H_2 : 2G) \neq R$, we get that $H_2 = 4G$ and $K = (4G : 2G)$.

Case (ii) Suppose that $H_1 \neq G$. Since $K$ is a maximal right ideal of $R$ and $K \subseteq (H_1 : G) \neq R$, we get that $H_1 = 2G$ and $K = (2G : G)$.

Therefore, either $K = (2G : G)$ or $(4G : 2G)$.\qed
Proposition 3.7. Let $R$ be the near-ring considered in Example 3.3. Let $U = (4G : 2G) = \{ f \in R \mid f(2G) \subseteq 4G \}$. Then $U$ is a maximal right ideal of $R$ and $R/U$ is a right $R$-group of type-2(e).

Proof. Clearly, $U$ is a right ideal of $R$. Consider the right $R$-group $R/U$. We prove that $R/U$ is a right $R$-group of type-2. Since $R$ has identity $1$, $1 + U$ is a generator of the right $R$-group $R/U$ and hence $R/U$ is a monogenic right $R$-group. Let $0 \neq f + U \in R/U$. So, $f \not\in U$. We get $0 \neq a \in 2G$ such that $b := f(a) \notin 4G$. So, $2G = \{ 0, b, 2b, 3b \}$ as 2 and 6 are generators of $2G$. Construct $r \in R$ by $r(b) = a$, $r(2b) = 0$, $r(3b) = a$ and $r = 0$ on $G - \{ 0, 1, 3, 5, 7 \}$. Now $(I - fr)(x) \in 4G$ for all $x \in 2G$. Therefore, $I - fr \in U$ and hence $(f + U)r = I + U$. This shows that $(f + U)R = R/U$. So, $R/U$ is a right $R$-group of type-2. We know that $P := (0 : 2G)$ is the only non-trivial ideal of $R$. Therefore, $P$ is the largest ideal of $R$ contained in $U = (4G : 2G)$ and hence $P$ is the largest ideal of $R$ contained in $(0 : R/U) = (U : R)$ $= \{ f \in R \mid fR \subseteq U \}$. Let $0 \neq s + U \in R/U$ and $f, h \in R$. Suppose that $(s + U)rf = (s + U)rh$ for all $r \in R$. So, $srf - srh \in U$ for all $r \in R$. We show that $f - h \in P$. If possible, suppose that $f - h \notin P$. We get $0 \neq a \in 2G$ such that $(f - h)(a) = f(a) - h(a) \neq 0$ with $h(a) \neq 0$. Let $s(c) \notin \{ 0, 4 \}$ for some $c \in 2G$. Choose $r \in R$ such that $r((f(a)) = 0$ and $r(h(a)) = c$. Now $(srf)(a) = 0$ and $(srh)(a) = s(c)$. So, $(sr - srh)(a) = 0 - s(c) \notin \{ 0, 4 \}$, a contradiction to the fact that $sr - srh \in U$. Therefore, $f(a) = h(a)$ for all $a \in 2G$. Hence $f - h \in P$. So, $R/U$ is a right $R$-group of type-2(e). □

Proposition 3.8. Let $R$ be the near-ring considered in Example 3.3. Then $J^r_{g}(R) = \{ 0 \}$ and $J^r_{g(\nu)}(R) = (0 : 2G) \neq \{ 0 \}$.

Proof. We know that $\{ 0 \}$ and $I := (0 : 2G) = \{ f \in R \mid f(2G) = \{ 0 \} \}$ are the only proper ideals of $R$. Let $K_1 := (2G : G) = \{ f \in R \mid f(G) \subseteq 2G \}$ and $K_2 := (4G : 2G) = \{ f \in R \mid f(2G) \subseteq 4G \}$. By Proposition 3.6, a maximal right ideal of $R$ is either $K_1$ or $K_2$. So, a right $R$-group of type-0 is isomorphic to $R/K_1$ or $R/K_2$. By Example 3.3, $R/K_1$ is a right $R$-group of type-2 but not of type-2(e). Since $\{ 0 \}$ is the largest ideal of $R$ contained in $K_1$, $\{ 0 \}$ is a right 2-primitive ideal of $R$ but not a right 2(e)-primitive ideal of $R$. By Proposition 3.7, $R/K_2$ is a right $R$-group of type-2(e). Since $I = (0 : 2G)$ is the largest ideal of $R$ contained in $K_2$, $I$ is a right 2(e)-primitive ideal of $R$. Therefore, $J^r_{g}(R) = \{ 0 \}$ and $J^r_{g(\nu)}(R) = (0 : 2G)$. □

Proposition 3.9. Let $R$ be the near-ring considered in Example 3.3. Then $J^\nu_{g_{\nu}}(R) = R$, $\nu \in \{ 0, 1, 2 \}$.

Proof. Let $R$ be the near-ring considered in the Example 3.3 and $K = (2G : G)$, $U = (4G : 2G)$. As seen above $K$, $U$ are the only maximal right ideals of $R$ and $R/K$ is a right $R$-group of type-2 but not of type-2(e), where as $R/U$ is a right $R$-group of type-2(e). If $f + K$ is a generator of the right $R$-group $R/K$, then the maximal right ideal $(0 : f + K)$ must be either $K$ or $U$. Since $0(K) = 2^0 \neq 2^9 = 0(U)$, and $R/(0 : f + K)$ is right $R$-isomorphic $R/K$, $(0 : f + K) = K$. Hence $R/K$ is not a right $R$-group of type-$g_{\nu}$, as $\{ 0 \}$, $(0 : 2G)$ and $R$ are the only ideals of $R$. By a similar argument we get that $R/U$ is not a right $R$-group of type-$g_{\nu}$. So $J^\nu_{g_{\nu}}(R) = R$. □
4. $J_{g_{\nu}}$-semisimple Near-rings, $\nu \in \{0, 1, 2\}$

In this section we present structure theorems for $J_{g_{\nu}}$-semisimple near-rings.

**Proposition 4.1.** Let $R (\neq \{0\})$ be a $J_{g_{\nu}}$-semisimple near-rings satisfying DCC on right ideals of $R$, $\nu \in \{0, 1, 2\}$. Then $\bar{R}$ is a finite direct sum of minimal right ideals which are right $R$-groups of type-$g_{\nu}$.

**Proof.** Let $P_i, i \in I$ be the collection of right $g_{\nu}$-primitive ideals of $R$. Since $R$ is a $J_{g_{\nu}}$-semisimple near-ring, $\bigcap\{P_i \mid i \in I\} = \{0\}$. We get a right $R$-group $G_i$ of type-$g_{\nu}$ such that $P_i = (0 : G_i) := \{r \in R \mid G_i r = \{0\}\}, i \in I$. Let $A_i$ be the set of generators of $G_i, i \in I$. Now $P_i = (0 : A_i) := \{r \in R \mid A_i r = \{0\}\}$. Note that for each $a \in A_i, (0 : a) := \{r \in R \mid ar = 0\}$ is a right $g_{\nu}$-module right ideal of $R$ and the right $R$-group $P_i/(0 : a)$ is right $R$-isomorphic to $G_i, i \in I$. Since each $P_i$ is an intersection of right $g_{\nu}$-module right ideals of $R$ and $\bigcap\{P_i \mid i \in I\} = \{0\}$, the intersection of all right $g_{\nu}$-module right ideal of $R$ is zero. We get a finite number of right $g_{\nu}$-module right ideals $K_1, K_2, ..., K_n$ of $R$ such that $\bigcap\{K_j \mid j = 1, 2, ..., n\} = \{0\}$. Let $T_i := K_i \cap K_2 \cap \cdots \cap K_{i-1} \cap K_{i+1} \cap \cdots \cap K_n, i = 1, 2, ..., n$. We may assume that $T_i \neq \{0\}$ for all $i = 1, 2, ..., n$. Now by Proposition 3.12[(2)] of [8], $R = T_1 \oplus T_2 \oplus ... \oplus T_n$, a direct sum of minimal right ideals $T_i$ of $R$ which are right $R$-groups of type-$g_{\nu}$. □

In [8](Definition 3.5), if $R$ is a direct sum of $n$ minimal right ideals of $R$, then the dimension of $R$ is defined as $n$ and is denoted by $\text{dim } R$.

**Definition 4.2.** A distributive idempotent $e$ of $R$ is called right $g_{\nu}$-primitive if $eR$ is a right $R$-group of type-$g_{\nu}$, $\nu \in \{0, 1, 2\}$.

**Theorem 4.3.** Let $R$ be a right $g_{\nu}$-primitive near-ring satisfying DCC on right ideals of $R$, $\nu \in \{0, 1, 2\}$. Then $R$ is a simple near-ring with identity and $R$ has a subnear-ring which is isomorphic to the matrix near-ring $M_n(S)$, where $S = eRe$, $e$ is a right $g_{\nu}$-primitive idempotent and $n = \text{dim } R$. If, in addition, $R$ is distributively generated, then $R$ is isomorphic to $M_n(S)$.

**Proof.** $R$ satisfies the hypothesis of Theorem 4.3 of [8] and hence the conclusion follows from it. □

**Theorem 4.4.** Let $R$ be a finite right $g_{2}$-primitive near-ring and $eRe$ be a non-ring. Then $R$ is (isomorphic to) the matrix near-ring $M_n(F)$, where $n = \text{dim } R$, $F := eRe$ is a near-field and $e$ is a right $g_{2}$-primitive idempotent in $R$.

**Proof.** Proof follows from Theorem 4.16 of [8]. □

**Theorem 4.5.** Let $R (\neq \{0\})$ be a $J_{g_{\nu}}$-semisimple near-rings satisfying DCC on right ideals of $R$, $\nu \in \{0, 1, 2\}$. Then $\bar{R}$ is a direct sum of minimal ideals which are simple right $g_{\nu}$-primitive near-rings with identity.
Proof. Let $P_i, i \in I$ be the collection of right $g_\nu$-primitive ideals of $R$, $\nu \in \{0, 1, 2\}$. Now $\bigcap \{P_i | i \in I\} = \{0\}$. Since $R$ has DCC on right ideals of $R$, we get a finite number of right $g_\nu$-primitive ideals of $P_1, P_2, ..., P_n$ of $R$ such that $P_1 \cap P_2 \cap ... \cap P_n = \{0\}$. We may assume that $K_j := P_1 \cap P_2 \cap ... \cap P_{j-1} \cap P_{j+1} \cap ... \cap P_n \neq \{0\}, j = 1, 2, ..., n$. By Theorem 4.3, $R/P_i$ is a simple near-ring with identity as $R/P_i$ is a right $g_\nu$-primitive near-ring with DCC on right ideals. Now by Theorem 2.50 of Pilz [4], $R = K_1 \oplus K_2 \oplus ... \oplus K_n$, $K_i$ are minimal ideals of $R$ and are simple right $g_\nu$-primitive near-rings with identity.

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