The Georgi “Avatar” of Broken Chiral Symmetry in Quantum Chromodynamics

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Abstract

We establish that in Quantum Chromodynamics (QCD) at zero temperature, $SU_{L+R}(N_F)$ exhibits the vector mode conjectured by Georgi and $SU_{L-R}(N_F)$ is realized in either the Nambu-Goldstone mode or else $Q_5^a$ is also screened from view at infinity. The Wigner-Weyl mode is ruled out unless the beta function in QCD develops an infrared stable zero.

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We shall establish that QCD at zero temperature, satisfying both asymptotic freedom and confinement, exhibits the following features: (a) $SU_{L+R}(N_F)$ exhibits the vector mode conjectured by Georgi [1] (Georgi-Goldstone mode). (b) $SU_{L-R}(N_F)$ exhibits either the Nambu-Goldstone mode or else the axial-vector charge $Q_5^a$ is also screened from view at infinity. If the latter case were to occur, then QCD confines without breaking chiral symmetry: both $SU_{L+R}(N_F)$ and $SU_{L-R}(N_F)$ are realized in the Higgs mode (Georgi-Wigner mode), with no scalar or pseudoscalar Nambu-Goldstone bosons and the vector and axial-vector mesons are degenerate. (c) The Wigner-Weyl mode corresponding to $Q^a | 0 \rangle = 0$, $Q_5^a | 0 \rangle = 0$ is ruled out: the Callan-Symanzik beta function has to turn over to yield an infrared stable fixed point at a finite value of $g$ if chiral symmetry is to be restored.

We now sketch the proof of these interesting assertions. We begin with the vector current $V_\mu^a$ and its conservation

$$\partial^\mu V_\mu^a (x, t) = 0.$$ (1)

This implies the local version

$$[Q^a (t), H (x, t)] = 0$$ (2)

where $H (x, t) = \Theta^{00}$ is the Hamiltonian density, if the surface terms at infinity can be discarded. This is clearly justified if the flavor vector charge annihilates the vacuum,

$$Q^a (t) | 0 \rangle = 0$$ (3)

which is guaranteed by the Vafa-Witten theorem [2,3] i.e., non-chiral symmetries cannot be spontaneously broken in vector-like gauge theory. Hence there are no scalar Nambu-Goldstone bosons to produce a long range interaction, which in turn would have resulted in a non-vanishing contribution to the surface terms.

The dilatation charge

$$Q_D (t) = \int d^3 x D_0 (x, t),$$ (4)

defined in terms of the dilatation current $D_\mu (x, t)$ satisfies [4] the trace anomaly

$$\partial^\mu D_\mu = \frac{\beta (g)}{2g} G^\alpha_{\mu \nu} G^\mu_{\alpha \nu}$$ (5)

in QCD in the chiral limit when the current quark mass is zero. It is well-known that scale invariance is broken both “spontaneously”, $Q_D (t) | 0 \rangle \neq 0$, and explicitly by the trace anomaly. Consequently, the states defined by successive repeated application of $Q_D (t)$ on the vacuum state are neither vacuum states nor are they necessarily degenerate [3]. Let the commutator

$$[Q_D (0), Q^a (0)] = - i d_Q Q^a (0)$$ (6)

define the scale dimension $d_Q$ of the charge $Q^a (0)$. By translation invariance, this can be put in the form
\[ [Q_D(t), Q_a(t)] = -i d_Q Q_a(t). \]  

(7)

It is important to stress that operator relations such as the above equation are unaffected by spontaneous symmetry breaking as emphasized by Weinberg [6]. Let us consider the double commutator which follows from Eq. (2),

\[ [Q_D(t), [Q_a(t), H]] = 0, \]  

(8)

where \( Q_D(t) \) is the dilation charge defined in Eq. (4). If we now invoke the Jacobi identity we can recast the above equation in the form

\[ [Q_a(t), [H, Q_D(t)]] + [H, [Q_D(t), Q_a(t)]] = 0. \]  

(9)

Since

\[ [H(x, t), Q_D(t)] = -i \partial_\mu D^\mu(x, t) \neq 0, \]  

(10)

by virtue of the trace anomaly, Eq. (3), and making use of Eqs. (2,7), we arrive at the operator relation

\[ [Q_a(t), \partial_\mu D^\mu(x, t)] = 0. \]  

(11)

Applying this relation on the vacuum state, we obtain

\[ [Q_a(t), \partial_\mu D^\mu(x, t)] \left| 0 \right. = 0. \]  

(12)

We may now invoke the result of Vafa-Witten theorem [2]

\[ Q_a(t) \left| 0 \right. = 0, \]  

(13)

and conclude that

\[ \mathcal{O}(x, t) \left| 0 \right. \equiv Q_a(t) \partial_\mu D^\mu(x, t) \left| 0 \right. = 0, \]  

(14)

where the operator \( \mathcal{O}(x, t) \) is local in space and time and commutes with itself for space-like intervals. Explicitly, we see that this condition reduces to

\[ [Q^a(t) \partial_\mu D_\mu(x, t), Q^a(t') \partial_\mu D_\mu(x', t')] = Q^a(t)Q^a(t') [\partial_\mu D_\mu(x, t), \partial_\mu D_\mu(x', t')] = 0 \]  

(15)

for space-like intervals, due to the time independence of \( Q^a(t) \) and Eq. (11). Here we have invoked the locality of \( \partial_\mu D_\mu(x, t) \).

We can now utilize the Federbush-Johnson theorem [7], which applies to any local operator, to Eq. (14) and immediately arrive at the key result,

\[ Q_a(t) \partial_\mu D_\mu(x, t) \equiv 0. \]  

(16)

Since \( \partial_\mu D_\mu(x, t) \) cannot vanish in a theory which exhibits both asymptotic freedom and confinement except at \( g = 0 \), we conclude that the vector flavor charges must be screened from view at infinity,
This important result is a manifestation of spontaneously broken local symmetry. The vector mesons become massive and the scalar would-be Nambu-Goldstone bosons disappear.

Let us now consider the axial-vector charges $Q_5^a$. The Vafa-Witten theorem does not apply in this case and therefore we proceed by the method of *reductio ad absurdum* as follows. We assume $Q_5^a|0\rangle = 0$ corresponding to the Wigner-Weyl mode of unbroken symmetry. We begin by defining the scale dimension of the axial-vector charge by

$$[Q_D(0), Q_5^a(0)] = -id_{Q_5}Q_5^a(0).$$

(18)

Repeating the earlier analysis now for the axial-vector charges, exactly as in Eqs. (7–15), we arrive at the conclusion

$$Q_5^a(t)\partial^\mu D_\mu(x, t) \equiv 0.$$ 

(19)

Since $Q_5^a(t) \neq 0$ for the assumed Wigner-Weyl mode, this requires $\partial^\mu D_\mu(x, t) = 0$. This would imply that QCD exhibiting both asymptotic freedom and confinement is free. Hence by *reductio ad absurdum* we are led to the conclusion: either $Q_5^a(t) |0\rangle \neq 0$ or $Q_5^a(t) \equiv 0$. The first alternative is the Nambu-Goldstone realization of chiral symmetry which must hold for $N_F = 3$. The second alternative in conjunction with the screening of the vector charges, *i.e.*, $Q^a(t) \equiv 0, Q_5^a(t) \equiv 0$, is the Higgs mode alternative (Georgi-Wigner mode): both scalar and pseudoscalar Nambu-Goldstone bosons have been devoured. This case corresponds to confinement with exact chiral symmetry. Such a mode is realized in Supersymmetric QCD.

In conclusion, it is interesting that the Wigner-Weyl mode is ruled out in QCD at $T = 0$, if both asymptotic freedom and confinement obtains. This follows from Eqs. (16,19). Hence the Wigner-Weyl mode can occur only if the beta function turns over to yield an infrared stable fixed point. Which of the above alternative realizations indeed occurs in QCD cannot be settled by this analysis. The answer may depend on the number of flavors.

An effective Lagrangian which realizes the Georgi-Goldstone mode is easily constructed, following the general procedure for building models with vector and axial vector mesons.

Finally, it is important to stress that we have not invoke the $SU_L(N_F) \times SU_R(N_F)$ charge algebra in our analysis. In view of the screened charges, *i.e.*, $Q^a(t) = 0$ for the Georgi-Goldstone mode and $Q^a(t) = Q_5^a(t) = 0$ for the Georgi-Wigner mode, one must revert back to current algebra which leads to the Weinberg sum rules in QCD.

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