Prognosis based on the Joint Parameter/State Estimation Using Zonotopic LPV Set-Membership Approach *

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Abstract: This paper addresses the problem of prognostics based on Joint Estimation of States and Parameters (JESP) using a zonotopic Linear Parameter-Varying (LPV) set-membership approach. The aim of the prognostics is to forecast the Remaining Useful Life (RUL) of systems with degraded components. Thus, this paper suggests transforming such systems with non-linear dynamical models into an LPV representation. Hence, a Zonotopic Set-Membership (ZSM) observer has been proposed to carry out the JESP for degradation assessment with a considerable focus on multiple-output systems. Despite the existence of different geometrical shapes, zonotopes have gained recently a lot of popularity due to their low computational complexity, in addition to the possibility of relating them with stochastic approaches. Moreover, an optimal correcting gain has been designed based on Linear Matrix Inequality (LMI) for robust observer tuning. Additionally, the LMI-based tuning can be solved offline with reduced computational effort while coping with a polytopic LPV representation of the model. Consequently, a Recursive ZSM (RZSM) approach has been employed for degradation prediction and guaranteed zonotopic RUL forecasting. Finally, the proposed approach is assessed using a degraded power electronics system, and the results are illustrated.

Keywords: prognostics, zonotopes, set-membership, linear parameter-varying, linear matrix inequality, parameter estimation.

1. INTRODUCTION

There has been a considerable interest in developing robust techniques for Prognostics and Health Management (PHM) approaches. The challenging task of predicting the Remaining Useful Life (RUL) of components or systems, is mainly related to the degradation assessment. Therefore, it is intended to employ model-based failure prognostics techniques for applications with slow degradation profiles. In broad, the model-based prognostics approaches depend on degradation estimation of the critical components which can be characterized by their failure precursors. The latter are parameters that indicate the level of deviation of a component or system from the desired operating point (Elattar et al., 2016). Moreover, most of the modern-engineering applications are systems with active and passive components. Thus, the internal components of these systems are mainly inaccessible for direct measurements, which necessitates the estimation of the parameters that represent the failure precursors. For this reason, we initiated in previous work (Al-Mohamad et al., 2020), the implementation of stochastic Extended Kalman Filter (EKF) observer for Joint Estimation of States and Parameters (JESP) of a DC-DC converter that represents a nonlinear dynamical system with degraded parameters such as MOSFET and electrolytic capacitor. The JESP is adopted after augmenting the critical parameters to the states for an overall estimation. Hence, in (Al-Mohamad et al., 2020), we upgraded our proposed RUL forecasting approach by employing a Zonotopic EKF (ZEKF) for JESP that propagate the estimated zonotopes and result in an online bounded RUL forecasting. Therefore, the remarkable equivalent features between the ZEKF and the Zonotopic Set-Membership (ZSM) under some conditions (Combastel, 2015), in addition to the obtained promising results, have led to inspect the ZSM observer for PHM applications. Hence, we have proposed to contribute to the investigation of the latter with robust tuning based on Linear Matrix Inequalities (LMIs) for JESP that is responsible of assessing the degradation state in the system based on the critical components. Thus, the RUL is
predicted by recursively implement a ZSM for zonotopic failure prediction and bounded RUL forecasting. In this context, zonotopes are specifically chosen for their efficient arithmetic operations compared to ellipsoids and the other polytopes such as parallelepipeds and boxes. Their rectangular geometrical shape allows the reduction of the computational complexity by employing simple vertices calculations while propagating the joint estimated sets (Alamo et al., 2005; Combastel, 2015; Wang et al., 2018). Additionally, these deterministic observers consider unknown-but-bounded noises and uncertainties unlike the Gaussian distribution in the stochastic approaches (Combastel, 2015).

Moreover, the nonlinear systems with varying parameters that describe the failure precursors of the degradation process require a linearized representation of the model. In the aforementioned previous work, we implemented the Jacobian linearization technique with the EKF at each time instant. Whereas, for problem generalization purposes in addition to decreasing the computational time and effort, we proposed to represent the system in a Linear Parameter Varying (LPV) model. Furthermore, the LPV representation contains the nonlinearities of the augmented system representation with varying parameters, and allows the extension of this problem to various engineering applications that could cope with the interest of PHM.

Furthermore, the main objective of PHM is to increase the reliability of systems. Thus, the proposed RUL forecasting approach is strongly based on the estimated failure precursors. In consequence, in order to guarantee a reliable zonotopic RUL forecasting, we propose to tune the ZSM observer with an LMI-based optimization. This framework with the LPV integration provide a robust estimation with less computational time that positively influence the online prognostics and RUL forecasting. Additionally, a classical online approach with an optimality criterion is also investigated for the validation of the proposed approach (Combastel, 2015; Wang et al., 2018).

This paper is structured as follows. Section 2 highlights the problem formulation. Section 3 investigates the design of the proposed ZSM observer for JESP. Moreover, Section 4 demonstrates the proposition of implementing the ZSM approach for RUL prediction. An assessment is illustrated in the case study of Section 5. Finally, the conclusions are drawn in Section 6.

2. PROBLEM SETUP

2.1 Preliminaries

The following material is an essential recap of background material that is used for formulating the PHM zonotopic approach presented in this paper.

Definition 1. (Zonotopes). A zonotope $Z = \langle c, H \rangle \subset \mathbb{R}^n$ with center $c \in \mathbb{R}^n$ and the generator matrix $H \in \mathbb{R}^{n \times m}$ is a polytopic set defined as a linear image of the unit hypercube $[-1, 1]^m$, as follows:

$$\langle c, H \rangle = \{c + Hs, \|s\|_{\infty} \leq 1\}. \quad (1)$$

Definition 2. (Sum of Zonotopes). The Minkowski sum of two zonotopes $Z_1 = \langle c_1, H_1 \rangle$ and $Z_2 = \langle c_2, H_2 \rangle$ is:

$$Z_1 \oplus Z_2 = \langle c_1 + c_2, [H_1, H_2] \rangle. \quad (2)$$

Definition 3. (Reduction operator). The weighted reduction operator $\downarrow_{q,w}$ is used to reduce the generator matrix $H$, where $q$ specifies the maximum number of columns in the reduced generator matrix $\downarrow_{q,w} H$ (Combastel, 2015).

Definition 4. ($F_w$-$F-$radius). $F_w$-$radius$ is an effective size criterion for zonotopes $Z = \langle c, H \rangle \subset \mathbb{R}^n$. It is calculated by a weighted Frobenius norm of $H$ as shown below (Combastel, 2015):

$$\|\langle c, H \rangle\|_{F_w} = \|H\|_{F_w}. \quad (3)$$

The $F-$radius criterion is similar to the $F_w$-radius with $w$ an identity matrix.

2.2 Problem Formulation

Harsh environmental conditions lead to electrically and thermally overstress electrical and mechanical components that could evolve towards catastrophic failures in critical applications such as autonomous vehicles, aircraft, huge industries, etc. These systems become degraded and can be characterized with nonlinearities caused by the degraded parameters. For these reasons, the PHM can improve the reliability of such systems due to the condition-based maintenance practices. Furthermore, the multidisciplinary nature of PHM burdens its characterization for wider and direct applications. Therefore, we propose in this paper to adopt the ZSM approach in order to assess the state of health of the degraded systems by estimating the internal parameters of the system, where no direct measurement is accessible. Then, the failure prediction is assessed by employing a Recursive ZSM (RZSM) to indirectly predict the degradation trajectories and forecast the RUL in a zonotopic framework. Consequently, for generalization purposes, a Multi Input Multi Output (MIMO) nonlinear dynamical system is modeled using an LPV representation as shown in the following discrete-time state-space model:

$$x_{k+1} = A(\theta_k)x_k + B(\theta_k)u_k + E_{\omega_k}, \quad (4a)$$

$$y_k = C(\theta_k)x_k + D(\theta_k)u_k + E_{\nu_k}, \quad (4b)$$

where $x_k \in \mathbb{R}^{n_x}$, $y_k \in \mathbb{R}^{n_y}$ and $u_k \in \mathbb{R}^{n_u}$ are the states, outputs, and inputs vectors of the system respectively. For the sake of simplicity, $A_k = A(\theta_k) \in \mathbb{R}^{n_x \times n_x}$, $B_k = B(\theta_k) \in \mathbb{R}^{n_x \times n_u}$, $C_k = C(\theta_k) \in \mathbb{R}^{n_y \times n_x}$ and $D_k = D(\theta_k) \in \mathbb{R}^{n_y \times n_u}$ denote the state matrix, input matrix, output matrix and feed-through matrix respectively. Moreover, $E_{\omega_k} \in \mathbb{R}^{n_x \times n_{\omega}}$ and $E_{\nu_k} \in \mathbb{R}^{n_y \times n_{\nu}}$ are the direction matrices for the process and measurement noises, uncertainties and perturbations $\omega_k \in \mathbb{R}^{n_{\omega}}$ and $\nu_k \in \mathbb{R}^{n_{\nu}}$ respectively. The latter are assumed to be unknown but bounded as follows:

$$\omega_k \in [-1, 1] \text{ and } \nu_k \in [-1, 1], \quad \forall k \in \mathbb{N} \quad (5)$$

2.3 Proposed PHM Approach

A reliable PHM approach is assessed by the efficiency of the RUL prediction. The proposed PHM approach is a threefold decision-making. Furthermore, the key feature of this paper is that the proposed PHM approach is completely based on optimally computed zonotopic scheme that covers system modeling, joint estimation and RUL prediction as shown in the PHM flowchart in Figure 1.

1. System modeling: The critical failure precursors of the system are included as varying parameters included as augmented states in order to allow the joint estimation of the overall system by only measuring
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Fig. 1. Proposed PHM Approach

input/output sensors, see (Al-Mohamad et al., 2020) for further explanation. Thus, the model is formulated in an LPV framework with embedded nonlinearities to cope with the following phase.

(2) **ZSM for JESP**: The ZSM is employed for JESP in order to estimate the varying parameters as well as the states of the system. We tune the observer via an LMI-based optimization for robust estimation. We also considered the MIMO case to cover realistic problems, by dealing with each measurement separately.

(3) **RZSM for RUL forecasting**: The efficiency of the previously-estimated varying parameters assess the reliability of the RUL forecasting of the system. By other meanings, in a homogeneous ZSM framework, the degradation trajectory is predicted based on a general exponential evolution model with unknown parameters. After estimating the latter parameters, the propagated zonotopic lifetime projection intersects with the physical threshold (TH) of the critical components. Hence, an online bounded RUL is obtained at each measurement.

### 3. ZONOTOPIC SET-MEMBERSHIP APPROACH FOR JOINT ESTIMATION

#### 3.1 ZSM Observer Design

The ZSM technique consists of three main steps in order to estimate the zonotopic sets of states and parameters. The prediction of sets of uncertain states and parameters are propagated. Hence, a strip is computed to ensure the minimal bounding of the uncertain states and parameters. Finally, the consistent estimation is obtained by intersecting the uncertain sets with the previous-time-instant measurement. The zonotopic estimation is guaranteed (Alamo et al., 2005), and the uncertainties are embedded due to the zonotopic inclusion property (Montes de Oca et al., 2012).

(1) **The prediction**: The first step computes the predicted states and the generator matrix as follows:

\[ \tilde{x}_{k+1} = A_k \tilde{x}_k + B_k u_k + E_w \omega_k, \]  
\[ \tilde{H}_{k+1} = A_k \tilde{H}_k, \]  

where the predicted zonotope \( \tilde{P}_{k+1} \) is calculated as follows (Alamo et al., 2005):

\[ \tilde{P}_{k+1} = \tilde{x}_{k+1} \oplus \tilde{H}_{k+1} B^m_u, \]  

where \( \tilde{x}_{k+1}^{ZSM} \) and \( \tilde{H}_{k+1}^{ZSM} \) are the center and the generator matrix of the zonotopic vertices of states and parameters \( \tilde{P}_{k+1} \) and \( B^m_u \) is a unitary box.

(2) **The strip**: The strip is expressed in the following equation:

\[ X_{y_k} = \{ x \in \mathbb{R} : |C_k x_k + D_k u_k - y_k^{meas} | < \upsilon_k \}, \]  

where \( y_k^{meas} \) is the measurement vector at time step \( k \). The strip is sequentially computed for each of the outputs. In MIMO systems, the intersection between the uncertain states and the measurement is carried with each of the measurement at a time and updated in the following intersection (Le et al., 2013).

(3) **The intersection**: The consistent estimated zonotopic sets are propagated by intersecting the predicted states with the measured outputs as follows:

\[ \tilde{P}_{k+1} \cap X_{y_k} = \hat{c}_{k+1}(\lambda) \oplus \hat{H}_{k+1}(\lambda) B^m_u, \]  

consequently, the center and the generator matrix of the estimated zonotopic sets are respectively shown below (Alamo et al., 2005; Combastel, 2015; Pouraghbar et al., 2016):

\[ \hat{c}_{k+1} = c_k + \lambda (y_k^{meas} - (C_k \tilde{c}_k + D_k u_k)), \]
\[ \hat{H}_{k+1} = [(I - \lambda C_k)H_k, \ (I - \lambda C_k)E_\omega, \ -\lambda E_u], \]  

where \( I \) is an identity matrix with proper dimensions, \( H \) is the reduced matrix that is obtained by applying the reduction operator as shown in Def. (3). Finally, the estimated zonotopes are determined as follows:

\[ \hat{X}_{k+1} = \hat{c}_{k+1} \oplus \hat{H}_{k+1} B^m_u. \]  

#### 3.2 LMI-based Optimization of \( \lambda \)

The role of the tuning parameter \( \lambda \) is crucial to guarantee the intersection of the states and the measurement in presence of uncertainties. Therefore, the existing classical tuning approach is applied online in order to compute the generator matrices. The latter is suitable for linear systems, whereas we propose an LMI-based optimization which is carried out online with reduced computations. Hence, it can be extended to nonlinear systems, and only a few arithmetic operations are computed online. In consequence, this real-time efficient approach guarantees a robust tuning of the ZSM that can be applied to real systems. The minimization criterion \( F \)–radius applied to \( H \) (see Def. (4)) can be formulated as the following LMI optimization problem:

\[ \text{minimize} \quad \gamma \]  
subject to

\[ \left[ \begin{array}{cc} \gamma I & I \\ I & \Gamma \end{array} \right] \preceq 0, \]  
\[ \left[ \begin{array}{cc} -\Gamma \Gamma A - WC & \Gamma Q^T W \\ * & -\Gamma \end{array} \right] \preceq 0, \]  

where \( Q = \sqrt{E_\omega} \) and \( R = E_u \) are the process and noise bounds respectively, and \( * \) denotes symmetrical elements. Then, the optimal \( \lambda ^* \) can obtained as follows:

\[ \lambda ^* = \Gamma ^{-1} W. \]
where $\Gamma$ and $W$ are the solution of the previous LMI optimization problem.

The previous LMI-based optimization is solved offline with reduced operations limited to the number of bounded varying parameters in the systems such as $[\theta, \bar{\theta}] = [\theta_0, \theta_{\text{TH}}]$, where $\theta_0$ is the rated value of the varying parameter $\theta$, and $\theta_{\text{TH}}$ is the maximum value of its failure TH which is defined and known. Furthermore, a polytopic representation of the LPV model is then computed by the bounding box approach considering the range of variation of the varying parameters as:

$$
x_{k+1} = \sum_{i=1}^{N} \mu_i(\theta_k) \left( A_i x_k + B_i u_k \right)
$$

$$
y_k = \sum_{i=1}^{N} \mu_i(\theta_k) \left( C_i x_k + D_i u_k \right).
$$

Hence, a varying value for the gain (13) is obtained online as follows:

$$
\lambda^*(\theta_k) = \sum_{i=1}^{N} \mu_i(\theta_k) \lambda^*_i,
$$

where $\lambda^*_i$ are obtained by solving (12c) at the vertices of the polytopic model (14). The main advantage of this LMI-based approach for tuning matrix calculation, is that the optimization problem (12c) is solved offline. Then, an interpolation based on the estimated parameters is calculated on-line in order to obtain the correct $\lambda^*$ with less computational effort.

It is worth mentioning that the same tuning matrices $\lambda^*$ are obtained if the LMI-based optimization was solved online or offline by interpolation. Nevertheless, it has been preferable to adopt the polytopic approach with less time and computational memory consumption.

On the other hand, there exists a classical online method to compute the tuning parameter $\lambda^*$ using (3) (Combastel, 2015; Wang et al., 2018):

$$
A_k^* = \frac{\hat{H}_k \hat{H}_k^\top C_k^\top}{C_k \hat{H}_k \hat{H}_k^\top C_k^\top + E_c E_c^\top}.
$$

Based on the aforementioned, we adopted the offline LMI-based tuning to reduce the computational efforts and time, in addition to the fact that the classical approach in (3) is applicable to linear systems only. A comparison between the two approaches is shown in Section 5.

3.3 ZSM Estimation Algorithm

The following algorithm highlights the implementation of the ZSM technique for the JESP.

**Algorithm 1. ZSM Algorithm for Joint Estimation**

**Initialize:** states $x_0$, inputs $u_0$, outputs $y_0$, disturbances $\omega_0$, noises $v_0$, distribution matrices $E_x$ and $E_v$, and statespace matrices $A_0$, $B_0$, $C_0$, $D_0$

**For** $k = 1 : N$

1- Identify the discrete-time LPV model (4)
2- Compute the predicted centered states and the generator matrices of the zonotope $\hat{P}_{k+1}$, as in (7)
3- Compute the strip of the consistent states $X_{\text{uk}}$ using the first measurement of $y_{\text{uk}}^{\text{meas}}$, as in (8)
4- Interpolate among the offline calculated tuning matrices $\lambda^*$ as explained in (3.2)
5- Compute the intersection between the consistent strip and the predicted states, as in (9)
6- Repeat steps 3, 4 and 5 with a new measurement strip
7- Return the estimated zonotope, as in (11)
8- Update the estimated zonotopic parameter by its estimated value and consequently the zonotopic LPV

Continue with the ZSM prediction in Algorithm 2

4. RZSM APPROACH FOR REMAINING USEFUL LIFE PREDICTION

The RUL prediction phase is the third step in this structured model-based PHM approach. Thus, it is essential to cope with the propagation of the zonotopic sets for the whole PHM application. Therefore, we have derived a RZSM in order to predict the degradation trajectories based on the estimation of the zonotopic varying parameters. Hence, each predicted trajectory will intersect with the proper physical TH of the parameters, and the End of Life (EoL) is then predicted. This approach provides the homogeneity between the JESP and the RUL forecasting under the zonotopic scheme. It also provides a prior knowledge of the degradation trajectory in the future, which allows a flexible time for maintenance practices.

In this context, since that the only available information about the degradation level is the estimated variation of the parameter, we proposed a general exponential evolution as a degradation model with unknown parameters that will be estimated using the RZSM:

$$
\Delta \theta(t) = \hat{\theta}(t) - \theta_0 = \alpha_1 e^{\alpha_2 t},
$$

where $\Delta \theta(t)$ is the variation of the varying parameter and $\hat{\theta}(t)$ is its estimated value obtained from the ZSM observer. $\alpha_1$ and $\alpha_2$ are the unknown parameters of the proposed exponential degradation model that mimic most of the degradation behaviors, which will be estimated using the RZSM in order to predict the degradation evolution in the future. Then, (17) is discretized and rewritten in a logarithmic form as shown below:

$$
\ln(\hat{\theta}_k - \theta_0) = \ln(\alpha_1) + \alpha_2 k T_s,
$$

where $k \in \mathbb{N}$ and $T_s$ is the sampling time.

Finally, a state-space model of the degradation model is obtained considering the parameters $\alpha_1$ and $\alpha_2$ to be estimated, and (18):

$$
A_{\text{deg}|k} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad C_{\text{deg}|k} = [1, k T_s],
$$

where $A_{\text{deg}|k}$ and $C_{\text{deg}|k}$ are designed with respect to the regressor equation, and the input $u = 0$. Therefore, the output and the states vectors are respectively defined as:

$$
y_{\text{deg}|k} = \ln(\hat{\theta}_k - \theta_0),
$$

$$
x_{\text{deg}|k} = [\ln(\alpha_1), \alpha_2]^\top,
$$

Hence, the second ZSM observer is applied to (19) with (20). Thus, the estimated $\alpha$ parameters are implemented in the general degradation model (17), and projected to the future. A bounded EoL is then obtained once the predicted zonotopic trajectory intersects with the predefined value TH.

**Algorithm 2. RZSM Prediction Algorithm**

**Input:** The estimated parameter $\hat{\theta}_k$ from Algorithm 1

**For** $i = k$

1- Calculate the measurement based on the estimated
parameter, as in (20a)
2- Apply a ZSM observer described in Algorithm 1 with proper modifications for α parameters estimation only
3- Return the zonotope $\hat{x}_{\text{deg}}[k]$, where $\hat{a}_1 = e^{\hat{x}_{\text{deg}}[1]}$, and $\hat{a}_2 = \hat{x}_{\text{deg}}[2]$.
4- Predict the degradation model starting at $k$ using (17)
5- Calculate the indirectly predicted zonotopic RUL ($\hat{RUL}_k$) with respect to the defined TH:
$$\hat{RUL}_k = \text{EoL} - t_{\text{current}}[k]$$
Return $\hat{RUL}_k$ in zonotopic form with lower and upper bounds: $[\hat{RUL}_k, \hat{RUL}_k]$, Repeat with the ZSM-based JESP in Algorithm 1
End

5. CASE STUDY
5.1 Converter Modeling
A 30 kW DC-DC converter with a degraded MOSFET is considered as a case study for the proposed approach. Thus, due to the limited space the degradation modeling is not detailed in this paper. The ON-resistance ($R_{\text{ON}}$) of the MOSFET plays an important role as a failure precursor. See (Al-Mohamad et al., 2020) for the detailed information, and (Celaya et al., 2011; Celaya, J. R. and Saxena, A. and Saha, S. and Goebel, K., 2011) for the accelerated aging measurements of a degraded MOSFET that we use to simulate a degraded converter. Furthermore, the DC-DC converter shown in Figure 2 is a switched-system which alternates between two subsystems that describe the working conditions of the converter (ON/OFF-State).

Fig. 2. Boost converter

The subsystems have been averaged according to $M(\text{avg}) = M(\text{sub 1})d + M(\text{sub 2})(1 - d)$, where $M$ denotes state-space matrices in general and $d = 0.35$ is the duty cycle of the switch (Al-Mohamad et al., 2020). In this case study, only the matrix $A$ is parameter-varying, and the rest remains Linear Time Invariant (LTI). The system has been first modeled in healthy operation, then augmented with the degraded parameters (Al-Mohamad et al., 2020), and transformed to an LPV model as shown below:
$$A = \begin{bmatrix}
-1 & -\text{ESR}_{\text{in}} & 0 & 0 \\
\frac{C_{\text{in}}(R_{\text{on}} + \text{ESR}_{\text{in}})}{L(R_{\text{on}} + \text{ESR}_{\text{in}})} & \frac{\text{ESR}_{\text{in}}}{C_{\text{in}}(R_{\text{on}} + \text{ESR}_{\text{in}})} & \frac{1}{L} & \frac{-\text{ESR}_{\text{in}}}{L} \\
0 & 0 & 0 & 1
\end{bmatrix}, \quad (21)$$
with
$$a_{22} = R_{\text{on}}(\text{ESR}_{\text{on}}(d - 1) - R_{\text{L}} - \hat{R}_{\text{ON}} d) - \text{ESR}_{\text{in}}R_{\text{in}},$$
and the states vector $x = [v_{\text{in}}, i_L, v_C, R_{\text{ON}}]^T$ is augmented with the varying parameter $\theta = R_{\text{ON}}$. The input and output vectors are $u = [v_{\text{in}} i_o]^T$ and $y = [i_{\text{in}} v_o]^T$ respectively. The zonotopic JESP has been investigated with unknown-but-bounded noises with the following direction matrices:
$$E_u = \text{diag}(0.01, 0.01, 0.01, 0.0005), \quad (22a)$$
$$E_v = \text{diag}(0.1, 0.1). \quad (22b)$$
Moreover, the state-space matrices are discretized with a 15000 Hz sampling frequency. $R_{\text{ON}}$ describes the embedded nonlinear degradation. Furthermore, 15000 cycles have been chosen to cover the accelerated life of a power MOSFET until reaching its EoL, denoting 150 accelerated minutes (Celaya, J. R. and Saxena, A. and Saha, S. and Goebel, K., 2011). For the reasons of space limitations, not all the numerical values are presented in this paper, see (Al-Mohamad et al., 2020) for further information.

5.2 ZSM for Joint Estimation Results
Figure 3 illustrates the estimated varying parameter using the ZSM observer which is tuned using the proposed offline robust LMI-based method and compared with the classical online approach. The estimated states are not illustrated due to size limitations, in addition to the fact that the RUL forecasting depends only on the estimated parameters.

![Zonotopic estimation of parameter 4 with $\lambda^*$ and $\Lambda^*$ tuning](image)

As shown in Figure 3, the degradation of the varying parameter is accurately estimated when comparing both tuning approaches. Interestingly, the proposed LMI-based approach ($\lambda^*$) has proved its reliability when compared to the classical online scheme ($\Lambda^*$). Moreover, Table 5.2 shows very high average Relative Accuracy (RA) for all the states and the parameter for both approaches side by side.

| States & Parameter | RA($\lambda^*$)% | RA($\Lambda^*$)% |
|-------------------|-----------------|-----------------|
| $X_1$             | 99.99489        | 99.99409        |
| $X_2$             | 99.77672        | 99.805779       |
| $X_3$             | 99.94787        | 99.93071        |
| $X_4$             | 92.20039        | 96.272235       |

It is essential to compute the RA of the estimated parameter and states in order to assess the reliability of the observer. Eventually, the proposed offline tuning is adopted due to its reduced on-line computational time. Nevertheless, the root objective of the PHM practices is
to forecast the RUL of the system. It is worth mentioning that the cascading degradation affects the internal components characterized by the states estimation, which are not illustrated in this paper due to size limitations.

5.3 RZSM for RUL Prediction Results

The zonotopic RUL is predicted online using the proposed robust approach with optimality criterion. Thus, due to the limited space, the parameters and the degradation prediction results could not be illustrated in this paper. However, the main objective of the overall PHM approach is to obtain a reliable RUL prediction. It is worth noting that the degradation prediction has shown accurate results with respect to the true evolution. Therefore, Figure 4 illustrates the zonotopic RUL forecasting, where the x-axis denotes the online measurements that covers after the EoL of the component. Thus, the y-axis shows the predicted RUL in cycles. The predicted bounds and center are compared to an empirical RUL trend shown in green. The system is assumed to start at a healthy condition, and the predicted RUL is always bounding the empirical model without any risky violation.

![Zonotopic RUL forecasting](image)

Here follows an example of a possible decision-making regarding the zonotopic RUL prediction: An online measurement at $k = 4500$, the empirical RUL indicates $RUL_{emp} = 9200$ cycles. Hence, the predicted RUL is obtained in the form of zonotope with center $RUL = 8741$ cycles and the bounds $[RUL, RUL] = [7830, 9453]$ cycles. Consequently, the decision should be made regarding the critical degree of each application.

6. CONCLUSION

This paper proposes a model-based PHM approach using a set-membership estimation approach based on zonotopes for LPV systems. The fact that the proposed PHM is based on degradation estimation, increases the complexity of the system under some conditions. Therefore, the first addressed issue of nonlinear dynamical system is carried out by transforming the nonlinear model into an LPV. Additionally, the ZSM approach was used as an observer in two complementary ways. The absence of degradation measurement has led to employ the JESP using ZSM with LMI-based tuning for robustness. Furthermore, the same estimated zonotopic parameter was estimated again in an indirect way to predict the degradation trajectory of the critical components in the system. Finally, the latter allows the RUL prediction in a zonotopic scheme. The proposed approach has shown promising results for prognostics. The ongoing research is being developed towards extending the proposed approach to a system-level prognosis methodology.

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