1. INTRODUCTION

Recent years, the models based on the hypothesis that our universe is a four-dimensional space-time hypersurface (3-brane) embedded in a fundamental multi-dimensional space [1, 2] have become quite popular, see, for example, the reviews [3–11] and the references therein. The number of extra dimensions, their characteristic size and the number of physical fields, which are spread out the bulk space, may be different in various approaches. At the same time it is assumed that the additional space size is large enough, and additional dimensions can be, in principle, detected in terrestrial experiments planned in the near future and/or in astrophysical observations. Four dimensions of our world can be ensured, in particular, by the localization mechanism of matter fields on three-dimensional hypersurfaces in multidimensional space, i.e. 3-branes. Different scenarios of domain walls description and their applications to elementary particle physics and cosmology can be found in a number of reviews [4–11]. The interplay between soft breaking of $O(2)$ symmetry and gravity influence is thoroughly investigated around the critical point of spontaneous $t$ symmetry breaking when the v.e.v. of the Higgs-type scalar field occurs. The possibility of (quasi)localization of scalar modes on such thick branes is examined.

In the present work the scalar matter is composed of two fields with $O(2)$ symmetric self interaction. One of them is mixed with gravity scalar modes and plays role of the brane formation mode (due to a kink background) and another one is of a Higgs-field type. The soft breaking of $O(2)$ symmetry by tachyon mass terms for both fields is introduced which eventually generates spontaneous breaking of translational symmetry due to formation of kink-type field v.e.v. Furthermore for special values of tachyon mass terms the critical point of spontaneous $\tau$ symmetry breaking exists when the v.e.v. of the FMG scalar field occurs. In the first phase the only nontrivial v.e.v. is given by a kink configuration. But the branon fluctuations around kink in the presence of gravity are suppressed by the universal repulsive centrifugal potential which survives in the zero gravity limit [22]. Thus gravity induces a discontinuity in the branon field spectrum. However the FMG field in this phase decouples from branons, is massive and exhibits a more regular weak gravity behavior. In the second phase the Higgs-type field obtains a localized v.e.v. to be used for generation de Sitter geometries on both sides of the brane. The formation of “thick” brane with the localization of light particles on it was obtained earlier in [24] with the help of a background scalar and the gravitational fields, when their vacuum configurations have non-trivial topology. Appearance of scalar states with (almost) zero mass on a brane has happened to be possible. However, as it was previously shown [22], the existence of the centrifugal potential in the second variation of scalar-field action may lead to absence of localized modes on a brane.

In the present work the scalar matter is composed of two fields with $O(2)$ symmetric self interaction. One of them (“branon” [25]) is mixed with gravity scalar modes and plays role of the brane formation mode (due to a kink background) and another one is a fermion mass generating (FMG) field (replacing a Higgs field). The soft breaking of $O(2)$ symmetry by tachyon mass terms for both fields is introduced which eventually generates spontaneous breaking of translational symmetry due to formation of kink-type field v.e.v. Furthermore for special values of tachyon mass terms the critical point of spontaneous $\tau$ symmetry breaking exists when the v.e.v. of the FMG scalar field occurs. In the first phase the only nontrivial v.e.v. is given by a kink configuration. But the branon fluctuations around kink in the presence of gravity are suppressed by the universal repulsive centrifugal potential which survives in the zero gravity limit [22]. Thus gravity induces a discontinuity in the branon field spectrum. However the FMG field in this phase decouples from branons, is massive and exhibits a more regular weak gravity behavior. In the second phase the Higgs-type field obtains a localized v.e.v. to be used for generation
of fermion masses [24]. Both fields, branons and FMG scalars, are mixed and the scalar mass spectrum and eigenstates must be found by functional matrix diagonalization.

The work starts (Section 2) with brief motivation of necessity for two scalar fields to provide fermion localization on domain wall [26–35] and to supply localized Dirac fermions with masses. In Section 3 the model of two scalar fields with their minimal coupling to gravity is formulated for arbitrary potential and the equations of motion are derived. In the subsection 3.2 the scalar potential is restricted with a quartic equation of motion are derived. In the subsection 3.2 the scalar potential is restricted with a quartic \( O(2) \) symmetric potential and soft breaking of \( O(2) \) symmetry quadratic in fields (as it could arise from the fermion induced effective action [23]). For this Lagrangian the gaussian normal coordinates are introduced and the appropriate equations of motion are obtained. The existence of two phases which differ in presence or absence of v.e.v. for the FMG field is obtained. The existence of two phases which differ in presence or absence of v.e.v. for the FMG field is obtained. The existence of two phases which differ in presence or absence of v.e.v. for the FMG field is obtained. The existence of two phases which differ in presence or absence of v.e.v. for the FMG field is obtained. The existence of two phases which differ in presence or absence of v.e.v. for the FMG field is obtained. The existence of two phases which differ in presence or absence of v.e.v. for the FMG field is obtained. The existence of two phases which differ in presence or absence of v.e.v. for the FMG field is obtained. The existence of two phases which differ in presence or absence of v.e.v. for the FMG field is obtained. The existence of two phases which differ in presence or absence of v.e.v. for the FMG field is obtained. The existence of two phases which differ in presence or absence of v.e.v. for the FMG field is obtained. 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cally constant, with \( C_\ast > 0 \) and \( \nu = 0 \) then there is a gap for the massive Dirac states.

In this paper we restrict ourselves with generating parity symmetric branes by field configurations of definite parity. The example of a parity odd topological configuration is realized by a kink-like scalar field background (of possibly dynamical origin, see below)

\[
\varphi^+ = M \tanh(M z) .
\]

(6)

The two mass operators have the following potentials

\[
m^2_\pm = -\partial_z^2 + M^2 [1 - 2 \sech^2(M z)];
\]

(7)

and the left-handed normalized zero-mode is localized around \( z = 0 \),

\[
\Psi_0 (x, z) = \Psi_L (x) \psi_0 (z),
\]

(8)

\[
\psi_0 (z) = \sqrt{M/2 \sech (M z)}.
\]

Evidently the threshold for the continuum is at \( M^2 \) and the heavy Dirac particles may have any masses \( m > M \). The corresponding wave functions are spread out in the fifth dimension.

But the fermions of the Standard Model are mainly massive and composed of both left- and right-handed spinors. Therefore, for light fermions on a brane one needs at least two five-dimensional fermions \( \psi_1 (X), \psi_2 (X) \) in order to generate left- and right-handed parts of a four-dimensional Dirac bi-spinor as zero modes. The required zero modes with different chiralities for \( \langle \Phi (X) \rangle_0 = \varphi^+ (z) \) arise when the two fermions couple to the scalar field \( \Phi (X) \) with opposite charges,

\[
[i \hat{\varphi} - \tau_3 \Phi (X)] \Psi (X) = 0, \quad \hat{\varphi} = \gamma_a \sigma^a,
\]

(9)

\[
\Psi (X) = \begin{pmatrix} \psi_1 (X) \\ \psi_2 (X) \end{pmatrix} ,
\]

where \( \gamma_a \otimes \mathbf{1}_2 \) are Dirac matrices and \( \tau_a = \mathbf{1}_4 \otimes \sigma_a \), \( a = 1, 2, 3 \) are the generalizations of the Pauli matrices \( \sigma_a \) acting on the bi-spinor components \( \psi_j (X) \).

In this way one obtains a massless Dirac particle on the brane and the next task is to supply it with a light mass. As the mass operator mixes left- and right-handed components of the four-dimensional fermion it is embedded in the Dirac operator (9) with the mixing matrix \( \tau_3 m_j \) of the fields \( \psi_j (X) \) and \( \psi_3 (X) \). If realizing the Standard Model mechanism of fermion mass generation by means of dedicated scalars, one has to introduce the second scalar field \( H(x) \), replacing the bare mass \( \tau_3 m_j \rightarrow \tau_3 H(x) \) in the Lagrangian density [24],

\[
\mathcal{L}^{(5)} (\bar{\Psi}, \Psi, \Phi, H) = \bar{\Psi} (i \hat{\varphi} - \tau_3 \Phi - \tau_3 H) \Psi .
\]

(10)

Both scalar fields may be dynamical and their self-interaction should justify the spontaneous symmetry breaking by certain classical configurations trapping light massive fermions on the domain wall. If the lagrangian of scalar fields is symmetric under reflections \( \Phi, -\Phi \) and \( H \rightarrow -H \) then the invariance may hold under discrete \( \tau \)-symmetry transformations,

\[
\Psi \rightarrow \tau_3 \Psi; \quad \Phi \rightarrow -\Phi ;
\]

(11)

\[
\Psi \rightarrow \tau_2 \Psi; \quad \Phi, H \rightarrow -\Phi, -H ;
\]

(12)

\[
\Psi \rightarrow \tau_1 \Psi; \quad H \rightarrow -H ,
\]

(13)

the \( \tau_3 \) symmetry in fact can be extended to the continuous \( U_\tau (1) \) symmetry under rotations,

\[
\Psi \rightarrow \exp \{ i \alpha \tau_2 / 2 \} \Psi; \quad \Phi \rightarrow \cos \alpha \Phi + \sin \alpha H , \quad H \rightarrow -\sin \alpha \Phi + \cos \alpha H ,
\]

(14)

which could be a high-energy symmetry if the scalar field lagrangian adopts it for large values of fields. But at full these symmetries do not allow the fermions to acquire a mass unless translational invariance is spontaneously broken in the scalar sector.

There may be several patterns of the partial \( \tau \) symmetry breaking by scalar field backgrounds. The first one is generated by a \( z \)-inhomogeneous v.e.v. of only one of the fields, say, the field \( \Phi (z) \) with \( H(z) = 0 \). Then the \( \tau_3 \) symmetry certainly survives but the \( \tau_1, \tau_2 \) symmetries are broken. Still if the function \( \Phi (z) \) is odd against reflection in \( z \) the latter symmetries can be restored being supplemented by reflection \( z \rightarrow -z \). But if \( \Phi (z) \) and \( H(z) \) are odd and even functions respectively, the \( \tau_3 P \) symmetry may again survive.

Thus one may anticipate a phase transition between the phases with different symmetry patterns which is presumably of the second order if the v.e.v. \( H(z) \) is continuous in coupling constants of the model. This realization is welcome to implement light fermion masses near a phase transition which are governed by a small deviation in parameters of the scalar field potential around a scaling point much less than the localization scale \( M \).

Further on we assume that the dynamics of fermions and scalar fields is \( \tau \) - and \( U_\tau (1) \)-symmetric (13), (14) at high energies whereas at low energies \( U_\tau (1) \)-symmetry is broken softly and \( \tau \)-symmetry is violated spontaneously. Accordingly the scalar field potential contains even powers of fields \( \Phi (z) \) and \( H(z) \) and its profile induces the required spontaneous symmetry breaking. A concrete model for two phases with broken translational invariance is presented in the next section.
3. FORMULATION OF THE MODEL IN BOSONIC SECTOR

3.1. General Two-boson Potentials: Conformal Coordinates

We want to examine the properties of scalar matter generating gravity. Therefore let us supply the five-dimensional space with gravity providing it with a pseudo Riemann metric tensor $g_{AB}$. This tensor is flat in space and for the rectangular coordinate system is reduced to $\eta^{ab}$. We define the dynamics of two real scalar fields $\Phi(x)$ and $H(x)$ with a minimal interaction to gravity by the following action functional,

\[ S[g, \Phi, H] = \int d^{5}x \sqrt{|g|} \mathcal{L}(g, \Phi, H), \tag{15} \]

\[ \mathcal{L} = \left\{-\frac{1}{2}M_*^{2}R + \frac{1}{2}(\partial_{A}\Phi\partial^{A}\Phi + \partial_{A}H\partial^{A}H) \right\}, \tag{16} \]

where $R$ is a scalar curvature, $\sqrt{|g|}$ is the determinant of the metric tensor, and $M_*$ denotes a five-dimensional gravitational Planck scale.

The equations of motion are

\[ R_{AB} - \frac{1}{2}g_{AB}R = \frac{1}{M_*^{2}}T_{AB}, \]

\[ D^{2}\Phi = -\frac{\partial V}{\partial \Phi}, \quad D^{2}H = -\frac{\partial V}{\partial H}, \tag{17} \]

where $D^{2}$ is a covariant D’Alambertian, and the energy-momentum tensor reads,

\[ T_{AB} = \partial_{A}\Phi\partial_{B}\Phi + \partial_{A}H\partial_{B}H - g_{AB}\left(\frac{1}{2}\partial_{C}\Phi\partial^{C}\Phi + \partial_{C}H\partial^{C}H - V(\Phi, H)\right). \tag{18} \]

In order to build a thick 3 + 1-dimensional brane we study such classical vacuum configurations which do not violate spontaneously 4-dimensional Poincare invariance. In this Section the metric is represented in the conformally flat form, $g_{AB} = A^{2}(x)\eta_{ab}$. This kind of metric suits well for interpretation of scalar fluctuation spectrum and their resonance effects (i.e. scattering states).

For this metric the equations of motion read,

\[ \left( A^{2}\Phi^{2}ight)' = \frac{\Phi^{2} + H^{2}}{3M_*^{2}A}, \tag{19} \]

\[ -2A^{5}V(\Phi, H) = 3M_*^{2}(A^{2}A'' + 2A(A')^{2}), \]

\[ (A^{3}\Phi)' = A^{5}\frac{\partial V}{\partial \Phi}, \quad (A^{3}H)' = A^{5}\frac{\partial V}{\partial H}. \tag{20} \]

One can prove [24], that only three of these equations are independent.

Following the arguments of the previous section we assume that the potential is analytic in scalar fields, exhibits the discrete symmetry under reflections $\Phi \rightarrow -\Phi$ and $H \rightarrow -H$ and has a set of minima for nonvanishing v.e.v. of scalar fields. Correspondingly there exist constant background solutions $\{\Phi_{\text{min}}, H_{\text{min}}\}$ which are compatible with the Einstein equations provided that $\langle V(\Phi, H) \rangle = V(\Phi_{\text{min}}, H_{\text{min}}) \equiv \lambda_{\text{cosm}}M_*^{2} < 0$, i.e. for positive cosmological constant $\lambda_{\text{cosm}}$. In this case the warped geometry will be of Anti-de-Sitter type, $1/A \sim \pm \kappa z$ with AdS curvature $\kappa = \sqrt{\lambda_{\text{cosm}}}/6$ as in the Randall-Sundrum model II [39].

3.2. Minimal Realization in $\phi^{4}$ Theory: Gaussian Normal Coordinates

In this Subsection we study the formation of a brane in the theory with a minimal stable potential admitting kink solutions. It possesses a quartic scalar self-interaction and wrong-sign mass terms for both scalar fields. This potential is designed with $U(1)$-symmetry of dim-4 vertices but with different quadratic couplings. The conveniently normalized effective action has the form,

\[ S_{\text{eff}}(\tilde{\Phi}, g) = \frac{1}{2}M_*^{2} \int d^{5}x \sqrt{|g|} \left\{-R + 2\lambda + \frac{3\kappa}{M^{2}}(\partial_{a}\tilde{\Phi}\partial^{a}\tilde{\Phi}) \right. \]

\[ + \partial_{a}\tilde{H}\partial^{a}\tilde{H} + 2M^{2}\tilde{\Phi}^{2} + 2\Delta_{H}\tilde{H}^{2} - (\tilde{\Phi}^{2} + \tilde{H}^{2}) + \left. - V_{0} \right\}, \tag{21} \]

where the normalization of the kinetic term of scalar fields $\kappa$ is chosen differently from (15) in order to simplify the Eqs. of motion (see below). They are connected as follows,

\[ [\Phi, H] = \left( \frac{3\kappa M_*^{2}}{M^{2}} \right)^{1/2} [\tilde{\Phi}, \tilde{H}]. \tag{22} \]

For relating it to the weak gravity limit we guess that $\kappa \sim M_*^{4}/M^{3}$ is a small parameter, which characterizes the interaction of gravity and matter fields. Let us take $M^{2} > \Delta_{H}$ then the true minima are achieved at $\tilde{\Phi}_{\text{min}} = \pm M$, $\tilde{H}_{\text{min}} = 0$ and a constant shift of the potential energy must be set $V_{0} = M^{4}$ in order to determine properly the cosmological constant $\lambda_{\text{cosm}}$.²

² It could be inherited from the low-energy effective action of composite scalar fields induced by the one-loop dynamics of five-dimensional pre-fermions [23].
Now we change the coordinate frame to the warped metric in gaussian normal coordinates,

\[ ds^2 = \exp(-2\rho(y))dx_\mu dx^\mu - dy^2, \]
\[ y = \int dz'A(z'). \] (23)

This choice happens to be more tractable for analytic calculations than the conformal one used for (20). With the definition (23) the function \( y(z) \) is monotonous and \( z \to -z \to y - y \).

The Eqs. of motion (20) for this metric take the form,

\[ \ddot{\Phi} = -2M^2 \dot{\Phi} + 4\rho' \dot{\Phi}' + 2 \dot{\Phi}(\dot{\Phi}^2 + \dot{H}^2), \] (24)

\[ \ddot{H} = -2\Delta_H \dot{H} + 4\rho' \dot{H}' + 2 \dot{H}(\dot{\Phi}^2 + \dot{H}^2), \] (25)

\[ \rho'' = \frac{\kappa}{M^2} (\dot{\Phi}^2 + \dot{H}^2), \] (26)

\[ \lambda_{\text{cosm}} = -6 \rho' + \frac{3\kappa}{2M^2} (\dot{\Phi}^2 + (\dot{H})^2) + 2M^2 \dot{\Phi}^2 + 2\Delta_H \dot{H}^2 - (\dot{\Phi}^2 + \dot{H}^2)^2 - M^4 \} , \] (27)

When compared to Eqs. (20) one finds that in the gaussian coordinates the Eqs. (24), (25), (26) are algebraically simpler being linear in the metric factor \( \rho' \). It allows to calculate few first orders in gravitational perturbation theory analytically.

As expected for constant background solutions \( \ddot{\Phi}_\text{min} = \pm M, \ddot{H}_\text{min} = 0 \) the cosmological constant \( \lambda_{\text{cosm}} \) completely determines the metric factor \( \rho' = \sqrt{-\lambda_{\text{cosm}}}/6 \). In general, for any classical solution, the right-hand side of (27) is an integration constant that can be proven by differentiating this equation. Thus \( \lambda_{\text{cosm}} \) is indeed a true constant at the classical level.

The above equations contain terms which have different orders in small parameter \( \kappa \), and accordingly they can be solved by perturbation theory assuming that,

\[ \left| \rho'(y) \right| = O(\kappa) \Rightarrow \left| \rho''(y) \right| = O(\kappa^2). \]

Then in the leading order in \( \kappa \) the equations for the fields \( \Phi(y), \dot{H}(y) \) do not contain the metric factor, and the metric is completely governed by matter order by order in \( \kappa \).

Depending on the relation between quadratic couplings \( M^2 \) and \( \Delta_H \) there are the two types of \( z \)-inhomogeneous solutions of the Eq. (27) which have the form of a two-component kink [24]. For gravity switched off the first one holds for \( \Delta_H \leq M^2/2 \),

\[ \Phi \to \Phi_\text{0} = \pm Mtanh(My) + O(\kappa), \quad \dot{H}(y) = 0, \] (28)

and therefore the conformal factor to the leading order in \( \kappa \) reads,

\[ \rho_1(y) = \frac{2\kappa}{3} \left\{ \ln \cosh(My) + \frac{1}{4} \tanh^2(My) \right\} + O(\kappa^2), \] (29)

which is chosen to be an even function of \( y \) in order to preserve the remaining \( \tau \) symmetry.

The second one arises only when \( M^2/2 \leq \Delta_H < M^2 \), i.e. \( 2\Delta_H = M^2 + \mu^2, \mu^2 < M^2 \),

\[ \Phi_\text{0}(y) = \pm Mtanh(\beta My), \quad H_\text{0}(y) = \pm \frac{\mu}{\cosh(\beta My)}, \] (30)

\[ \beta = \sqrt{1 - \frac{\mu^2}{M^2}} , \]

where from one can find the conformal factor to the leading order in \( \kappa \) in the following form,

\[ \rho_1(y) = \frac{\kappa}{3} \left\{ (3 - \beta^2) \ln \cosh(\beta My) \right\} + O(\kappa^2), \] (31)

as well symmetric against \( y \to -y \). One can see that the asymptotic AdS curvature \( \kappa \) (defined in the limit \( y \gg 1/M \) when \( \rho(y) \sim ky \)) is somewhat different in the \( r \) symmetry unbroken and broken phases,

\[ k_{\text{unbroken}} = \frac{2}{3}\kappa M \quad \text{vs.} \]
\[ k_{\text{broken}} = \frac{2}{3}\kappa M \left( 1 + \frac{\mu^2}{2M^2} \right) \frac{1 - \frac{\mu^2}{M^2}}{k_{\text{unbroken}}}. \] (32)

As the scalar potential is invariant under reflections \( \Phi(y) \to -\Phi(y) \) and \( \dot{H}(y) \to -\dot{H}(y) \) one finds replicas of the kink-type solutions which can be uniquely selected out from coupling to fermions if to specify their chirality (+\( M \) for left-handed ones) and the sign of induced masses (+\( \mu \) for positive masses). Let’s choose the positive signs further on.

Evidently the second solution generates the fermion mass in (10) whereas the first kink leaves fermions massless. The solution breaks \( \tau \) symmetry and is of main interest for our model building. Thus there are two phases with different scalar backgrounds and it can be shown (see below) that if \( \Delta_H < M^2/2 \) the first kink pro-
vides a local minimum but for some $M^2/2 < \Delta H < M^2$ it
gives a saddle point whereas the second kink with $\tilde{H} \neq 0$
guarantees a local stability.

3.3. Relationship to Conformal Coordinate Metric

To the leading order in $\kappa$ one can derive a simple
relation between conformal factor $A(z)$ and $\rho_{1}(y)$. Namely, with a certain ansatz for $A(z)$ the first equation
for the metric factor in (19) taken in the variables (22) is
linearized,

$$A(z) = \frac{1}{1 + f(z)}, \quad f(0) = 0;$$

$$f'' = \kappa \left( \Phi^2 \right)^2 + \left( \frac{\tilde{H}}{M} \right)^2 (1 + f).$$ (33)

Then the expansion in powers of gravitational coupling constant $\kappa$ is given by $f = \sum_{n=1}^{\infty} \kappa^{n} f_{n}$ and the
leading order $f_{1}$ obviously coincides in functional dependence with (26) for the $\tau$-symmetry unbroken phase or (31) for the broken phase,

$$\kappa f_{1}(z) = \rho_{1}(y - z)$$

$$= \kappa \left\{ (3 - \beta^2) \ln \cosh (\beta Mz) + \frac{1}{2} \beta^2 \tanh^2 (\beta Mz) \right\} + O(\kappa^3).$$ (34)

However the perturbative expansion in $\kappa$ is not valid for any $z$. Indeed for $\beta Mz \gg 1/\kappa \gg 1$ the asymptotic $f(z)$ is linearly growing and the second term in the
right-hand side of Eq. (33) dominates over the first one which generated the perturbation series. In spite of
that in conformal reference frame the coordinate asymptotic at $Mz \gg 1$ is given by the leading order in $\kappa$ as $f(z) \rightarrow k z$ the next orders in $k$ have a more complicated nonanalytic structure. Thereby the perturbation
theory in gaussian normal coordinates happens to be more tractable.

3.4. Next Approximation in $\kappa$: Unbroken $\tau$ Symmetry

Let us find the modifications of kink profiles and the shift of critical point under gravity influence. In
the unbroken phase (zero order in $\rho$) the expansion in $\kappa$ reads,

$$\Phi = M \sum_{n=0}^{\infty} \kappa^{n} \Phi_{n}, \quad \rho = \sum_{n=1}^{\infty} \kappa^{n} \rho_{n}.$$ (35)

In order to simplify the asymptotic behavior and analytic structure we introduce also the coupling depen-
dence into the argument of iterated functions similar
to Eq. (30), $\beta \rightarrow \beta(\kappa)$ with the expansion,

$$\frac{1}{\beta^2(\kappa)} = \sum_{n=0}^{\infty} \kappa^{n} \left( \frac{1}{\beta} \right)^{n}; \quad \left( \frac{1}{\beta} \right) = 1.$$ (36)

After rescaling $y = \tau/\beta M$, $\Phi \rightarrow M \Phi$ the next-to-leading order for $\Phi$ obeys the equation,

$$\frac{\partial_{\tau}^{2} + 2 - 6 \Phi_{0}}{\Phi_{0}} \Phi_{1} = 4 \rho_{1} \phi_{0} - 2 \kappa \left( \frac{1}{\beta^2} \right) + \cdots$$ (37)

where the definitions (28) and (29) have been used. Its real
parity-odd solution can be found by integration of (37),

$$\Phi_{1} = \frac{1}{\cosh^2 \tau \left( \frac{1}{\beta} \right)} \int \tau \cosh^4 \tau \int \Phi_{0} - \frac{1}{\cosh^2 \tau} \Phi_{1}(\tau).$$ (38)

It decreases at infinity for $\left( \frac{1}{\beta} \right) = 4/3$ and looks as follows,

$$\Phi_{1} = \frac{2}{9} \sinh \frac{\tau}{3}.$$ (39)

Therefrom the appropriately iterated function $&(\Phi)$ can be represented as,

$$\Phi(\tau) = M \tan \beta My \left( 1 - \kappa \frac{2}{9 \cosh^2 \beta My} \right) + O(\kappa^3);$$ (40)

$$\beta = 1 - \frac{2}{3} \kappa.$$ (41)

The second approximation of conformal factor $\rho_{2}^{\prime}$
derived directly from (39) obeys the equation,

$$\rho_{2}^{\prime} = 2 \Phi_{0} \Phi_{1},$$ (41)

which can be integrated to,

$$\rho_{2}^{\prime} = \frac{2M}{135} \tan \beta My \left( 38 + \frac{13}{\cosh^2 \beta My} + \frac{18}{\cosh^4 \beta My} \right).$$ (42)

Accordingly the iterated result could be assembled in,

$$\rho(\tau) = \frac{2}{3} \kappa \left( 1 - \frac{8}{45} \kappa \right) \log \cosh \tau + \frac{1}{6} \kappa \left( 1 - \frac{26}{45} \kappa \right)$$

$$- \frac{1}{6} \kappa \left( 1 - \frac{8}{45} \kappa \right) \log (1 + \tanh^2 \tau)$$

$$+ \frac{1}{6} \kappa \left( 1 - \frac{44}{45} \kappa \right) \tanh^2 \tau + \frac{1}{15} \kappa^2 \tanh^4 \tau + O(\kappa^3),$$ (43)

where the first expansion is ordered in accordance to its decreasing at large $y$ and the second one character-
izes better the vicinity of \( y = 0 \) where the normalization \( \rho(0) = 0 \) is employed.

### 3.5. Next Approximation in \( \kappa \):

**Broken \( \tau \) Symmetry Phase**

Above the phase transition point one discovers nontrivial solutions for \( \tilde{H}(\tau) \) which satisfy the properly normalized Eq. (25). When a weak gravity is present then all functions and constants are taken depending on \( \kappa \),

\[
\tilde{H}(\tau) = M \sum_{n, m = 0}^{\infty} \kappa^n \left( \frac{\mu}{M} \right)^{2m+1} H_{n, m}(\tau);
\]

\[
\tilde{\Phi}(\tau) = M \sum_{n, m = 0}^{\infty} \kappa^n \left( \frac{\mu}{M} \right)^{2m} \Phi_{n, m}(\tau);
\]

\[
\Phi_{n, 0} = \Phi_n, \quad \rho(\tau) = \kappa \sum_{n, m = 0}^{\infty} \kappa^n \left( \frac{\mu}{M} \right)^{2m} \rho_{n+1, m}(\tau);
\]

\[
\rho_{n, 0} = \rho_n, \quad \Delta_H = \Delta_{H, c}(\kappa) + \frac{1}{2} \mu^2,
\]

as well as,

\[
\frac{1}{\beta^2} = \sum_{n, m = 0}^{\infty} \kappa^n \left( \frac{\mu}{M} \right)^{2m} \left( \frac{1}{\beta^2} \right)_{n, m};
\]

\[
\left( \frac{1}{\beta^2} \right)_{0, 0} = 1; \quad \left( \frac{1}{\beta^2} \right)_{0, 1} = 1; \quad \left( \frac{1}{\beta^2} \right)_{1, 0} = \frac{4}{3}.
\]

The position of the critical point \( \mu = 0 \) is generically shifted,

\[
\Delta_{H, c}(\kappa) = \frac{1}{2} M^2 \sum_{n = 0}^{\infty} \kappa^n \Delta_{H} = \frac{1}{2} M^2 \left( 1 - \frac{44}{27} \kappa \right) + O(\kappa^3),
\]

which can be established from the consistency of integrated EoM. Indeed, in the leading approximation its normalization scale \( \mu \) the function \( \tilde{H}(\tau) \) satisfies the equation,

\[
(\partial^2 + 1 - 2 \Phi^2_{0, 0}) H_{1, 0} = -\kappa \left( \Delta_{H} + \left( \frac{1}{\beta} \right)_{1, 0} \right) H_{0, 0} + 4 \rho_1 H_{0, 0} + 2 \kappa \left( \frac{1}{\beta^2} \right)_{1, 0} H_{0, 0} \Phi_{0, 0} + 4 H_{0, 0} \Phi_{0, 0} \Phi_{1, 0}
\]

\[
= \mathcal{F}(\tau).
\]

Its solution can be found by integration of (48)

\[
H_{1, 0} = \frac{-1}{\cosh \tau} \left[ \int_{0}^{\tau} \int_{0}^{\tau} \frac{1}{\cosh \tau} \mathcal{F}(\tau) \right].
\]

and it is given by,

\[
H_{1, 0} = \frac{2}{27 \cosh \tau} (C_{1, 0}^H - 2 \log \cosh \tau + 3 \tanh^2 \tau),
\]

provided that (46) holds. The integration constant \( C_{1, 0}^H \) is not fixed at this order in \( \kappa, \mu \).

Mixed orders in \( \kappa \) and \( \mu^2/M^2 \) practically irrelevant as in realistic models \( \kappa \sim 10^{-15} \) and \( \mu^2/M^2 \sim 10^{-3} \) (see [23] and the Section 7). Correspondingly, \( \kappa \mu^2/M^2 \ll \kappa \ll \mu^2/M^2 \). Therefore the overlapping of classical solutions (30), (31) with solutions (40), (43), (49) provides our calculations with required precision in the case when the perturbation expansion works well. The latter seems to be flawless for classical EoM.

### 4. FIELD FLUCTUATIONS AROUND THE CLASSICAL SOLUTIONS

#### 4.1. Quadratic Action and Infinitesimal Diffeomorphisms

We consider small localized deviations of the fields from the average background values and find the action-square corresponding to them.

Action (15) is invariant under diffeomorphisms. Infinitesimal diffeomorphisms correspond to the Lie derivative along an arbitrary vector field \( \tilde{\xi}^A(X) \), defining the coordinate transformation \( X \rightarrow \tilde{X} = X + \tilde{\xi}(X) \).

Let us introduce the fluctuations of the metric \( h_{AB}(X) \) and the scalar fields \( \phi(X) \) and \( \chi(X) \) on the background solutions of the equations of motion,

\[
g_{AB}(X) = A^2(z) (\eta_{AB} + h_{AB}(X));
\]

\[
\Phi(X) = \Phi(z) + \phi(X); \quad H(X) = H(z) + \chi(X).
\]

Since 4D Poincaré symmetry is not broken, we select the corresponding 4D part of the metric \( h_{\mu \nu} \) and graviscalars \( h_{SS} = S \). By rescaling the vector fluctuations \( \tilde{\xi}_\mu = A^2 \xi_\mu \) and the scalar ones \( \xi_S = A \xi_S \), we obtain the following gauge transformations in the first order of \( \xi_A(X) \),

\[
h_{\mu \nu} \rightarrow h_{\mu \nu} - (\xi_{\mu, n} + \xi_{\nu, \mu} - \frac{2A'}{A} \eta_{\mu \nu} \xi_{55}),
\]

\[
v_\mu \rightarrow v_\mu + (\frac{1}{A} \xi_{\mu, 5} + \xi_{5, \mu}), \quad S \rightarrow S - \frac{2}{A} \xi_{55},
\]

\[
\Phi \rightarrow \Phi + \xi_S \Phi', \quad \chi \rightarrow \chi + \xi_S H',
\]

with an accuracy of order \( O(\xi^2, h^2, h\xi) \). Herein """" denotes a partial derivative.

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Now expand the action to quadratic order in fluctuations. The full action after this procedure is a sum,

\[ \mathcal{L}_{(2)} = \mathcal{L}_h + \mathcal{L}_{\phi, x} + \mathcal{L}_s + \mathcal{L}_v, \]

where

\[
\sqrt{\left(\mathcal{L}_h\right)} = -\frac{1}{2}M_a^3A^3\left\{h_{\alpha\beta}^\mu + \frac{1}{4}h_{\alpha\beta}^\mu h_{\gamma\delta}^\nu - \frac{1}{2}h_{\alpha}^\mu h_{\gamma\delta}^\nu \right\},
\]

\[
+ \frac{1}{2}h_{\alpha\gamma}^\mu h_{\beta\delta}^\nu + 4h_{\beta\delta}^\nu h_{\alpha\gamma}^\mu - \frac{1}{2}h_{\gamma\delta}^\nu h_{\alpha\beta}^\mu, \]

\[
\sqrt{\left(\mathcal{L}_{\phi, x}\right)} = \frac{1}{2}A^3\left(\phi \mu - \phi^2 + \chi_{\mu} \chi \right) - (\chi^2)
\]

\[
- \frac{1}{2}A^2\left(\frac{\partial^2 V}{\partial \phi^2} + 2 \frac{\partial^2 V}{\partial \phi \partial H} \phi \chi + \frac{\partial^2 V}{\partial H^2} \chi^2\right)
\]

\[
+ \frac{1}{2}A H^\mu + 2 \frac{\partial V}{\partial \phi} \phi \chi + \frac{\partial V}{\partial H} \chi^2\right)
\]

\[
\sqrt{\left(\mathcal{L}_s\right)} = \frac{1}{2}A^3\left(-S^2\right)
\]

\[
\sqrt{\left(\mathcal{L}_v\right)} = -\frac{1}{2}M_a^3 \left\{b_{\mu\nu, \sigma} - (b')_{\mu\nu} (b')^{\mu\nu} - f_{\mu\nu} \right\}.
\]

Then the vector fields are transformed as follows,

\[
F_{\mu} \rightarrow F_{\mu} - \zeta_{\mu}, \quad v_{\mu} \rightarrow v_{\mu} - \zeta_{\mu}, \quad F_{\mu} \rightarrow F_{\mu} - \zeta_{\mu}, \quad v_{\mu} \rightarrow v_{\mu} - \zeta_{\mu},
\]

i.e. the expression \( F_{\mu} - \zeta_{\mu} \) is gauge invariant. In turn, the scalar components \( \eta, E, \psi, S, \phi \) change under gauge transformations in the following way,

\[
\eta \rightarrow \eta - \frac{1}{A} \zeta_5 = C; \quad E \rightarrow E - 2C,
\]

\[
\psi \rightarrow \psi + \frac{2A'}{A} \zeta_5, \quad S \rightarrow S - \frac{2}{A} \zeta_5,
\]

\[
\phi \rightarrow \phi + \frac{A'}{A} \zeta_5, \quad \chi \rightarrow \chi + \frac{H'}{A} \zeta_5.
\]

Therefrom we can find four independent gauge invariants,

\[
\frac{1}{2} E^2 - \eta^2 - \frac{A'}{A} \psi; \quad -\psi + 2A' \phi;
\]

\[
\frac{1}{2} A^2 + \left(\frac{A'}{A}\right)^2; \quad H^\mu - \Phi^\mu \chi.
\]

Using the parametrization (57) we can calculate components of the quadratic action,

\[
h = h_{\mu} = \square E + 4 \psi; \quad h_{\alpha\beta}^\mu = \square (F^\mu + E^\alpha) + \psi^\alpha;
\]

\[
h_{\mu\nu}^\mu - h_{\mu} = -3 \square \psi; \quad h_{\mu\nu}^\nu - h_{\mu} = \square F - 3 \psi_{\mu}.
\]

Thus, the decomposition (57) entails a partial separation of degrees of freedom in the lagrangian quadratic in fluctuations,

\[
\sqrt{\left(\mathcal{L}_{(2)}\right)} = \frac{1}{8}M_a^3 A^3 \left\{b_{\mu\nu, \sigma} - (b')_{\mu\nu} (b')^{\mu\nu} - f_{\mu\nu} \right\}
\]

\[
+ \frac{3}{4} M_a^3 A^3 \left\{-\psi_{\mu} \psi^\mu + \psi_{\mu} S^\mu + 2 (\psi^2) + 4A' \psi^\mu S^\mu + \right\}
\]

\[
+ \frac{1}{2} A^3 \left[\phi_{\mu} \phi_{\mu} - (\phi^2) + \chi_{\mu} \chi^\mu - (\chi^2)
\]

\[
- A^2 \left[\frac{\partial^2 V}{\partial \phi^2} + 2 \frac{\partial^2 V}{\partial \phi \partial H} \phi \chi + \frac{\partial^2 V}{\partial H^2} \chi^2\right]
\]

\[
+ 4 \psi^2 (\Phi^\mu + H^\mu
\]

\[
+ S \left[-\Phi^\mu \chi + A^2 \left(\frac{\partial V}{\partial \phi} + \frac{\partial V}{\partial H^\mu} \chi \right)\right]
\]

\[
+ \frac{3}{4} M_a^3 A^3 \left(E^2 - 2 \eta \right) \left\{\frac{A'}{A} S + \psi^2 + \frac{2}{3} (\Phi^\mu + H^\mu)
\]

\[
\right\},
\]

where \( f_{\mu} = F_{\mu} - v_{\mu} \), \( f_{\mu\nu} = f_{\mu\nu} - v_{\mu\nu} \), \( f_{\mu\nu} \).
We see that some redundant degrees of freedom exist, one of vectors \( F_\mu \), \( v_\mu \) and one of scalars \( E, \eta \). They can be removed to provide \( v_\mu = 0 \). Obviously, in the quadratic approximation graviton, gravivector and graviscalar are decoupled from each other. From the last line it follows that the scalar \( E \) is a Lagrange multiplier and generates a gauge-invariant constraint,

\[
\frac{A'}{A} \dot{S} + \dot{\psi} = -\frac{2}{3 M_*} (\Phi' \dot{\phi} + H' \chi).
\]

(64)

From the last line it follows that the scalar field \( \eta \) is a gauge invariant Lagrange multiplier and generates a gauge invariant constraint,

\[
\frac{A'}{A} \dot{S} + \dot{\psi} = 0.
\]

(68)

Thus after taking this constraint into account only two independent scalar fields remain and the scalar action takes the following form,

\[
\sqrt{\frac{\lambda}{8}} [\mathcal{L}_{(2), \text{scal}} = \frac{A'}{A} \dot{R}^2 \{ \partial_\mu \dot{\psi} \partial^\mu \psi - (\partial_\mu \psi)^2 \}
\]

\[
- \frac{A'}{A} \dot{R} \partial^\gamma (\partial_\gamma \psi \chi + \frac{A^3}{4} \{ \partial_\mu \chi \partial^\mu \psi - (\partial_\mu \psi)^2 \}
\]

\[
- \left( \partial_\gamma \right)^2 + \frac{A^2}{R^2} \left( \psi^2 \right) \dot{\psi} \left( \chi^2 \right).
\]

(69)

To normalize kinetic terms the fields should be redefined \( \dot{\chi} = A^{1/2} \dot{\chi}, \dot{\psi} = \Omega \dot{\psi} \), where \( \Omega = A^{5/2} \sqrt{\mathcal{R}}/2 A' \).

\[
\sqrt{\frac{\lambda}{8}} [\mathcal{L}_{(2), \text{scal}} = \frac{1}{2} \partial_\mu \partial^\mu \dot{\psi} \dot{\psi} - (\partial_\mu \psi)^2 - \frac{\Omega''}{\Omega \psi^2}]
\]

\[
- 2 A^3 \chi (\partial_\gamma \partial_\gamma \Omega) \dot{\psi} + \frac{1}{2} \partial_\mu \chi \partial^\mu \psi - (\partial_\mu \psi)^2 - \frac{(A^{3/2} \Omega)'^2}{A^{3/2}}
\]

\[
- \left( \partial_\gamma \right)^2 + \frac{A^2}{R^2} \left( \psi^2 \right) \dot{\psi} \left( \chi^2 \right).
\]

(70)

5. SCALAR FIELD ACTION
IN GAUGE INVARIANT VARIABLES

The further analysis of the scalar spectrum is convenient to perform in gauge invariant variables. Let us perform the following rotation in \((\phi, \chi)\) sector:

\[
\phi = \phi \cos \theta + \chi \sin \theta, \quad \chi = -\phi \sin \theta + \chi \cos \theta,
\]

\[
\cos \theta = \frac{\Phi'}{R}, \quad \sin \theta = \frac{H'}{R}, \quad R^2 = (\Phi')^2 + (H')^2.
\]

(65)

While \( \dot{\chi} \) is gauge invariant \( \dot{\phi} \) is not. We can exclude redundant gauge invariance introducing three gauge invariant variables:

\[
\dot{\psi} = \psi - \frac{2 A'}{A} \dot{\phi}, \quad \dot{S} = S + \frac{2}{R} \dot{\phi} - \frac{2 A'}{A} \dot{\phi}', \quad \dot{\eta} = \dot{E} - 2 \eta - \frac{2}{R} \dot{\phi}.
\]

(66)

Accordingly the scalar part of the lagrangian quadratic in fluctuations takes the form:

\[
\sqrt{\frac{\lambda}{8}} [\mathcal{L}_{(2), \text{scal}} = \frac{3}{4} M_* A^3 \left\{ -\psi' \mu \psi'^\mu + \psi' \mu S'^\mu + 2 (\psi')^2 + 4 A' \psi \dot{S} \right\}
\]

\[
+ \frac{1}{2} \frac{A^3}{A} \left\{ \mu \nu \nabla^\mu \nabla^\nu (\chi')^2 - \left( \partial^\gamma \right)^2 + \frac{A^2}{R^2} \left( \psi^2 \right) \dot{\psi} \left( \chi^2 \right)
\]

\[
- 2 \frac{\partial^2 V}{\partial \phi \partial H} \Phi' H' + \frac{\partial^2 V}{\partial H^2} (\Phi')^2 \right \} \chi^2 - A^3 \Theta \partial^\gamma \dot{S}\chi
\]

\[
- \frac{1}{2} A^3 V(\Phi, H) S^2 + \frac{3}{4} M_* A^3 \nabla^\gamma \left[ \frac{A'}{A} S' + \dot{\psi} \right].
\]

(67)

where \( \Theta = (\arctan \frac{H}{\Phi})' = (H' \Phi' - \Phi'' H')/R^2 \).

6. FLUCTUATIONS IN DIFFERENT PHASES
AND AT CRITICAL POINT

6.1. Fluctuations Around a \( \tau \) Symmetric Background

When \( H(z) = 0 \) the two scalar sectors decouple because \( \theta = 0 \). The operator which describes the braneon mass spectrum,

\[
\tilde{m}_\psi^2 = -\partial_\zeta^2 + \frac{\Omega''}{\Omega} = \left( \partial_\zeta + \frac{\Omega}{\Omega} \right) \left( -\partial_\zeta + \frac{\Omega}{\Omega} \right),
\]

(71)

is positive on functions \( \dot{\psi} (z) \) normalizable along the fifth dimension \( z \). Indeed, the possible zero mode is singular \( \dot{\psi} (z) \sim 1/z_{\text{brane}} \). It corresponds to the centrifugal barrier in the potential \( \Omega''/\Omega \) at the origin [22]. Thus in the presence of gravity there is no a (normalizable) Goldstone zero-mode related to spontaneous breaking of translational symmetry. The cause is evident: the corresponding brane fluctuation represents, in fact, a gauge transformation (51) and does not appear in the invariant part of the spectrum. One could say that in the presence of gravity induced by a brane the latter becomes more rigid as only massive fluctuations are possible around it. Of course, the very
gauge transformation (51) leaves invariant only the quadratic action and thereby a track of Goldstone mode may have influence on higher order vertices of interaction between gravity and scalar fields. This option is beyond the scope of the present investigation.

As to the possible localized states with positive $m^2 > 0$ they may exist with masses of order $M$. However for the action (21) they happen to be unstable resonances as it will be evident from the spectral problem formulated in gaussian normal coordinates.

The fluctuations of the second, mass generating field $\hat{H}(\chi)$ do not develop any centrifugal barrier and as $\langle H \rangle = 0$ their mass spectrum is described by the operator,

$$\hat{m}^2_\chi = -\partial^2_z + \left. A^2(\Phi) \partial^2 \Phi / \partial \Phi^2 \right|_{\Phi = 0} \equiv -\partial^2_z + V(z).$$

Its potential is not singular and for background solutions delivering a minimum this operator must be positive. For the minimal potential with quartic self-interaction (21) in terms of the rescaled variables (22) one can come to more quantitative conclusions. Indeed, for gravity switched off the background $\Phi(z) = \Phi_0(z)$ (pay attention to $y \longrightarrow z$) is defined by (28).

Accordingly the mass spectrum operator receives the potential

$$V(z) = -2\Delta_H + 2\Phi_0^2 = (M^2 - 2\Delta_H)$$

$$+ M^2 \left( 1 - \frac{2}{\cosh^2 Mz} \right).$$

The only localized state of the mass operator $\hat{m}^2_\chi$ is $\chi_0 = 1/cosh(Mz)$ with the corresponding mass $m^2_0 = M^2 - 2\Delta_H$ as expected. Thus in the unbroken phase with $M^2 > 2\Delta_H$ the lightest scalar fluctuation in $\chi$ channel possesses a positive mass and the system is stable. In the critical point, $M^2 = 2\Delta_H$, a lightest fluctuation is massless and for $M^2 < 2\Delta_H \leq 2M^2$ the localized state $\chi_0$ represents a tachyon and brings instability providing a saddle point. Instead the solution (30) provides a true minimum (see [24]).

Qualitatively the spectrum pattern in the gravity background remains similar. But the derivation of localized eigenfunctions uniformly in coordinate $z$ encounters certain difficulties as explained in the Subsection 3.3 and therefore it will be done in gaussian normal coordinates.

6.2. Fluctuations in Gaussian Normal Coordinates

To simplify analytical calculations let us represent the quadratic action for scalar fields in the gaussian normal coordinates $x_\mu, y$,

$$ds^2 = \hat{A}^2(z) (dx_\mu dx^\mu - dz^2)$$

$$= \exp(-2\rho(y)) dx_\mu dx^\mu - dy^2.$$  \hspace{1cm} (74)

We remind the formulas for the transition,

$$z = \int \exp(\rho(y)) dy, \quad A(z) = \exp(-\rho(y)).$$

Below the prime denotes differentiation with respect to $y$. Further on we focus on the minimal potential with quartic self-interaction (21) in terms of the rescaled variables (22). To simplify the form of the action let us introduce $\hat{R} = \exp(\rho) R$ and in addition redefine the fields in order to normalize kinetic term, $\hat{\psi} = \exp(-\rho/2) \psi$, $\hat{\chi} = \exp(-\rho/2) \chi$.

$$S_{(2), \text{scal}} = \int d^4 y\left[ \frac{1}{2} \partial_\mu \hat{\psi} \partial^\mu \hat{\psi} + \frac{1}{2} \partial_\mu \hat{\chi} \partial^\mu \hat{\chi} - 2 \exp(-2\rho) \partial_\mu \hat{\psi} \partial^\mu \hat{\chi} \right]$$

$$- \partial_\mu \left[ \partial_\mu \hat{R} \right] \hat{\psi} - \frac{1}{2} \partial_\mu \hat{R} \partial^\mu \hat{\psi} - \frac{1}{2} \partial_\mu \hat{R} \partial^\mu \hat{\chi} + 2\rho \partial_\mu \hat{\chi} + 3(\rho')^2$$

$$+ 3\rho'' - 4\rho' \frac{\hat{R}}{\hat{\psi}} \hat{\psi} - \frac{1}{2} \exp(-2\rho) \hat{\chi} \left[ -\partial^2_\mu + (\theta')^2 + \frac{1}{\hat{\psi}^2} \right]$$

$$\times \left( \frac{\hat{H}}{\hat{\Phi}} \right) \partial^2 \mathcal{V} \left( \frac{\hat{H}}{\hat{\Phi}} \right) + 2\rho \partial_\mu \hat{\chi} + 3(\rho')^2 - \rho'' \right] \hat{\chi}.$$  \hspace{1cm} (75)

where the second variation of the field potential reads,

$$\partial^2 \mathcal{V} \left( \frac{\hat{H}}{\hat{\Phi}} \right) = \left[ -2\hat{H}^2 + 6\hat{\Phi}^2 + 2\hat{H} + 4\hat{\Phi} \hat{H} \right] / 4\hat{\Phi} \hat{H} - 2\Delta_H + 2\hat{\Phi}^2 + 2\hat{H}^2.$$  \hspace{1cm} (76)

Let us perform the mass spectrum expansion,

$$\hat{\psi}(X) = \exp(\rho) \sum_m \Psi^{(m)}(x) \psi_m(y),$$

$$\hat{\chi}(X) = \exp(\rho) \sum_m \Psi^{(m)}(x) \chi_m(y),$$  \hspace{1cm} (77)

$$\partial_\mu \partial^\mu \Psi^{(m)} = -m^2 \Psi^{(m)}.$$
where the factor $\exp(\rho)$ is introduced to eliminate first derivatives in the equations. We obtain the following equations,

\[
\left( -\partial + \frac{\rho''}{\rho} - \frac{\tilde{R}}{\tilde{R}} + 2\rho \right) \left( \partial + \frac{\rho''}{\rho} - \frac{\tilde{R}}{\tilde{R}} + 2\rho \right) \psi_m = \left( -\partial + \frac{\rho''}{\rho} - \frac{\tilde{R}}{\tilde{R}} + 2\rho \right) \psi_m,
\]

(78)

\[
-2\partial + \left( \frac{\rho''}{\rho} - \frac{\tilde{R}}{\tilde{R}} + 2\rho \right) \psi_m = \exp(2\rho)m^2\psi_m,
\]

(79)

This is a coupled channel equation of second order in derivative and with the spectral parameter $m^2$ as being a coupling constant of a part of potential (a non-derivative piece). The latter part is essentially negative for all $m^2 > 0$. Then as the exponent $\rho(y)$ is positive and growing at very large $y$ it becomes evident that the mass term in the potential makes it unbounded below. Thus any eigenfunction of the spectral problem (79) is at best a resonance state though it could be quasi-localized in a finite volume around a local minimum of the potential. In [23] the probability for quantum tunneling of quasi-localized light resonances with masses $m \ll M$ was estimated as $\sim \exp \left[ -\frac{3}{\kappa} \ln \frac{2M}{m} \right]$, which for phenomenologically acceptable values of $\kappa \sim 10^{-15}$ and $M/m \approx 30$ means an enormous suppression. Moreover in the perturbation theory the decay does not occur as the turning point to an unbounded potential energy is situated at $y \sim 1/\kappa$. Therefore one can calculate the localization of resonances following the perturbation schemes.

In the limit $\kappa \to 0$ we obtain,

\[
\left( -\partial + \frac{\rho''}{\rho_1} - \frac{\tilde{R}}{\tilde{R}} \right) \left( \partial + \frac{\rho''}{\rho_1} - \frac{\tilde{R}}{\tilde{R}} \right) \psi_m = \left( -\partial + \frac{\rho''}{\rho_1} - \frac{\tilde{R}}{\tilde{R}} \right) \psi_m,
\]

(80)

\[
-2\partial + \left( \frac{\rho''}{\rho} - \frac{\tilde{R}}{\tilde{R}} + \frac{\theta''}{\theta} \right) \psi_m = m^2 \psi_m,
\]

(81)

\[
\left( -\partial + \left( \frac{\rho''}{\rho} \right)^2 + \frac{1}{\tilde{R}} \left( -\frac{\tilde{R}}{\tilde{R}} \right) \right) \psi = m^2 \psi_m
\]

where $\rho_1$ is first order of $\kappa$.

\section{Phase Transition Point in the Presence of Gravity}

In the unbroken phase $\tilde{H}(y) = 0$ and the equation on $\chi$ takes the form,

\[
\left[ -\partial^2 + \frac{1}{\beta^2} M^2 e^{2\rho}(-2\Delta_H + 2\Phi^2) + 4(\rho')^2 - 2\rho'' \right] \chi_m = \frac{m^2}{M^2} e^{2\rho} \chi_m,
\]

(82)

where the variable $\tau = \beta MY$ is employed and the derivative is defined against it.

Let us perform the perturbative expansion in $\kappa$,

\[
\chi_m = \sum_{n=0}^{\infty} \kappa^n \chi_{m,n}, \quad \Delta_{H,c} = \frac{1}{2} M^2 \sum_{n=1}^{\infty} \kappa^n \Delta_{H}^n;
\]

(83)

and use also the expansions (35) and (36). The limit of turned off gravity is smooth and the differential operator on the left-hand side of (82) can be factorized,

\[
\left[ \frac{M^2}{4} - 2\Delta_H \right] \left( \partial + \tanh \tau \right) \left( \partial + \tanh \tau \right) \chi_{m,0} = \frac{(m^2)^0}{M^2} \chi_{m,0},
\]

(84)

which corresponds to $\Delta_{H,0} = 1$ for zero scalar mass (phase transition point). In general, for $M^2 - 2\Delta_H > 0$ one finds one localized state with positive $m^2$,

\[
\chi = \frac{1}{\cosh \tau} + O(\kappa), \quad m^2 = M^2 - 2\Delta_H + O(\kappa),
\]

(85)

as it was already established in the previous Subsection.

Let us now examine the phase transition point where $m^2 = 0$ and calculate the next approximation of $\kappa$:

\[
0 = \left[ -\partial^2 + \frac{1}{\beta^2} \left( 1 - \frac{2}{\cosh \tau} \right) \chi_1 \right.
\]

(86)

\[
+ \left[ \frac{1}{\beta^2} \left( 1 - \frac{2}{\cosh \tau} \right) - \Delta_{H,0} \right] \Phi_0 \Phi_1 - 2\rho'' \chi_0.
\]

At critical point $\Delta_{H,0} = 1/2M^2(1 - 44/27\kappa + O(\kappa^2))$ exactly as it has been obtained in (46). Accordingly there exists a normalizable solution of (86) which is a zero-mode corresponding to the second-order phase transition. In this case the corresponding first correction of $\chi$ takes the form,

\[
\chi_1 = \frac{1}{9} \frac{1}{\cosh \tau} \left[ \frac{1}{40 \ln(2 \cosh \tau)} + \frac{38}{3} + C_1 \right]
\]

(87)

where for the constant $C_1 = 0$ this correction is orthogonal to $\chi_0$. Thus in the scalar sector not mixed with branon (gravity) fluctuations the localization of mass-
When \( \Delta_H > \Delta_{H,c} \), the squared mass becomes negative signalling the instability of the unbroken phase. In broken phase mixing terms are nonzero and one has to study spectrum by perturbation theory near critical point. The calculations are not presented in this paper because of their high complexity but to the leading order in \( \kappa \) they provide the same mass for light scalar state as in the model [24] without gravity, namely, \( m^2 = 2\mu^2 + O(\mu^3/M^2) \). This state is associated with the fermion mass generation (Section 2) and substitutes the Higgs field of the Standard Model.

7. CONCLUSIONS: CONSISTENCY OF SCALES AND OF GRAVITATIONAL COUPLING WITH MODERN DATA

To consider phenomenological implications we have to study interaction of the scalar matter with fermions,

\[
\mathcal{L}_f = \overline{\psi}(i\partial - g_{\kappa}\tau_3\Phi\psi - g_{H}\tau_1H)\psi, \tag{88}
\]

where in general we can introduce different Yukawa constants for different fermions of the Standard Model (SM). The localization profile depends on the first coupling \( g_{\kappa} \),

\[
\psi_0 = \exp\left(-g_{\kappa}\int y\Phi(y')\right) \frac{1}{\cosh^a M_\beta y}, \quad \alpha = \frac{g_{\kappa}}{\beta} = g_{\kappa} + O\left(\frac{\mu^2}{M^2}\right). \tag{89}
\]

Correspondingly in the leading order in \( \mu \) and \( \kappa \) the fermion mass is described by

\[
m_f = \frac{\int_{-\infty}^{+\infty} \psi_0(y)^2 H(y)dy}{\int_{-\infty}^{+\infty} \psi_0(y)^2} = g_{H} \Gamma(\alpha + 1/2) \frac{\Gamma(\alpha + 1)}{\Gamma(\alpha + 1/2)}. \tag{90}
\]

As it was shown in Sections 3, 6 the scalar fluctuations have a single normalizable state associated with the fermion mass generation,

\[
\Phi = \Phi_0(y) + O\left(\frac{\mu}{M}\right),
\]

\[
H = H_0(y) + \chi_0(y)h(x) + O\left(\frac{\mu^2}{M^2}\right), \tag{91}
\]

\[
\chi_0 = \frac{1}{\cosh M_\beta y}.
\]

with the mass, \( m_h = \sqrt{2}\mu \left(1 + O(\mu^2/M^2)\right) \). For \( \mu \ll M \) low energy four-dimensional Lagrangian including only the lightest states takes the following form,

\[
\mathcal{L}_{\text{low}} = \frac{3\kappa M^2}{2M^2} \int \chi_0(y)^2 dy \cdot (\partial_\mu h^* h - m_h^2 h^2) + \int \psi_0(y)^2 dy \cdot \psi (i\partial - m_f) \psi - 2g_h \int \psi_0(y)^2 \chi_0(y)dy \cdot \overline{\psi} h \psi. \tag{92}
\]

After normalization,

\[
h \rightarrow h \left(\frac{2}{3\kappa M_h}\right)^{1/3} \int_{-\infty}^{+\infty} \chi_0(y)^2 dy, \tag{93}
\]

we obtain the following Yukawa coupling constant between Higgs-like boson and fermion,

\[
g_f = \frac{2}{3\kappa M_h} \frac{\int_{-\infty}^{+\infty} \chi_0(y)^2 \psi_0(y)^2 dy}{\int_{-\infty}^{+\infty} \chi_0(y)^2 dy \int_{-\infty}^{+\infty} \psi_0(y)^2 dy} \tag{94}
\]

\[
= \frac{2}{3\kappa M_h} \frac{\chi_0(y)h(x)}{\psi_0(y)^2}.
\]

We can compare it with similar couplings \( \lambda, g_{t,SM} \) in the standard Higgs model. We adopt the normalization of coupling constants in the Higgs potential of the Standard Model as follows,

\[
V_{SM}(h(x)) = -m_h^2 h^2 + \lambda h^4, \quad \langle h \rangle = \frac{\nu}{\sqrt{2}} = \frac{m}{\sqrt{2\lambda}}. \tag{95}
\]

The scale \( \nu = 246 \text{ GeV} \) stands for the v.e.v of the Higgs field \( h \) in the Standard Model [41]. For the top quark channel dominating for the Higgs boson decay via one-loop mechanism one obtains,

\[
m_h = \sqrt{2\lambda} \nu, \quad m_t = \frac{1}{\sqrt{2}} g_{t,SM} \cdot \nu \Rightarrow g_{t,SM} = 2\lambda \frac{m_t}{m_h}. \tag{96}
\]
Accordingly the relation between the Yukawa coupling constants is given by,

$$\lambda \frac{g_i^2}{g_{i,SM}} = \frac{1}{6\kappa} \left( \frac{M}{M_*} \right)^3.$$

(97)

Let us involve the gravity scales coming from reduction of five-dimensional Einstein–Hilbert action to the four-dimensional one [23],

$$M_*^3 = k M_p^2,$$

(98)

which can be derived from the graviton kinetic action (63) when taking the wave function $b_{i\mu\nu} = 0$ for massless graviton. It determines the four-dimensional gravity scale, the Planck mass, $M_p = 2.5 \times 10^{18}$ GeV [41].

From the experimental bounds on the AdS curvature in extra dimension [42] one can estimate the minimal value for the mass scales, $M_*$, $M$ as well as for the dimensional gravitational coupling $\kappa$. Indeed, if combining (98), (32) and (97) one gets,

$$M = \sqrt{\frac{3\sqrt{\kappa} M_p}{g_{i,SM}}}, \quad \kappa = \frac{1}{2\sqrt{\kappa}} \frac{M g_{i,SM}}{g_i}.$$  

(99)

The modern bound for the AdS curvature, $k > 0.004$ eV. As well the excess of $\gamma \gamma$ pair production observed recently on LHC [43] could be explained by the Higgs particle decay $h \rightarrow \gamma \gamma$ via virtual $t \bar{t}$ triangle loop if the Yukawa coupling is abnormally larger than the SM value, $g_i/g_{i,SM} = 1–1.5$. All together it entails the following bounds for the scales and couplings of our model,

$$M > 3.5 \text{ TeV;}$$

$$M_* > 3 \times 10^8 \text{ GeV;}$$

$$\kappa > 2 \times 10^{-15}.$$  

(100)

Thus we conclude that the gravitational corrections on localization mechanism are indeed very small except for branon spectrum. But the thickness of the brane may affect the high energy scattering processes already at the next LHC running and show up in appearance/disappearance processes, in particular in missing energy events [12, 44].

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