How Science and Inventing Play Together: Example of Averaging and Mixing Processes Design

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Abstract. The problem of poor systematic support of the stage of ideation in practical engineering design is addressed. Mathematical modelling of process's dynamics is used to generate alternative architecture of a technological unit. The approach is applied to the design of mixing-averaging complex, where the analysis of statistical dynamics equations paves the road to new design concepts. The parametric analysis and optimization of new solution demonstrates potential benefits and savings. The study suggests an iterative scheme of conceptual design, where quantitative and qualitative design tools work together.

1. Introduction
Applied science, by its purpose, bridges fundamental science and practice. However, in the field of controlled production processes, there is a growing gap between science and practice. Mathematical theory of control goes into the jungle of complex mathematical problems, often born in the bowels of science itself. At the same time, applied developments are evolving predominantly in line with the improvement of technical and software tools designed to solve quite primitive ideological tasks of automating individual fragments of technological processes (TP). Attempts to reach the level of solving complex problems of optimizing technology and management using inventions based only on heuristics and intuition far from always lead to positive results and remain unrealized on practice [1,2]. Another serious problem is that mathematics is generally used for parametric optimization only while the concept (architecture) of a project is defined without science, heuristically.

The study is to answer the question “Could quantitative modelling efforts support systematic inventing in engineering design?”. The main research methods are linear dynamic system theory and statistical dynamics of control processes.

By using an example of a specific type of TP we plan to demonstrate, how quantitative methods of science can interact with heuristic-based inventing. We are going to outline a general scheme for this interaction and shown how its results can lead to a significant increase in the efficiency (ultimately, economic) of technical solutions to practical problems.

2. Technology Process Description and Technical Problem Statement
The mechanical mixing of several (at least two) materials is the primary process for many continues flow industrial process. The mixture is to fit certain specification, for example, chemical, mineral or
granule’s contents [3]. Let us focus on two-component continues mixing process with the output chemical contents control hereafter for the sake of explicitly (Figure 1).

![Diagram](image)

**Figure 1.** Technological process scheme preparation of a two-component mixture.

Thus, the TP means the continuous dosing of two materials with various chemical element contents. These two materials are mixed in the mixer. To reduce the mixture contents fluctuations an averaging system is typically design at the output of the mixer. The later means a large volume tank with intensive forced homogenizing. The volume of the averaging device should be large enough to reduce the mixture contents variation magnitude. This concept seems to have resulted from inventive efforts, and immediately forces to think about the tank volume optimization. The latter problem is already accessible with mathematical modelling tools, although being very hard to solve.

We take the following dynamical model of the averaging device [4] that is called “perfect”

\[
\dot{\alpha}_{\text{out}}(t) = \left(\frac{1}{T}\right)[\alpha_{\text{in}}(t) - \alpha_{\text{out}}(t)]
\]

where \( T \) is the time required to fill the averaging tank. Its solution can be written as

\[
\alpha_{\text{out}}(t) = \int_0^\infty h(\theta)\alpha_{\text{in}}(t-\theta) d\theta
\]

(1)

where the weighing function \( h(\theta) \) is defined by

\[
h(\theta) = \left(\frac{1}{T}\right)\exp(-\theta/T)
\]

(2)

Let the variation of the chemical contents of the averaging input flow \( \alpha_{\text{in}}(t) \) be stochastic and defined by the correlation function \( r(\theta) \). Then the deviations of the output flow \( \alpha_{\text{out}}(t) \) can be found by

\[
D_{\text{out}} = \int_0^\infty \int_0^\infty h(\theta_1)h(\theta_2)r(\theta_1-\theta_2)d\theta_1d\theta_2
\]

(3)

Let the correlation function take the typical form \( r(\theta) = D_n \exp(-\xi|\theta|) \). Then, by taking into account (2) and (3), we get

\[
D_{\text{out}} = D_n \left(\frac{\xi T + 1}{\xi T}\right)
\]

(4)

Let the mixture mass flow rate be \( Q \), thus the averaging tank filling time \( T \) up to the mass \( P \) can be defined by \( P = QT \), from which with the help of (4) we obtain

\[
D_{\text{out}} = D_n Q \left(\frac{\xi P + Q}{\xi P}\right)
\]

Thus, if the required averaging factor \( \eta = \sigma_{\text{out}}/\sigma_{\text{in}} \) is set to \( \eta^{(req)} \), where \( \sigma \) is the mean square, then the volume of the averaging tank is to be defined by \( P = Q \left[\frac{1}{(\eta^{(req)})^2 - 1}\right] \).

3. Disadvantages of the Technology Processes and the Way to Fight Them

One of the serious disadvantages of the forced homogenizing concept described above is its substantial energy demand. How to reach the required averaging quality with smaller energy losses? Follow the logic of the Theory of Inventive Problem Solving (with the Russian acronym of TRIZ) ideal final result
inventive design support tool, we have to be even more aggressive in modelling the desired result [2].
Ideally, there is no averaging tank and all, \( T = 0 \), but the required averaging factor is maintained. What
can be changed in the existing design to eliminate the averaging operation? Let us first approximate the
integral in (1) by a finite sum

\[
\alpha_{av}(t) = \int_{0}^{\tau} h(\theta)\alpha_{in}(t-\theta)d\theta \approx \sum_{n=0}^{N} h(i\Delta)\alpha_{in}(t-i\Delta)\Delta
\]

where \( \Delta \ll T \), and \( N \) is big enough to ensure \( h(N\Delta) \approx 0 \).

The expression (5) is the result of pure mathematical manipulation. However, it can be seen as the
result of mixing of \( N+1 \) material flow that are extracted from the averaging input flow by the time shifts
of intervals, divisible by \( \Delta \). The shares of the materials in the mixture are defined by the mass shares
\( h(i\Delta)\Delta \). Figure 2 shows the scheme of this model.

\[ \sum \]

\[ \alpha_{in} \]

\[ h(0)\Delta \]

\[ h(\Delta)\Delta \]

\[ e^{-\alpha_0} \]

\[ h(2\Delta)\Delta \]

\[ e^{-2\alpha_0} \]

\[ h(N\Delta)\Delta \]

\[ e^{-N\alpha_0} \]

\[ \alpha_{av} \]

\textbf{Figure 2.} The scheme of dynamic averaging model.

Thus, a possible hardware realization of such an averaging concept means splitting of input flow into
\( N + 1 \) portions, delaying each part at its volume and unifying of all of them at the output. From the
derived above it follows that to ensure the averaging as in (1) the flow rate into the \( i \)-th volume \( Q_i \) and
the mass of the material in the \( i \)-th volume \( P_i \) are to be

\[
Q_i = Qh(i-1)\Delta\Delta, \quad P_i = Q_i \times [(i-1)\Delta], \quad i = 1, N+1
\]

by the total mass flow rate \( Q \).

The averaging device depicted in Figure 1 is able to ensure the same smothering as the “perfect”
averaging device (2). At the same time, it does not require forced mixing with high power consumption.

It is worth mentioning that the invention process has been supported here by the mathematical model
of the TP.

4. Averaging Optimization

The previous section described an approach that is capable of technical realization of any averaging
specification (1) by the system of parallel transporting volumes. The system of forces homogenizing is
not needed there that might save much costs. This leads to a question if the perfect averaging device
with exponential weighing (2) is the best one to attenuate stochastic deviations of chemical contents of
the raw materials. The negative answer to this question comes from the following simple example. Let
the deviations of the chemical contents of the input to the averaging tank follow the harmonic function

\[
\alpha_{in}(t) = a \sin(2\pi t / T_m)
\]

where \( a \) is the magnitude, and \( T_m \) is the period of variations. Then the chemical contents of the output
are perfectly stable if the averaging system divides the flow into two and delays one of them by the half
of period \( T_m/2 \) and then units them back. The similar effect can be reached in the stochastic variations
of input flow contents too if the spectral density of them has dominating resonance peak.

Having obtained the feasible method to ensure any averaging specification (1) by the system of
bypass volumes we can try to maximize the averaging capacity. For this, the averaging law \( h(\theta) \) should
correspond to the profile of the stochastic variations of the input flow. The following optimization problem emerges. We have to minimize the dispersion of the output material flow

$$D_{out} = \int_{0}^{\theta_2} \int_{0}^{\theta_1} h(\theta_1)h(\theta_2)r(\theta_1 - \theta_2)d\theta_1d\theta_2 \rightarrow \min_{h(\theta)}$$ (7)

subject to

$$\int_{0}^{\theta_2} h(\theta)d\theta = 1, \quad \int_{0}^{\theta_2} h(\theta)d\theta = T, \quad h(\theta) \geq 0$$ (8)

The first restriction of (8) is the condition of unbiasedness. It means that the input and output flows contents are the same if the constant input flow does not change. The second restriction links the averaging tank filling time and averaging law $h(\theta)$ [4]. The third restriction follows from the understanding of the averaging process as the mixing or material portions that enter the averaging tank at the various preceding moments of time. The weighting function $h(\theta)$ defines non-negative shares of the portions in the mixture. To approach this constrained extremum problem, we introduce the optimality criterion

$$J = \int_{0}^{\theta_2} \int_{0}^{\theta_1} h(\theta_1)h(\theta_2)r(\theta_1 - \theta_2)d\theta_1d\theta_2 + \gamma \int_{0}^{\theta_2} h(\theta)d\theta + \gamma_1 \int_{0}^{\theta_2} h(\theta)d\theta$$

and minimize it subject to $h(\theta) \geq 0$. The first variation of the functional is to be zero that delivers the equation

$$\int_{0}^{\theta_2} r(\theta_1 - \theta_2)h(\theta_1)d\theta_1 + \gamma_1 + \gamma_2 \theta_1 = 0$$

Now, as an example, let us return to the case where the input correlation is exponential. The optimal operator will be searched in the form

$$h(\theta) = \begin{cases} A\delta(\theta) + B + C\theta & \text{for } \theta < \tilde{\theta} \\ 0 & \text{for } \theta \geq \tilde{\theta} \end{cases}$$ (9)

Therefore, the best input fluctuation smothering is ensured if a certain part (defined by the parameter $A$) of the input flow bypasses the averaging system. The rest is to be averaged, and this operation should correspond the weighted average over the sliding time interval $\tilde{\theta}$. The weighing function is defined by the linear function $B + C\theta$. Then the bypassed and the averaged parts merge and this flow follows to the next operations. Omitting the derivations, we present the final expressions to define all the constants $A, B, C, \tilde{\theta}$:

$$A = 2/(2 + \varsigma), B = 2\varsigma(\varsigma + 1)/(2 + \varsigma)^2, C = -2\varsigma^2(2 + \varsigma)^2, \tilde{\theta} = \varsigma / \varsigma$$ (10)

$$\varsigma = 2(1 + \tilde{T}) \cos((1/3) \arccos(1 - 2/(1 + \tilde{T})^2)) \tilde{T} - 1, \quad \tilde{T} = \varsigma T$$

Substituting (10) in (9) and using (3) we also obtain the expressions to evaluate the dispersion at the optimal averaging system

$$D_{\text{out}} = D_{\text{in}}[8(6 + 12\varsigma + 6\varsigma^2 + \varsigma^3)]/[3(2 + \varsigma^2)^2]$$ (11)

Coming back to the point of the paper, we emphasize that the derived averaging law (9), (10) celebrate the “scientific” synthesis and by no means could be obtained by heuristic design.

If the correlation function $r(\theta)$ is general, the optimization problem can be approached numerically, by the discretization as in (5).

The optimality criterion takes the quadratic form and the optimization problem is reduced to quadratic programming procedure, that can be solved numerically by standard methods [5].
5. Adaptive Averaging System

One more practically reasonable assumption is that while the averaging system design takes place once and cannot be changed, the probabilistic properties of \( \alpha(t) \) can vary in time. This challenge can be addressed by the adaptive design approach. The adaptive averaging system should have the control and measurement subsystems. The measurement system should include the input material flow chemical contents analyzer, automatic dosing valves at the input and output of the flow systems where the flows are given the controllable delays, and the level sensors in these tanks. The analyzer’s data statistical processing would correct the correlation function \( r(\theta) \) approximation. Then we solve quadratic programming problem and define the optimal weighting function values \( h(i\Delta), i = 0, N \). Then the flow rates \( Q_i \) and masses \( P_i \) are corrected in accordance with (6). Simple single-input-single-output control loops can realize these corrections by dosing mechanisms and level sensors.

The presented design is an invention that is based on probability and control theories. At the same time, the concept idea of adaptive averaging system directly follows from TRIZ’s Trend of flexibility increase [6].

6. Mixture Contents Control

We have been discussing the concept solutions to the problem of reduction of mixture stochastic variations by the averaging operation so far. The join inventive and scientific efforts decreased the energy costs and the volume of averaging tanks. And still the cost of averaging installation can be too high form modern multi-ton production sites. Could we find an inventive solution to trim the averaging operation at all? Indeed, if the chemical analyzers continuously report the mixed materials’ contents variations \( \alpha_1(t) \) and \( \alpha_2(t) \), then the simplest model of mixing

\[
\alpha(t) = \alpha_1(t)u_1(t) + \alpha_2(t)u_2(t), \quad u_1(t) + u_2(t) = 1
\]

could help to evaluate the shares of \( u_1(t) \) and \( u_2(t) \) from the condition of control of the chemical contents of \( \alpha(t) \) at the required level. Then the model evaluations and careful dosing would control the shares of the mixed materials without any averaging system. The precise and permanent contents analysis of mixed materials and unavoidable dosing errors are technically difficult problems. Therefore, the practical realization of this ideal content control concept seems to be very problematic. At the same time, the idea of component share control made its way to the practice and widely used in industry together with finishing averaging operation [2]. Since the content analyzers are expensive and inaccurate, another inventive idea tells to replace the disturbance control by feedback control. The same single content analyzer that was introduced for adaptive averaging system design serves as feedback measurement sensor. The result is more sophisticated mixture contents process control design that includes subsystems for averaging and contents control (Figure 3).

![Figure 3. The scheme of controlled mixture contents preparation.](image-url)
of the controlled technology process design, that would ensure the acceptable smothering degree \( \eta \) at the minimal capital and operation costs. Or the maximal smothering degree \( \eta \) at the bounded costs.

7. Optimization of Automated Technological Complex (ATC)

Let \( H(p) \) and \( W(p) \) are the transfer functions describing the mixing plant and its feedback controller, respectively. We can approach nonparametric optimization problem of both components with the index (7). As we need to define not one but functions \( H(p) \) and \( W(p) \), this problem is much more difficult comparing to (7-8). According to the overview [6], the general solution to this nonparametric optimization problem of both plant and control design has not been found yet. Therefore, we have to reduce our ambitions to a simpler problem of parametric optimization of mixing-averaging ATC [7].

Evaluations made by (4) and (11) show that the averaging efficiency \( \eta \) of “perfect” design against the stochastic disturbances with exponential correlation is just 6% less optimal designs (9-10). Therefore, it can be considered as suboptimal.

Let us return to the “perfect” averaging plan scheme (2). We are interested how the averaging efficiency \( \eta = \sigma_{out} / \sigma_{in} \) change if we use the mixed components share control method together with the averaging. It can be shown that if the assumptions exponential correlation \( \alpha_{out} (t) \) hold, one can arrive at the following dimensionless expressions

\[
\eta^{(\text{uncont})}(T) = \frac{\sigma_{out}^{(\text{uncont})}(T)}{\sigma_{in}} = \frac{1}{(\xi T + 1)^{1/2}} \tag{12}
\]

\[
\eta^{(\text{cont})}(T) = \frac{\sigma_{out}^{(\text{cont})}(T)}{\sigma_{in}} = \lambda \eta^{(\text{uncont})}(T) \tag{13}
\]

\[
\lambda = \{1 - (1 - \xi T)^{-2} \times \{[\exp(-\xi \tau) - \xi T \exp(-\tau / T)]^2 - \xi T[\exp(-\xi \tau) - \exp(-\tau / T)]^2\}\}^{1/2}
\]

These expressions define the accessible stability boundaries of the output products of mixing and averaging units with and without the mixed material share control.

Let us evaluate the practical effect of the control and averaging system cooperation by using standard industrial cement manufacturing parameter values [8]. We assume \( \xi = 0.3 \) hour\(^{-1} \), \( \tau = 1 \) hour. We also assume that the required averaging efficiency is \( \eta^{(\text{req})} = 0.24 \). The Figure 4 presents the evaluations of (12) and (13) for the chosen parameters.

![Figure 4](image_url)

**Figure 4.** The dependence of averaging efficiency from the averaging tank volume in the uncontrolled and controlled systems.

These numerically extracted dependences can graphically chose the parameter \( T \) defining the required volume of averaging tank. The intersections of the horizontal line of \( \eta^{(\text{req})} = 0.24 \) with the curves of uncontrolled and controlled schemes yield the required value of \( T \). For our case \( T^{(\text{cont})} = 1.24 \) hours, \( T^{(\text{uncont})} = 54.7 \) hours. If the required production rate is 100 tons/hour the averaging tank volumes are \( P^{(\text{cont})} = 125 \) tons, \( P^{(\text{uncont})} = 5470 \) tons. The example shows that the heuristic idea to design the plan and the control simultaneously substantially reduced the averaging tank volume. It may save much when the design makes its way to reality [1, 9].
8. Conclusion
The study focuses on the interaction of inventive and optimization efforts in automation design. It demonstrates by the example of automated averaging-mixing unit design how mathematical modelling can be used to support heuristics of conceptual design in the sequence of qualitative (heuristic) and quantitative (scientific) improvements. The scheme of such a design process is generalized in three stages.

1) Landscape overview the existing solutions and problem formulation. Formulation of the ideal scenario solution-to-be.
2) New concept design based on heuristic and inventive design support methods.
3) New concept scientific (quantitative) feasibility analysis and verification. Parametric optimization.

These elementary three step sequence can be repeated many times during the design process, as the example demonstrates. It should be mentioned that, as the example shows, stages 2 and 3 can swap. It happens when the mathematical model supports inventing. The example also demonstrates the advantages of concurrent design when the plant and control are being developed simultaneously. This way of design seems to be as complex as promising.

References
[1] Yakovis L M, Chechurin L S 2015 Creativity and heuristics in process control engineering Chemical engineering research and design Official Journal of the European Federation of Chemical Engineering 103 40–49
[2] Chechurin L, Berdonosov V, Yakovis L, Kaliteevskii V 2019 Heuristic Problems in Automation and Control Design: What Can Be Learnt from TRIZ? Advances in Systematic Creativity 2019 45-69
[3] Gorenko I G, Doroganitch S K, Yakovis L M 1987 Multilevel process control in multicomponent mixtures blending Proceedings of the 10-th World Congress on Automatic Control (Munich, Germany) 2 218–222
[4] Fitzgerald T J 1974 Theory of blending in single inlet flow system Chemical Engineering Science 29 1019-1024
[5] Doroganitch S K, Edvabnik J A, Shtengel E G, Jakovis L M 1989 Optimization of parameters of automated process complex for raw mix preparation Proceedings of the 2-th NCB International seminar on Cement and Building Materials, (New Delhi, India, 30 January-3 February, 1989) vol 4 pp 56 – 63
[6] Sharifzadeh M 2013 Integration of process design and control: A review Chemical Engineering Research and Design 91(12) 2515-2549
[7] Yakovis L M, Chechurin L S 2015 Systematic design of automated processing complexes Proceedings of International Conference on Flexible Automation and Intelligent Manufacturing (June 23-26, 2015, Wolverhampton, West Midlands, United Kingdom)
[8] Duda W H 1988 Cement Data Book vol 3: Raw Material for Cement Production French & European Pubns 188 p
[9] Tsamatsoulis D 2014 Modelling and simulation of raw material blending process in cement raw mix milling installations Can J Chem Eng 92(11) 1882–1894