X-RAY/GeV EMISSIONS FROM CRAB-LIKE PULSARS IN THE LMC

J. Takata$^1$ and K. S. Cheng$^2$

$^1$School of Physics, Huazhong University of Science and Technology, Wuhan 430074, China; takata@hust.edu.cn
$^2$Department of Physics, The University of Hong Kong, Pokfulam Road, Hong Kong; hspksc@hku.hk

Received 2016 April 24; revised 2016 October 27; accepted 2016 November 9; published 2016 December 22

ABSTRACT

We discuss X-ray and gamma-ray emissions from Crab-like pulsars, PSRs J0537-6910 and J0540-6919, in the Large Magellanic Cloud. Fermi-LAT observations have resolved the gamma-ray emissions from these two pulsars and found pulsed emissions from PSR J0540-6919. The total pulsed radiation in the X-ray/gamma-ray energy bands of PSR J0540-6919 is observed with efficiency $\eta_{\text{J0540}} \sim 0.06$ (in $4\pi$ sr), which is about a factor of ten larger than $\eta_{\text{Crab}} \sim 0.006$ of the Crab pulsar. Although PSR J0537-6910 has the highest spin-down power among currently known pulsars, the efficiency of the observed X-ray emissions is about two orders of magnitude smaller than that of PSR J0540-6919. This paper mainly discusses what causes the difference in the radiation efficiencies of these three energetic Crab-like pulsars. We discuss electron/positron acceleration and high-energy emission processes within the outer gap model. By solving the outer gap structure with the dipole magnetic field, we show that the radiation efficiency decreases as the inclination angle between the magnetic axis and the rotation axis increases. To explain the difference in the pulse profile and in the radiation efficiency, our model suggests that PSR J0540-6919 has an inclination angle much smaller than that of the Crab pulsar (here we assume the inclination angles of both pulsars are $\alpha < 90^\circ$). On the other hand, we speculate that the difference in the radiation efficiencies between PSRs J0537-6910 and J0549-6919 is mainly caused by the difference in the Earth viewing angle, and that we see PSR J0537-6910 with an Earth viewing angle $\zeta \approx 90^\circ$ (or $< 90^\circ$) measured from the spin axis, while we see PSR J0540-6919 with $\zeta \sim 90^\circ$.

Key words: gamma-rays: stars – methods: numerical – pulsars: individual (J0537-6910, J0540-6919) – radiation mechanisms: non-thermal

1. INTRODUCTION

PSRs J0537-6910 and J0540-6919 are energetic young pulsars in the Large Magellanic Cloud (hereafter LMC), that were discovered by X-ray observations (Seward et al. 1984; Marshall et al. 1998). The spin-down powers of PSRs J0537-6910 and J0540-6919 are $L_{\text{sd}} \sim 5 \times 10^{38}$ erg s$^{-1}$ and $\sim 1.5 \times 10^{38}$ erg s$^{-1}$, respectively, which are similar to $L_{\text{sd}} \sim 4.5 \times 10^{38}$ erg s$^{-1}$ of the Crab pulsar. Among currently known pulsars, these three, PSR J0537-6910, Crab, and PSR J0540-6919, have the top three highest spin-down powers (see Table 1). In this paper, “Crab-like pulsars” is used to refer to all three. The Fermi Large Area Telescope (hereafter Fermi-LAT) resolved the gamma-ray emissions from the two Crab-like pulsars in the LMC, and furthermore detected the pulsed emissions from PSR J0540-6919 (Ackermann et al. 2015).

PSR J0540-6919 (spin period $P_s = 0.05$ s) is known as the “Crab-twin,” because not only the spin-down parameters but also the properties of the pulsed emissions in multi-wavelength bands are similar to those of the Crab pulsar. First, Fermi-LAT found that the ratio of X-ray luminosity and $>0.1$ GeV gamma-ray luminosity is $L_X/L_{\gamma} \sim 0.1$, which is similar to $L_X/L_{\gamma} \sim 5$ for the Crab pulsar (Abdo et al. 2010). This feature is clearly distinct from $L_X/L_{\gamma} < 10^{-2}-10^{-4}$ of the others (Abdo et al. 2013 for the Fermi-LAT pulsar catalog). Second, the pulse peaks in different wavelength bands are all in phase, just like the pulse profiles of the Crab pulsar. Furthermore, PSR J0540-6919 emits giant radio pulses that appear at the positions of the pulse peaks in the higher-energy bands (Johnston et al. 2004). This property is the same as for the Crab pulsar (Shearer et al. 2003). It is likely that these three features represent the nature of pulsars with $L_{\text{sd}} > 10^{38}$ erg s$^{-1}$.

While the Crab-like pulsars are similar in their spin-down properties, there are several remarkable differences in the observed radiation: (1) the pulse shape, and (2) the radiation efficiency, which is defined as the ratio of the radiation luminosity to the spin-down power $\eta \equiv L_{\text{rad}}/L_{\text{sd}}$. PSR J0540-6919 shows a broad pulse profile with a small dip at the center (Campana et al. 2008 for the X-ray pulse and Gradari et al. 2011 for the optical pulse), while the Crab pulsar shows a sharp double-peak structure with phase separation of $\delta \phi \sim 0.4$ (Abdo et al. 2010). The integrated luminosity of the pulsed X-ray/gamma-ray emissions from PSR J0540-6919 is $L_{\text{rad}} \sim 10^{37}(d/50$ kpc)$^2$ erg s$^{-1}$ (in $4\pi$ sr), which is about a factor of 3 larger than that of the Crab pulsar, $L_{\text{rad}} \sim 3 \times 10^{36}(d/2$ kpc)$^2$ erg s$^{-1}$. As a result, the radiation efficiency $\eta_{\text{J0540}} \sim 0.06$ of PSR J0540-6919 is a factor of ten larger than that of the Crab pulsar, $\eta_{\text{Crab}} \sim 0.006$. Fermi-LAT confirmed that the luminosity of the non-thermal radiation from a pulsar tends to increase as $L_{\gamma} \propto L_{\text{sd}}^{1/2}$ (Abdo et al. 2013), which yields $\eta \propto L_{\text{sd}}^{-1/2}$. This empirical relation cannot explain the ratio $\eta_{\text{J0537}}/\eta_{\text{Crab}} \approx 10$. For PSR J0537-6910, Fermi-LAT did not detect the pulsed emissions, and measured the spectrum fitted by a power-law function, suggesting the emissions originate from the pulsar wind and/or a supernova remnant. The pulsed X-ray emissions from PSR J0537-6910 were observed to be $F_X \sim 5 \times 10^{-13}$ erg cm$^{-2}$ s$^{-1}$ in 2–10 keV (Minoe et al. 2004), indicating the efficiency is $\eta_{\text{J0537}} \lesssim 10^{-3}$, which is much lower than those of the Crab and PSR J0540-6919. Since the distance to the Crab ($d \sim 2$ kpc), and the LMC ($d \sim 50$ kpc) are well determined, the uncertainty in the efficiency due to that in the distance should be small. The observed emission properties of the three energetic pulsars pose...
the high-energy emission process within the framework of the outer gap model. We will discuss the high-energy emissions from these systems.

Electron/positron acceleration and high-energy emission in the pulsar magnetosphere have recently been discussed within the framework of the slot gap model (Harding et al. 2008; Harding & Kalapotharakos 2015), outer gap model (Cheng et al. 2000; Hirotani 2015; Takata et al. 2016), current sheet of the force-free magnetosphere model (Spitkovsky 2006; Bai & Spitkovsky 2010), and pulsar wind model (Aharonian et al. 2012). In this paper, we will discuss the high-energy emission process within the framework of the outer gap accelerator model. For the Crab-like pulsars, the outer gap model has predicted that most >GeV photons from the outer gap are converted into pairs by the pair-creation process and cannot escape from the light cylinder (see Section 2, Cheng et al. 2000; Takata & Chang 2007; Tang et al. 2008). Synchrotron radiation and the inverse-Compton process of the secondary pairs can produce the observed emissions in the optical to TeV energy bands. The outer gap model predicts that the shape of the pulse profile is sensitive to the viewing angle and magnetic inclination angle measured from the spin axis. In the Fermi-LAT pulsar catalog (Abdo et al. 2013), ~75% of the sources show a double-peak structure in the pulse profile and ~40% show a wide phase (0.4 ~ 0.6) separation between the two peaks. The outer gap model explains the widely separated peaks by assuming a larger magnetic inclination angle and a larger Earth viewing angle (Takata et al. 2011; Watters & Romani 2011). On the other hand, Takata & Chang (2007) explain the pulse profile of PSR J0540-6919 by a smaller inclination angle $\alpha \sim 30^\circ$ and a larger viewing angle $\zeta \sim 90^\circ$. The observed geometry of the pulsar wind tori also suggests a viewing angle $\zeta \sim 90^\circ$ for PSR J0540-6919 (Ng & Romani 2004, 2008).

Previous studies of PSR J0540-6919 (Zhang & Cheng 2000; Takata & Chang 2007) mainly discussed the optical/X-ray emissions, since only the upper limit of the GeV flux had been reported before the launch of Fermi. Hence, it is not obvious why the efficiencies of the observed radiations among the Crab-like pulsars are so different. In this paper, we will revisit the non-thermal emission process of the Crab-like pulsars with the outer gap model. In Section 2, we describe our theoretical model for the Crab-like pulsars. In Section 3, we present our fitting spectrum for PSR J0540-6919 and discuss the differences between the Crab and this pulsar. In Section 4, we discuss the emissions from PSR J0537-6910.

### Table 1

| PSRs          | $P_s$ | $L_{ad,38}$ | $B_{k,6}$ | $d_{kpc}$ | $\eta_1$ | $\eta_2$ |
|--------------|-------|-------------|-----------|----------|----------|----------|
| J0537-6910*  | 0.016 | 4.9         | 2.07      | 50       | $3 \times 10^{-4}$ | $\cdots$ |
| Crab         | 0.033 | 4.5         | 0.96      | 2        | $5 \times 10^{-3}$ | 10^{-3}  |
| J0540-6919*  | 0.05  | 1.5         | 0.36      | 50       | 0.024    | 0.038    |
| J1813-1749a  | 0.045 | 0.59        | 0.25      | 4.7      | $\cdots$ | $\cdots$ |
| 11400-6325a  | 0.03  | 0.51        | 0.35      | 7        | $10^{-3}$ | $\times 1.5 \times 10^{-3}$ |

Notes: From the Left to Right Columns, Pulsar Name (PSR), Rotation Period ($P_s$) in Units of s, Spin-down Age ($L_{ad,38}$) in Units of $10^{38}$ erg s^{-1}, Magnetic Field Strength at the Light Cylinder ($B_{k,6}$) in Units of $10^6$ G, Distance to the Source ($d_{kpc}$) in Units of kpc, X-ray Efficiency, and Gamma-ray Efficiency (in $4\pi$ sr).

* The X-ray efficiency in 2~10 keV energy bands (Mineo et al. 2004).
* The X-ray efficiency in 0.3~10 keV and gamma-ray efficiency above 100 MeV (Abdo et al. 2013).
* The X-ray efficiency calculated with “absorbed” fluxes in 2~10 keV and 20~100 keV (Campana et al. 2008) and gamma-ray efficiency above 100 MeV (Ackermann et al. 2015).
* The X-ray efficiency in 20~100 keV, including pulsar and PWN, and upper limit of gamma-ray efficiency (Renaud et al. 2010).
surface X-rays. The new pairs created in the gap are accelerated by the electric field and emit the curvature photons. The pairs created outside the gap lose their energy via synchrotron radiation and the inverse-Compton scattering process. We denote as “primary” pairs the electrons/positrons accelerated inside the gap, and as “secondary” pairs those produced outside the gap. Since no measurements on the surface temperature of the Crab-like pulsars have been made, we assume $T_\gamma = 10^6$ K as the temperature of the entire stellar surface.

The main difference from the calculation in Takata et al. (2016) is that the current model of the Crab-like pulsars takes into account emission from the pairs created outside the outer gap. The X-rays produced by synchrotron radiation of the secondary pairs become the target soft-photon field for the photon–photon pair-creation process occurring outside the gap. One important difference in the circumstellar conditions between the Crab-like pulsars and other Fermi-LAT pulsars is the mean-free path of the pair-creation process between a >1 GeV photon and a background soft photon produced by the secondary pairs. The optical depth inside the light cylinder may be written as

$$\tau_p(r) \sim m_X \sigma_{\gamma \gamma} \sim \frac{1}{10^{35} \text{erg s}^{-1}} \left(\frac{r}{\omega_{\text{lc}}}ight)^{-1} \times \left(\frac{P_\gamma}{0.05 \text{ s}}\right)^{-1} \left(\frac{E_X}{0.1 \text{ keV}}\right)^{-1},$$

where $m_X$ is the number density of the soft photons, $\sigma_{\gamma \gamma} \sim 0.2 \sigma_\gamma$ is the cross-section, $L_X$ is the X-ray luminosity, $E_X$ is the light cylinder radius. The optical depth of the Crab-like pulsars is usually larger than unity with $L_X > 10^{35}$ erg s$^{-1}$. For the Crab-like pulsars, therefore, most of the primary gamma-rays with energy >1 GeV are absorbed by the pair-creation process and the secondary particles will emit X-rays via synchrotron emission. This explains the ratio of X-ray and gamma-ray luminosity $L_X/L_{\text{GeV}} \sim 1$ for the Crab-like pulsars, where $L_{\text{GeV}}$ is the apparent luminosity from the magnetosphere. For other Fermi-LAT pulsars, the mean-free path is of order $\tau_p \sim 10^{-3}$ with $L_X \sim 10^{32}$–$10^{33}$ erg s$^{-1}$, and results in $L_X/L_{\text{GeV}} \sim 10^{-3}$.

To calculate the emission from the pairs produced outside the outer gap, we trace the propagation of the >GeV photons and the pair-creation rates on the trajectory. We calculate the pair-creation mean-free path by assuming the number density of the soft-photons inferred from observations; that is,

$$\frac{dN_\gamma}{dE} = \left(\frac{d}{r}\right)^2 \frac{dN_{\text{obs}}}{dE},$$

where $dN_{\text{obs}}/dE$ is the observed spectrum in the optical to hard X-ray energy bands and $d = 50$ kpc is the distance to the LMC. The pitch angle, $\theta_p$, of the newborn pairs produced outside the outer gap is calculated from

$$\cos \theta_p = \mathbf{b}(r) \cdot \mathbf{n}_i(r_0),$$

where $\mathbf{b}$ is the unit vector of the magnetic field, $\mathbf{n}_i$ is the propagation direction of the gamma-rays, and $\mathbf{r}$ and $\theta_0$ represent the positions of pair-creation and the radiation, respectively. The emission direction is calculated from $\mathbf{n}_i(r_0) = \beta_\rho \mathbf{b} + \beta_\omega \mathbf{n}$, where $\beta_\rho$ is the co-rotation velocity, and $\beta_\omega$ is calculated from $|\mathbf{n}_i| = 1$. In the calculation, there is an uncertainty in the collision angle between the gamma-ray and magnetospheric soft photons. Since the latter are emitted by the secondary pairs, which has a pitch angle $\theta_p$, we may assume a collision angle of $\theta_\rho \sim 2\theta_p$.

Ground-based Cherenkov telescopes have observed pulsed emissions up to ~1 TeV from the Crab pulsar (Aliu et al. 2008, 2011; Abdo et al. 2010; Aleksić et al. 2011, 2012, 2014). The emissions between 10 GeV and 1 TeV are well fitted by a single power-law function. The standard curvature radiation process cannot easily explain the emissions above 100 GeV from the Crab pulsar, which suggests the inverse-Compton scattering process inside the magnetosphere (Aleksić et al. 2011; Harding & Kalapotharakos 2015) or at the pulsar wind region (Aharonian et al. 2012). Within the framework of the outer gap scenario, the >100 GeV emissions of the Crab pulsar are explained by the emission process of TeV primary pairs and/or secondary pairs that were produced by the pair-creation process of the TeV photons from the inverse-Compton scattering process of the primary pairs. If the infrared (IR) photons from the secondary pairs enter the outer gap, they are up-scattered by ~1 TeV electrons/positrons whose Lorentz factor is $\Gamma \sim 3 \times 10^3$, and become ~10 TeV gamma-rays. Most of the TeV gamma-rays from the outer gap are absorbed by the soft photons outside the gap, and create ~1 TeV electrons/positrons. The TeV secondary pairs also emit photons via synchrotron radiation and inverse-Compton scattering. Furthermore, the high-energy secondary photons also targets for pair-creation. In this paper, we also examine the pair-creation cascade outside the outer gap which is initiated by the TeV gamma-rays from the outer gap, and we will discuss its contribution to the observed emissions of PSR J0540-6919. Since the IR photons are produced above the outer gap, we assume that they irradiate the outer gap around the upper boundary, say ~10% of the gap thickness, with the number density estimated from Equation (2). Since the emission direction of the IR will be related to the pitch angle in Equation (3), we may roughly estimate the collision angle of the inverse-Compton process as $\cos \theta_{\text{IR}} \sim b \cdot n_i$ at the emission point.

3. RESULTS

3.1. Multi-wavelength Spectrum

In this section, we apply the model to PSR J0540-6919. Since there are two kinds of secondary pairs in our calculation, we define the terminology “low-energy secondary” which represents the pairs created by the primary curvature photons, and “high-energy secondary” for those created by primary TeV photons via the inverse-Compton scattering process.

Figure 1 shows the multi-wavelength spectrum of PSR J0540-6919 with the model fitting curves. In the figure, the dashed line shows the synchrotron and inverse-Compton scattering processes of the low-energy secondary pairs, and dashed–dotted line is the emissions from the high-energy secondary pairs. The results are for inclination angle $\alpha = 10^\circ$ and observer viewing angle $\zeta = 80^\circ$ (or 100$^\circ$). In addition, we assume that the injection rate at the inner and outer boundaries is 1% of the Goldreich–Julian value, $\dot{j}_\text{in} = \dot{j}_\text{out} = 10^{-2}$. 
Figure 2 shows the intrinsic spectra for the curvature radiation (solid line) and inverse-Compton scattering process (dashed line) inside the gap.

As we can see in Figure 1, the emissions (dashed line) from the low-energy secondary pairs explain the observed emissions in the 100 eV–1 GeV energy bands. However, the calculated spectrum above 1 GeV decays faster than the Fermi-LAT data. To reconcile these, therefore, the present model predicts that the residual curvature emissions (thin solid line) and/or the emissions from high-energy secondary pairs (dashed–dotted line) contribute to the Fermi-LAT observations. In the current calculation, a fraction of high-energy photons (>10 GeV) emitted by the secondary pairs created near the light cylinder can escape the pair-creation process. As we will argue in Section 4, the dynamic behavior of the outer gap discussed in Takata et al. (2016) will not be the main reason to explain the observed spectrum above the cut-off energy of PSR J0540-6919. With a small inclination angle $\alpha = 10^\circ$, the calculated gamma-ray light curve (solid line in Figure 3) shows a broad pulse with two narrow peaks separated by $\delta \phi \sim 0.2$, which is consistent with the observations.
3.2. Luminosity Versus Inclination Angle

The Fermi-LAT observations found that the efficiency, \( \eta \), of PSR J0540-6919 is about a factor of ten larger than the Crab pulsar, and this result is incompatible with the empirical relation \( \eta \propto L_{\text{sd}}^{-1/2} \) of the Fermi-LAT pulsars (Abdo et al. 2013). Here we suggest the smaller magnetic inclination of PSR J0540-6919 causes the larger radiation efficiency than the Crab pulsar, whose magnetic inclination angle will be relatively large.

Figure 4 shows the calculated gamma-ray luminosity as a function of inclination angle. In the figure, the vertical axis is normalized by the calculated luminosity at \( \alpha = 10^\circ \). We find that the calculation luminosity tends to decrease as the inclination angle increases. In the current model, this dependency was caused by the dependency on (1) the position of the null charge surface of the Goldreich–Julian charge density and on (2) the maximum gap current on the inclination angle. The gap power depends on the thickness of the gap in the poloidal plane, and it decreases with decreasing thickness. The electrodynamics of the conventional gap models expects the relation \( L_{\gamma} \sim f_{\text{gap}}^3 L_{\text{sd}} \), where \( f_{\text{gap}} \) is defined by the ratio between the size of the outer gap measured on the stellar surface and the polar cap size (Takata et al. 2010). For the Crab-like pulsars, the outer gap size may be controlled by the mean-free path of the pair-creation process between the curvature photons and soft X-rays from the neutron star surface (Wang et al. 2010).

Figure 3. Calculated gamma-ray pulse profiles for a viewing angle \( \zeta = 80^\circ \). The inclination angle is \( \alpha = 10^\circ \) for the solid line and \( \alpha = 70^\circ \) for the dashed line, respectively.

Figure 4. The calculated radiation luminosity as a function of inclination angle. The vertical axis is normalized by the luminosity at \( \alpha = 10^\circ \). The results are for \( j_{\text{in}} = j_{\text{out}} = 10^{-4} \).
open field lines approaches the stellar surface with the increase of inclination angle, the location of the outer gap is closer to the stellar surface for larger inclination angle. Since the number density of X-rays from the stellar surface is inversely proportional to the square of the radial distance, the pair-creation mean-free path inside the gap is shorter for the outer gap closer to the stellar surface. Hence, the outer gap becomes thinner and as a result the gap radiation power decreases with increasing inclination angle.

In the current calculation, the pair-creation process inside the gap is occurring due to collisions between GeV gamma-rays and surface X-rays. In this case, the mean free path of the pair-creation process around the light cylinder is estimated as \( \lambda(R_c) \sim 100R_c \). With this mean-free path and the injection rate \( j_{\text{out}} = 0.01 \), the gap thickness is determined to produce \( \sim 5 \times 10^4 \) curvature photons inside the outer gap by one particle injected at the outer boundary, and thus to make \( \sim 100 \) pairs inside the gap (see Section 4). With a constant mean-free path \( \lambda = 100R_c \), we find that the GeV photons have to travel a distance of \( \sim 0.15R_c \) inside the gap to screen the gap. The mean-free path actually depends on the position as \( \lambda(r) \propto r^2 \). For a smaller inclination angle, because the null charge surface is close to the light cylinder, a constant mean-free path with \( \lambda(r) = \lambda_0 \) is a good approximation. For a larger inclination angle, on the other hand, the null charge surface is closer to the stellar surface and the radial dependency of the mean-free path becomes more important, indicating the average mean-free path is shorter. Therefore, the required travel distance of the gamma-rays to create \( \sim 100 \) pairs becomes shorter than \( \sim 0.15R_c \) of the lower inclination case, and therefore the gap thickness reduces.

It has been suggested that the inner boundary of the middle part of the outer gap tends to be shifted toward the stellar surface as the gap current increases. The model suggests that the inner boundary will touch the stellar surface if the gap current is \( j_{\text{gap}} \sim \cos \alpha \) in units of the Goldreich–Julian value (Takata et al. 2004), which decreases with increasing inclination angle. This is because the charge density that is created by the gap current at the inner boundary should match the local value of the Goldreich–Julian charge density, which on the polar cap region is \( \sim \cos \alpha B_r / (P_c) \). One may expect that if the inner boundary of the outer gap touches the stellar surface, the latter supplies copious particles to close the outer gap. Therefore, the gap luminosity decreases with increasing inclination angle.

As described in Section 3.1, the smaller magnetic inclination angle of PSR J0540-6919 preferentially explains the observed small separation of the two peaks in the pulse profile. For a larger magnetic inclination angle (\( \alpha \geq 50^\circ \)) and a viewing angle \( \zeta \sim 90^\circ \), the phase-separation between two peaks is \( \delta \phi \sim 0.4–0.5 \), as shown in Figure 3, and this would be the case for the Crab pulsar. We emphasize, therefore, that the smaller inclination magnetic angle of PSR J0540-6919 can explain both the higher radiation efficiency and the narrower phase separations of the two peaks than those of the Crab pulsar.

4. DISCUSSION

4.1. PSR J0537-6910

Fermi-LAT resolved the gamma-ray emissions from the high spin-down powered pulsar, J0537-6910, in the LMC with a flux level of \( F_\gamma \sim 10^{-11} \text{erg cm}^{-2} \text{s}^{-1} \). However, the pulsed emissions in the Fermi-LAT data have yet to be confirmed, and the observed spectrum fitted by a single power-law function indicates the emissions to be from the pulsar wind nebula and/or a supernova remnant (Ackermann et al. 2015). Since PSR J0537-6910 has the largest spin-down power \( (L_{sd} \sim 5 \times 10^{38} \text{erg s}^{-1}) \) and the strongest magnetic field at the light cylinder \( (B_c \sim 2 \times 10^9 \text{G}) \) among the known pulsars (see the ATNF pulsar catalog, Manchester et al. 2005), it is likely that this pulsar produces gamma-rays in the magnetosphere, but they are buried under the background emission, or the gamma-ray beam is out of the line of sight. The observed emission properties of PSR J0537-6910 are very different from those of the Crab and J0537-6910: (1) the pulsed emissions have been discovered only in the X-ray bands, (2) the observed radiation efficiency in the X-ray is very low \( \eta_x \sim 3 \times 10^{-4} \), and (3) the pulse width, \( \sim 0.2 \), in the X-ray bands (Marshall et al. 1998) is narrower than those of the other pulsars.

No detection of the pulsed emissions by Fermi-LAT makes it difficult for us to discuss the electromagnetic spectrum in the wide energy bands, and to constrain the magnetic inclination and the Earth viewing angle. However, we may expect that the radiation process of PSR J0537-6910 is similar to those of the Crab and J0540-6919, and we may assume that the flux level of the pulsed gamma-rays measured on Earth is \( F_\gamma \sim F_X \), which is the case for the Crab and PSR J0540-6919. Under those assumptions, the observed radiation efficiency will be of the order of \( \eta_{J0537} \sim 10^{-3} \), which is about two orders of magnitude smaller than that of J0540-6919. As expected from Figure 4, we would say that it is difficult to explain \( \eta_{J0540}/\eta_{J0537} \sim 100 \) by the effect of the inclination angle. If both PSRs J0537-6910 and J0540-6919 have a viewing angle \( \zeta \sim 90^\circ \), it is also difficult to explain the difference in the pulse width by the difference in the inclination angle. We suggest therefore that the Earth viewing angle is very different between the two pulsars. Our model suggests that the Earth viewing angle of PSR J0540-6919 is close to \( \zeta \sim 90^\circ \) measured from the spin axis, which is also suggested by a study of the pulsar wind (Ng & Romani 2004, 2008). Since most of the pairs inside the gap are created around the null charge surface, the outer gap emission is stronger for an Earth viewing angle of \( \zeta \sim 90^\circ \). As the viewing angle deviates from \( \zeta \sim 90^\circ \), therefore, the observed gap emission rapidly decreases and hence the apparent radiation efficiency decreases (see Figures 3 and 4 in Takata et al. 2011); at the same time, the pulse width becomes narrower. On these grounds, we speculate that the main reason for the difference in the observed efficiencies and pulsed widths between PSRs J0540-6919 and J0537-6910 is the difference in the Earth viewing angle.

4.2. Dependency on \( j_\text{in} \) and \( j_\text{out} \)

In Figure 1, we assumed the same particle injection rates at the inner and outer boundaries. The assumption of equal injection rates at the gap boundaries is arbitrary, and it is not necessary for the real case. In the current local model, however, it would not be possible to consistently solve the injection particles at the gap boundaries, for which we would have to solve the global structure including the polar cap activities, outer gap activities, and pulsar wind region. To see the dependency on the choice of the injection current, we examined the case for \( j_\text{in} = 0 \) and \( j_\text{out} = 0 \), that is, no particles enter the gap from the inner boundary (star side) or outer boundary (light cylinder side), respectively.
Figure 5 summarizes the dependency of the emissions from the low-energy secondary pairs on the injection currents $j_{in}$ and $j_{out}$; the solid line, dashed line and dashed-dotted line are results for $(j_{in}, j_{out}) = (10^{-2}, 10^{-2}), (10^{-2}, 0)$ and $(0, 10^{-2})$, respectively. In addition, we assume $\alpha = 10^\circ$ and $\zeta = 80^\circ$.

Figure 6. Evolution of the number of the inwardly moving particles in the pair-creation regions along the magnetic field line from the outer boundary. The solid and dashed lines show the solutions for the pair-creation mean free path of $\lambda \propto (1 - s/R_0)^2$ (Equation (7)) and $\lambda$ = constant (Equation (6)).

Figure 5. Multi-wavelength spectrum of PSR J0540-6919. The lines show the spectra of synchrotron radiation (lower-energy part) and inverse-Compton scattering process (higher-energy part) of the low-energy secondary pairs. The solid lines, dashed lines, and dashed-dotted lines are the results for $(j_{in}, j_{out}) = (10^{-2}, 10^{-2}), (10^{-2}, 0)$ and $(0, 10^{-2})$, respectively. In addition, we assume $\alpha = 10^\circ$ and $\zeta = 80^\circ$.

Figure 6. Evolution of the number of the inwardly moving particles in the pair-creation regions along the magnetic field line from the outer boundary. The solid and dashed lines show the solutions for the pair-creation mean free path of $\lambda \propto (1 - s/R_0)^2$ (Equation (7)) and $\lambda$ = constant (Equation (6)).

Figure 5 summarizes the dependency of the emissions from the low-energy secondary pairs on the injection currents $j_{in}$ and $j_{out}$; the solid line, dashed line and dashed-dotted line are results for $(j_{in}, j_{out}) = (10^{-2}, 10^{-2}), (10^{-2}, 0)$ and $(0, 10^{-2})$, respectively. We find that the calculated spectra become harder for $j_{out} = 0$ (dashed line in Figure 5). This is related to the fact that most of the pairs are created by inwardly propagating gamma-rays. Collision with X-rays from the surface is a head-on process for inwardly propagating gamma-rays, while it is tail-on for outwardly propagating gamma-rays. Hence, the mean-free path of the former is shorter than that of the latter, and most of the pairs are created by inwardly propagating gamma-rays. This indicates that the gap size is mainly controlled by the pair-creation process of the inwardly propagating gamma-rays. For $j_{out} = 0$, therefore, the outer gap has to be thick to create enough pairs, and as a result the calculated spectrum becomes harder.

In our calculation, the gap structure is controlled by the magnitude of $j_{out}$, except for the case $j_{out} \ll j_{in}$. We quantitatively discuss how the gap size depends on the injection rate $j_{out}$. Since the gap thickness of the Crab-like pulsar is about 10% of the light cylinder radius, we can approximate that the propagation direction of the gamma-rays is the same as the direction of the particle’s motion. Under this approximation, the evolution of the number density of the inwardly moving particles (electrons) and gamma-rays in the
The pair-creation region is described by
\[ \frac{dn_\ast(s)}{ds} = \frac{g_\ast(s)}{\lambda(s)}, \tag{4} \]
and
\[ \frac{dg_\ast(s)}{ds} = P_\ast n_\ast(s), \tag{5} \]
respectively, where \( s \) is the distance from the outer boundary, and \( n_\ast \) and \( g_\ast \) are number density and photon number density normalized by the Goldreich–Julian density, respectively. We ignore the effect of pair-creation by gamma-rays propagating outward. In addition, \( \lambda(s) \) and \( P_\ast \) are the mean free path of the photon–photon pair-creation process and the rate of the curvature radiation. In the present calculation, since we assume the surface temperature \( T_s \sim 10^8 \) K, the mean-free path inside the gap at the light cylinder is estimated as \( \lambda_0 \sim 1/(\sigma_\gamma n_X) \sim 100R_c \), where we used \( \sigma_\gamma = 0.2\sigma_T \) and \( n_X \sim \sigma_{SB} R_c^2 T_s^3/(c k_B R_c^2) \sim 3 \times 10^{14} \text{ cm}^{-3} \) with \( \sigma_{SB} \) being the Stefan–Boltzmann constant. The rate of the curvature radiation is estimated as \( P_\ast \sim 3 \times 10^{44}/R_c (\Gamma/100)(R_c/R_c)^{-1} \), where \( R_c \) is the curvature radius, and \( \Gamma \) the Lorentz factor of the accelerated particles.

To solve Equations (4) and (5), we impose boundary conditions as \( n_\ast(0) = n_o \) and \( g_\ast(0) = 0 \), where \( s = 0 \) represents the outer boundary. We assume that the rate of the curvature radiation process, \( P_\ast \), is constant along the magnetic field line. By assuming that the mean free path is constant along the distance \( s \) from the outer boundary (\( \lambda(r) = \lambda_0 \)), we find the solution
\[ n_\ast(s) = \frac{n_o}{2} (e^{c_1 s} + e^{-c_1 s}), \tag{6} \]
where \( c_1 = (P_\ast/\lambda_0)^{1/2} \).

Since we consider the surface X-ray emission as the soft-photon field for the photon–photon pair-creation process inside the outer gap, the mean-free path will decrease as \( \lambda \propto r^2 \). To take into account this effect, we explore the solution with the mean-free path in the form \( \lambda(s) = \lambda_0 (1 - s/R_c)^2 \). The solution becomes
\[ n_\ast(s) = \frac{n_o}{b} \left[ a_+ \left( 1 - \frac{s}{R_c} \right)^{a_+ - 1} - a_- \left( 1 - \frac{s}{R_c} \right)^{a_- - 1} \right], \tag{7} \]
where \( a_{\pm} = (1 \pm b)/2 \) with \( b = \sqrt{1 + 4P_\ast R_c^2/\lambda_0} \). Figure 6 shows the evolution of the ratio of the local number density and that at the outer boundary of the inwardly moving particles, \( n(s)/n_o \), as a function of the distance from the outer boundary.

We find that the multiplicity of the particles injected at the outer boundary becomes \( \sim 100 \) if the gamma-rays travel \( \sim 0.15R_c \) from the boundary. This suggests that if the injection rate at the boundary is 1% of the Goldreich–Julian value (that is, \( J_{\text{out}} = 0.01 \)), the number density becomes the Goldreich–Julian value after the gamma-ray travels \( \sim 0.15R_c \) in the outer gap, and the created pairs will significantly screen the accelerating electric field. This is consistent with the gap structure solved in this paper. Figure 6 also indicates that for a smaller injection rate, the gamma-rays have to travel a greater distance to achieve the Goldreich–Julian number density of pairs by pair-creation, and hence the gap size becomes larger.

As Figure 1 shows, the current model with a constant injection rate \( (J_{\text{in}}, J_{\text{out}}) \) predicts that the residual curvature radiation of the primary particles and/or the emissions from the high-energy secondary pairs can explain the observed emissions above 1 GeV. Takata et al. (2016), on the other hand, argued that sub-exponential decays of the GeV spectra of the Fermi-LAT pulsars reflect the time-dependent emission process of the outer gap. They proposed that the injection rate at the gap boundaries is a time-dependent variable and the observed gamma-ray spectrum is emitted from different gap structures with different injection rates. In the model, the observed spectrum fitted better as the superposition of several power-law
plus exponential cut-off functions with varying cut-off energy, for which the different components are produced at the different injection rates at the gap boundaries.

We discuss the shape of the observed GeV spectrum of PSR J0540-6191 by Fermi-LAT using the dynamic model in Takata et al. (2016). We find that the calculated GeV spectra do not greatly affect the assumed extent of injection rate at the gap boundaries. This is because the GeV gamma-rays observed on Earth do not come from the primary pairs in the gap, but the secondary pairs that are decelerated by the radiation process. Figure 7 shows the spectra of the emissions from the low-energy secondary pairs calculated with different injection rates; $j_{\text{in}} = j_{\text{out}} = 10^{-2}$ (solid line), $10^{-3}$ (dashed line), and $10^{-4}$ (dotted line). We can see that the hardness (peak energy) of the “synchrotron emissions” (low-energy component) increases with decreasing injection rate. This is because the gap thickness increases and hence the electric field in the gap becomes stronger as the injection rate decreases. As a result, the energy distribution of the low-energy secondary pairs that emit synchrotron photons becomes harder for a gap with a smaller injection rate. On the other hand, the energy peak of the spectra by “inverse-Compton scattering” (high-energy component) does not greatly depend on the injection rate. This is because synchrotron cooling is more important for particles with a Lorentz factor $\Gamma > 200$ than radiation cooling, due to inverse-Compton scattering. As Figure 7 shows, therefore, the energy peak of the inverse-Compton scattering of low-energy secondary pairs always appears around $\sim 100$ MeV, regardless of the injection rates. As a result, it is obvious from Figure 7 that even if we superpose emissions calculated with different injection rates, the combined spectrum decays faster and still has a large discrepancy with the Fermi-LAT spectrum above 1 GeV. On these grounds, we conclude that the residual curvature photons and/or the emissions from the high-energy secondary pairs contribute to the observed emissions above 1 GeV.

In summary, we discussed the gamma-ray emissions from the Crab-like pulsars, PSRs J0537-6910 and J0540-6919, in the LMC. The pulsed emissions from PSR J0540-6919 are observed to have an efficiency that is a factor of ten larger than that of the Crab pulsar. By solving the electrodynamics of the outer gap accelerator, we concluded that the difference in the radiation efficiencies of PSR J0540-6919 and the Crab pulsar is caused by the difference in the inclination angle. Inferring from the observed X-ray emissions, the radiation efficiency of PSR J0537-6910 is about two orders of magnitude smaller than that of PSR J0540-6919. Because of the very narrow X-ray pulse and low radiation efficiency of PSR J0537-6910, we suspect that the Earth viewing angle of PSR J0537-6910 greatly deviates from $\zeta \sim 90^\circ$, which is the case for the Crab and PSR J0540-6919.

The authors thank K. Hirotani, the referee, for insightful comments and suggestions on the manuscript. We thank A. H. Kong, C. Y. Hui, P. H. T. Tam, M. Ruderman, and S. Shibata for useful discussions. J.T. is supported by the NSFC grants of China under 11573010. KSC is supported by a GRF grant of the Hong Kong Government under HKU17300814P. All calculations were done under the High Performance Computing Cluster (Hyperion) of the Institute of Particle Physics and Astrophysics, HUST.

REFERENCES
Abdo, A. A., Ackermann, M., Ajello, M., et al. 2010, ApJ, 708, 1254
Abdo, A. A., Ajello, M., Allafort, A., et al. 2013, ApJS, 208, 17
Acharya, B. S., Actis, M., Aghajani, T., et al. 2013, APh, 43, 3
Ackermann, M., Albert, A., Baldini, L., et al. 2015, Sci, 350, 801
Aharonian, F. A., Bogovalov, S. V., & Khangulyan, D. 2012, Natur, 482, 507
Aleksic, J., Alvarez, E. A., Antonelli, L. A., et al. 2011, ApJ, 742, 43
Aleksic, J., Alvarez, E. A., Antonelli, L. A., et al. 2012, A&A, 540, 69
Aleksic, J., Ansoldi, S., Antonelli, L. A., et al. 2014, A&A, 565, 12
Aliu, E., Anderhub, H., Antonelli, L. A., et al. 2008, Sci, 322, 1221
Aliu, E., Arlen, T., Aune, T., et al. 2011, Sci, 334, 69
Bai, X.-N., & Spitkovsky, A. 2010, ApJ, 715, 1282
Campana, R., Mineo, T., de Rosa, A., et al. 2008, MNRAS, 389, 691
Cheng, K. S., Ruderman, M., & Zhang, L. 2000, ApJ, 537, 964
de Plaa, J., Kuiper, L., & Hermens, W. 2003, A&A, 400, 1013
Gradari, S., Barbieri, M., Barbieri, C., et al. 2011, MNRAS, 412, 2689
Harding, A. K., & Kalapotharakos, C. 2015, ApJ, 811, 63
Harding, A. K., Stern, J. V., Dyks, J., & Frackowiak, M. 2008, ApJ, 680, 1378
Hirotani, K. 2015, ApJL, 798, 40
Johnston, S., Romani, R. W., Marshall, F. E., & Zhang, W. 2004, MNRAS, 355, 31
Manchester, R. N., Hobbs, G. B., Teoh, A., & Hobbs, M. 2005, AJ, 129, 1993
Marshall, F. E., Gotthelf, E. V., Zhang, W., Middleditch, J., & Wang, Q. D. 1998, ApJ, 499, 179
Mineo, T., Cussumano, G., & Massaro, E. 2004, NuPhS, 132, 632
Ng, C.-Y., & Romani, R. W. 2004, ApJ, 601, 479
Ng, C.-Y., & Romani, R. W. 2008, ApJ, 673, 411
Renaud, M., Marandon, V., Gotthelf, E. V., et al. 2010, ApJ, 716, 665
Seward, F. D., Harnden, F. R., Jr., & Helfand, D. J. 1984, ApJ, 287, 19
Shearer, A., Stappers, B., O’Connor, P., et al. 2003, Sci, 301, 493
Spitkovsky, A. 2006, ApJ, 648, 51
Takata, J., & Chang, H.-K. 2007, ApJ, 670, 67
Takata, J., Ng, C. W., & Cheng, K. S. 2016, MNRAS, 455, 4249
Takata, J., Shibata, S., & Hirotani, K. 2004, MNRAS, 348, 241
Takata, J., Wang, Y., & Cheng, K. S. 2010, ApJ, 715, 1318
Takata, J., Wang, Y., & Cheng, K. S. 2011, MNRAS, 415, 1827
Tang, A. P. S., Takata, J., Ji, J. F., & Cheng, K. S. 2008, ApJ, 676, 562
Wang, Y., Takata, J., & Cheng, K. S. 2010, ApJ, 720, 178
Watters, K. P., & Romani, R. W. 2011, ApJ, 727, 123
Zhang, L., & Cheng, K. S. 2000, A&A, 363, 575