EFFECTS OF CP-VIOLATING PHASES IN SUPERSYMMETRY

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Recent studies about the impact of the CP-violating complex parameters in supersymmetry on the decays of third generation squarks and about $T$-odd asymmetries in neutralino and chargino production and decay are reviewed. The CP-even branching ratios of the third generation squarks show a pronounced dependence on the phases of $A_t$, $A_b$, $\mu$ and $M_1$ in a large region of the supersymmetric parameter space. This could have important implications for stop and sbottom searches and the MSSM parameter determination in future collider experiments. We have estimated the expected accuracy in the determination of the parameters by global fits of measured masses, decay branching ratios and production cross sections. We have found that the parameter $A_t$ can be determined with an error of 2–3%, whereas the error on $A_b$ is likely to be of the order of 50–100%. In addition we have studied CP-odd observables, like asymmetries based on triple product correlations, which are necessary to unambiguously establish CP violation. We have analysed these asymmetries in neutralino and chargino production with subsequent three-body decays at the International Linear Collider with longitudinally polarised beams in the MSSM with complex parameters $M_1$ and $\mu$. The asymmetries, which appear already at tree-level because of spin correlation between production and decay, can be as large as 20% and will therefore be an important tool for the search for CP-violating effects in supersymmetry.

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1. Introduction

The small amount of CP violation in the Standard Model (SM), which is caused by the phase in the Cabibbo–Kobayashi–Maskawa matrix, is not sufficient to explain the baryon–antibaryon asymmetry of the universe [1].

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The Lagrangian of the Minimal Supersymmetric Standard Model (MSSM) contains several complex parameters, which can give rise to new CP-violating phenomena [2]. In the sfermion sector of the MSSM the trilinear scalar couplings \( A_f \) and the higgsino mass parameter \( \mu \) can be complex. The phase of one of the gaugino mass parameters is unphysical and can be eliminated. Conventionally the SU(2) gaugino mass parameter \( M_2 \) is chosen real, hence the chargino sector depends only on one complex parameter \( \mu \), whereas in the neutralino sector also the U(1) gaugino mass parameter \( M_1 \) can be complex.

The phases of the complex parameters are constrained or correlated by the experimental upper limits on the electric dipole moments of electron, neutron and the atoms \(^{199}\text{Hg}\) and \(^{205}\text{Tl} \) [3]. In a constrained MSSM the restrictions on the phases can be rather severe. However, there may be cancellations between the contributions of different complex parameters, which allow larger values for the phases [4]. For example, in mSUGRA-type models and if substantial cancellations are present, \( \varphi_\mu \), the phase of \( \mu \), is restricted to \( |\varphi_\mu| \lesssim 0.1\pi \), whereas \( \varphi_{M_1} \) and \( \varphi_A \), the phases of \( M_1 \) and the trilinear scalar coupling parameter, turn out to be essentially unconstrained, but correlated with \( \varphi_\mu \) [5]. Moreover, the restrictions are very model dependent. For example, when also lepton flavour violating terms are included, then the restriction on \( \varphi_\mu \) may disappear [6].

The study of production and decay of charginos (\( \tilde{\chi}_i^{\pm} \)) and neutralinos (\( \tilde{\chi}_i^0 \)) and a precise determination of the underlying supersymmetric (SUSY) parameters \( M_1, M_2, \mu \) and \( \tan\beta \) including the phases \( \varphi_{M_1} \) and \( \varphi_\mu \) will play an important role at the International Linear Collider (ILC) [7]. In [8] methods to determine these parameters based on neutralino and chargino mass and cross section measurements have been presented. In [5] the impact of the SUSY phases on chargino, neutralino and selectron production has been analysed and significances for the existence of non-vanishing phases have been defined. In [9] CP-even azimuthal asymmetries in chargino production at the ILC with transversely polarised beams have been analysed. However, CP-odd triple product correlations involving the transverse beam polarisations vanish, if at least one subsequent chargino decay is not observed. Further methods to probe the CP properties of neutralinos are described in [10].

In contrast to the parameters of the chargino and neutralino sector it is more difficult to measure the trilinear couplings \( A_f \) in the sfermion sector. In the real MSSM studies to determine \( A_f \) have been performed in [11]. In the complex MSSM the polarisation of final top quarks and tau leptons from the decays of third generation sfermions can be a sensitive probe of the CP-violating phases [12]. In [13] the effects of the CP phases of \( A_\tau, \mu \) and \( M_1 \) on production and decay of tau sleptons (\( \tilde{\tau}_{1,2} \)) and tau sneutrinos (\( \tilde{\nu}_\tau \))
have been studied. The branching ratios of $\tilde{\tau}_1, \tilde{\tau}_2$ can show a strong phase dependence. The expected accuracy in the determination of $A_\tau$ has been estimated to be 10% by a global fit of measured masses, branching ratios and production cross sections. The impact of the SUSY phases on the decays of the third generation squarks [14, 15] will be discussed in Sec. 2.

However, in order to unambiguously establish CP violation in supersymmetry, including the signs of the phases, the use of CP-odd observables is inevitable. In supersymmetry $T$-odd triple product correlations between momenta and spins of the involved particles allow the definition of CP-odd asymmetries already at tree level [16, 17]. Such asymmetries based on triple products in decays of scalar fermions have been discussed in [18]. $T$-odd asymmetries in neutralino and chargino production with subsequent two-body decays have been analysed in [19]. For leptonic two-body decays asymmetries up to 30% can occur. CP-odd observables involving the polarisation of final $\tau$ leptons from two-body decays of neutralinos have been studied in [20]. A Monte Carlo study of $T$-odd asymmetries in selectron and neutralino production and decay including initial state radiation, beamstrahlung, SM backgrounds and detector effects has been given in [21]. It has been found that asymmetries $O(10\%)$ are detectable after few years of running of the ILC. $T$-odd asymmetries in neutralino and chargino production with subsequent three-body decays [22, 23] will be discussed in Sec. 3.

2. Decays of third generation squarks

In [14, 15] we have studied the effects of the phases of the parameters $A_t, A_b, \mu$ and $M_1$ on the phenomenology of the third generation squarks, the stops $\tilde{t}_{1,2}$ and the sbottoms $\tilde{b}_{1,2}$ in the complex MSSM. We have focused especially on the effects of $\varphi_{A_t}$ and $\varphi_{A_b}$ in order to find possibilities to determine these parameters. The third generation squark sector is particularly interesting because of the effects of the large Yukawa couplings. The phases of $A_f$ and $\mu$ enter directly the squark mass matrices and the squark-Higgs couplings, which can cause a strong phase dependence of observables. The off-diagonal mass matrix element $M^2_{\tilde{t}_{\text{RL}}}$, which describes the mixing between the left and right squark states, is given by

$$M^2_{\tilde{t}_{\text{RL}}} = m_t \left( |A_t| e^{i \varphi_{A_t}} - |\mu| e^{-i \varphi_{\mu}} \tan \beta \right),$$

(1)

$$M^2_{\tilde{b}_{\text{RL}}} = m_b \left( |A_b| e^{i \varphi_{A_b}} - |\mu| e^{-i \varphi_{\mu}} \tan \beta \right)$$

(2)

for the stops and sbottoms, respectively. In the case of stops the $\mu$ term is suppressed by $1/\tan \beta$, hence the phase $\varphi_{\tilde{t}}$ of $M^2_{\tilde{t}_{\text{RL}}}$ is dominated by $\varphi_{A_t}$; in a large part of the SUSY parameter space with $|A_t| \gg |\mu|/\tan \beta$. $\varphi_{\tilde{t}} \approx \varphi_{A_t}$ enters the stop mixing matrix and in the following all couplings because
of the strong mixing in the stop sector. This can lead to a strong phase
dependence of many partial decay widths and branching ratios.

In the case of sbottoms the mixing is smaller because of the smaller
dominant in $M^2_{bRL}$. Hence the phase of $A_b$ has only minor impact on the
sbottom mixing in a large part of the SUSY parameter space. However, in
the squark-Higgs couplings, for example in the $H^±\tilde{t}_L\tilde{b}_R$ coupling
\begin{equation}
C(\tilde{t}_L^H\tilde{b}_R) \sim m_b (|A_b|e^{-i\varphi_{A_b}}\tan \beta + |\mu|e^{i\varphi_\mu}),
\end{equation}
the phase $\varphi_{A_b}$ appears independent of the sbottom mixing. This can lead
to a strong $\varphi_{A_b}$ dependence of sbottom and stop partial decay widths into
Higgs bosons.

2.1. Partial decay widths and branching ratios of stops and sbottoms

In this subsection we discuss the $\varphi_{A_t}$ and $\varphi_{A_b}$ dependence of stop and
tsbottom partial decay widths and branching ratios. We have analysed
fermionic decays $\tilde{q}_i \rightarrow \tilde{\chi}^±_1 q', \tilde{q}_i \rightarrow \tilde{\chi}^0_j q$ and bosonic decays $\tilde{q}_i \rightarrow \tilde{q}_j' H^±$, $\tilde{q}_i \rightarrow \tilde{q}_j' W^±$, $\tilde{q}_2 \rightarrow \tilde{q}_1 H$, $\tilde{q}_2 \rightarrow \tilde{q}_1 Z$ of $\tilde{t}_1,2$ and $\tilde{b}_{1,2}$. In the complex MSSM
the CP-even and CP-odd neutral Higgs bosons mix and form three mass
eigenstates $H_{1,2,3}$. Their masses and mixing matrices have been calculated
with the program FeynHiggs2.0.2 [24].

In the scenario of Fig. 1, where we show the partial decay widths $\Gamma$
and branching ratios $B$ of the $\tilde{t}_1$ decays, especially $\Gamma(\tilde{t}_1 \rightarrow \tilde{\chi}^+ \tilde{b})$ has a very
pronounced $\varphi_{A_t}$ dependence, which leads to a strong $\varphi_{A_t}$ dependence of the
branching ratios. This pronounced $\varphi_{A_t}$ dependence of $\Gamma(\tilde{t}_1 \rightarrow \tilde{\chi}^+ \tilde{b})$ is
caused by the phase $\varphi_t \approx \varphi_{A_t}$ of the stop mixing matrix which enters the
respective couplings. In the scenario of Fig. 2 with large $\tan \beta$ the decay
channel $\tilde{t}_1 \rightarrow H^±\tilde{b}_1$ is open. $\varphi_{A_b}$ in the $H^±\tilde{t}_L\tilde{b}_R$ coupling, Eq. (3), causes
a strong $\varphi_{A_b}$ dependence of $\Gamma(\tilde{t}_1 \rightarrow H^±\tilde{b}_1)$, which influences all branching
ratios. An interplay between the $\varphi_{A_t}$ dependence of the stop mixing matrix
and the $\varphi_{A_b}$ dependence of the $H^±\tilde{t}_L\tilde{b}_R$ coupling leads to the remarkable
correlation between $\varphi_{A_b}$ and $\varphi_{A_t}$. In the case of the heavy $\tilde{t}_2$ many decay
channels can be open and can show a strong $\varphi_{A_t}$ dependence.

In Fig. 3, where $\tilde{b}_1$ decays are discussed, only $\Gamma(\tilde{b}_1 \rightarrow H^-\tilde{t}_1)$ shows a
pronounced $\varphi_{A_b}$ dependence, which leads to a strong $\varphi_{A_b}$ dependence of the
branching ratios. This is again caused by $\varphi_{A_b}$ entering the $H^±\tilde{t}_L\tilde{b}_R$ coupling
Eq. (3). The other partial decay widths depend only very weakly on $\varphi_{A_b}$.
This is typical for the $\varphi_{A_b}$ dependence of the $\tilde{b}_{1,2}$ decays. Only the partial
decay widths into Higgs bosons ($\Gamma(\tilde{b}_1 \rightarrow H^-\tilde{t}_1)$ for $\tilde{b}_1$ and $\Gamma(\tilde{b}_2 \rightarrow H^-\tilde{t}_{1,2})$,
$\Gamma(\tilde{b}_2 \rightarrow H_{1,2,3}\tilde{b}_1)$ for $\tilde{b}_2$) can show a strong phase dependence for large $\tan \beta$. 
Fig. 1. (a) Partial decay widths $\Gamma$ and (b) branching ratios $B$ of the decays $\tilde{t}_1 \to \tilde{\chi}_1^\pm b$ (solid), $\tilde{t}_1 \to \tilde{\chi}_1^0 t$ (dashed) and $\tilde{t}_1 \to W^+ b_1$ (dashdotted) for $\tan \beta = 6, M_2 = 300 \text{ GeV}, |M_1|/M_2 = 5/3 \tan^2 \theta_W, |\mu| = 350 \text{ GeV}, |A_b| = |A_t| = 800 \text{ GeV}, \varphi_\mu = \pi, \varphi_{M_1} = \varphi_{A_b} = 0, \tilde{m}_t = 350 \text{ GeV}, \tilde{m}_{\tilde{t}_2} = 700 \text{ GeV}, \tilde{m}_{b_1} = 170 \text{ GeV}, M_{\tilde{Q}} > M_{\tilde{U}}$ and $m_{H^\pm} = 900 \text{ GeV}$. From [15].

Fig. 2. Contours of $B(\tilde{t}_1 \to \tilde{\chi}_1^0 b)$ for $\tan \beta = 30, M_2 = 300 \text{ GeV}, |\mu| = 300 \text{ GeV}, |A_b| = |A_t| = 600 \text{ GeV}, \varphi_\mu = \pi, \varphi_{M_1} = 0, \tilde{m}_t = 350 \text{ GeV}, \tilde{m}_{\tilde{t}_2} = 700 \text{ GeV}, \tilde{m}_{b_1} = 170 \text{ GeV}, M_{\tilde{Q}} > M_{\tilde{U}}$ and $m_{H^\pm} = 160 \text{ GeV}$. The shaded areas are excluded by the experimental limit $B(\bar{b} \to s \gamma) > 2.0 \times 10^{-4}$. From [15].
2.2. Parameter determination via global fit

In order to estimate the precision, which can be expected in the determination of the underlying SUSY parameters, we have made a global fit of many observables in [15]. In order to achieve this the following assumptions have been made: (i) At the ILC the masses of the charginos, neutralinos and the lightest Higgs boson can be measured with high precision. If the masses of the squarks and heavier Higgs bosons are below 500 GeV, they can be measured with an error of 1% and 1.5 GeV, respectively. (ii) The masses of the squarks and heavier Higgs bosons, which are heavier than 500 GeV, can be measured at a 2 TeV $e^+e^-$ collider like CLIC with an error of 3% and 1%, respectively. (iii) The gluino mass can be measured at the LHC with an error of 3%. (iv) For the production cross sections $\sigma(e^+e^\rightarrow \tilde{t}_i\bar{\tilde{t}}_j)$ and $\sigma(e^+e^\rightarrow \tilde{b}_i\bar{\tilde{b}}_j)$ and the branching ratios of the $\tilde{t}_i$ and $\tilde{b}_i$ decays we have taken the statistical errors, which we have doubled to be on the conservative side. We have analysed two scenarios, one with small $\tan\beta = 6$ and one with large $\tan\beta = 30$. In both scenarios we have found that $\Re(A_t)$ and $|\Im(A_t)|$ can be determined with relative errors of 2–3%. For $A_b$ the situation is considerably worse because of the weaker dependence of the observables on this parameter. Here the corresponding errors are of the order of 50–100%. For the squark mass parameters $M_{\tilde{Q}}, M_{\tilde{U}}, M_{\tilde{D}}$ the relative errors are of order of 1%, for $\tan\beta$ of order of 3% and for the other fundamental SUSY parameters of order of 1–2%.
3. T-odd asymmetries in neutralino and chargino production and decay

We have studied T-odd asymmetries in neutralino [22] and chargino [23] production with subsequent three-body decays

\[ e^+ e^- \rightarrow \tilde{\chi}_i + \tilde{\chi}_j \rightarrow \tilde{\chi}_i + \tilde{\chi}_1^0 f \bar{f}^\prime, \]

where full spin correlations between production and decay have to be included [25]. Then in the amplitude squared \(|T|^2\) of the combined process products like \(i\epsilon_{\mu
u\rho\sigma}p_i^\mu p_j^\nu p_k^\rho p_l^\sigma\), where the \(p_i^\mu\) denote the momenta of the involved particles, appear in the terms, which depend on the spin of the decaying neutralino or chargino. Together with the complex couplings these terms can give real contributions to suitable observables at tree-level. Triple products \(T_1 = \vec{p}_{e^-} \cdot (\vec{p}_f \times \vec{p}_{f^\prime})\) of the initial electron momentum \(\vec{p}_{e^-}\) and the two final fermion momenta \(\vec{p}_f\) and \(\vec{p}_{f^\prime}\) or \(T_2 = \vec{p}_{e^-} \cdot (\vec{p}_{\tilde{\chi}_j} \times \vec{p}_f)\) of the initial electron momentum \(\vec{p}_{e^-}\), the momentum of the decaying neutralino or chargino \(\vec{p}_{\tilde{\chi}_j}\) and one final fermion momentum \(\vec{p}_f\) allow the definition of T-odd asymmetries

\[ A_T = \frac{\sigma(T_i > 0) - \sigma(T_i < 0)}{\sigma(T_i > 0) + \sigma(T_i < 0)} = \frac{\int \text{sign}(T_i)|T|^2d\text{Lips}}{\int |T|^2d\text{Lips}}, \]

where \(\int |T|^2d\text{Lips}\) is proportional to the cross section \(\sigma\) of the process (4). \(A_T\) is odd under naive time-reversal operation and hence CP-odd, if higher order final-state interactions and finite-widths effects can be neglected.

3.1. T-odd asymmetry in neutralino production and decay

In neutralino production and subsequent leptonic three-body decay \(e^+ e^- \rightarrow \tilde{\chi}_i^0 + \tilde{\chi}_j^0 \rightarrow \tilde{\chi}_i^0 + \tilde{\chi}_1^0 \ell^+ \ell^-\) the triple product \(T_1 = \vec{p}_{e^-} \cdot (\vec{p}_{\ell^+} \times \vec{p}_{\ell^-})\) can be used to define \(A_T\). Then \(A_T\) can be directly measured without reconstruction of the momentum of the decaying neutralino or further final-state analyses. As can be seen in Fig. 4, asymmetries \(A_T = \mathcal{O}(10\%)\) are possible with corresponding cross sections \(\sigma = \mathcal{O}(10\text{ fb})\) for the associated production and decay of \(\tilde{\chi}_i^0\) and \(\tilde{\chi}_j^0\), \(e^+ e^- \rightarrow \tilde{\chi}_i^0 + \tilde{\chi}_j^0 \rightarrow \tilde{\chi}_i^0 + \tilde{\chi}_1^0 \ell^+ \ell^-\). For a centre of mass energy \(\sqrt{s} = 350\text{ GeV}\), i.e. closer to threshold of production, \(A_T\) is larger, which is typical for effects caused by spin correlations between production and decay. Also for the associated production and decay of \(\tilde{\chi}_2^0\) and \(\tilde{\chi}_3^0\), \(e^+ e^- \rightarrow \tilde{\chi}_3^0 + \tilde{\chi}_2^0 \rightarrow \tilde{\chi}_3^0 + \tilde{\chi}_1^0 \ell^+ \ell^-\), the asymmetry \(A_T\) has values \(\mathcal{O}(10\%)\) in large parameter regions, where the corresponding cross sections \(\sigma = \mathcal{O}(10\text{ fb})\) (see Fig. 5). However, for \(e^+ e^- \rightarrow \tilde{\chi}_4^0 + \tilde{\chi}_2^0 \rightarrow \tilde{\chi}_4^0 + \tilde{\chi}_1^0 \ell^+ \ell^-\) all couplings in the \(\tilde{\chi}_4^0\) pair production process are real, which leads to small asymmetries \(A_T = \mathcal{O}(1\%)\), whereas for \(e^+ e^- \rightarrow \tilde{\chi}_4^0 + \tilde{\chi}_2^0 \rightarrow \tilde{\chi}_4^0 + \tilde{\chi}_4^0 \ell^+ \ell^-\) with larger asymmetries \(A_T \approx 6\%\) the cross section \(\sigma \lesssim 1\text{ fb}\) is rather small.
Fig. 4. (a) $T$-odd asymmetry $A_T$ and (b) cross section $\sigma(e^+e^- \rightarrow \tilde{\chi}_0^0\tilde{\chi}_2^0 \rightarrow \chi_1^0\chi_2^0\ell^+\ell^-)$, summed over $\ell = e, \mu$, for $|M_1| = 150$ GeV, $M_2 = 300$ GeV, $|\mu| = 200$ GeV, $\tan \beta = 10$, $m_{\tilde{\ell}_L} = 267.6$ GeV, $m_{\tilde{\ell}_R} = 224.4$ GeV and $\varphi_{\mu} = 0$ at the ILC with beam polarisations $P_{e^-} = -0.8$, $P_{e^+} = +0.6$ and $\sqrt{s} = 500$ GeV (solid), $\sqrt{s} = 350$ GeV (dashed). From [22].

Fig. 5. Contours (a) of the $T$-odd asymmetry $A_T$ in % and (b) of the cross section $\sigma(e^+e^- \rightarrow \tilde{\chi}_3^0\tilde{\chi}_2^0 \rightarrow \chi_3^0\chi_2^0\ell^+\ell^-)$, summed over $\ell = e, \mu$, in fb, respectively, for $\tan \beta = 10$, $m_{\tilde{\ell}_L} = 267.6$ GeV, $m_{\tilde{\ell}_R} = 224.4$ GeV, $|M_1|/M_2 = 5/3 \tan^2 \theta_W$, $\varphi_{M_1} = 0.5\pi$ and $\varphi_{\mu} = 0$ with $\sqrt{s} = 500$ GeV and $P_{e^-} = -0.8$, $P_{e^+} = +0.6$. The dark shaded area marks the parameter space with $m_{\tilde{\chi}_2^\pm} < 103.5$ GeV excluded by LEP. The light shaded area is kinematically not accessible or in this area the analysed three-body decay is strongly suppressed because $m_{\tilde{\chi}_1^\pm} > m_{\tilde{\chi}_2^\pm}$ or $m_{\tilde{\chi}_2^0} > m_{\tilde{\ell}_R}$, respectively. From [22].
3.2. $T$-odd asymmetry in chargino production and decay

In chargino production and subsequent hadronic three-body decay $e^+e^- \rightarrow \tilde{\chi}_1^+ + \tilde{\chi}_1^- \rightarrow \tilde{\chi}_2^- + \tilde{\chi}_1^0 sc$ the triple product $T_1 = \vec{p}_{e^-} \cdot (\vec{p}_s \times \vec{p}_c)$ can be used to define $A_T$. In this case it is important to tag the $c$ jet to discriminate between the two jets and to measure the sign of $T_1$. For the associated production and decay of $\tilde{\chi}_1^+$ and $\tilde{\chi}_2^-$, $e^+e^- \rightarrow \tilde{\chi}_2^- + \tilde{\chi}_1^+ \rightarrow \tilde{\chi}_2^- + \tilde{\chi}_1^0 sc$, asymmetries $A_T = \mathcal{O}(10\%)$ are possible (Fig. 6). In the scenario of Fig. 6 the corresponding cross sections are in the range of $1$–$5$ fb. In Fig. 6(b) it is remarkable that large asymmetries $A_T \approx 10\%$ are reached for small complex $\varphi_\mu$ around $\varphi_\mu = \pi$. In the chargino sector even for the pair production and decay process $e^+e^- \rightarrow \tilde{\chi}_1^+ + \tilde{\chi}_1^- \rightarrow \tilde{\chi}_1^0 sc$ asymmetries $A_T \approx 5\%$ can appear, which can only originate from the decay process. This means that the contributions from the decay to $A_T$ play an important role in chargino production with subsequent hadronic decays, which can also be seen in Fig. 6(a) with large $A_T$ for $\varphi_\mu = 0$ and $\varphi_{M_1} \neq 0$. It is furthermore remarkable that $\sigma(e^+e^- \rightarrow \tilde{\chi}_1^+ + \tilde{\chi}_1^- \rightarrow \tilde{\chi}_1^0 sc)$ can be rather large, for example $117$ fb in the scenario $M_2 = 350$ GeV, $|\mu| = 260$ GeV and the other parameters as in Fig. 6(a), where $A_T \approx 4\%$.

If the momentum of the decaying chargino $\tilde{\chi}_1^+$ can be reconstructed, for example with help of information from the decay of the $\tilde{\chi}_1^-$, the process $e^+e^- \rightarrow \tilde{\chi}_1^- + \tilde{\chi}_1^+ \rightarrow \tilde{\chi}_1^- + \tilde{\chi}_1^0 \ell^+ \nu$ can be analysed, where the chargino decays leptonically. Then the triple product $T_2 = \vec{p}_{e^-} \cdot (\vec{p}_{\tilde{\chi}_1^+} \times \vec{p}_{e^+})$ can be used to define $A_T$. For the associated production and decay of $\tilde{\chi}_1^+$ and $\tilde{\chi}_2^-$, $e^+e^- \rightarrow \tilde{\chi}_2^- + \tilde{\chi}_1^+ \rightarrow \tilde{\chi}_2^- + \tilde{\chi}_1^0 \ell^+ \nu$, asymmetries $A_T \gtrsim 20\%$ can occur.

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**Fig. 6.** $T$-odd asymmetry $A_T$ for $e^+e^- \rightarrow \tilde{\chi}_2^- + \tilde{\chi}_1^+ \rightarrow \tilde{\chi}_2^- + \tilde{\chi}_1^0 sc$ in the scenario $M_2 = 150$ GeV, $|M_1|/M_2 = 5/3 \tan^2 \theta_W$, $|\mu| = 320$ GeV, $\tan \beta = 5$, $m_\tilde{\chi}_1 = 250$ GeV and $m_\tilde{\chi}_2 = 500$ GeV with $\sqrt{s} = 500$ GeV (a) for $\varphi_\mu = 0$ and (b) for $\varphi_{M_1} = 0$ and beam polarisations $P_{e^-} = -0.8$, $P_{e^+} = 0.6$ (solid), $P_{e^-} = +0.8$, $P_{e^+} = -0.6$ (dashed).
(Fig. 7). But in the region with largest asymmetries around $|\mu| = 320$ GeV and $M_2 = 120$ GeV the cross section is very small ($\sigma = \mathcal{O}(0.1 \text{ fb})$). However, for decreasing $|\mu|$ the cross section increases and reaches $\sigma = 2 \text{ fb}$ for $|\mu| = 220$ GeV and $M_2 = 120$ GeV. For pair production of $\tilde{\chi}_1^\pm$ and leptonic decays, $e^+e^- \rightarrow \tilde{\chi}_1^- + \tilde{\chi}_1^+ \rightarrow \tilde{\chi}_1^- + \tilde{\chi}_1^0 + \ell^+\nu$, the couplings in the production process are real which leads to small $A_T = \mathcal{O}(1\%)$, because the contributions from the decay are not large enough in this case.

![Figure 7](image-url)  

**Fig. 7.** Contours of the $T$-odd asymmetry $A_T$ in % for $e^+e^- \rightarrow \tilde{\chi}_2^- + \tilde{\chi}_1^+ \rightarrow \tilde{\chi}_2^- + \tilde{\chi}_1^0 + \ell^+\nu$ for $\varphi_\mu = 0.9\pi$, $\varphi_{M_1} = 1.5\pi$, $|M_1|/M_2 = 5/3\tan^2\theta_W$, $\tan \beta = 5$ and $m_\nu = 250$ GeV with $\sqrt{s} = 500$ GeV and $P_{e^-} = +0.8$, $P_{e^+} = -0.6$. The dark shaded area marks the parameter space with $m_{\tilde{\chi}_1^\pm} < 103.5$ GeV excluded by LEP. The light shaded area is kinematically not accessible or in this area the analysed three-body decay is strongly suppressed because $m_{\tilde{\chi}_1^\pm} > m_W + m_{\tilde{\chi}_1^0}$, respectively.

### 4. Conclusions

We have studied the impact of the complex parameters $A_t$, $A_b$, $\mu$ and $M_1$ on the decays of stops and sbottoms in the CP-violating MSSM. In the case of stop decays all partial decay widths and branching ratios can have a strong $\varphi_{A_t}$ dependence because of the large mixing in the stop sector. If $\tan \beta$ is large and decay channels into Higgs bosons are open, stop and sbottom branching ratios can show also a strong $\varphi_{A_b}$ dependence. This strong phase dependence of CP-even observables like branching ratios has
to be taken into account in SUSY particle searches at future colliders and the
determination of the underlying MSSM parameters. In order to estimate the
expected accuracy in the determination of the MSSM parameters we have
made a global fit of masses, branching ratios and production cross sections
in two scenarios with small and large $\tan \beta$. We have found that $A_t$ can be
determined with an error of 2–3%, whereas the error of $A_b$ is likely to be of
the order of 50–100%. Furthermore $\tan \beta$ can be determined with an error
of 3% and the other fundamental MSSM parameters with errors of 1–2%.

However, in order to unambiguously establish CP violation in supersym-
metry, including the signs of the phases, the use of CP-odd observables is
inevitable. We have studied $T$-odd asymmetries in neutralino and chargino
production with subsequent three-body decays, which are based on triple
product correlations between incoming and outgoing particles and appear
already at tree-level because of spin correlations between production and de-
cy. The $T$-odd asymmetries can be as large as 20% and will therefore be an
important tool for the search for CP violation in supersymmetry and the un-
ambiguous determination of the phases of the parameters in the neutralino
and chargino sectors.

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