Decoding Staircase Codes with Marked Bits

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Abstract—Staircase codes (SCCs) are typically decoded using iterative bounded-distance decoding (BDD) and hard decisions. In this paper, a novel decoding algorithm is proposed, which partially uses soft information from the channel. The proposed algorithm is based on marking certain number of highly reliable and highly unreliable bits. These marked bits are used to improve the miscorrection-detection capability of the SCC decoder and the error-correcting capability of BDD. For SCCs with 2-error-correcting BCH component codes, our algorithm improves upon standard SCC decoding by up to 0.30 dB at a bit-error rate of $10^{-7}$. The proposed algorithm is shown to achieve almost half of the gain achievable by an idealized decoder with this structure.

I. INTRODUCTION

Forward error correction (FEC) is required in optical communication systems to meet the ever increasing data demands in optical transport networks (OTNs), currently targeting data rates of 400 Gb/s and beyond [1], [2]. As the data rates increase, FEC codes that can boost the net coding gain (NCG) are of key importance. Soft-decision FEC codes provide large NCGs, however, they are not the best candidates for very high data rate applications due to their high power consumption and decoding delay. In this context, simple but powerful hard-decision (HD) FEC codes are a promising alternative, e.g., Reed-Solomon (RS) code [3] and concatenated codes consisting of two HD codes [4]. One popular family of HD-FEC codes is the so-called staircase codes (SCCs) [5], [6]. Compared to the best code from ITU-T standards [3], [4], SCCs offer an improvement of 0.42 dB NCG [5]. An implementation agreement has been reached for using an SCC as an outer code in the baseline draft of 400 Gb/s OTN [7].

Similar to classical product codes, SCCs are based on simple component codes, Bose-Chaudhuri-Hocquenghem (BCH) codes being the most popular ones. SCC decoding is done iteratively based on bounded-distance decoding (BDD) for the component codes. Although very simple, one drawback of BDD is that its error-correcting capability is limited to $t = \lfloor \frac{d_0 - 1}{2} \rfloor$, where $d_0$ is the minimum Hamming distance (MHD) of the component code [8]. BDD can detect more than $t$ errors, but cannot correct them. In some cases, BDD may also erroneously decode a received sequence with more than $t$ errors, a situation known as a miscorrection. Miscorrections are known to degrade the performance of iterative BDD. To prevent miscorrections, several methods have been studied in the literature [9]–[12].

The authors of [9] proposed rejecting bit-flips from the decoding of bit sequences associated with the last SCC block if they conflict with a zero-syndrome codeword from the previous block. However, the obtained gains are expected to be limited [10, Sec. I]. An anchor-based decoding algorithm has been proposed in [10], [11], where some bit sequences are labeled as anchor codewords. These sequences are thought to have been decoded without miscorrections. Decoding results that are inconsistent with anchor codewords are discarded. The algorithm in [11] outperforms [9], but it suffers from an increased complexity as anchor codewords need to be tracked during iterative BDD. Very recently, a modified iterative BDD for product codes was proposed in [12]. In this algorithm, channel reliabilities are used to perform the final HD at the output of BDD, instead of directly accepting the decoding result. Large gains are obtained, but it requires additional memory (and processing) as all the soft information needs to be saved. Moreover, its effectiveness for SCCs has not yet been reported in the literature.

In this paper, we propose a simple algorithm to improve the decoding of SCCs. This is achieved by marking highly reliable and highly unreliable bits. Unlike previous works, our proposed algorithm jointly increases the miscorrection-detection capability of the SCC decoder and the error-correcting capability of BDD. The proposed algorithm only requires modifications to the decoding structure related to the last block of each decoding window. Furthermore, the algorithm is based on marking bits only, and thus, no soft bits (log-likelihood ratios, LLRs) need to be saved. Marked bits do not need to be tracked during the iterative process either.

II. SYSTEM MODEL, SCCS, AND BDD

A. System Model

As shown in Fig. 1, information bits are encoded by a staircase encoder and then mapped to symbols $x_l$ taken from an equally-spaced $M$-ary PAM constellation $S =$...
\(\{s_1, s_2, \ldots, s_M\}\) with \(M = 2^m\) points, where \(l\) is the discrete time index. The bit-to-symbol mapping is the binary reflected Gray code. The received signal is \(y_t = \sqrt{p}x_t + z_t\), where \(z_t\) is zero-mean unit-variance additive white Gaussian noise.

The standard HD receiver structure for SCCs uses an HD-based demapper to estimate the code bits, which are then fed to the decoder (green block in Fig. 1). In this paper, we introduce a novel receiver architecture where the HD-FEC decoder uses partial soft information from the channel. This soft information is typically represented using LLRs, \(L_i = \log \frac{p_i}{1-p_i}\), where \(p_i = \Pr (\hat{b}_i = 1 | b_i = 0)\) for hard-decision decoding.

\[
\lambda_{l,k} = \sum_{\bar{b} \in \{0,1\}} (-1)^{\bar{b}} \log \sum_{i \in \mathcal{I}_{k,b}} \exp \left( \frac{(y_i - \sqrt{p}s_i)^2}{2} \right), \quad (1)
\]

with \(k = 1, \ldots, m\), and where \(\bar{b}\) denotes bit negation. In (1), the set \(\mathcal{I}_{k,b}\) enumerates all the constellation points in \(S\) whose \(k\)th bit \(c_{i,k}\) is \(b\), i.e., \(\mathcal{I}_{k,b} = \{i = 1, 2, \ldots, M : c_{i,k} = b\}\).

At the receiver side, SCCs are decoded iteratively using a sliding window covering \(L\) blocks. We use \(Y_i\) to indicate the received SCC block after HD-demapping corresponding to the transmitted block \(B_i\). The decoder first iterates over the blocks \(\{Y_0, Y_1, \ldots, Y_{L-1}\}\). When a maximum number of iterations is reached, the decoding window outputs the block \(Y_0\) and moves to decode the blocks \(\{Y_1, Y_2, \ldots, Y_L\}\). The block \(Y_1\) is then delivered and operation continues on \(\{Y_2, Y_3, \ldots, Y_{L+1}\}\). This process continues indefinitely.

Multiple decoding scheduling alternatives exist (see, e.g., [5, Sec. IV] [6, Sec. II]). We chose the most popular one, namely, alternated decoding of pairs of SCC blocks within a window, from the bottom right to the top left of the SCC window.

C. Bounded-Distance Decoding

BDD is used to decode (in Hamming space) the received bit sequence for the component code \(C\). To correct up to \(t\) errors, the MHD \(d_0\) of \(C\) must satisfy \(d_0 \geq 2t + 1\) (\(d_0 \geq 2t + 2\) for extended BCH codes). Thus, every codeword in the code \(C\) can be associated to a sphere of radius \(t\). Within such a sphere, no other codewords exist. If the received sequence \(r\) falls inside one of these spheres, BDD will decode \(r\) to the corresponding codeword. Otherwise, BDD will declare a failure. For a given transmitted codeword \(c\) and a received sequence \(r\), the BDD output \(\hat{c}\) is thus given by

\[
\hat{c} = \begin{cases} 
  c, & \text{if } d_H(r, c) \leq t, \\
  \hat{c} \in \mathcal{C}, & \text{if } d_H(r, c) > t \text{ and } d_H(r, \hat{c}) \leq t, \\
  r, & \text{if } d_H(r, \hat{c}) > t \quad \forall \hat{c} \in \mathcal{C}.
\end{cases}
\]

where \(d_H(\cdot, \cdot)\) represents the Hamming distance. In practice, BDD is often a syndrome-based decoder that uses syndromes to estimate the error pattern \(e\). If the syndromes are all zeros, no errors are present. For the first two cases in (2), BDD will declare decoding success and \(\hat{c} = r \oplus e\). In the second case, although BDD will still return an error pattern \(e\), this case corresponds to a miscorrection. In the next section we will show how to improve miscorrection detection (MD) using the underlying structure of SCCs and the marked HRBs.

III. THE PROPOSED ALGORITHM

The schematic diagram of the proposed algorithm is shown in Fig. 3 (red area). Compared to standard SCC decoding...
(green area), which always accepts the decoding result \( \hat{c} \) of BDD, the proposed algorithm further checks the decoding status of BDD. If BDD successfully decodes \( r \), miscorrection detection is performed. Furthermore, bit flipping (BF) is proposed as a way to handle decoding failures and miscorrections. In this section, we explain the steps in the proposed algorithm.

Our proposed algorithm can in principle be applied to all received sequences \( r \) within \( L \) SCC blocks. However, due to the iterative sliding window decoding structure applied to SCCs, most of the errors are known to be located in the last two blocks. To keep the complexity and latency low, we will therefore only use our algorithm on the received sequences from the last two blocks of the window. Therefore, from now on we only consider rows of the matrix \( \begin{bmatrix} Y_{i+L-2}^T \ Y_{i+L-1} \end{bmatrix} \).

A. Decoding Success: Improved Miscorrection Detection

To avoid miscorrections, it was suggested in [9] to reject the decoding result of BDD applied to \( \begin{bmatrix} Y_{i+L-2}^T \ Y_{i+L-1} \end{bmatrix} \) if the decoded codeword would cause conflicts with zero-syndrome codewords in \( \begin{bmatrix} Y_{i+L-3}^T \ Y_{i+L-2} \end{bmatrix} \). This method protects bits in \( Y_{i+L-2} \) but cannot handle bits in the last block \( Y_{i+L-1} \). We propose to enhance this method by using marked bits in \( Y_{i+L-1} \). In particular, we add one additional constraint to the algorithm in [9]: no HRBs in \( Y_{i+L-1} \) shall ever be flipped.

The reliability of a bit is given by the absolute value of its LLR, a high value indicating a more reliable bit. Therefore, a threshold \( \delta \) is set to decide if the bit is HR. If \( |\lambda_{i,k}| \geq \delta \), the corresponding bit is marked as an HRB. The decision of the staircase decoder will therefore be marked as a miscorrection if the decoded codeword causes conflicts with zero-syndrome codewords in \( \begin{bmatrix} Y_{i+L-3}^T \ Y_{i+L-2} \end{bmatrix} \), or if the decoded codeword flips a bit whose LLR satisfies \( |\lambda_{i,k}| \geq \delta \).

Example 1: Fig. 4 shows a decoding window with \( w = 6 \) and \( L = 5 \) and a component code \( C \) with \( t = 2 \) (\( d_0 = 6 \)). Following the notation of [11], a pair \((i,j)\) is used to specify the location of a component codeword in each window, where \( i \in \{1, 2, \ldots, L-1\} \) indicates the position relative to the current window and \( j \in \{1, 2, \ldots, w\} \) indicates the corresponding row or column index in the matrix of two neighbor blocks. A triple \((i, j, k)\) is used to indicate the \( k \)th bit in the component codeword \((i, j)\), where \( k \in \{1, 2, \ldots, 2w\} \). For example, the component codewords \((1, 2)\) and \((3, 1)\) are highlighted with light magenta, while bits \((1, 2, 11)\) and \((3, 1, 4)\) are highlighted with dark magenta. The bit sequence \((3, 1)\) is a codeword in \( \begin{bmatrix} Y_{i+2} \ Y_{i+3} \end{bmatrix} \) whose syndrome is equal to zero. The cells filled with dark yellow are the ones marked as HRBs.

After transmission, the received bit sequences for \((4, 1)\) and \((4, 3)\) have 5 and 4 errors (black crosses), resp. When applying BDD, miscorrections (red crosses) occur. For the received bits in \((4, 1)\), BDD detects bit \((4, 1, 1)\) as an error and suggests to flip it. However, because it is involved in the zero-syndrome codeword \((3, 1)\), it will be identified as a miscorrection by both our MD algorithm and the one in [9]. For the received bits in \((4, 3)\), however, the suggested flipping bit \((4, 3, 5)\) in \( Y_{i+L-2} \) is not involved in any zero-syndrome codewords, and thus, [9] would fail to detect this miscorrection. The bit \((4, 3, 9)\) is a HRB, and thus, our MD algorithm will successfully identify it as a miscorrection.

The MD algorithm in [9] does not always detect the miscorrections. The new rule we introduced (never flip HRBs in \( Y_{i+L-1} \)) is only heuristic and does not guarantee perfect MD either. For example, our MD algorithm fails when no bits are flipped by BDD because \( r = \hat{c} \in C \). Nevertheless, as we will see later, our MD algorithm combined with bit flipping (see next Sec.) gives remarkably good results with very small complexity increase.

B. Decoding Failures and Miscorrections: Bit Flipping

To deal with decoding failures and miscorrections, we propose to flip bits (see BF block in Fig. 3). The main idea is to flip certain bits in \( r \) and make the resulting sequence \( r' \) (after BF) closer to \( c \) in Hamming space. In particular, the proposed BF aims at making the Hamming distance between \( r' \) and \( c \) equal to \( t \) so that BDD can correct \( r' \) to the transmitted codeword \( c \). Two cases are considered by our proposed algorithm: (1) decoding failures, and (2) miscorrections.
Method in [11]

Standard decoding

Miscorrection-free decoding

Method in [9]

we flip the correct bits (flipping 3 errors, resp.). For the latter two bit sequences, provided that lighter yellow color indicates a smaller value of $|H|$ with the lowest reliability within that codeword. The marked bits will not bring $r$ close enough to $c$.

Example 3: Light yellow cells in Fig. 4 show the marked 3 HUBs with the lowest reliability within that codeword. The lighter yellow color indicates a smaller value of $|H|$. In this example, BDD fails to decode bit sequence $(4,5)$. Fortunately, $(4,5,8)$ corresponds to the marked HUB with smallest $|H|$. Thus, it will be flipped after BF, and then the remaining 2 errors $(4,5,3)$ and $(4,5,10)$ will be fully corrected by applying BDD again. This corresponds to Case 1.

For bit sequences $(4,1)$ and $(4,3)$, the decoding results of BDD are identified as miscorrections (as explained in Example 1) with $w_H(e) = 1$ and $w_H(e) = 2$, resp. According to the BF rule for miscorrections, 3 and 2 bits with smallest $|H|_{l,k}$ among the marked HUBs, i.e., $(4,1,8)$, $(4,1,10)$, $(4,1,11)$ in $(4,1)$, and $(4,3,7)$, $(4,3,10)$ in $(4,3)$, will all be flipped. As a result, only 2 errors are left in $(4,1)$ and $(4,3)$, which are within the error correcting capability of BDD. This corresponds to Case 2. △

BF will not always result in the correct decision. As shown in Example 2, this is the case for certain miscorrections (black diamonds in Fig. 4). Additionally, miscorrections for codewords at distances larger than $d_0$ are not considered either. Finally, marked LLRs might not correspond to channel errors. In all these cases, either decoding failures or miscorrections will happen. To avoid these cases, our proposed algorithm includes two final checks after BF and BDD (see lowest part of Fig. 3): successful decoding and MD.

IV. Simulation Results

The component codes used for simulations are extended BCH codes with $t = 2$. The decoding window size is $L = 9$, and the maximum number of iterations is 7. The LLR threshold $\delta$ in the MD algorithm is set to 10, which gives the best performance for $R = 0.87$ and 2-PAM. Optimization of $\delta$ for different SNRs could provide additional gains.

Example 4: Consider 2-PAM and a SCC with $R = 0.87$, i.e., with BCH codes $(256, 239, 2)$ ($w = 128$) as component code. These parameters are chosen to compare our algorithm with results presented in the literature. These results are shown in Fig. 6. Two baselines are: standard decoding where miscorrections are not dealt with (circles), and miscorrection-free decoding (stars). The latter is obtained via a genie BDD decoder which corrects the received sequence only when the number of errors is not more than $t$. This figure also shows the performance of previously proposed methods: [9] and [11].

The performance of our algorithm (squares) is also shown in Fig. 6, which is 0.3 dB better than standard decoding.
Our algorithm also outperforms both [9] and [11]. These two methods only prevent miscorrections, so their performance is bounded by the miscorrection-free case. Although our algorithm only deals with miscorrections related to the last block of each window, it outperforms the miscorrection-free case. This is due to its additional ability to better deal with miscorrections and decode even when BDD initially fails.

Fig. 6 also shows a lower bound for our proposed algorithm (triangles). This bound is obtained by a genie decoder which emulates a best-case scenario for our algorithm. This genie decoder is assumed to be able to ideally identify all miscorrections in the last two blocks of the window. This corresponds to having an idealized MD block in the top part of Fig. 3. The genie decoder also emulates an idealized assumption on what the BF block in Fig. 3 can do. For this, we assume that the decoder knows exactly which bits in the last two blocks are errors. If a given sequence has \( t + j \) errors (\( j = 1 \) for Case 1, or \( j = d_0 - w_1(e) - t \) for Case 2), and at least \( j \) errors in the last block, the genie decoder flips \( j \) errors in the last block, and then the received sequence is correctly decoded. If less than \( j \) errors are located in the last block, the genie decoder declares a failure. The results show that the maximum potential gain for our receiver structure (for 2-PAM, \( R = 0.87 \), and \( t = 2 \)) is 0.63 dB. Our algorithm achieved half of this gain with very small added complexity.

Example 5: Fig. 7 shows the simulation results of the proposed algorithm for 2-PAM, 4-PAM and 8-PAM. For each modulation format, two code rates are considered: \( R = 0.83 \) and \( R = 0.92 \) with code parameters \((228, 209, 2)\) and \((504, 485, 2)\), resp. These parameters are obtained by shortening the extended BCH code \((512, 493, 2)\) by 284 and 8 bits, resp. It can be seen from Fig. 7 that for different modulation formats and code rates, our proposed algorithm always outperforms the miscorrection-free case. When compared to standard staircase decoding, the achieved gains are between 0.20 dB and 0.29 dB, while the obtained maximum potential gains are between 0.46 dB and 0.62 dB. The results also show that the gains increase as the modulation size increases.

V. CONCLUSIONS

In this paper, a novel decoding algorithm for staircase codes was proposed. This algorithm is based on simple modification of the standard hard-decision-based staircase decoder and relies on the idea of marking bits. The algorithm consists of an improved miscorrection-detection mechanism and a bit-flipping operation to effectively increase the error correcting performance of bounded-distance decoding. Large gains compared to standard SCC decoding were obtained. A precise complexity evaluation of the algorithm is left for further investigation.

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