Learning to Speak on Behalf of a Group: Medium Access Control for Sending a Shared Message

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Abstract—The rapid development of Industrial Internet of Things (IIoT) technologies has not only enabled new applications, but also presented new challenges for reliable communication with limited resources. In this work, we define a deceptively simple novel problem that can arise in these scenarios, in which a set of sensors need to communicate a joint observation. This observation is shared by a random subset of the nodes, which need to propagate it to the rest of the network, but coordination is complex: as signaling constraints require the use of random access schemes over shared channels, each sensor needs to implicitly coordinate with others with the same observation, so that at least one of the transmissions gets through without collisions.

Unlike the existing medium access control schemes, the goal here is not to maximize total goodput, but rather to make sure that the shared message gets through, regardless of the sender. The lack of any signaling, aside from an acknowledgment or lack thereof from the rest of the network, makes determining the optimal collective transmission strategy a significant challenge. We analyze this coordination problem theoretically, prove its hardness, and provide low-complexity solutions. While a low-complexity clustering-based approach is shown to provide near-optimal performance in certain special cases, for the general scenarios, we model each sensor as a multi-armed bandit (MAB), and provide a learning-based solution. Numerical results show the effectiveness of this approach in a variety of cases.

Index Terms—Distributed coordination, multi-armed bandit, random access

I. INTRODUCTION

Over the past few years, the rise of the Industrial Internet of Things (IIoT) \cite{1} has opened new possibilities in the manufacturing, energy, and health sectors, as well as many others. The promise of 5G and beyond networks is to support massive numbers of sensors and machine-type devices, along with sporadic low-latency communications, without affecting human communication traffic \cite{2}. However, there are still many open problems in coordinating medium access for sporadically active sensors \cite{3}, particularly in remote deployments with very limited resources. In these cases, the lack of significant signaling requirements makes random access schemes, such as slotted ALOHA, the best choice for IIoT \cite{4}, but the drawbacks of these schemes are well-known, particularly for scenarios with a large number of nodes \cite{5}, in which collisions can become frequent. The use of multiple frequency bands and coding techniques \cite{6} can mitigate this phenomenon, but it is possible to exploit resources more efficiently by considering activity patterns, as well as the content of the observations, which might have a different value for the target application.

An interesting scenario in this context is the transmission of an observation or action information that is identical for all the devices in a given active set. For example, this could be the position of a target object, or an abnormal value of a parameter in a manufacturing process, which is shared by a subset of the sensors in the network and needs to be communicated to the rest of the sensor network or to an external controller. This scenario, which we call medium access control (MAC) with a shared message, is relevant in several applications pertaining to networked control and coordination, in which the whole network needs to perform an action synergistically. In our case, each of the active sensor nodes has the same piece of information, namely the shared message, which they want to deliver to the other nodes under limited bandwidth resources. At each time slot, the active set evolves in a random fashion, and each sensor node knows only about its own membership of the current active set. The challenge in terms of communication is that a node from the active set cannot coordinate with the other nodes about sending the shared message. The problem looks deceptively simple and familiar; yet, coordinating without signaling is extremely complex in this scenario as sensors cannot know which other sensors they might interfere with, and the set might change from one moment to the next. This gives rise to a rich set of problems with joint communication and control requirements, in which nodes with extremely limited communication capabilities must achieve some form of consensus with limited or no signaling, exploiting contextual information and the single bit of feedback to learn to coordinate their transmissions.

In this context, traditional throughput maximization-based schemes are extremely inefficient. The common assumption in random access schemes is that each packet is relevant, which makes it possible to exploit correlations in the activity of the sensors using classical \cite{7} or learning-based \cite{8} methods. A related, but different problem has been treated in \cite{9}, where the objective is for the controller to reliably receive a shared alarm message from the superposition of the individual signals; this is different from the collision model adopted here, and does not require coordination between the sensors.

In this work, we first prove that the coordination problem
for MAC with a shared message over multiple orthogonal channels is NP-hard for non-trivial scenarios. We then prove the existence of an optimal deterministic solution; that is, it is sufficient to assign a fixed transmission pattern representing the channels used for transmission to each sensor whenever it is active. To prove these results, we model the correlations in sensor activity as edge weights in a graph and different transmission patterns as colors, and show that the coordination problem with deterministic strategies is an instance of a slightly modified Weighted Graph Coloring Problem (WGCP) [10], which is a well-known NP-hard problem. Distributed solutions to the WGCP exist [11], and have been used in communications scenarios [12], but they either require signaling between the nodes or more extensive knowledge of the results of each configuration. On the other hand, the problem we face is due to the extremely limited nodes (which cannot sense the state of the system) or to an problem. Sec. III presents two alternative solutions to this fixed transmission pattern) and requires solving an NP-hard simulations in Sec. IV. Finally, we conclude the paper and approach, which are evaluated and compared through numerical scenarios, we propose a distributed learning solution using correlations in sensor activity as edge weights in a graph presenting the channels used for transmission to each sensor

\[ E[ξ|Φ] = \sum_{A ∈ P(N)} p_A(Φ) \sum_{x ∈ \{0, 1\}^N} ξ(x, A) \prod_a φ_a(x_a). \]  

(2)

where \( P(\cdot) \) is the power set operator, and \( φ_a(x_a) \) denotes the probability of sensor \( a \) choosing move \( x_a \). We then define our transmission optimization problem, whose solution is the strategy that maximizes the expected delivery probability:

\[ Φ^* = \arg \max_{Φ ∈ \{0, 1\}^N × 2^M} \mathbb{E}[ξ|Φ]. \]  

(3)

Let \( Φ^* \) denote one of the optimal solutions to the problem.

Theorem 1. At least one of the optimal solutions to the optimization problem is a deterministic strategy; that is,

\[ ∃ Φ ∈ \{0, 1\}^N × 2^M : E[ξ|Φ] = E[ξ|Φ^*]. \]  

(4)

Proof. As the value of \( ξ \) is between 0 and 1, its expected value is bounded in the compact interval \([0, 1]\), and the \([0, 1]^N × 2^M\) region specified by the constraints on \( Φ \) is also compact. Accordingly, there exists at least one global maximum. Assume that there is at least one sensor with a non-deterministic policy, which we denote as sensor 1. We can then look at all possible moves \( x_1 \) and compute the values of the deterministic strategies for sensor 1:

\[ E[ξ|Φ^*_1] = \sum_{A ∈ P(N)} p_A(Φ^*_1) E[ξ|Φ, A] + \sum_{A ∈ P(N)} p_A(Φ^*_1) \times \sum_{x ∈ \{0, 1\}^N} ξ((x_1, x_{-1}), A) Φ^*_1, \]  

(5)

where \( X_{-1} \) is matrix \( X \) without its first row. We can then substitute this into (2):

\[ E[ξ|Φ] = \sum_{x_1 ∈ \{0, 1\}^M} φ_1(x_1)E[ξ|Φ_1, Φ^*_{-1}]. \]  

(6)

Then, the strategy \( Φ^* \) that maximizes the expected value:

\[ Φ' = \left( \arg \max_{x_1 ∈ \{0, 1\}^M} E[ξ|Φ_1, Φ^*_{-1}] \right), \]  

(7)

II. SYSTEM MODEL

We consider a set \( N \) of \( N \) wireless sensors, which communicate over \( M \) shared orthogonal channels. In each time slot \( t \), a random set \( A(t) \) of sensors, with cardinality \( A(t) \), are active. The active set is drawn independently at each slot, according to probability mass function (PMF) \( p_A(A) \). The information to be transmitted by the active sensors is the same: we consider an alarm scenario in which the active nodes all have the same information, but need to send it to the inactive nodes (which cannot sense the state of the system) or to an external controller. The objective of the active sensors is to deliver at least one packet over the \( M \) channels, regardless of which sensor it comes from. The challenge lies in the fact that the active sensors have no knowledge of the other sensors in \( A \), and there is no way to explicitly coordinate. The sensors can only agree on a MAC protocol a priori.

It is clear that inactive sensors at each time slot must remain silent. Each active sensor \( a ∈ A(t) \) must decide on a transmission pattern, or a move, expressed as vector \( x_a ∈ \{0, 1\}^M \), where \( x_a,m = 1 \) denotes that sensor \( a \) transmits over channel \( m \), and is silent otherwise. We can represent the moves of all the sensors as an \( A(t) × M \) matrix \( X(t) \), with vector \( x_a \) as its \( a \)-th row. In the following, we omit the time index \( t \) for readability. We consider a simple collision channel, where the transmission is successful if condition \( ξ(X, A) \) is met:

\[ ξ(X, A) = \begin{cases} 1, & \text{if } ξ \in \{1, \ldots, M\} : \sum_{a ∈ A} x_a,m = 1; \\ 0, & \text{otherwise}. \end{cases} \]  

(1)

Note that there is a total of \( 2^M \) possible moves for each sensor at each time slot. We can define the strategy of node \( a \) as the PMF \( φ(x) \) over the set of possible moves, and represent the strategies of all the sensors in matrix \( Φ ∈ \{0, 1\}^N × 2^M \), where element \( φ_a(x) \) corresponds to the probability of sensor \( n \) choosing move \( x \) when it is active. We have \( \sum_{x=1}^M φ_n,x = 1, \forall n ∈ N \). By applying the law of total probability, we get:

\[ E[ξ|Φ] = \sum_{A ∈ P(N)} p_A(Φ) \sum_{x ∈ \{0, 1\}^N} ξ(X, A) \prod_a φ_a(x_a). \]  

(2)
will be a deterministic one due to the linearity of the objective with respect to \( \phi_1(x_1) \), and the compactness of the simplex.

By repeating this operation for all the sensors with non-deterministic strategies, we can find a deterministic solution \( \Phi^d \) that satisfies the condition in (4).

We can then concentrate on deterministic strategies over the discrete set of moves \( \{0, 1\}^{N \times M} \) instead of the continuous probability space \([0, 1]^{N \times 2^M}\) without loss of optimality:

\[
X^* = \arg \max_{X \in \{0, 1\}^{N \times M}} \mathbb{E}[\xi|X].
\]

(8)

The problem is trivial if only one user is active at a time, i.e., if \( A = 1 \), \( \forall A : p_A(A) > 0 \); in that case, there is no interference and the trivial solution \( x_{a,m} = 1, \forall a \in A(t), m \) is always successful. The same happens if \( M \geq N \), in which case each sensor can use a dedicated channel. However, the problem is extremely complex in the general case.

**Theorem 2.** The problem defined in (3) is NP-hard if \( A \geq 2 \) \( \forall A : p_A(A) > 0 \).

**Proof.** We will prove the NP-hardness of the problem if all the active sets have \( A = 2 \), \( \forall A : p_A(A) > 0 \), i.e., if exactly two nodes are active at a given time, by showing its equivalence to an instance of the WGCP [13]. The WGCP determines the nodes are active at a given time, by showing its equivalence is then the solution to the following weight minimization:

\[
e^* = \arg \min_{e \in \{1, \ldots, k\}^{|V|}} \sum_{(u,v) \in E} w_{u,v} I(c_u = c_v),
\]

(9)

where \( I(\cdot) \) is the indicator function, whose value is 1 if the condition is true and 0 otherwise.

In our case, we can consider a fully connected graph with \( V = N \). We assign weights equivalent to the probability of two sensors being active at the same time, i.e., \( w_{u,v} = p_A(\{u, v\}) \). We can then define the weight minimization as:

\[
X^* = \arg \min_{X \in \{0, 1\}^{N \times M}} \sum_{u,v \in N, u \neq v} p_A(\{u, v\}) \xi(X, \{u, v\}).
\]

(10)

As solving the communication problem is equivalent to solving a WGCP, it is NP-hard for \( A = 2 \). If there are sets with non-zero probability and size \( A > 2 \), the problem is equivalent to a Weighted Hypergraph Coloring Problem (WHCP), which models active sensor sets as weighted edges between two or more nodes and is also NP-hard [14]. The reader should also note that, although the problem is equivalent to a WHCP in terms of complexity, the definition is slightly different, as some combination of strategies might result in a successful transmission even if multiple sensors choose the same move, e.g., if two nodes choose to be silent, while the third transmits. The condition is then on \( \xi \), as in (10), and not on the colors (i.e., the strategies) themselves.

**III. SOLVING THE COORDINATION PROBLEM**

In the following, we present two solutions to this provably difficult problem. The first solution is based on clustering and requires full knowledge of \( p_A \). The second is based on MAB learning, and requires a training period in which the nodes attempt to communicate and learn how to coordinate. Unlike the clustering-based solution, MAB learning can be performed online, as it does not require an oracle view of the network: while knowing exactly which sensors are active at any given time is necessary to estimate \( p_A \), the sensors can independently implement MABs and try all strategies, using acknowledgments for correctly transmitted packets as their only feedback. The clustering-based solution is limited to the case with \( A = 2 \), and its generalization to larger active sets is non-trivial. Yet, its main advantage is its immediate applicability when the activation distribution \( p_A \) is known, with no training period required, as it is derived analytically.

**A. Optimal solution**

Although the problem of finding the optimal strategy is NP-hard, we can always find an optimal solution by brute force iteration: if the system is small enough, we can enumerate all possible strategies and go through the list, computing the success rate directly from the activation probability matrix \( p_A \) or measuring it from actual samples. Naturally, the complexity of this solution makes it impractical in most cases, as the number of strategy combinations that need to be evaluated is \( N^{2^M} \). However, we can still perform the computation for the small networks that we consider, in order to show the optimality gap of the other solutions.

**B. Clustering-based solution**

When \( A = 2 \), we can use the graph representation that we exploited to prove the NP-hardness of the problem to design a clustering-based solution. In this solution, we will group the sensors into \( 2^M \) clusters, and assign the same move to all the sensors in the same cluster. We will have a collision if and only if two sensors from the same cluster are active simultaneously. The solution achieved by this approach is suboptimal, as the optimal solution would scale exponentially, but the optimality gap in small scenarios is small.

To minimize the collision probability, we employ the probability of two sensors being together in the active set as a cost \( d_{u,v} = p_A(\{u, v\}) \), where the total cost of a cluster \( C \) is given by the sum of the pairwise costs in the cluster:

\[
d(C) = \sum_{u,v \in C, u \neq v} p_A(\{u, v\})/2.
\]

(11)

The \( \frac{1}{2} \) factor is due to the undirected nature of the cluster: as pairs of nodes are not ordered, each pair is counted twice. Note that the total cost is equivalent to the failure probability of the MAC scheme.

We employ a divisive clustering approach [15], which tries to minimize this total cost at each step in a greedy fashion. This approach significantly outperformed agglomerative and
K-means clustering in our experiments. We use a modified version of the DIANA clustering algorithm [16]. The steps for the algorithm are as follows:

1) Start with the whole set \( \mathcal{N} \) in the same cluster;
2) Among the current clusters, choose the one with the largest total cost \( d(C) \);
3) Split the cluster by choosing the sensor with the largest total cost from the rest and create a new cluster;
4) Place sensors that are more similar to the new cluster than the original one in the new cluster;
5) Iterate the splitting procedure to get \( 2^M \) clusters.

If \( A > 2 \), we can design a simple heuristic that assigns nodes to strategies one at a time, starting from the node with the highest probability and going down the list, so that every node is assigned to the strategy that maximizes the success probability for the subset of nodes that were already assigned.

C. Distributed learning approach

It is also possible to learn the optimal policy in a distributed fashion, implementing a MAB for each sensor. MABs are learning agents that have a number of arms, which correspond to possible actions, and learn by trying each arm and estimating its expected value over time. There are several sampling strategies to choose which action to use, balancing between exploration (i.e., choosing the action to gain new information) and exploitation (i.e., choosing the action with the highest expected value). As these deterministic policies are relatively stable over time, this round-based system greatly reduces the instability, allowing the system to converge.

After it receives the reward, the MAB for the \( n \)-th agent, which is an active one in this round, updates its estimate \( Q_n(x) \) of the value of the action it just chose:

\[
Q_n^{(t+1)}(x) = (1 - \alpha(t))Q_n^{(t)}(x) + \alpha(t)\xi(t),
\]

where parameter \( \alpha(t) \) is the learning rate, which gradually decreases over time while following the Robbins-Monro criterion [17]. The MAB also specifies the sampling policy, i.e., which actions are chosen to accelerate the learning process and maximize the expected reward.

Our problem is complicated by its distributed nature. In fact, every sensor implements a separate MAB, and distributed MABs often have instability issues [18]. In order to avoid any issues, we train the MABs individually, one at a time, following the procedure below:

1) The estimates \( Q_n^{(0)} \) are initialized with random values in \([0, 1]\) for each action. This is equivalent to starting with no information on the expected values of each action;
2) In each turn, a sensor is chosen using a shared criterion (e.g., round robin). If it is in the active set, the sensor uses a uniform random policy (which prioritises exploration over exploitation) to decide its action. Other sensors in the active set use a greedy policy, deterministically choosing the action with the highest expected value. As these deterministic policies are relatively stable over time, this round-based system greatly reduces the instability, allowing the system to converge;
3) If the chosen sensor was in the active set, its estimate of the action value is updated.
4) The procedure is repeated until convergence.

IV. Numerical results

In the following, we show the simulation results for the clustering and learning solutions. We consider a system with \( N = 10 \) sensor nodes, transmitting over \( M = 2 \) parallel channels. We limit the number of simultaneously active agents to the cases with \( A = 2 \) and \( A = 3 \). Although the clustering solution only works in the former, the MAB solution works equally well in both of these cases. Furthermore, we consider three different types of node activation distributions:

1) Deterministic activation: this is the simplest case, as each sensor is always paired with the same other sensor (or the same two, if \( A = 3 \)). In this case, the active sensor sets with non-zero probability represent a partition of \( \mathcal{N} \), and the sensors only have to coordinate over these smaller groups;
2) **Regular activation**: we consider a correlated activation pattern with significant regularity, which might be, e.g., due to the physical location of the sensors. Sensors that are closer together are often simultaneously active, while sensors that are farther apart have a smaller joint activation probability. This case is naturally more complex than the deterministic one;

3) **General case**: $p_A$ is a general PMF with no apparent regularity. Hand-designing a solution for this case is extremely difficult, as it requires to solve the overall problem and there are no regular features to exploit.

In each scenario, the training of the MAB agents is performed in rounds, which correspond to the transmission of $N$ packets, so that every sensor receives one possible update per round.

### A. Deterministic activation

In this case, which is illustrated in Fig. 1a, neighboring sensors with the same color are always active at the same time. The strategy is trivial: in each couple, one of the two nodes is elected as a leader and transmits every time it is active, while the other remains silent. This guarantees that there will be no collisions, even if only one communication channel is available. As Fig. 1b shows, the MAB solution converges to this leader-based procedure in about 100 steps. The graph clustering solution also has a 100% success rate.

In fact, the same result can be easily obtained with any number of activated sensors: the optimal strategy is extremely simple, as it replicates the same pattern as for $A = 2$. Each group of sensors that are active at the same time elects a leader, who then proceeds to speak for the whole group. The time until convergence increases linearly with $A$, but the endpoint is the same. Naturally, the extreme simplicity of this case makes it useful only as an example to illustrate different techniques, as the optimal solution is obvious and can be easily verified by hand.

### B. Regular activation

In this case, we can still use the geometric approximation, considering that sensors closer to each other on the circle are often active at the same time. In this case, the first sensor is picked at random with probability $\frac{1}{3}$, while the second sensor in the visible set is picked with probability $p(d)$, which depends on the distance along the circle: if $d = 1$, the probability is 0.275, if $d = 2$, it is 0.125, if $d = 3$, it is 0.075, and if $d = 4$, it is 0.025. This is shown graphically in Fig. 2a if sensor 1 is picked, the higher probability of picking closer sensors is depicted as a deeper blue. The node diametrically opposite to the first one is never picked. In the case in which $A = 3$, the third node is picked from the same distribution, considering distance from the second node and removing the first. The basic idea behind the strategy is still intuitive: nodes that often appear together should have different strategies, so as to avoid collisions and transmit the shared message through at least one of the 2 channels. If $A = 3$, the situation is slightly different, as it is possible to achieve an optimal solution only using strategies $(1, 0)$ and $(0, 1)$: the objective of the sensors in this case is to avoid a scenario in which all three transmit over the same channel, as any other combination leads to a collision on one channel and a successful transmission on the other. The results are shown in Fig. 2b: it is easy to see that the clustering solution and the MAB solution both achieve the optimum, although the latter required significant amount of training. However, the clustering solution might not be viable in a practical scenario, in which the activation patterns are unknown: in order to estimate the joint activation probabilities required to run the clustering, we would need an oracle view of the network, gathering information on exactly which sensor pairs have been active simultaneously at a given time slot. On the other hand, the MAB solution can be trained online with no knowledge beyond a shared acknowledgment signal, and is even robust to the loss of ACKs: performance during training is also already acceptable, with a success probability over 85% early in the training. Interestingly, the reward for $A = 3$ is slightly higher, although the training is slower: in this case, finding the correct strategy is more complex, but all three nodes need to be in the same set for the transmission to fail. In the $A = 2$ case, there are 4 different strategies, but only 2 active nodes at a time, which need to be more aggressive to avoid complete silence, thus reducing the overall reward.

### C. General case

Finally, we show the results for a general case, using a randomly drawn activation probability matrix. In this case, the $A = 3$ case is harder, as Fig. 2c shows: the success rate using
the optimal strategy is slightly over 0.85, while all other cases are well above 0.9. The training time for the MAB solution is correspondingly slightly longer, as the patterns are more complex. In both the \( A = 2 \) and \( A = 3 \) cases, graph-based approaches are not optimal anymore: the clustering solution with \( A = 2 \), and the heuristic one for \( A = 3 \), are based on the greedy approximation, whose optimality gap is negligible if there is regularity in the scenario, but becomes noticeable as the graph becomes a mesh with more complex combinations of weights. However, the MAB solution can still achieve an optimal performance, with a much smaller gap.

V. CONCLUSIONS AND FUTURE WORK

We have introduced and examined a novel distributed co-ordination and communication problem, in which multiple wireless sensors need to coordinate and find a common strategy to transmit a shared alarm message. The problem is fundamentally different from general error rate minimization, in which every packet contains different information; instead, we can accept collisions on one channel as long as the transmission of the message is guaranteed on another. We proved that this problem is both NP-hard and has a deterministic optimal solution, and proposed two heuristic solutions that can achieve a small optimality gap in practical conditions, each with its own advantages. The clustering-based solution is computationally fast, but requires a priori knowledge of the activation probability matrix. On the other hand, the MAB solution takes some time to converge to the optimum, but it can be trained distributedly with no additional signaling.

There are several possible avenues of future work on the topic, including the use of more advanced learning mechanisms, such as neural network-based bandits which can generalize experience and converge with fewer training samples. Another interesting research direction is the application of these principles to swarm control, in which the sensors are not only passively transmitting an alarm messages, but exchanging information about a shared environment, which they can directly modify by acting in concert.

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