Congruity of Crab pulsar’s gamma-ray spectrum with the spectral distribution of the radiation by the current sheet in its magnetosphere

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ABSTRACT
The spectrum derived here for the most tightly-focused component of the radiation generated by the superluminally moving current sheet in the magnetosphere of a non-aligned neutron star has a distribution function that fits the entire gamma-ray spectrum of the Crab pulsar on its own. This is the first time that the undivided breadth of this spectrum, from $10^2$ to $10^6$ MeV, is not only described by a single distribution function but is also explained by means of a single emission mechanism.

Key words: pulsars: individual: J0534+2200 – gamma-rays: stars – stars: neutron – methods: data analysis – radiation mechanisms: non-thermal – radiation: dynamics.

1 INTRODUCTION
Numerical computations based on the force-free and particle-in-cell formalisms have now firmly established that the magnetosphere of a non-aligned neutron star entails a current sheet outside its light cylinder whose rotating distribution pattern moves with linear speeds exceeding the speed of light in vacuum (see the review article Philippov & Kramer 2022, and the references therein). A study of the characteristics of the radiation that is generated by this superluminally moving current sheet has in turn provided an all-encompassing explanation for the salient features of the radiation received from pulsars: its brightness temperature, polarization, spectrum and profile with microstructure and with a phase lag between the radio and gamma-ray peaks (Ardavan 2021, 2022a).

The radiation field generated by a constituent volume element of the current sheet in the magnetosphere of a neutron star embraces a synergy between the superluminal version of the field of synchrotron radiation and the vacuum version of the field of Čerenkov radiation. Once superposed to yield the emission from the entire volume of the source, the contributions from the volume elements of the current sheet that approach the observation point with the speed of light and zero acceleration at the retarded time interfere constructively and form caustics in certain latitudinal directions relative to the spin axis of the neutron star. The waves that embody these caustics are more focussed the further they are from their source: as their distance from their source increases, two nearby stationary points of their phases draw closer and eventually coalesce at infinity (Ardavan 2021, Section 4). By virtue of their extremely narrow peaks in the time domain, such focused pulses have broad spectra that encompass X-ray and gamma-ray frequencies (Ardavan 2021, Table 1 and Section 5.3). In directions where the stationary points of their phases are isolated, the emitted pulses are not focussed as strongly and so are detectable at lower frequencies (Ardavan 2021, Section 5.4).

Here, the contribution toward the spectrum of this radiation from the vicinity of its tightly-focused caustics is derived in Section 2 and it is shown that the derived spectral distribution function fits the observed gamma-ray spectrum of the Crab pulsar (Abdo et al. 2010; Aleksić et al. 2011; Ansoldi et al. 2016) over its entire breadth from $10^2$ to $10^6$ MeV (Section 3). The contrast between the single radiation mechanism underlying the present spectral distribution function and the disparate radiation mechanisms and spectral distribution functions currently invoked in the literature for fitting the data in different sections of the Crab pulsar’s gamma-ray spectrum (see the review articles Zanin 2017; Amato & Olmi 2021, and the references therein) is briefly commented on in Section 4.

2 SPECTRAL DISTRIBUTION FUNCTION OF TIGHTLY-FOCUSED CAUSTICS
The frequency spectrum of the radiation that is generated by the superluminally moving current sheet in the magnetosphere of a non-aligned neutron star was presented, in its general form, in Ardavan (2021, Section 5.3). Here we derive the spectral distribution function of the most tightly-focused component of this radiation from the general expression given in equation (177) of that paper.

In a case where the magnitudes of the vectors denoted by $\mathcal{P}_l$ and $\mathcal{Q}_l$ in equation (177) of Ardavan (2021) are appreciably larger than those of their counterparts, $\mathcal{P}_1$ and $\mathcal{Q}_1$, and the dominant contribution towards the flux density $S_\nu$ of the radiation is made by only one of the two terms corresponding to $l = 1$ and $l = 2$, equation (177) of Ardavan (2021) can be written as

$$S_\nu = k_0 k^{-2/3} \left| \mathcal{P} \text{Ai}(-k^{2/3} \sigma^2) - ik^{-1/3} \mathcal{Q} \text{Ai}'(-k^{2/3} \sigma^2) \right|^2,$$  \hspace{1cm} (1)

where $\text{Ai}$ and $\text{Ai}'$ are the Airy function and the derivative of the Airy function with respect to its argument, respectively, $k = 2\pi \nu / \omega$ is the frequency $\nu$ of the radiation in units of the rotation frequency $\omega / 2\pi$ of the central neutron star, and the positive constants $k_0$ and $\sigma$ and...
the complex vectors $\mathbf{P}$ and $\mathbf{Q}$ are determined by the characteristics of the current sheet. The above spectrum is emblematic of any radiation that entails caustics (see Stammes 1986).

Evaluation of the right-hand side of equation (1) results in
\[
S_\nu = \kappa_1 k^{-2/3} \left[ A_2^2 \langle k^{-2/3} \sigma^2 \rangle + a^2 k^{-2/3} A_2^2 \langle k^{-2/3} \sigma^2 \rangle + 2a \cos \beta k^{-1/3} A_1 \langle k^{-2/3} \sigma^2 \rangle A_1' \langle k^{-2/3} \sigma^2 \rangle \right],
\]
where
\[
\kappa_1 = |\mathbf{P}|^2 k_0, \quad a = |\mathbf{Q}| / |\mathbf{P}|, \quad \cos \beta = \frac{\mathbf{Q} \cdot \mathbf{P}}{|\mathbf{Q}||\mathbf{P}|},
\]
and $\kappa$ and $a$ denote an imaginary part and the complex conjugate, respectively. The positive parameter $\kappa$ has a vanishingly small value at the peak of a high-frequency pulse (Ardavan 2021, Section 4.5). For the purposes of the present analysis, we may therefore replace $\mathbf{P}$ and $\mathbf{Q}$ by their limiting values for $k \to 1$ and $\sigma \ll 1$ and treat the quantities $\kappa_1$, $a$ and $\beta$ that appear in equation (2) as constant parameters.

To take account of the fact that the parameter $\kappa$ assumes a non-zero range of values (bordering on $\kappa = 0$) across the width of a high-frequency pulse (Ardavan 2022a, Section 3), we must integrate $S_\nu$ with respect to $\kappa$ over a finite interval $0 \leq \kappa \leq \kappa_0$ with $\kappa_0 \ll 1$.

Performing the integration of the Airy functions in equation (2) with respect to $\kappa$ by means of Mathematica, we thus obtain
\[
S_\nu = \int_0^{\kappa_0} S_\nu d\kappa = \kappa \left[ f_1(\chi) + \frac{b^2}{4\chi^3} f_2(\chi) - \frac{b \cos \beta}{2\sqrt{3} \chi} f_3(\chi) \right],
\]
where
\[
\kappa = \left( \frac{\kappa_0}{3\sigma} \right)^3 \kappa_1, \quad b = \sigma_0 a, \quad \chi = \frac{2}{3} \sigma_0^3 k,
\]
and
\[
f_1(\chi) = 3\left( \frac{7}{6} \chi^{-2/3} \int_0^1 F_3 \left( \frac{1}{2} \right) \frac{1}{2} \frac{1}{3} \frac{2}{3} \frac{5}{3} \chi^{-2} \right),
\]
\[
+ \pi^{1/2} \int_0^1 F_3 \left( \frac{1}{2} \right) \frac{1}{2} \frac{1}{4} \frac{3}{4} \frac{5}{3} \chi^{-2} \right),
\]
\[
+ \frac{9}{20} \left( \frac{5}{6} \chi^{-2/3} \int_0^1 F_3 \left( \frac{5}{6} \right) \frac{5}{6} \frac{5}{3} \frac{5}{3} \frac{11}{6} \chi^{-2} \right),
\]
and
\[
f_2(\chi) = \chi^{-4/3} 24G^{31} \left( -\chi^{-2/3} \right) \left( 5/6 \right) \frac{5}{6} \frac{7}{6} \frac{2}{3} \frac{3}{4} \frac{1}{3} \chi^{-1/6},
\]
\[
f_3(\chi) = \chi^{-4/3} 24G^{31} \left( -\chi^{-2/3} \right) \left( 5/6 \right) \frac{5}{6} \frac{7}{6} \frac{2}{3} \frac{3}{4} \frac{1}{3} \chi^{-1/6}.
\]
and $F_3$ and $G^{31}$ are respectively the generalised hypergeometric function (see Olver et al. 2010) and the generalised Meijer G-Function (see https://mathworld.wolfram.com/MeijerG-Function.html). The variable $\chi$ that appears in the above expressions is related to the frequency $\nu$ of the radiation via $\chi = 4\pi\sigma_0^3 \nu / (3c_a)$.

The scale and shape of the spectral distribution function $S_\nu(\nu)$ given in equation (4) depend on the four parameters $\kappa$, $b$, $\beta$ and $\sigma_0$; parameters whose values are dictated by the characteristics of the magnetospheric current sheet. The parameters $\kappa$ and $\sigma_0$ respectively determine the scale of the flux density and the range of frequencies over which the flux density has a detectable value, while the parameters $b$ and $\beta$ determine the shape of this distribution.

3 FITTING THE DERIVED DISTRIBUTION FUNCTION TO THE DATA ON THE GAMMA-RAY SPECTRUM OF THE CRAB PULSAR

The phase-averaged flux density of the gamma-rays received from the Crab pulsar is plotted (in units of erg cm$^{-2}$ s$^{-1}$ MeV$^{-1}$) versus photon energy (in units of MeV) in figure 1. The data points below 25 GeV in this figure are the ones detected by the FERMI Large Area Telescope (FERMI-LAT) which were reported in Abdo et al. (2010). The three data points between 25 and 100 GeV are those measured by the Major Atmospheric Gamma Imaging Cherenkov (MAGIC) telescope and were reported in Aleksic et al. (2011).

Having been derived by a Fourier decomposition of the pulse profile with respect to time, equation (1) and hence the expression for $S_\nu$ in equation (4) describe a spectral distribution that is averaged over the whole pulse period (see Ardavan 2021). On the other hand, the latest data by MAGIC on photon energies exceeding 100 GeV are reported by Ansoldi et al. (2016) for the main peak and the interpulse of the Crab pulsar separately. To estimate the phase-averaged fluxes at the five photon energies for which the fluxes associated with both the main peak and the interpulse are given in Table 3 of Ansoldi et al. (2016), we have here summed the contributions from these two components of the pulse profile. Given that the reported upper limits on the contribution of the main pulse toward the fluxes at 781 and 1211 GeV are comparable to the observational errors in the contribution of the interpulse to the fluxes at these energies (see Table 3 of Ansoldi et al. 2016), we have equated the phase averaged fluxes at 781 and 1211 GeV to the fluxes of the interpulse and have incorporated the upper limits on the contributions of the main pulse toward the phase averaged fluxes at these two photon energies in the error bars of the last two data points.

The data point with the highest photon energy (81 GeV) in the data set reported by Aleksic et al. (2011) and the data point with the lowest photon energy (87 GeV) in the data set reported by Ansoldi et al. (2016) correspond to the same photon energy to within the limits of their horizontal error bars. Hence, any discrepancy between the reported fluxes of these two data points must be due to the difference partly in the phase intervals over which these two group of authors have made their measurements and partly in the energy-dependent systematic uncertainties associated with their experiments (see Aleksic et al. 2012). Accordingly, we have removed the discrepancy between the phase-averaged fluxes of the lowest-energy data point extracted from Ansoldi et al. (2016) and the highest-energy data point given in Aleksic et al. (2011) by multiplying the former by the factor 9.77. Once the fluxes of each of the remaining six data points extracted from Ansoldi et al. (2016) are multiplied by the same factor, these points are collectively raised to the level shown in figure 1.

The curve in figure 1 depicts the spectral distribution function $S_\nu$ that is described by equation (4) for the following values of its free parameters:
\[
\kappa = 7.94 \times 10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ MeV}^{-1},
\]
\[
\chi = 5.62 \times 10^{-4} (h\nu) \text{ MeV},
\]
\[
\sigma_0 = 4.72 \times 10^{-8}, \quad b = 0.4 \quad \text{and} \quad \beta = 0.3,
\]
in which $h$ stands for the Planck constant and $2\pi/\omega$ has been set equal to the period of the Crab pulsar. As the photon energy $h\nu$ increases beyond 1.2 TeV, the slope of this curve continues to decrease at the relatively slow rate in which it decreases past 50 GeV. Thus the upper limits given by Ansoldi et al. (2016) for the flux density of gamma-rays whose energies exceed 1.2 TeV all lie above the continuation of the curve shown in figure 1.
Figure 1. Gamma-ray spectrum of the Crab pulsar. The FERMI-LAT data points below 25 GeV (coloured black) are those reported in Abdo et al. (2010). The MAGIC data points between 25 and 100 GeV (coloured blue) are those reported in Aleksić et al. (2011). The MAGIC data points beyond 100 GeV (coloured green) are extracted (by the procedure described in Section 3) from those reported by Ansoldi et al. (2016). The curve (coloured red) is a plot of the spectral distribution function described by equation (4) for the parameters given in equation (9).

4 CONCLUDING REMARKS

In the current literature on the gamma-ray spectrum of the Crab pulsar (see the review articles Zanin 2017; Amato & Olmi 2021, and the references therein), this spectrum is parametrised partly (between 0.1 and 100 GeV) by a power-law function with an exponential cut off and partly (between 100 GeV and 1 TeV) by a pure power-law with another exponent. The part of the emission between 0.1 and 100 GeV is commonly attributed to synchrotron, curvature or synchro-curvature processes. It is generally acknowledged, however, that the very high energy component of the emission is unlikely to be generated by such processes. The emission at higher than a few hundred GeV is thought to be produced either by inverse Compton scattering of lower energy photons or as a consequence of magnetic reconnection in the magnetospheric current sheet.

In contrast, we have here fitted the entire gamma-ray spectrum of the Crab pulsar by a single function representing the spectral distribution function of a single emission mechanism. That the spectral distribution function of the radiation generated by the superluminally moving current sheet in the magnetosphere of the Crab pulsar should provide such an all-encompassing fit to the observational data on its emission is not surprising given the firm theoretical grounds on which both the existence of the current sheet in question (see Philippov & Kramer 2022) and the characterisation of the radiation it generates (see Ardavan 2021) are based. Clearly, the relevancy of the spectral distribution function given in equation (4) is not limited to the Crab pulsar: this function describes the spectral distribution of the high frequency emission from any non-aligned neutron star.

As already mentioned, the caustics whose spectra are described by equation (4) are more focused the further they are from their source (Ardavan 2021, Section 4). As a result, flux densities of the pulses that are generated by the emission mechanism discussed here diminish with the distance $D$ from the star as $D^{-3/2}$ (rather than $D^{-2}$) in the latitudinal directions along which the high frequency pulses are detectable (Ardavan 2021, Section 5.5). This slower rate of decay of the flux density applies to the high frequency radiation from all non-aligned neutron stars, including the gamma-ray and/or X-ray emission from any pulsar or magnetar, thereby reducing the conventional estimates of their luminosities (see Ardavan 2022b,c).

DATA AVAILABILITY

The data used in this paper can be found in Abdo et al. (2010), Aleksić et al. (2011) and Ansoldi et al. (2016).

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