Spatial Qubit Entanglement Witness for Quantum Natured Gravity

Bin Yi,1 Urbasi Sinha,2 Dipankar Home,3 Anupam Mazumdar,4,5 and Sougato Bose1

1Department of Physics and Astronomy, University College London, Gower Street, WC1E 6BT London, United Kingdom.
2Raman Research Institute, C. V. Raman Avenue, Sudashivanagar, Bengaluru, Karnataka 560080, India
3Center for Astroparticle Physics and Space Science (CAPSS), Bose Institute, Kolkata 700 091, India
4University of Groningen, PO Box 72, 9700 Groningen, Netherlands
5Van Swinderen Institute, University of Groningen, 9747 AG Groningen, Netherlands

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Evidencing the quantum nature of gravity through the entanglement of two masses has recently been proposed. Proposals using qubits to witness this entanglement can afford to bring two masses close enough so that the complete $1/r$ interaction is at play (as opposed to its second order Taylor expansion), and micron sized masses separated by 10-100 microns (with/without electromagnetic screening) suffice to provide a $\sim 0.01 - 1$ Hz rate of growth of entanglement. Yet the only viable method proposed for obtaining qubit witnesses so far has been to employ spins embedded in the masses, whose correlations are used to witness the entanglement developed between masses during interferometry. This comes with the dual challenge of incorporating spin coherence preserving methodologies into the protocol, as well as a demanding precision of control fields for the accurate completion of spin aided (Stern-Gerlach) interferometry. Here we show that if superpositions of distinct spatially localized states of each mass can be created, whatever the means, simple position correlation measurements alone can yield a spatial qubit witness of entanglement between the masses. We find that a significant squeezing at a specific stage of the protocol is the principal new requirement (in addition to the need to maintain spatial quantum coherence) for its viability.

I. INTRODUCTION

The quantumness of gravity is an open question due to a lack of empirical evidence. A lot of research is performed in the setting of semi-classical gravity, in which matter and non-gravitational fields are treated quantum mechanically, while gravity is treated classically. A substantial community argues that gravity can be classical as quantum physics break down at macroscopic level, where gravitational effects become evident[1–4]. Various proposals have been made to avoid quantizing gravity, at the cost of introducing extra stochastic terms[5, 6] – but these are not ruled out by any current experiment such as measuring forces precisely. Therefore, testing the quantum nature of gravity experimentally is an open problem. Even if its quantumness is accepted from the point of view several existing successful theories [7, 8], its verification is still open.

In 2017, Bose and collaborators proposed a protocol to test whether the nature of gravity is quantum[9] (see also recorded online talk of 2016, where the same protocol is presented [10]). Two spatial superposition state of masses can not entangle via classical channel [11] (see also [12]). Locality in quantum field theory circumvents the non-local interaction between the two superposed test masses[13]. Entanglement between the masses can only be generated through local operations and quantum communication (in fact, quantum communication is necessary for operator valued interactions, [14] which in turn is necessary for the coherent interaction that generates entanglement). On the other hand, once quantum communication is proven through a witnessing of gravitationally generated entanglement, this unambiguously certifies the presence of off-shell/virtual graviton as this is the only way to get a continuous deterministic operator valued interaction[13, 14]. Alternatively, the witnessed entanglement can also be regarded as evidencing the quantum superposition of geometries inherent in superposition states of each mass [15, 16]. Moreover, from logical arguments, the quantum nature of the Newtonian component of the gravitational field automatically has a bearing on the quantum nature of other components [17–20].

The original proposal exploits spin-embedded masses[9]. Stern-Gerlach interferometers (SGIs) are used to prepare the spatial superpositions depend on embedded spins so that the motional and spin degree of freedom is entangled. The test masses then freely evolve subject to gravitational interaction. At the end of the proposed protocol, the Stern-Gerlach apparatus is exploited again to bring the superposition components back to the center. Spin correlations between the test masses then evidence the entanglement generated through the gravitational interaction during propagation. Alternatively, it has also been proposed to witness the gravitational entanglement growth between two initial delocalized gaussian states by position and momentum correlations [21–23] (see also [19, 20, 24] for similar entanglement via gravity between light and matter or between optical fields). However, a two-qubit witness for entanglement is applicable to a situation where masses are brought as close to each other as their delocalization (spatial superposition scale), so that the entanglement itself has a faster growth rate ($\sim 1$ Hz) even for smaller (micron sized) masses.

It is challenging to complete SGIs as an exact overlap has to be attained in both position and momentum of the wavepackets in the two interferometric arms [25, 26].
although this has been achieved very recently for small atomic interferometers [27]. Moreover, the spin can bring in an extra avenue of decoherence requiring extra dynamical decoupling procedures which could potentially complicate the interferometry even more requiring further dynamical decouplings [28, 29]. Although exceptionally long spin coherence have been shown, and these are the subject of qubits in quantum computers, it may be worthwhile to look for witnessing of the entanglement without spins. This may even be the case where we actually use spin dependent forces (Stern-Gerlach) to create the initial superposition. However, we do not worry about also completing the interferometry by appropriate matching of forces in the two arms. On the other hand, without spins, the advantage of two-qubit witnesses for quantum entanglement seems to disappear. We thus here look at the potential of using position measurements themselves to infer states of “spatial qubits” (which have been called Young qubits in photonic systems [30–33]) and use that as witness for evidencing gravitational entanglement. We find that aside the usual coherence requirements to be satisfied for maintaining the quantum superpositions [9, 34], which is unavoidable in any scheme, a spatial qubit witnessing of gravitational entanglement is possible if a challenging squeezing requirement can be met.

The spatial qubit methodology [35] encodes qubit in the spatial degree of freedom of a freely propagating test mass. The readout of the information stored in the qubit can be implemented by placing sets of spatial detectors at appropriate positions. Correlations generated between two such spatial qubits during evolution can then be tested by spatial detection. One advantage of this approach is its simplicity: free propagation followed by spatial detection. An interferometer with beam splitting elements such as Mach-Zhender, looks highly unfeasible because of the large mass that has to tunnel through such a system. Of course, a Stern-Gerlach interferometer is possible, but this requires a spin, as well as exquisite control in completing the interferometry. Both these requirements are completely avoided if spatial qubits are used. However, it requires the application of an additional squeezing operator in general. In this paper, we apply this methodology to witness quantum natured gravity.

II. SPIN ENTANGLEMENT WITNESS SETUP

Ref. [9] presents an unambiguous witness of quantum natured gravity. The scheme consists of two spin embedded test masses initially in spatially localized states, say, in respective traps. The test masses pass through a set of Stern-Gerlach (SG) interferometers so that the spatial degree of freedom entangles with the spin degree of freedom to prepare

$$|\psi_0\rangle = |L, \uparrow\rangle_j + |R, \downarrow\rangle_j (j = 1, 2)$$

with states $|L\rangle$ and $|R\rangle$ being separated by $d$, and the distance between the midpoint of each superposition (~ initial separation between the masses) being $D$. Next, let the coherent superposition state propagate for a time $\tau$ by switching off the magnetic field of the SG apparatus. If gravity were quantum, gravitational interaction can induce, through relative phases among the superposition components, an entanglement between the masses (classical gravity as mediator would not give an operator valued interaction [14], and hence will be unable to entangle the two masses). The final step refocuses the SG apparatus to bring the spatial superposition back to the center so that the final spin state reads

$$\frac{e^{i\phi}}{\sqrt{2}}(|\uparrow\rangle_1 \frac{1}{\sqrt{2}}(|\uparrow\rangle_2 + e^{i\Delta\phi_{\uparrow\downarrow}}|\downarrow\rangle_2)$$
$$+ |\downarrow\rangle_1 \frac{1}{\sqrt{2}}(e^{i\Delta\phi_{\uparrow\downarrow}}|\uparrow\rangle_2 + |\downarrow\rangle_2) \rangle$$

(2)

where $\phi = \frac{Gm_1m_2}{\hbar D}\tau, \Delta\phi_{\uparrow\downarrow} = \frac{Gm_1m_2}{\hbar(D+\Delta)}\tau - \phi$, $\Delta\phi_{\uparrow\downarrow} = \frac{Gm_1m_2}{\hbar(D-c\Delta)}\tau - \phi$. This is generically an entangled state as soon as $\frac{1}{\sqrt{2}}(|\uparrow\rangle_2 + e^{i\Delta\phi_{\uparrow\downarrow}}|\downarrow\rangle_2) \neq \frac{1}{\sqrt{2}}(e^{i\Delta\phi_{\uparrow\downarrow}}|\uparrow\rangle_2 + |\downarrow\rangle_2)$. By measuring spin correlations, one can then verify the entanglement induced during the propagation time $\tau$. That can only arise from the exchange of quantum coherent mediators. If gravity is the only interaction present, one can then conclude that gravity is quantum.

To produce observable relative phase, the original proposal considers massive objects with $m \sim 10^{-15}kg$. The required mass is restricted by the minimum distance between the two masses $D - d$. At micro-meter scale, Casimir-Polder force, which is another source of interaction between neutral objects, becomes dominant over gravitational interaction. A minimum distance $D - d \sim 100\mu m$ is necessary to ignore influence from Casimir-Polder interaction. Later, a revised scheme based on Casimir screening [34] relaxes the parameters to $D \sim 47\mu m, d \sim 23\mu m$ by placing a conducting plate, which acts as a Faraday cage, between the test masses to screen the mutual electromagnetic interaction. On the other hand, to test the spin entanglement witness also requires a delicate balancing of magnetic field gradient to bring the superposition components back to the center. In the revised scheme, the test mass $m \sim 10^{-15}kg$ and the total accumulated phase during interaction time $\tau \sim 1s$ is of order of 0.01rad.

III. MASSIVE SPATIAL QUBIT METHODOLOGY FOR WITNESSING GRAVITATIONAL ENTANGLEMENT

In this paper, we investigate the viability of testing the quantum nature of gravity with recently developed methodology: massive spatial qubit. This approach treats freely evolving spatially superposed masses as qubits, which does not call for spins. Therefore, it may relax the criteria for witnessing the quantum aspect
of gravity. The schematic diagram is shown in Fig.1. Two test masses $m_1, m_2$, each prepared in spatial superpositions of two well separated Gaussian states $|L\rangle$ and $|R\rangle$, are placed adjacent to each other then freely evolve, under their own Hamiltonian, as well as undergo a joint phase evolution under their mutual gravitational interaction as shown in Fig.1. The propagation of each individual wavepacket is modeled as the spreading of a Gaussian wavepacket due to free evolution. Each spatial superposition $\sqrt{2}(|L\rangle + |R\rangle)$ of test mass can be treated as a state of a qubit with the two qubit states identified with $|L\rangle$ and $|R\rangle$.

**Witnessing the quantum nature of gravity:** If quantum gravitational interaction is the only source of interaction, relative phase induced among the superposition components then reads:

$$|\psi(t = 0)\rangle = \frac{1}{\sqrt{2}}(|L\rangle_1 + |R\rangle_1) + \frac{1}{\sqrt{2}}(|L\rangle_2 + |R\rangle_2)$$  \hspace{1cm} (3)

$$\rightarrow |\psi(t = \tau)\rangle = \frac{e^{i\phi}}{\sqrt{2}}[|L\rangle_1 \frac{1}{\sqrt{2}}(|L\rangle_1 + e^{i\Delta \phi_L}|R\rangle_2) + |R\rangle_1 \frac{1}{\sqrt{2}}(e^{i\Delta \phi_R}|L\rangle_2 + |R\rangle_2)]$$  \hspace{1cm} (4)

It is true that the creation of a superposition of two localized states (a highly non-Gaussian state) is a pre-requisite for being able to use spatial qubits (we will not delve into that issue here in detail, but mention some possibilities in the next section). Here we concentrate on how to read-out spatial qubits in absence of (in this context impractical) beam-splitter elements, and thereby witness the entanglement of two masses without resorting to spins. Pauli measurements, which readout the encoded information, can be performed by placing spatial detectors at particular locations. Any entanglement induced during the propagation process can be witnessed by the correlations between two spatial qubits. All of Pauli-$x$, $y$, and $z$ measurements are involved in obtaining these correlations. Pauli-$z$ measurement $\sigma_z = |L\rangle\langle L| - |R\rangle\langle R|$ reads the probability amplitude of the encoded spatial qubit state, which has to be performed before $|L\rangle$ and $|R\rangle$ have spread so much as to be confused with each other (i.e., spread to about $\sim d$). On the other hand, Pauli-$x$ measurements project on to states $\frac{1}{\sqrt{2}}(|L\rangle \pm |R\rangle)$ and Pauli-$y$ measurements project on to states $\frac{1}{\sqrt{2}}(|L\rangle \pm i|R\rangle)$, which are only discernible in the interference plane as shown in Fig.1, which implies that they are performed after each of $|L\rangle$ and $|R\rangle$ have spread to a length $d$ so that they can overlap. At the interference plane, the probability of detecting the test mass at position $x$ is given by:

$$P(x) = |\langle \psi | k_x \rangle|^2 \propto \left| \exp\left[\frac{ik_x d}{\hbar t_{\text{meas}}} - k_x^2 \sigma_z^2 \langle \theta \psi \rangle \right] \right|^2$$  \hspace{1cm} (5)

where $|\theta\rangle = |0\rangle + |1\rangle e^{i\theta}$, $\theta = k_y d$ and $k_x = \frac{x}{\hbar t_{\text{meas}}}$ is the transverse wavevector. Projection of the spatial qubit state along $\sigma_x$, $\sigma_y$ ($\sigma_y$, $\sigma_z$) can then be implemented by placing spatial detectors corresponds to $\theta = 0, \pi(-\pi/2, \pi/2)$ respectively. Thus the Pauli-$x$, $y$ and Pauli-$z$ measurements would normally have to be performed at different times $t_{\text{meas}}$ (before spreading) and $\frac{1}{\hbar t_{\text{meas}}}$ (after spreading).

**Squeezing requirement** The masses have to entangle sufficiently first. This inevitably requires a time of $\tau$ over which the gravitational interaction should act between them. This is the earliest time any measurement can occur as we want to measure the two masses after they are entangled. Thus we set $t_{\text{meas}}^x = \tau$, being careful that hardly any spreading of the wavepackets $|L\rangle$ and $|R\rangle$ happens in time $\tau$. However, we want to measure Pauli-$x$, $y$ on the same state, i.e., the state evolved up to time $\tau$ as the gravitational interaction cannot be switched off. This demands that we must also measure Pauli-$x$, $y$ at (or very nearly) the same time. So we need to induce some process at $\tau$ so that the wavepackets immediately spread to $\sim d$ and overlap to interfere. This requires the application of an additional squeezing operator to the masses immediately after time $\tau$. This localizes the wavefunctions so much (each of $|L\rangle$ and $|R\rangle$ are now highly squeezed) that in subsequent free evolution for $t_{\text{meas}}^x - t_{\text{meas}}^\tau$, they spread rapidly to overlap and interfere. Thus the extra squeezing is $t_{\text{meas}}^x \approx t_{\text{meas}}^\tau$, thus both the Pauli-$z$ and the Pauli-$x$, $y$ measurements are essentially performed on the same entangled state of the two masses.

The spread of wavepackets scales approximately inversely proportional to its initial width $\sigma_d(0)$: $\sigma(t) \sim \frac{\hbar t}{\sigma_d(0)}$. Therefore, application of squeezing operator at $\tau$, squeezes width of the wavepacket and speeds up subsequent spreading. This technique can be accomplished by passing the test masses through local Harmonic potentials with jumps between frequencies. More precisely, $n$ sudden switches between frequencies $\omega_1$ and $\omega_2$, with a quarter period of harmonic evolution in each frequency, squeezes the wavepackets spatially by a factor $(\omega_1/\omega_2)^n$. However, our analysis assumes that $\omega_1 > \omega_2$ instead of the expected $\omega_2 > \omega_1$.
As, for the squeezing, we have to expend a quarter period of time $\pi/2\omega_j$ in each frequency $\omega_j$, this inevitably requires a time

$$t_{\text{squeeze}} = n(\pi/2\omega_1 + \pi/2\omega_2).$$

On the other hand, the amount of squeezing depends on satisfying $t_{\text{meas}}^z \approx t_{\text{meas}}^y = \tau$. We want the $t_{\text{squeeze}}$ to be a negligible time-scale in comparison to the other times of the problem as the gravitational entanglement is always happening (we cannot stop it). Thus $n, \omega_1, \omega_2$ must be so chosen that

$$t_{\text{squeeze}} << t_{\text{meas}}^z \approx t_{\text{meas}}^y = \tau$$

Squeezing thus places significant demands on this protocol.

**Casimir screening imposed constraints**

To shield the system from unwanted electromagnetic interaction, one places a conducting plate between the test masses to act as a Faraday cage. For simplicity we assume that the placement of a perfect conductor completely blocks the Casimir interaction between test masses. The Casimir screening scheme introduces additional Casimir force between the plate and test masses. The plate-mass Casimir interaction is now the dominant force at very small scale and places constraints on the system. The screen is closest to one of the components of the superposition ($|R\rangle$ for the mass to the left of the screen and $|L\rangle$ for the mass to the right of the screen). We do not want this component to be pulled so close to the screen that it prevents the overlap between the two components of each mass in order to enable the Pauli-X,Y measurements. Thus we require that the spread of wave packets (which we exploit in our scheme) dominates over the displacement due to Casimir force. This places a new constraint on minimum separation of a mass and the conducting plate. The Casimir force between a plate and sphere is given by

$$F_{ca} = -\frac{3\hbar c}{2\pi}\frac{(\epsilon - 1)}{\epsilon + 2}\frac{R^3}{s^5}$$

where $\epsilon$ is the dielectric constant of test masses, $R$ is the radius of test masses and $s = \frac{D-d}{2}$ is the mass-plate separation.

We require that the spread of wave packets dominates over the displacement due to Casimir force by at least one order of magnitude:

$$0.1\sigma_d(t_1) \geq D_{ca}$$

where $\sigma_d(t_1)$ is the width of the propagating wave packets, $t_1$ is the total time before the wave packets overlap. Thus, in terms of the notation of the previous part of the paper, $t_1 \sim t_{\text{meas}}^z + t_{\text{meas}}^y - t_{\text{squeeze}}$, as that is the time over which the wave packet expands after squeezing (We know that this spread is required to be of the order of the slit separation $d$ for the overlap to happen; moreover, actually $t_1 << t_{\text{meas}}^z, t_{\text{meas}}^y, t_{\text{squeeze}}$).

Classical treatment of Casimir force gives an estimate of its resulting displacement on test masses given by:

$$D_{ca} = -\frac{9\hbar c}{16\pi^2}\frac{(\epsilon - 1)}{(\epsilon + 2)}\frac{1}{\mu s^3}$$

where $\mu$ is the density of test mass.

On the other hand, the spread of Gaussian wave packet $\sigma_d(t)$ scales linearly with time by a factor of $\frac{d}{2\epsilon}$. Since wave overlaps at a time $t \sim 2n\sigma_d/\hbar$, the width of wave packets spread is of same order as the slit separation $\sigma_d(t_1) \sim d$. In the original setup, $d$ and $\frac{D-d}{2}$ are on the same scale. We may then take $\sigma_d(t_1) \sim d \sim s$. The above parameter domain is satisfied for mass density $\mu \sim 3 \times 10^3\text{kg/m}^3, \epsilon = 5.7$, and the minimum plate-mass separation $s \sim 12\mu m$.

**Induced phase**

With the same mechanism as in [9], we can prepare initial state given by eq3. Taking $D \sim 40\mu m, d \sim 10\mu m$, for mass $m \sim 10^{-15}\text{kg}$, gravitational interaction, if quantum, would induce relative phase $\Delta \phi \sim 1 \times 10^{-2}\text{rad}$ after $\tau \sim 3s$ of entangling time. To certify the induced relative phase, we estimate the entanglement witness [39]

$$W = I \otimes I - \sigma_x \otimes \sigma_x - \sigma_z \otimes \sigma_y - \sigma_y \otimes \sigma_z$$

The expectation value $\langle W \rangle = \text{Tr}(W \rho)$ would be negative, if the two masses are entangled. Using Eq.(4) for the state, and phases $\Delta \phi_{LR} = \Delta \phi_{\uparrow \downarrow}, \Delta \phi_{RL} = \Delta \phi_{\downarrow \uparrow}$ from below Eq.(2), the expected witness measure at $\tau \sim 3s$ is $\sim -0.0065$.

**Initial state preparation**

Witnessing the quantum nature of gravity through qubit correlations requires preparation of superposition state of large spatial splitting $d \sim 10 - 100\mu m$ of a micron scale mass, $m \sim 10^{-15} - 10^{-14}\text{kg}$ [9, 34]. This is considerably beyond what has already been realized (e.g., $10^{-22}\text{kg}$ mass over 0.5m [40], or $10^{-22}\text{kg}$ mass over 0.25m [41, 42]). However, there are several proposed schemes to achieve the required superpositions, with criteria in terms of temperature, pressure, acceleration/vibration noise well identified.

The protocol starts from preparing a mass in a pure quantum state in a harmonic trap, typically the ground state. This mass may or may not have a spin embedded in it (the protocol of creating the initial state $\frac{1}{\sqrt{2}}(|L\rangle + |R\rangle)$ will depend on that). We imagine these objects to be levitated in low frequency static magnetic traps [43–46]. Feedback cooling can be achieved by first shining light towards the particle. Scattered photons, which carries information of the particle’s position, are collected by photo-diode detector. For example, from detecting $n$ scattered photons, the position of a particle gets determined to the accuracy of $\lambda/\sqrt{n}$, where $\lambda$ is the wavelength of the scattered light [47]. This information is then used as feedback to cool the motion of the microsphere via an external damping force. Essentially, the information gathering rate from detecting scattered photons must overtake the entropy increase rate of the object from undetected scatterings (photons, atoms) and noises.
from the environment. Using the feedback cooling principle, a mechanical oscillator as massive as 10 kg has been prepared close to its ground state, the center-of-mass motion of which is cooled down to tens of nano-Kelvin\([48]\), which involved measuring position to the uncertainty of \(\sim 10^{-20}\text{m}/\sqrt{\text{Hz}}\). Cryogenic diamagnetic levitated micro-mechanical oscillator has also been realized with very low (\(\mu\) Hz) dissipation rate\([43, 44]\). For test mass \(m \sim 10^{-15}\text{kg}\) in a trap with frequency \(\sim \mathcal{O}(10 - 100)\text{Hz}\), its ground state spread is \(\sim \mathcal{O}(0.1 - 1)\text{nm}\). As this uncertainty being much larger than the precision to which position has been localized recently in feedback cooling\([48]\), it is reasonable to suppose that feedback cooling to nearly the ground state is also imminent for the above systems (for example, will require \(n \sim 10^8\) scattered photons to be detected, with undetected photons being much lower, in a time scale over which no collisions with air molecules take place). Following the above stage either spinless or spinful methods can be used to create the superposition.

If we have a spin embedded in the mass, a popular avenue to create a small superposition (\(\leq 100\text{nm}\)) is to use the Stern-Gerlach effect\([9, 29, 40–51]\). The test mass in an initially pure localized state \(|C\rangle\) (say, the ground state) is prepared with its spin in a state \(\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)\) by microwave pulses, and the trap is suddenly switched off. The spin state undergoes spatial splitting due to an inhomogeneous magnetic field gradient so that the system evolves as

\[
\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)|C\rangle \rightarrow \frac{1}{\sqrt{2}}(|L\uparrow\rangle + |R\downarrow\rangle). \tag{12}
\]

A subsequent measurement of the spin in a different basis, say, the \(|\pm\rangle\) basis, and, for example, getting the \(|+\rangle\) outcome, will prepare the mass in the spatial superposition

\[
\frac{1}{\sqrt{2}}(|L\rangle + |R\rangle). \tag{13}
\]

Care must be taken so that the spin measurement does not reveal the position of the mass itself to a better precision than the \(|L\rangle\) and \(|R\rangle\) difference. However, we now require to amplify this superposition as we require (in the case where there is a screening of electromagnetic interactions) a spatial splitting of \(\sim 10\text{µm}\), while the diamagnetism induced by the magnetic field gradient used for the splitting, restricts the splitting\([52, 53]\). Interestingly, the superposition could be amplified using a current carrying wire providing a diamagnetism induced repulsion between wire and each split component\([54]\). Alternatively, spins may be subject to nonlinear gradients to accumulate a velocity difference before catapulting to a large size\([55]\). With magnetic field gradient \(\sim 10^9\text{Tm}^{-1}\), the desired separation of \(10\text{µm}\) can be obtained after \(\sim 1\text{s}\) of flight time\([55]\). It is easy to verify that during such intervals of time, the wavepacket spread remains in the \(\sim \mathcal{O}(0.1 - 1)\text{nm}\) regime. During these protocols, spin coherence does not need to be retained during the amplifications (diamagnetic repulsion/catapulting) the electronic spin does not play an active role. So, just before the amplification stage, one could map it to much more coherent nuclear spins, or alternately, simply measured in a different basis as noted above, to obtain directly the \(\frac{1}{\sqrt{2}}(|L\rangle + |R\rangle)\) state. Noted above so as to have only a spatial superposition. Moreover, for the application herewith (using spatial qubits) we do not need to complete an interferometer – only create the large splitting superposition, which removes a significant challenge.

It is possible that even without spins the state \(\frac{1}{\sqrt{2}}(|L\rangle + |R\rangle)\) can be produced as far as \(\sim 1\mu\text{m}\) sized distances between \(|L\rangle\) and \(|R\rangle\) are concerned. For example, the first few stages of the on-chip interferometer of Ref.\([36]\) based on coherent inflation and a \(\hat{s}^2\) (position\(^2\)) measurement can be used (till the superposition is generated). One can then, in principle, combine with a diamagnetic repulsion aided further spatial splitting of the \(|L\rangle\) and \(|R\rangle\) terms\([54]\) so as to reach \(\sim 10\mu\text{m}\) size.

**Decoherence** During propagation, background gas collision and black body radiation unavoidably decoheres quantum coherence of the superposition state. Adopting the model in\([57]\), air molecule collision decoheres the \(10^{-15}\text{kg}\) test mass at a rate of \(\sim 0.776\text{s}^{-1}\) under pressure \(\sim 10^{-15}\text{Torr}\). Blackbody radiation arises from emission, absorption and scattering of thermal photons. The emission localization parameter typically dominates because the internal temperature is usually larger than the external temperature. If internal temperature can be cooled down to \(T_i = 4\text{K}\), black body radiation decoheres the test mass at a rate of \(\sim 0.72\text{s}^{-1}\), which corresponds to \(\sim 10\text{s}\) of coherence time. The required cooling to keep coherence of the superposition state is challenging with state of art technology.

**Squeezing challenges** Take initial width \(\sim 0.4\text{mm}\), the wavepacket expands to \(\sim 0.85\text{nm}\) after 3 seconds of flight. We will then have to squeeze the state by about 7 orders of magnitude to \(1.5 \times 10^{-18}\text{m}\), so that it expands to \(\sim 20\mu\text{m}\) in the next 0.03s, where interference occurs.

Optical squeezing is the most common technique for microscopic objects. However, momentum recoil due to photon scattering induce decoherence and heating on the levitated objects, the rate of which scales with the object size\([58–60]\). For large mass \(\sim 10^{-15}\text{kg}\), this approach becomes infeasible, the coherence time is only \(\sim 10^{-6}\text{s}\). Alternatively, one may adopt diamagnetic trapping for squeezing\([43–46]\). Diamagnetic trapping typically operates at much lower frequencies compare to its optical counterpart, therefore, making it advantageous to hold large masses. Potential energy per unit volume of a diamagnetic mass in trapped in magnetic field reads:

\[
U \approx -\frac{\chi_m}{2\mu_0}B^2 + pgr \tag{14}
\]

where \(\chi_m\) is the mass magnetic susceptibility, \(\mu_0\) is the magnetic permeability in vacuum, \(g\) is the gravitational acceleration, \(B\) is induction of the magnetic field when the particle is absent and \(r\) is the vertical displacement.
The mechanical frequency is therefore:

\[ \omega_m = \sqrt{\frac{\chi_m}{\mu_0} \frac{\partial B}{\partial r}} \]  

(15)

where \( \frac{\partial B}{\partial r} \) is the field gradient.

To control the motional superposition state during the squeezing procedure, the thermal decoherence rate \( \gamma_{th} = \bar{n}\gamma \) needs to be suppressed, where \( \gamma \) is the mechanical dissipation rate and \( \bar{n} = \frac{\hbar \omega_m}{k_B T} \) is the average phonon number. To keep coherence during \( n \) periods of quarter oscillations, the dissipation rate must, therefore, satisfy the following relation:

\[ \gamma < \frac{\hbar \omega_m^2}{nk_B T} \]  

(16)

In order to achieve the required squeezing, 7 times of successive changes between two periods of harmonic potentials of 100Hz and 1000Hz would suffice. The environmental temperature can be kept at \( \sim 10mK \) with commercial dilution refrigerator. The dissipation rate then must be kept below micro-hertz \( \gamma < 1 \mu \text{Hz} \).

For solid state systems, the main contribution to dissipation is the direct coupling between the system and the substrate. Ultra-low dissipation of micro hertz has been reported with diamagnetic levitated objects, where permanent magnets are used for trapping. In [44], where the levitated mass is similar to our scheme, damping rate of \( \mu \text{Hz} \) is achieved at pressure \( \sim 3 \times 10^{-7} \text{Torr} \) and room temperature. The major contribution of damping came from background gas collision, which scales linearly with pressure. Dissipation due to gas collision could be significantly reduced by lowering temperature and pressure. Comparing with the condition to keep coherence during the propagation stage, one finds that the required level of vacuum is much less demanding during the squeezing stage.

However, permanent magnet based scheme maybe unfeasible for squeezing since successive changes of trapping potential are required. Alternatively, one may use magnetic field generated by current-carrying coil, which introduces an additional type of dissipation due to field fluctuation. One has to consider specific current sources for that and we do not go to that technicality herewith. Moreover, there could be other sources of random forces at given times \( \delta F(t) \). Thus we herewith estimate the constraint those forces have to satisfy. Although the magnetic field noise will be switched off after attaining the required squeezing, and free spreading of wavepacket under propagation will ensue, we assume some random force noise being always present and constrain it (this will surely be an overestimate as far as forces from magnetic field noise are concerned). Random forces will give a decoherence rate of

\[ \Gamma \sim \frac{S_{FF}(\Omega)d^2}{\hbar^2} \]  

(17)

where \( S_{FF}(\Omega) = \int \delta F(0)\delta F(t)e^{i\Omega t}dt \) is the force noise spectrum at the frequency \( \Omega \sim 1/\tau \) of our experiment, and \( d \) is the spatial splitting of each superposition. Keeping \( \Gamma < 1 \text{Hz} \) gives us the constraint that random force noise should be kept below \( \sqrt{S_{FF}} \sim 10^{-29} \text{N}/\sqrt{\text{Hz}} \). Note that although this may sound challenging, faster frequency noise does not affect the experiment particularly, while noise at this Hz frequency should be determinable by precision measurements over a long duration of 1 s to be taken into account in the experiment. Note that this requirement is not unique to the spatial qubit method. Rather in the spatial qubit method we are using free propagation for a large fraction of the time when, at least randomness in superposition creating/delocalizing forces will be inactive.

**IV. CONCLUSIONS:**

We have analyzed the feasibility of applying the massive spatial qubit methodology to witness the quantum nature of gravity. This will enable a spinless witnessing of the gravitational entanglement growth between two masses \( \sim 10^{-15} \text{kg} \) each, essentially through position measurements on masses, but still using qubit-qubit correlations to measure the entanglement. The core property used here is the spreading of a free quantum wavepacket which brings two initially localized states \( |L\rangle \) and \( |R\rangle \) to interfere so that observables such as \( \sigma_z = |L\rangle\langle R|+|R\rangle\langle L| \) can be measured from the interference pattern. It is expedient work within the remit of a Faraday shielding scheme so as to block unwanted electromagnetic interactions between two masses, so that the masses can be brought closer and still interact only gravitationally. But here the necessity for the wavepacket spreading to dominate over the Casimir force of the Faraday screen to its nearest component arises. This, in turn, implies a minimum distance to the screen, which, in turn, dictates a minimum separation \( d \sim 10 \mu\text{m} \) between \( |L\rangle \) and \( |R\rangle \) to have a significant entanglement growth rate. This minimum \( d \) then also necessitates a wavefunction spreading from a very localized width \( \sigma_d \) of each of \( |L\rangle \) and \( |R\rangle \) rapidly to \( \sim d \) in an extremely short time-scale, so that the spatial qubit \( \sigma_z \) and \( \sigma_x \) measurements are accomplished at nearly the same time (on the same entangled state of the two masses). This is turn, necessitates squeezing the width immediately after significant gravitationally generated entanglement is attained between the masses (at a time \( \tau \)). The requirements and a method for achieving this squeezing is described along with the challenges (it requires position squeezing of the wavepacket by 7 orders of magnitude). As long as the above can be met, and spatial superpositions can be generated (for which one may use spins or other spinless methods), the evidencing through a qubit-qubit entanglement witness can take place purely through position measurements.
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