\( \epsilon' / \epsilon \) And Anomalous Gauge Boson Couplings

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Abstract

We study \( \epsilon' / \epsilon \) in the Standard Model and \( \epsilon' / \epsilon \) due to anomalous \( WW\gamma \) and \( WWZ \) interactions as a function of the top quark mass. In the Standard Model, \( \epsilon' / \epsilon \) is in the range \( 10^{-3} \sim 10^{-4} \) for the central value of top quark mass reported by CDF. The anomalous gauge couplings can have large contributions to the \( CP \) violating \( I = 2 \) amplitude in \( K \to \pi\pi \). Within the allowed regions for the anomalous gauge couplings, \( \epsilon' / \epsilon \) can be dramatically different from the standard model prediction.
I. INTRODUCTION

The $SU(2)_L \times U(1)_Y$ Standard Model (SM) of electroweak interactions is in very good agreement with present experimental data. The experimental data from LEP and SLC and the theoretical predictions in the SM for the gauge-fermion couplings agree at the 1% level or better \[1\]. However, one of the most direct consequences of the SM, the self-interaction of the gauge particles, the W, Z and photon, characteristic of nonabelian gauge theories, has not been directly tested. It is important to study these self-interactions to establish whether the weak bosons are gauge particles with interactions predicted by the SM, or gauge particles of some extensions of the SM which predict different interactions at loop levels, or even non-gauge particles whose self-interactions at low energies are described by effective interactions.

Large uncertainties are introduced into studies of physics beyond the SM due to our lack of knowledge of the top quark mass $m_t$. D0 has put the lower bound on $m_t$ to be 131 GeV \[2\]. CDF has announced evidence for the existence of top quark with a mass of $174 \pm 10^{+13}_{-12}$ GeV \[3\]. If confirmed, this information will allow us to make better predictions of new physics beyond the SM. In this paper we show how the information from CDF about the top quark mass helps the study of the effect of anomalous gauge couplings on the CP violating parameter $\epsilon'/\epsilon$ in comparison with the SM prediction.

In general there will be more gauge boson self-interaction terms than the tree level SM predicts. The most general $WWV$ interactions with the W boson on shell, invariant under $U(1)_{em}$, can be parametrized as \[4\]

$$L_V = -ig_V [\kappa V W^\mu W^- V^{\mu\nu} + \frac{\lambda V}{M_W^2} W^+_{\sigma\rho} W^-_{\rho\delta} V^\sigma_{\delta}]$$

$$+ \tilde{\kappa} V W^+_{\mu} W^-_{\nu} V^{\mu\nu} + \frac{\tilde{\lambda} V}{M_W^2} W^+_{\sigma\rho} W^-_{\rho\delta} \tilde{V}^\sigma_{\delta}$$

$$+ g_4 V (W^+_{\mu\nu} W^-_{\mu} - W^+_{\mu} W^-_{\nu}) V_{\nu} + g_4 W^+_{\mu} W^-_{\nu} (\partial^{\mu} V^{\nu} + \partial^{\nu} V^{\mu})$$

$$+ g_5 V \epsilon_{\mu\nu\alpha\beta} (W^+_{\mu\nu} \partial^\alpha W^-_{\nu} - \partial^\alpha W^+_{\mu\nu} W^-_{\nu}) V^{\beta}], \quad (1)$$

where $W^{\pm \mu}$ are the W boson fields; V can be the $\gamma$ or $Z$ fields; $W_{\mu\nu}$ and $V_{\mu\nu}$ are the $W$ and
$V$ field strengths, respectively; and $\tilde{V}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} V^{\alpha\beta}$. The terms proportional to $\kappa$, $\lambda$, and $g_{1,5}^Z$ are CP conserving and $\tilde{\kappa}$, $\tilde{\lambda}$ and $g_{4,5}^Z$ are CP violating. For $V = \gamma$, $g_V = e$ and for $V = Z$, $g_V = g \cos \theta_W$. $g_1^V$ defines the W boson charge, one can always set it to 1. In the SM at the tree level, $\kappa^V = 1$, $g_1^Z = 1$, and all other couplings in eq.(1) are zero. $\Delta \kappa^V = \kappa^V - 1$, $\Delta g_1^Z = g_1^Z - 1$, $\tilde{\kappa}^V$, $\tilde{\lambda}^V$, $g_4^V$ and $g_5^V$ are called the anomalous gauge boson couplings.

There have been many experimental and theoretical studies of the anomalous gauge boson couplings. Collider experiments at high energies have put constraints on some of these couplings \cite{5,6}. It has been shown that rare decays can provide important constraints \cite{7–10}. In Refs. \cite{9,10} using the recent data from CLEO on $b \rightarrow s\gamma$ \cite{12} and data on $K_L \rightarrow \mu^+\mu^-$ \cite{13}, constraints comparable or better than those obtained in collider physics were obtained. Rare $B$ decays may provide more stringent constraints \cite{11}. The constraints from rare decays are better than those obtained from the $g - 2$ of the muon \cite{14}. In the literature the most stringent constraints on the anomalous gauge boson couplings are from oblique corrections to the precision electroweak experiments \cite{15}. The anomalous gauge coupling contributions to the oblique corrections are some times quadratically or even quartically divergent. Care must be taken when evaluating these contributions. Strictly, one should return to the underlying theories to remove the quartic and quadratic divergences \cite{15}. For purely phenomenological studies, we think the constraints from direct $W$ pair productions \cite{8,9}, and rare decays \cite{9,10} (the divergences here are at most logarithmic) are more reliable. For the CP violating anomalous coupling, the best constraints are from neutron and electron electric dipole moments \cite{16}.

In obtaining the bounds on the anomalous gauge boson couplings, most of the analyses assumed only one coupling is different from the SM tree level predictions. A real underlying theory would produce more than just a single anomalous coupling. If the analyses were carried out including all anomalous couplings simultaneously, the bounds would be much weaker. It is nevertheless interesting to find out if, when these stringent bounds are applied, there are still large effects on other processes. In this paper we will show that there can be
still large effects on $\epsilon'/\epsilon$ from the anomalous $WW\gamma$ and $WWZ$ couplings.

The parameter $\epsilon'/\epsilon$ is a very important quantity to study. It measures direct CP violation in $K \rightarrow \pi\pi$. Experimental measurements are not conclusive at this stage [17],

$$Re(\epsilon'/\epsilon) = \begin{cases} (23 \pm 6.5) \times 10^{-4}, & \text{NA31} \\ (7.4 \pm 6.0) \times 10^{-4}, & \text{E731} \end{cases}$$

While the result of NA31 clearly indicates a non-zero $\epsilon'/\epsilon$, the value of E731 is compatible with zero. However, the two results are consistent at the 2 standard deviation level. The SM prediction for $\epsilon'/\epsilon$ depends on the value of the top quark mass. It has been shown that for a small top quark mass, the most important contributions to $\epsilon'/\epsilon$ are from the strong penguin and isospin breaking due to quark masses. For a large top quark mass, the electroweak penguins also become important [18,19]. In fact the sign of $\epsilon'/\epsilon$ may change for $m_t$ larger than 220 GeV. If the top quark mass is indeed about 174 GeV as reported by CDF, the electroweak penguin contribution will not cancel the other contributions completely. The predicted value for $\epsilon'/\epsilon$ is about $10^{-3} \sim 10^{-4}$ which will be within the reach of future experiment. We will then be able to find out if there are other contributions to $\epsilon'/\epsilon$. This illustrates the importance of knowing the $m_t$ in determining the physics beyond the SM.

The anomalous gauge interactions are purely electroweak, so their contributions to $\epsilon'/\epsilon$ will not affect the strong penguin but may have significant effects on the electroweak penguins. We will show that the anomalous gauge couplings can change the result dramatically.

**II. NEUTRAL FLAVOR CHANGING EFFECTIVE HAMILTONIAN**

The effective Hamiltonian $H_{eff}$ for flavour changing neutral currents with $\Delta F = 1$, at the one loop level, is given by

$$H_{eff} = H_{SM} + H_{AGC},$$

where $H_{SM}$ is the SM contribution which can be find in Ref. [20], and $H_{AGC}$ contains the contributions from anomalous gauge couplings. It is give by
\[ H_{AGC} = \frac{G_F}{2\sqrt{2}\pi} \sum_i V_{iq}V_{iq'}^* \frac{e}{8\pi} G(x_i) A q' (m_{q'}(1-\gamma_5) + m_q(1+\gamma_5)) \sigma_{\mu\nu} q F^{\mu\nu} \]
\[ + \alpha_{em} Q_f H(x_i) A q' \gamma_\mu (1-\gamma_5) q \bar{f} \gamma_\mu f \]
\[ + \alpha_{em} \cot^2 \theta_W F(x_i) A q' \gamma_\mu (1-\gamma_5) q \bar{f} \gamma_\mu (Q_f \sin^2 \theta_W - T^3 \frac{1-\gamma_5}{2}) f , \quad (4) \]

with
\[ G(x) = - (\Delta \kappa + i \tilde{\kappa}) \frac{x}{(1-x)^2} + \frac{x^2(3-x)}{2(1-x)^3} \ln x \]
\[ - (\lambda + i \tilde{\lambda}) \frac{x(1+x)}{2(1-x)^2} + \frac{x^2}{(1-x)^3} \ln x \]
\[ H(x) = \Delta \kappa \frac{x}{4} \ln \frac{\Lambda^2}{m_W^2} + \lambda \frac{x(1-3x)}{2(1-x)^2} - \frac{x^3}{(1-x)^3} \ln x , \quad (5) \]
\[ F(x) = - \Delta g_\xi \frac{3}{2} x \ln \frac{\Lambda^2}{m_W^2} + g_\xi \frac{3x}{1-x} + \frac{3x^2}{(1-x)^2} \ln x . \]

For terms which are divergent in the loop integral, we have just kept the leading terms. We used unitary gauge in our calculations. Our first term in \( H_A \) does not agree with Ref. [7] where the author obtained a cut-off independent result. The term in \( H_A \) proportional to \( \Delta \kappa \) is similar to the term in the SM with \( \kappa = 1 \). In \( R_\xi \) gauge, this term is gauge dependent [20]. In the unitary gauge this term diverges. This gauge dependent term is cancelled by terms from "box" and Z exchanges in physical processes. In our case because the coupling \( \Delta \kappa \) is anomalous, there are no terms coming from "box" and Z exchange to cancel it.

The Hamiltonian in eq. (3) is the lowest nonvanishing order contribution to flavor changing neutral current. It has been show that QCD corrections are important in the SM [18,19,21]. QCD corrections should be included in phenomenological analyses. To this end, we carry out the leading log QCD correction to the weak effective Hamiltonian. The effective Hamiltonian at the energy scale \( \mu \) relevant to us can be written as
\[ H_{eff}^{S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i(\mu) Q_i(\mu) , \quad (6) \]

where \( i = 1, \ldots, 10 \) and
\[ C_i(\mu) = z_i(\mu) + \tau \tilde{y}_i(\mu) , \tau = - V_{td} V_{ts}^* / V_{ud} V_{us}^* . \quad (7) \]
The coefficients $C_i$ satisfy the renormalization group equation to the first order in $\alpha_s$ and $\alpha_{em}$:

$$
(\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g}) C(\mu) = \frac{1}{2\pi} (\alpha_s(\mu) \gamma^{(s)} T + \alpha_{em}(\mu) \gamma^T) C(\mu),
$$

(8)

where $\gamma^{(s)}$ and $\gamma$ are the anomalous dimension matrices which were obtained in Ref. [22].

The Wilson coefficients at the scale $\mu$ is obtained by first calculating the coefficients at the scale $m_W$ and then using the renormalization group to evolve down to the scale $\mu$. In our calculation we will use experimental values for the CP conserving amplitudes. We only need to calculate the Wilson coefficients $\tilde{y}_i$ which are enter the calculation of CP violation. The four quark operators are defined as

$$
Q_1 = \bar{s} \gamma_\mu (1 - \gamma_5) d \bar{u} \gamma^\mu (1 - \gamma_5) u, \quad Q_2 = \bar{s} \gamma_\mu (1 - \gamma_5) u \bar{u} \gamma^\mu (1 - \gamma_5) d,
$$

$$
Q_3 = \bar{s} \gamma_\mu (1 - \gamma_5) d \sum_q \bar{q} \gamma^\mu (1 - \gamma_5) q, \quad Q_4 = \sum_q \bar{s} \gamma_\mu (1 - \gamma_5) q \bar{q} \gamma^\mu (1 - \gamma_5) d,
$$

$$
Q_5 = \bar{s} \gamma_\mu (1 - \gamma_5) d \sum_q \bar{q} \gamma^\mu (1 + \gamma_5) q, \quad Q_6 = -2 \bar{s} (1 + \gamma_5) q \bar{q} (1 - \gamma_5) d,
$$

$$
Q_7 = \frac{3}{2} \bar{s} \gamma_\mu (1 - \gamma_5) d \sum_q Q_q \bar{q} \gamma^\mu (1 + \gamma_5) q, \quad Q_8 = -3 \sum_q \bar{s} (1 + \gamma_5) q \bar{q} (1 - \gamma_5) d,
$$

$$
Q_9 = \frac{3}{2} \bar{s} \gamma_\mu (1 - \gamma_5) d \sum_q Q_q \bar{q} \gamma^\mu (1 - \gamma_5) q, \quad Q_{10} = \frac{3}{2} \sum_q Q_q \bar{s} \gamma_\mu (1 - \gamma_5) q \bar{q} \gamma^\mu (1 - \gamma_5) d.
$$

(9)

Among these operators there are only seven linearly independent ones. We use $Q_{1,2,3,5,6,7,8}$ as the independent operators. The corresponding coefficients $y_{1,2,3,5,6,7,8}$ are given by

$$
y_1 = \tilde{y}_1 - \tilde{y}_4 + \frac{3}{2} \tilde{y}_9 + \frac{1}{2} \tilde{y}_{10}, \quad y_2 = \tilde{y}_2 + \tilde{y}_4 + \tilde{y}_{10},
$$

$$
y_3 = \tilde{y}_3 + \tilde{y}_4 - \frac{1}{2} \tilde{y}_9 - \frac{1}{2} \tilde{y}_{10}, \quad y_i = \tilde{y}_i, \quad i = 5, 6, 7, 8.
$$

(10)

The boundary conditions at $m_W$ for the Wilson coefficients in the SM can be found in Ref. [18–20] which depend on the top quark mass. We will not display them here. When the anomalous gauge coupling contributions are included, due to the new contributions, the boundary conditions at $m_W$ for the Wilson coefficients are different from the SM. The new contributions will change $\tilde{y}_{3,7,9}$. From eq.(4) we obtain the anomalous gauge boson coupling contributions to the Wilson coefficients at the $m_W$ scale,
\[ y_3(m_W) = -\frac{\alpha_{em}}{24\pi} F_A(x_t), \]
\[ y_7(m_W) = -\frac{\alpha_{em}}{6\pi} (H_A(x_t) + \sin^2 \theta_W F_A(x_t)), \]
\[ y_8(m_W) = -\frac{\alpha_{em}}{6\pi} (H_A(x_t) - \cos^2 \theta_W F_A(x_t)). \]

The other coefficients are not changed from those of the SM. Note that the new contributions to the effective Hamiltonian depend only on \( \Delta \kappa, \lambda, \Delta g^Z_1 \) and \( g^Z_5 \). Contributions from the other anomalous couplings are suppressed by factors like \( O(m^2_{d,s}, m^2_K)/m^2_W \). We given in Table 1 and 2 the values for \( y_i \) as a function of \( m_t \) and the anomalous couplings.

### III. CONTRIBUTIONS TO \( \epsilon'/\epsilon \)

The parameter \( \epsilon'/\epsilon \) is a measure of CP violation in \( K_{L,S} \rightarrow 2\pi \) decays. It is defined as

\[
\frac{\epsilon'}{\epsilon} = i \frac{e^{i(\delta_2 - \delta_0)}}{\sqrt{2}(i\xi_0 + \bar{\epsilon})} \omega (\xi_2 - \xi_0),
\]

where \( \bar{\epsilon} \approx 2.26 \times 10^{-3} e^{i\pi/4} \) is the CP violating parameter in \( K^0 - \bar{K}^0 \), \( \delta_i \) are the strong rescattering phases, \( \omega = |ReA_2/ReA_0| \approx 1/22 \), and \( \xi_i = ImA_i/ReA_i \). Here \( A_0 \) and \( A_2 \) are the decay amplitudes with \( I = 0 \) and 2 in the final states, respectively.

To separate different contributions to \( \epsilon'/\epsilon \), we parametrize \( \epsilon'/\epsilon \) as

\[
\frac{\epsilon'}{\epsilon} = \left( \frac{\epsilon'}{\epsilon} \right)_6 (1 - \bar{\Omega}),
\]

where \( (\epsilon'/\epsilon)_6 \) is the contribution from \( y_6 \) which is given by

\[
\left( \frac{\epsilon'}{\epsilon} \right)_6 = \frac{\omega G_F}{2\epsilon |A_0|} y_6 < Q_6 > \text{Im}(V_{td}V_{ts}^*) .
\]

Here \( < Q_i >_I \) is defined as \( < Q_i >_I = < (\pi\pi)_I |Q_i|K > \). The parameter \( \bar{\Omega} \) contains several different contributions

\[
\bar{\Omega} = \Omega_{\eta+\eta'} + \Omega_{EW P} + \Omega_{octet} + \Omega_{27} + \Omega_{P}
\]

where \( \Omega_{\eta+\eta'} \) is the contribution due to isospin breaking in the quark masses which is estimated to be in the range \( 0.2 \sim 0.4 \) [22,23]. We will use \( \Omega_{\eta+\eta'} = 0.25 \) for illustration. The other contributions are defined as follows
\[ \Omega_{EWP} = \frac{1 - \sqrt{2} \omega y_7 < Q_7 >_2 + y_8 < Q_8 >_2}{\omega} , \]

\[ \Omega_{octet} = -\frac{y_1 < Q_1 >_0 + y_2 < Q_2 >_0}{y_6 < Q_6 >_0} , \]

\[ \Omega_{27} = \frac{1}{\omega} \frac{(y_1 + y_2) < Q_2 >_2}{y_6 < Q_6 >_0} , \]

\[ \Omega_{P} = -\frac{y_3 < Q_3 >_0 + y_5 < Q_5 >_0}{y_6 < Q_6 >_0} . \]  

(16)

The calculation of the hadronic matrix elements is the most difficult task [18,19,22,24,25]. There is no satisfactory procedure for this calculation at present. We will use the values in Ref. [19] in our tables and figures for illustration, and put our emphasis on the effects of the anomalous couplings. In Figure 1, we show the dependence of \( 1 - \bar{\Omega} \) as a function of \( m_t \) and the anomalous gauge boson couplings.

IV. DISCUSSION

We show in Table 1 the SM predictions for the Wilson coefficients as a function of top quark mass \( m_t \). In Table 2, we show the effects of anomalous couplings on \( y_{7,8} \) as functions of \( m_t \) and the anomalous couplings. It is clear that the anomalous couplings can dramatically change the SM predictions.

In our numerical analyses of the effects of anomalous coupling on the Wilson coefficients, we will assume that only one anomalous coupling is non vanishing. As have been mentioned before that this may not be true. We nevertheless carry out the analysis this way to illustrate the effects of anomalous couplings on \( \epsilon'/\epsilon \). We use some values of the anomalous couplings which are consistent with constraints from rare decays because they are all derived from the effective Hamiltonian in eq.(3). The constraints from rare decays are top quark mass \( m_t \) dependent. Using the recent CLEO bound on \( b \rightarrow s \gamma \) at the 95\% CL [12], the anomalous couplings \( \Delta \kappa^\gamma, \lambda^\gamma \) are constrained to be in the range \(-2.2 \sim 0.35 \) and \(-6.7 \sim 1.1 \) respectively for \( m_t = 174 \) GeV. For larger \( m_t \), the constraints are more stringent [3]. These constraints are cut-off scale \( \Lambda \) independent. However the constraints from \( K_L \rightarrow \mu^+\mu^- \) are cut-off dependent. For \( m_t = 174 \) GeV the experimental data on \( K_L \rightarrow \mu^+\mu^- \) constrain \( \Delta g^Z_1 \) to be...
in the range $-0.5 \sim 0.1$ for cut-off scale $\Lambda = 1$ TeV. For larger $\Lambda$ the constraint is more stringent [11]. $g_5^Z$ is constrained to be in the range $4 \sim -1$. the constraint on $g_5^Z$ is cut-off independent. The specific values for the anomalous couplings are given in Table 2. We used values for the anomalous couplings which are also consistent with the constraints from collider physics [5,6].

The anomalous couplings affect all the Wilson coefficients through renormalization. However, the effects on $y_{1,2,3,5,6}$ are less than $5\%$ and can be neglected. The effects on $y_{7,8}$ are large. In Table 2, we show the effects of anomalous couplings on $y_{7,8}$ as functions of $m_t$ and the anomalous couplings.

In Figure 1, we show the dependence of $1 - \bar{\Omega}$ as a function of $m_t$ and the anomalous gauge boson couplings. The anomalous gauge boson couplings have a large effect on $\Omega_{EW}$. The effect on other contributions to $\Omega$ can be neglected.

Using the value $<Q_6> = -0.255$ GeV$^3$ for $m_s = 0.175$ GeV, we have

$$\left(\frac{\epsilon'}{\epsilon}\right)_6 \approx 8 \text{Im}(V_{td}V_{ts}^*) , \quad (17)$$

Here we have used $y_6 \approx -0.09$. Using information from CP violation in $K - \bar{K}$ mixing and data from $B - \bar{B}$ mixings [13], the allowed range for $\text{Im}(V_{td}V_{ts}^*)$ is constrained to be in the region $3 \times 10^{-4} \sim 0.5 \times 10^{-4}$ for $m_t$ varying from 100 GeV to 250 GeV. We see that $\epsilon'/\epsilon$ in the SM is between $10^{-3}$ to $-3 \times 10^{-4}$. There is a strong dependence on the top quark mass $m_t$. For the hadronic matrix elements used here, $\epsilon'/\epsilon$ changes sign at about 230 GeV in the SM as mentioned before. If the top quark mass is determined, the uncertainties for $\epsilon'/\epsilon$ will be greatly reduced. The physical top quark mass observed by experiments are different from the running mass which we use in our calculation. A physical mass of 174 GeV corresponding to a running mass about 165 GeV. For $m_t = 165$ GeV, we find that $1 - \bar{\Omega} = 0.3$ and $\text{Im}(V_{td}V_{ts}^*)$ is in the range $2 \times 10^{-4}$ to $0.5 \times 10^{-4}$. Therefore $\epsilon'/\epsilon$ is in the range $5 \times 10^{-4} \sim 10^{-4}$ which will soon be accessible to experiments at CERN and Fermilab.

There are, of course, uncertainties due to our poor understanding of the hadronic matrix elements, error in the QCD scale $\Lambda_4$ for four flavor effective quarks. In Ref. [25], using a
different set of hadronic matrix elements, it is found that the value for $1 - \bar{\Omega}$ can vary a factor of two. It has recently been shown that the next-to-leading order QCD corrections can reduce the $\epsilon'/\epsilon$ about 10% to 20%. The uncertainty in $\Lambda_4$ is $\pm 30\%$. In the above analysis, we have neglected contributions from gluon dipole penguin operator of the form $\bar{q}\sigma_{\mu\nu}\lambda^{\alpha}(1-\gamma_5)qG^\mu\nu_{\alpha}$. It has been shown that to the leading order in chiral perturbation theory, this contribution vanishes \cite{27}. Higher order chiral perturbation calculations indicate that this contribution may enhance the value for $\epsilon'/\epsilon$ by about 10% for $m_t = 165$ GeV \cite{27,28}. When taking into account all the effects mentioned, we conclude that for $m_t = 165$ GeV, $\epsilon'/\epsilon$ is in the range $10^{-3} \sim 10^{-4}$.

From Figure 1 it can be easily seen that the anomalous gauge couplings can change the result dramatically. $\epsilon'/\epsilon$ can be much larger than the SM prediction and the value of $m_t$ where the sign change of $\epsilon'/\epsilon$ occurs can be significantly shifted. The change of sign for $\epsilon'/\epsilon$ can occur for $m_t$ as small as 120 GeV for allowed values for the anomalous gauge couplings. Future measurements on $\epsilon'/\epsilon$ will certainly provide useful information about the anomalous gauge couplings. In Figure 1, we also show $1 - \bar{\Omega}$ with the anomalous couplings set to be $\pm 0.1$ for $\Delta \kappa^\gamma$ and $\Delta g_1^Z$. We see that even with such small anomalous couplings, the effects on $\epsilon'/\epsilon$ are still sizeable. If we use the same bounds for $\lambda^\gamma$ and $g_2^Z$, the contributions are small (less than 5%).

We conclude that in the SM, $\epsilon'/\epsilon$ is predicted to be in the range $10^{-3}$ to $10^{-4}$ for $m_t = 165$ GeV. The predicted values are within the reach of future experiments. There can be large effects from the anomalous gauge boson interactions on $\epsilon'/\epsilon$, and hence measurement of $\epsilon'/\epsilon$ can provide useful information about the anomalous gauge boson couplings.

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TABLES

TABLE I. \( y_i \) as a function of \( m_t \) in the SM at \( \mu = 1 \text{ GeV} \) for \( \Lambda_4 = 0.25 \text{ GeV} \), \( m_b = 5 \text{ GeV} \), \( m_c = 1.35 \text{ GeV} \).

| \( m_t (\text{GeV}) \) | 140 | 165 | 180 | 200 | 240 |
|------------------------|-----|-----|-----|-----|-----|
| \( y_1 \)              | 0.041 | 0.039 | 0.038 | 0.037 | 0.033 |
| \( y_2 \)              | -0.049 | -0.049 | -0.048 | -0.048 | -0.047 |
| \( y_3 \)              | -0.020 | -0.019 | -0.019 | -0.018 | -0.017 |
| \( y_5 \)              | 0.012 | 0.012 | 0.012 | 0.013 | 0.013 |
| \( y_6 \)              | -0.091 | -0.092 | -0.093 | -0.093 | -0.094 |
| \( y_7/\alpha_{em} \)  | -0.003 | 0.029 | 0.051 | 0.083 | 0.155 |
| \( y_8/\alpha_{em} \)  | 0.081 | 0.121 | 0.149 | 0.188 | 0.278 |
TABLE II. $y_i$ as a function of $m_t$ and the anomalous gauge couplings at $\mu = 1$ GeV for
$
\Lambda_4 = 0.25$ GeV, $m_b = 5$ GeV and $m_c = 1.35$ GeV, and the cut-off $\Lambda = 1$ TeV.

|                   | $m_t$(GeV) | 140 | 165 | 180 | 200 | 240 |
|-------------------|------------|-----|-----|-----|-----|-----|
| $\Delta\kappa = 0.2$ | $y_7/\alpha_{em}$ | -0.036 | -0.016 | -0.003 | 0.0165 | 0.059 |
|                   | $y_8/\alpha_{em}$ | 0.039 | 0.064 | 0.080 | 0.104 | 0.157 |
| $\Delta\kappa = -0.5$ | $y_7/\alpha_{em}$ | 0.078 | 0.142 | 0.185 | 0.248 | 0.393 |
|                   | $y_8/\alpha_{em}$ | 0.184 | 0.264 | 0.319 | 0.398 | 0.580 |
| $\lambda = 1$     | $y_7/\alpha_{em}$ | -0.034 | -0.008 | 0.010 | 0.037 | 0.099 |
|                   | $y_8/\alpha_{em}$ | 0.042 | 0.074 | 0.096 | 0.129 | 0.207 |
| $\lambda = -3$    | $y_7/\alpha_{em}$ | 0.088 | 0.141 | 0.174 | 0.221 | 0.321 |
|                   | $y_8/\alpha_{em}$ | 0.197 | 0.263 | 0.305 | 0.363 | 0.489 |
| $\Delta g_1^Z = 0.05$ | $y_7/\alpha_{em}$ | 0.008 | 0.045 | 0.070 | 0.106 | 0.189 |
|                   | $y_8/\alpha_{em}$ | 0.095 | 0.142 | 0.173 | 0.218 | 0.321 |
| $\Delta g_1^Z = -0.5$ | $y_7/\alpha_{em}$ | -0.121 | -0.134 | -0.143 | -0.157 | -0.190 |
|                   | $y_8/\alpha_{em}$ | -0.066 | -0.082 | -0.094 | -0.111 | -0.153 |
| $g_5^Z = 3$       | $y_7/\alpha_{em}$ | -0.094 | -0.079 | -0.067 | -0.048 | -0.001 |
|                   | $y_8/\alpha_{em}$ | -0.033 | -0.014 | 0.001 | 0.025 | 0.086 |
| $\Delta g_5^Z = -0.5$ | $y_7/\alpha_{em}$ | 0.012 | 0.047 | 0.071 | 0.104 | 0.180 |
|                   | $y_8/\alpha_{em}$ | 0.099 | 0.144 | 0.173 | 0.215 | 0.310 |
Figure Captions

Figure 1. $1 - \bar{\Omega}$ as a function of $m_t$ and anomalous gauge boson couplings. Different values for $\Delta \kappa^\gamma$, $\lambda^\gamma$, $\Delta g_1^Z$ and $g_5^Z$ are used in Figures a,b,c, and d, respectively. In each of the figures all other anomalous couplings are set to be zero.
This figure "fig1-1.png" is available in "png" format from:

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