Non-linear and quantum optics of a type II OPO containing a birefringent element Part 1: Classical operation

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April 1, 2022

Abstract. We describe theoretically the main characteristics of the steady state regime of a type II Optical Parametric Oscillator (OPO) containing a birefringent plate. In such a device the signal and idler waves are at the same time linearly coupled by the plate and nonlinearly coupled by the $\chi^{(2)}$ crystal. This mixed coupling allows, in some well-defined range of the control parameters, a frequency degenerate operation as well as phase locking between the signal and idler modes. We describe here a complete model taking into account all possible effects in the system, i.e. arbitrary rotation of the waveplate, non perfect phase matching, ring and linear cavities. This model is able to explain the detailed features of the experiments performed with this system.

PACS. 42.65.-k Nonlinear optics – 42.65.Yj Optical parametric oscillators and amplifiers – 42.60.Fc Modulation, tuning, and mode locking – 42.25.Lc Birefringence

1 Introduction

In a type II OPO, signal and idler fields of crossed polarizations are generated when the pump exceeds a certain threshold. Energy conservation requires that $\omega_0 = \omega_1 + \omega_2$, where $\omega_0$, $\omega_1$ and $\omega_2$ are respectively the pump, signal and idler frequencies. The precise values of the signal and idler frequencies are set by the conditions of equal cavity detunings and minimum oscillation threshold, which depend on the value of the phase matching between the three waves, and on the vicinity of cavity resonances for the signal and idler modes. These frequencies are determined unambiguously when one knows the values of two control parameters of the OPO, namely the crystal temperature (which sets the value of the different indices) and the cavity length (which determines the cavity resonance conditions). Frequency degeneracy, i.e. $\omega_1 = \omega_2 = \omega_0/2$, occurs only accidentally since it corresponds to a single point in the parameter space. It cannot be achieved for a long time in real experimental conditions, as these parameters drift in time. Furthermore, even when the device is actively stabilized on the frequency degeneracy working point, the signal and idler fields still undergo a phase diffusion phenomenon, similar to the Schawlow-Townes effect in a laser \cite{112}, but acting on the difference between the phases of the signal and idler modes in the case of the type II OPO. As a result, the output field polarization direction slowly drifts with time.

In the context of quantum information and generation of EPR correlated bright beams, where both amplitude and phase correlations are involved, phase locking is inter-
esting since it allows a much simpler measurement of am-
pplitude and phase quantum correlations between the sig-
nal and idler beams \[3\] even above threshold: the measure-
ment of intensity quantum correlations between the sig-
nal and idler modes can be done even with non-frequency
degenerate beams \[4\] but the measurement of phase cor-
relations makes it necessary to use a local oscillator. Thus
a phase reference is defined and signal and idler must be
stable compared to this reference, which is not the case in
a regular, above threshold, type II OPO.

A few years ago, Wong et al. had the idea of achiev-
ing the frequency degenerate operation at the output of a
type-II OPO by introducing a linear coupling between the sig-
nal and idler fields. This coupling was made by way of
a birefringent quarter-wave plate placed inside the OPO
linear cavity which couples the two orthogonally polarized
signal and idler waves. In this way, they generated intense
and stable frequency degenerate signal and idler beams \[5\].
The theoretical model described in reference \[6\] was able
to account for the main features of this phenomenon, but,
for the sake of simplicity, it was made for a ring cavity,
for a small angle between the crystal neutral axes and the
birefringent plate neutral axes, and without any phase-
shifts introduced by the reflection on the cavity mirrors
or by non perfect phase matching.

Most experiments use linear cavities, whereas most
theoretical treatments assume ring cavities. For scalar fields
there is almost no difference between the two configura-
tions (the crystal in the ring cavity being taken twice as
long as the linear cavity), if one neglects the cavity mirror
differential phase-shifts. This is no longer the case when
polarization effects are taken into account: in this case,
a matrix formalism is needed, and the exact succession of
the different elements in the cavity is now important, as
they are described by non-commuting matrices. It is also
interesting to examine the regime when the birefringent
plate angle is not limited to small values. It seems also
important to take into account the mirror phase shifts,
which are known to induce a significant change in the
phase matching between the three waves and consequently
in the oscillation threshold of the linear cavity OPO \[7\].
The purpose of the present paper is to introduce all these
refinements in the theoretical model introduced in \[6\], and
also to discuss the properties of the phase-locked OPO
in terms of the actual control parameters of the device,
which are the cavity length and the crystal temperature.
This paper is followed by a second one \[8\] in which the
quantum fluctuations and correlations between the signal
and idler fields are determined and studied in the same
configuration.

In section \(2\), we introduce and describe the behavior
of the different elements placed inside the OPO cavity. We
then determine and discuss in section \(3\) the steady-state
regime in the ring cavity case. Finally, in section \(4\), we
examine and discuss the steady state regime in the linear
cavity case.

2 Linear and nonlinear elements in the OPO
cavity

We consider here a \(\chi^{(2)}\) crystal with a type II phase match-
ing, which means that the signal and idler fields have
orthogonal polarizations. The crystal length is \(l\) and its
indices of refraction are \(n_1\) and \(n_2\) respectively for the sig-
nal (ordinary) and idler (extraordinary) waves which are
supposed to be frequency degenerate. The non degenerate
case will be studied elsewhere \[8\].

Assuming a small variation of the various field ampli-
itudes inside the nonlinear medium, which is quite reason-
able in a c.w. OPO, one can solve in an approximate way
the propagation equations inside the crystal, and obtain
to the second order in the non-linearity, \(g\) :

\[
A_0(l) = A_0(0) - g \exp \left( -i \frac{\Delta k l}{2} \right) \sin \left( \frac{\Delta k l}{2} \right) A_1(0) A_2(0) \\
- \frac{g^2}{2} f^* \left( \frac{\Delta k l}{2} \right) \left( |A_1(0)|^2 + |A_2(0)|^2 \right) A_0(0)
\]

\[
A_1(l) = A_1(0) + g \exp \left( i \frac{\Delta k l}{2} \right) \sin \left( \frac{\Delta k l}{2} \right) A_0(0) A_2(0)
+ \frac{g^2}{2} f \left( \frac{\Delta k l}{2} \right) \left( |A_0(0)|^2 - |A_2(0)|^2 \right) A_1(0)
\]

\[
A_2(l) = A_2(0) + g \exp \left( i \frac{\Delta k l}{2} \right) \sin \left( \frac{\Delta k l}{2} \right) A_0(0) A_1(0)
+ \frac{g^2}{2} f \left( \frac{\Delta k l}{2} \right) \left( |A_0(0)|^2 - |A_1(0)|^2 \right) A_2(0)
\]

(1)
in which $A_0$ is the envelope amplitude and the $A_i$, $i = 1, 2$ are the envelope amplitudes of the interacting fields, assumed to be plane waves, along the crystal axes ($C_1$ : ordinary wave, $C_2$ : extraordinary wave); the envelopes are normalized in such a way that $|A_i|^2$ gives the photon flow (photon.m$^{-2}$.s$^{-1}$); $g$ is the nonlinear coupling coefficient given by

$$g = l\chi^{(2)} \sqrt{\frac{\hbar \omega_0 \omega_1 \omega_2}{2e^2 \varepsilon_0 n_0 n_1 n_2}}$$

and $f(x) = \frac{\exp(ix)}{ix} (\exp(ix) - \sin(x))$. The crystal input-output equations are then, when one expresses the pump field at the center of the crystal:

$$A_1(l) = A_1(0) + g' A_0(\frac{1}{2}) A_2^*(0)$$
$$A_2(l) = A_2(0) + g' A_0(\frac{1}{2}) A_1^*(0)$$

(3)

where $g' = g \exp(i \frac{\Delta k}{2}) \sin \frac{\Delta k}{2}$. The advantage of this expression is that it is valid to the second order in the non-linearity $g'$.

The second element in the cavity is the birefringent wave plate. It has a thickness $\varepsilon$ and its indices of refraction are $n_s$ and $n_f$ at frequency $\omega_0/2$ respectively for the slow and fast axes which make an angle $\rho$ with the crystal axes (see fig. 1).

Its effect will be described in the Jones matrices formalism [9] in the nonlinear crystal axes basis. The transmission through the wave plate can be written as the matrix :

$$M = e^{i\kappa n \varepsilon} \begin{pmatrix} \alpha & \epsilon \\ \epsilon^* & \alpha^* \end{pmatrix}$$

(4)

where

$$n = \frac{n_s + n_f}{2}$$

The moduli of the amplitude reflection coefficients of the coupling mirror are taken equal for the signal and idler waves (due to crystal absorption, surface scattering, other mirror finite transmission ...), assumed to be small. We define a generalized reflection coefficient $r' = r(1 - \mu) \approx 1 - (\mu + \kappa)$. We will call $\zeta_1$ and $\zeta_2$ the phase-shifts introduced by the reflection on the cavity mirrors for the signal and idler waves so that $r_j = r \exp(i \zeta_j)$, $j = 1, 2$. In all the article, we do not take into account the resonance of the pump mode: all equations are given for the pump field inside the crystal and we only calculate operating thresholds (not signal or idler intensities) normalized to the intracavity pump threshold.

### 3 Ring cavity type II OPO

We assume in this section that the cavity has a ring shape (fig. 2), and that the coupling mirror has large reflection coefficients for signal and idler modes ($r_1$ and $r_2$).

The moduli of the amplitude reflection coefficients of the coupling mirror are taken equal for the signal and idler modes: $|r_1| = |r_2| = r = 1 - \kappa$, with $\kappa \ll 1$, so that the transmission is $|t| \approx \sqrt{2\kappa}$, $\mu$ is the round-trip loss coefficient for the signal and idler waves (due to crystal absorption, surface scattering, other mirror finite transmission ...), assumed to be small. We define a generalized reflection coefficient $r' = r(1 - \mu) \approx 1 - (\mu + \kappa)$. We will call $\zeta_1$ and $\zeta_2$ the phase-shifts introduced by the reflection on the cavity mirrors for the signal and idler waves so that $r_j = r \exp(i \zeta_j)$, $j = 1, 2$. In all the article, we do not take into account the resonance of the pump mode: all equations are given for the pump field inside the crystal and we only calculate operating thresholds (not signal or idler intensities) normalized to the intracavity pump threshold.
of the OPO without the waveplate (standard OPO threshold), $\sigma_0$. The free propagation length inside the cavity is denoted $L$.

As the signal and idler fields are assumed to have the same frequency, the birefringent plate and the non-linear crystal couple the same fields, which are the ordinary and extraordinary waves at frequency $\omega_0/2$, and only three complex equations are needed to describe the system. From equations (3) and (4), one readily obtains the following steady state equations for the field amplitudes $A_1 = A_1(0)$ and $A_2 = A_2(0)$:

$$A_1 = r'\alpha_0 e^{i(\delta-\theta/2+\psi)} (A_1 + g' A_0 A_2^*) + r' e^{i(\delta+\theta/2)} (A_2 + g' A_0 A_1^*)$$

$$A_2 = r'\alpha_0 e^{i(\delta+\theta/2-\psi)} (A_2 + g' A_0 A_1^*) + r' e^{i(\delta-\theta/2)} (A_1 + g' A_0 A_2^*)$$

where $\delta = \omega_0/2c (\frac{n_1+n_2}{2l} + l) + \frac{\delta_1+\delta_2}{2}$ is the mean round-trip phase-shift, and $\theta = \omega_0/2c (n_1 - n_2)l + \zeta_1 - \zeta_2$ is the birefringent phase-shift introduced by the non-linear crystal and by the mirrors.

One immediately observes that these equations are not invariant under the gauge transformation $A_1 \rightarrow A_1 e^{i\varphi}$, $A_2 \rightarrow A_2 e^{-i\varphi}$, as is the case for the usual equations of a non-degenerate OPO without birefringent mixing. This implies that, unlike in the usual OPO, the phases of the signal and idler amplitudes solutions of equations (8), when they exist, are perfectly determined: phase-locking has occurred between the two oscillating modes, and there is no phase diffusion effect. This phase-locking phenomenon is common to all linearly coupled oscillators [10].

Since the effect of the different elements on the polarization is described by matrices which do not commute, one expects that the system depends on the plate position. However, it is straightforward to show that exchanging the positions of the waveplate and of the crystal amounts to a rotation of $\pi/2$ of the crystal which is equivalent to exchanging indices 1 and 2. This does not change the physics of the system so that we will place ourselves in the case where the waveplate is located after the crystal with respect to the input beam.

Equations (8) have been solved analytically in the small angle regime $\rho \ll 1$ and for small cavity detunings and losses in ref [6].

We will present here the properties of the more complex analytical solutions obtained without any approximations: we will not give the complicated expressions of the solutions, but instead give plots of the most striking results. The exact expressions for the different parameters in the case of a small angle are given in the appendix.

The real and imaginary parts of the first two equations of (8) form a set of two linear equations for the amplitude and phase of the field envelopes $A_1$, $A_2$. Thus, one obtains a set of four linear equations with four variables. A non-zero solution of this systems exists only when the corresponding $4 \times 4$ determinant is zero. This condition gives a real equation for the system parameters, which is fulfilled only in a specific operating range, or locking zone, for the self-phase-locked OPO. In the locking zone, this equation has two real solutions for the intracavity pump intensity, corresponding to two possible regimes of the system [5]. In this paper, we will focus our attention to the regime of lower threshold. These solutions give the oscillation threshold for the intracavity pump power as a function of the crystal temperature, the cavity length and the waveplate angle. We define $\rho$ as the ratio of the intracavity pump power to $\sigma_0$ and $\sigma^{th}$ as the ratio of the intracavity pump power threshold to $\sigma_0$. If, for a given set of parameters, $\rho$ is larger than $\sigma^{th}$, one obtains frequency degenerate oscillation. One can thus plot the values of cavity length and crystal temperature for which $\sigma^{th}$ is smaller than $\rho$ so that there is degenerate oscillation. Fig.(3a) displays the locking zones for two values of the wave plate angle $\rho$ as a function of $\delta T = T - T_{deg}$ and $\delta L = L - L_{deg}$, where $T_{deg}$ is the temperature for which the exact frequency-degenerate operation occurs without any birefringent coupling and and $L_{deg}$ the corresponding cavity resonance length. The locking zone consists of two surfaces which overlap for small values of $\rho$. Fig.(3b)
shows the cross section AA’ of the locking zone for a given value of $\delta T$, that is $\sigma^{th}$ as a function of $\delta L$. All curves in this paper are plotted in the case of KTP for which the index of refraction vary with the following dependance [11]:

$$\frac{dn_1}{dT} = 1.3 \times 10^{-5} \ K^{-1} \quad \text{and} \quad \frac{dn_2}{dT} = 1.6 \times 10^{-5} \ K^{-1} \quad (9)$$

For a given value of $\rho$, the coupling parameter $\epsilon = i \sin(\Delta\phi) \sin(2\rho)$ is maximized for $\Delta\phi = \pi$ that is for a $\lambda/2$ waveplate. As shown on fig. 4, a different value for $\Delta\phi$ will reduce the locking zone extension but does not change the general shape. In order to maximize $\epsilon$ and thus the locking zone extension, one can set $\Delta\phi = \pi$ and $\rho = 45^\circ$. In this case, the locking zone is infinite, in practice only limited by the phase matching.

Fig. 3. (a) : Locking zone as a function of the cavity length ($\delta L$) and of the crystal temperature ($\delta T$) for waveplate angle $\rho = 1^\circ$ (light grey) and $\rho = 5^\circ$ (dark grey). $\sigma = 3$. (b) : $\sigma^{th}$ as a function of the cavity length ($\delta L$) for $\delta T = 0.2 \ K$. For $\sigma = 3$, this corresponds to the cross section AA’ of the locking zone. $\Delta\phi = \pi$.

Fig. 4. Locking zone as a function of cavity length and crystal temperature. The thin dark grey line corresponds to a $\lambda/2$ waveplate while the light grey zone corresponds to a $\lambda/4$ waveplate. $\rho = 5^\circ$, $\sigma = 2$.

As the locking zone depends on the temperature, it may be important to take into account the phase matching. However for small values of the waveplate angle, the locking zone extension in $\delta T$ is small so that the effect of $\delta\kappa k \neq 0$ remains small. As $\rho$ is increased, this effect becomes noticeable and limits effectively the extension of the locking zone to a zone $\delta T \approx 10 \ K$. We have plotted on fig. 5 $\sigma^{res}$, the threshold on resonance : it corresponds to the minimum value of $\sigma$ as a function of $\delta L$ for a fixed value of $\delta T$. One notices on this figure that $\sigma^{res}$ is periodic in $\delta T$ if one does not take into account the phase matching : this is due to the periodicity in temperature of the crystal birefringence. When one takes into account the phase matching, this periodicity disappears (grey curve).
The phase of the reflection coefficient for signal and idler serves as a coupling mirror for signal and idler (fig 6). and serves as a coupling mirror for the pump while the cavity mirrors for the different interacting waves. One takes into account the reflection phaseshifts on the linear and ring cavity OPO have different behaviors when actually used in most experiments. We show here that the linear cavity case which is used in most experiments. We show here that the linear cavity OPO has distinct features when compared to the ring cavity while the triple resonance does not change the behavior of the system. In the linear cavity the beams undergo two interactions per round-trip. As the phase is important in a parametric interaction the phase-shift between signal and idler and the pump beam between the two nonlinear interactions in the crystal must be taken into account. The equations for the field enveloppes at face (a) of the crystal can be written to the first order in $\delta$:

$$A_1 = \alpha_0\rho^\prime e^{i(\delta-\delta')}[A_1 + (1 + e^{i\xi})g^\prime A_0 A_2'] + \epsilon\rho e^{i\delta}[A_2 + (1 - e^{i\xi})g^\prime A_0 A_1']$$

$$A_2 = \alpha_0\rho^\prime e^{i(\delta+\delta')}[A_2 + (1 + e^{i\xi})g^\prime A_0 A_2'] + \epsilon\rho e^{i\delta}[A_1 + (1 - e^{i\xi})g^\prime A_0 A_2']$$

where

$$\delta = \frac{\omega_0}{2c}(2n_0 + 2n_0 + 2L) + \zeta_1 + \zeta_2$$

$$\delta' = \theta - \psi$$

$$\delta_0 = \frac{\omega_0}{c}(2n_0 + 2L) + \zeta_0$$

$$\xi = \frac{\omega_0}{2c}(2n_0 - 2\tilde{n})l - \frac{\omega_0}{c}n(2c) + \zeta_0 - 2\zeta_2$$

2L is the total round-trip free propagation length. $\tilde{n}$ and $\theta$ have been defined in section 3.

One sees on the first two equations of expression 12 that when the phase-shift $\xi$ is taken equal to 0, the equations are similar to the ring cavity case, but with a crystal of double length (factor $2g^\prime$). This is no longer the case.

1 the free propagation and reflection on the coupling mirror simply shift the two waves by the same phase which does not change the effect of the waveplate

2 when one neglects the second order term in $\epsilon g$ if $\rho$ is taken to be small.
when this parameter is changed. A non-zero value of $\xi$ has been shown to increase the threshold of a standard OPO by a significant amount \[^7\]. In the case of a linear cavity with a birefringent element, a dissymmetry appears in the equations due to the terms $1 \pm e^{i\xi}$. Fig.7 shows an example of the results obtained: one notices the dissymmetry between the two locking zones.

Fig. 8 presents the value of the threshold on resonance $\sigma_{res}$ as a function of the crystal temperature $\delta T$ and the phase-shift $\xi$ for $\rho = 5^\circ$. Unshaded surfaces correspond to values of $(\delta T, \delta L)$ where frequency degenerate operation is not possible.

Fig. 9. Normalized threshold on resonance $\sigma_{res}$ as a function of the crystal temperature $\delta T$ and of the phase-shift $\xi$ for $\rho = 45^\circ$. Same curve optimized for $\delta T$. (top). 1.92 times the standard OPO threshold. This increase by a factor 1.92 is also found for $\xi = \pi$ in the case of the standard OPO \[^7\].
5 Conclusion

We have studied a system composed of an Optical Parametric Oscillator containing a birefringent waveplate inside the optical cavity. As shown previously [5,6], this system allows phase locking of the signal and idler fields. We have obtained equations that are valid for all wave plate angles as well as in different cavity configurations, namely ring or linear cavities. We have shown that the zone where phase locking occurs can be described by the cavity length and the crystal temperature and consists of two zones. As the waveplate angle is increased, the size of the locking zone increases. The optimal configuration is obtained by inserting a $\lambda/2$ waveplate in a ring cavity or a $\lambda/4$ waveplate in a linear cavity with a 45° angle with respect to the crystal’s axis. In the case of a ring cavity, the minimum threshold is obtained for a temperature such that the crystal birefringence compensates all the other birefringence in the cavity (waveplate and mirrors) and is equal to the standard OPO threshold. The effect of phase mismatch between the three waves is small for small values of the waveplate angle since the locking zone extension in temperature is small. As $\rho$ is increased, the effect of phase mismatch becomes noticeable and limits in practice the extension of the locking zone. In a linear cavity, the mirrors phase-shift modifies the minimum threshold which becomes dependent on the waveplate angle and can become twice as large as the standard OPO threshold. This increase is known even in standard OPOs but a linear cavity is usually chosen for experimental reasons (losses, mechanical stability . . . ). In both cases (standard and self-phase-locked OPO), this increase is accompanied by a shift in the optimal crystal temperature which may be large and must be taken into account to operate the OPO at low threshold.

Appendix

We give here the exact expression for the lower oscillation threshold in the case of a ring cavity:

$$I^{th} = \frac{u - \sqrt{v}}{g' r^{2} \rho^{2}}$$

(17)

with

$$u = \epsilon^{2} + r'^{2} - 2r'\alpha_{0} \cos(\delta) \cos \left( \frac{\theta}{2} - 2\psi \right) + \alpha_{0}^{2} \cos(\theta - 2\psi)$$

$$v = \left[ r'^{2} + \epsilon_{0}^{2} - 2r'\alpha_{0} \cos(\delta) \cos \left( \frac{\theta}{2} - \psi \right) + \alpha_{0}^{2} \cos^{2}(\theta - 2\psi) \right]^{2}$$

$$-1 - r'^{4} - 2r'^{2}\alpha_{0}^{2} - 2r' \left[ r' \cos(2\delta) + \alpha_{0} \times \left[ r' \alpha_{0} \cos(\theta - 2\psi) - 2 \left( 1 + r'^{2} \right) \cos(\delta) \cos \left( \frac{\theta}{2} - \psi \right) \right] \right]$$

(18)

(19)

with the parameters defined in the text.

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