Two–photon processes of pseudoscalar mesons
in a Bethe–Salpeter approach

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Abstract

We evaluate the $\pi^0\gamma^* \to \gamma$ transition form factor and $\gamma\gamma$ decay widths for $\pi^0, \eta_c$ and $\eta_b$ treated as $q\bar{q}$ bound states in the Bethe–Salpeter formalism, incorporating the dynamical chiral symmetry breaking and the Goldstone nature of the pion. In the chiral limit, the Abelian axial anomaly is incorporated analytically in our coupled Schwinger–Dyson and Bethe–Salpeter approach, which is also capable of quantitatively describing systems of heavy quarks, concretely $\eta_c$ and possibly $\eta_b$.

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Recently, Bando et al. [1] and Roberts [2] have demonstrated how the Adler-Bell-Jackiw axial anomaly can be incorporated in the framework of Schwinger–Dyson (SD) and Bethe–Salpeter (BS) equations, reproducing (in the chiral limit) the famous anomaly result for $\pi^0 \rightarrow \gamma\gamma$ analytically. This was also extended [3] to the off–shell case $\gamma^*\pi^0 \rightarrow \gamma$, a process of renewed experimental interest because of the plans to measure it with high precision at CEBAF (see refs. in [3,4]).

Refs. [2,3] avoided solving the SD equation for the dressed quark propagator $S$ by using an Ansatz quark propagator. Then, thanks to working in the chiral limit, they also automatically obtained the solution of the BS equation: in this limit, when the chiral symmetry is not broken explicitly but dynamically, and when pions must consequently appear as Goldstone bosons, the solution for the pion bound-state vertex $\Gamma_\pi$, corresponding to the Goldstone pion, is given (see Eq. (2), and, e.g., [5]) by the dressed quark propagator $S(q)$:

$$S^{-1}(q) = A(q^2)\gamma_\mu - B(q^2).$$

Concretely, [2,3] used the solution for $\Gamma_\pi$ that is of zeroth order in the pion momentum $p$, and which fully saturates the Adler-Bell-Jackiw axial anomaly [1,2]. It is proportional to $B(q^2)$ when (1) is the propagator of a massless quark, whereas its normalization is given [3] by the pion decay constant $f_\pi$:

$$\Gamma_\pi(q; p^2 = M_\pi^2 = 0) = 2\gamma_5 B(q^2)|_{m=0}/f_\pi,$$

(where the flavour structure of pions has been suppressed). Thanks to using the chiral-limit solution (2) (and to satisfying the electromagnetic Ward-Takahashi identity), [1,3] analytically reproduced the famous axial-anomaly result independently of what precise Ansatz has been used [2] for $S(q)$ and, consequently, independently of what interaction $G^{\mu\nu}(k)$ has formed the $\pi^0$ bound state.

In the present paper we argue that in the light of the recent experimental results on $\eta_c \rightarrow \gamma\gamma$ from CLEO [7], elements of these treatments [1,3], concerning the electromagnetic interactions of dressed quarks, appear essential also for the understanding of the electromagnetic processes of mesons in a totally different regime, far away from the chiral limit.
However, the above–described scheme which avoids solving BS equations \(^{2,3}\), is restricted to the chiral limit and very close to it. (Already when strange quarks are present, \(^2\) can be regarded only as an “exploratory” \(^4\) expression.) This scheme obviously must be abandoned in the case of \(c\)– and \(b\)–quarks, where the whole concept of the chiral limit is of course useless even qualitatively. Away from this limit, one must confront solving the pertinent bound–state equation, which is determined by the interaction between quarks. On the other hand, one should also have the axial anomaly incorporated correctly. This consistency requirement is satisfied by the coupled SD-BS approach to \(q\bar{q}\) bound states presented by Jain and Munczek \(^8\) (and reviewed in a broader SD-BS context by \(^3\)), because this treatment in the chiral limit yields pions as Goldstone bosons of dynamical chiral symmetry breaking (D\(\chi\)SB). On the other hand, it has also successfully reproduced \(^8\) almost the whole spectrum of meson masses, including those in the heavy-quark regime, and also the leptonic decay constants (\(f_P\)) of pseudoscalar mesons (\(P\)). We are therefore motivated to use this so far successful approach for calculating other quantities. In this paper we present the calculation of the \(\pi^0\), \(\eta_c\), and \(\eta_b\to\gamma\gamma \) decay widths, as well as of the \(\gamma^*\pi^0\)-to-\(\gamma \) transition form factor. (Of course, in the case of \(\eta_b\) we do not question the accuracy of the non–relativistic description, but want to see how well our approach does once the experimental data on \(\eta_b\) are obtained.)

2. To define the interaction kernel for the SD and BS equations, \(^8\) use the modeled Landau-gauge gluon propagator given as the sum of the following two parts:

\[
G^{\mu\nu}(k) = [G_{UV}(-k^2) + G_{IR}(-k^2)](g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}) ,
\]

which define the separation of the propagator into: a) the well–known perturbative part, correctly reproducing the ultraviolet (UV) asymptotic behaviour unambiguously required by QCD in its high–energy, perturbative regime, and b) the nonperturbative part, which should describe the infrared (IR) behaviour. The infrared behaviour of QCD is however still not well understood, so it is only this latter, nonperturbative part that is in fact modeled.
From the renormalization group, in the spacelike region \((Q^2 = -k^2)\),

\[
G_{UV}(Q^2) = \frac{16\pi}{3} \frac{\alpha_s(Q^2)}{Q^2} \approx \frac{16\pi^2 d}{Q^2 \ln(x_0 + \frac{Q^2}{\Lambda_{QCD}^2})} \left\{ 1 + b \frac{\ln[\ln(x_0 + \frac{Q^2}{\Lambda_{QCD}^2})]}{\ln(x_0 + \frac{Q^2}{\Lambda_{QCD}^2})} \right\},
\]

where the two–loop asymptotic expression for \(\alpha_s(Q^2)\) is employed, and where \(d = 12/(33 - 2N_f)\), \(b = 2\beta_2/\beta_1^2 = 2(19N_f/12 - 51/4)/(N_f/3 - 11/2)^2\), and \(N_f\) is the number of quark flavours. Following [10], we set \(N_f = 5\), \(\Lambda_{QCD} = 228\) MeV, and \(x_0 = 10\). For the modeled,

IR part of the gluon propagator, we choose \(G_{IR}(Q^2)\) also from Ref. [10]:

\[
G_{IR}(Q^2) = \frac{16\pi^2}{3} a Q^2 e^{-\mu Q^2},
\]

with their [10] parameters \(a = (0.387 \text{ GeV})^{-4}\) and \(\mu = (0.510 \text{ GeV})^{-2}\) adopted throughout.

We obtain the dressed quark propagators [1] for various flavours by solving the SD equation in the ladder (or “rainbow” [11]) approximation (i.e., with bare quark-gluon vertices):

\[
S^{-1}(q) = q - \tilde{m} - i \int \frac{d^4k}{(2\pi)^4} \gamma^\mu S(k) \gamma^\nu G_{\mu\nu}(q - k),
\]

where \(\tilde{m}\) is the bare mass term of the pertinent quark flavour, breaking the chiral symmetry explicitly. (\(I.e.,\) we do not use any Ansätze for \(A(q^2)\) or \(B(q^2)\).) The case \(\tilde{m} = 0\) corresponds to the chiral limit where the current quark mass \(m = 0\), and where the constituent quark mass \(B(0)/A(0)\) stems exclusively from \(\Delta\chi_{SB}\). For \(u\) and \(d\)-quarks, assumed massless, solving of (1) leads to \(B(0)/A(0) = 375\) MeV for the parameters from [10]. When \(\tilde{m} \neq 0\), the SD equation (3) must be regularised by a UV cutoff \(\Lambda\) [3,10], and \(\tilde{m}\) is in fact a cutoff–dependent quantity. In [10], \(\Lambda = 134\) GeV, while \(\tilde{m}(\Lambda^2)\) is 680 MeV for \(c\)-quarks, and 3.3 GeV for \(b\)-quarks. This gives us the constituent mass \(B(0)/A(0)\) of 1.54 GeV for the \(c\)-quarks, and 4.77 GeV for the \(b\)-quarks. In the chiral limit, Eqs. (1)–(2) reflect the fact that solving of (1) for \(S(q)\) with \(\tilde{m} = 0\) is already sufficient to give us the Goldstone pion bound-state vertex (2) saturating the anomalous \(\pi^0 \to \gamma\gamma\) decay, and that is how our \(\Gamma_\pi\) is obtained. Of course, we cannot avoid solving the BS equation like this for heavier \(q\bar{q}\) pseudoscalars \(P\), such as \(P = \eta_c, \eta_b\). Their bound-state vertices \(\Gamma_P\) must be obtained by explicit solving of

\[
\Gamma_P(q,p) = i \int \frac{d^4q'}{(2\pi)^4} \gamma^\mu S(q') + \frac{p}{2} \Gamma_P(q',p) S(q' - \frac{p}{2}) \gamma^\nu G_{\mu\nu}(q - q'),
\]
the homogeneous BS equation again in the (“generalized” [3]) ladder approximation, consistently with (6). (For \( \eta \) and \( \eta' \), their mixing complicates the situation, so we do not solve for \( P = \eta, \eta' \) at this point.) Note that \( S \) is the quark propagator obtained by solving the SD equation (5) with the same gluon propagator \( G_{\mu\nu} \). For pseudoscalar mesons, the complete decomposition of the BS bound state vertex \( \Gamma^P \) in terms of the scalar functions \( \Gamma^P_i \) is:

\[
\Gamma^P(q, p) = \gamma_5 \left\{ \Gamma^P_0(q, p) + q\Gamma^P_1(q, p) + q'\Gamma^P_2(q, p) + \left[ q, q' \right] \Gamma^P_3(q, p) \right\}.
\] (8)

(The flavour indices are again suppressed.) The BS equation (7) leads to a coupled set of integral equations for the functions \( \Gamma^P_i \) \((i = 0, ..., 3)\), which is most easily solved numerically in the Euclidean space. Solving for \( \Gamma^\eta_c \), we also get the \( \eta_c \) mass \( M_{\eta_c} = 2.875 \text{ GeV} \), whereas experimentally \( M_{\eta_c} = 2.979 \text{ GeV} \). Nevertheless, this is just one example of the successful reproduction of heavy-meson masses in this approach [8–10]. For \( \eta_b \), where there are no experimental results yet, we predict \( M_{\eta_b} = 9.463 \text{ GeV} \).

3. The transition matrix element for the processes \( \pi^0, \eta_c, \eta_b \rightarrow \gamma\gamma \), and \( \gamma^*\pi^0 \rightarrow \gamma \), is given by the famous triangle graph coupling two photons (with momenta \( k \) and \( k' \)) to a neutral pseudoscalar \( P \). Apart from the momentum-conserving delta-function ensuring \( p = k + k' \), and the photon polarization vectors \( \varepsilon^\mu(k, \lambda), \varepsilon^\nu(k', \lambda') \), it is in essence equivalent to a tensor amplitude \( T_{\mu\nu}(k, k') \), which is the time ordered product of two electromagnetic currents of quarks, evaluated between a pseudoscalar state \( |P\rangle \) and the vacuum state \( \langle 0 | \). By symmetry arguments, it can be written as

\[
T_{\mu\nu}(k, k') = \varepsilon^{\alpha\beta\mu\nu} k_\alpha k'_\beta T_P(k^2, k'^2),
\]

where \( T_P(k^2, k'^2) \) is a scalar. When both of the photons are on their mass shells, \( k^2 = 0 \) and \( k'^2 = 0 \), the decay width is

\[
\Gamma(P \rightarrow \gamma\gamma) = \pi \alpha^2 M_P^3 |T_P(0, 0)|^2/4. \quad (P = \pi^0, \eta_c, \eta_b.)
\]

On the other hand, for the \( \gamma^*\pi^0 \rightarrow \gamma \) transition, only the final photon \( \gamma \) is necessarily on the mass shell, \( k^2 = 0 \), whereas \( k'^2 = -Q^2 \neq 0 \) for the virtual photon \( \gamma^* \). The \( \gamma^*\pi^0 \rightarrow \gamma \) transition form factor \( F(Q^2) \) on the \( \pi^0 \) mass shell is then defined as

\[
F(Q^2) = T_{\pi^0}(0, -Q^2)/T_{\pi^0}(0, 0).
\] (9)

How to calculate \( T_P \), or, equivalently, \( T_{\mu\nu}^P \)? The framework for incorporating electromag-
netic interactions in the context of BS bound states of dressed quarks, advocated by (for example) [1–4] and called the generalized impulse approximation (GIA) by [2–4], is probably the simplest framework consistent with our coupled SD–BS approach. This means that in the triangle graph we use our dynamically dressed quark propagator $S(q)$, Eq. (1), and the pseudoscalar BS bound–state vertex $\Gamma_{P}(q, p)$ instead of the bare $\gamma_{5}$ vertex. Another essential element of the GIA is to use an appropriately dressed electromagnetic vertex $\Gamma^\mu(q', q)$, which satisfies the vector Ward–Takahashi identity (WTI),

$$
(q' - q)_{\mu} \Gamma^\mu(q', q) = S^{-1}(q') - S^{-1}(q) .
$$

Our dressed quark propagator contains the momentum-dependent functions $A(q^2)$ and $B(q^2)$, so that the bare vertex $\gamma^\mu$ violates (10), implying the nonconservation of the electromagnetic vector current and of the electric charge. Therefore, as in GIA, we must also use such a dressed, WTI-preserving quark–photon vertex. Solving the pertinent SD equation for the true, dressed quark-photon vertex $\Gamma^\mu$ is a difficult problem which has only recently begun to be addressed [13]. Therefore, it is customary to use realistic Ansätze [11]. Motivated by its successes in [2,3], we choose the Ball–Chiu (BC) [14] vertex for $\Gamma^\mu(q', q)$:

$$
\Gamma^\mu(q', q) = A_{\pm}(q^2, q'^2) \frac{\gamma^\mu}{2} + \frac{(q' + q)^\mu}{(q'^2 - q^2)} \left\{ A_{-}(q^2, q'^2) \frac{(q' + q)}{2} - B_{-}(q^2, q'^2) \right\} ,
$$

where $H_{\pm}(q^2, q'^2) \equiv [H(q^2) \pm H(q'^2)]$, for $H = A$ or $B$. Obviously, this Ansatz does not introduce any new parameters as it is completely determined for each flavour by the pertinent solution for the quark propagator (1). Its four chief virtues, however, are (i) that it satisfies the Ward–Takahashi identity (10), (ii) that it reduces to the bare vertex in the free-field limit as must be in perturbation theory, (iii) that its transformation properties under Lorentz transformations and charge conjugation are the same as for the bare vertex, and (iv) it has no kinematic singularities.

It is important to note that the correct axial-anomaly result cannot be obtained [2] analytically in the chiral limit unless a quark–photon–quark ($qq\gamma$) vertex that satisfies the Ward-Takahashi identity is used (even if $D\chi_{SB}$ is employed and the pion does appear as a Goldstone boson, as comparison with [15,16] shows).
In the case of \( \pi^0 \) for example, the GIA yields the amplitude \( T^{\mu\nu}_{\pi^0}(k,k') \):

\[
T^{\mu\nu}_{\pi^0}(k,k') = -N_c \frac{Q_u^2 - Q_d^2}{2} \int \frac{d^4q}{(2\pi)^4} \text{tr}\{\Gamma^\mu(q-p/2,k+q-p/2)S(k+q-p/2)\}
\times \Gamma^\nu(k+q-p/2,q+p/2)S(q+p/2)\Gamma_{\pi^0}(q,p)S(q-p/2)\} + (k \leftrightarrow k', \mu \leftrightarrow \nu).
\] (12)

(The analogous expressions for \( \eta_c \) and \( \eta_b \), or any other neutral pseudoscalar, are straightforward.) The number of colours \( N_c \) arose from the trace over the colour indices. The \( u \) and \( d \) quark charges, \( Q_u = 2/3 \) and \( Q_d = -1/3 \), appeared from tracing the product of the the quark charge operators \( Q \) and \( \tau_3/2 \), the flavour matrix appropriate for \( \pi^0 \):

\[
\text{tr}(Q^2 \tau_3/2) = (Q_u^2 - Q_d^2)/2.
\]

4. Let us first discuss the \( \pi^0 \gamma^* \rightarrow \gamma \) transition form factor \( F(Q^2) \) for space–like transferred momenta. The existing CELLO data [17], displayed (following [3]) as \( Q_2^2 F(Q_2^2) \) in Fig. 1, are described well by a monopole curve with the slope at \( Q_2^2 = 0 \) which is equivalent to the “interaction size” \( \left\langle r_{\gamma \pi^0 \gamma}^2/\right\rangle_{\text{exp}} = 0.65 \pm 0.03 \text{ fm} \) (defined in, e.g., [3] via \( \left\langle r_{\gamma \pi^0 \gamma}^2/\right\rangle = -6F'(Q_2^2)Q_2^2=0 \)). This was reproduced in various models. E.g., in the relativistic constituent quark model with an Ansatz for the pion wave function, Jaus [18] obtained a slope parameter equivalent to \( \left\langle r_{\gamma \pi^0 \gamma}^2/\right\rangle = 0.64 \text{ fm} \), although the transition amplitude at \( Q_2^2 = 0 \) (\( \pi^0 \rightarrow \gamma \gamma \) transition), is 20% too small. In a generalization of the Nambu-Jona-Lasinio model, Ito et al. [19] got \( F(Q^2) \) in excellent agreement with the data and with the monopole fit with \( \left\langle r_{\gamma \pi^0 \gamma}^2/\right\rangle = 0.65 \text{ fm} \). The amplitude obtained in the chiral perturbation theory gave (for small \( Q^2 \)’s) \( F(Q^2) = 1 - \left\langle r_{\gamma \pi^0 \gamma}^2/\right\rangle Q_2^2/6 \), with \( \left\langle r_{\gamma \pi^0 \gamma}^2/\right\rangle = 0.28 \text{ fm} \) [20], but introduction of vector mesons increased it to \( \left\langle r_{\gamma \pi^0 \gamma}^2/\right\rangle = 0.64 \text{ fm} \) [21]. Bando and Harada’s [22] Breit-Wigner forms for the propagators of \( \rho \) and \( \omega \) resonances, have infinite derivatives for \( Q^2 \rightarrow 0_+ \), so that one cannot give \( \left\langle r_{\gamma \pi^0 \gamma}^2/\right\rangle \), although their \( F(Q^2) \), when pure vector-meson dominance is assumed, almost coincides with the monopole curve with \( \left\langle r_{\gamma \pi^0 \gamma}^2/\right\rangle = 0.65 \text{ fm} \). (See Fig. 1.) We also display the Brodsky–Lepage interpolation curve [23] to the perturbative QCD factorization limit \( Q^2 F(Q^2) \rightarrow 8\pi^2 f_\pi = 0.67 \text{ GeV}^2 \). However, our results (the solid line in Fig.
1) should in the first place be compared with \( F(Q^2) \) of the closely related approach of Frank et al. \cite{3} (the dashed line). Our \( F(Q^2) \) is a little better, but it is in fact relatively similar to the curve of \cite{3}, considering that our \( B(q^2) \), obtained by solving of the SD equation \( (\ref{6}) \) (in this case in the chiral limit), is very different from the Ansatz for \( B(q^2) \) used by \cite{3}. Also, our \( \langle r_{\pi\gamma^0}^2 \rangle^{1/2} = 0.46 \text{ fm} \) is practically the same as 0.47 fm of \cite{3}, so that the “interaction size” discriminates even less between our respective \( B(q^2) \)'s, which describe the structure of the pion \( (\ref{2}) \) in the chiral limit. It seems, therefore, that the differences in the internal structure between our pion and that of \cite{3} play a relatively small role for \( \pi^0 \gamma^* \to \gamma \), at least in the chiral limit.

For the on-shell case \( \pi^0 \to \gamma\gamma \), the dependence on the pion structure falls out completely in this limit. Namely, our framework is in the chiral limit equivalent to \cite{1–3}, as demonstrated by the fact that, in this limit, with the solution \( (\ref{2}) \), we too can reproduce the famous anomaly result \( T_{\pi^0}(0,0) = 1/4\pi^2f_\pi \) analytically, in the closed form. The decay width is then given by \( \Gamma(\pi^0 \to \gamma\gamma) = (\alpha^2/64\pi^3)(M_\pi^2/f_\pi^2) \) \cite{12}, in excellent agreement with experiment – see Table I. Being a consequence of respecting the WTI \( (\ref{10}) \) and incorporating \( D\chi_{\text{SB}} \), this result is independent of our concrete choice of the interaction kernel and the resulting hadronic structure of \( \pi^0 \), i.e., of \( B(q^2) \). This is as it should be, because the axial anomaly, which dominates \( \pi^0 \to \gamma\gamma \), is of course independent of the structure. It is then not surprising that calculations for \( \pi^0 \to \gamma\gamma \) which rely on the details of the hadronic structure (instead on the axial anomaly) have problems to describe this decay accurately.

Of course, the situation is very different for quark masses of the order of \( \Lambda_{\text{QCD}} \) and higher: only the numerical evaluation of \( T_P(0,0) \) is reliable in this regime. Moreover, the details of the chosen interaction kernel and the resulting propagator functions \( A(q^2) \) and \( B(q^2) \), as well as the bound state solutions, do matter in that regime (which gets further and further away from the domination of the axial anomaly with growing quark masses, as demonstrated vividly by the amplitude ratios in the last column of Table I). This is naturally the case with the heavy-quark composites \( \eta_c \) and \( \eta_b \). Their two–photon widths are also given in Table I. In the case of \( \eta_b \), only predictions exist, as there are no experimental data yet. In
the case of $\eta_c$, our $\gamma\gamma$ width is (without any adjustment of model parameters) comfortably within the empirical error bars after the 1994 Particle Data Group average, and its 1995 electronic update [24], have been further updated with the new, 1995 CLEO [7] result of $(4.3\pm1.0\pm0.7\pm1.4)$ keV. As opposed to this, it seems that most of the other theoretical approaches predict $\eta_c \to \gamma\gamma$ decay widths that will be either too large or too small after inclusion of the 1995 CLEO [7] result in the Particle Data Group average.

To elaborate on this, let us remark that the results for $\Gamma(\eta_c \to \gamma\gamma)$ in the constituent, nonrelativistic potential models, once in good agreement with experiment, have recently been shown [25] to rise to far too large values of $11.8\pm0.8\pm0.6$ keV after the calculations have been improved by removing certain approximations. The estimates [25] of the relativistic corrections indicate that they can reduce the width back down to some 8.8 keV. This is not enough to bring the constituent quark model in agreement with experiment, although the relativistic corrections are substantial, corroborating the view that the relativistic approaches to bound states, such as ours, retain their importance on the mass scale of the $c$–quarks. This view has recently been strongly advocated by a broad overview [26] of the two–photon physics of mesonic $q\bar{q}$–composites in the BS approach (but without $D\chi\bar{\chi}$SB and without a dressed $qq\gamma$-vertex); it has concluded that the relativistic effects are important for any two–photon width, even for heavy quarkonia. It is interesting to note, however, that such BS calculations, typically yielding [26] $\Gamma(\eta_c \to \gamma\gamma)$ below 4 keV, are consistent only with the 1995 CLEO [7] result on $\Gamma^{\text{exp}}(\eta_c \to \gamma\gamma)$, whereas they are too low for all other [24] measurements, and even for the newly lowered average of $6.2\pm1.3$ keV in Table I. (E.g., see [26] and other related BS calculations discussed therein. They underestimate $\gamma\gamma$-widths in the light sector even more.)

The results of our approach are higher and in agreement with the empirical decay widths. Our results also reveal the importance of using appropriately dressed quark–photon vertices together with dressed quark propagators in yet another way, which has to do with internal consistency. To demonstrate the consequences of violating the WTI (10), and thereby of non–conservation of electric charge, we compare (in the fourth column of Table I) the decay
widths $\Gamma^{bare}(P \to \gamma\gamma)$ obtained inconsistently, i.e., in the coupled SD–BS approach [10] but using the bare electromagnetic vertex $\gamma_\mu$, with our results $\Gamma(P \to \gamma\gamma)$ using the WTI–preserving dressed vertex (11). While not so catastrophic as for the light $\pi^0$, the violation of the WTI (10) is still very bad for $\eta_c \to \gamma\gamma$, and very noticeable even for $\eta_b$ composed of very heavy $b$–quarks, for which the dynamical dressing is not as important as in the light sector.

The reader should note that we have not done any fine–tuning of the parameters, or of the gluon propagator form (3)-(5) which we used; these are the propagator and the parameters of Ref. [11], which achieved a broad fit to the meson spectrum and pseudoscalar decay constants. Therefore, in the consistently applied generalized impulse approximation, the coupled SD–BS approach genuinely, without fitting, leads to the adequate amplitude strength, which is otherwise too low. In other words, the BS approach [8–10] which is consistent with the ideas of [1–3] on dressed quark–photon interactions, seems to be able to describe electromagnetic processes well even in the heavy-quark sector without fine-tuning of model parameters. On the other hand, since $\Gamma(\eta_c \to \gamma\gamma)$ does depend on the interaction kernel and the parameters determining the internal structure of $\eta_c$, making the measurements of the processes such as $\eta_c \to \gamma\gamma$ more precise can, through our theoretical approach, contribute to determining the nonperturbative gluon propagator more accurately. In the first place, this pertains to the infrared part, which is at present poorly known, but is also of significance for the determination of the QCD running coupling $\alpha_s$ (contained in (4), the ultraviolet part of our gluon propagator), for example, at the scale $m_c$ naturally sampled by the $\eta_c$ system.

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TABLES

| $P$    | $\Gamma(P \to \gamma\gamma)$ | $\Gamma^{\text{exp}}(P \to \gamma\gamma)$ | $\Gamma^{\text{bare}}(P \to \gamma\gamma)$ | $T^{\pi_0}(0,0)$ | $T^{P}(0,0)$ |
|--------|--------------------------------|------------------------------------------|-------------------------------------------|-----------------|-------------|
| $\pi^0$ | 7.7                            | 7.74±0.56                                | 0.22                                      | 1               |             |
| $\eta_c$ | $5.3 \times 10^3$            | $(6.2 \pm 1.3) \times 10^3$             | 0.43                                      | 3.95            |             |
| $\eta_b$ | $0.15 \times 10^3$           | ?                                        | 0.73                                      | 133             |             |

TABLE I. Comparison of the calculated $\gamma\gamma$ decay widths (in eV) of $\pi^0$, $\eta_c$ and $\eta_b$ with their average experimental widths, where we have included the 1995 CLEO result [7]. The widths $\Gamma^{\text{bare}}(P \to \gamma\gamma)$, obtained with the bare electromagnetic vertices $\gamma\mu$, are significantly reduced with respect to $\Gamma(P \to \gamma\gamma)$ due to the non-conservation of charge even in the heavy–quark regime. At present, there are no experimental data on $\eta_b$, so the calculated mass of $\eta_b$ had to be used in the phase–space factors, unlike $\pi^0$ and $\eta_c$. 
The solid line represents our results. The dashed line represents the results of Ref. [3]. The dash–dotted line is the Brodsky–Lepage interpolation [23]. The line of open circles is the curve of [22], and open squares form the monopole curve corresponding to $\langle r_{\pi^0}\rangle^{1/2} = 0.65$ fm. Experimental points are the results of the CELLO collaboration [17].
$Q^2 F(Q^2)$ vs $Q^2 [GeV^2]$ plot.