Radiative type-I seesaw neutrino masses

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\textbf{Abstract}

We discuss a radiative type-I seesaw. In these models, the radiative generation of Dirac neutrino masses allows to explain the smallness of the observed neutrino mass scale for rather light right-handed neutrino masses in a type-I seesaw. We first present the general idea in a model independent way. This allows us to estimate the typical scale of right-handed neutrino mass as a function of the number of loops. We then present two example models, one at one-loop and another one at two-loop, in which we discuss neutrino masses and lepton flavour violating constraints in more detail. For the two-loop example, right-handed neutrino masses must lie below 100 GeV, thus making this class of models testable in heavy neutral lepton searches.
1 Introduction

The simplest possibility to generate the Weinberg operator \cite{1},

$$O^W = \frac{1}{\Lambda} LLHH,$$  \hspace{1cm} (1)

is certainly the type-I seesaw mechanism \cite{2–4} given by the diagram in figure 1. In the classical type-I seesaw the Yukawa vertices are point-like \(Y_\nu \bar{L}H\nu_R\) and the smallness of the neutrino masses is controlled by a large Majorana mass, \(\Lambda \sim M_R\), of the right-handed neutrinos \(\nu_R\).

After the electroweak symmetry breaking with the Higgs vacuum expectation value (vev), \(v \equiv \langle H^0 \rangle\), the Weinberg operator (1) leads to the light active neutrino Majorana mass terms. In one generation notation, the active neutrino mass is then given by the well-known relation

$$m_\nu \approx m_D^2/M_R, \text{ with } m_D = Y_\nu \langle H^0 \rangle.$$  \hspace{1cm} (2)

Assuming that the Yukawas entering \(m_D\) take values order \(O(1)\) current neutrino data \cite{5} would then point to \(M_R \sim 10^{14–15}\) GeV. This setup, apart from being able to explain neutrino oscillation data, leads only to one experimentally “testable” prediction: Neutrinoless double beta decay should be observed at some level, for reviews on \(0\nu\beta\beta\) decay see for example \cite{6, 7}.

Here, instead we discuss a simple idea that allows for a much lower scale \(M_R\), even for all involved Yukawa couplings order \(O(1)\), by generating the Dirac mass term corresponding to the Yukawa vertices in figure 1 effectively. To this end one can claim that the elementary Yukawa coupling is forbidden by some symmetry, which being softly broken allows one to generate these vertices at certain loop level directly or via higher dimension effective operators of the form

$$\frac{\kappa}{M^{2n}} \tilde{L}H \nu_R (H^\dagger H)^n,$$  \hspace{1cm} (3)

where \(M\) is the scale of new physics underlying these operators, supposedly somewhere above the electroweak scale, and \(\kappa\) is a loop suppression factor. The Dirac mass term is generated by the operator (3) after the electroweak symmetry breaking. We assume that only the SM Higgs acquires vev, though it is straightforward to generalize this to non-SM Higgses with vev’s as well. Then the resulting effective Yukawa couplings would be suppressed as

$$Y_\nu \sim \left( \frac{1}{16\pi^2} \right)^{\ell} \left( \frac{v^2}{M^2} \right)^n,$$  \hspace{1cm} (4)

where \(\ell\) is the number of loops in the diagram generating the operator (3).

As \(Y_\nu\) is generated effectively, it can be naturally small, while all couplings arising in the UV complete theory can take values order \(O(1)\).

As shown in the next section, right-handed neutrino masses of order of the electroweak scale are easily possible in this setup. Such moderately heavy right-handed neutrinos could be searched for in accelerator based experiments via displaced vertices. The topic of ‘long-lived light particles’ (LLLPs) has attracted much attention in the recent literature \cite{8}. A number of recent experimental proposals \cite{9–12} could search for this signal. Sensitivity estimates for right-handed neutrinos for these experiments can be found in \cite{13, 14}, for the LHC main experiments in \cite{15–17}.
As we already mentioned, in order to forbid tree-level Dirac Yukawa couplings, it is necessary to pos-
tulate some additional symmetry beyond the standard model (SM) gauge group. This symmetry could
be either gauged or discrete. For simplicity, in our model constructions we will concentrate on discrete
symmetries. Starting with a $\mathbb{Z}_4$ symmetry, which gets softly broken to an exact remnant $\mathbb{Z}_2$
symmetry. Thus, the same symmetry responsible for explaining the smallness of the neutrino mass is able to stabilize
a dark matter candidate too.

In our setup neutrinos are Majorana particles. However, our constructions have some overlap with
recent papers on Dirac neutrinos. Some general considerations on how to obtain small Dirac neutrino
masses have been discussed in [18]. Systematic studies of one-loop (and two-loop) Dirac neutrino masses
were given in [19], ([20]). Also the generation of $d = 6$ Dirac neutrino masses has been considered [21].

The rest of this paper is organized as follows. In the next section, we will discuss the radiative generation
of neutrino Dirac couplings from a model-independent point of view. This allows us to estimate the typical
scales for the Majorana mass of neutrinos as a function of the loop level, at which the Dirac couplings are
generated. In section 3 we will discuss then two concrete example models, at one-loop and two-loop level.
For these we will estimate in more detail the neutrino masses, discuss possible constraints from lepton flavor
violation and then turn briefly to dark matter. We will then close with a short summary and outlook.

2 Radiative type-I seesaw

In this brief section we will discuss the radiative generation of neutrino Dirac Yukawa couplings from a
model-independent point of view. Here we consider only the $d = 4$ Dirac mass operator $LH\bar{\nu}_R$ generated
via loops. The mass of the light active neutrinos arises then from the diagram depicted in figure 1 and is
given by eq. (2).

For simplicity, we will limit ourselves to discussing the phenomenologically unrealistic case of one massive
neutrino with no hierarchy or flavour structure for the Yukawas. This is sufficient for discussing the
parameter dependence, extending to three generations of active neutrinos is straightforward. The Dirac
Yukawa, $Y_\nu$, can be parametrized in general in terms of five exponents $(\ell, \alpha, \beta, \gamma, \delta) \in \mathbb{N}^+$, whose values
will depend on the specific UV complete realization of the operator $Y_\nu L H \bar{\nu}_R$, as:

$$Y_\nu \sim \left( \frac{1}{16\pi^2} \right)^\ell \left( \frac{m_\tau}{v} \right)^\alpha \left( \frac{M_F}{\Lambda} \right)^\beta \left( \frac{\mu}{\Lambda} \right)^\gamma \epsilon^\delta.$$  

This corresponds to generating effectively the Yukawa via a diagram with

- $\ell$ loops;
- $\alpha$ insertions of SM Yukawas. Unless the UV model allows for a top-quark in the loop, this corresponds
to a suppression of typically $\sim 10^{-2}$, from $Y_\nu^{SM}$ (or $Y_\nu^{SM}$);
- $\beta$ mass insertions of new (vector-like) fermions, not part of the SM, all set to $M_F$ for simplicity;
- $\gamma$ dimensionful couplings in the scalar sector, i.e. trilinear scalar couplings;
- $\delta$ dimensionless couplings, for instance Yukawas or four-point scalar couplings.

Not all possible sets of exponents can be realized in a UV complete model which is genuine, i.e. give the
dominant contribution to the neutrino mass. For example, for the most simple case of an one-loop Dirac
mass term, there are only two genuine diagrams [19] with one or two mass insertions and, at least, three
couplings. So, for $\ell = 1$ it is not possible to generate a genuine diagram with, for instance, $\alpha, \beta > 2$. The
possible combinations of $(\alpha, \beta, \gamma, \delta)$ can be deduced from the systematic studies of radiative Dirac models
given in [19, 20].

For our numerical estimates, we will assume that all couplings are in the perturbative regime, i.e.
$\epsilon \lesssim 1$.\footnote{It is often argued that perturbativity only requires Yukawa couplings to be $Y \lesssim \sqrt{4\pi}$. However, saturating this limit
would imply that higher order contributions are (at least) equally important than the leading order (that we consider), thus rendering estimates effectively inconsistent.} If $\mu$ is a trilinear coupling between some BSM scalar and the Higgs, it enters in the calculation

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of the stability of the Higgs potential, i.e. it will induce a modification of the quartic Higgs coupling at one-loop level. We will thus also assume that $\mu \lesssim m_S \equiv \epsilon m_S$, in order not to run into problems with the SM Higgs sector. With these considerations the light neutrino mass can be written in terms of the same five exponents, using the seesaw relation (2),

$$m_\nu \sim \left(\frac{1}{16\pi^2}\right)^{2\ell} \frac{v^2}{M_R} \left(\frac{m_\tau}{v}\right)^{2\alpha} \frac{(M_\nu)}{\Lambda}^{2\beta} \frac{(m_S)}{\Lambda}^{2\gamma} \epsilon^{2(\gamma+\delta)}.$$  \(\text{(6)}\)

As this equation shows, neutrino masses generated from this class of models will be very suppressed. If, for instance, the Dirac neutrino mass arises at two-loop order, then $m_\nu$ will effectively come from a four-loop diagram with an extra suppression due to the Majorana scale $M_R$. Thus, for relatively low masses of the order of TeV and couplings order one, a reasonable neutrino mass can be obtained easily.

A rough, but conservative limit on the Majorana mass scale, can be obtained setting all masses in the loop to the same scale $\Lambda = M_F = m_S$. Conservatively taking $\epsilon = 1$, we find

$$m_\nu \sim \left(\frac{1}{16\pi^2}\right)^{2\ell} \frac{v^2}{M_R} \left(\frac{m_\tau}{v}\right)^{2\alpha}.$$  \(\text{(7)}\)

Note, that the scale $\Lambda$ does not appear in this simple case in the expression for $m_\nu$. This is to be expected, given the $d = 4$ nature of the neutrino Dirac coupling. Taking as reference scale the atmospheric neutrino mass $\sqrt{\Delta m_{31}^2} \approx 0.05$ eV, we can set upper limits on $M_R$ as function of the exponents $\ell$ and $\alpha$. Limits are given in table 1 up to three-loops and two SM Yukawa insertions. The numbers given correspond to couplings order one.

| $M_R$ | $\alpha = 0$ | $\alpha = 1$ | $\alpha = 2$ |
|------|-------------|-------------|-------------|
| $\ell = 1$ | $2 \times 10^{10}$ GeV | $2 \times 10^{6}$ GeV | $2 \times 10^{2}$ GeV |
| $\ell = 2$ | $10^6$ GeV | $10^2$ GeV | $9 \times 10^{-3}$ GeV |
| $\ell = 3$ | $4 \times 10^1$ GeV | $4 \times 10^{-3}$ GeV | $4 \times 10^{-7}$ GeV |

Table 1: Estimated values for $M_R$ needed to fit a neutrino mass of 0.05 eV with couplings order one for different realizations of the Dirac mass operator $L^H\bar{\nu}_R$, considering $\ell$ loops and $\alpha$ SM Yukawa insertions. These mass scales constitute a rough, but conservative upper limit for $M_R$ for each class of models parametrized by the exponents $\ell$ and $\alpha$ in (7).

Obviously, $M_R$ decreases very fast as $\alpha$ or $\ell$ increase. This is due to the fact that for Majorana neutrinos $m_\nu$ depends quadratically on $Y_\nu$, rather than linearly. For $\alpha = 1$ and $l = 2$ one finds a scale of $M_R \sim 10^2$ GeV and similar numbers for $\alpha = 2$ and $l = 1$ or $\alpha = 0$ and $l = 3$. These are the phenomenologically most interesting cases.

Apart from the upper limit on the Majorana mass coming from the neutrino mass scale, lower limits on $M_R$ can be set from big bang nucleosynthesis [22] and the effective number of neutrinos in the early universe $\Delta N_{\text{eff}}$ [23]. These limits depend on the mixing angle between the right-handed and the active neutrinos (as a function of the mass $M_R$). For our class of models, as for the ordinary type-I seesaw, one expects $M_R \gtrsim (0.1 - 1)$ GeV, from these considerations [22, 23]. This constrains significantly the space of possible models to only those with three loops or less and at most two SM mass insertions (for the case of 1-loop). In the next section, we will therefore discuss two model examples in more detail: a one-loop and a two-loop model.

3 Examples of models

In this section we show two simple models where the Dirac mass is generated at one- and two-loops, both containing a stable dark matter candidate, which participates in the loop. We give an estimate of the
Table 2: Particle content of the example model that generates the one-loop diagram of figure 2 once the $Z_4$ is softly broken by the trilinear term $H \eta S^-$. After the breaking of $Z_4$ a remnant $Z_2$ is exactly conserved.

![Diagram](image)

Figure 2: One-loop Dirac neutrino mass. The diagram is realized when the $Z_4$ is softly broken (denoted by the symbol ⊗). As the symmetry is broken in two units, the diagram is still invariant under a remnant $Z_2$ of $Z_4$.

neutrino mass scale involved for a simplified benchmark, as well as an insight to the phenomenological constraints coming from charged lepton flavour violating processes.

3.1 One-loop Dirac mass

The particle spectrum of the model and their assignments under the SM gauge and the $Z_4$ discrete symmetry are shown in table 2. Notice that we have assumed a $Z_4$ symmetry, which is softly broken down to the preserved $Z_2$ symmetry, in order to guarantee that the Dirac neutrino mass matrix is generated at one-loop level. The scalar sector of the model is composed of the SM Higgs doublet $H$, the inert $SU(2)_L$ scalar doublet $\eta$ and the electrically charged gauge singlet scalar $S^-$. In addition, the SM fermion sector is extended by the inclusion of a right-handed Majorana neutrino $\nu_R$ and the vector like charged leptons $\chi_L$ and $\chi_R$. The relevant terms for the neutrino mass take the form,

$$\begin{align*}
-L_Y &= Y_e L H^\dagger e^c + Y_L L \eta^\dagger \chi_L + Y_R R S^+ \nu_R + h.c., \\
L_M &= M_R \nu_R^\dagger \nu_R + M_\chi \overline{\chi_R} \chi_L + h.c.,
\end{align*}$$

(8)

(9)

flavour indices and $SU(2)$ contractions have been suppressed for brevity.

The terms above generate the Dirac neutrino mass matrix at one-loop level through the diagram shown in figure 2 provided the following $Z_4$ trilinear soft breaking term is added to the scalar potential,

$$\mathcal{V} \supset \mu_S H \eta S^- + h.c.$$  

(10)

The softly broken $Z_4$ guarantees that the Dirac mass term is forbidden at tree-level but generated by loops, i.e. that the diagram is genuine (non-reducible) [19].

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$^2$We repeat, that we are interested here only in a rough estimate for the neutrino mass scale. For phenomenological reasons, one would need indeed at least two right-handed neutrinos that generate the solar and atmospheric neutrino mass. Since fits of the seesaw type-I to neutrino data are straightforward and have been done many times in the literature, we do not repeat these details here.
The Dirac mass term can be computed directly from the diagram in figure 2 given the Lagrangians eqs (8) and (9) and the soft-breaking term. In the mass insertion approximation and, for simplicity, setting all the masses of the internal scalars, as well as the soft-breaking parameter $\mu_S$, to $m_S$, one finds:

$$m_D \approx \frac{1}{16\pi^2} \frac{v m_S}{M_\chi} Y_L Y_R \mathcal{I}_1(m_S^2/M_\chi^2).$$

The loop integral $\mathcal{I}_1(x)$ can be written in terms of the Passarino-Veltman $B_0$ function [24] as,

$$\mathcal{I}_1(x) = \frac{1}{1-x} \left[ B_0(0,1,x) - B_0(0,x,x) \right].$$

The mass scale of the lightest active neutrino can be directly estimated through the seesaw approximation as,

$$m_\nu \sim \left( \frac{1}{16\pi^2} \right)^2 Y_L^2 Y_R^2 \frac{v^2 m_S^2}{M_\chi^2 M_R^2} [\mathcal{I}_1(m_S^2/M_\chi^2)]^2$$

This mass scale as a function of $M_R$ is plotted in figure 3. Two different benchmarks with $M_\chi = M_R$ and $m_S = M_\chi$ are represented by the solid and dashed lines respectively. For both cases, we can observe that the neutrino mass is strongly suppressed even for small values on the masses of the internal scalars, as well as the soft-breaking parameter $\eta$ and the soft-breaking term. In the mass insertion approximation and, for simplicity, setting all the parameters and (ii) LFV limits do not impose interesting limits on our one-loop model. Consequently, the points with a neutrino mass lying roughly below the atmospheric scale is phenomenologically non-viable, as it would require couplings larger than one (non-perturbative) to give a reasonable mass scale.

Current upper limits on lepton flavour violating (LFV) decays such as $\mu \rightarrow e\gamma$ can provide constraints on the parameters of our model. These depend on specific choices for the Yukawas $Y_L$ and $Y_R$. As eq. (13) shows, $m_\nu$ depends on the product of these couplings, while LFV decays are mostly sensitive to $Y_L$ only. There are then two extreme cases: (i) Choose $Y_L \simeq 1$ and fit $Y_R$ to $m_\nu$ as function of the other model parameters and (ii) $Y_R \simeq 1$ and fit $Y_L$. Case (i) is very similar to the situation in our two-loop model (see section 3.2), and thus we will discuss the details in the next section. For case (ii) on the other hand, we found that LFV limits do not impose interesting limits on our one-loop model.

The residual $Z_2$ symmetry ensures that the lightest of the fields running inside the loop will be stable. In order to not run into conflict with cosmology and to provide a good dark matter candidate, one should force the neutral component of the doublet $\eta$ to be the lightest of the loop particles. Similar DM candidates have been studied in the literature\cite{3}. Considering $\eta$ as the only source of dark matter, the observed relic density, together with direct detection limits and the constraints on the invisible width of the Higgs boson severely limit its mass to lie either around $m_\eta/2 \simeq 62.5$ GeV, in a small region around $m_\eta \simeq 72$ GeV or above $m_\eta \gtrsim 500$ GeV\cite{27}.

### 3.2 Two-loop Dirac mass

Analogously to the first example, we build a two-loop radiative seesaw model breaking softly a $Z_4$ discrete group to an exact $Z_2$ symmetry. The particle content and their transformation properties under the SM gauge and the $Z_4$ discrete symmetry are shown in table 3. We again include a right-handed Majorana neutrino $\nu_R$.

The relevant terms of the Lagrangian and the scalar sector invariant under $Z_4$ are,

$$-\mathcal{L}_Y = Y_e LH^\dagger e + Y_L \bar{F}_L \eta_2 \bar{e} + Y_R \bar{\nu}_R \eta_2 F_L + \text{h.c.},$$

$$\mathcal{L}_M = M_R \bar{\nu}_R \nu_R + M_F \bar{F}_R F_L + \text{h.c.},$$

$$\mathcal{V} \supset \lambda \eta_1^2 H \eta_2^2 H + \text{h.c.},$$
Figure 3: One-loop neutrino mass scale. The dashed line corresponds to the case where $m_S = M_X$, while the solid lines depict the case where $M_X = M_R$ for different scalar masses. The Yukawas $Y_L$ and $Y_R$ are set to 1. Big bang nucleosynthesis (BBN) excludes $M_R > (0.1 - 1) \text{ GeV}$, depending on mixing, for these class of models [22].

| Fields | $SU(3)_C \times SU(2)_L \times U(1)_Y$ | $Z_4$ | Residual $Z_2$ |
|--------|-------------------------------------|-------|----------------|
| $L$    | (1, 2, -1/2)                       | 1     | 1              |
| $e^c$  | (1, 1, 1)                          | 1     | 1              |
| $\nu_R$| (1, 1, 0)                          | −1    | 1              |
| $H$    | (1, 2, 1/2)                        | 1     | 1              |
| $(F_L, F_R)$ | (1, 2, -1/2)  | (i, i) | (−1, −1) |
| $\eta_1$ | (1, 2, 1/2)   | −i    | −1            |
| $\eta_2$ | (1, 2, 1/2)   | i     | −1            |

Table 3: Particle content of the example model that generates the two-loop diagram of figure 4 once the $Z_4$ is softly broken by the term $\eta_2^\dagger \eta_1$. After the breaking of $Z_4$ a remnant $Z_2$ is conserved.
the symbol \( \otimes \). As the symmetry is broken in two units, the diagram is still invariant under a remnant \( Z_2 \).

An effective Dirac term is generated once the \( Z_4 \) symmetry is softly broken in the scalar sector by the term,

\[
- \mathcal{L}_{\text{soft}} = \mu_{12}^2 \eta_2 \eta_1 + \text{h.c.} \tag{17}
\]

A Dirac mass appears at the two-loop level, as depicted in figure 4, which can be expressed in the mass insertion approximation, assuming no flavour structure in the Yukawa couplings, as

\[
m_D \approx \left( \frac{1}{16\pi^2} \right)^2 \lambda Y_e Y_L Y_R \frac{\mu_{12}}{M_F} \mathcal{I}_2 \left( \frac{m_S^2}{M_F^2} \right) \tag{18}
\]

with the \( \mathcal{I}_2(x) \) a dimensionless two-loop function. \( \mu_{12} \) is the soft breaking mass term depicted by \( \otimes \) in figure 4. For simplicity, we set all the masses of the new internal scalars to \( m_S \). Taking into account that the main contribution of the SM Yukawa \( Y_e \) would be \( m_e / v \), the mass scale of the lightest active neutrino is directly estimated through the seesaw approximation as,

\[
m_\nu \sim \left( \frac{1}{16\pi^2} \right)^4 \lambda^2 Y_e^2 Y_L^2 \frac{m_S^2}{M_F^2} \left[ \mathcal{I}_2 \left( \frac{m_S^2}{M_F^2} \right) \right]^2, \tag{19}
\]

where as before, we have set \( \mu_{12} = m_S \). \( \mathcal{I}_2 \) can be written in terms of simple two-loop integrals for which analytical solutions are known [28]. We do not give here its decomposition for brevity, though it can be found in the literature [29].

The neutrino mass scale, eq. (19), as a function of \( M_F \) is plotted in figure 5. We consider two different approximations: \( M_F = m_S \) and \( M_F = M_R \), represented by the dashed and solid lines respectively. As expected from table 1, the neutrino mass is more strongly suppressed compared with the one-loop model described previously. For the case \( m_S = M_F \) the Dirac Yukawa is independent of the scale, consequently the neutrino mass falls simply as \( \sim 1/M_R \). On the other hand, in the scenario where \( M_F = M_R \), this same behaviour is reproduced when \( m_S \) dominates, while for values of \( M_R > m_S \), the neutrino mass follows the curve \( 1/M_R^3 \).

Given the suppression factor \( (m_e / v)^2 \sim 10^{-4} \), and if we take into account the limit coming from cosmology (BBN), the range of allowed values of \( M_R \) which can fit the neutrino oscillation scale \( m_{\text{atm}} \sim 0.05 \) eV is considerably limited. For \( m_S > 10^2 \) GeV, \( M_R \) has to be \( M_R \lesssim 10^2 \) GeV. This makes the model testable in future heavy neutral lepton searches.

We mention again that the remnant \( Z_2 \) symmetry stabilises the lightest of the fields odd under this symmetry. Fermionic dark matter coming from a doubllet is ruled out by direct detection experiments [30], while for the scalar inert doublet the same limits described in the previous section apply.

Turning to LFV processes, figure 6 shows the \( \text{Br}(\mu \rightarrow e\gamma) \) as a function of \( M_R \) for two different scenarios, already mentioned in section 3.1: (i) \( Y_L \simeq 1 \) and fit \( Y_R \) to \( m_\nu \) or (ii) \( Y_R \simeq 1 \) and fit \( Y_L \). All other possibilities to choose Yukawas lie between these extremes. The dominant (one-loop) contribution to \( \text{Br}(\mu \rightarrow e\gamma) \) comes always from \( Y_L \), which directly connects the new particles with the SM leptons. For \( M_F = M_R \) and \( Y_R = 1 \) the branching is dominated by the fit of the neutrino mass, eq. (19). The branching increases as function of \( M_R \), as \( Y_L \) gets larger counteracting the suppression of \( 1/M_R^3 \) in the neutrino mass. We stop the calculation when \( Y_L \) grows larger than 1. In contrast, for \( Y_L = 1 \) there is no dependence from #See for instance the well-known Inert Doublet Model [25] or the Scotogenic model [26].
Figure 5: Two-loop neutrino mass scale assuming that $m_S = M_F$ and $M_F = M_R$, depicted in dashed and solid lines respectively. All dimensionless couplings are set to 1 and the BBN exclusion region is indicated in the left.
Figure 6: Estimate of the branching ratio of $\mu \rightarrow e\gamma$ as a function of $M_R$ for different values of $m_S$ fitting the neutrino mass to $m_{atm}$. The areas between the coloured lines are allowed in this model, see text. The grey lines represent the values of $M_R$ where one of the Yukawa couplings becomes non-perturbative in order to fit neutrino oscillation data. The shadowed region represents the experimentally excluded area for $Br(\mu \rightarrow e\gamma) > 4.2 \times 10^{-13}$ [31], while the purple line corresponds to the future prospect limit from MEG collaboration [32].
the neutrino mass fit, but a suppression of $1/M_R^4$ when this mass scale dominates over $m_S$ in the $\mu \rightarrow e\gamma$ loop function [33]. The regions in between those extremes are the regions allowed for this neutrino mass model.

4 Conclusions

We have constructed a new realization of the type-I seesaw mechanism based on radiatively generated Dirac neutrino masses. We showed that this class of models can naturally generate a small neutrino mass for order one couplings and relatively low mass scales. Compared to the standard type-I seesaw mechanism, for which the Majorana mass scale should be of the order of the GUT scale, we found viable models even for $M_R$ below 100 GeV. Parametrizing the neutrino mass in terms of five integers, we derived for each set of models a conservative limit on $M_R$ requiring only that they should fit the atmospheric neutrino mass scale. The strong suppression of the light neutrino mass with the number of loops, i.e. $(1/16\pi^2)^{2\ell}$, along with the seesaw Majorana mass suppression, allows remarkably low $M_R$ values. This fact makes models with large number of loops (or SM mass insertions) run into conflict with big bang nucleosynthesis and $\Delta N_{\text{eff}}$, which therefore significantly constrains the space of possible models.

To illustrate in further detail this idea, we presented two example models where the Dirac neutrino mass matrix is generated at one- and two-loop level. The latter lies at the edge of the excluded models. An extra $Z_4$ symmetry is incorporated to forbid a tree-level Dirac mass, but broken softly in order to generate the Dirac Yukawa radiatively. A remnant exact $Z_2$ symmetry is kept stabilising the lightest of the $Z_2$ charged fields and providing a good dark matter candidate.

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