Evaluation of the Elastic Properties of Highly Porous Alumina Foams using Finite Element Analysis

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Abstract. In this work, a two-dimensional approach is considered in the evaluation of the elastic properties of an alumina foam exhibiting a porosity of around 87%. Preliminary experimental procedures were performed for the determination of the Young’s modulus. The stiffness was evaluated with the help of microstructural models consisting of regular structures composed of tessellated patterns of different cell shapes (hexagons, circles, octagons/squares). The experimental data was compared with the numerical results.

1. Introduction

Cellular materials are used in a wide array of applications (aerospace, automotive, terrestrial and naval transportation, biomedicine, civil engineering etc.) due to their particular characteristics: good mechanical properties at reduced specific mass, good thermal and sonic insulation, high energy absorption capabilities [1, 2].

The microstructure of porous ceramics influences their properties: open-cell ceramic foams are used as filters, bioreactors, cancellous bone tissue substitutes etc. while closed-cell ceramic foams have applications in sonic and thermal insulation or as structural components [3].

The prediction of the mechanical properties of cellular structures through finite element analysis represents a useful tool in understanding and designing structural foams [4]. Both two-dimensional [5, 6] and three-dimensional [7] structures were considered in literature for the estimation of various mechanical characteristics.

The first step in the assessment of the mechanical behavior of the designed microstructures is the evaluation of the strength and stiffness in comparison with the experimental data [6]. Further studies can be extended to the studying the structures’ damping and dynamic response [8]. The development of accurate microstructures can also aid in the numerical prediction of the material fracture toughness [9].

This study investigates the mechanical characteristics of high-porosity alumina foam, which has applications in various fields such as thermal insulation, biomedical engineering or filtration [10].

The investigated ceramic foam exhibits a closed cell structure and has a porosity of 87%, a density of 465 kg/m³, the cell size varying between 30 and 250 µm, with cell wall thicknesses of around 5 µm (Figure 1). Three structures (hexagonal lattice, octagonal/quadratic lattice and squared cell lattice with circular orifices) were considered in predicting the mechanical behavior of the aforementioned foam structure.
2. Experimental results
The ceramic foam’s Young’s modulus and Poisson ratio were determined using ultrasonic echography tests with infinite medium transmission. 15mm x 13mm x 11mm specimens were cut from bricks provided by the manufacturer.

The principle of ultrasonic echography is based on acoustic wave emission in a material of a given thickness by means of two transducers: one sending longitudinal waves and the other sending transversal waves [11]. The contact between the transducers and the specimen is assured through an intermediary coupling agent. After the emission of the first waves, the values of the transversal \( V_T \) and longitudinal \( V_L \) propagation speeds through the thickness of the specimen is calculated through the delay propagation times, determined with the help of an oscilloscope. Knowing the density of the material \( \rho \), the values of the Young’s modulus \( E \), shear modulus \( G \) and Poisson ratio \( \nu \) are calculated with the relations:

\[
E = \rho \cdot \left( \frac{3V_L^2 - 4V_T^2}{V_L^2} \right) \\
G = \frac{\rho \cdot V_L^2}{V_T^2 - 1} \\
\nu = \frac{E}{2G} - 1
\]

The ultrasonic echography tests yielded a value of Young’s modulus of 13.23 ± 2.62 GPa and a value for the Poisson ratio of 0.24 ± 0.01.

3. Development of structures
The regular structures investigated in this work consist of tessellations of certain cell geometries: hexagonal cells, square cells with octagonal and rectangle orifices (equivalent to an octagon/square lattice) and square cells with circular orifices.

For each structure, several parameters were considered: cell wall length \( l \), cell wall thickness \( t \), chamfer radius \( r \) (for the hexagonal lattice and the octagonal/quadratic lattice), cell length \( L \) and orifice radii \( d \) and \( \delta \) (for the square cell with circular orifices). Considering the fact that, for regular structures, scaled versions of the same geometry behave identically, the structure variables were chosen as a ratio of the “primary” parameters (\( t, r, d \)) with respect to the “secondary” parameters, which determine the size of the cell (cell wall length \( l \) for the honeycomb and octagonal/quadratic lattice and cell length \( L \).
for the square cell with circular orifices. The geometries of each structure were generated using SolidWorks™.

The porosity $\Pi$ of a material is defined by the ratio of the volume of voids $V_{\text{void}}$ to the volume of the solid or the bulk material $V_{\text{solid}}$:

$$\Pi = \frac{V_{\text{void}}}{V_{\text{solid}}} [-]$$

For two-dimensional structures, the thickness can be equaled to the unit, thus, the porosity becomes a ratio of the surface of the voids $A_{\text{void}}$ to the surface of the circumscribed figure $A_{\text{solid}}$. In order to determine the porosities of each structure, the CAD software was used in evaluating the surface area of the structure $A_{\text{str}}$, the resulting porosity being calculated with the relation:

$$\Pi = \frac{A_{\text{solid}} - A_{\text{str}}}{A_{\text{solid}}}$$

a) Hexagonal lattice (honeycomb)

The honeycomb geometry represents a tessellation of regular hexagons, presented in Figure 2 for two models (with and without chamfer). The model has two variable parameters: the cell wall thickness to cell wall length ratio $t/l$ and the chamfer radius to cell wall length ratio $r/l$.

![Figure 2. Hexagonal lattice with no chamfer (a) and with a chamfer to cell wall length ratio of 0.6 (b)](image)

In order to determine the variation of the porosity of the structure with the variables, the two parameters $t/l$ and $r/l$ were varied in five steps. The values of the variables along with the resulting porosities are presented in Table 1 while the resulting surface is presented in Figure 3.

| Table 1. Variation in porosity with $t/l$ and $r/l$ for the hexagonal lattice |
|-------------------|---|---|---|---|---|
| $t/l$ [ ] | 0 | 0.1 | 0.2 | 0.4 | 0.6 |
| $r/l$ [ ] | 0 | 0.1 | 0.2 | 0.4 | 0.6 |
| $\Pi$ [%] | 0.1 | 0.782 | 0.781 | 0.777 | 0.762 | 0.73 |
| | 0.08 | 0.824 | 0.822 | 0.819 | 0.804 | 0.779 |
| | 0.06 | 0.866 | 0.865 | 0.861 | 0.846 | 0.821 |
| | 0.04 | 0.909 | 0.908 | 0.905 | 0.89 | 0.865 |
| | 0.02 | 0.954 | 0.953 | 0.949 | 0.942 | 0.909 |
A surface fitting equation was performed using the commercial software TableCurve 3D with the resulting data, obtaining in the relation:

\[
\Pi\left(\frac{t}{l}, \frac{r}{l}\right) = 1.33 \left(\frac{t}{l}\right)^2 - 2.31 \frac{t}{l} - 0.12 \left(\frac{r}{l}\right)^2 + 1
\]

Solving the equation for \(\Pi = 0.87\) and setting three values for the \(r/l\) ratio resulted in the pairs of parameters presented in Table 2.

**Table 2.** Value of the parameters \(t/l\) and \(r/l\) for \(\Pi = 0.87\) for the hexagonal lattice

| \(\Pi\) [%] | \(r/l\) [-] | \(t/l\) [-] |
|----------|-------------|-------------|
| 0        | 0.582       |             |
| 0.87     | 0.2         | 0.553       |
| 0.87     | 0.6         | 0.39        |

**Figure 3.** Graphical variation of porosity with \(t/l\) and \(r/l\) for the hexagonal lattice

b) Octagonal/quadratic lattice

The second investigated structure consists of a tessellated pattern of regular octagons and squares, presented in Figure 4 for models with and without chamfer. Similar to the honeycomb, the octagonal/quadratic lattice has two variable parameters: the cell wall thickness to cell wall length ratio \(t/l\) and the chamfer radius to cell wall length ratio \(r/l\).
Figure 4. Octagonal/quadratic lattice with no chamfer (a) and with a chamfer to cell wall length ratio of 0.4 (b)

The variation of the structure porosity with $t/l$ and $r/l$ was determined through the variation of each parameter in five steps. The obtained values are presented in Table 3 and the resulting surface in Figure 5.

| $\Pi$ [%] | $t/l$ [-] | $r/l$ [-] |
|-----------|-----------|-----------|
| 0.15      | 0.853     | 0.851     |
| 0.625     | 0.876     | 0.874     |
| 0.105     | 0.924     | 0.923     |
| 0.05      | 0.95      | 0.947     |

The surface fitting resulted in the equation:

$$\Pi\left(\frac{t}{l}, \frac{r}{l}\right) = -0.97\frac{t}{l} - 0.001\left(\frac{r}{l}\right)^2 + 0.99$$

Solving the equation for $\Pi = 0.87$ and setting three values for the $r/l$ ratio resulted in the pairs of parameters presented in Table 4.

| $\Pi$ [%] | $r/l$ [-] | $t/l$ [-] |
|-----------|-----------|-----------|
| 0.87      | 0.2       | 0.124     |
| 0.4       | 0.102     |           |
Figure 5. Graphical variation of porosity with \( t/l \) and \( r/l \) for the octagon/square lattice

c) Square cell with circular orifices

The third regular structure consists of a tessellation of a square cell with circular orifices, presented in Figure 6. In order to obtain high levels or porosity, two circle diameters were chosen. To simplify the generation of the models, a fixed ratio of 0.3 was set between the two diameters. Thus, the only variable for this geometry was the ratio between the cell length \( L \) and the diameter of the large circles \( d \).

The variation in structure porosity with \( d/L \) was determined by varying the parameter in five steps. The obtained values are presented in Table 5 and the resulting curve in Figure 7.

| \( d/L \) [-] | 4.9 | 4.8 | 4.7 | 4.6 | 4.5 |
|---------------|-----|-----|-----|-----|-----|
| \( \Pi \) [%] | 0.8969 | 0.8607 | 0.8252 | 0.7905 | 0.7564 |

The equation that describes the variation is:

\[
\Pi \left( \frac{d}{L} \right) = 3.57 \left( \frac{d}{L} \right)^2 + 0.15 \frac{d}{L} - 0.036
\]

Solving the equation for \( \Pi = 0.87 \) resulted in a \( d \) to \( L \) ratio of 0.482.
4. Numerical results

The structures described in the previous paragraphs (with the corresponding parameters for $\Pi = 0.87$) were analysed in compression using the commercial software Abaqus™. The material model for the solid was chosen as isotropic linear elastic, with a Young’s modulus of 350 GPa and a Poisson ratio of 0.24 (parameters for solid Alumina) [3]. The mesh was generated using second order quadratic elements in plane stress formulation (CPS8). The average size of the element was chosen so that a minimum of three elements per strut thickness would be obtained. As boundary conditions, an x-axis symmetry was imposed on the vertical edges of the model, a y-axis symmetry on the bottom edge and a displacement along the y-axis on the top edge.

The size effect was investigated using three models: one with 4x4 cells, one with 8x8 cells and one with 16x16 cells. The results, presented in Table 6, show very little variation between resulting stiffness.
Table 6. Relative error for stiffness results in size effect study

| Number of cells | Relative error [%] |
|-----------------|--------------------|
| 16              | 0.045              |
| 64              | 0.009              |
| 256             | 0                  |

The simulation results for a 4x4 lattice for each structure, along with the experimental results is presented in Table 7.

Table 7. Simulation results and comparison with the experimental data

| Hexagon | t/l [-] | r/l [-] | E Simulation [MPa] | E Experimental [MPa] | Error [%] |
|---------|---------|---------|--------------------|----------------------|-----------|
|         | 0.582   | 0       | 12566.64           | 13230                | -5.014    |
|         | 0.553   | 2       | 12600              |                      | -4.762    |
|         | 0.39    | 6       | 11625.41           |                      | -12.13    |

| Octagon with squares | t/l [-] | r/l [-] | E Simulation [MPa] | 13230 | Error [%] |
|----------------------|---------|---------|--------------------|-------|-----------|
|                      | 0.131   | 0       | 11170.62           |       | -15.56    |
|                      | 0.124   | 2       | 11077.51           |       | -16.27    |
|                      | 0.102   | 4       | 9897.95            |       | -25.18    |

| Square cell with circular orifices | d/L [-] | E Simulation [MPa] | 9306.92 | Error [%] |
|-----------------------------------|---------|--------------------|--------|-----------|
|                                   | 0.482   |                    |        | -29.65    |

5. Discussions and conclusion

In this study, the prediction of the elastic properties of high-porosity alumina foams was investigated for three different parametric geometries: hexagonal lattice, octagonal/quadratic lattice and squared cell lattice with circular orifices. For all the studied parameters, the stiffness resulted from the numerical analyses is lower than the value determined experimentally, with errors ranging from around 5% to 30%.

The structure that yielded the most accurate results was the one based on the hexagonal lattice. It was also observed that the lowest errors were determined for low values of the fillet radius to strut length ratio r/l, a value of 0.2 resulting in an error of -4.76%.

6. Bibliography

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