Suppressed-gap millimetre wave kinetic inductance detectors using DC-bias current

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(Dated: January 24, 2020)

In this study, we evaluate the suitability of using DC-biased aluminium resonators as low-frequency kinetic inductance detectors capable of operating in the frequency range of 50 - 120 GHz. Our analysis routine for supercurrent-biased resonators is based on the Usadel equations and gives outputs including density of states, complex conductivities, transmission line properties, and quasiparticle lifetimes. Results from our analysis confirm previous experimental observations on resonant frequency tuneability and retention of high quality factor. Crucially, our analysis suggests that DC-biased resonators demonstrate significantly suppressed superconducting density of states gap. Consequently these resonators have lower frequency detection threshold and are suitable materials for low-frequency kinetic inductance detectors.

Keywords: DC-bias, superconducting resonators, kinetic inductance detectors

I. INTRODUCTION

Kinetic inductance detectors (KIDs) are ultra-sensitive cryogenic detectors based on high-quality thin-film superconducting resonators. They can be straightforwardly multiplexed in the frequency domain, thereby allowing thousands of detectors to be read out by a common transmission line [1, 2]. These detectors are readily fabricated using conventional ultra-high vacuum deposition techniques, and have demonstrated potential to be extensively applied in the areas of astronomy observations across the electromagnetic spectrum [3–6], neutrinoless double-beta decay experiments [7, 8], and general-purpose terahertz imaging [11].

The detection mechanism of KIDs requires the incoming photon to have sufficient energy to break a Cooper pair into quasiparticles, i.e. $\hbar \omega \geq \hbar \omega_{\text{min}} = 2 \Delta_g$, where $\hbar$ is the reduced Planck constant, $\omega$ is the angular frequency of radiation, $\omega_{\text{min}}$ is the angular frequency detection threshold, and $\Delta_g$ is the superconducting density of states energy gap. As a result of the frequency detection threshold, significant difficulty arises when the KIDs technology is applied to the detection of low-frequency millimetre wave signals, such as Cosmic Microwave Background radiation in the frequency range of 70 - 120 GHz [12, 13], low red-shifted CO lines in the range of 100 – 110 GHz [14, 15], and O2 rotation lines at 50 – 60 GHz for atmospheric profiling [17, 18]. KIDs based on elemental superconductors are unable to simultaneously achieve high resonator quality factors as well as low frequency detection thresholds. Aluminium (Al), for example, has a strongly suppressed detector response below 100 GHz [13]. To address the scientific need for low frequency KIDs, two alternative solutions have been explored: the usage of alloy superconductors, such as aluminium manganese (AlMn) [21] and titanium nitride (TiN) [22, 23], as well as the usage of multi-metallic-layer superconductors [13, 14]. The alloy approach, in general, suffers from variations in material properties, even in a single deposition [21]. Significant improvement in material uniformity has been demonstrated through the use of multilayer in conjunction with alloys [23]. In contrast, the use of multi-elemental-metal-layer has the additional advantage of theoretical predictability in $\Delta_g$ through the application of the Usadel equations based on the BCS theory of superconductivity [16, 25], as well as predictability in electrical and optical properties through the application of the Mattis-Bardeen theory [27] (in contrast with TiN alloy which cannot be modelled using the Mattis-Bardeen theory [27]). In this paper, we explore a third approach to the problem of low-frequency detection through the introduction of DC-bias currents to KIDs.

Various design schemes have been proposed and studied to introduce DC-bias currents to superconducting microwave resonators [28, 33]. The context of these previous works include circuit quantum electro dynamics systems [34, 35], back-action-evading quantum measurement systems [36], and high sensitivity photo detection systems [1, 57]. These studies are motivated by the tuneable resonant frequencies [32, 33] and the tuneable Josephson junction inductances [29] of the biased devices. This present study is distinct from previous studies in exploiting the $\Delta_g$ suppression effect of bias currents. Introducing a DC-bias to a resonator such as KID without lowering its quality factor is an experimental challenge [29, 33]. Hitherto, studies of DC-biased resonators have been mainly focused on their experimental realizations. Successful schemes have been developed for coplanar waveguides [28, 32] as well as for microstrip transmission lines [33]. In this work, we present a numerical analysis of DC-biased KIDs in terms of density of states, complex conductivity, transmission line quality factor, and quasiparticle lifetime. We explain various features in previous experimental studies such as frequency tuneability and high quality factors across a wide range of bias currents. Our results show that DC-biased KIDs have lower frequency detection thresholds due to the suppression of $\Delta_g$ in the presence of supercurrents. This opens up the possibility of using DC-biased KIDs to fulfill the current scientific need for low-frequency ultra-sensitive detector systems.
II. ANALYSIS ROUTINE

TABLE I. Table of material properties.

| Property | Aluminium |
|----------|-----------|
| $T_c$ (K) | 1.25$^a$ |
| $\sigma_N$ (MS/m) | $132^a$ |
| $N_0$ (10$^{17}$/m$^3$) | 1.45$^d$ |
| $D$ (m$s^{-1}$) | 35$^d$ |
| $\xi$ (nm) | 189$^a$ |
| $\Theta_D$ (K) | 423$^d$ |
| $\tau_0$ (ns) | 395$^a$ |

$^a$ $T_c$ is the superconductor critical temperature. Value is measured.
$^b$ $\sigma_N$ is the normal state conductivity.
$^c$ $N_0$ is the normal state electron density of states, and is calculated from the free electron model.[25]
$^d$ Diffusivity constant $D$ is calculated using

\[ D = \frac{\sigma_N}{(N_0)^2}. \]

$^e$ Coherence length $\xi$ is calculated using

\[ \xi = \left[\frac{\hbar D}{(2\pi k_BT_c)}\right]^{1/2} \]

where $\hbar$ is the Boltzmann constant.

$^1$ $\Theta_D$ is the Debye temperature, and is given by $k_b\Theta_D = \hbar\omega_D$, where $\omega_D$ is the Debye frequency. Value is taken from [43].

$^2$ $\epsilon_{ph}$ is the characteristic electron-phonon coupling time. Value is taken from [42].

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The Usadel equations[25] are used to obtain the superconducting densities of states (DoSs) in the presence of supercurrent.

Nam’s equations[43] are used to compute the complex conductivities $\sigma = \sigma_1 - i\sigma_2$ from the superconducting DoSs.

Complex surface impedances $Z_s = R_s + j\omega L_s$ are computed using

\[ Z_s = \left(\frac{j\omega\mu_0}{\sigma}\right)^{1/2} \coth[(j\omega\mu_0\sigma)^{1/2}t], \]

where $j$ is the unit imaginary number, $t$ is the thickness of the superconducting film, $\mu_0$ is the vacuum permeability, and $\omega$ is the angular frequency of the signal of interest.[43][44]

Transmission line properties are computed using suitable conformal mapping results for the specific resonator geometry.[46] The series impedance and shunt admittance are given by

\[ Z = j(k_0\eta_0)g_1 + 2\sum_{n} g_{2,n} Z_{s,n} = R + j\omega L \]  \hspace{1cm} (2)

\[ Y = j \left(\frac{k_0}{\eta_0} \right) \left(\frac{\epsilon_{ph}}{g_1}\right) = G + j\omega C, \]  \hspace{1cm} (3)

where $k_0$ is the free-space wavenumber, $\eta_0$ is the impedance of free-space, subscript $n$ identifies superconductor surfaces, which are upper, lower conductor surfaces, and ground surfaces of the transmission line, denoted by subscripts $u, l$, and $g$ respectively, $\epsilon_{ph}$ is the effective modal dielectric constant, which is given by existing normal conductor transmission line theories, for example.[47][48] $g_1$ and $g_2$ are geometric factors which are calculated using appropriate conformal mapping results from[46]. $R, G, L, C$ are the resistance, conductance, inductance, and capacitance, per unit length, respectively.

5. Quasiparticle recombination lifetimes are computed using the low-energy expression given in[49], and the energy-averaged recombination lifetimes are then calculated according to the weighted-average procedure given in[16]. Explicit equations used in each numerical component are given with more details in[16][50]. The results in the next section are obtained by applying this analysis routine to DC-biased coplanar waveguide (CPW) KIDs based on Al. Here we have adopted a basic model for KIDs which assumes that the same superconducting material (Al) is responsible both for photon absorption as well as for readout resonance. The properties of Al used in this analysis are taken from a previous study[51], and are shown in Table I. The modelled CPW geometry has inner half-width $a = 1.0 \mu m$, gap width $b - a = 0.5 \mu m$, thickness $t = 20 \text{nm}$, dielectric height $h = 225 \mu m$, dielectric constant $\epsilon_r = 11.7$, and dielectric quality factor $Q_e = 10^5$ in accordance with measured values for silicon at cryogenic temperatures[52]. KIDs are typically read out by a microwave probe operating at $1 - 10 \text{GHz}$[16][53]. As such, results in the next section are calculated using a readout frequency $f_r$ of 10 GHz.

III. RESULTS

Figure[16] shows the superconducting density of states $N/N_0$ of Al against energy $E/k_B$ for different values of normalized supercurrent depairing factor $\Gamma/\Delta_0$, where $\Delta_0 = 1.764k_BT_c$. As seen in the figure, the shape of the DoS is broadened and the DoS gap is suppressed in the presence of bias current. This effect on the superconducting DoSs has been observed in previous experimental data[52]. In this study we express bias current in terms of supercurrent depairing factor $\Gamma/\Delta_0$. This is because $\Gamma/\Delta_0$ comes out naturally from the Usadel equations and
is not device geometry/material dependent. The conversion between $\Gamma/\Delta_0$ and physical supercurrent $I$ can be done using equation (7) of [53]. We have plotted this conversion in the inset of figure 1, which shows scaled supercurrent $I/I_r$ against supercurrent depairing factor $\Gamma/\Delta_0$. The critical current is given by $I_c = \sqrt{2} S \Delta_0 \sigma_N (e \xi)$, where $S$ is the cross-section area of the resonator, $\sigma_N$ is the normal state conductivity, $e$ is the electron charge, and $\xi$ is the material coherence length. The critical current is given by $I_c \approx 0.53 I_r$ [53]. For $T \ll T_c$, $I_r = 0$ when $\Gamma/\Delta_0 = 0$ and $I_r = I_c$ when $\Gamma/\Delta_0 = 0.25$. For Al, using parameters given in Table I, $I_r/S \approx 1.8 \times 10^{11} \text{A}/\text{m}^2$ and $I_c/S \approx 9.6 \times 10^{10} \text{A}/\text{m}^2$.

Figure 2 and figure 3 show the real (dissipative) and imaginary (reactive) components of the complex conductivity respectively against frequency, for different values of DC-bias. The shift in reactive conductivity $\sigma_2/\sigma_N$ at readout frequencies is responsible for the shift in resonant frequency. As seen in figure 3, $\sigma_2/\sigma_N$ is suppressed in the presence of supercurrent. This in turn results in a boost in surface inductance $L_s$ and transmission line inductance $L$ through equation (11,2). The increased $L$ then results in a lowering of the resonant frequency, central to the operation of frequency-tunable resonators experimentally demonstrated in [32, 33]. The shift in the gap of dissipative conductivity $\sigma_1/\sigma_N$, on the other hand, is responsible for the lowering of the frequency detection threshold as $\sigma_1/\sigma_N$ is the absorption ratio for electromagnetic radiation [53]. As seen in figure 2 photon detection is possible at lower frequencies in the presence of supercurrent.

Figure 4 shows the dependence of normalized $\sigma_2/\sigma_0$ (red line) and normalized $\Delta_g/\Delta_0$ (blue line) against $\Gamma/\Delta_0$ at readout frequency $f_r = 10 \text{GHz}$, with the normalization factor defined as $\sigma_0 = \sigma_2(\Gamma = 0 \text{K}) \approx \pi \Delta_0/(\hbar \omega)$ and $\Delta_0 = \Delta_0(\Gamma = 0 \text{K}) \approx 1.764 k_B T_c$. As seen in the figure, the extent of shift in $\Delta_g/\Delta_0$ is much greater compared to $\sigma_2/\sigma_0$. This is because the DoS gaps are the furthest shifted points on the DoSs, whereas the conductivities are energy integrals over functions of DoSs. From figure 4 we can predict the amount of gap suppression given a known tuneability on the resonant frequency. For example, [32] reports a 4% tuneability in frequency. Assuming the applicability of the Mattis-Bardeen theory, and that the kinetic inductance dominates the contributions to total device inductance, we estimate $\sigma_2/\sigma_0 \approx 0.92$. Using figure 4 we expect
\[ \Delta_g/\Delta_0 \approx 0.72, \text{ i.e. the frequency detection threshold is expected to be suppressed by almost 25\%. If similar or better performance could be translated to Al resonators, we expect DC-biasing to greatly extend the application of Al KIDs.} \]

Figure 4 shows resonant frequency \( f_{\text{res}} \) against supercurrent \( I \) for the Al CPW with dimensions described in the previous section. This tuneability in resonant frequency is the subject and motivation of previous studies on DC-biased resonators [32, 33]. It is important to note that the quantitative dependence is specific to the geometry of the resonator. The DC-bias affects only the kinetic inductance but not the geometric inductance. As such, a device with a higher kinetic inductance to geometric inductance ratio will, in general, demonstrate greater maximum tuneability. Design wise, transmission line theories such as [46] can be used to improve this ratio.

One important consideration in the design of low frequency KIDs for applications requiring high detector sensitivity is the quasiparticle recombination lifetime which governs the trade-off between detector response time and recombination noise [2, 55]. Figure 7 shows the recombination lifetime \( \tau_r \) against frequency for different values of bias current. The inset shows the energy averaged recombination lifetime \( \langle \tau_r \rangle_E \) against \( \Gamma/\Delta_0 \). This calculation is performed at \( T = 0.01 \text{ K} \), close to the saturation point of quasiparticle lifetime for Al [56]. The presence of supercurrent decreases the recombination lifetime across the energy range.

Another important consideration in evaluating the suitability of DC-biased KIDs is the quasiparticle recombination lifetime which governs the trade-off between detector response time and recombination noise [2, 55]. Figure 7 shows the recombination lifetime \( \tau_r \) against frequency for different values of bias current. The inset shows the energy averaged recombination lifetime \( \langle \tau_r \rangle_E \) against \( \Gamma/\Delta_0 \). This calculation is performed at \( T = 0.15 \text{ K} \), close to the saturation point of quasiparticle lifetime for Al [56]. The presence of supercurrent decreases the recombination lifetime across the energy range.
spectrum. The inset shows that the energy-averaged lifetime has an inverse exponential dependence on the depairing factor, which is proportional to the squared current. At $T << T_c$, the recombination lifetime also has an inverse exponential dependence on $T$ (57). This suggests the possibility of interpreting the effect of depairing current on quasiparticle lifetime as raising the effective temperature. It is important to note that the lifetime calculation presented here assumes BCS-like behavior from the superconductors. It is well documented that quasiparticle lifetime plateaus and deviates from BCS predictions at low temperatures $T/T_c < 0.15$ (59). Future experimental studies should be conducted to determine the low temperature lifetime behaviour of DC-biased KIDs: whether the DC-bias lowers the overall low-temperature lifetime curve (thereby lowering the low temperature saturation plateau), or whether the DC-bias effect can be account by an adjusted effective temperature along an existing lifetime curve (without changing the low temperature plateau height). In the first case more optimization may be needed to suit different applications, and in the second case the device can be operated simply as regular Al KIDs in terms of recombination lifetime.

To illustrate the gap-suppression characteristic of a biased KID, we have calculated the fractional power dissipated $P_d/P_i$ by a DC-biased Al CPW with length $l = 5$ mm fed by a non-biased Al CPW of the same dimensions, terminated at a matched load. Here $P_i$ is the incident power and $P_d$ is the dissipated power. $P_d/P_i$ is calculated from applying the transmission line dissipation propagation constant $\alpha$ over the length of the resonator (48), after taking into account the reflection off the interface between the non-biased feedline and the biased resonator. Figure 8 shows the signal frequency $f$ dependence of $P_d/P_i$. As seen in the figure, the power dissipated in the resonator increases sharply above the DoS gaps. The dissipated power $P_d$ increases with transmission line length $l$ according to $P_d/P_i = 1 - e^{-2\alpha l}$ (46, 58), where $P_i$ is the transmitted power across the boundary between the non-biased CPW and the biased CPW. This scaling relation has the important consequence that the steepness of the rise in dissipation with frequency can be further increased through the use of longer resonator lengths. The inset of figure 8 shows the bias current $I$ dependence of $P_d/P_i$ at frequency $f = 70$ GHz. The power dissipation is initially nearly zero due to the absence of significant superconductor loss at sub-pair-breaking frequency. As $I$ increases, pair-breaking frequency is reduced, and significant dissipation sets in when the pair-breaking frequency is suppressed below the signal frequency.

IV. CONCLUSION

We have studied theoretically and numerically the feasibility of operating KIDs below their unbiased density of states gaps through DC-biasing the superconducting thin-films used for photon detection. Our numerical analysis is based on the Usadel equations and evaluates detector performance in terms of density of states, complex conductivities, transmission line quality factors, and quasiparticle lifetimes. Our results confirm previous experimental observations on the tuneability of the resonant frequencies and the high quality factors of DC-biased resonators. Our analysis further predicts significant suppression in the frequency threshold of pho-
ton detection in the presence of DC-bias current. This phenomenon allows DC-biased resonators to be used as KIDs to fulfil the scientific need for high sensitivity, low-frequency threshold photon detectors. One important effect observed by previous experimental studies is the sharp onset of dissipation at high current values (but below the theoretical critical current), resulting in deviations from ideal calculations. This phenomenon places a limit on the maximum tuneability of resonant frequency and detection threshold. Future investigations should be conducted to extend the range of resonant frequency and detection threshold tuneability before the onset of significant dissipation, for examples, by decreasing the cross-sectional dimensions of the resonators. Lastly, our numerical analysis shows that low-frequency 50 – 120 GHz Al KIDs with high quality factors can be made by incorporating DC-bias schemes. In view of this, experimental realizations of DC-biased Al KIDs should be conducted to directly measure the suppression of frequency detection thresholds and to characterise their detector performance.
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