An Efficient of Chimera Grid Scheme with Spline Interpolation in FBTS Inversion Technique for Extremely Dense Breast Cancer Detection

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Abstract. Microwave imaging system is classified as non-invasive, simple to perform and inexpensive compared to MRI and X-Ray machine. Therefore, the novel idea of this research work is to develop a Chimera Grid Scheme (CGS) incorporate with the microwave inverse scattering technique in a low cost, non-ionising and safe short-range. The CGS with spline interpolation in Forward Backward Time-Stepping (FBTS) inversion technique can determined an accurate result especially for the biological anomalies like breast tumours at an early curable stage due to the high electrical properties contrast between malignant cells and normal cells. The findings showed that the proposed method successfully detected and reconstructed the breast structure in relative permittivity profiles. The quantitative information of reconstructed images, such as location, shape, size and internal composition also can be obtained. Furthermore, the normalised functional error for proposed method was also lower than the FDTD method in FBTS. At 150\(^{th}\) iteration, the difference of normalised functional error between these two methods was 1.74 x 10\(^{-5}\). The result shows that the CGS method in FBTS inversion technique would reconstruct breast composition more precisely.

1. Introduction

The World Health Organisation (WHO) stated that an approximately of 18.1 million new cases and 9.6 million deaths annually caused by global cancer burden [1]. Breast cancer is a most commonly health problem and leading cause of death for women [2]. In a biological aspect, human body cells divide, grow and die in an orderly manner, whereas cancer cells grow due to uncontrolled causes such as heredity, disease as well as external and internal factors [3]. These kinds of cells will form a mass or lump known as a tumour that can be either benign or malignant. There are two main factors that cause cancer-related diseases, specifically: external factors such as infectious organisms, cigarette smoking, excess body weight and unhealthy diet; or internal factors such as immune conditions, inherited genetic changes, and hormones.

As shown in Figure 1(a), benign tumours are an abnormal growth of breast tissues and are not cancerous (e.g. fibro-adenoma, cyst, abscess, fibrocystic disease, and others). These types of tumours can be cured permanently after removal. However, malignant tumours are cancerous. This kind of tumour is an arbitrary shape with a dimpled skin as illustrated in Figure 1(b). Malignant tumours can spread to different organs beyond the original tumour and may be difficult to diagnose due to confusion with other breast problems.
Screening and diagnosis methods are very important to finding out the cause of a health problem. Breast cancer screening should be done by every women whether symptomatic or asymptomatic to detect the growth of breast cancer earlier. The most common screening and diagnosis tools to detect the breast diseases are X-ray Mammography, Ultrasound, MRI, and Scintimammography. X-ray Mammography is the specialised non-invasive medical imaging and as a primary tool that used to screen the breasts disease. National Institute for Health and Care Excellence (NICE) suggested an annually screening once diagnosed with primary breast cancer for women less than 50 years old, whereas, if diagnosed at the age of older than 50 years old screening has to be done annually for minimum of 5 years. Although X-ray Mammography has the ability to detect a small abnormalities in breast tissue, it has difficulties in detecting an early stage breast tumours and in a dense breast tissue of young patients [5]. According to the reports, X-ray mammography has a high misses and false-detection rates. The rate ranging for false positive is from 2.6% to 15.9%, and false negative is from 12% to 37%. Besides that, the patients will feel uncomfortable and perhaps painful when compression of the breast to get the clearest image. Additionally, mammography suffers from variability of interpretation by radiologist, and potentially induce cancer due to the ionising x-rays radiations, so frequent screening is not advisable.

Microwave imaging techniques have shown excellent capabilities in various fields especially for biomedical diagnostic. This technique has a significant difference in dielectric properties between healthy and cancer tissues, so it is suitable for use in the detection of breast tumours [6-8]. Furthermore, this technique also is low cost, comfort, safety, non-ionising and allow recurring screening without any impact on the patient health. Microwave imaging system able to generate the breast structure that includes skin, fat and fibro-glandular regions. This research focuses on the development of a new numerical method for extremely dense breast cancer detection. Therefore, the Chimera Grid Scheme method with spline interpolation in FBTS inversion technique was proposed extremely dense breast cancer detection. By overlapping the unstructured minor-grid on a major-grid, each grid is determined alternately by using spline interpolation technique. The unknown values at boundary points in the overlapping area are calculated. This interpolation technique is more effective as compared to other interpolations due to it can produce more accurate, high resolution and efficient interpolation [9, 10]. Thus, it able to determine the curved boundaries and small features for imaging the breast composition.
2. Method

2.1 Electromagnetic Computational Techniques

The techniques for solving the Electromagnetic (EM) field problems can be classified as analytical, experimental or numerical. The numerical solution approximates an analytical solution through the transformation of the continuous domain into a discrete domain. The most popular numerical techniques to solve the EM field problems are Finite-Difference method (FDM) [11], Finite Element method (FEM) [12], Method of Moments (MoM) [13] and Partial Element Equivalent Circuit (PEEC) method [14]. The main features of the FDM, FEM, MoM and PEEC can be found in [15]. FDM was proposed in this research due to it has several advantages. It is widely used within EM modelling mainly due to its easy to implement numerical method [16]. FDM allows local mesh condensation and promotes the discretization of boundary conditions in the case of arbitrary domain shapes. It is applicable in both the time and frequency domain [17].

The novelty of Finite-Difference Time-Domain method was introduced by Kane S. Yee in [18]. The advantages of FDTD method are simplicity, generality, robustness and most powerful numerical tool for modelling computational electrodynamics [19]. However, the major drawback of the FDTD method is based on a Cartesian coordinate system, and it is hard to exactly generate meshes for electromagnetic structures with curved boundaries and small features due to its restriction to inherent orthogonal grids [20]. Hence, the Chimera Grid Scheme method with spline interpolation in the FBTS technique have been introduced as a new numerical approach to address the limitation of the FDTD system. Details regarding this new method will be discussed in section 2.2.

2.2 CGS-FDTD method with spline interpolation

The Chimera Grid Scheme (CGS) is a grid embedding technique also known as Overset Grid Generation (OGG), which provides a conceptually simple method for the decomposition of domains [21]. The Chimera grid scheme is generally classified into the composite structured grid category, which uses overset grids to solve complex geometry or flow features. A major grid is generated for the main component; a minor grid is then overridden on the main grid to resolve interesting configuration features as shown in Figure 2(a). The four points of biquadratic spline interpolation technique is illustrated in Figure 2(b). The points of \( \varnothing_1, \varnothing_2, \varnothing_3 \) and \( \varnothing_4 \) are the four known values points from major grid while the point \( \varnothing_s \) is unknown value of the overset minor grid. The unknown values can be obtained via biquadratic spline or \( B_2 \)-spline interpolation at the boundary points in the overlapping region as Equation (1):-

\[
\varnothing_s = \left[ \frac{d_{i+1} - d_i}{2(y_B - y_A)} \right] (y_s - y_A)^2 + d_i (y_s - y_A) + \varnothing_A \\
+ \frac{d_i}{2(y_B - y_A)} x_s - x_A + \varnothing_A
\]

where,

\[
d_j = 0, d_{i+1} = \frac{2(\varnothing_B - \varnothing_A)}{y_B - y_A} - d_j, \quad j = 0, 1, 2, \ldots, m
\]

\[
\varnothing_A = \left[ \frac{d_i + d_{i-1}}{2(x_2 - x_1)} \right] (x_A - x_1)^2 + d_i (x_A - x_1) + \varnothing_1
\]

\[
d_i = 0, d_{i+1} = \frac{2(\varnothing_2 - \varnothing_1)}{x_2 - x_1} - d_i, \quad i = 0, 1, 2, \ldots, n
\]

\[
\varnothing_B = \left[ \frac{d_{i+1} - d_i}{2(x_B - x_3)} \right] (x_B - x_3)^2 + d_i (x_B - x_3) + \varnothing_3
\]

\[
d_i = 0, d_{i+1} = \frac{2(\varnothing_4 - \varnothing_3)}{x_4 - x_3} - d_i, \quad i = 0, 1, 2, \ldots, n
\]
Figure 2(c) describes the procedural steps used for the CGS method with spline interpolation. Details regarding CGS method with spline interpolation can be found in [22].

![Diagram of CGS method with spline interpolation]

**Figure 2.** CGS method with spline interpolation technique

### 2.3 CGS method with spline interpolation in FBTS inversion technique

Figure 3 shows the block diagram of CGS method with spline interpolation in FBTS inversion technique. All parameters required in this simulation were declared and the values were set. Then, the process of measurement for the scattered field was setup. The actual profiles were contained the dielectric properties for realistic breast phantom and Region of Interest (ROI) as shown in Figure 4. Sixteen (16) point sources were represented as antennas that encircled the ROI. When the transmitter excited Gaussian pulse to the test subject, the measurement data at the receiving point is collected and stored.

Forward-Backward Time-Stepping (FBTS) is a unique technique for solving EM inversion scattering issues in time-domain by utilising broadband microwave signals. FBTS has the potential to generate images that provide useful quantitative information about the internal structure of the breast.
such as the shape and composition of fibroglandular tissue regions [23]. The use of a broadband signal in FBTS enable large amount of scattering information about the breast to be collected and allowing stable reconstructions of relatively high-resolution images.

The FBTS inversion technique begins with the process of forward stepping, two-dimensional CGS lattice in Transverse Magnetic z-plane (TMz) wave numerical simulations are carried out to obtain synthetic scattering data as measured scattering data. 16 points represented as antennas enircles the breast phantom, each of the 16 points acts as transmitter sequentially represented by a simple soft source in CGS simulation with the electric fields being sampled at the remaining 15 points. This set of scattering calculations representing 240 transmitter/receiver combinations formed a set of measured data. When the backward step of the FBTS inversion technique, the CGS method with biquadratic spline interpolation was used to calculate the adjoint fields when the different of the forward time-stepping reconstructions data and measurement data at the receiving points is propagated back to the estimated profile of the breast imaging. Then, the adjoint fields are utilised for the Conjugate Gradient (CG) method. The conjugate gradient minimisation methods by Fletcher-Reeves and Polak-Ribiere-Polyak were utilising in this research. It can be used for optimal control of machine computation problems and also to minimise the functional error [24].

As shown in Equation (2), the optimisation problem is formulated in the form of cost functional for the assumed set of dielectric parameters p.

$$Q(p) = \int_0^T \sum_{m=1}^M \sum_{n=1}^N K_{mn}(t) |v_m(p; r_n^r, t) - \tilde{v}_m(r_n^r, t)|^2 dt$$

where $K_{mn}(t)$ is a non-negative weighting function to give fixed reduction influence on the cost functional. $v_m(p; r_n^r, t)$ is the measured scattering E-field data at a receiving antennas $n$ due to a transmitter $m$. $\tilde{v}_m(p; r_n^r, t)$ is the computed scattering E-field data for the assumed set of parameters $p$. The cost functional was computed by comparing the estimated profiles to the actual profiles until the difference between both profiles are small. The FBTS method was repeated until the cost functional was convergence. Lastly, the two dimensional reconstruction images for Relative Permittivity was plotted by using MATLAB.

3. Numerical model setup

In this paper, the extremely dense breast phantom was selected to evaluate the validation of the proposed numerical method on breast tumour detection. The Class D breast is also recognised as extremely dense breast which contains a lot of dense glandular and fibrous tissue. This may make it hard to see a cancer on a mammogram because the cancer can blend in with the normal tissue.
In this research, the numerical extremely dense breast phantom was retrieved from University of Wisconsin-Madison Computer Electromagnetics (UMCEM) numerical phantom repository [25]. The retrieved three-dimensional (3D) numerical breast phantom is pre-processed in order to extract a suitable coronal slice of two-dimensional (2D) model for the investigations. The major grid was set to 190 mm×190 mm grids and the overset minor grid was set to 80 mm×120 mm grids. The overset minor grid was modelled as an entire numerical breast phantom. A tumour of 4.0 mm radius was added in the fibroglandular region of the numerical breast model. Reconstructions were conducted utilising a 1.0 mm×1.0 mm CGS grid size. The optimisation was carried out up to 150 iterations in order to reconstruct the microwave breast images. The actual value of relative permittivity for tumour is $\varepsilon_r = 63.00$ F/m. The feasibility of the extremely dense breast is immersed in a free space. The utilisation of free space for background medium is necessary for easy maintenance for microwave mammography equipment and simple process during the breast screening process. Table 1 demonstrates the non-dispersive of electronic property parameter for breast model. Based on these parameters, the difference of dielectric properties between tumour and fibroglandular tissue is significantly higher.

**Table 1.** Electronic property parameters utilised for breast model [24].

| Tissue               | Non-dispersive $\varepsilon_r$ (2 GHz) | $\sigma$ (2 GHz) |
|----------------------|----------------------------------------|------------------|
| Fibroglandular       | 21.45                                  | 0.46             |
| Fat                  | 9.98                                   | 0.18             |
| Skin                 | 36.73                                  | 1.43             |
| Tumour/chest wall    | 53.62                                  | 1.19             |
4. Results and discussion

![Figure 5](image)

**Figure 5.** Original and reconstructed profiles of the extremely dense breast phantom: (a) original relative permittivity profile, (b) Reconstructed relative permittivity profile for FDTD method in FBTS, (c) Reconstructed relative permittivity profile for CGS method with spline interpolation in FBTS.

The original profiles of non-dispersive relative permittivity profile of an extremely dense realistic breast phantom as shown in Figure 5(a). Figure 5(b) shows the reconstructed relative permittivity profile of the FDTD method in FBTS. This inversion technique is unable to reconstruct the tumour embedded in the fibroglandular region. To address this issue, a new numerical method based on the CGS method with spline interpolation in FBTS inversion technique was proposed to obtain the presence and position of malignant tumour in the breast. A shown in Figure 5(c), the proposed method successfully detected and reconstructed the breast structure in relative permittivity profile includes skin, adipose and fibroglandular regions. Reconstructed image clearly indicated the value of the dielectric parameter with a radius of 4 mm tumour placed within the fibroglandular region as well as its location and size.

![Figure 6](image)

**Figure 6.** Cross-sectional view of original and reconstructed relative permittivity profile along y-axis at x=118

Figure 6 illustrates a cross-sectional view of original and reconstructed profiles. This figure has shown that the high contrast of dielectric properties between the healthy and cancerous breast tissues.
at position \( y = 105 \) mm. The peak value of tumour for FDTD method in FBTS is \( \epsilon_{\text{max}} = 53.16 \, \text{F/m} \) while the CGS method with spline interpolation in FBTS is \( \epsilon_{\text{max}} = 68.16 \, \text{F/m} \). Hence, the peak value of tumour for the proposed numerical method is more similar to the actual values of \( \epsilon_{\text{max}} = 63.00 \, \text{F/m} \) as compared to the FDTD method in FBTS.

The normalised functional error versus number of iterations is shown in Figure 7. The gradient optimisation was carried out until 150 iterations in order to reconstruct the extremely dense breast model. The normalised functional error will be decreased, when the number of iterations is increased. The normalised functional error for proposed numerical method is less than the FDTD method FBTS. At 150\(^{\text{th}}\) iteration, the difference of normalised functional error between these two methods was \( 1.74 \times 10^{-5} \). The results show that the CGS method with spline interpolation in FBTS inversion technique can reconstruct the breast composition accurately.

![Figure 7](image.png)

**Figure 7.** Normalised functional error versus number of iterations

5. **Conclusion**

In this research, the CGS method with spline interpolation in FBTS inversion technique has been developed for imaging the interior of an extremely dense breast composition. The potential for using microwaves in the detection of breast tumours is based on the concept of tissue-dependent microwave scattering and absorption in the breast to exploit the contrast in the dielectric properties of malignant and healthy breast tissues. The numerical results showed that the proposed method has the ability to accurately determine the useful quantitative information such as location, shape, dimension and dielectric properties of the breast composition. This proposed numerical method can improve the quality of reconstructed image and the location for the tumour also can be satisfactorily detected.

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References

[1] American Cancer Society 2020 Cancer facts & figures 2020, viewed 8 July 2020, <https://www.cancer.org/content/dam/cancer-org/research/cancer-facts-and-statistics/annual-cancer-facts-and-figures/2020/cancer-facts-and-figures-2020.pdf>.

[2] DeSantis C E, Ma J, Gaudet M M, Newman L A, Miller K D, Goding Sauer A, Jemal A and Siegel R L 2019 Breast cancer statistics CA: a cancer journal for clinicians 69(6) 438-451.

[3] Kome S G 2013 Facts for life: what is breast cancer, viewed 2 July 2020, <http://www5.komen.org/uploadedFiles_Komen/Content/About_Breast_Cancer/Tools_and_Resources/Fact_Sheets_and_Breast_Shelf_Awareness_Cards/What%20is%20Breast%20Cancer.pdf>.

[4] Cart P H 2012 Different types of breast lumps, viewed 24 September 2019, <http://www.personalhealthcart.com/different-types-of-breast-lumps>.

[5] Brennan M and Houssami N 2016 Discussing the benefits and harms of screening mammography Maturitas 92 150-153.

[6] Nguyen P T, Abbosh A and Crozier S 2017 Three-dimensional microwave hyperthermia for breast cancer treatment in a realistic environment using particle swarm optimization IEEE Transactions on Biomedical Engineering 64(6) 1335-1344.

[7] Irishina N, Moscoso M and Dorn O 2009 Microwave imaging for early breast cancer detection using a shape-based strategy IEEE Transactions on Biomedical engineering 56(4) 1143-1153.

[8] Hossain M D and Mohan A S 2017 Cancer detection in highly dense breasts using coherently focused time-reversal microwave imaging IEEE Transactions on Computational Imaging 3(4) 928-939.

[9] Boor C D 1978 A practical guide to spline1978 (New York: Springer-Verlag).

[10] Singh M R and Bhide A S 2016 A review of image retrieval using different types of interpolation techniques International Research Journal of Engineering and Technology (IRJET) 3(12) 1423-1426.

[11] Erfanian M and H Zeidabadi 2020 Finite difference method for solving partial integro-differential equations Mathematical Researches 6(1) 399-402.

[12] Zuo S, Doñoro DG, Zhang Y, Bai Y and Zhao X 2019 Simulation of challenging electromagnetic problems using a massively parallel finite element method solver IEEE Access 7 20346-20362.

[13] Orujov G, Anderson E, Swidinsky A and Streich R 2019 Towards modeling the electromagnetic response of complex pipelines using the Method of Moments Society of Exploration Geophysicists 1070-1074.

[14] Voltolina D, Bettini P, Aloatto P, Moro F and Torchio R 2019 High-performance PEEC analysis of electromagnetic scatterers IEEE Transactions on Magnetics 55(6) 1-4.

[15] Ekman J 2003 Electromagnetic modeling using the partial element equivalent circuit method Thesis PhD in Computer Science and Electrical Engineering, University of Technology: Lulea, Sweden.

[16] Ji J, Yu M and Guo R 2020 Fast finite-difference time-domain (FDTD) method of two dimensional target scattering calculation by two-level hierarchical approach Optik 203 1-11.

[17] VonNeumann J and Richtmyer R D 1950 A method for the numerical calculation of hydrodynamic shocks Journal of applied physics 21(3) 232-237.

[18] Yee K S 1966 Numerical solution of initial boundary value problems involving Maxwell’s equations in isotropic media IEEE Transactions on Antennas and Propagation 14(3) 302-307.

[19] Schneide J B 2016 Understanding the finite-difference time-domain method Ph.D Thesis in electrical engineering and computer science, Washington State University.
[20] Jiménez-Mejia E and Herrera-Murcia J 2015 Validation of a non-uniform meshing algorithm for the 3D-FDTD method by means of a two-wire crosstalk experimental set-up Ingeniería e Investigación 35 98-103.

[21] Jiang W, Zhang Y and Yang A 2019 Numerical simulations of complex aircraft configurations using structured overset grids with implicit hole-cutting Aerospace Science and Technology 94 1-13.

[22] Wee B S, Sahrani S and Ping K A H 2018 $B_2$-spline interpolation technique for Overset Grid Generation and Finite-Difference Time-Domain Method Progress In Electromagnetics Research C 86 177-190.

[23] Wee B S, Ping K A H and Sahrani S 2020 Arbitrary shaped objects detection and reconstruction through Overset Grid Generation Method with $B_2$-spline interpolation in Forward-Backward Time-Stepping inverse scattering Applied Computational Electromagnetics Society Journal 35(3) 295-304.

[24] Johnson J E, Takenaka T, Ping K A H, Honda S and Tanaka T 2009 Advances in the 3-D Forward-Backward Time-Stepping (FBTS) inverse scattering technique for breast cancer detection IEEE Transactions on Biomedical Engineering 56(9) 2232-2243.

[25] University of Wisconsin Cross-Disciplinary Electromagnetics Laboratory 2020 Numerical Breast Phantom Repository, viewed 8 July 2020 <https://uwcem.ece.wisc.edu/phantomRepository.html>.