A Novel Multi-Camera Calibration Method based on Flat Refractive Geometry

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Abstract. Multi-camera calibration plays an important role in many field. In the paper, we present a novel multi-camera calibration method based on flat refractive geometry. All cameras can acquire calibration images of transparent glass calibration board (TGCB) at the same time. The application of TGCB leads to refractive phenomenon which can generate calibration error. The theory of flat refractive geometry is employed to eliminate the error. The new method can solve the refractive phenomenon of TGCB. Moreover, the bundle adjustment method is used to minimize the reprojection error and obtain optimized calibration results. Finally, the four-cameras calibration results of real data show that the mean value and standard deviation of the reprojection error of our method are 4.3411e-05 and 0.4553 pixel, respectively. The experimental results show that the proposed method is accurate and reliable.

1. Introduction

Multiple cameras calibration system is a key problem in computer vision research, and it has been become more and more attention recently. As camera calibration is indispensable step from two-dimensional (2D) images to extract three-dimensional (3D) space information, so this technology has been widely used in 3D space object measurement, virtual reality, 3D reconstruction, and other fields. At present, many kinds of calibration methods have been proposed in multi-camera system. For example, Pribanić used an orthogonally contacted 3-axis wand for multi-calibration [1,2]. Jia proposed an approach to multiple cameras calibrate based on LCD template [3]. Svoboda developed a useful multi-camera self-calibration method [4]. Hui presents a series of scanning calibration methods using 3D patterns [5,6]. Sun developed a global calibration of multiple cameras based on sphere targets, which can be easily manipulated and observed from different directions [7]. However, each method has its own limitation of the accurate and feasible.

To solve above problem, a new multi-camera calibration method based on flat refractive geometry is proposed in the paper. Besides, calibration targets usually use planar boards in metric calibration. In the paper, the TGCB is used with the help of calibration question. Compared with Traditional calibration methods, the new multi-camera calibration method has a higher accuracy and feasible and it can also effectively improve the quality of 3D reconstruction.

2. Camera model

2.1. camera projection model
The camera imaging model is the basis of the camera calibration. In the camera calibration, first of all, we need to know the image pixel coordinate system, image physical coordinate system, the camera coordinate system and the world coordinate system position relationship. \( uOv \) is established in the image coordinate system, the coordinates of each point on the image \((u, v)\) on behalf of each pixel in the image of the number of columns and rows. As \((u, v)\) corresponds to the value of the image at this point is the grey value, so \((u, v)\) called pixel coordinates, which \((u_0, v_0)\) is the pixel coordinates of the center point of the image.

Figure 1 shows the \(O_c - X_cY_cZ_c\) represents the camera coordinate system, where \(O_c\) is the optical center of the camera. \(X_c, Y_c\) are parallel to the \(x\) axes and \(y\) axes of the image coordinate system, respectively. The choice of the world coordinate system is generally based on the actual situation to carry on the reasonable choice, the \(O_w - X_wY_wZ_w\) represents the world coordinate system. The world coordinate system and the camera coordinate system transformation relations as follows:

\[
\begin{bmatrix}
X_c \\
Y_c \\
Z_c \\
1
\end{bmatrix} =
\begin{bmatrix}
R & t \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
X_w \\
Y_w \\
Z_w \\
1
\end{bmatrix}
\]

where, the vector \(t\) is the coordinate value of the origin of the world coordinate system in the camera coordinate system. \(R\) is the rotation matrix of the world coordinate system to the camera coordinate system.

The world coordinate system 3D point to image coordinate system 2D point is described by:

\[
\begin{bmatrix}
u \\
v \\
1
\end{bmatrix} = K
\begin{bmatrix}
R & t \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
X_w \\
Y_w \\
Z_w \\
1
\end{bmatrix}, K = \begin{bmatrix} f_u & 0 & u_0 \\ 0 & f_v & v_0 \\ 0 & 0 & 1 \end{bmatrix}
\]

where, the vector \(f_u\) and \(f_v\) are the focal length in the image axes \(u\) and \(v\).

2.2. transparent glass calibration board (TGCB)

The TGCB can provide the 3D coordinates of feature points, which are used as corner points. Several corner points are uniformly distributed on the TGCB.
In this paper, the TGCB shown in the Figure 2 is a standard planar template. Moreover, we can calibrate cameras with Zhang’s method [8]. This TGCB is simple to manufacture and easy to carry, therefore, it could be effectively applied to site calibration projects.

![Transparent glass calibration board](image)

**Figure 2.** Transparent glass calibration board (TGCB).

### 2.3. Flat refractive geometry

The flat refractive geometry is introduced to solve the refractive problem of the TGCB. The TGCB is used as calibration targets, which contains the refractive surface and non-refractive surface. Figure 3 shows the flat refractive projection model. The black and white grids pattern were printed on the side \( \lambda \) of the TGCB, and the camera of this side can directly observe black and white grids pattern. The camera on the other side can observe black and white grids pattern through the glass. Where, \( P_1 \) is the intersection of refractive surface and imaging ray. \( P_2 \) is the spatial 3D point coordinates, which assumed to be known. \( d_0 \) indicates the distance from the camera to the TGCB, and \( d_1 \) represents the thickness of the TGCB. \( r_0 \) denotes the direction vectors of light path in the air, and \( r_1 \) denotes the direction vectors of refractive light path in the glass. The camera's extrinsic parameter \([R \ t]\) is transformation between the world coordinate system \( O_w \) and the camera coordinate system \( C \).

![Flat refractive projection model](image)

**Figure 3.** Flat refractive projection model.

The transformed from the world coordinate system to the camera coordinate system is presented as:

\[
p_t = K P_t = K [R \ t] P_2 + d_1 \left( \frac{r_0}{R r_1} \right)
\]

where, \( K \) represents the camera's intrinsic parameters. When the ray \( r_0 \) of the camera don’t pass through refractive surface of the TGCB, and directly intersect at the 3D point \( P_2 \) of the TGCB.

### 3. Proposed method
3.1. multi-camera calibration based on refraction model

In order to obtain accurate calibration parameters of the multi-camera system, we present a novel method based on flat refractive geometry. The multi-camera system consists of four cameras which distribute on both sides of the calibration target. Each camera captures an image of the calibration target from different positions and orientations. The proposed method can accurately calculate the parameters of multi-camera system by using Zhang’s method [8] and the flat refractive geometry.

![Diagram of Calibration Scene](image)

Figure 4. The calibration scene of multi-camera.

The specific forms of the multi-camera refractive projective model are defined in this section. Figure 4 depicts the calibration scene of multi-camera, which distributes on both sides of the calibration target, where one side of the camera has a refractive phenomenon.

Assume that the thickness of the TGCB is accurately measured. The non-reflective surface of the TGCB is defined as the world coordinate system, and the world coordinates of 3D point is denoted by \( \mathbf{P} = (X \ Y \ Z)^T \). When the point \( \mathbf{P} \) is projected to four cameras at the same time, the \( C_1, C_2 \) camera receive projection rays through refractive surface, while the \( C_3, C_4 \) camera will receive the projective rays without going through refractive surface. The projection rays of the camera without through the refractive surface by using the traditional calibration method, and through the refractive surface by using the refractive projection model for calibration.

3.2. Nonlinear optimization

The camera parameters are optimized by using the bundle adjustment method to minimize the reprojection error. The objective function for bundle adjustment is expressed as:

\[
\min_{D,K,R,t,\gamma} \quad \omega = \sum_{k=1}^{L} \sum_{j=1}^{n} w_j \left\| \tilde{p} - \bar{p} \left( D, K, R, t \right) \right\|^2 + (1 - w_j) \left\| \tilde{p}_\gamma - \bar{p}_\gamma \left( D, K, R, t, \gamma \right) \right\|^2
\]

where, \( \tilde{p} \), \( \tilde{p}_\gamma \) are actual pixels point and measured point without refraction. \( \bar{p}, \bar{p}_\gamma \) are actual pixels point and measured point with refraction. Projection parameters contain distortion parameter \( D \), intrinsic parameter \( K \), rotation matrix \( R \), translation vector \( t \) and refractive index \( \gamma \). \( w \) is the refractive weight, which indicates whether the ray is refracted. \( L \) represents camera number, \( n \) indicates the number of captured images, and \( m \) represents the number of 3D point. The equation (4) is solved by the bundle adjustment method based on the Levenberg-Marquardt algorithm. Then, we get the optimal value of the multi-camera system obtained by minimizing the reprojection error.

4. Experiments
In this experiment, an actual four-camera system is applied to calculate the extrinsic parameters of the system which is similar to Figure 4. The four-camera system is displayed in Figure 5. Six groups of images are captured and one of them is shown in Figure 6. The calibration results of four cameras are relatively complex, so the intrinsic and extrinsic parameters of the cameras are not shown.

![Figure 5. The setup for four cameras system for calibration experiments.](image)

![Figure 6. A group of sample images. (a) Calibration image of the first camera. (b) Calibration image of the second camera. (c) Calibration image of the third camera. (d) Calibration image of the fourth camera.](image)

In order to prove the accuracy and reliability of the proposed method, the frequency distribution of reprojection error of the four cameras is counted. From Figure 7 we can see that the mean and standard deviation of reprojection errors change from -0.3378 and 2.954 pixel to 4.3411e-05 and 0.4553 pixel. The calibration accuracy of the four-camera system is significantly improved by refinement. Therefore, the real data of four-camera system inspect the accuracy and reliability of the proposed method.

![Figure 7. Frequency distribution histogram of the reprojection error without refinement (four camera).](image)
5. Conclusion
In this paper, we developed a novel calibration method for multi-camera system based on flat refractive geometry. This method achieves a higher accuracy and feasibility. The TGCB is applied as calibration target and all cameras can acquire calibration images at the same time from different orientations. Due to the refractive phenomenon of TGCB, the proposed calibration method also solves the phenomenon that can be used for one-shot measurement application. The four-camera calibration results of real data show that the mean value and standard deviation of the reprojection error of our method are $4.3411 \times 10^{-05}$ and 0.4553 pixel, respectively. The result of the experiments also confirmed the accuracy and feasibility of the proposed method.

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Figure 8. Frequency distribution histogram of the reprojection error with refinement (four camera).