Effective quark model with chiral $U(3) \times U(3)$ symmetry for baryon octet and decuplet

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Abstract

We suggest an effective quark model for low–lying baryon octet and decuplet motivated by QCD with a linearly rising confinement potential incorporating the extended Nambu–Jona–Lasinio (ENJL) model with linear realization of chiral $U(3) \times U(3)$ symmetry. Baryons are considered as external heavy states coupled to local three–quark currents with fixed spinorial structure and to low–lying mesons through quark–meson interactions defined in the ENJL–model. In the constituent quark loop representation we have calculated the coupling constants of the $\pi NN$, $\pi N\Delta$ and $\gamma N\Delta$ interactions and the $\sigma_{\pi N}$–term. The obtained results are in reasonable agreement with experimental data and other effective field theory approaches.

PACS number(s): 11.30.Hv, 11.30.Rd, 12.38.Aw, 12.39.Ki, 12.39.Fe, 13.40.Hq, 13.75.Gx, 14.20.Dh, 14.20.Jn
Key words: effective quark model, confinement potential, QCD, baryon, chiral symmetry, pion, photon
1 Introduction

It is well–known that low–energy properties of low–lying mesons and baryons can be suitably described by phenomenological chiral Lagrangians in the tree approximation [1–4]. The step beyond the tree approximation has been done by Weinberg [5] and then systematized by Gasser and Leutwyler within Chiral perturbation theory (CHPT) [6]. The application of CHPT to the description of low–lying baryons coupled to low–lying mesons showed [7,8] the necessity of the consideration of the baryons as very heavy states and the development of large $M_B$, the baryon mass, expansion. The numerous applications of this approach have been done by Bernard et al. and collected in the review [9].

The success of CHPT had inspired the development of different approaches to the description of low–energy interactions of low–lying hadrons based on the effective chiral Lagrangians derived within the extended Nambu–Jona–Lasinio (ENJL) model [10] motivated by QCD with a non–linear [11] and linear [12] realization of chiral $U(3) \times U(3)$ symmetry. The relation of the ENJL model with linear realization of chiral $U(3) \times U(3)$ symmetry to QCD with linearly rising confinement potential has been shown in [13]. The properties of octet and decuplet of low–lying baryons within QCD with linearly rising confinement potential have been investigated in [14]. There has been shown that these baryons are only three–quark states [15], this means that they do not contain the contributions of bound diquark states and so on, the spinorial structure of quark currents of which is defined as products of axial–vector diquark densities and a quark field transforming under $SU(3)_f \times SU(3)_c$ group like $(\bar{6}_f, \bar{3}_c)$ and $(\bar{3}_f, \bar{6}_c)$ multiplets, respectively. The former agrees with the structure of baryon quark currents suggested by Ioffe [16], Pascual and Tarrach [17], and Reinders, Rubinstein and Yazaki [18].

This paper is to apply the dynamical constraints on the structure of baryons obtained in Ref.[14] to the construction of an effective quark model for the description of low–energy interactions of the baryon octet and decuplet coupled to low–lying mesons. According to these dynamical constraints imposed by a linearly rising interquark confinement potential [14] the spinorial structure of the three–quark currents should be defined by the products of axial–vector diquark densities and a quark field transforming under $SU(3)_f \times SU(3)_c$ group like $(\bar{6}_f, \bar{3}_c)$ and $(\bar{3}_f, \bar{6}_c)$ multiplets, respectively. This allows to construct the three–quark currents with quantum numbers of the baryon octet and decuplet in the following form [14–18]:

\[
\begin{align*}
\eta^N(x) &= -\varepsilon^{ijk} [\bar{u}^c_i(x) \gamma^\mu u_j(x)] \gamma_\mu \gamma^5 d_k(x), \\
\eta^\Sigma(x) &= \varepsilon^{ijk} [\bar{u}^c_i(x) \gamma^\mu u_j(x)] \gamma_\mu \gamma^5 s_k(x), \\
\eta_\Lambda(x) &= -\sqrt{2} \varepsilon^{ijk} \{ [\bar{u}^c_i(x) \gamma^\mu s_j(x)] \gamma_\mu \gamma^5 d_k(x) - [\bar{d}^c_i(x) \gamma^\mu s_j(x)] \gamma_\mu \gamma^5 u_k(x) \}, \\
\eta^\Xi(x) &= \varepsilon^{ijk} [\bar{s}^c_i(x) \gamma^\mu s_j(x)] \gamma_\mu \gamma^5 u_k(x), \\
\eta^{\mu}_\Xi(x) &= \varepsilon^{ijk} [\bar{u}^c_i(x) \gamma^\mu u_j(x)] u_k(x), \\
\eta^{\mu}_\Sigma(x) &= \sqrt{\frac{1}{3}} \varepsilon^{ijk} \{ 2 [\bar{u}^c_i(x) \gamma^\mu s_j(x)] u_k(x) + [\bar{u}^c_i(x) \gamma^\mu u_j(x)] s_k(x) \},
\end{align*}
\]
$$\eta^p_{\mu}(x) = \sqrt{\frac{2}{3}} \varepsilon^{ijk} \left\{ 2 \left[ \bar{s}_i(x) \gamma^\mu u_j(x) \right] s_k(x) + \left[ \bar{s}_i(x) \gamma^\mu s_j(x) \right] u_k(x) \right\},$$
$$\eta^{\Omega}_{\mu}(x) = \varepsilon^{ijk} \left[ \bar{s}_i(x) \gamma^\mu s_j(x) \right] s_k(x),$$

(1.2)

where \(i, j\) and \(k\) are colour indices, then \(\bar{q}c(x) = q(T)C\) and \(C = -C^T = -C^\dagger\) is a matrix of a charge conjugation, \(T\) is a transposition.

In our approach baryons are external heavy states \([8,9]\) coupled to three–quark currents defined by Eq.(1.1) and Eq.(1.2) as

$$L_{\text{int}}(x) = \frac{1}{\sqrt{2}} g_B \bar{p}(x) \eta_N(x) + g_B \bar{\Delta}^{++}(x) \eta^{\Delta}_{\mu}(x) + \ldots + \text{h.c.},$$

(1.3)

where \(p(x)\) and \(\Delta^{++}_{\mu}(x)\) are the fields of the proton and the \(\Delta\)–resonance, respectively, the ellipses denote the contribution of interactions of other components of octet and decuplet, \(g_B\) is a phenomenological coupling constant, and a factor \(1/\sqrt{2}\) is introduced due to a non–relativistic quark model with \(SU(6)\) symmetry \([19]\). We should emphasize that in our consideration the \(\Delta\)–resonance and other components of decuplet are the Rarita–Schwinger fields obeying the subsidiary conditions

$$\partial^\mu \Delta_{\mu}(x) = \gamma^\mu \Delta_{\mu}(x) = 0, \text{ etc.}$$

(1.4)

To low–lying mesons baryons couple through quark–meson interactions induced within the ENJL–model with linear realization of chiral \(U(3) \times U(3)\) symmetry \([12,13]\).

The paper is organized as follows. In Section 2 we calculate the \(g_{\pi NN}\) and \(g_{\pi N\Delta}\) coupling constants. We find the ratio \(g_{\pi N\Delta}/g_{\pi NN} = 2\) in agreement with both experimental data and other approaches \([9]\). In Section 3 we calculate the coupling constant \(g_{\gamma N\Delta}\) of the \(\Delta \rightarrow N + \gamma\) decays. In Section 4 we calculate chiral corrections to the mass of the proton related to the \(\sigma_{\pi N}\)–term of the \(\pi N\)–scattering and estimate the value of the \(\sigma_{\pi N}\)–term. In Conclusion we discuss the obtained results.

2 \(g_{\pi NN}\) and \(g_{\pi N\Delta}\) coupling constants

The coupling constant \(g_{\pi NN}\) of the \(\pi NN\)–interaction we define by a phenomenological \(\pi^0pp\)–interaction \([20]\):

$$L^{\pi^0pp}_{\text{eff}}(x) = g_{\pi NN} \left[ \bar{p}(x)i\gamma^5 p(x) \right] \pi^0(x),$$

(2.1)

where \(\pi^0(x)\) is the \(\pi^0\)–meson field. The pion fields couple to current quark fields through the interactions \([12]\):

$$L^{\pi^0qq}(x) = \frac{g_{\pi qq}}{\sqrt{2}} \left[ \bar{u}(x)i\gamma^5 u(x) - \bar{d}(x)i\gamma^5 d(x) \right] \pi^0(x) + g_{\pi qq} \left[ \bar{u}(x)i\gamma^5 d(x) \right] \pi^+(x) + \text{h.c.}$$

(2.2)

The quark–meson coupling constant \(g_{\pi qq} = \sqrt{2m}/F_0\) \([12]\) is given in terms of the constituent quark mass \(m = 0.33\) GeV and the PCAC constant \(F_0 = 0.092\) GeV calculated.
in the chiral limit [12]. The effective Lagrangian $L_{\text{eff}}^{\pi^{0}pp}(x)$ defined by the interactions Eq.(1.3) and Eq.(2.2) reads

$$\int d^4x L_{\text{eff}}^{\pi^{0}pp}(x) = -\left(\frac{g_{\pi qq}}{\sqrt{2}}\right)\left(\frac{g_{B}}{2}\right)\int d^4xd^4x_1d^4x_2\pi^0(x_2) \rho(x) < 0|T(\eta_N(x)[\bar{u}(x_2)i\gamma^5u(x_2) - \bar{d}(x_2)i\gamma^5d(x_2)]\bar{\eta}_N(x_1))|0 > p(x_1), \quad (2.3)$$

where $T$ is a time–ordering operator, $\bar{\eta}_N(x) = \varepsilon^{ijk} \bar{d}_i(x)\gamma_\nu\gamma^5[\bar{u}_j(x)\gamma^\nu u^c_k(x)]$ and $u^c(x) = C\bar{u}(x)^T$. By applying formulae of quark conversion [12] (Ivanov) we can represent the r.h.s. of Eq.(2.3) in terms of constituent quark diagrams. In the momentum representation they read

$$\int d^4x L_{\text{eff}}^{\pi^{0}pp}(x) = -\frac{3}{2}\left(\frac{g_{\pi qq}}{\sqrt{2}}\right)^2\frac{g_{B}}{8\pi^2}\int d^4x \int \frac{d^4x_1d^4p_1}{(2\pi)^4} \frac{d^4x_2d^4p_2}{(2\pi)^4} e^{-i(x-x_2)\cdot p_1} e^{-i(x-x_2)\cdot p_2}\pi^0(x_2) \int \frac{d^4k_1}{\pi^2i} \frac{d^4k_2}{\pi^2i} \bar{\rho}(x) \left[\gamma^\mu\gamma^5\frac{1}{m - k_1 - k_2 - \hat{p}_1 - \hat{p}_2}i\gamma^5\frac{1}{m - k_1 - k_2 - \hat{p}_1}\gamma^\nu\gamma^5\right] p(x_1). \quad (2.4)$$

The calculation of the momentum integrals representing constituent quark diagrams should be carried out keeping only the divergent parts and dropping the contributions of the parts finite in the infinite limit of the cut–off. Such a prescription realizes a naive description of the quark confinement. Indeed, dropping the finite parts of quark diagrams we remove the imaginary parts of them and suppress by this the appearance of quarks in the intermediate states of low–energy hadron interactions. This naive description of confinement has turned out to be rather useful for the derivation of Effective Chiral Lagrangians [10–12]. As has been shown in Ref. [13] this prescription can be justified in QCD with a linearly rising interquark confinement potential. Thereby, within such a naive approach to the quark confinement mechanism one can bridge quantitatively the quark and the hadron level of the description of strong low–energy interactions of hadrons. The cut–off $\Lambda_\chi = 0.94$ GeV and the constituent quark mass $m = 0.33$ GeV [12] can be considered in such an approach as input parameters and fixed at one–loop approximation via the experimental values of the $\rho\pi\pi$ coupling constant $g_\rho$ and the leptonic pion constant $F_\pi$ [12]. Thus, we should accentuate that in the effective quark models based on the NJL approach, or equivalently QCD with a linearly rising interquark confinement potential [13], quark diagrams lose the meaning of quantum field theory objects and only display how quark flavours can be transferred form an initial hadron state to a final hadron state in hadron–hadron low–energy transitions. The coupling constants of such transitions, described in terms of divergent parts of quark diagrams and depending of the cut–off $\Lambda_\chi = 0.94$ GeV and the constituent quark mass $m = 0.33$ GeV, can be expressed in terms of effective phenomenological coupling constants of low–energy hadron interactions given by Effective Chiral Lagrangians [4].

For the computation of the momentum integrals we assume following Jenkins and Manohar [8] that the proton is a very heavy state and its mass is much larger than other
momenta in the integrand such as $p_1^2 = M_N^2$, where $M_N$ is an averaged mass of the baryon octet. For numerical estimates we set below $M_N = 940$ MeV. In this picture a very heavy source (the proton) is surrounded by a cloud of light (almost massless) particles [9]. This resembles completely the methods used in the Heavy Quark Effective theory (HQET) [21].

Thus, keeping the leading order in large $M_N$ expansion we reduce the r.h.s. of Eq. (2.4) to the form

$$\int d^4x \mathcal{L}_{\text{eff}}^\alpha p\pi(x) =$$

$$= -\frac{3}{2} \left( \frac{g_{\pi qq}}{\sqrt{2}} \right)^2 \int d^4x \int d^4x_1 d^4p_1 d^4x_2 d^4p_2 e^{-i(x-x_1) \cdot p_1} e^{-i(x-x_2) \cdot p_2} \pi^0(x_2)$$

$$\int \frac{d^4k_1}{\pi^2} \frac{d^4k_2}{\pi^2} \tilde{p}(x) \left[ \gamma^\mu \gamma_5 \frac{\not{k}_1}{M_N^2} i \gamma^5 \frac{\not{k}_2}{M_N^2} \gamma^\nu \gamma_5 \right] p(x_1) \text{tr} \left\{ \frac{1}{m + \not{k}_1} \not{\gamma}_\nu \frac{1}{m - \not{k}_2} \not{\gamma}_\mu \right\}. \quad (2.5)$$

The replacement of the constituent quark Green function

$$\frac{1}{m - \hat{k}_1 - \hat{k}_2 - \hat{p}_1} \rightarrow -\frac{\hat{p}_1}{M_N^2} \quad (2.6)$$

agrees with the heavy baryon [8,9] and HQET [21] approaches. Indeed, in accordance with [8,9] and HQET [21] we obtain

$$\frac{1}{m - \hat{k}_1 - \hat{k}_2 - \hat{p}_1} = \frac{m + \hat{k}_1 + \hat{k}_2 + \hat{p}_1}{m^2 - (k_1 + k_2 + p_1)^2 - i0} =$$

$$-\frac{m + \hat{k}_1 + \hat{k}_2 + \hat{p}_1}{M_N^2 + 2(k_1 + k_2) \cdot p_1 - m^2 + (k_1 + k_2)^2 + i0} =$$

$$-\frac{1}{M_N} \frac{M_N}{1 + \frac{2(k_1 + k_2) \cdot v_1}{M_N} - \frac{m^2}{M_N^2} + \frac{(k_1 + k_2)^2}{M_N^2} + i0}. \quad (2.7)$$

where we have set $p_1^\mu = M_N v_1^\mu$ [8,9,21]. In the case of $m = M_N$ and in the limit $M_N \rightarrow \infty$ [8,9,21] we arrive at the well-known expression for the Green function of a heavy baryon (or a heavy quark in HQET [21]) [8,9]:

$$\frac{1}{m - k_1 - k_2 - \hat{p}_1} \rightarrow \frac{1}{M_N - k_1 - k_2 - M_N \hat{v}_1} \rightarrow -\left( \frac{1 + \hat{v}_1}{2} \right) \frac{1}{(k_1 + k_2) \cdot v_1 + i0}. \quad (2.8)$$

In our case $m \ll M_N$, therefore, in the limit $M_N \rightarrow \infty$ [8,9] we arrive at the expression Eq. (2.8).

Let us discuss the choice of variables in the momentum integrals of Eq. (2.4). These integrals represent the constituent quark diagrams with three virtual quark lines two of which are confined by the trace and the third is joined to the initial and the final proton fields. For convenience we would call this third quark a spectator. The main problem of
the computation of the momentum integrals like those of Eq. (2.4) is in the overlap of the virtual momenta \( k_1 \) and \( k_2 \). The former makes rather complicated the computation of the momentum integrals. Therefore, the choice of variables should help to disconnect the momenta \( k_1 \) and \( k_2 \) in the limit \( M \to \infty \). This can be reached assuming that the proton momentum transfers itself from the initial state to the final one by the spectator. This factorizes the momentum integrals over \( k_1 \) and \( k_2 \). For the sake of self-consistency of the approach we should use this choice of the virtual momenta, when the momentum of the initial baryon transfers itself to the final one by the spectator, for any application of the model to the description of low–energy interactions of the baryon octet and decuplet.

The result of the integration over momenta \( k_1 \) and \( k_2 \) can be represented in terms of the quark condensate [12]

\[
\int \frac{d^4k_1 \, d^4k_2}{\pi^2 i} \text{tr} \left\{ \frac{1}{m + k_1} \gamma_\nu \frac{1}{m - k_2} \gamma_\mu \right\} = \left( \frac{8 \pi^2}{3} \right)^2 < \bar{q} q >^2 g_{\mu \nu}. \tag{2.9}
\]

After some algebra of Dirac matrices we obtain the effective Lagrangian in the form

\[
L_{\pi \text{pp}}^{\text{eff}}(x) = 6 \left( \frac{g_{\pi q}}{\sqrt{2}} \right) \left( \frac{g_B}{8 \pi^2} \right)^2 \left( \frac{8 \pi^2}{3} \right)^2 \frac{< \bar{q} q >^2}{M_N^2} \bar{p}(x) i\gamma^5 p(x) \pi^0(x) =
\]

\[
= \left( \frac{g_B^2}{3} \frac{2}{F_0} \frac{m}{M_N^2} \right) \bar{p}(x) i\gamma^5 p(x) \pi^0(x). \tag{2.10}
\]

This yields the coupling constant \( g_{\pi NN} \):

\[
g_{\pi NN} = \frac{g_B^2}{3} \frac{2}{F_0} \frac{m}{M_N^2} < \bar{q} q >^2. \tag{2.11}
\]

Now let us proceed to the computation of the coupling constant \( g_{\pi N\Delta} \) of the \( \pi N\Delta \)–interaction.

The most general form of the \( \pi N\Delta \)–interaction compatible with requirements of chiral symmetry reads [22]

\[
L_{\pi N\Delta}^{\text{eff}}(x) = \frac{g_{\pi N\Delta}}{2 M_N} \Delta_\mu^a(x) \Theta^{\mu \nu} N(x) \partial_\mu \pi^a(x) + \text{h.c.}, \tag{2.12}
\]

where \( \Delta_\mu^a(x) \) is the \( \Delta \)–resonance field, the isotopical index \( a \) runs over \( a = 1, 2, 3 \),

\[
\Delta^1 = \frac{1}{\sqrt{2}} \left( \frac{\Delta^{++} - \Delta^0/\sqrt{3}}{\Delta^+ / \sqrt{3} - \Delta^-} \right), \quad \Delta^2 = \frac{i}{\sqrt{2}} \left( \frac{\Delta^{++} + \Delta^0/\sqrt{3}}{\Delta^+ / \sqrt{3} + \Delta^-} \right),
\]

\[
\Delta^3 = - \sqrt{3} \left( \frac{\Delta^+}{\Delta^0} \right). \tag{2.13}
\]

The nucleon field \( N(x) \) is the isotopical doublet with components \( N(x) = (p(x), n(x)) \). The tensor \( \Theta^{\mu \nu} \) is given by [22]: \( \Theta^{\mu \nu} = g^{\mu \nu} - (Z + 1/2) \gamma^\mu \gamma^\nu \), where the parameter \( Z \) is fully arbitrary. It describes the \( \pi N\Delta \)–interaction off–mass shell of the \( \Delta \)–resonance. The propagator of the \( \Delta \)–field is defined [23]:

\[
< 0 | T(\Delta_\mu(x_1) \Delta_\nu(x_2)) | 0 > = -i S_{\mu \nu}(x_1 - x_2). \tag{2.14}
\]
In the momentum representation \( S_{\mu\nu}(x) \) reads [23,24]:
\[
S_{\mu\nu}(p) = \frac{1}{M_{\Delta} - p} \left( -g_{\mu\nu} + \frac{1}{3} \gamma_\mu \gamma_\nu + \frac{1}{3} \gamma_\mu p_\nu - \gamma_\mu p_\nu + \frac{2}{3} \frac{p_\mu p_\nu}{M_{\Delta}^2} \right),
\] (2.15)
where \( M_{\Delta} \) is an averaged mass of the baryon decuplet. It can be kept of order \( M_N \). The Green function of the free \( \Delta \)--field Eq.(2.14) is related to the free Lagrangian given by [25]
\[
\mathcal{L}_{\text{kin}}^\Delta(x) = \tilde{\Delta}_\mu(x)[-(i\gamma^\alpha \partial_\alpha - M_\Delta)g^{\mu\nu} + \frac{1}{4} \gamma^\mu \gamma^\beta (i\gamma^\alpha \partial_\alpha - M_\Delta)\gamma_\beta \gamma_\nu]\Delta_\nu(x).
\] (2.16)
In the component form the \( \pi N\Delta \)--interaction Eq.(2.12) reads
\[
\mathcal{L}_{\text{eff}}^{\pi N\Delta}(x) = \frac{g_{\pi N\Delta}}{2M_N} \Delta_\mu^{++}(x)\Theta^{\mu\nu}(x)\partial_\nu \pi^+(x) + \ldots.
\] (2.17)
The effective Lagrangian \( \mathcal{L}_{\text{eff}}^{\pi N\Delta} \) defined by Eq.(1.3) and Eq.(2.2) reads
\[
\int d^4x \mathcal{L}_{\text{eff}}^{\pi N\Delta}(x) = -\left( \frac{g_{\pi qq}}{\sqrt{2}} \right) g_B^2 \int d^4xd^4x_1d^4x_2 \nonumber
\]
\[
\tilde{\Delta}_\mu^{++}(x) < 0|T(\eta_\Delta^A(x)[\bar{u}(x_2)i\gamma^5d(x_2)]\bar{v}(x_1))|0 > p(x_1) \pi^+(x_2).
\] (2.18)
The computation of the \( g_{\pi N\Delta} \) coupling constant we perform on--mass shell of the \( \Delta \)--resonance. The application of formulae of quark conversion represents the r.h.s. of Eq.(2.18) in the form of constituent quark diagrams determined by the momentum integrals as follows:
\[
\int d^4x \mathcal{L}_{\text{eff}}^{\pi N\Delta}(x) = \nonumber
\]}
\[
= 3\left( \frac{g_{\pi qq}}{\sqrt{2}} \right) \left( \frac{g_B}{8\pi^2} \right)^2 \int d^4x \int \frac{d^4p_1d^4p_2d^4x_1d^4x_2}{(2\pi)^4} \frac{1}{(2\pi)^4} e^{-i(x-x_1)\cdot p_1} e^{-i(x-x_2)\cdot p_2} \nonumber
\]
\[
\int \frac{d^4k_1d^4k_2}{\pi^2i^2} \tilde{\Delta}_\mu^{++}(x) \left[ \frac{1}{m - k_1 - k_2 - \hat{p}_1 - \hat{p}_2} i\gamma^5 \frac{1}{m - k_1 - k_2 - \hat{p}_1} \gamma_\nu \gamma^5 \right] p(x_1) \nonumber
\]
\[
\text{tr}\left\{ \frac{1}{m + k_1} \gamma_\nu \frac{1}{m - k_2} \gamma^\mu \right\} \pi^+(x_2).
\] (2.19)
In the large \( M_N \) expansion due to the constraints Eq.(1.4) the nontrivial contribution to the effective \( \pi N\Delta \)--interaction appears at first order in the pion momentum expansion. This contribution reads
\[
\mathcal{L}_{\text{eff}}^{\pi N\Delta}(x) = 6\left( \frac{g_{\pi qq}}{\sqrt{2}} \right) \left( \frac{g_B}{8\pi^2} \right)^2 \frac{2}{3} \frac{\bar{q}q}{M_N^3} \tilde{\Delta}_\mu^{++}(x) p(x) \partial^\mu \pi^+(x) = \nonumber
\]
\[
= \frac{1}{2M_N} \left[ \frac{g_B}{F_0} \frac{4}{3} \frac{m < \bar{q}q >}{M_N^2} \right] \tilde{\Delta}_\mu^{++}(x) p(x) \partial^\mu \pi^+(x)
\] (2.20)
and gives the \( g_{\pi N\Delta} \) coupling constant:
\[
g_{\pi N\Delta} = g_B \frac{4}{3} \frac{m < \bar{q}q >}{M_N^2}.
\] (2.21)
Thus, in our approach the ratio of the coupling constants \( g_{\pi N\Delta} \) and \( g_{\pi NN} \) is defined \( g_{\pi N\Delta}/g_{\pi NN} = 2 \). This agrees with the experimental data and other effective field theory approaches [9]. The parameter \( Z \) is left undefined, since we have computed \( g_{\pi N\Delta} \) on--mass shell of the \( \Delta \)--resonance, while \( Z \) describes the \( \pi N\Delta \)--interaction off--mass shell.
3 $g_{\gamma N\Delta}$ coupling constant

Assuming that the transition $\Delta \to N + \gamma$ is primarily a magnetic one [26,27] the effective Lagrangian describing the $\Delta \to N + \gamma$ decays can be defined on–mass shell of the $\Delta$–resonance as [26,27]:

$$\mathcal{L}_{\text{eff}}^{\gamma N\Delta}(x) = -i e \frac{g_{\gamma N\Delta}}{2M_N} \bar{\Delta}_\mu^+(x) \gamma_\mu \gamma^5 N(x) F^{\mu\nu}(x) + \text{h.c.} =$$

$$= i e \sqrt{\frac{2}{3}} \frac{g_{\gamma N\Delta}}{2M_N} \bar{\Delta}_\mu^+(x) \gamma_\mu \gamma^5 p(x) F^{\mu\nu}(x) + \ldots + \text{h.c.,} \quad (3.1)$$

where $F^{\mu\nu}(x) = \partial^\mu A^\nu(x) - \partial^\nu A^\mu(x)$ and $A^\mu(x)$ is the photon field, $e$ is the electric charge of the proton and $g_{\gamma N\Delta}$ is the coupling constant, which we calculate below in our effective quark model by example the mode $\Delta^+ \to p + \gamma$.

The computation of effective photon–hadron interactions in effective quark models is a rather sensitive procedure demanding careful summation of all possible diagrams containing both point–like photon–quark and photon–hadron vertices. In order to simplify calculations we suggest to replace the derivation of the $\gamma N\Delta$–interaction by the $\rho^0 N\Delta$–interaction, as photons in the decays $\Delta \to N + \gamma$ are isovectors, and then to make a change $\rho^0_\mu(x) \to (e/g_\rho) A_\mu(x)$, where $g_\rho$ is the $\rho\pi\pi$–coupling constant, according to the Vector Dominance hypothesis.

It is well–known [4,28,29] that the Vector Dominance hypothesis is applicable not only to high energy processes but mainly at low energies to the description of low–energy interactions of low–lying mesons within the Effective Chiral Lagrangian approach [4,28]. For example, the amplitude of the famous decay $\pi^0 \to \gamma + \gamma$ can be completely described within the Vector Dominance hypothesis. Indeed, the amplitude of the $\pi^0 \to \gamma + \gamma$ decay defined by the Adler–Bell–Jackiw anomaly reads [30]:

$$\mathcal{A}(\pi^0 \to \gamma\gamma) = -\frac{e^2}{8\pi^2 F_0}.$$  

The same expression can be obtained via the intermediate $(\rho^0 + \omega^0)$–state, i.e. $\pi^0 \to \rho^0 + \omega^0 \to \gamma + \gamma$, by virtue of the direct transitions $\rho^0 \to \gamma$ and $\omega^0 \to \gamma$ with the coupling constants $\rho^0_\mu(x) \to (e/g_\rho) A_\mu(x)$ and $\omega^0_\mu(x) \to (e/3g_\rho) A_\mu(x)$, respectively [28–30]:

$$\mathcal{A}(\pi^0 \to \gamma\gamma) = -g_{\pi\rho\omega} \frac{e}{g_\rho} \frac{e}{3g_\rho} = -\frac{3g_\rho^2}{8\pi^2 F_0} \frac{e}{g_\rho} \frac{e}{3g_\rho} = -\frac{e^2}{8\pi^2 F_0}.$$  

Referring to this experience [4,28–30] the Vector Dominance hypothesis can be readily applied to the description of the $\Delta^+ \to p + \gamma$ transition.

Since the $\rho^0$–meson couples to the quark current through the interaction

$$\mathcal{L}^{\rho^0 q}(x) = \frac{g_\rho}{2} [\bar{u}(x) \gamma^\nu u(x) - \bar{d}(x) \gamma^\nu d(x)] \rho^0_\nu(x), \quad (3.2)$$

the effective Lagrangian $\mathcal{L}_{\text{eff}}^{\rho^0 p\Delta^+}(x)$ of the $\Delta^+ \to p + \rho^0$ transition is defined by

$$\int d^4 x \mathcal{L}_{\text{eff}}^{\rho^0 p\Delta^+}(x) = -\left(\frac{g_\rho}{2}\right) \left(\frac{g_B^2}{\sqrt{2}}\right) \int d^4 x d^4 x_1 d^4 x_2 \rho^0_\nu(x_2)$$

$$\bar{\Delta}^+_\mu(x) < 0 |T(g^\nu_{\Delta^+}(x)[\bar{u}(x_2) \gamma^\nu u(x_2) - \bar{d}(x_2) \gamma^\nu d(x_2)]\bar{\eta}_N(x_1))|0 > p(x_1), \quad (3.3)$$

8
In this section we calculate current quark mass corrections to the mass of the proton. Fix the parameter of the shift removing the gauge non–invariant contributions [31]. The gauge invariant term turns out to be independent of the parameter of the shift and reads

\[
\int d^4x \mathcal{L}_{\text{eff}}^{\rho^0 p \Delta^+}(x) = -\left( \frac{g_\rho}{2\sqrt{2}} \right)^2 \left( \frac{g_B}{8\pi^2} \right)^2 \int d^4x \int \frac{d^4x_1 d^4p_1}{(2\pi)^4} \frac{d^4x_2 d^4p_2}{(2\pi)^4} e^{-i(x-x_1) \cdot p_1} e^{-i(x-x_2) \cdot p_2} \rho_\nu(x_2) \rho_\mu^0(x_1) \frac{1}{m - k_1 - k_2 - \hat{p}_1 - \hat{p}_2} \gamma_\nu \gamma_\mu \gamma_5 \rho_\nu^0(x_2) \rho_\mu^0(x_1)
\]

Since we use a cut–off regularization [12], the derivation of the gauge invariant \(\rho^0 N \Delta–\)interaction from the r.h.s. of Eq.(3.4) is not straightforward procedure. In order to extract a gauge invariant contribution we suggest to make an arbitrary shift of virtual momenta proportional to the 4–momentum \(p_2\) of the \(\rho^0\)–meson. When applying then large \(M_N\) expansion and keeping only linear terms in \(\rho^0\)–meson momentum expansion [12] one can fix the parameter of the shift removing the gauge non–invariant contributions [31]. The gauge invariant term turns out to be independent of the parameter of the shift and reads

\[
\int d^4x \mathcal{L}_{\text{eff}}^{\rho^0 p \Delta^+}(x) = -\frac{3}{2} \left( \frac{g_\rho}{\sqrt{2}} \right)^2 \left( \frac{g_B}{3} \right)^2 \frac{<\bar{q}q>^2}{M_N^2} \int d^4x \int \frac{d^4x_1 d^4p_1}{(2\pi)^4} \frac{d^4x_2 d^4p_2}{(2\pi)^4} e^{-i(x-x_1) \cdot p_1} e^{-i(x-x_2) \cdot p_2} \Delta_\mu^+(x) \left( -\frac{2}{M_N} \right) (p_2^\mu \gamma_\nu - \hat{p}_2 g^{\mu\nu}) \gamma_5 \rho_\nu^0(x_1) \rho_\mu^0(x_2).
\]

This yields the effective Lagrangian \(\mathcal{L}_{\text{eff}}^{\rho^0 p \Delta^+}(x)\):

\[
\mathcal{L}_{\text{eff}}^{\rho^0 p \Delta^+}(x) = i \frac{g_\rho}{2M_N} \left[ \frac{\sqrt{2}}{3} \frac{g_B^2 <\bar{q}q>^2}{M_N^2} \right] \Delta_\mu^+(x) \gamma_\nu \gamma_5 \rho_\nu^0(x) \mathcal{F}^{\mu\nu}(x) + \text{h.c.},
\]

where \(\mathcal{F}^{\mu\nu}(x) = \partial_\mu \rho_\nu^0(x) - \partial_\nu \rho_\mu^0(x)\). Making then a shift \(\rho_\mu^0(x) \rightarrow (e/g_\rho) \partial_\mu A(x)\) caused by the Vector Dominance hypothesis we derive the effective Lagrangian of the \(\gamma N \Delta–\)interaction

\[
\mathcal{L}_{\text{eff}}^{\gamma^0 p \Delta^+}(x) = i \frac{e}{2M_N} \left[ \frac{\sqrt{2}}{3} \frac{g_B^2 <\bar{q}q>^2}{M_N^2} \right] \Delta_\mu^+(x) \gamma_\nu \gamma_5 \rho_\nu^0(x) \mathcal{F}^{\mu\nu}(x) + \text{h.c.}
\]

Thus, the coupling constant \(g_{\gamma N \Delta}\) is given by \(g_{\gamma N \Delta}/g_{\pi N \Delta} = (\sqrt{3}/4)(F_0/m) = 0.12\). This ratio agrees with the \(SU(6)\) prediction \(g_{\gamma N \Delta}/g_{\pi N \Delta} = 0.14\) [32]. The agreement of our result \(g_{\gamma N \Delta}/g_{\pi N \Delta} = 0.12\) with other phenomenological approaches [26,27] is only qualitative.

### 4 \(\sigma_{\pi N–}\)term

In this section we calculate current quark mass corrections to the mass of the proton \(M_p\) and the \(\sigma_{\pi N–}\)term, defined as

\[
\sigma_{\pi N} = m_0 < p|\bar{u}(0)u(0) + \bar{d}(0)d(0)||p >
\]

(4.1)
and related to these chiral corrections through the Feynman–Hellmann theorem

$$\sigma_{\pi N} = m_0 \frac{\partial M_p(m_0)}{\partial m_0}, \quad(4.2)$$

where $m_0 = (m_{0u} + m_{0d})/2 = 5.5 \text{ MeV}$ is an averaged current quark mass at $m_{0u} = 4 \text{ MeV}$ and $m_{0d} = 7 \text{ MeV}$ [33]. The calculation of the $\sigma_{\pi N}$-term has a long history [34]. The current magnitudes of $\sigma_{\pi N}$ obtained on lattice are $40 \div 60 \text{ MeV}$ [35] and $50 \text{ MeV}$ [36].

In our effective quark model current quark mass corrections to the mass of the proton can be defined by the effective Lagrangian

$$\int d^4x \delta L_{\text{eff}}^{pp}(x) = m_0 \left( \frac{g_B^2}{2} \right) \int d^4x d^4x_1 d^4x_2 \bar{p}(x) \left< 0 \right| T(\eta_N(x)[\bar{u}(x_2)u(x_2) + \bar{d}(x_2)d(x_2)]\bar{\eta}_N(x_1)) \left| 0 > p(x_1). \quad(4.3)$$

By using the formulae of quark conversion [12] (Ivanov) the matrix element in the r.h.s. of Eq.(4.3) can be represented by the constituent quark diagrams and is given in terms of the momentum integrals as follows:

$$\int d^4x \delta L_{\text{eff}}^{pp}(x) = m_0 \left( \frac{g_B^2}{8\pi^2} \right)^2 \int d^4x \int \frac{d^4x_1 d^4p_1}{(2\pi)^4} e^{-i(x-x_1)\cdot p_1}$$

$$\frac{1}{\pi^2 i} \frac{1}{\pi^2 i} \bar{p}(x) \left[ \gamma_\mu \gamma_5 \frac{1}{m - k_1 - k_2 - \hat{p}_1} \frac{1}{m - k_1 - k_2 - \hat{p}_1} \gamma_\mu \gamma_5 \right] p(x_1)$$

$$\text{tr} \left\{ \frac{1}{m + k_1} \frac{1}{m - k_2} \gamma_\mu \frac{1}{m - k_2} \gamma_\mu \right\} + m_0 \frac{3}{4} \left( \frac{g_B^2}{8\pi^2} \right)^2 \int d^4x \int \frac{d^4x_1 d^4p_1}{(2\pi)^4} e^{-i(x-x_1)\cdot p_1}$$

$$\left( \frac{\bar{v}}{4m} \right) \text{tr} \left\{ \frac{1}{m + k_1} \frac{1}{m + k_1} \gamma_\mu \frac{1}{m - k_2} \gamma_\mu + \frac{1}{m + k_1} \frac{1}{m - k_2} \gamma_\mu \right\}, \quad(4.4)$$

where $\bar{v} = - < \bar{q} q > / F_0^2 = 1.92 \text{ GeV}$ [12] and the factor $\bar{v}/4m$ appears due to the contribution of the exchange of the isoscalar scalar $\sigma$-meson with the quark structure $(\bar{u}u + \bar{d}d)/\sqrt{2}$ and the mass $M_\sigma = 2m$ [12,37]. The calculation of the integrals entering Eq.(4.4) is analogous to that having been performed in preceding sections. The resultant effective Lagrangian $\delta L_{\text{eff}}^{pp}(x)$ is given by

$$\delta L_{\text{eff}}^{pp}(x) =$$

$$= - \left[ m_0 \frac{2}{3} g_B^2 < \bar{q} q >^2 \left( \frac{\bar{v}}{4m} \right) + m_0 \frac{3}{4} \frac{M_N^2}{m} g_B^2 < \bar{q} q >^2 \left( \frac{\bar{v}}{4m} - 1 \right) \right] \bar{p}(x)p(x) =$$

$$= \left[ m_0 g_{\pi NN} \frac{F_0}{m} \left( \frac{\bar{v}}{4m} \right) + m_0 g_{\pi NN} \frac{9}{8} \frac{F_0}{m} M_N \left( \frac{\bar{v}}{4m} - 1 \right) \right] \bar{p}(x)p(x). \quad(4.5)$$

This yields the proton mass as a function of $m_0$:

$$M_p(m_0) = M_N + m_0 g_{\pi NN} \frac{F_0}{m} \left( \frac{\bar{v}}{4m} \right) \left[ 1 + \frac{9}{8} \frac{M_N}{m} \left( 1 - 4m/\bar{v} \right) \right]. \quad(4.6)$$
Due to Eq. (4.2) we arrive at the $\sigma_{\pi N}$-term:

$$\sigma_{\pi N} = \frac{m_0 g_{\pi NN}}{m} \frac{F_0}{4m} \left( \frac{\bar{v}}{4m} \right) \left[ 1 + \frac{9M_N}{8m} \left( 1 - \frac{4m}{\bar{v}} \right) \right] = 60 \text{ MeV.} \quad (4.7)$$

The numerical value of the $\sigma_{\pi N}$-term is obtained at $M_N = 940 \text{ MeV}$ and $g_{\pi NN} = 13.4$. Our estimate $\sigma_{\pi N} = 60 \text{ MeV}$ agrees with the numerical data obtained on lattice [35,36].

## 5 Conclusion

We have suggested an effective quark model for octet and decuplet of low–lying baryons. This model is some kind of baryon extension of the ENJL–model with linear realization of chiral $U(3) \times U(3)$ symmetry [12]. Baryons are included as external heavy states coupled to the three–quark currents the spinorial structure of which is fixed within QCD with linearly rising interquark potential [14] incorporating the ENJL–model [13]. As has been shown in [13] the mass spectra of low–lying mesons in QCD with linearly rising confinement potential coincide with the mass spectra of mesons in the ENJL model. Then, the constituent quark mass $m$ can be expressed in terms of the string tension $\sigma$ as follows $m = 2\sqrt{\sigma}/\pi$. At $\sigma \sim 440 \text{ MeV}$ we get $m \sim 300 \text{ MeV}$. For the description of low–energy dynamics of baryons we have added two new low–energy parameters with respect to the ENJL–model. These are the phenomenological coupling constant $g_B$ and the averaged mass of the octet $M_N$. The averaged mass of the decuplet $M_\Delta$ can be kept of order $M_N$. In terms of these parameters and the parameters of the ENJL–model, fixed by the low–energy phenomenology of low–lying mesons [12], we have described the $\pi NN$, $\pi N\Delta$ and $\gamma N\Delta$ interactions and the $\sigma_{\pi N}$–term related to the amplitude of the elastic low–energy $\pi N$–scattering. The obtained results $g_{\pi N\Delta}/g_{\pi NN} = 2$, $g_{\pi N\Delta}/g_{\pi \Delta} = 0.12$ and $\sigma_{\pi N} = 60 \text{ MeV}$ agree reasonably well with the data on low–energy phenomenology of baryons obtained both experimentally and within other effective field theory approaches.

Concluding the discussion we would like to make the relation of our approach to QCD much more obvious. It is well-known that QCD at low energies should realize the main non–perturbative phenomena such as hadronization, spontaneous breaking of chiral symmetry (SB$\chi$S) and confinement. At present there is a consensus that the quark confinement realizes itself through a linearly rising interquark potential. As has been shown in Ref.[13] a linearly rising interquark potential is also responsible for SB$\chi$S. The formation of a linearly rising interquark potential is caused by the colour electric flux induced by the low–energy gluon field configurations. Integrating out gluon degrees of freedom around the low–energy gluon field configurations providing the formation of a linearly rising interquark potential one should arrive at an effective theory, an effective low–energy QCD, containing only quark degrees of freedom. The quark system described by this effective theory possesses chirally invariant and chirally broken phases. The transition to the chirally broken phase, i.e. the phenomenon of SB$\chi$S, accompanies itself the hadronization, i.e. bosonization and baryonization, describing the appearance of quark bound states with quantum numbers of low–lying mesons $q\bar{q}$, baryons $qqq$ and so on. Due to the quark confinement all observed quark bound states should be colourless. As in such an effective low–energy QCD the gluon degrees of freedom have been integrated out, all low–energy interactions of hadrons should be defined in terms of quark–loop diagrams.
Since nowadays in continuum space–time formulation of QCD the integration over gluon degrees of freedom can be hardly carried out explicitly, the approximate schemes of this integration admitting an analytical investigation are rather actual and play an important role for the understanding of the behaviour of the effective QCD at low energies. In our effective quark model with chiral $U(3) \times U(3)$ symmetry the result of the integration over gluon degrees of freedom concerning bosonization and baryonization is represented by the effective Lagrangian incorporating the effective local four–quark interactions responsible for $SB\chi S$, the linearized version of which contains quark–meson interactions like Eq.(2.2) and Eq.(3.2), and the quark–baryon interactions like Eq.(1.3). The former corresponds to the phenomenological description of the appearance of the tree–quark bound states – baryons. We emphasize that the spinorial structure of the three–quark currents $\eta_{B^8}(x)$ and $\eta_{B^{10}}(x)$ of the baryon octet and decuplet, respectively, is not unambiguously defined if we would follow only the $SU(3)$ or $SU(3) \times SU(2)_{\text{spin}} \to SU(6)$ flavour symmetry. Indeed, the three–quark currents with quantum numbers of the baryon octet and decuplet can be constructed from the scalar, pseudoscalar, vector and axial–vector diquark densities transforming like $(\tilde{3}_f, \tilde{3}_c)$ and $(6_f, \tilde{3}_c)$, respectively. The general form of the three–quark currents should contain the contributions of these diquark densities with four arbitrary coupling constants. However, as has been shown in Ref.[13], due to a linearly rising interquark potential, i.e. a dynamics induced by low–energy gluon field configurations, the spinorial structure of the three–quark currents with quantum numbers of the baryon octet and decuplet is fixed unambiguously and defined by Eq.(1.1) and Eq.(1.2). Thus, the structure of the three–quark currents, which we have used in the approach, is completely caused by the low–energy properties of QCD leading to the quark confinement.

The phenomenological character of the result of the integration over low–energy gluon configurations, responsible for the formation of a linearly rising interquark potential and leading to the baryonization of the effective low–energy QCD, is accentuated by the inclusion of the phenomenological coupling constant $g_B$ which is an analogy of the quark–meson coupling constants $g_{\pi qq}$ and $g_\rho$ describing phenomenologically the bosonization of QCD. The coupling constants $g_B$, $g_{\pi qq}$ and $g_\rho$ define the vertices of low–energy meson–baryon interactions in terms of the constituent quark–loop diagrams, the virtual momenta of which are restricted by the $SB\chi S$ scale $\Lambda_{\chi} = 0.94$ GeV. As has turned out the effective coupling constants of the $\pi NN$, $\pi N\Delta$ and $\gamma N\Delta$ interactions depend on $g_B$ and the quark condensate being a quantitative measure of $SB\chi S$ in the effective low–energy QCD.

Therefore, the spinorial structure of the three–quark currents with quantum numbers of the baryon octet and decuplet and the proportionality of the effective coupling constants of the meson(photon)–baryon–baryon interactions to the quark condensate testify the direct relation of all low–energy interactions of the baryon octet and decuplet to non–perturbative phenomena of low–energy QCD. Thus, in our effective quark model with chiral $U(3) \times U(3)$ symmetry the existence and the low–energy interactions of the observed baryon octet and decuplet are completely caused by low–energy non-perturbative properties of QCD leading to the quark confinement.
6 Acknowledgment

This work was supported in part by the Slovak Grant Agency for Science, Grant No. 2/4111/97 (M.Nagy).
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