Extremal limits and Bañados-Silk-West effect

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Abstract

A fascinating property of extremal Kerr black hole (BH) is that it could be act as a particle accelerator with infinite high center-of-mass (CM) energy [1]. In this note, we would like to discuss about such fascinating result and to point out that this infinite energy at the event horizon comes solely due to the singular nature of the extremal limit. We also show that a non-extremal Kerr BH can not transform into extremal Kerr BH by the Bañados-Silk-West mechanism. Moreover, we discuss about three possible geometries (near extremal, purely extremal and near horizon of extremal Kerr) of this mechanism. We further prove that near extremal geometry and near horizon geometry, precisely extremal geometry of extremal Kerr BHs are qualitatively different. Near extremal geometry and near horizon geometry gives the CM energy is finite, whereas precisely extremal geometry gives the diverging energy. Thus, we can argue that extremal Kerr BH and non-extremal Kerr BH are quite distinct objects. Finally, we show that the CM energy of collisions of particles not only diverges at infinite red-shift surface ($r_+$) but it could also diverges at the ISCO ($r_{isco}$) or at the circular photon orbit ($r_{cpo}$) or at the marginally bound circular orbit ($r_{mbco}$) or at the Cauchy horizon i.e. at $r \equiv r_{isco} = r_{cpo} = r_{mbco} = r_+ = r_- = M$. 

1 Introduction

In 2009, Bañados, Silk and West (BSW) [1] proposed an interesting mechanism for extremal rotating chargeless BH is that it could be act as a particle accelerators with infinite amount of CM energy, when two massive dark matter particles falling from rest into the infinite red-shift surface of the BH for some critical values of angular momentum of the in-falling particles. Whereas, this energy is finite for non-extremal Kerr BH. For non-rotating Schwarzschild BH the CM energy is also finite [2].

This proposal soon criticized by several researchers. Particularly in [3], the authors showed that there is an astrophysical bound i.e. maximal spin, back reaction effect and gravitational radiation etc. on that CM energy due to the Thorn’s bound [4] i.e. $\frac{a}{M} = 0.998$ ($M$ is the mass, $a = \frac{J}{M}$ is the spin parameter and $J$ is the angular momentum of the BH). Also in [5], the author derived that the CM energy in the near extremal situation for rotating Kerr BH is $E_{cm} \sim \frac{4.06}{(1-a)^{1/4}} + O((1-a)^{1/4})$. Lake [6] computed that the CM energy at the Cauchy horizon of a static RN BH and Kerr BH which are bounded. Grib and Pablov [7] investigated the CM energy for Kerr BH using multiple scattering method. The collision in the ISCO particles was investigated by the Harada et al. [8] for Kerr BH. On the other hand in [9], the author suggested that extremal Kerr BHs are neither particle accelerators nor dark matter probes.

In the present study, we shall prove that the CM energy would be diverging not only at the extremal infinite red-shift surface ($r_+$). Also at the direct ISCO [innermost stable circular orbit ($r_{isco}$)] or at the direct CPO [circular photon orbit ($r_{cpo}$)] or at the direct MBCO [marginally bound circular orbit ($r_{mbco}$)], the CM energy is also diverging i.e.

$$E_{cm} \mid_{r_{isco}} = E_{cm} \mid_{r_{mbco}} = E_{cm} \mid_{r_{cpo}} = E_{cm} \mid_{r_+} = E_{cm} \mid_{r_-} \to \infty$$

Thus we may conclude that the CM energy of collision of particles at $r \equiv r_{isco} = r_{cpo} = r_{mbco} = r_+ = r_- = M$ could be arbitrarily large for extremal Kerr space-time. On the other hand it can be
interpreted as if we choose the different collision point, say ISCO or CPO or MBCO then we get the
same diverging CM energy for each cases.

We will also prove that the disparity between precisely extremal geometry and non- extremal
geometry by using the BSW effect. One can not transform a non-extremal Kerr space-time to extremal
Kerr space-time via this process. This can be manifested by computing the the CM energy and it
is diverging for extremal Kerr space-time while the CM energy is finite for non-extremal space-time.
Moreover we also suggest that this diverging energy for precisely extremal kerr space-time comes
solely due to the singular nature of the extremal limit, whereas in [1] the author suggested that this
diverging collision energy comes solely due to the gravitational acceleration.

It has long been known that the extremal limit is singular and it is also discontinuous [10, 11, 12, 13]
has been examined from various aspect. In our earlier work [13], we have shown that geodetically an
extremal space-time and non-extremal space-time are quite distinct objects. For example, in case of
precisely extremal space-time the direct ISCO which lies on the event horizon which coincides with
the principal null geodesic generator of the horizon while the non-extremal space-time do not possess
such types of feature.

There are numerous key features which have been present in the extremal space-time while they
are completely absent in the non-extremal space-time. For instance, the Wald’s [14] bifurcation S2
present in the non-extremal space-time while the extremal space-time do not possess such S2. There
are three regions (Region I, Region II and Region III) in the Carter-Penrose diagram of non-extremal
space-times while the extremal space-times consists of only two regions (Region I and Region III)
[15, 16]. This implies that Region II is completely absent in extremal geometry while non-extremal
geometry do possesses. This could be interpreted as purely geometric discontinuous nature of the
extremal space-time and non-extremal space-time due to the lack of the “Region II”.

Indeed it is true that extremal space-time don’t have any trapped surfaces inside the event horizon
while the non-extremal space-time filled up with the trapped surfaces [17, 12, 13].

The fact that the proper radial distances between any two points in the extremal space-time always
diverges while the proper distances between any two points are finite in the non-extremal situation
[10]. Another interesting feature of extremal geometry is that both surface gravity and Hawking
temperature are zero while they are non-zero in the non-extremal geometry.

It is also known [7] that both coordinate time interval \( \Delta t \) as well as proper time interval \( \Delta \tau \)
diverges for extremal BH while they are finite in the non-extremal regime. This is another way to
prove the discontinuity between extremal geometry and non-extremal geometry. It was also mentioned
there that the angle \( \Delta \phi \) of the in-falling particle is found to be diverge for extremal Kerr BH while
they are finite for the non-extremal Kerr BH.

In an earlier work [18], we have proved that for spherically symmetric extremal string BH could
be act as a particle accelerator with diverging energy at \( r \equiv r_{isco} = r_{cpo} = r_{mbco} = r_+ = 2M \). The
main motivation in the present work comes from this work and from Bardeen et al. [19].

2 Review of ISCOs in Extremal Kerr BH:

Let us consider the Kerr metric in Boyer-Lindquist coordinates [19],

\[
\begin{align*}
    ds^2 &= -\frac{\Delta}{\rho^2} \left[ dt - a \sin^2 \theta d\phi \right]^2 + \frac{\sin^2 \theta}{\rho^2} \left[ (r^2 + a^2) d\phi - adt \right]^2 + \rho^2 \left[ \frac{dr^2}{\Delta} + d\theta^2 \right]. \\
\end{align*}
\]

where

\[
\begin{align*}
    a &\equiv \frac{J}{M}, \quad \rho^2 \equiv r^2 + a^2 \cos^2 \theta \\
    \Delta &\equiv r^2 - 2Mr + a^2 \equiv (r - r_+)(r - r_-) \\
\end{align*}
\]

\( M, a \) are the mass and angular momentum per unit mass or Kerr parameter respectively. It may be
noted that we have used geometrized units through out this work, where \( G = c = 1 = M \).
Now the radius of the event horizon (infinite red-shift surface) and Cauchy horizon are
\[ r_{\pm} = 1 \pm \sqrt{1 - a^2} \] (4)
which are the roots of the equation \( \Delta = 0 \). The extremal case defined as when \( r_+ = r_- \) or \( a = M = 1 \) or \( J = M^2 = 1 \).

The angular velocity of the horizons are
\[ \Omega_{\pm} = \frac{a}{r_{\pm}^2 + a^2} = \frac{a}{2r_{\pm}} \] (5)
The ergo-sphere is occur at
\[ r \equiv r_{\text{ergo}} = 1 + \sqrt{1 - a^2 \cos^2 \theta} \] (6)
Now let us consider \( x^\mu(\tau) \) represents the trajectory of the moving particles. \( \tau \) is the proper time of the moving particles. We restrict ourselves the geodesic motion of the particles confined on the equatorial plane i.e. \( u^\theta = 0 \) or \( \theta = \pi/2 \). Thus the equatorial time-like geodesics for Kerr space-times are
\[ u^t = \frac{dt}{d\tau} = \frac{1}{\Delta} [(r^2 + a^2 + 2a^2 \tau)E - \frac{2a}{r}L] \] (7)
\[ u^r = \frac{dr}{d\tau} = \pm \sqrt{E^2 + \frac{2}{r^3} (aE - L)^2 + \frac{a^2 E^2 - L^2}{r^2} + \frac{\Delta}{r^2} \sigma} \] (8)
\[ u^\theta = \frac{d\theta}{d\tau} = 0 \] (9)
\[ u^\phi = \frac{d\phi}{d\tau} = \frac{1}{\Delta} [(1 - \frac{2}{r})L + \frac{2a}{r} E] \] (10)
where \( \sigma = -1 \) for time-like geodesics and \( \sigma = 0 \) for null geodesics. The quantities \( E \) and \( L \) are the specific energy and the angular momentum of the particles respectively. The radial equation for the massive particles moving along the geodesics is described in terms of effective potential [21] as :
\[ \frac{1}{2} (u^r)^2 + V_{\text{eff}}(r) = 0 \] (11)
Thus one obtains \( V_{\text{eff}}(r) \) as :
\[ V_{\text{eff}}(r) = -\frac{1}{r} + \frac{L^2 - a^2(E^2 - 1)}{2r^2} - \frac{(L - aE)^2}{r^3} - \frac{E^2 - 1}{2} \] (12)
The circular orbit of the particle is defined by
\[ V_{\text{eff}}(r) = 0 \] (13)
and
\[ \frac{dV_{\text{eff}}(r)}{dr} = 0 \] (14)
Thus we may obtain the energy and angular momentum of the test particle associated with this circular motions evaluated at \( r = r_0 \) are given by
\[ E_0 = \frac{r_0^2 \pm 2r_0^\frac{3}{2} \pm a}{r_0^\frac{3}{2} \sqrt{r_0^\frac{3}{2} - 3r_0^\frac{1}{2} \pm 2a}} \] (15)
\[ L_0 = \pm \frac{r_0^\frac{3}{2} \sqrt{r_0^\frac{3}{2} - 3r_0^\frac{1}{2} \pm 2a}}{r_0^\frac{3}{2} \sqrt{r_0^\frac{3}{2} - 3r_0^\frac{1}{2} \pm 2a}} \] (16)
The upper (lower) sign holds for direct (retrograde) orbits. The innermost stable circular orbit (ISCO) equation can be obtained by solving the second derivative of the effective potential:

\[
\frac{d^2 V_{\text{eff}}(r)}{dr^2} = 0
\]

Thus one may obtain the ISCO equation for Kerr BH is:

\[
r^2 - 6r \mp 8a\sqrt{r} - 3a^2 = 0
\]

The solution of the equation gives the radius of ISCO for non-extremal Kerr space-times [19] are:

\[
r_{\text{isco}} = 3 + z_2 \mp \sqrt{(3 - z_1)(3 + z_1 + 2z_2)}
\]

where

\[
\begin{align*}
z_1 &= 1 + (1 - a^2)^{1/3}[(1 - a)^{1/3} + (1 + a)^{1/3}] \\
z_2 &= (3a^2 + z_1^2)
\end{align*}
\]

The circular photon orbit occurs at [19]

\[
r_{\text{cpo}} = 2\{1 + \cos\frac{2}{3}\cos^{-1}(\pm a)\}
\]

and the radius of the marginally bound circular orbit is given by

\[
r_{\text{mbco}} = 2 \mp a + 2\sqrt{(1 \mp a)}
\]

For extremal BH the direct ISCO occurs at \(r_{\text{isco}} = M = 1\), the direct photon orbit is at \(r_{\text{cpo}} = M = 1\) and the direct MBCO occurs at \(r_{\text{mbco}} = M = 1\). Thus three radii coincides with the horizon [19] i.e.

\[
r_{\text{isco}} = r_{\text{cpo}} = r_{\text{mbco}} = r_+ = r_- = M = 1
\]

3 BSW Effect:

Now we are going to compute the energy in the CM frame for the collision of two neutral particles coming from infinity with \(\frac{E_1}{m_0} = \frac{E_2}{m_0} = 1\) and approaching the BH with different angular momenta \(\ell_1 = L_1/M\) and \(\ell_2 = L_2/M\). The CM energy is derived by using the formula [1] which is valid in both flat and curved space-time given by

\[
\left(\frac{E_{\text{cm}}}{\sqrt{2m_0}}\right)^2 = 1 - g_{\mu\nu}u_1^\mu u_2^\nu.
\]

where \(u_1^\mu\) and \(u_2^\nu\) are the 4-velocities of the two particles, which can be determine from the following equation(10). Therefore one can compute the CM energy using the formula (25) [1]

\[
\left(\frac{E_{\text{cm}}}{\sqrt{2m_0}}\right)^2 = \frac{1}{r(r^2 - 2r + a^2)} \left[ 2a^2(r + 1) - 2a(\ell_1 + \ell_2) - \ell_1\ell_2(r - 2) + 2r^2(r - 1) - \sqrt{2(a - \ell_2)^2 - \ell_2^2r + 2r^2} \sqrt{2(a - \ell_1)^2 - \ell_1^2r + 2r^2} \right].
\]

Thus the limiting value of CM energy \(E_{\text{cm}}\) at the horizon \(r = r_+\) is given by

\[
E_{\text{cm}} = 2m_0\sqrt{1 + \frac{(\ell_1 - \ell_2)^2}{2r_-(\ell_1 - \ell_H)(\ell_2 - \ell_H)}}.
\]
The ranges of the angular momentum per unit rest mass for in-falling geodesics are given by

\[-2 \left(1 + \sqrt{1 + a} \right) \leq \ell \leq 2 \left(1 + \sqrt{1 - a} \right).\]  
(28)

Substituting the values of \(\ell_1 = 2 \left(1 + \sqrt{1 - a} \right)\) and \(\ell_2 = -2 \left(1 + \sqrt{1 + a} \right)\) in (27), and \(\ell_H = \frac{2m_0}{a} = \frac{2(1 + \sqrt{1 - a})}{a}\), we get the CM energy for non-extremal Kerr BH reads as [7]

\[E_{cm} = \frac{2m_0}{(1 - a^2)^{1/4}} \sqrt{\frac{(1 - a^2) + (1 + \sqrt{1 + a + \sqrt{1 - a}})^2}{1 + \sqrt{1 - a^2}}}.\]  
(29)

This is indeed a finite quantity. When \(a = J\) then CM energy reads as

\[E_{cm} = \frac{2m_0}{(1 - J^2)^{1/4}} \sqrt{\frac{(1 - J^2) + (1 + \sqrt{1 + J + \sqrt{1 - J}})^2}{1 + \sqrt{1 - J^2}}}.\]  
(30)

It follows from the above analysis the CM energy depends on the spin of the BH. Now if we take the extremal limit \(a = J = 1\), CM energy diverges. Therefore we have drawn the following conclusions:

(a) We cannot obtain the extremal Kerr space-time by taking the extremal limit of a non-extremal Kerr space-time. (b) BSW mechanism can not transform non-extremal Kerr BH to extremal Kerr BH. (c) The infinite amount of CM energy for extremal BH solely due to the singular nature of the extremal limit. The geometric discontinuity between two space-time gives the diverging energy.

Let us now discuss briefly about the BSW effect for the three possible geometries namely, near extremal geometry, extremal geometry and near horizon geometry of extremal Kerr space-time.

(a) Near Extremal Geometry:

It is astro-physically relevant to compute such geometry for the limiting [22] behavior of \(r_{isco}, r_{cpo}, r_{mbco}\). Taking \(a = 1 - \chi\); then

\[r_+ = 1 + (2\chi)^{1/2} + O(\chi^{3/2}), r_- = 1 - (2\chi)^{1/2} + O(\chi^{3/2})\]

\[r_{cpo} = 1 + \sqrt{\frac{8\chi}{3}} + O(\chi^{3/2}), r_{mbco} = 1 + 2\sqrt{\chi}\]

\[r_{isco} = 1 + \frac{3}{4\chi}, r_{ergo} = 2\]

\[\ell_1 = 2(1 + \sqrt{\chi}), \ell_2 = -2(1 + \sqrt{2 - \chi})\]  
(31)

Now calculating the CM energy for the above cases we get,

\[\left(\frac{E_{cm}}{\sqrt{2m_0}}\right)^2|_{r=r_{isco}} = \frac{F(r_{isco})}{G(r_{isco})}\]  
(32)

where

\[F(r_{isco}) = 2a^2(r_{isco} + 1) - 2a(\ell_1 + \ell_2) - \ell_1 \ell_2 (r_{isco} - 2) + 2r_{isco}^2 (r_{isco} - 1) - 2(a - \ell_1)^2 - \ell_2^2 r_{isco} + 2r_{isco}^2 \]

\[G(r_{isco}) = r_{isco}^2 (r_{isco}^2 - 2r_{isco} + a^2)\]  
(33)

\[E_{cm} \mid_{r_{isco}} \propto \frac{1}{\chi^{1/3}}\]  
(35)

Similarly, we can obtain for \(r_\pm \approx 1 \pm (2\chi)^{1/2}\), the CM energy:

\[E_{cm} \mid_{r_\pm} \propto \frac{1}{\chi^{1/4}}\]  
(36)
Similarly, we get for \( r_{mb} \approx 1 + 2\sqrt{\chi} \), the CM energy:

\[
E_{cm} \mid r_{mb} \propto \frac{1}{\chi^{1/2}}
\]  

(37)

and for \( r_{cop} \approx 1 + \sqrt{2\chi} \), the CM energy:

\[
E_{cm} \mid r_{cop} \propto \frac{1}{\chi^{1/2}}
\]  

(38)

In each cases, we have seen that the CM energy \( E_{cm} \) is finite for any generic values of \( \ell_1 \) and \( \ell_2 \).

The interesting phenomenon could be occur at the extremal limit \( \chi = 0 \).

\[
E_{cm} \mid r_{+} = E_{cm} \mid r_{-} = E_{cm} \mid r_{isco} = E_{cm} \mid r_{mbox} = E_{cm} \mid r_{cop} \rightarrow \infty
\]  

(39)

Interestingly, the CM energy at each collision point is diverging. In fact this infinite energy comes solely due to the singular nature of the extremal limit and due to the purely geometric discontinuity between the two space-time.

It is noteworthy to mentioned here that the expression for CM energy at the ergo-sphere \( r_{ergo} = 2 \) is found to be

\[
\left( \frac{E_{cm}}{\sqrt{2m_0}} \right)^2 = \frac{F(r_{ergo})}{G(r_{ergo})}
\]  

(40)

where

\[
F(r_{ergo}) = 8 - 2(1 - \chi)(\ell_1 + \ell_2) + 6(1 - 2\chi) - \sqrt{2(1 - \chi - \ell_2)^2 - 2\ell_2^2 + 8(1 - \chi - \ell_1)^2 - 2\ell_1^2 + 8}
\]  

(41)

\[
G(r_{ergo}) = 2(1 - 2\chi)
\]  

(42)

Now we may turn to the precisely extremal case to see what happens there.

(b) Precisely Extremal Geometry:

For the extremal cases \( a = M = 1 \) and at the extremal horizon \( r = M = 1 \), we obtain the CM energy [1]:

\[
E_{cm} \mid r_{+} = M = 1 = \sqrt{2m_0} \sqrt{\frac{\ell_2 - 2}{\ell_1 - 2} + \frac{\ell_1 - 2}{\ell_2 - 2}}
\]  

(43)

This is also finite for any generic values of \( \ell_1 \) and \( \ell_2 \). Here the CM energy strongly depends on the critical values of the angular momentum. When \( \ell_1 = 2 \) and \( \ell_2 = 2 \), the CM energy is diverging i.e.

\[
E_{cm} \rightarrow \infty
\]  

(44)

On the other hand using equations (29) and (30), at the extremal limit \( a = M = J = 1 \), it can be directly seen that

\[
E_{cm} \rightarrow \infty
\]  

(45)

Thus it proves that one can not obtain the extremal geometry via taking the extremal limit of a non-extremal/near-extremal geometry. The fact that CM energy is diverging in the extremal limit suggests that one should regard non-extremal and precisely extremal BHs as qualitatively distinct objects. Thus one can not turn a non-extremal Kerr BH into a extremal Kerr BH via limiting procedure because the extremal limit is singular. Alternatively, we can say that the diverging energy comes due to the singular nature of the extremal limit.
Whereas at the ergo-sphere the CM energy is found to be
\[ E_{cm} \big|_{r_{ergo}=2} = m_0 \sqrt{14 + 4\sqrt{2} - \sqrt{17 + 8\sqrt{2}}} \]  

(46)

It is indeed finite for both near-extremal case and extremal case. For our record, we also discuss the near horizon geometry of extremal Kerr (NHEK) BH.

(c) Near Horizon Geometry of Extremal Kerr BH:
In [20], the author computed the CM energy in the near horizon setup for Kerr-Newman BH. We apply the same formalism here for calculating the CM energy for the near horizon geometry of extremal Kerr background. Then we compare the results obtained in three possible geometries, near extremal geometry, extremal geometry and near horizon of extremal geometry.

In this geometry, the new coordinates are
\[ r \to 1 + \epsilon r_0 r, t \to \frac{t r_0}{\epsilon}, \phi \to \phi + \frac{ta}{\epsilon r_0} \]  

(47)

where \( r_0^2 = 1 + a^2 \), in such a way that in the new frame the horizon is situated at \( r = 0 \). Then one takes the limit \( \epsilon \to 0 \), which yields
\[ ds^2 = \rho_0^2 (-r^2 dt^2 + \frac{dr^2}{r^2} + d\theta^2) + \frac{(1 + a^2)^2 \sin^2 \theta}{\rho_0^2} (d\phi + \frac{2a}{1 + a^2} r dt)^2 . \]  

(48)

where \( \rho_0^2 = 1 + a^2 \cos^2 \theta \).

This is also a vacuum solution of the Einstein’s equations. Now restrict our attention to the equatorial geodesics, one obtains

\[ u^t = \frac{dt}{d\tau} = \frac{1}{r^2} - \frac{2a\ell}{(1 + a^2)r} \]  

(49)

\[ u^r = \frac{dr}{d\tau} = \pm \sqrt{1 - \frac{4a\ell}{(1 + a^2)r} - \frac{[\ell^2(1 - 4a^2) + (1 + a^2)^2]}{(1 + a^2)^2} r^2} \]  

(50)

\[ u^\theta = \frac{d\theta}{d\tau} = 0 \]  

(51)

\[ u^\phi = \frac{d\phi}{d\tau} = -\frac{(1 - 4a^2)\ell}{(1 + a^2)^2} - \frac{2a}{(1 + a^2)r} . \]  

(52)

Now deriving the CM energy near the horizon \( r \to 0 \) is given by
\[ E_{cm} = m_0 \sqrt{\frac{16a^2 + (\ell_1 - \ell_2)^2}{4a^2}} . \]  

(53)

This is in-fact CM energy of near horizon of non-extremal Kerr BH. This is indeed finite for any values of \( \ell_1 \) and \( \ell_2 \).

In the extremal case \( a = M = 1 \), the CM energy is found to be
\[ E_{cm} = m_0 \sqrt{\frac{16 + (\ell_1 - \ell_2)^2}{4}} . \]  

(54)

It is indeed finite for any generic values of \( \ell_1 \) and \( \ell_2 \).

4 Discussion

Thus we have argued that extremal Kerr BHs are fundamentally different class from non-extremal Kerr BHs because they have diverging CM energy. This is why we can claim that the non-extremal Kerr BH can not be transformed into extremal one by the BSW mechanism. The other reasons
are the topology of two space-times are drastically different. Thus extremal Kerr geometry can not be obtained as limits of non-extremal Kerr geometry. Since the extremal limit itself is a singular. Therefore the divergence energy generates due to this fact. The other aspect we have studied that the diverging energy in the CM frame due to the singular nature of the extremal limit. We also showed that the near extremal geometry, precisely extremal geometry and near horizon geometry of extremal Kerr BHs are qualitatively different. Near extremal geometry and near horizon geometry gives the CM energy is finite whereas purely extremal geometry gives the diverging energy.

Another way we can explained as extremal BH and non-extremal Kerr BHs are physically quite distinct objects and it is impossible to transform the former to the latter by the BSW mechanism. The another feature of this work is that for extremal BH three radii namely ISCO, CPO and MBCO coalesces to both the horizons namely event horizon \[ r = r_{\text{ISCO}} = r_{\text{CPO}} = r_{\text{MBCO}} = r^+ = r^- = M = 1 \] is arbitrarily large. This diverging CM energy for the colliding particles can only be attained when the BH is precisely extremal and only at infinite coordinate time as well as infinite proper time. To sum up, we suggests the infinite amount of collision energy at different collision point comes solely due to the singular nature of the extremal limit.

References

[1] M. Bañados et al., Phys. Rev. Lett. 103, 111102 (2009).
[2] A. N. Baushev, Int. J. Mod. Phys. D 18, 1195 (2009).
[3] E. Berti et al., Phys. Rev. Lett. 103, 239001 (2009).
[4] K. S. Thorn, Astrophys. J. 191, 507 (1974).
[5] T. Jacobson and T. P. Sotiriou, Phys. Rev. Lett. 104, 021101 (2010).
[6] K. Lake, Phys. Rev. Lett. 104, 211102 (2010); 104, 259903 (2010).
[7] A. Grib and Y. Pavlov, Astropart. Phys. 34, 581 (2011).
[8] T. Harada and M. Kimura, Phys. Rev. D 83, 024002 (2011).
[9] S. McWilliams, Phys. Rev. Lett., 110, 011102 (2012).
[10] S. Das et al., Mod. Phys. Lett. A12 (1997) 3067.
[11] S. M. Carroll et al., JHEP 0911 (2009) 109.
[12] P. Pradhan and P. Majumdar, Phys. Lett. A 375 (2011) 474-479.
[13] P. Pradhan and P. Majumdar, Eur. Phys. J. C 73, 2470 (2013).
[14] R. M. Wald, Phys. Rev. D 48 (1993) 3427.
[15] B. Carter, Phys. Rev. Lett. 141(4) (1966) 1242.
[16] B. Carter, Phys. Rev. Lett. 174(5) (1968) 1559.
[17] R. Penrose, Phys. Rev. Lett. 14 (1965) 57.
[18] P. Pradhan Astrophys. and Space Sci., DOI: 10.1007/s 10509-014-1896-9.
[19] J. M. Bardeen et al., The Astrophys. J. 178, 347 (1972).
[20] A. Galajinsky, Phys. Rev. D 88, 027505 (2013).
[21] R. M. Wald, *General Relativity* (Univ. Chicago Press, Chicago, 1984).

[22] S. Chandrashekar, *The Mathematical Theory of Black Holes*, Clarendon Press, Oxford (1983).