The Hall instability of weakly ionized, radially stratified, rotating disks

Edward Liverts and Michael Mond

Department of Mechanical Engineering, Ben-Gurion University of the Negev,
P.O. Box 653, Beer-Sheva 84105, Israel

eliverts@bgu.ac.il

and

Arthur D. Chernin

Sternberg Astronomical Institute, Moscow State University,
University Prospect 13, Moscow 119899, Russia

ABSTRACT

Cool weakly ionized gaseous rotating disk, are considered by many models as the origin of the evolution of protoplanetary clouds. Instabilities against perturbations in such disks play an important role in the theory of the formation of stars and planets. Thus, a hierarchy of successive fragmentations into smaller and smaller pieces as a part of the Kant-Laplace theory of formation of the planetary system remains valid also for contemporary cosmogony. Traditionally, axisymmetric magnetohydrodynamic (MHD), and recently Hall-MHD instabilities have been thoroughly studied as providers of an efficient mechanism for radial transfer of angular momentum, and of density radial stratification. In the current work, the Hall instability against nonaxisymmetric perturbations in compressible rotating fluid in external magnetic field is proposed as a viable mechanism for the azimuthal fragmentation of the protoplanetary disk and thus perhaps initiating the road to planet formation. The Hall instability is excited due to the combined effect of the radial stratification of the disk and the Hall electric field, and its growth rate is of the order of the rotation period. Such family of instabilities are introduced here for the first time in astrophysical context.

Subject headings: protoplanetary disks — instabilities — MHD
1. Introduction

Linear mode analysis provides a useful tool for gaining important insight into the relevant physical processes that determine the stability of rotating fluid configurations. The importance of magnetic fields in rotating disks has been demonstrated by the rediscovery of the magneto rotational instability (MRI) in which hydrodynamically stable flows with an angular velocity that is decreasing outwards are highly unstable when threaded by a weak magnetic field (Balbus & Hawley 1991). That investigation has been carried out in the magnetohydrodynamic (MHD) limit and invoked a number of approximations appropriate to the study of the evolution of long-wavelength perturbations in the weak magnetic field limit. The inclusion of the Hall electric field (Hall MHD) is a relatively recent development (Wardle 1999; Balbus & Terquem 2001; Sano & Stone 2002a; Sano & Stone 2002b; Salmeron & Wardle 2003; Desch 2004; Urpin & Rudiger 2005; Rudiger & Kitchatinov 2005). The Hall electric field plays an important role in the disk’s dynamics when the coupling between the electrons and neutral components of the fluid is low. In such cases, the inertial length of the ions is longer than the characteristic perturbation’s length scale and consequently the motions of the ions and electrons are decoupled. Indeed, it has been shown that the Hall electric field has a profound effect on the structure and growth rate of unstable modes like the MRI’s. Furthermore, as will be shown below, the Hall term gives rise to new branches of unstable modes. In particular, it will be shown that the Hall term, in the presence of radial stratification, excites non axisymmetric instabilities.

As an astrophysical interest, we mention magnetically supported cool molecular clouds and its dynamics. In the disk of typical spiral galaxy, the magnetic field strength is usually estimated to be of several to more than 10µG while in some regions of spiral galaxies the magnetic field strength may be higher than several tens of microgauss (see Beck et al. 1996). By that estimations and others, it is almost certain that MHD density waves should also play an important role in the dynamic and evolution of structures within a magnetized gas disk (Fan & Lou 1996). As is well known in classical nebular hypothesis by Kant-Laplace the condensation in protoplanetary rotating disk plays an important part in forming stars and planets. That part of the Kant-Laplace theory remains valid also for a contemporary cosmogony. The “standard” theory of the multistage accretionary formation of planets, or the so-called core accretion mechanism (Savronov 1972; Pollack et al. 1996) remained the most popular until recently, when it was criticized by (Boss 2002; Boss 2003) and others. The main problem of the latter is the timescale, which is longer than estimates of the lifetime of many planet-forming disks (Taylor 1962; Feigelson & Montmerle 1999). In any case all theories rely on instabilities as a mechanism to transform a relatively uniform rotating gaseous disk into a planetary system. That is, at an early stage, the protosolar nebula are formed by fragments that separated from a molecular cloud. Planetary formation is thought
to start with inelastically colliding gaseous and dust particles settling to the central plane of the rotating nebula to form a thin layer around the plane. During the early evolution of the disk it is believed that the dust particles coagulate also into comets-planetesimals. On attaining a certain critical thickness (and, correspondingly, low temperature) small in comparison with the size of the disk, as a result of a local gravitational collapse the nebula disintegrates into the central body and a number of separate protoplanets. Instabilities arise as the thickness of the disk is reduced (Gurevich & Chernin 1978, Savronov 1972). If a rotating gaseous disk has a large vertical thickness owing to a high internal temperature, then it is stabilized against gravitational instabilities by thermal motion (Gurevich & Chernin 1978). In (Boss 2004) it is demonstrated that convective cooling is able to cool the disk midplane at the desired rate to produce clumps in marginally unstable disks. The physical phenomena treated in the current paper occur during the stage of evolution of the protoplanetary cloud when the dust and gas in the disk start to condense into planetesimals and a star with current luminosity emerges at the center of the nebula.

The rest of paper is organized as follows. In Sec.2.1 we present basic equations and state our assumptions. In Sec.2.2 we present the dispersion relation to be solved. In Sec.2.3 we pay particular attention to the conditions of the existing of complex-conjugate roots of the dispersion relation. We present our conclusions and discussion in Sec.3.

2. Hall MHD equations and the dispersion relation

2.1. Basic equations

We consider a thin rotating gaseous disk with angular velocities $\Omega(r)$, where $G$ is the gravitational constant and $M$ is the mass of the central body, and $r$ is the distance from the center of the rotating disk. The thickness of the disk can be estimated by $c_s/\Omega(r)$ where $c_s$ is the sound speed. The disk is made of partially ionized plasma where ions are well coupled to the neutrals while the electrons are not. However, charge neutrality, $n_e = n_i$, is assumed to be valid. The disk is immersed in a magnetic field directed along the rotation axis (defined as the $z$-axis in our frame of reference). Following (Braginskii 1965), the equations that govern the evolution of the two fluid system, namely the heavy particles (ions and atoms) and the electrons, are:

the momentum equation

$$nm_i \frac{d\vec{v}}{dt} = -\nabla p + \frac{1}{c} \vec{j} \times \vec{B} - nm_i \frac{GM}{r^3} \vec{r}, \quad (1)$$

here $n$ is the number density of the heavy particles (ions and atoms), and the generalized
Ohm’s law

\[ m_e \frac{d\vec{u}}{dt} = e\vec{E} + \frac{e}{c} \vec{v} \times \vec{B} - \frac{1}{n_e c} \vec{j} \times \vec{B} + T_e \frac{\nabla n_e}{n_e} - \frac{e\vec{j}}{\sigma_R}, \]

(2)

where \( \sigma_R \) is the electrical conductivity. The generalized Ohm’s law as given in Eq.(2) differs from the corresponding equation of MHD theory by the term on the left-hand side that describes the effect of electron inertia, by the third term on the right-hand side, which is the Hall effect, by the fourth term, which describes the effect of the electrons’ pressure and by the last term that represents the drag force acting on the electrons. In addition, it is convenient to write the induction equation by substituting the electric field from Eq.(2) into Faraday’s equation which than becomes

\[ \frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) - \nabla \times \left( \frac{\vec{j} \times \vec{B}}{en_e} \right) - \frac{m_e c}{e} \nabla \times \frac{d\vec{u}}{dt} + \eta \Delta \vec{B}, \]

(3)

where \( \eta = c^2/4\pi \sigma_R \) is magnetic diffusivity. The relative importance of the various terms in Eq.(3) may be investigated by considering appropriate length and time scales. Thus, consider, for example, the second term on the right-hand side of Eq.(3) (the Hall term). If we assume that \( \nabla \approx 1/L, \ j \approx cB/4\pi L \) (displacement current is neglected), where \( L \) is a typical length-scale of the density inhomogeneity, then that term will be of the same order as the convective electric field if \( \ell_i \approx \sqrt{\zeta}L \) where

\[ \ell_i = \frac{c}{\omega_{pi}} = \frac{c\sqrt{m_i}}{\sqrt{4\pi e^2 n_i}}. \]

In other words, in order for the Hall term to be important the length-scale \( L \) should be of the same order of magnitude or less than the ions inertial length divided by \( \sqrt{\zeta} \) where \( \zeta \) is ionization degree, i.e.,

\[ L \leq \ell_i / \sqrt{\zeta}. \]

(4)

Similarly, the ratio of the electron inertia term [the last term on right-hand side of Eq.(3)] to the convective term is given by:

\[ \frac{\omega}{\Omega_e} \frac{c}{4\pi en_e L} \approx \frac{1}{\sqrt{4\pi nm_i}} \]

Thus, the electron’s inertial term is important for inhomogeneities which are characterized by \( L \approx c/\omega_{pe} = \ell_e \). As far as the time-scale is concerned, it is seen from the relationship \( v_A / \ell_i = \sqrt{\zeta} \Omega_i \) (\( v_A = B/\sqrt{4\pi nm_i} \) is the Alfvén velocity) that for the Hall term to be important the frequency should be \( \omega > \sqrt{\zeta} \Omega_i \) while the electron inertia should be retained if \( \omega > \Omega_e \). Note that in case of \( \omega >> \sqrt{\zeta} \Omega_i \) the electrons drift in the wave’s electric field, while the ions are immobile. Thus, if \( \sqrt{\zeta} \Omega_i << \omega << \Omega_e \) the second term on the right-hand side of Eq.(3),
namely the Hall term, is the leading term. This is the reason to term such approximation HMHD and waves in that regime Hall waves. It is seen from Eq. (3) that in such case

$$\omega \approx \frac{c^2 \Omega_i}{\omega_{pi} L^2} = \frac{v_{Hd}}{L}$$

where \(v_{Hd} = \frac{v^2_A}{\zeta \Omega_i L}\) is the phase velocity of the Hall waves in the presence of density gradients whose scale length is \(L\). Such waves exist merely due to electron drift in the electric field of the Hall waves. It should be noted from the discussion above that the conditions for the Hall term to be significant are more easily satisfied as the degree of ionization is decreasing. Finally, it should be noted that for low enough densities the Ohmic dissipation term in Eq. (3) (the last term on the right hand side) is negligible in comparison with the Hall term (Balbus & Terquem 2001). Additional support to that point of view is provided by (Jin 1996) who estimated the ratio of the rotation period to Ohmic dissipation time to be of the order of \(10^{-3} k^2 H^2\). As will be seen in the subsequent sections, the growth rates of the Hall instability are of the order of the rotation period and the relevant wave lengths satisfy \(kH < 1\). Hence, the effect of Ohmic dissipation will be neglected from now on bearing in mind that it may nonetheless lower the growth rates of the investigated instabilities.

2.2. The linearized equations and the dispersion relation

We consider a differentially rotating disk with angular velocity \(\Omega(r)\) where \(r\) is the distance from the disk’s center, and \(r\)-dependent density \(\rho(r)\). The disk is threaded by and axial magnetic field \(\vec{B} = B_0(r) \hat{z}\). The equations governing the linear stability of the rotating disk may be derived from Eqs. (1)-(3) by assuming that the perturbations of the steady rotation are of the following form

$$f(r, \theta) = f(r) exp[i(m\theta - \omega t)]$$

where \(f(r, \theta)\) stands for the perturbation of any of the physical variables that describe the system. It should be noted that strictly speaking expression (5) may be used only for rigid rotation, as differential rotation results in non exponential perturbations. Nonetheless that fact accentuates the nondependence of the Hall instability on the rotation shear, in contrast to MHD instabilities like MRI’s. A full description of the temporal evolution of perturbations in differentially rotating disk will be described in a forthcoming publication. The amplitudes of the perturbed \(\theta\)-component of the velocity, density and \(z\)-component of the magnetic field can be expressed in terms of the amplitude of the radial component of the perturbed velocity. This results in an ordinary differential equation for \(u_r(r)\) that should be solved with appropriate boundary conditions thus yielding an eigenvalue problem. However for purposes
of demonstration we first consider the simplified case of local approximation by assuming that \( k_r \ll k \) where \( k_r \sim \frac{1}{u_r} \frac{\partial u_r}{\partial r} \) and \( k \sim \frac{m}{r} \). After linearization, Eqs.(1)-(3) assume the following form:

\[
i(\omega - m\Omega)u_r + 2\Omega u_\theta \cong 0
\]

\[
(\omega - m\Omega)u_\theta + \frac{X^2}{2\Omega} u_r = \frac{m}{r} \left( \frac{c_s^2}{\rho} + \frac{m}{r} V_A^2 \right) \frac{b_z}{B_0}
\]

\[
- \frac{1}{\zeta \rho \Omega_i} \frac{m}{r} \nabla_r \frac{B_0^2}{8\pi \rho} + \frac{m}{r} u_\theta - [\omega - m(\Omega + \frac{\ell_2 \Omega_i}{Lr})] \frac{b_z}{B_0} = 0
\]

\[
(\omega - m\Omega) \frac{\sigma}{\rho} - \frac{m}{r} u_\theta = 0
\]

where \( \sigma \) is the perturbation of the density, and \( \chi \) is the epicyclic frequency given by \( \chi = \sqrt{4\Omega^2 + 2r\Omega d\Omega/dr} \). The system of equations (6)-(9) yields following dispersion relation:

\[
\tilde{\omega}^3 - \tilde{\omega}^2 \omega_{Hd} - \tilde{\omega} [\chi^2 + k^2 (c_s^2 + V_A^2)] + \omega_{Hd} (\chi^2 + k^2 \frac{L}{\rho} \frac{\partial P}{\partial r}) = 0
\]

where \( \omega \) is the Hall drift frequency. It is a direct result of the assumed form of the perturbation, i.e., Eq.(5) that the axial and radial components of the perturbed magnetic field as well as the axial component of the perturbed velocity are zero. In addition, the derivatives of the equilibrium profiles have been neglected in deriving the linearized equations above, except in the axial component of Faraday’s law (Eq.(8)) where, due to the Hall term, they are of the same order of the rest of the terms.

### 2.3. The instability against non axisymmetric perturbations

In the case of homogeneous density and magnetic field strength \( (L \to \infty) \) the two roots of Eq.(10) represent two stable branches of density waves that originate due to both the rotation of the disk as well as the external magnetic field. However in the case of density or magnetic field inhomogeneity the roots of Eq.(10) with real coefficients are real if, and only if, the following conditions are satisfied:

\[
D = \frac{\omega_{Hd}^6}{108} (27X^2 + 4X(1 - 9Y^2) - 4Y^2 + 8Y^4 - 4Y^6) \leq 0
\]

where

\[
X = -\frac{k^2 L \nabla_r P}{\rho \omega_{Hd}^2} + \frac{k^2 (c_s^2 + V_A^2)}{\omega_{Hd}^2}
\]
and

$$Y^2 = \frac{\chi^2}{\omega_{Hd}^2} + \frac{k^2(c_s^2 + V_A^2)}{\omega_{Hd}^2}$$

. If the last condition is not fulfilled, Eq.(10) has two complex-conjugate roots one of which signifies instability. The conditions for that to happen are:

$$X \geq \frac{2}{27}(-1 + 9Y^2 + \sqrt{1 + 9Y^2 + 27Y^4 + 27Y^6})$$ (12)

or

$$X \leq \frac{2}{27}(-1 + 9Y^2 - \sqrt{1 + 9Y^2 + 27Y^4 + 27Y^6})$$ (13)

Further insight into the onset of the Hall instability may be gained by denoting the total pressure radial derivative by $(p + B_0^2/8\pi)/L_P$ where $L_P$ is the inhomogeneity length of the total pressure. Next, the well known relation $c_s = H\Omega$ is recalled, and finally normalizing frequencies to $\Omega$, and wavelengths to $H$ the following dispersion relation is obtained:

$$\tilde{\omega}^3 - q\alpha\tilde{\omega}^2 - \tilde{\omega}[\hat{\chi}^2 + \tilde{q}^2(1 + \beta^{-2})] + q\alpha[\hat{\chi}^2 + \xi\tilde{q}^2(1 + \beta^{-2}/2)] = 0$$ (14)

where

$$\alpha = \frac{1}{\beta L\sqrt{\zeta}},$$

$$\beta = \frac{c_s}{V_A}, q = kH, \hat{\chi} = \chi/\Omega, \text{and} \xi = L/L_P.$$ It is first noted that for $\alpha \to 0$ the MHD regime is recovered and the roots of Eq. (14) represent the stable combination of the fast magnetosonic waves and the epicyclic oscillations. However, as $\alpha$ is increased (which means that $L$ is decreased relative to the inertial length of the ions) the system enters into the Hall MHD regime. In that case, elementary analysis of Cardano’s solution of cubic equations reveals that the nature of the roots of the dispersion equation (14) hinges upon the value of $\mu$ which is given by:

$$\mu = \frac{\hat{\chi}^2 + \xi\tilde{q}^2(1 + \beta^{-2}/2)}{\hat{\chi}^2 + \tilde{q}^2(1 + \beta^{-2})},$$ (15)

and the roots of the following quadratic equation:

$$4\mu S^2 + (1 + 18\mu - 27\mu^2)S + 4 = 0,$$ (16)

where

$$S = \frac{q^2\alpha^2}{\hat{\chi}^2 + q^2(1 + \beta^{-2})}.$$ 

Thus, Eq. (14) has two complex roots and hence the system is unstable in the following two cases:
1. $\mu > 1$, for $S_1 < S < S_2$, where $S_1$ and $S_2$ are the roots of Eq. (16).
   In this case $\xi$ must be positive which means that the density and the total pressure change radially in the same direction. It is therefore obvious that regions of instability occur where the total pressure changes more rapidly than the density ($\xi > 1$). This is indeed the case in polytropic disks for which $L/L_p = \gamma > 1$ where $L_p$ is the inhomogeneity length associated with the pressure. Hence $\xi > \gamma$ and consequently $\mu > 1$, which means that the disk is unstable under the Hall instability if $\beta$ is such that $S$ is between the two roots of Eq. (16). Exact values of the growth rate, obtained by the numerical solution of Eq. (14), are depicted in Fig. 1 for Keplerian rotation ($\hat{\chi} = 1$), with $\ell_i/L\sqrt{\zeta} = 10$, and $\xi = 5/3$, for various values of $\beta$. It is found that the disk is unstable for $1 < \beta < 20$.

2. $\mu < 0$, for $S > \max(S_1, S_2)$.
   In this case it is obvious that the gradients of the density and the total pressure must have opposite signs (i.e. $\xi < 0$). Such situations may occur when radial inflow plays an important role in the dynamics of the evolving disk, such as in young protoplanetary clouds (Hogerheijde 2003), or when radial rings of non monotonic density profiles are formed due to gravitational instabilities (Mayer et al. 2005). In these cases, in the limit $\alpha \gg 1$ one of the solutions of Eq.(14) is approximated by equating the second and fourth terms in Eq.(14) and is given by:

$$\omega = \pm i\gamma,$$  \hspace{1cm} (17)

where

$$\gamma = \Omega \sqrt{|\xi| q^2(1 + \beta^{-2}/2) - \hat{\chi}^2}.$$ \hspace{1cm} (18)

It is clear that in this case, the rotation plays a stabilizing role. Also, since $S \propto 1/\beta$, there is an upper bound on $\beta$ for instability to occur but not a lower bound. Furthermore, in the limit of small $\beta$ the growth rate grows without bound as $\beta$ is decreased. Numerical solutions of Eq. (14) for Keplerian rotation, and for $\ell_i/L\sqrt{\zeta} = 10$ and $\xi = -1.5$ are depicted in Fig. 2. The instability exists for $\beta < 9$ and the growth rate is indeed a growing function of $1/\beta$.

The unstable branch of the dispersion relation (14), termed the Hall instability, has been investigated in detail in (Liverts & Mond 2004) where it has been shown that it represents a quasi-electrostatic slow mode in which the perturbation in the ions’ density and velocity play a crucial role while the perturbed electrons’ density and magnetic field are negligible. Thus, the Hall instability provides a powerful mechanism for the azimuthal breaking of radially stratified disks into small fragments of size comparable to the disk’s thickness and on time scale of the order of the rotation period.
3. Summary and discussion

This paper examined the instability of a weakly ionized, thin disks threaded by an external magnetic field within the Hall-MHD model. The vertical stratification as well as the azimuthal variations of the disks properties were ignored whereas radial distribution was considered. In particular, in astrophysical context such weakly ionized gaseous nebulae are relevant to the protoplanetary disks. The conditions in protoplanetary disks were discussed in (Savronov 1972, Gurevich & Chernin 1978). At a certain stage of its evolution the star nebula is believed to have a characteristic disk size up to order of 30-100 AU and the total mass of the disk is believed to be less than roughly 0.1 of the mass of the central star. This yields an integrated column density of $\Sigma \approx 3 \times 10^2 g \cdot cm^{-2}$. Assuming that the mean mass of the particles is $m_p = \Lambda m_H$, the number density is given by $n = \Sigma / \Lambda m_H H(r) \approx 2 \times 10^{14} cm^{-3}$ where $H(r)$ is the scale thickness of the disk. To estimate the value of the factor $\Lambda$ it should be noted that within this system, lighter elements such as hydrogen and helium were driven out of the central regions by star wind and radiation pressure during a highly active phase, leaving behind heavier elements like Na, Al, and K and dust particles. Thus in the outer part of the star nebula, ice and volatile gases were able to survive. As a result, the inner planets are formed of minerals, while the outer planets are more gaseous or icy. Concerning thickness estimation one can use $c_s(r) / \Omega(r)$, which yields $H(r) \approx 0.002$AU. It should be noted also that due to very low temperature of the protoplanetary disks the only sources of ionization are non-thermal, e.g., cosmic rays, X-rays and the decay of radioactive elements. Following (Sano & Stone 2002b) the ionization degree at the midplane of gaseous disk is estimated as $\zeta = n_e / n \approx 10^{-12}$. So the disk material is a partially ionized plasma where ions and charged small dust grains are well coupled to the neutrals but electrons are not. Following this assumption we can estimate that $\ell_i / \sqrt{\zeta}$ is up to order of 1 AU. It should be noted that such big values are obtained mostly due to low degree of ionization, however one should keep in mind that existence of positively charged grain particles increases the inertial length (see definition after Eq.(3)) and thus enhances the effect of the Hall term. In Sec.2 it has been demonstrated that the Hall term is important if $H(r) < L < \ell_i / \sqrt{\zeta}$. Thus, due to the very small values of the ionization degree in protoplanetary disks the HMHD model has to be employed when studying the stability of structures with realistic radial density inhomogeneities. Indeed, radial stratification with length scale $L > H$ may exist in the disk due to such mechanisms as axisymmetric density waves that give rise to alternating high and low density rings. On the other hand, $L$ is bounded from below by $H$ due to thermal pressure (Gurevich & Chernin 1978). Hence, as has been further shown in Sec.2, protoplanetary disks with such radial density distributions are susceptible to strong nonaxisymmetric instabilities whose growth rates are of the order of the rotation period of the disk. Such instabilities result in breaking of the density rings into fragments that may be identified as planetesimals.
Following widely adopted standpoint, an accumulation of planetesimals may lead to the next stage of evolution of protoplanetary disk, which is the coalescence of the planetesimals into protoplanets. Such planetesimals may survive the thermal pressure if their characteristic size, i.e., $1/k$, is bigger than the disk’s thickness $H$ (Gurevich & Chernin 1978). On the other hand, the linear analysis presented in Sec.2.2 is valid if $kL < 1$. Combining those two conditions, and taking into account condition (11) results in the following limitations on $k$:

$$\sqrt{\zeta \frac{\Omega_i}{\Omega}} < kH < 1$$

(19)

where value of $\beta$ has been taken as 1 for simplicity. Thus, for typical values in protoplanetary disks, namely, $\zeta = 10^{-12}$, $\Omega_i = 10^4 \times B_0/Gs^{-1}$, $\Omega \approx 1.9 \times 10^{-7}s^{-1}$ the left hand side of the second inequality in (19) is of the order of or smaller than unity. It is therefore again the small ionization degree that enables the onset of the Hall instability, in the small magnetic field limit, and by thus providing a mechanism of initiation of the standard scenario of planet formation.

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Fig. 1.— Growth rates of the Hall instability for Keplerain rotation, and $\ell_i/L\sqrt{\zeta} = 10$ and $\xi = 1.66$. 
Fig. 2.— Growth rates of the Hall instability for Keplerain rotation, and $\ell_i/L\sqrt{\zeta} = 10$ and $\xi = -1.5$. 