Performance bounds of non-adiabatic quantum harmonic Otto engine and refrigerator under a squeezed thermal reservoir

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We analyze the performance of a quantum Otto cycle, employing time-dependent harmonic oscillator as the working fluid undergoing sudden expansion and compression strokes during the adiabatic stages, coupled to a squeezed reservoir. First, we show that the maximum efficiency that our engine can achieve is 1/2 only, which is in contrast with the earlier studies claiming unit efficiency under the effect of squeezed reservoir. Then, we obtain analytic expressions for the upper bound on the efficiency as well as on the coefficient of performance of the Otto cycle. The obtained bounds are independent of the parameters of the system and depends on the reservoir parameters only. Additionally, with hot squeezed thermal bath, we obtain analytic expression for the efficiency at maximum work which satisfies the derived upper bound. Further, in the presence of squeezing in the cold reservoir, we specify an operational regime for the Otto refrigerator otherwise forbidden in the standard case.

I. INTRODUCTION

The concept of Carnot efficiency (ηC) is one of the most important results in physics, which led to the formulation of the second law of thermodynamics [1]. It puts a theoretical upper bound on the efficiency of all macroscopic heat engines working between two thermal reservoirs at different temperatures. However, with the rise of quantum thermodynamics [2–5], many studies have showed that this sacred bound may be surpassed by quantum heat machines exploiting exotic quantum resources such as quantum coherence [6–9], quantum correlations [10–14], squeezed reservoirs [15–25], among others. In such cases, the second law of thermodynamics has to be modified to account for the quantum effects, and the notion of generalized Carnot bound is introduced which is always satisfied [10–14, 26–27]. In this context, different theoretical studies have been carried out to study the implications of work extraction when quantum heat machines are coupled to nonequilibrium stationary reservoirs [27–31]. In particular, it is instructive to look into the working of heat machines coupled to squeezed thermal reservoirs. The use of squeezed thermal reservoir allows us to extract work from a single reservoir [19] and obtain generalized version of Curzon-Ahlborn efficiency [15]. Manzano et. al studied a modified version of HQOC and discussed the effect of squeezed hot bath in different operational regimes [19]. Extending the analysis to the quantum refrigerators, Long and Liu optimized the performance of a HQOC in contact with low temperature squeezed thermal bath and concluded that the coefficient of performance (COP) can be enhanced by squeezing [21].

With the exception of Refs. [20, 24], all the aforementioned studies involving squeezed reservoirs are confined to the study of quasi-static Otto cycle in which adiabatic steps are performed quasi-statically, thus producing vanishing power output. In this work, we fill this gap by confining our focus to the highly non-adiabatic (dissipative) regime corresponding to the sudden switch of frequencies (sudden compression/expansion strokes) during the adiabatic stages of the Otto cycle. We obtain analytic expressions for the upper bounds on the efficiency and COP of the HQOC coupled to a squeezed thermal reservoir.

The paper is organized as follows. In Sec. II, we discuss the model of HQOC coupled to a hot squeezed thermal reservoir. In Sec. III, we obtain analytic expression for the upper bound on the efficiency of the engine operating in the sudden switch limit. We also obtain analytic expression for the efficiency at maximum work and compare it with the derived upper bound. In Sec. IV, we repeat our analysis for the Otto refrigerator coupled to a cold squeezed reservoir and obtain upper bound on the COP of the refrigerator. We conclude in Sec. V.

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II. QUANTUM OTTO CYCLE WITH SQUEEZED RESERVOIR

We consider quantum Otto cycle of a time-dependent harmonic oscillator coupled to a hot squeezed thermal bath while cold bath is still purely thermal in nature. It consists of four stages: two adiabatic and two isochoric. These processes occur in the following order: (1) Adiabatic compression $A \rightarrow B$: To begin with, the system is at inverse temperature $\beta_1$. The system is isolated and the frequency of the oscillator is increased from $\omega_1$ to $\omega_2$. Work is done on the system in this stage. The evolution is unitary and von Neumann entropy of the system remains constant. (2) Hot isochore $B \rightarrow C$: During this stage, the oscillator is coupled to the squeezed thermal heat reservoir at inverse temperature $\beta_2$ at fixed frequency ($\omega_2$) and allowed to thermalize. No work is done in this stage, only heat exchange between the system and reservoir takes place. After the completion of the hot isochoric stage, the system relaxes to a nondisplaced squeezed thermal state $|\beta_2, r\rangle$ with mean photon number $\langle n(\beta_2, r)\rangle = \langle n\rangle + (2\langle n\rangle + 1)\sinh^2 r$, where $r$ is the squeezing parameter and $\langle n\rangle = 1/(e^{\beta_2 \omega_2} - 1)$ is the thermal occupation number (we have set $\hbar = k_B = 1$ for simplicity). (3) Adiabatic expansion $C \rightarrow D$: The system is isolated and the frequency of the oscillator is unitarily decreased back to its initial value $\omega_1$. Work is done by the system in this stage. (4) Cold isochore $D \rightarrow A$: To bring back the working fluid to its initial state, the system is coupled to the cold reservoir at inverse temperature $\beta_1$ ($\beta_1 > \beta_2$), and allowed to relax back to the initial thermal state $A$.

The average energies $\langle H \rangle$ of the oscillator at the four stages of the cycle read as follows:

$$\langle H \rangle_A = \frac{\omega_1}{2} \coth\left(\frac{\beta_1 \omega_1}{2}\right),$$

$$\langle H \rangle_B = \frac{\omega_2}{2} \coth\left(\frac{\beta_1 \omega_1}{2}\right),$$

$$\langle H \rangle_C = \frac{\omega_2}{2} \coth\left(\frac{\beta_2 \omega_2}{2}\right) \Delta H(r),$$

$$\langle H \rangle_D = \frac{\omega_1}{2} \lambda \coth\left(\frac{\beta_2 \omega_2}{2}\right) \Delta H(r),$$

where $\Delta H(r) = \langle n(\beta_2, r)\rangle/\langle n\rangle = 1 + (2 + 1/\langle n\rangle)\sinh^2 r$ reflects the effect of the squeezed hot thermal bath on the mean energy of the oscillator, $\lambda$ is the dimensionless adiabaticity parameter $[43]$. For the adiabatic process, $\lambda = 1$; for non-adiabatic expansion and compression strokes, $\lambda > 1$. The expression for mean heat exchanged during the hot and cold isochores can be evaluated, respectively, as follows:

$$\langle Q_2 \rangle = \langle H \rangle_C - \langle H \rangle_B$$

$$= \frac{\omega_2}{2} \left[\Delta H(r) \coth\left(\frac{\beta_2 \omega_2}{2}\right) - \lambda \coth\left(\frac{\beta_1 \omega_1}{2}\right)\right],$$

$$\langle Q_4 \rangle = \langle H \rangle_A - \langle H \rangle_D$$

$$= \frac{\omega_1}{2} \left[\coth\left(\frac{\beta_1 \omega_1}{2}\right) - \lambda \Delta H(r) \coth\left(\frac{\beta_2 \omega_2}{2}\right)\right].$$

Here, we are employing a sign convention in which heat absorbed (rejected) from (to) the reservoir is positive (negative) and work done on (by) the system is positive (negative).

Since after one complete cycle, the working fluid comes back to its initial state, the extracted work in one complete cycle is given by, $\langle W_{\text{ext}} \rangle = \langle Q_2 \rangle + \langle Q_4 \rangle > 0$. In this work, we are interested in the sudden switch case for which $\lambda = (\omega_1^2 + \omega_2^2)/2\omega_1\omega_2$ $[43][45]$. Substituting above expression for $\lambda$ in Eqs. (5) and (6), we obtain the following expressions for the extracted work, $\langle W_{\text{ext}} \rangle$, and efficiency, $\eta = \langle W_{\text{ext}} \rangle/\langle Q_2 \rangle$, of the engine, respectively:

$$\langle W_{\text{ext}} \rangle = \langle Q_2 \rangle + \langle Q_4 \rangle = \frac{\omega_2^2 - \omega_1^2}{4\omega_1\omega_2} \left[\omega_1 \Delta H(r) \coth\left(\frac{\beta_2 \omega_2}{2}\right) - \omega_2 \coth\left(\frac{\beta_1 \omega_1}{2}\right)\right].$$
Now, efficiency, $\eta$, can attain its maximum when the expression inside the square bracket attains its minimum value. The minimum value of the first term can be inferred as follows: $\text{Min}[A_1] = \text{Min}[2/(1 - \omega_1^2/\omega_2^2)] = 2$, as $\omega_1 < \omega_2$ for the engine operation. Similarly, $\text{Min}[A_2] = \text{Min}[1/2 \Delta H(r) \coth(\beta_2 \omega_2/2) \tanh(\beta_1 \omega_1/2)] = 0$, which can be inferred from the positive work condition, $\langle W_{\text{ext}} \rangle > 0$ [see Eq. (7)], which implies that $\Delta H(r) \coth(\beta_2 \omega_2/2) \tanh(\beta_1 \omega_1/2) > 1$. Thus, we can conclude that the efficiency of a harmonic quantum Otto engine, operating in the sudden switch limit, is bounded from above by one-half the unit value, i.e.,

$$\eta \leq \frac{1}{2} \equiv \eta_{\text{max}}.$$  \hfill (9)

This is our first main result. The result is very interesting as it implies that even in the presence of very very large squeezing ($r \to \infty$), the efficiency of the engine can never surpass $1/2$. This is in contrast with the previous studies, valid for the quasi-static regime, implying that the thermal engine fueled by a hot squeezed thermal reservoir asymptotically attains unit efficiency for large squeezing parameter ($r >> 1$) \cite{15, 19, 26}. We attribute this to the highly frictional nature of the sudden switch regime as explained below. In the sudden switch regime, the sudden quench of the frequency of the harmonic oscillator induces non-adiabatic transitions between its energy levels, thereby causing the system to develop coherence in the energy frame. In such a case, the energy entropy increases and an additional parasitic internal energy is stored in the working medium. The additional energy corresponds to the waste (or excess) heat which is dissipated to the heat reservoirs during the proceeding isochoric stages of the cycle \cite{46, 47}. This limits the performance of the device under consideration.

### III. UPPER BOUND ON THE EFFICIENCY

In order to obtain analytic expression in closed form for the efficiency, we will work in the high-temperature regime \cite{48, 50}. In this regime, we set $\coth(\beta_i \omega_i/2) \approx 2/(\beta_i \omega_i)$ ($i = 1, 2$) and $\Delta H(r) = \cosh(2r)$. Then, the expressions for the extracted work ($W_{\text{ext}}$) [Eq. (7)] and the efficiency [Eq. (8)] take the following forms:

$$\langle W_{\text{ext}} \rangle = \frac{(1 - z^2) \left[ z^2 \cosh(2r) - \tau \right]}{2z^2\beta_2}, \hfill (10)$$

and

$$\eta = \frac{(z^2 - 1) \left[ z^2 \cosh(2r) - \tau \right]}{\tau - z^2 \cosh(2r) - \tau}, \hfill (11)$$

where we have defined $z = \omega_1/\omega_2$ and $\tau = \beta_2/\beta_1 = 1 - \eta_C$. From Eq. (10), the positive work condition, $\langle W_{\text{ext}} \rangle > 0$, implies that

$$z^2 \cosh(2r) > 1 - \eta_C. \hfill (12)$$

Using the expression for efficiency in Eq. (11), $z^2$ can be written in terms of $\eta$ and $\eta_C$, and is given by

$$z^2 = \frac{1}{2} \left\{ (1 - \eta)(1 + \eta) + (1 - 2\eta) \cosh(2r) - \sqrt{[(1 - \eta)(1 + \eta) + (1 - 2\eta) \cosh(2r)]^2 - 4(1 - \eta)(1 + \eta) \cosh(2r)} \right\}. \hfill (13)$$

Using the above expression for $z$ in Eq. (12), we obtain following upper bound on the efficiency of the engine:

$$\eta < \frac{[1 - \eta_C - \cosh(2r)] \left[-1 + \eta_C - 2 \cosh(2r) + 2\sqrt{2[1 - \eta_C] \cosh(2r)}\right]}{[1 - \eta_C - 2 \cosh(2r)]^2} \equiv \eta_{\text{up}}. \hfill (14)$$

This is our second main result. Notice that the above derived bound is independent of the parameters of the model under consideration and depends on the reservoir parameters $r$ and $\eta_C$ (or $\tau$) only. For $r \to \infty$, $\eta_{\text{up}} \to 1/2$, which reconfirms our earlier result [Eq. (9)] that the maximum efficiency that our engine can attain is one-half the unit efficiency; it never reaches unit efficiency unlike the engines operating in the quasi-static regime \cite{15, 19, 26}.

Further, we derive analytic expression for the efficiency at maximum work by optimizing Eq. (10) with respect to $z$, and it is given by:

$$\eta_{\text{MW}} = \frac{1 - \sqrt{1 - \eta_C} \text{sech}(2r)}{2 + \sqrt{1 - \eta_C} \text{sech}(2r)}. \hfill (15)$$

We have plotted Eqs. (14) and (15) in Fig. 2 as a function of $r$ for different fixed values of Carnot efficiency.
For the given values of \( \eta_C \) smaller than 1/2, both \( \eta_{up} \) (solid red and blue curves) and \( \eta_{MW} \) (dashed red and blue curves) can surpass corresponding Carnot efficiency (dotted curves with same color) for some value of squeezing parameter \( r \) and approach 1/2 for relatively larger values of \( r (r > 5) \). From the inset of Fig. 2, it is clear that \( \eta_{MW} \) always lies below \( \eta_{up} \), which should be the case as for the given temperature ratio (\( \eta_C \)), \( \eta_{up} \) is the upper bound on the efficiency.

One more comment is in order here. Although, for given values of \( \eta_C \) (\( \eta_C < 1/2 \)), \( \eta_{up} \) and \( \eta_{MW} \) may surpass standard Carnot efficiency, they can never surpass generalized Carnot efficiency (not shown in Fig. 2) [15] [30],

\[
\eta_{gen}^C = 1 - \frac{\beta_2}{\beta_1 \cosh(2r)} \equiv 1 - \frac{T_1}{T_2 \cosh(2r)},
\]

which follows from the second law of thermodynamics applied to the nonequilibrium situations [27]. The concept of generalized Carnot efficiency can be understood as follows. We can always assign a frequency dependent local temperature to a squeezed thermal reservoir characterized by its genuine temperature \( T \) and squeezing parameter \( r \) [29] [30]. The expression for this frequency dependent local temperature can be obtained from the following relation [29] [30]:

\[
\exp \left( -\frac{\omega}{T(\omega, r)} \right) = \frac{\langle n \rangle + (2\langle n \rangle + 1) \sinh^2 r}{1 + \langle n \rangle + (2\langle n \rangle + 1) \sinh^2 r}.
\]

In the high-temperature limit, the effective temperature of the squeezed hot bath reads as,

\[
T_{2}^{\text{eff}}(r) = T_2(1 + 2 \sinh^2 r) = T_2 \cosh(2r).
\]

Hence, for positive values of \( r \), engine may be assumed to be operating between temperatures \( T_1 \) and \( T_{2}^{\text{eff}}(r) \). The actual (generalized) Carnot efficiency should then be given by Eq. [19].

Finally, we discuss the special case when \( r \to 0 \). This corresponds to the case in which our harmonic quantum engine is working between two purely thermal reservoirs. Thus, for \( r \to 0 \), Eqs. [14] and [15] reduce to the following forms, respectively:

\[
\eta < \frac{[3 - 2\sqrt{2(1 - \eta_C)} - \eta_C] \eta_C}{(1 + \eta_C)^2} \equiv \eta_{up}^{th},
\]

\[
\eta_{RK} = 1 - \frac{\sqrt{1 - \eta_C}}{2 + \sqrt{1 - \eta_C}}.
\]

The above bound, \( \eta_{up}^{th} \), is much tighter than the classical Carnot bound, even tighter than \( \eta_C/2 \) (see Fig. 3). Eq. [20], which we derived as a special case of our more general result Eq. [14], was first derived by Rezek and Kosloff (RK) for the optimization of a harmonic quantum Otto engine undergoing sudden switch of frequencies in the adiabatic stages [35]. Again, it is clear from Fig. 3 that \( \eta_{RK} \) (dashed red curve) always lies below \( \eta_{up}^{th} \) (solid blue curve), which should be the case.

### IV. Upper Bound on the Coefficient of Performance

Here, we discuss the operation of QHOC as a refrigerator. In the refrigeration process, heat is absorbed from the cold bath, \( \langle Q_4 \rangle > 0 \), and dumped into the hot bath, \( \langle Q_2 \rangle < 0 \). The net work investigated in the system is positive, \( \langle W_{in} \rangle = -\langle Q_2 \rangle + \langle Q_4 \rangle > 0 \). Here, we will first discuss the case when refrigerator is coupled to two
purely thermal reservoirs. We follow the same procedure as done for the heat engine in Sec. III. Since the calculations are straightforward, we merely present our results here. For the refrigerator running between two purely thermal reservoirs, positive cooling condition, \( \langle Q_4 \rangle > 0 \), implies that

\[
\zeta_C > 1 \quad \text{and} \quad \zeta = 1 + 3\zeta_C - 2\sqrt{2\zeta(1 + \zeta_C)} \equiv \zeta_{up}, \quad (21)
\]

where \( \zeta = \omega_1 / (2 - \omega_1) \) and \( \zeta_C = \beta_2 / (\beta_1 - \beta_2) \) are the COP and Carnot COP, respectively. The condition \( \zeta_C > 1 \) implies that \( \tau > 1/2 \), which in turns implies that cold reservoir cannot be cooled below the temperature \( T_2/2 \), thus putting a restriction to the operation of the refrigerator operating in sudden-switch regime. The upper bound \( \zeta_{up} \) derived here is dependent of the parameters of the system and depends on ratio of the reservoir temperatures only, which makes it quite general in nature. Similar to the heat engine case, the obtained upper bound is much tighter than the corresponding Carnot bound.

Now, we will discuss the effect of coupling the refrigerator to the cold squeezed reservoir. In the high-temperature regime, the mean energies at points \( A, B, C \) and \( D \) are given by:

\[
\begin{align*}
\langle H \rangle_A &= \omega_1 \coth(\beta_1 \omega_1 / 2) \cosh(2r) / 2, \\
\langle H \rangle_B &= \omega_2 \lambda \coth(\beta_1 \omega_1 / 2) \cosh(2r) / 2, \\
\langle H \rangle_C &= \omega_2 \coth(\beta_2 \omega_2 / 2), \quad \langle H \rangle_D = \omega_1 \lambda \coth(\beta_2 \omega_2 / 2) / 2.
\end{align*}
\]

The positive cooling condition, \( \langle Q_4 \rangle > 0 \), yields the following expressions:

\[
\frac{1}{2} \text{sech}(2r) < \tau < \text{sech}(2r) \quad \text{and} \quad \zeta < \frac{3}{1 - \tau \cosh(2r)} - 2 - 2\sqrt{2\zeta(c)} \equiv \zeta_{up}. \quad (22)
\]

Eq. (22) along with the equation (21) is our third main result. As expected, \( \zeta_{up} \) reduces to \( \zeta_{up}^{th} \) for the vanishing squeezing parameter, \( r = 0 \). To discuss the physical significance of condition given in Eq. (22), we invert it in terms of lower and upper limits on squeezing parameter \( r \):

\[
0 < r < \frac{1}{2}, \quad \frac{1}{2} \cosh^{-1} \left( \frac{1}{2\tau} \right) < r < \frac{1}{2} \cosh^{-1} \left( \frac{1}{\tau} \right) \quad \text{or} \quad \frac{1}{2} < \tau < 1, \quad 0 < r < \frac{1}{2} \cosh^{-1} \left( \frac{1}{\tau} \right). \quad (23)
\]

It is clear from the above equation that we can extract heat from squeezed cold reservoir even for \( \tau < 1/2 \), which is otherwise impossible with the refrigeration operation with purely thermal reservoirs. Again this can be explained on the basis of effective temperature of the cold reservoir [see Eq. (18)]. For \( r = \frac{1}{2} \cosh^{-1} \left( \frac{1}{2\tau} \right) \) and \( r = \frac{1}{2} \cosh^{-1} \left( \frac{1}{\tau} \right) \), the effective temperatures of the cold reservoir become \( T_2/2 \) and \( T_2 \), respectively. As per the original positive work condition (1/2 < \( \tau \)) without cold squeezed reservoir, \( T_1 > T_2/2 \), hence in case of cold squeezed reservoir this condition is satisfied for the given range of squeezing parameter \( r \) in Eq. (23). Eventually, the refrigeration stops when effective temperature of cold squeezed reservoir approaches \( T_2 \), which is temperature of the thermal hot reservoir. Finally, for \( \tau = 1/2 \) or \( T_2 = 2T_1 \), the allowed range of \( r \) is: \( 0 < r < \frac{1}{2} \cosh^{-1}(2) \), which implies that effective temperature of cold reservoir should be smaller than \( 2T_1 \) which is natural.

V. CONCLUSIONS

We have investigated the performance of a HQOC, operating in the sudden switch limit, coupled to a squeezed thermal reservoir. First, we showed that even in the presence of very large squeezing (\( r \rightarrow \infty \)), the maximum efficiency of the engine is 1/2 only. This is due to the frictional effects caused by the non-adiabatic transitions when we operate in the sudden switch regime. Our study is in contrast with the previous studies which claim that the efficiency can reach unity for large squeezing. Then we obtained closed form expression for the upper bound on the efficiency of the engine operating in the high-temperature regime. The result is interesting in the sense that the obtained bound is independent of the parameters of the model under consideration and depends on the ratio of the reservoir temperatures and squeezing parameter \( r \) only. Additionally, we also derive the analytic expression for the efficiency at maximum work and showed that it satisfies the derived upper bound. As a special case of our more general setup, when squeezing parameter \( r \rightarrow 0 \), our results correspond to the case in which engine is running between two purely thermal reservoirs. Further, we have also obtained upper bounds for the Otto refrigerator working between two purely thermal reservoirs as well as for the case when cold reservoir is taken to be squeezed thermal reservoir. Finally, we showed that squeezing can help in cooling process otherwise impossible in standard setup with thermal reservoirs.

[1] D. Kondepudi and I. Prigogine, Modern thermodynamics: from heat engines to dissipative structures (John Wiley & Sons, 2014).
[2] S. Vinjanampathy and J. Anders, Contemp. Phys. 57, 545 (2016).
[3] G. Mahler, Quantum thermodynamic processes: Energy and information flow at the nanoscale (Jenny Stanford Publishing, 2014).
[4] S. Deffner and S. Campbell, Quantum Thermodynamics (Morgan & Claypool Publishers, 2019).
[5] R. Alicki and R. Kosloff, in Thermodynamics in the Quantum Regime (Springer, 2018) pp. 1–33.
[6] M. O. Scully, Phys. Rev. Lett. 87, 220601 (2001).
[7] M. O. Scully, M. S. Zubairy, G. S. Agarwal, and H. Walther, Science 299, 862 (2003).
[8] D. Türkpençe and O. E. Müstecaplıoğlu, Phys. Rev. E 93, 012145 (2016).
[9] P. Skrzypczyk, A. J. Short, and S. Popescu, Nat. Commun. 5, 4185 (2014).
[10] M. N. Bera, A. Riera, M. Lewenstein, and A. Winter, Nat. Commun. 8, 1 (2017).
[11] J. J. Park, K.-H. Kim, T. Sagawa, and S. W. Kim, Phys. Rev. Lett. 111, 230402 (2013).
[12] N. Brunner, M. Huber, N. Linden, S. Popescu, R. Silva, and P. Skrzypczyk, Phys. Rev. E 89, 032115 (2014).
[13] M. Perarnau-Llobet, K. V. Hovhannisyan, M. Huber, P. Skrzypczyk, N. Brunner, and A. Acín, Phys. Rev. X 5, 041011 (2015).
[14] F. Altintas, A. U. C. Hardal, and O. E. Müstecaplıoğlu, Phys. Rev. E 90, 032102 (2014).
[15] J. Roßnagel, O. Abah, F. Schmidt-Kaler, K. Singer, and E. Lutz, Phys. Rev. Lett. 112, 030602 (2014).
[16] X. L. Huang, T. Wang, and X. X. Yi, Phys. Rev. E 86, 051105 (2012).
[17] R. Kosloff and Y. Rezek, Entropy 19, 136 (2017).
[18] B. K. Agarwalla, J.-H. Jiang, and D. Segal, Phys. Rev. B 96, 104304 (2017).
[19] G. Manzano, F. Galve, R. Zambrini, and J. M. Parrondo, Phys. Rev. E 93, 052120 (2016).
[20] B. Xiao and R. Li, Phys. Lett. A 382, 3051 (2018).
[21] R. Long and W. Liu, Phys. Rev. E 91, 062137 (2015).
[22] J. Klaers, S. Faelt, A. Imamoglu, and E. Togan, Phys. Rev. X 7, 031044 (2017).
[23] L. A. Correa, J. P. Palao, D. Alonso, and G. Adesso, Sci. Rep. 4, 3949 (2014).
[24] R. J. de Assis, J. Sales, U. C. Mendes, and N. G. de Almeida, arXiv:2003.12664 (2020).
[25] J. Wang, J. He, and Y. Ma, Phys. Rev. E 100, 052126 (2019).
[26] W. Niedenzu, V. Mukherjee, A. Ghosh, A. G. Kofman, and G. Kurizki, Nat. Commun. 9, 1 (2018).
[27] O. Abah and E. Lutz, EPL (Europhysics Letters) 106, 20001 (2014).
[28] W. Niedenzu, D. Gelbwaser-Klimovsky, A. G. Kofman, and G. Kurizki, New J. Phys. 18, 083012 (2016).
[29] R. Alicki and D. Gelbwaser-Klimovsky, New J. Phys. 17, 115012 (2015).
[30] R. Alicki, arXiv:1401.7865 (2014).
[31] A. Ghosh, D. Gelbwaser-Klimovsky, W. Niedenzu, A. I. Lvovsky, I. Mazets, M. O. Scully, and G. Kurizki, Proc. Natl. Acad. Sci. USA 115, 9941 (2018).
[32] G. Manzano, Phys. Rev. E 98, 042123 (2018).
[33] H. T. Quan, Y.-x. Liu, C. P. Sun, and F. Nori, Phys. Rev. E 76, 031105 (2007).
[34] T. D. Kieu, Phys. Rev. Lett. 93, 140403 (2004).
[35] Y. Rezek and R. Kosloff, New J. Phys. 8, 83 (2006).
[36] O. Abah, J. Roßnagel, G. Jacob, S. Deffner, F. Schmidt-Kaler, K. Singer, and E. Lutz, Phys. Rev. Lett. 109, 203006 (2012).
[37] G. Thomas and R. S. Johal, Phys. Rev. E 83, 031135 (2011).
[38] S. Chand and A. Biswas, EPL (Europhysics Letters), 60003 (2017).
[39] J. P. Peterson, T. B. Batalhão, M. Herrera, A. M. Souza, R. S. Sarthour, I. S. Oliveira, and R. M. Serra, Phys. Rev. Lett. 123, 240601 (2019).
[40] O. Abah and E. Lutz, EPL (Europhysics Letters) 113, 60002 (2016).
[41] M. Kim, F. De Oliveira, and P. Knight, Phys. Rev. A 40, 2494 (2009).
[42] P. Marian and T. A. Marian, Phys. Rev. A 47, 4474 (1993).
[43] K. Husimi, Prog. Theor. Exp. Phys. 9, 238 (1953).
[44] S. Deffner and E. Lutz, Phys. Rev. E 77, 021128 (2008).
[45] S. Deffner, O. Abah, and E. Lutz, Chem. Phys. 375, 200 (2010).
[46] Y. Rezek, Entropy 12, 1885 (2010).
[47] F. Plastina, A. Alecce, T. J. Apollaro, G. Falcone, G. Francica, F. Galve, N. L. Gullo, and R. Zambrini, Phys. Rev. Lett. 113, 260601 (2014).
[48] R. Kosloff, J. Chem. Phys. 80, 1625 (1984).
[49] R. Uzdin and R. Kosloff, EPL (Europhysics Letters) 108, 40001 (2014).
[50] V. Singh and R. S. Johal, Phys. Rev. E 100, 012138 (2019).