On the Decay of Unparticles

Arvind Rajaraman

Department of Physics and Astronomy, University of California, Irvine, CA 92697, USA

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We show that when the unparticle sector is coupled to the Standard Model, unparticle excitations can decay to Standard Model particles. This radically modifies the signals of unparticle production. We present a method for the calculation of the decay lifetimes of unparticles. In a particular model, we show that depending on their lifetime, unparticles can manifest themselves through monojets, delayed events or prompt decays.

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I. INTRODUCTION

Recently Georgi [1, 2] has suggested the possibility that there is a new, almost conformal, sector coupled to the Standard Model through interactions of the form \(O_\mu O_{SM}\), where \(O_\mu\) is an operator of the conformal field theory, and \(O_{SM}\) is a Standard Model operator. This conformal sector can have experimental signals radically different from those of normal particles, and hence was dubbed the unparticle sector. There has since been a great deal of work studying both the theoretical and experimental aspects of the unparticle sector.

Unparticles manifest themselves through two kinds of processes. In the first class of processes, the unparticle propagator mediates processes like rare decays of Standard Model particles. In a particular model, we show that depending on their lifetime, unparticles can manifest themselves through monojets, delayed events or prompt decays.

II. UNPARTICLE DECAYS

The propagator for unparticles in an exactly conformal theory is fixed by conformal invariance to be [2, 3]

\[
iB_d D_0(p^2) = iB_d(p^2)^{d-2}
\]

where

\[
B_d = A_d \frac{\left(e^{-i\pi}\right)^{d-2}}{2\sin d\pi}, \quad A_d = \frac{16 \pi^{5/2} \Gamma(d + \frac{1}{2})}{(2\pi)^{2d} \Gamma(d - 1) \Gamma(2d)},
\]

and \(1 \leq d < 2\) is the dimension of the operator \(O_\mu\).

The coupling to the Standard Model modifies the propagator. The most important coupling is the relevant operator \(O_\mu H^2\), which breaks conformal invariance [15-17] and introduces a scale \(\mu\) into the conformal field theory. The precise modification to the propagator is model dependent [18]; here we will use a simple model proposed in [19], where the modified propagator is

\[
iB_d D(p^2) = iB_d(p^2 - \mu^2)^{d-2}
\]

which has the feature that at high \(p^2\) we recover the original propagator, and for \(p^2 < \mu^2\), the propagator vanishes since there are no states.

It appears that the propagator is complex. This can be understood by writing the propagator using a dispersion relation as

\[
iB_d D(p^2) = i \frac{A_d}{2\pi} \int_0^\infty dM^2 \frac{\rho(M^2)}{p^2 - \mu^2 - M^2 + i\epsilon}
\]
with \( \rho(M^2) = (M^2)^{d-2} \). In this representation it is manifest that the complex nature of the propagator is not related to a decay, but rather is because the propagator is a sum over resonances [20].

There are also other couplings to other fields through irrelevant operators like \( O_d(F_{\mu\nu})^2, O_d\bar{\psi}\psi \) (along with \( O_dH^2 \)). These will modify the propagator through loop contributions. We can resum the contributions to obtain the full propagator

\[
\int e^{i\rho} \langle 0| T(O_d(x)O_d(0)) |0\rangle \, d^4x = iB_d D(p^2) + iB_d^2 D(p^2)\Sigma(p^2)D(p^2) + \ldots
\]

\[
\equiv \frac{iB_d}{(p^2 - \mu^2)^{2-d} - B_d\Sigma(p^2)}
\]

where the loop diagram is \(-i\Sigma(p^2)\).

Our main concern will be with the effects on the unparticle of \( \Sigma(p^2) \). In a particle propagator (i.e. \( d = 1 \)), an imaginary part of \( \Sigma(p^2) \) leads to a width for the particle, signalling that it can decay. The decay rate is related to the imaginary part of the propagator through the Cutkosky rules. We expect a similar effect for unparticles.

To make this explicit, we will attempt to express the modified propagator as a dispersion integral. We will first do this assuming that \((p^2 - \mu^2)^{2-d} \gg |B_d\Sigma(p^2)|\), so that the loop term can be treated as a perturbation. In this regime, the propagator is approximately

\[
iB_d\]

\[
\frac{1}{(p^2 - \mu^2 - B_d\Sigma(p^2))^{2-d}}
\]

\[
i\frac{A_d}{2\pi} \int_0^\infty dM^2 \frac{(M^2)^{d-2}}{p^2 - \mu^2 - M^2 - B_d\Sigma(p^2)}
\]

\[
\equiv \frac{i\rho(M^2)}{(2-d)M^2 - \mu^2)^{d-1}}
\]

\[
\frac{1}{2} \cot(\pi d)
\]

\[
\Gamma(M) = \frac{\Sigma_I(M^2)}{(2-d)M^2 - \mu^2)^{d-1}} A_d \frac{1}{2} \cot(\pi d)
\]

The width of the resonance of mass \( M \) may then be read off to be

\[
\Gamma(M) = \frac{\Sigma_I(M^2)}{(2-d)M^2 - \mu^2)^{d-1}} A_d \frac{1}{2} \cot(\pi d)
\]

We thus return to the description [20] of the unparticle as a set of resonances with a continuous distribution of masses; the new feature is that these resonances decay, with a lifetime \( \Gamma^{-1}(M) \). The position space propagator then has the form

\[
0| T(O_d(x)O_d(0)) |0\rangle \propto \int dM^2 \rho(M^2) e^{-\Gamma(M)t}
\]

which explicitly shows that the unparticle propagator has a decay.

We note that our expression does not work if \( d > 1.5 \), where the width is negative. This is a failure of our perturbation expansion. We have not been able to find a way to extend the deconstructed expression to the region \( d > 1.5 \); henceforth we will only consider the situation where \( d \leq 1.5 \).

This perturbation expansion also fails if \( \Sigma_I(p^2) \) is too large. Since the loop contribution is small, this only happens very close to the mass gap at the point \( p^2 \leq p_0^2 \) with \( \langle p_0^2 - \mu^2 \rangle^{d-2} = |B_d\Sigma_I(\mu^2)| \). In this regime, we may approximate

\[
iB_d\]

\[
\frac{1}{(p^2 - \mu^2)^{2-d} - iB_d\Sigma_I(p^2)}
\]

\[
\equiv \frac{iB_d}{(p^2 - \mu^2)^{d-2}}
\]

\[
\frac{1}{p_0^2 - \mu^2)^{d-2} - iB_d\Sigma_I(\mu^2)}
\]

\[
\Sigma_I(\mu^2) = \frac{\rho(\mu^2)}{\rho(1)^{d-2}}
\]

\[
\Gamma(M) = \frac{\rho(M^2)}{\rho(1)^{d-2}}
\]

It would be interesting to find a more accurate representation of the propagator for this regime.

To summarize, we have shown that unparticles have decays, just like normal particles. The unparticle can be regarded a sum over several particle propagators, where the particles have a continuously distributed mass \( M \), and a width \( \Gamma(M) \). The width is related to the imaginary part of the loop correction as required by unitarity.

Before turning to the experimental consequences, we briefly comment on previous arguments in the literature that unparticles do not decay. The argument comes from looking at the deconstructed form of the propagator; each resonance of mass \( M \) has an infinitesimal coupling which would appear to preclude decay. The resolution to this paradox is that the mass of the unparticle is not a well defined quantity. The unparticle should be treated as having a fixed momentum, and one should sum over all resonance masses, keeping \( p^2 \) fixed. The sum over the infinite set of resonances compensates for the infinitesimal coupling, and we get a finite decay rate.
III. EXPERIMENTAL SIGNATURES

We will now examine how this decay affects the experimental signals of unparticles. We shall here consider in detail one particular model, where the unparticle couples mainly to massless vector bosons, and analyze the signatures of such unparticles at the LHC. We take the couplings to be

\[ L_{int} = \frac{O_d F_{\mu\nu} F^{\mu\nu}}{\Lambda_F^2} + \frac{O_d G_{\mu\nu} G^{\mu\nu}}{\Lambda_G^2} \]  

where \( F_{\mu\nu}, G_{\mu\nu} \) are the electromagnetic and color field strengths respectively, and \( \Lambda_F, \Lambda_G \) are scales parametrizing the couplings. We will take for simplicity \( \Lambda_F \sim \Lambda_G = \Lambda \).

We first calculate the widths using the formalism above. There is a crossover point for these widths, which occurs at a scale \( p_0 \) set by

\[ \left( \frac{p_0^2 - \mu^2}{\Lambda^2} \right)^{2-d} = |B_d| \frac{\mu^4}{2\pi \Lambda^4} \] 

For \( M^2 > p_0^2 \) we find

\[ \rho(M^2) = (M^2 - \mu^2)^{d-2} \] 
\[ \Gamma(M) = \left( \frac{M^2 - \mu^2}{\Lambda^2} \right)^{d-1} \frac{M^3 A_d \cot(\pi d)}{4\pi \Lambda^2} (2-d) \] 

while for \( M^2 < p_0^2 \) we have

\[ \rho(M^2) = \frac{2\pi}{A_d} |B_d| (p_0^2 - \mu^2)^{d-1} \delta(M^2 - \mu^2) \] 
\[ \Gamma(M) = \left( \frac{p_0^2 - \mu^2}{\Lambda^2} \right)^{d-1} \frac{\mu^3}{2\pi \Lambda^2} |B_d| \] 

Unparticles with these interactions can be produced at colliders through gluon fusion, in processes like \( gg \to \Omega U, gg \to gO_U \). If one does not consider unparticle decay, the unparticle will escape, and this leads to missing energy signals; in particular the process \( gg \to gO_U \) leads to monojet signals.

However, if unparticle decay is considered, then the signals are very different. The unparticle can decay through the processes \( O_{\Omega U} \to gg \) and \( O_{\Omega U} \to \gamma\gamma \), leading to multijet events, or events with two photons plus jets. In particular, there will be few or no missing energy events or monojets, unless the lifetime is very long.

We henceforth focus on the process \( gg \to gO_{\Omega U} \). The cross-section for this process is found to be

\[ \frac{d\sigma}{dt dM^2} = \frac{1}{16\pi \Lambda^2 \tilde{s}^2} \frac{A_d}{2\pi} \rho(M^2) |\mathcal{M}| \] 

with

\[ |\mathcal{M}| = \frac{1536\pi \alpha_s (M^2)^4 + (s^2 + t^2 + \tilde{u}^2)^2}{4.8.8 \tilde{s} \tilde{t} \tilde{u}} \]  

FIG. 1: Number of events with 10 fb\(^{-1}\) of LHC data as a function of \( \mu \). The solid (red) line corresponds to the number of prompt events, dot-dashed (blue) corresponds to the number of monojet events, and dashed (green) is the number of delayed events. We have taken \( d = 1.1 \) and \( \Lambda = 10000 \) GeV.

The produced unparticles will decay either to gluons or photons. The resulting signals are of several types.

\hspace{1cm} \textit{a. Monojets:} If the unparticle decays outside the detector, it is effectively missing energy, and we get a monojet signal.

To estimate the number of monojet events, we need the cross-section for events in which the unparticle decays after moving a distance \( r \). This is

\[ \sigma = \int dM^2 \frac{d\sigma}{dM^2 dt} \exp\left(-\frac{M \Gamma(M) r}{p}\right) \] 

We assume the detector size to be 10 cm. Therefore, the number of monojets is the number of events where the displacement of the vertex is greater than 10 cm. We will in addition require the gluon jet to have \( E > 100 \) GeV, and to have rapidity \( \eta < 2.5 \).

\hspace{1cm} \textit{b. Delayed jets/photons:} A more striking signal is provided by the situation where the unparticle decays before exiting the detector. The decay will produce either photons or gluons which are detectable, and with a time delay given by the lifetime of the unparticle. The total cross-section for such events where the unparticle has a lifetime \( t \) is

\[ \sigma = \int dM^2 dt \frac{d\sigma}{dM^2 dt} \exp\left(-\frac{M \Gamma(M) t}{E}\right) \] 

We note that the unparticle will usually be strongly boosted, and the decay products will then be almost collinear and will appear as a single photon/jet. The signal will be a delayed photon/jet, accompanied by a hard jet.

The detection threshold is set by the timing resolution of the detectors. According to [21], the ATLAS calorimeter has a timing resolution of 100 ps, and the CMS EM calorimeter has a comparable resolution. We therefore
require the time delay to be greater than 100 ps. We will in addition require the gluon jet to have $E > 100$ GeV, and to have rapidity $\eta < 2.5$.

c. Prompt decays: If the lifetime of the unparticle is less than 100 ps, the decay is prompt. We then get two photons with an extra hard jet. These are similar to the virtual unparticle processes.

We now calculate the number of signal events of each type as a function of $\mu$. We will assume 10 fb$^{-1}$ of LHC data. The numbers of such events is shown in Fig. 1 for $d = 1.1$ and Fig. 2 for $d = 1.4$. We see that for larger values of $\mu$, the decays are almost all prompt. For small $\mu$, more unparticles with a long lifetime can be produced, and we get a large number of monojets. In the intermediate range ($\mu \sim 1$ GeV), we find a significant number of delayed events. This provides a new type of signal of unparticles.

We have ignored issues of efficiencies and backgrounds; these must of course be included in a realistic analysis.

IV. SUMMARY AND FUTURE DIRECTIONS

We have shown that unparticles can decay to standard model particles, and found an expression for their decay rate. We have applied this to a particular model, and shown that such effects can have striking signals. In particular for a range of parameters, we can have delayed events, where the unparticle travels a significant time before decaying.

These results drastically affect many other unparticle analyses in the literature. For example, the seminal paper [1] considered the coupling $O_d t u$ which can mediate top decay through the process $t \to u O_d$. If the unparticle does not decay, this would be observed as a missing energy+jet signal. However, the unparticle can decay through $O_d \to bW u$, and this implies that a different possible decay mode is $t \to u b W u$. Furthermore, if the unparticle has a significant lifetime, the unparticle decay products may come from a significantly displaced vertex. It would be very interesting to examine the experimental constraints on this process, and on similar decay modes like $b \to s O_d$.

Finally, it would be very interesting to understand how to extend our formulae to the case $d > 1.5$. We will leave this for future work.

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[1] H. Georgi, Phys. Rev. Lett. 98, 221601 (2007).
[2] H. Georgi, Phys. Lett. B 650, 275 (2007) [arXiv:0704.2457 [hep-ph]].
[3] K. Cheung, W. Y. Keung and T. C. Yuan, Phys. Rev. D 76, 055003 (2007) [arXiv:0706.3155 [hep-ph]].
[4] M. Bander, J. L. Feng, A. Rajaraman and Y. Shirman, Phys. Rev. D 76, 115002 (2007) [arXiv:0706.2677 [hep-ph]].
[5] J. R. Mureika, Phys. Lett. B 660, 561 (2008) [arXiv:0712.1786 [hep-ph]].
[6] P. Mathews and V. Ravindran, Phys. Lett. B 657, 198 (2007) [arXiv:0705.4599 [hep-ph]].
[7] J. L. Feng, A. Rajaraman and H. Tu, Phys. Rev. D 77, 075007 (2008) [arXiv:0801.1534 [hep-ph]].
[8] T. G. Rizzo, JHEP 0710, 044 (2007) [arXiv:0706.3025 [hep-ph]].
[9] K. Cheung, W. Y. Keung and T. C. Yuan, Phys. Rev. Lett. 99, 051803 (2007) [arXiv:0704.2588 [hep-ph]].
[10] M. Luo and G. Zhu, Phys. Lett. B 659, 341 (2008) [arXiv:0704.3532 [hep-ph]].
[11] Y. Liao, Phys. Rev. D 76, 056006 (2007) [arXiv:0705.0837 [hep-ph]].
[12] A. Lenz, Phys. Rev. D 76 (2007) 065006 [arXiv:0707.1535 [hep-ph]].
[13] S. L. Chen, X. G. He and H. C. Tsai, JHEP 0711, 010 (2007) [arXiv:0707.0187 [hep-ph]].
[14] M. J. Strassler, arXiv:0801.0629 [hep-ph].
[15] P. J. Fox, A. Rajaraman and Y. Shirman, Phys. Rev. D 76, 075004 (2007) [arXiv:0705.3902 [hep-ph]].
[16] A. Delgado, J. R. Espinosa and M. Quiros, JHEP 0710, 094 (2007) [arXiv:0707.4309 [hep-ph]].
[17] T. Kikuchi and N. Okada, Phys. Lett. B 661, 360 (2008) [arXiv:0707.0893 [hep-ph]].
[18] A. Delgado, J. R. Espinosa, J. M. No and M. Quiros, JHEP 0804, 028 (2008) [arXiv:0802.2680 [hep-ph]].
[19] B. Grinstein, K. Intriligator and I. Z. Rothstein, Phys. Lett. B 662, 367 (2008) [arXiv:0801.1140 [hep-ph]].
[20] M. A. Stephanov, Phys. Rev. D 76, 035008 (2007).
[21] S. Vigano and A. De Min, “Detection methods for long lived particles at the LHC,” Prepared for IFAE 2006 (in Italian), Pavia, Italy, 19-21 Apr 2006 See also www.pv.infn.it/ ifae2006/talks/NuovaFisica/Vigano.ppt