Features of the stress-strain state in the corner zones of structures

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Abstract. Corner regions of structures are characterized by a complex stress-strain state (SSS), caused by both the shape of the boundary or the “geometric factor”, and the finite rupture of the specified forced deformations, mechanical properties that go to the contact surface of the elements of composite structures. The relevance of the work consists in the analysis of local SSS in the corner zone of the region boundary with various study approaches: theoretical, experimental. For the theoretical analysis of the SSS of the region, we consider the resolving system of equations of the problem of the theory of elasticity in the zone of the cutout of the boundary and elements of the similarity theory; the photoelasticity method is used for experimental studies. An experimental solution was obtained on polymer composite models with an angle cut of the boundary under the action of forced deformations in one of the regions. The analysis of possibilities of theoretical, experimental approaches of SSS studies in zones with irregular point of border allows to reveal zones of applicability of solutions of linear elasticity problem.

Keywords: stress-strain state, corner zone of the boundary of structure, similarity, homogeneity of the solution functions, experimental solution.

1. Introduction
The stress-strain state (SSS) of composite structures and constructions is characterized by the concentration of stresses at the junctions of elements with a complex shape of the boundaries under the action of forced deformations. The most complex SSS arises in the region of stress concentration due to both the shape of the region boundary or the “geometric factor” and the final rupture of the given forced deformations and mechanical properties that contact surface of elements of composite structures.

The need to study local SSS in the composite region of significant structural heterogeneity arises in the calculation and design of structural elements with an abrupt change in the shape of the boundary of the structure, for example, with an angular or step shape.

The purpose of this paper consists in the analysis of local SSS in the corner zone of the region boundary with various study approaches: theoretical and experimental.

The objectives of the paper are to determinate characteristic features of SSS distribution in the zone of corner cutout of the region boundary at the complex research method: theoretical and experimental.

The study focuses on a composite region of the corner cutout of the region boundary, at the top of which a failure of forced deformations along the contact line of the regions exists.

SSS studies in the zone of corner cutout of the region boundary are given on the basis of theoretical and experimental approaches. Theoretical studies are based on the analysis of the local SSS in the area of the region boundary cutout top. Experimental studies by the photoelasticity method are based on the analysis of a strip pattern in the corner zone of the region boundary. The feature of the problem statement is determined by the fact that the failure of the given forced deformations along the boundary of the region junction goes to the top of the corner cutout of the region.

Solutions of Laplace, Poisson and elliptical equations for regions with not smooth boundaries are
considered in the works of Kondratiev V A, Williams V L, Uflyand Ya S, Kalandia A I, Cherepanov G P, Bodzhi D B, Aksentian O K, Aleksandrov A Ya [1-7] and others.

In the works of Kondratiev V A, Williams V. L, Uflyand Ya S, and other authors, the solution of the boundary elliptical problem in the neighbourhood of irregular points of the boundary of the region is presented as an asymptotic series and infinitely differentiable function [1, 4, 6, 7]. The components of this range are solutions of homogeneous edge problems for model areas: a wedge or a cone.

In works [3-5] for the study of SSS in the neighbourhood of an irregular point on a special line of the elastic body boundary, a local curvilinear coordinate system is introduced, in which the Lame equations are recorded. When approaching an irregular boundary point from within a region, the solution of the elastic problem is reduced to the solution of two homogeneous plane problems: a flat deformation and an antiplane deformation or a transverse shift.

Theoretical and experimental studies of stress concentrations due to the form of the boundary are presented in the works of Neubert, Petterson R, Savin N G and Tulchii V I, Ushakov B N, Fomin I P, Makhutov N A, Vasiliev V V and many others [8-17].

The method of photoelasticity [8, 9, 12, 13], which is a continuum method, and the method of deformation “defrosting”, as its unit, allow to obtain SSS in the region of irregular point, boundary lines on models from optically-responsive material.

SSS of structures in zones of concentration of stresses caused by “structural heterogeneity” and breaking forced deformations, are defined experimentally on polymeric models of a method of photoelasticity [8, 9] as concentrators of stresses.

2. Materials and methods

We consider the resolving system of equations of the theory of elasticity problem in the zone of the cutout of the boundary and elements of the similarity theory for the theoretical analysis of the SSS of the region; the photoelasticity method is used for experimental studies. An experimental solution was obtained on polymer composite models with a corner cutout of the boundary under the action of forced deformations in one of the regions. The analysis of possibilities of theoretical, experimental approaches of SSS studies in zones with irregular point of boundary allows to reveal zones of applicability of solutions of linear elasticity problem.

2.1. Representation of SSS in areas of the corner cutout of the region boundary

A plane wedge-shaped area with a solution is considered. $2\alpha > \frac{\pi}{2}$, $2\alpha \in (\pi, 2\pi)$, which consists of two symmetrical regions: $\Omega_1$, $\theta \in [0, \alpha]$ and $\Omega_2$, $\theta \in [-\alpha, 0]$.

In one region, forced temperature deformations $\varepsilon_{ij} = \alpha T \delta_{ij}$ occur. A deformation discontinuity (jump) $\Delta \varepsilon_{ij} = \alpha T \delta_{ij}$ appears along the region contact boundary $L = \Omega_1 \cap \Omega_2$ entering the region boundary angle cutout vertex. Concentrated forces are not considered.

The analysis of the resolving system of equations of the elasticity theory problem [2-6, 12] in the region with an irregular point of the boundary in which the finite rupture (jump) of forced deformations comes out shows that the SSS in this region of the boundary is presented as:

$$\eta = \eta^0 + \eta^l,$$

where $\eta^0$ is a solution of a homogeneous boundary value problem in the neighbourhood of an irregular point of the boundary value of the region (“own” or singular component of the solution), characterizes the power singularity of the solution. In the neighbourhood of an irregular point on the special boundary line of the region, the solution of a homogeneous boundary problem is presented as solutions to two plane homogeneous problems: a plane deformation and an antiplane deformation;

$\eta^l$ is a solution of the resolving system of equations of the elasticity problem under the action of the given loads: forced deformations, in particular, temperature deformations.

Let’s introduce a polar coordinate system with the polar pole O (0,0) at the boundary angle cutout vertex and polar axis along the wedge symmetry axis. The solution of the elastic problem in this area will be recorded in the following form:
\[ u_r = \sum_{r=0}^n r^{-\lambda_r} \{ A_r \cos[(1 + \lambda_r)\theta] + C_r \cos[(1 - \lambda_r)\theta] \} + \] 
\[ + \sum_{\lambda_{r+}^{i},\lambda_{r-}^{i}} r^{-\lambda_{r}^{i}} \{ B_{r} \sin[(1 + \lambda_{r}^{i})\theta] + D_{r} \sin[(1 - \lambda_{r}^{i})\theta] \} \] 
\[ u_\theta = \sum_{r=0}^n r^{-\lambda_r} \{ -A_r \sin[(1 + \lambda_r)\theta] - v_2^r \ C_r \sin[(1 - \lambda_r)\theta] \} + \] 
\[ + \sum_{\lambda_{r+}^{i},\lambda_{r-}^{i}} r^{-\lambda_{r}^{i}} \{ B_{r} \cos[(1 + \lambda_{r}^{i})\theta] + v_2^r \ D_{r} \cos[(1 - \lambda_{r}^{i})\theta] \} + u_\theta^s, \] 
\[ \sigma_\theta = \mu \sum_{r=0}^n r^{\lambda_r-1} \{ -2\lambda^- A \cos[(1 + \lambda^-)\theta] - (1 + \lambda^-)(1 - \nu_2) C \cos[(1 - \lambda^-)\theta] \} + \] 
\[ + \mu \sum_{\lambda_{r+}^{i},\lambda_{r-}^{i}} r^{\lambda_{r}^{i}-1} \{ -2\lambda^+ B \sin[(1 + \lambda^+)\theta] - (1 + \lambda^+) \cos[(1 - \lambda^+)\theta] \} ] + \] 
\[ \tau_{r\theta} = \mu \sum_{r=0}^n r^{\lambda_r-1} \{ -2\lambda^- A \sin[(1 + \lambda^-)\theta] - (1 + \lambda^-)(1 - \nu_2) C \sin[(1 - \lambda^-)\theta] \} + \] 
\[ + \mu \sum_{\lambda_{r+}^{i},\lambda_{r-}^{i}} r^{\lambda_{r}^{i}-1} \{ 2\lambda^+ B \cos[(1 + \lambda^+)\theta] + (1 + \lambda^+) \sin[(1 - \lambda^+)\theta] \} + \] 
\[ \tau_{\theta r} = 2\mu \lambda^- \sum_{r=0}^n r^{\lambda_r-1} \{ (A \cos[(1 + \lambda^-)\theta] + \frac{3-\lambda^-}{k-\lambda^-} C \cos[(1 - \lambda^-)\theta] \} + \] 
\[ + 2\mu \lambda^+ \sum_{\lambda_{r+}^{i},\lambda_{r-}^{i}} r^{\lambda_{r}^{i}-1} \{ B \sin[(1 + \lambda^+)\theta] + \frac{3-\lambda^+}{k-\lambda^+} D \sin[(1 - \lambda^+)\theta] \} + \sigma_{\theta}^s, \] 
where \( u_\theta^s, \sigma_{\theta}^s \) are displacements and stresses caused by the action of given loads: forced, temperature deformations or the total field of displacements and stresses;
where \( \mu = \frac{E}{2(1+\nu)}, E, \nu \) – an elastic modulus, Poisson’s ratio of the material of the region, respectively,
\( \nu_2 = \frac{3+\lambda-4\nu}{3-\lambda-4\nu}, 1 - \nu_2 = \frac{2\lambda}{k-\lambda}, k = 3 - 4\nu, \lambda \) are the homogeneous boundary-value problem eigenvalues. The values \( \lambda \) are determined as the solution of the characteristic equation of a homogeneous boundary value problem, in the general case they depend on the type of homogeneous boundary conditions, the mechanical characteristics of the material, the angle of the wedge solution. When solving the characteristic equation, a root system \( \lambda_{r}^{i}, \lambda_{r}^{-i} \) is obtained, for which in the general solution (2) it is necessary to keep various constants \( A, C \) or \( B, D \).

By introducing a geometric parameter characterizing the “degree of approximation” to the singular point \( O \) of the boundary of the region, it is possible to analyze the relations of the summands of the form (1) and highlight the characteristic regions of the SSS.

i) When tending to an irregular boundary point from within the region, there is a neighborhood of an irregular boundary point of a plane domain in which a singular solution of the homogeneous boundary value problem is valid: \( \sigma_{ij} \rightarrow \sigma_{ij}^0, \sigma_{ij}^0 \rightarrow 0 \). Feature of inherent stresses, \( \sigma_{ij}^0 \) (deformations \( \epsilon_{ij}^0 \)) has a power form \( r^{-\alpha_{ij}} \), where eigenvalues \( \lambda \in [0,0.5] \) are determined numerically.

ii) When tending to an irregular boundary point from within the region, there is a neighborhood in which \( \sigma_{ij} \approx \sigma_{ij}^\circ, \sigma_{ij}^\circ \approx 0 \) and the nonsingular boundary elastic problem with the same eigenvalue \( \min Re \lambda \) is valid, as in the singular value problem. The domain of a nonsingular solution does not contain a neighborhood of the singular solution and the irregular point itself, but is adjacent to it. In case of external tendency toward the boundary of the region of a singular stress solution, deformations change continuously, their values are large, but finite.

iii) At a sufficient distance from the irregular point of the boundary, there is a region in which \( \sigma_{ij}^\circ = 0, \sigma_{ij}^\circ \neq 0 \) and stresses are due to specified loads (common stress field).

In the region of a nonsingular solution of a boundary-value elastic problem close to an irregular point of the boundary, it is possible to compare the regularities of the distribution of SSS obtained using
similarity criteria and experimentally. This allows us to determine the applicability zone of the solutions of the linear boundary value problem of elasticity in the region of an irregular boundary point and to reveal the regularities of the distribution of the SSS in the zone of the singular point. For this zone, we apply the analysis of the system of equations of the elasticity theory problem using the similarity theory.

2.2. Analysis of SSS using the theory of similarity and dimensions

The resolving system of equations of the problem of the theory of elasticity in a small neighborhood of an irregular point $O_\delta(0)$ on a special boundary line of the elastic body is reduced to a dimensionless form using the relations:

$$\xi = \xi_0 \bar{x},$$

where $\xi$ is a value under study, $\xi_0$ is a characteristic value of the considered quantity, $\bar{x}$ is a dimensionless value of this quantity. The following relations are considered:

$$\sigma = \sigma_0 \bar{\sigma}; \quad \varepsilon = \varepsilon_0 \bar{\varepsilon}; \quad U = U_0 \bar{U};$$

$$x = \frac{l_0}{t} \bar{x}; \quad y = \frac{l_0}{t} \bar{y}; \quad z = l_0 \bar{z},$$

where $t$ is a dimensionless parameter of the similarity group, it is introduced to analyze the “degree of approximation to an irregular point”. In order to obtain a homogeneous boundary-value problem and pass to a self-similar solution, it is necessary and sufficient that, in accordance with the resolving system of equations, the criteria associated with the effects and the criteria due to the similarity theory and experimentally. This allows us to determine the applicability zone of the solutions of the linear boundary value problem and to reveal the regularities of the distribution of the SSS in the zone of the singular point.

According to the criteria (7), the given forced deformations $\varepsilon_{01}, \sigma_{01}T_0$ in $O_\delta(0)$ neighborhood change much more slowly when approaching an irregular point or with increasing parameter $t$, than deformations $\varepsilon_0$, due to the irregularity of the boundary at the point $O$. Therefore when $t \rightarrow \infty$, i.e. the closer to irregular point $O$, the smaller the set deformation $\varepsilon_{01}$.

Form condition: $\sigma_0 \sim \varepsilon_0$ when $t \rightarrow \infty$ means that stresses and deformations have the same order of change in coordinates in the neighbourhood of a singular point $O$, i.e. in $O_\delta(0)$ neighborhood, we can talk about the equivalence of the order of change in coordinates for stresses and deformations.

As it moving away from irregular point $O$, i.e. when $t \rightarrow 1$, specified forced deformations $\varepsilon_{01}$, $\sigma_0T_0$ according to (7) have the same change order as deformations $\varepsilon_0$.

The following criteria must be met:

$$\frac{\sigma_{01}}{\sigma_0} = 1$$

$$\frac{\sigma_{01}}{\sigma_0} \ll \sigma_0, \; t \rightarrow \infty,$$

$$\frac{\sigma_{01}}{\sigma_0} = 1$$

$$\frac{\sigma_{01}}{\sigma_0} \sim \sigma_0, \; t \rightarrow 1,$$

where $\sigma_{01}, \varepsilon_{01}$ is a characteristic value for stresses, deformations in the cross section of a subregion $O_\delta(0)$, containing an irregular boundary point.

Criterion (5) means that when approaching an irregular point of the boundary from the inside, the stress region $\sigma_0$, due to the singular solution of the problem change much faster than stress $\sigma_{01}$, due to exposure to initial loads, i.e. “common stress field” in this neighborhood can be neglected.

When moving away from the irregular point of the boundary of the region, the orders of variation of stresses $\sigma_0, \sigma_{01}$ are aligned and at some position of the section or parameter value $t$ of stress $\sigma_0$ and $\sigma_{01}$ become equivalent according to (6).

For self-similarity of the solution of the boundary-value problem of elasticity in the region of an irregular point of the boundary, the following criteria must be met:

$$\left\{ \frac{\sigma_0}{\varepsilon_0} \right\} = 1; \quad \left\{ \frac{\varepsilon_{01}}{\varepsilon_0} \right\} = 0; \quad \left\{ \frac{\sigma_{01}}{\varepsilon_0} \right\} = 0; \quad (7a)$$

or

$$\text{at } t \rightarrow \infty, \sigma_0 \sim \varepsilon_0; \quad \varepsilon_{01} \ll \varepsilon_0; \quad \alpha_0T_0 \ll \varepsilon_0, \quad (7b)$$

where $\varepsilon_0$ is a characteristic value of given forced deformations $\varepsilon_{01}$.

According to the criteria (7), the given forced deformations $\varepsilon_{01}, \sigma_0T_0$ in $O_\delta(0)$ neighborhood change much more slowly when approaching an irregular point or with increasing parameter $t$, than deformations $\varepsilon_0$, due to the irregularity of the boundary at the point $O$. Therefore when $t \rightarrow \infty$, i.e. the closer to irregular point $O$, the smaller the set deformations compared with the “singular” deformations.

As it moving away from irregular point $O$, i.e. when $t \rightarrow 1$, specified forced deformations $\varepsilon_{01}$, $\sigma_0T_0$ according to (7) have the same change order as deformations $\varepsilon_0$.

The following criteria must be met:

$$\left\{ \frac{u_{0T}}{\varepsilon_{01}} \right\} = 1$$

or

$$U_0t \sim \varepsilon_0l_0 \quad \text{or} \quad U_0 \sim \frac{\varepsilon_0l_0}{t}, \quad (8)$$

According to criteria (8), in $O_\delta(0)$ neighborhood of an irregular boundary point, the order of change of the displacement functions from the coordinates of the point is one greater than the order of change...
of deformations, and according to the equivalence condition (7) and the order of change of stresses.

If displacements in $O_8(0)$ neighborhood have a change order $r^\lambda$, where $r$ is a distance to an irregular point $O(0,0)$ to the given one, then the deformation, and therefore the stresses have the order of change $r^{\lambda-1}$. When $\lambda \in (0,1)$, displacements are limited, and stresses and deformations have a feature of the order $|\lambda - 1|$, in this case estimates are possible:

$$U_0 = O(r^\lambda); \quad \sigma_0 = O((r - r_0)^{\lambda-1}),$$

where $r \neq r_0$ and $O_8(0)$ neighborhood without point $O$ itself and some of its small neighborhood is considered.

It can be shown that, due to the linearity of the homogeneous boundary-value problem of elasticity, the order of change of displacements depending on the coordinates of the point $(x, y) \in O_8(0)$ have the following form:

$$U(tx) \sim C(tx)^\lambda \text{ or } U(x_1) \sim Cx_1^\lambda; \quad V(x_1) \sim Cx_1^\lambda,$$

where $C$ is an arbitrary constant, $x_1 = tx$, $y_1 = ty$, $x_1, y_1 \in O_8(0)$.

When $\lambda \in (0,1)$, $U(x_1, y_1)$, $V(x_1, y_1)$ displacements in $O_8(0)$ neighborhood are continuous, bounded, according to relations (10) they have the property of homogeneity.

According to relations (7, 9), stresses and deformations have the order of changing the coordinates of a point in $O_8(0)$ neighborhood, which is one less than the displacement:

$$\sigma_{ij} \sim C_1 \lambda x_1^{\lambda-1}; \quad \varepsilon_{ij} \sim C_1 \lambda x_1^{\lambda-1},$$

$x_1 = (x_1, y_1) \in O_8(0)$; $x_1 = tx$; $y_1 = ty$. When $\lambda \in (0,1)$, stresses and deformations in $O_8(0)$ neighborhood have a feature of order $|\lambda - 1|$.

It is advisable to search for the solution of homogeneous plane problems in the neighborhood of an irregular point on a singular line in the polar coordinate system in the following form:

$$U = r^\lambda f(\theta); \quad V = r^\lambda g(\theta); \quad W = r^\lambda p(\theta),$$

which is consistent with the solutions given in [2, 15-20].

Due to the self-similarity of the solution to the problem of the theory of elasticity of stress, deformation, displacement in a certain neighborhood of an irregular point of the boundary of the region, they admit a similarity group and possess the property of homogeneity of functions, characteristic that such functions are representable in the form of power complexes (10), (11), (12).

The same properties (similarity, homogeneity) shall be possessed by the experimental solution obtained on the model in the form of a strip pattern by the photoelastic method.

2.3. Experimental solution in the corner cutout region

The problem of thermoelasticity is experimentally solved by the method of photoelasticity and defrosting of free temperature deformations [8, 9, 12-17]. The polymer model of the beam consists of two regions, in one of the regions, free temperature deformations $\alpha T \delta_{ij}$ are created, and the other region is free of loads. After defrosting, the desired thermoelastic state is created in the model. Forced deformation jump $\Delta \varepsilon_{ij} = \alpha T \delta_{ij}$ along the contact line of the regions making up the model reaches an irregular point $O(0,0)$ of the boundary – the top of the corner cutout of the beam end. View of one of the strip patterns with the expansion of the corner cutout 100\(^0\) of the region boundary is shown in figure 1. In the region of the end of the beam with an angular cutout of the boundary, diagrams of the orders of the strips (isochrom) are plotted for several radial sections. The presence of zones corresponding to the “singular” and non-singular solutions of the problem is established, the similarity of the diagrams of the orders of the strips is established.
According to the theoretical analysis of SSS, there exists a neighborhood in which a singular solution of the homogeneous boundary value problem is valid: \( \sigma_{ij} \to \sigma_{ij}^0, \sigma_{ij}^l \to 0 \). The orders of the strips in the region of the stress concentrator on the model (regions of singular solution) cannot be read at any increase in the neighborhood of an irregular point.

According to the theoretical analysis of SSS, there is a neighborhood in which \( \sigma_{ij} \approx \sigma_{ij}^0, \sigma_{ij}^l \approx 0 \) and the nonsingular boundary elastic problem with the same eigenvalue \( \min Re \lambda \) as in the singular problem is valid. The orders of the strips on the model corresponding to the non-singular domain of the solution are read with the possible exception of some. The existence of such a self-balanced radial SSS explains the growth of strip orders (isochrom) observed from the inside of the stress concentration region, and not at the very top of the notch of the region. The absence of a zero strip is explained by the existence of another self-balanced SSS due to the common stress field. For this region, distribution (2) is applicable, and criteria (7), (8) are valid, i.e. similarity and homogeneity of the solution functions are observed.

According to the theoretical and experimental analysis of SSS in the neighbourhood of an irregular point of the boundary of a plane region, into which a jump in forced deformations enters, a formula for the similarity of the orders of the strips is established

\[
m_{i+1} = \left( \frac{r_{i+1}}{R} \right)^{1-\lambda_0} m_i \text{ or } m_{i+1}(r_{i+1})^{1-\lambda_0} = m_i(r_i)^{1-\lambda_0}, \tag{13}
\]

where \( m_i \) are strip orders according to experiment in the calculated section \( r_i \) in a neighborhood of a nonsingular solution of a homogeneous boundary value problem, \( m_{i+1} \) are orders of strips in a smaller radius section \( r_{i+1} < r_i \), located in a region with an unreadable or “poorly readable” picture of an isochrome model, \( \lambda_0 = \min Re \lambda \) is a minimum value of the real part of the complex root of the characteristic equation of a homogeneous boundary value problem for a model wedge. Relation (13) determines the power dependence of the order of isochrom on the coordinates of a point in the cutout area of the boundary of the region \( m \sim C \lambda r^\lambda r^{\lambda-1} \).

The experimental solution obtained by photoelasticity on a model in the form of a strip pattern confirms the existence of a zone for which the similarity and homogeneity of the solution functions are valid.

3. Results
An analysis of the SSS in the area of the corner cut-out of the boundary is carried out on the basis of the resolving system of equations of the problem of the theory of elasticity, similarity theory and
experimental solution on models of the photoelasticity method.

As follows from the theoretical analysis, the stress-strain state (SSS) in the area of irregular boundary point, into which the finite rupture (jump) of forced deformations occurs is given as (1) as a sum of two components: singular SSS and SSS under set loads: forced deformations, in particular temperature deformations. The singular SSS component in the corner zone of the region boundary is defined as a solution to a homogeneous boundary-value problem in the neighbourhood of an irregular point of the region boundary, characterizing the power singularity of the solution. The obtained SSS is determined according to the formula (2). It is worth noting that the eigenvalues of the problem \( \lambda \) are determined as the solution of the characteristic equation of a homogeneous boundary value problem, in the general case they depend on the type of homogeneous boundary conditions, the mechanical characteristics of the material, the angle of the wedge solution. It has been noted that the solution of the characteristic equation results in a root system \( \lambda_1^+ \), \( \lambda_1^- \), for which SSS with appropriate coefficients should be written down.

A geometric parameter is introduced in the zone of corner cutout of the boundary, changes of which characterize the ratios of SSS components in formula (1). The introduced parameter allows determining the area in which the SSS singular component is considered.

The obtained SSS representation (1) in the zone of corner cutout of the area boundary as a set of components: singular SSS and SSS under given loads, the introduction of a geometric parameter characterizing the “approximation” of the cross section to an irregular point, allow us to determine the area of applicability of solutions to the linear boundary-value elasticity problem in the area of an irregular boundary point.

An analysis of the equation system of the elasticity theory problem using the similarity theory has been applied in order to reveal the regularities of SSS distribution in the area of solutions to the linear elasticity problem. A dimensionless similarity group parameter (4) is introduced for this analysis, characterizing the “degree of approximation to an irregular point”.

Criteria (5) to (8) related to the exposure and criteria based on the given equations have been formulated, which allow to obtain a homogeneous boundary-value problem and move on to a self-similar solution of the boundary corner cutout zone problem.

As a result of the application of similarity theory, displacement change procedures (10), stresses and deformations (11) depending on the coordinates of the point in the zone of the corner cutout of the boundary have been established. Conclusion: In some neighbourhoods of an irregular point of the region boundary, due to the self-similarity of the solution the problem of the theory of elasticity of stress, deformation and displacements allow for a similarity group, have homogeneous functions, and are represented as power complexes (10), (11), (12).

The theoretical analysis results in the area of the boundary corner cutout are consistent with the trial solution obtained from a strip pattern model by photoelasticity method. The analysis of the interference fringes order in the boundary corner cutout zone shows the domain of applicability of the distribution (2), fairness of criteria (7), (8).

The research results confirm the presence of zones within the boundaries in which the property of similarity and homogeneity of solution functions is valid. The orders of the strips in some neighborhood of an irregular point of the model boundary are representable in the form of power complexes: \( m \sim C \lambda r^{k-1} \), which corresponds to the results of a theoretical analysis of SSS in this zone.

4. Discussion
The top zone of the corner cutout of the boundary of the region consists of several subregions: the region of plastic deformations, where the finite deformations shall be taken into account, the region of elastic (linear and nonlinear) deformations, for which, in the framework of the linear problem of the theory of elasticity, SSS of the form is written down (2) with a power feature of the form (10), (11) or (12) of the same order as for the SSS region of a singular solution. In the presence of a small zone of nonlinear material properties in the neighbourhood of an irregular boundary point, there exists a neighborhood in which the SSS expressions (2) quite accurately determine the distribution of stresses and deformations,
which is confirmed by experimental studies of solutions in this area. The existence of a self-similarity region for solving the elastic problem is theoretically and experimentally confirmed by the fact that stresses, deformations, and displacements in a certain neighborhood of an irregular point of the boundary of the region admit a similarity group and possess the property of homogeneity of functions, characteristic of the fact that such functions can be represented as power complexes.

5. Conclusion
A theoretical analysis of the resolving system of equations of the elasticity theory problem, similarity criteria, and experimental solution data confirms the presence of a region in which the linear elasticity problem solution and the self-similarity of the problem solution are true. The zones close to an irregular point on the boundary of the region shall be considered with plastic and finite deformations in mind, which is the subject of independent research.

6. References
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