THE STRANGE QUARK DISTRIBUTION

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\textbf{ABSTRACT}

We discuss the latest CCFR determination of the strange sea density of the proton. We comment on the differences with a previous, leading–order, result and point out the relevance of quark mass effects and current non–conservation effects. By taking them into account it is possible to solve the residual discrepancy with another determination of the strange quark distribution. Two important sources of uncertainties are also analyzed.

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1 Introduction

Until few years ago the problem of the strange sea distribution was rather controversial, despite the impressive amount of knowledge on the internal structure of nucleons accumulated in the last decade. The best available fits in 1993 [1, 2] differed by almost a factor 2 for the strange density. As we pointed out in [3, 4], this puzzling situation was mainly determined by an erroneous interpretation of the experimental results, which ignored important physical effects already investigated in [5, 6].

There are two ways to extract the strange sea distribution $s(x)$. The first method consists in subtracting the structure functions $F_2$ measured in muon ($\mu$) and neutrino ($\nu$) deep inelastic scattering (DIS). We call this the $\nu - \mu$ determination of $s(x)$. The second, more direct, method consists in selecting $\nu$DIS events with charm production: the signature of these events is the presence of opposite–sign dimuons in the final state.

When the NMC $\mu$DIS data [7] and the CCFR $\nu$DIS data [8, 9] made both these determinations possible, it was found, rather surprisingly, that the two results for $s(x)$ were largely different (see however the anticipations in [5, 6]): the strange density extracted from dimuon data was considerably smaller than the one obtained by subtracting $\nu$DIS and $\mu$DIS data.

In trying to solve this discrepancy all the available global parton parametrizations ran into serious difficulties. The CTEQ group, trusting the $\nu - \mu$ result, was led to advocate a very large strange sea content $\kappa = 2S/(\bar{U} + \bar{D}) \simeq 0.9$ ($S \equiv \int dx \, xs(x)$, etc.), i.e. an almost $SU(3)$–symmetric light sea. At the same time, however, the CTEQ strange distribution lied well above the dimuon data. On the other hand, the MRS $D^0$ fit stuck to the more conservative value $\kappa = 0.5$ (which was an input) but still overshot the dimuon data, while lying below the $\nu - \mu$ result. (We shall see a posteriori that the MRS compromise between the two data sets and the value of $\kappa \simeq 0.5$ are
–accidentally– much closer to the truth than the CTEQ picture.)

The solution to the strange sea puzzle proposed in [3, 4] was very simple: there is no real puzzle because the two determinations of \( s(x) \) measure in fact different quantities, none of which coincides with the true strange density. This is due to the fact that, up to moderately large \( Q^2 \) (of order of 30 GeV\(^2\) or so, \textit{i.e.} not much above the charm threshold), the relevant diagrams for charm production are the vector–boson–gluon fusion subprocesses [10]. These are, in the common massless QCD terminology, \textit{next–to–leading} order diagrams and hence they are often mistakenly forgotten as if they were \textit{subleading} corrections. When the gluon–fusion diagrams are taken into account, two effects arise which explain why the two determinations of \( s(x) \) do not really provide \( s(x) \) (at least directly, as a naive leading–order analysis would suggest) and should indeed give different results. They are: \textit{i)} quark mass corrections; \textit{ii)} non conservation of weak currents (yielding large longitudinal contributions). These effects are calculable in perturbative QCD [5, 6] and are relevant up to moderate \( Q^2 \) values (not much larger than the heavy quark mass scale). We called them \textit{non–universality} effects because they make the heavy flavour contributions to \( \nu \text{DIS} \) and to \( \mu \text{DIS} \) structure functions \textit{intrinsically} different.

Near threshold, massless QCD is inappropriate to describe heavy quark production and mass contributions must be kept. The gluon–fusion process is the dominant one. Since the mass thresholds in the transitions \( W^+ \bar{s} \rightarrow c, \gamma^* \rightarrow \bar{s}s(\bar{c}c), Z^0 \rightarrow \bar{s}s(\bar{c}c), \) are different, and since the longitudinal structure functions in weak DIS are larger than in electromagnetic DIS because weak currents are non conserved, we expect [5, 6]

\[
F_2^{\nu, \bar{s}c} \neq F_2^{\nu, \bar{s}s} \neq F_2^{\nu, \bar{c}c} \neq F_2^{\mu, \bar{s}s} \neq F_2^{\mu, \bar{c}c},
\]

where we denote by \( F_2^{\nu, \bar{s}c} \) the \( \bar{s}c \) contribution to the \( \nu \text{DIS} \) structure function \( F_2 \) with the electroweak coupling factored out (differently stated, \( F_2^{\nu, \bar{s}c} \) reduces to \( x(\bar{s} + c) \) at
leading order), and we use an analogous notation for the other quantities.

Due to the importance of the gluon–fusion processes, we must expect a considerable difference between a leading order (LO) and a next–to–leading (NLO) extraction of \( s(x) \) from the dimuon experiment [3, 4]. This expectation proves correct. The CCFR collaboration has recently released [11] a new determination of \( s(x) \) based on an analysis of the dimuon data which takes into account the gluon–fusion processes. The ‘new’ strange density [11] is considerably larger than the ‘old’ one [9] and partially bridges up the gap with the \( \nu - \mu \) result.

The purpose of this paper is to discuss and clarify our present knowledge of the strange density. We shall comment on the recent CCFR determination of \( s(x) \), on its difference with the previous one, and on the important physics behind such a difference. On the quantitative side, we shall show that the residual discrepancy existing at small \( x \) between the CCFR NLO strange density and the \( \nu - \mu \) data is easily accounted for by the non–universality (or, differently said, by next-to-leading order) effects related to the gluon fusion processes. Practically, our calculations endorse an MRS–like strange sea density [12] with a value \( \kappa \simeq 0.5 \). We shall also discuss some subtleties and some sources of uncertainties in the analysis of the neutrino data, and propose a more convenient way to present the dimuon results.

2 The strange density from dimuon data

The most direct mechanism to measure the strange density is charm production in charged current neutrino deep inelastic scattering.
The $\nu$DIS cross section reads \[2\]

$$\frac{d^2\sigma^{\nu}}{dx dy} = \frac{G^2 m_N E_\nu}{\pi(1 + \frac{Q^2}{M_W^2})^2} \left[ y^2 F_1^{\nu N}(x) + (1 - y) F_2^{\nu N}(x) + (y - \frac{y^2}{2}) x F_3^{\nu N}(x) \right].$$ \[2\]

If we restrict ourselves to charm excitation, only the transitions $W^+ d \to c$, $W^+ s \to c$ contribute to the structure functions. At leading order we have to consider only the quark scattering terms \[3\] and the cross section for charm production in $\nu$DIS can be expressed in terms of the LO parton densities as

$$\frac{d^2\sigma(\nu N \to cX)}{dx dy} = \frac{G^2 m_N E_\nu}{\pi(1 + \frac{Q^2}{M_W^2})^2} x \left\{ u(x, Q^2) + d(x, Q^2) \right\} |V_{cd}|^2 + 2s(x, Q^2)|V_{cs}|^2 \right\}$$ \[3\]

where $V_{cd}, V_{cs}$ are the Cabibbo–Kobayashi–Maskawa matrix elements. Since $V_{cs} \gg V_{cd}$ the measure of $\sigma(\nu N \to cX)$ provides an excellent determination of $xs(x)$.

In order to take into account effects connected with the non negligible mass of the charm quark, it has been customary for many years to adopt the slow–rescaling procedure (this is what the CCFR collaboration also did in \[9\]).

In practice, the slow rescaling method consists in replacing Bjorken’s $x$ by the new variable $\xi = x(1 + m_c^2/Q^2)$, which is (naively) expected to take into account the effects of the heavy quark mass. If the Callan–Gross relation is enforced in terms of $\xi$, namely $F_2(\xi) = 2\xi F_1$, an extra factor appears in the $\nu$DIS cross section which then reads

$$\frac{d^2\sigma(\nu N \to cX)}{d\xi dy} = \frac{G^2 m_N E_\nu}{\pi(1 + \frac{Q^2}{M_W^2})^2} \times \xi \left\{ u(\xi, Q^2) + d(\xi, Q^2) \right\} |V_{cd}|^2 + 2s(\xi, Q^2)|V_{cs}|^2 \right\} \left( 1 + \frac{m_c^2}{2m_N E_\nu \xi} \right).$$ \[4\]

Slow rescaling lacks a solid theoretical foundation. It is a sensible method if one considers only the quark scattering term. In this case, in the $W^+ s \to c$ transition, the $s$ quark is taken on shell, as usual in the parton model, and its mass is neglected with \[5\] For the sake of clarity we stick hereafter to the terminology of Ref. \[14\], which is used by the CCFR collaboration in their analysis.
respect to the charm mass: if we call $\xi$ the fraction of proton’s momentum carried by the strange quark, the substitution $x \rightarrow \xi$ follows straightforwardly.

However, considering higher order, gluon fusion, diagrams (which are actually the dominant ones) it is no longer possible to assume that the $s$ quark is on shell: the whole procedure thus breaks up and a more sophisticated treatment is called for. Of course, one could still think that slow rescaling mimicks rather faithfully the real world and that it accounts for quark mass effects in an effective way. This attitude has been quite popular and it is still widely believed that slow rescaling is at least a very good approximation. One of the conclusions of the first, leading order, CCFR analysis was that the data supported the slow rescaling model of charm production (although with a very small charm mass, $m_c \simeq 1.31\, \text{GeV}^2$). The NLO analysis has completely reversed the situation: a large difference is found between the LO determination with slow rescaling and the new one, which is on a better theoretical ground and leads to a more realistic charm mass, $m_c \simeq 1.70\, \text{GeV}^2$. The first conclusion we can draw from the new results is indeed that the slow rescaling, which is intrinsically a LO procedure, fails to provide a realistic picture of charm production and therefore must simply be abandoned (this criticism was anticipated in [5], well before both CCFR determinations).

In the region of small and moderate values of $Q^2$ (where most of the CCFR data lie), i.e. not much above the charm threshold $m_c^2$, it is not legitimate to retain only the quark scattering LO diagrams. As a matter of fact, near threshold, the whole contribution of a heavy quark to structure functions\footnote{Remember that, in charged current $\nu$DIS, probing the $s$ quark means at the same time exciting the $c$ quark.} is given by the vector–boson–gluon fusion process (Fig. 1), which are conventionally classified as a next–to–leading order term (although the leading term is not the dominant one).

In the case at hand we have for $F_2$, in a formal notation ($\otimes$ means convolution)
where $g(x, Q^2)$ is the gluon distribution and $C_2$ is the unsubtracted Wilson coefficient, i.e. the full cross section for the $W^+g \to \bar{s}c$ process, which is made of the two inseparable diagrams shown in Fig. 1.

It is worthwhile to draw a comparison with another approach \[14\]. In the formalism of Ref. \[14\] the $\bar{s}c$ contribution to structure functions is expressed as (we omit all the electroweak couplings)

$$F_2^{\bar{s}c} = x(\bar{s} + c) - C_{\text{subtr}}^{s} + \left(\frac{\alpha_s}{2\pi}\right) g \otimes C_2(W^+g \to \bar{s}c),$$

(6)

where $\bar{s}(x, Q^2)$ and $c(x, Q^2)$ are NLO parton densities, and $C_{\text{subtr}}^{s}$ is a subtraction term proportional to $\alpha_s \log(\mu_f^2/m_c^2)$ ($\mu_f$ is the factorization scale). In this approach the logarithmic term in the Wilson coefficient, which explodes at $Q^2$ much larger than $m_c^2$, giving eventually rise to collinear singularities, is subtracted out; at the same time, the quark scattering term is introduced so that, when the physical scale is very large compared to the heavy quark mass, the massless QCD parton model is regained.

Up to $Q^2$ of order $10 m_c^2$ the two formulas for $F_2^{\bar{s}c}$ given above, eqs. (5) and (6), are equivalent \[14\]. In fact, the two extra-terms in eq. (6) are approximately equal and cancel out. All the relevant physics (mass effects and current non conservation effects producing large longitudinal structure functions) is thus contained in the $W$–gluon fusion diagrams.

When these are taken into account the relation between the $\nu$DIS cross section with charm production and the parton densities is much more involved than eqs. (3) or (4). In fact, the simplicity of the leading order formulas for $d^2\sigma(\nu N \to cX)/d\xi dy$ is determined by the (fortuitously) virtuous combination of two elements: i) the relation
(valid for the structure function components relevant to charm excitation)

\[ F_2^{\nu N} = 2xF_1^{\nu N} = xF_3^{\nu N} = x(u + d + 2s), \]

(or the corresponding one in terms of \(\xi\)), and ii) the consequent, accidental, cancellation of all factors in \(y\) in front of the structure functions. None of these two circumstances occur in the next–to–leading order case (we keep using this terminology although we stressed above that the \(Wg\) fusion contribution is the dominant one and cannot be, strictly speaking, called next–to–leading). The expression on the r.h.s. of eq. (4) is therefore much more complicated and, of course, the slow rescaling substitution becomes completely meaningless.

Hence it is expected, from a theoretical viewpoint, that there should be a large difference between the leading order strange density determined from (5) and the next–to–leading order strange density extracted from (2) upon use of eq. (5). This is indeed what has been recently found [11] in the new, NLO, CCFR analysis of the dimuon data. As one can see in Fig. 2, at moderate \(Q^2\) values, the difference between the LO result for \(xs(x)\) and the NLO result is rather large (more than 50% at \(Q^2 \approx 10 GeV^2\)). This is clearly not a mere higher–order correction: the LO analysis hides and neglects all the important physics contained in the \(W\)–gluon fusion diagrams. The slow rescaling method, while theoretically ill–founded, is not even an effectively successful way to account for heavy quark masses. Moreover, mass effects are not the whole story. Effects of current non conservation in \(\nu\)DIS scattering are quantitatively as much (or even more) important [6]. In \(\nu\)DIS the ratio \(R = \sigma_L/\sigma_T\) in the heavy quark sector near threshold is larger than 1. Moreover it is different in charged current processes (where both the vector and the axial currents are not conserved) and in neutral current processes (where only the axial current is not conserved). The large longitudinal contribution leads to a strong violation of the Callan–Gross relation. Neither the use
of this relation nor its correction by a value of $R$ taken from electromagnetic scattering (as it was done in the LO analysis [9]) are then legitimate. The inclusion of gluon fusion diagrams allows considering all important physical effects automatically and in a QCD computable way.

The new CCFR strange quark distribution is in much better agreement with the MRS–A fit [12] (see Fig. 2) and leads to a strange sea content $\kappa = 0.477 \pm 0.050$ at the average $\langle Q^2 \rangle \simeq 22\, GeV^2$. There is a $(10 - 15)$% error on the result due to the factorization scale uncertainty (see sec. 4). The measured quark mass is $m_c = 1.70 \pm 0.19\, GeV^2$ and is larger than the unrealistic value $m_c = 1.31\, GeV^2$ found in the LO analysis of Ref. [9] because of the slow rescaling method adopted there.

Finally, notice that, in passing from LO to NLO, a variation is expected (and detected [8]) also for the non strange distributions, which are Cabibbo suppressed in the cross section [9], because the $u$ and $d$ contributions to structure functions are mixed with the charm contribution in the $W^+ g \to \bar{u}c, \bar{d}c$ processes.

3 Comparison with another determination of the strange density

There is another way to extract the strange sea density from deep inelastic scattering. It combines measurements of $\nu$DIS and $\mu$DIS structure functions: we shall call it the $\nu - \mu$ determination of $s(x)$.

Let us resort once more to the parton model or, equivalently, to leading order QCD. The decompositions of $F_2$ for muonic and neutrino probes and an isoscalar nucleon are

$$F_2^{\mu N} = \frac{5}{18}x(u + \bar{u} + d + \bar{d}) + \frac{8}{9}xc + \frac{2}{9}xs ,$$ (8)
\[ F_2^{\nu N} = x(u + \bar{u} + d + \bar{d} + 2s + 2\bar{c}). \]  

(9)

In eq. (8) we used \( s = \bar{s}, c = \bar{c} \) and eq. (9) refers to charged current scattering.

By appropriately combining \( F_2^{\nu N} \) and \( F_2^{\mu N} \) one can select the (leading order) strange distribution, under the assumption \( c \ll s \):

\[
\frac{5}{6} F_2^{\nu N}(x, Q^2) - 3F_2^{\mu N}(x, Q^2) = xs(x, Q^2).
\]

(10)

It is immediately evident what is the main experimental difficulty with this determination. The quantity on the l.h.s. of eq.(10) is obtained by subtracting data from two different experiments and is very sensitive to the relative normalization. Besides, data are not taken at the same \( x \) and \( Q^2 \) values. Large uncertainties thus arise in the \( \nu - \mu \) difference.

On the other hand, the \( \nu - \mu \) determination presents at least two advantages. First of all, on the theoretical side, its parton density content is simpler and much easier to reconstruct than that of the quantity in curly brackets in eq. (2). Second, when \( F_2^{\nu N} \) is measured there is no spurious, acceptance–dependent, separation of the two gluon fusion diagrams in Fig. 1, as in the dimuon measurement (see next section), and the whole \( \bar{s}c \) contribution to the structure function is determined.

Now, after the first (LO) CCFR determination of the strange density came out [9] it was clear that there existed a big discrepancy (see Fig. 2) with the result from the \( \nu - \mu \) difference, obtained by combining NMC [7] and CCFR [8] data, although the latter was affected by large errors. As we have already recalled, this discrepancy was mainly due to the leading order analysis of the dimuon data which hided important physical effects. With the new CCFR determination of \( xs(x) \) which includes the contribution of the gluon fusion diagrams the situation is considerably less puzzling. Still, there seems to be a residual discrepancy with the \( \nu = \mu \) result, which deserves some explanation (see Fig. 2). This is simple if one remembers that the non–universality effects taken
into account in the NLO dimuon result are not included in the LO formula (10) on which the identification of the difference \( (5/6) F_{2}^{\nu N} - 3 F_{2}^{\mu N} \) with the strange density is based. Otherwise stated, NLO effects invalidate eq. (10) which must be replaced by

\[
\frac{5}{6} F_{2}^{\nu N} - 3 F_{2}^{\mu N} = \frac{5}{3} F_{2}^{\bar{s}c} - \frac{1}{3} F_{2}^{s\bar{s}} ,
\]

where (we are interested in the \( Q^2 \) region around 10 GeV\(^2\))

\[
F_{2}^{\bar{s}c} = \left( \frac{\alpha_s}{2\pi} \right) g \otimes C_{2}(W^+ g \to \bar{s}c) ,
\]

\[
F_{2}^{s\bar{s}} = x(s + \bar{s}) + \left( \frac{\alpha_s}{2\pi} \right) g \otimes C_{2}^{massless}(\gamma^* g \to s\bar{s}) \approx x(s + \bar{s}) .
\]

The second equality in eq. (13) is valid when the physical scale \( Q^2 \) is sufficiently higher than the strange threshold.

Were the non–universality ratio \( F_{2}^{\bar{s}c}/F_{2}^{s\bar{s}} \) equal to 1/2, as in the LO case, eq. (11) would reduce to eq. (10). However, for not too large \( Q^2 \), this ratio is largely different from 1/2 (see [3, 4] and, for a more systematic study, [13]). Therefore the difference \( (5/6) F_{2}^{\nu N} - 3 F_{2}^{\mu N} \) does not coincide with the strange quark distribution \( x_s(x) \). We evaluated the r.h.s. of eq. (11) by resorting to eqs. (12,13), with factorization scale \( \mu_f^2 = m_c^2 \), and using the MRS–A parton densities [12]. Since the MRS–A strange density reproduces rather well the CCFR NLO data, our calculation will also clarify whether there is a real contradiction between the dimuon and the \( \nu - \mu \) determinations.

The result for the \( \nu - \mu \) difference (11) is displayed in Fig. 3 and compared to the data. The good agreement found shows that the dimuon and the \( \nu - \mu \) determinations are compatible in the whole \( x \) range, and is a check of the goodness of the MRS–A parametrization of \( x_s(x) \). Notice that, accounting for NLO effects, the \( \nu - \mu \) difference turns out to be larger than \( x_s(x) \) (dashed line) at low \( x \) and smaller at high \( x \), with a crossover at \( x \approx 0.07 \).

With the (important) caveat illustrated in the next section, we have now a trustworthy picture of the strange density. If we believe in the recent CCFR determination and
we assume the reliability of the $\nu - \mu$ data, no discrepancy whatsoever is at present detected and all experimental determinations converge to an $s(x)$ well fitted by the MRS curve with a conservative value of $\kappa \simeq 0.5$.

4 Uncertainties in the extraction of the strange density

We have seen that the strange density recently determined by the CCFR Collaboration has been obtained by a next–to–leading order QCD analysis of the dimuon data, which considers all the relevant effects previously overlooked. The result, however, is affected by a relatively large inherent uncertainty (more than 10%).

In this section we want to discuss two important sources of systematic uncertainties in the extraction of the strange sea density: one is specific of the dimuon determination, the other is more generally related to the kinematical region considered, close to a heavy quark threshold.

The first correction was already discussed in [4], where it was pointed out that in the first CCFR extraction of the strange density an important acceptance effect had been neglected, namely the experimentally different weight of the two diagrams in Fig. 1 due to the energy cut on the second muon.

In order to limit the background (mostly due to kaon and pion decays) a lower cutoff is set on the momentum $p_{\mu_2}$ of the muon coming from the semileptonic decay of charm ($p_{\mu_2} \gtrsim 5 \text{ GeV}$). This reflects itself into a cut on the momentum of the produced $c$ quark (we shall call $z$ the fraction of the light–cone momentum of the $W$ boson carried by the charm). Given that the low–$z$ region is dominated by the subprocess where

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the gluon splits into a $c\bar{c}$ pair with the $\bar{s}$ produced in the $W$–hemisphere (u channel), and, viceversa, the high–$z$ region is dominated by the process where the gluon splits into an $s\bar{s}$ pair with the $c$ produced in the $W$–hemisphere, it is clear that the cut on $z$ introduces an acceptance correction on the final result for the strange density. A way to compute such correction is to look at the $z$–distribution $\sigma_{Wg}(z)$ for the $W$–gluon fusion process \[4\]. This is strongly asymmetric: the two peaks at $z \to 0$ (backward peak) and at $z \to 1$ (forward peak) are not delta–like at small $Q^2$ (they tend to become so only at asymptotically large $Q^2$) and have a different $Q^2$ evolution: the forward peak appears sooner. The $p_{\mu_2}$ cut can be taken to generate an effective cutoff $z_c$ on $z$. The $W$–gluon fusion contribution to the structure function $F_2^{s\bar{c}}$ really measured is then proportional to $\int_{z_c}^1 \sigma_{Wg}(z)dz$ and hence smaller than the whole contribution $\int_0^1 \sigma_{Wg}(z)dz$. Were $z_c$ known, the correction would be theoretically predictable. On the other hand a possible uncertainty on $z_c$ may be a non irrelevant source of error on the extracted strange structure function. In Fig. 4 we show the results of the model of Ref. \[4\] for $F_2^{s\bar{c}}$ with two choices for $z_c$: $z_c = 0.5$ (solid curve) and $z_c = 0.8$ (dashed curve). The area below these two curves amounts to about 75% and 55%, respectively, of the whole integral (corresponding to $z_c = 0$).

An experimental evaluation of this acceptance correction can be performed by folding the $z$–cross section $\sigma_{Wg}(z)$ with an empirically determined acceptance function, which is zero at $z \to 0$ and rises to 1 at large $z$ (the method sketched in the previous paragraph corresponds to a step–function choice for the acceptance curve). In its latest analysis \[11\], the CCFR collaboration has carried out such computation and found that the acceptance correction is 60% with an estimated error of 10%. These values correspond approximately, in our approach, to the situation depicted in Fig. 4 and confirm the importance of the effect (and also, incidentally, the educated guess on the
acceptance uncertainty made in [4]).

A more relevant (and fundamental) uncertainty on the strange quark density comes from the fact that at $Q^2$ not much below a heavy quark threshold it is generally unsafe to extract the distributions of the vector–boson–gluon fusion products, instead of their contributions to structure functions.

To explain this point, let us look for instance at the expression (6) for $F_2^{sc}$. Since the $W$–quark fusion terms are negligible with respect to the $W$–gluon fusion ones, the (anti)quark distributions are contained only in the quark scattering term. This is small near threshold and undergoes a subtle and rather precise cancellation with the subtraction term: therefore its extraction is a delicate matter. More importantly, the quark scattering contribution is strongly dependent on the factorization scale, as it is theoretically predicted [14] and experimentally observed [11]: most of the overall $(10–15)\%$ uncertainty in the NLO CCFR result for $x_s(x)$ comes from the arbitrariness in the choice of $\mu^2$. By contrast, the structure function $F_2^{sc}$ is rather stable against various choices of the factorization scale and is therefore the best quantity to explore, at least as far as the quark scattering terms do not become the dominant contribution – which happens at $Q^2$ well above threshold.

It would thus be desirable, at least near heavy quark thresholds, to get from experiments data on structure functions instead of data on parton distributions.

5 Conclusions

Let us summarize the main points of this work.

We have by now a much better knowledge of the strange content of the nucleon,
coming mostly from neutrino deep inelastic scattering.

The next–to–leading order determination of $x s(x)$ performed by CCFR supersedes
the leading–order one: the latter should be recorded as a result which has little to do
with the real world. It relied on two assumptions: \( i \) the gluon fusion diagrams are neg-
ligible corrections; \( ii \) the quark mass effects are accounted for by slow rescaling. In the
\( Q^2 \) region of present experimental interest, we pointed out that: the former assump-
tion is simply wrong (as the explicit calculations show); the latter is ill–established
and the slow rescaling procedure does not even mimic the correct treatment of heavy
quarks. Not far from threshold, the next–to–leading order term is not a correction of
the leading–order term and one should be very careful in using such a terminology
which could induce into dangerous misunderstandings.

The discrepancy between the dimuon and the $\nu - \mu$ results for $x s(x)$, which was
worrisome at the time of the first CCFR determination, is now solved. The higher–
order analysis correctly takes into account the effects which were the physical cause of
such an apparent puzzle (we dubbed them “non–universality effects”): different mass
thresholds in neutral and charged current DIS and large longitudinal contributions to
$\nu$DIS structure functions. In other terms, the dimuon data and the $\nu - \mu$ data measure
different quantities, which coincide only when the two classes of effects just mentioned
are neglected, \( i.e. \) only when a LO analysis is performed. By accounting for these
non–universality (or NLO) effects \( i.e. \) for the non negligible difference between $2F_{2c}^s$
and $F_{2c}^{ce} + F_{2s}^{ss}$, we have explicitly shown that even the residual gap with the new
dimuon data at small $x$ is fictitious.

Although greatly improved, our present knowledge of the strange distribution is
not yet free from uncertainties. The NLO extraction of the strange sea density from
dimuon data near charm threshold is intrinsically unsafe, because the quark scattering
term (that is, the parton density) is a small contribution subjected to cancellation by the subtraction term and, at the same time, has a relatively strong dependence on the factorization scale. This dependence weakens if one considers the whole structure function (including the dominant gluon fusion term). Thus, at least at moderate values of $Q^2$, the strange and charm structure functions (and not their parton distributions) should be experimentally extracted to be object of theoretical study.
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Figure captions

Fig. 1 \( W \)-gluon fusion process for charm production: a) \( u \) channel diagram; b) \( t \) channel diagram.

Fig. 2 The strange quark distribution \( x_s(x) \). The boxes are the LO CCFR determination \[9\] at \( Q^2 \simeq 10 \text{GeV}^2 \). The circles are the quantity \( (5/6) F_2^{\nu N} - 3 F_2^{\mu N} \) at \( Q^2 \simeq 7 \text{GeV}^2 \), obtained by combining NMC \[7\] and CCFR data \[8\]. The shaded area represents the new NLO CCFR result \[11\] at \( Q^2 = 10 \text{GeV}^2 \). The continuous line is the MRS–A fit \[12\] for \( x_s(x) \) at \( Q^2 = 7 \text{GeV}^2 \).

Fig. 3 The difference \( (5/6) F_2^{\nu N} - 3 F_2^{\mu N} \) at \( Q^2 \simeq 7 \text{GeV}^2 \). The circles are the experimental results obtained by combining NMC \[7\] and CCFR data \[8\]. The solid line is the next–to–leading order QCD prediction described in the text. The dashed line is the MRS–A fit \[12\] for \( x_s(x) \) at \( Q^2 = 7 \text{GeV}^2 \).

Fig. 4 The charm–strange structure function \( F_2^{sc} \) at \( Q^2 = 10 \text{GeV}^2 \) with two different values of the cutoff \( z_c \) on the light–cone momentum of the charmed quark (see text): the solid line is for \( z_c = 0.5 \), the dot–dashed line for \( z_c = 0.8 \).
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