The Clustering of Faint Galaxies and the Evolution of $\xi(r)$

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ABSTRACT

The two-point angular correlation function, \( \omega(\theta) \), is constructed from a catalog of 13,000 objects in 24 fields distributed over an area of 4 deg\(^2\) and complete to a limit of \( R = 23.5 \). The amplitude and slope of our correlation function on arcminute scales are in broad agreement with recent CCD results in the literature and decreases with depth. No evidence is found for a flattening of the slope of the correlation function away from \( \delta \sim 0.8 \). Using the redshift distribution from the recent I-band selected Canada-France Redshift Survey, the observed \( w(\theta) \) implies a value of the clustering length \( r_0 = 1.86 \pm 0.43h^{-1}\) Mpc \((q_0 = 0.5)\) at \( z = 0.48 \). This is generally consistent with the possible rates of clustering evolution expected for optically selected galaxies. We finally discuss the implications of our results for the nature of the faint galaxy population.

Subject headings: galaxies: clustering galaxies:evolution
1. Introduction

Understanding the strength and evolution of field galaxy clustering is a key ingredient to synthesizing several areas of current extragalactic research. The evolution of the galaxy correlation function will likely reflect both the evolution of large scale structure in the Universe and the effects of evolution on the galaxy population. The large-scale distribution of galaxies is a product of the initial density fluctuations present in the early Universe and the growth of these fluctuations is strongly $\Omega$ dependent. The strength of clustering provides clues as to the nature of the galaxies themselves, for example, whether they are dwarf galaxies, normal galaxies or low-mass haloes in the process of forming galaxies. Finally, as merging may be one of the drivers of galaxy evolution, there may be a direct link between the evolution of clustering and the evolution of galaxies.

As new data on galaxies at cosmological distances is acquired, we are able for the first time to examine the correlation function at redshifts that are sufficiently large that the evolution in the correlation function should be unambiguously detectable.

An observationally convenient statistic to quantify galaxy clustering is the two-point correlation function, $\xi(r)$, and its two-dimensional projection onto the surface of the sky, $\omega(\theta)$, both of which measure the strength of clustering in comparison to a random population. From bright surveys, $\omega(\theta)$ has been found to obey a power-law of the form $\omega(\theta) = A\omega\theta^{-\delta}$ where $\delta = 0.8$ (Peebles 1980). More recently, Maddox et al. (1990) have found $\delta = 0.66$ for galaxies between $17.5 \leq b_J \leq 20.5$ in the APM survey. Regardless of any cosmic evolution in the clustering, the amplitude of $\omega(\theta)$ will decrease with increasing magnitude (i.e. depth) because the increased path length means that an increasing fraction of pairs at a given angular separation are unrelated chance projections.

In this paper, the angular correlation function is determined for a sample of galaxies
selected in the R-band, $19 < R < 23.5$. At the median redshift of this sample ($z \sim 0.56$), the R-band samples the rest-frame B-band light of the galaxy population and the selection of this sample is therefore well-matched to that of local samples of optically-selected galaxies. Recent papers on the correlation function in the R-band include those of Infante and Pritchet (1995) who used photographic plates to measure correlations of 39000 galaxies over an area of 2 deg$^2$ and to a depth of $F = 23.0$. They found evidence for a flattening of the slope of the power-law with increased depth between $F = 21$ and $F = 23$. Also, clustering amplitudes were closer to no-evolution model predictions in the red filter (F) sample than in the blue (J) filter sample. Couch et al. (1993) examined $\omega(\theta)$ for more than 116,000 galaxies over an area of 3.6 deg$^2$ in a hybrid “VR” band to an equivalent depth of $R \sim 23$ where they found no evidence for changes in the slope with depth. Roche et al. (1993) used 3627 galaxies distributed over 331 arcmin$^2$ to measure correlations $\omega(\theta)$ to a depth of $R = 23.5$, obtaining reasonable agreement with a $\theta^{-0.8}$ power law. Efstathiou et al. (1991) studied correlations between $23 < R < 25$ (as well as $24 < B_J < 26$) and found a surprisingly low clustering amplitude, i.e. their correlations at $\theta \leq 0^\circ.06$ showed little or no signal. The deepest survey thus far is that of Brainerd et al. (1995) who used Keck data to $r = 26$. They observed 5700 galaxies on a single 90 arcmin$^2$ CCD field and found the familiar $\theta^{-0.8}$ power law along with a low but non-zero value for the correlation amplitude. Lower than expected correlation amplitudes can be produced by a broadening of the $N(z)$ (Efstathiou et al. 1991, Roche et al. 1993) or if the sample is dominated by dwarf galaxies that are more weakly clustered than bright galaxies (Efstathiou et al. 1991, Brainerd et al. 1995).

In this paper we use high quality images taken in sub-arcsecond seeing with the 3.6m Canada-France-Hawaii Telescope to investigate the angular correlation function from arcsecond to arcminute scales to a depth of $R = 23.5$. In an important development, the results from the deep I-selected, Canada-France Redshift Survey (see e.g. Lilly et al. 1995a, Le Fèvre et al. 1995, Crampton et al. 1995) are used to determine the $N(z)$ for our R-band
selected sample. This information enables us to estimate the true three-dimensional correlation function $\xi(r)$ at a fiducial redshift of our sample, $z \sim 0.48$ (see Section 4.2).

The layout of this paper is as follows. In Section 2, the data reduction and image analysis which generated the photometric galaxy catalogs is described. In Section 3, our methods of correlation estimation and bias correction are described and our estimate of the $\omega(\theta)$ on angular scales up to a few arcminutes presented. These results are compared with those of others in section 4, and using our new knowledge of $N(z)$, an estimate for the $\xi(r)$ at $z = 0.48$ is derived. The implications of this measurement are discussed in in Section 5. Throughout the paper, we take $H_0$ to be $100 \text{ h km s}^{-1}\text{Mpc}^{-1}$ and assume $q_0 = 0.5$ unless otherwise indicated. Length scales, such as correlation lengths, are quoted as physical (or “proper”), rather than comoving, lengths.

2. Data

Characteristics of the data and its calibration are described below. Further details may be found in Hudon (1995).

2.1. Observations and Data Reduction

The primary dataset for this project consists of R-band CCD images obtained with the Faint Object Camera (FOCAM) at the Prime Focus of the 3.6m Canada-France-Hawaii Telescope on Aug. 10-12, 1991. Twenty-four fields randomly distributed across an area of 4 deg$^2$ were observed, with each field covering an area of approximately 50 arcmin$^2$. Assuming a redshift interval $0.3 < z < 0.7$ (see below), the survey samples a volume of $\sim 10^6 h^3 \text{ Mpc}^3$. 
The observations were made under good seeing conditions (0.′6 − 0.′9) using the LICK2 chip with pixel size 0.207 arcsec. For reasons unconnected with this project, each of the 24 fields was imaged on three occasions over the course of three nights for a total exposure time of 15 minutes. The separation in time between exposures thus varied between 10 minutes and 2 days. Photometric calibration through the run was carefully monitored with repeated observations of three Landolt (1993) standards, together with a reference star that was located near to our fields. The color term was found to be essentially zero. The survey is centered on the SSA-22 field (2215+00) of Lilly, Cowie and Gardner (1991, see also Lilly 1993) which is also one of the five fields of the Canada-France Redshift Survey (see e.g. Lilly et al. 1995b).

The CCD images were each pre-processed by subtracting a median bias frame and flat-fielded using first dome flats and then a sky-flat generated from the whole data set. After this procedure, a low-amplitude fringe pattern of variable amplitude persisted, caused primarily by an imperfect coating on the CCD chip. We developed an iterative scheme to effectively separate out and remove the pattern from each of the images.

Each image was photometrically calibrated individually, and then the three images for each field were coadded using a 3-sigma clipping algorithm. This was effective in removing essentially all cosmic rays. The photometric zeropoint for the three observations of each field was found, from photometry of stellar images in each field, to be consistent to better than 2 % rms.

### 2.2. Catalog Generation

Object detection and photometry on the final co-added images was done using FOCAS (Valdes 1982). We used detection criteria of 3σ per pixel and a six pixel minimum
contiguous area. As we believe has been seen before, various tests such as generating catalogs from rotated images or after the addition of a constant background level produced slightly different catalogs. Image rotation produces different catalogs because the image detection algorithm proceeds through an image row by row and thus the local sky at any object depends on the history of pixels leading up to that object. However, it was found that merging together the catalogs from images rotated by $0^\circ, 90^\circ, 180^\circ$ and $270^\circ$ with the background sky level determined independently from FOCAS produced a more complete catalog in the sense that fewer objects were missed. Approximately 1000 objects were found on each image down to our magnitude limit of 24.2. The catalogs were examined on an interactive display using the Picture Processing Package (Yee 1991) and spurious objects removed. These were typically due to haloes around bright stars or multiple identifications of faint amorphous objects, and, together, these comprised a few per cent of the catalogs. A handful of obvious objects missed by the image detection algorithm were also added at this stage. Due to a small spatial shift between the three images in each field, we have used only objects that are located more than 48 pixels from the edge of the image as the image edges suffer from dead pixels, cosmic rays, and greater photometric errors due to the coadding. Using this finding list, magnitudes were determined using a fixed aperture of 5″. Magnitudes determined in this way are within 0.2 of “total” magnitudes. The $1\sigma$ photometric noise on these large apertures is equivalent to an R-magnitude of approximately 24.2.

### 2.3. Number Counts

The number counts of deep objects provide a good consistency check between our catalogs and others who have examined the counts at similar depths. Following Couch et al. (1993) we have plotted our counts together with the compilation of Metcalfe et al. (1991, their Figure 11) in Figure 1. Our counts are seen to be in good agreement with
other recent determinations in both slope and absolute number. The best-fit slope of the solid line in Figure 1 is 0.36. Stars dominate the catalogs at magnitudes brighter than 19. This is seen in the data (starred symbols) as well as the predicted star counts of Bahcall and Soneira (1980) (see Section 2.4 below).

The turnover in our counts in the faintest two bins is due to incompleteness. For the correlation analysis below, we decided to take a conservative magnitude cut-off of $R = 23.5$, which is above the incompleteness roll-over in all of the individual fields. The standard deviation of the number of objects observed from one field to the next down to $R = 23.5$ is 15%. This did not correlate with any obvious observational parameter such as the seeing, airmass etc., and is most likely to be dominated by large scale galaxy clustering (see Section 3.3 below).

2.4. Stellar Contamination

Foreground stars are randomly distributed on deep sky images and have the effect of diluting the amplitude of $\omega(\theta)$ relative to that which would be measured for the galaxies alone. It can easily be proved that the stellar-corrected correlation function is equal to the observed correlation function divided by the square of the galaxy fraction. For reasons unconnected with this project, the survey fields are at relatively low Galactic latitude $b_{II} = +46^\circ$, so stellar contamination is relatively important. This problem may be addressed either by removing stars from the sample or by correcting the $\omega(\theta)$ determined from a composite sample that includes the stars. Although at this seeing and S/N the vast majority of galaxies are distinguishable morphologically from stars (see Crampton et al. 1995), a danger in attempting to remove stars from the sample is that the criteria that separate stars from galaxies can vary with magnitude and, especially, from one field
to the next due to variations in the point spread function. There is also the possibility of systematic effects if the most compact galaxies have different clustering properties. Accordingly, the second approach was adopted in this investigation.

Estimation of the stellar contamination proceeded as follows. First, at bright magnitudes, where there are relatively few objects, a star-galaxy separator was applied which compares the intensity of the peak pixel for each object to the average intensity of the remaining pixels in a 3 arcsecond aperture. On our images, the distribution of stars and galaxies in the plane overlaps at a magnitude of $R \gtrsim 22.0$, as shown in Figure 2 for four of the 24 different fields. Beyond this limit, we used the galaxy model of Bahcall and Soneira (1980) to estimate the faint star counts. Finally, we checked our star counts with those derived from the I-band selected Canada-France Redshift Survey (which obtained spectra of all objects regardless of morphology) in this same region of the sky (Lilly et al. 1995b) and found good agreement. Our estimates for the galaxy fraction, $f_g$, as a function of depth are presented in Table 1.

2.5. N(z) Information

The redshift distribution is a critical element in interpreting the observed amplitude of the projected 2-d correlation function $w(\theta)$. The Canada-France Redshift Survey (CFRS, see e.g Lilly et al. 1995a, Le Fèvre et al. 1995, Crampton et al. 1995) is a new spectroscopic survey of over 1000 red-selected objects with isophotal $17.5 < I_{AB} < 22.5$, comparable in depth to the galaxy sample considered here. The overall success rate in securing redshift measurements for the galaxies was 80%. However, as discussed by Crampton et al. (1995), the redshifts of half of the unidentified objects are known statistically, and it is likely, based on their colors, that the remaining 10% of galaxies do not have a very dissimilar redshift
distribution. The fraction of the sample for which redshift information is not available is therefore less than 10%.

Since \((V - I)\) colours are available for all objects, and \((I - K)\) and \((B - I)\) for most of them, it is relatively straightforward to produce a predicted \(N(z)\) for an \(R\)-band selected sample. This procedure, described by Lilly et al. (1995c), uses the \(1/V_{\text{max}}\) formalism. For each galaxy, the absolute magnitude and spectral energy distribution is defined, and thus the comoving volume throughout which it would have been within the apparent magnitude limits of the original I-band selected sample \((17.5 \leq I_{AB} \leq 22.5)\) is calculated:

\[
V_{\text{max}} = \left( \frac{c}{H_0} \right)^3 \int_{z_{17.5}}^{z_{22.5}} \frac{Z_q^2(z)}{(1 + z)(1 + 2q_0z)^{\frac{2q_0}{q_0 - 1}}} d\Omega dz, \tag{1}
\]

\(d\Omega\) is the effective solid angle of the survey, 112 arcmin\(^2\) (see Le Fèvre et al. 1995) and

\[
Z_q(z) = q_0z + (q_0 - 1)\left[\sqrt{1 + 2q_0z} - 1\right] \frac{q_0^2}{q_0^2 + 1}. \tag{2}
\]

The spectral energy distribution is determined by interpolating the spectral energy distributions given by Coleman et al. (1980) on the basis of the observed \((V - I)_{AB}\) colours, available for all objects.

Using the same absolute magnitude and spectral energy distribution, the apparent \(R\) magnitude as a function of redshift of each galaxy is calculated and the \((R, z)\) plane is populated with

\[
dn = \frac{dV}{dz} \frac{1}{V_{\text{max}}}. \tag{2}
\]

Integration over \(R\) then gives the \(N(z)\) distribution for a given \(R\)-selected sample. The procedure should account fully for the different weighting of galaxy types in the \(R\)-band sample, and for the effects of K-corrections and volume elements as the redshift increases. The procedure does not, however, attempt to account for any evolutionary effects (i.e. it is
assumed that the population does not evolve at higher or lower redshifts than the galaxies observed, but for samples well-matched in redshift (i.e. with similar $< z >$), these effects should be small.

In constructing the R-band N(z) we used photometrically estimated redshifts for all the CFRS galaxies for which reliable spectroscopic redshifts were not available (the “best estimate” sample of Lilly et al. 1995b). The redshift distribution derived in this way for our photometric sample $19.0 < R < 23.5$ and several other magnitude ranges is shown in Figure 3. We note in passing that although the predicted $N(m, z)$ must reproduce the number-magnitude counts at the depth of the original survey, it does not do so at substantially fainter magnitudes. This is presumably due to continuing evolution of the luminosity function at higher redshifts (see Lilly et al. 1995c for a discussion).

The effect of placing galaxies with unknown redshifts at some arbitrary redshift is to dilute the projected correlation function, leading to an underestimate of the true 3-d clustering in the sample. The maximum effect would be observed if the unidentified objects were, like the foreground stars, intrinsically unclustered (perhaps because they lay at very high redshift). A 10% contamination by unclustered galaxies, the maximum that we could conceive, would have a 20% effect on the amplitude of the correlation function and therefore a 10% effect on the estimation of $r_0(\theta)$.

3. Calculation of the Correlation Function

3.1. The Estimators

The power of the projected correlation function, $\omega(\theta)$, is its ability to deliver clustering information for a magnitude-limited sample of galaxies simply by taking images of the sky,
without recourse to time-consuming individual distance measurements. In addition, since \(\omega(\theta)\) is not affected by distortion due to the peculiar motions of galaxies, and photometric surveys probe deeper than spectroscopic surveys, \(\omega(\theta)\) is effectively as useful as \(\xi(s)\) itself. The correlation function \(\omega(\theta)\) is defined as the excess probability (above random) of finding a galaxy in an angular element \(\delta \Omega\) at a distance \(\theta\) away from some reference galaxy:

\[
\delta P = N (1 + w(\theta)) \delta \Omega,
\]

where \(N\) is the mean density of objects on the sky. Over the last two decades, several methods have been developed to estimate the above probability. These are discussed in detail by Sharp (1979), Hewett (1982) and Infante (1994) amongst others. For this investigation, we have used both the counts-in-cells method in which the form of the estimator is:

\[
w(\theta) = \frac{\langle N_i N_j \rangle}{\langle N_i \rangle \langle N_j \rangle} - 1
\]

(3)

where \(N_i\) and \(N_j\) denote the counts in cells i and j and the brackets denote averages over pairs of cells separated by \(\theta \pm \delta \theta / 2\).

In practice, each image was divided up into cells 32 pixels on a side (i.e. 6.′4 x 6.′4) and \(w(\theta)\) was calculated for bins that were separated by 0.15 in log\(\theta\). The range is 5.′5 \(\leq \theta \leq 204′\), the upper limit corresponding to half the size of each individual CCD image. The large bins cause some “digitisation” effects at the smallest scales where the effective separation is no longer the nominal separation between the cell centers. The smallest two bins have been adjusted to smaller effective separations to reflect this and on these small scales we have also constructed traditional ring counts which agree well.

3.2. Errors
The internal errors in our correlation function can be estimated in three different ways: (1) Modified Poisson errors from the number of pairs at any separation (e.g. Peebles 1980, § 48): \[ \delta \omega(\theta) = \sqrt{\frac{1 + \omega(\theta)}{N_{\text{pairs}}(\theta)}}, \]
(2) by taking the standard error of the 24 averaged correlation functions (i.e. from the “local” method, discussed below), or (3) from the bootstrap method (e.g. Ling et al. 1986). Since the bootstrap method is a general methodology for assessing the accuracy of a given estimator (Efron and Tibshirani 1986), we choose it to evaluate our correlation uncertainties. For the bootstrap method, given the original data set, a pseudo-data set is generated by choosing N data points with replacement from the original set of N data points and the correlation function is redetermined. This process is repeated a minimum of 300 times. At each separation, \( \theta \), we take the error as the standard deviation of the distribution of points in the pseudo-correlation functions. We note that our bootstrap errors are approximately 40% larger than the error determined from the standard deviation of the correlation function for individual fields or Poisson errors.

### 3.3. Biasses in \( \omega(\theta) \)

A key issue in the construction of \( \omega(\theta) \) is the estimation of the background density of galaxies. The estimator for \( \omega(\theta) \) may be biased for two reasons: (1) the “integral constraint” (Peebles 1980) and (2) possible variations in the surface density of galaxies introduced by spurious observational effects especially from field to field. The integral constraint arises because the galaxy surface density is estimated on finite angular scales. If \( \omega(\theta) \) is still positive on these scales, the galaxy surface density, and hence the expected number of pairs, is biased. Thus, the observed \( \omega(\theta) \) is reduced from its true value and forced to become negative at some large angle, roughly half the angular extent of the sample. The integral constraint operates as a scaling factor, \( B \) (with \( B < 1 \)), introduced into the estimator to
correct for this bias, \( \text{viz.} \),

\[
w(\theta) = \frac{\langle N_i N_j \rangle}{B \langle N_i \rangle \langle N_j \rangle} - 1.
\] (4)

Note that for convenience, we have allowed the integral constraint to operate as a multiplicative scaling factor; since \( \omega(\theta) \ll 1 \), this is not significantly different from the usual additive way of accounting for the integral constraint (Peebles 1980, §32). The value of \( B \) may be estimated if the true \( \omega(\theta) \) is known (e.g. from the double integral of \( \omega(\theta) \) over the whole sky). For a power-law angular correlation function with a cutoff in correlation power beyond some angle \( \theta_c \), \( B \) is given by (Pritchet and Infante 1986),

\[
B = \left[ 1 + \frac{2 A_\omega}{2 - \delta} \left( \frac{\theta_c}{\theta_o} \right)^2 \theta_c^{-\delta} \right]^{-1},
\] (5)

where \( \theta_o \) is the radius of the field being observed, \( A_\omega \) is the amplitude at 1° and \( \delta \) is the slope. (For \( \theta_c > \theta_o \), \( \theta_c \) should be replaced with \( \theta_o \) in the above formula.)

If there are spurious field-to-field variations (e.g. from errors in the photometric zeropoint) then the effect is to bias \( \omega(\theta) \) to higher values (because the spurious variations mimic galaxy clustering on large scales). This introduces a second parameter \( B' \) (with \( B' > 1 \)), given by,

\[
B' = 1 + \left( \frac{N_{\text{inst}}}{N_{\text{avg}}} \right)^2,
\] (6)

where \( N_{\text{inst}} \) is the rms variation in the number of objects due to spurious instrumental (photometric) error and \( N_{\text{avg}} \) is the average number of objects per frame.

As noted above, the numbers of galaxies down to a fixed magnitude limit varies by 15% across our sample. The photometric calibration is known to be constant to within 2% rms. Hence, purely photometric uncertainties would only produce a \( \sim 2\% \) variation in the expected numbers of galaxies (given the slope of the number counts) and thus a negligible \( 4 \times 10^{-4} \) bias to \( \omega(\theta) \). Although we could find no correlation between the numbers of galaxies observed in each field and other observational parameters such as the seeing
or airmass of observation, we have no way, a priori, of knowing what fraction of this 15% variation is due to observational effects and what fraction is due to true galaxy clustering.

In the light of this, we have taken two approaches to the construction of $\omega(\theta)$. First, we compute $\omega(\theta)$ in each field individually, i.e. the sum over cells is carried out over each field separately, and the resulting $\omega(\theta)$ is then averaged. This is effectively estimating the background galaxy density from each field separately which obviously eliminates any problems due to spurious field-to-field variations, but produces a relatively large integral constraint bias. We refer to this as the “local” determination of $\omega(\theta)$. Second, the sum over cells is carried out over all cells in the sample, effectively setting the background galaxy density to be the average of the whole sample, determined over a scale almost 20 times larger. We refer to this as our “global” $\omega(\theta)$. It has a very small integral constraint bias but may suffer from spurious field-to-field variations, although, as noted above, in our sample these should be small (at least from photometric variations).

The correlation function for the counts-in-cells method is shown in Figure 4 for three magnitude samples: $19 \leq R \leq 22.5$, $19 \leq R \leq 23.0$ and $19 \leq R \leq 23.5$. We have presented both the local and global averaging methods along with a power-law, $\theta^{-0.8}$. The data points are uncorrected for the integral constraint, spurious field to field variations or for the stellar contamination. As discussed above, at larger scales the integral constraint pushes the local function down while spurious field-to-field variations may push the global function up. The best fit slopes to the global $\omega(\theta)$ are $\delta = -0.82$ for $19 < R < 23$ and $\delta = -0.73$ for $19 < R < 23.5$. The decrease in the last half magnitude is almost certainly due to the effect of small spurious field-to-field variations entering towards the bottom of the sample and introducing a non-zero $B'$ term, thus raising the correlation function at the largest scales. These slopes give little support to the suggestion that the slope of $\omega(\theta)$ flattens with depth (c.f. Infante and Pritchet 1995).
Together, the local and global methods must bracket the true (unbiased) correlation function. To obtain an estimate for this true correlation function for $19 < R < 23.5$, we follow other recent work and force the slope of the true unbiased $\omega(\theta)$ to be $\delta = -0.8$ – the observed slope from local surveys (Peebles 1980) and as observed in our $19 < R < 23$ sample. This effectively determines both $B$ and $B'$ empirically (subject, of course, to the assumption about the slope).

In the $19 < R < 23.5$ sample, we find $B = 0.987$ for the locally estimated $\omega(\theta)$ and $B' = 1.004$ for the global estimate. The value of $B$ agrees exactly with the Pritchet and Infante formula (as it must), while the small value of $B'$ implies that the observed field to field variations of 15% rms are dominated by true clustering effects. The component arising from observation effects is (again, assuming the slope of $\delta = -0.8$) about 6% rms in the faintest sample. This is about 3 times larger than that expected from photometric errors above, but is not too surprising and, as noted above, these spurious effects disappear entirely for the samples limited at slightly brighter magnitudes, $19 < R < 23.0$, for which the observed variations are entirely accounted for by the expected correlations of galaxies from field to field with $\delta = -0.8$.

We summarize our results in Table 1.

4. The correlation function $\omega(\theta)$ on arcminute scales

4.1. Comparisons with the results of others

In order to compare our correlation function with those of other recent investigations, we have determined the amplitude of the correlation function extrapolated to a scale of
\( \theta = 1^\circ \) for \( \delta = -0.8 \) slope. It should be noted, however, that this amplitude is based on the amplitude of \( \omega(\theta) \) on scales up to 2 arcmin, i.e. on Mpc scales at \( z = 0.48 \).

The amplitude at \( \theta = 1^\circ \) is compared with other recent determinations, extrapolated with the same \( \delta = 0.8 \) slope, in Figure 5, where the general decline of correlation amplitude with limiting survey depth is clearly evident. Data at the bright end is taken from Stevenson et al. (1985) who used 1.2m UKST plates to carry the correlation analyses to \( r_F < 20 \) (we have neglected to plot their AAT data as they report anomalously low number-magnitude counts for one of their fields). Our amplitudes agree well with those of Infante and Pritchet (1995) and Roche et al. (1993), but we are approximately two times higher than Couch et al. (1993). Fainter than \( R = 23.5 \), the correlation amplitude appears to drop off more steeply with the low results of Brainerd et al. (1995) and the single measurement of Efstathiou et al. (1991). All points in the figure have been corrected (either by the original authors or by ourselves) for contamination by foreground stars.

4.2. Inversion to yield \( \xi(r) \)

With the completion of the I-band selected Canada-France Redshift Survey and the definition of \( N(z) \) for red-selected faint galaxy samples, it is possible to invert the projected \( \omega(\theta) \) to yield an estimate for the three-dimensional \( \xi(r) \) at some fiducial redshift.

Since the spatial correlation function is observed to be a power law locally (Davis and Peebles 1983) and to have essentially the same slope at fainter magnitudes (see above), it is convenient to parametrise the cosmic evolution of clustering by a parameter, \( \epsilon \), such that (Groth and Peebles 1977, Phillipps et al. 1978):
\[ \xi(r, z) = \left( \frac{r_0(0)}{r} \right)^\gamma (1 + z)^{-(3+\epsilon)}, \]  

(7)

where \( r_0 \) and \( r \) are measured in physical units, and \( r_0(0) \) is the correlation length at \( z = 0 \).

The change in physical correlation scale length with redshift, \( r_0(z) \) is thus:

\[ r_0(z) = \frac{r_0(0)}{(1 + z)^{(3+\epsilon)/\gamma)}. \]  

(8)

The quantity \( r_0(z) \) is simply the correlation length that would be measured, in physical units, by an observer at the epoch in question. The value of \( r_0(z) \) that would be expected at some earlier epoch clearly depends on the correlation length of that population at the present epoch, \( r_0(z = 0) \), the slope of the correlation function, \( \gamma \), and the value of the evolutionary parameter, \( \epsilon \).

The evolutionary parameter \( \epsilon \) is interpreted as follows: \( \epsilon = -1.2 \) corresponds to the case where clustering is fixed in comoving coordinates – galaxy clusters expand with the Universe, there is no relative motion of galaxies and clustering does not grow with time; \( \epsilon = 0 \) is produced by clustering models that have bound units of constant physical size (i.e. in the same way that galaxies are bound objects that do not grow with the expanding Universe). The clustering grows in this case because the background density of galaxies is diluted by the expansion (while the density in the clusters is constant) - effectively it is the voids that are growing. As an example of an evolutionary model with hierarchical growth, we have examined the mass correlation function from the Cold Dark Matter simulations of Davis et al (1985). The \( \Omega = 1 \) EdS models have \( \epsilon \sim +0.8 \), since \( \xi \) in comoving terms evolves as \( (1 + z)^{-2} \) as expected from the linear growth of perturbations.

Once the spatial distribution of galaxies in a sample is determined through \( dN/dz \) (see Section 2.4), integrating along the lines of sight gives the projected two-dimensional distribution (Peebles 1980):

\[ \omega(\theta) = A \omega \theta^{1-\gamma}, \]  

(9)
where $\theta$ is in radians and $A_\omega$ is given by (e.g. Phillipps et al. 1978):

$$A_\omega = Cr_0(0)^\gamma \int_0^\infty D_\theta^{1-\gamma}(z)g^{-1}(z)(1+z)^{-(\epsilon+3)} \left(\frac{dN}{dz}\right)^2 dz \left[ \int_0^\infty \left(\frac{dN}{dz}\right) dz \right]^{-2}$$

where $D_\theta(z)$ is the angular diameter distance, $g(z)$ is the scale factor multiplied by the element of comoving distance $d\omega/dz$:

$$g(z) = \frac{c}{H_o}((1+z)^2(1+\Omega_o z)^{\frac{1}{2}})^{-1}$$

and $C$ is a constant involving only numerical factors:

$$C = \sqrt{\pi} \frac{\Gamma[(\gamma-1)/2]}{\Gamma(\gamma/2)}.$$  

Note that the above amplitude depends (strongly) on the shape of the redshift distribution, $dN/dz$ (i.e. effectively, the “width” squared and the median redshift), but not on the overall normalization. To perform this inversion we have assumed that $\omega(\theta)$ is a perfect power law, i.e. on all scales, and that the slope, $\gamma$ does not change with epoch, consistent with our observations.

In principle, knowing $N(m, z)$ at all magnitudes, one could fit all of the data in Figure 5 and determine $r_0(z)$ at all $z$. However, there are at least two effects to consider: the rest-frame waveband of the observed R-band shifts with redshift, and different populations of galaxies may evolve quite differently in luminosity (see e.g. Lilly et al. 1995c). Together these imply that the morphological composition of the sample changes with depth. As clustering characteristics can vary with galaxy type (e.g. Loveday et al. 1995), any apparent evolution in clustering may reflect a change in the galaxy population rather than the evolution of density perturbations in the Universe. Therefore, in producing an evolving correlation function, the evolution of large scale structure and the evolution of the galaxy population are likely to be inextricably linked. We are thus determining an “effective” $\epsilon$ that reflects both effects.
In order to reduce our measurement of $\omega(\theta)$ at $R = 23.5$ to a single useful number, we seek simply to estimate the value of $r_o(z)$ at some typical redshift of the sample. Determining the best value of $r_0(z)$ at high $z$, our procedure was to find all combinations of $r_0(0)$ and $\epsilon$ that are consistent with the correlation amplitude for our galaxy sample at $19 < R < 23.5$. It is found that these different “consistent” $r_0(z)$’s all intersect at a redshift of $z = 0.48$ at a value $r_0(0.48) = 1.86h^{-1}\text{Mpc}$ for $q_0 = 0.5$ and $r_0(0.48) = 2.16h^{-1}\text{Mpc}$ for a low-density Universe, $q_0 = 0.05$. We note that an intersection of $z \sim 0.48$ is convenient since, at this redshift, the observed R-band is approximately equivalent to the rest-frame B-band, allowing a ready comparison between our faint sample and local B-selected samples. It should be noted that, the intersection is somewhat less than the median redshift of the sample $\langle z \rangle \sim 0.56$ due to the higher weighting of lower-redshift objects in Equation 4.

4.3. Errors in the correlation length

There are three contributions to the error in the correlation length. The first is the error in the amplitude of $\omega(\theta = 1^\circ)$ at $R = 23.5$ that we are trying to fit. This error is determined by fitting each of the 300 bootstrap realizations to a slope of $\delta = -0.8$ and taking the standard deviation of the resulting amplitude distribution. We have found $\omega(\theta = 1^\circ) = 0.001386 \pm 0.00024$ which corresponds to a fractional error in $r_0(z)$ of 18%.

The second contribution to the error in the correlation length is from the galaxy fraction. An estimate of the error in the galaxy fraction is 10% (from the relative numbers of spectroscopically identified stars and galaxies in the 22hr field of the CFRS), corresponding to a fractional error in $r_0(z)$ of 14%. This was determined by varying the amplitude $\omega(\theta = 1^\circ)$ by 10% and examining the range in $r_0(0)$ for $\epsilon = 0$, and hence $r_0(z)$, that this implies.
The third contribution to the error is from the redshift distribution, \( N(z) \). A total of 730 galaxies from the CFRS were used in the R-band \( N(z) \) determination, which was then put into Limber’s equation, with \( r_0(0) \) and \( \epsilon \) as free parameters, to determine \( r_0(z) \). We have investigated the statistical error introduced by errors in \( N(z) \) by resampling the galaxies that went into the \( N(z) \) determination and comparing the fit to the correlation amplitude for a given \( r_0(0) \) and \( \epsilon \). We have performed 20 resamplings and find that the contribution to the fractional error in \( r_0(0) \) due to purely statistical uncertainties in the redshift distribution is approximately 3%, much smaller than the above two sources of error.

Adding these three contributions in quadrature gives \( \delta r_0(z) = \pm 0.43 \). It is noted that the dominant source of error in \( r_0(z) \) is in the amplitude of \( \omega(\theta) \). This error in \( r_0(z) \) corresponds to an uncertainty in \( \epsilon \) of \( \pm 1.1 \).

An additional, systematic, effect to note is that redshifts were unobtainable for 15% of the objects in the CFRS. Crampton et al. (1995) have argued that at least half of the unidentified 15% must be distributed like the CFRS sample as a whole. If the other half of the unidentified 15% were located at high redshifts, they would act to dilute the predicted correlation. Hence, in comparing to observations, a higher predicted correlation length must be chosen to account for the extra dilution. We find that such an effect would increase the deduced correlation length by approximately 10%.

5. Discussion

The deduced \( r_0(0.48) = 1.86h^{-1}\) Mpc can be produced by the following combinations: \( \epsilon = -1.2, \, r_0(0) = 2.75h^{-1}\) Mpc; \( \epsilon = 0, \, r_0(0) = 3.60h^{-1}\) Mpc; \( \epsilon = 1, \, r_0(0) = 4.45h^{-1}\) Mpc. These possibilities are shown in Figure 5. As discussed above, although the high \( \epsilon \) curves nominally provide a better fit to the data over a wide magnitude range, the fact that the
galaxy population may possibly be changing with depth (i.e. redshift) means that a firm conclusion to this effect can not be drawn.

The range of possible $r_0(0)$ and $\epsilon$ combinations is illustrated in Figure 6, which shows the three representative models which are constrained to go through the $r_0(z = 0.48) = 1.86h^{-1}$ Mpc. These are compared to several local surveys tabulated in Table 2. The CfA survey has a limiting magnitude of $B \sim 14.5$ and contains 1840 galaxies in the North zone, of which 1230 were used in the correlation analysis (Davis and Peebles 1983). The APM/Stromlo survey is complete to $b_J = 17.15$ and contains 1769 galaxies (Loveday et al. 1992). The “Durham” survey represents three surveys with 676 redshifts to a limiting depth of $b_J \sim 16.8$ (Hale-Sutton et al. 1989). The KOS survey consists of redshifts for 164 field galaxies brighter than $J = 15.0$ in the north and south galactic caps. The IRAS redshift survey represents results from both the $S_{60} \geq 0.6$ Jy (Saunders et al. 1992) and $S_{60} \geq 1.2$ Jy (Fisher et al. 1994) catalogs.

As a first statement, we can conclude from Figure 6 that our faint galaxies at $z = 0.48$ can “easily” evolve into the local population (with correlation length between $r_0(0) = 4.0$ and $5.5h^{-1}$ Mpc) if the clustering evolution is given by $0 < \epsilon < 2$, i.e. with mild to rapid evolution. In other words, the galaxies observed down to $R = 23.5$ cluster like “normal” galaxies. At a more detailed level, evolution to a CfA-like population requires $\epsilon \sim +2$ and evolution to an IRAS-like population requires $\epsilon \sim 0$. It is important to note that clustering evolution with $\epsilon \sim -1.2$ (i.e. fixed in comoving space) would result in a local population that is considerably less clustered ($r_0(0) \sim 3h^{-1}$ Mpc) than any of the local samples plotted in Figure 6. While low values of $r_0(0)$ have been measured for local dwarfs (e.g. Santiago and da Costa, 1990), these have yet to be confirmed (e.g. Thuan et al. 1991). These conclusions generally remain intact when a low-$\Omega$ Universe is considered. Changing $\Omega$ from 1.0 to 0.1 affects the line element and the angular diameter distance thereby increasing the
correlations at large distances and hence reducing the implied clustering evolution to local samples. In this case, the range in local correlation length, \( r_0(0) = 4.0 - 5.5h^{-1}\) Mpc is satisfied by \( \epsilon \sim -0.5\) to 1.0.

Can our knowledge of the galaxy population be used to distinguish between these possibilities? Two results from the analysis of the luminosity function of the I-band-selected Canada-France redshift survey (Lilly et al. 1995c), selected to have \( 17 < I < 22 \) and thus broadly equivalent to the present R-band selected sample, are relevant. First, the luminosity function of the redder galaxies (redder than a typical local Sbc galaxy) shows no change back to \( z \sim 1 \), suggesting that these galaxies have been relatively stable over the time interval relevant to the present study. These galaxies are presumably the same galaxies that appear in local B-selected samples such as the CfA. On the other hand, the luminosity function of the bluer galaxies shows substantial evolution. At \( z = 0.48 \) there are about three times more blue galaxies with luminosities comparable to present-day L* than are found at lower redshifts. These galaxies, which dominate the galaxy population at \( z \sim 0.6 \), are responsible for the steep number counts of galaxies in the B-band. Their nature and the identity of their local descendents (and thus their expected \( r_0(0) \)) are not yet determined. Important clues as to the nature of these galaxies are given by Le Fèvre et al. (1996) who have determined correlation lengths for the blue and red populations separately. At \( z > 0.5 \) blue and red galaxies are found to cluster similarly while at lower redshifts red galaxies cluster more strongly than blue galaxies. This indicates that environment may be playing a role in the relationship between blue and red galaxies as the Universe expands.

Returning to Figure 6, we now comment on the much fainter Efstathiou et al. (1991) and Brainerd et al. (1995) results. We note that they lie well below the predicted curves for any of the models in Figure 6 which were constructed from the expected \( N(z) \) and the observed \( \omega(\theta) \) at \( R < 23.5 \). That is, the expected levelling off in the amplitude
of the correlation function with depth (due to the fact that \( N(z) \) is expected to cease changing with depth because of the effects of volume constriction at high redshifts) is not observed. This may reflect the fact, noted above, that the \( N(m, z) \) analysis (derived from the Canada-France Redshift Survey sample at \( I < 22 \)) fails to reproduce the number counts at the faintest magnitudes (see Lilly et al. 1995c), presumably because of evolution in the galaxy sample at these faintest magnitudes relative to that seen in the CFRS at \( I \sim 22 \) or in our sample at \( R < 23.5 \).

6. Summary

The angular correlation function has been determined to a limiting magnitude of \( R = 23.5 \) for a galaxy catalog containing 13000 objects in a region 4 deg\(^2\) using high quality sub-arcsecond seeing CFHT images. The main results are:

- The amplitude of the correlation function at arcmin-scale separations is in accord with most other recent determinations.
- No evidence for a significant decrease in the slope of \( \omega(\theta) \) away from \( \delta \sim 0.8 \) is seen.
- Using the \( N(z) \) information from the I-band selected Canada-France Redshift Survey to predict an \( N(z) \) for this sample, we estimate the correlation length at \( z = 0.48 \) to be \( r_0(0.48) = 1.86 \pm 0.43h^{-1} \) Mpc (for \( q_0 = 0.5 \)) and \( r_0(0.48) = 2.16 \pm 0.49h^{-1} \) Mpc (for \( q_0 = 0.05 \)).
- This is consistent with local samples of normal, optically selected galaxies if clustering has grown in the Universe since \( z \sim 0.6 \). Evolution to CfA-like samples with \( r_0(0) \sim 5.5h^{-1} \) Mpc would require clustering evolution with \( \epsilon \sim +2 \), stronger than
that seen in CDM-like hierarchical models, while evolution to IRAS-like samples with $r_0(0) \sim 4h^{-1}$ Mpc would require evolution with $\epsilon \sim 0$, which can be obtained with clustering of fixed physical size. Clustering that is fixed in comoving space, i.e. $\epsilon \sim -1.2$, would result in a local sample with very weak clustering, $r_0(0) \sim 3h^{-1}$ Mpc.

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FIGURES

**Figure 1:** Galaxy and star counts for our data compared to the recent compilation of results in Metcalfe *et al.* (1991) shown as the dotted lines. Poisson errors are too small to be seen. Also shown are the stars determined from our star-galaxy separator for magnitudes $R < 22.0$ (starred symbols) and the predictions from the galactic model of Bahcall and Soneira ($x$’s).

**Figure 2:** Results for our star-galaxy separator for four different fields. Stars occupy the horizontal sequence which is well-defined until $R = 22.0$. This was used to estimate the stellar contamination for $R \leq 22.0$.

**Figure 3:** The $N(z)$ distribution inferred from the I-band selected CFRS for magnitude bins (of constant width 4.5 mag) above $R = 18.0, 18.5, 19.0$, etc., up to $R = 25.0$. The distribution used in the present work $R = 19.0 - 23.5$ is highlighted.

**Figure 4:** The angular correlation function from the counts-in-cells method for three magnitude ranges. Both the global (filled circles) and local (open circles) averaging methods are shown. The data is uncorrected for the integral constraint, any spurious field to field variations or for stellar contamination. The dotted line represents a slope of $\delta = -0.8$.

**Figure 5:** The observed correlation amplitudes scaled to $\theta = 1^\circ$ for this work (filled circles) along with several recent studies. A slope of $\delta = -0.8$ has been used for all points and stellar contamination has been corrected for. Three evolutionary models described in the text which reproduce our $\omega(\theta)$ at $R = 23.5$ are also shown These have combinations of $[r_0(0), \epsilon]$ of $[4.45,+1]$ (steepest), $[3.6,0]$ (middle) and $[2.75,-1.2]$ (shallowest). See text for discussion.
Figure 6: The correlation length for our sample at \( z = 0.6 \) \((q_0 = 0.5)\) compared to local results, with four values of the evolutionary parameter, \( \epsilon = -1.2, 0, 1, 2 \) (shallowest to steepest). Setting \( q_0 = 0.05 \) gives the open circle.
Table 1: Correlation function parameters

| mag. interval | $N_{obj}$ | $N_{galaxies}$ | $N_{gal}deg^{-2}$ | $f_g$ | $A_8^{0.8}$ | $\log A_{corr}^\omega$ |
|---------------|-----------|----------------|-------------------|------|------------|-------------------|
| $19.0 \leq R \leq 22.5$ | 6209 | 4153 | 14726 | 0.67 | 0.00128 $\pm$ 0.00017 | -2.54 $\pm$ 0.06 |
| $19.0 \leq R \leq 23.0$ | 8856 | 6323 | 22421 | 0.71 | 0.00105 $\pm$ 0.00011 | -2.68 $\pm$ 0.05 |
| $19.0 \leq R \leq 23.5$ | 12757 | 9710 | 34432 | 0.76 | 0.00080 $\pm$ 0.00014 | -2.86 $\pm$ 0.08 |

Table 2: Local Correlation Results

| Survey       | $z_{med}$ | $N_{galaxies}$ | $r_0(0)$  | $r_0(z_{med})$ | References                   |
|--------------|-----------|----------------|-----------|----------------|-------------------------------|
| CfA          | 2500 km s$^{-1}$ | 1230           | 5.4       | 5.3            | Davis and Peebles (1983)      |
|              | $z = 0.0083$ |                |           |                |                               |
| APM/Stromlo  | 15,200 km s$^{-1}$ | 1769           | 5.7       | 5.2            | Loveday et al. (1992)         |
|              | $z = 0.05$ |                |           |                |                               |
| Durham       | $\sim 13,000$ km s$^{-1}$ | 676            | 4.5       | 4.2            | Hale-Sutton et al. (1989)     |
|              | $z = 0.043$ |                |           |                |                               |
| KOS          | $\sim 8000$ km s$^{-1}$ | 164            | 4.0       | 3.8            | Kirschner et al. (1978)       |
|              | $z = 0.027$ |                |           |                |                               |
| IRAS (0.6Jy) | $\sim 5000$ km s$^{-1}$ | 9080           | 3.8       | 3.7            | Saunders et al. (1992)        |
| (QIGC)       | $z = 0.017$ |                |           |                |                               |
| IRAS (1.2Jy) | $\sim 5000$ km s$^{-1}$ | 5313           | 3.8       | 3.7            | Fisher et al. (1994)          |
|              | $z = 0.017$ |                |           |                |                               |
