Cavity soliton mobility in semiconductor microresonators above laser threshold

Reza Kheradmand, Mansour Eslami
Photonics Group, Research Institute for Applied Physics and Astronomy, Tabriz University, Tabriz, Iran
E-mail: r_kheradmand@tabrizu.ac.ir

Abstract. In this paper we investigated cavity soliton switching in a VCSEL type semiconductor laser which has been driven by homogeneous holding beam. Using quadratic fitting of the gain curve we studied the incoherent switching technique for the controlling of cavity solitons in a VCSEL-type laser and we show the cavity soliton spontaneous mobility in this type of injection.

1. Introduction
Cavity Solitons (CS), which are classified as localized structures, are isolated intensity peaks formed on a homogeneous dark background of radiation [1]. In optical systems that can exhibit bistability in some parameter range, one can find a region in which the homogeneous solution is present simultaneously with pattern solution. In this case the CS can be created by applying a transient Gaussian beam [2], or similarly by local carrier injection [3]. These bright spots are then self-sustained, in that they are stable once created by either method. Their ability to remain stationary in time and space, and the possibility of switching them on and off, make them a suitable candidate for applications in information technology. When making use of them in optical storage, they appear as a bit of information, and when carrying information processing tasks they appear as logic gates [4-6]. In account of being able to control the position and mobility, performance of these structures can be parallel. For instance, in the case of logic gates, it is possible to locate any number of CS arrays in which logical operations can be performed independently.

Theoretical prediction of CSs in optical resonators is mainly based on the interplay among dispersive/absorptive nonlinearities, paraxial diffraction, and dissipation/feedback [7]. In most cases, and ours as well, the energy of the cavity is supplied by an external plane-wave field. In addition, we make use of an electric current as the pump source to provide laser threshold, beyond which the dynamics of the system is explored. Experimentally, CSs have been realized both above and below laser threshold in VCSEL (Vertical Cavity Surface Emitting Laser) [8, 9].

In this paper, by inserting a parameter which represents the quadratic fitting of the gain curve, we proposed a more realistic description for these kinds of systems, besides; we were able to write CSs in this system by two methods, i.e. both coherent and incoherent. However, we observed a spontaneous motion of the CSs which has been analyzed due to its novelty.

In section 2, full description of the system and the governing dynamical equations are presented. Section 3.1, is devoted to the solutions of the system and instability analysis of them. In section 3.2, the results obtained from simulations will be considered, including the switching process and description of movement of the CS.

2. Model
Cavity Solitons are usually produced by means of optical resonators containing nonlinear medium. But as to miniaturization purposes and efficient use in circuits due to fast response, it is desired to use semiconductor materials as the nonlinear medium. The VCSEL type resonator which is considered in this paper contains GaAs/AlAs multiple quantum well nanostructure as the active medium which is shown schematically in figure 1. Bragg reflectors are used at both ends of the cavity and the energy is
provided to the system by a broad-area and stationary holding beam (HB) and also by an electric current.

**Figure 1.** Schematic illustration of the microresonator with multiple quantum-well nanostructure active material filling the cavity.

We used full set of Maxwell-Bloch equations to describe the dynamics of the system, however, what makes this system distinguished from previous models is the introduction of a parameter to take into account the quadratic fitting of the gain curve, $\beta$, in order to make the model more realistic. So, the full set of effective Maxwell-Bloch equations describing the system will be:

$$\begin{align}
\frac{\partial E}{\partial t} &= \sigma [P + E_t - (1 + i\theta)E + i\nabla^2E] \\
\frac{\partial D}{\partial t} &= -b[1/2 (E^*P + P^*E) + D - d\nabla^2D] \\
\frac{\partial P}{\partial t} &= \Gamma(1 + i\Delta)[(1 - i\alpha)D(1 - \beta D)E - P]
\end{align} \quad (1)$$

where $E, P$ are the slowly varying envelopes of electric field and of the effective macroscopic polarization, and $D$ is the population variable proportional to the excess of carries with respect to transparency. It has to be mentioned that time is scaled to the dephasing rate $\tau_d$ of microscopic dipoles. The decay rates are defined as \( \sigma = \tau_d/\tau_p \), \( b = \tau_d/\tau_c \) where $\tau_p, \tau_c$ are the photon life time and carrier recombination time, respectively. The difference between the cavity longitudinal mode frequency and of the injected field multiplied by $\tau_d$, detuning, is represented by $\theta$, which can be one of the important parameters. Amplitude of the injected field, holding beam, is denoted by $E_t$, and $J$ is the pump current which is normalized in such a way that the threshold for the system can be obtained from $J_{th} = \frac{1 - \sqrt{1 - 4\beta}}{2\beta}$, so for our model it will be $J_{th} = 1.171$ with $\beta = 0.125$. Finally $d$ is the diffusion coefficient for carriers.

The two real parameters, $\Gamma$ and $\Delta$, determines the shape of the effective susceptibility and assumed to depend on the population variable, that can be derived phenomenologically. Taking into consideration the quadratic fitting of the gain curve, we set $\Gamma(D) = \frac{2.308D + 1.206}{\sqrt{1 + \sigma^2}}$, $\Delta = -\alpha + \frac{2\beta}{\Gamma}$. $\Gamma(D)$ is related to the gain line width, $\delta(D)$ is the detuning between the reference frequency and the frequency where the gain is maximum. The values used for physical parameters throughout this paper are summarized in table.1.

| d  | $\alpha$ | $b$   | $\sigma$ | $\tau_d$ | $\tau_p$ |
|----|---------|-------|----------|----------|----------|
| 0.052 | 4 | $10^{-4}$ | $4 \times 10^{-2}$ | 100 fs | 2.5 ps |
Thus, the free parameters of the system will be the intensity of the HB, the pump parameter $J$ and the detuning $\theta$.

3. Results and discussion

3.1. Homogeneous solution and linear stability analysis

Homogeneous stationary solutions of the equations are obtainable by setting $\frac{\partial}{\partial t} = 0$ and $\nabla^2 = 0$, we arrive at:

$$|E_l|^2 = |E_s|^2\{(1 - Ds + \beta Ds^2)^2 + (\theta + aDs - a\beta Ds^2)^2\},$$  \hfill (2)

$$Ds = \frac{(1 + |Es|^2) - \sqrt{(1 + |Es|^2)^2 - 4|Es|^2\beta}}{2|Es|^2\beta}.$$  \hfill (3)

As can be seen in the figure 2, the homogeneous stationary solution exhibits bistability in some parameter region, and this allows the system to perform switching operations.

![Figure 2.](image)

Figure 2. Stationary intensity $|E_s|^2$ versus the injected intensity $|E_l|^2$, which exhibits bistability for some specific range of parameters. $J = 1.288$, $\theta = -2$, $\alpha = 4$.

The linear stability of the homogeneous stationary solution is analyzed by studying the response of the system to small fluctuations around the steady state. We carry linear stability analysis by adding an ansatz of the form below and determine the instability domains for Turing and Hopf instabilities.

$$\begin{aligned}
E &= E_s + \delta E_s \ e^{\lambda t - i(k_x x + k_y y)} \\
P &= P_s + \delta P_s \ e^{\lambda t - i(k_x x + k_y y)} \\
D &= Ds + \delta Ds \ e^{\lambda t - i(k_x x + k_y y)}
\end{aligned}$$  \hfill (4)
The set of equations which are obtained from applying such a fluctuation gets a nontrivial solution only if the eigenvalue $\lambda$ satisfies equation below:

$$\lambda^5 + a_4 \lambda^4 + a_3 \lambda^3 + a_2 \lambda^2 + a_1 \lambda + a_0 = 0$$

$\lambda$ in general is a complex number, however, we put Real part of $\lambda$ zero and separate Real and Imaginary parts of the resultant equation. Thus we obtain the Turing and Hopf instability conditions. The instability curves for pump current of about 10% above threshold are shown in figure 3, 4.

![Figure 3. Instability domain for the Turing instability, $K$ is the modulus of the transverse wave vector against which the stability of homogeneous solution is analyzed. $J = 1.288, \theta = -2, \alpha = 4.$](image1)

![Figure 4. Instability domain for the Hopf instability, $K$ is the modulus of the transverse wave vector against which the stability of homogeneous solution is analyzed. $J = 1.288, \theta = -2, \alpha = 4.$](image2)

### 3.2. Simulations

As it is apparent from the set of equations used for describing the system, they involve both time derivative and Laplacian terms. Thus it is needed to use a specific method of integration, named split-step method. We used split-step method with periodic boundary conditions which consists in separating the algebraic and the Laplacian terms on the right-hand part of the equations; the algebraic term is integrated using a Runge-Kutta algorithm, while for the Laplacian operator a fast Fourier transform is adopted. This implies that the number of points for each side of the grid must be a power of 2 and we mostly assumed a $64 \times 64$ grid.

The next step is to switch on a CS and then analysing its behaviour in transverse plane. To do this we have several alternatives including Coherent and Incoherent switching methods. In Coherent method one uses a Gaussian beam as the writing beam in phase with holding beam. In contrast, the Incoherent method uses local injection of carriers to enhance the light emission rate. In this regard we preferred incoherent switching technique to coherent one because of several reasons. First of all, this technique requires shorter time interval compared with coherent switching. Second, and more important one, is the advantage of possibility of switching on a CS with almost any energy able to generate carriers in that specific region in transverse plane.

Figure 4 shows a typical switching on of a CS with injection field intensity of about $|E_i|^2 = 0.6$. The process of injection started at time 4 tu and continued until 4.92 tu. This is done through using a Gaussian function of the form $E_{inj} = -e_{inj} \exp \left(-\frac{(i-i_{loc})^2 + (j-j_{loc})^2}{2\Re(w)^2}\right)$, where $e_{inj}$ is the amplitude, $i_{loc}, j_{loc}$ is the coordinate of injection point and $w$ is the width of injection. As can be seen in Figure 5, simulations show that the cavity soliton is created and is stable even in absence of injection process.
Figure 6 shows the bistability curve together with solitonic branch which is indicated by filled triangles.

![Bistability curve and solitonic branch](image)

**Figure 5.** Switching on a cavity soliton in presence of holding beam intensity of about $|E_1|^2=0.6$.

**Figure 6.** Bistability curve and solitonic branch for the same parameters as the previous graphs.

Of all the phenomena which are associated with CSs, mobility of them in transverse direction is of high interest. Cavity Solitons can have some instabilities, e.g. start moving, breathing, and oscillating. Moving of the CSs in some specific systems has been appointed to instabilities such as Hopf which, in the case of a system with thermal effects, arises as the result of presence of thermal effects [10]. For this system simulations show a spontaneous motion of CS along vertical direction with a velocity of about 0.442, in terms of time unit. Results implies a roughly periodic motion involving an interval of time in which the CS is stable in its position and then it undergoes a back and forth motion between its current position and the lower position which is due to the noise, and eventually it rests back on the lower position. This fact is shown in figure 7.

![Movement of CS along vertical direction](image)

**Figure 7.** Movement of CS along vertical direction in presence of noise. The inset shows the effect of noise which results in bouncing of CS between two places before coming a step down.
4. Conclusion

In this paper, first the electric field, carrier density and polarization equations have been investigated by quadratic fitting of the gain curve in semiconductor microresonator above laser threshold. The existence of cavity solitons in a driven vertical cavity semiconductor lasers has been demonstrated theoretically. We have studied the current injection technique for the control of a CS, which is called incoherent injection. We have analyzed the switching dynamics and its behaviour which have been observed. Furthermore, a spontaneous mobility in arbitrary direction has been found.

References

[1] S. Barland, J. R. Tredicce, M. Brambilla, L. A. Lugiato, S. Balle, M. Giudici, T. Maggipinto, L. Spinelli, G. Tissoni, T. Knoll, M. Miller, And R. Jager, “Cavity solitons as pixels in semiconductor microcavities,” Nature, vol. 419, pp. 699–702, London, 2002.

[2] X. Hachair, L. Furfaro, J. Javaloyes, M. Giudici, S. Balle, and J. Tredicce, “Cavity-solitons switching in semiconductor microcavities,” Phys. Rev. A 72,013815, 2005.

[3] S. Barbay, Y. Ménesguen, X. Hachair, L. Leroy, I. Sagnes, and R. Kuszelewicz, ‘Incoherent and coherent writing and erasure of cavity solitons in an optically pumped semiconductor amplifier,’ OPTICS LETTERS, Vol.31, No.10, 2006.

[4] D. Gomila, M. A. Matias, and P. Colet Phys. Rev. Lett. 94, p. 063905, 2005.

[5] D. Gomila, A. Jacobo, M. A. Matias, and P. Colet Phys. Rev. E 75, p.026217, 2007.

[6] A. Jacobo, D. Gomila, M. A. Matias, and P. Colet, Phys. Rev. A 78, 053821, 2008.

[7] X. Hachair, F. Pedaci, E. Caboche, S. Barland, M. Giudici, R. Tredicce, F. Prati, G. Tissoni, R. Kheradmand, L. A. Lugiato, I. Protsenko and M. Brambilla, IEEE J, vol 12, No 3, 2006.

[8] Barland, S., et al., ‘Cavity solitons as pixels in semiconductor microcavities,” Nature, Vol.419, 699–702, 2002.

[9] Hachair, X., et al., ‘Cavity solitons in a driven VCSEL above threshold,” Journal of Selected Topics in Quantum Electronics, Vol.12, 339-351, 2006.

[10] L. Spinelli, G. Tissoni, L. A. Lugiato and M. Brambilla, “Thermal effects and transverse structures in semiconductor microcavities with population inversion,” Phys. Rev. A 66, 023817, 2002.

[11] S. Barland, M. Brambilla, L. Columbo, L. Furfaro, M. Giudici, X. Hachair, R. Kheradmand, L. A. Lugiato, T. Maggipinto, G. Tissoni, and J. Tredicce, “Cavity solitons in a VCSEL: Reconfigurable micro pixel arrays,” Euro-phys. News, vol. 34, pp. 136–139, 2003.

[12] Giovanna Tissoni, Lorenzo Spirielli, Luigi A. Lugiato, Massimo Brambilla, ‘Thermal and electronic nonlinearities in semiconductor cavities,’ Proceedings of SPIE, Vol. 4283, 2001.

[13] S. Barbay, Y. Ménesguen, X. Hachair, L. Leroy, I. Sagnes, and R. Kuszelewicz, ‘Incoherent and coherent writing and erasure of cavity solitons in an optically pumped semiconductor amplifier OPTICSLETTERS, Vol.31, No.10, May15, 2006.

[14] S. Barbay and R. Kuszelewicz, ‘Physical model for the incoherent writing/erasure of cavity solitons in semiconductor optical amplifiers’ OPTICS EXPRESS, Vol. 15, No. 19, 2007.