Kinematic Method of Determining Safe Fairway Bend Widths

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ABSTRACT: This article presents a dedicated kinematic method of determining a safe fairway bend width with a specific turn angle and arc radius as the function of ship parameters and prevailing navigational conditions on the fairway. The assumed approach takes into consideration manoeuvring and navigational components of the safe fairway bend width. The method is based on an analysis of the results of numerical tests conducted on a model representing all physically possible movements of ship's centre of gravity in the bend. The developed method was initially verified on the Íńskie bend, part of the Świnoujście-Szczecin fairway.

1 INTRODUCTION

The dimensioning of bends in existing fairways for preset turn angles and arc radiuses comes down to the determination of their safe widths. Such problems are usually solved when the existing waterways are to be modernized to allow navigation by ships with larger parameters. This means the conditions of safe ship operation in a specific waterway are to be changed. Although in such situations the existing infrastructure of the area restricts the possibility of changing turn angles and radiuses of bend arcs, the problem can be solved using simulation or empirical methods.

Simulation methods in which the simulation experiment is conducted on full mission bridge simulators (FMBS) following specific research procedures [Gucma S. et al. 2015] enable the determination of bend widths with high accuracy. However, these methods are relatively cost-intensive due to the need to carry out many (reliable sample size) simulated passages in real time, executed by properly qualified fairway pilots [Gucma S. et al. 2008].

Empirical methods of determining safe bend widths are much less costly approximate methods. An analysis has been made to examine the accuracy of safe widths of five bends on the Świnoujście-Szczecin fairway, determined by using these methods:
- PIANC [PIANC 2014]
- Spanish [Puertos del Estado 2007]
- Japanese [Japan Institute of Navigation 2003]
- USACE [USACE 2006, USACE 2008]
- Canadian [Canadian Coast Guard 1999]
- Polish INM [Gucma S. 2001]
- Polish MTEC [Gucma S. et al. 2015]

The analysis has shown a relatively low accuracy of determining safe fairways widths depending on the parameters of the bend and of the manoeuvring ship [Gucma S. et al. 2017].

In empirical methods such as MTEC (Marine Traffic Engineering Center) and INM (Institute of Marine Navigation) the width of a safe manoeuvring area in bends has two components: manoeuvring and...
navigational. These components depend on the parameters of the bend and the ship manoeuvring therein, while they do not depend on the fairway coordinates of the bend. For a specific bend and ship, the components assume constant values, while the manoeuvring component is an empirically determined quantity, and the navigational component is probabilistic (ship positioning error in the bend).

As the simulation method is costly, and empirical methods are not accurate in the determination of safe bend widths for modernization or changed ship operational conditions, a kinematic method for the determination of the manoeuvring component of safe fairway bend widths with a known turn angle and bend arc radius has been developed. The method is based on an analysis of the results of numerical tests conducted on a model representing all physically possible movements of ship’s centre of gravity in the bend. The model is analytical, solved by numerical procedures [Dzwonkowski J. 2018; Dzwonkowski J., Przywarty M. 2017].

The dimensioning of bends in existing fairways for preset turn angles and arc radii comes down to the determination of their manoeuvring areas. The safe manoeuvring area in the bend must meet the basic condition of navigation:

\[
d_{a(i-\alpha)} \subseteq D_i(t) \cap h_{xy}(t) \geq T_k + \Delta a_{i(i-\alpha)}
\]

where
- \(d_{a(i-\alpha)}\) = safe area in i-th bend for k-th ship performing a simulated manoeuvre specified at the level of confidence 1-\(\alpha\);
- \(D_i(t)\) = navigable area of i-th bend (the condition of safe depth at instant \(t\) is satisfied);
- \(h_{xy}(t)\) = water depth at point (x, y) at instant \(t\);
- \(T_k\) = maximum draft of k-th ship;
- \(\Delta a_{i(i-\alpha)}\) = underkeel clearance in the i-th bend determined for k-th ship at the level of confidence 1-\(\alpha\).

In pilot navigation, ship’s position in the bend is determined in the path coordinates [Gucma S. et al. 2015]. In the kinematic method of determining safe bend widths the following fairway coordinate system is adopted:

- the x axis corresponds to the adopted centre line of the fairway;
- the y axis is perpendicular to the tangent of the fairway centre line at a given point. The adopted direction of y-axis outward is (+) and inward is (−).

Further in the article the the width of the safe manoeuvring area of the bend at j-th point of the fairway centre line at a specific confidence level will be denoted as \(d_{(1-\alpha)}(j)\).

In the fairway bend for one-way traffic the width of a safe area at the confidence level (1-\(\alpha\)) is determined by relation:

\[
d_{(1-\alpha)}(j) = d_{m(1-\alpha)}(j) + 2d_{n(1-\alpha)}
\]

where
- \(d_{(1-\alpha)}(j)\) = safe width for j-th point of the bend at the confidence level (1-\(\alpha\));
- \(d_{m(1-\alpha)}(j)\) = manoeuvring component of safe width for j-th point of the bend at the confidence level (1-\(\alpha\));
- \(d_{n(1-\alpha)}\) = navigational component of safe width of a given bend at the confidence level (1-\(\alpha\)).

The manoeuvring component of the safe manoeuvring area width is calculated using a specially developed kinematic method of determining the manoeuvring component of safe bend widths. In the developed method, the manoeuvring component of safe bend width is divided into a safe width of the manoeuvring area of ship’s centre of gravity in the bend and an additional margin for ship parameters and a drift angle:

\[
d_{m(1-\alpha)}(j) = d_{m(1-\alpha)}(j) + \Delta d_z + \Delta d_w
\]

where
- \(d_{m(1-\alpha)}(j)\) = width of the manoeuvring area of ship’s centre of gravity at j-th point of the bend at the level of confidence (1-\(\alpha\));
- \(\Delta d_z\) = additional margin of the manoeuvring component of the bend width taking into account ship’s parameters (Lc, B) and its drift angle on the external part of the bend;
- \(\Delta d_w\) = additional margin of the manoeuvring component of the bend width taking into account the ship’s parameters and its drift angle on the internal part of the bend.

The safe width of the manoeuvring area of ship’s centre of gravity in the bend is determined using the kinematic method presented in this article.

The additional margin of the manoeuvring component can be determined by either of the two methods:

- the drift angle method
  \[
  \Delta d_z = \frac{L}{2} \cdot \sin \alpha + \frac{B}{2} \cdot \cos \alpha
  \]
  \[
  \Delta d_w = \frac{B}{2}
  \]
- empirical method [Puertos Del Estado 2007]
  \[
  \Delta d_z = \frac{K^2 \cdot L^2}{2R}
  \]
  \[
  \Delta d_w = \frac{B}{2}
  \]

where
- B = breadth of the ship;
- \(\alpha\) = drift angle;
- R = radius of ship movement in the bend;
- K = coefficient dependent on the depth h to draft T ratio (for h/T ≤ 1.2 K = 1/2; for h/T ≥ 1.5 K = 2/3).
The navigational component of safe bend width is the directional error of the bow or stern position (point farther from the observer on the bridge) determined at a specified confidence level. It is the directional error perpendicular to the centre line of the fairway, equal to:

\[ d_{a(\alpha)} = p_{\beta D(1-\alpha)} = \sqrt{p_{\beta(1-\alpha)}^2 + \left( \frac{m_{KR(1-\alpha)} L_D}{57.3} \right)^2} \] [m]

where

- \( p_{\beta D(1-\alpha)} \) – directional error of ship’s bow at the confidence level \((1-\alpha)\) [m];
- \( p_{\beta(1-\alpha)} \) – directional error of ship’s position determination (observer’s position) at the confidence level \((1-\alpha)\) [m];
- \( m_{KR(1-\alpha)} \) – heading determination error in a bend at the confidence level \((1-\alpha)\) [°].

For large ships \((Lc \geq 150 \text{ m})\) with the superstructure aft it is assumed [Gucma S. et al. 2015] that:
- \( m_{KR(0.95)} = \pm 2° \) (Pilot Navigation System)
- \( m_{KR(0.95)} = \pm 4° \) (other position determination systems)
- LD=0.75 \( Lc \)

For various methods of ship position determination, types of fairway (inner or outer) and the aids to navigation (marked fairway or not, types of seakmarks), directional errors of the bow/stern position \( p_{\beta D(1-\alpha)} \) are determined using dedicated algorithms [Gucma S. et al. 2017].

2 THE KINEMATIC METHOD OF DETERMINING THE MANOEUVRING COMPONENT OF FAIRWAY SAFE BEND WIDTHS.

The kinematic method for determining the manoeuvring component of safe areas in a fairway bend makes use of a ship movement model, whose concept requires that multiple simulations are made of the ship’s centre of gravity passage through the bend, divided longitudinally into sectors and transversely into segments. The model represents the entire physically possible ship movement in the bend. Paths of ship’s centre of gravity consist of, numerically calculated in each sector, circle arcs or sections that are further regarded as separate manoeuvring events (Fig. 1). Thus created sets of manoeuvring events are subject to further analysis, searching for the distribution of movement density and the tolerance of rudder settings applied [Dzwońkowski J. 2018].

The method requires the introduction of the following assumptions and definitions:

1. The research is retrospective and starts from the position, in which the ship’s centre of gravity completes negotiating the bend, which in the analysis of the paths facilitates the achievement of the goal.

2. The movement model in the method refers to the centre of gravity, which results in the following:
   - width of ship’s swept path differs from the width of the path followed by the centre of gravity by an additional value depending on drift and ship’s length and breadth,
   - rate of turn (ROT) of the centre of gravity is the angular speed of movement along an arc of circle radius \( r \).

3. Internal and external boundaries of the fairway are defined in the model by arcs of concentric circles. The fairway boundary line may in reality have another shape, but it has to lie outside the arcs. The problem considered herein refers to a part of the fairway, a single bend (turn), whose boundaries can be defined by arcs of two concentric circles.

4. For sets of events to describe the entire physically possible movement and the movement model to be similar to reality, the arcs along which the ship’s centre of gravity moves have to result from the maximum rudder settings the captain or pilot use. In addition, the ship will not turn in the direction opposite to the one followed, therefore the centre of gravity ROT0 = 0. Besides, for the movement in the bend to be considered as safe, the maximum rudder angle has to enable the same change in the position of the centre of gravity from the axis towards the external and internal boundaries. Hence, ROT2 is twice greater than ROT1.

5. The other requirement for a set of manoeuvring events to describe the whole physically possible movement is to determine the magnitude of the sector so that it corresponds to a minimum time of making decisions concerning rudder settings [PRS 2013]]. The time was defined as 10 seconds, which corresponds to classification society requirements for the rudder to move from ‘midships’ to the recommended [PIANC 2015] maximum value of rudder angle in a bend, i.e. 20°. The reduction of sector length is restricted by the assumed time of numerical computing, because the number of arcs in a sector increases at least 2.5 times, so that in sector 13, for instance, 100,000 arcs have to be calculated.

The input data for the movement model are: the position and the course over ground at the end of the
bend, outside and inside fairway boundaries, sector and segment sizes, longitudinal speed and ROT.

The outcome of calculations using the kinematic model are different sets of arcs (manoeuvring events) in each sector. The proposed method for determining a safe manoeuvring area for the tested ship in the bend of known arc radius uses the following sets (Fig. 2):
- successful manoeuvring events (SME), i.e. those that do not go outside the fairway,
- non-rectilinear manoeuvring events, i.e. all those, whose ROT is different from 0.

![Figure 2. Distributions of manoeuvring events in a sector.](image)

Along the external part of the bend where the ship’s centre of gravity can move 95% of the field under the curve of SME distribution defines the external boundary of the safe manoeuvring area of the ship’s gravity centre at the confidence level \((1-\alpha) = 0.95\).

Along the internal part of the bend that the ship’s centre of gravity can move the boundary of the safe manoeuvring area is determined by the ratio of non-rectilinear events to SME in each segment. The segment in which the ship’s centre of gravity moves non-rectilinearly at 95% probability makes up an internal border of the safe manoeuvring area of the ship’s centre of gravity, determined at the confidence level \((1-\alpha) = 0.95\).

The safe manoeuvring area of the ship’s centre of gravity in a bend at the level of confidence \((1-\alpha) = 0.95\) is determined by conducting numerical research and an analysis of the results for four cases, i.e. for both directions of passing through the bend and for two positions of the end of the turn. These positions are defined by the boundaries of the manoeuvring areas of adjacent straight sections of the fairway determined at the confidence level \((1-\alpha) = 0.95\).

The safe manoeuvring area of ship’s centre of gravity in the bend is determined by summing up four component manoeuvring areas of the ship’s centre of gravity calculated at the level of confidence \((1-\alpha) = 0.95\) (Fig. 3):

\[
d_{\text{ma}(1-\alpha)} = d_{pz(1-\alpha)} \cdot d_{pn(1-\alpha)} \cdot d_{\zeta(1-\alpha)} \cdot d_{\nu w(1-\alpha)}
\]

where
- \(d_{pz(1-\alpha)}\) – manoeuvring area of the gravity centre of a ship proceeding in the bend ‘to the right’, completing the manoeuvre at a point of external boundary (Z1) of the adjacent manoeuvring area of a straight fairway section, \((1-\alpha) = 0.95\);
- \(d_{pn(1-\alpha)}\) – manoeuvring area of the gravity centre of a ship proceeding in the bend ‘to the right’, completing the manoeuvre at a point of internal boundary (W1) of the adjacent manoeuvring area of a straight fairway section, \((1-\alpha) = 0.95\);
- \(d_{\zeta(1-\alpha)}\) – manoeuvring area of the gravity centre of a ship proceeding in the bend ‘to the left’, completing the manoeuvre at a point of external boundary (Z2) of the adjacent manoeuvring area of a straight fairway section, \((1-\alpha) = 0.95\);
- \(d_{\nu w(1-\alpha)}\) – manoeuvring area of the gravity centre of a ship proceeding in the bend ‘to the left’, completing the manoeuvre at a point of internal boundary (W2) of the adjacent manoeuvring area of a straight fairway section, \((1-\alpha) = 0.95\);

![Figure 3. The diagram of manoeuvring areas of the gravity centre of a ship proceeding in the bend to the right \(d_{\text{ma}(1-\alpha)}\) and \(d_{\text{pw}(1-\alpha)}\).](image)

Taking into account the movement of a specific ship along the external and internal boundaries of the bend, the component manoeuvring areas allow to determine a safe manoeuvring area of the ship’s center of gravity within this bend at the assumed confidence level \((1-\alpha) = 0.95\) (Fig. 4).

With additional margins of the manoeuvring component of the bend width for ship parameters and drift \((\Delta z\) and \(\Delta w\)), we determine the safe manoeuvring area of the bend, and after summing up the navigational components we obtain the safe area of the bend for the examined ship. On this basis we can determine the safe width at j-th point of the bend at an appropriate level of confidence \(d_{\text{ma}}(j)\). It should be noted at this point that the determination of a safe area of the bend by the kinematic method is done at the confidence level \(1-\alpha = 0.95\) for a maximum ship expected to sail through the given fairway.

The allowable wind speed expected in the conditions of safe fairway operation is accounted for by choosing a specific drift angle while defining margins \(\Delta z\) and \(\Delta w\).
3 SHIP RATE OF TURN IN FAIRWAY BENDS.

The rate of turn, i.e. angular speed at which ships negotiate fairway bends depends primarily on the parameters of the bend, although other factors come into play:
- ship size and manoeuvring characteristics,
- rudder angle,
- initial longitudinal speed of the ship,
- external conditions (wind, waves, current).

The presented method allows the determination of the ROT in the fairway bend for ships of different sizes and for different input parameters. To simplify the method and extend its universality it was assumed that ship’s ROT in the bend depends on ship length and block coefficient of the hull. Additionally, the impact of initial longitudinal speed and rudder angle were taken into account. The reduction of ROT due to shallow water was regarded as negligibly small and was not taken into account [Nowicki A. 1999]. Due to the variability of external conditions (wind, waves, current) it was decided not to take them into account in the determination of ship’s ROT in the bend. The ship’s ROT can be calculated using the following relationship:

\[ ROT = ROT_{NOM} + \Delta ROT_{SOG} + \Delta ROT_a \]

where
- \( ROT \) – ship’s ROT in fairway bend;
- \( ROT_{NOM} \) – nominal ROT determined from manoeuvring data;
- \( \Delta ROT_{SOG} \) – change in ROT due to other than nominal initial longitudinal speed of the ship;
- \( \Delta ROT_a \) – change in ROT due to the change in rudder angle.

The impact of selected factors on ship’s ROT in the bend was defined upon detailed review of some publications [Elcott K. et al. 2018; Fossen T. 2011; Nowicki A. 1999] and gathered manoeuvring data from ships of various types and sizes. First, the relationship was determined between the ship size (length) and ROT read out from available manoeuvring data for a 90 degree turn (Fig. 5). To assess the dependence between chosen factors polynomial regression was used. The analysis was made for ships with various sizes, different loading state and for longitudinal speeds corresponding to full speed ahead, at maximum rudder angle (35°-45°). The ships were divided into two groups by block coefficient criterion: large (Cb>0.75) and small (Cb<0.75). Nominal ROT values for ships of various sizes and block coefficients were determined through an analysis of the gathered data (Table 1).

![Figure 4. A safe manoeuvring area of ship’s centre of gravity in the bend.](image)

![Figure 5. ROT of ships of different lengths and block coefficients.](image)

| ship length [m] | \( ROT_{NOM} \) [deg/min] \( C_b \geq 0.75 \) | \( C_b < 0.75 \) |
|-----------------|---------------------------------|-----------------|
| 60              | 101.362                         | 108.722         |
| 80              | 91.626                          | 103.846         |
| 100             | 82.61                           | 99.05           |
| 120             | 74.314                          | 94.334          |
| 140             | 66.738                          | 89.698          |
| 160             | 59.882                          | 85.142          |
| 180             | 53.746                          | 80.666          |
| 200             | 48.33                           | 76.27           |
| 220             | 43.634                          | 71.954          |
| 240             | 39.658                          | 67.718          |
| 260             | 36.402                          | 63.562          |
| 280             | 33.866                          | 59.486          |
| 300             | 32.05                           | 55.49           |

A ROT change that results from other speed than the nominal initial longitudinal speed of the ship was calculated from gathered manoeuvring data and literature review [Przywarty M. et al. 2013]. It was found that no significant differences exist for ships with large and small block coefficients, so the analysis was combined for both ship groups (Fig. 6).
Figure 6. Change in ROT due to a change in ship’s initial longitudinal speed.

Based on an analysis of the results it was assumed that the percentage change of ROT is equal to the percentage change of initial longitudinal speed. It can therefore be calculated by using the following relationship:

$$\Delta ROT_{SOG} = \left[ \frac{SOG - SOG_{NOM}}{SOG_{NOM}} \right] \cdot ROT_{NOM}$$

where

- $\Delta ROT_{SOG}$ – change in ROT due to speed other than the initial longitudinal speed adopted for the determination of ROT$_{NOM}$;
- $SOG$ – initial longitudinal speed of the ship;
- $SOG_{NOM}$ – initial longitudinal speed adopted for the determination of ROT$_{NOM}$;
- ROT$_{NOM}$ – nominal ROT.

The additional decrease in ROT due to less than the maximum rudder angle can be calculated thus:

$$\Delta ROT_\alpha = -k \left( ROT_{NOM} + \Delta ROT_{SOG} \right)$$

The values of the coefficient $k$ determined on the basis of literature review [Fossen T. 2011]; [Nowicki A. 1999] are presented in Table 2.

Table 2. Value of the coefficient $k$ for different rudder angles.

| Rudder angle [deg] | k   |
|-------------------|-----|
| >=35              | 0   |
| 30                | 0.09|
| 25                | 0.24|
| 20                | 0.29|
| 15                | 0.39|
| 10                | 0.5 |
| 5                 | 0.57|

The determination of safe widths in the Irskie bend, in the currently modernized Swinoujście–Szczecin fairway, for a bulk carrier $L_c = 195\text{ m}, B = 29.0\text{ m}, T = 11.0\text{ m}$; lateral windage 1200 m2; power of main engine 8,500 kW.

The developed kinematic method was used to determine safe widths (safe area) of the Irskie bend, in the currently modernized Swinoujście–Szczecin fairway, for a bulk carrier $L_c = 195\text{ m}, B = 29.0\text{ m}, T = 11.0\text{ m}$ (Fig. 7). The figure also depicts safe widths in the Irskie bend determined by the computer simulation method and the deterministic-probabilistic MTEC method [Analiza ... 2015].

The following conclusions can be drawn from the results:

1 Safe widths of the bend determined by a simulated experiment conducted on a Kongsberg-made full-mission bridge simulator at the Maritime University of Szczecin, with a participation of highly qualified pilots, who executed a reliable number of bend passages, can be regarded as model ones due to their high accuracy.
2 The kinematic method, like the simulation method, estimates the safe bend width as a function of the turn angle. The MTEC method determines a safe bend width as a constant quantity.

3 The kinematic method overestimates safe bend width by roughly 30% compared to the simulation method, which is related to the human factor involved in the simulation method.

4 The kinematic method is more accurate than the MTEC method by approximately 40%.

Given the moderate costs and high accuracy of the kinematic method it should be recommended for use in the preliminary design of marine traffic engineering systems.

REFERENCES

Analiza nawiągacyjna modernizacji toru wodnego Świnoujście – Szczecin (pogłębienie do 12,5m), Praca naukowo-badawcza zlecona przez Europrojekt Gdańsk S.A. Akademia Morska w Szczecinie 2015.

Canadian Coast Guard (1999): Canadian Waterways National Maneuvering Guidelines – Channel Design Parameters, Waterways development, marine navigation services, Ottawa.

Dzwonkowski J. (2018): Kinematyczna metoda określenia bezpiecznych parametrów zakłóń na torach wodnych. Rozprawa doktorska Wydział Nawiągacyjny, Akademia Morska w Szczecinie.

Dzwonkowski J., Przywarty M. (2017): Analysis of vessel traffic flows on a waterway bend. Scientific Journals of the Maritime University of Szczecin, no. 50(122)

Eloit K., Delefortrie G., Mostaert F. (2018): Maneuvering characteristics: Sub report 7 – Comparison of the Maneuvering Characteristics of the COSCO 20.000 TEU and other Ultra Large Container Ships, Flanders hydraulics Research, Antwerp.

Fossen T. (2011): Handbook of marine craft hydrodynamics and motion control, John Wiley & Sons Ltd.

Gucma S. (2001): Inżynieria ruchu morskiego. i Żeglugi, Gdańsk.

Gucma S. (2015): Optymalizacja parametrów systemu morskich dróg wodnych pogłębionych do zadanej głębokości na przykładzie przebudowy toru wodnego Świnoujście – Szczecin. Archives of Transport No 4/2016, Vol. 40, pp.29-38.

Gucma S. i inni (2015): Morskie drogi wodne – projektowanie i eksploatacja w ujęciu inżynierii ruchu morskiego. Fundacja Promocji Przemyślu Okrętowego i Gospodarki Morskiej, Gdańsk.

Gucma S. i inni (2017): Inżynieria Ruchu Morskiego – wytyczne do projektowania morskich dróg wodnych i portów oraz warunków ich bezpiecznej eksploatacji. Fundacja Promocji Przemyślu Okrętowego i Gospodarki Morskiej, Gdańsk.

Gucma S., Gucma L., Zalewski P. (2008): Symulacyjne metody badań w inżynierii ruchu morskiego. Monografia pod redakcją Stanisława Gucmy. Wydawnictwo Naukowe Akademii Morskiej w Szczecinie.

Japan Institute of Navigation (2003): Design Standard for Fairway in Next Generation, Japan Institute of Navigation (Standard committee), Ministry of Land, Infrastructure and Transport (National Institute for Land an Infrastructure Management, Port and Harbor Department), Tokyo.

Nowicki A. (1999): Wiedza o manewrowaniu statkami morskimi, Wydawnictwo Trademar, Gdynia.

PIANC (2014): Setting the Course, Harbour Approach Channels – Design Guidelines, Report n° 121 – 2014, Groupa PIANC i IAPH, w kooperacji z IMPA i IALA, Bruksela.

PIANC (2015): Workshop on Design Guidelines For Inland Waterways (PIANC INCOM WG 141) Smart Rivers 2015, Bundesanstalt fur Wassbau, PIANC.

PRS (2013): Przepisy Klasifikacji I Budowy Statków Morskich Część III Wyposażenie Kadłubowe, Polski Rejestr Statków, Gdańsk.

Przywarty M., Gucma L., Perkovic M. (2013): Influence of speed reduction on navigational safety of container ships, Scientific Journals of the Maritime University of Szczecin, no. 36 / 2013.
Puertos Del Estado (2007): ROM 3.1-99, Design of the Maritime Configuration of Ports, Approach Channels and Harbour Basins, Spanish National Ports & Harbours Authority, Madrid.

USACE (2006): Hydraulic Design of Deep-Draft Navigation Projects, Department of the Army, US Army Corps of Engineers, Washington DC.

USACE (2008): Coastal engineering manual - Part V, chapter 5: Navigation projects, Department of the Army, US Army Corps of Engineers, Washington DC.