Asymptotically Free Non-Abelian Gauge Theories
With Fermions and Scalars As Alternatives to QCD

N.D. Hari Dass†
Institute of Mathematical Sciences, C.I.T Campus, Chennai 600 113, INDIA

V. Soni‡
National Physical Laboratory, New Delhi, INDIA

Abstract
In this paper we construct non-Abelian gauge theories with fermions and
scalars that nevertheless possess asymptotic freedom. The scalars are taken
to be in a chiral multiplet transforming as \((2, 2)\) under \(SU(2)_L \otimes SU(2)_R\)
and transforming as singlets under the colour \(SU(3)\) group. We consider
two distinct scenarios, one in which the additional scalars are light and an-
other in which they are heavier than half the Z-boson mass. It is shown that
asymptotic freedom is obtained without requiring that all additional cou-
plings keep fixed ratios with each other. It is also shown that both scenarios
can not be ruled out by what are considered standard tests of QCD like R-
parameter, \(g-2\) for muons or deep inelastic phenomena. The light mass sce-
nario is however ruled out by high precision Z-width data (and only by that
one data). The heavy mass scenario is still viable and is shown to naturally
pass the test of flavour changing neutral currents. It also is not ruled out
by precision electroweak oblique parameters. Many distinctive experimental
signatures of these models are also discussed.

1 Introduction
Quantum chromodynamics (QCD) is today believed to be the correct theory
of strong interactions. The reasons for this belief are, on the one hand, the
fact that the global symmetries of QCD and that of the observed hadronic
world are the same, and on the other hand, the phenomenon of asymptotic
freedom (AF). Indeed, asymptotic freedom (AF) allows one to make experimental comparisons with the predictions of QCD in the deep inelastic regime and the evidence in favour of QCD appears to be quite compelling. Nevertheless, it is extremely important to analyse the experimental data within a broader theoretical framework that contains QCD as a special case.

The need for a framework to analyse data that is larger than the theory one is trying to uphold is fairly obvious. Otherwise the data analysis is likely to be biased by elements of the theory itself. Of course, no analysis of the data is possible without some theoretical framework. But such a theoretical framework should be as broad as possible. For example, analysis of gravitational phenomena in the framework of Brans-Dicke theory, parametrised by an additional parameter $\omega$ over and above those of General Theory of Relativity (GTR), leading to very large values of $\omega$ (GTR is the $\omega \to \infty$ limit) is a superior vindication of GTR than the one done entirely within the framework of GTR. Speaking in the modern language of Renormalisation Group and Fixed Points, several RG-trajectories could, for example, be flowing towards the same infra-red fixed point and in the vicinity of that fixed point several flows could be practically indistinguishable (this is illustrated in fig. 1). A wider analysis of the data would be in terms of the universality class to which the fixed point belongs while an analysis based on a particular theory would correspond to one of the many trajectories flowing into the fixed point.

Such a broader framework must necessarily possess the property of AF without which it would be impossible to make any credible statements about deep inelastic phenomena. Also, on theoretical grounds it is believed today that only those theories possessing AF can be consistent. While the relationship between AF and scaling is not straightforward, as argued by Gross, non-AF theories are unlikely to lead to scaling. From the works of Gross and Coleman it is well known that non-Abelian gauge fields are

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1 Seiler and Patrasciou have, however, presented a dissenting perspective on Asymptotic Freedom and reliability of perturbative methods in non-abelian gauge theories

2 In a pragmatic sense one could argue that since we do not believe even theories like QCD or the Standard Model to be valid at arbitrarily large energy scales, the issue of whether a theory should possess AF is a moot one though the presence of AF in the sense of a region where couplings are small and get smaller with scale, would be important from a phenomenological point of view. Even such a pragmatic view would perhaps admit that the "ultimate theory" would possess AF.
indispensable, at least in four dimensions, for realising AF. It is also by now well known that the matter content of gauge theories has important repercussions for the survival of AF. For example, for SU(3) colour gauge theories, more than 16 families of quarks in the fundamental representation of the gauge group would spoil AF. However, relatively less is known about the influence of scalar fields on AF. In some specific cases, depending on the choice of the scalar representation AF could be obtained by fine-tuning the scalar self-coupling, but did not occur naturally as in QCD. To paraphrase G. t’Hooft: ”In gauge theories with fermions and scalars the situation is complicated. If one imposes the condition that all beta functions come out negative one usually finds that all coupling constants should keep fixed ratios with each other”[5]. However, in this paper we show that the situation need not be as restrictive as that.

In this paper we consider alternatives to QCD by enlarging the matter sector to include elementary scalar fields. We are using the phrase ”matter fields” in a generalised sense to include all fields that couple directly to quarks or to gluons. One may argue that what we have considered here are really alternatives to the Standard Model. We prefer to view our attempts as alternatives to QCD because the symmetry structure of the extended sector is $SU(2)_L \otimes SU(2)_R$. Of course inclusion of the $U(1)$-sector of electroweak interactions eventually reduces the $SU(2)_R$ part of this symmetry.

We restrict our attention to cases where the additional scalars are colour singlets. This restriction can obviously be removed and an even larger class of theories studied. We postpone such an analysis to the future. Of course, if the scalars were not colour singlets, even the one loop $\beta$-function for the QCD coupling constant would change and it is doubtful whether the nice features obtained here would survive. In fact it is believed that in QCD it will be hard to add scalars (coupled to gluons) while keeping AF because the quarks are in the fundamental representation and that is the reason why fundamental scalars that couple to the strong gauge group can not exist[6].

This choice of the colour singlet scalars was strongly motivated for us by the fact that the Gell-Mann-Levy linear $\sigma$-model with quarks substituted for nucleons has been found to provide a very reasonable description of the nucleon as well as of the strong interactions at finite temperature and density. While the linear model was not asymptotically free and could therefore be considered only as a low energy effective theory, the tantalising new question that emerges is that of the AF of the alternative theories with a chiral multi-
plet of scalar fields. It was established in [6] that such a model can indeed be AF in certain regions of the parameter space. What was however not clear at that time was the relation of this model to QCD itself; even the possibility that this theory is indistinguishable from QCD could not be ruled out.

In the present work we consider this issue at length by first establishing that there are indeed classes of such theories that are asymptotically free in all the additional couplings and that AF is stable against inclusion of the electro-weak sector $^3$ and that not all of them are equivalent. We then proceed to confront these classes with existing experimental data to see their viability. Our analysis shows that for deep inelastic scattering the leading behaviour of this theory in the ultra violet is identical to that of QCD. This is so as the Yukawa coupling and the scalar self coupling go to zero faster than the QCD coupling. In fact, the numerical factors that arise are such as to make even the subleading QCD corrections to be marginally larger than the contributions of these extra couplings. This is important as some high energy experiments are already sensitive to these subleading corrections.

We further find that the usual strong interaction tests for QCD based on the properties of quarks and gluons will all go through for this theory. The other set of tests for the particle content of QCD include the precision measurements like the R parameter and the g-2 for the muon. Here we will have extra contributions from our new scalar color singlet partons which couple to the photon. However, we find both these precision tests cannot confidently rule out our theory in favour of QCD.

A careful analysis of the flavour changing neutral currents of our model reveals more or less uniform coupling to all flavours if significant flavour changing neutral currents are to be avoided. In contrast, extensions of the standard model available in the literature like ‘two Higgs Doublet models’ (THDM), supersymmetric extensions of standard model like SSM, MSSM etc generically predict dominant coupling to heavy flavours as well as coupling to associated leptons. Our model naturally avoids coupling of the chiral scalars with leptons.

The most important conclusions we have reached are that while the alternatives with massless or nearly massless chiral multiplets are ruled out by $^3$This is of course true only when the U(1) couplings are ignored. Also, the Higgs-Yukawa and the Higgs self-coupling of the standard model violate AF. These features are not altered in our alternative models.
high precision Z-width data[4] (interestingly they are ruled out at present only by this one observable), those with suitably massive chiral multiplets can not be ruled out by current experimental data. We then propose a variety of experimental tests for such extended models that have hitherto not been performed.

In such a theories, a distinctive signal will be the appearance of an excess of four jet events in $e^+e^-$ - collisions over what is to be expected from the standard model. Such four jet signals arise as the massive scalars eventually decay into $\bar{q}q$ pairs.

Though the ALEPH collaboration [20, 21, 22] had reported seeing such an excess of four jet events in $e^+e^-$ collisions at $\sqrt{s} = 130,136,161$ and 172 Gev respectively, they have subsequently attributed the excess to fluctuations and do not see any excess in the later runs. Nevertheless, it is intertesting that so many features of the additional four jet events that follow naturally and in a parameter-free manner in our theories[8] were reported by ALEPH in their earlier claims. The disappearance of the ALEPH events does not repudiate our model. It only raises the limit on the mass of the scalars which is also consistent with other precision searches for additional heavy particles. The present lower limit on the masses of such particles is around 100 GeV.

The precision tests of the standard model based on the so called oblique parameters S,T&U [24, 25, 26] also do not rule out the extensions considered here.

## 2 The Model

Our model is described by the lagrangian

\begin{equation}
\mathcal{L} = - \frac{1}{2} (\partial _\mu \sigma )^2 - \frac{1}{2} (\partial _\mu \vec{\pi})^2 - \lambda ^2 (\sigma ^2 + \vec{\pi}^2 - f_{\pi}^2)^2 \\
- \overline{\Psi}_q [D_\mu + g_y (\sigma + i \gamma_5 \vec{\tau} \cdot \vec{\pi})] \Psi_q - \frac{1}{4} G_{\mu \nu}^a G^{\mu \nu a}
\end{equation}

where $D_\mu = \partial _\mu - ig_3 A^a_\mu T^a$ and $G_{\mu \nu}^a = \partial _\mu A^a_\nu - \partial _\nu A^a_\mu + g_3 f_{abc} A^b_\mu A^c_\nu$. $A^a_\mu$ is the gluon field and $T^a$ the SU(3) generator in the fundamental representation. $g_y, g_3$ and $\lambda$ are the Yukawa, QCD, and scalar self- couplings respectively. $\overline{\Psi}_q$ is the quark field.

The $\sigma, \vec{\pi}$ in eqn (1) should not be confused with the chiral multiplet occurring in the low lying spectrum of strongly interacting particles containing the
pion \((m_\pi \approx 140\text{MeV})\) though they have the same quantum numbers. While the latter are bound states the \(\sigma, \vec{\pi}\) of eqn (1) are elementary, to be thought of as additional "partons" of the model. The important property of eqn (1) is that it is invariant under the Global \(SU(2)_L \otimes SU(2)_R\) transformations that affect the quark fields and chiral multiplet. Introducing

\[
U = \sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi}
\]

(2)

these transformations are given by

\[
\begin{align*}
\psi'_L &= g_L \psi_L \\
\psi'_R &= g_R \psi_L \\
U' &= g_L U g_R^\dagger
\end{align*}
\]

(3)

It is further assumed that in the extended sector there is no spontaneous breaking of \(SU(2)_L \otimes SU(2)_R\).

As shown in [7], a model described by eqn(1) automatically implies that the chiral multiplet couples to the electroweak gauge bosons. Representing the chiral multiplet as a complex doublet

\[
\Phi_{ch}^T = (\sigma + i\pi_0, i\sqrt{2}\pi_-)
\]

(4)

The coupling of the chiral multiplet to the electroweak gauge bosons is given by the minimal coupling

\[
\mathcal{L}_{\text{gauge, chiral}} = \frac{1}{2} |(\partial_\mu \Phi_{ch} - i\gamma_5 \vec{\tau} \cdot \vec{A}_\mu \Phi_{ch} - i\frac{g'}{2} Y B_\mu \Phi_{ch})|^2
\]

(5)

with \(Y = -1\) and \(g, g'\) be the \(SU(2)_L\) and \(U(1)\) couplings respectively. For example, the linear couplings that result are

\[
\begin{align*}
\mathcal{L}_{\text{lin}}^{\text{neut}} &= e(A_\mu - \frac{\gamma}{2} Z_\mu)(\vec{\pi} \times \partial_\mu \vec{\pi})_3 - \frac{e}{2cs} Z_\mu (\pi_0 \partial_\mu \sigma - \sigma \partial_\mu \pi_0) \\
\mathcal{L}_{\text{lin}}^{\text{char}} &= \frac{g}{2} W^\mu [(\pi_+ \partial_\mu \sigma - \sigma \partial_\mu \pi_+) + i(\pi_+ \partial_\mu \pi_0 - \pi_0 \partial_\mu \pi_+)] + c.c
\end{align*}
\]

(6)

where \(\gamma = (1 - 2s^2)/cs\) and \(c, s\) are \(\cos \theta_W, \sin \theta_W\) respectively. Likewise, the quadratic gauge couplings are given by

\[
\mathcal{L}_{\text{quad}} = \frac{e^2}{8s^2c^2} Z_\mu Z^\mu (\sigma^2 + \pi_0^2) + \frac{g^2}{2} W^+ W^- \pi_+ \pi_-
\]
\[\begin{align*}
+ i \frac{eg}{4sc} & \left[ Z^\mu W^-_\mu (\sigma + i\pi_0) \pi_+ - Z^\mu W^+_\mu (\sigma - i\pi_0) \pi_- \right] \\
+ & \frac{g^2}{4} W^-_\mu W^{+\mu} (\sigma^2 + \pi_0^2) \\
+ & e^2 (A_\mu A^\mu + \gamma^2/4 Z_\mu Z^\mu - \gamma A_\mu Z^\mu) \pi_+ \pi_- \\
+ & ieg/2 (A_\mu - \gamma/2 Z_\mu) W^-\mu (\sigma + i\pi_0) \pi_+ \\
- & ieg/2 (A_\mu - \gamma/2 Z_\mu) W^+\mu (\sigma - i\pi_0) \pi_- 
\end{align*}\]

(7)

2.1 Higgs Sector

Since the additional multiplet has to be massive, it is economical to have their masses generated by the Higgs mechanism. The simplest way to do this is by introducing the coupling

\[\mathcal{L} = \frac{\bar{\lambda}}{2} |\Phi|^2 (\sigma^2 + \vec{\pi}^2)\]  

(8)

Then \(\lambda\) is fixed to be

\[\bar{\lambda} = \frac{m_c^2}{v^2}\]  

(9)

where \(m_c\) is the chiral multiplet mass and \(v\) the vacuum expectation value of the Higgs field. Now, in the broken phase, the physical Higgs field couples to \(\sigma\) and \(\vec{\pi}\) according to

\[\mathcal{L} = \frac{m_c^2}{v} H (\sigma^2 + \vec{\pi}^2)\]  

(10)

Note that both eqn (1) and eqn (10) are invariant under \(SU(2)_L \otimes SU(2)_R\) of eqn (3). If the Higgs is not too light but lighter than the top quark, this channel would be the most dominant mode for Higgs to decay and would open a new window into Higgs search. This additional coupling to Higgs raises new issues; for example, the renormalisation group eqns for the flow of both the Yukawa coupling \(g_y\) and the scalar self coupling \(\lambda\) will now change due to the additional coupling \(\bar{\lambda}\). More importantly, the renormalisation group flow for the Higgs coupling parameters of the standard model will change due to the coupling of the Higgs to the chiral multiplet considered here. These issues will be addressed elsewhere. It also raises the issue of radiative mixing between the chiral multiplet and Higgs particle, which by design is absent at tree level. This is discussed in detail later in the text.
2.2 Unitarity Constraints

One of the key properties of the standard model is the unitarity of all scattering amplitudes. The standard model Higgs kills the bad high-energy behaviour of processes like $WW \rightarrow WW$ and $WW \rightarrow \bar{f}f$ where $f$ is any fermion. Introduction of new bosons into the theory that couple to fermions and gauge bosons should not spoil this. The necessary and sufficient conditions for this are [23]

$$\Sigma_i g_{VV}^2 = g_{hVV}^2$$

where $h_{VV}$ are all the neutral scalar fields of the theory including the analog of $h$. In our theory, since there is no SSB in the chiral-multiplet sector, no $\chi_{VV}$ coupling is introduced and the above conditions are trivially satisfied.

3 The RNG Flows

The one-loop $\beta$ function for the QCD coupling, $\alpha$, is

$$\frac{\partial \alpha}{\partial t} = -\left(\frac{33 - 2N_F}{3}\right) \frac{\alpha^2}{8\pi^2} \quad (g_3^2 = \alpha)$$

where $t = \ln(p/\mu)$. To this order the $\beta$-function for the QCD coupling does not receive any contribution from $g_y$, $\lambda$, $g$ or $g'$ as neither the chiral multiplet nor the electroweak gauge bosons couple directly to gluons.

The situation with the beta functions for $g_y$, $\lambda$ is, however, more intricate. If the $U(1)$-coupling $g'$ is large, radiative corrections will spoil the $SU(2)_L \times SU(2)_R$ structure. Instead of a single Yukawa coupling as in eqn(1), one would have

$$\mathcal{L}_{Yukawa}^{broken} = g_{y1}(\bar{u}_L \bar{d}_L)\Phi u_R + g_{y2}(\bar{u}_L \bar{d}_L)\Phi' d_R + h.c$$

where $\Phi' = C\Phi^*$ is the charge-conjugate of $\Phi$. We will show shortly that the $U(1)$-coupling can be neglected in the one-loop analysis and one can continue to use the form as in eqn(1). Before doing that we derive the full beta
functions(one-loop). In the minimal subtraction scheme, the renormalisation constants relevant for this discussion are given by:

\[
\begin{align*}
Z_{\sigma}^{wf} &= 1 + \frac{1}{8\pi^2\epsilon}[-6N'_y(g_{y1}^2 + g_{y2}^2) + \frac{3g^2 + g'^2}{2}] \\
Z_{uL}^{wf} &= 1 + \frac{1}{8\pi^2\epsilon}[-(g_{y1}^2 + g_{y2}^2) - \frac{27g^2 + g'^2}{36} - \frac{4g_3^2}{3}] \\
Z_{uR}^{wf} &= 1 + \frac{1}{8\pi^2\epsilon}[-2g_{y1}^2 - \frac{4g'^2}{3} - \frac{4g_3^2}{3}] \\
Z_{dL}^{wf} &= 1 + \frac{1}{8\pi^2\epsilon}[-(g_{y1}^2 + g_{y2}^2) - \frac{27g^2 + g'^2}{36} - \frac{4g_3^2}{3}] \\
Z_{dR}^{wf} &= 1 + \frac{1}{8\pi^2\epsilon}[-2g_{y2}^2 - \frac{g'^2}{9} - \frac{4g_3^2}{3}] \\
Z_{g_{y1}}^{w} &= 1 + \frac{1}{8\pi^2\epsilon}[2g_{y2}^2 + \frac{3g^2}{4} + \frac{25g'^2}{36} + \frac{16g_3^2}{3}] \\
Z_{g_{y2}}^{w} &= 1 + \frac{1}{8\pi^2\epsilon}[2g_{y1}^2 + \frac{3g^2}{4} + \frac{g'^2}{36} + \frac{16g_3^2}{3}]
\end{align*}
\]

(14)

Here 'wf' and 'v' refer to the wave-function and vertex renormalisations respectively. $N'_y$ is the effective number of flavours to which the chiral multiplet couples (see the discussion on flavour changing neutral currents in sec.5.1 which forces equal coupling of the chiral multiplet to all flavours). The resulting beta-functions are

\[
\begin{align*}
\frac{dg_{y1}}{dt} &= -\frac{g_{y1}}{8\pi^2}[-3N'_y(g_{y1}^2 + g_{y2}^2) + \frac{3}{2}(g_{y2}^2 - g_{y1}^2) + \frac{27g^2 + 17g'^2}{24} + 4g_3^2] \\
\frac{dg_{y2}}{dt} &= -\frac{g_{y2}}{8\pi^2}[-3N'_y(g_{y1}^2 + g_{y2}^2) - \frac{3}{2}(g_{y2}^2 - g_{y1}^2) + \frac{27g^2 + 5g'^2}{24} + 4g_3^2]
\end{align*}
\]

(15)

In these equations, the terms in bold are the ones arising out of loops with $M_X$ as the largest mass (except the cases where the one of the internal fermion lines is that of top quark). The significance of these terms will be clarified shortly. Already at this stage it is clear that the $U(1)$-coupling $g'$ has a negligible influence. This follows from the fact that $g^2 \simeq g'^2$ and that $g^2$ is already about a fourth of $g_3^2$ (say, near the scale of the Z-mass; at lower scales, due to the non-asymptotically free nature of $g'$, the importance of it becomes
even less). But this would be a little hasty though correct observation. To see this, let us introduce, following [9],
\[ \rho_1 = g_{y1}^2/\alpha, \rho_2 = g_{y2}^2/\alpha, \xi = g^2/\alpha \] and \( \xi' = g'^2/\alpha \), it is easy to obtain
\[
\frac{\partial \rho_1}{\partial \alpha} = -\frac{\rho_1}{A\alpha} \left[ 6N'_g(\rho_1 + \rho_2) - 3(\rho_2 - \rho_1) - \left( 8 + \frac{17}{12}\xi' + \frac{9}{4}\xi - A \right) \right] (16)
\]
where \( A = 11 - 2N_F/3 \). Thus we see that the contributions due to \( \xi' \) should really be negligible in comparison to \( A - 8 - 9\xi/4 \) in order that we can ignore the \( U(1) \)-couplings as it is the sign of \( A - 8 - 9\xi/4 - c\xi' \) in eqns(14) that determines whether the flows are asymptotically free or not. It is also significant for this discussion that all the gauge couplings \( g_3, g, g' \) enhance the tendency towards asymptotic freedom. For \( N_F = 6, 5, A - 8 = 1, 1/3 \) respectively and one can have AF regions for \( \rho_1, \rho_2 \) irrespective of the values of \( \xi, \xi' \). At a scale \( \simeq M_Z \) where \( A - 8 = -1/3, 9\xi/4 \simeq 0.6 \) and \( 17\xi' \simeq 0.12 \) and thus neglecting the \( \xi' \) contribution leads to an error of about 10%. In the eqn for the average of \( \rho_1, \rho_2 \) this error is in fact only about 8%. It should however be kept in mind that eventually the \( U(1) \)-coupling grows in the asymptotia because of the lack of asymptotic freedom in this coupling. But in practical terms this coupling begins to equal the other couplings only around \( \simeq 10^{14} GeV \) or so, which is a scale far beyond the scope of our model. We therefore emphasise that our discussion of asymptotic freedom is within the natural framework where asymptotia means a scale much larger than all the natural scales of the theory and for our model should be thought of as in the range 100-1000 GeV.

It should be observed that if we neglect the \( \xi' \) contribution, a consistent solution to eqn(14) is \( \rho_1 = \rho_2 \). In the Wilson RNG spirit we take this branch as defining our theory. Then the terms in boldface in these eqns vanish with the consequence that as we cross the mass threshold at \( M_X \), the flow eqns of the theory are unmodified. We shall return to a discussion of the effects of various mass thresholds shortly.

Thus we can return to analysing the theory in terms of a single \( g_y \) and in terms of \( \rho = g_y^2/\alpha \), we have
\[
\frac{\partial \rho}{\partial \alpha} = -\frac{\rho}{A\alpha} \left[ 12N'_g\rho - \left( 8 + \frac{9}{4}\xi - A \right) \right] (17)
\]
As it stands, the above eqn can not be integrated analytically because $\xi$ is a function of $\alpha$. It is however illuminating to consider eqn(15) after neglecting the $\xi$-dependent term, as in that case the eqn is analytically soluble and possesses qualitatively similar features as the full equation. Thus the equation we consider is

$$\frac{\partial \rho}{\partial \alpha} = -\frac{\rho}{A\alpha}[12N_g\rho - 8 + A]$$  \hspace{1cm} (18)

By defining $\tilde{\rho} = N_g\rho$, we can rewrite this as

$$\frac{\partial \tilde{\rho}}{\partial \alpha} = -\frac{\tilde{\rho}}{A\alpha}[12\tilde{\rho} - 8 + A]$$  \hspace{1cm} (19)

For the $N_F = 6$ case where $A = 7$, calling $\tilde{\rho}_c = 1/12$, there are two regimes:

**The Region** $0 < \tilde{\rho} < \tilde{\rho}_c$

In this case $\partial \tilde{\rho}/\partial \alpha > 0$. This implies that $\tilde{\rho}$ decreases as $\alpha$ decreases, that is, $\tilde{\rho}$ will decrease with increasing momentum scale. We can integrate the $\tilde{\rho}$ equation to get

$$\tilde{\rho} = \frac{\alpha^{1/7}K}{(1 + 12\alpha^{1/7}K)}$$  \hspace{1cm} (20)

where $K$ is a positive integration constant that is set by initial data on $\alpha$ and $g_y^2$. Specifically

$$K = \frac{\tilde{\rho}_0\alpha_0^{-1/7}}{1 - 12\tilde{\rho}_0}$$  \hspace{1cm} (21)

It is therefore clear that there is a whole family of solutions corresponding to different K’s. Deep in the ultraviolet when $\alpha \to 0$

$$\tilde{\rho} \sim K\alpha^{1/7}$$

which further implies that in the ultra violet

$$N_g^2g_y^2 \sim K\alpha^{8/7}$$

This means that $g_y^2$ is asymptotically free and vanishes faster than $\alpha$. Therefore, the leading behaviour of this theory in the ultraviolet is given by that of standard QCD, with the yukawa coupling contributing only in sub leading order.

The region $\tilde{\rho} > 1/12$ is also of interest, though from a different physical perspective. However, the theory is not AF in this region. Therefore we shall not consider it anymore in this paper.
The above analysis is valid for \( q^2 \geq m_t^2 \). For the region \( m_b \leq q \leq m_t \), the relevant behaviours are: \( \tilde{\rho}_c = 1/36, \tilde{\rho} \simeq K\alpha^{1/3} \). For \( N_F \leq 4 \) this approximate eqn does not admit any AF regime. But by the time the effective \( N_F \) reaches 4, \( \alpha \simeq 2.74 \) and the one-loop analysis need not be trusted.

Now let us return to the more precise eqn(15). As mentioned earlier, the explicit dependence of \( \xi \) makes it difficult to analyse this equation analytically. However, the precise form of \( \xi \) can be obtained by integrating the flow equation for \( \xi \). The flow eqn for the SU(2)_L coupling constant is given by

\[
\frac{dg_2}{dt} = \frac{g_2^3}{16\pi^2} \left[ -\frac{11}{3} t_2(V) \frac{4}{3} + n_F t_2(F) + \frac{1}{3} n_S t_2(S) \right]
\]

(22)

where \( n_F, n_S \) are the number of fermion and (complex) scalar multiplets in the fundamental representation of SU(2)_L. In the standard model as also in our model, if \( N_g \) is the number of quark-lepton generations, \( n_F = 2N_g \). However, it is more instructive to separate the lepton and quark contributions as all the lepton thresholds would be contributing, but not all the quark thresholds. Then, \( n_F = \frac{3}{2} + \frac{3N_F}{4} \) where \( N_F \) as before denotes the number of effective quark flavours. Again, in the standard model, \( n_S = 1 \) (the standard Higgs doublet) while in our model it is \( 1 + N_\chi \) where \( N_\chi \) is the effective number of chiral multiplets. Further, \( t_2(V) = 2 \) and \( t_2(f) = t_2(S) = 1/2 \) for SU(2).

Thus the relevant flow equation for our model is

\[
\frac{dg_2}{dt} = \frac{g_2^3}{16\pi^2} \left[ -\frac{37}{6} + \frac{N_F}{2} + \frac{N_\chi}{6} \right]
\]

(23)

thus

\[
\frac{d\xi}{dt} = -\frac{A_L}{8\pi^2} \xi^2
\]

(24)

where \( A_L = \frac{37}{6} - \frac{N_F}{2} - \frac{N_\chi}{6} \). On the other hand

\[
\frac{d\alpha}{dt} = -\frac{A}{8\pi^2} \alpha^2
\]

(25)

The solutions to these equations are, respectively,

\[
\frac{1}{\alpha} = \frac{A}{8\pi^2} t + \text{const}
\]

\[
\frac{1}{\xi} = \frac{A_L}{8\pi^2} t + \text{const}
\]

(26)
hence the quantity $\frac{A}{\xi} - \frac{A_L}{\alpha}$ is a RNG invariant. As a consequence we have

$$\frac{A}{\xi} - \frac{A_L}{\alpha} = \frac{A}{\xi_0} - \frac{A_L}{\alpha_0} = A(N_F, N_\chi, \xi_0, \alpha_0)$$  \hspace{1cm} (27)

This can be used to solve for $\xi$ as a function of $\alpha$:

$$\xi = \frac{A(N_F)}{A_L(N_F, N_\chi) + A\alpha}$$  \hspace{1cm} (28)

Substituting this in eqn (6) one gets

$$\frac{\partial \rho}{\partial \alpha} = -\frac{\rho}{A\alpha} [12N'_g\rho - (8 + \frac{9\xi}{4} - A)]$$  \hspace{1cm} (29)

This can be integrated numerically. But already some trends can be gleaned from this eqn; in the absence of the $\xi$ term, $\rho_c$ would be given by $\rho_c = (8 - A)/(12N'_g)$. But $\xi$ changes it to $\rho_c = (8 + \frac{9\xi}{4} - A)/(12N'_g)$. In the regime where $N_F = 5, N_\chi = 1$ (scales in the range 90 GeV-175 GeV), the $\xi$ contribution triples the value of $\rho_c$! This is clearly of great relevance to the phenomenology of this model. An accurate numerical solution of these eqns will be provided elsewhere.

### 3.1 Threshold Effects

We have treated the flow equations without regard to the various mass thresholds. A precise treatment of this is very complicated and is not warranted right now. However, the qualitative trends can be analysed. As one comes down from deep asymptotia, the first mass threshold to be encountered (in our model) is the top mass threshold at $\simeq 175 GeV$. Below this (actually much below this, but we shall not go into such finer details), the effective $N_F = 5$ and it remains at this value till we reach the bottom threshold at 5 GeV where it jumps to 4. In regions where $N_F \leq 4$ one loop results need not be reliable. As one goes below the threshold at $M_Z$, the $\xi$ contributions to the flow equations drop out. Of course, the $\xi'$ contributions arising out of the massless photon exchanges are always present, but due to the combined effects of the asymptotically free nature of $\alpha$ (which therefore keeps increasing towards the infrared) and the non-asymptotically free nature of $g'$ (which therefore keeps decreasing towards the infrared), $\xi'$ actually decreases very
rapidly. As we have already remarked, the threshold at $M_\chi$ has no effect. This is a very important property of the model which allows the flow eqns derived above both for the scenario in which the chiral multiplet is light mass as well as the case where they are considered to be at least as massive as $M_Z/2$. Thus for $N_F \leq 4$, our one-loop flow eqn reduces to the approximate form of eqn(16) which has no asymptotically free solutions.

For the purposes of our paper (in the context of the discussion on $g-2$ for muons) it is important that regions where QCD is perturbatively treatable, our theory is too. AF for the scale $> m_b$ implies $\rho \leq 1/36$. Thus at $q = m_b$, the Yukawa couplings are small. Now the lack of AF for $q < m_b$ has the desired effect of making $\rho$ even smaller as we go to smaller energy scales. Thus $g_y$ is perturbatively treatable wherever QCD is.

## 3.2 Scalar Self-coupling

So far we have not addressed the question of the RNG flows of the self-coupling $\lambda$. We leave all details of the calculation and merely present the full $\beta$-function for $\lambda$ in our model:

$$\frac{d\lambda}{dt} = \frac{1}{8\pi^2} \left[ 2\lambda^2 + 24N_g'\lambda(g_y^2 - \frac{3g^2}{8} - \frac{g'^2}{8}) - 144N_g'g_y^4 + \left( \frac{27g^4}{8} + \frac{9g^2g'^2}{4} + \frac{9g'^4}{8} \right) \right]$$

It is again clear that the $U(1)$-coupling can be neglected. Again we can introduce the ratio $R = \lambda/g_y^2$ following [9] and convert this flow equation to

$$[12N'g\rho - (8 + \frac{9\xi}{4} - A)] \frac{dR}{d\rho} = 2R^2 + 12N'gR + \frac{8R}{\rho} - 144N'g + (\frac{9}{4} - 9N')\frac{\xi}{\rho}R + \frac{27\xi^2}{8\rho^2}$$

As before, let us first consider an approximation to this eqn by dropping the $\xi$-dependent terms:

$$[12N'g\rho - 8 + A] \frac{dR}{d\rho} = 2R^2 + 12N'gR + \frac{8R}{\rho} - 144N'g$$

This is the same type of eqn that was considered in [9]; it depends only on $\rho$ and can be easily solved analytically. It is found that only on a single trajectory in the $[R, \rho]$ parameter space, that is the so called invariant line [11]-[18], the behaviour of $R$ for the regime $N'g\rho < 1/12$ is such that $R \to 0$ in the
ultraviolet. It follows further that \( \lambda \rightarrow 12N_g \rho \) in the extreme ultraviolet even faster than \( g^2_y \). Note that such a possibility is not available with eqn(21).

In figs. 3 and 4 we have shown the flow diagrams for \( R \) according to eqn (30) for the case \( N - g' = 3(A = 7) \). Of particular importance is the function \( R_+(\rho) \) which is the positive root of

\[
R^2 + 18R + 4R/\rho - 216 = 0
\]

Fig. 3 shows \( R_+(\rho) \) as well as the unique trajectory (the invariant line) on which \( R \) is regular everywhere. Every point on this line flows to \( R = 0 \) in the UV as \( R = 48\rho \) and approaches \( R_\infty = -9 + \sqrt{279} \) in the IR. For points such that \( R > R_+ \), \( \frac{dR}{d\rho} \) is positive. The point \( \rho = 1/36 \) at which the value of \( R \) on the invariant line is \( R_c = -81 + \sqrt{6777} \) is shown by the cross. The typical flows are shown in fig. 4. The curve above the invariant line but to the right of \( \rho = 1/36 \) is such that all points on it flow to \( R = \infty \) in the IR, but approach the invariant line in the UV. The flow to the right of \( \rho = 1/36 \) but below the invariant line is such that all points on it flow to \( R = 0 \) in the IR at some finite scale and eventually to negative \( R \), but converge to the invariant line in UV. Likewise, the flow to the left of \( \rho = 1/36 \) which is above the invariant line is such that all points on it move to \( R_\infty \) in the IR but to \( R = \infty \) at a finite scale in the UV. The flow to the left of \( \rho = 1/36 \) below the invariant line is such that it flows to \( R_\infty \) in the IR but goes to \( R = 0 \) at a finite scale in the UV and eventually to negative values of \( R \).

On the other hand, the structure of the eqn(29) shows that there can be no solution for \( R \) that is regular as \( \rho \rightarrow 0 \). But there is no reason for \( R \) to be regular for \( \lambda \) to be asymptotically free because \( \lambda \) could be vanishing slower than \( g^2_y \) and yet be asymptotically free. In fact an inspection of eqn(29) shows that as \( \rho \rightarrow 0 \) \( R \) must approach \( c/\rho \) where \( c \) satisfies

\[
2c^2 + 8c - \frac{27}{4} A \frac{c}{A_L} + \frac{27}{8} A^2_L - (8 - A + \frac{9A}{4A_L}) = 0
\]

Deep in asymptotia, \( N_F = 6 \) and \( N_\chi = 1 \) and consequently \( A = 7 \) and \( A_L = 3 \) and this eqn becomes

\[
2c^2 - 14c + \frac{147}{8} = 0
\]

with solutions \( c = \frac{21}{4}, \frac{7}{4} \). The nature of these solutions is completely different from the solutions we got without \( \xi \). It should be emphasised that \( R = c/\rho \) means \( \lambda = c\alpha \).
Thus we have classes of theories that are not only AF in all their couplings, but become increasingly indistinguishable from QCD at high energies. As far as AF is concerned one loop analysis is stable against higher loop corrections. Since these classes of theories are AF, they are all candidates for a consistent theory of strong interactions.

3.3 Higgs- Chiral Scalar Mixing

Since both the chiral multiplet and the standard model Higgs doublet couple directly to quarks and electroweak gauge bosons, quantum fluctuations generically lead to a mixing between them. The typical loop diagrams that contribute to such mixing is shown in fig. 5. These diagrams are also divergent. Thus renormalisability of the model would require the most general Higgs-Chiral Scalar coupling and not just the coupling in eqn (8). It is also clear that the mixing generated by these diagrams reduces $SU(2)_L \otimes SU(2)_R$ to $SU(2)_L \otimes U(1)$. In current literature one usually finds such couplings under the assumption of some discrete symmetries. Such discrete symmetries which in our case are present at tree level, are broken by quantum corrections. The most general such potential without assuming any discrete symmetries is

$$V = \mu_1^2 \Phi_1^\dagger \Phi_1 + \mu_2^2 \Phi_2^\dagger \Phi_2 + (\mu_3 \Phi_1^\dagger \Phi_2 + h.c) + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + (\lambda_4 (\Phi_1^\dagger \Phi_2)^2 + \lambda_5 (\Phi_1^\dagger \Phi_1)^2 + \lambda_6 (\Phi_1^\dagger \Phi_1)(\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)(\Phi_2^\dagger \Phi_2) + \lambda_8 (\Phi_1^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2) + (h.c)$$

(36)

Our model at tree level is assumed to be such that only $\mu_1, \mu_2, \lambda_1, \lambda_2, \lambda_3$ are non-vanishing. When quantum fluctuations are taken into account, all these parameters become scale dependent and one can not consistently put them to zero at all scales. But the scale evolution of these parameters could be such that for all practical purposes the parameters chosen to be zero at tree level remain very small for all scales of interest. To establish this completely satisfactorily requires a lot of work. Instead, we shall show the reasonableness of this by estimating the sizes of various diagrams after they have been renormalised to keep the renormalised values of these parameters at zero at some scale.

In the self energy type of graphs of fig. 5, the dominant contribution comes from the top quark loop. The Higgs- top coupling is taken to be $\eta \simeq 3/4$. Taking $\alpha_s = \alpha/(4\pi) \simeq 0.1$ and $\rho = 1/36$, one gets $g_y \simeq 1/5.5$. 

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The self-energy graph at top mass scale can thus be estimated to be \( \frac{\eta g y}{8 \pi^2} m_t^2 \approx m_t^2 / 500 \). Thus the expected magnitude of \( \mu_2^2 \approx m_{H^+}^2 \). Thus the mixing effect here is indeed small. The mixing terms that are quartic in fields can be of the type \( H^3 \chi, H^2 \chi^2, \) or \( H \chi^3 \). Of these, \( H^3 \chi \) can only be induced by quark loop diagrams and here too the top loop would dominate. The expected \( H^3 \chi \)-coupling is \( \frac{g g y (3/4)^2}{8 \pi^2} \approx 10^{-3} \). Likewise the \( H^2 \chi^2 \) induced by W-loop is \( \approx \frac{(g^2)^2 \ln m_H^2 / M_W^2}{8 \pi^2} \approx 1/1600 \) while that induced by top loop is \( \approx \frac{(g y)^2 (3/4)^2}{8 \pi^2} \approx 1/3600 \). Thus we conclude that the induced mixing between Higgs and the chiral multiplet can be safely neglected.

4 Massless Scenario

In this section we consider the scenario where \( M_\chi \) is small. Physically, this circumstance could either arise when the \( SU(2)_L \otimes SU(2)_R \) is spontaneously broken giving rise to Goldstone bosons with small masses due to some explicit breaking by quark mass terms, or when the \( SU(2)_L \otimes SU(2)_R \) symmetry is manifest with a small chiral invariant mass term for the scalars. In the following we analyse how this scenario is constrained by data on R-parameter, \( g - 2 \) for muons and high precision Z-width data.

4.1 The R parameter

The R parameter measures the ratio of \( \sigma(e^+e^- \rightarrow \text{hadrons}) \) to \( \sigma(e^+e^- \rightarrow \mu^+\mu^-) \). In QCD, at high energies, the former is approximated by \( \sigma(e^+e^- \rightarrow \Sigma q\bar{q}) \). When the energies are well below the Z mass we can neglect the contribution of the Z mediated processes to R. To leading order, then, the R parameter measures the total number of operational flavours (of course, multiplied by the number of colors of the quarks), modulated by their charge squared.

As we move to higher generations the R parameter changes rapidly at quark mass thresholds. At thresholds we also encounter many resonances which also give rapid changes in the R parameter. However, away from thresholds R can be quite stable. Therefore in the region above \( bb \) threshold we expect R to be stable apart from QCD corrections. But as we approach
\( \sqrt{s} = M_Z \), a new class of diagrams become operative and subsequently the R parameter has a steady rise to the Z peak.

There is thus an energy region, 20-40 GeV, where the effect of the Z is yet very small, where R is relatively stable. To leading order, that is, in the absence of QCD corrections, here \( R_0 = 11/3 \).

The R parameter will change for our theory as we have new scalar charged partons, the \( \tilde{\pi}^+\tilde{\pi}^- \) that couple to the photon which will contribute to the hadronic cross section. For the the zero mass partons considered by us the contribution to the R parameter is exactly calculable and is given by \( R = 1/4 \). This additional contribution should be clearly visible, particularly in the region: 20 - 40 GeV. The contribution of Z - exchanges are negligible in this region.

The R parameter receives QCD corrections and the QCD corrected R parameter in this region is: \( R(s) = R_0(1 + \alpha_s/\pi + ...) \). The measured value of the R parameter in this region has a world average of 4.02. The difference between this number, 4.02, and \( R_0 = 11/3 \) is supposed to come from the QCD corrections and yield a value for \( \alpha_s \) at this energy scale.

However if we add the extra contribution of our new partons, \( R_0 = 11/3 + 1/4 = 3.91 \), leaving only a deficit of .11 to be accounted for by the QCD corrections. The corresponding value of the QCD coupling will then be too low to be admissable. However the systematic errors at Amy, Topaz etc are not so small, of the order of 5% . Also, different groups have reported R in this region to be as high as 4.2 or even as low as 3.8 . This circumstance means that our theory cannot be ruled out as the effect we are considering is \( \Delta R/R = 6 \% \). It is worth pointing out that experimentally low multiplicity (\( \approx 5 \)) jets are not counted as these are very unlikely in QCD. On the other hand, for the color neutral pionic partons of our theory, we expect to have only low multiplicity jets most of which would have been excluded by experimental cuts.\(^4\) Of course, two prong events would have been counted as \( \mu^+\mu^- \)-pairs and could have shown up as anomalies which could be distinguished by their different angular dependence as compared to leptons.

Given these facts the R parameter is not at present a definitive test for this theory versus QCD though reduced systematic errors could put things

\(^4\) This exemplifies the point made in the introduction whereby biases based on a particular theoretical framework (QCD, in this case) can surreptiously influence the way the data is analysed.
4.2 g-2 for the muon

The contribution to g-2, the muon magnetic moment, can be potentially disturbed by the presence of additional charged particles. Our theory, in the light mass scenario considered in this section, has particles with zero or light masses coupling to photons thereby contributing additionally to the photon propagator. Such a contribution to the propagator can be related to the contribution to the R parameter we have just considered. This is precisely how g-2 is calculated in the literature [19]. In this work the contribution to g-2 for the muon, $a_v$, is given by

$$a_v = \int_{4m^2} dt K(t) R(t) \quad (37)$$

where $R(t)$ is the R-ratio and $K(t)$ is a known function.

However, the manner in which zero mass scalar partons enter the low energy description of $R(t)$ (where $\sqrt{t}$ is the centre of mass energy) is subtle. At low energy there is mixing between our zero mass partonic pions and the pseudoscalar channel. This will generate a higher mass state with quantum numbers of the pion in addition to the usual 'goldstone pion' associated with the spontaneous breaking of chiral symmetry. Since we cannot calculate the mass of this non-perturbatively generated extra state we can only use experiment to glean its mass. The particle data book lists the first additional state with pion quantum numbers at 1.3 Gev. At low energies then we must use this state to calculate the extra contribution to the photon self energy or the R parameter as the usual pion's contribution is already accounted for in the standard treatment of hadronic contributions to g-2. The threshold for the contribution of this state then starts at $\simeq 2.6$ Gev. This puts us in the perturbative QCD regime.

Before computing the extra contributions a few remarks are in order:

1) The low energy regime, 0.8-2.0 Gev, has to be gleaned from experiment as perturbative QCD can not be used. The contribution to $a_v$ from this region as listed in Table 2 of [19] is $(1404 \pm 100) \cdot 10^{-11}$. As this is evaluated from experimental data, one cannot differentiate the contributions from QCD and our theory.

2) Perturbative QCD is used for $t > 2 Gev^2$ (Table 1 of [19]) except for the threshold regions which are populated by numerous
resonances, where again one has to only rely on experimental data, and can not differentiate between our theory and QCD. These regions are: 3.3-3.6 Gev, 3.6-4.9 Gev and 9-14 Gev.

Thus it is only for regions for which estimates are made via perturbative QCD that comparison between this theory and QCD is possible. In respect of the foregoing discussion this region for us must begin at $\sqrt{t} > 2.6$ Gev. We briefly sketch how the additional contribution can be evaluated for our theory in the region 2.6-3.1 Gev: i) $K(t)$ goes as $1/t^2$. Assuming $R(t) = R_0$ and using eqn(10) we can get the two contributions for 1.4-2.6 Gev and 2.6-3.1 Gev for QCD. The partial contribution for the region, 2.6-3.1 Gev is found to be 1/10 of the total. ii) The extra contribution to $R$ assuming zero mass pionic partons is $\delta R = 1/4$ whereas the QCD contribution is $R_0 = 2$. Thus the fractional extra contribution is 1/8. iii) This is further down when we take into account the mass of the excited state (1.3 Gev). A rough estimate is provided by multiplying by the phase space factor $(1 - 4M^2/t) = 0.17$ taken at the average value 2.85 Gev for $\sqrt{t}$. The total extra contribution for the region 2.6-3.1 Gev works out to $1.2 \cdot 10^{-11}$. Below we display the contributions and errors for various regions:

| $\sqrt{t}$ in Gev | QCD multiplet | Chiral Theo. error | error(5% ) |
|-------------------|---------------|-------------------|------------|
| 2.6 - 3.1         | 56            | 1.2               | ±1         | ± 2.8 |
| 4.9 - 9           | 67.5          | 4                 | ±1         | ± 3.5 |
| > 14              | 13            | 0.9               | ±2         | ± 0.65 |

It should be noted that: i) The theoretical error in [19] is arbitrarily estimated as half the $\alpha_s^2$ correction to $R$. ii) we have taken the systematic error to be 5% of $R$ (see Sec 4.1)

The extra contribution of our theory falls within the sum of the theoretical and the systematic errors. The error in the low energy region, 0.8-2 Gev is roughly $100 \cdot 10^{-11}$. By comparison, all our extra contributions are negligible.

We are therefore led to the conclusion that $g$-2 for the muon despite being a very high precision measurement of the charged-particle content of theories cannot differentiate between QCD and our theory - a rather non trivial result.
4.3 Z width

The Z-width data on the other hand is known with great accuracy. The minimal coupling of the chiral multiplet to $Z_\mu$ is (see eqn (6,7)):

$$\mathcal{L}^{\text{neut}}_{\text{lin}} = e(A_\mu - \frac{\gamma}{2}Z_\mu)(\vec{\pi} \times \partial_\mu \vec{\pi})_3 - \frac{e}{2cs}Z_\mu(\bar{\pi}_0 \partial_\mu \bar{\sigma} - \bar{\sigma} \partial_\mu \pi)$$

(38)

where $\gamma = (1-2s^2)/cs$ with $s$ being $\sin \theta_W$ and $c^2 = 1-s^2$. The contribution to the hadronic width of Z-boson due to the extra scalars can be calculated easily:

$$\frac{\Delta \Gamma_Z}{\Gamma_{\text{had}}} = \frac{9((1-2\sin^2\theta)^2 + 1)}{N_c(90-168\sin^2\theta + 176\sin^4\theta)}$$

(39)

At $\sin^2\theta \simeq .25$, this works out to roughly 4.5% of the total width $\Gamma_Z$. The high precision LEP data only allows total uncertainty of about .3%.

This immediately rules out the extended version where the chiral symmetry in the extended sector is spontaneously broken or where the chiral symmetry in the extended sector is manifest with low mass for the multiplet, or where chiral symmetry is fully broken but with light masses for the chiral scalars.

In conclusion, by explicit construction of an alternative theory to QCD for the strong interactions we have found that all precision tests for QCD except for the Z width cannot select between the two theories. Only by extending the theories to the FULL electroweak standard model do we find an unambiguous support in favour of QCD from the Z width. This underscores the fact that most tests and vindications of QCD that are to be found in archival references in the literature are just not adequate and that only the Z width is precise enough to be the final arbiter.

A different version of this theory where the chiral multiplet mass of more than one half the Z mass will be exempt from this problem. This is considered in the next section.

5 Massive Scenario

The main conclusion from our analysis of the massless scenario is that $M_\chi < M_Z/2$ is not viable. In this section we consider the opposite scenario that $M_\chi > M_Z/2$. Before looking at the signatures for such a scenario, we first
investigate the constraints on the model by Flavour Changing Neutral Currents (FCNC) as well by the oblique parameters S, T & U.

5.1 Flavour Changing Neutral Currents

In the standard model $\Delta F = 2$ processes like the ones mediated by $G_F \alpha$, they actually turn out to be of order $G_F^2$. This remarkable suppression, borne out by data, is due to the unitarity of the so-called CKM matrix for three generations. In our model too FCNC processes must be likewise suppressed; otherwise they are immediately ruled out. A natural question to ask is whether there is an analogue of the CKM matrix for our model too. As in the standard model we introduce mass-eigenstate quark-fields through

$$\psi_R = T^\dagger \psi'_R$$
$$\psi_L = S^\dagger \psi'_L$$

Now it is important to see whether FCNC are introduced in our model. We must have

$$\mathcal{L}_{\text{yukawa}} = F_{AB} \bar{\Psi}_A (\sigma = i\gamma_5 \vec{\tau} \cdot \vec{\pi}) \Psi'_B$$

in order that the interaction preserve $SU(2)_L$- invariance. Here $\Psi$ is the Dirac field. On splitting the Dirac field into its left and right-handed components and rotating them into the mass-eigenstate basis one obtains

$$(S_{(p)}^\dagger F T_{(p)})_{AB} \bar{p}_L^A (\sigma + i\pi^0) p_R^B + (S_{(n)}^\dagger F T_{(n)})_{AB} \bar{n}_L^A (\sigma - i\pi^0) n_R^B$$

$$+ \ i\pi^-(S_{(n)}^\dagger F T_{(n)})_{AB} \bar{n}_L^A p_R^B + \ i\pi^+(S_{(p)}^\dagger F T_{(p)})_{AB} \bar{p}_L^A n_R^B$$

Now there are potential FCNC terms even at tree level because of the first two terms in the above expression. However, the contributions to tree level from FCNC due to $\sigma$ and $\pi^0$ exactly cancel, and there are no tree level FCNC in this model. This is true both for $\Delta F = 2$ and $\Delta F = 1$ processes. This is unlike the situation in generic two Higgs doublet models, where often the couplings have to be fine-tuned to keep tree-level FCNC under control. This is because in such models, the analogs of $\sigma$ and $\pi^0$ are not mass-eigenstates; instead, the mass-eigenstates are linear combinations of $(H_0, \sigma, \pi^0)$ where $H_0$ is the higgs of the minimal standard model. This is again due to the fact...
that the most general Higgs potential that is $SU(2) \times U(1)$-invariant, allows such a mixing (of course the analogs of $\pi^{\pm}$ are mass-eigenstates even in such models). On the other hand in our model the Higgs-chiral multiplet coupling term has to be $SU(2)_L \times SU(2)_R$-invariant, which only allows

$$|\Phi|^2 \cdot (\sigma^2 + \pi^2)$$

and this keeps $(\sigma, \pi^0)$ as mass eigenstates even after SSB, except for the small mixing effects described in sec. 3.3.

This cancellation of FCNC due to the tree-level $\Delta F = 1$ vertices persists even for the one-loop contributions of the type shown below. The FCNC generated by the last two terms of eqn(4) are more complicated and involve deeper analysis. The generic contributions to $\bar{d}s \rightarrow \bar{s}d$ involve box diagrams of two types: a) $W\pi$ exchange and b) $\pi\pi$ exchanges shown below. In addition, unlike the standard model case, there are several $\Delta F = 2$ operators to deal with. We shall illustrate this for the particular case of $\bar{d}s \rightarrow \bar{s}d$ by adopting the notation that $\bar{d}_{\lambda_1}s_{\lambda_2} \rightarrow \bar{s}_{\lambda_3}d_{\lambda_4}$ shall be denoted by $(\lambda_1, \lambda_2, \lambda_3, \lambda_4)$. Thus the case of fig. 1 is $(L, R, L, R)$. In principle 16 possibilities are possible. But for the $w\pi$ exchange, at the W-vertex only L’s are allowed. This way the number of possibilities is cut down considerably. Also, diagrams where the internal i-quark propagator connects quark fields of different helicities are suppressed by $m_i^2/M_W^2$. Thus the independent classes to be counted are: for the $W\pi$-exchange, $(L, R, L, R)$ and for the $\pi\pi$-exchanges, $(L, L, \lambda, \lambda)$ and $(R, R, \lambda, \lambda)$.

The analysis is facilitated on noting that there are four coupling matrices in eqn(4) denoted by

$$V_{\text{neut}}^p = S_{(p)}^T F_{(p)}$$
$$V_{\text{neut}}^n = S_{(n)}^T F_{(n)}$$
$$V_{\text{char}}^p = S_{(p)}^T F_{(n)}$$
$$V_{\text{char}}^n = S_{(n)}^T F_{(p)}$$

(44)

It should be noted that $V_{\text{char}}^p$ is associated with outgoing $p_L$ and incoming $n_R$ (the indices are also ordered this way), while $V_{\text{char}}^n$ is associated with outgoing $n_L$ and incoming $p_R$. It is further useful to note

$$V_{\text{char}}^p = V_{\text{CKM}} \cdot V_{\text{neut}}^n$$
$$V_{\text{char}}^n = V_{\text{CKM}}^\dagger \cdot V_{\text{neut}}^p$$

(45)
The contributions to various $\Delta S = 2$ processes can be summarised thus

\begin{align*}
(L, R, \lambda_1, \lambda_2)_\pi & \simeq (V^\dagger_{CKM} V^p_{\text{char}})_{12} = (V^n_{\text{neut}})_{12} \\
(R, R, \lambda_1, \lambda_2)_{\pi\pi} & \simeq (V^p_{\text{char}} \dagger V^p_{\text{char}})_{12} = (V^n_{\text{neut}} \dagger V^n_{\text{neut}})_{12} \\
(L, L, \lambda_1, \lambda_2)_{\pi\pi} & \simeq (V^n_{\text{char}} \dagger V^n_{\text{char}})_{12} = (V^p_{\text{neut}} \dagger V^p_{\text{neut}})_{12}
\end{align*}

(46)

If we had considered FCNC for $\bar{u}_R c_R \rightarrow \bar{c}_R u_R$, the first of these eqns would have been modified by the replacement $V^n_{\text{neut}} \rightarrow V^p_{\text{neut}}$.

Thus we see that natural suppression of all FCNC operators would require that the matrices $V^p_{\text{neut}}, V^n_{\text{neut}}$ are both diagonal. Now we use two theorems (which are quite easily proved) to show that this can happen only when the matrix $F$ is proportional to the Identity-matrix.

**Theorem 1**

If a hermitean matrix $M$ is diagonalised by a bi-unitary transformation

$$VMU^\dagger = M_d$$

then

$$VU^\dagger = W_d$$

where $W_d$ is some diagonal unitary matrix.

**Theorem 2**

If a Hermitean matrix $M$ is brought to diagonal form by two sets of bi-unitary transformations

\begin{align*}
V_1 MU_1^\dagger & = W_1^d H_d \\
V_2 MU_2^\dagger & = W_2^d H_d
\end{align*}

(49)

and if

$$V_1 V_2^\dagger \neq W_3^d$$

(50)

or

$$U_1 U_2^\dagger \neq W_4^d$$

(51)

where $W_d^{(3,4)}$ are some diagonal unitary matrices, then

$$M = mI$$

(52)
Since in our case $F$ is a hermitean matrix, theorem-1 immediately implies

$$S_p T_{(p)}^\dagger = W_d^{(1)}$$

$$S_n T_{(n)}^\dagger = W_d^{(2)}$$

(53)

As stressed earlier, natural suppression of all FCNC in this model would require that $V_{neut}^p, V_{neut}^n$ are both diagonal. From the theorem proved above it is also clear that these have to be diagonal unitary matrices.

In our case $S_p^\dagger S_n$ is the CKM-matrix which is not diagonal; hence theorem-2 implies $F$ must be a multiple of unit matrix.

This in turn implies that

$$V_{neut}^p = W_d^1$$

$$V_{neut}^n = W_d^2$$

$$V_{char}^p = V_{CKM} \cdot W_d^2$$

$$V_{char}^n = V_{CKM}^\dagger \cdot W_d^1$$

(54)

where $W_d^{1,2}$ are some diagonal unitary matrices.

This is the unique solution to natural FCNC suppression in this model. It is instructive to pause and reflect whether this manner of FCNC suppression amounts to fine-tuning in this model. A particular solution is not a fine-tuned solution if it enhances the symmetries of the system. In the absence of $SU(2) \times U(1)$ couplings our solution indeed enhances the symmetry from $SU(2)_L \times SU(2)_R$ to $SU(3)_{\text{hor}} \times SU(2)_L \times SU(2)_R$. Thus if the standard model is not truly fundamental but only an effective description, then there could be an intermediate phase (at scales above the weak scale) where nature may have preferred $SU(3)_{\text{hor}} \times SU(2)_L \times SU(2)_R$. If that is so, the chiral-multiplet may be even more fundamental than what we may have been thinking.

Lastly, we see that our solution to the suppression of FCNC can not generically yield CP-conservation. The minimal CP-nonconservation can be achieved by taking $W_d^{(1,2)}$ to be unit matrices.
5.2 The electroweak precision parameters S, T and U for the extended theory

The so called oblique parameters\(^5\) as precision tests for electroweak theories are defined by \([24, 25, 26]\)

\[\begin{align*}
\alpha \tilde{T} M_W^2 &= \tilde{\pi}_{WW}(0) - c^2 \tilde{\pi}_{ZZ}(0) \\
\alpha \tilde{S} M_Z^2 &= 4 c s [c s (-\tilde{\pi}_{ZZ}(0) - \tilde{\pi}_{\gamma\gamma}(M_Z^2)) + (s^2 - c^2) \tilde{\pi}_{\gamma Z}(M_Z^2)] \\
\alpha \tilde{U} M_W^2 &= 4 s^2 [c^4 \tilde{\pi}_{ZZ}(0) - \tilde{\pi}_{WW}(0) \\
&\quad - c^2 (s \tilde{\pi}_{\gamma\gamma}(M_Z^2) + 2 c \tilde{\pi}_{\gamma Z}(M_Z^2))] 
\end{align*}\] (55)

In these equations \(\tilde{\pi}_{VV}\) refers to the propagator function for the vector boson \(V\) after mass and wavefunction renormalisations. For the mixed propagator function \(\tilde{\pi}_{\gamma Z}\), the mass and wavefunction renormalisations are understood to be carried out on the photon pole; \(c\) and \(s\) stand for \(\cos \theta_W\) and \(\sin \theta_W\) respectively.

It turns out that all propagator functions \(\tilde{\pi}_{AB}\) before mass and wavefunction renormalisations are proportional to each other, so in this section we shall first consider the details of that common function.

Denoting the generic one-particle irreducible 2-point function by \(\mathcal{M}_{\mu\nu}\), one eventually finds

\[\mathcal{M}_{\mu\nu} = -i A g_{\mu\nu} + i B P_{\mu} P_{\nu}\] (56)

with

\[A = B P^2 = \Pi(P^2)\] (57)

where

\[\begin{align*}
\Pi(P^2) &= -\frac{1}{48 \pi^2} (\gamma_E + \ln M_\pi^2) P^2 \\
&\quad + \frac{P^2}{16 \pi^2} \left(\frac{2}{9} - \frac{8}{3} a^2 + \frac{16}{3} a^3 \arctg\left(\frac{1}{2a}\right)\right)
\end{align*}\] (58)

Thus the full two point function has the expected gauge-invariant structure

\[\mathcal{M}_{\mu\nu} = -i (g_{\mu\nu} - \frac{P_{\mu} P_{\nu}}{P^2}) \Pi(P^2)\] (59)

---

\(^5\)The results of this section were obtained in collaboration with Dr. Rahul Sinha. We are also indebted to him for many useful discussions.
Recall that the various $\tilde{\pi}(P^2)$ are mass and wavefunction renormalised. Because $\Pi(0) = 0$, mass renormalisations for $\tilde{\pi}_{\gamma\gamma}, \tilde{\pi}_{\gamma Z}$ are automatically taken care of. To discuss wavefunction renormalisation, we compute $\Pi(P^2)'$ where $'$ denotes differentiation wrt $P^2$. The result is

$$\Pi(P^2)' = -\frac{1}{48\pi^2}(\gamma_E + \ln M^2_\pi) + \frac{1}{18\pi^2} + \frac{f_2(a)}{16\pi^2}$$

where

$$f_2(a) = \frac{8a}{3}[-(a^2 + 3/4)\arctg(\frac{1}{2a}) + \frac{a}{2}]$$

introducing

$$f_1(a) = 2/9 - 8/3a^2 + 16/3a^3 \arctg(1/2a)$$

we can recast $\Pi(P^2)$ as

$$\Pi(P^2) = \frac{P^2}{16\pi^2}[-1/3(\gamma_E + \ln M^2_\pi) + f_1(a)]$$

It should be noted that $f_1(a)_{a\to\infty} \to 0$. The fully renormalised quantities renormalised at $P^2 = M^2$ denoted by $\tilde{\pi}(P^2)^{(M)}$ are given by

$$\tilde{\pi}(P^2)^{(M)} = \Pi(P^2) - \Pi(M^2) - (P^2 - M^2)\Pi(M^2)'$$

The case $M = 0$ will be explicitly worked out as the limit $a \to \infty$ could be problematic for a numerical evaluation. The result is

$$\tilde{\pi}(P^2)^{(0)} = \frac{P^2}{16\pi^2}[f_1(a) - 2/3]$$

The result for the generic case is

$$\tilde{\pi}(P^2)^{(M)} = \frac{P^2}{16\pi^2}f_1(a) - \frac{M^2}{16\pi^2}f_1(a_M) - \frac{(P^2 - M^2)}{16\pi^2}(f_2(a_M) + 8/9)$$

where $a^2_M = M^2_\pi/M^2 - 1/4$. Now we only have to identify the coupling constants that multiply the various functions:

$$\tilde{\pi}_{WW} = \frac{e^2}{2s^2}\tilde{\pi}^{(MW)}$$

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\[ \tilde{\pi}_{ZZ} = e^2 \left( \frac{\gamma^2}{4} + \frac{1}{4s^2c^2} \right) \tilde{\pi}^{(M_Z)} \]  
(68)

\[ \tilde{\pi}_{Z\gamma} = -\frac{e^2 \gamma}{2} \tilde{\pi}^{(0)} \]  
(69)

\[ \tilde{\pi}_{\gamma\gamma} = e^2 \tilde{\pi}^{(0)} \]  
(70)

\( e^2 = 4\pi\alpha \) and \( \gamma = (1 - 2s^2)/sc \) with \( s^2 = 0.23 \).

It should be noted that the structure of eqn (52) is such that \( \tilde{\pi}^{(0)} \) is evaluated at \( M_Z^2 \) and \( \tilde{\pi}^{M_Z,M_W} \) is evaluated at \( P^2 = 0 \). Hence we can use

\[ \tilde{\pi}^{(0)}(M_Z^2) = \frac{M_Z^2}{16\pi^2} [f_1(a_Z) - 2/3] \]

\[ \tilde{\pi}^{(M)}(0) = \frac{M^2}{16\pi^2} [f_2(a_M) - f_1(a_M) + 8/9] \]  
(71)

We finally give here the corrections to these parameters:

| Scalar Mass (GeV) | \( \delta \tilde{T} \) | \( \delta \tilde{S} \) | \( \delta \tilde{U} \) |
|------------------|----------------|----------------|----------------|
| 50               | -.010          | -.027          | .005           |
| 55               | -.005          | -.018          | .002           |
| 60               | -.003          | -.013          | .001           |

these corrections are much smaller than the uncertainties in even the most precise LEP measurements [23].

### 5.3 Other Precision Tests for the Model

The coupling of the chiral multiplet to the electroweak bosons given by eqns (6-7) leads to additional contributions to various processes of the electroweak theory. One of the sensitive tests for QCD is the value of the R-parameter. As the cross-section for the pair production of the scalars at LEP energies is of the order of a 1pb, the R-parameter is not very sensitive to the presence of the scalars. The other important precision test is the g-2 for muons. There are two types of additional contributions to g-2 that arise. One is due to the enhanced ultraviolet degrees of freedom and
the other due to additional hadronic interactions. The contributions due to
the former arise out of the modification of the photon propagation function.
The analytic result is $\Delta g = \frac{\alpha}{2\pi} \frac{\alpha}{180\pi} \frac{m^2}{m^2_{\tilde{\pi}}}$ For $m_{\tilde{\pi}} = 45 GeV$ this amounts to
$\Delta g = 4 \cdot 10^{-14}$ and hence insignificant. The shift due to the modification
of hadronic interactions is much harder to estimate precisely. One should
expect very little difference between QCD and the extended theories here
because of the expected decoupling of massive particles. One expects the
additional interactions to produce changes in g-2 at the level of $\frac{\alpha^2}{4\pi^2} \frac{1}{2\pi}$ times
the dominant hadronic contributions. This amounts to less than 2 parts in
1000 of the dominant hadronic contributions and is hence much less than the
known theoretical uncertainties in g-2.

6 Four Jet Events

As mentioned in the introduction, though the ALEPH collaboration has now
retracted its earlier claims of having seen excess four jet events, many features
reported by them earlier follow naturally and in a virtually parameter-free
manner from our model. We find it worthwhile to present them here as
generic features of four jet events for our model.

In our model one expects excess four jet events identified with the decay
products of the scalars which have no couplings to the leptons. Hence one
of the characteristic features of such four jet events in our model are: a) no
leptons with high transverse momentum wrt the jets are expected. b) no
events of the type $\tau^+ \nu_\tau \bar{c}s$ and $\tau^+ \nu_\tau \tau^- \bar{\nu}_\tau$ are expected.

The following table gives in pb the cross-sections for $e^+ e^- \rightarrow \tilde{\sigma} \tilde{\pi}$ ($\tilde{\sigma}_{neut}$) and
to $\tilde{\pi}^+ \tilde{\pi}^- (\tilde{\sigma}_{char})$ as a function of $\sqrt{s}$ and scalar mass. Taking into account
the FCNC constraint that the scalars couple nearly equally to all flavours,
the branching ratios for having at least 2 b($\bar{b}$) jets ($R_{2b}$) and four b($\bar{b}$) jets
($R_{4b}$) are also given.
In our model the widths of the scalars are: $\Gamma_{\tilde{\sigma}} = \frac{g_s^2}{4\pi} \frac{3M_{\tilde{\sigma}}}{4}$ and $\Gamma_{\tilde{\pi}^0} = \frac{g_s^2}{4\pi} 3M_{\tilde{\sigma}}$. At $\rho = 1/36$, the former amounts to about 0.375 Gev/flavour, and with a top mass of 170 Gev, only five flavours contribute to the decay of neutrals, leading to a width of about 1.88 Gev. Likewise, the latter works out to 0.75 Gev/flavour, and the width of charged scalars is about 1.5 Gev. These should be the expected widths for the di-jet mass-sum distribution.

We present a comparison of the expected properties of four jet events in our model with what one would expect from some other models in the following Table.

### Table

| $\sqrt{s}$ (GeV) | $M_{\tilde{\sigma}}$ | $\bar{\sigma}_{\text{neut}}$ | $\bar{\sigma}_{\text{char}}$ | $R_{2b}$ | $R_{4b}$ |
|-----------------|----------------------|-----------------------------|---------------------------|----------|----------|
| 130             | 50                   | .63                         | .54                       | 1/5      | 1/46     |
| 130             | 55                   | .37                         | .31                       | 1/5      | 1/46     |
| 130             | 60                   | .14                         | .12                       | 1/5      | 1/46     |
| 161             | 50                   | .43                         | .57                       | 2/13     | 1/58     |
| 161             | 55                   | .35                         | .46                       | 2/13     | 1/58     |
| 161             | 60                   | .27                         | .35                       | 2/13     | 1/58     |
| 172             | 50                   | .38                         | .55                       | 1/7      | 1/61     |
| 172             | 55                   | .32                         | .46                       | 1/7      | 1/61     |
| 172             | 60                   | .26                         | .37                       | 1/7      | 1/61     |

In this section we shall discuss some other experimental signatures for the extended theories that appear feasible at the moment.

### 7 Other Experimental Signatures

In electron-proton(ep) scattering experiments, by restricting to suitable regions of scattered electron angles and energies, one can get almost real photons. We now describe the new events predicted by the extended model in those regions.
Table 1: COMPARISON

| **SUPERSYMMETRY** | **TWO HIGGS** | **OUR MODEL** |
|-------------------|----------------|--------------|
| **Additional Particles** | Charginos, Squarks | $HA, H^+H^-$ | $\tilde{\sigma}, \tilde{\pi}$ |
| **Additional Parameters** | many | 2 Yukawa, 5 Self | 1 Yukawa, 2 Self |
| $\sigma_{\text{tot}}(\text{pb})(\text{Lower} \sqrt{s})$ | 1 to 7 | $\sim 1$ | $\sim 1.17$ |
| (Higher $\sqrt{s}$) | similar | similar | $\sim 1$ |
| **Width Of Mass Dist** | Large | Depends on Yukawa coupling | 1 to 2 Gev |
| **Final State Leptons** | Expected | Expected | No leptons |
| $R_{2b}$ | Expected | Predominant | 1/5-1/7 with 100% eff |
| $R_{4b}$ | Expected | Predominant | 1/45-1/63 |
| FCNC | Fine tuning | Fine Tuning | Naturally fulfilled |

The dominant process in ep-scattering is essentially electron-quark scattering (eq) which manifests itself as a 2 jet event with activity throughout the rapidity region. The advantages of working with nearly real photons is that this dominant process is kinematically forbidden. Indeed, the leading order process now is $\gamma q \rightarrow qg$ which experimentally manifests itself as 2+1 (the gluon jet, the scattered quark jet and the beam jet) jet topology with no rapidity gaps. In the extended theory we now have the additional process $\gamma q \rightarrow q\pi(\sigma)$. These events are characterised by 2+2 jet topologies where the last two jets arise out of the decay of $\pi(\sigma)$, and there will be significant rapidity gaps in the $(q, \pi), (\pi, \text{beam})$ regions. Now the relative fraction of these events to the dominant QCD process is $\sim N_g^2 \rho = 1/12$ apart from some phase space suppression. This is a sizeable effect.

### 7.2 ep scattering

In ep scattering the dominant QCD process is the 2 jet event with no rapidity gap produced by the subprocess $eq \rightarrow eq$. Of the 2 jets, the quark jet is produced back to back with the electron in the eq centre of mass frame. In the extended model, the dominant process is $eq \rightarrow eq\pi(\sigma)$. The distinguishing
features of these new events are that only the rapidity region between the
beam jet and the quark jet is filled. Also, the quark jet is no longer back
to back with the electron in the eq centre of mass frame and can in fact be
produced at sufficiently small angles.

7.3 pp(\bar{p}) scattering

In addition to the dominant one gluon exchange, now one can exchange the
scalar particles leading to a slightly different angular distribution for jets.
The scalar admixture will roughly be a fraction $N'_g/\rho$ and could easily be
about 3%. Another way this could manifest is in a slightly different estimate
for $\alpha_s$ in pp($\bar{p}$) reactions compared to $e^+e^-$ or ep reactions.

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[†] electronic address : dass@imsc.ernet.in

[‡] electronic address : vsoni@ren.nicnet.in

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Figure 1: Flows with an Infra-red Fixed Point
Figure 2: The renormalisation group flows for rho vs alpha.
Figure 3: The Invariant-line and the curve $R_+(\rho)$
Figure 4: The Renormalisation Group Flows for R vs rho.
Figure 5: Typical diagrams contributing to Higgs-Chiral Scalar mixing

Figure 6: Typical box diagram contribution to $\Delta F = 2$ processes
Figure 7: Tree Level contribution to $\Delta F = 2$ processes

Figure 8: Neutral exchange contribution to $\Delta F = 2$ processes