Multifractality and scale invariance in human heartbeat dynamics

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Human heart rate is known to display complex fluctuations. Evidence of multifractality in heart rate fluctuations in healthy state has been reported [Ivanov et al., Nature 399, 461 (1999)]. This multifractal character could be manifested as a dependence on scale or beat number of the probability density functions (PDFs) of the heart rate increments. On the other hand, scale invariance has been recently reported in a detrended analysis of healthy heart rate increments [Kiyono et al., Phys. Rev. Lett. 93, 178103 (2004)]. In this paper, we resolve this paradox by clarifying that the scale invariance reported is actually exhibited by the PDFs of the sum of detrended healthy heartbeat intervals taken over different number of beats, and demonstrating that the PDFs of detrended healthy heart rate increments are scale dependent. Our work also establishes that this scale invariance is a general feature of human heartbeat dynamics, which is shared by heart rate fluctuations in both healthy and pathological states.

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I. INTRODUCTION

The heartbeat interval in human is known to display complex fluctuations, referred to as heart rate variability (HRV). In the past decade, many analyses [1, 2, 3, 4, 5, 6, 7, 8] have been carried out to characterize the statistical features of human HRV, with an aim to gain understanding of human heartbeat dynamics. An intriguing finding is the multifractality in healthy heart rate fluctuations in both healthy and pathological states. Such multifractal complexity in healthy HRV was further shown to be related to the intrinsic properties of the control mechanisms in human heartbeat dynamics and is not simply due to changes in external stimulation and the degree of physical activity [4].

In another complicated phenomenon of fluid turbulence, physical measurements are also known to be multifractal [9]. In fluid turbulence, it is common to study structure functions, which are the statistical moments of the increments of the signals at different scales, and their scaling behavior. Multifractality manifests itself as a nonlinear dependence of the scaling exponents on the order of the structure functions. This nonlinear dependence is equivalent to the scale dependence of the probability density functions (PDFs) of the increments of the signals at different scales. These ideas of structure functions in fluid turbulence were employed to analyse healthy HRV and similar multifractality, with a scale dependence of the PDFs of the heart rate increments at different scales or between different number of beats, was indeed found [10]. This analogy of human HRV to fluid turbulence was further exploited and a hierarchical structure found in fluid turbulence [11] was shown to exist also in human HRV, with different parameters for heart rate fluctuations in healthy and pathological states [12]. The different values of the parameters can thus be used to quantify the multifractal character of healthy HRV and its loss in pathological HRV. On the other hand, in a recent detrended analysis [13] that aims to eliminate the non-stationarity of heartbeat data, “scale invariance of the PDFs of detrended healthy heart rate increments” was reported, and interpreted as an indication that healthy heartbeat dynamics are in a critical state. This finding appears to be in contradiction to the multifractal character of healthy HRV and needs clarification.

In this paper, we resolve this apparent paradox and further establish that human heartbeat dynamics exhibit a general scale invariance, which is shared by heart rate fluctuations in both healthy and pathological states.

This paper is organized as follows. We first review the statistical character of multifractality in healthy HRV in Sec. II. In Sec. III, we clarify that the scale invariance reported for healthy HRV in Ref. [13] is actually exhibited by the PDFs of the sum of detrended heartbeat intervals and demonstrate explicitly that the PDFs of detrended healthy heart rate increments are indeed scale dependent and thus consistent with the multifractal character of healthy HRV. Then we show that pathological heart rate fluctuations in patients with congestive heart failure also display this scale invariance. Our finding thus shows that such scale invariance cannot be an indication of healthy human heartbeat dynamics being in a critical state. In Sec. IV, we show that this general scale invariance in human heartbeat dynamics is non-trivial in that it is absent in multifractal turbulent temperature measurements in thermal convective flows. In Sec. V, we show that the essential effect of the detrended analysis
is to take out the local mean from the data. Finally, we summarize and conclude our paper in Sec. VI.

II. STATISTICAL SIGNATURE OF MULTIFRACTALITY

For completeness, we first review how the multifractality of healthy human HRV can be studied using the ideas of structure functions in fluid turbulence. Consider a dataset of human heartbeat intervals $b(i)$, where $i$ is the beat number. The beat-to-beat interval is also known as RR interval as it is the time interval between successive “R” peaks in the electrocardiogram (ECG) time signal. The value of $b(i)$ varies from beat to beat and this variation is the human HRV. Following the ideas of structure functions in turbulent fluid flows, one defines the heart rate increments between an interval of $n$ beats as,

$$\Delta_n b(i) = b(i + n) - b(i) ,$$  

(1)

which are differences between the heart rate intervals separated by $n$ beats. The $p$-th order structure functions are the $p$-th order statistical moments of the increments:

$$S_p(n) = \langle |\Delta_n b(i)|^p \rangle$$  

(2)

It was found that $S_p(n)$ for heart rate fluctuations in healthy state exhibits power-law dependence on $n$:

$$S_p(n) \sim n^{\zeta_p}$$  

(3)

for $n \approx 8 - 2048$. This power-law or scaling behavior is analogous to that found for velocity or temperature structure functions in turbulent fluid flows. Moreover, the scaling exponents $\zeta_p$ were found to depend on $p$ in a nonlinear fashion as in turbulent fluid flows. This implies that for healthy HRV, the standardized PDFs (with mean subtracted then normalized by the standard deviation) of $\Delta b_n$ changes with the scale or the number of beats $n$, and are thus scale dependent. In fact when Eq. (3) holds, the standardized PDFs of $\Delta b_n$ are scale invariant, i.e., independent of $n$, if and only if $\zeta_p$ is proportional to $p$. Hence the nonlinear dependence of $\zeta_p$ on $p$ or, equivalently, the scale dependence of the standardized PDFs of $\Delta b_n$ on $n$ is a characteristic signature of the multifractality of healthy HRV, in analogy to turbulent fluid flows.

III. SCALE INVARIANCE IN THE DETRENDED ANALYSIS

In contrast to physical measurements in turbulent fluid flows, human heartbeat interval data are often non-stationary. This non-stationarity is one possible reason for the relatively poor quality of scaling in HRV as compared to that in turbulent fluid flows. To eliminate the non-stationarity, a “detrended fluctuation analysis” has been introduced. This analysis was further developed to study detrended heart rate increments. The procedure of this detrended analysis consists of the following steps. First, $B(m)$, which is the sum of $b(j)$:

$$B(m) = \sum_{j=1}^{m} b(j) ,$$  

(4)

is calculated. Second, the dataset of $B(m)$ is divided into segments of size $2n$, and the datapoints in each segment is fitted by the best $q$th-order polynomial. This polynomial fit represents the “trend” in the corresponding segment. Third, these polynomial fits are subtracted from $B(m)$ to get $B^*(m)$, which are then “detrended”. Finally, the standardized PDFs of the increments of $B^*$:

$$\Delta_n B^*(i) = B^*(i + n) - B^*(i)$$  

(5)

for different values of $n$ are studied. Note that in Refs. $[13, 15]$, $\Delta_n B^*(i)$ was denoted as $\Delta_n B(i)$ and the symbol $s$ was used in place of $n$. The standardized PDFs of $\Delta_n B^*(i)$ for healthy heartbeat data were found to be independent of $n$, and this was referred to as “scale-invariance in the PDFs of detrended healthy human heart rate increments” in Ref. $[13]$. We shall show below that this conclusion is inaccurate.

Let us denote the detrended heartbeat interval by $b^*(i)$. From the detrended procedure described above, it is natural to define $b^*(i)$ by

$$B^*(m) = \sum_{j=1}^{m} b^*(j)$$  

(6)

Then

$$\Delta_n B^*(i) = \sum_{j=i+1}^{i+n} b^*(j)$$  

(7)

Thus the detrended heart rate increment between $n$ beats should be defined as

$$\Delta_n b^*(i) = b^*(i + n) - b^*(i)$$  

(8)

Hence $\Delta_n B^*(i)$ is the sum of detrended heartbeat intervals taken over $n$ beats rather than detrended heart rate increments. As a result, the observation of scale-invariant or $n$-independent standardized PDFs of $\Delta_n B^*$ does not necessarily imply that the standardized PDFs of $\Delta_n b^*$ are also $n$-independent. Indeed, one expects the contrary, namely, the standardized PDFs of $\Delta_n b^*$ should depend on $n$ as healthy human HRV is multifractal.

To investigate this issue, we study the scaling behavior of the statistical moments of $\Delta_n B^*$ and $\Delta_n b^*$. As the standardized PDFs of $\Delta_n B^*$ are $n$ independent, the scaling exponents for the statistical moments of $\Delta_n B^*$ should be proportional to the order of the moments. On the other hand, we expect the scaling exponents for the statistical moments of $\Delta_n b^*$ to have a nonlinear dependence on the order of the moments. We analyze healthy
heartbeat data that are taken from a database of 18 sets of daytime normal sinus rhythm data downloaded from public domain [16]. We follow the detrended procedure described above to get $\Delta_n B^*(i)$. We find that a polynomial of degree 3 $(q = 3)$ is sufficient to fit the "trend" as found in Ref. [13]. To get the detrended heart rate increment $b^*(i)$, we use Eq. (6) to get

$$b^*(i) = B^*(i) - B^*(i - 1)$$

for both $B^*(i - 1)$ and $B^*(i)$ belonging to the same segment and skip that datapoint when $B^*(i - 1)$ and $B^*(i)$ fall into different (consecutive) segments. Next, we evaluate the statistical moments

$$\hat{S}_p(n) = \langle |\Delta_n B^*(i)|^p \rangle$$

for $n$ between 16 to 1024 and $p$ between 0.2 to 3. On the other hand, $S^*_p(n)$ exhibits better scaling behavior with $n$ with exponents $\xi_p^*$:

$$S^*_p(n) \sim n^{\xi_p^*}$$

for $n$ between 32 to 1024 and $p$ between 0.2 to 3 (see Fig. 2).

In Fig. 3, we plot the scaling exponents $\hat{\xi}_p$ and $\xi_p^*$ as a function of $p$. It can be seen that $\hat{\xi}_p$ is proportional to $p$ confirming that the standardized PDFs of $\Delta_n B^*$ are scale invariant as reported in Ref. [13]. On the other hand, $\xi_p^*$ is not proportional to $p$ but changes with $p$ in a nonlinear manner, as expected from the multifractal character of healthy human HRV. Thus we have clarified that for healthy human HRV that is multifractal, the standard deviation of detrended heart rate increments between $n$ beats depend on $n$ while those of the sum of detrended heartbeat intervals taken over $n$ beats are $n$-independent.

We have also calculated the scaling exponents $\zeta_p$ of $S_p(n)$ of the statistical moments of untreated heart rate increments and the results are shown in Fig. 3 too. It can be seen that the detrended procedure does not change much the scaling exponents of the heart rate increments. We shall return to this in Sec. V.

It was suggested that this scale invariance of the standardized PDFs of $\Delta_n B^*$, the sum of detrended heartbeat intervals taken over $n$ beats, is an indication of healthy human heartbeat dynamics being in a critical state [13]. To check this suggestion, it would be useful to perform the same analysis to human HRV in pathological state. Thus, we perform the same analysis using 45 sets of daytime data from congestive heart failure patients, also downloaded from the same public domain [16].

The results for $\hat{\xi}_p$ and $\xi_p^*$ in this case are shown in Fig. 4. Note that $\xi_p^*$ is now approximately proportional to $p$, showing that the multifractality is lost in pathological HRV, consistent with earlier report [3]. On the other hand, $\hat{\xi}_p$ is again proportional to $p$, demonstrating that the scale invariance of the standardized PDFs of the sum of detrended heartbeat intervals is not restricted to healthy HRV but also exhibited by pathological HRV in congestive heart failure patients. Moreover, we find that the scale-invariant standardized PDFs are approximately exponential for both the healthy and pathological heartbeat data as shown in Figs. 5 and 6.

Since this scale invariance is found generally in heart rate fluctuations in both healthy and pathological state, it could not be an indication that healthy heartbeat dynamics are in a critical state. Common feature for both healthy and diseased human HRV was also reported before [20]; it would be interesting to explore whether this earlier feature and the present one are related.

**IV. DETRENDED ANALYSIS FOR TURBULENT TEMPERATURE MEASUREMENTS**

It is natural to ask whether this general scale invariance found in human heartbeat dynamics is trivial, i.e., whether it exists for any fluctuating data. In this section, we shall see that such scale invariance is absent in temperature data in turbulent flows so the answer to the above question is no.
frequency of 320 Hz such that in the experiment, the measurements were sampled at a constant rate intervals [see Eq. (11) for definition] for healthy heartbeat data for \( p \) ranges from 0.2 to 3.0. Same symbols as in Fig. 1. The curves have been shifted vertically for clarity.

The exponents \( \zeta_\phi/\zeta_2 \) (plusses) and \( \zeta'_\phi/\zeta'_2 \) (crosses) as a function of \( p \) for pathological heartbeat data from congestive heart failure patient. Both of them are close to the solid line of \( p/2 \).

The standardized PDFs of the temperature increments \( \Delta_n \theta(t_i) = \theta(t_{i+n}) - \theta(t_i) \) have been studied and found to change with \( n \) thus the temperature data in turbulent thermal convection are multifractal. Also, the temperature structure functions, \( R_p(n) \equiv \langle |\Delta_n \theta(t_i)|^p \rangle \) have been studied and found to have good relative scaling [13]:

\[
R_p(n) \sim [R_2(n)]^{\xi_p/\xi_2} \label{14}
\]

We calculate \( \Theta(t_m) = \sum_{j=1}^{m} \theta(t_j) \) and repeat the procedure of the detrended analysis, as discussed in Sec. III with \( B(m) \) replaced by \( \Theta(t_m) \), to obtain \( \Delta_n \Theta^*(t_i) \) and \( \Delta_n \theta^*(t_i) \). We then calculate the corresponding statistical moments \( \hat{R}_p(n) \equiv \langle |\Delta_n \Theta^*(t_i)|^p \rangle \) and \( \hat{R}_p^*(n) \equiv \langle |\Delta_n \theta^*(t_i)|^p \rangle \) and their respective relative exponents \( \xi_p/\xi_2 \) and \( \xi'_p/\xi'_2 \), defined by:

\[
\hat{R}_p(n) \sim [\hat{R}_2(n)]^{\xi_p/\xi_2} \label{15}
\]
\[
\hat{R}_p^*(n) \sim [\hat{R}_2^*(n)]^{\xi'_p/\xi'_2} \label{16}
\]

Our results are shown in Fig. 7. Again we find that \( \xi'_p/\xi'_2 \) deviates from \( p/2 \), as expected from the multifractal character of the turbulent temperature measurements. However, interestingly \( \xi_p/\xi_2 \) deviates from \( p/2 \) too, showing that the standardized PDFs of \( \Delta_n \Theta^* \) are scale dependent and changing with \( n \). To show this deviation more clearly, we plot \( \xi_p/\xi_2 - p/2 \) versus \( p \) in the inset of Fig. 7.

Indeed, the standardized PDFs of \( \Delta_n \Theta^* \) changes from stretched-exponential to exponential to Gaussian as \( n \) increases from 4 to 4096, as shown explicitly in Fig. 8. This

**FIG. 2:** The statistical moments \( S_p^*(n) \) of detrended heart rate intervals [see Eq. (11) for definition] for healthy heartbeat data for \( p \) ranges from 0.2 to 3.0. Same symbols as in Fig. 1. The curves have been shifted vertically for clarity.

**FIG. 3:** The exponents \( \zeta_\phi/\zeta_2 \) (plusses), \( \zeta'_\phi/\zeta'_2 \) (crosses), and \( \zeta_\phi/\zeta_2 \) (circles) as a function of \( p \) for healthy heartbeat data. It can be seen that \( \zeta_\phi/\zeta_2 \) is close to \( p/2 \) which is shown as the solid line.

**FIG. 4:** The exponents \( \zeta_\phi/\zeta_2 \) (plusses) and \( \zeta'_\phi/\zeta'_2 \) (crosses) as a function of \( p \) for pathological heartbeat data from congestive heart failure patient. Both of them are close to the solid line of \( p/2 \).

Specifically, we apply the detrended analysis to temperature measurements taken in turbulent thermal convective flows [17]. In place of \( b(i) \), we now have \( \theta(t_i) \), the temperature measurement taken at time \( t_i \). In the experiment, the measurements were sampled at a constant frequency of 320 Hz such that \( t_i = i \delta t \) with \( \delta t = 1/320 \) s.
change of the standardized PDFs of \( \Delta_n \Theta^* \), the sum of detrended temperature measurements taken over \( n \) sampling intervals, with \( n \) is similar to the change of the standardized PDFs of the temperature increments \( \Delta_n \theta \) with \( n \) as reported in Ref. [13].

As can be seen in Fig. 5, \( \xi_p^*/\xi_2^* \) are close to \( \xi_p/\xi_2 \), indicating again that the detrended procedure does not affect the scaling exponents of the temperature increments. We shall understand this in the next section.

V. THE ESSENTIAL EFFECT OF THE DETRENDED ANALYSIS

In this section, we shall explore and understand what the detrended procedure does to the data. As discussed in Sec. III, the “trend” in the data is estimated by a polynomial fit in each segment of the dataset of \( B(m) \), and we have used a polynomial of degree 3. We check that our results do not change much when a polynomial of a lower degree is used instead. In particular, we obtain similar results by using a linear fit of the different segments of \( B(m) \). In the following, we shall get explicit results for “detrended” \( B^* \) when the “trend” is estimated by a linear fit.

Let us focus on the \( l \)th segment of \( B(m) \) with \( m_1 \leq m \leq m_2 \), where \( m_1 = (l-1)(2n) + 1 \) and \( m_2 = l(2n) \) for some \( l \). \( l \) runs from 1,2,3,... for all the segments. Denote the best linear fit to this segment by \( a_l \) \( m + c_l \) where the fitting constants \( a_l \) and \( c_l \) depend on \( l \). The fitting constant \( a_l \) can be reasonably well approximated by the slope in this segment:

\[
a_l \approx \frac{B(m_2) - B(m_1)}{2n - 1} \quad (17)
\]

Using Eq. (1), we have

\[
a_l \approx \frac{\sum_{j=m_1+1}^{m_2} b(j) - \bar{b}_l}{2n - 1} = \bar{b}_l \quad (18)
\]

where \( \bar{b}_l \) is the local average of \( b(j) \) in the \( l \)th segment. Recall from Sec. III that \( B^* \) is \( B \) subtracting the best linear fit and use Eq. (4), we have

\[
B^*(m) \approx \sum_{j=1}^{m} [b(j) - \bar{b}_l] - c_l \quad (19)
\]

and

\[
B^*(m+n) \approx \begin{cases} 
\sum_{j=1}^{m+n} [b(j) - \bar{b}_l] - c_l & m+n \leq m_2 \\
\sum_{j=1}^{m+n} [b(j) - \bar{b}_{l+1}] - c_{l+1} & m+n > m_2
\end{cases} \quad (20)
\]

Thus using Eq. (3), we get

\[
\Delta_n B^*(m) \approx \sum_{j=m+1}^{m+n} [b(j) - \bar{b}_l] \quad (21)
\]

for \( m+n \leq m_2 \) and

\[
\Delta_n B^*(m) \approx \sum_{j=m+1}^{m_2} [b(j) - \bar{b}_l] + \sum_{j=m_2+1}^{m+n} [b(j) - \bar{b}_{l+1}] \quad (22)
\]

for \( m+n \geq m_2 \).
that the approximation that the two linear fits of the
pare the results obtained with those from the detrended
heartbeat data with the local mean subtracted and com-
subtract the local average from the data. Hence what the detrended analysis essentially does is to
of and study the scaling behavior of the statistical moments
for $m + n > m_2$. To obtain Eq. (22), we make use of
the approximation that the two linear fits of the $l$th and
$(l + 1)$th segments intersect at $m = m_2$:
\[
\begin{align*}
b_l m_2 + c_l & \approx b_{l+1} m_2 + c_{l+1} \\
\Rightarrow c_{l+1} - c_l & \approx \tilde{b}_{l+1} - \tilde{b}_l
\end{align*}
\]
(23)

Comparing Eqs. (21) and (22) with (11), we see immedi-
ently that the detrended heartbeat interval $b^\ast$ is given
approximately by
\[
b^\ast(j) \approx b(j) - \tilde{b}_l
\]
(24)

Hence what the detrended analysis essentially does is to
subtract the local average from the data.

To verify this directly, we redo the analysis for the
heartbeat data with the local mean subtracted and com-
pare the results obtained with those from the detrended
analysis. We define
\[
\begin{align*}
\tilde{b}(j) & = b(j) - \tilde{b}_l \\
\tilde{B}(m) & = \sum_{i=1}^{m} \tilde{b}(i)
\end{align*}
\]
(25)
(26)

and study the scaling behavior of the statistical moments
of
\[
\begin{align*}
\Delta_n \tilde{b}(j) & = \tilde{b}(j+n) - \tilde{b}(j) \\
\Delta_n \tilde{B}(j) & = \tilde{B}(j+n) - \tilde{B}(j) = \sum_{i=j+1}^{j+n} \tilde{b}(i)
\end{align*}
\]
(27)
(28)

FIG. 7: The three relative exponents $\hat{\xi}_p/\hat{\xi}_2$ (plusses),
$\hat{\xi}_p^{\ast}/\hat{\xi}_2^{\ast}$ (crosses), and $\hat{\xi}_p/\hat{\xi}_2$ (circles) for temperature measurements in turbulent convective flows. All the three relative exponents deviate from $p/2$ (the solid line). The deviation $\hat{\xi}_p/\hat{\xi}_2 - p/2$ is plotted versus $p$ in the inset to show clearly that $\hat{\xi}_p/\hat{\xi}_2$ is not proportional to $p$.

FIG. 8: The standardized PDFs of $\Delta_n \Theta^\ast$ for temperature measurements taken in turbulent thermal convective flows with $n = 4$ (solid), $n = 32$ (dashed), $n = 256$ (dot-dashed) and $n = 4096$ (dotted). The dependence of the standardized PDFs on $n$ is clearly seen.

We compare $\alpha_p$ and $\hat{\alpha}_p$ with $\zeta_p^\ast$ and $\hat{\zeta}_p$ respectively. As seen from Fig. 3, the results are in good agreement confirming that the essential effect of the detrended analysis is to eliminate the local average from the data.

As a result, $b^\ast(j + n) - b^\ast(j) \approx \tilde{b}(j + n) - \tilde{b}(j)$ will be close to $b(j + n) - b(j)$, that is, the heart rate increments are not affected much by the detrended analysis. This explains why $\zeta_p^\ast$ are close to $\zeta_p$ (see Fig. 3) and similarly why $\hat{\zeta}_p$ are close to $\hat{\zeta}_p$ (see Fig. 7). On the other hand, $B^\ast(j + n) - B^\ast(j) \approx \tilde{B}(j + n) - \tilde{B}(j)$ can be different from $B(j + n) - B(j)$, and thus the sum of detrended heartbeat intervals could have different statistical features from those of the sum of untreated heartbeat intervals.

VI. SUMMARY AND CONCLUSIONS

Understanding the nature of the complicated human
HRV and thus human heartbeat dynamics has been the
subject of many studies. An interesting and intrigu-
ing finding reported in earlier studies [3] is that in the
healthy state, human heart rate fluctuations display mul-
tifractality, and that this multifractal character is lost

with $n$. The corresponding exponents are denoted by $\alpha_p$
and $\hat{\alpha}_p$, which are defined by
\[
\begin{align*}
\langle |\Delta_n \tilde{b}(i)|^p \rangle & \sim n^{\alpha_p} \\
\langle |\Delta_n \tilde{B}(i)|^p \rangle & \sim n^{\hat{\alpha}_p}
\end{align*}
\]
(29)
(30)
for heart rate fluctuations in pathological state such as congested heart failure. Based on an analogy with measurements in turbulent fluid flows, which are known to have multifractal character as well, such multifractality in healthy HRV can be manifested as a scale dependence of the standardized PDFs of the increment of heartbeat intervals at different scales or between different number of beats. A detrending analysis aiming to eliminate the non-stationarity of heartbeat data has been performed, and "scale invariance of the PDFs of detrended healthy human heart rate increments" reported [13]. We have resolved this paradox by clarifying that the scale invariance found in the detrended analysis is actually exhibited by the PDFs of the sum of detrended heartbeat intervals taken over different number of beats, and demonstrating explicitly that the PDFs of detrended healthy heart rate increments are scale dependent. We have understood the essential effect of this detrended analysis is to eliminate the local average from the heartbeat data. We have further found that this scale invariance of the PDFs of the sum of heartbeat intervals, with the local mean subtracted, is displayed also by heart rate fluctuations of congestive heart failure patients. In both the healthy and pathological states, such scale-invariant PDFs are close to an exponential distribution. On the other hand, this scale invariance is absent in the multifractal temperature measurements in turbulent thermal convective flows. Hence we have found an interesting scale invariance of exponential PDFs in human heartbeat dynamics, which is exhibited generally by heart rate fluctuations in both healthy and pathological states. Since this scale invariance is a general feature, it cannot be an indication of the healthy state being critical, in contrast to what was claimed in earlier studies [13].

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