Interaction between collisionless galactic discs and nonaxissymmetric dark matter haloes

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ABSTRACT

Using N-body simulations ($N \sim 10^6 - 10^7$), we examine how a non-axisymmetric dark halo affects the dynamical evolution of the structure in collisionless (stellar) discs. We demonstrate how the model parameters such as mass of the halo, initial conditions in the disc and the halo axes ratio affect morphology and kinematics of the stellar discs. We show that a non-axisymmetric halo can generate a large-scale spiral density pattern in the embedded stellar disc. The pattern is observed in the disc for many periods of its revolution, even if the disc is gravitationally over-stable. The growth of the spiral arms is not accompanied by significant dynamical heating of the disc, irrelevant to its initial parameters. We also investigate transformation of the dark halo’s shape driven by the long-lived spiral pattern in the disc. We show that the analysis of the velocity field in the stellar disc and in the spiral pattern gives us a possibility to figure out the spatial orientation of the triaxial-shaped dark halo and to measure the triaxiality.

Key words: Galaxies: evolution; galaxies: haloes; galaxies: kinematics and dynamics; galaxies: spiral

1 INTRODUCTION

Both dynamic models of real galaxies and cosmological simulations give a strong evidence that the stellar discs of spiral galaxies are embedded into massive dark haloes. The halo mass and its influence on the disc evolution is the matter of debates. Among the different ways to put constraints on the halo mass it is worth mention the use of gravitational stability criteria in the disc’s plane (Safronov 1964; Toomre 1964; Lynden-Bell and Ostriker 1964; Khoperskov et al. 2004), a condition of stellar disc stability to bending perturbations (Zasov et al. 1991; Sotnikova et al. 2003; Khoperskov et al. 2010), simulation of polar rings (Whitmore et al. 1987; Sackett and Sparke 1992; Reshetnikov and Sotnikova 2000; Iodice et al. 2003), and the gravitational lensing (Koopmans et al. 1998; Matter et al. 2004; Oguri et al. 2003).

Morphology of galactic spiral patterns also indicates significant amount of the dark matter within the optical radius (Athanassoula et al. 1987; Korchagin et al. 2000; Griv and Gedalin 2004). Estimates of the dark mass contribution can also be obtained from the vertical structure of stellar discs analysis (e.g. Bahcall 1984; Zasov et al. 1991; Bizyaev and Mitronova 2002; Zasov and Bizyaev 2002; Korchagin et al. 2003; Bizyaev and Mitronova 2009). Maximum disc models suggest that $M_b/(M_d + M_h) \approx 0.5 - 1$, where $M_b$ is the halo mass within the optical radius, $M_b$ is the mass of stellar bulge, $M_d$ is the mass of stellar disc (Bottinelli 1993). Note that some galactic discs (especially low luminous ones) are less massive than that required by the maximum disc assumption (Zasov et al. 2004; Bershady et al. 2011; Westfall 2011; Saburova and Zasov 2012). At any case, one may conclude that in most galaxies (at least of normal surface brightness) masses $M_b$ and $M_d$ are comparable within the optical borders.

Modern cosmological experiments suggest a triaxial distribution of the dark matter in the haloes (e.g. Zentner et al. 2005; Kuhlen et al. 2007; Muoz-Cuartas et al. 2011; Klypin et al. 2011, and references therein). Intriguing problem is a possibility to explain the generation and support for the long-lived and extended spiral structure in the stellar discs by triaxial potential of the dark halo. True mechanism of the formation of the spiral structure is still a subject of discussions. Although a bar or a close satellite may induce the spiral density wave, these events provide a relatively short term support. In contrast, the dark halo can keep its triaxiality and support the spiral structure over much longer time scale. The problem is to estimate whether the existing...
with a large number of particles (as the Via Lactea
Hayashi et al. 2007; Widrow 2008). Simulations conducted
and distance to the centre (Bailin and Steinmetz 2005;
c and
more asymmetric at their central regions than at the periph-
Figure 1. Evolution of the stellar velocity dispersion $c_\gamma$ at dif-
ferent radii $r$ in a model with spherically symmetric halo. The
curves $1$ demonstrates the central zone of the disc, the curve $8$ –
the periphery of the disc. The arrow indicates the time interval
corresponding to one period of rotation $\tau_0$ at the disc’s periphery.

development from the axial symmetry is quantitatively enough
to support the spiral pattern in collisionless gravitationally
stable discs.

Usually the distribution of the volume density in the
halo is approximated as follows:

$$g_\nu(x, y, z) = g_\nu(\xi), \quad \xi \equiv \sqrt{\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}},$$

(1)

where the $a = b = c$ case corresponds to the spherical sym-
metry in the density distribution. The model with $a = b \neq c$
corresponds to an axisymmetric halo coplanar with the stellar
disc. Note that in this labeling convention, the relative
ordering of $a$, $b$, and $c$ is unconstrained. The ratios $q = b/a$, $s = c/a$ describe the deviation of the density profiles front the spherical symmetry. For simplicity, our further consid-
eration of the dark matter (DM) haloes assumes that $a \geq b$
and $c = a$, where $a$ and $b$ are the halo major and minor axes
in the disc plane.

Cosmological $N$-body simulations show that the tria-
xiality parameters $s$ and $q$ depend on the halo mass
and distance to the centre (Bahe and Steinmetz 2003;
Havashi et al. 2007; Widrow 2008). Simulations conducted with
a large number of particles (as the Via Lactea
with $N = (1 - 2.3) \cdot 10^8$. Diemand, Kuhlen and Madau
(2007); Kuhlen et al. (2007); Debattista et al. (2008);
Diemand and Moore (2009)) reveal that the dark haloes are
more asymmetric at their central regions than at the periph-
ery where the ratios $b/a$ and $c/a$ are close to to unit.
For the topic of this paper the most important parameter of the
halo’s shape is the axes ratio $q$.

Observing constraints on the dark halo geometrical pa-
rameters come from studies of globular clusters, variable
stars (such as RR Lyrae), blue horizontal branch stars, low-
metallicity giants, red giants, Cepheids (W Vir type) and
F-dwarfs (Keller et al. 2008). Anisotropy of the velocity dis-
tribution of objects in the stellar halo of our Galaxy indicates
an asymmetric distribution of DM (Morrison et al. 2009).
Stellar haloes in other spiral galaxies also demonstrate sig-
nificant deviation from spherical symmetry in the haloes;
$c/a \sim 0.3$ is suggested by Cooper et al. (2010). According to
the Sloan Digital Sky Survey (SDSS) and Sloan Extention,
our galactic spheroid has a triaxial shape with $s = 0.63,
q = 0.75$ (Newberg et al. 2007).

The polar ring galaxies are unique objects for investigation
of the DM halo shape (Combes 2006). Whitmore et al.
(1987) analysed the polar ring kinematics in several galaxies
and found significant deviation of the DM distribution from
the spherical shape: $c/a \sim 0.5$ (Sackett and Pogge 1993).
$c/a \sim 0.3 - 0.4$ (Sackett et al. 1994). A prolate shape
of the dark matter haloes was inferred by Codice et al. (2003)
based on the analysis of location of the polar ring galaxies
on the Tully-Fisher diagram.

Dynamics of decaying satellites also helps to constrain the
dark halo properties, such as its shape, orientation, mass
distribution (Ibata et al. 2001B; Helmi 2004; Fellhauer et al.
2006). According to Sesar et al. (2011), SDSS data suggest
that the Milky Way’s dark halo is oblate ($s = 0.7$) and non-
axisymmetric in the disc plane ($q = 0.98$) within 28 kpc of
the centre.

An assumption of the axissymmetric potential for the
Milky Way halo applied to the Sgr dSph tidal stream yields
controversial results: while the leading arm of the stream
requires an oblate halo, the radial velocity distribution of stars
in the arm agrees better with a prolate halo (Vivas et al.
2003). Note that the introduction of a triaxial potential helps
to resolve this ambiguity (Law et al. 2009).

Statistical analysis of shapes of the spiral galaxies
also reveals some deviation from the disc axial symmetry,
which in turn may evidence the halo triaxiality. According
to Ryder (2006), the disc ellipticity is $0.02 - 0.3$ in the
$K_s$ photometric band. The ellipticity varies in dependence
of the photometric band and of the galaxy morphological
type (Ryder 2006).

Despite direct and non-direct evidences of the halo
triaxiality mentioned above, its shape and ellipticity in
specific galaxies remains poorly known. Understanding
of the role that the asymmetry of the halo potential
is not easy. Dynamics of decaying satellites can play in dynamical evolution of stellar discs in
galaxies remains incomplete. El-Zant and Haberl (1999);
Ibata and Hozumi (2005); Tutukov and Fedorova (2000;
Berentzen and Shlosman (2000); Hayashi and Navarro
2006); Hayashi et al. (2005) studied different aspects of
how a non-axisymmetric gravitational potential affects
the galactic disc’s dynamics. The triaxial halo allows to
explain the observed shape of rotation curves in low surface
brightness (LSB) galaxies (Hayashi and Navarro
2005; Widrow 2005).

Galaxies with warped discs were successfully repro-
duced within the models with triaxial dark matter (DM)
halo (Dubinski and Chakrabarty 2005; Roskar et al. 2010).
The effects of the bar response to the triaxial halo perturba-
tions and bar dissolution in prolate halo were studied by
Machado and Athanassoula (2011) and Kazantzidis et al.
(2010). Numerous $N$-body simulations are usually used
to study the formation and evolution of stellar bars em-
bedded in triaxial dark matter haloes. Athanassoula (2002;
Berentzen et al. 2004; Dubinski et al. 2009). The triaxial
halo can destroy stellar bars due to the angular momentum
exchange (Berentzen and Shlosman 2004). The decreasing
of the bar pattern speed was demonstrated in models with
triaxial dark matter halo (Ceverino and Klypin 2007).

Beck et al. (2002) explained the spiral struc-
ture extended far beyond optical limits in a blue compact
dwarf galaxy NGC 2915 by the presence of a triaxial dark halo, Masset and Bureau (2003) modeled the spiral structure in the outer region of the gaseous disc of NGC 2915 generated by a triaxial dark matter potential. The main result of these papers is that triaxial DM halo is able to produce a two-arm spiral pattern in external regions beyond the optical radius. Tinker and Ryden (2002) investigated effects of rotating triaxial haloes in the disc galaxies using $N$-body simulations with the rigid halo potential and found that the average pitch angle of the spiral structure strongly anticorrelates with the total mass of the system (disc+halo). Dubinski and Chakrabarty (2009) applied $N$-body simulations to demonstrate the formation of warps and spirals in isolated discs embedded in triaxial haloes. They showed that the spiral pattern may emerge in gravitational stable stellar disc through interactions with a non-axisymmetric halo. Note that the evolution of the grand design spiral structure in stellar discs embedded in the triaxial dark matter haloes remains poorly studied.

In our previous paper (Khoperskov et al. 2012) we studied the dynamics of a gaseous galactic disc in a non-axisymmetric gravitational potential in general. In the present paper we perform $N$-body modelling (with $N \sim 10^6 - 10^7$) of stellar collisionless discs embedded in a non-axisymmetric dark halo with focusing on the formation of the spiral pattern, its morphology and kinematics. In Section 2 we describe our modelling technique and the initial conditions in the our galactic discs. The spiral structure formation and the kinematics of the density waves are analysed in Section 3. Discussion and main conclusions are given in Sections 4 and 5, respectively.

2 MODELLING COLLISIONLESS STELLAR DISCS

Throughout this paper we study the dynamics of the galactic discs in the frames of a self-consistent $N$-body problem by solving the motion equations for equal mass test particles:

$$\frac{d^2 \mathbf{r}_j}{dt^2} = -m \sum_{i=1 \atop i \neq j}^N \frac{(\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^3} \frac{\partial \Psi_b}{\partial \mathbf{r}_j} + \frac{\partial \Psi_h}{\partial \mathbf{r}_j}, \quad j = 1, 2, \ldots, N,$$

here $\mathbf{r}_j = \{x, y, z\}$ is the coordinate vector of $j$-th particle, $M_d$ is the total stellar disc mass, $\Psi_b(x, y, z)$ is the bulge potential, $\Psi_h(x, y, z, t)$ is the halo potential and $m = M_d/N$ is the mass of a test particle.

The force of gravity in the disc is determined by the summation over all particles in Equation (2) and calculations via the TreeCode Top Down algorithm (Barnes and Hut 1986). Initial distribution functions in the equilibrium stellar disc are described in detail in our previous papers (Khoperskov et al. 2003, 2010). The schemes of the 2nd and 4th orders we apply provide a good accuracy of integration of Equations (2).

We introduce the following dimensionless model units: Our spatial distance unit is assumed to be 10 kpc, the gravitational constant is chosen to be $G = 1$, the time unit is 91 Myr, the velocity is expressed in the units of 110 km s$^{-1}$. Assumptions $G = 1, V = 1$ lead to the total mass of model galaxy equal to about $10^{11} M_\odot$ within $R = 1$.

The number of particles in our models is $N = (1 - 10) \cdot 10^6$. The potential for $i$-th particle at $j$-th point is $\Psi_{ij} = -m/\sqrt{r_{ij}^2 + r_0^2}$, where the cut off radius is $r_0 \leq 10^{-3}$ (which corresponds to $\leq 10$ pc). The opening angle parameter $\Theta$ is assumed to be $0.1 - 0.5$, although we did not find significant variations of the results between models with different $\Theta$. The results presented in this paper were obtained assuming $\Theta = 0.1$, if alternative value is not specified. This values of $r_0$ and $\Theta$ determine the accuracy of the calculations. The integration step is chosen to be $2 \cdot 10^{-3}$, which approximately corresponds to $2 \cdot 10^5$ yr.

Basic model parameters are constrained in the range suggested by observations as follows:

- Relative dark halo mass $1 \leq \mu = M_h/M_d \leq 4$ within the limits of the stellar disc $R = 4r_d$, where $r_d$ is the radial scalelength of the disc.
- $\mu_h = M_h/M_d = 0.3$ is the relative stellar mass of the bulge with scalelength $r_0 = 0.1 - 0.3$.
- $\tau_v = 3$ is the period of rotation at the periphery of the disc.
- Nonaxisymmetric halo parameter $\varepsilon = 1 - b/a = 0.25$ in the disc plane.
- $\Omega_\varepsilon = 0.3$ is the angular rotation velocity of the halo.
- $\tau_\varepsilon = 0 - 20$ is the time scale of variation of the non-axisymmetry parameter from 0 to $\varepsilon$.

1 It should be emphasised that the value $\varepsilon = 1 - b/a$ calculated for the dark halo volume density distribution differs from that for the gravitational potential $\varepsilon \psi$. Approximately, $\varepsilon \psi \approx 3\varepsilon$ (Binney and Tremaine 2008).
We assume the exponential surface density distribution in the disc as an initial condition: \( \sigma(r) = \sigma_0 \exp(-r/r_d) \), where \( r_d = 0.25 \) of dimensionless distance units is the radial scale length in the disc. At the initial stage, all stellar disc particles are situated within the radius \( r \leq 5r_d = 1.25 \). We assume a flat rotation curve with the maximum velocity \( V_{\text{max}} \approx 2 \) in dimensionless units, which corresponds to the rotation period of \( \tau_d \approx 3 \) at the disc’s periphery.

Following Begeman et al. (1991), we use a pseudo-isothermal model of the density distribution in the halo:

\[
\rho_h(r) = \frac{\rho_{h0}}{1 + (r/a)^2}. \tag{3}
\]

This distribution of the dark mass provides a constant circular velocity \( V_c \) for \( r \gg a \). The gravitational potential can be written

\[
\Psi_h(x, y, z) = 4\pi G \rho_{h0} a^2 \left\{ \ln(\xi) + \frac{\arctg(\xi)}{\xi} + \frac{1}{2} \ln \left( 1 + \xi^2 \right) \right\}, \tag{4}
\]

(Khoperskov et al. 2012), which corresponds to the density law (3) for the case of \( a = b = c \).

Note that the cosmological simulations predict that virialised dark matter haloes should have a universal density profile. For a spherically averaged density distribution it can be written (Navarro et al. 1997):

\[
\rho_h(r) = \frac{\rho_{h1}}{(r/a)(1 + (r/a))^2}. \tag{5}
\]

Its distinctive feature is the inner density cusp, which however may be dissolved due to its interaction with baryonic matter in the central region of galaxies (see i.e. Gustafsson et al. (2005); Maccio et al. (2012); Khoperskov et al. (2012)). Moreover, inevitable singularity emerges at \( r = 0 \) in the case of transformation from a spherical NFW profile to the triaxial shape. Another widely used dark matter density distribution is Burkert’s profile (Burkert 1995), which usually fits observing data very well (Gentile et al. 2003). It is very close to the NFW profile: it has a cored density distribution in the inner part \( r < a \). Several more halo density profiles were introduced, such as exponential profile (Fux 1997) and Einasto profile (Einasto 1965). All density distribution functions mentioned above (except 4) allow generalization of the spherical dark matter distribution shape to the triaxial case using the formal replacement \( r \rightarrow \xi \) (see eq. 1). Note however, that the choice of the accepted law for \( \rho_h(r) \) makes only a small effect on the results described below.

Rotation pattern in the dark haloes predicted by \( \Lambda \)CDM cosmological models is similar to that in giant elliptical galaxies, where the kinetic energy of the rotation is much smaller than that in the stellar chaotic motions. The origin and properties of angular momentum of DM haloes were investigated by Vitvitska et al. (2002); Maccio et al. (2007); Bett et al. (2007). For the disc galaxies cosmological simulations suggest slow rotation of their dark matter haloes within \( R_{\text{opt}} \). The rotation (tumbling) of the overall shape of the halo in \( \Lambda \)CDM simulations was discussed by Bailin and Steinmetz (2004); Bryan and Cress (2007).

Authors conclude that the tumbling rates of the haloes are less than one km s\(^{-1}\) kpc\(^{-1}\). Evidently, a disc subsystem rotates much faster than a halo. Hereafter we use a non-rotating or slow rigidly rotating model halo with the angular velocity \( \Omega_h \), which places the corotation radius at the disc’s periphery \( \Omega h \approx V_{\text{max}}/R_{\text{opt}} \). Our principal results are shown for the case of rigid DM halo, while the situation with the live halo is considered in Section 3.4.

### 3 DISC DYNAMICS: PROPERTIES OF THE SPIRAL STRUCTURE

Propagation of the density waves of gravitational nature in a self-gravitating stellar disc requires that the stellar velocity dispersion were close to the stability condition \( Q_r \approx 2 \) or \( Q_r \approx 3 \) at the periphery of the disc, see Khoperskov et al. (2003), where \( Q_r \) is the Toomre stability parameter (Toomre 1964). The case \( Q_r = 1 \) is unstable due to non-radial perturbations. In the following section we consider gravitationally stable discs \( Q_r \approx 2 - 4 \) at \( r \sim (1 - 2) \cdot r_d \) and study the pure effects from the non-axisymmetric dark matter distribution. We also compare the properties of the spiral pattern generated by triaxial haloes in the cases of stable and unstable discs (see Section 3.3).

#### 3.1 Dynamically stable stellar discs

The initial distributions of the radial \( c_r \), azimuthal \( c_\phi \) and vertical velocities dispersions \( c_z \) are chosen to be high enough to prevent the disc gravitational instability, which is able to generate spiral waves in the initially axisymmetric model by its own. The stellar velocity dispersion should be high enough to support the gravitational stability against
ident that in all models with \( \varepsilon > 0.05 \) the presence of the non-axisymmetric dark halo excites the formation of a two-arm spiral pattern in the stellar disc. In figure 2 we show snapshots of the typical evolution of the surface density in the model with \( \varepsilon = 0.2 \). Within 1-2 periods of rotation \( t_0 \), a two-armed spiral structure forms in the initially axisymmetric exponential disc. A nested \( \Theta \)-like structures and a central bar can be identified in the inner disc region \( (r \lesssim 0.2 \lesssim r_d = 0.25) \). The rigid triaxial halo models (models without the disc feedback to the halo) generate a long lasting global spiral pattern that extends from the centre to the disc periphery. It is important to note that interactions between the disc and non-axisymmetric dark halo’s potential do not introduce any significant systematic variations to the azimuthally-averaged disc parameters.

Evolution of \( c_\ell(r) \), \( V(r) \) and \( \sigma(r) \) is shown in figure 3. Note that the centre of the disc (curves 1 and 2) remains unperturbed over the course of the simulations, whereas quasi-periodic radial oscillations can be observed, especially in the outer part of the disc. In contrast to the global gravitational stochastic spiral structure that develops in the slightly unstable discs [De Simone et al. 2004], here the heating of the stellar disc is insignificant (except for the short initial stage of the experiment).

The model parameters \( \mu, \mu_h, r_h \) and \( r_0 \) significantly affect the growth increment of the spiral structure. The amplitude of the stellar density wave at early stages in the simulations is primarily determined by the relative halo mass parameter \( \mu \) (see figure 4). However, after several \( t_0 \) two-arm spirals reach the similar amplitudes at a fixed value of \( \varepsilon \), independently of \( \mu \). Our modelling demonstrates the dependence of the eventual amplitude \( A_2 \) of the spiral waves on the triaxiality parameter \( \varepsilon \) for the spiral mode \( m = 2 \) (see figure 5). It seems that \( \varepsilon \) is a crucial parameter, which defines the properties of the strength of the spiral structure as a whole. Despite the fact that the growth rate of the spiral waves depends on the relative mass of the halo (see figure 4), the final amplitude of the waves is determined by the non-axisymmetric parameter \( \varepsilon \) (see figure 5).

There are two distinctive properties of the spiral structure generated by the triaxial halo. First, the spirals are generated in a sufficiently hot (i.e. gravitationally stable) disc with the Toomre parameter \( Q_T \sim 2 - 4 \), in contrast to the standard density waves theory. Therefore, this mechanism can produce spirals even in the over-stable stellar discs, e.g. in the outer regions of galaxies far beyond their optical size limits, where the disc surface density is much lower than the gravitational stability threshold. Second, the formation of the spiral structure leads to a weak stellar disc warming-up during the initial stage of the spiral pattern development, and after that the velocity dispersion components only slightly increases even in the case of high amplitude spiral waves (figure 3). These properties are significantly different from those seen in \( N \)-body simulations of gravitationally unstable discs, in which the development of the gravitational instability is accompanied by significant growth of the velocity dispersion during one period of rotation. It should be noted that a slight disc heating for large amplitude waves that we observe in the our modelling may be a result of limited accuracy in the numerical solution of the motion equations (2). Indeed, we found that the disc heating decreases with the increasing of the number of test
particles. The heating up effect may also be attributed to the lack of perfect disc equilibrium at the beginning of the simulations. The final conclusion about the low disc heating requires modelling with higher spatial resolution.

3.3 Simulations with different stellar disc initial state

While we started simulations with the initially stable disc in the previous section, we should consider whether the initial gravitational instability of the disc can qualitatively change the results of section 3.2. To reveal the effect of self-gravity we have performed simulations for initially unstable disc by assuming the relative halo mass \( M_R/M_d = 1 \) and \( Q_t = 1 \). We started with the axisymmetric halo to be sure that the evolution of the disc and spiral structure is fully determined by the global gravitational instability of the entire disc, and that the formation of the spiral waves is accompanied by the disc heating up to the marginal stability condition (see Figure 6). After that we ran another simulation with the same initial conditions, but assuming the halo with \( \kappa = 0.1 \). Relatively low halo mass led to slower growth of the wave amplitude than that shown in Figure 4 for \( \mu = 1 \). Curiously, the final amplitude of the spiral waves is practically the same independently on the mechanism of their generation (either non-axisymmetric halo + gravitational instability, or pure gravitational instability). A general character of the evolution of the velocity dispersion is also very similar in both models. It confirms that the non-axisymmetric halo does not contribute essentially to the eventual disc heating.

To analyze the direct role of a triaxial halo and to separate it from effects of the disc initial state, below we compare different numerical models of evolution of gravitationally stable stellar discs in the halo field. It was assumed that the disc always remains in a quasi-equilibrium state even it is non-axisymmetric, since the halo shape changes slowly and adiabatically in our models. Model (a) is a “reference” model for the initially round-shaped disc described in Section 3.2. In model (b) we introduce a very large timescale for the halo triaxiality growth (\( \tau_h = 15 \)) for initially round disc to be sure that the disc has enough time to follow the slowly changing halo potential. In our third model (c) we use the initially elliptical disc with the same spatial major axis orientation as the non-axisymmetric halo. In our last model (d) we took the results of long-time evolution of a “reference” model of round disc in the non-axisymmetric halo at \( t = 20 \). After that the spiral structure was radially smoothed (which introduced slight ellipticity to the disc), and next simulation was started from this equilibrated, spiral-free initial state.

Results of modelling are shown in Figure 7. In all four models the spiral structure emerges. The main difference between the models is observed at the central part of the disc. In models (a) and (b) a \( \Theta \)-like structure forms. In the model (c) of initially elliptical disc a large bar was formed, and beyond the bar there is a tightly wound spiral structure. In the case (d) there is a small bar and very even spiral arms. Formation of a bar in the potential of triaxial DM halo links our models with those developed by [Berentzen et al. (2004)], where the bar generation and bar-halo interaction are also described. It is essential that our models demonstrate the emergence of the spiral arms in the very outer regions of the disc driven by the halo triaxiality, independently of the disc gravitational stability and its initial shape.

To analyze the properties of the spiral structure without considering the central disc regions, we considered the evolution of the Fourier harmonics and velocity dispersion beyond the radius \( r = 0.5 \) (the solid circle in Figure 7). De-
Discs to DM haloes interaction

3.4 Kinematics of the spiral structure

The non-stationary evolution of the spiral waves is characterised by quasi-periodic oscillations that are caused by the radial propagation of spiral waves (figure 3). The spiral wave pattern rotates with the angular velocity $\Omega_p$, which slightly changes with time. Figure 9 shows the evolution of the wave amplitude at different radii. Estimation of the phase shift along the radius enables us to find the pitch angle of the spiral pattern, which ranges from $5^\circ$ to $30^\circ$. For illustration, we assume it to be $\sim 12^\circ$ in our model with $\varepsilon = 0.1$. In our simulations, the pitch angle is a function of radius: it grows from the centre to the periphery in the galactic disc, so the spiral arms become more open. This radial dependence may be used for the revealing of the halo non-axisymetry signature in the real galaxies.

3.5 Models with the live halo

To analyse the influence of possible interior substructure in the dark matter, e.g. of presence of subhaloes predicted by the standard cosmology theory (Klypin et al. 1999; Bullock et al. 2001; Rocha et al. 2012), we ran our N-body simulations of interaction between the triaxial live halo and the stellar collisionless disc. We performed the simulations in several steps. First, we construct a quasi-steady triaxial live halo with a fixed stationary disc (a "frozen disc") inside. The halo is characterised by the parameter $\varepsilon \approx 0.2 - 0.3$ as described before, which is initially assumed
to be constant along the radius. All disc particles were positioned within the radius $R_{\text{opt}} = 1.5$, and the halo particles were extended to $R_h = 5$. The total number of particles was $4 \cdot 10^6$ for the halo and $2 \cdot 10^6$ for the disc. The total halo mass in our modelling was $M_h = 4$. After several dynamical times the halo particles reached the state of relaxation and the initial numerical noise dissolved. Then we “unfreeze” the stellar disc particles instantly. Right after that, a two-arm spiral structure started forming, similar in its kinematics and morphological properties to that appeared in the case of rigid halo (see figure 12).

It is important to consider the effects of the stellar disc feedback on the dark halo shape in projection to the disc plane for a wide range of the non-axisymmetry parameter $\varepsilon$. In figure 12 we demonstrate the isodensity contours of the dark halo projected into the disc’s plane at $t = 12$ (about 4 rotation periods at $R_{\text{opt}}$). We trace the evolution $\varepsilon$ of the halo at three regions separately: 1) the central region $r_1 \leq r_d = 0.25$, 2) the disc periphery at $r_3 \geq 1$, and the intermediate region $r_1 < r < r_3$ (see figure 13). Up to time $t = 10$, the amplitude of the spiral perturbations remains small and there is no noticeable stellar disc feedback on the halo shape. Some energy initially transfers from the triaxial halo particles to the disc, causing the spiral pattern generation. After that the central halo region becomes more axisymmetric, which means some energy is returned back to the halo due to the interaction with the spiral pattern. Local non-axisymmetric parameter in the intermediate zone also decreased down to $\simeq 0.15$. The influence of spiral waves on the halo shape in the outer region remains insignificant.

We executed several simulations with live and symmetrical halo to investigate whether the disc and halo particle interactions can produce the disc heating. Such dynamical interaction could also generate some transient structures. Our modelling demonstrates the lack of the dynamical disc heating, at least in the marginally stable stellar disc in the presence of the live DM halo. In early moments of the modelling some oscillations of the disc velocity dispersion caused by relaxation of a small initial imbalance in particles distribution is observed (see the bottom panel in figure 11). After the relaxation is finished, the increasing of the velocity dispersion stops. Transient structures forming in the disc have an insignificant amplitude $< 1\%$ in the case of the symmetric halo (see e.g. top panel in figure 11). Note that its amplitude is several times larger than in the model with rigid halo model.

4 DISCUSSION

Cosmological simulations show that dark matter plays significant role in formation and evolution of galaxies of all morphological types. In this paper we consider a possibility of generation of a spiral pattern in the presence of non-axisymmetric dark halo.

The key question related to the spiral structure in the observed galaxies is whether the spirals are transient (Sellwood 2011) or long-lived features (Bertin and Lin 1996). In this paper we bring the existing evidences of non-spherical and, in general case, a triaxial dark matter distribution within and around the galaxies. Our numerical simulations suggest that the long-lived spiral patterns in stellar discs may be generated not only by the internal mechanisms (as it is traditionally assumed) but also via gravitational
interaction between a triaxial dark halo and the galactic disc. Large-scale spirals emerge in our modelling even if the halo’s triaxiality is as small as $\varepsilon \sim 0.02 - 0.05$. We demonstrate that mechanism of the long-lived spiral pattern formation discussed above acts very effectively even in dynamically hot stellar discs (initial value of the Toomre parameter $Q_T \sim 2 - 4$). Our preliminary simulations show that the presence of a gaseous component does not change significantly this picture [Khoperskov et al. 2013]. Note that similar mechanism can explain the existence of grand design spiral pattern observed in some dwarf early type galaxies [Jerjen et al. 2000, Lisker et al. 2006]; we can assume that their spherical components are slightly triaxial.

We also infer that the halo axes orientation can be found from kinematic studies of the spiral pattern. As we showed above, the pattern angular velocity cannot be described by a single constant value, as in the classical axisymmetric models: at any radial distance it changes periodically twice for the period of rotation, as a result of nonaxisymmetry of the gravitational field of the halo (see figure 10). In principle, detailed measurements of the stellar velocity field can help detect the jumps of the phase velocity in the pattern (see figure 10), which spatially should point us at the major axis of the halo. The amplitude of the angular velocity perturbations can trace the deviation from the symmetry of the halo. The best candidates to this halo diagnostics are isolated non-barred spiral galaxies with small pitch angle of their spiral arms ($\sim 20^\circ$). Apparently the present day accuracy of the $\Omega_p$ estimation makes tracing its variation along the radial and azimuthal direction a difficult problem. Note however that there are some observational evidences of multiplicity of the pattern angular velocity in the inner and in the outer regions of galactic discs (see e.g. Buta and Zhang 2009). A combined estimation of the bar pattern speed in the Milky Way obtained from the gas dynamics by Gerhard (2011) gives $\Omega_b \approx 52 \pm 10$ km s$^{-1}$ kpc$^{-1}$ and places the corotation radius near the end of the bar at $3.5 - 5$ kpc). On the other hand, the Milky Way spiral arms rotate much slower: observations constrain the speed of the spiral pattern in the range of $\Omega_s \approx 17 - 28$ km s$^{-1}$ kpc$^{-1}$ (Chakrabarty 2007), what corresponds to the corotation radius position near the solar neighborhood ($\sim 8$ kpc). Thus, it is possible that the outer spiral structure in the Milky Way is relevant mostly to the triaxial halo-disc interaction, whereas the inner spiral arms are connected with the Milky Way bar.

We demonstrate in figure 13 how the dark matter halo becomes more spherical in the centre due to the spiral structure formation in the case of the initially triaxial halo. However, we bring detailed consideration of effects of the angular momentum feedback from the disc to the halo beyond the scope of this research. The dynamical interaction between the stellar disc and the DM halo occurs at resonances, and the result is the increasing of the halo rotation [Athanassoula 2003]. It is worth mentioning that the angular velocity of the bar in barred galaxies decreases in time due to dynamical frictions against the dark matter halo (see e.g. Debattista and Sellwood 2000). Numerical cosmological simulations reveal the possibility of variations of the axes ratios in the galactic haloes during their evolution. Here we note that parallel to the secular processes (e.g. see Debattista et al. 2008) the exchange by the angular momentum and energy between the live halo and density waves it creates may also lead to the quasi-stationary periodical variation of parameters of the spiral arms (including their pitch angle) with time. The latter, in turn, may cause a variation of the morphological type of a spiral galaxy during its evolution (see Mazzei and Curir 2003).

5 CONCLUSIONS

We conducted high resolution N-body simulations to study interactions between the stellar disc and triaxial massive dark halo in terms of the spiral structure formation in disc for different initial disc and halo parameters. Our results are most relevant to the galaxies with mass-geometric properties similar to the Milky Way. The main conclusions are as follows:

1. A triaxial dark matter halo can produce and maintain a long-lasting (at least a few Gyr) grand design spiral structure in a stellar disc even if the disc is gravitationally stable.
2. As in the case of gaseous discs (see Khoperskov et al. 2013), the stellar discs respond to the dark halo asymmetry via the global spiral pattern formation, even if triaxiality in the halo is rather small ($\varepsilon \approx 0.02 - 0.05$).
3. The resulting amplitude of the spiral waves ($m = 2$) depends on the triaxiality $\varepsilon$ and is practically independent of the initial disc conditions.
4. Morphology of the spiral structure in our models is rather similar to that formed in classical density wave theory. The growing spiral pattern does not lead to significant redistribution of mass in the disc in the radial and azimuthal directions in the case of a non-axisymmetric massive halo.
5. Triaxial halo does not result to significant dynamic heating of the stellar disc (except, perhaps, the very initial stages of the interaction), in contrast to the case of a spiral pattern formed due to gravitational instability mechanisms.
6. Dynamics of the spiral pattern cannot be described by a single pattern angular velocity. Instead, we observe obviously two pattern angular velocities at any fixed radius. Most of the time the spiral pattern at given radius rotates slowly with the angular velocity $\Omega_{p1}$, but twice for it rotation cycle its angular velocity jumps up to $\Omega_{p2} \approx (4 - 5)\Omega_{p1}$ and then returns back. These jumps is caused by passing through the elongated potential well of the dark matter halo.
7. In general, morphology and kinematics of the spiral pattern in a stellar disc are quite similar to those obtained in modelling of gaseous discs embedded in non-axisymmetrical dark matter haloes [Bekki and Freeman 2002, Khoperskov et al. 2013].
8. There is a tendency for the halo to become more symmetric in the disc plane in the models that take into account self-consistent interaction between the disc and the dark halo. In this case a formation of spiral density waves remains effective in the outer regions of disc even when the inner regions of the halo have become axisymmetric.

6 ACKNOWLEDGMENTS

The authors would like to thank the anonymous referee for helpful comments and advises that improved the paper. Authors express their gratitude for valuable discussions to
REFERENCES

Allgood B., Flores R.A., Primack J.R., Krvatsov A.V., Wechsler R.H., Faltenbacher A., Bullock J.S., 2006, MNRAS, 367, 1781

Athanassoula E., Bosma A., Papaiouannou S., 1987, A&A, 179, 23

Athanassoula E., 2002, ApJ, 569, L83

Athanassoula E., 2003, MNRAS, 341, 1179

Bahcall J.N., 1984, ApJ, 276, 156

Bailin J., Steinmetz M., 2004, ApJ, 616, 27

Bailin J., Steinmetz M., ApJ, 2005, 627, 647

Barnes J., Hut P. 1986, Nature, 324, 446

Begeman K.G., Broeils A.H., Sanders R.H. 1991, MNRAS, 249, 523

Berentzen I., Shlosman I., 2006, ApJ, 637, 582

Berentzen I., Shlosman I., Jogee S. 2006, ApJ, 648, 807

Bershady M.A., Martinsson T.P.K., Verheijen M.A.W., 2006, EAS Publ. Ser., 20, 97

Bekki K., Freeman K.C., 2002, ApJ, 574, 857

Bekki K., Freeman K.C., 2002, ApJ, 574, 21

Berti G., Lin C.C., 1996, MA MIT Press, p. 271

Bett P., Eke V., Frenk C. S., Jenkins A., Helly J., Navarro J., 2007, MNRAS, 376, 215

Binney, J. & Tremaigne, S., 2008, Galactic Dynamics. Princeton Univ. Press, Princeton, p.856

Bottema, R., 1993, A&A, 275, 16

Bryan S. E., Cress C. M., 2007, MNRAS, 380, 657

Bruzual A., Charlot S., 2003, ApJ, 585, 73

Bruzual A., Charlot S., 2003, ApJ, 585, 739, 47

Bruzual A., Charlot S., 2003, ApJ, 585, 739, 47

Ceverino D., Klypin A. 2007, MNRAS, 379, 1155

Chakrabarty, D. 2007, A&A, 145, 162

Combos F., 2006, EAS Publ. Ser., 20, 97

Cooper A.P., Cole S., Frenk C.S., White S.D.M., Helly J., Benson A.J., De Lucia G., Helmi A., Jenkins A., Navarro J.F., Springel V., Wang J., 2010, MNRAS, 406, 744

De Simone R., Wu X., Tremaine S., 2004, MNRAS, 350, 627

Debattista V.P., Sellwood J.A., 2000, ApJ, 543, 704

Debattista V.P., Mayer L., Carollo C.M., Moore B., Wadsley J., Quinn T., 2006, ApJ, 645, 209

Debattista V.P., Moore B., Quinn T., Kazantzidis S., Maas R., Mayer L., Read J., Stadel J., 2006, ApJ, 681, 1076

Diemand J., Kuhlen M., Madau P., 2007, ApJ, 657, 262

Diemand J., Moore B., 2011, Advanced Science Letters, 4, 297

Dubinski J, Chakrabarty D., 2009, ApJ, 703, 2068

Dubinski J., Berentzen I., Shlosman I., 2009, ApJ, 697, 293

Einasto J., 1965, Trudy Inst. Astrofiz. Alma-Ata, 51, 87

El-Zant A.A., Habler B., 1998, New Astronomy, 3, 493

Fux R., 1997, A&A, 327, 983

Fellhauer M., Belokurov V., Evans N. W., Wilkinson M. I., Zucker D. B., Gilmore G., Irwin M. J., Bramich D. M., Vidrih S., Wyse R. F. G., Beers T. C., Brinkmann J., 2006, ApJ, 651, 167

Garbari S., Read J.L., Lake G., 2011, MNRAS, 416, 2318

Gentile G., Burkert A., Salucci P., Klein U., Walter F., 2005, ApJ, 634, 145

Gerhard O., 2011, Mem. S.A.It., 18, 183

Griv E., Gedalin M., 2004, AJ, 128, 1965

Gustafsson M., FairBairn M., Sommer-Larsen J., 2006, Phys. Rev. D 74, 123522

Hayashi E., Navarro J.F., 2006, MNRAS, 373, 1117

Hayashi E., Navarro J.F., Springel V., 2007, MNRAS, 377, 50

Heller C.H., Shlosman I., Athanassoula E., 2007, ApJ, 671, 226

Helmi A., 2004, MNRAS, 351, 643

Ibata R., Lewis G.F., Irwin M., Totten E., Quinn T., 2001, ApJ, 551, 294

Ideta M., Hozumi S., 2000, ApJ, 535L, 91

Iodice E., Arnaboldi M., Bournaud F., Combes F., Sparke L.S., van Driel W., Capaccioli M., 2003, ApJ, 585, 730

Jerjen H., Kalnajs A., Binggeli B., 2000, A&A, 358, 845

Just A., Jahrei H., 2010, MNRAS, 402, 461

Kazantzidis S., Abadi M.G., Navarro J.F., 2010, ApJ, 720L, 62

Keller S. C., Murphy S., Prior S., Da Costa G., Schmidt B., 2008, ApJ, 678, 851

Khoperskov A.V., Zasov A.V., Shustov B.M., Khoperskov A.V., 2012, ARep, 56, 664

Khoperskov A.V., Zasov A.V., Tiurina N.V., 2003, ARep, 56, 664

Klypin A., Klypin A.V., Kravtsov A.V., Prada F., 1999, Phys. Rev. D 74, 123522

Klypin A. V., Dutton A.A., van den Bosch F.C., Moore B., Potter D., Stadel J., 2007, MNRAS, 397, 62
Macciò A. V., Stinson G., Brook C. B., Wadsley J., Couchman H. M. P., Shen S., Gibson B. K., Quinn T., 2012, ApJ, 744, L9
Machado R.E.G., Athanassoula E., 2010, MNRAS, 406, 2386
Maller A.H., Simard L. Guhathakurta P., Hjorth J., Jaunsen A.O., Flores R.A., Primack J.R., 2000, ApJ, 533, 194
Mazzei P., Curir A., 2003, ApJ, 591, 784
Masset F.S., Bureau M., 2003, ApJ, 586, 152
Morrison H., Helmi A., Sun J., Liu P., et al., 2009, ApJ, 694, 130
Muñoz-Cuartas J.C., Macciò A.V., Gottlöber S., Dutton A.A., 2011, MNRAS, 411, 584
Navarro J.F., Frenk C.S., White S., 1997, ApJ, 490, 493
Navarro J.F., Ludlow A., Springel V., Wang J., Vogelsberger M., White S.D.M., Jenkins A., Frenk C.S., Helmi A., 2010, MNRAS, 402, 21
Newberg H.J., Yanny B., Cole N., 2007, ApJ, 668, 221
Oguri M., Lee J., Suto Y., 2003, ApJ, 599, 7
Romano-Diaz E., Shlosman I., Keller C., Hoffman Y., 2008, ApJ, 687, 13
Roskar R., Debattista V.P., Brooks A.M., Quinn T.R., Brook C.B., Governato F., Dalcanton J.J., Wadsley J., 2010, MNRAS, 408, 783
Ryden B.S., 2006, ApJ, 641, 773
Saburova A.S., Zasov A.V., 2012, Astron. Letters, 38, 139
Ryden B.S., Sparke L.S., 1990, ApJ, 361, 408
Ryden B.S., Pogge R.W., 1995, AIP Conf. Proc., 336, 141
Safronov V. S., 1960, Ann. d’Astrophysique, 23, 979
Sellwood J., 2011, MNRAS, 410, 1637
Sesar B., Juric M., Ivezic Z., 2011, ApJ, 731, 4
Sotnikova N.Ya., Rodionov S.A., 2005, Astr. Letter., 31, 15
Tinker J.L., Ryden B.S. 2002, [arXiv:astro-ph/0209165]
Toomre A., 1964, ApJ, 139, 1217
Tutukov A.V., Fedorova A.V., 2006, ARep., 50, 785
Vivas A.K., Zinn R., Gallart C., 2005, AJ, 129, 189
Vitvitska M., Klypin A.A., Kravtsov A.V., Wechsler R.H., Primack J.R., Bullock J.S., 2002, ApJ, 581, 799
Whitmore B.C., McElroyD.B., Schweizer F., 1987, ApJ, 314, 439
Westfall K. B., Bershady M. A., Verheijen M. A. W., Andersen D. R., Martinsson T.PK, Swaters R. A., Schechtman-Rook A., 2011, ApJ, 742, 18
Widrow L.M., 2008, ApJ, 679, 1232
Zasov A.V., Makarov D.I., Mikhailova E.A., 1991, PAZh, 1991, 17, 884
Zasov A.V., Bizyaev D.V., Makarov D.I., Tyurina N.V., 2002, Astr. Lett, 28,527
Zasov A.V., Khoperskov A.V., Tiurina N.V., 2004, Astr. Lett., 30, 593
Zentner A. R., Kravtsov A. V., Gnedin O. Y., Klypin A. A., 2005, ApJ, 629, 219
Zibetti S., White S., Brinkmann J., 2004, MNRAS, 347, 556
