The GRB luminosity function: predictions from the internal shock model and comparison with observations

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ABSTRACT

We compute the expected luminosity function of gamma-ray bursts (GRBs) in the context of the internal shock model. We assume that GRB central engines generate relativistic outflows characterized by the respective distributions of injected kinetic power $E$ and contrast in Lorentz factor $\kappa = \Gamma_{\text{max}}/\Gamma_{\text{min}}$. We find that if the distribution of contrast extends down to values close to unity (i.e. if both highly variable and smooth outflows can exist), then the luminosity function has two branches. At high luminosity it follows the distribution of $E$ while at low luminosity it is close to a power law of slope $\sim -0.5$. We then examine if existing data can constrain the luminosity function. Using the log $N$–log $P$ curve, the $E_p$ distribution of bright Burst and Transient Source Experiment (BATSE) bursts and the X-ray flash (XRF)/GRB ratio obtained by High Energy Transient Explorer 2 (HETE2), we show that single and broken power laws can provide equally good fits of these data. Present observations are therefore unable to favour one form or the other. However, when a broken power law is adopted they clearly indicate a low-luminosity slope $\sim -0.6 \pm 0.2$, compatible with the prediction of the internal shock model.

Key words: methods: statistical – stars: luminosity function, mass function – gamma-rays: bursts.

1 INTRODUCTION

The isotropic luminosity of long gamma-ray bursts (GRBs) is known to cover a wide range from underluminous, nearby bursts such as GRB 980425 or GRB 060218 (with $L \lesssim 10^{47}$ erg s$^{-1}$) to ultrabright objects like GRB 990123 $(L \gtrsim 10^{53}$ erg s$^{-1}$). While it has been suggested that the weakest GRBs could simply be normal events seen off-axis (Yamasaki, Yonetoku & Nakamura 2003), this possibility has been recently discarded both from limits on afterglow brightness and for statistical reasons (Soderberg et al. 2004; Daigne & Mochkovitch 2007). The difference of six orders of magnitude between the brightest and weakest GRBs is therefore probably real. The parameters (stellar rotation, metallicity etc.) which are responsible for this diversity in radiated power are not known. However, in the restricted range $10^{51} \lesssim L \lesssim 10^{53}$ erg s$^{-1}$ the value of the isotropic luminosity is possibly fixed by the opening angle of the jet which may always carry the same characteristic energy (Frail et al. 2001).

The purpose of this paper is to see how basic theoretical ideas and existing data can be used to constrain the GRB luminosity function (LF) $p(L)$. First, we should insist that $p(L)$ here represents the ‘apparent’ LF which includes viewing angle effects and beaming statistics (i.e. bursts with narrow jets are more likely seen off-axis and therefore underrepresented in the distribution). It is therefore different from the ‘intrinsic’ LF $p_{\text{in}}(L)$ which would be obtained with all GRBs seen on-axis. In the lack of a complete, volume-limited sample of GRBs with known redshift, only indirect observational indicators such as the log $N$–log $P$ plot can constrain the LF. These indicators, however, depend not only on $p(L)$ but also on the GRB rate and spectral energy distribution. The simplest possible form for $p(L)$ is a single power law $p(L) \propto L^{-\delta}$ between $L_{\text{min}}$ and $L_{\text{max}}$. Together with the parameters describing the GRB rate and spectral shape, $\delta, L_{\text{min}}$ and $L_{\text{max}}$ can be adjusted to provide the best possible fit of the available indicators. Considering the mixing of the LF with other quantities in the fitting process it is remarkable that studies using different observational constraints have converged to a similar value of the slope $\delta \sim 1.5–1.7$ (e.g. Firmani et al. 2004; Daigne & Mochkovitch 2007).

In a second step one can consider the more general case of a broken power-law LF with now five parameters: $L_{\text{min}}, L_{\text{max}}, \delta_1, \delta_2$, the two slopes $\delta_1$ and $\delta_2$ and the break luminosity $L_0$. We will see in Section 2 that there is some indication that the internal shock model of GRBs can produce a broken power-law LF and we want to check if it is also favoured by the existing observational data. As in our previous study we have used a Monte Carlo method to generate a large number of synthetic events where the parameters defining the burst...
properties are varied within fixed intervals. Preferred values of the parameters are those which yield the minimum \( \chi^2 \) for a given set of observational constraints. We summarize these constraints and present the Monte Carlo simulations in Section 3. We discuss our results and conclusions in Sections 4 and 5, respectively.

2 THE GRB LUMINOSITY FUNCTION IN THE INTERNAL SHOCK MODEL

The internal shock model (Rees & Meszaros 1994) is the most discussed solution to explain the prompt gamma-ray emission in GRBs. In this section, we demonstrate that it naturally leads to a broken power-law LF with a low-luminosity slope close to \(-0.5\).

2.1 Internal shock efficiency and luminosity function

In the context of the internal shock model (Rees & Meszaros 1994), the prompt gamma-ray emission is produced by relativistic electrons accelerated in internal shocks propagating within a relativistic outflow. The (isotropic equivalent) radiated luminosity \( L \) is then a fraction of the (isotropic equivalent) rate of kinetic energy injected in the flow \( \dot{E} \):

\[
L = f(\kappa) \dot{E}.
\]

The efficiency \( f(\kappa) \) is the product of three terms:

\[
f(\kappa) \simeq f_{\text{rad}} \epsilon_e f_{\text{dyn}}(\kappa),
\]

where (i) \( f_{\text{dyn}}(\kappa) \) is the fraction of the kinetic energy which is converted by internal shocks into internal energy ("dynamical efficiency"). This fraction depends mainly on the contrast \( \kappa = \Gamma_{\text{max}}/\Gamma_{\text{min}} \) of the Lorentz factor distribution in the relativistic outflow. (ii) \( \epsilon_e \) is the fraction of this dissipated energy which is injected into relativistic electrons. We assume that \( \epsilon_e \) is close to the equipartition value \( \epsilon_e = 1/3 \), as it is a necessary condition to have an efficient mechanism. (iii) \( f_{\text{rad}} \) is the fraction of the electron energy which is radiated. To explain the observed variability timescales in GRB light curves and to maintain a reasonable efficiency, the relativistic electrons must be in the fast cooling regime, which means that their radiative time-scale is very short compared to the hydrodynamical timescales in the outflow. In this case, we have \( f_{\text{rad}} \simeq 1 \).

The GRB LF in the internal shock model is therefore related to the physics of the relativistic ejection by the central engine. We assume that the contrast \( \kappa \) is distributed between \( \kappa_{\text{min}} \) and \( \kappa_{\text{max}} \) with a density of probability \( \psi(\kappa) \) and that \( \dot{E} \) is distributed between \( \dot{E}_{\text{min}} \) and \( \dot{E}_{\text{max}} \) with a density of probability \( \phi(\dot{E}) \). The minimum and maximum radiated luminosities are therefore

\[
L_{\text{min}} = f(\kappa_{\text{min}}) \dot{E}_{\text{min}}
\]

and

\[
L_{\text{max}} = f(\kappa_{\text{max}}) \dot{E}_{\text{max}}.
\]

For \( L_{\text{min}} \leq L \leq L_{\text{max}} \), the probability to have a radiated luminosity in the interval \( [L; L + dL] \) is \( p_0(L) dL \), where the intrinsic LF \( p_0(L) \) is given by

\[
p_0(L) = \int_{\kappa_{\text{min}}}^{\kappa_{\text{max}}} \psi(\kappa) \frac{L}{f(\kappa)} \phi(\dot{E}) d\kappa.
\]

2.2 The case of a power-law distribution of kinetic energy flux

We assume that the injection rate of kinetic energy in the relativistic outflow follows a power-law distribution

\[
\phi(\dot{E}) \simeq \delta - 1 \frac{\dot{E}}{\dot{E}_{\text{min}}}^{-\delta}
\]

with \( \dot{E}_{\text{max}} \gg \dot{E}_{\text{min}} \). Then, the GRB LF given by equation (5) becomes

\[
p_0(L) \simeq \frac{\delta - 1}{L_{\text{max}} - L_{\text{min}}} \left( \frac{L}{L_{\text{max}}} \right)^{-\delta} \int_{\kappa_{\text{min}}}^{\kappa_{\text{max}}} \psi(\kappa) \left( \frac{f(\kappa)}{f(\kappa_{\text{max}})} \right)^{-1 \delta} d\kappa,
\]

where the luminosity \( L_{\text{c}} \) is defined by

\[
L_{\text{c}} = f(\kappa_{\text{max}}) \dot{E}_{\text{min}}.
\]

Let us now consider first the case where GRB central engines can produce all kinds of outflows, from highly variable to perfectly smooth. In this case, the minimum contrast is \( \kappa_{\text{min}} = 1 \), corresponding to a minimum efficiency \( f(\kappa_{\text{min}}) = 0 \), as no internal shocks can develop in an outflow with a constant Lorentz factor. The first limit in the integral in equation (7) is then always given by \( f(\kappa_{\text{min}}) \dot{E}_{\text{max}} \) and the LF is made of two branches.

(i) High-luminosity branch. For \( L_{\text{c}} \leq L \leq L_{\text{max}} \), the second limit in the integral is \( f(\kappa_{\text{max}}) \dot{E}_{\text{min}} \), which leads to

\[
p_0(L) \simeq \frac{\delta - 1}{L_{\text{max}} - L_{\text{min}}} \left( \frac{L}{L_{\text{max}}} \right)^{-\delta} \int_{L_{\text{min}}}^{L_{\text{max}}} d\kappa \int_{\kappa_{\text{min}}}^{\kappa_{\text{max}}} \psi(\kappa) \left( \frac{f(\kappa)}{f(\kappa_{\text{max}})} \right)^{-1 \delta} d\kappa.
\]

(ii) Low-luminosity branch. For \( L_{\text{min}} = 0 \leq L \leq L_{\text{c}} \), the second limit in the integral is \( \int_{L_{\text{min}}}^{L_{\text{max}}} d\kappa \int_{\kappa_{\text{min}}}^{\kappa_{\text{max}}} \psi(\kappa) \left( \frac{f(\kappa)}{f(\kappa_{\text{max}})} \right)^{-1 \delta} d\kappa \), which leads to

\[
p_0(L) \simeq \frac{\delta - 1}{L_{\text{max}} - L_{\text{min}}} \left( \frac{L}{L_{\text{max}}} \right)^{-\delta} \int_{L_{\text{min}}}^{L_{\text{max}}} d\kappa \int_{\kappa_{\text{min}}}^{\kappa_{\text{max}}} \psi(\kappa) \left( \frac{f(\kappa)}{f(\kappa_{\text{max}})} \right)^{-1 \delta} d\kappa.
\]

The LF is determined by the behaviour of the efficiency \( f(\kappa) \) at very low contrast. We assume that \( f(\kappa) \simeq f_{\text{0}}(\kappa - 1)^{\alpha} \) for \( \kappa \to 1 \), and we define \( L_{\text{0}} = f_{\text{0}} \dot{E}_{\text{min}} \). Then, for \( L \ll L_{\text{0}} \), we have

\[
p_0(L) \simeq \frac{\delta - 1}{L_{\text{max}} - L_{\text{min}}} \left( \frac{L}{L_{\text{0}}} \right)^{\delta - 1 \alpha} \psi(1) \left( 1 + \alpha (\delta - 1) \right).
\]

We therefore find that for \( L \ll L_{\text{c}} \) and \( \kappa_{\text{min}} = 1 \), the LF is also a power law, with however a slope \( (1/\alpha) - 1 \) which does not depend on the slope of the intrinsic distribution of injected kinetic power.

Then, the predicted intrinsic GRB LF in the internal shock model is a broken power law of slope \( (1/\alpha) - 1 \) at low luminosity and \( -\delta \) at high luminosity, with a break luminosity \( L_{\text{c}} = f(\kappa_{\text{max}}) \dot{E}_{\text{min}} \). The
The GRB luminosity function in the internal shock model

Figure 1. The intrinsic GRB LF in the internal shock model. The function $p_0(L)$ is plotted, assuming an intrinsic distribution of injected kinetic power $E$ that is a power-law of slope $-\delta = -1.7$ between $E_{\text{min}} = 10^{52}$ erg s$^{-1}$ and $E_{\text{max}} = 10^{54}$ erg s$^{-1}$. This function $\phi(E)$ is plotted as a thin line in the left-hand panel. Left: effect of the minimum contrast $\kappa_{\text{min}}$. We assume that log $\kappa$ is uniformly distributed between log $\kappa_{\text{min}}$ and log $\kappa_{\text{max}}$ with $\kappa_{\text{min}} = 1, 1.01, 1.01, 1.1$ and $2$, and $\kappa_{\text{max}} = 10$. Right: effect of the shape of the distribution of contrast. We fix $\kappa_{\text{min}} = 1$ and $\kappa_{\text{max}} = 10$ and consider three different shapes for the distribution of contrast $\psi(\kappa)$: (i) log $\kappa$ uniformly distributed (solid line); (ii) $\psi(\kappa) \propto \exp(-\kappa/4)$ (dotted line); (iii) $\kappa$ uniformly distributed (dashed line). An inset shows the transition at $L_\ast = 1.4 \times 10^{51}$ erg s$^{-1}$ in more details.

shape of $p_0(L)$ at the transition ($L \sim L_\ast$) is related to the distribution of the contrast $\psi(\kappa)$. If $\kappa_{\text{min}} \neq 1$ this result remains valid as long as very low contrasts can be achieved ($\kappa_{\text{min}} \lesssim 1.1$; see Fig. 1, left-hand panel). Note that the analysis of the internal shock model parameter space made by Barraud et al. (2005) shows that very low contrasts are necessary to produce soft GRBs such as X-ray rich GRBs (XRRs) and X-ray flashes (XRFs). However, if it happens that GRB central engines never produce smooth outflows (i.e. if $\kappa_{\text{min}}$ is not close to unity), the calculation made above remains valid for the high-luminosity branch, which is still a power law of slope $-\delta$, but the low-luminosity branch is much reduced and no more a power law.

Finally, we have briefly considered the case where the distribution of injected kinetic power $\phi(E)$ is not a power law. For example, for a lognormal distribution peaking at $E_\ast$, we again obtain that the intrinsic LF follows $\phi(E)$ at high luminosity and is a power law of slope $-0.5$ at low luminosity, with a transition at $L_\ast \sim f(\kappa_{\text{min}})E_\ast$.

2.3 A simple model for the internal shock efficiency

To investigate more precisely the GRB LF, we need to know the form of the efficiency $f(\kappa)$. As shown in Daigne & Mochkovitch (1998), it is a priori a complex function of the initial distribution of the Lorentz factor and the kinetic energy in the relativistic outflow. However, one can make a simple estimate by using the toy model developed in Daigne & Mochkovitch (2003) and Barraud et al. (2005) where we only consider direct collisions between two equal-mass relativistic shells. In this case, the efficiency is simply given by

$$f(\kappa) \simeq \epsilon_\ast \left(\sqrt{\kappa} - 1\right)^2 / \kappa + 1.$$  

For low contrast, it behaves as $f(\kappa) \simeq \epsilon_\ast(\kappa - 1)^2/8$, which corresponds to $f_0 = \epsilon_\ast/8$ and $\alpha = 2$. In addition to this toy model which gives an explicit expression for the efficiency, we have used our detailed internal shock code (Daigne & Mochkovitch 1998) and checked the quadratic dependence of $f(\kappa)$ in $(\kappa - 1)$. Also notice that the result $\alpha = 2$ will remain valid even if $\epsilon_\ast$ is not strictly constant, as long as it does not vary as some power of $(\kappa - 1)$ at low $\kappa \epsilon_\ast \propto (\kappa - 1)$ leading for example to $\alpha = 3$ and $p_0(L) \propto L^{-(3/2)}$ at low luminosity.

Assuming a constant $\epsilon_\ast$, we have plotted the resulting GRB LF in Fig. 1. The intrinsic distribution of injected kinetic power is defined between $E_{\text{min}} = 10^{52}$ erg s$^{-1}$ and $E_{\text{max}} = 10^{54}$ erg s$^{-1}$ and has a slope $-\delta = -1.7$. We have fixed the maximum value of the contrast to $\kappa_{\text{max}} = 10$, so that $L_\ast \simeq 1.4 \times 10^{51}$ erg s$^{-1}$. In the left-hand panel, we have assumed that the logarithm of the contrast $\kappa$ is uniformly distributed between log $\kappa_{\text{min}}$ and log $\kappa_{\text{max}}$, with $\kappa_{\text{min}} = 1, 1.01, 1.01, 1.1$ or 2. For all values of $\kappa_{\text{min}}$, the high-luminosity branch is the same power law of slope $-\delta = -1.7$. For $\kappa_{\text{min}} = 1$, the low-luminosity branch extends down to $L = 0$ and is the expected power law of slope $(1/\alpha) - 1 = -1/2$. This branch is still clearly visible for $\kappa_{\text{min}} = 1.001, 1.01$ and 1.1 but nearly disappears for $\kappa_{\text{min}} = 2$. For even higher values of $\kappa_{\text{min}}$, only the high-luminosity power law remains.

We have tested in the right-hand panel of Fig. 1 that for other choices of the distribution of contrast $\psi(\kappa)$, the GRB LF is not affected (the two slopes remain unchanged) except for the shape of the transition at $L \sim L_\ast$. A low-luminosity branch of slope $-0.5$ in the intrinsic LF is therefore a robust prediction of the internal shock model, as long as GRB central engines can produce smooth outflows (very low contrasts). The low-luminosity branch will however manifest itself only if $L_\ast = f(\kappa_{\text{max}})E_{\text{min}}$ is large enough; otherwise the observationally accessible part of the LF of cosmological GRBs will behave as a single power law (in Fig. 1, $L_\ast \simeq 1.4 \times 10^{51}$ erg s$^{-1}$).

2.4 Apparent GRB luminosity function

The GRB LF that has been derived from the internal shock model is intrinsic. If GRB ejecta have a jet-like geometry with an opening
angle $\Delta \theta$ which is not correlated to the kinetic energy flux $\dot{E}$, the apparent LF above $L_a$ has the same shape as the intrinsic one since the fraction of observed GRBs does not depend on $E$. At lower luminosities, two effects are in competition: low-luminosity bursts can be due to a low internal shock efficiency and/or a large viewing angle. Close to $L_a$, the first effect dominate and the slope is still very close to $-0.5$ as predicted above. At very low luminosity, the second effect takes over. It can be shown that the slope then becomes close to $-7/6$. Observing this final slope seems difficult as it involves the detection of very faint bursts. However, viewing angle effects already modify the low-luminosity slope below the break where it progressively departs from its intrinsic value $-0.5$ (see Fig. 2).

If it happens that the opening angle $\Delta \theta$ is correlated with $\dot{E}$, the apparent $p(L)$ will be also different from the intrinsic one at high luminosity. A possible correlation could be

$$E (1 - \cos \Delta \theta) = \text{cst} = E_{\text{min}},$$

meaning that the true kinetic energy rate is the same for all bursts. Such an assumption is motivated by the evidence that there might be a standard energy reservoir in GRBs (Frail et al. 2001). In this case, the high-luminosity branch (above $L_a$) of $p(L)$ has a slope $-1 + \delta$, where $-\delta$ is the slope of the intrinsic LF. This is illustrated in Fig. 2 (right-hand panel).

3 CONSTRaining THE GRB Luminosity Function

3.1 Monte Carlo simulations

We use Monte Carlo simulations to constrain the GRB LF from observations. The method is described in details in Daigne, Rossi & Mochkovitch (2006). We recall here the main lines.

(a) Properties of the long GRB population. We characterize this population by the intrinsic distribution of several physical properties: (1) the comoving rate. We define $R_{\text{GRB}}(z)(\text{Mpc}^{-3} \text{yr}^{-1})$ as the GRB comoving rate at redshift $z$. We assume that the GRB comoving rate is proportional to the star formation rate (SFR). Following Porciani & Madau (2001), this can be written as $R_{\text{GRB}} = k R_{\text{SN}}$, where $R_{\text{SN}}$ is the comoving rate of collapses of massive stars above 8 $M_\odot$. We consider three possible scenarios (see fig. 1 in Daigne et al. 2006 and reference therein) that all fit the observed SFR up to $z \sim 2-3$: SFR1, where the SFR decreases for $z \gtrsim 2$, SFR2, where it is constant for $z \gtrsim 2$ and SFR3, where it increases for $z \gtrsim 2$. In the last case, we have to assume a maximum redshift for star formation. We adopt $z_{\text{max}} = 20$. (2) The LF. In this paper, for simplicity, we do not discuss evolutionary effects. Therefore, the probability density $p(L)$ of the isotropic equivalent luminosity $L$ does not depend on $z$. In Daigne et al. (2006) we only considered the case of a power-law distribution, with $p(L) \propto L^{-\alpha}$ for $L_{\text{min}} \leq L \leq L_{\text{max}}$. Here, we test more complicated LFs, i.e. broken power laws defined by

$$p(L) \propto \begin{cases} L^{-\alpha_1} & \text{for } L_{\text{min}} \leq L \leq L_b, \\ L^{-\alpha_2} & \text{for } L_b \leq L \leq L_{\text{max}}. \end{cases}$$

(3) The distributions of intrinsic spectral parameters. We assume that the GRB photon spectrum is given by a broken power law with a break at energy $E_p$ (Band et al. 1993) and a slope $-\alpha (-\beta)$ at low (high) energy. We checked that the use of the more realistic spectrum shape proposed by Band et al. (1993) does not affect our conclusions. The slopes $\alpha$ and $\beta$ are given the distributions observed in a sample of Burst and Transient Source Experiment (BATSE) bright long GRBs (Preece et al. 2000). For the peak energy $E_p$ we consider two possible cases, either a lognormal distribution, with a mean value $E_{p,0}$ and a dispersion $\sigma = 0.3$ dex (hereafter 'lognormal $E_p$ distribution') or an intrinsic correlation between the spectral properties and the luminosity (hereafter 'Amati-like relation'), as found by Amati et al. (2002) and Amati (2006). We assume in this case that

$$E_p = 380 \text{ keV} \left( \frac{L}{1.6 \times 10^{52} \text{ erg s}^{-1}} \right)^{0.43},$$

with a normal dispersion $\sigma \approx 0.2$ dex in agreement with observations (Yonetoku et al. 2004; Ghirlanda et al. 2005, see however Band & Preece 2005; Nakar & Piran 2005 who tested this
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relation against BATSE data and concluded that selection effects were dominant).

(b) Criteria of detection by several instruments. With the assumptions listed above, a GRB in the simulation is characterized by a redshift \( z \), a peak luminosity \( L \) and a spectrum defined by \( E_p, \alpha \) and \( \beta \). It is therefore possible to compute the observed peak flux in any spectral band. Using the known sensitivity of several instruments (see Daigne et al. 2006 for the detailed thresholds we use), we can determine if a given burst is detected by the following experiments:

(1) BATSE, for which we define two samples in our synthetic bursts (all BATSE bursts and bright BATSE bursts); (2) \textit{High Energy Transient Explorer 2} (HETE2), for which we test the detection either by the gamma-ray [French Gamma-Ray Telescope (FREGATE)] or the X-ray [Wide Field X-Ray Monitor (WXM)] instruments and (3) \textit{Swift}, for which we test the detection by the gamma-ray instrument only (BAT) and we also define two samples (all \textit{Swift} bursts and bright \textit{Swift} bursts). It is then possible to compute simulated observed distribution of various quantities to compare them with real data.

(c) Observational constraints. We use three different kinds of observations: (1) the log \( N - \log P \) diagram of BATSE bursts (Stern et al. 2000, 2001); (2) the observed peak energy distribution of long bright GRBs (Preece et al. 2000) and (3) the observed fraction of soft GRBs (SXR and SXR) in the sample of GRBs detected by HETE2 (Sakamoto et al. 2005). The log \( N - \log P \) diagram is broadly used for such kind of studies but we have shown in Daigne et al. (2006) that (2) and (3) are good complements to better constrain the parameters of the GRB population.

Depending on the assumptions on the spectral properties, we have four or five free parameters for a single power-law LF \((k, L_{\text{min}}, L_{\text{max}}, \delta \) and \( E_{p,0} \) in the case of a lognormal distribution). The observational constraints correspond to 41 data points (see Fig. 4: 30 data points in the log \( N - \log P \) diagram published by Stern et al. 2001; 10 points for the \( E_p \) distribution published by Preece et al. 2000 which has been rebinned in 10 logarithmic bins of size 0.2 dex between 15.8 keV and 1.58 MeV; one point for the fraction of soft GRBs). We have therefore 37 or 36 degrees of freedom. The numerical procedure is the following: for a set of parameters, we generate randomly a population of \( 10^5 \) GRBs using the distribution defined above, we then compute the simulated distributions of observed peak flux, peak energy etc. and we compare them to real data, by computing a \( \chi^2 \). We do that for a large number of sets of parameters, randomly chosen to explore a large space. We always find a clear minimum \( \chi^2_{\text{min}} \) of \( \chi^2 \) and we define as ‘best models’, all models with \( \chi^2_{\text{min}} \leq \chi^2 \leq \chi^2_{\text{min}} + \Delta \chi^2 \), where \( \Delta \chi^2 \) defines the 1\( \sigma \) level and is computed from the number of degrees of freedom.

The main results obtained in Daigne et al. (2006) are (1) that SFRs is strongly favoured by the observed redshift distribution of \textit{Swift} bursts. However, a SFR rising at large \( z \) appears unlikely as it would overproduce metals at early times. This is therefore an indirect indication in favour of a GRB rate that does not directly follow the SFR, for instance due to an evolution with redshift of the efficiency of massive stars to produce GRBs.\(^1\) (2) with this SFR, both the ‘Amati-like relation’ and the ‘lognormal \( E_p \) distribution’ give good fits to the observational constraints listed above. Best model parameters of the LF do not vary too much from one scenario to another: the slope is well determined, \( \delta \sim 1.5 - 1.7 \), but the minimum and maximum luminosities are not so well constrained, with \( L_{\text{min}} \sim 0.8 - 3 \times 10^{50} \text{erg s}^{-1} \) and \( L_{\text{max}} \sim 3 - 5 \times 10^{53} \text{erg s}^{-1} \). It is interesting to note that with a different methodology, several groups have confirmed our conclusions on the GRB comoving rate (Guetta & Piran 2007; Le & Dermer 2007; Kistler et al. 2008).

In this paper, we present the results of additional simulations that we have carried out to test if the GRB LF can be a broken power law. With two supplementary parameters (the break luminosity \( L_b \) and the second slope), we have now six (seven) free parameters in the Amati-like relation case (the case of a lognormal peak energy distribution). It is difficult to constrain accurately so many parameters with Monte Carlo simulations. Therefore, we have chosen to keep the maximum luminosity constant in all our simulations. We adopt \( L_{\text{max}} = 10^{53.5} \text{erg s}^{-1} \), according to our previous study (Daigne et al. 2006). We also keep \( L_{\text{min}} \) constant, and equal to a low value corresponding to weak GRBs that cannot be detected at cosmological distance. We usually adopt \( L_{\text{min}} = 10^{50} \text{erg s}^{-1} \) but we have also tested other values (see next section). Keeping these two luminosities constant in our Monte Carlo simulation, we have the same number of degrees of freedom than in the model with a simple power-law LF.

3.2 Results

In our whole new set of simulations, we always find a clear minimum of \( \chi^2 \). The parameters of the best model as well as 1\( \sigma \) error bars are listed in Table 1. As can be seen, we focus on the scenario where the comoving GRB rate follows SFR3 and the peak energy is given by the Amati-like relation. For comparison, we also give the parameters of two reference models with a single power-law LF (Daigne et al. 2006). Fig. 3 illustrates, in the case SFR3 and Amati-like relation, the position of the best models in the parameter space of the LF. As can be seen, the low-luminosity slope is strongly constrained to be small, \( \delta_1 \lesssim 1 \), with a mean value \( \delta_1 \sim 0.6 \), while the high-luminosity slope is larger, \( \delta_2 \gtrsim 1 \), with a mean value \( \delta_2 \sim 1.7 \). The break luminosity (right-hand panel) is not so well constrained with best models having \( L_b \sim 4 \times 10^{50-6} \times 10^{53} \text{erg s}^{-1} \). Fig. 4 compares the fit of the data points (log \( N - \log P \) diagram and \( E_p \) distribution) with the best model obtained either with a power-law or a broken power-law LF. Both models are in good agreement with data, without a preference for one or the other. This is also indicated by the value of the reduced minimum \( \chi^2 \) in both cases: 1.4 (power law) and 1.3 (broken power law) for 37 degrees of freedom.

Fig. 5 shows – for the best model – the LF as well as the luminosity distribution of bursts detected by BATSE, HETE2 and \textit{Swift}. It appears that the high-luminosity branch above \( L_b \) is extremely close to the best model single power-law LF (thin line). Below a few \( 10^{49} \text{erg s}^{-1} \) (corresponding to the lowest values of the contrast, \( \kappa \lesssim 1.2 \)), the fraction of detected GRBs is extremely low (less than \( 10^{-4} \) of the total). The two models (power law versus broken power law) differ in the \( 10^{49}-10^{50} \text{erg s}^{-1} \) range, where the fraction of detected bursts is still small (less than 20 per cent of the total) with therefore little effect on the observable quantities.

These results indicate that present data are not sufficient to distinguish between a power-law and a broken power-law LF. Both models can provide equally good fits to the observations. It is however interesting that a broken power law remains allowed, as there are good theoretical arguments in favour of such a shape (see

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\(^1\) In Daigne et al. (2006), we have tested whether an evolution of the LF could reconcile the SFR1 or SFR2 scenario with \textit{Swift} data. We found that this is very unlikely as the evolution should be strong \( L \propto (1+z)^{\nu} \) with \( \nu > 2 \). We therefore conclude that the evolution of the GRB rate (i.e. the evolution of the stellar efficiency to produce GRBs) is dominant compared to a possible evolution of the LF.
on 30 July 2018

Table 1. Best models: parameters.

| SFR        | log $L_{\text{min}}$ (erg s$^{-1}$) | log $L_0$ (erg s$^{-1}$) | log $L_{\text{max}}$ (erg s$^{-1}$) | $\delta_1$ | $\delta_2$ | log $E_{p,0}$ (keV) | log $k$ | log $\rho_0$ (GRB Gpc$^{-3}$ yr$^{-1}$) |
|------------|------------------------------------|--------------------------|-------------------------------------|------------|------------|----------------------|--------|-------------------------------|
| 3rd        | 50.3 ± 0.7                         | 53.5 ± 0.4               | 1.54 ± 0.18                         |            |            | -6.0 ± 0.2            | -0.8 ± 0.2 |
| 1          | 45 50.4 ± 0.5                       | 53.5                     | 0.65 ± 0.22                         | 1.71 ± 0.07 | -5.2 ± 0.3  | 0.0 ± 0.3             |
| 2          | 45 50.5 ± 0.4                       | 53.5                     | 0.62 ± 0.20                         | 1.71 ± 0.09 | -5.3 ± 0.2  | -0.1 ± 0.2            |
| 3          | 45 51.2 ± 0.6                       | 53.5                     | 0.60 ± 0.22                         | 1.71 ± 0.22 | -5.9 ± 0.2  | -0.7 ± 0.2            |
| 3          | 46 51.1 ± 0.7                       | 53.5                     | 0.64 ± 0.20                         | 1.70 ± 0.29 | -5.9 ± 0.2  | -0.7 ± 0.2            |
| 3          | 47 51.2 ± 0.6                       | 53.5                     | 0.69 ± 0.24                         | 1.70 ± 0.30 | -5.8 ± 0.2  | -0.6 ± 0.2            |
| 3          | 48 51.3 ± 0.5                       | 53.5                     | 0.74 ± 0.27                         | 1.75 ± 0.38 | -5.8 ± 0.2  | -0.6 ± 0.2            |
| 3          | 49 51.6 ± 1.1                       | 53.5                     | 1.02 ± 0.45                         | 1.98 ± 0.72 | -5.7 ± 0.2  | -0.5 ± 0.2            |
| 3          | 50 52.0 ± 1.2                       | 53.5                     | 1.47 ± 0.65                         | 2.05 ± 0.86 | -5.9 ± 0.1  | -0.7 ± 0.1            |

Lognormal peak energy distribution

| SFR        | log $L_{\text{min}}$ (erg s$^{-1}$) | log $L_0$ (erg s$^{-1}$) | log $L_{\text{max}}$ (erg s$^{-1}$) | $\delta_1$ | $\delta_2$ | log $E_{p,0}$ (keV) | log $k$ | log $\rho_0$ (GRB Gpc$^{-3}$ yr$^{-1}$) |
|------------|------------------------------------|--------------------------|-------------------------------------|------------|------------|----------------------|--------|-------------------------------|
| 3rd        | 50.5 ± 1.3                         | 53.7 ± 0.9               | 1.52 ± 0.48                         |            |            | 2.79 ± 0.08           | -6.2 ± 0.2 | -1.0 ± 0.2 |
| 1          | 45 51.2 ± 0.4                       | 53.5                     | 0.67 ± 0.23                         | 1.80 ± 0.16 | 2.80 ± 0.11 | -5.5 ± 0.2            | -0.3 ± 0.2 |
| 2          | 45 51.5 ± 0.5                       | 53.5                     | 0.71 ± 0.21                         | 2.04 ± 0.57 | 2.75 ± 0.09 | -5.6 ± 0.3            | -0.5 ± 0.3 |
| 3          | 45 52.1 ± 0.5                       | 53.5                     | 0.66 ± 0.20                         | 2.37 ± 0.71 | 2.80 ± 0.08 | -6.1 ± 0.2            | -0.9 ± 0.2 |

*Preferred model in the case of a single power-law LF (Daigne et al. 2006).

Figure 3. Parameter space (SFR3 and Amati-like relation): we plot the location of the best models (1σ level) for a broken power-law LF with fixed minimum and maximum luminosities $L_{\text{min}} = 10^{45}$ erg s$^{-1}$ and $L_{\text{max}} = 10^{53.5}$ erg s$^{-1}$ (big dots). The range of parameters explored in the Monte Carlo is indicated with a box. In the right-hand panel, the best models for a single power-law LF (Daigne et al. 2006) are also plotted with small dots (in this case the $x$-axis stands for the minimum luminosity and the $y$-axis for the slope).

Section 2. Table 1 shows that the properties of the broken power-law LF remain very stable as long as $L_{\text{min}}$ is kept to a low value ($L_{\text{min}} \lesssim 10^{38}$ erg s$^{-1}$): the position of the break and especially the values of the two slopes are not changing much, even for different GRB rates (SFR1 and SFR2 have also been tested). It seems however that the broken power-law LF is more sensitive to the assumptions on the spectral parameters. In the case where the spectral properties are not correlated to the luminosity (lognormal peak energy distribution), the low-luminosity slope is not too different from the ‘Amati-like relation’ case ($\delta_1 \approx 0.7$ instead of 0.6), but the break luminosity is larger ($L_B \approx 10^{51–52}$ instead of $10^{50–51}$ erg s$^{-1}$) and the high-luminosity branch is steeper ($\delta_2 \approx 1.8–2.4$ instead of 1.7).

These results can be partially compared to other studies. Based on an analysis of the BATSE log $N$–log $P$ diagram, Stern, Tikhomirova & Svensson (2002a) have tested several shapes of GRB LFs, including a power law and a broken power law. Their assumptions concerning GRB spectra are different from those chosen in the present paper but are very close to our ‘lognormal peak energy distribution’ scenario. For a GRB rate similar to our SFR3, they find $\delta_1 \approx 1.3–1.6$, $\delta_2 \gtrsim 3$ and $L_B \approx 10^{51–6} \times 10^{52}$ erg s$^{-1}$. Whereas we are in reasonable agreement for the high-luminosity slope, there is a large discrepancy for the low-luminosity branch, which is much steeper in their study. To understand the origin of this discrepancy, we made an additional simulation where we force $\delta_1 = 1.3$ and let $\delta_2$ and $L_B$ free. We find that a good fit to the log $N$–log $P$ diagram can be found but that the peak energy distribution is not reproduced, which stresses the importance of this additional constrain in our study. Firmani et al. (2004) have presented a set of Monte Carlo...
The GRB luminosity function in the internal shock model

Figure 4. Best model (SFR₃ and Amati-like relation). Left: the simulated log N–log P diagram of BATSE is plotted as well as BATSE data (Stern, Atteia & Hurley 2002b). Right: the simulated peak energy distribution of bright BATSE bursts is plotted as well as the observed distribution (Preece et al. 2000). In both panels, the best model for a broken power-law LF with fixed minimum and maximum luminosities $L_{\text{min}} = 10^{45}$ erg s$^{-1}$ and $L_{\text{max}} = 10^{53.5}$ erg s$^{-1}$ is plotted in thick line. For comparison the best model for a single power-law LF obtained in Daigne et al. (2006) is plotted in thin line.

Figure 5. LF in the scenario SFR₃ and Amati-like relation. The apparent LF, as well as the luminosity distribution of bursts detected by BATSE, HETE₂ and Swift are plotted in thick line for the best model using a broken power-law with fixed minimum and maximum luminosities $L_{\text{min}} = 10^{45}$ erg s$^{-1}$ and $L_{\text{max}} = 10^{53.5}$ erg s$^{-1}$. All other parameters can be found in Table 1. For comparison the best model for a single power-law LF obtained in Daigne et al. (2006) is plotted in thin line. Despite the fact that this figure was obtained with a special run simulating 10⁹ GRBs with the best model parameters, the curves are very noisy at low luminosity, as these events are very rare.

simulations with assumptions on the intrinsic GRB properties that are very similar to ours but different observational constraints, as they fit the distribution of BATSE pseudo-redshifts obtained from the luminosity–variability correlation (Fenimore & Ramirez-Ruiz 2000). For the case with an intrinsic correlation between the luminosity and the spectral properties (Amati-like case) they find a break at $L_b \simeq 3 \times 10^{52}$ erg s$^{-1}$ with the low- and high-luminosity slopes equal to $\delta_1 \simeq 0.8$ and $\delta_2 \simeq 2.1$. Taking into account the differences in the two approaches, the agreement between this study and our result is satisfactory, especially for the slopes. More recently, Guetta, Piran & Waxman (2005) and Guetta & Piran (2007) have also studied the GRB rate and LF using the recent results from Swift. Their analysis is based on the use of the log N–log P diagram only and they assume a very simplified GRB spectral shape, that is a power-law spectrum of photon index $-1.6$. For SFR₃, they find a break luminosity $L_b \simeq 4 \times 10^{51}$ erg s$^{-1}$ and low- and high-luminosity slopes $\delta_1 \simeq 0.1$ and $\delta_2 \simeq 2$, with large error bars. There is therefore a discrepancy for the low-luminosity slope which is flatter in their case. We believe however that the use of a more realistic spectral shape together with a constraint on the peak energy distribution leads in our case to a better determined LF at low luminosity.

The two important results from this new set of Monte Carlo simulations are that (1) a broken power law is compatible with present data but is not preferred compared to a single power law (equally good fits of the observations); (2) if the LF is indeed a broken power law, the low-luminosity slope is constrained to be $\delta_1 \simeq 0.6 \pm 0.2$ (Amati-like relation) or $\delta_1 \simeq 0.7 \pm 0.2$ (lognormal peak energy distribution), i.e. compatible with the prediction of the internal shock model derived in Section 2.

3.3 The rate of underluminous GRBs

A by-product of this study is an estimate of the local GRB rate, which is given in Table 1 and is typically, in the SFR₃ scenario, 0.1–0.3 GRB Gpc$^{-3}$ yr$^{-1}$ in the Amati-like case, and 0.08–0.2 GRB Gpc$^{-3}$ yr$^{-1}$ for a lognormal peak energy distribution. This is in good agreement with the results of Guetta & Piran (2007). Despite the fact that our broken power-law LF can in principle extend to very low luminosity, such a low local rate corresponds to less
that $10^{-3}$ GRB yr$^{-1}$ within 100 Mpc, which cannot explain the observation of GRB 980425 at $z = 0.008$ or GRB 060618 at $z = 0.03$. As shown in Daigne & Mochkovitch (2007), such underluminous bursts are well explained in the framework of the internal shock model by mildly relativistic/mildly energetic outflows. This would then indicate that the collapsing stars capable to generate such outflows, less extreme than those required to produce standard GRBs, are very numerous and should then produce an additional component in the LF, dominant at very low luminosity. This new branch cannot be the simple continuation of the LF of standard GRBs derived in this paper. Recently, Liang et al. (2007) have studied such a two component LF and found that the local rate corresponding to the low-luminosity component has to be several orders of magnitude above that of standard GRBs but can still represent only a fraction of all type Ib/c supernovae.

4 CONCLUSION

We have demonstrated that in the framework of the internal shock model, a two branch LF is naturally expected, with a predicted low-luminosity branch which is a power law of slope close to $-0.5$. This result is robust as long as the central engine responsible for GRBs is capable to produce a broad diversity of outflows, from highly variable to very smooth.

Using a set of Monte Carlo simulations, we have then shown that current observations (log $N$–log $P$ diagram, peak energy distribution, fraction of XRRs and XRFs) are compatible with a broken power-law LF but still do not exclude a single power-law distribution. The low-luminosity slope of the broken power law is strongly constrained to be $\delta_1 \simeq 0.4$–0.9, compatible with the prediction of the internal shock model.

These results are encouraging but only preliminary. A better determination of the GRB LF would provide an interesting test of the internal shock model when the low-luminosity branch becomes more easily accessible. This will however require the difficult task of detecting many bursts at the threshold of current instruments and measuring their redshift and spectral properties.

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