110,116 Cd(α, α)110,116 Cd elastic scattering and systematic investigation of elastic α scattering cross sections along the Z = 48 isotopic and N = 62 isotonic chains

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The elastic scattering cross sections for the reactions 110,116Cd(α, α)110,116Cd at energies above and below the Coulomb barrier are presented to provide a sensitive test for the alpha-nucleus optical potential parameter sets. Additional constraints for the optical potential are taken from the analysis of elastic scattering excitation functions at backward angles which are available in literature. Moreover, the variation of the elastic alpha scattering cross sections along the Z = 48 isotopic and N = 62 isotonic chain is investigated by the study of the ratios of the 106,110,116Cd(α, α)106,110,116Cd scattering cross sections at Ecm ≈ 15.6 and 18.8 MeV and the ratio of the 110Cd(α, α)110Cd and 112Sn(α, α)112Sn reaction cross sections at Ecm ≈ 18.8 MeV, respectively. These ratios are sensitive probes for the alpha-nucleus optical potential parameterizations. The potentials under study are a basic prerequisite for the prediction of α-induced reaction cross sections, e.g. for the calculation of stellar reaction rates in the astrophysical p- or γ-process.

I. INTRODUCTION

Most of the nuclei heavier than iron are built up via neutron capture reactions in the so-called s and r processes. However, on the proton-rich side of the valley of stability there are 35 proton-rich nuclei not created by neutron capture processes. These mostly even-even proton-rich, stable isotopes between 72Se and 196Hg are the so called p nuclei [1]. Their natural isotopic abundance is 10 - 100 times less than that of the more abundant neutron-rich isotopes which were synthesized in the s- or r-processes.

In the production of the p nuclei, photon-induced reactions at temperatures around a few GK are playing a crucial role. It is generally accepted that the main stellar mechanism synthesizing these nuclei — the so called γ process — is initiated by (γ,n) photodisintegration reactions on preexisting neutron-rich s and r seed nuclei. Photons with high energy and high flux — necessary for the γ - induced reactions — are available in explosive nucleosynthesis scenarios like in the Ne/O burning layer in type II supernovae [1, 2]. As the neutron separation energy increases along the (γ,n) path toward more neutron deficient isotopes, (γ,p) and (γ,α) reactions become more important and process the material toward lower atomic numbers [2, 5]. Recently, consistent studies of p nucleosynthesis have become available, employing theoretical reaction rates in large reaction networks [3, 4]. These studies confirmed that, in the case of the heavy p nuclei (140 ≤ A ≤ 200), (γ,n) and (γ,α) reactions play the dominant role.

Modeling the synthesis of the p nuclei and calculating their abundances requires an extended reaction network involving more than 10⁴ reactions on 2000 mostly unstable nuclei. The reaction rates are usually based on the Hauser-Feshbach statistical model. Because of the experimental challenges very few (γ,α) studies have been performed until now: in a pioneer experiment the cross section of the 144Sm(γ,α)140Nd has been measured recently [5]. However, in such an experiment the target nucleus is always in its ground state, whereas in stellar environments thermally populated excited states also contribute to the reaction rate. Thus theoretical considerations cannot be avoided [6]. Alternatively, the (γ,α) rates can be determined experimentally by measuring the inverse (α,γ) reaction cross section and converting the results by using the detailed balance theorem. In this direction the influence of thermally excited states remains relatively small [7, 8]. In recent years a range of (α,γ) reaction cross sections on 70Ge, 96Ru, 106Cd, 112,117Sn, 113In, 144Sm, 151Eu and 169Tm has been measured using the activation method, and the results have been compared with model predictions [9, 17].

It was generally found that the (γ,α) and (α,γ) reaction cross section calculations are very sensitive to the choice of the α-nucleus potential which is a sum of a Coulomb and a nuclear part (the latter one consists of a
real and an imaginary part). The cross section predictions using different global alpha-nucleus optical potential parameterizations can differ within an order of magnitude [12]. Since the parameters of the global alpha-nucleus optical potentials are usually determined from the analysis of the angular distributions of elastically scattered alpha particles (and are adjusted to alpha-induced cross sections if experimental data exist), the elastic alpha scattering cross sections on several $p$ nuclei had been measured in recent years at ATOMKI [18–22] and similar experiments are ongoing at Notre Dame University [23].

In order to increase our knowledge on the alpha-nucleus optical potential parameterizations the energy and the mass dependence of the potential parameters has to be understood. Although it would be helpful to perform systematic investigations on the alpha-nucleus optical potential parameterizations in the whole mass range of the $p$ nuclei (i.e. from about $A \approx 70$ up to almost $A \approx 200$), the fact that most of these nuclei have low-lying first excited state makes a study of elastic scattering experimentally very difficult. Experimental studies are well accessible in the region of the lower mass $p$ nuclei in the $A \approx 100$ mass range and around $A \approx 140–150$ where relatively high-lying first excited states are found. Here the features of the optical potential parameterizations should be as well understood as possible. However, as a word of caution, one should to keep in mind that high-lying first excited states are related to shell closures (e.g. $Z = 50$ and $N = 82$ in these mass regions), and the imaginary part of the optical potential is typically smaller for closed-shell nuclei than for nuclei off closed shell. It is one motivation of the present investigation to study the optical potential for nuclei without closed shells.

From the astrophysical point of view, the potential parameters should be derived in the relevant energy region, in the so called Gamow window. However, at those sub Coulomb energies the elastic scattering cross sections are practically not deviating from the Rutherford cross section, and for this reason it is not possible to derive reliable optical potential parameters for these energies. Consequently, the experiments have to be performed at slightly higher energies just below and above the Coulomb barrier and then the resulting optical potential parameters have to be extrapolated down to the relevant energy region. Contrary to the real part of the nuclear potential which has a smooth energy-dependence, the imaginary part changes drastically around the Coulomb barrier.

A global alpha-nucleus optical potential must be able not only to provide a correct prediction for the alpha elastic scattering angular distributions but also to describe the variation of the angular distributions along isotopic and isotonic chains. This is especially important for the extrapolation to unstable nuclei where no measured alpha-induced reaction data are available and the potential cannot be derived from experimental scattering data. Recently, the variation of the scattering cross sections along the $Z = 50$ isotopic and $N = 50$ isotonic chain has been investigated. The ratio of the measured cross sections of the $^{112}$Sn($\alpha,\alpha)^{112}$Sn/$^{124}$Sn($\alpha,\alpha)^{124}$Sn ($Z = 50$) and $^{89}$Y($\alpha,\alpha)^{89}$Y/$^{92}$Mo($\alpha,\alpha)^{92}$Mo ($N = 50$) reactions showed an oscillation pattern at backward angles. It was found that both regional and global alpha-nucleus optical potential parameterizations failed to reproduce these oscillation patterns [22, 24].

In order to further investigate the variation of the elastic alpha scattering cross sections along isotopic and isotonic chains, in the present work the $^{110,116}$Cd($\alpha,\alpha)^{110,116}$Cd reactions are studied at energies above and below the Coulomb barrier. This paper is organized as follows. In Sec. II we describe our experimental procedure. The measured angular distributions are compared to predictions using local (Sec. III) and global excitation functions taken from literature [25, 27] provide further information on the potentials; the experimental excitation functions are compared to the results from the local, regional, and global potentials in Sec. IV. Additionally, all calculations are used to predict the ratio of angular distributions along the cadmium ($Z = 48$) isotopic and $N = 62$ isotonic chains (Sec. VI). The elastic alpha scattering cross sections of the $^{106}$Cd($\alpha,\alpha)^{106}$Cd and the $^{112}$Sn($\alpha,\alpha)^{112}$Sn are taken from [29, 22]. A further detailed study on $^{106}$Cd($\alpha,\alpha)^{106}$Cd elastic scattering and the influence of the chosen potential on $\alpha$-induced cross sections of $^{106}$Cd will be presented in a separate paper [29].

### Table I: Charge and neutron number, energy of the first excited state of the target nuclei, enrichment, and $E_{lab.}$ and $E_{c.m.}$ energies for each of the angular distributions studied in the present work, the $^{106}$Cd and $^{112}$Sn data are taken from [20, 22].

| target nuclei | proton number | neutron number | 1st excited state [keV] | enrichment [%] | $E_{lab.}$ [MeV] | $E_{c.m.}$ [MeV] | Ref. |
|---------------|---------------|----------------|------------------------|---------------|-----------------|-----------------|-----|
| $^{110}$Cd    | 48            | 62             | 657.76                 | 95.7          | 16.14, 19.46    | 15.6, 18.8      | this work       |
| $^{116}$Cd    | 48            | 68             | 513.49                 | 98.3          | 16.14, 19.46    | 15.6, 18.8      | this work       |
| $^{108}$Cd    | 48            | 58             | 632.64                 | 96.5          | 16.13, 19.61    | 15.6, 18.9      | [20, 21]        |
| $^{112}$Sn    | 50            | 58             | 1256.85                | 99.8          | 19.51           | 18.8            |       |
II. EXPERIMENTAL PROCEDURE

The experiment was carried out at the cyclotron laboratory of ATOMKI, Debrecen. A similar experimental setup was used also in the previous experiments [18, 19, 21, 22, 24] and is described in more detail in [29–30]. The proton and neutron number, the energy of the first excited states of the target nuclei and the energies of the measured angular distributions are summarized in Table I. The following paragraphs provide a short description of the experimental setup.

The targets were produced by evaporating highly enriched (≥ 95%) $^{110,116}$Cd onto thin carbon foils (≈ 20 µg/cm²). The target thickness was approximately 200 µg/cm², determined via alpha particle energy loss measurement using radioactive sources. The targets were mounted on a remotely controlled target ladder in the center of the scattering chamber. Figure 1 illustrates the scattering chamber.

The energies of the alpha beam were 16.14 and 19.46 MeV with typical beam currents of 150-200 pnA. An aperture of 2 x 6 mm was mounted on the target ladder to check the beam position and size of the beamspot before and after every change of the beam energy or current. We optimized the beam until not more than 1% of the total beam current could be measured on this aperture. As a result of the procedure, the horizontal size of the beamspot was below 2 mm during the whole experiment which is crucial for the precise determination of the scattering angle. Since the imaginary part of the optical potential depends sensitively on the energy, it is important to have a well-defined beam energy. Therefore the beam was collimated by tight slits (1 mm wide) after the analyzing magnet; this corresponds to an overall energy spread of around 100 keV which is the dominating contribution of the energy resolution of the spectra (see Fig. 2).

Six ion implanted silicon detectors with active areas of 50 mm² were used to measure the angular distributions. Their solid angles varied between $1.45 \times 10^{-4}$ and $1.87 \times 10^{-4}$. The detectors were mounted on two turntables. Two detectors with angular distance of 10° were mounted onto the upper turntable and were used to measure the scattering cross sections at forward angles. To measure the cross sections at backward angles four detectors with angular distance of 5° were used. The ratio of their solid angles was determined by measurements at overlapping angles with good statistics (≤1% uncertainty). Typical spectra are shown in Fig. 2. As can be seen, the relevant peaks from elastic $^{116}$Cd-α and $^{110}$Cd-α scattering are well resolved from the inelastic events and from both the $^{12}$C-α and $^{16}$O-α elastic scattering.

Knowledge of the exact angular position of the detectors is of crucial importance for the precision of a scattering experiment since the Rutherford cross section depends sensitively on the angle specially at forward directions. The uncertainty in the angular distribution is dominated by the error of the scattering angles in the for-
ward region. To determine the scattering angle precisely, we measured kinematic coincidences between elastically scattered alpha particles and the corresponding $^{12}$C recoil nuclei using a pure carbon backing as target. One detector was placed at $\theta = 70^\circ$, and the signals from the elastically scattered alpha particles on $^{12}$C were selected as gates for the other detector which moved around the expected $^{12}$C recoil angle $\theta = 45.83^\circ$. We repeated this process for all detector pairs. Figure 3 shows the relative yield of the $^{12}$C recoil nuclei in coincidence with elastically scattered alpha particles as a function of the $^{12}$C recoil angle. The final angular uncertainty was found to be $\Delta \theta \leq 0.12^\circ$.

Complete angular distributions between $20^\circ$ and $175^\circ$ were measured at energies of $E_{\text{lab}} = 16.14$ and 19.46 MeV in $^1$S ($20^\circ \leq \theta \leq 100^\circ$) and $2.5^\circ$ ($100^\circ \leq \theta \leq 175^\circ$) steps. The statistical uncertainties varied between 0.1% (forward angles) and 4% (backward angles). The count rates $N(\theta)$ have been normalized to the yield of the monitor detectors $N_{\text{Mon.}}(\theta = 15^\circ)$:

$$\left( \frac{d\sigma}{d\Omega} \right) (\theta) = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mon.}} \frac{N(\theta)}{N_{\text{Mon.}}} \frac{\Delta\Omega_{\text{Mon.}}}{\Delta\Omega},$$

with $\Delta\Omega$ being the solid angles of the detectors. Whereas the Rutherford normalized cross sections cover only about two orders of magnitude between the highest (forward angles at $E_{\text{lab}} = 16.14$ MeV) and the lowest cross sections (backward angles at $E_{\text{lab}} = 19.46$ MeV), the underlying cross sections cover more than four orders of magnitude. Over this huge range of cross sections almost the same accuracy of about 4-5% total uncertainty could be achieved. This error is mainly caused by the uncertainty of the determination of the scattering angle in the forward region and from the statistical uncertainty in the backward region.

The origin of the above uncertainties has to be studied in further detail. The uncertainty of the scattering angle is composed of two fractions. First, a systematic uncertainty comes from the alignment of the angular scale and the beam direction; it affects all data points in the same direction. This uncertainty is partly compensated by the absolute normalization of the data (see below) where the data are adjusted to Rutherford scattering at forward angles. Second, the accuracy of setting/reading of the angle leads to a statistical uncertainty, obviously different for each data point. The combination of both remains below 4-5%. From the small scatter of the data points (see Fig. 4) it may be estimated that the systematic contribution dominates the real uncertainties. Because the statistical uncertainties are smaller than the shown error bars, it must be expected that the resulting $\chi^2/F$ may be even below 1.0 for the locally adjusted potentials (see Sec. III).

The absolute normalization is done in two steps. In a first step the absolute normalization is taken from experiment, i.e. from the integrated beam current, the solid angle of the detectors, and the thickness of the target. This procedure has a relatively large uncertainty of the order of 10% which is mainly based on the uncertainties of the target thickness. In a second step a “fine-tuning” of the absolute normalization is obtained by comparison to theoretical calculations at very forward angles. It is obvious that calculated cross sections from any reasonable potential do practically not deviate from the Rutherford cross section at the most forward angles of this experiment; typical deviations are below 0.5% for all potentials listed in Sec. III and IV (including those potentials that do not describe details of the angular distributions at backward angles). This “fine-tuning” changed the first experimental normalization by only 2.5% and thus confirmed the first normalization within the given errors.

The measured angular distributions are shown in Fig. 4. The $^{106}$Cd($\alpha,\alpha$)$^{106}$Cd data, are taken from [20][21]. The lines are the results of optical model predictions using local, regional and global $\alpha$-nucleus potentials (see discussion in the following Sec. III and IV).

III. THE LOCAL $\alpha$-NUCLEUS OPTICAL POTENTIAL

The complex optical model potential (OMP) $U(r)$ is given by:

$$U(r) = V_C(r) + V(r) + iW(r),$$

where $V_C(r)$ is the Coulomb potential, $V(r)$, and $W(r)$ are the real and the imaginary parts of the nuclear potential, respectively. The volume integrals per interacting nucleon pair $J_R$ and $J_I$ are defined as usual; although
$J_R$ and $J_I$ are negative (attractive real potential and absorption by the imaginary potential), in the discussion the negative signs are omitted (as usual).

The $V(r)$ real part of the local optical potential is derived from the double-folding model. For calculating the $V_F(r)$ folding potential the density-dependent M3Y interaction \([35,37]\) was used

$$V(r) = \lambda V_F(r/w)$$  

(3)

where $\lambda \approx 1.1 - 1.4$ is the potential strength parameter \([38]\) and $w = 1.0 \pm 0.05$ is the width parameter that slightly modifies the potential width. (Larger deviations of the width parameter $w$ from unity would indicate a failure of the folding potential.) The nuclear densities are derived form the compilation of charge densities measured by electron scattering \([39]\). Thus, we have only two adjustable parameters ($\lambda$ and $w$) in the real part of the potential (e.g., compared to three parameters for Woods-Saxon potentials), and in addition the range of these parameters is very restricted from the systematics of volume integrals $J_R$ \([38]\) and the above requirement $w \approx 1$.

The Coulomb potential is taken in the usual approximation of a homogeneously charged sphere. The Coulomb radius $R_C$ is equal to the root-mean-square (rms) radius of the folding potential with $w = 1$.

The imaginary part $W(r)$ of the potential is taken in the usual Woods-Saxon parametrization. For the fits to the experimental data we use volume and surface potentials:

$$W(r) = W_V \times f(x_V) + 4 W_S \times \frac{df(x_S)}{dx_S}$$  

(4)

with the potential depths $W_V$ and $W_S$ of the volume and surface parts and

$$f(x_i) = \frac{1}{1 + \exp (x_i)}$$  

(5)

and $x_i = [r - R_i (A_T^{1/3})]/a_i$ with the radius parameters $R_i$ in the light-ion convention, the diffuseness parame-
It is well-known that there are ambiguities in the determination of the optical potential at energies around and especially below the Coulomb barrier. We do not consider here the so-called “family problem” which means that almost identical angular distributions are calculated from potentials where the depth of the real part is increased or decreased in discrete steps by about 30%. This problem has been discussed in detail in [18], and its influence on $\alpha$-induced reaction cross sections for $^{106}$Cd will be one focus of the separate study of $^{106}$Cd [28]. We restrict ourselves here to real potentials with $J_R \approx 350\,\text{MeV}\,\text{fm}^3$; these volume integrals are consistent with results which are derived at higher energies without ambiguities [38].

With the above restriction for the volume integral $J_R$, the angular distributions at 19 MeV can be fitted satisfactorily. The reproduction of the data is excellent, and as expected, $\chi^2/F$ values below 1.0 are found. The width parameters remain very close to unity (deviation less than 1%). The strength parameters $\lambda$ and the resulting volume integrals $J_\lambda$ are slightly larger by a few per cent than found for neighboring semi-magic nuclei. The imaginary parts have volume integrals around $J_I \approx 90 - 90\,\text{MeV}\,\text{fm}^3$, again somewhat larger than for neighboring semi-magic nuclei. Such a behavior is expected from the larger absorption and increased reaction cross section; e.g., the total reaction cross sections $\sigma_{\text{reac}}$ around 19 MeV are about 10% smaller for the semi-magic even-even nuclei $^{112,124}$Sn (see [10]) compared to non-magic even-even $^{106,110,116}$Cd.

Unfortunately, the situation changes for the angular distributions at 16 MeV. Here the best-fit potentials require width parameters $w$ which deviate by about 5% from unity ($w = 1.046$ for $^{110}$Cd, $w = 0.955$ for $^{116}$Cd). However, the fit quality remains almost the same if the width parameter $w$ is kept fixed at $w = 1.0$. So it must be noted that the angular distributions at 16 MeV are not sufficiently sensitive to the width parameter $w$ of the potential. Instead, the fits provide a so-called “one-point potential” [18, 20, 41, 42]. The smaller (larger) width parameter $w$ is compensated by a larger (smaller) strength parameter $\lambda$ leading to a fixed potential depth at a large radius (e.g., a value $R_{0.2}$ where the real potential depth is 0.2 MeV is derived in [20] from the analysis of elastic scattering excitation functions). We show two calculations in Fig. 4 using the adjusted values for $w$ and using the fixed value $w = 1.0$; the parameters of both calculations are also listed in Table I. Although the differences between the two calculations are small, it must be noted that the derived total reaction cross sections $\sigma_{\text{reac}}$ differ by about 10%. Because the width parameter $w$ is nicely determined to be close to unity from the 19 MeV data, we prefer the total reaction cross sections from the calculations with $w = 1.0$ ($\sigma_{\text{reac}} = 506\,\text{mb}$ for $^{110}$Cd and 536 mb for $^{116}$Cd), and we assign an uncertainty of 10% for $\sigma_{\text{reac}}$ from the 16 MeV data. The total cross sections at 19 MeV are well-defined with an uncertainty of about 3% (discussion of uncertainties see also [40]).

In addition to the above two calculations with the adjusted width parameter $w$ and the fixed $w = 1.0$, a third calculation has been performed using the potential which was derived at the higher energy of 19 MeV. For both $^{110}$Cd and $^{116}$Cd it is found that the calculated cross sections at backward angles are slightly larger than the experimental values. This clearly indicates that a slight energy dependence of the potential is required to reproduce the angular distributions at both energies.

Further restrictions on the $\alpha$-nucleus potential can be derived from the analysis of excitation functions, see Sec. V.

IV. GLOBAL OPTICAL MODEL PREDICTIONS

In the present work the following open access regional and global alpha nucleus optical potential parameterizations are considered: the recent regional potential of Avrigeanu et al. [43] and the global potential of McFadden and Satchler [45].

The regional optical potential (ROP) of Avrigeanu et al. [43] was derived starting from a semi-microscopic analysis, using the double folding model [49], based on alpha-particle elastic scattering on $A \approx 100$ nuclei at energies below 32 MeV. The energy-dependent phenomenological imaginary part of this semi-microscopic optical potential takes into account also a dispersive correction to the microscopic real potential. A small revision of this ROP and especially the use of local parameter sets were able to describe the variation of the elastic scattering cross sections along the Sn isotopic chain [50]. A further step to include all available $\alpha$-induced reaction cross sections below the Coulomb barrier has recently been carried out [43]. First, the ROP based entirely on $\alpha$-particle elastic scattering [48] was extended to $A \approx 50-120$ nuclei and energies from $\approx 13$ to 50 MeV. Secondly, an assessment of available $(\alpha, \gamma), (\alpha, n)$ and $(\alpha, p)$ reaction cross sections on target nuclei ranging from $^{45}$Sc to $^{118}$Sn at incident energies below 12 MeV was carried out. In the present study we use the potential from a review paper [43]. A minor revision of this potential has been
TABLE II: Parameters of the local optical potential. The alpha optical potential parameters of the $^{106}\text{Cd}$ and $^{112}\text{Sn}$ nuclei are taken from [20, 22].

| Nucleus | $E_{\text{c.m.}}$ | $\lambda$ | $\varphi$ | $J_R$ | $r_{R,\text{rms}}$ | $W_V$ | $r_V$ | $a_V$ | $W_S$ | $r_s$ | $a_s$ | $J_I$ | $r_{I,\text{rms}}$ | $\sigma_{\text{reac}}$ | $\chi^2/F$ |
|---------|-------------------|------------|-----------|-------|------------------|-------|-------|-------|-------|-------|-------|-------|----------------|----------------|-------|
| $^{110}\text{Cd}$ | 15.6 | 1.955 | 1.046 | 362.3 | 5.495 | - | - | 32.1 | 1.563 | 0.344 | 7.14 | 7.617 | 456 | 0.51 |
| $^{112}\text{Sn}$ | 18.8 | 1.348 | 1.001 | 356.1 | 5.310 | - | - | 39.4 | 1.366 | 0.472 | 7.37 | 6.926 | 832 | 0.61 |
| $^{106}\text{Cd}$ | 15.6 | 1.378 | 0.987 | 367.9 | 5.164 | -2.9 | 1.748 | 0.347 | 84.8 | 1.263 | 0.207 | 90.9 | 6.127 | 749 | 0.77 |
| $^{116}\text{Cd}$ | 15.6 | 1.602 | 0.955 | 367.8 | 5.226 | - | - | 19.2 | 0.613 | 1.391 | 37.9 | 5.914 | 609 | 0.22 |

*fixed

The global potential of McFadden and Satchler [45] is fitted to the numerous alpha elastic scattering experiments done on nuclei between O and U at alpha energies of 24.7 MeV in the 60’s. Fits were obtained using a four-parameter Woods-Saxon potential. This simple potential is widely used for reaction rate calculations and for $p$ process reaction flow simulations [47].

The results of the model calculations are compared with the experimental data in Figure 4. For a strict comparison between the potentials a $\chi^2$ analysis has been done. The resulting $\chi^2$ parameters can be found in Table IV. It is interesting to note that the ROP of $^{110}\text{Cd}$ is almost perfect for the $^{106}\text{Cd}$ case whereas it slightly overestimates the elastic scattering cross sections at backward angles for $^{110,116}\text{Cd}$. This is also seen in the analysis of the backward angle excitation functions (see next Sec. V). It is obvious that evolution of the cross sections along the cadmium isotopic chain cannot be reproduced exactly by the ROP under these circumstances. Nevertheless, as discussed in detail in Sec. VI, the cross section ratios are very sensitive to the chosen potential, and thus these ratios are able to provide some hints on possible improvements of the ROP.

V. EXCITATION FUNCTIONS AT BACKWARD ANGLES

Excitation functions of elastic scattering at very backward angles have been measured by three groups [25-27]. Numerical data are not available, and all data have to be read from the given figures in [25-27]. Ref. [25] studies only $^{110}\text{Cd}$, and the figure is hard to decipher. So we restrict ourselves here to the excitation functions in Refs. [26-27]: Badawy et al. [26] have measured excitation functions at the very backward angle of $\vartheta_{\text{lab}} \approx 178.6^\circ \approx \vartheta_{\text{c.m.}}$, from about 10.5 MeV to 15.5 MeV in the c.m. system. Miller et al. [27] show data at $\vartheta_{\text{lab}} \approx 175^\circ$ from about 9.5 MeV to 16.5 MeV. Because the Rutherford-normalized cross section at very backward angles is almost constant, we present both data sets in a common figure (the calculated deviations because of the different angle remain below 1% in the whole energy range shown in Fig. 5). In addition, we add the most backward data point from our angular distributions which is also at approximately $175^\circ$.

As already pointed out in [26], it is impossible to derive an optical potential from an excitation function at one particular angle. Instead, it is only possible to determine an approximate strength of the imaginary potential and a so called “one-point” potential for the real part. Nevertheless, the excitation functions provide additional information, and global potentials should be able to reproduce the measured excitation functions. In the following we compare the predictions from the local potentials (without further adjustment to the experimental data of the excitation function) and from the global potentials to the experimental data [26, 27]. For $^{110}\text{Cd}$ the agreement between the experimental data of [26] and [27] is not good; the data of [26] are slightly higher than the data of [27]. The result from the present analysis of the full angular distribution is a few per cent lower than [27]. It must be noted that the above discrepancies may – at least partly – be assigned to the
TABLE III: Regional optical potential parameters calculated from Table 3 of [43].

| Nucleus | E_{c.m.} | V_{R} | r_{R} | a_{R} | W_{V} | r_{V} | a_{V} | W_{S} | r_{S} | a_{S} | \sigma_{reac} |
|---------|----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-----------|
| 110Cd   | 15.6     | -134.2| 1.367 | 0.636 | -6.2  | 1.34  | 0.50  | 21.6  | 1.52  | 0.374 | 377       |
|         | 18.8     | -125.7| 1.405 | 0.602 | -9.8  | 1.34  | 0.50  | 16.9  | 1.52  | 0.374 | 773       |
| 116Cd   | 15.6     | -134.0| 1.367 | 0.637 | -6.0  | 1.34  | 0.50  | 21.9  | 1.52  | 0.368 | 418       |
|         | 18.8     | -125.6| 1.406 | 0.602 | -9.6  | 1.34  | 0.50  | 17.3  | 1.52  | 0.368 | 821       |
| 106Cd   | 15.6     | -134.4| 1.367 | 0.635 | -6.4  | 1.34  | 0.50  | 21.4  | 1.52  | 0.379 | 348       |
|         | 18.9     | -125.5| 1.407 | 0.600 | -10.1 | 1.34  | 0.50  | 16.5  | 1.52  | 0.379 | 754       |
| 112Sn   | 18.8     | -125.9| 1.406 | 0.601 | -9.8  | 1.34  | 0.50  | 17.0  | 1.52  | 0.372 | 706       |

TABLE IV: \chi^2_{red} of predictions using different global and regional parameterizations compared with the angular distributions studied in the present work.

| Parameterization | 110Cd(\alpha,\alpha) | 116Cd(\alpha,\alpha) | 106Cd(\alpha,\alpha) | 112Sn(\alpha,\alpha) | Ref. |
|------------------|-----------------------|-----------------------|-----------------------|-----------------------|------|
|                  | 15.6 MeV              | 18.9 MeV              | 15.6 MeV              | 18.8 MeV              | 18.8 MeV |
| Local            | 0.51-0.70             | 0.32                  | 0.22-0.27             | 0.61                  | 1.21  |
|                  |                      |                      |                      |                      | 1.43  |
|                  |                      |                      |                      |                      | 0.77  |
|                  |                      |                      |                      |                      | this work |
| Avrigeanu        | 40.8                  | 23.8                  | 16.1                  | 42.1                  | 2.44  |
|                  |                      |                      |                      |                      | 2.14  |
|                  |                      |                      |                      |                      | 2.25  |
|                     |                       |                       |                       |                       | [43] |
| McFaddeden        | 85.8                  | 62.9                  | 45.8                  | 103.0                 | 38.8  |
|                  |                      |                      |                      |                      | 54.2  |
|                  |                      |                      |                      |                      | 61.3  |
|                     |                       |                       |                       |                       | [45] |

FIG. 5: (Color online.) Excitation function of 110Cd(\alpha,\alpha)110Cd (left) and 116Cd(\alpha,\alpha)116Cd (right) reactions. Experimental data are taken from [26, 27]. Further discussion see text.
uncertainty of the extraction of the data from figures in [26, 27].

The 19 MeV local potential reproduces the excitation function in general quite well, but slightly overestimates our lower data point at 15.6 MeV. The 16 MeV local potential with the unusual width parameter \( w = 1.046 \) can be excluded because of its strange energy dependence and the strong overestimation of our 18.8 MeV data point. The 16 MeV local potential with the standard width \( w = 1 \) slightly underestimates the whole excitation function but shows a regular energy dependence (similar to the 19 MeV local potential).

Both global and regional potentials also provide a regular energy dependence; as already seen in the analysis of the angular distributions in Fig. 4, both potentials overestimate the experimental data at backward angles and thus also the excitation functions. The new potential of [43] is closer to the data than the old standard potential from McFadden and Satchler [45].

For \(^{116}\text{Cd}\) the data of [26] and [27] are in better agreement. Again, our data point at 15.6 MeV is slightly lower than the excitation function by [27]. The theoretical results are very similar to the \(^{116}\text{Cd}\) case. Again, the 19 MeV local potential nicely reproduces the data. The 16 MeV local potential with the unusual width \( w = 0.955 \) shows an oscillatory energy dependence which is not visible in the experimental data, and it underestimates the excitation function at very low energies. The 16 MeV local potential with \( w = 1 \) reproduces the smooth energy dependence and gives slightly smaller cross sections than the 19 MeV potential.

Both global and regional potentials reproduce again the smooth energy dependence of the data but overestimate the absolute scale. The new potential of [43] is again closer to the data than [45].

VI. VARIATION OF THE SCATTERING CROSS SECTION ALONG ISOTOPIC AND ISOTONIC CHAINS

Recently, the variation of the elastic scattering cross sections along the Sn isotopic chain had been studied by Galaviz et al. [22]. Complete angular distributions of the \(^{112,124}\text{Sn}(\alpha,\alpha)\) \(^{112,124}\text{Sn} \) reactions at \( E_{\text{lab}} = 19.51 \) MeV were measured. It was found that the ratio of the elastic alpha scattering cross sections of the \(^{112}\text{Sn} \) and \(^{124}\text{Sn} \) at backward angles shows an oscillation feature. It was evident that the global alpha-nucleus potentials failed to reproduce either the amplitude and/or the phase of the oscillation pattern for backward angles [22]. This behavior is very similar to the ratio of the Rutherford normalized cross sections of the \(^{92}\text{Mo}(\alpha,\alpha)^{92}\text{Mo} \) and \(^{89}\text{Y}(\alpha,\alpha)^{89}\text{Y} \) derived by Kiss et al., [24].

In the present work, first, the behavior of the elastic alpha scattering cross sections along the Cd isotopic \((Z = 48)\) chain is investigated at \( E_{\text{c.m.}} \approx 15.6 \) and 18.8 MeV. Although there are small differences in the center-of-mass energies (< 120 keV), the ratios of the Rutherford normalized cross sections are well defined because the dominating \( 1/E^2 \) dependence of the scattering cross section is taken into account during Rutherford normalization. Therefore, the ratios of Rutherford normalized cross sections are a very sensitive test for local, regional and global alpha-nucleus potential parameterizations. It is found that the ratio of the normalized scattering cross sections shows an oscillation pattern at backward angles (see Fig. 5) similarly to the variation of the elastic scattering cross sections along the Sn isotopic chain [22]. The large number of experimental points and the low uncertainties on all data sets provide a unique probe to understand the evolution of the alpha nucleus potential along the Cd isotopic chain.

Moreover, the variation of the elastic alpha scattering cross sections along the \( N = 62 \) isotonic chain is also studied by investigating the ratio of the \(^{110}\text{Cd}(\alpha,\alpha)^{110}\text{Cd} \) and \(^{112}\text{Sn}(\alpha,\alpha)^{112}\text{Sn} \) reaction cross sections at \( E_{\text{c.m.}} \approx 18.8 \) MeV (see Fig. 6). The \(^{112}\text{Sn}(\alpha,\alpha)^{112}\text{Sn} \) is taken from [22]. It was found that the ratio of the elastic scattering cross sections along the \( N = 62 \) isotonic chain shows a similar behavior to the one reported in [24].

In Fig. 6 and 7 the experimental ratio of the Rutherford normalized elastic scattering cross sections is compared to the corresponding results of the regional potential of Avrigeau et al. [43] and the global potential of McFadden and Satchler [45]. The grey shaded error band is a very conservative estimate which is derived from the total errors (statistical and systematic, see discussion in Sec. II) of the measured cross sections in Fig. 1. If we consider only the statistical uncertainties, it can be clearly seen that the oscillatory patterns in the cross section ratios are well defined by the experimental data. (Note that the systematic uncertainty cancels out in the ratio to a large extent.) Figs. 6 and 7 show that no regional or global parameterization can describe correctly the amplitude and the phase of the oscillation pattern of the experimental data at backward angles. This fact clearly indicates that the available theoretical alpha nucleus optical potential parameterizations have to be further improved.

A closer look at the shown ratios in Figs. 6 and 7 and the underlying cross sections in Fig. 4 provides deeper insight into the reasons for the failure of the potentials and should lead to suggestions for improvements, in particular for the regional potential by Avrigeau et al. [43] with its careful parameterization of all the parameters of the potential in dependence on the target mass number \( A \), charge number \( Z \), and energy \( E \). It is obvious that the mass- and energy-independent global potential by McFadden and Satchler [45] does a good job, but improvements within this very limited parameter space are
almost impossible. Therefore, the following discussion focuses mainly on the ROP potential.

The ROP potential is able to reproduce the angular distributions for $^{106}$Cd almost perfectly, and thus it also reproduces the total reaction cross section $\sigma_{\text{reac}}$ for $^{106}$Cd. The elastic scattering cross sections of $^{110}$Cd and $^{116}$Cd are significantly smaller at backward angles; this corresponds to a significantly larger $\sigma_{\text{reac}}$. The increase of $\sigma_{\text{reac}}$ with neutron number can be understood easily because of the dominance of the ($\alpha$,n) reaction channel and its increasing cross section with increasing neutron number. However, it is surprising that there is a strong change from $^{106}$Cd to $^{110}$Cd and only a much smaller change from $^{110}$Cd to $^{116}$Cd: the ratio of elastic scattering cross sections at backward angles is about 1.5 for $^{106}$Cd/$^{110}$Cd whereas it is only about 1.25 for $^{110}$Cd/$^{116}$Cd. Any global potential with a smooth mass dependence like e.g. [43] or missing mass dependence [45] will fail to reproduce the cross section ratios in Fig. 6. The calculated ratios are about 1.1 in all cases and smaller than the experimental results. The apparently different behavior of $^{106}$Cd may be understood from a weak subshell closure of the $g_{7/2}$+ neutron shell at $N = 58$. Although one textbook reference of the shell model [46] shows in its Fig. 1 that the $d_{5/2}^+$ subshell is slightly lower than the $g_{7/2}^+$ subshell (and thus one should find a subshell closure at $N = 56$ instead of $N = 58$), there is some evidence from the ground state spins of $J = 5/2^+$ for neighboring $N = 59$ nuclei like $^{105}$Pd, $^{107}$Cd, $^{109}$Sn that the $g_{7/2}^+$ neutron subshell is filled at $N = 58$. Note that the lowering of the $g_{7/2}^+$ subshell below the $d_{5/2}^+$ subshell is well-established for the proton subshells [46]. This weak subshell closure may explain the relatively small total reaction cross section of $^{106}$Cd.

The experimental ratio of elastic scattering cross sections at backward angles is about 0.5 between $^{110}$Cd and $^{112}$Sn. Again, this can be understood, but the argument is different: the neutron numbers are the same for $^{110}$Cd...
FIG. 7: (Color online.) Rutherford normalized elastic scattering cross sections of $^{112}\text{Sn}(\alpha,\alpha)^{112}\text{Sn}$ reaction at $E_{\text{c.m.}} = 18.8$ MeV (left side). Experimental ratio of the scattering cross sections $(\sigma/\sigma_{\text{RUTH}})^{110}\text{Cd} / (\sigma/\sigma_{\text{RUTH}})^{112}\text{Sn}$ at $E_{\text{c.m.}} \approx 18.8$ MeV (gray area with taking into account the experimental uncertainty) versus the angle in center-of-mass frame. The cross sections of the $^{112}\text{Sn}(\alpha,\alpha)^{112}\text{Sn}$ are taken from [22]. The lines correspond to the predictions using the present local, regional [43] and global [45] optical potential parameter sets. For more information see Sec. III and IV.

and $^{112}\text{Sn}$ and cannot have strong impact on the ($\alpha,n$) or total reaction cross section. However, $^{112}\text{Sn}$ is a semi-magic nucleus with $Z = 50$, and thus the total reaction cross section $\sigma_{\text{reac}}$ is smaller compared to neighboring nuclei, and the elastic scattering cross section is larger. Again, such a behavior cannot be reproduced by any potential with a smooth (or even missing) mass dependence.

This leads to the following recommendations for improvements. In addition to a smooth dependence on the mass number $A$ and charge number $Z$, a further dependence on shell closures (e.g. parametrized by the distance to a closed shell) should be included in global parameterizations of $\alpha$-nucleus potentials. This may be complemented by a further dependence of the neutron-to-proton ratio $N/Z$. The above recommendation may be also interpreted as a guide to the experimentalist for further experiments on non-magic nuclei. Note that only very few data on non-magic nuclei have entered into the determination of the global potential [43] above $A > 80$ (see their Table 2); this may also explain that [43] nicely reproduce the data for the semi-magic $^{112}\text{Sn}$ but is not able to describe the data for the non-magic $^{110,116}\text{Cd}$ with the same accuracy.

VII. SUMMARY

In the present work angular distributions of elastically scattered alpha particles on $^{110,116}\text{Cd}$ have been measured at $E_{\text{lab.}} = 16.14$ MeV and 19.46 MeV to provide a sensitive test for global parameterizations of the $\alpha$-nucleus potential used in $p$ process network calculations. The measured data cover the full angular range and have small uncertainties of about $3-4\%$ over the whole angular range.

A local fit to the angular distributions using a folding potential in the real part and a surface Woods-Saxon imaginary part reproduces all measured angular distributions with high accuracy ($\chi^2/F < 1$). The volume integrals are slightly higher than for neighboring semi-magic nuclei. The best-fit potential at 16 MeV shows an unusual width parameter for $^{110}\text{Cd}$ and $^{116}\text{Cd}$ and does not describe the measured excitation functions at backward angles. Very similar fits (also with $\chi^2/F < 1$) can be obtained using the standard width $w = 1$.

The regional and global potentials by [43] and [45] are able to describe the angular distributions with relatively small deviations although both global potentials overestimate the data at backward angles. In all cases the new potential by [43] is closer to the experimental data than the potential of [45]. The same conclusion is found for
the excitation functions at backward angles which are available from literature [26, 27]. However, the situation becomes worse for the evolution of the potentials along isotopic and isotonic chains. The measured ratio of cross sections cannot be reproduced by any regional and global potential because the deviations at backward angles are amplified in the ratios. A reason for this problem may be the influence of shell closures which are not taken into account in the parameterizations of [19] or [20].

Since modeling explosive nucleosynthesis scenarios requires reaction rates on large number of reactions involving thousands of nuclei, the α-nucleus potential has to be known in a wide region. The reliability of the extrapolation to unstable nuclei has to be tested by measuring the elastic scattering cross sections on several nuclei along isotopic and isotonic chains. The ratio of Rutherford normalized cross sections along isotopic or isotonic chains is a very sensitive observable for the quality of α-nucleus potentials, and it should be used in further work to restrict global parameterizations of α-nucleus potentials.

Further systematic experimental elastic alpha scattering studies at energies around the Coulomb barrier are essential, in particular on intermediate mass and heavy nuclei without shell closures. The experimental scattering data should be complemented by data on α-induced reaction cross sections in the same energy region. Scattering and reaction data have to enter into theoretical studies leading eventually to a robust global α-nucleus potential which is able to describe all observables with reasonable accuracy.

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[1] S. E. Woosley and W. M. Howard, Astrophys. J. Suppl. 36, 285 (1978).
[2] M. Arnould and S. Goriely, Phys. Rep. 384, 1 (2003).
[3] T. Rauscher, Phys. Rev. C 73, 015804 (2006).
[4] W. Rapp, J. Görres, M. Wiescher, H. Schatz and F. Käppeler, Astrophys. J. 653, 474 (2006).
[5] T. Rauscher, International Journal of modern Physics E (arXiv: 1010.4283v1).
[6] C. Nair, A. R. Junghans, M. Erhard, D. Bemmerer, R. Beyer, E. Grosse, K. Kosev, M. Marta, G. Rusev, K. D. Schilling, R. Schwengner, and A. Wagner, Phys. Rev. C 81, 055806 (2010).
[7] P. Mohr, Zs. Fülöp and H. Utsunomiya, Eur. Phys. J. A 32, 357 (2007).
[8] T. Rauscher, G. G. Kiss, Gy. Gyürky, A. Simon, Zs. Fülöp, E. Somorjai, Phys. Rev. C 70, 035801 (2009).
[9] Zs. Fülöp, Á. Z. Kiss, E. Somorjai, C. E. Rolfs, H.P. Trautvetter, T. Rauscher and H. Oberhummer, Z. Phys. A 355, 203 (1996).
[10] W. Rapp, M. Heil, D. Hentschel, F. Käppeler, R. Reifarth, H. J. Brede, H. Klein and T. Rauscher, Phys. Rev. C 66, 015803 (2002).
[11] Gy. Gyürky, G. G. Kiss, Z. Elekes, Zs. Fülöp, E. Somorjai, A. Palumbo, J. Görres, H. Y. Lee, W. Rapp, M. Wiescher, N. Özkan, R. T. Güray, G. Efè and T. Rauscher, Phys. Rev. C 74, 025805 (2006).
[12] N. Özkan, G. Efè, R. T. Güray, A. Palumbo, J. Görres, H. Y. Lee, L. O. Lamm, W. Rapp, E. Stech, M. Wiescher, Gy. Gyürky, Zs. Fülöp and E. Somorjai, Phys. Rev. C 75, 025801 (2007).
[13] I. Cata-Danil, D. Filipescu, M. Ivascu, D. Bucurescu, N. V. Zamfir, T. Gлодариу, L. Stroe, G. Cata-Danil, D. G. Ghita, C. Mihai, G. Suliman and T. Sava, Phys. Rev. C 78, 035803 (2008).
[14] C. Yalcin, R. T. Güray, N. Özkan, S. Kutlu, Gy. Gyürky, J. Farkas, G. G. Kiss, Zs. Fülöp, A. Simon, E. Somorjai and T. Rauscher, Phys. Rev. C 79, 065801 (2009).
[15] E. Somorjai, Zs. Fülöp, Á. Z. Kiss, C.E. Rolfs, H.P. Trautvetter, U. Greife, M. Junker, S. Goriely, M. Arnould, M. Rayet, T. Rauscher and H. Oberhummer, Astron. Astrophys. 333 1112 (1998).
[16] Gy. Gyürky, Z. Elekes, J. Farkas, Zs. Fülöp, Z. Halász, G. G. Kiss, E Somorjai, T Szics, R T Güray, N Özkan, C Yalcin and T Rauscher, J. Phys. G 37, 115201 (2010).
[17] G. G. Kiss, T. Rauscher, T. Szics, Zs. Kertész, Zs. Fülöp, Gy. Gyürky, C. Fröhlich, J. Farkas, Z. Elekes and E. Somorjai, Phys. Lett. B 695, 419 (2011).
[18] P. Mohr, T. Rauscher, H. Oberhummer, Z. Máté, Zs. Fülöp, E. Somorjai, M. Jaeger and G. Staudt, Phys. Rev. C 55, 1523 (1997).
[19] Zs. Fülöp, Gy. Gyürky, Z. Máté, E. Somorjai, L. Zohrai, D. Galaviz, M. Babilon, P. Mohr, A. Zilges, T. Rauscher, H. Oberhummer and G. Staudt, Phys. Rev. C 64, 065805 (2001).
[20] D. Galaviz, Ph.D. thesis, TU Darmstadt (2004).
[21] G. G. Kiss, Zs. Fülöp, Gy. Gyürky, Z. Máté, E. Somorjai, D. Galaviz, A. Kretschmer, K. Sonnabend and A. Zilges, Eur. Phys. J. C 27, 197 (2006).
[22] D. Galaviz, Zs. Fülöp, Gy. Gyürky, Z. Máté, P. Mohr, T. Rauscher, E. Somorjai, and A. Zilges, Phys. Rev. C 71, 065802 (2005).
[23] A. Palumbo, W. Tan, J. Grres, M. Wiescher, Zs. Fülöp, Gy.
Gyürky, G. G. Kiss, E. Somorjai, D. Galaviz, N. Özkan, R.T. Güray, POS (NIC X) 046 (2008).

[24] G. G. Kiss, P. Mohr, Zs. Fülöp, D. Galaviz, Gy. Gyürky, Z. Elekes, E. Somorjai, A. Kretschmer, K. Sonnabend, A. Zilges, and M. Avrigeanu, Phys. Rev. C 80, 045807 (2009).

[25] Y. Eisen, E. Abramson, G. Engler, M. Samuel, U. Smilansky, Z. Vager, Nucl. Phys. A236, 327 (1974).

[26] I. Badawy, B. Berthier, P. Charles, M. Dost, B. Fernandez, J. Gastebois, S. M. Lee, Phys. Rev. C 17, 978 (1978).

[27] M. Miller, A. M. Kleinfeld, A. Bockisch, K. Bharuth-Ram, Z. Phys. A 300, 97 (1981).

[28] D. Galaviz et al., to be published.

[29] Z. Máte, S. Szilágyi, L. Zolnai, Á. Bredbacka, M. Brenner, K.-M. Källmann, and P. Maungärd, Acta Phys. Hung. 65, 287 (1989).

[30] G. G. Kiss, D. Galaviz, Gy. Gyürky, Z. Elekes, Zs. Fülöp, E. Somorjai, K. Sonnabend, A. Zilges, P. Mohr, J. Görres, M. Wiescher, N. Özkan, T. Güray, C. Yalcin and M. Avrigeanu, AIP conf. proc. 1016, 221 (2008).

[31] D. De Frenne and A. Negret, Nuclear Data Sheets 109, 943 (2008).

[32] D. De Frenne and A. Jacobs, Nuclear Data Sheets 89, 481 (2000).

[33] J. Blachot, Nuclear Data Sheets 92, 455 (2001).

[34] D. De Frenne and E. Jacobs, Nuclear Data Sheets 79, 639 (1996).

[35] A. M. Kobos, B. A. Brown, R. Lindsay, and G. R. Satchler, Nucl. Phys. A425, 205 (1984).

[36] G. R. Satchler and W. G. Love, Phys. Rep. 55, 183 (1979).

[37] H. Abele and G. Staudt, Phys. Rev. C 47, 742 (1993).

[38] U. Atzrott, P. Mohr, H. Abele, C. Hillenmayer, G. Staudt, Phys. Rev. C 53, 1336 (1996).

[39] H. de Vries, C. W. de Jager, and C. de Vries, Atomic Data and Nuclear Data Tables 36, 495 (1987).

[40] P. Mohr, D. Galaviz, Zs. Fülöp, Gy. Gyürky, G. G. Kiss, E. Somorjai, Phys. Rev. C 82, 047601 (2010).

[41] C. Signorini, A. Andrighetto, M. Ruan, J. Y. Guo, L. Stroe, F. Soramel, K. E. G. Löbner, L. Müller, D. Pieroutsakou, M. Romoli, K. Rudolph, I. J. Thompson, M. Trotta, A. Vitturi, R. Gernhäuser, A. Kastenmüller, Phys. Rev. C 61, 061603(R) (2000).

[42] J. P. Fernández-García, M. Rodríguez-Gallardo, M. A. G. Alvarez, A. M. Moro, Nucl. Phys. A840, 19 (2010).

[43] M. Avrigeanu, A.C. Obreja, F.L. Roman, V. Avrigeanu, W. von Oertzen, At. Data Nucl. Data Tables 95, 501 (2009).

[44] M. Avrigeanu and V. Avrigeanu, Phys. Rev. C 82, 014606 (2010).

[45] L. McFadden and G. R. Satchler, Nucl. Phys. 84, 177 (1966).

[46] P. A. Klinkenberg, Rev. Mod. Phys. 24, 63 (1952).

[47] T. Rauscher, code NON-SMOKERWEB, http://nucastro.org/websmoker.html

[48] M. Avrigeanu, W. von Oertzen, A.J.M. Plompen and V. Avrigeanu, Nucl. Phys. A723, 104 (2003). M. Avrigeanu, W. von Oertzen and V. Avrigeanu, Nucl. Phys. A764, 246 (2006).

[49] D. T. Khoa, W. von Oertzen and H. G. Bohlen, Phys. Rev. C 49, 1652 (1994).

[50] M. Avrigeanu and V. Avrigeanu, Phys. Rev. C 73, 038801 (2006).