The Equation of State of an Interacting Tachyon

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Abstract

We examine the cosmological solutions of a tachyon field non minimally coupled to gravity through an effective Born-Infeld interacting Lagrangian with a power law potential, in order to investigate its equation of state as related to tracking properties. We find exact solutions in the case of a tachyon dominated universe and when the dominant component of the stress energy tensor is determined by some other perfect fluid.

1 Introduction

Experimental observations performed in the last few years in the context of Cosmic Microwave Background [1] and of the type Ia Supernovae [2] strongly suggest that the Universe has been going through a phase of accelerated expansion for the last 5-10 Gy. The main candidates usually introduced in order to explain this behavior are a cosmological constant and a scalar field which begins to dominate at the latest cosmological phase. One of the possibilities taken into account in the literature is that scalar field is a tachyonic one. This kind of field appears in the context of string theory [3] and has been extensively applied to Cosmology [4]. Usually the tachyon is taken without explicit coupling to gravity (in analogy with a minimally coupled ordinary scalar field). We will extend the discussion of the properties of the system when an explicit non-minimal coupling is introduced. In particular we will be interested in solutions of the field equations in the case when the tachyon stress energy density is the dominant part of the content of energy of the Universe and, on the opposite, when the total stress energy density is dominated (and coherently the scale factor evolution) by some other fluid. In particular we will discuss the equation of state of the tachyon field, as it is at the base of the determination of the tracking feature ([6, 7]). We will start with a Born-Infeld Lagrangian assuming a power-law

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tachyon potential. This will be done in analogy to the analysis of [8] in order to determine the qualitative and quantitative effects of the gravitational interaction term. In fact one of the many interesting results obtained in [8] is the presence of two attracting solutions with a dust and a cosmological constant like behavior. We will look at the solutions of the coupled equations of motion for the tachyon and for the scale factor of a FRW metric in order to explicitly determine the behavior of the field and thus its equation of state to see if the tracking behavior is still present in the interacting case. In Section 2 we introduce the basic framework for the tachyon effective action. In Section 3 we consider the case when the tachyon energy density is the dominant one in the Friedmann equations and in Section 4 the case when a different fluid drives the expansion of the scale factor. In Section 5 we discuss the results.

2 The Basic Framework

The action describing the non-interacting tachyon field is usually (see i.e. [8]) taken in a Born-Infeld form:

\[
L_{BI} = -\sqrt{-g} V(\phi) \sqrt{1 - \frac{\partial^\mu \phi \partial_\mu \phi}{M^4}}. \tag{2.1}
\]

This scalar field was proposed in connection with string theory, as it seems to represent a low-energy effective theory of D-branes and open strings, and has been conjectured to play a role in cosmology [3, 5]. The energy and pressure densities of the tachyon, in the homogeneous case, are:

\[
\rho_T = \frac{V(\phi)}{\sqrt{1 - \frac{\dot{\phi}^2}{M^4}}}, \tag{2.2}
\]

\[
p_T = -V(\phi) \sqrt{1 - \frac{\dot{\phi}^2}{M^4}}. \tag{2.3}
\]

The tachyon fluid is also characterized by the ratio between pressure and energy (the equation of state) \(w_T\) and sound speed \(c_T^2\):

\[
w_T = -1 + \frac{\dot{\phi}^2}{M^4}, \quad c_T^2 = -w_T. \tag{2.4}
\]

Since the equation of state is necessarily nonpositive because of the square root in the action (2.1), the theory is stable — energy and pressure are real, and inhomogeneous perturbations have a positive sound speed. Moreover, because \(w_T \leq 0\), the tachyon is a natural candidate for dark energy and inflation. Another interesting property is that the equation of state and sound speed of tachyons are equal, but with opposite signs, irrespective of the form for the potential. Canonical scalar fields, on the other hand, obey the Klein-Gordon equations, hence their fluctuations travel with sound speed equal to unity (in units where \(c = 1\)). Therefore, tachyon fluctuations are fundamentally different from the fluctuations of a canonical scalar field, irrespective of the shape of the potential.

One might expect drastic changes by taking a non-minimal coupling of \(\phi\), as encoded in the Lagrangian

\[
L_\phi = \sqrt{-g} \left[ -V(\phi) \sqrt{1 - g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi} + \xi R \phi^2 \right]. \tag{2.5}
\]
where $R$ is the Ricci scalar, $\xi$ is the non-minimal coupling constant and $V(\phi)$ is the potential, so that the total Lagrangian may be written as

$$L_{\text{tot}} = \sqrt{-g} \left[ -V(\phi)\sqrt{1-g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \xi R \phi^2} + \frac{M^2}{2}R \right], \quad (2.6)$$

where $M$ is the Planck mass.

In the case of a tachyon living in a Universe where some kind of matter, effectively described as a perfect fluid, is present, Einstein’s field equations are given as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G [T_{\mu\nu}(\phi) + T_{\mu\nu}(m)] \quad (2.7)$$

with energy-momentum tensor components of tachyon and matter as

$$T_{\mu\nu}(\phi) = (\rho_\phi + p_\phi)u_\mu u_\nu - p(\phi)g_{\mu\nu} \quad (2.8)$$

and

$$T_{\mu\nu}(m) = (\rho_m + p_m)u_\mu u_\nu - p_m g_{\mu\nu} \quad (2.9)$$

respectively, where $u^\mu = (1, 0, 0, 0)$. The diagonal components are defined as

$$T_{\mu\mu}(\phi) = (\rho_\phi, -p_\phi, -p_\phi, -p_\phi) \quad (2.10)$$

and can be obtained from the lagrangian as

$$T_{\mu\nu}(\phi) = -V(\phi)[1 - \nabla^\rho \phi \nabla_\rho \phi + \xi R \phi^2]^{-1/2} \times \left[ -\nabla_\mu \phi \nabla_\nu \phi + \xi R_{\mu\nu} \phi^2 \right. \quad (2.11)$$

$$+ \xi (\nabla_\mu \nabla_\nu - g_{\mu\nu} \nabla^2 \phi)^2 - g_{\mu\nu}(1 - \nabla^\rho \phi \nabla_\rho \phi + \xi R \phi^2). \quad (2.12)$$

Here $\nabla_\mu$ stands for the covariant derivative and $R_{\mu\nu}$ are Ricci tensor components. The field equations for $\phi$ are obtained as

$$\nabla^2 \phi + \frac{2(\nabla^\mu \phi)(\nabla_\mu \phi)(\nabla^\rho \phi)(\nabla_\rho \phi) - 2\xi R \phi \nabla^\rho \phi \nabla_\rho \phi - \xi \phi^2 g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi}{2(1 - \nabla^\rho \phi \nabla_\rho \phi + \xi R \phi^2)}$$

$$+ \xi R \phi + \frac{V'}{V}(1 + \xi R \phi^2) = 0, \quad (2.14)$$

from the lagrangian. Here $V'(\phi) = \frac{d}{d\phi} V(\phi)$ and

$$\nabla^2 \phi = \nabla^\rho \nabla_\rho \phi = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} (\sqrt{-g} g^{\mu\nu} \frac{\partial}{\partial x^\nu}) \phi. \quad (2.15)$$

According to cosmological observations [1, 2], we currently live in a spatially flat and speeding up universe, such that $\ddot{a}/a > 0$ for the scale factor $a(t)$, of a Friedmann-Robertson-Walker metric

$$ds^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2]. \quad (2.16)$$
In the following we will deal only with the homogeneous mode of the tachyon field and thus assume

\[ \phi(x,t) = \phi(t). \]  

(2.17)

The Friedmann and the acceleration equations are as usually written in the form

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho \quad ; \quad \left( \frac{\ddot{a}}{a} \right) = -\frac{4\pi G}{3} (\rho + 3p)
\]  

(2.18)

and the equation of motion for the tachyon is

\[
\ddot{\phi} + 3H \dot{\phi} + \frac{2\ddot{\phi} \dot{\phi}^2 - 2\xi R \dot{\phi} \ddot{\phi} - \xi \dot{\phi}^2 \dot{R} \dot{\phi}}{2(1 - \phi^2 + \xi R \phi^2)} + \xi R \phi + \frac{V'}{V} (1 + \xi R \phi^2) = 0,
\]  

(2.19)

Of course, only two of these three equations are independent when the universe is driven by a single source. In the following we will make the following ansatz for the tachyon field

\[ \phi(t) = At \]  

(2.20)

where \( A \) is a dimensionless constant. This allows us to convert differential equations into algebraic ones, which is the only way we were able to solve the field equations for a generic value of the gravitational coupling. The potential is assumed to be in a power-law form

\[ V(\phi) = m^{2-\gamma} \phi^{-2-\gamma} \]  

(2.21)

where \( m \) has the dimension of a mass, which is the one used, in addition to the exponential one, in all the phenomenological models.

### 3 Tachyon Driven Universe

In a Friedmann Universe, with the assumptions discussed in the previous section, the energy density and pressure of the tachyon field are given by

\[
\rho_\phi = - \frac{V(\phi)}{\left[6\xi \phi^2 \left(\dot{H} + 2H^2\right) - 1\right] \sqrt{1 - 6\xi \phi^2 \left(\dot{H} + 2H^2\right) - \dot{\phi}^2}} \left\{ 1 + 3\xi \left[36\xi \phi^3 H^4 + 3\phi \left(18\xi \phi^2 \dot{H} + 3\dot{\phi}^2 - 7\right) - H \left(2\dot{\phi} + 3\xi \phi^3 \dot{H}\right) H^3 \right]\right\}
\]  

(3.22)
and

\[ p_\phi = - \frac{V(\phi)}{\left[6\xi\phi^2 (\dot{H} + 2H^2) - 1\right]} \sqrt{1 - 6\xi\phi^2 (\dot{H} + 2H^2) - \dot{\phi}^2} \] \begin{align*}
-5184(\gamma + 1)\xi^4 H^7 \phi^7 \dot{\phi} + 432\xi^3 H^6 \phi^6 \left[-13 + 8(\gamma + 1)\xi + 66\xi\phi^2 \dot{H}\right] \\
+(6 + 4(2\gamma^2 + \gamma - 1)\xi)\phi^2 \left[-1 + 2(\gamma + 2)\xi + (1 + (2\gamma + 7\gamma + 8)\xi)\phi^2\right] \\
-324(\gamma + 1)\xi^4 \phi^7 \dot{\phi} H^2 \dot{H} - 54\xi^3 \phi^5 \dot{\phi} H H \left(\gamma \dot{\phi}^2 - 2\gamma - 5\right) + 3\xi^2 \phi^3 \dot{\phi} \left[(8 + 3\gamma)\phi^2\right] \\
-3(\gamma + 4) + 144\xi^3 \phi^5 H^5 \left[\left(14 + 9\gamma - 90(\gamma + 1)\phi^2 \dot{H}\right) \dot{\phi} + (6\gamma - 3)\phi^3 - 18\xi\phi^3 \dot{H}\right] \\
+12\xi^2 \phi^3 H^3 \left[\dot{\phi} \left(-19 + 9\gamma - 756(\gamma + 1)\xi^2 \phi^4 \dot{H}^2 - (12\gamma + 13)\phi^2 + 9\gamma\phi^4 + 12\phi^2 \dot{H}\right) \right] \\
\times \left(29 + 15\gamma + 3(\gamma - 1)\phi^2\right) - 6\phi^3 \left[7\dot{\phi}^2 - 6\right] \dot{H} + \xi \phi H \left[-\left(-1 + 6\xi\phi^2 \dot{H}\right) (8 + 3\gamma)\right] \\
+6\xi\phi^2 \dot{H} \left(-35 - 12\gamma + 54(\gamma + 1)\phi^2 \dot{H}\right) - 2 \left(-8 - 3\gamma + 9\phi^2 \dot{H}\right) \left(-1 + 2\gamma + 6\xi\phi^2 \dot{H}\right)\right) \dot{\phi}^3 \\
+3 \left(-8 - 3\gamma + 18\gamma \dot{\phi}\right) \dot{\phi}^5 + 6\phi^3 \left(-3 + 7\phi^2 + 6\left(3\phi^2 \dot{H}^2 - \phi^2\right) (6\xi\phi^5 \dot{H} + \dot{\phi}^2)\right) \dot{H} \\
+54\xi^4 \phi^8 \dot{H} \left(12\dot{H}^3 + 3\dot{H}^2 - 2\dot{H} H^3\right) + \xi \phi^2 \left[\dot{H} \left(-23 + 60\xi + 36\gamma \xi + (35 + 6\right)\right] \\
\times (6\gamma^2 + 11\gamma + 1)\xi) \dot{\phi}^2 - 12 (1 + 2(\gamma + 2)\xi) \dot{\phi}^4 - \dot{\phi} \phi \left(\dot{H}^3\right) + 3\xi^2 \phi^4 \left[2\dot{H}^2 \left(31\right)\right] \\
-12(4 + 3\gamma)\xi - 2 (17 + 3(6\gamma^2 + 7\gamma - 4)\xi) \phi^2 + 6 \left(1 + \gamma (1 + 2\gamma) \xi \phi^2\right) - H^3 + 4\phi \phi \left(\dot{H}^3\right) \\
+9\xi^3 \phi^6 \left[4\dot{H}^3 \left(-17 + 12(\gamma + 1)\xi + (11 + 6(-1 + \gamma + 2\gamma)\xi) \phi^2\right) + \left(2\phi^2 - 3\right) \dot{H}^2\right] \\
+4\dot{H} H^3 - 4\phi \phi \left(\dot{H}^3\right) + 36\phi^2 \phi^4 \left(21 - 8(3\gamma + 4)\xi - 4 \left(5 + (6\gamma^2 + 7\gamma - 4)\xi\right) \phi^2\right) \right) \\
(9 + 4\gamma(2\gamma + 1)\xi) \phi^4 - 36(\gamma + 1)\xi^2 \phi^3 \dot{\phi} H + 12\xi^2 \phi^4 \left(57\dot{H}^2 - H^3\right) + 4\xi \phi^2 \\
\times \left(-56 + 36(\gamma + 1)\xi + 3(5 + 6(2\gamma^2 + \gamma - 1)\xi) \phi^2\right) - \dot{\phi} \phi \left(\dot{H}^3\right) + 3\xi^2 \phi^2 H^2 \left[-15\right] \\
+8(3\gamma + 5)\xi + (22 + 4(6\gamma^2 + 11\gamma + 1)\xi) \phi^2 - (13 + 16(\gamma + 2)\xi) \phi^4 + 6\phi^6 \\
-432(\gamma + 1)\xi^3 \phi^5 \dot{\phi} H H - 36\xi^2 \phi^3 \phi \left(\phi \phi^2 - 2\gamma - 5\right) H + 36\xi^3 \phi^6 \left(78\dot{H}^3 + 3\dot{H}^2 - 4\dot{H} H^3\right) \\
+2\xi \phi^2 \left(\dot{H} \left(125 - 48(3\gamma + 4)\xi - 4(25 + 6(6\gamma^2 + 7\gamma - 4)\xi) \phi^2 + 3(5 + 8\gamma(2\gamma + 1)\xi) \phi^4\right) \right) \\
+4\phi \phi \left(\dot{H}^3\right) + 12\xi^2 \phi^4 \left(\dot{H}^2 (-119 + 72(\gamma + 1)\xi + 6(7 + 6(2\gamma^2 + \gamma - 1)\xi) \phi^2\right) \\
+2\dot{H}^3 - 4\phi \phi \left(\dot{H}^3\right)\right] \right) \] \quad (3.23)

where \( f^{(3)}(t) \equiv d^3 f(t)/dt^3 \). It is convenient to define a time dependent parameter \( w(t) \) by the relation \( w(t) \equiv p_\phi(t)/\rho_\phi(t) \). The equation of motion for the scalar field, written in the form \( d(\rho a^3) = -wd\tau d(a^3) \) can be integrated to give

\[ (\dot{\rho}_\phi/\rho_\phi) = -3H(1 + w) \] \quad (3.24)
The Friedmann equation, on the other hand, gives $\rho_\phi \propto H^2$ so that $(\dot{\rho}_\phi/\rho_\phi) = 2(\dot{H}/H)$. Combining the two relations we get

$$w(t) = -1 - \frac{2}{3}\frac{\dot{H}}{H^2}$$

(3.25)

and thus determine $w(t)$ when the time dependence of the Hubble parameter is known and $\rho_\phi$ is the dominant energy in the Universe. (Note that we have not used the specific form of the source so far, so this equation will be satisfied by any source in a FRW model.)

If such a dependence is unknown, from the Einstein equations one can easily write

$$\frac{R_1^1 - \frac{1}{2}R}{R_0^0 - \frac{1}{2}R} \simeq \frac{-p_\phi}{\rho_\phi}$$

(3.26)

and consider dominance of tachyon energy over matter.

In this case the Friedmann equation may be solved with our ansatz for the field $\phi(t) = At$ and for the scalar curvature $H(t) = q/t$ only in the case of a potential with $\gamma = 0$ and this is that case we will discuss later on. The equation of state for the tachyon is determined by the equation

$$w_\phi \simeq \frac{2\dot{H} + 3H^2}{3H^2} = \frac{3q - 2}{3q}$$

(3.27)

as derived from the Einstein equation (2.7) whenever one can solve the system of coupled equations

$$w_\phi = \frac{p_\phi}{\rho_\phi}$$

(3.28)

$$3A^2q[1 + 2\xi(2q - 1)(\gamma + 1)] - \gamma - 2 = 0$$

(3.29)

for $q$ and $A$. When $\gamma = 0$, in particular, $\rho_\phi(\phi)$ and $p_\phi(\phi)$ in eq. (3.28) are given by

$$\rho_\phi = \frac{1 - 9A^2q\xi(q - 1)}{A^4t^4\sqrt{1 - A^2[1 + 6q\xi(2q - 1)]]}}$$

(3.30)

$$p_\phi = \frac{-1 + A^2[1 + 3\xi(3q^2 - 3q + 2)]}{A^4t^4\sqrt{1 - A^2[1 + 6q\xi(2q - 1)]}}$$

(3.31)

We solved the equations numerically and the results are quite interesting. The first aspect to note is that in the limit $\xi \to 0$ one recovers the results obtained in the non-interacting tachyon (see [8]) as expected. In fact for every value of the ratio $M / m$, by taking the limit $\xi \to 0$ one obtains for $w_\phi$ the couple of values $(-1, 0)$. One can thus infer that the attractor structure of the non interacting case, where a dust and a cosmological constant-like values are obtained, is reproduced. One should then ask what is the effect of the interaction term. In the case $M / m > 1$ (see Fig. 1) one can see that there is a maximum value for the coupling constant $\xi$ in order to obtain real values for the equation of state. As one approaches this limiting value for $\xi$ the cosmological constant and dust behavior are modified and approach a value of $w_\phi \simeq 0.25$.

Even in the case $M / m < 1$ (see Fig. 2) one can see that there is a maximum value for the coupling constant $\xi$ in order to obtain real values for the equation of state. As one approaches this limiting value for $\xi$ both the solutions for the equation of state parameter approach the
Figure 1: The two plots above show $w_\phi$ obtained solving eqs. \((3.28-3.29)\) for different values of $\xi$ when $M/m = 2$ (l.h.s) and $M/m = 10^2$ (r.h.s). The straight line lying above the horizontal axis in both the plots represents $\xi$ as a function of the discrete index $n$ which labels the solutions obtained. The solid line which bifurcates is the real part of $w_\phi$ and the dashed line is the imaginary part of the solutions. Physical solutions are admissible only in the $\text{Im} w_\phi = 0$ region.

cosmological constant case. One can thus deduce that in this case even as one takes a very small but fixed value for $\xi$ there are values of the ratio $m/m$ for which the structure of the non-interacting case is not recovered and the only possible tachyon behavior is that of a cosmological constant. One can thus conclude that the introduction of an interaction term completely modifies the conclusion one might draw in the non-interacting case even for a very weakly interacting theory.

One would need to introduce a formalism similar to that used in \[7\], where a phase plane analysis of the flow defined by the equations of motion was performed. This is beyond the scope of this preliminary analysis as the coupled interacting equations cannot be solved analytically as in \[7\].

4 Tachyon in a Fluid-Driven Universe

In the case where the Universe expansion is driven by a fluid with equation of state

$$p_f = w_f \rho_f$$

(4.32)

for a power-law evolution of the scale factor

$$a(t) = a_0 \left( \frac{t}{t_0} \right)^q$$

(4.33)

one can show that the Hubble parameter is given by

$$H(t) = \frac{2}{3(1 + w_f)t} = \frac{q}{t}$$

(4.34)

The equation of motion for the tachyon field in a potential of the form $V(\phi) = m^{4-\alpha} \phi^{-\alpha}$ (with $\alpha = 2 + \gamma$ of eq. \[2.21\]) reduces to

$$12(\alpha - 1)\xi A^2 q^2 - 3(1 + 2(\alpha - 1)\xi) A^2 q + \alpha = 0$$

(4.35)
Figure 2: The two plots above show the real part (l.h.s.) and the imaginary part (r.h.s.) of $w_\phi$ when $M/m = 2 \cdot 10^{-3}$. The two graphs were obtained solving eqs. (3.28-3.29) for different values of $\xi$ and the straight line lying above the horizontal axis in the plot on the right represents $\xi$ as a function of the discrete index $n$ which labels the solutions obtained. The solid line which bifurcates in the l.h.s. plot is the real part of $w_\phi$ and the dashed line in the r.h.s. plot is the imaginary part of the solutions. Physical solutions are admissible only in the $\text{Im} w_\phi = 0$ region.

where $q = 2/3(1 + w_f)$ as determined by (4.34). Now one has

$$A^2 = -\frac{\alpha}{12(\alpha - 1)q^2 - 3[1 + 2(\alpha - 1)q]} \bigg|_{q=2/(3(1+w_f))}$$

By substituting the expression for $A$ in the expression of the energy density $\rho_\phi$ and pressure $p_\phi$ one obtains the equation of state for the tachyon. The result of the above procedure is very interesting: the equation of state for the tachyon is independent of $\xi$

$$w_\phi = -1 + \frac{1 + w_f}{2}.$$  

The introduced coupling to gravity does not show any effect on the equation of state for a non-dominant tachyon.

5 Conclusion

We have examined the equation of state for a tachyon field coupled to gravity through the Born-Infeld Lagrangian (2.5) in order to determine the equation of state for a tachyon coupled to gravity. This generalizes the case described by [8]. When the homogeneous tachyon field is the dominant source in the Friedmann equation, even for small values of the coupling ($\xi$), the system may behave very differently from the uncoupled case. The dust attractor, present in the uncoupled case, may disappear in favor of a cosmological constant type of behavior. On the contrary, when some other fluid is driving the expansion of the scale factor, the gravitational coupling shows no effect on the tachyon equation of state. We feel that these results are quite intriguing and would need a deeper, thought not simple, numerical phase space analysis in order to be fully understood.

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