DISAGREEMENT BETWEEN STANDARD MODEL AND EXPERIMENT FOR MUON G-2 ?

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Abstract. In a recent experimental paper reporting a precise measurement of the muon anomalous magnetic moment, Brown et al. claim disagreement of their result with evaluations based on the standard model. This claim is based on comparison with a single theoretical evaluation. It is pointed out that other equally legitimate theoretical evaluations exist in the literature, with which the new experimental figure is quite compatible. The claim of indications for new physics beyond the standard model does not appear justified. In this version we take into account corrections to one of the works quoted, we include a new discussion at the end, and also minor errors are corrected. The conclusions remain unchanged.

In a recent paper[1] a new measurement is reported for the value of the muon anomalous magnetic moment ($g - 2$), $a_{\mu}$. After averaging with older determinations one has then the precise experimental result,

$$10^{11} \times a_{\mu}(\text{experiment}) = 116 592 023 \pm 140 \pm 60.$$  \hspace{1cm} (1)

Brown et al. then compare this with a theoretical evaluation of the same quantity and conclude that there is discrepancy at the 2.7 standard deviations level, and thus indication of new physics. This has triggered a deluge of non-standard model explanations of the ‘discrepancy’, of which here we only mention ref. 2.

In order to clarify the issue, we first very briefly review the theoretical calculation of $a_{\mu}$. According to Hughes and Kinoshita[3] we have the following theoretical contributions, apart from that of hadronic vacuum polarization (h.v.p.), that we discuss later:

$$10^{11} \times a_{\mu}(\text{Pure QED}) = 116 584 705.7 \pm 1.8$$
$$10^{11} \times a_{\mu}(\text{Weak}) = 151 \pm 4.$$  \hspace{1cm} (2)

$$10^{11} \times a_{\mu}(\text{Hadronic light by light}) = -79.2 \pm 15.4$$
$$10^{11} \times a_{\mu}(\text{Other higher order hadronic}) = -101 \pm 6$$

Excluding the hadronic contributions to vacuum polarization, and compositing errors quadratically, we then have

$$10^{11} \times a_{\mu}(\text{Except h.v.p.}) = 116 584 677 \pm 17.$$  \hspace{1cm} (3)

There is reasonable consensus on these contributions, although I personally think the errors on the hadronic pieces to be somewhat underestimated; for example, the authors of ref. 2 give

$$10^{11} \times a_{\mu}(\text{Hadronic light by light}) = -86 \pm 25,$$  \hspace{1cm} (4)

but we will use the figure given by Hughes and Kinoshita here.

We now we pass to the more controversial hadronic vacuum polarization which, moreover, provides the bulk of the theoretical error. There are a number of evaluations of the h.v.p. corrections; we quote now four recent ones:

$$10^{11} \times a_{\mu}(\text{h.v.p.}) = 6 924 \pm 62 \hspace{1cm} (\text{DH})$$
$$10^{11} \times a_{\mu}(\text{h.v.p.}) = 7 250 \pm 158 \hspace{1cm} (EJ)$$
$$10^{11} \times a_{\mu}(\text{h.v.p.}) = 6 988 \pm 111 \hspace{1cm} (J)$$
$$10^{11} \times a_{\mu}(\text{h.v.p.}) = 7 113 \pm 103 \hspace{1cm} (AY)$$  \hspace{1cm} (5)
of the errors are very important, in particular at high energy ππ use of dispersive methods, that allow use of e in the spacelike region, so curbing systematic errors of e other evaluation (see, for example, fig. 3 in Davier’s review, ref. 9, or fig. 1 here).

Perhaps by chance, falls dead on top of the new experimental result: 116 592 024

The meaning of this is: DH stands for the analysis of ref. 4, EJ for that of ref. 5 and J for that, as yet unpublished, of F. Jegerlehner, quoted in ref. 2, and private communication. Finally, AY indicates the result of ref. 6.

The figure quoted for EJ contains an error in the sense that it contains a string of hadron corrections to the photon propagator and thus leads to double counting when including higher hadronic corrections. So we will here consider only the figure denoted by J.

Adding (2) and (3), the theoretical predictions are then,

\[
\begin{align*}
10^{11} \times a_\mu(\text{theory}) &= 116591601 \pm (\sqrt{172^2 + 62^2} = 67) \quad (\text{DH}) \\
10^{11} \times a_\mu(\text{theory}) &= 116591665 \pm (\sqrt{172^2 + 111^2} = 112) \quad (\text{J}) \\
10^{11} \times a_\mu(\text{theory}) &= 116591790 \pm (\sqrt{172^2 + 103^2} = 104) \quad (\text{AY}); \\
10^{11} \times a_\mu(\text{theory}) &= 116591904 \pm (\sqrt{172^2 + 112^2} = 115) \quad (\text{CLY}); \\
10^{11} \times a_\mu(\text{exp.}) &= 116592023 \pm (\sqrt{140^2 + 60^2} = 152).
\end{align*}
\]

In the line before last (CLY) we have included the ‘old’ result of ref. 7, corrected for the new favoured value of higher order hadronic corrections,* and in the last line we have repeated the recent experimental value of ref. 1, to facilitate the comparison.

Subtracting from (1) we find that the ‘discrepancies’ between theory and experiment are thus:

\[
\begin{align*}
10^{11} \times \Delta a_\mu(\text{exp – th}) &= 422 \pm 152(\text{exp.}) \pm 77(\text{th}) \quad (\text{DH}) \\
10^{11} \times \Delta a_\mu(\text{exp – th}) &= 358 \pm 152(\text{exp.}) \pm 112(\text{th}) \quad (\text{J}) \\
10^{11} \times \Delta a_\mu(\text{exp – th}) &= 233 \pm 152(\text{exp.}) \pm 104(\text{th}) \quad (\text{AY}) \\
10^{11} \times \Delta a_\mu(\text{exp – th}) &= 119 \pm 152(\text{exp.}) \pm 115(\text{th}) \quad (\text{CLY})
\end{align*}
\]

with, I believe, self-explanatory notation (we have included the new values of the higher hadronic corrections for the CLY figure). The first theoretical evaluation (DH) is more distant from experiments than what the errors, at 1σ level, would allow. J is slightly outside the 1σ region, and CLY and AY are perfectly compatible with experiment (considering that the evaluations CLY, EJ and AY antedate the experiment, one should perhaps say that experiment validates the standard model theory). We will discuss very briefly DH, CLY and AY as representative calculations: each is based on a different method of evaluation.

All these evaluations use essentially old (pre-1985) experimental data in the critical s \(1/2 \leq 5\) GeV region for \(e^+e^- \rightarrow \) hadrons since 1985, with the exception of what one can get indirectly from the hadronic decays of the τ. The variation of DH with respect to CLY is to use these τ decay results to supplement low energy \(e^+e^- \rightarrow \) hadrons data. The improvement of CLY (and AY) with respect to older calculations lies in the use of dispersive methods, that allow use of \(\pi\pi\) scattering phase shifts, and data on the pion form factor in the spacelike region, so curbing systematic errors of \(e^+e^- \rightarrow \) hadrons (at low energy). These systematic errors are very important, in particular at ‘high’ energy \(s^{1/2} \geq 1.5\) GeV; in some regions, as large as 10%, as demonstrated e.g. in ref. 8 comparing various experiments with one another and with QCD predictions. Because of these systematic errors, AY use a QCD calculation, plus resonances, above \(s^{1/2} = 1.5\) GeV. This is what lowers a little the result of AY as compared to CLY.

To summarize: for the h. v. p. contributions, EJ use \(e^+e^- \rightarrow \) hadrons data essentially; AY supplement this by \(\pi\pi\) phase shifts and data for the pion form factor in the spacelike region (so does CLY) at low energy, and QCD at high energies (CLY uses experimental data at higher energies). Finally, DH take τ decay data to improve the evaluation for low energy, and use essentially experimental \(e^+e^- \rightarrow \) hadrons data at higher energies. Although based on different methods, all four determinations are compatible with one another, within errors. The value reported by DH is slightly lower than the other three, and indeed than almost any other evaluation (see, for example, fig. 3 in Davier’s review, ref. 9, or fig. 1 here).

* If we had taken the value actually reported in CLY, eq. 1.4, we would have had a figure that, perhaps by chance, falls dead on top of the new experimental result: 116 592 024 \(\pm (\sqrt{27^2 + 111^2} = 115)\) (CLY, uncorrected).
On the face of the results of eq. (5), what one is tempted to conclude is that experiment favours the evaluations of CLY, AY which are in fact in perfect agreement among themselves and in good agreement with the new experimental result. This is seen very clearly in the figure 1 (where I have added also the result of Brown and Worstell (BW), ref. 10, just to show that there are other evaluations that those by myself and my collaborators that are in strict agreement with the new data).

Of course, such a conclusion would not be totally correct; the discrepancy between the theoretical result of DH and experiment is not really large enough to discriminate. But, by the same token, it follows that, unless one can prove that the evaluation of Davier and Höcker is clearly superior to the others, and until substantially more precise theoretical evaluations and experimental data become available, what is completely unjustified is to claim any disagreement between theory and experiment.

To advertise evidence for SUSY or any other kind of nonstandard physics on such basis is, to put things in as mild a way as possible, misleading.

Further discussion. It has been claimed in a number of places, in particular by some of the proponents of the “harbinger of new physics” interpretation, that the result of DH is the best because it gives smallest errors. Unfortunately, however, small errors are not always a sign of improved evaluation; many times they just reflect unjustified optimism. The fact that the error in DH is a factor about 2 or more smaller than all the others, including the very recent evaluation of Jegerlehner (J) that uses all data available to DH, and a few more, should make one suspicious about the claims of accuracy made by Davier and Höcker.

Acknowledgements I am grateful to W. Lucha and B. Gavela who brought this problem to my attention, and to several other members of the Department in UAM and IFT for discussions. Correspondence with F. Jegerlehner, who clarified some points of his analysis is acknowledged, as is a remark of M. Davier that allowed me to correct a typo in the previous version of this note.

Thanks are also due to CICYT for financial support.
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