The Relic Neutrino Composition as Seen from Earth

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Abstract—Being generated, the relic neutrino background contained equal fractions of electron $\nu_e$, muon $\nu_\mu$, and tauon $\nu_\tau$ neutrinos. We show that the gravitational field of our Galaxy and other nearby cosmic objects changes this composition near the Solar System. As a result, the relic background becomes enriched with tauon and particularly muon neutrinos. The electron relic neutrinos are the rarest for a terrestrial observer.

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1. INTRODUCTION

A very common cosmological consideration shows that, along with the relic radiation, there should be relic neutrinos as well. Neutrinos were in the thermodynamical equilibrium with the other substance in the early Universe and decoupled much earlier than the radiation did (when the temperature of the Universe was $T_0 \sim 2$ MeV). Soon after that the electron-positron pairs annihilated, boosting the temperature of the photons, but not of the neutrinos. As a result, the temperature of the neutrinos after that became of the photon one $T_\nu \approx 13\, 0714 T_\gamma$. Thus, the relic neutrinos occur in a very general cosmological scenario and their number density is comparable with that of the relic photons.

There are three known types of neutrinos: electron $\nu_e$, muon $\nu_\mu$, and tauon $\nu_\tau$ ones. The spin of all the neutrinos is $\frac{1}{2}$; however, only their state with left-handed helicity seems to interact with anything else, and therefore only this state appears in any elementary particle interaction. Modern theoretical models suggest that there can be more exotic neutrino-like particles. For instance, heavy sterile neutrinos may form the main part of the dark matter. However, we will consider only the three above-mentioned Standard Model neutrinos in this paper, and we will hereafter imply only them by the term “neutrino”.

Neutrino oscillations show that all the neutrinos have non-zero mass, and that $\nu_e$, $\nu_\mu$, $\nu_\tau$ are not the eigenstates of the hamiltonian, and therefore they have no certain mass. We denote the three mass states of neutrinos by $\nu_1$, $\nu_2$, $\nu_3$ and their masses by $m_1$, $m_2$, $m_3$.

The oscillations allow to measure the neutrino mass squared differences [2, Table 14.1]:

\begin{equation}
\Delta m^2_1 = 7.37 \times 10^{-5} \text{ eV}^2, \\
\Delta m^2_2 = 2.54 \times 10^{-3} \text{ eV}^2.
\end{equation}

This means that there should always be at least one neutrino with $m \gtrsim \sqrt{(\Delta m^2_3)^{1/2} + (\Delta m^2_1)^{1/2}} = 0.06 \text{ eV}$.

The idea that cosmology sets a limit on the neutrino sum mass was offered by [3]. Recent data on the Baryon Acoustic Oscillations [4, Eq. (63b)] say that

\begin{equation}
m_1 + m_2 + m_3 < 0.12 \text{ eV}, \quad 95\% \text{ CL.}
\end{equation}

Unfortunately, we do not know the masses of $\nu_1$, $\nu_2$, $\nu_3$: Eqs. (1), (2) set only constraints on $m_1$, $m_2$, $m_3$. Historically, three limiting scenarios were considered [2, p. 254]:

- **Normal Hierarchical (NH):** $m_1 < m_2 \ll m_3$, $m_2 \approx (\Delta m^2_1)^{1/2} \approx 0.0086 \text{ eV}$, $m_3 \approx |\Delta m^2_3|^{1/2} \approx 0.0506 \text{ eV}$;
- **Inverted Hierarchical:** $m_3 \ll m_1 < m_2$, $m_1 \approx (\Delta m^2_3 - \Delta m^2_1)^{1/2} \approx 0.0497 \text{ eV}$, $m_2 \approx |\Delta m^2_{23}|^{1/2} \approx 0.0504 \text{ eV}$;
- **Most Degenerated:** $m_1 \approx m_2 \approx 0.05 \text{ eV}$, $m_3 \approx 0.07 \text{ eV}$.

From the experimental point of view, the most degenerated scenario is now excluded, and the inverted hierarchical one is highly unlikely [5]. We will consider only the normal hierarchical scenario in this paper, though the final result can be trivially generalized for the inverted hierarchical case. We will make a short comment about that in the end of the letter.

It is curious that, though the Universe is much more transparent for the relic neutrinos than for relic
photons, the relic photons may come to us from larger distances: if a neutrino is heavier than $2 \times 10^{-4}$ eV, it moves significantly slower than light and passes shorter distance since the Big Bang [6].

Though the neutrino masses have not measured yet, recent experiments allow us to estimate them and other neutrino parameters rather reliably. The values that we will use in this paper are not very precise, since the paper is estimative in general. If the hierarchy is normal, $\nu_3$ is the heaviest neutrino, and $\nu_1$ is the lightest one. From (1) and (2) we obtain $0.08 > m_3 > 0.06$ eV, $0.02 > m_2 > 0.0086$ eV, $0.02 > m_1 > 0$ eV. All the neutrinos, $\nu_1$, $\nu_2$, and $\nu_3$, are mixtures of $\nu_e$, $\nu_\mu$, and $\nu_\tau$. Here we skip the details of calculation (which can be found, for instance, in [2]), presenting only the final result: $\nu_1$ contains ~67% of $\nu_e$, ~8% of $\nu_\mu$, and ~24% of $\nu_\tau$; $\nu_2$ contains ~30% of $\nu_\mu$, ~35% of $\nu_e$, and ~35% of $\nu_\tau$; $\nu_3$ contains ~2% of $\nu_\tau$, ~57% of $\nu_\mu$, and ~41% of $\nu_e$. The fact that the heavy neutrino, $\nu_3$, contains only ~2% of the electron neutrino is the most important for us.

2. THE SPECTRUM AND DENSITY OF THE RELIC NEUTRINOS IN THE ABSENCE OF LARGE-SCALE STRUCTURE IN THE UNIVERSE

First of all, we need to calculate the spectrum and density of the relic neutrinos in the absence of structures in the Universe. Then the Universe evolution can be described by the Friedmann metric $ds^2 = c^2dt^2 - a^2(t)dr^2$, where $a(t)$ is the scale factor. We accept that $a = 1$ at the moment of the neutrino decoupling (we may do it, since the Universe is flat).

Let us consider one type of neutrinos (or antineutrinos) of mass $m$. When they were in the equilibrium in the early Universe, their number $dN$ in a volume $W$ with the absolute momentum between $p$ and $p + dp$ was [7]

$$dN = \frac{Wp^2 dp}{2\pi^2 \hbar^3 (\exp[(\sqrt{m^2 c^4 + p^2 c^2} - \omega)/T] + 1)}, \quad (3)$$

where $\omega$ is the chemical potential, $T$ is the temperature. This equation can be significantly simplified, since $mc^2 \ll T$, $\omega \ll T$ in the early Universe [1]. If we denote the particle momentum at the decoupling temperature $T_d$ by $p_d$, we obtain for the decoupling moment:

$$dN = \frac{W_d p_d^2 dp_d}{2\pi^2 \hbar^3 (\exp(p_p c/T_d) + 1)}. \quad (4)$$

After the decoupling the neutrinos propagate freely in the Universe. The only two changes experienced by the neutrinos are the red shift and number density decreasing as a result of the Universe expansion. Consequently, in order to obtain the neutrino distribution at any moment of time (or at any $a(t)$), we need to substitute $W_d = W/a^3(t)$, $p_d = pa(t)$ into (4):

$$dN = \frac{W_p^2 dp}{2\pi^2 \hbar^3 (\exp(pc/T) + 1)}, \quad (5)$$

$$\text{where } T = \frac{T_d}{a(t)}.$$ 

This equation would describe the relic neutrino distribution in the present-day Universe, if it was perfectly uniform. Formally, it coincides with the thermal distribution of ultrarelativistic fermions with $T = T_\nu$. However, though we will name $T_\nu$ “the neutrino temperature”, it is important to understand that actually $T_\nu$ is no temperature. For instance, some of neutrino types are now non-relativistic (at least, one of them has $m > 0.05$ eV, while $T_\nu \approx 2$ K $= 2 \times 10^{-4}$ eV now), and distribution (5) is non-equilibrium for this sort of particles. There is no reason to be surprised: after the decoupling at $T = T_d$, the neutrinos are perfectly collision-less and need not have any real thermodynamical temperature, being out of thermodynamical equilibrium with anything.

It is best to determine $T_\nu$ from the well-measured temperature of the relic photons $T_i \approx 2.7255$ K [2]. Then the present-day value $\tau$ of $T_\nu$ is $\tau = (4/11)^{1/3} T_i \approx 1.945$ K $= 1.677 \times 10^{-8}$ eV, and $T_\nu$ at a red shift $z$ is equal to

$$T_\nu(z) = (z + 1)\tau. \quad (6)$$

Since $\tau^2$ is much smaller than $\Delta m_1^2$ and $\Delta m_2^2$, at least two (or, possibly, all three) neutrinos are now non-relativistic. The case of a relativistic neutrino is rather trivial: its distribution cannot be significantly influenced by gravitational fields of the large-scale structure, and it is still adequately described by (5) and (6).

Let us consider the case of non-relativistic neutrinos of mass $m$. Then $p = mv$, and we may rewrite (5) as

$$f(\nu) = \frac{m^3}{h^3 (\exp(\nu/\xi) + 1)}, \quad (7)$$

$$\text{where } \xi \equiv \frac{c(1 + z)\tau}{mc^2}.$$ 

Here we introduce the distribution function $f$ in the 6-dimensional phase space, i.e., the number of the neutrinos of a certain flavor in a 6-dimensional volume $dx^3 dv^3$ is $\int dx^3 dv^3 f$. The velocity parameter $\xi$ defines the characteristic speed of the relic neutrinos: the root-mean-square speed is $\approx 3.6615$. Substituting the mass ranges of the neutrinos $\nu_1$, $\nu_2$, and $\nu_3$, we find...
that at the present epoch ($z = 0$) the most massive neutrino $v_3$ has 630 km/s < $\xi$ < 840 km/s and $\xi > 2500$ km/s for $v_1$ and $v_2$.

3. THE RELIC NEUTRINO SEPARATION
BY SPACE STRUCTURES

3.1. The Galaxy

Distribution (7) is valid only in an absolutely homogenous universe: the structures of the real Universe disturb function $f$ by their gravitational field. Let us start from an estimation of the influence of our Galaxy. At the epoch of the Milky Way formation ($z = 4$), $\xi > 3000$ km/s even for $v_3$. The capturing speed of the protogalaxy can be estimated as $v_{protogalaxy}^2/2 = \sqrt{GM_{protogalaxy}/R_{protogalaxy}}$, where $M_{protogalaxy} \sim 10^{12} M_\odot$, $R_{protogalaxy} \sim 200$ kpc are the Galaxy mass and radius. We obtain $v_{protogalaxy} \sim 200$ km/s, and integrating distribution (7) we find that only $\sim 2.7 \times 10^{-3}$% of neutrinos can be captured, if $\xi = 3000$ km/s. Even taking into account that the neutrino density during the Galaxy formation was $(z + 1)^3 = 125$ times higher than the present-day value, we obtain that the number of neutrinos $v_3$ captured inside the Galaxy is $\sim 300$ times lower than the background given by (7). For the lighter neutrinos, $v_1$ and $v_2$, the captured fraction is absolutely negligible.

However, the gravitational field of the formed Galaxy attracts neutrinos and increases their density. Let us estimate the efficiency of this process. For simplicity, we consider a toy (but rather close to realistic) model of our Galaxy as a single, stationary, spherically-symmetric halo immersed into the field of neutrinos with distribution (7). We choose two concentric spheres of radii $r_1$ and $r_2$ around the Galaxy center. The radius $r_1 = 8$ kpc, i.e., $r_1$ is equal to the radius of the Sun orbit around the Galaxy center. We choose $r_2 \gg R_{protogalaxy}$, i.e., it is so large that we may neglect the influence of the gravitational field of the Galaxy at $r_2$. We denote the distribution function $f$, the angle $\theta$ between the direction to the center and the particle trajectory, and the particle speed at $r_1$ and $r_2$ by $f_1$, $\theta_1$, $v_1$ and $f_2$, $\theta_2$, $v_2$, respectively.

Let us consider the number of neutrinos $f_1 dx_1 dv_1^3$ crossing the sphere $r_1$ in an angle interval $[\theta_1, \theta_1 + d\theta_1]$ in a speed interval $[v_1, v_1 + dv_1]$ in a time interval $dt$. They occupy the volume $dx_1^3 = 4\pi r_1^3 v_1 \cos \theta_1 dt$ and the velocity volume $dv_1^3 = 2\pi v_1^2 d(\cos \theta_1) dv_1$. The neutrinos cross the sphere $r_2$ in an angle interval $[\theta_2, \theta_2 + d\theta_2]$ in a speed interval $[v_2, v_2 + dv_2]$. Since the system is stationary, they cross $r_2$ in the same time interval $dt$. We obtain

$$f_1 2\pi v_1^2 d(\cos \theta_1) dv_1 4\pi r_1^3 v_1 \cos \theta_1 dt = f_2 2\pi v_2^2 d(-\cos \theta_2) dv_2 4\pi r_2^3 v_2 \cos \theta_2 dt.$$

One may easily simplify this equation to

$$f_1 r_1^3 v_1^2 \sin^2 \theta_1 dv_1 = f_2 r_2^3 v_2^2 \sin^2 \theta_2 dv_2.$$  

Describing the neutrino motion, we may use the energy and angular momentum conservation:

$$v_1^2 = v_2^2 + 2\phi,$$  

$$\eta v_1 \sin \theta_1 = r_2 v_2 \sin \theta_2,$$

where $\phi$ is the gravitational potential at $r_1 = 8$ kpc, at the Sun orbit. From (10) we obtain $v_1 dv_1 = v_2 dv_2$. Substituting it and (11) to (9), we find that $f_1 = f_2$.

One of course, this result is predictable: it is just a consequence of the Liouville’s theorem, the phase-space distribution function is constant along the trajectories of the particles. However, the derivation directly demonstrates that the distribution function near the Earth is isotropic. Moreover, since $r_2 \gg r_1$, only particles with $\theta_2 = 0$ may reach $r_1$. Thus, even if the distribution function $f_2$ far from the center is strongly anisotropic, the distribution near Earth $f_1$ is most likely isotropic: it is sufficient to have $f_2(\theta_2 = 0) = \text{const}$ at the sphere $r_2$. The isotropization of the distribution function of collisionless particles towards the center of a spherical halo is a general property [8].

For convenience, we denote the distribution function of the neutrinos at the Sun orbit in the Galaxy (i.e., $f_2$) by $F$, and the neutrino velocity at this radius (i.e., $v_1$) by $V$. Since $f_1 = f_2$, we need just to substitute (10) into (7) to obtain $F$:

$$F(V) = \frac{m^3}{h^3} \exp \left( \frac{\sqrt{V^2 - 2\phi}}{\xi} \right) + 1$$  

for $V \geq \sqrt{2\phi},$

$$F(V) = 0$$  

for $V < \sqrt{2\phi}.$

Instead of the gravitational potential $\phi$, one may use the escape speed $v_{esc} = \sqrt{2\phi}$. The density of neutrinos corresponding to the undisturbed distribution $F$ is

$$n = \int f(\tilde{v}) d\tilde{v}^3,$$

the density of neutrinos near the Earth is $n_0 = \int F(V) dV/V^3.$ We may introduce the boost factor

$$b \equiv n_0 / n,$$  

for $V = \sqrt{2\phi},$  

$$b = \int_{v_{esc}}^{\infty} F(V) dV^3 / \int_{0}^{\infty} f(\tilde{v}) d\tilde{v}^3.$$
The boosting factor $b$ depends only on the ratio \( \frac{\sqrt{2}\phi/\xi}{v_{\text{esc}}/\xi} \), i.e., the ratio between the escape speed and the characteristic speed $\xi$ of the neutrinos. The dependence is presented in Fig. 1.

The physical mechanism of the boosting is apparent: the gravitational field of the Galaxy accelerates the neutrinos, increasing the volume, which they occupy in the velocity space. However, the multiplication $dx^3dv^3$ should be conserved in accordance with the Liouville’s theorem. Thus, the space volume occupied by the neutrinos shrinks, and their density grows.

The galactic escape speed near the Solar System indubitably exceeds 525 km/s [9] and in principle may be much larger [10, 11]. If we accept $v_{\text{esc}} = 600$ km/s and $m = 0.08$ eV (i.e., $\xi \approx 630$ km/s), we obtain $b \approx 1.15$. The obtained value of the gravitational boosting of neutrino density are in good agreement with the $N$-body estimations of this quantity [12, 13].

### 3.2. The Laniakea Supercluster

It follows from (13) that the density boosting is defined by the gravitational potential $\phi$ of the observer. The contribution of the gravitational field of the Galaxy into the potential near the Solar System is large, but probably not dominant. The Galaxy is surrounded by huge extragalactic structures (like the Great Attractor), which are less dense, but much more massive, and their gravitational potential can be at least comparable with the Galactic one. Unfortunately, the influence of the nearest superclusters on the relic neutrino distribution cannot be calculated precisely: the structure shape is rather complex, and numerical simulations seem to be the only method. The properties of neutrinos differ significantly from the properties of the cold dark matter, while even the single-component $N$-body simulations of the cold dark matter suffer from essential numerical issues [14, 15]. Moreover, the superclusters are dominated by the dark matter, and the observational determination of their masses and space structures are not very precise, since the main component is invisible.

However, we may estimate the influence of the nearby objects with the help of Eq. (12): we need just to estimate the total gravitational potential created by the large scale structures, add it to the galactic potential, and substitute the obtained value to (12). Let us justify this approach.

First of all, we need to specify that we use the comoving frame of reference (the standard Friedmann coordinates). Then by a “velocity” is meant a “peculiar velocity”, i.e., the velocity with respect to the relic radiation field. This specification is not important in the case of the Galaxy, but is essential for large structures.

Let us follow the derivation of (12). We may choose $r_2$ so large that the nearby galaxy clusters are inside it. Then $f_2$ is defined by Eq. (7), and $\phi$ in (12) is defined not only by the gravitational field of the Galaxy, but by the nearby galaxy clusters as well. Equation (11) in the derivation of (12) is not valid anymore, since the system is not spherically symmetric. However, the final conclusion, $f_3 = f_1$, does not depend on the spherical symmetry and remains correct: it is just a consequence of the Liouville’s theorem.

The distribution $f_2$ at the infinity depends only on the particle speed, and the change of the particle speed depends only on the potential difference between $r_2$ and the observational point $r_1$. In the case of the Galaxy, we may conclude on this ground that the distribution $f_1$ at the observational point is also isotropic and depends only on the particle speed (which directly leads to distribution (12)). However, contrary to the case of the Galaxy, we cannot be sure that $f_1$ is isotropic. The principle difference is that the gravitational field of the present Galaxy is rather stationary, while a supercluster significantly evolves in the time necessary for a relic neutrino to cross it. As a result, the potential $\phi$ changes as the neutrino moves from the sphere $r_2$ to the observer, and the final neutrino speed depends on the neutrino trajectory. So Eq. (12) is much less accurate in this case, than in the case of the Galaxy. However, Eq. (12) is quite applicable as an approximation, since the logic of its derivation remains valid: the gravitational field accelerates the neutrinos, decreasing the space value $dx^3$ that they occupy.

Thus, we need to find the total gravitational potential $\phi$ near the Solar System. Today this task cannot be
solved precisely, but we need only a rough estimation of this quantity. The gravitational potential is additive, and we just sum the potentials from various massive objects, neglecting the contributions that are significantly smaller than the galactic one \( \sqrt{2\phi_{MW}} = 600 \text{ km/s} \). It is easy to see that the potentials of the other members of the Local Group or of the Council of Giants are negligible with respect to \( \phi_{MW} \). However, there are at least two objects that should undoubtedly be taken into account.

First of them is the Virgo cluster. Its mass inside 2.2 Mpc from the center is estimated as \( M = 1.2 \times 10^{15} M_\odot \) [16], and the distance to its center is \( d = 16.5 \text{ Mpc} \) [17]. Assuming that all the Virgo mass lies inside 2.2 Mpc, we estimate the potential created by the Virgo cluster near the Local Group as \( \phi_{V} = GM/d \) or \( v_{esc} = \sqrt{2\phi_{V}} = 790 \text{ km/s} \).

The Local Group, as well as the Virgo cluster, are parts of the Laniakea Supercluster. The mass distribution of this Supercluster is poorly known: some of its area projects on the Zone of Avoidance, making the objects there essentially undetectable. As the second massive object we consider the Great Attractor, a gravitational anomaly, which is supposed to be a massive galaxy cluster obscured by the Milky Way disk. Its mass and the distance to this object are estimated \(^1\) as \( M \sim 5 \times 10^{17} M_\odot \) and \( d \approx 70 \text{ Mpc} \) [19]. It is important to underline that the total mass inside 70 Mpc from the Great Attractor center far exceeds this value. However, calculating the potential, we should take into account only the overdensities, i.e., “additional” masses with respect to the homogeneous Friedmann’s universe. We obtain \( v_{esc} = \sqrt{2\phi_{GA}} = \sqrt{2GM/d} \approx 2500 \text{ km/s} \).

The total gravitational potential \( \phi \) near the Solar System can be obtained by summation \( \phi_{MW} + \phi_{V} + \phi_{GA} \). Since the last quantity is very uncertain, we may conclude that the total \( v_{esc} \) lies most probably between \( \sqrt{2(\phi_{MW} + \phi_{V})} \approx 1000 \) and 3000 km/s. For \( \nu_e \) (if we accept \( m_\nu = 0.08 \text{ eV} \)) these values correspond to the boost factors \( b = 1.34 \) and \( b = 2.66 \), respectively. The boost factors for the light neutrinos \( \nu_e \) and \( \nu_\mu \) are much less definite, because their masses and characteristic speeds \( \xi \) vary in a wide range, but even if we take the biggest possible mass 0.02 eV and the highest speed \( v_{esc} = 3000 \text{ km/s} \), we obtain \( b = 1.22 \), i.e., the boosting factors for the light neutrinos are much lower than for \( \nu_e \), and we will neglect them in this letter. However, they also may be important, if \( v_{esc} \) is high.

Finally, a question appears: if the escape speed is so high, is it correct to neglect the relic neutrino capturing by the above-mentioned structures? The answer is positive, if we discuss the relic neutrinos near the Solar System. Indeed, the Virgo cluster potential \( \phi_{V} \) is comparable with \( \phi_{MW} \), and we could see that the fraction of the captured neutrinos is negligible in this case. The gravitational potential of the Laniakea Supercluster is much larger, but it is not a gravitationally bound object. We take into account only the peculiar velocities in our calculations, while the Laniakea Supercluster participates in the general Hubble-Lemaître expansion of the Universe, and the Great Attractor, for instance, moves away from us with the speed 5000 km/s. No capturing is possible under these conditions.

A significant quantity of relic neutrinos should be captured in the central areas of rich galaxy clusters (like the Virgo cluster), where the escape speed is very high, and the captured neutrinos may even dominate. However, even if the experimentalists ever manage to detect the relic neutrinos, it will hardly happen in the center of a rich galaxy cluster, which reduces the practical importance of this fact.

### 4. THE FLAVOR COMPOSITION OF RELIC NEUTRINOS

A detection of relic neutrinos is a longed-for, but a notoriously difficult task. The cross-section of neutrino interactions increases with the neutrino energy \( \nu \) as \( \nu^2 \), but the Earth is transparent even for neutrinos with \( \nu \sim 1 \text{ MeV} \), and one may imagine, how small the cross-section is for the relic ones. Despite that, the neutrino community is offering new ideas about possible ways of the relic neutrino detection, and some of them are even close to the practical implementation (see, for instance, [20]).

All the possible experimental methods have one common feature: they are based on the neutrino flavors. The experimentalists observe \( \nu_e \), \( \nu_\mu \), \( \nu_\tau \), and not the mass states, \( \nu_1 \), \( \nu_2 \), and \( \nu_3 \). As we could see, the density of \( \nu_3 \) is significantly boosted by the cosmic structures \( b = 1.34-2.66 \), while the density of the two others is boosted much less, if any. So the fraction of \( \nu_3 \) is significantly boosted in the relic neutrinos. But \( \nu_3 \) is almost free of the electron neutrino (only \( \sim 2\% \) of \( \nu_e \)) and contains more muon component than the tau one.

Being generated, the relic neutrinos have equal fractions of \( \nu_e \), \( \nu_\mu \), \( \nu_\tau \), and we can see that the gravitational field of the cosmic structures changes this com-
position. Figure 2 shows the relic neutrino composition as a function of the total escape speed $v_{\text{esc}}$. We accepted $m_3 = 0.08$ eV and neglected the boosting of the two light neutrinos, $\nu_1$ and $\nu_2$. One may see that the fraction of $\nu_\tau$ remains almost constant (this is a result of the fact that the fraction of $\nu_\tau$ in $\nu_3$ is $41\% = 1/3$), while the fraction of $\nu_e$ significantly decreases, and the fraction of $\nu_\mu$ significantly increases with the $v_{\text{esc}}$ growth. If $v_{\text{esc}} = 1000$ km/s, the relic neutrinos contain $\sim 30\%$ of $\nu_e$, $\sim 36\%$ of $\nu_\mu$, and $\sim 34\%$ of $\nu_\tau$; if $v_{\text{esc}} = 3000$ km/s, the relic neutrinos contain $\sim 22\%$ of $\nu_e$, $\sim 42\%$ of $\nu_\mu$, and $\sim 36\%$ of $\nu_\tau$.

Since the mass of a particle is always equal to the mass of the antiparticle, the densities of relic neutrinos of any mass or flavor and the densities of corresponding antineutrinos remain exactly equal.

Unfortunately, it is the rarest electron neutrino $\nu_e$ that is the most accessible to observation. To comfort the experimentalists, we underline that the decreasing of the $\nu_e$ fraction is not a result of the $\nu_e$ density decreasing (the low limit on this density is given by Eq. (7) for the undisturbed relic background), being a result of higher densities of the two others (mainly of $\nu_\mu$).

In conclusion, let us discuss the highly unlikely case of the inverted hierarchy of neutrinos. Then the situation is the opposite: $\nu_1$ and $\nu_2$ are heavy, $m_2 \gtrsim m_1 \sim 0.05$ eV, and $\nu_3$ is much lighter. As a result, the densities of $\nu_1$ and $\nu_2$ are boosted by the gravitational effects, and $\nu_3$ is the most abundant. The effect on the flavor composition is also the opposite in this case: the relic neutrinos contain more electron neutrinos and less muon neutrinos. Unfortunately, this type of hierarchy is now almost excluded experimentally.

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