Resolving an ambiguity of Higgs couplings in the FSM, greatly improving thereby the model’s predictive range and prospects

José BORDES
jose.m.bordes@uv.es
Departament Fisica Teorica and IFIC, Centro Mixto CSIC, Universitat de Valencia, Calle Dr. Moliner 50, E-46100 Burjassot (Valencia), Spain

CHAN Hong-Mo
hong-mo.chan@stfc.ac.uk
Rutherford Appleton Laboratory, Chilton, Didcot, Oxon, OX11 0QX, United Kingdom

TSOU Sheung Tsun
tsou@maths.ox.ac.uk
Mathematical Institute, University of Oxford, Radcliffe Observatory Quarter, Woodstock Road, Oxford, OX2 6GG, United Kingdom

Abstract

We show that, after resolving what was thought to be an ambiguity in the Higgs coupling, the FSM gives, apart from two extra terms (i) and (ii) to be specified below, an effective action in the standard sector which has the same form as the SM action, the two differing only in the values of the mass and mixing parameters of quarks and leptons which the SM takes as inputs from experiment while the FSM obtains as a result of a fit with a few parameters. Hence, to the accuracy that these two sets of parameters agree in value, and they do to a good extent as shown in earlier work [1], the FSM should give the same result as the SM in all the circumstances where the latter has been successfully applied, except for the noted modifications due to (i) and (ii). If so, it would be a big step forward for the FSM. The correction terms are: (i) a mixing between the SM’s $\gamma - Z$ with a new vector boson in the hidden sector, (ii) a mixing between the standard Higgs

\footnote{Work supported in part by Spanish MINECO under grant PID2020-113334GB-I00 and PROMETEO 2019-113 (Generalitat Valenciana).}
with a new scalar boson also in the hidden sector. And these have been shown a few years back to lead to (i') an enhancement of the $W$ mass over the SM value \cite{2}, and (ii') effects consistent with the $g - 2$ and some other anomalies \cite{3}, precisely the two deviations from the SM reported by experiments \cite{4,5} recently much in the news.
So as to place the present problem in an appropriate context, let us recall first that the framed standard model (FSM) [6] is obtained by framing the standard model (SM). The new ingredients are thus the framons, that is, frame vectors in respectively the flavour and colour symmetry space promoted into fields. The frame vectors themselves can be taken as the columns of the transformation matrices relating the local flavour or colour frames to global reference frames. Therefore, frame vectors and framons transform under both su(2), su(3) changes in the local frames and \( \tilde{su}(2), \tilde{su}(3) \) changes in the global reference frames, and so are representations of \( su(2) \times su(3) \times \tilde{su}(2) \times \tilde{su}(3) \). The FSM has thus to be invariant under this doubled symmetry.

The flavour framons is just the Higgs scalar of the SM multiplied by a global (spacetime independent) real unit 3-vector \( \alpha \) transforming under dual colour \( \tilde{su}(3) \). The colour framons are the really new ingredients. They give rise to a hidden sector comprising particles over and above those we know in the SM, the latter henceforth referred to as the standard sector. Though of interest by itself, the hidden sector interacts but little with the standard sector, and will not figure much in this paper except in giving rise in the standard sector to the following FSM modifications [1, 7] to the SM action:

- **(a)** the Yukawa coupling in which \( \alpha \) appears, where \( \alpha \), though global and therefore not directly renormalized, is coupled to the colour vacuum and rotates with changing scale because of the renormalization of the colour vacuum in the hidden sector by colour framon loops;

- **(b)** a modified Weinberg mixing of \( \gamma - Z^0 \) into a new vector boson in the hidden sector [2];

- **(c)** a mixing of the standard Higgs boson \( h_W \) into a new scalar boson again in the hidden sector [1, 7].

The Yukawa coupling of quarks and leptons in the FSM, when stripped down to essentials, appears as follows:

\[
A_{YK} \sim (m_T/\zeta_W) \alpha H \alpha^\dagger, \tag{1}
\]

where \( H \) is the the standard Higgs scalar field, \( \zeta_W \) its vacuum expectation value, \( m_T \) a normalization constant depending on the quark or lepton type. The vector \( \alpha \) coming from the flavour framon is independent of the quark/lepton type but changes (or rotates) with changing scale. This
changed Yukawa coupling is giving most of the known FSM modifications to the SM scheme.

First, it gives the fermion mass matrix as:

\[ m = m_T \alpha \alpha^\dagger, \]  

which immediately leads to [1]:

- (1) the prediction of 3 (and only 3) generations of quarks and leptons.

Next, the fact that \( m \) in (2) has zero modes implies that at any scale, any theta-angle term in the action can be removed by a suitable chiral transformation on a zero quark mode, hence

- (2) solution to the strong CP problem (without axions) [1] as well as to a similar problem for leptons [8].

The fact that \( \alpha \) rotates with scale then gives:

- (3) the mass hierarchical pattern of quarks and leptons [1],

- (4) the characteristic pattern of mixing between up and down states for both quarks and leptons [1],

- (5) CP-violating phases in both the CKM and PMNS matrices [1, 8],

the last via the chiral transformations made to get (2). Indeed, on this basis,

- (6) a good fit, mostly to within present experimental error, is obtained to the mass and mixing parameters of quarks and leptons with far fewer parameters [1], effectively replacing 17 empirical parameters of the SM by 7 in the FSM.

However, all these apparent successes (2) - (6) of the FSM have so far been limited to single particle properties, namely the mass and state vector of each quark or lepton. Indeed, to go beyond that is this paper’s main concern. The difficulty is that when more than one particle is involved, it seems often to become unclear at what scale to evaluate \( \alpha \), which occurs in most expressions and changes with changing scales. And since \( \alpha \) turns out to be a rather rapidly varying function of \( \mu \) in certain ranges of \( \mu \), this can be a very serious ambiguity in practice.
The simplest example where this problem seems to occur is the decay of the Higgs boson into fermion-antifermion pairs. Expanding, in the Yukawa coupling in (1), the Higgs field $H$ around its vacuum expectation value thus:

$$H \rightarrow \zeta W + h_W,$$

one obtains to zeroth order in $h_W$ the fermion mass matrix as in (2), and to first order in $h_W$, the Higgs coupling to fermions as:

$$g_h = (m_T/\zeta_W)\alpha\alpha^\dagger.$$  

(4)

The coupling (4) depends on $\alpha$, and $\alpha$ depends on the scale $\mu$. So one has to specify at what value of $\mu$ to take $\alpha$ for calculating the said decay widths. There seems thus to be an ambiguity, which we did not previously know how to resolve, but now we think we do, as follows.

Specializing to the decay mode $\mu^+\mu^-$, sandwiching the Yukawa term (1) between muon states $|\mu\rangle$, and expanding the Higgs field about its vacuum expectation value as in (3), one has:

$$(m_\tau/\zeta_W)\langle\mu|\alpha(\mu)\rangle(\zeta_W + h_W)\langle\alpha(\mu)|\mu\rangle,$$

(5)

where in the scenario that one works in [1], only $\alpha$ depends on the scale $\mu$. For any given value of $\mu$ we get then the running mass as:

$$m_\mu(\mu) = m_\tau|\langle\mu|\alpha(\mu)\rangle|^2,$$

(6)

and the (running) coupling of $h_W$ to a muon pair as:

$$g_{h\mu}(\mu) = (m_\tau/\zeta_W)|\langle\mu|\alpha(\mu)\rangle|^2.$$  

(7)

In other words, for any value of $\mu$,

$$g_{h\mu}(\mu) = m_\mu(\mu)/\zeta_W,$$

(8)

or that the Higgs coupling of the muon is proportional to the muon mass.

Next, we have to specify at what value of $\mu$ we are to take the “physical” values $m_\mu^{\text{phys}}$ and $g_{h\mu}^{\text{phys}}$ for our calculation of the decay width. Notice that although two physical quantities are to be specified, there is actually only one question to answer. Once a $\mu$ is given, then $\alpha$ is specified and both $m_\mu^{\text{phys}}$ and $g_{h\mu}^{\text{phys}}$ can be evaluated via (6) and (7). In other words, only one condition
is required to fix the physical value of both the muon mass and its Higgs coupling within the framework we are operating. And the one condition needed we have at hand. In the FSM, the physical mass $m^\text{phys}_X$ of a particle $X$ is defined to be the running renormalized mass $m_X(\mu)$ taken at the scale equal to the physical mass of $X$ itself, or in other words, as the solution of the fixed-point equation:

$$m_X(\mu) = \mu.$$ (9)

This fixes the scale for determining its Higgs coupling $g^\text{phys}_{h\mu}$ as well.

Put in another way, the Yukawa term (1) is split into two terms by expanding the Higgs scalar $H$ around its expectation value as in (3), giving respectively the mass (2) and the Higgs coupling (4) to the fermion. Now in the original term (1) the only scale dependence comes from the factors $\alpha$, which are shared between the two split terms (2) and (4) and therefore identical. This means that whatever scale $\mu$ one chooses for finding the physical mass of the fermion $f$, the same scale has to be used in evaluating its coupling to the Higgs.

One might question the FSM definition (9) of the physical mass of particles although it seems quite widely accepted. For a particle, such as the $b$ quark, the mass of which can be studied perturbatively, this definition of the physical mass is justified usually as giving the best convergence of the perturbation series [9]. In the FSM, however, the definition is applied to all particles, although a theoretical justification in general is, to our knowledge, lacking. But, since it has been built into the FSM right from the beginning and played its part in most FSM results obtained earlier [1], such as on the mass and mixing patterns of quarks and leptons, it should be taken as part of the FSM itself. The conclusion reached in the preceding paragraph can thus be claimed by the FSM as a derived consequence.

Applying then this definition to the muon, we obtain as a direct consequence of the FSM the physical mass $m^\text{phys}_\mu$ of the muon as the solution to the equation:

$$m_\mu(\mu) = \mu.$$ (10)

Once we solve this equation for $m^\text{phys}_\mu$, we obtain the value for $\alpha$ at $\mu = m^\text{phys}_\mu$, the substitution of which into (7) will then give us the physical value of the coupling $g^\text{phys}_{h\mu}$ as well. Explicitly:

$$m^\text{phys}_\mu = m_\tau |\langle \mu | \alpha(\mu = m^\text{phys}_\mu) | \tau \rangle|^2$$ (11)
and
\[ g_{\mu\mu}^{\text{phys}} = (m_\tau / \zeta_W) |\langle \mu | \alpha(\mu = m_\mu^{\text{phys}}) \rangle|^2. \] (12)

And we have
\[ g_{\mu\mu}^{\text{phys}} = m_\mu^{\text{phys}} / \zeta_W, \] (13)

which is precisely the same conclusion as is obtained in the SM.

Further, the state vector of the muon \( \mu \) in the FSM is defined as the projection of \( \alpha(\mu = m_\mu) \) on to the plane orthogonal to \( \tau \), so that \( e \) the state vector of the electron, which is by definition orthogonal to both \( \mu \) and \( \tau \), is orthogonal to \( \alpha(\mu = m_\mu) \) as well. There is thus no flavour-violating coupling of \( h_W \) to \( \mu, e \) according to (11). The other coupling \( h_W \) to \( \mu \tau \) can be ignored, because the scale \( \mu = m_\mu \) being below the physical \( \tau \) mass, the mass matrix, and hence the coupling matrix (11) too, by FSM rules based on unitarity, has to be truncated [1]. There are therefore no flavour-violating (generation-changing) decays.

Clearly, the same arguments can be applied to the other charged leptons, and to quarks as well provided that physical conditions are such that the quarks can be treated as freely propagating particles with each a definite (Dirac) mass. We can conclude then that, apart possibly from neutrinos which may need more delicate handling (for example, possibly a see-saw mechanism), Higgs couplings of the quarks and leptons in the FSM are all proportional to their masses, as they are in the SM.

When put in this way, it seems that what was flagged earlier as an ambiguity in the Higgs coupling is actually, within the context of the FSM, quite straightforwardly resolved. Or, rather, that there has never been a genuine ambiguity in the first place. Indeed, one may well wonder why it has taken us such a long time to arrive at this conclusion. We think that we were misled by a result we had obtained previously in a different context, in what is called R2M2 (rotating rank-one mass matrix) phenomenology, which predated and contained some ideas later incorporated into the FSM. That result was at variance with the present FSM result obtained above but was thought wrongly at one stage [7] (Section 7.1, eq. (83)) to apply to the FSM as well. Since this bit of history is of some interest to the problem at hand, it seems worth a little digression here for its clarification.

The R2M2 scheme postulated a mass matrix of form (2) for quarks and leptons, with \( \alpha \) rotating with scale, as was later obtained as a result by the FSM. It assumed also the form (11) for the Higgs coupling, as was also derived later by the FSM. But how \( \alpha \) changes with scale was in R2M2 simply left to
be fitted to experiment, while in the FSM it comes about as a consequence of renormalisation by framion loops. Now, as described above, it is this further piece of information supplied by the FSM which forces the $\alpha$ appearing in both the mass and Higgs coupling of any quark or lepton to be evaluated at the same scale $\mu$, namely as the solution to (9). In contrast, for the R2M2 scheme, only the $\alpha$ appearing in the mass of a quark or lepton was specified as to be evaluated at this scale, not its Higgs coupling. Hence the scale in the R2M2 at which the $\alpha$ appearing in the Higgs coupling is to be evaluated remains to be specified, that is, there is an ambiguity. Appealing then partly to intuition and partly to folklore, R2M2 chose this scale to be the mass of the Higgs boson. And this led in [10] to the prediction that the decay rate of the Higgs decay into $\mu^+\mu^-$ to be much suppressed compared to that predicted by the SM and that predicted above by the FSM, and apparently also to the recent data from the LHC [11, 12].

In a sense, our intuition in choosing in [10] the Higgs mass as scale for Higgs decay in R2M2 was not so much wrong as misapplied. Our intuition was for the decay of the Higgs boson as a whole but we were drawing conclusions from it on particular modes, including those with very small branching ratios. The coupling of $h_W$ to $f\bar{f}$ pairs is dominated by $\bar{t}t, \bar{b}b, \tau\bar{\tau}$ in that order. Given that the state vectors for these, namely $t = \alpha(\mu = m_t), b = \alpha(\mu = m_b), \tau = \alpha(\mu = m_\tau)$, which are all close in value to $\alpha$ taken at the Higgs mass scale of 125 GeV because the rotation of $\alpha$ at these high scales is slow, the couplings obtained by putting all scales at the Higgs mass would make very little numerical difference. Indeed, if one were to take only one value of $\mu$ for evaluating $\alpha$ for Higgs decay, as the question was initially posed, one could not have come up with a much better answer than the Higgs mass for evaluating the total width, or the partial widths of the dominant modes. It is only when one asks for decays into the lower generations such as $\mu^+\mu^-$ with miniscule widths whose contribution to the total is negligible that the error from [10] becomes prominent.

So much then for the digression into history, and back now to the FSM proper. Although the FSM result posted above for Higgs decay turns out to be unexcitingly the same as the SM, it is nevertheless a breakthrough for the FSM, being the first example where it has managed to make an assertion, without any added assumption, on a physical quantity involving more than just a single fermion, that is, other than masses and state vectors of quarks and leptons studied previously. In other words, a breach has been made, and as it sometimes happens, with luck, a breach once made can turn into a
floodgate to irrigate and vitalize a whole wide area behind, and this seems to be the case here. Indeed, as will be shown, this breach has so greatly enhanced both the FSM’s predictive range and prospects that the model will appear in a much more favourable light as it has appeared before.

To see this, let us go back to the beginning where we noted that, as far as the standard sector of known particles are concerned, the FSM action differs from the SM action only through (a), (b), and (c). Further, when renormalization of the vacuum by colour framon loops is taken into account, the global vector $\alpha$ turns out to depend on the renormalization scale. Although this scale-dependence of $\alpha$ has been put to good use before to explain the masses and mixing parameters taken as empirical parameters in the SM, it seemed to have left the FSM a rotating mass matrix to calculate with, which we have found hard to manipulate.

What we have missed before but have now learned, however, is that by merely accounting for the scale dependence of $\alpha$, the renormalization of the vacuum by framon loops is not yet complete. We have still to specify at what scale we are to evaluate this $\alpha$ wherever this $\alpha$ occurs, namely here, as noted in (a), in the Yukawa coupling term. We have learned also that the scale to be chosen depends on the fermion type, and if that correct scale is chosen, then not only the mass but also the Higgs coupling for that fermion-type will turn out to be the same as in the SM. This means that the FSM, after renomalization by colour framon loop, gives to each individual quark and lepton the same Yukawa coupling that each possesses in the SM. And, as already noted, there will be no flavour-violating or generation-changing off-diagonal terms.

Explicitly, repeating basically the same argument as before but more succinctly, let us start anew from (1) the initial Yukawa coupling of the FSM, and turn on the renormalization by colour framon loops in the hidden sector as before in [1] leading to the rotation of $\alpha$ with changing scale. Since the result depends so far on scale, one has yet to specify at what scale or scales it is to be evaluated. The answers, we have now learned, are different for the different diagonal elements of the matrix in (1). Suppose we write out specifically for the up-type quarks the 3 diagonal elements of the Yukawa coupling (1) as:

$$
\begin{align*}
(m_T/\zeta_W)\bar{\psi}_t(t|\alpha(\mu))H(\alpha(\mu)|t)\psi_t, & \quad (m_T/\zeta_W)\bar{\psi}_c(c|\alpha(\mu))H(\alpha(\mu)|c)\psi_c, \\
(m_T/\zeta_W)\bar{\psi}_u(u|\alpha(\mu))H(\alpha(\mu)|u)\psi_u.
\end{align*}
$$

(14)

Our conclusion is that the first $t$ element is to be evaluated at $\mu = m_t$, the
second $c$ element at $\mu = m_c$, and the last $u$ element at $\mu = m_u$. Following
the same procedure above as worked out explicitly for the charged leptons,
one sees that in the way in which physical fermion masses are obtained in
the FSM:

$$A_{YK} \rightarrow (m_t/\zeta_W) \bar{\psi}_t H \psi_t + (m_c/\zeta_W) \bar{\psi}_c H \psi_c + (m_u/\zeta_W) \bar{\psi}_u H \psi_u,$$

(15)

namely, exactly as in the SM. The same conclusion applies also to the down-
type quarks, the charged leptons and the neutrinos, the last subject to the
same reservations as the SM neutrinos are subject to.

This means that after the rotation of $\alpha$ and proper choice of scales are
taken into account, the FSM gives for the standard sector an effective action
which is formally the same as the SM action, except for the terms (b) a
modified Weinberg mixing and (c) analogous mixing of the SM Higgs $h_W$.

It follows then that:

- (7) to the accuracy that the FSM can reproduce the mass and mixing
  parameters of quarks and leptons on which the SM action depends, it
  would share the successes of the SM in all of the latter’s applications
  within the standard sector, except for corrections due to (b) and (c).

We recall from (6) that for the mass and mixing parameters of quarks and
leptons, which the SM takes as inputs from experiment, the FSM has repro-
duced, in a fit with a much smaller set of adjustable parameters, some quite
accurate values already at a one-framon-loop level, the accuracy for which
can probably be improved with closer future studies. This means therefor:

- [A] A great extension of the FSM’s predictive range in the standard
  sector which, we recall, was limited before to single-particle processes.
  Now, one can apply all the techniques developed for the SM to this
effective action from the FSM, avoiding all our previous difficulties in
manipulating the rotating mass matrix and related questions.

- [B] A great improvement in the FSM’s prospects for surviving future
  experimental tests, given that the SM has been tested already in great
details and to great accuracy within the standard sector. The devi-
ations of the FSM from experiment will themselves be limited then
only to within the smallish errors in reproducing the mass and mixing
parameters of the quarks and leptons. Indeed, it would seem more
practical to leave the SM as it is with mass and mixing parameters
taken from experiment while charging the FSM with reproducing them to greater accuracy with more sophisticated applications and judging it by its results therein. Meanwhile one concentrates on the corrections that (b) and (c) can give.

And what corrections can we expect from (b) and (c)? Investigations done a few years back have already revealed that:

- (8) as a result of modified Weinberg mixing, a $W$ mass larger than is predicted by the SM [2],
- (9) as result of the standard Higgs $h_W$ mixing with a low mass member of the hidden sector, the accommodation in FSM [3] of the $g - 2$ [5, 13, 14] and Lamb shift [15, 16] anomalies.

And the two last items, namely enhanced $W$ mass and the $g - 2$ anomaly, are precisely the two hot experimental issues, recently very much in the news [4, 5, 13, 14].

Further, to this list can be added the intriguing FSM prediction of:

- (10) a hidden sector interacting little with particles in the standard sector we know, thus harbouring some, or perhaps even most, of the dark matter in the universe [7, 17].

In other words, in brief, besides offering an answer to the old generation puzzle (1), solutions to the CP problems (both strong (2) and weak (3)), and an explanation for the mass and mixing patterns of quarks and leptons (4), (5), (6), the FSM now claims (7) also to share the SM’s successes in almost all the range the latter has been applied, except for some few areas: (8) $W$ mass, (9) $g - 2$ and several other anomalies, (10) dark matter, that is, exactly where experiment seems to show departures from SM expectations. It is a scoresheet, if it can indeed survive future scrutiny, that one would wish to extend but otherwise largely maintain.

References

[1] José Bordes, Chan Hong-Mo and Tsou Sheung Tsun, Int. J. Mod. Phys. A30 (2015) 1550051, doi:10.1142/S0217751X15500517; arXiv:1410.8022.
[2] José Bordes, Chan Hong-Mo and Tsou Sheung Tsun, Int. J. Mod. Phys. A33 (2018), 1850190, doi:10.1142/S0217751X18501907; arXiv:1806.08271

[3] José Bordes, Chan Hong-Mo and Tsou Sheung Tsun, Int. J. Mod. Phys. A34 (2019), 1950140, doi:10.1142/S0217751X19501409; arXiv:1906.09229

[4] T. Aaltonen et al. (CDF), Science 376, 170 (2022).

[5] B. Abi et al. (Muon g-2 Collaboration), Phys. Rev. Lett. 126, 141801; arXiv:2104.03281

[6] For a recent review, see: Chan Hong-Mo and Tsou Sheung Tsun, Invited contribution to the "Festschrift for the Yang Centenary". edited FC Chen et al., 2022 (to appear); arXiv:2201.12256

[7] José Bordes, Chan Hong-Mo and Tsou Sheung Tsun, Int. J. Mod. Phys. A33 (2018) 1850195, doi: 10.1142/S0217751X18501956; arXiv:1806.08268

[8] José Bordes, Chan Hong-Mo and Tsou Sheung Tsun, Int. J. Mod. Phys. A 36 (2021) 31n32, 2150236; arXiv:2107.05420

[9] Aida X El-Khadra and Michael Luke, Ann.Rev.Nucl.Part.Sci.52:201-251,2002; arXiv:hep-ph/0208114.

[10] J. Bordes, Chan Hong-Mo and Tsou Sheung Tsun, Eur. Phys. J. C 65 (2010) 537; arXiv:0908.1750.

[11] ATLAS Collaboration, Phys. Lett. B 812 (2021) 135980; arXiv:2007.07830

[12] CMS Collaboration, JHEP 01 (2021) 148; arXiv:2009.04363

[13] Editorial A moment for muons. Nat. Phys. 17, 541 (2021). https://doi.org/10.1038/s41567-021-01251-x

[14] P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020) and 2021 update.

[15] N. Bezginov, T. Valdez, M. Horbatsch, A. Marsman, A. C. Vutha and E. A. Hessels Science 365, 1007-1012 (2019)
[16] Yong-Hui Lin, Hans-Werner Hammer and Ulf-G. Meiße\,ner Phys. Rev. Lett. 128, 052002 (2022); \texttt{arXiv:2109.12961}.

[17] José Bordes, Chan Hong-Mo and Tsou Sheung Tsun, Int. J. Mod. Phys. A33 (2018) 1830034, doi:10.1142/S0217751X1830034X; \texttt{arXiv:1812.05373}.

[18] Michael J Baker and Tsou Sheung Tsun, Eur.Phys.J.C70:1009-1015,2010; \texttt{arXiv:1005.2676[hep-ph]}. 

11