Observation of thermalization and information scrambling in a superconducting quantum processor

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Understanding various phenomena in non-equilibrium dynamics of closed quantum many-body systems, such as quantum thermalization, information scrambling, and nonergodic dynamics, is a crucial for modern physics. Using a ladder-type superconducting quantum processor, we perform analog quantum simulations of both the XX ladder and one-dimensional (1D) XX model. By measuring the dynamics of local observables, entanglement entropy and tripartite mutual information, we signal quantum thermalization and information scrambling in the XX ladder. In contrast, we show that the XX chain, as free fermions on a 1D lattice, fails to thermalize, and local information does not scramble in the integrable channel. Our experiments reveal ergodicity and scrambling in the controllable qubit ladder, and opens the door to further investigations on the thermodynamics and chaos in quantum many-body systems.

Whether the out-of-equilibrium dynamics of a quantum many-body system can present thermalization [1, 2] and information scrambling [3] is a fundamental issue in statistical mechanics. The occurrence or absence of ergodicity and information scrambling depends on whether integrability is broken or not. A nonintegrable system thermalizes when it evolves, where the quenched state can be described by the Gibbs distribution [4]. However, thermalization is absent in integrable systems due to infinitely many conserved quantities [2, 5]. Similarly, information scrambling cannot occur in 1D free fermions as an integrable system, while a generic nonintegrable system scrambles information [3, 6]. Experiments on quantum thermalization have been demonstrated in cold atoms [19] and trapped ions [8] with time-independent Hamiltonians, as well as periodic Floquet systems [9, 10]. In addition, information scrambling can be identified by out-of-time-order correlators (OTOCs) [3, 11], which have been directly measured using time-reversal operations [12, 13]. Nevertheless, the experimental implementation of both integrable and nonintegrable systems on the same quantum processor, where distinguishable characteristics of ergodicity and information scrambling can be observed, remains limited.

Recent numerical works have shown that ergodicity and scrambling can occur in the XX ladder [14, 15], but the 1D XX model is a typical integrable system [16] that exhibits the characteristics of free fermions. Here, we realize the XX chain and the XX ladder with a superconducting qubit chain and ladder, respectively, on a programmable quantum processor consisting of 24 qubits. Through the measurements of local observables and von Neumann entanglement entropy, we observe two distinct non-equilibrium dynamical behaviors of the qubit chain and ladder. Specifically, during the dynamics of the qubit ladder, the results of local observables validate the predictions of the Gibbs ensemble. Moreover, entanglement entropy saturates the maximum value corresponding to the average entropy of subsystems in random pure states [17]. However, with these signatures of thermalization, the dynamics of the XX chain is verified to be nonergodic due to its integrability. Furthermore, without the need of time-reversal operations for measuring the widely explored OTOCs, by performing efficient and accurate quantum state tomography (QST), we monitor the quench dynamics of the tripartite mutual information (TMI) as a genuine quantification of information scram-

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blinking [3]. For the first time, we present a critical experimental evidence of scrambling, characterized by a stable negative value of TMI in the XX ladder.

Our experiments are performed on a ladder-type superconducting circuit comprised of 24 transmon qubits (see Fig. 1). The superconducting circuit can be described by a Bose-Hubbard Hamiltonian [1]

\[ H = \sum_{m \in \{1,2\}} \sum_{n=1}^{11} J_{1m}^1 (\hat{a}_{m,n}^{\dagger} \hat{a}_{m,n+1} + \text{h.c.}) \]

\[ + \sum_{n=1}^{12} J_{2n}^0 (\hat{a}_{1,n}^{\dagger} \hat{a}_{2,n} + \text{h.c.}) + \sum_{m \in \{1,2\}} \sum_{n=1}^{12} \mu_{mn} \hat{N}_{m,n} \]

\[ + \sum_{m \in \{1,2\}} \sum_{n=1}^{12} U_{mn} \frac{1}{2} \hat{N}_{m,n} (\hat{N}_{m,n} - 1), \]

where \( J_{1m}^1 \) and \( J_{2n}^0 \) are the hopping amplitudes in the chain and ladder, respectively. The \( \mu_{mn} \) and \( U_{mn} \) are the chemical potential and the on-site energy, respectively.

With \( m \) denoting the number of rung, \( \hat{a}_{m,n} \) (\( \hat{a}_{m,n}^{\dagger} \)) are the bosonic annihilation (creation) operator, \( \hat{N}_{m,n} = \hat{a}_{m,n}^{\dagger} \hat{a}_{m,n} \) is the bosonic number operator, and \( J_{1m}^1 \) and \( J_{2n}^0 \) are the on-site chemical potential and nonlinear interaction, respectively.

Since \( |U|/\bar{U} \approx 19 \) with \( |U| \) and \( \bar{U} \) being the average value of nonlinear and hopping interactions (see Supplementary Information), the system (1) approximates to the XX spin model where the bosonic annihilation and creation operator are mapped to the spin lowering and raising operator, i.e., \( \hat{a}^{\dagger} \rightarrow \hat{\sigma}^{\dagger}, \hat{a} \rightarrow \hat{\sigma} \) [2]. Thus, the qubit chain can be described by the Hamiltonian

\[ H_C = \bar{U} \sum_{n}(\hat{\sigma}_n^{\dagger} \hat{\sigma}_{n+1} + \text{h.c.}). \]

Figure 1C and D respectively. (A) False-color optical micrograph of the superconducting quantum circuit. The up and down arrows indicate that the initial state of the qubit is |1⟩ and |0⟩, respectively. The qubits Q1–Q12 are employed to the quantum simulation of XX chain. The qubits Q1–Q6 and Q3–Q18 are employed to the quantum simulation of XX ladder. (C) and (D) are the experimental pulse sequence of the quantum simulation of XX chain and ladder, respectively.

The pulse sequences consist of initialization, evolution and readout. In the initialization, all qubits are at |0⟩, and the X gates are applied on the qubits whose chosen initial state is |1⟩. Next, the qubits are tuned to the working point via Z pulses, and the time evolution is realized. Finally, the measurements are performed after tuning the qubits back to their idle points.

We then study ergodicity via the operator distance \( d(\rho_A(t), \rho_A^{(eq)}(T)) \) as the maximum eigenvalue of \( \rho_A(t) - \rho_A^{(eq)}(T) \), where \( \rho_A(t) \) is the single-site reduced density matrix at time \( t \) measured using the QST, and \( \rho_A^{(eq)}(T) \) is the Boltzmann density operator with temperature \( T \rightarrow \infty \) (see Supplementary Information). When the dynamics is ergodic, it can be predicted that \( d(\rho_A(t), \rho_A^{(eq)}(T)) = 0 \) for a long time \( t \) [4, 19]. Figure 2C shows the time evolutions of the average over all qubits. The distance shows a value smaller than 0.05 for the ladder, while it exhibits a strong oscillation between 0.1 and 0.2 for the chain, providing an evidence of the occurrence and absence of ergodicity in the XX ladder and chain, respectively.

We also investigate the entanglement entropy (EE), as a quantification of bipartite entanglement, characterizing ergodicity via the volume law extracted from its dependence on the subsystem size \( l \) [17, 20]. Indirect methods of measuring the second Rényi EE, including quantum
interference [19] and randomized measurements [21], have been developed. Nonetheless, the measurement of the von Neumann EE requires the accurate and efficient QST. We perform a 6-qubit state tomography to obtain the reduced density matrix $\rho_A(t)$ with the subsystem $A$ comprised of $Q_1$–$Q_6$, and then calculate the EE $S_A = -\text{Tr}[\rho_A(t) \ln \rho_A(t)]$. By partially tracing the 6-qubit density matrix, we also obtain the EE of smaller subsystems.

Figure 3A–F shows the dynamics of the EE in the qubit chain and ladder. We observe that the temporal fluctuations of EE become more dramatic in the chain than that in the ladder. Furthermore, we study the time-averaged EE (after $t = 60$ ns) as a function of the subsystem size $l$. As depicted in Fig. 3G, the volume law of EE $S_A \propto l$ is satisfied for the quenched states in both qubit chain and ladder. However, the value of EE is larger for the ladder, which approaches to the Page value for random pure states [17]. In short, the experimental data of EE are consistent with the results in Ref. [20], where stronger fluctuations and a smaller volume-law slope in integrable systems than those in non-integrable cases are revealed.

Next, we study information scrambling by considering...
Tripartite mutual information (TMI) [3]:

\[ I_3 = S(\rho_A) + S(\rho_B) + S(\rho_C) + S(\rho_{ABC}) \]
\[ - S(\rho_{AB}) - S(\rho_{AC}) - S(\rho_{BC}) , \]

where \( S(\rho) \) is the von Neumann entropy, and \( A, B \) and \( C \) refer to three subsystems. Experimentally, to calculate TMI, we measure \( \rho_{ABC} \) using QST, and obtain the density matrix of smaller subsystems by partially tracing \( \rho_{ABC} \).

The schematic experimental pulse sequence for measuring TMI in the qubit chain is depicted in Fig. 4A. Different from the previous pulse sequences (Fig. 1C and D), the qubits \( Q_1 \) and \( Q_2 \) are prepared in a Einstein-Podolsky-Rosen (EPR) pair \( |\text{EPR}\rangle_{12} = \frac{1}{\sqrt{2}} (|0\rangle_1 |0\rangle_2 + |1\rangle_1 |1\rangle_2) \) by the \( X_\pi \) and a CNOT gates (see the frames in Fig. 4A and B). Subsystems \( A \) and \( B \) are chosen as \( Q_1 \) and \( Q_2 \), respectively, and the subsystem \( C \) consists of \( Q_3 \sim Q_5 \). A similar scheme of scrambling in the qubit ladder is plotted in Fig. 4B with the same choice of subsystem \( A, B \) and \( C \), but the remainder becomes \( Q_{14} \sim Q_{17} \). The initialization protocol in Fig. 4A and B are enlightened by the quantum teleportation and information retrieval from black holes [11, 22, 23], and the dynamics of TMI can characterize how the local information encoded by the EPR pair scrambles.

Figure 4C shows the experimental results of the quench dynamics of TMI for the qubit chain and ladder. In the qubit chain as an integrable case, TMI recovers zero after the decreasing period, while in the qubit ladder, TMI saturates to a stationary negative value. Moreover, for the \( XX \) ladder, the value of time-averaged TMI (after \( t = 60 \) ns), smaller than that in the chain, reflects a stronger information scrambling.

The measurement of TMI characterizing information scrambling lays the foundation for further experimental studies on TMI in other systems such as digital quantum circuits simulating black holes [11]. The ladder-type superconducting processor, where ergodicity is observed, can be a suitable platform for experimentally probing the phenomena of ergodicity breaking, such as many-body localization [4], measurement-induced disentangling phase [24], and quantum many-body scars [25].

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**Data availability:** All relevant data are available from the corresponding authors upon request.
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Supplementary Materials for ‘Observation of thermalization and information scrambling in a superconducting quantum processor’

I. DEVICE

A. Architecture

Our device is a 24-qubit superconducting quantum processor arranged into two rows of 12 Transmon qubits [1]. The simplified circuit diagram of the device is shown in Fig. S5a. We use 18 of them for this experiment. The qubits $Q_1$ - $Q_{12}$ are used for the quantum simulation of the 1D $X$ chain, and the qubits $Q_{13}$ - $Q_{18}$ are used for the quantum simulation of the $X$-$X$ ladder. With the hard-core boson limit [2], the Hamiltonian of the qubit chain can be written as

$$\hat{H}_C = \sum_{n=1}^{11} J_{n}^\parallel (\hat{\sigma}_n^+ \hat{\sigma}_{n+1}^- + \text{h.c.})$$

with $J_{n}^\parallel$ as the intrachain hopping interaction. Similarly, the Hamiltonian of the qubit ladder can be written as

$$\hat{H}_L = \sum_{m \in \{1,2\}} \sum_{n=1}^{5} J_{m,n}^\parallel (\hat{\sigma}_{m,n}^+ \hat{\sigma}_{m,n+1}^- + \text{h.c.}) + \sum_{n=1}^{6} J_{n}^\perp (\hat{\sigma}_{1,n}^+ \hat{\sigma}_{2,n}^- + \text{h.c.}),$$

where $m$ is the number of rung, and $J_{n}^\parallel$ and $J_{m,n}^\parallel$ refer to the rung and intrachain interactions.

Each qubit is capacitively coupled to its nearest neighbors with a fixed coupling strength. The qubit energy relaxation time $T_1$, and dephasing time $T_2$ are presented in Table S1, which are measured at the idle frequency $\omega_{\text{idle}}/2\pi$. The coupling strength $J/2\pi$ measured at the working frequency 4.863 GHz by two-qubit resonant oscillations are presented in Table S1, which is about 12.3 MHz for intrachain interactions ($J_{0}^\parallel$) and 13.6 MHz for rung interactions ($J_{1}^\perp$). We define the rate of correctly measuring $|1\rangle$ ($|0\rangle$) when the qubit is prepared at $|1\rangle$ ($|0\rangle$) as $f_{11}$ ($f_{00}$). As an example, the single shot events of $Q_4$ is shown in Fig. S5c. After the integration time of 1100 ns, the fidelity of $f_{00}$ and $f_{11}$ is determined as 99.6 %, and 96.2 %, respectively.

B. Readout and bandpass filter

For state readout we dispersively couple each qubit to a $\lambda/4$ readout resonator with coupling strengths $g_{\text{res}}/2\pi$ designed to be about 100 MHz. The measured resonator frequencies $\omega_{\text{read}}/2\pi$ and coupling strengths $g_{\text{res}}/2\pi$ are listed in Table S1. Resulting from the increasing of the coupling strength, the Purcell effect formed by the readout line is non-negligible [3]. To mitigate that side effect, we insert a $\lambda/2$ bandpass filter [4–6] between the readout line and the resonators. The bandpass filter suppresses the coupling of qubit frequencies while enhances the coupling of readout frequencies, as show in Fig. S5b. As a result, the bandpass filter allows for fast and high fidelity readout while maintaining the high-quality qubit performance by reducing the environmental damping. The bandpass filter is designed with a bandwidth of about 300 MHz, which covers the spanning of six readout resonators. The leakage time $1/\kappa_r$ of the readout resonator after coupled with the bandpass filter is designed to be about 100 ns. However, as a result of the frequency drift in fabrication, those resonators whose frequencies are far away from the center frequency of the bandpass filter have larger values of $1/\kappa_r$. More detailed parameters can be found in Table S1.

C. Fabrication of airbridge

We use HF airbridges [7], instead of crossovers, to connect the control lines separated by bandpass filters. In addition, the HF airbridges are used across the control lines, readout resonators and bandpass filters to suppress parasitic slotline modes. The device is fabricated in the same way as previous sample [8, 9], except for the process in the fabrication of HF airbridges.

Here we briefly describe the fabrication of HF airbridges. First, a 500 nm SiO$_2$ dielectric layer is defined by laser lithography followed by electron-beam evaporation. Second, the upper 500 nm aluminum electrodes are fabricated with laser lithography and electron-beam evaporation. Lastly, HF airbridges are fabricated with a dry VHF etcher to remove the dielectric layer. In short, the fabrication of HF airbridges is the same as that of crossovers, except for the final step which removes the dielectric layer. A scanning electron micrograph (SEM) photograph of HF airbridges is shown in Fig. S6.

II. EXPERIMENTAL WIRING SETUP

The experimental wiring setup for qubit control and frequency-multiplexed readout at different stages of the cryogenic system and the schematic of room temperature electronics are shown in Fig. S7. The quantum processor device is installed under the mixing chamber of the dilution refrigerator (DR), whose base temperature is about 12 mK, with a magnetic field shield. The readout and control waveforms are generated by Digital-to-
Analog Converters (DAC) at room temperature, and then attenuated by different attenuators installed at different stages of the DR. The signals are finally filtered by different low-pass filters installed under the mixing chamber plate. The DC signals are damped by 10 KΩ resistors installed at 4K plate. Before arriving at the control lines of the quantum device, XY, Z and DC controls are combined together by bias-tees. To obtain higher signal-noise ratio (SNR) in state readout, we use Josephson parametric amplifiers (JPA) [10] as the first stage amplification of readout signals. The high-electron-mobility transistors (HEMT) and low-noise microwave amplifiers, working at 4 K stage and room temperature, are used as the second and third stage amplifications, respectively. The average gain of the JPAs is 8.8 dB, 14.1 dB, and 13.2 dB, respectively. The signal carrying qubits information are finally demodulated and digitized by Analog-to-Digital Converters (ADC).

The room temperature electronics used in this experiment includes 80 DAC channels, 6 ADC channels, 28 DC channels and 7 microwave source channels. Among them, 6 DC channels are employed to keep the unused qubits (Q_{19}, Q_{20}, Q_{21}, Q_{22}, Q_{23}, Q_{24}) idling below 4.2 GHz.

### III. GATE PERFORMANCE

#### A. Single-qubit gate

We use the cross entropy benchmarking (XEB) to benchmark the fidelity of single-qubit X/2 gate [11–13]. In the single-qubit gate benchmarking process, many cycles of random single-qubit gates are applied. Each cycle

| | Q_1 | Q_2 | Q_3 | Q_4 | Q_5 | Q_6 | Q_7 | Q_8 | Q_9 | Q_{10} | Q_{11} | Q_{12} | Q_{13} | Q_{14} | Q_{15} | Q_{16} | Q_{17} | Q_{18} |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\omega_{\text{read}}/2\pi$ (GHz) | 6.688 | 6.729 | 6.790 | 6.832 | 6.885 | 6.701 | 6.855 | 6.908 | 6.954 | 6.384 | 6.423 | 6.482 | 6.521 | 6.576 | 6.637 |
| $\omega_{\text{max}}/2\pi$ (GHz) | 4.928 | 5.536 | 4.962 | 5.600 | 4.887 | 5.600 | 4.941 | 5.562 | 4.904 | 5.602 | 5.807 | 5.562 | 4.89 | 5.571 | 4.902 | 5.525 | 4.928 |
| $\omega_{\text{eff}}/2\pi$ (GHz) | 4.835 | 5.31 | 4.693 | 5.39 | 4.82 | 5.23 | 4.68 | 5.32 | 4.77 | 5.25 | 4.67 | 5.42 | 5.37 | 4.74 | 5.47 | 4.88 | 5.29 | 4.76 |
| $T_1$ (μs) | 24.3 | 22.8 | 26.5 | 24.0 | 28.8 | 25.9 | 19.5 | 28.5 | 20.5 | 17.9 | 31.8 | 13.1 | 16.6 | 22.4 | 12.4 | 24.4 | 23.9 | 21.4 |
| $T_2^*$ (μs) | 5.2 | 2.0 | 1.8 | 2.2 | 6.2 | 1.8 | 5.5 | 2.3 | 4.1 | 2.0 | 10.4 | 2.3 | 2.7 | 2.9 | 4.6 | 10.4 | 2.5 | 3.2 |
| $U/2\pi$ (MHz) | -238 | -230 | -230 | -230 | -238 | -230 | -240 | -230 | -236 | -230 | -236 | -230 | -226 | -236 | -226 | -236 | -228 | -236 |
| $g_{\text{res}}/2\pi$ (MHz) | 111 | 112 | 115 | 112 | 110 | 119 | 109 | 110 | 108 | 113 | 116 | 116 | 103 | 105 | 106 | 107 | 106 | 110 |
| $1/\kappa_r$ (ns) | 65 | 63 | 92 | 65 | 99 | 125 | 67 | 62 | 69 | 111 | 172 | 263 | 62 | 63 | 75 | 64 | 83 | 149 |
| $n_{\text{photons}}$ | 19 | 11 | 25 | 11 | 32 | 38 | 24 | 13 | 15 | 22 | 47 | 156 | 13 | 13 | 5 | 22 | 11 | 62 |
| $f_{\text{100}}$ (%) | 99.0 | 99.8 | 99.6 | 99.9 | 99.5 | 99.7 | 99.9 | 99.9 | 99.7 | 99.8 | 98.1 | 99.7 | 99.8 | 99.5 | 97.5 | 97.5 |
| $f_{\text{11}}$ (%) | 89.0 | 94.2 | 95.0 | 96.2 | 91.7 | 93.9 | 94.2 | 93.7 | 91.4 | 96.7 | 95.5 | 93.0 | 90.7 | 90.9 | 94.7 | 93.8 | 92.4 | 94.1 |
| Integration time (ms) | 1700 | 1100 | 1200 | 1100 | 1400 | 1500 | 1400 | 1250 | 1200 | 1200 | 1000 | 1300 | 1500 | 1100 | 1200 | 900 | 900 | 1000 |
| 1Q XEB fidelity (%) | 99.91 | 99.87 | 99.92 | 99.96 | 99.91 | 99.88 | 99.87 | 99.88 | 99.84 | 99.75 | 99.88 | 99.87 | 99.89 | 97.0 | 99.72 | 99.85 | 99.85 | 99.94 |
| 1Q SPB fidelity (%) | 99.92 | 99.89 | 99.92 | 99.86 | 99.99 | 99.87 | 99.88 | 99.86 | 99.84 | 99.84 | 99.76 | 99.91 | 99.88 | 99.89 | 99.74 | 99.74 | 99.90 | 99.90 | 99.94 |

**TABLE S1. Parameters of the device:** $\omega_{\text{read}}/2\pi$ is the frequency of the readout resonator; $\omega_{\text{max}}/2\pi$ is the maximum frequency of the qubit; $\omega_{\text{eff}}/2\pi$ is the idle frequency of the qubit; $T_1$ and $T_2^*$ are the energy relaxation and dephasing time of the qubit, respectively, which are measured at the idle frequency; $U$ is the anharmonicity of qubit measured at the idle frequency; $1/\kappa_r$ is the leakage time of the readout resonator; $J/2\pi$ is the coupling strength of the corresponding qubit-pair measured at the working frequency (4.863 GHz); $n_{\text{photons}}$ represents the number of photons occupied in the resonator calibrated by measuring the ac Stark shift of the qubit; $f_{\text{100}}$ ($f_{\text{11}}$) is the probability of correctly identifying the qubit state when it is initially prepared in $|1\rangle (|0\rangle$); The 1Q XEB fidelity is the average gate fidelity of single-qubit X/2 gate measured at idle frequency. The 1Q SPB fidelity describes the effect of decoherence on the single-qubit X/2 gate. The length of X/2 gates for all qubits is 30 ns.
FIG. S5. **Superconducting quantum circuit and the state readout.**

a Circuit diagram of the device. Each qubit is capacitively coupled to its nearest neighbors, inductively coupled to a control line for XY and Z control, and dispersively coupled to a $\lambda/4$ resonator for readout. Specially, every six readout resonators are inductively coupled to a $\lambda/2$ bandpass filter for fast and high fidelity multi-qubit readout. Each bandpass filter has a capacitively coupled input line and a inductively coupled output line. 

b Transmission spectrum of the bandpass filter for the readout resonators of $Q_1 - Q_6$, measured with a vector network analyzer. The $\lambda/2$ bandpass filter $|S21|$ data has a shape of Lorentzian type, which is designed to enhance the readout frequencies and suppress the qubit frequencies.

c Single shot readout events for $Q_4$. The readout process is repeated for 10,000 times after the qubit is successfully initialized in $|0\rangle$ (blue dots) and $|1\rangle$ (red dots), respectively. The fidelity $f_{00}$ is 99.6 %, and $f_{11}$ is 96.2 %.

consists of one single instance of gate sequence sampled from 80 random circuits. The circuits use a single-qubit gate set formed by the $\pi/2$ rotations around the eight axes in the Bloch representation: $\pm X$, $\pm Y$, and $\pm (X\pm Y)$, and end with a random single-qubit gate before measurement.

We apply a linear XEB [13] to compare the measured state probabilities with the ideal probabilities, and then
FIG. S6. **SEM photographs of HF airbridges.** a HF airbridges are used between those control lines separated by bandpass filters to connect them, and across the control lines, readout resonators and bandpass filter to suppress parasitic slotline modes. b An HF airbridge across the control line. c One end of the HF airbridge in b.

FIG. S7. **Schematic of room temperature electronics and cryogenic wiring setup.**

acquire the sequence fidelity \( \alpha \) as

\[
\alpha = \frac{\sum_q p_m(q)Dp_s(q) - 1}{D\sum_q p_s(q)^2 - 1},
\]

(5)

with \( D = 2^N \) (\( N = 1 \) for single-qubit XEB), \( q \) as one of bitstrings (for single-qubit XEB, \( q \) is 0 or 1), \( p_s(q) \) as the ideal probability of \( q \), and \( p_m(q) \) as the measured probability of \( q \). The over lines in Eq. (5) refer to the average of the 80 random circuits in each cycle. The sequence fidelity \( \alpha \) decaying with the number of cycles \( m \) is shown in Fig. S8a. The fitting function is \( \alpha = Ap_m^m + B \), where \( A \) and \( B \) represent the state preparation and measurement errors, respectively. The average error \( r \) of single-qubit gate is obtained according to

\[
r = (1 - p) \frac{D - 1}{D},
\]

(6)

and the average XEB fidelity of single-qubit gate is \( F = 1 - r \).

Meanwhile, we use the speckle purity benchmarking (SPB) [13] to calibrate the effect of decoherence error,

\[
\text{Purity} = \frac{\text{Var}(P_m)}{D} \frac{D^2(D + 1)}{D - 1},
\]

(7)

where \( \text{Var}(P_m) \) is the variance of the experimental probabilities extracted from the XEB experiment. The fitting function of Purity versus cycle number \( m \) is the same as that of XEB. As an example, the XEB and SPB results of the single-qubit X/2 gate on \( Q_3 \) are presented in Fig. S8a and b, respectively.

**B. Two-qubit gate**

In the experiment of probing information scrambling, the qubit \( Q_1 \) and \( Q_2 \) are prepared in a Einstein-Podolsky-
Rosen (EPR) pair state. The CNOT gate in realizing the entanglement state is realized by a two-qubit controlled-phase (CZ) gate and one single-qubit Y_{\pi/2} gate on control qubit \(Q_1\) and two single-qubit Y_{\pi/2} gates on target qubit \(Q_2\). The two-qubit CZ gate is implemented by tuning the \(|11\rangle\) state close to \(|02\rangle\) state following a fast adiabatic trajectory, generating a \(\pi\) phase shift on the \(|11\rangle\) state [14]. Specifically, we tune \(Q_2\) from 5.31 GHz to 5.085 GHz, while keeping \(Q_1\) at the idle point 4.835 GHz all the time. The length of the CZ gate is 55 ns, and the fidelity is 98.7% determined by the quantum process tomography (QPT), which is shown in Fig. S8c and d. A completely positive and trace-preserving (CPTP) [15] protocol is used to ensure the physical estimation of the \(\chi\) matrix from QPT.

IV. CALIBRATE ALL QUBITS TO WORKING FREQUENCY

Adjusting all qubits to the same working frequency plays an important role in this work, as the mismatch of qubit frequencies will induce an unwanted disorder. Although the calibration of Z pulse distortion and Z pulse crosstalk [8, 9] have been performed, the imperfect calibration of the Z pulse crosstalk still results in a drift of frequency when detuning the qubits to working points. Here, we use multi-qubit excitation propagation to calibrate and correct the frequency drift. The calibration process is listed below:

1. Prepare \(Q_m\) among the \(N\) qubits to \(|1\rangle\) and leave the others in \(|0\rangle\). Then we tune all qubits to the target frequency \(f_m\). After an evolution time \(t\), we measure the population of all sites, i.e., \(\langle \hat{n}_i \rangle = \langle \hat{\sigma}_z^+ \hat{\sigma}_z^- \rangle\). Here, in the 12-qubits chain case, we set
FIG. S9. Calibrate qubits to working frequency. a Experimental results of the time evolution of $\{n_i\}$ when $Q_6$ is excited in the qubit chain. b Experimental results of the time evolution of $\{n_i\}$ when $Q_6$ is excited in the qubit ladder. c The numerical simulation of the data in a. d The numerical simulation of the data in b.

We use QuTiP [16, 17] to simulate the evolution. The Hamiltonian used in the simulation is

$$\hat{H} = \hat{H}_C + \frac{1}{2} \sum_{n=1}^{12} n \times s + \delta_f(n)(1 - 2\hat{\sigma}_n^-\hat{\sigma}_n^+)$$

(9)

with $\hat{\sigma}_n^-$ (or $\hat{\sigma}_n^+$) as the annihilation (creation) operator of the $n$-th qubit, $\hat{H}_C$ as the Hamiltonian (3) and $\delta_f$ as the independent variables which refer to the frequency drifts in tuning qubits. For the 24 time-dependent distribution $Z_{\text{exp}}$ (ii) (ii=1 to 24), the population propagations are numerically simulated with corresponding initial states and $f_m$, and then the expected values $Z_{\text{sim}}$ for all qubits can be obtained. The distance between the numerical and experimental results is defined as $Z_{\text{diff}}(ii) = (Z_{\text{sim}} - Z_{\text{exp}})^2$, and the distance of all 24 evolutions is $Z_{\text{diffall}} = \sum_{i=1}^{24} Z_{\text{diff}}(ii)$. By changing $\delta_f$, we use Nelder-Mead optimization algorithm to minimize the distance $Z_{\text{diffall}}$, and finally get a $\delta_f$ array which referred to the frequency drift.

(3) After $\delta_f$ is obtained, we add $\delta_f$ to $f_m$ as the offset calibration to correct the drift, and then repeat the steps (1)-(3) until the absolute values of frequency drift are all small and the distance $Z_{\text{diffall}}$ without optimization is close to the optimized distance in the previous cycle.

Fig. S9a shows a part of the calibration results for the qubit chain, in which $Q_6$ is excited and then evolved for about 100 ns. After two cycles of calibration, the simulation pattern of $Q_6$ is quite similar with the experiment result (Fig. S9c), and the final distance is $Z_{\text{diffall}} = 10$. For the 12-qubit chain, the relative frequency drift is smaller than that presented in Table S2 according to the final calibration.

We use the same method to calibrate the 12-qubits ladder, except for the alternation of the Hamiltonian

$$\hat{H} = \hat{H}_L + \frac{1}{2} \sum_{m\in\{1,2\}} \sum_{n=1}^{6} \Delta_{mn}(1 - 2\hat{\sigma}_{m,n}^-\hat{\sigma}_{m,n}^+)$$

(10)

with the last term involves the arrangement of alignment frequencies and $\Delta_{mn} = s(m + 2(n - 1)) + \delta_f(n + 6(m - 1))$. Fig. S9b shows a part of the calibration results for the 12-qubit ladder, in which $Q_6$ is excited. After two cycles of calibration, the simulation pattern is also similar with the experiment result (Fig. S9d). The final distance is $Z_{\text{diffall}} = 21$, and the relative final frequency drift is smaller than that presented in Table S3 for the qubit ladder.

| Qubit number | $Q_1$ | $Q_2$ | $Q_3$ | $Q_4$ | $Q_5$ | $Q_6$ |
|--------------|------|------|------|------|------|------|
| Final frequency draft $\delta_f/2\pi$ (MHz) | 0.3 | 0.2 | 0.3 | 0.3 | 0.2 | 1.5 |

| Qubit number | $Q_1$ | $Q_2$ | $Q_3$ | $Q_4$ | $Q_5$ | $Q_6$ |
|--------------|------|------|------|------|------|------|
| Final frequency draft $\delta_f/2\pi$ (MHz) | 0.0 | 0.3 | 0.5 | 0.3 | 0.1 | 2.2 |

TABLE S2. The final frequency draft in the qubit chain.
The Hubbard model reads

\[ \hat{H}_{\text{BH}} = J \sum_{i=1}^{L} \left( \hat{a}_i \hat{a}_{i+1} + \hat{a}_i^{\dagger} \hat{a}_{i+1}^{\dagger} \right) + \frac{U}{2} \sum_{i=1}^{L} \hat{n}_i \left( \hat{n}_i - 1 \right) \]  

with \( J \) and \( U \) as the standard hopping and nonlinear interaction parameters, and \( \hat{n}_i = \hat{a}_i^{\dagger} \hat{a}_i \) as the bosonic number operator. The Hamiltonian (11) can describe a superconducting qubit chain. The qubit chain used in this work satisfies \( |U|/J \approx 19 \). Since the dynamics of the local densities \( n_{|00\rangle} \) under the unitary evolution \( \exp(-i \hat{H}_{\text{BH}} t) \) can be analytically derived when \( U/J \to \infty \) [2], we can compare the analytical results of an ideally noninteracting model with the experimental data and show that the influence of finite \( U/J \) is negligible.

When \( U/J \to \infty \), the system reaches the hard-core limit of the Bose-Hubbard model, which can be mapped to a free-fermionic spinless model via the Jordan-Wigner transformation

\[ \hat{a}_n = \exp(-i \pi \sum_{m<n} \hat{f}_m \hat{f}_m) \hat{f}_n \]  

where \( \hat{f}_n(\hat{f}_n^{\dagger}) \) refers to the fermionic annihilation (creation) operator. Using a Fourier transformation, we can obtain

\[ \hat{H}_{\text{BH}}^{U/J \to \infty} = \sum_{k=1}^{L} \lambda_k \hat{a}_k^{\dagger} \hat{a}_k \]  

with \( \hat{a}_k(\hat{a}_k^{\dagger}) \) as the fermionic operator in the momentum space, and

\[ \lambda_k = 2J \cos(2\pi k/L), \]

where \( k = 1, 2, ..., L \). With an initial state \( |\psi(0)\rangle = |1, 0, 1, 0, ..., 1, 0\rangle = \hat{f}_1 \hat{f}_3 \hat{f}_5 ... |0\rangle \) and a diagonal Hamiltonian (13), we can directly calculate the local densities as [2]

\[ n_{|00\rangle}(t) = \frac{1}{2} + \frac{1}{2L} \sum_{k=1}^{L} \exp[-4itJ \cos(2\pi k/L)]. \]

When \( L \to \infty \), the term \( \frac{1}{2} \sum_{k=1}^{L} \exp[-4itJ \cos(2\pi k/L)] \) can be rewritten as a Bessel function \( J_0(-4Jt) \). The experimental data of \( n_{|00\rangle} \) and the analytical results according to Eq. 16 are plotted in Fig. S10a, showing that the short-time behavior of the experimental data is consistent with the analytical results. The oscillation of the experimental data at later time can be regarded as a finite-size effect since the fluctuation of \( n_{|00\rangle} \) becomes stronger in smaller system (see Fig. S10b).

### TABLE S3. The final frequency draft in the qubit ladder.

| Qubit number | Final frequency draft \( \delta_f/2\pi \) (MHz) |
|--------------|-----------------------------------------------|
|              | \( Q_{13} \)   | \( Q_{14} \)   | \( Q_{15} \)   | \( Q_{16} \)   | \( Q_{17} \)   | \( Q_{18} \)   |
|              | 0.1  | 0.3  | 0.7  | 0.3  | 0.4  | 0.2  |

### VI. THE TEMPERATURE IN THE BOLTZMANN DENSITY OPERATOR

Ergodic dynamics suggests that for a subsystem \( A \) in the long-time and large scale limit [18],

\[ \rho_A(t) = \rho_A^\text{eq}(T), \]

where \( \rho_A(t) = \text{Tr}_B[\rho(t)], \rho_A^\text{eq}(T) = \text{Tr}_B[\rho^\text{eq}(T)], \) and \( \rho(t) \) and \( \rho^\text{eq}(T) \) refer to the quenched state at time \( t \) and Boltzmann density operator with temperature.
Entanglement entropy

The time evolution of entanglement entropy in the qubit chain with subsystem length $l = 6$. The green diamonds are the experimental data. The solid line is the numerical result for an isolated system, and the dashed line is the numerical result taking the dephasing effect into consideration.

$T$, respectively. According to Eq. (17), the distance $d(\rho_A(t), \rho_A^n(T))$ can characterize the ergodicity.

The temperature $T$ in Eq. (17) can be determined by the initial state, which satisfies $[19, 20]$

$$\text{Tr}\{[\rho^n(T) - |\psi_0\rangle\langle\psi_0|] \hat{H}\} = 0,$$  

(18)

where $|\psi_0\rangle$ is the initial state, and $\hat{H}$ is the Hamiltonian of the qubit chain or ladder. For the chosen initial states in the qubit chain and ladder, it can be directly calculated that $\langle\psi_0|\hat{H}|\psi_0\rangle \approx 0$ and thus $1/T = 0$ satisfies Eq.(18). For the following experimental and numerical results and the results in the main text, we consider the distance $d(\rho_A(t), \rho_A^n(T))$ with $T \to \infty$.

VII. THE IMPACT OF DECOHERENCE ON THE ENTANGLEMENT ENTROPY

In the main text, the presented numerical data are calculated by considering the unitary evolution of the superconducting qubits as an isolated system. However, the coupling of the qubits to the environment is unavoidable, leading to the decoherence that may affects the dynamics of entanglement entropy.

To quantitative estimate the effect of decoherence, we can solve the Lindblad master equation for the reduced density matrix obtained from partially tracing the environment, i.e.,

$$\dot{\rho}(t) = -i[\hat{H}, \rho(t)]$$

$$+ \frac{1}{2} \sum_n [2\hat{C}_n \rho(t) \hat{C}_n^\dagger - \{\hat{C}_n^\dagger \hat{C}_n, \rho(t)\}],$$  

(19)

with $\hat{C}_n = \sqrt{\frac{T}{\pi t}}\hat{A}_n$ as the collapse operators. There are two effects of the decoherence, i.e., the energy relaxation effect and the dephasing effect, characterized by the $T_1$ and $T_2^*$ in Table S1, respectively. Since $T_2^* \ll T_1$, it is predicted that the dephasing effect is stronger than the energy relaxation effect. Hence, we can study the dephasing effect to explain the discrepancy between the experimental and numerical results in Fig. 3e. The collapse operator of the dephasing effect is $\hat{C}_n = \hat{\sigma}_n^z/\sqrt{2T_{2,n}}$, where $T_{2,n}$ refers to the dephasing time of the $n$-th qubit. With Eq. (20), we can numerically simulate the time evolution of entanglement entropy with the dephasing effect. As shown in Fig. S11, the numerics considering the dephasing effect have a better agreement with the experimental data.

VIII. NUMERICAL RESULTS OF THE TRIPARTITE MUTUAL INFORMATION

In this section, we present more numerical details of the tripartite mutual information (TMI). The definition of TMI is

$$I_3 = S(\rho_A) + S(\rho_B) + S(\rho_C) + S(\rho_{ABC})$$

$$- S(\rho_{AB}) - S(\rho_{AC}) - S(\rho_{BC}),$$

(20)

which actually consists of the von Neumann entropy of different subsystems. Below, we will show the dynamics of entanglement entropy for different subsystems. For both the qubit chain and ladder, the subsystem $A$ and $B$ is chosen as $Q_1$ and $Q_2$ respectively, and the subsystem $C$ is comprised of the qubit $Q_3, Q_4$ and $Q_5$.

The results are plotted in Fig. S12 and S13. The finite value of $S(\rho_B)$ at the initial time $t = 0$ indicates that the information about the qubit $Q_1$ is locally encoded in the qubit $Q_2$ through entanglement by the CNOT gate (Fig. S12a and S13a). Moreover, in Fig. S12 and S13, it is seen that although the values of entanglement entropy are influenced by the decoherence, the overall non-equilibrium behaviors of entanglement entropy does not significantly affected by the decoherence. Consequently, the experimental data of TMI comprised of the results in Fig. S12 and S13 can reveal the distinct difference between the information scrambling in the qubit chain and ladder. We then present the numerical results of the TMI in comparison with the experimental data (Fig. S14).
FIG. S12. Experimental and numerical results of the entanglement entropy for different subsystems in the qubit chain. a The dynamics of $S(\rho_A)$ ($Q_1$) and $S(\rho_B)$ ($Q_2$). b The dynamics of $S(\rho_C)$. c The dynamics of $S(\rho_{AB})$. d The dynamics of $S(\rho_{AC})$. e The dynamics of $S(\rho_{BC})$. f The dynamics of $S(\rho_{ABC})$.

FIG. S13. Experimental and numerical results of the entanglement entropy for different subsystems in the qubit ladder. a The dynamics of $S(\rho_A)$ ($Q_1$) and $S(\rho_B)$ ($Q_2$). b The dynamics of $S(\rho_C)$. c The dynamics of $S(\rho_{AB})$. d The dynamics of $S(\rho_{AC})$. e The dynamics of $S(\rho_{BC})$. f The dynamics of $S(\rho_{ABC})$.

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FIG. S14. Experimental and numerical results of tripartite mutual information (TMI) in the qubit chain and ladder. a The numerical results of the TMI in the qubit chain and ladder without considering decoherence. b The numerical results of the TMI in the qubit chain and ladder with the dephasing effect. c The experimental data of the TMI in the qubit chain and ladder.

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