Abstract

In this paper I examine black hole and cosmological space-times in Born-Infeld-Einstein theory with electric and magnetic charges. The field equations are derived and written in the form $G_{\mu\nu} = -\kappa T_{\mu\nu}$ for spherically symmetric space-times. The energy-momentum tensor is not the Born-Infeld energy-momentum tensor, but can be obtained from Born-Infeld theory by letting $a \rightarrow ia$, where $a$ is the Born-Infeld parameter. It is shown that there is a curvature singularity in spherically symmetric space-times at a nonzero radial coordinate and that, as in Reissner-Nordstrom space-times, there are zero, one or two horizons. Charged black holes have either two horizons and a timelike singularity or one horizon with a spacelike, timelike, or null singularity. Anisotropic cosmological solutions with electric and magnetic fields are obtained from the spherically symmetric solutions.
Introduction

Born-Infeld electrodynamics follows from the Lagrangian [1]
\[ L = -\frac{1}{4\pi b} \left\{ \sqrt{-\text{det}(g_{\mu\nu} + bF_{\mu\nu})} - \sqrt{-\text{det}(g_{\mu\nu})} \right\}, \]
where \( g_{\mu\nu} \) is the metric tensor and \( F_{\mu\nu} \) is the electromagnetic field tensor. In the weak field limit this Lagrangian reduces to the Maxwell Lagrangian plus small corrections. For strong fields the field equations deviate significantly from Maxwell’s theory and the self energy of the electron can be shown to be finite. The Born-Infeld action also appears in string theory. The action for a D-brane is of the Born-Infeld form with two fields, a gauge field on the brane and the projection of the Neveu-Schwarz B-field onto the brane [2].

In recent papers [3, 4] I considered using a Palatini variational approach to derive the field equations associated with the Lagrangian
\[ L = -\frac{1}{\kappa b} \left\{ \sqrt{-\text{det}(g_{\mu\nu} + bR_{\mu\nu} + \kappa bM_{\mu\nu})} - \sqrt{-\text{det}(g_{\mu\nu})} \right\}, \]
where \( R_{\mu\nu} \) is the Ricci tensor, \( M_{\mu\nu} \) is the matter contribution, and \( \kappa = 8\pi G \). This Lagrangian, with \( M_{\mu\nu} = 0 \), has also been examined using a purely metric variation by Deser and Gibbons [5], Feigenbaum, Freund and Pigli [6] and Feigenbaum [7].

In this paper I examine black hole and cosmological space-times in Born-Infeld-Einstein theory with electric and magnetic charges. The field equations are derived for spherically symmetric space-times and it is shown that they can be written in the form \( G_{\mu\nu} = -\kappa T_{\mu\nu} \). The energy-momentum tensor is not the Born-Infeld energy-momentum tensor, but can be obtained from Born-Infeld theory by letting \( a \rightarrow ia \), where \( a \) is the Born-Infeld parameter. The field equations are solved and it is shown that the space-time has a curvature singularity at a nonzero radial coordinate and that, as in Reissner-Nordstrom space-times, there are zero, one or two horizons. Thus, black holes in this theory will have one or two horizons. Anisotropic cosmological solutions with electric and magnetic fields are obtained from the spherically symmetric solutions.

The Field Equations

In this section I will derive the field equations for the Born-Infeld-Einstein theory with an electromagnetic field and a cosmological constant. The action is given by
\[ L = -\frac{1}{\kappa b} \left\{ \sqrt{-\text{det}(g_{\mu\nu} + bR_{\mu\nu} + \kappa bF_{\mu\nu} + b\lambda g_{\mu\nu})} - \sqrt{-\text{det}(g_{\mu\nu})} \right\}, \]
where \( R_{\mu\nu} \) is the Ricci tensor, \( \kappa = 8\pi G \), \( b \) is a constant, \( F_{\mu\nu} \) is the electromagnetic field tensor and \( \lambda \) is a constant. The Ricci tensor is given by
\[ R_{\mu\nu} = \partial_\nu \Gamma^\alpha_{\mu\alpha} - \partial_\alpha \Gamma^\alpha_{\mu\nu} - \Gamma^\alpha_{\beta\alpha} \Gamma^\beta_{\mu\nu} + \Gamma^\alpha_{\beta\mu} \Gamma^\beta_{\alpha\nu} \]
and the connection is taken to be symmetric. Note that $R_{\mu\nu}$ is not symmetric in general. The electromagnetic field term appears in the action multiplied by the constant $(\kappa b)^{1/2}$ instead of $\kappa b$ since the lowest order term in the expansion of the Lagrangian is quadratic in $F_{\mu\nu}$.

Varying the action with respect to $g_{\mu\nu}$ gives

$$(1 + b\lambda)\sqrt{P} (P^{-1})^{(\mu\nu)} = \sqrt{g} g^{\mu\nu},$$

where $P_{\mu\nu} = (1 + b\lambda)g_{\mu\nu} + bR_{\mu\nu} + (\kappa b)^{1/2}F_{\mu\nu}$, $P^{-1}$ is the inverse of $P$, $(P^{-1})^{(\mu\nu)}$ is the symmetric part of $P^{-1}$, $P = -\det(P_{\mu\nu})$ and $g = -\det(g_{\mu\nu})$.

In the above action I have included the cosmological constant term in the first determinant along with the electromagnetic source term. However, the cosmological term could be added to the second determinant instead giving $-\sqrt{-\det((1 + b\lambda)g_{\mu\nu})}$ for this term. The resulting field equations, written in terms of $\bar{g}_{\mu\nu}$ and $\bar{\lambda}$ can be obtained from the above field equations by making the following substitutions

$$g_{\mu\nu} = (1 + b\bar{\lambda})\bar{g}_{\mu\nu} \quad \text{and} \quad 1 + b\lambda = \frac{1}{1 + b\bar{\lambda}}. \quad (6)$$

A cosmological term could also be added to both terms in various ways, but I will not consider these possibilities here.

Varying with the action with respect to $\Gamma^{\alpha}_{\mu\nu}$ gives

$$\nabla_\alpha \left[ \sqrt{P} (P^{-1})^{(\mu\nu)} \right] - \frac{1}{2} \nabla_\beta \left\{ \sqrt{P} \left[ \delta^\mu_{\alpha} (P^{-1})^{\beta\nu} + \delta^\nu_{\alpha} (P^{-1})^{\beta\mu} \right] \right\} = 0. \quad (7)$$

In an earlier paper [3] I showed that this equation together with the electromagnetic field equation, which will be discussed next, implies that the connection is the Christoffel symbol.

The field equations for the electromagnetic field are derived by varying the action with respect to $A_\mu$ and are given by

$$\nabla_\mu \left[ \sqrt{P} (P^{-1})^{[\mu\nu]} \right] = 0. \quad (8)$$

The above field equations are difficult to solve in general. However, it is possible to solve them for spherically symmetric electric and magnetic fields. In this case

$$P_{\mu\nu} = \begin{bmatrix}
q_{00} & E' & 0 & 0 \\
-E' & q_{11} & 0 & 0 \\
0 & 0 & q_{22} & B' \\
0 & 0 & -B' & q_{33}
\end{bmatrix} \quad (9)$$
and

\[
(P^{-1})^{\mu\nu} = \begin{bmatrix}
\frac{q_{11}}{\Delta_1} - \frac{E'}{\Delta_1} & 0 & 0 \\
\frac{E'}{\Delta_1} & \frac{q_{00}}{\Delta_1} & 0 \\
0 & 0 & \frac{q_{33}}{\Delta_2} - \frac{B'}{\Delta_2} \\
0 & 0 & \frac{B'}{\Delta_2} & \frac{q_{22}}{\Delta_2}
\end{bmatrix},
\]

(10)

where \( q_{\mu\nu} = (1 + b\lambda)g_{\mu\nu} + bR_{\mu\nu} \), \( E' = \sqrt{k'bE} \), \( B' = \sqrt{k'bB} \), \( \Delta_1 = q_{00}q_{11} + E'^2 \) and \( \Delta_2 = q_{22}q_{33} + B'^2 \). If \( \Delta_1 < 0 \) and \( \Delta_2 > 0 \) at some point in space-time then this must be true everywhere, if \( \Delta_1 \) and \( \Delta_2 \) are continuous, since \( P = -\Delta_1\Delta_2 \neq 0 \). This will be the case for the space-times examined in this paper and I will therefore take \( \Delta_1 < 0 \) and \( \Delta_2 > 0 \).

Taking the determinant of both sides of (5) gives

\[
g = -(1 + b\lambda)^4q_{00}q_{11}q_{22}q_{33}.
\]

(11)

The field equations can then be written as

\[
(P^{-1})^{(\mu\nu)} = (1 + b\lambda)^4 \frac{q_{00}q_{11}q_{22}q_{33}}{\Delta_1\Delta_2} g^{\mu\nu}
\]

(12)

and give the following four equations

\[
q_{00} = -(1 + b\lambda)^4 \frac{q_{00}q_{11}q_{22}q_{33}\Delta_1}{\Delta_2} g^{11},
\]

(13)

\[
q_{11} = -(1 + b\lambda)^4 \frac{q_{00}q_{11}q_{22}q_{33}\Delta_1}{\Delta_2} g^{00},
\]

(14)

\[
q_{22} = (1 + b\lambda)^4 \frac{q_{00}q_{11}q_{22}q_{33}\Delta_2}{\Delta_1} g^{33},
\]

(15)

and

\[
q_{33} = (1 + b\lambda)^4 \frac{q_{00}q_{11}q_{22}q_{33}\Delta_2}{\Delta_1} g^{22}.
\]

(16)

From (13) and (14) we find that

\[
(1 + b\lambda)^2 q_{22}q_{33}\Delta_1 = g_{00}g_{11}\Delta_2 \quad and \quad g_{00}R_{11} = g_{11}R_{00}
\]

(17)

and from (15) and (16) we find that

\[
(1 + b\lambda)^2 q_{00}q_{11}\Delta_2 = g_{22}g_{33}\Delta_1 \quad and \quad g_{22}R_{33} = g_{33}R_{22}.
\]

(18)
Using these equations gives
\[ g_{11}b^2 R_{00}^2 + 2(1 + b\lambda)g_{00}g_{11}bR_{00} + \frac{g_{00}E^2 - g_{00}g_{11}[(1 + b\lambda)^{-2} - (1 + b\lambda)^2(1 - a^2\tilde{B}^2)]}{(1 - a^2\tilde{B}^2)} = 0, \]  
(19)
where
\[ \tilde{B}^2 = (1 + b\lambda)^2 g^{22} g^{33} B^2. \]  
(20)
and \( \kappa b = a^2 \). The solution to this equation is
\[ R_{00} = -\frac{g_{00}}{b} \left[(1 + b\lambda) - (1 + b\lambda)^{-1} \sqrt{1 + a^2 \tilde{E}^2} \right]^{-1}, \]  
where \( \tilde{E}^2 = -(1 + b\lambda)^2 g^{00} g^{11} E^2 \). To find \( R_{11} \) we can use \( R_{11} = g_{11} R_{00} / g_{00} \). A calculation similar to the one above gives
\[ R_{22} = -\frac{g_{22}}{b} \left[(1 + b\lambda) - (1 + b\lambda)^{-1} \sqrt{1 + a^2 \tilde{B}^2} \right]^{-1}, \]  
(22)
and \( R_{33} \) is given by \( R_{33} = g_{33} R_{22} / g_{22} \). The solution can be expressed in terms of \( \tilde{F}_{\mu\nu} = (1 + b\lambda)F_{\mu\nu} \) and the invariants \( \tilde{F}^2 = \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu}, \tilde{S}^2 = \frac{1}{4} \star \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} \) and \( \star \tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} \tilde{F}_{\alpha\beta} \). It is given by
\[ G_{\mu\nu} = -(1 + \tilde{\lambda})^{-1} \kappa \left\{ \frac{\tilde{F}_{\mu}^{\alpha} \tilde{F}_{\nu\alpha}}{\sqrt{1 - \frac{1}{2} a^2 \tilde{F}^2 - a^4 \tilde{S}^4}} - \frac{g_{\mu\nu}}{a^2} \left[ (1 + \tilde{\lambda})^2 - \frac{1 - \frac{1}{2} a^2 \tilde{F}^2}{\sqrt{1 - \frac{1}{2} a^2 \tilde{F}^2 - a^4 \tilde{S}^4}} \right] \right\}, \]  
(23)
where \( \tilde{\lambda} = a^2 \lambda / \kappa \). Note that we require \( 1 + \tilde{\lambda} > 0 \) so that the effective Einstein constant \( (1 + \lambda)^{-1} \kappa \) is greater than zero. Also note that if \( \lambda = 0 \) the energy-momentum tensor is identical to the Born-Infeld energy-momentum tensor, except for the sign in front of the \( \tilde{F}^2 \) term in the square roots and the sign in front of the metric. Thus, the Einstein field equations in the absence of a cosmological constant can be obtained from the Born-Infeld theory by replacing \( a \) by \( ia \). The energy-momentum tensor that appears on the right hand side of equation (23) can be separated into a cosmological constant term plus a contribution that vanishes if \( F^{\mu\nu} = 0 \):
\[ T_{\mu\nu} = -\frac{\Lambda}{\kappa} g_{\mu\nu} + (1 + \tilde{\lambda})^{-1} \left\{ \frac{\tilde{F}_{\mu}^{\alpha} \tilde{F}_{\nu\alpha}}{\sqrt{1 - \frac{1}{2} a^2 \tilde{F}^2 - a^4 \tilde{S}^4}} - \frac{g_{\mu\nu}}{a^2} \left[ 1 - \frac{1 - a^2 \tilde{F}^2}{\sqrt{1 - \frac{1}{2} a^2 \tilde{F}^2 - a^4 \tilde{S}^4}} \right] \right\}, \]  
(24)
where
\[ \Lambda = \lambda \left[ \frac{2 + b\lambda}{1 + b\lambda} \right]. \]  
(25)
The electromagnetic field equations given in (8) can be written as
\[
\nabla_\mu \left\{ \frac{\tilde{F}^{\mu\nu} + a^2 \tilde{S}^2 \star \tilde{F}^{\mu\nu}}{\sqrt{1 - \frac{1}{2} a^2 \tilde{F}^2 - a^4 \tilde{S}^4}} \right\} = 0.
\]
(26)

Once again, in the absence of a cosmological constant these equations follow from Born-Infeld theory by replacing \(a\) by \(ia\).

In flat space-time with \(B = 0\) and \(\lambda = 0\) the square root in Born-Infeld theory is given by \(\sqrt{1 - a^2E^2}\), so that the maximum magnitude of the electric field is \(1/a\). In the theory presented here, with \(\lambda = B = 0\), the square root is \(\sqrt{1 + a^2E^2}\), so that the square root does not, by itself, constrain the magnitude of the electric field (see [3] for a more detailed discussion). The situation is reversed if \(E = 0\) and \(B \neq 0\).

The electromagnetic field equations (26) and the electromagnetic contribution to the energy momentum tensor (24) can be derived from the Lagrangian
\[
L = \frac{1}{a^2}(1 + \tilde{\lambda})^{-1} \left[ \sqrt{1 - \frac{1}{2} a^2 \tilde{F}^2 - a^4 \tilde{S}^4} - 1 \right].
\]
(27)

It is convenient to introduce the field \(\tilde{D}^{\mu\nu}\) defined by
\[
\tilde{D}^{\mu\nu} = -2 \frac{\partial L}{\partial F^{\mu\nu}}.
\]
(28)

It is given by
\[
\tilde{D}^{\mu\nu} = \frac{\tilde{F}^{\mu\nu} + a^2 \tilde{S}^2 \star \tilde{F}^{\mu\nu}}{\sqrt{1 - \frac{1}{2} a^2 \tilde{F}^2 - a^4 \tilde{S}^4}}.
\]
(29)

The field equations (26) can be written as
\[
\nabla_\mu \tilde{D}^{\mu\nu} = 0
\]
(30)

and the electric induction is defined to be \(\tilde{D}_k = \tilde{D}_{k0}\).

**Black hole space-times**

Now consider spherically symmetric black hole solutions. The general spherically symmetric solution for any theory with a Lagrangian of the form
\[
-\frac{1}{2\kappa} R + L(F^2, S^2),
\]
(31)
is given by [8, 9, 10, 11, 12, 13]
\[
ds^2 = -\left[1 - \frac{2m(r)}{r}\right] dt^2 + \left[1 - \frac{2m(r)}{r}\right]^{-1} dr^2 + r^2 d\Omega^2,
\]
(32)
\[ D = \frac{Q}{r^2} \, dt \wedge dr \quad \text{and} \quad B = P \sin \theta \, d\theta \wedge d\phi , \] (33)

where
\[ \frac{dm(r)}{dr} = \frac{1}{2} \Lambda r^2 + 4\pi r^2 H(r) , \] (34)

and \( H \) is given by
\[ H = (1 + \tilde{\lambda})^{-1} \tilde{E} \cdot \tilde{D} - L . \] (35)

From (29) and (35) it is easy to show that
\[ H = \frac{1}{a^2} (1 + \tilde{\lambda})^{-1} \left[ 1 - \sqrt{1 - \frac{a^2 \tilde{Z}^2}{r^4}} \right] . \] (36)

and equation (34) can be written as
\[ \frac{dm(r)}{dr} = \frac{1}{2} \Lambda r^2 + \frac{4\pi}{a^2} (1 + \tilde{\lambda})^{-1} \left[ r^2 - \sqrt{r^4 - a^2 \tilde{Z}^2} \right] , \] (37)

where \( \tilde{Z}^2 = \tilde{Q}^2 + \tilde{P}^2, \tilde{Q} = (1 + \tilde{\lambda})Q \) and \( \tilde{P} = (1 + \tilde{\lambda})P \). Thus, \( r \geq \sqrt{aZ} \).

To see if the space-time becomes singular \( r \to \sqrt{aZ} \) consider the Ricci tensor which is given by
\[ R = -2 \left[ \frac{rm'' + 2m'}{r^2} \right] . \] (38)

From (37) we have
\[ R = -4\Lambda - \frac{32\pi}{a^2} (1 + \tilde{\lambda})^{-1} \left[ 1 - \frac{r^2 - \frac{a^2 \tilde{Z}^2}{2r^2}}{\sqrt{r^4 - a^2 \tilde{Z}^2}} \right] . \] (39)

Thus, there is a curvature singularity at \( r = \sqrt{aZ} \).

To understand the horizon structure of a charged black hole we need to look for zeros of \( 1 - 2m(r)/r \). From (37) we see that
\[ \frac{2m(r)}{r} = \frac{2m_0}{r} + \frac{\Lambda}{3} r^2 - \frac{8\pi}{a^2r} (a\tilde{Z})^{3/2} (1 + \tilde{\lambda})^{-1} \int_{\sqrt{aZ}}^{\infty} \left[ u^2 - \sqrt{u^4 - 1} \right] du , \] (40)

where \( m_0 \) is a constant. The second term is the usual cosmological constant term and \( m_0 \) is the mass of the black hole. The last term in this expression drops off as \( 1/r^2 \) at large \( r \) and is related to the electric and magnetic charges of the black hole. In fact, for \( r \gg \sqrt{aZ} \)
\[ \frac{8\pi}{a^2r} (a\tilde{Z})^{3/2} (1 + \tilde{\lambda})^{-1} \int_{\sqrt{aZ}}^{\infty} \left[ u^2 - \sqrt{u^4 - 1} \right] du \approx \frac{4\pi (1 + \tilde{\lambda})^{-1} (\tilde{Q}^2 + \tilde{P}^2)}{r^2} . \] (41)
Thus, the electric and magnetic charges of the black hole are given by

\[ Q_{BH} = \sqrt{1 + \tilde{\lambda}} \, Q \quad \text{and} \quad P_{BH} = \sqrt{1 + \tilde{\lambda}} \, P . \]  

(42)

The zeros of \( g_{tt} \) can be found by solving the equation

\[ h(r) = r - 2m_0 + \frac{8\pi}{a^2} (aZ)^{3/2} \int_{\sqrt{aZ}}^{\infty} \left[ u^2 - \sqrt{u^4 - 1} \right] du = 0 , \]  

(43)

where I have taken \( \Lambda = 0 \) for simplicity. First find the critical points of \( h(r) \). The derivative of \( h(r) \) is given by

\[ h'(r) = 1 - \frac{8\pi}{a^2} \left[ r^2 - \sqrt{r^4 - a^2Z^2} \right] . \]  

(44)

The solution to \( h'(r) = 0 \) for \( r > \sqrt{aZ} \) is given by

\[ r = \sqrt{\frac{a^2 + 64\pi^2Z^2}{16\pi}} \quad \text{if} \quad Z > \frac{a}{8\pi} . \]  

(45)

If \( Z < a/8\pi \) there is no solution. Thus, if \( Z > a/8\pi \) the function \( h(r) \) has one local minimum (one can show that \( h'' > 0 \) at the critical point) and if \( Z < a/8\pi \) it has no local maxima or minima. Note that as \( a \to 0 \) we obtain the Reissner-Nordstrom result \( r = \sqrt{4\pi Z} \) for the minimum of \( r g_{tt} \). Next consider \( h(r) \) evaluated at the endpoints of the interval it is defined on:

\[ h(\sqrt{aZ}) = \sqrt{aZ} - 2m_0 + \frac{8\pi}{a^2} (aZ)^{3/2} \int_{1}^{\infty} \left[ u^2 - \sqrt{u^4 - 1} \right] du \]  

(46)

and

\[ h(r) \to \infty \quad \text{as} \quad r \to \infty . \]  

(47)

Now consider fixing the electric and magnetic charges of the black hole and letting its mass vary. If \( m_0 \) is large enough \( h(\sqrt{aZ}) \) will be negative and there will be only one horizon. In this case the singularity will be spacelike. If \( m_0 \) is sufficiently small, so that \( h(\sqrt{aZ}) \geq 0 \), there can be zero, one or two horizons and the singularity will be timelike in all three cases. If \( h(\sqrt{aZ}) = 0 \) the singularity is null and there will be either zero or one horizon outside the singularity. It is interesting to note that this horizon structure is the same as in the Reissner-Nordstrom space-time if we look for horizons outside some fixed coordinate radius \( r_0 \). If \( r_0 \) is a timelike coordinate in the Reissner-Nordstrom space-time the singularity in the Born-Infeld-Einstein space-time is spacelike and if \( r_0 \) is timelike the singularity is spacelike.

This implies that charged black holes have either two horizons and a timelike singularity or one horizon with a spacelike, timelike, or null singularity.
Cosmological space-times

If \( m(r) > \frac{1}{2}r \) for \( r > \sqrt{aZ} \), then \( r \) is a timelike coordinate and \( t \) is a spacelike coordinate [14]. Relabeling \( r \) and \( t \) and denoting the spacelike variable by \( x \) gives

\[
ds^2 = -\left[\frac{2m(t)}{t} - 1\right]^{-1}dt^2 + \left[\frac{2m(t)}{t} - 1\right]dx^2 + t^2d\Omega^2,
\]

(48)

\[
D = \frac{Q}{t^2} dx \wedge dt \quad \text{and} \quad B = P \sin \theta \, d\theta \wedge d\phi,
\]

(49)

\[
\frac{dm(t)}{dt} = \frac{1}{2} \Lambda t^2 + 4\pi t^2 H(t),
\]

(50)

and

\[
H(t) = \frac{1}{a^2}(1 + \Lambda)^{-1}\left[1 - \sqrt{1 - \frac{a^2Z^2}{t^4}}\right].
\]

(51)

Note that constant timelike surfaces have the topology \( R \times S^2 \) and the two sphere has radius \( t \). The condition \( \frac{2m(t)}{t} - 1 > 0 \) requires that

\[
\frac{\alpha}{t} + \frac{\Lambda}{3} t^2 - 1 + \frac{8\pi}{a^2}(a\tilde{Z})^{3/2}(1 + \tilde{\lambda})^{-1}\int_1^{\sqrt{a\tilde{Z}}} [u^2 - \sqrt{u^4 - 1}] \, du > 0,
\]

(52)

where \( \alpha \) is a constant. Since the integral is positive (\( \tilde{\lambda} > -1 \)) for all \( t > \sqrt{a\tilde{Z}} \) the inequality will be satisfied if \( \alpha > \frac{2}{3} \Lambda^{-1/2} \), with \( \Lambda > 0 \). This condition ensures that the first three terms in (52) are greater than zero on \( t \in (0, \infty) \). Thus, this condition is sufficient but not necessary. By computing the Ricci tensor, as was done in the previous section, it is easy to show that the space-time is singular at \( t = \sqrt{a\tilde{Z}} \).

Note that as \( t \to \sqrt{a\tilde{Z}} \) the radius of the two sphere approaches \( \sqrt{a\tilde{Z}} \) and

\[
g_{xx} \to \frac{\alpha}{\sqrt{a\tilde{Z}}} + \frac{1}{3}a\Lambda\tilde{Z} - 1.
\]

(53)

Thus, all the metric components generally are nonzero and finite as the initial singularity is approached. At large \( t \) the cosmological constant term will dominate and the Universe will approach a de Sitter space-time.

Conclusion

In this paper I examined black hole and cosmological space-times in Born-Infeld-Einstein theory with electric and magnetic charges. It was shown that the field equations can be written in the form \( G_{\mu\nu} = -\kappa T_{\mu\nu} \) for spherically symmetric space-times. The energy-momentum tensor is not the Born-Infeld energy-momentum tensor, but can be obtained from Born-Infeld theory by letting \( a \to ia \), where \( a \) is the Born-Infeld parameter. It
was shown spherically symmetric space-times have a curvature singularity at a nonzero radial coordinate and that, as in the Reissner-Nordstrom space-time, there are zero, one or two horizons. It was also shown that charged black holes have either two horizons and a timelike singularity or one horizon with a spacelike, timelike, or null singularity. Anisotropic cosmological solutions with electric and magnetic fields were obtained from the spherically symmetric solutions. These cosmological space-times have topology $R \times S^2$ and as the initial singularity is approached all the metric components generally remain finite and nonzero.

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