Dynamical breaking of shift-symmetry in supergravity-based inflation

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Shift-symmetry is essential to protect the flatness of the potential, even beyond the super-Planckian vacuum expectation value (VEV) for an inflaton field. The breaking of the shift-symmetry can yield potentials suitable for super-Planckian excursion of the inflaton. The aim of this paper is to illustrate that it is indeed possible to break the shift-symmetry dynamically within 4 dimensional supergravity prior to a long phase of inflation. Thanks to the shift-symmetry, the leading contribution to the inflaton potential is free from the dangerous exponential factor even after its breaking, which is the main obstacle to realizing the super-Planckian inflation in supergravity. But, in our simple model, the resulting inflaton potential is a cosine type potential rather than the power-law one and it is difficult to realize a super-Planckian breaking scale unfortunately.

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I. INTRODUCTION

The observations of the cosmic microwave background (CMB) temperature anisotropies1,2 now strongly support the occurrence of primordial inflation3 in the early Universe. The observed temperature anisotropy can be well fitted by the primordial perturbations generated during inflation and the anti-correlation of the temperature (T) and E-mode polarization at large angular scale suggests that the primordial perturbations have been stretched on superhorizon scales1,2. In addition, very recently, BICEP2 reported the detection of the primordial tensor perturbations through the B-mode polarization as4

\[
r = 0.20^{+0.07}_{-0.05} \quad (68\% \text{ CL}),
\]

where \(r\) is the tensor-scalar ratio. To explain this large tensor-to-scalar ratio is challenging for cosmology and particle physics because of the Lyth bound5, one would expect a super-Planckian excursion of the inflaton field in order to generate large \(r\). Of course, the current data can also be explained by the sub-Planckian excursion of the inflaton field3,6, or via assisted inflation8 with many copies of the inflaton field, where the field displacement \(\Delta \phi \simeq 0.1 M_p \leq M_p\), where \(\phi\) is the inflaton and \(M_p \simeq 2.44 \times 10^{18} \text{ GeV}\), but here in this paper we are interested in studying the opposite limit, when \(\Delta \phi > M_p\).

Generally speaking, the super-Planckian excursion of the inflaton is problematic from the effective field theory (EFT) point of view of particle physics and string theory10. In particular, within string theory there are many scales, the string scale, \(M_s\), the compactification scale, \(M_c\) and the derived 4 dimensional Planck scale, with a spectrum, \(M_s \leq M_c \leq M_p\). Beyond \(M_s\) there are quantum corrections not only to the inflaton potential but also to the inflaton kinetic term which can lead to various complications, see11. One would require a full non-perturbative completion of gravity, which we lack sorely within string theory as well. Even if we assume that we have only one fundamental scale, such as \(M_p\), there are many issues pertaining to the validity of an EFT when the field’s VEV goes beyond \(M_p\).

In principle, a gauge singlet inflaton can couple to many degrees of freedom, including the Standard Model and the hidden sector degrees of freedom, see12. Typically, the individual inflaton’s couplings to matter has to be smaller than \(10^{-3}\) to maintain the flatness of the inflaton potential and also to match the density perturbations created during inflation. Of course there is no fundamental justification to make such couplings smaller other than matching the current constraints arising from the CMB.

Furthermore, there are higher derivative corrections to the inflaton kinetic term, see11, if we do not take all infinite higher derivative terms into consideration, there are potential problems with ghosts and quantum instability during inflation. One cannot ignore the higher derivative terms, because a priori one does not know what should be the inflaton’s kinetic energy, i.e. the inflaton need not slow rolling throughout the phase of inflation11.

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In spite of all these challenges, we wish to ask the question - whether can we explain at least a such small inflaton couplings to matter, and large inflaton’s VEV during inflation within an EFT approach by invoking some symmetry such as *shift-symmetry*. Within EFT, one has to ensure that the inflaton’s and all other field’s kinetic terms are small, and here we simply assume so in some patch of the Universe just to be within the EFT regime \[13\], though this still relies on an anthropic arguments.

In principle, one could imagine a *shift-symmetry* as a fundamental symmetry of nature, which would forbid masses and couplings to an inflaton field. Such a *shift-symmetry* has been for the first time introduced in the context of chaotic inflation in supergravity (SUGRA) \[14,15\]. However, based on the same token if *shift-symmetry* remains unbroken inflation would never occur in our patch of the universe. The *shift-symmetry* has to be broken, but in such a way that the breaking remains *soft*, which could be understood via some dynamics of the fields. A hard breaking can be introduced \[14,15\], but the predictions can be lost or one has to resort to some anthropic arguments.

The purpose of this paper is to illustrate a concrete model of dynamical shift symmetry breaking. Our model is described within 4 dimensional $\mathcal{N} = 1$ SUGRA setup and the effective inflaton potential results in cosine type potential \[16\] without the dangerous exponential factor. Unfortunately, in our simple model, it is difficult to realize the super-Planckian breaking scale like natural inflation. ¹

The organization of the paper is as follows. In the next section, we will introduce *shift-symmetry*, then we will construct a simple scenario of dynamically breaking of the *shift-symmetry* in SUGRA and explain how it works. In the final section, we will give our conclusions and discussions.

## II. A BRIEF DISCUSSION ON *SHIFT-SYMMETRY*

Let us explain how the *shift-symmetry* allows the super-Planckian variation of the inflaton field. Note that this argument is not confined to a supersymmetric (SUSY) theory but applies to a non-SUSY theory. A *shift-symmetry* is characterized by a symmetry under the following transformation of a (real) inflaton field $\phi$,  
\[
\phi \to \phi + c \quad (c : \text{real constant}).
\]  
As long as this symmetry is exact, the potential of the inflaton is completely flat and any field variation even beyond the reduced Planck scale $M_p$ is allowed. This is an essential idea. However, inflation must end to reheat our Universe, then the *shift-symmetry* must be broken to generate the gradient of the potential.

As far as we know, in all of the models considered so far, the *shift-symmetry* is broken simply by hand or by introducing an auxiliary field, spurion field, with no kinetic term, whose non-zero VEV is given by hand. For example, in SUGRA models, it is often assumed that the Kähler potential respects the *shift-symmetry* while the superpotential breaks the *shift-symmetry*. In such a case, any kind of superpotential can appear because there is no founding principle behind the breaking of *shift-symmetry*. The introduction of a spurion field might cure such ambiguity because the original action before giving a non-zero VEV to the spurion field respects the *shift-symmetry* in this approach. Then, the interactions, or the forms of the Kähler potential and the superpotential, can be constrained. For example, let us introduce a spurion field $S$ and extend the *shift-symmetry* as \[15\]
\[
\phi \to \phi + c \quad (c : \text{real constant}),
\]
\[
S \to S \frac{\phi}{\phi + c}.
\]  
Then, the combination $S\phi$ is invariant under the *shift-symmetry*. Once this spurion field $S$ takes a non-zero VEV, i.e. $\langle S \rangle = m$, the *shift-symmetry* is broken and the potential is generated. The key points are, that

- the inflaton field $\phi$ always appears in the combination: $\langle S \rangle \phi = m \phi$, where $m \ll M_p$. As long as $m \phi \ll M_p^2$, the EFT treatment is still justified, in spite of the fact that the cutoff scale of the inflaton is now raised to $M_p^2/m$.

- no super-Planckian masses of fermions and bosons appear, because any interactions of the inflaton including Yukawa and four-point interactions are suppressed by the small scale $m \ll M_p$.

- if we take the $m \to 0$ limit, the *shift-symmetry* is restored. In this sense, this model is technically natural. Thus, chaotic inflation can be naturally realized in this setup and the model given in Refs. \[14,17\] is a concrete realization in the context of SUGRA.

¹ See Refs. \[17,18\] for recent works on natural inflation. Also see e.g. Refs. \[12,19\] for other inflation models in supergravity.
However, even in this setup, the non-zero VEV of the field $S$, i.e. the breaking of the *shift-symmetry* has been introduced by hand unfortunately, by assuming that it is a spurion field. Needless to say, it is better to break the *shift-symmetry* dynamically because, otherwise, we cannot control the whole dynamics of the system, or evaluate the effects of the *shift-symmetry* breaking adequately. In this paper, we address this issue and propose a concrete model of the dynamical breaking of the shift symmetry in SUGRA.

### III. DYNAMICAL BREAKING OF *SHIFT-SYMMETRY*.

In this section, we are going to construct a concrete model of dynamical breaking of the *shift-symmetry* in $\mathcal{N} = 1$ SUGRA. The key observation is that the following superpotential,

$$ W = e^{a\Phi} $$

is invariant (up to a constant phase) under the *shift-symmetry*,

$$ \Phi \rightarrow \Phi + \frac{C}{a}, $$

where $a$ and $C$ are real constants. In fact, the scalar potential in the global SUSY limit is given by

$$ V(\Phi) = a^2 e^{a(\Phi + \Phi^*)}, $$

which depends only on the real part of $\Phi$. Thus, the *shift-symmetry* on the imaginary part of $\Phi$ remains. On the other hand, the following superpotential,

$$ W = e^{a\Phi} + e^{-a\Phi} $$

is *not* invariant under the *shift-symmetry*. In fact, the scalar potential in the global SUSY limit is given by

$$ V(\phi, \chi) = a^2 \left[ e^{\sqrt{2a}\chi} + e^{-\sqrt{2a}\chi} - 2 \cos\left(\sqrt{2a}\phi\right) \right], $$

where

$$ \Phi = \frac{1}{\sqrt{2}} (\chi + i\phi). $$

It should be noticed that the scalar potential depends not only on the real part of $\Phi$, i.e. $\chi$, but also on the imaginary part of $\Phi$, i.e. $\phi$. In order to recover the *shift-symmetry* for the second type of the superpotential, see Eq. (8), we need to introduce a pair of superfields, $S$ and $\tilde{S}$, and another superfield $X$. Let us now consider the following superpotential,

$$ W_I = v \left( S^n e^{a\Phi} + \tilde{S}^n e^{-a\Phi} \right) X, $$

where $v \ll 1$ is a constant (in Planck units), and $n$ is a positive integer number. This superpotential is invariant under the following *shift-symmetry*,

$$ \begin{align*}
\Phi & \rightarrow \Phi + \frac{nC}{a}, \\
S & \rightarrow Se^{-iC}, \\
\tilde{S} & \rightarrow \tilde{S}e^{iC}, \\
X & \rightarrow X.
\end{align*} $$

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2 From here onwards, we denote the scalar components of the superfields by the same symbols as the corresponding superfields.

3 The superfield $X$ is not necessary only for recovery of the *shift-symmetry*. It is also useful to guarantee the positivity of the potential during inflation [14, 15, 20].
However, in order to realize inflation, this shift-symmetry must be broken. For this purpose, we introduce another superfield $T$ and add the following superpotential,

$$W_B = \lambda T \left( S\bar{S} - \mu^2 \right),$$  \hspace{1cm} (13)

where $\lambda \leq \mathcal{O}(1)$ and $\mu \leq \mathcal{O}(1)$ are constants. Then, the total superpotential, which is given by:

$$W = W_I + W_B,$$  \hspace{1cm} (14)

is invariant under the shift-symmetry, Eq. (12), along with

$$T \rightarrow T.$$  \hspace{1cm} (15)

One can easily understand that, once the scalar components of the superfields $\langle S \rangle \neq 0$ and $\langle \bar{S} \rangle \neq 0$, acquire non-zero VEVs, the shift-symmetry is broken dynamically. Let us consider the following Kähler potential of the type \footnote{The linear term of $\Phi + \Phi^*$ can appear in the Kähler potential because of the absence of the $Z_2$ symmetry. Such an effect causes two effects. First one is additional contribution to the D-term. Second one is a slight deviation of the minimum of $\chi$ field during inflation from the global minimum. This deviation is still compatible with the D-flat condition because its deviation exactly cancels out the additional contribution to the D-term. So, the essential dynamics remains unchanged and we omit it for simplicity.}:

$$K = \frac{1}{2} (\Phi + \Phi^*)^2 + \left| S \right|^2 + \left| \bar{S} \right|^2 + \left| T \right|^2 + \left| X \right|^2,$$  \hspace{1cm} (16)

which is invariant under the shift-symmetry, Eqs. (12) and (13), and generates the canonical kinetic terms for all of the fields. Note that shift-symmetry will also allow higher order terms, such as $(\Phi + \Phi^*)^4$, $(\Phi + \Phi^*)^6$, $|S|^4$, $|\bar{S}|^4$, $|T|^4$, $|X|^4$, $|S_n e^{\alpha \Phi} + \bar{S}_n e^{-\alpha \Phi}|$, etc., where ... contain higher order terms to all infinite orders. These terms will give corrections to the canonical kinetic terms. But, as long as $(\Phi + \Phi^*), |S|, |\bar{S}|, \cdots \ll 1$, which can be realized dynamically in our model, these corrections are negligible. Of course, at initial period, we assume the presence of at least one patch of the Universe, in which the kinetic energies of all of the fields are smaller than the Planck energy density and subdominant.

The Higher derivative terms like $D_{\nu} \Phi D^{\nu} \Phi^*$ in the Kähler potential are also allowed from our symmetry, see \footnote{The linear term of $\Phi + \Phi^*$ can appear in the Kähler potential because of the absence of the $Z_2$ symmetry. Such an effect causes two effects. First one is additional contribution to the D-term. Second one is a slight deviation of the minimum of $\chi$ field during inflation from the global minimum. This deviation is still compatible with the D-flat condition because its deviation exactly cancels out the additional contribution to the D-term. So, the essential dynamics remains unchanged and we omit it for simplicity.}. Unless these higher order terms are suppressed by $(\Phi + \Phi^*)^2$ for example, the derivative expansion may not be justified because of the super-Planckian value of $\phi$ (see again Ref. \footnote{The linear term of $\Phi + \Phi^*$ can appear in the Kähler potential because of the absence of the $Z_2$ symmetry. Such an effect causes two effects. First one is additional contribution to the D-term. Second one is a slight deviation of the minimum of $\chi$ field during inflation from the global minimum. This deviation is still compatible with the D-flat condition because its deviation exactly cancels out the additional contribution to the D-term. So, the essential dynamics remains unchanged and we omit it for simplicity.}). One would need to take all infinite higher derivative corrections in order to avoid ghosts and instability of the vacuum \footnote{The linear term of $\Phi + \Phi^*$ can appear in the Kähler potential because of the absence of the $Z_2$ symmetry. Such an effect causes two effects. First one is additional contribution to the D-term. Second one is a slight deviation of the minimum of $\chi$ field during inflation from the global minimum. This deviation is still compatible with the D-flat condition because its deviation exactly cancels out the additional contribution to the D-term. So, the essential dynamics remains unchanged and we omit it for simplicity.}. This would require a complete ultraviolet completion of inflaton and gravitational sector, which we do not aim to address in this paper. Instead, we make an assumption that the inflationary patch is always within an EFT regime.

Further note that the present model possesses $U(1)_R$ symmetry, under which

$$\Phi(\theta) \rightarrow \Phi(\theta e^{i\alpha}),$$
$$S(\theta) \rightarrow S(\theta e^{i\alpha}),$$
$$\bar{S}(\theta) \rightarrow \bar{S}(\theta e^{i\alpha}),$$
$$X(\theta) \rightarrow e^{-2i\alpha} X(\theta e^{i\alpha}),$$
$$T(\theta) \rightarrow e^{-2i\alpha} T(\theta e^{i\alpha}).$$  \hspace{1cm} (17)

The scalar potential in $\mathcal{N} = 1$ SUGRA is given by

$$V = e^K \left[ v_0 \left( S^n e^{\alpha \Phi} - \bar{S}_n e^{-\alpha \Phi} \right) X + (\Phi + \Phi^*) W \right]^2 + n v S^{n-1} e^{\alpha \Phi} X + \lambda T \bar{S} + S^* W \right|^2 \hspace{1cm} (18)$$

$$+ n v S^{n-1} e^{-\alpha \Phi} X + \lambda T S + \bar{S}^* W \right|^2 + \left| \lambda S S - \mu^2 \right|^2 + \left| T^* W \right|^2 + V_D.$$
From here onwards we set $M_p = 1$. In the above potential, $V_D$, represents the D-term contribution, which is given by

$$V_D = \frac{e^2}{2} \left( |S|^2 - |S|^2 + \frac{n}{a}(\Phi + \Phi^*) \right)^2,$$

with $e$ being a gauge coupling constant. Such a term can be present if the shift-symmetry is gauged by changing the constant parameter $C$ to a spacetime dependent one $C(x)$.

### IV. INFLATIONARY POTENTIAL

Now, let us take a closer look at the dynamics of this system. First, let us assume that the energy scale of the shift-symmetry breaking sector, i.e. $W_B$, is much higher than that of the inflation sector, i.e. $W_I$, which requires:

$$\lambda^2 \mu^4 \gg v^2 \mu^{2n} \iff \lambda \gg v \mu^{n-2}. \quad (20)$$

Under this assumption, the potential energy is roughly given by $V \simeq \lambda^2 \mu^4$ at the onset of inflation, and the Hubble expansion rate: $H^2 \simeq V/3 \simeq \lambda^2 \mu^4/3$. At such higher energies, hybrid-type inflation \[22, 23\] can occur, where $T$ cannot take a value larger than unity (in Planck units) due to the exponential factor $e^K$ in the potential, see Eq. (18). Then, the mass squared of the field $X$ is estimated to be:

$$m_X^2 \simeq \lambda^2 \mu^4 (1 + |T|^2) \simeq 3H^2 (1 + |T|^2), \quad (21)$$

which dynamically drives the field $X$ to the zero VEV. It can be easily confirmed that, even after this inflation, $m_X^2$ is always positive, so that $X$ stays at the origin for ever. By inserting $X = 0$ to the scalar potential, Eq. (18) yields

$$V|_{X=0} = e^K \left[ \lambda^2 |T|^2 (\Phi + \Phi^*)^2 \left| S\bar{S} - \mu^2 \right|^2 + \lambda^2 |T|^2 \left( |S|^2 (1 + |S|^2) - \mu^2 S^a \right)^2 + \left| S \left( 1 + |S|^2 \right) - \mu^2 \bar{S}^a \right|^2 \right]$$

$$+ \lambda^2 (1 - |T|^2 + |T|^4) \left| S\bar{S} - \mu^2 \right|^2 + v^2 \left| S^a e^{a\Phi} + \bar{S}^a e^{-a\Phi} \right|^2 \right] + \frac{e^2}{2} \left[ \left| S\bar{S} - |S|^2 \right|^2 + \frac{n}{a}(\Phi + \Phi^*) \right]^2, \quad (22)$$

and the mass terms for $S$ and $\bar{S}$ are estimated as

$$m_{S, \bar{S}}^2 \simeq -\lambda^2 \mu^2 \left( |S\bar{S} + S^a \bar{S}^a \right) + \lambda^2 |T|^2 \left( 1 + \mu^4 \right) \left( |S|^2 + |\bar{S}|^2 \right)$$

$$= \lambda^2 \left[ (1 + \mu^4) |T|^2 + \mu^2 \right] |\Psi|^2 + \lambda^2 \left[ (1 + \mu^4) |T|^2 - \mu^2 \right] |\overline{\Psi}|^2, \quad (23)$$

where we have defined

$$\Psi = \frac{1}{\sqrt{2}} \left( S - \bar{S}^* \right), \quad \overline{\Psi} = \frac{1}{\sqrt{2}} \left( S + \bar{S}^* \right), \quad (24)$$

and we have taken $n \geq 2$ in Eq. (11). Since $m_{S, \bar{S}}^2 \gg \lambda^2 \mu^4 \simeq 3H^2$, the $\Psi$ field has a Hubble-induced mass and quickly settles down to the zero VEV within one Hubble time or so, which implies $S = \bar{S}^*$ and $|S| = |\bar{S}|$. This condition is compatible with the D-term flatness condition, $V_D = 0$, along with $\Phi + \Phi^* = 0$, which holds true for almost all periods. At this point, we can discuss the dynamics of the fields for two particular scenarios:

- **$|T| \gtrsim T_c$, dynamically preserving shift-symmetry:**

  As long as the VEV of $T$ is such that $|T| \gtrsim T_c \simeq \mu$, or, $m_{\Psi}^2 \gg \lambda^2 \mu^4 \simeq 3H^2$, which also leads dynamically to $\overline{\Psi} = 0$. Therefore, for $|T| \gtrsim T_c$, $S$ and $\bar{S}$ stay at the origin and the potential $V$ is dominated by $\lambda^2 \mu^4$, leading to the hybrid inflation \[22, 23\].

  The SUGRA effects and the one-loop potential coming from the SUSY breaking effects could drive the inflaton field $T$ like in the case of standard hybrid inflation. It should be noticed that, during this inflation, the effective mass squared of the real part of $\Phi$, $\chi$, is approximately $3H^2$. Therefore, $\chi = (\Phi + \Phi^*)/\sqrt{2}$ quickly rolls down to its minimum, that is, the zero as well. On the other hand, the imaginary part of $\Phi$, $\phi$, is still arbitrary. That is, the shift-symmetry which is preserved at this stage.
• \( |T| \lesssim T_c \), dynamically breaking shift-symmetry.

In this case the effective mass squared \( m_T^2 < 0 \), with its magnitude is larger than the Hubble parameter squared, the \( \Psi \) field becomes unstable so that the fields \( S \) and \( \tilde{S} \) quickly roll down to the minimum of the potential with \( SS = \mu^2 \) and \( |S| = |\tilde{S}| \) together with \( \Phi + \Phi^* = 0 \), which can be parametrized as

\[
S = \mu e^{i\beta}, \quad \tilde{S} = \mu e^{-i\beta}
\]

with \( \beta \) being a real constant. Thus, the fields \( S \) and \( \tilde{S} \) acquire the non-zero VEVs, which dynamically breaks the shift-symmetry.

Further note that, for \( SS \approx \mu^2 \), the effective mass squared of \( T \), \( m_T^2 \), is estimated as

\[
m_T^2 \approx 2\lambda^2 \mu^2,
\]

which mainly comes from the second and third terms in the right hand side of the first line in Eq. (22). Thus, after the end of hybrid inflation, \( T \) quickly settles down to its minimum, i.e. \( \langle T \rangle = 0 \). Then, the effective scalar potential

\[
V|_{X=T=0} = e^K \left[ \lambda^2 \left| SS - \mu^2 \right|^2 + v^2 \left| S_n e^{i\Phi} + \tilde{S}_n e^{-i\Phi} \right|^2 \right] + \frac{\epsilon^2}{2} \left( \left| SS \right|^2 - |S|^2 + \frac{n}{a} (\Phi + \Phi^*) \right)^2
\]

with \( K = \chi^2 + |S|^2 + |\tilde{S}|^2 \). It is manifest that this effective potential is positive definite and its global minimum is given by the conditions

\[
SS - \mu^2 = 0, \\
S_n e^{i\Phi} + \tilde{S}_n e^{-i\Phi} = 0, \\
|S|^2 - |\tilde{S}|^2 + \frac{n}{a} (\Phi + \Phi^*) = 0.
\]

These conditions lead to the global minimum for the fields, as

\[
S_{\text{min}} = \mu e^{i\beta}, \quad \tilde{S}_{\text{min}} = \mu e^{-i\beta}, \\
\chi_{\text{min}} = 0, \\
\phi_{\text{min}} = -\frac{\sqrt{2n\beta}}{a} + \frac{(2m-1)\pi}{2\sqrt{a}},
\]

where \( m \) being an integer number.

However, when hybrid inflation ends and the shift-symmetry is broken with \( SS = \mu^2 \), the imaginary part of \( \Phi \) does not necessarily stay at the minimum, because before the breaking of the shift-symmetry all the values of the imaginary part of \( \Phi, \phi \), are equally distributed, thanks to the shift-symmetry. Thus, the initial condition of \( \phi \) is determined accidentally. The effective potential is given by

\[
V_{\text{eff}} = e^{K} v^2 \left| S_{\text{min}}^n e^{i\Phi} + \tilde{S}_{\text{min}}^n e^{-i\Phi} \right|^2, \\
= e^{\chi^2 + 2\mu^2 + n^2 \mu^2} \left[ e^{\sqrt{2}\chi/M} + e^{-\sqrt{2}\chi/M} + 2 \cos \left( 2n\beta + \frac{\sqrt{2}\phi}{M} \right) \right],
\]

with \( M = 1/a \). Here, let us identify the inflaton and the Nambu-Goldstone (NG) boson correctly, which are given by

\[
\phi_{\text{inf}} = \phi + \frac{nM}{\mu} \beta_c, \\
\phi_{\text{NG}} = \phi - \frac{nM}{\mu} \beta_c.
\]
with $\beta_c \equiv \sqrt{2}\mu\beta$. Then, the covariant kinetic terms are given by

$$
\frac{1}{2} (D_\mu \phi)^2 + \frac{1}{2} (D_\mu \beta_c)^2
= \frac{1}{2} \left( \frac{1}{4} \left( 1 + \frac{\mu^2}{n^2 M^2} \right) \right) \left\{ (\partial_\mu \phi_{\text{inf}})^2 + (\partial_\mu \phi_{\text{NG}})^2 \right\} + \left( 1 - \frac{\mu}{n M} \right) \partial_\mu \phi_{\text{inf}} \partial_\mu \phi_{\text{NG}}
+ \sqrt{2} n M A_\mu \left\{ \left( 1 - \frac{\mu^2}{n^2 M^2} \right) \partial_\mu \phi_{\text{inf}} + \left( 1 + \frac{\mu^2}{n^2 M^2} \right) \partial_\mu \phi_{\text{NG}} \right\} + 2(n^2 M^2 + \mu^2) A_\mu A^\mu .
$$

(37)

where

$$
D_\mu \phi \equiv \partial_\mu \phi + \sqrt{2} n M A_\mu , \quad D_\mu \beta_c \equiv \partial_\mu \beta_c - \sqrt{2} \mu A_\mu ,
$$

(38)

with $A_\mu$ being the gauge field. The NG boson $\phi_{\text{NG}}$ is eaten by the gauge field, so the remaining kinetic terms in the unitary gauge become

$$
\frac{1}{2} \left[ \frac{1}{4} \left( 1 + \frac{\mu^2}{n^2 M^2} \right) (\partial_\mu \phi_{\text{inf}})^2 + \sqrt{2} n M \left( 1 - \frac{\mu^2}{n^2 M^2} \right) A_\mu \partial_\mu \phi_{\text{inf}} + 2(n^2 M^2 + \mu^2) A_\mu A^\mu \right]
= \frac{1}{2} \left[ \frac{1}{4} \frac{n M - \frac{\mu^2}{n M}}{\sqrt{2}(\mu^2 + n^2 M^2)} \partial_\mu \phi_{\text{inf}} \right] + (\mu^2 + n^2 M^2) \tilde{A}_\mu \tilde{A}^\mu
= \frac{1}{2} \left( \partial_\mu \tilde{\phi}_{\text{inf}} \right)^2 + (\mu^2 + n^2 M^2) \tilde{A}_\mu \tilde{A}^\mu ,
$$

(39)

where

$$
\tilde{A}_\mu \equiv A_\mu + \frac{n M - \frac{\mu^2}{n M}}{\sqrt{2}(\mu^2 + n^2 M^2)} \partial_\mu \phi_{\text{inf}},
$$

(40)

$$
\tilde{\phi}_{\text{inf}} \equiv \sqrt{ \frac{1}{1 + \frac{n^2 M^2}{\mu^2}} },
$$

(41)

Thus, the effective potential for the canonically normalized inflaton $\tilde{\phi}_{\text{inf}}$ is given by

$$
V_{\text{eff}}(\tilde{\phi}_{\text{inf}}) = 2e^{2\mu^2} \cdot v^2 \mu^{2n} \cos \left( \sqrt{\frac{2}{M}} \sqrt{1 + \frac{n^2 M^2}{\mu^2}} \tilde{\phi}_{\text{inf}} \right),
$$

(42)

where the decay constant $f$ is given by

$$
f = \frac{M}{\sqrt{2}} \frac{1}{\sqrt{1 + \frac{n^2 M^2}{\mu^2}}} \rightarrow \frac{\mu}{\sqrt{2} n} \quad \text{for} \quad n M \gg \mu .
$$

(43)

Thus, since $n$ is an integer number and larger than unity in this simple example, the decay constant $f$ cannot be super-Planckian scale as long as $\mu$ is sub-Planckian scale. So, inflation becomes hilltop type one.

In order to reheat the Universe after inflation, we introduce the following superpotential,

$$
W_R = y S^n e^{\phi} NN,
$$

(44)

where $y$ is a (Yukawa) coupling constant and $N$ is the right-handed neutrino superfield. This superpotential with the canonical Kähler potential for $N$ is manifestly invariant under the shift-symmetry Eqs. (12), (15), and $N \rightarrow N$. Once $S$ acquires the non-zero VEV, this superpotential leads to a Yukawa coupling between the inflaton $\phi$ and the right-handed neutrino $N$. Therefore, the leptogenesis through the inflaton decay and the reheating of the Universe through the decay of the right handed neutrino to the standard particles are possible by tuning the parameters adequately.

V. CONCLUSIONS AND DISCUSSIONS

In this paper, we constructed a concrete example of the dynamical breaking of the shift-symmetry in SUGRA. By taking the exponential type of the superpotential for $\Phi$, which might appear through some non-perturbative effects,
we first consider the superpotential invariant under the shift-symmetry. Then, by arranging the GUT Higgs-like superpotential as well, the shift-symmetry is dynamically broken. The inflaton has no dangerous exponential factor at the leading order in the scalar potential even after the shift symmetry breaking. Such an exponential factor is the main obstacle to realizing super-Planckian inflation in supergravity. Unfortunately, in our simple model, the potential obtained for the inflaton is a cosine type potential rather than power-law one and it is difficult to realize a super-Planckian decay constant. One possible way to obtain the super-Planckian decay constant with $\mu$ being the sub-Planckian scale is to make $n$ smaller than unity. Of course, $n$ is an integer number and larger than unity in this simple example. However, for example, if we start from the higher order Kähler potential for $S$ like $|S|^{2n}$ ($\langle S \rangle^{2n}$) instead of the canonical Kähler potential (with $\langle S \rangle$ replaced by $\langle S \rangle^{n}$ in the superpotential $W_B$ at the same time), then such a model becomes equivalent to our simple model with the effective $n_{\text{eff}} = n/m$ by field redefining $S' = S^{m} (\langle S \rangle = S^{m})$. Thus, if we take $n_{\text{eff}} \lesssim \mu/M_p \lesssim 1$, the decay constant $f \gtrsim M_p$, which may realize super-Planckian inflation like natural inflation. We leave more realistic realization of natural and chaotic inflation as a future work.

We have restricted ourselves within the regime of EFT, where the fields have masses and energy densities below the cut-off in spite of the fact that the inflaton VEV could be large and above the Planck scale. We have also pointed out that it is possible to generate small inflaton couplings to matter in order to avoid some of the quantum corrections to the inflaton potential [11]. In this paper we have explicitly assumed that all the fields are slow rolling initially in some patch of the Universe, such that the kinetic energy is indeed sub-dominant to be well within the regime of EFT.

Note added. While we finalized the paper, Ref. [18] appeared, in which a similar breaking of shift symmetry is given, though the model is rather different and more complicated. We thank the authors of [18] for noticing us that fact.

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