Inflaton and metric fluctuations in the early universe from a 5D vacuum state

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Abstract

In this letter we complete a previously introduced formalism to study the gauge-invariant metric fluctuations from a noncompact Kaluza-Klein theory of gravity, to study the evolution of the early universe. The evolution of both, metric and inflaton field fluctuations are reciprocally related. We obtain that $\langle \delta \rho \rangle / \rho_0$ depends on the coupling of $\Phi$ with $\delta \varphi$ and the spectral index of its spectrum is $0.9483 < n_1 < 1$.

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I. INTRODUCTION AND MOTIVATION

The key property of the laws of physics that makes inflation possible is the existence of states of matter that have a high-energy density which cannot be rapidly lowered. In the original version of the inflationary theory [1], the proposed state was a scalar field in a local minimum of its potential energy function. A similar proposal was advanced by Starobinsky [2], in which the high-energy density state was achieved by curved space corrections to the energy-momentum tensor of a scalar field. The scalar field state employed in the original version of inflation is called a false vacuum, since the state temporarily acts as if it were the state of lowest possible energy density. Chaotic inflation is driven by a scalar field called inflaton, which in its standard version, is related to a potential with a local minimum or a gently plateau [3]. In this relativistic theory, cosmological perturbations are a cornerstone in our understanding of the early universe and are indispensable in relating early universe scenarios [4].

Higher dimensional spacetime is now an active field of activity in both general relativity and particle physics in its attempts to unify gravity with all other forces of nature [5]. Higher dimensional unified-field theories include Kaluza-Klein, induced matter, super string

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supergravity and string theory. In these \((4 + d)\)-dimensional models the \(d\)-spacelike dimensions are generally spontaneously compactified and the symmetries of this space appear as gauge symmetries of the effective 4D theory. In higher-dimensional theories of gravity with large extra dimensions, the cylinder condition of the old Kaluza-Klein theory is replaced by the conjecture that the ordinary matter and fields are confined to a 4D subspace usually referred to as “3-brane” [6]. Randall and Sundrum showed, for \(d = 1\), that there is no contradiction between Newton’s \(1/r^2\) law of gravity in 4D and the existence of more than 4 noncompact dimensions if the background metric is nonfactorizable [7]. This has motivated a great interest in brane-world models where our 4D universe is embedded in a 5D noncompact spacetime [8]. On the other hand, another noncompact theory is the so called space-time-matter (STM) theory. In STM the conjecture is that the ordinary matter and fields that we observe in 4D result from the geometry of the extra dimension [9].

This paper is devoted to study a 4D de Sitter expansion of the universe from a non-compact Kaluza-Klein (NKK) theory of gravity, taking into account in a consistent manner both, the gauge-invariant scalar metric and inflaton field fluctuations. It was done, but in a partial manner, in a previous work [10].

II. REVIEW AND EXTENSION OF THE FORMALISM

In the framework of a NKK theory we shall consider an action for a scalar field \(\varphi\), which is minimally coupled to gravity on a 5D manifold

\[
I = -\int d^4x \, d\psi \left( \sqrt{\left|\bar{g}\right|} \left( \frac{\bar{R}}{16\pi G} + L(\varphi, \varphi, A) \right) \right),
\]

where \(G = M_p^{-2}\) is the gravitational constant and \(M_p = 1.2 \times 10^{19} \text{ GeV}\) is the Planckian mass. To describe a manifold in apparent vacuum we shall consider a Lagrangian density \(L\) in (1), which is only kinetic

\[
L(\varphi, \varphi, A) = \frac{1}{2} g^{AB} \partial_\varphi A \varphi B.
\]

Here, \(A, B\) can take the values 0, 1, 2, 3, 4 and the Ricci scalar \((5)\bar{R} = 0\) in (1) is evaluated on the background metric [11]

\[
\left( dS^2 \right)_b = \psi^2 dN^2 - \psi^2 e^{2N} dr^2 - d\psi^2,
\]

which is 3D spatially isotropic, homogeneous and flat [12]. Furthermore, it is globally flat (i.e., \(\bar{R}_{bcd}^A = 0\)). Here, \(N, x, y, z\) are dimensionless and \(\psi\) has spatial units. This background metric describes an apparent vacuum, because \(G_{AB} = 0\). For the metric (3), \(\left|\bar{g}\right| = \psi^8 e^{6N}\) is the absolute value for the determinant of \(\bar{g}_{AB}\) and \(\left|\bar{g}_0\right| = \left|\left(\bar{g}\right)|_{N = N_0, \psi = \psi_0} = \psi_0^8 e^{6N_0}\right|\) is a dimensional constant. In this work we shall consider \(N_0 = 0\), so that \(\left|\bar{g}_0\right| = \psi_0^8\).

In order to describe scalar metric fluctuations we must consider a symmetric energy-momentum tensor. In a longitudinal gauge the perturbed line element is given by [10]

\[
dS^2 = \psi^2 (1 + 2\Phi) dN^2 - \psi^2 (1 - 2\Phi) e^{2N} dr^2 - (1 - 2\Phi) d\psi^2,
\]
where the field $\Phi(N, \vec{r}, \psi)$ is the scalar metric perturbation of the background 5D metric (3).

The contravariant metric tensor, after a $\Phi$-first order approximation, is given by

$$g^{AB} = \text{diag} \left[ \frac{1 - 2\Phi}{\psi^2}, -\frac{1 + 2\Phi}{\psi^2}, -\frac{1 + 2\Phi}{\psi^2}, e^{2N}/\psi^2, -(1 - 2\Phi) \right],$$

which can be written as $g^{AB} = \bar{g}^{AB} + \delta g^{AB}$, being $\bar{g}^{AB}$ the contravariant background metric tensor.

If we use the semiclassical approximation $\varphi(N, \vec{r}, \psi) = \varphi_b(N, \psi) + \delta \varphi(N, \vec{r}, \psi)$, the Lagrange equations (we use a linear approximation on $\delta \varphi$ and $\Phi$) for $\varphi_b$ and $\delta \varphi$ are

$$\frac{\partial^2 \varphi_b}{\partial N^2} + 3 \frac{\partial \varphi_b}{\partial N} = 0,$$

$$\frac{\partial^2 \delta \varphi}{\partial N^2} + 3 \frac{\partial \delta \varphi}{\partial N} - e^{-2N} \nabla^2 \varphi_b - \psi \left[ 4 \frac{\partial \delta \varphi}{\partial \psi} + \psi \frac{\partial^2 \delta \varphi}{\partial \psi^2} \right]$$

$$- 2 \frac{\partial \varphi_b}{\partial N} \frac{\partial \Phi}{\partial N} - 2\psi^2 \left[ \frac{\partial \varphi_b}{\partial \psi} \frac{\partial \Phi}{\partial \psi} + \left( \frac{\partial^2 \varphi_b}{\partial \psi^2} + 4 \frac{\partial \varphi_b}{\partial \psi} \right) \right] = 0,$$

where $\varphi_b$ is the solution of equation of motion for the inflaton field in absence of the inflaton and metric fluctuations [i.e., for $\Phi = \delta \varphi = 0$]. On the other hand, one obtains [10]

$$\left( \frac{\partial \varphi_b}{\partial N} \right)^2 + \psi^2 \left( \frac{\partial \varphi_b}{\partial \psi} \right)^2 = 0.$$

From the diagonal first order Einstein’s equations $\delta G_{AA} = -8\pi G T_{AA}$, and taking into account the eq. (7), we obtain

$$\frac{\partial^2 \Phi}{\partial N^2} + 3 \frac{\partial \Phi}{\partial N} - e^{-2N} \nabla^2 \Phi - 2\psi^2 \frac{\partial^2 \Phi}{\partial \psi^2} = 0.$$

This equation was obtained in [10], and describes the evolution for the 5D scalar metric fluctuations $\Phi(N, \vec{r}, \psi)$.

Once we know $\Phi$ from eq. (8) we must calculate the inflaton field fluctuations from the equation (6). To do it, firstly, we must solve the equation (5) for the background inflaton field $\varphi_b$. If we propose that $\varphi_b(N, \psi) = \varphi_1(N) \varphi_2(\psi)$, we obtain the following equations

$$\frac{1}{\varphi_1} \frac{\partial^2 \varphi_1}{\partial N^2} + \frac{3}{\varphi_1} \frac{\partial \varphi_1}{\partial N} = 4\psi \frac{1}{\varphi_2} \frac{\partial \varphi_2}{\partial \psi} + \psi^2 \frac{1}{\varphi_2} \frac{\partial^2 \varphi_2}{\partial \psi^2} = -M^2,$$

where $M^2$ is a constant of integration. The solution for these equations are

$$\varphi_1(N) = e^{-3N/2} \left[ A_1 e^{\sqrt{-4M^2} \cdot N/2} + A_2 e^{-\sqrt{-4M^2} \cdot N/2} \right],$$

$$\varphi_2(\psi) = \left( \frac{\psi}{\psi_0} \right)^{-3/2} \left[ B_1 \left( \frac{\psi}{\psi_0} \right)^{\sqrt{-4M^2}/2} + B_2 \left( \frac{\psi}{\psi_0} \right)^{-\sqrt{-4M^2}/2} \right],$$

This is the solution for the 5D scalar metric fluctuations $\Phi(N, \vec{r}, \psi)$.
where \((A_1, A_2, B_1, B_2)\) are constants of integration. From the second equation in (9), we obtain

\[
4\psi \frac{\partial \varphi_b}{\partial \psi} + \psi^2 \frac{\partial^2 \varphi_b}{\partial \psi^2} = -M^2 \varphi_b,
\] (12)

which means that the eq. (6) can be written as

\[
\frac{\partial^2 \delta \varphi}{\partial N^2} + 3 \frac{\partial \delta \varphi}{\partial N} - e^{-2N} \nabla_r^2 \delta \varphi + \psi \left[ 4 \frac{\partial \delta \varphi}{\partial \psi} + \psi \frac{\partial^2 \delta \varphi}{\partial \psi^2} \right] = -4M^2 \varphi_b \Phi + 2 \frac{\partial \Phi}{\partial N} \frac{\partial \varphi_b}{\partial N} + 2\psi \frac{\partial \varphi_b}{\partial \psi} \Phi.
\] (13)

This equation is very important because shows how the gauge-invariant metric fluctuations and the inflaton field fluctuations are related. However, as we shall see later, the right hand of this equation could be zero for a constant \(\varphi_b\) and \(M = 0\). This is the case in a effective 4D de Sitter expansion, which is the subject of study in this letter. Using the fact that \(\delta \varphi(N, \vec{r}, \psi) = e^{-3N/2} \left( \frac{\psi}{\Psi} \right)^2 \phi(N, \vec{r})\) and \(\Phi(N, \vec{r}, \psi) = \left( \frac{\psi}{\Psi} \right)^2 \chi(N, \vec{r})\), we obtain that the eq. (13) only is consistent for \(M = 0\). Hence, if we require that \(\varphi_b\) be consistent with (7), the solution for \(\varphi_b\) holds

\[
\varphi_b(N, \psi) = \varphi_b^{(0)},
\] (14)

where \(\varphi_b^{(0)} = A_1B_1 = \varphi_b(N = 0, \psi = \psi_0)\) is a constant. Hence, the equation (13) becomes

\[
\frac{\partial^2 \phi}{\partial N^2} - e^{-2N} \nabla_r^2 \phi - \left( \psi^2 \frac{\partial^2 \phi}{\partial \psi^2} + \frac{1}{4} \phi \right) = 0,
\] (15)

where \(\phi(N, \vec{r})\) can be expanded as

\[
\phi(N, \vec{r}) = \frac{1}{(2\pi)^{3/2}} \int d^3k_r \int d\psi \left[ A_{k_r, \psi} e^{ik_r \cdot \vec{r}} \Sigma_{k_r, \psi}(N) + c.c. \right].
\] (16)

Therefore, the \(N\)-dependent modes \(\Sigma_{k_r, \psi}(N)\) comply with the differential equation

\[
\frac{\partial^2 \Sigma_{k_r, \psi}(N)}{\partial N^2} + \left[ k_r^2 e^{-2N} - \left( \frac{k_r^2 \psi^2}{4} + 1 \right) \right] \Sigma_{k_r, \psi}(N) = 0.
\] (17)

The general solution for the eq. (17) is

\[
\Sigma_{k_r, \psi}(N) = \alpha_1 \mathcal{H}_{\mu}^{(1)}[x(N)] + \alpha_2 \mathcal{H}_{\mu}^{(2)}[x(N)],
\] (18)

where \((\alpha_1, \alpha_2)\) are constants of integration, \(\mu = \sqrt{1 + 4(k_r \psi)^2}/2\) and \(x(N) = k_r e^{-N}\).

To normalize the solution (18), we can take only the homogeneous solution, such that for a generalized Bunch-Davies vacuum [13], we obtain \(\alpha_1 = 0\) and \(\alpha_2 = i \sqrt{\frac{3}{4}}\).

\textbf{III. EFFECTIVE 4D DE SITTER EXPANSION}

In this section we shall study the effective 4D dynamics in a de Sitter expansion of both, the metric and the inflaton field fluctuations, which are reciprocally related.
A. Effective 4D metric

To study the dynamics of the system on an effective 4D de Sitter expansion, we shall consider the transformation

\[ t = \psi_0 N, \quad R = \psi_0 r, \quad \psi = \psi. \]  \hspace{1cm} (19)

With this transformation the 5D background metric (3) becomes

\[ (dS^2)_b \rightarrow (ds^2)_b = dt^2 - e^{2t/\psi_0} dR^2 - d\psi^2. \]  \hspace{1cm} (20)

This effective 4D metric describes a 4D expansion of a 3D spatially flat, isotropic and homogeneous universe that expands with a constant Hubble parameter \( H = 1/\psi_0 \) and a 4D scalar curvature \((^{(4)} R) = 12H^2\). Furthermore, the background energy density is given by

\[ \rho_b = \frac{3H^2}{8\pi G}. \]  \hspace{1cm} (21)

Once we know the modes \( \xi_{kRk\psi_0}(t) = i\sqrt{\frac{\pi}{4H}}H_\mu^{(2)}[y(t)] \) [with \( \mu = \sqrt{\frac{9+16k^2_{\psi_0}/H^2}{2}} \) and \( y(t) = (kR/H)e^{-Ht} \)] [10], we can study the dynamics for the modes \( \Sigma_{kRk\psi_0}(t) \) for a de Sitter expansion. If we take into account the transformations (19), the equation (17) assume the form

\[ \ddot{\Sigma}_{kRk\psi_0}(t) + \left[ e^{-2Ht}k^2_R - \left( \frac{k^2_{\psi_0} + H^2}{4} \right) \right] \Sigma_{kRk\psi_0}(t) = 0, \]  \hspace{1cm} (22)

where \( 4k^2_{\psi_0} \) plays the role of the squared mass of the inflaton field in 4D conventional models of inflation. Note that in our model this mass is geometrically induced by the fifth coordinate. Moreover, in a more realistic model (like a power-law expansion of the universe), the equation for the modes (22) should be inhomogeneous. The origin of this inhomogeneity should be in the right hand of the equation (13).

If we normalize the solution of the differential equation (22), we obtain \( \Sigma_{kRk\psi_0}(t) = \lambda \left[ \frac{\pi}{2} \right] H_\lambda^{(2)}[y(t)] \), where \( \lambda = \frac{1}{2} \sqrt{1 + 4k^2_{\psi_0}/H^2} \). This solution give us the time dependent modes for the scalar field fluctuations \( \delta\varphi \) related to the modes of the gauge-invariant scalar metric fluctuations \( \Phi \), for an effective 4D de Sitter expansion of the universe.

B. Energy density fluctuations and spectrum

In order to make an analysis for the amplitude of the energy density fluctuations and its spectrum we must calculate \( \delta\rho = \tilde{g}^{NN}\delta T_{NN} \bigg|_{N=Ht,\psi=\psi_0=1/H} \), where \( \delta T_{NN} = -\frac{1}{2}\delta g_{NN}\varphi_L\varphi^L \). After make a semiclassical approximation for \( \varphi \), we obtain
\[ \delta \rho = 2 \Phi \psi^2 \left( \frac{\partial \varphi}{\partial \psi} \right)^2 = 8H^2 \Phi \delta \varphi^2 + 24H^2 \varphi_0 \Phi \delta \varphi, \]  

(23)

where \( \varphi_0 \) is the effective 4D background inflaton field. In a de Sitter expansion, this field is a constant of \( t \) so that the slow-roll conditions [16] for this field are guaranteed.

The expectation value for \( \delta \rho \) will be (we assume the commutation relation \( [\Phi, \delta \varphi] = 0 \))

\[ \langle \delta \rho \rangle = 24H^2 \varphi_0 \langle \Phi \delta \varphi \rangle, \]

(24)

where \( \langle \Phi \delta \varphi \rangle = \frac{e^{-3Ht}}{2\pi^2} \int dk_R k_R^2 \xi_{kkRk_0}(t) \xi_k(t) \). On the infrared sector (i.e., for \( k_R \ll k_H = He^{Ht} \)), we obtain

\[ \langle \delta \rho \rangle \approx \frac{8G}{\pi^2} H \varphi_0 2^{\mu+\lambda} \Gamma(\lambda) \Gamma(\mu) \frac{\epsilon^{3-\lambda-\mu}}{3-\mu-\lambda}, \]

(25)

where \( \epsilon = \frac{k_{IR}}{k_p} \ll 1 \) is a dimensionless parameter. Here, \( k_{IR} \) is the wavenumber related to the Hubble radius at the moment \( t_i \) (the time when the horizon enters) and \( k_p \) is the Planckian wavenumber. If fact, we choose \( k_p \) as a cut-off scale of all the spectrum. Note that (25) takes into account both, the metric and inflaton fluctuations during an effective 4D de Sitter expansion. The relative amplitude for the energy density fluctuations are \( \langle \delta \rho \rangle / \rho_b \), where \( \rho_b \) is the background energy density for a de Sitter expansion given by the eq. (21). To analyze the spectrum of \( \langle \delta \rho \rangle / \rho_b \) on cosmological scales we can make use of the result obtained in a previous work [10]. From the experimental data [18], for the energy perturbation spectral index \( n_s = 4 - \sqrt{9 + 16k_{\psi_0}^2/H^2} = 0.97 \pm 0.03 \), we obtain

\[ 0 < \frac{k_{\psi_0}}{H} < 0.15. \]

(26)

This result was obtained from the spectrum of \( \langle \Phi^2 \rangle \) for a de Sitter expansion and is analogous to the obtained from the requirement of scale invariance for a de Sitter expansion in the standard 4D inflationary models: \( \left( \frac{m}{M} \right)^2 \ll 1 \).

The spectral index \( n_1 = 3 - \mu - \lambda \) for \( \langle \Phi \delta \varphi \rangle \) in (25), is

\[ 0.9483 < n_1 < 1, \]

(27)

which is almost scale invariant as the spectrum of \( \langle \Phi^2 \rangle \). In a more general model like power-law inflation (where the background field \( \varphi_0 \) depends on the time), one would expect other two spectral indices \( n_2 \) and \( n_3 \), which, could not be nearly scale invariant. This case will be subject of study in a forthcoming work.

In the figure (1) was plotted \( \langle \delta \rho \rangle / \rho_b \) as a function of \( \epsilon \) for \( k_{\psi_0}/H = 0.15 \) (continuous line) and \( k_{\psi_0}/H = 0.00015 \) (pointed line), respectively. The \( \epsilon \)-values here plotted corresponds to \( 10^4 \) or \( 10^3 \) times the typical galactic size today (called cosmological scales). Note that in both cases \( \langle \delta \rho \rangle / \rho_b \) increases with \( \epsilon \) (more exactly, increases as \( \epsilon^{3-\lambda-\mu} \)) and \( \langle \delta \rho \rangle / \rho_b \big|_{k_{\psi_0}/H = 0.15} \approx 41.8 \left( \frac{\Phi}{H} \right) \epsilon^{0.97} \) always is greater (and its slope, too) than \( \langle \delta \rho \rangle / \rho_b \big|_{k_{\psi_0}/H = 0.00015} \approx 42.7 \left( \frac{\Phi}{H} \right) \epsilon^{0.99} \), because \( \epsilon \ll 1 \). The discrepancy between them increases on smallest scales, which is a manifestation of how the universe becomes more and more inhomogeneous as we approach to astrophysical scales. This is not surprising, because is well known that \( \langle \delta \rho \rangle / \rho_b \) on astrophysical scales is bigger than on cosmological scales.
IV. FINAL COMMENTS

In this paper we have studied the dynamics of 4D gauge-invariant metric fluctuations (which are reciprocally related to inflaton field fluctuations), from a 5D vacuum state. This topic was examined also in [17], but in a rather different setting. In particular, we have examined these fluctuations in an effective 4D de Sitter expansion for the universe using a first-order expansion for the metric tensor. It was done in a previous work, but in a partial manner. In the complete approach here developed, we require that the spectrum of $\langle \Phi^2 \rangle$ be nearly scale invariant, in agreement with the experimental data [18]. Once the cut (26) is obtained, it is possible to find the power of the spectrum for $\langle \delta \rho \rangle / \rho_b$ [see the cut for $n_1$ in eq. (27)]. We obtain that

$$\frac{\langle \delta \rho \rangle}{\rho_b} \simeq \frac{64}{3\pi^2} 2^{\mu+\lambda} \varphi_0 \Gamma(\lambda) \Gamma(\mu) \frac{\epsilon^{3-\mu-\lambda}}{(3-\mu-\lambda)},$$

take values of the order of $10^{-5}$ for $H \simeq 10^{-9} \, M_p$ and $\varphi_0 \simeq 10^{-12} \, M_p$, so that the model provides a sufficient number of e-folds (i.e., $N_e > 60$) required to solve the horizon/flatness problem for $t > 6 \times 10^{10} \, G^{1/2}$. It is very important that $\varphi_0$ can take sub-Planckian values. It solves one of the problems of standard 4D chaotic inflation, in which the scalar field remains always with trans-Planckian values. The consequences of this complete approach are very important, because we find that the spectrum of $\langle \delta \rho \rangle / \rho_b$ now depends on the coupling of $\Phi$ with $\delta \phi$. Moreover, this spectrum is almost scale invariant (as the spectrum of $\langle \Phi^2 \rangle$), for $k_\psi/\dot{H} \ll 1$. However, one would expects additional spectral indices in a more realistic inflationary model where $\varphi_0$ depends on the time. This case will be studied in a forthcoming work.

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Fig. 1) Evolution of $\langle \delta \rho \rangle / \rho_b$ as a function of $\epsilon$ for $k_{\psi_0}/H = 0.15$ (continuous line) and $k_{\psi_0}/H = 0.00015$ (pointed line).
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