Infrared Conductivity of Cuprate Metals: Detailed Fit Using Luttinger Liquid Theory

P.W. ANDERSON

Joseph Henry Laboratories of Physics
Jadwin Hall, Princeton University
Princeton, NJ 08544

ABSTRACT

Measurements of infrared conductivity in the normal state of the cuprate layer metals show a characteristic behavior in the plane of the layers which is in essential agreement among many experiments. A simple parametrization of this behavior, proposed originally by Collins and Schlesinger, and exploited by N. Bontemps and her group, which gives an adequate fit over frequencies from a few hundred cm$^{-1}$ to $> 5000$ cm$^{-1}$, is that the phase angle of the complex conductivity is independent of frequency. This fit is shown to be a natural consequence of Luttinger Liquid theory with charge-spin separation, and determines the exponent of the singularity at the Fermi surface to be $\sim .15 \pm .05$.

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The infrared conductivity of the high-$T_c$ cuprates in the normal state has a characteristic deviation from the normal “Drude” behavior of metals, which has sometimes been described as an additional, distinct “mid-infrared absorption” and sometimes as an extended tail of the low-frequency peak. Schlesinger, some years ago, analyzed his data on the reflectivity of single crystals of $YBCO_7$ in terms of the conventional expression

$$\sigma = \frac{ne^2}{m(\omega + \frac{1}{\tau})}$$

(1)

with frequency-dependent parameters $m(\omega)$ and $1/\tau(\omega)$, which showed remarkably simple behavior (see Fig. 1): $1/\tau$ is proportional to $\omega$, and $m$ has a slow, approximately logarithmic variation. There is in fact little difference in the data among actual experiments, as opposed to interpretations, on good materials, so we may take Fig. 1 as typical of optimally doped cuprates, since it is in essence a heuristic description of the data.

N. Bontemps and collaborators have used a similar plot to describe data over a wide range of frequencies, up to around 8000 cm$^{-1}$, using transmission and reflection data on films of a number of cuprates, most but not all closely related to YBCO (see Figs. 2 and 3.) With this wide frequency range the family resemblance of all of the data becomes striking, particularly plotted using Schlesinger’s parameters. I believe that there would be little disagreement as to the general characteristics of the actual data among these and other experimentalists, except that less highly doped YBCO samples show “spin gap” deviations at the lower end of the range ($\leq 500$ cm$^{-1}$), and that quite impure samples may have a small residual resistivity.

We describe a detailed fit to the data of figs. 2 and 3 using the “Luttinger liquid” hypothesis for the electronic state of the 2D normal metal. This result depends only on rather general properties of the theory but is totally dependent on its non-Fermi liquid nature.

The basis of the fit is the remark that the cuprates are in the “holon non-drag regime” of Luttinger liquid transport theory. This is the regime where charge excitations (“holons”) are scattered sufficiently rapidly that they do not recohere with the spinons after the accelerated electron decays into charge and spin excitations (The condition for this regime is $\omega > (1/\tau)_{\text{holon}} \sim \frac{E_F}{\sqrt{\tau_{\text{holon}}}}$; the source of $(\frac{1}{\tau_{\text{holon}}})$ is probably impurity scattering at small $\omega$ and phonons at large $\omega$.)

Under these circumstances vertex corrections are damped out by holon scattering and the conductivity is given by the simple one-loop diagram (Fig. 3)

$$\sigma(\omega) \propto \frac{1}{\omega} \int dx \int dt G^e(x,t) G^h(x,t) e^{i\omega t}$$

(2)

$G^e$ and $G^h$ are the exact (interacting) one-electron Green’s functions for electrons and holes respectively. The physical process which controls the rate of entropy production is the decay of the electron and hole into spin and charge excitations, but this is enabled to act as a resistivity mechanism by the fact that the momentum
decays because the charge is then scattered by the lattice. The process is analogous to phonon scattering in the phonon non-drag regime, where the momentum decay occurs by the scattering of the phonons by the lattice which prevents phonon drag, while the entropy production is caused by phonon emission which is momentum-conserving and controls the observed resistivity. Ogawa has shown that the vertex corrections which would invalidate (2) and restore the Ward identities in a pure sample are cut off by the mean free path for charge scattering in this regime. Note that in this regime neither the conventional residual resistivity nor phonon resistivity appear, and they are replaced by the “linear $T$” resistivity when the sample is “impure enough”.

We can evaluate (2) very simply using the fact that $G_1(x, t)$ is a homogeneous function of $(x, t)$ considered as a single variable. This is the consequence of the fact that all excitations have a finite Fermi velocity. For the Fermi liquid,

$$G_{FL} \propto e^{ik_F x \over x - v_F t}$$

homogeneous of order $(-1)$, while for the 1D Luttinger liquid,

$$G_{LL} \propto e^{ik_F x \over \sqrt{(x - v_s t)(x - v_c t)(x^2 - v_s^2 t^2)}}$$

which is homogeneous of order $(-1 - \alpha)$. For the 2D liquid $G$ is an average of an expression like (3) or (4) over the Fermi surface. For the Fermi liquid, the relevant $G$ in momentum and frequency space may be approximated by

$$G(p, \omega) \simeq \frac{1}{\hbar \omega - (p - p_F) \cdot v_F}$$

where $p_F, v_F$ are at the projection of $p$ on the Fermi surface along $v_F$, $p$ assumed close to the Fermi surface. A similar construction for the Luttinger liquid will give a pair of variables $\Delta p = p - p_F, \omega$, in which the Green’s function will again be homogeneous of order $-(1 - \alpha)$, but this function has no simple formal expression. Nonetheless we may in general write, as the appropriate law for scaling of the low-frequency excitation spectrum,

$$G(x, t) = \frac{1}{t^{1+\alpha}} F \left( \frac{x}{v_F t} \right) \quad \text{(L.L.)}$$

$$G(p, \omega) = \frac{1}{\omega^{1-\alpha}} F \left( \frac{(p - p_F) v_F}{\omega} \right)$$

where $F$ will depend on the parameters $v_c/v_F, v_s/v_F$ as functions of position on the Fermi surface. (5) reduces to the Fermi liquid expression (3) if $\alpha \to 0$. By a simple scaling argument, we find

$$\sigma(\omega) = \text{const} \left( i\omega \right)^{1-2\alpha}$$

(7) holds up to an upper frequency cutoff $\Omega = \\wedge / \hbar$ of the order of the electron band width $\wedge$. The sum rule
on conductivity will be satisfied if the coefficient in (7) is set so that

\[ \sigma(\omega) = \frac{ne^2}{i\omega m_0} \left( \frac{i\omega}{\Omega} \right)^{2\alpha} \frac{2\alpha}{\sin \pi\alpha} \]  

(8)

Here \( m_0 \) is the sum rule mass,

\[ \int \sigma(\omega) d\omega = \frac{ne^2}{m_0} \]

which should be not far from the band mass: (8) contains all intraband mass renormalization effects.

I would remind the reader that for the Fermi liquid the integral (2) is not convergent without a finite lifetime or giving an imaginary part \( \hbar/\tau \) to the energy denominator in \( G \). This gives the characteristic “Drude” behavior of ordinary metals, with \( \sigma \) falling off as \( 1/\omega^2 \) at high frequencies. The Luttinger liquid is qualitatively different from a Fermi liquid with small \( Z \).

(8) contains only two free parameters, \( n/m_0 \) and \( \alpha \) (the upper cutoff \( \Omega \) merely scales \( m_0 \) and is not independent.) Neither can vary much: \( m_0 \) must not be much bigger than the band mass, and \( c \)-axis Hall data \(^5\) among others tell us that \( n \) is the conventional band filling \( \propto 1 - \delta \). \( \alpha \) for the 1D Hubbard model is \( \leq 1/8 \), but models with \( \alpha > 1/8 \) exist. There is no fundamental theory of \( \alpha \) in 2D. Vague indications from gauge theory \(^6\) suggest \( 1/6(2\alpha = 1/3) \), while the tomographic picture \(^7\) might suggest agreement with 1D.

The data give two independent measures of \( 2\alpha \), one from the slope of \( 1/\tau \) vs. \( \omega \) and one from the dependence of \( m \) on \( \omega \). These two numbers are unrelated in the “marginal Fermi liquid” theory \(^8\), and their agreement argues against that theory, as well as does the relatively large value of \( 2\alpha \) we find. The slope of \( 1/\tau \) (dashed line in Fig. 2a) is \( \sim .7 \pm .1 \) which gives \( \alpha \approx .15 \pm .05 \). The median slope is used for the dashed line in Fig. 2b, which as you can see is an adequate fit, although the power-law form is not much constrained by the data. On the other hand, the analytic properties of \( \sigma \) require that if it really has constant phase angle (as Fig. 2a shows) it must be a power of \( (i\omega) \) (or logarithmic in the limit \( \alpha = 0+ \)).

Let us summarize the achievements of the Luttinger liquid hypothesis, coupled with the concept of the holon-non-drag regime. (A second way to think of this regime is as one in which the transport is by spinons \(^9\) and is relaxed by holon emission and reabsorption). The original motivation which was satisfied by this idea was to explain the absence of phonon scattering effects or, in most cases, of residual impurity scattering, both of which should be large in most of these materials. Let it be explicit that the separation of charge and spin, though it fails to appear in the formal expression (5) or (6), which depends only on the “Fermi surface” exponent, \( \alpha \), is essential to the entire theory, because of the concept of “holon drag”.

Now we see that the theory leads to a unique scaling form for the conductivity which holds over almost 2 decades of frequency and for a number of cuprates. Particularly important, in my view, is the fact that the expression scales from \( > 5000\text{cm}^{-1} \) to \( < 500\text{cm}^{-1} \), a property which no alternative theory motivates in any natural way.
It is interesting that other groups (especially Bozovic$^{10}$) see indications of similar behavior in the “mid-infrared conductivity” of a number of other materials, mostly those with other symptoms of strong correlation phenomena. With considerable caution because of the existence of other transport regimes, we would consider a Luttinger liquid explanation for some of these cases.

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FIGURE CAPTIONS

(1) Schlesinger’s original data on the IR spectrum of YBCO. This is repeated in Fig. (2) as the crossed square points. (Ref. (1)).

(2) (a) The Bontemps group’s data on a group of cuprates from Ref. (2): $1/\tau$ vs $\omega$. Further, more recent data are given in (2b) and reported in Ref. (11).

(2b) (a)+(b) Same set of data, $m^*$ vs. $\omega$.

(3) Primitive diagram for the conductivity. Vertex corrections are omitted for reasons given in the text.
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