Threat from Within: Excitation of Venus’s Co-orbital Asteroids to Earth-crossing Orbits

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Abstract

All five currently known asteroids in the 1:1 mean-motion resonance (co-orbital) with Venus cross Earth’s orbit. We explore a scenario in which these near-Earth asteroids originate in a reservoir of asteroids co-orbiting with Venus on low-eccentricity orbits. Such an asteroid reservoir was proposed as the only viable source of Venus’s co-orbital dust ring. So far, efforts to discover low-eccentricity Venus co-orbital (LEVCO) asteroids have been unsuccessful; however, their existence has not been ruled out. We show that LEVCO asteroids, stable for millions to billions of years, eventually evolve into Earth-crossing configurations, where they may pose a threat to Earth. We find that the orbits of these asteroids cross Earth’s orbit for 22.5 million yr, on average, an order of magnitude longer than the corresponding Earth-crossing time of most asteroids escaping from the main belt. Using the results of the latest survey of LEVCO asteroids, we conclude that, given their absolute magnitudes, $H$, most of the observed Venus co-orbitals likely do not originate from the hypothetical population of LEVCO asteroids. However, we infer that there are up to $\sim$500 asteroids originating from the LEVCO region with $H < 26.3$ (10–40 m in diameter) that currently cross the orbit of Earth. Up to $\sim$20 of those have $H < 24.1$ (30–100 m in diameter), easily detectable by various near-Earth asteroid surveys. We estimate the current mass of the LEVCO reservoir as $M \approx 10^{13} \ldots 10^{16}$ kg, 3–6 orders of magnitude lower than the current mass of the main belt, depending on their size–frequency distribution.

Unified Astronomy Thesaurus concepts: Near-Earth objects (1092); Orbital evolution (1178); Orbital resonances (1181); Venus (1763); Asteroids (72); Asteroid dynamics (2210); Atira group (111); Aten group (110); Dynamical evolution (421)

Supporting material: animation

1. Introduction

Currently, there are five objects identified as co-orbitals of Venus; i.e., they have approximately the same orbital period as Venus (see Table 1) and librate in the 1:1 mean-motion resonance (MMR) with Venus. All five of these known Venus co-orbitals have high eccentricities, high enough that they are currently crossing the orbit of Earth. The origin of these objects is unknown, but Morais & Morbidelli (2006) showed that the near-Earth asteroids (NEAs) can evolve into orbits that are temporarily captured in the 1:1 MMR with Venus. Their work also showed that these temporarily resonant asteroids reside in the 1:1 MMR with Venus for, on average, around 340 kyr. In addition to these known Earth-crossing high-eccentricity Venus co-orbitals, there may also be a population of low-eccentricity Venus co-orbital (LEVCO) asteroids. Pokorný & Kuchner (2019) hypothesized that the LEVCO asteroids are primordial and showed that LEVCO asteroids that are stable for 4.5 Gyr have eccentricities below 0.1 and inclinations below 10°. The authors found that a population of dust-generating LEVCO asteroids is necessary to explain the origin of Venus’s co-orbital dust ring. There are currently no other theories or scenarios that could explain the existence of Venus’s co-orbital dust ring. Could this inferred population of LEVCO asteroids be the source of the observed Venus co-orbitals on high-eccentricity orbits that cross Earth’s orbit?

Although LEVCO asteroids have not yet been detected, some meaningful upper limits on their numbers exist. Pokorný et al. (2020) analyzed the observability of asteroids residing in the 1:1 MMR with Venus using the simulation results from Pokorný & Kuchner (2019). Using the Dark Energy Camera mounted on the 4 m telescope at the Cerro Tololo Inter-American Observatory, they conducted a twilight survey that probed approximately 5% of the hypothetical LEVCO population members. Pokorný et al. (2020) were able to constrain the number of LEVCO asteroids brighter than r-band magnitude $m_r < 21$ to $N = 18^{+30}_{-14}$ and also determined the limiting diameter of LEVCO asteroids based on their types (using six different groups: S, M, E, C, P, and D) to be $D \approx 400–900$ m. The Zwicky Transient Facility survey with a limiting magnitude in the r band of 19.5 ($D \approx 800–1800$ m) found no LEVCO asteroids as well (Ye et al. 2020).

Several works have examined the stability of this hypothetical LEVCO population and ultimately found it to be stable. The initial study, Scholl et al. (2005), concluded that Venus Trojans were not stable on gigayear timescales, and that the Yarkovsky effect further reduces the Trojan stability. Several years later, Ćuk et al. (2012) showed that horseshoe orbits near Venus are significantly more stable than tadpole orbits and found some Venus co-orbitals to be stable for at least 1 Gyr. Most recently, Pokorný & Kuchner (2019) showed that approximately 8% of Venus co-orbitals released with eccentricity $e < 0.2$ and inclination $i < 20°$ remain in a stable 1:1 MMR with Venus for the age of the solar system, preferentially on horseshoe orbits.
In this paper, we analyze the dynamical evolution of populations of Venus co-orbitals that begin in the LEVCO reservoir and then escape from the 1:1 MMR into dynamically unstable orbits. Our goals are to (1) quantify the probability that LEVCO asteroids evolve into Earth-crossing orbits and the time they spend in these Earth-crossing orbits, (2) determine the dynamical pathways involved, and (3) derive constraints for the currently unobserved population of Venus co-orbitals.

2. Integration Setup

We began our numerical exploration with the simulations of Pokorný & Kuchner (2019), who analyzed the dynamical stability of a population of LEVCO asteroids for the age of the solar system. The Pokorný & Kuchner (2019) simulations provided the time when each simulated LEVCO asteroid left the co-orbital region, which allows us to specifically focus on objects that escape in the early stages of the simulation (<150 Myr) and save the computational time that would be needed to track co-orbital asteroids that were long-term stable. In Pokorný & Kuchner (2019), the co-orbital region was defined using a range of semimajor axes of 0.7088 au < a < 0.7378 au. This semimajor axis range was wider than the range of LEVCO asteroids stable for 4.5 Gyr (0.716 au < a < 0.729 au) that the authors found (Figure 4 in Pokorný & Kuchner 2019) and provided an acceptable measure to identify long-term stable co-orbitals. The asteroids that left this range in a were removed from the simulation. In order to track the dynamical evolution of asteroids escaping from Venus’s co-orbital region, we performed new numerical integrations of these objects using Swift_RMVS_4 (Levison & Duncan 2013), modeling all asteroids as massless test particles influenced by the gravitational forces of the Sun and eight planets. In all simulations, we used a 3.6525 day integration time step, the same as the one used in Pokorný & Kuchner (2019). We used Pokorný & Kuchner (2019) only to obtain initial conditions for LEVCO asteroids. At first, we did not incorporate any nongravitational effects; see Section 6 for a discussion of the Yarkovsky effect.

In our new simulations, we examined two different populations of unstable LEVCOS chosen for two reasons: (A) they contain similar numbers of asteroids on unique orbits providing two complementary probes of the general escape mechanism, and (B) the sample sizes and integration times suited our computational capabilities (<50,000 CPU hr). We examined the two populations separately and also combined both of them together, providing three scenarios.

### Scenario I: 1685 objects that co-orbited with Venus for 5 Myr from the start of the simulation and left the 1:1 MMR with Venus within the next 20 Myr; i.e., they are LEVCO asteroids for 5–25 Myr.

**Scenario II:** 1396 objects that co-orbited with Venus for at least 25 Myr and left the 1:1 MMR with Venus within the next 125 Myr, i.e., LEVCO lifetime of 25–150 Myr.

**Scenario III:** both sets of objects combined.

By comparing these two populations (three scenarios), we analyzed the general behavior of asteroids escaping the 1:1 MMR resonance with Venus and considered whether we can extrapolate such behavior to the present time. We followed the orbital evolution of these bodies for an additional 2 Gyr after they left the 1:1 MMR with Venus in order to sample their dynamical pathway through the solar system.

2.1. Defining the Stable Co-orbital Zone

During our initial analysis of scenarios I and II, we discovered that the Venus’s stable co-orbital zone was crudely estimated in Pokorný & Kuchner (2019). So, in order to determine the boundaries of Venus’s stable co-orbital zone more precisely, we recorded the orbital elements of all particles in scenarios I and II every 25 kyr. Figure 1 shows the distributions of the semimajor axes, a, of all 3,247,369 records in the simulation of scenario I within the first 250 Myr using 0.0001 au binning for three different ranges of a. There are spikes in the left panel representing particles temporarily captured in various MMRs with the terrestrial planets and 1:1 resonance with Venus at a ≈ 0.7232 au. Zooming in shows that there are no additional well-populated resonances in the region within 0.05 au of that 1:1 MMR with Venus (middle panel of Figure 1). In the right panel of Figure 1, which is zoomed in even more, we see clear transitions at the inner and outer edges of the 1:1 resonance, which we use to define the co-orbital zone: 0.7154 au < a < 0.7308 au (the gray area in the figure). The results for scenario II are similar to those from scenario I; the only difference is that the particles spend, on average, 10 times more time in the Venus co-orbital zone in scenario II, as expected from the definitions of scenarios I and II.

2.2. LEVCO Asteroid Impact Rates on Terrestrial Planets

We detected collisions of asteroids with planets in our simulations using the mean planetary radii in Swift_RMVS_4. To estimate the uncertainties in the properties of our samples, we performed a simple bootstrapping analysis with resampling. We ran our bootstrapping process 10,000 times for each scenario, randomly picking N asteroid IDs from the pool of all.

| ID       | a (au) | e     | i (deg) | Q (au) | \(\Omega\) (deg) | \(\omega\) (deg) | H   | References |
|----------|--------|-------|---------|--------|------------------|------------------|-----|------------|
| (322756) | 0.7250 | 0.3826 | 8.1333  | 1.036  | 109.4107         | 234.0448         | 18.94| Brasser et al. (2004) |
| (524522) | 0.7237 | 0.4101 | 9.0345  | 1.166  | 231.4881         | 355.4197         | 20.5 | Mikkola et al. (2004) |
| 2012 XE86| 0.7232 | 0.4326 | 6.7262  | 1.020  | 281.0317         | 337.0751         | 23.3 | DLFM 2013 |
| 2013 ND15| 0.7236 | 0.6116 | 4.7946  | 1.002  | 95.8087          | 19.7582          | 21.4 | DLFM 2014 |
| 2015 WZ12| 0.7220 | 0.4123 | 3.6302  | 1.020  | 251.3515         | 346.145          | 26.3 | DLFM 2017 |

Note. Here \(a\) is the semimajor axis in au, \(e\) is the eccentricity, \(i\) is the inclination in degrees, \(Q\) is the aphelion distance, \(\Omega\) is the longitude of the ascending node in degrees, and \(\omega\) is the argument of periastron in degrees. We use the following reference acronyms: DLFM 2013 is de la Fuente Marcos & de la Fuente Marcos (2013), DLFM 2014 is de la Fuente Marcos & de la Fuente Marcos (2014), and DLFM 2017 is de la Fuente Marcos & de la Fuente Marcos (2017). All currently known Venus co-orbitals are crossing the orbit of Earth. The orbital elements were obtained from https://www.mpcworld.org/iau/MPCORB/NEA.txt on 2021 February 23.
and $q$ (

Figure 1. Semimajor axis distribution of all (3,247,369) orbital element records of asteroids in the scenario I simulation collected in the first 250 Myr. All asteroids in scenario I leave the co-orbital zone within the first 25 Myr of the simulation, and the asteroids then roam the inner solar system freely. The gray shaded area ($0.7154 < a < 0.7308$ au) shows our adopted definition of the Venus co-orbital region. Left: entire range of semimajor axes recorded in the simulation. Middle: zoomed view with the semimajor axis range $0.6732 < a < 0.7322$ au. Right: more magnified view of Venus’s co-orbital zone clearly showing the transition between the nonresonant and resonant regions.

IDs available in the simulation, where $N$ equals the total number of simulated asteroids. Since we allow for resampling, we obtain a mean value and standard deviation for each scenario. The combined scenario (scenario III) gives the following rates for the escape from the co-orbital zone after 2 Gyr of integration time: $40.7\% \pm 2.6\%$ of the objects impact Venus, $18.0\% \pm 1.3\%$ impact Earth, $7.0\% \pm 0.5\%$ impact Mercury, and $29.3\% \pm 1.4\%$ come closer than 0.05 au to the Sun (the disruption limit in our simulations). The final $5.0\% \pm 0.4\%$ either hit one of the remaining planets or their heliocentric distance exceeds 10,000 au, i.e., the outer limit of our simulations. From these standard deviations for the impact rates on each planet in scenarios I and II, we conclude that the two scenarios show similar dynamical fates for asteroids escaping from the 1:1 MMR with Venus. Only four bodies from our total sample of 3081 (scenario III) remained in the simulation for more than 500 Myr after leaving Venus’s co-orbital region.

3. From Venus Co-orbitals to Earth-crossing Orbits

We are primarily interested in the detailed dynamical evolution during the Earth-crossing region insertion regardless of an asteroid’s total dynamical age. However, the time the particles spend in the Venus co-orbital zone (up to billions of years) can be thousands of times longer than the dynamical timescales of the postescape evolution (tens of millions of years). So we analyzed the behavior of each asteroid escaping the LEVCO reservoir for only a select time interval around the time it first entered the Earth-crossing region, which we define as

$$a(1 - e) = q \leq 1 \text{ au} \leq Q = a(1 + e). \quad (1)$$

Here $a$ and $e$ are the asteroid’s semimajor axis and eccentricity, and $q$ and $Q$ are the asteroid’s perihelion and aphelion distances. For each asteroid, we identified the time when it first entered the Earth-crossing region. Then we examined a 100 kyr window in time around that moment. We refer to the time before or since the moment of an asteroid’s first Earth crossing as $\tau$; the asteroid first enters the Earth-crossing zone at $\tau = 0$.

Figure 2 shows the dynamical evolution of asteroids escaping from the LEVCO reservoir into Earth-crossing orbits for scenario I (top row), scenario II (middle row), and their combination (scenario III; bottom row). Figure 2 shows two primary escape routes for asteroids leaving the co-orbital configuration and getting to Earth-crossing space: (A) they stay in the 1:1 MMR with Venus, but their eccentricity increases until their aphelion distance reaches 1 au; and (B) they leave the co-orbital region with their semimajor axes and eccentricities both gradually increasing until their aphelion distances reach 1 au. In case B, the eccentricities mostly remain below $0.369$, which, for asteroids in the Venus co-orbital zone, corresponds to $Q < 1$ au. Overall (i.e., scenario III), at the time the asteroids cross $Q = 1$ au ($\tau = 0$), $5.7\%$ of them have semimajor axes smaller than the inner boundary of the co-orbital zone ($a < 0.7154$ au), while $12.4\%$ are in the co-orbital zone $0.7154 < a < 0.7308$.

The simulated asteroids in both scenarios remain in the Venus-crossing region (the area denoted by red dashed lines in Figure 2) even after being in the Earth-crossing zone for 10 kyr (Figures 2(c), (f), and (i)). This dynamical link to Venus gets weaker with time due to frequent close encounters with Earth. After 10 kyr, there are $78.2\%$ and $80.6\%$ Venus crossers in scenarios I and II, respectively. These values decrease to $69.6\%$ and $70.4\%$ for $\tau = 100$ kyr. At $\tau = 700$ kyr, we see another $10\%$ decrease of Venus crossers to $59.6\%$ and $58.9\%$. Even at $\tau = 2$ Myr, we still record $49.3\%$ and $50.6\%$ Venus crossers in the simulation. The number of Earth-crossing asteroids in our
simulation is rather steady and fluctuates around 45% for both scenarios for the same period of time, 2 Myr.

At first glance, Figure 2 shows no obvious difference between scenarios I and II and their combination (scenario III). While there is an order-of-magnitude difference in the asteroids’ ages (co-orbital lifetimes) between scenarios I and II, the dynamical pathways of the asteroids are almost identical prior to entering the Earth-crossing region. This similar behavior at two different times supports the idea of extrapolating these results, which are for primordial asteroids, to estimate the behavior of asteroids escaping the 1:1 MMR with Venus just recently.

In Scenario I, there are four asteroids outside the Venus-crossing zone (denoted by red dashed lines) just 2.5 kyr before they enter the Earth-crossing zone (Figure 2(b)). These asteroids increase their aphelion distance $Q$ from $Q < 0.7233$ to $Q > 1.0$ au while being on orbits that do not cross any planet. These four asteroids provide interesting insight into a rapid dynamical development inside Venus’s orbit. In Figure 3, we show the evolution of the semimajor axis $a$, eccentricity $e$, inclination $i$, aphelion distance $Q$, argument of pericenter $\omega$, and longitude of the ascending node $\Omega$ for these four peculiar asteroids 50 kyr before and after they enter the Earth-crossing zone for the first time, i.e., $\tau \in (-50, 50)$ kyr. As expected, all four asteroids have $Q < 0.7233$ at $\tau = -2.5$ kyr and then experience a sudden increase in both the $a$ and $e$ that pushes them at $\tau = 0$ kyr to $Q > 1.0$ au. During those 2.5 kyr, the inclination of all four asteroids does not show any abrupt changes. Asteroids with IDs 714 and 1581 in scenario I were in internal MMRs (9:8V for ID 714 and 15:14V for ID 1581) with

Figure 2. Dynamical evolution of LEVCOs into Earth-crossing orbits. Each plot shows the distribution of asteroid eccentricities with respect to their semimajor axes color coded by their inclination for a selected $\tau$ shown in the upper part of each plot. The gray area with a dashed border shows the Earth-crossing zone defined in Equation (1). The red dashed lines show the Venus-crossing zone, defined as $q < 0.7233 < \bar{Q}$. The red rectangle denotes the Venus co-orbital zone based on Figure 1. In each row, the time sequence goes from left ($\tau = -50,000$ yr) to right ($\tau = 10,000$ yr). Top row: scenario I particles escaping Venus’s co-orbital zone between 5 and 25 Myr. Middle row: scenario II particles escaping between 25 and 150 Myr. Bottom row: all particles in both scenarios together (scenario III). An animation of this figure is available in the online journal. The animated version traces scenarios I (left), II (middle), and III (right) from $\tau = -50,000$ to 50,000 yr in 250 yr increments. (An animation of this figure is available.)
Figure 3. Evolution of orbital elements of four asteroids from scenario I that had $Q < 0.7233$ au at $\tau = -2.5$ kyr, i.e., the last recorded output before entering the Earth-crossing zone. The panels show the asteroids’ orbital elements: (a) the asteroid semimajor axis $a$ in au, (b) the eccentricity $e$, (c) the inclination $i$ in degrees, (d) the aphelion distance $Q$ in au, (e) the perihelion distance $q$ in au, and (f) the argument of pericenter $\omega$ in degrees. The color coding key is in panel (a).
Venus before they jumped into the Earth-crossing zone. Only the asteroid with ID 1581 was not in the 1:1 MMR with Venus 50 kyr before entering the Earth-crossing zone (Figure 3). Figure 3 shows how abruptly asteroids completely inside Venus’s orbit can evolve into Earth-crossing orbits.

For scenario I, 1553 asteroids out of 1685 became Earth-crossers \((P_{\text{Earth}}(I) = 92.2\%)\). Of those Earth-crossing asteroids, 701 were at some point crossing Earth’s orbit while they were in Venus’s co-orbital zone \((P_{\text{MMR}}(I) = 41.6\%)\). For scenario II, the fraction of Earth-crossers was slightly smaller and consistent within the Poisson noise \((P_{\text{Earth}}(II) = 1268/1396 = 90.8\%)\), as was the fraction of asteroids crossing Earth while within Venus’s co-orbital zone \((P_{\text{MMR}}(II) = 596/1396 = 42.7\%)\). The combined scenario (scenario III) gives \(P_{\text{Earth}}(III) = 91.5\% \pm 0.9\%\) and \(P_{\text{MMR}}(III) = 42.1\% \pm 0.8\%\). Both scenario I and scenario II yield comparable probabilities of producing Earth-crossing asteroids with surprisingly large fractions of asteroids that become Earth-crossers while still residing in Venus’s co-orbital zone. The difference between the portion of asteroids that enter the Earth-crossing region for the first time while still in the co-orbital zone (12.4%) and the total number of asteroids that are Earth-crossers and in Venus’s co-orbital zone at the same time \(P_{\text{MMR}}(III) = 42.1\% \pm 0.8\%\) stems from the fact that asteroids can exit and reenter the co-orbital zone at high eccentricities.

4. Earth-crossing Times

One of the important metrics in our analysis is the length of time the asteroids spend in Earth-crossing orbits. In our simulations, we checked the Earth-crossing condition from Equation (1) every 250 yr (one integration record), and for each asteroid, we calculated the number of integration records when the Earth-crossing condition is fulfilled. Then we multiplied the total number of records by 250 yr to estimate the total Earth-crossing time \(T_{\text{Earth}}\) for each asteroid. We chose this sampling frequency for two reasons: (1) more than 90% of all asteroids have Earth-crossing times at least 1 order of magnitude longer than the sampling frequency, and (2) asteroids in our simulation do not frequently jump in and out of the co-orbital/Earth-crossing zone during one record. We also calculated the number of records satisfying both Equation (1) and 0.7154 \(\text{au} < a < 0.7308 \text{au}\), i.e., to monitor the Earth-crossers in Venus’s co-orbital zone to obtain the total Earth-crossing time of asteroids in the 1:1 MMR with Venus \(T_{\text{MMR}}\).

Figure 4 shows the cumulative distributions of \(T_{\text{MMR}}\) (Figure 4(a)) and \(T_{\text{Earth}}\) (Figure 4(b)) for scenarios I and II and their total (scenario III). All three data sets provide almost identical results for both \(T_{\text{MMR}}\) and \(T_{\text{Earth}}\). To find an analytic description of the cumulative distributions of \(T_{\text{MMR}}\) and \(T_{\text{Earth}}\) for scenarios I and II, we tried 88 different continuous distributions available in SciPy. A (4) generalized gamma distribution yielded the best fit,

\[
F_x(x) = \frac{\gamma(a, (x/s)^\gamma)}{\Gamma(a)},
\]

where \(a\), \(c\), and \(s\) are nonnegative free parameters; \(\gamma(a, (x/s)^\gamma)\) is the lower incomplete gamma function; and \(\Gamma(a)\) is the gamma function. Note that the normalization factor \(\Gamma(a)\) ensures that \(F_x(x)\) is unity for \(x \to \infty\).

Granvik et al. (2018) studied the dynamical behavior of asteroids escaping the main belt and their evolution into near-Earth object (NEO) orbits. In their study, Granvik et al. (2018) estimated the average lifetimes of NEOs originating in different escape routes, such as 3:1J, \(\nu_6\), or asteroids escaping from the Hungaria family. Granvik et al. (2018) defined NEOs as asteroids having \((q < 1.3 \text{ au}, a < 4.2 \text{ au}, e < 1, i < 180^\circ)\), which labels all LEVCO asteroids as NEOs even though they are not on Earth-crossing orbits. Since our \(T_{\text{Earth}}\) definition is more restrictive and does not take into account the time the simulated asteroids spend in the LEVCO reservoir, we can compare them to the lifetimes of asteroids escaping from the main belt \(L\). The average Earth-crossing times of asteroids originating from the LEVCO reservoir \(T_{\text{Earth}} \approx 22.5 \text{ Myr}\) (Table 2) are an order of magnitude longer than the lifetimes of asteroids escaping from the main belt: 3:1J complex \(L \approx 2 \text{ Myr}\), \(\nu_6\) complex \(L \approx 6 \text{ Myr}\), 5:2J complex \(L \approx 1 \text{ Myr}\), and 2:1J complex \(L \approx 0.4 \text{ Myr}\). The mean Earth-crossing times of objects after leaving Venus’s 1:1 MMR resemble those of asteroids escaping from the Phocaea (\(L \approx 11 \text{ Myr}\)) and Hungaria (\(L \approx 37 \text{ Myr}\)) families.

5. Constraining the Current and Initial Number and Mass of the Venus Co-orbital Asteroid Population

In this section, we will combine our knowledge of the five known Venus co-orbitals with the Earth-crossing duration of asteroids inside Venus’s co-orbital zone to estimate the number of asteroids in the LEVCO reservoir and their mass. Assume that the LEVCO reservoir currently contains \(N_{\text{LEVCO}}\) asteroids, where the flux of these asteroids escaping from the reservoir is \(F_{\text{esc}}\). Here \(P_{\text{MMR}}\) is the fraction of these escapees that are in the co-orbital zone and also become Earth-crossers, which we estimated in Section 3. The average time that one of these asteroids is in Venus’s co-orbital zone and crossing the orbit of Earth is \(\bar{T}_{\text{MMR}}\) (Section 3). Combining these numbers then gives us the number of asteroids that currently cross the orbit of Earth and are in the Venus co-orbital zone:

\[
N_{\text{MMR}} = N_{\text{LEVCO}} F_{\text{esc}} P_{\text{MMR}} \bar{T}_{\text{MMR}}.
\]

In other words, \(N_{\text{MMR}}\) represents the typical number of Earth-crossing asteroids in the Venus co-orbital zone at any given moment that arose from the LEVCO reservoir, i.e., a quantity that we can observe. From a statistical point of view, \(N_{\text{MMR}}\) follows Poisson statistics.
Table 1 lists five Earth-crossing asteroids in the Venus co-orbital zone. We do not know whether these five originated in the LEVCO reservoir, the main belt, or elsewhere. However, let us now hypothesize that one of these five known Venus co-orbitals originated in the LEVCO reservoir and examine the implications of this hypothesis.

We can also write a similar equation to describe the number of asteroids from the LEVCO population that are crossing Earth’s orbit at any given time, regardless of whether or not they are in the co-orbital zone of Venus:

$$N_{Earth} = N_{LEVCO} \mathcal{F}_{esc} P_{Earth} T_{Earth}. \quad (4)$$

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**Figure 4.** Cumulative distributions of the duration of Earth-crossing configurations for asteroids that cross Earth’s orbit while their orbits are inside Venus’s co-orbital zone $T_{MMR}$ (a) and all asteroids in our simulations, including those that have left Venus’s co-orbital zone $T_{Earth}$ (b). In both panels, scenario I is represented by purple crosses, scenario II is represented by green crosses, and orange crosses depict the combination of both scenarios (scenario III). The black lines show general gamma distributions fit to the data, with the best-fit parameters of these distributions shown in the figure key.
The Planetary Science Journal, 2:193 (13pp), 2021 October

Pokorný & Kuchner

Table 2
Median and Mean Values of Total Earth-crossing Times for All Simulated Asteroids ($T_{Earth}$) and Just for Asteroids within the Venus Co-orbital Zone ($\bar{T}_{MMR}$)

| Scenario | Median $T_{MMR}$ (Myr) | Mean $T_{MMR}$ (Myr) | Median $T_{Earth}$ (Myr) | Mean $T_{Earth}$ (Myr) |
|----------|------------------------|----------------------|--------------------------|-------------------------|
| I        | 0.127                  | 0.379                | 9.175                    | 23.285                  |
| II       | 0.106                  | 0.415                | 8.811                    | 21.921                  |
| III      | 0.114                  | 0.395                | 9.026                    | 22.672                  |

Note. All values are in millions of years. See Figure 4 for the cumulative distributions used to derive $T_{Earth}$ and $\bar{T}_{MMR}$.

where $P_{Earth}$ is the fraction of LEVCO escapes that evolve into Earth-crossing orbits, and $T_{Earth}$ can be found in Table 2. Later on, we will use Equation (4) to help us study this second important population.

5.1. Transport Rates from LEVCO Reservoir to Earth-crossing Orbits

Let us work our way through the factors in Equation (3), starting with $F_{esc}$. Using the Venus co-orbital population decay investigated in Pokorný & Kuchner (2019), who derived that the fraction of survivors in Venus’s co-orbital zone follows a logarithmic trend $F_{surv} \approx -0.187 \log_{10}(t/\text{Gyr}) + 0.2$, where $t$ is the age of the LEVCO reservoir, we can estimate the probability that a Venus co-orbital leaves the co-orbital region per unit time as

$$F_{esc} = \frac{dF_{surv}}{dt} \approx \frac{0.187}{\ln 10} \frac{1}{t/\text{yr}},$$

(5)

which results in $F_{esc} = 1.8 \times 10^{-11}$ asteroids escaping the LEVCO reservoir per year, using $t = 4.5 \times 10^9$ yr.

We already calculated $P_{MMR}$ and $P_{Earth}$ in Section 3. We determined that the percentage of asteroids that become Earth-crossers while being in Venus’s co-orbital zone is $P_{MMRIII} = 42.1 \pm 0.8$; that is the next factor in Equation (3). We also determined that the total percentage of all asteroids that evolve into Earth-crossing orbits is $P_{EarthIII} = 91.5 \pm 0.9$. These quantities are synonymous with $P_{MMR}$ and $P_{Earth}$.

The final factor in Equation (3) is the average time asteroids spend in Earth-crossing orbits while they are inside the Venus co-orbital zone, $\bar{T}_{MMR}$. In the previous section, we showed that the asteroids in all scenarios have a large range of Earth-crossing times, whose cumulative distributions are well represented by general gamma distributions (see Figure 4 with the average crossing times summarized in Table 2). For the rest of this paper, we will use $\bar{T}_{MMR} = 395$ kyr, estimated from scenario III and listed in Table 2.

Now let us assume that there is a single Earth-crossing asteroid that originated from and currently resides in the Venus co-orbital zone and ask how many LEVCO asteroids are required to produce this one object. In other words, let us assume $N_{MMR} = 1$. Using Equation (3) and parameters $F_{esc} = 1.8 \times 10^{-11}$ yr$^{-1}$, $P_{MMR} = 0.421$, and $\bar{T}_{MMR} = 395$ kyr, we estimate that to obtain $N_{MMR} = 1$, we need $N_{LEVCO} = 333,700$. In other words, a reservoir of 333,700 LEVCO asteroids produces, on average, a steady state with one Earth-crossing asteroid in the Venus co-orbital zone. This process is size-independent, since we assume that all asteroids are much smaller than planets and treat them as test particles. Note that the transport from the LEVCO reservoir into Earth-crossing orbits is a stochastic process, yet as we showed in Section 3, its efficiency does not depend on the dynamical age of LEVCO asteroids.

Now we will use Equation (4) to examine the steady-state production of all varieties of Earth-crossing asteroids by the LEVCO population. Using $P_{Earth} = 0.915$ and $T_{Earth} = 22.7 \times 10^6$ yr (from Table 2) and setting $N_{Earth} = 1$, we find that 2685 LEVCO asteroids are required to produce a steady state with an average of one Earth-crossing asteroid that arose from the LEVCO population. In other words, the process of creating a generic Earth-crosser from the LEVCO population is 125 (333,700/2685 = 125) times as efficient as the process of creating an Earth-crosser that is in co-orbital resonance with Venus.

5.2. Size–Frequency Distribution of LEVCO Asteroids

In the previous discussion, we made no assumptions about the size–frequency distribution of asteroids escaping the LEVCO reservoir. Now let us consider the full range of asteroid sizes that may be present in the LEVCO population. As we consider the size–frequency distribution of these bodies, we will keep in mind that any new estimate on $N_{LEVCO}$ we derive will likely be influenced most by our assumptions about the smallest bodies in the distribution.

In general, the absolute magnitude, $H$, is related to the geometric albedo, $p_V$, and asteroid diameter, $D$, according to the relation (e.g., Muinonen et al. 2010)

$$\log_{10} D = 3.1236 - 0.2H - 0.5\log_{10} p_V.$$

(6)

Table 3 shows geometric albedos and asteroid diameters corresponding to $H = 24.1$ and 26.3 considering six taxonomic types from Shevchenko et al. (2016). The same table shows the estimated absolute magnitude limit and inferred limiting diameter for each of these types from the Pokorný et al. (2020) survey. We use these numbers to derive an upper limit on the number of asteroids in the LEVCO reservoir based on the nondetection in the Pokorný et al. (2020) survey.

Let us assume that the asteroids in stable co-orbital configuration with Venus follow a single power-law size–frequency distribution with the following parameters: (1) a maximum asteroid size $D_{\text{max}}$, (2) a minimum size $D_{\text{min}}$, and (3) a differential size–frequency distribution index $\alpha$. With these assumptions, the total number of asteroids $N_{LEVCO}$ in the size range $(D_{\text{min}}, D_{\text{max}})$ is

$$N_{LEVCO}(D_{\text{min}} < D < D_{\text{max}}) = \begin{cases} N_{\alpha} \left[ 1 - \left( \frac{D_{\text{min}}}{D_{\text{max}}} \right)^{\alpha+1} \right] & \text{if } \alpha \neq -1 \\ N_{\alpha} \ln \left( \frac{D_{\text{max}}}{D_{\text{min}}} \right) & \text{if } \alpha = -1 \end{cases},$$

(7)

where $N_{\alpha}$ is the scaling factor.

Let us choose parameter 1, $D_{\text{max}}$, to be 100 km, i.e., much larger than the Pokorný et al. (2020) observation limit (400–900 m). Using $D_{\text{max}} = 1000$ or 300 km yields an insignificant difference to $N_{LEVCO}$, considering $-4 < \alpha < -3$. For parameter 2, $D_{\text{min}}$, we will explore, in turn, all of the values of $D_H = 24.1$ and $D_H = 26.3$ listed in Table 3, i.e., the diameters of the two smallest known Venus co-orbital
Table 3
Overview of Six Asteroid Types from Shevchenko et al. (2016)

| Type | $p_V$ | $D_{H<24.1}$ (m) | $D_{H<26.3}$ (m) | $D_{\text{Det limit}}$ (km) | $H_{\text{Det limit}}$ |
|------|-------|------------------|------------------|-----------------|------------------|
| S    | 0.22 ± 0.05 | 42.89 ± 4.87 | 15.57 ± 1.77 | 0.408 ± 0.036 | 19.21 ± 0.19 |
| M    | 0.17 ± 0.07 | 48.80 ± 10.05 | 17.72 ± 3.65 | 0.478 ± 0.072 | 19.28 ± 0.28 |
| E    | 0.45 ± 0.07 | 29.99 ± 2.33 | 10.89 ± 0.85 | 0.245 ± 0.017 | 19.54 ± 0.17 |
| C    | 0.061 ± 0.017 | 81.46 ± 11.35 | 29.58 ± 4.12 | 0.898 ± 0.097 | 18.89 ± 0.22 |
| P    | 0.042 ± 0.008 | 98.17 ± 9.35 | 35.64 ± 3.39 | 1.036 ± 0.081 | 18.98 ± 0.17 |
| D    | 0.049 ± 0.022 | 90.89 ± 20.40 | 33.00 ± 7.41 | 0.927 ± 0.154 | 19.06 ± 0.31 |

Note. These numbers are based on the upper limit $H = 24.1$ and 26.3. We also show the smallest detectable asteroid diameter in the Pokorný et al. (2020) survey, $D_{\text{Det limit}}$, and their limiting absolute magnitude, $H_{\text{Det limit}}$. These values are used to estimate the number of asteroids in the LEVCO reservoir and their mass.

Table 4
The Upper Limit on the Number of Asteroids Currently Residing in the Low-$\epsilon$ Venus Co-orbital Reservoir $N_{\text{LEVCO}}$ and the Number of Asteroids in the Venus Co-orbital Zone Currently Crossing the Orbit of Earth $N_{\text{MMR}}$, for Two Values of the Asteroid Absolute Magnitude $H$, Six Asteroid Taxonomic Types, and Three Values of the Differential Size Index $\alpha$

| $\alpha$ | Type | $N_{\text{LEVCO}}(H < 26.3)$ | $N_{\text{MMR}}(H < 26.3)$ | $N_{\text{LEVCO}}(H < 24.0)$ | $N_{\text{MMR}}(H < 24.0)$ |
|---------|------|-------------------------------|--------------------------|--------------------------|--------------------------|
| $-3.0$  | S    | 12,836                        | 0.038                    | 1543                     | 0.005                    |
|         | M    | 11,971                        | 0.036                    | 1439                     | 0.004                    |
|         | E    | 9446                          | 0.028                    | 1136                     | 0.003                    |
|         | C    | 17,194                        | 0.052                    | 2067                     | 0.006                    |
|         | P    | 15,751                        | 0.047                    | 1894                     | 0.006                    |
|         | D    | 14,732                        | 0.044                    | 1771                     | 0.005                    |
| $-3.5$  | S    | 66,332                        | 0.199                    | 4696                     | 0.014                    |
|         | M    | 60,787                        | 0.182                    | 4303                     | 0.013                    |
|         | E    | 45,208                        | 0.135                    | 3200                     | 0.010                    |
|         | C    | 95,579                        | 0.286                    | 6766                     | 0.020                    |
|         | P    | 85,657                        | 0.257                    | 6064                     | 0.018                    |
|         | D    | 78,786                        | 0.236                    | 5578                     | 0.017                    |
| $-4.0$  | S    | 342,780                       | 1.027                    | 14,289                   | 0.043                    |
|         | M    | 308,687                       | 0.925                    | 12,868                   | 0.039                    |
|         | E    | 216,371                       | 0.648                    | 9020                     | 0.027                    |
|         | C    | 531,344                       | 1.592                    | 22,150                   | 0.066                    |
|         | P    | 465,860                       | 1.396                    | 19,420                   | 0.058                    |
|         | D    | 421,382                       | 1.263                    | 17,566                   | 0.053                    |

Note. These numbers are based on the upper limit $N_{\text{obs}} = 18.7^{+30}_{-14}$ from the Pokorný et al. (2020) survey, which was able to detect LEVCO asteroids as small as $D \approx 400-900$ m. Note that the uncertainty of all values is dominated by the uncertainty of the upper limit from the Pokorný et al. (2020) survey, $N = 18.7^{+30}_{-14}$, i.e., all numbers in this table have an uncertainty of a factor of 2.7–4.5.

For parameter 3, since we lack a priori information about the LEVCO population, we will begin with a differential size–frequency distribution $\alpha = -3.5$ (Dohnanyi 1969) but also consider $\alpha = -4$ and $-3$.

Presently, we have only two constraints on the size of the Venus co-orbital population. (1) Pokorný et al. (2020) estimated $N_{\text{obs}} = 18.7^{+30}_{-14}$ to be the upper limit for the number of LEVCO asteroids with $r$-band magnitudes brighter than 21, which corresponds to an absolute magnitude $H \leq 18.9$ for C-type asteroids to $H \leq 19.5$ for E-type asteroids (see Table 3). Note that Ye et al. (2020) performed a similar survey, but the upper limits from this survey were not as strong as those from Pokorný et al. (2020). (2) There are five known Venus co-orbitals (see Table 1), which may or may not have originated in the LEVCO reservoir.

We can use Equation (7) together with $N_{\text{obs}} = 18.7^{+30}_{-14}$ to calculate the normalization constant $N_\alpha$, and then extrapolate the results to asteroid diameters of known Venus co-orbitals and obtain the number of LEVCO asteroids $N_{\text{LEVCO}}(H)$ with $D > D_{\text{min}}$. Here $N_\alpha$ is calculated by assuming that there are $N_{\text{obs}}$ in the diameter range denoted by $D_{\text{Det limit}}$ from Table 3:

$$N_\alpha = N_{\text{obs}}(\alpha + 1) \left[ 1 - \left( \frac{D_{\text{Det limit}}}{D_{\text{max}}} \right)^{\alpha + 1} \right]^{-1} .$$

Now that we have derived an upper limit on $N_\alpha$ from the Pokorný et al. (2020) survey, we can use Equation (3) to determine an upper limit on the number of asteroids currently in Venus’s co-orbital zone that are on Earth-crossing orbits that arose from the LEVCO population, that is, an upper limit on $N_{\text{MMR}}$. Our estimates for these quantities for two different values of the asteroid absolute magnitude $H$ and three different values of $\alpha$ for the six taxonomic types are shown in Table 4. To calculate the numbers in Table 4, we used a maximum diameter of $D_{\text{max}} = 100$ km. Note that the numbers in Table 4 are not sensitive to $D_{\text{max}}$ unless $D_{\text{max}} < 5$ km.
Venus is highly unlikely to be from the LEVCO reservoir. Most of the numbers in these columns ($N_{\text{MMR}}$) are less than 1 by 1–2 orders of magnitude; producing even one asteroid in this category would require a much larger LEVCO asteroid population than is currently permitted by surveys. However, at the bottom of Table 4, we see that steeper power-law distributions ($\alpha = -4.0$) are consistent with the production of the smallest Venus co-orbitals, such as 2015 WZ$_{12}$, which has $H = 26.3$.

In Section 5.1, we determined that one Earth-crossing asteroid in the 1:1 MMR that originated in the LEVCO reservoir implies, on average, a total of 125 Earth-crossing asteroids originating from the LEVCO reservoir. Multiplying this factor by the numbers in Table 4, we find, for example, that for $\alpha = -4.0$, the upper limit $N_{\text{Earth}}(H < 26.3) = 199^{+332}_{-155}$ for C-type asteroids with $H < 26.3$ currently in the Earth-crossing orbits originating from the LEVCO reservoir. These numbers are considerably lower for larger asteroids; for example, for $H < 24.0$, $N_{\text{Earth}}(H < 24.0) = 8^{+4}_{-3}$.

As of 2021 July 15, there are 25,850 known NEOs, where 15,053 of them are in Earth-crossing orbits defined by Equation (1). In other words, current upper limits on the population of LEVCO asteroids suggest that this population is not likely to be a major source of Earth-crossing asteroids.

Based on the estimated total number of asteroids in the LEVCO reservoir (Table 4), only 2015 WZ$_{12}$ could plausibly have originated from the LEVCO population. The other known Venus co-orbital asteroids are more likely to be migrants from the main belt, as suggested by Morais & Morbidelli (2006).

### 5.3. Total Mass of the Venus Co-orbital Asteroids

Knowledge of the number of LEVCO asteroids, $N_{\text{LEVCO}}$, allows us to calculate the total mass, $M_{\text{LEVCO}}$, of the bodies in this reservoir. Using the same parameters as above, $N_{v}$, $D_{\text{min}}$, $D_{\text{max}}$, $\alpha$, we can derive the current mass of LEVCO asteroids using the following equation:

$$
M_{\text{LEVCO}} = \begin{cases} 
M_{\text{max}} \frac{N_v}{\alpha + 4} \left[ 1 - \left( \frac{D_{\text{min}}}{D_{\text{max}}} \right)^{4+\alpha} \right] & \text{if } \alpha = -4 \\
M_{\text{max}} N_v \ln \left( \frac{D_{\text{max}}}{D_{\text{min}}} \right) & \text{if } \alpha = -4 
\end{cases}
$$

where $M_{\text{max}} = \pi \rho D_{\text{max}}^3 / 6$ is the mass of the largest assumed asteroid in the population with diameter $D_{\text{max}}$ and bulk density $\rho$. Having an upper limit on the total number of asteroids larger than a certain diameter, e.g., that from Pokorný et al. (2020), allows us to calculate the normalization constant $N_v$ and then extrapolate upper limits on the size and mass distributions in a desired size range.

We calculated upper limits on $M_{\text{LEVCO}}$ for the six different asteroid taxonomic types and three different values of the differential size index $\alpha = [-3.0, -3.5, -4.0]$ using Equation (9); Table 5 summarizes these calculations. For the calculation of these upper limits on $M_{\text{LEVCO}}$, we used $D_{\text{min}} = D_{\text{MB},26.3}$ from Table 3 and $D_{\text{max}} = 100$ km. For $\alpha = -3.0$ and $-3.5$, the upper limit on $M_{\text{LEVCO}}$ is strongly dependent on $D_{\text{max}}$ because $D_{\text{max}} / D_{\text{min}} \gg 1$ and $(\alpha + 4) > 0$; i.e., in this case, most of the mass resides in the largest asteroids. Here $M_{\text{LEVCO}}$ scales as $D_{\text{max}}^{-\alpha - 4}$ for $\alpha > -3.5$. For $\alpha = -4.0$, the mass is distributed log-uniformly for different diameters; thus, in this case, changing $D_{\text{max}}$ and $D_{\text{min}}$ by $\pm 1$ order of magnitude yields only a 20%–30% difference from the values presented in Table 5.

Besides the unknown $D_{\text{max}}$, the main source of uncertainty in our upper limits on $M_{\text{LEVCO}}$ lies in the observational upper limit estimate $N_{\text{obs}} = 18^{+36}_{-14}$. Assuming the upper limit $N_{\text{obs}} \lesssim 48$, the maximum value of $M_{\text{LEVCO}} \lesssim 1.12 \times 10^{16}$ kg corresponds to P-type asteroids with $\alpha = -3.0$ due to the smallest albedo of P-types assumed here, $p_V = 0.042$. This value is 5 orders of magnitude smaller than the estimated mass of the main belt, $M_{\text{MB}} = 2.64 \pm 0.04 \times 10^{21}$ kg (Kuchyna & Kuehner 2013). Interestingly, if we assumed that (322756) 2001 CK32 originated from the LEVCO reservoir, this origin would imply an $M_{\text{LEVCO}} \sim 10^{17}$–$10^{20}$ kg reservoir of LEVCO asteroids, i.e., almost as much mass in the LEVCO reservoir as in the main belt.

Let us compare our mass estimates for the LEVCO reservoir to this mass region would have if it simply had the same spatial density as the main belt. Using the model LEVCO asteroid maximum inclination of $I_{\text{max}} = 10^\circ$ and the maximum eccentricity $e_{\text{max}} = 0.1$ from Pokorný & Kuchner (2019), we can estimate the volume of the LEVCO reservoir as

$$
V_{\text{LEVCO}} = \frac{4}{3} \pi (r_3^3 - r_1^3) \sin(I_{\text{max}}) = \frac{4}{3} \pi (Q^3 - q^3) \sin(I_{\text{max}}),
$$

where $r_1 = Q = a_{\text{max}} (1 + e_{\text{max}}) = 0.804$ au is the outer boundary of the reservoir, and $r_1 = q = a_{\text{max}} (1 - e_{\text{max}}) = 0.644$ au is the inner boundary. Using these parameters, we find that the volume of the LEVCO reservoir is $V_{\text{LEVCO}} = 0.18$ au$^3$. Similarly, we can estimate the volume of the main belt using $r_1 = 3.5$ and $r_1 = 2.0$ au and $I_{\text{max}} = 20^\circ$, obtaining $V_{\text{MB}} = 49.96$ au$^3$. The ratio between these two volumes is $V_{\text{MB}} / V_{\text{LEVCO}} = 272$. If we scale the mass of the current main belt to the volume of the LEVCO reservoir using the volume ratio of 272, this would yield a mass of $\sim 10^{16}$ kg, about 1000 times greater than the maximum value of $M_{\text{LEVCO}}$ we calculated above based on the observed upper limits.

Pokorný & Kuchner (2019) showed that 8% of low-e Venus co-orbital asteroids can be stable for 4.5 Gyr assuming that the planetary configuration was similar to today’s solar system architecture, i.e., ignoring early solar system planetary migration (see, e.g., Section 3.3 in Raymond & Morbidelli 2020). Therefore, we can estimate upper limits on the primordial total mass of the low-e Venus co-orbital reservoir $M_{4.5 \text{ Ga}}$ by simply dividing $M_{\text{LEVCO}}$ by 8%. Table 5 lists values of $M_{4.5 \text{ Ga}}$ for different taxonomic types and values of $\alpha$, estimated in this manner.

In summary, based on the currently available surveys of the LEVCO asteroid population, we estimate upper limits on the current total mass of this population of $M_{\text{LEVCO}} \lesssim 10^{13}$–$10^{16}$ kg. These upper limits on the present-day mass translate into upper limits on the primordial mass of $M_{4.5 \text{ Ga}} \lesssim 10^{14}$–$10^{17}$ kg. 4.5 Ga based on how the population decays via dynamical evolution (Pokorný & Kuchner 2019).

### 6. Significance of the Yarkovsky Effect

Pokorný & Kuchner (2019) analyzed the stability of the asteroid population in Venus’s co-orbital zone while omitting
To estimate the values in this table, we consider asteroid diameters between the values from our simulation. The bootstrapping analysis shows higher survival rates than those recorded in the simulation results. The solid lines show the mean remaining population based on our bootstrapping analysis, where the shaded areas represent the 1σ interval. The dashed lines show the values from our simulation. The bootstrapping analysis shows higher survival rates than those recorded in the simulations. Note that the simulation results (dashed lines) are outliers in the 1σ interval (shaded area) in that they tend to produce results lower than the bootstrapping mean.

The Yarkovsky effect, which can significantly change the semimajor axis of an asteroid (see Bottke et al. 2006, for a review) over timescales of millions of years. In short, the Yarkovsky effect is a force caused by anisotropic thermal emission of photons on a rotating body in space, which results in a momentum change. This effect depends on many material- and shape-related properties of the asteroid, such as the thermal conductivity $K$, specific heat at constant pressure $C_p$, bond albedo $A$, thermal emissivity $ε$, asteroid diameter $D$, and material density $ρ$. The asteroid’s shape, spin-axis orientation, and rotation period all affect the impact of the Yarkovsky effect on perturbations of the asteroid’s $da/dt$. There are currently no observed LEVCO asteroids to suggest constraints on these parameters.

While hampered by the lack of constraints on the asteroid parameters, we nonetheless forged ahead with an illustrative simulation to show the order of magnitude of the effect, with parameters borrowed from the known NEAs. We took 100 random asteroids from the initial population in the Pokorný & Kuchner (2019) simulation and followed their dynamical evolution over 1 Gyr with an added drag force that causes secular changes in their semimajor axes. We tried six different values of the drag force, resulting in six different drag rates: $da/dt = \pm 4 \times 10^{-4} \text{ au Myr}^{-1}$ at a heliocentric distance of $r = 1 \text{ au}$. These values represent the currently observed distribution of Yarkovsky effect–induced $da/dt$ among NEAs (Greenberg et al. 2020). The semimajor axis drift scales with the heliocentric distance $r$ as $\propto 1/r$, so the drift is $\sim 1.38$ times faster for the asteroids in our simulation that are co-orbiting with Venus on low-eccentricity orbits. We used the same numerical integrator as in Pokorný & Kuchner (2019), i.e., SWIFT (Levison & Duncan 2013), adding the drag force at each time step of 3.6525 days in the simulation. Using the same LEVCO asteroid removal condition as in Pokorný & Kuchner (2019), we removed asteroids from the simulation when their semimajor axes were outside 0.7088 au $< a < 0.7378$ au.

Figure 5(a) summarizes the results from our simulation; each curve shows how the population evolves in time, given a different assumed drag force. The simulation with no drag force at all exceeds the expectation that $<10\%$ of the asteroids are still here at the end of 1 Gyr. A random choice of $\pm 40\%$ in $da/dt$, however, results in roughly $\sim 1\%$ of asteroids surviving to the end of our time slice. The bootstrapping mean values show much lower survival rates than the expectations.
(black curve) had \(S_{1\text{Gyr}} = 16\%\) of survivors after 1.0 Gyr. The two strongest drift values, \(\frac{da}{dt} \leq 4 \times 10^{-4} \text{ (au Myr}^{-1})\) continued beyond 1 Gyr, and simulations with \(\frac{da}{dt} = \pm 4 \times 10^{-4} \text{ (au Myr}^{-1})\) had \(S_{1\text{Gyr}} = 3\%\) (negative drag) and 7\% (positive drag) of survivors. From the population decay record in Figure 5, we see that the positive drag in \(a\) is less destabilising for the LEVCO population, leading to approximately 70\% longer survival times than the negative drag in \(a\).

These survival rates, modeled by only a handful of particles, suffer from substantial Poisson noise. To reveal the underlying statistics of our simulations, we performed bootstrapping with replacement (Figure 5(b)), resulting in the following survival rates and uncertainties: \(S_{1\text{Gyr}} = 20.00\% \pm 3.99\%\) (for no drag), \(S_{1\text{Gyr}} = 13.01\% \pm 3.36\%\) (for \(\frac{da}{dt} = 4 \times 10^{-4}\) (au Myr\(^{-1}\))), and \(S_{1\text{Gyr}} = 5.99\% \pm 2.3\%\) (for \(\frac{da}{dt} = -4.0 \times 10^{-4}\) (au Myr\(^{-1}\))). We can extrapolate these population decay rates to 4.5 Gyr by fitting the population decay curves with \(f(x) = a \log_{10}(x) + b\). These fits show that two of the cases we examined yield positive survival rates after 4.5 Gyr of evolution. Here \(S_{10\text{Gyr}} = 5.4\%\) for the no drag simulation, and \(S_{1\text{Gyr}} = 0.5\%\) for \(\frac{da}{dt} = 4.0 \times 10^{-4}\) (au Myr\(^{-1}\)). For comparison, Pokorný & Kuchner (2019) reported that 8.2\% of the asteroids remained stable for 4.5 Gyr using 10,000 asteroids, a number that is consistent with our extrapolation given the Poisson noise. Crucially, note that our experiment with Yarkovsky drag at the level of \(\frac{da}{dt} = 4 \times 10^{-4}\) (au Myr\(^{-1}\)) also yielded survivors at 1 Gyr and when extrapolated to 4.5 Gyr. Data from Greenberg et al. (2020) show that 15 out of 247 NEAs (6\%) in their sample have \(\frac{da}{dt} < 4 \times 10^{-4}\) (au Myr\(^{-1}\)), where the number of retrograde rotators \(\frac{da}{dt} < 0\) is 2.7 times higher than the number of prograde rotators.

We emphasize that our simple model, with a small sample, is far from the rigorous treatment of the Yarkovsky and YORP effects required to correctly assess their influence on the dynamical stability of LEVCO asteroids. Yet it does suggest that for drag forces that are plausible based on our understanding of NEAs, LEVCO asteroids can remain dynamically stable in the Venus co-orbital region for 4.5 Gyr. Future work could improve our understanding of this issue using full-fledged numerical integrators (e.g., SWIFT_MVS2_PP_YE_YORP; Brož et al. 2011) and a broad suite of physical and thermal characteristics of asteroids.

### 7. Conclusions

In this paper, we explored the efficiency of the dynamical transport of asteroids from the LEVCO reservoir into Earth-crossing orbits. We found that more than 90\% of asteroids escaping the Venus co-orbital reservoir become Earth-crossing asteroids at some point in their dynamical evolution, and that they reside in the Earth-crossing zone for 22.5 million yr, on average. This Earth-crossing time is an order of magnitude longer than the dynamical lifetimes of asteroids escaping the main belt, with the exception of the asteroids escaping the Hungaria and Phocaea families (Granvik et al. 2018).

If we assume that LEVCO asteroids are the source of Earth-crossing Venus co-orbitals, we find that, on average, one Venus co-orbital Earth-crosser points to 333,700 currently existing LEVCO asteroids of the same size. If we look instead at all Earth-crossers originating in Venus’s co-orbital zone (not just those co-orbiting with Venus), we see that for every 2685 asteroids currently in the LEVCO reservoir, there should be one asteroid currently on an Earth-crossing orbit that escaped from the LEVCO reservoir.

Using the latest results from the Venus co-orbital asteroid survey (Pokorný et al. 2020), we estimated the upper limit on the total number of asteroids with \(H < 26.3\) currently co-orbiting Venus at low-eccentricity orbits to be between 10,000 and 1,000,000. This number is potentially high enough to suggest that 2015 WZ\(_{12}\), the smallest currently known Venus co-orbital, originated in the LEVCO asteroid population. We estimated that <500 of 10–40 m asteroids (\(H < 26.3\)) originating from the LEVCO reservoir are currently on Earth-crossing orbits. All four larger known Venus co-orbitals are unlikely to have originated from this population, since they would require orders of magnitude more LEVCOs to exist than what is consistent with current surveys.

We estimated the mass of the LEVCO population to be \(10^{13}–10^{16}\) kg, which translates into \(10^{12}–10^{17}\) kg of the primordial mass for the LEVCO reservoir, 4.5 billion yr ago.

Our tests of the influence of the Yarkovsky effect showed that both the positive and negative Yarkovsky drag have destabilizing effects on the LEVCO population, where the population decays faster, the stronger the Yarkovsky effect. However, our extrapolated results showed that even with the Yarkovsky effect, 0.5\% of LEVCOs survive for the age of the solar system, assuming they rotate in the retrograde direction and have an average drag \(\frac{da}{dt} < 4 \times 10^{-4}\) (au Myr\(^{-1}\)). Despite these upper limits on the prevalence of LEVCO asteroids, this population remains fascinating; there could be an interesting subset of NEAs that originated in low-eccentricity orbits near Venus bearing mineralogical records of the inner solar system.

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**References**

Botke, W. F. J., Vokrouhlický, D., Rubincam, D. P., & Nesvorny, D. 2006, *AREPS*, 34, 157.
Brasser, R., Inman, K. A., Connors, M., et al. 2004, *Icar*, 171, 102.
Brož, M., Vokrouhlický, D., Morbidelli, A., Nesvorny, D., & Botke, W. F. 2011, *MNRAS*, 414, 2716.
