Solar Flare Modified Complex Network

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Received 2019 December 10; revised 2020 March 23; accepted 2020 March 23; published 2020 May 6

Abstract

A constructive approach is developed to build the solar flare complex network by utilizing a visibility graph condition alongside the Abe–Suzuki method. Solar flare information such as position, start time, and peak flux is used for this purpose. The obtained characteristics of the topological features (such as the characteristic path length, power-law behavior of the probability distribution function of degrees, and the clustering coefficient) demonstrate the scale-free and small-world properties of the solar flare modified network. To explain the complexity of the constructed network, Omori’s law as well as the universal scaling features are investigated. Furthermore, a nonextensive modification of the Gutenberg–Richter law is examined for the solar flare modified network using a q-stretched exponential model. Establishing a two-dimensional map for the configuration of 118 energetic main flares observed between 2006 and 2016, it is found that the main flares are located within the regions consisting of hubs (high-connectivity regions) of the network. A fractal dimension of the solar flare network of about 0.79 is also obtained.

Unified Astronomy Thesaurus concepts: Solar flares (1496); Solar physics (1476); The Sun (1693); Astronomy data modeling (1859); Astronomy databases (83); Astronomy data analysis (1858); Time series analysis (1916); Neural networks (1933); Solar active regions (1974); Computational methods (1965); Computational astronomy (293); Astronomical simulations (1857)

1. Introduction

Solar flares are magnetic features on the Sun’s surface that are accompanied with highly energetic explosions involving huge and rapid releases of energy. Studies show that during solar flares, the complexity of the coronal magnetic field significantly increases within the active regions (ARs; Priest & Forbes 2002; Aschwanden 2006). The frequency–size distribution of solar flares is observed to follow a power-law behavior, which is a key characteristic of complex systems. Due to the stochastic nature of solar flares and the fact that their underlying physical mechanism has not yet been fully understood, probabilistic methods are widely applied in order to study the solar flares’ statistics (Wheatland 2005; Barnes & Leka 2008; Raboony et al. 2016; Alipour & Safari 2019).

Owing to the fact that complex systems generally follow the same empirical laws (DeArcangelis et al. 2006), the complex network approach has recently been applied to study the properties of these systems in a variety of areas (e.g., earthquakes, neuroscience, business, physics, chemistry, biology, text mining, and solar physics). Watts et al. (1998) performed a detailed study of regular, random, scale-free, and small-world networks. The time series of the seismic phenomena were analyzed using the complex network theory by Pastor et al. (2018). The Abe–Suzuki model (hereafter the AS model) was applied to extract various properties of the earthquake networks (Abe & Suzuki 2004a, 2004b, 2005, 2007; Darooneh & Dadashinia 2008; Abe & Suzuki 2009; Darooneh & Mehr 2010; Abe et al. 2011; Lotfi & Darooneh 2012; Rezaei et al. 2017). Complexity features such as nonlinearity, universal scaling, self-organized criticality, limited predictability, and long-range interaction with long-temporal correlation in different systems were also investigated (Lu & Hamilton 1991; Carreras et al. 2004; Dobson et al. 2007; Barnes & Leka 2008; Lippiello et al. 2008; Alipour & Safari 2015; Aschwanden et al. 2016), as well as the waiting-time distribution of earthquakes (Corral 2004) and solar flares (Wheatland et al. 1998).

A probabilistic prediction model for solar flares based on their spatial location on the Sun’s surface has not yet been developed. The context of the complex network theory may be useful to study the properties of the solar flare system, since the extracted information of an appropriate network can reveal many features of the system (Barabási et al. 1999; Albert & Barabási 2002; Dorogovtsev & Mendes 2003; Daei et al. 2017; Gheibi et al. 2017). In this study, a modified model is implemented to construct the solar flare complex network and to examine its topological properties. To this aim, the latitudes and longitudes of the main flaring events observed between 2006 and 2016 are mapped. The AS approach is adopted to construct the modified network based on the spatial locations and start times of solar flares. The visibility graph (hereafter VG) condition is applied with regard to the occurrence time and size of the flaring events (Lacasa et al. 2008; Rezaei et al. 2017). The complexity of the modified network is highlighted by performing a survey on the two well-known classical laws, namely the Gutenberg–Richter (hereafter G–R) law (Gutenberg & Richter 1954) and Omori’s law (Omori 1894).

A modified version of the G–R law is investigated applying the nonextensive statistical mechanics approach for the complementary cumulative distribution of the number of links. This approach is established based on the maximization of the Tsallis entropy (Tsallis 1988; Tsallis & Brigatti 2004) under particular physical constraints and could predict various features of the subject system (Telesca 2010). Furthermore, the fractal dimension of the modified solar flare network is calculated using the Box–Counting method. The resulting...
probability distributions are investigated using both maximum
the likelihood estimation (hereafter MLE) technique in
the Bayesian framework and the genetic algorithm (Farhang et al.
2018).

In Section 2 the solar flare data is introduced. The model and
the applied techniques are described in Section 3. The results
are discussed in Section 4. Finally, the conclusion is presented
in Section 5.

2. Data

The information of 14,395 solar flares registered in the
Lockheed Martin Solar and Astrophysics Laboratory (LMSAL)
Latest Event Archive between 2006 January 1 and 2016 July
21 are studied. The data is available online at https://www.
lmsal.com/solarsoft/latest_events_archive.html. The LMSAL
Latest Event Archive includes information like start time, peak
time, end time, flare classification (according to the Geoosta-
ionary Operational Environmental Satellites (GOES) flare list,
solar flares are classified into five categories, A, B, C, M, and
X, based on their strength), NOAA number (a particular four-
digit number assigned to each sunspot, which helps to track the
sunspot group as it rotates), and peak flux. The positions of
flares span from 90 N to −90 S in latitude and 180 E to −180W
in longitude. Corrections concerning the differential rotation
of the Sun are applied to the data (Alipour et al. 2012; Alipour &
Safari 2015; Honarbakhsh et al. 2016; Yousefzadeh et al.
2016). These corrections are applied on latitude and longitude
of the flares relative to the coordinate of the first flare registered
in the catalog.

3. Model and Methods

3.1. Solar Flare Modified Complex Network

Following the AS model, here we construct the solar flare
modified complex network using the location (latitude and
longitude) of flares and their occurrence time. The solar
spherical surface is divided into smaller grids (same-sized cells,
namely resolution) iso-longitudinally. There is no overlap
between the regions on the solar coordinate. The coordinates of
each cell in the spherical coordination (θ, φ) are given by:

\[
\phi_{i+1} = \phi_i + \frac{2\pi}{n}, \quad \phi \in [-180^\circ, 180^\circ],
\]

\[
\sin(\theta_{j+1}) = \sin(\theta_j) - \frac{2\pi}{n}, \quad \theta \in [-90^\circ, 0^\circ],
\]

\[
\sin(\theta_{j+1}) = \sin(\theta_j) + \frac{2\pi}{n}, \quad \theta \in [0^\circ, 90^\circ],
\]

where \(n\) is the resolution. Then, the flare network is constructed
using edges (links) and loops. Empty cells with no events
are neglected. Therefore, the solar flare network is developed
based on the occurrence time and energies of flaring events under
the VG condition. The filling factor of nodes over the solar surface,
\(N/n^2\) in which N is the number of nodes, varies between 0.59
and 0.45.

Also following the Telesca–Lovallo model (hereafter the TL
model), the VG condition is considered as (Lacasa et al. 2008;
Telesca 2010; Rezaei et al. 2017):

\[
F_c < F_b + (F_a - F_b) \frac{t_b - t_c}{t_b - t_a},
\]

In this condition, any event, \(a\), with peak energy, \(F_a\),
representing the class of flare at time, \(t_a\), is connected to event
\(b\) with peak energy, \(F_b\), at time, \(t_b\). Event \(c\) with peak energy,
\(F_c\), is located between these two events if \(t_a < t_c < t_b\).

Therefore combining the AS and TL models, the new
modified model is developed for the solar flare network. Using
the modified model, the statistical properties of the flaring
events are investigated. As will be discussed in detail in the
following, it is observed that applying the modified model in
construction of the solar flare network results in both the G–R
and Omori’s laws. The VG condition leads to an increase in the
number of links, while the number of nodes remains unchanged.
This means that there is not much difference in the
computation time compared to Gheibi et al. (2017).

We also calculated the adjacency matrix (a two-dimensional
matrix in which the rows and columns represent the
graph vertices). In Figure 1, a small part of the solar flare
modified network is presented. The loops and links between
the nodes of the graph are shown. A similar graph is presented by
Gheibi et al. (2017) for the same flare information including the
same node numbers (see Gheibi et al. 2017, Figure 4 therein).

The main difference between the two models is the application
of the VG condition. For example, as shown in Figure 1, node
980 with EName gev 20101115 030 is connected to node
1264 with EName – 0303 is connected to node 1264 with EName gev – 20150511 – 0345 while they were not connected in the Gheibi et al. model. Here, we used the undirected graph to present a modified small-world
network.

3.2. Modeling the Probability Distributions

The power-law behavior is a well-known characteristic of the
distribution of events in scale-free systems. The ideal
power-law distribution for the frequency of events with sizes
\(S = s_1, s_2, \ldots, s_M\) is:

\[
P(S) \sim S^{-\gamma},
\]

where \(P(S)\) is the probability distribution, and \(\gamma\) is the power
index. In practice, the power-law behavior only holds over a
limited range of sizes due to factors like lack of information at
small sizes and background contamination. An alternative
approach to model distributions that depart from the ideal
power law is to examine them against the threshold power-law
distribution (Aschwanden 2015; Farhang et al. 2018):

\[
P(S) dS = P_0 (S + S_0)^{-\gamma} dS,
\]

where \(P_0, S_0,\) and \(\gamma\) are the normalization constant, threshold,
and power index, respectively. Considering the possibility of
the finite size effect, \(P_0\) is extracted from normalization of
Equation (6) over the finite range \(S_1 \leq S \leq S_2\), where \(S_2\) is the
largest size in the distribution. Therefore,

\[
P_0 = (\gamma - 1) [(S_1 + S_0)^{-\gamma + 1} - (S_2 + S_0)^{-\gamma + 1}]^{-1}.
\]

Further to this approach, distributions with power-law
behavior in their tails could be described by a \(q\)-exponential
function (Tsallis & Brigatti 2004; Lotfi & Darooneh 2013;
Farhang et al. 2019):

\[
P_e(S) = \frac{e_{-\beta}^q}{(1 + \beta (q - 1) S)^{1/q}},
\]

where \(\beta, q,\) and \(P_e(S)\) are the events rate, nonextensive
parameter, and the complementary cumulative distribution for
events with sizes larger than $S$, respectively. The simple exponential distribution is achieved as $q$ approaches unity.

In the present study, the thresholded power-law function is used as well as the $q$-exponential function to model the distribution of degree of nodes. The distributions are modeled against the thresholded power-law function by applying both the chi-square test with genetic algorithm and MLE in the Bayesian framework (see Farhang et al. 2018 for further details). The first method estimates the model parameters through an iterative cycle of search and selection (Mitchell 1996; Haupt & Haupt 1998; Kramer 2017), whereas the later method avoids the graphical errors of the linear least-square fit (Bai 1993; Goldstein et al. 2004; Newman 2005; Barnes & Leka 2008; Clauset et al. 2009; Giles et al. 2013).

4. Result

We developed a modified model to construct the solar flare network based on the AS model, which also satisfies the VG condition. For this purpose, the solar surface is divided into equally sized areas. Various resolutions are studied. The time series of flare energies for 14,395 flaring events between 2006 January 1 and 2016 July 21 is presented in Figure 2. Based on the VG condition, larger events could connect to both small and intermediate events, which results in greater degrees of connectivity for the vertex of the main flares.

Applying the detrended fluctuation analysis technique (Javaherian et al. 2017), the Hurst exponent is calculated equal to 0.86. This value emphasizes the existence of a long-temporal correlation (self-affinity) in the system of flares.

The properties of the modified model are shown in the adjacency matrix. The number of links are expressed by counting the nondiagonal entries. All possible tadpoles (links from a node to itself) in the network are omitted. Therefore, the adjacency matrix is constructed with diagonal entries equal to zero. The adjacency matrix is a cornerstone in calculation of the topological properties of the solar flare network. Properties such as clustering coefficient, characteristic length, and degree distribution together with some of the statistical features of networks like Omori’s law, G–R law, and fractal dimension are studied.

The dependency of the network properties on the grid sizes is also investigated by performing the same analysis for different resolutions ($n = 44$–$n = 88$). The number of nodes available in the resulting network is obtained to vary from 1147 to 3505 and 47,808 to 64,154 links. In Figure 3, the relation between the number of nodes (links) and resolutions is displayed. It seems that the number of links saturates as the

Figure 1. A small part of the solar flare network is presented. The nodes of the network are chosen regarding the position, start times, and energies of the flaring events.
resolution increases. This issue has already been investigated for seismic phenomena (Lotfi & Darooneh 2012).

Another important topological parameter is the clustering coefficient (Watts et al. 1988):

$$c_i = \frac{2t_i}{k_i(k_i - 1)},$$

where $k_i$ and $c_i$ are the numbers of neighbors and the local clustering coefficient, respectively. The number of links available between node $i$ and its neighbors is stated by $t_i$. The average clustering coefficient is given by:

$$C = \frac{1}{N} \sum_{i=1}^{N} c_i,$$

where $N$ is the number of nodes.

Furthermore, the clustering coefficients of a random network and a regular network are (Albert & Barabási 2002):

$$C_{\text{rand}} \simeq \frac{\langle k \rangle}{N},$$

$$C_{\text{reg}} \simeq \frac{3\langle k \rangle - 1}{4\langle k \rangle - 1},$$

respectively, where $\langle k \rangle$ indicates the average of the degree of nodes. The study of the average local clustering coefficient helps us to distinguish a scale-free network from a random network. According to Equations (11) and (12), the clustering coefficient of a random/regular network is independent of the degree of nodes.

In Figure 4, the clustering coefficients of the solar flare modified network and its equivalent random network (with the same size) $C_{\text{rand}}$ versus the resolution are presented. As shown in the figure, the ratio $\frac{C_{\text{rand}}}{C_{\text{reg}}}$ increases with the network resolution. A comparison between the ratio of the clustering coefficient of the modified network and its equivalent random network demonstrates that the modified network is not random. Moreover, we want to compare the clustering coefficient of the solar flare modified network with the previously reported clustering coefficient (Gheibi et al. 2017). In the modified network, the number of links is higher than the previous research due to application of the VG condition. Therefore, the clustering coefficient of the modified solar flare network is larger.

Another way to determine a scale-free network from a random network is to study the behavior of the ratio $k/k_{\text{max}}$. This parameter could ensure that the flare network contains...
hubs, namely, the primary property of the scale-free networks. Also, the obtained power-law exponent ($\gamma > 3$) indicates that the flare network is small-world (Cohen & Havlin 2003; Bollobás & Riordan 2004). In Figure 5, a power-law behavior is observed for the average degree of nodes versus the resolution. The power index is obtained equal to 1.14 ± 0.04. Similarly, the maximum degree of nodes is observed to increase with the resolution with a degree exponent of about 0.72 ± 0.08. Moreover, the ratio $k/k_{\text{max}}$ increases with resolution, which indicates the scale-free behavior of the solar flare modified network with an exponent of about 1.89 ± 0.06. The goodness of fit in the Figure 5 is 0.99.

The characteristics path length is a suitable parameter to investigate the scaling property of the solar flare complex network, which is defined by:

$$\Lambda = \frac{1}{N(N-1)} \sum_{i,j=1}^{N} l_{ij},$$

where $l_{ij}$ is the distance between a pair of nodes.

Figure 6 shows the dependency of the characteristic path length on resolution for different networks. In panel (a), the degree exponent for the characteristic path length of the modified network is calculated to be 0.29 ± 0.01. The characteristic path length of a random network of the same size is presented in panel (b). In this case, the degree exponent is obtained 0.46 ± 0.01. The degree exponent of a regular network, panel (c), is equal to 2.84 ± 0.04. The ratio of the average path length to its equivalent random network is shown in panel (d), which follows a power law. The power index is
calculated to be $0.16 \pm 0.01$. The comparison between the average path length of the solar flare modified network and its equivalent random network indicates the scale-free behavior in the solar flare modified network (Boccaletti et al. 2006).

Previously, Aschwanden (2015) and Farhang et al. (2018) showed that the frequency–size distribution of solar and stellar flares, or any other distribution with departures from the ideal power-law behavior at small sizes, could be well-described by a thresholded power-law distribution. Here, we apply the thresholded power-law function of Equation (6) to study the obtained probability distribution function (PDF) of the degree of nodes for different resolutions. The results are shown in Figure 7. The power indices are calculated by minimization of the chi-square function through an iterative cycle of search and selection (due to the application of the genetic algorithm). As seen in the figure, the power indices are larger than 3 for all the resolutions, which clearly represents the small-world characteristic of the modified network.

The same procedure is applied to study the PDFs of the clustering coefficients for different resolutions (Figure 8). The power indices obtained for resolutions $n = 44, 62, 74,$ and $88$ are $0.44 \pm 0.01, 0.59 \pm 0.01, 0.63 \pm 0.01,$ and $0.78 \pm 0.01$, respectively. The observed power-law behavior in the clustering coefficient distribution confirms the scale-free nature of the modified solar flare network. The clustering coefficients of the hubs for the modified network are obtained and have small values.

Omori’s law is a well-known law in various fields such as solar phenomena, seismology, etc. (Omori 1894; UTSU 1955). It states that the frequency of events decays after the occurrence of a main event. In other words,

$$\frac{dn}{dt} \sim (t - t_r)^{-m},$$

where $t_r$ is the occurrence time of the main event. We investigate Omori’s law by calculating the decay rate in the number of links after a main flare occurs with respect to its occurrence time. Hence, we consider the solar events as nodes in the TL network. Empirically, it is observed that the intensity of the main events is greater than the subsequent events. Therefore, applying the VG condition, all subsequent events are connected to the main event. In other words, each link represents an event after the main burst.

Rezaei et al. (2017) calculated the rate of changes (decay rate) in the weight of links for the earthquakes and obtained that it follows Omori’s law. Pursuing the same procedure, we calculated the rate of change in the cumulative number of links

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Figure6.png}
\caption{(a) Dependency of the characteristic path length of the solar flare modified network, (b) a random network with the same size, (c) and a regular network with resolution in a log–log scale. (d) The ratio of the characteristic path length of the solar flare modified network to its equivalent random network.}
\end{figure}
over the elapsed time $t$ starting from a main event. We performed this analysis for different choices of the main flare at different resolutions. The results for some of the $X$-type main flares are shown in Figures 9 and 10. In Figure 9, the ratio of the number of links to the elapsed time, $n(t)/t$, is displayed where $n(t)$ is the cumulative number of links per unit time and $t$.
represents days elapsed from the main flare. The start times of some of the selected main flares for \( n = 44 \) are illustrated in this figure. The selected main flares at each panel are \( X = 6.9 \), \( X = 1.5 \), \( X = 2 \), \( X = 1.8 \), and \( X = 2.1 \), respectively. The power indices in all cases approximately approach unity, which confirms Omori’s law. This means that in the modified model the frequency of large events decays after the main flare, expectedly.

**Figure 8.** PDFs of the clustering coefficient for the different resolutions of the solar flare modified network. The PDFs are modeled against a thresholded power-law function applying the genetic algorithm. The power indices obtained are about \( 0.44 \pm 0.01, 0.59 \pm 0.01, 0.63 \pm 0.01 \), and \( 0.78 \pm 0.01 \), for \( n = 44, 62, 74 \), and \( 88 \) respectively.
In Figure 10, the dependency of the adopted resolutions on the choice of the main flare is shown. It is observed that the power index is approximately constant with the resolution. For instance, the power index obtained for the main flare with magnitude $X = 2.1$ that occurred on 2015 March 3 is about 0.995 at resolutions $n = 44, 62, 74,$ and 88. This indicates the...
convergence of the universality of the scaling features for solar flares as well as the Earth’s seismic events (Rezaei et al. 2017, Figure 6 therein).

As the modified network grows with time, new nodes and links are added to the network. In Figure 11, the cumulative number of links versus time (days elapsed from the main flare) are shown for two X-type flares ($X = 2$ and $X = 2.1$ in panels (a) and (b), respectively) at resolutions $n = 62$, $n = 74$, and $n = 88$, respectively.

The G–R law is an empirical equation that explains the relation between the size of a seismic event and the number of events with sizes higher than $M$:

$$
\log_{10} N(M) = a - bM,
$$

where $N(M)$ is the number of events with size equal to or greater than $M$, $b$ is a scaling parameter and $a$ is a constant. The G–R law describes the statistics of systems with exponentially shaped frequency–size distributions (Gutenberg & Richter 1944). The

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**Figure 10.** Ratio of the number of the links to days elapsed from a main flare, $n(t)/t$, for some of the selected main flares with magnitude (a) $X = 6.9$, (b) $X = 1.5$, (c) $X = 2$, (d) $X = 1.8$, and (e) $X = 2.1$ at different resolutions ($n = 44$, $n = 62$, $n = 74$, and $n = 88$).
G–R law is one of the most famous paradigms of self-similarity (Davidsen & Baez 2016).

The thresholded power-law model is investigated in the modified model and the power index is estimated by using the MLE technique. The results are shown in Figure 13. The power indices obtained are about 4.19 ± 0.27, 3.33 ± 0.05, 2.18 ± 0.05, and 3.11 ± 0.11, for \( n = 44, 62, 74, \) and 88, respectively.

The scale-free behavior (also referred to as power-law behavior) manifests the inverse functionality of the frequencies on the event sizes. This characteristic is usually investigated by performing a comparison between a power-law (or a thresholded power-law) model and the experimental distribution. In this study, further to this approach, we have also applied the nonextensive Tsallis statistics (Tsallis 1988) to study the time series of solar flares (Equation (8)). In the nonextensive context, the temporal distributions of successive shocks could be described by a \( q \)-exponential function (Farhang et al. 2019). This approach is also found to be consistent with the Earth’s seismic phenomena in various seismic regions (Silva et al. 2006; Darooneh & Dadashnia 2008; Telesca 2010; Telesca & Chen 2010). Applying the nonextensive statistical mechanics for solar flare sequences, the value of \( q \) is obtained to be 1.78 and 1.71 for \( n = 62 \) and \( n = 88 \), respectively (Figure 12).

Figure 14 represents the result of mapping the positions of 118 energetic main flares specified by GOES classification on the solar surface in two dimensions. The studied flares accumulate within two bands between latitudes 10 and 20 (north and south). The positions (longitude and latitude) of these flares are located on regions consisting of hubs of the network. The probability/occurrence rate of flaring events on cells consisting of hubs is considerably greater than other cells.

The solar flare network model presented by Gheibi et al. (2017) could predict the probability of occurrence for only a few flares of M and X types in the hubs. However, application of the VG condition in construction of the modified network (AS + VG) provides the capability to compare the exact regions of all 118 studied main solar flares with positions of the hubs (Figure 14).

The fractal dimension of the modified network is calculated by using the Box–Counting method. In this method, the dimension \( D \) is (Falconer 2003):

\[
D = \frac{\log(N)}{\log(r)},
\]

where \( N \) is the number of boxes that cover the pattern, and \( r \) is the magnification (inverse of the size of box). The value of the fractal dimension lies between 1 and 2, corresponding to a straight line and a completely wiggly line (in a 2D plane), respectively. We studied the changes of the fractal dimension for different resolutions. The size of cells at each studied case is the area of the Sun in the spherical coordinate \( S_\odot \) divided by the square of the corresponding resolution. Figure 15 displays the number of filled cells (the number of nodes) against cell sizes for different resolutions. A power-law behavior is observed. The fractal dimension is equal to the slope of the fitted red line, which is equal to 0.79 ± 0.01.

5. Conclusion

In this study, we presented a modified model to construct the solar flare complex network and investigated the utility of the model. We showed how the solar flare modified network theory can reproduce some of the empirical laws. The developed model has two main advantages compared to previous models (Gheibi et al. 2017). First, the mapping method divides the Sun’s surface into equally sized nonoverlapping cells. In this approach, each cell is considered as a node in the network only if a flare has taken place therein. Therefore, applying the AS model, two nodes/flares would connect if successive flares possess the same location.

Second, the VG condition is added to the modeling. In other words, we established the solar flare modified network by using the energy of flaring events further to the constraints concerning their positions and occurrence times. To be more specific, two nodes of the modified network are connected if the VG condition holds between them. This approach allows us to retrieve the empirical laws such as the G–R and Omori’s laws.

The application of the VG condition, which is associated with the time series of solar flare energies, increases the number of links in the network. The modified network is more appropriate to investigate the empirical laws compared to...
previous models since the modified model provides a greater number of links needless of changing the number of nodes.

The analysis of the topological features of the network (such as the degree distribution, the characteristic path length, and the clustering coefficient) while constructing the solar flare network only considering the VG condition is found to be very time consuming. This is due to the fact that the number of nodes is equal to the solar flares in such circumstances. On the
Figure 13. Complementary cumulative number of links vs. size of flaring events in log–log scale for different resolutions (n = 62 and n = 88). The red dashed line represents the $q$-exponential fit. $q$ is obtained to be greater than unity. Threshold power-law exponent ($\gamma$) is also shown in the figure for different resolutions (n = 62 and n = 88).

Figure 14. Configuration of 118 energetic main flares specified by the GOES classification and mapped in two dimensions. Most of these flares are located on regions consisting of hubs of the network (also see Gheibi et al. 2017, Figure 8 therein).

Figure 15. Number of nodes vs. cell size for different resolutions ($\frac{E}{n^k}$). The slope (fractal dimension) obtained is about $0.79 \pm 0.01$. 
other hand, constructing the network applying both the AS + VG models (the modified network), the number of nodes is limited since the number of nodes depends on the mapping method.

The main results of this study are briefly listed below:

1. A solar flare modified network is constructed in order to investigate the empirical laws for the solar flares. In the new approach, the AS and TL models are combined to extract the topological features of the network.
2. The obtained PDFs for the degree of nodes (at different resolutions) are observed to follow a power law with power indices greater than 3.
3. The ratio of the clustering coefficient of the modified network to the clustering coefficient of an equivalent random network with the same size is observed to increase with resolution. This indicates that the modified network is not random.
4. Furthermore, the study of the clustering coefficient confirms that the modified network is scale-free as the modified network consists of hubs and its PDF follows a power law.
5. The G–R and Omori’s laws are investigated for the modified network and the power index obtained is about 1. We conclude that our empirical results are in fairly good agreement with the function $t^{-1}$. 
6. We applied the nonextensive statistical mechanics to investigate the modified network. We described the temporal distribution of successive flaring events with a $q$-exponential model.
7. The solar flare modified network can locate the exact regions of all the energetic flaring events. Moreover, it is found that the main flares play the role of hubs in the modified network as they are connected to both small and intermediate events.
8. The fractal dimension of the modified network obtained is about $0.79 \pm 0.01$.
9. Our results show that the universal scaling features of the solar flare complex system and the Earth’s seismic events are similar.

The authors acknowledge the use of data from the Lockheed Martin Space and Astrophysics Laboratory Latest Events Archive, which uses GOES data provided by the NOAA National Geophysical Data Center. We would especially like to thank Prof. Hossein Safari for his helpful suggestions and discussions. The authors also gratefully acknowledge the anonymous referee for useful comments and suggestions.

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