Cosmological Constraints on B-L Violation

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Abstract

This is a review of the cosmological bounds on $B-L$ violating interactions, and the loopholes in the argument that gives these constraints. If one assumes that the baryon asymmetry we observe today was present above the electroweak phase transition in equilibrium with the non-perturbative $B + L$ violating processes, then interactions that violate all three of $\{B/3 - L_i\}$ cannot simultaneously be in equilibrium. Otherwise the baryon asymmetry would be washed out. Therefore violation of at least one of the $B/3 - L_i$ must be small. This argument can be evaded by not having the observed baryon asymmetry present in the thermal bath (for instance, make it at the electroweak phase transition), by using a non-standard cosmological model, or possibly by some mass effects in models where the difference between lepton flavour asymmetries is conserved.

1 introduction

It is observed that at least up to the scale of our local galaxy cluster, the Universe we see is made of matter rather than anti-matter. It is difficult to build a cosmological model where the Universe contains equal numbers of baryons and anti-baryons separated on scales larger than galaxy clusters (There is a problem with causality [1]), so one usually assumes that the Universe as a whole contains a net excess of baryons over anti-baryons, and that at some time in the early history of the Universe, this asymmetry was generated. This is the Baryon Asymmetry of the Universe, or BAU (for reviews, see, e.g. [2, 3, 4].)

There are three ingredients required to create a baryon asymmetry [5]: baryon number violation, C and CP violation, and some out of equilibrium dynamics. The Standard Model (SM) does contain all these ingredients, but not in the right quantities to generate the BAU.

The Standard Model violates C maximally (because it is chiral), and contains small amounts of CP violation in the CKM matrix (probably not enough to generate the BAU though). It has no perturbative baryon number violation. The lowest dimensional baryon number violating operator allowed by standard model gauge symmetries and involving SM fields is $qqql$ [6], which is of dimension six. However the Standard Model does contain non-perturbative baryon number violation [7]: the current associated with the quantum number $B + L$ (baryon + lepton number) is anomalous, so certain gauge field configurations can induce the $B + L$ violating vertex $(qqql)^3$ (see, for instance, [8] for a review and references). This operator allows processes such as

$$u_g d_b d_r c_g s_b s_r t_g b_b b_r \rightarrow \bar{\nu}_e \bar{\nu}_\mu \bar{\nu}_r$$

(1)

where $g, r, b$ are SU(3) colour labels, not summed, and the quarks are SU(2) doublet members. At zero temperature the rate for such $B + L$ non-conserving processes is exponentially suppressed by the action of the weak gauge field configuration, so one does not expect to see low energy $B + L$
violation. At finite temperature the rate is much less suppressed; the rate above the electroweak phase transition is

\[ \Gamma_{B^+ L} = \kappa n^2 T \]

where \( \kappa \) is a constant of order 1, and \( n \) is 4 or 5 [9]. This is clearly in thermal equilibrium in the early Universe (\( \Gamma > H \equiv \text{the expansion rate of the Universe} \); if \( \Gamma > H \) the timescale for the interaction is less than the age of the Universe.). After the phase transition, the rate is Boltzmann suppressed (\( \Gamma \sim \exp\{-m_W/(\alpha T)\} \)), so rapidly drops out of equilibrium. This is potentially very interesting for baryogenesis (as was noted by [8]): a source of baryon number violation that is efficient in the early Universe when the asymmetry needs to be generated, and then turns off at low energy in perturbative processes where we see no baryon number violation.

Unfortunately the Standard Model lacks the third ingredient required to generate the baryon asymmetry. There is not a big enough departure from equilibrium at the electroweak phase transition; the \( B + L \) violating processes turn off gradually and no asymmetry in generated [10]. The observation of the baryon asymmetry can therefore be interpreted as evidence for new physics, since it does not appear possible to generate an asymmetry within the Standard Model.

The new physics can have two effects on the generation of the baryon asymmetry. It may simply add enough CP violation and make the phase transition sufficiently strongly first order that the baryon asymmetry is generated at the phase transition using the SM \( B + L \) violation. Alternatively, the beyond-the-Standard Model physics may involve new sources of \( B \) and/or \( L \) violation which generate the observed asymmetry before the electroweak phase transition. In the first case there are no cosmological bounds on \( B - L \) violation. So for most of this review, I will assume the second possibility: suppose that the baryon asymmetry we observe today was generated early in the history of the Universe and is present as an asymmetry in the number density of quarks and antiquarks in the thermal bath just above the electroweak phase transition. The anomalous \( B + L \) violating processes are also operating in this plasma, so if the asymmetry generated previously wishes to survive in their presence, it must carry at least one of \( B/3 - L_e \), \( B/3 - L_\mu \) or \( B/3 - L_\tau \). These are the three global quantum numbers exactly conserved in the Standard Model. If there are simultaneously interactions in equilibrium that violate all three of these quantum numbers, all asymmetries will be washed out. Since we are assuming that the baryon asymmetry we observe today was generated before the electroweak phase transition, this is undesirable. One can avoid this outcome by requiring that processes violating one of the \( \{B/3 - L_i\} \) be out of equilibrium:

\[ \Gamma_{B/3-L_i} < H \quad \text{for one } i \]

where \( H \) is the expansion rate of the Universe, and \( i \) is one of \( e, \mu \) or \( \tau \). This condition should hold for all temperatures during which the baryon asymmetry is at risk: \( 100 \text{ GeV} \lesssim T \lesssim 100 \text{ TeV} \). The reasons for this range of temperatures will be discussed in section 3.

For a yukawa-type \( B - L \) violating interaction \( \lambda \phi \psi \bar{\psi} \) (eg R-parity violating trilinears in Supersymmetry), equation (3) becomes

\[ 10^{-2} \lambda^2 T < \frac{25T^2}{m_{pl}} \]

which gives a bound of order \( \lambda < 10^{-7} \). I use \( H = 10T^2/m_{pl} \) in the SM, and \( H = 25T^2/m_{pl} \) for SUSY particle content. One can estimate the rate associated with a majorana neutrino mass to be of order

\[ \Gamma_{m_\nu} \sim \frac{10^{-3}m^2}{T} < H \]

which gives \( m_\nu < 30 \text{ keV} \) in one generation. The numerical factors in these bounds will be discussed more carefully in section 2. Bounds can also be set on non-renormalisable \( B/3 - L_i \) violating operators.
of dimension $D = 4 + n > 4$; if all the coupling constants are absorbed into the mass scale $M$, the rate can be estimated as

$$\Gamma \sim 10^{-3} \frac{T^{2n+1}}{M^{2n}} < \frac{10T^2}{m_{pl}}$$

(6)

For $D = n + 4 = 6$, this gives $M > 10^{4.3/4}T_{\text{GeV}}^3$ GeV. It is clear that non-renormalisable interaction rates increase faster with temperature than the expansion rate, so are more likely to be in equilibrium at higher temperatures. The maximum temperature at which the bounds can be applied, which gives the best bounds ($T \sim 100$ TeV), will be discussed in section 3.

These constraints are comparatively stringent, so it is useful to know exactly where they apply and how they can be avoided. These bounds apply to only one of the $\{B/3 - L_i\}$, so lepton number violation in one family and baryon number violation must be small. In section 2 I will review estimates of interaction rates for various coupling constants, and the bounds one can derive from these estimates. The most obvious loophole is to generate the asymmetry at the electroweak phase transition, or more precisely, to not have the baryon asymmetry present in the plasma just above the EPT. In section 3 I will show that the BAU only needs to be absent from the thermal soup for temperatures between $\sim 100$ TeV and the EPT, and discuss various ways of accomplishing this. Section 4 contains various other mechanisms for avoiding the bounds such as low $T_{\text{reheat}}$ inflationary models. There is a summary of the constraints and loopholes in section 5. The remaining part of the introduction is a short overview of previous work on this topic.

It has been known for a long time that if the baryon asymmetry is generated by the out-of-equilibrium decay of heavy GUT particles, then baryon number violating processes mediated by these particles must be out of equilibrium after the asymmetry is generated [11]. Otherwise they will wipe out the asymmetry generated in the decay. For the baryon asymmetry to survive in the presence of the $B + L$ violating electroweak effects, it must be an asymmetry in $B - L$, which makes it difficult (but not impossible, see section 4.1) to preserve the baryon asymmetry in $B - L$ conserving GUTs. An attractive alternative, introduced by Fukugita and Yanagida [12], is to generate a lepton asymmetry in the decay of heavy singlet neutrinos, and use the electroweak $B + L$ violation to transform this into a baryon asymmetry. Lepton number violating interactions mediated by the heavy singlet neutrinos must be out of equilibrium after the lepton asymmetry is generated, to avoid washing out the asymmetry [13]. This bound on lepton number violating processes is not limited to leptogenesis scenarios; Barr and Nelson [14], and [15], observed that the baryon asymmetry, the $B + L$ violating processes, and interactions that violate the three $\{B/3 - L_i\}$ cannot simultaneously coexist in equilibrium, and used this to set bounds on majorana neutrino masses in one generation. This argument was then extended to R parity violating interactions in the Supersymmetric version of the Standard Model [13, 16] and to generic $B - L$ nonconserving interactions [16]. The constraints and loopholes were then discussed by many people [17, 18, 19, 20, 21].

## 2 rate estimates and constraints

The argument that gives cosmological bounds of $B - L$ violation goes as follows: assume that the observed baryon asymmetry was generated before the EPT. To survive in the thermal soup, it must carry a quantum number that is “effectively conserved” — i.e. conserved by the interactions that are in chemical equilibrium. The SM interactions are all in chemical equilibrium just above the EPT, and the three global quantum numbers that they conserve are the $\{B/3 - L_i\}$. To protect the BAU, any additional interactions present must also “effectively conserve” at least one of the $\{B/3 - L_i\}$, or equivalently, interactions violating at least one of the $\{B/3 - L_i\}$ must be out of equilibrium.

In the first part of this section, I will estimate interaction rates associated with coupling constants of different mass dimensions. It follows from these estimates that all the SM interactions come into
equilibrium before the EPT. In the second part, I will discuss the equations of chemical equilibrium and charge conservation for the SM interactions. These are interesting because there are small “second order” terms proportional to lepton masses that can be used to protect an asymmetry. Finally in the third subsection I will use the rate estimates from section 2.1 to set bounds on neutrino masses, $R$-parity violating coupling constants, etc.

As one can see from the estimated bounds in equations (4), (5) and (6), the best constraints are on renormalisable couplings. With the SM particle content, there are no $B - L$ non-conserving operators of dimension less than 5. However, in the supersymmetric extension of the SM, when $R$-parity is not imposed, baryon and lepton number violating masses and trilinear terms are possible. In this proceedings, I will discuss constraints on $B - L$ violation with SM particle content at the electroweak scale, and also with SUSY particle content.

The MSSM superpotential is

$$W = \mu H_1 H_2 + h_u^p H_2 Q_p U^c_q + h_d^p H_1 Q_p D^c_q + h_e^i H_1 L_i E^c_j \ .$$

(7)

The Lagrangian also contains kinetic terms, gauge interactions, D-terms and soft SUSY breaking terms of the form

$$\text{soft masses} + B H_1 H_2 + A_u^p H_2 Q_p U^c_q + A_d^p H_1 Q_p D^c_q + A_e^i H_1 L_i E^c_j \ .$$

(8)

I am abusively using capital letters for both superfields (as in eqn 7) and scalar component fields (as in eqn 8). Quark generation indices are $p,q,r,s...$ and lepton indices are $i,j,k...$. Whether indices are up or down makes no difference.

The above SUSY Lagrangian is with the symmetry $R$-parity imposed. Other renormalisable interactions can be constructed with the MSSM particle content, but these new interactions violate either $B$ or $L$, and are often removed by imposing a symmetry like $R$-parity [22]. See [23] for other possibilities. These are examples of the interactions I am interested in setting bounds on. There are possible new superpotential terms

$$W_R = \epsilon^i H_2 L_i + \lambda^{ijk} L_i L_j E^c_k + \lambda^{ipq} L_i Q_p D^c_q + \lambda^{''pq} U^c_p D^c_q D^c_r$$

(9)

and new $B$ or $L$ violating soft terms

$$\text{soft masses mixing } L^1 \text{ and } H_1$$

$$+ B^i H_2 L_i + A^{ijk} L_i L_j E^c_k + A^{ipq} L_i Q_p D^c_q + A^{''pq} U^c_p D^c_q D^c_r \ .$$

(10)

I assume that SUSY masses are of order 100 GeV $\sim T_c$. The bounds on trilinears are not substantially weakened by larger soft masses—if one sets the constraints at $T \sim m_{SUSY} \sim 1$ TeV, they are weakened by a factor of about 3. The effects of larger SUSY masses were discussed in [18, 20].

### 2.1 rate estimates

In this section I will roughly estimate interaction rates for a selection of operators. See for instance [17, 18, 20] for more thorough discussions that disagree in details but all get approximately the same constraints. For a detailed calculation of the interaction rate associated with a specific coupling constant see, e.g. [24].

The rate for a boson to scatter off a fermion via the yukawa interaction $y \phi \bar{\psi}_1 \psi_2$ can be estimated using kinetic theory (see e.g. [4] for an introduction) to be

$$\Gamma(\phi \psi_1 \rightarrow \phi \psi_1) \simeq< \sigma v n_{\psi_1} > \sim 4 \times 10^{-4} y^4 T$$

(11)
where $\sigma$ is the $T = 0$ cross section, $n_{\psi_1}$ is the equilibrium thermal distribution of fermions $\psi_1$, $v$ is the relative velocity of the boson and the fermion ($\simeq c = 1$), and the brackets represent averaging the cross section over thermal distributions in momentum space.

For a gauge boson, the matrix element, and therefore the numerical coefficient, will be different, but if one substitutes the gauge coupling $g$ for the yukawa $h$ in equation (11), it is clear that gauge interactions are in equilibrium up to temperatures of order $\lesssim m_{\text{GUT}}$.

One can similarly estimate the rate for $\phi \psi_1 \to \psi_1 \gamma$ where $\gamma$ is an arbitrary gauge boson, to be of order

$$\Gamma(\phi \psi_1 \to \gamma \psi_1) \sim 5 \times 10^{-3} y^2 \alpha T$$

(12)

The rate for a scalar (Higgs or spartner) to decay to two fermions via the interaction $y \phi \bar{\psi} \psi$ can be estimated as

$$\Gamma \sim 1.4 \times 10^{-2} \frac{y^2 m^2}{T}$$

(13)

$m_{\phi}$ is the decaying particles mass in the finite temperature effective potential. Equation (13) neglects final state phases space suppresion due to fermion masses, which can be significant if the mass of the scalar is of order $gT$. (The final state fermions can also have thermal masses of order $gT$.)

To estimate a rate associated with a yukawa, one therefore has a choice between scattering and decay. The scattering rate has an extra power of $\alpha$, but the decay may be suppressed by the small amount of final state phase space, depending on the (thermal) masses of the participating particles. I will use the decay rate estimate to set bounds on $B - L$ violating couplings, because the decaying particle is a SUSY spartner with a zero temperature mass of order 100 GeV.

In the SM case, $y \simeq \sqrt{2} m_f / v$ where $v = 246$ GeV is the Higgs vev, and $m_f$ is a fermion mass. (This is not entirely accurate for the quarks, because the finite temperature mass eigenstates will not be the same as the zero temperature ones, but this should be a reasonable estimate). Taking $H \sim 10 T^2 / m_{\text{pl}}$, one finds that the electron yukawa is in thermal equilibrium below $T_{eR} \sim 10 - 100$ TeV [13], the $u_R$ quark will be in chemical equilibrium below $T_{u_R} \sim 100$ TeV, and so on. (See [24] for a careful discussion of when the electron yukawa comes into equilibrium, taking into account the thermal masses in the decay rate.). This means that all SM interactions are in equilibrium just above the EPT.

To estimate a rate associated with a majorana neutrino mass, I need to consider how it is generated. If the zero temperature mass arises from the dimension 5 operator $y^2 (\ell \ell HH) / M$, where $M$ is a heavy right-handed singlet neutrino mass, then at temperatures $T_{\ell} < T \ll M$ this operator generates two-fermion-two-Higgs scattering. The rate can be estimated [14, 15] as in equation (11) to be

$$\Gamma(\ell H \to \ell H) \sim 4 \times 10^{-4} \frac{y^4 T^3}{M^2}$$

(14)

This is a conservative estimate of the lepton number violating rate, because the latter should also include the processes $\ell \ell \to HH$, $HH \to \ell \ell$ and so on.

Alternatively, if the majorana mass is present above the EPT as a mass, then it generates lepton number violating processes such as $\nu W \to \bar{\nu} W$. Majorana mass insertions on the external neutrino legs contribute the lepton number violation. The mass corrections to the scattering process can be estimated from (11) as

$$\Gamma(\nu W \to \bar{\nu} W) \simeq 10^{-3} g^4 T \left(\frac{m_\nu}{T}\right)^2$$

(15)

and to a decay as

$$\Gamma(W \to \nu \nu) \sim 3 \times 10^{-2} g^2 T \left(\frac{m_\nu^2}{T^2}\right)$$

(16)
(To make this estimate, I took $m_W^2(T) \sim g^2T^2/3$ and $m_\nu^2(T) \sim g^2T^2/8$. For these masses, the decay $W \rightarrow \nu\nu$ is kinematically not allowed, so I ought not to use this estimate. It is nonetheless in the literature.)

### 2.2 chemical equilibrium equations

There are two approaches in the literature to the equations of chemical equilibrium as applied to cosmological contraints on $B-L$ violation. The more common one uses kinetic theory [25, 18], and is reliable insofar as it agrees with the more esotheric calculations that start from the free energy [26]. I will use something in between, that I think gives the right answers as long as one is not to near the EPT (See [26] for an analysis that is valid for all temperatures.)

If an interaction is in chemical equilibrium, for example the decay $\phi \rightarrow \psi_1\psi_2$, then the sum of the chemical potentials of the participating particles is zero:

$$\mu_\phi - \mu_{\psi_1} - \mu_{\psi_2} = 0 \quad . \quad (17)$$

Above $T_c$, all the gauge bosons have zero chemical potential [25, 18]. From equation (11) it is clear that SM gauge interactions are in equiliibrum up to very high temperatures, so all members of a gauge multiplet have the same chemical potential.

At $T \simeq T_c$, all the SM processes are in equilibrium. If the chemical potential for a gauge multiplet is written as the name of the multiplet (so the chemical potential for $e_R$ is $e_R$), then the Yukawas imply

$$-e_R^i + \ell^i + H = 0 \quad ,$$
$$-u_R^i + q^i - H = 0 \quad ,$$
$$-d_R^i + q^i + H = 0 \quad . \quad (18)$$

The rate estimates in section 2.1 say that the electron yukawa is in equilibrium below $T_{e_R} \sim 10 - 100$ TeV. So equations (18) apply for $T_c < T < 10 - 100$ TeV. Above this temperature, the $e_R$ have a thermal distribution (they have gauge interactions) but any asymmetry in the $e_R$ is decoupled from the rest of the plasma. In the absence of an interaction that can transfer the $e_R$ asymmetry to other particles, asymmetries must remain among the other particles (in particular, a baryon asymmetry) to cancel the hypercharge carried by the $e_R$ asymmetry.

The non-perturbative $B + L$ violating processes give

$$9q_L + \sum_i \ell^i = 0 \quad (19)$$

Note that these interactions eat three units of baryon number and three units of lepton number, so they violate $B + L$ but not $B - L$. However, when they are in equilibrium with SM interactions in the early Universe, they imply $n_B = -\frac{28}{51}n_L$ and not $n_B = -n_L$. This is because the interactions in equilibrium try to minimise the free energy, and the minimum is not necessarily where all the quantum numbers are zero. The “sphalerons” of equation (19) do not take $n_{B+L}$ to zero because they only eat SU(2) doublets, and the SU(2) singlets (who also carry $B$ or $L$) have different hypercharge. In a plasma with $Y = 0$ but non-zero $n_{B-L}$, the free energy minimum is at $n_B = -(28/51)n_L$ rather than at $n_{B+L} = 0$.

In the case of the MSSM, there are many new particles. The gauginos have majorana masses (sufficiently large to be in chemical equilibrium) so can carry no asymmetry and must have zero chemical potential. This means that the sfermion and fermion chemical potentials are equal and opposite to each other, because the gaugino interaction $\tilde{g}\phi\psi$ imply

$$\phi + \psi + \tilde{g} = \phi + \psi = 0 \quad (20)$$
The two Higgs have opposite chemical potentials due to the mass term $\mu H_1 H_2$, so one gets the same chemical potentials as in the SM, subject to the same constraints due to yukawa couplings.

If the flavour off-diagonal slepton masses are large enough for lepton flavour violation to be in equilibrium, then only $B - L$ is conserved, and not the three $B/3 - L_i$ separately. I will assume for the moment that the off-diagonal masses are “small enough” $(m_{ij}/m_{ii} < 10^{-2})$, and come back to this constraint when I am discussing bounds on $B - L$ violating operators in supersymmetric theories.

This gives six equations for the 10 unknown chemical potentials. The four free parameters correspond to the four conserved charges: electric charge (or hypercharge) and the three $\{B/3 - L_i\}$. The electric charge must be zero (imposing $Y = 0$ gives an equivalent constraint):

$$\sum_{\text{particles}} Q_{\text{em}}(n - \bar{n}) = 0 \quad (21)$$

where $(\bar{n})$ is the (anti-) particle number density. $n - \bar{n} = \partial F/\partial \mu$ where $F$ is the free energy and $\mu$ the particle chemical potential. One finds

$$n - \bar{n} = \frac{2g}{\pi^2 \mu T^2} \left\{ \begin{array}{ll} 1 + O(m^2/T^2) & \text{fermions} \\ 2 + O(m^2/T^2) & \text{bosons} \end{array} \right. \quad (22)$$

for $m \ll T$. $g$ here is the number of degrees of freedom of the field—2 for a chiral fermion and a charged scalar. One can correctly get the first term of equation (22) using kinetic theory, by expanding the equilibrium particle distributions in $\mu$ and $m^2$ (see section 4). However, this expansion does not always give the right coefficient for the $O(\mu m^2)$ terms. Note that I am working above the EPT, so the masses in (22) are thermal masses or soft SUSY masses, but are not due to the Higgs vev. Neglecting for the moment the mass effects, one finds in the SM (with $n_H$ Higgs doublets)

$$Q_{\text{em}} \propto 3q_L + 6u_R - 3d_R - \sum_i [\bar{\ell}^i + e^i_R] - 2n_{H} H = 0 \quad (23)$$

In the MSSM, the numerical coefficients change slightly, assuming $m_{\text{SUSY}} \lesssim T_c$, because there are now two Higgs and spartners as well as partners in the soup. The net effect is to change the number of Higgs $n_H$ from one to two in (23).

There are now seven homogeneous equations for the 10 unknown chemical potentials. If one fixes the asymmetries in the three $\{B/3 - L_i\}$s, all the chemical potentials are determined.

The baryon asymmetry carried by the quarks is

$$n_B = \frac{\sum_{\text{col}} \sum_{\text{gen}} (2 \times \frac{4}{3}(n_{q_L} - n_{q_L}) + \frac{1}{3}(n_{u_R} - n_{u_R}) + \frac{1}{3}(n_{d_R} - n_{d_R}))}{12q_L T^2} \quad (24)$$

where I have used equations (18) and neglected $O(\mu m^2)$ corrections. Quarks of different families have the same chemical potential so quark mass effects in equation (22) cannot be used to preserve an asymmetry. Equation (24) can be written using equations (18), (19) and (23) as

$$n_B = \frac{24 + 4n_H}{66 + 13n_H} \sum_i n_{B/3-L_i} + \frac{47y_r^2}{1896\pi^2} (n_{L_e-L_r} + n_{L_\mu-L_r}) \quad (25)$$

where $n_H$ is the number of Higgs doublets. I have included here the lepton mass effects—specifically the $\tau$ yukawa $y_r = m_\tau/(175 \text{ GeV})$, because it is largest. These small corrections, which are present below [27] and above [28] the EPT, will be discussed in section four. It is clear that if $B - L = 0$, one needs very large lepton flavour asymmetries $(n_{L_i-L_j}/s \sim 10^{-3}$, where $s$ is the entropy density) to preserve the baryon asymmetry via lepton mass differences. (As noted in [18], slepton mass differences could be more effective.)
2.3 bounds on $B - L$ violating coupling constants

Equation (25) says, as expected, that a baryon asymmetry will remain in the plasma if an asymmetry remains in any one of the $\{B/3 - L_i\}$. Neglecting the lepton mass effects, it is clear that we need an asymmetry in at least one of the $\{B/3 - L_i\}$ to preserve the asymmetry in the quarks. This means that interactions violating one of the $\{B/3 - L_i\}$ need to be out of equilibrium. Suppose, for instance that it is $B/3 - L_e$ that is not washed out. Then one requires

$$\Gamma_{B/3 - L_e} < H$$

for $T_c < T < T_{\epsilon R} \sim 100$ TeV.

I can translate (26) into a constraint on coupling constants using the rate estimates from section 2.1.

For majorana neutrino masses that are present in the thermal soup above the EPT as a mass, requiring $\Gamma$ from equations (16) or (15) to be less than $H$ at $T = 100$ GeV gives

$$m_\nu < 10 - 100 \text{ keV}$$

(The weaker bound corresponds to the scattering interaction rate).

In the case where $m_\nu \sim m^2_D/M$, above the EPT there is a lepton number violating scattering process mediated by the dimension 5 operator $(\ell\ell HH)/M$. As discussed in the introduction, $\Gamma_L/H$ increases with the temperature for $D > 4$ operators, so the best bound is at the highest possible temperature. This is $T_{\epsilon R} \sim 100$ TeV [14]. Requiring equation (14) to be out of equilibrium at $T \sim 100$ TeV gives the bound

$$m_\nu \lesssim \text{keV}$$

These are bounds on a neutrino mass that violates electron lepton number. So in the basis where the charged lepton mass matrix is diagonal the majorana neutrino mass matrix entries $m^{ei}_\nu < 10$ keV for $i = e, \mu, \tau$.

The $R$-parity violating masses $\epsilon_i \tilde{h} L_i$ are slightly trickier [29, 30], because one can set them to zero by redefining the higgsino $\tilde{h}_1$. The “problem” here is that the $H_1$ superfield has the same gauge quantum numbers as the lepton superfields $L_i$. In a lepton number conserving theory, $L_i$ are distinct from $H_1$ because they carry lepton number. However if the $i$th lepton number is not conserved, then there is no unique definition of which linear combination of $L_i$ and $H_1$ is “the Higgs”.

For setting bounds on lepton number violation, it should not matter which basis we calculate in. Whether an asymmetry in, for instance $L_e$, survives in the plasma is a physical question that should not be basis dependent. It is possible to construct basis independent “invariants” that parametrise the amount of $R$ parity violation between different coupling constants (analogous to Jarlskog invariants for $CP$ violation), and to set bounds on the invariants [29, 30, 31]. However, one can get the right answer with a less formal approach.

If we rotate the superpotential term $\epsilon_i H_2 L_i$ into $\mu H_1 H_2$ by redefining $H_1$:

$$H_1' = \frac{1}{\sqrt{\mu^2 + \epsilon^2}} (\mu H_1 + \epsilon_i L_i)$$

then we generate new trilinears:

$$\frac{h^{ij}_q \epsilon_i}{\sqrt{\mu^2 + \epsilon^2}} L_i Q_p D^c_q \frac{h^{jk}_e \epsilon_i}{\sqrt{\mu^2 + \epsilon^2}} L_i L_j E^c_k$$

If the soft masses $B_H H_1 H_2 + B_i H_2 L_i$ are parallel (in $H_1, L_i$ space) with $(\mu, \epsilon_i)$, (i.e. $B_i/B_H = \epsilon_i/\mu$), then they are simultaneously rotated away. If in this basis the soft masses mixing $H_1$ and the
lem 

\( L_i \) are also zero, then there is no lepton violation in the masses. In this case, there are two possible bounds one can set on \( \epsilon_i \): one could require that the rate associated with the mass \( \epsilon_i \) be out of equilibrium, or that the trilinear interaction \( (h_d^{33}_e \epsilon_i/\sqrt{\mu^2 + \epsilon_i^2}) T < H \). The first bound would imply \( \epsilon_i < 10 \text{ keV} \), the second \( \epsilon_i < 10^{-5} \mu \approx \text{MeV} \). So it matters which is the right one to impose. It turns out that the second, weaker constraint is the correct one. One can see this in a one generation model; the stronger rate due to the mass term chooses the Higgs to be the direction of equation (29), so the lepton is the orthogonal direction, and lepton number is violated by the \( \lambda' LQD^c \) term in this basis. One can check that in the basis (29), the trilinear \( \lambda^{'33} \) can be written in terms of the coupling constants of the original basis as \( (\epsilon_i h_d^{33} - \mu \lambda^{'33})/\sqrt{\mu^2 + \epsilon_i^2} \). Equation (37) gives the bound

\[
(\epsilon_i h_d^{33} - \mu \lambda^{'33})/\sqrt{\mu^2 + \epsilon_i^2} < 10^{-7} \quad \text{for one } i
\]  

or \( \epsilon_i < \text{MeV} \) for \( \mu \sim 100 \text{ GeV} \).

It is unlikely in realistic models that \( (\mu, \epsilon_i) \) should be exactly parallel to \( (B_H, B_i) \), or that the soft masses \( m^2_{H,L} \) should be zero in this basis. Small lepton number violating masses parametrising the misalignment should remain. For simplicity, let me neglect the \( m^2_{H,L} \), and just consider \( \epsilon_i \) and \( B_i \). The rate for these masses can be estimated as a mass correction times a gauge interaction rate, like the estimate for the neutrino majorana mass, that is

\[
\Gamma \sim \min \left\{ \frac{\epsilon_i^2}{\mu^2 + g^2 T^2}, \frac{(B_i)^2}{(B_H + g^2 T^2)^2} \right\} \times 10^{-2} g^2 T
\]  

where in this formula \( \epsilon_i \) (\( B_i \)) is taken in the basis where \( B_i \) (\( \epsilon_i \)) is zero. The denominator contains a crude attempt to include thermal mass effects. To see why equation (32) is the right bound, imagine sitting in the early Universe above the EPT. As the temperature drops, more and more renormalisable interactions come into equilibrium. At some point the larger of the two mass terms \( \sqrt{(B_H)^2 + (B_i)^2} \) and \( \mu^2 + \epsilon_i^2 \) comes into equilibrium, and thereby chooses the Higgs direction—i.e. if \( \sqrt{(B_H)^2 + (B_i)^2} > \mu^2 + \epsilon_i^2 \) the Higgs will be combination \( B_H H + B_i L_i \). Then in this basis chosen by the interactions, one needs the lepton number violating rate associated with the mass \( \epsilon_i \) to be out of equilibrium, which is equation (32).

The bound (32) can be expressed in a more basis independent way as:

\[
\frac{B_H \epsilon_i - \mu B_i}{\sqrt{(B_H)^2 + (B_i)^2} \sqrt{\mu^2 + \epsilon_i^2}} < 2 \times 10^{-7}
\]  

or, in the basis where \( B \epsilon_i \) is zero, \( \epsilon_i/\mu < 2 \times 10^{-7} \). So the cosmological bounds require that \( (\mu, \epsilon_i) \) be more aligned with \( (B_H, B_i) \) than with the yukawas.

In the supersymmetric Standard Model, lepton flavours are unlikely to be separately conserved. The slepton mass matrix is unlikely to be exactly diagonal in the lepton mass eigenstate basis. If lepton flavour violating processes due to flavour off-diagonal slepton masses are in thermal equilibrium, then all \( B-L \) violating processes need to be out of equilibrium, not just interactions violating on of the \( B/3 - L_i \), i.e. the bounds (31), (37) and (33) would apply to all \( B-L \) violating coupling constants, and not just those involving one lepton generation. So if one wants to use the “flavour loophole”, and only impose the bounds on one generation, the flavour changing soft masses must be out of equilibrium.

The rate associated with a flavour off-diagonal soft mass can be estimated analogously to the previous discussion about \( \epsilon_i H^2 L_i \). There are three interactions that choose a basis in lepton flavour space: the lepton yukawa \( h^{ij}_e \), the \( A \) term \( A^{ij}_e \), and the soft masses \( m^2_{ij} \). If I want to separately conserve lepton flavours in the thermal soup, then all the interactions in equilibrium must agree on the basis in lepton flavour space. Suppose I start in the basis where \( h^{ij}_e \) is diagonal, because this is
a familiar basis from phenomenology, and I put a hat on the coupling constants in this basis. There will be off-diagonal terms \([\hat{m}^2]_{ij}, \hat{A}^2_{ij}, i \neq j\) in this basis. Since the interaction rate associated with \(m^2\) is larger at \(T \sim 100\) GeV than the rate for \(h_e\), this is probably not a sensible basis. I should rotate to the basis where \(m^2\) is diagonal, because the stronger interaction will choose the lepton flavour basis. In this new basis, there will be flavour off-diagonal yukawas \(\hat{h}_{ij}^e \sim \hat{h}_{ii}^e \hat{m}_{ij}^2 / \hat{m}_{ii}^2\) \((i \neq j, \text{and no sums on repeated indices})\). These yukawas need to be out of equilibrium:

\[
\Gamma \sim 10^{-2} \left( \frac{\hat{h}_{ii}^e \hat{m}_{ij}^2}{\hat{m}_{ii}^2} \right)^2 T < H
\]  

(34)

If one wants to preserve \(B/3 - L_e\), this gives the bound \(\hat{m}_{ej}/\hat{m}_{ee} < 5 \times 10^{-2}\). Preserving a different lepton flavour would give a stronger bound (the yukawa is larger) on different elements of \(\hat{m}^2\).

Now consider the \(A\) term. Suppose I estimate the associated rate from equation (13) to be

\[
\Gamma_A \sim 10^{-2} \frac{A^2_e}{T}
\]

(35)

This is equivalent to assuming that the \(A\) term mediates a decay of one scalar into the other two. It is not clear that this is kinematically allowed, but I use this estimate anyway (I estimate the scattering rate \(\Gamma(\phi \phi \rightarrow \phi \gamma) \simeq 10^{-3} g^2 A^2 / T\), which gives a bound on \(A\) that is about an order of magnitude weaker.). In the basis where \(m^2\) is diagonal, I need

\[
\Gamma_A \sim 10^{-2} \frac{(A^2_{ij})^2}{T} < H \quad (i \neq j)
\]

(36)

for whichever lepton flavour \(i\) one wishes to conserve. This gives \(A^2_{ij} < 10^{-5}\) GeV. This is not a particularly strict bound; recall that I have absorbed the yukawa into \(A\) (see equation (8)). Similar bounds apply to the \(R\)-violating \(A\) terms of equation (10).

Finally I need to calculate a bound on \(B - L\) violating trilinear interactions. There are no basis confusions for these interactions, so to conserve, for instance, \(B/3 - L_e\), I need all the trilinears that violate \(L_e\) and \(B\) to satisfy

\[
10^{-2} \lambda^2 T < H \simeq \frac{25T^2}{m_{pl}}
\]

(37)

This means that \(\lambda^\nu_{pqr} < 10^{-7}\) for all \(p, q, r\), \(\lambda^e_{pq} < 10^{-7}\) for all \(p, q\), and \(\lambda_{ejk}, \lambda_{jke} < 10^{-7}\) for \(j, k \neq e\).

### 2.4 summary of bounds on \(R\) violating interactions

Suppose that I define the Higgs such that the three \(B_i\) are zero. Then to conserve \(B/3 - L_e\), I need

\[
\epsilon_e < 10 - 100 \text{ keV}, \quad m_{H_1}^2 < (60 - 90 \text{ MeV})^2
\]

(38)

for \(\mu^2 \simeq m^2 \simeq B \simeq 100\) GeV.

I need all the trilinears and \(A\) terms that violate \(B/3 - L_e\) to respectively satisfy

\[
\lambda < 10^{-7}, \quad A < 10 - 100 \text{ keV}
\]

(39)

I also need \(L_e\) flavour violation mediated by the \(R\) conserving soft masses to be out of equilibrium, which will be the case if

\[
\frac{m_{2i}^2}{m_{ii}^2} \lesssim 5 \times 10^{-2}, \quad A^e_i \lesssim 10 - 100 \text{ keV}
\]

(40)
3 if the BAU was not there...

I wrote in the introduction that I would assume that the BAU was present in the thermal soup above the EPT. This is of course one of the loopholes in the constraints that I am discussing; if the BAU is not there, then there are no constraints on $B - L$ violation. So in this section I will relax this assumption.

First I would like to work out over what temperature range the BAU needs to be “not there”. The lower end of this range is straightforward to find. The $B + L$ violating electroweak effects drop out of equilibrium at or shortly after the electroweak phase transition. Once they are gone, the BAU is no longer at risk. So the baryon asymmetry can safely be present in the plasma at $T \ll T_c$.

It can also be present at high temperatures, before the electron yukawa comes into equilibrium. The interaction rate associated with a dimensionless couplings constant $\lambda$ scales linearly with the temperature: $\Gamma \sim \lambda^2 T$. Since $H \sim T^2/m_{pl}$, this means that yukawa interactions are out of thermal equilibrium at sufficiently high temperatures, and come into equilibrium as the temperature drops. Since the electron yukawa is the smallest coupling constant in the SM, it is the last to come into equilibrium at $T_{e_R} \sim 100$ TeV. The $e_R$ carries hypercharge, and the Universe ought to be hypercharge neutral; so if there is an asymmetry in the $e_R$, and no interaction in equilibrium that can transfer this asymmetry to other particle species, then asymmetries must remain in other particles to ensure that $Y = 0$. It is easy to check from the relevant equations of chemical equilibrium that this means a baryon asymmetry will remain in the plasma. At $T \gg T_{e_R}$, there will be other yukawa interactions out of equilibrium, and additional symmetries [22] that can protect the baryon asymmetry, but the singlet $e_R$ is the last particle species to come into chemical equilibrium, so can protect the baryon asymmetry for the longest [3]. Any $B - L$ violating operator not involving the $e_R$ can be in equilibrium above $T_{e_R}$, without being able to eat the BAU. This was not fully realised in the earlier papers discussing bounds on non-renormalisable $B - L$ violating operators, where strong bounds were set on $D > 4$ operators by requiring that they be out of equilibrium up to $T_{\text{reheat}}$, or the temperature when the BAU was generated, or the temperature when the $B + L$ violation came into equilibrium.

Note that this whole argument assumes that there is an asymmetry stored in the $e_R$. This is maybe not so easy to arrange—for instance the decay of heavy singlet (“right-handed”) majorana neutrinos generates an asymmetry in the lepton doublets, but not the charged singlets. However, for setting generic bounds, one must allow for the possibility that some baryogenesis mechanism does generate an $e_R$ asymmetry, which can then protect the baryon asymmetry down to $T_{e_R}$.

To approximately determine $T_{e_R}$, one can assume that the $e_R$ comes into chemical equilibrium when the decay of the Higgs into a left handed and a right-handed lepton is of order $H$ [14]:

$$\Gamma \approx 10^{-2} h_e^2 T \approx H$$

which gives $T_{e_R} \approx 100$ TeV. See [24] for a careful determination of $T_{e_R}$, that includes the various decay and scattering interactions mediated by $h_e$, and the thermal masses of the participating particles.

So the baryon asymmetry is really only “at risk” over three decades in temperature: $T_c < T < 100$ TeV. If the asymmetry is present in the thermal soup during this period, it needs to be an asymmetry in $B - L$, and interactions violating at least one of the $B/3 - L_i$ need to be out of equilibrium during this period. This gives the bounds of section 2.

Alternatively, the BAU can be not in the soup between $T_c < T < 100$ TeV. The most obvious way to do this is to generate the baryon asymmetry at the electroweak phase transition (see, e.g [4, 3] for reviews of electroweak baryogenesis). But this is not the only possibility; if we turn out to be living in a corner of SUSY parameter space that does not allow electroweak baryogenesis, there are models where the baryon asymmetry is generated in the late decay of some particle, after the EPT [15].

Another possibility is to generate the baryon asymmetry early, and then “hide it” between $T_{e_R}$ and $T_c$. One way to do this [34], is to add additional particles that carry baryon number to the SM...
or the SSM, decouple them from the rest of the SM particles at $T > T_{e_R}$ and have them decay after the EPT, returning the baryon asymmetry that they carry to the plasma of SM particles. Another possibility is to store some hypercharge in a coherent hypercharge magnetic field for $T_c < T < T_{e_R}$ [35]. If there is a sufficiently large asymmetry in the $e_R$, it can act as a source for the hypercharge magnetic field before $T_{e_R}$, and this field may survive until the EPT.

Another somewhat related loophole is to take the up-quark yukawa to be considerably smaller than is usually assumed [36]. If $h_u < \sim 10^{-7}$ ($m_u < \sim 20$ keV!), then an asymmetry in the $u_R$s could play the same role as the $e_R$ asymmetry does, but all the way down to the EPT. If $h_u$ did not come into equilibrium until after the EPT, and if the $u_R$s carried a primordial asymmetry, then this asymmetry will be present after the EPT, irrespective of how much $B - L$ violation is in equilibrium above the EPT (providing of course that the $B - L$ violating operators do not involve $u_R$).

4 other loopholes

In this section, I list the remaining loopholes.

4.1 lepton mass effects

In models where $n_{B-L} = 0$, lepton mass effects can protect the baryon asymmetry from SM $B + L$ violation, provided that there are lepton flavour asymmetries [27]. A small baryon asymmetry will remain below [27] and above [28] the EPT, proportional to lepton mass differences and flavour asymmetries. Since I am here discussing what happens above the EPT, the numerical factors in equation (25) are for this case, where the lepton masses are “thermal”. In this subsection, I neglect most of the numerical factors.

The number density for particles in thermal equilibrium is

$$n = \frac{2g}{\pi^2} \int \frac{p^2 dp}{e^{(E-\mu)/T} \pm 1}$$

where the $\pm$ is for fermions or bosons, and the number density $\bar{n}$ for anti-particles has the sign in front of the chemical potential $\mu$ flipped. Expanding $n - \bar{n}$ in $\mu/T$ and $m^2/T^2$, assuming that both are small, gives equation [22]. As previously noted, to correctly determine the $O(\mu m^2)$ term, one must calculate the free energy, and take the derivative with respect to $\mu$. However, I am not here interested in the exact coefficient; see [26, 28] for numerical factors.

The reason that these ($\mu m^2$) contributions are interesting is that they can preserve a baryon asymmetry in the presence of the $B + L$ violating interactions, when $B - L = 0$, and there are flavour asymmetries among leptons (also when $B + L$ and $B$ violation are in equilibrium, and there are flavour asymmetries). They do not protect the BAU if there are interactions in equilibrium that take all three lepton chemical potentials to zero.

So suppose that the $B + L$ violating interactions are in equilibrium (equation [49], and that $n_{B-L} = 0$:

$$12q_L - 3 \sum_i \ell_i \left[ 1 + O \left( \frac{m_i^2}{T^2} \right) \right] - 3H = 0$$

This could be the case if the BAU was generated in the out-of-equilibrium decay of heavy particles from a $B - L$ conserving GUT. There are two types of constraint imposed on the plasma: the sum of the chemical potentials participating in each interaction must be zero, and certain charge densities ($Y, B - L$) must be zero. If one includes the $O(\mu m^2)$ terms in the charge densities, one finds that there is a small BAU in equilibrium. This is due to the non-zero chemical potentials and not equal masses for the different lepton generations.
From the condition of charge neutrality, equation (23), I can solve for the Higgs chemical potential in terms of $q_L$ and $\sum \ell_i$:

$$H \sim \frac{3q_L - \sum \ell_i[1 + O(m_i^2/T^2)]}{7} \sim \frac{12q_L - \sum \ell_i \times O(m_i^2/T^2)}{7}$$

(This assumes SM particle content, and I have dropped $H m_i^2$ terms). Substituted into (43), and using (19) one finds

$$n_B \propto q_L T^2 \sim \sum \ell_i m_i^2$$

$\sum \ell_i m_i^2 \neq 0$, if there are lepton flavour asymmetries ($n_{L_i - L_j} \neq 0$), so the remaining baryon asymmetry (15) is proportional to lepton mass differences, and flavour asymmetries, see equation (25). It is clear from (25), where all the numerical factors are included, that one needs very large lepton flavour asymmetries ($\sim 10^{-2} - 10^{-3}$) to preserve a sufficiently large baryon asymmetry. An interesting possibility suggested in [18] is to use the slepton mass differences in a SUSY model, because these could be much larger.

### 4.2 modified cosmologies

In this subsection I briefly review some modifications of cosmology that allow one to escape the bounds discussed in section 2.

One possibility is low $T_{\text{reheat}}$ models; if the Universe after inflation reheats to a temperature $T_{\text{reheat}} < T_c$, then one never has to worry about having $B + L$ and $B - L$ violating operators simultaneously present. However, one does have to worry about how to generate the baryon asymmetry in such models. This is not necessarily a problem; one can, for instance, generate an asymmetry from the decay of the inflaton, or in some cases via the Affleck-Dine mechanism [37]. In fact, the reheat temperature after Affleck-Dine baryogensis is often low.

Another possibility is to modify the expansion rate of the Universe. If the Universe expanded somewhat faster, the $e_R$ could protect the BAU all the way down to $T_c$. This can be accomplished by, for instance, adding a component to the energy density of the Universe that scales as $1/a^6$ [38]. (This is not as far-fetched as it might sound; a scalar field in the right shaped potential can behave this way.)

It is also possible to “get rid of” the $B + L$ violating electroweak interactions. For instance, if there is a SU(2) doublet with a large vev above the EPT, it would give a mass to the $W$, so the rate for $B + L$ violation would be exponentially suppressed. This could occur in an Affleck-Dine baryogenesis scenario, when the condensate carries a large baryon asymmetry [39], and therefore does not decay until $T \sim 100$ GeV.

### 5 summary

The argument leading to cosmological bounds on $B - L$ violating interactions goes as follows. Assume that the baryon asymmetry was created before the electroweak phase transition as an asymmetry in at least one of the three $B/3 - L_i$, and that it is present in the thermal soup above the electroweak phase transition. The electroweak $B + L$ violating interactions are operating in this plasma. If there are also interactions in equilibrium that violate all of the $\{B/3 - L_i\}$, the baryon asymmetry will be washed out. Since it is supposed to be around today, this is undesirable. Therefore one requires that interactions violating one of the $B/3 - L_i$ be out of thermal equilibrium, so they cannot eat the baryon asymmetry.
This generically gives bounds of order $\lambda < 10^{-7}$ on “yukawa-type” coupling constants, and $m \lesssim$ keV-100 MeV for different masses. The mass bounds can be slightly tricky to calculate. These constraints apply to interactions that violate $B$ and one particular lepton flavour.

There are various loopholes to this argument. One can avoid them completely by not having the BAU present in the plasma for $100 \text{ TeV} > T > 100 \text{ GeV}$. This is approximately when it is at risk from the $B + L$ and $B - L$ violating interactions. The most obvious way to not have the baryon asymmetry present is to generate it at the electroweak phase transition. There are other models where the BAU is generated after the phase transition, or where it is generated at very high temperatures and hidden for $100 \text{ TeV} > T > 100 \text{ GeV}$.

It is also possible to modify the cosmological model to avoid these bounds. If the reheat temperature after inflation is below that of the electroweak phase transition, there is no opportunity to wash out the BAU. (One has fewer ways of making it though). If there electroweak $B + L$ violating interactions are turned off above the phase transition, then an asymmetry in $B + L$ can survive. If the expansion rate of the Universe is increased, the slowest Standard Model interaction, which is the electron yukawa, might not come into equilibrium until after the phase transition. If there is an asymmetry in the $e_R$, this means that an asymmetry must remain in the baryons, because the Universe should be hypercharge neutral.

It is possible to avoid the strong bounds on $B$ violating operators, or the conclusion that the asymmetry must carry $B - L$, via lepton mass effects. If there are large lepton flavour asymmetries, and lepton flavours are conserved, then a small baryon asymmetry proportional to $m_c^2/T^2 \times n_{L_i}-L_i$ will remain, even if $n_{B-L} = 0$ or if there are $B$ violating interactions in equilibrium.

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