Analytical Kinematics and Coupled Vibrations Analysis of Mechanical System Operated by Solar Array Drive Assembly

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Abstract. To address the impact of solar array (SA) anomalies and vibrations on performance of precision space-based operations, it is important to complete its accurate jitter analysis. This work provides mathematical modelling scheme to approximate kinematics and coupled micro disturbance dynamics of rigid load supported and operated by solar array drive assembly (SADA). SADA employed in analysis provides a step wave excitation torque to activate the system. Analytical investigations into kinematics is accomplished by using generalized linear and Euler angle coordinates, applying multi-body dynamics concepts and transformations principles. Theoretical model is extended, to develop equations of motion (EoM), through energy method (Lagrange equation). The main emphasis is to research coupled frequency response by determining energies dissipated and observing dynamic behaviour of internal vibratory systems of SADA. The disturbance model captures discrete active harmonics of SADA, natural modes and vibration amplifications caused by interactions between active harmonics and structural modes of mechanical assembly. The proposed methodology can help to predict true micro disturbance nature of SADA operating rigid load. Moreover, performance outputs may be compared against actual mission requirements to assess precise spacecraft controller design to meet next space generation stringent accuracy goals.

1. Introduction
SADA is a particular spacecraft component which rotates SA to maintain a perpendicular position to incidence direction of sunlight in order to obtain maximum solar power. Over the course of sun-tracking drive process vibration disturbances caused by low-and-dense natural frequencies of SA, attached to SADA, simultaneously react on spacecraft body thus influencing its dynamic environment [1]. These must be mitigated effectively to achieve high pointing accuracy [1]. Current expedition is to research & apply some cost effective and reliable methods of analysis that may help to understand disturbance behaviour of SADA. Most of research in SADA analysis domain is focused on its electric and electronic systems [2]. Xia and Bodson worked on mechanism of electromagnetic vibration [3, 4] of SADA and methods to reduce its disturbance amplitude using appropriate drive circuit [3]. Other contributions include simulation procedures to study slewing phenomenon [5] and design of controllers to enhance sensitivity [6], improve resolution [7] and dynamics of drive assembly [8]. However, SA is composed of mechanical systems [9, 10] which get influenced by external force / torque provided by drive assemblies. It is verified experimentally [11] that emitted vibration forces can degrade performance of precision instruments. In [12] and [13] coupled disturbance produced by SADA driving a rigid load through a transmission shaft of variable stiffness is modelled and simulated. Mariyam proposed theoretical formulations [13] and experimental analysis procedures [14] to
investigate disturbance behaviour of different subdivisions (SD) SADA operated without and with rigid load. However, there are few limitations associated with experimental tests and analytical techniques. Testing of interruptions generated by SADA in atmospheric environment / ground is hard to carry out, costly and time consuming task. The constraint of formerly proposed theoretical disturbance models is the assumption that systems operated by it possess single degree of freedom (DoF) and moves only along the spin axis which restricts research to obtain its realistic behaviour during space mission. In actual scenario, SADA controlled systems generate multi-DoF motion in which residual rotations and lateral translations also exist besides an ideal movement along spin axis. To the author’s knowledge, this gap has not been covered yet in any of its proposed dynamic analysis methods. This research attempts to bridge short comings in earlier projected evaluation schemes by providing comprehensive analytical investigation strategies to determine SADA coupled vibration behaviour. The proceeding sections provide SADA specifications and problem description, then analytical kinematics and coupled dynamics for SADA disruption is conversed, simulation method to validate analytical model is discussed next, followed by conclusion and future scope.

2. SADA details & problem description
SADA, an equipment of Electric Power Subsystem, [15] consists of Solar Array Drive Mechanism (SADM) and Solar Array Drive Electronics that drive SA. In this investigation SADA consists of 50 rotor teeth (r), 4 beats, 32 subdivisions, current amplitude (C_{amp}) of 0.1 A and electromagnetic torque coefficient (E_n) of 10. When an electric pulse signal is contributed to SADA stepper it induces electronic and electromagnetic coupling vibrations [13, 14] in system as stated in (1);

$$f_{32} = \sum_{m=1}^{\infty} 7.41m \quad ; \quad k_{em} = E_n C_{amp}$$

where; m = 1, 2, ..., \infty , f_{32} is active frequency and k_{em} is the electromagnetic stiffness. The problem geometry for analytical model consists of SADA operating rigid rectangular plate of 600*500*90 mm³ attached to transmission shaft, of length 300 mm and diameter of 100 mm, fixed to top of motor rotor figure (1). SADA ideally provides a step wave excitation torque to mounted assembly along spin axis. k_{em} in motor is approximated as a torsional spring between rotor and stator. After input of excitation force, mechanical system is under influence of lateral disturbances and angular motions about roll (x), pitch (y) and yaw (z) axis because of shaft flexibility. Analytical modelling for disturbance analysis of SADA provides an estimate of vibrations that are possibly aroused by drive mechanism in actual space mission when it is made to operate flexible SA.

3. SADA disturbance (analytical model): Transformations and coordinate systems
For kinematic analysis, geometry is defined by 3 cartesian and 3 angular coordinates. From figure (1), XYZ is inertial coordinate system (CS), X’Y’Z’ is fixed to point O of body, x_{0y}y_{0z} is body fixed CS with origin at O. x_{0y}y_{0z} is related to X’Y’Z’ system through a set of Euler angles i.e. α, β and γ which help to prescribe body axis with respect to (wrt) X’Y’Z’ and hence wrt XYZ system. Intermediate systems i.e., l_i’l_{i+1}’, and s_i s_{i+1} s_{i+2} s_{i+3} are used to facilitate computation of composite axis transformations. α, β and γ define an orthogonal CS i.e. \hat{i} \hat{j} \hat{k} that result from three successive rotations from a fixed CS i.e. \hat{I} \hat{J} \hat{K}. Three successive positive rotations i.e., about Z’ axis, new y-axis i.e., l_i’ and about final x-axis i.e., s_i axis result in final body fixed system. x_{0y}y_{0z} is related to XYZ by orthogonal transformation provided below. C & S in (2) correspond to Cos & Sine functions respectively.

$$\begin{bmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{bmatrix} = \begin{bmatrix} C\beta C\gamma & C\beta S\gamma & -S\beta \\ S\alpha S\beta C\gamma - S\gamma C\alpha & S\alpha S\beta S\gamma + C\alpha C\gamma & S\alpha C\beta \\ C\alpha S\beta C\gamma + S\gamma S\alpha & C\alpha S\beta S\gamma - S\alpha C\gamma & C\alpha C\gamma \end{bmatrix} \begin{bmatrix} I \\ J \\ K \end{bmatrix}$$

4. Kinematic analysis of SADA operated system
Suppose the body undergoes small rotation \text{d}y in time \text{d}t. The position of every point in rotated body is determined if position of a single point in it is specified and orientation of x_{0y}y_{0z} body axis wrt fixed XYZ axis. An infinitesimal rotation of body during time interval \text{d}t is;
Using figure (1) and (2) and putting (3.1) in (3) gives of \( \omega_b \) in \( x, y, z \) components w.r.t fixed frame.

\[
\varepsilon_a = \dot{a} \quad ; \quad \varepsilon_\beta = \cos \alpha \dot{a} - \sin \alpha k \quad ; \quad \varepsilon_\gamma = -\sin \beta \dot{a} + \sin \alpha \cos \beta j + \cos \alpha \cos \beta k
\]

\[
\omega_b = \omega_{ab} i + \omega_{ub} j + \omega_{ub} k \implies \begin{bmatrix} p \\ q \\ r \end{bmatrix} \begin{bmatrix} \omega_{ab} \\ \omega_{ub} \\ \omega_{ub} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin \beta \\ 0 & \cos \alpha \cos \beta & \cos \alpha \sin \beta \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \{ \alpha \beta \gamma \} \quad \text{(4)}
\]

\[ p, q, r \text{ are the components of angular velocity vector. } \alpha, \beta, \gamma \text{ are the angles associated with roll, pitch and yaw. An assumption of constant rotational motion, } (\dot{x} = x, Cx = 1) \text{, reduces to: (5):}
\]

\[
\begin{bmatrix} C \beta \gamma \cr C \gamma \cr S \alpha \end{bmatrix} = \begin{bmatrix} 1 & -\gamma & \beta \\ -\gamma & 1 & \alpha \\ \beta & \alpha & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \{ \alpha \beta \gamma \} \quad \text{(5)}
\]

Considering figure (3) for position vectors i.e., \( \text{R}_o \), position vector of transmission shaft attachment point to rigid load with respect to origin of inertial frame then:

\[
\text{R} = x \dot{i} + y \dot{j} + z \dot{k} \quad ; \quad \text{R} = x \dot{i} + y \dot{j} + z \dot{k} \quad ; \quad \dot{R} = x \dot{i} + y \dot{j} + z \dot{k}
\]

\[ X, Y, Z, \text{ denote components of lateral and linear motion of position } O \text{ w.r.t inertial } XYZ \text{ axis. These three linear independent coordinates in addition to } \alpha, \beta, \gamma \text{ completely specify position and orientation of mechanical system. Let point } P \text{ } (x_p, y_p, z_p), \text{ in figure (3), be any fixed point to lateral / linear end of rigid load wrt position } (X, Y, Z). \text{ R}_p \text{ is position vector of point P w.r.t inertial frame.}
\]

\[
\dot{R} = \dot{R}_o + \dot{L}_p \quad ; \quad \dot{L}_p = \dot{R}_o + \dot{R}_p = \dot{x}_p i + \dot{y}_p j + \dot{z}_p k \quad ; \quad \dot{R}_p = x \dot{i} + y \dot{j} + z \dot{k}
\]

\[ \ddot{R} = \ddot{R}_o + \ddot{L}_p \quad ; \quad \ddot{L}_p = \ddot{x}_p i + \ddot{y}_p j + \ddot{z}_p k \implies \dot{L}_p = \{x, y, z\} \quad \text{(8)}
\]

Using \( \dot{i}, \dot{j}, \dot{k} \) from (5) in (8), and formulation for \( \dot{R}_p \) from (7) gives acceleration of shaft at point P.

\[
\ddot{R}_p = \ddot{L}_p + \ddot{R}_o + \ddot{R}_p = \ddot{x}_p i + \ddot{y}_p j + \ddot{z}_p k
\]

Part I and II show lateral acceleration, while III is along bounce axis. \( x_p, y_p, \) and \( z_p \) are distances of point P from centre O along the \( x_b, y_b, \) and \( z_b \) axis respectively. The acceleration equation of any point \( P \) reflects influence of pitch, roll and yaw motion in addition to translational accelerations. Since, point \( R_o \) is defined w.r.t to centre O on rigid body and geometry is mechanically coupled so, the displacement equation of point \( R_o \) can be used to show deflections generated in the whole system or the coupled oscillations because of input motion by rotor. So, deflection equation \( \Delta_o, \) at point P on rigid body is;

\[
\Delta_o = (X_o + z_p \beta - y_p \gamma) + (Y_o + x_p \gamma - z_p \alpha) + (Z_o + y_p \alpha - x_p \beta)
\]

\[ X, Y, Z, \text{ are the lateral displacements and } Z_b \text{ is bounce motion of point } O, \gamma \text{ is angular displacement of body about } z_b \text{ axis, while } \beta \text{ and } \alpha \text{ are angular displacements of body about } y_b \text{ and } x_b \text{ axis respectively.}
\]
5. Dynamic analysis

Figure (4) provides approximate dynamic model of SADA supporting rigid load. Rotor strictly rotates along spin axis while mechanical system mounted atop shows coupled motion as explained in kinematic analysis. The electromagnetic effect between rotor and stator of stepper motor is approximated as torsional spring possessing electromagnetic stiffness. Moment of inertias of rigid load and SADA rotor are considered to determine coupled frequencies of mechanical system. The seven EoM of system are derived using Lagrange’s method which in its generalized form is;

\[
\frac{d}{dt} \left( \frac{\partial KE}{\partial q_i} \right) - \frac{\partial KE}{\partial \dot{q}_i} + \frac{\partial PE}{\partial q_i} + \frac{\partial DE}{\partial \dot{q}_i} = Q_i
\]

(11)

\[KE = \frac{1}{2} I (\dot{X}_o^2 + \dot{Y}_o^2 + \dot{Z}_o^2) + \frac{1}{2} I_o \dot{\gamma}_o^2 + \frac{1}{2} I_p \dot{\beta}^2 + \frac{1}{2} I_o \dot{\alpha}^2\]

(12)

\[PE = \frac{1}{2} K_e \dot{y}_o \beta - y_p (\dot{\gamma}_o - \gamma_o) + \{Y_o + x_p (\dot{\gamma}_o - \gamma_o) - z_p \dot{\alpha}\} + \{Z_o + y_p \alpha - x_p \dot{\beta}\}\]

(13)

\[DE = \frac{1}{2} C \dot{y}_o \beta - y_p (\dot{\gamma}_o - \gamma_o) + \{Y_o + x_p (\dot{\gamma}_o - \gamma_o) - z_p \dot{\alpha}\} + \{Z_o + y_p \alpha - x_p \dot{\beta}\}\]

(14)

Where; KE, PE and DE are kinetic, potential and dissipation function / dissipation energies respectively, Q_i is generalized external force acting on system, K is stiffness and C is damping coefficient. Equations (12~14) govern coupled torsional-lateral motion behaviour of SADA supporting rigid load with transmission shaft. For an initial approximation of system’s coupled frequency response, Part-I inside square brackets of PE, DE and corresponding acceleration functions from KE are used to formulate EoM for yaw, pitch and lateral motion in x-direction using Lagrange method.

\[
\begin{bmatrix}
I_o & 0 & 0 & 0 & \dot{\gamma}_o & 0 & 0 \\
0 & I_o & 0 & 0 & 0 & \dot{\beta} & 0 \\
0 & 0 & I_o & 0 & 0 & 0 & 0 \\
0 & 0 & m_o & I_x & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\dot{\gamma}_o \\
\dot{\beta} \\
\dot{X}_o \\
\dot{Y}_o \\
\dot{Z}_o \\
\end{bmatrix}
= \begin{bmatrix}
\gamma_o \\
\beta \\
X_o \\
Y_o \\
Z_o \\
\end{bmatrix}
\]

(15)

Moment of inertia of SADA rotor, electromagnetic torque coefficient, stiffness of transmission shaft and damping coefficient are obtained by experimental testing. System is designed in solid works to get moment of inertia and mass of rigid load. The values of x_p, y_p and z_p depend upon dimensions of rigid load. Later, program will be written in MATLAB / MuPad to determine vibration behaviour of SADA driven assembly. Equation (16) provides probable disturbance response of SADA influenced by structural coupling. It reveals that both electronic harmonics of SADA and natural frequencies of mechanical system contribute to SADA disturbance.

\[\int_{SADA} = \{f_{electronic}, f_{natural}\}\]

(16)

6. Conclusion & future scope

This work has proposed a mathematical SADA disturbance analysis scheme for incorporation into a performance assessment and enhancement framework of satellite motion. In that context, disturbance model is used to drive model of spacecraft and outputs are compared against requirements to assess controller design. The development of disturbance model is an important step in this process because the accuracy of original performance results is highly dependent on quality of developed vibration model. The problem geometry analysed in this paper consisted of rigid load supported on transmission shaft and functioned by SADA. Background study is completed to extract least targeted areas. Based on short comings a new disturbance model for SADA controlled assembly is proposed. The analytical kinematic model captures motion behaviour by considering translation along and rotation about 3 coordinate axes. The kinematics of system utilizes Euler transformations and relative motion analysis. The feasibility of mathematical model & stability of system is ensured by developing dynamic model using Lagrange approach and determining energies dissipated in system. Dynamic model provides coupled torsional lateral vibrations induced in the system as a result of input step excitation torque.

In order to validate analytical dynamics, MATLAB / Simulink toolbox is used to simulate disturbance model of SADA figure (5). This model will be based on dynamic equations of stepper motor dragging rigid load with transmission shaft. The Simulink model has advantage that results
from it can be represented in either time or frequency domain and is applicable to any geometry of rigid loading. The input step excitation torque will be considered from [11, 13]. Earlier studies have considered SADA supporting rigid load as 2-DoF system. The disturbance response consisted of natural frequencies and discrete active harmonics of SADA [12, 13]. However, present investigation is an effort to contribute its more complete and realistic disruption behaviour by considering influence of coupled torsional and lateral / linear vibrations in analytical model. The disturbance feedback will consist of mode excited by electromagnetic frequency, three natural mechanical frequencies of coupled equivalent SADA driven system and harmonics of its electronic frequencies. The expected coupled frequency response for 2-DoF system (active harmonics and two natural frequencies) is depicted in figure 6. Analysis provides a theoretical basis for prediction of instabilities emitted by SADA during in orbit operation. The Simulink model will verify that fundamental active harmonic of SADA has the highest disturbance amplitude and it will decrease in higher frequency zone. In addition, at any point during analysis, interference of electronic and natural frequencies will tend to multiply disturbance amplitude observed by SADA supported system.

Researchers have worked on the influence of electromagnetic stiffness and increase in subdivisions number on the disturbance amplitude of SADA operated systems [13]. The same factors can be applied in this work to evaluate influence of coupled torsional lateral dynamics of system supported, on, and driven by SADA. Models of SADA induced disturbances can be used in jitter analysis to predict effects of vibrations on spacecraft, allow development of drive circuits and design of suitable isolation and damping technologies.

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