The rapidity dependence of the average transverse momentum in p + Pb collisions at the LHC: The Color Glass Condensate versus hydrodynamics

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We show that in proton–lead (p + Pb) collisions at the LHC, the Color Glass Condensate (CGC) and hydrodynamics lead to qualitatively different behavior of the average transverse momentum, \( \langle p_t \rangle \), with the particle rapidity. In hydrodynamics, the \( \langle p_t \rangle \) decreases as one goes from zero rapidity, \( y = 0 \), to the proton fragmentation region since the number of particles decreases. In contrast, in the CGC the saturation momentum increases as one goes from \( y = 0 \) to the proton fragmentation region, and so the \( \langle p_t \rangle \) increases. At the LHC, the difference between the two models may be large enough to be tested experimentally.

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1. Introduction

Recent experiments with proton–lead (p + Pb) collisions at the LHC on 2- and 4-particle correlations [1–6] give rise to various theoretical interpretations. The two-dimensional correlation functions in relative pseudorapidity and relative azimuthal angle demonstrate the ridge-like structures, elongated in pseudorapidity, with in relative pseudorapidity and relative azimuthal angle demonstrating the ridge-like structures, elongated in pseudorapidity, with enhanced emission of particle pairs in same \( \Delta \phi \simeq 0 \) and away-side \( \Delta \phi \simeq \pi \) directions. The Color Glass Condensate (CGC) approach leads to the long-range correlations in rapidity [7] with the same-side structure coming from the interference diagrams enhanced in the saturation regime [8,9]. The measured elliptic and triangular harmonic coefficients of azimuthal distributions can also be explained by the hydrodynamic expansion of the dense small fireball, see Refs. [10–15]. A recently proposed measurement of the femtoscopy radii in p + Pb interactions at different centralities could disentangle between the two scenarios [16,12], as the collective expansion leads generally to a larger size of the system. Another observable sensitive to the collective expansion is the average transverse momentum \( \langle p_t \rangle \) of the emitted particles [17–19]. The average momentum is larger in proton–proton than in p + Pb collisions for events of the same multiplicity. As was shown in Ref. [20], the model that treats a p + Pb collision as a superposition of independent p + p interactions explains this trend. According to this model the average transverse momentum is smaller in p + Pb than in p + p collisions, at the same multiplicity. Moreover, the model results lie below the experimental data [20]. This leaves room for an additional collective push, which is naturally present in hydrodynamics. Also the relative excess of \( \langle p_t \rangle \) for different masses over various model predictions in p + Pb interactions can be understood in hydrodynamics [15,18]. On the other hand, the mass hierarchy of the transverse momentum at central rapidity may appear due to the color reconnection or the geometrical scaling discussed in Refs. [21,22]. To resolve this ambiguity in p + Pb collisions, the measurement of the number of charged particles at central rapidity as a function of the number of participants was proposed in Ref. [23].

In this Letter, we propose to study the rapidity, \( y \), dependence of the average transverse momentum of charged particles. In the CGC the average transverse momentum is determined by the nucleus saturation momentum. The evolution of the saturation momentum with rapidity towards the proton direction yields a growth of the average transverse momentum, quite in opposite to what is expected from a collective expansion. Namely, the hydrodynamic model predicts a decrease of the average transverse momentum when going from midrapidity, \( y = 0 \), to the proton side, owing to a decreasing number of produced particles.
2. Rapidity dependence of transverse momentum

In the CGC the dependence of the average transverse momentum on rapidity can be deduced from quite general arguments. First of all, the relation between the average transverse momentum of final particles, \( \langle p_{\perp} \rangle \), and the average transverse momentum of produced gluons, \( \langle k_{\perp} \rangle \), is

\[
\langle p_{\perp} \rangle = \frac{\int d^{2}p'_{\perp} f_{g}^{2}(\vec{p}_{\perp}) f_{g}(\vec{p}_{\perp}')}{\int d^{2}p'_{\perp} f_{g}^{2}(\vec{p}_{\perp})} = \langle z \rangle \langle k_{\perp} \rangle, \tag{1}
\]

where \( f_{g}(\vec{p}_{\perp}) \) is the gluon distribution function, and \( D(z) \) is the gluon fragmentation function. We defined the first moment of the gluon fragmentation function \( z \) as

\[
\langle z \rangle = \frac{\int_{0}^{1} \! dz D(z) z}{\int_{0}^{1} \! dz D(z)}. \tag{2}
\]

Deriving Eq. (1) we assumed that the gluon fragmentation function is independent of the transverse momentum. This assumption may not be justified for very soft gluons.\(^1\)

The information available about the gluon fragmentation function is rather limited. Therefore it is important to construct an observable for which \( \langle z \rangle \) cancels out. In this Letter, we adopt the ratio of the transverse momentum at a given rapidity \( y \) to the value at \( y = 0 \):

\[
\frac{\langle p_{\perp} \rangle_{y}}{\langle p_{\perp} \rangle_{y=0}} = \frac{\langle k_{\perp} \rangle_{y}}{\langle k_{\perp} \rangle_{y=0}} \tag{3}
\]

The gluon distribution function can be obtained within the \( k_{\perp} \)-factorization formalism, according to which the cross-section for inclusive gluon production reads \([24]\):

\[
\frac{d\sigma^{p+Ag \rightarrow g}}{d^{2}k_{\perp} \, dy} = \frac{2\alpha_{s}}{C_{F}} \frac{1}{k_{\perp}^{2}} \int d^{2}q_{\perp} \phi_{p}(q_{\perp}^{2})\phi_{A}(\vec{k}_{\perp} - \vec{q}_{\perp})^{2}, \tag{4}
\]

where \( \phi_{p,A} \) are the unintegrated gluon distribution (UGD) functions for the proton and the nucleus, respectively, and the Casimir operator in the fundamental representation of SU(3) is given by \( C_{F} = 4/3 \). Here and in what follows, to lighten the notation we suppress dependence of the UGD and saturation momentum on \( x \).

The gluon distribution is then

\[
f_{g}(k_{\perp}) = \frac{dN}{d^{2}k_{\perp} \, dy} = \frac{1}{\sigma_{\text{inel}}} \frac{d\sigma^{p+Ag \rightarrow g}}{d^{2}k_{\perp} \, dy}.
\]

Using the McLerran–Venugopalan model for the classical gluon distribution function one gets (see Ref. \([25]\) for details)\(^2\)

\[
\langle k_{\perp} \rangle \approx 2 Q_{A} \frac{\ln(Q_{A}^{2}/Q_{p}^{2})}{\ln^{2}(Q_{A}^{2}/Q_{p}^{2})}. \tag{5}
\]

Neglecting logarithmic corrections we have for the gluon distribution function

\[
\frac{f_{g}(k_{\perp})}{S_{\perp}} \propto \begin{cases} 1, & k_{\perp} < Q_{p}, \\ \frac{Q_{p}^{2}}{k_{\perp}^{2}}, & Q_{p} < k_{\perp} < Q_{A}, \\ \frac{Q_{A}^{2}}{k_{\perp}^{2}}, & k_{\perp} > Q_{A}. \end{cases} \tag{7}
\]

\(^1\) The average transverse momentum of produced pions in high multiplicity \( p+\text{Pb} \) collisions is approximately 0.6 GeV, thus \( \langle k_{\perp} \rangle \) is expected to be around a few GeV.

\(^2\) This relation is valid in the regime \( Q_{p} < k_{\perp} < Q_{A} \).

This formula captures the general features of the CGC description of \( p+\text{Pb} \) collisions, see discussions in Refs. \([25,26]\). In this framework the system is characterized by two different saturation scales: \( Q_{p} \), the saturation momentum of the proton, and \( Q_{A} \), the saturation momentum of the nucleus. In our discussion we assume that \( Q_{A} \gg Q_{p} \), which seems to be justified for central \( p+\text{Pb} \) collisions. Performing straightforward integrations we obtain

\[
\langle k_{\perp} \rangle \approx 2 Q_{A} - \frac{2}{1 + \ln(Q_{A}^{2}/Q_{p}^{2})} \approx 2 Q_{A}. \tag{6}
\]

As expected the average transverse momentum of gluons is roughly proportional to the saturation momentum of the nucleus. Taking into account certain logarithmic corrections to Eq. (7), one obtains a more accurate expression Eq. (6).

The rapidity dependence of the average transverse momentum follows from the standard relations (see, e.g., Ref. [27])

\[
Q_{p}^{2} \sim Q_{A}^{2} e^{-\lambda y}, \\
Q_{p}^{2} \sim Q_{A}^{2} e^{-\lambda y}. \tag{9}
\]

In this article we choose \( \lambda \approx 0.2 \), following Ref. [28]. It is worth noticing that our results for \( \langle p_{\perp} \rangle_{y}/\langle p_{\perp} \rangle_{y=0} \) are insensitive to the value of \( Q_{0} \).

Substituting above relations to Eqs. (8), (6) we obtain the results presented in Fig. 1. The black band corresponds to calculations based on Eq. (8) with several values of \( N_{\text{part}} \), ranging from 10 to 25. The red band is based on Eq. (6). As expected the \( \langle p_{\perp} \rangle \) in the CGC is increasing when going from \( y = 0 \) towards the proton fragmentation region owing to increasing \( Q_{A} \), see Eq. (9). It is worth mentioning that \( \langle p_{\perp} \rangle_{y}/\langle p_{\perp} \rangle_{y=0} \) very weakly depends on \( N_{\text{part}} \) since \( \langle p_{\perp} \rangle_{y} \sim Q_{A} \) and the number of participants cancels in the ratio. We emphasize that our results are not sensitive to the specific form of \( D(z) \), see Eq. (3).

In hydrodynamics the dependence of \( \langle p_{\perp} \rangle_{y}/\langle p_{\perp} \rangle_{y=0} \) on rapidity is expected to be quite opposite. The average transverse momentum of particles emitted in the hydrodynamic model is composed of two contributions, the thermal motion at the freeze-out and the collective velocity acquired during the expansion. Unlike in heavy-ion collisions, in \( p+\text{Pb} \) interactions the matter density

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Fig. 1. The average transverse momentum of produced particles as a function of rapidity, divided by the average transverse momentum at \( y = 0 \). The CGC results for different values of \( N_{\text{part}} \) (black and red bands corresponding respectively to Eqs. (8), (6)) differ qualitatively from those obtained in the hydrodynamical framework (the dashed band covers three centralities 0–3, 5–15, and 40–60%). We plot two curves for the CGC to demonstrate the robustness of our predictions under the logarithmic corrections. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)
depends strongly on rapidity. Experimental results show a larger multiplicity on the lead side than on the proton side [29], and the asymmetry increases with centrality of a collision [30]. In hydrodynamics the collective flow velocity results from the action of pressure gradients in the fireball [31]. The initial energy deposition in the fireball should increase as function of space–time rapidity when going to the lead side in order to match the observed asymmetry of charged particle density in pseudorapidity.

In Figs. 1 and 2 we present the results obtained from state-of-the-art (3 + 1)-dimensional event-by-event hydrodynamic simulations [32]. In this calculation, the asymmetry of the initial density of the fireball is imposed following the experimental observations in deuterion–gold collisions at $\sqrt{s_{NN}} = 200$ GeV [33–35]. The initial entropy profile is determined by the positions of the participant nucleons obtained from the Glauber Monte Carlo model. The entropy deposited at the transverse position $x, y$ and space–time rapidity $\eta_i$ by a participant located at the position $x_i, y_i$ is

$$s_i(x, y, \eta_i) = f_{\pm}(\eta_i) \exp\left(-\frac{(x-x_i)^2+(y-y_i)^2}{2\sigma_0^2}\right),$$

where $\sigma_0 = 0.4$ fm. The profiles $f_{\pm}(\eta_i)$ are of the form

$$f_{\pm}(\eta_i) = \left(1 \pm \frac{\eta_i}{\eta_\text{beam}}\right) f(\eta_i)$$

and the longitudinal profile

$$f(\eta_i) = \exp\left(-\frac{(|\eta_i| - \eta_0)^2}{2\sigma_0^2}\theta(|\eta_i| - \eta_0)\right),$$

where $\sigma_0 = 1.4$, $\eta_0 = 2.4$, and $\eta_\text{beam} = 8.5$ is the beam rapidity. The total entropy is the sum of the contribution of the incoming proton and $N_{\text{part}}$ nucleons from the lead nucleus, defined with the signs “+” and “−” respectively in Eq. (12). With the increasing number of participants the asymmetry of the fireball increases, yielding the charged particle pseudorapidity distributions in semi-quantitative agreement with experiment [30].

The parameters for the hydrodynamic calculation are chosen as in Ref. [15], so that it reproduces reasonably the transverse momentum of identified particles in central and semi-central collisions as well as the elliptic and the triangular flow in the most central collisions. As seen in Figs. 1 and 2 the transverse momentum for various centralities decreases when going from $y = 0$ to the proton side. The precise form of the charged particle density and of the average transverse momentum obtained from the hydrodynamic model depends on the parameters of the initial profile in Eq. (13) and on the details of the Glauber model used [11], but qualitatively the same dependence of the average transverse momentum on rapidity is observed. In practice the LHC experiments cannot measure identified particles in a wide enough range of rapidities. The transverse momentum of charged particles as function of pseudorapidity from hydrodynamical calculations is shown in Fig. 2. The change to the pseudorapidity variable causes a reduction of $(p_{\perp}(\eta))/(p_{\perp}(\eta = 0))$ when going away from midrapidity, as compared to $(p_{\perp}(y))/(p_{\perp}(y = 0))$. The effect is noticeable, but would not drive the value of $(p_{\perp}(\eta \simeq 2))/(p_{\perp}(\eta = 0))$ below one for the CGC case, so that experimentally the dependence of the average transverse momentum on pseudorapidity can be used to distinguish between the two scenarios.

As seen in Fig. 1, going from midrapidity, $y = 0$, towards the proton fragmentation region we increase $(p_{\perp})$ in the CGC owing to the increasing saturation momentum of the nucleus. On the contrary, $(p_{\perp})$ is decreasing in the hydrodynamics picture owing to the decreasing number of particles. This is the main result of our Letter.

Finally we would like to make several remarks on the CGC expectations presented in this Letter. The above results are reliable only for large $N_{\text{part}}$ to ensure that we have the separation of the two scales $Q_A \gg Q_p$, which is implicitly assumed by applying the $k_t$-factorization formalism. Our CGC results are based on quite general arguments and to obtain more precise predictions a detailed model calculations should be performed, however this could be very challenging. For example, recently the NLO calculations in the CGC framework were performed in Ref. [36]. For the forward hadron production, these calculations demonstrated that the NLO corrections seem to be dominant at high transverse momentum. This implies that even higher order corrections should play an important role.

3. Conclusions

In conclusion, we investigated the rapidity dependence of the average transverse momentum of charged particles in proton–lead collisions at the LHC. We noticed, based on the general arguments and simplified analytical calculations, that, in the CGC, the transverse momentum is slightly increasing with the increasing rapidity (going from $y = 0$ towards the proton fragmentation region) owing to the increasing saturation momentum of the nucleus. On the contrary, the $(p_{\perp})$ in the hydrodynamic framework is decreasing owing to the decreasing number of particles. The collective expansion scenario cannot lead in a simple way to an increase of the average transverse momentum on the proton side.

We would like to point an interesting possibility, namely that $(p_{\perp}(y))$ decreases with $y$ around midrapidity, according to the collective expansion picture, but it starts to increase for larger $y$, in a region where collectivity switches off and possibly saturation becomes dominant for the dynamics of the system.

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