Compositionality for Presuppositions over Tableaux*

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Abstract

Tableaux originate as a decision method for a logical language. They can also be extended to obtain a structure that spells out all the information in a set of sentences in terms of truth value assignments to atomic formulas that appear in them. This approach is pursued here. Over such a structure, compositional rules are provided for obtaining the presuppositions of a logical statement from its atomic subformulas and their presuppositions. The rules are based on classical logic semantics and they are shown to model the behaviour of presuppositions observed in natural language sentences built with if . . . then, and and or. The advantages of this method over existing frameworks for presuppositions are discussed.

Keywords: presupposition, compositionality, tableaux, natural language representation.

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1 The Problem of Compositionality for Presuppositions

This work is concerned with the problems of adding the concept of presupposition to a logical language. Although presupposition originates as a natural language phenomenon, for the purposes of the present work sentences will be represented as propositions of a logical language. At this level of granularity, presupposition can simply be represented as a relation between sentences. For instance, the sentence *The typewriter is working* can be said to presuppose: *There is a typewriter.*

Presupposition has the additional property (often used to characterize it) that the negation of a sentence has the same presuppositions as the sentence itself. For example, the sentence *The typewriter is not working* has the same presupposition as its positive counterpart given above (namely, that there is a typewriter). The reader is referred to the literature on presupposition for a wider analysis of the implications of this property.

A formal language has the property of compositionality if it is possible to describe the meaning of a complex expression of the language in terms of the meaning of its parts. It is considered a desired property for any formal language. When a logical statement is composed from propositions that presuppose other propositions, it should be possible to describe the presuppositions of the resulting complex expression in terms of its parts and the presuppositions of its parts. If one takes the natural language connectives *if*...*then*, *and*, and *or* to be related to material conditional, conjunction, and disjunction, natural language examples provide some clues as to what the behaviour of presupposition should be. Sentences (1), (2) and (3) presuppose *There is a typewriter*; while sentences (4), (5) and (6) do not.

1. *If the typewriter is blue then Sue will be happy.*
2. *If you are in Sue’s office then the typewriter is blue.*

The most important consequence is that attempts to model presupposition as an entailment lead to characterizations where all presuppositions are tautologies in the classical sense, which is an undesirable result. This is based on the argument that if $\alpha \vdash \beta$ and $\neg \alpha \vdash \beta$ then it must be the case that $\vdash \beta$. Because presuppositions are observed to be informative in their actual use by language speakers, this way of modelling them is not useful.
(3) Either the typewriter is blue or the chair is.
(4) If there is a typewriter then the typewriter is blue.
(5) Either there isn’t a typewriter or the typewriter is blue.
(6) Either the typewriter is blue or there isn’t a typewriter.

The behaviour of presuppositions of sentences of this form has traditionally been studied as part of the projection problem for presuppositions, which is concerned with describing the presuppositions of a sentence in terms of the presuppositions of its subordinate clauses. The constructions considered in the projection problem involve nested subordination (verbs of propositional attitude, factive verbs) beyond the natural language connectives treated here.

2 Previous Work

The framework presented in this work is compared with three other approaches.

Beaver [1, 2] presents a solution based on update semantics. This account originates from an approach to the projection problem that concentrates on how the presuppositions of a compound sentence are obtained from the presuppositions of its components (Karttunen [9, 10], Karttunen and Peters [11] and Heim [7, 8]). The projection problem is rephrased in terms of what it takes for a context (represented as a set of possible worlds) to satisfy a presupposition. This question is answered by defining for each connective updates of compounds in terms of sequences of simpler updates that involve only their components. However, there are several problems: no satisfactory account for the behaviour of disjunction is provided; the defeasibility of presuppositions is not addressed; and presupposition as an informative operation is only allowed as a repairing modification to the original framework.

Mercer [12, 13] applies a default logic approach. This account originates from an approach to the projection problem that concentrates on how presupposition as an inference is defeated when inconsistent with more firmly established information (Gazdar [4, 5]). Presuppositions (of negative sentences) are represented as normal defaults. Mercer’s method follows Gazdar in taking into account the internal structure of sentences only in terms of Gricean pragmatic implicatures of the sentence. In order to avoid excessive
production of presuppositions, Mercer needs to take into account that an assertion of $\alpha \lor \beta$ or $\alpha \rightarrow \beta$ carries with it the assumption that the speaker does not know $\alpha, \neg \alpha, \beta,$ or $\neg \beta$, that is, that he considers an information state where all of them are open possibilities. The consistency checks required for defaults must consider these implicatures. The method obscures the issue of how the compositionality for truth values and the compositionality for presupposition are related.

There is an alternative approach (van der Sandt [14]) within the framework of DRT. This approach considers presupposition as anaphora. In this approach, interpretation of presupposition is reformulated as a search for an anaphoric referent. This solution is essentially linguistic. It captures an essential aspect of the nature of presupposition (its anaphoric nature), but it fails to address its direct relation with the semantics required for interpreting these connectives in a logical sense.

3 The Tableau Interpretation

3.1 The Basic Framework

Where language is simplified to a set of sentences so that the internal construction of each sentence does not play a role, the representation of presupposition can be restricted to defining the ordered pairs of sentences for which this relationship holds. I assume that such a relation of presupposing is given for the atomic formulas of the language. To make this information conspicuous without introducing too many definitions, I notate the fact that ‘$\alpha$ presupposes $\beta$‘ by writing each instance of $\alpha$ as $\alpha^\beta$.

In terms of this notation, the behaviour of presupposition concerning negation has the following implication: $\neg(\alpha^\beta) \equiv (\neg \alpha)^\beta$ (For simplicity, I leave out the parentheses in these cases from now on.)

For complex sentences the relation of presupposing has to be worked out, ideally in a compositional way. This is achieved in the next section by defining rules that govern the compositionality. In order to obtain a simpler formulation of these rules, a specific representation of the logical structure of the connectives is required. This representation is described in the present section.

The logical connectives I am considering have their own definition of
compositionality with respect to truth value. In the present framework, these
definitions are represented as tableaux expansion rules.

Tableau expansion rules:

\[
\begin{array}{c}
\alpha - rules) \\
\alpha_1 \land \alpha_2 \\
\hline
\alpha_1 \\
\alpha_2 \\

\neg(\alpha_1 \rightarrow \alpha_2) \\
\hline
\neg\alpha_1 \\
\neg\alpha_2 \\

\neg(\alpha_1 \lor \alpha_2) \\
\hline
\neg\alpha_1 \\
\neg\alpha_2 \\
\end{array}
\]

Classical negation operates over the language wherever reasoning about
negated sentences is required.

Some additional definitions are needed to introduce this way of under-
standing the representation. Apart from the expansion rules, these defini-
tions follow existing tableau frameworks for propositional logic. They include
definitions of: a tableau, a branch of a tableau, branch closure, and closure
for a tableau. (For standard definitions, see Fitting [3]).

### 3.2 Coverage Property

The definition of tableau expansion rules ensures that these tableau obey a
special property.

**Coverage Property:**

If one branch of a tableau holds a sentence \( \delta \), then every (open)
branch of that tableau will hold either \( \delta \) or \( \neg\delta \).

This property can be seen to hold: 1) it applies to each one of the expa-
sion rules, 2) the procedure for adding sentences to a tableau is defined in
terms of adding the new sentence to every (open) branch (and then applying
the expansion rules to it).

A consequence of this property in terms of the semantics, is that each
branch of the tableau contains a complete atomic truth-value assignment that
makes the sentence true. The structures that result are equivalent to classical
truth tables. A tableau formulation is retained in spite of this fact because it takes into account that lines of the truth table that become inconsistent as more information is added are dropped out of the reckoning (as an effect of branch closure).

3.3 The Formalism as a Decision Method for the Logic

The present framework differs from traditional tableaux only in the definition of $\beta$ expansion rules. The traditional definition of $\beta$ expansion rules is not suitable for presupposition because it specifies the alternatives in the minimal form that preserves soundness and completeness of the logical calculus. This is done in order to simplify the computation of logical consequences. The information that is not specified explicitly as a result of this policy does not affect logical consequences, but it is relevant to presupposition behaviour as described by the compositionality rules given here.

The tableaux given here (presuppositional tableaux or PT) can be used as a decision method in the same way as traditional tableaux (semantic tableaux or ST). A simple way of showing this may be to show that PT tableaux are equivalent to ST tableaux. Given that the definitions of tableau, tableau for a set of sentences, closure, proof and refutation are the same in the PT framework as in ST tableaux, the discussion concerns only the expansion rules. Of these, only $\beta$-rules differ from one framework to the other. It is enough to show that under the circumstances under which ST tableau for a $\beta$-formula becomes closed, the PT tableau for the same $\beta$-formula also becomes closed (and viceversa).

4 Compositionality of Presupposition over Logical Connectives

4.1 Compositionality Rules for Presupposition

Over the representation of the logical language given in the previous section, compositionality of presupposition can be defined.

Two issues need to be dealt with: 1) how the presuppositions of a branch are ascertained from the presuppositions of the atomic propositions in it, and 2) how the presuppositions of branches of the same tableau interact.
I notate a branch as $\Delta$. I use $\beta \in \Delta$ as shorthand for ‘the proposition $\beta$ appears in the branch $\Delta$. The notation for presupposition is extended to branches so that $\Delta^\beta$ stands for ‘the branch $\Delta$ presupposes $\beta$’.

The compositionality rules for presupposition do not take into account presuppositional information from branches of a tableau that are closed.

**Rule 1**

For an open branch $\Delta$ such that $\alpha^\beta \in \Delta$, $\Delta^\beta$ unless i) $\neg \beta \in \Delta$, or ii) $\beta \in \Delta$, or iii) there is some $\delta^\beta \in \Delta$.

**Rule 2**

For a tableau $\Gamma$ with at least one open branch $\Delta$, $\Gamma^\beta$ iff $\Delta^\beta$.

The determination of presuppositions of a branch is handled in a simple way by considering that presupposition is weaker than assertion, so that it survives only in cases where it does not overlap or clash with any other information. Conditions 1.i and 1.ii capture the intuitive observation that presupposition is only informative when it concerns a proposition that does not appear in any literal already in the branch. Condition 1.iii allows the presuppositions of a branch to survive only when there is no conflict between them.

The interaction between presuppositions of different branches can be shown to be trivial in the present framework by application of the Coverage Property. By the Coverage Property, different branches will have the same atomic propositions and they will differ only in that each atomic proposition may appear negated in one branch and appear unnegated in another. In terms of Rule 1, these differences would affect only $\alpha$ and $\delta$ as possible origins of presuppositions and $\beta$ in conditions i) and ii). Because the relation of presupposing is given for atomic propositions, if $\delta^\beta$ in one branch, it is not possible to have $\delta^\beta$ in a different branch. The results of applying rule 1 are the same when any of these propositions is exchanged for its negation.\[\text{[1]}\]

This can be used to show that if a tableau has a branch that presupposes a proposition, then every branch of the tableau will presuppose that proposition. Conversely, if a tableau has a (open) branch that does not presuppose a proposition, then no branch of the tableau will presuppose that proposition.

\[\text{[2]}\] For $\alpha$ and $\delta$, the negated and the unnegated version have the same presuppositions; for $\beta$, substituting with the negation is equivalent to interchanging conditions i) and ii).
4.2 Simple Sentences

I give the first few examples in detail to show how the rules operate.

Example (1) is a case of behaviour of presuppositions originating in the antecedent of a conditional. Sentence (1) *If the typewriter is blue then Sue will be happy* corresponds to the following representation.

\[
\alpha^3 \to \gamma
\]

By application of rule 1, every one of the alternatives presupposes \( \beta \). None of the conditions holds for any branch. By application of rule 2 to any one of them, \( \beta \) is obtained as a presupposition of the compound. This matches expected behaviour.

The sentence (7) *Bill regrets that there is no hot water left* presupposes (8) *There is no hot water left*. The problem is to determine what the presuppositions are for sentence (9) *If Mary has had a bath, then Bill regrets that there is no hot water left*. Assume sentence (9) has the form \( \alpha \to \delta^3 \). The representation for this sentence in this framework would be:

\[
\alpha \to \delta^3
\]

Application of the rules predicts a presupposition \( \beta \) for the sentence.

Example (4) is a case of behaviour of presuppositions originating in the consequent of a conditional, in the particular case where the presupposition itself forms the antecedent. Sentence (4) *If there is a typewriter then the typewriter is blue* corresponds to the following representation.

\[
\beta \to \alpha^3
\]

By rule 1, none of the alternatives under the compound presuppose \( \beta \) (condition 1.i holds for the first two columns, condition 1.ii holds for the third one). This agrees with the intuitive behaviour that had to be modelled.

Disjunction presented the most problems in update semantics attempts to model intuitive behaviour in terms of compositionality. Example (5) *Either there isn’t a typewriter or the typewriter is blue* shows the case where
the presuppositions of one of the disjuncts are not presuppositions of the disjunction. This sentence corresponds to a structure that is equivalent to that for the conditional of example (4), and gives no presupposition for the compound. This matches the required behaviour.

Unlike update semantics methods, this approach can also handle the symmetrical version of the disjunct with no problems. For the sentence in example (6) *Either the typewriter is blue or there isn’t a typewriter* the corresponding expansion contains the same literals in each branch but in a different order. Since order in the branch plays no role in the compositionality rules, the predictions are the same as for the previous case.

The method can also handle the case of disjunctions with contradictory presuppositions. Take for instance the sentence in example (10) *Either Bill has started smoking or Bill has stopped smoking.* This has the following representation:

\[
\begin{array}{c}
\delta \land \beta \\
\delta \land \beta \\
\neg \alpha \land \beta \\
\alpha \land \beta \\
\end{array}
\]

In this case, the method predicts (condition 1.iii) no presupposition for any column. Application of rule 2 gives no presupposition for the whole compound.

### 4.3 Complex Sentences

Complex sentences are treated as follows. First the sentence is expanded into a tableaux by application of the expansion rules. Then the compositionality rules are applied to the resulting tableau in order to obtain the presuppositions of the sentence.

Sentence (11) *If John is married and he has children, then his children are at school* can act as an example. Assuming a logical form for this sentence \((\alpha \land \beta) \rightarrow \delta\), the tableau for this sentence would be:

\[
\begin{array}{c}
\delta \land \alpha \\
\delta \land \neg \alpha \\
\neg \delta \land \alpha \\
\neg \delta \land \neg \alpha \\
\end{array}
\]

\(^3\) Ordering does seem to play a role in the intuitive validity of conjunctions (*There is a typewriter and the typewriter is blue* is acceptable, *The typewriter is blue and there is a typewriter* is harder to accept). This issue is discussed further in \(\)
The presupposition rules applied to this tableau predict no presupposition. (Rule 1 fails for all branches).

4.4 Discourses

When logical statements of the type considered so far are strung into a sequence of assertions, a discourse is obtained. An appropriate treatment of discourses is required to study the effect of context on presupposition interpretation. A discourse is represented by the tableaux for the set of formulas in it.

A problematic case studied by Beaver [2] is based on example (9) above. Suppose it is understood in the context that whenever Mary has a bath, she uses up all the hot water. This can be represented by assuming that the context already holds sentence (12) If Mary has had a bath, then there is no hot water left, with a logical form $\alpha \rightarrow \beta$. A representation for the discourse (12), (9) or $\alpha \rightarrow \beta, \alpha \rightarrow \delta^\beta$ would be the following tableau:

Over this representation it can be seen that the rules correctly predict no presupposition for the discourse as a whole, even though sentence (9) on its own did have a presupposition.
4.5 Traditional Presuppositional Concepts in Terms of Tableaux

The tableaux framework allows definition of some of the traditional concepts that surround presupposition.

The presupposition $\phi$ of a presuppositional sentence $\delta^\phi$ added to a tableau $\Gamma$ is satisfied if the tableau $\Gamma \cup \{\neg \phi\}$ is closed.

The presupposition $\phi$ of a presuppositional sentence $\delta^\phi$ added to a tableau $\Gamma$ is canceled if the tableau $\Gamma \cup \{\phi\}$ is closed.

These two definitions correspond to the intuitive concepts of satisfaction and cancelation. However, it is clear that there will be cases when some branches of a tableau are closed by $\phi$ and some by $\neg \phi$. These hybrid cases between satisfaction and cancelation escape the simpler analysis and give rise to the need for projection rules. In their simplest manifestation, hybrid cases occur as the traditional cases of problematic projection. These involve sentences (4), (5) and (6) given above. More complex manifestations concern discourses where the effect of context plays a role in the interpretation of presupposition. The discourse constructed with sentences (12) and (9) is an instance of these cases. In all these examples it holds that for any of the tableau representations some branches of the tableau are closed by the presupposition involved and some by its negation. Under those circumstances, the traditional definitions of satisfaction and/or cancelation could not account for the resulting presuppositional behaviour.

The present framework achieves this by allowing presupposition to be blocked locally by either $\beta$ or $\neg \beta$ (conditions 1.i and 1.ii).

5 Critical Analysis

5.1 Advantages over van der Sandt

The framework presented here is closely related to that proposed by van der Sandt. Both can be interpreted as a branching structure for a sentence based on the connective words that appear in it. The advantages of this framework are that 1) it makes explicit the semantics that are attributed to the connectives, and 2) logical consistency (both at sentence level and at branch level) and logical consequence are explicitly taken into account in the
framework.

5.2 Advantages over Beaver

The behaviour of all the connectives (including disjunction) is described satisfactorily by the rules given. No specific rules for each connective are required. The compositionality rules rely on the semantics in general terms. As a result, the same compositionality rules may be applied to other connectives if their semantics can be represented in the same framework in a way that preserves the Coverage Property.

The general approach that starts with Karttunen and leads to Beaver is criticized for not being able to justify their choice of method for obtaining presuppositions of compounds on grounds other than that it describes the behaviour. In the present framework the method is given simply by attributing a weak informative status to presupposition (so that it is overridden whenever it overlaps or conflicts with explicit information or other presuppositions).

5.3 Advantages over Mercer

In the present framework the information that Mercer must introduce as pragmatic implicatures can be read off the representation. An expansion rule like:

\[
\frac{\beta_1 \lor \beta_2}{\beta_1 
\beta_1 \neg \beta_1 \beta_1 
\beta_2 \beta_2 \neg \beta_2}
\]

captures the idea that each of \( \beta_1 \), \( \neg \beta_1 \), \( \beta_2 \) and \( \neg \beta_2 \) is valid in at least one of the possible alternatives given. This property of the present framework is a direct result of the insistence on taking the semantics – as given by the internal structure of the sentence in terms of connectives – into account.

The method followed by the compositional rules is quite close to Gazdar’s method for computing presuppositions simply in terms of consistency with the context. However, it presents two major innovations: 1) it allows both satisfaction and cancelation (presuppositions disappear when inconsistent and/or when already present), and 2) it applies the procedure locally. The combination of these two innovations allows adequate treatment of hybrid cases.
6 Conclusions

Presuppositional behaviour in sentences can be described in relation to the implicit logical structure represented by the appearance of natural language connectives if . . . then, and, and or in the sentences to be interpreted.

The framework presented here provides a method for determining the behaviour of presuppositions of complex logical statements in a compositional manner.

The predictions of the proposed method match the observed behaviour in traditional examples.

The semantics used in the present framework are chosen to ensure that all the different valid alternatives implied by a sentence are listed explicitly in a tableau for that sentence, and all atomic formulas involved appear (either negated or not) in every branch (Coverage Property). These constraints on the semantics allow all the predictions of the framework to be explained in terms of two basic assumptions: presuppositions are only considered informative where they do not overlap with or contradict asserted information, and where they do not conflict with other presuppositions.

There are some problems left unsolved. The present framework covers both information explicitly represented in the logic and presuppositional information, but does not allow a unified treatment; and it does not address the defeasible nature of certain presuppositions. In Gervás [3], these problems are addressed by providing tableau expansion rules for presupposition. This makes presuppositional information explicit in the framework. Because the issue of defeasibility is closely related to consistency checking, the framework proposed here presents advantages for this purpose by having the semantics of each connective made fully explicit.

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