Cosmic implications of a low-scale solution to the axion domain wall problem

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The post-inflationary breaking of Peccei-Quinn (PQ) symmetry can lead to the cosmic domain wall catastrophe. In this Letter we show how to avoid domain walls implementing the Instanton Interference Effect (IIE) with a new interaction which itself breaks PQ symmetry and confines at an energy scale smaller than $\Lambda_{QCD}$. We give a general description of the mechanism and consider its cosmological implications and constraints within a minimal model. Contrary to other mechanisms we do not require an inverse phase transition neither fine-tuned bias terms. Incidentally, the mechanism leads to the introduction of new self-interacting dark matter candidates and the possibility of producing gravitational waves in the frequency range of SKA. Unless a fine-tuned hidden sector is introduced, the mechanism predicts a QCD axion in the mass range $1\text{ meV} – 15\text{ meV}$.

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INTRODUCTION AND MOTIVATION

The axion solution to the strong CP problem \cite{1–6} is a well-known paradigm where domain walls bounded by strings\textsuperscript{1} emerge \cite{9, 10}. This is because as the Universe cools down two different phase transitions occur. In the first one the PQ scalar

$$\Phi = ve^{in/v}$$

develops a non-zero vacuum expectation value (VEV), $|\langle \Phi \rangle| = v$, breaking $U(1)_{PQ}$ spontaneously, and cosmic strings form. This is expected to happen at very high temperatures, around $T \sim v$. When the cosmic temperature reaches the QCD scale, QCD instantons generate an effective potential for the axion field and domain walls form. At this point, cosmic strings get attached to domain walls. The potential will have a periodicity determined by $N_{QCD}$, the QCD coefficient anomaly, computed as

$$N_{QCD} = 2 \sum q_R t_R,$$

where $t_R$ is the Dynkin index and $q_R$ are the PQ charge of the fermions. Notice that the axion decay constant and the VEV, $|\langle \Phi \rangle| = v$, are related as

$$f_a = \frac{v}{N_{QCD}}.$$

It is well-known that for $N_{QCD} = 1$ the string-wall system is not stable because the tension of the walls make strings to collapse to a point and the network decays fastly into non-relativistic axions \cite{11}. However, for $N_{QCD} > 1$ each string gets attached to $N_{QCD}$ walls and the network cannot decay: the walls are topologically protected and stable. Their evolution with cosmic expansion is slower than that of matter or radiation and will eventually dominate the energy density of the Universe. To avoid such a cosmological catastrophe one needs to make them disappear or evade their formation. Actually, as showed in Ref.\cite{12}, simple KSVZ axion models with different representations for the exotic quarks generating the PQ anomaly generically have $N_{QCD} > 1$ and, therefore, do suffer from DW problem. Therefore, simply neglecting the problem and assume $N_{DW} = 1$ sounds too simplistic.

A straightforward way to solve the domain wall problem is to invoke cosmic inflation \cite{13–15}. Domain walls are, in this case, pushed beyond the horizon and will not harm our cosmic evolution. This is the case of a PQ symmetry spontaneously broken before inflation and never restored after reheating. This case, however, can be constrained by isocurvature fluctuations in the CMB \cite{16}.

Here instead we consider a scenario in which PQ symmetry is broken after inflation. Different post-inflation mechanisms have been proposed since the ’80s. In the Lazarides-Shafi mechanism \cite{17}, for example, one asso-

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\textsuperscript{1} This kind of topological defect was first proposed in \cite{7} and has been experimentally found in superfluid $^3$He \cite{8}.
ciates the discrete symmetry unbroken by instantons to the center of a gauge symmetry\(^2\). Other solutions such as primordial black hole (PBH) formation [19] or the Witten effect have been explored more recently [20].

As a guideline, to make the walls unstable one needs to remove their topological protection by explicitly breaking the discrete symmetry that relates the degenerated set of vacua. The simplest solution following this philosophy is almost as old as the DW problem and is known as the bias term solution [10]. It consists in adding to the scalar potential an ad-hoc term

\[
\Delta V_{\text{bias}} = \Xi f_a^3 (\Phi e^{i\delta} + h.c.) .
\]

This term breaks explicitly PQ symmetry and produces a potential for the axion field that generates a effective theta term which is constrained to be \(\theta \leq 10^{-10}\). Then, if one assumes natural values for the phase, \(\delta \sim O(1)\), the dimensionless parameter \(\Xi\) is constrained to be [21]

\[
\Xi \leq 2 \cdot 10^{-45} N_{DW}^2 \left( \frac{10^{10} GeV}{f_a} \right)^4 .
\]

In this article we will consider a natural (and not fine-tuned) realization of the bias term by using a low scale version of the IIE. The introduction of a new confining interaction and its associated instantons will generate the explicit breaking of the \(Z_{N_{DW}}\) symmetry, unbroken by QCD instantons.

The paper is organized as follows. We first introduce the IIE and explain how it solves the domain wall problem. Then, we explore the constraints on the confinement scale of the HC sector we introduce. Once we set the scale of the new sector, we specify to a minimal \(SU(N)\) model and explore its cosmological implications. Finally, we conclude and comment on future directions to follow.

### THE INSTANTON INTERFERENCE EFFECT

The Instanton Interference Effect (IIE) is a compelling mechanism to avoid the cosmic domain wall problem in a inflation independent way (it applies in both, pre and post-inflationary PQ breaking scenarios) [22, 23]. In this mechanism one adds to the invisible QCD axion model a new non-abelian gauge group \(HC\) which is also anomalous under PQ

\[
HC \times U(1)_{PQ} \times SM,
\]

with SM the usual Standard Model gauge symmetry \(SU(3)_C \times SU(2)_L \times U(1)_Y\). Instantons associated to the new group will in general break the PQ symmetry down to a \(Z_{N_{HC}}\) subgroup. On the other hand, QCD instantons break the same symmetry to a \(Z_{N_{QCD}}\) discrete symmetry. The full axion potential reads

\[
V(a) = V_{QCD}(a) + V_{HC}(a) = \kappa \Lambda_{QCD}^4 \left( 1 - \cos \left( \frac{a}{v N_{QCD}} \right) \right) + \Lambda_{HC}^4 \left( 1 - \cos \left( \frac{a}{v N_{HC} - \delta} \right) \right),
\]

with \(\kappa \approx 1.2 \times 10^{-3}\), in which we recognize the QCD term, with anomaly coefficient \(N_{QCD}\), and the new term due to the HC interaction which confines at some scale \(\Lambda_{HC}\) and has its own anomaly coefficient, \(N_{HC}\). When \(N_{QCD}\) and \(N_{HC}\) are co-prime numbers, the PQ symmetry is completely broken by the combination of explicit symmetry breaking by instantons and the domain wall problem is solved. This is the Instanton Interference Effect (IIE). We notice that a similar periodic bias term with \(N_{HC} = 1\) has been considered in [24]; there the authors focus on the generation of primordial black holes and did not consider neither the origin nor its cosmological implications (and constraints) which are intriguing and diverse.

In its high-scale realization [22, 23], the IIE mechanism requires a confinement scale much larger than the QCD scale. In fact, if PQ is spontaneously broken by hypercolor HC condensates, as in [23], \(f_a\) and \(\Lambda_{HC}\) scales coincide. Therefore one needs to turn off the interaction below a critical temperature, once PQ is spontaneously broken and the axion field sits on the same vacuum everywhere, in order to allow the standard PQ mechanism to work and drive dynamically the axion field to a CP conserving minimum. To allow this, a new fermion in the HC sector and a new scalar with an inverted phase transition are needed.

It is attractive to explore the possibility of a low-scale version of the IIE with \(\Lambda_G \ll \Lambda_{QCD}\), which is indeed the goal of the present work. As we will show, this scenario does not require any of the complications related

\(^2\) See [18] for a different realization in the context of family symmetries.
to the inverse phase transition and can have peculiar cosmological implications, such as the presence of new self-interacting dark matter candidates and a strong first order phase transition which could produce detectable gravitational waves.

**SETTING THE CONFINEMENT SCALE OF THE DARK SECTOR**

In this section we derive upper and lower bounds to the HC scale, $\Lambda_{HC}$. To this end, we require not to spoil the strong CP problem solution and not to overclose the Universe producing too much axion dark matter through domain wall decay.

**Lower bounds to $\Lambda_{HC}$**

Two different lower bounds appear for the confinement scale, $\Lambda_{HC}$, of the new interaction. When the walls decay to DM, as it is our QCD axion, the BBN constraints are relaxed and the only requirement is that they must disappear before matter-radiation equality, given roughly by $T_{eq} \sim 1$ eV. The reason is that since these domain walls can only decay into axions or gravitational waves (gravitons) the interaction of their decay products with the SM is not efficient and they do not spoil BBN. In general, for $N_{DW} > 1$, domain walls will disappear when the discrete symmetry $Z_{N_{QCD}}$ unbroken by QCD instantons starts to feel the effects of HC instantons. Then, due to the IIE the $Z_{N_{QCD}}$ symmetry gets explicitly broken and the domain walls are not topologically protected. This occurs because the initially degenerated vacua get a small splitting from the bias term. Therefore the new term in the potential causes an energy difference between the false and the true minima $\Delta \rho \sim \Lambda_{HC}^4$ leading to a pressure, $p_V \sim \Delta \rho$, which acts against the domain wall. We remark that, in our case, the bias term comes from the potential $V_{HC}(a)$ (generated by HC instantons). The condition that the vacuum pressure generated from the energy difference exceeds the wall tension reads

$$p_V > p_{\text{tension}}$$

and therefore

$$V_{\text{bias}} > H_{\text{decay}} \sigma,$$

with $\sigma = 8m_a f_a^2$. This allow us to write the Hubble parameter when the walls decay as:

$$H_{\text{decay}} = \frac{V_{\text{bias}}}{\sigma} \approx \frac{\Lambda_{HC}^4}{8 m_a f_a^2}.$$  \hspace{1cm} (10)

We can impose that the domain walls decay before matter domination, obtaining the lower bound to the confinement scale

$$\Lambda_{HC} \gg 3.36 \times 10^{-10} \left[8 \Lambda_{QCD}^2 f_a \right]^{1/4} \text{GeV}^{1/4}.$$ \hspace{1cm} (11)

A different, and actually more stringent, bound can be obtained by the requirement that DW never dominate the energy density of the Universe. The contribution of domain walls to the energy density of the Universe is

$$\rho_w \sim \frac{\sigma t^2}{t^3} \sim \frac{\sigma}{t}.$$ \hspace{1cm} (12)

The Universe becomes dominated by domain walls when they reach relativistic speeds at the time $t_c \sim (G\sigma)^{-1}$. Consequently one has to impose that the time associated to the decay is much shorter:

$$t_c \sim \frac{1}{G\sigma} \gg H^{-1}_{\text{decay}}.$$ \hspace{1cm} (13)

Using equation (10) we get

$$\Lambda_{HC} \gg 0.3 \left( \frac{f_a}{M_P} \right)^{1/2} \Lambda_{QCD},$$ \hspace{1cm} (14)

which looks more stringent than Eq.(11) for reasonable values of $f_a$.

**Upper bound to $\Lambda_{HC}$**

The axion potential (see Eq. (7)) presents two contributions from QCD and HC instantons. The QCD contribution is the one that turns on first. Then, below a critical temperature, the HC potential also turns on and a small $\theta_{\text{eff}}$ will be generated. We can minimize the above potential to get

$$\frac{\partial V}{\partial a} = 0 \rightarrow \langle a \rangle \approx \frac{-\Lambda_{HC}^4 \sin \delta}{\kappa \Lambda_{QCD}^4 N_{QCD} + \Lambda_{HC}^4 N_{HC} \cos \delta}.$$ \hspace{1cm} (15)

and impose the solution of the Strong CP problem is not spoiled.

We see that in the limit $\Lambda_{HC} \ll \Lambda_{QCD}$, the effective CP violating phase behaves as $\theta_{\text{eff}} \propto \frac{\Lambda_{HC}}{\Lambda_{QCD}}$, as one would
naively expect. If we assume the parameter $\delta$ not to be unnaturally small, $\delta \gtrsim O(10^{-2})$, we find that in order not to spoil the solution of the strong CP problem the new scale needs to be

$$\Lambda_{HC} \lesssim 1\,\text{MeV}. \quad (16)$$

Fig.1 shows the minimum of the potential as a function of $\Lambda_{HC}$ for different values of the phase, $\delta$. When the HC potential is switched off, the minimum of the potential lies at $a = 0$, as the solution to the strong CP problem requires. Then, when the HC potential is switched on, the minimum is shifted proportionally to $\sim (\Lambda_{HC}/\Lambda_{QCD})^4$. Requiring $|a| \lesssim 10^{-10}$ implies, as already stated, an upper bound $\Lambda_{HC} \lesssim 1\,\text{MeV}$ for $\delta \gtrsim O(10^{-2})$. Of course, the smaller (and fine tuned) is the shift $\delta$, the larger will be the upper limit (see yellow curve in Fig.1).

**AXION DARK MATTER ABUNDANCE**

One of the most attractive feature of the PQ solution of the Strong CP problem is that it provides a new dark matter candidate, the axion, to which a great experimental and theoretical effort has been devoted [25–47]. In usual models, when DW are short lived the three non-thermal contributions (decay of DW, coherent oscillations and cosmic string radiation) are comparable [21]. However, if DW survive for a finite amount of time their contribution gets an enhancement. This is because while string decay and coherent oscillations occur when the Hubble parameter becomes comparable to the axion mass, $H \sim 3 m_a$, DW will be still topologically protected and will continue evolving with cosmic expansion. When the axion starts to feel the IIE, around $H \sim H_{\text{decay}}$, the walls will decay into non-relativistic axions. If $H_{\text{decay}} \ll m_a$, the contribution from domain wall decay dominates over the other contributions and, by imposing $\Omega_{\text{DW}} h^2 \lesssim 0.12$, we can therefore constrain $H_{\text{decay}}$. This will automatically constrain $\Lambda_{HC}$ and $f_a$ due to Eq.(10).

Assuming the so-called exact scaling regime we can write the relic abundance of axions today due to walls decay as

$$\rho_a(t_0) = \left( \frac{a(t_{\text{decay}})}{a(t_0)} \right)^3 \rho_a(t_{\text{decay}}), \quad (17)$$

where

$$\rho_a(t_{\text{decay}}) = \frac{\sigma}{t_{\text{decay}}} = \sigma H_{\text{decay}}, \quad (18)$$

is the energy of the domain walls, then converted into axion, at the time of their decay. The relic density of cold axions from DW is given by

$$\Omega_a h^2 = \frac{\rho_a(t_0) h^2}{\rho_c} = \left( \frac{a(t_{\text{decay}})}{a(t_0)} \right)^3 \rho_a(t_{\text{decay}}) h^2 / \rho_c. \quad (19)$$

It is useful to write the ratio

$$\frac{a(t_{\text{decay}})}{a(t_0)} = \frac{a(t_{\text{eq}})}{a(t_0)} \frac{a(t_{eq})}{a(t_{eq})}, \quad (20)$$

and, then, use the relations

$$\frac{a(t_{\text{decay}})}{a(t_{eq})} = \left( H(t_{eq})^2 / 2 H(t_{dec})^2 \right)^{1/4},$$

$$\frac{a(t_{eq})}{a(t_0)} = 4.15 \times 10^{-5} \left( \Omega_{\text{CDM}} h^2 \right)^{-1}. \quad (21)$$

The Hubble parameter at matter-radiation equality and the critical density are given by

$$H(t_{eq}) = 1.13 \times 10^{-26} (\Omega_{\text{CDM}} h^2)^2 \text{eV},$$

$$\rho_c = 1.053 \times 10^{-11} \left( H / 100 \right)^2 \text{eV}^4. \quad (22)$$

Finally we find

$$\Omega_{a,w} = 0.12 \times \left( \frac{f_a}{1.8 \times 10^9 \text{GeV}} \right)^{3/2} \left( \frac{\Lambda_{HC}}{\text{MeV}} \right)^{-2}$$

where we used

$$m_a \sim 5.7 \left( \frac{10^9 \text{GeV}}{f_a} \right) \text{meV}. \quad (23)$$
In Fig. 2 we show the relation between the scale $\Lambda_{HC}$ and the decay constant $f_a$ for $\Omega_{a,w} h^2 = 0.12$ in a generic model with $N_{HC} = 2$. Axions from coherent oscillations and radiated by strings can be estimated as [21]

$$\Omega_{coh} h^2 \sim 0.0009 \times \left( \frac{f_a}{1.8 \times 10^9 \text{ GeV}} \right)^{7/6},$$

$$\Omega_{string} h^2 \sim 0.0007 \times N_{DW}^2 \left( \frac{f_a}{1.8 \times 10^9 \text{ GeV}} \right)^{7/6}. \quad (24)$$

One can easily see that the axions coming from the decay of the walls will dominate the relic density today unless we have a large value of $N_{DW}^2$. In this case axions radiated from strings become important.  

Then, to obtain a lower bound for $\Lambda_{HC}$ from DM abundance one has to distinguish between different axion models. This is because different lower bounds or constraints for the axion decay constant $f_a$ will pose different lower bounds for $\Lambda_{HC}$. If one considers the DFSZ, processes involving the axion coupling to electrons contribute to fast stellar cooling. From the constrain $g_{ae} \leq 2.6 \times 10^{-13}$ [46] one gets

$$f_a \geq 6.5 \sin^2 \beta \times 10^8 \text{ GeV}, \quad (25)$$

where $\tan \beta = v_u/v_d$ is the ratio of up-type and down-type Higgs doublets in the DFSZ model. For hadronic KSVZ models one has the constraints coming from SN1987A, where processes like $N N \rightarrow N N a$ generate a more efficient energy-loss channel, resulting in a reduced neutrino burst duration. This constraints the axion decay constant to be $f_a \geq 4 \times 10^8$ GeV [36]. For the sake of clarity, in the rest of the paper we will consider a general KSVZ-like model with $N_{QCD} \neq 1$ and heavy, vector-like quarks which we denote as $Q$.

**ON THE HC SECTOR**

The dark HC sector can have a rich structure and many groups can lead to the desired interference effect, making this solution quite generic (up to simple model building).

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3 We warn the reader that the subject of axions from topological defects is of course a technically complicated one, to which a lot of effort has been dedicated (for an incomplete list of works see [48–55]), therefore it follows that any estimate has to be taken with a grain of salt.
we explore a more general situation in which the fermion is in the adjoint representation, $F \sim (\text{Adj}, 1, 1, 0)$. In such a case the spectrum will be composed of HC glueballs and glueballinos as in [62] and the HC anomaly coefficient will be $N_{HC} = N$. Glueballinos are composite states of a HC gluon and a fermion in the adjoint representation, $(gF)$, which are protected by gauge and Lorentz symmetry from its decay. In both, fundamental and adjoint cases, the hadron-like states are expected to be much heavier than the HC scale, $M_{\text{hadron}} \sim m_F$, and self-interact strongly by glueball exchange of masses $m \sim \Lambda_{HC}$. The relic density today of glueballinos or HC hadrons can be estimated in a canonical way as it is done for other WIMP candidates (taking into account only S-wave annihilation)[62, 65]:

$$\Omega_F h^2 \sim \frac{s_0}{\rho_{c,0}/h^2} \frac{25\epsilon}{\rho_{c,0}} \frac{m^2}{m_F^2} \langle \sigma v \rangle. \quad (29)$$

where we take also $\langle \sigma v \rangle \sim \alpha^2_{HC}(m_F/m^2_F)$. This expression neglects the effect of re-annihilation considered in [56]. A more sophisticated expression should take this effect into account, investigating both de-excitation and dissociation processes. However, in the present setup glueballs do not decay and therefore we expect re-annihilation to be inefficient. Consequently, we will neglect its effects.

Concerning the glueballino relic density, we note that, as usually happens for stable relics that were in thermal equilibrium, they can overclose the Universe if its mass is larger than $\mathcal{O}(100)$ TeV [66]. In addition, our scenario also suffers from the glueball problem and, therefore, one needs $\epsilon \ll 1$. In such a case, when $m_F \ll 100$ TeV and $\epsilon \ll 1$, the relic density of dark matter will be QCD axion dominated. In a following section we will identify the regions of the parameter space where dark matter is composed by axions and those where dark matter is made of glueballs and glueballino.

Finally, let us comment that the suppression of the dark sector temperature respect to the Standard Model also allows to evade bounds from the number of relativistic degrees of freedom $N_{\text{eff}}$. Results from Planck give $N_{\text{eff}} = 2.99 \pm 0.17$ [67]. We have then to worry if our framework implies non-standard values of $N_{\text{eff}}$. The temperature of the Standard Model when the dark sector
confines is given by

\[ T_{\text{conf}} = 100 \text{MeV} \left( \frac{T_{\text{HC}}}{\text{MeV}} \right) \left( \frac{0.01}{\epsilon} \right) . \]  

(30)

Therefore, for \( \epsilon \lesssim 10^{-2} \) there are no relativistic, massless species to act as the hidden-sector bath during BBN because HC confinement occurs well before BBN and structure formation.

\[ \text{FIG. 4: Here we show the constraints on the parameters } f_a, M_F \text{ for fixed value of } \Lambda_{\text{HC}} \text{ and } \epsilon. \]  

The bound from stellar cooling [36] is of course independent of these latters (and also of \( M_F \)). For large values of the decay constant, \( f_a \gtrsim 2 \times 10^9 \text{GeV} \), axions are overproduced and overclose the Universe. Similarly for large values of the dark fermion mass, \( M_F \gtrsim 13.5\text{TeV} \), glueballinos overclose the Universe.

**Thermal equilibrium between the HC sector and the standard model**

In the previous section we introduced a new parameter \( \epsilon = T_{\text{HC}}/T \). Glueball and/or glueballino relic abundance in the considered model turn out to be an important problem unless \( \epsilon \) is small. This means that the HC sector needs to have a smaller temperature than the SM thermal bath. We make the assumption that the two sectors start with different equilibrium temperatures\(^4\). Still, we have then to make sure that the two sectors never enter thermal equilibrium. The strongest constraint on the parameters of the theory comes from the scattering process

\[ g_{\text{HC}} + F \rightarrow a + F \]  

(31)

mediated by an heavy fermion, \( F \). The rate for this process at high temperatures is given by

\[ \Gamma = T^3 \frac{\alpha_{\text{HC}}}{T^2} \left( \frac{M_F}{f_a} \right)^2 , \]  

(32)

which has to be compared with the Hubble rate

\[ \Gamma_H = H = \sqrt{\frac{4\pi^3 G g_s}{45}} T^2 = \sqrt{\frac{g^*}{100}} \frac{T^2}{7.36 \times 10^{17} \text{GeV}} . \]  

(33)

In the above equation \( G \) is the Newton constant, \( T \) is the temperature of the standard model and \( g^* \) the number of relativistic degrees of freedom, which is given by

\[ g^*(t) = g^*_{\text{SM}}(t) + g^*_{\text{HC}}(t) \left( \frac{T_{\text{HC}}(t)}{T(t)} \right)^4 \sim g^*_{\text{SM}}(t) . \]  

(34)

The comparison can be made for \( T_{\text{HC}} = m_F = \epsilon T \), that is to say when \( T = m_F/\epsilon \). Below this temperature the number density of heavy fermions, \( F \), is rapidly suppressed by Boltzmann factor. Expressing \( m_F = Y_1 f_a \) we find that in order not to enter thermal equilibrium

\[ Y_1 \lesssim 10^{-5} \left( \frac{0.01}{\alpha_{\text{HC}}} \right) \left( \frac{0.01}{\epsilon} \right) \left( \frac{f_a}{10^9 \text{GeV}} \right) , \]  

(35)

which also implies

\[ m_F \lesssim 10 T \text{eV} \left( \frac{0.01}{\alpha_{\text{HC}}} \right) \left( \frac{0.01}{\epsilon} \right) \left( \frac{f_a}{10^9 \text{GeV}} \right)^2 . \]  

(36)

Notice that \( \alpha_{\text{HC}} \equiv \alpha_{\text{HC}}(m_F) \) is the HC gauge coupling at the \( F \) mass. We also checked other possible processes such as scattering of colored and dark fermions mediated by axion (or by the heavy scalar), scattering of dark and visible gluons, scattering of Higgses and heavy scalars, etc. For all of these we still find Eq.35 to give the strongest constraint on \( Y_1 \).

**DARK MATTER CANDIDATES**

As we have seen, the solution of domain wall problem actually introduces new dark matter candidates, namely glueballs and glueballinos. It is therefore interesting to explore the range of parameters to see where axions dominate, or not, as dark matter candidates. Indeed, even if

\(^4\) This might occur due to their coupling with the inflaton sector.
we added new parameters, most of them are highly constrained: the new confinement scale has been found to be \( \Lambda_{HC} \sim 0.1 - 1 \, \text{MeV} \), the ratio of the temperatures of the two sectors has to be \( \epsilon \lesssim 10^{-2} - 10^{-3} \) in order not to overproduce glueballs, and the Yukawa of the HC fermion is \( Y_1 \lesssim 10^{-5} \left( 0.01 \frac{\Lambda_{HC}}{\text{MeV}} \right)^{0.1} \left( \frac{\phi_{HC}}{10^4 \text{GeV}} \right) \) in order not to spoil thermal (non-)equilibrium. To explore the parameter space we fix \( \Lambda_{HC} = 1 \, \text{MeV}, \epsilon = 10^{-3} \). The latter value makes glueball relic density almost negligible. On the one hand, for large values of the axion decay constant

\[
f_a \gtrsim 1.8 \times 10^9 \left( \frac{\Lambda_{HC}}{\text{1MeV}} \right)^{4/3} \text{GeV},
\]

axions from domain wall decay overclose the Universe (see Eq.(23)). On the other hand, SN1987A gives a lower limit \( f_a \gtrsim 4 \times 10^8 \text{GeV} \) [36]. It follows that the value of \( f_a \) is restricted to be in a small (and testable in the near future) window, which is thinner for smaller values of the confinement scale (which correspond to larger values of the phase \( \delta \)). In addition, large values for dark fermions masses, \( M_F \gtrsim 13.5 \, \text{TeV} \), are also excluded (see figure 4) in order not to overclose the universe. In the remaining parameter space one can arrange \( M_F \) and \( f_a \) to have dark matter mostly in axions or glueballinos. Notice also that the allowed region is always far from the thermalization bound (dotted blue line in Fig.4), which ensures the treatment to be consistent. As a simple rule of thumb, regions where \( f_a \) is large and \( M_F \) is small are axion-dominated, while regions with large \( M_F \) and small \( f_a \) are glueballino-dominated. It is also interesting to notice that the solution of the domain wall problem and the consequent value of the confinement scale \( \Lambda_{HC} \) naturally pointed us to an interesting region for glueballs and glueballino, extensively studied in [62] with the goal of introducing self-interacting dark matter as a solution to the small-scale formation woes in astrophysics [68, 69]. As showed in [62], it is also possible to arrange the parameters of the model, just by slightly increasing \( \epsilon \), to have most of the dark matter in the form of glueballs instead of glueballinos. Ultimately what matters is the fact that the dark sector does not communicate with the visible one and the entropy in the dark sector is separately conserved. Then one can play to have more dark matter in one dark species than another, but this does not alter the overall picture.

**GRAVITATIONAL WAVES FROM THE DARK**

Violent phenomena in the early Universe can lead to large anisotropic fluctuations in the energy momentum tensor and therefore gravitational waves (GW). Examples of such phenomena are first order phase transitions [70–72]. Within the Standard Model there are at least two phase transitions, one associated with the breaking of the electroweak symmetry and the other with the breaking of chiral symmetry. Both of them are known to proceed through a smooth cross-over, that is to say they are not first order and cannot be sources of GW. One may wonder if in our model the HC sector, \( SU(N)_{HC} \), phase transition can lead to a GW signal in the frequency range of future detectors [73]. The situation under consideration, \( m_F \gg \Lambda_{HC} \), is analogous to a pure Yang-Mills theory, which is well known to exhibit a strong first order phase transition [74]. However, a GW signal is proportional to the energy density of the dark sector, which is proportional to the fourth power of the temperature \( T_h \). As we have seen, the latter is smaller than the temperature of the visible world and the GW signal will be consequently suppressed. Using the same input parameters and results of [73], properly taking into account the suppression of energy density due to the low HC sector temperature, we find that the signal frequency will fall in the range of frequency of SKA [75] (thank to the difference in temperatures between dark and visible sectors)

\[
f_{\text{peak}} \approx 3.33 \times 10^{-9} \, \text{Hz} \left( \frac{\Lambda_{HC}}{1 \, \text{MeV}} \right) \left( \frac{10^{-2}}{\epsilon} \right)
\]

but just outside its sensitivity for the considered values of \( \epsilon \). Nevertheless this is an interesting situation that deserves further study and will be presented elsewhere.

Finally, we also checked the possible GW production from domain wall decay into gravitons [76], which unfortunately turns out to be subdominant and always negligible in the explored parameter space.

**CONCLUSIONS**

We have explored a scenario where the domain wall problem of axion models is naturally solved introducing a new gauge group which confines at a scale \( \Lambda_{HC} \) smaller than \( \Lambda_{QCD} \). This scenario does not require neither extremely small parameters nor inverse phase transitions.
and results to be quite general. Constraints from domain wall energy density and the requirement not to spoil the solution to the strong CP problem fix the HC scale to be around the MeV.

Interestingly enough, this particular scale and the mechanism itself have been shown to have a number of non-trivial consequences. For a simple and minimal model in which $HC = SU(N)_{HC}$ we found that having $\Lambda_{HC} \sim 1 \text{ MeV}$ naturally provides new self-interacting dark matter candidates, in the form of glueballs or glueballinos (or both) with masses below $\sim 10 \text{ TeV}$. Moreover, the HC sector turns out to have a different temperature ($T_H$) from the visible one, making possible for $\Lambda_{HC} \sim 1 \text{ MeV}$ to produce gravitational waves in the frequency range of SKA due to the strong first order phase transition of $SU(N)_{HC}$. Finally, it is noteworthy that axions are mainly produced from domain wall decay and, unless one assumes unnaturally small values for the phase $\delta$, the decay constant, $f_a$, is constrained to be $4 \cdot 10^8 \text{GeV} \lesssim f_a \lesssim 2 \cdot 10^9 \text{GeV}$. This corresponds to an axion mass range of $1 \text{ meV} - 15 \text{ meV}$. It is remarkable that this lies close to the expected sensitivity of axion antennas [77], dielectric haloscopes [78] or ARIADNE [79]. Axion experiments will therefore be able to test this scenario in the near future.

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