Risk-Based Robust Bidding Strategies for EVs’ Aggregators in Day-ahead Markets with Uncertainty

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Abstract

In the recent electricity market framework, the profit of the generation companies depends on the decision of the operator on the schedule of its units, the energy price, and the optimal bidding strategies. Due to the expanded integration of uncertain renewable generators which is highly intermittent such as wind plants, the coordination with other facilities to mitigate the risks of imbalances is mandatory. Accordingly, coordination of wind generators with the evolutionary Electric Vehicles (EVs) is expected to boost the performance of the grid. In this paper, we propose a robust optimization approach for the coordination between the wind-thermal generators and the EVs in a virtual power plant (VPP) environment. The objective of maximizing the profit of the VPP Operator (VPPO) is studied. The optimal bidding strategy of the VPPO in the day-ahead market under uncertainties of wind power, energy prices, imbalance prices, and demand is obtained for the worst case scenario. A case study is conducted to assess the effectiveness of the proposed model in terms of the VPPO’s profit. A comparison between the proposed model and the scenario-based optimization was introduced. Our results confirmed

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that, although the conservative behavior of the worst-case robust optimization model, it helps the decision maker from the fluctuations of the uncertain parameters involved in the production and bidding processes. In addition, robust optimization is a more tractable problem and does not suffer from the high computation burden associated with scenario-based stochastic programming. This makes it more practical for real-life scenarios.

**Keywords:** Wind uncertainty, Electric Vehicles (EVs), robust optimization, wind-thermal coordination, V2G, scenario-based optimization.

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**Nomenclature**

*Uncertainty Parameters*

- $\lambda_t$: Realized energy price at time $t$ in $\$.  
- $\theta_w, \theta_\lambda, \theta_L$: Parameters of the uncertainty set size for wind, energy price, demand respectively.  
- $L_t^{RT}$: Realized demand at time $t$ in MW.  
- $W_{d,t}^{RT}$: Realized output of wind unit $d$ at time $t$ in MW.

*Indices*

- $c$: Index for EV.  
- $d$: Index for wind units.  
- $e$: Index for segments.  
- $g$: Index for thermal generators.  
- $s$: Index for scenarios.  
- $t$: Index for time periods.

*Other Variables*
\( \alpha \)  Confidence level.
\( \Gamma \)  Risk-attituude parameter.
\( \Omega \)  Dummy variable.
\( CVaR_{\alpha} \)  Conditional value at risk at the \( \alpha \) confidence interval.
\( Imb_{t}^{dn} \)  Under-generation power from the scheduled at time \( t \) in MW.
\( Imb_{t}^{up} \)  Over-generation power from the scheduled at time \( t \) in MW.

**Parameters**

\( \eta \) and \( \zeta \)  Auxiliary variables for \( CVaR \) s.
\( \rho \)  Incentive for the EV owners.
\( a, b, c \)  Parameters of thermal heat rate curve.
\( Av_{c,t} \)  The availability of EV \( c \) for charging at time \( t \).
\( BrkPt_{e,g} \)  Break point of a segment \( e \) for thermal unit \( g \) for the heat rate curve.
\( C_{g} \)  Operation cost of the thermal generator \( g \).
\( FC_{g} \)  Fuel cost of thermal unit \( g \) in $.
\( I_{0}^{g} \)  Initial state of thermal unit \( g \).
\( InitUP_{g} \)  Initial minimum up-time for thermal unit \( g \).
\( K_{g} \)  Offset of thermal unit \( g \) for the heat rate curve.
\( L_{t} \)  Scheduled demand at time \( t \) in MW.
\( MC_{c} \)  Maximum capacity of charge for EV \( c \) in KWh.
\( MinUP_{g} \)  Minimum up-time for thermal unit \( g \).
\( MP_{c} \)  Maximum charging power for EV \( c \).
\( N_{C} \)  Number of EVs.
$N_D$ Number of wind units.

$N_E$ Number of segments.

$N_G$ Number of thermal generator units.

$N_S$ Number of scenarios.

$N_T$ Number of time periods.

$p_g^{\text{max}}, p_g^{\text{min}}$ Max/min power generation of thermal unit $g$, in MW.

$Pr$ The probability of the scenario $s$.

$R$ Utility rate charged for the customers $\$/MWh.

$r^u_t, r^o_t$ Under-and over-generation imbalance penalties as multipliers of the energy price at time $t$.

$RU_g, RD_g$ Ramp up/down rate of thermal unit $g$, in MW/hour.

$Slope_{e,g}$ Slope of a segment $e$ for thermal unit $g$ for the heat rate curve.

$SOC_{c}^{\text{reduce}}$ Reduction in SOC of the $c^{th}$ EVs’ battery as a result of driving.

$SOC_{I_c}$ Initial state of charge of EV $c$ in KWh.

$SU_{g,t}$ Startup cost of thermal unit $g$ at time $t$ in $\$$. 

$T_{Dep}(x)$ Time of the $x^{th}$ EV departure.

$W_{d}^{\text{max}}$ Rated output of wind unit $d$ in MW.

Uncertainty Variables

$\gamma^+$ Binary variables its value is 1 if the uncertain parameter at the upper bound of its set.

$\gamma^-$ Binary variables its value is 1 if the uncertain parameter at the lower bound of its set.
\(\delta_{g,e,t}\) Output of thermal unit \(g\) at time \(t\) corresponding to segment \(e\) of the thermal heat curve in MW.

\(AP_{c,t}^{\text{max}}\) Maximum scheduled increase power for EV \(c\) at time \(t\) in KW.

\(AP_{c,t}^{\text{min}}\) Maximum scheduled reduction power for EV \(c\) at time \(t\) in KW.

\(I_{g,t}\) State of thermal unit \(g\) at time \(t\), 1 = ON , 0 = OFF.

\(P_{g,t}^{\text{bid}}\) The optimal bidding of thermal unit \(g\) at time \(t\) in MW.

\(P_{g,t}^{\text{RT}}\) The real time realized power of thermal unit \(g\) at time \(t\) in MW.

\(PD_{c,t}\) awarded energy bid for EV \(c\) at time \(t\) in KWh.

\(POP_{c,t}\) Preferred operating point for EV \(c\) at time \(t\) in KW.

\(W_{d,t}^{\text{bid}}\) The optimal bidding of wind unit \(d\) at time \(t\) in MW.

1. Introduction

Recently, the focus of researchers was directed towards stochastic programming approach to handle uncertainty in decision making process [1]. However, stochastic programming has been proven to be more computationally challenging due to the need of large number of scenarios which is indispensable to capture the real nature of the random variable. Furthermore, the complete knowledge of the probability distribution of the random variable is necessary but it is not always available [2]. Recently, another alternative for the stochastic programming has attracted the attention of researchers is Robust Optimization (RO). Although robust optimization field is considered a relatively young research area, there have been many publications which reflect the advantages of the RO approach in many of research areas such as finance, energy, supply chain management, circuit design and scheduling problems. The basic concept behind robust optimization is that it is not a probabilistic model, the uncertainty is handled based on a construction of an uncertainty set where the solution is robust for all realizations of the uncertain parameter within the defined set [3, 4, 5, 6, 7, 8, 9].

Robust optimization was extensively discussed in models such as Unit Commitments (UC) and Security Constrained Unit Commitment (SCUC) when
the decision maker is faced with uncertainty in energy prices or/and output of another intermittent generator such as wind. In [10] the UC problem was formulated as a two-stage model. At the first stage, the scheduling of the thermal generators was decided, while at the second stage, the economic dispatch of the units and the actual output of the power were decided. The objective function was to minimize the total cost of the system; i.e., startup cost in the two stages. Two formulations of the UC problem were proposed, which are the risk constrained and the expanded robust formulations. The results proved the economic benefits of the robust solution of the UC problem over the opposed scenario based solution. Again in [11], the UC problem was coupled with the wind uncertainty and demand response in a two-stage problem. Minimizing the total operation cost of the power system was aimed by the authors. The results show that there is a significant reduction in the system cost using the proposed robust UC model. The concept of the adjustable uncertainty set was introduced by the authors in the problem of the UC with wind power penetration in [12]. The problem of the robust risk-constrained UC was formulated as a two-level problem, the decomposition method was used to solve the optimization problem when the the sub-problem was solved for certain values for the decision variables of the master problem. Then, the relaxed master problem is solved based on the convergence of the sub-problem. The objective is to minimize the total operation cost of the system. Although the results show that the model is effective in reducing the cost, the proposed model is hardly applicable in the current energy market structure. Coordination of the wind power with another sources of power to mitigate the risks and to handle the uncertainty under the robust optimization umbrella was also proposed in the literature. In [13], a conditional value at risk (CVaR) approach was used as a risk measure linked with the uncertainty set of the robust model, to optimally bid the wind power plant which was coordinated with a storage system. Maximizing the profit of the wind power producer was aimed under the uncertainty of the wind and the energy prices. In [14] and [15] the coordination with hydro power generators was proposed. In [14], a two-stage robust model was formulated to maximize the profit of the virtual power plant operator gained from the forward contracts and the day-ahead market. Bender’s decomposition was used to facilitate the solution of the min-max problem which was reformulated as a mixed-integer linear program (MILP). The results show that the proposed robust model was effective to mitigate the risks resulted form the uncertainties of the wind output and the energy price and to maximize the profit.
Despite the effectiveness of the thermal generators as a source of wind power balancing, the need for a more flexible source which matches both positive and negative imbalances without extra cost is raised. Exploiting the V2G services through the EVs is considered the most suitable solution for such a case [16, 17]. In [18], a robust model for aggregating the EVs under the uncertainty of the arrival and departure time coupled with the energy prices’ uncertainty was proposed. In [8, 9], the interval optimization was proposed to model the uncertainty of the energy market prices, while the uncertainty of the hydrogen storage and the EVs fleet offering in the pool market under uncertain real-time price was proposed [9]. Bi-objective optimization was used to formulate the solution to the problem in both models. However, our model includes more general formulation since the uncertainties of the demand were considered, the demand uncertainty is crucial for the decision-making process at the energy market. The robust optimization approach for optimal scheduling of the VPP and the optimal bidding strategy under the energy price’s uncertainty was proposed in [19] and [20], respectively. When making decisions and optimizing models based on the deterministic values of the parameters whiles those parameters are uncertain in nature, the obtained solutions would be figured out to lead to sub-optimal or even infeasible solutions after the realization of uncertain parameters. Robust optimization based on the worst-case, which is the most conservative optimization technique, guarantees that the obtained optimal solutions will remain feasible for all realizations of the uncertain parameters. Another methodology to hedge against parameters uncertainty is to formulate models based on risk averse measures. The CVaR risk measure is a coherent risk measure that never conflicts with optimizing the expectation of any risk-averse utility function [4, 21, 22]. Based of the investigated literature and for the best of our knowledge, considering the uncertainties in wind output power, energy prices, imbalance prices, and the demand in one model under the robust optimization was not tackled before. To fill this gap, we propose a robust model and a CVaR-based model for a VPPO maximization profit under a number of uncertainties, where coordination between the EVs and the wind-thermal generators to mitigate the risk of the uncertainties is considered. Therefore, the main contributions of the manuscript are as follow:

- A robust thorough mathematical formulation of virtual power plant and electric vehicles that describes and model in details the behavior of the electric vehicles, wind plant, and thermal generation units.
Providing a systematic approach to decide on the uncertainty set of the robust optimization in for the application of coordination of virtual power plants and electric vehicles.

Formulating the problem under the CVaR measure of risk considering a scenario tree for energy prices, imbalance prices, and wind power output generated using the auto-regressive integrating moving average (ARIMA) technique and reducing the set of generated scenarios based on the Fast-forward methodology.

A comparative study between robust optimization and stochastic optimization showing the pros and cons of each methodology in the context of virtual power plant and electric vehicles.

The rest of the paper is organized as follows. In section (2), the system model is introduced. Section (3) introduces the problem formulation. The stochastic programming formulation is discussed in section (4). A case study is presented in section (5). Simulation results are discussed in section (6). We conclude our work and give some insights for the future work in section (7).

2. System Model

We consider a small Virtual Power Plant (VPP) with wind generation output \( W_{d,t} \) from unit \( d \) at time \( t \), thermal units with output \( P_{g,t} \) form unit \( g \) at time \( t \), and a load consisting of two types controllable load such as EVs and uncontrollable load, as shown in Fig. 1.
The VPPO is considered as a price taker who submits a certain amount of energy to the day-ahead market 24 hours before the time market clearing process. The VPPO faces uncertainties of the actual wind power, the energy and imbalance prices, and the loads. The coordination between the EVs and thermal generators to balance the wind deviation is presented in Section 3.

3. Problem Formulation

The profit of the VPPO from participating in the day-ahead market when facing uncertainties can be maximized as:

$$
\max_{x} \sum_{t=1}^{N_T} \sum_{g=1}^{N_G} \tilde{\lambda}_t P^{bid}_{g,t} - C_g(P^{RT}_{g,t}) - (SU_{g,t}(I_{g,t} - I_{g,t-1}))^+ \\
+ \sum_{d=1}^{N_D} \tilde{\lambda}_t W^{bid}_{d,t} + \tilde{\lambda}_t r^u_{t} I m b^{up}_{t} - \tilde{\lambda}_t r^u_{t} I m b^{dn}_{t} \\
+ R_t(L^{RT}_{t} + \rho \sum_{c=1}^{N_C} PD_{c,t}) - \tilde{\lambda}_t (L_t + \sum_{c=1}^{N_C} POP_{c,t}) 
$$

where

$$
x \in \{ I_{g,t}, P^{RT}_{g,t}, P^{bid}_{g,t}, W^{bid}_{d,t}, POP_{c,t}, AP^{max}_{c,t}, AP^{min}_{c,t} \}
$$
$PD_{c,t}, \delta_{g,c,t}$, and $(\ldots)$ defines the uncertainty in the input parameter.

The first line of (1) defines the profit from thermal generation expressed as the income from the amount bid of power in the day-ahead market minus the cost of production and the thermal generation start-up costs. The second line represents the profit from bidding the wind power into the market and the profit gained from positive imbalances minus the penalties the VPPO might face as a result of the negative imbalances. The last line defines; in its first term, the revenue from the loads. Noting that $\rho$ which is less than one represents an incentive for the EVs to participate in the coordination, and $R$ is the utility rate which reflects the energy price plus a fixed amount as a revenue for the VPPO. While, the second term is the cost of purchasing the scheduled energy from the market.

The imbalance up term defines the running long status of the producer, where the imbalance down term defines the running short status. At least one of them is zero at any time period $t$. The producer is running long when the generation is larger than the load [16]:

$$Imb_{up}^t = \begin{cases} -\Delta P_t & \text{if } \Delta P_t \leq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$Imb_{dn}^t = \begin{cases} \Delta P_t & \text{if } \Delta P_t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

and,

$$\Delta P_t = \sum_{d=1}^{N_D} W_{d,t}^{bid} - \sum_{d=1}^{N_D} \tilde{W}_{d,t}^{RT} + \sum_{g=1}^{N_G} P_{g,t}^{bid} - \sum_{g=1}^{N_G} P_{g,t}^{RT} - \sum_{c=1}^{N_C} (POP_{c,t} - PD_{c,t}) - (L_t - \tilde{L}_{RT})$$

The objective function in (1) is constrained by:

- Imbalance constraints

$$\Delta P_t = Imb_{dn}^t - Imb_{up}^t$$
\[ 0 \leq I_{mbi}^{dn} \leq \sum_{g=1}^{N_G} P_{g,t}^{max} \cdot I_{g,t} + \sum_{d=1}^{N_D} W_{d,t}^{max} - \sum_{c=1}^{N_C} A P_{c,t}^{min} - L_t \]  
(6)

\[ 0 \leq I_{mbi}^{up} \leq \sum_{g=1}^{N_G} P_{g,t}^{RT} + \sum_{d=1}^{N_D} \bar{W}_{d,t}^{RT} - \sum_{c=1}^{N_C} P_{D,c,t} - \bar{L}_{t}^{RT} \]  
(7)

- Operating limits

\[ 0 \leq W_{d,t}^{bid} \leq W_{d}^{max} \]  
(8)

\[ I_{g,t} \cdot P_{g}^{min} \leq P_{g,t}^{RT} \leq P_{g,t}^{max} \cdot I_{g,t} \]  
(9)

\[ I_{g,t} \cdot P_{g}^{min} \leq P_{g,t}^{bid} \leq P_{g,t}^{max} \cdot I_{g,t} \]  
(10)

- Ramping up/down limits

\[ -RD_g \leq P_{g,t}^{RT} - P_{g,t-1}^{RT} \leq RU_g \]  
(11)

- Minimum up/down times

\[ \sum_{t=1}^{InitUp_{p_g}} (1 - I_{g,t}) = 0 , \]  
(12)

\[ \sum_{n=t}^{t+MinUp_{p_g}-1} I_{g,n} \geq MinUp_{p_g} \cdot (I_{g,t} - I_{g,t-1}) , \]  
\[ \forall g, \forall t = InitUp_{p_g} + 1,...,N_T - MinUp_{p_g} + 1 , \]  
(13)
\[ \sum_{n=t}^{N_T} I_{g,n} - (I_{g,t} - I_{g,t-1}) \geq 0, \]
\[ \forall g, \forall t = N_T - \text{MinUp}_g + 2 \ldots N_T . \quad (14) \]

Another three equivalent constraints to (12) - (14) can be included to model the minimum down times.

- **Production cost constraints**

\[ C_g(P_{g,t}^{RT}) = FC_g \left( I_{g,t} \cdot K_g + \sum_{e=1}^{N_E} \text{Slope}_{g,e} \cdot \delta_{g,e,t} \right) , \quad (15) \]

\[ P_{g,t}^{RT} = P_{g}^{\text{min}} \cdot I_{g,t} + \sum_{e=1}^{N_E} \delta_{g,e,t}, \quad \forall g \ , \quad (16) \]

\[ 0 \leq \delta_{g,e,t} \leq \text{BrkPt}_{g,e} - \text{BrkPt}_{g,e-1} \quad \forall g,e,t \ , \quad (17) \]

\[ K_g = a_g + b_g P_{g}^{\text{min}} + c_g P_{g}^{\text{min}^2} . \quad (18) \]

- **EV charging power limits**

\[ 0 \leq POP_{c,t} \leq MP_{c,t} \quad (19) \]

- **EV charge requirement limits**

\[ \sum_{t=1}^{T_{\text{Dep}(1)}} PD_{c,t} \geq 0.9 MC_c - SOC_{c}^{I} \ , \quad (20a) \]

\[ \sum_{t=1}^{T_{\text{Dep}(1)}} PD_{c,t} \leq MC_c - SOC_{c}^{I} \ , \quad (20b) \]

\[ \sum_{t=1}^{T_{\text{Dep}(x)_{c}}} PD_{c,t} - (x-1)SO\text{c}_{c}^{\text{reduc}} \leq MC_c - SOC_{c}^{I} \ , \quad (20c) \]

\[ \sum_{t=1}^{N_T} PD_{c,t} - 4SO\text{c}_{c}^{\text{reduc}} \leq MC_c - SOC_{c}^{I} \ , \quad (20d) \]

\[ \sum_{t=1}^{N_T} PD_{c,t} - 4SO\text{c}_{c}^{\text{reduc}} \geq 0 \ . \quad (20e) \]
• EV additional power limits

\[ 0 \leq AP_{c,t}^{\text{max}} \leq MP_{c,t} \times Av_{c,t} - POP_{c,t} \quad \forall c \in N_C, t \in N_T \]  \hspace{1cm} (21)

\[ 0 \leq AP_{c,t}^{\text{min}} \leq POP_{c,t} \quad \forall c \in N_C, t \in N_T \]  \hspace{1cm} (22)

• EV actual power draw limits

\[ POP_{c,t} - AP_{c,t}^{\text{min}} \leq PD_{c,t} \leq POP_{c,t} + AP_{c,t}^{\text{max}} \quad \forall c \in N_C, t \in N_T \]  \hspace{1cm} (23)

In general, the constraints from (19) - (23) are presented to formulate the charging power limits, the commuting profiles and the battery capacities before and after the charging and the commuting. In (19), the charging cannot exceed the maximum charging limits. The constraints (20a-20e) formulate the charging requirements for the commuting profile of the EVs owners and ensure that the battery will not exceed its maximum power capacity. For example, constraint (20a) aims to maintain 90% or more of the battery before the first departure. While constraint (20d) aims to guarantee for the EV owner at the end of the day that SOC must be at least equal to SOC\text{I}. The constraints in (21)-(22) aim to set the max. and the min. limits for the additional power that can be bid which is related to the POP. Last constraint in (23) aims to set the limits of the actual power draw.

3.1. Deterministic Model

In the deterministic model, the VPPO takes his decision with no consideration of the uncertainties such that all parameters are assumed to be accurate. Under the deterministic model, the VPPO will not benefit form the imbalance income or incur any imbalance cost. Hence, the part of the imbalance profit in the objective function will be removed as the constraints (2) - (7).

3.2. Robust Optimization Model

In the robust optimization literature, the uncertainty is handled by defining the uncertain parameters over uncertainty sets which represent all possible realizations of the uncertain parameter. More details of the uncertainty set are introduced in section (3.3).
3.3. Uncertainty Set

The selection of the uncertainty set usually relies on the available information about the uncertain parameter [3]. In this paper, we consider the case of the decision making process by the VPPO without waiting for the realization of the uncertain parameters. In other words, the problem is formulated for the worst-case realization of the uncertainty. Hence, the uncertainty set is defined as $Z = Z_W \cup Z_\lambda \cup Z_L$ where $Z_W$ is the set of the wind uncertainty, and $Z_\lambda, Z_L$ are the sets of the energy price uncertainty and the demand uncertainty, respectively. The uncertainty set of the wind power gives the relation between the nominal value of the generation and the lower and upper bounds of the deviation as:

$$Z_W = \left\{ W_t = \hat{W}_t + \gamma_t^+ W_t - \gamma_t^- W_t, \right. \right.$$

$$\left. W_t \geq 0, \gamma_t^+, \gamma_t^- \in \{0, 1\}, \forall t \right\} \quad (24)$$

where $\hat{W}$ is the nominal value for the wind generation, while $W$ and $\bar{W}$, are the lower and upper bounds of the deviation, respectively. Clearly for the $\gamma_t^+$ and $\gamma_t^-$, at least one of them is equal to zero at each time period $t$ since there is only a positive or a negative deviation at a certain time period $t$. Similarly, the sets $Z_\lambda$ for the price and $Z_L$ for the demand are:

$$Z_\lambda = \left\{ \lambda_t = \hat{\lambda}_t + \gamma_t^+ \bar{\lambda}_t - \gamma_t^- \Delta_t, \right. \right.$$  

$$\left. \lambda_t \geq 0, \gamma_t^+, \gamma_t^- \in \{0, 1\}, \forall t \right\} \quad (25)$$

$$Z_L = \left\{ L_t = \hat{L}_t + \gamma_t^+ \bar{L}_t - \gamma_t^- L_t, \right. \right.$$  

$$\left. L_t \geq 0, \gamma_t^+, \gamma_t^- \in \{0, 1\}, \forall t \right\} \quad (26)$$

It is worthy noting that the uncertainty in the imbalance price is considered as a function of the energy price with the parameters $r^o, r^u$ where $r^u > 1$ and $r^o < 1$. For more details about the formulation of this variables, please refer to [23] and [24].
3.4. Robust Counterpart

One known method of dealing with uncertainty in the objective function is to reformulate it as a constraint-wise uncertainty [25]. It is known that:

$$\max_x \hat{c}^T x : Ax \leq b \quad \forall x \in X$$

is equivalent to

$$\max_{x, m} m : \hat{c}^T x \geq m \quad \forall c \in C \quad Ax \leq b \quad \forall x \in X$$

where $C$ is the uncertainty set for the parameter $c$, $m$ is an auxiliary variable. Hence, the uncertainty of the objective function is reformulated as a constraint-wise uncertainty. Similarly, the uncertainty in the energy price in problem (1) and the uncertainty of the other constraints can be reformulated as [26]:

$$\max_{x, \Omega} \quad \Omega$$

s.t.

$$\sum_{t=1}^{N_T} \sum_{g=1}^{N_G} (\hat{\lambda}_t - \bar{\lambda}_t) P_{g,t}^{bid}$$

$$- C_g (P_{g,t}^{RT}) - (S U_{g,t} (I_{g,t} - I_{g,t-1}))^+$$

$$+ \sum_{d=1}^{N_D} (\hat{\lambda}_t - \bar{\lambda}_t) W_{d,t}^{bid} + (\hat{\lambda}_t - \bar{\lambda}_t) r_t^{o} Imb_{t}^{op}$$

$$- (\hat{\lambda}_t + \bar{\lambda}_t) r_t^{a} Imb_{t}^{dn} + R_t ((\hat{L}_t^{RT} - L_t^{RT}) + \rho \sum_{c=1}^{N_C} PD_{c,t})$$

$$- (\hat{\lambda}_t + \bar{\lambda}_t) (L_t + \sum_{c=1}^{N_C} POP_{c,t}) \geq \Omega$$

(27)
Since it’s not favorable for robust optimization to deal with equality constraints \([25]\), constraint \((4)\) will be replaced with its equivalence:

\[
\text{Imb}_{tn} - \text{Imb}_{up} - \sum_{d=1}^{N_D} W_{d,t}^{bid} - \sum_{g=1}^{N_G} P_{g,t}^{bid} + \sum_{g=1}^{N_G} P_{g,t}^{RT} \\
+ \sum_{c=1}^{N_C} (POP_{c,t} - PD_{c,t}) + L_t \leq \left( \hat{L}_t^{RT} - L_t^{RT} \right) \\
- \sum_{d=1}^{N_D} (\hat{W}_t^{RT} - W_t^{RT}) \times \beta
\]

\tag{28}

\[
\text{Imb}_{tn} - \text{Imb}_{up} - \sum_{d=1}^{N_D} W_{d,t}^{bid} - \sum_{g=1}^{N_G} P_{g,t}^{bid} + \sum_{g=1}^{N_G} P_{g,t}^{RT} \\
+ \sum_{c=1}^{N_C} (POP_{c,t} - PD_{c,t}) + L_t \geq \left( \hat{L}_t^{RT} + L_t^{RT} \right) \\
- \sum_{d=1}^{N_D} (\hat{W}_t^{RT} - W_t^{RT}) \times (1 - \beta)
\]

\tag{29}

Where \(\beta\) is a binary auxiliary variable to guarantee that only one of \((28)\) and \((29)\) is active at each time. Furthermore, constraint \((7)\) can be transformed into its robust counterpart as:

\[
0 \leq \text{Imb}_{up} \leq \sum_{g=1}^{N_G} P_{g,t}^{RT} + \sum_{d=1}^{N_D} (\hat{W}_t^{RT} - W_t^{RT}) \\
- \sum_{d=1}^{N_D} PD_{c,t} - (\hat{L}_t^{RT} + L_t^{RT})
\]

\tag{30}

To this end, we considered all uncertainties and completed the formulation of the robust counterpart of our model.

4. Stochastic Optimization Model

The uncertainties in obtaining the maximum profit from day-ahead market include wind power output, hourly LMP, imbalance prices, and the loads.
The problem of maximizing the total profit of the VPPO is formulated as a stochastic mixed-integer linear program (MILP). As most of the problems under the stochastic framework the uncertainties in optimization are handled through a two-stage decision-making process. Here and now decisions in the first stage which are made before the realization of the stochastic parameters. Wait and see decisions in the second stage which are affected by those in the first stage. The second stage decision variables are then scenario-based [24, 27].

4.1. Scenario generation and reduction

Scenario tree is a model where the values of the random variables are arranged. The scenario tree defines the two stages of making the decision by two different nodes. The link between the root at the first stage and the leaf at the second stage is called scenario. Many techniques have been developed to generate the scenario trees. Since the size of the problem is proportional to the number of the scenarios which were generated, the reduction of the number of scenarios is important to overcome the huge computational burden.

Monte Carlo Simulation (MCS) is one of the most common techniques to generate the scenario tree for energy prices and wind output. However, the expected values and the standard deviation of the random variable are needed. One of the disadvantage of MCS is that it does not count for the coupling among the consecutive hours. In this work, a seasonal auto-regressive integrating moving average (ARIMA) technique is used to generate the scenario tree for energy prices, imbalance prices, and wind power output. Then, a fast-forward reduction method is used to reduce the number of generated scenarios. Fast-forward methodology is based on an iterative method to minimize the distance between the original set and the reduced set of the scenarios.

In our model, we consider the wind power bid, thermal power bid, and the EVs scheduling as the here and now decision variables. We also consider the the actual power draw of the EVs, the actual output of the thermal generators, and the CVaR auxiliary variables as the wait and see decision variables. The form of the optimization problem under stochastic optimization framework can be shown as:

$$\max_x \ E[\text{PROFIT}] + \Gamma.\text{CVaR}_\alpha$$

(31)
where
\[ x \in \{ I_{g,t}, P^{RT}_{g,t,s}, P^{bid}_{g,t}, W^{bid}_{d,t}, POP_{c,t}, AP^{max}_{c,t}, AP^{min}_{c,t}, PD_{c,t,s}, \delta_{g,c,t,s}, \zeta, \eta_s \} \].

\{PROFIT\} is the objective function at (1)

s.t. (5) - (23), additional constraints needed to be added to the problem such:

1. \[ \text{CVaR}_\alpha = \zeta - \frac{1}{1-\alpha} \sum_{s=1}^{N_s} P_{r_s, \eta_s} \] (32)
2. \[-[\text{PROFIT}]_s + \zeta - \eta_s \leq 0, \quad \forall s \] (33)
3. \[ \eta_s \geq 0, \quad \forall s \] (34)

where \( \alpha \) is the confidence level which is considered here as 95%.

5. Case Study

A case study for a VPP that serves a small urban area is considered. We aim from this study to assess the benefits of the proposed coordination for both the VPPO and EVs’ owners under the considered uncertainties. VPP consists of one wind power plant with installed capacity of 200 MW, five thermal generators with total installed capacity of 340 MW. For detailed characteristics of the wind and the thermal generators, we refer the reader to [16] and [24]. A group of 10 Thousands EVs with 50 similar driving profile representing the Spanish commuting behavior are used. A sample of 10 patterns are illustrated in Table (1) with average commuting distance of 10 KM, where it is assumed that the EVs are charging at home and work as well. The batteries capacities, EV’s characteristics are such that in [16].

A set of 1000 scenario for the wind output, energy price, and the imbalance price, are generated using seasonal auto-regressive integrated moving average technique (ARIMA) [24]. Three load profiles (low, nominal, and high) are used. Hence, reducing the scenarios-tree from 1000 to 3 is used by the fast-forward method [28]. The final size of the scenario-tree is \( 3^4 = 81 \) scenarios.

The reduced scenario-tree is used to construct the uncertainty sets for the wind output, energy price, imbalance price, and demand as shown in Fig. 2 to Fig. 4.

The robust optimization formulation is very rich based on the choice of the uncertainty set construction. Many uncertainty sets such as Box uncertainty set, interval, Ellipoidcal, and polyhedral uncertainty set were discussed.
Table 1: Travel profiles of 10 EVs

| Time / EVs | Commute to Work | Lunch | return to Work | Back to Home |
|------------|-----------------|-------|----------------|--------------|
| 8 AM       | EV1 1 0 0 1 1 0 1 0 |
| 9 AM       | EV2 0 1 0 1 1 0 0 1 |
| 10 AM      | EV3 0 0 1 1 1 1 0 0 |
| 2 PM       | EV4 0 1 0 1 1 0 1 0 |
| returned at Lunch | EV5 0 0 1 1 1 0 0 1 |
| 7 PM       | EV6 0 1 0 1 1 0 1 0 |
| 8 PM       | EV7 1 0 0 1 1 1 0 0 |
| 9 PM       | EV8 1 0 0 1 1 0 0 1 |
| 10 PM      | EV9 0 1 0 1 1 1 0 0 |
| Back Home  | EV10 0 0 1 0 0 1 0 |

in the literature. The box uncertainty set is one of the most common and simple uncertainty sets used on the robust optimization framework, where the uncertain parameter is considered in the uncertainty set as [nominal value - lower bound, nominal value + upper bound], then the upper and lower bounds are modeled as a percentage of the nominal value to represent the uncertainty set as (nominal value × (1 ± θ)), where θ is chosen by the VPPO according to his/her preference as a risk-averse or risk-taker decision maker. Three different values for each uncertainty set are considered θ₇, θ₈, θ₉ for wind output, energy price, and demand, respectively. θ is an adjustable parameter that controls the uncertainty set size, and hence controlling the conservativeness of the solution. The interval uncertainty set is resulting if θ₇ = 1 [5, 29].

For the uncoordinated case, the thermal-wind coordination is still considered. However, the EVs are not coordinated to mitigate the wind uncertainties. Furthermore, it is assumed that the EVs are charged according to the opportunistic charging. In this model the EVs are charged by the maximum charging rate once they are plugged in. Moreover, the decision variables of the EVs (i.e., POP, AMₘₐₓ, AMₘᵢₙ) are set to zero, and the total amount of
energy needed for the EVs charging is added as a constant load.

The solution algorithm was modeled using IBM ILOG CPLEX [30] and executed on a PC with an Intel(R) core™ i3 @ 2.53 GHz CPU and 6 GB of RAM, the processing time is about 15 seconds.

6. Simulation Results

Two cases are considered to assess the effectiveness of the proposed model. In case 1, the robust optimization is used to model the uncertainties of the uncoordinated case between the EVs and wind-thermal generators. In case 2, the coordination between the EVs and the thermal-wind generators is considered under both deterministic and robust models. The profit of the VPPO at the deterministic case is compared with both the coordinated and the uncoordinated profit of the robust model under different levels of uncertainty. Table (2) shows the results of the VPPO’s profit for the worst-case scenario of the realization of the uncertain parameters when the uncertainty set is at its extreme bound compared to the stochastic and deterministic case. Although both the deterministic and Stochastic models outperforms the robust model in terms of profits by \((26072 - 24996)/26072 = 4.13\%\) and \((25310 - 24996)/25310 = 1.24\%\), respectively, the robust model counts for the worst case realization of the uncertain parameters which keeps the decision maker safe with any realization of the uncertainty. Moreover, moving from the most conservative situation i.e., worst case might increase the
profits of the VPPO while considering the uncertainties. Essentially, expected profits of the three algorithms compared to the actual realized profits are shown in Fig. 5, the actual profits is obtained by applying an actual day prices and units production. Indeed, CVaR results in higher profits but with a higher risk compared to the robust optimization that provides profits also and a safe decision-making environment for the decision maker. The benefits of the coordination is very evident on the profit of the VPPO compared to the uncoordinated case. The coordination gain be given as $(24996 - 24404)/24996 = 2.36\%$.

Table 2: VPPO’s profits with EVs coordination

| Model              | Profits $ |
|--------------------|-----------|
| Deterministic      | 26072     |
| Stochastic         | 25312     |
| Coordinated Robust | 24996     |
| Uncoordinated Robust | 24404    |
Tables 3 and 4 show the commitment of the thermal generation units in both coordinated and uncoordinated cases. Note that the status which changes was highlighted by bold font.
7. Conclusion

In this paper, we proposed a robust optimization model for the coordination between the wind-thermal generators and the EVs aggregators in a VPP. The objective of maximizing the profit of the VPP operator was studied. The optimal bidding strategy of the VPPO in the day-ahead market under uncertainties of wind power, energy prices, imbalance prices, and demand was obtained for the worst case scenario. A case study was conducted to assess the effectiveness of the proposed model in terms of the VPPO’s profit. The simulation results confirmed our argument regarding the profit maximization in case of coordination considering the worst-case scenarios. Although, the coordination gain is about 2%, this gain is expected to increase with the raised demand of EVs in last years. Although robust optimization gives less profitable operational schedules, but it is more reliable, dependable, involves less risk and computationally more efficient than stochastic programming.

There are many directions for further extensions of the proposed work, the uncertainties about the EVs fleet can be included such as the uncertainties about the battery state of charge (SOC) and also the commuting patterns which all considered deterministic in that work. The correlation between the uncertainty sets can be studied also, where each uncertainty set to be considered independent at the proposed work. Furthermore, the coordination of the EV aggregator with other types of energy producer such as Hydro-plants or solar energy; which introduces more uncertainties, can be studied.
to pave the way for a safe, profitable bidding at the energy markets for the decision-maker.

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