KINETIC THEORY OF ANOMALOUS TRANSPORT OF SUPRATHERMAL PARTICLES.

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Abstract

Investigation of behavior of fast electrons in toroidal discharges was performed. The kinetic equation, describing evolution of fast particle distribution was derived and analyzed. Semi-analytical solution of the kinetic equation was obtained into suprathermal energy region. External electric field, anomalous transport, nonuniformity of mean magnetic field and collisions was shown to be an important factors, affecting distribution function. The role of ambipolar electric field was established and identified as an essential factor of the process of diffusion of fast electrons. The effect of strong influence of density profile on diffusion of fast particles was clearly demonstrated. Comparison with the experimental data, obtained on ZT-40M device, was carried out. The agreements with the results of these experiments is observed.

1 Introduction

Plasma confinement in toroidal devices is an important problem of thermonuclear fusion research. It is well known that various instabilities leading to discharge turbulization are readily excited in plasma. Experiments have shown that electron heat conductivity is by several orders of magnitude higher than the limit predicted by the neoclassical theory [1]. Such a strong energy transfer to a discharge wall is the main energy loss channel in toroidal systems. This is the reason why this problem is of great interest, see the review [2].

The theory suggests two possible mechanisms of anomalous transport. One of these mechanisms is due to turbulence generated by potential electric field oscillations caused by drift instabilities [3, 4, 5, 6]. The other mechanism is due to turbulence induced by magnetic fluctuations. On exceeding the threshold value (determined by overlapping of mean resonance modes), magnetic field fluctuations
lead to a stochastic wandering of magnetic field lines about the discharge and, as a consequence, to a transport of particles moving along these field lines. The qualitative picture of this phenomenon was first presented in the paper [7] as a stochastic description of a diffusion of magnetic field lines. The theory of this mechanism of anomalous transport was further on comprehensively studied by many authors [8, 9, 10, 11, 12]. According to the theory, the effective diffusion coefficient in the former mechanism is inversely proportional to the velocity of particle motion

\[ D_{\perp e} \sim \frac{\langle e^2 \rangle c^2 L_e}{B_0^2 v_e} \]  

whereas in the latter case the particle diffusion coefficient increases with increasing particle velocity

\[ D_{\perp b} \sim \frac{\langle b^2 \rangle}{B_0^2} L_e v_e \]  

The role of each of these mechanisms in the formation of energy flux has not yet been finally established. Measurements of the excitation level in tokamaks have shown that the first type transport mechanism dominates in the vicinity of discharge boundary [13, 14, 15, 16], while inside a discharge (where the level of magnetic fluctuations is difficult to measure) the transport may be due to both excitation mechanisms. As distinct from the tokamak, in pinch type devices with a reversed field, where the magnetic fluctuation level is by two orders of magnitude greater than that in tokamaks [17], predominant is a transport due to large-scale magnetic fluctuations [18].

Along with a thermal particle flux, there also exists a superthermal fast particle flux. Moreover, if the particle lifetime in a discharge increases, the number of superthermal particles may increase appreciably. This effect is obviously of particular importance in devices in which plasma is heated in an ohmic way. One of such devices is a reversed field pinch (RFP) where, according to experimental data, a considerable part of energy is transported by fast particles [19, 20, 21].

To estimate the energy confinement efficiency in RFP type devices, it is necessary to determine the distribution function of fast particles in a discharge. Furthermore, fast particles bear information on the processes proceeding inside a discharge, and
so the investigation of their distribution may be used to diagnose the state of the plasma \[19, 22\].

In the paper \[22\], the authors formulated a consistent kinetic theory of anomalous transport processes in a turbulized plasma and derived the kinetic equation describing the averaged particle distribution function in these conditions. A relaxed state of a turbulent plasma and the anomalous transport processes under RFP conditions were analyzed in the papers \[23, 24\]. Here we are considering the superthermal electron distribution function in RFP. As has already been mentioned above, predominant in RFP is an anomalous transport due to magnetic fluctuations. Therefore, the influence of this particular mechanism of transport upon the distribution function of superthermal electrons will be our prime concern in the sequel.

The anomalous transport produces a strong effect on fast particles because the diffusion coefficient \( \mathcal{D} \) grows linearly with increasing particle velocity. Another factor affecting strongly the distribution function of electrons is an electric field applied to the plasma.

It is known that even a weak electric field applied to a plasma induces the formation of a tail of runaway electrons in the energy range exceeding the critical one, \( \epsilon > \epsilon_c \), where \( \epsilon_c = \frac{E_c}{E} T_e \), and \( E_c \) is the critical field \[25, 26\]. In RFP, an applied field is high, that is, the ratio \( E/E_c \) is significantly larger than that in tokamaks, and therefore the distribution function distortion due to the action of the field \( E \) is much stronger here. In addition to the vortex electric field, applied to plasma, a potential electric field, a so-called "ambipolar field is generated in the discharge due to the difference in the ion and electron coefficients of diffusion. Producing an immediate effect upon fast electron diffusion, this field is responsible for the dependence between the ejection of fast electrons and the transport of the ion plasma component, which have typically been considered independently. On the other hand, as the number of fast particles increases, they themselves may start affecting the macroscopic state of the plasma, which is for example the case with convective transfer in a rippled field \[27\]. We shall not consider here the effects due to corrugation, but this role will be played by anomalous transport in a turbulent plasma. Thus, an examination of the distribution function of electrons is necessary for a correct self-consistent analysis of
Besides the factors listed above, the distribution function is noticeably affected by inhomogeneity of the mean magnetic field and temperature. For example, nonuniformity of electron temperature results in thermal runaway and appearance of hot particle tail in the cold plasma region [28].

It should be noted that the influence of anomalous transport in RFP on the distribution function of fast electrons was examined in the paper [30], but the authors proceeded from the model kinetic equation which disregards a number of essential factors affecting the distribution function. In particular, the effect of Coulomb collisions, a potential ambipolar field and inhomogeneity of the mean magnetic field were neglected. It is therefore necessary to investigate the distribution function of superthermal electrons more thoroughly making allowance for the influence of all essential factors. This is just the goal of the present paper. In section 2 we derive the master kinetic equation with account of the influence of the applied electric field, an anomalous diffusion, a potential ambipolar field, collisions and magnetic field inhomogeneity. The principal parameters determining the distribution function of fast electrons are discussed. Bearing in mind complicity of studying the complete problem, at the beginning of section 3 we analyze the distribution function of fast electrons in a homogeneous magnetic field. The strong influence of inhomogeneity of the electric field applied to plasma and the dependence of the electron diffusion rate on the profile of the thermal particle density are resolved. In section 4 we investigate the influence of inhomogeneity of the mean magnetic field upon the distribution function of electrons and point out a substantial deformation of this function. Finally, in section 5 we estimate the influence of the indicated effects in specific RFP conditions and compare the developed theory with available experimental data. The results of experiments are seen to be in agreement with the theory.

2 The kinetic equation for superthermal electrons

Let us consider a magnetized plasma with a magnetic field $\vec{B}(\vec{r})$ which has a regular $\vec{B}_0(\vec{r})$ and a fluctuational $\vec{b}$ components. The amplitude of fluctuations will be
assumed small as compared with the mean field $\vec{B}_0$, $|\vec{b}| \ll |\vec{B}_0|$. The basic quantities characterizing the fluctuations (the correlation length and the correlation time) will be thought of as large as compared to the Larmor radius of particles and their inverse gyrofrequencies as in [22]. Within such a statement of the problem and disregarding toroidality effects, the authors derived the general kinetic equation for the distribution function of particles $f(r, u, \tilde{\mu})$ averaged in the ensemble of fluctuations, which, by virtue of cylindrical symmetry about the angle $\theta$ and the direction $z$ along the cylinder axis depends only on the radius $r$ [23]:

$$\frac{\partial f}{\partial t} + \frac{e}{m_e} E_e \frac{\partial f}{\partial u} = St(f) + I(f)$$

(3)

where the collision integral of particles with fluctuations is given by

$$I(f) = \frac{u}{B(r)} \frac{1}{r} \frac{\partial}{\partial r} \{r K\} + \frac{\partial}{\partial u} \left\{ \left( \frac{e}{m_e} \frac{E_e}{B} - \frac{\tilde{\mu} dB}{2 \frac{dr}{dr} B} \right) K \right\}$$

$$K = \left[ \frac{|u|}{u} \frac{F \partial f}{\partial r} + \frac{F}{|u|} \frac{\partial f}{\partial u} \left( \frac{e}{m_e} \frac{E_e}{B} - \frac{\tilde{\mu} dB}{2 \frac{dr}{dr} B} \right) \right]$$

Here the ambipolar electric field $eE_a = (\frac{dn}{dr} \frac{T_e}{n} + \frac{1}{2} \frac{dT_e}{dr}) \{1 - 2\delta_m\}$ due to the difference between the diffusion rates of electrons and ions, the constant $\delta_m = \sqrt{m_e/m_i} \ll 1$ gives correction related to ion diffusion (assuming that transport of ions is defined by magnetic fluctuations too), $n(r)$ is the particle number density, $F = \int_0^\infty dL(b_r b_r')$ - is the correlation function of fluctuations $b_r$, the integration over $L$ goes along the trajectory of particle motion, $b_r' = b_r(r'(L), \theta'(L), z'(L))$, $u$ is the particle velocity along the magnetic field line, $\tilde{\mu} = u^2 / B$ is the adiabatic invariant of particle motion, $E_e$ is the external field, $E_e = \vec{E}_e \cdot \vec{h}$, $\vec{h} = \vec{B}_0 / B_0$, $St(f)$- is the Coulomb collision integral of particles.

In plasma heating devices, the number of fast particles $N_f$ is always small as compared to the concentration $N$ of main particles. That is why the kinetic equation (3) can be linearized in the small parameter $N_f/N \ll 1$. Equation (3) is convenient to write in a spherical coordinate system in the velocity space $\epsilon$, $\mu$, where $\epsilon = u^2 + u_\perp^2$ is the total energy and $\mu = u / \sqrt{u^2 + u_\perp^2}$ is the cosine of pitch angle (below for brevity we’ll use simply pitch-angle without cosine). Linearizing we obtain

$$\frac{\partial f}{\partial r} + E_e(r) \delta \{ 2\mu \epsilon \frac{\partial f}{\partial \epsilon} + (1 - \mu^2) \frac{\partial f}{\partial \mu} \} = 4T(r) \frac{\partial^2 f}{\partial \epsilon^2} + 2 \frac{\partial f}{\partial \epsilon} +$$
\[
\frac{Z_{\text{eff}}}{\epsilon} \frac{\partial}{\partial \mu} \{ (1 - \mu^2) \frac{\partial f}{\partial \mu} \} + \epsilon \mu \delta_2 \{ \frac{1}{rB} \frac{\partial}{\partial r} \{ rK \} + 2E_a \{ \frac{\partial K}{\partial \epsilon} + \frac{1 - \mu^2}{2\epsilon \mu} \frac{\partial K}{\partial \mu} \} - \frac{dB}{dr} \frac{1}{B^2} \frac{1 - \mu^2}{2\mu} \frac{\partial K}{\partial \mu} \}
\]

Here \(E_e(r)\) is the profile of the external longitudinal electric field normalized to the critical electron runaway field \(E_c\) \[\text{[23]}\], \(E_c = \frac{4\pi e^2 \Lambda n}{T_e}\), \(Z_{\text{eff}}\) is the effective ion charge, \(\Lambda\) is Coulomb logarithm, \(\delta_1 = E_{e0}/E_c\) is the dimensionless parameter characterizing the magnitude of the longitudinal field \(E_{e0}\) relative to the critical field \(E_c\), \(\delta_2 = \nu_a/\nu_0\) is a dimensionless parameter characterizing the particle-fluctuation collision frequency \(\nu_a = \frac{F_{\text{max}}}{a^2 B_0^2 \sqrt{T_e/m_e}}\) as compared with the Coulomb collision frequency of electrons \(\nu_0 = \frac{4\pi e^4 n \Lambda}{m_e^{3/2} T_e^{3/2}}\), \(a\) is the characteristic system dimension, \(T(r)\) - is the profile of the electron temperature normalized to the temperature at the center \(T_e\). Furthermore, dimensionless quantities \(r = \tilde{r}/a\), \(F = \tilde{F}/F_{\text{max}}\), \(B = \tilde{B}/B_0\), \(E_a = \tilde{E}_a e a/T_e\), \(\epsilon = \tilde{\epsilon}/(T_e/m_e)\), \(\tau = \nu_0 t\) are introduced, the sign \(-\) marks the corresponding dimensional quantities, \(\delta_1\) and \(\delta_2\) are small parameters of our problem. In what follows we shall consider steady-state solutions of equation (4) to determine the established distribution function of superthermal particles. We shall assume the distribution function of the main particles in plasma to be stationary and equilibrium. In such a statement, it will be a source of superthermal plasma particles. As boundary conditions for the distribution function \(f\) it is natural to require that \(f\) be regular as \(r \to 0\) and that all the particles die on the boundary for \(r = 1\), that is,

\[
\frac{\partial f}{\partial r} |_{r=0} = 0 \quad f |_{r=1} = 0
\]

The investigation of the complete problem is difficult because of a simultaneous action of such factors as an applied electric field \(E_e\), an anomalous diffusion and inhomogeneity of the mean magnetic field. We shall therefore begin with examining a joint effect of the external electric field \(E_e\) and the fluctuations assuming the magnetic field gradient to be small

\[
\frac{dB}{dr} \frac{a}{B} \ll 1
\]

and then proceed to the case of an inhomogeneous magnetic field \(B(r)\).
3 Distribution function of superthermal electrons in a homogeneous magnetic field.

Provided that the condition (5) is satisfied, equation (4) has the form

\[
E_e(r) \delta_1 \{2 \mu_e \frac{\partial f}{\partial \epsilon} + (1 - \mu^2) \frac{\partial f}{\partial \mu}\} = 4T(r) \frac{\partial^2 f}{\partial \epsilon^2} + 2 \frac{\partial f}{\partial \epsilon} +
\]

\[
\frac{Z_{eff}}{\epsilon} \frac{\partial}{\partial \mu} \{(1 - \mu^2) \frac{\partial f}{\partial \mu}\} + \epsilon \mu \delta_2 \left\{ \frac{1}{r B} \frac{\partial}{\partial r} \{r K\} +
\right.
\]

\[
2E_a \left\{ \frac{\partial K}{\partial \epsilon} + \frac{1 - \mu^2}{2 \epsilon \mu} \frac{\partial K}{\partial \mu} \right\} \}
\]

K = \left| \frac{\mu}{\mu} \right| F \left\{ \frac{\partial f}{\partial r} + 2E_a \left\{ \frac{\partial f}{\partial \epsilon} + \frac{1 - \mu^2}{2 \epsilon \mu} \frac{\partial f}{\partial \mu} \right\} \right\}
\]

E_a = \left( \frac{dp}{dr} \frac{T(r)}{n} + \frac{dT}{dr} \right) \{1 - \delta_m\}, \delta_m \ll 1
\]

Equation (6) describes the established distribution function of electrons in the presence of the field \(E_e\) and plasma turbulence. The method of solving equation (6) depends on the energy range within which we seek the solution. Therefore we shall first examine the energy range immediately adjoining the equilibrium region, which is henceforth referred to as a polynomial region of solution

\[
1 \leq \epsilon \leq \delta_1^{-1/2}, \delta_1 \ll 1
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\[
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\]

We shall not consider here the thermal runaway of particles [28, 29]. According to the results of [29] one may estimates the critical value of energy \(y_k\),

\[
y = \epsilon \delta_1^{1/2}
\]

when the temperature profile will relax to homogeneous one, \(y_k = \delta_1^{1/2} \delta_2^{-1/3} \left( \frac{a}{L_{||e}} \right)^{2/3}\), where \(a\) - is a scale of the system, \(L_{||e}\) - is a correlation length of fluctuations along magnetic field line. As usually \(\frac{a}{L_{||e}} \ll 1\) and we assume that,

\[
y_k \ll 1
\]

And therefore, in accordance with (8) in equation (6) we have put \(T_e = \text{const}\) and for the sake of simplicity \(T_i = \text{const}\), because ion temperature profile contributes only to a correction term in ambipolar field. So, as is seen inhomogeneity of electron temperature contributes only to ambipolar field.
3.1 The behavior of solution in the polynomial region

To begin with, we consider the case of a constant field \( E_e(r) = \text{const} \). We imply at first to clear up the effects of ambipolar field, that \( \frac{dT}{dr} \ll \delta_m \) and neglect contribution from temperature gradient in ambipolar field. In equation (6) we pass over to a new variable \( y = \epsilon \delta_1^{1/2} \) and seek the solution as a series of eigenfunctions of the Sturm-Liouville problem

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r F \frac{\partial \chi_l}{\partial r} \right) + \eta \chi_l = 0 \quad (9)
\]

\[
\frac{\partial \chi_l}{\partial r} \big|_{r=0} = 0, \quad \frac{\partial \chi_l}{\partial r} \big|_{r=1} = 0
\]

that is,

\[
f = C n(r) \exp\left(-\Psi_0/\delta_1^{1/2}\right) \sum_l \chi_l(r) R_l(y, \mu) \quad (10)
\]

As the boundary condition for \( y \to 0 \) we require the condition of sewing with the equilibrium distribution function

\[
f_0 = C n(r) \exp\left(-y/\delta_1^{1/2}\right)
\]

\[
n(r) \big|_{r=1} = 0
\]

Substituting (10) into (6) and collecting terms with the same power \( \delta_1^{1/2} \) in zero approximation, we obtain \( \Psi_0 = y/2 \). In the next order of perturbation theory, neglecting correction of the order of \( \delta_m \ll 1 \) in the expression for the ambipolar field (6), we obtain a system of equations for the functions \( R_l(y, \mu) \)

\[
2 \frac{\partial R_l}{\partial y} = \frac{Z_{eff}}{y} \frac{\partial}{\partial \mu} \left[ \left(1 - \mu^2\right) \frac{\partial R_l}{\partial \mu} \right] - y |\mu| \beta \eta R_l + 2 \mu y R_l \quad (11)
\]

with boundary conditions as \( y \to 0 \)

\[
R_0(0, \mu) = 1, \quad R_l(0, \mu) = 0 \quad l \neq 0
\]

where \( \beta = \delta_2/\delta_1 \). Since zero eigenvalue of the problem (6) \( \eta_0 = 0 \), for the function \( R_0(y, \mu) \) we obtain an equation containing no contribution of anomalous diffusion,
and it will have a solution similar to the one obtained in the paper [25], where the distribution function was distorted only by the electric field $E_e$ and had a directed character for $\mu \simeq 1$ (Fig. 1, the dashed line). The solution of the system (11) for $l \neq 0$, which satisfies the boundary conditions (because for $l \neq 0$ all the eigenvalues $\eta_l > 0$ are nonnegative), will be $R_l(y, \mu) = 0$. So, up to terms of the order of $\delta_1^{1/2}$ (terms of the order of $\delta_1^{1/2} \delta_m \ll 1$ and $\delta_1 \ll 1$ are neglected) the initial equilibrium distribution function $f_0$ will not be distorted by anomalous transport. This means, as is readily seen, that the contribution from the anomalous diffusion is completely compensated by the ambipolar field $E_a$ in exactly the same way as in the case of diffusion of thermal plasma particles. As is well known, electrons and ions diffuse together as a single whole with a doubled ion diffusion coefficient, which in our case is $\delta_m$ times smaller than the electron one (provided that the predominant ion diffusion mechanism is also an anomalous transport caused by magnetic fluctuations).

To determine the distortion of the distribution function by an anomalous transport, we have to examine the solution of equation (6) with allowance for terms of the order of $\delta_1^{1/2} \delta_m$ and $\delta_1$, which under certain conditions (to be discussed below) become appreciable. Now we are in a position to consider the case where the parameter $\delta_m$ satisfies the condition $\delta_m \gg \delta_1^{1/2}$. This means that we shall take into account the contribution from the anomalous diffusion, proportional to the anomalous ion diffusion coefficient.

With allowance for corrections of the order of $\delta_m$, the solution of equation (6) is convenient to seek as before in the form of an eigenfunction series of the Sturm-Liouville problem

$$\frac{1}{r} \frac{\partial}{\partial r} \{ r F \frac{\partial g_l}{\partial r} \} + \lambda_l g_l = 0$$

$$\left. \frac{\partial g_l}{\partial r} \right|_{r=0} = 0$$

$$\left. g_l \right|_{r=1} = 0$$

$$f = C \exp(-\Psi_0/\delta_1^{1/2}) \sum_l g_l(r) \Theta_l(y, \mu)$$

$$n(r) = \sum_l A_l g_l$$

with the boundary condition as $y \to 0$:

$$\Theta_l(0, \mu) = A_l$$
In order that we might pass over to a system of ordinary differential equations, it is also convenient to expand the functions \( \Theta_l(y, \mu) \) in a power series of Legendre polynomials \( P_m(\mu) \):

\[
\Theta_l(y, \mu) = \sum_m F_l^m(y) P_m(\mu) \quad (15)
\]

Substituting (13) into (6) with account of (15), we ultimately arrive at a system of linking equations for the functions \( F_l^m(y) \), \( \Psi_0 = y/2 \):

\[
2 \frac{dF_l^m}{dy} = - \frac{Z_{ell}}{y} m(m + 1) F_l^m + y F_l^m \alpha_{kl} \gamma_{mn} \beta + 2y \left\{ \frac{F_l^{m-1}}{2m - 1} + \frac{(m + 1) F_l^{m+1}}{2m + 3} \right\} \quad (16)
\]

with boundary conditions:

\[
F_l^0(0) = A_l \\
F_l^m(0) = 0 \quad m \neq 0
\]

where

\[
\alpha_{kl} = \int_0^1 r g_l \left\{ \frac{1}{r} \frac{\partial}{\partial r} \{ r \Pi_k \} - E_a \Pi_k \} dr \right\} / \int_0^1 r g_l^2 dr \\
\Pi_k = F \left\{ \frac{\partial g_k}{\partial r} - E_a g_k \right\} \\
\gamma_{mn} = (2m + 1) \int_1^1 P_m P_n |\mu| d\mu, \quad \beta = \delta_1/\delta_2
\]

The system of equations (16) is solved numerically by cutting off the chain of equations on the term \( L \) in the expansion in \( g_l \) and on the term \( M \) in the expansion in \( P_m \), so that a doubling of \( L \) and \( M \) changes the solution by less than 10%. Clearly, strong distribution function distortions by an anomalous transport will take place only when the parameter \( \Delta = 2\beta \lambda_m \delta_m \) is of the order of unity \( \Delta \sim 1 \), where \( \lambda_m \) is the maximum eigenvalue corresponding to the eigenfunctions \( g_l(r) \) which contribute to the expansion (14). The smallest distortions may be expected in the case \( n(r) = g_0(r) \).

We are examining the behavior of the system (16) in the model case \( F = \text{const} \), when the eigenfunctions of the problem (12) are the Bessel functions \( g_l(r) = J_0(\xi_l r) \).

We begin with examining the case \( n(r) = g_0(r) \). The solution of the system (16) depending on the pitch angle \( \mu \) and the energy \( y \) is presented in Figs. 1, 2. Figure 1
shows solutions obtained for various values of the parameter $\beta$. For small $\beta$, when $\Delta \ll 1$ and the effect of the electric field $E_e$ dominates, the solution is close to that obtained in the paper [25] (the dashed line in Fig. 1). As $\beta$ increases, the influence of the anomalous transport (when $\Delta \geq 1$) becomes predominant. In this case the solution becomes almost symmetric in $\mu$. As should be expected, it is concentrated in the vicinity of $\mu = 0$, where the diffusion coefficient (2) vanishes and falls symmetrically for $\mu \Rightarrow \pm 1$, where the diffusion coefficient is maximal. Depending on the energy (Fig.2), the distribution function falls exponentially $\ln(f/f_0) \sim -\Delta_0 y^2$, where $\Delta_0 = 2\delta_m \beta \lambda_0$ ($\lambda_0$ is a nonzero eigenvalue of the problem (12)). The function $f$, which is shown in fig.2, was averaged over $\mu$.

It should be emphasized that the initial distribution over discharge $n(r)$ will not be deformed considerably.

We shall now see what will happen if as the initial profile $n(r)$ we shall choose an arbitrary function $\tilde{n}(r)$ such that the expansion (14) will involve (and with a substantial contribution) harmonics with $l \neq 0$, as is shown in Fig.3. The harmonics with eigenvalues $\lambda_l \gg \lambda_0$ should be expected to damp faster than the zero component does already for $y < 1$. As a result, the distribution over discharge will rather rapidly relax to $g_0(r)$, which will immediately lead to balance violation between the ambipolar field and anomalous diffusion. In this case, an effective increase of the diffusion rate may be expected. Indeed, Fig.3 shows the distribution over discharge radius for $y = 1$, obtained in the solution of the system (16) with the profile $n(r) = \tilde{n}(r)$, is almost coincident with $g_0(r)$ (dashed line in the same figure). On the other hand, Fig.2 presents the dependencies obtained for one and the same value of $\beta$ for $n(r) = g(r)$ and the profile $n(r) = \tilde{n}(r)$ depicted in Fig. 3. It is seen that in the latter case the decrement is considerably larger, which testifies to an effective increase of the diffusion rate.

The result was obtained make allowance us to conclude, that the rate and character of diffusion (and as a consequence lost of energy due to diffusion of fast particles) essentially depends on profile of mean particle density. The minimum of lost will be as $n(r) = g_0(r)$.

The density profile in a device $n(r)$ is defined by many factors: anomalous trans-
port, neutral particles flow from the camera wall and external sources, accelerated ions injection, convective transport processes. So by means of injection, for example, one may to drive profile \( n(r) \) and respectively process of fast particle transport.

The result obtained suggests that the compensation of anomalous transport by an ambipolar field is a consequence of equilibrium in the leading term of the distribution function of electrons, that is \( \Psi_0(y) = y/2 \), and the absence of distortion of the initial electron distribution \( n(r) \). As shown above, the strict balance is violated by the anomalous transport itself when \( n(r) \neq g_0(r) \). In the presence of an electric field such a balance will also be violated for \( y \gg 1 \) since according to [25] the electric field \( E_e \) induces strong deviations of electron distribution from an equilibrium one.

We may point out another mechanism of balance violation already for \( 0 < y < 1 \), namely, inhomogeneity of the applied field \( E_e(r) \) which in this case plays the role of a fast particle source nonuniform in space (note that it is exactly the case realized in RFP). We shall consider the solution of equation (6) in the polynomial region using the methods presented above. As before, we put \( F = \text{const.} \) Figure 2 presents the dependence of the distribution function \( \ln(f/f_0) \) on the energy \( y \) for one and the same value of the parameter \( \beta \) and \( n(r) = g_0(r) \) in two cases:

a) for \( E = \text{const} \) and b) for \( E_e(r) = J_0(\kappa_0 r) \), \( \kappa_0 = 3.0 \), (such a profile is close to the one observed in RFP discharges). It can be readily seen that in case b) the distribution function fall with energy is much higher than in case a), which shows an acceleration of the diffusion process. It is noteworthy that the electron temperature inhomogeneity (which we do not consider here) will obviously play a role similar to that played by the inhomogeneity of the external field \( E_e(r) \) and will also lead to a violation of strict balance, see Fig.2.

We have assumed above that \( \delta_m \gg \delta_1^{1/2} \), and the correction of the order of \( \delta_m \delta_1^{1/2} \) has led to strong distribution function distortion when \( \Delta \geq 1 \). Clearly, in the converse case, that is when \( \delta_m \ll \delta_1^{1/2} \), there will proceed an analogous process which is not distinct qualitatively from the one investigated above with the only difference that in this case the role of the parameter \( \Delta \) will be played by the parameter \( \Delta_1 = 2\beta\lambda_m\delta_0^{1/2} \). As a result, as before for \( \Delta_1 \sim 1 \) the distribution function will relax rapidly to the profile \( g_0(r) \).
Thus we have completely investigated the behavior of the solution of equation (6) in the polynomial region. We have obtained that in a special case where the profile $n(r)$ is chosen in the form $n(r) = g_0(r)$ and the external field $E_e$ is homogeneous, the ambipolar field $E_a$ completely damps the anomalous transport of fast electrons with an accuracy of $\delta_m \ll 1$, so that the effective diffusion is determined by the parameter $\Delta$ or $\Delta_1$ which is substantially smaller than $\beta\lambda_0$, $\Delta \ll \beta\lambda_0$. On the other hand, such a strict balance in higher terms of expansion may be violated provided that $n(r) \neq g_0(r)$ or the external field $E_e$ and temperature $T(r)$ are inhomogeneous. We note that the latter always takes place in RFP. An essential result is here the fact that for $\Delta > 1$ (or $\Delta_1 > 1$) the distribution function of fast particles over discharge relaxes with increasing energy to the universal profile $g_0(r)$ independent of the initial distribution. This fact will simplify appreciably our analysis in the remaining part of this section, where we consider the energy range $y >> 1$.

3.2 The behavior of the solution in the exponential region.

We shall now consider the domain of solution for high energy values $\epsilon \gg \delta^{-1/2}$, where $\delta = \delta_1, \delta_2$. The distortions of the distribution function in this domain are known to be of exponential character [26]. Therefore, a polynomial expansion is not effective here. As we have seen above, practically for $\epsilon \gg \delta^{-1/2}$, the distribution function of fast electrons over discharge is coincident with the zero eigenfunction $g_0(r)$ of the problem (12). Therefore, in this energy range, in the expansion of the distribution function it is natural to make allowance only for terms containing $g_0(r)$:

$$f = C \sum_l g_l \exp(-\Psi_l)$$

Let us consider the case $E_e = 0$. We shall pass over to a new variable $z = \epsilon\delta_2$ and represent $\Psi$ as:

$$\Psi = \Psi_0/\delta_2 + \Psi_1/\delta_2^{1/2} + \Psi_2 + \ldots$$

Substituting (17) and (18) into (1) and collecting terms with the same powers $\delta_2^{1/2}$, we obtain the system of equations

$$\frac{\partial \Psi_0}{\partial \mu} = 0$$
\( \frac{\partial \Psi_0}{\partial \mu} \frac{\partial \Psi_1}{\partial \mu} = 0 \) \tag{20}

\[
4\left( \frac{\partial \Psi_0}{\partial z} \right)^2 - 2 \frac{\partial \Psi_0}{\partial z} + \frac{Z_{\text{eff}}}{z} (1 - \mu^2) \left( \frac{\partial \Psi_1}{\partial \mu} \right)^2 - z|\mu| \{ \lambda_0 + B \left( \frac{\partial \Psi_0}{\partial z} \right)^2 \} = 0 \tag{21}
\]

Where \( B = -4 \int_0^1 \rho r^2 F(r) E_a^2(r) dr / \int_0^1 \rho r^2 dr \)

From (19) it follows that \( \Psi_0 = \Psi_0(z) \), and (21) holds identically. From the condition of the absence of a jump of the derivative \( \frac{\partial \Psi}{\partial \mu} \) for \( \mu = 0 \) there follows a natural condition (in view of symmetry under a substitution of \( \mu \) for \( -\mu \) in (3) for \( E_e = 0 \))

\[ \frac{\partial \Psi_1}{\partial \mu} \bigg|_{\mu=0} = 0 \]

Taking into consideration this condition, as well as the fact that \( \Psi_0 = \Psi_0(z) \), from equation (21) for \( \mu = 0 \) we obtain

\[ \Psi_0(z) = \frac{z}{2} \]

Substituting the expression found for \( \Psi_0(z) \) back into (21), we obtain the equation for determining \( \Psi_1(z, \mu) \).

\[ \frac{\partial \Psi_1}{\partial \mu} = z \sqrt{\frac{\mu |\lambda_0^*|}{z_{\text{eff}} (1 - \mu^2)}} \tag{22} \]

where \( \lambda_0^* = \lambda_0 + B/4 \) is an effective eigenvalue with account of the influence of the ambipolar field \( E_a \). From (22) we see however that the asymptotical expansion obtained is violated in the vicinity of \( \mu = 0 \) because the second derivative \( \frac{\partial^2 \Psi}{\partial \mu^2} \) contains a singularity. Indeed, \( \frac{\partial^2 \Psi}{\partial \mu^2} \rightarrow \infty \) as \( \mu \rightarrow 0 \), which indicates of the presence of a boundary layer near \( \mu = 0 \). To obtain a correct expansion, it is necessary to investigate the behavior of the solution in the vicinity of \( \mu = 0 \). To this end, the small term with a second derivative \( \frac{\partial^2 \Psi}{\partial \mu^2} \delta_1^{1/2} \) in equation (21) should be retained.

Omitting terms of the order of \( \mu^2 \) as \( \mu \rightarrow 0 \) in (21) and making a substitution \( \Psi_1 = -\ln(\Theta) \delta_2^{1/2} \), we come to the Airy equation

\[ \frac{\partial^2 \Theta}{\partial \xi^2} = \xi \Theta \tag{23} \]
\[ \xi = \frac{\lambda^* z^2 \mu - \varphi(z)}{z_{\text{eff}}^{1/3} \delta_{2}^{1/3} \lambda^{2/3}} \]

\[ \varphi(z) = \left\{ 4 \left( \frac{\partial \Psi_0}{\partial z} \right)^2 - 2 \frac{\partial \Psi_0}{\partial z} \right\} \]

\[ \lambda^* = \lambda_0 + B \left( \frac{\partial \Psi_0}{\partial z} \right)^2 \]

Equation (23) has two linearly independent fundamental solutions, one of which grows exponentially as \( \xi \to \infty \) and may be discarded for being limited. The other solution \( \Theta = C_0 \text{Ai}(\xi) \) is shown in Fig. 4, \( \xi_0 \) is the point where the function \( \Theta \) and \( \Psi_1 \) has an extremum, \( \frac{\partial \Psi_1}{\partial \mu} = 0 \). Then from the condition \( \frac{\partial \Psi_1}{\partial \mu} \big|_{\mu=0} = 0 \) we obtain an additional relation that allows us to determine the function \( \Psi_0(z) \)

\[ 4 \left( \frac{\partial \Psi_0}{\partial z} \right)^2 - 2 \frac{\partial \Psi_0}{\partial z} = |\xi_0| z_{\text{eff}}^{1/3} \lambda^{2/3}(z) \delta_{2}^{1/3} z^{1/3} \quad (24) \]

In the general case, to find \( \Psi_0(z) \) it is necessary to solve the nonlinear equation (24), but for \( z \ll \delta_{2}^{-1} \) expanding equation (24) in the small parameter \( \delta_{2} \) we obtain

\[ \Psi_0(z) = \frac{z}{2} + \frac{3}{4} |\xi_0| z_{\text{eff}}^{1/3} \lambda_0^{2/3} \delta_{2}^{1/3} z^{4/3} \quad (25) \]

\[ \lambda_0^* = \lambda_0 + B/4 \]

Taking into account (23), we may determine the angular dependence of the distribution function:

\[ \Psi_1(\mu, z) = \int_{\mu_0}^{\mu} \sqrt{\frac{z \{ \lambda^*(z)z|\mu| - |\xi_0| z_{\text{eff}}^{1/3} \lambda^*(z)^{2/3} z^{1/3} \delta_{2}^{1/3} \}}{z_{\text{eff}}(1 - \mu^2)}} d\mu + \tilde{\Psi}_1(z) \quad (26) \]

where \( \tilde{\Psi}_1(z) \) is an unknown function. As \( \mu \to 0 \), it is necessary to sew (26) with the solution of equation (23) to find the constant \( C_0 \). It is readily seen that as \( \delta_{2} \to 0 \), (25) transforms into \( \Psi_0(z) = z/2 \) and (26) into (22). As \( z \to 0 \), the solution found here must pass over to the solution obtained in the polynomial region. Figure 5 shows dependencies of the distribution function on the pitch angle in the polynomial region for \( y > 1 \) and the solution obtained in the exponential region when \( z \to 0 \).

One can see good agreement between the two solutions. The solution obtained by us shows that the leading term of the asymptotical expansion for \( \delta_{2} \to 0 \) is close to equilibrium, and deviations from it occur only for \( z \sim \delta_{2}^{-1/2} \). This result is in
close agreement with the results of the direct numerical simulations [31] carried out for TOKAMAK conditions in the absence of the applied electric field $E_e$ and experiments [32], [33]. It should be noted that the role of an ambipolar field in this range of energy values comes down to renormalization of the eigenvalue $\lambda_0$ which determines the anomalous transport, namely, the effective eigenvalue with allowance for the influence of the ambipolar field $E_a$ will be $\lambda_0^* = \lambda_0 + B/4$, $\lambda_0^* < \lambda_0$ since $B < 0$. That is, as expected, the ambipolar field $E_a$ damps the anomalous transport of fast electrons. It is clear that, this effect is important when the effective temperature of fast electrons is of the order of equilibrium temperature at the center of the discharge.

The field $E_a$ also affects the behavior of the distribution function in the far region of energy values as $z \to \infty$. So, when $B \neq 0$ one can readily obtain an asymptotical expression for $\Psi_0(z)$ as $z \to \infty$:

$$\Psi_0(z) \simeq z \sqrt{\lambda_0 / B}$$

At the same time when $B = 0$ and $z \to \infty$, from (24) we have

$$\Psi_0(z) \simeq \frac{3}{14} z^{7/6} \lambda_0^{1/3} z_{eff}^{1/6}$$

that is, in the absence of the ambipolar field $E_a$ the distribution function falls stronger with energy.

We shall now consider the case with a nonzero external field $E_e$. As in the previous case in the expansion (17), we shall take into account only the contribution from the terms $g_0(r)$. In equation (6) we pass over to a new variable $z = \epsilon \delta_1$ and represent the index of the exponential in (17) in the form

$$\Psi = \Psi_0 / \delta_1 + \Psi_1 / \delta_1^{1/2} + \Psi_2 + \ldots$$

Substituting (27) into (6) and collecting terms with the same powers $\delta_1^{1/2}$, we obtain a chain of connected equations for the functions $\Psi_0, \Psi_1, \Psi_2, \ldots$:

$$\frac{\partial \Psi_0}{\partial \mu} = 0$$

$$\frac{\partial \Psi_0}{\partial \mu} \frac{\partial \Psi_1}{\partial \mu} = 0$$
\[4 \left( \frac{\partial \Psi_0}{\partial z} \right)^2 - 2 \frac{\partial \Psi_0}{\partial z} + \frac{Z_{\text{eff}}}{z} (1 - \mu^2) \left( \frac{\partial \Psi_1}{\partial \mu} \right)^2 - z |\beta \lambda^*| + 2 \mu z E \frac{\partial \Psi_0}{\partial z} = 0 \] (30)

\[8 \frac{\partial \Psi_0}{\partial z} + \frac{\partial \Psi_1}{\partial z} + \frac{Z_{\text{eff}}}{z} \{ 2 (1 - \mu^2) \frac{\partial \Psi_1}{\partial \mu} - \frac{\partial^2 \Psi_1}{\partial \mu^2} + 2 \mu \frac{\partial \Psi_1}{\partial \mu} \} - 2z |\mu| B \frac{\partial \Psi_0}{\partial z} + 2 \mu z E \frac{\partial \Psi_1}{\partial z} + E (1 - \mu^2) \frac{\partial \Psi_1}{\partial \mu} = 0 \] (31)

\[B = -4 \int_0^1 r g_0^2(r) F(r) E^2(r) dr / \int_0^1 r g_0^2(r) dr \]

\[E = \int_0^1 r g_0^2(r) E_e(r) dr / \int_0^1 r g_0^2(r) dr \]

\[\beta = \delta_2 / \delta_1; \quad \lambda^*(z) = \lambda_0 + B \left( \frac{\partial \Psi_0}{\partial z} \right)^2 \]

From the first equation (28) it follows that \(\Psi_0 = \Psi_0(z)\). Given this, equation (29) holds automatically. Then, in the absence of a singularity for \(\mu = 1\), from (30) we obtain the equation with the help of which we may determine \(\Psi_0\):

\[4 \left( \frac{\partial \Psi_0}{\partial z} \right)^2 - 2 \frac{\partial \Psi_0}{\partial z} - z \beta \lambda^*(z) + 2 z E \frac{\partial \Psi_0}{\partial z} = 0 \] (32)

This implies

\[\frac{\partial \Psi_0}{\partial z} = \frac{(1 - z E) + \sqrt{(1 - z E)^2 + z \beta \lambda_0 (4 - \beta_z B)}}{4 - z \beta B} \] (33)

Integrating (33) we arrive at

\[\Psi_0(z) = \frac{E z \beta_B}{B \beta} - \frac{\sqrt{1 + a_1 z + a_2 z^2}}{B \beta} - \frac{(8a_2 + a_1 B \beta) \ln(a_1 + 2a_2 z + 2 \sqrt{a_2 \sqrt{1 + a_1 z} + a_2 z^2})}{2 \sqrt{a_2^2 B^2 \beta^2}} + \frac{(4E - B \beta) \ln(S_1)}{B^2 \beta^2} \] (34)

\[S_1 = -4a_1 B^3 \beta^3 - 2B^4 \beta^4 - 8a_2 B^3 \beta^3 z - a_1 B^4 \beta^4 z - 2B^3 \beta^3 (4E - B \beta) \times \]

\[\sqrt{1 + a_1 z + a_2 z^2} \]

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\[ a_1 = 4\beta\lambda_0 - 2E; \quad a_2 = E^2 - \beta^2\lambda_0 B \]

Taking into consideration (33), we obtain from (30)

\[
\Psi_1(\mu, z) = -z \int_1^\mu \sqrt{\frac{\beta(\lambda_0 + B(\partial\Psi_0/\partial z)^2)(|\mu| - 1) - 2\partial\Psi_0/\partial z E(\mu - 1)}{z_{eff}(1 - \mu^2)}} d\mu + \tilde{\Psi}_1(z) \quad (35)
\]

It should be noted that (35) holds for

\[ \beta\{\lambda_0 + B/4\} < E \quad (36) \]

In this case, the solution will not have a singularity for \( \mu = 1 \). Substituting (35) into (31), provided that there is no singularity for \( \mu = 1 \), the unknown function \( \tilde{\Psi}_1(z) \) may be determined:

\[
\tilde{\Psi}_1(z) = \sqrt{Z_{eff}(-1/\sqrt{2} + 1)} \int_0^z \sqrt{\frac{2\partial\Psi_0/\partial z E - \beta(\lambda_0 + B(\partial\Psi_0/\partial z)^2)}{4\partial\Psi_0/\partial z + z\beta B \partial\Psi_0/\partial z + Ez - 1}} dz \quad (37)
\]

The expressions (34), (35) and (37) determine in the exponential approximation the dependence of the distribution function in the exponential region on the energy and the pitch angle \( \mu \). As \( z \to 0 \), the obtained solution must coincide with the solution in the polynomial region for \( y > 1 \). Figure 6 shows both the solutions which are seen to be almost coincident.

To establish the asymptotical behavior of the distribution function as \( z \to \infty \), we represent the index of the exponential in (17) in the form

\[
\Psi = \Psi_0 z + \Psi_1 + \ldots
\]

\[
\Psi_0 = \varphi_0/\delta_1 + \varphi_1/\delta_1^{1/2} + \varphi_2 + \ldots
\]

\[ z = \epsilon\delta_1 \quad (38) \]

Substituting (38) into (3) with allowance for (17), keeping terms of the order of \( z \) and collecting terms with the same powers \( \delta_1^{1/2} \), we obtain the system of equations
for $\varphi_0, \varphi_1, \ldots$:

$$\frac{\partial \varphi_0}{\partial \mu} = 0 \quad (39)$$

$$\frac{\partial \varphi_0}{\partial \mu} \frac{\partial \varphi_1}{\partial \mu} = 0 \quad (40)$$

$$(\frac{\partial \varphi_1}{\partial \mu})^2 Z_{eff} (1 - \mu^2) - |\mu| \beta \{\lambda_0 + B \varphi_0^2\} + 2\mu \varphi_0 E = 0 \quad (41)$$

$$2Z_{eff} (1 - \mu^2) \frac{\partial \varphi_1}{\partial \mu} \frac{\partial \varphi_2}{\partial \mu} - |\mu| \beta B \varphi_0 \varphi_1 + 2\mu E \varphi_1 + (1 - \mu^2) \frac{\partial \varphi_1}{\partial \mu} E = 0 \quad (42)$$

where $\beta = \delta_2/\delta_1, \ldots$, and $B$ and $E$ are similar to (30), whence $\varphi_0 = \text{const}$. Then in the absence of singularity for $\mu = 1$ we have from (41):

$$\varphi_0 = \sqrt{\frac{E^2 - \beta^2 \lambda_0 B - E}{-\beta B}} \quad (43)$$

Knowing $\varphi_0$ from (41), we obtain

$$\frac{\partial \varphi_1}{\partial \mu} = 0 \quad \mu > 0$$

$$\frac{\partial \varphi_1}{\partial \mu} = -\sqrt{\frac{\mu (4E/\beta B)(\sqrt{E^2 - \beta^2 \lambda_0 B - E})}{Z_{eff} (1 - \mu^2)}} \quad \mu < 0 \quad (44)$$

$$\varphi_1 = \text{const} \quad \mu > 0$$

$$\varphi_1 = \int_1^{\mu} \sqrt{\frac{\mu (4E/\beta B)(\sqrt{E^2 - \beta^2 \lambda_0 B - E})}{Z_{eff} (1 - \mu^2)}} \, d\mu + \tilde{\varphi}_1 \quad \mu < 0 \quad (45)$$

where $\tilde{\varphi}_1$ is an unknown constant. Then in the absence of singularity we obtain from (42) for $\mu = 1$ that $\tilde{\varphi}_1 = 0$.

The expressions (43) and (45) determine the asymptotical behavior of the distribution function for $E_e(r) \neq 0$ as $z \to \infty$. It can be readily seen that in the limit $z \to \infty$ (44) goes over to (43) and (35) into (45). So, the obtained solution (44), (45) in the exponential approximation describes the particle distribution in the entire range of energy values for $z > \delta_1^{1/2}$. It should be noted that the asymptotical expression for the distribution function for $z \to \infty$ (44), (45) holds also under the condition inverse of (36), but obtaining this asymptotes is apparently not described by the simple expansion (17). The solution obtained in this section describes completely the distribution function of fast particles in the presence of an external field $E_e$ and anomalous diffusion in a homogeneous magnetic field. We have investigated
here a rather general case in the model assumption $F = \text{const}$. In section 5 we shall analyze the distributions of fast electrons in specific experimental conditions, and now we proceed to the question of the influence of inhomogeneity of a mean magnetic field.

4 Distribution function of fast particles in the presence of a finite magnetic field gradient.

We are now ready to consider the influence produced by an inhomogeneous magnetic field upon the distribution function of fast electrons. From the point of view of physics, the occurrence of terms proportional to the magnetic field gradient in the kinetic equation (3) is associated with keeping the adiabatic invariant $u^2_\perp/B(r) = \text{const}$ upon a diffusion particle motion along a discharge. In this case, particle diffusion is responsible for particle energy redistribution from the transverse degree of freedom to the longitudinal one and vice versa, depending on the sign of the derivative $dB/dr$. Taking into consideration the fact that on the average particles move from the center to periphery of a discharge, in the case $dB/dr < 0$ part of the transverse particle energy is transferred into the longitudinal and conversely in the case $dB/dr > 0$. Such dynamics must in turn affect the diffusion process itself. Indeed, in case the amount of particles with high longitudinal energy increases (when $dB/dr < 0$) or on the contrary decreases (when $dB/dr > 0$) because the anomalous diffusion coefficient (3) is proportional to the longitudinal velocity, the diffusion rate will respectively either increase or on the contrary decrease.

Let us consider the stationary solution of equation (3) in the range $1 \leq \epsilon \leq \delta^{-1/2}$, where $\delta = \delta_1, \delta_2$. Not to complicate the picture, we put the external field $E_e(r) = 0$. In order to obtain the solution, we shall use the technique we applied in the solution of equation (3) in this range, that is, the polynomial expansion (13). The function $F(r)$ will put $F = \text{const}$, and $n(r) = g_0(r)$ of the problem (12). Figure 7 shows the dependencies of the distribution of fast electrons obtained in the solution of equation (3) depending on the electron energy $y$ in two cases: $dB/dr > 0$ and $dB/dr < 0$. The figure also presents for comparison the result for $dB/dr = 0$. As has been
expected, the greatest decrement of $\ln(f/f_0)$ appears in the case $dB/dr < 0$, and the smaller in the case $dB/dr > 0$. It would also be of interest to trace the behavior of the solution obtained depending on the longitudinal $u_\parallel^2$ and the transverse $u_\perp^2$ energies. In Fig.8 we can see the dependencies of $\ln(f/f_0)$ on the transverse and the longitudinal particle energy in the cases $dB/dr < 0$, $dB/dr > 0$ and $dB/dr = 0$. We see that both for $dB/dr = 0$ and $dB/dr > 0$ there exists anisotropy in particle distribution over transverse and longitudinal energy, so that $T_\perp > T_\parallel$. However for $dB/dr < 0$ the anisotropy changes sign and then $T_\perp < T_\parallel$, which is naturally connected with a substantial energy redistribution from the transverse degree of freedom into the longitudinal one. This effect is most clearly pronounced in the dependence of the distribution function on the pitch-angle $\mu$, Fig.9, which differs substantially from that obtained in the case $dB/dr = 0$, Fig.1. The fall in the center (for $\mu \sim 0$) in Fig. 9 and the appearance of maxima near $\mu \sim 1$ is just due to particle efflux from the region $\mu \sim 0$ towards the region $\mu \sim 1$.

5 Behavior of the distribution function of fast electrons in RFP.

In the preceding sections of the paper we have analyzed the influence of various factors upon the distribution function of fast particles. It should be noted here that just in RFP discharges the contribution from each of them will be substantial because all of them are present in this type of devices. An RFP discharge possesses a force-free magnetic configuration with components of the mean magnetic field (the toroidal $B_z(r)$ and poloidal $B_\theta(r)$) which are of the same order of magnitude and are determined by one and the same parameter $\Theta = 2\pi a I/\Phi$, where $I$ is total current, $a$ is small radius of the torus, and $\Phi$ is the total toroidal magnetic field flux. Magnetic field components are well enough described by Bessel functions of the form

$$B_z(r) = B_0 J_0(2\Theta r)$$

(46)
\[ B_\Theta(r) = B_0 J_1(2\Theta r) \]

see Fig.10, [34]. So, the scale of the magnetic field gradient in RFP will be of the order of the small radius \( a \) of the torus:

\[
\frac{dB}{dr}a \sim 1
\]

Since plasma in RFP is heated in an ohmic way, present in the discharge is a sufficiently strong longitudinal electric field \( E_e \). Since an applied electric field \( \vec{E} = E \hat{e}_z \) is toroidal, its projection \( E_e(r) \) onto a magnetic field line is described by the relation

\[
E_e(r) = E \frac{B_z(r)}{B(r)}
\]  \( \text{(47)} \)

From (47) it becomes clear that the applied field is strongly inhomogeneous about the space.

Direct measurements of distribution of superthermal electrons in steady-state of RFP were carried out on ZT-40M devices [19, 35, 36, 37]. For comparison with the theory, we shall consider the experimental data obtained in [35]. Performed in the experiment were direct measurements of energy distribution of fast electrons in the near-boundary region of discharge with a temperature \( T_w \approx 20\text{eV} \) near the boundary and \( T_0 \approx 220\text{eV} \) in the center of discharge. The characteristic value of the parameter \( \Theta = 1.4 \). Observations were carried out for particles of energy up to \( 1.5\text{keV} \). Against the background of cold plasma with temperature \( T_w \approx 20\text{eV} \) one could see a tail of energetic electrons which moving in the direction of the magnetic field and have Maxwell distribution with characteristic temperature \( T_1 \approx 530\text{eV} \) and a flux of particles that moved backward and had a temperature \( T_2 \approx 330\text{eV} \). The anisotropy in the distribution of particles in the longitudinal and transverse energies was observed, the temperature being \( T_\perp < T_\parallel \). Under conditions of the experiment described above, the parameters \( \delta_1, \delta_2 \) of our problem were \( \delta_1 = 0.21, \delta_2 = 0.036 \) for \( T_0 = 220\text{eV}, \ n = 2,4 \cdot 10^{13}\text{cm}^{-3}, \ E_e = 10\text{V/m} \) and the amplitude of fluctuations \( |b/B| \sim 1\%, \ \frac{F_{\max}}{B_0^2a} \sim 4 \cdot 10^{-4} \).

The solution we obtained in section three (formulae (34), (35), (37)) gives qualitative agreement with experiment: exponential distribution of particles on energy
and anisotropy in particles distribution over forward and backward direction along magnetic field lines.

An important comparison with experimental data is given by modeling superthermal electron current \( I_{\text{eea}} \), collected by the electrostatic energy analyzer (EEA), as function of applied retarding potential \( V \). The current is related to the electron distribution function \( f(r, \mu, \epsilon) \) by expression:

\[
I_{\text{eea}}(r, V) \propto \int_{v_0}^{\infty} \int_{0}^{\infty} f(r, \mu, \epsilon)v_{\perp}dv_{\perp}v_{\parallel}dv_{\parallel}
\]

where

\[
v_0 = \sqrt{2eV/m_e}
\]

Figure 11 shows the dependencies of current \( I_{\text{eea}} \) calculated by formulae (34), (35), (37), (48) for the given values of the parameters \( \delta_1 \) and \( \delta_2 \) for particles flying along the direction of the magnetic field and backward. The profile of the correlation function \( F(r) \) shown in Fig.12 was borrowed from ref.[35]. The graph of the zero eigenfunction of the problem (12) \( g_0(r) \) for a given \( F(r) \) is shown in Fig.13, where \( \lambda_0 = 1.2 \). The profile \( n(r) \) was chosen in the form \( n(r) = g_0(r) \) and \( T(r) = 1 - r^2 \). The calculated current \( I_{\text{eea}} \) was normalized on experimental value. The absolute value of \( I_{\text{eea}} \) seems to be rather uncertain because of a lack of information on the correlation function near the boundary.

Figure 11 testifies to not only qualitative agreement between the theory and experiment (Maxwell distribution and the anisotropy observed in the distribution of particles flying in the direction of the magnetic field and backward), but to quantitative one as well. So, the characteristic temperature \( T_{1\text{teor}} \) for particles moving along the magnetic field makes up \( T_{1\text{teor}} = 526 \, eV \). But for particles moving in the backward direction it was \( T_{2\text{teor}} = 183 \, eV \), i.e. less than experimental one. Backward flowing component of fast electrons is seen to be a much smaller than forward one (less than 10 \% ). This value is also close to experimental observations [35].

The behavior of distribution of fast electrons on longitudinal and transverse energy is shown on fig.14, at the parameters mentioned above. In this case for particles moving along the magnetic field line \( T_\perp < T_\parallel \) and \( T_\parallel = 513 \, eV, T_\perp = 385 \, eV \).
6 Summary

In conclusion we shall present the main results obtained in this paper. In the entire energy range above thermal energies, we have constructed the solution of the kinetic equation describing the distribution function of fast particles in the presence of an external electric field, collisions, and anomalous diffusion due to magnetic fluctuations. The ambipolar electric field resulting from the difference in the ion and electron diffusion rates is shown to play essential role in the process of diffusion and to be responsible for the fact that the character of the diffusion depends strongly on the density profile of the main plasma particles and on the profile of an external electric field applied to the plasma. The influence of inhomogeneity of the magnetic field of a discharge upon the distribution function of fast particles is investigated. The analysis performed suggests that the magnetic field inhomogeneity is an important factor affecting the distribution function of fast particles. In the polynomial region of energy values, which is an immediate neighbor of the thermal region, the semi-analytical solution is constructed with allowance for all the factors mentioned above. On the basis of comparison with experimental data we have shown that the theory describes the basic details of the distribution function of fast particles. We have referred our analysis mainly to the case of RFP discharges, but the results obtained in the absence of strong electric field seems to be closely connected with anomalous process in TOKAMAK discharges.

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References
[1] H.P.Furth, Nuclear Fusion 15 (1975) 487.

[2] P.S.Liewer, Nuclear Fusion 25 (1985) 543.

[3] V.V Parail, O.P.Pogutse, Plasma Physics and Controlled Nuclear Fusion Research, (Proceedings of Int. Conf., 1980, Brussels) 1 (1980) (IAEA, Vienna, 1981) 67.

[4] C.Z.Cheng, H.Okuda, Nuclear Fusion 18 (1978) 587.

[5] M.Wakatami, A.Hasegawa, Phys.Fluids 27 (1984) 611.

[6] R.Waltz, Phys.Fluids 26 (1983) 169.

[7] A.B.Rechester, M.N.Rosenbluth, Phys.Rev.Letters 40 (1978) 38.

[8] E.Mazzucato, Phys.Rev.Letters 48 (1982) 1829.

[9] C.M.Surko, R.E.Slusher, Phys.Fluids 23 (1980) 2425.

[10] J.O.Strachan, N.Bretz, E.Mazzucato, C.W.Barnes, D.Boyd et al. Nuclear Fusion 22 (1982) 1145.

[11] P.B.White in: Statistical Physics and Chaos in Fusion Plasmas, p.209. Edited by C.W.Horton and L.E.Reichl (Wiley, New York, 1984).

[12] R.Balescu, J.H.Misguich, R.Nakach. Diffusion of charged particles in a stochastic magnetic field. Preprint EUR-CEA-FC-1463, July 1992.

[13] R.J.Goldston, E.Mazzucato, R.E.Slusher, C.M.Surko, Plasma Physics and Controlled Nuclear Fusion Research, (Proceedings of Int. Conf., 1976, Berchtesgaden) 1 (1976) (IAEA, Vienna, 1977) 371.

[14] J.Hugil, Nuclear Fusion 23 (1983) 331.

[15] C.M.Surko, R.E.Slusher, Science 221 1983 817.

[16] S.J.Levinson, J.M.Beall, E.J.Powers, R.D.Bengston, Nuclear Fusion 24 1984 527.
[17] D.Brotherton - Ratcliffe, C.G.Gimblet, J.H.Hutchinson, Plasma Phys. 29 (1987) 161.

[18] S.C.Prager, Plasma Physics and Controlled Fusion 32 1990 903.

[19] J.C.Ingraham et al. Phys.Fluids B 2 (1990) 143.

[20] S.Ortolani, Invited Lecture at the 1992 Inter. Conference on Plasma Physics, Insbruck, Austria 29 June 1992.

[21] A.A.Newton, E.Evans, H.Y.W. Tsui, Proceedings of 16th EPS Conference on Contr. Fusion and Plasma Physics, Venice, 2 (1989) 721.

[22] A.V.Gurevich, K.P.Zybin, Ya.N.Istomin, Nuclear Fusion 27 (1987) 453.

[23] A.V.Gurevich, A.V.Lukyanov, K.P.Zybin, Physics of alternative magnetic confinement schemes (Proceedings of the Workshops, Held at Villa Monastero 15 October 1990, Varena, Italy) 421.

[24] A.V.Gurevich, A.V.Lukyanov, K.P.Zybin, R.Nebel, Sov.Phys. JETP 73 (1991) 976.

[25] A.V.Gurevich, Sov.Phys. JETP 12 (1961) 904.

[26] A.V.Gurevich, Ya.S.Dimant, Nuclear Fusion 18 (1978) 629.

[27] A.V.Gurevich, Ya.S.Dimant, Nuclear Fusion 21 (1981) 159.

[28] A.V.Gurevich, Ya.N.Istomin, Sov.Phys. JETP 77 (1979) 933.

[29] A.V.Gurevich, Ya.N.Istomin, K.P.Zybin, Sov.Phys. JETP 57 (1983) 51.

[30] K.F.Schoenberg and R.W.Moses, Physics of Fluids 3 B (1991) 1467.

[31] G.Giruzzi, I.Fidone, X.Garbet. Kinetic effects of magnetic turbulence in tokamaks, Preprint, DRFC-CAD, EUR-CEA-FC-1440, November 1991.

[32] K.Molvig, J.Rice, M.Tenyla, Phys.Rev.Letters 41 (1978) 1240.

[33] J.Rice, K.Molvig, H.Helawa, Phys.Rev. A25 (1982) 1645.
[34] H.A.B. Bodin, A.A. Newton, Nuclear Fusion 20 (1980) 1255.

[35] J.C. Ingraham et al. Physics of alternative magnetic confinement schemes (Proceedings of the Workshops, Held at Villa Monastero 15 October 1990, Varena, Italy) 859

[36] J.C. Ingraham et al. Bull. Am. Phys. Soc. 34 (1989) 2105.

[37] K.F. Schoenberg, J.C. Ingraham et al. in 14 European Conference on Controlled Fusion and Plasma Physics, Vol. IID (European Physical Society, Madrid 1987) 481.
Captions

Fig.1 The dependencies of fast particles distribution function on pitch angle $\mu$ into polynomial area of solution under various parameters $\beta$, $y = 2$, $r = 0.5$. Dashed line - solution, obtained in [25] under $\beta = 0$.

Fig.2 The dependencies of fast particles distribution function $\ln(f/f_0)$ averaged over $\mu$ on energy $y$ into polynomial solution area. 1) - $n(r) = g_0(r)$, $T(r) = \text{const}$, $E_e(r) = \text{const}$ 2) - in case of nonuniform electric field $n(r) = g_0(r)$, $T(r) = \text{const}$ 3) - $n(r) = \tilde{n}(r)$, plot $\tilde{n}(r)$ is shown on figure 3, $T(r) = \text{const}$, $E_e(r) = \text{const}$ and 4) $E_e(r) = \text{const}$, $n(r) = g_0(r)$, $T(r) = 1 - r^2$. $\beta = 0.3$, $r = 0.5$.

Fig.3 The distribution over discharge of initial density profile $\tilde{n}(r)$, distribution function $f(r)$ at $y = 1$, obtained in solving the system (16) and eigenfunction of the problem (12) (dashed line).

Fig.4 The solution of equation (23) as a function of $\xi$.

Fig.5 The fast electrons distribution function $f/f_0$ as a function of $\mu$ in the absence of applied electric field $E_e$. Solid line - the polynomial solution at $y > 1$, dashed line - the exponential solution at $z = y \delta_2^{1/2} \ll 1$.

Fig.6 The fast electrons distribution function $f/f_0$ as a function of $\mu$ with the applied electric field $E_e$ present. Solid line - the polynomial solution at $y > 1$, dashed line - the exponential solution at $z = y \delta_2^{1/2} \ll 1$.

Fig.7 The dependencies of fast electrons distribution function, averaged over $\mu$, $\ln(f/f_0)$ on energy $y$ with finite gradient of mean magnetic field present. 1) $dB/dr > 0$, 2) $dB/dr = 0$, 3) $dB/dr < 0$. $r = 0.95$

Fig.8 The dependencies of fast electrons distribution function $\ln(f/f_0)$ on longitudinal (solid line) and transverse (dashed line) energy, in the present of finite gradient of mean magnetic field. 1) $dB/dr > 0$, 2) $dB/dr = 0$, 3) $dB/dr < 0$. $r = 0.95$

Fig.9 The dependencies of fast electrons distribution $f/f_0$ on $\mu$ under various positions over radius. $dB/dr < 0$.

Fig.10 Radial distribution of magnetic field components across the minor radius.

Fig.11 The comparison of theoretical versus experimental current $I_{e ea}$, collected
by electron energy analyzer as a function of retarding potential $V$ for the case of ZT-40M. Solid line - for particles moving along the magnetic field line, dashed line - for particles moving backward. Experimental data are shown by labels.

Fig.12 Correlation function of fluctuation in RFP.

Fig.13 Eigenfunction of the problem $[12]$ for the correlation function, depicted on figure 12.

Fig.14 The dependencies of fast electrons distribution function $\ln(f)$ on longitudinal (solid line) and transverse (dashed line) energy for particles moving along the magnetic field line. $\beta = 0.17$, $\delta_1 = 0.21$, $\delta_2 = 0.036$. 