A Study of the Requirements of $p-^{11}\text{B}$ Fusion Reactor by Tokamak System Code

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Abstract — Most tokamak devices including ITER exploit the deuterium-tritium reaction due to its high reactivity, but the wall loading caused by the associated 14-MeV neutrons will limit the further development of fusion performance at high beta. To explore the $p-^{11}\text{B}$ fusion cycle, a tokamak system code is extended to incorporate the relativistic bremsstrahlung since the temperature of electrons approaches the electron rest energy. By choosing an optimum $p-^{11}\text{B}$ mix and ion temperature, some representative sets of parameters of the $p-^{11}\text{B}$ tokamak reactor, whose fusion gain exceeds 1, have been found under the thermal wall loading limit and beta limit when synchrotron radiation loss is neglected. However, the fusion gain greatly decreases when the effect of synchrotron radiation loss is considered. Helium ash also plays an important role in the fusion performance, and we have found that the helium confinement time must be below the energy confinement time to keep the helium concentration ratio in an acceptable range.

Keywords — Tokamak system code, $p-^{11}\text{B}$ fusion reactor, Lawson criterion.

Note — Some figures may be in color only in the electronic version.

1. INTRODUCTION

A successful design of a tokamak device is based on a good understanding of the dependence of performance on key plasma parameters, such as density, temperature, magnetic field, current, etc. To study the dependence of these parameters, a tokamak system code (TSC) based on a one-dimensional (1-D) model that considers a simple profile assumption has been developed.

The TSC was first developed in the design of a tokamak reactor in the 1980s (Refs. 1 and 2). Stambaugh et al. gave a more concise description of the system code to study the physical and engineering limits that restrict the design of a fusion power reactor. Then, it was extended by Costley et al. to study the regime of steady-state reactors with high fusion gain, and they found that the fusion gain depends strongly on the fusion power and energy confinement and weakly on the size of the device while the steady-state reactor operates at fixed fractions of the density and beta limits.

The TSC has been used successfully on the deuterium-tritium (D-T) fusion reactor, while 14-MeV neutron-induced damage and degradation would limit the useful lifetime of reactor components. The neutron production in $p-^{11}\text{B}$ fuel is lower than that in D-T and deuterium- deutrium (D-D) fuel by orders of magnitude. Furthermore, hydrogen and boron are abundant and fairly accessible on earth while $^3\text{He}$, which is one of the ingredients of deuterium-$^3\text{He}$ ($^3\text{He}$) fuel, might be mined and transported from the moon. Therefore, there are sufficient reasons for us to pay more attention to the proton-$^{11}\text{B}$ ($p-^{11}\text{B}$) fusion cycle. Since the core temperature of electrons approaches the electron rest energy, a relativistic effect has to be considered in the extended

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TSC to incorporate the relativistic bremsstrahlung. This extended TSC is designed to include a physics module, an engineering module, and an economy module, the latter two of which are still under development and will be finished in future work.

Of course, the ion temperature and \( n_iT_F \) being as high as 300 keV and \( 10^{22} \text{m}^{-3} \text{s} \) for \(^{11}\text{B}\) fuel are far beyond the current experiment condition. In this paper, we hypothesize that the fuel ions could be heated up to the temperatures that the \(^{11}\text{B}\) fusion reaction requires by a certain method, the mechanism of which is not the main point this paper discusses. Under this premise, we could further explore the \(^{11}\text{B}\) fusion cycle and study the requirements of the \(^{11}\text{B}\) fusion reactor under some optimistic assumptions.

This paper is organized as follows. Section I introduces \(^{11}\text{B}\) fusion and the TSC. Section II comprises the main equations used in this code. Section III gives the results derived by the extended TSC. Section IV gives the conclusion and discussion.

II. EQUATIONS OF THE TSC

In this TSC, a simple parabolic radial profile has been considered. Here, a zero-dimensional (0-D) model does not include the profile effects of density and temperature whereas a 1-D model considers some certain density and temperature radial profiles.

The extended TSC is developed from the models described in the papers by Stambaugh et al.,\(^3\),\(^4\) Costley et al.,\(^5\) and Petty et al.\(^6\) In terms of the fusion power of \(^{11}\text{B}\) fuel and the relativistic bremsstrahlung, the model has been revised accordingly, and the confinement enhancement factor \( H \) has been used here to measure the confining capacity needed for the \(^{11}\text{B}\) reactor compared with the existing capabilities.

II.A. Geometry of Tokamak

The geometry of the tokamak size is described by parameters like aspect ratio \( A \), major radius \( R_0 \), elongation \( \kappa \), and triangularity \( \delta \), from which other geometry parameters can be deduced:

**Minor radius:**

\[
a = R_0/A ;
\]

**Elongation:**

\[
\kappa = 0.9\kappa_{\text{max}} = 0.9(2.4 + 65\exp(-A/0.376)) ;
\]

Plasma volume:

\[
V_p = \left( 2\pi^2\kappa^4 + \frac{16\pi^3\delta}{3} \right) a^3 ; \tag{3}
\]

Wall area:

\[
S_w = (4\pi^2 A\kappa^{0.65} - 4\kappa\delta) a^2 . \tag{4}
\]

Since a low-aspect-ratio tokamak has the advantage of burning plasmas in a compact geometry at a lower cost than in a conventional tokamak, we consider a low-aspect-ratio tokamak device, \( A = 1.4 \) and \( \delta = 0.5 \), as shown in Fig. 1, which will prove to be a convenient choice later in the paper.

II.B. Pressure and Beta

In the \(^{11}\text{B}\) plasma, the density-related parameters are as follows: the proton density \( n_p \), the density of the boron ion \( n_B \), the ion density of the \( p-B \) fuel \( n_i,pB \), the electron density \( n_e \), and the density of the helium ion \( n_{He} \), the profiles of which are described by the same parabolic form:

\[
n(x) = n_0(1 - x^2)^{S_n}, \tag{5}
\]

where

\[
x = r/a = \text{normalized radial distance} \]

\[
0 = \text{core density} \]

\[
S_n = \text{exponent of density profile}.
\]

In this paper, densities are in units of \( 10^{20} \text{m}^{-3} \), and in what follows, the units are generally meter, second, tesla, megawatt, megaampere, and kilo-electron-volt.

In addition, the fractional ion densities are defined as \( f_{pB} = n_p/n_B, f_B = n_B/n_{i,pB}, \) and \( f_{He} = n_{He}/n_e \). From proportional relation, we have

\[
f_{pB}f_B + f_B = 1 .
\]

From charge balance, we have

\[
f_{pB}f_Bn_{i,pB} + 5f_Bn_{i,pB} + 2f_{He}n_e = n_e ,
\]

and core electron and ion density can be expressed as

\[
n_{e0} = \left( f_{pB}f_Bn_{i,pB} + 5f_Bn_{i,pB} \right)/(1 - 2f_{He}) \tag{6}
\]
and

\[ n_{0} = n_{0,\text{pB}} + n_{\text{He}0} . \]  

(7)

The line-averaged density is

\[ \bar{n}_{e} = \frac{1}{a} \int_{0}^{a} n_{e0} (1 - x^2)^{S_{n}} dx . \]  

(8)

The effective charge is

\[ Z_{\text{eff}} = \frac{f_{\text{pB}} n_{\text{pB}} + 25f_{\text{B}} n_{\text{pB}} + 4n_{\text{He}}}{n_{e}} , \]  

and the average mass is

\[ M = \frac{f_{\text{pB}} + 5f_{\text{B}}}{f_{\text{pB}} + f_{\text{B}}} . \]  

(10)

The temperature profiles are assumed to be the similar parabolic form:

\[ T(x) = T_{0} (1 - x^2)^{S_{T}} , \]  

and \( S_{T} \) is the exponent of the temperature profile, indicating this is a 1-D model. The cross section is considered to be elliptic in the integral computation, and triangularity is only used while calculating the volume and area. Refer to the previous papers\(^{5,6}\); the density profile has taken \( S_{n} = 0.5 \) for a broad H-mode profile with a pedestal, and the temperature profile was assumed steeper, \( S_{T} = 1.5 \), which is shown in Fig. 1.

The profile effect has been considered in the calculation of the triple product to compare with the result of the 0-D mode, which is shown in Fig. 2. Figure 2 illustrates that the Lawson criterion of the D-T, D-^3^He, and D-D fusion reactor could meet at the same density of fuel ions and at the same ion and electron temperature whether in the 0-D model or in the 1-D model. But, in the p-\( ^{11}^\text{B} \) fusion reactor, the optimum mix of proton and boron is not 1:1, and the ion temperature and electron temperature must also be different while the profile effect has been considered. Here, we define the ratio of ion temperature to electron temperature as

\[ \frac{T_{i}}{T_{e}} = f_{T} . \]  

(12)

The optimum mix of proton and boron and the critical ratio of ion temperature to electron temperature will be

Fig. 1. (a) Sketch of a low-aspect-ratio tokamak geometry, (b) a broader profile of density, and (c) a steeper profile of temperature. In this paper, \( S_{n} = 0.5 \) and \( S_{T} = 1.5 \).
In p-\textsuperscript{11}B fuel, the fusion power element is written as

\[ dP_{\text{fus}} = (1.6 \times 10^{15}) \times 8.7 \times n_{p}n_{B}(\overline{\sigma V})dV, \]

in which the fusion reactivity is used from the recent evaluation by Sikora and Weller.\(^8\) Using \( x = r/a \), the fusion power can be represented as

\[ P_{\text{fus}} = (1.6 \times 10^{15}) \times E_{pB} \times n_{p}\bar{n}_{B}T_{i0}^{-2} \]
\[ \times 2V_{p}^{1/2}(\overline{\sigma V})^{1} \left( 1 - x^{2} \right)^{2S_{n}}xdx. \]

The value \( \overline{\sigma V} \) is determined by \( x \) since the fusion reactivity is a function of ion temperature and ion temperature is a function of \( x \). We use \( \phi \) to replace with the fusion reactivity integral:

\[ \phi = \frac{1}{2}(\overline{\sigma V})^{1/2}(1 - x^{2})^{2S_{n}}xdx, \]

and once the core ion temperature and the profile exponent have been given, the fusion reactivity integral is determined. The core ion density of p-B fuel can be represented as

\[ n_{\phi pB} = T_{i0}^{-1}(f_{pB})^{-1} \left( \frac{P_{\text{fus}}}{27.84 \times 10^{15}V_{p}} \right)^{0.5}. \]

The fusion gain equals fusion power divided by auxiliary heating power:

\[ n_{\phi pB} = T_{i0}^{-1}(f_{pB})^{-1} \left( \frac{P_{\text{fus}}}{27.84 \times 10^{15}V_{p}} \right)^{0.5}. \]
\[ Q = \frac{P_{\text{Aux}}}{P_{\text{Aux}}} \cdot \] (18)

The condition of \( Q = 1 \), when the fusion power is equal to the auxiliary heating power, is referred to as scientific breakeven. Since this paper is mostly focused on the physics module, the engineering breakeven and economic breakeven are not discussed here.

**II.D. Plasma Current**

In the \( p^{11}B \) reactor, an empirical current-driven efficiency formula derived in the tokamak has been used, and the driven current can be calculated by

\[ I_{\text{cd}} = \frac{0.031 \xi_{\text{cd}} P_{\text{cd}} T_{e0}}{R_{0} n_{e0}} \cdot \] (19)

where \( \xi_{\text{cd}} \) is the dimensionless current drive efficiency, which is taken as 0.2 here. The safety factor can be calculated from

\[ q_{\text{eng}} = \frac{5B_{T0} a^{2} \kappa}{R_{0} I_{p}} \cdot \] (20)

In addition to the driven current, the rest is the bootstrap current:

\[ I_{\text{cd}} = (1 - f_{\text{bs}}) I_{p} \cdot \] (21)

where \( f_{\text{bs}} \) is the bootstrap current fraction. From Andrade and Ludwig, the bootstrap current fraction \( f_{\text{bs}} \) can be deduced from

\[ f_{\text{bs}} = \frac{5C_{\text{bs}} A^{0.5} C_{p} n_{\text{eng}} (R_{0}/R_{m})}{100 I_{L}^{0.2}} \cdot \] (22)

where

\[ C_{\text{bs}} = 0.1558 \pm 0.0005 \]

\[ C_{p} = (1 + S_{n} + S_{T}) \]

\[ I_{L} = 0.5 = \text{internal inductance} \]

\[ \frac{P_{\text{bs}}}{P_{m}} = 0.8 \cdot \] (23)

We define a constant \( \text{const} = \frac{5C_{\text{bs}} C_{p} \sqrt{a}}{I_{L}^{0.2}} = 7.16C_{p} \) and then substitute the equation of \( \beta_{N} \) and \( q_{\text{eng}} \) into Eq. (22), after which the bootstrap current fraction could be expressed as

\[ f_{\text{bs}} = \frac{\text{const} A^{0.5} a B_{T0} \beta_{T}}{100 I_{p}} \left( \frac{5B_{T0} a^{2} \kappa}{R_{0} I_{p}} \right) = \frac{\text{const} F}{I_{p}^{2}} \cdot \] (23)

where

\[ F = A^{0.5} \left( \frac{a B_{T0} \beta_{T} \kappa}{R_{0}} \right) \cdot \] (24)

The plasma current and bootstrap current can be deduced by Eqs. (21) and (23), and the solution is

\[ I_{p} = \frac{I_{cd}}{2} + \frac{1}{2} (f_{\text{cd}} + 4\text{const} F)^{0.5} \cdot \] (25)

Hence, the bootstrap current fraction could also be deduced. The Greenwald density limit is expressed as

\[ n_{GW} = \frac{I_{p}}{\pi a^{2}} \cdot \] (25)

**II.E. Radiation**

One of the main difficulties for \( p^{11}B \) fuel is that the bremsstrahlung radiation power loss is even higher than fusion power without synchrotron radiation. In the non-relativistic case, the bremsstrahlung radiation power per unit volume is given by

\[ \frac{dP_{\text{brem}}}{dV} = 0.00534 Z_{\text{eff}} n_{e}^{2} T_{e}^{1/2} \cdot \] (26)

In the \( p^{11}B \) reaction, the relativistic electrons have to be considered since the electron temperature is approaching the electron rest energy. The relativistic electron-ion bremsstrahlung power per unit volume is given by

\[ \frac{dP_{\text{brem}}^{\text{rel}}}{dV} = n_{e}^{2} Z_{\text{eff}} W_{e}^{2} m_{e} c^{3} \left\{ \begin{array}{l} \frac{32}{3} \sqrt{\frac{2}{\pi}} (1 + 1.78 t^{1.34}) , \alpha^{2} \leq t < 1 \\
\frac{12}{t} \ln(2n_{e} t^{2} + 0.42) + \frac{t}{2} , t > 1 \end{array} \right\} \cdot \] (27)

where

\[ r_{e} = 2.818 \times 10^{-15} \text{ m} = \text{classical electron radius} \]

\[ \alpha = 1/137 = \text{fine structure constant} \]
\[ m_e = 9.11 \times 10^{-31} \text{kg} = \text{electron mass} \]
\[ c = 2.998 \times 10^8 \text{m/s} = \text{light speed} \]
\[ e = 1.6 \times 10^{-19} \text{C} = \text{elementary charge} \]
\[ t = \frac{1}{m_e c^2} = \text{normalized electron temperature with respect to the electron rest energy} \]

and where \( \eta_e \approx 0.5616 \) (Refs. 11 and 12).

At this temperature, the bremsstrahlung radiation caused by the electron-electron scattering is comparable to that caused by the electron-ion scattering. And, the relativistic electron-electron bremsstrahlung power per unit volume is \(^{11,12}\)

\[
\frac{dP^{\text{ee}}_{\text{brem}}}{dV} = n_e^2 \alpha_e^2 m_e c^3
\]

\[ \left\{ \begin{array}{ll}
20 \alpha_e^2 \left( 44 - 3\pi^2 \right) \left( 1 + 1.1 t + t^2 - 1.25 t^3 \right), & 0 \leq t < 1 \\
24 t \ln \left( 2 \eta_e t + \frac{3}{2} \right), & t \geq 1
\end{array} \right. \]

(28)

Hence, the total bremsstrahlung power is

\[
P_{\text{brem}} = 2V_p \frac{1}{0} \left( \frac{dP^{\text{ee}}_{\text{brem}}}{dV} + \frac{dP^{\text{ei}}_{\text{brem}}}{dV} \right) dx.
\]

(29)

In addition to bremsstrahlung radiation, synchrotron radiation is one of the important energy loss mechanisms in magnetically confined p-\( ^{11}\)B plasmas, whose critical role has been discussed in the previous work,\(^{13,14}\) showing that the p-\( ^{11}\)B fuel cannot generate net power in a magnetic confinement device with the synchrotron radiation loss. The synchrotron radiation loss could be obtained from

\[
P_{\text{cycl}} = 4.14 \\
\times 10^{-7} n_{\text{eff}}^{0.5} T_{\text{eff}}^{2.5} B_{70}^{0.5} (1 - R_w)^{0.5} a_{\text{eff}}^{0.5} \left( 1 + \frac{2.5 T_{\text{eff}}}{511} \right) V,
\]

(30)

where

\[ n_{\text{eff}} = n_{\text{e0}}/\left(1 + S_n\right) = \text{volume-averaged density} \]
\[ T_{\text{eff}} = \frac{1}{0} T_e(x) dx = \text{effective electron temperature} \]
\[ R_w = \text{wall reflectivity} \]
\[ a_{\text{eff}} = a \kappa^{0.5} = \text{effective minor radius} \]
\[ V = 2\pi R_0 \pi a_{\text{eff}}^2 = \text{approximation of the plasma volume}.\]

In this paper, the cases of considering the effect of synchrotron radiation loss and neglecting synchrotron radiation loss are both discussed.

II.F. Confinement

The confinement time could be obtained by

\[
\tau_E = \frac{(0.024 n_{0,T_{\phi}} + 0.025 n_{e,T_{\phi}}) V_p}{(1 + S_m + S_T)(P_{\text{fus}} + P_{\text{aux}} - P_{\text{brem}})},
\]

(31)

and a confinement enhancement factor \( H \) is used here to compare the confinement time needed for p-\( ^{11}\)B fuel compared with the confinement time predicted by ITER98y2 high-confinement mode (H-mode) scaling\(^7\):

\[
\tau_{98y2} = 0.145 \frac{p_{0.93} R_0^{1.30} a^{0.58} \kappa^{0.78} \eta_e^{0.41} B_{10}^{0.15} M^{0.19}}{(P_{\text{fus}} + P_{\text{aux}})^{0.69}}
\]

(32)

and

\[
H = \frac{\tau_E}{\tau_{98y2}}.
\]

(33)

III. RESULTS

In the D-T, D-\( ^{3}\)He, and D-D fusion reactors, one of the main goals is to study the dependence of the fusion performance on which plasma and device parameters, especially fusion gain \( Q \). But, in the p-\( ^{11}\)B fusion reactor, consideration of many optimistic assumptions and optimum parameters are first needed to achieve \( Q = 1 \), which is described by a simple logic diagram as shown in Fig. 3.

In the p-\( ^{11}\)B fusion reactor, the hydrogen-boron mix needs to be carefully evaluated first to maximize the ratio of fusion power to radiation power. The range of ion temperature and \( n_i \tau_E \) could be confirmed by the break-even condition. In addition, the constraints of beta and thermal wall loading, which are related to the magnetic field limit and ion density limit, respectively, are also important. Combined with the ITER98y2 scaling of energy confinement time, the effect of the \( H \) factor and major radius on fusion performance can be studied.

III.A. Concentration Ratio of Hydrogen to Boron

The ratio of fusion power to radiation power \( \frac{P_{\text{fus}}}{P_{\text{brem}}} \) is a function of the ion temperature, the electron temperature, and the concentration ratio of hydrogen to boron \( f_{\text{h}B} \). To
obtain the optimum value of $f_{pB}$ that maximizes Eq. (34), the ratio of ion to electron temperature $f_T$ has been extracted, and the extremum problem has been simplified to maximizing $F(f_{pB}, T_e)$ by choosing the optimum value of $f_{pB}$:

$$\frac{P_{\text{fus}}}{P_{\text{brem}}} = F(f_{pB}, T_i, T_e) = f_T^2 F(f_{pB}, T_e).$$

A color map of $\frac{P_{\text{fus}}}{P_{\text{brem}}}$ versus electron temperature and $f_{pB}$ is shown in Fig. 4. The optimal value of ratio $f_{pB} = 9:1$, which maximizes $\frac{P_{\text{fus}}}{P_{\text{brem}}}$ with fixed electron temperature. Nevertheless, the maximum value of $\frac{P_{\text{fus}}}{P_{\text{brem}}}$ obtained in this simulation is still below 0.5, which indicates that a higher ratio of ion to electron temperature is needed in order to obtain higher $\frac{P_{\text{fus}}}{P_{\text{brem}}}$.

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**Fig. 3.** The logic diagram of requirements for the $p^{11}$B fusion reactor.

**Fig. 4.** The color map of $\frac{P_{\text{fus}}}{P_{\text{brem}}}$ versus $T_{i0}$ and $f_{pB}$ for the constant $f_T = 1$. To maximize output of fusion power, the optimal value of $\frac{n_p}{n_B}$ is chosen as 9.
III.B. Ratio of Ion to Electron Temperature

One of the main difficulties of p-11B fuel is that the bremsstrahlung radiation power is much higher than the fusion power produced by fuel ions of p-11B, as shown in Fig. 4, with equal ion and electron kinetic temperature:

$$P_{\text{fus}} + P_{\text{aux}} = 2P_{\text{fus}} > P_{\text{brem}} + \frac{W_{\text{dia}}}{\tau_E}.$$  

To study the dependence of the Lawson criterion on $T_i/T_e$, we scan the $T_i/T_e$ ratio, and we plot the minimum value of $n_i\tau_E$ and the corresponding ion temperature to meet fusion gain $Q = 1$ as Fig. 5a shows, which indicates that when $T_i/T_e < 1.12$, there are no positive values of $n_i\tau_E$, and to obtain a fusion gain greater than 1, the ion temperature is at least 1.12 times the electron temperature. In the following of the present paper, the value of $T_i/T_e$ is assumed to be 2.5 in order to get a higher gain fusion while the value of $n_i\tau_E$ could be as low as possible.

III.C. Constraint of Thermal Wall Loading

The minimum value of $n_i\tau_E$ meeting the Lawson criterion of p-11B fusion with $T_i/T_e = 2.5$ is $2.3 \times 10^{21} \text{ m}^{-3}\text{s}$, which is still much higher than the current experiment.

Fig. 5. (a) The minimum value of $\bar{n}_i\tau_E$, which meets the Lawson criterion and the corresponding volume-averaged ion temperature $\bar{T}_i$ versus $T_i/T_e$ in the 0-D model and in the 1-D model. When $T_i/T_e < 1.12$, no positive solution that meets the Lawson criterion with $Q = 1$ can be found. (b) The minimum value of $n_{io}\tau_E$, which meets the Lawson criterion and the corresponding core ion temperature $T_{i,0}$ versus $T_i/T_e$ in the 1-D model.
parameters. To make a further study on the parameters of p-\(^{11}\)B fusion devices, a higher confinement enhancement factor \(H\) not limited by the ITER98y2 energy scaling is assumed.

The p-\(^{11}\)B fusion reactor may not be limited by neutron irradiation, but thermal loading due to the significantly high bremsstrahlung radiation power at the ion temperature of 300 to 500 keV would restrict the regime of ion density. In this paper, we take the maximum value of the thermal wall loading caused by radiation as 10 MW/m\(^2\), which is the maximum tolerable steady-state perpendicular power flux density onto the ITER divertor plate. This wall-loading limit might further increase within the near-term development of materials technology.

If we assume all radiation power must be brought out through the first wall, we have the constraint of thermal wall loading:

\[
\phi_W = \frac{P_{\text{brem}}}{S_W} \leq 10 ,
\]

where \(\phi_W\) is the wall-loading limit. The thermal wall loading is a function of the ion temperature, ion density, and major radius, and for a given ion temperature, a larger major radius would restrict the increase of density.

In Fig. 6, the magenta and blue lines are the bremsstrahlung radiation power curves corresponding to the wall-loading limit with \(R_0 = 2\) m and \(R_0 = 3\) m, which is also a limit of ion density for a given ion temperature. The parabola describes the value of \(n_i\tau_E\) versus ion temperature satisfying fusion gain \(Q = 1\) while \(T_i/T_e = 2.5\), where the minimum value of \(n_i\tau_E = 2.3 \times 10^{21}\) m\(^{-3}\)s can be found at \(T_{i0} = 380\) keV. Since the fusion power density is directly related to the economic benefits, one approach to achieving maximum economic benefits is to operate at marginal ion density, which is shown by the circles in Fig. 6.

III.D. Constraint of Beta

A strong magnetic field is required to confine p-\(^{11}\)B plasma with extremely high pressure, which might exceed the engineering constraints greatly. In this paper, the on-axis magnetic field is designed below 20 T considering the foreseeable development of technology. The normalized beta \(\beta_N\), which is related to economic benefits and plasma stability, is designed to be the maximum allowed for stability to confine the high-pressure p-\(^{11}\)B plasma.

To study the dependence of the magnetic field on \(\beta_N\), we combine Eqs. (13) and (14); substitute the value \(T_i/T_e = 2.5\), \(n_e/n_i = 1.4\) derived before into it; and obtain the relation of the plasma current, magnetic field, and \(\beta_N\):

\[
\frac{2.08n_{i0}T_{i0}a}{\beta_N} = I_pB_{T0} .
\]

Recalling the safety factor limit \(q_{\text{eng}} > 2\), we have

\[
B_{T0}^2 > \frac{4.16n_{i0}T_{i0}R_0}{5a\alpha\phi_N} .
\]

Fig. 6. The left \(y\)-axis shows the ion density versus ion temperature for the bremsstrahlung radiation power density limit, and the right \(y\)-axis shows \(n_i\tau_E\) versus ion temperature meeting the Lawson criterion with \(Q = 1\). The magenta and blue lines are the wall-loading limit by radiation when \(R = 2\) m and \(R = 3\) m, and the circles intersected by the optimum temperature of \(T_{i0} = 380\) keV and the curve of the bremsstrahlung radiation power limit indicate the marginal ion density.
To minimize the magnetic field at fixed fusion power density and tokamak geometry, the normalized beta has been set to be the maximum $\beta_N \approx \frac{2}{3}$ (Refs. 4 and 5). In Table I, $n_\| = 6 \times 10^{20} \text{ m}^{-3}$, $T_\| = 380 \text{ keV}$, and $R_0 = 3 \text{ m}$ are assumed, and the minimum value of the magnetic field needed in p-\textsuperscript{11}B fuel is 10.7 T for a low-aspect-ratio tokamak with $A = 1.4$ while it turns out to be 23.2 T for a conventional tokamak with $A = 2.4$.

### III.E. Size Effects and $H$ Effects on Fusion Gain

As discussed above, the optimal values of $n_p/n_b = \frac{2}{7}$, $T_i/T_e = 2.5$, and $T_\| = 380 \text{ keV}$ have been found, and two constraint conditions $\Phi_w = 10 \text{ MWm}^{-2}$ and $\beta_N = 6.4$ have been confirmed.

To study the dependence of fusion gain on $H$ and major radius $R$ under the input and constraint condition, we plot the fusion gain $Q$ versus major radius $R$ at four different confinement cases, $H = 1.5$, $H = 3$, $H = 5$, and $H = 10$, while ion temperature, normalized beta, and wall loading are fixed. Figure 7 indicates that the fusion gain is strongly dependent on major radius and $H$, both of which have a positive effect on $Q$ by increasing the energy confinement time.

The confinement enhancement factor that can be achieved by the current experiments is $H = 1.5$, and the minimum value of the major radius that could meet $Q = 1$ is 5 m. In p-\textsuperscript{11}B reactions, the core ion temperature could be higher than 300 keV, in which the neoclassical transport decreases significantly. Once a new method has been found to effectively suppress the anomalous transport, confinement enhancement factors of $H = 3$, 5, and 10 might be possible in the future. In any case, $H = 3$ is an overly optimistic but still foreseeable confinement enhancement factor and could reach $Q = 1.5$ at $R_0 = 3 \text{ m}$ as shown in Fig. 7.

### III.F. Impurity Effect

In the preceding discussion, helium concentration has been ignored. In an actual reactor, helium as a producer of the p-\textsuperscript{11}B reaction would decrease the fusion power output by diluting the fuel and increasing the

| $A$   | 1.4 | 1.9 | 2.4 |
|-------|-----|-----|-----|
| $B_{0,\min}$ (T) | 10.7 | 17.3 | 23.2 |
| $I_p$ (MA) | 147 | 91.2 | 68.2 |

![Fig. 7. Fusion gain $Q$ versus major radius $R$ at four different confinement cases, $H = 1.5$, $H = 3$, $H = 5$, and $H = 10$.](image-url)
bremsstrahlung radiation power, which would result in the termination of the fusion reaction eventually.

The number of helium ions could be obtained from the continuity equation:

$$\frac{N_{\text{He}}}{\tau_{\text{He}}} = \dot{N}_{\text{He}},$$

where

- $N_{\text{He}} = \text{number of helium ions}$
- $\dot{N}_{\text{He}} = \text{helium ion generation speed}$
- $\tau_{\text{He}} = \text{helium confinement time}$.

The helium ion generation speed is obtained by the fusion power:

$$\dot{N}_{\text{He}}(10^{20}/s) = \frac{3P_{\text{fus}}}{8.7 \times 16}.$$ 

And, the core helium ion density is

$$n_{\text{He0}} = \dot{N}_{\text{He}}(1 + S_n)\tau_{\text{He}} / V_P.$$ 

If we assume a breakeven case, $n_{i0} = 6 \times 10^{20} \text{ m}^{-3}$, $T_i = 380 \text{ keV}$, $P_{\text{fus}} = 5400 \text{ MW} \cdot \text{m}^{-3}$, $\tau_e = 5 \text{ s}$, and $\tau_{\text{He}} = 10\tau_e \approx 50 \text{ s}$, we could get $n_{\text{He0}} = 9.5 \times 10^{20} \text{ m}^{-3}$, which is even higher than the fuel ion density. Since the helium ash is poisonous to the fusion performance, excessive density of helium ions should be prevented.

In Fig. 8, we plot $Q$ versus the ratio of helium density to ion density at three different confinement cases, $H = 3$, $H = 5$, and $H = 10$, while ion temperature, normalized beta, and wall loading are set constant. The dependence of fusion gain on $f_{\text{He}}$ is illustrated in Fig. 8, which indicates that $f_{\text{He}}$ does play an important role in fusion gain. At the $H = 3$ case, the highest ratio of $f_{\text{He}}$ that the device could tolerate is 5%, which means that the divertor ash removal efficiency should be high enough to keep the helium confinement time below the energy confinement time.

### III.G. Effect of Synchrotron Radiation Loss

In the previous discussion, all these optimistic predictions have been done without considering the synchrotron radiation loss, whose critical role effect can hardly be ignored in the relatively high strong magnetic field up to 10 T. As one of the most important energy loss mechanisms, its effect has to be taken into consideration in the actual reactor, and in this section, we will discuss the effect of synchrotron radiation loss on $p^{11}\text{B}$ fuel.

Equation (30) gives the volume-integrated synchrotron radiation power loss, which is determined by the averaged density, toroidal magnetic field on axis, effective electron temperature, wall reflectivity, effective minor radius, and
approximated plasma volume. The range of parameters such as the electron temperature, electron density, toroidal magnetic field, and device size in the p-\(^{11}\)B tokamak have been already discussed in Secs. III.B through III.E; hence, one possible way to reduce the synchrotron power loss density is to increase the wall reflectivity high enough to reflect most of the radiation back and get reabsorbed in the core plasma.

To give better insight into the effect of the synchrotron radiation loss on p-\(^{11}\)B fusion, the synchrotron radiation loss has been considered in the model predictions while other assumptions and constraints are still the same, and the helium concentration is set as 5%. The change of the fusion gain and ratio of synchrotron radiation to bremsstrahlung radiation power loss in three different high-confinement cases, \(H = 5\), \(H = 10\), and \(H = 20\), with wall reflectivity varied from 0.5 to 0.99 is shown in Fig. 9.

In Fig. 9, the fusion gain of the three high-confinement cases is even less than 0.5 with a low wall reflectivity of 0.5. When the wall reflectivity increases to 90%, the fusion gain of the three high-confinement cases cannot meet the breakeven condition, and the synchrotron radiation power loss is slightly higher than the bremsstrahlung power loss. When the wall reflectivity is greater than 0.96, the breakeven condition can be obtained in the high-confinement case of \(H = 20\). However, in this case the wall reflectivity and confinement enhancement factor are both unrealistically high for the existing technology.

The results reveal that the fusion gain of p-\(^{11}\)B fuel is strongly affected by synchrotron radiation loss, and one of the biggest challenges to the p-\(^{11}\)B fusion reactor is the reduction of synchrotron radiation loss.

### III.H. Parameters of Future Device Design

In the actual reaction, helium, which is poisonous to fusion performance, and synchrotron radiation loss, which greatly decreases the fusion gain, should be taken into consideration. Based on the calculations of the system code, five representative sets of the p-\(^{11}\)B tokamak device parameters that set the helium concentration to 5% are given in Table II. Among these sets, the wall reflectivity is assumed as 1 in sets A through D, and in set E the wall reflectivity is assumed as 0.95 as a comparison. To keep the helium concentration ratio in an acceptable range, the helium confinement time is assumed to be less than the energy confinement time, which also could be seen in Table II.

A normal-aspect-ratio tokamak with \(A = 2.5\) in set A could be found, in which a high magnetic field of 36.8 T is needed, indicating a low-aspect-ratio tokamak would be a better choice for p-\(^{11}\)B tokamak with extremely high pressure. In addition, the plasma-stored energy is much higher in the low-aspect-ratio tokamak from sets B, C, and D than that in set A with the same size of major radius.

![Fig. 9. Fusion gain (left y-axis) and the ratio of synchrotron radiation to bremsstrahlung radiation power loss (right y-axis) versus wall reflectivity at three different confinement cases, \(H = 5\), \(H = 10\), and \(H = 20\).](image-url)
Three low-aspect-ratio tokamaks with $A = 1.4$ from sets B, C, and D are shown in Table II. Compared with sets B and C, when one increases the ratio $f_T$ from 2 to 2.5, the requirement for the confinement condition is reduced to meet the breakeven condition. When one considers a hypothetical confinement enhancement factor $H = 10$ in set D, fusion gain $Q = 4.14$ could be obtained, which indicates that a more optimistic confinement condition would bring a greater economic benefit. In set E, the effect of synchrotron radiation loss is considered by changing the wall reflectivity from 1 to 0.95, and the fusion gain is decreased from 4.14 to 0.84, which shows that the synchrotron radiation loss would greatly decrease the fusion gain.

| IV. CONCLUSION AND DISCUSSION |

Because of the low reactivity of the p-$^{11}$B reaction, meeting the Lawson criterion is difficult. However, if we choose the optimum proton-boron mix and ion temperature and assume the ratio of the ion to electron temperature to be 2.5, the fusion power produced by the fuel ions of p-$^{11}$B can be comparable to heating power meeting the Lawson criterion with $Q = 1$.

The helium is poisonous to the fusion performance by diluting the fuel and increasing bremsstrahlung radiation loss, and excessive density of helium ions would result in the termination of the fusion reaction. In order to keep the helium concentration ratio in an acceptable

| TABLE II |

Five Typical Sets of p-$^{11}$B Tokamak Device Parameter Based on TSC

| Parameter | Set A | Set B | Set C | Set D | Set E |
|-----------|-------|-------|-------|-------|-------|
| $A$       | 2.5   | 1.4   | 1.4   | 1.4   | 1.4   |
| $f_T$ ($T/T_o$) | 2.5   | 2     | 2.5   | 2.5   | 2.5   |
| $H$       | 3     | 5     | 3     | 10    | 10    |
| $f_{ie}$  | 0.05  | 0.05  | 0.05  | 0.05  | 0.05  |
| $\eta_w$  | 1     | 1     | 1     | 1     | 0.95  |
| $R_0$ (m) | 3     | 3     | 3     | 3     | 3     |
| $A$ (m)   | 1.2   | 2.14  | 2.14  | 2.14  | 2.14  |
| $V_p$ ($m^3$) | 185 | 919 | 919 | 919 | 919 |
| $S_v$ ($m^3$) | 233.3 | 548 | 548 | 548 | 548 |
| $\kappa$  | 2.24  | 3.57  | 3.57  | 3.57  | 3.57  |
| $\delta$  | 0.5   | 0.5   | 0.5   | 0.5   | 0.5   |
| $Q$       | 0.8   | 1.17  | 1.06  | 4.14  | 0.84  |
| $P_{jux}$ (MW) | 2311 | 4424 | 5427 | 5427 | 5427 |
| $P_{aux}$ (MW) | 2878 | 3771 | 5316 | 1311 | 6426 |
| $P_{cd}$ (MW) | 576  | 754  | 1063 | 262  | 1292 |
| $P_{brem}$ (MW) | 2333 | 5480 | 5480 | 5480 | 5480 |
| $P_{cycl}$ (MW) | 0    | 0    | 0    | 0    | 4762 |
| $\phi_w$ (MW/m$^2$) | 10   | 10   | 10   | 10   | 10   |
| $B_{T0}$ (T) | 36.8 | 11.3 | 11.4 | 12.9 | 11    |
| $I_p$ (MA)  | 63   | 140  | 140  | 124  | 145  |
| $W_{dia}$ (MJ) | 7685 | 25864 | 26256 | 26256 | 26256 |
| $f_{ls}$   | 0.77  | 0.72  | 0.72  | 0.92  | 0.67  |
| $Z_{eff}$  | 2.4   | 2.4   | 2.4   | 2.4   | 2.4   |
| $T_{e0}$ (keV) | 152  | 190  | 152  | 152  | 152  |
| $T_{i0}$ (keV) | 380  | 380  | 380  | 380  | 380  |
| $n_{e0}$ ($10^{20}$ m$^{-3}$) | 12.4 | 7.72 | 8.55 | 8.55 | 8.55 |
| $n_{i0}$ ($10^{20}$ m$^{-3}$) | 8.70 | 5.40 | 5.98 | 5.98 | 5.98 |
| $\tau_E$ (s) | 2.69 | 9.52 | 4.88 | 20.8 | 15.9 |
| $\tau_{He}$ (s) | 1.08 | 1.73 | 1.57 | 1.57 | 1.57 |
| $\beta_T$ (%) | 5.12 | 36.7 | 36.8 | 28.8 | 39.5 |
| $\beta_p$  | 1.90  | 1.9   | 1.9   | 2.4   | 1.8   |
| $\beta_N$  | 3.6   | 6.4   | 6.4   | 6.4   | 6.4   |
| $q_{eng}$  | 3.14  | 2.22  | 2.2   | 2.8   | 2.06  |
range, the technique of active ash removal needs to be developed in order to reduce the helium confinement time less than the energy confinement time.

Based on the calculation of the TSC, the fusion gain increases with the increasing major radius and confinement enhancement factor at a fixed wall-loading limit and beta limit. A major radius of 5 m is needed to obtain fusion gain $Q = 1$ with the current confinement condition of $H = 1.5$ without helium concentration. After considering the helium concentration, if one can achieve a more optimistic confinement enhancement factor of $H = 10$, the fusion gain of $Q = 4.14$ at $R_0 = 3$ m could be found, which is of economic benefit.

Although the ion temperature of $>300$ keV and energy confinement time of 10 s required for the $p^{11}$B reaction are not achievable by existing technologies, the main point of this paper is to study the requirements needed for the $p^{11}$B fusion reactor—not to give engineering solutions to the requirements. In this point, this paper could achieve its aim if it calls colleagues’ attention to $p^{11}$B fusion.

Finally, we should bear in mind that all these optimistic assumptions have been done by neglecting the synchrotron radiation loss, which is relatively high in the strong magnetic field. If we consider synchrotron radiation loss and assume a high wall reflectivity of 95% in the calculations of the case $H = 10$, the fusion gain would decrease from 4.14 to 0.84. The results shows the $p^{11}$B fusion reactor will not come true unless some techniques are found in the future to avoid excessive synchrotron radiation loss.

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No potential conflict of interest was reported by the author(s).

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References

1. R. REID et al., “Tokamak Systems Code,” Oak Ridge National Laboratory (1985).
2. W. BARR et al., “ETR/ITER Systems Code,” Oak Ridge National Laboratory (1988).
3. R. D. STAMBAUGH et al., “The Spherical Tokamak Path to Fusion Power,” Fusion Technol., 33, 1 (1998); https://doi.org/10.13182/FST33-1.
4. R. D. STAMBAUGH et al., “Fusion Nuclear Science Facility Candidates,” Fusion Sci. Technol., 59, 279 (2011); https://doi.org/10.13182/FST59-279.
5. A. E. COSTLEY, J. HUGILL, and P. F. BUXTON, “On the Power and Size of Tokamak Fusion Pilot Plants and Reactors,” Nucl. Fusion, 55, 033001 (2015); https://doi.org/10.1088/0029-5515/55/3/033001.
6. C. C. PETTY et al., “Feasibility Study of a Compact Ignition Tokamak Based upon Gyrobohm Scaling Physics,” Fusion Sci. Technol., 43, 1 (2003); https://doi.org/10.13182/FST03-A245.
7. W. M. NEVINS, “A Review of Confinement Requirements for Advanced Fuels,” J. Fusion Energy, 17, 25 (1998); https://doi.org/10.1023/A:1022513215080.
8. M. SIKORA and H. WELLER, “A New Evaluation of the $^{11}$B(p,$\alpha$)$\alpha$ Reaction Rates,” J. Fusion Energy, 35, 538 (2016); https://doi.org/10.1007/s10894-016-0069-y.
9. T. LUCE et al., “Generation of Localized Noninductive Current by Electron Cyclotron Waves on the DIII-D Tokamak,” Phys. Rev. Lett., 83, 4550 (1999); https://doi.org/10.1103/PhysRevLett.83.4550.
10. M. ANDRADE and G. LUDWIG, “Scaling of Bootstrap Current on Equilibrium and Plasma Profile Parameters in Tokamak Plasmas,” Plasma Phys. Contr. Fusion, 50, 065001 (2008); https://doi.org/10.1088/0741-3335/50/6/065001.
11. R. SVENSSON, “Electron-Positron Pair Equilibria in Relativistic Plasmas,” Astrophys. J., 258, 335 (1982); https://doi.org/10.1086/160082.
12. S. PUTVINSKI, D. RYUTOV, and P. YUSHMANOV, “Fusion Reactivity of the pB11 Plasma Revisited,” Nucl. Fusion, 59, 076018 (2019); https://doi.org/10.1088/1741-4326/ab1a60.
13. D. C. MOREAU, “Potentiality of the Proton-Boron Fuel for Controlled Thermonuclear Fusion,” Nucl. Fusion, 17, 13 (1977); https://doi.org/10.1088/0029-5515/17/1/002.
14. A. KUKUSHKIN and V. KOGAN, “Relativistic Boron-Hydrogen Plasma as a Fusion Fuel,” Soviet J. Plasma Phys., 5, 708 (1979).
15. A. KUKUSHKIN and P. MINASHIN, “Generalization of Trubnikov Formula for Electron Cyclotron Total Power Loss in Tokamak-Reactors,” *Proc. XXXVI Int. Conf. Plasma Physics and Controlled Fusion*, Zvenigorod, Russia, 2009.

16. A. KUKUSHKIN, P. MINASHIN, and V. NEVEROV, “Electron Cyclotron Power Losses in Fusion Reactor-Grade Tokamaks: Scaling Laws for Spatial Profile and Total Power Loss,” *Proc. 22nd International Atomic Energy Agency Fusion Energy Conf.*, Geneva, Switzerland, 2008, Citeseer (2008).

17. ITER PHYSICS EXPERT GROUP ON CONFINEMENT AND TRANSPORT et al., “Plasma Confinement and Transport,” *Nucl. Fusion*, 39, 2175 (1999); https://doi.org/10.1088/0029-5515/39/12/302.

18. R. PITTS et al., “Status and Physics Basis of the ITER Divertor,” *Phys. Scr.*, 2009, 014001 (2009); https://doi.org/10.1088/0031-8949/2009/T138/014001.