We study electromagnetic mass splittings of charmed baryons. We point out discrepancies among theoretical predictions in non-relativistic potential models; none of these predictions seems supported by experimental data. A new calculation is presented.

Quite a successful phenomenology has been obtained using non-relativistic potential models built in order to describe the low-energy limit of QCD. Among the various observables studied in such models, a large amount of work was devoted to the electromagnetic mass differences [1]. In general a good agreement was obtained for nucleon, Σ and Ξ baryons and predictions for charmed baryons were supplied (see table 1). Albeit the predictions for charmed baryons were quite dispersed, some general feature was anyway common to most of the models, as $\Sigma^+_c - \Sigma^0_c \lesssim \Sigma^{++}_c - \Sigma^0_c$.

Surprisingly enough, when data about charmed baryons finally appear, none of these models came out in agreement with them. Because of the poor amount of experimental data and large errors, we cannot really exclude that future experimental determination could change the situation. However, aimed by this failure, we have decided to investigate isospin-violating mass differences of baryons in a successful potential model, including all possible contributions. We wish to understand whether the failure of potential models arises from technical problems (as having neglected some contributions, or having used perturbative procedures improperly, etc.) or reveals some intrinsic limitation of this approach.

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In the following we will use the potential model AL1 [2] supplemented with electric and dipole–dipole magnetic interactions. The difference of masses between $u$ and $d$ quarks has been fixed to reproduce the $n – p$ and $\Sigma^- – \Sigma^+$ mass splittings.

Our result is that, while $\Sigma^- – \Sigma^0$, $\Xi^- – \Xi^0$ and $\Delta^0 – \Delta^+$ and even (within large errors) the splitting of excited states $\Sigma(1385)$ and $\Xi(1530)$ come out in good agreement with the experimental data [3], some problems appear for charmed baryons (see table 2). In fact, while the experimental datum $\Sigma^{++} c – \Sigma^0 c = 0.8 \pm 0.4$ MeV is well reproduced, one finds a negative $\Sigma^+_c – \Sigma^0_c$, at variance with the experimental datum $1.4 \pm 0.6$ MeV. The result $\Xi^+_c – \Xi^0_c = 2.2$ MeV is smaller than the PDG average $6.3 \pm 2.3$ [3], but agrees rather well with a new determination at CLEO, $2.5 \pm 1.7 \pm 1.1$ [4]. Reasonable changes of light quark masses do not modify substantially this situation.

The problem with the $\Sigma_c$ multiplet raises the question whether some contribution has been neglected. For example some models, though reproducing quite well the excitation energies, fail in providing good absolute mass predictions unless an empirical three-body interaction is introduced. Its form is rather arbitrary and is chosen only for the sake of simplicity as $D_3 + A_3(m_1m_2m_3)^{-\alpha}$. As it depends on masses, this term gives a contribution to electromagnetic mass splittings as well. However, how is evident by inspecting this term, the contribution to $\Sigma^+_c – \Sigma^0_c$ goes in the wrong direction for solving the splitting problem. In fact, our numerical study shows that one obtains reasonable electromagnetic mass splittings for light baryons, but the situation for charmed baryons slightly deproves with $\Sigma^+_c – \Sigma^0_c \simeq – 0.7$ MeV, and $\Xi^+_c – \Xi^0_c \simeq 2$ MeV. One could think of more complicated three-body interactions, but their form remain completely arbitrary and somehow the need of introducing complicate multi-body interactions would cast doubts on the applicability of potential models.

One can consider also the running of $\alpha_s$, which is smaller when heavy quarks appear, for the scale is proportional to the masses involved. Such an effect would decrease the coupling of the spin–spin term and does not go in the right direction for changing the order within the $\Sigma_c$ multiplet.

We also ask ourselves whether instantonic interactions could improve the situation. The
non-relativistic form of this contribution has be evaluated in [3] (the value of the coupling must be fixed phenomenologically). However the effect of this interaction is inversely proportional to the quark masses and vanishes for a quark pair with spin 1, thus will not contribute substantially to $\Sigma_c$ mass splittings; it gives, however, a positive contribution to $\Xi_c^+ - \Xi_c^0$.

In conclusion[4] we find that, albeit a good agreement with light quark baryons, it is practically impossible to explain the splitting pattern of charmed baryons in potential models based on one–gluon exchange. More precise data are however required before drawing firm conclusions about the relevance of such models to describe the confining region of QCD.

References

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\[^2\] We will not discuss potentials related to meson exchange, for which an extension to charmed baryons is problematic.
Table 1. Predictions of different models for charmed baryons electromagnetic mass splittings

| Model                  | $\Sigma^{++} - \Sigma^{0}$ | $\Sigma^{+} - \Sigma^{0}$ | $\Xi^{0} - \Xi^{+}$ |
|------------------------|-----------------------------|-----------------------------|----------------------|
| Itoh                   | 6.5                         | 2.4                         | 2.51                 |
| Ono                    | 6.1                         | 2.24                        | 1.77                 |
| Lane and Weinberg      | -6                          | -4                          | 4                    |
| Chan                   | 0.4                         | -0.7                        | 3.2                  |
| Lichtenberg            | 3.4                         | 0.8                         | 1.1                  |
| Kalman and Jakimow     | -2.7                        | -2.24                       | 3.6                  |
| Isgur                  | -2                          | -1.8                        |                      |
| Richard and Taxil I    | 3                           | 1                           | 0                    |
| II                     | -2                          | -2                          | 2                    |

Table 2. Our predictions of electromagnetic mass splittings (in MeV, upper row) with $m_u = 327$ MeV and $m_d = 338$ MeV compared with experimental data (lower row).

| $n - p$ | $\Sigma^{-} - \Sigma^{0}$ | $\Sigma^{-} - \Sigma^{+}$ | $\Xi^{-} - \Xi^{0}$ | $\Delta^{0} - \Delta^{++}$ | $\Delta^{+} - \Delta^{++}$ |
|---------|-----------------------------|-----------------------------|----------------------|-----------------------------|-----------------------------|
| 1.24    | 5.24                        | 8.67                        | 7.46                 | 2.54                        | 0.36                        |
| 1.293318 ± 0.000009 | 4.88 ± 0.08                 | 8.09 ± 0.16                 | 6.4 ± 0.6             | 2.7 ± 0.3                   |

| $\Delta^{-} - \Delta^{++}$ | $\Sigma^{0} - \Sigma^{*+}$ | $\Sigma^{*-} - \Sigma^{0}$ | $\Xi^{*-} - \Xi^{*0}$ |
|-----------------------------|-----------------------------|-----------------------------|-----------------------|
| 6.55                        | 1.9                         | 3.8                         | 3.6                   |
| -4 to 4                     | 2.0 ± 2.4                   | 3.2 ± 0.6                   |

| $\Sigma^{++} - \Sigma^{0}$ | $\Sigma^{+} - \Sigma^{0}$ | $\Xi^{0} - \Xi^{+}$ | $\Sigma^{+}_{b} - \Sigma^{-}_{b}$ | $\Sigma^{0}_{b} - \Sigma^{-}_{b}$ | $\Xi^{-}_{b} - \Xi^{0}_{b}$ |
|-----------------------------|-----------------------------|----------------------|-------------------------------------|-----------------------------------|-----------------------------|
| 1.20                        | -0.36                       | 2.83                 | -3.58                               | -1.94                             | -5.39                       |
| 0.8 ±0.4                    | 1.4 ± 0.6                   | 6.3 ± 2.3            |

The $u - d$ mass difference is somehow larger than the common wisdom current quarks one, however the constituent quarks $u - d$ mass difference can be in principle different from the former.