Modeling the stock price returns volatility using GARCH(1,1) in some Indonesia stock prices

S A Awalludin¹, S Ulfah¹ and S Soro¹

¹Mathematics Education, Universitas Muhammadiyah Prof. DR. HAMKA, Jakarta Indonesia

E-mail: subhanajiz@uhamka.ac.id

Abstract. In the financial field, volatility is one of the key variables to make an appropriate decision. Moreover, modeling volatility is needed in derivative pricing, risk management, and portfolio management. For this reason, this study presented a widely used volatility model so-called GARCH(1,1) for estimating the volatility of daily returns of stock prices of Indonesia from July 2007 to September 2015. The returns can be obtained from stock price by differencing log of the price from one day to the next. Parameters of the model were estimated by Maximum Likelihood Estimation. After obtaining the volatility, natural cubic spline was employed to study the behaviour of the volatility over the period. The result shows that GARCH(1,1) indicate evidence of volatility clustering in the returns of some Indonesia stock prices.

1. Introduction

In the financial field, volatility is one of the key variables to make an appropriate decision. According to [6], the volatility can be defined as a degree of fluctuation in asset price which can be going up or down. In fact, the volatility has taken place in different areas in financial theory and practice, such as risk management, portfolio selection and derivative pricing [2]. In many cases, the volatility is shown by low fluctuation in some period, then following by high fluctuation and vice versa. It indicates that volatility is not constant over time. Estimating the volatility as accurate as possible is needed since return can be obtained from volatility and price can be computed based on the return. We can employ time series model to capture the volatility of returns asset.

The time series model that will be used must agree with heteroscedasticity property. Heteroscedasticity describes the volatility changes over the time horizon. One of the heteroscedasticity models is Generalized Autoregressive Conditional Heteroscedasticity (GARCH) which was proposed by Bollerslev [3]. Estimating volatility using the GARCH has been frequently studied by many researchers. Kamau et al.[6] used GARCH(1,1) to estimate the volatility of stock return in Kenyan stock markets. Their finding is that the returns stylized facts including volatility clustering, non-normal distribution and mean. Volatility clustering is the situation that high fluctuations in the returns of an asset are often followed by other high fluctuations, likewise low fluctuations are followed by other low fluctuations.

Their finding is similar to a study by Namugaya et al.[8] which showed that Uganda Securities Exchange (USE) returns have a non-normal distribution, positively skewed and stationary. In fact, those returns attributes usually appear in financial time series data. It is well known that the volatility...
series give important information about the data. Thus, we need to investigate the volatility behavior through the period.

In order to simplify investigation of the volatility behaviour, we need to smooth the volatility series. Numerical method can be employed to do the job. This leads to natural cubic spline function which is a widely used technique for piecewise smooth curve fitting. This function is simply piecewise cubic polynomial which can be constructed so that the connections between adjacent cubic splines are visually smooth [4]. Further, we note that volatility signal is obtained due to the natural cubic spline fitting of the volatility series.

The rest of the paper is organized as follows: (1) theory and methods; (2) results and discussion and (3) conclusions.

2. Methodology

This section describes mathematical and statistical methods which were used for analyzing of the volatility of stock returns in this study. These methods comprise of obtaining the returns from stock price data, estimating volatility series using GARCH(1,1), using Maximum Likelihood Method to Estimate Parameters of GARCH(1,1) and fitting volatility series using natural cubic spline function. The details will be explained as follows.

2.1. Obtaining Returns from Stock Price

We involve data from daily closing prices of the seven companies of Indonesia from July 2007 to September 2015. We can obtain returns series from stock prices data by differencing log of the price from one day to the next. Returns can be defined as the continuously compounded return during day \( t \) (between the end of day \( t - 1 \) and the end of day \( t \) )\[5\], as:

\[
R_t = \ln \frac{S_t}{S_{t-1}},
\]

where \( S_t \) is the price at day \( t \). Commonly, continuously compounded return, \( R_t \), is called log return. After obtaining the returns, the information of return fluctuation (volatility series) over time can be estimated by fitting GARCH(1,1) to the returns.

2.2. Estimating Volatility Series Using GARCH(1,1)

Financial data contain non-constant variance over time. It is well known as heteroscedasticity. Capturing heteroscedasticity can be done by GARCH model. We involve the definition of the general process of GARCH which is GARCH \((p,q)\).

Definition 1.

Let \((w_t)_{t \geq 0}\) be a sequence of independent and identically distributed (i.i.d) random variables such that \(w_t \sim N(0,1)\). The \(R_t\) is called the generalized autoregressive conditionally heteroscedasticity or GARCH \((p,q)\) process \[9\] if

\[
R_t = \sigma_t w_t, \quad t \in \mathbb{N},
\]

where \(\sigma_t\) is a nonnegative process such that,

\[
\sigma_t^2 = \gamma V_L + \alpha_1 R_{t-1}^2 + \ldots + \alpha_q R_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \ldots + \beta_p \sigma_{t-p}^2, \quad t \in \mathbb{N},
\]

and

\[
\gamma > 0, \quad \alpha_i \geq 0 \quad i = 1, \ldots, q, \quad \beta_i \geq 0 \quad i = 1, \ldots, p,
\]
where integers $p$ and $q$ are orders of $\sigma_i^2$ and $R_i^2$, respectively. In particular, GARCH(1,1) is the simplest and frequently useful model [2] which is given by

$$\sigma_i^2 = \gamma V_L + \alpha u_{i-1}^2 + \beta \sigma_{i-1}^2,$$

where $\gamma$, $\alpha$ and $\beta$ are the weight assigned to long-run average variance rate $V_L$, returns squared $R_{i-1}^2$, and variance $\sigma_{i-1}^2$, respectively. The weights $\gamma$, $\alpha$ and $\beta$ must sum to unity, that is

$$\gamma + \alpha + \beta = 1.$$

Now, we set $\omega = \gamma V_L$, the GARCH(1,1) model can also be written

$$\sigma_i^2 = \omega + \alpha u_{i-1}^2 + \beta \sigma_{i-1}^2,$$  \hspace{1cm} (1)

where $\omega > 0$, $\alpha \geq 0$ and $\beta \geq 0$. In order to guarantee the variance to be positive, we set $\alpha + \beta < 1$. The formula (1) is often used for the purpose of estimating the volatility. After that, we estimate the parameters $\alpha$ and $\beta$ by maximum likelihood method.

2.3. Using Maximum Likelihood Method to Estimate Parameters of GARCH(1,1)

The method gives values of the parameters that maximize the likelihood function of the variable of interest [5]. Now, we have the transformed returns $R_t$ which are approximately normal with mean zero and variance $\sigma_t^2$ as required in definition 1. Initially, we determine the probability density function of $R_t$, $t = 1, 2, 3, ..., n$. Since for each $t$ we have

$$f_{r_t} = \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left(-\frac{r_t^2}{2\sigma_t^2}\right),$$

then the likelihood function $L_{r_t} = f_{r_1}f_{r_2}...f_{r_n}$. For each $t$, $R_t$ is independence so that

$$L_{r_t} = \prod_{i=1}^{n} f_{r_t}$$

$$= \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left(-\frac{r_t^2}{2\sigma_t^2}\right)$$

by monotonicity of logarithm function, maximizing likelihood function can be done by maximizing its logarithm [8]. Therefore, we now can maximize (2) by taking natural logarithm. Then we have,
Ignoring constant multiplicative factors of \( l_{r_t} \) gives

\[
\hat{l}_{r_t} = \sum_{i=1}^{n} \left[ -\ln \sigma_i^2 - \frac{r_t^2}{\sigma_i^2} \right], \tag{3}
\]

where \( r_t \) and \( \sigma_i^2 \) are the returns and the variance at day \( t \), respectively. The parameters that maximize \( l(r_t) \), also maximize \( \hat{l}(r_t) \). Furthermore, we solve formula (3) numerically by damped Newton’s method. In summary, fitting the GARCH (1,1) volatility series of the stock returns. In order to simplify investigation of the volatility in many situations, we need to smooth the volatility series. The volatility series will be smoothed by the natural cubic spline.

2.4. Fitting Volatility Series Using Natural Cubic Spline Function

According to the preceding section, estimating volatility series using the GARCH(1,1), the model gives daily volatility series over the period. In order to study the behavior of volatility, we employ the natural cubic spline to fit volatility series obtaining from GARCH(1,1). It is because the natural cubic spline has such attractive properties as smoothness, continuity of the first and second derivative so that many financial institutions use the method for curve fitting \([1]\). Therefore, we can get the information on the rate of change and cumulative change of volatility series over the period.

Let \((t_1, y_1), (t_2, y_2), \ldots, (t_n, y_n)\) where \( t_1 < t_2 < \ldots < t_n \) and \( s(t) \) be a series of knot points and cubic spline function which fits consecutive knot points, respectively. We employed a natural cubic spline which easily to apply in the data. It was improved by McNeil \textit{et al.} \([7]\). The cubic spline function is defined as:

\[
s(t) = a + bt + \sum_{k=1}^{p} c_k (t - t_k)_+^3, \tag{4}
\]

where \( t \) denotes time, \( t_1 < t_2 < \ldots < t_p \) are specified knots and \( (t - t_k)_+ \) is \( t - t_k \) for \( t > t_k \) and 0 otherwise. Since the formula (4) is a linear function of the coefficients \( a, b \) and \( c_k \), it is fitted to the data using linear regression. However, linearity in the future means that the quadratic and cubic coefficients are 0 for \( t > t_p \) by setting \( s^n(t) = 0 \). Therefore the formula (4) can also be written as

\[
s(t) = a + bt + \sum_{k=1}^{p-2} c_k \left[ (t - t_k)_+^3 - \frac{(t - t_k)_+}{(t_{p-1} - t_{p-2})_+} (t - t_{p-1})_+^3 + \frac{(t_{p-1} - t_k)_+}{(t_{p-1} - t_{p-2})_+} (t - t_{p-1})_+^3 \right].
\]

3. Results and Discussion

This study concerns two objectives related to modeling the volatility of stock returns and studying the behaviour of volatility over the period. Firstly, we show the behaviour of stock returns of seven
companies. After that, we present the volatility series obtaining from the GARCH(1,1). Finally, we show the result of fitting the natural cubic spline through the volatility series.

3.1 Stock Price Returns
In figure 1, the data are plotted on the y-axis and corresponding quantiles from a standardize normal distribution on the x-axis. It clearly can be seen from all panels that the stock returns are normal in the middle, but have stretched tails on both sides. Points distant from a fitted line indicated non-normality. In other words, the returns distributions contain fat tail (heavy tail). After obtaining the returns, the information of return fluctuation (volatility series) over time can be estimated by fitting GARCH(1,1) to the return.

Figure 1. Quantile-Quantile plots of stock returns.

3.2 Volatility of Stock Price Returns
Fitting the GARCH(1,1) gives volatility series of the seven stock returns plotted in figure 2 which describes the return fluctuation over the period.

Figure 2. Volatility series of seven stock returns.
It is clear from figure 2 that all stock returns have higher volatility during the end of 2008 but subsequently remained relatively stable. The seven companies are big companies that can possibly reflect the economy of Indonesia. The increasing volatility at the end of 2008 corresponds to the economic crisis in Indonesia at that time. As in figure 2, the volatility series is very fluctuating, we need to smooth the volatility series in order to simplify investigation of their change in many situations. The volatility series will be smoothed using natural cubic spline.

3.3 Fitted Volatility

In summary, we fitted the volatility series of seven stock returns using eight-knot natural cubic spline, the results are graphed in figure 3. It shows the volatility fitted by natural cubic spline which reflects the volatility signals. The lower right panel shows the volatility signals of stock returns for each of the seven stocks on the same axes. It can be seen that the seven volatility signals have the same trends, particularly during the end of 2008. In addition, foods and telecom might simply reflect flat volatility over the period.

4. Conclusions

Modelling the volatility is needed in order to predict the volatility of stock price returns. The returns could be obtained from stock price data by differencing log price from one day to the next. GARCH(1,1) was employed to fit the returns over the period. This is done through the estimation of parameters of the model by the Maximum Likelihood Estimation (MLE). After obtaining the volatility, natural cubic spline was used to study the behavior of the volatility. The result shows that GARCH(1,1) indicates evidence of volatility clustering in the returns of some Indonesia stock prices.

Acknowledgments

We would like to address our gratitude to Emeritus Prof. Dr. Don McNeil and Dr. Rattikan Saelim for invaluable assistance that greatly improved the manuscript.

References

[1] Alexander C 2008 Quantitative Methods in Finance (West Sussex: John Wiley & Son Ltd) pp 19-21
[2] Arowolo W B 2003 The International J. Eng. and Sci.2 32-37
[3] Bollerslev T 1986 J. Eco 31 307-327
[4] Chapra S C and Canale R P 2010 Numerical Method for Engineers 6th ed (New York:
McGraw-Hill Companies Inc) pp 528-529
[5] Hull J C 2009 *Option Future and Other Derivative Securities* 7th ed (New Jersey: Pearson Prentice Hall) pp 469-477
[6] Kamau K J, Mwita P N and Nassiuma D K 2015 *Kabarak J. Research and Innov.* 3 48-54
[7] McNeil N, Odton P and Ueranantasun A 2011 *Songklanakarin J. Sci. and Technol.* 33 117-120
[8] Namugaya J, Weke P G and Charles W M 2014 *International J. Sci.: Basic and App. Research* 16 216-223
[9] Posedel P 2005 *Metodoloskizvezki* 2 243-257