Analysis and comparison of Gaussian noise denoising algorithms

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Abstract. Collecting and processing various images has become an irreversible trend. It is meaningful to conduct more in-depth research on image denoising algorithms. We proposed mean filtering, median filtering, wiener filtering and wavelet filtering to denoise the image with Gaussian noise separately. And the objective image quality assessments are used to evaluate the quality of the images after denoising. Among them, wavelet filtering and Wiener filtering have a better effect on weaken Gaussian noise. Mean filtering and median filtering can also weaken Gaussian noise to some extent but the effect is limited. At the same time, it is equally important to select the appropriate denoising block for the diverse mean and variance of Gaussian noise. In wavelet filtering, the number of layers to be decomposed and the choice of threshold will also affect the effect of image denoising.

1. Introduction

With the continuous development of image processing and the improvement of human living standards, images and human life have been inseparable, and people are more and more dependent on images. Collecting and processing various images has become an irreversible trend. However, since the image data is often interfered by the system itself or the external environment during the process of generation and transmission, the digital image obtained by people cannot accurately reflect the real information, which affects the using in the actual life and production. Therefore, it is meaningful to conduct more in-depth research on image denoising algorithms[1].

Digital image noise, is a deviation signal that disturbs with human visual perception or interferes with sensor acceptance of image source information, and can also be understood as the bias between real signals and ideal signals. Noise can theoretically be defined as ‘unpredictable and random errors’ that can only be identified by probabilistic methods. Therefore, it is completely reasonable to regard image noise as a multi-dimensional random signal[2].

The noise of digital images mainly comes from the acquisition of images(the digitization of images) and the transmission process[3]. The main source of noise during transmission is the interference of the used channel. For example, images transmitted over a radio network must be contaminated by light or other atmospheric factors. Another large part also comes from electronic components, such as thermal noise caused by resistance, electronic fluctuation noise of photocells, and optical quantum noise. Noise can be simply divided into two types: one is the additive noise, and the image signal which is not related to the strength, such as “channel noise” from TVs or cameras during the transmission. Another one is multiplicative noise, which is related to the image signal and often varies with changes in the image, such as television scanning gratings, film, grain, and etc.

Digital images we obtained tend to be noisy, and denoising reasonably can enhance the quality of
images. The method of image denoising are two major categories: spatial domain methods and frequency domain methods, which are currently emerging mathematical morphology and wavelet transform image processing, as well as independent component analysis. The behavior and effects of analog noise are at the key of image denoising. And also, after denoising the quality of images need to be evaluated. At present, there are two main methods for image quality evaluation, namely subjective assessment and objective assessment. Although the most ideal image quality assessment is to be able to find a quantitative description of image fidelity and intelligibility as a basis for evaluating images and designing image systems. However, due to the current lack of sufficient understanding of the human visual system, there is no quantitative description method for human visual factors. Hence, objective quality evaluation method is an objective quality assessment which is more used and more authoritative. It is commonly applied like Mean Square Error (MSE), Peak Signal to Noise Ratio (PSNR) and structural similarity index method (SSIM)[4]. These will also serve as the evaluation criteria for this paper.

In this paper, Gaussian noise is used to pollute the picture, which is the main noise source of digital image. It is a kind of random noise and is distributed according to the Gaussian probability law. Through the implementation and comparison of several denoising algorithms, such as mean filtering median filtering, wiener filtering and wavelet filtering, an effective denoising algorithm is obtained.

2. Method
In order to compare the denoising effects of different filters, a Gaussian noise with a mean of 0 and a variance of 0.01 is added to the original picture as interference. Original image and Gaussian noise image can be seen in Fig.1. Then comparing the denoising effects of different filtering algorithms, we use the same size filtering template and perform average filtering, median filtering, Wiener filtering and wavelet filtering on the same Gaussian noise images. The filtering results are shown in Fig.2. From the simulation results, the image quality has been greatly improved than the original picture, where the Wiener filter and wavelet filtering effect are better, but not as good as the original picture. It is difficult for the human eye to distinguish the quality of each image directly and accurately, which embodies the limitations of the subjective evaluation system. Therefore, we introduce an objective evaluation model to quantify the processed image, and get a more powerful conclusion. There are many common objective evaluation algorithms, and we have adopted three more general ones.

2.1. Mean squared error (MSE)
Mean squared error is given by

$$\text{MSE} = \frac{1}{MN} \sum_{i,j} (y_{ij} - x_{ij})^2$$

where M and N are width and height of the image; $x_{ij}$ and $y_{ij}$ are the values of the distorted image and the original image separately. MSE is the energy average of the difference between distortion images and standard images. The smaller MSE, the better the quality of the image.

2.2. Peak signal to noise ratio (PSNR)
Peak Signal to Noise Ratio is the ratio of the energy of the peak signal to the average energy of the noise. It is usually expressed as the log variable component (dB), which can be described as the ratio of peak signal energy to MSE. The definition is as follows

$$\text{PSNR} = 10 \log_{10} \frac{255^2}{\text{MSE}}$$

In contrast to MSE, larger PSNR means better quality of image, which can be easily inferred in former formula. 255 represents the maximum pixel value.

2.3. Structural similarity index method (SSIM)
Structural similarity index is proposed by Wang Zhou et al., which is widely used in the objective quality assessment of images. The surface brightness of an object is determined by the light and reflected light, while the structural properties of the object are independent of the light. Using the
structural features of the object as one of the similarity comparison criteria can further improve the robustness of the similarity criterion. Those factors, loss of correlation, luminance distortion and contrast distortion, are redefined by

\[
c(x, y) = \frac{2\sigma_x\sigma_y + c_1}{\sigma_x^2 + \sigma_y^2 + c_1} \\
l(x, y) = \frac{2\bar{xy} + c_2}{\bar{x}^2 + \bar{y}^2 + c_2} \\
s(x, y) = \frac{\sigma_{xy} + c_3}{\sigma_x\sigma_y + c_3}
\]

where \(c_1, c_2\) and \(c_3\) are constants, in order to avoid the denominator being zero to maintain stability. These three coefficients are combined to give a complete value of SSIM:

\[
SSIM = l(x, y) c(x, y) s(x, y)
\]

3. Performance evaluation

The denoising image by each algorithm has been detected by each objective evaluation model. The results of detection are shown in Table 1. It can be found that the wavelet filtering and Wiener filtering algorithms are obviously better than another algorithms, and the median filtering is slightly stronger than mean filtering. Overall conclusions are reflected in Figure 3. It can be seen that the size of templates also determines denoising effect for different noises.

| Noise image | MSE | PSNR (dB) | SSIM |
|-------------|-----|-----------|------|
| Mean filtering | 121 | 27.28 | 0.88 |
| Median Filtering | 97  | 28.24 | 0.86 |
| Wiener filtering | 78  | 29.18 | 0.89 |
| Wavelet Filtering | 86  | 28.52 | 0.85 |

![Figure 1](image1.png)  
(a) Original image. (b) Gaussian noise image with a mean of 0 and a variance of 0.01, MSE=626, PSNR=20.16, SSIM=0.68.
Figure 2. Comparison of original image and Gaussian noise image.
(a) Mean filtering with [5X5], MSE = 121, PSNR = 27.28, SSIM = 0.88.
(b) Median filtering with [5X5], MSE = 97, PSNR = 28.24, SSIM = 0.86.
(c) Wiener filtering with [5X5], MSE = 78, PSNR = 28.18, SSIM = 0.89.
(d) Wavelet filtering, MSE = 86, PSNR = 28.52, SSIM = 0.85.

Figure 3. The comparison of image quality with different filters under different block sizes (MSE as the image objective assessment)

4. Discussion
Image is an important source of information, but for a variety of reasons the acquired images are accompanied by noise pollution. These noises are various and random. We focus on the common noise, Gaussian noise, in real life, and use the filtering algorithm to denoise the distorted image. The denoising results are compared by using various filters, specifically mean filtering, median filtering,
wiener filtering and wavelet filtering. The denoising image improves obviously compared with the original image, but the human eyes cannot accurately measure the quality of the image, which means that it is necessary to introduce objective image quality assessment methods. We adopt three common types named MSE, PNSR and SSIM. The comparison of the three indicators has made a certain discrimination on the quality of images, which is more objective and accurate. From the simulation results, Wiener filtering and wavelet filtering have better effects on denoising Gaussian noise, and the results of median filtering and mean filtering are poor. However, for the specific filtering algorithm, the size of the block is also important. It is necessary to select the appropriate block according to the characteristics of the noise.

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