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After a brief review of the theoretical basis of void scaling function properties of hierarchical structure, we analyze the phenomenological consequences at single jet level in Monte Carlo $e^+e^-$ annihilation events. We find an interesting alternative approach for characterizing quark and gluon jets.

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ABSTRACT

After a brief review of the theoretical basis of void scaling function properties of hierarchical structure, we analyze the phenomenological consequences at single jet level in Monte Carlo $e^+e^-$ annihilation events. We find an interesting alternative approach for characterizing quark and gluon jets.

1. Introduction

In a field like multiparticle dynamics, where the underlying theory (QCD) is as yet too difficult to be useful for predictive calculations at final particle level, the study of experimental regularities and their interpretation is of uttermost importance. As is widely known, regularities which have been discovered at complex levels include the NB regularity within final charged hadron multiplicity distributions (MD’s). More recently, it was found experimentally \textsuperscript{1} that separating the full sample of events into subsamples with a fixed number of jets, NB regularity is satisfied with better accuracy. Our studies of single jets\textsuperscript{2,3} shows that the same regularity is reproduced at this very elementary level.

In order to deepen our understanding of single jets MD’s, we carried out the analysis of the void structure in $e^+e^-$ annihilation Monte Carlo events. The interesting result is that the differences between quark and gluon jets seen in the clan analysis of MD’s \textsuperscript{2,3} appear relevant also in the ‘void’ analysis, which becomes also an alternative approach to characterizing jets of different origin.

In Sec. 2 the present theoretical framework for the ‘void’ function is very briefly reviewed; in Sec. 3 our results on Monte Carlo events are commented.

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\textsuperscript{*} Talk presented by R. Ugoccioni
2. Hierarchical Correlation Functions and Void Scaling

The void probability $P_0(\Delta y)$ is defined as the probability of detecting no particles in the region of phase space identified by the symbol $\Delta y$; here we make explicit reference to a central rapidity interval, but it should be clear that any other cut can be used: e.g., in Sec. 3 we will discuss the results in rapidity and transverse momentum intervals; as another example, in astrophysics $P_0$ is the probability that a region of real space is empty of galaxies.

Knowing the void probability is formally equivalent to a knowledge of the full MD, since it can easily be shown that

$$P_n(\Delta y) = \frac{(-\bar{n})^n}{n!} \frac{\partial^n}{\partial \bar{n}^n} P_0(\Delta y).$$

(1)

In practice however it is more useful to consider the void function

$$\mathcal{V}(\Delta y) \equiv -\frac{1}{\bar{n}(\Delta y)} \log P_0(\Delta y)$$

$$= \sum_{n=1}^{\infty} \frac{(-\bar{n}(\Delta y))^{n-1}}{n!} \kappa_n(\Delta y)$$

(2)

where $\kappa_n(\Delta y)$ is the reduced $n$-order cumulant in the interval $\Delta y$ defined in the standard way as the $n$-fold integral of the $n$-order reduced correlation function:

$$\kappa_n(\Delta y) = \int_{\Delta y} dy_1 \ldots \int_{\Delta y} dy_n c_n(y_1 \ldots y_n).$$

(3)

Notice that there is no average over different intervals of the same size, contrary to what is sometimes done experimentally. It will be seen in fact that the regularity is better satisfied in central intervals, and degrades when moving away.

Eq. 3 is very helpful in connection with hierarchical models: these assume that reduced correlation functions of any order can be expressed as products of two-particle reduced correlation functions, summed over all combinations of pairs,

$$c_n(y_1, \ldots, y_n) = \sum_{\alpha_t} A_{n,\alpha_t} \sum_{\sigma} c_2(y_{i_1}, y_{i_2}) \ldots c_2(y_{i_{n-1}}, y_{i_n}).$$

(4)

Here the two-particle reduced correlation functions $c_2(y_i, y_j)$, linking particles of rapidity $y_i$ and $y_j$, are summed over all non-symmetric relabelings $\sigma$ of the particles, and then summed over all distinct topologies $\alpha_t$, with weights $A_{n,\alpha_t}$, which depend only on the topology and not on energy or the phase space interval considered. In Table 1 the graphical representations of two different hierarchical models are shown: edges correspond to two-particle reduced correlation functions, each line corresponds
Table 1. The table shows the allowed shapes for a graphical representation of correlation functions up to fourth order in the Linked-Pair Ansatz (LPA), and in the Van Hove Ansatz (VHA).

to a different topology, and different graphs in the same line correspond to different relabelings. Two models are shown in Table 1 which differ in the allowed topologies: the Linked Pair Ansatz\(^5\) (LPA) requires that a particle appears at most twice in a given term of Eq. 4 (‘snake’ graphs in Table 1), which then becomes simply a sum over permutations:

\[
c_n(y_1, \ldots, y_n)|_{\text{LPA}} = A_n \sum_\mathcal{P} c_2(y_{i_1}, y_{i_2}) \cdots c_2(y_{i_{n-1}}, y_{i_n})
\]  

(5)

On the other hand, the Van Hove Ansatz\(^6\) (VHA), which was motivated by the occurrence of NB regularity, allows in addition connections with three particles (‘star’ diagrams in Table 1); in this case one can establish a recurrence relation among correlation functions:

\[
c_n(y_1, \ldots, y_n)|_{\text{VHA}} = (n - 1) \{c_{n-1}(y_1, \ldots, y_{n-1})c_2(y_i, y_n)\}_{\text{symm.}}
\]  

(6)

where the right-hand term is symmetrized over all particles. Both these models however, coincide at the reduced cumulant level, because integrating Eq. 4 over a central rapidity interval \(\Delta y\) one finds the hierarchical structure for cumulants:

\[
\kappa_n(\Delta y) = A_n \kappa_2^{n-1}(\Delta y)
\]  

(7)

It also worth noting that other, non-hierarchical models can approximate relation 7, thus making it impossible to go backwards from the cumulant level to the correlation functions level. One of these models, developed in the framework of the \(1/N\) expansion, is examined in detail in reference 7.
3. Monte Carlo Results

The considerations developed in the previous section will now be applied to $e^+e^-$ annihilation events in different rapidity and transverse momentum ($p_T$) intervals, with focus on the analysis of single jet properties.

$e^+e^-$ annihilation events have been generated by using JETSET 7.2 (parton shower)\textsuperscript{8} with Lund string fragmentation as hadronization prescription; the values of the parameters of JETSET different from the default ones are listed in reference 2. 2-, 3- and 4-jet events have been selected by using the Luclus algorithm, and single jets have been identified within each sample by the same algorithm; for each sample 40000 events have been considered at c.m. energies $\sqrt{s} = 91$ GeV, $\sqrt{s} = 200$ GeV, $\sqrt{s} = 500$ GeV and $\sqrt{s} = 1000$ GeV. The analysis has been performed in central rapidity intervals $|y| < y_{\text{cut}}$ with $y_{\text{cut}}$ from 0.25 up to the kinematically available value and in $p_T$ intervals $p_T < p_{T\text{cut}}$ with $p_{T\text{cut}}$ starting from 0.125 GeV/c. Rapidity and $p_T$ are defined with respect to the single jet axis.

In Fig. 1a we look at the sample of single jets obtained from 2-jet events: each event therefore contributes with 2 jets. The figure shows the void scaling function
Figure 2. Same as in Fig. 1, but for a) $h$-jets and b) $l$-jets from 3-jet events. In the figure are indicated the jet energies $E_j$ corresponding to the c.m. energies $\sqrt{s} = 91$ GeV (squares), $\sqrt{s} = 200$ GeV (circles), $\sqrt{s} = 500$ GeV (triangles) and $\sqrt{s} = 1000$ GeV (diamonds).

$V$ vs the product $\bar{n}\kappa_2$. Each point represents the value of the void function for particles in a jet of given c.m. energy, given rapidity interval and given $p_T$ interval. For clarity, points referring to different $p_T$ intervals have been plotted displaced from one another by a fixed amount. The dotted lines, which represent NB behavior, become therefore superimposed when this shift is removed. Different center-of-mass energies are represented by different symbols, and each point refers to a different rapidity interval, up to $y_{\text{cut}} = 2.0$. We notice firstly that there is a good scaling behavior, as the dependence of the function $V$ on energy, rapidity and transverse momentum is confined to its dependence on the product $\bar{n}\kappa_2$. Secondly, we observe there is good agreement with the dotted curve which represents NB behavior; this was expected on the ground of our previous studies on MD’s in restricted domains of phase space$^{2,3}$. As for large rapidity intervals, the observed violations to the scaling, which are not shown in this figure, are due to the lack of translational invariance for the two-particle correlation function which invalidates the hierarchical structure of cumulants. Since 2-jet events are composed of a quark and an antiquark jet of the same energy, one can conclude that quark jets fulfill NB regularity, and $V$-scaling behavior.

We now compare Fig. 1b, where we show the same quantities as in Fig. 1a, but the single jets here come from 3-jet events. Each event contributes to the sample
with a quark jet, an antiquark jet and a gluon jet, all of different energies. In this figure, all contributions are superimposed. We see that scaling behavior is less good than in the 2-jet sample (a fact which also was expected from previous analysis), but still acceptable for small $p_T$ intervals, and that the data points are spread to larger values of $\bar{n}\kappa_2$ and at smaller values of $\mathcal{V}$ than in the 2-jet sample. Since 3-jet events contain a gluon jet, we can say that void analysis seems sensitive to the presence of a jet originated by a gluon. This is confirmed by the analysis of 4-jet events in which the spread is even larger.

In order to specify better the role of the gluon jet in these events, an energy scan of the 3-jet event sample was performed. From each event of the 3-jet sample the lowest ($l$-jet), the intermediate and the highest ($h$-jet) energy jet have been collected in three separated samples. This separation is indeed suggested by perturbative QCD: according to conventional wisdom, the behavior of $h$-jet samples is expected to reflect that of quark (antiquark) jets and the behavior of $l$-jet samples that of gluon jets. The jet energy dependence of the $h$- and $l$-jet samples has been studied by further collecting the jets in energy intervals 2 GeV wide. The energy of each jet is the kinematically reconstructed energy. For this program we analyzed 100000 3-jet events at c.m. energies $\sqrt{s} = 91$ GeV, $\sqrt{s} = 200$ GeV, $\sqrt{s} = 500$ GeV and $\sqrt{s} = 1000$ GeV.

Figure 2 shows the results of void analysis on the above sample. In Fig. 2a we see the $h$-jet sample, and Fig. 2b the $l$-jet sample. The points are now labeled with jet-energy instead of c.m. energy, but the same rapidity and $p_T$ intervals of the other figures have been used. One finds a good scaling behavior and good agreement with expectation of NB regularity at all energies, all $y_{cut} < 2.0$ and all $p_T^{cut}$. It should be noticed that these two samples ($h$-jet and $l$-jet) are part of the samples that were shown superimposed in Fig. 1b. Isolating them and separating them in energy has much improved their scaling behavior. In Fig. 3 we compare the two samples, the $h$-jet and the $l$-jet at the same jet-energy of 43-45 GeV (of course the samples come from events of different c.m. energies). The main feature here is the different spread of points, because $h$-jets appear to yield larger values of $\mathcal{V}$ than $l$-jets. One should remember that this implies that clans are larger in the case of $l$-jets.

In order to be assured that $h$-jets are actually quark jets, one can compare results from the 2-jet sample (where only quark jets appear) and from the $h$-jet sample at the same jet-energy. It is apparent from Fig. 4 that the identification of $h$-jets with quark jets can be made safely. We have no pure gluon-jet sample with which to compare $l$-jets. However we noticed in Fig. 2b that $l$-jets also satisfy NB regularity, and with different parameters from $h$-jets. One expects that if the $l$-jet sample were a mixture of gluon and quark jets, i.e., a mixture of two NB like samples with different parameters, NB regularity would not be satisfied. Since on the contrary it is, we conclude that in the $l$-jet sample the contamination of quark jets is negligible, and that we can continue to treat $l$-jets as gluon jets.
We can therefore characterize $h$-jets by a little amount of branching which grows with energy (clans are small): this situation is consistent with the identification of the $h$-jet as a quark (antiquark) jet within which gluon bremsstrahlung is the dominant mechanism. For the $l$-jet sample branching plays a more relevant role than for the $h$-jet one: data points spread along the NB curve showing more deviation from Poissonian behavior. This result is consistent with the identification of the $l$-jet as a gluon jet within which gluon self-interaction is the dominant mechanism.

Further support to this result comes from the study of single particle inclusive rapidity and transverse momentum distributions separately for $h$-jets and for $l$-jets; for this program 50000 events have been generated. The inclusive rapidity distributions $dn/dy$ are shown vs rapidity $y$ in Fig. 5 at different jet energies: rapidity is with respect to the ancestor of each jet, event by event; distributions are
Figure 5. a) Single particle inclusive rapidity distributions $dn/dy$ vs. $y$ for $h$-jets from 3-jet events at $E_J = 43$-45 GeV from $\sqrt{s} = 91$ GeV (solid line) and $E_J = 248$-250 GeV from $\sqrt{s} = 500$ GeV (dashed line); rapidity is defined in the ancestor frame. Plots are normalized to average multiplicity $\bar{n} = 8.8$ at c.m. energy $\sqrt{s} = 91$ GeV and $\bar{n} = 14.5$ at $\sqrt{s} = 500$ GeV. b) Same as in (a) but for $l$-jets from 3-jet events at $E_J = 8$-10 GeV from c.m. energy $\sqrt{s} = 91$ GeV (solid line) and $E_J = 43$-45 GeV from $\sqrt{s} = 500$ GeV (dashed line). Plots are normalized to average multiplicity $\bar{n} = 7.7$ at $\sqrt{s} = 91$ GeV and $\bar{n} = 11.8$ at $\sqrt{s} = 500$ GeV.

Figure 6. a) Single particle normalized inclusive rapidity and $p_T$ distributions $1/\bar{n}(d^2n/dydp_T)$ vs. $(y, p_T)$ for $h$-jets from 3-jet events at jet energy $E_J = 43$-45 GeV from c.m. energy $\sqrt{s} = 91$ GeV; rapidity is defined in the c.m. frame. b) Same as in (a) but for $l$-jets from 3-jet events at jet energy $E_J = 43$-45 GeV from $\sqrt{s} = 500$ GeV.
normalized to the average multiplicity. The distribution for $l$-jets (Fig. 5b) is peaked at $y = 0$, while the distribution for $h$-jets (Fig. 5a) is more flat. Both distributions grow slowly with jet energy. The inclusive $(y, p_T)$ distributions are shown in Fig.s 6a ($h$-jets) and 6b ($l$-jets): here the distributions can be compared directly, because both have the same jet energy ($E_J = 43-45$ GeV), both are normalized to 1 and rapidity is taken with respect to the c.m. frame of the annihilation. The $l$-jet is seen to be less extended in rapidity but more spread out in $p_T$, as generally expected for a gluon jet with respect to a quark jet.

In conclusion, all results on single particle inclusive distributions support the idea that particles inside a gluon jet, where branching plays a relevant rôle, show more correlation than in a quark jet, where gluon-bremsstrahlung emission is dominant. This picture is strengthened finally by Fig. 7, where the average distance in rapidity between particles is plotted against jet energy, separately for $h$-jets, which lie all within the less dense area, and for $l$-jets, which lie all within the more dense area. $l$-jets show more concentration in rapidity (and therefore more correlation) than $h$-jets, as particles inside a gluon jet are closer together than particles inside a quark jet, and in agreement with the fact that the aggregation parameter (NB parameter $k^{-1}$) is larger for $l$-jets than for $h$-jets.

Figure 7. Average distance in rapidity between particles for $h$- and $l$-jets from 3-jet events as a function of jet energy; dotted less dense area shows $h$-jets, dotted more dense area shows $l$-jets.
4. Conclusions

The analysis of the structure of voids has been presented as a tool to explore the hierarchical structure of correlations in multiparticle dynamics. Its application to samples of single jets obtained from $e^+e^-$ annihilation Monte Carlo events has shown that quark and gluon jets can be characterized by means of the respective void properties. It should be noticed that the results obtained are consistent with the presence of NB regularity at single jet level, and suggest a hierarchical structure for correlations within single jets.

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6. References

1. P. Abreu et al., DELPHI Collaboration, Z. Phys. C56 (1992) 63
2. F. Bianchi, A. Giovannini, S. Lupia and R. Ugoccioni, Z. Phys. C58 (1993) 71
3. F. Bianchi, A. Giovannini, S. Lupia and R. Ugoccioni, “Single Jet Multiplicity Distributions in $e^+e^-$ Annihilation at High Energies”, DFTT 49/92, to be published in the Proceedings of the XXII International Symposium on Multiparticle Dynamics, Santiago de Compostela, Spain, 13–17 July 1992
4. J.N. Fry, Ap. J. 306 (1986) 358
5. P. Carruthers and I. Sarcevic, Phys. Rev. Lett. 63 (1989) 1562
6. L. Van Hove, Phys. Lett. B242 (1990) 485
7. S. Lupia, A. Giovannini and R. Ugoccioni, Z. Phys. C59 (1993) 427
8. M. Bengtsson and T. Sjöstrand, Nucl. Phys. B289 (1987) 810;
   T. Sjöstrand and M. Bengtsson, Computer Physics Commun. 43 (1987) 367
9. A. Giovannini and L. Van Hove, Z. Phys. C30 (1986) 391
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