Mathematical description of elastic mechanisms of force compensating manipulator

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Abstract. Improvement of control systems for manipulators and robots was required implementation complex automation of production processes. It is necessary to have a reliable mathematical description of elastic mechanisms interacting during operation to solve problems of analysis and synthesis of control devices of force compensating manipulators. The work goal is the development of the mathematical description of elastic executive mechanisms of the ShBM-150M balanced manipulator. Studies have shown that low-frequency components of moved weight elastic vibrations are due to flexural deformation of a manipulator's shoulder and arm. Inertia equivalent values of manipulator links are changed by 2.2 and 37 times during manipulator operation with a constant load value. Total flexural deformation of shoulder and arm deflection is 25 mm during changing mass per 100 kg. And parameters of the mathematical model can be changed by 11 times, which needs adaptive control use.

1. Introduction

Currently, in different industries, manipulators and robots are the basis for complex automation of production processes [1]. Improvement of modern industrial manipulators (IM) is required to solve tasks of mechanization and automation of production processes. Analysis of the technical characteristics and capabilities of modern IM showed that the negative effect of elastic structures and mechanical links increases on quality and productivity of work because of complications of tasks and new approaches to design focused on reducing materials [2-4]. The dynamic loads of mechanisms and electric drives are increased during IM work stimulation. The vibrations of controlled axes are grown [2].

The requirements of manipulators control systems are raised at implementation of modern intelligent production technology. It is necessary to have a real mathematical description of multilink elastic mechanisms in realization.

2. Setting of the problem

The operation of modern manipulators and robots has negative effect by the flexibility of structures and mechanical transmissions.

Because vibrations of executive mechanism (EM) are increased, quality of IM work is degraded, the accuracy of weight position is lowered.

Recently, force compensating manipulators (FCM) become a frequent practice. The electric drives (ED) control systems compensating weight of links and moved load are used in them. This allows a
worker to move and position objects with minimal force values between 2 and 5% of the weight. It is necessary to create systems with high-precision force control in elastic manipulators EM.

It is necessary to have a reliable mathematical description allowing to study power energy and kinematic effects in links, mechanisms, and gears of a manipulator for the design of FCS electromechanical system. The elasticity of mechanisms and structures, nonlinearity of characteristics, changes in a structure, and parameters values of control object are needed to use in FCM mathematical description [5].

A difficult task is the development of FCM mathematical models with multilink elastic mechanisms with cross internal feedbacks and nonlinear characteristics. Therefore, it is important to substantiate using hypotheses, formulate correct assumptions and determine rational approaches and research methods during solving development of a mathematical description of complex manipulator mechanisms and perform its correct simplification at the conceptual stage of FCM study.

3. Papers’ review determining relevance
Various approaches and methods are used during developing FCS mathematical description [6]. At the same time, a difficult task is creation of real mechanisms mathematical description with an elastic multilink structure resulting in large elements deformations [7].

Recently, a mathematical description with Hamilton principle is used for study of energy power and kinematic mechanisms action of multilink manipulators. Based on its method Lagrange equations of second kind are obtained:

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial \dot{q}_i} + \frac{\partial D}{\partial q_i} = Q_i, \quad i = 1, 2, n, \tag{1}
\]

where \( n \) is number of degrees of system freedom; \( q_i, \dot{q}_i \) are generalized coordinates of speed; \( Q_i \) is generalized force acting on system.

Influence of inertia, stiffness coefficients and mechanisms damping on motion of FCS EM are determined by analysis of Lagrange equations components describing change in kinetic (\( T \)) and potential (\( U \)) energy of studied system [8]. Damping mechanisms properties are considered using the dissipative dispersion function (\( D \)) determined from results of experimental studies. It allows a demonstration that real values of internal viscous friction forces in system with structural damping [9].

Following assumptions were made during developing a mathematical description of FCM [10]:

- wave processes were not taken into account in study of mechanisms deformations, it possible to use discrete mathematical mechanisms models in which concentrated masses are connected by weightless elastic links;
- deformations and stresses in elements of mechanical transmissions and structures of a manipulator are followed Hooke’s law;
- forces and torques acting on mechanical system elements exerted on non-deflecting point mass;
- elastic elements (EE) have constant stiffness characterized proportionality constant between force and deformation;
- twisting of motor shaft and gearbox is neglectfully small and it is neglected;
- stiffness of a manipulator stand fixing to a base is neglected.

4. Goal and tasks of paper
The goal of the work is justification of rational approaches and methods for completing a reliable a mathematical description of FCM with elastic mechanical transmissions and parameters changing during operation. The solved problem is focused on developing a real mathematical model of manipulator multilink mechanisms, making analysis and synthesis of control systems requiring their functioning quality.
5. Approaches and methods of theoretical study

Justification of approaches and methods used in development of the FCM mathematical description will be considered by example of the ShBM-150M type manipulator. Its construction consists of two pantograph mechanisms [11]. The main advantage of such mechanisms is performing large vertical and horizontal motions of unit with minimal moving actuator providing movement with saved required orientation [12]. However, IM mathematical description is heavily complicated with such mechanism.

Features of the ShBM-150M manipulator construction will be considered affecting on development of a mathematical description of elastic mechanisms. The kinematic structure of the ShBM-150M manipulator mechanisms is shown in figure 1.

![Figure 1. Kinematic structure of the ShBM-150M manipulator mechanisms.](image-url)

A lift actuator is mounted in the manipulator head. It consists of an electric motor compensating required load weight during vertical movement. Sew-Eurodrive frequency-controlled electric drive consists of Movitrac B frequency converter (0.75 kW, 4.2 A), a DRS80S4 synchronous gearmotor (0.75 kW; 5.1 Nm; 1400 rpm) [13].

The gearmotor is jointed to two gear wheels rotating a rack through a helical gearbox. Required object movement is provided rack motion through the lever system by shoulder and arm. Structurally, the levers are located inside squaring shoulder and arm. Structural elements and mechanical transmissions of a multilink manipulator are got complex elastic deformations by Hooke law during operation. It possible to analyze their components separately.

A moved cargo orientation is provided by two parallelogram mechanisms with a rack, levers, shoulder and arm. The parallelogram mechanism structure is led by working levers in tension and compression, and shoulder and arm of the ShBM-150M manipulator are mainly got bending deformations.

Studies have shown that deflection value is greater than tension or compressive deformations of same elements with same forces values acted to long levers and structures of the FCM [12]. This is due to the following main factors:

a) the tensile stretch of a long structural element is determined by sectional area, and the deflection is dependent on value of axial moment of inertia;

b) for structural steels, Young’s modulus determining tension and compressive deformation has many folds larger value than the shear modulus.

For the ShBM-150M manipulator, studies of structural elements and transmission mechanisms using the Ansys Mechanical software package [14] showed that tensile-compression deformation of levers is 0.04 mm, the box-type arm is 0.012 mm, and the arm is 0.034 mm with an acting force of 1000 N. The deflection (bend) of the shoulder is 6.9 mm, and the arm is 2 mm with same force value. Thus, arm deflection is 166 times greater than stretching deformation, and shoulder deflection are 202 times greater than tension deformation. Studies have shown that equivalent deformation of mechanical transmissions
and manipulator structure have resulted in cargo shift in the vertical position by 25 mm with a change in cargo mass per 1000 N.

A collaborative Ansys Mechanical software analysis of the ShBM-150M manipulator mechanism driving forces from a motor to a cargo showed that equivalent stretching mechanism elements can be ignored during studying low-frequency IM vibrations. And vibration processes can be studied with only deformations of the shoulder and arm. In this case, the shoulder bend must be studied about the C point, and the arm bend will be studied about the F point with levers action of parallelogram mechanisms.

A design model is importantly drawn up at a conceptual stage of development of a mathematical model of the manipulator ShBM-150M mechanisms. This helps to determine elastic influence of mechanisms. Mechanisms design models are developed with various levels of detail depending on solved tasks.

The construction analysis of the ShBM-150M manipulator showed that maximal inertia of a synchronous motor with a gearbox, the shoulder, arm and a cargo is had. Minimal stiffness coefficients are determined by bending deformations of the shoulder and arm. Therefore, mechanisms design model characterizing bending deformations of the shoulder and arm should be presented in a three-mass model form showed in figure 2.

Figure 2. The three-mass design model of the ShBM-150M manipulator mechanism.

Figure 3. The construction of the manipulator executive mechanism.

In figure 2 $J_1$ inertia consists of inertial properties of motor rotor, gearbox and rack gear, $J_2$ is shoulder inertia, $J_3$ is arm and cargo with gripper inertia. Stiffness coefficients $C_{12}, C_{23}$ characterize equal bending deformation of the shoulder and arm. Damping coefficients $b_{12}, b_{23}$ describe dissipative properties of the shoulder and arm.

The inertia values of structural elements of the ShBM-150M manipulator were refined using the SolidWorks Motion software [15]. And the stiffness coefficients values of mechanical transmissions were refined using the Ansys Mechanical software.

The mathematical description is developed to study the electromechanical system of the ShBM-150M manipulator. Therefore, the parameters of design model must put into motor shaft. The reduction of masses and inertia is completed with constant kinetic energy, and equivalent values of elements stiffness coefficients are determined from constant potential energy condition of the system [16]. Studies have shown that the parameters of the design model of the manipulator can change with changing cargo weight and shoulder and arm configuration. The range of changing inertial properties and elastic deformations values of mechanical transmissions of the ShBM-150M manipulator is determined by cargo weight value also values of masses and inertia of levers, shoulder and arm put into motor shaft.

The gear ratio and reduction radius are must to know for putting parameters of design model into motor shaft. When determining these dependencies, shoulder rotates the center of mass about the C point.
in vertical cargo movement, and arm complex moves in linear direction with cargo and rotation about the F point.

The method of parameters putting of design model into motor shaft is explained in figure 3. In the figure, the radial direction of cargo movement $X_C$ coincides with the X-axis, and its vertical movement $Z_C$ coincides with the Z-axis.

The generalized basic parameter of the $\varphi_2$ angle was chosen at determination of manipulator links configuration influence in different cargo positions. The angle determines shoulder down about horizontal position and it uniquely determines position of moved shoulder, arm and cargo.

In figure 3, the distance between the rotation axis of the shoulder point $C$ and horizontal and vertical move of cargo is characterized by variables $X_C$ and $Z_C$. The length of arm and shoulder is determined by the parameter $L = 1.5$ m.

The analytical equations for reduction radius $\rho_C$ and gear ratio $i_3$ describing motion of a mass centers of a cargo, arm, and shoulder are got nonlinear. It is far more complex study.

For example, the gear ratio putting rotational arm movement into a motor shaft is:

$$i_3 = \left[ \frac{L \sin \varphi_2 - \frac{C C}{L} \left( \cos \varphi_2 - \frac{X_C}{C L} \right)}{\sqrt{1 - \left( \cos \varphi_2 - \frac{X_C}{C L} \right)^2}} \right]^{-1} \quad \rho_C \sin \left( \arccos \left( \frac{X_C - \cos \varphi_2}{C L} \right) \right)$$

They were taking a Taylor series expansion for linearization of obtained nonlinear dependences. The simplified linearized analytical equations were got using only linear components of a Taylor series.

The linearized gear ratios of the shoulder and arm, the radius of linear arm move and putting angle of the shoulder rotation into a motor shaft are determined by the following equations:

$$i_2 = (\gamma + \beta \varphi_2)^{-1}; \quad i_3 = \rho_C^{-1} \cdot i_3 \cdot \rho_3 = \rho_C + \lambda \varphi_2; \quad \varphi_2 = \lambda + \gamma \varphi_2,$$

where $\gamma = \frac{\rho_C}{L}; \ \beta = \left( \rho_C \left( L - X_C \right) \right) \left( 2 \left( L X_C - X_C^2 \right)^{1/2} \right); \ \theta = -\rho_C \left( 2 L X_C - X_C^2 \right)^{-1/2}; \ \lambda = \rho_C \left( L - X_C \right) \left( 2 \left( L X_C - X_C^2 \right)^{1/2} \right); \ \Lambda = \sqrt{2 L X_C - X_C^2} \cdot L^{1/2}.$

The analysis showed that the range between nonlinear and linearized characteristics does no more than 10%. At the same time, it was found that the greatest linearization errors are got in out points of a manipulator working area, and minimal values are got in area middle.

The dependences of the equivalent total inertia of the shoulder, arm, and cargo putting into a motor shaft were determined due to the linearized equations. The equations were got in changing configuration of the manipulator links determined by cargo position.

Studies have shown that parameters of the mathematical model putting into a motor shaft with moved cargo are changed in the following ranges: shoulder inertia $J_2 = (4.2 - 10^{-7} - 1.5 - 10^{-5})$ kg·m², arm inertia at vertical cargo motion with 10 kg mass $J_3 = (1.1 - 10^{-5} - 2.5 - 10^{-5})$ kg·m² and changing mass between 10 and 150 kg $J_3 = (1.1 - 10^{-5} - 1.1 - 10^{-5})$ kg·m². Changing stiffness of shouldner is $C_{12} = (0.03 - 0.15)$ Nm/rad and arm is $C_{23} = (0.3 - 0.8)$ Nm/rad. Damping coefficients $b_{12}, b_{23}$ change in range between 0.00008 Nm/rad and 0.00025 Nm/rad. Notice that the inertia, stiffness and damping coefficients are constants for the elements. The parameters change is due to give into a motor shaft.

The analysis showed that mechanism parameters can change of ten times. Must be factored into development of control systems for the manipulator [17].

The mathematical model was developed differential equations form describing the movement of the mechanism manipulator and cargo. The model was got with considered approaches and assumptions based on Lagrange equations, and variable values of mass reduction radius and stiffness coefficients given to a motor shaft in equation (4).

The block diagram of three-mass mathematical model of mechanical part of the ShBM-150M manipulator was obtained based on the equations of cargo motion (4). This model makes allowances for
analyzed correlation and changes in the mechanism parameters. The simplified block diagram is shown in figure 4. Only basic correlations and parameters presented in more complex model are given in it.

\[
\begin{align*}
\dot{\phi}_1 J_1 (y + \beta(\Lambda + \gamma \phi_1))^2 + \dot{\phi}_2 J_2 \rho_C \beta (y + \beta(\Lambda + \gamma \phi_2)) - \\
-\phi_1^2 J_1 \rho_C \beta (y + \beta(\Lambda + \gamma \phi_1))^2 - \\
-C_{23} (\phi_2 - \phi_1) \theta^2 \left[ y(\Lambda + \gamma \phi_2)(\phi_2 - \phi_1) + (\Lambda + \gamma \phi_2)^2 \right] + \\
+C_{23} (\phi_1 - \phi_2) \left[ y(\Lambda + \gamma \phi_2)^2 + \beta \theta (y + \beta(\Lambda + \gamma \phi_2)(\phi_2 - \phi_1)) \right] = 0; \\
\phi_1 \left[ m_{\text{arm}} (\rho_C + \lambda(\Lambda + \gamma \phi_1))^2 + m_c \rho_C^2 + J_{\text{arm}} \theta^2 (\Lambda + \gamma \phi_2)^2 \right] + \\
+2 \phi_1 \phi_2 J_{\text{arm}} \theta^2 (\Lambda + \gamma \phi_2) + m_{\text{arm}} \lambda (\rho_C + \lambda(\Lambda + \gamma \phi_1)) - \\
-C_{23} (\phi_2 - \phi_1) \theta^2 \left[ (\Lambda + \gamma \phi_2)^2 \right] = F_w \rho_C - m_c g \rho_C; \\
\dot{\phi}_1 J_1 + C_{23} (\phi_1 - \phi_2) \left[ y + \beta(\Lambda + \gamma \phi_1) \right]^2 = M_1 - M_\omega.
\end{align*}
\]

Figure 4. The block diagram for study of the ShBM-150M manipulator mechanism.

The \( M_w \) component of motor torque compensating a weight of the shoulder and arm, the force \( F \) acted by a worker for cargo are marked in figure 4. The mathematical model validity of the ShBM-150M mechanisms has been confirmed by large experimental studies with real manipulator. For example, experimental and simulated oscillograms are presented in figure 5. The "Tenzo-M" type load cell [18] with 2500 N capacity was jointed between the gripper and cargo for experimental studies of elastic vibrations of the manipulator mechanisms. In figure 5 the force oscillogram was got at dead stop of a cargo with 100 kg mass.

Figure 5. The force oscillogram in manipulator mechanism at shock pulse input on cargo with 100 kg mass with maximum go out of cargo from the stand.

Figure 6. The force oscillogram in manipulator mechanism at shock pulse input on cargo with 100 kg mass with at simulating manipulator operation.
In figure 5, there is the oscillogram with maximum go out of cargo from the stand. Its analysis showed that the cargo frequency vibration is 2.5 Hz. In this case, the amplitude of first force vibration over the final value is 176 N, the vibration decay time is more than 5 s. The force oscillograms taken in other positions showed the damping decrement because of position varies in the range between 0.12 and 0.44. Then if cargo is placed farther from a stand the damping decrement is smaller.

In figure 6 there is the oscillogram at simulating manipulator operation with similar conditions of experiment with the given parameters: \( J_1 = 0.000178 \text{ kg} \cdot \text{m}^2 \), \( J_2 = 8.4 \cdot 10^{-6} \text{ kg} \cdot \text{m}^2 \), \( J_3 = 6.39 \cdot 10^{-5} \text{ kg} \cdot \text{m}^2 \), \( C_{12} = 0.07 \text{ Nm/rad} \), \( C_{23} = 0.56 \text{ Nm/rad} \), \( b_{12} = 0.00008 \text{ Nms/rad} \), \( b_{23} = 0.0001 \text{ Nms/rad} \).

The simulation was made with a cargo mass of 100 kg and maximum distance of a cargo from a stand. Into the system, pulsed disturbance was inputted every 0.3 seconds with 200 N amplitude. In this case, the cargo vibration frequency is 2.52 Hz, the amplitude of first force vibration over the final value is 181 N, damping decrement is 0.11, vibration decay time is 6.3 s. The imprecision between compared research results was: 0.8% for frequencies, 2.8% for maximum amplitude over final value, 18% for damping decrements of the transient.

6. Analysis of research results

The effect research of changing parameters of mechanical transmissions on manipulator operation was obtained using the mathematical model of the ShBM-150M. In figure 7 bode plots of changing torque in mechanism EE of the ShBM-150M were shown. Bode plots were given with different cargo positions about stand with cargo weight of 100 kg.

![Figure 7. Bode plot of lift mechanism of the ShBM-150M.](image)

Line 1 is links position in minimum distance of a cargo about stand. It is stiff configuration of mechanisms. Line 2 is position at middle distance of a cargo with vertical position of the arm. Line 3 is position at maximum distance of cargo about the stand. In figure 7 values of resonant frequencies are marked about maximum values of all bode plots. Values are different at cargo position. They differ 1.67 times for low frequency, and 1.75 times for high frequency. Research has shown that low frequency resonances are primarily determined by tension shoulder, and high frequency resonances are determined by tension arm. The difference between low frequency and high frequencies is from 4.8 to 14.6 times depending on a cargo position. However, if a cargo is placed near a stand (line 1) then difference is smaller.

The study of changing parameters of mechanisms for joint balanced manipulator operation showed that it is necessary to take on change of mechanisms parameters and complete adaptive control with the mathematical model of the ShBM-150M during solving problem of synthesizing a control system [19].
7. Conclusion
The dynamic loads of mechanisms are increased, there are large EM vibrations, the transients properties are degraded, the accuracy of cargo positioning is decreased because of elastic mechanical transmissions. This is because the FCM is realized with light-weight construction and high drive velocity.

- A valid mathematical description of multilink elastic mechanisms in various conditions and operations is importantly got during solve problems of analysis and synthesis of force control systems. It is necessary to improve the quality of the FCM function.
- Studies have shown that for the ShBM-150M manipulator, the low-frequency components of cargo vertical elastic vibrations are mainly due to bending deformations of the shoulder and arm. They are 25 mm in cargo weight changing by 100 kg.
- The parameters changing of the mathematical model of the ShBM-150M manipulator mechanisms is dependent on configuration of the shoulder and arm of the manipulator, and weight value and cargo position.
- At the ShBM-150M manipulator operation the equivalent value of the shoulder inertia putted into a motor shaft can change by 37 times. The arm inertia can change by 2.2 times. A cargo inertia can change by 11 times. Therefore, using of an adaptive control system is needed.

Acknowledgment
The reported study was funded by RFBR, project number 19-38-90047.

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