SHOCK ACCELERATION MODEL WITH POSTSHOCK TURBULENCE FOR GIANT RADIO RELICS

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Abstract: We explore the shock acceleration model for giant radio relics, in which relativistic electrons are accelerated via diffusive shock acceleration (DSA) by merger-driven shocks in the outskirts of galaxy clusters. In addition to DSA, turbulent acceleration by compressive MHD mode downstream of the shock is included as well as energy losses of postshock electrons by Coulomb scattering, synchrotron emission, and inverse Compton scattering off the cosmic background radiation. Considering that only a small fraction of merging clusters host radio relics, we favor the reacceleration scenario in which radio relics are generated preferentially when shocks encounter the regions containing low-energy ($\gamma_e \lesssim 300$) cosmic ray electrons (CRe). We perform time-dependent DSA simulations of spherically expanding shocks with physical parameters relevant for the Sausage radio relic, and calculate the radio synchrotron emission from the accelerated CRe. We find that significant level of postshock turbulent acceleration is required in order to reproduce broad profiles of the observed radio flux densities of the Sausage relic. Moreover, the spectral curvature in the observed integrated radio spectrum can be explained, if the putative shock should have swept up and exited out of the preshock region of fossil CRe about 10 Myr ago.

Key words: acceleration of particles — cosmic rays — galaxies: clusters: general — shock waves

1. INTRODUCTION

The Sausage relic is a giant radio relic detected in the outskirts of the merging cluster CIZA J2242.8+5301 located at the redshift, $z = 0.188$ (van Weeren et al. 2010). It is an arc-like radio structure whose spectral index increases away from the edge of the relic toward the cluster center. Its volume-integrated radio spectrum has a power-law form with a steep spectral curvature above $\sim 2$ GHz (Stroe et al. 2013, 2014). In van Weeren et al. (2010), the observed radio spectrum at the relic edge was interpreted as a power-law with the slope, $\alpha_{\text{sh}} \approx 0.6$, which can be translated into the ‘radio Mach number’, $M_{\text{rad}} = [(3 + 2\alpha_{\text{sh}})/(2\alpha_{\text{sh}} - 1)]^{1/2} \approx 4.6$, based on the diffusive shock acceleration (DSA) model. On the other hand, the Mach number inferred from the Suzaku X-ray observations of Akamatsu et al. (2015) indicates a much lower ‘X-ray Mach number’, $M_X = 2.7^{+0.4}_{-0.7}$. However, recent radio observations by Stroe et al. (2016) showed that the spectral index between 153 and 608 MHz may be fitted by $\alpha_{\text{sh}}^{608} \approx 0.7$ slightly downstream of the hypothesized shock location, if we ignore the flattest point with $\alpha_{\text{sh}}^{608} \approx 0.54$ near the relic edge (see Figure 3 below). This gives a much lower radio Mach number, $M_{\text{rad}} \approx 3.3$, which is more comparable to $M_X$. The spectral steepening at high frequencies in the observed integrated spectrum, $J_{\nu}$, of the Sausage relic is not consistent with a single power-law energy spectrum of relativistic electrons that are expected to be accelerated by a steady planar shock (Stroe et al. 2014, 2016). So Kang & Ryu (2016) suggested that such a spectral curvature could be explained, if the relic is generated by the shock that sweeps through and moves out of a finite-size cloud with preexisting cosmic ray electrons (CRe). Lack of seed electrons outside of the cloud results in softening of the volume-integrated electron spectrum beyond radiative cooling alone. Moreover, it was pointed out that the ubiquitous presence of radio galaxies, AGN relics and radio phoenix implies that the intralcluster medium (ICM) may contain fossil CRe left over from radio jets (Ensslin 1999; Slee et al. 2001; Clarke et al. 2013; Pinzke et al. 2013; de Gasperin et al. 2013; Kang 2016a).

In addition to the integrated spectrum, radio flux density, $S_\nu$, can be used to constrain the shock model parameters such as the shock speed and magnetic field strength. The transverse length scale of the radio relic at high frequencies, for instance, is related with the cooling length of the electrons with synchrotron peak frequency, $\nu_{\text{peak}} \approx 0.3(3eB/4\pi n_eC)^{1/2} \text{eV}$ (in cgs units):

$$\Delta l_{\text{cool}} = t_{\text{rad}}(\gamma_e) \cdot u_{2,3} \approx 100 \text{ kpc} \cdot u_{2,3} \cdot Q \left( \frac{\nu_{\text{obs}}(1 + z)}{0.63 \text{GHz}} \right)^{-1/2},$$

where $t_{\text{rad}}$ is the radiative energy loss time scale of CRe, $u_{2,3} = v_d/10^3 \text{ km s}^{-1}$ is the downstream flow speed, $\nu_{\text{obs}}$ is the observation frequency and $z$ is the redshift of the host cluster (Kang 2016a). Here, the factor $Q$ is defined as

$$Q(B, z) \equiv \left[ \frac{5 \mu G^2}{B^2 + B_{\text{rad}}(z)^2} \right] \left( \frac{B}{5 \mu G} \right)^{1/2},$$
where $B_{\text{rad}} = 3.24 \mu G (1 + z)^2$ takes account for energy losses due to inverse Compton (iC) scattering off the cosmic background radiation and $B$ is expressed in units of $\mu G$ [Kang 2011]. Besides $\Delta t_{\text{cool}}$, the radio flux density profiles projected in the sky plane are also affected by the geometrical shape of the downstream volume and the viewing orientation (see Figure 1 of Kang 2015).

In Kang (2016b) (Paper I), we attempted to reproduce the observed radio flux density profiles and the integrated spectrum by the reacceleration model in which a shock of the sonic Mach number, $M_s \approx 3$, sweeps through a finite-sized cloud with a preexisting population of CRe, $f_{\text{pre}} \propto p^{-s} \exp[-(p/p_{c,e})^2]$ with $s = 4.1$. A few shortcomings of this scenario are (1) the preexisting CRe is required to have a flat energy spectrum with high cutoff energy ($\gamma_{c,e} = p_{c,e}/m_e c \approx 3 \times 5 \times 10^3$), (2) the dimension of the pre-shock region with such CRe should be as large as $\sim 400$ kpc across the width of the relic and $\sim 2$ Mpc along the length of the relic, and (3) the adopted temperatures of the preshock and postshock region, $kT_1 = 3.4$ keV and $kT_2 = 10.7 - 13.1$ keV are higher than the observed values, $kT_{1,\text{obs}} = 2.7^{+0.7}_{-0.4}$ keV and $kT_{2,\text{obs}} = 8.5^{+0.6}_{-0.4}$ keV [Akamatsu et al. 2015], respectively. Hereafter, the subscripts 1 and 2 identify the upstream and downstream states of a shock, respectively.

The first and second requirements may be considered somewhat unrealistic, as CR electrons need to be accelerated to $\gamma_{c,e} > 3 \times 10^4$ in the cold radiative environment (30 Myr) in $\mu G$-level magnetic fields. So it would be challenging to maintain or replenish CRe with such a high cutoff energy ($\gamma_{c,e} \lesssim 300$) that have long cooling times ($t_{\text{cool}} \gtrsim 3.5$ Gyr), so they merely provide seed electrons to be injected to the Fermi I process. (2) The radio spectral index of the relic at the shock location is determined by the shock Mach number, $M_s \approx 3$, instead of the energy spectrum of preexisting CRe (i.e. $s$ and $\gamma_{c,e}$). (3) The shock-accelerated electrons are further accelerated by the Fermi II process due to postshock turbulence, delaying the spectral aging of CRe behind the shock.

In the next section, the numerical simulations and the shock models are described. The comparison of our results with observations is presented in Section 3, followed by a brief summary in Section 4.

2. NUMERICAL CALCULATIONS

The numerical setup for our DSA simulations was described in detail in Paper I and Kang et al. (2017). Some basic features are repeated here in order to make this paper self-contained.

2.1. DSA Simulations for 1D Spherical Shocks

We assume that the Sausage relic can be represented by a wedge-like patch of a spherical shell shown in Figure 1 of Kang (2015), whose depth along the line-of-sight is specified by the extension angle $\psi$. The spherical shell that contains radio-emitting electrons is assumed to be generated by a spherically expanding shock and its downstream volume.

We follow the electron acceleration by DSA at the shock, and radiative cooling and turbulent acceleration in the postshock region by solving the diffusion-convection equation in the one-dimensional (1D) spherically symmetric geometry:

$$\frac{\partial g_e}{\partial t} + \frac{\partial g_e}{\partial r} = \frac{1}{3r^2} \frac{\partial}{\partial r} \left( r^2 \kappa(r,p) \frac{\partial g_e}{\partial r} \right) + p \frac{\partial}{\partial y} \left( \frac{D_{pp}}{p^2} \frac{\partial g_e}{\partial y} - 4g_e \right) + \frac{\partial}{\partial y} \left( \frac{b}{p^2} g_e \right),$$

where $f_e(r,p,t) = g_e(r,p,t)p^{-4}$ is the pitch-angle-averaged phase space distribution function for CRe, $u(r,t)$ is the flow velocity, $y = \ln(p/m_e c)$, $m_e$ is the electron mass, and $c$ is the speed of light [Skilling 1973]. Here $r$ is the radial distance from the cluster center.

We adopt a Bohm-like spatial diffusion coefficient, $\kappa(p) = \kappa_N \cdot (p/m_e c)$ for relativistic electrons, where the normalization factor, $\kappa_N = k_B \cdot m_e c^3/(3eB) = k_B \cdot 1.7 \times 10^{19}\text{cm}^2\text{s}^{-1}/B_{1G}$, with $B_{1G}$ expressed in units of $\mu G$. The numerical factor, $k_B$, depends on the strength of turbulent magnetic fields, $\delta B$ and becomes $k_B = 1$ for Bohm diffusion that represents the particle diffusion in completely random fluctuating fields. The electron energy loss term, $b(p) = b_{\text{Coul}} + b_{\text{em+cIC}}$, accounts for Coulomb scattering, synchrotron emission, and iC scattering off the cosmic background radiation (e.g., Sarazin 1999).
Here we explore a scenario in which the postshock electrons gain energy from turbulent waves via Fermi II acceleration, thus abating spectral aging downstream of the shock. As in Kang et al. (2017), we consider a simple model based on TTD resonance with compressive fast-mode MHD turbulence, since that is likely to be the most efficient turbulent acceleration process in the ICM (Brunetti & Lazarian 2007, 2011). The momentum diffusion coefficient for TTD resonance can be approximated as

\[ D_{pp} = \frac{p^2}{4 \tau_{\text{acc}}}, \]

where \( \tau_{\text{acc}} \) is an effective acceleration time scale for turbulent acceleration. In order to model the decay of turbulence behind the shock, we assume the turbulent acceleration time increases behind the shock on the scale of \( r_{\text{dec}} \) as

\[ \tau_{\text{acc}} = \tau_{\text{acc,0}} \cdot \exp \left( \frac{(r_s - r)}{r_{\text{dec}}} \right) \]

with, in most of our simulations, \( \tau_{\text{acc,0}} \approx 10^8 \) yr and \( r_{\text{dec}} \approx 100 \) kpc.

### 2.2. Injection versus reacceleration Model

In this study, we consider the two kinds of DSA models: (1) in the injection model, suprathermal particles are generated via plasma kinetic processes near the shock and injected into the Fermi I process at the shock, while (2) in the reacceleration model, preexisting low energy CRe are injected into the Fermi I process. In both models, the injected electrons are accelerated via DSA at the shock, and then they cool radiatively while being accelerated via turbulent acceleration behind the shock.

In the injection model, the electron population injected in situ from the background suprathermal population and then accelerated by DSA is modeled as

\[ f_{\text{inj}}(r_s, p) = f_N \left( \frac{p}{p_{\text{inj}}} \right)^{-q \exp \left( -\left( \frac{p}{p_{\text{eq}}} \right)^2 \right)} , \]

where \( f_N, q = 4M_s^2/\left( M_s^2 - 1 \right), p_{\text{inj}}, \) and \( p_{\text{eq}} \) are the normalization factor, the standard test-particle DSA power-law slope, the injection momentum, and the cutoff momentum, respectively. The injection momentum represents the low momentum boundary above which particles have mean free paths large enough to cross the shock transition and thus participate in the Fermi I acceleration process. Here, we adopt a simple model in which the electron injection depends on the shock strength as \( p_{\text{inj}} \approx (6.4/\sigma) m_s u_s \) (where \( \sigma = p_s/p_i \) is the shock compression ratio), in effect, resulting in \( p_{\text{inj}} \sim 150 m_s u_s \). The cutoff momentum can be estimated from the condition that the DSA acceleration rate balances the synchrotron/jC loss rate (Kang 2011). For typical parameters for the ICM shocks, \( u_s \sim 3 \times 10^3 \) km s\(^{-1}\) and \( B_1 \sim 1 \mu G \), if we assume Bohm diffusion, it becomes \( p_{\text{eq}}/m_s c \sim 10^9 \).

The factor \( f_N \) depends on the suprathermal electron population in the background plasma, which is assumed to be energized via kinetic plasma processes at the shock and form a \( \kappa \)-distribution, rather than a Maxwellian distribution (Pierrard & Lazarian 2010). The value of \( \kappa \) is expected to depend on the shock parameters such as the obliquity angle and the sonic and Alfvénic Mach numbers, in addition to the plasma parameters of the background medium. For instance, the electron energy spectrum measured in the interplanetary medium near the Earth orbit can be fitted with the \( \kappa \)-distribution with \( \kappa \sim 2 - 5 \) (Pierrard & Lazarian 2010). Here we adopt a somewhat flatter value of \( \kappa \sim 1.6 \) to maximize the electron injection rate. Figure 1 illustrates the \( \kappa \)-distribution (dot-dashed line) for \( p < p_{\text{inj}} \) and \( f_{\text{inj}}(r_s, p) \) (solid line) for \( p > p_{\text{inj}} \) in one of the models considered below. For \( p > p_{\text{inj}} \), the \( \kappa \)-distribution can be approximated as \( f_{\kappa} \propto p^{-2(\kappa + 1)} \), so the amplitude \( f_N \) becomes smaller for a larger value of \( \kappa \). For example, the factor \( f_N \) for \( \kappa = 2.5 \) is smaller by a factor of about 200 than that for \( \kappa = 1.6 \).

In the case of the reacceleration model, the preexisting seed CRs are assumed to have a power-law spectrum with exponential cutoff as follows:

\[ f_{\text{pre}}(p) = f_o \cdot p^{-s} \exp \left( -\left( \frac{p}{p_{\text{c,e}}} \right)^2 \right) , \]

where the slope \( s = 4.6 \) and the cutoff \( \gamma_{c,e} = 300 \) are adopted for all models considered here (see the red dotted line in Figure 1). But the exact shape of \( f_{\text{pre}}(p) \) is not important, because the only significant role of these low-energy CRe is to provide seed particles to be injected to the DSA process. Note that the electrons with...
\( \gamma_{e,c} < 300 \) cool on the time scales longer than 3.5 Gyr, so they could represent fossil electrons in the ICM that are left over from AGN jets ejected early on. The normalization factor, \( f_o \), can be parameterized with the ratio of the preexisting CRe pressure to the gas pressure in the preshock region, \( N \equiv P_{\text{CRe,1}}/P_{\text{gas,1}} \propto f_o \) for a given set of \( s \) and \( p_{e,c} \). In the models considered here, typically, the models with \( N \sim 10^{-4} \) produce the radio flux profiles that can match the amplitude of observed flux in the Sausage relic.

The reaccelerated population of \( f_{\text{pre}}(p) \) at the shock can be calculated by

\[
  f_{\text{reac}}(r_s, p) = q \cdot p^{-q} \int_{p_{\text{eq}}}^{p} p'^{q-1} f_{\text{pre}}(p') dp' \tag{8}
\]

[Drury 1983]. Note that if the DSA slope, \( q \), is flatter (smaller) than the slope, \( s \), of the preexisting population, the downstream energy spectrum does not have any memory of the upstream spectrum other than its amplitude. As can be seen in the red dashed and black solid lines in Figure 1, both \( f_{\text{reac}}(r_s, p) \) and \( f_{\text{inj}}(r_s, p) \) have the same power-law form.

Since the time scale for DSA at the shock is much shorter than the electron cooling time scale, we can assume that electrons are accelerated almost instantaneously to \( p_{\text{eq}} \) at the shock front. Moreover, the minimum diffusion length scale to obtain converged solutions in simulations for diffusion-convection equation is much smaller than the typical downstream cooling length of \( \sim 100 \text{ kpc} \). Taking advantage of such disparate scales, we adopt analytic solutions for the electron spectrum at the shock location as \( f(r_s, p) = f_{\text{inj}}(r_s, p) \) or \( f_{\text{reac}}(r_s, p) \), while Equation (8) is solved outside the shock. So, basically we follow the energy losses and turbulent acceleration of electrons behind the shock, while the DSA analytic solutions are applied to the zone containing the shock.

### 2.3. Shock Parameters

It is not well understood how merger-driven shocks evolve dynamically as they propagate in the cluster periphery. In a major binary merger, shocks are launched after core passage of the two subclumps and propagate beyond the virial radius of the newly formed cluster ([van Weeren et al. 2011]). It is expected that in general shock speeds increase during the initial launch period and may decrease later during the expansion phase. In a realistic cluster merger, however, the merger is likely to involve subsequent infall of more subclumps along the filaments connected with the cluster. So the dynamical evolution of a merger shock can be quite complex ([e.g. Paul et al. 2011]).

Here we assume that the shock dynamics can be approximated by a self-similar blast wave that propagates through the isothermal ICM with the density profile of \( n_{\text{H}} = 10^{-4} \text{ cm}^{-3} (r/0.8 \text{ Mpc})^{-2} \), where \( n_{\text{H}} \) is the number density of hydrogen atom. So the shock radius and velocity evolves roughly as \( r_s \propto t^{2/3} \) and \( u_s \propto t^{-1/3} \), respectively, where \( t \) is the time since the point explosion for the spherical blast wave ([e.g. Ryu & Vishniac 1991]). During the simulation time period of \( \sim 200 \text{ Myr} \), the model shock speed decreases by a factor of \( \lesssim 1.3 \).

The ICM temperature upstream and downstream of the relic edge is observed to be \( kT_1 = 2.7^{+0.7}_{-0.4} \text{ keV} \) and \( kT_2 = 8.5^{+0.8}_{-0.6} \text{ keV} \), respectively, which indicates the sonic Mach number of \( M_s \approx 2.7 \) ([Akamatsu et al. 2015]). Since the shock speed decreases in time our

### Table 1. Model Parameters for the Sausage Radio Relic

| Model  | \( M_{\text{inj}} \) | \( kT_1 \) (keV) | \( B_1 \) (\( \mu \text{G} \)) | \( L_{\text{cloud}} \) (kpc) | \( t_{\text{exit}} \) (Myr) | \( t_{\text{obs}} \) (Myr) | \( M_{\text{obs}} \) | \( kT_{2,\text{obs}} \) (keV) | \( u_{s,\text{obs}} \) (km s\(^{-1}\)) | \( N \) | remarks |
|--------|-------------------|-----------------|-----------------|-------------------|-----------------|-----------------|----------------|-----------------|-----------------|--------|----------|
| M3.5a  | 3.5               | 2.5             | 1               | 420               | 144             | 155             | 2.97           | 9.0             | \( 2.4 \times 10^4 \) | 1.6 \times 10^{-4} | seed CRe |
| M3.5b  | 3.5               | 2.5             | 1               | 485               | 167             | 177             | 2.93           | 8.8             | \( 2.4 \times 10^4 \) | 1.6 \times 10^{-4} | seed CRe |
| M3.5c  | 3.5               | 2.5             | 1               | 581               | 200             | 214             | 2.86           | 8.5             | \( 2.4 \times 10^4 \) | 1.6 \times 10^{-4} | seed CRe |
| M4.0a  | 4.0               | 2.1             | 1               | 451               | 144             | 159             | 3.34           | 9.1             | \( 2.5 \times 10^4 \) | 1.2 \times 10^{-4} | seed CRe |
| M4.0b  | 4.0               | 2.1             | 1               | 520               | 167             | 180             | 3.28           | 8.9             | \( 2.4 \times 10^4 \) | 1.2 \times 10^{-4} | seed CRe |
| M4.0c  | 4.0               | 2.1             | 1               | 624               | 200             | 211             | 3.21           | 8.6             | \( 2.4 \times 10^4 \) | 1.2 \times 10^{-4} | seed CRe |
| M4.0cl | 4.0               | 2.1             | -               | 200               | 211             | 3.21           | 8.6             | \( 2.4 \times 10^4 \) | -              | injection |
| M4.0cB | 4.0               | 2.1             | 2.5             | 624               | 200             | 211             | 3.21           | 8.6             | \( 2.4 \times 10^4 \) | 1.2 \times 10^{-4} | stronger B |
| M4.0eN | 4.0               | 2.1             | 1               | 624               | 200             | 211             | 3.21           | 8.6             | \( 2.4 \times 10^4 \) | 1.2 \times 10^{-4} | No TA    |
According to Akamatsu et al. (2015), the discontinuity in the X-ray temperature distribution agrees well with the outer edge of the Sausage relic within the angular resolution of the Suzaku X-ray observation (2 arcmin≈ 384 kpc). In the case of the Toothbrush relic, on the other hand, the spatial offset of ≈ 1 arcmin between the X-ray shock and the relic edge was indicated in the XMM-Newton observation by Ogrean et al. (2013). However, such discrepancy was rebutted by van Weeren et al. (2016) where the refitted XMM-Newton observation by Ogrean et al. (2013) shows the model parameters for the DSA simulations considered here.

At the onset of the simulations (t = tₖ), the initial shock speed, u₀, is specified by Mₖ and KT₁, while the shock location is assumed to be rₖ = 0.8 Mpc. This fixes the initial time t₀ when the shock encounters the cloud of preexisting CRs, and the scaling factors for the similarity solution, ρᵣ, uᵣ, and t₀. We define the “shock age”, tₕ = t - t₀, as the time since the onset of the simulations.

As in Paper I, in order to reproduce the spectral steepening about 2 GHz, we assume that, in the reacceleration model, at the onset of the simulations the shock encounters a cloud of size L₋₀ containing preexisting seed CRs, and then exits out of it at tₑₓ. So the size L₋₀ affects the postshock profiles of radio flux densities. The ‘time of observation’, tₑₓ, is chosen when both the simulated brightness profiles and the integrated spectra become consistent with the observations reported by Stroe et al. (2016). Between the exit time, tₑₓ, and tₑₓ, the shock sweeps the region devoid of preexisting CRs, which results in steepening of the volume-integrated electron energy spectrum. As a result, the elapsed period of (tₑₓ - tₑₓ) ≈ 10 – 15 Myr controls the spectral curvature of the integrated radio spectrum.

The fiducial value of the preshock magnetic field strength is set to be B₁ = 1µG, which is assumed to be uniform in the upstream region. As in Paper I, the postshock magnetic field strength is modeled as B₂(t) = B₁√1/3 + 2σ(t)/3 ≈ 2.5 – 2.7µG, which decreases slightly as the shock compression ratio, σ, decreases in time in response to shock evolution. For the downstream region (r = rₛ), we assume a simple model in which the magnetic field strength scales with the gas pressure as Bₛ(r,t) = B₂(t) · [P(r,t)/P₂(t)]²/3, where P₂(t) is the gas pressure immediately behind the shock (see Figure 2).

We adopt the model naming convention in Table 1, where the number after the first letter 'M' corresponds to Mₖ. This is followed by a sequence label (a, b, c) as the size of the cloud containing preexisting CRs, L₋₀, increases. The M₄.0c model is the reacceleration model with fossil CRs with M₅ = 4.0 and
$L_{\text{cloud}} = 624$ kpc. The M4.0cI model is the injection model in which only in situ injection from background suprathermal electrons is included, while the M4.0cB model adopts a preshock magnetic field strength higher than that of the rest of the models. In M4.0cI, the in situ injection is turned on at the onset of the simulation, and then it is turned off at $t_{\text{exit}} = 200$ Myr to create a spectral curvature at high frequencies. In the M4.0cN model, turbulent acceleration is turned off to demonstrate its effects on the postshock spectral aging.

The eighth and ninth columns of Table 1 show the shock Mach number and the postshock temperature at $t_{\text{obs}}$: $M_{s,\text{obs}} = 2.9 - 3.3$ and $kT_{\text{obs}} = 8.5 - 9.1$ keV, which are reasonably consistent with the X-ray observations reported by Akamatsu et al. (2013).

We note here that $M_X$ inferred from X-ray observations could be lower than $M_{\text{radio}}$ estimated from radio spectral index, since a radio relic may be associated with multiple shocks. According to mock observations of cluster shocks formed in structure formation simulations, X-ray observations tend to pick up shocks with lower $M_s$ along a given line-of-sight, while radio emissions come preferentially from shocks with higher $M_s$ (e.g., Hong et al. 2015). In the case of the Toothbrush relic, it was shown that $M_s \approx 3.0$ is required to reproduce the radio data, while the X-ray data indicate $M_X \approx 1.2 - 1.5$ (Kang et al. 2017).

Finally, the eleventh column shows, $N \approx 10^{-4}$, the ratio of the preexisting CRe pressure to the upstream gas pressure that can generate radio flux densities consistent with the observations reported by Stroe et al. (2016).

3. RESULTS OF DSA SIMULATIONS

Figure 2 shows the DSA simulation results for the M4.0c model at three epochs: just before the shock exits out of the cloud at $t_{\text{exit}} \approx 200$ Myr (red dashed lines), at the time of observation, $t_{\text{obs}} = 211$ Myr (black solid), and at $t_{\text{age}} = 225$ Myr (blue dot-dashed). The upper left panel shows the profiles of the magnetic field strength, which contain a discontinuous jump at the shock location, $r_s(t) = 1.33 - 1.4$ Mpc. Note that the shock expands radially outward in the left-hand panels of Figure 2.

3.1. Surface Brightness and Spectral Index Profiles

Using the CRe energy spectrum and the magnetic field strength in the model DSA simulations, we first calculate the synchrotron emissivity $j_{\nu}(r)$ of each spherical shell. The lower left panel of Figure 2 demonstrates that the outermost edge of the synchrotron emissivity at 153 MHz, $j_{153\text{MHz}}(r)$, lags behind the shock location after the shock moves out of the cloud at $t_{\text{exit}}$.

The radio surface brightness, $I_{\nu}(R)$, is calculated by integrating $j_{\nu}(r)$ along a given line-of-sight, where a wedge-like postshock volume of radio-emitting electrons is adopted, as in Figure 1 of Kang (2013). Here $R$ is the distance behind the projected shock edge in the plane of the sky. This volume is specified by the extension angle, $\psi$, which is assumed to be about $10^\circ$ in the case of the Sausage relic (e.g., van Weeren et al. 2010, Kang et al. 2012). The upper right panel of Figure 2 shows $I_{153\text{MHz}}(R)$ at 153 MHz (in arbitrary units), using $\psi = 10^\circ$. Note that the shock faces to the left in the right-hand panels of Figure 2, so the region of $R < 0$ is the preshock region. Again, one can see that the edge of the radio relic is located behind the shock (at $R = 0$) after $t_{\text{exit}}$. The spectral index profile, $\alpha_{153}(R)$ in the lower right panel is calculated with the ratio between $I_{153\text{MHz}}(R)$ and $I_{608\text{MHz}}(R)$.

In order to obtain beam-convolved flux density, the intensity $I_{\nu}(R)$ is smoothed by Gaussian smoothing with 51.7 kpc width (equivalent to 16.14") for 153 MHz and 13.4 kpc width (equivalent to 4.2") for 608 MHz. For the profile of the spectral index, $\alpha_{153}(R)$, both $I_{153\text{MHz}}(R)$ and $I_{608\text{MHz}}(R)$ are smoothed with the same width of 51.7 kpc.

Figure 3 shows the time evolution of $S_{153\text{MHz}}(R)$, $S_{608\text{MHz}}(R)$, and $\alpha_{153}(R)$ in the M4.0a, b, c models with different cloud size $L_{\text{cloud}}$. The times of observation, $t_{\text{obs}} = 159, 180,$ and 211 Myr (black solid lines) are chosen for the M4.0a, b, c models, respectively. At the times earlier (red dashed lines) or later (blue dot-dashed lines) than $t_{\text{obs}}$, are shown for comparison. Since the shock slows down and moves out of the cloud of preexisting CRe at $t_{\text{exit}}$ (given in Table 1), the amplitude of $S_{\nu}$ decreases in time.

The observed flux density at 153 MHz for the beam of 16.14"×13.75" is $S_{153\text{MHz}} \approx 0.014$ Jy at $R \approx 55$ kpc (Stroe et al. 2016). In Figure 3, the normalization factor of $S_{\nu}$ and its peak location are chosen so that the black solid lines match the observed data represented by the magenta solid circles. The amount of preexisting CRe that matches the observed flux density corresponds to $N \approx 1.6 \times 10^{-4}$ (see Table 1).

Although the three models can reproduce reasonably well both $S_{153\text{MHz}}(R)$ and $S_{608\text{MHz}}(R)$, the M4.0c model (at 211 Myr) can fit best the observed profile of $\alpha_{153}(R)$. So we take the M4.0c model as the ‘fiducial’ model in this discussion. In the M4.0a and M4.0b models, $L_{\text{cloud}}$ is smaller, so the shock exits out of the cloud earlier, resulting in less spectral aging for $R > 150$ kpc at $t_{\text{obs}}$, compared to the M4.0c model (see the bottom panels of Figure 3). Of course, if we were to choose a later epoch for $t_{\text{obs}}$ for these two models, the spectral index profile would become more comparable to the observations. As we will show in Figure 5 below, however, the time interval of $t_{\text{obs}} - t_{\text{exit}}$ becomes longer in those cases, resulting in the spectral curvature of the integrated radio spectrum much steeper than observed.

Note that we do not attempt to fit the flattest data for $\alpha_{153} \approx 0.54$ at $R \approx 0$ kpc. This allows us to choose a much smaller shock Mach number, i.e., $M_{s,\text{obs}} \approx 3.2$ for the M4.0c model, instead of $M_{\text{radio}} \approx 4.6$ suggested in earlier papers (e.g., van Weeren et al. 2010).

The right-hand panels of Figure 4 show the results at $t_{\text{obs}} = 155$, 177, and 214 Myr for M3.5a, b, c models, respectively. The M3.5c model at $t_{\text{obs}} = 214$ Myr seems to give the best fit to the observations. But the profiles of $\alpha_{153}(R) \gtrsim 0.8$ for $R < 80$ kpc are slightly steeper.
Figure 3. Beam convolved brightness profiles $S_{\nu}(R)$ at 153 MHz (top panels) and at 608 MHz (middle panels), and the spectral index $\alpha_{608}^{153}$ between the two frequencies (bottom panels) are plotted for three shock ages (red, black, and blue lines), specified in the bottom panels. Here $R$ is the projected distance behind the shock in units of kpc. The extension angle, $\psi = 10^\circ$, is adopted. The results are shown for the M4.0a model with $L_{\text{cloud}} = 451$ kpc (left-hand panels), M4.0b model with $L_{\text{cloud}} = 520$ kpc (middle panels), and M4.0c model with $L_{\text{cloud}} = 624$ kpc (right-hand panels). The simulated brightness profiles, $I_{\nu}(R)$, are smoothed with Gaussian smoothing with $51.7$ kpc (equivalent to the beam angle $\theta_1 = 16.14^\prime$) for 153 MHz and with $13.4$ kpc (equivalent to $\theta_1 = 3.42^\prime$) for 608 MHz to be compared with the observed flux profiles of Stroe et al. (2016) (magenta filled circles).

than the observed profile in these three models with $M_{s,\text{obs}} = 2.9 - 3.0$.

The left-hand panels of Figure 4 compare the M4.0cB (stronger $B_1$, red dashed lines) with the M4.0c model. Stronger magnetic fields enhance the synchrotron emission and cooling, resulting in higher radio flux densities and a steeper profile of $\alpha_{608}^{153}(R)$. So we reduce $S_{\nu}$ by a factor of 0.23 for the M4.0cB model in order to plot both models with the same normalization scaling.

In the middle panels of Figure 4, the M4.0cI (injection only, blue dot-dashed lines) and M4.0cN (no turbulent acceleration, red dashed lines) are compared with the fiducial M4.0c model. As mentioned in Section 2.2, the normalization for the injection model depend on the value of the $\kappa$ index for the suprathermal electrons in the background plasma (see Figure 1). With the adopted value $\kappa = 1.6$ in M4.0cI, the peak value becomes $S_{153\text{MHz}} \approx 0.018$ Jy, so the radio flux densities are scaled down by a factor of 0.77 for this model in order to compare them with the observational data in the Figure. We note, however, a more realistic value would be $\kappa > 2$, so the amplitude of $S_{153\text{MHz}}$ in the injection model might be much smaller than observed. In the M4.0cN model without turbulent acceleration, $S_\nu$ is smaller and $\alpha_{608}^{153}$ is steeper, compared to the M4.0c model.

Figures 3 and 4 show that the predictions of the M4.0c model convolved with appropriate beam widths are in reasonable agreement with the observations, providing that there exist fossil CRE with $N \approx 10^{-4}$ in the ICM. This exercise demonstrates that the profiles of observed radio flux density, $S_{\nu}(R)$, at multi frequencies can provide strong constraints on the model parameters for radio relics.
Figure 4. Same as Figure 2 except that the M4.0c and M4.0cB models with the extension angle $\psi = 10^\circ$ are compared in the left-hand panels, the M4.0c, M4.0cN, and M4.0cI models with $\psi = 10^\circ$ are compared in the middle panels, and the M3.5a, M3.5b, and M3.5c models with $\psi = 12^\circ$ are compared in the right-hand panels. The radio flux density, $S_\nu$, is multiplied by a factor of 0.23 for M4.0cB and 0.77 for M4.0cI with respect to $S_\nu$ for the fiducial model, M4.0c. 

3.2. Integrated Spectrum

As shown in Paper I, the spectral curvature in the observed integrated radio spectrum of the Sausage relic cannot be reproduced by a simple DSA model for a steady planar shock. But it can be explained if we adopt an addition condition for a finite size of the cloud with preexisting CRe. We note that in the \textit{in situ} injection model (M4.0cI), the same kind of curvature can be created somewhat artificially by turning off the injection after $t_{\text{exit}} = 200$ Myr. Figure 5 shows the time evolution of the integrated spectrum, $\nu J_\nu$, for six different models. The red dashed lines for each model show the spectrum at the first epoch just before the shock exits out of the cloud. They follow roughly the predictions based on the postshock radiative cooling, i.e., steepening of $J_\nu$ from $\nu^{-\alpha_s}$ to $\nu^{-(\alpha_s+0.5)}$ at $\sim$GHz. Such description is only approximate here, because additional turbulent acceleration operates in the postshock region.

Then the green dot-dashed lines, black solid lines, magenta dot-long dashed lines, and blue long dashed lines present the spectra with progressively steeper curvatures at four later epochs in chronological order. The open magenta squares and the error bars are observational data taken from Table 3 of [Stroe et al. (2016)]. Basu et al. (2016) calculated that the amount of the Sunyaev-Zeldovich (SZ) decrement in the observed radio flux for several well-known radio relics, based on models for the ICM electron density profile and the radio flux profile. Although we know such predictions depend sensitively on those model details, we adopt their estimates for the SZ contamination factor for the Sausage relic given in their Table 1. Then, the SZ correction factors, $F$, for the fluxes at 16 GHz and 30 GHz are about 1.1 and 1.96, respectively. The two solid circles in each panel of Figure 5 correspond to the flux levels so-corrected at the two highest frequencies.

Note that Stroe et al. (2016) suggested that the observed spectrum could be fitted by a broken power-law: $\alpha = 0.90 \pm 0.04$ below 2 GHz and $\alpha = 1.77 \pm 0.13$ above 2 GHz. The black solid lines at $t_{\text{obs}}$ for each model are chosen as the best fits to the observed spectrum in the range of $1 - 3$ GHz. All six models seem to generate similar integrated spectra, although the simulated profiles of $\alpha_{608}^{153}(R)$ are rather different as shown in Figures.
Figure 5. Time evolution of volume-integrated radio spectrum at five shock ages, specified in each panel, are shown in chronological order by the red dashed, green dot-dashed, black solid, magenta dot-long dashed, and blue long dashed lines. The open magenta squares and the error bars are the observational data taken from Stroe et al. (2016). The solid black circles at 16 GHz and 30 GHz are the data points, multiplied by factors of 1.11 and 1.96, respectively, which could represent the SZ-corrected fluxes (Basu et al. 2016).

3 and 4. Considering that the observation errors in the flux data is only 10% for $\nu \lesssim 3$ GHz, it seems somewhat difficult to fit very well the observational data both below and above 1 GHz simultaneously with our model predictions. We conclude the fiducial model M4.0c is the best case, in which the predictions for both $\alpha_{608}^{\nu J}(R)$ and $\nu J_{\nu}$ are in reasonable agreement with the observations.

Note that in previous studies including Paper I the integrated spectrum was often presented in the form of $J_{\nu}$ typically over four orders of magnitudes, so it gave much better visual impressions for the comparison between the predicted and the observed spectra.

4. SUMMARY

Many of observed features of giant radio relics are thought to be explained by the shock acceleration model: elongated shapes on scales of Mpc, radio spectral index steepening toward the cluster center, and high polarization levels (van Weeren et al. 2010; Stroe et al. 2016). Among some remaining puzzles concerning the shock acceleration model, in the case of the Sausage relic, we notice (1) the steep spectral curvature above GHz in the volume-integrated spectrum (Stroe et al. 2016) and (2) the discrepancy between the X-ray based shock Mach number, $M_{s} \approx 2.7$ and the radio based value, $M_{\text{radio}} \approx 4.6$ (Akamatsu et al. 2015; van Weeren et al. 2010). To understand these features, in earlier studies we explored the reacceleration scenario, in which a weak shock with $M_{s} \approx 3$ propagates through a finite-size cloud of the ICM gas, containing a flat spectrum of preexisting CRe (Kang & Ryu 2016; Kang 2016d). Considering the short cooling time of GeV electrons, however, it remains challenging to explain how to maintain such a flat population of high-energy electrons over a large preshock volume (Kang et al. 2017).

In this study, we explore an alternative model in which a shock of $M_{s} \approx 3 - 4$ sweeps through a preshock cloud containing low-energy fossil electrons and the electron aging is delayed by Fermi II acceleration by postshock turbulence. Here preexisting CRe with $\gamma_{e} \lesssim 300$ provide only seed electrons to Fermi I process, so the slope of the electron spectrum at the shock is determined by the sonic Mach number, i.e., $q = 4M_{s}^{2}/(M_{s}^{2} - 1)$. This eliminates the unrealistic requirements for a flat power-law spectrum ($s = 4.1$) with a high energy cutoff ($\gamma_{e,c} \approx 3 - 5 \times 10^{4}$) adopted in Kang (2016d) (Paper I). Stochastic acceleration via transit time damping resonance off compressive MHD turbulence in the postshock region is adopted, since the observed width of the Sausage relic is somewhat too broad to be explained solely by the electron cooling.
length (see the M4.0cN model in Figure 4). We find that turbulent acceleration with $\tau_{acc} \approx 10^8$ yr is required in order to match the observed broad profiles of the radio flux density, $S_\nu(R)$, of the Sausage relic. We note such a strength of turbulence acceleration is similar to what is required to reproduce the radio flux profiles of the Toothbrush radio relic (Kang et al. 2017).

Here we attempt to reproduce the observed profiles of $S_{1.53\text{MHz}}$, $S_{608\text{MHz}}$, and the spectral index $\alpha_{608}$ as well as the volume-integrated spectrum $J_v$ of the Sausage radio relic (Stroe et al. 2016). In the best fitting fiducial model, M4.0c (see Table 1 for the model parameters), the spherical shock with the initial Mach number $M_{s,i} = 4.0$ and the radius $r_{s,i} = 0.8$ Mpc encounters the cloud of preexisting CRe and then sweeps out of the cloud after $t_{exit} \approx 200$ Myr. It turns out that the degree of the spectral steepening above GHz in $J_v$ strongly constrains the duration, $t_{obs} - t_{exit} \approx 10$ Myr, during which the shock propagates in the preshock region without fossil CRe. At the time of observation, the model shock weakens to $M_{s,obs} \approx 3.2$ and the postshock temperature becomes $kT_{2,obs} \approx 8.6$ keV, which are in reasonable agreements with X-ray observations (Akamatsu et al. 2015). Note that the M4.0c model does not reproduce the flattest observed index, $\alpha_{153} \approx 0.54$, at the relic edge, which requires a much stronger shock with $M_s \approx 6.0$.

As shown in Figures 3 and 4, the spectral index profile, $\alpha_{153}(R)$, provides the most stringent constraints to the model parameters such as $M_{s,i}$, $L_{\text{cloud}}$, $B_1$, and $\tau_{acc}$. The amount of fossil low-energy CRe that can produce the observed radio flux density corresponds to the pressure ratio $N = P_{\text{CRe}}/P_k \approx 10^{-4}$, which is dynamically insignificant. Considering that the observational error in $J_v$ is about 10%, we could argue that the model predictions in Figure 5 (log $\nu J_\nu$ versus log $\nu$) only marginally fit the observed integrated spectrum.

This study demonstrates that it is possible to explain most of the observed properties of the Sausage relic by the shock reacceleration model with fossil relativistic electrons and an additional postshock Fermi II acceleration. This scenario is consistent with the observational fact that only a small fraction (~10%) of merging clusters host radio relics (Ferretti et al. 2012). Thus we favor the DSA reacceleration model in which radio relics are generated preferentially when merger-driven shocks encounter the regions containing preexisting low-energy CRe.

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