On the possibility of determining antineutrino mass by measuring relative characteristics in β-decay processes

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Abstract. An influence of the neutrino (antineutrino) mass on the characteristics of nuclei and neutron β-decay is demonstrated. It has been shown that measuring relative characteristics allows one to get rid of the unknown nuclear structure, which makes it possible to determine the magnitude of the neutrino (antineutrino) mass in the β-decay processes with greater accuracy. An analysis was carried out based on general theoretical formulas for differential probabilities of the β-decays of the free neutron and tritium nucleus. As a result, it was possible to obtain simple formulas from which a theoretical estimation for the neutrino (antineutrino) rest mass can be obtained.

1. Introduction
Currently, most of the experiments that are carried out to determine the magnitude of the neutrino (antineutrino) mass are mainly focused on obtaining its value by studying the kinematic factor in the energy spectrum expression for the electrons probability emission with their energies very close to the maximum value (just near at the end of the β−-spectrum). However, it is desirable to take into account the influence of the neutrino mass in the lepton and nucleon parts of the matrix element (through the conservation energy and momentum laws). Because the nuclear weak current depends on the unknown nuclear structure, this is an extremely difficult task. In the present work, experiments are proposed to measure relative characteristics, such as the longitudinal polarization degree of the electrons (positrons) and the “forward-backward” electron (positron) emission asymmetry, which makes it possible to minimize the influence of not well known effects of the nuclear structure.

2. Materials and methods of research
Beta-decay becomes possible when the replacement of a neutron in a nucleus with a proton (or, conversely, a proton with a neutron) is energetically favorable and the resulting new nucleus has a smaller rest mass, and, therefore, a large binding energy. That is, the essence of the phenomenon is that the nucleus emits leptons (an electron and an electron antineutrino, or a positron and an electron neutrino), thereby passing into the nucleus with the same mass number A, but with a different atomic number Z, greater or less by unit depending on β±-decay. Excess energy is distributed between the reaction products.
2.1. The Lagrangian of the weak interaction

Based on the fact that each of the particles participating in the process satisfies the Dirac equation, Fermi constructed for $\beta^-$-decay of a neutron inside the nucleus

$$n \rightarrow p + e^- + \bar{\nu}_e$$

(1)

the interaction Lagrange function (invariant with respect to the Lorentz transformations having “vector by vector” form) [1]:

$$L = G_F \langle \bar{\Psi}_p \gamma_\mu \Psi_n \rangle \langle \bar{\Psi}_e \gamma_\mu \Psi_\nu \rangle$$

(2)

Here: $G_F = 1.026 \cdot 10^{-5} m_p^{-2}$ is the Fermi constant; $\bar{\Psi}_p$ is the proton creation operator; $\Psi_n$ is the neutron annihilation operator; $\Psi_\nu$ is the antineutrino creation operator; $\bar{\Psi}_e$ is the electron creation operator; $\gamma_\mu$ are the Dirac matrices. Later it turned out that in the $\beta^-$-decay of the neutron the parity is violated [2] and, in the current-current theory of the weak interaction Lagrangian for process (1), in the case of 4-component massive antineutrinos, can be written as [3]:

$$L = \sum_{j=\alpha,\nu} C_j \langle \bar{\Psi}_p O_j \Psi_n \rangle \langle \bar{\Psi}_e O_j \Psi_\nu \rangle$$

(3)

Here $O_{\nu} = \gamma_\mu$; $O_\alpha = \gamma_\mu \gamma_5$; $C_{\nu}, C_\alpha$ are the vector and axial-vector coupling constants [3].

In this case the squared module of the matrix element of the process (1) can be written as:

$$|M|^2 = M^+ M = C_j C_{j'} \langle \bar{u}_e O_{j'}^\dagger u_e \rangle \langle \bar{u}_n O_{\alpha}^\dagger u_n \rangle \langle \bar{u}_\nu O_{\alpha}^\dagger u_\nu \rangle$$

(4)

Hereinafter, summation over repeated indices is implied. In addition, we omitted in this formula the upper indices of the summation $j, j'$, which are nonetheless implied. In formula (4), the lepton and nucleon tensors are determined by the expressions: $L_{\alpha\beta} = \langle \bar{u}_e O_{\alpha}^\dagger u_e \rangle \langle \bar{u}_n O_{\beta}^\dagger u_n \rangle$ is the lepton tensor; $N_{\alpha\beta} = \langle \bar{u}_\nu O_{\alpha}^\dagger u_\nu \rangle \langle \bar{u}_\nu O_{\beta}^\dagger u_\nu \rangle$ is the nucleon tensor. In this expression (4) $u_i, \bar{u}_i$ are the Dirac spinor amplitudes of the corresponding wave functions, $i = e, \bar{\nu}, n, p$.

In the case of massive antineutrinos, the left and right states are present in the wave function:

$$\Psi_\nu \equiv \frac{1}{2} (1 + \gamma_5) \Psi_\nu + \frac{1}{2} (1 - \gamma_5) \Psi_\nu$$

(5)

In the form (3), the Lagrangian is quite symmetric and does not contain terms that violate P-parity. However, if one at the end of calculation fixes the antineutrino helicity to be only positive namely ($s_\nu = +1$), we should discard the second term in expression (5), then this immediately leads to a violation of spatial parity.

It follows from formula (4) that, in the general case, the square of the matrix element is not factorized (we cannot represent it as the product of scalar functions), since it is the product of lepton and nucleon tensors with summation over repeated indices. The expression for the differential probability of neutron decay can be represented by the formula [3]:

$$dW = (2\pi)^{-5} K |M|^2 E_e P_e E_\nu P_\nu d\Omega_e d\Omega_\nu$$

(6)

Here $K$ is the kinematic factor depending on the momentum $p$, energy $E$, and mass $m$ of the particles. The lepton tensor $L_{\alpha\beta}(E_\nu, m_\nu, \kappa, \theta, s_\nu)$ depends on the energy, mass of the antineutrino, the
admixture of the right currents (parameter $\kappa = a_d/a_v \neq 1$), the scattering angle $\theta$ of electron or antineutrino emission, and the electron helicity $s_e$. The nucleon tensor $N_{a\beta}(E_\nu, m_\nu, \kappa, G_2, \text{nucl. str.})$, depends on the magnitude of the neutrino mass (through the energy-momentum conservation laws), and on the nuclear structure and possible admixture of Weinberg type II currents $G_2$ [4, 5]:

$$N_{a\beta} = N_{a\beta}(E_\nu, m_\nu, \kappa, G_2, \text{nucl. str.})$$  \hspace{1cm} (7)

All these factors introduce uncertainties into the behavior of the electron energy spectrum curve at its end. In the general case, the decay probability is a product of tensors, but for pure Fermi transitions (when the nucleon current is mainly a vector) and for pure Gamow-Teller transitions [6] (when the nucleon current is mainly only axial), the tensor product in (4) can be factorized, thereby represented it as a product of two scalar functions:

$$L_{a\beta} \cdot N_{a\beta} = L(E_\nu, m_\nu, \kappa, \theta, s_e) \cdot N(E_\nu, m_\nu, \kappa, G_2, \text{nucl. str.})$$  \hspace{1cm} (8)

But even in this case, the probability of beta-decay is still proportional to the not well known nuclear structure contained in the nucleon part of this expression. Uncertainties contained in the nucleon function $N$ of the nuclear matrix element can be eliminated by measuring relative values: various asymmetries in the angular distribution, particle polarization, etc. Then the term $N$ in the numerator and denominator is reduced, which will be demonstrated in part 2.4 of this paper. For example, the degree of longitudinal polarization of electrons and the coefficient of angular electron-antineutrino correlation will be represented by the formulas:

$$P_{e^-} = \frac{[L(s_e = +1) - L(s_e = -1)]N(\kappa, G_2, \text{nucl. str.})}{[L(s_e = +1) + L(s_e = -1)]N(\kappa, G_2, \text{nucl. str.})} = \frac{L(s_e = +1) - L(s_e = -1)}{L(s_e = +1) + L(s_e = -1)}$$  \hspace{1cm} (9)

$$A_{e\nu} = \frac{[L(0^{\circ}) - L(180^{\circ})]N(\kappa, G_2, \text{nucl. str.})}{[L(0^{\circ}) + L(180^{\circ})]N(\kappa, G_2, \text{nucl. str.})} = \frac{L(0^{\circ}) - L(180^{\circ})}{L(0^{\circ}) + L(180^{\circ})}$$  \hspace{1cm} (10)

2.2 The dependence of the kinematic factor on the neutrino mass

Fermi believed that the main dependence on $m_\nu$ is concentrated in the kinematic factor, which has the form [1]:

$$W \sim K = \left(W_0 - E_e + m_\nu\right)\left(W_0 - E_e + m_\nu\right)^2 - m_\nu^2 \right)^{1/2}$$  \hspace{1cm} (11)

Here $W_0$ is the maximum electron energy; $m_\nu$ is the neutrino (antineutrino) rest mass; $E_e$ is the electron energy. It was assumed that the nuclear matrix element very weakly depends on $m_\nu$, that is, it can be considered a constant, which is true with a relative accuracy of $10^{-3}$. He analyzed the effect of the antineutrino mass at the end of the spectrum in the case of three different antineutrino mass values:

1. $m_\nu = 0$;
2. $m_\nu \gg 0$;
3. $m_\nu \gg 0$. 

As can be seen from figure 1 of [1], all three graphs converge at one point on the abscissa $E_e$, regardless of the antineutrino mass, which is not consistent with the law of energy conservation. In his reasoning, Fermi did not take into account the fact that, in the presence of a neutrino rest mass, the end of the electron spectrum graph should shift to the left precisely by the mass value, as shown in figure 2. In fact, part of the decay energy is spent on the formation of the antineutrino mass.

2.3 The dependence of the differential probability on the nuclear structure in the tritium $\beta$-decay

The differential probability of the experimentally observed $\beta$-decay process:

$$\frac{3}{1}H \rightarrow \frac{3}{2}He + e^- + \bar{\nu}_e$$

(12)

can be written as:

$$dW = N(E_e) dE_e$$

(13)

The energy spectrum of electrons $N(E_e)$ is given by the formula [7]:

$$N(E_e) = 4(2\pi)^{-3} G_F^2 \beta^2 \nu p_e E_e K f_{\text{WM}} F(Z, E_e) \xi F(F_i, E_e, W_0, m_\nu)$$

(14)

Here $\xi = a_e^2 (1 + \kappa^2)$; $\kappa = \frac{a_A}{a_V}$ and where $a_{\nu}, a_{A_{\nu}} (\beta_{\nu}, \beta_A)$ are the vector and axial-vector coupling constants of the lepton (hadron) currents; $p_e, E_e$ is the momentum and the energy of an electron; $f_{\text{WM}}$ - function taking into account the influence of the center of mass, $F(Z, E_e)$-coulomb correction factor.

In the nuclear shell model the Curie function can be written [7]:

$$\left[N(E_e)/p_e E_e\right]^2 = \frac{3}{2} (2\pi)^{-3} G_F^2 \beta^2 \nu p_e E_e K F(Z, E_e) \xi F(F_i, E_e, W_0, m_\nu)$$

(15)

where $F(F_i, E_e, W_0, m_\nu) = A_0 + A_1/M + A_2/M^2$ is a function that takes into account the influence of the nuclear structure.

Here

$$A_0 = F_A^2 (1 - C) + 3\lambda^2 F_A^4 (1 + C)$$

$$A_1 = -2\lambda F \nu (\beta_\nu p_\nu - \beta_e p_e) + \lambda^2 (F_A + q_0 F_P - 2MF_F) F_A (\beta_\nu p_\nu + \beta_e p_e)$$

$$A_2 = \frac{\mu^2}{\nu} \left[ (1 + C) p^2 + \frac{4}{3} \beta_\nu p_\nu \beta_e p_e \right] + \frac{\lambda^2}{4} (F_A + q_0 F_P - 2MF_F)^2 \left[ (1 - C) p^2 + \frac{2}{3} \beta_\nu p_\nu \beta_e p_e \right]$$

(16)
\[ p^2 = p_e^2 + p_\nu^2 \]
\[ C = \frac{m_e m_\nu(1 - \kappa^2)}{E_\nu E_e(1 + \kappa^2)} \]

where \( \lambda = \left| \frac{\beta_A}{\beta_\nu} \right|, \beta_e = \frac{p_e}{E_e}, \beta_\nu = \frac{p_\nu}{E_\nu}, \mu = F_1 + 2MF_2 \cdot (F_i - \text{form factors of the first (second) kind currents, } i = 1,2,A,P,(T,S)). \)

The first term \( A_0 \) takes into account the contribution of the right lepton currents and the antineutrino mass. The second term \( A_1 \) and the third term \( A_2 \) in formula (16) take into account the contribution of the nuclear structure through the form factors \( F_i \). In the Fermi approximation the matrix element \( F(F_i, E_e, W_0, m_\nu) \) can be given by a constant \( \sim (1 + 3\lambda^2) \).

**Figure 3.** Curie plot for tritium \( \beta \)-decay taking into account the nuclear matrix element \( F(F_i, E_e, W_0, m_\nu) \) in the form of a constant and for two antineutrino mass values \( m_\nu = 0 \) (dashed line), \( m_\nu \neq 0 \) (solid line).
Figure 4. Curie plot for tritium β-decay taking into account the uncertainties in the nuclear matrix element $F\left(F_l, E_e, W_0, m_\nu\right)$, which is different from the constant for two antineutrino mass values: $m_\nu = 0$ (dashed line), $m_\nu \neq 0$ (solid line). Two dashed lines define the range of uncertainties due to the influence of nuclear structure (formfactors of the first (second) class currents) depending on the sign of formfactors.

The contribution of second class currents turn out to be almost constant over the entire energy range of the β-spectrum, since it is determined by the form factor $F_T$ times the function $\left(\beta_e p_e + \beta_e p_v\right)$ in formula (16), which is almost constant and is very weakly dependent on the energy of an electron. The mass of the electron antineutrino leads, besides the well-known curve at the end of the spectrum, to an additional shift.

2.4 Antineutrino mass effects in the free unpolarized neutron β-decay

The squared module of the matrix element for β-decay of a free nonpolarized resting neutron, taking into account the longitudinal polarization of the electron and antineutrino, has the following form [3]:

$$|M|^2 = \frac{G_F^2 a_V^2}{4} \left\{ \eta_1 C_+ - \frac{1}{2} \eta_3 C_- - s_\nu s_\nu [2\eta_4 D_- - \eta_3 D_+] \cos \theta + m_p^{-1} \lambda (2\eta_1 + \eta_3) p (s_e p_e^0 + s_v p_v^0) \ight.$$  

$$m_p^{-1} \eta_2 D_+ p (s_e p_e^0 - s_v p_v^0) \right\}$$

$$C_\pm = 1 \pm 3\lambda^2$$

$$D_\pm = 1 \pm \lambda^2$$

$$\eta_1 = \frac{1}{4} [\delta_e^{(-)} \delta_v^{(+) + (1 + \kappa)^2} + \delta_e^{(+) \delta_v^{(-)}} (1 - \kappa)^2] \quad \cos \theta = (p_e^0 p_v^0)$$

$$\eta_2 = \frac{1}{4} [\delta_e^{(+) \delta_v^{(-)}} (1 + \kappa)^2 - \delta_e^{(-)} \delta_v^{(+) (1 - \kappa)^2}]$$

$$p = -(p_e + p_v)$$

$$\eta_3 = (1 - \kappa^2) \frac{m_e m_\nu}{E_e E_v}$$

$$\delta_l^{(\pm)} = 1 \pm s_l \beta_l \quad (l = e, v)$$
Here $s_e = \pm 1$, $s_\nu = \pm 1$ are the helicities of the electron and the massive antineutrino; $p$ and $m_p$ are the momentum and the mass of the recoil proton; $p_e^0, p_\nu^0$ are unit vectors in the direction of the electron and antineutrino momentum. Substituting (17) into (6), we obtain that the degree of longitudinal polarization electrons and the “forward-backward” asymmetry of the electron emission will have the following form:

$$P_{e^-} = \frac{dW(s_e = +1) - dW(s_e = -1)}{dW(s_e = +1) + dW(s_e = -1)} = \frac{L(s_e = +1) - L(s_e = -1)}{L(s_e = +1) + L(s_e = -1)}$$

(18)

$$A_{e\nu} = \frac{dW(0^\circ) - dW(180^\circ)}{dW(0^\circ) + dW(180^\circ)} = \frac{L(0^\circ) - L(180^\circ)}{L(0^\circ) + L(180^\circ)}$$

(19)

Substituting expression (17) into (18) and (19), and after reducing the nucleon function and the kinematic factor, one carries out a summation over the neutrino (antineutrino) spin states and integrates over the solid angles $d\Omega_e, d\Omega_\nu$. Then neglecting the small terms $\sim \frac{m_\nu}{m_p}, \frac{p_e}{m_p}, \frac{p_\nu}{m_p}$, one obtains as a result the expressions of the following form [3, 8]:

$$P_{e^-} \approx -\beta_e \frac{2\kappa}{1 + \kappa^2} \left[1 + \left(\frac{1 - 3\lambda^2}{1 + 3\lambda^2}\right)\left(\frac{1 - \kappa^2}{1 + \kappa^2}\right) \frac{m_em_\nu}{E_eE_\nu}\right]$$

(20)

$$A_{e\nu} \approx a\beta_e\beta_\nu \left[1 + \left(\frac{1 - 3\lambda^2}{1 + 3\lambda^2}\right)\left(\frac{1 - \kappa^2}{1 + \kappa^2}\right) \frac{m_em_\nu}{E_eE_\nu}\right]$$

(21)

where $a = \frac{1-\lambda^2}{1+3\lambda^2}$, is the coefficient of the angular electron-antineutrino correlation at zero rest mass of the antineutrino ($m_\nu = 0, \kappa = 1$). Near the upper boundary of the $\beta$-spectrum decay electrons, neglecting the second term in the square bracket (21), we have:

$$A_{e\nu} = a\beta_e\beta_\nu = a \left\{ \frac{E_e^k}{E_e^k + \frac{m_e^2}{Z}} \left[1 - \frac{m_\nu^2}{(W_0 + m_\nu - E_e)^2}\right] \right\}^{\frac{1}{2}}$$

(22)

Here $W_0$ is the upper boundary value of the total electron energy, $E_e^k$ is the kinetic energy of an electron, $\lambda=1.262$ [9]. The greatest effect of the antineutrino mass should be observed at the end of the spectrum. We will illustrate this in the graphs shown in figure 5 and figure 6.
From figure 5 and figure 6 it is seen that for $m_\nu = 0$, in the case of a pure V-A structure, the coefficient $A_{e\nu}$ takes a value equal to $-0.094113$ (curve 1). For $m_\nu \neq 0$, the coefficient $A_{e\nu}$ at the end of the $\beta$-spectrum tends to zero and the boundary itself shifts to the left (curve 3); for $\kappa \neq 1$, the slope of the curve changes. For $\kappa < 1$ (curve 4), the slope becomes more gentle, and for $\kappa > 1$ (curve 2) the slope becomes steeper. Thus, at the end of the spectrum ($E_e \to W_0$), it follows from formula (22) that the asymmetry of the electron emission disappears. The existence of a minimum the function $A_{e\nu}$ can be used to determine the rest mass of an electron antineutrino using the following formula:

$$m_\nu \approx m_e \left( \frac{\Delta E - E_e^*}{E_e^*} \right)^3 \left[ 1 + \frac{m_e^2}{E_e^{*2}} \left( \frac{\Delta E - 2E_e^*}{E_e^*} \right) \right]^{-\frac{1}{2}}$$  

(23)

Here $E_e^*$ is the total energy of the electron, at which the coefficient $A_{e\nu}$ takes on an extreme (minimal) value.

2.5 The polarized neutron $\beta$-decay

The expression for the differential probability $\beta$-decay of a polarized neutron in the nonrelativistic approximation:

$$E_p \approx m_p$$
$$E_\nu \ll m_p$$
$$E_e \ll m_p$$

has the form [3]:

$$dW = d\Omega_e d\Omega_\nu dE_e E_e p_e (\Delta E - E_e) \left[ (\Delta E - E_e)^2 - m_\nu^2 \right] F$$  

(25)

where $\Delta E - E_e = E_\nu$, 

Figure 5. The dependence of the asymmetry coefficient $A_{e\nu}$ on the energy of electrons.

Figure 6. The behavior of the asymmetry coefficient $A_{e\nu}$ at the end of the spectrum.
\[ F = \frac{e^2}{2(2\pi)^2} A_0 \left\{ 1 + A_{ev}\left( \vec{p}_e^0 \cdot \vec{p}_\nu^0 \right) + A_{ne}\left( \vec{s}_n \cdot \vec{p}_e^0 \right) + A_{n\nu}\left( \vec{s}_n \cdot \vec{p}_\nu^0 \right) + D_{nev}(\vec{s}_n \times \vec{p}_e^0) \right\} \]  

(26)

Here vector \( \vec{s}_n \) determines the direction of the neutron spin.

\[ A_0 = |\beta_v|^2 \left( 2 - \frac{m_p}{E_p} \right) + |\beta_A|^2 \left( 2 + \frac{m_p}{E_p} \right) + \beta_e^2 \frac{E_e}{E_p} |\beta_v + \beta_A|^2 + \beta_v^2 \frac{E_v}{E_p} |\beta_v - \beta_A|^2 \]  

(27)

\[ A_{ev} = \beta_e \beta_v \frac{1 - \lambda^2}{1 + 3\lambda^2} \]  

(28)

\[ A_{ne} = \beta_e \frac{2\lambda(1-\lambda)}{1 + 3\lambda} \]  

(29)

\[ A_{n\nu} = \beta_v \frac{2\lambda(1+\lambda)}{1 + 3\lambda^2} \]  

(30)

\[ D_{nev} = \beta_e \beta_v \frac{2\lambda}{1 + 3\lambda^2} \sin \alpha \]  

(31)

Formulas (28)-(31) determine the angular correlation coefficients \( A_{ev} \) and the asymmetry of electron emission \( A_{ne} \) and antineutrino emission \( A_{n\nu} \), as well as the triple correlation \( D_{nev} \) during \( \beta^- \)-decay of a polarized neutron. For example, we can obtain a simple expression for the neutrino mass from (28):

\[ m_\nu = (\Delta E - E_e) \left[ 1 - a^{-2} \beta_e^{-2} A_{ev}^2 \right]^\frac{1}{2} \]  

(32)

3. Conclusion

By measuring relative characteristics such as polarizations of particles, emission asymmetries, it is possible to obtain an estimate of the neutrino (antineutrino) mass in the \( \beta^- \)-decay processes. In the case of the Gamov-Teller and Fermi transitions, this allows us to get rid of the unknown nuclear structure contained in the nucleon tensor. It is desirable that the relative characteristics depend directly proportionally to the mass neutrino (antineutrino) and the term that contains the dependence on neutrino (antineutrino) mass give a significant contribution (that is not an infinitesimal quantity of a high order of smallness).

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