Hyperon polarization in $e^- p \rightarrow e^- HK$ with polarized electron beams

Liang Zuo-tang and Xu Qing-hua

Department of Physics, Shandong University, Jinan, Shandong 250100, China

Abstract

We apply the picture proposed in a recent Letter for transverse hyperon polarization in unpolarized hadron-hadron collisions to the exclusive process $e^- p \rightarrow e^- HK$ such as $e^- p \rightarrow e^- \Lambda K^+$, $e^- p \rightarrow e^- \Sigma^+ K^0$, or $e^- p \rightarrow e^- \Sigma^0 K^+$, or the similar process $e^- p \rightarrow e^- n \pi^+$ with longitudinally polarized electron beams. We present the predictions for the longitudinal polarizations of the hyperons or neutron in these reactions, which can be used as further tests of the picture.
1. Introduction

Hyperon polarization in different reactions have attracted much attention recently (see, e.g. [1-15], and the references given there). This is, on the one hand, because the polarization can easily be determined in experiments by measuring the angular distribution of their decay products thus provide us with a powerful tool to study the spin effects in different cases. On the other hand, it is also triggered by the surprisingly large transverse hyperon polarization discovered already in 1970s in unpolarized hadron-hadron and hadron-nucleus reactions. Understanding the origin(s) of such puzzling transverse hyperon polarizations has been considered as a challenge to the theoretician working in this field and should provide us with useful information on spin structure of baryon and spin dependence of strong interaction.

In a recent Letter, triggered by the similarities of the corresponding data, we pointed out that the transverse polarization observed in unpolarized hadron-hadron collisions should be closely related to the left-right asymmetries observed in singly polarized \(pp\) collisions. They should have the same origin(s). By using the single-spin left-right asymmetries for inclusive \(\pi\) production as input, we can naturally understand the transverse polarization for hyperon which has one valence quark in common with the projectile, such as \(\Sigma^-, \Xi^0\) or \(\Xi^-\) in \(pp\)-collisions, or \(\Lambda\) in \(K^-p\)-collisions. To further understand the puzzling transverse polarization of \(\Lambda\) in \(pp\)-collisions, which has two valence-quarks in common with the projectile, we need to assume that the \(s\) and \(\bar{s}\), which combine respectively with the valence-(ud)-diquark and the remaining \(u\)-valence quark to form the produced \(\Lambda\) and the associatively produced \(K^+\) in the fragmentation region, should have opposite spins. Under this assumption, we obtained a good quantitative fit to the \(x_F\)-dependence of \(\Lambda\) polarization in \(pp\) collisions (where \(x_F \equiv 2p_\parallel/\sqrt{s}\), \(p_\parallel\) is the longitudinal component of the momentum of the produced hyperon, and \(\sqrt{s}\) is the total center of mass energy of the \(pp\) system.) The obtained qualitative features for the polarizations of other hyperons are all in good agreement with the available data. We further pointed out, in a recent Brief Report, that \(\Lambda\) polarization in the
exclusive process $pp \rightarrow p\Lambda K^+$ can be used to give a special test of the picture since $|P_\Lambda|$ in this channel, which is the simplest one for $pp \rightarrow \Lambda X$, should take the maximum among all the different channels. It is encouraging to see that also this result is in agreement with the data obtained by R608 Collaboration at CERN, which shows that $|P_\Lambda|$ in this channel is indeed much larger than that for $pp \rightarrow \Lambda X$. It should be interesting to see whether the assumption that $s$ and $\bar{s}$ have opposite spins can also be applied to other reactions. In this connection, it is encouraging that the measurements on $\Lambda$ polarization in $e^-p \rightarrow e^-\Lambda K^+$ is being carried out at Jefferson Laboratory.

In this paper, we would like to apply the picture in [2] to $e^-p \rightarrow e^-\Lambda K^+$ to give the prediction on $\Lambda$ polarization in reactions with longitudinally polarized electron beams. We compare the results with those obtained in other cases for the spin states of the $s$ and $\bar{s}$ needed for the production of $\Lambda$ and $K^+$ in this process. These will be given in Section 2. After that, in section 3, we extend the calculations to other similar processes such as $e^-p \rightarrow e^-\Sigma K$ or $e^-p \rightarrow e^-n\pi^+$ and present the predictions on the polarizations of the hyperons (or neutron) in these reactions. We will also discuss the influence from the sea and that due to the mass differences between the $J^P = (1/2)^+$ and $J^P = (3/2)^+$ baryons in section 4. We will summarize the results in section 5. We found out that these polarizations are very sensitive to the spin states of the $s$ and $\bar{s}$ thus provide us with an ideal tool to study the spin correlations between them.

2. Polarization of $\Lambda$ in $e^-p \rightarrow e^-\Lambda K^+$

We now consider the process $e^-p \rightarrow e^-\Lambda K^+$, and discuss the problem in the center of mass frame of the produced hadronic system. We suppose that the energy and momentum transfer in the reaction are high enough so that the parton picture can be used. We thus envisaged with the following picture. In the collision process, a $u_v$ quark is knocked out by the virtual photon. The knocked $u_v$ combines with a $\bar{s}_s$ to form the observed $K^+$ and the remaining $(u_vd_v)$-diquark combines with the $s_s$ to form the observed $\Lambda$. This process is
illustrated pictorially in Fig.1.

Since we are now considering the longitudinal polarizations of hyperons in $e^- p \rightarrow e^- HK$, two points of the picture in [2] apply: (i) The $q_s$ and $\bar{q}_s$ that combine with the diquark from the incoming proton to form the hyperon and the remaining quark to form the associatively produced meson should have opposite spins. (ii) The SU(6) wave function can be used to relate the spins of the quarks to that of the hadrons. Using these two points, we obtain immediately the following: Because $K^+$ is a spin zero object, the $\bar{s}_s$ has to have the spin opposite to the scattered $u_v$-quark. Hence, according to (i), the $s_s$ quark should be polarized in the same direction as the scattered $u_v$-quark. According to the SU(6) wave function, which states that $|\Lambda\uparrow\rangle = (ud)_{0,0}^s\uparrow$, [where the subscripts denote the spin and its third component of the $(ud)$-diquark], the $(ud)$-diquark has to be in the spin zero state and the $\Lambda$ spin just equals to that of the $s$ quark. Since the momentum of $\Lambda$ is opposite to that of $K^+$ or that of the scattered $u_v$-quark, the longitudinal polarization of $\Lambda$ is opposite to that of the scattered $u_v$-quark in the helicity basis. Hence, we obtain,

$$P_{\Lambda}^{(a)} = -P_u = -P_e D(y).$$  \hspace{1cm} (1)

Here, $P_e$ is the polarization of the incoming electron beam. We use the superscript $(a)$ to denote that the result is obtained in the case that the spins of $s_s$ and $\bar{s}_s$ are opposite for the convenience of comparison with other cases which will be discussed in the following. $D(y)$ is the spin transfer factor from $e^-$ to $q$ in the scattering process $e^- q \rightarrow e^- q$. This is an electromagnetic process thus $D(y)$ can easily be calculated and can be found in different publications. It is a function of only one variable $y$, which is defined as $y \equiv p \cdot q / p \cdot k$, where $p$, $k$ and $q$ are the four momenta for the incoming proton, $e^-$ and the virtual photon exchanged in the scattering. $D(y)$ is given as,

$$D(y) = \frac{1 - (1 - y)^2}{1 + (1 - y)^2}. \hspace{1cm} (2)$$

It is a monotonically increasing function of $y$ which increases from 0 to 1 when $y$ goes from 0 to 1. Since $y$ is related to $x_B$ and $Q^2$ by $Q^2 = sx_By$, (where $x_B$ is the Bjorken-$x$, $Q^2 \equiv -q^2$,}
\[ s = 2p \cdot k \text{ is the total } e^- p \text{ center of mass energy squared}, \] it is clear that we can also express \( P_\Lambda \) for \( \Lambda \) in \( e^- p \rightarrow e^- \Lambda K^+ \) as a function of \( x_B \) and \( Q^2 \) at given \( \sqrt{s} \).

Eq.(1) is the result obtained based on the picture in Ref.[2] under the assumption that the spins of \( s_s \) and \( \bar{s}_s \) are opposite to each other. To compare, we now consider the other two cases for the spins of \( s_s \) and \( \bar{s}_s \), i.e., (b) the spins of \( s_s \) and \( \bar{s}_s \) are parallel to each other, and (c) the spins of \( s_s \) and \( \bar{s}_s \) are completely uncorrelated, i.e., they have the same probability to be parallel or anti-parallel. Apparently, in case (b), we have,

\[ P^{(b)}_\Lambda = P_u = P_e D(y), \quad (3) \]

and in case (c),

\[ P^{(c)}_\Lambda = 0. \quad (4) \]

We see that, the results are very much different from each other. Hence, measuring \( P_\Lambda \) in this process can distinguish these three different cases with good efficiency.

**3. Polarization of \( \Sigma \) or \( n \) in \( e^- p \rightarrow e^- \Sigma K \) or \( e^- p \rightarrow e^- n\pi^+ \)**

We can extend the calculations to other similar processes such as \( e^- p \rightarrow e^- \Sigma^0 K^0 \), \( e^- p \rightarrow e^- \Sigma^0 K^+ \), or \( e^- p \rightarrow e^- n\pi^+ \). As we can see from Fig.1, since in all these processes, the produced meson \( K \) or \( \pi \) is a spin zero object, we can obtain the same relation between the polarization of the \( q_s \) shown in the figure and that of the scattered \( q_v \) from the incoming proton as we obtained in \( e^- p \rightarrow e^- \Lambda K^+ \). However, in contrast to the case for \( \Lambda \) production where only spin zero \((ud)\)-diquark contributes and if the \((ud)\)-diquark is in the spin zero state, the produced \((ud)s\) can only be a \( \Lambda \), in reactions such as \( e^- p \rightarrow e^- \Sigma K \), the diquarks in different spin states can contribute and given the spin state of the diquark \((q_v q_v)\) and that of the quark \( q_s \), the produced \((q_v q_v)q_s\) can still be a \( J^P = (1/2)^+ \) or a \( J^P = (3/2)^+ \) baryon. All these make the calculations more complicated. To obtain the polarization of the produced hyperon, we need to know the probabilities for the diquark which participates in the reaction to be in different spin states and the relative probabilities for the the produced
$(q_vq_v)q_s$ to evolve into a $J^P = (1/2)^+$ or a $J^P = (3/2)^+$ baryon. The former can be calculated using the SU(6) wave-function of the proton. To do this, we note that, due to the helicity conservation, the scattered quark has the same longitudinal polarization before and after the scattering. Given the polarization of the scattered quark and the wave function of the proton, we can calculate the probabilities for the remaining diquark in different spin states. The latter can be determined by the projection of the spin wave function for the produced $(q_vq_v)q_s$ to that of the corresponding baryon, if we neglect the influence from the mass difference of the corresponding $(1/2)^+$ and $(3/2)^+$ baryons.

To show how these calculations are carried out, we now take $e^-p \rightarrow e^-\Sigma^+K^0$ as an explicit example. In this reaction, the scattered quark has to be a $d$ quark. We recall that the SU(6) wave function for proton is,

$$|p^\uparrow\rangle = \frac{1}{\sqrt{3}}[\sqrt{2}(uu)_{1,1}d^\downarrow - (uu)_{1,0}d^\uparrow],$$

(5)

where we use the up or down arrow to denote the helicity “+” or “−” state. We now consider the case where the scattered $d_v$ quark is in the helicity + state. We see that if the incoming proton is in the helicity “+” state, the remaining $(u_vu_v)$-diquark should be in the $(u_vu_v)_{1,0}$ state, and the relative weight is $1/3$. The incoming proton can also be in the helicity “−” state. The corresponding wave function can be obtained by reversing the spins of the quarks in Eq.(5). In this case, since the scattered $d_v$ is in the helicity “+” state, the remaining $(u_vu_v)$-diquark should be in the $(u_vu_v)_{1,-1}$ state, and the relative weight is $2/3$. Hence, we obtain that, if the the scattered $d_v$ quark is in the helicity “+” state, the $(u_vu_v)$-diquark can be in $(uu)_{1,0}$ and $(uu)_{1,-1}$ state, and the relative probabilities are $1/3$ and $2/3$ respectively.

Next, we consider that the produced $(u_vu_v)$-diquark combine together with a $s_s$ quark to form a $\Sigma^+$ or a $\Sigma^*$. We first consider the case (a) that $s_s$ and $\bar{s}_s$ have opposite spins. Since the $s_s$ should be polarized in the same direction as the scattered $d_v$ and moves in the opposite direction of the scattered $d_v$, it should be in the helicity “−” state. We consider the case that the $(u_vu_v)$-diquark is in the state $(u_vu_v)_{1,0}$. The spin state of the produced $(uus)$
is given by \((uu)_{1,0}s^\downarrow\). The projections of this state \((uu)_{1,0}s^\downarrow\) to the SU(6) wave function of \(\Sigma\) and that of \(\Sigma^*\) are \(1/\sqrt{3}\) and \(\sqrt{2}/3\) respectively. We thus obtain the relative probabilities for the production of these two hyperons in this case are \(1/3\) and \(2/3\) respectively. The results in other cases can be obtained in the same way. The results in case (c) are shown in Tables 1-3 for the different reactions. Those in case (a) and (b) can also be read out from the tables by looking only at the results for \(s^\downarrow\) or \(s^\uparrow\) respectively.

From Tables 1-3, we can calculate the hyperon polarizations in different cases for the spin states of \(s_s\) and \(\bar{s}_s\). The results are shown in Table 4. We see that the produced \(\Sigma^0\), \(\Sigma^+\) and \(n\) are all polarized significantly. We also see that, similar to \(\Lambda\) in \(e^-p \rightarrow e^-K^+\Lambda\), the polarizations for \(n\) in \(e^-p \rightarrow e^-\pi^+n\) obtained in the three cases differ quite significantly from each other. But the differences between the results for \(\Sigma^0\), \(\Sigma^+\) are not so significant. Hence, to distinguish between the three cases, we should study \(P_\Lambda\) and \(P_n\) in the corresponding reactions. We note that although the polarization of \(\Sigma^0\) cannot be measured but it implies a polarization of \(\Lambda\), \(P_\Lambda = -P_{\Sigma^0}/3\) in \(e^-p \rightarrow e^-\Sigma^0K^+ \rightarrow e^-\Lambda\gamma K^+\) which can be measured experimentally.

4. Influences from the sea and the mass effects

We should emphasize that the results in Table 4 are obtained under the following two assumptions and/or approximations: (1) only valence quarks are taken into account in the scattering of \(e^-\) with \(p\) while the sea contribution is neglected; (2) the influence of the mass differences between the \(J^P = (1/2)^+\) and the \(J^P = (3/2)^+\) baryons on the relative production rates are neglected. In practice, both of them should be taken into account. We now discuss their influences on the polarization results obtained above.

It is clear that, in deeply inelastic \(e^-p\)-scattering, the exchanged virtual photon \(\gamma^*\) can also be absorbed by a sea-quark or an anti-sea-quark. However, since we consider only the exclusive reactions \(e^-p \rightarrow e^-HK\) or the \(2 \rightarrow 2\) process \(\gamma^*p \rightarrow KH\), both of produced hadrons have very large momentum fractions in the center of mass frame of the hadronic
system. The sea contributions to such process should be very small. To see this, we take \( e^-p \rightarrow e^-\Lambda K^+ \) as an explicit example. Besides the case shown in Fig.1, this process can also happen via the absorption of the \( \gamma^* \) by a \( \bar{s}_s \) from the sea of the proton. After the absorption, the \( \bar{s}_s \) flies in the direction of the \( \gamma^* \) while the rest of the proton moves in the opposite direction. This \( \bar{s}_s \) may pick up a valence \( u_v \) from the proton and combine into a \( K^+ \) which flies in the direction of the \( \bar{s}_s \) while the rest of the proton combines together into a \( \Lambda \). Since the valence quark \( u_v \) usually takes a relatively large fraction of the momentum of the proton, the probability for this to happen should be small in particular at high energies. Furthermore, the ratio of the probability for the virtual photon to be absorbed by \( \bar{s}_s \) to that by \( u_v \) is given by \( s(x_B,Q^2)/[4u_v(x_B,Q^2)] \). At, e.g., the CEBAF energies, it is only of the order of \( 10^{-3} \). Hence we expect that the sea contribution should be negligible.

In contrast, there should be some influences from the mass differences of the \( J^P=(1/2)^+ \) and the \( J^P=(3/2)^+ \) baryons. The relative production weights have to be influenced by such mass differences. However, it is very interesting to note that in case (a), there is completely no influence from the mass difference on the hyperon polarizations in all the reactions discussed above and the \( \Lambda \) polarizations in all the three cases (a), (b) and (c) are not influenced by the mass effect. This is because in case (a), only \( (q_vq_v)_{1,0}q_s^\perp \) contributes to the production of \( \Sigma \) or \( n \). It always gives rise to \( \Sigma^{0\downarrow} \), \( \Sigma^{+\downarrow} \) or \( n^\downarrow \), i.e. completely negatively polarized \( \Sigma \) or \( n \). The mass effect can affect the production rate, but does not influence the polarization in the corresponding exclusive reaction. For \( \Lambda \) production, only \( (ud)_{0,0} \) contributes to \( \Lambda \) production and it contributes only to \( \Lambda \) production, so the \( \Lambda \) polarization in all cases are not influenced by such mass effect.

In cases (b) and (c), there are indeed some influences from the mass difference on the polarizations of \( \Sigma^0 \), \( \Sigma^+ \) and \( n \). To see how large they may influence the polarization of the hyperons, we multiply the production rate by a corresponding exponential factor \( \exp(-\lambda m^2) \) for the production of each baryon, where \( m \) is the mass of the hyperon. We assume that the production processes can be treated as two steps, i.e., first the production of the corresponding quarks states then the evolution to the corresponding baryons. In this way, we
need to normalize the production weights of the produced baryons to the total production weight of the corresponding quark states. We obtain that the above polarization should be modified into the following. In case (b), we have

\[ P_{\Sigma^+}^{(b)} = -P_eD(y) \cdot \frac{2e^{-\lambda m_\Sigma^2} + 7e^{-\lambda m_0^2}}{6e^{-\lambda m_\Sigma^2} + 9e^{-\lambda m_0^2}}. \]  

(6)

\[ P_{\Sigma^0}^{(b)} = -P_eD(y) \cdot \frac{2e^{-\lambda m_\Sigma^2} + 7e^{-\lambda m_0^2}}{6e^{-\lambda m_\Sigma^2} + 9e^{-\lambda m_0^2}}. \]  

(7)

\[ P_n^{(b)} = P_eD(y) \cdot \frac{2e^{-2\lambda m_n^2} + 36e^{-2\lambda m_\Delta^2} + 19e^{-\lambda(m_n^2+m_\Delta^2)}}{3e^{-2\lambda m_n^2} + 36e^{-2\lambda m_\Delta^2} + 27e^{-\lambda(m_n^2+m_\Delta^2)}}. \]  

(8)

Similarly, in case (c), we have

\[ P_{\Sigma^+}^{(c)} = -P_eD(y) \cdot \frac{2e^{-\lambda m_\Sigma^2} + 4e^{-\lambda m_0^2}}{4e^{-\lambda m_\Sigma^2} + 5e^{-\lambda m_0^2}}. \]  

(9)

\[ P_{\Sigma^0}^{(c)} = -P_eD(y) \cdot \frac{2e^{-\lambda m_\Sigma^2} + 4e^{-\lambda m_0^2}}{4e^{-\lambda m_\Sigma^2} + 5e^{-\lambda m_0^2}}. \]  

(10)

\[ P_n^{(c)} = -P_eD(y) \cdot \frac{e^{-2\lambda m_n^2} + 8e^{-\lambda(m_n^2+m_\Delta^2)}}{11e^{-2\lambda m_n^2} + 144e^{-2\lambda m_\Delta^2} + 100e^{-\lambda(m_n^2+m_\Delta^2)}}. \]  

(11)

To get a feeling of how strongly they depend on \( \lambda \), we show the results in case (b) as functions of \( \lambda \) in Fig.2. We see that there are indeed some influences on \( \Sigma \) or \( n \) polarization in the corresponding reactions but the influences are not very large.

The following point should be noted here. All the polarizations obtained above are functions of only one variable \( y \). They are independent of the energies under the condition that the energies are high enough that the parton picture can be used.

5. Summary

In summary, we have calculated the polarizations of the baryons in the exclusive processes \( e^-p \rightarrow e^-\Lambda K^+, e^-p \rightarrow e^-\Sigma^+ K^0, e^-p \rightarrow e^-\Sigma^0 K^+, \) and \( e^-p \rightarrow e^-n\pi^+ \) with longitudinally polarized electron beams. We used the picture proposed in [2], where it is assumed that the
$q_s$ and $\bar{q}_s$ needed in these processes to combine with the valence-diquark and struck quark to form the hyperons and the associated produced meson have the opposite spins. The results show that all these baryons are longitudinally polarized and the polarizations are functions of only one variable $y$. We compared the results with those obtained in other possible cases for the spin states of $q_s$ and $\bar{q}_s$. We found out that the magnitudes of hyperon polarizations are considerably large in all the different cases and that they are quite different from each other. Hence, they can be used as a good probe to study the spin correlations between the $q_s$ and $\bar{q}_s$ in future experiments.

We thank Li Shi-yuan, Xie Qu-bing and other members of the theoretical particle physics group in Shandong University for helpful discussions. This work was supported in part by the National Science Foundation of China (NSFC) and the Education Ministry of China under Huo Ying-dong Foundation.
REFERENCES

1. For a review of the data, see e.g., A. Bravar, in Procs. 13th Inter. Symp. on High Energy Spin Physics, eds. Tuyrin et al., World Scientific, Singapore, 1999.

2. Liang Zuo-tang and C. Boros, Phys. Rev. Lett. 79, 3609 (1997).

3. For a dedicated review, see e.g., Liang Zuo-tang and C. Boros, Inter. J. Mod. A15, 927 (2000).

4. Liang Zuo-tang and C. Boros, Phys. Rev. D61, 117503 (2000).

5. R608 Collab., T. Henkes et al., Phys. Lett. B283, 155 (1992).

6. M. Mestayer, private communication (2001).

7. ALEPH Collab., D. Buskulic et al., Phys. Lett. B374, 319 (1996); OPAL Collab. K. Ackerstaff et al., Eur. Phys. J. C2, 49 (1998).

8. NOMAD Collab., P. Astier et al., Nucl. Phys. B588, 3 (2000); ibid B605, 3 (2001).

9. HERMES Collab., A. Airapetian et al., Phys. Rev. D64, 112005 (2001); E665 Collab., M.R. Adams et al., Euro. Phys. J. C17, 263 (2000).

10. R.L. Jaffe and X. Ji, Phys. Rev. Lett. 67, 552 (1991); Nucl. Phys. B375, 527 (1992).

11. C. Boros and Liang Zuo-tang, Phys. Rev. D57, 4491 (1998).

12. A. Kotzinian, A. Bravar and D. von Harrach, Eur. Phys. J. C2, 329 (1998).

13. M. Anselmino, M. Boglione, and F. Murgia, Phys. Lett. B362, 164 (1995); Phys. Rev. D60, 054027 (1999); M. Anselmino, M. Boglione, U. D’Alesio, and F. Murgia, Eur. Phys. J. C21, 501(2001); M. Anselmino, D. Boer, U. D’Alesio, and F. Murgia, Phys. Rev. D63, 054029 (2001).

14. B.Q. Ma, I. Schmidt and J.J. Yang, Phys. Rev. D61, 034017 (2000); B.Q. Ma, I. Schmidt, J. Soffer and J.J. Yang, Phys. Rev. D62, 114009 (2000); Phys. Rev. D64,
014017 (2001), erratum-ibid, D64, 099901 (2001); hep-ph/0107157.

15. Liu Chun-xiu and Liang Zuo-tang, Phys. Rev. D62, 094001 (2000); Liu Chun-xiu, Xu Qing-hua and Liang Zuo-tang, Phys. Rev. D64, 073004 (2001); Xu Qing-hua, Liu Chun-xiu and Liang Zuo-tang, Phys. Rev. D63, 111301(R) (2001); Phys. Rev. D (2002) in press.
**TABLES**

Table 1. Possible states for the produced \((uuu)\), their relative production weights, the possible corresponding products and their weights in the reaction \(e^-p \rightarrow e^-\Sigma^+(or \Sigma^{*+})K^0\) in the case that the spins of \(s_s\) and \(\bar{s}_s\) have same probability to be parallel or anti-parallel if the scattered \(d_v\) is in the helicity “+” state.

| Possible spin states for \((u_vu_v)s_s\) | \((u_vu_v)_{1,0}s^\downarrow_s\) | \((u_vu_v)_{1,-1}s^\uparrow_s\) | \((u_vu_v)_{1,0}s^\downarrow_{\bar{s}}\) | \((u_vu_v)_{1,-1}s^\uparrow_{\bar{s}}\) |
|----------------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| Corresponding relative weights         | 1/6                           | 1/3                           | 1/6                           | 1/3                           |
| Possible products                      | \(\Sigma^+\uparrow\)          | \(\Sigma^{*+}\uparrow\)      | \(\Sigma^+\downarrow\)       | \(\Sigma^{*+}\downarrow\)    |
| Corresponding relative weights         | 1/3                           | 2/3                           | 2/3                           | 1/3                           |
| The final relative weights             | 1/18                          | 1/9                           | 2/9                           | 1/18                          |

Table 2. Possible states for the produced \((uds)\), their relative production weights, the possible corresponding products and their weights in the reaction \(e^-p \rightarrow e^-\Lambda\(or \Sigma^0\or \Sigma^{*0}\) \(K^+\) in the case that the spins of \(s_s\) and \(\bar{s}_s\) have same probability to be parallel or anti-parallel if the scattered \(u_v\) is in the helicity “+” state.

| Possible spin states \((u_vd_v)_{0,0}s^\downarrow_s\) | \((u_vd_v)_{1,0}s^\uparrow_s\) | \((u_vd_v)_{1,-1}s^\downarrow_{\bar{s}}\) | \((u_vd_v)_{0,0}s^\uparrow_{\bar{s}}\) | \((u_vd_v)_{1,0}s^\downarrow_{\bar{s}}\) | \((u_vd_v)_{1,-1}s^\uparrow_{\bar{s}}\) |
|---------------------------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| Their weights                                      | 3/8                           | 1/24                          | 1/12                          | 3/8                           | 1/24                          | 1/12                          |
| Possible products                                  | \(\Lambda^\uparrow\)         | \(\Sigma^{0\uparrow}\)       | \(\Sigma^{*0\uparrow}\)      | \(\Sigma^{0\downarrow}\)     | \(\Sigma^{*0\downarrow}\)     |
| Relative weights                                   | 1                             | 1/3                           | 2/3                           | 2/3                           | 1                             | 1/3                           |
| Final weights                                      | 3/8                           | 1/72                          | 1/36                          | 1/18                          | 3/8                           | 1/72                          | 1/36                          | 1/12                          |
Table 3. Possible states for the produced ($udd$), their relative production weights, the possible corresponding products and their weights in the reaction $e^-p \rightarrow e^-n(\text{or } \Delta^0)\pi^+$ in the case that the spins of $d_s$ and $\bar{d}_s$ have the same probability to be parallel or anti-parallel if the scattered $u_v$ is in the helicity “+” state.

| Possible spin states $(u_v d_v)_{0,0} d_s^\uparrow$ $(u_v d_v)_{1,0} d_s^\uparrow$ $(u_v d_v)_{1,-1} d_s^\downarrow$ $(u_v d_v)_{0,0} d_s^\downarrow$ $(u_v d_v)_{1,0} d_s^\downarrow$ $(u_v d_v)_{1,-1} d_s^\downarrow$ | Their weights $3/8$ $1/24$ $1/12$ $3/8$ $1/24$ $1/12$ |
| Possible products $n^\uparrow$ $n^\uparrow$ $\Delta^0^\uparrow$ $n^\downarrow$ $\Delta^0^\downarrow$ | Relative weights $1$ $1/9$ $8/9$ $1/3$ $2/3$ $1$ $1/9$ $8/9$ $1$ |
| Final weights $3/8$ $1/216$ $1/27$ $1/36$ $1/18$ $3/8$ $1/216$ $1/27$ $1/12$ |

Table 4. Polarizations of $\Lambda$, $\Sigma^0$, $\Sigma^+$, and $n$ in the reactions $e^-p \rightarrow e^-\Lambda(\text{or } \Sigma^0)K^+$, $e^-p \rightarrow e^-\Sigma^+K^0$ and $e^-p \rightarrow e^-n\pi^+$ in the three different cases for the spin states of the $q_s$ and $\bar{q}_s$, i.e., case (a): the spins of $q_s$ and $\bar{q}_s$ are anti-parallel, case (b): the spins of $q_s$ and $\bar{q}_s$ are parallel, case (c): the spins of $q_s$ and $\bar{q}_s$ have equal probability to be anti-parallel or parallel.

| Polarizations | $P_\Lambda/P_e D(y)$ | $P_{\Sigma^0}/P_e D(y)$ | $P_{\Sigma^+}/P_e D(y)$ | $P_n/P_e D(y)$ |
|---------------|---------------------|------------------------|-----------------------|----------------|
| case (a)      | -1                  | -1                     | -1                    | -1             |
| case (b)      | 1                   | -3/5                   | -3/5                  | 19/22          |
| case (c)      | 0                   | -2/3                   | -2/3                  | -3/85          |
Fig. 1. Illustrating graph showing the process $e^-p \rightarrow e^-HK$.

Fig. 2. Ratio of $P_{\Lambda}$ or $P_n$ to $P_eD(y)$ as a function of $\lambda$ for case (b) in different processes from Eqs.(6-8).
Table captions

Table 1: Possible states for the produced ($uus$), their relative production weights, the possible corresponding products and their weights in the reaction $e^-p \rightarrow e^-\Sigma^+(or \Sigma^{*+})K^0$ in the case that the spins of $s_s$ and $\bar{s}_s$ have same probability to be parallel or anti-parallel if the scattered $d_v$ is in the helicity “$+$” state.

Table 2: Possible states for the produced ($uds$), their relative production weights, the possible corresponding products and their weights in the reaction $e^-p \rightarrow e^-\Lambda(or \Sigma^0 or \Sigma^{*0})K^+$ in the case that the spins of $s_s$ and $\bar{s}_s$ have same probability to be parallel or anti-parallel if the scattered $u_v$ is in the helicity “$+$” state.

Table 3: Possible states for the produced ($udd$), their relative production weights, the possible corresponding products and their weights in the reaction $e^-p \rightarrow e^-n(or \Delta^0)\pi^+$ in the case that the spins of $d_s$ and $\bar{d}_s$ have the same probability to be parallel or anti-parallel if the scattered $u_v$ is in the helicity “$+$” state.

Table 4: Polarizations of $\Lambda$, $\Sigma^0$, $\Sigma^+$, and $n$ in the reactions $e^-p \rightarrow e^-\Lambda(or \Sigma^0)K^+$, $e^-p \rightarrow e^-\Sigma^+K^0$ and $e^-p \rightarrow e^-n\pi^+$ in the three different cases for the spin states of the $q_s$ and $\bar{q}_s$, i.e., case (a): the spins of $q_s$ and $\bar{q}_s$ are anti-parallel, case (b): the spins of $q_s$ and $\bar{q}_s$ are parallel, case (c): the spins of $q_s$ and $\bar{q}_s$ have equal probability to be anti-parallel or parallel.

Figure captions

Figure 1: Illustrating graph showing the process $e^-p \rightarrow e^-HK$.

Figure 2: Ratio of $P_\Sigma$ or $P_n$ to $P_eD(y)$ as a function of $\lambda$ for case (b) in different processes from Eqs.(6-8).