The Extraction of the Gluon Density
from Jet Production in Deeply Inelastic Scattering‡

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Abstract

The prospects of a direct extraction of the proton’s gluon density in next-to-leading order via jet rates in deeply inelastic scattering are studied. The employed method is based on the Mellin transform, and can be applied, in principle, to all infra-red-safe observables of hadronic final states. We investigate the dependence of the error band on the extracted gluon distribution on the statistical and systematic error of the data.
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Abstract: The prospects of a direct extraction of the proton’s gluon density in next-to-leading order via jet rates in deeply inelastic scattering are studied. The employed method is based on the Mellin transform, and can be applied, in principle, to all infra-red-safe observables of hadronic final states. We investigate the dependence of the error band on the extracted gluon distribution on the statistical and systematic error of the data.

1 Introduction

The extraction of the proton’s gluon density over a wide kinematical range is one of the central issues at HERA. This distribution is important for phenomenological applications at \textit{pp}-colliders, as well as at small \(x\), \(x \lesssim 0.005\) say, for studies related to parton dynamics, since higher-order terms of the type \([\alpha_s \ln(1/x)]^k\) in the perturbative expansion may become important here. At HERA, this region is mainly covered by the scaling violations and (with increasing luminosity) more directly by the charm content of the structure function \(F_2\). The classical gluon constraint at larger \(x\) \(x > 0.01\), direct photon production in \textit{pp} collisions, is plagued by sizeable theoretical uncertainties, see Ref. \textsuperscript{[1]}. It is therefore desirable to have a direct determination of the gluon density from HERA also at \(x \gtrsim 0.01\), complementing the observables so far employed in global fits (see, for example, Ref. \textsuperscript{[2]}).

Experimental analyses of hadronic final states use very complicated sets of cuts. The theoretical predictions for the employed infra-red-safe observables are in general obtained by means of time-consuming Monte-Carlo integrations of parton cross-sections. This is contrary to the case of DIS structure functions, for example, where the convolution kernels can be given in a compact analytical form. Parametrizations of the gluon density involve some free parameters — for a decent description at, say, a scale of a few GeV\textsuperscript{2} at least 3 – 4 parameters are necessary \textsuperscript{[1, 2]} — so that a fitting procedure with a large number of iterations is unavoidable. This leads to the problem of the repeated evaluation of the theoretical cross-section for various gluon density parametrizations. It turns out that the direct method is not easily feasible due to its
prohibitive need of computer time. A method based on the Mellin transform technique\textsuperscript{1} to circumvent this problem has been presented in Ref. [3]. It allows the rapidly repeated evaluation of the convolution of a cross-section and a parton density, even if the cross-section is not of the Mellin-factorizing form. In fact, this is always the case when acceptance cuts are applied, even for “factorizing” jet definition schemes \textsuperscript{[4]}.

Jet observables are particularly suitable for the extraction of the gluon density. The reason for this is that the lowest-order gluon-induced subprocess is the photon–gluon fusion process (Fig. 1b), which can give rise to (2+1) jet final states. The cross-section of the competing quark-initiated “QCD Compton”-subprocess (Fig. 1a) is either small (at small x) or well-known (at large x). Hence it can be determined theoretically and then subtracted from the experimental cross-sections. In leading-order QCD, the momentum fraction of the incident parton can be reconstructed from the final-state jet momenta, and therefore a direct unfolding of the gluon density is possible \textsuperscript{[5]}. This procedure does no longer work beyond leading order, where the mass-factorized hard scattering cross-section is a distribution (in the mathematical sense) to be convoluted with the parton densities.

For general factorization schemes, the parton-level cross-section contains subtractions, thus “the momentum fraction of the incident gluon” in a naive probabilistic interpretation no longer makes sense. The physical origin of this phenomenon is initial-state radiation (see Fig. 1c; in this example, the antiquark $\bar{q}$ is assumed to be radiated into the forward direction, giving a contribution to the hard process of Fig. 1a). The separation of the calculable short-distance subprocess from the long-distance physics of the proton state (whose onset appears in the form of collinear and soft divergences of matrix elements) requires the renormalization of the parton densities. A change of the factorization scheme amounts to a redefinition of both the hard scattering cross-section as well as of the parton densities — in other words, the notion of an

\textsuperscript{1} The Mellin transform $F_n = \int_0^1 \, dx \, x^{n-1} \, F(x)$ maps a convolution $\sigma(x_B) = \int_{x_B}^1 \, (d\xi/\xi) \, f(\xi) \, \sigma^P(x_B/\xi)$ of a parton density $f$ and a parton-level cross-section $\sigma^P$ into the product $\sigma_n = f_n \, \sigma_n^P$ of the respective moments. For a specific set of acceptance cuts, the time-consuming Monte Carlo calculation of the moments $\sigma_n^P$ has to be done only once. The cross-section $\sigma$ can be evaluated repeatedly for varying parton densities by means of an inverse Mellin transform. A restriction of the method is that the factorization scale in every bin of analyzed data is assumed to be constant. However, owing to scale-compensating terms in the parton-level cross-section, the procedure is always accurate to the order of perturbation theory for which the cross section is calculated. For details, in particular on how to treat the non-factorizable case, see Ref. [3].
“incident parton” becomes a factorization-scheme dependent concept — such that, to all orders, observable quantities are unaffected. For general factorization schemes, the determination of the gluon density has to rely on a fitting method.

2 Two Scenarios at HERA

In this section, we present the explicit results for a study based on jet rates in the modified JADE jet definition scheme \[6\] with a jet cut parameter of \(y_{\text{cut}} = 0.02\). We use the \textsc{projet} next-to-leading-order Monte-Carlo program \[7\] with the matrix elements from Ref. \[8\]. The program is applied in a phase space region (jets in the very forward direction are excluded) where the approximations made in the calculation underlying \textsc{projet} should be justified. In Fig. 2a the error bands for a typical statistical-, systematic- and theoretical-error scenario are shown. The employed acceptance cuts are discussed in Ref. \[9\]. A luminosity of about 3 pb\(^{-1}\) has been assumed. The inner shaded region shows the statistical plus experimental systematic errors, added in quadrature. The outer shaded region displays the total error, obtained by subsequently adding the theoretical error quadratically, including a scale variation in the range of \([Q/2, 2Q]\). Only the region with \(x > y_{\text{cut}}\) is covered by data\[2\], but the extrapolation to smaller \(x\) exhibits a reasonable behaviour of the gluon density parametrization.

Figure 2: Fit result on the proton’s gluon density with typical error bands for statistical, systematic (and theoretical) errors added in quadrature compared with recent parametrizations \[2\]. (a) for 1994 HERA luminosity, (b) high-luminosity scenario. For details see the text.

In Fig. 2b it is assumed that, owing to a much higher luminosity (of the order of 250 pb\(^{-1}\)), the systematic error can be halved by much tighter cuts leaving the same statistical error in the sample, and that the progress in the understanding of theoretical uncertainty will allow for neglecting its influence against the remaining experimental error. Under these assumptions the

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\[2\] The use of the modified JADE algorithm restricts the accessible range in the gluon momentum fraction to values larger than the jet cut, because the invariant mass of the outgoing hadronic system is constrained. It is possible to extend this range by using other jet algorithms, such as the cone or the \(k_T\) schemes.
error is reduced dramatically, enabling a discrimination between different present-day parton parametrizations [2]. It should be noted that the spread of these parametrizations does not represent the real uncertainty on the gluon distribution, due to the use of similar data samples and theoretical assumptions. This emphasizes the discriminating power and constraining effect of a DIS jet data sample at HERA with much reduced systematic error in the region $0.01 < x < 0.1$.

3 Summary and Conclusions

We have studied the prospects of a direct determination of the proton’s gluon density via jet rates in deeply inelastic scattering at HERA. The application of the Mellin transform method [3] allows for an efficient fitting procedure with several parameters, the input data of the fit being the jet rates in bins of $Q^2$. We have illustrated that high luminosity at HERA and, consequently, smaller systematic errors owing to tight acceptance cuts, can permit a useful constraint of the gluon density, complementary to (and finally includible in) global fits. Forthcoming high-statistics runs of HERA will also allow for a binning in other variables more closely related to the (unobservable) momentum fraction of the gluon, which should have a direct impact on the quality of the fit.

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References

[1] W. Vogelsang and A. Vogt, Nucl. Phys. B453 (1995) 334.
[2] A. Martin, R. Roberts and W.J. Stirling, Phys. Lett. B354 (1995) 155; M. Glück, E. Reya and A. Vogt, Z. Phys. C67 (1995) 433; H.L. Lai et al., CTEQ Collaboration, Phys. Rev. D51 (1995) 4763.
[3] D. Graudenz, M. Hampel, A. Vogt and Ch. Berger, Z. Phys. C70 (1996) 77.
[4] S. Catani, Y.L. Dokshitzer and B.R. Webber, Phys. Lett. B285 (1992) 291; B.R. Webber, J. Phys. G19 (1993) 1567.
[5] H1 Collaboration, S. Aid et al., Nucl. Phys. B449 (1995) 3.
[6] D. Graudenz and N. Magnussen, Proceedings of the 1991 HERA Workshop, eds. W. Buchmüller and G. Ingelman (DESY, Hamburg, 1991), p. 261.
[7] D. Graudenz, Comput. Phys. Commun. 92 (1995) 65.
[8] D. Graudenz, Phys. Lett. B256 (1991) 518; Phys. Rev. D49 (1994) 3291.
[9] M. Hampel, Ph.D. thesis (1996), in preparation.