On analytical and numerical solutions of inverse problems of the mechanics of composites

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Abstract. A number of statements of inverse problems of rational and optimal design of composite structures are considered. The problem of maintenance of a momentless state of a composite shell with respect to parameters’ choice of reinforcement is solved. The problem of optimum design of composite pressure vessels is investigated.

1. Introduction
Composite plates and shells are the most important structural elements in aviation and rocket–space technology, shipbuilding and automotive industry, energy and chemical engineering [1, 2]. Several mathematical problems arise while researching the behaviour of modern composite and hybrid structures.

One of the problem is caused by the need to develop structural models of composite materials (CM) that take into account the features of their real internal structure, nonlinear processes of deformation and fracture. To date, a large number of structural models of composites have been developed. A comparative analysis of various structural models of CM is given in [3–5]. Carbon fiber reinforced plastics (CFRP) are the most perspective class of modern composites that combine high strength and stiffness with low specific gravity. In [6, 7] a comprehensive approach to the construction of mathematical models of nonlinear-elastic deformation of CFRP under bending taking into account the effect of the multi-modulus behaviour under tension and compression is presented. Mathematical model for calculating and analyzing the stress-strain state (SSS) of composite materials on the basis of an ice matrix is presented in [8].

Another problem is attributable to the need to consider refined models of the behaviour of composite and hybrid structures. Currently, there are various methods for obtaining equations of theories of plates and shells. The hypothesis method has become more widespread in practice. A detailed description of the classical theory and various refined theories of plates and shells can be found, in particular, in the monographs [4, 9–17].

The next problem concerns the development of effective numerical methods for solving boundary value problems for nonlinear systems of partial differential equations with variable coefficients and with small parameters in the leading derivatives. Note that the transition from the classical theory of plates and shells to certain refined theories is accompanied not only by an increase in the order of systems of differential equations. It also includes qualitative changes in the structure of solutions, as a result, there is a need to develop new effective numerical methods for solving problems of the mechanics of composites.
An important place among the approaches to solving of boundary value problems (BVPs) in the theory of plates and shells is taken by various methods of reducing the dimension of the problem, for example, the spline interpolation method of functions in one of the coordinate directions [18], the method of separation of variables using the trigonometric representation of functions [19–22], the least-squares collocation method [23–26]. Some results of the numerical calculation of hybrid and anisogrid composite structures are presented in [27–30].

And finally the problem is related to the development of effective methods for solving the inverse problems of multicriteria optimization of composite and hybrid structures. A number of monographs are devoted to various problem statements and methods for solving them [1, 2, 4, 35–39].

2. Basic functionals and quality criteria

For thin-walled structures used in aerospace, machine-building, power plants and the civil engineering, their weight or cost characteristics play a major role. The mass of hybrid composite structures is determined by the expression

$$B = \sum_{n=1}^{N} \int_{h_{n-1}}^{h_{n}} \left( \int_{S} \bar{p}^{(n)} dS \right), \quad \bar{p}^{(n)} = a^{(n)} p_{c}^{(n)} + \sum_{k=1}^{K^{(n)}} \omega_{k}^{(n)} p_{k}^{(n)}, \quad a^{(n)} = 1 - \sum_{k=1}^{K^{(n)}} \omega_{k}^{(n)}, \quad (2.1)$$

where $p_{c}^{(n)}$, $p_{k}^{(n)}$ — volumetric densities of matrix materials and fibers of $k$-th family in $n$-th layer of the structure; $K^{(n)}$ — number of fiber families in $n$-th layer; $N$ — total number of layers. The cost of a structure is determined by the expression

$$C = \sum_{n=1}^{N} \int_{h_{n-1}}^{h_{n}} \left( \int_{S} \bar{c}^{(n)} dS \right), \quad \bar{c}^{(n)} = a^{(n)} c_{c}^{(n)} + \sum_{k=1}^{K^{(n)}} \omega_{k}^{(n)} c_{k}^{(n)}, \quad (2.2)$$

where $c_{c}^{(n)}$, $c_{k}^{(n)}$ — the specific costs of matrix materials and reinforcement of the $k$-th family in the $n$-th layer of the structure; $a^{(n)}$ — is determined by the expression from (2.1).

In many cases, a certain (etalon) structure for these operating conditions already exists. If its weight and cost are equal, respectively, $B_{0}$, $C_{0}$, then the relative characteristics $B = B_{0}/B$, $C = C_{0}/C$ are quality measures for improved structures. New projects will be considered rational if $B < 1$, $C < 1$; conditionally rational if $B < 1$, $C \geq 1$ or $B \geq 1$, $C < 1$, and nonrational if $B > 1$, $C > 1$. Optimal projects are contained in a class of rational and meet the requirements of

$$B^* = \min B < 1, \quad C^* = \min C < 1. \quad (2.3)$$

In some cases, operating conditions require limitations on overall or local structure compliance. The measure of general compliance can be the functional corresponding to the work of the given surface loads $\bar{F}$:

$$\bar{J}_{1} = \int_{S} \bar{F} \bar{u} dS, \quad \bar{J}_{01} = \int_{S} \bar{F} \bar{u}_{0} dS, \quad J_{1} = \bar{J}_{1}/\bar{J}_{01}, \quad (2.4)$$

where $\bar{u}$, $\bar{u}_{0}$ — vectors of displacements (here and further the values with zero in the indexes refer to the etalon project). The measure of local compliance can be the value of the displacement (deflection) vector at a given point:

$$\bar{J}_{2} = w(x_{1}^*, x_{2}^*), \quad \bar{J}_{02} = w_{0}(x_{1}^*, x_{2}^*), \quad J_{2} = \bar{J}_{2}/\bar{J}_{02}, \quad (x_{1}^*, x_{2}^*) \in S, \quad (2.5)$$
where \((x^*_1, x^*_2)\) — coordinates of the maximum deflection point. Due to the difficulty of obtaining solutions for such local functionals, they are roughly replaced by integral functionals:

\[
J_3 = \left( \frac{1}{\text{mes} V} \iiint_V |w|^r dV \right)^{1/r}, \quad J_{03} = \left( \frac{1}{\text{mes} V_0} \iiint_V |w_0|^r dV \right)^{1/r},
\]

\[
J_3 = J_{3}/J_{03}, \quad r \ll 1.
\]

(2.6)

Total potential energy can also be a measure of structure compliance

\[
J_4 = \sum_{n=1}^{N} \int_{h_n}^{h_{n-1}} \left( \int_S E_{\alpha\beta\gamma\delta}^{\varepsilon\alpha\beta\varepsilon\gamma\delta} dS \right) dz, \quad J_{04} = \sum_{n=1}^{N_0} \int_{h_n}^{h_{n-1}} \left( \int_S E_{\alpha\beta\gamma\delta}^{0\varepsilon\alpha\beta\varepsilon\gamma\delta} dS \right) dz,
\]

\[
J_4 = J_4/J_{04},
\]

(2.7)

then projects for which the following can be considered rational in terms of compliance

\[
J_1 < 1, \quad J_2 < 1, \quad J_3 < 1, \quad J_4 < 1.
\]

In many cases an important project characteristic is the initial failure load, which is defined as follows:

\[
p^* = \min( p^*_1, p^*_2, p^*_3, \ldots ) \quad p^* = p^*_0/p^*;
\]

(2.8)

where \(p^*_1, p^*_2, p^*_3, \ldots\) — initial fracture load for the corresponding phase materials; \(p^*_0\) — initial fracture load for the etalon structure. The control parameters must meet the additional requirements of mass constancy \((B = 1)\) or structure cost \((C = 1)\). Structures for which \(p^* < 1\) should be considered rational.

In arbitrarily loaded layered shells, the fields of stresses and deformations in the direction of normal to the reference surface are essentially heterogeneous, which leads to uneven behaviour of the material in the cross-section, premature cracking on the outermost surfaces of the structure and bundles from transverse shears. In order to reduce such unfavourable behaviours, it is purposeful to consider classes of projects of membrane composite shells, which correspond to the formulation of additional requirements for curvature changes

\[
k_{11} = 0, \quad k_{22} = 0, \quad k_{12} = 0.
\]

(2.9)

In the case of single-layer, quasi-homogeneous in thickness, reinforced shells, the requirement of membrane deformation is equivalent to the requirement of implementation of projects of strictly momentless shells.

In the case of fiber composites, the condition of equally stressed-strained fibers, i.e. execution of equations, is relevant

\[
\sigma_k = E_k e_k = \text{const}, \quad e_k = e_{11} \cos^2 \psi_k + e_{22} \sin^2 \psi_k + e_{12} \sin 2\psi_k.
\]

(2.10)

3. General formulation of rational design

Let’s assume that based on operational or economic considerations, \(K\) requirements from (2.1) — (2.10) are presented to the shell, formulated in the form

\[
\Phi_k (s, \varphi, \bar{y}(s, \varphi), \bar{p}(s, \varphi), \bar{q}(s, \varphi)) = 0, \quad (k = 1, \ldots, K),
\]

(3.1)

where \(s, \varphi\) — independent variables; \(\bar{y}\) — vector-function describing the SSS of the shell; \(\bar{p}\) — vector-function defining geometry and thickness of the shell, structure and properties of its material; \(\bar{q}\) — vector-function of external influences.
Then the relations (3.1) together with the initial system of \( N \) equations of a thin-walled reinforced shell form a system of \((N + K)\) equations overdetermined relative to \( N \) unknown parameters included into the vector-function of the state \( \varphi \). The resolution of this overdetermined system of equations can be attempted by the parameters included into the design functions \( \varphi \) and the load of \( \varphi \). At least two ways are possible.

The first consists in closing the resulting system of equations by introducing \( K \) additional unknown parameters from \( \varphi \) and \( \varphi \) and the subsequent numerical solution of nonlinear boundary value problems of the system \((N + K)\) equations with \((N + K)\) unknown parameters, which leads to significant mathematical difficulties.

The second, more promising way, consists in a preliminary study of the compatibility of the overdetermined system of equations and obtaining, after exclusion of \( N \) unknown parameters state functions, of its solvability conditions in the form of

\[
\Psi_k(s, \varphi, \varphi(s, \varphi), \varphi(s, \varphi)) = 0, \quad (k = 1, \cdots, K),
\]  
(3.2)

on the basis of which it is possible to formulate a wide class of various statements of problems of rational design.

In the most general form the problem about realization of the rational SSS in a reinforced shell can be formulated as follows: it is necessary to define such laws of loading and changes of thickness, such structures of a composite material and the form of a median surface of a shell at which conditions of solvability (3.2) will be executed identically.

4. Momentless reinforced shells

Let’s consider a shell of rotation, loaded and fixed axially and reinforced with identical pairs of bars, located along the trajectories symmetrical along the meridian. Following [4], let’s express forces and displacements in a momentless shell in terms of the following formulas:

\[
T_1 = \frac{1}{r \sin \varphi} \left[ \int_{\varphi_0}^{\varphi} r R_1 (p_3 \cos \varphi - p_1 \sin \varphi) d\varphi + c_1 \right], \quad c_1 = r_0 \sin \varphi_0 T_1^0,
\]

\[
T_2 = R_2 p_3 - \frac{1}{R_1 \sin^2 \varphi} \left[ \int_{\varphi_0}^{\varphi} r R_1 (p_3 \cos \varphi - p_1 \sin \varphi) d\varphi + c_1 \right],
\]  
(4.1)

\[
u = \sin \varphi \left[ \int_{\varphi_0}^{\varphi} \frac{R_1}{\sin \varphi} \left( \varepsilon_1 - \frac{R_2}{R_1} \varepsilon_2 \right) d\varphi + c_2 \right], \quad c_2 = \frac{u_0}{\sin \varphi_0},
\]

\[
\omega = - \cos \varphi \left[ \int_{\varphi_0}^{\varphi} \frac{R_1}{\sin \varphi} \left( \varepsilon_1 - \frac{R_2}{R_1} \varepsilon_2 \right) d\varphi + c_2 \right] + R_2 \varepsilon_2.
\]  
(4.2)

Here \( p_1 \) and \( p_3 \) — distributed load components along the meridian and normal, \( T_1^0 \) — values of \( T_1 \) at \( \varphi = \varphi_0 \), \( u_0 \) — value of \( u \) at \( \varphi = \varphi_0 \). Values \( \varepsilon_1, \varepsilon_2 \) are associated with \( T_1, T_2 \) expressions

\[
\varepsilon_1 = (2H)^{-1} (b_{11} T_1 + b_{12} T_2), \quad \varepsilon_2 = (2H)^{-1} (b_{12} T_1 + b_{22} T_2),
\]  
(4.3)

\[
||b_{ij}|| = ||a_{ij}||^{-1} \quad (i, j = 1, 2).
\]

Let the axially symmetrical external load, the law of thickness distribution, the nature of reinforcement and momentless boundary conditions be defined. It is necessary to find such a law of additional reinforcement, which would provide a momentless axially symmetric state
in the given shell. Additional reinforcement is represented by two identical bar families with reinforcement angles $\psi^*$ and $\pi - \psi^*$. In this case we get

\[
\begin{aligned}
  a_{11}^o \varepsilon_1 + a_{12}^o \varepsilon_2 + 2 \omega^* E^* (aE)^{-1} e^* \cos^2 \psi^* &= (2HaE)^{-1} T_1, \\
  a_{12}^o \varepsilon_1 + a_{22}^o \varepsilon_2 + 2 \omega^* E^* (aE)^{-1} e^* \sin^2 \psi^* &= (2HaE)^{-1} T_2, \\
  \omega^* E^* &= \omega_0^2 E + \omega_1^2 E_1^* \\
  \varepsilon^* &= \varepsilon_1 \cos^2 \psi^* + \varepsilon_2 \sin^2 \psi^*, \\
  a_{ii}^o &= a_{ii}(aE)^{-1} \\
  a_{12}^o &= a_{12}(aE)^{-1},
\end{aligned}
\]

where $a_{ii}$, $a_{12}$ — functions that describe preliminary reinforcement, $E_1^*$ — Young modulus of additional fiber families, $\omega_i^*$ — their specific volume content.

From (4.4), multiplying the first equation (4.4) by $a_{12}^o \sin^2 \psi^*$, the second equation by $a_{11}^o \cos^2 \psi^*$ and subtracting the second from the first one, we find:

\[
2 \omega^* E^* / aE = \left\{ (2HaE)^{-1} (a_{11}^o T_2 - a_{12}^o T_1) - \left[ a_{11}^o a_{22}^o - (a_{12}^o)^2 \right] \varepsilon_2 \right\} (e^* \Delta)^{-1}
\]

where

\[
\begin{aligned}
  \Delta &= \lambda a_{11}^o - (1 - \lambda) a_{12}^o \\
  \lambda &= \sin^2 \psi^* \\
  \varepsilon_1 &= \left\{ (2HaE)^{-1} \left[ \lambda T_1 - (1 - \lambda) T_2 \right] - \left[ \lambda a_{12}^o - (1 - \lambda) a_{22}^o \right] \varepsilon_2 \right\} \Delta^{-1}
\end{aligned}
\]

By substituting the expression for $\varepsilon_1$ from (4.6) into the continuity equation of deformations, we get the equation relative to $\varepsilon_2$:

\[
\frac{rd\varepsilon_2}{dr} + \{ 1 + [\lambda a_{12}^o - (1 - \lambda) a_{22}^o] \Delta^{-1} \} \varepsilon_2 = (2HaE)^{-1} [\lambda T_1 - (1 - \lambda) T_2]
\]

From here, integrating:

\[
\varepsilon_2 = \frac{1}{r} \exp \left[ -\Phi(r) \right] \left\{ \varepsilon_2^0 r_0 \frac{1}{2aE} \int_{r_0}^{r} \frac{\lambda T_1 - (1 - \lambda) T_2}{H \Delta} \exp \left[ \Phi(r) \right] dr \right\},
\]

where

\[
\Phi(r) = \int_{r_0}^{r} \frac{1}{r \Delta} \left[ \lambda a_{12}^o - (1 - \lambda) a_{22}^o \right] dr.
\]

The expression (4.5), after the value substitution for $\varepsilon_2$ from (4.7), under the given angle $\psi^*(r)$ defines the law of additional reinforcement, at which a membrane state of stress is realized in the considered shell. Displacements in such a shell are determined by the relations (4.2) when substituting in them the values of $\varepsilon_1$, $\varepsilon_2$ from (4.6), (4.7).

If there is no preliminary reinforcement, then

\[
\begin{aligned}
  a_{ii}^o &= (1 - \nu^2)^{-1} \\
  a_{12}^o &= \nu(1 - \nu^2)^{-1} \\
  \Delta &= \lambda(1 + \lambda) - \nu
\end{aligned}
\]

and the expression (4.5) is given as follows

\[
2 \omega^* E^* / aE = C^{-1} \left\{ A(r) + B(r) \right\} (D |\lambda T_1 - (1 - \lambda) T_2| + 2HaE \varepsilon_2)^{-1} - C^{-1}
\]

\[
\begin{aligned}
  A(r) &= \lambda(1 - \lambda)(1 + \nu)^2 - \nu \\
  B(r) &= C - (1 - \nu^2)(1 - \lambda)^2 \\
  C(r) &= 1 - 2\lambda(1 - \lambda)(1 + \nu) \\
  D(r) &= (1 - \lambda)(1 - \nu^2).
\end{aligned}
\]
For the ellipsoid of revolution of constant thickness, loaded with internal uniform normal pressure, the values of $2\omega^*E^*/aE$ from (4.8), are shown in Fig. 1 on the left depending on the ellipsoid’s elongation ($\alpha = 1, 2, 3, 4$) for $\psi^* = 90^\circ$. The value of $2\omega^*E^*/aE$ for ellipsoids with $\alpha = 2$ (dotted curves) and $\alpha = 4$ (solid curves) depending on the angles $\psi^* = 60^\circ, 70^\circ, 80^\circ, 90^\circ$ (1-4, respectively) are shown in Fig. 1 to the right.

Since the reinforcement law can be set within wide limits, so as can be seen from these data, by choosing the nature of reinforcement we can reduce the thickness of a momentless shell or to implement a technologically acceptable character of its change in comparison to a shell made of homogeneous isotropic material.

5. Composite pressure vessels

Composite pressure vessels (CPV) are used in the rocket and spacecraft making industry due to their high strength and lightweight. CPV have been one of the most actual and perspective directions of research, supported especially by NASA [31,32].

Let’s consider a multilayer composite pressure vessel at a state of equilibrium under equidistributed inner pressure. We need to determine the parameters of structure and CM meeting the following requirements:

$$V \geq V_0, \quad P \geq P_0, \quad M \leq M_0,$$

where $V$ is the volume of the vessel, $P$ is inner pressure and $M$ is the vessel’s mass and they are constrained by some preset values $V_0, P_0, M_0$.

We define the optimization problems the following way: to find extremum of one functional from (Eq. (5.1)) under other constraints.

The structures optimization problem statement includes selection of objective functional, formulation of constitutive equations and constraints on performance and design variables.

The mathematical models describing the vessel’s state are based on the following assumptions: the vessel is a multilayer thin-walled structure, the vessel’s layers can have different mechanical characteristics, the reinforced layer’s material is quasi-homogeneous, the vessel’s main loading is high inner pressure.

These assumptions allow us to reduce dimension of the corresponding mathematical problem and to build the mathematical vessel’s models based on the different theories of multilayer non-isotropic shells [4]. The Kirchhoff—Love shell theory [33] (KLST) and the improved Timoshenko [16] (TiST) and Andreev-Nemirovskii [11] (ANST) theories are used to solve the
direct calculation problems of multilayer composite vessels, to analyse their behavior and to verify optimization problem solutions.

The full systems of equations were described in the paper [3]. Relations between stresses and strains are described by the structural models [4]. The main idea of these models is that CM parameters are calculated through matrix and fibers mechanical parameters, fibers volume content and winding angles. The SSS of matrix and fibers are evaluated through stresses and strains of the composite shell. A failure criterion is applied for every component of CM. Here we use the von Mises criterion to determine the first stage of failure.

The objective function whose minimum is required is the minimum mass:

$$M = 2\pi \int_{\theta_0}^{\theta_1} r R_1 h d\theta \left[ \rho_m (1 - \omega_r) + \rho_r \omega_r \right] \rightarrow \min,$$

where $\rho_m, \rho_r$ are the densities of matrix and reinforcing fibers, $\omega_r$ is the volume content of reinforcement.

We chose the following design functions: the curvature radius $R_1(\theta)$ to define the generatrix, the thickness of the shell $h(\theta)$, and the reinforcement angle $\psi(\theta)$.

The solution has to satisfy the constraints on the shell’s inner volume:

$$\pi \int_{\theta_0}^{\theta_1} r^2 R_1 \sin \theta d\theta = V_0$$

and the strength requirement:

$$\max\{b_{sr}, b_{sm}\} \leq 1,$$

where $b_{sr}, b_{sm}$ are the normalized von Mises stresses in the matrix and fibers [4]. Note that the factor of safety is widely used while solving engineering problems. It can be considered by correction of the right-hand member of the inequality (Eq. (5.4)).

We used the following constraints on the design functions:

$$0 \leq \psi \leq 90, \ h^* \leq h \leq h_1^*, \ R_0^* \leq R_1 \leq R_1^*.$$

Estimation of the SSS of composite vessels using offered models leads to the solution of boundary value problems for rigid systems of differential equations. These problems are ill-conditioned, and their solutions have pronounced character of thin boundary layers. Numerical analysis was performed by the spline collocation and discrete orthogonalization methods, realized in the COLSYS [34] and GMDO [20] software. These computing tools have proved to be effective in numerical solving of wide range of problems of composite shell mechanics [4].

We investigated the vessel’s deformations by computing its stress-strain state based on the different shell theories. The vessel’s shape was a part of a toroid: $R_1 = 2.46$ m, $\theta_0 = 0.108^\circ$, $\theta_1 = 90^\circ$ (the computed half), $r(\theta_0) = 0.04$ m. The carbon composite parameters were: $E_m = 3 \cdot 10^9$ Pa, $\nu_m = 0.34$, $E_r = 300 \cdot 10^9$ Pa, $\nu_r = 0.3$, $\omega_r = 0.55$, $V_0 = 350$ liters where $E_m, E_r$ are the Young’s modulus of the matrix and fibers, $\nu_m, \nu_r$ — their Poisson’s ratio.

Figure 2 shows the SSS characteristics of the vessel with the thickness $h = 0.6$ cm, reinforced in the circumferential direction ($\psi = 90^\circ$) under the load of 170 atm. On the left the displacements of the reference surface along the generatrix $u(r)$ (dashed curves) and the normal displacement of these surface $w(r)$ (solid curves) are shown. On the right are the distribution of normalized von Mises stress (nVMS) along the thickness in the matrix $b_{sm}(r)$. The solid curves correspond to a slice at the shell edge, the dashed curves — to a slice at $\theta = 0.1$.

It’s easy to see that the basic kinematic characteristics coincide both qualitatively and quantitatively. Small differences are observed only for the stresses and deformations near the
Figure 2. The stress-strain state characteristics of the composite vessel computed using different shell theories. The curves without symbols correspond to KLST simulations, the curves marked with △ — to those using TiST, and □ — to ANST.

Figure 3. The winding angle’s influence on the composite vessel stress-strain state. KLST’s results are drawn without marks, TiST – with symbols △, ANST – with □.

Compressed edge. The maximum results and qualitative difference were obtained for ANST. This is due to accounting for the transverse shears by non-linear distribution in a thickness of a shell. Earlier it was shown [4] that ANST’s based results were closest to the ones of 3D elastic theory in most cases.

The winding angle’s influence on the CPV performance was investigated using parametric analysis. Dependence of the maximum nVMS in the matrix $bs_m$ (dashed curves) and the fibers $bs_r$ (dash–dotted curves), and the maximum size of the displacement vector $||\vec{v}||$ (solid curves) are shown in Figure 3.

The calculated values are very close in the area of their minima (Figure 3 left side). The graphs of kinematic function $||\vec{v}||$ coincide qualitatively. Some noticeable quantitative difference are revealed only for KLST’s results.
The range $\psi \in (42; 45)$ corresponds to the zones of minimum values (Figure 3 right side), which practically coincide ($\min_\psi b_{s_m} \approx 0.65$, $\min_\psi b_{s_r} \approx 1.05$, $\min_\psi ||v|| \approx 5 \cdot 10^{-3}$ m), as well as the angles, where these values are obtained ($\psi \approx 43.2^\circ$ for $b_{s_m}$ and $b_{s_r}$, $\psi \approx 43.8^\circ$ for $||v||$).

It was revealed that the winding angles of minimum stresses values were almost insensitive to the thickness variation. The change of $h$ from 0.6 to 1.6 cm corresponded to the angle’s change about 0.2°.

Additionally we investigated stress-strain state of the vessel (the thickness $h = 0.6$ cm, the winding angles at $\psi = \pm 43.2$), when nVMS in the matrix and the fibers were near their minimum. The adopted notation is the same as in Figure 2.

And again the difference is visible only in a very small region near the edge but now this difference is small enough to be neglected. Moreover the displacement values of the reference surface, the efforts and the moments completely coincide for all the theories.

All the theories (KLST, TiST, ANST) provided similar estimated characteristics of SSS. This vessel was characterized not only by essential decrease of the maximal nVMS in the matrix and the fibers, but also by their uniform distribution along the generatrix. At the same time the values of bending moments significantly reduced bringing vessel’s SSS close to momentless.

One can see that the winding angle as a design parameter gives an opportunity to increase the vessel’s strength significantly. The difference between the ”best” and ”worst” designs can reach 20–35 times comparing their nVMS in the matrix and fibers. The ”worst” designs have the winding angle close to 90°. In this case are considerable transverse shears near the compressed edge, and the loading is redistributed to a rather weak matrix while the fibers remain unloaded.

Conclusions

• The problem of maintenance of a momentless state of a composite shell with respect to parameters’ choice of reinforcement is solved.

• The solutions of the inverse problem of composite pressure vessels have been received and verified by solving the direct problems with obtained design parameters using the classical and refined shell theories.

• The performed analysis showed that the optimizing problem can be solved using rather simple shell theories (Kirchhoff–Love and Timoshenko). These theories are characterized by a lower computational complexity of corresponding boundary value problem if compared to Andreev–Nemirovskii theory. It takes from 10 to 20 times less resources.

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