Distribution of Instanton and Monopole Clustering

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Abstract

We study the relation between the instanton distribution and the monopole loop length in the SU(2) gauge theory with the abelian gauge fixing. We measure the monopole current from the multi-instanton ensemble on the 16$^4$ lattice using the maximally abelian gauge. When the instanton density is dilute, there appear only small monopole loops. On the other hand, in the dense case, there appears one very long monopole loop, which is responsible for the confinement property, in each gauge configuration. We find a clear monopole clustering in the histogram of the monopole loop length from 240 gauge configurations.

I Topological Objects in QCD

In QCD, there are two non-trivial topological objects, which are important for understanding non-perturbative properties. QCD is reduced to an abelian gauge theory with QCD-monopole after performing the abelian gauge fixing [1]. The monopole appears corresponding to the non-trivial homotopy group, $\pi_2(SU(N_c)/U(1)^{N_c-1}) = Z_{N_c-1}^\infty$, and monopole condensation plays an essential role on color confinement and chiral symmetry breaking [2,3]. On the other hand, an instanton appears as a classical non-trivial solution in Euclidean 4-space corresponding to the homotopy group, $\pi_3(SU(N_c)) = Z_\infty$ [4]. Also, instantons are very important for the phenomena related to the $U_A(1)$ anomaly and chiral symmetry breaking. Until now, however, there has been no evidence the instanton has anything to do with color confinement.

Thus, it seems that instantons and monopoles are belonging completely different sectors of physics, and are hardly related to one another. However, both of them should be essential key for non-perturbative QCD. Therefore, we investigate the relation between instantons and monopoles in terms of the color confinement mechanism.

II Strong Correlation between Instanton and Monopole

Recently, the strong correlation between instantons and monopoles has been found both in the analytical framework and the lattice QCD [5-11]. There are two remarkable points. First, from the analytical studies [6,8-11], each instanton seems to accompany a monopole trajectory near its center. Such a correlation may lead to the linearity on the relation between the instanton number and the total monopole loop length, which is clearly observed in the lattice QCD [11]. Second, apart from the centers of instantons, monopole trajectories are very unstable against small fluctuation on the instanton’s size and
location [8,11]. Therefore, these fluctuations tend to combine isolated monopole trajectories into one longer trajectory. We then expect that monopole trajectories would combine neighboring instantons as the instanton density increases. Due to the above two facts, there would appear very long and complicated monopole trajectories in the non-perturbative QCD vacuum characterized by the dense topological pseudoparticles (instantons and anti-instantons).

The monopole clustering is observed in the lattice QCD simulation [11]. In the confinement phase, which includes many instantons, there appears one very long monopole loop which covers the entire lattice space in each gauge configuration [12]. This long and complicated monopole loop is a signal of monopole condensation, which is responsible for color confinement [13]. On the other hand, in the deconfinement phase, there appear only small monopole loops, which would not contribute to the confinement force.

We study the multi-instanton system in terms of the monopole clustering in order to understand the essence of the non-perturbative QCD vacuum.

### III Multi-Instanton Configuration

The multi-instanton ensemble is characterized by the density and the size distribution of instantons [14]. It has been analytically shown that the linear confinement potential can be obtained if the instanton size distribution falls off as $1/\rho^3$ at large $\rho$. While, the ordinary instanton liquid models suggest that the distribution behaves as $1/\rho^5$ at the large size. As for the small size, the distribution has to follow the one loop result, $f(\rho) \sim \rho^{b-5}$ with $b = 11N_c/3$. Therefore, we adopt the size distribution as

$$f(\rho) = \frac{1}{(\rho/\rho_1)^\nu + (\rho/\rho_2)^{\nu-5}}$$

with size parameters $\rho_1$ and $\rho_2$, which should satisfy the normalization condition, $\int_0^\infty d\rho f(\rho) = 1$. The maximum of the distribution is fixed to the standard probable size $\rho_0$. We calculate for the two cases with $\nu = 3, 5$ for the large size distribution.

The gauge configuration of an instanton with the size $\rho$ and the center $z$ in the singular gauge is expressed as

$$A_I^\mu(x; \rho, O) = \frac{i\tau^a \rho^2 O^{ab} \tilde{\eta}^b_{\mu\nu} (x-z) \nu}{(x-z)^2 [(x-z)^2 + \rho^2]}$$

where $O$ denotes the SU(2) color orientation matrix and $\tilde{\eta}^a_{\mu\nu}$ is the 't Hooft symbol. For an anti-instanton $A_{\bar{I}}^\mu$, one has to replace the $\tilde{\eta}$ symbol with $\eta^a_{\mu\nu}$.

The multi-instanton configurations are approximated as the sum of instanton and anti-instanton solutions,

$$A_{\mu}(x) = \sum_k A_I^\mu(x; z_k, \rho_k, O_k) + \sum_k A_{\bar{I}}^\mu(x; z_k, \rho_k, O_k).$$

We generate ensemble of pseudoparticles with random color orientations $O_k$ and centers $z_k$. The instanton sizes $\rho_k$ are randomly taken according to Eq.(1). These procedures are performed in the
continuum theory. We then introduce a lattice on this gauge configuration and define the link variables, $U_\mu = \exp(i a A_\mu)$. We apply the maximally abelian gauge fixing \cite{8,10-12}, which maximizes $R = \sum_{\mu,s} \text{Tr}(U_\mu(s) \tau^3 U_\mu(s)^\dagger \tau^3)$, in order to extract monopole trajectories in the multi-instanton ensemble. We measure the monopole loop lengths and make the histograms of the monopole loop length.

IV Numerical Results

We take a $16^4$ lattice with the lattice spacing $a = 0.15 \text{fm}$, which means that the total volume is equal to $V = (2.4 \text{fm})^4$. The probable instanton size is fixed as $\rho_0 = 0.4 \text{fm}$ in our calculation. We show two typical cases with the total pseudoparticle number $N = 20$ and 60, which correspond to the density $(N/V)\simeq 174$ and 228 MeV, respectively. Fig.1 shows the histograms of monopole loop lengths for two density cases with $\nu = 3$. At low instanton density (Fig.1(a)), only relatively short monopole loops remain. At high density (Fig.1(b)), there appears one very long monopole loop in each gauge configuration. As the density increases, a monopole trajectory tends to combine the overlapping pseudoparticles, and this mechanism generates one very long and highly complicated monopole loop. The appearance of the monopole clustering can be interpreted as a Kosterlitz-Thouless-type phase transition \cite{13}. The critical density is found to be about 200 MeV in the $\nu = 3$ case. For the $\nu = 5$ case, the similar tendency is obtained qualitatively, although the critical density is a little higher.

V Discussion and Concluding Remarks

Finally, we compare our results with those of the true SU(2) lattice QCD with $16^3 \times 4$ at different temperatures ($\beta = 2.2$ and 2.35) \cite{11}. The monopole clustering at the high density case (Fig.1(b)) resembles that in the confinement phase ($\beta = 2.2$), where the instanton density is relatively high as suggested in lattice QCD. On the other hand, the disappearance of the long monopole loop at the dilute case (Fig.1(a)) is similar to the result obtained in the deconfinement phase ($\beta = 2.35$). As the temperature increases, the instanton density is largely reduced \cite{11} due to the instanton and anti-instanton pair annihilation. Therefore, each instanton becomes isolated, and the monopole loop originating from the instanton tends to be localized around each instanton. As a result, there remain only small monopole loops at high temperature.

Thus, the instanton density would play an essential role on the deconfinement phase transition through the monopole clustering, which is a signal of monopole condensation \cite{13}.

As interesting related subjects, we are now studying the local correlation between instantons and monopoles, and are calculating the Wilson loop in order to clarify the role of these topological objects on the color confinement mechanism directly.

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Fig.1(a): dilute case \[ (N/V)^{1/4} = 174\text{MeV} \]

Fig.1 Histograms of monopole loop lengths in the multi-instanton system:
(a) dilute case and (b) dense case.
Fig. 1(b): dense case \[(N/V)^{1/4} = 228\text{MeV}\]