Compare between rotational and translational seismic response spectra

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Abstract. Rotational response spectra are still absent due to the substantial lack of direct measurements of rotational seismic waves. The work develops a relationship between rotational response spectra and translational ones. By the elastic wave theory, seismic rotations can be calculated from translational waves. In this paper, 113 translational waves are used to calculating rotational waves. And then the translational response spectra and rotational response spectra are calculated and compared. Factors such as the site condition and frequency are considered. The results show that from lower frequency to higher frequency, the ratio of normalized response spectra between rotational and translational waves increases. The ratios are also related to the site condition.

1. Introduction

Many earthquake damages have shown that buildings may be destroyed due to large torsion. Even symmetric structures could be expected to undergo substantial torsional excitations during an earthquake [1]. It has been believed that rotational ground motions must exist and it may cause heavily structural damage. However, rotational input is not considered directly due to a lack of knowledge of the rotational waves. Until now, there are not enough recordings of rotational seismic waves because most current seismometers could record translational motions only.

Some methods have been developed to estimate the seismic rotations. One of the methods is the two point difference theory [2]. The rotations have also been estimated using the elastic wave theory of wave propagation [3]. In civil engineering, a simplified theoretical method for simulating the rotations, which is called the elastic wave theory in this paper, has been widely studied [4]. Newmark established a simple relation between the rotational and translational components [5]. Later on, this idea was pursued further by other investigators. Hart and DiJulio [6] differentiated numerically two orthogonal translational records and obtained the associated free-field rotational motions. An improved approach for obtaining the rotations was presented by Li et al. [7].

Rotational response spectra, which are very important for rotational seismic design of buildings, have not been widely researched. Tso and Hsu [8] studied some rotational spectra of a few waves. The rotations were calculated by the elastic wave theory. Lee and Trifunac [9] used a computer program to simulate the rotation on the site at Westmoreland in Imperial Valley. The rotations were also calculated by the elastic wave theory. The ratio of rotational to translational response spectra at different periods was discussed. Castellani and Zembaty [10] pointed out that the cross-correlation of the surface accelerations at nearby stations was directly related to the power spectra of rotations. This relation was applied to evaluate rotation spectra on the basis of empirical cross-correlations derived from records collected at three arrays of strong motion. The research was dependent on some specific situations. The rotational spectra developed by Rutenberg and Heidebrecht [11] were related to the
corresponding translational spectra through a harmonic relationship, which was derived from pure theoretical studies. More statistical analyses of rotational spectra involving a great number of rotations are needed. Kalkan and Graizer [12] presented another way to calculate the title spectra, but torsions (rotations about vertical axes) and torsional spectra can’t be generated with his method.

This paper tries to develop the rotational response spectra for civil engineers. The study involves a large number of strong earthquake waves to find the characteristics of the rotational response spectra.

2. Relation of rotational and translational spectra

2.1. Description of rotation

As shown in figure 1, in an elastic body, an element OBAC deformed to O’B’A’C’. It displacement in x and y direction is \( u \) and \( v \) respectively. The rotational angle of OB is \( \theta_1 \) and the rotational angle of OC is \( \theta_2 \). And then the rotation of the body about z axis can be expressed as equation (1)

\[
\phi_z = \frac{1}{2} (\theta_2 - \theta_1) = \frac{1}{2} (\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y})
\]

2.2. Relationship between rotational and translational wave

In civil engineering, by the elastic wave theory for simulating the rotations, the relationship between rotational seismic ground motion and translational seismic ground motion is given as follows.

Figure 2 shows the case of SH wave propagation in direction \( x \) in soil. The SH wave oblique incident in direction \( x \) to a surface. And it will be reflected. Assume that the medium of wave propagation is homogeneous, isotropic and elastic half-space. The \( x \), \( y \) and \( z \) axis in coordinate system stand, respectively, for two horizontal axes and a vertical axis. \( u \) and \( v \) is the horizontal motion in \( x \) and \( y \) direction, and \( \phi_z \) is rotation about \( z \) axis. The SH wave only produces the displacement in the \( y \) direction while displacements in \( x \) and \( z \) directions are both zero. The incident and reflect SH waves are given by equation (2) and (3), respectively.

\[
S_I = A \exp(i\omega t) (\sin \theta \frac{x}{\beta} + \cos \theta \frac{z}{\beta})
\]

\[
S_R = A \exp(i\omega t) (\sin \theta \frac{x}{\beta} - \cos \theta \frac{z}{\beta})
\]

Where \( S_I \) and \( S_R \) is incident and reflect SH wave, \( A \) is wave amplitude, \( \theta \) is incidence angle, \( \beta \) is velocity of the wave, \( \omega \) is frequency of the wave.

On the surface of the space, the displacement is \( u = 0 \)
Substituting equation (4) and equation (5) in equation (1), one may obtain

\[ \varphi_x = 2 \frac{1}{\beta} \left( \frac{\partial \varphi}{\partial t} - \frac{\partial \psi}{\partial x} \right) = -\sin \theta \frac{\partial v}{\partial t} - \frac{\sin \theta}{2} 2A \omega \exp \left( t - \frac{\sin \theta}{\beta} x \right) \]

Where \( \beta/\sin \theta \) is called apparent wave velocity.

3. Wave data
The translational waves in this paper come from the PEER database [13]. The database contains a large number of translational waves. The wave forms, hypocenters, epicenter distances, stations and site conditions [14] of the waves are all available in the database. 59 waves in site B and 54 waves in site E in the database are used. Their epicenter distances are shorter than 100km to avoid surface waves. Their apparent wave velocities are assumed to be 1 m/s.

As pointed out by Graizer, the translational recording is contaminated by rotational component. But as well known, the error is usually acceptable for civil engineers. Even though the rotation calculated by elastic wave theory based on the recording is also contaminated, the error of the calculated rotation is acceptable too. In addition, the baseline correction of the translations may suppress the recording error.

Rotational response spectra are calculated based on the estimated rotations. The spectra here are pseudo acceleration response spectra with damping ratio 0.05, as shown in Figure 3, for illustration.

4. Ratio of normalized spectra
From the seismic ground motion, rotational or translational, one can get seismic response spectra. Translational response spectra of strong seismic waves have been well developed. This paper tries to establish the relationship between rotational and translational spectra. Before calculate response spectra, the peak amplitude of all waves, rotational or translational, are scaled to 1. It means the response spectra discussed here is normalized response spectra. Let \( r_p \) be

\[ r_p = \frac{P(\bar{v})}{P(\bar{\varphi})} \]

Where \( P \) is the response spectra of wave, \( \bar{v} \) is normalized translational wave, \( \bar{\varphi} \) is normalized
rotational wave, $r_p$ is the ratio between normalized rotational and translational spectra, it depends on frequencies.

The average spectra are presented together in Figure 4 to make a comparison. It shows that the rotational spectra are affected by site condition as well as the translational spectra. The site effects on rotational and translational spectra seem not to be obviously different. For frequencies smaller than a certain frequency (call the frequency $\Omega$ at present), the normalized spectral amplitudes for hard site are substantially lower than those for soft site, whereas, they are higher if the frequencies are greater than $\Omega$. $\Omega$ for the rotational spectra is about 5-7.5Hz, and about 2.5-5Hz for the translational spectra. $\Omega$ is also the frequency where the spectral amplitude is maximum. So $\Omega$ is named to be the peak spectra frequency (PSF). From lower frequency to higher frequency, the translational spectra and rotational spectra increase before PSF. For frequencies higher than PSF, the translational spectra decline obviously, but the amplitude of rotational spectra extends over a larger frequency region. That is because the rotations contain more high frequency components compared with translations. The frequencies where $r_p$ equal to 1 are close to the average of translational and rotational PSF. From lower frequency to higher frequency, $r_p$ increase from 0.1 to about 1.4 (Figure 5). So, buildings with short natural period are more sensitive to rotational waves. $r_p$ tend to be smaller a little in the order of sites B and E.

\begin{center}
\begin{figure}[h]
\begin{subfigure}{0.45\textwidth}
\includegraphics[width=\textwidth]{translational_spectra}
\caption{translational spectra}
\end{subfigure}
\begin{subfigure}{0.45\textwidth}
\includegraphics[width=\textwidth]{rotational_spectra}
\caption{rotational spectra}
\end{subfigure}
\end{figure}
\end{center}

5. Conclusion
The rotations of a large number of strong earthquakes are estimated and the rotational response spectra are calculated and analysed. Factors such as the site condition and frequency are considered. The results show that from lower frequency to higher frequency, the ratio of normalized response spectra between rotational and translational waves increases. The ratios are also related to the site condition. The work may be helpful for establishing seismic codes of building design.
Figure 5. $r_p$

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