Intensity Mapping as a Probe of Axion Dark Matter

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ABSTRACT

Intensity mapping (IM) of spectral lines has the potential to revolutionize cosmology by increasing the total number of observed modes by several orders of magnitude compared to the cosmic microwave background (CMB) anisotropies. In this paper, we consider IM of neutral hydrogen (HI) in the redshift range 0 \( \lesssim z \lesssim 3 \) employing a halo model approach where HI is assumed to follow the distribution of dark matter (DM) halos. If a portion of the DM is composed of ultralight axions then the abundance of halos is changed compared to cold dark matter below the axion Jeans mass. With fixed total HI density, \( \Omega_{\text{HI}} \), assumed to reside entirely in halos, this effect introduces a scale-independent increase in the HI power spectrum on scales above the axion Jeans scale, which our model predicts consistent with N-body simulations. Lighter axions introduce a scale-dependent feature even on linear scales due to its suppression of the matter power spectrum near the Jeans scale. We use the Fisher matrix formalism to forecast the ability of future HI surveys to constrain the axion fraction of DM and marginalize over astrophysical and model uncertainties. We find that a HIRAX-like survey is a very reliable IM survey configuration, being affected minimally by uncertainties due to non-linear scales, while the SKA1MID configuration is the most constraining as it is sensitive to non-linear scales. Including non-linear scales and combining a SKA1MID-like IM survey with the Simons Observatory CMB, the benchmark “fuzzy DM” model with \( m_a = 10^{-22} \text{ eV} \) can be constrained at the 10% level. For lighter ULAs this limit improves below 1%, and allows the possibility to test the connection between axion models and the grand unification scale across a wide range of masses.

Key words: cosmology: theory, dark matter, elementary particles

1 INTRODUCTION

Measurements of the power spectrum of the cosmic microwave background (CMB) anisotropies establish the precision cosmological standard model (Planck Collaboration et al. 2016, 2018). Intensity mapping (IM) of spectral lines has great potential as a future cosmological probe (Loeb & Wyithe 2008; Bull et al. 2015; Kovetz et al. 2017, 2019; Parsons et al. 2019; Padmanabhan et al. 2019; Bernal et al. 2019), since the frequency dependence due to redshift gives a tomographic three dimensional map, vastly increasing the number of accessible modes compared to the CMB (Mao et al. 2008).

Hydrogen is the most abundant element in the Universe: according to measurements and the standard theory of Big Bang Nucleosynthesis, it makes up approximately 75% of all ordinary matter (Planck Collaboration et al. 2018). In the intergalactic medium at \( z \lesssim 6 \) hydrogen is ionized by UV radiation from galaxies, while neutral hydrogen (HI) resides inside galactic halos (Wolfe et al. 2005; Zwaan et al. 2005; Lah et al. 2007, 2009; Villaescusa-Navarro et al. 2018). Therefore HI traces the structure of galaxies and their host dark matter (DM) halos, and IM of the hyperfine, or 21cm, HI transition is a probe of DM clustering. An empirical (data-driven) framework for the HI power spectrum is provided by the HI halo model (Padmanabhan et al. 2017; Padmanabhan & Refregier 2017), which maps between the theoretical DM halo mass function (e.g. Sheth & Tormen 2002), and the HI halos which trace it (see e.g. Bagla et al. 2010; Marin et al. 2010, for other HI halo prescriptions).

DM is a key ingredient in the cosmological standard model, yet its nature is a mystery. The only DM candidate in the standard model of particle physics is the neutrino, which is known to make up less than 1% (but more than 0.5%) of the total DM abundance because the relativistic velocity of the neutrino background make it too “hot” to account for observed structure formation (Planck Collaboration et al. 2018; Alam et al. 2017; Tanabashi et al. 2018). Ob-
observations are consistent with the majority of the remaining DM being composed of a single species of cold, collisionless DM (CDM). Of relevance to the present study, CMB anisotropies constrain the density parameter of ultralight axions (ULAs) to be $\Omega_{a} h^2 \lesssim 0.003$ over the mass range $10^{-32} \text{eV} \lesssim m_a \lesssim 10^{-25} \text{eV}$ (Hložek et al. 2018).

The existence of multiple species of light axions is a generic and well-established prediction of string/M-theory (Svrcek & Witten 2006; Conlon 2006; Arvanitaki et al. 2010; Acharya et al. 2010; Demirtas et al. 2018) and many other extensions of the standard model (e.g. Peccei & Quinn 1977; Weinberg 1978; Wilczek 1978; Banks et al. 2003; Kim & Marsh 2016). The axion density parameter is determined by the axion mass, $m_a$, and symmetry breaking scale (or “decay constant”), $f_a$, as well as a single random number, $\theta \in [0, \pi]$, related to the initial conditions (see Marsh 2016, for a review). In the mass range of interest to cosmology, the decay constant is constrained to be in the range $10^9 \text{GeV} \lesssim f_a \lesssim 10^{18} \text{GeV}$, with some models preferring the grand unified scale, $f_a \sim 10^{16} \text{GeV}$. Taking $\theta \sim \theta (1)$ it is thus predicted that ULAs with $m_a \lesssim 10^{-22} \text{eV}$ contribute sub-dominantly to the DM density (consistent with observations). On the other hand those with $m_a \sim 10^{-22} \text{eV}$, known as “fuzzy DM” (Hu et al. 2000), can make up a significant fraction of the DM, and furthermore have a host of interesting phenomenological consequences on galaxy formation and other astrophysical systems (see e.g. Marsh 2016; Hui et al. 2017; Schive et al. 2014; Arvanitaki et al. 2010; Niemeyer 2019).

In the present work we use the HI halo model to explore the effects of a ULA sub-species of DM in the mass range $10^{-32} \lesssim m_a \lesssim 10^{-22} \text{eV}$ on the HI power spectrum at $z \lesssim 6$. Due to the large ULA de Broglie wavelength, small scale structure formation is suppressed relative to CDM, which manifests in an increase in the HI power on large scales. We show that, due to this effect, future IM surveys in conjunction with the CMB are sensitive to a percent level ULA component of DM, and can thus be used as a precision test of the predictions related to fuzzy DM and the grand unified scale for $f_a$.

This paper is organized as follows: First, we introduce the formalism in Sec. 2. This includes a description of the axion physics at play (Sec. 2.1) and the HI halo model and how axions are accommodated to it (Sec. 2.2). We present the results on the HI power spectrum and compare them to those of pure ULA numerical simulations in Sec. 2.3. In Sec. 2.4 the configurations for the IM surveys are described and in Sec. 2.5 the Fisher forecast formalism is introduced. We proceed by presenting the main results and constraints gained within this framework in Sec. 3 and discuss our main findings in Sec. 4.

2 THE HI POWER SPECTRUM

21 cm IM measures the integrated intensity of the spin-flip transition of neutral hydrogen across the sky and redshift (see e.g. Bull et al. 2015, and references therein). The redshifted 21 cm radiation is well into the radio regime which means that matter along the line of sight does not interfere with the signal. The redshifted signal can be detected from the “Dark Ages”, through the epoch of reionization (EoR) and the post-reionization Universe. After reionization the remaining neutral hydrogen is expected to reside to great extent in comparatively dense clouds, which shields them from ionizing UV radiation. These clouds are known as damped Lyman-$\alpha$ (DLA) systems and are confined to galaxies. Therefore, neutral hydrogen is expected to trace the galaxy distribution in the current, post-reionization epoch ($z \lesssim 6$). Over recent years, it became apparent that using IM to measure the large-scale structure in the late-time Universe is a promising cosmological probe (Bharadwaj & Sethi 2001; Chang et al. 2010; Battye et al. 2013; Bull et al. 2015; Villaescusa-Navarro et al. 2015). In this work we especially focus on the prospect of these surveys to constrain cosmologies including ultra-light axions (ULAs). To do this we employ and modify an empirical framework to describe the 21 cm signal conceived and further constrained by Padmanabhan et al. (Padmanabhan et al. 2015, 2016; Padmanabhan & Refregier 2017; Padmanabhan et al. 2017). This formalism effectively treats the neutral hydrogen as a biased tracer of the underlying matter distribution and the model parameters are entirely constrained by the compilation of the latest observations on neutral hydrogen systems over $z \sim 0$ – 5. The astrophysical priors thus obtained are realistic since they are grounded in present-day observations. The advantages of using such an empirical model are manifold: Due to the computational simplicity, it allows us to consider many models, vary physics easily, and it can be conveniently implemented within a Fisher matrix analysis. Furthermore, our model reproduces the qualitative features of simulations with ULA DM by Carucci et al. (2017).

Table 1. Fiducial cosmological and astrophysical parameters and step size for calculating the Fisher derivatives. The cosmological parameters are the same as in the forecast by Hložek et al. (2017) and are within current CMB data constraints in Hložek et al. (2018). The astrophysical parameters have been adopted from Padmanabhan et al. (2019) and are the best-fit values found in Padmanabhan et al. (2017).

| Parameter   | Fiducial value | Step size |
|-------------|----------------|-----------|
| $h$         | 0.69           | 0.01      |
| $\Omega_d$  | 0.25142        | 0.004     |
| $\Omega_b$  | 0.04667        | 0.004     |
| $\Omega_a/\Omega_d$ | 0.02 | 0.005 |
| $\sum m_i \ [\text{eV}]$ | 0.06 | fixed |
| $N_{\text{eff}}$ | 3.046 | fixed |
| $A_s$       | $2.1955 \times 10^{-9}$ | $10^{-13}$ |
| $n_s$       | 0.9655         | 0.0005    |
| $k_{\text{min}} \ [\text{Mpc}^{-1}]$ | 0.05 | fixed |
| $m_a \ [\text{eV}]$ | $10^{-32} < m_a < 10^{-22}$ | fixed per run |
| $v_{c,0} \ [\text{km/s}]$ | 36.3 | 0.01 |
| $\beta$     | $-0.58$        | 0.003     |
| $\gamma$    | 1.45           | fixed     |
| $\Omega_{\text{m},0}$ | 0.2865 | fixed |
| $\Omega_{\text{b},0}$ | 2.45 $\times 10^{-4}$ | fixed |

1 This treatment is supported by results of large magneto-hydrodynamic simulations, e.g. using TNG100 by the IllustrisTNG project (Villaescusa-Navarro et al. 2018).

2 Publicly available at https://github.com/dgrin1/axionCAMB.

3 In summary, we treat the axions with potential $a \sim (\theta)$ and the model

2.1 Axion Physics

In this section we shortly recapitulate the relevant linear physics of axions. It is included in axionCAMB (Hložek et al. 2015), which we use to perform the calculations. ULAs are described as a pseudo-scalar field obeying the Klein-Gordon equation for temperatures below the global symmetry breaking and non-perturbative scales. This classical treatment of the axion field is justified due

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to huge occupation number of a condensate with cosmological density.

Axion DM is produced by vacuum realignment of the classical field (Preskill et al. 1983; Abbott & Sikivie 1983; Dine & Fischler 1983). The axion field at early times in the Universe, shortly after inflation, is overdamped and therefore mimics the vacuum energy with equation-of-state parameter \( w = -1 \). Later, when \( m_a \sim H \) (with \( H := a/\dot{a} \) and \( a \) being the scale factor in the FLRW metric) the axion field starts to oscillate, defining \( a_{osc} \). From that time on the energy density scales as \( \rho_a \sim a^{-3} \), just as ordinary matter (and the pressure \( p_a = \frac{\rho_a}{\rho_{crit}} \) oscillate rapidly around zero). This makes the axion field a suitable candidate for CDM. The axion density parameter depends on the value of the axion energy density \( \Omega_a \), \( \rho_a \sim \rho_{crit} \) being the cosmological critical energy density today. A useful approximate formula for the axion density parameter is given by (Hložek et al. 2015):

\[
\Omega_a \approx \begin{cases} 
3.3 \times 10^{-3} \left( \frac{\Omega_0}{0.30} \right)^{3/4} \\
\times \left( \frac{m_a}{10^{-28}\text{eV}} \right)^{1/2} \left( \frac{\phi}{10^{14}\text{eV}} \right)^2, & \text{if } a_{osc} < a_{eq} \\
7.6 \times 10^{-6} \left( \frac{\Omega_0}{0.30} \right) \left( \frac{\phi}{10^{14}\text{eV}} \right)^2, & \text{if } a_{eq} < a_{osc} < 1.
\end{cases}
\] (1)

In the following we will parametrize the axion abundance relative to the total DM density parameter with \( \Omega_Q \) and \( \Omega_a = \Omega_{Q} + \Omega_a \).

Perturbation in this axion field can be solved with help of a WKB-like ansatz, once the scalar field is in its oscillary phase. One finds (Hwang & Noh 2009) that \( \langle w \rangle \equiv \langle w' \rangle = 0 \) with a non-negligible sound speed arising from the large de Broglie wavelength of the axion:

\[
c^2_s = \frac{k^2}{4m_a^2c^2},
\] (2)

Thus, perturbations in the axion field are subject to a pressure induced by the uncertainty principle (relevant at cosmological scales due to the tiny mass of the axion). The equations of motion in synchronous gauge for the perturbed axion energy density is that of a general fluid with the above parameters (when \( a \gg a_{osc} \)) (Marsh 2016)

\[
\delta_a' = -3k_a^2 - \frac{3}{2}H - 3\mathcal{H}c^2_s\delta_a - 9\mathcal{H}^2c^2_s u_a/k, \\
u_a' = -3\mathcal{H} u_a + c^2_s k\delta_a + 3c^2_s \mathcal{H} u_a.
\] (3)

Here, \( \mathcal{H} \) denotes the conformal Hubble rate. The additional non-canonical terms on the right-hand side of Eqs. (3) and (4) account for Eq. (2) applying only in the axion comoving gauge (Hložek et al. 2015).

From Eqs. (3) and (4) it is evident that for large scales the pressure terms go to zero, \( c^2_s \rightarrow 0 \), and the dynamics of the axion equations of motion match those for CDM. For smaller scales, however, the sound speed term becomes relevant, giving rise to a scale-dependent, oscillating solution, ultimately responsible for the suppression of structure when compared to CDM.

To get an idea how this scale- and time-dependent sound speed term affects the evolution of axion density perturbations, let us first define the scale at horizon crossing,

\[
k_c = aH,
\] (5)

and the scale where oscillations roughly start, \( k_r \sim m_a \). Modes with \( k > k_r \) are thus relativistic modes with \( c^2_s \approx 1 \), while modes with \( k < k_r \) are non-relativistic with \( c^2_s \rightarrow k^2/(4m^2a^2) \). As \( k_r \) increases with time, more and more modes become non-relativistic.

If a mode is already non-relativistic when it enters the horizon \( (k_r < k_r \Rightarrow H < m_a) \), the sound speed term is negligibly small and the mode will behave as ordinary DM (‘long modes’). If, however, the mode is relativistic when entering the horizon \( (m_a > H) \) and becomes non-relativistic later on, the sound speed term cannot be neglected and the axions “free-stream” (“short modes”). These modes will decay until some later time (when the gravitational pull is dominant), given by the comoving Jeans wavenumber \( k_J = a\sqrt{H\Omega_m} \). The comoving Jeans wavenumber is time-independent in a radiation-dominated epoch. So, relativistic modes entering the horizon when radiation domination will decay until after matter-radiation equality.

Let us define the minimal scale \( k_m \) at which suppression, i.e. no growth of modes, sets in for given mass \( m_a \) (cf. Fig. 1). The axions therefore introduce a step-like feature to the matter transfer function, which is due to its scale-dependent sound speed term (Arvanitaki et al. 2010). The width of the step is given by \( k_m \) and \( k_J \). The different scales at play are shown in Fig. 1 for an axion mass of \( m_a = 10^{-28} \text{eV} \) and other parameters as given in Table 1. The hatched region shows the scales inside the Hubble horizon (Eq. 5) and the purple-shaded region indicates non-relativistic modes (where \( c^2_s \approx k^2/(4m^2a^2) \)).

Finally, note that this behavior is conceptually similar to that of massive neutrinos. Due to their large thermal velocities, massive neutrinos also introduce an effective sound speed term \( c_s = T^0_\nu/(m_\nu) \) for \( m_\nu \geq T^0_\nu \), where \( T^0_\nu \) is the neutrino temperature today and \( m_\nu \) the neutrino mass (e.g. Amendola & Barbieri 2006; Marsh et al. 2012). This, equivalently to \( k_m \) & \( k_J \) for axions defines a “free-streaming” scale below which the pressure term dominates and clustering is prohibited. Analogous to axions, this also introduces a step-like feature to the transfer function, i.e. a suppression of the matter power spectrum above \( k_m \) when compared to CDM.

### 2.2 Modeling HI

In the following we introduce the HI halo model exploited for the present study (Sec. 2.2.1) and specifically comment on the inclusion of ULAs in Sec. 2.2.2. Halo models are concerned about amplitudes of matter fluctuations larger than one (Cooray & Sheth 2002) and matter components with small variance can approxi-
2.2.1 The Halo Model and Angular Power Spectrum

We exploit spherical harmonic tomography to analyze the two-point correlation of the HI fluctuation. This choice circumvents the need to assume a specific comoving distance relation $r(z)$ and, therefore, a specific cosmology when analyzing the data. Spherical harmonic tomography discretizes the redshift range and decomposes the signal in each redshift bin with spherical harmonics. The measured brightness temperature $\delta T(x, z)$ is projected on the sky with the commonly used projection kernel (Battye et al. 2013)

$$W_i(z) = \begin{cases} \frac{1}{2}, & \text{if } z_i - \frac{\Delta z}{2} \leq z \leq z_i + \frac{\Delta z}{2} \\ 0, & \text{otherwise.} \end{cases}$$

(6)

The dimensionless 2D angular power spectrum (dividing by the mean brightness temperature) is ultimately given by

$$C_i(z_i, z_j) = \frac{2}{\pi} \int dz W_i(z) \int dz' W_i(z')$$

$$\times \int k^2 dk P_{\text{HI}}(k, z_i, z_j) j_2(k r(z')) j_2(k r(z)),$$

where $j_2(x)$ is the spherical Bessel function of the first kind and $P_{\text{HI}}(k, z_i, z_j)$ denotes the unequal-time HI power spectrum.\(^5\) The comoving distance to redshift $z$ is given by (with $c$ being the speed of light)

$$r(z) = c \int_0^z \frac{dz'}{H(z')}.$$

(8)

In the Limber approximation (Limber 1953), the spherical Bessel function of the first kind is approximated by $j_2(x) \rightarrow \sqrt{\pi x} \delta_0(x - (\ell + \frac{1}{2}))$, and correlations between different redshift bins cancel. Thus, we can write the dimensionless angular power spectrum by:

$$C_i(z_i) \simeq \frac{1}{c^2} \int dz \frac{W_i^2(z) H(z)}{r^2(z)} P_{\text{HI}} \left( \frac{r_i}{r(z)} \right),$$

(9)

where $P_{\text{HI}}(k, z)$ denotes the Cartesian HI power spectrum. LoVerde & Afshordi (2008) showed that in the case of narrow redshift bins the approximation is expected to be accurate within 1% above $\ell \sim 10$ for $P(k, z) = P(k) D(z)$. In this study, similar narrow redshift bins are used (see below). If the above factorization of wavenumber $k$ and redshift $z$ does not hold, second order corrections to the Limber approximation arise. However, this factorization does hold in the redshift range considered presently for ULAs with mass $>10^{-32}$ eV. Out of this reason and given that ULAs leave the matter power spectrum unchanged on large scales exactly where the Limber approximation is known to be less accurate, we do not expect that the accuracy changes significantly for $C_i$ in the present case (except for the marginal case $m_0 \sim 10^{-32}$ eV). Moreover, Olivi et al. (2017) showed that the difference between the exact formula and the Limber approximation is small: the loss in information from the cross-correlation of different redshift bins is roughly compensated in the enhancement of the auto-correlation.

Note that in deriving Equations (7) and (9) the effect of peculiar velocities was neglected (Battye et al. 2013). The latter will lead to redshift-space distortions (RSDs), which manifest themselves e.g. in the Kaiser effect (Kaiser 1987) and the Sachs-Wolfe effects (for a thorough discussion, see e.g. Bonvin & Durrer 2011). Seehars et al. (2016) showed by a semi-numerical approach (i.e. taking a N-body simulations and populating the halos with neutral hydrogen via HI halo mass relation) that the RSDs may be significant for large scales ($\ell \lesssim 200$). Nonetheless, we choose to neglect the RSDs in the present study. Ignoring the RSDs is a conservative assumption in that including them would provide additional cosmological constraints (Bull et al. 2015) and increase the 21 cm signal (cf. Seehars et al. 2016).

To calculate the HI power spectrum we exploit the neutral hydrogen halo model, elaborated on in several papers (Padmanabhan et al. 2016; Padmanabhan & Refregier 2017; Padmanabhan et al. 2017). The authors constrained it with the combination of all existing low-redshift 21 cm observations, DLA data and HI galaxy surveys (e.g. Switzer et al. 2013; Zafar, T. et al. 2013; Martin et al. 2012, respectively). Specifically, the halo model assumes that a halo of mass $M$ at redshift $z$ contains a number of galaxies which in total host neutral hydrogen of mass $M_{\text{HI}}$. Compactly, this statement is condensed in the deterministic function

$$M_{\text{HI}}(M, z) = \alpha f_{\text{HI}}(M) \left( \frac{M}{10^{11} h^{-1} M_{\odot}} \right)^{\beta} \exp \left[ - \left( \frac{v_{c,0}}{v_c(M, z)} \right)^3 \right].$$

(10)

This relation features three free parameters. $\alpha$ is the overall normalization, $\beta$ is the slope of the HI halo mass relation and $v_{c,0}$ is the physical, lower virial velocity cut-off, above which a halo can host neutral hydrogen. $f_{\text{HI}}$ is the cosmic hydrogen fraction: $f_{\text{HI}} = \Omega_h / \Omega_m (1 - Y_p)$, where $Y_p = 0.24$ is the helium abundance. The virial velocity of the halo, $v_c(M, z)$, is related to its mass $M$ by

$$v_c(M, z) = \sqrt{\frac{GM}{R_{\text{vir}}}}.$$

(11)

where the virial radius in physical coordinates is

$$R_{\text{vir}} = \left( \frac{3}{4\pi} \frac{M}{\Delta v^2 \rho_0} \right)^{1/3} \frac{1}{1 + z}.$$

(12)
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\[ \rho_0 = \Omega_0 \rho_{\text{crit}} \] is the background density, also used to calculate the halo mass function and bias (this choice will be discussed in Sec. 2.2.2). The virial parameter \( \Delta_0 \), takes the following form (Bryan & Norman 1998):

\[ \Delta_0 = 18 \pi^2 + 82d - 39d^2, \] (13)

\[ d := \frac{\Omega_m (1+z)^3}{E(z)^2} - 1, \] (14)

where \( E(z) \) is given by \( H(z) = H_0 E(z) \). The HI-halo mass relation is shown in Fig. 2, where the normalization \( \alpha \) was fixed to match the neutral hydrogen density for a cosmology with \( n_0 = 10^{-24} \text{eV} \) and other cosmological parameters as given in Table 1.

Furthermore, to make fully use of the halo model an exponential profile for the HI is assumed, which is well motivated and commonly used (e.g. Binney & Tremaine 1987; Cormier et al. 2016):

\[ \rho_{\text{HI}}(r,M) = \rho_{\text{HI},0} e^{-r/r_s} \] (15)

where \( r_s = R_{\text{vir}}(M,z)/c_{\text{HI}}(M,z) \), \( \rho_{\text{HI},0} \) is fixed such that integrating the radial profile over a sphere of radius \( R_{\text{vir}} \) matches \( M_{\text{HI}}(M,z) \). \( R_{\text{vir}} \) is the virial radius given in Eq. (12) and \( c_{\text{HI}}(M,z) \) is the HI concentration parameter given by (Padmanabhan et al. 2017)

\[ c_{\text{HI}}(M,z) = c_{\text{HI},0} \left( \frac{M}{10^{11} M_\odot} \right)^{-0.109} \frac{4}{(1+z)^2}. \] (16)

The radial profile, therefore, has two free parameters \( c_{\text{HI},0} \) and \( \gamma \). HI IM constrains these parameters only poorly, since the specific profile is only relevant at very small scales (for a halo of mass \( M = 10^{13} M_\odot/h \) at redshift \( z = 2 \) one finds that \( 1/r_s \approx 25 h/\text{Mpc} \) and 1M is mostly sensitive to larger scales (Padmanabhan et al. 2019). Thus, these parameters are assumed to be fixed throughout the study. The form of the concentration parameter assumes the same halo mass dependence for CDM as for neutral hydrogen. This choice is discussed in more detail in Sec. 2.2.2.

Ultimately, the Fourier transform of the radial density profile will be of importance and is given by

\[ u_{\text{HI}}(k|M) = \frac{4 \pi}{M_{\text{HI}}(M)} \int_0^{R_{\text{vir}}} \rho_{\text{HI}}(r,M) \sin kr kr^2 dr. \] (17)

For large \( c_{\text{HI}} \), it is well approximated by

\[ u_{\text{HI}}(k|M) \simeq \frac{1}{(1 + (kr_s)^2)^2}. \] (18)

All quantities in the present model are given in comoving coordinates (with exception of the virial velocity in Eq. 10). So, \( r_s \) must be in comoving coordinates too and the physical virial radius in Eq. (12) has to be converted, which in effect cancels the redshift dependence in the virial radius.

With this at hand, the Cartesian power spectrum is expressed by a one-halo term and two-halo term:

\[ P_{\text{HI}}(k,z) = P_{\text{1h,HI}} + P_{\text{2h,HI}} \] (19)

with

\[ P_{\text{1h,HI}}(k,z) = \frac{1}{P_{\text{HI}}} \int dM n(M,z) M_{\text{HI}}(M,z) u_{\text{HI}}(k|M)^2 \] (20)

and

\[ P_{\text{2h,HI}}(k,z) = b_{\text{HI}}^2(k,z) P_{\text{HI}}(k). \] (21)

The HI bias relates to the halo bias \( b(M,z) \) and the halo mass function (HMF) \( n(M,z) \) via

\[ b_{\text{HI}}(k,z) = \frac{1}{P_{\text{HI}}} \int dM M_{\text{HI}}(M,z) n(M,z) b(M,z) |u_{\text{HI}}(k|M)| \] (22)

and

\[ \overline{P}_{\text{HI}} = \int dM M_{\text{HI}}(M,z) n(M,z). \] (23)

Assuming \( \overline{P}_{\text{HI}} = \Omega_0 \rho_{\text{crit}} \) can be accomplished by fixing the normalization \( \alpha \). Note however that this normalization cancels for the above quantities and fixing \( \overline{P}_{\text{HI}} = \Omega_0 \rho_{\text{crit}} \) is computationally unnecessary. For the calculation of the above quantities, we used the Sheth-Tormen HMF and bias (Cooray & Sheth 2002). Both (HMF, \( n \), and halo bias, \( b \)) are computed from the variance of matter fluctuations:

\[ \sigma^2(R,z) = \frac{1}{2\pi^2} \int_0^\infty P_{\text{HI}}(k,z) W(kR)^2 k^2 dk. \] (24)

For a spherical top-hat window function in real space, mass and radius are related by \( M = 4\pi R_b R^3 / 3 \) and the Fourier transform of that top-hat function is given by

\[ W(kR) = \frac{1}{(kR)^3} (\sin(kR) - kR \cos(kR)). \] (25)

The linear (matter) power spectra explicitly appearing in Eq. (24) & (21) were obtained from axionCAMB.

2.2.2 Axions and HI

In the previous sections the modeling of the 21 cm signal was discussed in rather general terms. In this section we describe the way ULAs are accommodated in this formalism. In principle to model their impact we have to consider their influence on the large scale structure (i.e. specifically the HMF and halo bias), on small scales where they could influence the HI density profile as well as onto the \( M_{\text{HI}}(M) \) relation.

(i) For the LSS the main idea we employ is to treat ULAs below a certain mass similarly to massive neutrinos. For massive neutrinos considerable effort has been put into investigating their influence on the HMF and bias: Large N-body simulations were employed and analyzed in a series of papers (e.g. Villaescusa-Navarro et al. 2014; Castorina et al. 2014; Costanzi et al. 2013) and the spherical, top-hat collapse model was revisited for massive neutrinos in Ichiki & Takada (2012). Both studies found the halo mass function and halo bias are better fit if one considers only the baryon and CDM field, instead of the total matter field including neutrinos for their computation. Consequently, to model the HI signal one should take the HI as a tracer of the CDM and baryon field, but not the total matter field including massive neutrinos. Note that neutrinos and axions are included in the dynamics of the perturbations and the background, and so affect the CDM + baryon fluid indirectly.

Villaescusa-Navarro et al. (2015) included massive neutrinos in modeling the HI signal for low redshifts (\( z < 3 \)) in such a way and could show that constraints on the sum of the neutrino masses from late-time 21 cm observations are possible. To be precise, \( \rho_0 = \rho_{\text{crit}}(\Omega_{\text{CDM}} + \Omega_\nu) \) was set throughout the calculation of the HMF and halo bias and for the computation of the variance (Eq. 24) and the 2-halo term (Eq. 21) the power spectrum of the CDM and baryon component, \( P_{\text{CDM-baryon}}(k) \), was used. Furthermore, in the present case \( \rho_0 \) was considered for the calculation of the virial radius (Eq. 12), too. The choice on \( \rho_0 \) affects the HI density profile (which is not significant as we will argue below) and the cut-off in the \( M_{\text{HI}}(M) \) relation.

To relate axions of mass \( m_a \) to massive neutrinos, we choose to compare the axion field variance to the neutrino field variance (see Fig. 3): For massive neutrinos this is much lower than unity.
and, therefore, does not contribute significantly to the collapse of a halo. This justifies the linear approximation for neutrinos as not bound within halos, as discussed. Based on this observation, we choose to approximate ULAs in the same way as massive neutrinos whenever the variance is less than unity. In Fig. 3, we observe that this is true for $m_a < 10^{-27} \text{eV}$ at $z = 0$. For axions heavier than this boundary mass, $m_a \geq 10^{-27} \text{eV}$, we choose to treat them as usual CDM (i.e. collapsed into halos following the Sheth-Tormen model for the mass function), setting $\rho_0 = \rho_{\text{crit}}(\Omega_{\text{CDM}} + \Omega_\gamma + \Omega_a)$ and $P(k, z) = B_{\text{CDM + a}}(k, z)$.

(ii) The HI halo mass relation is subject to several (astrophysical) effects and absorbs those with a deterministic function of a few parameters. It can be constrained empirically (fairly immune to the HMF and cosmology) and - although not specifically including the possible impact of axions to it - allowing it to vary with respect to its parameters. A subtle issue is the choice of $\rho_0$ mentioned above which affects the cut-off of HI in Eq. (10). To make the background density (and redshift) dependence on that cut-off clear, the exponential in Eq. (10) can be rewritten in terms of the halo mass $M$ as $\exp[-M_{\text{min}}(\rho_0, z)/M]$ with

$$M_{\text{min}} = 2.45 \times 10^{11} M_\odot / h \left( \frac{V_z}{363 \text{km/s}} \right)^3 \left( \frac{0.3}{\Omega_0} \right)^{1/2} (1 + z)^{-3/2}. \quad (26)$$

In effect, taking only the CDM+baryon component increases the cut-off mass when translating the exponential in the $M_{\text{HI}}(M)$ relation (Eq. 10) to halo masses compared to the case where $\rho_0$ includes the axions in addition. This is so, because of the fixed lower virial velocity $V_z$. Note however that this shift in the cut-off mass is small (for small axion and neutrino contributions) and a minor effect compared to the general impact of axions onto the HI halo model and the overall uncertainty of $V_z$. The fact that we marginalize over it in the Fisher forecast analysis should account for potential modeling uncertainties this choice introduces.

(iii) The HI profile is affected by axions in its specific shape (for example, axions will alter the HI profile on scales of order the de Broglie wavelength, where the ULA condenses into a soliton Veltmaat et al. 2019). Furthermore, the concentration parameter of HI is assumed to be identical to that for cold dark matter and, thus, the specific value of it might depend weakly on axions, too. We note, however, that this form has a universal applicability in the description of low-$z$ surface density profiles and high-$z$ DLA observations (Padmanabhan et al. 2019). Generally, the HI profile becomes relevant only at small scales (cf. Sec. 2.2.1). Since typically HI IM surveys are concerned about larger scales, they are mostly insensitive to the specific HI density profile and the instrumental noise is typically larger than the signal on those scales. Therefore, we neglect the specific impact of axions onto the HI profile.

Generally, we expect that the HI halo model becomes less accurate on smaller, non-linear scales. To account for this potential shortcoming, it is expedient to compare the results with a computation where the wavenumbers above a cut-off scale, $k_{\text{cut}}$, are not considered. We adopt the redshift scaling as in Bull et al. (2015):

$$k_{\text{cut}} = 0.14 \text{Mpc}^{-1}(1 + z)^{2/3 + n_s}, \quad (27)$$

where $n_s$ is the scalar spectral index.

2.3 Results on the HI Power Spectrum for ULAs

The HMF, the HI bias and the HI power spectrum calculated with the current model are shown in Figures 4 to 6, respectively. We restrict ourselves to two exemplary axion masses each exhibiting the
In contrast, heavier ULAs $m_a \sim 10^{-24} \, eV$ suppress the formation of halos below their Jeans mass as shown in Fig. 4. Due to the reduced number of low mass halos, assuming a fixed HI density parameter, the HI has to reside in more massive halos which are more strongly biased. Thus, the suppression of low mass halos effectively leads to an enhancement in the HI bias, which increases at higher redshift (cf. Fig. 5). This explains the main impact of axions in that mass range onto the HI power spectrum (Fig. 6): The suppression of the matter power spectrum is present only at small, non-linear scales for ULAs in that mass range. Hence, when considering the 2-halo term (Eq. 21) dominant on large scales, the main impact is the boost in the HI bias. This makes their imprint possibly degenerate with the astrophysical parameters controlling the HI bias and $\Omega_{HI}$ in general, which we discuss in more detail in Sections 3.2 and 4.

In contrast, when $m_a = 10^{-28} \, eV$ an enhancement of power is seen on large scales but a suppression on small scales is present compared to the ΛCDM case (cf. Fig. 6). These low mass ULAs act Dark Energy (DE)-like, in the sense that they suppress matter fluctuations on almost all relevant scales. This shifts the HMF towards lighter halo masses, which leads to a significant decrease of intermediate halo masses ($\gtrsim 10^{11} \, M_{\odot}/h$) at redshift $z = 0.5$. Thus, more HI needs to reside in each halo (similar to the effect of heavier ULAs) and the halo bias is increased too, leading to an enhancement in the HI bias. This increase competes with the overall suppression of the linear power spectrum, which appears on already linear scales and reaches its saturation at the Jeans scale $k_J$ (cf. Sec. 2.1). This gives rise to the scale-dependent imprint of lighter ULAs.

To summarize, we find that axions of all masses increase the HI bias, albeit lighter ones out of a qualitatively different reason than heavier, fuzzy DM benchmark axions. For lighter axions ($m_a \lesssim 10^{-25} \, eV$) the suppression of the matter power spectrum competes with the boost in the HI bias on already linear scales for the considered redshift range (i.e. 0 to 3 for realistic surveys), while for heavier ones the suppression is “hidden” by the dominant 1-halo term. Physically this distinct behavior can be qualitatively understood by their different de Broglie wavelength. Lighter axions have a larger de Broglie wavelength and, therefore, smooth out matter power fluctuations on larger scales, ultimately given by $k_m$ and $k_J$ (cf. Fig. 1).

Lastly, we compare our present findings with Carucci et al. (2015), who investigated the impact of warm dark matter (WDM) on the 21 cm power spectrum with the help of N-body simulations (similar to Seehars et al. 2016). A subsequent paper with similar methodology from Carucci et al. (2017) considers the impact of

**Figure 6.** The 21 cm power spectrum (Eq. 19) for $m_a = 10^{-28} \, eV$ and $10^{-24} \, eV$ and different axion fractions at redshift zero. For higher axion masses an overall enhancement is seen, whereas for lower axion masses power is suppressed at small scales due to the larger de Broglie wavelength (cf. 2.1). The enhancement can be understood from the HI bias in Fig. 5. The dotted lines show the non-linear scale ($k_{nl} = 0.14 \, hMpcc^{-1}$) as defined in Eq. 27. The relative difference between the ΛCDM scenario and axion fractions at the 2σ exclusion limits obtained by the SKA1MID+CMB-SO surveys is on the percent-level.

https://mnras.org
ULAs, when they are the only component in the DM sector. Although we consider a mixed DM sector in this work, their findings are useful as they give guidance to what one should expect for high ULA density parameters for the fuzzy DM benchmark. Reassuringly similarly an overall increase of the 21 cm power spectrum for these heavier ULAs is found by Carucci et al. (2017) due to the fact that the formation of low mass halos is suppressed and the HI has to reside in the more massive halos which are more strongly biased.

Fig. 7 shows the relative difference between ΛCDM and ULA only dark matter models at redshifts $z = 1$ and compares to Carucci et al. (2017, Fig. 7). While we observe a similar trend along $k$ and axion mass (not shown), our model underestimates the relative difference by a factor of a few relative to Carucci et al. (2017). Note that the $k$ range is already in the non-linear regime as given by Eq. (27). The increase of the relative difference for larger $k$ in our model comes about, because of the 1-halo term which is dominant at those scales. Since for this term $M_{\Pi}(M)$ goes in squared, the effect of ULAs is more pronounced. It is reassuring that the 1-halo and 2-halo term seem to capture the trend observed by the simulation even in the mild non-linear regime. On the other hand we underestimate the relative difference between ΛCDM and a factor of a few. Differences from our model are a different HI halo mass relation $M_{\Pi}(M)$ (which is redshift independent in the case of Carucci et al. 2017), slightly different cosmological parameters and the inclusion of RSDs. Fig. 7 shows how the relative difference varies in our model for different $v_{c,0}$. It increases upon decreasing $v_{c,0}$ such that the suppression of low mass halos becomes more important and might well explain some discrepancies between Carucci et al. (2017) and our model. In short, the results are broadly consistent, given the overall methodological differences and that the simulations may have ingredients that are not necessarily tuned to match all the relevant data. Since our model underestimates the impact of ULAs compared to that of Carucci et al. (2017), our forecasted constraints on the parameters are conservative.

2.4 IM Observations

In recent years several radio telescopes have been designed, planned (and constructed) to conduct 21 cm IM surveys. In this work we specifically consider the future Square Kilometer Array (SKA; Dewdney 2013) telescope, its already built precursor, the MeerKAT telescope (Jonas 2009), as well as the HIRAX (Newburgh et al. 2016) and BINGO (Battye et al. 2013) telescopes. The important survey specifications for this work are listed in Table 2.

A significant advantage of radio antennas compared to receivers in the optical range is that they can measure the phase of the incoming electromagnetic (EM) wave. Modern radio telescopes make heavily use of that and, therefore, typically consist of multiple dishes. Broadly, these can be run in two different modes:

- **Single dish**: One can auto-correlate the signal for each individual dish. This, effectively, increases the observation time for each pixel by the number of dishes (and number of beams).
- **Interferometer**: The signal for each antenna is cross-correlated with another antenna, separated by a given baseline $d$. This, in result, increases the effective dish size for the antennas run in interferometric mode by the baseline, such that a much higher angular resolution is obtained.

The radio telescopes are subject to thermal noise depending on the mode in which they are run. The equations used for the noise power spectrum of both modes are summarized shortly hereafter. Important specifications to calculate the noise of a radio telescope are the dish diameter of a single dish $D_0$, the number of dishes $N_d$, the number of beams $N_b$ (which includes the number of different polarization channels $n_{pol}$ which generally equals two), the frequency channel width (here corresponding to the redshift bin width) $\Delta v$, the solid angle sky coverage of the survey $\Omega_{\text{surv}}$ and the observed wavelength of the incoming EM wave $\lambda$. Furthermore, the total system temperature is taken $T_{\text{sys}} = T_{\text{sky}} + T_{\text{inst}}$, with $T_{\text{sky}} = 60 \text{K (350MHz/}$ν$)^{2.5}$ (Padmanabhan et al. 2019).

2.4.1 Single dish mode

For the single dish mode we used the noise expression given in Knox (1995). Together with the beam smearing term, the dimensionless noise power spectrum is given by

$$N_l = \frac{1}{\bar{T}_b^2} \Omega_{\text{pix}} \sigma_{\text{pix}}^2 \exp \left( \frac{\ell^2 \theta_b^2}{8 \ln 2} \right).$$

where $\theta_b$ is the beam full-width at half maximum $\theta_b \approx \frac{\lambda}{2D_0}, \Omega_{\text{pix}}$ is the solid angle beam area and $\bar{T}_b$ is the mean brightness temperature of expected HI signal given by (Villaescusa-Navarro et al. 2016)

$$\bar{T}_b \approx 190 \Omega_{\text{HI}} B \left( 1 + \frac{z^2}{E(z)} \right) \text{mK .}$$

The thermal noise per pixel is given by the radiometer equation. For a perfect receiver, one has (Battye et al. 2013):

$$\sigma_{\text{pix}} = \frac{T_{\text{sys}}}{\sqrt{t_{\text{pix}} \Delta v}}.$$  

The time of observation for each pixel depends on the total observation time divided by the number of pixels in the map and multiplied by the number of dishes and beams (if the radio telescope is run in an autocorrelation mode). Therefore, it is

$$t_{\text{pix}} = t_{\text{obs}} N_d N_b \frac{\Omega_{\text{pix}}}{\Omega_{\text{surv}}}.$$  

Note that this noise expression equals the noise expression given in Bull et al. (2015) upon converting to angular scales except of a different prefactor $\sim 3$ due to the inclusion of the effective dish.
Table 2. Instrumental parameters for different surveys from Bull et al. (2015) and for HIRAX from Alonso et al. (2017). The numbers below in parenthesis in column 6 & 7 (for \(v_{\text{max}}\) and \(v_{\text{min}}\)) are the corresponding redshifts. We have combined band 1 and 2 of SKA1-MID as in Padmanabhan et al. (2019). The total observation time is set to \(t_{\text{tot}} = 10000\) h for all surveys.

| Experiment | \(T_{\text{int}} \) [K] | \(D_d\) [m] | \(D_{\text{min}}\) [m] | \(D_{\text{max}}\) [m] | \(N_f\) (dual pol.) | \(N_d\) | \(v_{\text{max}}\) [MHz] | \(v_{\text{min}}\) [MHz] | \(\Omega_{\text{surv}}\) [sq. deg.] |
|------------|----------------|---------|----------------|----------------|----------------|-------|----------------|----------------|----------------|
| BINGO      | 50             | 25      | -              | -              | 50 × 2          | 1     | 1260           | (0.13)         | 2000           |
| MeerKAT (B1) | 29            | 13.5    | -              | -              | 1 × 2           | 64    | 1015           | (0.4)          | 25000          |
| SKA1-MID (B1+B2) | 28          | 15      | -              | -              | 1 × 2           | 190   | 1420           | (0)            | 20000          |
| HIRAX      | 50             | 6       | 6              | 300            | 1 × 2           | 1024  | (0.78)         | (2.55)         | 15000          |

To assess the viability of future 21 cm IM surveys to constrain the cosmological parameters in general and the fractional axion density parameter \(\Omega_a/\Omega_d\) in particular we develop Fisher matrix forecasts. The inverse Fisher matrix \(F^{-1}\) is the covariance matrix of the probability distribution of the parameter and gives an estimate on the best possible, minimal error on the parameters (Tegmark 1997).

Assuming a Gaussian likelihood with covariance matrix \(C\), one can show that the Fisher information matrix takes the following form (Tegmark 1997)

\[
F_{ij} = \frac{1}{2} \text{tr} \left[ C^{-1} \frac{\partial C}{\partial p_i} C^{-1} \frac{\partial C}{\partial p_j} + \frac{\partial \mu}{\partial p_i} C^{-1} \frac{\partial \mu}{\partial p_j} \right],
\]

where \(\mu\) denote the model parameters, and \(\mu = \langle x \rangle\). In the present case \(x\) are the temperature fluctuations expanded in spherical harmonics and the forecasted model parameters are \(p = \{\ln A_1, n_s, \Omega_b, \Omega_c, \Omega_m/\Omega_d, h, \tau, n_s, \beta\}\). We assume isotropy such that

\[
\mu = 0
\]

\[
C_{ij} = \delta_{ij} \left[ C_{\ell} + N_{\ell} \right].
\]

With that at hand, the Fisher matrix is given by

\[
F_{ij} = \sum_\ell \frac{1}{(2\ell+1)f_{\text{sky}}(C_{\ell} + N_{\ell})^2},
\]

where the factor \(f_{\text{sky}}\) is included to account for the fact the survey is scanning only a fraction of the sky \(f_{\text{sky}} = \Omega_{\text{surv}}/(4\pi)\). In the above expression, it was assumed that the noise \(N_{\ell}\) does not depend on any parameters which shall be constrained/forecasted. This is not strictly true in the dimensionless framework as the mean brightness temperature depends weakly on cosmological parameters (cf. Eq. 29). We will, however, assume that the mean brightness temperature is fixed, e.g. determined from other surveys/probes and neglect its information to the Fisher matrix in the present analysis (cf. Padmanabhan et al. 2019; Chen et al. 2019).

As mentioned above the redshift range will be divided into several bins. For each bin \(i\), a Fisher matrix \(F^{(i)}\) is obtained. In the Limber approximation cross-correlations between different redshift bins have been effectively neglected. In this picture, one can then simply add the Fisher matrices from each redshift bin to obtain the cumulative Fisher matrix, containing all the information which can be gained from the survey in the current formalism:

\[
F_{\text{cumul}} = \sum_{i=1}^{N_{\text{bin}}} F^{(i)}.
\]

We take equal-sized redshift bin width of \(\Delta z = 0.05\) and calculate...
the $C_l$'s in Eq. (9) at the midpoints of each. Furthermore, we restrict ourselves to $\ell \leq 1000$ when forecasting.

The partial derivatives in Eq. (38) have been calculated numerically by using the central finite difference

$$\frac{\partial f(x)}{\partial x} = \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x}. \quad (40)$$

The step sizes for calculating the derivatives have been carefully chosen such that a sufficient level of convergence was reached. The fiducial values for the forecasted parameters, as well as the step size for calculating the derivatives are listed in Table 1. The general outline of the code written for this analysis is based on the code presented in Bull et al. (2015).\footnote{The adapted version for this analysis is publicly available at \url{https://github.com/JurekBauer/axion21cmIM.git}.} We conclude by listing the main assumptions taken into account for the model presented in this section:

(i) All angular power spectra are calculated using the Limber approximation.

(ii) No peculiar velocities, i.e. RSDs, are included in the calculation for the HI angular power spectrum.

(iii) $\Omega_{HI}$ is supposed to be fixed and redshift-independent.

(iv) Foregrounds can be removed efficiently and do not intercept with the HI signal.

(v) Axions can be modeled like massive neutrinos for $m_a < 10^{-27}$ eV and as a CDM-like component for $m_a \geq 10^{-27}$ eV.

(vi) No cosmological information gained from $\tilde{T}_b$.

(vii) Cosmological information can be obtained from $h_{100}$ (the HI bias is not treated as model-independent or a “nuisance” parameter).

(viii) Axions do not change the neutral hydrogen profile.

(ix) No further inclusion of nuisance parameters, which are marginalized over, than those given in above and listed in Table 1.

3 RESULTS

3.1 Survey Comparison

To compare different surveys, we look at how they constrain $m_a = 10^{-26}$ eV and $m_a = 10^{-24}$ eV on the axion fraction. We choose the first mass since it is the most constrained bin in the CMB analyses (Hložek et al. 2018) and the second to highlight the mass-dependent impact discussed in Sec. 2.3. The estimated marginal error on $\Omega_a/\Omega_d$ is shown for different configurations in Fig. 9. CV1 and CV2 denote cosmic variance limited surveys (the noise is set to zero) for redshift ranges $[0, 3]$ and $[0, 5]$, respectively, and $f_{sky} = 1$. Depending on the instrumental noise properties CV2 might be comparable to PUMA (Bandura et al. 2019). We also consider the case of making each survey in turn “cosmic variance limited” by switching off instrumental noise. The different surveys are then only distinguished by the redshift range and sky coverage.

The filled markers in Fig. 9 indicate all scales accessible by the survey, while the empty ones use the non-linear cut-off scale in Eq. (27): The instrumental noise is set to infinity for $\ell > k_{\text{max}}(z)$. In this way the non-linear scales are excluded and do not pass any information to the Fisher matrix. Commonly, radio telescopes run in the interferometric mode are more constraining on the axion fraction and come close to cosmic variance limited surveys when non-linear scales are included (this, however, might change when allowing for $\ell > 1000$). Specifically, SKA1MID is much more effective if run in the interferometric mode (cosmic variance limited) than in the single-dish mode. This is even more marked for MeerKAT. Generally and in realistic instrumental noise scenarios, HIRAX and SKA1MID run in the interferometric mode are the most constraining surveys.

As shown in Fig. 8 radio telescopes, run in an interferometric mode, typically scan smaller and, thus, potentially non-linear scales. Evidently, excluding non-linear scales leads to an increase for the error on the axion fraction for all experiments and both axion masses. However, the magnitude strongly depends on the axion mass: While constraints for $m_a = 10^{-26}$ eV are barely affected, constraints for $m_a = 10^{-24}$ eV increase significantly upon excluding non-linear scales. This can be understood by the fact that degeneracies with other parameters are already broken on linear scales for the lighter ULAs. Interestingly, the HIRAX survey is an exception to that observation, such that we conclude that the HIRAX survey...
is only mildly sensitive to non-linear scales. To shed more light onto the degeneracy structure, it is discussed in the next section for heavy mass ULAs in more detail.

### 3.2 Degeneracy Structure

To study the degeneracy structure of the HI IM survey with ULAs, the derivatives \( \partial_a C_\ell \) in the definition of the Fisher matrix (Eq. 38) are of importance. Fig. 10 shows the normed derivatives defined by

\[
(C'_\ell)_{\text{norm}} = \frac{(C'_\ell)}{\left( \max_i (C'_\ell) \right)^{-1}} \text{ if } \max_i (C'_\ell) > \left| \min_i (C'_\ell) \right|
\]

\[
(C'_\ell)_{\text{norm}} = \left( \min_i (C'_\ell) \right)^{-1} \text{ else},
\]

for \( m_a = 10^{-24} \text{ eV} \) and \( \Omega_b/\Omega_d = 0.02 \) at \( z = 0.5 \). While the derivative with respect to parameters \( \Omega_d, \Omega_b, h, n_s \) have distinct shapes, the four parameters \( A_s, \Omega_a/\Omega_d, \beta \) and \( v_{c,0}, \beta \) resemble each other closely. These degeneracies are expected: The astrophysical parameters \( \beta \) and \( v_{c,0} \) dictate the specific value of \( b_{HI} \) (and for large scales less importantly the 1-halo term). Similarly for \( m_a \gtrsim 10^{-24} \text{ eV} \) and on the considered scales, \( \Omega_a/\Omega_d \) engenders a scale-independent enhancement in the 21 cm signal, because of the boost in \( b_{HI} \) (cf. Fig. 5 and the discussion in Sec. 2.3). \( A_s \) does alter \( b_{HI} \) and the matter power spectrum linearly, and thus an increase in \( A_s \) results in a scale-independent increase in the 21 cm signal, too. Subsequently, all these parameters induce – if one is only concerned about large, linear scales – a scale-independent imprint and the normed derivatives are similar.

There are several ways to break these degeneracies: (i) Providing a prior from other observations for the cosmological parameters (e.g. the CMB fluctuations for \( A_s \) and \( \Omega_d/\Omega_b \)), (ii) including RSDs (isolating \( A_s \); Chen et al. 2019, and possibly \( \Omega_a/\Omega_d \), depending on the influence of axions onto RSDs), (iii) probing non-linear scales (i.e. the 1-halo term), (iv) the different redshift scaling of the impact of each parameter or (v) independent observations to constrain \( v_{c,0} \) and \( \beta \) tightly. Probing non-linear scales breaks the degeneracy, because on smaller scales the 1-halo term becomes relevant which is affected differently for the parameters (e.g. the magnitude in increase is larger for \( A_s \) and \( \Omega_a/\Omega_d \) than for the astrophysical parameters) and depending on the axion mass and abundance the suppression at the Jeans scale in the 2-halo term is visible. Since RSDs are not included at present, degeneracies between those four parameters are (partly) broken by (i), (iii), (iv) and (v) in what follows.

Note that Fig. 10 also shows that degeneracies with other parameters are likely, although less marked: for example a degeneracy with \( \Omega_b \) is present if one is only concerned about smaller scales and the BAOs are washed out.

The degeneracy of axions with CDM is relevant for other cosmological probes (e.g. the CMB Amendola & Barbieri 2006; Hložek et al. 2015) and has an interesting structure for HI IM, too. For axions of high mass \( m_a \gtrsim 10^{-24} \text{ eV} \) a degeneracy with \( \Omega_{CDM} \) (if varied along with \( \Omega_A \)) is expected on large, linear scales. Fig. 11 shows the error ellipses for two different masses and surveys: In the left panel for a mass of \( m_a = 10^{-28} \text{ eV} \) and in the right panel a high mass of \( m_a = 10^{-24} \text{ eV} \) for SKA1MID run in the interferometric and in the single-dish mode. SKA1MID in the single-dish mode probes smaller \( \ell \) and, thus, larger scales. SKA1MID in the interferometric mode, on the other hand, probes smaller, mostly non-linear scales. The error ellipses for a mass \( m_a = 10^{-28} \text{ eV} \) show that, indeed, a degeneracy if run in the single-dish mode and in the interferometric mode when only linear scales are included. For the full interferometric SKA1MID survey the overall error on the axion fraction reduces significantly, because degeneracies are broken with other parameters as mentioned above and additional scales are included to the Fisher analysis.

For an axion of mass \( m_a = 10^{-28} \text{ eV} \) a degeneracy with CDM is observed too, but in the opposite direction. This can be understood from the HI power spectra in Fig. 6. For \( m_a = 10^{-28} \text{ eV} \) on the range of scales where there is good signal-to-noise ratio, the axion just suppresses power, so it has the opposite degeneracy with CDM as for the heavier case. This suppression is due to the suppression of matter fluctuations on already linear scales and relates to the lower Jeans wavenumber compared to higher axion masses. The degeneracy is less prominent for the full interferometric SKA1MID survey.

### 3.3 Constraints on the Axion Fraction

This section presents exclusion limits of the axion fraction by the IM surveys. They are estimated by the minimum axion fraction which one is able to resolve with its \( \sigma \) marginal error. Explic-
Figure 12. Error ellipses of the cosmological parameters from the HIRAX, CMB-SO surveys and when combined. Fiducial parameters were used as in Table 1 with $m_a = 10^{-23} \text{eV}$ and fiducial axion fraction $\Omega_a/\Omega_d = 0.9$.

Figure 13. Expected axion fraction for different initial field values $\phi_i$ and axion masses $m_a$. The CMB-SO survey has $\Omega_b/\Omega_d = 0.9$.

---

In this paper, we have investigated the imprint of ULAs on late-time 21 cm IM surveys in a mixed DM scenario. To do so we exploited the accurate, data-driven halo model introduced by Padmanabhan et al. (2015). Axions were accommodated to this model by reference to massive neutrinos. Both were compared by looking at their variances. Thereby, a critical axion mass of $10^{-27} \text{eV}$ was identified below which ULAs are treated similar to neutrinos (HI as a tracer of the CDM and baryon field, but not the total matter field) and above which they are incorporated as a CDM-like component. The halo model was roughly checked against numerical results for heavier ULAs from Carucci et al. (2017). The present model adequately captures their main findings, providing further confidence for the proposed framework. In contrast to the earlier work by Carucci et al. (2017) the present study allows the axion fraction to be a free parameter (which is to our knowledge not studied in the literature at present). This is important because it is predicted that these ULAs depending on their mass occur in sub-dominant abundances related to the GUT scale.

Heavier ULAs, $m_a \gtrsim 10^{-24} \text{eV}$, suppress the formation of small mass halos below the Jeans mass. Assuming a fixed amount of neutral hydrogen, more HI needs to reside in heavier halos which
are more strongly biased than their low mass counterparts. This leads to an increase in the HI bias compared to ΛCDM. Note that Fig. 7 shows that at least some knowledge on the cut-off mass in the HI halo mass relation is necessary to constrain those ULAs: If the cut-off mass is larger than the Jeans mass of the ULA, the suppression of halos below the Jeans mass does not leave a “fingerprint”.

For lighter ULAs, $m_a \sim 10^{-28} \text{eV}$ an increase in the HI bias is observed, too, albeit out of different reasons: These ULAs suppress structure on almost all relevant scales (when compared to ΛCDM), which makes their influence DE-like. The halo bias is increased for most halo masses and intermediate sized halos are suppressed.

For ULAs with $m_a \lesssim 10^{-25} \text{eV}$ the suppression in the power spectrum starting at scale $k_m$ and saturating at the Jeans scale $k_J$ becomes relevant even at large, linear scales in the HI power spectrum (cf. Eq. 21), leading to a salient scale-dependent imprint of those ULAs. For heavier ULAs the increase in the HI bias is the main imprint on large, linear scales. The scale-dependence is partly “hidden”: The suppression in the matter power spectrum happens at non-linear scales and for $m_a \gtrsim 10^{-23} \text{eV}$ at scales where the 1-halo term is dominant.

The scale-independent impact for ULAs of $m_a \gtrsim 10^{-24} \text{eV}$ is degenerate with the astrophysical parameters, $v_{c,0}$ and $H_0$, which effectively control the HI bias, and $A_s$. When only probing large, linear scales at a single redshift, this degeneracy cannot be broken.

Disentangling these parameters should be possible by (i) providing priors from other observations, (ii) including RSDs, (iii) probing non-linear scales or (iv) including the different redshift scalings of the impact of each parameter (cf. Sec. 3.2).

Forecasts for different surveys were run with help of a Fisher matrix analysis. Degeneracies were partly broken by priors (CMB or fixing astrophysics), the different redshift scaling of the impact of different parameters and for HIRAX only mildly probing non-linear scales (cf. Fig. 9). The results give an exciting and promising impression: within this framework combined future CMB and IM observations should be able to test axion fractions to the percent level or even below. It is possible to probe the interesting region near the GUT scale in the mass region where the ULA imprint the scale-dependent feature onto the 21 cm signal. If astrophysics are known precisely, current and future constraints could be greatly improved for ULAs with $m_a \sim 10^{-22} \text{eV}$.

These exciting results call for other studies to check the robustness of the present results, which includes calibrating the halo model in the mixed ULA-CDM scenario. The following list includes the most important assumptions with respect to HI IM which could be checked in more detail and give guidance to future studies.

(i) Fixed and redshift-independent $\Omega_{\text{HI}}$.
(ii) No foreground contamination.
(iii) Cosmological information from $b_{\text{HI}}$. 

Figure 14. 2σ exclusion limits for the HIRAX and SKA1MID survey run in the interferometric mode. Dotted lines indicate the IM survey alone, while solid ones are for those combined with the forecasted CMB-SO constraints. Depending on the precision of the astrophysical parameters SKA1MID and HIRAX probe the interesting region near the GUT scale for ULAs with $m_a \gtrsim 10^{-24} \text{eV}$. While axions with $m_a \lesssim 10^{-25} \text{eV}$ leave a salient, scale-dependent imprint on linear scales due to their large de Broglie wavelength, heavier ones are degenerate with the astrophysical parameters on those scales. Breaking this degeneracy is possible by including smaller non-linear scales or by a more precise knowledge of the astrophysical parameters. With SKA1MID being more sensitive to smaller, non-linear scales than HIRAX, it is able to constrain the heavy fuzzy DM axions more strongly.
(iv) No inclusion of RSDs.
(v) No specific impact of axions onto the neutral hydrogen profile.
(vi) Use of Limber approximation.

Firstly, given the current poor constraints on $\Omega_{\text{HI}}$, this is an optimistic assumption at the present day. Also, HI IM surveys alone measuring the large, linear scales can only estimate the quantity $\Omega_{\text{HI}}$. Because the main effect of ULAs with $m_a \gtrsim 10^{-24} \text{eV}$ is the increase in HI bias, it is necessary to know the precise value of $\Omega_{\text{HI}}$. A possible resolution is to determine $\Omega_{\text{HI}}$ with other observations or to break this degeneracy by the inclusion of RSDs into our analysis. The latter requires a model of RSDs which includes axions and might provide by itself additional information on the axion fraction, too. Hence, the inclusion of RSDs to the analysis is an important extension.

Secondly, foregrounds have been neglected and a perfect foreground removal has been assumed. It is expected that foregrounds are the leading systematic factor for HI IM surveys. Thus, relaxing this assumption is a relevant next step.

Thirdly, the information from modeling the bias parameter have been included in the analysis. This touches a central point for any large-scale structure survey: it is necessary to have a sufficient knowledge on the exact behavior and modeling of the bias parameter to rely on the results. If not, deviations from an expected (LCDM) signal of LSS survey could be alleviated by an inaccurate modeling of the bias. This works also the other way round: A deviation from LCDM could be hidden by an inaccurate model of the bias, giving rise to a potential psychological confirmation bias. Two of such potential pitfalls when modeling the bias shall be stated. Firstly, it could be expected that the neutrino-like ULAs introduce a slight scale-dependence to the bias as simulations show for massive neutrinos (Villaescusa-Navarro et al. 2014), which the present model does not capture. Note however that this effect is reduced by our choice to only consider the CDM and baryon field. Secondly, to what extent it can be expected that the bias parameter is scale-independent on large scales even for the LCDM case, is an ongoing question and large simulations are employed to investigate the HI bias relation further (Villaescusa-Navarro et al. 2018). To give an idea on the needed accuracy in the modeling of the HI bias in the present case for the heavier ULAs ($m_a \gtrsim 10^{-24} \text{eV}$) Fig. 5 gives a rough estimate: The HI bias changes at $\sim 10\%$ level for axion fractions at $m_a = 10^{-24} \text{eV}$ on the $\sim 1\%$. Thus, precision in HI bias modeling needs to be below the $10\%$ ballpark for constraints on the percent-level (and in that mass region) to be robust. A more conservative approach could relax the assumption of precise modeling knowledge of the HI bias, e.g. by introducing nuisance parameters in the redshift dependence.

Also, this model is partly ignorant of the influence of axions to the exact HI radial profile. Some clear deviations from pure CDM could be expected (Veltmaat et al. 2019) and modeled more accurately. This would also provide additional information on the axion fraction, although the inclusion of highly non-linear scales is necessary. Apart from that, the effect of the Limber approximation could be checked against more accurate calculations of the angular power spectrum. This study could include a thorough investigation of the effect of (un)equal-time correlators (which might become important), too.

In short, all of the above points are of importance to adequately capture future 21 cm IM data and while relaxing the assumptions (i) to (iii) will likely weaken the present constraints, the opposite will be true for a more accurate modeling of (iv) and (v). An important future step is to investigate the HMF and halo bias in the mixed CDM-axion DM scenario with simulations and calibrating the model. We leave these extensions and simulations to future work.

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In Appendix D of Bull et al. (2015) a formula for the temperature noise is given by the Gaussian root mean square width

\[ \sigma_T \approx \frac{T_{\text{sys}}}{\sqrt{n_{\text{pol}}AV_{\text{obs}}}} \frac{\lambda^2}{\theta_B^2} \frac{1}{N_d N_B} \sqrt{\frac{\Omega_{\text{surv}}}{2}} \sqrt{\frac{1}{N_d N_B}}. \]

If one ignores the beam responses for the moment, the 3D power spectrum is given by \( P_N = \sigma_T^2 V_{\text{pix}} \), where \( V_{\text{pix}} = (r_B \theta_B^2)^2 \times (r_v \Delta v/V_{\text{pix}}) \) is the 3D volume of each volume element with \( r_v = c(1+z)^2 \).

To convert \( P_N \) to \( N_l \), we take

\[ N_l(z_i, z_j) = \frac{2}{\pi} \int dz W_i(z) \int dz' W_j(z') \times \int dk k^2 P_N(k, z, z') j_j(kr(z'))j_j(kr(z')). \]

(A1)

where \( W_i \) and \( W_j \) are the window function for the redshift bins \( z_i \) and \( z_j \), \( j_j \) denotes the spherical Bessel function of rank \( \ell \) and \( r(z) \) is the comoving distance.

The expression above in Eq. (A1) can be simplified upon assuming the instrumental noise power spectrum is k-independent:

\[ N_l = \int dz \frac{H(z)}{c} \frac{W_i^2(z)}{W_j^2(z)} P_N(z_i) \approx \Delta \frac{W^2(z)}{c} \frac{H(z)}{cr^2(z)} V_{\text{pix}} \sigma_T^2 \]

\[ \approx \theta_B^2 \sigma_T^2 \]

This yields:

\[ N_l = \frac{T_{\text{sys}}^2 \Omega_{\text{surv}}}{n_{\text{pol}}AV_{\text{obs}}} \theta_B^4 \frac{1}{N_d N_B} \frac{\lambda^4}{\theta_B^2} \frac{1}{N_d N_B} \]

The beam frequency and angular responses is given by

\[ B^{-1} = B^{-1} B^{-1} \]

(A2)

with

\[ B_{\parallel} = \exp \left[ \frac{(-k_i r_B \Delta v/V_{\text{pix}})^2}{16 \lambda^2} \right] \]

\[ B_{\perp} = \exp \left[ \frac{(-k l r_B \Delta v/V_{\text{pix}})^2}{16 \lambda^2} \right] \]

Assuming \( k_l \) to be small, one can take \( \ell + \frac{1}{2} = r k_l \) such that for large \( \ell \)

\[ B^{-1} \approx \exp \left[ \frac{\ell^2 \theta_B^2}{8 \lambda^2} \right]. \]

We conclude for the dimensionless noise expression:

\[ N_l = \frac{T_{\text{sys}}^2 \Omega_{\text{surv}}}{T_{\text{sys}}^2 n_{\text{pol}} AV_{\text{obs}}} \frac{\lambda^4}{\theta_B^2} \frac{1}{N_d N_B} \exp \left[ \frac{\ell^2 \theta_B^2}{8 \lambda^2} \right]. \]

(A3)

Upon redefining the number of beams in the upper expression to include the number of polarization modes, the difference between this expression and the expression adapted from Knox (1995) in Eq. (28) resides in the inclusion of the effective area of the dish. This results in a different prefactor of \( a \sim 3 \).
