Gluon emission from heavy quarks in dense nuclear matter

Le Zhang,1,2 De-Fu Hou,2 and Guang-You Qin2

1College of Physics and Electronic Science, Hubei Normal University, Huangshi, 435002, China
2Institute of Particle Physics and Key Laboratory of Quark and Lepton Physics (MOE), Central China Normal University, Wuhan, 430079, China
(Dated: December 31, 2018)

We study the medium-induced gluon emission process experienced by a hard jet parton propagating through the dense nuclear matter in the framework of deep inelastic scattering off a large nucleus. We work beyond the collinear rescattering expansion and the soft gluon emission limit, and derive a closed formula for the medium-induced single gluon emission spectrum from a heavy or light quark jet interacting with the dense nuclear medium via transverse and longitudinal scatterings. Without performing the collinear rescattering expansion, the medium-induced gluon emission spectrum is controlled by the full distribution of the differential elastic scattering rates between the propagating partons and the medium constituents. We further show that if one utilizes heavy static scattering centers for the traversed nuclear matter and takes the soft gluon emission limit, our result can reduce to the first order in opacity Djordjevic-Gyulassy-Levai-Vitev formula.

I. INTRODUCTION

High-energy heavy-ion collisions at the Large Hadron Collider (LHC) and the Relativistic Heavy-Ion Collider (RHIC) can create the strongly-interacting quark-gluon plasma (QGP), a state of matter consisting of deconfined quarks and gluons. Hard partonic jets, which are produced from early hard scatterings, provide useful tools to study such highly-excited QCD matter. During their passage through the QGP matter, jets may interact with the medium constituents via elastic and inelastic collisions. The interaction between jets and medium not only causes the energy loss of hard partons, but also changes the internal structure of full jet shower. Such phenomenon is generally referred to as jet quenching [1–4]. There have been a wealth of experimental evidences and phenomenological studies on jet-medium interaction and jet quenching in the LHC and RHIC heavy-ion collision programs, such as the suppressions of high transverse momentum ($p_T$) hadron productions [5–7] and jet productions [10–13], and the nuclear modification of jet-related correlations [14–18], jet shape and jet substructure observables [19–22].

From the theoretical side, tremendous effort has been devoted to our understanding of parton energy loss and jet modification in both cold and hot dense nuclear matter. For example, the effects of binary elastic collisions of hard partons with medium constituents have been studied in various literatures [23–27]. The medium-induced radiative process experienced by hard partons in dense nuclear matter have been investigated in details as well. There currently exist a few theoretical schemes on radiative parton energy loss based on the perturbative QCD framework: Baier-Dokshitzer-Mueller-Peigne-Schiff-Zakharov (BDMPS-Z) [28–32], Djordjevic-Gyulassy-Levai-Vitev (DGLV) [33–35], Armesto-Salgado-Wiedemann (ASW) [36–38], Arnoldi-Moore-Yaffe (AMY) [39, 40] and higher twist (HT) [41–44] formalisms. The other hand, much recent phenomenological effort [see e.g., Ref. 8] has been devoted to quantitative extraction of various jet transport coefficients, such as $q$ which quantifies the transverse momentum squared transferred per unit time between hard jet partons and soft nuclear medium [20].

While the study of jet quenching in heavy-ion collisions has already entered quantitative era, various systematic uncertainties still exist from the theory side, such as different implementations of jet-medium interaction effects and various approximations made in the foundations of jet quenching formalisms. Regarding medium-induced radiative parton energy loss, some jet quenching formalisms (e.g., GLV and ASW) take the soft approximation for the radiative gluons, while others (e.g., BDMPS-Z and HT) take collinear expansion for the rescatterings with the medium constituents. These issues have already been pointed out in Ref. [13] in which a detailed comparison of different jet energy loss schemes is also provided. Some recent studies have been performed on relaxing various theoretical approximations. For example, Ref. [46] includes the non-eikonal corrections for medium-induced emission within the path integral formalism. Refs. [47, 48] generalize the HT formalism beyond the collinear rescattering expansion; it is interesting that such generalized HT formalism reduces to the GLV formalism in the soft gluon emission limit. Ref. [49, 50] reinvestigate the GLV formalism by relaxing the soft gluon emission approximation.

In this work, we study the medium-induced gluon emission from a massive quark jet which scatters off the medium constituents during its passage through the dense nuclear medium, within the framework of deep-inelastic scattering (DIS) off a large nucleus. In particular, we work beyond the collinear rescattering expansion and soft gluon emission approximation, and derive a closed formula for the medium-induced single gluon emission spectrum from a heavy quark jet interacting with medium constituents via both transverse and longitudinal scatterings. The work is an extension of Ref. [47], which studies the medium-induced gluon emission from a massless light quark jet. Without performing the
collinear expansion, our medium-induced gluon emission spectrum is controlled by the full distribution of the differential elastic scattering rates between the propagating partons and medium constituents. Our study can be viewed as a generalization of the higher twist formalism [11, 14] for both heavy and light flavor medium-induced radiative process. We further show that if one utilizes heavy static scattering centers for the traversed dense nuclear matter. Some details of our main results are presented in the Appendix. The last section contains our summary.

The paper is organized as follows. In Sec. II, we present the gluon emission from a heavy (or light) quark jet at leading twist in the DIS framework. In Sec. III, we derive the medium-induced single gluon emission spectrum for a hard quark jet propagating through the dense nuclear matter. Some details of our main results are presented in the Appendix. The last section contains our summary.

II. GLUON EMISSION AT LEADING TWIST

Consider the process in which a heavy or light quark jet is produced in the framework of semi-inclusive deep inelastic scattering (DIS) off a large nucleus,

\[ l(L_1) + A(Ap) \rightarrow \nu_l(L_2) + q(l_q) + X(P_X). \]  

(1)

Here \( L_1 \) and \( L_2 \) represent the momenta of the incoming lepton and outgoing neutrino. \( Ap \) is the momentum of the incoming nucleus with the nucleon number \( A \), and \( p = [p^+, p^-, p_\perp] \approx [p^+, 0, 0_\perp] \) is the momentum of each nucleon in the nucleus (\( m_N \) is the mass of the nucleon and is neglected in the high energy limit). The outgoing quark jet has the mass \( M \) and carries the momentum \( l_q \). In this work, we focus on the charged current interaction channel since it allows us to study the medium-induced gluon emission for heavy and light quarks together on the same footing. In the charge current interaction channel, the four-momentum of the exchanged \( W \) boson is: \( q = L_2 - L_1 = \{x_Bp^+, q^-, 0_\perp\} \), with \( x_B = Q^2/(2p^+q^-) \) being the Bjorken fraction variable and \( Q^2 = -q^2 \) being the invariant mass of the exchanged \( W \) boson. Here we assume \( Q^2 \ll m_W^2 \).

The differential cross section for the above semi-inclusive DIS process can be written as:

\[ E_{L_2} \frac{d\sigma_{DIS}}{d^3L_2} = \frac{G_F^2}{(4\pi)^3s} L_{\mu\nu} W^{\mu\nu}. \]  

(2)

Here \( G_F \) is the four fermion coupling and \( s = (p + L_1)^2 \) is the center-of-mass energy of the lepton-nucleon collision system. The leptonic tensor \( L_{\mu\nu} \) is given as:

\[ L_{\mu\nu} = \frac{1}{2} \text{Tr}[L_1(1 + \gamma^5)\gamma_\mu L_2\gamma_\nu(1 - \gamma^5)]. \]  

(3)

The semi-inclusive hadronic tensor \( W^{\mu\nu} \) is given as:

\[ W^{\mu\nu} = \frac{1}{2} \sum_X (2\pi)^4 \delta^4(q + Ap - P_X - l_q) \times (A|J_\mu(0)|X)(X|J_\nu^\dagger(0)|A), \]  

(4)

where \( |A\) denotes the initial state of the incoming nucleon \( A \), and \( |X\) represent the final hadronic (or partonic) states. The sum \( \sum_X \) runs over all possible final states except the produced hard quark jet and the radiated gluon. \( J_\mu = \bar{\psi}_l\gamma_\mu(1 - \gamma^5)\psi_l \) is the charged current and \( V_{ij} \) is the Cabibbo-Kobayashi-Maskawa flavor mixing matrix [53]. In the following, we concentrate on the hadronic tensor \( W^{\mu\nu} \) which contains the detailed information about the final state interaction between the propagating quark jet and the traversed nuclear medium.

![FIG. 1. Leading twist contribution to gluon emission from a heavy (or light) quark.](image)

Figure 1 shows the leading twist DIS process in which a \( W \) boson carrying momentum \( q \) strikes a light quark from a nucleus constituent at location \( y_0 = 0 \) (\( y_0 \) in the complex conjugate) and produces a heavy (or light) quark with mass \( M \). Here the light quark from the nucleus carries momentum \( p_0^\prime \) (\( p_0 \) in the complex conjugate) and the struck heavy quark carries momentum \( l_q \). For the DIS process in Figure 1 the hadronic tensor can be written as follows:
\[
W_0^{A\mu
u} = \int \frac{d^4l}{(2\pi)^4} \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \delta(l^2) \int d^4y_0 e^{-ip_0 y_0} \int d^4z \int d^4z' \int d^4q_1 \int d^4q_2 \times e^{-i q_1 \cdot (y_0 - z)} e^{-i q_2 \cdot (z' - y_0)} e^{-i \epsilon (z' - z)} |V_{ij}|^2 \langle A| \bar{\psi}_i(y_0) \gamma^\mu (1 - \gamma^5) \frac{-i (\not{q}_1 + M)}{q_1^2 - M^2 - i\epsilon} \rangle \times \frac{i (\not{q}_1 + M)}{q_1^2 - M^2 + i\epsilon},
\]

where \(\tilde{G}^{\alpha\beta}(l)\) is the sum of the gluon polarizations. In the light-cone gauge, \(n \cdot A = A^- = 0\), where the unit four vectors \(n = [1, 0, 0, \perp]\), the polarization sum \(\tilde{G}^{\alpha\beta}(l)\) reads:

\[
\tilde{G}^{\alpha\beta}(l) = -g^{\alpha\beta} + \frac{n^\alpha l^\beta + n^\beta l^\alpha}{n \cdot l}.
\]

In the limit of very high energy, we can ignore the (\perp\) component of the quark field operators and factor out one-nucleon state from the nucleon state as follows:

\[
\langle A| \bar{\psi}_i(y_0) \gamma^\mu (1 - \gamma^5) \hat{O}(1 + \gamma^5) \gamma^\nu \psi_i(0)|A\rangle 
\approx AC_p^A \langle p| \bar{\psi}_i(y_0) \frac{\gamma^+}{2} \psi_i(0)|p\rangle \frac{1}{4p^+ q^+} \text{Tr}[\rho \gamma^\mu (1 - \gamma^5) \{\not{g} + (x_B + x_M) p\} (1 + \gamma^5) \gamma^\nu] \text{Tr} \left[ \frac{\gamma^\nu}{2} \hat{O} \right],
\]

where \(C_p^A\) denotes the probability of finding a nucleon state with momentum \(p\) inside a nucleus (with \(A\) nucleons). Here the momentum fraction \(x_M = \frac{q^+}{2p^+ q^-}\) is defined for convenience. Now we perform the integration over gluon emission positions \(z, z'\) and obtain two \(\delta\) functions. Then we integrate over momenta \(q_1, q_2\), and obtain \(q_1 = q'_1 = l + l_q\). Also for convenience, we re-introduce the momentum variable \(p_0 = q_1 - l - q\) and perform the integration over \(l_q\), which implies \(l_q = q + p_0 - l = q + p_0' - l\). After the above simplifications, the hadronic tensor can be organized as follows:

\[
W_0^{A\mu
u} = g^2 C_F \int \frac{d^4l}{(2\pi)^4} \frac{d^4p_0}{(2\pi)^4} e^{-ip_0 y_0} \frac{1}{q_1^2 - M^2 - i\epsilon} \frac{1}{q_2^2 - M^2 + i\epsilon} \delta(l^2 - M^2) \times |V_{ij}|^2 AC_p^A \langle p| \bar{\psi}_i(y_0) \frac{\gamma^+}{2} \psi_i(0)|p\rangle \frac{1}{4p^+ q^+} \text{Tr}[\rho \gamma^\mu (1 - \gamma^5) \{\not{g} + (x_B + x_M) p\} (1 + \gamma^5) \gamma^\nu] \times \text{Tr} \left[ \frac{\gamma^\nu}{2} (\not{q}_1 + M) \gamma_\alpha (l_q + M) \gamma_\beta (\not{q}_2 + M) \right] \tilde{G}^{\alpha\beta}(l).
\]

Now we investigate the on-shell condition for the final outgoing heavy quark:

\[
\delta(l^2 - M^2) = \frac{1}{2p^+ q^- (1 - y)} \delta(x_0 - x_B - x_M - \hat{x}_L),
\]

where \(y = l^-/q^-\) is the fraction of the forward momentum carried by the radiated gluon with respect to the parent heavy quark. Here for convenience, we also define the momentum fractions \(x_0 = p_0^+ / p^+\) and \(\hat{x}_L = x_L^2 + y^2 M^2 / 2p^+ q^- (1 - y)\). The above \(\delta\) function may be utilized to carry out the integration over \(p_0^+ = x_0 p^+\). One may further perform the integration over \(y_0^+\) and \(y_0\perp\), yielding three-dimensional \(\delta\) functions for carrying out three-dimensional integrations over \(p_0^+\) and \(p_0\perp\). Now the differential hadronic tensor for the leading twist DIS process can be expressed as follows:

\[
\frac{dW_0^{A\mu
u}}{dy d^4l} = |V_{ij}|^2 AC_p^A \langle x_B + x_M + \hat{x}_L | \frac{1}{4p^+ q^+} \text{Tr}[\rho \gamma^\mu (1 - \gamma^5) \{\not{g} + (x_B + x_M) p\} (1 + \gamma^5) \gamma^\nu] \times C_F \frac{\alpha_s}{2\pi} P(y) \frac{l_2^2 + \frac{\alpha_s}{1 - y} + M^2}{(l_1^2 + y^2 M^2)^2},
\]

In the above expression, \(f_i(x)\) is the light quark distribution function in the nucleon of the incoming nucleus,

\[
f_i(x) = \int \frac{dy_0}{2\pi} e^{-ixp^+ y_0} \langle p| \bar{\psi}_i(y_0) \frac{\gamma^+}{2} \psi_i(0)|p\rangle.
\]

where \(x\) is the fraction of the forward momentum carried by the light quark from the nucleon, and \(P(y) = \frac{1}{1+y^2}\) is the leading order light quark to gluon (photon) splitting function (Note that the color factor \(C_F\) for light quark to
gluon splitting vertex have been factor out). From the above expression, one may read off the differential single gluon emission spectrum from a heavy quark jet in vacuum as:

$$\frac{dN_g^{vac}}{dy d^2l} = C_F \frac{\alpha_s}{2\pi^2}P(y) \frac{l_p^2 + \frac{y^4}{1+y^2} M^2}{(l_p^2 + y^2 M^2)^2}. \tag{12}$$

One can see that when the mass $M$ of the quark jet is set to zero, the above formula reduces to the single gluon emission spectrum from a light quark jet in vacuum.

### III. MEDIUM-INDUCED GLUON EMISSION IN DENSE NUCLEAR MATTER

In the previous section, we have studied the DIS process in which a heavy quark is first produced, then emits a gluon and exits the medium without further interaction. In this section, we consider the medium modification to the above gluon emission process from a heavy quark jet. In particular, we study how the rescattering of the heavy quark or the radiated gluon with the medium constituents influences the single gluon emission spectrum. In this work, we focus on the single rescattering process.

Figure 2 shows one central-cut diagram describing the process in which the radiated gluon from a heavy quark jet experiences a single rescattering with the medium constituents in both the amplitude and the complex conjugate. It contributes to the hadronic tensor at the so-called twist-four level. The other 20 diagrams at twist-four level are listed in Appendix. In this section, we present the details for the computation of the DIS hadronic tensor and the medium-induced single gluon emission spectrum for Figure 2. The computations for the other 20 diagrams are completely analogous and we list their key results in the Appendix.

In Figure 2, a $W$ boson carrying momentum $q$ strikes a light quark from the nucleus at the location $y_0' = 0$ ($y_0$ in the complex conjugate) and produces a heavy quark with mass $M$. The light quark from the nucleus carries momentum $p_0'$ ($p_0$ in the complex conjugate) and the struck heavy quark carries momentum $q_0'$ ($q_1$ in the complex conjugate). The produced heavy quark propagates through the dense nuclear medium and emits a gluon with momentum $l_0'$ ($l_1$ in the complex conjugate) at the location $z'$ ($z$ in the complex conjugate). The radiated gluon then scatters off the gluon field of the medium constituent at the location $z_0'$ ($z_1$ in the complex conjugate) and exchanges momentum $k_0'$ ($k_1$ in the complex conjugate) with the nuclear medium. The final emitted real gluon carries momentum $l_q$. The hadronic tensor for the DIS process in Figure 2 can be expressed as follows:

$$W^{A\nu B}_{2} = \frac{1}{N_c} \int \frac{d^4l}{(2\pi)^4} 2\pi \delta(l^2) \int \frac{d^4l_2}{(2\pi)^4} 2\pi \delta(l_2^2 - M^2) \int d^4y_0 e^{iq.y_0} \int d^4z \int \frac{d^4q_1}{(2\pi)^4} e^{-iq.(y_0 - z)} \times \int d^4z_0 \int d^4l_1 \int \frac{d^4l_2}{(2\pi)^4} e^{-il.(z_0 - z)} \int d^4z' \int \frac{d^4q_2}{(2\pi)^4} e^{-iq.(z'' - y_0)} e^{-il.(z_0 - z')} \times |V_{ij}|^2(A)|\tilde{u}(y_0)|\gamma^\mu(1 - \gamma^5) -i(\gamma_1 + M) \frac{-i}{q_2^2 - M^2 - i\epsilon} (-ig\gamma_0 T^{\alpha\beta})(l_q + M) -i\delta_{\alpha\beta}(l_1) \times [g f^{b_1 c_1 a_1} \Gamma_{b_1, \gamma_1, a_1} (-l_1, -k_1, l) A_1^{\gamma_1}(z_1)] [\delta_{a_1 a_2} G^{\alpha_1, \alpha_2}(l)] \times \frac{i\gamma_5}{q_1^2 + i\epsilon} (ig\gamma_0 T^{\alpha_1}) \frac{i(\gamma_1 + M) \frac{-i}{q_1^2 - M^2 + i\epsilon} (1 + \gamma^5) \gamma^\nu \psi_1(0)|A). \tag{13}$$

Here $f^{b_1 c_1 a_1}$ is the anti-symmetric structure constant of the $SU(3)$ group, and $\Gamma_{b_1, \gamma_1, a_1}$ is the kernel of three-gluon vertex,

$$\Gamma_{b_1, \gamma_1, a_1}(-l_1, -k_1, l) = g_{b_1 \gamma_1}(-l_1 + k_1)_{a_1} + g_{\gamma_1 a_1}(-k_1 - l)_{b_1} + g_{a_1 b_1}(l + l_1)_{\gamma_1}. \tag{14}$$
Now we simplify the hadronic tensor $W_{A\mu}^{B\nu}$. First, one may isolate the phase factors associated with two gluon insertions, $e^{-i(l_0 + l_1 - q_1) \cdot z} e^{i(l'_0 + l'_1 - q'_1) \cdot z'}$. Integrating out the coordinate variables $z$ and $z'$, one may obtain two $\delta$ functions, which can be utilized to carry out the integrations over the momenta $q_1$ and $q'_1$ and obtain two momentum-conservation relations: $q_1 = l_q + l_1$ and $q'_1 = l_q + l'_1$ at vertices $z$ and $z'$. From momentum conservations in Figure 2, we may also obtain the following relations for various momenta:

$$p_0 = q_1 - q, \quad k_1 = l - l_1, \quad p'_0 = q'_1 - q, \quad k'_1 = l - l'_1.$$  \hfill (15)

Here for convenience, we re-introduce the momentum variable $p_0$, with $p_0 = l_q + l - q - k_1$ from momentum conservation. We also change the integration variables $l_1 \to k_1$ and $l'_1 \to k'_1$. Then the phase factor for $W_{A\mu}^{B\nu}$ can be expressed as: $e^{-ip_0 \cdot y_0 - ik_1 \cdot z - ik'_1 \cdot z'}$.

In the limit of very high energy, one may factor out the quark field and the gluon field operators of the nucleus state and ignore the transverse ($\perp$) component of the quark field operators,

$$\langle A|\bar{\psi}_1(y_0)\gamma^\mu(1 - \gamma^5)\hat{O}(1 + \gamma^5)\gamma^\nu\psi_0)|A \rangle$$  

$$\approx AC^\dagger F(p|\bar{\psi}_1(y_0)\gamma^\nu/2\gamma^\mu\psi_0(p)|p\rangle \frac{1}{4p^+q^-} \text{Tr}[p^\mu(1 - \gamma^5)(g + (x_B + x_M)p\beta)(1 + \gamma^5)\gamma^\nu]\text{Tr}[\gamma^\mu/2\langle A|\hat{O}|A \rangle].$$  \hfill (16)

Then the gluon propagators together with the three-gluon vertices can be simplified as follows:

$$\hat{G}^{\alpha_0\beta_1}(l_1)\tilde{\Gamma}_{\beta_1\gamma_1\alpha_0}(-l_1, -k_1, l)A_{\gamma_1}^a(z_1)\hat{G}^{\alpha_1\beta'_1}(l)\tilde{\Gamma}_{\alpha_1\gamma'_1\beta'_1}(-l, k'_1, l')A_{\gamma'_1}^a(z'_1)\hat{G}^{\beta'_1\alpha'_0}(l')$$  

$$= \hat{G}^{\alpha_0\beta_1}(l_1)\left[\bar{g}_{\alpha_0\beta_1}(l_1 - k_1)A_{\gamma_1}^a(z_1)\right]\hat{G}^{\alpha_1\beta'_1}(l)\left[\bar{g}_{\alpha_1\beta'_1}(l + l'_1)A_{\gamma'_1}^a(z'_1)\right]\hat{G}^{\beta'_1\alpha'_0}(l').$$  \hfill (17)

Here we have only kept the dominant forward ($+$) component of the scattered gluon field in very high energy limit. With the above simplifications, the hadronic tensor for Figure 2 may be written as:

$$W_{A\mu}^{B\nu} = g^4 \int \frac{d^4l}{(2\pi)^4} 2\pi\delta(l^2) \int \frac{d^4k_1}{(2\pi)^4} \delta^4(l + l_0 - k_1 - q) \int d^4y_1 \int d^4z_1 \int d^4z'_1$$  

$$\times \left[ \int \frac{d^4p_0}{(2\pi)^4} \int \frac{d^4k'_1}{(2\pi)^4} \left( e^{-ip_0 \cdot y_0 - ik_1 \cdot z - ik'_1 \cdot z'} \right) \frac{1}{q^2 - M^2 - i\epsilon} \frac{1}{l^2 - i\epsilon} \frac{1}{q^2 - l^2 + i\epsilon} \frac{1}{l^2 = M^2 + i\epsilon} \times \frac{1}{2} \text{Tr}[p^\mu(1 - \gamma^5)(g + (x_B + x_M)p\beta)(1 + \gamma^5)\gamma^\nu] \times \frac{1}{N_c} \text{Tr}[\hat{T}^{\alpha_0\beta_1\gamma_1\alpha_0}(l_1)\hat{G}^{\alpha_0\beta_1}(l_1)\hat{G}^{\alpha_1\beta'_1}(l)\hat{G}^{\beta'_1\alpha_0}(l')] \right].$$  \hfill (18)

We now look at the internal quark and gluon propagators and the final external quark line,

$$q_1^2 - M^2 = (q + x_0)^2 - M^2 = 2p^+q^- (1 + x_0) [-x_B + x_0 - \zeta_M],$$  \hfill (19)

$$l_1^2 = (l - k_1)^2 = 2p^+q^- (y - \lambda_1)[\bar{x}_L(1 - y) - y x_M - \lambda_1 - \lambda_D],$$  \hfill (20)

$$l'_0^2 - M^2 = (q_0 + k_1 - l)^2 - M^2 = 2p^+q^- (1 + x_0^+ + \lambda_1 - y)[-x_B + x_0 + \lambda_1 - \bar{x}_L(1 - y) + y x_M - \eta_M],$$  \hfill (21)

where for convenience we have defined the momentum fractions,

$$x_0 = \frac{x_0}{p^+}, \quad \lambda_1 = \frac{k_1^+}{p^+}, \quad x_0^- = \frac{x_0^-}{q^-}, \quad \lambda_1^- = \frac{k_1^-}{q^-},$$

$$\zeta_M = \frac{p_{0\perp}^2 + M^2}{2p^+q^- (1 + x_0)} = \frac{(l_1 - k_1 \perp)^2}{2p^+q^- (y - \lambda_1)}, \quad \eta_M = \frac{(l_1 - k_1 \perp - p_{0\perp})^2 + M^2}{2p^+q^- (1 + x_0 + \lambda_1 - y)}. \hfill (22)$$

From the above relations, the contributions from the internal quark and gluon propagator and the on-shell condition for the final outgoing heavy quark can be obtained as:

$$D_q = \frac{C_q}{8(p^+q^-)^2} \frac{y - \lambda_2}{y - \lambda_2^-} \frac{y - \lambda_1^-}{-x_B + x_0 - \zeta_M - \lambda_D - \zeta^\ell} \frac{1}{x_B + x_0 - \bar{x}_L(1 - y) - y x_M - \lambda_1 - \lambda_D - \epsilon} \frac{1}{-x_B + x_0 - \zeta_M - \lambda_D - \epsilon} \frac{1}{(2\pi)^5} \delta[-x_B + x_0 + \lambda_1 - \bar{x}_L(1 - y) + y x_M - \eta_M],$$  \hfill (23)
where
\[ C_q = \frac{1}{(1 + x_0)(1 + x_0') + (1 + x_0 + \lambda_1 - y)}. \] (24)

Performing the trace part of the hadronic tensor [the last line of Eq. (18)], we obtain:
\[ N_q = \frac{4q^-}{C_q} \frac{1 + \left(1 - \frac{y - \lambda_1^-}{1 + x_0}\right) \left(1 - \frac{y - \lambda_1^-}{1 + x_0'}\right)}{(y - \lambda_1^-)(y - \lambda_1^-') \left(1 - \frac{y - \lambda_1^-}{1 + x_0}\right) \left(1 - \frac{y - \lambda_1^-}{1 + x_0'}\right)} \times \left[ \left(1 - k_{1,\perp} - \frac{y - \lambda_1^-}{1 + x_0} p_{0,\perp}\right) \cdot \left(1 - k'_{1,\perp} - \frac{y - \lambda_1^-'}{1 + x_0'} p'_{0,\perp}\right) \right] \times \frac{(y - \lambda_1^-)^2}{1 + \left(1 - \frac{y - \lambda_1^-}{1 + x_0}\right) \left(1 - \frac{y - \lambda_1^-}{1 + x_0'}\right)} \right]. \] (25)

With the above simplifications, the hadronic tensor for Figure 2 now reads:
\[ W_{\lambda_{10}} = \frac{g^4}{16\pi^3} \int \frac{dy}{y} \int \frac{d\lambda_1}{2\pi} \int \frac{d\lambda_1'}{2\pi} \int \frac{d\lambda}{2\lambda_1 - \lambda_1'\delta} \int \frac{d^3k_1}{(2\pi)^3} \int \frac{d^3k_1'}{(2\pi)^3} e^{-ix_0p_0'} y_0 e^{-ik_1 z_1 x_1} |V_{ij}|^2 A C_p^* |p| \psi_i(y_0) \frac{\gamma^+}{2} \psi_j(p) \]
\[ \times \frac{1}{4p^+ q^2} \text{Tr} \left[ p \gamma^\mu \left(1 - \gamma^5\right) \{g + (x_0 \pm A_1) p\} \right] (1 + \gamma_5) \gamma^\nu |A| A_{ci}^* (z_1) \frac{1}{N_c} \text{Tr} |T^a q f_{a c i} a_{0} f_{a 1} z_1 T^a 0^| \]
\[ \times \frac{1}{2(p^+ q^2)^2} \frac{1}{y - \lambda_1^- (y - \lambda_1^-') \left(1 - \frac{y - \lambda_1^-}{1 + x_0}\right) \left(1 - \frac{y - \lambda_1^-}{1 + x_0'}\right) \left(1 - \frac{y - \lambda_1^-}{1 + x_0}\right) \left(1 - \frac{y - \lambda_1^-}{1 + x_0'}\right)} \times \left[ \left(1 - k_{1,\perp} - \frac{y - \lambda_1^-}{1 + x_0} p_{0,\perp}\right) \cdot \left(1 - k'_{1,\perp} - \frac{y - \lambda_1^-'}{1 + x_0'} p'_{0,\perp}\right) \right] \times \frac{(y - \lambda_1^-)^2}{1 + \left(1 - \frac{y - \lambda_1^-}{1 + x_0}\right) \left(1 - \frac{y - \lambda_1^-}{1 + x_0'}\right)} \right]. \] (26)

Here for convenience, the three-vector notations for coordinate and momentum are used: \( z = (z^+, z^\perp) \) and \( k = (k^-, k^\perp) \). Their dot product reads: \( k \cdot z = k^- z^+ + k^\perp \cdot z^\perp \).

Now we perform the integrations over the momentum fractions \( x_0, \lambda_1 \) and \( \lambda_1' \). The \( \delta \) function from the on-shell condition of the outgoing heavy quark can be used to carry out the integration over \( x_0 \),
\[ \int \frac{dx_0}{2\pi} \frac{1}{x_B + x_0 - \zeta_M - i\epsilon - x_B + x_0' - \zeta_{M0} + i\epsilon} \left(2\pi\right)^2 \delta[-x_B + x_0 + \lambda_1 - \tilde{x}_L(1 - y) + y x_M - \eta M1] \]
\[ = \frac{1}{x_L(1 - y) - y x_M - \lambda_1 + \eta M1 - \zeta M0 - i\epsilon} \frac{1}{x_L(1 - y) - y x_M - \lambda_1' + \eta M1' - \zeta M0' + i\epsilon}. \] (27)

The integration over \( \lambda_1 \) may be performed by closing the contour with a counterclockwise semicircle in the upper half of the \( \lambda_1 \) complex plane:
\[ \int \frac{d\lambda_1}{2\pi} e^{-i\lambda_1 p^+ (z^- - y_0)} \]
\[ = i\theta(z^- - y_0) e^{-i\tilde{x}_L(1-y)-x_0 p^+ (z^- - y_0)} e^{i(\eta M1 - \zeta M0) p^+ y_0} e^{i\lambda D1 p^+ z^-} \frac{e^{-ix M10 p^+ y_0}}{\chi M10}, \] (28)

where the momentum fraction \( \chi M10 = \eta M1 + \lambda D1 - \zeta M0 \) has been defined for convenience. The integration over the momentum fraction \( \lambda_1' \) (in the complex conjugate) is completely analogous. After carrying out the integration over
the exchanged gluon field is initiated by the on-shell quark, we have
\[ W^{A_{\mu
u}} = \frac{g^4}{16\pi^2} \int \frac{dy}{y_0} \int \frac{d^2p_0}{(2\pi)^3} e^{-ip_0 \cdot y_0} e^{-i(x_B + x_M + \bar{x}_L)p_y y_0} |V_{ij}|^2 AC_p A^\dagger (p|\bar{v}_i(y_0) \frac{\gamma^\nu}{2} \psi_i(0)|p) \]
\[ \times \int dz_1 \int dz'_1 \int [i\theta(z'_1 - y_0)][-i\theta(z_1 - y_0)] \int d^3z_1 \int d^3z'_1 \int \frac{d^3k_1}{(2\pi)^3} \int \frac{d^3k'_1}{(2\pi)^3} e^{-ik_1 \cdot z_1 e^{ik'_1} \cdot z'_1} \]
\[ \times \langle A|A_{c_1}^+(z_1) A_{c'_1}^+(z'_1)|A \rangle \]
\[ \times \left[ 1 + \frac{1}{\lambda_{M10}} \frac{1}{(p^+ q^-)^2} \frac{y - \lambda_{1}^{-}}{1 + x_0} - \frac{y - \lambda_{1}^{-}}{1 + x_0} (y - \lambda_{1}^{-})(y - \lambda_{1}^{-}) \left(1 + \frac{1}{\lambda_{M10}} \frac{y - \lambda_{1}^{-}}{1 + x_0} \right) \frac{M^2}{1 + \frac{1}{\lambda_{M10}} \frac{y - \lambda_{1}^{-}}{1 + x_0}} \right] \] (29)

Now we look at the correlations of two gluon field operators in the nucleus state \( \langle A|A_{c_1}^+(z_1) A_{c'_1}^+(z'_1)|A \rangle \). One may first factor out the color factor for the gluon field correlator as follows:
\[ \langle A|A_{c_1}^+(z_1) A_{c'_1}^+(z'_1)|A \rangle = \frac{1}{d(R)} \text{Tr}[T_{c_1}(R)T_{c'_1}(R)] \langle A|A^+(z_1) A^+(z'_1)|A \rangle = \delta_{c_1 c'_1} \frac{C_2(R)}{N_c^2 - 1} \langle A|A^+(z_1) A^+(z'_1)|A \rangle, \] (30)

where \( d(R) \) and \( C_2(R) \) are the dimensions and the Casimir factors for the representation \( R \) of the \( SU(3) \) group. If the exchanged gluon field is initiated by the on-shell quark, we have \( d(R) = 3 \) and \( C_2(R) = C_F \). If the exchanged gluon field is initiated by the on-shell gluon, \( d(R) = 8 \) and \( C_2(R) = C_A \). Since the two gluon insertions from the amplitude and the complex conjugate carry the same color \( (\delta_{c_1 c'_1}) \), one may evaluate the color factor for the hadronic tensor \( W^{A_{\mu
u}} \) as follows:
\[ \delta_{c_1 c'_1} \frac{1}{N_c} \text{Tr}[T_{c_1}(R)T_{c'_1}(R)] = C_F C_A. \] (31)

In addition, we change the coordinate variables \( (z_1, z'_1) \) to their mean value and difference \( (Z_1, \delta z_1) \):
\[ Z_1 = \frac{z_1 + z'_1}{2}, \quad \delta Z_1 = z_1 - z'_1. \] (32)

By using the translational invariance, the gluon field correlations only depend on \( \delta Z_1 \),
\[ \langle A|A^+(z_1) A^+(z'_1)|A \rangle \approx \langle A|A^+(\delta Z_1) A^+(0)|A \rangle. \] (33)

Now we may carry out the following integration for the phase factor,
\[ \int d^3Z_1 \int d^3Z'_1 e^{-ik_1 \cdot Z_1 e^{ik'_1} \cdot Z'_1} = (2\pi)^3 \delta^3(k_1 - k'_1) \int d^3Z_1 e^{-i(k_1 \cdot \delta Z_1)} \frac{2\pi}{Z_1}. \] (34)

The above \( \delta \) function comes from the integration over the three-coordinate \( Z_1 \) (the mean value of two gluon insertions), and means that two gluon field insertions carry the same momentum, \( k'_1 = k_1 \). We also perform the integration over the three-coordinate \( y_0 \), which gives \( p_0 = 0 \), thus \( p'_0 = p_0 + k_1 - k'_1 = 0 \). With the above simplifications, the hadronic
tensor for Figure 2 may be written as:

\[ W_{\mu \nu}^{A} = \frac{g^4}{16 \pi^3} \int \frac{dy}{y} \int d^2 \mathbf{l}_\perp |V_{ij}|^2 A_C^A (2\pi) f_i(x_B + x_M + \mathbf{x}_L) \frac{1}{4p^+ q^-} \text{Tr}[\gamma^\mu (1 - \gamma^5) \{ \mathbf{g} + (x_B + x_M) \mathbf{p} \} (1 + \gamma^5) \gamma^\nu] \]

\[ \times \int dZ_1^- \int d\delta_1^- \int d^3 \mathbf{z}_1 e^{-ik_1^- \cdot \delta_1^-} C_2(R) N_c^2 - 1 \langle A|A^+ (\delta_1^-, \delta_1^-) A^+ (0)|A \rangle \]

\[ \times e^{i(\mathbf{x}_L + x_M - \zeta_M0) p^+ y_0} e^{-i(\mathbf{x}_L (1-y) - x_M - \lambda_{D1}) p^+ \delta Z_1^-} (e^{-i\chi_{M10} p^+ y_0} - e^{-i\chi_{M10} p^+ (Z_1^- + \frac{1}{2} \delta Z_1^-)})(1 - e^{i\chi_{M10} p^+ (Z_1^- - \frac{1}{2} \delta Z_1^-)}) \]

\[ \times C_F C_A \left[ 1 - \frac{1}{(2p^+ q^-)^2} \frac{1}{(y - \lambda_1^-)^2} \left( \frac{y - \lambda_1^-}{y - \lambda_1^-} \right)^2 \left[ (l_1^- - k_1^-)^2 + \frac{(y - \lambda_1^-)^4 M^2}{1 + (1 + \lambda_1^- - y)^2} \right] \right]. \quad (35) \]

Now we simplify the second last line of Eq. (35), which we refer to as the phase factor \( S_2 \). We first note that the coordinate \( Z_1^- \) is the mean value of two gluon insertion points and can span over the whole nucleus size, while the coordinates \( y_0 \) and \( \delta Z_1^- \) are confined within the nucleon size. Therefore, we may suppress the contributions from \( y_0 \) and \( \delta Z_1^- \) as compared to the contribution from \( Z_1^- \). Then the phase factor \( S_2 \) can be greatly simplified,

\[ S_2 = e^{i(\mathbf{x}_L + x_M - \zeta_M0) p^+ y_0} e^{-i(\mathbf{x}_L (1-y) - x_M - \lambda_{D1}) p^+ \delta Z_1^-} (e^{-i\chi_{M10} p^+ y_0} - e^{-i\chi_{M10} p^+ (Z_1^- + \frac{1}{2} \delta Z_1^-)})(1 - e^{i\chi_{M10} p^+ (Z_1^- - \frac{1}{2} \delta Z_1^-)}) \approx 2 - 2 \cos(\chi_{M10} p^+ Z_1^-). \quad (36) \]

Recalling the definitions of the momentum fractions \( \eta_{M11}, \lambda_{D1} \) and \( \zeta_M0 \), one may obtain the momentum fraction variable \( \chi_{M10} = \eta_{M1} + \lambda_{D1} - \zeta_M0 \) as follows:

\[ \chi_{M10} = \frac{(1 + (1 - \lambda_1^- - y)^2 M^2}{2p^+ q^- (y - \lambda_1^-)} \frac{x_L}{(y - \lambda_1^-)^2} \frac{(1 - k_1^-)^2 + (y - \lambda_1^-)^2 M^2}{l_1^2 + y^2 M^2} \quad (37) \]

The above expression may be used to evaluate the last line of Eq. (35), which we refer to as the hard matrix element \( T_2 \),

\[ T_2 = 2yP(y) C_F C_A \left[ 1 + \frac{1}{(1 + (1 - \lambda_1^- - y)^2 M^2}{2(1 - \lambda_1^- - y)^2} \frac{(1 - k_1^-)^2 + (y - \lambda_1^-)^2 M^2}{l_1^2 + y^2 M^2} \right] = 2yP(y) \tilde{T}_2, \quad (38) \]

where we have also defined the kernel of the hard matrix element \( \tilde{T}_2 \) for convenience. With the above simplifications, the differential hadronic tensor for Figure 2 now reads:

\[ \frac{dW_{\mu \nu}^{A}}{dy d^2 \mathbf{l}_\perp} = |V_{ij}|^2 A_C^A (2\pi) f_i(x_B + x_M + \mathbf{x}_L) \frac{1}{4p^+ q^-} \text{Tr}[\gamma^\mu (1 - \gamma^5) \{ \mathbf{g} + (x_B + x_M) \mathbf{p} \} (1 + \gamma^5) \gamma^\nu] \]

\[ \times \int dZ_1^- \int d\delta_1^- \int d^3 \mathbf{z}_1 e^{-ik_1^- \cdot \delta_1^-} \frac{1}{(2p^+ q^-)^2} \frac{1}{(y - \lambda_1^-)^2} \frac{(1 - k_1^-)^2 + (y - \lambda_1^-)^2 M^2}{l_1^2 + y^2 M^2} \]

\[ \times C_A \left[ 1 + \frac{1}{(1 + (1 - \lambda_1^- - y)^2 M^2}{2(1 - \lambda_1^- - y)^2} \frac{(1 - k_1^-)^2 + (y - \lambda_1^-)^2 M^2}{l_1^2 + y^2 M^2} \right] \quad (39) \]

where \( \tilde{T}_\text{form} = 1/(\mathbf{x}_L p^+) \) is the formation time for gluon radiation from a heavy quark jet. Now we define the following distribution function \( \mathcal{D}(k_1^-, k_1) \) for the exchanged three-momentum \( k_1 = (k_1^-, k_1) \) with the nuclear medium,

\[ \mathcal{D}(k_1^-, k_1) = \int d\delta Z_1^- \int d^3 \mathbf{z}_1 e^{-ik_1^- \cdot \delta_1^-} \frac{1}{(2p^+ q^-)^2} \frac{(1 - k_1^-)^2 + (y - \lambda_1^-)^2 M^2}{l_1^2 + y^2 M^2} \quad (40) \]

In fact, the distribution function \( \mathcal{D}(k_1^-, k_1) \) is, up to a constant factor, the differential elastic scattering rate \( dP_{el}/dk_1^- d^2k_1^- dZ_1^- \) between a light quark and medium constituents,

\[ \mathcal{D}(k_1^-, k_1) = (2\pi)^3 \frac{dP_{el}}{dk_1^- d^2k_1^- dZ_1^-}. \quad (41) \]
One may read off the medium-induced single gluon emission spectrum for Figure 2 as follows:

\[
\frac{dN_{g}^{med}}{dyd^2k_{\perp}} = \frac{\alpha_s}{2\pi^2} P(y) \int dZ_1^- \int \frac{dk_{1\perp}^- d^2k_{1\perp}}{(2\pi)^3} D(k_1^-, k_{1\perp})
\]

\[
\times \left[ 2 - 2 \cos \left( \frac{y(1-y)}{(y-\lambda_1)(1+\lambda_1^- - y)} \right) \left( \frac{(1-\lambda_1^- + k_{1\perp})^2 + (y-\lambda_1^-)^2 M^2}{l_{1\perp}^2 + y^2 M^2} \right) Z_1^- \right]
\]

\[
\times C_A \left[ 1 + \left( 1 + \lambda_1^- - y \right)^2 \left( \frac{y - \lambda_1^-}{y - \lambda_1^-} \right)^2 \left( \frac{(1-\lambda_1^- + k_{1\perp})^2 + (y-\lambda_1^-)^2 M^2}{l_{1\perp}^2 + y^2 M^2} \right) \right]
\]

\[
+ \left[ 2 - 2 \cos \left( \frac{Z_1^-}{\bar{\tau}_{form}} \right) \right]
\]

\[
\times C_F \left[ \frac{l_{1\perp}^2 + \frac{4y^2}{1+y^2} M^2}{l_{1\perp}^2 + y^2 M^2} \right] - C_A \left[ \frac{(y - \lambda_1^-)^2}{2} + \frac{y^2}{1+y^2} M^2 \right]
\]

\[
+ \left( \frac{C_A}{2} - C_F \right) \frac{1 + (1-y) \left( 1 - \frac{y}{1+\lambda_1^-} \right) \left( 1 - \frac{y}{1+\lambda_1^-} \right)}{1 + (1-y)^2} \left( l_{1\perp}^2 + y^2 M^2 \right) \left( 1 \right. \left[ \frac{l_{1\perp}^2 + \frac{4y^2}{1+y^2} M^2}{l_{1\perp}^2 + y^2 M^2} \right]
\]

\[
\left. + \frac{l_{1\perp}^2 + \frac{4y^2}{1+y^2} M^2}{l_{1\perp}^2 + y^2 M^2} \right]
\]

The above formula is quite general in the sense that the property of the nuclear medium that the hard jet parton probes is contained in the distribution function \( D(k_1^-, k_{1\perp}) \). In other words, as we work beyond the collinear re-scattering expansion, the medium-induced emission spectrum is controlled by the full distribution of the differential elastic scattering rate \( dF_{el}/dk_1^- d^2k_{1\perp} dZ_1^- \) between the propagating hard partons and medium constituents. In addition, the
contributions from both transverse and longitudinal momentum exchanges with the medium constituents are included in the above medium-induced single gluon emission spectrum.

In the following, we model the traversed nuclear medium by heavy static scattering centers and take the static screened potential for the exchanged gluon field, 

\[ A^\mu(p) = g^\mu(2\pi)\delta(p^\mu)\frac{-g}{p_1^2 + \mu^2}, \tag{44} \]

where \( \mu \) is the screened mass of the exchanged gluon. We note that the color factor of the gluon field has been factored out. Note that for scattering with the static potential, only transverse momentum is exchanged between the hard partons and medium constituents. By performing the Fourier transformation, the gluon field correlation may be obtained:

\[ \langle A|A^\mu(0)|A \rangle = \delta_\perp^\mu_\perp \rho^- \delta(z_7^-) \int \frac{d^2p_\perp}{(2\pi)^2} e^{ip_\perp \cdot z_7^-} g^2 \frac{(p_1^2 + \mu^2)^2}{(p_1^2 + \mu^2)^2}. \tag{45} \]

where \( \rho^- \) is the light-cone density of the medium constituents (scattering centers) that the hard jet parton interacts. One may also compute the differential elastic cross section for a light quark scattering with the above static screened potential as follows:

\[ \frac{d\sigma_{el}}{d^2k_\perp} = \frac{C_F C_2(R) |gA_+(p_\perp)|^2}{N_c^2 - 1} \frac{4\alpha_s^2}{4\pi^2} = \frac{C_F C_2(R) 4\alpha_s^2}{N_c^2 - 1} \frac{4\alpha_s^2}{(p_1^2 + \mu^2)^2}. \tag{46} \]

Using the above expressions, the distribution function \( D(k_1^-, k_\perp) \) can be obtained as follows:

\[ D(k_1^-, k_\perp) = (2\pi)\delta(k_1^-)(2\pi)^2 \rho^- \frac{d\sigma_{el}}{d^2k_\perp} = (2\pi)\delta(k_1^-)D_\perp(k_1 \perp). \tag{47} \]

Here for convenience, we have also defined the distribution function \( D_\perp(k_1 \perp) \) for transverse momentum exchange,

\[ D_\perp(k_1 \perp) = (2\pi)^2 \rho^- \frac{d\sigma_{el}}{d^2k_\perp} = (2\pi)^2 \frac{dP_{el}}{d^2k_\perp dZ_1^-}, \tag{48} \]

where \( dP_{el}/(d^2k_\perp dZ_1^-) \) is the differential elastic scattering rate with only transverse momentum exchange. Given the above distribution function, the light quark (light-cone) transport coefficient \( \tilde{q}_{lc} \) may be computed,

\[ \tilde{q}_{lc} = \frac{d<k_1 \perp_d L^-}{dL^-} = \int d\mathbf{k}_1^- d\mathbf{k}_\perp \mathbf{D}(k_1^-, k_\perp) = \int \frac{d^2k_\perp d^2k_\perp \mathbf{D}_\perp(k_1 \perp)}{(2\pi)^2} = \int d^2k_\perp d^2k_\perp \rho^- \frac{d\sigma_{el}}{d^2k_\perp}. \tag{49} \]

Using the above static screened potential for the exchanged gluon field, the medium-induced single gluon emission spectrum can be simplified as follows:

\[ \frac{dN^\text{med}_{y}}{dy d^2k_\perp} = \frac{\alpha_s}{2\pi^2} P(y) \int dZ_1^- \int d^2k_\perp \frac{dP_{el}}{d^2k_\perp dZ_1^-} \times \left\{ C_A \left[ 2 - 2 \cos \left( \frac{(l_\perp - k_1 \perp)^2 + y^2 M^2}{l_\perp^2 + y^2 M^2} \frac{Z_1^-}{\tilde{\tau}_{\text{form}}} \right) \right] \times \left[ \frac{(l_\perp - k_1 \perp)^2 + \frac{y^4}{1 + (1-y)^2} M^2}{l_\perp^2 + \frac{y^4}{1 + (1-y)^2} M^2} \right] \right. \]

\[ \times \left[ \frac{1}{2} l_\perp \cdot (l_\perp - k_1 \perp) + \frac{y^4}{1 + (1-y)^2} M^2 \right] \left[ \frac{1}{2} (l_\perp \cdot k_1 \perp)^2 + y^2 M^2 \right] - \frac{1}{2} \left[ (l_\perp - k_1 \perp) \cdot (l_\perp - y k_1 \perp) + \frac{y^4}{1 + (1-y)^2} M^2 \right] \right. \]

\[ + \left. \left[ \frac{C_A}{2} - C_F \right] \left[ 2 - 2 \cos \left( \frac{Z_1^-}{\tilde{\tau}_{\text{form}}} \right) \right] \right\} \frac{1}{\left[ l_\perp^2 + y^2 M^2 \right]} \left[ (l_\perp - y k_1 \perp)^2 + y^2 M^2 \right] \left[ (l_\perp - k_1 \perp)^2 + y^2 M^2 \right] - \frac{l_\perp^2 + \frac{y^4}{1 + (1-y)^2} M^2}{l_\perp^2 + y^2 M^2} \right. \]

\[ + \frac{C_F}{\left[ l_\perp^2 + y^2 M^2 \right]} \left\{ \frac{1}{\left[ l_\perp^2 + y^2 M^2 \right]} \left[ (l_\perp - y k_1 \perp)^2 + y^2 M^2 \right] - \frac{l_\perp^2 + \frac{y^4}{1 + (1-y)^2} M^2}{l_\perp^2 + y^2 M^2} \right\}, \tag{50} \]

The above formula represents the medium-induced single gluon emission spectrum from a heavy quark jet when interacting with the static screened potential. Since only transverse momentum is exchanged with the nuclear medium,
the gluon emission spectrum is controlled by the distribution of transverse momentum exchange \( D_\perp(k_\perp) \), or the differential elastic scattering rate \( dP_{\text{el}}/d^2k_\perp dZ_1^- \), between the propagating partons and the medium constituents. Note that the above formula is still beyond the collinear rescattering expansion and the soft gluon emission limit. One may further perform the simplification by taking the soft gluon emission limit (\( y = l^-/q^- \ll 1 \)). For our case, if we take the limit \( y^2M \ll yM \sim l^- \sim k_\perp \), the above result can be reduced to the following form:

\[
\frac{dN_{\text{med}}}{dyd^2l_\perp} = \frac{\alpha_s}{2\pi^2} P(y) \int dZ_1^- \int d^2k_\perp dZ_1^- \frac{dP_{\text{el}}}{d^2k_\perp dZ_1^-} \times C_A \left[ 2 - 2\cos \left( \frac{(l^- - k_\perp)^2 + y^2M^2}{l^2_\perp + y^2M^2} Z_1^-/\tau_{\text{form}} \right) \right]
\]

This means that when we only consider the contribution from the transverse scattering and take the limit of soft gluon emission, our result for medium-induced gluon emission reduces to the first order in opacity DGLV formula [35, 54].

IV. SUMMARY

In this work, we have studied the medium-induced gluon emission process experienced by a hard jet parton propagating through the dense nuclear matter. Using the framework of deep inelastic scattering off a large nucleus, we have derived a general formula for the medium-induced single gluon emission from a heavy (or light) quark jet interacting with the dense nuclear medium via both transverse and longitudinal scatterings. As we have not performed the collinear rescattering expansion, our medium-induced gluon emission spectrum is controlled by the full distribution of the differential elastic scattering rates between the propagating hard partons and the medium constituents. We have further shown that our medium-induced gluon radiation result can reduce to the first order in opacity Djordjevic-Gyulassy-Levai-Vitev formula if one utilizes heavy static scattering centers for the traversed nuclear matter and also takes the soft gluon emission limit. By going beyond the collinear rescattering expansion and soft gluon emission approximation, our study presents a significant progress on our understanding of the medium-induced radiative process experienced by the hard jet partons traversing and interacting with the dense nuclear matter. The application of our formalism to phenomenological jet quenching studies in the context of high-energy nuclear collisions will be performed in the future effort.

ACKNOWLEDGMENTS

We thank X.-N. Wang for discussions. This work is supported in part by Natural Science Foundation of China (NSFC) under grant Nos. 11775095, 11375072, 11890711. D.-F.H. is supported by Ministry of Science and Technology of China (MSTC) under “973” project No. 2015CB856904(4) and by NSFC under grant Nos. 11735007, 11521064, 11375070.

APPENDIX

In this Appendix, we present the main calculation results for the other 20 cut diagrams, as shown in Figures (3-13).

![Fig. 3. Two central-cut diagrams: one rescattering in the amplitude and one in the complex conjugate.](image-url)
The phase factor for Figure 3 reads:

\[ S_{3a} = e^{-i\xi_{M0} y_0} e^{-i(\tilde{\xi}_L(1-y) - y x M - \lambda_{D1}) p^+ z_i^+} e^{-i\xi_{M0} y_0} e^{-i\xi_{M10} p^+ z_i^+} \left( e^{-ix_{M10} p^+ z_i^+} - e^{-ix_{M10} p^+ z_i^+} \right) \]

\[ \approx e^{i\xi_{M10} p^+ Z_1^+} - 1, \]

\[ S_{3b} = e^{-i\xi_{M0} y_0} e^{-i(\tilde{\xi}_L(1-y) - y x M + \eta_{M1}) p^+ z_i^+} e^{-i(\tilde{\xi}_L(1-y) - y x M - \lambda_{D1}) p^+ z_i^+} \left( 1 - e^{i\xi_{M10} p^+ z_i^+} \right) \]

\[ \approx e^{-i\xi_{M10} p^+ Z_1^+} - 1, \]

\[ S_3 = S_{3a} + S_{3b} \approx \left[ 2 \cos(\chi_{M10} p^+ Z_1^+) - 2 \right]. \tag{52} \]

The matrix element for Figure 3 reads:

\[
\tilde{T}_{3a} = \tilde{T}_{3b} = C_F \frac{C_A}{2} \frac{1 + (1 + \lambda^-_1 - y) \left( 1 - \frac{y}{1 + \lambda^-_1} \right)}{1 + (1 - y)^2} \frac{y - \lambda^-_1}{y - \lambda^-_1} \frac{1}{1 + (1 + \lambda^-_1 - y) \left( 1 - \frac{y}{1 + \lambda^-_1} \right)} \]

\[ \times \left[ (1_\perp - k_{1\perp}) \cdot \left( 1_\perp - \frac{y}{1 + \lambda^-_1} k_{1\perp} \right) + \frac{\left( \frac{y}{1 + \lambda^-_1} \right)^2 (y - \lambda^-_1)^2}{1 + (1 + \lambda^-_1 - y) \left( 1 - \frac{y}{1 + \lambda^-_1} \right)} M^2 \right] \alpha \left[ (1_\perp - k_{1\perp})^2 + (y - \lambda^-_1)^2 M^2 \right] \]

\[ \frac{(1_\perp - k_{1\perp}) \cdot \left( 1_\perp - \frac{y}{1 + \lambda^-_1} k_{1\perp} \right) + \frac{\left( \frac{y}{1 + \lambda^-_1} \right)^2 (y - \lambda^-_1)^2}{1 + (1 + \lambda^-_1 - y) \left( 1 - \frac{y}{1 + \lambda^-_1} \right)} M^2 }{1 + (1 - y)^2} \frac{y - \lambda^-_1}{y - \lambda^-_1} \frac{1}{1 + (1 + \lambda^-_1 - y) \left( 1 - \frac{y}{1 + \lambda^-_1} \right)} \]

\[ \times \left[ (1_\perp - k_{1\perp}) \cdot \left( 1_\perp - \frac{y}{1 + \lambda^-_1} k_{1\perp} \right) + \frac{\left( \frac{y}{1 + \lambda^-_1} \right)^2 (y - \lambda^-_1)^2}{1 + (1 + \lambda^-_1 - y) \left( 1 - \frac{y}{1 + \lambda^-_1} \right)} M^2 \right] \alpha \left[ (1_\perp - k_{1\perp})^2 + (y - \lambda^-_1)^2 M^2 \right] \]

\[ \frac{(1_\perp - k_{1\perp}) \cdot \left( 1_\perp - \frac{y}{1 + \lambda^-_1} k_{1\perp} \right) + \frac{\left( \frac{y}{1 + \lambda^-_1} \right)^2 (y - \lambda^-_1)^2}{1 + (1 + \lambda^-_1 - y) \left( 1 - \frac{y}{1 + \lambda^-_1} \right)} M^2 }{1 + (1 - y)^2} \frac{y - \lambda^-_1}{y - \lambda^-_1} \frac{1}{1 + (1 + \lambda^-_1 - y) \left( 1 - \frac{y}{1 + \lambda^-_1} \right)} \]

\[ \times \left[ (1_\perp - k_{1\perp}) \cdot \left( 1_\perp - \frac{y}{1 + \lambda^-_1} k_{1\perp} \right) + \frac{\left( \frac{y}{1 + \lambda^-_1} \right)^2 (y - \lambda^-_1)^2}{1 + (1 + \lambda^-_1 - y) \left( 1 - \frac{y}{1 + \lambda^-_1} \right)} M^2 \right] \alpha \left[ (1_\perp - k_{1\perp})^2 + (y - \lambda^-_1)^2 M^2 \right] \]

\[ \frac{(1_\perp - k_{1\perp}) \cdot \left( 1_\perp - \frac{y}{1 + \lambda^-_1} k_{1\perp} \right) + \frac{\left( \frac{y}{1 + \lambda^-_1} \right)^2 (y - \lambda^-_1)^2}{1 + (1 + \lambda^-_1 - y) \left( 1 - \frac{y}{1 + \lambda^-_1} \right)} M^2 }{1 + (1 - y)^2} \frac{y - \lambda^-_1}{y - \lambda^-_1} \frac{1}{1 + (1 + \lambda^-_1 - y) \left( 1 - \frac{y}{1 + \lambda^-_1} \right)} \]

\[ \times \left[ (1_\perp - k_{1\perp}) \cdot \left( 1_\perp - \frac{y}{1 + \lambda^-_1} k_{1\perp} \right) + \frac{\left( \frac{y}{1 + \lambda^-_1} \right)^2 (y - \lambda^-_1)^2}{1 + (1 + \lambda^-_1 - y) \left( 1 - \frac{y}{1 + \lambda^-_1} \right)} M^2 \right] \alpha \left[ (1_\perp - k_{1\perp})^2 + (y - \lambda^-_1)^2 M^2 \right] \]

\[ \frac{(1_\perp - k_{1\perp}) \cdot \left( 1_\perp - \frac{y}{1 + \lambda^-_1} k_{1\perp} \right) + \frac{\left( \frac{y}{1 + \lambda^-_1} \right)^2 (y - \lambda^-_1)^2}{1 + (1 + \lambda^-_1 - y) \left( 1 - \frac{y}{1 + \lambda^-_1} \right)} M^2 }{1 + (1 - y)^2} \frac{y - \lambda^-_1}{y - \lambda^-_1} \frac{1}{1 + (1 + \lambda^-_1 - y) \left( 1 - \frac{y}{1 + \lambda^-_1} \right)} \]

\[ \times \left[ (1_\perp - k_{1\perp}) \cdot \left( 1_\perp - \frac{y}{1 + \lambda^-_1} k_{1\perp} \right) + \frac{\left( \frac{y}{1 + \lambda^-_1} \right)^2 (y - \lambda^-_1)^2}{1 + (1 + \lambda^-_1 - y) \left( 1 - \frac{y}{1 + \lambda^-_1} \right)} M^2 \right] \alpha \left[ (1_\perp - k_{1\perp})^2 + (y - \lambda^-_1)^2 M^2 \right] \]

\[ \frac{(1_\perp - k_{1\perp}) \cdot \left( 1_\perp - \frac{y}{1 + \lambda^-_1} k_{1\perp} \right) + \frac{\left( \frac{y}{1 + \lambda^-_1} \right)^2 (y - \lambda^-_1)^2}{1 + (1 + \lambda^-_1 - y) \left( 1 - \frac{y}{1 + \lambda^-_1} \right)} M^2 }{1 + (1 - y)^2} \frac{y - \lambda^-_1}{y - \lambda^-_1} \frac{1}{1 + (1 + \lambda^-_1 - y) \left( 1 - \frac{y}{1 + \lambda^-_1} \right)} \]

\[ \times \left[ (1_\perp - k_{1\perp}) \cdot \left( 1_\perp - \frac{y}{1 + \lambda^-_1} k_{1\perp} \right) + \frac{\left( \frac{y}{1 + \lambda^-_1} \right)^2 (y - \lambda^-_1)^2}{1 + (1 + \lambda^-_1 - y) \left( 1 - \frac{y}{1 + \lambda^-_1} \right)} M^2 \right] \alpha \left[ (1_\perp - k_{1\perp})^2 + (y - \lambda^-_1)^2 M^2 \right] \]

\[ \frac{(1_\perp - k_{1\perp}) \cdot \left( 1_\perp - \frac{y}{1 + \lambda^-_1} k_{1\perp} \right) + \frac{\left( \frac{y}{1 + \lambda^-_1} \right)^2 (y - \lambda^-_1)^2}{1 + (1 + \lambda^-_1 - y) \left( 1 - \frac{y}{1 + \lambda^-_1} \right)} M^2 }{1 + (1 - y)^2} \frac{y - \lambda^-_1}{y - \lambda^-_1} \frac{1}{1 + (1 + \lambda^-_1 - y) \left( 1 - \frac{y}{1 + \lambda^-_1} \right)} \]

\[ \times \left[ (1_\perp - k_{1\perp}) \cdot \left( 1_\perp - \frac{y}{1 + \lambda^-_1} k_{1\perp} \right) + \frac{\left( \frac{y}{1 + \lambda^-_1} \right)^2 (y - \lambda^-_1)^2}{1 + (1 + \lambda^-_1 - y) \left( 1 - \frac{y}{1 + \lambda^-_1} \right)} M^2 \right] \alpha \left[ (1_\perp - k_{1\perp})^2 + (y - \lambda^-_1)^2 M^2 \right] \]
FIG. 5. A central-cut diagram: one rescattering in the amplitude and one in the complex conjugate.

The matrix element for Figure 5 reads:

$$\tilde{T}_{(5)} = C^2 \frac{p^2 L^2 + \frac{y^4 M^2}{1+(1-y)^2}}{[l^2 + y^2 M^2]^2}.$$  (58)

FIG. 6. A central-cut diagram: one rescattering in the amplitude and one in the complex conjugate.

The phase factor for Figure 6 reads:

$$S_{(6)} = e^{-i\zeta p^+ y_0} e^{-i\xi \lambda p^+ z^+} e^{-i\xi \lambda p^- z^-} \approx 1.$$  (59)

The matrix element for Figure 6 reads:

$$\tilde{T}_{(6)} = C^2 \frac{1 + \left(1 - \frac{y}{1 + \lambda_1}\right)^2 \left(l_1 - \frac{y}{1 + \lambda_1} k_1 \right)^2 + \left(\frac{y}{1 + \lambda_1}\right)^4 M^2}{1 + (1-y)^2 \left[ \left(l_1 - \frac{y}{1 + \lambda_1} k_1 \right)^2 + \left(\frac{y}{1 + \lambda_1}\right)^2 M^2 \right]^2}.$$  (60)

FIG. 7. Two central-cut diagrams: one rescattering in the amplitude and one in the complex conjugate.
The phase factor for Figure 9 reads:

$$S_{(a)} = e^{-i(\eta_{M1} - \eta_{M0})p^+ z_1^-} e^{-i\hat{x}_L \hat{p}^+ z_1^-} (1 - e^{i\hat{x}_L \hat{p}^+ z_1^-} \approx e^{-i\hat{x}_L \hat{p}^+ Z_1^-} - 1),
$$

$$S_{(b)} = e^{-i(\eta_{M0} - \eta_{M0})p^+ y_0^-} e^{-i(\eta_{M1} - \eta_{M0})p^+ z_1^-} (e^{-i\hat{x}_L \hat{p}^+ y_0^-} \approx e^{-i\hat{x}_L \hat{p}^+ Z_1^-} - 1),
$$

$$S_{(a)} = S_{(a)} + S_{(b)} \approx 2 \cos(\hat{x}_L p^+ Z_1^-) - 2. \quad (61)$$

The matrix element for Figure 7 reads:

$$\tilde{T}_{(a)} = \tilde{T}_{(b)} = C_F \left( C_F - \frac{C_A}{2} \right) \frac{1 + (1 - y) \left( 1 - \frac{y}{1 + \lambda_{1\perp}} \right)}{1 + (1 - y)^2} \frac{1 + (1 - y) \left( 1 - \frac{y}{1 + \lambda_{1\perp}} \right) + y^2 \left( \frac{y}{1 + \lambda_{1\perp}} \right)^2 M^2}{(l_2^2 + y^2 M^2) \left( 1 - \frac{y}{1 + \lambda_{1\perp}} k_{1\perp} \right)^2 + \left( \frac{y}{1 + \lambda_{1\perp}} M^2 \right)} \quad (62)$$

FIG. 8. Two non-central-cut diagrams: two rescatterings in the amplitude and zero in the complex conjugate (or vice versa).

The phase factor for Figure 6 reads:

$$S_{(a)} = \frac{1}{2} e^{-i(\hat{x}_L (1-y) - y x_M - \lambda_{D1})} e^{i(\hat{x}_L (1-y) - y x_M - \lambda_{D1})} e^{-i(\hat{x}_L (1-y) - y x_M - \lambda_{D1}) p^+ z_1^-} (e^{-i\hat{x}_L \hat{p}^+ Z_1^-} - 1) \approx \frac{1}{2} (e^{-i\hat{x}_L \hat{p}^+ Z_1^-} - 1),
$$

$$S_{(b)} = \frac{1}{2} e^{-i(\eta_{M0} - \eta_{M0})p^+ y_0^-} e^{-i(\eta_{M1} - \eta_{M0})p^+ z_1^-} e^{-i(\hat{x}_L (1-y) - y x_M - \lambda_{D1}) p^+ z_1^-} (e^{-i\hat{x}_L \hat{p}^+ y_0^-} - e^{-i\hat{x}_L \hat{p}^+ Z_1^-} \approx \frac{1}{2} (e^{-i\hat{x}_L \hat{p}^+ Z_1^-} - 1),
$$

$$S_{(a)} = S_{(a)} + S_{(b)} \approx 2 \cos(\hat{x}_L p^+ Z_1^-) - 1 \quad (63)$$

The matrix element for Figure 6 reads:

$$\tilde{T}_{(a)} = \tilde{T}_{(b)} = C_F \left( C_A \frac{y - \lambda_{1\perp}}{y(y - \lambda_{1\perp})} \right) \frac{l_2^2 + y^2 M^2}{(l_2^2 + y^2 M^2)^2} \quad (64)$$

The phase factor for Figure 6 reads:

$$S_{(a)} = e^{-i(\eta_{M0} - \eta_{M0})p^+ z_1^-} e^{-i(\hat{x}_L (1-y) - y x_M - \lambda_{D1})} e^{-i(\hat{x}_L (1-y) - y x_M - \lambda_{D1})} e^{-i(\hat{x}_L (1-y) - y x_M - \lambda_{D1}) p^+ z_1^-} (e^{-i\hat{x}_L \hat{p}^+ y_0^-} + \frac{1}{2} e^{-i\hat{x}_L \hat{p}^+ Z_1^-} - \frac{1}{2} e^{-i\hat{x}_L \hat{p}^+ Z_1^-}) \approx e^{-i\hat{x}_L \hat{p}^+ Z_1^-} - e^{-i(\hat{x}_L \hat{p}^+ Z_1^-)} \quad (65)$$

$$S_{(b)} = S_{(a)} + S_{(b)} \approx 2 \cos(\hat{x}_L p^+ Z_1^-) - 2 \cos(\hat{x}_L - \chi_{M10}) p^+ Z_1^- \quad (65)$$

The matrix element for Figure 6 reads:

$$\tilde{T}_{(a)} = \tilde{T}_{(b)} = C_F \frac{1 + (1 + \lambda_{1\perp} - y) (1 - y) y - \lambda_{1\perp}}{1 + (1 - y)^2} \frac{1 + (1 - y) \left( 1 - \frac{y}{1 + \lambda_{1\perp}} \right) + y^2 \left( \frac{y}{1 + \lambda_{1\perp}} \right)^2 M^2}{(l_2^2 + y^2 M^2) \left( 1 - \frac{y}{1 + \lambda_{1\perp}} k_{1\perp} \right)^2 + \left( \frac{y}{1 + \lambda_{1\perp}} M^2 \right)} \quad (66)$$
FIG. 9. Two non-central-cut diagrams: two rescatterings in the amplitude and zero in the complex conjugate (or vice versa).

FIG. 10. Two non-central-cut diagrams: two rescatterings in the amplitude and zero in the complex conjugate (or vice versa).

The phase factor for Figure 10 reads:

\[ S_{10} = \frac{1}{2} e^{-i\chi M_{01} p^+ + y_0} e^{-i(\xi L_{11} - \xi M_{01}) p^+ + z_1} e^{-i(\xi L_{11} - \xi M_{01}) p^+ + z_2} (e^{-i\chi M_{11} p^+ + z_1} - e^{-i\chi M_{11} p^+ + z_2}) \approx 0, \]

were \( \chi M_{11} = \eta_{M1} + \lambda_{D1} - \xi M_{11} \). The matrix element for Figure 11 reads:

\[ \hat{T}_{11} = \frac{C_F}{2} \left[ 1 - \left( \frac{1 - y + \lambda_{11}^{11} + 1}{1 + \lambda_{11}^{11}} \right) \frac{(1 - y)}{2} \right] \left( 1 - y - \lambda_{11}^{11} \right) \left( \frac{1}{2} + \frac{y^2}{2} M^2 \right) \]

FIG. 11. Two non-central-cut diagrams: two rescatterings in the amplitude and zero in the complex conjugate (or vice versa).
The phase factor for Figure 11 reads:

\[
S_{11(a)} = -\frac{1}{2} e^{i(\zeta_{M1} - \zeta_{M0})p^+ z_1^-} e^{-i(\zeta_{M1} - \zeta_{M0})p^+ x_1^+} \approx -\frac{1}{2} e^{i\bar{z}_1^- p^+ z_1^-},
\]

\[
S_{11(b)} = -\frac{1}{2} e^{-i\zeta_{M0} p^+ y_0} e^{-i(\zeta_{M1} - \zeta_{M0})p^+ z_1^-} e^{i(\zeta_{M1} - \zeta_{M0})p^+ z_1^-} e^{-i\bar{z}_1^- p^+ z_1^-} \approx -\frac{1}{2} e^{i\bar{z}_1^- p^+ Z_1^-},
\]

\[
S_{11} = S_{11(a)} + S_{11(b)} \approx -\cos(\bar{z}_1^+ p^+ Z_1^-).
\] (69)

The matrix element for Figure 11 reads:

\[
\hat{T}_{11(a)} = \delta \hat{T}_{11(b)} = C_F \frac{p^2 + \frac{y^4 M^2}{1 + (1 - y)^2}}{M^2}.
\] (70)

FIG. 12. Two non-central-cut diagrams: two rescatterings in the amplitude and zero in the complex conjugate (or vice versa).

The phase factor for Figure 12 reads:

\[
S_{12(a)} = -\frac{1}{2} e^{i(\eta_{M1} - \eta_{M0})p^+ z_1^-} e^{-i(\eta_{M1} - \eta_{M0})p^+ x_1^+} (1 - e^{i\bar{z}_1^- p^+ z_1^-}) \approx -\frac{1}{2} (1 - e^{i\bar{z}_1^- p^+ Z_1^-}),
\]

\[
S_{12(b)} = -\frac{1}{2} e^{-i\eta_{M0} p^+ y_0} e^{-i(\eta_{M1} - \eta_{M0})p^+ z_1^-} e^{i(\eta_{M1} - \eta_{M0})p^+ z_1^-} (e^{-i\bar{z}_1^- p^+ y_0} - e^{-i\bar{z}_1^- p^+ z_1^-}) \approx -\frac{1}{2} (1 - e^{-i\bar{z}_1^- p^+ Z_1^-}),
\]

\[
S_{12} = S_{12(a)} + S_{12(b)} \approx \cos(\bar{z}_1^+ p^+ Z_1^-) - 1.
\] (71)

The matrix element for Figure 12 reads:

\[
\hat{T}_{12(a)} = \hat{T}_{12(b)} = C_F \frac{p^2 + \frac{y^4 M^2}{1 + (1 - y)^2}}{M^2}.
\] (72)

FIG. 13. Two non-central-cut diagrams: two rescatterings in the amplitude and zero in the complex conjugate (or vice versa).

The phase factor for Figure 13 reads:

\[
S_{13(a)} = -\frac{1}{2} e^{-i\eta_{M0} p^+ y_0} e^{i\eta_{M1} p^+ z_1^-} e^{i\eta_{M2} p^+ z_2^-} \left( e^{i\bar{z}_1^- p^+ z_1^-} - e^{i\bar{z}_1^- p^+ z_2^-} \right) \approx 0,
\]

\[
S_{13(b)} = -\frac{1}{2} e^{-i\zeta_{M0} p^+ y_0} e^{-i\eta_{M1} p^+ z_1^-} e^{-i\eta_{M2} p^+ z_2^-} \left( e^{-i\bar{z}_1^- p^+ z_1^-} - e^{-i\bar{z}_1^- p^+ z_2^-} \right) \approx 0,
\]

\[
S_{13} = S_{13(a)} + S_{13(b)} \approx 0.
\] (73)
The matrix element for Figure 13 reads:

\[
\hat{t}_{13a} = \hat{t}_{13b} = C_F \left( C_F - \frac{C_A}{2} \right) \frac{1 + (1 - y) \left( 1 - \frac{y}{1 + \lambda_1} \right)}{1 + (1 - y)^2} \left[ \frac{1}{1 + \lambda_1} \cdot \left( \frac{1}{1 + \lambda_1} - \frac{y}{1 + \lambda_1} k_{1\perp} \right) + \frac{y^2 \left( \frac{y}{1 + \lambda_1} \right)^2 M^2}{1 + (1 - y) \left( 1 - \frac{y}{1 + \lambda_1} \right)^2} \right].
\]

where

\[
\eta_{M2} = \frac{t_{1\perp}^2 + M^2}{2p^+ q^- (1 - y)} = \eta_{M0}.
\]

The matrix element for Figure 13 reads:
[46] L. Apolinrio, N. Armesto, J. Milhano, and C. A. Salgado, JHEP 02, 119 (2015), arXiv:1407.0599.
[47] L. Zhang, D.-F. Hou, and G.-Y. Qin, Phys. Rev. C98, 034913 (2018), arXiv:1804.00470.
[48] Y. Zhang, G.-Y. Qin, and X.-N. Wang, to be published.
[49] B. Blagojevic, M. Djordjevic, and M. Djordjevic, (2018), arXiv:1804.07593.
[50] M. D. Sievert and I. Vitev, (2018), arXiv:1807.03799.
[51] M. Gyulassy, I. Vitev, and X. N. Wang, Phys. Rev. Lett. 86, 2537 (2001), arXiv:nucl-th/0012092.
[52] M. Djordjevic and U. Heinz, Phys. Rev. C77, 024905 (2008), arXiv:0705.3439.
[53] M. A. G. Aivazis, F. I. Olness, and W.-K. Tung, Phys. Rev. D50, 3085 (1994), arXiv:hep-ph/9312318.
[54] M. Djordjevic and U. W. Heinz, Phys.Rev.Lett. 101, 022302 (2008), arXiv:0802.1230.