Modeling of Tire Vertical Behavior Using a Test Bench

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ABSTRACT Tire models are of great importance for precise investigations of vertical vehicle dynamics, including vehicular safety and riding comfort. An adequate modeling of the tire is crucial to properly reproduce vehicle vertical behavior in simulations and to evaluate the influence of the tire in the overall performance of the suspension system. This paper introduced vehicle single-point tire models and, thereafter, investigated the influence of the excitation frequency and inflation pressure on the damping and stiffness coefficients of the proposed tires experimentally. In this manner, a test-bench was used to obtain the model parameters of a light vehicle tire and a motorcycle tire. Given the obtained results, it has been observed that both factors have a significant effect on the parameters of the proposed tire models. Moreover, a quarter car suspension model was investigated using the modelled tires to illustrate the influence of the correct characterization of the tire on the vertical suspension performance.

INDEX TERMS Automotive testing, hysteresis, road vehicles, single point models, tire models, vehicle vertical dynamics, viscoelasticity.

I. INTRODUCTION

The proper modeling of the tire is essential to adequately reproduce vehicle vertical behavior in vehicle dynamics simulations. The study of the characteristics of the tire is complex due to its geometry, the great deformation it suffers and the diversity of materials of which it is composed. For this reason, many research groups devote great effort in developing models that make it possible to reproduce tire forces and moments [1]–[4]. Vehicle vertical dynamics is mainly determined by the suspension system and the tire. Good vehicle vertical behavior is desirable because it will not only improve the comfort of the occupants, but it will also contribute to a better performance of the vehicle in traction and braking processes. To this end, the use of accurate vertical tire models is decisive.

Most research papers related to suspension systems and vehicle vertical dynamics resort to single point contact tire models using lumped parameters. In the well-known Kelvin-Voigt model [5], the tire is simply modelled using a spring with constant stiffness or, at best, using a spring and a damper connected in parallel, also with constant stiffness and damping coefficients. However, this model does not reproduce the viscoelastic behavior of the tire and does not accurately represent the physics of the contact between the tire and the road. A model that takes into account the viscoelastic behavior of the tire is the Maxwell model [5], [6]. Other authors studied the influence of hysteresis on both the longitudinal and vertical modeling of tires [7], [8]. There are also tire models based on the ‘flexible ring theory’ that characterize their frequency response [9]–[12]. Finally, some researchers focus on the characterization of the frequency behavior of the tire through experimental tire modal parameters to calculate the static vertical tire stiffness on both drum and flat road surfaces and the distribution of vertical and shear forces [13], [14]. Interestingly, due to the importance of the tire in the improvement of driver comfort in most agricultural wheeled tractors, which commonly lack a suspension system,
several studies model the vertical behavior of the tire and determine which factors influence tire stiffness and damping coefficients [15], [16].

All these models need to obtain the characteristic stiffness and damping coefficient through tire bench tests. There are different methods to measure tire stiffness and damping coefficient. According to the proposed methodology, they can be grouped as follows [9], [16]:

- Static load-deflection. Static stiffness of a non-rotating tire on a drum or flat road surface by increasing the static load and measuring the static loaded radius.
- Non-rolling vertical free vibration. Dynamic stiffness of a tire by dropping the tire from a known height and obtaining the vertical position versus time data.
- Non-rolling equilibrium load-deflection. Dynamic stiffness of a non-rotating tire on a drum or flat road surface by applying a vibration or variable displacement and measuring the dynamic loaded radius.
- Rolling vertical free vibration. The method is a vertical free vibration but applied to a rolling tire.
- Rolling equilibrium load-deflection. Dynamic stiffness of a rotating tire on a drum or flat road surface by applying a variable displacement.

In some cases, test benches used to determine tire vertical stiffness and damping do not only take into account the deformation of the tire, but are also influenced by the deformation of the rim and by the effects of the unsprung mass. This situation is commonly found in test benches in which the tire is rotating. To solve this problem, a test bench in which only the tire is taken into account in the measurement of stiffness and damping coefficients has been developed in this work. Besides, this test bench makes it possible to study other effects such as tire viscoelasticity and hysteresis. Consequently, this bench allows obtaining tire parameters and more accurate single point contact models as well as studying the factors that influence on the value of these parameters.

In this paper, the proposed test bench has been used for the modeling of two tires of size 205/65 R15 and 90/58 R17. The first is a tire commonly used in light four-wheeled vehicles while the second is designed to be used in motorcycles. Data obtained from tests are used to fit single point contact tire model parameters. Thus, tire stiffness and damping, viscoelastic behavior and hysteresis are determined. Furthermore, the influence of the excitation frequency and inflation pressure is evaluated. It has been found that there is a considerable difference between the parameters obtained for both the motorcycle tire and for the vehicle. These results are of great interest, since there is not much information about the modeling of motorcycle tires in literature. This way, this work will contribute to improving motorcycle dynamics models and the safety of this type of vehicles. The main contributions of this paper are the following:

- to evaluate the influence of excitation frequency and inflation pressure on the damping and stiffness coefficients of single point tire models.

This paper is organized as follows: the test bench developed to obtain the vertical behavior of tires is described in section II. In section III, single point contact models used to reproduce tire behavior are explained. Section IV is devoted to describing the experimental tests carried out and the methodology proposed to obtain tire model parameters. Tire model parameters obtained from tests conducted with the aforementioned tire are also included. The influence of the models obtained in the previous section on vehicle vertical dynamics is evaluated in section V. Finally, conclusions are drawn in section VI.

II. TEST BENCH

To carry out this work, a test bench designed by this research team was built. This test bench makes it possible to study and characterize the vertical behavior of tires and vehicle suspension systems and components (see Figure 1). Thus, this versatile test bench can be used to model the behavior of two- and four-wheeled vehicle suspensions. The shock absorber and the spring response can be tested independently or, if necessary, the whole suspension system can be tested simultaneously. Furthermore, the tire response can also be studied independently by fixing the tire to the structure of the test bench and applying a displacement of controlled frequency and amplitude. To remove the influence of rim deformation, the deformation of the rim where the tire is mounted is measured with a sensor and the different displacements are normalized to obtain the tire deformation. This way, only the tire response is measured and modeled.

The bench is composed of a frame, which must withstand the forces and vibrations transmitted by the tested elements. These efforts are transmitted to the bench through a linear guide, where a fixed and a mobile plate are connected.
TABLE 1. Test bench general technical characteristics.

| Parameter                                      | Value     |
|------------------------------------------------|-----------|
| Maximum measurable force                      | ±10 kN    |
| (Traction - compression)                      |           |
| Testing speed                                 | from 0.05 mm/s to 2.1 m/s with continuous variation |
| Maximum cylinder rod stroke                   | 350 mm    |
| Test frequency range                          | 0 to 25 Hz with continuous variation             |
| Control loop frequency                        | 1 Khz     |
| Maximum data recording frequency              | 10 Khz    |
| DAQ resolution                                | Up to 24 bits                                    |

TABLE 2. Measurement system.

| Number, sensor type and model                  | Accuracy | Measurement principle | Range         |
|------------------------------------------------|----------|-----------------------|---------------|
| 1 x displacement sensor Schreiber SM42         | ±0.05 mm | Inductive - LVDT      | 0-360 mm      |
| 2 x displacement sensor OMRON ZX-LD40          | ±2 μm    | Laser                 | 30-60 mm      |
| 2 x displacement sensor OMRON ZX-LD100         | ±16 μm   | Laser                 | 60-140 mm     |
| 1 x Load cell HBM U93 10kN                    | ±1 N     | Strain gauge          | ±10 kN        |
| 1 x Load cell Honeywell 125-100kg              | ±0.1 N   | Strain gauge          | ±1 kN         |
| 2 x accelerometer KYOWA AS-2G                  | ±0.01 m/s² | Piezo - resistive | ±2 g          |
| 2 x accelerometer KYOWA AS-5G                  | ±0.02 m/s² | Piezo - resistive | ±5 g          |
| 2 temperature sensors OPTRIS CS                 | ±0.1°C   | Infrared              | -20°. 150°C   |

Depending on the type of test, the forces on the unsprung mass, the damping system or the tire will be transmitted and measured. The movable plate is the component that transmits the movement or vibration input to the system. In this work, the tire is mounted on a rim that is fixed to the bedplate.

The excitation to the parts to be tested is generated by a hydraulic cylinder controlled by a servo-valve. This way, a controlled displacement or force is transmitted to the plate where the tire is placed. The control system makes it possible to simulate different types of inputs, such as sinusoidal, ramp, bump and random. The characteristics of the test bench are summarized in Table 1. The measurement system installed in the test bench is described in Table 2.

The control of the test bench is carried out by an algorithm programmed in a CompactRio-9066@real time system. Input and output modules are connected to an FPGA. Real time loop communicates with the FPGA to perform data transmission and to establish the desired control action (see Figure 2).

III. TIRE VERTICAL MODELING

There are numerous mathematical models described in literature [5], [17], [18] to model the hysteresis and viscoelastic behavior of materials and components. Among them, some of the most widely used models in the case of tires are the so-called ‘single point contact models’. In these models, the entire part of the tire in contact with the road is reduced to a single point contact, so that neither the length nor the contact area is taken into account. These models are composed of a combination of ideal elastic elements (springs) and energy dissipators (dampers). Their main field of application is related to modeling the behavior of suspension systems and the vertical dynamics of vehicles. In this work, the vertical behavior of the tire is modeled with three different single point contact model approaches (see Figure 3).

The first one is a Kelvin-Voigt model in which a damper element and two spring elements [5], [17] are included (Figure 3a). All components are arranged in parallel. The first spring ($K_s$) represents the static stiffness of the tire. The second spring ($K_d$) is used to model the change in stiffness with the frequency of the excitation [14]. The damper ($C_d$) allows evaluating the damping of the tire and its variation with the frequency of the excitation ($\omega$). The basic equation for this model is the following:

$$F = C_d \cdot (\dot{x}_t - \dot{x}_r) + K_d (\omega) \cdot (x_t - x_r - \delta) + K_s \cdot (x_t - x_r)$$

where $F$ is the total force provided by each model, $(x_t - x_r)$ is the tire displacement ($x_r$ being equal to zero in our test) and $\delta$ is the initial displacement.

The second model is composed of a Maxwell model in parallel with a spring element [5], [17]. This model is also called Standard Linear Model (Figure 3b). This model is governed by the following equations:

$$\dot{F} = K_s \cdot (x_t - x_r) + f_v$$

$$\dot{\hat{F}} + \frac{K_d}{C_d} \cdot f_v = (\dot{x}_t - \dot{x}_r)$$

where $f_v$ represents the viscoelastic force.

Finally, the third model is the simple Bouc model [18]. This model is similar to the Kelvin-Voigt model, but incorporates a fourth element that reproduces the hysteresis of the material (Figure 3c). The equations that describe this model are:

$$F = C_d \cdot (\dot{x}_t - \dot{x}_r) + K_d (\omega) \cdot (x_t - x_r - \delta) + K_s \cdot (x_t - x_r) + \alpha \cdot z$$

$$\dot{z} = A \cdot (\dot{x}_t - \dot{x}_r) + \mu \cdot (\dot{x}_t - \dot{x}_r) \cdot |z|^\gamma$$

$$-\gamma \cdot |x_t - x_r| \cdot |z|^{\gamma-1}$$

where $z$ is an auxiliary term that models the hysteresis by means of parameters $A$, $\mu$ and $\gamma$ and $\alpha$ is a scaling factor.

Once the equations that determine the force in the tire for the three proposed models have been obtained, data obtained in the bench can be used to obtain the fundamental parameters of each model, such as the static stiffness ($K_s$), dynamic stiffness ($K_d$) and dynamic damping ($C_d$) coefficients.

The main reason why these parameters and models were chosen to represent the vertical behavior of the tire was derived from the observation of Figure 4a. In the first place, this figure shows the static stiffness of the tire that was measured by conducting tests at very low speed and measuring the vertical force. The tests included the data measured during the loading and unloading processes. Next, the vertical forces that were measured in dynamic tests at different excitation
frequencies (1, 10 and 20 Hz) are also included in the figures. Two series of tests were conducted with 3500 N and 1500 N of preload in the tire (Figures 4b and 4c respectively). A local increase of the stiffness in both series of tests can be observed, i.e., the static stiffness curve has a lower slope compared to the dynamic stiffness. Furthermore, the higher the excitation frequency, the higher the increase in the slope of the dynamic stiffness. For this reason, the static and dynamic stiffness has been modelled separately in each of the previous models. Dynamic damping has also been modelled to obtain the energy dissipation behavior of the tire as a function of the excitation frequency.

\[ f(t) = K_s \cdot x(t) + C_s \cdot \dot{x}(t) + K_d \cdot \ddot{x}(t) \]

IV. TESTS

The goal of these tests was to evaluate the influence of the frequency of the excitation and the inflation pressure on the parameters of tire single point contact models. A light vehicle and a motorcycle tires were used in this series of tests. First, the static stiffness was measured in a quasi-static test. Next, sinusoidal movements of frequency between 1 and 20 Hz were reproduced. Amplitudes of 3 and 2 mm were programmed for the light vehicle and the motorcycle tire respectively. As described before, control was performed with a real time system. The control system loop frequency and data acquisition sampling rate were fixed at 1000 Hz. Actual bedplate displacement and rim deformation were measured using a high accuracy linear variable differential transducer and a laser displacement sensor respectively. Forces were measured using a load cell located in the fixed plate. An optimization algorithm provided the parameters that best fit the tire model output to the experimental data. Finally, results were analyzed and interpreted.

As mentioned before, static rigidity \( K_s \) was obtained through an experimental test in which the tire was deformed progressively from zero to maximum deflection (Figure 4). The vertical force was measured during this process. These data allowed obtaining the change of rigidity vs displacement of the tires.

Next, tests were carried out in which a sinusoidal movement was generated in the bedplate. Tests were conducted at different frequencies, amplitudes and preloads. This way, the change of parameters with the frequency was determined.

An algorithm based on Particle Swarm optimization was used to perform parameter identification [19]. Data measured in the test bench were fitted to each model. The algorithm yielded the parameters that best modelled the experimental data. To do so, a goal function was defined. In this case, the average of the quadratic error between the measured force and the force provided with the model was selected as the goal function to be minimized. The following parameters were used in the optimization algorithm: number of particles per group = 30, number of groups = 2, number of iterations = 200, acceleration factors = 0.49445 and initial and final weights equal to 0.9 and 0.4 respectively. This way, two
FIGURE 5. Test data fitting using the Kelvin-Voigt model. (a) Frequency 10 Hz, preload 1500 N  (b) Frequency 20 Hz, preload 1500 N  (c) Frequency 10 Hz, preload 2500 N  (d) Frequency 20 Hz, preload 2500 N.

(K_d, C_d) or six (K_d, C_d, \alpha, A, \mu, \gamma) additional parameters were obtained to model the vertical behavior of the tire.

A. TESTS WITH A TIRE OF SIZE 205/65 R15

First, tests were conducted using a 205/65 R15 tire. This tire size is commonly used in light passenger cars. An amplitude of 3 mm was set in these tests. Test frequencies were 1, 5, 10, 15 and 20 Hz. Finally, three preload conditions were simulated, 1500, 2500 and 2500 N. In these tests, inflation pressure (P_n) was set at 1.8 bar.

1) KELVIN-VOIGT MODEL

The following plots show the results obtained at two frequencies, 10 and 20 Hz, with two preloads, 1500 and 2500 N (Figure 5) using the Kelvin-Voigt model. Model parameters were obtained by means of the aforementioned optimization algorithm. Two plots are generated with each test. The first one includes the force measured (Force_m), the force provided by the fitted model (Force_f) and the displacement (Disp) plotted vs time. In addition, the second plot includes the measured and the fitted forces plotted vs the displacement. It can be seen that the force provided by the model fits the measured force measured properly. Furthermore, the model reproduces the measured data adequately for all frequencies and preload conditions in the tire. Similar performance was observed for all tests. Finally, results using the Kelvin-Voigt model equation (eq. 1) for all tests are shown in Table 3.

Table 3 includes the test conditions, i.e. preload and excitation frequency, static rigidity and the model optimized parameters. Minimum error obtained after the optimization process is also included. For the sake of clarity, Figure 6 shows the evolution of the dynamic stiffness and damping coefficients as a function of the excitation frequency.

In this case, dynamic stiffness increased with the frequency of excitation whatever preload condition was imposed. On the other hand, dynamic damping decreased with frequency, being relatively high at low frequencies. However, low damping coefficients were observed for frequencies above 5 Hz.

2) MAXWELL MODEL

Similarly, the following plots show the results obtained at two frequencies, 10 and 20 Hz, with two preloads, 1500 and 2500 N using the Maxwell model (Figure 7). Plots show the
FIGURE 8. Stiffness and damping coefficients of the Maxwell model vs excitation frequency. Tire 205/65 R15 (1.8 bar).

TABLE 4. Summary of results using the Maxwell model. Tire size: 205/65 R15. Inflation pressure = 1.8 bar.

| Preload [N] | Frequency [Hz] | $K_d$ [N/m] | $C_d$ [Ns/m] | Error [N] |
|-------------|----------------|------------|-------------|-----------|
| 1500        | 1              | 151227     | 36302       | 36.20     | 106.92    |
| 1500        | 5              | 151227     | 44970       | 44.39     | 258.46    |
| 1500        | 10             | 151227     | 51654       | 50.52     | 182.16    |
| 1500        | 15             | 151227     | 55670       | 53.80     | 324.57    |
| 1500        | 20             | 151227     | 67131       | 65.02     | 181.65    |
| 2500        | 1              | 169325     | 43091       | 42.95     | 308.15    |
| 2500        | 5              | 169325     | 47258       | 46.82     | 200.58    |
| 2500        | 10             | 169325     | 53384       | 52.04     | 286.55    |
| 2500        | 15             | 169325     | 60311       | 58.29     | 289.03    |
| 2500        | 20             | 169325     | 69115       | 66.42     | 639.09    |
| 3500        | 1              | 182145     | 46217       | 46.07     | 244.67    |
| 3500        | 5              | 182145     | 54521       | 53.77     | 281.81    |
| 3500        | 10             | 182145     | 60549       | 59.02     | 213.08    |
| 3500        | 15             | 182145     | 68138       | 65.78     | 451.97    |
| 3500        | 20             | 182145     | 79222       | 77.21     | 404.01    |

FIGURE 9. Test data fitting using the Bouc model. (a) Frequency 10 Hz (b) Frequency 20 Hz. Preload 2500 N.

FIGURE 10. Stiffness and damping coefficients of the Bouc model vs excitation frequency. Tire 205/65 R15 (1.8 bar).

FIGURE 11. Comparison of three single point models. Tire 205/65 R15. (a) Kelvin-Voigt (b) Maxwell (c) Bouc.

data reproduced by the model according to equations (2) and (3) with the obtained optimized parameters.

As for the previous model, the fitting of test data with the model is suitable both for the case of frequency variation and when the preload is changed. Optimized parameters using the Maxwell model for all tests are shown in Table 4.

In the case of the Maxwell’s model, it can be seen in Figure 8 and Table 4 that the dynamic stiffness values are of the same order of magnitude as those obtained in the previous model. On the contrary, the dynamic damping values are much smaller. It can also be observed that both stiffnesses and dynamic damping coefficients increase with the vibration frequency.

3) BOUC MODEL

This more complex model, which is commonly used for reproducing hysteresis, was also used. According to equations (4) and (5), the Bouc model requires the estimation of four additional parameters ($\alpha, A, \gamma$ and $\mu$). Parameter $n$ was set at 1. As a consequence, the computational time to obtain the estimation of the model parameters was considerably longer.

The following plots show the results obtained at two frequencies, 10 and 20 Hz with 2500 N of preload using the Bouc Model (Figure 9).

Optimized parameters using the Bouc model for all tests are shown in Table 5. Interestingly enough, errors are lower compared to the other models. However, the improvement is small. Due to the higher computational cost that this model requires in the optimization process and during its use in simulations, this model is only recommended when the accuracy required in the simulations is very high.

Figure 10 shows the value of main parameters $K_d$ and $C_d$ provided by the model according to equations (4) and (5). Dynamic stiffness gradually increases with respect to frequency, except for the frequency of 15 Hz. Damping coefficients have similar behavior to the one obtained for the Kelvin-Voigt model.

4) MODELS COMPARATIVE

Once the results for the three single point models have been shown, a comparison of the three models used is shown.
TABLE 5. Summary of results using the Bouc model. Tire size: 205/65 R15. Inflation pressure = 1.8 bar.

| Preload [N] | Frequency [Hz] | $K_s$ [N/m] | $K_d$ [N/m] | $C_d$ [Ns/m] | $\alpha$ | $\Delta$ | $\gamma$ | $\mu$ | Error $[N^2]$ |
|-------------|----------------|-------------|-------------|-------------|---------|---------|---------|-------|---------------|
| 2500        | 1              | 169325      | 31750       | 2545        | 9       | -11     | 60      | 1000  | 15.15        |
| 2500        | 5              | 169325      | 40438       | 398         | -33     | 113     | 81      | 829   | 12.23        |
| 2500        | 10             | 169325      | 45562       | 324         | 51      | -26     | 200     | 11    | 52.18        |
| 2500        | 15             | 169325      | 38757       | 218         | 237     | 103     | 38      | 671   | 108.22       |
| 2500        | 20             | 169325      | 55728       | 158         | 176     | 78      | 143     | 697   | 262.27       |

FIGURE 12. Comparison at different inflation pressures, 1.8 and 2.1 bar vs excitation frequency. (a) Kelvin-Voigt (b) Maxwell.

TABLE 6. Summary of results using the Kelvin-Voigt model. Tire size: 205/65 R15. Inflation pressures 1.8 and 2.1 bar.

| Preload [N] | Frequency [Hz] | $K_s$ [N/m] | $K_d$ [N/m] | $C_d$ [Ns/m] | $\alpha$ | $\Delta$ | $\gamma$ | $\mu$ | Error $[N^2]$ |
|-------------|----------------|-------------|-------------|-------------|---------|---------|---------|-------|---------------|
| 3500        | 1              | 182145      | 36311       | 2826        | 210282  | 52000   | 2797    | 657   | 210282 41475   |
| 3500        | 5              | 182145      | 45772       | 657         | 210282  48035  | 394     | 210282  55319  | 252     | 210282 60835   | 189     |

in Figure 11. This figure has been obtained from tests conducted with a preload of 2500 N, an excitation frequency of 20 Hz, an excitation amplitude of 3 mm and an inflation pressure of 1.8 bar. It can be seen that the three models provided similar results, obtaining error values of the same order in all cases. For comparison purposes, plots include the static stiffness ($K_s$) measured in the static test. It can be observed that higher dynamic stiffness was obtained compared to the static stiffness. Similar performance was observed in all tests.

5) TIRE INFLATION PRESSURE INFLUENCE
It is well known that static tire stiffness increases with tire inflation pressure. Consequently, for comparative purposes, a second series of tests was carried out with a tire inflation pressure of 2.1 bar and a preload of 3500 N to evaluate its influence on the model parameters. Results are summarized in Table 6. Next, Figure 12 includes the results of this comparison for the Kelvin-Voigt and Maxwell models.

As it can be seen in Figure 12(a) and Table 5, dynamic stiffness decreases when the inflation pressure increases. However, dynamic damping coefficients remain almost independent of the inflation pressure for the Kelvin-Voigt model. In contrast, in the Maxwell model, both the dynamic stiffness and damping coefficients are lower when the pressure is increased (see Figure 12b). As expected, Table 6 also shows that static stiffness is higher when the inflation pressure is increased.

B. TESTS WITH A TIRE OF SIZE 90/58 R17
A series of tests was conducted using a 90/58 R17 competition tire. This tire size is commonly used in two-wheeled vehicle races. An amplitude of 2 mm was set in these tests. Test frequencies were 1, 5, 10, 15 and 20Hz. Three preload conditions of 500, 1000 and 1500 N were simulated. Finally, tire inflation pressure was set at 1.9 bar.

1) KELVIN-VOIGT MODEL
The following plots show the results obtained using the Kelvin-Voigt model at two frequencies, 1 and 15 Hz, and preloads of 500 and 1500 N (Figure 13). Plots show the data reproduced by the model according to equations (2) and (3) with the obtained optimized parameters. As in the case of the light passenger car tire, it can be seen that the model reproduced the measured data adequately for all frequencies and preload conditions in all the cases.

Estimated parameters using the Kelvin-Voigt model for all tests carried out with the 90/58 R17 tire are shown in Table 7.

It can be seen in Figure 14 that stiffness and the estimated dynamic damping coefficient have the same behavior as found with the light passenger car tire. That is, stiffness coefficient increases with frequency and damping coefficient decreases towards a low constant value.
Table 7. Summary of results using the Kelvin-Voigt model. Tire size: 90/58 R17. Inflation pressure = 1.9 bar.

| Preload [N] | Frequency [Hz] | $K_a$ [N/m] | $C_a$ [N/s/m] | Error [N$^2$] |
|-------------|----------------|-------------|---------------|---------------|
| 500         | 1              | 101570      | 21148         | 2188          | 6.18          |
| 500         | 5              | 101570      | 32100         | 663           | 8.30          |
| 500         | 10             | 101570      | 36700         | 366           | 51.31         |
| 500         | 15             | 101570      | 39927         | 284           | 155.00        |
| 500         | 20             | 101570      | 44430         | 153           | 160.12        |
| 1000        | 1              | 120470      | 32749         | 3513          | 23.95         |
| 1000        | 5              | 120470      | 44296         | 869           | 43.83         |
| 1000        | 10             | 120470      | 50937         | 361           | 95.49         |
| 1000        | 15             | 120470      | 54038         | 293           | 220.46        |
| 1000        | 20             | 120470      | 57646         | 265           | 213.81        |
| 1500        | 1              | 142950      | 35086         | 3935          | 19.44         |
| 1500        | 5              | 142950      | 49827         | 941           | 27.27         |
| 1500        | 10             | 142950      | 55839         | 505           | 75.05         |
| 1500        | 15             | 142950      | 60633         | 360           | 200.92        |
| 1500        | 20             | 142950      | 65188         | 306           | 228.33        |

FIGURE 14. Stiffness and damping coefficients of the Kelvin-Voigt model vs excitation frequency. Tire 90/58 R17 (1.9 bar).

Table 8. Summary of results using the Maxwell model. Tire size: 90/58 R17. Inflation pressure = 1.9 bar.

| Preload [N] | Frequency [Hz] | $K_a$ [N/m] | $C_a$ [N/s/m] | Error [N$^2$] |
|-------------|----------------|-------------|---------------|---------------|
| 500         | 1              | 101570      | 28369         | 28.26         | 93.12         |
| 500         | 5              | 101570      | 44431         | 43.55         | 165.45        |
| 500         | 10             | 101570      | 50649         | 48.26         | 93.22         |
| 500         | 15             | 101570      | 56850         | 53.70         | 222.14        |
| 500         | 20             | 101570      | 62052         | 49.64         | 260.16        |
| 1000        | 1              | 120470      | 49016         | 48.81         | 149.36        |
| 1000        | 5              | 120470      | 60918         | 59.79         | 137.92        |
| 1000        | 10             | 120470      | 59601         | 58.35         | 352.86        |
| 1000        | 15             | 120470      | 67769         | 65.90         | 256.71        |
| 1000        | 20             | 120470      | 77578         | 72.88         | 256.56        |
| 1500        | 1              | 142950      | 52458         | 52.22         | 305.25        |
| 1500        | 5              | 142950      | 65350         | 64.17         | 470.01        |
| 1500        | 10             | 142950      | 74117         | 71.77         | 193.30        |
| 1500        | 15             | 142950      | 80090         | 76.34         | 147.11        |
| 1500        | 20             | 142950      | 88904         | 83.32         | 310.10        |

FIGURE 15. Test data fitting using the Maxwell model. (a) Frequency 1 Hz, preload 500 N (b) Frequency 15 Hz, preload 500 N (c) Frequency 1 Hz, preload 1500 N (d) Frequency 15 Hz, preload 1500 N.

2) MAXWELL MODEL

Figure 15 shows the results obtained using the Maxwell model at two frequencies, 1 and 15 Hz and preloads of 500 and 1500 N. The plots show the data reproduced by the model according to equations (2) and (3) with the obtained optimized parameters.

V. INFLUENCE OF THE TIRE MODEL IN THE VEHICLE VERTICAL DYNAMICS

The influence of the models obtained in the previous section on the behavior of the vehicle suspension system is evaluated next. Most research papers about vehicle vertical dynamics and control algorithms for active and semi-active suspension systems do not take into account the influence of tire damping [20]–[23], or, at best, it is assumed to have a very low constant value [24]–[26]. As shown before, tire damping and tire stiffness depend on the frequency of the excitation. As a consequence, inappropriate modeling of the tire in suspension system models can lead to deterioration in the expected suspension performance [14], [27]. Furthermore, tire stiffness is also modelled with a constant parameter. However, it has also been shown that tire stiffness changes with tire deformation and tire pressure.

The vehicle vertical model shown in Figure 17 was used to evaluate the influence of tire modeling on the performance of the suspension system. $K_a$ and $C_a$ represent suspension stiffness and damping coefficients respectively.
FIGURE 17. Vertical model of a quarter vehicle.

For comparison purposes, the transfer functions of the movement of the sprung mass \( H_{SM} \), unsprung mass \( H_{UM} \), sprung mass vertical acceleration \( H_A \), rattle space \( H_{RS} \) and tire deflection \( H_{TD} \) are used. These transfer functions are defined next:

\[
H_{SM} = \frac{x_s}{x_r} = \frac{B_1 \cdot B_4}{M_s \cdot s^2 \cdot B_1 + B_2 \cdot B_3}
\]

\[
H_{UM} = \frac{x_t}{x_r} = \frac{B_1 \cdot B_4}{M_s \cdot s^2 \cdot B_1 + B_2 \cdot B_3}
\]

\[
H_A = \frac{\ddot{x_s} - \dot{x}_r}{\dot{x}_r} = \frac{B_1 \cdot B_4 - B_2 \cdot B_3}{M_s \cdot s^2 \cdot B_1 + B_2 \cdot B_3}
\]

\[
H_{RS} = \frac{x_s - x_t}{x_r} = \frac{B_2 \cdot B_4 - M_s \cdot s^2 \cdot B_1 + B_2 \cdot B_3}{M_s \cdot s^2 \cdot B_1 + B_2 \cdot B_3}
\]

\[
H_{TD} = \frac{x_t - \dot{x}_r}{\dot{x}_r} = \frac{B_2 \cdot B_4 - M_s \cdot s^2 \cdot B_1 + B_2 \cdot B_3}{M_s \cdot s^2 \cdot B_1 + B_2 \cdot B_3}
\]

where \( B_1 = C_a \cdot s + K_a \), \( B_2 + M_s \cdot s^2 + C_a \cdot s + K_u \), \( B_3 = M_u \cdot s^2 + C_d \cdot s + K_s + K_d \) and \( B_4 = C_d \cdot s + K_s + K_d \).

Three simulations were performed using the Kelvin-Voigt tire model. The following default parameters were used: \( M_s = 250 \text{ kg}, M_u = 35 \text{ kg}, K_a = 16000 \text{ N/m}, C_a = 1600 \text{ N/s/m} \) and \( C_d = 170 \text{ N/s/m} \). In the first simulation, tire stiffness was set at \( K_s = 160875 \text{ N/m} \). This value was obtained from the static test of the car tire with 2500 N of vertical load and a tire pressure of 1.9 bar (see Figure 18, dashed line). In the second simulation, the same parameters were used but tire stiffness was now modelled fitting the data of the previous test to a second order polynomial curve (Figure 18, dash-dotted line). Again, no change in tire stiffness and damping coefficients with the frequency of the excitation were modelled. Finally, in the third simulation, tire damping and stiffness were modelled using the tire model parameters obtained in the test with the car tire with a 2500 N vertical load. In this simulation, both tire damping and stiffness changed with the frequency of the excitation. The following figures show the result of these simulations.

As it can be seen in Figure 19a, unsprung mass movement reduces slightly when tire stiffness is not constant (polynomial stiffness) due to the lower tire stiffness in the low range of tire deflections. On the other hand, variable tire stiffness and damping with frequency has a big influence on unsprung mass movement at high frequencies. In addition, the unsprung mass resonant frequency increases due to the increase of tire stiffness with frequency. As a consequence, higher unsprung mass resonant frequencies are observed at frequencies above the resonant frequency. Sprung mass movement remains almost unaffected with the changes in the parameters (see Figure 19b).
Vertical acceleration and rattle space transfer functions remain almost unmodified no matter if constant or variable static stiffness is used (Figure 20a and 20b). The polynomial stiffness assumption yields lower transfer ratios for frequencies close to and above the unsprung mass resonant frequency while variable damping and stiffness leads to the opposite effect. However, the tire deflection transfer function is remarkably affected by the changes in the parameters. In the first case, it shows higher magnitude levels for frequencies up to the unsprung mass resonant frequency (Figure 20c). On the other hand, the variable damping and stiffness assumption yields lower tire deflections, which is related to lower changes in the tire vertical force and, as a consequence, with better vehicle maneuverability and safety. This way, in this example, the vertical force remains more constant with frequency than expected.

As a summary, it has been demonstrated that the proper modeling of tire stiffness and damping is required to evaluate the performance of vehicle suspension systems and to design vehicle control algorithms. Otherwise, results obtained from simulations can differ significantly from reality.

VI. CONCLUSION

In this paper, the hysteretic and viscoelastic behavior of tires has been evaluated. Three well-known models have been used to reproduce this behavior. To perform this task, model parameters have been obtained from data gathered using a tire test bench using an optimization fitting algorithm. It has been observed that the Kelvin-Voigt and Maxwell lumped models can properly model tire response. The Bouc model yields a slightly better fitting but it requires a higher optimization time and computational cost. The change of model parameters with the frequency of the excitation has been evaluated. It has been demonstrated that tire stiffness and damping coefficients are highly dependent on the frequency of the excitation. Tire stiffness increases with the frequency of the excitation while tire damping is high for low frequencies but it reduces sharply with the increase of frequency. The influence of tire pressure on model parameters has also been studied. It has been shown that as the pressure increases, so does tire stiffness. In this case, little change in tire damping is observed.

Finally, a quarter-car model has been used to evaluate the influence of the previous results on vehicle vertical dynamics. It has been shown that the change of damping and stiffness coefficients with frequency modifies vehicle vertical dynamics. This way, car vertical models should take into account this effect to properly reproduce vehicle movements.

Further works will include a wider range of tires and will focus on new factors, such as excitation amplitude, tire temperature and tire age, related to the determination of tire parameters.

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