Heavy quark expansion of $\Lambda_b \to \Lambda^*$ form factors beyond leading order

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Abstract

I review the parametrisation of the full set of $\Lambda_b \to \Lambda^*(1520)$ form factors in the framework of Heavy Quark Expansion, including next-to-leading order $O(\alpha_s)$ and for the first time next-to-leading power $O(1/m_b)$ corrections. The unknown hadronic parameters are obtained performing a fit to a recent lattice QCD calculations. I investigate the compatibility of the HQE and the current lattice data, finding a tension between these two approaches in the case of tensor and pseudo-tensor form factors, whose origin could come from an underestimation of the current lattice QCD uncertainties and higher order terms in the HQE.

1 Introduction

The flavour changing neutral current (FCNC) mediated $b \to s\ell^+\ell^-$ transition plays an important role in the search of physics beyond the Standard Model (SM). Its potential has been extensively studied through the $B \to K^{(*)}\ell^+\ell^-$ decays. Interestingly, the LHCb experiment found some discrepancies with respect to the SM predictions in a few observables: $R_K$ and $R_{K^*}$, which test universality between the muon and electron final states and the angular coefficient $P_5'$ in the $B \to K^{*}\mu^+\mu^-$ angular distribution [1–6]. These hints, together with all available $b \to s\ell^+\ell^-$ data, form a coherent pattern of discrepancies which can be addressed introducing New Physics (NP) effects. Low-energy fits point toward a breaking of lepton flavour universality with a combined significance for the NP hypothesis higher than $5\sigma$ [7–10]. Whether these data show without doubts first signs of NP is not clear yet. Only further measurements with higher statistics or measurements of new processes able to corroborate these data will give a final answer.

A possibility to better understand these data is studying further $b \to s\ell^+\ell^-$-mediated decays, among which baryon decays are promising candidates. The decay channel involving ground state...
baryons $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ has already received attention both from the experimental and theoretical point of view. The LHCb experiment measured its angular distribution [11, 12], finding good agreement with the SM predictions [13–15] for the angular observables. Even though this might be discouraging for NP searches, it has been shown in Refs. [14,15] that this is still consistent with the NP hypothesis. This is due to the fact that baryon decays are described by a very different angular distributions than the ones in the meson cases, and NP affects them differently.

Another possibility is studying excited $\Lambda^*$ states. The LHCb experiment used the decay chain $\Lambda_b \rightarrow \Lambda^*(\rightarrow pK^-)\ell^+ \ell^-$ to measure $R(pK)$, the universality ratio between muons and electrons, finding results consistent with both the SM expectation and the measured values of $R_{K^*(\gamma)}$ [16]. In this analysis, the various $\Lambda^*$ resonances below a certain mass threshold are not distinguished. However, in Ref. [17] it is shown that the $\Lambda^*(1520)$ is expected to be the most frequent among the $\Lambda^*$ resonances, with a quite narrow mass distribution. Therefore, it is motivated to study in details the $\Lambda_b \rightarrow \Lambda^*(1520)\ell^+ \ell^-$ decay from both experimental and theoretical point of view and the latter is the main focus of this paper.

The determination of the form factors of the $\Lambda_b \rightarrow \Lambda^*(1520)$ decays had already been object of study in the literature. The Heavy Quark Expansion (HQE) of the form factors up to next-to-leading (NLO) order in $\alpha_s$ and leading power in $1/m_b$ has been employed [18,19], using Quark Models to constrain the unknown hadronic parameters [20]. Recently, a lattice QCD determination of the full base of form factors has become available [21], but constrained to the low-recoil region. In this work I investigate the compatibility between the HQE form factors and the recent lattice QCD determination. At this scope, I perform a HQE of form factors including NLO $\alpha_s$ corrections and next-to-leading power (NLP) $1/m_b$ corrections, being the latter not known so far in the literature. The results are then matched onto the lattice QCD calculation. In the HQE the numbers of independent, hadronic parameters is reduced compared to the lattice QCD case, introducing strict correlations among the form factors. I perform a fit to the lattice QCD results to obtain the central values, uncertainties and correlations among the HQE parameters. The comparison between the lattice QCD results and the HQE predictions show a tension between the two methods in the case of tensor and pseudo-tensor form factors, whose origin is not yet completely determined.

This paper is organised as follows: in Sect. 2 I present the HQE of the form factors; in Sect. 3 I discuss the fit to lattice data; in Sect. 4 I conclude. Appendix A and Appendix B report details on the calculation of the form factors and Appendix C contains the covariance matrix for the fitted values of the HQE parameters.

## 2 Setup

In the following I investigate the form factors mediating the transition $\Lambda_b(p, s_b) \rightarrow \Lambda^*(1520)(k, \eta(\lambda_\Lambda), s_\Lambda)$, where $p$ and $k$ are the momenta of the initial and final states, respectively, $s_b$ and $s_\Lambda$ are the rest-frame helicities of the two baryons and $\eta(\lambda_\Lambda)$ is the polarisation vector of the $\Lambda^*$ for each polarisation state $\lambda_\Lambda$. Given that in this work I refer only to the $\Lambda^*(1520)$, I simply denote it as $\Lambda^*$ in the following.
I define the helicity form factors for $\Lambda_b(p, s_b) \rightarrow \Lambda^*(k, \eta(\lambda_A), s_A)$ as

$$\langle \Lambda^*(k, \eta(\lambda_A), s_A)| \bar{s} \gamma^\mu b |\Lambda_b(p, s_b) \rangle = +\bar{u}_\alpha(k, \eta(\lambda_A), s_A) \left[ \sum_i F_i(q^2) \Gamma^\mu_{V,i} \right] u(p, s_b),$$

$$\langle \Lambda^*(k, \eta(\lambda_A), s_A)| \bar{s} \gamma^\mu \gamma_5 b |\Lambda_b(p, s_b) \rangle = -\bar{u}_\alpha(k, \eta(\lambda_A), s_A) \left[ \sum_i G_i(q^2) \Gamma^\mu_{A,i} \right] u(p, s_b),$$

$$\langle \Lambda^*(k, \eta(\lambda_A), s_A)| \bar{s} s \sigma^{\mu\nu} q_\nu b |\Lambda_b(p, s_b) \rangle = -\bar{u}_\alpha(k, \eta(\lambda_A), s_A) \left[ \sum_i T^\mu_i (q^2) \Gamma^\mu_{T,i} \right] u(p, s_b),$$

$$\langle \Lambda^*(k, \eta(\lambda_A), s_A)| \bar{s} s \sigma^{\mu\nu} q_\nu \gamma_5 b |\Lambda_b(p, s_b) \rangle = -\bar{u}_\alpha(k, \eta(\lambda_A), s_A) \left[ \sum_i T^5_i (q^2) \Gamma^\mu_{T5,i} \right] u(p, s_b),$$

where $\bar{u}_\alpha$ is the spin 3/2 projector of a Rarita-Schwinger object [22]. The Dirac structures $\Gamma^\mu_{L,i}$, with $L = V, A, T, T5$ are given in Appendix A. In the cases $L = V, A$ I adopt the parametrisation in Ref. [23], while for $L = T, T5$ I adapt the parametrisation in Refs. [18, 21], with the convention $\sigma^{\mu\nu} = \frac{i}{2} (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu)$.

### 2.1 The Heavy Quark Expansion

In the low-recoil limit, a HQE of the $\Lambda_b \rightarrow \Lambda^*$ form factors can be performed. At leading power in $1/m_b$ and leading order in $\alpha_s$, the hadronic matrix element for $\Lambda_b \rightarrow \Lambda^*$ transitions reads:

$$\langle \Lambda^*(k, \eta, s_A)| \bar{s} \gamma^\mu b |\Lambda_b(p, s_b) \rangle = \sqrt{4} \bar{u}_\alpha(k, \eta, s_A) \zeta^\mu(k, \eta, s_A) u(m_{\Lambda_b} v, s_b),$$

where $v$ is the velocity of the initial state and $\Gamma^\mu$ denotes a Dirac structure. In the following, I will focus on the cases $\Gamma^\mu = \gamma^\mu, \gamma^\mu \gamma_5, i\sigma^{\mu\nu} q_\nu, i\sigma^{\mu\nu} q_\nu \gamma_5$. The most general decomposition for the leading-order and leading-power contribution $\zeta^\mu$ reads

$$\zeta^\mu = v^\alpha [\zeta_1 + \zeta_2],$$

where $\zeta_1$ and $\zeta_2$ are the leading Isgur-Wise (IW) functions. The discussion of $1/m_b$ and $\alpha_s$ corrections closely follows Refs. [24–26]. In this spirit I replace the (axial-)vector and (pseudo-)tensor currents with:

$$\bar{s} \gamma^\mu b \mapsto \bar{s} J^\mu_{V(A)} h_v = (1 + C_0^{(v)}(\mu)) \bar{s} \gamma^\mu (\gamma_5) h_v \pm C_1^{(v)}(\mu) v^\mu \bar{s} (\gamma_5) h_v + \frac{1}{2m_b} \bar{s} \Delta J^\mu_{V(A)} h_v, \quad (2.4)$$

$$\bar{s} (\gamma_5) i\sigma^{\mu\nu} q_\nu b \mapsto \bar{s} J^\mu_{T5(5)} h_v = (1 + C_0^{(t)}(\mu)) \bar{s} i\sigma^{\mu\nu} q_\nu h_v + \frac{1}{2m_b} \bar{s} \Delta J^\mu_{T5(5)} h_v. \quad (2.5)$$

The matching coefficients at NLO read [24, 25]

$$C_0^{(v)}(\mu) = -\frac{\alpha_s C_F}{4\pi} \left[ 3 \log \left( \frac{\mu}{m_b} \right) + 4 \right] + \mathcal{O}(\alpha_s^2),$$

$$C_1^{(v)}(\mu) = +\frac{\alpha_s C_F}{2\pi} + \mathcal{O}(\alpha_s^2),$$

$$C_0^{(t)}(\mu) = -\frac{\alpha_s C_F}{4\pi} \left[ 5 \log \left( \frac{\mu}{m_b} \right) + 4 \right] + \mathcal{O}(\alpha_s^2). \quad (2.6)$$

For numerical purposes, the scale of the Wilson coefficients is set to $\mu \sim 2\text{ GeV}$. The NLP $1/m_b$ corrections due to the expansion of the current are parametrised as

$$\langle \Lambda^*(k, \eta, s_A)| \bar{s} \Delta J^\mu_{V(A,T,T5)} b |\Lambda_b(p, s_b) \rangle = \sqrt{4} \sum_i \bar{u}_\alpha(k, \eta, s_A) \zeta^{\alpha \beta} [O^{V(A,T,T5)}]^{\mu}_{\beta} u(m_{\Lambda_b} v, s_b),$$

$$3$$
where

\[ \zeta^{\alpha\beta} = g^{\alpha\beta} \left( \zeta^{SL}_1 + \phi^{SL}_2 \right) + v^\alpha v^\beta \left( \zeta^3 + \phi^{\gamma}_4 + \phi^{\gamma}_5 \right) - \zeta^{\gamma\beta} \left( \zeta^6 + \phi^{\gamma}_6 \right) \tag{2.8} \]

The functions \( \zeta^{SL}_{1,6} \) are the subleading Isgur-Wise functions and they correspond to all the independent Dirac structured that can appear in \( \zeta^{\alpha\beta} \). The possible operators \( O_i^{\mu\nu} \) are listed in Ref. [24]. Out of the possible six of them, only the operator \( [O_1^{\mu\nu}]^\alpha \) arises at order \( 1/m_b \), while the others are suppressed by \( O(\alpha_s/m_b) \) and therefore are beyond the precision here required. Therefore, the only contributions that I consider for this analysis come from:

\[ \left[ O_1^V \right]^\mu_\beta = + \gamma^\mu \gamma_\beta, \quad \left[ O_1^A \right]^\mu_\beta = - \gamma_5 \gamma^\mu \gamma_\beta, \quad \left[ O_1^T \right]^\mu_\beta = + i \gamma_5 \sigma^{\mu\nu} q_\nu \gamma_\beta \tag{2.9} \]

By means of Dirac algebra, and using the properties of Rarita-Schwinger objects in Ref. [22], inserting Eq. (2.9) in Eq. (2.8) yields:

\[ \langle \Lambda^*(k, \eta, s_\Lambda) | \bar{s} \Delta J^{\mu}_{J=4} | \Lambda_b(p, s_b) \rangle = 2 \left[ 2 \bar{u}_\alpha(k, \eta, s_\Lambda) u(m_{\Lambda_b}, s_b) g^{\alpha\mu}(\zeta^{SL}_1 + \zeta^{SL}_2) + \bar{u}_\alpha(k, \eta, s_\Lambda) u(m_{\Lambda_b}, s_b) v^\alpha(\zeta^3 + \zeta^4 - 2\zeta^5 - 2\zeta^6) \right. \]

\[ + 2 \bar{u}_\alpha(k, \eta, s_\Lambda) u(m_{\Lambda_b}, s_b) v^\alpha(\zeta^5 + 2\zeta^6) \right], \tag{2.10} \]

\[ \langle \Lambda^*(k, \eta, s_\Lambda) | \bar{s} \Delta J^{\mu}_{T=5} | \Lambda_b(p, s_b) \rangle = 2 \left[ - \bar{u}_\alpha(k, \eta, s_\Lambda) \gamma_5 \gamma^\mu u(m_{\Lambda_b}, s_b) v^\alpha \right. \]

\[ \times (2m_{\Lambda_b} \zeta^1 + 2m_{\Lambda_b} \zeta^2 + (m_{\Lambda_b} + m_{\Lambda_b}^*) (\zeta^3 + 2\zeta^6) - \frac{m_{\Lambda_b}^2 + m_{\Lambda_b} m_{\Lambda_b}^* - q^2}{m_{\Lambda_b}} \zeta^1) \]

\[ + 2 \bar{u}_\alpha(k, \eta, s_\Lambda) u(m_{\Lambda_b}, s_b) v^\alpha(\zeta^3 + 2\zeta^5 + 2\zeta^6) \right], \tag{2.11} \]

\[ \langle \Lambda^*(k, \eta, s_\Lambda) | \bar{s} \Delta J^{\mu}_{T=5} | \Lambda_b(p, s_b) \rangle = 2 \left[ - i \bar{u}_\alpha(k, \eta, s_\Lambda) \gamma_5 \gamma^\mu u(m_{\Lambda_b}, s_b) v^\alpha \right. \]

\[ \times (2m_{\Lambda_b} \zeta^1 + 2m_{\Lambda_b} \zeta^2 + (m_{\Lambda_b} - m_{\Lambda_b}^*) (\zeta^3 + 2\zeta^6) - \frac{m_{\Lambda_b}^2 + m_{\Lambda_b} m_{\Lambda_b}^* - q^2}{m_{\Lambda_b}} \zeta^2) \]

\[ + 2 \bar{u}_\alpha(k, \eta, s_\Lambda) u(m_{\Lambda_b}, s_b) v^\alpha(\zeta^3 + 2\zeta^5 + 2\zeta^6) \right], \tag{2.12} \]

\[ \langle \Lambda^*(k, \eta, s_\Lambda) | \bar{s} \Delta J^{\mu}_{T=5} | \Lambda_b(p, s_b) \rangle = 2 \left[ - \bar{u}_\alpha(k, \eta, s_\Lambda) \gamma_5 \gamma^\mu u(m_{\Lambda_b}, s_b) v^\alpha \right. \]

\[ \times (2m_{\Lambda_b} \zeta^1 + 2m_{\Lambda_b} \zeta^2 + (m_{\Lambda_b} - m_{\Lambda_b}^*) (\zeta^3 + 2\zeta^6) - \frac{m_{\Lambda_b}^2 + m_{\Lambda_b} m_{\Lambda_b}^* - q^2}{m_{\Lambda_b}} \zeta^2) \]

\[ + 2 \bar{u}_\alpha(k, \eta, s_\Lambda) u(m_{\Lambda_b}, s_b) \gamma_5 u(m_{\Lambda_b}, s_b) k^\mu(\zeta^1 + \zeta^2) \]

\[ + \bar{u}_\alpha(k, \eta, s_\Lambda) u(m_{\Lambda_b}, s_b) v^\alpha k^\mu(\zeta^3 + 2\zeta^5 + 2\zeta^6) \]

\[ + 4 \bar{u}_\alpha(k, \eta, s_\Lambda) u(m_{\Lambda_b}, s_b) v^\alpha k^\mu \zeta^2 \]

\[ + \bar{u}_\alpha(k, \eta, s_\Lambda) u(m_{\Lambda_b}, s_b) v^\alpha k^\mu \zeta^6 \]

\[ \times (2m_{\Lambda_b} \zeta^1 + 2m_{\Lambda_b} \zeta^2 + (m_{\Lambda_b} - m_{\Lambda_b}^*) (\zeta^3 + 2\zeta^6) - \frac{m_{\Lambda_b}^2 + m_{\Lambda_b} m_{\Lambda_b}^* - q^2}{m_{\Lambda_b}} \zeta^2) \right], \tag{2.13} \]
The number of independent subleading IW functions can be reduced by using equations of motions. In particular, for this decay, the relation \( \psi_{\beta}^{\alpha} = 0 \) gives the following conditions:

\[
\begin{align*}
\zeta_1^\text{SL} + \zeta_3^\text{SL} + \zeta_6^\text{SL} &= 0, \\
\zeta_2^\text{SL} + \zeta_4^\text{SL} + \zeta_5^\text{SL} &= 0.
\end{align*}
\]

I choose to retain as independent quantities the subleading IW functions \( \zeta_1^\text{SL}, \zeta_2^\text{SL}, \zeta_3^\text{SL} \) and \( \zeta_4^\text{SL} \).

Corrections to the form factors arise also by of non-local insertions of the heavy quark Lagrangian at order \( 1/m_b \). Following the discussions in Refs. [24, 27, 28], non-local insertion of the kinetic operator yields a universal shift proportional to the tree-level matrix elements. Hence I reabsorb such shift in a redefinition of the leading order IW function \( \zeta_1^\text{SL} \). Non-local insertion of the chromomagnetic operator can be parametrised as [29]

\[
R_{\mu\nu}^\alpha \bar{u}_a(k, \eta, s_A) \Gamma \frac{1 + \frac{i}{2} \sigma^{\mu\nu}}{\sqrt{m_b^2 - m_A^2}} u(m_A, s_b),
\]

where the object \( R_{\mu\nu}^\alpha \) is an antisymmetric tensor in the indices \( \mu \) and \( \nu \) and contains the velocity \( v \). \( \Gamma \) symbolises all the possible Dirac structures which mediate \( A_b \to A^* \) transitions. Using equations of motion, it can be shown that no possible form of \( R_{\mu\nu}^\alpha \) gives a non-zero contribution for the chromomagnetic operator.

The expressions of the form factors in terms of the leading and subleading IW functions are obtained by matching the helicity amplitudes with their HQE expansion. For the vector current, by comparing Eqs. (A.17)–(A.20) to Eqs. (A.33)–(A.36), I get

\[
\begin{align*}
F_{1/2,0} &= \frac{\sqrt{s_+}}{2m_{A_b}^2 m_{A^*_b}^2} \left\{ \zeta_1 s_- \left[ (1 + C_0^{(v)}) + C_1^{(v)} \frac{s_+}{2m_{A_b}(m_{A_b} + m_{A^*_b})} \right] \\
&\quad + \zeta_2 s_- \left[ (1 + C_0^{(v)}) \frac{m_{A^*_b}^2 + m_{A^*_b}m_{A_b} - q^2}{m_{A_b}(m_{A_b} + m_{A^*_b})} + C_1^{(v)} \frac{s_+}{2m_{A_b}(m_{A_b} + m_{A^*_b})} \right] \\
&\quad - \zeta_3^{\text{SL}} \frac{m_{A^*_b}^2 + m_{A^*_b} - m_{A_b}^2 + q^2}{m_{A_b}^2(m_{A^*_b} + m_{A_b})} - \zeta_4^{\text{SL}} \frac{m_{A^*_b}^2 + m_{A^*_b} - m_{A_b}^2 + q^2}{m_{A_b}^2(m_{A^*_b} + m_{A_b})} \\
&\quad - s_- \left[ \zeta_5^{\text{SL}} \frac{2m_{A^*_b}^2 + 3m_{A^*_b}m_{A_b} + m_{A_b}^2 - 2q^2}{2m_{A_b}^2 m_{A^*_b}^2} - \zeta_6^{\text{SL}} \frac{m_{A^*_b}^2 + 3m_{A^*_b}m_{A_b} + 2m_{A_b}^2 - q^2}{2m_{A_b}^2 m_{A^*_b}^2} \right] \right\},
\end{align*}
\]

\[
\begin{align*}
F_{1/2,\pm} &= \frac{\sqrt{s_+}}{2m_{A_b}^2 m_{A^*_b}^2} \left\{ \zeta_1 \left[ (1 + C_0^{(v)}) - C_1^{(v)} \frac{m_{A^*_b}^2 - m_{A_b}^2 - q^2}{2m_{A_b}(m_{A_b} - m_{A^*_b})} \right] \\
&\quad - \frac{1}{2m_{A_b}^2 (m_{A_b} - m_{A^*_b})} \left[ 2(1 + C_0^{(v)})(m_{A^*_b}^2 - m_{A^*_b}m_{A_b} - q^2) + C_1^{(v)}(m_{A^*_b}^2 - m_{A_b}^2 - q^2) \right] \\
&\quad - \frac{1}{2m_{A_b}^2 (m_{A_b} - m_{A^*_b})} \left[ \frac{s - m_{A_b}^2}{m_{A_b}^2} \zeta_1^{\text{SL}} - \zeta_2^{\text{SL}} \right] \\
&\quad + \frac{2m_{A^*_b}^2 - m_{A^*_b}m_{A_b} - m_{A_b}^2 - 2q^2}{2m_{A_b}^2 (m_{A_b} - m_{A^*_b})} \zeta_3^{\text{SL}} - \frac{m_{A_b}^2 + m_{A^*_b}m_{A_b} - 2m_{A_b}^2 - q^2}{2m_{A_b}^2 (m_{A_b} - m_{A^*_b})} \zeta_4^{\text{SL}} \right\},
\end{align*}
\]

\[
\begin{align*}
F_{1/2,\pm} &= \frac{\sqrt{s_+}}{2m_{A_b}^2 m_{A^*_b}^2} \left\{ s_- (1 + C_0^{(v)})(\zeta_1 - \zeta_2) - m_{A^*_b}(\zeta_3^{\text{SL}} + \zeta_2^{\text{SL}}) + \frac{s_-}{2m_{A_b}^2} (\zeta_3^{\text{SL}} + \zeta_4^{\text{SL}}) \right\},
\end{align*}
\]
\[ F_{3/2,\perp} = -\frac{\sqrt{s_+}}{2m^3_{A_b}m_{1/2}^a} \left\{ \zeta^{\text{SL}}_1 + \zeta^{\text{SL}}_2 \right\}, \quad (2.20) \]

and for the axial vector current, by matching Eqs. (A.21)–(A.24) to Eqs. (A.37)–(A.40) I obtain

\[ G_{1/2,0} = \frac{\sqrt{s_-}}{2m^3_{A_b}m_{1/2}^a} \left\{ \zeta s_+ \left[ (1 + C^{(v)}_0) + C^{(v)}_1 \frac{s_-}{2m_{A_b}(m_{A_b} - m_{A^*})} \right] - \zeta_2 s_+ \left[ (1 + C^{(v)}_0) \left( \frac{m^2_{A^*} - m_{A^*}m_{A_b} - q^2}{m_{A_b}(m_{A_b} - m_{A^*})} \right) - C^{(v)}_1 \frac{s_-}{2m_{A_b}(m_{A_b} - m_{A^*})} \right] - \zeta_1 \frac{\Lambda + m^2_{A^*} - m^2_{A_b} + q^2}{m^2_{A_b}(m_{A_b} - m_{A^*})} + \zeta^{\text{SL}}_2 \frac{(m^2_{A^*} - m^2_{A_b} + q^2)}{(m_{A_b} - m_{A^*})} \right) \}, \quad (2.21) \]

\[ G_{1/2,\perp} = \frac{\sqrt{s_-s_+}}{2m^3_{A_b}m_{1/2}^a} \left\{ \zeta_1 \left[ (1 + C^{(v)}_0) - C^{(v)}_1 \frac{m^2_{A^*} - m^2_{A_b} - q^2}{2m_{A_b}(m_{A_b} + m_{A^*})} \right] + \frac{2}{m_{A_b} + m_{A^*}} \left[ 2(1 + C^{(v)}_0)(m^2_{A^*} + m_{A^*}m_{A_b} - q^2) - C^{(v)}_1(m^2_{A^*} - m^2_{A_b} - q^2) \right] \right\} - \frac{1}{m_{A_b} + m_{A^*}} \left[ s_+ \left[ (1 + C^{(v)}_0)(\zeta_1 + \zeta_2) + m_{A^*}(\zeta^{\text{SL}}_1 - \zeta^{\text{SL}}_2) + \frac{s_-}{2m_{A_b}}(\zeta^{\text{SL}}_3 - \zeta^{\text{SL}}_4) \right] \right\} \], \quad (2.22) \]

\[ G_{3/2,\perp} = -\frac{\sqrt{s_-}}{2m^3_{A_b}m_{1/2}^a} \left\{ \zeta^{\text{SL}}_1 - \zeta^{\text{SL}}_2 \right\} \]. \quad (2.24) \]

For the tensor current, the comparison between Eqs. (A.25)–(A.27) and Eqs. (A.41)–(A.43) yields

\[ T_{1/2,0} = \frac{\sqrt{s_+}}{m^3_{A_b}m_{1/2}^a} \left\{ (1 + C^{(t)}_0)(\zeta_1 - \zeta_2)s_- - \frac{m^2_{A^*} + m^2_{A_b} - q^2}{m_{A_b}}(\zeta^{\text{SL}}_1 - \zeta^{\text{SL}}_2) \right\} \], \quad (2.25) \]

\[ T_{1/2,\perp} = \frac{\sqrt{s_+}}{m^3_{A_b}m_{1/2}^a} \left\{ (1 + C^{(t)}_0) \left[ \zeta_1 - \frac{m_{A^*}(m_{A_b} + m_{A^*}) - q^2}{m_{A_b}(m_{A^*} + m_{A_b})} \zeta_2 \right] s_- + \frac{m_{A^*}m_{A_b} - q^2}{m_{A_b}(m_{A^*} + m_{A_b})} \zeta^{\text{SL}}_1 \right\} + \frac{s_-}{2m_{A_b}} \left\{ -\zeta^{\text{SL}}_3 + \frac{m_{A^*}(m_{A_b} + m_{A^*}) - q^2}{m_{A_b}(m_{A^*} + m_{A_b})} \zeta^{\text{SL}}_2 \right\} \], \quad (2.26) \]

\[ T_{3/2,\perp} = -\frac{\sqrt{s_+}}{m^3_{A_b}m_{1/2}^a} \left\{ -\zeta^{\text{SL}}_1(m_{A_b} - m_{A^*}) + \frac{m^2_{A^*} - m_{A^*}m_{A_b} - q^2}{m_{A_b}} \zeta^{\text{SL}}_2 \right\} \], \quad (2.27) \]

while for the axial-tensor form factors the comparison between Eqs. (A.29)–(A.31) and Eqs. (A.44)–
(A.46) gives

\[ T_{1/2,0} = \frac{\sqrt{s}}{m_{A_b}^3 m_{A^*}^{1/2}} \left\{ (1 + C_0^{(l)}) (\zeta_2 + \zeta_1) s_+ - \frac{m_{A^*}^2 + m_{A_b}^2 - q^2}{m_{A_b}} (\zeta_1^{SL} - \zeta_2^{SL}) + \frac{s_+}{2m_{A_b}} (\zeta_3^{SL} - \zeta_4^{SL}) \right\}, \]

\[ T_{1/2,\perp} = \frac{\sqrt{s}}{m_{A_b}^3 m_{A^*}^{1/2}} \left\{ (1 + C_0^{(l)}) \left[ \zeta_1 - \frac{m_{A^*} (-m_{A_b} + m_{A^*}) - q^2}{m_{A_b} (m_{A_b} - m_{A^*})} \right] s_+ + m_{A^*} (m_{A_b} + m_{A^*}) - q^2 \right\} s_+ + \frac{m_{A^*} (m_{A_b} + m_{A^*})}{m_{A_b} (m_{A_b} - m_{A^*})} \zeta_2^{SL} - \frac{s_+}{2m_{A_b}} \left\{ \zeta_3^{SL} + \frac{m_{A^*} (-m_{A_b} + m_{A^*}) - q^2}{m_{A_b} (m_{A_b} - m_{A^*})} \right\}, \]

\[ T_{3/2,\perp} = - \frac{\sqrt{s}}{m_{A_b}^3 m_{A^*}^{1/2}} \left\{ \zeta_1^{SL} (m_{A_b} + m_{A^*}) + \frac{m_{A^*}^2 + m_{A_b} m_{A^*} - q^2}{m_{A_b}} \zeta_2^{SL} \right\}. \]

The expressions in Eqs. (2.17)–(2.30) have been checked against the results in Ref. [19], where the HQE for the form factors including and NLO \( \alpha_s \) corrections are presented. With respect to the results therein, I find a sign difference in the contribution proportional to \( C_1^{(v)} \) in \( G_{1/2,l} \).

3 Form Factors relations and comparison with lattice QCD results

The first lattice QCD calculation for the full basis of \( \Lambda_b \rightarrow \Lambda^* \) form factors is presented in Ref. [21]. The calculation is performed in the low-recoil region, very close to the zero-recoil point \( q_{\text{max}}^2 = (m_{A_b} - m_{A^*})^2 \). Two lattice points per form factor are obtained, allowing to determine the normalisation and the slope of each form factor. In the kinematical limit where the lattice QCD computation is valid, it is more convenient to substitute the variable \( q^2 \) with the adimensional variable \( w = p \cdot k/(2m_{A_b} m_{A^*}) = (m_{A_b}^2 + m_{A^*}^2 - q^2)/(2m_{A_b} m_{A^*}) \), where the zero-recoil point corresponds to \( w = 1 \).

The continuum extrapolation of the results in Ref. [21] is performed using the following functional form for each of the form factor \( f_i \):

\[ f_i = F_i + A_i (w - 1). \]

Values for the coefficients \( F_i \) and \( A_i \) and their covariance matrix can be found in ancillary files of Ref. [21].

Given that both the results in Ref. [21] and the parametrisation based on HQE in Sect. 2 are valid in the low-recoil region, the former are suitable to extract the unknown, hadronic parameters for the leading and sub-leading IW functions introduced in Sect. 2. I want to stress that it is not possible to extrapolate these results to the high-recoil region without any further information on the form factors valid at low \( q^2 \).

The form factor base employed in Ref. [21] differs from the one presented in Sect. 2 and the matching between the two is given in Appendix B. In the following I denote with capital letters the HQE base and with lower cases the base of Ref. [21].

3.1 Relations in the zero-recoil point

I study the form factors first at the zero-recoil point. At this particular kinematical configuration, all the axial-vector and pseudo-tensor HQE form factors become zero because they are weighted by the factor \( s_- \). The remainder is further simplified since the terms associated with \( \zeta_1, \zeta_2, \zeta_3^{SL}, \zeta_4^{SL} \), are always proportional to \( s_- \), hence vanish, leaving only \( \zeta_1^{SL} \) and \( \zeta_2^{SL} \) to determine the form factors in the zero-recoil point. Even more interestingly, from Eqs. (2.17)–(2.30) it can be seen that only the
The central values showed above are the medians of the distributions. Concerning the ratios, principles. Given that the formalism of HQE is valid mainly in the low-recoil region for $b \rightarrow w$ transitions, the form factors can be expanded around the zero-recoil point. Substituting $q^2$ with $w$, the IW functions $\zeta_i$ can be expanded as

\[
\zeta_i = \sum_{n=0}^{N} \frac{\zeta_i^{(n)}}{n!} (w - 1)^n. \tag{3.4}
\]

The truncation order $N$ of the expansion depends on the precision required and how far from $w = 1$ the form factors are evaluated. The parameters $\zeta_i^{(n)}$ are unknown and have to be fixed using some dynamical information. Notice that since the $\Lambda^*$ is not a ground state baryon, no normalisation for the leading IW functions is predicted from HQE.

For convenience, I perform a fit to the lattice QCD data using the base in Ref. [21]. Since the lattice QCD data do not provide information on the curvature of the form factors, it is useful to express the HQE form factors as in the form of Eq. (3.1). At this scope, I use the parametrisation in Eq. (3.4) up to $N = 1$ and then re-expand the full form factors up to the first order in $w - 1$. After this procedure, it can be noticed that $i)$ the parameters $\zeta_1^{(1)}$ and $\zeta_2^{(1)}$ appear always in the combination $\zeta_1^{(1)} + \zeta_2^{(1)}$ and $ii)$ the parameters $\zeta_3^{SL,(1)}$ and $\zeta_4^{SL,(1)}$ appear always in the combination $\zeta_4^{SL,(1)} - \zeta_3^{SL,(1)}$. Therefore these parameters cannot be determined on their own, but only the combinations $\zeta_1^{(1)} + \zeta_2^{(1)}$ and $\zeta_4^{SL,(1)} - \zeta_3^{SL,(1)}$ are determined. This makes the number of independent, unknown HQE parameters
to be 10.

Before discussing the fit results, a couple of technical comments are in order:

1. Given the available information in Ref. [21], it is possible to use two pseudo-points for each form factor. I choose to evaluate them at \( w = 1.02 \) and \( w = 1.04 \). Given that the HQE parametrisation depends on fewer parameters than the lattice QCD one, I choose to perform a fit to only a subset of the lattice QCD data. In particular I choose to fit to the data on the vector and axial-vector form factors and provide predictions for the tensor and pseudo-tensor form factors based on the fit results. I comment on the consequences of this choice in the following. I stress that this is a common procedure and has been already employed for example in Ref. [30].

2. The HQE form factors are affected by uncertainties due to the unknown contributions from higher orders of expansion. By naive dimensional arguments these contributions are expected to be roughly \( \mathcal{O}(\text{few } \%) \). Hence, I introduce an uncorrelated 1\% uncertainty on all HQE expressions of the form factors to take this effects into account. Comments on this choice can be found later in the text.

| Parameter          | Best fit point         |
|--------------------|------------------------|
| \( \zeta_1^{(0)} \) | 0.454 ± 0.070          |
| \( \zeta_2^{(0)} \) | 0.0303 ± 0.0552        |
| \( \zeta_1^{(1)} + \zeta_2^{(1)} \) | 0.113 ± 0.024         |
| \( \zeta_1^{\text{SL}(0)} \) | 0.125 ± 0.038         |
| \( \zeta_1^{(1)} \) | 0.0487 ± 0.0614        |
| \( \zeta_2^{\text{SL}(0)} \) | 0.0110 ± 0.0363       |
| \( \zeta_1^{\text{SL}(1)} \) | 0.00362 ± 0.06184     |
| \( \zeta_2^{\text{SL}(1)} \) | 0.228 ± 0.190         |
| \( \zeta_3^{\text{SL}(0)} \) | 0.0883 ± 0.185        |
| \( \zeta_4^{\text{SL}(1)} - \zeta_3^{\text{SL}(1)} \) | −0.0267 ± 0.0773     |

Table 3.1: Best fit points for the HQE parameters.

The fit is performed with a \( \chi^2 \) minimisation, yielding at the minimum \( \chi^2_{\text{min}} / \text{d.o.f.} \sim 0.25 \). This low value is a direct consequence of the poor current knowledge of the exact size of the theory uncertainties and their correlations. If the theory uncertainties were uncorrelated, the fit procedure would indicate that their natural size is smaller than the one inferred from HQE. However, such a low value for \( \chi^2_{\text{min}} / \text{d.o.f.} \) could also indicate that strong correlations among unknown higher order terms in the HQE have been neglected. At the current status it is not possible to obtain more precise estimation on both size and correlations of the theory uncertainties; therefore, I just retain the conservative choice of an uncorrelated 1\% uncertainty on all the form factors in the analysis.
Figure 3.1: Comparison between the lattice results in Ref. [21] (grey band) and the fit results for the HQE form factors (red band) for the vector and axial-vector form factors. The two bands represent the 68% interval.

The best fit points for the hadronic parameters and their uncertainties are shown in Table 3.1, and their correlation matrix is given in Appendix C. With these results, I compare the HQE form factors to the lattice QCD ones. The comparison is given in Fig. 3.1, showing excellent agreement.
The results of the fit are used to obtain predictions for the tensor and pseudo-tensor form factors. The comparison between them and the lattice QCD computation is shown in Fig. 3.2.

The form factors $h_{\perp}$ and $\tilde{h}_{\perp}$ show a tension between lattice QCD data and HQE predictions. This is also reflected in the fact that introducing the tensor and pseudo-tensor form factors in the fit makes the $\chi^2$/d.o.f. to be much higher than 1. This corroborates the choice of excluding them from the fit procedure.

Sources of these tensions can be looked for in the lattice QCD data and in the HQE. The hypothesis are mainly two: i) the uncertainties on the lattice QCD parameters describing tensor and pseudo-tensor form factors are underestimated, and ii) missing corrections in the HQE cause a shift in the hadronic

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1The lattice QCD results employed here for tensor and pseudo-tensor form factors differ of a global sign with respect to Ref. [21]. This sign inconsistency will be fixed in a forthcoming update of Ref. [21].
parameters. In the case of \( i) \), I checked explicitly the result of inflating lattice QCD uncertainties by 20% for \( h_{1\perp} \) and \( \tilde{h}_{1\perp} \). In Fig. 3.3 the results of this test are shown, proving that the compatibility slightly improves, even if it is still poor in the case of \( \tilde{h}_{1\perp} \). Concerning \( ii) \), the most important corrections in the HQE beside the ones already discussed in this work are at order \( \mathcal{O}(\alpha_s/m_b) \), which could produce a \( \mathcal{O}(\text{few \%}) \) shift in the central value of the form factors parameters. However, assessing the impact of these corrections quantitatively requires understanding how they affect the form factors in a correlated way.

From these estimates it seems that neither \( i) \) nor \( ii) \) can explain the tension on their own, but most likely a combination of the two effects might be the key to reconcile the HQE for the tensor and pseudo-tensor form factors and the current lattice QCD determination.

4 Conclusions

I revisit the Heavy Quark Expansion of the \( \Lambda_b \to \Lambda^*(1520) \) form factors including next-to-leading order \( \alpha_s \) corrections and for the first time next-to-leading power \( 1/m_b \) corrections. In this framework, form factors are described by unknown hadronic parameters which are obtained by fitting the form factor parametrisation here discussed to a recent lattice QCD computation [21]. I perform the fit using data for vector and axial-vector form factors, finding rather good agreement between the lattice QCD calculation and the Heavy Quark Expansion predictions. The fit results are used to predict tensor and pseudo-tensor form factors. In this case, tensions between the Heavy Quark Expansion based predictions and the lattice QCD data are observed. I discuss two possible sources of the tensions: an underestimation of the uncertainties on the lattice QCD parameters involved in these form factors and missing higher order terms in the Heavy Quark Expansion, e.g. at order \( \mathcal{O}(\alpha_s/m_b) \). Most likely, only a combination of these two effects could reconcile lattice QCD determination and Heavy Quark Expansion based parametrisation of tensor and pseudo-tensor form factors. Until then, it is not possible to claim high precision in the Heavy Quark Expansion parametrisation of \( \Lambda_b \to \Lambda^*(1520) \) form factors.

Beside this, I want to point out the need of extending the calculation of the \( \Lambda_b \to \Lambda^*(1520) \) form factors to the high-recoil region. Quark models [20] are available, although without a consistent treatment of uncertainties. It is therefore needed to perform up-to-date calculations of the \( \Lambda_b \to \Lambda^*(1520) \) form factors using e.g. sum rules at \( q^2 \lesssim 0 \). Estimates of this type can allow to extrapolate the form factors to the high-recoil region and to assess the magnitude of their curvature. These studies will be crucial for future LHCb analysis of \( \Lambda_b \to \Lambda^*(1520)\ell^+\ell^- \) decays.
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A Details on the form factor parametrisation

Concerning the vector and the axial vector currents, we follow the notation of Ref. [23]. For the vector current we have:

\begin{align}
\Gamma_{V,(1/2,t)}^{\alpha\mu} &= \frac{\sqrt{4m_{A}m_{A^*}}}{\sqrt{s}} \frac{2m_{A^*}}{\sqrt{s^+s^-}} p^\alpha \left( m_{A^*} - m_{A^*} \right) \frac{q^\mu}{\sqrt{q^2}}, \\
\Gamma_{V,(1/2,0)}^{\alpha\mu} &= \frac{\sqrt{4m_{A}m_{A^*}}}{\sqrt{s}} \frac{2m_{A^*}}{\sqrt{s^+s^-}} p^\alpha \left( m_{A^*} + m_{A^*} \right) \frac{q^\mu}{\sqrt{q^2}}, \\
\Gamma_{V,(1/2,\perp)}^{\alpha\mu} &= \frac{\sqrt{4m_{A}m_{A^*}}}{\sqrt{s}} \frac{2m_{A^*}}{\sqrt{s^+s^-}} p^\alpha \left[ (p + k)^\mu - \frac{m_{A^*}^2 - m_{A^*}^2}{q^2} q^\mu \right], \\
\Gamma_{V,(3/2,\perp)}^{\alpha\mu} &= \frac{\sqrt{4m_{A}m_{A^*}}}{\sqrt{s}} \frac{2m_{A^*}}{\sqrt{s^+s^-}} \gamma_5 + \Gamma_{V,(1/2,\perp)},
\end{align}

while for the axial vector current:

\begin{align}
\Gamma_{A,(1/2,t)}^{\alpha\mu} &= \frac{\sqrt{4m_{A}m_{A^*}}}{\sqrt{s}} \frac{2m_{A^*}}{\sqrt{s^+s^-}} p^\alpha \left( m_{A^*} + m_{A^*} \right) \frac{q^\mu}{\sqrt{q^2}}, \\
\Gamma_{A,(1/2,0)}^{\alpha\mu} &= \frac{\sqrt{4m_{A}m_{A^*}}}{\sqrt{s}} \frac{2m_{A^*}}{\sqrt{s^+s^-}} p^\alpha \left( m_{A^*} - m_{A^*} \right) \frac{q^\mu}{\sqrt{q^2}}, \\
\Gamma_{A,(1/2,\perp)}^{\alpha\mu} &= \frac{\sqrt{4m_{A}m_{A^*}}}{\sqrt{s}} \frac{2m_{A^*}}{\sqrt{s^+s^-}} p^\alpha \left[ (p + k)^\mu - \frac{m_{A^*}^2 - m_{A^*}^2}{q^2} q^\mu \right], \\
\Gamma_{A,(3/2,\perp)}^{\alpha\mu} &= \frac{\sqrt{4m_{A}m_{A^*}}}{\sqrt{s}} \frac{2m_{A^*}}{\sqrt{s^+s^-}} \gamma_5 - \Gamma_{A,(1/2,\perp)}.
\end{align}

Concerning the tensor currents, we modify the parametrisation in Ref. [18] by rescaling each structure with suitable factors. We have:

\begin{align}
\Gamma_{T,(1/2,0)}^{\alpha\mu} &= \frac{\sqrt{4m_{A}m_{A^*}}}{\sqrt{s}} \frac{q^2}{s^+s^-} p^\alpha \left[ (p + k)^\mu - \frac{m_{A^*}^2 - m_{A^*}^2}{q^2} q^\mu \right], \\
\Gamma_{T,(1/2,\perp)}^{\alpha\mu} &= \frac{\sqrt{4m_{A}m_{A^*}}}{\sqrt{s}} \frac{m_{A^*}}{s^-} p^\alpha \left[ \gamma^\mu - \frac{2m_{A^*}^2}{s^+} p^\mu - \frac{2m_{A^*}k^\mu}{s^+} \right], \\
\Gamma_{T,(3/2,\perp)}^{\alpha\mu} &= \frac{\sqrt{4m_{A}m_{A^*}}}{\sqrt{s}} \frac{m_{A^*}}{s_-} p^\alpha \left[ \gamma^\mu - \frac{1}{m_{A^*}} k^\mu + \frac{2m_{A^*}}{s^+} \frac{p^\mu}{s^+} + \frac{2m_{A^*}}{s^-} k^\mu \right],
\end{align}

and

\begin{align}
\Gamma_{T5,(1/2,0)}^{\alpha\mu} &= \frac{\sqrt{4m_{A}m_{A^*}}}{\sqrt{s}} \frac{q^2}{s^+s^-} p^\alpha \left[ (p + k)^\mu - \frac{m_{A^*}^2 - m_{A^*}^2}{q^2} q^\mu \right],
\end{align}
\[ \Gamma_{T_5,(1/2,\perp)}^{\alpha \mu} = \frac{\sqrt{4m_{\lambda_b}m_{\lambda^*}}}{s_-} \frac{m_{\lambda_b} - m_{\lambda^*}}{s_+} \rho^\alpha \left[ \gamma^\mu + \frac{2m_{\lambda^*}}{s_-} p^\mu - \frac{2m_{\lambda_b} k^\mu}{s_-} \right], \quad (A.13) \]

\[ \Gamma_{T_5,(3/2,\perp)}^{\alpha \mu} = \frac{\sqrt{4m_{\lambda_b}m_{\lambda^*}}}{s_-} \left[ g^\alpha \mu - \frac{m_{\lambda^*}}{s_+} \rho^\alpha \left( \gamma^\mu + \frac{2}{m_{\lambda^*}} k^\mu - \frac{2m_{\lambda_b} p^\mu + 2m_{\lambda_b} k^\mu}{s_-} \right) \right]. \quad (A.14) \]

I define the helicity amplitudes as

\[ A_\Gamma(s_b, s_\lambda, \lambda_\Lambda, \lambda_q) = \langle \Lambda^*(s_\Lambda, \eta(\lambda_\Lambda)) | s \Gamma^\mu \epsilon_\mu^*(\lambda_q) b | \Lambda_b(s_b) \rangle, \quad (A.15) \]

where \( \epsilon_\mu^*(\lambda_q) \) are a basis of polarisation vectors for the virtual W exchange with polarisation states \( \lambda_q = \{t, 0, +1, -1\} \). For details see Ref. [23]. The physical helicity amplitudes are identified by the following set:

\[ A_\Gamma(+1/2, +3/2, +1) \equiv A_\Gamma(+1/2, +1/2, +1, +1), \]
\[ A_\Gamma(+1/2, +1/2, 0) \equiv \sqrt{\frac{2}{3}} A_\Gamma(+1/2, +1/2, 0, 0) + \sqrt{\frac{1}{3}} A_\Gamma^{(3/2)}(+1/2, -1/2, +1, 0), \]
\[ A_\Gamma(+1/2, +1/2, t) \equiv \sqrt{\frac{2}{3}} A_\Gamma(+1/2, +1/2, 0, t) + \sqrt{\frac{1}{3}} A_\Gamma^{(3/2)}(+1/2, -1/2, +1, t), \quad (A.16) \]
\[ A_\Gamma(+1/2, -1/2, -1) \equiv \sqrt{\frac{2}{3}} A_\Gamma(+1/2, -1/2, 0, -1) + \sqrt{\frac{1}{3}} A_\Gamma^{(3/2)}(+1/2, +1/2, -1, -1), \]

where \( \Gamma \) represents the four possible currents. In the case of the vector and axial vector current, the helicity amplitudes are already calculated in Ref. [23]. For convenience, I adapt them to this case and report them here:

\[ A_V(+1/2, +3/2, +1) = -4\sqrt{m_{\lambda_b}m_{\lambda^*}} F_{3/2,\perp}, \quad (A.17) \]
\[ A_V(+1/2, +1/2, 0) = 2\sqrt{\frac{2}{3} m_{\lambda_b}m_{\lambda^*}} (m_{\lambda_b} + m_{\lambda^*}) F_{1/2,0}, \quad (A.18) \]
\[ A_V(+1/2, +1/2, t) = 2\sqrt{\frac{2}{3} m_{\lambda_b}m_{\lambda^*}} (m_{\lambda_b} - m_{\lambda^*}) F_{1/2,t}, \quad (A.19) \]
\[ A_V(+1/2, -1/2, -1) = -\frac{4}{3} \sqrt{m_{\lambda_b}m_{\lambda^*}} F_{1/2,\perp}, \quad (A.20) \]

and for the axial vector current:

\[ A_A(+1/2, +3/2, +1) = -4\sqrt{m_{\lambda_b}m_{\lambda^*}} G_{3/2,\perp}, \quad (A.21) \]
\[ A_A(+1/2, +1/2, 0) = 2\sqrt{\frac{2}{3} m_{\lambda_b}m_{\lambda^*}} (m_{\lambda_b} - m_{\lambda^*}) G_{1/2,0}, \quad (A.22) \]
\[ A_A(+1/2, +1/2, t) = 2\sqrt{\frac{2}{3} m_{\lambda_b}m_{\lambda^*}} (m_{\lambda_b} + m_{\lambda^*}) G_{1/2,t}, \quad (A.23) \]
\[ A_A(+1/2, -1/2, -1) = \frac{4}{3} \sqrt{m_{\lambda_b}m_{\lambda^*}} G_{1/2,\perp}. \quad (A.24) \]

In the case of tensor currents, with the definitions in Eqs. (A.11)–(A.14), I find

\[ A_T(+1/2, +3/2, +1) = -2\sqrt{m_{\lambda_b}m_{\lambda^*}} T_{3/2,\perp}, \quad (A.25) \]
\[ A_T(+1/2, +1/2, 0) = -\sqrt{\frac{2}{3}} \sqrt{\frac{m_{\lambda_b}m_{\lambda^*}}{m_{\lambda^*}}} q^2 T_{1/2,0}, \quad (A.26) \]
\[ A_T(+1/2, -1/2, -1) = +\frac{2}{\sqrt{3}} \sqrt{m_{\lambda_b}} (m_{\lambda_b} + m_{\lambda^*}) T_{1/2,\perp}, \quad (A.27) \]
\[ A_T(+1/2, +1/2, t) = 0, \]  
\[ A_T(+1/2, +3/2, +1) = +2 \sqrt{m_{\Lambda_b} m_{\Lambda^*}} T^5_{3/2, 1}, \]  
\[ A_T(+1/2, +1/2, 0) = + \sqrt{\frac{2}{3}} \sqrt{\frac{m_{\Lambda_b}}{m_{\Lambda^*}}} q^2 T^5_{1/2, 0}, \]  
\[ A_T(+1/2, -1/2, -1) = + \frac{2}{\sqrt{3}} \sqrt{\frac{m_{\Lambda_b}}{m_{\Lambda^*}}} (m_{\Lambda_b} - m_{\Lambda^*}) T_{1/2, 1}, \]  
\[ A_T(+1/2, +1/2, t) = 0. \]  

In the heavy quark expansion, the helicity amplitudes concerning the vector current read:

\[ A_V(+1/2, +3/2, +1) = 2 \frac{s_+}{m_{\Lambda_b}} (\zeta_1^{\text{SL}} + \zeta_2^{\text{SL}}), \]  
\[ A_V(+1/2, +1/2, 0) = \frac{\sqrt{2s_+}}{m_{\Lambda_b} m_{\Lambda^*} \sqrt{3q^2}} \left\{ \frac{s_+}{m_{\Lambda_b}} \left[ (1 + C_0^{(v)})(m_{\Lambda^*}^2 + m_{\Lambda_b} m_{\Lambda^*} - q^2) + \frac{1}{2} C_1^{(v)} s_+ \right] \zeta_2 + s_- \left[ (1 + C_0^{(v)})(m_{\Lambda^*} + m_{\Lambda_b}) + \frac{1}{2m_{\Lambda_b}} C_1^{(v)} s_+ \right] \zeta_1 - \left[ m_{\Lambda_b}(m_{\Lambda^*}^2 - m_{\Lambda_b}^2 + q^2) + s_- s_+ \right] \zeta_1^{\text{SL}} - (m_{\Lambda^*}^2 - m_{\Lambda_b}^2 + q^2) \zeta_2^{\text{SL}} \right\} + \frac{s_-}{2m_{\Lambda_b}} \left[ (2m_{\Lambda^*}^2 + 3m_{\Lambda_b} m_{\Lambda^*} + m_{\Lambda_b}^2 - 2q^2) \zeta_3^{\text{SL}} - (m_{\Lambda^*}^2 + 3m_{\Lambda_b} m_{\Lambda^*} + 2m_{\Lambda_b}^2 - q^2) \zeta_4^{\text{SL}} \right], \]  
\[ A_V(+1/2, +1/2, t) = \frac{\sqrt{2s_- s_+}}{3m_{\Lambda_b} m_{\Lambda^*} \sqrt{3q^2}} \left\{ \frac{1}{m_{\Lambda_b}} \left[ (1 + C_0^{(v)})(-m_{\Lambda^*}^2 + m_{\Lambda_b} m_{\Lambda^*} + q^2) + \frac{1}{2} C_1^{(v)} (-m_{\Lambda^*} + m_{\Lambda_b}^2 + q^2) \right] \zeta_2 + \left[ (1 + C_0^{(v)})(m_{\Lambda_b} - m_{\Lambda^*}) + C_1^{(v)} \frac{-m_{\Lambda^*}^2 + m_{\Lambda_b}^2 + q^2}{2m_{\Lambda_b}} \right] \zeta_1 + \left[ \frac{m_{\Lambda^*}^2 - q^2}{m_{\Lambda_b}^2} \zeta_1^{\text{SL}} + \zeta_2^{\text{SL}} \right] \right\} + \frac{1}{2m_{\Lambda_b}} \left[ (2m_{\Lambda^*}^2 - m_{\Lambda_b} m_{\Lambda^*} - m_{\Lambda_b}^2 - 2q^2) \zeta_3^{\text{SL}} - (m_{\Lambda^*}^2 + m_{\Lambda_b} m_{\Lambda^*} - 2m_{\Lambda_b}^2 - q^2) \zeta_4^{\text{SL}} \right], \]  
\[ A_V(+1/2, -1/2, -1) = \frac{2 \sqrt{s_+}}{3m_{\Lambda_b} m_{\Lambda^*}} \left\{ s_- (1 + C_0^{(v)})(\zeta_2 - \zeta_1) + m_{\Lambda^*} (\zeta_1^{\text{SL}} + \zeta_2^{\text{SL}}) - \frac{s_-}{2m_{\Lambda_b}} (\zeta_3^{\text{SL}} + \zeta_4^{\text{SL}}) \right\}, \]

and for the axial vector current:

\[ A_A(+1/2, +3/2, +1) = 2 \frac{s_-}{m_{\Lambda_b}} (\zeta_1^{\text{SL}} - \zeta_2^{\text{SL}}), \]  
\[ A_A(+1/2, +1/2, 0) = \frac{\sqrt{2s_-}}{3q^2 m_{\Lambda_b} m_{\Lambda^*}} \left\{ \frac{s_+}{m_{\Lambda_b}} \left[ (1 + C_0^{(v)})(-m_{\Lambda^*}^2 + m_{\Lambda_b} m_{\Lambda^*} + q^2) + \frac{s_-}{2} C_1^{(v)} \right] \zeta_2 + s_+ \left[ (m_{\Lambda_b} - m_{\Lambda^*})(1 + C_0^{(v)}) + \frac{s_-}{2m_{\Lambda_b}} C_1^{(v)} \right] \zeta_1 + \zeta_1^{\text{SL}} \right\} + \frac{m_{\Lambda_b}^2 (m_{\Lambda^*}^2 - m_{\Lambda_b}^2 - q^2) - s_- s_+}{m_{\Lambda_b}^2} \zeta_2^{\text{SL}} + \frac{m_{\Lambda^*}^2 - m_{\Lambda_b}^2 + q^2}{m_{\Lambda_b}^2} \zeta_2^{\text{SL}}, \]  
\[ \left[ (2m_{\Lambda^*}^2 - 3m_{\Lambda_b} m_{\Lambda^*} + m_{\Lambda_b}^2 - 2q^2) \zeta_3^{\text{SL}} + (m_{\Lambda^*}^2 - 3m_{\Lambda_b} m_{\Lambda^*} + 2m_{\Lambda_b}^2 - q^2) \zeta_4^{\text{SL}} \right]. \]
\[ A_A(+1/2, +1/2, t) = \frac{2^{s_+ - s_-}}{\sqrt{3}m_A m_A^*} \left\{ m_A^* + m_A^* m_A - q^2 \right\} \left( 1 + C_0^{(v)} \right) + \frac{1}{2} \left( m_A^2 - m_A^* + q^2 \right) C_1^{(v)} \right\} \zeta_2, \]

\[ + \left[ (m_A^2 + m_A^* m_A - q^2) \right] \left( 1 + C_0^{(v)} \right) + \frac{m_A^2 m_A^* m_A - q^2}{2m_A} \right\} \zeta_4 \]

\[ = \frac{1}{2m_A^2} \left[ 2m_A^2 + m_A^* m_A - m_A^2 \right] \zeta_3 + (m_A^2 - m_A^* m_A - 2m_A^2 - q^2) \zeta_4 \]

(A.39)

\[ A_A(+1/2, -1/2, -1) = \frac{2^{s_+}}{\sqrt{3}m_A m_A^*} \left\{ (1 + C_0^{(v)})(\zeta_1 + \zeta_2) + m_A^* (\zeta_1 - \zeta_2) \right\} + \frac{s_+}{2m_A} (\zeta_3 - \zeta_4) \right\} \zeta_2, \]

(A.40)

In the case of the tensor current, the non-zero helicity amplitudes in the heavy quark limit are

\[ A_T(+1/2, +3/2, +1) = \frac{2^{s_+ - s_-}}{m_A} \left\{ m_A^* - m_A \right\} \zeta_1^{SL} + \frac{m_A^* m_A^* m_A - m_A^* m_A^*}{m_A} \zeta_2 \right\} \zeta_2, \]

(A.41)

\[ A_T(+1/2, +1/2, 0) = \frac{2^{s_+}}{m_A} \left\{ s_-(\zeta_2 - \zeta_1) + m_A^* \left( \zeta_2 - \zeta_1 \right) \right\} + \frac{s_+}{2m_A} (\zeta_3 + \zeta_4) \right\} \zeta_2, \]

(A.42)

\[ A_T(+1/2, -1/2, -1) = \frac{2^{s_+ - s_-}}{m_A} \left\{ m_A^* + m_A \right\} \zeta_3 + \frac{m_A^* m_A^* m_A - m_A^* m_A^*}{m_A} \zeta_1 \right\} \zeta_2, \]

(A.43)

and for the tensor axial current:

\[ A_T(+1/2, +3/2, +1) = \frac{2^{s_+}}{m_A} \left\{ m_A^* - m_A \right\} \zeta_1^{SL} + \frac{m_A^* m_A^* m_A - m_A^* m_A^*}{m_A} \zeta_2 \right\} \zeta_2, \]

(A.44)

\[ A_T(+1/2, +1/2, 0) = \frac{2^{s_+}}{m_A} \left\{ s_+(\zeta_1 + \zeta_2) + m_A^* \left( \zeta_1 + \zeta_2 \right) \right\} + \frac{s_+}{m_A} (\zeta_3 - \zeta_4) \right\} \zeta_2, \]

(A.45)

\[ A_T(+1/2, -1/2, -1) = \frac{2^{s_+}}{m_A} \left\{ m_A - m_A^* \right\} \zeta_3 + \frac{m_A^* m_A^* m_A - m_A^* m_A^*}{m_A} \zeta_1 \right\} \zeta_2, \]

(A.46)


\begin{table}
\begin{tabular}{cccccccccccc}
1  & 0.879  & 0.440  & 0.0458  & 0.0460  & 0.120  & 0.0619  & 0.363  & 0.337  & 0.0312  \\
0.879  & 1  & 0.160  & 0.0109  & 0.0585  & 0.130  & 0.0936  & 0.325  & 0.343  & 0.121  \\
0.440  & 0.160  & 1  & 0.00723  & 0.00712  & 0.0101  & 0.0465  & 0.211  & 0.139  & 0.218  \\
0.0458  & 0.0109  & 0.00723  & 1  & 0.861  & 0.512  & 0.382  & 0.221  & 0.0500  & 0.611  \\
0.0460  & 0.00585  & 0.00712  & 0.861  & 1  & 0.406  & 0.435  & 0.214  & 0.0603  & 0.707  \\
0.120  & 0.130  & 0.0101  & 0.512  & 0.406  & 1  & 0.887  & 0.0941  & 0.119  & 0.783  \\
0.0619  & 0.0936  & 0.0465  & 0.382  & 0.435  & 0.887  & 1  & 0.160  & 0.0287  & 0.871  \\
0.363  & 0.325  & 0.211  & 0.214  & 0.214  & 0.0941  & 0.160  & 1  & 0.966  & 0.164  \\
0.337  & 0.343  & 0.139  & 0.0500  & 0.0603  & 0.119  & 0.0287  & 0.966  & 1  & 0.0606  \\
0.0312  & 0.121  & 0.218  & 0.611  & 0.707  & 0.783  & 0.871  & 0.164  & 0.0606  & 1  \\
\end{tabular}
\caption{Correlation matrix for the HQE parameters.}
\end{table}

\section{Relations with lattice form factors}

The definitions of the form factors here employed differ from other conventions in the literature. In particular, the translation with the Lattice determination in Ref. [21] is needed. I find

\begin{align}
F_{1/2,\ell} &= \frac{1}{2} \sqrt{\frac{s_-}{4m_{\Lambda_b}m_{\Lambda^*}}} f_0, \\
F_{1/2,\perp} &= \frac{1}{2} \sqrt{\frac{s_+}{4m_{\Lambda_b}m_{\Lambda^*}}} f_\perp, \\
F_{3/2,\ell} &= \frac{1}{2} \sqrt{\frac{s_+}{4m_{\Lambda_b}m_{\Lambda^*}}} f_+, \\
F_{3/2,\perp} &= -\frac{1}{2} \sqrt{\frac{s_-}{4m_{\Lambda_b}m_{\Lambda^*}}} f_\perp, \\
G_{1/2,\ell} &= \frac{1}{2} \sqrt{\frac{s_+}{4m_{\Lambda_b}m_{\Lambda^*}}} g_0, \\
G_{1/2,\perp} &= \frac{1}{2} \sqrt{\frac{s_-}{4m_{\Lambda_b}m_{\Lambda^*}}} g_\perp, \\
G_{3/2,\ell} &= \frac{1}{2} \sqrt{\frac{s_-}{4m_{\Lambda_b}m_{\Lambda^*}}} g_+, \\
G_{3/2,\perp} &= \frac{1}{2} \sqrt{\frac{s_+}{4m_{\Lambda_b}m_{\Lambda^*}}} g_\perp,
\end{align}

(B.1)

\section{Correlations between the fit parameters}

The correlation matrix for the HQE parameters is reported in Table C.1. The order of the various correlation coefficients is the same as in Table 3.1.

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17
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