In this paper, we introduce a $q$-analogue of the Tricomi expansion for the incomplete $q$-gamma function. A general method is described for converting a power series into an expansion in incomplete $q$-gamma function. Also, we use the $q$-Tricomi expansion for giving a formal proof of the relation between the incomplete gamma function and the exponential integral. Finally, we formally deduce the $q$-Tricomi expansion via the $q$-Taylor expansion.

Keywords: Incomplete $q$-gamma function; exponential integral; the $q$-Tricomi expansion; basic hypergeometric function; $q$-binomial theorem; $q$-Taylor formula.

Mathematics Subject Classification 2000: 33B20, 33E20, 33D15.

1. Introduction

The incomplete gamma function is given by [3, 12]:

$$
\gamma(\alpha, x) = \int_0^x t^{\alpha-1} e^{-t} \, dt = x^{\alpha} \frac{\alpha - 1}{\alpha} 1_F_1 \left( \alpha; \frac{\alpha + 1}{1 - x} \right). \tag{1.1}
$$

In 1950, Francesco G. Tricomi [11] stated without proof the following expansion:

$$
\gamma(\alpha, \omega x) = \omega^\alpha \sum_{n=0}^{\infty} \frac{\gamma(\alpha + n, x)}{n!} (1 - \omega)^n, \tag{1.2}
$$

which is a special case of the multiplication theorem [3, 6.14(1)]

$$
1_F_1 \left( \frac{a}{b}; \omega z \right) = \sum_{n=0}^{\infty} \frac{(a)_n}{(b)_n} \frac{1}{n!} \left( \frac{a + n}{b + n} \right) \left( -1 - \omega \right)^n \frac{n!}{z^n} \tag{1.3}
$$

for the confluent hypergeometric functions. In a search for better methods of evaluating the exponential integral

$$
E_1(x) = \int_x^{\infty} t^{-1} e^{-t} \, dt \tag{1.4}
$$

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which occurs widely in applications, most notably in quantum-mechanical electronic structure calculations), Gautschi et al. \[6\] discovered the following equation
\[\begin{align*}
E_1(x) &= -\gamma - \ln x + \sum_{n=1}^{\infty} \frac{\gamma(n,x)}{n!} 
\end{align*}\]
from the Tricomi expansion as a limiting case, where \(\gamma\) denotes the Euler constant. Also, Shy-Der Lin et al. \[10\] presented a rather elementary demonstration of Eq. (1.5) without using the Tricomi expansion.

Gupta \[7\] defined a \(q\)-analogue of the incomplete gamma function by
\[\begin{align*}
\Gamma_q(a, x) &= \frac{1}{1-q} \int_0^x t^{a-1}(tq; q)_\infty dt 
\end{align*}\]
and studied its important analytical properties and gave an application of it in statistical distribution theory.

The organization of this paper is as follows. In Sec. 2, we give some \(q\)-notations. In Sec. 3, we introduce a \(q\)-analogue of the incomplete gamma function, essentially the same as (1.6), and some of its identities. Some of the properties obtained in this section were already obtained in \[7\] with different proofs. Section 4 is devoted to proving a \(q\)-analogue of the Tricomi expansion. In Sec. 5, we give a general method to express any power series in a specific form as a series in the incomplete \(q\)-gamma function. In Sec. 6, we present a \(q\)-analogue of an important relation of the incomplete gamma function. Finally, Sec. 7 presents another proof of the \(q\)-Tricomi expansion by the \(q\)-Taylor expansion.

2. Preliminaries
In this paper we will always assume that \(0 < q < 1\). The basic hypergeometric function is defined by \[9\]
\[\begin{align*}
\phi^{(s)}_{\omega}(a_1, \ldots, a_r; b_1, \ldots, b_s; q \mid z) &= \sum_{k=0}^{\infty} \frac{(a_1; q)_k \cdots (a_r; q)_k}{(b_1; q)_k \cdots (b_s; q)_k} \frac{(zq^{k+1}; q)_k}{(q)_k}, 
\end{align*}\]
where the \(q\)-shifted factorials are defined by \[9\]
\[\begin{align*}
(a; q)_0 &= 1, \\
(a; q)_k &= \prod_{j=0}^{k-1} (1 - aq^j), \quad k = 1, 2, \ldots, \\
(a_1, \ldots, a_r; q)_k &= (a_1; q)_k \cdots (a_r; q)_k, \\
(a; q)_\infty &= \prod_{k=0}^{\infty} (1 - aq^k).
\end{align*}\]
The classical exponential function \(e^x\) has two different natural \(q\)-extensions denoted by \(e_q(x)\) and \(E_q(x)\) and defined by \[9\]
\[\begin{align*}
e_q(x) &= \phi^{(0)}_0 \left(0 \mid q \mid x \right) = \sum_{k=0}^{\infty} \frac{x^k}{(q)_k} = \frac{1}{(x; q)_\infty}, \quad |x| < 1 \\
E_q(x) &= \phi^{(0)}_0 \left(-1 \mid q \mid -x \right) = \sum_{k=0}^{\infty} \frac{q^{-k}x^k}{(q)_k} = (-x; q)_\infty, 
\end{align*}\]
where \(x \in \mathbb{C}\).
The q-difference operator $D_q$ is defined by [9]
\[
D_q f(x) = \begin{cases} 
\frac{f(x) - f(qx)}{(1 - q)x}, & x \neq 0 \\
\frac{df(0)}{dx}, & x = 0 
\end{cases}
\]  
(2.4)

where
\[
\lim_{q \to 1} D_q f(x) = \frac{df(x)}{dx}.
\]

The q-integral on $(0, x)$ is defined by [9]
\[
\int_0^x f(t) dq_t = (1 - q)x \sum_{k=0}^{\infty} q^k f(x q^k).
\]  
(2.5)

3. A q-Analogue of the Incomplete Gamma Function

Definition 1. The incomplete $q$-gamma function is defined by
\[
\gamma_q(\alpha, x) = \int_0^x t^{\alpha-1} E_q(-(1 - q)t) dq_t, \quad \text{Re}(\alpha) > 0.
\]  
(3.1)

As a special case of (3.1), we have
\[
\gamma_q(\alpha, x) = x^{\alpha-1} q^\alpha \sum_{k=0}^{\infty} q^{k \alpha} (1 - q)^k.
\]

Lemma 1.
\[
\gamma_q(\alpha, x) = x^{\alpha-1} q^\alpha \left[ \psi_{\alpha+1} \left( \frac{q}{q-1}, q^{-1}(q-1)x \right) \right],
\]  
(3.2)

where the quantum number $[\alpha]_q = \frac{1 - q^\alpha}{1 - q}$.

Proof.
\[
\gamma_q(\alpha, x) = \int_0^x t^{\alpha-1} E_q(-(1 - q)t) dq_t
\]
\[
= (1 - q)x \sum_{k=0}^{\infty} q^k (x q^k)^{\alpha-1} E_q(-(1 - q)x q^k)
\]
\[
= (1 - q)x^{\alpha-1} (1 - q)x_q \sum_{k=0}^{\infty} \frac{q^{\alpha k}}{(1 - q)x q^k}.
\]
Then
\[ \gamma_{q}(a, x) = (1-q)x^{\alpha}(1-q)x^{\alpha} 2\varphi_{1}\left( \begin{array}{c|c} q, 0 \\ (1-q)x & q^{\alpha} \end{array} \right). \] (3.3)

By using the relation [9]
\[ 2\varphi_{1}\left( \frac{a, 0}{e}, \frac{q, z}{q} \right) = \frac{(aq, qz)}{(e, qz); q} 1\varphi_{1}\left( \frac{z}{aq}, \frac{q, e}{q} \right), \]
we get
\[ \gamma_{q}(a, x) = \frac{1-q}{q}x^{\alpha}(q^{\alpha}+1)q^{\alpha} \sum_{k=0}^{\infty} \frac{(-1)^{k}q^{\alpha+1}}{(q^{\alpha}; q)_{k+1}((1-q)x)^{k+1}} \frac{1-q^{\alpha}}{(1-q^{\alpha})}. \]

If we take the limit as \( q \to 1 \), then we get the second equality in (1.1).

**Lemma 2.**
\[ D_{q}\gamma_{q}(a, x) = x^{\alpha-1}E_{q}(-1-q)x. \] (3.4)

**Proof.** We get the proof immediately from the observation that \((D_{q}F)(x) = f(x)\) if \( F(x) = \int_{q}^{x} f(t) \, dt \).

If we take the limit as \( q \to 1 \), then we obtain [3, 9.2(8)]
\[ \frac{d}{dx} \gamma_{q}(a, x) = x^{\alpha-1}e^{-x}. \]

**Lemma 3.**
\[ D_{q}(x^{\alpha-\gamma_{q}(a, x)}) = -(qx)^{-\alpha-1}\gamma_{q}(a + 1, qx). \] (3.5)

**Proof.**
\[ D_{q}(x^{\alpha-\gamma_{q}(a, x)}) = \frac{1}{[\alpha]_{q}(1-q)x^{\alpha}} \sum_{k=1}^{\infty} \frac{(-1)^{k}q^{\alpha+1}((1-q)x)^{k+1}}{(q^{\alpha}; q)_{k+1}((1-q)x)^{k+1}} 1-q^{\alpha} \]
\[ = \frac{1}{[\alpha]_{q}} \sum_{k=1}^{\infty} \frac{(-1)^{k-1}q^{\alpha+1}((q^{\alpha+1}; q)_{k-1}((1-q)x)^{k-1} \frac{1-q^{\alpha}}{(1-q^{\alpha})} - (qx)^{-\alpha-1}\gamma_{q}(a + 1, qx). \]

Also, we can prove the following lemma by induction:

**Lemma 4.**
\[ D_{q}^{n}(x^{\alpha-\gamma_{q}(a, x)}) = (-1)^{n}q^{\alpha}((q^{\alpha}x)^{-\alpha-\gamma_{q}(a + n, qx^{n}x)}). \] (3.6)
If we take the limit as $q \to 1$, then we have [3, 9.2(9)]
\[
\frac{d^n}{dx^n}(x^{-\alpha}g(\alpha, x)) = (-1)^n x^{-\alpha-n}\gamma(\alpha + n, x).
\]

Lemma 5.

\[
\gamma(\alpha + 1, qx) = -q^{\alpha+1}\gamma(\alpha, x) - qx^n E_n(-1 - q)x. \tag{3.7}
\]

Proof. By using Eq. (3.5), we have
\[
\gamma(\alpha + 1, qx) = -(qx)^{\alpha+1}D_q(x^{-\alpha}\gamma(\alpha, x)).
\]

By using the $q$-Leibniz' rule [9]
\[
(D_q(fg))(x) = (D_qf)(x)g(x) + f(x)(D_qg)(x)
\]
and Eq. (3.4), we obtain
\[
\gamma(\alpha + 1, qx) = -(qx)^{\alpha+1}\gamma(\alpha, x) - qx^n E_n(-1 - q)x.
\]

If we take the limit as $q \to 1$, we get [2]
\[
\gamma(\alpha + 1, x) = \alpha'\gamma(\alpha, x) - x^\alpha e^{-x}.
\]

Lemma 6.

\[
D_q((1 - q)x^{\alpha}g(\alpha, x)) = e_q((1 - q)x^{\alpha})\gamma(\alpha, x) + x^\alpha(x^{-1} - 1 + q). \tag{3.9}
\]

Proof.
\[
D_q((1 - q)x^{\alpha}g(\alpha, x)) = \frac{1}{x(1 - q)}[e_q((1 - q)x^{\alpha})\gamma(\alpha, x) - [1 - (1 - q)x]e_q((1 - q)x)]\gamma(\alpha, x)
\]
\[
- (1 - q)x^{\alpha}E_n(-1 - q)x)
\]

and by using the relation [9]
\[
e_q(z)E_n(-z) = 1, \tag{3.10}
\]
then
\[
D_q((1 - q)x^{\alpha}g(\alpha, x)) = e_q((1 - q)x^{\alpha})\gamma(\alpha, x) + x^\alpha(x^{-1} - 1 + q).
\]

Also, we can prove the following lemma by induction:

Lemma 7.

\[
D_q^n((1 - q)x^{\alpha}g(\alpha, x)) = e_q((1 - q)x^{\alpha})\gamma(\alpha, x) + \sum_{i=0}^{n-1} D_q^i(x^\alpha(x^{-1} - 1 + q)) \tag{3.11}
\]
\[
= e_q((1 - q)x^{\alpha})\gamma(\alpha, x) + x^{\alpha-1} - (1 - q)x^{\alpha} + \sum_{i=1}^{n-1} \frac{x^{\alpha-(i+1)}[1 - q^{\alpha-1}]}{x(1 - q)(1 - q^i)} \tag{3.12}
\]
Theorem 1. \textit{(q-Tricomi expansion)}

\[
\gamma_{\alpha}(x, q) = \omega^x \sum_{n=0}^{\infty} \frac{\gamma_{\alpha+n}}{\omega^n} \left( \frac{\omega}{q} \right)_n, \quad 0 \leq x < \frac{1}{1-q}, \quad 0 \leq \omega \leq 1; \quad \alpha > 0, \tag{4.1}
\]

where \( \left[ \frac{n}{q} \right]_n \) is given by

\[
\sum_{k=0}^{\infty} \frac{\omega^k}{\omega^k} \left( \frac{\omega}{q} \right)_k. \tag{4.2}
\]

This double series will have positive terms if \( x \geq 0, (1-q)xq^k < 1 \) for \( k \geq 0 \) and \( \omega^k \leq 1 \) for \( k \geq 0 \). Then, the double series (4.2) is of positive terms if 0 \( \leq x < \frac{1}{1-q} \) and 0 \( \leq \omega \leq 1 \).

Then, the double series (4.2) is given by

\[
\sum_{k=0}^{\infty} \frac{\omega^k}{\omega^k} \left( \frac{\omega}{q} \right)_k. \tag{4.3}
\]

By using the q-binomial theorem [9]

\[
\sum_{n=0}^{\infty} \frac{\omega_n}{\omega_n} \left( \frac{\omega}{q} \right)_n |z| < 1, \tag{4.4}
\]

then we get

\[
\sum_{n=0}^{\infty} \frac{\omega_n}{\omega_n} \left( \frac{\omega}{q} \right)_n = \frac{q^k(1-q)xq^k}{(1-q)xq^k} |z| < \frac{1}{1-q}
\]

Then

\[
\sum_{k=0}^{\infty} \frac{\omega^k}{\omega^k} \left( \frac{\omega}{q} \right)_k. \tag{4.5}
\]
By using the relation [9]
\[
\frac{(x; q)_m}{(n; q)_m} = (aq^m; q)_\infty, \quad m \in \mathbb{C},
\]
then we get
\[
\sum_{k=0}^{\infty} \left( \sum_{n=0}^{\infty} \alpha_n \right) = (1 - q)(x; q)_\infty \sum_{k=0}^{\infty} (1 - q)x^k q^{k} = \gamma(x, \omega x).
\]
Then, the series \(\sum_{k=0}^{\infty} \sum_{n=0}^{\infty} \alpha_n\) converges.

Now, the double series (4.2) is of positive terms and one of its repeated series is convergent, so also is the other and also the double series; and the three sums are the same. Also, the interchanging of summation order is always true [1]. Then
\[
\gamma(x, \omega x) = \omega^n \sum_{k=0}^{\infty} \left( \sum_{n=0}^{\infty} \alpha_n \right) \left( (1 - q)x^k q^{k} \right) \sum_{n=0}^{\infty} \frac{q^n}{(1 - q)x^n q^n} \gamma(x, \omega x).
\]

Replace \(x\) by \((q - 1)x\) and take the limit when \(q \rightarrow 1\), then we have formally
\[
\gamma(x, \omega x) = \omega^n \sum_{n=0}^{\infty} \frac{\alpha_n + n x}{n!} (1 - \omega)^n.
\]
which is the Tricomi expansion in the case of the incomplete gamma function.

By using (4.1) and (3.2) with \(x\) replaced by \(x q\), we obtain

Lemma 9.

\[
i \Gamma \left( q; \omega \right) = \left[ q \right]_q \Gamma \left( q; \omega q \right) = \left[ q \right]_q \Gamma \left( q; \omega q \right) = 0 \leq x < 1, \quad 0 \leq \omega \leq 1.
\]

Replace \(x\) by \((q - 1)x\) and take the limit when \(q \rightarrow 1\), then we have formally
\[
i F_1 \left( \frac{a}{a + 1} \right) = \sum_{n=0}^{\infty} \frac{(a)_n}{(a + 1)_n} i F_1 \left( \frac{a + n}{a + n + 1} \right)[(1 - \omega)^n],
\]
which is a special case of Eq. (1.3) (Erdélyi multiplication formula).

5. Some Expansions in \(\gamma(x, \omega x)\)

We can write the q-Tricomi expansion in the form
\[
\frac{\gamma(x, \omega x)}{\omega^n} = \gamma(x, \omega x) = \sum_{n=1}^{\infty} \frac{\gamma(x, \omega x)}{[n]_q^2} \gamma(x, \omega x).
\]
and by using Eq. (3.2), we get
\[
(1 - q)x^k ((1 - q)x^k q^k) = \sum_{k=0}^{\infty} \frac{q^k}{(1 - q)x^k q^k} - \gamma(x, \omega x) = \sum_{n=1}^{\infty} \frac{\gamma(x, \omega x)}{[n]_q^2} \gamma(x, \omega x).
\]
When \(\omega \rightarrow 0\), we have
\[
\gamma(x, \omega x) = \sum_{n=1}^{\infty} \frac{\gamma(x, \omega x)}{[n]_q^2}.
\]
or

\[ x^n_{\alpha q} = \sum_{k=0}^{\infty} \gamma_q(\alpha + n, x), \quad 0 \leq x < \frac{1}{1-q} \]  

(5.2)

Then

\[ x^{n+k}_{\alpha + kq} = \sum_{k=0}^{\infty} \gamma_q(\alpha + n, x), \quad k \geq 0. \]  

(5.3)

We can choose arbitrary \( d_k \), subject to convergence, to form the following series

\[
\sum_{k=0}^{\infty} x^{n+k}_{\alpha + kq} d_k = \sum_{k=0}^{\infty} \sum_{\alpha = 0}^{\infty} \gamma_q(\alpha + n, x) d_k
\]

\[ = \sum_{\alpha = 0}^{\infty} \gamma_q(\alpha + n, x) \sum_{k=0}^{\infty} d_k, \]

where \([n]_q = \frac{q^n - 1}{q - 1}\) (q-binomial). Then

\[
\sum_{k=0}^{\infty} x^{n+k}_{\alpha + kq} d_k = \sum_{\alpha = 0}^{\infty} \sum_{k=0}^{\infty} \gamma_q(\alpha + n, x) d_k
\]

(5.4)

where \( c_{\alpha, q} = \sum_{k=0}^{\infty} \gamma_q(\alpha + n, x) d_k \). So, the power series which take the form given in the left side of Eq. (5.4) can therefore be written as a series in incomplete q-gamma functions.

For example, if we take \( d_k = (-1)^k \frac{\omega^k}{\alpha q^k} \), then

\[
c_{\alpha, q} = \omega^\alpha \sum_{k=0}^{\infty} \left( \frac{-\omega}{\alpha q} \right)^k \frac{\omega^{k+1}}{\alpha q} \omega^k.
\]

By using (4.4) if we replace \( a \) by \( q^{-n} \) and \( z \) by \( -\frac{z}{\alpha q} \), we get

\[
x^n(-q; x; \omega)_n = \sum_{k=0}^{\infty} \left( \frac{\omega^{k+1}}{\alpha q} \right)^N \omega^{k+1}.
\]

(5.5)

which is called Gauss's q-binomial formula. Replace \( x \) by \( 1/\omega \) and put \( a = -1 \), then

\[
(\omega; q)_n = \sum_{k=0}^{\infty} \left( \frac{\omega^{k+1}}{\alpha q} \right)^N \omega^{k+1}.
\]

(5.6)

Hence,

\[
c_{\alpha, q} = \omega^n (\omega; q)_n,
\]

and

\[
\sum_{k=0}^{\infty} (-1)^k \frac{\omega^{k+1}}{\alpha q^k} \omega^{k+1} = \sum_{\alpha = 0}^{\infty} \gamma_q(\alpha + n, x) \omega^\alpha (\omega; q)_n.
\]

(5.7)

In view of (3.2), (4.1) is the special case of \( d_k = (-1)^k \frac{\omega^k}{\alpha q^k} \omega^{k+1} \) of (5.4).

6. A Formal Proof of the Expansion Formula in Eq. (1.5)

It is well known that \[3, 9.7(5)\]

\[
E_1(x) = -\gamma - \ln x + \sum_{k=1}^{\infty} \frac{(-1)^{k-1} x^k}{k n^k},
\]

(6.1)

so it would suffice to prove that

\[
\sum_{k=1}^{\infty} \frac{(-1)^{k-1} x^k}{k n^k} = \sum_{k=1}^{\infty} \frac{\gamma_q(\alpha + n, x)}{\alpha q^k}.
\]
Now, in (5.4) with \( \alpha > 0 \) if we take
\[
d_k = \frac{(-1)^k q^{\frac{k(1-k)}{2}}}{[k+1]_q},
\]
then
\[
e_{n,q} = \sum_{k=0}^{n} (-1)^k q^{\frac{k(k-1)}{2}} - \frac{1}{[n+1]_q} \sum_{k=0}^{n} (-1)^k q^{\frac{k(k-1)}{2}} q^{k-1}.
\]
By using Eq. (5.5) at \( x = q \) and \( a = -1 \), we get
\[
e_{n,q} = \frac{-q^n}{[n+1]_q} \left[ e^{n+1}(1/q)_{n+1} - q^{n+1} \right]
\]
and \( e_{n,q} = 1 \).

By using Eq. (5.4), we have
\[
\sum_{k=0}^{\infty} \frac{(-1)^k q^{\frac{k(k+1)}{2}} e^{k+1}}{k!} = \sum_{n=1}^{\infty} \frac{q^n (\alpha + n, x)}{[n+1]_q^n} + \gamma_q(\alpha, x).
\]
Put \( \alpha = 1 \) and take the limit when \( q \to 1 \), then formally
\[
\sum_{k=1}^{\infty} \frac{(-1)^k e^{k+1}}{k!} = \sum_{n=1}^{\infty} \frac{n(n, x)}{n!},
\]
which proves the expansion formula in Eq. (1.5).

7. \( q \)-Tricomi Expansion and \( q \)-Taylor Expansion

In this section, we will formally deduce the \( q \)-Tricomi expansion via the \( q \)-Taylor expansion. Let \( f(z) \) be a continuous function on some interval \((a, b)\) and \( c \in [a, b] \). Then the \( q \)-Taylor expansion ([4, 8] and [5]) is given by the formal series
\[
f(z) = \sum_{n=0}^{\infty} z^n (c/z, q)_n (D_q^n f)(c), \quad z \in (a, b).
\]
By using the substitution \( q \to q^{-1} \), we get the formal series
\[
f(z) = \sum_{n=0}^{\infty} (-1)^n c^n (c z^{-1}, q)_n (D_q^n f(q^{-n} c)).
\]
Then
\[
f(\omega z) = \sum_{n=0}^{\infty} \frac{(-1)^n \omega^n (\omega, q)_n (D_q^n f)(q^{-n} \omega z)}{[n]_q!}.
\]
Now, let \( f(z) = z^{-\gamma_0}(\alpha, x) \) and by using the relation (3.6), we obtain
\[
(\omega z)^{-\gamma_0}(\alpha, \omega x) = \sum_{n=0}^{\infty} \frac{(-1)^n \omega^n (\omega, q)_n (-1)^n q^{\frac{n(1-n)}{2}} z^{-\gamma_0}(\alpha, z)}{[n]_q!},
\]
thereby recovering Eq. (4.1).
Acknowledgments

I am very grateful to the referee(s) for valuable comments and useful suggestions, which greatly helped improve the presentation and the quality of the paper.

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