Nature of the Chiral Restoration Transition in QCD

Gerald E. Brown, a,1 Loïc Grandchamp, a,b,2 Chang-Hwan Lee, c,3 Mannque Rho d,e,4

aDepartment of Physics and Astronomy, State University of New York, Stony Brook, NY 11794, USA
bIPN Lyon, IN2P3-CNRS et UCBL, 43 Bd. du 11 Novembre 1918, 69622 Villeurbanne Cedex, France
cDepartment of Physics, Pusan National University, Pusan 609-735, Korea
dService de Physique Théorique, CEA/DSM/SPhT. Unité de recherche associée au CNRS, CEA/Saclay, 91191 Gif-sur-Yvette cédex, France
eSchool of Physics, Korea Institute for Advanced Study, Seoul 130-722, Korea

Abstract

As the chirally restored phase ends with $T$ coming down to $T_c$, a phase resembling a mixed phase is realized, during which the hadrons (which are massless at $T_c$ in the chiral limit) get their masses back out of their kinetic energy. The gluon condensation energy is fed into the system to keep the temperature (nearly) constant. Lattice results for the gluon condensation are matched by a Nambu-Jona-Lasinio calculation. The latter shows that below $T_c$ the chiral symmetry is barely broken, so that with an $\sim 6\%$ drop in the scalar coupling $G$ it is restored at $T_c$. Nearly half of the glue, which we call epoxy, is not melted at $T_c$.

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1 Ellen.Popenoe@sunysb.edu
2 loic@tonic.physics.sunysb.edu
3 clee@pusan.ac.kr
4 rho@spht.saclay.cea.fr

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1 Introduction

As the chirally restored phase of the QGP plasma ends, it is commonly assumed that there is a mixed phase of that plasma with hadronic excitations with constant temperature before freezeout of the hadrons. We will show, however, that in the chiral limit as one approaches $T_c$, the plasma consists of nearly massless hadrons. As these go back on mass shell, chiefly using their kinetic energy to do so, the temperature drops only slightly - at least the freeze-out temperature is only a few MeV below $T_c$. In fact, the system is assumed to be equilibrated at $T_c$, and also at freezeout, where the interactions are strong enough to equilibrate it. It may well go free-flow between $T_c$ and $T_{\text{freeze out}}$, the condensation energy of soft glue furnishing the “dark energy” to produce the scalar field energy in the hadronic phase.

2 Nature of the Scale Anomaly

Harada & Yamawaki [1] have shown in putting into renormalization group (RG) equations the hidden local symmetry theory matched at a suitable scale to QCD that in the chiral limit hot/dense matter flows to the “vector manifestation (VM) fixed point” at $m_\rho^* = 0$ and $f_\pi^* = 0$ as chiral symmetry is restored. This provides a support for “Brown-Rho (BR) scaling” [2] which claims that light-quark hadron masses go to zero with chiral restoration $^5$. Shuryak & Brown [6] have recently discussed evidence for the decrease in medium of the

$^5$ It is perhaps worthwhile to give more precision in light of the more recent development to the notion of BR scaling. The original formulation [2] was based on the skyrmion Lagrangian implemented with the scale anomaly of QCD embedded in hadronic medium. The scaling behavior obtained there was at a mean field level and at that level, the “parametric mass” of the Lagrangian and the “pole mass” inside the medium, both of which are dependent on the background (temperature and/or density), were the same. As recently explained [3,4], the important development by Harada and Yamawaki shows however that what represents BR scaling is the parametric dependence in the bare Lagrangian which governs the background dependence of the parametric masses (and coupling constants) when quantum effects and thermal or dense loop corrections are taken into account. At the vector manifestation (VM) fixed point, the pole mass of the vector meson does vanish because the parametric mass and coupling constant vanish at that point. This modern interpretation of BR scaling is reformulated in terms of the skyrmion description of dense matter in [5].

Another important point that we should keep in mind is that it does not make sense to consider a system “sitting” right on top of the VM point [1]: It makes sense only to approach it from the broken symmetry sector. It is in this sense that we shall refer to “massless hadrons” in what follows.
\( \rho \)-meson mass by the STAR experiments [7].

In this note we wish to understand what happens to the scale anomaly of QCD with restoration of chiral symmetry. As laid out first by Adami, Hatsuda & Zahed [8] and later by Koch & Brown [9], this anomaly consists of about 50/50 of soft glue, which condenses as the temperature of the quark-gluon plasma moves downwards through \( T_c \) and hard glue, or “epoxy”, which condenses slowly over a wide range of temperatures, and has little effect on the phase transition. The epoxy can be described as residing in instanton molecules, as we discuss below.

We use the Gross-Neveu model in four dimensions, that is, essentially Nambu-Jona-Lasinio without pion, as in Brown, Buballa & Rho [11] that we shall refer to as BBR. The Lagrangian is

\[
L = \bar{\psi}(i\partial + g\sigma)\psi - \frac{1}{2}m_\sigma^2\sigma^2. \tag{1}
\]

Here \( \sigma \) is an auxiliary (scalar) field. Its vacuum expectation value can be obtained by setting the variation of the expectation value of \( L \) to zero

\[
\delta\langle L \rangle/\delta\sigma = 0 \tag{2}
\]

from which we obtain

\[
\sigma = \frac{g}{m_\sigma^2}\langle \bar{\psi}\psi \rangle. \tag{3}
\]

Eq. (1) with eq.(3) looks very much like Walecka theory at mean-field level, except that negative-energy states are included in the \( \langle \bar{\psi}\psi \rangle \) of Eq. (3) and one must take their kinetic energy into account.

As noted in BBR, the proper variables for \( \rho = 0, T = 0 \) are nucleons. However, the lowering of the energy of the negative energy sea because of the mass generation (cut off at \( \Lambda \)) is [10]

\[
B.E.(\text{glue}) = -E_{vac} = 4 \int_0^\Lambda \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + m_N^2} - \frac{\Lambda^4}{2\pi^2}. \tag{4}
\]

This equation differs from Eq. (4.10) of BBR, which had an additional term \(-\frac{1}{2}m_\sigma\sigma^2\), which subtracted off the field energy of the \( T = 0, \rho = 0 \) configuration. This \( \frac{1}{2}m_\sigma^2\sigma^2 \) will be fed back into the system as \( \sigma \) goes to zero, but will chiefly heat up the pions and other particles, and will not have any appreciable effect on the constituent quarks.
With $\Lambda = 660$ MeV and the NJL $G\Lambda^2 = 4.3$, which give $T_c = 170$ MeV, we find $B.E.(\text{glue}) = 0.012$ GeV$^4$. This is the correct total gluon condensate. However, it does not agree with the Miller lattice calculation, which we discuss in the next section. The reason for this may be understood in terms of the random instanton vacuum structure [12]. In the random instanton vacuum picture, there is considerable rearrangement of the glue in the vacuum at low temperatures into instanton molecules by $T_c$. These molecules with nearest neighbor instanton just fill the compacted time dimension of $\pi/T$ for $T_c$. The $\sim 50\%$ glue in the molecules does not melt with chiral restoration, making up the epoxy background. As discussed by BBR, at some point in temperature or density, nucleons dissociate smoothly into constituent quarks, with $m^*_Q \sim m^*_N/3$. When they are fully dissociated, we will have the NJL correlation energy (which we identify with bag constant)

$$B.E.(\text{soft glue}) = 12 \int_0^{\Lambda} \frac{d^3k}{(2\pi)^3} \sqrt{k^2 + m^2_Q} - \frac{3\Lambda^4}{2\pi^2}$$

the $\Lambda$ being the same. The $\Lambda$ will be chosen so as to give $T_c \sim 170$ MeV under the conditions for RHIC experiments.

Now we assume that appreciable melting of the soft glue occurs at temperatures high enough that the nucleons have dissociated into constituent quarks. We find in the case of lattice calculations of charmonium[13] that their curve bends flat at $T_c$, with a drop of 640 MeV, indicating that the constituent quark mass of the nonstrange quark is $\sim 320$ MeV. (See also Ref. [14] for this interpretation.) With $\Lambda = 660$ MeV and $m_Q = 320$ MeV we find from Eq. (5) that

$$B.E(\text{soft glue}) = 0.0058 \text{ GeV}^4,$$

roughly 50% of the total glue.

Now the only way that NJL knows about the gluon interactions is through $G$ as the glue degrees of freedom have been integrated out. As the soft glue is melted, $G$ should decrease from the value at which it can break chiral symmetry. However, at that point a new NJL-type interaction, chiefly driven by the instanton-anti-instanton molecules arises [12]. Since about half of the original glue, the epoxy, is retained in those molecules, one would not expect the original $G$, which below $T_c$ broke chiral symmetry enough to condense the other half of the glue, to be much larger than the $G$ slightly above $T_c$.

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6 We thank M. Prakash for stressing this point.
3 Lattice Results

Calculations including both light dynamical quarks and heavier ones have been carried out by D.E. Miller [15] and show that the trace anomaly decreases from 0.012 GeV$^4$ at $T \sim 122$ MeV to 0.0056 GeV$^4$ at $T \sim 170$ MeV. We take the latter to be $T_c$ for his unquenched calculations. In other words, the drop in total bag constant at $T_c$ is 53% of the total gluon condensate, the remainder being “hard glue” or “epoxy”.

In Fig. 1 we see that an $\sim 6\%$ drop in $G$ in the neighborhood of $T_c$ allows us to fit Miller’s lattice results of Fig. 2 quite well. Although it may be a coincidence, this $\sim 6\%$ is roughly how much greater the soft glue is than the epoxy in Miller’s results.

Our scenario is, then:

(1) At $T \sim 125$ MeV, the nucleons have begun to “loosen” into constituent quarks. About half of the binding energy of the negative energy nucleons goes into the formation of instanton molecules (46% in Miller’s lattice calculation which are shown below). The remaining binding energy which is the negative energy of constituent quarks is melted as soft glue.

(2) As $T$ increases above 150 MeV, the coupling constant $G$ which contains the information about the gluonic interactions drops, so that by $T_{f.o.} \sim 170$ MeV it is below the value needed to break chiral symmetry. Only an $\sim 6\%$ drop is needed for the interaction to no longer break chiral symmetry. Thus, the new Nambu-Jona-Lasinio [12] formulated about the instanton molecules which no longer break chiral symmetry would be expected to have coupling constants for the scalar, pseudoscalar, vector and axial-vector excitations which are just about as large as those of the original NJL in the symmetry-broken sector. The fact that such a small change in $G$ is needed in going from the chirally broken to restored phase is compatible with the $\sim 50/50$ division of glue into soft and hard.

The hard glue is melted at temperatures $T > T_c$ over a much greater scale, and does not concern our arguments for $T < T_c$ here.

4 Chiral Symmetry Breaking

As noted earlier, Harada & Yamawaki [1] have shown that hadron masses go to zero in the chiral limit. Brown & Rho [4] showed that the scaling of hadron masses could be realized in terms of the scaling of the NJL $\langle \bar{q}q \rangle^*$, the star denoting the in-medium condensate.
Fig. 1. (a) Temperature dependence of the coupling $G$ and (b) $B.E. (soft \ glue)$ with varying $G$. In this calculation $\Lambda = 695$ MeV and $m_{Q, current} = 0$ are used.

The $G' \equiv GA^2$ in NJL is, within factors,\(^7\) $(g_{\sigma QQ}/m_\sigma^2)\Lambda^2$ in the constituent quark sector. As suggested by Brown & Rho\([4]\) the $\Lambda$ may be identified as the Wilsonian matching scale for constituent quarks, and should therefore be taken to be independent of density or temperature. However $m_\star^*$ should scale with the masses of the other mesons. Thus, to the extent that $g_{\sigma QQ}^*/m_\sigma^*$ does not change, we see that $g_{\sigma QQ}^*$ scales roughly as $m_\star^*$, the ratio dropping at most only a few percent with chiral restoration. The NJL in the instanton molecule sector for $T > T_c$ \([12]\) has, therefore, nearly the same coupling constants as the NJL in the broken symmetry sector, although those in the former are not

\(^7\) In fact, from the particle data tables $m_\sigma = m_{f(0)} = 600$ MeV is not far from our $\Lambda = 660$ MeV.
Fig. 2. Gluon condensates taken from Miller[15]. The lines show the trace anomaly for SU(3) (solid) and the ideal gluon gas (broken) in comparison with that of the light dynamical quarks denoted by the open circles and the heavier ones by filled circles. The $T_c$ marked in the figure is that for quenched QCD, whereas we deal with unquenched QCD in this note.

quite strong enough to break chiral symmetry. In fact, the total amount of glue, 0.12 GeV$^4$, assumed by Miller [15] has been superseded by somewhat larger values [16]. However Miller’s determination of the soft glue which is melted by $T_c$ remains valid. What is changed is the amount of hard glue left unmelted and left possibly in the form of epoxy.

In the instanton description [12], the hard glue is in the form of instanton molecules. Our development above suggests that at chiral restoration, which brings the system into a “liquid” of instanton molecules for $T \gtrsim T_c$, the interactions change little. In the instanton-molecule NJL, the quark masses from the breaking of chiral symmetry below $T_c$ are replaced by thermal masses (given in perturbation theory by $m_Q = gT/\sqrt{6}$ [17]) above $T_c$.

In a recent publication, Shuryak and Zahed [18] have suggested that the evolution of the color Coulomb interaction is such that light quark states are likely to be Coulomb-bound up to a temperature of $T_{\bar{q}q} \approx 1.45 \ T_c \approx 250$ MeV. This color Coulomb interaction is not incorporated in the zero-ranged instanton-molecule NJL four-Fermi interactions and should therefore be added in the theory. The attractive four-Fermi interactions strengthen the binding such as to support light-quark bound states up to an even higher temperature. We are currently in the process of implementing the color Coulomb interaction to the NJL Lagrangian, but the general result is already clear; i.e., RHIC has found the Instanton Molecule Liquid, not the quark-gluon plasma.
The $q \bar{q}$ and $gg$ bound states for $T \gtrsim T_c$ will have the very important effects that Shuryak and Zahed have emphasized. As the fire-ball formed in RHIC collisions expands and the initial temperature decreases, these bound states will begin to form. Just as they go through zero binding energy, their scattering amplitude $a$ will go to infinity, and their scattering cross section $\sigma = 4\pi a^2$, also. The net result of these large cross sections will be a low viscosity and good equilibration. Instead of a quark-gluon plasma, we will have “sticky molasses.” Note that the Shuryak & Zahed $\bar{q}q$ and $gg$ bound states are colorless, so that color is filtered out in their formation; i.e., the Instanton Molecule Liquid is colorless.

Now going from $T_c$ downwards in temperature, the phase in which the hadrons get their masses back has many of the properties of a mixed phase, but it is not a genuine mixed phase, since it is composed of (“off-shell” ⁸) hadrons. Nonetheless, the temperature changes only little until freezeout, although the hadrons get most of their mass back, because “dark energy” is fed back through the dropping field energy (equivalently, from the gluon condensation).

One of the remarkable results from RHIC physics [19] is the common freezeout temperature of $T_{fo} = 174 \pm 7$ MeV for all hadrons. For a $\rho$-meson of mass $m_\rho = 770$ MeV this means a total energy of 1090 MeV, including thermal energy, at 174 MeV. We neglect the difference between this and the 170 MeV we obtain from LGS.

At $T_c$, each of the massless quark and antiquark coming together to make up the $\rho$ will have asymptotic energy $3.15T$, so the $\rho$ is formed with energy $6.3T$. For $T_c = 175$ MeV this means $E_\rho = 1103$ MeV, in other words only slightly – if at all – more energy than it freezes out at. Thus, the “dark energy” fed in from the field energy keeps the temperature essentially constant.

The critical temperature is the fixed point not only for the masses but also for the vector coupling $g_V$ which goes to zero there. Thus hadrons have only weakly interacting, essentially perturbative coupling until they get most of their masses back; i.e. go nearly on shell. Thus, they move relatively freely until nearly back on shell, but kinetic energy is converted into mass.

We believe that the Harada-Yamawaki work shows that the nature of the chiral symmetry breaking as $T$ goes below $T_c$ is described well by NJL-type mean field. This does not mean that the phase-transition is mean field. The fact that the vector meson masses go to zero is highly suggestive of an $\omega$ condensation [20]; i.e., a density discontinuity in the transition, so that it is probably first order over most of the phase diagram.

⁸ Note that the hadrons are off-shell with respect to the low-temperature environment in which the masses are measured. The hadrons are actually “on-shell” in the heat bath, their masses being the pole masses in the thermal Green’s functions.
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A Appendix

The trace (conformal) anomaly is

\[ \theta \equiv T_\mu^\mu = -\langle 0 | \left[ \beta(g)/2g \right] (G_{\mu\nu}^a)^2 | 0 \rangle \]  \hspace{1cm} (A.1)

where \( \beta(g) = -(g^3/16\pi^2)(11 - \frac{2}{3} N_f) \) is the beta function of QCD at one loop, and \( N_f \) is the number of flavors. The value of the gluon condensate

\[ \frac{g^2}{4\pi^2} \langle 0 | (G_{\mu\nu}^a)^2 | 0 \rangle = (330 \text{ MeV})^4 = 0.012 \text{ GeV}^4 \]  \hspace{1cm} (A.2)

is well pinned down from the states of charmonium using the QCD sum rules [21]. We work in the chiral limit, but compare with calculations made with both light and heavier dynamical quarks, so we add the contribution from the quarks which derives from explicit breaking

\[ \sum m_q \bar{q}q. \]  \hspace{1cm} (A.3)

The lighter quarks in Miller’s LGS of Fig. 2 had the MILC collaboration’s bare mass of \( \sim 6 \text{ MeV} \) and the heavier ones, \( \sim 24 \text{ MeV} \). Since \( \langle \bar{q}q \rangle \sim -(250 \text{ MeV})^3 \),

\[ \frac{m_q \langle \bar{q}q \rangle}{\frac{g^2}{4\pi^2} \langle 0 | (G_{\mu\nu}^a)^2 | 0 \rangle} \sim 0.03 \]  \hspace{1cm} (A.4)

explaining why no difference is seen from Miller’s curve.

It has been somewhat of a mystery why the order parameter of the chiral restoration transition is \( \langle \bar{q}q \rangle^* \), the quark density condensate, whereas the masses of hadrons, e.g., the mass of the nucleon is given by the trace anomaly

\[ m_N = \langle N|\theta|N \rangle. \]  \hspace{1cm} (A.5)
The trace anomaly must be connected by QCD to $\langle \bar{q}q \rangle^*$ in such a way that: 

*In driving a car the speedometer, i.e. $\langle \bar{q}q \rangle^*$, tells one how fast one is driving, but the car actually moves because the wheels, i.e. $\theta$, turn.*

In our note above we have described in a simple way the conduit coupling $\langle \bar{q}q \rangle$ and $\theta$, in terms of the condensation energy (i.e., bag constant). Essentially the soft glue is the glue connected with quarks; the quarks would not have their masses were the symmetry not broken by the soft glue. Chiral symmetry restoration implies the melting of the soft glue (the restoration of scale invariance in the constituent quark sector). This is precisely what has been found in the skyrmion description of dense matter in [5].

Note that the lattice calculation melting the soft glue is carried out for temperature well within the (effective) hadron sector. Thus, from the standpoint of the part of the trace anomaly which vanishes at $T_c$, the order parameter is the hadron mass, in the case discussed $m_N^*$, but equally well the $\rho$-meson mass $m_{\rho}^*$ as used by Adami & Brown[22]. The connection of the hadron mass with $\langle \bar{q}q \rangle^*$ is described by the Harada and Yamawaki theory, in that the scaling of $m_{\rho}^*$ towards the fixed point $m_{\rho}^* = 0$ at chiral restoration is given by $\langle \bar{q}q \rangle^* \rightarrow 0$.

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