Quantum Zeno Effect in the Quantum Non-Demolition Detection of Itinerant Photons

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We analyze the detection of itinerant photons using a quantum non-demolition (QND) measurement. We show that the backaction due to the continuous measurement imposes a limit on the detector efficiency in such a scheme. We illustrate this using a setup where signal photons have to enter a cavity in order to be detected dispersively. In this approach, the measurement signal is the phase shift imparted to an intense beam passing through a second cavity mode. The restrictions on the fidelity are a consequence of the Quantum Zeno effect, and we discuss both analytical results and quantum trajectory simulations of the measurement process.

Quantum non-demolition (QND) measurements are ideal projective measurements that reproduce their outcome when repeated [1, 2]. Using them, it is possible to measure the state of a system with the minimal disruption required by quantum mechanics. Recent successful experimental demonstrations of QND detection for superconducting qubits and microwave photons [3, 4, 5, 6] are both of fundamental interest and crucial for the development of quantum communication and information processing. When QND detection is applied continuously to a system that would otherwise undergo some intrinsic dynamics, quantum jumps are observed, tracing the quantum evolution in real-time [7, 8, 9, 10, 11]. As a consequence, the dynamics tends to be frozen, a result now known as the Quantum Zeno effect [12, 13, 14, 15, 16, 17].

In the present paper, we show that the interplay of these phenomena may put interesting constraints on the detection of itinerant quanta. The specific minimal example we will discuss concerns the continuous dispersive QND detection of single photons passing through a cavity. The crucial distinction to be recognized is the following: For localized quanta (e.g. a photon already created inside a cavity [8, 18]), the Quantum Zeno effect could presumably only enhance the detection by suppressing the decay. However, this no longer holds for the detection of itinerant quanta, if we require that our detector is always working and can detect the quantum without knowing the arrival time in advance. As we will show, in this case the unavoidable back-action of the measurement device produces a Quantum Zeno effect, suppressing the fidelity of measurements.

We investigate a QND scheme utilizing the non-linear, Kerr-type coupling [19, 20, 21] of two discrete localized modes of a bosonic field. The presence of a quantum inside the signal mode gives rise to a frequency shift of the detection mode, which can be observed dispersively via the phase shift of a beam transmitted through that mode (see Fig. 1). In turn, the signal mode frequency fluctuates due to the detection beam’s shot noise. As a consequence, the incoming signal photon will be reflected with a probability that rises with coupling strength and detection beam intensity.

This incarnation of the Quantum Zeno effect generates a trade-off that yields the highest detection efficiency at intermediate coupling strengths. In that way, such dispersive schemes for itinerant quanta turn out to be similar to weak measurements using general linear detectors and amplifiers [2].

We proceed as follows: (i) We numerically evaluate quantum jump trajectories for the phase shift signal in a minimal model of a QND photon detector and analyze the fraction of detected photons, observing the trade-off described above. (ii) We interpret these findings using an analytical approximation. (iii) Finally, we briefly comment on possible experimental realizations.

Model. – We consider a system of two cavity modes with a Kerr-type coupling of strength $g$:

$$
\hat{H} = \hbar \omega \left( \hat{n} + \frac{1}{2} \right) + \hbar \omega_{\text{det}} \left( \hat{n}_{\text{det}} + \frac{1}{2} \right) + \hbar g \hat{n} \hat{n}_{\text{det}} + \hat{H}_{\text{drive+decay}}.
$$

(1)

These modes might represent two different electromagnetic field modes inside an optical or microwave cavity,
the modes of two adjacent cavities, or even two anharmonically coupled modes of a nanomechanical resonator. Photons in the signal mode (frequency $\omega$, number operator $\hat{n}$) and the detector mode ($\omega_{\text{det}}$, $\hat{n}_{\text{det}}$) decay by leaking out of the cavity. The anharmonic Kerr-type coupling arises generically when introducing any nonlinear medium, such as an atom, a qubit or a quantum dot, into a cavity and has been studied for the purpose of QND measurements in quantum optics [19, 20, 21]. It induces a phase shift in the strong detection beam ($\langle\hat{n}_{\text{det}}\rangle \gg 1$) upon presence of a signal photon.

We are interested in analyzing individual realizations of the phase shift signal as a function of time. The phase shift can be observed by continuously measuring an appropriate field quadrature of the detection beam (e.g. in a homodyne setup). As the beam passes through the cavity, the beam becomes entangled weakly with the cavity’s state. Thus, the stochastic measurement outcomes reveal information about that state, feedback into the time-evolution of the cavity’s density matrix. This physics is described by a stochastic Lindblad master equation [2, 18, 22, 23, 24] for the density matrix $\hat{\rho}$ conditioned on the output signal (see [18]):

$$\dot{\hat{\rho}} = -i\sqrt{\frac{N_{\text{in}}^\ast}{2}}(\hat{a} + \hat{a}^\dagger, \hat{\rho}) + \kappa(\hat{a}\hat{\rho}\hat{a}^\dagger - \frac{1}{2}\hat{a}\hat{\rho} - \frac{1}{2}\hat{\rho}\hat{a}^\dagger) - 2\Gamma[\hat{n}, [\hat{n}, \hat{\rho}]] - \sqrt{4\Gamma}(\hat{n}\hat{\rho} + \hat{\rho}\hat{n} - 2\hat{\rho}(\hat{n}(t))(t). (2)$$

We analyze a situation with a continuous weak coherent beam of photons entering at a rate $N_{\text{in}}$ into the signal mode, whose decay-rate is $\kappa$ (first line of Eq. (2)). We have chosen to work in the limit of a large detector mode decay rate, $\kappa_{\text{det}} \gg \kappa$, which is favorable for the detection process and makes it possible to adiabatically eliminate that mode [18], keeping only the signal mode $\hat{n} = \hat{a}^\dagger\hat{a}$ and drastically reducing the numerical effort. After adiabatic elimination, the coupling strength $g$ and the detection beam intensity are combined into the measurement rate [18] $\Gamma \equiv g^2\langle\hat{n}_{\text{det}}\rangle/(4\kappa_{\text{det}})$, where $1/\Gamma$ is the time-scale needed to resolve different photon numbers. The last, stochastic term in Eq. (2), describes the measurement back action. The (suitably normalized) phase shift signal reads:

$$X(t) \equiv \langle\hat{n}(t)\rangle + \frac{1}{4}\sqrt{\frac{\Gamma}{1}}\xi(t). (3)$$

It contains a systematic term depending on the average number of signal photons, as well as a stochastic term representing the unavoidable vacuum noise, where $\langle\xi\rangle = 0$ and $\langle\xi(t)\xi(t')\rangle = \delta(t - t')$. In deriving Eq. (3), we have assumed that the transmitted and reflected signals are superimposed symmetrically to extract the maximum information. As in any measurement of field quadratures, temporal filtering is required to suppress the noise. We average over a timespan $\tau_{\text{avg}}$, which should be as large as possible while still remaining smaller than the expected temporal extent of the phase shift signal due to a single photon, i.e. $\tau_{\text{avg}} \ll \kappa^{-1}$. We denote the averaged signal as $\bar{X}(t)$.

Numerical Results. – We numerically solve the master equation, using it to compute the signal $\bar{X}(t)$ and the occupation of the signal mode $\langle\hat{n}\rangle(t)$ as a function of time. We then implement the minimal model of a threshold detector: Time-points when the quantum jump trajectory $\bar{X}(t)$ first exceeds the threshold $X_{\text{thr}}$ are counted as detection events, and the detector is then set insensitive for a dark-time $\tau_{\text{dark}}$, suitably chosen to avoid multiple detection, i.e. $N_{\text{in}}^\ast \gg \tau_{\text{dark}} \gg \kappa^{-1}$. Our discussion will focus on small values of $N_{\text{in}}$, making the results independent of $\tau_{\text{dark}}$, while we will analyze the dependence on $X_{\text{thr}}$ in some detail. In figure 2 we show two example trajectories. Whereas the expected number of signal photons is the same for both cases, the increase in the

Figure 2: (Color online) Quantum jump trajectories illustrating dispersive photon detection: The observable homodyne signal $\bar{X}(t)$ (red lines) and the corresponding signal mode occupation $\langle\hat{n}\rangle(t)$ (blue lines), for two different values of the measurement rate $\Gamma/\kappa$ at a fixed input rate $N_{\text{in}}$. Photon detection events are indicated as filled circles. The relative noise strength $\propto (\Gamma\tau_{\text{avg}})^{-1/2}$ is suppressed with increasing $\Gamma/\kappa$, but the number of photons actually detected also decreases, due to the Quantum Zeno effect (see main text). The size of the noise floor, the detector threshold $X_{\text{thr}}$ (green dashed line), and the dark time are indicated. Here and in the following plots $\kappa\tau_{\text{avg}} = 2$. 

\[ \rho(t) = \begin{cases} 
\frac{1}{\kappa}\hat{T} d\hat{T}^\dagger, & \text{if } \tau_{\text{dark}} \leq t < \tau_{\text{avg}} \\
\rho(t), & \text{if } \tau_{\text{avg}} \leq t < \tau_{\text{thr}} \\
0, & \text{if } \tau_{\text{thr}} \leq t < \tau_{\text{dark}} - \tau_{\text{thr}} \\
\frac{1}{\kappa}\hat{T} d\hat{T}^\dagger, & \text{if } \tau_{\text{dark}} - \tau_{\text{thr}} \leq t < \tau_{\text{dark}} \\
0, & \text{if } \tau_{\text{dark}} \leq t.
\end{cases} \]
measurement rate \( \Gamma/\kappa \) decreases the number of photons actually detected, while at the same time enhancing the signal to noise ratio in the trajectory. This is an indication of the Quantum Zeno effect in the detection of itinerant quanta, which we now want to study in more quantitative detail.

We plot the rate of photon detection events \( N_{\text{det}} \) versus the rate of incoming photons \( N_{\text{in}} \) [Fig. 3(a)]. The detection efficiency \( \eta \) is naturally defined as the ratio of detected vs. incoming photons, obtained at small input rates \( N_{\text{in}} \):

\[
\eta \equiv \frac{dN_{\text{det}}}{dN_{\text{in}}} \bigg|_{N_{\text{in}}=0}.
\]

Fig. 3(c) displays the efficiency \( \eta \) as a function of \( \Gamma/\kappa \) and \( X_{\text{thr}} \). The statistics for this figure were obtained from extensive numerical simulations by generating \( O(10^4) \) trajectories of length \( 10^2/\kappa \) for seven different rates \( N_{\text{in}} \) at each value of \( \Gamma/\kappa \). Apparently, the detection efficiency \( \eta \) is strongly suppressed both for \( \Gamma/\kappa \ll 1 \) (low signal to noise ratio) and \( \Gamma/\kappa \gg 1 \).

**Analytical Results** - To interpret these results, we now calculate the total transmission probability through the signal mode, whose frequency fluctuates due to the shot noise in the detection mode, which is treated as classical noise. We start from the semiclassical equation of motion for the complex field amplitude \( \alpha(t) \) in the signal mode,

\[
\dot{\alpha}(t) = \left(-i \frac{\delta \omega(t) - \kappa}{2}\right) \alpha(t) + \sqrt{\frac{\kappa}{2}} \alpha_{\text{in}}.
\]

Here \( \alpha_{\text{in}} \) is the amplitude of the signal photon field entering the cavity from the left side, and \( \delta \omega(t) \equiv g n_{\text{det}}(t) \) is the fluctuating frequency shift (\( n_{\text{det}} \gg 1 \)). The correlator of the noise is given by

\[
\langle \delta \omega(t) \delta \omega(0) \rangle - \langle \delta \omega \rangle^2 = g^2 n_{\text{det}} e^{-\kappa_{\text{det}}|t|/2}.
\]

To obtain an expression for the transmission probability, we write down the formal solution for \( \alpha(t) \),

\[
\frac{\alpha(t)}{\sqrt{\kappa L} \alpha_{\text{in}}^2} = \int_{-\infty}^{t} dt' \exp \left[-i \int_{t'}^{t} \delta \omega(t'') dt'' - \frac{\kappa}{2} (t - t') \right].
\]

Figure 3: (Color online) (a) Detector profile: Rate of detected vs. incoming photons, at \( \Gamma/\kappa = 0.6 \), for two different thresholds \( X_{\text{thr}} \). The blue data points display \( \eta \) defined from the slope at \( N_{\text{in}} = 0 \), and the saturation for large \( N_{\text{in}} \sim \tau^{-1}\text{dark} \). (b) Suppression of the signal photon number inside the cavity as a function of measurement rate \( \Gamma/\kappa \). (c) Detector efficiency \( \eta \), obtained from quantum trajectory simulations, as a function of \( \Gamma \) and \( X_{\text{thr}} \) (3D-inset), and comparison with the analytical results (main plot). The blue data points display \( \eta \) for fixed \( X_{\text{thr}} = 0.5 \) (blue cut in 3D-inset). When maximizing the efficiency over \( X_{\text{thr}} \) for any given \( \Gamma/\kappa \), the red data points are obtained (labeled “max(\( \eta \))”, coinciding with the analytical asymptote (green thin line) at higher values of \( \Gamma/\kappa \). Small inset: The dependence of the efficiency \( \eta \) on the averaging time \( \tau_{\text{avg}} \) is shown for \( \Gamma/\kappa = 4 \).

Note that the fluctuations \( \delta \omega(t) \) themselves are non-Gaussian. Still, the integral in the exponent is approximately Gaussian for time-intervals that fulfill \( \kappa_{\text{det}} |t - t'| \gg 1 \), due to the central limit theorem. These times yield the main contribution under our assumption of a “fast detector”, \( \kappa_{\text{det}} \gg \kappa \). Thus, we can evaluate \( \langle |\alpha|^2 \rangle \) using the formula \( \langle \exp[-iY]\rangle = \exp[-i\langle Y \rangle - \frac{1}{2} \text{Var}Y] \) for a Gaussian random variable \( Y \), and inserting Eq. (6).

From this, we obtain the average transmitted intensity

\[
\langle |\alpha_{\text{R}}|^{2} \rangle = \frac{\kappa}{2} |\alpha|^2 = \langle T \rangle |\alpha_{\text{in}}|^2,
\]

and the average transmission probability

\[
\langle T \rangle = \left(1 + 4 \frac{\Gamma}{\kappa} \right)^{-1}.
\]

Before we can correlate the suppression of the transmission with the reduction of the detector efficiency \( \eta \) in the limit of \( \Gamma/\kappa \gg 1 \), one more consideration is necessary.
In this limit, any photon that has entered the cavity will almost certainly be detected. Once detected, the photon loses the coherence with the incoming beam, which is needed for perfect transmission on resonance in the ideal, coherent case. As a consequence, it acquires an equal probability to leave the cavity through the left or the right port. This means that, on average, the number of detected photons is twice the number of transmitted photons. The expected relation is thus \( \eta = 2\langle T \rangle \), which is indeed observed nicely when comparing with the numerical data (Fig. 3(c)).

The reduction of detector efficiency at \( \Gamma / \kappa \) thus has found its explanation in the Quantum Zeno effect: many photons remain undetected, because they are reflected due to detector back-action. As low values of \( \Gamma / \kappa \) are also unfavorable, due to a bad signal-to-noise ratio, the maximum efficiency is found at an intermediate value, namely near \( \Gamma / \kappa = 1/2 \) (see Fig. 3).

Possible realizations. - Cavity QED setups in superconducting circuits [12, 22, 24, 27] have been used to implement ideas of quantum optics on the chip, and are considered a promising candidate for scalable, fault tolerant quantum computing [28]. While proposals for generating nonclassical photon states exist or have been implemented [1, 21, 29, 30], the on-chip single-shot detection of itinerant photons is still missing. Building on recent experiments that demonstrated dispersive qubit detection [22] and measurements of photon statistics [23], one could employ the superconducting qubit to induce a nonlinear coupling between two modes of the microwave transmission line resonator (or coupling two cavities [32]), thus creating a dispersive photon detector of the type discussed here. These experiments realize a Jaynes-Cummings coupling between qubit and resonator of up to \( 2\pi \cdot 100 \text{MHz} \), resonators with frequencies of about \( 2\pi \cdot 5 \text{GHz} \), and a large spread of resonator decay rates \( \kappa \) between 1MHz and 100MHz. Given this wide parameter range, it is possible to cover the full range of \( \Gamma / \kappa \) explored here, assuming around 10 to 100 photons in the detection mode. The detector efficiency, although limited by the Quantum Zeno effect as shown before, could then reach values of about 40\% even without considering more elaborate detector and signal analysis schemes. Another experiment in which essentially the same physics could be observed is the detection of single photons in a microwave cavity by employing the dispersive interaction with a stream of Rydberg atoms [8].

Conclusions. - In this paper we have analyzed a rather generic scheme for the detection of itinerant photons in a QND measurement process, employing quantum trajectory simulations. We have shown how the Quantum Zeno effect enters the detection efficiency, a result that will be relevant to many other situations, such as the detection of electrons tunneling through a quantum dot by current passing through a nearby quantum point contact [31], the detection of itinerant phonons entering a micromechanical cantilever or membrane (e.g. in an optomechanical setup [32]), and other similar settings in mesoscopic physics, quantum optics, and atomic physics.

Acknowledgements. - We thank S. M. Girvin, J. M. Gambetta, A. A. Houck, M. Blencowe, A. Blais and J.-M. Raimond for discussions. Support from the SFB 631, NIM, and the Emmy-Noether program (F.M.) of the DFG and EuroSQIP (E.S.) are gratefully acknowledged.

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