Finding the Displacement of Elastically Bent Pipeline Disposed on Roller Supports Under the Action of the Axial Shear Force

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Abstract. The authors of this article have developed a spatial position determination method of a real steel pipeline free segment located above the ground surface during pipeline pullback using a pipeline pusher for the horizontal directional drilling technology. Values of elastically bent pipeline displacement were found in different points of the pipeline profile with the aim of defining the admissible values of the shear force applied to the pipeline by additional equipment during the pipeline pullback process. The developed methods allow finding the points of the curved pipeline profile in which it is necessary to place additional fixing connections above the pipeline, as this will expand the range of the shear force applied to the pipeline.

1. Introduction

When using the horizontal directional drilling technology, the values of the axial shear force applied to the pipeline as a subsidiary during pipeline pullback can have a wide range and depend on the potential of the equipment model used [2]. Nowadays, various types of approaches to applying the shear force to the pipeline exist. At the moment, there are several types of pipeline pushing machines [8] and obviously their number will continue to grow.

The behavior of elastically bent pipelines under the action of the axial shear force should be investigated in the following way. It is necessary to determine the pipeline stress-strain state considering the force application point and the values of roller supports height. This article solves the problem of finding different displacement points of elastically bent pipeline terminal section pushed by the pipeline shear force applied to the end of the pipeline [4].

2. Research object (model, process, structure, synthesis, experimental part, etc.)

We will use a system consisting of such interacted objects as pipeline, roller supports and pipeline pusher to solve our problem. Main physical parameters of a pipeline are taken into account during the solution process. They are elastic modulus of pipeline material, uniformly distributed load from its own weight, flexural stiffness, tensile stiffness, etc. A shear force application point and different values of the force are taken in account as well.

Being in a curved position, the pipeline researched, having in view the regulatory compliance of the bending values [5] and the most natural profile for real pipelines [1] of the terminal portion, is placed on roller supports. The problem is solved by dividing the terminal section into different parts and finding general solutions of equilibrium equation systems for each part located between the roller supports that separated these parts.

3. Methods.

A free pipeline segment located above the ground surface during pullback will have the profile shown in figure 1. It refers to a steel pipeline that has a much larger allowed elastic bending radius [6], in contradistinction to plastic pipelines [7]. We divided the elevated pipeline segment by O, A, B, C and D into individual parts with different values of its curvature.
Free pipeline segment profile located above the ground surface. At the same time, the pipeline is located on roller supports and we will consider that the supports are installed at equal intervals (figure 2). In this case, the pipeline in each section $P_iP_{i+1}$ can represent a clamped rod loaded with its own weight, that is a uniformly distributed load $q$.

Let us find the general solution of differential equilibrium equations for each pipeline section described above. For such a case, we will have six equations in a set [3]. The solution will depend on six specific constants. As we have divided the pipeline into $n$ different parts, we have $6 \cdot n$ specific constants: $C_1 \ldots C_{6n}$.

Now we have two boundary conditions at $O$, six matching conditions at intermediate points and two boundary conditions at $D$. In total, there are $6 \cdot n$ conditions for determination of constants $C_1 \ldots C_{6n}$. Matching conditions for supports without connection and supports with connection will be different. Let $K$ be the pipeline section number located to the left of the support, and $K + 1$ – to the right (figure 3).

Continuity conditions of all stress-strain state parameters must be met in those points where supports without connection are located:
where $S$ – the curvilinear coordinate; $S_K$ – the $K$th support coordinate; $U(S)$, $W(S)$ – the tangential and axial movements; $\Theta(S)$ – the angle of rotation; $N(S)$, $Q(S)$ – the axial and shear forces; $M(S)$ – the bending moment.

In its turn, the following conditions should be met in supports with connections:

\[
\begin{align*}
U_K(S_K) &= U_{K+1}(S_K) \\
W_K(S_K) &= W_{K+1}(S_K) \\
\Theta_K(S_K) &= \Theta_{K+1}(S_K) \\
N_K(S_K) &= N_{K+1}(S_K) \\
Q_K(S_K) &= Q_{K+1}(S_K) \\
M_K(S_K) &= M_{K+1}(S_K)
\end{align*}
\]

The transverse force $Q$ has a gap in the support, so the continuity conditions for it should not be met.

Now we will find the general solution of this problem. The equilibrium equations for pipeline segments curved convex up (figure 4) and curved convex down (figure 5) will be different.

**Figure 4.** Design scheme of pipeline segments on roller supports curved convex up: $q$ - distributed load; $R$ – pipeline segment radius of curvature; $\varphi$ - central arc angle; $n$ - normal to the pipeline surface; $\tau$ - tangential force.

The equation for pipeline segments curved convex up will be as follows:
\[
\begin{align*}
(N + 1) \begin{cases}
\frac{1}{R} \frac{dN}{d\varphi} - \frac{1}{R^2} \frac{dM}{d\varphi} &= q \cdot \sin \varphi \\
N - \frac{1}{R} \frac{d^2M}{d\varphi^2} &= q \cdot \cos \varphi
\end{cases}
\end{align*}
\]

where \( q \) – the vertical load distributed uniformly;
\( \varepsilon \) - the axial deformation;
\( \chi \) - the flexural deformation (curvature change);
\( D = E \cdot A \) – the tensile stiffness;
\( E \) – the elastic modulus;
\( A \) – the cross-sectional area of pipeline;
\( H = E \cdot I \) – the flexural stiffness;
\( I \) – the pipeline cross-section moment of inertia.

\[
(N + 2) \begin{cases}
N = D \cdot \varepsilon \\
M = H \cdot \chi
\end{cases}
\]

\[
(N + 3) \begin{cases}
\varepsilon = \frac{1}{R} \left( \frac{dN}{d\varphi} + W \right) \\
\chi = \frac{1}{R^2} \left( \frac{d^2W}{d\varphi^2} - \frac{dU}{d\varphi} \right)
\end{cases}
\]

\( \Phi < 0 \), \( \Phi > 0 \)

\( q \), \( \bar{n} \), \( \varphi \)

Figure 5. Design scheme of pipeline segments on roller supports curved convex down.

For pipeline segments curved convex down, the equation will be:

\[
\begin{align*}
\left( N + 1 \right) \begin{cases}
\frac{1}{R} \frac{dN}{d\varphi} - \frac{1}{R^2} \frac{dM}{d\varphi} &= -q \cdot \sin \varphi \\
N - \frac{1}{R} \frac{d^2M}{d\varphi^2} &= q \cdot \cos \varphi \\
Q &= -\frac{1}{R} \frac{dM}{d\varphi}
\end{cases}
\end{align*}
\]
\[
\begin{align*}
\varepsilon &= \frac{1}{R} \left( \frac{dU}{d\phi} - W \right) \\
\Theta &= \frac{1}{R} \left( \frac{dW}{d\phi} + U \right) \\
\chi &= \frac{1}{R^2} \left( \frac{d^2W}{d\phi^2} + \frac{dU}{d\phi} \right)
\end{align*}
\]  

(7)

And for straight segments of a pipeline without tangential distributed loading the general solution is:

\[
\begin{align*}
\frac{dN}{dS} &= 0 \\
\frac{d^2M}{dS^2} &= q \\
Q &= -\frac{dM}{dS} \\
\varepsilon &= \frac{dU}{dS} \\
\Theta &= \frac{dW}{dS} \\
\chi &= \frac{d^2W}{dS^2}
\end{align*}
\]  

(8)

(9)

4. Results and discussion

The general solution of the system \((N+1) \ldots (N+3)\) for pipeline segments curved convex up was derived.

The tangential movement for this case:

\[
U = C_1 \left( A \cdot \phi \cdot \sin \phi + B \cdot \cos \phi \right) + C_2 \left( A \cdot \phi \cdot \cos \phi - B \cdot \sin \phi \right) - \\
- C_3 \cdot \frac{R^3}{H} \cdot \phi + C_4 \cdot \frac{R^3}{H} \cdot \cos \phi - C_5 \cdot \frac{R^3}{H} \cdot \sin \phi + C_6 \cdot \frac{R^3}{H} - \\
- q \cdot R \cdot \frac{R^3}{H} \cdot \frac{3 \cdot \left( \sin \phi - \phi \cdot \cos \phi \right)}{2} - \\
- q \cdot R \cdot \frac{A}{4} \cdot \left( 2 \cdot \phi \cdot \cos \phi - 2 \cdot \phi^2 \cdot \sin \phi + \sin \phi \right)
\]  

(10)

The axial movement:

\[
W = -C_1 \cdot A \cdot \phi \cdot \cos \phi + C_2 \cdot A \cdot \phi \cdot \sin \phi + C_3 \cdot \frac{R^3}{H} + C_4 \cdot \frac{R^3}{H} \cdot \sin \phi + \\
+ C_5 \cdot \frac{R^3}{H} \cdot \cos \phi + q \cdot R \cdot \frac{R^3}{H} \cdot \phi \cdot \sin \phi \left( \frac{2}{2} \right) \left( 2 \cdot \phi \cdot \sin \phi - 2 \cdot \phi^2 \cdot \cos \phi + \cos \phi \right)
\]  

(11)

The angle of rotation:
\[ \Theta = -C_1 \cdot \frac{R^2}{H} \cdot \cos \varphi + C_2 \cdot \frac{R^2}{H} \cdot \sin \varphi + C_3 \cdot \frac{R^2}{H} \cdot \varphi - C_6 \cdot \frac{R^2}{H} + q \cdot R \cdot \frac{R^2}{H} \cdot (2 \cdot \sin \varphi - \varphi \cdot \cos \varphi) \]  
(12)

The axial force:
\[ N = C_1 \cdot \sin \varphi + C_2 \cdot \cos \varphi + q \cdot R \cdot \varphi \cdot \sin \varphi \]  
(13)

The shear force:
\[ Q = -C_1 \cdot \cos \varphi + C_2 \cdot \sin \varphi - q \cdot R \cdot \varphi \cdot \cos \varphi \]  
(14)

The bending moment:
\[ M = C_1 \cdot R \cdot \sin \varphi + C_2 \cdot R \cdot \cos \varphi + C_3 \cdot R + q \cdot R^2 (\varphi \cdot \sin \varphi + \cos \varphi) \]  
(15)

In these equations:
\[ \gamma = \frac{H}{R^2 D} << 1 \]  
(16)

\[ A = \frac{R^3}{2 \cdot H} (1 + \gamma) \approx \frac{R^3}{2 \cdot H} \]  
(17)

\[ B = \frac{R^3}{2 \cdot H} (1 - \gamma) \approx \frac{R^3}{2 \cdot H} \]  
(18)

And the general solution for pipeline segments curved convex down was derived, too.

The tangential movement:
\[ U = C_1 (A \cdot \varphi \cdot \sin \varphi + B \cdot \cos \varphi) + C_2 (A \cdot \varphi \cdot \cos \varphi - B \cdot \sin \varphi) + + C_3 \cdot \frac{R^3}{H} \cdot \varphi - C_4 \cdot \frac{R^3}{H} \cdot \cos \varphi + C_5 \cdot \frac{R^3}{H} \cdot \sin \varphi + C_6 \cdot \frac{R^3}{H} + - q \cdot R \cdot \frac{R^3}{H} \cdot \frac{3 \cdot (\sin \varphi - \varphi \cdot \cos \varphi)}{2} + + q \cdot R \cdot \frac{A}{4} \cdot (2 \cdot \varphi \cdot \cos \varphi - 2 \cdot \varphi^2 \cdot \sin \varphi - \sin \varphi) \]  
(19)

The axial movement:
\[ W = C_1 \cdot A \cdot \varphi \cdot \cos \varphi - C_2 \cdot A \cdot \varphi \cdot \sin \varphi + C_3 \cdot \frac{R^3}{H} \cdot \sin \varphi + + C_4 \cdot \frac{R^3}{H} \cdot \sin \varphi + + C_5 \cdot \frac{R^3}{H} \cdot \cos \varphi + q \cdot R \cdot \frac{R^3}{H} \cdot \frac{\varphi \cdot \sin \varphi}{2} + + q \cdot R \cdot \frac{A}{4} \cdot (2 \cdot \varphi \cdot \sin \varphi - 2 \cdot \varphi^2 \cdot \cos \varphi + \cos \varphi) \]  
(20)

The angle of rotation:
\[ \Theta = C_1 \cdot \frac{R^2}{H} \cdot \cos \varphi - C_2 \cdot \frac{R^2}{H} \cdot \sin \varphi + C_3 \cdot \frac{R^2}{H} \cdot \varphi + C_6 \cdot \frac{R^2}{H} + + q \cdot \frac{R^3}{H} \cdot (2 \cdot \sin \varphi - \varphi \cdot \cos \varphi) \]  
(21)

The axial force:
\[ N = C_1 \cdot \sin \varphi + C_2 \cdot \cos \varphi - q \cdot R \cdot \varphi \cdot \sin \varphi \]  
(22)

The shear force:
\[ Q = C_1 \cdot \cos \varphi - C_2 \cdot \sin \varphi - q \cdot R \cdot \varphi \cdot \cos \varphi \]  
(23)

The bending moment:
\[ M = -C_1 \cdot R \cdot \sin \varphi - C_2 \cdot R \cdot \cos \varphi + C_3 \cdot R + q \cdot R^2 \left( \varphi \cdot \sin \varphi + \cos \varphi \right) \]  
(24)

And finally, for the case of straight pipeline segments the results are:
\[ U = C_1 \cdot \frac{x}{D} + C_2 \cdot \frac{l}{D} \]  
(25)

\[ W = 0 + 0 + C_3 \cdot \frac{x^3}{6H} + C_4 \cdot \frac{l^2}{2H} + C_5 \cdot \frac{l^3}{H} + C_6 \cdot \frac{l^4}{24H} + q x^4 \]  
(26)

\[ \Theta = 0 + 0 + C_3 \cdot \frac{x^3}{2H} + C_4 \cdot \frac{l x}{H} + C_5 \cdot \frac{l^2}{H} + 0 + \frac{q x^3}{6H} \]  
(27)

\[ N = C_1 \]  
(28)

\[ M = 0 + 0 + C_3 \cdot x + C_4 \cdot l + 0 + 0 + \frac{q x^2}{2} \]  
(29)

\[ Q = 0 + 0 - C_3 + 0 + 0 + 0 - q x \]  
(30)

5. Conclusion

The general solutions developed in this article are an important step in determining the pipeline stress-strain state when the force is exerted on the pipeline. The solutions can be applied to pipelines of different lengths and diameters for special cases of calculating the elastic bending and shear force combinations.

We get a possibility to find the points of the curved pipeline profile for placing additional fixing connections above the pipeline using the obtained solutions to find the pipeline displacement from the initial position. This will allow increasing the admissible shear force and will prevent the pipeline from falling off the roller supports in case of excessive increase in the axial shear force.

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