Steady-state transalfvenic MHD flows in coaxial channels with longitudinal magnetic field

E V Stepin

Keldysh Institute of Applied Mathematics, Russian Academy of Sciences, Miusskaya sq. 4, 125047 Moscow, Russia

eugene.v.stepin@gmail.com

Abstract. Transalfvenic magnetohydrodynamic (MHD) flows in coaxial channels of plasma accelerators in the presence of a longitudinal magnetic field are considered. Transalfvenic flows are characterized by the transition of a plasma flow through the alfvenic velocity corresponding to the longitudinal field. The main results are obtained for narrow nozzle-type flow tubes of curvilinear geometry in the quasi-one-dimensional approach. The analytical expressions for plasma parameters in the transition point and for its geometric place are presented. The numerical researches of flow regime changing from superalfvenic to subalfvenic in a narrow flow tube of two configuration variants are provided.

1. Introduction

1.1. Classification of MHD flows in channels of plasma accelerators with a longitudinal magnetic field

Plasma acceleration in crossed electric and magnetic fields [1] is one of the most promising effects in terms of its research and applied importance. This made it possible to develop a new type of devices – plasma accelerators [2]. Various acceleration schemes and a wide range of its parameters allow to use them across several actual and perspective technological directions.

In the present work, plasma is considered as a continuous medium, which behavior is described in terms of magnetic hydrodynamics (MHD). Plasma motion in an accelerator channel is due to the interaction between an electric current flowing in it and a transverse magnetic self-field generated in the plant. The acceleration mechanism under consideration can be enhanced by creation of a longitudinal magnetic field using external electric current conductors.

Depending on the relation between the plasma velocity $v$ and the alfvenic one $C_A$ (corresponding to the longitudinal magnetic field), steady-state MHD flows in an accelerator channel belong to one of the two classes [3]: superalfvenic ($v > C_A$) or subalfvenic ($v < C_A$). A general classification of steady-state flow types in a coaxial channel of a plasma accelerator is provided on figure 1 [4].

The notion of the transalfvenic flows in which the transition through the alfvenic velocity $C_A$ takes place is introduced in the present paper. In this case, combined flow regimes simultaneously containing areas of superalfvenic and subalfvenic classes are realized in an accelerator channel.
Figure 1. Classification of steady-state flows in plasma accelerator with longitudinal field

1.2. Objective of the work
The main attention of the previous researches was focused on the study of the superalfvenic flow regime with acceleration (type 3 on figure 1). The analyses of how electrode geometry and a longitudinal magnetic field influence on properties of this flow type are presented in [5, 6].

In the paper [7], features of the relaxation process and characteristics of the alfvenic regime (type 5) in which the plasma velocity coincides with the alfvenic one along the channel had been determined. This flow type can exist only in a narrow tube with a horizontal boundary.

The objective of the present work is the research of the previously unexplored transalfvenic flows with the transition through the alfvenic velocity in the accelerator channel space. Coaxial narrow nozzle-type tubes formed by close flow trajectories are considered, therefore the formulation of the corresponding problem is provided in the quasi-one-dimensional approach (see, e.g., [8]).

2. Mathematical model

2.1. Object of modelling
Preshaped electrodes connected to an external electric current source create the coaxial nozzle-type channel of the plasma accelerator (figure 2). Working medium in the plasma state enters the channel. Plasma as an electrically conductive medium completes the electric circuit formed, and the discharge current with the density $j$ arises in it. The electric current flowing through the anode creates the transverse magnetic self-field $H_\perp$. Thus, the Ampere force $F_\perp = (1/c) j \times H_\perp$ appears in the system and causes plasma acceleration along the channel axis. External electric current conductors induce the longitudinal magnetic field $H_\parallel$. It causes rotation of the plasma flux in the transverse direction of the channel with the force $F_\parallel = (1/c) j \times H_\parallel$.

Figure 2. Coaxial plasma accelerator scheme

Figure 3. Coaxial narrow flow tube scheme
2.2. Statement of the steady-state problem

The axisymmetric \((\partial/\partial \varphi = 0)\) MHD flows in narrow tubes with a ring cross-section \(S(z)\) formed by close trajectories of the flux motion (figure 3) in the plasma accelerator channel are considered.

Plasma enters the channel of the length \(L\) with the density \(\rho_0\), the temperature \(T_0\) and the pressure \(p_0\) in the absence of rotation \(\nu_0 = 0\). The azimuthal magnetic field \(H_\varphi\) is specified in the accelerator input section. The longitudinal field can be defined by a some constant value at the channel entrance.

Steady-state flows of a continuous infinitely conducting medium consisting of ions and electrons with common macro-parameters in the quasi-one-dimensional approach are described by the following system of algebraic equations – conservation laws (in the dimensionless form):

\[
\rho \nu_z S = C_1, \quad H_z S = C_2, \quad \frac{p}{\rho^\gamma} = C_3, \quad \left(\frac{C_1 H_\varphi - C_2 \nu_\varphi}{\rho}\right) r = C_4,
\]

where \(S = r(z)h(z)\), \(W = \gamma p/(\gamma - 1)\rho\) – entalphy.

The system of algebraic equations (1) completely describes steady-state plasma flows in coaxial narrow tubes. The values of all constants, except \(C_1\), are determined by the input boundary conditions:

\[
\rho = 1, \quad T = 1, \quad H_\varphi = r_0/r(0), \quad \nu_\varphi = 0, \quad H_z = H_{z0},
\]

where \(r_0\) – the average accelerator channel radius.

According to the classification (figure 1), the flows of the superalfvenic subsonic (type 4) and subalfvenic supersonic (type 6) regimes are adjacent to the alfvenic velocity. Therefore, the transalfvenic flows located between these types are considered in the present paper.

These flow types are required an additional boundary condition at the channel exit to be defined. Since the main interest is focused on the process of the transition through the alfvenic velocity, it makes sense to specify the following relation at the tube exit:

\[
\nu_t = C_{Al} + \delta,
\]

where \(\nu_t = \nu_c \cos \alpha = \nu_c \sqrt{1 + r'(z)^2}\), \(C_{Al} = H_z \sqrt{1 + r'(z)^2}/\rho\) – the plasma velocity and the alfvenic one along the flow tube.

The value \(\delta\) acts as a parameter in the problem under consideration. It allows to vary the difference between the plasma velocity and the alfvenic one at the tube output.

It is known [4, 7] that in a coaxial narrow nozzle-type tube symmetrical relative to its center, distributions of the basic plasma parameters of the flow types 4 and 6 are also symmetrical. Therefore, the constant \(C_1\) (that equals to the flow input velocity to within a factor) is defined by the condition (3) on the plasma output velocity.

Excluding all the variables, except \(\rho\), the system (1) is reduced to the single algebraic equation:

\[
\Phi(\rho, z, C_1) = F_0(\rho, z, C_1) + F_1(\rho, z, C_1) - \frac{C_1^2 (1 + r'(z)^2)}{2S^2(z)} = 0,
\]

where

\[
F_0 = \rho^2 \left( C_6 - \gamma^{-1} C_5 \rho^{-1} - \rho \left(\frac{C_5}{C_1}\right)^2 r(z)^2 \right)
\]

\[
F_1 = \left(1 - k^2 \rho^2 \right) \left( - C_4 - C_2 r(z) - \rho r(z) C_5 \frac{C_4}{C_1} + \rho r(z) C_5 \frac{3}{2} \rho k^2 \right)
\]

After solving the equation (4), all other flux parameters are determined through \(\rho(z)\) by using the equations (1).
3. Results
The results of the calculations are represented by two series, differing from each other by a value of the longitudinal field and by geometry of the flow tubes identified in the channels of two different configurations: convex outward and inward relative to the channel axis (figure 4). The dimensionless parameter, the ratio of the characteristic gas and magnetic pressures, $\beta = 0.05$ corresponds to a strong azimuthal field and electric current.

Figure 4. Narrow nozzle-type flow tubes convex outward (a) and inward (b) relative to $z$–axis

3.1. Plasma parameters in the transition point through the alfvenic velocity and its place
The function $\Phi(\rho, z, C_s)$ has a pole of order 2 in the transition place through the alfvenic velocity. A numerical solution of the equation (4) therefore doesn’t exist in this point.

Operating with the system (1), the analytical expressions for the desired plasma flux parameters in the transition point had been obtained:

$$\rho_{\text{alf}} = \frac{1}{k^2}, \quad \nu_{\text{alf}} = k \frac{C_2}{S(z_{\text{alf}})}, \quad \nu_{\phi \text{alf}} = k \left( \frac{C_4}{C_5 r(z_{\text{alf}})} + H_{\text{alf}} \right)$$

$$H_{\phi \text{alf}} = \frac{1}{k} \sqrt{2C_6 - k^2 \left( \frac{C_2}{S(z_{\text{alf}})} \right)^2 \left( 1 + r'(z_{\text{alf}})^2 \right) + \left( \frac{C_4}{C_5 r(z_{\text{alf}})} \right)^2 + 2C_3 \frac{\gamma}{(\gamma-1)k^{\gamma}}}$$

The geometric place of the transition is defined as follows:

$$r(z_{\text{alf}}) = \sqrt{-k \frac{C_4}{C_5}} \Rightarrow z_{\text{alf}} = f^{-1} \left( \sqrt{-k \frac{C_4}{C_5}} \right), \text{ where } r = f(z)$$

3.2. Narrow flow tube convex outward
In this section, the results of the calculations of the transalfvenic flows in the narrow flow tube convex outward (figure 4a) are presented.

Positive values of $\delta$ correspond to the superalfvenic subsonic flow regime (type 4) with distributions of plasma parameters symmetrical relative to the tube center. Decrease of $\delta$ results in the plasma velocity decreasing along the channel.

When $\delta < 0$, the flow transition through the alfvenic velocity takes place. The areas of the subalfvenic regime begin to form in the input and output sections of the tube. With increasing $\delta$ towards negative values, the subalfvenic areas expand to the channel center until the flow type will not be completely changed to the subalfvenic supersonic one (type 6).

It is important to note that the opportunity of the transition through the alfvenic velocity is entirely based on curvilinear geometry of the tubes under consideration. In a narrow tube with a horizontal boundary, this transition is impossible. In this case, the plasma velocity can only coincide with the alfvenic one along the entire channel length, and so-called alfvenic regime (type 5) is realized.
The plasma density and the azimuthal magnetic field, meanwhile, increase in the channel center, but the rotational velocity decreases (figure 6).

3.3. Narrow flow tube convex inward
In this section, the reverse geometry of the tube is considered: now, it is convex inward relative to the channel axis (figure 4b).
The plasma density and rotational velocity increase in the central part of the channel (figure 7), while the azimuthal field is remained almost unchanged (i.e. insignificantly decreases).

Contrary to the previous case, while reducing $\delta$, i.e. reducing the output velocity, the transition through the alfvenic velocity initially takes place in the tube center. The flow regime changing arises here and then symmetrically moves to the tube boundaries (figure 8).

![Figure 8. Process of transition through alfvenic velocity in narrow tube convex inward](image)

4. Conclusions

In the present study, the results of the theoretical and numerical researches of transalfvenic flows in coaxial channels of plasma accelerators with a longitudinal magnetic field are presented. The steady-state quasi-one-dimensional MHD model in narrow flux tubes bounded by close flow trajectories is considered.

The following results are highlighted as the most significant:

- The analytical expressions for the plasma parameters in the transition point through the alfvenic velocity and for its geometric place are obtained;
- The characteristics of the transalfvenic flows in the coaxial narrow nozzle-type tubes of the two configurations are numerically studied.

Acknowledgments

The reported study was funded by RFBR according to the research project № 18-31-00351.

References

[1] Morozov A I 1957 *JETP* **5** (2) 215–20
[2] Artsimovich L A, Luk’ianov S Iu, Podgornyj I M, Chuvatin S A 1958 *JETP* **6** (1) 1–5
[3] Morozov A I and Solov’yev L S 1980 vol 8 ed by M A Leontovich *Reviews of Plasma Physics* (NY, London: Consultants Bureau) pp 1–103
[4] Brushlinskii K V 2009 *Mathematical and Computational Problems in Magnetohydrodynamics* (Moscow: Binom) [in Russian]
[5] Brushlinskii K V and Styopin E V 2017 *J. Phys.: Conf. Ser.* **937** 012007
[6] Brushlinskii K V, Zhananova N S, Stepin E V 2018 *Comp. Math. and Math. Phys.* **58** (4) 593–603
[7] Styopin E V 2015 *J. Plasma Phys.* **81** 905810309
[8] Brushlinskii K V and Styopin E V 2017 *J. Phys.: Conf. Ser.* **788** (1) 012009