A model for precessing helical vortex in the turbine discharge cone

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Abstract. The decelerated swirling flow in the discharge cone of hydraulic turbine develops various self-induced instabilities and associated low frequency phenomena when the turbine is operated far from the best efficiency regime. In particular, the precessing helical vortex (“vortex rope”) developed at part-load regimes is notoriously difficult and expensive to be computed using full three-dimensional turbulent unsteady flow models. On the other hand, modern design and optimization techniques require robust, tractable and accurate a-priori assessment of the turbine flow unsteadiness level within a wide operating range before actually knowing the runner geometry details. This paper presents the development and validation of a quasi-analytical model of the vortex rope in the discharge cone. The first stage is the computing of the axisymmetrical swirling flow at runner outlet with input information related only to the operating point and to the blade outlet angle. Then, the swirling flow profile further downstream is computed in successive cross-sections through the discharge cone. The second stage is the reconstruction of the precessing vortex core parameters in successive cross-sections of the discharge cone. The final stage lies in assembling 3D unsteady flow field in the discharge cone. The end result is validated against both experimental and numerical data.

1. Introduction

The vortex rope in a hydro turbine draft tube is one of the main and strong sources of pressure pulsations in non-optimal modes of hydro turbine operation. The most comprehensive analysis of the pulsations phenomena was done by Dörfler et al. [1]. The development of the vortex rope in the turbine discharge cone is directly related to instability of the decelerated swirling flow downstream the turbine runner. This conclusion was supported in recent papers by Ruprecht et al. [2], Sick et al. [3], Ciocan et al. [4], Zhang et al. [5] devoted to the three-dimensional unsteady computations of the flow in hydro turbine. As was shown, flow at runner outlet is practically axi-symmetric, but due to deceleration in the turbine discharge cone a swirling becomes unstable, with the development of a precessing helical vortex. Although such numerical simulations are in good agreement with experimental data for both unsteady pressure and velocity fields, the computing efforts are too large.
both in terms of computing resources and computing time for turbine design and especially for optimization purposes. A more attractive approach for investigating the swirling flow in the turbine discharge cone is to use a simplified axi-symmetric flow model since the geometry of conical part of the draft tube is axial-symmetric. Such approach developed in Susan-Resiga et al. [6] showed that calculated turbulent, axi-symmetric swirling flow can recover a circumferentially averaged flow field, in very good agreement with experimental data for axial and circumferential velocity components. Moreover, this simplified model captures a central stagnant region in agreement with the qualitative model proposed by Nishi et al. [7], and the vortex rope is located in the region of large shear between the stagnant region and the annular swirling flow. In comparison with three-dimensional unsteady flow simulation, the axi-symmetric flow computation is obviously several orders of magnitude less expensive in terms of computing time and resources. However, the main drawback of this approach is that the circumferentially averaged flow provides a steady velocity and pressure fields, without any direct indication on the real flow unsteadiness. A stability analysis of this axi-symmetric base flow could eventually provide additional information with respect to the most amplified perturbations and their dominant frequency, but it cannot estimate the level of pressure fluctuations associated with the flow unsteadiness.

The theory of precessing helical vortices in swirling flows was developed by Alekseenko et al. [8, 9] up to analytical solutions for velocity and pressure fields in cylindrical geometry. A further step toward practical applications in hydraulic machines is presented in [10], where it is shown that the vortex rope geometry, precession frequency, as well as the wall pressure fluctuations can be computed given a set of swirling flow integral quantities.

In hydraulic turbines the liquid leaving the runner enters conical part of the draft tube. When the vortex rope behind the runner takes helical form, its supporting surface looks like cone rather than cylinder. Okulov [11] extends approach [8, 9] to a conical vortex, for a small opening angle, and found an approximate solution for the velocity field induced by the conical helical vortex filament. There were no validations made. Moreover the cone angle limitation is a strong constraint to applying this model to turbine vortex ropes.

In the present paper we suppose that the model of cylindrical helical vortex is valid for every successive cross-sections of the discharge cone providing conservation of the integral flux parameters (first of all, flow rate and flux of vorticity). Of course, the precession frequency should be the same for all cross-sections.

2. Axisymmetric swirling flow computation

The axisymmetric swirling flow model is employed by Susan-Resiga et al. [6] to investigate the swirling flow downstream a Francis runner, when the turbine is operated at partial discharge corresponding to 70% best efficiency discharge. It is shown that when a stagnant region model is implemented on top of the FLUENT 2D axisymmetric turbulent swirling flow solver, the numerical results match the time-averaged experimental data. Figure 1 shows a detail of the computational domain corresponding to the actual discharge cone. The inlet section is located immediately downstream the runner blades trailing edge, and the inlet velocity field is obtained from a 3D runner flow computation. Also, we show in figure 1, with dashed lines, the six survey sections further used for vortex rope reconstruction. The computed axisymmetric velocity and pressure fields practically correspond to the circumferentially averaged three-dimensional flow with precessing vortex rope. Figure 1 displays the Stokes’ streamlines, showing that the flow is confined in an annular region close to the cone wall, while a central stagnant region is developed. The methodology developed in this paper takes such 2D axisymmetric swirling flow fields as input, and reconstruct the three-dimensional vortex rope using the axial and radial velocity profiles.

The numerical results are presented and further used in dimensionless form, using as reference length the model runner outlet radius $R_{ref} = 0.2 m$, and as reference velocity the transport velocity $\Omega \Omega_{ref}$, where the runner angular velocity is $\Omega = \pi/30$ with $n = 750 rpm$. 

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Figure 1. Axisymmetric swirling flow computation in the discharge cone. Left half-plane: computational domain in the meridian half-plane and the survey sections (dashed lines; \(z = 1.25, 1.50, 1.75, 2.00, 2.25\) and \(2.50\)) for vortex rope reconstruction. Right half-plane: streamlines for the meridian velocity components and the central stagnant region.

Figure 2 shows an example of swirling flow configuration for a Francis turbine operated at 70\% the discharge at best efficiency point, and nominal head. We can conclude that the simplified mathematical model captures the main features of the swirling flow at runner outlet. First, it correctly predicts the extent of the stalled region. Second, it agrees well with the axial and circumferential velocity profiles. Third, we can provide a good approximation of the swirl at runner outlet, without actually computing the runner flow, thus evaluating the flow behavior in the discharge cone for a wide range of operating range. This is quite useful in the early design and optimizations stage, since it provides a quantitative tool to assess various design choices with respect to the blade trailing edge.

Further, in section 3 we analyze the calculated velocity profiles in the discharge cone successive cross-sections, shown in figure 1, to reconstruct the vortex rope characteristics.

3. Precessing vortex rope
The axi-symmetric approach can correctly predict the circumferentially averaged velocity profiles, but it cannot describe three-dimensional vortex structure arising behind the turbine runner operating at partial load. To construct a proper model one can use the theory of helical vortices [8, 9]. Kuibin et al. [10] demonstrated advantages of this theory by finding both the frequency and amplitude of pressure pulsations generated by the precessing vortex rope. The problem solution was based on given integral characteristics: vortex intensity, liquid flow rate, momentum and moment of momentum fluxes. Here we present a model for describing the vortex rope in conic part of the hydroturbine draft tube taking into account both conical form of the helix supporting surface and expanding conical tube itself.
Instead of using the integral fluxes we will estimate the geometrical and dynamical vortex characteristics from direct comparison of the measured circumferentially averaged velocity profiles with dependencies inherent to the model of helical vortex as well as known experimental data on the helix form.

3.1. Model of helical vortex with cylindrical supporting surface

The model of helical vortex developed in [8, 9] allows presentation of averaged circumferential, $u_\theta$, and axial, $u_z$, velocity profiles, dependent on the radial coordinate $r$, through a single function $G(r)$:

$$u_\theta = \frac{\Gamma}{2\pi r} G(r), \quad u_z = u_{z\text{axis}} + \frac{\Gamma}{2\pi l} G(r), \quad G(r) = \frac{1}{S_z} \begin{cases} 0, & r < r' \\ 1, & r \geq r' \end{cases} dS . \quad (1)$$

Here $h = 2\pi l$ is the helix pitch, $\Gamma$ is the vortex intensity and $u_{z\text{axis}}$ is the value of axial velocity at $r = 0$. For helix (of radius $a$) presented by tube of circular cross-section the integral in (1) equals the intersection area of the circle of radius $r$ with the horizontal cross-section $S_z$ of the helical tube. The ratio of areas in (1) does not change, if both figures are projected on the plane normal to the helical axis of the vortex. The core cross-section in this plane is the circle of radius $\epsilon$ and instead of the circle of radius $r$ we obtain the ellipse prescribed in polar coordinates $(\sigma, \theta)$ by the formula

$$(a + \sigma \cos \theta)^2 / l^2 + (\sigma \sin \theta)^2 / (a^2 + l^2)^2 = r^2 / l^2 .$$

Therefore, $G(r) = S^{(0)}/\pi \epsilon^2$, where $S^{(0)}$ is the intersection area of this ellipse with the circle $\sigma = \epsilon$.

Comparison of the model profiles (1) with the measured or calculated velocity profiles yields the vortex rope parameters: $\Gamma$, $l$, $a$, $\epsilon$ and $u_{z\text{axis}}$. The result of such procedure made in [12] with found parameters $\Gamma = 0.531$, $l = 0.311$, $a = 0.349$, $\epsilon = 0.126$, $u_{z\text{axis}} = 0$, is shown in figure 3. One can see here good approximation of the inner flow structure by the model.

![Figure 3](image)

Figure 3. Axial and circumferential velocity profiles at the runner outlet and computed with the model of helical vortex. Experimental data are the same as in figure 2.

3.1.1. Frequency of helical vortex precession. Now we can calculate the frequency of the vortex precession with formula derived by Kuibin and Okulov [13] rewritten here in the following form

$$\omega = \frac{u_{z\text{axis}}}{l} - \frac{\Gamma}{2\pi l^2} \left[ \left( \frac{1}{a^2} - 1 \right) - \frac{3r}{12} \frac{r^3}{r' \eta^3} + \frac{9\eta^3 r}{\eta^3} \log \left(1 - \tilde{\omega}^2\right) \right] \left(1 + r^2\right) \left[ \frac{r^2}{12} + \frac{r}{2r'} \left[ g_1(r) + \log \left(\frac{\|\|}{\epsilon} + g_2(\chi)\right) \right] \right] .$$

\[ (2) \]
where \( g_i(\tau) = \frac{1 + 1.455 \tau + 1.723 \tau^2 + 0.711 \tau^3 + 0.616 \tau^4}{\tau + 0.486 \tau^2 + 1.176 \tau^3 + \tau^4} - \frac{1}{4}. \)

The parameter \( \tau = l/a \) means the relative torsion of a helical line; \( \tau' = (1 + \tau^2)^{1/2}, \eta = l/r, \eta' = (1 + \eta^2)^{1/2}, \bar{a} = \exp[(\tau' - \tau)/(\eta' - \eta)/(\tau' + \tau)] \). The function \( g_2(\chi) \) describes the impact into the frequency from the non-uniform vorticity distribution in the core,

\[
g_2(\chi) = \frac{1}{4} - \frac{4\pi^2}{R^2} \int_0^\pi \sigma w^2 d\sigma,
\]

when a fraction \( \chi \) of the total vorticity is concentrated inside the core. Here \( w \) denotes the local circumferential velocity in the vortex core. At uniform vorticity distribution \( g_2(\chi) \equiv 0 \). For the Scully vortex [9] with local distribution of type \( (r^2 + \epsilon^2)^{-2} \) we have

\[
g_2(\chi) = \frac{1}{4} + \frac{\chi}{2} + \frac{1}{2} \log(1 - \chi).
\]

One can consider some other structures of the vortex core. Nonetheless, the Scully vortex is the most appropriate model for highly turbulent flow [14] which takes place in the hydroturbine draft tube.

For the helix’s parameters determined in [12] we find that the uniform core frequency equals 0.184, much smaller than the measured dimensionless frequency \( \omega_{\text{cope}} = 0.3 \). In the same time applying equation (3) with 90% of the total vorticity in the core yields \( \omega_{\text{cope}} = 0.305 \), which is practically the same as in the experiment [4]. Note that the dimensionless rope frequency is defined here as \( \omega_{\text{cope}} = \Omega_{\text{cope}} / \Omega \), i.e. it represents the ratio between the rope precession angular speed and the runner angular speed.

3.1.2. Pressure oscillations induced by the helical vortex. The model approach for evaluation of amplitude of the pressure pulsations due to precession of the helical vortex in cylindrical tube was developed by Kuibin et al. [10]. Authors used assumptions on inviscid flow in the tube in the presence of helical vortex with uniform core. This allowed integrating the Euler equations and to derive the analytical relation between the pressure distribution and the velocity field and stream function. In particular, the following formula for pressure pulsations at the tube wall was obtained

\[
\tilde{p} = \text{const} - \left( \omega_\text{v}_z + \frac{v^2 + v^2}{2} \right)_{\theta=0}.
\]

The final formula for peak-to-peak dimensionless pressure pulsations reads

\[
\Delta \tilde{p} = \frac{\Gamma^2}{2\pi^2} \left[ \frac{2\bar{a}}{1 - \bar{a}^2} - g_1(\tau, \eta) \log \left( 1 - \bar{a} + \frac{1}{1 + \bar{a}} \right) \left( \frac{2\pi^2}{\Gamma} \omega + \eta + \eta' + \sqrt{\eta^2 + \eta^2} \right) \right] = \frac{2\bar{a}}{1 - \bar{a}^2} - g_1(\tau, \eta) \log \left( 1 - \bar{a}^2 \right),
\]

where \( g_1(\tau, \eta) = \frac{1}{24} \left[ \frac{7\tau^3}{\tau^4} - \frac{9\tau}{\eta^2} + \frac{9\eta^3}{\eta} \right]. \)

The result of calculation with these formulae for the case considered above yields \( \Delta \tilde{p} = 0.027 \). This value can be compared with measurement. Taking into account that \( \Delta \tilde{p} = 0.5 \varphi^2 \Delta \epsilon_p \), for the discharge coefficient \( \varphi = 0.264 \) and evaluated from experimental data \( \Delta \epsilon_p \equiv 0.9 \) we obtain the experimental value \( \Delta \tilde{p}_{\text{exp}} = 0.028 \), in excellent agreement with prediction in the model.

3.2. Model of helical vortex with conical supporting surface
The swirling flow with vortex rope modeled in this paper has been investigated experimentally by Ciocan and Iliescu [15] using a 3D-PIV method. They identified the vortex rope core from velocity measurements and proposed a geometrical description as a conical logarithmic spiral of the vortex
rope shape, in very good agreement with experimental data. As one can see from figure 4 the actual vortex rope develops in the draft tube cone, and it is wrapped on a cone with half-cone angle of $\gamma_{\text{rope}} = 17^\circ$. This cone originates on the runner crown, where it has a radius of $r_0 = 0.09$ for the axial coordinate value $z_0 = -0.615$. As a result, the distance between the vortex rope core and cone axis (the rope radius) increases downstream as

$$r_{\text{rope}}(z) = r_0 - (z - z_0) \tan \gamma_{\text{rope}}$$  \hspace{1cm} (4)

The discharge cone shown in figure 4 starts at $z = -1$ and ends at $z = -1 - \sqrt{3} \approx -2.732$, and has a half-cone angle of $\gamma_{\text{cone}} = 8.5^\circ$, [6, 15]. For the survey section located at $z = -1.426$ where velocity measurements shown in figure 2 were performed, the dimensionless radial distance of the vortex rope core from the axis is $r_{\text{rope}} = 0.338$, quite close to the stagnant region radius in the simplified model $r_s = 0.363$. This confirms the conclusion of Susan-Resiga et al. [6] that the vortex rope is located at the boundary between the main swirling flow and the central stagnant region.

The axial pitch of the conical vortex rope shown in figure 4 increases as the flow evolves downstream into the turbine discharge cone. From the mathematical model of the conical vortex rope [15] we find that the pitch increases with $z$ as

$$h_{\text{rope}}(z) = r_{\text{rope}}(z) \ln(b) / \tan \gamma_{\text{rope}},$$  \hspace{1cm} (5)

where $b = 3.2$ is the rate of radial growth for a complete rotation. With the data above, the rope pitch increases linearly as $h_{\text{rope}}(z) = |0.373 + 1.163z|$. Constructing the model of conical helical vortex is based on the velocity field calculation [6] with the axisymmetrical model in successive cross-sections of the discharge cone. The results of analysis are presented in tables 1, 2. The value of velocity at axis $u_{\text{axis}} = 0$ for all cross-sections considered. Flow rate $Q$ corresponds to flow induced by the helical vortex. It slightly differs from the flow rate determined on the base of calculated velocity profiles ($Q_{\text{calc}} = 0.808$). Frequency $\omega$ is calculated in accordance with equation (2) for the model of vortex with uniform vorticity distribution in the core ($g_2 \equiv 0$). To provide the same frequency in all cross-sections, moreover considering frequency measured in experiment [4] $\omega = 0.3$ we take into account smooth vorticity distribution with parameter $\chi$ denoting part of vorticity concentrated inside the core of radius $\varepsilon$. The required values of $\chi$ are shown in the last column of table 1. It looks like diffusion of the vorticity concentration downstream the cone. The dependencies from table 1 are presented graphically in figure 5. Finally, we can compare the radius $a$ of helix as well as its pitch $h$ with experimental data on vortex rope [4] (see figure 6). As seen, the model data overestimate the measured ones. Nonetheless, taking into account simplified calculations and idealized model of conical helical vortex such qualitative and quantitative correspondence should be considered as very good.
Table 1. Parameters of the helical vortex determined in the successive cross-sections of the discharge cone.

| \( x \) | \( l \) | \( a \) | \( \varepsilon \) | \( \Gamma \) | \( Q \) | \( \omega \) | \( \chi \) |
|-------|------|------|------|------|------|------|------|
| 1.25  | 0.346| 0.349| 0.114| 0.100| 0.858| 0.155| 0.909|
| 1.50  | 0.354| 0.436| 0.128| 0.103| 0.873| 0.181| 0.902|
| 1.75  | 0.364| 0.520| 0.144| 0.105| 0.870| 0.207| 0.890|
| 2.00  | 0.394| 0.600| 0.167| 0.113| 0.858| 0.227| 0.879|
| 2.25  | 0.422| 0.671| 0.195| 0.120| 0.847| 0.250| 0.852|
| 2.50  | 0.458| 0.736| 0.227| 0.129| 0.831| 0.269| 0.817|

Figure 5. Evolution of the helix parameters in successive cross-sections of the discharge cone.

Figure 6. Comparison of the helix parameters obtained in the model (circles) and parameters of vortex rope in experiment [4] (lines).

4. Conclusions

The approach developed in this paper presents next step in creating new instruments for assessment of turbine behavior far from the design operating point, in the early design stage, with several orders of magnitude smaller computing effort than the full three-dimensional unsteady flow simulation.

First, we use axisymmetrical approach for computing the swirling flow at Francis turbine runner outlet when turbine operates at partial discharge. We obtain the axial and circumferential velocity profiles in successive cross-sections of the discharge cone, as well as the radial extent of the central quasi-stagnant region, in good agreement with the experimental data available for a Francis model turbine. It is remarkable that we do not need to actually compute the three-dimensional flow in the turbine runner in order to assess the swirling flow at runner outlet.

Second, we employ a precessing helical vortex model fitting the model velocity profiles into averaged axial and circumferential velocities plots taken in every successive cross-sections of the discharge cone. Thus we determine helix parameters in every cross-sections and construct a model of “conical” helical vortex. This allows to assess the precession frequency and wall pressure fluctuation level associated with the vortex rope developed in the turbine discharge cone at partial discharge. We show that this helical vortex model can correctly predict the swirling flow unsteadiness, with respect to precession frequency and level of pressure fluctuation at the wall, in comparison with available experimental data.
Table 2. Comparison of the calculated axial and swirl velocity profiles with averaged velocities for model of helical vortex for six cross-sections within the discharge cone. Symbols correspond to calculation, lines - to model. Swirl velocity is plotted by boxes and solid lines, axial velocity - by circles and dashed lines.
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Nomenclature

Symbols

| Symbol | Description |
|--------|-------------|
| $a$ | Dimensionless distance from axis to vortex core center (radius of helix) |
| $b$ | Rate of helix radial growth for a complete rotation, dimensionless |
| $F,f$ | Flow force $[N]$, dimensionless |
| $G,g_1,g_2$ | Functions, dimensionless |
| $h$ | Dimensionless axial pitch of the vortex rope |
| $l$ | Dimensionless characteristic length for helical symmetry |
| $M,m$ | Flux of moment of momentum $[m^2/s^3]$, dimensionless |
| $P,p$ | Pressure $[Pa]$, dimensionless |
| $Q$ | Volumetric discharge $[m^3/s]$ |
| $R,r$ | Radius $[m]$, dimensionless |
| $U,u$ | Velocity $[m/s]$, dimensionless |
| $w$ | Local circumferential velocity in the vortex core, dimensionless |
| $\gamma$ | Dimensionless modified radial coordinate (dimensionless radius squared) |
| $\varphi$ | Discharge coefficient, dimensionless |
| $\beta$ | Relative flow angle |
| $\Gamma$ | Dimensionless vortex intensity |
| $\gamma$ | Half-cone angle, grad |
| $\varepsilon$ | Vortex core radius, dimensionless |
| $\Omega, \omega$ | Angular speed $[rad/s]$, dimensionless |
| $\psi$ | Energy coefficient, dimensionless |
| $\rho$ | Density $[kg/m^3]$ |
| $\sigma$ | Dimensionless local radius in the core |
| $\tau, \eta$ | Relative torsion of helical lines, dimensionless |
| $\chi$ | Fraction of the total vorticity concentrated inside the vortex core, dimensionless |

Subscript and superscript

| Subscript | Superscript | Description |
|-----------|-------------|-------------|
| 0 | cone | rope | Refers to swirl-free conditions, cone, rope |
| s | wall | w | Stagnant region, wall |
| ref | $\theta$ | Circumferential direction |

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