Proton radius puzzle and quantum gravity at the Fermi scale

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received 28 August 2013; accepted in final form 17 October 2013
published online 15 November 2013

PACS 04.60.Bc – Phenomenology of quantum gravity
PACS 12.10.-g – Unified field theories and models
PACS 31.30.jr – QED corrections (Lamb shift) in muonic hydrogen and deuterium

Abstract – We show how the “proton radius puzzle” emerging from the measurement of the Lamb shift in muonic hydrogen may be solved by means of a binding energy contribution due to an effective Yukawian gravitational potential related to charged weak interactions. The residual discrepancy from the experimental result should be mainly attributable to the need for the experimental determination of the gravitational radius of the proton. The absence of an analogous contribution in the Lamb shift of electronic hydrogen should imply the existence of generation-dependent interactions, corroborating previous proposals. Muonic hydrogen plays a crucial role to test possible scenarios for a gravitoweak unification, with weak interactions seen as manifestations of quantum gravity effects at the Fermi scale.

Hydrogen spectroscopy has played a major role for understanding the microscopic world in terms of quantum mechanics and quantum field theory [1]. Detailed studies of hydrogen have now reached an accuracy level limited by the proton size, expressed as the root-mean square charge radius \( r_p = \langle r^2 \rangle^{1/2} \). To test quantum electrodynamics at the highest precision level, the proton size should then be determined with high precision from independent experiments. In the analysis of electron scattering experiments, a value of \( r_p = (0.897 \pm 0.018) \) fm has been determined [2,3]. Higher-precision determinations are possible using muonic hydrogen [4]. Since the more massive muon has a smaller Bohr radius and a more significant overlap with the proton, the correction due to the finite size of the latter is more significant than in usual hydrogen. However, a recent measurement [5] reported a value of \( r_p = (0.84184 \pm 0.00067) \) fm, which differs by seven standard deviations from the CODATA 2010 value of \((0.8775 \pm 0.0051)\) fm, obtained by a combination of hydrogen spectroscopy and electron-proton scattering experiments. Barring back-reaction measurement effects on the proton radius due to the use of a different leptonic probe, the radius of the proton is expected to be an invariant, constant quantity, even considering the underlying assumed lepton universality for electromagnetic interactions. This has generated what is called the “proton radius puzzle” [6,7]. If the proton radius is kept at its CODATA value, this anomaly can be rephrased as if there is an excess of binding energy for the \( 2s \) state with respect to the \( 2p \) state equal to \( \Delta E_{2s-2p} = 0.31 \) meV, a 0.15% discrepancy on the Lamb shift theoretical expectation.

This anomaly has elicited a number of theoretical hypotheses, including some invoking new degrees of freedom beyond the standard model [8–11]. In this context, pioneering papers have already discussed high-precision spectroscopy as a test of extra-dimensional physics [12,13] both for hydrogen [14,15], helium-like ions [16], and muonium [17], giving bounds on the number of extra-dimensions and their couplings. Values for the coupling constant necessary to explain the proton radius puzzle in extra-dimensional models were determined in a recent paper [18]. The idea that extra-dimensions may be in principle tested with atomic physics tools is appealing also considering the paucity of viable experimental scenarios to test quantum gravity [19–21]. However, it would be most compelling to have a setting in which this may be achieved in an economic fashion, that is without necessarily introducing new free parameters conveniently chosen to accommodate a posteriori the experimental facts.

In this letter, we try to provide such an approach through a tentative unification between gravitation and weak interactions conjectured in [22]. We first discuss an
effective potential energy between two point-like masses which recovers Newtonian gravity at large distances while morphing into an inverse square-law interaction with strength equal to the one of weak charged interactions at the Fermi scale, coinciding with the Planck scale. We then generalize this gravitational potential to the case of an extended structure like the proton, and evaluate the gravitational contribution to the Lamb shift in muonic structure like the proton, and evaluate the gravitational contribution to the Lamb shift in muonic hydrogen below is the observed value within a factor of three, i.e. we derive \( \Delta E_{2s2p} = 0.106 \text{ meV} \) vs. the experimentally determined value of \( \Delta E_{2s2p} = 0.31 \text{ meV} \). One potential source of discrepancy between our prediction and the experimental result is then discussed in more detail. This is then followed by a qualitative discussion of the possible nature of the Yukawian potential of gravitowork origin, including its selectivity towards the flavor of the fundamental fermions. Finally we stress that a more accurate evaluation calls for the measurement of the gravitational radius of the proton, which is expected to significantly differ from the charge radius due to the gluonic energy density distribution for which no experimental access seems available.

While we refer to [22] for more details, we briefly recall here that the idea we have explored is that what we call weak interactions, at least in their charged sector, should be considered as empirical manifestations of the quantized structure of gravity at or below the Fermi scale. This opens up a potential merging between weak interactions and gravity at the macroscopic scale, a possibility supported by earlier formal considerations on the physical consequences of the Einstein-Cartan theory [23]. Various attempts have been made in the past to introduce gravitowork unification schemes [24–28], and a possible running of the Newtonian gravitational constant in purely four-dimensional models has been recently discussed [29,30]. The conjecture discussed in [22] relies upon identification of a quantitative relationship between the Fermi constant of weak interactions \( G_F \) and a renormalized Newtonian universal gravitational constant \( \tilde{G}_N \),

\[
G_F = \sqrt{2} \left( \frac{h}{c} \right)^2 \tilde{G}_N.
\]  

This expression holds provided that we choose \( \tilde{G}_N = 1.229 \times 10^{33} \) \( G_N = 8.205 \times 10^{22} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \). Equation (1) differs from eq. (2) in [22] since we have adopted in this letter a more rigorous definition of Planck mass as the one corresponding to the equality between the Compton wavelength and the Schwarzschild radius, i.e. \( h/(M_P c) = 2G_N M_P / c^2 \), the factor of 2 in the Schwarzschild radius having been omitted in the first analysis presented in [22].

The identification of the Fermi constant with a renormalized Newtonian universal constant via fundamental constants \( h \) and \( c \) allows to identify Fermi and Planck scales as identical, \( \tilde{E}_F = v \), where \( \tilde{E}_F \) and \( v \) are, respectively, the renormalized Planck energy and the vacuum expectation value of the Higgs field, sometimes called the Fermi scale, avoiding then any hierarchy issue. As discussed in [22], there are also a number of possible tests of this conjecture that can span a wide range of energy, from the ones involved in the search for gravitational-like forces below the millimetre range [31,32], to the ones explored at the Large Hadron Collider, with the spectroscopy of exotic atoms in between. We have envisaged in [22] the possibility that muonic hydrogen provides a suitable candidate, and this is now discussed in detail evaluating the generalized gravitational contribution to the Lamb shift in muonic hydrogen.

First, we need to interpolate between the two regimes of weak gravity at macroscopic distances and the conjectured strong gravity/weak interactions at the microscale. Newtonian gravitation at large distances with (coupling strength \( G_N \)) can morph into weak interactions corresponding to a renormalized universal gravitational constant \( \tilde{G}_N \) at small distances by means of a generalized potential energy \( \nu_{\text{eff}} \) for the gravitational interaction between two point-like particles of mass \( m_1 \) and \( m_2 \)

\[
\nu_{\text{eff}}(r) = -\frac{G_N m_1 m_2}{r} \left[ 1 + \left( \frac{\tilde{G}_N}{G_N} - 1 \right) e^{-r/\tilde{\Lambda}_P} \right],
\]  

where we have assumed as Yukawa range the renormalized Planck length \( \tilde{\Lambda}_P = \sqrt{2\hbar \tilde{G}_N / c^3} = 8.014 \times 10^{-19} \text{ m} \).

Equation (2) reduces to ordinary gravity for \( r \gg \tilde{\Lambda}_P \), whereas in the opposite regime of \( r \ll \tilde{\Lambda}_P \) continues to have a \( 1/r \) behaviour, but with coupling strength proportional to \( \tilde{G}_N \). In both limits, the evaluation in perturbation theory of the average gravitational energy should give no difference between the 2s and 2p states since any 1/r potential is degenerate for states with the same principal quantum number and different angular momenta. Therefore, the difference we may evidence in this analysis will reflect the genuine deviation from an inverse square law characteristic of Yukawian potentials. To simplify the notation, in the following equations we will consider the parameter \( \alpha = \tilde{G}_N / G_N - 1 \) and \( \lambda = \tilde{\Lambda}_P \), as customary in the analysis of Yukawian components of gravity [31,32]. Notice that the second term on the right-hand side of eq. (2) becomes comparable to the first one at a length scale \( R \) such that \( \tilde{G}_N / G_N e^{-R/\tilde{\Lambda}_P} \simeq 1 \), i.e. \( R \simeq \tilde{\Lambda}_P \ln(\tilde{G}_N / G_N) \simeq 76\tilde{\Lambda}_P \). Thus, the effect of the Yukawian component may be evidenced at length scales much larger than the renormalized Planck length \( \tilde{\Lambda}_P \), depending on the precision available in the specific experimental scheme used to test quantum gravity effects of this nature.

The evaluation of the gravitational energy between two particles is quantitatively different if one of them has an extended structure, as in the case of the proton. We schematize the proton as a spherical object with uniform mass density only within its electromagnetic radius \( R_p \), related to the rms charge radius through \( R_p = \sqrt{3/4 \pi} \).
assuming a uniform charge density inside the proton, \( \rho_p = 3m_p/(4\pi R_p^3) \). With this assumption, the evaluation of the Newtonian potential energy—the first term on the right-hand side of eq. (2)—between the proton and a generic lepton of mass \( m_\ell \) \((\ell = e, \mu, \tau)\) yields

\[
V_{NI}(r) = G_N \frac{m_\ell m_p}{2R_p^2} - \frac{3}{2} G_N \frac{m_\ell m_p}{R_p} \quad (0 < r < R_p),
\]

\[
V_{NI}(r) = -G_N \frac{m_\ell m_p}{r} \quad (r > R_p).
\]

The calculation of the energy contribution due to the Newtonian potential may be performed by means of time-independent perturbation theory applied to \( 2s \) and \( 2p \) states. Due to space isotropy we focus only on the radial (normalized) components of the unperturbed wave functions which are, respectively,

\[
R_{2s}(r) = \frac{1}{(2a_\ell)^{3/2}} \left( 2 - \frac{r}{a_\ell} \right) e^{-r/a_\ell},
\]

\[
R_{2p}(r) = \frac{1}{\sqrt{3}(2a_\ell)^{3/2}} e^{-r/\sqrt{3}a_\ell},
\]

where \( a_\ell \) is the Bohr radius, which also takes into account the reduced mass of the lepton-proton bound system.

By simple algebraic manipulations we obtain a compact expression for the Newtonian potential energy difference between the \( 2s \) and \( 2p \) states as

\[
\Delta(V_{NI})_{2s2p} = (V_{NI})_{2s} - (V_{NI})_{2p} = \frac{G_N m_\ell m_p}{R_p}
\times \left[ 6(e^\beta - 1)\beta^{-2} - 6\beta^{-1} - 3 - \beta - \frac{\beta^2}{4} \right] e^{-\beta},
\]

where we have introduced \( \beta = R_p/a_\ell \). It is easy to see that, in the \( \beta \to 0 \) limit of a point-like proton, \( \Delta(V_{NI})_{2s2p} \to G_N m_\ell m_p \beta e^{-\beta}/(20R_p) \to 0 \). For a proton of finite size, this contribution is different for hydrogen and muonic hydrogen since the gravitational mass \( m_\ell \) appears both as a factor and in the expression for the Bohr radius \( a_\ell \), on which \( \beta \) depends. We notice also that this contribution tends to increase the energy of the \( 2s \) state, as this state is more sensitive to the finite size of the proton with respect to the \( 2p \) state, analogously to the case of the Coulombian attraction. Moreover, it is easy to check that the difference is absolutely negligible in our context, being about 37 orders of magnitude smaller than the experimentally observed anomaly in muonic hydrogen, so it cannot play any role in its understanding. This is also consistent with a simple estimate obtained just considering a point-like proton. From the estimate of the absolute Newtonian potential energy \( V_N \approx G_N m_\ell m_p/a_\ell \) we obtain a value of \( V_N \approx 4.6 \times 10^{-34} \) eV for muonic hydrogen. The presence of an extended proton structure and the fact that the Lamb shift contribution is given by the difference between the potential energy in the \( 2s \) and \( 2p \) states further suppress the expected Newtonian contribution. However, this shows that by boosting the gravitational term using

\[
\tilde{G}_N \text{ instead of } G_N, \text{ i.e. by more than 33 orders of magnitude, the value of the estimate leads to } V_N \approx 565 \text{ meV}.
\]

Based on this promising estimate, we now evaluate the Yukawian component, i.e. the second term on the right-hand side of eq. (2). The first step is the evaluation of the effective potential due to the interaction of the muon with an extended proton. Assuming a semiclassical picture, we expect that if the muon is outside the proton by at least an amount \( \lambda \) there will be a negligible Yukawian gravitational contribution, and the same will occur if the muon is completely inside the proton, since the uniform distribution of the proton mass will exert isotropic interactions averaging out to zero. A non-zero value for the proton-lepton interaction instead occurs while the muon is partially penetrating inside the proton radius within a layer of order \( \lambda \), reaching its maximum intensity when the muon is located at the proton radius, i.e. when point P coincides with point K in fig. 1. The exact calculation for the Yukawian potential felt by the lepton should proceed by integrating the Yukawian potential contributions due to each infinitesimal volume element inside the proton. This calculation involves exponential integral functions and is not trivially performed analytically. A brute-force numerical integration is also subtle since the problem involves length scales, like \( \lambda, R_p \), and \( a_\ell \), differing by several orders of magnitude. We then proceed with an analytical evaluation by introducing two approximations. First, we truncate the Yukawian potential between two point-like masses at distance \( r \) in such a way that \( V_Y(r) = -\alpha G_N m_\ell m_p/\lambda \) if \( 0 < r < \lambda \) and \( V_Y(r) = 0 \) if \( r > \lambda \). This means that the evaluation of the potential is carried out only in the region of intersection between two spheres, one of radius equal to \( R_p \),
the other of radius equal to the Yukawa range $\lambda$. In using this approximation, the amplitude of Yukawian potential is then overestimated in the $0 < r < \lambda$ region, while it is underestimated in the region corresponding to $r > \lambda$. Second, we assume that the intersection region is a spherical cap, an approximation corresponding to $R_p \gg \lambda$, which we have found a posteriori well satisfied for the value of $\lambda$ able to justify the anomalous Lamb shift.

The potential felt by the lepton at a distance $r$ from the proton center is then evaluated by integrating the infinitesimal potential energy contributions over the volume of the spherical cap of the sphere of radius $\lambda$ centered on the lepton location. With reference to fig. 1, and using spherical coordinates for the infinitesimal volume $d\rho$, this leads to a Yukawian potential energy as follows:

$$V_Y(r) = -2\pi\alpha G_N m_p \rho^2,$$

$$V_Y(r) = \pi\alpha G_N m_p [(r - R_p)^2 + 2\lambda^2 (r - R_p) - \lambda^2],$$

$$V_Y(r) = \pi\alpha G_N m_p [(r - R_p)^2 - 2\lambda^2 (r - R_p) + \lambda^2],$$

$$V_Y(r) = 0,$$

respectively in the four regions $[0, R_p - \lambda], [R_p - \lambda, R_p], [R_p, R_p + \lambda]$, and $[R_p + \lambda, +\infty]$, with continuity of the Yukawa potential enforced for all boundaries. As discussed above, this potential corresponds to a net attractive force only in the range $R_p - \lambda < r < R_p + \lambda$.

We evaluate the expectation value of the Yukawian potential energy in the 2s and 2p states, $\langle V_Y \rangle_{2s}$ and $\langle V_Y \rangle_{2p}$, according to perturbation theory. The Yukawian component of the gravitational potential is still much smaller that the Coulombian potential even if it is coupled through $\tilde{G}_N$ at short distances, since $\tilde{G}_N m_p m_\ell \ll e^2/(4\pi\epsilon_0)$. The difference between the two contributions gives, after lengthy algebraic simplifications, the expression

$$\Delta(V_Y)_{2s2p} = \langle V_Y \rangle_{2s} - \langle V_Y \rangle_{2p} = -\frac{\alpha G_N m_p m_\ell}{16 R_p^3} \times \{ \lambda^2 f(y) + a_7^2 [g_+(y, \beta) + g_+(\beta, z)] \},$$

where $f(y) = 2 \beta^2 e^{-y} - \frac{1}{2} y^2 e^{-y}, g_+(a, b) = \sum_{n=0}^{a-1} h_n^+ G_n(a, b)^n$, $G_n(a, b) = a^n e^{-a} - b^n e^{-b}, y = (R_p - \lambda)/a_\ell, z = (R_p + \lambda)/a_\ell$, $y = \lambda/a_\ell$, and

$$h_0^+ = h_1^+ = \pm 2(12 \pm 2 \beta),$$

$$h_2^+ = 2(12 \pm 2 \beta),$$

$$h_3^+ = 2(12 \pm 2 \beta),$$

$$h_4^+ = 2(12 \pm 2 \beta),$$

$$h_5^+ = 2(12 \pm 2 \beta).$$

The overall contribution is negative, i.e. $\Delta E_{2s2p} < 0$, indicating that the 2s state gets more bounded than the 2p state. This outcome differs from the case of the long-range Newtonian component since in the latter case the 2s state, exploring more the proton interior, gets a weaker binding, as only the inner mass is relevant for the gravitational potential. This feature is not shared by the Yukawian potential as it does not fulfill the Gauss theorem, only the proton mass nearby the lepton matters regardless of its location with respect to the proton center of mass. We will indicate from now on the absolute value of $\Delta E_{2s2p}$ with the implicit understanding that it is negative.

The result of this analysis is shown in fig. 2, where $\Delta E_{2s2p}$ is plotted as a function of the Yukawa range $\lambda$ for a value of $\alpha$ corresponding to $\tilde{G}_N$. For a value of $\lambda = \tilde{\Lambda}_P$, we obtain a value of $\Delta E_{2s2p} \approx 0.106$ meV, about 2.8 times smaller than the measured value. The experimental value of $\Delta E_{2s2p}$ is instead obtained from eq. (8) by assuming a value of $\lambda = 1.35 \times 10^{-18}$ m at the effective coupling strength and proton radius assumed above. The prediction of the model for $\lambda = \tilde{\Lambda}_P$, corresponding to $\Delta E_{2s2p} = 0.106$ meV (blue square), is also shown. The prediction of the anomaly for muonic deuterium at the same $\lambda$ accommodating the anomaly for muonic hydrogen is $42.6 \mu$eV (green dot).
Fig. 3: (Color online) Locus in the \((\lambda, r_p)\)-plane of all solutions to proton radius puzzle, giving rise to a Lamb shift excess of \(0.309\ \text{meV} \leq \Delta E_{2s2p} \leq 0.311\ \text{meV}\), as shown by the monotonically increasing curve. The range of values allowed, within one standard deviation, for the proton radius according to the CODATA value is delimited by the two vertical lines, the horizontal line corresponding instead to \(\lambda = \tilde{\Lambda}_P\). The experimental value for the anomalous Lamb shift is reproduced, for instance, by choosing \(\lambda \approx 1.7\tilde{\Lambda}_P\), while having \(r_p\) at its CODATA value (solution A), or by choosing the gravitational radius of the proton equal to \(0.7\) times the CODATA value for the charge radius while keeping \(\lambda = \tilde{\Lambda}_P\) (solution B).

CODATA 2010 range of values for \(r_p\), and the horizontal line corresponds to the value of \(\lambda = \tilde{\Lambda}_P\). Exact validation of our model corresponds to a single intersection point among the three curves, which is not achieved within \(80\%\) in \(\lambda\) and \(40\%\) in \(r_p\).

Like in the Newtonian case, it is possible to give a “back-of-the-envelope” estimate of the effect which is consistent with the lengthy calculation resulting in equation (8). By thinking of a muon trajectory as in a semiclassical, Bohr-Sommerfeld approach, we argue that the \(2p\) state does not acquire basically any Yukawian contribution as the corresponding trajectory is far away from the proton radius. The muon in the \(2s\) state instead penetrates inside the proton, and then the Lamb shift basically coincides with the shift expected by the \(2s\) state alone. Due to the short-range nature of the Yuwawa potential, the estimate of the excess energy in the \(2s\) state due to the Yukawan component will involve the Yukawa range and the effective gravitational mass of the proton actually felt by the muon, in such a way that the absolute Yukawan potential energy in the \(2s\) state is \(V_Y \approx \tilde{G}_N m_p \, m_p^\text{eff} / \lambda\). The effective mass of the proton participating to the Yukawa interaction is only the one intercepted by the muon within its Yukawa range (in the truncation approximation we have adopted), \(m_p^\text{eff} \approx (\lambda / R_p)^3 m_p\), so we get

\[ V_Y \approx \tilde{G}_N \frac{m_p \, m_p^\text{eff} \lambda^2}{R_p^3} \approx 0.18 \text{ meV}, \tag{10} \]

which is within a factor less than two from both the comprehensive analytical evaluation and the experimental value of the anomaly.

All possible refinements of the calculations are limited by the fact that in this approach the knowledge of the mass density distribution is essential. In particular, no information is currently available on the density distribution for the gluonic fields, the most important component of the proton at the level of determining its mass. This, at the moment, seems the most critical issue preventing a more quantitative comparison of the model with the experimental result. It seems plausible that gluons, only sensitive to the attractive color interaction, tend to cluster more than valence quarks which are also sensitive to the electromagnetic interaction acting both attractively and repulsively depending on the quark flavors, thereby decreasing the effective gravitational radius of the proton below its electromagnetic value. A smaller gravitational radius could bring the prediction more in line with the experimental value assuming \(\lambda = \tilde{\Lambda}_P\) as visible in fig. 3.

In spite of the limitation arising from the lack of knowledge of the gravitational proton radius, it is worth proceeding with the analysis of other systems in which this putative Yukawian potential may also give rise to observable effects and predictions. As visible in fig. 2, the expected contribution in muonic deuterium is suppressed since the approximate doubling of the gravitational mass of the nucleus cannot compensate for the cubic dependence on the larger deuteron radius. By repeating the evaluation for the Lamb shift in (electronic) hydrogen, we obtain a value which is too large, since it corresponds to \(0.52\ \mu\text{eV}\) at \(\lambda = \tilde{\Lambda}_P\) and \(1.47\ \mu\text{eV}\) at \(\lambda = 1.35 \times 10^{-18}\ m\), if using the same CODATA 2010 proton radius. Basically, due to the fact that \(a_\ell \gg R_p, \lambda\) for both electrons and muons, the leading difference between electrons and muons is the direct effect of their gravitational mass, with the form factors due to the extended structure of the proton expressed through the functions \(f(y), g_{\pm}(a, b)\) in eq. (8) being quite similar. The gravitational contribution in electronic hydrogen is therefore suppressed with respect to the one of muonic hydrogen by their mass ratio. This is in principle a big issue for the validation of the proposed model. The Lamb shift in hydrogen is known with an extraordinary precision which cannot incorporate such a large contribution, corresponding to an anomalous frequency shift of about \(0.2\ \text{GHz}\) against an absolute accuracy of \(9.0\ \text{KHz}\), or \(8.5 \times 10^{-6}\) relative accuracy on the experiment-theory comparison.

Among possible ways to go around this issue while continuing to pursue this approach is to assume that the effective interaction corresponding to the Yukawan part in eq. (2) does not act among fundamental fermions belonging to the same generation. Such a flavor-dependent interaction naturally spoils the universality characteristic of gravitation. However, it should be considered that in a possible gravitoweak unification scheme, the emerging structure should presumably incorporate features of both
weak and gravitational interactions. The former interaction is manifestly flavor dependent, as shown in the presence of CKM and PMNS mixing matrices for the charged current, so it is not a priori impossible that the interaction corresponding to the Yukawa component in eq. (2) is highly selective in flavor content. Our assumption is aligned with recent attempts to justify the muonic hydrogen anomaly in terms of interactions differentiating between leptons, thereby violating their assumed universality [8–11]. This solution could inspire searches for models in which a “hidden sector” of the standard model includes intermediate bosons of mass in the range of the Higgs vacuum expectation value mediating interactions which have mixed features between the usual charged weak interactions and gravity, for instance heavier relatives of the Z° boson. A flavor-dependent interaction could also potentially contribute to the understanding of the mass difference between charged leptons, an unsolved puzzle since the famous question by Isidor Rabi.

In conclusion, we have shown that a solution to the “proton radius puzzle” is potentially available by means of an effective Yukawan potential originated by the muonic hydrogen anomaly in terms of interactions differentiating between leptons, thereby violating their assumed universality [8–11]. This solution could inspire searches for models in which a “hidden sector” of the standard model includes intermediate bosons of mass in the range of the Higgs vacuum expectation value mediating interactions which have mixed features between the usual charged weak interactions and gravity, for instance heavier relatives of the Z° boson. A flavor-dependent interaction could also potentially contribute to the understanding of the mass difference between charged leptons, an unsolved puzzle since the famous question by Isidor Rabi.

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