Simulation on Terminal Penetration Trajectory of Anti-ship Missile under Interception Conditions

Lin-ping FENG, Zuo-e FAN* and Yong-fang NIE
Department of Strategic Missile and Underwater Weapon, Navy Submarine Academy, Qingdao, China
*Corresponding author

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Abstract. When the anti-ship missile is attacked by the enemy's anti-aircraft missile, in order to compare the penetration effect of the anti-ship missile with snake maneuver and without end maneuver, the model of the anti-ship missile against the ship-to-air missile is established. In the background of interception and confrontation, the model considers the influence of many factors on the anti-ship missile, including interference, modeling error, sea condition, uncertainty, etc. At the same time, it also considers the forward angle constraint and dead zone constraint of the ship-to-air missile. Based on this model, the terminal penetration trajectory of anti-ship missile is simulated under different interception conditions. The simulation results show that the penetration effect of the anti-ship missile with longitudinal snake-like maneuver is the best, followed by the anti-ship missile with heading snake-like maneuver, and the penetration effect of the anti-ship missile without terminal maneuver is the worst.

Introduction

Anti-ship missile is the main weapon in modern naval warfare. Its tactical performance will directly affect the development of combat situation. With the advanced air defense weapon system equipped with surface warships, the air defense capability of warships is gradually enhanced. With the advanced air defense weapon system equipped with surface warships, the air defense capability of warships is gradually enhanced. At present, in some countries, anti-ship missiles have already possessed certain terminal maneuvering ability, such as Russian "white maggot" and Taiwan's "Xiongfeng II" anti-ship missiles, which can make snake-shaped trajectory maneuver several kilometers away from the target. Therefore, it is necessary to study the simulation of the anti-ship missile with terminal snake-like maneuver against the enemy ship-to-air missile under interception conditions, so as to study the penetration effect of terminal snake-like maneuver[1-2].

From the point of view of Engineering application, this paper establishes a snake-like maneuvering trajectory model of anti-ship missile terminal. Considering as many practical factors as possible, the terminal trajectory simulation of anti-ship missile is carried out in the case of ship-to-air missile interception, and the penetration effect of snake-like maneuvering trajectory and terminal non-maneuvering trajectory is compared.

Snake-like Maneuvering Trajectory Model

In order to realize the snake-like maneuver at the end of an anti-ship missile, it is necessary for the missile to have sufficient maneuverability to change the value and direction of flight speed. The maneuverability of missile is usually expressed by overload, so the theory of overload control can be used to control the maneuverability of missile well[3].

Reference [4] in this paper, the control system of anti-ship missile adopts overload control for terminal snake maneuver. The control signal includes overload control signal and cancroids control signal. The inner loop adopts overload control signal and the outer loop adopts cancroids control
signal. The two signals cooperate and coordinate to control the anti-ship missile to realize the terminal snake-like maneuvering trajectory.

**Cancroids Control Model**

The terminal maneuvering trajectory of an anti-ship missile is the trajectory of the mass center of the missile in the ground coordinate system \(O_{xyz}\), which can be controlled by the flight distance \(x\). The equation of terminal maneuvering trajectory can be described by variable \(x\).

\[
\begin{align*}
  x &= x \\
  y &= y_1 + l_y \left[ 1 + \sin \left( \frac{\xi_0 + k_y \pi}{A} \left( x - x_1 \right) \right) \right] \\
  z &= z_1 + l_z \cos \left( \frac{\xi_0 + k_z \pi}{A} \left( x - x_1 \right) \right)
\end{align*}
\]  

In the formula, \((x_1, y_1, z_1)\) is the initial position for the terminal maneuver of the anti-ship missile, \(\xi_0\) is the initial phase angle, here \(\xi_0 = -\pi/2\), \(l_y, l_z, k_y, k_z\) and \(A\) are parameters describing terminal maneuver, which are selected according to different maneuvering modes (course serpentine and longitudinal serpentine). \(l_y\) and \(l_z\) are the radii of projection curves of flight trajectories in longitudinal and heading planes, respectively. \(k_y, \pi\) and \(k_z, \pi\) are the phase change values of longitudinal and heading projection curves in a maneuvering cycle respectively. \(A\) refers to the distance of an anti-ship missile flying a maneuvering cycle along the ox axis of the ground coordinate system. \(n\) is defined as the number of maneuvering cycles that an anti-ship missile can complete in the terminal maneuvering range. Assuming that the anti-ship missile stops maneuvering when it flies to \(x = x_2\), there are \(n = (x_2 - x_1)/A\).

Eq. (1) shows that the maneuvering trajectory of an anti-ship missile is decoupled on both the longitudinal and the heading planes and expressed by trigonometric functions respectively. Different three-dimensional maneuvering flight trajectories are synthesized by choosing different parameters of trigonometric functions in two planes. Since the trajectory of the anti-ship missile in the course plane is the same as that in the longitudinal plane, this paper only establishes a snake-like maneuver cancroids control model in the course plane, that is \(l_y = 0\). Therefore, the desired centroid trajectory of a snake-like maneuver becomes

\[
\begin{align*}
  x^* &= x \\
  y^* &= y_1 \\
  z^* &= z_1 + l_z \cos \left( \frac{\xi_0 + k_z \pi}{A} \left( x - x_1 \right) \right)
\end{align*}
\]  

Eq. (2) is the centroid control signal of the anti-ship missile designed in this paper when it performs serpentine maneuver at the end of flight trajectory.

**Overload Control Model**

According to formula 1, the desired flight trajectory of anti-ship missile during snake maneuver can be obtained. Then, according to the relationship between the mass center and overload of the missile, the expected overload required by the missile at this time can be deduced. In this paper, to simplify the discussion, the following assumptions are made:

Hypothesis 1: The projection component \(R\) of the flight velocity of an anti-ship missile on the OX axis of the ground coordinate system is a constant.

Hypothesis 2: Ignore the interference on anti-ship missiles.
Under the above two assumptions, the flight velocity component of the anti-ship missile in terminal maneuver can be obtained by deriving Eq. (1)

\[
\begin{align*}
\dot{x} &= V_{mx} \\
\dot{y} &= V_{my} = 0 \\
\dot{z} &= V_{mz} = -\frac{1}{A} l_l k_l \pi V_{mx} \sin \left( \frac{\xi_0 + k_l \pi x - x_i}{A} \right)
\end{align*}
\]  

Eq. (3) continuing to derive the time, the components of the maneuvering acceleration of the anti-ship missile along the three axes of the ground coordinate system can be obtained, respectively

\[
\begin{align*}
a_x &= 0 \\
a_y &= 0 \\
a_z &= -l_z \left( \frac{k_l \pi V_{mx}}{A} \right)^2 \cos \left( \frac{\xi_0 + k_l \pi x - x_i}{A} \right)
\end{align*}
\]  

According to the definition of missile overload, \( n = N/G \), \( G \) is the gravity of the missile, \( N \) is the sum of all external forces (i.e. control forces) that a missile receives except gravity. Assuming that the maneuvering acceleration vector of the missile is \( \mathbf{a} = (N + G)/m \). The components of overload \( n \) along the axes of the ground coordinate system are as follows

\[
\begin{align*}
n_x &= N_x/G = a_x/g \\
n_y &= N_y/G = a_y/g + 1 \\
n_z &= N_z/G = a_z/g
\end{align*}
\]  

From Eq. (4) and Eq. (5),

\[
\begin{align*}
n_x &= 0 \\
n_y &= 1 \\
n_z &= -\frac{l_z}{g} \left( \frac{k_l \pi V_{mx}}{A} \right)^2 \cos \left( \frac{\xi_0 + k_l \pi x - x_i}{A} \right)
\end{align*}
\]  

Eq. (6) is the expected overload control signal \( n_{x2} \) and \( n_{z2} \) in the terrestrial coordinates system. There are the following transformations between the ground coordinate system and the ballistic coordinate system\(^5\)

\[
\begin{bmatrix}
n_x \\
n_y \\
n_z
\end{bmatrix}
= \begin{bmatrix}
\cos \theta \cos \psi_v & \sin \theta & -\cos \theta \sin \psi_v \\
-\sin \theta \cos \psi_v & \cos \theta & \sin \theta \sin \psi_v \\
\sin \psi_v & 0 & \cos \psi_v
\end{bmatrix}
\begin{bmatrix}
n_x \\
n_y \\
n_z
\end{bmatrix}
\]

Combined with \( n_x = 0 \), normal overload control signals \( n_y^* \) and \( n_z^* \) can be designed as follows:

\[
\begin{align*}
n_y^* &= n_y = n_z \sin \psi_v \\
n_z^* &= n_z = n_z \cos \psi_v
\end{align*}
\]  

Eq. (2) and Eq. (8) are the control models of anti-ship missile snake-like maneuvering trajectory. Under the interaction of two signals, the maneuvering trajectory of anti-ship missile can be realized. It should be noted that this paper mainly studies the terminal penetration trajectory simulation of anti-ship missile under interception conditions, rather than the accuracy of anti-ship missile hitting
target. Therefore, in this paper, the anti-ship missile is only guided to the target, not hit the target accurately.

**Interception Model of Ship-to-Air Missile**

**Relative Motion Model**

![Figure 1. Relative motion between anti-ship missile and ship-to-air missile.](image)

The three-dimensional interception relationship between anti-ship missile and ship-to-air missile is similar to the decoupling in the course plane and the longitudinal plane. In this paper, the relative motion relationship between them in the course plane is modeled, as shown in Fig. 1, the corresponding symbols are defined as follows\(^8\). The coordinate system OXYZ is the terrestrial coordinate system. \((x \ y \ z)\) and \((x_I \ y_I \ z_I)\) are coordinate positions of anti-ship missile M and ship-to-air missile I. \(R_{wz}\) is the distance between the missile and the target. \(q_{wz}\) is the line-of-sight angle. \(v\) and \(v_I\) are the velocities of anti-ship missiles and ship-to-air missiles, respectively. \(\psi_M\) and \(\psi_I\) are respectively the trajectory deflection angles of anti-ship missiles and ship-to-air missiles, which are defined as counterclockwise positive. \(\eta_M\) and \(\eta_I\) are the leading angles of anti-ship missiles and ship-to-air missiles respectively.

According to Fig. 1, the relative motion models of anti-ship missile and ship-to-air missile in the course plane are established as follows:

\[
\dot{R}_{wz} = v \cos(q_{wz} - \psi_M) - v_I \cos(q_{wz} - \psi_I) \tag{9}
\]

\[
R_{wz} \dot{q}_{wz} = v_I \sin(q_{wz} - \psi_I) - v \sin(q_{wz} - \psi_M) \tag{10}
\]

\[
R_{wz} = \sqrt{(x - x_I)^2 + (z - z_I)^2} \tag{11}
\]

\[
q_{wz} = \pi - \arctan\left(\frac{z - z_I}{x - x_I}\right), (x < x_I) \tag{12}
\]

When \(x > x_I\), that is, when the ship-to-air missile is tail-tracking and intercepting the anti-ship missile

\[
q_{wz} = -\arctan\left(\frac{z - z_I}{x - x_I}\right) \tag{13}
\]

**Guidance System Model**

Since we can not obtain the specific tactical decision of the enemy ship-to-air missile intercepting our anti-ship missile, the following assumptions are made for the guidance system of the ship-to-air missile:

1) Enemy Ship-borne Radar can always reliably detect and track anti-ship missiles entering the radar horizon.

2) Ship-to-air missiles can always intercept targets reliably;
3) Ship-to-air missiles are regarded as controllable particles.
4) The flight speed of the ship-to-air missile remains unchanged.

Based on the above assumptions, the centroid motion equation of ship-to-air missile is as follows:

\[
\begin{align*}
\dot{x}_t &= v_t \cos \theta_t \cos \psi_t \\
\dot{y}_t &= v_t \sin \theta_t \\
\dot{z}_t &= -v_t \cos \theta_t \sin \psi_t
\end{align*}
\]

The guidance system of ship-to-air missile is proportional guidance law, that is, the angular rate \( \dot{\psi}_t \) of missile velocity vector rotation is proportional to the angular rate \( \dot{\theta}_{sc} \) of line-of-sight angle rotation, as shown in Eq. (15)

\[
\dot{\psi}_t^* = N_1 \dot{\theta}_{sc}
\]

(14)

Among them, \( N_1 \) is the proportional guidance coefficient of ship-to-air missile, usually between 3 and 6.

The damage of ship-to-air missile to target depends on the warhead power and the vulnerability of anti-ship missile and other factors. In order to analyze the problem conveniently, this paper simplifies and judges whether the ship-to-air missile successfully intercepts the anti-ship missile, mainly by judging the miss distance \( R_m \). If \( R_m \leq R_e \) (\( R_e \) is the warhead killing radius of the ship-to-air missile), the ship-to-air missile intercepts successfully. In the process of studying the simulation of ship-to-air missile intercepting anti-ship missile, the following disturbances and errors are considered, including:

1) Heading error, that is, the difference between the actual heading angle and the ideal heading angle of the missile at any given time;
2) Seeker errors, including system errors caused by seeker installation errors, servo hysteresis, machining defects or manufacturing tolerances, and random errors caused by thermal noise, target echo fluctuations, multi-path and other reasons.

The above errors are simulated in the simulation process. The course error is simulated by adding or subtracting the predicted error in the ideal course of the missile. It is assumed that the line-of-sight angular velocity has errors and obeys normal distribution, and it is added to the theoretical calculation of line-of-sight angular velocity.

**Simulation**

When the anti-ship missile flies to 240 km along the Ox axis in the ground coordinate system, the snake maneuver is carried out with the radius \( R_z = 200 \text{m} \), and maneuver parameters \( A = 7000 \text{m} \). The initial phase angle of the anti-ship missile when it starts snake maneuver is \( \xi_0 = -\pi/2 \). Ship-to-air missile intercept anti-ship missile from position \( (x_I, z_I) \) at flight speed \( V_I = 850 \text{m/s} \), and the killing radius of the warhead of a ship-to-air missile is \( R_w = 5 \text{m} \).

Under the above simulation conditions, combined with the different initial conditions of ship-to-air missile, three kinds of anti-ship missile trajectory simulation are carried out, including heading snake-shaped maneuvering trajectory, longitudinal snake-shaped maneuvering trajectory and non-maneuvering trajectory. The simulation data are shown in Table 1 and the simulation curve is shown in Figure 2-3 (heading plane only).
Table 1. Miss distance of ship-to-air missile.

| Interception position | xl (km) | zl (km) | Miss distance of longitudinal snake-shaped maneuvering (m) | Miss distance of heading snake-shaped maneuvering (m) | Miss distance of non-maneuvering (m) |
|-----------------------|---------|---------|----------------------------------------------------------|-----------------------------------------------------|-------------------------------------|
| position I            | 265     | 1       | 18.455                                                   | 1.85                                                | 0.500                               |
| position II           | 244     | 19.66   | 13.453                                                   | 9.45                                                | 0.493                               |

From the simulation data, it can be seen that under the same simulation conditions, the miss distance of ship-to-air missile is quite different from that of anti-ship missile with or without terminal snake maneuver. When anti-ship missile carries out snake maneuver in the longitudinal plane, the miss distance of ship-to-air missile is the largest, and when it maneuvers in the course plane, the miss distance is the second. When the anti-ship missile does not maneuver at the end, the miss distance is the smallest. The miss distance of ship-to-air missile also proves the penetration effect of anti-ship missile from the reverse side. That is, under the same interception condition, the penetration effect of anti-ship missile with longitudinal snake-like maneuver is the best, followed by anti-ship missile with heading snake-like maneuver, and the penetration effect of anti-ship missile without terminal maneuver is the worst.

Summary

In order to compare the penetration effect of anti-ship missile with and without terminal snake maneuver, this paper establishes centroid control model and overload control model, combines them organically to form terminal snake maneuver trajectory model, and establishes a model of ship-to-air missile intercepting anti-ship missile. The influence of system disturbance, modeling error, sea condition, uncertainty and so on is considered, and the forward angle constraint and dead zone constraint of ship-to-air missile are also considered. Based on this confrontation model, under different interception conditions, the simulation study is carried out for the snake-like maneuver at the end of the interception belt and the non-terminal maneuver of the ship-to-air missile respectively. The simulation results show that the penetration effect of the anti-ship missile with longitudinal snake-like maneuver is the best, followed by the anti-ship missile with heading snake-like maneuver, and when the end is not maneuvering. The penetration effect is the worst.

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