Relativistic Shells and Gamma-Ray Bursts

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Abstract

In many models of Gamma-Ray Bursts (GRBs) relativistic shells are responsible for the overall envelope of emission. We use kinematics and symmetry to calculate the time history and spectral evolution expected from a relativistic shell including effects from intrinsic variations in the shell’s intensity and spectra. We find that the decay phase of an envelope is produced by photons delayed by the shell’s curvature. These delayed photons are produced by regions that are off-axis such that the spectra evolve according to a universal function ($\propto T^{-1}$) regardless of intrinsic variations in the rest frame of the shell. We compare our predictions to the overall envelope of emission of GRBs. The observed spectra evolve faster. Intrinsic variations cannot make the spectra evolve that fast, which adds strength to the “shell symmetry” problem: models, in particular, the external shock model, that involve relativistic shells must either confine the material to narrow pencil beams, be very inefficient, or break the local spherical symmetry so that the shell acts like a parallel slab. In the case of the internal shock models involving winds (i.e., central engines), it will probably be easier to break the local spherical symmetry, but then one must postulate nearly continuous energy generation at $10^{51}$ erg s$^{-1}$ lasting up to hundreds of seconds at the central site.

Key words: gamma-ray bursts, relativistic shells

1. Introduction

Gamma-ray Bursts (GRBs) have defied explanation since their discovery 25 years ago (Klebesadel, Strong, & Olsen 1973). The mystery was deepened by the observation by the Burst and Transient Source Experiment (BATSE) that GRBs are isotropic, yet there are fewer dim bursts than expected relative to the bright bursts (Meegan et al. 1992). Two conflicting explanations have arisen. In the “cosmological” explanation (see, e.g., Paczyński 1995), bursts originate at redshifts the order of $\sim 1$ and literally flood the entire universe with gamma-rays. In the alternative “galactic” explanation (see e.g., Lamb 1995), the bursts originate on high-velocity neutron stars which have traveled sufficient distances ($\sim 100$ kpc) from the disk of the Galaxy that we appear to be at the center of their spatial distribution.

In both explanations, the lack of apparent photon-photon attenuation of high energy photons ($\sim 1$ Gev) implies substantial bulk relativistic motion. The bulk Lorentz factor, $\Gamma$, must be the on order of $10^2$ to $10^4$ for the cosmological explanation, and $10$ to $10^2$ for the galactic explanation.

Two major scenarios involving relativistic shells have been developed. In the “external” shock models (see, Mészáros & Rees 1993, Piran, Shemi, & Narayan 1993), a relativistic shell is generated by a single, short-duration explosion at a central site. The shell coasts in a gamma-ray quiet phase for a time $t_0$. Eventually, the shell becomes gamma-ray active, perhaps due to shocks formed as the shell sweeps up the ISM. (Hence, the name “external” shock.) Variations in the shell explain the subpeaks observed in GRBs, but the overall envelope of emission should follow that for a single expanding shell. There is a single, very thin, relativistic shell, and its properties predict the overall envelope of emission of the GRB.

Instead of an impulsive, single injection of energy, “internal” shock models (Rees & Mészáros 1994) feature a “central engine” that releases energy on timescales comparable to the observed duration of the GRBs (up to hundreds of seconds). In some models, the central engine produces a relativistic wind which develops instabilities that generate gamma-ray producing shocks. In others, it produces a series of relativistic shells, and the fast shells collide with the slow shells resulting in a shock. In either case, the emitting region need not be a single, thin shell as in the external models. The duration of the GRB comes from the activity of the central engine (perhaps hundreds of seconds) and the individual peaks come from inhomogeneities or instabilities in that activity.

2. Expected Evolution from a Relativistic Shell

In Fenimore, Madras, and Nayakshin (1996), we calculated the time history and spectral evolution expected from a relativistic shell under the assumption of “local
spherical symmetry” (LSS). By local, we mean that only regions lying within angles the order of $\Gamma^{-1}$ contribute. By spherical, we emphasize that the curvature of the shell within $\Gamma^{-1}$ is the dominant feature that produces the envelope. By symmetry, we emphasize that the shell is seen head-on, rather than edge-on, as in many scenarios involving jets. Each of these assumptions is justified in Fenimore, Madras, and Nayakshin (1996).

We denote the rest frame time of the detector as $t$ and the arrival time as $T$. While the shell is gamma-ray active, the emitting surface keeps up with the emitted gamma-rays so they arrive at the detector over a much shorter period of time than they were emitted. If the material directly on axis emits for time $t$, the photons arrive on a time scale $T = (c - v)t/c \approx t/(2\Gamma^2)$, where $v$ is the bulk velocity, $\Gamma = (1 - \beta^2)^{-1/2}$, and $\beta = v/c$. Thus, in the case of the external shock model, the energy is released at the central site on a time scale of less than $10^7$ s, emitting for a time scale somewhat shorter than $10^7$ s, but the photons arrive within $\sim 10^5$ s (if $\Gamma = 10^3$).

Let $P(\theta, \phi, R)$ give the rate of gamma-ray production on the shell as a function of spherical coordinates. Individual bursts might have temporal structure caused by variations in $P$ as a function of $(\theta, \phi)$; however, on average, all off-axis regions should behave like the on-axis regions. Since we are considering the average properties of bursts in this paper, we assume $P(\theta, \phi, R)$ is independent of $(\theta, \phi)$. It is a single, thin shell, so $R = vt$ such that $P(\theta, \phi, R) = P(t)$. The choice of a thin shell is justified by the observations. If $\Delta T$ is the typical time scale for variability in a GRB with duration $T_{\text{dur}}$, then the thickness relative to the size of the shell is $\Delta R/\gamma_0 \sim \Delta T (2\Gamma^2 T_{\text{dur}})^{-1}$ (see Fenimore, Madras, and Nayakshin 1996).

### 2.1. Temporal Structure from Emission at a Single Time

Assume the shell expands for a time ($T_e$) in a photon quiet phase and then emits for a short period of time (i.e., $P(t) = P_0\delta(t - t_e)$). In terms of arrival time, the on-axis emission will arrive at $T_e = t_e/(2\Gamma^2)$ and the off-axis emission will arrive later with an expected shape

$$V_{\delta}(T, T_e) = \begin{cases} V_0(T, T_e) = 0 & \text{if } T < T_e \quad (1a) \\ = \psi_1 T_e \left( \frac{T}{T_e} \right)^{-\alpha - 1} & \text{if } T > T_e \quad (1b) \end{cases}$$

where we have assumed that the rest frame photon-number spectrum is isotropic with a power law with index $-\alpha$, and $\psi_1$ is a constant. Note in Fenimore, Madras, & Nayakshin, this was incorrectly derived as a power law with $-\alpha - 2$. This envelope is similar to a “FRED” (fast rise, exponential decay) where the slow, power law decay depends only on the time that the shell expands before it emits ($T_e$). The decay phase is due to photons delayed by the curvature.

GRB spectra can often be fit by the so-called “Band” model (Band et al. 1993) which consists of two power laws ($\alpha_1, \alpha_2$) and a connecting energy. If $E'_{\text{c}}$ is the connecting energy in the rest frame of the shell, the observed connecting energy is boosted, $E_c = B_0(T, T_e)E'_{\text{c}}$. The delayed photons are boosted less than at the peak because they originate from regions moving at the angle $\theta$ relative to the on-axis regions (see Katz, 1994). The boost as a function of arrival time is (see eq. 5 in Fenimore, Madras, Nayakshin 1996)

$$B_0(T, T_e) = \Lambda^{-1} = \Gamma(1 - \beta \cos \theta) = 2\Gamma(T/T_e)^{-1} \quad (2)$$

This is the most important equation in this paper since it establishes a one-to-one relationship between the angle at which photons originate and the time at which they arrive at the detector if $T_c$ can be determined. It is from this equation that we determine how the observed spectra should evolve relative to how the time history evolves.

### 2.2. Temporal Structure from Emission from a Range of Times

More complex envelopes can be found from weighted sums of $V_{\delta}(T, T_e)$. For example, the shell might emit for a range of times starting at $t_0$. We model $P(T_e, T_0)$ to be non-zero from $t_0$ to $t_{\text{max}}$ and, we assume $P(T_e, T_0)$ can be approximated as

$$P(T_e, T_0) = P_0(T_e/T_0)^{-\eta}. \quad (3)$$

In terms of arrival time, the on-axis emission will arrive between $T_0 = t_0/(2\Gamma^2)$ and $T_{\text{max}} = t_{\text{max}}/(2\Gamma^2)$. The expected envelope, $V(T)$, is (see eq. 11 in Fenimore, Madras, & Nayakshin 1996):

$$V(T) = \int_{T_0}^{T} V_{\delta}(T, T_e)P(T_e, T_0) dT_e \quad (4)$$

Due to the curvature of the shell, off-axis photons will be delayed, and most emission will arrive later:

$$V(T) = \begin{cases} V(T) = 0 & \text{if } T < T_0 \quad (5a) \\ = \frac{\psi_1 P_0}{\omega T_0^{-\eta}} \frac{T^{\omega} - T_0^{\omega}}{T_0^{\omega + 1}} & \text{if } T_0 < T < T_{\text{max}} \quad (5b) \\ = \frac{\psi_1 P_0}{\omega T_0^{-\eta}} \frac{T_{\text{max}}^{\omega} - T_0^{\omega}}{T_0^{\omega + 1}} & \text{if } T > T_{\text{max}} \quad (5c) \end{cases}$$

where

$$\omega = \alpha + 3 - \eta \quad (6)$$

Here, as a simplification to show the nature of the envelope, we have used a power law spectral shape with
a number index of $\alpha$ ($\sim 1.5$). The deviations from the
Band model are not important and $\alpha \sim (\alpha_1 + \alpha_2)/2$.

The envelope in equation (5) is also similar to a
"FRED" where the rise depends mostly on the duration
of the photon active phase $(T_{\text{max}} - T_0)$ and the slow,
power law decay depends mostly on the final overall size
of the shell $(T_{\text{max}})$.

The spectrum of the shell might change while it is emitting. We model the possible change in $E_c$ as

$$E'_c(T_e, T_0) = E'_0(T_e/T_0)^{-\nu}$$  \hspace{1cm} (7)

The observed connecting energy as a function of arrival time is

$$E_c(T) = \int_{T_0}^{T} \frac{V_b(T, T_e)P(T_e, T_0)E'_c(T_e, T_0)B_3(T, T_e)\,dT_e}{V(T)}$$  \hspace{1cm} (8)

such that

$$E_c(T) = \frac{2\Gamma E'_0}{T_0^{-\nu}} \omega' T^\omega' - T_0^\omega' \frac{T_{\text{max}}}{T_0}$$ \hspace{1cm} if $T_0 < T < T_{\text{max}}$ \hspace{1cm} (9a)

$$= \frac{2\Gamma E'_0}{T_0^{-\nu}} \omega' T^\omega' - T_0^\omega' \frac{T_{\text{max}}}{T_0}$$ \hspace{1cm} if $T > T_{\text{max}}$ \hspace{1cm} (9b)

where

$$\omega' = \omega - \nu + 1$$  \hspace{1cm} (10)

3. Comparison to Observations

In the external shock models, there is a single relativistic shell which should provide the overall GRB emission envelope. From the temporal structure of GRBs, it is clear that some process is required to break the local spherical symmetry and produce a chaotic time history rather than a smooth profile as given by equation (5). However, on average, GRB time histories should conform to the envelope if a single relativistic shell is responsible. Figure 1a is the average time history of 35 bright long GRBs (from Fenimore, 1997b). It was generated by scaling the measured duration to a standard length which we call $T_{100}$ and then adding the 35 bursts together. $(T_{100}$ is an average of $T_{50}/0.5$ and $T_{90}/0.9$) Adding together in this fashion is very reasonable for the external shock models. Each burst appears to have a different duration in arrival time because arrival time is compressed by $\Gamma^{-2}$. Normalizing each burst to a standard duration is equivalent to normalizing to a standard $\Gamma$.

The average profile is very similar to a FRED, which is remarkable because only 3 or 4 of the 35 bursts are clearly FRED-like. It is difficult to detect the FRED-like envelope in individual bursts because the shell must be patchy in order to make a long complex burst (Fenimore, Madras, & Nayakshin 1996). However, the envelope becomes evident when many bursts are averaged together.

The dotted line in Figure 1a is the best fit of equation
(5) to the average profile. Besides an overall scale factor
and $\alpha$, there are only two free parameters: $T_0$ and $T_{\text{max}}$. Note we have assumed that $\Gamma$ is constant. From the fit in Figure 1a, $T_0 = 0.78T_{100}$ and $T_{\text{max}} = 0.857T_{100}$. We have shifted the $T$ scale such that zero is at the time
(relative to $T_{100}$) when the central explosion occurred. Thus, if a relativistic shell is responsible for the shape in Figure 1a, we learn that the gamma-ray quiet phase for the shell is $\sim 1.5\Gamma^2T_{100}$ and the gamma-ray active phase is short: $0.07\Gamma^2T_{100}$.

Figure 1a implies that a relativistic shell can explain the average envelope of emission of GRBs. However, the single relativistic shell paradigm also predicts how the spectra should evolve. The dotted curve in Figure 1b uses the fit parameters from Figure 1a in equation (9). Since there are no additional free parameters, the spectral evolution is entirely determined by the fit to the time history. We have averaged the 16 channel BATSE MER data in the same manner as the averaging in Figure 1a. The MER data were then summed into five samples of duration $0.15T_{100}$ starting at $T = 1.0T_{100}$. The resulting spectra were fit with the “Band” spectral shape. We determined $E_c$ in a manner similar to how Liang and Kargatis (1996) determined spectral evolution in GRB pulses. We first fit the entire time history to determine an average $\alpha_1$ and $\alpha_2$. The five spectra were analyzed with these average power law indexes such that $E_c$ was the only free parameter. In Figure 1b, we have plotted the ratio of $E_c$ for each sample to $E_c$ of the first sample. The first sample was chosen to be in the decay phase of the envelope because the spectral evolution (eq. [9]) is effectively independent of the intrinsic variation during the decay phase. The average GRB envelope has spectra that evolve somewhat faster than a relativistic shell.

It could be argued that the averaging of the 35 bursts is uncertain because each burst is chaotic, and it is difficult to properly identify how to scale each one. Since FRED-like bursts have clear envelopes and are the shape expected from a relativistic shell, they should be easy to scale properly. In Figure 1c we averaged six FRED-like BATSE bursts, stretched by their durations to a standard time. The decay phase fits better, probably because the chaotic nature of the long complex bursts only allows a rough determination of how much stretching is needed before averaging. For the FRED-like bursts, the time of expansion from the central site is somewhat smaller, $T_0 \sim 0.53T_{100}$, and the rise is very rapid, implying that $P(T_e, T_0)$ is non-zero only for a short range of times. In Figure 1d we show the expected evolution of the connecting energy (dash curve) and the observed evolution (squares). The conflict between the observations and the predictions from a single relativistic shell is even stronger than observed with the 35 long bright bursts.
4. Intrinsic Spectral Variations

In Figure 1, we assumed no intrinsic variations in the emission as a function of time, that is, \( \nu = \eta = 0 \) such that \( P(T) = P_0 \) and \( E'_c = E'_0 \). In this section we show that intrinsic variations cannot provide softening as rapid as seen in Figure 1. In Figure 2a, we use a range of parameters in equation (5) to demonstrate the range of possibilities. (All curves have \( T_0 = 80 \).) The solid curve uses an intermediate value of \( T_{\text{max}} (=145) \), and a value of \( \eta \) of about 9 was used to obtain a fast rise and decay phase commensurate with the observed characteristics of GRBs. The dot-short-dash curve was found as in Figure 1 by fitting to the simulated data from the solid curve with \( \eta = 0 \). The resulting \( T_{\text{max}} \) is 98. The short-dash curve used \( T_{\text{max}} = 101 \) and \( \eta = 5 \). Finally, the long-dash curve uses \( T_{\text{max}} = \infty \) and \( \eta = 12 \). In all cases, the decay phase is approximately independent of the parameters used and scales roughly as \( (T/T_0)^{-\alpha-1} \).

Figure 2b shows the expected variation in \( E_c \) based on the parameters used for the curves in Figure 2a. For the solid curve and the dot-short-dash curve, we used no intrinsic variation in \( E'_c \) (i.e., \( \nu = 0 \)). For the short-dash and long-dash curves, we used \( \nu = -2 \) and +2, respectively. During the regions where \( P(T_c, T_0) \) is non-zero, the observed \( E_c \) can be influenced by intrinsic variation in \( E'_c \). However, in the decay phase, \( E_c \) is almost independent of \( \nu \) and varies as \( (T/T_0)^{-1} \) (eq. 9, Fig. 2b), whereas the observations (Fig. 1b, d) vary faster.

One assumption that we have implicitly made is that a weighted sum of Band spectra gives the appearance of having a value of \( E_c \) that is similar to the average of the values of individual \( E_c \). We tested this assumption by producing simulated spectra using the parameters of the long-dash curve in Figure 2. Simulated spectra were produced for each time bin. We processed the simulated spectra in the same manner as the MER data and the
results are shown as the squares in Figure 2. In the decay phase, they closely follow the expected relationship.

5. Discussion
The only approximations that we have made are that $P$ and $E'_c$ are approximately power laws (eqs. [3] and [7]). This allowed analytic solutions which show the nature of equations (4) and (8). These equations use only kinematic considerations to demonstrate that the decay phase is influenced only by the curvature of the shell. During the decay phase, the delayed off-axis photons are strongly beamed, and they are Lorentz-shifted out of the bandpass of the instrument. The combination of these two effects causes the time histories to decay as $\sim \Lambda^{-\alpha-1} \sim (T/T_0)^{-\alpha-1}$. In contrast, the characteristic energy of the spectra varies as $\sim \Lambda^{-1} \sim (T/T_0)^{-1}$. For example, by the time $T = 2T_0$, the time history will have returned almost to its baseline, while the characteristic energy will have changed only by a factor of 2. These relative rates of change are inconsistent with the observed overall envelope of GRB evolution. Indeed, it is well known that the characteristic energy of GRB spectra can change by factors the order of 4 throughout a burst.

The fundamental reason that the spectra evolve as $T^{-1}$ can be seen in equations (8) and (9). Notice that the only dependency on $T$ in equation (8) comes from $B_3(T, T_c)$ and $V_3(T, T_c)$. To first order, the dependency on $V_3(T, T_c)$ is cancelled by the $V(T)$ in the denominator.
so that \( E_c(T) \sim B_\delta(T, T_{\text{max}}) \). If \( T > T_{\text{max}} \), then equation (9b) states that \( E_c(T) = k T^{-1} \) where \( k \) is a constant. In fact, when \( T > T_{\text{max}}, E_c(T) \) is precisely a constant times \( T^{-1} \) for all functions \( P(T, T_e) \) and \( E_c(T, T_e) \), regardless of their functional form. Even if \( P(T, T_e) \) has a form that allows \( T_{\text{max}} \) to formally go to infinity, there must be an effective \( T_{\text{max}} \) where \( P(T, T_e) \) approaches zero. Otherwise, \( V(T) \) would diverge or GRB profiles would not have fast rises. Thus, in all cases, \( E_c(T) \) follows \( T^{-1} \) in the decay phase.

From a physical perspective, this universal spectral evolution can be understood on the basis of symmetry. We are not claiming that the intrinsic variations in \( P \) and \( E_c \) are not manifested in the observations. Indeed, while \( P \) and \( E_c \) are changing, the observations track \( P \) and \( E_c \) (between \( T = 50 \) and \( T = 100 \) in Fig. 2). The universal spectral evolution only occurs in the decay phase. During the decay phase, photons are delayed by the curvature of the shell, so they come from some off-axis annulus at an angle \( \theta = \theta_1 \). By symmetry, the same averaging that occurs at \( \theta \approx 0 \) also occurs at \( \theta = \theta_1 \); therefore, the two spectra are identical, except that the average spectrum from \( \theta = \theta_1 \) is Lorentz shifted by the boost \( \Gamma(1 - \beta \cos \theta_1) \) rather than \( \Gamma(1 - \beta) \). Equation (2) gives a one-to-one relationship of the boost to the arrival time. It is that boost that gives the universal spectral evolution.

The disagreement between the time history evolution and the spectral evolution is further evidence that something must break the local spherical symmetry if single relativistic shells are involved in GRBs. For example, if the shell can act as a parallel slab or if the GRB producing region is confined to narrow pencil beams, then the envelope and spectra can evolve uncoupled. In Fenimore, Madras, & Nayakshin (1996), we presented several other arguments that implied that LSS must be broken or GRBs must be central engines. For example, precursors and the fact that many long complex bursts do not follow the expected envelope imply that many parts of the shell never emit photons, resulting in a low efficiency for converting the kinetic energy of the shell into gamma-rays. Sari & Piran (1997) argue that the efficiency is too low (~1%) to be consistent with merging neutron stars and, thus, one must resort to internal shock models (however, see Fenimore 1997a).

In this paper, we show the need to break the LSS is also supported by the spectral evolution. We have shown that intrinsic variations alone are not sufficient to explain the observed spectral evolution. This reinforces the “shell symmetry” problem we introduced in Fenimore, Madras, and Nayakshin: GRB models that involve a single relativistic shell must explain how the material is confined to pencil beams narrower than \( \Gamma^{-1} \), how LSS is broken, or how the shell can be very inefficient without excessive energy requirements.

In this paper, we did not vary \( \Gamma \) with time. Varying \( \Gamma \) would not introduce any additional intrinsic spectral evolution since it would be equivalent to varying \( E'_c \), which is already accounted for in equation (7). However, in a future paper we will show how changes in \( \Gamma \) in time can affect the shape of the envelope and, perhaps, result in the breaking of LSS.

If the “shell symmetry” problem cannot be solved for the external shock models, those models might have to be abandoned in favor of the internal shock models, although these might have similar problems. In the internal shock models, peaks in the GRB time history arise from variations in the central activity. We have fit equations (5) and (9) to individual peaks in GRBs and the spectra evolve faster than \( (T/T_0)^{-1} \). In general, characteristic energies in peaks can evolve by factors of 4 whereas spectra from relativistic shells should only evolve by a factor \( \sim 2 \) across a peak.

If the internal shocks occur because fast shells run into slow shells (Rees and Mészáros 1994) resulting in a shell with curvature, then the internal shock models will also have to solve the “shell symmetry” problem. The internal shock models also have to explain how a source can produce \( (\Omega/4\pi) 10^{51} \text{ erg sec}^{-1} \) of activity for hundreds of seconds. (Here, \( \Omega/4\pi \) is the fraction of the sky into which the burst beams its shells.)

References
Band, D. L., et al. 1993, ApJ, 413, 281
Fenimore, E. E., Madras, C. D., & Nayakshin, S., 1996, ApJ 473, 998, \texttt{astro-ph 9607163}
Fenimore, E. E., 1997a, Proc. of 18th Texas Symposium on Relativistic Astrophysics, eds. A. Olinto, J. Frieman, & D. Schram, \texttt{astro-ph/9705028}
Fenimore, E. E., 1997b, ApJ, to be submitted
Klebesadel, R. W., Strong, I., & Olsen, R., ApJ, 1973, 182 L85
Katz J. I., 1994, ApJ, 422, 248
Liang, E., & Kargatis 1996 Nature 381, 49
Lamb D. Q., 1995, PASP 107, 1152
Meegan C. A., et al., 1992, Nature, 335, 143
Mészáros P., & Rees M. J., 1993, ApJ, 405, 278
Paczynski B., 1995, PASP 107, 1167
Piran T., Shemi A. & Narayan R., 1993 MNRAS 263, 861
Rees M. J. & Mészáros P., 1994, ApJ, 430, L93
Sari, R., & Piran, T., 1997, ApJ, in press