Efficient Compressive Sensing Using Mixed Adaptive-Random Measurements

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Abstract—A novel framework to construct an efficient sensing (measurement) matrix, called mixed adaptive-random (MAR) matrix, is introduced for directly acquiring a compressed image representation. The mixed sampling (sensing) procedure hybridizes adaptive edge measurements extracted from a low-resolution image with uniform random measurements predefined for the high-resolution image to be recovered. The mixed sensing matrix seamlessly captures important information of an image, and meanwhile approximately satisfies the restricted isometry property. To recover the high-resolution image from MAR measurements, the total variation algorithm based on the compressive sensing theory is employed for solving the Lagrangian regularization problem. Both peak signal-to-noise ratio and structural similarity results demonstrate the MAR sensing framework shows much better recovery performance compared to completely random sensing one. The work is particularly helpful for high-performance and lost-cost data acquisition.

Index Terms—Data acquisition, Mixed adaptive-random sampling, Total variation, Compressive sensing.

I. INTRODUCTION

A novel and revolutionary sensing (sampling) paradigm, compressive sensing (CS) theory has attracted much interest over the past few years. Now one can recover certain signals and images after directly acquiring far fewer samples or measurements in comparison with massive amounts of data collected in traditional data acquisitions [1]–[4]. The sensing (measurement) matrix is essential to CS framework and must capture important information about the object of interest.

Literature [5] proves that the sensing matrix satisfying the Johnson-Lindenstrauss lemma also holds true for the restricted isometry property (RIP) in compressive sensing. Furthermore, a sparse random projection [6], which is almost as accurate as the conventional random projection, is proposed to reduce computational cost in the measurement process. Moreover, a structurally random matrix (SRM) [7] is also constructed for fast and efficient CS, where SRM and sparsifying transforms have a comparable incoherence to that of completely random sensing matrix and the transforms. However, all the measurement processes above are nonadaptive, i.e., the sensing matrix is predefined or fixed. On one hand, the predefined random sampling obeys the incoherence condition (RIP); and is a fascinating character of CS as well, where all the measurements are equally important. On the other hand, a completely random sampling pattern capturing information of object aimlessly becomes inefficient for reconstructing high-resolution data. For example, the completely uniform random sampling [4], [8] in k-space for magnetic resonance imaging application performs worse than the nonuniform random sampling, where low frequency components are densely sampled. Physically, the adaptive and nonadaptive properties of data acquisition can be regarded as an analogy of wave-particle duality of light.

Could one acquire more important information of object in spatial (time) domain with fewer measurements? Could one improve the completely random sampling in spatial domain? Motivated by previous literatures and the analogy of wave-particle duality, we involve object-dependent and ‘most important’ edge information of object, which can be extracted from a low-cost sampling procedure with much lower sampling rate, into the sensing matrix. In this work, the novel mixed adaptive-random (MAR) sensing hybridizes adaptive edge measurements obtained from a low-resolution image with uniform random measurements predefined for the high-resolution image to be reconstructed. This is the basic idea of the paper and also our new contribution. To the best of our knowledge, it is the first time we have introduced the mixed sensing framework over the completely random sensing one.

II. MIXED SAMPLING PROTOCOL

Fig. 1. The schematic diagram for the mixed adaptive-random sampling protocol.

Fig.1 shows the schematic diagram for the MAR sampling protocol. Here we assume the image \( f(x, y) \) as a function in
2D Hilbert space $L(R) \times L(R)$. The MAR sensing matrix can be constructed by the following procedures:

**Step 1**, sampling a low-resolution image $f_{l}$ with extremely low cost to predict the edge information of the high-resolution image $f$ to be recovered. Regarding practical hardware implementation, the low-resolution image requires much fewer photosensitive elements (such as $128 \times 128$ instead of millions (such as $1024 \times 1024$) required in convolutional data acquisitions for high-resolution image. Mathematically, we have

$$
\Gamma(f) \approx \Gamma(f_{p}) = \Gamma(I(f_{l}))
$$

(1)

where $\Gamma(f) = 1$ for the edge pixels of $f$, otherwise $\Gamma(f) = 0$. The interpolation operator $I$ maps the low-resolution image $f_{l}$ to the predicted high-resolution one $f_{p}$. The $\Delta$ denotes the edge detection operator that can be implemented with the Sobel edge detector [9, 10] and binary thresholding. As a result, real edges of the high-resolution image $\Gamma(f)$ can be approximated by the predicted edges $\Gamma(f_{p})$.

**Step 2**, due to possible inaccuracy of the predicted edges, morphology operations can be used to generate an adaptive sampling pattern around the edges of $f$.

$$
S_{a} = M_{p}(\Gamma(f_{p}))
$$

(2)

where $S_{a}$ is the adaptive sampling pattern and $M_{p}$ is the binary morphology operator on the edges of the predicted image $f_{p}$. The morphology operator involves dilation $M_{d}$ and closing $M_{c}$ (dilation followed by erosion). Additionally, $M_{p}$ suggests no morphology operation is executed. After understanding the function of image edges in computer vision and image processing, we suppose that the image pixels located at edges or near the edges are more important than those located at smooth regions. Consequently, involve the adaptive sampling pattern into the sensing procedure is highly reasonable.

**Step 3**, generating the random sampling pattern $S_{r}$ with a 2D uniform distribution $U(0, 1) \times U(0, 1)$ and binary thresholding. The completely random sampling, which acquires pixels at edges and smooth regions uniformly, captures the image profile information and guarantees the RIP and incoherence condition.

**Step 4**, we mix the random and adaptive sampling patterns via a union operation to get the new MAR sampling pattern (sensing matrix with $0/1$ elements).

$$
S_{m} = S_{a} \cup S_{r} \cup S_{l}
$$

(3)

where $S_{l}$ is the low-resolution sampling pattern corresponding to $f_{l}$. In other words, we reuse (do not resample) the pixels of $f_{l}$ obtained at the Step 1 for saving the measurements.

To physically acquire the pixels corresponding to the MAR sensing matrix $S_{m}$, we may use integrated circuits to control reset transistors (or switches) in complementary metal-oxide-semiconductor (CMOS) camera. As a result, only a portion of photodetectors and amplifiers (with respect to $S_{l}$ and $S_{m}\setminus S_{l}$) are turned on. Compared to traditional image acquisitions, the MRA sensing saves electrical power and extends lifetime of image sensors. More importantly, the MRA sensing can be generalized to other data acquisitions where the most important information of object is adaptively extracted and reused via a low-cost sampling.

For convenience, the sensing ratio of the MAR sensing matrix $\eta_{1}$ is defined as the number of nonzero elements of $S_{m}$ over the dimension of $S_{m}$ (i.e. image size of $f$). The adaptive sampling ratio $\eta_{2}$ is defined as the number of nonzero elements of $S_{m}\setminus S_{r}$, which is the complement of $S_{r}$ in $S_{m}$, over that of $S_{m}$.

$$
\eta_{1} = \frac{\sum_{i,j} S(i,j)}{\text{Dim}(S_{m})}, \quad \eta_{2} = 1 - \frac{\sum_{i,j} S_{r}(i,j)}{\sum_{i,j} S_{m}(i,j)}
$$

(4)

The sensing ratio $\eta_{1}$ could be considerably smaller and thus measurement cost can be reduced. In addition, the adaptive sampling ratio $\eta_{2}$ cannot be too large to satisfy the RIP and incoherence condition.

**III. Recovery Algorithm**

After using the MAR sensing matrix to directly acquire a compressed image representation, the recovery algorithm plays a key role to reconstruct a high-quality image with a high resolution. The greedy pursuit algorithm [11–13] offers a $\ell_{0}$ minimization for sparse reconstruction. Linear programming [8] and other convex optimization algorithms [8, 14–16] have been proposed to solve the $\ell_{1}$-minimization also. The TV regularizer was introduced by Rudin, Osher and Fatemi in [17] and became popular in recent years [18–19].

For reconstructing the high-resolution image $f$ from the measurements (compressed image representation) $g$, a Lagrangian regularization problem should be solved, i.e.

$$
\min_{f} \left\{ \int (g - S_{m}f)^{2} dx dy + \alpha \int \sqrt{\left(\frac{d f}{d x}\right)^{2} + \left(\frac{d f}{d y}\right)^{2}} \ dx dy + \beta \int \sqrt{(T f)^{2}} dx dy \right\}
$$

(5)

where $S_{m}$ is the MAR sensing operator, and $\alpha$ and $\beta$ are Lagrangian multipliers. The second term is the TV regularizer; and the third-term relates to $\ell_{1}$-minimization with a sparsifying transform operator $T$. According to the variational principle, we have

$$
\frac{\delta O(f)}{\delta f} = 2 S_{m}^{*}(g - S_{m}f) - \alpha \frac{d}{d x} \left( \frac{d f / d x}{\sqrt{(d f / d x)^{2} + (d f / d y)^{2}}} \right) - \frac{d}{d y} \left( \frac{d f / d y}{\sqrt{(d f / d x)^{2} + (d f / d y)^{2}}} \right) + \beta T^{*} \left( T f \right)
$$

(6)

where $O(f)$ is the objective functional given in (5); and $S_{m}^{*}$ and $T^{*}$ are adjoint operators of $S_{m}$ and $T$, respectively. In this work, we did not focus on the recovery algorithm and set $\beta$ to zero for fast and simple reconstruction. With the help of nonlinear conjugate gradient method [17, 20] and (6), the Lagrangian regularization problem (5) can be solved.

**IV. Numerical Results**

In this section, numerical performances of the proposed MAR sensing matrix will be evaluated. Without loss of generality, we assume $\text{Dim}(S_{m}) = \text{Dim}(f) = 256 \times 256$. The
sensing ratio $\eta_1$ and adaptive sampling ratio $\eta_2$ defined in [4] can be tunable with modifying binary thresholds in Steps 1 and 3 of Section 2. We will demonstrate that incorporation of edge information to the sensing procedure can pronouncedly improve the recovery performance. In the beginning, the MAR sensing performance for different edge extraction methods are investigated. Then, we compare recovery results by the MAR sensing matrix to those by the completely random sensing matrix. Finally, we will discuss the influence of $\eta_2$ on the recovery performance.

The low-resolution image $f_l$ is numerically generated by downsampling the original high-resolution image $f$ by a factor of 4, i.e. Dim$(f_l)$ = 64 × 64. Using the bicubic interpolation method [21], we can get the predicted image $f_p$ (Step 1 of Section 2). The edges of $f$ and $f_p$ can be extracted by the Sobel method (Step 1 of Section 2). For simple notations, $M_m$, $M_d$ and $M_c$ correspond to the edges of $f$ with null morphology operation, dilation and closing. Similarly, $M_m$, $M_d$ and $M_c$ correspond to the edges of $f_p$ (Step 2 of Section 2). Moreover, we use abbreviations of $S_r$ and $S_m$ to denote sensing methods using the completely random matrix and MAR matrix, respectively (Step 4 of Section 2).

Using the Phantom image, Fig. 2 shows the peak signal-to-noise ratio (PSNR) as a function of the sensing ratio $\eta_1$. We observe: (1) the convergence of all the methods are comparable; (2) the performance of $S_m$ sensing is much better than that of $S_r$; (3) the best PSNR is achieved by the $S_m$ sensing involving the dilated edge information. This also suggests the pixels around edges contain very important information of image. Fig. 3 shows the sensing performance of the Phantom. After comparing Figs. 2(g.k.o) to Figs. 3(h.l.p), $S_m + M_{n,d,c}$ shows better recovery results than $S_m + M_{n,d,c}$. However, the MAR sensing matrix incorporating predicted edges followed by $M_{n,d,c}$ operations still achieves impressively high PSNR values (such as 29.93 dB with the sensing ratio $\eta_1 = 29.94\%$) in contrast to completely random sensing matrix (21.77 dB with the sensing ratio $\eta_1 = 30.24\%$). Using standard images, Fig. 4 and Table 1 demonstrate significant advantages of the MAR sensing matrix not only on PSNR but also on structural similarity (SSIM).

![Fig. 2. PSNR performance as a function of the sensing ratio $\eta_1$ for the completely random sensing $S_r$ and MAR sensing $S_m$.](image)

![Fig. 3. Reconstructed Phantom images with sensing ratios of 30.24%, 29.41%, 29.16%, 29.85%, 29.94%, 28.87% and 28.01% respectively for $S_r$, $S_m + M_{n,d,c}$, $S_m + M_d$, $S_m + M_d$, $S_m + M_{n,d,c}$ and $S_m + M_{n,d,c}$.](image)

| Image | PSNR (dB) | SSIM | $\eta_1$ | $\eta_2$ |
|-------|-----------|------|----------|----------|
| Phantom | $S_r$ | 21.7653 | 0.9486 | 30.24% |
| | $S_m + M_{n,d,c}$ | 72.4857 | 1.0000 | 29.81% | 43.76% |
| | $S_m + M_d$ | 25.5346 | 0.9712 | 30.24% | 26.64% |
| Fruits | $S_r$ | 25.6011 | 0.8336 | 30.46% |
| | $S_m + M_{n,d,c}$ | 33.6908 | 0.8767 | 30.30% | 68.11% |
| | $S_m + M_d$ | 27.0509 | 0.8409 | 30.28% | 23.00% |
| Lena | $S_r$ | 25.5683 | 0.8634 | 30.18% |
| | $S_m + M_{n,d,c}$ | 31.3728 | 0.8845 | 29.82% | 68.55% |
| | $S_m + M_d$ | 27.5257 | 0.8714 | 29.82% | 37.38% |
| Boat | $S_r$ | 24.4288 | 0.7433 | 29.85% |
| | $S_m + M_{n,d,c}$ | 28.7316 | 0.7715 | 29.81% | 84.69% |
| | $S_m + M_d$ | 25.0184 | 0.7513 | 29.82% | 36.02% |

Too much edge information in the MAR sensing matrix will destroy the RIP and incoherence condition of the CS framework. This situation appears at extremely high adaptive sampling ratios $\eta_2$, where the MAR sensing breaks down.
Fig. 4. Reconstructed Phantom, Fruits, Lena and Boat images using $S_r$, $S_m + M_d^p$ and $S_m + M_c^p$ strategies. (a–d): The original images; (e–h), (i–l) and (m–p) are corresponding reconstructed images for $S_r$, $S_m + M_d^p$ and $S_m + M_c^p$, respectively. The corresponding key parameters are listed in Table II. Finally, we evaluate the optimum value of $\eta_2$ for different $\eta_1$ as illustrated in Fig. 5. Using morphology operations, better reconstruction results are achieved by the MAR sensing even if the number of adaptive sampling is larger than that of random sampling (the optimum $\eta_2^{opt} \approx 80\%$ for $S_m + M_d^p$).

V. CONCLUSION

A novel MAR sensing protocol is proposed to acquire a compressed image representation in space domain. Incorporating adaptive edge information that can be trivially extracted from a low-resolution sampling, the MAR measurements show much better reconstruction results in comparison with the completely random measurements. The RIP and incoherence condition of the MAR sensing matrix can be satisfied by balancing the number of adaptive sampling with that of completely random sampling. The mixed sensing concept opens up a bright and unexplored way for high-resolution and lost-cost data acquisition.

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