Full one-loop electroweak and NLO QCD corrections to the associated production of chargino and neutralino at hadron colliders *

Sun Hao², Han Liang², Ma Wen-Gan¹,², Zhang Ren-You², Jiang Yi², and Guo Lei²
¹ CCAST (World Laboratory), P.O.Box 8730, Beijing 100080, P.R.China
² Department of Modern Physics, University of Science and Technology of China (USTC), Hefei, Anhui 230027, P.R.China

Abstract

We study the process of the association production of chargino and neutralino including the NLO QCD and the complete one-loop electroweak corrections in the framework of the minimal supersymmetric standard model (MSSM) at the Fermilab Tevatron and the CERN Large Hadron Collider (LHC). In both the NLO QCD and one-loop electroweak calculations we apply the algorithm of the phase-space slicing (PSS) method. We find that the NLO QCD corrections generally increase the Born cross sections, while the electroweak relative corrections decrease the Born cross section in most of the chosen parameter space. The NLO QCD and electroweak relative corrections typically have the values of about 32% and −8% at the Tevatron, and about 42% and −6% at the LHC respectively. The results show that both the NLO QCD and the complete one-loop electroweak corrections to the processes $pp/\bar{p}p \rightarrow \tilde{\chi}_1^\pm \tilde{\chi}_2^0 + X$ are generally significant and should be taken into consideration in precision experimental analysis.

PASC:12.60.Jv, 14.80.Ly, 12.15.Lk, 12.38.Bx

*Supported by National Natural Science Foundation of China.
I Introduction

People believe that the minimal supersymmetric standard model (MSSM) is a very attractive extension of the standard model (SM). The direct discovery of the supersymmetric particles is one of the most important endeavors of present and future high energy experiments. The MSSM theory predicts many new particles, such as: squarks, sleptons, neutral CP-even Higgs-bosons $h^0$, $H^0$ and CP-odd Higgs boson $A^0$, charged Higgs-bosons $H^\pm$, charginos $\tilde{\chi}_i^\pm (i = 1, 2)$, which are the mixtures of charged winos and charged higgsinos, and four neutralinos $\tilde{\chi}_j^0 (j = 1 - 4)$ being the mixtures of the neutral wino, bino and two neutral higgsinos.

In most of the constrained MSSM scenarios, such as the R-conserving minimal supergravity (mSUGRA), the lighter chargino ($\tilde{\chi}_1^\pm$) and the neutralinos ($\tilde{\chi}_1^0, \tilde{\chi}_2^0$) are considerably less massive than the gluinos and squarks over most of the parameter space, they may belong to the class of the relative light supersymmetric particles. The association production of $\tilde{\chi}_1^\pm$ and $\tilde{\chi}_2^0$, which seems to be one of the primary source of the trilepton events, is a promising channel for supersymmetric particle searches at hadron colliders. In the association production of the lighter chargino ($\tilde{\chi}_1^\pm$) and the second lightest neutralino ($\tilde{\chi}_2^0$), both final supersymmetric particles could dominantly decay to leptons, i.e., $\tilde{\chi}_1^\pm \rightarrow \tilde{\chi}_1^0 \ell^\pm \nu_\ell$ and $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 \ell^+ \ell^-$, which leads to a gold-plated $\ell^\pm \ell^+ \ell^-$ trilepton signature\cite{1,2}. We expect that the lighter charginos and neutralinos to be detected in their above mentioned decays at the Tevatron and the LHC. Actually, such trilepton signature was exploited primarily in the CDF and D0 experiments at the Tevatron and got the similar bounds to those
from LEP2 in the MSSM parameter space\[3\].

To investigate the discovering potential of hadron colliders, not only a proper understanding of the hadron production mechanisms is necessary, but also accuracy theoretical predictions of the signature should be provided. As we know, the impact of higher order electroweak and QCD contributions normally grows with increasing colliding energy and would become to be more obvious at very high energies. In Ref.\[4\], V. Barger and Chung Kao investigated the prospects for detecting trilepton events ($l = e$ or $\mu$) from neutralino-chargino($\tilde{\chi}^0_2\tilde{\chi}^\pm_1$) associated production at the upgraded Tevatron in the mSUGRA model. They found that if there is a large integrated luminosity (for example $L = 30 \text{ fb}^{-1}$) and the decay modes $\tilde{\chi}^\pm_1 \to \tilde{\chi}^0_1\ell^\pm\nu_\ell$ and $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1\ell^+\ell^-$ are kinematically dominant, the value of statistical significance ($N_S \equiv S/\sqrt{B}$, $S =$ number of signal events, and $B =$ number of background events) can reach 36.9 when we use suitable cuts and take $\tan \beta = 3$. Then one can expect the accuracy of the cross section measurement of the neutralino-chargino($\tilde{\chi}^0_2\tilde{\chi}^\pm_1$) associated production at the upgraded Tevatron can reach few percent. Therefore, for the precise experiments at $TeV$ scale hadron colliders, both the higher order QCD and electroweak corrections should be considered in the theoretical predictions, and thereby one can improve experimental mass bounds and exclusion limits for the new particles. Moreover, the consideration of higher order QCD contributions can reduce the dependence of the cross sections on the renormalization and factorization scales in the LO. And the cross sections in NLO are under much better theoretical control than the leading order estimates.

There have been many works which present the theoretical calculations of the
production of SUSY particles in hadron collisions at NLO QCD level, such as, Refs. [5] [6] [7] [8]. In Ref. [8] it presents the complete next-to-leading order SUSY QCD analysis for the production of all possible pairs of noncolored supersymmetric particles, including \( pp/\bar{p}p \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0 + X \). In their calculations of the NLO QCD corrections, the infrared and collinear singularities were extracted by applying the dipole subtraction method[9].

In our work, we are to calculate and discuss the complete one-loop electroweak radiative corrections to the processes \( pp/\bar{p}p \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0 + X \) at the Tevatron and the LHC. And for the completeness of our investigation we present also the NLO QCD corrections to the \( pp/\bar{p}p \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0 + X \) processes by applying the phase-space slicing method(PSS)[10]. At the same time we compare our NLO QCD numerical results from the PSS method with those from dipole subtraction method in Ref. [8].

The structure of this paper is organized as follows: In Section 2, we calculate the cross section of the leading order results for the subprocess \( u\bar{d} \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0 \). In Section 3, we give the analytical and numerical calculations and discussions for the NLO QCD corrections to the processes \( pp/\bar{p}p \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0 + X \) at the Tevatron and the LHC. In Section 4, we present the calculations of the complete one-loop electroweak corrections and discussions for the process \( pp/\bar{p}p \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0 + X \). Finally, a short summary is given.
II The leading order calculation for subprocess $u \bar{d} \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0$

Since the cross sections for the subprocess $u \bar{d} \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0$ and its charge-conjugate subprocess $\bar{u}d \rightarrow \tilde{\chi}_1^- \tilde{\chi}_2^0$ in the CP-conserved MSSM are the same, we present here only the calculation of the subprocess $u \bar{d} \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0$. The tree-level diagrams for the subprocess $u \bar{d} \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0$ are shown in Fig.1.

Figure 1: The tree-level Feynman diagrams for the subprocess $u \bar{d} \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0$.

In our calculation, we neglect the small masses of the light-quarks and there is no contributions from the Feynman diagrams which involve the couplings $H^+/G^+ - u - d$.

We denote the subprocess as

$$u(p_1) + \bar{d}(p_2) \rightarrow \tilde{\chi}_1^+(k_3) + \tilde{\chi}_2^0(k_4) \quad (2.1)$$

where $p_i$ ($i = 1, 2$) and $k_i$ ($i = 3, 4$) are the four-momenta of incoming up-quark/anti-down-quark and outgoing $\tilde{\chi}_1^+$ and $\tilde{\chi}_2^0$, respectively. All these four-momenta satisfy the on-shell conditions: $k_3^2 = m_{\tilde{\chi}_1^+}^2$, $k_4^2 = m_{\tilde{\chi}_2^0}^2$, $p_1^2 = p_2^2 = 0$. The center-of-mass energy squared is denoted by $\hat{s} = E_{cm}^2 = (p_1 + p_2)^2$. 

The amplitude of $u \bar{d} \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0$ subprocess can be divided into three parts and expressed as

$$\mathcal{M}^0 = \mathcal{M}^s + \mathcal{M}^t + \mathcal{M}^u$$

(2.2)

where $\mathcal{M}^0$ is the tree-level amplitude and $\mathcal{M}^s$, $\mathcal{M}^t$, $\mathcal{M}^u$ represent the amplitude parts arising from the s-channel, t-channel and u-channel diagrams shown in Fig. II(a-c), respectively. Then the lowest order cross section for the subprocess $u \bar{d} \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0$ in the MSSM is obtained by using the following formula:

$$\hat{\sigma}_0(\hat{s}, u \bar{d} \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0) = \frac{1}{16\pi \hat{s}^2} \int_{\hat{t}_{\text{min}}}^{\hat{t}_{\text{max}}} dt \sum |\mathcal{M}^0|^2.$$  

(2.3)

where

$$\hat{t}_{\text{min}}(\hat{t}_{\text{max}}) = \frac{1}{2} \left\{ (m_{\tilde{\chi}_1^+}^2 + m_{\tilde{\chi}_2^0}^2 - \hat{s}) \pm \sqrt{(\hat{s} - m_{\tilde{\chi}_1^+}^2 - m_{\tilde{\chi}_2^0}^2)^2 - 4m_{\tilde{\chi}_1^+}^2 m_{\tilde{\chi}_2^0}^2} \right\}.$$  

(2.4)

The summation is taken over the spins and colors of initial and final states, and the bar over the summation denotes averaging over the spins and colors of initial partons.

III NLO QCD corrections

In considering the NLO QCD corrections to the subprocesses $u \bar{d}(\bar{u}d) \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0$, we should involve the gluon emission subprocess $u \bar{d} \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0 + g$ to cancel the soft IR divergence arising from the virtual QCD corrections of the subprocesses $u \bar{d}(\bar{u}d) \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0$. The light-quark emission subprocesses $gu(\bar{d}) \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0 + d(\bar{u})$ should be also included for a consistent and complete mass factorization.
III.1 Virtual corrections

The complete one-loop Feynman diagrams of QCD corrections in the MSSM, which are built up with gluon, gluino, quark and squark exchanging loops, are shown in Fig. 2. We use the fermion flow prescription for the calculation of the matrix elements including Majorana particles. Same as in the tree-level calculation, we neglect the masses of the light-quarks in the calculation of the virtual QCD corrections for the subprocess $u\bar{d} \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0$. There exist both ultraviolet (UV) divergences and soft/collinear IR singularities in the amplitudes from these QCD one-loop diagrams. We use the method of the phase-space slicing (PSS) to treat the soft and collinear divergences. The PSS method is intuitive, simple to implement, and relies on a minimum of process dependent information and has been used in many works.

In the NLO QCD calculation, we adopt the ’t Hooft-Feynman gauge and dimensional regularization method with $n = 4 - 2\epsilon$ to evaluate the one-loop contributions. The expressions for the relevant renormalization constants can be found in next section, except all the relevant self-energies including only the QCD parts instead of the electroweak parts. We have verified the cancellation of the UV divergence in the virtual QCD corrections analytically. Then we get an UV finite amplitude for the $\mathcal{O}(\alpha_s)$ virtual radiative corrections.

The virtual correction to the cross section can be written as

$$\hat{\sigma}^V(\hat{s}, u\bar{d} \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0) = \frac{1}{16\pi \hat{s}^2} \int_{\hat{t}_{\text{min}}}^{\hat{t}_{\text{max}}} dt \ 2Re \sum [(\mathcal{M}^V)^\dagger \mathcal{M}^0]. \hspace{1cm} (3.1)$$

where $\mathcal{M}^V$ is the UV renormalized amplitude for virtual corrections, and again the
Figure 2: The one-loop Feynman diagrams of virtual QCD corrections for the subprocess $u\bar{d} \rightarrow \tilde{\chi}_1^+\tilde{\chi}_2^0$ in the MSSM.
summation with bar over head means the same operation as before. The expressions for \( t_{\text{max}} \) and \( t_{\text{min}} \) can be found in Eq. (2.4)

After the renormalization procedure, \( \hat{\sigma}^V \) is UV-finite. However, it still contains the soft/collinear IR singularities. The IR divergence part in \( d\hat{\sigma}^V \) can be obtained as

\[
d\hat{\sigma}^V|_{IR} = d\hat{\sigma}^0 \left[ \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi\mu^2}{s} \right)^\epsilon \right] \left( \frac{A_2^V}{\epsilon^2} + \frac{A_1^V}{\epsilon} \right),
\]

(3.2)

where

\[
A_2^V = \frac{8}{3}, \quad A_1^V = -4.
\]

(3.3)

The soft divergences can be cancelled by adding the soft real gluon emission corrections. The collinear divergences together with those coming from the real light-quark emission corrections are absorbed into the parton distribution functions, which will be discussed in the next subsection.

III.2 Real emission corrections

The real emission subprocesses, which present NLO QCD corrections to the \( u\bar{d} \rightarrow \tilde{\chi}_1^+\tilde{\chi}_2^0 \) subprocess, include real gluon emission subprocess \( u\bar{d} \rightarrow \tilde{\chi}_1^+\tilde{\chi}_2^0 + g \), and real light-quark emission subprocesses \( gu \rightarrow \tilde{\chi}_1^+\tilde{\chi}_2^0 + d \), \( gd \rightarrow \tilde{\chi}_1^+\tilde{\chi}_2^0 + \bar{u} \). The later two subprocesses will make additional contributions to the final two-body and the three-body cross sections.

1. Real gluon emission corrections

The real gluon emission subprocess \( u\bar{d} \rightarrow \tilde{\chi}_1^+\tilde{\chi}_2^0 + g \) presents \( \mathcal{O}(\alpha_s) \) corrections to the subprocess \( u\bar{d} \rightarrow \tilde{\chi}_1^+\tilde{\chi}_2^0 \). It is also one of the origins of IR singularities. Its
IR singularities can be either of soft or collinear nature and can be conveniently isolated by slicing the $u \bar{d} \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0 + g$ phase space into different regions defined by suitable cutoffs, a method which goes under the general name of phase-space slicing (PSS)\[10\]. The soft IR singularity part from subprocess $u \bar{d} \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0 + g$ cancels exactly the analogous singularity presented in the $\mathcal{O}(\alpha_s)$ virtual corrections calculated in above subsection.

![Feynman diagrams](image)

Figure 3: The tree-level Feynman diagrams for the real gluon emission subprocess $u \bar{d} \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0 + g$.

We denote the $2 \rightarrow 3$ subprocess $u \bar{d} \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0 + g$ as

$$u(p_1) + \bar{d}(p_2) \rightarrow \tilde{\chi}_1^+(k_3) + \tilde{\chi}_2^0(k_4) + g(k_5), \quad (3.4)$$

The Mandelstam variables are defined as

$$\hat{s} = (p_1 + p_2)^2, \quad \hat{t} = (p_1 - k_3)^2, \quad \hat{u} = (p_1 - k_4)^2, \quad \hat{s}_{45} = (k_4 + k_5)^2,$$

$$\hat{t}_{15} = (p_1 - k_5)^2, \quad \hat{t}_{25} = (p_2 - k_5)^2, \quad \hat{t}_{45} = (k_4 - k_5)^2 \quad (3.5)$$

By introducing an arbitrary small soft cutoff $\delta$, we separate the phase space of the $2 \rightarrow 3$ subprocess $u \bar{d} \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0 + g$ into two regions, according to whether the energy
of the emitted gluon is soft, i.e. \( E_5 \leq \delta_s \sqrt{\hat{s}}/2 \), or hard, i.e. \( E_5 > \delta_s \sqrt{\hat{s}}/2 \). The partonic cross section can be written as

\[
\hat{\sigma}_g^R(u \bar{d} \to \bar{\chi}_1^+ \chi_2^0 + g) = \hat{\sigma}_g^S(u \bar{d} \to \bar{\chi}_1^+ \chi_2^0 + g) + \hat{\sigma}_g^H(u \bar{d} \to \bar{\chi}_1^+ \chi_2^0 + g),
\]

where \( \hat{\sigma}_g^S \) is obtained by integrating over the soft gluon emission phase space region, and contains all the soft IR singularities. In order to isolate the remaining collinear singularities from \( \hat{\sigma}_g^H \), we further decompose \( \hat{\sigma}_g^H \) into hard-collinear (HC) and hard non-collinear (\( \text{HC} \)) parts by introducing another cutoff \( \delta_c \) named collinear cutoff,

\[
\hat{\sigma}_g^H(u \bar{d} \to \bar{\chi}_1^+ \chi_2^0 + g) = \hat{\sigma}_g^{HC}(u \bar{d} \to \bar{\chi}_1^+ \chi_2^0 + g) + \hat{\sigma}_g^\text{HC}(u \bar{d} \to \bar{\chi}_1^+ \chi_2^0 + g).
\]

The HC regions of the phase space are those in collinear condition, where any invariant, \( \hat{t}_{15}, \hat{t}_{25}, \hat{t}_{45} \), becomes smaller in magnitude than \( \delta_c \sqrt{\hat{s}} \), while at the same time the emitted gluon remains hard. \( \hat{\sigma}_g^{HC} \) contains the collinear divergences. In the soft and HC regions, \( \hat{\sigma}_g^S \) and \( \hat{\sigma}_g^{HC} \) can be obtained by performing the phase space integration in \( n = 4 - 2\epsilon \) dimensions analytically. But in the \( \text{HC} \) region, \( \hat{\sigma}_g^\text{HC} \) is finite and can be evaluated in four dimensions by using standard Monte Carlo techniques\(^{[12]} \). The cross sections, \( \hat{\sigma}_g^S, \hat{\sigma}_g^{HC} \) and \( \hat{\sigma}_g^\text{HC} \), depend on the arbitrary parameters, \( \delta_s \) and \( \delta_c \).

With the arbitrary small cutoff \( \delta_s \), the differential cross section in the soft region is given as

\[
d\hat{\sigma}_g^S = d\hat{\sigma}^0 \left[ \frac{\alpha_s}{2\pi} \frac{\Gamma(1 - \epsilon)}{\Gamma(1 - 2\epsilon)} \left( \frac{4\pi\mu_r^2}{\hat{s}} \right)^\epsilon \right] \left( \frac{A_2^S}{\epsilon^2} + \frac{A_1^S}{\epsilon} + A_0^S \right),
\]

with

\[
A_2^S = \frac{8}{3}, \quad A_1^S = -\frac{16}{3} \ln \delta_s, \quad A_0^S = \frac{16}{3} \ln^2 \delta_s.
\]
In the limit where two of the partons are collinear, the three body phase space is greatly simplified. And at the same limit, the leading pole approximation of the matrix element is valid. According to whether the collinear singularities are initial or final state in origin, where \( \hat{\sigma}^{HC}_g \) can be separated into two pieces

\[
\hat{\sigma}^{HC}_g = \hat{\sigma}^{HC}_{g,i} + \hat{\sigma}^{HC}_{g,f}
\]

(3.10)

Since in the process \( u\bar{d} \rightarrow \chi^+ \chi^0 + g \), only initial particles are involved in strong interaction and massless in the limit, we only need to calculate the cross section \( \hat{\sigma}^{HC}_{g,i} \) which arises from the case that the emitted gluon is collinear to the initial partons, i.e., \( 0 \leq t_{15}, t_{25} \leq \delta_c \hat{s} \).

In \( HC \) phase space region, the initial state partons \( i(i = u, \bar{d}) \) is considered to split into a hard parton \( i' \) and a collinear gluon, \( i \rightarrow i' g \), with \( p_{i'} = zp_i \) and \( k_5 = (1 - z)p_i \). The cross section \( \sigma^{HC}_{g,i} \) can be written as

\[
d\sigma^{HC}_g = d\sigma^{HC}_{g,i} = d\sigma^0 \left[ \frac{\alpha_s}{2\pi} \frac{\Gamma(1 - \epsilon)}{\Gamma(1 - 2\epsilon)} \left( \frac{4\pi \mu_r^2}{\hat{s}} \right)^\epsilon \left( -\frac{1}{\epsilon} \right) \delta_c^{-\epsilon} \right] \left[ P_{uu}(z, \epsilon)G_{u/A}(x_A/z)G_{d/B}(x_B) + P_{d\bar{d}}(z, \epsilon)G_{d/A}(x_A/z)G_{u/B}(x_B) + (A \leftrightarrow B) \right] \frac{dz}{z} \left( \frac{1 - z}{z} \right)^{-\epsilon} dA dB \]

(3.11)

where \( G_{u,d/A,B} \) are the bare parton distribution functions. A and B refer to protons at the LHC, and proton, antiproton at the Tevatron. \( P_{uu}(z, \epsilon) \) and \( P_{d\bar{d}}(z, \epsilon) \) are the n-dimensional unregulated \( (z < 1) \) splitting functions related to the usual Altarelli-Parisi splitting kernels\[13\]. \( P_{ii}(z, \epsilon)(i = u, \bar{d}) \) can be written explicitly as

\[
P_{ii}(z, \epsilon) = P_{ii}(z) + \epsilon P'_{ii}(z)
\]

\[
P_{ii}(z) = C_F \frac{1 + z^2}{1 - z}
\]

\[
P'_{ii}(z) = -C_F (1 - z) \quad (i = u, \bar{d}),
\]

(3.12)
with $C_F=4/3$.

2. Real light-quark emission corrections of $gu(\bar{d}) \to \tilde{\chi}_1^+ \tilde{\chi}_2^0 + d(\bar{u})$

In addition to the real gluon emission subprocess, the subprocesses $gu(\bar{d}) \to \tilde{\chi}_1^+ \tilde{\chi}_2^0 + d(\bar{u})$ should also be included. The contributions from these two subprocesses contain only the initial state collinear singularity. These subprocesses will make additional contributions to the two-body term cross section $\sigma^{(2)}$ and the three-body term cross section $\sigma^{(3)}$. The Feynman diagrams for these two subprocesses at the tree-level are shown in Fig.4 and Fig.5 respectively.

By using the PSS method described above, we split the phase space into two regions: collinear region and non-collinear region.

$$\hat{\sigma}^R_q(gu(\bar{d}) \to \tilde{\chi}_1^+ \tilde{\chi}_2^0 + d(\bar{u})) = \hat{\sigma}^{HC}_q(gu(\bar{d}) \to \tilde{\chi}_1^+ \tilde{\chi}_2^0 + d(\bar{u}))$$

$$+ \hat{\sigma}^{HC}_q(gu(\bar{d}) \to \tilde{\chi}_1^+ \tilde{\chi}_2^0 + d(\bar{u})).$$

(3.13)
Figure 5: The tree-level Feynman diagrams for the real light-quark emission subprocess $g\bar{d} \to \tilde{\chi}_1^+\tilde{\chi}_2^0 + \bar{u}$.

Also $\hat{\sigma}^{HC}_q$ in hard non-collinear region is finite and can be evaluated in four dimensions using Monte Carlo method. The differential cross sections of $d\sigma^{HC}_q$ for subprocesses $gu(\bar{d}) \to \tilde{\chi}_1^+\tilde{\chi}_2^0 + d(\bar{u})$ can be written as

\[
d\sigma^{HC}_q(gu(\bar{d}) \to \tilde{\chi}_1^+\tilde{\chi}_2^0 + d(\bar{u})) = \left. d\hat{\sigma}^0 \right[ \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi\mu^2}{s} \right)^\epsilon \left( \frac{-1}{\epsilon} \right) \delta^{\epsilon-\epsilon} [P_{d(\bar{u})g}(z,\epsilon)G_{g/A}(x_A/z)G_{u(\bar{d})/B}(x_B) \\
+ (A \leftrightarrow B)] \frac{dz}{z} \left( \frac{1-z}{z} \right)^{-\epsilon} dx_A dx_B. \right]
\]

with $P_{ij}(z,\epsilon)(i = \bar{u}, d, j = g)$ expressed explicitly as

\[
P_{d(\bar{u})g}(z,\epsilon) = P_{d(\bar{u})g}(z) + \epsilon P'_{d(\bar{u})g}(z), \\
P_{d(\bar{u})g}(z) = \frac{1}{2} [z^2 + (1-z)^2], \\
P'_{d(\bar{u})g}(z) = -z(1-z). \tag{3.15}
\]

3. **NLO QCD corrected cross section for $pp/p\bar{p} \to \tilde{\chi}_1^+\tilde{\chi}_2^0 + X$**

After adding the renormalized virtual corrections and the real corrections, the partonic cross sections still contain the collinear divergences, which can be absorbed into the redefinition of the distribution functions at NLO. Using the $\overline{\text{MS}}$ scheme,
the scale dependent NLO parton distribution functions are given as \[^{[14]}\]

\[ G_i/A (x, \mu_f) = G_i/A (x) + \left( -\frac{1}{\epsilon} \right) \left[ \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi\mu_r^2}{\mu_i^2} \right)^\epsilon \right] \int_z^1 \frac{dz}{z} P_{ij}(z) G_j/A (x/z). \]

(3.16)

By using above definition, we get a NLO QCD parton distribution function counter-terms which are combined with the hard collinear contributions (Eq. (3.11) and (3.14)) to result in the \( O(\alpha_s) \) expression for the remaining collinear contributions:

\[ d\sigma_{\text{coll}} = d\hat{\sigma}^0 \left[ \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi\mu_r^2}{\hat{s}} \right)^\epsilon \right] \left\{ \tilde{G}_{u/A}(x_A, \mu_f) G_{\bar{d}/B}(x_B, \mu_f) + G_{u/A}(x_A, \mu_f) \tilde{G}_{\bar{d}/B}(x_B, \mu_f) \right. \]

\[ + \sum_{\alpha=u,d} \left[ A_{sc}^{\epsilon}(\alpha \to \alpha g) + A_0^{sc}(\alpha \to \alpha g) \right] G_{u/A}(x_A, \mu_f) G_{\bar{d}/B}(x_B, \mu_f) \]

\[ + (A \leftrightarrow B) \} dx_A dx_B, \]

(3.17)

where \( A \) and \( B \) are proton and antiproton for the Tevatron, and proton and proton for the LHC, respectively.

\[ A_{1sc}^{\epsilon}(u(d) \to u(d) g) = C_F (2 \ln \delta_s + 3/2), \quad A_{0sc}^{\epsilon} = A_{1sc}^{\epsilon} \ln \left( \frac{\hat{s}}{\mu_f^2} \right), \]

(3.18)

and

\[ \tilde{G}_{\alpha/A,B}(x, \mu_f) = \sum_{\epsilon=\alpha,g} \int_x^{1-\delta_s} \frac{dy}{y} G_{c'/A,B}(x/y, \mu_f) \tilde{P}_{\alpha c'}(y), \quad (\alpha = u, \bar{d}) \]

(3.19)

with

\[ \tilde{P}_{\alpha c'}(y) = P_{\alpha c'} \ln \left( \delta_c \frac{1-y}{y} \frac{\hat{s}}{\mu_f^2} \right) - P_{\alpha c'}(y). \]

(3.20)

We can observe that the sum of the soft (Eq. (3.8)), collinear (Eq. (3.17)), and ultraviolet renormalized virtual correction (Eq. (3.2)) terms is finite, i.e.,

\[ A_2^S + A_2^V = 0, \]

\[ A_1^S + A_1^V + A_1^{sc}(u \to ug) + A_1^{sc}(\bar{d} \to \bar{d} g) = 0. \]

(3.21)
The final result for the total $\mathcal{O}(\alpha_s)$ correction consists of two parts of contributions: a two-body term $\sigma^{(2)}$ and a three-body term $\sigma^{(3)}$. The two-body correction term $\sigma^{(2)}$ is expressed as

$$
\sigma^{(2)} = \frac{\alpha_s}{2\pi} \int dx_A dx_B d\hat{\sigma}^0 \{G_{u/A}(x_A, \mu_f) G_{\bar{d}/B}(x_B, \mu_f) [A_0^S + A_0^V + A_{0}^{sc}(u \rightarrow ug) + A_0^{sc}(d \rightarrow \bar{d}g)] 
+ \tilde{G}_{u/A}(x_A, \mu_f) G_{\bar{d}/B}(x_B, \mu_f) + G_{u/A}(x_A, \mu_f) \tilde{G}_{\bar{d}/B}(x_B, \mu_f) + (A \leftrightarrow B)\}.
$$

(3.22)

And the three-body correction term $\sigma^{(3)}$ is written as

$$
\sigma^{(3)} = \sigma^{(3)}(pp/p\bar{p} \rightarrow u\bar{d} \rightarrow \tilde{\chi}_1^{+}\tilde{\chi}_2^{0} + g) 
+ \sigma^{(3)}(pp/p\bar{p} \rightarrow gu \rightarrow \tilde{\chi}_1^{+}\tilde{\chi}_2^{0} + d) + \sigma^{(3)}(pp/p\bar{p} \rightarrow g\bar{d} \rightarrow \tilde{\chi}_1^{+}\tilde{\chi}_2^{0} + \bar{u}) 
= \int dx_A dx_B [G_{u/A}(x_A, \mu_f) G_{\bar{d}/B}(x_B, \mu_f) + (A \leftrightarrow B)]d\hat{\sigma}^{(3)}(u\bar{d} \rightarrow \tilde{\chi}_1^{+}\tilde{\chi}_2^{0} + g) 
+ \int dx_A dx_B [G_{g/A}(x_A, \mu_f) G_{u/B}(x_B, \mu_f) + (A \leftrightarrow B)]d\hat{\sigma}^{(3)}(gu \rightarrow \tilde{\chi}_1^{+}\tilde{\chi}_2^{0} + d) 
+ \int dx_A dx_B [G_{g/A}(x_A, \mu_f) G_{\bar{d}/B}(x_B, \mu_f) + (A \leftrightarrow B)]d\hat{\sigma}^{(3)}(g\bar{d} \rightarrow \tilde{\chi}_1^{+}\tilde{\chi}_2^{0} + \bar{u}).
$$

(3.23)

Finally, the NLO total cross section for $pp/p\bar{p} \rightarrow \tilde{\chi}_1^{+}\tilde{\chi}_2^{0} + X$ is

$$
\sigma^{NLO} = \sigma^0 + \sigma^{(2)} + \sigma^{(3)}.
$$

(3.24)

In the cross section part of $\hat{\sigma}^{(2)} + \hat{\sigma}^{(3)}$, the dependence on the arbitrary cutoffs $\delta_s$ and $\delta_c$ should be vanished. This constitutes an important check of our calculation.

### III.3 Numerical results involving NLO QCD corrections

In Ref. [8], W. Beenakker, et al, presented the NLO QCD calculations of the processes $pp/p\bar{p} \rightarrow \tilde{\chi}_1^{+}\tilde{\chi}_2^{0} + X$. There the infrared and collinear singularities of the three
parton cross sections are extracted by applying algorithm of the dipole subtraction method\cite{9}. As a check of our numerical calculation by adopting the two cutoff PSS method, we take the same mSUGRA input parameters $m_{1/2} = 150$ GeV, $m_0 = 100$ GeV, $A_0 = 300$ GeV, $\mu > 0$, $\tan \beta = 4$ and reproduce the Fig.2 shown in Ref.\cite{8} with the coincident numerical results. From this comparison we have verified the correctness of our calculations of the NLO QCD corrections. We take the colliding energies of $p\bar{p}/pp$ at the Tevatron Run II and the LHC are 2 TeV and 14 TeV, respectively.

In the following numerical calculation for NLO QCD corrections, we use the package FormCalc\cite{15} to get all the masses of the supersymmetric particles by inputting $\tan \beta$, $m_{A^0}$, $M_{\text{susy}}$, $\mu$, $M_2$ and $A_f$ parameters. In the package FormCalc, the grand unification theory (GUT) relation $M_1 = (5/3) \tan^2 \theta_W M_2$ is adopted\cite{16}, and $M_Q = M_U = M_D = M_E = M_L = M_{\text{susy}}$ is assumed in the sfermion sector for simplification. We use the one-loop formula for the running strong coupling constant $\alpha_s$ with $\alpha_s(m_Z) = 0.1187$, and take CTEQ6L and the CTEQ6M parton distribution functions in calculating the LO and the NLO cross sections, respectively\cite{17}.

Fig.6 shows the independence of the NLO QCD corrected cross sections on the arbitrary cutoffs $\delta_s$ and $\delta_c$ by applying the two cutoff PSS method. The two-body correction term ($\sigma^{(2)}$) and three-body correction term($\sigma^{(3)}$) and the NLO QCD corrected total cross section $\sigma^{NLO}$ at the Tevatron and the LHC, are shown as the functions of the soft cutoff $\delta_s$ with the collinear cutoff $\delta_c = \delta_s/50$ (shown in Fig.6(a),(c)), and as the functions of the collinear cutoff $\delta_c$ with the soft cutoff $\delta_s = 50\delta_c$ (shown in Fig.6(b),(d)). The input supersymmetric parameters are taken
as \( \tan \beta = 4 \), \( m_{A^0} = 300 \text{ GeV} \), \( M_{\text{susy}} = 250 \text{ GeV} \), \( \mu = 278 \text{ GeV} \), \( M_2 = 123 \text{ GeV} \) and \( A_f = 450 \text{ GeV} \). We can see the NLO QCD corrected total cross section \( \sigma^{NLO} = \sigma^0 + \delta \sigma = \sigma^0 + \sigma^{(2)} + \sigma^{(3)} \) is independent of the cutoffs. This is an important check of the correctness of our calculation. In the following numerical calculations, we set \( \delta_s = 10^{-5} \) and \( \delta_c = \delta_s/50 \).

During our numerical calculation we investigate also the dependence of the LO and NLO QCD corrected total cross sections at the LHC and the Tevatron on the renormalization and factorization scales (\( Q = \mu_r = \mu_f \)), and find that the theoretical predictions including NLO QCD corrections become stable, being nearly independent of the factorization/renormalization scales for the processes \( pp/p\bar{p} \rightarrow \tilde{\chi}_1^\pm \tilde{\chi}_2^0 + X \) as concluded in Ref.[8].

In Fig.7, the dependence of the Born, NLO QCD corrected cross sections (shown in Fig.7(a) and (d)) and the relative corrections \( \delta \) (shown in Fig.7(b) and (d)) for the processes \( pp/p\bar{p} \rightarrow \tilde{\chi}_1^\pm \tilde{\chi}_2^0 + X \) at the LHC and the Tevatron on the gaugino mass parameter \( M_2 \) are plotted. There we take the input parameters as \( m_{A^0}=300 \text{ GeV} \), \( M_{\text{susy}}=500 \text{ GeV} \), \( \mu=400 \text{ GeV} \) and \( A_f=450 \text{ GeV} \), with \( \tan \beta = 4 \), \( \tan \beta = 15 \) and \( \tan \beta = 40 \), respectively. We can see the Born and NLO QCD corrected cross sections decrease rapidly to a small value with the increment of \( M_2 \). At the Tevatron, the NLO QCD relative correction for \( \tan \beta = 4 \) decreases from 37.7\% to 29.0\% with
Figure 6: The dependence of the total cross sections for \(pp/\bar{p}p \to \tilde{\chi}_1^\pm \tilde{\chi}_2^0 + X\) processes at the Tevatron and the LHC as the functions of the cutoff \(\delta_s\) with \(\delta_c = \delta_s/50\) (see Fig.\(\theta\)(a), (c)) and \(\delta_c\) with \(\delta_s = 50\delta_c\) (see Fig.\(\theta\)(b), (d)), respectively.
Figure 7: The dependence of the Born, NLO QCD corrected cross sections (shown in Fig.12(a)) and the corresponding relative corrections $\delta$ (shown in Fig.12(b)) for the processes $pp/\bar{p}p \rightarrow \tilde{\chi}_1^{\pm} \tilde{\chi}_2^0 + X$ at the LHC and the Tevatron on the gaugino mass parameter $M_2$. There we take the input parameters as $m_{\tilde{A}^0}=300 \text{ GeV}$, $M_{\text{susy}}=500 \text{ GeV}$, $\mu=400 \text{ GeV}$ and $A_f=450 \text{ GeV}$, with $\tan \beta = 4$, $\tan \beta = 15$ and $\tan \beta = 40$ respectively.
the increment of $M_2$ from 120 GeV to 200 GeV. While at the LHC, the relative corrections decrease at the region $M_2 < 160$ GeV, and then go down slowly in the region $M_2 > 160$ GeV. The NLO QCD relative corrections for $\tan \beta = 4, 15, 40$ at the LHC have the values in the range between 45.3% to 41%, when $M_2$ runs from 120 GeV to 200 GeV.

In Fig. 8(a) and (c), we present the Born cross sections and NLO QCD corrected cross sections for the processes $p\bar{p}/pp \rightarrow \tilde{\chi}_1^\pm \tilde{\chi}_2^0 + X$ at the Tevatron and the LHC as the functions of the supersymmetric soft breaking mass parameter $M_{\text{susy}}$, on the conditions of $m_{A^0}=300$ GeV, $M_2=123$ GeV, $\mu=278$ GeV and $A_f=450$ GeV, with $\tan \beta = 5$, $\tan \beta = 20$ and $\tan \beta = 40$, respectively. The solid and dotted curves present the Born and NLO QCD corrected cross sections, respectively. We can see the cross sections, especially the corrected ones, increase with the increment of $M_{\text{susy}}$. Fig. 8(b) and (d) present the relative corrections $\delta$ as the functions of $M_{\text{susy}}$ at the LHC and the Tevatron, respectively. The relative correction at the Tevatron for $\tan \beta = 4$ increases rapidly from about 29.7% to 38.4% when $M_{\text{susy}}$ goes from 250 GeV to 600 GeV, and becomes to be a constant value about 38.4% when $M_{\text{susy}}$ varies in the region beyond 600 GeV. While at the LHC, the relative correction decreases rapidly when $M_{\text{susy}}$ goes up from 250 GeV to 400 GeV, and keeps the value about 45% in the region of $M_{\text{susy}} > 400$ GeV.

We also calculate the cross sections of the processes $pp/p\bar{p} \rightarrow \tilde{\chi}_1^\pm \tilde{\chi}_2^0 + X$ as the functions of higgsino-mass parameter $\mu$(with $m_{A^0} = 300$ GeV, $M_2 = 120$ GeV,
Figure 8: The Born cross sections, NLO QCD corrected cross sections and the relative NLO QCD corrections $\delta$ for the processes $p\bar{p}/pp \to \tilde{\chi}_1^\pm \tilde{\chi}_2^0 + X$ at the Tevatron and the LHC as the functions of the supersymmetric mass parameter $M_{\text{susy}}$, on the conditions of $m_{A^0}=300$ GeV, $M_2=123$ GeV, $\mu=278$ GeV and $A_f=450$ GeV, with $\tan \beta = 4$, $\tan \beta = 20$ and $\tan \beta = 40$, respectively.
$M_{\text{susy}} = 500 \text{ GeV}, A_f = 450 \text{ GeV, tan} \beta = 4 \text{ or } 15 \text{ and } \mu \in [250 \text{ GeV}, 1000 \text{ GeV}],$
and $\tan \beta (\text{with } m_{A^0} = 300 \text{ GeV}, M_2 = 160 \text{ GeV}, M_{\text{susy}} = 500 \text{ GeV}, A_f = 450 \text{ GeV,} \mu = 450 \text{ GeV and } \tan \beta \in [5, 40])$ at the LHC/Tevatron and find the result does not depend much on these parameters. And the relative corrections have the typical values of about 32\% and 42\% at the Tevatron and the LHC, respectively.

IV Full one-loop electroweak corrections to the $u \bar{d} \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0$ subprocess

For a precise analysis of the association production processes of neutralino and chargino, higher order electroweak corrections should be included. The one-loop level UV renormalized electroweak virtual corrections to the $u \bar{d} \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0$ subprocess can be expressed in the form as

$$\Delta \hat{\sigma}_{\text{vir}}(\hat{s}, u \bar{d} \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0) = \frac{1}{8\pi \hat{s}^2} \int_{\hat{t}_{\text{min}}}^{\hat{t}_{\text{max}}} d\hat{t} \text{Re} \sum [(\mathcal{M}^V)^\dagger \mathcal{M}^0]. \quad (4.1)$$

And again the summation with bar in the equation recalls the same operations as appeared in Eq.(2.3). $\hat{t}_{\text{max}}$ and $\hat{t}_{\text{min}}$ are the same as expressed in Eq.(2.4). $\mathcal{M}^V$ is the the UV renormalized amplitude of virtual Feynman diagrams including self-energy, vertex, box and counterterm diagrams. We use FeynArts and FormCalc\cite{18} packages to generate Feynman diagrams. For the numerical evaluation of the loop integrals we use the developed package LoopTools\cite{19}.

IV.1 Renormalization scheme

As we know, the contributions of the electroweak one-loop diagrams contain both ultraviolet(UV) and infrared(IR) divergences. The UV divergence can be regu-
larized by adopting the dimensional reduction (DR) regularization scheme \cite{20} and
the relevant fields are renormalized by using the on-shell (OS) conditions \cite{21} \cite{22} \cite{26} (neglecting the finite widths of particles). In treating the QED soft and collinear
divergences we use again the two cutoff PSS method \cite{10}, the IR divergencies are
cancelled in complete analogy to the calculation for the corresponding QCD radiative
corrections as shown in Section 3. The amplitudes are performed by adopting
the ’t Hooft-Feynman gauge and \( n = 4 - 2\epsilon \) space-time dimensions to isolate the UV
and IR singularities. There is no QED induced collinear IR singularity from final
state radiation due to chargino being massive, but there exists collinear IR singulari-
ities from the initial state radiation. Similar with that as declared in Section 3, in
the electroweak correction calculation we use again the fermion flow prescription to
deal with the matrix elements including Majorana particles \cite{11}. The quark mixing
matrix is assumed to be diagonal. The bare parameters are split into renormalized
parameters and their counter terms.

1. Gauge sector

The definitions and the explicit expressions of the renormalization constants for
the gauge boson sector are written as \cite{22} \cite{27}:

\[
m_W^2 \rightarrow m_W^2 + \delta m_W^2, \quad m_Z^2 \rightarrow m_Z^2 + \delta m_Z^2, \quad W^\pm \rightarrow (1 + \frac{1}{2} \delta Z_W) W^\pm, \quad (4.2)
\]

\[
\begin{pmatrix} Z \\ A \end{pmatrix} \rightarrow \begin{pmatrix} 1 + \frac{1}{2} \delta Z_{ZZ} \\ \frac{1}{2} \delta Z_{AZ} \\ 1 + \frac{1}{2} \delta Z_{AA} \end{pmatrix} \begin{pmatrix} Z \\ A \end{pmatrix}. \quad (4.3)
\]

The renormalization conditions are taken as that the renormalized mass param-
eters of the physical particles are fixed by the requirement that they are equal to the physical masses, i.e., to the real parts of the poles of the corresponding propagators. Then the relevant fields and mass parameters are properly normalized. It yields the following results for their counter terms.

\[ \delta m_Z^2 = \tilde{\text{Re}} \Sigma_T^{ZZ}(m_Z^2), \quad \delta m_W^2 = \tilde{\text{Re}} \Sigma_T^{WW}(m_W^2), \]  

\[ \delta Z_{VV} = -\tilde{\text{Re}} \frac{\partial \Sigma^{VV}(p^2)}{\partial p^2} |_{p^2=0}, \quad V = A, Z, W, \]  

\[ \delta Z_{AZ} = -\frac{2 \tilde{\text{Re}} \Sigma_T^{AZ}(m_Z^2)}{m_Z^2}, \quad V = A, Z, W, \]  

where \( \Sigma_T \) denotes the transverse self-energy and \( \tilde{\text{Re}} \) means only taking the real part of the loop integrals. The weak mixing angle is defined as \( s_W^2 = 1 - \frac{m_W^2}{m_Z^2} \).

2. Fermion sector

The relevant fermion field renormalization constants are defined as:

\[ m_{f,0} = m_f + \delta m_f, \quad f_0^L = (1 + \frac{1}{2} \delta Z_{f,ii}^L) f^L, \quad f_0^R = (1 + \frac{1}{2} \delta Z_{f,ii}^R) f^R. \]

For the SM fermions, the normalized constants can be expressed as:

\[ \delta m_f = \frac{1}{2} \tilde{\text{Re}} \left[ m_f \Sigma_f^L(m_f^2) + m_f \Sigma_f^R(m_f^2) + \Sigma_f^S(m_f^2) + \Sigma_f^S(m_f^2) \right], \]  

\[ \delta Z_{f,ii}^L = -\tilde{\text{Re}} \Sigma_f^{L}(m_f^2) - m_f \frac{\partial}{\partial p^2} \tilde{\text{Re}} \left\{ m_f \Sigma_f^{L}(p^2) + m_f \Sigma_f^{R}(p^2) + \Sigma_f^S(p^2) + \Sigma_f^S(p^2) \right\} |_{p^2=m_f^2}, \]  

\[ \delta Z_{f,ii}^R = -\tilde{\text{Re}} \Sigma_f^{R}(m_f^2) - m_f \frac{\partial}{\partial p^2} \tilde{\text{Re}} \left\{ m_f \Sigma_f^{L}(p^2) + m_f \Sigma_f^{R}(p^2) + \Sigma_f^S(p^2) + \Sigma_f^S(p^2) \right\} |_{p^2=m_f^2}. \]
\[
\delta Z_{f,ij}^L = \frac{2}{m_{f_i}^2 - m_{f_j}^2} \tilde{R}e \left[ m_{f_j} \Sigma_{f,ij}^L(m_{f_j}^2) + m_{f_i} \Sigma_{f,ij}^R(m_{f_j}^2) \right] + m_{f_i} \Sigma_{f,ij}^S(m_{f_j}^2) + m_{f_j} \Sigma_{f,ij}^S(m_{f_i}^2), \quad (i \neq j), \tag{4.12}
\]

\[
\delta Z_{f,ij}^R = \frac{2}{m_{f_i}^2 - m_{f_j}^2} \tilde{R}e \left[ m_{f_i} m_{f_j} \Sigma_{f,ij}^L(m_{f_j}^2) + m_{f_j}^2 \Sigma_{f,ij}^R(m_{f_i}^2) \right] + m_{f_i} \Sigma_{S,f,ij}^L(m_{f_i}^2) + m_{f_j} \Sigma_{S,f,ij}^R(m_{f_j}^2), \quad (i \neq j). \tag{4.13}
\]

The one-particle irreducible two-point function \(i\Gamma_{f,ij}(p^2)\) for fermions is decomposed as

\[
\Gamma_{f,ij}(p^2) = \delta_{ij}(\not{p} - m_f) + \left[ \not{p} \Sigma_{f,ij}^L(p^2) + \not{p} \Sigma_{f,ij}^R(p^2) \right] + P_L \Sigma_{f,ij}^{S,L}(p^2) + P_R \Sigma_{f,ij}^{S,R}(p^2). \tag{4.14}
\]

In our calculation we use an improved scheme to make the perturbative calculation more reliable. That means we use the effective \(\overline{MS}\) fine structure constant value at \(Q = m_Z\) as input parameter, \(\alpha_{ew}(m_Z^2)^{-1}|_{\overline{MS}} = 127.918^{[34]}\). This results in the counter-term of the electric charge expressed as \([20, 30, 31]\):

\[
\delta Z_e = \frac{e^2}{6(4\pi)^2} \left\{ 4 \sum_f N_C^f e_f^2 \left( \Delta + \log \frac{Q^2}{x_f^2} \right) + \sum_f \sum_{k=1}^2 N_C^f e_f^2 \left( \Delta + \log \frac{Q^2}{m_{f_k}^2} \right) \\
+ 4 \sum_{k=1}^2 \left( \Delta + \log \frac{Q^2}{m_{\tilde{\chi}_k}^2} \right) + \sum_{k=1}^2 \left( \Delta + \log \frac{Q^2}{m_{H_k}^2} \right) \\
- 22 \left( \Delta + \log \frac{Q^2}{m_W^2} \right) \right\}, \tag{4.15}
\]

where we take \(x_f = m_Z\) when \(m_f < m_Z\) and \(x_t = m_t\). \(e_f\) is the electric charge of (s)fermion and \(\Delta = 2/\epsilon - \gamma + \log 4\pi\). \(N_C^f\) is color factor, which equal to 1 and 3 for (s)leptons and (s)quarks, respectively. In our calculation we take \(Q = (m_{\tilde{\chi}_1^\pm} + m_{\tilde{\chi}_1^0})/2\) in using Eq.(4.15).
3. Supersymmetric sector

In the MSSM theory the physical chargino mass eigenstates $\tilde{\chi}^{\pm}_{1,2}$ are the combinations of charged gauginos and higgsinos. Their physical masses can be obtained by diagonalizing the corresponding mass matrix $X$, which has the form as \[37\]:

$$X = \begin{pmatrix} M_{SU(2)} & \sqrt{2} m_W \sin \beta \\ \sqrt{2} m_W \cos \beta & \mu \end{pmatrix}. $$

$X$ is diagonalized with two unitary matrices $U$ and $V$ according to

$$U^* X V^\dagger = \text{diag}(m_{\tilde{\chi}^+_1}, m_{\tilde{\chi}^+_2}), \quad (4.16)$$

which yields the chargino masses.

The neutralinos are the mixtures of the neutral gauginos and higgsinos. Their physical masses can be obtained by diagonalizing the corresponding mass matrix $Y$ \[37\].

$$Y = \begin{pmatrix} M_{U(1)} & 0 & -m_Z s_W \cos \beta & m_Z s_W \sin \beta \\ 0 & M_{SU(2)} & m_Z c_W \cos \beta & -m_Z c_W \sin \beta \\ -m_Z s_W \cos \beta & m_Z c_W \cos \beta & 0 & -\mu \\ m_Z s_W \sin \beta & -m_Z c_W \sin \beta & -\mu & 0 \end{pmatrix}. \quad (4.17)$$

$Y$ is diagonalized with a unitary matrix $N$ according to

$$N^* Y N^\dagger = \text{diag}(m_{\tilde{\chi}^0_1}, m_{\tilde{\chi}^0_2}, m_{\tilde{\chi}^0_3}, m_{\tilde{\chi}^0_4}), \quad (4.18)$$

which yields the four neutralino mass eigenstates.

We follow the renormalization definitions for chargino, neutralino and sfermion sectors as in Ref.\[23\]. The chargino, neutralino wave functions and mass counter terms in the mass eigenstate basis are introduced as

$$\tilde{\chi}_i \rightarrow (\delta_{ij} + \frac{1}{2} \delta Z^{0}\widetilde{e}_{L} P_{L} + \frac{1}{2} \delta Z^{0}\widetilde{e}_{R} P_{R}) \tilde{\chi}_j, \quad m_{\tilde{\chi}_i} \rightarrow m_{\tilde{\chi}_i} + \delta m_{\tilde{\chi}_i}, \quad (4.19)$$

27
where $\tilde{\chi}$ stands for both charginos and neutralinos, $i, j = 1, 2$ for chargino sector, $i, j = 1, 2, 3, 4$ for neutralino sector. These counter terms can be expressed as the functions of the corresponding self-energies similar with the equations of Eqs. (4.19)-(4.14) with the replacements of $f_{i,j} \rightarrow \tilde{\chi}_{i,j}$.

The wave function and mass counter terms for scalar quarks and scalar leptons are defined as

$$\left( \begin{array}{c} \tilde{q}^{(B)}_1 \\ \tilde{q}^{(B)}_2 \end{array} \right) \rightarrow \left( \begin{array}{cc} Z_{11}^{\tilde{q}} & \frac{1}{2} Z_{12}^{\tilde{q}} \\ \frac{1}{2} Z_{21}^{\tilde{q}} & Z_{22}^{\tilde{q}} \end{array} \right) \left( \begin{array}{c} \tilde{q}_1 \\ \tilde{q}_2 \end{array} \right) = \left( 1 + \frac{1}{2} \delta Z^{\tilde{q}} \right) \left( \begin{array}{c} \tilde{q}_1 \\ \tilde{q}_2 \end{array} \right), \quad (4.20)$$

$$\left( \begin{array}{c} \tilde{l}^{(B)}_1 \\ \tilde{l}^{(B)}_2 \end{array} \right) \rightarrow \left( \begin{array}{cc} Z_{11}^{\tilde{l}} & \frac{1}{2} Z_{12}^{\tilde{l}} \\ \frac{1}{2} Z_{21}^{\tilde{l}} & Z_{22}^{\tilde{l}} \end{array} \right) \left( \begin{array}{c} \tilde{l}_1 \\ \tilde{l}_2 \end{array} \right) = \left( 1 + \frac{1}{2} \delta Z^{\tilde{l}} \right) \left( \begin{array}{c} \tilde{l}_1 \\ \tilde{l}_2 \end{array} \right), \quad (4.21)$$

where

$$\delta Z^{\tilde{q}, \tilde{l}} \rightarrow \left( \begin{array}{cc} \delta Z_{11}^{\tilde{q}, \tilde{l}} & \delta Z_{12}^{\tilde{q}, \tilde{l}} \\ \delta Z_{21}^{\tilde{q}, \tilde{l}} & \delta Z_{22}^{\tilde{q}, \tilde{l}} \end{array} \right). \quad (4.22)$$

The corresponding counter terms for the scalar fermions are given as

$$\delta m_{\tilde{q}, \tilde{l}}^2 = \tilde{R} e \sum \tilde{q}_{i, j} (m_{\tilde{q}_{j, i}}^2), \quad \delta Z_{\tilde{q}, \tilde{l}}^{\tilde{q}, \tilde{l}} = -\tilde{R} e \frac{\partial}{\partial k^2} \sum_{\tilde{q}_{i, j}} (k^2)_{k^2 = m_{\tilde{q}_{j, i}}^2}. \quad (4.23)$$

$$\delta Z_{\tilde{q}, \tilde{l}}^{\tilde{q}, \tilde{l}} = -\tilde{R} e \frac{2 \sum \tilde{q}_{i, j} (m_{\tilde{q}_{j, i}}^2)}{m_{\tilde{q}_{j, i}}^2 - m_{\tilde{q}_{i, j}}^2} \quad (i, j = 1, 2, \quad i \neq j). \quad (4.24)$$

We introduce the counter terms for unitary matrices $N, U, V$ and $R$ as follow:

$$N \rightarrow N + \delta N, \quad U \rightarrow U + \delta U, \quad V \rightarrow V + \delta V, \quad R \rightarrow R + \delta R, \quad (4.25)$$

where $N$ and $R$ are the rotation matrices of neutralino and squark(slepton) sectors, respectively. $U$ and $V$ are the diagonal unitary matrices for chargino sector.
The counterterms $\delta U, \delta V, \delta N$ and $\delta R$ can be fixed by requiring that the counterterms $\delta U, \delta V, \delta N$ and $\delta R$ cancel the antisymmetric parts of the wave function corrections. We get the expressions of the counter terms for the neutralino, chargino and sfermion rotation matrices $N$, $U$, $V$ and $R$ as below:

$$
\delta N_{ij} = \frac{1}{4} \sum_{k=1}^{4} (\delta \tilde{Z}^{0,L}_{ik} - \delta \tilde{Z}^{0,R}_{ki}) N_{kj}, \quad (4.26)
$$

$$
\delta U_{ij} = \frac{1}{4} \sum_{k=1}^{4} (\delta \tilde{Z}^{+,R}_{ik} - \delta \tilde{Z}^{+,L}_{ki}) N_{kj}, \quad \delta V_{ij} = \frac{1}{4} \sum_{k=1}^{4} (\delta \tilde{Z}^{+,L}_{ik} - \delta \tilde{Z}^{+,R}_{ki}) N_{kj} \quad (4.27)
$$

$$
\delta R^{\tilde{f}}_{ij} = \frac{1}{4} \sum_{k=1}^{4} (\delta \tilde{Z}^{\tilde{f}}_{ik} - \delta \tilde{Z}^{\tilde{f}}_{ki}) R^{\tilde{f}}_{kj} \quad (4.28)
$$

As we expected, the UV divergence induced by the one-loop diagrams can be cancelled by that contributed by the counterterm diagrams exactly. While the soft and collinear IR divergences still exist.

**IV.2 Real photon emission**

The soft IR singularity in the $\mathcal{M}^V$ is originated from virtual photonic loop correction. It can be cancelled by the contribution of the real soft photon emission process. The real photon emission Feynman diagrams for the subprocess $u \bar{d} \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0 \gamma$ are shown in Fig.9.

We denote the real photon emission process as

$$
u(p_1) + \bar{d}(p_2) \rightarrow \tilde{\chi}_1^+(k_3) + \tilde{\chi}_2^0(k_4) + \gamma(k_5), \quad (4.29)
$$

We adopt the general PSS method to separate the soft photon emission singu-
larity from the real photon emission process. By introducing an arbitrary small soft cutoff $\delta_s$ we separate the phase space of the subprocess $u \bar{d} \to \tilde{\chi}_1^+ \tilde{\chi}_2^0 \gamma$ into two regions, according to whether the energy of the emitted photon is soft, i.e. $E_5 \leq \delta_s \sqrt{s}/2$, or hard, i.e. $E_5 > \delta_s \sqrt{s}/2$, where $\sqrt{s}/2$ is the incoming parton beam energy in the c.m.s. frame. Then the correction of the real photon emission is broken down into corresponding soft and hard terms

$$\Delta \hat{\sigma}_{\text{real}} = \Delta \hat{\sigma}_{\text{soft}} + \Delta \hat{\sigma}_{\text{hard}} = \hat{\sigma}_0 (\delta_{\text{soft}} + \delta_{\text{hard}}).$$  \hspace{1cm} (4.30)$$

Although both $\Delta \hat{\sigma}_{\text{soft}}$ and $\Delta \hat{\sigma}_{\text{hard}}$ depend on the soft photon cutoff $\delta_s \sqrt{s}/2$, the real correction $\Delta \hat{\sigma}_{\text{real}}$ is cutoff independent. If we use the soft photon emission approximation[22], we can set $k_5^\mu = 0$ in the delta function of the phase space element (i.e., $\delta^{(n)} (p_1 + p_2 - k_3 - k_4)$) up to corrections of $\mathcal{O}(\delta_s)$. Then we can take the n-momenta of the initial and final particles in the $p_1 + p_2$ rest frame as

$$p_1 = \frac{\sqrt{s}}{2} (1, \ldots, 0, 0, 1), \quad p_2 = \frac{\sqrt{s}}{2} (1, \ldots, 0, 0, -1),$$

$$k_3 = (E_3, \ldots, p \sin \theta, 0, p \cos \theta), \quad k_4 = (E_4, \ldots, -p \sin \theta, 0, -p \cos \theta),$$

$$k_5 = E_5 (1, \ldots, \sin \theta_1 \sin \theta_2, \sin \theta_1 \cos \theta_2, \cos \theta_1).$$  \hspace{1cm} (4.31)$$

Then the integral over the soft photon phase space can be implemented analytically.
We get the expression of the differential cross section for the subprocess $u\bar{d} \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_2^0$ as [13]

$$d\Delta\hat{\sigma}_{soft} = d\hat{\sigma}_0 \left[ \frac{\alpha_{ew}}{2\pi} \frac{\Gamma(1 - \epsilon)}{\Gamma(1 - 2\epsilon)} \left( \frac{4\pi\mu^2_\tau}{\tilde{s}} \right)^\epsilon \right] \left( \frac{A^S_2}{\epsilon^2} + \frac{A^S_1}{\epsilon} + A^S_0 \right),$$

where

$$A^S_2 = \frac{5}{9},$$
$$A^S_1 = -\frac{4}{9}C^1_{12} + \frac{12}{9}C^1_{13} + \frac{6}{9}C^1_{23} - C^1_{33},$$
$$A^S_0 = -\frac{4}{9}C^0_{12} + \frac{12}{9}C^0_{13} + \frac{6}{9}C^0_{23} - C^0_{33},$$

with

$$C^1_{12} = -2 \ln \delta_s,$$
$$C^0_{12} = 2 \ln^2 \delta_s,$$
$$C^1_{13} = -\ln \delta_s - \frac{1}{2} \ln \left( \frac{E_3 - p \cos \theta}{E_3 - p} \right)^2,$$
$$C^0_{13} = \frac{1}{2} \left[ \ln^2 \left( \frac{E_3 - p}{E_3 - p \cos \theta} \right) - \frac{1}{2} \ln^2 \left( \frac{E_3 + p}{E_3 - p} \right) + 2Li \left( \frac{p \cos \theta - p}{E_3 - p} - 2Li \left( \frac{p \cos \theta - p}{E_3 - p} \right) \right) \right]$$
$$+ \ln \delta_s \ln \left( \frac{(E_3 - p \cos \theta)^2}{E_3^2 - p^2} \right) + \ln^2 \delta_s,$$
$$C^1_{23} = -\ln \delta_s - \frac{1}{2} \ln \left( \frac{E_3 + p \cos \theta}{E_3} \right)^2,$$
$$C^0_{23} = \frac{1}{2} \left[ \ln^2 \left( \frac{E_3 - p}{E_3 + p \cos \theta} \right) - \frac{1}{2} \ln^2 \left( \frac{E_3 + p}{E_3 - p} \right) + 2Li \left( \frac{p \cos \theta - p}{E_3 - p} - 2Li \left( \frac{p \cos \theta - p}{E_3 + p \cos \theta} \right) \right) \right]$$
$$+ \ln \delta_s \ln \left( \frac{(E_3 + p \cos \theta)^2}{E_3^2 - p^2} \right) + \ln^2 \delta_s,$$
$$C^1_{33} = -1,$$
$$C^0_{33} = 2 \ln \delta_s - \frac{E_3}{p} \ln \frac{E_3 + p}{E_3 - p},$$

(4.33)
In above equations we have following relations between some variables:

\[ E_3 = \frac{2m_{\tilde{\chi}_1^+}^2 - \hat{t} - \hat{u}}{2\sqrt{s}}, \quad p \cos \theta = \frac{\hat{t} - \hat{u}}{2\sqrt{s}}, \]

\[ p = \frac{1}{2\sqrt{s}} \sqrt{(2m_{\tilde{\chi}_1^+}^2 + 2\sqrt{s}m_{\tilde{\chi}_1^+} - \hat{t} - \hat{u})(2m_{\tilde{\chi}_1^+}^2 - 2\sqrt{s}m_{\tilde{\chi}_1^+} - \hat{t} - \hat{u})}. \]

(4.34)

Since the incoming light-quarks are assumed to be massless in the parton model and the outgoing particles are massive, there exist only collinear IR singularities induced by initial state hard photon radiation. To isolate the collinear singularities from \( \Delta \tilde{\sigma}_{\text{hard}} \), we further decompose \( \Delta \tilde{\sigma}_{\text{hard}} \) into a sum of hard collinear(HC) and hard non-collinear(\( \overline{\text{HC}} \)) terms by introducing another cutoff \( \delta_c \) named collinear cutoff

\[ \Delta \tilde{\sigma}_{\text{hard}} = \Delta \tilde{\sigma}_{\text{HC}} + \Delta \tilde{\sigma}_{\overline{\text{HC}}}. \]

(4.35)

where the HC regions of the phase space are those any one of the Lorentz invariants \( \hat{t}_{15}(\equiv (p_1 - k_5)^2), \hat{t}_{25}(\equiv (p_2 - k_5)^2) \) becomes smaller in magnitude than \( \delta_c \hat{s} \) and the emitted photon remains hard. \( \Delta \tilde{\sigma}_{\text{HC}} \) contains collinear divergences. As mentioned above, the soft IR divergence of the virtual photonic corrections can be cancelled exactly by that of soft real corrections. The remaining collinear singularities are absorbed by a redefinition(renormalization) of the parton distribution functions PDFs\[32\]. This is done in analogy to the calculation of QCD radiative correction.

In the \( \overline{\text{HC}} \) region, \( \Delta \tilde{\sigma}_{\overline{\text{HC}}} \) is finite and can be evaluated in four-dimensions by applying standard Monte Carlo method. We can see that \( \Delta \tilde{\sigma}_{\text{soft}}, \Delta \tilde{\sigma}_{\text{HC}} \) and \( \Delta \tilde{\sigma}_{\overline{\text{HC}}} \) depend on the two arbitrary parameters \( \delta_s \) and \( \delta_c \). However, in the total electroweak corrected hadronic cross section, after mass factorization, the dependence on these
arbitrary cutoffs($\delta_s$ and $\delta_c$) cancels, as will be explicitly shown in numerical calculation (see Section IV.4). This constitutes an important check of our calculation. Finally, we get an UV and IR finite corrections $\Delta\sigma$ of the processes $p\bar{p}/pp \rightarrow \tilde{\chi}_1^\pm\tilde{\chi}_2^0 + X$.

IV.3 The cross sections of processes $p\bar{p}/pp \rightarrow \tilde{\chi}_1^\pm\tilde{\chi}_2^0 + X$

The total electroweak corrected cross sections of the parent processes $p\bar{p}/pp \rightarrow u\bar{d}(\bar{u}d) \rightarrow \tilde{\chi}_1^\pm\tilde{\chi}_2^0 + X$ at the Tevatron and the LHC can be calculated from the cross sections of subprocesses $u\bar{d} \rightarrow \tilde{\chi}_1^\pm\tilde{\chi}_2^0$ and $\bar{u}d \rightarrow \tilde{\chi}_1^\pm\tilde{\chi}_2^0$.

By summing the UV renormalized electroweak virtual corrections and the real photon emission corrections, the remaining collinear divergences are absorbed into the redefinition of the distribution functions. Using the $\overline{\text{MS}}$ scheme, the scale dependent parton distribution functions including $\mathcal{O}(\alpha_{\text{ew}})$ corrections are given as

$$G_{i/A}(x, \mu_f) = G_{i/A}(x) + \left(-\frac{1}{\epsilon}\right) \left[ \frac{\alpha_{\text{ew}}}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu_F^2}{s}\right)^\epsilon \right] \int_x^1 \frac{dz}{z} P_{ij}(z) G_{j/A}(x/z).$$

(4.36)

By using above definition, we get a $\mathcal{O}(\alpha_{\text{ew}})$ parton distribution function counterterms which are combined with the hard collinear contributions to result in the $\mathcal{O}(\alpha_{\text{ew}})$ expression for the remaining collinear contributions:

$$d\sigma^{\text{coll}} = d\hat{\sigma}^{0} \left[ \frac{\alpha_{\text{ew}}}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu_F^2}{s}\right)^\epsilon \right] \{ G_{u/A}(x_A, \mu_f) G_{d/B}(x_B, \mu_f) + G_{u/A}(x_A, \mu_f) \tilde{G}_{d/B}(x_B, \mu_f) \} + \sum_{\alpha=u,d} \left[ A^{\text{ie}}_{\alpha}(\alpha \rightarrow \alpha\gamma) + A^{\text{sec}}_{\alpha}(\alpha \rightarrow \alpha\gamma) \right] G_{u/A}(x_A, \mu_f) G_{d/B}(x_B, \mu_f) + (A \leftrightarrow B) \} dx_A dx_B,$$

(4.37)

where $A/B$ are proton/antiproton for the Tevatron, and proton/proton for the LHC,
respectively.

\[ A_1^{sc}(u(d) \to u(d)\gamma) = q_f^2(2 \ln \delta_s + 3/2), \quad A_0^{sc} = A_1^{sc} \ln \left(\frac{\hat{s}}{\mu_f^2}\right), \tag{4.38} \]

and

\[ \tilde{G}_{\alpha/A,B}(x, \mu_f) = \sum_{c' = \alpha, \gamma} \int_{1-\delta_c}^{1} dy \frac{y q_f^2 G_{c'/A,B}(x/y, \mu_f) P_{\alpha c}(y)}{y} \tilde{G}_{c'A,B}(x/y, \mu_f), \quad (\alpha = u, \bar{d}) \tag{4.39} \]

with

\[ \tilde{P}_{\alpha c}(y) = P_{\alpha c} \ln \left(\frac{1 - y \hat{s}}{y \mu_f^2}\right) - P_{\alpha c}'(y). \tag{4.40} \]

where \( \tilde{P}_{qq} = \frac{3}{4} P_{qq} \) and \( P_{q\gamma} = \frac{1}{3} P_{qq} \). \( P_{qq} \) and \( P_{qg} \) have the expressions as shown in Eqs. (3.12) and (3.15), respectively. \( q_f (f = u, \bar{d}) \) are the charges of quarks.

The final result for the total \( \mathcal{O}(\alpha_{ew}) \) correction consists of two parts of contributions: a two-body term \( \sigma^{(2)} \) and a three-body term \( \sigma^{(3)} \). The two-body correction term \( \sigma^{(2)} \) is expressed as

\[ \sigma^{(2)} = \frac{\alpha_{ew}}{2\pi} \int dx_A dx_B d\hat{\sigma}^0 \{ G_{u/A}(x_A, \mu_f) G_{\bar{d}/B}(x_B, \mu_f) [A_0^S + A_0^V + A_0^{sc}(u \to u\gamma) + A_0^{sc}(\bar{d} \to \bar{d}\gamma)] + G_{u/A}(x_A, \mu_f) G_{\bar{d}/B}(x_B, \mu_f) + G_{u/A}(x_A, \mu_f) \tilde{G}_{\bar{d}/B}(x_B, \mu_f) + (A \leftrightarrow B) \}. \tag{4.41} \]

And the three-body correction term \( \sigma^{(3)} \) is written as

\[ \sigma^{(3)} = \sigma^{(3)}(pp/\bar{p}p \to u\bar{d} \to \tilde{\chi}^+_1 \tilde{\chi}^-_2 + \gamma) \]

\[ = \int dx_A dx_B [G_{u/A}(x_A, \mu_f) G_{\bar{d}/B}(x_B, \mu_f) + (A \leftrightarrow B)] d\hat{\sigma}^{(3)}(u\bar{d} \to \tilde{\chi}^+_1 \tilde{\chi}^-_2 + \gamma), \tag{4.42} \]

where \( G \) is the proton/antiproton distribution function. Finally, the full one-loop electroweak corrected cross section for \( pp/\bar{p}p \to \tilde{\chi}^+_1 \tilde{\chi}^-_2 + X \) is

\[ \sigma^{EW} = \sigma^0 + \sigma^{(2)} + \sigma^{(3)} = \sigma^0 + \Delta \sigma. \tag{4.43} \]
The cross section part of $\hat{\sigma}^{(2)} + \hat{\sigma}^{(3)}$ should be independence of the cutoff parameters $\delta_s$ and $\delta_c$. The electroweak one-loop relative correction is defined as $\delta = \Delta_{\sigma}/\hat{\sigma}_0$.

**IV.4 Numerical results including electroweak corrections**

In this subsection, we present some numerical results for the one-loop $\mathcal{O}(\alpha_{ew})$ electroweak corrections to the processes $p\bar{p}/pp \rightarrow \tilde{\chi}_1^{\pm} \tilde{\chi}_0^0 + X$. We take the SM input parameters as $m_Z = 91.1876$ GeV, $m_W = 80.425$ GeV, $m_t = 178.1$ GeV, $m_b = 4.7$ GeV\textsuperscript{33} and neglect the light-quark masses in the numerical calculation. The fine structure constant is taken having the value at the $Z$-pole, $\alpha_{ew}(m_Z^2)|_{\overline{\text{MS}}} = 1/127.918$\textsuperscript{34}. The new MRST 2004-QED parton distribution functions including $\mathcal{O}(\alpha_{ew})$ corrections to the parton evolution are adopted in calculating the Born and the one-loop order corrected cross sections\textsuperscript{35}. The renormalization and factorization scales are taken to be equal for simplicity ($Q = \mu_r = \mu_f$), and have the value being the average of the final particle masses in analogy to the NLO QCD calculation. We use again the package FormCalc to obtain all the masses of supersymmetric particles by inputting the supersymmetric parameters $\tan \beta$, $m_{A^0}$, $M_{\text{susy}}$, $\mu$, $M_2$ and $A_f$. Among these six input supersymmetric parameters, the CP-odd Higgs-boson mass $m_{A^0}$ and $\tan \beta$ with the constraint $\tan \beta \geq 2.5$ are for the Higgs sector. In FormCalc package the radiative corrections to Higgs-boson masses up to two-loop contributions have been involved\textsuperscript{36}. While the tree-level Higgs-boson masses can be obtained by using the equations

$$m_{H^0, H^0}^2 = \frac{1}{2} \left( m_{A^0}^2 + m_{Z^0}^2 + \sqrt{(m_{A^0}^2 + m_{Z^0}^2)^2 - 4m_{A^0}^2 m_{Z^0}^2 \cos^2(2\beta)} \right),$$

$$m_{H^\pm}^2 = m_W^2 + m_{A^0}^2.$$

(4.44)
In order to keep the gauge invariance during our numerical calculation, we adopt the tree-level Higgs-masses obtained from Eq. (4.44), but not the Higgs-masses from the output of FormCalc package throughout the tree-level and one-loop calculations.

As mentioned above, the final results should be independent on cut-offs $\delta_s$ and $\delta_c$. For demonstration, we present the cross section corrections of the $p\bar{p}/pp \to \tilde{\chi}_1^\pm \tilde{\chi}_2^0 + X$ processes as the functions of the soft cut-offs $\delta_s$ and $\delta_c$ in Figs.10(a-d) on conditions of $\tan \beta = 4$, $m_{A^0} = 300$ GeV, $M_{susy} = 250$ GeV, $\mu = 278$ GeV, $M_2 = 127$ GeV and $A_f = 450$ GeV at the Tevatron and the LHC. The dashed, solid and dotted lines correspond to the total correction $\Delta \sigma = \sigma^{(2)} + \sigma^{(3)}$, three-body correction $\sigma^{(3)}$ and two-body correction $\sigma^{(2)}$, respectively. As shown in these figures, the full $\mathcal{O}(\alpha_{ew})$ correction $\Delta \sigma$ is independent of the soft cutoff $\delta_s(\delta_c)$, as $\delta_s(\delta_c)$ running from $10^{-6}(10^{-6})$ to $10^{-1}(10^{-3})$ and $\delta_c = \delta_s/50(\delta_s = 50\delta_c)$. In the further numerical calculations, we set $\delta_s = 10^{-5}$ and $\delta_c = \delta_s/50$, if there is no other statement.

In Fig.11 the dependence of the Born cross sections, full one-loop $\mathcal{O}(\alpha_{ew})$ corrected cross sections and the corresponding relative corrections of processes $pp/p\bar{p} \to \tilde{\chi}_1^\pm \tilde{\chi}_2^0 + X$ at the LHC and the Tevatron on the gaugino mass parameter $M_2$ are depicted. There we take the input parameters as $m_{A^0} = 300$ GeV, $M_{susy} = 350$ GeV, $\mu = 550$ GeV and $A_f = 450$ GeV, with $\tan \beta = 4$, $\tan \beta = 15$ and $\tan \beta = 40$, respectively. We can see that the Born and electroweak corrected cross sections decrease
Figure 10: The dependence of the full one-loop electroweak corrected cross sections for $p\bar{p}/pp \rightarrow \tilde{\chi}_1^\pm \tilde{\chi}_2^0 + X$ processes at the Tevatron and the LHC as the functions of the cutoff $\delta_s$ with $\delta_c = \delta_s/50$ (see Fig.10(a),(c)) and $\delta_c$ with $\delta_s = 50\delta_c$ (see Fig.10(b),(d)), respectively.
Figure 11: The dependence of the Born, full one-loop electroweak corrected cross sections (shown in Fig 11(a,c)) and the corresponding relative corrections $\delta$ (shown in Fig 11(b,d)) for the processes $pp/\bar{p}p \rightarrow \tilde{\chi}^\pm \tilde{\chi}^0_2 + X$ at the LHC and the Tevatron on the gaugino mass parameter $M_2$. There we take the input parameters as $m_{A^0}=300$ GeV, $M_{\text{susy}}=500$ GeV, $\mu=400$ GeV and $A_f=450$ GeV, with $\tan \beta = 4$, $\tan \beta = 15$ and $\tan \beta = 40$ respectively.
rapidly to small values with $M_2$ going from 120 GeV to 260 GeV at the Tevatron (shown in Fig.11(c)) and the LHC (shown in Fig.11(a)) due to kinematical effects, because the masses of the final-state chargino and neutralino are roughly proportional to $M_2$. While the relative correction $\delta$ (shown in Fig.11(b)(d)) increases clearly. At the Tevatron the relative correction $\delta$ is in the range from -8.8% to 1.2% with our chosen parameters, while at the LHC it can reach -7.9%. Here we have the masses of chargino and neutralino in the ranges of $m_{\tilde{\chi}_1^\pm} \in [111.756 \text{ GeV}, 252.504 \text{ GeV}]$, $m_{\tilde{\chi}_2^0} \in [112.067 \text{ GeV}, 252.538 \text{ GeV}]$.

In Figs.12(a,c), we show the dependence of the Born and the full one-loop electroweak corrected cross sections of processes $pp/p\bar{p} \rightarrow \tilde{\chi}_1^\pm \tilde{\chi}_2^0 + X$ on the SUSY soft breaking mass parameter $M_{\text{susy}}$ at the LHC and the Tevatron, on the conditions of $m_{A_0}=300$ GeV, $M_2=127$ GeV, $\mu=278$ GeV and $A_f=450$ GeV, with $\tan \beta = 4(m_{\tilde{\chi}_1^\pm}=104.428 \text{ GeV}, m_{\tilde{\chi}_2^0}=106.632 \text{ GeV}), \tan \beta = 20(m_{\tilde{\chi}_1^\pm}=113.353 \text{ GeV}, m_{\tilde{\chi}_2^0}=113.780 \text{ GeV})$ and $\tan \beta = 40(m_{\tilde{\chi}_1^\pm}=114.577 \text{ GeV}, m_{\tilde{\chi}_2^0}=114.839 \text{ GeV})$, respectively. The solid curves represent the Born cross sections and the dotted curves represent the one-loop electroweak corrected cross sections. We can see the total cross sections have the values from 0.34 pb to 1.26 pb and from 3.33 pb to 11.65 pb at the Tevatron and the LHC respectively, as the SUSY soft breaking mass parameter $M_{\text{susy}}$ runs from 250 GeV to 950 GeV. Figs.12(b,d) present the one-loop electroweak relative corrections as the functions of $M_{\text{susy}}$, and we can see the relative corrections generally decrease with the increment of $M_{\text{susy}}$. And the relative corrections are between
Figure 12: The dependence of the Born and the electroweak corrected cross sections of process $pp/\bar{p}p \rightarrow \tilde{\chi}_1^{\pm} \tilde{\chi}_2^{0} + X$ (see in Fig.12(a)(c)) and the corresponding relative corrections in Fig.12(b)(d) on the SUSY soft breaking mass parameter $M_{\text{susy}}$ at the LHC and the Tevatron on the conditions of $m_{A^0}=300\text{GeV}$, $M_2=127\text{ GeV}$, $\mu=278\text{ GeV}$ and $A_f=450\text{GeV}$, with $\tan \beta = 4$, $\tan \beta = 20$ and $\tan \beta = 40$, respectively.
−9.33% and −7.64% at the Tevatron and between −7.46% and −5.03% at the LHC with our chosen parameters.

We also calculate the Born cross sections, the one-loop electroweak corrected cross sections and the corresponding relative corrections as the functions of µ (with $m_{A^0} = 300 \text{ GeV}$, $M_2 = 127 \text{ GeV}$, $M_{\text{susy}} = 250 \text{ GeV}$, and $A_f = 450 \text{ GeV}$, tan $\beta = 4$ or 15 and $\mu \in [250 \text{ GeV}, 1000 \text{ GeV}]$), and tan $\beta$ (with $m_{A^0} = 300 \text{ GeV}$, $M_2 = 200 \text{ GeV}$, $M_{\text{susy}} = 350 \text{ GeV}$, $A_f = 450 \text{ GeV}$, $\mu = 550 \text{ GeV}$ and tan $\beta \in [5, 40]$). We find that the results do not depend much on those parameters. The relative corrections have the typical values of about −8% and −6% at the Tevatron and the LHC, respectively.

V Summary

In this paper, we present the calculations of the NLO QCD and the full one-loop electroweak corrections to the processes $p\bar{p}/pp \to \tilde{\chi}_1^\pm \tilde{\chi}_2^0 + X$ at the Tevatron and the LHC in the framework of the MSSM. In the calculations of both the NLO QCD and one-loop electroweak corrections we apply the algorithm of the phase-space slicing (PSS) method. We analyze the numerical results and investigate the dependence of the cross sections and corresponding relative corrections for the processes on several supersymmetric parameters. We find that the NLO QCD corrections generally increase the Born cross section, while the electroweak corrections decrease the Born cross sections in most of the chosen parameter space. The contributions from the NLO QCD corrections make the theoretical predictions nearly independent of the renormalization and factorization scales. Our results show that the NLO QCD and
electroweak relative corrections typically have the values of about 32%(42%) and 
-8%(-6%) at the Tevatron(LHC), respectively. We conclude that the NLO QCD 
and complete one-loop electroweak corrections to the processes $p\bar{p}/pp \rightarrow \tilde{\chi}^{\pm}_1\tilde{\chi}^0_2 + X$
are generally significant and should be considered in high precision analysis.

Acknowledgments:

This work was supported in part by the National Natural Science Foundation of 
China and a special fund sponsored by Chinese Academy of Science.

References

[1] S. Nandi and X. Tata, Phys. Lett. B129 (1983) 451; A.H. Chamseddine, P. 
Nath and R. Arnowitt, Phys. Lett. B129, (1983) 445; H. Baer and X. Tata, 
Phys. Lett. B155, (1985) 278; H. Baer, K. Hagiwara and X. Tata, Phys. Rev. 
D35, (1987) 1598; P. Nath and R. Arnowitt, Mod. Phys. Lett. A2, (1987) 331.

[2] H. Baer and X. Tata, Phys. Rev. D47 (1993) 2739; H. Baer, C. Kao and X. 
Tata, Phys, Rev. D48 (1993) 5175; H. Baer, C.-H. Chen, C. Kao and X. Tata, 
Phys. Rev. D52 (1995) 1565; H. Baer, C.-H. Chen, F. Paige and X. Tata, 
Phys. Rev. D54 (1996) 5866; H. Baer, C.-H. Chen, M. Drees, F. Paige and X. 
Tata, Phys. Rev. D58 (1998) 075008.

[3] D0 Collaboration, S. Abachi et al., Phys. Rev. Lett. 76 (1996) 2228; CDF 
Collaboration, F. Abe et al., Phys. Rev. Lett. 80, (1998) 5275.
[4] V. Barger and Chung Kao, Phys. Rev. D60 (1999) 115015, Fermilab-Pub-98/342-T, MADPH-98-1085, hep-ph/9811489.

[5] W. Beenakker, R. H"opker, Nucl.Phys.Proc.Suppl. 51C (1996) 261-266, W. Beenakker, R. H"opker, M. Spira, P.M. Zerwas, Nucl. Phys. B492 (1997) 51-103, W. Beenakker, M. Kr"amer, T. Plehn, M. Spira, P.M. Zerwas, Nucl.Phys. B515 (1998) 3-14.

[6] T. Plehn, Ph.D, Thesis, University Hamburg 1998, DESY-THESIS-1998-24, arXiv:hep-ph/9809319.

[7] E. L. Berger, T. M. P. Tait, M. Klasen, ANL-HEP-CP-01-002, arXiv:hep-ph/0101164.

[8] W. Beenakker, M. Klasen, M. Kr"amer, T. Plehn, M. Spira, P.M. Zerwas, Phys. Rev. Lett. 83 (1999) 3780-3783.

[9] S. Catani and M.H. Seymour, Nucl. Phys. B485(1997) 291 and Erratum ibid B510(1975)03.

[10] K. Fabricius, I. Schmitt, G. Kramer and G. Schierholz, Z. Phys. C11, 315(1981); G. Kramer and B. Lampe, Fortschr. Phys. 37, 161(1989); W. T. Giele and E.W.N. Glover, Phys. Rev. D46, 1980(1992); W. T. Giele, E.W.N. Glover and D.A. Kosower, Nucl. Phys. B403, 633 (1993).

[11] A. Denner, H. Eck, O. Hahn and J. Küblbeck, Nucl. Phys. B387 (1992)467.

[12] G. P. Lepage, J. Comput. Phys. 27 (1978) 192.
[13] G. Altarelli and G. Parisi, Nucl. Phys. B126 298 (1997).

[14] B.W. Harris, J.F. Owens, Phys. Rev. D65 (2002) 094032.

[15] Thomas Hahn and Christian Schappacher, Comput. Phys. Commun. 143 (2002) 54, arXiv:hep-ph/0105349.

[16] J. F. Gunion, H. E. Haber, Nucl. Phys. B272 (1986) 1.

[17] J. Pumplin, D.R. Stump, J. Huston, H.L. Lai, P. Nadolsky, W.K. Tung, JHEP 0207 (2002) 012.

[18] J. Kühlbeck, M. Böhm, A. Denner, Comput. Phys. Commun. 60 (1990) 165; T. Hahn, Comput. Phys. Commun. 140 (2001) 418; T. Hahn, C. Schappacher, Comput. Phys. Commun. 143 (2002) 54; T. Hahn, M. Perez-Victoria, Comput. Phys. Commun. 118 (1999) 153.

[19] G. J. Oldenborgh, Comput. Phys. Commun. 66 (1991) 1; T. Hahn, Acta Phys. Polon. B30 (1999) 3469.

[20] D. M. Copper, D. R. T. Jones, and P. van Nieuwenhuizen, Nucl. Phys. B167, 479 (1980); W. Siegel, Phys. Lett. B84, (1979) 193.

[21] D. A. Ross and J. C. Taylor, Nucl. Phys. B51, (1979)25.

[22] A. Denner, Fortschr. Phys. 41 (1993) 307.

[23] H. Eberl, M. Kinkel, W. Majerotto, Y. Yamada, Phys. Rev. D64 (2001) 115013, arXiv:hep-ph/0104109.
[24] A. Denner, T. Sack, Nucl. Phys. B347, (1990) 203.

[25] B. A. Kniehl, A. Pilaftsis, Nucl. Phys. B474 (1996) 286.

[26] J. Guasch, W. Hollik, J. Sola, JHEP 10 (2002) 040, arXiv:hep-ph/0207364.
   W. Hollik, E. Kraus, M. Roth, C. Rupp, K. Sibold and D. Stöckinger, Nucl. Phys. B639 (2002) 3, arXiv:hep-ph/0204350.

[27] D. Pierce and A. Papadopoulos, Phys. Rev. D47 (1993)222; Zhang Ren-You, Ma Wen-Gan, Wan Lang-Hui and Jiang Yi, Phys. Rev. D65 (2002)075018.

[28] A. Sirlin, Phys. Rev. D22 (1980) 971; W.J. Marciano and A. Sirlin, Phys. Rev. D22 (1980) 2695; A. Sirlin and W.J. Marciano, Nucl. Phys. B189 (1981) 442.

[29] C. Weber, H. Eberl, W. Majerotto, Phys. Lett. B572 (2003) 56, arXiv:hep-ph/0305250.

[30] H. Eberl, M. Kincel, W. Majerotto and Y. Yamada, Nucl. Phys. B625 (2002) 372, arXiv:hep-ph/0111303.

[31] K. Kovařík, C. Weber, H. Eberl, W. Majerotto, Phys.Lett. B591 (2004) 242-254.

[32] A. de Rújula, R. Petronzio, and A. Savoy-Navarro, Nucl. Phys. B154 (1979) 394.

[33] Particle Data Group, Eur. Phys. J. C15 2000.

[34] S. Eidelman, et al., Phys. Lett. B592 (2004) 1.
[35] A.D. Martin, R.G. Roberts, W.J. Stirling, and R.S. Thorne, Eur. Phys. J. C39 (2005) 155, arXiv:hep-ph/0411040.

[36] S. Heinemeyer, W. Hollik, G. Weiglein, Phys. Lett. B455 (1999) 179.

[37] H. Haber, G. Kane, Phys. Rep. 117 (1985) 75; J. Gunion, H. Haber, Nucl. Phys. B272 (1986) 1.