NORMAL AND IRREDUCIBLE ADIC SPACES, 
THE OPENNESS OF FINITE MORPHISMS, AND 
A STEIN FACTORIZATION

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Abstract. We transfer several elementary geometric properties of rigid-analytic spaces to the world of adic spaces, more precisely to the category of adic spaces which are locally of (weakly) finite type over a non-archimedean field. This includes normality, irreducibility (in particular, irreducible components), and a Stein factorization theorem. Most notably, we show that a finite morphism in our category of adic spaces is automatically open if the target is normal and both source and target are of the same pure dimension. Moreover, our version of the Stein factorization theorem includes a statement about the geometric connectedness of fibers which we have not found in the literature of rigid-analytic or Berkovich spaces.

§1. Introduction

The goal of this paper is to provide some elementary geometric properties of adic spaces which are locally of (weakly) finite type over a field \( K \), such as normality, irreducibility, and a Stein factorization theorem. In [14], we apply these results, which were previously only stated in the realm of rigid-analytic and Berkovich spaces, to show that integral models of local systems descend along proper surjections—here, it is crucial to work with the adic theory, even if at the end one is only interested in rigid-analytic varieties or Berkovich spaces. We hope that the results of the present paper will turn out to be useful for other purposes as well.

While many of the desired results could easily be transferred from the rigid-analytic world to the adic world via the usual equivalences of categories, this is not the case for statements that make direct references to the underlying topological spaces. One example of such a statement is the following main result, around which the paper evolved.

**Theorem 1.1.** Let \( K \) be a non-archimedean field, let \( X \) and \( Y \) be adic spaces which are locally of (weakly) finite type over \( K \), and let \( f : Y \to X \) be a finite morphism. Assume that both \( X \) and \( Y \) are of the same pure dimension \( d \) and that \( X \) is normal. Then \( f \) is open.

A similar result for Berkovich spaces is found in [1, Lem. 3.2.4], but Theorem 1.1 cannot be derived from it. Instead, we will use Huber’s approach in [7, Lem. 1.7.9] to reduce the claim to a similar claim about schemes, which is well known to be true. Our proof is given in Theorem 2.20.

The second main result of this paper is the following adic version of the Stein Factorization Theorem.
Theorem 1.2. Let $K$ be a non-archimedean field, let $X$ and $Y$ be adic spaces which are locally of (weakly) finite type over $K$, and let $f : Y \to X$ be a proper map of finite type. Then $f$ factors as

$$Y \xrightarrow{h} Z \xrightarrow{g} X$$

with the following properties:

(i) The map $g$ is finite.
(ii) The map $h$ is proper and has geometrically connected fibers.

Similar results can be found in the rigid-analytic setting (see [2, Prop. 9.6.3/5]) and in Berkovich’s theory (see [1, Prop. 3.3.7]), but our result is slightly more general by allowing adic spaces which are only weakly of finite type over $K$. Moreover, both of the mentioned references do not show that the fibers of $h$ are geometrically connected, so we add that fact as well. Our proof is presented in Theorem 3.9.

This paper is structured as follows. In §2, we introduce the notions of normal and irreducible adic spaces of (weakly) finite type over a non-archimedean field $K$, thereby explaining all of the terminology occurring in Theorem 1.1. Although one could transfer most of the definitions and results from the rigid-analytic world, we instead decided to work directly in the adic world and build everything from ground up; we found this approach more elegant while not requiring much more effort. It also allows us to work in a slightly more general setting than what the rigid-analytic world provides us with (e.g., we allow canonical compactifications). For most of the basic definitions and results, we follow [3], with the necessary modifications to the adic world supplied where necessary. The main result of §2 is Theorem 1.1.

In §3, we prove the Stein Factorization Theorem 1.2. This is done by first transferring the rigid-analytic Stein Factorization Theorem to the adic world and then providing an additional argument (analogous to the scheme case) to show that the fibers of $h$ are geometrically connected.

Notation and conventions.
Throughout the paper, we fix a non-archimedean field $K$, that is, $K$ is equipped with a nontrivial non-archimedean valuation under which it is complete. We let $\text{wFinType}(K)$ denote the category of adic spaces $X$ which are locally of weakly finite type over $K$ (see [7, Def. 1.2.1(i)]). If $X = \text{Spa}(A, A^+)$ is affinoid, then this finiteness hypothesis means precisely that $A$ is of topologically finite type over $K$. (There is no assumption on $A^+$.)

§2. Normal and irreducible adic spaces

Let $K$ and $\text{wFinType}(K)$ be as in the conventions. We will introduce the notions of normality, irreducibility, and irreducible components for objects $X \in \text{wFinType}(K)$. We also study the interactions of these notions with each other and with the purity of dimensions. Most of the ideas are taken from the rigid-analytic analogue, specifically from [3]. At the end of the section, we prove Theorem 1.1.

The main technical idea is to reduce all questions about an adic space $X \in \text{wFinType}(K)$ to questions about the local rings at those points corresponding to maximal ideals of an affinoid covering. This idea is made more precise by the following definition (cf. Definition 3.2 in [13]) and lemma.
Definition 2.1. Let $X \in \text{wFinType}(K)$. The Jacobson–Gelfand spectrum of $X$ is the subset

$$\text{JG}(X) \subset X$$

of all rank-1 points $x \in X$ such that there is an affinoid open neighborhood $U = \text{Spa}(A, A^+)$ of $x \in X$ with $\text{supp}\, x \subset A$ being a maximal ideal. We endow $\text{JG}(X)$ with the subspace topology.

Lemma 2.2. Let $X \in \text{wFinType}(K)$.

(i) The Jacobson–Gelfand spectrum is local, that is, for every open subset $U \subset X$, we have $\text{JG}(U) = \text{JG}(X) \cap U$. Similarly, for every closed adic subspace $Z \subset X$, we have $\text{JG}(Z) = \text{JG}(X) \cap Z$. Moreover, $\text{JG}(X) \subset X$ is dense.

(ii) Suppose that $X = \text{Spa}(A, A^+)$ is affinoid. Then $\text{supp} : \text{JG}(X) \to \text{MaxSpec}A$ is a continuous bijection.

(iii) In the setting of (ii), let $x \in \text{JG}(X)$ with corresponding maximal ideal $m = \text{supp}\, x \subset A$. Then $\mathcal{O}_{X,x} = \hat{A}_m$.

(iv) $X$ is connected if and only if $\text{JG}(X)$ is connected. If $X = \text{Spa}(A, A^+)$ is affinoid, then $X$ is connected if and only if $\text{Spec}A$ is connected.

Proof. Clearly, $K$ is a Jacobson–Tate ring in the sense of [13, Def. 3.1]; hence, $X$ is a Jacobson adic space by [13, Prop. 3.3(1)]. Thus, (ii), (iii), and the density of $\text{JG}(X) \subset X$ follow from [13, Prop. 3.3(2) and (3)]. The claim $\text{JG}(U) = \text{JG}(X) \cap U$ reduces easily to the case that $X$ is affinoid and $U$ is a rational open subset. Then the claim follows from (ii) and [13, Prop. 3.3(2)]. The corresponding statement for $Z$ follows easily by reducing to the affinoid case and then using (ii) and [16, Lem. 00G9].

It remains to prove (iv). Clearly, if $\text{JG}(X)$ is connected, then so is $X$. Conversely, assume that $\text{JG}(X) = U'_1 \cup U'_2$ for some open subsets $U'_1, U'_2 \subset X$. For $i = 1, 2$, let $U_i$ be the union of all open subsets $U \subset X$ with $\text{JG}(U) \subset U_i$. Clearly, $U_1$ and $U_2$ are disjoint open subsets of $X$, so we only have to check that they cover $X$ in order to finish the proof. Let $x \in X$ be given, and let $U = \text{Spa}(A, A^+)$ be an affinoid open neighborhood of $x$. Assume that $U$ is not contained in $U_1$ or $U_2$. Then $U'_1$ and $U'_2$ produce a disconnection of $\text{MaxSpec}A$, and since $A$ is a Jacobson ring (by [13, Prop. 3.3(3)]), this produces a disconnection of $\text{Spec}A$ and hence of $U$. (This also proves the second part of the claim.) Then one part of this disconnection is contained in $U_1$, and the other part is contained in $U_2$, so that $x$ lies in $U_1$ or $U_2$.

Apart from the Jacobson–Gelfand spectrum, the other main tool for working with $X \in \text{wFinType}(K)$ is the fact that if $X = \text{Spa}(A, A^+)$ is affinoid, then $A$ is an excellent ring by [12, Th. 3.3]. This allows us to make the following definition of normality. Note that in the following, a ring $A$ is called normal if all its localizations at prime ideals are domains which are integrally closed in their quotient fields; in particular, normal rings are reduced but not necessarily domains.

Definition 2.3. Let $X \in \text{wFinType}(K)$. We say that $X$ is normal (resp. reduced) if $X$ can be covered by affinoid adic spaces of the form $\text{Spa}(A, A^+)$ such that $A$ is a normal (resp. reduced) ring.
LEMMA 2.4. For $X \in \text{wFinType}(K)$, the following are equivalent:

(i) $X$ is normal (resp. reduced).
(ii) For every affinoid open subspace $\text{Spa}(A,A^+) \hookrightarrow X$, $A$ is a normal (resp. reduced) ring.
(iii) For every $x \in \text{JG}(X)$, $\hat{O}_{X,x}$ is a normal (resp. reduced) ring.

Proof. Let $U = \text{Spa}(A,A^+) \hookrightarrow X$ be an open subspace. Then $A$ is an excellent ring by [12, Th. 3.3]. Thus, by [16, Lems. 0FIZ, 0C23, 07QV, and 07NZ], the ring $A$ is normal (resp. reduced) if and only if for all maximal ideals $m \subset A$, the completed localization $\hat{A}_m$ is normal (resp. reduced). Now, the claim follows easily from Lemma 2.2.

Next, we want to construct the normalization. For any ring $A$, we denote by $\hat{A}$ its normalization, that is, the integral closure of the associated reduced ring $A_{\text{red}}$ inside its total ring of fractions.

LEMMA 2.5. Let $X = \text{Spa}(A,A^+) \in \text{wFinType}(K)$. Let $U = \text{Spa}(B,B^+) \hookrightarrow X$ be an affinoid open subset. Then the natural map $B \rightarrow \hat{A} \otimes_A B$ is a normalization.

Proof. As in the proof of Lemma 2.4, $A$ is an excellent ring and in particular a Nagata ring (cf. [16, Lem. 07QV]). We can, moreover, replace $A$ by $A_{\text{red}}$ and thus assume that $A$ is reduced. We want to apply [3, Th. 1.2.2], for which we need to verify the following three properties:

(a) $B$ is flat over $A$: this is [7, Lem. 1.7.6].
(b) The ring $\hat{A} \otimes_A B$ is normal: to see this, let $\bar{X} := \text{Spa}(\hat{A}, \hat{A}^+)$ and $\bar{U} := \text{Spa}(\hat{A} \otimes_A B, B'^{\times})$, where $\hat{A}^+$ is the integral closure of $A^+$ in $\hat{A}$ and $B'^{\times}$ is the integral closure of $B^+$ inside $\hat{A} \otimes_A B$. (Note that $A \rightarrow \hat{A}$ is finite because $A$ is Nagata [see [16, Lem. 035S]].) Then $\bar{U} = U \times_X \bar{X}$ is an open subset of $\bar{X}$. However, $\bar{X}$ is normal because $A$ is normal, hence $\bar{U}$ and therefore also $\hat{A} \otimes_A B$ must be normal by Lemma 2.4.
(c) For every minimal prime $p \subset A$, $B/pB$ is reduced. This follows by a similar argument as in (b): letting $X' = \text{Spa}(A/p,A'^{\times})$ and $U' = \text{Spa}(B/pB,B'^{\times})$ (where $A'^{\times}$ and $B'^{\times}$ are the integral closures of $A^+$ and $B^+$), we have $U' = U \times_X X'$, which is an open subset of $X'$, so that the reducedness of $X'$ implies the reducedness of $U'$ by Lemma 2.4.

This finishes the proof.

DEFINITION 2.6. Let $X \in \text{wFinType}(K)$.

(a) If $X = \text{Spa}(A,A^+)$ is affinoid, then the normalization of $X$ is the space $\bar{X} := \text{Spa}(\hat{A}, \hat{A}^+)$, where $\hat{A}^+$ is the integral closure of (the image of) $A^+$ in $\hat{A}$.
(b) For general $X$, note that if $V \subset U$ is an inclusion of affinoid open subsets of $X$, then by Lemma 2.5, there is a unique isomorphism $\tilde{V} \cong V \times_U \bar{U}$ over $V$. We can thus glue the $\bar{U}$’s to get an adic space $\bar{X}$, called the normalization of $X$.

LEMMA 2.7. For $X \in \text{wFinType}(K)$, the normalization $\bar{X}$ is a normal adic space and the natural projection $\bar{X} \rightarrow X$ is finite surjective.

Proof. It follows directly from the definition that $\bar{X}$ is a normal adic space. The finiteness of the map $\bar{X} \rightarrow X$ follows from [16, Lem. 035S] using that all coordinate rings are Nagata as in previous proofs. It remains to show that the map $\bar{X} \rightarrow X$ is surjective. It is enough to show that $\text{JG}(\bar{X}) \rightarrow \text{JG}(X)$ is surjective because $\bar{X} \rightarrow X$ is a closed map and $\text{JG}(X) \subset X$ is dense. This reduces to the affinoid case and then to the analogous result for (affine) Noetherian schemes, which is [16, Lem. 035Q(2)].
Next, we want to study dimensions.

**Lemma 2.8.** Let $X \in \text{wFinType}(K)$.

(i) We have

$$\dim X = \sup_{x \in \text{JG}(X)} \dim \hat{O}_{X,x},$$

where the dimension on the left is the dimension of the underlying spectral space $|X|$ (see [7, Def. 1.8.1]) and the dimension on the right denotes the Krull dimension.

(ii) The function

$$\dim_X : \text{JG}(X) \to \mathbb{Z}_{\geq 0}, \quad x \mapsto \dim_X x := \dim \hat{O}_{X,x},$$

is upper semicontinuous, that is, for every $x \in \text{JG}(X)$, there exists some open neighborhood $U \subseteq X$ of $x$ such that $\dim_X x \leq \dim_X y$ for all $y \in \text{JG}(U)$.

(iii) $X$ is of pure dimension $d$ (i.e., every nonempty open subset of $X$ has dimension $d$) if and only if $\dim \hat{O}_{X,x} = d$ for all $x \in \text{JG}(X)$.

**Proof.** In all cases, we can assume that $X = \text{Spa}(A,A^+)$ is affinoid.

We start with (i). By [7, Lem. 1.8.6], we have $\dim X = \dim A$, where $\dim A$ denotes the Krull dimension of $X$. On the one hand, we have $\dim A = \sup_{m \in \text{MaxSpec} A} \dim A_m$, and on the other hand, $\dim A_m = \dim \hat{A}_m = \dim \hat{O}_{X,x}$ for all $m$ (see [16, Lem. 07NV]), where $x \in \text{JG}(X)$ is the point of $X$ associated with $m$. This proves (i).

To prove (ii), let $x \in \text{JG}(X)$ be given and let $m = \text{supp}(x) \in \text{MaxSpec} A$ be the corresponding maximal ideal in $A$. Let $I = \{p_1, \ldots, p_n\}$ be the collection of minimal prime ideals in $A$, and let $I_m \subseteq I$ be the subset of minimal prime ideals that lie inside $m$. For every $p \in I$, let $Z_p = \text{Spa}(A/p, A^{+p})$ (where $A^{+p}$ is the integral closure of $A^+$ in $A/p$), which is a closed adic subspace of $X$ (cf. [7, (1.4.1)]). Let $U := X \setminus \bigcup_{p \in I \setminus I_m} Z_p$.

Now, choose any $u \in \text{JG}(U)$. We claim that $\dim_X u \leq \dim_X x$. To see this, let $n \subseteq A$ be the maximal ideal corresponding to $u$, so that $\dim_X u = \dim A_n$ (using [16, Lem. 07NV]). By the definition of $U$, $n = \text{supp} u$ cannot contain any $p \in I \setminus I_m$, so that every maximal chain of prime ideals $p_0 = q_0 \subseteq q_1 \subseteq \cdots \subseteq n$ has to start with some $p_i \in I_m$. We may thus replace $A$ by $A/p_i$ and assume that $A$ is an integral domain. Then [3, Lem. 2.1.5] implies that $A$ is equidimensional and we are done.

Part (iii) follows easily from (i) and (ii).

**Corollary 2.9.** If $X \in \text{wFinType}(K)$ is normal and connected, then it is pure-dimensional.

**Proof.** If $X = \text{Spa}(A,A^+)$ is affinoid (so that $A$ is normal), then $A$ is an integral domain, so it is equidimensional by [3, Lem. 2.1.5] and hence $X$ is pure-dimensional by Lemma 2.8(i). To handle the general case, for every $d \in \mathbb{Z}_{\geq 0}$, let $U_d \subseteq X$ be the union of all open subspaces which are of pure dimension $d$. From the affinoid case, one deduces that the $U_d$’s form a disjoint open partition of $X$. As $X$ is connected, all but one of the $U_d$’s must be empty.

**Lemma 2.10.** Let $f : Y \to X$ be a finite morphism in $\text{wFinType}(K)$. Then $\dim Y \leq \dim X$. If $f$ is surjective, then, for every $x \in \text{JG}(X)$, there is some $y \in \text{JG}(f^{-1}(x))$ such that $\dim_Y y = \dim_X x$; in particular, $\dim Y = \dim X$.

**Proof.** The claim $\dim Y \leq \dim X$ follows immediately from the analogous statement for schemes (see [16, Lem. 0ECG]) and [7, Lem. 1.8.6(ii)]. Similarly, the second claim reduces
We claim that if \( A \to B \) is a finite morphism of rings such that \( \text{Spec} B \to \text{Spec} A \) is surjective and \( m \subseteq A \) is a maximal ideal, then \( \dim A_m = \dim(A_m \otimes_A B) \) (see, e.g., [16, Lem. 0ECG]).

We can finally come to the definition and study of irreducible spaces.

**Definition 2.11.** We say that \( X \in \mathsf{wFinType}(K) \) is **irreducible** if it cannot be written as the union of two proper closed adic subspaces.

**Lemma 2.12.** If \( X \in \mathsf{wFinType}(K) \) is normal and connected, then \( X \) is irreducible.

**Proof.** It is enough to prove the following: let \( Z \subset X \) be a closed adic subspace and suppose that \( Z \) contains some open subspace \( U \subset X \); then \( Z = X \). We can assume that both \( Z \) and \( U \) are connected. By Corollary 2.9, we know that \( X \) is pure-dimensional, say \( \dim X =: d \). Now, let

\[
Z'_1 := \{ x \in JG(Z) \mid \dim_Z x = d \}, \quad Z'_2 := \{ x \in JG(Z) \mid \dim_Z x < d \}.
\]

We claim that if \( V \subset X \) is any affinoid connected open subset with \( V \cap Z'_1 \neq \emptyset \), then \( V \subset Z \). Indeed, let \( V = \text{Spa}(B, B^+) \), so that \( Z \cap V = \text{Spa}(B/J, B'^+) \) for some ideal \( J \subset B \) (where \( B'^+ \) is the integral closure of \( B^+ \) in \( B/J \)). But then, \( \text{Spec} B \) is normal and connected (cf. Lemmas 2.2(iv) and 2.4), hence irreducible, and there is some maximal ideal \( m \subset B/J \) (the one corresponding to \( x \)) such that \( \dim B = d = \dim(B/J)_m \). This implies that \( \dim B/J = \dim B \) and hence \( J = (0) \), as desired.

From the previous paragraph, we deduce that \( Z'_1 \) is open in \( JG(Z) \) and by Lemma 2.8(ii) the same is true for \( Z'_2 \). However, \( JG(Z) \) is connected by Lemma 2.2(iv) so that \( Z'_2 = \emptyset \) because \( U \subset Z \). Hence, \( JG(Z) = Z'_1 \) and now the previous paragraph shows that \( Z \) is open in \( X \). But then, \( Z \) is open and closed in \( X \) and thus \( Z = X \).

**Lemma 2.13.** Let \( X, Y \in \mathsf{wFinType}(K) \), and let \( f : Y \to X \) be a finite morphism. Then \( \text{im}(f) \subset X \) is (the image of) a closed adic subspace.

**Proof.** We can assume that \( X = \text{Spa}(A, A^+) \) is affinoid; then \( Y = \text{Spa}(B, B^+) \) is also affinoid, \( f \) is given by a finite map \( f^* : A \to B \), and \( B^+ \) is the integral closure of \( A^+ \) in \( B \). Let \( J := \ker(f^*) \), and let \( Z = \text{Spa}(A/J, A'^+) \), where \( A'^+ \) is the integral closure of \( A^+ \) in \( A/J \). We claim that \( \text{im}(f) = Z \). Clearly, \( \text{im}(f) \subset Z \) and \( \text{im}(f) \) is closed (because \( f \) is proper, e.g., by [7, Lem. 1.4.5(ii)])

so it is enough to show that \( \text{im}(f) \) contains all \( x \in JG(Z) \). Using Lemma 2.2, this boils down to the fact that \( f^* \) induces a surjective map \( \text{MaxSpec} B \to \text{MaxSpec}(A/J) \), which is clear.

The following definition is analogous to [3, Def. 2.2.2].

**Definition 2.14.** Let \( X \in \mathsf{wFinType}(K) \), let \( p : \tilde{X} \to X \) be the normalization of \( X \), and let \( \tilde{X}_i \subset \tilde{X} \) be the connected components of \( \tilde{X} \). The **irreducible components** of \( X \) are the (reduced) closed adic subspaces \( X_i := p(\tilde{X}_i) \subset X \) (cf. Lemma 2.13).

**Lemma 2.15.** Let \( X \in \mathsf{wFinType}(K) \), and let \( (X_i)_{i \in I} \) be the irreducible components of \( X \). Then the \( X_i \)'s are irreducible and form a locally finite cover of \( X \). Moreover, for all \( i \in I \), we have

\[
X_i \not\subseteq \bigcup_{i' \neq i} X_{i'}.
\]
Proof. Let \( \rho: \tilde{X} \to X \) be the normalization of \( X \) with connected components \((\tilde{X}_i)_{i \in I}\). Each \( \tilde{X}_i \) is irreducible by Lemma 2.12 and maps surjectively onto \( X_i \). It follows easily that \( X_i \) is irreducible. (Any \( Z_1 \cup Z_2 \subset X_i \) can be lifted to \((Z_1 \times X) \cup (Z_2 \times X) \) isomorphic to \( \tilde{X}_i \).) The local finiteness of the cover follows from the fact that for every quasi-compact subset \( U \subset X \), the preimage \( \rho^{-1}(U) \subset \tilde{X} \) is still quasi-compact and can therefore only meet finitely many connected components.

To prove the second part of the claim, let \( i \in I \) be given and let \( U := \tilde{X}_i \) and \( V := \tilde{X} \setminus \tilde{X}_i \). It is enough to show that \( \rho^{-1}(\rho(V)) \) does not contain \( U \). In fact, we claim that the closed adic subspace \( \rho^{-1}(\rho(V)) \cap U \subset U \) has strictly lower dimension than \( U \) at all points. This can be checked locally, so we can assume that \( X \) and \( \tilde{X} = X \cup V \) are affinoid. Then the claim follows easily from the analogous statement for schemes.

Corollary 2.16. Let \( X \in \text{wFinType}(K) \) with normalization \( \tilde{X} \). Then the following are equivalent:

(i) \( X \) is irreducible.
(ii) \( \tilde{X} \) is irreducible.
(iii) \( \tilde{X} \) is connected.

Proof. The equivalence of (ii) and (iii) is Lemma 2.12. As in the proof of Lemma 2.15, if \( \tilde{X} \) is irreducible, then so is \( X \). Conversely, assume that \( \tilde{X} \) is reducible, that is, disconnected. Then \( X \) has more than one irreducible component and is therefore reducible by Lemma 2.15.

Proposition 2.17. If \( X \in \text{wFinType}(K) \) is irreducible, then it is pure-dimensional.

Proof. Let \( \rho: \tilde{X} \to X \) be the normalization of \( X \). Then \( \tilde{X} \) is connected by Corollary 2.16, hence pure-dimensional by Corollary 2.9. However, by Lemma 2.10, for every \( x \in JG(X) \), there is some \( \tilde{x} \in JG(\tilde{X}) \) with \( \dim_{\tilde{X}} x = \dim_{\tilde{X}} \tilde{x} = \dim \tilde{X} \).

Most of the properties of \( X \in \text{wFinType}(K) \) discussed so far are stable under étale extensions of \( X \).

Lemma 2.18. Let \( X, Y \in \text{wFinType}(K) \), and let \( Y \to X \) be an étale map.

(i) If \( X \) is normal (resp. reduced), then so is \( Y \).
(ii) Let \( \tilde{X} \to X \) be the normalization of \( X \). Then \( \tilde{X} \times_X \tilde{X} \) is a normalization of \( Y \).
(iii) If \( X \) is of pure dimension \( d \), then so is \( Y \).

Proof. Since all of the discussed properties are local (see Lemma 2.4 for normality and reducedness), we can use \([7, \text{Lem. } 2.2.8]\) to reduce the claim to \( X = \text{Spa}(A, A^+) \) and \( Y = \text{Spa}(B, B^+) \) affinoid with \( A \to B \) finite étale. Then (i) and (iii) follow directly from the analogous property for schemes (see, e.g., \([16, \text{Lems. } 033B, 033C, \text{and } 039S]\)). Moreover, (ii) follows from (i) and \([3, \text{Th. } 1.2.2]\).

We now come to the proof of Theorem 1.1. The following proof essentially reduces the result to a similar result for schemes. We start with the affinoid version, from which the global result will be derived more or less formally.

Lemma 2.19. Let \( f: Y = \text{Spa}(B, B^+) \to X = \text{Spa}(A, A^+) \) be a finite morphism of affinoid Noetherian adic spaces. Assume that both \( A \) and \( B \) are integral domains, that \( A \) is normal and that the associated map \( f^*: A \to B \) is injective. Then \( f \) is open.
Assume that both $X$ and $Y$ are of the same pure dimension $d$ and that $X$ is normal. Then $Y$ and moreover, by [5, Prop. 2.6(iii)], there are retractions $A$ and $B$. Open. But then, the above diagram immediately implies that $f$ and hence $A$ and $B$, are reduced, because passing to the reduction does not change the topology.

Letting $(X_i)_{i \in I}$ be the irreducible components of $Y$, it is enough to show that each $Y_i \to X$ is open; we can thus assume that $Y$ is irreducible as well. Since $f$ is finite and $X$ affinoid, $Y$ must be affinoid as well, say $Y = \text{Spa}(B, B^+)$. We can furthermore assume that $A$ and $B$ are reduced, because passing to the reduction does not change the topology. Then $A$ and $B$ are integral domains of dimension $d$, the induced map $f^*: A \to B$ is finite, and $A$ is normal. In order to apply Lemma 2.19, it only remains to verify that $f^*: A \to B$ is injective. However, this is clear, as otherwise $f^*$ factors over $A/I$ for some ideal $0 \neq I \subset A$, but $A/I$ has lower dimension than $A$ and the dimension of $B$ is at most the one of $A/I$; contradiction!

**Proof**. The argument is similar to [7, Lem. 1.7.9]. As in the reference, we define

$$X' := \{ v \in \text{Spv} A \mid v(a) \leq 1 \text{ for } a \in A^+ \text{ and } v(a) < 1 \text{ for } a \in A^\infty \},$$

$$Y' := \{ v \in \text{Spv} B \mid v(b) \leq 1 \text{ for } b \in B^+ \text{ and } v(b) < 1 \text{ for } b \in B^{\infty} \}.$$\[\]

Moreover, by [5, Prop. 2.6(iii)], there are retractions $r_X: \text{Spv} A \to \text{Spv}(A, A^\infty A)$ and $r_Y: \text{Spv} B \to \text{Spv}(B, B^{\infty} B)$. From [5, Th. 3.1], we deduce that $X' \cap \text{Spv}(A, A^\infty A) = X$ and $Y' \cap \text{Spv}(B, B^{\infty} B) = Y$. It follows easily that, by restricting $r_X$ and $r_Y$, we obtain retractions $s_X : X' \to X$ and $s_Y : Y' \to Y$. Let $f' : Y' \to X'$ be the restriction of the natural map $\text{Spv}(f^*) : \text{Spv} B \to \text{Spv} A$. We obtain the following commutative diagram:

$$\begin{array}{ccc}
Y & \xrightarrow{id_Y} & Y' \xrightarrow{s_Y} & Y \\
\downarrow f & & \downarrow f' & \downarrow f \\
X & \xleftarrow{id_X} & X' \xrightarrow{s_X} & X
\end{array}$$

By [8, Cor. 2.1.7(iii)], the assumptions on $A$, $B$, and $f^*: A \to B$ are enough to guarantee that $\text{Spv}(f^*) : \text{Spv} B \to \text{Spv} A$ is open. Note also that $B^+$ is the integral closure of $A^+$ in $B$ and $B^{\infty} = A^{\infty} B^+$ by finiteness of $f$ (see [7, (1.4.2)]). This implies that

$$Y' = \{ v \in \text{Spv} B \mid v(b) \leq 1 \text{ for } b \in A^+ \text{ and } v(b) < 1 \text{ for } b \in A^{\infty} \},$$

and hence $f'^{-1}(X') = Y'$. Together with the openness of $\text{Spv}(f^*)$, we deduce that $f'$ is open. But then, the above diagram immediately implies that $f$ is open as well.

**Theorem 2.20.** Let $X, Y \in \text{wFinType}(K)$, and let $f: Y \to X$ be a finite morphism. Assume that both $X$ and $Y$ are of the same pure dimension $d$ and that $X$ is normal. Then $f$ is open.

**Proof.** Since the claim is local on $X$, we can assume that $X = \text{Spa}(A, A^+)$ is affinoid and irreducible. Letting $(Y_i)_{i \in I}$ be the irreducible components of $Y$, it is enough to show that each $Y_i \to X$ is open; we can thus assume that $Y$ is irreducible as well. Since $f$ is finite and $X$ affinoid, $Y$ must be affinoid as well, say $Y = \text{Spa}(B, B^+)$. We can furthermore assume that $A$ and $B$ are reduced, because passing to the reduction does not change the topology. Then $A$ and $B$ are integral domains of dimension $d$, the induced map $f^*: A \to B$ is finite, and $A$ is normal. In order to apply Lemma 2.19, it only remains to verify that $f^*: A \to B$ is injective. However, this is clear, as otherwise $f^*$ factors over $A/I$ for some ideal $0 \neq I \subset A$, but $A/I$ has lower dimension than $A$ and the dimension of $B$ is at most the one of $A/I$; contradiction!

**§3. Stein factorization**

We will now prove the Stein Factorization Theorem for adic spaces. Again, we fix a non-archimedean field $K$ and work with the category $\text{wFinType}(K)$ of adic spaces which are locally of weakly finite type over $K$ (see the introduction).

The first major ingredient for the Stein Factorization Theorem is the fact that the direct image of a coherent sheaf along a proper morphism of finite type is again coherent. This
has been proved by Kiehl in the rigid-analytic setting, and can easily be generalized to adic spaces in \( \text{wFinType}(K) \).

**Lemma 3.1.** Let \( j: U \hookrightarrow X \) be an open immersion of locally strongly Noetherian analytic adic spaces such that for every open affinoid \( V = \text{Spa}(A,A^+) \subset X \), we have \( U \cap V = \text{Spa}(A,A') \) for some \( A' \supseteq A^+ \). Then \( j_*\mathcal{O}_U = \mathcal{O}_X \) and the functors \( j_* \) and \( j^{-1} \) induce an equivalence of categories of coherent sheaves on \( U \) and \( X \):

\[
j_*: \text{Coh}(\mathcal{O}_U) \cong \text{Coh}(\mathcal{O}_X) : j^{-1}.
\]

**Proof.** The claim is local on \( X \), so we can assume that \( X = \text{Spa}(A,A^+) \) and \( U = \text{Spa}(A,A') \) for some \( A' \supseteq A^+ \). Then, clearly, \( \mathcal{O}_X(X) = A = \mathcal{O}_U(U) \), and using the same reasoning for every rational open subset of \( X \) shows \( j_*\mathcal{O}_U = \mathcal{O}_X \). On the other hand, by [4, Satz 3.3.18] (or [10, Th. 2.3.3]), the categories \( \text{Coh}(\mathcal{O}_U) \) and \( \text{Coh}(\mathcal{O}_X) \) are both equivalent to the category of finite \( A \)-modules, and using \( j_*\mathcal{O}_U = \mathcal{O}_X \), one checks easily that \( j_* \) provides the required equivalence.

**Proposition 3.2.** Let \( X,Y \in \text{wFinType}(K) \), let \( f: Y \to X \) be a proper map of finite type, and let \( M \) be a coherent sheaf of \( \mathcal{O}_Y \)-modules on \( Y \). Then \( f_*M \) is a coherent sheaf of \( \mathcal{O}_X \)-modules on \( X \).

**Proof.** The claim is local on \( X \), so we can assume that \( X = \text{Spa}(A,A^+) \) is affinoid. Let us first consider the case \( A^+ = A^0 \). Then \( X \) is quasi-separated and of finite type over \( K \), and hence so is \( Y \). By [6, Prop. 4.5(iv)], both \( X \) and \( Y \) are induced from rigid-analytic varieties and the same is true for all rational subsets. Thus, the claim follows from the analogous claim in rigid-analytic geometry (see [11, Th. 3.3]).

Now, let \( A^+ \) be general, and let \( X' := \text{Spa}(A,A^0) \). Then we have an open immersion \( j_1: X' \to X \). Let \( Y' := Y \times_X X' \), so that we also have an open immersion \( j_2: Y' \to Y \). Letting \( f': Y' \to X' \) denote the restriction of \( f \), we get the diagram

\[
\begin{array}{ccc}
Y' & \xrightarrow{j_2} & Y \\
\downarrow f' & & \downarrow f \\
X' & \xrightarrow{j_1} & X
\end{array}
\]

Both \( j_1 \) and \( j_2 \) satisfy the hypothesis of Lemma 3.1, as one checks easily using the explicit construction of fiber products of adic spaces. Thus, the claim follows from Lemma 3.1 and the fact that it holds for \( f' \) by the first paragraph of the proof.

The second major ingredient for our Stein Factorization Theorem is the geometric connectedness of certain fibers. Our proof is analogous to the proof for schemes.

**Definition 3.3.** Let \( f: Y \to X \) be a morphism of analytic adic spaces.

(a) Let \( x: \text{Spa}(k,k^+) \to X \) be a morphism, where \( k \) is an analytic field with open and bounded valuation subring \( k^+ \). The fiber of \( f \) at \( x \) is the space \( Y_x := \text{Spa}(k,k^+) \times_X Y \). We say that \( x \) is a geometric point of \( X \) if \( k \) is algebraically closed. Moreover, by abuse of notation, we will often identify a point \( x \in X \) with the injection \( x: \text{Spa}(k(x),k(x)^+) \hookrightarrow X \) (cf. [7, Def. 1.1.7]).

(b) We say that \( f \) has connected fibers if for every \( x \in X \), the fiber \( Y_x \) is connected.

(c) We say that \( f \) has geometrically connected fibers if for every geometric point \( \varpi \) of \( X \), the fiber \( Y_{\varpi} \) is connected.
Lemma 3.4. Let $X,Y \in \wFinType(K)$, and let $f : Y \to X$ be a morphism. Then the following are equivalent:

(i) $f$ has geometrically connected fibers.

(ii) For every $x \in X$ and every $x'$: $\Spa(k,\mathcal{O}_k) \to \Spa(k(x),k(x)^+) \hookrightarrow X$ such that $k$ is a finite separable extension of $k(x)$, the fiber $Y_{x'}$ is connected.

(iii) For every étale map $U \to X$, the base-changed morphism $f_U : Y \times_X U \to U$ has connected fibers.

Proof. We first note that whether $f$ has connected fibers or not can be checked on the rank-1 points of $X$: if $x \in X$ is any point, let $x_0 \in X$ be the maximal generalization of $x$. Then there is a natural open immersion $Y_{x_0} \hookrightarrow Y_x$ and each point of $Y_x$ has a generalization in $Y_{x_0}$. Thus, every nontrivial disjoint open cover of $Y_x$ produces a nontrivial disjoint open cover of $Y_{x_0}$; therefore, if $Y_{x_0}$ is connected, then so is $Y_x$. We immediately deduce that (ii) implies (iii) (cf. [7, Prop. 1.7.5]).

We next show that (iii) implies (ii): given $x'$: $\Spa(k,\mathcal{O}_k) \to X$ with image $x \in X$, it is enough to show that there is an étale map $\alpha : U \to X$ with a point $u \in U$ such that $x' = \alpha \circ u$. To see this, we can assume that $X = \Spa(A,A^+)$ is affinoid. Then $\hat{k}(x)$ is the completion of the quotient field $\kappa(p) = \Quot(A/p)$, where $p = \sup p x$ (see [9, Lem. 2.4.17(a)]). By Krasner’s lemma (cf. [2, Prop. 3.4.2/5]), there is a finite separable extension $k'$ of $\kappa(p)$ such that $k = k' \otimes_{\kappa(p)} k(x)$. By the proof of [16, Lem. 00UD], after possibly replacing $A$ by the localization $A_f$ for some $f \in A$ (which corresponds to an open subspace in the adic world), there is a finite étale $A$-algebra $A'$ and a prime ideal $p' \subset A'$ lying above $p$ such that $\kappa(p') = k'$. Letting $U = \Spa(A',A'^+)$, where $A'^+$ is the integral closure of $A^+$ in $A'$, one sees easily that there is a canonical injection $u: \Spa(k,\mathcal{O}_k) \to U$ whose image point is the rank-1 valuation with support $p'$.

It is clear that (i) implies (ii): if $Y_{x'}$ is disconnected for any $x'$ as in (ii), then $Y_{x'} = Y_{x'} \times_{x'} \bar{x}'$ is also disconnected, where $\bar{x}' : \Spa(\hat{k},\mathcal{O}_{\hat{k}}) \to X$ is the obvious map which factors over $x'$.

Lemma 3.5. Let

\[
\begin{array}{ccc}
Y' & \to & Y \\
\downarrow f' & & \downarrow f \\
X' & \to & X
\end{array}
\]

be a Cartesian diagram of adic spaces in $\wFinType(K)$ and assume that $g$ is étale and $f$ is proper of finite type. Let $\mathcal{M}$ be a coherent sheaf of $\mathcal{O}_Y$-modules on $Y$. Then the natural base change map

$g^* f_\ast \mathcal{M} \iso f'_\ast g^* \mathcal{M}$

is an isomorphism of sheaves on $X'$. 


Remark 3.6. One should see Lemma 3.5 as a weak analogue of the flat base change theorem in algebraic geometry. From the proof, it is easy to see that one can weaken the hypothesis on \( g \) to just being flat instead of étale (then we cannot assume that \( A' \) is finite over \( A \) in the proof and hence need to add completions, but Noetherian completions are exact)—we chose not to do that because we did not want to introduce flat morphisms and only need the base change result in case \( g \) is étale.

It is, however, crucial for our proof that \( f \) is proper and of finite type, because the proof (similar to the case of schemes) makes use of the fact that the direct image of a coherent sheaf along \( f \) is (quasi-)coherent (by Proposition 3.2). In order to remove the properness assumption, one needs to have a good theory of quasi-coherent sheaves on adic spaces, which does not seem to exist in the classical language due to topological issues. Very recently, Clausen and Scholze managed to fix this problem in their new theory of analytic spaces [15]. In fact, the new formalism provides a very general base change result (see [15, Prop. 12.14]). The new requirement is that \( f \) is qcqs (quasicompact and quasiseparated) and that \( g \) is “steady”; the latter condition is closely related to \( \Gamma(Y, \mathcal{M}) \times_A A' = \Gamma(Y', g^* \mathcal{M}) \).

Proof of Lemma 3.5. The claim is local on \( X' \), and it clearly holds if \( g \) is an open immersion, so by using [7, Lem. 2.2.8], we can assume that \( X = \text{Spa}(A, A^+) \) and \( X' = \text{Spa}(A', A'^+) \) are affinoid with \( A \to A' \) being finite étale. By Proposition 3.2, the sheaf \( f_\ast \mathcal{M} \) is coherent, and hence equals the coherent sheaf associated with the finite \( A \)-module \( M_A := \Gamma(Y, \mathcal{M}) \). Then \( g^* f_\ast \mathcal{M} \) is the coherent sheaf on \( X' \), which is associated with the finite \( A' \)-module \( M_A \otimes_A A' \) (cf. [16, Lem. 01BJ]). Using a similar treatment for \( f'_\ast g'^* \mathcal{M} \), we see that the claim boils down to showing

\[
\Gamma(Y, \mathcal{M}) \otimes_A A' = \Gamma(Y', g'^* \mathcal{M}).
\]

Again, by [16, Lem. 01BJ], we have \( \Gamma(Y', g'^* \mathcal{M}) = \Gamma(Y, \mathcal{M}) \otimes_{\mathcal{O}_Y(Y)} \mathcal{O}_{Y'}(Y') \), which reduces the claim to the case \( \mathcal{M} = \mathcal{O}_Y \). Choose a finite cover \( Y = \bigcup_{i=1}^n U_i \) with all \( U_i = \text{Spa}(B_i, B_i^+) \) affinoid. For every \( i = 1, \ldots, n \), let \( U'_i := U_i \times_X X' = \text{Spa}(B_i \otimes_A A', B_i'^+) \), where \( B_i'^+ \) is the integral closure of \( B_i^+ \) in \( B_i \otimes_A A' \) (cf. proof of [7, Prop. 1.2.2]). The sheaf property of \( \mathcal{O}_Y \) and \( \mathcal{O}_{Y'} \) provides exact sequences

\[
0 \to \mathcal{O}_Y(Y) \to \bigoplus_{i=1}^n \mathcal{O}_Y(U_i) \to \bigoplus_{i,j=1}^n \mathcal{O}_Y(U_i \cap U_j),
\]

\[
0 \to \mathcal{O}_{Y'}(Y') \to \bigoplus_{i=1}^n \mathcal{O}_{Y'}(U'_i) \to \bigoplus_{i,j=1}^n \mathcal{O}_{Y'}(U'_i \cap U'_j).
\]

Note that \( \mathcal{O}_{Y'}(U'_i) = \mathcal{O}_Y(U_i) \otimes_A A' \) by the explicit description of \( U_i \) and \( U'_i \). Moreover, since \( f \) is separated, all the intersections \( U_i \cap U_j \) are affinoid (by the same argument as in the case of schemes, using that fiber products and closed subspaces of affinoids are affinoid; for the latter, see [4, Prop. 3.6.27]), and hence satisfy the similar relation \( \mathcal{O}_{Y'}(U'_i \cap U'_j) = \mathcal{O}_Y(U_i \cap U_j) \otimes_A A' \). Therefore, the second of the above exact sequences reads

\[
0 \to \mathcal{O}_{Y'}(Y') \to \bigoplus_{i=1}^n \mathcal{O}_Y(U_i) \otimes_A A' \to \bigoplus_{i,j=1}^n \mathcal{O}_Y(U_i \cap U_j) \otimes_A A'.
\]
Comparing this to the first exact sequence and noting that $A'$ is flat over $A$, we deduce that $\mathcal{O}_Y(Y') = \mathcal{O}_Y(Y) \otimes_A A'$, as desired.

**Lemma 3.7.** Let $f: Y \to X$ be a proper morphism of analytic adic spaces, and let $x \in X$ such that $Y_x$ is disconnected, say the union of disjoint open subsets $V_1$ and $V_2$. Then there is an open neighborhood $U \subset X$ of $x$ such that $f^{-1}(U) = U_1 \sqcup U_2$ for open subsets $U_1, U_2 \subset Y$ with $V_i \subset U_i$ for $i = 1, 2$.

**Proof.** In the following argument, we denote by $\mathcal{O}[Z]$ the topological space underlying an adic space $Z$. We also use the associated Berkovich spectrum of all the spaces involved: for any analytic adic space $Z$, let $|Z|^B$ be the subset of all maximal points of $Z$. There is a natural continuous projection $|Z| \to |Z|^B$ by [7, Lem. 8.1.7(ii)], and if $Z$ is taut (e.g., qcqs), then $|Z|^B$ is Hausdorff by [7, Lem. 8.1.8(ii)].

The claim is local on $X$, so we can assume that $X$ is qcqs (e.g., affinoid); then $Y$ is also qcqs, so in particular $X$ and $Y$ are taut. Clearly, $|Y_x|^B = |Y_x| \cap |Y|^B$ and $|Y_x|^B = |V_1|^B \cup |V_2|^B$ with $|V_1|^B, |V_2|^B \subset |Y_x|^B$ open subsets. By the discussion at the beginning of the proof, $|Y|^B$, $|V_1|^B$, and $|V_2|^B$ are quasi-compact Hausdorff spaces and in particular T4 spaces. Hence, there exist disjoint open neighborhoods $U_1''$ and $U_2''$ of $|V_1|^B$ and $|V_2|^B$ inside $|Y|^B$. Let $U_1', U_2' \subset Y$ be their preimages under the projection $|Y| \to |Y|^B$. Then $U_1'$ and $U_2'$ are disjoint open subsets of $Y$ such that $V_i \subset U_i'$ for $i = 1, 2$.

Let $Z := |Y| \setminus (|U_1''| \cup |U_2''|)$, which is a closed subset of $|Y|$. By the properness of $f$, $f(Z) \subset |X|$ is closed. Now, let $U := |X| \setminus f(Z)$. Then $U$ is an open neighborhood of $x$ and $f^{-1}(U) = U_1 \sqcup U_2$ for $U_i := f^{-1}(U) \cap U_i'$, as desired.

**Proposition 3.8.** Let $X$ and $Y$ be analytic adic spaces, and let $f: Y \to X$ be a proper map of finite type such that $f_* \mathcal{O}_Y = \mathcal{O}_X$ (via the natural morphism). Then $f$ has geometrically connected fibers.

**Proof.** Let $U \to X$ be any étale morphism, and let $f_U: Y_U := Y \times_X U \to U$ be the base change. By Lemma 3.4, it is enough to show that $f_U$ has connected fibers. However, by Lemma 3.5, we have $f_U^* \mathcal{O}_{Y_U} = \mathcal{O}_U$, and hence we can replace $U$ by $X$ and reduce to showing that $f$ has connected fibers. Let $x \in X$ be given and assume that $Y_x$ is disconnected. By Lemma 3.7, there is an open neighborhood $V \subset X$ of $x$ such that $f^{-1}(V) = V_1 \sqcup V_2$, where both $V_1$ and $V_2$ are nonempty over $x$. In particular, there are idempotents $e_1, e_2 \in \mathcal{O}_Y(f^{-1}(V)) = (f_* \mathcal{O}_Y)(V) = \mathcal{O}_X(V)$ corresponding to $V_1$ and $V_2$. These idempotents satisfy $e_1 = 1 - e_2$ and hence correspond to two disjoint subsets of $X$. On the other hand, both of these subsets must contain $x$ because both $e_1$ and $e_2$ are nonzero in any neighborhood of $x$ (since $V_1$ and $V_2$ are nonempty over $x$). Contradiction!

We can finally prove a version of the Stein Factorization Theorem.

**Theorem 3.9.** Let $X, Y \in \wFinType(K)$, and let $f: Y \to X$ be a proper map of finite type. Then $f$ factors as

$$Y \xrightarrow{h} Z \xrightarrow{g} X$$

with the following properties:

(i) The map $g$ is finite.

(ii) The map $h$ is proper and has geometrically connected fibers.
Proof. Given any coherent sheaf $\mathcal{M}$ on $X$, there is a unique adic space $M$ with a (necessarily finite) map $m: M \to X$ such that for any affinoid $U \subset X$ with preimage $V = m^{-1}(U)$, we have $\mathcal{O}_M(V) = \mathcal{M}(U)$. Indeed, if $X$ is affinoid, this is clear, and the general case is easily obtained by glueing. Now, by Proposition 3.2, the sheaf $f_*\mathcal{O}_Y$ is a coherent sheaf of $\mathcal{O}_X$-modules on $X$. Applying the gluing construction to $\mathcal{M} = f_*\mathcal{O}_Y$, we obtain a finite map $g: Z \to X$ with $g_*\mathcal{O}_Z = f_*\mathcal{O}_Y$. From the construction of $Z$, one sees easily that there is a map $h: Y \to Z$ such that $f = g \circ h$ and $h_*\mathcal{O}_Y = \mathcal{O}_Z$. Then Proposition 3.8 implies that $h$ has geometrically connected fibers.

Acknowledgment. I am grateful to Annette Werner for helpful discussions and many comments on this paper, and to Torsten Wedhorn for suggesting to look into Lemma 1.7.9 of Huber’s book.

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