Computation of spatial significance of mountain objects extracted from multiscale digital elevation models

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Abstract. The derivation of spatial significance is an important aspect of geospatial analysis and hence, various methods have been proposed to compute the spatial significance of entities based on spatial distances with other entities within the cluster. This paper is aimed at studying the spatial significance of mountain objects extracted from multiscale digital elevation models (DEMs). At each scale, the value of spatial significance index \( SSI \) of a mountain object is the minimum number of morphological dilation iterations required to occupy all the other mountain objects in the terrain. The mountain object with the lowest value of \( SSI \) is the spatially most significant mountain object, indicating that it has the shortest distance to the other mountain objects. It is observed that as the area of the mountain objects reduce with increasing scale, the distances between the mountain objects increase, resulting in increasing values of \( SSI \). The results obtained indicate that the strategic location of a mountain object at the centre of the terrain is more important than its size in determining its reach to other mountain objects and thus, its spatial significance.

1. Introduction
A spatially significant entity can be defined as an entity from which it is easy to reach all of its neighbouring entities (e.g., objects, hotspots, zones). A spatially significant entity has greater geometric proximity to the other entities, and should be at a strategic location and possess a relatively larger size [1-4]. The derivation of spatial significance is an important aspect of geospatial analysis and hence, various methods have been proposed to compute the spatial significance of entities based on spatial distances with other entities within the cluster, including kernel density estimation [5-7], spatial proximity matrix [8, 9], local indicators of spatial association [10], vulnerable localities index [11] and local clustering index [12]. These methods only consider the proximities of the entities, but not their geometric properties.

To this end, Sagar et al. [1] proposed a mathematical morphological based algorithm to compute the spatial significance of entities. The algorithm employed iterative dilations between origin and destination entities to determine the dilation distance. The computed dilation distances are then used to determine the spatial significance index \( SSI \) of each entity. As the effects of the morphological operations have intuitive interpretations using geometric terms of shape, size and location, the computed values of \( SSI \) are affected by the entities’ boundary points that are furthest out with respect to the other entities. The algorithm was demonstrated using a cluster of water bodies and states in India.
In this paper, the algorithm proposed in Sagar et al. [1] is employed to study the spatial significance of mountain objects extracted from multiscale digital elevation models (DEMs). Scale variations can constrain the detail with which information can be observed, represented and analysed. The term scale refers to a combination of both spatial extent, and spatial detail or resolution [13-16]. Changing the scale without first understanding the effects of such an action can result in the representation of patterns or processes that are different from those intended due to loss of detail, and variations in terrain parameters and landforms [15-18]. Hence, feature detection and characterisation often need to be performed at different scales of measurement. Wu et al. [19], Wood [20], Drăguţ et al. [21] and Poulos et al. [22] demonstrated that analysis of a location at multiple scales allows for a greater amount of information to be extracted from a digital elevation model (DEM) about the spatial characteristics of a feature.

2. Computation of spatial significance

Mathematical morphology is a branch of image processing that deals with the extraction of image components that are useful for representational and descriptive purposes. It has a well developed mathematical structure that is based on set theoretic concepts. The fundamental morphological operators are discussed in Matheron [23], Serra [24], Soille [25], and Najman and Talbot [26]. Morphological operators generally require two inputs; the input image \( A \), which can be in binary or grayscale form, and the kernel \( B \), which is used to determine the precise effect of the operator. Each cell in \( A \) is compared with \( B \) by moving \( B \) so that its centre hits the cell. Depending on the type of morphological operator employed, the cell value is reset to the value or average value of one or more of its neighbours.

Dilation (Equation 1) sets the cell values within the kernel to the maximum value of the cell neighbourhood. Binary dilation gradually enlarges the boundaries of regions of foreground cells, resulting in areas of foreground cells growing in size, and holes within those regions becoming smaller. Dilation can be performed iteratively for \( n \) times as in Equation 2.

\[
A \oplus B = \{a + b : a \in A, b \in B \} \\
A \oplus nB = (A \oplus B) \oplus B...n \text{ iterations} 
\]  

(1)
(2)

Let a cluster of entities \( A \) be consisted of \( N \) number of entities denoted as \( A_1, A_2, A_3, \ldots, A_N \). An entity is designated as spatially most significant if its distance to the other entities in the cluster is the shortest. The dilation distance \( d \) from an origin entity \( A_i \) and a destination entity \( A_j \) is minimum number of dilation iterations required for \( A_i \) to occupy \( A_j \):

\[
d(A_j) = \min\{n : A_j \subseteq (A_i \oplus nB)\} 
\]

(3)

Similarly, the dilation distance from \( A_j \) to \( A_i \) is computed as follows:

\[
d(A_i) = \min\{n : A_j \subseteq (A_i \oplus nB)\} 
\]

(4)

If both \( A_i \) and \( A_j \) have the same size, shape and orientation, \( d(A_i) = d(A_j) \). If there is a shape-size dissimilarity between the two entities, \( d(A_i) \neq d(A_j) \). A smaller entity would require a greater number of dilation iterations to occupy a larger entity and hence, the larger entity would have a smaller dilation distance.
SSI determines the spatial significance on an entity based on the distance from the other entities in the cluster. The lower the value of SSI of an entity, the higher is its significance, indicating that it has shorter distance to the other entities. The entity with the lowest value of SSI is the spatially most significant entity. The value of SSI for $A_i$ is the minimum number of dilation iterations required for $A_i$ to occupy all the other entities in the cluster, which would be the largest value of $d(A_i)$:

$$SSI(A_i) = \max_{\forall j} \left( \min \left\{ n : (A_j \subseteq (A_i \oplus nB)) \right\} \right)$$

(5)

3. Methodology

3.1. Data set

The DEM in figure 1 shows the area of Great Basin, Nevada, USA, which is bounded by latitude 38° 15’ to 42° N and longitude 118° 30’ to 115° 30’W. The DEM was resampled to 925 m in both x and y directions. It is a Global Digital Elevation Model (GTOPO30) and was downloaded from the USGS GTOPO30 website [27]. GTOPO30 DEMs are available at a global scale, providing a digital representation of the Earth’s surface at a 30 arc-seconds sampling interval. The land data used to derive GTOPO30 DEMs are obtained from digital terrain elevation data (DTED), 1-degree DEMs for USA and the digital chart of the world (DCW). The accuracy of GTOPO30 DEMs varies by location according to the source data. The DTED and the 1-degree dataset have a vertical accuracy of ±30 m while the absolute accuracy of the DCW vector dataset is ±2,000 m horizontal error and ±650 m vertical error [28]. Tensional forces on the terrain’s crust and thins by normal faulting cause the formation an array of tipped mountain blocks that are separated from broad plain basins, producing a basin-and-range physiography [29-33].

![Figure 1](image_url)

**Figure 1.** The GTOPO30 DEM of Great Basin. The elevation values of the terrain (1,005 to 3,651 m) are rescaled to the interval of 0 to 255 (the brightest cell has the highest elevation). The scale is approximately 1:3,900,000.

3.2 Generation of multiscale DEMs

In this paper, multiscaling is performed using the lifting scheme [34-35], which has proven to be a powerful multiscale analysis tool in image and signal processing [36-39], and has received recent attention in geospatial analysis [40-45]. This is due to its ability to preserve accurate surface profiles, in terms of waveform, shape and amplitude, without causing boundary destruction [46-48]. It is used to decompose an original dataset into low and high frequency subsets using the following three steps:
Step 1: Split
The original dataset $x[n]$ is divided into two disjoint subsets, even $x_e[n]$ and odd $x_o[n]$ indexed points.

Step 2: Predict
The odd and even subsets are often highly correlated. As this correlation structure is typically local, one subset can be used to predict the other subset. In this case, the even indexed subset is used to compute a course approximation of the odd indexed subset using the prediction operator $P$ (Equation 6). The predicted odd indexed subset is smoother than the original odd indexed subset, with the high frequency components removed. Hence, the difference between the two subsets gives the high frequency subset $d[n]$ (Equation 7). The even indexed subset is left unchanged to become the input for the next step in the transform.

$$P(x_e[n]) = \frac{1}{2}(x_e[n] + x_e[n+1])$$  \hspace{1cm} (6)

$$d[n] = x_o[n] - P(x_o[n])$$  \hspace{1cm} (7)

Step 3: Update
The update operator $U$ (Equation 8) uses $d[n]$ to compute the difference between the original dataset and its coarse approximation, which consists of the low frequency components of the original dataset. This is then applied to the even indexed subset to obtain the low frequency subset $c[n]$ (Equation 9).

$$U(d[n]) = \frac{1}{4}(d[n-1] + d[n+1])$$  \hspace{1cm} (8)

$$c[n] = x_e[n] + U(d[n])$$  \hspace{1cm} (9)

The above three steps form a lifting stage. Using a DEM as the input, an iteration of the lifting stage generates the complete set of multiscale DEMs $c_s[n]$ and the elevation loss caused by the change of scale $d_s[n]$. At each iteration, $c_s[n]$ only contains half of the points of the input for the iteration, and hence, the resolution of the generated multiscale DEM is reduced by half. At each iteration, the cells of the DEM that are modified are curvatures regions, while the unmodified cells are planar regions. The iterations are repeated until all curvatures in the DEM are removed, leaving only planar regions. For varying DEMs, the number of iterations required would be dependent on the surface profile; a rougher surface profile would require more iterations, while a smoother surface profile would require fewer iterations.

Multiscale DEMs of the Great Basin terrain are generated by implementing the lifting scheme for scales $s$ of 1 to 20. As shown in figure 2, as the scale increases, the merging of small regions into the surrounding grey level regions increases, causing removal of fine detail in the DEM. As a result, the generated multiscale DEMs possess lower resolutions at higher degrees of scaling.
3.3 Mountain extraction
Mountains are portions of a terrain that are sufficiently elevated above the surrounding land (greater than 300 up to 600 m) and have comparatively steep sides. In a mountain, two parts are distinctive [49]:

1) The summit, the highest point (the peak) or the highest ridges
2) The mountainside, the part of a mountain between the summit and the foot.

Figure 2. Multiscale DEMs generated using scales of (a) 1 (b) 3 (c) 5 (d) 10 (e) 15 (f) 20.
The mountains of the generated multiscale DEMs are extracted using the mathematical morphological based algorithm proposed in Dinesh [50]. First, ultimate erosion is performed on the DEM to extract the peaks of the DEM. Conditional dilation is performed on the extracted peaks to obtain the mountains of the DEM. As shown in figure 3, the merge of small regions into the surrounding grey level regions and removal of fine detail in the DEM cause a reduction in the area of the extracted mountains.

Figure 3. Mountains (the cells in white) extracted from the corresponding multiscale DEMs in figure 2.
4. Results & Discussion

The computed $SSI$ values of the mountain objects extracted from the original DEM (figure 3(a)) are shown in table 1. In general, larger mountain objects, which have better reach to the other mountains objects, have lower value of $SSI$, indicating higher significance. The spatially most significant mountain object, with the lowest value of $SSI$, is mountain object 5 (figure 4(a)), which is the largest mountain object and is located at the centre of the terrain.

| Label | Area (cells) | $SSI$ |
|-------|--------------|-------|
| 1     | 1227         | 281   |
| 2     | 10422        | 195   |
| 3     | 1353         | 273   |
| 4     | 298          | 284   |
| 5     | 14232        | 106   |
| 6     | 432          | 246   |
| 7     | 6444         | 172   |
| 8     | 311          | 267   |
| 9     | 1119         | 221   |
| 10    | 219          | 233   |
| 11    | 3574         | 247   |
| 12    | 3058         | 252   |
| 13    | 494          | 282   |
| 14    | 261          | 289   |

The $SSI$ values for the spatially most significant mountain objects of the multiscale DEMs (figure 4) are shown in table 2. As the area of the mountain objects reduce with increasing scale, the distances between the mountain objects increase, resulting in increasing values of $SSI$. At scales 2 to 7, 9 to 11 and 13 to 20, the corresponding spatially most significant mountain objects have the same values (130, 151 and 154 respectively). In these cases, the spatially most significant mountain objects are objects with corresponding locations, requiring the same minimum number of iterative dilations to reach the boundary points that are furthest out with respect to the objects. This highlights the advantages of iterative dilations in computing $SSI$ values as compared to proximity based algorithms that do not consider the geometric properties of the objects.

For scales 1 to 7 and 13 to 20, larger mountain objects located at the centre of the terrain are classified as spatially most significant. However, for scales 8 to 12, significantly smaller mountain objects located at the centre of the terrain are classified as spatially most significant. This indicates that the strategic location of a mountain object at the centre of the terrain is more important than its size in determining its reach to other mountain objects and hence, its spatial significance. For example, for scale 10 (figure 4(d)), the strategic location of mountain object 10 allows it better reach to the other mountain objects as compared to the larger mountain objects (Table 3).
Figure 4. Spatially most significant mountain object of the mountains extracted from the corresponding multiscale DEMs in figure 2.
Table 2. Spatially most significant mountain object of each scale.

| Scale | Num. of objects | Spatially most significant object | Area (cells) | SSI |
|-------|-----------------|-----------------------------------|--------------|-----|
| 1     | 14              | 5                                 | 14232        | 106 |
| 2     | 20              | 8                                 | 9581         | 130 |
| 3     | 21              | 8                                 | 5518         | 130 |
| 4     | 19              | 7                                 | 5350         | 130 |
| 5     | 19              | 7                                 | 5282         | 130 |
| 6     | 18              | 7                                 | 5280         | 130 |
| 7     | 18              | 7                                 | 5040         | 130 |
| 8     | 19              | 12                                | 288          | 145 |
| 9     | 16              | 12                                | 263          | 151 |
| 10    | 14              | 10                                | 263          | 151 |
| 11    | 14              | 10                                | 263          | 151 |
| 12    | 15              | 11                                | 263          | 140 |
| 13    | 12              | 5                                 | 2670         | 154 |
| 14    | 10              | 3                                 | 2599         | 154 |
| 15    | 10              | 3                                 | 2183         | 154 |
| 16    | 10              | 2                                 | 2025         | 154 |
| 17    | 9               | 2                                 | 1946         | 154 |
| 18    | 8               | 2                                 | 1912         | 154 |
| 19    | 7               | 1                                 | 1402         | 154 |
| 20    | 8               | 3                                 | 629          | 154 |

Table 3. Computed SSI values of the mountain objects extracted from the DEM of scale 10.

| Label | Area (cells) | SSI |
|-------|--------------|-----|
| 1     | 571          | 242 |
| 2     | 1967         | 218 |
| 3     | 266          | 253 |
| 4     | 255          | 217 |
| 5     | 3330         | 154 |
| 6     | 1298         | 173 |
| 7     | 1420         | 182 |
| 8     | 2176         | 172 |
| 9     | 1152         | 187 |
| 10    | 263          | 151 |
| 11    | 221          | 207 |
| 12    | 803          | 214 |
| 13    | 229          | 236 |
| 14    | 1048         | 260 |
The results obtained thus far are based on analysis of a mountainous terrain with rough surface profile. The analysis is further extended for the GTOPO30 DEM of Great Plains, Nebraska (bounded by latitude 39° to 43° N and longitude 98° to 101° W) with a smoother surface profile (figure 5). The smoother surface profile of the terrain results in fewer multiscaling iterations (11) required to remove all the curvature regions. It is observed in figure 6 and table 4 that for scales 1 to 7, smaller mountain objects located at the centre of the terrain are classified as spatially most significant. For scales 8 to 11, with no strategically located mountain objects at the centre of the terrain, larger mountain objects located at the corner of the terrain are classified as spatially most significant. For both cases, the corresponding spatially most significant mountain objects have the same values (165 for scales 1 to 7, and 169 for scales 8 to 11), as the objects require the same minimum dilation distances to the boundary points that are furthest out with respect to the objects.

![Figure 5. The GTOPO30 DEM Great Plains. The scale is approximately 1:3,900,000.](image)

5. Conclusion
In this paper, iterative morphological dilations were used to study the spatial significance of mountain objects extracted from multiscale DEMs. It was observed that as the area of the mountain objects reduce with increasing scale, the distances between the mountain objects increase, resulting in increasing values of SSI. The results obtained indicate that the strategic location of a mountain object at the centre of the terrain is more important than its size in determining its reach to other mountain objects and thus, its spatial significance.

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Figure 6. Spatially most significant mountain objects for the multiscale DEMs of Great Plains at scales of: (a) 1  (b) 3  (c) 5  (d) 11.

Table 4: Spatially most significant mountain objects for the multiscale DEMs of Great Plains.

| Scale | Num. of objects | Spatially most significant object | Area (cells) | SSI |
|-------|----------------|----------------------------------|--------------|-----|
| 1     | 8              | 5                                | 743          | 165 |
| 2     | 8              | 5                                | 743          | 165 |
| 3     | 9              | 7                                | 701          | 165 |
| 4     | 8              | 6                                | 587          | 165 |
| 5     | 9              | 7                                | 474          | 165 |
| 6     | 7              | 5                                | 397          | 165 |
| 7     | 6              | 5                                | 333          | 165 |
| 8     | 5              | 1                                | 4269         | 169 |
| 9     | 6              | 1                                | 4244         | 169 |
| 10    | 6              | 1                                | 4170         | 169 |
| 11    | 5              | 1                                | 3951         | 169 |
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