Two kinds of basic problems in the interference of light

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Abstract. In the teaching of light interference, we talked about that the interference occurs when two beams of light have the same vibration direction, the same frequency and the same phase or constant phase difference. However, the temporal coherence dominated by the length of the wave train, and the spatial coherence dominated by the size of the light source are usually neglected. We will take Thomas Young's tow-slit interference as an example, introduce the temporal and spatial coherence, as well as the interference intensity dominated by the temporal and spatial coherence.

1. Introduction

Since Thomas Young, a British Physicist, successfully observed the phenomenon of light interference in the laboratory in 1801, the techniques of holography, lithography, and coherence measurement based on the principle of light interference have played an important role in our life and production. The course of “College Physics” has mentioned that interference occurs when two beams of light have the same vibration direction, frequency, and phase or a constant phase difference. When the phase difference between the two beams is equal to an integral multiple of 2π, that is, the optical path difference is equal to an integral multiple of the wavelength of the light, the coherence is strengthened, and bright fringes appear. When the phase difference between the two beams is equal to an odd multiple of π, that is, the optical path difference is equal to an odd multiple of half of the wavelength of the light, the coherence is weakened, and dark fringes appear. However, even though the above conditions are satisfied, bright or dark fringes do not necessarily appear. For example, when the light is incident on the glass sheet, a film is formed on its front and back surfaces. But the interference fringes have never been observed, because the same vibration direction, frequency, and phase or a constant phase difference are only the necessary conditions; in order to obtain the light and dark fringes, the temporal and spatial coherence of light must be satisfied. This paper mainly introduces the temporal and spatial coherence of light by taking Young's double-slit interference as an example.

2. Time coherence

![Figure 1. Young's double-slit interference device](image)
According to the atomic luminescence mechanism of the light source, the light wave of atomic radiation is not an infinite continuous electromagnetic wave, but a series of intermittent, spaced electromagnetic wave trains, and there is no constant phase difference relationship between different wave trains. The wave trains emitted at different times have no constant phase difference relationship, and they will not interfere. Therefore, interference only occurs between two beams split from one wave train emitted by the atom at the same time, because that is the only way for the phase difference between the two waves to be constant, which is dependent on the optical path difference. As for the different wave trains emitted by the atom at different times, there exists no interference relationship between them.

The experimental device for introducing the temporal coherence of light based on Young’s double-slit interference is shown in Figure 1. The whole device is placed in the air. The light emitted by the light source S is a series of intermittent and spaced wave trains. The wave train emitted at a certain time is divided into wave trains a1 and a2 after passing through the slits S1 and S2. At the point O of the receiving screen, a1 and a2 are completely overlapped at the beginning and the end, so the two beams are strengthened by interference. It is assumed that the intensity of the two beams passing through the slits is I0, then based on the formula of the intensity distribution of the double-beam interference, the intensity of the point O is:

\[ I = I_0 + I_0 + 2\sqrt{I_0I_0}\cos\Delta\Phi = 4I_0 \]  

The phase difference between the two beams is \( \Delta\Phi = 0 \). At the point P of the receiving screen, a1 and a2 cannot overlap completely at the beginning and the end, so there may be two situations: (1) a1 and a2 partially overlap; if so, the overlapping part will be coherently superimposed according to Formula (1). The interference enhancement occurs when the phase difference between the overlapping part is equal to ±2kπ, while the interference weakening occurs when the phase difference between the overlapping part is equal to ±(2k+1)π. Interference does not occur at the non-overlapping part. Therefore, even if the two wave trains partially overlap, and the optical path difference between the overlapping part is equal to an integral multiple of the wavelength, and the point P is coherently strengthened, the light intensity the point P is still weaker than that of the point O. That is also why the bright fringe in the center is always brighter than others; (2) When the optical path difference between S1P and S2P is greater than the length of the wave train, a1 and a2 do not overlap at all while they overlap with the wave trains emitted at other times with no fixed phase difference relationship. So no interference occurs at point P. That is to say that the occurrence of interference requires that the wave trains emitted at the same time be divided into two that are wholly or partially overlapped.

In fact, only when the optical path difference between S1P and S2P is smaller than the length of the wave train emitted by the light source can a1 and a2 have the possibility to be spatially overlapped, thereby causing interference. The length of the wave train is called the coherence length. Assume that the wave train length is \( L_0 \) and the average duration of the light source illumination is \( \tau_c \), then

\[ L_0 = c\tau_c \]  

In the formula, c is the speed of light and \( \tau_c \) is the coherence time. When the difference between the time for a1 and a2 to reach the point P is smaller than the coherence time, the optical path difference between S1P and S2P is smaller than the length of the wave train emitted by the light source, and a1 and a2 overlap at the point P, then interference occurs. In the opposite situation, a1 and a2 do not overlap at the point P, and no interference occurs. Therefore, both coherence time and coherence length can be used to characterize the temporal coherence of light. The often-said monochromatic light is actually quasi-monochromatic light, and its light intensity changes with the wavelength, as shown in Figure 2. \( \lambda \) is the wavelength of light emitted by the light source, and \( \Delta\lambda \) is the width of the line at the half of the intensity of the center wavelength. Then the coherence length of the light source is:

\[ L_0 = \frac{\lambda^2}{\Delta\lambda} \]
3. Spatial coherence

3.1. Principle of spatial coherence

In the Young's double-slit interference experiment, as shown in Figure 1, the light source is assumed to be a point source; however, the light source we often use has a certain degree of width, as shown in Figure 3. According to the principle that bright fringe is generated when two beams of light have the same optical path, the light emitted from the center of the light source \( O' \) produces a central bright fringe at the center of the screen \( O \), the light emitted from the places above \( O' \) generates the central bright fringe below \( O \), while the light emitted from the places below \( O' \) generates the central bright fringe above \( O \). Take point \( c \), which is \( \Delta s \) below \( O' \) as an example, the emitted light reaches the point \( P \), \( \Delta x \) above \( O \), and the optical path difference can be expressed as:

\[
\delta = (R_1 + \Delta r) - (R_2 + \Delta r) \tag{4}
\]

When the optical path difference is 0, the central bright fringes appears:

\[
R_1 - R_2 = \Delta r - \Delta r \tag{5}
\]

However, the width of the light source and \( \Delta s \) are very small. According to the geometric relationship in Figure 3, it is obtained that:

\[
\frac{\Delta x}{\Delta s} \approx \frac{D}{2d} \tag{6}
\]

Therefore:

\[
\frac{\Delta x}{\Delta s} = \frac{D}{2d} \tag{7}
\]

When the central bright fringe generated by light from point \( C \) overlaps at point \( P \) with the first-level dark fringe, which is above point \( O \) and is generated by the light from point \( O' \), then point \( P \) is no longer a dark fringe, and it satisfies:

\[
\Delta x = \frac{D\lambda}{2d} = \frac{D}{\Delta s} \tag{8}
\]

Or:

\[
\Delta s = \frac{R\lambda}{2d} \tag{9}
\]
At this point, the total width of the light source is \( b = 2\Delta s = \frac{R\lambda}{d} \) \hspace{1cm} (10)

Since the light source is not a point source, bright fringes still appear in the place where there is supposed to be dark fringes. This phenomenon is caused by the spatial coherence of light.

### 3.2 Relationship between fringe contrast and light source width

![Young's double-slit interference device](image)

Figure 4. Young’s double-slit interference device

As shown in Figure 4, it is assumed that the intensity of the light emitted by the light source of unit width passing through any slit is \( i \). Take \( ds \), an infinitesimal line element, from the light source, then the light intensity of the light emitted by \( ds \) is \( ids \). According to Formula (1), the interference light intensity on the screen is:

\[
dI = ids + ids + 2ids \cos \Delta \Phi = 2ids \left[ 1 + \cos \left( \frac{2\pi}{\lambda} \delta \right) \right]
\] \hspace{1cm} (11)

By combining Formulas (4), (6) and (11), the total intensity of the light emitted by the entire light source after passing through the two slits can be obtained by integration:

\[
I = \left[ \frac{\beta}{2} \frac{\pi \lambda}{d} \right] \left[ 1 + \cos \left( \frac{2\pi}{\lambda} \left( \frac{d}{R} s + \frac{d}{D} x \right) \right) \right] ds

= 2i \delta \left[ 1 + \frac{\sin \left( \frac{\pi \lambda d}{R \lambda} \right)}{\pi \lambda d} \cos \left( \frac{2\pi d}{D \lambda} x \right) \right]
\] \hspace{1cm} (12)

Let \( \alpha = \frac{\pi \lambda d}{R \lambda} \)

Then, the maximum and minimum light intensity are:

\[
I_{\text{max}} = 1 + \left| \frac{\sin \alpha}{\alpha} \right| \quad I_{\text{min}} = 1 - \left| \frac{\sin \alpha}{\alpha} \right|
\] \hspace{1cm} (13)

The contrast of the interference fringes is:

\[
\gamma = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} = \left| \frac{\sin \alpha}{\alpha} \right|
\] \hspace{1cm} (14)

Based on the above theoretical models, the relationship between the contrast of the interference fringes and the source width \( b \) is obtained, as shown in Figure 5. When the light source is a point source (\( b=0 \)), the maximum contrast is 1. When the width of the light source is an integral multiple of \( R\lambda/d \), the contrast is 0. The contrast changes periodically as the light source increases, and the maximum contrast in each cycle decreases.
Figure 5. Relationship between the contrast of interference fringes and the source width

4. Conclusion
This paper takes Yang's double-slit interference experiment as an example to introduce the temporal and spatial coherence of light, and the interference light intensity determined by two types of coherence. The two beams emitted by the point source have the same vibration direction, frequency, and phase or a constant phase difference, which is only a necessary condition for the occurrence of interference. For the interference to occur, the temporal coherence must also be satisfied. If the light source has a certain degree of width, the contrast of the interference fringes changes with the size of the light source, and it changes periodically with the increase of the width of the light source, and tends to decrease gradually. Our findings may be important for the teaching and the application of the interference of light.

Acknowledgments
This work is partially supported by the Project of Information Teaching of Qingdao University of Science and Technology, as well as the National Natural Science Foundation of China under grant no.11504194.

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