Interesting coupling phenomena of heavy and light holes in a \((GaAs/AlAs)^n\) superlattice

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Abstract

An appropriate combination of the scattering theory and the transfer matrix formalism, for the solution of a \((4 \times 4)\) Kohn-Luttinger model, allow us to study the multichannel-multiband transmission process of heavy and light holes through a \((GaAs/AlAs)^n\) superlattice. Appealing effects and interesting channel coupling phenomena, mediated by quasi-bond states, are clearly foreseen.

Key words: Hole tunneling, Scattering theory, Transfer Matrix.
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1 Theory: an outline of the Transfer Matrix-Scattering Matrix approach

Following the theory of finite periodic systems \([1]\), we combine the transfer matrix (TM) and the scattering matrix (SM) formalisms to describe, with clear advantages and transparency, multichannel-multiband transmission processes. To study the multichannel transport of heavy \((hh)\) and light \((lh)\) holes through a semiconductor superlattice (SL) we consider a model in the framework of the \((4 \times 4)\) Kohn-Luttinger (KL) approach. At variance with the usual restrictions where the incoming flux is a pure hole state \([2,3,4,5,6]\), we have here the possibility of a mixed-state input. Once the KL eigenvalue problem is solved, we obtain the TM in the propagating mode representation and, hence, the \(n\)-cell transmission amplitude\([1]\)

\[ t_n = \alpha_n - \beta_n \delta_n^{-1} \gamma_n, \]

with \(\alpha_n, \beta_n, \delta_n, \gamma_n\), \(4 \times 4\) \((n\text{-cell})\) TM blocks. Various transport coefficients can now be evaluated. Among them, we will calculate the following global

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\[ T_{nij} = \left| t_{nij} \right|^2; \quad T_n = \sum_j \left| t_{nj} \right|^2 \quad \text{and} \quad g_n = Tr \left( t_n t_n^\dagger \right). \tag{2} \]

Here \( i, j = hh_{+3/2}, lh_{-1/2}, lh_{+1/2}, hh_{-3/2} \) (ordered as in Ref. [2]), \( T_{nij} \) is the transmission coefficient from channel \( i \) to channel \( j \), \( T_n \) the total transmission probability (through channel \( i \)), and \( g_n \) the two-probe superlattice Landauer Conductance (in units of \( e^2/\pi \hbar \)).

### 2 Transmission Results

We shall illustrate the just sketched approach with specific calculations of \( hh \) and \( lh \) transmission coefficients through a \((GaAs/AlAs)^n\) superlattice, where the GaAs well thickness will be \( L_w = 50 \) Å, the band off-set \( V_b = 0.55 \) eV and the Lütttinger parameters as in Ref. [7]. For the AlAs barrier thickness \( L_b \) and for the number of cells \( n \), we will take different values. To keep an easy notation, we will drop the subindex \( n \) from the physical quantities we have plotted.

In figure 1(a), “direct path” transmission coefficients \( T_{ii} \) and the “crossed path” \( T_{ij} \) (with \( i \neq j \)), are plotted for the especial \( n = 1 \) case, and for transverse wave number \( \kappa = 0 \). In figure 1(b) the transmission coefficients \( T_i \) (through the indicated channels \( i \) ) and the conductance \( g \) are plotted. The conductance \( g \) is shown also for other values of \( \kappa \). For \( \kappa = 0 \), all \( T_{ij} \) but the crossed path \( lh_{+1/2} \rightarrow hh_{-3/2} \) (dotted line) are null[6]. This result suggests some, low-level interference, and the passage of flux from \( lh_{+1/2} \) to \( hh_{-3/2} \). This is congruous with the \( T_i \) plots. There, the \( T_{lh_{+1/2}} \) contribution remains around 0.5 eV. The \( \kappa \) dependance of \( g \) (see r.h.s. panel), shows that increasing \( \kappa \) the interaction and coupling between channels (either opened or closed) gets enhanced. Since the conductance is a global magnitude it does not reflect fine variations.

In Figures 2 we plot various transmission coefficients \( T_{ij} \) for \( n = 2 \), and \( i \) and \( j \) as indicated in the right hand side panel. At low energies, discrete quasi-stationary hole levels in the quantum well are correctly resolved [3,4,8] ( levels are labelled as in reference [9]). At these energies (see fig. 2(b)), clear resonant transmission of flux from direct to crossed paths is seen. Looking at the crossed paths \( lh_{+1/2} \rightarrow hh_{-3/2} \) (full line) and \( hh_{+3/2} \rightarrow lh_{-1/2} \) (thick full line), for energies around 0.48 eV and 0.56 eV, interesting coupling phenomena between heavy- and light-hole propagating modes are also apparent. As a consequence, it is possible that a hole entering this region as \( hh_{\pm3/2} \) or as \( lh_{\pm1/2} \) can exit in the same or a different state. Nevertheless, what is more appealing is the fact that such resonant transport takes place without account of the effective masses.

In figure 3 we plot a direct path transmission probability for different number of cells of the SL. The most visible effect is the formation of a subband structure as \( n \) grows [1]. In this case, the band spectrum reveals itself reasonably well defined [1] when \( n \approx 5 \). This number depends of course on the
Fig. 1. Transmission coefficients and conductance of AlAs/GaAs/AlAs, as functions of the incident particle energy. In (a), the Transmission coefficients $T_{ij}$ for $n = 1$, $L_w + L_b = 60 \text{ Å} \kappa = 0$ and $i$ and $j$ as indicated. In (b), the total transmission coefficients $T_i$ through the indicated channel $i$, and the conductance $g$ for $\kappa = 0, 0.01,$ and $0.02 \text{ Å}^{-1}$.

SL parameters. As for 1D one channel periodic system, the band structure can be strongly modified when these parameters are changed. To illustrate this, we plot in figure 4 the transmission probability for different values of the barrier thickness $L_b$, while the remaining parameters are kept fixed. Reducing the barrier width, the transmission coefficient grows (as it should) and, at the same time, the gaps become less pronounced. Simultaneously the resonant peaks experience a kind of blue shift. It is worth noticing however that, even though the band structure is a consequence of and will emerge once the phase coherence and the periodicity have been combined, the single-cell multichannel-multiband transfer matrix contains all the information of this fundamental property [1,6].

3 Summary

We have presented a more realistic TM-SM calculations for hole tunnelling across different realizations of the (GaAs/AlAs)$_n$ superlattice. The system exhibits clear channel interference effects between light- and heavy-hole quasi-particles. The resonant transition from $lh$-like modes to $hh$-like modes is much stronger than the other way around. Global quantities such as the conductance $g$ and total transmission coefficient $T_i$, show that our model distinguishes pretty well the four input-output channels, as well as, all the contributions to the tunnelling process. Some of the features that we have presented are accessible only when one works in the full $(4 \times 4)$ KL space within the proposed scheme of multichannel-multiband transmission, of the 4 available input channels.

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Fig. 2. (a) The transmission coefficients $T_{ij}$ through a $(GaAs/AlAs)^n$ superlattice, with $n = 2$ and total length of 120 Å, as a function of the incident particle energy. At low energies the discrete quasi-stationary hole levels are visible as isolated resonances. The height of the barrier is pointed with an arrow in the energy axis. In (b) we have a zoom of the lowest energy resonances. It is clear the channel coupling effect. The legend displayed at the right hand panel is valid for both panels.

![Fig. 2](image)

Fig. 3. The Transmission probability for a “direct” path $hh_{+3/2} \rightarrow hh_{+3/2}$ through a $(GaAs/AlAs)^n$ superlattice, as a function of the incoming particle energy, for different number of cells $n$. Other parameters are $\kappa = 0.01$ Å$^{-1}$ and $V_b = 0.55$ eV. The subband spectrum is visible when $n$ is of the order of 5. The single cell length is 60 Å.

![Fig. 3](image)

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Fig. 4. The same as Figure 3, for three barrier thickness keeping fixed the well width at $\kappa = 0$ for $n = 3$. As the length of the barrier is raised the structure of the spectrum is better defined.

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