Pure annihilation decays of $B^0_s \to a_0^+ a_0^-$ and $B^0_d \to K_0^{*-} K_0^{*+}$ in the PQCD approach

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We study the $CP$-averaged branching fractions and the $CP$-violating asymmetries in the pure annihilation decays of $B^0_s \to a_0^+ a_0^-$ and $B^0_d \to K_0^{*-} K_0^{*+}$, where $a_0$ [$K_0^*$] denotes the scalar $a_0(980)$ and $a_0(1450)$ [$K_0^*(800)$] (or $\kappa$) and $K_0^*(1430)]$, with the perturbative QCD factorization approach under the assumption of two-quark structure for the $a_0$ and $K_0^*$ states. The numerical results show that the branching ratios of the $B^0_s \to K_0^{*-} K_0^{*+}$ decays are in the order of $10^{-6}$, while the decay rates of the $B^0_d \to a_0^+ a_0^-$ modes are in the order of $10^{-5}$. In light of the measured modes with the same quark components in the pseudoscalar sector, namely, $B^0_s \to K^+ K^-$ and $B^0_s \to \pi^+ \pi^-$, the predictions for the considered decay modes in this work are expected to be measured at the Large Hadron Collider beauty and/or Belle-II experiments in the near future. Meanwhile, it is of great interest to find that the twist-3 distribution amplitudes $\phi^S$ and $\phi^D$ with inclusion of the Gegenbauer polynomials for the scalar $a_0(1450)$ and $K_0^*(1430)$ states in scenario 2 contribute slightly to the branching ratios while significantly to the $CP$ violations in the $B^0_s \to K_0^*(1430)^+ K_0^*(1430)^-$ and $B^0_s \to a_0(1450)^+ a_0(1450)^-$ decays, which indicates that, compared to the asymptotic $\phi^S$ and $\phi^D$, these Gegenbauer polynomials could change the strong phases evidently in these pure annihilation decay channels. These predictions await for the future confirmation experimentally, which could further provide useful information to help explore the inner structure of the scalars and shed light on the annihilation decay mechanism.

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In the heavy flavor $B$ physics, the annihilation diagrams are highly important in understanding the (non-)perturbative dynamics involved in the related decays, although which are generally considered as the power suppressed ones and then ever neglected because of not knowing how to effectively calculate them at the early stage of investigating the $B$ meson decays [1–3] 1. However, to interpret well the fundamental quantity, namely, the $CP$-violating asymmetry, one realized that the annihilation diagrams should be included essentially. Although, to date, both of the soft-collinear effective theory [8] and the perturbative QCD factorization(PQCD) approach [9–11] believed that the annihilation diagrams could be perturbatively calculated, the extremely different observations still made this issue controversial, namely, an almost real amplitude with a tiny strong phase was obtained in the former framework by introducing the zero-bin subtraction [12]; and in contrast, an almost imaginary one with a large strong phase appeared in the latter formalism naturally by keeping the transverse momentum $k_T$ of valence quark [13]. Furthermore, it is stressed that more recent works based on the QCD factorization approach [3, 14], one of the popular factorization methods in the current market, claimed that a complex contribution arising from the annihilation diagrams with significant imaginary parts should be essential in the $B_{(s)} \to PP$, $PV$ and $VV$ decays by fitting to experimental data [15–23]. Phenomenologically speaking, they supported the viewpoint of the PQCD approach on the effective calculations of the annihilation diagrams to some extent. And what is more, the measurements from the Large Hadron Collider beauty(LHCb) experiment on the pure annihilation modes [24–26], i.e., $B^0_s \to K^+ K^-$ and $B^0_s \to \pi^+ \pi^-$, confirmed the predictions of their branching ratios in the PQCD approach at leading order [27, 28]. Undoubtedly, the good agreement between theory and experiment is exciting and inspiring. Therefore, in order to provide solid foundation to understand the annihilation decay mechanism, more and more investigations on the annihilation diagrams in the PQCD approach are necessary.

Motivated by this success, we shall study the $B^0_s \to a_0^+ a_0^-$ and $B^0_d \to K_0^{*-} K_0^{*+}$ decays within the PQCD approach at leading order in this work, where $a_0$ and $K_0^*$ denote the light scalar states $a_0(980)$ and $a_0(1450)$, and $K_0^*(800)$ (or $\kappa$) and $K_0^*(1430)$, respectively. Before proceeding, it is essential to give a short review on the current status about the light scalar states $a_0$ and $K_0^*$. It is well known that the description on the inner structure of light scalars is still in controversial (for a review, see e.g., Refs. [29–31]). The states $a_0(980)$ and $\kappa$ with masses below or close to 1 GeV are classified into one nonet, while those ones $a_0(1450)$ and $K_0^*(1430)$ with masses above 1 GeV belong to the other nonet. It has been stressed that these two nonets are hard to be considered as the low-lying $q\bar{q}$ states simultaneously due to the major difficulties, e.g., see Ref. [32] for detail. Therefore, two typical scenarios are suggested for the classification of these light scalar states [33]: In scenario 1(S1), $a_0(980)$ and $\kappa$ are the lowest-lying $q\bar{q}$ states, while $a_0(1450)$ and $K_0^*(1430)$ are the first excited $q\bar{q}$ states correspondingly; In scenario 2(S2), $a_0(980)$ and $\kappa$ are treated as four-quark states, and then $a_0(1450)$ and $K_0^*(1430)$ are considered as the lowest-lying two-quark states. In the present work, we shall consider the decays of $B^0_s \to a_0^+ a_0^-$ and $B^0_d \to K_0^{*-} K_0^{*+}$ with the scalars in the two-quark model. Then, it is easily found that the considered decays have the same quark components as those measured ones, i.e., $B^0_s \to \pi^+ \pi^-$ and $B^0_d \to K^+ K^-$ in the pseudoscalar sector. Therefore, the LHCb and/or Belle-II experiments could potentially make a

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1 It is worth stressing that the annihilation diagrams seem much more important in understanding the dynamics contained in the charmed meson decays, e.g., see Refs. [4–7] for detail.
sound examination on the predictions about the branching ratios and/or the CP violations of the considered $B_s^0 \rightarrow a_0^+ a_0^-$ and $B_d^0 \rightarrow K_0^{*+} K_0^{-}$ decays in the PQCD approach.

At the quark level, the considered $B_s^0 \rightarrow a_0^+ a_0^-$ and $B_d^0 \rightarrow K_0^{*+} K_0^-$ decays are induced by the $b \rightarrow s$ and the $b \rightarrow d$ transitions, respectively. The related weak effective Hamiltonian $H_{\text{eff}}$ can be written as [34],

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}^* V_{uQ} [C_1(\mu)O_{1u}(\mu) + C_2(\mu)O_{2u}(\mu)] - V_{tb}^* V_{tQ} \sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right\},$$

(1)

with the Fermi constant $G_F = 1.16639 \times 10^{-5}$ GeV$^{-2}$, the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements $V_{u(t)b}$ and $V_{u(t)Q}$, the light $Q = d, s$ quark, and the Wilson coefficients $C_i(\mu)$ at the renormalization scale $\mu$. The local four-quark operators $O_i (i = 1, \cdots, 10)$ are written as

- **Tree operators**

$$O_{1u} = (\bar{Q}_\alpha u_\beta)_{V-A} (\bar{u}_\beta b_\alpha)_{V-A}, \quad O_{2u} = (\bar{Q}_\alpha u_\beta)_{V-A} (\bar{u}_\beta b_\alpha)_{V-A},$$

(2)

- **QCD penguin operators**

$$O_3 = (\bar{Q}_\alpha b_\beta)_{V-A} \sum_q \langle \bar{q}_q' q_\beta \rangle_{V-A}, \quad O_4 = (\bar{Q}_\alpha b_\beta)_{V-A} \sum_q \langle \bar{q}_q' q_\beta \rangle_{V-A},$$

$$O_5 = (\bar{Q}_\alpha b_\beta)_{V-A} \sum_q \langle \bar{q}_q' q_\beta \rangle_{V+A}, \quad O_6 = (\bar{Q}_\alpha b_\beta)_{V-A} \sum_q \langle \bar{q}_q' q_\beta \rangle_{V+A},$$

(3)

- **Electroweak penguin operators**

$$O_7 = \frac{3}{2} (\bar{Q}_\alpha b_\beta)_{V-A} \sum_{q'} e_{q'} \langle \bar{q}_q' q_\beta \rangle_{V+A}, \quad O_8 = \frac{3}{2} (\bar{Q}_\alpha b_\beta)_{V-A} \sum_{q'} e_{q'} \langle \bar{q}_q' q_\beta \rangle_{V+A},$$

$$O_9 = \frac{3}{2} (\bar{Q}_\alpha b_\beta)_{V-A} \sum_{q'} e_{q'} \langle \bar{q}_q' q_\beta \rangle_{V-A}, \quad O_{10} = \frac{3}{2} (\bar{Q}_\alpha b_\beta)_{V-A} \sum_{q'} e_{q'} \langle \bar{q}_q' q_\beta \rangle_{V-A},$$

(4)

with the color indices $\alpha$, $\beta$ and the notations $(\bar{q} q')_{V, A} = \bar{q} \gamma_\mu (1 \pm \gamma_5) q'$. The index $q'$ in the summation of the above operators runs through $u, d, s, c$, and $b$. Note that we will use the leading order Wilson coefficients since we work in the framework of the PQCD approach at leading order. For the renormalization group evolution of the Wilson coefficients from higher scale to lower scale, we will adopt straightforwardly the formulas as given in Refs. [9, 10].

![Feynman diagrams](image.png)

**FIG. 1.** Leading order Feynman diagrams for $B_s^0 \rightarrow a_0^+ a_0^-$ decays in the PQCD formalism.

The Feynman diagrams of the $B_s^0 \rightarrow a_0^+ a_0^-$ decays at leading order in the PQCD formalism are illustrated in Fig. 1: Figs. 1(a) and 1(b) describe the factorizable annihilation($fA$) diagrams, while Figs. 1(c) and 1(d) describe the nonfactorizable annihilation($nA$) ones. By replacing the $s$ and $d$ quarks in the $B_s^0 \rightarrow a_0^+ a_0^-$ decays with the $d$ and $s$ ones correspondingly, then we can obtain the annihilation modes $B_d^0 \rightarrow K_0^{*+} K_0^-$ directly. As we know, several $B \rightarrow SS$ decays with $S$ denoting
the scalar mesons have been studied in the PQCD approach [35–39]. Therefore, the analytic expressions for the decay amplitudes of the considered \( B_d^0 \rightarrow a_0^+ a_0^- \) and \( B_d^0 \rightarrow K_0^{*+} K_0^{*-} \) decays can be found easily, for example, in the Refs. [37–39]. The \( B_d^0 \rightarrow K_0^{*+} K_0^{*-} \) decay amplitudes have been presented in the PQCD approach [39]. Then, we just need to replace the \( B_d^0 \) and \( K_0^* \) states in Ref. [39] with the \( B_s^0 \) and \( a_0 \) states, as well as the related CKM matrix elements, to obtain easily the corresponding information of the \( B_s^0 \rightarrow a_0^+ a_0^- \) decays in the PQCD approach. Hence, for simplicity, we will not collect the aforementioned formulas in this paper. The interested readers can refer to Ref. [39] for detail.

As presented in Ref. [39], the \( B_d^0 \rightarrow K_0^{*+} K_0^{*-} \) decay amplitude is as the following,

\[
A(B_d^0 \rightarrow K_0^{*+} K_0^{*-}) = V_{ub} V_{ud} \left[ M_{nfa} C_2 \right] - V_{tb} V_{td} \left[ M_{nfa} (C_4 + C_{10}) + M_{nfa}^2 (C_6 + C_8) + M_{nfa} [K_0^{*+} \leftrightarrow K_0^{*-}] (C_4 - \frac{1}{2} C_{10}) + M_{nfa}^2 [K_0^{*+} \leftrightarrow K_0^{*-}] (C_6 - \frac{1}{2} C_8) \right],
\]

(5)

where \( M_{nfa} \) and \( M_{nfa}^2 \) stand for the nonfactorizable annihilation amplitudes arising from the \( (V-A)(V-A) \) and \( (S-P)(S+P) \) currents [39], respectively. Moreover, the almost exact cancellation of the factorizable annihilation amplitudes appear in the considered two decay modes due to the very small flavor symmetry breaking effects, which can be seen in the numerical calculations later. Similarly, we can easily obtain the \( B_s^0 \rightarrow a_0^+ a_0^- \) decay amplitude by the corresponding replacement of \( d \leftrightarrow s \) in the \( B_d^0 \rightarrow K_0^{*+} K_0^{*-} \) one, that is,

\[
A(B_s^0 \rightarrow a_0^+ a_0^-) = V_{ub} V_{us} \left[ M_{nfa} C_2 \right] - V_{tb} V_{ts} \left[ M_{nfa} (C_4 + C_{10}) + M_{nfa}^2 (C_6 + C_8) + M_{nfa} [a_0^+ \leftrightarrow a_0^-] (C_4 - \frac{1}{2} C_{10}) + M_{nfa}^2 [a_0^+ \leftrightarrow a_0^-] (C_6 - \frac{1}{2} C_8) \right].
\]

(6)

Then, we can turn to the numerical calculations of the \( CP \)-averaged branching ratios and the \( CP \)-violating asymmetries of the \( B_s^0 \rightarrow a_0^+ a_0^- \) and \( B_d^0 \rightarrow K_0^{*+} K_0^{*-} \) decays in the PQCD approach. Some comments on the nonperturbative inputs are listed essentially as follows:

(a) For the heavy \( B_d^0 \) and \( B_s^0 \) mesons, the wave functions and the distribution amplitudes, and the decay constants are same as those utilized in Ref. [39], but with the updated lifetimes \( \tau_{B_d^0} = 1.52 \text{ ps} \) and \( \tau_{B_s^0} = 1.509 \text{ ps} \) [40]. It is worth mentioning that, due to its highly small effects, namely, the power-suppressed \( 1/m_B \) contributions to \( B \) decays in final states with energetic light particles [10, 41], the high twist contributions from the \( B \) meson wave function in the considered pure annihilation channels have to be left for future studies associated with the precise measurements. For recent development about the \( B \) meson wave function and/or distribution amplitude, please see references, e.g., [41–46] for detail.

(b) For the considered light scalar \( a_0 \) and \( K_0^* \) states, the decay constants and the Gegenbauer moments in the distribution amplitudes \(^2\) have been derived at the normalization scale \( \mu = 1 \text{ GeV} \) in the QCD sum rule method [33]:

\[
\tilde{f}_K = 0.340 \pm 0.020 \text{ GeV}, \quad f_K = 0.050 \pm 0.003 \text{ GeV},
\]

(7)

\[
B_1 = -0.92 \pm 0.11, \quad B_3 = 0.15 \pm 0.09;
\]

(8)

\[
\tilde{f}_{a_0}(980) = 0.365 \pm 0.020 \text{ GeV}, \quad f_{a_0}(980) \sim 0.0011 \text{ GeV},
\]

(9)

\[
B_1 = -0.93 \pm 0.10, \quad B_3 = 0.14 \pm 0.08;
\]

(10)

\[
\tilde{f}_{K_0^*}(1430) = \begin{cases} 
-0.300 \pm 0.030 \text{ GeV}, \\
0.445 \pm 0.050 \text{ GeV}, 
\end{cases} \quad f_{K_0^*}(1430) = \begin{cases} 
-0.044_{-0.005}^{+0.004} \text{ GeV}, \\
0.066_{-0.008}^{+0.007} \text{ GeV}, 
\end{cases}
\]

(11)

\[
B_1 = \begin{cases} 
0.58 \pm 0.07, \\
-0.57 \pm 0.13,
\end{cases} \quad B_3 = \begin{cases} 
-1.20 \pm 0.08, \\
-0.42 \pm 0.22,
\end{cases}
\]

(12)

\[
\tilde{f}_{a_0}(1450) = \begin{cases} 
-0.280 \pm 0.030 \text{ GeV}, \\
0.460 \pm 0.050 \text{ GeV}, 
\end{cases} \quad f_{a_0}(1450) = \begin{cases} 
\sim -0.0009 \pm 0.0002 \text{ GeV}, \\
0.0014 \pm 0.0002 \text{ GeV}, 
\end{cases}
\]

(13)

\[
B_1 = \begin{cases} 
0.89 \pm 0.20, \\
-0.58 \pm 0.12,
\end{cases} \quad B_3 = \begin{cases} 
-1.38 \pm 0.18, \\
-0.49 \pm 0.15,
\end{cases}
\]

(14)

\(^2\) It is necessary to mention that we firstly adopt the asymptotic form of the twist-3 distribution amplitudes \( \phi^S \) and \( \phi^T(T3A) \) in the numerical calculations here as usual [33, 47]. And then we will estimate the effects in this work arising from the twist-3 distribution amplitudes with inclusion of the Gegenbauer polynomials(T3G) in (S2) later. It is noted that only the T3G form in (S2) is available currently [48].
Note that the scale-dependent scalar decay constant \( \bar{f}_S \) and the vector decay constant \( f_S \) are related with each other through the following relation [33],

\[
\bar{f}_S = \mu_S f_S = \frac{m_S}{m_q(\mu) - m_q(\mu)} f_S,
\]

in which \( m_S \) is the mass of the scalar meson, and \( m_q \) are the running quark masses. It is worth pointing out that for the \( a_0 \) states, the isospin symmetry breaking effects from the \( u \) and \( d \) quark masses are considered. Therefore, the running quark masses for the strange quark and the nonstrange light quarks can be read as \( m_s = 0.128 \) GeV, \( m_d = 0.006 \) GeV, and \( m_u = 0.003 \) GeV, respectively, which are translated from those at the \( \overline{\text{MS}} \) scale \( \mu \approx 2 \) GeV [40]. For the masses of the \( a_0 \) and \( K^*_0 \) states, the values \( m_{a_0} = 0.824 \) GeV, \( m_{a_0}(980) = 0.980 \) GeV, \( m_{K^*_0}(1430) = 1.425 \) GeV, and \( m_{a_0(1450)} = 1.474 \) GeV will be adopted in the numerical calculations \(^3\).

(c) For the CKM matrix elements, we also adopt the Wolfenstein parametrization at leading order, but with the updated parameters \( A = 0.836, \lambda = 0.22453, \bar{\rho} = 0.122^{+0.018}_{-0.017} \), and \( \bar{\eta} = 0.355^{+0.012}_{-0.011} \) [40].

Now, we present the numerical results of the \( B_s^0 \to a_0^+a_0^- \) and \( B_d^0 \to K^{*+}K^{*-} \) decays in the PQCD formalism. Firstly, the PQCD predictions of the CP-averaged branching ratios can be read as follows:

\[
\text{Br}(B_d^0 \to \kappa^+\kappa^-) = 2.84^{+0.39}_{-0.32} \times 10^{-6},
\]

\[
\text{Br}(B_s^0 \to a_0(980)^+a_0(980)^-) = 2.64^{+0.30}_{-0.28} \times 10^{-5},
\]

and

\[
\text{Br}(B_d^0 \to K^0(1430)^+K^0(1430)^-) = \begin{cases} 
3.08^{+0.00}_{-0.02}(\omega_B)^{+0.84}_{-0.95}(B_{m0})^{+0.16}_{-0.15}(V) \times 10^{-6}, \\
2.45^{+0.50}_{-0.51}(\omega_B)^{+0.18}_{-0.19}(B_{m0})^{+0.09}_{-0.09}(V) \times 10^{-6};
\end{cases}
\]

\[
\text{Br}(B_s^0 \to a_0(1450)^+a_0(1450)^-) = \begin{cases} 
5.63^{+0.00}_{-0.10}(\omega_B)^{+0.83}_{-0.96}(B_{m0})^{+0.04}_{-0.05}(V) \times 10^{-5}, \\
3.55^{+0.75}_{-0.57}(\omega_B)^{+0.34}_{-0.63}(B_{m0})^{+0.34}_{-0.31}(V) \times 10^{-5};
\end{cases}
\]

where, as clearly seen from the above results, the major errors are mainly induced by the uncertainties of the scalar decay constants \( \bar{f}_{a_0} \) and \( \bar{f}_{K^*_0} \), and the combined CKM matrix elements \( V_{ub} \) and \( V_{cd} \) in the leading twist distribution amplitudes of the scalar mesons. The other errors induced by the shape parameter \( \omega_B \) in the \( B_d^0 \) meson distribution amplitude and by the combined CKM matrix elements \( V \) are much smaller. Frankly speaking, these mentioned hadronic parameters of the scalar mesons are currently less constrained from the experiments and/or Lattice QCD calculations. Therefore, we have to adopt those available parameters calculated in the QCD sum rule method to give a rough estimation preliminarily. Of course, both the essential measurements at the experimental aspects and the Lattice QCD computation at the theoretical aspects on the above-mentioned nonperturbative inputs for the scalar mesons are urgently demanded, which is expected to help better understand the related hadron dynamics and provide more precise predictions. Note that all the errors from various parameters as specified above have been added in quadrature, which can be seen from the results presented in the square brackets.

Based on the numerical results with large theoretical uncertainties as shown in the Eqs. (16)-(19), several remarks are in order:

(a) For the considered \( B_d^0 \to K^{*+}K^{*-} \) decays, the CP-averaged branching ratios presented in Refs. [35, 39] could be reproduced in this work but with slight differences, which are mainly because of the updated parameters such as the values of the CKM matrix elements, the \( B_d^0 \) meson lifetime, and the running quark masses of the light quarks. As aforementioned, the \( B_d^0 \to K^{+}K^{-} \) decay has been measured experimentally and the decay rate is read as \( 7.8 \pm 1.5 \times 10^{-8} \) [24, 25], which confirmed the PQCD calculation, that is, \( 1.56^{+0.56}_{-0.52} \times 10^{-7} \) [28] about this channel theoretically. As the counterpart with the same quark structure in the scalar sector, the \( B_d^0 \to K^{*+}K^{*-} \) modes with large decay rates in the order of \( 10^{-6} \) predicted with the same PQCD formalism could be examined at the LHCb and/or Belle-II experiments in the (near) future. The confirmation from the experimental side would help explore the inner structure especially for the scalar \( K^*_0(1430) \) meson.

(b) For the \( B_s^0 \to a_0^+a_0^- \) modes, they have the same quark structure as that of the measured one \( B_s^0 \to \pi^+\pi^- \), whose decay rate is read as \( 7.0 \pm 1.0 \times 10^{-7} \) [24] and agrees well with the prediction about the decay rate \( 5.10^{+2.26}_{-1.89} \times 10^{-7} \) [28] in the PQCD approach within theoretical errors. Therefore, the large \( B_s^0 \to a_0^+a_0^- \) decay rates in the order of \( 10^{-5} \) could be accessed with much more probabilities experimentally with respect to those of the \( B_d^0 \to K^{*+}K^{*-} \) ones. Furthermore,
(i) Recently, the $B^0_s \rightarrow a_0(980)^+a_0(980)^-$ mode has been investigated in the PQCD approach by Liang and Yu [37]. However, it is a bit strange that the CP-averaged branching ratio of the $B^0_s \rightarrow a_0(980)^+a_0(980)^-$ channel is predicted as small as $5.17^{+2.36}_{-1.94} \times 10^{-6}$. In fact, as naively estimated from the $B^0_s \rightarrow \pi^+\pi^-$ decay rate, the $B^0_s \rightarrow a_0(980)^+a_0(980)^-$ branching ratio could be, as far as the central values are concerned, \[ \left[ \frac{f_{a_0(980)}}{f_{\pi}} \right]^2 \cdot Br(B^0_s \rightarrow \pi^+\pi^-)^{\text{Exp}} \sim 3.43 \times 10^{-5} \] and \[ \left[ \frac{f_{a_0(980)}}{f_{\pi}} \right]^2 \cdot Br(B^0_s \rightarrow \pi^+\pi^-)^{\text{PQCD}} \sim 3.16 \times 10^{-5} \] singly induced by the decay constants $f_\pi = 0.13$ GeV and $f_{a_0(980)} = 0.365$ GeV, apart from the possible enhancement arising from the asymmetric behavior of the only odd terms involved in the leading twist distribution amplitude of the scalar meson.

(ii) We study the $B^0_s \rightarrow a_0(1450)^+a_0(1450)^-$ decay in two scenarios within the PQCD framework for the first time. The predicted branching ratios are large around $10^{-5}$ and are expected to be tested in the near future at LHCb and/or Belle-II experiments. The understanding of the scalar $a_0(1450)$ meson could provide useful information to uncover the nature of the $K^*_0(1430)$ meson through the $SU(3)$ flavor symmetry (breaking) effects, and vice versa.

(c) In light of the large theoretical errors, a precise ratio of the related branching ratios would be more interested because, generally speaking, the theoretical errors resulted from the hadronic inputs could be cancelled to a great extent. Therefore, we define the following ratios to be measured at the relevant experiments of $B$ meson decays, which would help to study the QCD dynamics, even the decay mechanism of these considered pure annihilation decays.

\[
R_{sd}^{(980)/\kappa} = \frac{Br(B^0_s \rightarrow a_0(980)^+a_0(980)^-)}{Br(B^0_d \rightarrow \kappa^+\kappa^-)} = 9.30^{+0.40}_{-0.33}(\omega_B)_{-0.31}(B_m)_{+0.17}(\bar{f}_M)_{+0.44}(V)_{+0.44} \quad (20)
\]

\[
R_{sd}^{(1450)/K^*_0(1430)} = \frac{Br(B^0_s \rightarrow a_0(1450)^+a_0(1450)^-)}{Br(B^0_d \rightarrow K^*_0(1430)^+K^*_0(1430)^-)} = \left\{ \begin{array}{l}
18.25_{-7.21}^{+7.97} (\omega_B)_{-0.67}^{+0.45} (B_m)_{-0.16}^{+0.16} (\bar{f}_M)_{+0.51}^{+0.75} (V)_{+0.48}^{+0.75} \\
14.40_{-7.57}^{+7.63} (\omega_B)_{+0.58}(B_m)_{-0.47}(\bar{f}_M)_{-0.47}(V)_{+0.49} \end{array} \right. \quad (S1) \quad (S2) \quad (21)
\]

\[
R_{sd}^{S1/S2}(K^*_0(1430)) = \frac{Br(B^0_d \rightarrow K^*_0(1430)^+K^*_0(1430)^-)}{Br(B^0_d \rightarrow K^*_0(1430)^+K^*_0(1430)^-)}_{S1/S2} = 1.26_{-0.22}^{+0.23}(\omega_B)_{-0.37}(B_m)_{-0.14}(\bar{f}_M)_{-0.02}(V)_{+0.07} \quad (22)
\]

\[
R_{sd}^{S1/S2}(a_0(1450)) = \frac{Br(B^0_d \rightarrow a_0(1450)^+a_0(1450)^-)}{Br(B^0_d \rightarrow a_0(1450)^+a_0(1450)^-)}_{S1/S2} = 1.59_{-0.34}^{+0.36}(\omega_B)_{+0.07}(B_m)_{+0.01}(\bar{f}_M)_{+0.01}(V)_{+0.37} \quad (23)
\]

\[
R_{sd}^{(1430)/\kappa} = \frac{Br(B^0_d \rightarrow K^*_0(1430)^+K^*_0(1430)^-)}{Br(B^0_d \rightarrow \kappa^+\kappa^-)} = 1.08_{-0.06}^{+0.06}(\omega_B)_{+0.16}(B_m)_{+0.17}(\bar{f}_M)_{+0.01}(V)_{+0.28} \quad (24)
\]

\[
R_{sd}^{(1430)/\kappa} = \frac{Br(B^0_d \rightarrow K^*_0(1430)^+K^*_0(1430)^-)}{Br(B^0_d \rightarrow \kappa^+\kappa^-)} = 0.86_{-0.09}^{+0.12}(\omega_B)_{+0.18}(B_m)_{+0.27}(\bar{f}_M)_{+0.02}(V)_{+0.04} \quad (25)
\]

\[
R_{sd}^{(1450)/a_0(980)} = \frac{Br(B^0_d \rightarrow a_0(1450)^+a_0(1450)^-)}{Br(B^0_d \rightarrow a_0(980)^+a_0(980)^-)} = 2.13_{-0.22}^{+0.19}(\omega_B)_{-0.30}(B_m)_{-0.47}(\bar{f}_M)_{-0.00}(V)_{+0.00} \quad (26)
\]
\[ R_{s,S2}^{a_0(1450)/a_0(980)} = \frac{Br(B_0^0 \to a_0(1450)^+a_0(1450)^-)_{S2}}{Br(B_0^0 \to a_0(980)^+a_0(980)^-)_{S2}} = 1.34^{+0.19}_{-0.15}(\omega_B)^{-0.20}_{-0.24}(B_m)^{-0.30}_{-0.28}(f_M)^{-0.01}_{-0.00}(V)[1.34^{+0.41}_{-0.40}]. \]

Of course, it is found that the uncertainties in some of the above ratios are not small, for example, \( R_{d}^{S1/S2}(K_0^*(1430)) = 1.26^{+0.69}_{-0.45} \) in Eq. (22). The underlying reason is that the large uncertainties induced by the Gegenbauer moments \( B_1 \) and \( B_3 \) cannot be cancelled correspondingly, unlike the exact cancellation of the uncertainties resulted from the scalar decay constants that can be isolated from the distribution amplitudes.

(d) Another eight more interesting ratios could be obtained and are expected to be examined at the future experiments, if we take the already measured \( B_0^0 \to K^+K^- \) and \( B_0^0 \to \pi^+\pi^- \) decays as referenced channels. By combing the branching fractions of the \( B_0^0 \to K^+K^- \) and \( B_0^0 \to \pi^+\pi^- \) decays from both of the PQCD predictions [28] and the experimental measurements [24] sides, and the decay rates of the \( B_0^0 \to K_0^*(1430)^+K_0^*(1430)^- \) and \( B_0^0 \to a_0^+a_0^- \) modes in this work, they are read as follows,

\[ R_{d,\text{Exp}}^{K/K} = \frac{Br(B_0^0 \to K^+K^-)_{\text{PQCD}}}{Br(B_0^0 \to K^+K^-)_{\text{Exp}}} = 36.41^{+11.55}_{-10.70}, \]

\[ R_{d,\text{Exp}}^{K^*(1430)/K} = \frac{Br(B_0^0 \to K_0^*(1430)^+K_0^*(1430)^-)_{\text{PQCD}}}{Br(B_0^0 \to K^+K^-)_{\text{Exp}}} = \begin{cases} 39.49^{+1.14}_{-1.15} & (S1) \\ 31.41^{+1.35}_{-1.08} & (S2) \end{cases}, \]

\[ R_{d,\text{Th}}^{K/K} = \frac{Br(B_0^0 \to K^+K^-)_{\text{PQCD}}}{Br(B_0^0 \to K^+K^-)_{\text{PQCD}}} = 18.21^{+2.83}_{-2.63}. \]

\[ R_{d,\text{Th}}^{K^*(1430)/K} = \frac{Br(B_0^0 \to K_0^*(1430)^+K_0^*(1430)^-)_{\text{PQCD}}}{Br(B_0^0 \to K^+K^-)_{\text{PQCD}}} = \begin{cases} 19.74^{+2.62}_{-1.82} & (S1) \\ 15.71^{+7.63}_{-7.63} & (S2) \end{cases}, \]

and

\[ R_{s,\text{Exp}}^{a_0(980)/\pi} = \frac{Br(B_0^0 \to a_0(980)^+a_0(980)^-)_{\text{PQCD}}}{Br(B_0^0 \to \pi^+\pi^-)_{\text{Exp}}} = 37.18^{+13.07}_{-10.85}. \]

\[ R_{s,\text{Exp}}^{a_0(1450)/\pi} = \frac{Br(B_0^0 \to a_0(1450)^+a_0(1450)^-)_{\text{PQCD}}}{Br(B_0^0 \to \pi^+\pi^-)_{\text{Exp}}} = \begin{cases} 80.43^{+42.82}_{-39.43} & (S1) \\ 50.71^{+28.48}_{-28.48} & (S2) \end{cases}, \]

\[ R_{s,\text{Th}}^{a_0(980)/\pi} = \frac{Br(B_0^0 \to a_0(980)^+a_0(980)^-)_{\text{PQCD}}}{Br(B_0^0 \to \pi^+\pi^-)_{\text{PQCD}}} = 51.70^{+2.86}_{-2.54}. \]

\[ R_{s,\text{Th}}^{a_0(1450)/\pi} = \frac{Br(B_0^0 \to a_0(1450)^+a_0(1450)^-)_{\text{PQCD}}}{Br(B_0^0 \to \pi^+\pi^-)_{\text{PQCD}}} = \begin{cases} 110.39^{+23.58}_{-34.75} & (S1) \\ 69.61^{+20.88}_{-27.87} & (S2) \end{cases}. \]

These large values of the above ratios with still large theoretical errors could be easily tested when the related samples are collected with good precision experimentally.

(e) As mentioned in the above, the isospin symmetry breaking effects from the \( u \) and \( d \) quark masses have been considered in the \( B_0^0 \to a_0^+a_0^- \) decays. Therefore, the factorizable annihilation decay amplitudes are not exact zero, which can be seen clearly from the numerical results as shown in the Tables I and II in both scenarios S1 and S2. However, they are still tiny and could be neglected safely. It means that the contributions to the pure annihilation decays considered in this work are absolutely from the nonfactorizable annihilation decay amplitudes. It is noticed that, relative to the \( B_0^0 \to K^+K^- \) and \( B_0^0 \to \pi^+\pi^- \) decays, the antisymmetric QCD behavior of the leading twist distribution amplitude could make the destructive interferences in the pseudoscalar sector become the constructive ones in the scalar sector to the nonfactorizable annihilation diagrams between the Fig. 1(c) with hard gluon radiating from light \( d(s) \) quark and the Fig. 1(d) with hard gluon radiating from heavy anti-\( b \) quark, which eventually result in the large \( CP \)-averaged decay rates of the considered decays, as presented in the Eqs. (16)-(19).
TABLE I. The factorization decay amplitudes (in units of $10^{-3}$ GeV) in S1 of the pure annihilation $B_{s}^{0} \to K_{s}^{0}a_{0}$ and $B_{s}^{0} \to a_{0}a_{0}$ decays in the PQCD approach, where only the central values are quoted for clarifications.

| Modes                  | $A_{\nu,T}(T3A)$ | $A_{\nu,T}(T3G)$ |
|------------------------|-------------------|-------------------|
| $B_{s}^{0} \to K_{s}^{0}(800)^{+}K_{s}^{0}(800)^{-}$ | -0.610 $\pm$ i1.974 | 0.0008 $\pm$ i0.0004 |
| $B_{s}^{0} \to K_{s}^{0}(1430)^{+}K_{s}^{0}(1430)^{-}$ | 1.877 $\pm$ i2.658 | 0.001 $\pm$ i0.003 |
| $B_{s}^{0} \to a_{0}(980)^{+}a_{0}(980)^{-}$ | 6.892 $\pm$ i1.325 | 0.0003 $\pm$ i0.002 |
| $B_{s}^{0} \to a_{0}(1450)^{+}a_{0}(1450)^{-}$ | -1.588 $\pm$ i10.057 | -0.006 $\pm$ i0.002 |

TABLE II. Similar to Table I but in S2 for the $B_{s}^{0} \to K_{s}^{0}(1430)^{+}K_{s}^{0}(1430)^{-}$ and $B_{s}^{0} \to a_{0}(1450)^{+}a_{0}(1450)^{-}$ decays.

| Modes                  | $A_{\nu,T}(T3A)$ | $A_{\nu,T}(T3G)$ |
|------------------------|-------------------|-------------------|
| $B_{s}^{0} \to K_{s}^{0}(1430)^{+}K_{s}^{0}(1430)^{-}$ | -1.940 $\pm$ i0.188 | -0.0005 $\pm$ i0.0006 |
| $B_{s}^{0} \to a_{0}(1450)^{+}a_{0}(1450)^{-}$ | 5.038 $\pm$ i6.778 | -0.002 $\pm$ i0.0006 |

Next, we will discuss the $CP$-violating asymmetries of the $B_{s}^{0} \to K_{0}^{+}K_{0}^{-}$ and $B_{s}^{0} \to a_{0}^{+}a_{0}$ decays in the PQCD approach. Similar to the $CP$ violations discussed in Ref. [39], we will present the direct and the mixing-induced $CP$ violations $A_{\text{dir}}$ and $A_{\text{mix}}$ for the $B_{s}^{0} \to K_{0}^{+}K_{0}^{-}$ decays. While, except for $A_{\text{dir}}$ and $A_{\text{mix}}$, the third $CP$ asymmetry $A_{\Delta\Gamma_{s}}$, should be calculated simultaneously for the $B_{s}^{0} \to a_{0}^{+}a_{0}$ decays because of the nonzero ratio ($\Delta\Gamma/\Gamma$) for the $B_{s}^{0} \to B_{s}^{0}$ mixing, where $\Delta\Gamma$ is the decay width difference of the $B_{s}^{0}$ meson mass eigenstates [51, 52]. Then the numerical results of the $CP$ asymmetries $A_{\text{dir}}$, $A_{\text{mix}}$, even $A_{\Delta\Gamma_{s}}$, in the PQCD approach can be read as follows,

(a) For the $B_{s}^{0} \to K_{0}^{+}K_{0}^{-}$ decays,

$$A_{\text{dir}}(B_{s}^{0} \to K_{0}^{+}K_{0}^{-}) = 15.5 \pm 0.0 (\omega_{B})^{+3.7 (B_{m})^{+0.8 (f_{K_{s}})}^{+0.7 (V)}{[15.5^{+3.5}] \times 10^{-2}},$$

$$A_{\text{mix}}(B_{s}^{0} \to K_{0}^{+}K_{0}^{-}) = -80.4 \pm 0.0 (\omega_{B})^{+3.5 (B_{m})^{+0.0 (f_{K_{s}})}^{+3.8 (V)}{[-80.4^{+5.2}] \times 10^{-2}};$$

and

$$A_{\text{dir}}(B_{s}^{0} \to K_{0}^{+}(1430)^{+}K_{0}^{0}(1430)^{-}) = \begin{cases} -73.9 \pm 0.0 (\omega_{B})^{+3.8 (B_{m})^{+0.0 (f_{K_{s}})}^{+3.8 (V)}{[-73.9^{+5.2}] \times 10^{-2}}; (S1) \\ 21.4 \pm 0.0 (\omega_{B})^{+8.9 (B_{m})^{+0.0 (f_{K_{s}})}^{+8.9 (V)}{[21.4^{+8.9}] \times 10^{-2}}; (S2) \end{cases};$$

$$A_{\text{mix}}(B_{s}^{0} \to K_{0}^{+}(1430)^{+}K_{0}^{0}(1430)^{-}) = \begin{cases} -39.6 \pm 0.0 (\omega_{B})^{+3.8 (B_{m})^{+0.0 (f_{K_{s}})}^{+3.8 (V)}{[-39.6^{+5.2}] \times 10^{-2}}; (S1) \\ 97.5 \pm 0.0 (\omega_{B})^{+3.8 (B_{m})^{+0.0 (f_{K_{s}})}^{+3.8 (V)}{[97.5^{+5.2}] \times 10^{-2}}; (S2) \end{cases};$$

(b) For the $B_{s}^{0} \to a_{0}^{+}a_{0}$ decays,

$$A_{\text{dir}}(B_{s}^{0} \to a_{0}(980)^{+}a_{0}(980)^{-}) = -0.7 \pm 0.0 (\omega_{B})^{+0.5 (B_{m})^{+0.1 (f_{a_{0}})}^{+0.1 (V)}{[-0.7^{+0.5}] \times 10^{-2}},$$

$$A_{\text{mix}}(B_{s}^{0} \to a_{0}(980)^{+}a_{0}(980)^{-}) = 13.6 \pm 0.0 (\omega_{B})^{+0.5 (B_{m})^{+0.0 (f_{a_{0}})}^{+0.5 (V)}{[13.6^{+1.1}] \times 10^{-2}},$$

$$A_{\Delta\Gamma_{s}}(B_{s}^{0} \to a_{0}(980)^{+}a_{0}(980)^{-}) = 99.1 \pm 0.0 (\omega_{B})^{+0.1 (B_{m})^{+0.0 (f_{a_{0}})}^{+0.0 (V)}{[99.1^{+1.1}] \times 10^{-2}};$$

The definitions of $A_{\text{dir}}$, $A_{\text{mix}}$, and $A_{\Delta\Gamma_{s}}$ are as follows: $A_{\text{dir}} = |\lambda_{CP}|^{1/2}, A_{\text{mix}} = 2\text{Im}(\lambda_{CP})^{1/2},$ and $A_{\Delta\Gamma_{s}} = 2\text{Re}(\lambda_{CP})^{1/2},$ respectively, where the CP-violating parameter $\lambda_{CP} \equiv \eta_{f} \frac{V_{ub}^{\ast}V_{cb}}{V_{ub}V_{cb}} \frac{(\bar{f}_{B_{s}}^{\ast}\bar{f}_{f})^{2}}{(\bar{f}_{B_{s}}^{\ast}\bar{f}_{f})^{2}}$ with $\eta_{f}$ being the CP-eigenvalue of the final states.
in which the Gegenbauer moments in the scalar meson distribution amplitudes and the parameters in the CKM matrix elements contribute to the major errors theoretically, as clearly seen from the above Eqs. (36)-(45).

Some comments are in order:

• It is clear to see that these predicted CP violations are generally insensitive to the variation of the scalar decay constant \( f_M = a_0, K_S^0 \). The underlying reason is that the decay amplitudes of the considered decays are nearly proportional to the scalar decay constant, due to the vanishing vector decay constant (See Eqs. (7)-(14) for detail) in the leading twist distribution amplitude [33],

\[
\phi_S(x, \mu) = \frac{3}{\sqrt{6}} x(1-x) \left\{ f_S(\mu) + \tilde{f}_S(\mu) \sum_{m=1}^{\infty} B_m(\mu) C_{3/2}^m(2x-1) \right\},
\]

where \( f_S(\mu) \) and \( \tilde{f}_S(\mu) \), \( B_m(\mu) \), and \( C_{3/2}^m(t) \) are the vector and the scalar decay constants, the Gegenbauer moments, and the asymptotic forms of the twist-3 distribution amplitudes of the scalar \( a_0 \) and \( K_S^0 \) mesons. While, the CP asymmetries of the \( B_d^0 \rightarrow K_S^0(1430)^+K_S^0(1430)^- \) decays are more sensitive to the \( \bar{f}_{K_S^0(1430)} \) than those of the \( B_d^0 \rightarrow a_0(1450)^+a_0(1450)^- \) ones to the \( f_{a_0(1450)} \). The fact is that the isospin symmetry breaking effect from the \( u \) and \( d \) quark masses leads to the tiny and negligible vector decay constant \( f_{a_0(1450)} \), i.e., Eq. (13), and the \( SU(3) \) flavor symmetry breaking effect from the \( u \) and \( s \) quark masses results in the small but non-negligible \( \bar{f}_{K_S^0(1430)} \), i.e., Eq. (11).

• It is easy to find that, apart from the \( A_{dir}(B_ s^0 \rightarrow a_0(980)^+a_0(980)^-)_S1 \) and \( A_{dir}(B_d^0 \rightarrow a_0(1450)^+a_0(1450)^-)_S2 \) with few percent, the rest CP-violating asymmetries for the considered pure annihilation decays of \( B_d^0 \rightarrow K_0^+K_0^- \) and \( B_s^0 \rightarrow a_0^+a_0^- \) in the PQCD approach are large, which means that the contributions from the penguin diagrams are generally sizable. To see this point explicitly, we present the decay amplitudes classified as the tree diagrams and the penguin ones, respectively, in the Tables III-IV, where only the central values are quoted for clarifications. Then it is expected that these predictions of the CP violations, associated with the predicted large decay rates, could be confronted with the relevant experiments at LHCb and/or Belle-II in the (near) future. Of course, the \( A_{dir}(B_d^0 \rightarrow a_0(980)^+a_0(980)^-)_{S1} \) and \( A_{dir}(B_d^0 \rightarrow a_0(1450)^+a_0(1450)^-)_{S2} \) are too small to be measured easily in the near future, though the corresponding branching ratios are as large as \( 10^{-5} \).

At last, we shall discuss the effects arising from the twist-3 distribution amplitudes of the scalar \( a_0(1450) \) and \( K_S^0(1430) \) mesons by including the Gegenbauer polynomials in S2, as mentioned in the above footnote 2. It is noted that the twist-3 distribution amplitudes with Gegenbauer polynomials of the scalar mesons in S2 have been investigated in Ref. [48],

\[
\phi_{K_S^0(1430)}(x) = \frac{\bar{f}_{K_S^0(1430)}}{2\sqrt{N_c} x} \left\{ 1 + a_1(2x-1) + \frac{1}{2} b_2 \left( 3(2x-1)^2 - 1 \right) \right\}, \quad \phi_{K_S^0(1430)}(x) = \frac{\bar{f}_{K_S^0(1430)}}{2\sqrt{N_c} x} \left\{ x(1-x) \left[ 1 + \frac{3}{2} b_2 \left( 5(2x-1)^2 - 1 \right) \right] \right\},
\]

with the Gegenbauer moments

\[
a_1 = 0.030 \pm 0.012, \quad a_2 = -0.18 \pm 0.15, \quad b_1 = 0.046 \pm 0.009, \quad b_2 = 0.075 \pm 0.075.
\]
TABLE III. The tree and penguin decay amplitudes (in units of $10^{-3}$ GeV$^3$) in S1 of the pure annihilation $B_d^0 \to K^*_0(1430)^+ + K^*_0(1430)^-$ and $B_s^0 \to a_0^+(1450)^- + a_0^-(1450)^-$ decays in the PQCD approach, where the results in the parentheses are the corresponding amplitudes of the $B_d^0 \to K^*_0(1430)^+ + K^*_0(1430)^-$ and $B_s^0 \to a_0^+(1450)^- + a_0^-(1450)^-$ decays, and only the central values are considered for clarifications.

| Modes | Tree diagrams(T3A) | Penguin diagrams(T3A) |
|-------|---------------------|-----------------------|
| $B_d^0 \to K^*_0(800)^+ + K^*_0(800)^-$ | $0.551 - i 1.920$ | $-1.160 - i 0.054$ |
| | $(1.615 + i 1.76)$ | $(-0.779 + i 0.861)$ |
| $B_d^0 \to K^*_0(1430)^+ + K^*_0(1430)^-$ | $1.633 - i 1.433$ | $0.245 - i 1.222$ |
| | $(-2.169 + i 0.126)$ | $(1.040 - i 0.687)$ |
| $B_s^0 \to a_0(980)^+ + a_0(980)^-$ | $-0.013 - i 0.504$ | $6.906 + i 1.827$ |
| | $(-0.300 + i 0.406)$ | $(6.906 + i 1.827)$ |
| $B_s^0 \to a_0(1450)^+ + a_0(1450)^-$ | $0.738 - i 0.619$ | $-2.332 + i 10.678$ |
| | $(-0.963 + i 0.035)$ | $(-2.332 + i 10.678)$ |

TABLE IV. Similar to Table III but in S2 for the $B_d^0 \to K^*_0(1430)^+ + K^*_0(1430)^-$ and $B_s^0 \to a_0(1450)^+ + a_0(1450)^-$ decays.

| Modes | Tree diagrams(T3A) | Penguin diagrams(T3A) | Tree diagrams(T3G) | Penguin diagrams(T3G) |
|-------|---------------------|-----------------------|-------------------|-----------------------|
| $B_d^0 \to K^*_0(1430)^+ + K^*_0(1430)^-$ | $-1.081 - i 1.178$ | $-0.859 + i 1.365$ | $-0.566 + i 0.658$ | $-1.079 + i 1.675$ |
| | $(0.129 + i 1.593)$ | $(-1.574 + 0.352)$ | $(0.850 - i 0.171)$ | $(-1.949 + 0.415)$ |
| $B_s^0 \to a_0(1450)^+ + a_0(1450)^-$ | $-0.328 - i 0.048$ | $5.364 - i 6.730$ | $-0.097 + 0.188$ | $4.954 - i 6.998$ |
| | $(0.229 + i 0.239)$ | $(5.364 - i 6.730)$ | $(0.192 - i 0.088)$ | $(4.954 - i 6.998)$ |

at $\mu = 1$ GeV for the $K^*_0(1430)$ state, and

$$\phi^S_{a_0(1450)}(x) = \frac{f_{a_0(1450)}}{2 \sqrt{2} N_c} \left\{ 1 + \frac{1}{2} a_2 \left( 3(2x - 1)^2 - 1 \right) + \frac{1}{8} a_4 \left( 35(2x - 1)^4 - 30(2x - 1)^2 + 3 \right) \right\},$$

(50)

$$\phi^T_{a_0(1450)}(x) = \frac{\tilde{f}_{a_0(1450)}}{2 \sqrt{2} N_c} \frac{d}{dx} \left\{ x(1 - x) \left[ 1 + \frac{3}{2} b_2 \left( 5(2x - 1)^2 - 1 \right) + \frac{15}{8} b_4 \left( 21(2x - 1)^4 - 14(2x - 1)^2 + 1 \right) \right] \right\},$$

(51)

with the Gegenbauer moments

$$a_2 = -0.255 \pm 0.075 \ , \quad a_4 = 0.14 \pm 0.25 \ , \quad b_2 = 0.029 \pm 0.029 \ , \quad b_4 = 0.135 \pm 0.065 \ ,$$

(52)

at $\mu = 1$ GeV for the $a_0(1450)$ state, respectively. Then, by including the contributions from these twist-3 distribution amplitudes, the numerical results for the branching ratios and the CP-violating asymmetries in S2 could be obtained as follows,

$$Br(B_d^0 \to K^*_0(1430)^+ + K^*_0(1430)^-) = 2.38 \times 10^{-5},$$

(53)

$$Br(B_s^0 \to a_0(1450)^+ + a_0(1450)^-) = 3.60 \times 10^{-5},$$

(54)

in which the central value of the branching fraction $Br(B_d^0 \to K^*_0(1430)^+ + K^*_0(1430)^-)_{S2}$ becomes slightly smaller/larger than that obtained in the case with adopting the asymptotic form of the twist-3 distribution amplitudes correspondingly. Of course, the CP-averaged branching ratios almost remain unchanged within the still large theoretical errors.

$$A_{\text{dir}}(B_d^0 \to K^*_0(1430)^+ + K^*_0(1430)^-) = -73.1^{-11.0}_{+8.1}(\omega_B)_{-13.3}(B_m)^{+52.9}_{-11.2}(ab)^{+0.0}_{-0.3}(\bar{f}_{K^*_0})^{+2.0}_{-2.1}(V) [-73.1^{+10.4}_{-19.2} \times 10^{-2}],$$

(55)

$$A_{\text{mix}}(B_d^0 \to K^*_0(1430)^+ + K^*_0(1430)^-) = -3.5^{-0.7}_{+0.6}(\omega_B)_{-50.9}(B_m)^{+57.2}_{-95.1}(ab)^{+0.3}_{-0.0}(\bar{f}_{K^*_0})^{+1.4}_{-1.3}(V) [-3.5^{+113.8}_{-116.5} \times 10^{-2}],$$

(56)

It is necessary to mention that the Gegenbauer moments for the twist-3 distribution amplitudes of $K^*_0(1430)$ and $a_0(1450)$ states are originally presented as [48] (1) $a_1 = 0.018 \sim 0.042$, $a_2 = -0.33 \sim -0.025$, $b_1 = 0.037 \sim 0.055$, and $b_2 = 0 \sim 0.15$ at $\mu = 1$ GeV for the $K^*_0(1430)$ state, and (2) $a_2 = -0.33 \sim -0.18$, $a_4 = -0.1 \sim 0.39$, $b_2 = 0 \sim 0.058$, and $b_4 = 0.070 \sim 0.20$ for the $a_0(1450)$ state, respectively. To give the numerical results as central values with uncertainties, we adopt the form of the Gegenbauer moments in the twist-3 distribution amplitudes as presented in the Eqs. (49) and (52) for convenience.
\[
    A_{\text{dir}}(B_s^0 \to a_0(1450)^+a_0(1450)^-) = 4.6^{+0.4}_{-0.2}(\omega_B)^{+2.1}_{-1.3}(B_m)^{+1.1}_{-0.3}(ab)^{+0.0}_{-0.5}(f_{a_0})^{+0.1}_{-0.2}(V)[4.6^{+2.6}_{-1.7}] \times 10^{-2},
\]
\[
    A_{\text{mix}}(B_s^0 \to a_0(1450)^+a_0(1450)^-) = 8.6^{+0.7}_{-0.3}(\omega_B)^{+23.3}_{-26.4}(B_m)^{+28.3}_{-26.6}(ab)^{+0.1}_{-0.0}(f_{a_0})^{+0.3}_{-0.2}(V)[8.6^{+36.7}_{-37.9}] \times 10^{-3},
\]
\[
    A_{\Delta \Gamma}(B_s^0 \to a_0(1450)^+a_0(1450)^-) = 99.9^{+0.0}_{-0.0}(\omega_B)^{+0.1}_{-0.1}(B_m)^{+0.0}_{-0.1}(ab)^{+0.0}_{-0.0}(f_{a_0})^{+0.0}_{-0.0}(V)[99.9^{+0.1}_{-0.1}] \times 10^{-2},
\]

where the direct \( CP \)-violating asymmetry and the mixing-induced one are changed significantly for the \( B_d^0 \to K_s^0(1430)^+K_s^0(1430)^- \) and \( B_s^0 \to a_0(1450)^+a_0(1450)^- \) decays in S2. Note that the parameter \((ab)\) in the above Eqs. (55)-(59) denotes the combined Gegenbauer moment of \( a_i \) and \( b_i \) in the twist-3 distribution amplitudes with Gegenbauer polynomials. In terms of the central value, the \( A_{\text{dir}} \) varies from 21.4\% in the T3A form to -73.1\% in the T3G form for the former mode, while from 1.4\% in the T3A form to 4.6\% in the T3G form for the latter one; and the \( A_{\text{mix}} \) changes from -97.5\% to -3.5\% for the former channel, while from 7.2\% to 0.86\% for the latter one, which indicate that the strong phases in these two pure annihilation decays could be dramatically affected by the twist-3 distribution amplitudes with inclusion of the Gegenbauer polynomials. In order to show the significant changes of the mentioned strong phases, we present the decay amplitudes of these two pure annihilation modes, i.e., \( B_d^0 \to K_s^0(1430)^+K_s^0(1430)^- \) and \( B_s^0 \to a_0(1450)^+a_0(1450)^- \), in S2 explicitly by considering the twist-3 distribution amplitudes in the T3A and T3G forms respectively, as given in the Tables II and IV. It is clearly seen that the magnitudes of the dominant contributions from the nonfactorizable annihilation diagrams vary slightly, but the strong phases in both of the factorizable and nonfactorizable annihilation diagrams change remarkably, which means that the considered T3G contributions are also important to the strong phases in the annihilation diagrams of this work. Therefore, the clear understanding of the hadron dynamics about the scalar mesons considered in this work could be very helpful to provide precise predictions theoretically, even to explore the presently unknown annihilation decay mechanism.

In short, it is well known that the annihilation diagrams, although which are power suppressed and with currently unknown mechanism, could play important roles in the heavy flavor \( B \) and \( D \) meson decays. As one of the popular tools theoretically, the PQCD approach has the advantages in computing the annihilation contributions. Therefore, by assuming the scalar \( a_0 \) and \( K_0^* \) being the \( q\bar{q} \) mesons in two different scenarios, we have investigated the \( CP \)-averaged branching ratios and the \( CP \)-violating asymmetries for the pure annihilation decays of \( B_d^0 \to K^+K^- \) and \( B_s^0 \to a_0^+a_0^- \) in the PQCD approach, which embrace the same quark structure as \( B_d^0 \to K^+K^- \) and \( B_s^0 \to \pi^+\pi^- \) in the pseudoscalar sector. At the same time, for the \( B_d^0 \to K_s^0(1430)^+K_s^0(1430)^- \) and \( B_s^0 \to a_0(1450)^+a_0(1450)^- \) decays, we also studied the contributions from the twist-3 distribution amplitudes in the Gegenbauer polynomial forms of the \( K_0^*(1430) \) and \( a_0(1450) \) in S2. Based on the numerical results within still large theoretical errors and the phenomenological analyses, the predictions in the PQCD approach showed that:

- the large decay rates in the order of \( 10^{-6} \) and \( 10^{-5} \) have been obtained in the PQCD calculations for the pure annihilation \( B_d^0 \to K_s^0(1430)^+K_s^0(1430)^- \) and \( B_s^0 \to a_0^+a_0^- \) decays, respectively, but which suffer from large theoretical uncertainties mainly arising from the hadronic parameters of the scalar \( a_0 \) and \( K_0^* \) mesons, such as the Gegenbauer moments, the scalar decay constants, etc. Undoubtedly, these large predictions in the PQCD approach could be examined at the LHCb and/or Belle-II experiments in the (near) future.

- except for the \( A_{\text{dir}}(B_s^0 \to a_0(980)^+a_0(980)^-)_{S1} \) and \( A_{\text{dir}}(B_s^0 \to a_0(1450)^+a_0(1450)^-)_{S2} \) in the order of 1\%, the large \( CP \)-violating asymmetries could be found in the rest \( B_d^0 \to K_s^0(1430)^+K_s^0(1430)^- \) and \( B_s^0 \to a_0(1450)^+a_0(1450)^- \) modes. These asymmetries could also be tested in the future measurements with the help of the predicted large branching ratios.

- the \( B_s^0 \to K_s^0(1430)^+K_s^0(1430)^- \) and \( B_s^0 \to a_0(1450)^+a_0(1450)^- \) decays are investigated in S2 with two different kinds of twist-3 distribution amplitudes, i.e., the asymptotic form and the Gegenbauer polynomial form. And the latter form could change the strong phases evidently, and subsequently change the \( CP \)-violating asymmetries apparently.

- the unknown inner structure or QCD dynamics of the scalar states demands the reliable studies from the experimental examinations, or in the nonperturbative techniques, for example, Lattice QCD. The clear understanding about the QCD dynamics of the scalars must constrain the errors of the predictions in the PQCD approach of the pure annihilation modes in this work, which would provide opportunities to shed light on the mechanism of the annihilation decays.

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