In the present article, the authors intend to propose a new theory which potentially allows the propagation of the formation and the evolution of quarkonium in a thermal BIon. When quarks are close to each other, quarkonium behaves like a scalar and by their getting away, it transits to a fermionic system. In order to analyze this particular behaviour, a new outlook approach needs to be adopted as the concurrent view is found deficient to analyse the aforesaid behaviour. Therefore, the authors’ post deliberation accept the fermions and fermionic being cognate. We need to accept a theory that the origin of fermions and bosons be the same. However, in $M$-theory, these particles are independent and for this reason, we use a new broader theory based on Lie-$N$-Algebra and we call it BLNA (Broad Lie-$N$-Algebra) theory. Thus, the BLNA in a way the $M$-theory with 11 dimensions. In this model, two types of energies with opposite signs emerge from nothing such as the sum over them becomes zero. They produce two types of branes with opposite quantum numbers and bosonic fields, which interact with each other and get compact. By compacting branes, the quarks and anti-quarks are produced on branes and exchange the graviton and the gravitino. These particles produce two types of wormholes which act opposite to each other. They preclude from closing or getting away of branes from each other and also occurrence of confinement. This confined potential which emerges from these wormholes depends on the separation distance between quarks and anti-quarks and also on temperature of system and is reduced to predicted potential in experiments and QCD. Also, total entropy of this system grows with increasing temperature and produces a repulsive force which leads to the separation of quarks and anti-quarks and also to the emergence of deconfinement.

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I. INTRODUCTION

One of the main puzzles in QCD is the reason for occurring confinement between quarks and anti-quarks which prevents quarks from going much away from each other or coming very close to each other. Although, a transition from hadronic matter to a plasma of deconfined quarks and gluons for high temperature can be observed \[1\]. Many scientists have tried to resolve this problem. For example, in one investigation, authors have obtained the quark-antiquark potential in \(d\)-dimensions by using the explicit solution of \(d+1\)-dimensional dilatonic gravity. Their results were consistent with the predicted potential following IIB supergravity and Ads/CFT \[2\]. In another research, authors have argued that quark-anti-quark potential depends on the curvature in IIB super-gravity background with non-trivial dilaton and with curved four-dimensional space. This potential has shown the appearance of the confinement for the geometry type of hyperbolic (or de Sitter universe) and proposed the standard area law for large separations \[3\]. In another paper, it has been discussed that the entropy of heavy quarkonium in strongly coupled quark-gluon plasma, grows with the inter-quark distance \(r\) and thus, the entropic force \(F = T \frac{\partial S}{\partial r}\) (\(T\) is temperature) results in an anomalously strong quarkonium suppression in high temperatures. This means that for high temperature, the entropic force leads to delocalization of the bound hadron states; and this delocalization can be the mechanism indicating deconfinement \[4\]. Some other authors have discussed that color confinement leads to formation of an event horizon for quarks and gluons which may be crossed only by quantum tunneling similar to black holes. The radiation of this horizon is thermal; and its temperature is calculated by the chromodynamic theory which be in accordance with experiments, however no substantial progress could be made in the direction, owing to the limitations of the concurrent propounded theories. In our proposed theory, all parameters of QCD can be the mechanism indicating deconfinement \[4\].

To consider quarkonium, we need a Bion whose branes play the role of quarks and can’t go very close or distant from anti-branes which plays the role of anti-quarks. By closing of branes to each other, system behaves like scalars and by getting away, it transits to two separated fermions. To construct this system, it is needed that the origin of fermions and bosons be the same; and they can be changed to each other. In \(M\)-theory, fermions are completely independent of bosons and for this reason, we propose the new BLNA theory where the number of its degrees of freedom and dimensions is more than \(M\)-theory and is reduced to 11 dimensions. In this model, first, there is nothing with zero energy and degrees of freedom. Then, two energies, one with positive sign and other with negative sign are produced and each of them has its own degrees of freedom. These degrees of freedoms produce two branes with opposite quantum numbers which are known as branes and anti-branes. On these branes, only bosons like scalars and gravitons live which interact with fields of other branes and lead to the compacting of branes. The compacting of branes produces fermions like quarks, antiquarks and gravitino. Quarks are placed on branes and exchange gravitons and gravitinos with anti-quarks that are located on anti-branes. Gravitons produce one bosonic wormhole subject to distance between quarks and antiquarks, leads to the emergence of attractive force and confinement between these particles and prevents them getting away from each other. Gravitino creates the fermionic wormhole which causes the production of repulsive force in small separation distance between quarks and anti-quarks and prevents the proximity to each other. These results are in agreement with predictions of QCD that quarks and anti-quarks can’t go much away or much close to each other. By increasing temperature, the entropy of bosonic wormhole increases and one of fermionic wormhole decreases and one repulsive force emerges which leads to deconfinement.

According to Nojiri and Odintsov \[6, 10\], there is some ground to believe that gravitational alternative to dark energy (which may be called the effective dark energy) may be produced by the modification of General Relativity which is dictated by string/\(M\)-theory. Modified gravity presents very natural modification of the early-time inflation and late time acceleration. The authors \[9, 10\] concluded that such a model which seems to eliminate the need of dark energy may have the origin in \(M\)-theory. In 11-dimensional \(M\)-theory and 10-dimensional string theory, there are many puzzles and questions remain unanswered. The limitations of the aforesaid framework create an extensive scope for researchers to work for rather more encompassing theory. For example, in \(M\)-theory, only there are two stable objects, \(M2\) and \(M5\). While for constructing four-dimensional universe, we need to \(M3\)-brane which is unstable in it. On the other hand, for both \(M2\) and \(M5\)-branes, a lie-3-algebra has been suggested (see for example, \[11, 12\]). Extending this algebra to \(N\)-dimensions and dimensions from 11 to \(M\), we search for a bigger theory that solves the puzzles in \(M\)-theory. Additionally, the origin of potentials and confinement in QCD is also unclear. Specially, strong coupling constant is a free parameter and its value can be determined through experiments. Attempts have been made by various scientists to locate its origin in the string theory and also to obtain the value of this constant in this theory which be in accordance with experiments, however no substantial progress could be made in the direction, owing to the limitations of the concurrent propounded theories. In our proposed theory, all parameters of QCD can
be determined without adding any value manually. Also, our theory explains the origin of confinement.

In section II of the article, the process of formation of BIon in BLNA-theory and construct quarkonium on it is considered, while in section III the relation between potential and entropy of quarkonium with temperature in BIon is studied. The last section is devoted to the summary and conclusion.

Units throughout the paper are: \( \hbar = c = 8\pi G = 1 \).

II. EMERGENCE OF QUARKONIUM IN BIONIC SYSTEM

In this section, the process of formation of a BIon in Lie-N-algebra is considered in the beginning itself. We assume that there is a null at first. Then, two positive and negative energies emerge such that the sum over them becomes zero. After that, these energies get excited and produce a brane and an anti-brane. Bosons which live on these branes, interact with those on anti-branes and produce a bosonic wormhole between two branes. Also, fermions which are placed on these branes create a fermionic wormhole. This system behaves like a BIon with two separate wormholes which may act reverse to each other. We will construct the quarkonium in this system and extract the predicted potential in QCD.

To construct BIonic system in Lie-N-algebra, first, we should extend the usual actions in string theory and M-theory to a world with \( M \) dimensions and Lie-N-algebra. Previously, it has been shown that all Dp-branes in string theories are constructed from D0-branes \([7, 8, 13–20]\). Also, all Mp-branes in M-theory are built from M0-branes \([22–25]\). The difference between D0-branes and M0-branes is in the dimensions of their actions and their algebra. The action of M0-branes contains three dimensional brackets which obey rules of Lie-three-algebra, while the action of D0-branes has two dimensional brackets which obey that of Li-two-algebra. In these theories, by joining M0/D0-branes and formation of higher dimensional branes, gauge fields emerge and by compacting branes-anti-symmetrically, fermions emerge. Now, we extend these theories by increasing dimensions of brackets in action of M0 and D0 to \( N \) and using of Lie-N-algebra. We name this new theory as BLNA-theory. In BLNA-theory, we can show that the origin of bosons and fermions are the same and for this reason, it can be applied for considering the behaviour of some systems like quarkonium. In addition to this, BLNA-theory can be reduced to M-theory, only by putting \( N = 3 \) and to string theory, by putting \( N = 2 \).

First, we will show that the action of Dp-branes can be constructed by multiplying the action of D0-branes. Then, we generalize this mechanism to eleven dimensional M-theory and calculate the action of Mp-branes by multiplying the action of M0-branes. The action for D1-brane is \([7, 8, 13–20]\):

\[
S = -T_{D1} \int d^2 \sigma \ STr \left( -\det(P_{ab}[E_{mn}E_{mi}(Q^{-1} + \delta)^{ij}E_{jn}] + \lambda F_{ab})\det(Q^j_i)^{1/2} \right),
\]

where

\[
E_{mn} = G_{mn} + B_{mn}, \quad Q^j_i = \delta^j_i + i\lambda[X^j, X^k]E_{kj}
\]

where \( \lambda = 2\pi l_s^2 \), \( G_{ab} = \eta_{ab} + \partial_a X^i \partial_b X^i \) and \( X^i \) are attached scalar strings to branes. In this relation, \( a, b = 0, 1, ..., p \) refer to the world-volume indices of the Dp-branes, \( i, j, k = p + 1, ..., 9 \) denote indices of the transverse space, and \( m, n \) are corresponded to ten-dimensional spacetime indices. Also, \( T_{Dp} = \frac{1}{g_s(2\pi)^{p+1}} \) denotes the tension of Dp-brane, \( l_s \) refers to the string length and \( g_s \) is the string coupling. To obtain the action for Dp-brane, we should use the below relations \([7, 8, 13–20]\):
\[ \Sigma^p_{\alpha=0} \Sigma^9_{\beta=0} \rightarrow \frac{1}{(2\pi l_s)^p} \int d^{p+1}\sigma \Sigma^9_{m=p+1} \Sigma^p_{\alpha=0} \quad \lambda = 2\pi l_s^2 \]

\[ i, j = p+1, \ldots, 9 \quad a, b = 0, 1, \ldots, p \quad m, n = 0, 1, \ldots, 9 \]

\[ i, j \rightarrow a, b \Rightarrow \left[ X^a, X^i \right] = i\lambda \partial_a X^i \quad \left[ X^a, X^b \right] = \frac{i\lambda F^{ab}}{2} \]

\[ \frac{1}{Q} \rightarrow \frac{1}{Q} (\partial_a X^i \partial_b X^i + \frac{\lambda^2}{4} (F^{ab})^2)^n \]

\[ \det(Q^i_j) \rightarrow \det(Q^i_j) \prod_{n=1}^{p} \det(\partial_{a_n} X^i \partial_{b_n} X^i + \frac{\lambda^2}{4} (F^{a_n b_n})^2) \] (3)

Applying above relations in action given by (1), we obtain the action for \( D_p \)-brane \([7, 8, 13–20]\):

\[ S = -\frac{T_{D_p}}{2} \int d^{p+1}\sigma \beta_n \beta^\nu \chi_{\mu_0 \mu_1 \ldots \mu_n} \]

where

\[ \chi_{\nu}^\mu = \sqrt{g^{\mu \rho} \partial_{\rho} X^i \partial_{\nu} X^j \eta_{ij} + \frac{\lambda^2}{4} (F^{ij})^2} \] (5)

and \( X^a \)'s refer to scalars, \( \mu, \nu = 0, 1, \ldots, p \) denote to the world-volume indices of the \( M \)-branes, \( i, j, k = p+1, \ldots, 9 \) refer to indices of the transverse space and \( \beta \) is a constant. Also, \( T_{D_p} = \frac{1}{g_s (2\pi)^{p+1}} \) denotes the tension of \( D_p \)-brane, \( l_s \) refers to the string length and \( g_s \) is the string coupling. Now, we can assert that this action can be built by multiplying over the actions for \( D_0 \)-branes. Applying the mechanism in ref \([8]\), we can obtain the below relations \([7, 8, 13–20]\):

\[ \left[ X^a, X^i \right] = i\lambda \partial_a X^i \quad \left[ X^a, X^b \right] = \frac{i\lambda F^{ab}}{2} \]

\[ \Sigma^9_{m=0} \rightarrow \Sigma^9_{a,b=0} \Sigma^9_{j=p+1} \] (6)

Using (5) in (6), we obtain:

\[ S_{D_p} = -(T_{D_0})^p \int dt \sum_{n=1}^{p} \beta_n \left( \sigma^{a_1 a_2 \ldots a_n} \epsilon_{b_1 b_2 \ldots b_n} L_{a_1} \ldots L_{a_n} \right)^{1/2} \]

\[ (L)^{b}_a = Tr \left( \Sigma^9_{a,b=0} \Sigma^9_{j=p+1} \left( [X^b, X^j][X_a, X_j] + [X^b, X^j][X_a, X_b] + [X^b, X^j][X_i, X_j]\delta^{b}_{a} \right) \right) , \] (7)

where we have defined the antisymmetric properties for \( \delta \) and used of the action of \( D_0 \)-brane \([7, 8, 13–20]\):

\[ S_{D_0} = -T_{D_0} \int dt Tr (\Sigma^9_{m=0} [X^m, X^n]^2) \] (8)

Equation (7) indicates that each \( D_p \)-brane can be built from linking \( p \) \( D_0 \)-branes. This mechanism can be applied in \( M \)-theory, and each \( M \)-brane can be constructed by joining \( M_0 \)-branes. By substituting three dimensional Nambu-Poisson bracket for \( M \)-branes instead of two one in action and using the \( L_i \)-algebra \([22, 23]\), we can obtain the relevant action for \( M_0 \)-brane \([7, 8, 13–20, 22, 23]\):

\[ S_{M_0} = T_{M_0} \int dt Tr (\Sigma^9_{M,N,L=0} (\left[ X^M, X^N, X^L \right], \left[ X^M, X^N, X^L \right]) ) , \] (9)

where \( X^M = X_{\alpha}^M T^\alpha \) and
by applying the following mappings \[8, 13–20, 22–25\]:

\[ [T^{\alpha}, T^{\beta}, T^{\gamma}] = f_{\gamma}^{\alpha \beta} T^{\eta} \]
\[ (T^{\alpha}, T^{\beta}) = h^{\alpha \beta} \]
\[ [X^{M}, X^{N}, X^{L}] = [X_{a}^{M} T^{\alpha}, X_{b}^{N} T^{\beta}, X_{c}^{L} T^{\gamma}] \]
\[ (X^{M}, X^{M}) = X_{a}^{M} X_{b}^{M} (T^{\alpha}, T^{\beta}) \]

(10)

where \( X^{M} (i = 1, 3, \ldots 10) \) refer to scalars which are attached to M0-brane. Replacing the action of \( D0 \) by \( M0 \) in the action (7), we get \[7, 8, 13–20, 22–25\]:

\[ S_{M0} = -(T_{M0})^{\rho} \int dt \sum_{n=1}^{p} \delta_{a_{1}}^{a_{2}} a_{n} L_{a_{1}}^{b_{1}} \ldots L_{a_{n}}^{b_{n}} \]
\[ (L)^{\rho}_{b} = Tr \left( \sum_{n=1}^{p} \Sigma_{j=0}^{n^{10}} [X^{a}, X^{i}, X^{j}] + [X^{a}, X^{c}, X^{j}] + [X^{b}, X_{c}, X_{j}] + [X^{k}, X^{i}, X^{j}] + [X^{k}, X_{c}, X_{j}] + [X^{k}, X_{c}, X_{j}] \right) \]

(11)

Following the mechanism for \( Dp \)-branes in string theory, different \( Mp \)-branes can be constructed from \( M0 \)-brane by applying the following mappings \[8, 13, 20, 22, 25\]:

\[ \langle [X^{a}, X^{b}, X^{i}], [X^{a}, X^{b}, X^{i}] \rangle = \frac{1}{2} (\partial_{b} \partial_{a} X^{i}, \partial_{b} \partial_{a} X^{i}) \]
\[ \langle [X^{i}, X^{b}, X^{i}], [X^{i}, X^{b}, X^{i}] \rangle = \frac{1}{2} \sum_{j} (X^{i})^{2} (\partial_{b} X^{i}, \partial_{b} X^{i}) \]
\[ \langle [X^{a}, X^{b}, X^{c}], [X^{a}, X^{b}, X^{c}] \rangle = \frac{\lambda^{2}}{6} (F_{abc}^{r}_{\alpha \beta \gamma}) (F_{abc}^{r}_{\alpha \beta \gamma}) ([T^{\alpha}, T^{\beta}, T^{\gamma}], [T^{\alpha}, T^{\beta}, T^{\gamma}]) = \frac{\lambda^{2}}{6} (F_{abc}^{r}_{\alpha \beta \gamma}) (F_{abc}^{r}_{\alpha \beta \gamma}) \delta^{r \sigma} (T^{\gamma}, T^{\gamma}) = \frac{\lambda^{2}}{6} (F_{abc}^{r}, F_{abc}^{r}) \]
\[ \langle [X^{i}, X^{b}, X^{c}], [X^{i}, X^{b}, X^{c}] \rangle = \frac{\lambda^{2}}{4} \sum_{j} (X^{i})^{2} (F_{abc}^{r}, F_{abc}^{r}) \]

\[ \Sigma_{m} \rightarrow \frac{1}{(2\pi)^{p}} \int d^{p+1} \sigma \Sigma_{m-p-1}^{i, j = p+1, \ldots 10} a, b = 0, 1, \ldots p \quad m, n = 0, \ldots 10 \]

(12)

where

\[ F_{abc} = \partial_{a} A_{bc} - \partial_{b} A_{ca} + \partial_{c} A_{ab} \]

(13)

and \( A_{ab} \) is 2-form gauge field. Using the mappings of Eq. (12) in action (11), we can obtain the relevant action for \( Mp \)-brane

\[ S_{Mp} = -(T_{M0})^{\rho} \int dt \sum_{n=1}^{p} \delta_{a_{1}}^{a_{2}} a_{n} L_{a_{1}}^{b_{1}} \ldots L_{a_{n}}^{b_{n}} \]
\[ (L)^{\rho}_{b} = \delta_{b}^{a} Tr \left( \sum_{n=1}^{p} \Sigma_{i=0}^{n^{10}} [X^{a}, X^{i}], [X^{a}, X^{i}] + [X^{a}, X^{c}, X^{i}] + [X^{b}, X_{c}, X_{i}] + [X^{k}, X^{i}, X^{i}] + [X^{k}, X_{c}, X_{i}] + [X^{k}, X_{c}, X_{i}] \right) \]
\[ \frac{\lambda^{2}}{6} (F_{abc}^{r}, F_{abc}^{r}) + \frac{\lambda^{2}}{4} \sum_{j} (X^{i})^{2} (F_{abc}^{r}, F_{abc}^{r}) - \frac{1}{4} \left[ X^{i}, X^{i}, X^{i}, X^{i}, X^{i}, X^{i} \right] \]

(14)

This action is in good agreement with previous predictions of M-theory \[3, 8, 13, 20, 22, 25\]. In this theory, rank of fields change from zero to 2, which rank zero is related to scalar \( (X) \), rank one is the vector \( (A^{a}) \) and rank two is corresponded to tensor fields \( (A^{ab}) \) like gravitons.
Now, by replacing the brackets in actions of $S$ and $G$ by $N$-dimensional brackets and extending dimensions to $M$, we define the action of $G_0$-brane as:

$$S_{G0} = T_{G0} \int dt \operatorname{Tr} \left( \sum_{L=1}^{M} \left[ X^{L_1}, X^{L_2}, \ldots, X^{L_N} \right], [X^{L_1}, X^{L_2}, \ldots, X^{L_N}] \right),$$

(15)

where $X^M = X^M \alpha T^\alpha$ and

$$\left[ T^{\alpha_1}, T^{\alpha_2}, \ldots, T^{\alpha_N} \right] = f^{\alpha_1 \ldots \alpha_N}_{\alpha L} T^L$$

$$\left( T^\alpha, T^\beta \right) = h^{\alpha \beta}$$

$$\left[ X^{L_1}, X^{L_2}, \ldots, X^{L_N} \right] = \left[ X^{L_1} T^{\alpha_1}, X^{L_2} T^{\alpha_2}, \ldots, X^{L_N} T^{\alpha_N} \right]$$

$$\left( X^M, X^M \right) = X^a \alpha X^j \beta \left( T^\alpha, T^\beta \right)$$

(16)

This action can be reduced to the action of $M$-branes by putting $N = 3$ and $M = 10$ and to the action of $D0$-brane for $N = 2$ and $M = 9$. By replacing three dimensional brackets with $N$-dimensional brackets and increasing dimensions from 11 to $M$ in action (11), we can calculate the action of $G_0$-brane:

$$S_{G0} = -(T_{G0})^p \int dt \sum_{n=1}^{p} \delta^{a_1 \ldots a_n}_{b_1 \ldots b_n} \left( \delta_{b_1 b_2 \ldots b_n} L_{a_1} b_{a_1} \ldots L_{a_n} b_{a_n} \right)^{1/2}$$

$$\left( L \right)^{a_n}_{b_n} = \delta^{a_n}_{b_n} \operatorname{Tr} \left( \sum_{L=0}^{N} \sum_{H=0}^{L} \sum_{a_1 \ldots a_L} \sum_{a_1 \ldots a_L} \left( \left[ X^{j_1}, \ldots, X^{j_H-1}, X^{a_1}, \ldots, X^{a_L}, X^{j_H} \right], \left[ X^{j_1}, \ldots, X^{j_H-1}, X^{a_1}, \ldots, X^{a_L}, X^{j_H} \right] \right) + \right.$$

$$\sum_{L=0}^{M} \sum_{H=0}^{L} \sum_{a_1 \ldots a_L} \sum_{a_1 \ldots a_L} \left( \left[ X^{j_1}, \ldots, X^{j_H}, X^{a_1}, \ldots, X^{a_L} \right], \left[ X^{j_1}, \ldots, X^{j_H}, X^{a_1}, \ldots, X^{a_L} \right] \right) \right)$$

(17)

Generalizing the laws given by Eq. (12) for $M$-theory to $N$-dimensional brackets in BLNA-theory, we can write following mappings:

$$\sum_{L=0}^{M} \sum_{H=0}^{L} \sum_{a_1 \ldots a_L} \sum_{a_1 \ldots a_L} \left( \left[ X^{j_1}, \ldots, X^{j_H-1}, X^{a_1}, \ldots, X^{a_L}, X^{j_H} \right], \left[ X^{j_1}, \ldots, X^{j_H-1}, X^{a_1}, \ldots, X^{a_L}, X^{j_H} \right] \right) =$$

$$\frac{1}{2} \sum_{L=0}^{M} \sum_{H=0}^{L} \sum_{a_1 \ldots a_L} \sum_{a_1 \ldots a_L} \left( X^{j_1}, \ldots, X^{j_H-1} \right)^2 \left( \partial_{a_1}, \partial_{a_2} X^i, \partial_{a_1}, \partial_{a_2} X^i \right)$$

$$\sum_{L=0}^{M} \sum_{H=0}^{L} \sum_{a_1 \ldots a_L} \sum_{a_1 \ldots a_L} \left( \left[ X^{j_1}, \ldots, X^{j_H}, X^{a_1}, \ldots, X^{a_L} \right], \left[ X^{j_1}, \ldots, X^{j_H}, X^{a_1}, \ldots, X^{a_L} \right] \right) =$$

$$\frac{1}{2} \sum_{L=0}^{M} \sum_{H=0}^{L} \sum_{a_1 \ldots a_L} \sum_{a_1 \ldots a_L} \left( \frac{\lambda^2}{1.2^{N}} \left( X^{j_1} \ldots X^{j_H} \right)^2 \left( F_{a_1 \ldots a_L}, F_{a_1 \ldots a_L} \right) \right)$$

$$F_{a_1 \ldots a_n} = \partial_{a_1} A_{a_2 \ldots a_n} = \partial_{a_1} A_{a_2 \ldots a_n} - \partial_{a_2} A_{a_1 \ldots a_n} + \ldots$$

$$\Sigma_m \rightarrow \frac{1}{(2\pi)^p} \int d^{p+1} x \Sigma_{m-p+1}$$

(18)

Substituting mappings of Eq. (13) in action (17), we obtain the following action for $G_0$-branes:

$$S_{G0} = -(T_{G0})^p \int dt \sum_{n=1}^{p} \delta^{a_1 \ldots a_n}_{b_1 \ldots b_n} \left( \delta_{b_1 b_2 \ldots b_n} L_{a_1} b_{a_1} \ldots L_{a_n} b_{a_n} \right)^{1/2}$$

$$\left( L \right)^{a_n}_{b_n} = \delta^{a_n}_{b_n} \operatorname{Tr} \left( \frac{1}{2} \sum_{L=0}^{M} \sum_{H=0}^{L} \sum_{a_1 \ldots a_L} \sum_{a_1 \ldots a_L} \left( X^{j_1}, \ldots, X^{j_H} \right)^2 \left( \partial_{a_1}, \partial_{a_2} X^i, \partial_{a_1}, \partial_{a_2} X^i \right) + \right.$$

$$\sum_{L=0}^{M} \sum_{H=0}^{L} \sum_{a_1 \ldots a_L} \sum_{a_1 \ldots a_L} \left( \frac{\lambda^2}{1.2^{N}} \left( X^{j_1} \ldots X^{j_H} \right)^2 \left( F_{a_1 \ldots a_L}, F_{a_1 \ldots a_L} \right) \right)$$

(19)
This action is reduced to action of $Dp$-brane [7] for $N = 2$ and $M = 9$ and action of $Mp$-brane [14] for $N = 3$ and $M = 10$. This action is not complete, because, we have ignored fermions in it. In fact, in cosmology and other systems like quarkonium, we need supersymmetry and in order for it to be produced, we need certain degrees of freedom for bosons and fermions to be the same. Previously, it has been shown that by compacting part of brane, systems like quarkonium, we need supersymmetry and in order for it to be produced, we need certain degrees of freedom for bosons and fermions and generators of the $S^2 (N)$ that include both degrees of freedom for bosons and fermions and generators of $\delta_m$. We use the mechanism in [13], and compactify $M^{th}$ dimension of branes on a circle with radius $R$ by choosing $<X^M> = \frac{iR}{p}T^M$ for boson and $<\psi^{L,M}> = \frac{1}{(2\pi)^p} T^{L,M}$ for fermions in action of [15]. We obtain the following action for $G\phi$-brane:

$$S_{G0} = S_{G0, non-compact} + S_{G0, compact} =$$

$$T_{G0} \int dt \text{Tr} \left( \frac{\delta_{a_1, a_2, \ldots, a_n}^N \Sigma_{L_1, \ldots, L_N=0} (\sum_{L_{j=1}^{N}} L_{a_1} \cdots L_{a_n})}{(2\pi)^p} \right)^{1/2}$$

(20)

We can choose $\gamma_{L_1} = T \gamma_{L_1} \frac{R^2}{p}$ where $\gamma_{L_1}$'s are the Pauli matrices in $M$ dimensions and rewrite action of $G\phi$-brane as follows:

$$S_{G0} = T_{G0} \int dt \text{Tr} \left( \frac{\delta_{a_1, a_2, \ldots, a_n}^N \Sigma_{L_1, \ldots, L_N=0} (\sum_{L_{j=1}^{N}} L_{a_1} \cdots L_{a_n})}{(2\pi)^p} \right)^{1/2}$$

(21)

It is clear that brackets in this action include both degrees of freedoms for bosons and fermions and generators of algebra behave like the Pauli matrices in $M$ dimensions. Replacing these brackets with brackets of Eq. [17], we can calculate the action of $G\phi$-brane:

$$S_{Gp} = -(T_{G0})^p \int dt \sum_{n=1}^p \beta_n \left( \frac{\delta_{a_1, a_2, \ldots, a_n}^N \Sigma_{L_1, \ldots, L_N=0} (\sum_{L_{j=1}^{N}} L_{a_1} \cdots L_{a_n})}{(2\pi)^p} \right)^{1/2}$$

(22)

To obtain the general form of action in terms of spinor fields, we add following laws to equation [18]:

$$\Sigma_{L_1=0, \ldots, L_N=0} \Sigma_{L_{j=1}^{N}} (\sum_{L_{j=1}^{N}} L_{a_1} \cdots L_{a_n}) =$$

$$\sum_{L_1=0, \ldots, L_N=0} \Sigma_{L_{j=1}^{N}} (\sum_{L_{j=1}^{N}} L_{a_1} \cdots L_{a_n})^2 \frac{1}{2} \left( \sum_{L_{j=1}^{N}} L_{a_1} \cdots L_{a_n} \right)$$

(23)
where $\tilde{A}_{a_2...a_n}$ are fermionic superpartners of gauge bosons $A_{a_2...a_n}$ and $\psi$ are the fermionic superpartner of scalar strings $X$. Replacing rules of Eq. (24) in action of (22), we get:

$$S_{Gp} = -(T_{Gp}) \int dt \sum_{n=1}^{p} \beta_n \left( \delta_{b_1b_2...b_n} A_{a_1...a_n} L_{a_1...a_n} \right)^{1/2}$$

$$\left( L \right)_{a_n}^{b_n} = \delta_{b_n}^{a_n} Tr \left( \frac{1}{2} \sum_{L=0}^{N} \sum_{H=0}^{N-L} \sum_{a_1...a_L=0}^{M} \sum_{j_1...j_H=0}^{M} \lambda^2 \frac{1}{12...N} (X^{j_1}...X^{j_H})^2 (F^{a_1...a_L}, F^{a_1...a_L}) - \frac{1}{2} \sum_{L=0}^{N} \sum_{H=0}^{N-L} \sum_{a_1...a_L=0}^{M} \sum_{j_1...j_H=0}^{M} \sum_{j_1...j_H=p+1}^{M} \lambda^2 \frac{1}{12...N} (X^{j_1}...X^{j_H})^2 (\tilde{F}^{a_1...a_L}, \tilde{F}^{a_1...a_L}) \right)$$

This action is reduced to supersymmetric actions for $M_p$-brane by putting $N = 3$ and $M = 10$ in 11-dimensions. It is clear that number of degrees of freedom for bosons and fermions are the same and all particles and their superpartners appear in this action. Also, the exact wave equations for scalars, Dirac fields, gauge fields and higher dimensional spinors can be observed in this action.

Now, we like to extract gravity from actions in BLN-theory. For this reason, we assume that $A^{ab}$ plays the role of graviton and $\tilde{A}^{ab}$ has the role of gravitino and other higher dimensional fields have the following relations with these fields:

$$A_{a'b'} \rightarrow g_{a'b'}$$

$$F^{a'b',c'} = \partial_{[a'} g_{b',c']} = \partial_{a'} g_{b',c'} + \partial_{b'} g_{c',a'} \rightarrow \Gamma_{a'b',c'}$$

$$\tilde{A}_{a'b'} \rightarrow \tilde{g}_{a'b'}$$

$$\tilde{F}^{a'b',c'} = \partial_{[a'} \tilde{g}_{b',c']} = \partial_{a'} \tilde{g}_{b',c'} + \partial_{b'} \tilde{g}_{c',a'} \rightarrow \Gamma_{a'b',c'}$$

where $(g_{a'b'})$ and $(\tilde{g}_{a'b'})$ are graviton and gravitino respectively. Substituting equations of (25),(26),(27),(28),(29) in action of (24), we obtain the action of $Gp$-brane in terms of curvatures and metrics:
\[ S_{G\sigma} = -\langle T_{G\sigma} \rangle \int dt \sum_{n=1}^{P} \beta_n \left( \delta_{n_1 n_2 \ldots n_n} L_{a_1}^{b_1} \ldots L_{a_n}^{b_n} \right)^{1/2} \]

\[ (L)^{a_n}_{b_n} = \delta^{a_n}_{b_n} Tr \left( \frac{1}{2} \Sigma_{L=0}^{N} \Sigma_{H=0}^{N-L} \Sigma_{a_1 \ldots a_L=0}^{M} \Sigma_{j_1 \ldots j_H=p+1}^{M} (X^{j_1} \ldots X^{j_H})^2 \partial_{a_1} \ldots \partial_{a_L} X^i, \partial_{a_1} \ldots \partial_{a_L} X^i \right) + \]

\[ \Sigma_{L=0}^{N} \Sigma_{H=0}^{N-L} \Sigma_{a_1 \ldots a_L=0}^{M} \Sigma_{j_1 \ldots j_H=p+1}^{M} (X^{j_1} \ldots X^{j_H})^2 \partial_{a_5} \ldots \partial_{a_L} \bar{R}^{a_1 a_2 a_3 a_4}, \partial_{a_5} \ldots \partial_{a_L} \bar{R}^{a_1 a_2 a_3 a_4} \right) - \]

\[ \frac{i}{2} \Sigma_{L=0}^{N} \Sigma_{H=0}^{N-L} \Sigma_{a_1 \ldots a_L=0}^{M} \Sigma_{j_1 \ldots j_H=p+1}^{M} (X^{j_1} \ldots X^{j_H})^2 \partial_{a_1} \ldots \partial_{a_L} \psi^i, \partial_{a_1} \ldots \partial_{a_L} \psi^i \right) - \]

\[ \langle \partial_{a_5} \ldots \partial_{a_L-1} \bar{R}^{a_1 a_2 a_3 a_4}, \partial_{a_5} \ldots \partial_{a_L} \bar{R}^{a_1 a_2 a_3 a_4} + \rangle \right) \] (31)

This action contains various orders of curvatures and its derivatives and is reduced to related actions in \( F(R) \)-gravity in [26, 27] in four dimensional universe. In addition to this, gravity includes both curvatures of gravitons and gravitinoes and shows the role of spinor gravity in evolutions of branes. On the other hand, curvatures which are produced by gravitinoes have opposite sign respect to curvatures that are created by gravitons; and thus, they may cancel their effects and universe seems to be flat.

Now, two significant questions emerge: what is difference between branes and anti-branes physically? And how are they produced? To respond to these questions, we assume that there is nothing at the beginning. Then, two energies with opposite sign are emerged such as the sum over them is zero again. After that, these energies produce 2\( M \) degrees of freedom which each two of them leads to creation of new dimension. At the fourth stage, \( M - N \) of degrees of freedom are removed by compacting half of \( M-N \) dimensions on a circle to produce Lie-N-algebra. During this compactification, the behaviour of one dimension is different with other dimensions for one of initial energies which is known as time. Also, for second energy the behaviour of two dimensions is different which leads to the appearance of two time coordinates. After compactification, usual action of branes emerge with one time coordinates, however the physics of anti-branes is different and they have two time coordinates.

First, we show that two oscillating energies are produced from nothing and expanded in \( M^{th} \) dimension. We write:

\[ E \equiv 0 \equiv E_1 + E_2 \equiv 0 \equiv N_1 + N_2 \equiv k((X^M)^2 - (X^M)^2) = k \int d^2x \left( \frac{\partial}{\partial x} \right)^2 ((X^M)^2 - (X^M)^2), \] (32)

where \( N_{1/2} \) denote the number of degrees of freedom for first and second energies. These energies oscillate, excite and create \( M \) dimensions with 2\( M \) degrees of freedom. We can show this by rewriting Eq. (32) as follows:

\[ E \equiv 0 \equiv k \int d^2x \epsilon^{i_1 i_2 \ldots i_M} \epsilon^{i_1 i_2 \ldots i_M} \left( \frac{\partial}{\partial x_{i_1}} \frac{\partial}{\partial x_{i_2}} \ldots \frac{\partial}{\partial x_{i_{M-1}}} \right)^2 (X^M)^2 - \]

\[ k \int d^2x \epsilon^{i_1 i_2 \ldots i_M} \epsilon^{i_1 i_2 \ldots i_M} \left( \frac{\partial}{\partial x_{i_1}} \frac{\partial}{\partial x_{i_2}} \ldots \frac{\partial}{\partial x_{i_{M-1}}} \right)^2 (X^M)^2, \] (33)

where, we use of \( \epsilon^{i_1 i_2 \ldots i_M} \epsilon^{i_1 i_2 \ldots i_M} = -1 \). In Eq. (33), each integral and derivative \( \int dx \frac{\partial}{\partial x} \) shows one degree of freedom and thus we have \( M \) dimensions and 2\( M \) degrees of freedom. Also, each derivative with respect to special dimension of initial energy, produces a new force \( (F = \frac{\partial V}{\partial x}) \) and leads to expansion of energy and creation of new degrees of freedom in that direction. We can replace derivatives by brackets as follows [7, 8, 13, 20, 22, 25]:
\[ \frac{\partial}{\partial x_i} X^M = [X^{i_1}, X^{i_2}] \]
\[ \frac{\partial}{\partial x_{i_1}} \frac{\partial}{\partial x_{i_2}} X^M = [X^{i_1}, X^{i_2}, X^M] \]
\[ \left( \frac{\partial}{\partial x_{i_1}} \frac{\partial}{\partial x_{i_2}} \frac{\partial}{\partial x_{i_{M-1}}} \right) (X^M) = [X^{i_1}, X^{i_2}, \ldots, X^{i_{M-1}}, X^M] \]
\[ \epsilon^{i_1 i_2 \ldots i_M} \epsilon^{i_1' i_2' \ldots i_M'} \left( \frac{\partial}{\partial x_{i_1}} \frac{\partial}{\partial x_{i_2}} \frac{\partial}{\partial x_{i_{M-1}}} \right) (X^M)^2 = \]
\[ \epsilon^{i_1 i_2 \ldots i_M} \epsilon^{i_1' i_2' \ldots i_M'} \left( \left[ \left( \frac{\partial}{\partial x_{i_1}} \frac{\partial}{\partial x_{i_2}} \frac{\partial}{\partial x_{i_{M-1}}} \right) (X^M) \right] \left[ \left( \frac{\partial}{\partial x_{i_1'}} \frac{\partial}{\partial x_{i_2'}} \frac{\partial}{\partial x_{i_{M-1}'}} \right) (X^M) \right] \right) = \]
\[ (\{X_{i_1}, X_{i_2}, \ldots, X_{i_M}\}, \{X_{i_1}, X_{i_2}, \ldots, X_{i_M}\}) \]

Using the mappings of Eq. (34) in Eq. (33), we obtain:

\[ E \equiv 0 \equiv E_1 + E_2 \equiv \]
\[ E_1 = k \int d^2M x (\{X_{i_1}, X_{i_2}, \ldots, X_{i_M}\}, \{X_{i_1}, X_{i_2}, \ldots, X_{i_M}\}) \]
\[ E_2 = -k \int d^2M x (\{X_{i_1}, X_{i_2}, \ldots, X_{i_M}\}, \{X_{i_1}, X_{i_2}, \ldots, X_{i_M}\}) \]

These energies are similar to action of \( G_p \)-branes, however it is expected that algebra be of order \( N \), however they are or order of \( M \) and for this reason, we should remove \( M - N \) degrees of freedom by compacting. To this end, we use of the mechanism in [22–25] and replace \( X_{i_{M+1},M-N} = i T^a \) where \( l_P \) is the Planck length. We obtain the following action for first energy:

\[ E_1 \equiv k \int d^2M x (\{X_{i_1}, X_{i_2}, \ldots, X_{i_M}\}, \{X_{i_1}, X_{i_2}, \ldots, X_{i_M}\}) = \]
\[ k \int d^2M x \epsilon^{i_1 i_2 \ldots i_M} \epsilon^{i_1' i_2' \ldots i_M'} X_{i_1} X_{i_2} \ldots X_{i_M} X_{i_1'} X_{i_2'} \ldots X_{i_M'} = \]
\[ (i)^{2(M-N)} k \int d^N x (\frac{R^{M-N}}{l_p^{(M-N)/2}})^{i_1 \ldots i_M} \epsilon^{i_1' \ldots i_M'} X_{j_1} \ldots X_{j_N} X_{j_1'} \ldots X_{j_N'} = \]
\[ (i)^{2(M-N)} k \int d^N x (\frac{R^{M-N}}{l_p^{(M-N)/2}})^{(i_1 \ldots i_M)} (\{X_{j_1}, X_{j_2}, \ldots, X_{j_N}\}, \{X_{j_1}, X_{j_2}, \ldots, X_{j_N}\}) = \]
\[ k \int d^N x (\frac{R^{M-N}}{l_p^{(M-N)/2}})^{(i_1 \ldots i_M)} (\{i X_{j_1}, i X_{j_2}, \ldots, i X_{j_{M-N+1}}, X_{j_N}\}, \{i X_{j_1}, i X_{j_2}, \ldots, i X_{j_{M-N+1}}, X_{j_N}\}) \]

where we have used \( \epsilon^{i_1 i_2 \ldots i_M} \epsilon^{i_1' i_2' \ldots i_M'} = (-1)^{N-M} \epsilon^{i_1 \ldots i_M} \epsilon^{i_1' \ldots i_M'} \). Scalars which are different from other scalars by one extra \((i)\), are located in time directions. Thus, in BLNA-theory, we can have \( M - N \) time coordinates where \( M \) is dimension of world and \( N \) is dimension of algebra. For an observer on the brane, we can put \((M = p)\) where, \( p \) is dimension of brane. For example, in \( M \)-theory, for a four dimensional brane like our universe, we have 4 dimensions and 3 dimensional algebra. Thus, we observe only one time coordinate, however for branes with higher dimensions, we observe more time coordinates. Also, this equation shows that \( N \) should be equal or less of \((\frac{M}{2})\) which is consistent with Lie-two algebra in string theory and Lie-three-algebra in \( M \)-theory. For second energy which is different from first one in its sign, we have one extra time coordinate, because we have:

\[ E_2 = -E_1 = (i)^2 E_1 = \]
\[ k \int d^N x (\frac{R^{M-N}}{l_p^{(M-N)/2}})^{(i_1 \ldots i_M)} (\{i X_{j_1}, i X_{j_2}, \ldots, i X_{j_{M-N+1}}, X_{j_N}\}, \{i X_{j_1}, X_{j_2}, \ldots, i X_{j_{M-N+1}}, X_{j_N}\}) \]

(37)
Thus, physics of branes which is produced by this energy is different and we have more difference between dimensions. For example, in our universe, length of one object can be obtained by \( l^2 = -t^2 + x_1^2 + x_2^2 + x_3^2 \) where \( t \) is time and \( x_1 \) are coordinates of space. However in anti-universe, length is defined by \( l^2 = -t^2 - x_1^2 + x_2^2 + x_3^2 \). Also, energy and momentums which have the relation with mass \( (m^2 = -E^2 + P_1^2 + P_2^2 + P_3^2) \), now their relation is different for anti-universe \( (m^2 = -E^2 - P_1^2 + P_2^2 + P_3^2) \).

We can correct action in Eq. (22) by regarding \((i^2(p-N))\) for branes and \((i^2(p-N+1))\) for anti-branes and assuming \( R = l_{(1)}^4 \). We obtain:

\[
S_{Gp} = -(T_{Gp})^p \int dt \sum_{n=1}^{p} \beta_n \left( \delta_{b_1b_2...b_n} L_{a_1}...L_{a_n} \right)^{1/2} \\
(L)^{a_n}_{b_n} = \delta^{a_n}_{b_n} T \left( \sum_{L=0}^{N} \sum_{H=0}^{N} \right) (\cdot)_{j_H,j_H = p+1} + \\
i^{2(p-N)} \left( \left[ X^{j_1},...X^{j_{H-1}}, X^{a_1},...X^{a_L}, X^{j_H} \right], \left[ X^{j_1},...X^{j_{H-1}}, X^{a_1},...X^{a_L}, X^{j_H} \right] \right) + \\
i^{2(p-N)} \sum_{L=0}^{N} \sum_{H=0}^{N} \left( a_{1...a_L} = 0 \right) \sum_{j_H,j_H = p+1} ^{N} (\cdot)_{j_H,j_H = p+1} + i^{2(p-N)} \left( \left[ X^{j_1},...X^{j_{H-1}}, X^{a_1},...X^{a_L}, X^{j_H} \right], \left[ X^{j_1},...X^{j_{H-1}}, X^{a_1},...X^{a_L}, X^{j_H} \right] \right) - \\
i^{2(p-N)+1} \sum_{L=0}^{N} \sum_{H=0}^{N} \left( a_{1...a_L} = 0 \right) \sum_{j_H,j_H = p+1} ^{N} (\cdot)_{j_H,j_H = p+1} + i^{2(p-N)} \left( \left[ X^{j_1},...X^{j_{H-1}}, X^{a_1},...X^{a_L}, X^{j_H} \right], \left[ X^{j_1},...X^{j_{H-1}}, X^{a_1},...X^{a_L}, X^{j_H} \right] \right) - \\
i^{2(p-N)+1} \sum_{L=0}^{N} \sum_{H=0}^{N} \left( a_{1...a_L} = 0 \right) \sum_{j_H,j_H = p+1} ^{N} (\cdot)_{j_H,j_H = p+1} + i^{2(p-N)} \left( \left[ X^{j_1},...X^{j_{H-1}}, X^{a_1},...X^{a_L}, X^{j_H} \right], \left[ X^{j_1},...X^{j_{H-1}}, X^{a_1},...X^{a_L}, X^{j_H} \right] \right) \right) (38)
\]

Using the laws in Eq. (25) and replacing gauge fields by mappings in Eqs. (25,30) we can rewrite action of \((31)\) and obtain the action of \(G_p\)-brane and anti-\(G_p\)-brane in terms of curvatures:

\[
S_{Gp} = -(T_{Gp})^p \int dt \sum_{n=1}^{p} \beta_n \left( \delta_{b_1b_2...b_n} L_{a_1}...L_{a_n} \right)^{1/2} \\
(L)^{a_n}_{b_n} = \delta^{a_n}_{b_n} T \left( \sum_{L=0}^{N} \sum_{H=0}^{N} \right) (\cdot)_{j_H,j_H = p+1} + \\
i^{2(p-N)} \sum_{L=0}^{N} \sum_{H=0}^{N} \left( a_{1...a_L} = 0 \right) \sum_{j_H,j_H = p+1} ^{N} (\cdot)_{j_H,j_H = p+1} + i^{2(p-N)} \left( \left[ X^{j_1},...X^{j_{H-1}}, X^{a_1},...X^{a_L}, X^{j_H} \right], \left[ X^{j_1},...X^{j_{H-1}}, X^{a_1},...X^{a_L}, X^{j_H} \right] \right) - \\
i^{2(p-N)+1} \sum_{L=0}^{N} \sum_{H=0}^{N} \left( a_{1...a_L} = 0 \right) \sum_{j_H,j_H = p+1} ^{N} (\cdot)_{j_H,j_H = p+1} + i^{2(p-N)} \left( \left[ X^{j_1},...X^{j_{H-1}}, X^{a_1},...X^{a_L}, X^{j_H} \right], \left[ X^{j_1},...X^{j_{H-1}}, X^{a_1},...X^{a_L}, X^{j_H} \right] \right) - \\
i^{2(p-N)+1} \sum_{L=0}^{N} \sum_{H=0}^{N} \left( a_{1...a_L} = 0 \right) \sum_{j_H,j_H = p+1} ^{N} (\cdot)_{j_H,j_H = p+1} + i^{2(p-N)} \left( \left[ X^{j_1},...X^{j_{H-1}}, X^{a_1},...X^{a_L}, X^{j_H} \right], \left[ X^{j_1},...X^{j_{H-1}}, X^{a_1},...X^{a_L}, X^{j_H} \right] \right) \right) \right) (40)
\]
where $\hat{R}$ and $\hat{\mathcal{R}}$ are curvatures of graviton and gravitino in anti-branes respectively. These curvatures are different from curvatures in branes, because they contain more derivatives respect to time coordinates. In addition to this, the sign of curvatures in branes are reversed with respect to anti-branes, which means that the lines of gravity go outside the branes, while these lines go inside the anti-branes and thus these objects attract each other. For four dimensional universe in $11$ dimensional $M$-theory with Lie-three-algebra the action is reduced to known actions for $F(R)$-gravity within $^{[26, 27]}$.

Now, we will show that gravitons and gravitino produce two different wormholes that act reverse to each other. The wormhole which is produced by gravitons prevents the getting away of quarks and anti-quarks from each other and generates confluence, while the wormhole, which is produced by gravitinoes, prevents quarks and anti-quarks from coming close to each other and creates deconfluence. For this reason, in quarkonium, quarks and anti-quarks don’t go away from each other or come close to each other. To obtain the shape of wormholes, we use the method in $^{[28]}$ and obtain the momentum density. However before doing it, we define bosonic and fermionic Lagrangian as:

\[
S_{Anti-GP} = -(T_{Anti-GP}) \int dt \sum_{n=1}^{P} \beta_n \left(\delta_{b_1 b_2 \ldots b_n} L_{a_1} \ldots L_{a_n} \right)^{1/2}
\]

\[
(L)_{a_n}^{a_n} = \delta_{a_n}^{a_n} T_{\epsilon} \left(\frac{1}{2} i^{2(p-N+1)} \sum_{L=0}^{N} \sum_{H=0}^{N-L} \sum_{a_1 \ldots a_L, \bar{a}_1 \ldots \bar{a}_L} (X^{j_1} \ldots X^{j_H-1})^2 \langle \partial_{a_1} \ldots \partial_{a_L} X^i, \partial_{a_1} \ldots \partial_{a_L} X^i \rangle + \right)
\]

\[
i^{2(p-N+1)} \sum_{L=0}^{N} \sum_{H=0}^{N-L} \sum_{a_1 \ldots a_L, \bar{a}_1 \ldots \bar{a}_L} (X^{j_1} \ldots X^{j_H-1})^2 \langle \partial_{a_5} \ldots \partial_{a_L-1} \hat{R}^{a_1, a_2, a_3, a_4}, \partial_{a_5} \ldots \partial_{a_L} \hat{R}^{a_1, a_2, a_3, a_4} \rangle + \ldots \right).
\]

\[
S_{GP} = -(T_{GP}) \int dt \sum_{n=1}^{P} \beta_n \left(\delta_{b_1 b_2 \ldots b_n} L_{a_1} \ldots L_{a_n} \right)^{1/2}
\]

\[
(L)_{b_n, brane} = (L)_{b_n, bosonic, brane} + (L)_{b_n, fermionic, brane}
\]

\[
(L)_{b_n, bosonic, brane} = \delta_{b_n}^{b_n} T_{\epsilon} \left(\frac{1}{2} i^{2(p-N)} \sum_{L=0}^{N} \sum_{H=0}^{N-L} \sum_{a_1 \ldots a_L, \bar{a}_1 \ldots \bar{a}_L} (X^{j_1} \ldots X^{j_H-1})^2 \langle \partial_{a_1} \ldots \partial_{a_L} X^i, \partial_{a_1} \ldots \partial_{a_L} X^i \rangle + \right)
\]

\[
i^{2(p-N)} \sum_{L=0}^{N} \sum_{H=0}^{N-L} \sum_{a_1 \ldots a_L, \bar{a}_1 \ldots \bar{a}_L} (X^{j_1} \ldots X^{j_H-1})^2 \langle \partial_{a_5} \ldots \partial_{a_L-1} \hat{R}^{a_1, a_2, a_3, a_4}, \partial_{a_5} \ldots \partial_{a_L} \hat{R}^{a_1, a_2, a_3, a_4} \rangle + \ldots \right).
\]

\[
(L)_{b_n, fermionic, brane} = \delta_{b_n}^{b_n} T_{\epsilon} \left(\frac{1}{2} i^{2(p-N)+1} \sum_{L=0}^{N} \sum_{H=0}^{N-L} \sum_{a_1 \ldots a_L, \bar{a}_1 \ldots \bar{a}_L} (X^{j_1} \ldots X^{j_H-1})^2 \langle \partial_{a_1} \ldots \partial_{a_L-1} \hat{R}^{a_1, a_2, a_3, a_4}, \partial_{a_5} \ldots \partial_{a_L} \hat{R}^{a_1, a_2, a_3, a_4} \rangle + \right)
\]

\[
\langle \partial_{a_5} \ldots \partial_{a_L-1} \hat{R}^{a_1, a_2, a_3, a_4}, \partial_{a_5} \ldots \partial_{a_L} \hat{R}^{a_1, a_2, a_3, a_4} \rangle + \ldots \right).
\]
and

\[ S_{\text{Anti-Gp}} = -(T_{\text{Anti-Gp}}) \int dt \sum_{n=1}^{P} \beta_n \left( \delta_{b_1 b_2 \ldots b_n} T_{a_1 \ldots L_{a_n}}^b \right)^{1/2} \]

\[(L)^b_{b_n,\text{anti-brane}} = (L)^b_{b_n,\text{bosonic,anti-brane}} + (L)^b_{b_n,\text{fermionic,anti-brane}} \]

\[(L)^b_{b_n,\text{bosonic,anti-brane}} = \delta_{b_n} T_F \left( \frac{1}{2} (p-N+1) \Sigma_{L=0}^{N-L} \Sigma_{L=0}^{N-L} \Sigma_{L=0}^{M} \delta_{a_1 \ldots a_L=0} (X^{j_1 \ldots X^{j_H}})^2 \partial_{a_5} \partial_{a_L} \tilde{R}^{a_1 \ldots a_3, a_4} + \ldots \right) \]

\[(L)^b_{b_n,\text{fermionic,anti-brane}} = \delta_{b_n} T_F \left( -\frac{1}{2} (p-N+1) \Sigma_{L=0}^{N-L} \Sigma_{L=0}^{M} \gamma_{a_1 \ldots a_L=0} (X^{j_1 \ldots X^{j_H}})^2 \partial_{a_5} \partial_{a_L} \tilde{R}^{a_1 \ldots a_3, a_4} + \ldots \right) \]

(43)

First, we should calculate the momentum densities for bosonic part and fermionic part of Lagrangian in Eq. (42) respect to derivatives of curvature. To this end, we begin with derivatives of order of \( p - 4 \), where \( p \) is dimension of brane and 4 denotes four indices of curvature. We obtain

\[ \Pi_{\text{bosonic,brane},p-4} \approx \frac{-2(p-N) \Sigma_{L=0}^{N-L} \Sigma_{L=0}^{M} \gamma_{a_1 \ldots a_L=0} (X^{j_1 \ldots X^{j_H}})^2 \partial_{a_5} \partial_{a_L} \tilde{R}^{a_1 \ldots a_3, a_4}}{\sqrt{(L)^b_{b_n,\text{bosonic,brane}}}} \]

(44)

\[ \Pi_{\text{fermionic,brane},p-4} \approx \frac{-2(p-N+1) \Sigma_{L=0}^{N-L} \Sigma_{L=0}^{M} \gamma_{a_1 \ldots a_L=0} (X^{j_1 \ldots X^{j_H}})^2 \partial_{a_5} \partial_{a_L} \tilde{R}^{a_1 \ldots a_3, a_4}}{\sqrt{(L)^b_{b_n,\text{bosonic,brane}}}} \]

(45)

\[ \Pi_{\text{bosonic,anti-brane},p-4} \approx \frac{-2(p-N) \Sigma_{L=0}^{N-L} \Sigma_{L=0}^{M} \gamma_{a_1 \ldots a_L=0} (X^{j_1 \ldots X^{j_H}})^2 \partial_{a_5} \partial_{a_L} \tilde{R}^{a_1 \ldots a_3, a_4}}{\sqrt{(L)^b_{b_n,\text{bosonic,brane}}}} \]

(46)

\[ \Pi_{\text{fermionic,anti-brane},p-4} \approx \frac{-2(p-N+1) \Sigma_{L=0}^{N-L} \Sigma_{L=0}^{M} \gamma_{a_1 \ldots a_L=0} (X^{j_1 \ldots X^{j_H}})^2 \partial_{a_5} \partial_{a_L} \tilde{R}^{a_1 \ldots a_3, a_4}}{\sqrt{(L)^b_{b_n,\text{bosonic,brane}}}} \]

(47)

We assume that all coordinates are the same (\( x^2 \cdot P = \sigma \)) and construct a \( p \) dimensional sphere. The Hamiltonian for this system can be obtained as:
\[ H_{p-4}^1 \approx 4\pi \int d\sigma^{p-1}\sum_{L=0}^{N-L}\sum_{H=0}^{N-L}\sum_{a_1,\ldots,a_L=0}^{M} \frac{\lambda^2}{2^{N-N}} (X_{j_1} \ldots X_{j_H})^2 \partial_{a_1} \ldots \partial_{a_L} \tilde{R}_{a_1,a_2,a_3,a_4}^{1,2} \Pi_{\text{bosonic,brane,}p-4}^{2(p-N)} \]
\[ -4\pi \int d\sigma^{p-1}\sum_{L=0}^{N-L}\sum_{H=0}^{N-L}\sum_{a_1,\ldots,a_L=0}^{M} \frac{\lambda^2}{12^{N-N}} (X_{j_1} \ldots X_{j_H})^2 \partial_{a_1} \partial_{a_2} \tilde{R}_{a_1,a_2,a_3,a_4}^{1,2} \Pi_{\text{fermionic,brane,}p-4}^{2(p-N)+1} \]
\[ +4\pi \int d\sigma^{p-1}\sum_{L=0}^{N-L}\sum_{H=0}^{N-L}\sum_{a_1,\ldots,a_L=0}^{M} \frac{\lambda^2}{12^{N-N}} (X_{j_1} \ldots X_{j_H})^2 \partial_{a_1} \partial_{a_2} \tilde{R}_{a_1,a_2,a_3,a_4}^{1,2} \Pi_{\text{bosonic,anti-brane,}p-4}^{2(p-N+1)} \]
\[ -4\pi \int d\sigma^{p-1}\sum_{L=0}^{N-L}\sum_{H=0}^{N-L}\sum_{a_1,\ldots,a_L=0}^{M} \frac{\lambda^2}{12^{N-N}} (X_{j_1} \ldots X_{j_H})^2 \partial_{a_1} \partial_{a_2} \tilde{R}_{a_1,a_2,a_3,a_4}^{1,2} \Pi_{\text{fermionic,anti-brane,}p-4}^{2(p-N+1)+1} \]
\[ -L_{1,\text{bosonic,brane,}p-4}^1 - L_{1,\text{fermionic,brane,}p-4}^1 = \]
\[ 4\pi \int d\sigma^{p-1}\sum_{L=0}^{N-L}\sum_{H=0}^{N-L}\sum_{a_1,\ldots,a_L=0}^{M} \frac{\lambda^2}{12^{N-N}} (X_{j_1} \ldots X_{j_H})^2 \partial_{a_1} \ldots \partial_{a_L} \tilde{R}_{a_1,a_2,a_3,a_4}^{1,2} \Pi_{\text{bosonic,brane,}p-4}^{2(p-N)} \]
\[ -4\pi \int d\sigma^{p-1}\sum_{L=0}^{N-L}\sum_{H=0}^{N-L}\sum_{a_1,\ldots,a_L=0}^{M} \frac{\lambda^2}{12^{N-N}} (X_{j_1} \ldots X_{j_H})^2 \partial_{a_1} \partial_{a_2} \tilde{R}_{a_1,a_2,a_3,a_4}^{1,2} \Pi_{\text{fermionic,brane,}p-4}^{2(p-N)+1} \]
\[ +4\pi \int d\sigma^{p-1}\sum_{L=0}^{N-L}\sum_{H=0}^{N-L}\sum_{a_1,\ldots,a_L=0}^{M} \frac{\lambda^2}{12^{N-N}} (X_{j_1} \ldots X_{j_H})^2 \partial_{a_1} \partial_{a_2} \tilde{R}_{a_1,a_2,a_3,a_4}^{1,2} \Pi_{\text{bosonic,anti-brane,}p-4}^{2(p-N+1)} \]
\[ +4\pi \int d\sigma^{p-1}\sum_{L=0}^{N-L}\sum_{H=0}^{N-L}\sum_{a_1,\ldots,a_L=0}^{M} \frac{\lambda^2}{12^{N-N}} (X_{j_1} \ldots X_{j_H})^2 \partial_{a_1} \partial_{a_2} \tilde{R}_{a_1,a_2,a_3,a_4}^{1,2} \Pi_{\text{fermionic,anti-brane,}p-4}^{2(p-N+1)+1} \]
\[ \partial_{a_1} (\sigma^{p-1} \Pi_{\text{bosonic,brane,}p-4}^{1,2} \Pi_{\text{fermionic,brane,}p-4}^{2(p-N)}) \]
\[ -4\pi \int d\sigma^{p-1}\sum_{L=0}^{N-L}\sum_{H=0}^{N-L}\sum_{a_1,\ldots,a_L=0}^{M} \frac{\lambda^2}{12^{N-N}} (X_{j_1} \ldots X_{j_H})^2 \partial_{a_1} \partial_{a_2} \tilde{R}_{a_1,a_2,a_3,a_4}^{1,2} \Pi_{\text{fermionic,anti-brane,}p-4}^{2(p-N+1)+1} \]
\[ -L_{1,\text{bosonic,brane,}p-4}^1 - L_{1,\text{fermionic,brane,}p-4}^1 \]
\[ \text{where we have used in the second step integrated by parts. We can impose the constraint} \]
\[ \text{(} \partial_{a_1} (\sigma^{p-1} \Pi_{\text{bosonic/fermionic,brane/anti-brane,}p-4}^{1,2(p-N)}) = 0 \text{)} \]
\[ \text{and obtain the momentum densities:} \]
\[ \Pi_{\text{bosonic,brane,}p-4} = \frac{i^{2(p-N)} k_{\text{bosonic,brane,}p-4}}{\sigma^{p-1}} \]
\[ \Pi_{\text{fermionic,brane,}p-4} = \frac{-i^{2(p-N)+1} k_{\text{fermionic,brane,}p-4}}{\sigma^{p-1}} \]
\[ \Pi_{\text{bosonic,anti-brane,}p-4} = \frac{i^{2(p-N)+1} k_{\text{bosonic,anti-brane,}p-4}}{\sigma^{p-1}} \]
\[ \Pi_{\text{fermionic,anti-brane,}p-4} = \frac{-i^{2(p-N)+1} k_{\text{fermionic,anti-brane,}p-4}}{\sigma^{p-1}} \]

Using momentum densities in Eqs. (49) and (48), we can calculate the Hamiltonian as:
\[ H_{p=4}^{1} \approx 4 \pi \int d\sigma^{p-1} \left( \frac{1}{2} i^{2(p-N)} \sum_{L=0}^{N} \sum_{H=0}^{L} \sum_{\alpha_{1} \ldots \alpha_{L}, a_{1} \ldots a_{L}=0}^{M} \sum_{j_{1} \ldots j_{H}=p+1}^{M} (X^{j_{1}} \ldots X^{j_{H}-1})^{2} \partial_{a_{1}} \ldots \partial_{a_{L}} X^{i_{1}} \partial_{a_{1}} \ldots \partial_{a_{L}} X^{i_{1}} \right) \]

\[ \frac{i^{2(p-N)} \sum_{L=0}^{N} \sum_{H=0}^{L} \sum_{\alpha_{1} \ldots \alpha_{L}=0}^{M} \sum_{j_{1} \ldots j_{H}=p+1}^{M} \lambda^{2} \frac{1}{2 \ldots N} (X^{j_{1}} \ldots X^{j_{H}})^{2} \partial_{a_{1}} \partial_{a_{L}} \tilde{R}^{a_{1}a_{2}a_{3}a_{4}} \partial_{a_{1}} \partial_{a_{L}} \tilde{R}^{a_{1}a_{2}a_{3}a_{4}} + \ldots)^{1/2} \times \]

\[ \sqrt{1 + \left( \frac{i^{2(p-N)} \kappa_{bosonic,brane,p=4}}{\sigma^{p-1}} \right)^{2}} + \]

\[ \left[ -\frac{1}{2} i^{2(p-N)+1} \sum_{L=0}^{N} \sum_{H=0}^{L} \sum_{a_{1} \ldots a_{L}=0}^{M} \sum_{j_{1} \ldots j_{H}=p+1}^{M} (X^{j_{1}} \ldots X^{j_{H}-1})^{2} \gamma^{a_{1} \ldots a_{L}-1} \partial_{a_{1}} \partial_{a_{L}} \psi^{i} \partial_{a_{1}} \partial_{a_{L}} \psi^{i} \right] \]

\[ \frac{i^{2(p-N)+1} \sum_{L=0}^{N} \sum_{H=0}^{L} \sum_{\alpha_{1} \ldots \alpha_{L}=0}^{M} \sum_{j_{1} \ldots j_{H}=p+1}^{M} \lambda^{2} \frac{1}{2 \ldots N} (X^{j_{1}} \ldots X^{j_{H}})^{2} \partial_{a_{1}} \partial_{a_{L}} \tilde{R}^{a_{1}a_{2}a_{3}a_{4}} \partial_{a_{1}} \partial_{a_{L}} \tilde{R}^{a_{1}a_{2}a_{3}a_{4}} + \ldots)^{1/2} \times \]

\[ \sqrt{1 + \left( \frac{i^{2(p-N)+1} \kappa_{fermionic,brane,p=4}}{\sigma^{p-1}} \right)^{2}} + \]

\[ \left[ -\frac{1}{2} i^{2(p-N)+1} \sum_{L=0}^{N} \sum_{H=0}^{L} \sum_{a_{1} \ldots a_{L}=0}^{M} \sum_{j_{1} \ldots j_{H}=p+1}^{M} (X^{j_{1}} \ldots X^{j_{H}-1})^{2} \gamma^{a_{1} \ldots a_{L}-1} \partial_{a_{1}} \partial_{a_{L}} \psi^{i} \partial_{a_{1}} \partial_{a_{L}} \psi^{i} \right] \]

\[ \frac{i^{2(p-N)+1} \sum_{L=0}^{N} \sum_{H=0}^{L} \sum_{\alpha_{1} \ldots \alpha_{L}=0}^{M} \sum_{j_{1} \ldots j_{H}=p+1}^{M} \lambda^{2} \frac{1}{2 \ldots N} (X^{j_{1}} \ldots X^{j_{H}})^{2} \partial_{a_{1}} \partial_{a_{L}} \tilde{R}^{a_{1}a_{2}a_{3}a_{4}} \partial_{a_{1}} \partial_{a_{L}} \tilde{R}^{a_{1}a_{2}a_{3}a_{4}} + \ldots)^{1/2} \times \]

\[ \sqrt{1 + \left( \frac{i^{2(p-N)+1} \kappa_{fermionic,anti-brane,p=4}}{\sigma^{p-1}} \right)^{2}} \]  

(50)

We use of previous mechanism again and obtain momentum densities for curvature of order \( p = 5 \):

\[ \Pi_{bosonic,brane,p=5} \approx \frac{i^{2(p-N)} \sum_{L=0}^{N} \sum_{H=0}^{L} \sum_{\alpha_{1} \ldots \alpha_{L}=0}^{M} \sum_{j_{1} \ldots j_{H}=p+1}^{M} \lambda^{2} \frac{1}{2 \ldots N} (X^{j_{1}} \ldots X^{j_{H}})^{2} \partial_{a_{1}} \partial_{a_{L}} \tilde{R}^{a_{1}a_{2}a_{3}a_{4}}}{H_{p=4}^{1}} \]  

(51)

\[ \Pi_{fermionic,brane,p=5} \approx -\frac{i^{2(p-N)+1} \sum_{L=0}^{N} \sum_{H=0}^{L} \sum_{\alpha_{1} \ldots \alpha_{L}=0}^{M} \sum_{j_{1} \ldots j_{H}=p+1}^{M} \lambda^{2} \frac{1}{2 \ldots N} (X^{j_{1}} \ldots X^{j_{H}})^{2} \partial_{a_{1}} \partial_{a_{L}} \tilde{R}^{a_{1}a_{2}a_{3}a_{4}}}{H_{p=4}^{1}} \]  

(52)

\[ \Pi_{bosonic,anti-brane,p=5} \approx \frac{i^{2(p-N)+1} \sum_{L=0}^{N} \sum_{H=0}^{L} \sum_{\alpha_{1} \ldots \alpha_{L}=0}^{M} \sum_{j_{1} \ldots j_{H}=p+1}^{M} \lambda^{2} \frac{1}{2 \ldots N} (X^{j_{1}} \ldots X^{j_{H}})^{2} \partial_{a_{1}} \partial_{a_{L}} \tilde{R}^{a_{1}a_{2}a_{3}a_{4}}}{H_{p=4}^{1}} \]  

(53)

\[ \Pi_{fermionic,anti-brane,p=5} \approx \frac{-i^{2(p-N)+1} \sum_{L=0}^{N} \sum_{H=0}^{L} \sum_{\alpha_{1} \ldots \alpha_{L}=0}^{M} \sum_{j_{1} \ldots j_{H}=p+1}^{M} \lambda^{2} \frac{1}{2 \ldots N} (X^{j_{1}} \ldots X^{j_{H}})^{2} \partial_{a_{1}} \partial_{a_{L}} \tilde{R}^{a_{1}a_{2}a_{3}a_{4}}}{H_{p=4}^{1}} \]  

(54)

We replace derivatives of order \( p = 5 \) by these momentums and obtain the Hamiltonian as follows:
\[ H_1^{p-5} \approx 4\pi \int d\sigma^{p-1} \sum_{L=0}^{N} \sum_{H=0}^{N-L} \sum_{a_1L=0}^{M} \sum_{j_{1H}=p+1}^{+1} \frac{\lambda^2}{1.2...N} (X_{j_1}^a X_{j_1}^b)^2 \partial_{a_5} \cdot \partial_{a_6} \hat{R}_{a_5 a_6 a_3 a_4} \Pi_{bosonic,brane,p-5} s(p-N) \times \]
\[ \sqrt{1 + \left( \frac{i^{2(p-N)} k_{bosonic,brane,p-4}}{\sigma^{p-1}} \right)^2} \]
\[ -4\pi \int d\sigma^{p-1} \sum_{L=0}^{N} \sum_{H=0}^{N-L} \sum_{a_1L=0}^{M} \sum_{j_{1H}=p+1}^{+1} \frac{\lambda^2}{1.2...N} (X_{j_1}^a X_{j_1}^b)^2 \partial_{a_5} \cdot \partial_{a_6} \hat{R}_{a_5 a_6 a_3 a_4} \Pi_{fermionic,brane,p-5} s(p-N+1) \times \]
\[ \sqrt{1 + \left( \frac{i^{2(p-N)+1} k_{fermionic,brane,p-4}}{\sigma^{p-1}} \right)^2} \]
\[ +4\pi \int d\sigma^{p-1} \sum_{L=0}^{N} \sum_{H=0}^{N-L} \sum_{a_1L=0}^{M} \sum_{j_{1H}=p+1}^{+1} \frac{\lambda^2}{1.2...N} (X_{j_1}^a X_{j_1}^b)^2 \partial_{a_5} \cdot \partial_{a_6} \hat{R}_{a_5 a_6 a_3 a_4} \Pi_{bosonic,anti-brane,p-5} s(p-N+1) \times \]
\[ \sqrt{1 + \left( \frac{i^{2(p-N)+1} k_{bosonic,anti-brane,p-4}}{\sigma^{p-1}} \right)^2} \]
\[ -4\pi \int d\sigma^{p-1} \sum_{L=0}^{N} \sum_{H=0}^{N-L} \sum_{a_1L=0}^{M} \sum_{j_{1H}=p+1}^{+1} \frac{\lambda^2}{1.2...N} (X_{j_1}^a X_{j_1}^b)^2 \partial_{a_5} \cdot \partial_{a_6} \hat{R}_{a_5 a_6 a_3 a_4} \times \]
\[ \partial_{a_5} \left( \sqrt{1 + \left( \frac{i^{2(p-N)} k_{bosonic,brane,p-4}}{\sigma^{p-1}} \right)^2} \sigma^{p-1} \Pi_{bosonic,brane,p-5} s(p-N) \right) \]
\[ -4\pi \int d\sigma^{p-N} \sum_{L=0}^{N} \sum_{H=0}^{N-L} \sum_{a_1L=0}^{M} \sum_{j_{1H}=p+1}^{+1} \frac{\lambda^2}{1.2...N} (X_{j_1}^a X_{j_1}^b)^2 \partial_{a_5} \cdot \partial_{a_6} \hat{R}_{a_5 a_6 a_3 a_4} \times \]
\[ \partial_{a_5} \left( \sqrt{1 + \left( \frac{i^{2(p-N)+1} k_{fermionic,brane,p-4}}{\sigma^{p-1}} \right)^2} \sigma^{p-1} \Pi_{fermionic,brane,p-5} s(p-N+1) \right) \]
\[ -4\pi \int d\sigma^{p-N} \sum_{L=0}^{N} \sum_{H=0}^{N-L} \sum_{a_1L=0}^{M} \sum_{j_{1H}=p+1}^{+1} \frac{\lambda^2}{1.2...N} (X_{j_1}^a X_{j_1}^b)^2 \partial_{a_5} \cdot \partial_{a_6} \hat{R}_{a_5 a_6 a_3 a_4} \times \]
\[ \partial_{a_5} \left( \sqrt{1 + \left( \frac{i^{2(p-N)+1} k_{bosonic,anti-brane,p-4}}{\sigma^{p-1}} \right)^2} \sigma^{p-1} \Pi_{bosonic,anti-brane,p-5} s(p-N+1) \right) \]
\[ -L^1_{bosonic,brane,p-4} - L^1_{fermionic,brane,p-4} - L^1_{bosonic,anti-brane,p-4} - L^1_{fermionic,anti-brane,p-4} \]

Similar to previous stage, we use the constraints \( (\partial_{a_5} \left( \sqrt{1 + \left( \frac{i^{2(p-N)} k_{bosonic,brane,p-4}}{\sigma^{p-1}} \right)^2} \sigma^{p-1} \Pi_{bosonic,brane,p-5} s(p-N) \right) = 0) \) and obtain the following momentums:
\[ \Pi^1_{\text{bosonic,brane},p-5} = \frac{i^{2(p-N)} k_{\text{bosonic,brane},p-5}}{\sigma^{p-1}} \left[ 1 + \frac{2^{(p-N)} k_{\text{bosonic,brane},p-4}}{\sigma^{p-1}} \right]^2 \]

\[ \Pi^1_{\text{fermionic,brane},p-4} = \frac{i^{2(p-N)+1} k_{\text{fermionic,brane},p-4}}{\sigma^{p-1}} \left[ 1 + \frac{2^{(p-N)+1} k_{\text{fermionic,brane},p-4}}{\sigma^{p-1}} \right]^2 \]

\[ \Pi^1_{\text{bosonic,anti-brane},p-5} = \frac{i^{2(p-N+1)} k_{\text{bosonic,anti-brane},p-5}}{\sigma^{p-1}} \left[ 1 + \frac{2^{(p-N+1)} k_{\text{bosonic,anti-brane},p-4}}{\sigma^{p-1}} \right]^2 \]

\[ \Pi^1_{\text{fermionic,anti-brane},p-5} = \frac{i^{2(p-N+1)+1} k_{\text{fermionic,anti-brane},p-5}}{\sigma^{p-1}} \left[ 1 + \frac{2^{(p-N+1)+1} k_{\text{fermionic,anti-brane},p-5}}{\sigma^{p-1}} \right]^2 \] (56)

Substituting these momentums in Hamiltonian (55), we derive the following Hamiltonian:

\[ H^1_{p-4} \approx 4\pi \int d\sigma^{p-1} \left[ \frac{1}{2} \sqrt{2(p-N)} \sum_{L=0}^{N} \sum_{j=0}^{j-1} \sum_{j=1}^{l+1} (X_{i\dot{i}} X_{\dot{j}h-1})^2 \langle \partial_{a_{1}} \ldots \partial_{a_{L}} X_{i} \rangle^{\dot{i},1} \ldots \partial_{a_{L}} X^{1} \right] \]

\[ \left[ 1 + \left( \frac{i^{2(p-N)} k_{\text{bosonic,brane},p-5}}{\sigma^{p-1}} \right)^2 \right] \]

\[ \left[ 1 + \left( \frac{i^{2(p-N)+1} k_{\text{fermionic,brane},p-4}}{\sigma^{p-1}} \right)^2 \right] \]

\[ \left[ 1 + \left( \frac{i^{2(p-N+1)} k_{\text{bosonic,brane},p-5}}{\sigma^{p-1}} \right)^2 \right] \]

\[ \left[ 1 + \left( \frac{i^{2(p-N+1)+1} k_{\text{fermionic,brane},p-5}}{\sigma^{p-1}} \right)^2 \right] \]

\[ \left[ 1 + \left( \frac{i^{2(p-N+1)+1} k_{\text{fermionic,brane},p-5}}{\sigma^{p-1}} \right)^2 \right] \]

\[ \left[ 1 + \left( \frac{i^{2(p-N+1)+1} k_{\text{fermionic,brane},p-5}}{\sigma^{p-1}} \right)^2 \right] \]

After doing some mathematical calculations, we can remove all derivatives respect to curvatures and obtain the Hamiltonian as follows:
$H_t = 4\pi \int d\sigma \sigma^{p-1} \left[ \left( \frac{1}{2} \gamma^{p} \gamma^{N} \right)^{N-L} \sum_{i=1}^{N} \sum_{a=L}^{p} \sum_{j=1}^{j_H} \sum_{a_L=1}^{j_H} (X^{i_H} - X^{i_H-1})^2 (\partial_{a} \partial_{a} X^{i_H}, \partial_{a_{L}} \partial_{a_{L}} X^{i_H}) + ... \right]^{1/2} \times$

$F_{bosonic,brane,tot} + \left[ -\frac{1}{2} \gamma^{p} \gamma^{N} \gamma^{N-L} \sum_{i=1}^{N} \sum_{a=L}^{p} \sum_{j=1}^{j_H} \sum_{a_L=1}^{j_H} (X^{i_H} - X^{i_H-1})^2 (a_{L} \partial_{a} \partial_{a} \psi^{i_H}, \partial_{a} \partial_{a} \psi^{i_H}) + ... \right]^{1/2} \times$

$F_{fermionic,brane,tot} + \left[ -\frac{1}{2} \gamma^{p} \gamma^{N} \gamma^{N-L} \sum_{i=1}^{N} \sum_{a=L}^{p} \sum_{j=1}^{j_H} \sum_{a_L=1}^{j_H} (X^{i_H} - X^{i_H-1})^2 (a_{L} \partial_{a} \partial_{a} \psi^{i_H}, \partial_{a} \partial_{a} \psi^{i_H}) + ... \right]^{1/2} \times$

$F_{bosonic,anti-brane,tot} + \left[ -\frac{1}{2} \gamma^{p} \gamma^{N} \gamma^{N-L} \sum_{i=1}^{N} \sum_{a=L}^{p} \sum_{j=1}^{j_H} \sum_{a_L=1}^{j_H} (X^{i_H} - X^{i_H-1})^2 (a_{L} \partial_{a} \partial_{a} \psi^{i_H}, \partial_{a} \partial_{a} \psi^{i_H}) + ... \right]^{1/2} \times$

$F_{fermionic,anti-brane,tot}$

where functions of $F$ are defined as follows:

$F_{bosonic,brane,tot} = 1 + \left( \frac{i^2(p-N)}{\sigma^{p-1}} \right) \left( \frac{i^2(p-N)}{\sigma^{p-1}} \right)^2$

$F_{fermionic,brane,tot} = 1 + \left( \frac{i^2(p-N) + 1}{\sigma^{p-1}} \right) \left( \frac{i^2(p-N) + 1}{\sigma^{p-1}} \right)^2$

$F_{bosonic,anti-brane,tot} = 1 + \left( \frac{i^2(p-N) + 1}{\sigma^{p-1}} \right) \left( \frac{i^2(p-N) + 1}{\sigma^{p-1}} \right)^2$

$F_{fermionic,anti-brane,tot} = 1 + \left( \frac{i^2(p-N) + 1}{\sigma^{p-1}} \right) \left( \frac{i^2(p-N) + 1}{\sigma^{p-1}} \right)^2$

These results are the very same as Hamiltonians of Blon in [7, 8, 28]. It is clear from the above equations that curvatures of bosonic gravitons and fermionic gravitinoes produce two types of wormholes in which their signatures and couplings are different. These wormholes can act against each other and also cancel the effect of each other. In addition to that, the sign of Hamiltonians of wormholes which are created by bosonic gravitons and fermionic gravitinoes on anti-branes is opposite. This means that the potential energy of one brane has negative sign and it attracts particles and the potential energy of another brane has positive sign and it repels particles and thus particles move from one brane to another and a wormhole is formed between two branes. Now, we simplify calculations by choosing $x^0 = i t, x^1 = z, X^0 = t, X^1 = z, X^1 = 0, i \neq 0, 1, \psi^{0} = t, \psi^{1} = y+iy, \psi^{i} = 0, i \neq 0, 1, \psi^{2(p-N)} = 1$ and $\gamma^{a_{L-1}} = \gamma^{2(N-L-1)}$ where indices $\pm$ denote the fields on brane and anti-branes respectively. Also, $\sigma$ denotes the separation between quarks on one brane, $z$ and $y$ refer to lengths of bosonic and fermionic wormholes between branes and their $n^{th}$ derivatives are shown by $z^{n(\psi)}, y^{n(\psi)}$. Putting these assumption in Eq. (58), we rewrite Hamiltonian as:
\[
H_{\text{tot}} \approx 4\pi \int d\sigma \sigma^{p-1} \left( 1 + \sum_{n=1}^{N-1} \left( \frac{(N-n-1)z_+^{(n)}}{z_+ (n)} \right)^{1/2} F_{\text{bosonic,brane,tot}} + \left[ -1 + \sum_{n=1}^{N-1} (-iz_+)^{2(N-n-1)}y_+ y_+^{(n)} \right]^{1/2} F_{\text{fermionic,brane,tot}} + \left[ -1 + i2N-2\sum_{n=1}^{N-1} (z_+^{(n)} n_+^{(n)}) \right]^{1/2} F_{\text{bosonic,anti-brane,tot}} + \left[ 1 + i2N-2\sum_{n=1}^{N-1} (-iz_-)^{2(N-n-1)}y_- y_-^{(n)} \right]^{1/2} F_{\text{fermionic,anti-brane,tot}} \right)
\]

Now, we can obtain the equation of motion for \(z\) and \(y\):

\[
\left( \sum_{n=1}^{N-1} (-1)^n (z_+^{(n)} x_+^{(n)} z_+^{(n)})^{1/2} \right) = \left[ 1 + \sum_{n=1}^{N-1} \left( \frac{z_+^{(n)} z_+^{(n)}}{z_+^{(n)}} \right)^{1/2} F_{\text{bosonic,brane,tot}} \right]
\]

\[
\left( \sum_{n=1}^{N-1} (-iz_+)^{2(N-n-1)}y_+ y_+^{(n)} \right) = \left[ 1 + \sum_{n=1}^{N-1} (-iz_+)^{2(N-n-1)}y_+ y_+^{(n)} \right]^{1/2} F_{\text{fermionic,brane,tot}}
\]

\[
\left( \sum_{n=1}^{N-1} (z_+^{(n)} x_+^{(n)} z_+^{(n)})^{1/2} \right) = \left[ 1 + \sum_{n=1}^{N-1} \left( \frac{z_+^{(n)} z_+^{(n)}}{z_+^{(n)}} \right)^{1/2} F_{\text{bosonic,anti-brane,tot}} \right]
\]

\[
\left( \sum_{n=1}^{N-1} (iz_-)^{2(N-n-1)}y_- y_-^{(n)} \right) = \left[ 1 + i2N-2\sum_{n=1}^{N-1} (z_-^{(n)} n_+^{(n)}) \right]^{1/2} F_{\text{fermionic,anti-brane,tot}}
\]

Solving these equations, we obtain:

\[
z_+ = z_{+,0} \sum_{n=0}^{N-1} e^{- \int d^n \sigma F_{\text{bosonic,brane,tot}}(\sigma) \frac{1}{F_{\text{bosonic,brane,tot}}(\sigma_0, \text{bosonic,brane}) - F_{\text{bosonic,brane,tot}}(\sigma)} \left( 1 + \int d^n \sigma F_{\text{bosonic,brane,tot}}(\sigma) (F_{\text{bosonic,brane,tot}}(\sigma_0, \text{bosonic,brane}) - F_{\text{bosonic,brane,tot}}(\sigma)) \sin(n\sigma) \right)}
\]

\[
y_+ = y_{+,0} \sum_{n=0}^{N-1} e^{- \int d^n \sigma F_{\text{fermionic,brane,tot}}(\sigma) \frac{1}{F_{\text{fermionic,brane,tot}}(\sigma_0, \text{fermionic,brane}) - F_{\text{fermionic,brane,tot}}(\sigma)} \left( 1 + \int d^n \sigma F_{\text{fermionic,brane,tot}}(\sigma) (F_{\text{fermionic,brane,tot}}(\sigma_0, \text{fermionic,brane}) - F_{\text{fermionic,brane,tot}}(\sigma)) \cos(n\sigma) \right)}
\]

\[
z_- = z_{-,0} \sum_{n=0}^{N-1} e^{- \int d^n \sigma F_{\text{bosonic,anti-brane,tot}}(\sigma) \frac{1}{F_{\text{bosonic,anti-brane,tot}}(\sigma_0, \text{bosonic,anti-brane}) - F_{\text{bosonic,anti-brane,tot}}(\sigma)} \left( 1 + \int d^n \sigma F_{\text{bosonic,anti-brane,tot}}(\sigma) (F_{\text{bosonic,anti-brane,tot}}(\sigma_0, \text{bosonic,anti-brane}) - F_{\text{bosonic,anti-brane,tot}}(\sigma)) \sin(n\sigma) \right)}
\]

\[
y_- = y_{-,0} \sum_{n=0}^{N-1} e^{- \int d^n \sigma F_{\text{fermionic,anti-brane,tot}}(\sigma) \frac{1}{F_{\text{fermionic,anti-brane,tot}}(\sigma_0, \text{fermionic,anti-brane}) - F_{\text{fermionic,anti-brane,tot}}(\sigma)} \left( 1 + \int d^n \sigma F_{\text{fermionic,anti-brane,tot}}(\sigma) (F_{\text{fermionic,anti-brane,tot}}(\sigma_0, \text{fermionic,anti-brane}) - F_{\text{fermionic,anti-brane,tot}}(\sigma)) \cos(n\sigma) \right)}
\]
between quarks and anti-quarks. Thus, these solutions show that at $\sigma = 0$, the length of gravitonic wormholes is zero, by increasing the separation distance between two quarks ($\sigma$), this length grows, turns over a maximum and reduces to zero at throat $\sigma_0$ and then one new fermionic wormhole is born, it’s length increases with increasing $\sigma$ and tends to infinity at $\sigma = \infty$. On the other hand, the length of fermionic wormholes is $\infty$ at $\sigma = 0$ and reduces to zero at throat $\sigma_0$, then one new fermionic wormhole is formed, it’s length grows with increasing $\sigma$, turns over a maximum and reduces to zero at $\infty$. Thus, fermionic and bosonic wormholes act against to each other and this prevents the closing in and getting away of quarks from each other.

Using Eqs. (65-68) in Eq. (60), we obtain the potential for this system:

$$H_{\text{tot}} \approx V_{\text{tot}} = V_{\text{bosonic,brane,tot}} + V_{\text{fermionic,brane,tot}} + V_{\text{bosonic,anti-brane,tot}} + V_{\text{fermionic,anti-brane,tot}}$$

(69)
\[ V_{\text{bosonic,anti-brane,tot}} \]
\[ \approx 4\pi \int d\sigma \left( \left( \sum_{n'=1}^{N-1} z_{-0} \sum_{n=0}^{N-1} e^{-f} \right) F_{\text{bosonic,anti-brane,tot}}(\sigma) \right) \]
\[ \left( 1 + \int d^{n'} \sigma F_{\text{bosonic,anti-brane,tot}}(\sigma)(F_{n}^{m} - F_{n}^{m}) F_{\text{bosonic,anti-brane,tot}}(\sigma) - F_{\text{bosonic,anti-brane,tot}}(\sigma) \right) \times \]
\[ \left( F_{n}^{m} - F_{n}^{m} \right) F_{\text{bosonic,anti-brane,tot}}(\sigma) \times \]
\[ e^{-f} \sigma F_{\text{bosonic,anti-brane,tot}}(\sigma) \]
\[ e^{-f} \sigma F_{\text{bosonic,anti-brane,tot}}(\sigma) \]
\[ F_{n}^{m} \sigma F_{\text{bosonic,anti-brane,tot}}(\sigma) \]
\[ F_{n}^{m} \sigma F_{\text{bosonic,anti-brane,tot}}(\sigma) \]
\[ F_{n}^{m} \sigma F_{\text{bosonic,anti-brane,tot}}(\sigma) \]

For simplicity, we assume that \( \sigma_{0,\text{bosonic,brane}} = \sigma_{0,\text{bosonic,anti-brane}} \) and \( \sigma_{0,\text{fermionic,brane}} = \sigma_{0,\text{fermionic,anti-brane}} \) and obtain the potentials and their relative forces between quarks and anti-quarks approximately:

\[ V_{\text{tot}} = V_{\text{bosonic,brane+anti-brane,tot}} + V_{\text{fermionic,brane+anti-brane,tot}} \]

\[ V_{\text{bosonic,brane+anti-brane,tot}} \approx -\sum_{m=1}^{P} \sum_{n=1}^{N-1} [k_{\text{bosonic,brane}} - k_{\text{bosonic,anti-brane}}] m \sigma^{m} \left( \sigma_{0,\text{bosonic,brane}} - \sigma_{nm} \right) \]

\[ F_{\text{bosonic}} \approx \sum_{m=1}^{P} \sum_{n=1}^{N-1} [k_{\text{bosonic,brane}} - k_{\text{bosonic,anti-brane}}] m \sigma^{m} \sigma_{nm} \]

\[ F_{\text{bosonic}} \approx -\sum_{m=1}^{P} \sum_{n=1}^{N-1} [k_{\text{bosonic,brane}} - k_{\text{bosonic,anti-brane}}] m \sigma^{m} \sigma_{nm} \]

(75)
\[ V_{\text{fermionic,brane}+\text{anti-brane,tot}} \approx \]
\[ \Sigma_{m=1}^{n-1} \Sigma_{n=1}^{m-1} [k_{\text{fermionic,brane}} - k_{\text{fermionic,anti-brane}}] \left( \frac{1}{\sigma_m} \sigma_{nm} - \sigma_{nm}^{\text{fermionic,brane}} \right) \]

For \( \sigma \ll \sigma_{0,\text{fermionic,brane}} \):
\[ F_{\text{fermionic}} \approx \Sigma_{m=1}^{n-1} \Sigma_{n=1}^{m-1} [k_{\text{fermionic,brane}} - k_{\text{fermionic,anti-brane}}] \left( \frac{m + nm}{\sigma_{m+1} \sigma_{0,\text{fermionic,brane}}} \right) \]

For \( \sigma \gg \sigma_{0,\text{fermionic,brane}} \):
\[ F_{\text{fermionic}} \approx -\Sigma_{m=1}^{n-1} \Sigma_{n=1}^{m-1} [k_{\text{fermionic,brane}} - k_{\text{fermionic,anti-brane}}] \left( \frac{m \times m + 1}{\sigma_{m+1} \sigma_{0,\text{fermionic,brane}}} \right) \sigma_{nm} + \sigma_{nm+1} \]

These results show that when quarks and anti-quarks are very close (\( \sigma = 0 \)), the potential of gravitonic wormholes is zero. By increasing the separation distance between these particles, bosonic wormhole produces a repulsive potential and anti-gravity force which first grows, turns over a maximum and then shrinks to zero at \( \sigma_{0,\text{bosonic,brane}} \). After this distance, bosonic potential becomes attractive, gravity emerges and prevents the getting away of quarks from anti-quarks. On the other hand, the gravitino produces a wormhole which leads to creation of repulsive potential and anti-gravity for small separation distance. This potential is \( \infty \) at \( \sigma = 0 \) and causes the quarks and anti-quarks to get away from each other. By increasing the separation distance between these particles, repulsive gravity decreases and shrinks to zero at \( \sigma_{0,\text{fermionic,brane}} \). Then, the sign of potential reverses and anti-gravity changes to gravity. This gravity grows, turns over a maximum and shrinks to zero at \( \infty \).

### III. THERMAL QUARKONIUM IN A THERMAL BION

Until now, we have shown that for small separation distance between quarks and anti-quarks (\( \sigma < \sigma_{0,\text{bosonic/fermionic,brane}} \)), the gravitational potentials which are produced by bosonic and fermionic wormholes, are repulsive and thus, one repulsive force causes the getting away of particles from each other. However, for large distance between quarks and anti-quarks (\( \sigma > \sigma_{0,\text{bosonic/fermionic,brane}} \)), the gravitational potential is attractive and thus, attractive force leads to closing particles toward each other. In this section, we show that by increasing temperature, the boundary between repulsive and attractive potential (\( \sigma_{0,\text{bosonic/fermionic,brane}} \)) tends to infinity (\( \infty \)) and consequently, attractive force is removed and quarks and anti-quarks become free.

To show this, we use the method in [29] and assume that one gauge field like one photon moves between quarks and anti-quarks. The wave equation for this particle is:
\[ \frac{\partial^2 A^i}{\partial t^2} + \frac{\partial^2 A^i}{\partial z^2} = 0 \]  
(77)

Here, \( z \) is the length of wormhole which connects quark and anti-quarks. Using the below re-parameterizations [29]:
\[ \rho = \frac{z^2}{w}, \]
\[ w = \frac{V_{\text{tot}}}{2E_{\text{system}}}, \]
\[ \tilde{\tau} = \gamma \int_0^t \frac{w}{\tilde{w}} - \gamma \frac{z^2}{2} \]  
(78)

and doing the below calculations:
\[ \left\{ \left( \frac{\partial^2}{\partial \tilde{\tau}^2} - \frac{\partial^2}{\partial \tilde{\tau} \partial z} \right) + \left( \frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial \rho^2} \right) \right\} X^i = 0 \]  
(79)

we get [29]:
\[ (-g)^{-1/2} \frac{\partial}{\partial x_\mu} \left[ (-g)^{1/2} g^{\mu \nu} \right] \frac{\partial}{\partial x_\nu} X^i = 0 \]  
(80)
where \( x_0 = \tau, x_1 = \rho \) and the line elements are obtained by:

\[
g^{xx} \sim \frac{1}{\beta^2} \left( \frac{w'}{w} \right)^2 \frac{(1 - \left( \frac{w}{w'} \right)^2)}{1 + \left( \frac{w}{w'} \right)^2 (1 + \gamma^{-2})}^{1/2} \]

\[
g^{\rho\rho} \sim -(g^{xx})^{-1} \quad (81)
\]

At this stage, we compare above elements with the metric of a thermal BIon \([29]\):

\[
ds^2 = D^{-1/2} \bar{H}^{-1/2} (-f \, dt^2 + dx_1^2) + D^{1/2} \bar{H}^{-1/2} (dx_2^2 + dx_3^2) + D^{-1/2} \bar{H}^{1/2} (f^{-1} \, dr^2 + r^2 d\Omega_5)^2
\]

where

\[
f = 1 - \frac{r_0^4}{r^4}, \quad \bar{H} = 1 + \frac{r_0^4}{r^4} \sinh^2 \alpha \]

\[
D^{-1} = \cos^2 \varepsilon + \bar{H}^{-1} \sin^2 \varepsilon \]

\[
\cos \varepsilon = \frac{1}{\sqrt{1 + \frac{\beta^2}{\sigma^2}}}
\]

Comparing the metric of \([33]\) with the metric of \([31]\), we derive the following relations \([29]\):

\[
f = 1 - \frac{r_0^4}{r^4} \sim 1 - \left( \frac{w'}{w} \right)^2 \frac{1}{z^4},
\]

\[
\bar{H} = 1 + \frac{r_0^4}{r^4} \sinh^2 \alpha \sim 1 + \left( \frac{w}{w'} \right)^2 \frac{(1 + \gamma^{-2})}{z^4}
\]

\[
D^{-1} = \cos^2 \varepsilon + \bar{H}^{-1} \sin^2 \varepsilon \sim 1
\]

\[
\Rightarrow r \sim z, r_0 \sim \left( \frac{w}{w'} \right)^{1/2}, (1 + \gamma^{-2}) \sim \sinh^2 \alpha
\]

\[
\cosh^2 \alpha \sim \frac{3 \cos^4 \frac{\delta}{2} + \sqrt{3} \sin^2 \frac{\delta}{2}}{\cos \delta}
\]

\[
\cos \delta \equiv \frac{T}{F_{\text{Total}}}, \quad T \equiv \frac{T}{T_c}
\]

\[
F_{\text{Total}} = F_{\text{bosonic,brane,tot}} + F_{\text{fermionic,brane,tot}} + F_{\text{bosonic,anti-brane,tot}} + F_{\text{fermionic,anti-brane,tot}} \quad (84)
\]

The Bionic temperature is defined by \( T = \frac{1}{\pi r_0 \cosh \alpha} \). Consequently, the temperature of a BIon has the following relation with the potential:

\[
T = \frac{1}{\pi r_0 \cosh \alpha} = \frac{\gamma}{\pi} \left( \frac{w'}{w} \right)^{1/2} \sim \frac{\gamma}{\pi} \left( \frac{V_{\text{tot}}'}{V_{\text{tot}}} \right)^{1/2} \sim
\]

\[
\frac{\gamma}{\pi} \left[ -\sum_{m=1}^{P-1} \sum_{n=1}^{N-1} \left[ k_{\text{bosonic,brane}, - k_{\text{bosonic,anti-brane}}} \right]^m \sigma_{m-1}^{\text{m}} \left( \sigma_{0, \text{bosonic,brane}} - \sigma_{0, \text{norm}} \right) + \right.
\]

\[
\left. \sum_{m=1}^{P-1} \sum_{n=1}^{N-1} \left[ k_{\text{bosonic,brane}, - k_{\text{bosonic,anti-brane}}} \right]^m \sigma_{m}^{\text{m}} \left( \sigma_{0, \text{norm}} - \sigma_{0, \text{fermionic,brane}} \right) - \right.
\]

\[
\left. \sum_{m=1}^{P-1} \sum_{n=1}^{N-1} \left[ k_{\text{fermionic,brane}, - k_{\text{fermionic,anti-brane}}} \right]^m \sigma_{m}^{\text{m}} \left( \sigma_{0, \text{fermionic,brane}} - \sigma_{0, \text{norm}} \right) \right] \right]^{1/2} \times
\]

\[
\left[ -\sum_{m=1}^{P-1} \sum_{n=1}^{N-1} \left[ k_{\text{bosonic,brane}, - k_{\text{bosonic,anti-brane}}} \right]^m \sigma_{m}^{\text{m}} \left( \sigma_{0, \text{bosonic,brane}} - \sigma_{0, \text{norm}} \right) + \right.
\]

\[
\left. \sum_{m=1}^{P-1} \sum_{n=1}^{N-1} \left[ k_{\text{fermionic,brane}, - k_{\text{fermionic,anti-brane}}} \right]^m \sigma_{m}^{\text{m}} \left( \sigma_{0, \text{norm}} - \sigma_{0, \text{fermionic,brane}} \right) \right]^{-1/2} \quad (85)
\]
From this point of view that $\gamma$ has the relation the temperature, we get \cite{20}:

\[
\gamma = \frac{1}{\cos \alpha} \sim \frac{2\cos \delta}{3\sqrt{3} - \cos \delta - \frac{2}{3}\cos^2 \delta} \sim \frac{2T^4}{3\sqrt{3} - T^4} F_{\text{Total}} - \frac{\sqrt{3}T^3}{3T^4} F_{\text{Total}}^2
\]

(86)

To similarity, we assume that throats of bosonic and fermionic wormholes have the same size ($\sigma_{0, \text{bosonic,brane}} = \sigma_{0, \text{fermionic,brane}} = \sigma_{0, \text{fermionic,anti-brane}}$) and also ($k_{\text{bosonic,brane}} - k_{\text{bosonic,anti-brane}} = k_{\text{fermionic,brane}} - k_{\text{fermionic,anti-brane}}$). Using Eqs (84, 85 and 86), we can obtain the approximate form of the separation distance between quarks and anti-quarks in terms of temperature:

\[
\sigma \approx \sum_{m=1}^{P-1} \sum_{n=1}^{N-1} [k_{\text{bosonic,brane}} - k_{\text{bosonic,anti-brane}}]^{-m} \left[ \left( \frac{\sqrt{3}}{6} T^2 \right)^{\frac{1}{m+\gamma+1}} + \left( \frac{T}{2} \right)^{\frac{1}{m+\gamma+1}} \right] \times \left[ \left( \frac{\sqrt{3}}{6} T^2 \right)^{\frac{1}{m+\gamma+1}} + \left( \frac{T}{2} \right)^{\frac{1}{m+\gamma+1}} \right] \times \left[ \frac{T}{T_c - T} \right]^{\frac{1}{m+\gamma+1}}
\]

(87)

This equation shows that by increasing temperature, the place of boundary between repulsive and attractive force ($\sigma_{0, \text{bosonic/fermionic,brane/anti-brane}}$) changes and goes to infinity at a critical temperature and thus, quarks and anti-quarks become free at this point. This result is in agreement with experiments. In fact, by increasing temperature, energy of particles increases and they can overcome attractive force and deconfinement emerges. We will demonstrate this by calculating the bosonic and fermionic potentials in terms of temperature. The relation between entropies and potentials are as follows \cite{19}:

\[
F_{\text{tot}} = M_{\text{tot}} - T \tilde{S}
\]

(88)

where $F_{\text{tot}}$ is the free energy for this system which has direct relation with Hamiltonian and potential ($F_{\text{tot}} \approx V_{\text{tot}}$). Also, $M_{\text{tot}}$ is total mass of system which is related to total energy of system $E_{\text{tot}} = M_{\text{tot}}$, $T$ is temperature and $S$ is entropy. In this mechanism, all things are produced from nothing as discussed already and after Eq. (32) and thus $E_{\text{tot}}$ is zero and $\tilde{S} \approx \frac{V_{\text{tot}}}{T}$. Substituting Eq. (87) in Eq. (74), we obtain potentials and entropies as:

\[
V_{\text{tot}} = V_{\text{bosonic,brane+anti-brane,tot}} + V_{\text{fermionic,brane+anti-brane,tot}}
\]

\[
\tilde{S}_{\text{tot}} = \tilde{S}_{\text{bosonic,brane+anti-brane,tot}} + \tilde{S}_{\text{fermionic,brane+anti-brane,tot}} \approx \frac{V_{\text{bosonic,brane+anti-brane,tot}}}{T} + \frac{V_{\text{fermionic,brane+anti-brane,tot}}}{T}
\]

(89)

\[
\tilde{S}_{\text{bosonic,brane+anti-brane,tot}} \approx \frac{V_{\text{bosonic,brane+anti-brane,tot}}}{T} \approx \frac{1}{T} \sum_{m=1}^{P-1} \sum_{n=1}^{N-1} [k_{\text{bosonic,brane}} - k_{\text{bosonic,anti-brane}}]^{-m} \times \left[ \left( \frac{\sqrt{3}}{6} T^2 \right)^{\frac{1}{m+\gamma+1}} + \left( \frac{T}{2} \right)^{\frac{1}{m+\gamma+1}} \right] \times \left[ \left( \frac{\sqrt{3}}{6} T^2 \right)^{\frac{1}{m+\gamma+1}} + \left( \frac{T}{2} \right)^{\frac{1}{m+\gamma+1}} \right] \times \left[ \frac{T}{T_c - T} \right]^{\frac{1}{m+\gamma+1}}
\]

(90)
This entropy also consists of negative terms which reverse to bosonic one, produce repulsive force and positive terms it is also clear that for high temperature the repulsive force between bosonic states of quarks are more than attractive agreement with experimental data and previous predictions from QCD in [1–6]. All above dependences are describe

\[
F_{\text{bosonic, repulsive}} \approx \sum_{m=1}^{P} \sum_{n=1}^{N} [k_{\text{bosonic, brane}} - k_{\text{bosonic, anti-brane}}]^m m \times \\
[\sum_{m=1}^{P} \sum_{n=1}^{N} [k_{\text{bosonic, brane}} - k_{\text{bosonic, anti-brane}}]^{-m} \left( \frac{\sqrt{3} T^2}{6} T \right)^{n-m+1} + \left( \frac{T}{2} \right)^{n-m+1}]^{m-1} \times \\
[\sum_{m=1}^{P} \sum_{n=1}^{N} [k_{\text{bosonic, brane}} - k_{\text{bosonic, anti-brane}}]^{-m-nm} \times \\
\left( \frac{\sqrt{3} T^2}{6} T \right)^{\frac{n-m+1}{n-m+1}} + \left( \frac{T}{2} \right)^{\frac{n-m+1}{n-m+1}} \frac{T}{T_c - T} \right)^{\frac{n-m+1}{n-m+1}} \right]^{nm} 
\]

(91)

\[
F_{\text{bosonic, attractive}} \approx -\sum_{m=1}^{P} \sum_{n=1}^{N} [k_{\text{bosonic, brane}} - k_{\text{bosonic, anti-brane}}]^m m(m+1) \times \\
[\sum_{m=1}^{P} \sum_{n=1}^{N} [k_{\text{bosonic, brane}} - k_{\text{bosonic, anti-brane}}]^{-m} \left( \frac{\sqrt{3} T^2}{6} T \right)^{n-m+1} + \left( \frac{T}{2} \right)^{n-m+1}]^{m-1-nm} 
\]

(92)

\[
S_{\text{fermionic, brane + anti-brane, tot}} \approx -\frac{V_{\text{fermionic, brane + anti-brane, tot}}}{T} \approx \\
-\frac{1}{T} \sum_{m=1}^{P} \sum_{n=1}^{N} [k_{\text{fermionic, brane}} - k_{\text{fermionic, anti-brane}}]^{-m} \times \\
[\sum_{m=1}^{P} \sum_{n=1}^{N} [k_{\text{bosonic, brane}} - k_{\text{bosonic, anti-brane}}]^{-m} \left( \frac{\sqrt{3} T^2}{6} T \right)^{n-m+1} + \left( \frac{T}{2} \right)^{n-m+1}]^{-m-nm} \\
\left( \frac{\sqrt{3} T^2}{6} T \right)^{\frac{n-m+1}{n-m+1}} + \left( \frac{T}{2} \right)^{\frac{n-m+1}{n-m+1}} \frac{T}{T_c - T} \right)^{\frac{n-m+1}{n-m+1}} \right]^{nm} 
\]

(93)

\[
F_{\text{fermionic, repulsive}} \approx \sum_{m=1}^{P} \sum_{n=1}^{N} [k_{\text{fermionic, brane}} - k_{\text{fermionic, anti-brane}}]^m (m+nm) \times \\
[\sum_{m=1}^{P} \sum_{n=1}^{N} [k_{\text{bosonic, brane}} - k_{\text{bosonic, anti-brane}}]^{-m} \left( \frac{\sqrt{3} T^2}{6} T \right)^{n-m+1} + \left( \frac{T}{2} \right)^{n-m+1}]^{-m-nm} 
\]

(94)

\[
F_{\text{fermionic, attractive}} \approx -\sum_{m=1}^{P} \sum_{n=1}^{N} [k_{\text{fermionic, brane}} - k_{\text{fermionic, anti-brane}}]^m (m+1) \times \\
[\sum_{m=1}^{P} \sum_{n=1}^{N} [k_{\text{bosonic, brane}} - k_{\text{bosonic, anti-brane}}]^{-m} \left( \frac{\sqrt{3} T^2}{6} T \right)^{n-m+1} + \left( \frac{T}{2} \right)^{n-m+1}]^{-m-1} \times \\
[\sum_{m=1}^{P} \sum_{n=1}^{N} [k_{\text{bosonic, brane}} - k_{\text{bosonic, anti-brane}}]^{-m-nm} \times \\
\left( \frac{\sqrt{3} T^2}{6} T \right)^{\frac{n-m+1}{n-m+1}} + \left( \frac{T}{2} \right)^{\frac{n-m+1}{n-m+1}} \frac{T}{T_c - T} \right)^{\frac{n-m+1}{n-m+1}} \right]^{nm} 
\]

(95)

These results show that bosonic entropy is zero near $T = 0$, which grows with temperature and tends to infinity at $\infty$. This entropy includes two types of terms—some with positive sign and some with negative sign. Terms with positive sign produce repulsive force and those with negative sign create the attractive force. With increasing temperature, repulsive terms grow and tend to $\infty$ at $T = T_c$, while, attractive terms increase with lower velocity. On the other hand, the fermionic entropy is infinity near $T = 0$, decrease and shrink to zero at higher temperatures. This entropy also consists of negative terms which reverse to bosonic one, produce repulsive force and positive terms which reverse to the case of bosonic wormhole, create the attractive force. The attractive terms decrease faster than repulsive terms and shrink to zero at $T = T_c$. Thus, the repulsive force which is produced by both fermionic and bosonic wormholes overcomes attractive force at this point and produces deconfinement. These results are in good agreement with experimental data and previous predictions from QCD in [16]. All above dependences are describe on Figures 1-4. From figure 3, it is clear that for low temperature the attractive force between bosonic states of quarks are more than repelling force, while in figure 4, repulsive force for fermionic states of quarks is more than attractive force. If we sum over these forces, we observe that for low temperature quarks repel each other. This is in agreement with previous prediction of QCD that in a quarkonium quarks can’t become very close to each other. From figure 3, it is also clear that for high temperature the repulsive force between bosonic states of quarks are more than attractive force, while in figure 4, attractive force for fermionic states of quarks is more than repulsive force. If we sum over these
forces, we observe that for low temperature quarks attract each other. This is in agreement with previous prediction of QCD that in a quarkonium quarks can’t become very distant from each other. Thus, in our model, totally quarks can’t become very close or very distant from each other and are approximately free in middle distant in quarkonium. This is in agreement with QCD.

FIG. 1: The total entropy $\bar{S}_{\text{tot}}$ from Eq. (59).

FIG. 2: (left) The bosonic entropy $\bar{S}_{\text{bosonic,brane+anti-brane,tot}}$ from Eq. (50); (right) the fermionic entropy $\bar{S}_{\text{fermionic,brane+anti-brane,tot}}$ from Eq. (63).

In figure 5, we have obtained the bosonic and the fermionic potentials in terms of temperature for $N=3$, $P=4$ and $T_c = 1 GeV$. It is clear that when temperature of a quark and an anti-quark becomes zero in a quarkonium, they meet each other and both bosonic and fermionic potentials become infinite. In the middle temperature that quarks and anti-quarks are separated approximately. The negative bosonic potential and bosonic fermionic potential cancel the effect of each other and therefore existed a freedom like the same as predicted in QCD. By achieving temperature to critical temperature $(1 - 3 GeV)$, total potential becomes zero and a real deconfinement is appeared. These results are in agreement with previous prediction for deconfinement in [30–32]. By increasing temperature, two quarks become far from each other and both potentials grow and tend to large positive values. In these conditions, the deconfinement of system increases as can be seen in the energies of LHC (See for example [33]). Thus, our model gives true value for critical temperature which really has been seen in experiments.
FIG. 3: (left) The repulsive force $F_{bosonic, repulsive}$ from Eq. (91); (right) the attractive force $F_{bosonic, attractive}$ from Eq. (92).

FIG. 4: (left) The repulsive force $F_{fermionic, repulsive}$ from Eq. (94); (right) the attractive force $F_{fermionic, attractive}$ from Eq. (95).

FIG. 5: The bosonic and fermionic potential between quark and anti-quark for $N=3$, $P=4$ and $T_c = 1 GeV$. 
IV. SUMMARY AND CONCLUSION

In this paper, we have considered the process of birth of quarks, anti-quarks and confiding potential between them which leads to formation of quarkonium in a thermal BIon. Quarkonium is constructed of one quark and one anti-quark that are confined to each other and can’t become very close to each other or go much away from each other. By closing quarks to anti-quarks, they are paired and form an scalar system. However, by getting away of these particles, the fermionic properties overcome. Thus, we need a theory that fermions and bosons have the same origin and transit to each other in it. In M-theory, these two types of particles are completely independent and for this reason, we introduce BLNA-theory that has higher dimensions respect to M-theory and is reduced to it in 11-dimensions. In this theory, at the beginning, there is no degree of freedom and energy. Then, two types of energies emerge that are only different in their sign and sum over them is zero. Each of these energies creates some degrees of freedom which lead to production of two types of branes with opposite quantum numbers. Coinciding with the birth of these branes, some bosonic tensor fields are born with their rank is changed from zero to dimension of brane and appear as scalar fields and gravitons in four dimensions. These fields interact with fields of other branes and cause the branes to be compacted. By compacting of branes, fermions emerge which some of them with lower spins play the role of quarks and anti-quarks and some other with higher spins have the role of gravitino. In this system, gravitons create a bosonic wormhole that leads to attractive potential in large separation distance between quarks and anti-quarks and prevents them from getting away from each other. Also, gravitinos produce fermionic wormhole that causes to the emergence of repulsive force in small separation distance between quarks and anti-quarks and prevents them from closing into each other. The confiding potential which is produced by these wormholes can be reduced to previous predicted potential in QCD and experimental data. With increasing temperature, these two wormholes produce two types of entropies which lead to the emergence of repulsive force at higher temperatures. The bosonic entropy is zero near \( T = 0 \), which grows with temperature and tends to infinity at \( \infty \). This entropy contains two types of terms, positive terms which create repulsive force and negative terms which produce the attractive force. With increasing temperature, repulsive terms grow and tend to \( \infty \) at \( T = T_c \), while, attractive terms increase with lower velocity. Also, the fermionic entropy is infinity near \( T = 0 \), decreases and shrinks to zero at higher temperatures. This entropy also includes negative terms which reverse to bosonic one, produces repulsive force and positive terms which reverse to the case of bosonic wormhole, create the attractive force. The attractive terms decreases faster than repulsive terms and shrinks to zero at \( T = T_c \). Thus, total entropy produces repulsive force which overcomes to attractive force in higher temperature and leads to the deconfinement.

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