Nuclear matter properties in fragmentation reactions

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Abstract. Nuclear multifragmentation is closely related to the properties of nuclear matter. In particular, the possibility of strong cluster correlations is explored based on the antisymmetrized molecular dynamics calculations. We introduce the two-nucleon collision process to the final states in which the scattered nucleons may form clusters (such as deuterons, tritons and α particles). This cluster correlation has strong impacts on the whole collision dynamics. The multifragmentation data at 50 MeV/nucleon are well reproduced with this strong cluster correlation if pairs of clusters with small relative velocities are assumed to form light nuclei. Effects of the density dependence of symmetry energy are discussed by analyzing the compression-expansion dynamics and the emission of nucleons, clusters and fragment nuclei.

1. Introduction
In medium-energy heavy-ion collisions, the system is compressed at an early stage and then expands so that the whole system disintegrates into many intermediate mass fragments and light particles. The reaction dynamics can be regarded as a manifestation of nuclear matter properties, such as the equation of state and the liquid-gas phase transition. In addition to the mean-field aspect, many-body correlations are also an important aspect of matter properties.

The experimental data shows that the clusters are actually an important part of multifragmenting systems. In central Xe + Sn collisions at 50 MeV/nucleon [1], only about 10 % of the total protons in the system are emitted as free protons, while the other protons are bound in light particles (d, t, \(^{3}\)He, α) and heavier fragments in the final state. The number of protons in α particles (20 %) is more than the number of free protons (10 %). In theoretical studies, there are large discrepancies in the number of emitted nucleons predicted by different transport models (see Ref. [2] for example), and most transport models tend to overestimate the experimental data of the proton multiplicity. Therefore the cluster correlations may be much stronger than usually assumed by transport models.

We are recently extending the antisymmetrized molecular dynamics (AMD) approach in order to properly incorporate cluster correlations in the final states of two-nucleon collisions induced by the residual interaction. This model can explain the multiplicities of protons and clusters (such as α particles) as well as the fragment size distribution in central collisions at 50 MeV/nucleon. Currently there is a lot of interest in the equation of state of asymmetric nuclear matter at various densities [3, 4]. The model is also applied to the study on how the density dependence of symmetry energy is reflected in the collision dynamics and final observables such as the triton/\(^{3}\)He spectra and the isotopic yields of fragments.
2. Model

AMD [5] solves the time evolution of many-nucleon system starting with two boosted nuclei with a given impact parameter and a suitable distance between them. To describe the state at each time, AMD employs a single Slater determinant of Gaussian wave packets for \( A \) nucleons

\[
\langle \mathbf{r}_1 \ldots \mathbf{r}_A | \Phi (Z) \rangle \propto \det_{ij} \left[ \exp \left\{ -\nu (\mathbf{r}_i - \mathbf{Z}_j)/\sqrt{\nu} \right\} \chi_{\alpha_i} (i) \right],
\]

where \( \chi_{\alpha_i} \) are the spin-isospin states with \( \alpha_i = \uparrow, \downarrow, \uparrow \downarrow, \) or \( \downarrow \uparrow \). Thus the many-body state \( |\Phi (Z)\rangle \) is parametrized by a set of complex variables \( Z = \{ \mathbf{Z}_i \}_{i=1,\ldots,A} \). The width parameter \( \nu = (2.5 \text{ fm})^{-2} \) is treated as a constant parameter common to all the wave packets. The time evolution of the wave packet parameters \( Z \) is determined, up to the extensions described later, by applying the time-dependent variational principle from which the equation of motion for \( Z \) is obtained,

\[
i\hbar \sum_{j\tau} C_{\sigma j\tau} \frac{dZ_{j\tau}}{dt} = \frac{\partial \mathcal{H}}{\partial Z_{\sigma}^*},
\]

where \( \sigma, \tau = x, y, z \). The Hamiltonian \( \mathcal{H} \) is the expectation value of the Hamiltonian operator with an additional correction for the spurious zero-point energies of the center-of-mass motions of fragments. A positive definite Hermitian matrix \( C_{\sigma j\tau} \) appears in the equation of motion, suggesting that the variables \( Z \) are not canonical coordinates. The effective interactions for the mean-field calculations, such as the Gogny force and the Skyrme force, have been usually employed. The calculated results presented in this paper were obtained with the Skyrme SLy4 force [6] unless otherwise mentioned. The equation of motion can be interpreted intuitively as representing the motion of individual wave packets in the mean-field potential.

In addition to the mean-field effect, the residual interaction plays important roles in collisions at energies of more than 10 MeV/nucleon. This effect of two-nucleon collision is treated as a stochastic transition form an AMD state to one of the possible other AMD states. The probability is decided based on the differential cross section \((d\sigma/d\Omega)_{\text{NN}}\) of the two-nucleon scattering, which depends on the scattering energy and may be modified in nuclear medium. In the calculations presented in this paper, we employ the in-medium density-dependent total cross sections by Li and Machleidt [7] for the scattering energies \( E > 35 \text{ MeV} \) in the nucleon-nucleon center-of-mass system, while the cross sections at \( E = 35 \text{ MeV} \) are employed at lower energies. Angular distributions are assumed to be similar to the experimental data in the free space.

As the AMD variables \( Z \) do not have an intuitive meaning of nucleon coordinates due to the effect of antisymmetrization, we have introduced physical coordinates \( W = \{ \mathbf{W}_1, \ldots, \mathbf{W}_A \} \) which are nonlinear functions of \( Z \) [5]. Let us consider a two-nucleon collision between the 1st and the 2nd nucleons. The assumption, which is usually taken for granted but is essential for the discussion in this paper, is that only the two coordinates are changed by the collision with a randomly generated scattering angle, so that the physical coordinates after the collision are written as \( W' = \{ \mathbf{W}_1', \mathbf{W}_2', \mathbf{W}_3', \ldots, \mathbf{W}_A' \} \). The Pauli-blocking for the final state of the collision is considered with the physical coordinates.

The two-nucleon collision process in AMD is a transition from an AMD wave function \(|\Psi_f\rangle\) to one of possible final states \(|\Psi_f\rangle\) which is assumed to occur instantaneously and to conserve the energy expectation value \( \langle \Psi_f | H | \Psi_f \rangle = \langle \Psi_f | H | \Psi_f \rangle \). As the transition is induced by the residual interaction \( V \) between the two colliding nucleons, the transition rate may be expressed as

\[
W_{i \to f} = \frac{2\pi}{\hbar} |\langle \Psi_f | V | \Psi_i \rangle|^2 \delta (E_f - E_i)
\]

which is similar to Fermi’s golden rule. However, some care should be taken since the transition is considered here between states which are not eigenstates of the unperturbed Hamiltonian.
An important question here is how to choose the complete basis \( \{ \ket{\Psi_f} \} \) of final states. If the correlations exist in the final states between the scattered nucleons and other nucleons, it is reasonable to construct the final states \( \{ \ket{\Psi_f} \} \) taking account of the correlations.

Cluster correlations can exist in nuclear medium under some conditions [8, 9]. We assume here that clusters with \( A = 2, 3 \) and 4 can propagate in the medium if it is allowed by the Pauli principle. In AMD, if several wave packets (with different spins and isospins) are placed at the same phase space point, these wave packets will tend to move together as a cluster by the equation of motion. On the other hand, if the wave packets are placed randomly in the phase space, the chance for these nucleons to form a cluster after propagation is small. Therefore, in order to respect the possibility of forming a cluster, the set of final states \( \{ \ket{\Psi_f} \} \) should be suitably constructed.

To obtain the transition probabilities to clustered states, we employ an approximation similar to Ref. [8]. As an example of possible final states \( \ket{\Psi_f} \) for a collision of two nucleons \( N_1 \) and \( N_2 \) with the initial relative velocity \( v_{NN} \), let us consider a case that \( N_1 \) forms a cluster \( C_1 \) with another nucleon \( B_1 \) in the final state. The partial differential cross section to this final channel is given by

\[
v_{NN}d\sigma(N_1B_1N_2B_2 \rightarrow C_1C_2) = \frac{2\pi}{\hbar} |\langle \varphi'_1|\varphi^+_{1q}\rangle|^2 |\langle \varphi'_2|\varphi^-_{2q}\rangle|^2 |M|^2 \delta(H - E)\frac{p_{rel}dp_{rel}d\Omega}{(2\pi\hbar)^3},
\]

where \( M \) is the matrix element for the two-nucleon scattering to the final state with the relative momentum \( p_{rel} \) and the scattering angle \( \Omega \) in the two-nucleon center-of-mass system. The overlap matrix \( \langle \varphi'_1|\varphi^+_{1q}\rangle \) is taken between \( |\varphi^+_{1q}\rangle = e^{i\mathbf{q}\cdot\mathbf{r}_1}|\varphi_1\rangle \) and \( |\varphi'_1\rangle \), where \( |\varphi_1\rangle \) and \( |\varphi'_1\rangle \) are the initial and final states of the \( N_1 + B_1 \) system, respectively, and the operator \( e^{i\mathbf{q}\cdot\mathbf{r}_1} \) gives the momentum transfer to the nucleon \( N_1 \). The clusterized states \( |\varphi'_1\rangle \) and \( |\varphi'_2\rangle \) are approximated by the simple harmonic oscillator \( (0s)^n \) configuration with the oscillator constant associated with the wave packet width \( \nu \) in AMD, so that any final state of the collision is represented by an AMD wave function. By identifying the two-nucleon matrix element \( M \) with that of the usual two-nucleon scattering, the cluster-forming cross section can be expressed by using the two-nucleon collision cross section that is an input to the calculation.

The actual situation of a two-nucleon collision requires more considerations because there are many possible ways of forming a cluster with different nucleons (B’s) in the system for each of the scattered nucleons \( N_1 \) and \( N_2 \). It is important to take care of the non-orthogonality of the final states corresponding different ways of cluster formation.

The procedure is repeated for the cluster formation with nucleons (B’s) with other spin and isospin directions until formation of an \( \alpha \) cluster is considered. The particle N should be regarded as a cluster, instead of a nucleon, if a (sub)cluster has been already formed in previous steps of the repetition.

There is a different direction of extending AMD by introducing wave packet splitting [10, 11]. The extension for cluster correlations can be introduced together with wave packet splitting as reported in Ref. [12]. In this paper, however, the results without wave packet splitting are presented below. Surprisingly, conclusion does not depend on wave packet splitting.

### 3. Effects of cluster correlations in multifragmentation

It is known that the early version of AMD with usual two-nucleon collisions does not reproduce multifragmentation data except for collisions of light nuclei. In the left part of Fig. 1, the fragment charge distribution calculated with the usual AMD without special care of cluster correlations is compared with the INcura data [1]. The number of fragments with \( Z > 20 \) is seriously overestimated while the number of intermediate mass fragments is underestimated.

The proton multiplicity \( M_p = 40.2 \) is too large while the \( \alpha \)-particle multiplicity \( M_\alpha = 2.5 \) is too small compared to the experimental data \( (M_p = 8.4 \) and \( M_\alpha = 10.1) \) [1].
When cluster formation is turned on in the final states of two-nucleon collisions, many clusters are created in heavy-ion collisions as soon as two nuclei overlap and hard two-nucleon collisions occur. The fragment charge distribution has been drastically changed by cluster correlations as shown in the middle part of Fig. 1. The proton multiplicity has decreased from $M_p = 40.2$ to 10.9 and the $\alpha$-particle multiplicity has increased from $M_\alpha = 2.5$ to 23.2 by the incorporation of cluster correlations. Although the proton multiplicity is now consistent with the experimental data [1], the $\alpha$ particles are produced too much and heavier fragments are not sufficient. In short, the tendency of turning into cluster gas is too strong compared to experimental data.

The reason of the unphysically strong tendency of cluster gas may be due to the fact that the motion of clusters is almost classical in AMD calculations when the equation of motion for the wave packet centroids is solved. In reality, a few clusters can form a bound state corresponding to the ground state of a light nucleus such as the Li and Be isotopes. The binding of clusters in many of these nuclei is weak so that there is only one bound intrinsic state. Even though the AMD wave function can describe the ground state, the phase space of classically bound configurations does not necessarily have a correct volume corresponding to a single quantum state. When the binding is weak, the classical phase space is much smaller than the quantum phase space, which suppresses the binding probability of clusters in AMD.

To test this idea, some inter-cluster correlation is introduced as a stochastic process of bonding clusters. The relative momentum between clusters is replaced by zero if moderately separated clusters ($R_{rel} < 5$ fm) are moving away from each other with a small relative momentum ($R_{rel} \cdot P_{rel} < 0$ and $P_{rel}^2 / 2\mu < 8$ MeV). Thus this cluster bonding process works to select bound states of two or several clusters during the time evolution of reactions.

This inter-cluster correlation certainly reduces the tendency of cluster gas and increases the yields of relatively heavy fragments as shown in the right part of Fig. 1. The $\alpha$-particle multiplicity is reduced from $M_\alpha = 23.2$ to 10.7 by the inter-cluster correlation, while the the proton multiplicity changes only slightly from $M_p = 10.9$ to 10.8. This calculated result is now consistent with the data in many aspects.

**4. Symmetry energy effects in multifragmentation**

The results in the previous section demonstrate that the global dynamics and fragmentation are reasonably described by AMD with cluster and inter-cluster correlations, at least for central collisions of $A \approx 120$ nuclei at 50 MeV/nucleon. Based on it, we investigate here how the collision...
The density dependence of the symmetry energy is sensitive to the symmetry energy parameter $L$, as well as the mean value of the distribution (for carbon isotopes produced in $^{124}\text{Sn}+^{124}\text{Sn}$ collisions, $\langle N-Z \rangle = 0.75$ for $L = 46$ MeV, and $\langle N-Z \rangle = 0.99$ for $L = 108$ MeV). Comparison with data [13, 14] for these observables suggests that the interior (and therefore low energy) part of the expanding system should be more neutron rich even than in the case of $L = 46$ MeV.

Acknowledgments
This work was supported by JSPS KAKENHI Grant Number 21540253. The numerical calculations were partly carried out on SR16000 at YITP in Kyoto University.

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Figure 2. Yield distribution of carbon isotopes in the $^{124}\text{Sn}+^{124}\text{Sn}$ central collision at 50 MeV/nucleon.

Figure 3. Spectra of tritons and $^3\text{He}$ in central collisions of Sn isotopes at 50 MeV/nucleon, calculated with the symmetry energy parameter $L = 46$ (left) and $108$ (right) MeV.