An Analysis of Errors in Solving Limits of Algebraic Function

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Abstract. By knowing types of mistakes made by students when solving mathematics problems, teacher can choose right learning strategy to avoid similar errors. This study aims to identify students' errors in solving problems on limits of algebraic function. A total of 30 students in Yogyakarta vocational school who have studied concept of limits algebraic function, were asked to complete six questions without looking at the book in 45 minutes. Documents on student work were corrected by three mathematics teachers and identified the type of error that appeared. The results of the study are categorized into two types of errors: (1) procedural errors, including: errors not writing $\lim$ appropriately during executing limits, errors in substituting values into variables, errors not simplifying the final results, or no attempts; (2) conceptual error, including: errors in clearing fractions, errors in factoring, errors in rationalisation.

1. Introduction

Education is one of the leading factors in the process of improving human resources. The 21st century requires teachers and educators to always improve the quality of learning in accordance with the development of science and technology. This development of science and technology takes place quickly and provides its own challenges to each individual. Every individual is required to always make improvements to be able to adjust well.

Therefore, the development of human resources is crucial so that each individual can adjust to science and technology. One effort to keep up with the mentioned development is to improve the quality of education. One of the aspects included in the efforts to improve the quality of education is by looking at the teaching and learning process. The process of teaching, learning and creative thinking is one of educational components. Teachers carry out the teaching process while students perform the learning process [1]. In terms of education, mathematics is one of the subjects learned from early age to college. There are several aspects that underlie the importance of learning mathematics, which is a means to think logically and clearly, to solve everyday problems, to connect patterns of problems, to generalize experience, to develop creativity and to develop awareness of culture. Meanwhile, learning mathematics with understanding makes students and educators increasingly reach one of the important goals of mathematics education, the realization of this goal has been problematic for a long time. Many factors might explain this, such as teacher knowledge and pedagogy, curriculum, etc. [2].

When studying mathematics, concepts Mathematics must be understood in advance so that it can easily solve existing problems, because the completion of mathematical questions depends on understanding mathematical concepts. In solving the metamorphic problems there are various tiger strategies that can facilitate students in solving problems[3]. Developing understanding in mathematics is an important but difficult goal. Recognizing student difficulties and the sources of those difficulties and designing instructions to reduce them are important steps in achieving this goal. Students' difficulties
in learning symbols, concepts, and written procedures can be reduced by creating learning environments that help them build relationships between formal and informal mathematics knowledge; use the right representation depending on the given context of the problem; and help them connect procedural and conceptual knowledge [4]. Students' low mathematical abilities can be seen from student learning outcomes which still often do not meet expectations. Students' mistakes in working on the problem or solving the problem can be one of the instructions to find out the extent to which students master the material [5]. Understanding of mathematical concepts is one of the cognitive aspects that aims to add the level of knowledge of students to the subject matter that previously did not know to be aware and which originally did not understand to understand [6].

Most of the students cannot avoid the difficulties of learning mathematics in school. It must be realized that students in general experience different difficulties in learning mathematics. Avoiding mathematical difficulties for pragmatic purposes will be faced with other greater difficulties. Therefore, students need to motivate themselves to be more interested in learning mathematics. Students need to put in their minds that mathematics is important. One way to reduce these difficulties is to help students make connections between both conceptual and procedural knowledge. The construction of conceptual knowledge requires identifying the characteristics of concepts, recognizing similarities and differences between concepts according to these characteristics, and building relationships between them. On the other hand, procedural knowledge requires building skills, strategies or algorithms that mean to achieve goals [7]. The student errors discussed above are considered as difficulties in learning mathematics, since that seems to involve potential misunderstandings, misdirected procedures and misinterpretations. The fact that students, according to the teacher, do not even seem aware or able to understand the nature of the mistakes made underpinning this interpretation. Errors made by them depend on the thinking process carried out in solving problems.

Algebra is one material that is considered difficult. In mathematics at the vocational level, algebra is one of the subjects that makes students tend to experience mistakes. Algebra is seen as a difficult subject when a student lacks of understanding concepts coupled with lack of training. However, such exercises might further extend the misconception that says that algebra is a connecting requirement for handling different contexts such as "do the same thing on both sides", "change sides, change marks", etc. [8]. Based on observations made in one of the Vocational Schools in Yogyakarta, Indonesia, where the authors have been teaching, the limit of algebra function material is one of the complicated material, the material is included in the National Examination. Students in Yogyakarta City, achieved an average of 46.33 in 2018 (less than the minimum criterion that is 50.50).

The number of mistakes made by students in working on the limit problem of algebraic functions can be a guide to the extent to which students master the concept of limit algebra functions. By seeing mistakes made by students, it can be examined more deeply on the causes of students making these mistakes. The cause of mistakes made by students is to find a complete problem solving. This solution is taken by analyzing the problems that cause students to make these mistakes, then find an alternative to solve the problem, so that later the same problem can be avoided so that it does not happen again.

2. Research Method
This research was conducted in Vocational Schools in Yogyakarta, in class XI of the Department of Industrial Chemistry that applies 2013 Curriculum. Using a qualitative approach, researchers are the main instruments. To collect data, researchers compiled observation sheets, tests, and interview guides. The research subjects are students studying the limit of algebra function material. The students are assumed to have just learned the concept of the limit of algebraic functions when the study was conducted. The unit of analysis in this study is the result of the completion of the test questions, which consists of 6 items in the description. The test results identify what kinds of errors made by students, in terms of conceptual, procedural and technical knowledge. As many as 6 items consisting of the limit questions of algebraic functions, students are asked to complete 2 pieces of questions by factoring, 2 questions using root friends and 2 questions solved by dividing the highest rank. Tests are given immediately after students complete learning. Tests are given in 45 minutes, students are not allowed to open books, ask questions, or see friends' answers, which work is easy to do first. Student seating is not determined in the order of absence so the student position is random. The test results were corrected by three mathematics teachers as data triangulation techniques. If there are different corrections, discussions
will be conducted. The researcher then lists all types of errors that appear, simplifying / reducing data, then interpreting findings based on these data.

3. Results

Data is obtained after students complete a written test. The data obtained is in the form of a written answer sheet which is the result of the student's work on the given problem. This data can also be used to identify the types of student errors. The form of errors made by students in solving the limit problem of algebra functions can be seen in Table 1.

| Student number | Type of error | Student number | Type of error |
|---------------|---------------|---------------|---------------|
| 1             | B, C          | 16            | B, C          |
| 2             | M             | 17            | M             |
| 3             | M             | 18            | B             |
| 4             | A             | 19            | B             |
| 5             | A             | 20            | N             |
| 6             | A             | 21            | B             |
| 7             | A             | 22            | M             |
| 8             | M             | 23            | B             |
| 9             | B, C          | 24            | M             |
| 10            | C             | 25            | C             |
| 11            | M             | 26            | A             |
| 12            | M             | 27            | C             |
| 13            | C             | 28            | M             |
| 14            | M             | 29            | A             |
| 15            | N             | 30            | C             |

Note:
A: Error in understanding concepts
B: Procedural errors
C: Calculation error
D: Error in final answer
M: No difficulties encountered
N: No attempts

By looking at Table 1, it can be seen that four students still experience conceptual misconceptions, nine students experience procedural errors, seven students experience calculation errors, five people experience errors in determining the final answer, eight students answer correctly and two of them do not answer. The following is a table of percentage recapitulation performed by students:

| Type of error                              | Percentage |
|--------------------------------------------|------------|
| Errors in conceptual understanding         | 18%        |
| Procedural errors                          | 14%        |
| Errors in calculation                      | 27%        |
| Errors in the final answer                 | 8%         |

From Table 2, it can be seen that the most mistakes experienced by students occur during process skills, where students can have to be able to solve the problem in the right way, then as many as 18% still experience errors in determining the steps to work on the problem, 14% of students experience errors with do not follow the settlement procedure and as many as 8% of students have difficulty in determining the final answer.

Figure 1 is an example of students’ works representing each type of errors.
Figure 1(a) Example of errors in conceptual understanding

Figure 1(b) Example of procedural errors

Figure 1(c) Example of calculation errors
4. Discussion
From the results of the observation, it is clearly seen how the level of student mastery of the concept of limit algebra functions. There are still students who still do not really understand the concept of limit algebraic functions, this is shown in Table 2 as many as 18% of students in the class. At each group's presentation not all students can focus on the material being studied, it is also included in the weaknesses of group discussion. For students with a good understanding of the concept, the method can accept the concepts given so that in solving problems have sufficient knowledge, both conceptual knowledge and procedural knowledge.

When conducting examinations students tend to remain silent for a long period of time to find a step that they think is right in solving the problem. Examination of the exam is done in a closed book, sometimes it seems that students are anxious by glancing at the answers of their next friends hoping to see steps that can be taken to solve the exam questions. By looking at students' anxieties in working on exam questions, it shows how far the concept has not been well embedded in the learning process. From the results of interviews with several students who made mistakes, from the beginning they acknowledged their lack of understanding of the concept. From the concepts they have, it is sometimes difficult to determine what kind of problem how to do it, so it takes a lot of time to solve the problem. It was also found that some students still did not understand the results of the completion of the limit of algebraic functions, where when they worked and had managed to find answers sometimes did not respond to answers, even though the answer he got was an indeterminate form that must be avoided in solving the limit of algebraic functions. During the interview, several students mentioned weaknesses in factoring in the quadratic form that should have been mastered since junior high school.

By knowing the various difficulties, it is hoped that it can help the teacher to understand a possibility to describe learning, identify the abilities of each student, identify students who may be weak in certain material concepts, identify zones of proximal development for each student so they can adjust planning and learning, and identify diversity mathematical knowledge in class [9]. Teachers who use the framework can use this information to teach each child more effectively. One way to reduce these difficulties is to help students make connections between both conceptual and procedural knowledge. The construction of conceptual knowledge requires identifying the characteristics of concepts, recognizing similarities and differences between concepts according to these characteristics, and building relationships between them. On the other hand, procedural knowledge requires building skills, strategies or algorithms that are meaningful to achieve goals. To help students overcome all the mistakes made, it is necessary to make a complete location error indicator, namely an error in determining the initial condition, stating the final answer, error editorial, and process errors in getting the final answer [10].

5. Conclusion
Students errors can be used as guidelines to determine the extent to which they understand and master the given concepts. By following Polya's problem solving steps, it can be seen that errors in the first stage are fact errors, habitual errors, and interpretation errors. Errors in the second stage are concepts and facts errors. The errors in the third stage are errors in principle and procedure. From the results of the discussion, several weaknesses are found and should be strengthened before the learning process of limit algebraic functions, including the form of quadratic numbers and factoring. Factors that influence students in making errors are: determining the right way to solve the problem, students are less adept at manipulating the completion step, students are less careful in carrying out the sum calculation operation, reducing multiplication and division in the algebraic form.

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