1. Introduction

A principal operation in the production of plant growing products is the sowing of seeds by using seeders. Timely and qualitative execution of technological operation of sowing directly affects crop yields. Increasing the operating speeds of sowing machines in order to improve the unit’s performance and to meet the agricultural terms, along with the existence of surface irregularities, enhance oscillatory processes, increase the dynamic strain in the parts and nodes of
seeders [1]. In addition, such situation adversely affects the uniformity of introduction and seeding depth of fertilizers and seeds.

Technical and technological level of existing equipment does not adequately meet modern requirements that ensure the implementation of technologies for producing competitive agricultural products. The use of certain seeding machines in modern technologies of crop production is not always rational in terms of natural-climatic and economic conditions, for example, in Ukraine.

Modern means of mechanization actively contribute to the compaction of soil layers and atomization of soil, to increasing the heterogeneity of its structure and density, to the formation of lumps of soil with elevated density, to increasing fuel consumption, to reducing the effectiveness of use of fertilizers.

The result is a decrease in the yields of agricultural crops and a low coefficient of the biopotential utilization [2]. The procedure for the development of such tasks of agricultural mechanics typically comes down to the application of differential equations, which greatly complicates their solution and requires simplifications, assumptions when forming the boundary conditions [3]. Moreover, when designing new equipment and technologies, attention should be paid to the task of soil fertilization preservation [4]. Attempts by some scientists to explore the stability and to define the rational parameters for seeding machinery, for example, a seeder for sub-soil-spread sowing with mounted arrow-paw coulters [5], did not lead to their wide application. This is explained by the lack of an effective mathematical apparatus for designing the structures of seeding machines based on the stability criterion.

Thus, it is necessary to analyze existing theoretical approaches to addressing the challenges related to the problems of stability for mechanical systems and to work out an appropriate procedure. At the same time, one should take into consideration the features of the design and functionality of various types of seeders. This would make it possible to substantiate parameters of mechanical systems in terms of their stability, accurate enough for practical implementation. The development of a procedure for studying the stability of mechanical systems is a relevant task for the agricultural, construction, and other industries.

2. Literature review and problem statement

Research into scientific-technological foundations of mechanical-technological processes and development of appropriate technical means, which meet agricultural and ecological-economic requirements of agricultural production are described in [6, 7]. In the search for solutions, papers [8, 9] categorized systems of interaction between the working bodies of tillage machines and soil; substantiated methods for determining the parameters of elastic mechanisms of machines. However, considering the oscillations of working bodies separately from the operation of an entire unit does not make it possible to identify the stability of the whole system. In addition, the equations derived are not finite. The difficulty is to determine the coefficients and magnitudes in the equations, for example, the reduced mass, the frequency of natural oscillations and the nonlinear rigidity of the system.

To substantiate the technological parameters of mechanical systems, agricultural mechanics employs theoretical and experimental methods [10]. Mechanical systems operate in accordance with the laws of mechanics; therefore, theoretical studies apply the laws and principles of mechanics. Theoretical description of operation of mechanical systems using mathematical correlations implies working out the appropriate models rather than technical objects. Therefore, to determine the degree of adequacy of the models that are accepted in theoretical studies, it is necessary to verify them experimentally.

In agricultural mechanics, there are three types of models, physical models, estimation models, and mathematical models [7]. The first ones describe the phenomena and processes in accordance with their physical nature. Estimation models imply the use of modern methods of mathematics and computational science. Using the estimation models helps describe some of the components that have the greatest significance, and the processes themselves in the examined technological process in general. The mathematical models, for the chosen physical and estimation models, also take into consideration the quality criteria in the implementation of a technological process. For example, the timing of execution of a certain technological operation in a predefined area.

Physical models are developed based on scientific hypotheses, results of experimental studies, as well as the theories that describe the essence of the physics of processes and phenomena. Scientific hypotheses that are put forward require experimental identification in terms of their adequacy to actual processes and phenomena.

When building the estimation models, one defines the mathematical dependences between the examined parameters of the studied mechanical systems, the impact of external environment on the performance of the latter, etc. Most widely used in estimation models of the functioning of mechanical systems are the differential equations. A generalized procedure for building the estimation models of dynamic systems is given in paper [7]. However, solving these differential equations by numerical methods presents certain difficulties. Models have a large number of parameters. At the same time, parameters accept variable values, for example, a changing mass of the unit during its operation, etc.

Development of recommendations on solving the systems of differential equations implies the use of numerical methods, methods to study mechanical systems for the stability of their operation, as well as examples of the application of the theory of stability in solving the systems of differential equations [11]. This makes it possible to solve the problems on agricultural mechanics.

An analysis of results of theoretical studies into stability processes of mechanical systems allows us to argue on that all of them are compared to a certain solution to the Cauchy problem for the systems of differential equations. At the same time, the models consider the following problems [2, 3, 11]:

- stability of variables applied in the construction of the appropriate mathematical model;
- stability of equilibrium and stationary movements;
- establishing the classes (classification) of stable solutions to the systems of differential equations;
- methods for comparing the system parameters in terms of studying the problems of stability;
- methods for building special functions for certain classes of differential equations.

Since mechanical systems are composed of solid bodies, which execute the translational or rotational movements, their number is relatively small. Therefore, based on the
theorem about motion of the center of masses, the dynamics of such systems can be considered to be the motion of systems of material points. The study into dynamics of systems of solid bodies could also be reduced to studying the motion of material points under condition of their small quantity with relatively simple forms of bonds.

Problems on the stability of mechanical systems, including sowing machines, were considered in papers [12–14]. It was established that the differential equations of the derived mathematical models have a large number of variables that change over time. That is why the identification of such variables is difficult.

The emergence of promising working bodies with new designs also leads to the complexity of employing the standard procedures for solving equations on stability. Thus, paper [15] proposes to use, in order to improve the quality of sowing, the double-disc anchor coulter. A given design ensures the uniformity for depth with the help of a spring, which enables stable operation of the working body. How-

ever, due to the lack of substantiated parameters for the working body, the absence of studies into its operation under different agricultural conditions (background), it is difficult to determine its stability based on the standardized procedures. Furthermore, there is a lack of the comprehensive study into its impact on the stability of other factors.

The possibility of improving the quality of sowing through the variation in the structurally-kinematic parameters was outlined in [16]. The authors derived variation coefficients for the seed arrangement in a furrow – 19.25...25% during operation of 3 seeders of varying mass, motion speed, and design features. However, the modeling approach is of a specific nature, predetermined by the operating conditions and design of the seeder.

A procedure for determining the effectiveness of operation of sowing machines based on the popular no-till system was addressed in [17]. The authors investigated performance indicators of seeders with different working bodies (including disc), against the agricultural background with different stubble. They explored the influence of organic residues on the quality of operation of seeders; its deterioration was caused, among other factors, by the compromised stability of the unit. It should be noted that the aim of the research was to study the stability of working bodies of specific models of seeders with a variation in the conditions of seeders’ operation. Thus, the results obtained have limited applicability and cannot be used to model different types of seeders.

The results of a study into the quality of operation of seeders with direct seeding were described in [18]. The authors examined the influence of the seeder motion speed, as well as the soil density and moisture content, on a deviation in the corn seeding depth. They proved the need to adjust basic working bodies of the seeder to ensure stable operation and maximum yield of corn. However, considering the parameters of working bodies only, excluding the magnitudes of a seeding unit mass, the length of its tongue hitch, etc., does not make it possible to obtain adequate results for the stability of operation of the entire system.

Paper [19] reports mathematical expressions on the stability of motion of the seeder with boxes for seeds and fertilizers, milling coulters with drive. The study helped to establish that ensuring the stability requires the use of specialized stabilizers-rippers. The authors determined the presence of additional resistance from those devices, which depends on the quantity and design parameters of active working bodies, taking into consideration their modes of operation as well. However, the work does not account for the elastic elements (springs) of the working bodies of most seeders that define the deviations of seeders’ coulters and regulate the quality of a technological operation involving the sowing of crops.

Thus, existing methods for solving the tasks on the operational stability of sowing units require a systemic simplification that would produce practical results. An analysis revealed the factors that influence stability, which need to be taken into consideration during research: elastic elements (springs), a shoulder to fix the coulters, a seeder fastening shoulder and its weight, hitches that must be taken into consideration when estimating the stability. In addition, models should be universal, with their application to be extended to the promising designs of seeders, for example, for direct sowing.

3. The aim and objectives of the study

The aim of this study is to construct mathematical models for the operation of mechanical systems of sowing machines of different types, which would make it possible to improve their stability.

To accomplish the aim, the following tasks have been set:

- to analyze the processes of interaction between operation of the working bodies of sowing machines and soil, to elucidate the prospects for their theoretical modelling and for improving their design;
- to substantiate the principles, boundary conditions, and to determine an efficient algorithm for modeling the stability of dynamic systems of sowing machines;
- to define the rational parameters for the working bodies of sowing machines.

4. Materials and methods to study the stability of mechanical systems

When building mathematical models of operation of mechanical systems, there arise certain difficulties. Thus, given that the intensities of external influences are random variables, it is not possible to take into consideration all influences and their level of influence. Under field conditions, the most probabilistic character of external influence on machine units is exerted by constantly repeated discrete jumps. To examine the extent of this influence on the dynamic systems, the stability of its motion is investigated, which characterizes the capability of mechanical systems to maintain the assigned motion trajectory. The required and sufficient conditions for the stable motion were established by scientists in papers [11, 12].

In theory [13, 20], a system is defined to be stable if, at a deviation in the action of external forces on it or initial conditions that vary within certain limits, changes in the motion trajectory of the system would be insignificant. Definition of the motion stability of a machine-tractor unit is given in [21]; the author understands it as the capability to ensure, over time, small deviations in the perturbed motion of the unit from the unperturbed motion without intervention of the tractor driver that would use control systems. It should be noted that the magnitudes of the perturbing efforts that act on agricultural machines and tools are small in comparison with the mass of the energy means. At the same time, these units are equipped with control mechanisms. The study into
differential equations is then advisable as determining the asymptotic stability. An asymptotic stability indicator defines the capability of a system to return to the unperturbed law of motion. A measure of such a capability could be time or distance, required to restore the regularity of the undisturbed motion of the system.

The experience of modeling the dynamic systems in agricultural mechanics shows that it is, as a rule, impossible to derive the general and analytical solutions to differential equations or systems of equations. Therefore, in order to solve the problems of stability, it is advisable to use a second Lyapunov method [12, 14, 20], which implies the construction and investigation of functions of the perturbed motion.

A decrease in the order in the system of differential equations will be denoted \( \dot{q}_j = \omega \). We obtain then:

\[
\begin{cases}
\dot{q}_j = \omega, \\
\dot{\omega} = q_j (q_j; \omega; Q_{ao}; t),
\end{cases}
\]

where \( Q_{ao} \) is the number of forces; \( q_j \) is the derivative from forces; \( t \) is time; \( q_j \) is the generalized coordinate; \( \dot{q}_j \) is the derivative from the generalized coordinate.

As a result of the influences of external environment on mechanical systems, they do not lose their constructive relationships and do not lead to a change in them. The differential equations that describe the unperturbed and perturbed motion are then identical analytically.

The equations of the perturbed motion of a mechanical system will take the form:

\[
\dot{q}_j + \delta = \omega + \beta
\]

or

\[
\begin{cases}
\dot{\delta} = \beta, \\
\dot{\beta} = q_j (q_j + \delta; \omega; Q_{ao}; t),
\end{cases}
\]

where \( \delta \) is the gain of the generalized coordinate as a result of perturbation influence on the system; \( \beta \) is the speed gain of the generalized coordinate as a result of perturbation influence on the system.

We decompose a system of equations (2) into a Taylor series with an accuracy up to a first-order smallness:

\[
\begin{align*}
\omega & + \dot{\beta} = q_j (q_j; \omega; Q_{ao}; t) + \\
& + q''_j (q_j; \omega; Q_{ao}; t) \delta + q''_j (q_j; \omega; Q_{ao}; t) \beta,
\end{align*}
\]

where \( q''_j \) is the derivative from the generalized coordinate.

The difference between the desired equations of the perturbed and the unperturbed motion of a mechanical system defines a system of differential equations of perturbation:

\[
\begin{cases}
\dot{\delta} = \beta, \\
\dot{\beta} = q''_j (q_j; \omega; Q_{ao}; t) \delta + q''_j (q_j; \omega; Q_{ao}; t) \beta.
\end{cases}
\]

The degree of asymptotic stability of a system is defined by the magnitude of time required to return it to the unperturbed motion, or close to it. To this end, we solve in a combination the systems of differential equations of the unperturbed motion of a system and the perturbation (3).

The first system of differential equations produces at a preset time \( t_0 \) the value for the generalized coordinate \( q_j \) and the rate of its change \( \dot{q}_j = \omega \).

The data obtained are employed to solve the system of equations (3) that characterizes the intensity of perturbations over time.

## 5. Stability model of operation of the coulter system with a support-packer roller

The quality of the technological operation of seeding is determined by the uniformity of seeding depth and their contact with the solid phase of soil. These indicators can be improved by installing a support-packer roller on the disk system (Fig. 1).

When such a mechanical system operates, its elements are exposed to the action of the following forces: the mass of hitch \( G_t \), the mass of coulter \( G_c \), the mass of packer roller \( G_r \), the mass of sowing unit \( G_s \), the mass of coulter discs \( G_d \), and the mass of packer \( G_p \). The distance, required to restore the regularity of the undisturbed motion, is determined by the magnitude of time required to return it to the law of motion. A measure of such a capability could be time or distance, required to restore the regularity of the undisturbed motion of the system.

As a result of the influences of external environment on mechanical systems, they do not lose their constructive relationships and do not lead to a change in them. The differential equations that describe the unperturbed and perturbed motion are then identical analytically.

The equations of the perturbed motion of a mechanical system will take the form:

\[
\dot{q}_j + \delta = \omega + \beta
\]

or

\[
\begin{cases}
\dot{\delta} = \beta, \\
\dot{\beta} = q_j (q_j + \delta; \omega; Q_{ao}; t),
\end{cases}
\]

where \( \delta \) is the gain of the generalized coordinate as a result of perturbation influence on the system; \( \beta \) is the speed gain of the generalized coordinate as a result of perturbation influence on the system.

We decompose a system of equations (2) into a Taylor series with an accuracy up to a first-order smallness:

\[
\begin{align*}
\omega & + \dot{\beta} = q_j (q_j; \omega; Q_{ao}; t) + \\
& + q''_j (q_j; \omega; Q_{ao}; t) \delta + q''_j (q_j; \omega; Q_{ao}; t) \beta,
\end{align*}
\]

where \( q''_j \) is the derivative from the generalized coordinate.

The difference between the desired equations of the perturbed and the unperturbed motion of a mechanical system defines a system of differential equations of perturbation:

\[
\begin{cases}
\dot{\delta} = \beta, \\
\dot{\beta} = q''_j (q_j; \omega; Q_{ao}; t) \delta + q''_j (q_j; \omega; Q_{ao}; t) \beta.
\end{cases}
\]

The degree of asymptotic stability of a system is defined by the magnitude of time required to return it to the

We assume that the displacement of a seeder frame is parallel to the soil surface at a constant motion speed.

The coulter system is considered as a mechanical one with a single degree of freedom. We accept as the generalized coordinate a rotation angle of hitch \( \phi \) around the point of suspension.

Taking into consideration the dissipation of energy to overcome the forces of friction between the coulter discs and soil as a result of oscillations in the system, we record the equation in the form:

\[
\begin{align*}
\phi & \left( \frac{m_f l^2}{2} + 2m_f l_f^2 + 2m_f l_z^2 + 2m_l l_f^2 + I_1 + I_2 + I_3 + I_4 \right) = \\
& = - (h_s + I_s \sin \phi \cos \phi - 0.5G_c + G_r) \times \\
& \times l_s \sin (\phi_0 + \phi) + R l_s \sin (\psi - f \cos \psi) - \\
& - G_f l_s \sin (\phi_0 + \phi) - G_f l_f \sin (\phi_0 + \phi) + \\
& + R l_f \sin (\phi_0 + \phi) + f \cos (\phi_0 + \phi) - \\
& - K_f \dot{\phi} l_f^2 \sin (\phi_0 + \phi),
\end{align*}
\]
where $K_d$ is a coefficient that determines the value of the dissipative frictional force of coulters’ discs against soil; $R_c$ is the resistance of soil:

$$R_c = k(h + l_3(\cos(\phi_0 + \phi) - \cos(\phi_0)))$$

where $k = k_1(1 + \rho \sin(\pi t))$ is the coulter resistance for a preset depth of its motion, which varies in line with the sinusoidal law; $k_1$ is the averaged soil resistance to the coulter; $\rho, \nu$ are the sinusoid parameters; $h$ is the depth of seeding; $\nu$ is the vertical component of the force that acts on a packer roller; $m_1, \ldots, m_3$ are the masses of the hitch, coulter, lever, packer roller, respectively; $l_1$ is the distance from the point of suspension to the point of application of the action of force of the spring of the push rod; $l_2$ is the length of the coulter hitch; $l_3$ is the distance between the point of suspension and the center of mass of the lever connecting a packer roller to the lever; $l_4$ is the distance from the point of suspension to the center of mass of the roller; $I_1, \ldots, I_4$ are the moments of inertia of the hitch, coulter, lever, packer roller, respectively; $G_1, \ldots, G_4$ are the masses of the hitch, coulter, lever, packer roller, respectively; $f_s$ is the coefficient of rolling; $f$ is a friction coefficient; $h_{\text{app}}$ is the spring deformation at the initial effort of spring compression.

After the decomposition of equation (4) into a Taylor series with an accuracy of the first-order smallness, we obtain a system of differential equations that describe the perturbation of the studied mechanical system:

$$\begin{align*}
\dot{\delta} &= \beta; \\
\ddot{\delta} &= -\mu l_1^3 \sin(\phi_0 + \phi) + f_s \cos(\phi_0 + \phi) + \beta.
\end{align*}$$

(5)

where:

$$B_i = \frac{m_i l_i^2}{2} + 2m_2 l_i^2 + 2m_3 l_i^2 + 2m_1 l_i^2 + I_i + l_1 + l_2 + l_3 + l_4;$$

$$R_i = \left[ l_i (\cos(\phi_0 + \phi + \delta) - \cos(\phi_0 + \phi)) + h_1 \right] C_s - \mu l_1 \omega \sin(\phi_0 + \phi + \delta);$$

$$\frac{\partial R_i}{\partial \phi} = -l_i \sin(\phi_0 + \phi + \delta) C_s - \mu l_1 \omega \cos(\phi_0 + \phi + \delta);$$

$\mu$ is a damping factor; $h_1$ is the soil deformation; $C_s$ is the soil elasticity; $\delta$ is the angle of oscillations of the perturbed motion of the system.

The value of the coefficient, which defines the magnitude of dissipative force of friction between the coulters’ discs against soil $K_d$ was considered to be equal to $1 \text{ Ns/m}$ [20].

Asymptotic stability of the system was estimated based on the magnitude of the distance covered by the system from the perturbed motion to its returning it to the unperturbed motion, according to [20]. When solving a system of differential equations (3), the perturbation was assigned based on the following parameters: $\delta = 3^\circ$, $\beta = 9 \text{ deg/s}$. The obtained character of the damping of oscillations of the perturbed motion of the system is shown in Fig. 2. Results of solving a system of equations (5) at its different parameters are shown in Fig. 3.

Fig. 2. Character of damping the perturbed motion

Fig. 3. Dependence of the perturbed path of the coulter system on the length of its hitch ($l_2$), rigidity of the spring ($C_{\text{app}}$), the inclination angle of its hitch ($\phi_0$), distance to the attachment point of rod of the push spring ($l_1$)

Further study implied the analysis of dependences of the perturbed path of the coulter system of sowing machines on its parameters (Fig. 2). It was established that an increase in the length of the hitch (longer than 1.01 m), in the distance from the hitch fastening axis on a seeder frame to the push spring rod attachment point (longer than 0.9 m), leads to a decrease in the coefficient of variation of maximum deviations. The motion stability of the mechanical system increases. The same character is observed at the rational values for the rigidity of a push spring of 1,200...1,650 N/m, the initial inclination angle of the hitch of 80°...85°, the coulter mass of 4...6 kg.
In order to test the validity of theoretical studies into determining the influence of the coulter mass, the initial inclination angle, and the rigidity of the push rod spring on the motion stability of the system, experimental investigations were conducted in a special soil channel. Variation levels for the factors were chosen based on the results of theoretical studies. The encoding of factors and their levels of variation are given in Table 1.

Prior to the experiments, soil in the channel was brought to natural conditions by loosening and levelling. Next, in order to perturb the system, we laid in the channel, at a distance of 1 m, the perturbers for the coulter system, the 25-mm thick wooden planks.

After enabling the drive of the soil channel cart, the examined system was led to perturbation. When the system drove over the wooden planks, its motion was perturbed to be eventually returned to its initial state. We used video recording to register the process of system operation and determined distance \( S \) based on the time that it required to return from the perturbed state to the original state. This distance, required for the system to return to the unperturbed motion, was accepted as the optimization criterion \( y \).

Experiments were carried out at motion speed of the coulter system of 1 m/s with a soil moisture content of 20.5 %. The matrix of experiments and their results are given in Table 2.

The regression equation takes the following form:

\[
C = 0.91 - 0.13x_1 - 2.00x_2 - 0.18x_3.
\]

Statistical processing of the data obtained indicates their credibility with a 95 % probability. Evaluation of the degree of difference between different variants of the experiment was conducted by using the least significant difference (LSD). For a given mechanical system, LSD = 0.031.

An analysis of the regression equation reveals that an increase in the parameters studied contributes to improving the stability of the mechanical system composed of a disk coulter system support-packer wheel, specifically the reduction of its perturbed distance \( y \). The greatest influence on the motion stability at the predefined limits of change in the values of factors is exerted by the initial inclination angle of the section’s hitch.

6. Model of operational stability of the coulter system in a direct seeding seeder

To test the proposed algorithm for modeling the stability, we verified it based on the mechanical system of a direct seeding seeder, which is relevant given its low cost [22].

A modeling object was the assembly in the form of an attachment with soil-cultivating disks, attached through hinges to a grain seeder, the type of SZ-5,4-0,6. The disks of the attachment form a cultivated strip over unprepared soil, along which a seeder’s coulters move and seeds are sown.

However, when such mechanical systems move, the action of external factors results in the system being driven to the oscillatory motion. This leads to a decrease in the quality of sowing, deterioration in the reliability of a seeder’s nodes; that requires appropriate calculations for stability.

When the assembly moves, the coulters are exposed (Fig. 4) to the components of resultant force \( R_{x_{cm}}, R_{y_{cm}}, R_{x_{cl}}, R_{y_{cl}} \), the seeder’s wheels – to \( R_{x_{kn}}, R_{y_{kn}} \), the attachment’s discs – to \( R_{x_{dn}}, R_{y_{dn}}, R_{x_{dl}}, R_{y_{dl}} \).

The system executes translational motion in the \( xOy \) plane; the center of masses of a seeder points \( C_1 \) and \( O_2 \) is the place of attaching the machine to the tractor and uniform motion in the direction of the \( x \) axis. We accept: \( \phi_1 \) is the angle of rotation of the seeder’s tongue hitch relative to point \( O_1 \), \( \phi_2 \) is the rotation angle of the attachment’s discs tongue hitch relative to point \( O_2 \).

![Fig. 4. Estimation scheme of the coulter system in a direct seeding seeder](image)
In order to study the system for motion stability with respect to the dissipative energy, we obtain the following system of equations:

\[ M_{y}\left[l_{y}^{2} \ddot{\psi}_{y} + l_{y} \dot{\psi}_{y} D_{1} + l_{y} \dot{\theta}_{y} D_{2}\right] + \dot{\psi}_{y} = Q_{y}\,; \]

\[ M_{x}\left[l_{x}^{2} \ddot{\psi}_{x} + l_{x} \dot{\psi}_{x} D_{1}\right] + \left(M_{x} l_{x}^{2} + I_{x}\right) \ddot{\theta}_{x} = Q_{x}\,; \]

\[ (8) \]

where

\[ D_{y} = \sin \phi_{y} \cos \phi_{y} - \cos \phi_{y} \sin \phi_{y} \; ; \]

\[ D_{x} = \sin \phi_{x} \cos \phi_{x} - \cos \phi_{x} \sin \phi_{x} \; ; \]

\[ Q_{y} = -R_{y}\left(0.5l_{y} \cos \phi_{y} + l \sin \phi_{y}\right) - \sum_{i=1}^{21} R_{y}\left(0.25l_{y} \cos \phi_{y} \sin \phi_{i} - 0.5l_{y} \sin \phi_{i}\right) - \sum_{i=1}^{24} \left(0.25l_{y} \cos \phi_{y} - l \sin \phi_{i} - R_{y}\left(0.5l_{y} \cos \phi_{y} + 0.5l_{y} \sin \phi_{i}\right) - \sum_{i=13}^{24} R_{y}\left(l \cos \phi_{y} - 0.25l_{y} \sin \phi_{i}\right) - K_{y} \dot{\theta}_{y}\,; \]

\[ Q_{x} = -R_{x}\left(l_{x} \sin \phi_{x} + R_{x} l \cos \phi_{x}\right) + \sum_{i=13}^{21} R_{x}\left(0.25l_{x} \sin \phi_{i} + l_{x} \sin \phi_{2}\right) - \sum_{i=13}^{21} R_{x}\left(l_{x} \cos \phi_{x} + 0.25l_{x} \sin \phi_{i}\right) - \sum_{i=13}^{21} R_{x}\left(0.25l_{x} - l_{x} \sin \phi_{2}\right) - \sum_{i=13}^{21} R_{x}\left(l \cos \phi_{x} - 0.25l_{x} \sin \phi_{2}\right) - K_{x} \dot{\theta}_{x}\,; \]

where \( K_{y}; K_{x}\) are the coefficients that determine the value of dissipative frictional forces for, respectively, a seeder and a disk attachment on soil; \( l_{i}\) is the attachment’s frame length in longitudinal plane; \( l\) is the distance between point \( O_{1}\) and the center of mass of the seeder.

After finding the difference between the perturbed and unperturbed motion equations of the system and decomposing the former into a Taylor series with an accuracy of the first-order smallness, we obtain a system of differential perturbation equations:

\[ \dot{\delta}_{1} = \frac{1}{M_{y}} + \left[Q_{y}\left(M_{y} l_{y}^{2} + I_{y}\right) - M_{y} l_{y} \left(Q_{y} D_{1} - M_{y} \omega_{y}^{2} D_{2}\right)\right] \frac{\partial D_{y}}{\partial \phi_{y}} \; ; \]

\[ - D_{y} \left[Q_{y} D_{1} + M_{y} \omega_{y} \frac{\partial D_{2}}{\partial \phi_{y}}\right] + M_{y} \omega_{y} \frac{\partial D_{2}}{\partial \phi_{y}} \; ; \]

\[ + \left[Q_{x}\left(M_{x} l_{x}^{2} + I_{x}\right) - M_{x} l_{x} \left(Q_{x} D_{1} - M_{x} \omega_{x}^{2} D_{2}\right)\right] \frac{\partial D_{x}}{\partial \phi_{x}} \; ; \]

\[ + M_{x} l_{x} \left(Q_{x} D_{1} + M_{x} \omega_{x}^{2} D_{2}\right) \left[\frac{\partial D_{x}}{\partial \phi_{x}} - \frac{\partial D_{x}}{\partial \phi_{y}}\right] \; ; \]

\[ + \frac{\partial Q_{y}}{\partial \omega_{y}} \left(M_{x} l_{x}^{2} + I_{x}\right) \dot{\delta}_{1} + \frac{\partial Q_{x}}{\partial \omega_{x}} \left(M_{x} l_{x}^{2} + I_{x}\right) \dot{\delta}_{2}\,; \]

\[ (10) \]

where \( M_{y}\) is the center of mass of the seeder; \( \delta_{1}; \delta_{2}\) are the perturbation increment of the generalized coordinate in, respectively, the first and second differential equations; \( \dot{\delta}_{1}; \dot{\delta}_{2}\) is the increment of perturbation speed of the generalized coordinate in, respectively, the first and second differential equations.

After solving the system of equations (10) when \( \delta_{1} = \delta_{2} = 3\% ; \beta_{1} = \beta_{2} = 0\), we obtain the character of dependence for the distance traveled by a direct seeding seeder from the moment of perturbation to the unperturbed motion at its motion speed of 3 m/s (Fig. 5).

Dependences for the distance travelled by a direct seeding seeder from the moment of perturbation to the unperturbed motion are shown in Fig 6, 7. Comparison of the results obtained when solving the systems of differential equations has showed their difference. An increase in the length of the tongue hitch of a seeder, a decrease in the rear bracket of the soil-cultivating attachment and in the length of its tongue hitch, lead to the improvement in the system’s stability. It was also established that increasing the tongue hitch of a soil-cultivating attachment over 3 m does not significantly affect the stability of its motion. In this case, it was found that increasing the mass of a seeder (\( M_{c1}\)) and an attachment (\( M_{c2}\)) contributes to that the stability of the system deteriorates by 5.8—20\%.
To validate the mathematical model adopted, we carried out additional experimental study [20]. In this case, we chose four factors that had the most significant impact. Before conducting the experimental study, we constructed a planning matrix for full factorial experiment $2^4$ using 1/2 - replica at two levels of varying factors, marked by signs $+\rightarrow$ and $-\rightarrow$ that correspond to the respective levels.

Experiments were performed under field conditions, repeated at least three times. To perform this task, the field was divided into plots with a length of 48 m and a width of 10 m, which were not cultivated in advance. We stretched a cord along the edges of the plot, adopted as a reference line. We placed pegs in chessboard order from the middle of the plot at a distance of 1.5 m for width and 8 m for length in order to set a trajectory of the assembly motion in the form of a sine wave. The distance in front of the first plot, after the eighth plot, and between the plots, was 8 m. This is explained by the necessity to re-equip a direct deeding seeder, as well as the continuity in the assembly’s motion trajectory. The assembly was driven at the field along a predetermined motion trajectory with a guide on the hood of a tractor at a constant speed.

Next, to assess the amplitude of the seeder’s oscillations, we accepted the standard deviation in the obtained trajectory of the assembly motion at peak points, from a theoretical trajectory.

When the assembly moved over the respective plots, working bodies of the direct seeding seeder left a trace. Next, we measured the distance from the reference line to the extreme coulters at peak sine wave points to the left and right along the assembly motion. A pair of peak points from the sinusoid midline constituted the repeatability of the experiment.

Next, based on peaks in the repeatability, we determined the average value of distance from the reference line, which corresponds to the value for a distance to the line of the motion trajectory of the seeder’s center. We established deviations from the theoretical trajectory of the seeder’s center. The impact of variants was verified based on a Fisher criterion.

The optimization criterion, similar to the previous model, was the distance travelled by a direct seeding seeder from the moment of perturbation to the unperturbed motion of mechanical system. Experiments were performed with a soil moisture content of 23.3 % soil and the assembly motion speed of 3 m/s.

Accepted coded designations and levels of variation for the experiment are given in Table 3 [20].

Planning matrix and results of the experiment are given in Table 4.

Based on experimental data, we derived a regression equation, which describes the influence of parameters of the direct seeding seeder on the amplitude of oscillations at motion:

$$y = 95.82 - 5.54x_1 + 21.47x_2 - 13.19x_3 + 12.03x_4 - 28.82x_1x_2 - 2.84x_2x_3 + 11.74x_3x_4 - 2.84x_4x_4 - 28.82x_1x_4.$$  

The adequacy of the model was tested based on the Fisher criterion. The significance of coefficients in the regression equation was determined based on the Student’s criterion. The calculated value of $t$-criterion for factors $x_1, x_2, x_3, x_4$ exceeds the tabular value for a 75–99 % level of significance, indicating significance of the coefficients.

An analysis of regression coefficients shows that among the four factors the largest influence (41.4 %) on the deviation of the seeder from the predefined motion trajectory of the sowing unit is exerted by an increase in the length of the seeder’s tongue hitch. It was also established that reducing the mass of the attachment’s discs reduces the deviation of the seeder by 25.3 % and reducing the mass of the seeder increases the deviation up to 23 %. The length of the attachment’s tongue hitch exerts the least influence of all these factors; it is 10.6 %. Thus, we can conclude that the experimental data confirm results of theoretical studies into the impact of factors.

### Table 3

| Coded designation of factor | Title of factor          | Measurement unit | Level of variation |
|----------------------------|--------------------------|------------------|-------------------|
| $X_1$                      | Attachment’s tongue hitch length | m                | 2.20              |
| $X_2$                      | Seeder’s tongue hitch length | m                | 3.13              |
| $X_3$                      | Mass of attaching the discs | kg               | 810               |
| $X_4$                      | Mass of seeder            | kg               | 1,580             |
Planning matrix and results of the experiment

| Experiment No. | Factors | $S$, cm |
|----------------|---------|---------|
|                | $X_1$   | $X_2$   | $X_3$ | $X_4$ |
| 1              | +       | +       | +     | +     | 90.67  |
| 2              | +       | +       | -     | -     | 75.16  |
| 3              | +       | -       | -     | +     | 134.4  |
| 4              | +       | -       | +     | -     | 57.84  |
| 5              | -       | -       | +     | +     | 41.01  |
| 6              | -       | -       | -     | -     | 61.12  |
| 7              | -       | +       | -     | +     | 162.29 |
| 8              | -       | +       | +     | -     | 141.02 |

7. Discussion of results of studying the stability of sowing machines

The study conducted has made it possible to implement a method of mathematical modelling of mechanical systems’ stability using sowing machines as an example. To address these problems on stability, we employed the second Lyapunov method, which implies the construction and investigation of functions of a perturbed motion.

A distinctive feature of our research is the unification of the method for modeling the operational stability of different types of seeders. The derived mathematical expressions have allowed us to determine the perturbation of working bodies in the common coulter systems of direct seeding seeders and those with a support-packer roller, which account for more than 90% of all cultivation machines. At the same time, the models take into consideration the excitation conditions of the system, which are characteristic of sowing machines. The calculation of the derived equations of systems’ perturbation has made it possible to identify those factors that are relevant in terms of their stability.

By solving the constructed mathematical expressions for the operational stability of a coulter system with a support-packer roller, by conducting the experimental study, we have derived the rational values for significant factors: the mass of a coulter (4...6 kg); the initial inclination angle of a hitch (80...85 degrees); the mass of a seeder (1,380...1,580 kg); the mass of attaching the discs (610...810 kg); the mass of a seeder’s tongue hitch (1.8...3.13 m), the mass of a seeder (1,380...1,580 kg). These established factors have the greatest influence on the motion stability of given mechanical systems.

We established in the course of a multi-factorial experiment the rational parameters for the mechanical systems of sowing machines based on the optimization criterion of the distance from the moment of perturbation to the unperturbed motion of working bodies.

The resulting simulation algorithm could be applied to study the stability of mechanical systems in agricultural, construction, and other machines.

8. Conclusions

1. An analysis of influences on the working bodies of sowing machines allowed us to define a system of differential equations of stability as the difference between the desired equations of the perturbed and the unperturbed motion of a mechanical system.

2. We have constructed mathematical expressions of asymptotic stability of sowing machines of different types. We identified damping patterns of the perturbed systems that suggest potential techniques to improve the stability of mechanical systems. That made it possible to derive the dependences of paths of their coulter systems on the structural-kinematic parameters of a seeder.

3. Using the constructed mathematical expressions and by conducting an experimental study, we derived rational parameters for sowing machines. It was established that the best stability of the coulter system with a supporting-packer roller (at a minimum coefficient of variation in maximum deviations) was observed at the following rational parameters: the rigidity of a push spring is 1,200...1,650 N/m; the initial inclination angle of a hitch is 80...85 degrees, the mass of a coulter is 4...6 kg. Research into stability of a direct seeding seeder established that the greatest influence (up to 41.4%) on the seeder deviating from the predefined motion trajectory of the sowing assembly is exerted by an increase in the length of the seeder’s tongue hitch. Reducing the mass of attaching the discs reduces the seeder’s deviation by 25.3%, while reducing the mass of the seeder leads to an increase in the deviations by 23%. Of all the specified factors, the length of the attachment’s tongue hitch has the least influence; it is 10.6%.

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