Effect of magnetic impurity correlations on Josephson tunneling

A. Bill\textsuperscript{a}, S.A. Wolf\textsuperscript{b}, Yu.N. Ovchinnikov\textsuperscript{c}, and V.Z. Kresin\textsuperscript{a}

\textsuperscript{a}Lawrence Berkeley Laboratory, University of California, Berkeley, CA 94720, USA
\textsuperscript{b}Naval Research Laboratory, Washington D.C. 20375-5343
\textsuperscript{c}L.D. Landau Institute for Theoretical Physics, Russian Academy of Sciences, Kosygin 2, Moscow, 11733V, Russia

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The ordering trend of magnetic impurities at low temperature results in the frustration of the pair-breaking effect and induces a “recovery” of superconducting properties. We show that this effect manifests itself in the deviation of the Josephson current amplitude from the values obtained within the Ambegaokar-Baratoff and the Abrikosov-Gor’kov models. We consider both weak and strong-coupling cases. The theory is applied to describe the experimental data obtained for the low-\(T\) superconductor SmRh\(_4\)B\(_4\). We further predict a “recovery” effect of the Josephson current in high-temperature superconductors.

I. INTRODUCTION

The interaction between mobile charge-carriers and magnetic impurities is accompanied by a spin-flip process. In superconductors, this process leads to the pair-breaking effect, that is, to the destruction of Cooper pairs. As a result, superconducting properties are strongly affected by the presence of magnetic impurities. For example, an increase of magnetic impurity concentration \(n_M\) leads to a decrease of the critical temperature \(T_c\) and to gaplessness, \(\Delta = 0\), to a decrease of the critical field \(H_c(0)\) or the Josephson current amplitude \(J_c(0)\) (at given temperature).

Most studies including the effect of magnetic impurities consider the magnetic moments as independent from one another. Because of the conservation of the total spin (disregarding spin non-conserving couplings such as the dipole-dipole interaction) scattering processes involve the spin-flip of both the charge carrier and the localized magnetic moment. However, in the region near \(T = 0\) one should take into account the correlations between impurity spins. The ordering trend of the magnetic moments frustrates the spin-flip process and decreases, therefore, the pair breaking. This decrease leads to the “recovery” effect of superconductivity. As shown by two of the authors in Ref. \[\text{4}\] the account of this effect explains the unusual temperature dependence of \(H_c(0)\) in high-temperature superconductors and leads to excellent agreement with all experimental data available so far.

The interplay of superconductivity and ordering of impurity spins has also been studied in the context of ternary compounds containing rare earth elements (\(\text{RERh}_4\text{B}_4\) with \(\text{RE} = \text{Sm, Ho, Er}\) or \(\text{Lu}\)). In some of these materials (\(\text{RE} = \text{Sm, Ho, Er}\)) the impurity spin-spin correlations lead to a magnetic phase transition into an (anti)ferromagnetic state below a critical temperature \(T_m\). \(\text{SmRh}_4\text{B}_4\) is of special interest because it shows coexistence of superconductivity and antiferromagnetism and displays an unusual temperature dependence of \(H_c(0)\). Anomalies in the Josephson critical current have also been observed in this material. The experiment shows that the Josephson amplitude is enhanced in the antiferromagnetic region as compared to the value extrapolated from the paramagnetic phase.

In the following, we calculate the critical-current amplitude \(J_c\) of a Josephson junction in the presence of magnetic impurities and study the influence of the low-temperature ordering trend. Our analysis shows that including impurity spin-spin correlations leads to a sizeable and experimentally observable increase of \(J_c\) at low temperatures with respect to the saturated value obtained when neglecting these correlations. It would be interesting to carry out experiments verifying our predictions. The experiment done on \(\text{SmRh}_4\text{B}_4\) is also discussed in the light of our theory.

II. MAIN EQUATIONS

A. The Josephson current

Based on the method of thermodynamic Green’s functions, one can obtain a set of equations for the superconducting order parameter and the renormalization function in the presence of magnetic impurities:

\[
\Delta_n Z_n = \lambda \pi T \sum_{n'} D_{nn'} \frac{1}{\sqrt{u_{n'}^2 + 1}}, \quad (1)
\]

\[
Z_n = 1 + \lambda \pi T \sum_{n'} \frac{u_{n'}}{\sqrt{u_{n'}^2 + 1}}, \quad (2)
\]

where \(D_{nn'} = \Omega^2 / [\Omega^2 + (\omega_n - \omega_{n'})^2]\) is the phonon Green’s function, \(\Omega\) is a characteristic phonon frequency and \(\lambda\) is the coupling constant. Here and in the following we define \(\Delta_n \equiv \Delta(k_F, \omega_n)\) and \(Z_n \equiv Z(k_F, \omega_n)\) as the order parameter and renormalization function of the isotropic superconductor \([\omega_n = (2n + 1)\pi T]\) with \(n = \ldots, -1, 0, 1, \ldots\) \(k_F\) is the Fermi wave-vector). The function \(u_n\) that appears in Eqs. (1) and (2) is solution of the equation (see Refs. \[\text{1, 3}\]):

\[
\frac{\omega_n}{\Delta_n} = u_n \left(1 - \frac{1}{\Delta_n \sqrt{u_n^2 + 1}}\right). \quad (3)
\]
\( \Gamma_s \sim n_M \) is the spin-flip scattering amplitude \((n_M \) is the magnetic impurity concentration). In the absence of magnetic impurities \((\Gamma_s = 0)\) the previous expression reduces to \(u_n = \omega_n/\Delta_n\). Solving Eqs. (1)-(3) for a given concentration of impurities (that is, for a given value of the parameter \(\Gamma_s\)) and for a given coupling constant \(\Lambda\) (or \(T_c\), see below) one obtains \(u_n\), which allows us to calculate the Josephson current.

Consider a symmetrical Josephson junction and let us focus on the stationary Josephson effect. The amplitude \(J_c\) of the Josephson critical current is given by (see, e.g., Ref. 5,6)

\[
J_c = \frac{\pi T}{eR} \sum_n \frac{1}{u_n^2 + 1},
\]

where \(R\) is the normal state resistance of the barrier. This relation applies for both weak and strong-coupling superconductors. The current depends on the temperature and on the concentration of magnetic impurities through \(u_n\).

In general, the sum over \(n\) cannot be evaluated analytically. This is especially true for the strong-coupling case for which the dependence of \(u_n\), Eq. (6), on \(n\) appears not only through \(\omega_n\), but also through \(\Delta_n\) and is thus non-trivial. However, there are two limiting cases, that are of interest for the present work, in which an analytical expression can be derived. Both are within the weak-coupling BCS theory \((\Delta_n \equiv \Delta)\). In the absence of magnetic impurities \((\Gamma_s = 0)\) one obtains the well-known Ambegaokar-Baratoff result:

\[
J_c^{AB}(T) = \frac{\pi}{2eR} \Delta(T) \tanh \left( \frac{\Delta(T)}{2k_B T} \right),
\]

where \(k_B\) is Boltzmann’s constant. In the numerical part we compare this temperature dependence with the one obtained for a superconductor containing (un)correlated magnetic impurities. Eq. (6) has two important features. The current saturates as \(T \to 0\), and is linear near \(T_c\). These features remain when uncorrelated magnetic impurities are added to the system. In the second limiting case we allow for the presence of magnetic impurities, but set \(T = 0\). Then,

\[
\Delta \left[ \frac{\pi}{2} \tau - \frac{3}{2} \bar{\Gamma} \right]
\]

\[
\Delta \left[ \frac{\pi}{2} - \arctan \sqrt{1 - \frac{4}{9} \bar{\Gamma}} + \frac{\tau/2 - 1}{\sqrt{1 - \frac{4}{9} \bar{\Gamma}} (2\bar{\Gamma}^2 + 1)} \right]
\]

with \(\bar{\Gamma} = \Gamma_s/\Delta\). This expression is useful for the determination of \(J_c\) at \(T = 0\) in the weak-coupling case. For strong-coupling superconductors one has to use Eqs. (1) and (3). The numerical solution near \(T = 0\) can then be obtained using the Poisson Formula.

### B. Spin Ordering of Magnetic Impurities

The equations of the previous section describe the pair-breaking effect induced by the time-reversal non-invariant perturbation due to magnetic impurities. The theory accounts for such a perturbation through the spin-flip scattering amplitude \(\Gamma_s = 1/\tau_s\) (see Eq. 3; \(\tau_s\) is the relaxation time). In the usual picture of independent magnetic moments, the amplitude \(\Gamma_s\) is a temperature independent parameter. As already stressed in the introduction, the assumption of independent impurity spins is a good approximation at high temperatures and low impurity concentrations, but is not valid in the other cases. One example where the independent impurity-spin assumption is not valid is given by systems undergoing a magnetic phase transition at a critical temperature \(T_m\). The observed deviations from the expected behaviour of superconducting properties near and below the (anti)ferromagnetic transition temperature of ternary compounds mentioned in the introduction point towards a change in the value of \(\Gamma_s\) due to spin-spin correlations. We stress, however, that spin-spin correlations are important even if one does not observe a magnetic phase transition. For example, the formation of a spin “glass” is related to these correlations. The importance of magnetic impurity correlations increase with decreasing temperature. The anomalous behaviour of the critical field \(H_{c2}\) observed in the cuprates can be explained by such correlations. Another manifestation of such a “recovery” effect is the decrease in microwave losses observed under application of an external magnetic field. In this case, the frustration of the spin-flip process is due to the external field.

Because of the spin-spin correlations (the ordering trend of magnetic impurities) the spin-flip scattering amplitude depends on temperature. This dependence \(\Gamma_s(T)\) was studied by two of the authors in Ref. 4 and was applied to the evaluation of \(H_{c2}\). According to this study, \(\Gamma_s(T)\) can be represented in the form:

\[
\Gamma_s(T) = \begin{cases} 
\Gamma_0 \left( 1 + \frac{\beta \tau}{1 + \tau} \right) & : \; T > T_1 \\
\Gamma_0 \left( 1 + \frac{\tau}{1 + \tau} \right) & : \; T < T_1 
\end{cases}
\]

where \(\tau = (T - T_1)/\theta, \; T_1 = T_m - \delta T\) with \(\delta T = \alpha \theta\) and \(\alpha \approx 1\). This expression is valid both for systems that do and do not undergo a magnetic phase transition. In the former case \(T_m\) is the temperature of the phase transition and \(\delta T\) corresponds to the width of the critical region around \(T_m\) (it is such that \(T_m - \delta T > 0\)). On the other hand, if there is no phase transition (e.g., when dealing with the formation of a spin “glass”), one sets
\[ T_1 = \delta T = T_m = 0 \] and \( \theta \) is the characteristic parameter of the ordering.

The expression (\ref{recovery}) introduced in Ref. \cite{ref} describes the “recovery” effect in a phenomenological way (a microscopic treatment will be presented elsewhere). As can be seen from Eq. (\ref{recovery}), the spin-flip scattering amplitude is constant for \( T > \theta \) (or \( T_m \)) and decreases continuously near and below \( T \approx \theta \) (or \( T_m \)). Below \( T_1 \) the scattering amplitude \( \Gamma_s \) is a constant independent of temperature. A change of only several percent of the value of \( \Gamma_s(T) \) at low temperatures can strongly affect the temperature dependencies of properties such as \( H_c2 \) or \( J_c(T) \) (see Refs. \cite{ref} and below).

Generally speaking, the theory contains three parameters: \( \Gamma_0 \), \( \beta \) and \( \theta \). To determine their value one has to consider separately the case of conventional and high-\( T_c \) superconductors. In the first case, the only existing experiments studying the effect of ordering on pair-breaking were done on systems that are (anti)ferromagnetically ordered. A reasonable value for the characteristic temperature \( \theta \) is thus given by the critical temperature of the phase transition which is of the order of 1K. Furthermore, the value of \( \Gamma_s(T_c) \) can be obtained within the Abrikosov-Gor’kov theory, once the experimental value of the critical temperature \( T_c \) in the presence of magnetic impurities is given. Its value corresponds to the high-temperature limit of the spin-flip scattering amplitude (when \( \theta \ll T_c \)). We are thus left with one free parameter.

In the case of high-temperature superconductors the analysis of \( H_{c2} \) performed in Ref. \cite{ref} delivers the values for all parameters. There is thus no free parameter left for the high-\( T_c \) materials studied in Ref. \cite{ref}. For example, for overdoped \( \text{Tl}_2\text{Ba}_2\text{CuO}_6 \) studied experimentally in Ref. \cite{ref} one has \( \Gamma_0 = 95\text{K}, \beta = 1.26, \theta = 1\text{K} \), whereas for \( \text{Bi}_2\text{Sr}_2\text{CuO}_6 \) studied in Refs. \cite{ref} one has \( \Gamma_0 = 105\text{K}, \beta = 1.38 \) and \( \theta = 1.6\text{K} \).

Based on Eqs. (\ref{recovery}), (\ref{amplitude}) and (\ref{amplitude}) we can calculate the amplitude of the Josephson critical current in the presence of magnetic-impurity ordering at low temperatures.

### III. RESULTS AND DISCUSSION

We consider the weak and strong coupling cases separately. Since, to our knowledge, no experimental study of the ordering trend on \( J_c(T) \) has been undertaken either on conventional or high-temperature superconductors for systems that do not undergo a magnetic phase transition, we chose arbitrary but realistic values of the parameters and discuss qualitatively what can be expected for the two types of superconductors.

#### A. Weak Coupling

For a weak-coupling superconductor \( Z \equiv 1, \Delta(i\omega_n) \equiv \Delta \) and one neglects the dependence on \( \omega_n \) of \( D_n^\text{AB} \). Eqs. (\ref{recovery})-(\ref{amplitude}) reduce to the Abrikosov-Gor’kov theory. For the numerical calculations we determine \( \lambda \) and \( \Gamma_s \) from the experimental values of \( T_c \) and \( T_{c0} \) (the critical temperature with and without magnetic impurities). \( \lambda \) is obtained from Eqs. (\ref{recovery})-(\ref{amplitude}) for \( \Gamma_s = 0 \), i.e. \( \lambda = \ln^{-1}(\Omega/T_{c0}) \). On the other hand, \( \Gamma_s(T_c) \) can be extracted from the Abrikosov-Gor’kov equation for \( T_c \) in the presence of magnetic impurities.

\[
\ln\left(\frac{T_{c0}}{T_c}\right) = \psi\left(\frac{1}{2} + \gamma_s\right) - \psi\left(\frac{1}{2}\right),
\]

where \( \gamma_s = \Gamma_s(T_c)/2\pi T_c \) is the pair-breaking parameter and \( \psi \) is the psi function.

Fig. 1 shows the temperature dependence of the Josephson amplitude for different concentrations of magnetic impurities, that is for different values of \( \Gamma_0 \). The temperature and amplitude of the current are normalized to the critical temperature \( T_{c0} \) and the amplitude \( J_c(\Omega = 0) \) (see Eq. (\ref{amplitude}) in the absence of magnetic impurities, respectively. As seen in the figure, the presence of magnetic impurities lowers the value of \( T_c \) and \( J_c(T) \) with respect to the Ambegaokar-Baratoff result. As stated in the introduction this is a consequence of the pair-breaking effect. One notes further that \( J_c(T) \) does not saturate as \( T \rightarrow 0 \) but increase in a temperature range given by \( \theta \).

The upturn is the direct signature of the impurity-spin ordering trend, that is, the temperature dependency of the spin-flip scattering amplitude. There is a “recovery” effect due to impurity-spin ordering opposing the saturation trend of \( J_c \) for uncorrelated spins. Note that the recovery is not complete \( [J_c(T = 0) < J_c^\text{AB}(0)] \) since \( \Gamma_s(T \rightarrow 0) \neq 0 \). This is even true for a fully ordered state. The mechanisms impeding the total recovery of the Ambegaokar-Baratoff result (as, e.g., dipole-dipole interactions; see also Ref. \cite{ref}) will be described elsewhere.

The ordering trend of local moments leads to a positive curvature of \( J_c(T) \) at low temperatures. Fig. 1 shows that, the high-temperature linear part becomes dominant as one increases the concentration of magnetic impurities. The account of correlations leads thus to a behaviour of \( J_c(T) \) that is qualitatively different from the result obtained for \( H_{c2}(T) \). Indeed, in the absence of spin correlations, \( J_c \) and \( H_{c2} \), considered as functions of temperature, both have a negative curvature up to \( T_c \) and are linear in the vicinity of \( T_c \). Adding magnetic impurities extends the high-temperature linear part of \( J_c(T/T_c) \) to lower temperatures to the expense of the low-temperature region (that has positive curvature if impurity correlations are taken into account; see Fig. 1). The effect on \( H_{c2}(T) \) is opposite. For high enough magnetic impurity concentrations, \( H_{c2}(T) \) displays a positive curvature over the whole temperature range when correlations are taken into account. The concentration of magnetic impurities
at which a positive curvature of $H_{c2}$ was observed experimentally corresponds to the case $\Gamma_\circ = 8$ of Fig. 1 (i.e. to the gapless regime).

Fig. 2 gives another representation of the situation presented in Fig. 1. Compare first the solid and dashed lines. The dashed curve corresponds to the case $\Gamma_\circ/T_{c0} = 0.6$, $\theta/T_{c0} = 0.1$ and $\beta = 1.3$ (same parameters as the dotted line of figure 1) whereas the solid line corresponds to the case of uncorrelated spins and $\Gamma_s/T_{c0} \simeq 0.73$. The latter value was chosen in such a way that the critical temperature $T_c$ is the same for both curves. One can see that the result obtained by taking into account the correlation of impurity spins strongly deviates from the curve expected when the spins are uncorrelated. This deviation is already significant at temperatures near $T_c$ for high enough impurity concentrations, but is largest near $T = 0$. Furthermore, the amplitude of the deviation is such that it should be experimentally observable.

Another conclusion can be drawn when comparing the dashed and dotted lines of Fig. 2. These curves were obtained for different values of the parameter $\beta$ [see Eq. (7)]. Thus, both curves include the spin-correlations of the magnetic impurities, but the strength of the correlations is different. The dashed line corresponds to $\beta = 1.3$ whereas the dotted line is for $\beta = 1.5$. Because $\Gamma_\circ$ remained unchanged, the two curves have slightly different $T_c$'s. The increase of $\beta$, which corresponds to an increase of the influence of magnetic-impurity correlations, implies a stronger temperature dependency of $J_s(T)$. Fig. 2 shows that impurity correlations reduce the high and intermediate temperature negative curvature of $J_s(T)$. Note, finally, that the value at $T = 0$ is independent of $\beta$.

It would be interesting to study experimentally the same high-temperature superconductors as those for which $H_{c2}$ was measured and described in Ref. 1. Indeed, the values of the parameter for the expression of $\Gamma_s(T)$ are given by the analysis of $H_{c2}(T)$, leaving no free parameter (see previous section). Of special interest would be the case of $\text{YBa}_2(\text{Cu}_{1-x}\text{Zn}_x)_3\text{O}_{6-\delta}$ since there are evidences that the depression of $T_c$ and other superconducting properties is mainly due to the increase in magnetic impurity concentration (see also section on strong-coupling superconductors).

Note, finally, that our calculations suggest a possible application of Josephson junctions based on superconductors doped with magnetic impurities (as e.g. high-$T_c$ materials). Indeed, because the temperature dependence of the Josephson current is linear for superconductors in the gapless regime (see dash-dotted line of Fig. 1), the measure $\Delta J_s(T)$ for such systems can be used as a thermometer.

**B. Ternary compounds**

Up to now we have assumed that the magnetic moments are correlated but remain disordered. In the case of a magnetic phase transition, the result differs from above in that the temperature dependency of $\Gamma_s$ vanishes below $T_m$ [see Eq. (8)]. This is shown on Fig. 3 for parameters corresponding to the experimental results on $\text{SmRh}_3\text{B}_4$. The critical temperature in the absence of magnetic impurities is set to $T_{c0} = 11.4K$, the value of $T_{c0}$ for $\text{LuRh}_4\text{B}_4$ which has the same structure as the $\text{Sm}$ compound, is also superconducting but has no magnetic moments. The values $T_c = 1.815K$, $T_m = 0.87K$, $\delta T_\circ = 0.21K$ were taken from the experiment. The value $\Gamma_\circ = 9.75K$ was determined from the measure of $J_s(T_1 = T_m - \delta T)$. Finally, the remaining parameter $\beta = 1.029$ was fixed by the value $\Gamma_s(T_c)$ chosen in such a way that $T_c \simeq 1.815K$ from Eqs. (7) and (8). The values of all parameters have thus been taken from the experiment and there is no free fitting parameter left. Regarding the determination of these parameters, one notes first that we are in the gapless regime ($\Gamma = \Gamma_s(T)/\Delta(T) > 1$) and thus in the situation of the dash-dotted curve of Fig. 1. In addition, the value of $\beta$ that arises because of the spin-spin correlations is close to one (value at which the spins are uncorrelated) and thus much smaller than most of the high-$T_c$ superconductors studied in Ref. 2 (which are of the order $\beta \sim 1.3$).

Comparing our result with the experiment on the ternary compound, one notes that the behaviour above the temperature $T_1$ (at which the transition to the antiferromagnetic state is complete) is well reproduced by our calculation. This demonstrates that the increase of the current observed near $T_1$ in Ref. 2 can indeed be explained by the ordering trend of the magnetic impurities. Below $T_1$, however, the situation is less clear. If we use Eq. (8) then we observe a distinct discrepancy between experiment and theory. In the experiment, the current continues to increase as $T$ is lowered, whereas the current saturates in our calculation (see dashed line below $T_1$ in Fig. 3). One reason for this discrepancy lies in the fact that in metallic systems containing magnetic impurities several processes occur that were not accounted for in the present analysis. One of them is connected with symmetry breaking caused by the impurities. Another reason for the discrepancy is related to the direct dipole-dipole or RKKY type interaction between the impurity spins. In the specific case studied in Ref. 2, the occurrence of a proximity effect due to unoxidized parts of the Lu layer of the junction may also be responsible for the linear behaviour of $J_s(T)$ for $T < T_1$ (see Ref. 2). Further experimental studies should be carried out on this system to determine which mechanism is the most relevant to explain the data.

To take into account the additional temperature dependence of the spin-spin correlations induced by the above processes, we have considered a temperature de-
dependent $\Gamma_s$ below $T_1$ as well, but with modified values of the parameters. The experimental value $J_c(T_1)$ gives $\Gamma_s(T_1)$, $\theta$ is kept unchanged and $\beta = 1.08$ gives the best fit to the data. As seen on Fig. 3 (solid line), the temperature dependence of $\Gamma_s$ below $T_1$ can account well for the experimental result. This is not a trivial statement because, as shown in the previous figures, the temperature dependency $J_c(T)$ cannot be modified at case by changing the values of the parameter. The fact that one has to include a temperature dependency of $\Gamma_s$ in the antiferromagnetic region leads us to the conclusion that not only the contact interaction but also the other mechanisms (see above and Ref. 16) are important in this material.

C. Strong Coupling

We study the influence of strong coupling onto the results of the last section. This case is numerically more difficult to handle, due to the fact that the renormalization function has to be considered and that all three unknown functions $\Delta_s$, $Z_n$ and $u_n$ depend on $n$. As before, we determine $\Gamma_s$ and $\lambda$ from the equations for $T_c$ and $T_{co}$. For $T_c$ one linearizes Eq. 3. As for $T_{co}$ ($\Gamma_s(0)$) we use the expression derived by one of the authors:

$$T_{co} = \frac{0.25\Omega}{(e^{2/\lambda} - 1)^{1/2}},$$

where $\Omega$ is the characteristic phonon frequency. Fig. 4 shows a generic case for the strong coupling limit. There is no qualitative difference between the weak and strong coupling results. The parameters in Fig. 4 were chosen so as to correspond to the case of YBa$_2$(Cu$_{1-x}$Zn$_x$)$_3$O$_{7-\delta}$ with $x \approx 0.04$ ($T_c \approx 55K$). This result leads us to conclude that the overall qualitative behaviour obtained for weak-coupling superconductors is not modified by increasing the strength of the coupling. In particular, one expects a nearly linear behaviour of $J_c(T)$ for high-$T_c$ materials where $T_c$ has been strongly reduced by the presence of magnetic impurities and are in the gapless regime. Examples of such systems are Zn-doped YBa$_2$Cu$_3$O$_7$ and La$_{1.85}$Sr$_{0.15}$CuO$_4$ as well as other under- and overdoped materials studied in Ref. 3. (Tl$_2$Ba$_2$CuO$_6$, Bi$_2$Sr$_2$CuO$_6$, or Sm$_{1.85}$Ce$_{0.15}$CuO$_{4-y}$). It would be interesting to perform measurements on these (or other) systems to verify our predictions. As stated earlier, the above mentioned materials are of special interest, since all parameters necessary to calculate $\Gamma_s(T)$ and $J_c(T)$ [Eqs. 3 and 4, respectively] were determined from the study of H$_2$.

IV. CONCLUSION

In summary, we have calculated the amplitude of the Josephson current $J_c(T)$ in the presence of magnetic impurities taking into account their ordering trend at low temperatures. We have shown that the “recovery” effect introduced in Ref. 1 is also occurring for the Josephson current $J_c(T)$. The effect manifests itself in an upturn of $J_c(T)$ at low temperatures (instead of the saturation occurring for uncorrelated magnetic moments). The effect is sizeable and qualitatively similar in weak and strong-coupling superconductors. The theory was used to explain the experimental results obtained for SmRh$_2$B$_4$ and leads to good agreement for temperatures above the magnetic phase transition. We have also presented results for high-$T_c$ materials.

It would be interesting to measure the effect in both conventional and high-temperature superconductors so as to verify our predictions. The effect might be larger in the latter (for overdoped or Zn-doped systems) as indicated by the unusual temperature dependence of $H_c2$, since most of these materials are in the gapless regime.

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Figure captions:

Fig. 1: Temperature dependence of the Josephson current amplitude $J_c(T)$ for different magnetic impurity concentrations. The temperature and the current amplitude are normalized to the values $T_{c0}$ and $J_c^{AB}(T=0)$ in the absence of magnetic impurities, respectively. $\beta = 1.3$, $\theta/T_{c0} = 0.1$ and $\Gamma_0/T_{c0} = 0$ (Ambegaokar-Baratoff; solid line), $\Gamma_0/T_{c0} = 0.4$ (dashed), $\Gamma_0/T_{c0} = 0.6$ (dotted) $\Gamma_0/T_{c0} = 0.8$ (dotted-dashed).

Fig. 2: Temperature dependence of the Josephson current for different values of $\beta$. Solid line: $\beta = 1$, dashed line: $\beta = 1.3$, dotted line $\beta = 1.5$. $\Gamma_0/T_{c0} = 0.6$, $\theta/T_{c0} = 0.1$.

Fig. 3: Josephson current for SmRh$_4$B$_4$. Solid line: theoretical result with $\Gamma_s(T)$, dashed line: $J_c(T)$ for $\Gamma_s(T < T_1) = \Gamma_0$, dots: experimental points from Ref. 7. The parameters are given in the text.

Fig. 4: Influence of impurity-spin correlations on the Josephson current amplitude for strong-coupling superconductors. solid line: uncorrelated spins, dashed line: correlated impurity spins. The parameters correspond to the case of YBa$_2$(Cu$_{1-x}$Zn$_x$)$_3$O$_{7-\delta}$ with $x \simeq 0.04$ (see text).
FIG. 1.
FIG. 2.
FIG. 3.
FIG. 4.

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