The effective theory of type IIA AdS$_4$ compactifications on nilmanifolds and cosets

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ABSTRACT: We consider string theory compactifications of the form AdS$_4 \times \mathcal{M}_6$ with orientifold six-planes, where $\mathcal{M}_6$ is a six-dimensional compact space that is either a nilmanifold or a coset. For all known solutions of this type we obtain the four-dimensional $\mathcal{N} = 1$ low energy effective theory by computing the superpotential, the Kähler potential and the mass spectrum for the light moduli. For the nilmanifold examples we perform a cross-check on the result for the mass spectrum by calculating it alternatively from a direct Kaluza-Klein reduction and find perfect agreement. We show that in all but one of the coset models all moduli are stabilized at the classical level. As an application we show that all but one of the coset models can potentially be used to bypass a recent no-go theorem against inflation in type IIA theory.

KEYWORDS: anti-de Sitter vacua, cosets, effective theory, inflation.
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1. Introduction and summary

Motivation

String compactifications with non-trivial $p$-form fluxes provide a class of backgrounds in which the stabilization of moduli fields – a phenomenologically problematic feature of traditional compactifications on spaces such as Calabi-Yau manifolds – can be realized in a controlled way within the classical supergravity approximation. Motivated by this and other applications, the derivation of the four-dimensional (4D) low energy effective actions of flux compactifications has been an active area of research during the past decade (for reviews and a more complete list of references, see, e.g., [1, 2, 3]).

One aspect that complicates the derivation of these effective actions is that the $p$-form fluxes generally back-react on the geometry of the compact manifolds, deforming them away from well-understood classes such as Calabi-Yau spaces. This back-reaction can be rather mild, as, e.g., in type IIB orientifolds with D3/D7-branes [4], where the internal space is still conformal to a Calabi-Yau manifold. In these comparatively simple models, however, the fluxes turn out to stabilize only the dilaton and the complex structure moduli, while the Kähler moduli stabilization requires the use of quantum effects, e.g., along the lines of [5].

In the present paper, we will instead be interested in a different class of flux compactifications for which the back-reaction of the fluxes on the geometry is less trivial. Concretely, we will study $\mathcal{N} = 1$ supersymmetric flux compactifications of type IIA string theory on background spaces of the form $\text{AdS}_4 \times \mathcal{M}_6$, where $\mathcal{M}_6$ is a six-dimensional compact space.
that is in general not a Calabi-Yau manifold. Being compactifications to AdS space-time, these models do not appear realistic as such, but they can serve as starting points for the construction of more realistic setups or have other applications. More specifically, some of the motivations for studying this class of string vacua are:

- In certain AdS\textsubscript{4} type IIA vacua, it is possible to stabilize all moduli at tree level in a controlled supergravity regime. It is then an interesting question whether these compactifications can be of phenomenological use, e.g., after the inclusion of an additional uplifting potential so as to construct meta-stable de Sitter vacua in the spirit of the IIB models discussed in [5]. But even without an explicit uplift potential, one can investigate whether the potential already has meta-stable de Sitter vacua away from the AdS vacuum or whether there are regions suitable for slow-roll inflation.

- Although no longer Calabi-Yau spaces, the manifolds \( M_6 \) we discuss in this paper have a surprisingly simple and explicitly known geometry, which makes them in some sense even more tractable than a Calabi-Yau manifold. In particular, it is possible to derive the low energy spectrum on AdS\textsubscript{4} directly from an explicit Kaluza-Klein reduction, as we demonstrate for some of the models. For these models, we find complete agreement with the results from the more commonly used, but less direct, \( \mathcal{N} = 1 \) supergravity techniques, which provides a valuable consistency check.

- Replacing fluxes by branes, the above AdS vacua can be potentially obtained as near-horizon geometries of intersecting branes [6]. In view of recent developments following the Bagger-Lambert-Gustavsson theory [7], AdS\textsubscript{4} flux vacua of the type considered here may then admit a full non-perturbative definition via a dual three-dimensional CFT. The above-mentioned brane solutions also correspond to domain walls that interpolate between different flux vacua. The existence of these domain walls may correspond to interesting transitions in the landscape of flux vacua.

The first examples of type IIA models where all the moduli were stabilized at tree level in a controlled classical supergravity regime were the torus orientifold models of [8, 9, 10, 11]. Possible cosmological applications along the lines described in the first item above were subsequently explored in a number of papers, with surprisingly little success. In [12], for instance, a simple F-term uplift to a meta-stable de Sitter vacuum based on an effective O’Raifeartaigh sector was found to be impossible. Using similar arguments, the authors of [13, 14] could also formulate a no-go theorem against slow-roll inflation and de Sitter vacua for general type IIA models with only 3-form NSNS-flux, RR-fluxes, D6-branes and O6-planes. As additional ingredients that can circumvent this no-go theorem, the authors of [14] identified geometric fluxes, NS5-branes and/or the more exotic non-geometric fluxes.\footnote{Recent progress obtaining inflation with these ingredients appeared in [15, 16].} We will give a short discussion of this no-go theorem in the context of the models considered in this paper, as they contain one of these additional ingredients, namely geometric fluxes.
Models we will study

As was pointed out in [17], the model of [10] belongs to a larger class of supersymmetric strict SU(3)-structure type IIA AdS\(_4\) vacua, the conditions for which were presented in [18], generalizing the earlier work of [19]. To obtain a ten-dimensional supergravity description of the model of [10], a fine-tuned smeared orientifold source must be introduced. In fact, this procedure works for any Calabi-Yau space (including K3\(\times\)T\(^2\) and the torus). More generally, however, the vacua of [18] need not be Calabi-Yau: certain torsion classes of the SU(3)-structure can be non-zero, which in the physics literature is sometimes called geometric flux. It is then natural to ask whether any of these more general IIA AdS\(_4\) vacua could be used in order to succeed where the model of [10] has failed; in particular whether they can be used to bypass the above-mentioned no-go theorem of [14].

To the best of our knowledge, there are only very few explicitly known examples of \(\mathcal{N} = 1\) supersymmetric type IIA AdS\(_4\) compactifications with geometric fluxes. They all can be seen to belong to the class of solutions discussed in [18] (or to T-duals thereof). More explicitly, the known examples are of the form AdS\(_4\) \(\times\) \(\mathcal{M}_6\) with the internal space \(\mathcal{M}_6\) being one of the following:

- **The Iwasawa manifold and T-duals:**

  The Iwasawa is a particular nilmanifold (or ‘twisted torus’ in the physics literature), proposed as a type IIA solution in [18]. As for the torus examples mentioned above, smeared orientifold sources have to be introduced. In fact, for a certain regime of the parameters it is T-dual to a six-torus compactification. Essentially, the Iwasawa solution is the twisted torus T\(^6\)/(\(\mathbb{Z}_2 \times \mathbb{Z}_2\)) example examined in [8, 9, 11]. As shown in [6] and reviewed in section 3.4, for these spaces it is possible to replace all fluxes by their corresponding brane sources (D-branes, NS5-branes, KK-monopoles) so that the original backgrounds arise as near-horizon geometries of the intersecting branes.

  We will also study a type IIB example with static SU(2)-structure on the nilmanifold 5.1 (according to the labelling of Table 4 of [20]), which is the intermediate node in the T-duality web between the torus and the Iwasawa example.

  For completeness, we mention that the Iwasawa solution is in fact a singular degeneration of a model on K3\(\times\)\(t\)T\(^2\), where the subscript \(t\) denotes a geometric twist in the T\(^2\)-fiber. In this paper we will not study this solution, which was also proposed in [18], any further, although we expect it to be similar to the Iwasawa solution.

- **The coset spaces:**

  The group manifold SU(2)\(\times\)SU(2) and the coset spaces \(\frac{SU(3)}{SU(2)}\), \(\frac{Sp(2)}{SU(2)\times U(1)}\), \(\frac{SU(3)}{U(1)\times U(1)}\), and \(\frac{SU(3)\times U(1)}{SU(2)}\) provide the remaining known examples. It will be convenient to henceforth refer to all these as ‘the coset models’ even though SU(2)\(\times\)SU(2) is a trivial coset. We remark that \(\frac{SU(3)\times U(1)}{SU(2)}\) is somewhat special in that it is the only case in which the coset models are not T-dual to the torus.

\(^2\)In the Iwasawa model in this paper there are four orientifolds. These can be equivalently described as a single orientifold supplemented with its images under a certain geometric \(\mathbb{Z}_2 \times \mathbb{Z}_2\) group acting on the internal manifold.
coset model in the above list that does not have a nearly-Kähler limit and that does not allow for a type IIA solution without orientifold sources. While examples in other contexts have already appeared some time ago [21, 22], solutions of type IIA string theory on the cosets with nearly-Kähler limit were proposed only in the more recent works [23, 19, 24, 25, 26]. Finally, in [27], from which we will start the analysis in this paper, a systematic search for coset solutions was performed, and the non-nearly-Kähler example of \( \frac{\text{SU}(3) \times U(1)}{\text{SU}(2)} \) was added to the list of solutions. As explained in [27], each of these supersymmetric vacuum solutions has a number of parameters describing its scale, shape and orientifold charge. In the full string theory these parameters are quantized.

In terms of the internal components, \( \eta^{(1,2)} \), of the 10D supersymmetry generators, a compactification on a manifold with strict SU(3)-structure corresponds to the case when \( \eta^{(1)} \) and \( \eta^{(2)} \) are proportional (see appendix B for our terminology and notation on internal spinors and the G-structure of manifolds). More general compactification ansätze for the supersymmetry generators are also conceivable. The corresponding general framework is commonly referred to as SU(3)×SU(3)-structure. The conditions for supersymmetric solutions with this ansatz were derived in [28]. In appendix B.2 we will provide a no-go theorem against supersymmetric AdS\(_4\) compactifications in both IIA and IIB with a left-invariant SU(3)×SU(3)-structure that is neither a strict SU(3) nor a static SU(2). A way out would be to consider \( e^{2A-\Phi} \eta^{(2)\dagger} \eta^{(1)} \) non-constant in type IIA (where \( A \) and \( \Phi \) denote, respectively, the warp factor and the dilaton), while in type IIB we need a genuine type-changing dynamic SU(3)×SU(3)-structure. This is beyond the scope of this paper.

**The four-dimensional effective theory**

In this paper, we will study in detail the effective 4D field theory that is obtained from the compactification on the above-mentioned nilmanifolds and coset spaces. To render the analysis tractable, we will only consider SU(3)-structures and fluxes which are constant in the basis of left-invariant one-forms, as in [27]. Thanks to these simplifications, we will be able to straightforwardly construct the effective action using 4D effective supergravity techniques. In more detail, we use the expression for the Kähler potential and the superpotential of [29, 30, 31, 32]. For more work see also [33, 34, 35, 36].

A general problem of this supergravity approach is that an explicit computation of the low energy theory of a given compactification requires a suitable choice of expansion basis for the ‘light’ fluctuations. Unfortunately, it is still unclear how to construct such a basis in general. In generic flux compactifications, the set of harmonic forms would be unsuitable as expansion forms, as the forms \( J \) and \( \Omega \) that define the SU(3)-structure (and which enter the supergravity expressions for the Kähler and superpotential) are no longer closed (see e.g. [37, 30, 34] for a few proposals). A detailed discussion of the general constraints on such a basis appeared in [38]. In the special case of nilmanifolds and coset manifolds, however, the set of left-invariant forms (with the appropriate behaviour under the orientifold action) readily presents itself as the natural choice and obeys the requirements of [38].\(^3\)

\(^3\)Since the left-invariant forms are constant over the moduli space, this basis satisfies requirements *7-*9
Interestingly, for our models, it is also possible to derive the low energy effective action using an alternative approach, which does not rely on supersymmetry: direct Kaluza-Klein reduction (for a review of this approach in eleven-dimensional supergravity see [39]). In section 5, we will provide an important consistency check by calculating the mass spectrum for the six-torus and the Iwasawa examples, both by Kaluza-Klein reduction as well as in the effective supergravity approach, obtaining exactly the same result in both cases (see also [9] for related work). Having performed this consistency check for the six-torus and the Iwasawa examples, we will restrict ourselves to the effective supergravity approach for the coset examples of section 6.

Orientifolds

In this paper, we introduce orientifold sources for a number of reasons. The first reason is that, in some of the models we study, the Bianchi identities cannot be satisfied without orientifolds (to be specific, this concerns the nilmanifold examples and the $SU(3) \times U(1)$ model). Secondly, as we discuss further in section 2.2, the orientifolds potentially allow for a hierarchy of scales between the size of the internal manifold and the AdS$_4$ curvature, thereby providing a possibility to decouple the tower of Kaluza-Klein modes from the light modes. The third reason is that we are interested in 4D, $\mathcal{N} = 1$ supersymmetric low energy effective theories, for which the orientifold sources are necessary.\footnote{For a discussion of the $\mathcal{N} = 2$ theory arising from IIA on nearly-Kähler manifolds without orientifolds see [40].}

A somewhat delicate feature of our models is that the orientifolds have to be smeared. The reason for this is that the supersymmetry conditions of [18] (for constant Romans mass) force the warp factor to be constant. Considering the back-reaction of a localized orientifold, on the other hand, one would expect a non-constant warp factor, at least close to the orientifold source. A possible way around this contradiction is that taking into account $\alpha'$-corrections might allow for a non-constant warp factor (see also [41] for an alternative discussion). A helpful interpretation of the smearing of a localized source, whose Poincaré dual is given roughly-speaking by a delta-function, is that it corresponds to Fourier-expanding the delta-function and discarding all but the zero mode. In this paper, we will adopt the pragmatic point of view that the smeared orientifolds are an unavoidable feature of our models that is consistent with a Kaluza-Klein reduction in the approximation where only the lowest modes are kept.

The question of how to associate orientifold involutions to a smeared source turns out to be somewhat subtle. We will make the natural assumption that the different orientifolds correspond to the decomposable (simple) terms in the orientifold current; the rationale and details behind this are explained in appendix C.

Outline of the paper and summary of results

In section 2, we review the general properties of the AdS$_4$ solutions of [18] on which all the type IIA examples of the present paper are based. We also discuss the issue of the

\footnote{Note that left-invariant forms are not in general harmonic: they can be combined into eigenmodes of the Laplacian to eigenvalues of the order of the geometric flux.}
separation of scales and the self-consistency of our analysis. This requires that we can
decouple the higher Kaluza-Klein modes in a regime with small string coupling and a
sufficiently large internal manifold (in units of the string length) so as to be able to use the
classical supergravity approximation.

Section 3 examines the geometry of the nilmanifold examples, namely the six-torus
and the Iwasawa manifold in the IIA theory, as well as the nilmanifold 5.1 in type IIB.
If, for the Iwasawa manifold, we restrict to the branch of moduli space where the Romans
mass is zero, these three manifolds can be shown to be T-dual to one another.

Interestingly, as shown in section 3.4, for the same range of the parameter space for
which the T-dualities above are valid, the solutions admit an interpretation as near-horizon
geometries of intersecting brane configurations, as in [6]. From this point of view, the
nilmanifold vacua in this range are nothing but near-horizon geometries of intersections of
KK-monopoles with other branes in flat space. This nice feature of the ‘brane picture’ is
summarized in Table 1. Each solution in this table is related to the one in the column next
to it by a T-duality.

\[
\begin{array}{|c|c|c|}
\hline
 & \text{IIA} & \text{IIB} \\
\hline
\text{T}^6 & \text{nilmanifold 5.1} & \text{Iwasawa} \\
D4/D8/NS5 & D3/D5/D7/NS5/KK & D2/D6/KK \\
\hline
\end{array}
\]

\textbf{Table 1: Brane picture}

In section 4, we come to the analysis of the geometry of the coset examples; this is
mostly review material in which we follow closely reference [27]. The four-dimensional low
energy physics of our models is then analysed in the subsequent two sections: section 5
discusses the nilmanifolds, whereas section 6 deals with the coset models. In each case, we
compute the superpotential and the Kähler potential, as well as the mass matrix of the
scalars. As mentioned earlier, in the case of the six-torus and the Iwasawa manifold, the
masses of the lightest excitations are obtained both by a direct Kaluza-Klein reduction as
well as by using the general expressions for the superpotential and Kähler potential based
on the effective supergravity approach, with agreement in both cases.

In Table 2 we list the nilmanifold geometries of section 3, indicating the number of light
real scalar fields in each case. We also indicate how many real moduli remain unstabilized
and whether, according to the analysis of section 2.2, it is possible to decouple the Kaluza-
Klein tower. As we show in section 5, in each of the above models, three axions remain

\[
\begin{array}{|c|c|c|c|}
\hline
 & \text{T}^6 & \text{nilmanifold 5.1} & \text{Iwasawa} \\
\hline
\text{Light fields} & 14 & 14 & 14 \\
\text{Unstabilized} & 3 & 3 & 3 \\
\text{Decouple KK} & \text{yes} & \text{yes} & \text{yes} \\
\hline
\end{array}
\]

\textbf{Table 2: Results for the nilmanifolds}
massless\textsuperscript{5}. In [11], it was argued that upon introducing space-time filling D-branes, the massless axions may be ‘eaten’ via a Stückelberg mechanism to provide masses for pseudo-anomalous abelian gauge fields on the world-volume of the branes; we do not pursue this here any further.

Table 3 lists the coset geometries of section 4, indicating in each case the number of light real scalar fields, how many of them stay massless, whether it is possible to decouple the tower of Kaluza-Klein modes and whether it is possible to get the internal curvature scalar $R < 0$. As we will see in section 7, the latter property is important for possible circumventions of the no-go theorem of [14]. As we show in section 6, all moduli are

\begin{table}[h]
\centering
\begin{tabular}{|l|c|c|c|c|c|}
\hline
 & $G_2$ & Sp(2) & SU(3) & SU(2) & SU(3) \times SU(1) \\
\hline
Light fields & 4 & 6 & 8 & 14 & 8 \\
Unstabilized & 0 & 0 & 0 & 1 & 0 \\
Decouple KK & no & yes & yes & yes & no \\
$R < 0$ possible & no & yes & yes & yes & yes \\
\hline
\end{tabular}
\caption{Results for the coset spaces}
\end{table}

stabilized in each model except for SU(2) \times SU(2). However, it turns out to be rather hard to decouple the tower of Kaluza-Klein modes, and in only three models there is a limit where this happens. As we will explain, for two of these three models we will have to analytically continue the shape parameters of the model to negative values, so that strictly speaking they do not describe a left-invariant SU(3)-structure on a coset anymore, but a related model based on a twistor bundle over a hyperbolic space [26, 42]. For the third model, SU(2) \times SU(2), if we take the limit where the Kaluza-Klein modes should decouple ($W_{-1} \rightarrow 0$), the other relevant torsion class $W_{-2}$ blows up just as the lower bound for the orientifold charge. We were not able to derive anything interesting in this singular limit.

As a general remark, we note that none of our models above contains light bulk gauge fields in the spectrum.

Section 7 discusses an application of some of our results in the context of type IIA inflation. More concretely, we show that the recent no-go theorem of [14] is no longer applicable when the scalar curvature $R$ can be made negative, which is the case for all the coset models except for $G_2 / SU(3)$. Finally, in section 8 we conclude with a discussion of open questions and future directions.

In appendix A, we summarize our conventions on supergravity, whereas appendix B explains our conventions and terminology regarding SU(3), static SU(2) and SU(3) \times SU(3)-structures. In appendix B, we present our no-go theorem against constant intermediate SU(2)-structures on AdS$_4$-compactifications. Appendix C discusses in detail how one can associate orientifold involutions to a smeared orientifold current. In appendices D and E, we present the details on the Kaluza-Klein reduction and the effective supergravity approach, respectively. In appendix F, finally, we discuss a special point in the moduli

\textsuperscript{5}The meaning of mass in AdS is somewhat subtle, however, as we review in section 5.1.
space of the coset model $\frac{SU(3) \times U(1)}{SU(2)}$ and show that in fact the supersymmetry there is extended to $\mathcal{N} = 2$.

### 2. Supersymmetric type IIA AdS$_4$ compactifications

To date all our explicit ten-dimensional examples of $\mathcal{N} = 1$ supersymmetric compactifications to AdS$_4$ fall within the class of type IIA SU(3)-structure compactifications and T-duals thereof. In this section we review this class of ten-dimensional solutions. We also discuss how to obtain a controlled parameter regime in which the string coupling is small, supergravity is valid and the tower of Kaluza-Klein modes decouples. For additional background material and a summary of our conventions the reader is referred to appendices A, B and C.

#### 2.1 Conditions for a supersymmetric vacuum

The most general form of $\mathcal{N} = 1$ compactifications of IIA supergravity to AdS$_4$ with the ansatz $\eta^{(1)} \propto \eta^{(2)}$ for the internal supersymmetry generators (the strict SU(3)-structure ansatz) was given by two of the present authors in [18]. These vacua must have constant warp factor and constant dilaton, $\Phi$. Setting the warp factor to one, the solutions of [18] are given by$^6$:

\begin{align*}
H &= \frac{2m}{3} e^{\Phi} \text{Re} \Omega, \quad \text{(2.1a)} \\
F_2 &= \frac{f}{9} J + F'_2, \quad \text{(2.1b)} \\
F_4 &= f \text{vol}_4 + \frac{3m}{10} J \wedge J, \quad \text{(2.1c)} \\
W e^{i\theta} &= -\frac{1}{5} e^\Phi m + \frac{i}{3} e^\Phi f. \quad \text{(2.1d)}
\end{align*}

where $H$ is the NSNS three-form, and $F_n$ denote the RR forms. Furthermore, $(J, \Omega)$ is the SU(3)-structure of the internal six-manifold, i.e. $J$ is a real two-form, and $\Omega$ is a decomposable complex three form such that:

\begin{align*}
\Omega \wedge J &= 0, \quad \text{(2.2a)} \\
\Omega \wedge \Omega^* &= \frac{4i}{3} J^3 \neq 0. \quad \text{(2.2b)}
\end{align*}

$f, m$ are constants parameterizing the solution: $f$ is the Freund-Rubin parameter, while $m$ is the mass of Romans’ supergravity [43] – which can be identified with $F_0$ in the ‘democratic’ formulation [44]. $e^{i\theta}$ is a phase associated with the internal supersymmetry generators: $\eta^{(2)}_+ = e^{i\theta} \eta^{(1)}_+$. The constant $W$ is defined by the following relation for the AdS$_4$ Killing spinors, $\zeta_{\pm}$,

\[ \nabla_\mu \zeta_- = \frac{1}{2} W \gamma_\mu \zeta_+ , \quad \text{(2.3)} \]

$^6$As opposed to [18] we do not use superspace conventions. Furthermore we use here the string frame and put $m = -2m_{\text{there}}, H = -H_{\text{there}}, J = -J_{\text{there}}, F_2 = -2m_{\text{there}} B'$ and $F_4 = -G$. 

---

"---
so that the radius of $\text{AdS}_4$ is given by $|W|^{-1}$. The two-form $F'_2$ is the primitive part of $F_2$ (i.e. it is in the $8$ of $\text{SU}(3)$). It is constrained by the Bianchi identity:

\[
dF'_2 = \left( \frac{2}{27} f^2 - \frac{2}{5} m^2 \right) e^\Phi \text{Re} \Omega - j^6 ,
\]

(2.4)

where we have added a source, $j^6$, for D6-branes/O6-planes on the right-hand side.

The general properties of supersymmetric sources and their consequences for the integrability of the supersymmetry equations were recently discussed by two of the present authors in [45] within the framework of generalized geometry. It was shown in this reference that, under certain mild assumptions, supersymmetry guarantees that the appropriately source-modified Einstein equation and dilaton equation of motion are automatically satisfied if the source-modified Bianchi identities are satisfied. For this to work the source must be supersymmetric, which means it must be generalized calibrated as in [46].

Finally, the only non-zero torsion classes of the internal manifold are $W_1^-, W_2^-$ which are defined such that (see also (B.6)):

\[
dJ = -\frac{3}{2} i W_1^- \text{Re} \Omega ,
\]

(2.5a)

\[
d\Omega = W_1^- J \wedge J + W_2^- \wedge J .
\]

(2.5b)

These torsion classes are given by:

\[
W_1^- = -\frac{4i}{9} e^\Phi f , \quad W_2^- = -i e^\Phi F'_2 .
\]

(2.6)

For the following it will be convenient to also introduce $c_1 := -\frac{3}{2} i W_1^-$, which appears in (2.5a). In addition, for vanishing sources or for sources proportional to $\text{Re} \Omega$, we have $dW_2 \propto \text{Re} \Omega$. We define the proportionality constant $c_2$ by

\[
dW_2^- = ic_2 \text{Re} \Omega .
\]

(2.7)

As we review below (B.9), one can show that

\[
c_2 = -\frac{1}{8} |W_2^-|^2 .
\]

(2.8)

For a given geometry to correspond to a vacuum without orientifold sources, we find from (2.4), (2.6)-(2.8) that the following bound on $(W_1^-, W_2^-)$ has to be satisfied

\[
\frac{16}{5} e^{2\Phi} m^2 = 3 |W_1^-|^2 - |W_2^-|^2 \geq 0 ,
\]

(2.9)

where we have defined $|\Theta|^2 := \Theta^m_{mn} \Theta^m_{mn}$, for any two-form $\Theta$. Incidentally, let us note that condition (2.9) turns out to be too stringent to be satisfied for any nilmanifold whose only non-zero torsion classes are $W_{1,2}^- [47]$. This implies that without orientifolds there are no solutions on nilmanifolds.

The constraint (2.9) can however be relaxed by allowing for an orientifold source, $j^6 \neq 0$. As a particular example, let us consider:

\[
j^6 = -\frac{2}{5} e^{-\Phi} \mu \text{Re} \Omega ,
\]

(2.10)
where $\mu$ is an arbitrary, discrete, real parameter of dimension $(\text{mass})^2$, so that $-\mu$ is proportional to the orientifold/D6-brane charge ($\mu$ is positive for net orientifold charge and negative for net D6-brane charge). The addition of this source term was first considered in [17]. Eq. (2.10) above guarantees that the calibration conditions, which for D6-branes/O6-planes read

$$ j^6 \wedge \text{Re} \Omega = 0, \quad j^6 \wedge J = 0, \quad (2.11) $$

are satisfied and thus the source wraps supersymmetric cycles. The bound (2.9) changes to

$$ e^{2\Phi} \mu^2 = \mu + \frac{5}{16} (3|W_1|^2 - |W_2|^2) \geq 0. \quad (2.12) $$

Since $\mu$ is arbitrary the above equation can always be satisfied, and therefore no longer imposes any constraint on the torsion classes of the manifold.

Let us also note that it is possible to consider the inclusion of more general supersymmetric orientifold six-plane sources that do not satisfy eq. (2.10). In that case, the constraint that $dW_2^-$ should be proportional to $\text{Re} \Omega$ is relaxed. Requiring this source to satisfy the calibration conditions (2.11), we find that it is now of the following form:

$$ j^6 = -\frac{2}{5} e^{-\Phi} \mu \text{Re} \Omega + w_3, \quad (2.13) $$

with $w_3$ a primitive $(2,1)+(1,2)$-form. From the Bianchi identity (2.4) we find

$$ w_3 = -ie^{-\Phi} dW_2^-|_{(2,1)+(1,2)}, \quad (2.14) $$

and (2.12) still unchanged.

In appendix C we will explain how to associate orientifold involutions to a smeared source. Under each orientifold involution the dilaton, metric and fluxes must transform as follows:

Even: $\sigma^* e^\Phi = e^\Phi, \quad \sigma^* F_0 = F_0, \quad \sigma^* F_4 = F_4,$

Odd: $\sigma^* H = -H, \quad \sigma^* F_2 = -F_2,$

whereas the SU(3)-structure transforms as

Even: $\sigma^* \text{Im} \Omega = \text{Im} \Omega,$

Odd: $\sigma^* \text{Re} \Omega = -\text{Re} \Omega, \quad \sigma^* J = -J.$ \hspace{1cm} (2.15b)

### 2.2 Hierarchy of scales

In the full quantum theory, all fluxes have to be quantized according to

$$ \frac{1}{l_p^{-1}} \int_{C_p} F_p = n_p, \quad (2.16) $$

where $l := 2\pi \sqrt{\alpha'}$, $C_p$ is a cycle in the internal manifold, and $n_p \in \mathbb{Z}$. The NSNS three-form turns out to be exact in our models, hence its integral over any internal three-cycle
vanishes; it therefore suffices to impose (2.16) for the RR fluxes. The issue of quantization is studied in more detail in [26].

For the analysis of the present paper to be valid, we need to show that we can consistently take the string coupling constant to be small ($g_s = e^\Phi \ll 1$), so that string loops can be safely ignored, and that the volume of the internal manifold is large in string units ($L_{\text{int}}/l \gg 1$, where $L_{\text{int}}$ is the characteristic length of the internal manifold), so that $\alpha'$-corrections can be neglected. This can be seen by essentially employing the following scaling argument: Let $f_p/(g_n L_{\text{int}})$ be the norm of the flux density $F_p$, for some numbers $f_p$ depending on the internal geometry (but not on the overall scale $L_{\text{int}}$). The quantization conditions (2.16) imply:

$$g_s = (f_0^3 f_4)^{1/4} (n_0^3 n_4)^{-1/4}; \quad \frac{L_{\text{int}}}{l} = \left(\frac{f_0}{f_4}\right)^{1/4} \left(\frac{n_4}{n_0}\right)^{1/4}; \quad \frac{n_2}{n_0 n_4} = \frac{f_2}{f_0 f_4}; \quad \frac{n_0 n_6}{n_2 n_4} = \frac{f_0 f_6}{f_2 f_4}. \quad (2.17)$$

It can then be easily verified that, given a solution $\{n_p\}$ to the quantization conditions (2.16), there are several different possible scalings $n_p \to N^\lambda p n_p$, for $N, \lambda_p \in \mathbb{N}$, which leave the $f_p$’s invariant and at the same time ensure that $g_s$ is parametrically small while $L_{\text{int}}/l$ is parametrically large (with large parameter $N$). This schematic argument can be made precise, by taking into account the specifics of the geometry of each internal manifold, as in [26]. Despite the fact that we are allowing for large flux quanta, it can be shown that higher-order flux corrections can also be neglected. Indeed it is not difficult to see that the parameter $|g_s F_p|^2$, which controls the size of these corrections, scales with a negative power of the large parameter $N$.

A further consistency requirement is that the Kaluza-Klein tower can be decoupled. Since the Compton wavelength of the lightest excitations above the Breitenlohner-Freedman bound in four dimensions is of the order of the AdS$_4$ radius, we need to show that the Compton wavelength of the Kaluza-Klein excitations (which is proportional to $L_{\text{int}}$) satisfies:

$$|\Lambda_{\text{AdS}}|^2 L_{\text{int}}^2 \ll 1, \quad (2.18)$$

where $\Lambda_{\text{AdS}}$ is the four-dimensional cosmological constant. In models without orientifolds this is impossible to achieve, since the characteristic length of the internal manifold turns out to be of the same order as the radius of AdS$_4$. This is the problem of separation of scales which, for example, plagues the compactifications of eleven-dimensional supergravity on the seven-sphere. Ultimately we would like to uplift our models to a de Sitter space with a small, positive cosmological constant, and the position could be taken that the question of the mass spectra should be re-addressed only after this uplifting. However, let us now study whether it is possible to tune the orientifold source such that there is a hierarchy between the two scales even before the uplifting and (2.18) is obeyed.

Taking into account $|\Lambda_{\text{AdS}}| \sim |W|^2$ and using (2.1d), we find that to decouple the Kaluza-Klein scale we must impose

$$|W|^2 L_{\text{int}}^2 = \frac{1}{25} (g_s)^2 m^2 L_{\text{int}}^2 + \frac{1}{9} (g_s)^2 f^2 L_{\text{int}}^2 \ll 1, \quad (2.19)$$

- 12 -
which means that each of the two terms on the right-hand side of the equal sign must be separately much smaller than one. Tuning the orientifold charge we can accomplish 

$$e^{2\Phi} m^2 L_{\text{int}}^2 \ll 1.$$  

Indeed, we just need to show that we can choose \( \mu \) so that it is close to its bound (2.12):

$$\mu L_{\text{int}}^2 + \frac{5}{16} (3|W_1^-|^2 - |W_2^-|^2) L_{\text{int}}^2 \ll 1.$$  

(2.20)

In our conventions the discrete parameter \( \mu \), which is proportional to the net number of orientifold planes \( n_{O6} \), is given by (up to numerical factors of order one):

$$\mu \sim g_s n_{O6} L_{\text{int}}^{-3}.$$  

Taking into account that the torsion classes are given by (again up to numerical factors of order one):

$$|W_i^-|^2 \sim L_{\text{int}}^{-2},$$

we can rewrite the above equation schematically as follows:

$$n_{O6} g_s \left( \frac{l}{L_{\text{int}}} \right) + a \ll 1,$$

(2.21)

where \( a \) is a number of order one. Since \( g_s (\frac{l}{L_{\text{int}}}) \ll 1 \), we can then satisfy this bound by choosing some large integer \( n_{O6} \). Note that in the examples where we study this limit, \( a \) turns out to be negative so that we can accomplish this with positive \( n_{O6} \), which corresponds to net orientifold charge (as opposed to net D-brane charge).

However, we must also make sure that the second square in (2.19) is small, which means that \( f g_s L_{\text{int}} \propto |W_i^-|^2 |L_{\text{int}}| \) is small. Manifolds for which \( W_1^- \) vanishes (and only \( W_2^- \) is possibly non-zero) are called ‘nearly Calabi-Yau’ (NCY) see e.g. [42]; hence for the bound (2.18) to be satisfied, the internal manifold must admit an SU(3)-structure which is sufficiently close to the NCY limit. For the torus model we have \( W_1^- = 0 \), while in section 6.2-6.3 we will argue that both the \( \text{Sp}(2) \) and \( \text{SU}(3) \times U(1) \) models have continuations that admit NCY SU(3)-structures.

Once a solution for \( n_{O6} \) is obtained in this way, we are free to rescale \( n_{O6} \rightarrow N^q n_{O6} \) leaving (2.21) invariant, provided we take: \( q = (\lambda_0 + \lambda_4) / 2 \in \mathbb{N} \). For example, the reader can verify that the rescaling \( \{ n_0 \rightarrow N^4 n_0, n_2 \rightarrow N^6 n_2, n_4 \rightarrow N^8 n_4, n_6 \rightarrow N^{10} n_6, n_{O6} \rightarrow N^6 n_{O6} \} \) leave eq. (2.21) and all the \( f_p's \) in eq. (2.17) invariant, so that:

$$g_s \sim N^{-5}, \quad \frac{L_{\text{int}}}{l} \sim N, \quad |\Lambda_{\text{AdS}}| L_{\text{int}}^2 = \text{fixed} \ll 1,$$

(2.22)

where we can take \( N \) large.

3. Ten-dimensional geometries I: nilmanifolds

By taking the internal six-dimensional space to be a nilmanifold \( \mathcal{M} \), it turns out that one can construct explicit examples of the type of compactifications reviewed in section 2, as we will show in the present section. As follows from the discussion of section 2, it suffices to look for all possible six-dimensional nilmanifolds whose only non-zero torsion classes are \( W_{1,2}^- \). A systematic scan yields exactly two possibilities in type IIA, namely the six-torus and the nilmanifold 4.7 of Table 4 of [20] (also known as the Iwasawa manifold), which (for some values of the parameters) turn out to be related by T-duality along two directions. We also found a type IIB solution with static SU(2)-structure on the nilmanifold
5.1, as described in appendix B.2, which forms the intermediate step after one T-duality. Unfortunately, in the case of type IIB we were not able to make a systematic scan so there might be more solutions of this type.

Before we turn to the description of each of those possibilities in the next section, let us also mention that closely related solutions can be obtained by replacing the six-torus by a direct product $K3 \times T^2$, the Iwasawa manifold by the $T^2$ fibration over $K3$ constructed in [18], and the 5.1 nilmanifold by an $S^1$ fibration over $K3 \times S^1$. This relation arises from the fact that on the boundary of its moduli space, the $K3$ surface degenerates to a discrete quotient of $T^4$. Just as the previous cases, all these three solutions are connected by T-dualities.

### 3.1 The type IIA $T^6$ solution

Our first IIA solution is obtained by taking the internal manifold to be a six-dimensional torus. Let us define a left-invariant basis $\{e^i\}$ such that:

$$\text{de}^i = 0, \quad i = 1, \ldots, 6.$$  \hspace{1cm} (3.1)

On the torus we can just choose $e^i = dy^i$, where $y^i$ are the internal coordinates. The SU(3)-structure is given by

$$J = e^{12} + e^{34} + e^{56},$$  
$$\Omega = (ie^1 + e^2) \wedge (ie^3 + e^4) \wedge (ie^5 + e^6),$$  \hspace{1cm} (3.2)

which can indeed be seen to satisfy eqs. (B.2, B.3) and (2.5) for $f = 0$, putting $\text{vol}_6 = e^{1\ldots6}$. It readily follows that all torsion classes vanish in this case. Note, however, that there are non-vanishing $H$ and $F_4$ fields given by (2.1)

$$H = \frac{2}{5} e^\Phi m \left(e^{246} - e^{136} - e^{145} - e^{235}\right),$$
$$F_4 = \frac{3}{5} m \left(e^{1234} + e^{1256} + e^{3456}\right).$$  \hspace{1cm} (3.3)

From (2.12) we find that there is an orientifold source of the type (2.10) with $\mu = e^{2\Phi} m^2$, which corresponds to smeared orientifolds along (1,3,5), (2,4,5), (2,3,6) and (1,4,6). The corresponding orientifold involutions are

$$O_6 : \quad e^2 \rightarrow -e^2, \quad e^4 \rightarrow -e^4, \quad e^6 \rightarrow -e^6,$$
$$O_6 : \quad e^1 \rightarrow -e^1, \quad e^3 \rightarrow -e^3, \quad e^6 \rightarrow -e^6,$$
$$O_6 : \quad e^1 \rightarrow -e^1, \quad e^4 \rightarrow -e^4, \quad e^5 \rightarrow -e^5,$$
$$O_6 : \quad e^2 \rightarrow -e^2, \quad e^3 \rightarrow -e^3, \quad e^5 \rightarrow -e^5.$$  \hspace{1cm} (3.4)

### 3.2 The type IIA Iwasawa-manifold solution

The second IIA solution is obtained by taking the internal manifold to be the Iwasawa manifold. The left-invariant basis is defined by:

$$\text{de}^a = 0, \quad a = 1, \ldots, 4,$$
$$\text{de}^5 = e^{13} - e^{24},$$
$$\text{de}^6 = e^{14} + e^{23}.$$  \hspace{1cm} (3.5)
and is usually denoted by $(0, 0, 0, 0, 13 - 24, 14 + 23)$. Up to basis transformations there is a unique SU(3)-structure satisfying the supersymmetry conditions of section 2:

$$J = e^{12} + e^{34} + \beta^2 e^{56},$$
$$\Omega = \beta (ie^5 - e^6) \wedge (ie^1 + e^2) \wedge (ie^3 + e^4),$$

(3.6)

In the left-invariant basis, the metric is given by $g = \text{diag}(1, 1, 1, 1, \beta^2, \beta^2)$, and the torsion classes can be read off from $dJ, d\Omega$, taking eqs. (2.5) into account:

$$W^-_1 = -\frac{2i}{3} \beta,$$
$$W^-_2 = -\frac{4i}{3} \beta \left( e^{12} + e^{34} + 2 \beta^2 e^{56} \right),$$

(3.7)

while all other torsion classes vanish. The fluxes can be read off from (2.1) by plugging in $f = \frac{3}{2} e^{-\Phi} \beta$, while we can find $m$ from (2.12). We can verify that $dW^-_2$ is proportional to $\text{Re} \Omega$:

$$dW^-_2 = -\frac{8i}{3} \beta^2 \text{Re} \Omega.$$

(3.8)

From the second line of (3.7) we can read off: $|W^-_2|^2 = 64 \beta^2 / 3$. Comparing with eq. (2.12), taking $|W^-_1|^2 = 4 \beta^2 / 9$ into account – as follows from the first line of (3.7) – we therefore find a non-zero net orientifold six-plane charge:

$$\mu \geq \frac{25}{4} \beta^2.$$  

(3.9)

The solution (3.6) has one continuous parameter, $\beta$, corresponding essentially to the first torsion class $W^-_1$. An additional second parameter can be introduced by noting that the defining SU(3)-structure equations (B.2b) are invariant under the rescaling $J \rightarrow \gamma J; \ \Omega \rightarrow \gamma^3 \Omega$.

(3.10)

The additional scalar $\gamma$ is related to the volume modulus via $\text{vol}_6 = -\gamma^6 \beta^2 e^{1\ldots6}$, as can be seen from eq. (B.3).

For the case $m = 0$, for which the bound (3.9) is saturated, the above example can also be obtained by performing two T-dualities on the torus solution of section 3.1, as can be checked explicitly. We find then that $\beta = \frac{2}{3} m_T e^\Phi$ where $m_T$ is the mass parameter of the dual torus solution.

### 3.3 The type IIB nilmanifold 5.1 solution

This solution is related, via a single T-duality, to both $T^6$ and the Iwasawa manifold of 3.2. Indeed, let us perform a T-duality on the six-torus example of section 3.1 using the T-duality rules of e.g. [48] (see also [49] for a discussion of the action of T-duality on the pure spinors of a SU(3)×SU(3)-structure).\footnote{Note that it does not matter along which direction one performs the T-duality since all six perpendicular directions are equivalent. For the second T-duality (from which we obtain the Iwasawa solution of the previous section), only one direction leading to a geometric background is possible. We will not pursue the interesting case of non-geometric backgrounds in the present paper.}

After rescaling and relabelling the left-invariant
forms we find the nilmanifold 5.1 described by \((0,0,0,0,12+34)\). For the SU(2)-structure quantities described in appendix B.2 we obtain

\[
\begin{align*}
  e^{i\theta}V &= \frac{1}{2} (\beta e^6 + ie^5), \\
  \omega_2 &= e^{13} - e^{24}, \\
  \Omega_2 &= -ie^{i\theta}(e^1 + e^3) \wedge (e^4 + e^2).
\end{align*}
\]

The metric is given by \(g = \text{diag}(1,1,1,1,\beta^2,\beta^2)\), and for the fluxes we have

\[
\begin{align*}
  H &= -\beta (e^{235} + e^{145}), \\
  e^\Phi F_1 &= \frac{5}{2} \beta^2 e^6, \\
  e^\Phi F_3 &= \frac{3}{2} \beta (e^{135} - e^{245}), \\
  e^\Phi F_5 &= \frac{3}{2} \beta^2 e^{12346}.
\end{align*}
\]

Again we find that \(\beta\) is related to the mass parameter of the torus example via \(\beta = \frac{2}{5} m_T e^\Phi\).

### 3.4 The brane picture\(^8\)

Following [6], it is possible to interpret the solutions presented in sections 3.1-3.3, from the perspective of intersecting branes. Namely, we would like to recover these solutions as near-horizon limits of domain walls in four noncompact dimensions, corresponding to systems of (orthogonally) intersecting branes (we will henceforth use the term ‘brane’ to refer to either a Dp-brane, an NS5-brane, or a KK-monopole).

More specifically, we will impose the following requirements on our brane configurations:

1. All configurations should consist of branes in ten-dimensional flat space, of which four directions are noncompact and six directions form a six-torus.
2. All branes should have exactly the same two spatial directions along the noncompact space.
3. All branes should intersect orthogonally, and we do not consider world-volume gauge fields.
4. The resulting configuration should preserve \(\mathcal{N} = 1\) supersymmetry in D=3, and should admit a regular near-horizon geometry with an AdS\(_4\) factor.
5. Each configuration should include the maximum number of branes compatible with requirements 1-4.

Before we come to the description of explicit configurations satisfying the above requirements, let us note that, as we will see in the following, only brane configurations

\(^8\)This section is somewhat outside the main line of the paper and may be omitted in a first reading.
that lead to strict SU(3)-structure (as well as their T-dual configurations leading to static SU(2)-structures) arise in this way; this is the same class of backgrounds considered in section 2. The easiest way to arrive at this conclusion is to first determine which types of SU(3)×SU(3)-structure are compatible with each brane separately. Indeed, using their corresponding \( \kappa \)-symmetry projectors, it is straightforward to analyse what relations between the internal supersymmetry generators \( \eta^{(1)} \) and \( \eta^{(2)} \) of (B.1) are possible, which leads to the following table of branes and their corresponding compatible types of structure:

\[
\begin{array}{|c|c|}
\hline
\text{Brane} & \text{Structure type} \\
\hline
D2 & \text{strict SU(3)} \\
D3 & \text{static SU(2)} \\
D4 & \text{SU(3)×SU(3)} \\
D5 & \text{SU(3)×SU(3)} \\
D6 & \text{SU(3)×SU(3)} \\
D7 & \text{static SU(2)} \\
D8 & \text{strict SU(3)} \\
\text{NS5} & \text{SU(3)×SU(3)} \\
\text{KK} & \text{SU(3)×SU(3)} \\
\hline
\end{array}
\]

See appendix B for the terminology. It turns out, that the configuration always needs to have D-branes to get a regular near-horizon AdS limit. From the above table it follows, that if one of these D-branes is a D2, D3, D7 or D8 we already find strict SU(3)- or static SU(2)-structure. If not, let us consider the SU(3)-structure associated to \( \eta^{(1)} \) as in (B.5). Let us also define the complex coordinates \( z^i \) associated with this SU(3)-structure as well as their real and imaginary parts: \( z^i = x^i + iy^i \). Because all the branes defining this SU(3)-structure intersect orthogonally (requirement 3), for each brane the \( x^i \) and \( y^i \) directions will be either along or perpendicular to the brane, i.e., there are no angles other than right angles. Now the relation between \( \eta^{(1)} \) and \( \eta^{(2)} \), which we can get from the \( \kappa \)-symmetry conditions of one of the D-branes, will contain gamma-matrices for directions that are also parallel or orthogonal to the \( x^i \) and \( y^i \) directions. Exhausting then all possibilities for the resulting structure shows that it can only be strict SU(3) or static SU(2). It follows that if one is interested in constructing a configuration with general SU(3)×SU(3)-structure, one should restrict to D4, D6, D5, NS5 and KK-branes and put these branes at non-orthogonal angles.

Let us make a few comments concerning the requirements 1-5 above. The first one anticipates the fact that, as it will turn out, the internal nilmanifolds in the solutions of section 3 can be thought of as intersections of KK-monopoles in flat space. It therefore suffices to consider branes in flat space. The second requirement is of course just the requirement that the configuration should correspond to a domain wall in four space-time dimensions. The requirement of orthogonality was imposed for simplicity. It would be

\[\text{We also refer to Table 1 of [45] which represents the allowed types of structure too, but now for space-filling orientifolds. Orientifolds have the same supersymmetry properties as D-branes with vanishing world-volume gauge field, however the difference of space-filling versus domain wall basically shifts the table.}\]
interesting to consider branes/monopoles intersecting at angles, but it would be quite difficult to construct the corresponding geometry because one could no longer use the harmonic superposition rules for branes [50]. The first part of the fourth requirement is equivalent to demanding that the domain wall, viewed from the point of view of four-dimensional space-time, should be supersymmetric. Indeed, the minimal supersymmetry a domain wall in four dimensions can preserve, is one-half of \( \mathcal{N} = 1 \) in \( D = 4 \). This is equal to two real supercharges, i.e. \( \mathcal{N} = 1 \) in \( D = 3 \). Note that this implies that exactly one-sixteenth of the original supersymmetry of type II supergravity in \( D = 10 \) should be preserved. As each brane breaks supersymmetry by (at most) one-half, there will be (at least) four branes in the configuration. The final requirement is imposed because a configuration that does not include the maximum number of branes compatible with requirements 1-4, turns out not to have a regular AdS\(_4\) near-horizon limit.

The rules for supersymmetric, orthogonally-intersecting branes were formulated some time ago [50, 51]. For the type of configurations we are considering in the present paper, they can be summarized as follows:

| intersecting branes | \# of relative transverse directions |
|---------------------|-------------------------------------|
| Dp/Dq               | 0 mod 4                             |
| NS5/NS5             | 0 mod 4                             |
| Dp/NS5              | 7 \(-p\) or 11 \(-p\)               |
| Dp/KK               | 5 \(-p\) or 9 \(-p\)               |
| KK/KK               | 0 mod 4                             |
| NS5/KK              | 4 or 8                              |

The requirements 1-5 listed above severely restrict the set of admissible intersecting-brane configurations. It is in fact straightforward to show that all possible such configurations are related to each other by T-dualities. The brane configurations comprising the ‘nodes’ of this T-duality web, listed in Table 1, are analysed in the following\(^{10}\).

**D4/D8/NS5**

This is the IIA solution given in [6] and corresponds to the following system of intersecting D4/NS5/D8-branes:

---

\(^{10}\)Without the second part of the fourth requirement there are three more configurations connected to each other by T-duality: D5/NS5, D6/D4/NS5/KK and D5/KK. Because they do not admit a regular near-horizon limit with AdS\(_4\) factor they are not of interest to us here and we do not consider them.
The full solution of [6] patches two asymptotic regions: a near-horizon AdS$_4 \times$T$^6$ region and a flat region at infinity. Here we will concentrate on the near-horizon limit of the solution where the brane system above is replaced by fluxes. After rescaling of the coordinates, it can be written as:

\[ ds_{10}^2 = ds_{\text{AdS}_4}^2 + \sum_{i=1}^{6} (dy^i)^2; \quad \Phi = \text{const.}; \]

\[ H_{y^2y^4y^5} = H_{y^2y^3y^4} = H_{y^1y^5y^4} = a, \]

\[ F_{y^2y^4y^5} = F_{y^1y^2y^5} = F_{y^1y^2y^3} = \frac{3}{2} e^{-\Phi} a, \quad F_0 = \frac{5}{2} e^{-\Phi} a, \]

where \( a \) and \( e^\Phi \) are given in terms of the brane quanta in [6], and the SU(3)-structure is given by:

\[ J = dy^1 \wedge dy^2 \wedge dy^3 \wedge dy^4 \wedge dy^5 \wedge dy^6, \]

\[ \Omega = (idy^1 + dy^2) \wedge (idy^3 + dy^4) \wedge (idy^5 + dy^6). \]

We can readily see that, in the language of section 2, the present solution corresponds to setting \( F_2^2 = 0, \ f = 0 \) and \( m = a \) with a source term:

\[ j^{O6} = -\frac{2a^2}{5} e^{-\Phi} \text{Re} \Omega. \]

So while the original brane configuration has disappeared in the near-horizon limit we have to introduce a set of smeared orientifold sources in order to satisfy the tadpole conditions:

Indeed, as follows from (2.6), in this limit, all torsion classes of the internal manifold vanish, as they should for T$^6$. Moreover, this is exactly the solution of section 3.1.
D3/D5/D7/NS5/KK

By applying a T-duality on the solution of the previous subsection, we obtain the following configuration (we do not display the noncompact directions anymore, but let us keep in mind that they form domain walls):

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
 & y^1 & y^2 & y^3 & y^4 & y^5 & y^6 \\
\hline
D7 & & & & & & \\
D3 & & & & & & \\
D5' & & & & & & \\
D5'' & & & & & & \\
NS5 & & & & & & \\
NS5' & & & & & & \\
KK'' & & & & & & \\
KK''' & & & & & & \\
\hline
\end{array}
\]

Without loss of generality, we have taken the T-duality to be along $y^1$. Let us only describe the salient features of this model.

First of all, an analysis of the $\kappa$-symmetry conditions of the D-branes reveals that for this configuration the internal spinors satisfy:

\[
\eta_+^{(2)} = -e^{-i\theta} \eta_1^{(1)},
\]

where $e^{-i\theta}$ is a phase describing the supersymmetry preserved by the domain wall in four dimensions, or, after taking the near-horizon limit, the phase of the superpotential $W$ of AdS. So we see that we have static SU(2)-structure, which is also the only possibility for type IIB as explained in appendix (B.2).

Secondly, when one goes to the near-horizon limit, the effect of the KK-monopoles is to twist the $S^1$ of direction 1 over the $T^4$ corresponding to the directions $(3, 4, 5, 6)$, which is indicated with a bullet in the tables. This means that we find for the metric, after rescaling,

\[
ds_{10}^2 = ds_{AdS_4}^2 + \sum_{i=1}^{6} (e^i)^2,
\]

with

\[
e^1 := dy^1 + a(y^6dy^3 + y^5dy^4),
\]

\[
e^i := dy^i ; \quad i = 2, \ldots, 6,
\]

where $a$ is the same parameter as in the T-dual. This means we have

\[
de^1 = a(e^{63} + e^{54}),
\]

\[
de^i = 0,
\]

which, in fact, is equivalent to nilmanifold 5.1. So we see that just like the other branes the KK-monopoles disappear in the near-horizon limit and are replaced by flux, in this case the geometric flux $a$. 

\[\text{-- 20 --}\]
It turns out that in addition to the fluxes we have O5/O7 orientifold planes along the following directions:

|   | $x^0$ | $x^1$ | $x^2$ | $x^3$ | $y^1$ | $y^2$ | $y^3$ | $y^4$ | $y^5$ | $y^6$ |
|---|---|---|---|---|---|---|---|---|---|---|
| O5 | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ |
| O5' | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ |
| O7 | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ |
| O7' | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ |

After appropriate rescaling and relabelling, this solution corresponds to the solution on the nilmanifold 5.1 of section 3.3.

**D2/D6/KK**

Starting from the configuration of section 3.4, there is exactly one possibility left for a T-duality, i.e. along $y^2$. This is because T-dualizing along a direction perpendicular to a KK-monopole would result in a nongeometric background.

|   | $y^1$ | $y^2$ | $y^3$ | $y^4$ | $y^5$ | $y^6$ |
|---|---|---|---|---|---|---|
| D6 |   | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ |
| D2 |   |   |   |   |   |   |
| D6' | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ |
| D6'' | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ |
| KK | $\otimes$ | $\bullet$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ |
| KK' | $\otimes$ | $\bullet$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ |
| KK'' | $\bullet$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ |
| KK''' | $\bullet$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ | $\otimes$ |

An analysis of the $\kappa$-symmetry conditions of the branes reveals that this model has again strict SU(3)-structure. The four KK-monopoles result in a near-horizon geometry for which the $T^2$ along the directions $(1, 2)$ is twisted over the base $T^4$ along $(3, 4, 5, 6)$. The metric reads

$$ds_{10}^2 = ds_{AdS_4}^2 + \sum_{i=1}^{6} (e^i)^2,$$  \hspace{1cm} (3.20)

where we have defined

$$e^1 := dy^1 + a(y^6 dy^3 + y^5 dy^4),$$
$$e^2 := dy^2 + a(y^5 dy^3 - y^6 dy^4),$$
$$e^i := dy^i \ ; \ i = 3, \ldots, 6 ,$$

such that

$$de^1 = a(e^{63} + e^{54}),$$
$$de^2 = a(e^{53} + e^{46}),$$
$$de^i := dy^i \ ; \ i = 3, \ldots, 6 .$$

(3.22)
After rescaling and relabelling we find the solution of section 3.2 for \( m = 0 \). For \( m \neq 0 \) the latter solution does not have a dual brane picture.

Finally note that in order to satisfy the tadpole conditions we have again O6-planes along the following directions:

\[
\begin{array}{cccccccc}
\times & \times & \times & \times & \times & \times & \times & \times \\
O6 & O6' & O6'' & O6''' & & & & \\
\end{array}
\]

This completes the overview of brane configurations of Table 1.

4. Ten-dimensional geometries II: coset spaces

A large class of IIA solutions of the type described in section 2 was given recently in [27], also incorporating solutions that were already known [21, 19, 26] into a single unifying framework of left-invariant SU(3)-structures on coset spaces. In other words, the solutions described in [27] are all of the form \( \text{AdS}_4 \times \mathcal{M}_6 \) where the internal manifold is a coset, \( \mathcal{M}_6 = G/H \), equipped with a left-invariant SU(3)-structure. In Tomasiello’s recent work [26] an alternative description in terms of twistor bundles is used for the cosets of sections 4.2 and 4.3. Although this description does not allow to describe the complete parameter space on the coset \( \frac{SU(3)}{U(1) \times U(1)} \), it is more accurate for the nearly Calabi-Yau limit in which, as we will see, the shape parameters take negative values and the coset description is not valid anymore.

Before we come to the description of the individual coset solutions listed in [27], let us review some well-known facts about coset spaces. For more details see, e.g., [52, 53].

In dealing with coset spaces of the form \( G/H \) it suffices for our purposes to examine the corresponding Lie algebras \( \mathfrak{g}, \mathfrak{h} \). Let \( \{ \mathcal{H}_a \} \) be a basis of generators of the algebra \( \mathfrak{h} \), and let \( \{ \mathcal{K}_i \} \) be a basis of the complement \( \mathfrak{k} \) of \( \mathfrak{h} \) inside \( \mathfrak{g} \), i.e. \( a = 1, \ldots, \text{dim}(H) \) and \( i = 1, \ldots, \text{dim}(G)-\text{dim}(H) \). We define the structure constants as follows:

\[
\begin{align*}
[\mathcal{H}_a, \mathcal{H}_b] &= f^{c}_{ab} \mathcal{H}_c, \\
[\mathcal{H}_a, \mathcal{K}_i] &= f^{i}_{ai} \mathcal{K}_j + f^{b}_{ai} \mathcal{H}_b, \\
[\mathcal{K}_i, \mathcal{K}_j] &= f^{k}_{ij} \mathcal{K}_k + f^{a}_{ij} \mathcal{H}_a.
\end{align*}
\]

(4.1)

If \( H \) is connected and semisimple, or compact, one can always find a basis of generators \( \{ \mathcal{K}_i \} \) such that the structure constants \( f^{b}_{ai} \) vanish [52]. In other words: \( [\mathcal{H}, \mathcal{K}] \subset \mathcal{K} \), and in this case the coset \( G/H \) is called \textit{reductive}.

Let \( y^m, m = 1, \ldots, \text{dim}(G)-\text{dim}(H) \), be local coordinates on \( G/H \) and let \( L(y) \) be a coset representative. The decomposition of the Lie-algebra valued one-form

\[
L^{-1} dL = e^i \mathcal{K}_i + \omega^a \mathcal{H}_a,
\]

(4.2)

– 22 –
defines a coframe $e^i(y)$ on $G/H$. Moreover, using the commutation relations (4.1), we find

$$de^i = -\frac{1}{2} f^i_{jk} e^j \wedge e^k - f^i_{aj} \omega^a \wedge e^j .$$

(4.3)

We are interested in expanding in forms that are \textit{left-invariant} under the action of $G$ on $G/H$. One can show that this is the case if and only if for a $p$-form

$$\phi = \frac{1}{p!} \phi_{i_1 \cdots i_p} e^{i_1} \wedge \cdots \wedge e^{i_p} ,$$

(4.4)

its components $\phi_{i_1 \cdots i_p}$ are constants and

$$f^j_{a[i_1} \phi_{i_2 \cdots i_p] j} = 0 .$$

(4.5)

If we then take the exterior derivative $d\phi$, condition (4.5) ensures that the part coming from the second term in (4.3) drops out and we get again a left-invariant form. One can show that harmonic forms must be left-invariant and thus the cohomology of the coset manifold is isomorphic to the cohomology of left-invariant forms.

Similarly, a metric $g = g_{ij} e^i \otimes e^j$ is left-invariant if and only if its components $g_{ij}$ are constants and

$$f^k_{a(i} g_{j)k} = 0 .$$

(4.6)

The Riemann tensor for such a metric is calculated in e.g. [53]. We display here the Ricci scalar, which we find by contracting indices:

$$R = -g^{ij} f^k_{ai} f^a_{kj} - \frac{1}{2} g^{ij} f^k_{il} f^l_{kj} - \frac{1}{4} g_{ij} g^{kl} g^{mn} f^i_{km} f^j_{ln} .$$

(4.7)

If we introduce orientifolds the structure constant tensor

$$f = \frac{1}{2} f^i_{jk} E_i \otimes e^j \wedge e^k + f^i_{aj} E_i \otimes \omega^a \wedge e^j + \frac{1}{2} f^a_{ij} U_a \otimes e^i \wedge e^j + \frac{1}{2} f^a_{bc} U_a \otimes \omega^b \wedge \omega^c ,$$

(4.8)

where the $E_i, U_a$ are dual to the $e^i, \omega^a$, has to be even under the orientifold involution (for some suitable extension of the involution to the $\omega^a$) in order to ensure that the exterior derivative is even.

We are now ready to proceed to the description of the individual coset solutions listed in [27].

4.1 The $G_2 \times SU(3)$ solution

The $G_2$ structure constants are given by:

$$f^1_{63} = f^1_{45} = f^2_{53} = f^2_{64} = \frac{1}{\sqrt{3}} ,$$

$$f^7_{36} = f^7_{45} = f^8_{53} = f^8_{46} = f^9_{56} = f^9_{34} = f^{10}_{16} = f^{10}_{52}$$

$$= f^{11}_{51} = f^{11}_{62} = f^{12}_{41} = f^{12}_{32} = f^{13}_{31} = f^{13}_{24} = \frac{1}{2} ,$$

(4.9)

$$f^{14}_{43} = f^{14}_{56} = \frac{1}{2\sqrt{3}} , \quad f^{14}_{21} = \frac{1}{\sqrt{3}} ,$$

$$f^{i+6}_{j+6,k+6} = f_{GMijk} .$$
where \( f_{GMijk} \) are the Gell-Mann structure constants.

The \( G \)-invariant two-forms and three-forms are spanned by
\[
\{e^{12} - e^{34} + e^{56}\}, \quad \{\rho = e^{245} + e^{135} + e^{146} - e^{236}, \hat{\rho} = -e^{235} - e^{246} + e^{145} - e^{136}\},
\]
respectively, and there are no invariant one-forms. With only these two invariant three-forms\(^{11}\) there is no room for a source not proportional to Re\(\Omega\).

The most general solution is then given by
\[
J = a(e^{12} - e^{34} + e^{56}), \\
\Omega = d\left[(e^{245} + e^{146} + e^{135} - e^{236}) + i(e^{145} - e^{246} - e^{235} - e^{136})\right],
\]
with \( a \), the overall scale, the only free parameter, and
\[
a > 0, \quad \text{metric positivity}, \\
d^2 = a^3, \quad \text{normalization of } \Omega, \\
c_1 := -\frac{3i}{2} \mathcal{W}_1^- = -\frac{2}{3} e^f f = -\frac{\sqrt{3} a}{d}, \\
\mathcal{W}_2^- = 0, \\
e^{2\Phi} m^2 - \mu = \frac{5}{12} c_1^2.
\]

We conclude that the only possibility for this coset is the nearly-Kähler geometry. It will be convenient to isolate the scale \( a \) and introduce the reduced flux parameters
\[
\tilde{m} = a^{1/2} e^f m, \quad \tilde{f} = a^{1/2} e^f f, \quad \tilde{\mu} = a \mu, \quad \tilde{c}_1 = a^{1/2} c_1,
\]
in terms of which the background fluxes take the form:
\[
H = \frac{2\tilde{m}}{5} a(e^{245} + e^{135} + e^{146} - e^{236}), \\
e^{\Phi} F_2 = a^{1/2} \frac{2}{2\sqrt{3}} (e^{12} - e^{34} + e^{56}), \\
e^{\Phi} F_4 = a^{-1/2} \tilde{f} \text{vol}_4 - \frac{3}{5} \tilde{m} a^{3/2} (e^{1234} - e^{1256} + e^{3456}), \\
e^{\Phi} j^6 = -\frac{2}{5} a^{1/2} \tilde{\mu} (e^{245} + e^{135} + e^{146} - e^{236}).
\]

As mentioned before, \( \mu > 0 \) corresponds to net orientifold charge. Solutions with \( \mu \leq 0 \) — i.e. with net D-brane charge — are possible, but in that case we still assume that smeared orientifolds are present, which then should be compensated by introducing enough smeared D-branes. It can be easily read off from \( j^6 \) that the orientifolds are along the directions \((1,3,6),(2,4,6),(2,3,5)\) and \((1,4,5)\), leading to four orientifold involutions. One can check that all fields and the SU(3)-structure transform as in (2.15) under each of the orientifold involutions. Also, the structure constant tensor (4.8) is even.

\(^{11}\)\(\hat{\rho}\) can be found by lowering one index of the purely \( K \)-part of the structure constant tensor with the Cartan-Killing metric and \( \rho \) is its Hodge dual, so they are both left-invariant. Moreover, since the structure constant tensor should be even under all orientifold involutions and the Hodge dual is odd, we find that \( \hat{\rho} \) is even and \( \rho \) odd. We can immediately conclude that they should be proportional to Im\(\Omega\) and Re\(\Omega\) respectively. Of course a priori there could have been more left-invariant three-forms.
4.2 The $\text{Sp}(2)_{\text{SU}(2) \times \text{U}(1)}$ solution

The structure constants are totally antisymmetric. The non-zero ones are given by:

\[
\begin{align*}
 f_{541}^5 &= f_{32}^5 = f_{13}^6 = f_{42}^6 = \frac{1}{2}, &
 f_{56}^7 &= f_{10}^8 = -1, \\
 f_{21}^7 &= f_{43}^7 = f_{14}^8 = f_{32}^9 = f_{13}^9 = f_{24}^{10} = f_{34}^{10} = f_{21}^{10} = \frac{1}{2}, \\
\end{align*}
\]

(4.15)
corresponding to the nonmaximal embedding. The $G$-invariant two-forms and three-forms are spanned by

\[
\{e^{12} + e^{34}, e^{56}\}, \quad \{\rho = e^{245} - e^{135} - e^{146} - e^{236}, \hat{\rho} = e^{235} + e^{246} + e^{145} - e^{136}\},
\]

(4.16)
respectively, and there are no invariant one-forms. The source (if present) must be proportional to $\text{Re} \Omega$.

The most general solution is then given by

\[
\begin{align*}
 J &= a(e^{12} + e^{34}) - ce^{56}, \\
 \Omega &= d \left[ (e^{245} - e^{136} - e^{146} - e^{236}) + i(e^{246} + e^{235} + e^{145} - e^{136}) \right], \\
\end{align*}
\]

(4.17)
with $a$ and $c$ two free parameters and

\[
\begin{align*}
 a > 0, & \quad c > 0, \quad \text{metric positivity}, \\
 d^2 &= a^2 c, \quad \text{normalization of } \Omega, \\
 c_1 &:= -\frac{3i}{2} W_1^{-} = -\frac{2}{3} e^{\Phi} f = \frac{2a + c}{2d}, \\
 W_2^{-} &= -\frac{2i}{3d} \left[ a(a - c)(e^{12} + e^{34}) + 2c(a - c)e^{56} \right], \\
 c_2 &:= -\frac{1}{8} \|W_2^{-}\|^2 = \frac{2}{3a^2 c} (a - c)^2, \\
 \frac{2}{5} (e^{2\Phi} m^2 - \mu) &= c_2 + \frac{1}{6} c_1^2 = \frac{1}{8a^2 c} \left( -4a^2 - 5c^2 + 12ac \right).
\end{align*}
\]

(4.18)
The nearly-Kähler limit corresponds to setting $a = c$. The two parameters correspond to the overall scale $a$ and a parameter $\sigma \equiv c/a$ that measures the deviation from the nearly-Kähler limit, and we can make contact with the results of [26] as in [27].

For the background fluxes and source we find in terms of the reduced flux parameters (4.13):

\[
\begin{align*}
 H &= \frac{2m}{5} a^1 \sigma^{1/2} (e^{245} - e^{135} - e^{146} - e^{236}), \\
 e^{\Phi} F_2 &= \frac{3}{4} \sigma^{-1/2} \left[ (2 - 3\sigma)(e^{12} + e^{34}) + (6\sigma - 5\sigma^2)e^{56} \right], \\
 e^{\Phi} F_4 &= a^{-1/2} f_{\text{vol}} + \frac{3}{5} a^{3/2} m (e^{1234} - \sigma e^{1256} - e^{3456}), \\
 e^{\Phi} j^6 &= -\frac{2}{5} a^{1/2} \mu_1 / \sigma^{1/2} (e^{245} - e^{135} - e^{146} - e^{236}).
\end{align*}
\]

(4.19)
We introduce the same orientifold involutions as in section 4.1 and check that all fields and the structure constants transform appropriately.
4.3 The SU(3)/U(1) solution

We choose a basis such that the structure constants of SU(3) are given by

\[ f_{154} = f_{136} = f_{246} = f_{235} = f_{347} = f_{576} = \frac{1}{2}, \quad f_{127} = 1, \quad f_{348} = f_{568} = \frac{\sqrt{3}}{2}, \quad \text{cyclic}. \] (4.20)

The G-invariant two-forms and three-forms are spanned by

\[ \{e^{12}, e^{34}, e^{56}\}, \quad \{\rho = e^{245} + e^{135} + e^{146} - e^{236}, \hat{\rho} = e^{235} + e^{136} + e^{246} - e^{145}\}, \] (4.21)

respectively, and there are no invariant one-forms. The source (if present) must again be proportional to ReΩ.

The most general solution is then given by

\begin{align*}
J &= -ae^{12} + be^{34} - ce^{56}, \\
\Omega &= d \left[ (e^{245} + e^{135} + e^{146} - e^{236}) + i(e^{235} + e^{136} + e^{246} - e^{145}) \right],
\end{align*}

with \(a, b\) and \(c\) three free parameters and

\[ a > 0, b > 0, c > 0, \quad \text{metric positivity}, \]
\[ c_1 := \frac{3i}{2} W_1 = -\frac{2}{3} \frac{\Phi f}{a+b+c}, \]
\[ W_2 = \frac{2i}{3d} \left[ a(2a - b - c)e^{12} + b(a - 2b + c)e^{34} + c(-a - b + 2c)e^{56} \right], \]
\[ c_2 := \frac{1}{8} \left| W_2 \right|^2 = \frac{2}{3abc} \left( a^2 + b^2 + c^2 - (ab + ac + bc) \right), \]
\[ \frac{2}{5}(e^{2\Phi}m^2 - \mu) = c_2 + \frac{1}{6}c_1^2 = \frac{1}{8abc} \left[ -5(a^2 + b^2 + c^2) + 6(ab + ac + bc) \right]. \]

Putting \(a = b\) we end up with a model that is very similar to the one of section (4.2), while further putting \(a = b = c\) corresponds to the nearly-Kähler limit. Next to the overall scale \(a\) we have this time two shape parameters \(\rho \equiv b/a\) and \(\sigma \equiv c/a\). For a comparison with the results of [26] see [27].

Introducing again the reduced flux parameters (4.13) we find for the fluxes and source

\begin{align*}
H &= \frac{2\tilde{m}}{5} a(\rho \sigma)^{1/2} (e^{245} + e^{135} + e^{146} - e^{236}), \\
e^{\Phi} F_2 &= \frac{a^{1/2}}{4} (\rho \sigma)^{-1/2} \left[ (5 - 3\rho - 3\sigma)e^{12} + (3\rho - 5\rho^2 + 3\rho\sigma)e^{34} + (-3\sigma - 3\rho\sigma + 5\sigma^2)e^{56} \right], \\
e^{\Phi} F_4 &= a^{-1/2} f_{\text{vol}4} - \frac{3}{5} a^{3/2} \tilde{m} \left( \rho e^{1234} - \sigma e^{1256} + \rho \sigma e^{2456} \right), \\
e^{\Phi} f^6 &= \frac{2}{5} a^{1/2} \tilde{\mu}(\rho \sigma)^{1/2} (e^{135} + e^{146} + e^{245} - e^{236}),
\end{align*}

(4.24)

while the orientifold involutions are still as in section 4.1.
4.4 The SU(2) × SU(2) solution

The structure constants in this case are
\[ f^1_{23} = f^4_{56} = 1, \quad \text{cyclic} \quad (4.25) \]

The most general solution to eqs. (2.4), (2.5), (2.6), (2.12) and (2.13) is
\[ J = ae^{14} + be^{25} + ce^{36}, \]
\[ \Omega = -\frac{1}{c_1} \left\{ a(e^{234} - e^{156}) + b(e^{246} - e^{135}) + c(e^{126} - e^{345}) \right. \\
\quad \left. - \frac{i}{h} \left[ -2abc(e^{123} + e^{456}) + a(b^2 + c^2 - a^2)(e^{234} + e^{156}) + b(a^2 + c^2 - b^2)(e^{153} + e^{426}) \right. \\
\quad \left. + c(a^2 + b^2 - c^2)(e^{345} + e^{126}) \right\} \quad (4.26) \]

with \( a, b \) and \( c \) three free parameters and \( abc > 0 \), metric positivity,
\[ h = \sqrt{2a^2b^2 + 2b^2c^2 + 2a^2c^2 - a^4 - b^4 - c^4}, \]
and thus \( 2a^2b^2 + 2b^2c^2 + 2a^2c^2 - a^4 - b^4 - c^4 > 0 \),
\[ c_1^2 = \frac{4}{9}e^{2\Phi}f^2 = \frac{h}{2abc}, \]
\[ W^-_2 = -\frac{2i}{3hc_1} \left\{ \frac{(b^2 - c^2)^2 + a^2(-2a^2 + b^2 + c^2)}{bc} e^{14} + \frac{(c^2 - a^2)^2 + b^2(-2b^2 + c^2 + a^2)}{ac} e^{25} \right. \\
\quad \left. + \frac{(a^2 - b^2)^2 + c^2(-2c^2 + a^2 + b^2)}{ab} e^{36} \right\}. \quad (4.27) \]

By a suitable change of basis we can always arrange for \( a > 0, b > 0 \) and \( c > 0 \), which we will assume from now on. In terms of the reduced flux parameters (4.13), to which we add \( \tilde{h} = \frac{a}{\sqrt{2}}h \),
\[ \tilde{h} = a^{-2}h, \quad (4.28) \]

we find for the fluxes
\[ H = \frac{2\tilde{m}}{5c_1} \left\{ (e^{156} - e^{234}) + \rho (e^{135} - e^{246}) + \sigma (e^{345} - e^{126}) \right\}, \]
\[ F_2 = \frac{c_1 a^{1/2}}{2h^2} \left\{ [3(\rho^4 + \sigma^4) - 5 + 2(\rho^2 + \sigma^2) - 6\rho^2 \sigma^2] e^{14} \right. \\
\quad \left. + \rho \left[ 3(1 + \sigma^4) - 5\rho^4 + 2\rho^2(1 + \sigma^2) - 6\sigma^2 \right] e^{25} \right. \\
\quad \left. + \sigma \left[ 3(1 + \rho^4) - 5\sigma^4 + 2\sigma^2(1 + \rho^2) - 6\rho^2 \right] e^{36} \right\}, \quad (4.29) \]
\[ F_4 = a^{-1/2}f\text{vol}_4 - a^{3/2} \frac{3\tilde{m}}{5} (\rho e^{1245} + \sigma e^{1346} + \rho \sigma e^{2356}). \]
Computing $j^6$ gives
\[
e^{\Phi}j = -idW_2 + \left(\frac{2}{27}f^2 - \frac{2}{3}m^2\right)e^{2\Phi}\text{Re}\Omega,
\]
with $j_1, j_2$ and $j_3$ some complicated factors depending on $a, b$ and $c$ whose exact form does not matter for the moment. It contains the same terms as $\text{Re}\Omega$ but with different coefficients. In fact, one can check that $j^6$ is not proportional to $\text{Re}\Omega$ unless $|a| = |b| = |c|$, which reduces the solution to a nearly-Kähler geometry. This time it is not immediately obvious how to choose the orientifold projection. Choosing them naively along the six terms leads to the fields and structure constants having the wrong transformation properties. In appendix C we outline how to find the orientifold involutions associated to a smeared source in general and then apply the procedure to the case at hand. In order to present the resulting involutions, it is convenient to define complex one-forms as follows
\[
e^{x_1} = \pm \frac{e^{\frac{\pi i}{4}}}{2c_1\sqrt{bc(2bc - h)}} \left\{ [2bc - h + i(a^2 - b^2 - c^2)]e^1 + [a^2 - b^2 - c^2 + i(2bc - h)]e^4 \right\},
\]
\[
e^{x_2} = \pm \frac{e^{\frac{\pi i}{4}}}{2c_1\sqrt{ac(2ac - h)}} \left\{ [2ac - h + i(b^2 - a^2 - c^2)]e^2 + [b^2 - a^2 - c^2 + i(2ac - h)]e^5 \right\},
\]
\[
e^{x_3} = \pm \frac{e^{\frac{\pi i}{4}}}{2c_1\sqrt{ab(2ab - h)}} \left\{ [2ab - h + i(c^2 - a^2 - b^2)]e^3 + [c^2 - a^2 - b^2 + i(2ab - h)]e^6 \right\},
\]
where the signs must be chosen such that $\Omega = e^{x_1}z^2x^2$. Defining further the associated $x$ and $y$ one-forms $e^{x_i} = e^{x_i} - ie^{y_i}$, the orientifold involutions are given as in (C.10).

### 4.5 The $\frac{\text{SU}(3) \times U(1)}{\text{SU}(2)}$ solution

We construct the algebra by taking
\[
E_i = G_{i+3}, \quad i = 1, \ldots, 5; \quad E_6 = M; \quad E_7 = G_1; \quad E_8 = G_2; \quad E_9 = G_3,
\]
where the $G_i$'s are the Gell-Mann matrices generating $\text{su}(3)$, $M$ generates a $\text{u}(1)$, and the $\text{su}(2)$ subalgebra is generated by $E_7, E_8$ and $E_9$. It follows that the $\text{SU}(2)$ subgroup is embedded entirely inside the $\text{SU}(3)$, so that the total space is given by $\frac{\text{SU}(3)}{\text{SU}(2)} \times \text{U}(1) \simeq S^5 \times S^1$. The structure constants are
\[
f^7_{89} = 1, \quad f^7_{14} = f^7_{32} = f^8_{13} = f^8_{24} = f^9_{12} = f^9_{43} = 1/2, \quad f^{5}_{12} = f^{5}_{34} = \frac{\sqrt{3}}{2}, \quad \text{cyclic}.
\]
Invariant one-forms are generated by $\{e^5, e^6\}$, invariant two-forms by
\[
\{e^{12} + e^{34}, e^{13} - e^{24}, e^{14} + e^{23}, e^{56}\},
\]
and invariant three-forms are given by
\[
\{e^{145} + e^{235}, e^{135} - e^{245}, e^{126} + e^{346}, e^{146} + e^{236}, e^{136} - e^{246}, e^{125} + e^{345}\}.
\]

There is a solution for non-zero source:
\[
J = -a(e^{13} - e^{24}) + b(e^{14} + e^{23}) + ce^{56},
\]
\[
\Omega = -\frac{\sqrt{3}}{2c_1}\left\{2a(e^{145} + e^{235}) + 2b(e^{135} - e^{245}) + c(e^{126} + e^{346})\right\}
\]
\[
- \frac{i}{\sqrt{a^2 + b^2}} \left[ac(e^{146} + e^{236}) + bc(e^{136} - e^{246}) - 2(a^2 + b^2)(e^{125} + e^{345})\right],
\]
with \(a, b\) and \(c\) three free parameters and
\[
c > 0, \quad a^2 + b^2 \neq 0, \quad \text{metric positivity},
\]
\[
\frac{1}{(c_1)^2} = \frac{2}{3} \sqrt{a^2 + b^2}, \quad \text{normalization of } \Omega,
\]
\[
c_1 := -\frac{3i}{2} \mathcal{W}_1 = -\frac{2}{3} e^f,
\]
\[
\mathcal{W}_1 = \frac{i}{2c_1 \sqrt{a^2 + b^2}} \left[-a(e^{13} - e^{24}) + b(e^{14} + e^{23}) - 2ce^{56}\right],
\]
\[
d\mathcal{W}_2 = -\frac{i \sqrt{3}}{2c_1 \sqrt{a^2 + b^2}} \left[a(e^{145} + e^{235}) + b(e^{135} - e^{245}) - c(e^{126} + e^{346})\right],
\]
\[
3|\mathcal{W}_1|^2 - |\mathcal{W}_2|^2 = 0.
\]

By a suitable change of basis we can always arrange for \(a > 0\) and \(b > 0\), which we will assume from now on. Note that \(d\mathcal{W}_2\) is not proportional to \(\text{Re} \Omega\), hence the source is not of the form \((2.10)\). Interestingly, if we take the part of the source along \(\text{Re} \Omega\) to be zero, i.e. \(j^6 \wedge \text{Im} \Omega = 0\), we find from the last equation in \((4.35)\) that \(m = 0\). This would amount to a combination of smeared D6-branes and O6-planes such that the total tension is zero. Allowing for negative total tension (more orientifolds), we could have \(m > 0\). For an arbitrary \(m\) we find the background
\[
H = -\frac{\sqrt{3} \tilde{m}}{5c_1} a \left[2(e^{145} + e^{235}) + 2\rho(e^{135} - e^{245}) + \sigma(e^{126} + e^{346})\right],
\]
\[
e^f F_2 = \frac{1}{2} a^{1/2} \tilde{c}_1 \left[(e^{13} - e^{24}) - \rho(e^{14} + e^{23}) + \sigma e^{56}\right],
\]
\[
e^f F_4 = a^{-1/2} \tilde{f} \text{vol}_4 + \frac{3}{5} a^{3/2} \tilde{m} \left[(1 + \rho^2)e^{1234} - \sigma(e^{1356} - e^{2456}) + \rho \sigma(e^{1456} + e^{2356})\right],
\]
\[
(4.36)
\]
where we defined \(\rho = b/a\) and \(\sigma = c/a\) and used again \((4.13)\). From \((2.4)\) we compute for the source
\[
e^f J^{O6} = -\frac{\sqrt{3}}{10c_1} a^{1/2} (5\tilde{c}_1^2 - 4\tilde{m}^2) \left[e^{145} + e^{235} + \rho(e^{135} - e^{245})\right]
\]
\[
+ \frac{\sqrt{3}}{20c_1} a^{1/2} \sigma (5\tilde{c}_1^2 + 4\tilde{m}^2) \left(e^{126} + e^{346}\right),
\]
\[
(4.37)
\]
One can check that for the background the source satisfies the calibration conditions (2.11). If we make the following coordinate transformation

\[ e'^1 = e^1, \quad e'^2 = e^2, \quad e'^3 = e^3 + \rho^{-1} e^4, \quad e'^4 = e^3 - \rho e^4, \quad e'^5 = e^5, \quad e'^6 = e^6, \quad (4.38) \]

we see clearly that \( j \) is a sum of four decomposable terms

\[ e^\Phi j^6 = \frac{-\sqrt{3}}{10 c_1} a^{1/2} (5 e_1^2 - 4 m^2) (e^{2\prime 4\prime 5\prime} + \rho e^{1\prime 3\prime 5\prime}) \]
\[ + \frac{\sqrt{3}}{20 c_1} a^{1/2} \sigma (5 e_1^2 + 4 m^2) \left( e^{1\prime 2\prime 6\prime} - \frac{\rho}{1 + \rho^2} e^{3\prime 4\prime 6\prime} \right), \quad (4.39) \]

to which we can associate four orientifold involutions.

5. Low energy physics I: nilmanifolds

In this section, we will first explicitly perform the Kaluza-Klein reduction on the torus solution of section 3.1 and the Iwasawa solution with \( m = 0 \) of section 3.2 and calculate the mass spectrum. Next, we will use the effective supergravity approach and construct the Kähler potential and the superpotential. From there we can get the potential and compare the mass spectrum in both approaches. We find exact agreement. From then on, we will only use the effective supergravity approach and study the Iwasawa solution with \( m \neq 0 \) and the type IIB solution of section 3.3 in this section as well as the coset models in the next section.\(^\text{12}\)

5.1 Kaluza-Klein reduction

We are interested in performing a Kaluza-Klein reduction on each of the \( \text{AdS}_4 \times \mathcal{M}_6 \) solutions described in sections 3.1 and 3.2. Let \( x \) and \( y \) be space-time and internal-manifold coordinates, respectively. Moreover, let \( \Phi(x, y) \) be a ‘vacuum’, i.e. a particular solution of the equations of motion of ten-dimensional supergravity. The Kaluza-Klein reduction (see [39] for a review) consists in expanding all ten-dimensional fields \( \Phi(x, y) \) in ‘small’ fluctuations around the vacuum:

\[ \Phi(x, y) = \hat{\Phi}(x, y) + \delta \Phi(x, y), \quad (5.1) \]

keeping only terms up to linear order in \( \delta \Phi(x, y) \) in the equations of motion (corresponding to at most quadratic terms in the Lagrangian). From now on the hats indicate background quantities and the \( \delta \)s fluctuations. The fluctuations are Fourier-expanded in the internal space:

\[ \delta \Phi(x, y) = \sum_n \phi_n(x) \omega_n(y), \quad (5.2) \]

\(^{12}\)As a general remark, we will not consider blow-up modes associated to the fixed points of the orientifold involutions. Ideally, we would like to argue that the blow-up modes will be stabilized by flux through the blown-up cycle at a size much smaller than the size of the internal manifold. Unfortunately, such an analysis is beyond the scope of the present paper. It may be possible, however, to argue for the stabilization of the blow-up modes using a local analysis of the singularities as in [10].
where $\phi_n(x)$ are four-dimensional space-time fields, and the $\omega_n(y)$’s form a basis of eigenforms of the Laplacian operator $\Delta = dd^\dagger + d^\dagger d$ in the six-dimensional space $\mathcal{M}$ (the internal part of the vacuum solution).

In the following we will truncate all the higher Kaluza-Klein modes in the harmonic expansion (5.2) and keep only those $\omega_n(y)$’s in (5.2) that are left-invariant on $\mathcal{M}_6$. The resulting modes are not in general harmonic, but can be combined into eigenvectors of the Laplacian whose eigenvalues are of order of the geometric fluxes.

Plugging the ansatz (5.1)-(5.2) into the ten-dimensional equations of motion and keeping at most linear-order terms in the fluctuations, one can read off the masses of the space-time fields, i.e. the ‘spectrum’. In the present case, this is accomplished by comparing with the equations of motion for non-interacting fields propagating in AdS$_4$. Let $M$ and $\Lambda$ be the mass of the field and the cosmological constant of the AdS space, respectively, such that

Scalar: $\Delta \phi + \left( M^2 + \frac{2}{3} \Lambda \right) \phi = 0$, \hspace{1cm} (5.3a)

Vector: $\Delta \phi_\mu + \nabla_\mu \nabla^\nu \phi_\nu + M^2 \phi_\mu = 0$, \hspace{1cm} (5.3b)

Metric: $\Delta_L h_\mu\nu + 2\nabla_\mu \nabla_\nu h_\rho^\rho - \nabla_\nu h_\mu_\rho \left( M^2 - 2\Lambda \right) = 0$, \hspace{1cm} (5.3c)

where $\Delta_L$ is the Lichnerowicz operator defined by:

$$
\Delta_L h_\mu\nu = -\nabla^2 h_\mu\nu - 2R_{\mu\rho\nu\sigma} h^{\rho\sigma} + 2R_{(\mu}^{\rho} h_{\nu)\rho}.
$$

With the above definitions, the Breitenlohner-Freedman bound [54] is simply

$$
M^2 \geq 0,
$$

for the metric and the vectors. For the scalars, however, a negative mass-squared is allowed:

$$
M^2 \geq \frac{\Lambda}{12} = -\frac{|W|^2}{4},
$$

where $W$ was defined in eq. (2.3). Actually, we will present the results for the mass spectrum of the scalars in terms of

$$
\tilde{M}^2 = M^2 + \frac{2}{3} \Lambda,
$$

for which the Breitenlohner-Freedman bound reads

$$
\tilde{M}^2 \geq -\frac{9 |W|^2}{4}.
$$

We will take $\tilde{M} = 0$ as the definition of an unstabilized modulus since from (5.3a) we see that then, if it were not for the boundary conditions of AdS$_4$, a constant shift of $\phi$ would be a solution to the equations of motion. Therefore a constant shift of $\phi$ leads to a new vacuum solution.
We would also like to express the fluctuations of the RR field strengths $\delta F$ in terms of the fluctuations of the potentials $\delta C$ in such a way that the Bianchi identity $d_H F = -j$ is automatically satisfied. Indeed, as explained in appendix D, this is achieved for

$$e^{\delta B} \delta F = (d + \hat{H}) \delta C - (e^{\delta B} - 1) \hat{F},$$

where we have set $\delta F_0 = 0$. For the NSNS flux we can just write

$$H = \hat{H} + \delta H = \hat{H} + d\delta B.$$  \hspace{1cm} (5.10)

### 5.1.1 IIA on $\text{AdS}_4 \times T^6$

By direct computation of the Kaluza-Klein reduction on the six-torus solution of section 3.1, we obtain the following mass eigenvalues $\tilde{M}^2/|W|^2$ for the scalar fields:\textsuperscript{13}

| Complex structure       | $-2, -2, -2$ |
|-------------------------|--------------|
| Kähler & dilaton        | 70, 18, 18, 18 |
| Three axions of $\delta C_3$ | 0, 0, 0 |
| $\delta B$ & one more axion | 88, 10, 10, 10 |

Although the Kaluza-Klein procedure, as outlined in 5.1 is straightforward, many of the intermediate steps are rather subtle. The interested reader may consult appendix D for more details on the derivation and on the exact mass eigenvectors.

Even without these details we can make a number of interesting observations. First of all three axions correspond to massless moduli. This is a feature that is also discussed in [11]. It is argued there that, when one introduces D6-branes, these axions can provide Stückelberg masses to some of the U(1) gauge fields on the D-brane. In any case, we will see later that most of the coset examples do have all moduli stabilized. Secondly, we notice that some masses are tachyonic, which is allowed because they are still above the Breitenlohner-Freedman bound (5.8). And finally, scalars that are in the same supermultiplet, like the complex structure moduli and the three corresponding axions, the dilaton and the remaining axion, the Kähler moduli and the $B$-field moduli have different masses. This is in fact a subtlety of the supersymmetry algebra of AdS$_4$ that no longer allows a definition for the mass as an invariant Casimir operator.

For this model, we can decouple the tower of Kaluza-Klein masses (see the discussion below (2.18) in section 2.2) when we take $m^2(e^{2\Phi}L_{\text{int}}^2) \ll 1$.

### 5.1.2 IIA on the Iwasawa manifold

As explained in detail in appendix D, performing the Kaluza-Klein reduction on the Iwasawa manifold we obtain the exact same mass spectrum as in the case of the Kaluza-Klein reduction on the six-torus solution of the previous section. This is of course the expected result, since the two solutions are related by T-duality. The limit for decoupling the Kaluza-Klein tower corresponds to taking $\beta \ll 1$.

\textsuperscript{13}The calculations in appendix D.3 were made in the ten-dimensional Einstein frame, while the effective supergravity approach followed in later sections will lead to a result in the four-dimensional Einstein frame. By dividing out with $|W|^2$ we avoid conversion problems, since $\tilde{M}$ and $|W|^2$ transform in the same way under change of frame.
5.2 Effective supergravity

In this section we derive the masses of the scalar fields by means of the superpotential and Kähler potential for the three explicit examples of compactification manifolds we found. Comparing these results with the results of the explicit Kaluza-Klein reduction in the previous section may be seen as a cross-check for the expressions for the superpotential and Kähler potential.

5.2.1 Superpotential and Kähler potential

The superpotential and Kähler potential of the effective $\mathcal{N} = 1$ supergravity have been derived in various ways in [30, 31, 32] (based on earlier work of [55, 29]). Here we summarize the main formulæ which will be used in the following; more details on the derivation can be found in appendix E. We present first the superpotential and Kähler potential appropriate for general $SU(3) \times SU(3)$-structure and then specialize to strict $SU(3)$ and static $SU(2)$-structure.

The part of the effective four-dimensional action containing the graviton and the scalars reads:

$$S = \int d^4x \sqrt{-g_4} \left( \frac{M_P^2}{2} R - M_P^2 K_{ij} \partial_i \phi^j \partial^j - V(\phi, \bar{\phi}) \right),$$

(5.11)

where $M_P$ is the four-dimensional Planck mass. The scalar potential is given in terms of the superpotential via:

$$V(\phi, \bar{\phi}) = M_P^{-2} e^K \left( K^{ij} D_i W_E D^j W_E^* - 3 |W_E|^2 \right),$$

(5.12)

where the superpotential in the Einstein frame $W_E$ reads

$$W_E = -\frac{i}{4 \kappa_{10}^2} \int_M \langle \Psi_2, F + i d_H(\text{Re} T) \rangle,$$

(5.13)

and $\langle \cdot, \cdot \rangle$ indicates the Mukai pairing (B.19), $\text{Re} T = e^{-\Phi} \text{Im} \Psi_1$, and $\Psi_1$ and $\Psi_2$ are the pure spinors describing the geometry. Using the expansion in background and fluctuations of (5.9) and (5.10) we can rewrite this as

$$W_E = -\frac{i}{4 \kappa_{10}^2} \int_M \langle \Psi_2 e^{\delta B}, \hat{F} + i d_H(e^{\delta B}\text{Re} T - i \delta C) \rangle,$$

(5.14)

where we used property (B.20). This shows how the fields organize in complex multiplets $\Psi_2 e^{\delta B}$ and $\text{Re} T - i \delta C$, which will be clearer in concrete examples.

The Kähler potential reads

$$K = -\ln i \int_M \langle \Psi_2, \bar{\Psi}_2 \rangle - 2 \ln i \int_M \langle t, \bar{t} \rangle + 3 \ln(8 \kappa_{10}^2 M_P^2),$$

(5.15)

where we defined $t = e^{-\Phi} \Psi_1$. Note that $\text{Re} t$ should be thought of as a function of $\text{Im} t$ so that $t$ can be seen as (non-holomorphically) dependent on $T$. This is explained in more detail in appendix E.

\textsuperscript{14}In [36] the scalar potential was for general type II $SU(3) \times SU(3)$ compactifications directly derived from dimensional reduction of the action.
IIA SU(3)

Specializing to the IIA SU(3) case with pure spinors (B.29), the superpotential takes the form

$$W_E = -ie^{-i\theta} \int_M \langle e^{i(J-i\delta B)} - i\delta B e^{-\Phi} \text{Im} \Omega + i\delta C \rangle, \quad (5.16)$$

and the Kähler potential is given by

$$K = -\ln \int_M \langle J^3 \rangle - 2\ln \int_M 2 e^{-\Phi} \text{Im} \Omega \wedge e^{-\Phi} \text{Re} \Omega + 3\ln(8\kappa_{10}^2 M_P^2), \quad (5.17)$$

where $e^{-\Phi} \text{Re} \Omega$ should be seen as a function of $e^{-\Phi} \text{Im} \Omega$. On the fluctuations we must impose the orientifold projections (2.15). It turns out that for all our examples (except for a special case of the SU(3)×U(1)SU(2)-model):

$$\delta B \wedge \text{Im} \Omega = 0, \quad (5.18)$$

since there are no odd five-forms. By expanding in a suitable basis of even and odd expansion forms (which have to be identified separately for each case), we find that the fluctuations organize naturally in complex scalars

$$J_c = J - i\delta B = (k^i - ib^i)Y_i^{(2-)} = t^i Y_i^{(2-)}, \quad (5.19a)$$

$$e^{-\Phi} \text{Im} \Omega + i\delta C_3 = (u^i + ic^i) e^{-\Phi} Y_i^{(3+)} = z^i e^{-\Phi} Y_i^{(3+)}, \quad (5.19b)$$

where we took out the background $e^{-\Phi}$ from the definition of $z^i$ for further convenience.

IIB SU(2)

Specializing to the case of type IIB SU(2) with pure spinors (B.35), the superpotential becomes

$$W_E = \frac{i}{4\kappa_{10}^2} \int_M \langle 2 V \wedge e^{i(\omega_2 - i\delta B)} - i\delta B e^{-\Phi} \text{Im} (e^{2V \wedge \bar{V}} \wedge \Omega_2) + i\delta C \rangle, \quad (5.20)$$

and the Kähler potential

$$K = -\ln \left( -2i \int_M 2 V \wedge 2 \bar{V} \wedge \omega_2^2 \right) - 2\ln \int_M 2 \langle \text{Ret}, \text{Im} t \rangle + 3\ln(8\kappa_{10}^2 M_P^2), \quad (5.21)$$

where again Ret should be considered as a function of Im$t$ as $-\text{Im} \left( e^{-\Phi} e^{2V \wedge \bar{V}} \Omega_2 \right)$.

Under the orientifold projections we find from eq. (3.5a) and (3.5b) of [45] for the NSNS-sector

$$O5: \quad \sigma^* V = -V, \quad \sigma^* \omega_2 = -\omega_2, \quad \sigma^* \Omega_2 = -\Omega_2^*, \quad \sigma^* \delta B = -\delta B, \quad (5.22a)$$

$$O7: \quad \sigma^* V = V, \quad \sigma^* \omega_2 = -\omega_2, \quad \sigma^* \Omega_2 = \Omega_2^*, \quad \sigma^* \delta B = -\delta B, \quad (5.22b)$$

and for the RR-sector

$$O5: \quad \sigma^* \delta C_2 = \delta C_2, \quad \sigma^* \delta C_4 = -\delta C_4, \quad (5.23a)$$

$$O7: \quad \sigma^* \delta C_2 = -\delta C_2, \quad \sigma^* \delta C_4 = \delta C_4. \quad (5.23b)$$
Again we find that the fluctuations organize naturally in complex scalars

\[ \omega_c = \omega_2 - i\delta B = (k^i - ib^i)Y_i^{(2-)} = t^iY_i^{(2-)}, \]

\[ e^{-\Phi}\text{Im} \Omega_2 + i\delta C_2 = (u^i + ic^i)e^{-\Phi}Y_i^{(2-)} = z^i e^{-\Phi}Y_i^{(2-)}, \]

\[ -ie^{-\Phi}2V \wedge \bar{V} \wedge \text{Re} \Omega_2 + i\delta C_4 = (u^i + ih^i)e^{-\Phi}Y_i^{(4+)} = w^i e^{-\Phi}Y_i^{(4+)}, \]

\[ 2V = C(iY_1^{(1-)} - \tau Y_2^{(1-)}), \]

where we define \( \tau = x + iy \), and each time the first/second sign of the \( Y_i \) indicates the behaviour under the O5/O7-involution. Note that \( C \) is a complex overall factor that can be scaled away together with the warp factor and the arbitrary \( U(1) \) phase.

5.2.2 IIA on \( \text{AdS}_4 \times \text{T}^6 \)

For convenience we choose a slightly different expansion basis as in appendix D.3:

\[ Y_i^{(2-)}: e^{12}, e^{34}, e^{56}; \]

\[ Y_i^{(3+)}: -e^{135}, e^{146}, e^{236}, e^{245}. \]

We then find the superpotential

\[ W_{\text{E,Torus}} = \frac{e^{-\theta}}{4\kappa_{10}^2} V_s \ln \left[ -t_1 t_2 t_3^3 + \frac{3}{5}(t_1 + t_2 + t_3^3) - \frac{2}{5}(z_1 + z_2 + z_3 + z_4) \right], \]

where \( V_s \) is a standard volume \( V_s = \int e^{1...6} \), which does not depend on the moduli. Moreover, the Kähler potential reads:

\[ K = K_k + K_c + 3 \ln(8\kappa_{10}^2 M_P^2 V_s^{-1} e^\Phi/3), \]

where

\[ K_k = -\ln \left( \prod_{i=1}^{3}(t^i + \bar{t}^i) \right) \]

is the Kähler potential in the Kähler-moduli sector and

\[ K_c = -\ln \left( \prod_{i=1}^{4}(z^i + \bar{z}^i) \right) \]

is the Kähler potential in the complex structure moduli sector.

Using the expressions for the superpotential and the Kähler potential it is straightforward to calculate the masses for the scalar fields from the quadratic terms in the potential. To perform this calculation we made use of [56]. Upon noting that in the Kaluza-Klein analysis we set the background values for the warp factor and the dilaton equal to zero and \( \text{Vol} = V_s \), we find exactly the same result as in section 5.1.1.

\[ \text{– 35 –} \]
5.2.3 IIA on the Iwasawa manifold

We choose the following expansion basis:

\[ Y^{(2-)} : \beta^2 e^{65}, e^{12}, e^{34}; \]
\[ Y^{(3+)} : -\beta e^{135}, -\beta e^{146}, -\beta e^{236}, \beta e^{245}. \]

This implies that \( dY^{(3+)}_i = -\beta e^{1234} \) for all \( i = 1, \ldots, 4 \). We find the superpotential

\[ W_{E, Iwasawa} = -\frac{i e^{-i\theta}}{4\kappa_{10}^2} m_T V_s \left[ \frac{3}{5} - \frac{2}{5} t^1 (z^1 + z^2 + z^3 + z^4) + \frac{3}{5} (t^1 t^2 + t^1 t^3) - t^2 t^3 \right], \]

where \( V_s = \int -\beta e^{1\ldots6} \) is again a standard volume and \( m_T = \frac{5}{2} e^{-\hat{\Phi}} \beta \) the Romans mass of the T-dual torus solution. We note here the following relation

\[ W_{E, Iwasawa} = -i t^1 W_{E, Torus} (t^1 \to \frac{1}{t^1}), \]

which follows from T-duality. The Kähler potential for the Iwasawa manifold is the same as in (5.27).

In the end, we find exactly the same masses as on the torus, as expected from T-duality, and thus also the same masses as in the Kaluza-Klein approach for the Iwasawa. This provides a consistency check on the ability of the superpotential/Kähler potential approach to handle geometric fluxes.

If we now turn on \( m \neq 0 \) in the Iwasawa solution, we get extra terms in the superpotential that look exactly like the torus superpotential, so we find:

\[ W_{E, Iwasawa, m \neq 0} = W_{E, Iwasawa} (m_T) + W_{E, Torus} (m). \]

The mass spectrum is the same upon replacing \( m_T^2 \to m^2 + m_T^2 \). Also, this time it is possible to decouple the Kaluza-Klein tower: in the limit \( (m^2 + m_T^2)(e^{2\Phi} L^2_{int}) \ll 1 \).

5.2.4 IIB on the nilmanifold 5.1

For our analysis we will need expansion forms with the following behaviour under \( O5 \) and \( O7 \)-planes

| type under O5/O7 | basis | name |
|-------------------|-------|------|
| odd/even 1-form   | \( e^5, \beta e^6 \) | \( Y_i^{(1-+)} \) |
| even/odd 2-form   | \(-e^{23}, e^{14}\) | \( Y_i^{(2+-)} \) |
| odd/odd 2-form    | \(-e^{13}, e^{24}\) | \( Y_i^{(2--)} \) |
| odd/even 4-form   | \( e^{1256}, e^{3456}\) | \( Y_i^{(4-+)} \) |

and choose the standard volume \( V_s = \int \beta e^{123456} \).

So together with (5.24) we see that there are two complex “four-dimensional” Kähler moduli in \( \omega_c \), two complex moduli in \( \delta C_2 - ie^{-\Phi} \text{Im} \Omega_2 \) and two in \( e^{-\Phi} 2V \wedge \bar{V} \wedge \text{Re} \Omega_2 + i\delta C_4 \).
These four moduli include the axions, the dilaton, the “four-dimensional” complex structure moduli and the “two-dimensional” Kähler modulus in $2V \wedge \bar{V}$.

The superpotential is given by:

$$W_{\text{E,nil}} = \frac{m_T V_s C}{4\kappa_{10}^2} \left( \frac{3}{5} - \frac{2}{5} \tau (z^1 - z^2 + w^1 + w^2) + \frac{3}{5} \tau (t^1 + t^2) - t^1 t^2 \right),$$

(5.32)

where $V_s = \int \beta e^{123456}$ is the standard volume. The Kähler potential reads:

$$K = \ln \left( \tau_i + \bar{\tau}_i \right)^2 \prod_{i=1}^{2} (t^i + \bar{t}^i) - \ln \left( -4 \prod_{i=1}^{2} (z^i + \bar{z}^i) \prod_{i=1}^{2} (w^i + \bar{w}^i) \right) - 3 \ln (8\kappa_{10} M_p^2 V_s^{-1} e^{4\Phi/3}) - \ln |C|^2.$$

(5.33)

We can eliminate the complex scalar $C$ by performing a Kähler transformation (E.18). Using the above, we derive the expected (due to T-duality) result that the masses for the scalar fields are the same as for the T$^6$ and the Iwasawa manifold.

### 6. Low energy physics II: coset spaces

In this section we study the low energy effective theory of the coset spaces described in section 4.

#### 6.1 IIA on $G_{3,3}$

We choose the expansion forms in (5.19) as follows:

$$Y^{(2-)} : \quad a(e^{12} - e^{34} + e^{56});$$

$$Y^{(3+)} : \quad a^{3/2}(-e^{25} - e^{246} + e^{145} - e^{136}),$$

(6.1)

and the standard volume $V_s = -\int a^3 e^{123456}$.

The superpotential reads:

$$W_E = \frac{i e^{-\theta} e^{-\phi}}{4\kappa_{10}^2} V_s a^{-1/2} \left( -\frac{3\sqrt{3}}{2} + \frac{8\tilde{m}_i}{5} z^0 - \frac{9\tilde{m}_i}{5} t^1 + 4\sqrt{3} z^0 t^1 - \frac{\sqrt{3}}{2} (t^1)^2 + i\tilde{m}(t^1)^3 \right),$$

(6.2)

whereas the Kähler potential is

$$K = -\ln \left( (t^1 + \bar{t}^1)^3 \right) - \ln \left( 4(z^0 + \bar{z}^0)^4 \right) + 3 \ln (8\kappa_{10}^2 M_p^2 V_s^{-1} e^{4\Phi/3}).$$

(6.3)

If we plot $\tilde{M}^2/|W|^2$ the overall scale $a$ drops out and the only parameter is the reduced orientifold tension $\tilde{\mu}$: see Figure 1, where the dashed and solid red line represent the Breitenlohner-Freedman bound (5.8) and the bound (2.9) for $\tilde{\mu}$, respectively. We see that all four moduli masses are above the Breitenlohner-Freedman bound. Moreover, all masses are positive for $\tilde{\mu} > -0.82$. For $\tilde{\mu} \to \infty$ the masses asymptote to $\tilde{M}^2/|W|^2 = (10, 18, 70, 88)$, which are the same as for the torus in section 5.1.1 (except there are no complex structure moduli and corresponding axions). In fact, this is universal behaviour for all models we
studied. Indeed, for \( \tilde{\mu} \to \infty \) we find from (2.12) that \( m \to \infty \) regardless of the details \( W_1^\tau, W_2^\tau \) of the model, and exactly only the same terms as in the torus example are relevant in the superpotential.

In section 2.2 we have seen that \( |W_1^\tau| L_{\text{int}} \ll 1 \) is one way to obtain a separation of scales between the light masses and the Kaluza-Klein masses even before the uplifting. However, as can be seen from eq. (4.12), this is impossible to achieve for this coset.

6.2 IIA on \( \text{Sp}(2)_{\text{SU}(2)\times U(1)} \)

We choose the expansion forms in (5.19) as follows:

\[
\begin{align*}
Y^{(2-)} & : \quad a(e^{12} + e^{34}), -ae^{56}; \\
Y^{(3+)} & : \quad a^{3/2}(e^{235} + e^{246} + e^{145} - e^{136}),
\end{align*}
\]

and the standard volume \( V_s = -\int a^3 e^{123456} \). We find the following superpotential

\[
W_k = \frac{ie^{-i\theta}e^{-\hat{\Phi}}}{4\kappa_1^{10}} V_s a^{-1/2} \left( -\tilde{f}\sigma + \frac{8\tilde{m}i}{5} \sigma^{1/2} z_0 - \frac{3\tilde{m}i}{5} (2\sigma t^1 + t^2) - 2(2t^1 + t^2)z_0 + i\tilde{m}(t) t^2 \right)
+ \sigma^{1/2} \left( \frac{3}{2} - \frac{5}{4} \sigma \right) (t^1)^2 - \left( \sigma^{-1/2} - \frac{3}{2} \right) (t^1 t^2),
\]

and Kähler potential

\[
K = -\ln \left( (t^1 + \bar{t}^1)(t^2 + \bar{t}^2) \right) - \ln \left( 4(z_0 + \bar{z}_0)^4 \right) + 3\ln(8\kappa_1^{10} M_{P}^2 V_s^{-1} e^{4\hat{\Phi}/3}).
\]

This time the solution has next to the overall scale \( a \) two free parameters: the “shape” \( \sigma = c/a \) and the orientifold tension \( \tilde{\mu} \). In Figure 2 we display plots for several values of \( \sigma \): \( \sigma = 1 \) is the nearly-Kähler point while for \( \sigma = 2/5 \) and \( \sigma = 2 \) the lower bound for \( \tilde{\mu} \) from (2.12) is exactly zero. These were extreme points in [26] since outside the interval \([2/5, 2]\) the lower bound is above zero and solutions without orientifolds are no longer possible. Moreover, for \( \tilde{\mu} = 0 \) also \( m = 0 \) and these solutions can be lifted to M-theory. We also
display a plot for large $\sigma$, here $\sigma = 13$. We see that the lower bound for $\tilde{\mu}$ is indeed positive so that there must be net orientifold charge. The behaviour is however already like the universal behaviour for $\tilde{\mu} \to \infty$. Again we see that in all cases all masses are above the Breitenlohner-Freedman bound and by choosing $\tilde{\mu}$ large enough they are all positive.

Again we would like to get $|W_{-1}| L_{\text{int}} \ll 1$ to decouple the Kaluza-Klein modes. From eq. (4.18) we see that this can be formally obtained by putting $\sigma \to -2$, i.e. we need to analytically continue to negative values for $\sigma$. From [42] we learn that $\sigma < 0$ is indeed possible, but the model cannot be described as a left-invariant SU(3)-structure on the coset $\text{Sp}(2) \text{S}(U(2) \times U(1))$ anymore. Rather it is a twistor bundle on a four-dimensional hyperbolic space. The precise agreement between the results of [26] (which is based on [42]) and [27] (wherever they overlap) suggests that the analytic continuation is possible. Strictly speaking, however, one should check that also the mass spectrum can be analytically continued to negative values for $\sigma$. Although this seems plausible to us, verifying it directly would require using entirely different technology, and lies beyond the scope of the present paper. In deriving the plot of Figure 3 for $\sigma = -2$, we have assumed that such analytic continuation of the mass spectrum is possible. We see that two mass eigenvalues stay light, while the others blow up if $W_{-1} \to 0$ and join the Kaluza-Klein masses. In this limit the light modes have $\tilde{M}^2/|W|^2 = (-38/49, 130/49)$.
Figure 3: Mass spectrum of the continuation of $\text{Sp}(2)_{\text{SU}(2) \times \text{U}(1)}$ to negative $\sigma$.

6.3 IIA on $\text{SU}(3)_{\text{SU}(1) \times \text{U}(1)}$

In this case we choose the expansion forms in (5.19) as follows:

$$Y^{(2-)}: -ae_{12}, ae_{34}, -ae_{56};$$

$$Y^{(3+)}: a_{3/2}(e_{235} + e_{246} + e_{136} - e_{145}),$$

and the standard volume $V_s = \int a^3 e_{123456}$.

Using the expression (5.16) for the superpotential in the SU(3) case and the expansion given in (5.19), we derive the superpotential

$$W_E = -ie^{-i\theta}e^{-\Phi} \frac{4\kappa_{10}^2}{\kappa_{10}^2} V_s a^{-1/2} \left( \tilde{f} \rho \sigma - \frac{8\tilde{m}i}{5} (\rho \sigma)^{1/2} z^0 + \frac{3\tilde{m}i}{5} (\rho \sigma t^1 + \sigma t^2 + \rho \rho^3) + \frac{1}{4}(\rho \sigma)^{-1/2} \left( (3\sigma + 3\rho \sigma - 5\sigma^2)t^1 t^2 + (3\rho - 5\rho^2 + 3\rho \sigma)t^1 t^3 + (-5 + 3\rho + 3\sigma)t^2 t^3 \right) - 2z^0(t^1 + t^2 + t^3) - i\tilde{m}t^1 t^2 t^3 \right).$$

(6.8)

The Kähler potential is evaluated as in section 5.2 and reads

$$K = -\ln \left( \prod_{i=1}^{3}(t^i + \tilde{t}^i) \right) - \ln \left( 4(z^0 + \tilde{z}^0)^4 \right) + 3\ln(8\kappa_{10}^2 M_P^2 V_s^{-1} e^{4\Phi/3}).$$

(6.9)

The model has this time two shape parameters: $\rho = b/a$ and $\sigma = c/a$. We display the mass spectrum for a number of selected values of these parameters in Figure 4. There is a symmetry under permuting $(a, b, c)$ which translates into a symmetry under $\rho \leftrightarrow \sigma$ and $(\rho, \sigma, \tilde{\mu}) \leftrightarrow (\rho/\sigma, 1/\sigma, \tilde{\mu})$. Applying these symmetries leads to identical mass spectra. Moreover, the mass spectra for $\rho = 1$ are apart from two more eigenvalues identical to the mass spectra of $\text{Sp}(2)_{\text{SU}(2) \times \text{U}(1)}$. We also display an example with $\sigma, \rho \neq 1$.

In the plots of Figure 5 we have analytically continued to $\rho < 0, \sigma < 0$ in order to approach the NCY limit, which we obtain for $\rho + \sigma = -1$. Again, two eigenvalues stay light with $\tilde{M}_2^2/|W|^2 = (-38/49, 130/49)$ in the limit while the other eigenvalues blow up to the Kaluza-Klein scale.
(a) $\rho = \sigma = 1$: nearly-Kähler. Lines indicated with 2 have multiplicity 2.

(b) $\rho = 1$ and $\sigma = \frac{2}{5}$.

(c) $\rho = 1$ and $\sigma = 2$.

(d) $\rho = \frac{5}{2}$ and $\sigma = \frac{1}{2}$.

Figure 4: Mass spectrum of $\mathbb{SU}(3)/\mathbb{U}(1) \times \mathbb{U}(1)$.

Figure 5: Mass spectrum of $\mathbb{SU}(3)/\mathbb{U}(1) \times \mathbb{U}(1)$ for negative $\sigma$ and $\rho$. 

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6.4 IIA on SU(2)×SU(2)

The expansion forms are given by

\[ Y_{1}^{2-} = ae^{14}, \quad Y_{2}^{2-} = be^{25}, \quad Y_{3}^{2-} = ce^{36}, \]
\[ Y_{1}^{3+} = e^{x_{1}^{2}y^{3}} = \frac{-h}{4c_{1}(a + b + c)}(e^{(123 + e^{456} + e^{126} + e^{345} + e^{315} + e^{264} + e^{156} + e^{234})}, \]
\[ Y_{2}^{3+} = e^{x_{1}^{3}y^{2}x^{3}} = \frac{h}{4c_{1}(-a + b + c)}(e^{(123 + e^{456} - e^{126} - e^{345} - e^{315} - e^{264} + e^{156} + e^{234})}, \]
\[ Y_{3}^{3+} = e^{y^{3}x^{2}x^{3}} = \frac{-h}{4c_{1}(a - b + c)}(-e^{123} - e^{456} + e^{126} + e^{345} - e^{315} - e^{264} + e^{156} + e^{234}), \]
\[ Y_{4}^{3+} = -e^{y^{3}y^{2}x^{3}} = \frac{h}{4c_{1}(a + b - c)}(e^{123} + e^{456} + e^{126} + e^{345} - e^{315} - e^{264} - e^{156} - e^{234}), \]

and the standard volume \( V_s = -\int_{M} abc e^{1-6} \). One finds for the superpotential:

\[
\mathcal{W} = \frac{ie^{-i\theta} e^{-\Phi}}{4\kappa_{10}^2} V_{s} a^{-1/2} \left\{ \frac{3}{2} \tilde{c}_{1} + i \tilde{m} \left( t^{1} t^{2} t^{3} - \frac{3}{5} (t^{1} + t^{2} + t^{3}) - \frac{2}{5} (z^{1} + z^{2} + z^{3} + z^{4}) \right) \\
+ \frac{3}{2} \tilde{c}_{1} (t^{1} t^{2} + t^{2} t^{3} + t^{1} t^{3}) \\
+ \frac{c_{1}}{h^{2}} \left[ 4 \left[ t^{1} t^{2} (1 - \rho^{2} - \sigma^{2}) + t^{1} t^{3} \rho^{2} (1 - \rho^{2} - \sigma^{2}) + t^{1} t^{2} \sigma^{2} (1 - \rho^{2} + \sigma^{2}) \right] \\
+ \left[ t^{1} (1 + \rho^{2} + \sigma^{2}) + t^{2} \rho^{2} (1 + \rho^{2} + \sigma^{2}) + t^{2} \sigma^{2} (1 + \rho^{2} - \sigma^{2}) \right] (z^{1} + z^{2} + z^{3} + z^{4}) \\
+ \rho \sigma \left[ -2t^{1} t^{2} (1 + \rho^{2} - \sigma^{2}) + t^{1} (1 + \rho^{2} + \sigma^{2}) \right] (z^{1} + z^{2} - z^{3} - z^{4}) \\
+ \sigma \left[ t^{1} (1 + \rho^{2} - \sigma^{2}) - 2\rho^{2} t^{2} + t^{3} (1 + \rho^{2} + \sigma^{2}) \right] (z^{1} - z^{2} + z^{3} - z^{4}) \\
+ \rho \left[ t^{1} (1 - \rho^{2} + \sigma^{2}) + t^{2} (-1 + \rho^{2} + \sigma^{2}) - 2\sigma^{2} t^{3} \right] (z^{1} - z^{2} - z^{3} + z^{4}) \right]\right\}. \]

(6.11)

The Kähler potential reads:

\[
\mathcal{K} = -\ln \left( \prod_{i=1}^{3} (t^{i} + \tilde{t}^{i}) \right) - \ln \left( 4 \prod_{i=1}^{4} (z^{i} + \tilde{z}^{i}) \right) + 3 \ln(8\kappa_{10}^{2} M_{P}^{2} V_{s}^{-1} e^{4\Phi/3}) . \]

(6.12)

There are again two shape parameters \( \rho = b/a \) and \( \sigma = c/a \) and the same symmetries \( \rho \leftrightarrow \sigma, (\rho, \sigma, \tilde{\mu}) \leftrightarrow (\rho/\sigma, 1/\sigma, \tilde{\mu}) \) as in the previous model. In Figure 6 we display the mass spectrum for some values of the parameters. This time there will always be one unstabilized massless axion (\( \tilde{M}^{2} = 0 \)) and a corresponding tachyonic complex structure modulus with \( \tilde{M}^{2}/|W|^{2} = -2 \).

In the limit \( W_{1}^{-} \to 0, W_{2}^{-} \) blows up just as the lower bound for \( \tilde{\mu} \). So in principle we could decouple the Kaluza-Klein modes this way, however it is quite difficult to study this singular limit.
6.5 IIA on $\frac{SU(3) \times U(1)}{SU(2)}$

We display the general results here and comment on the special case $5c_1^2 - 4e^{26}m^2 = 0$ in appendix F. We choose the expansion forms in (5.19) as follows:

\begin{align*}
Y^{(2-)} : & \ -a[(e^{13} - e^{24}) - \rho(e^{14} + e^{23})], \ a e^{56}; \\
Y^{(3+)} : & \ a^{3/2}[(e^{13} - e^{24}) + \rho^{-1}(e^{14} + e^{23})] \wedge e^6, \ a^{3/2}(e^{125} + e^{345}),
\end{align*}

and the standard volume $V_s = \int a^3(1 + \rho^2)e^{123456}$. The superpotential and Kähler potential read:

\begin{align*}
W_E & = -\frac{ie^{-i\theta} e^{-\Phi}}{4\kappa^2_{10}}V_s a^{-1/2} \left( \tilde{f}\sigma + \frac{3i\tilde{m}}{5}\sigma(2t^1 + \frac{1}{\sigma}) \right. \\
& \quad + \sqrt{\frac{3}{2}}(1 + \rho^2)^{-\frac{1}{4}} \left( -t^1 t^2 + \frac{\sigma}{2}(t^1)^2 \right) - i\tilde{m}(t^1)^2 t^2 \\
& \quad - \frac{4\sqrt{2i\tilde{m}}}{5\rho}(1 + \rho^2)^{\frac{1}{4}}z^1 + \frac{2\sqrt{2i\tilde{m}}}{5\sigma}(1 + \rho^2)^{-\frac{3}{4}}z^2 + \frac{2\sqrt{3}}{\rho}z^1 t^1 - \sqrt{3}(1 + \rho^2)^{-1}t^2 z^2 \right), \tag{6.14}
\end{align*}

and

\begin{align*}
K & = -\ln \left( (t^1 + i\tilde{t}^1)(t^2 + \tilde{t}^2) \right) - \ln \left( \frac{1}{\rho^2(1 + \rho^2)}(z^1 + \tilde{z}^1)^2(z^2 + \tilde{z}^2)^2 \right) \\
& \quad + 3\ln(8\kappa^2_{10}M^2_P V_s e^{4\overline{\Phi}/3}). \tag{6.15}
\end{align*}

This model has two shape parameters $\rho = b/a$ and $\sigma = c/a$, and a symmetry under $(\rho, \sigma, \tilde{\mu}) \leftrightarrow (1/\rho, \sigma/\rho, \rho\tilde{\mu})$. In Figure 7, we show the mass spectrum for some values of the parameters. The mass spectrum at $\mu = 0$ turns out to be independent of the parameters $\rho, \sigma$. There always seem to be two negative $\tilde{M}^2$ eigenvalues.
7. Application to inflation in type IIA

In the previous two sections, we have derived the low energy effective actions for the AdS$_4$ compactifications studied in this paper. While an extensive analysis of the physical properties of these low energy effective actions is beyond the scope of this paper, we would nevertheless like to take a first look at our models in the context of the recent interesting work [14]. Extending the earlier work [13], the authors of [14] proved a no-go theorem against a period of slow-roll inflation in type IIA compactifications on Calabi-Yau manifolds with standard RR and NSNS-fluxes, D6-branes and O6-planes at large volume and with small string coupling. More precisely, they show that the slow-roll parameter $\epsilon$ is at least $\frac{27}{13}$ whenever the potential is positive, ruling out slow-roll inflation in a near-de Sitter regime, as well as meta-stable dS vacua. As emphasized in [14], however, the inclusion of other ingredients such as NS5-branes, geometric fluxes and/or non-geometric fluxes evade the assumptions that underly this no-go theorem\textsuperscript{15}. Our coset models could thus be candidates for circumventing the no-go theorem as they all have geometric fluxes. So let us study this in some more detail.

The proof of this no-go theorem is remarkably simple and uses only the scaling properties of the scalar potential with respect to the volume modulus

$$\rho = \left( \frac{\text{Vol}}{V_s} \right)^{1/3},$$

(7.1)

where $V_s = | \int e^{123456} |$ is a standard volume, and the dilaton modulus

$$\tau = e^{-\Phi} \sqrt{\text{Vol}},$$

(7.2)

as well as the signs of the various contributions to the potential. Concretely, if one denotes by $V_3$, $V_p$, $V_{D6}$ and $V_{O6}$ the potential contributions due to, respectively, $H_3$-flux, $F_p$-flux,

\textsuperscript{15}In [15] de Sitter vacua in type IIA were found using some of these additional ingredients. Furthermore a concrete string inflationary model on nilmanifolds with D4-branes was presented in [16].
D6-branes and O6-planes, the full potential has the schematic form

\[ V = V_3 + \sum_p V_p + V_{D6} + V_{O6} \]

\[ = \frac{A_3(\phi_i)}{\rho^3 \tau^2} + \sum_p \frac{A_p(\phi_i)}{\rho^{p-3} \tau^4} + \frac{A_{D6}(\phi_i)}{\tau^3} - \frac{A_{O6}(\phi_i)}{\tau^3} \]  

(7.3)

with positive coefficients, \( A_i \), that depend on the other moduli, \( \phi_i \). One can check that our potentials have indeed this behaviour. This implies

\[ -\rho \frac{\partial V}{\partial \rho} - 3\tau \frac{\partial V}{\partial \tau} = 9V + \sum_p pV_p \geq 9V. \]  

(7.4)

From this inequality and using

\[ \epsilon \equiv \frac{K^0V_3V_3}{V^2} \geq \frac{M_p^2}{2} \left[ \left( \frac{\partial \ln V}{\partial \rho} \right)^2 + \left( \frac{\partial \ln V}{\partial \tau} \right)^2 \right], \]  

(7.5)

where the hatted fields are the canonically normalized volume modulus and dilaton, one derives the bound [14]

\[ \epsilon \geq \frac{27}{13} \quad \text{whenever } V > 0. \]  

(7.6)

This forbids slow-roll inflation everywhere in moduli space. Moreover, for a vacuum, the right-hand side of eq. (7.4) should vanish so that de Sitter vacua are ruled out (as well as Minkowski vacua whenever \( V_p > 0 \) for at least one of \( p = 2, 4, 6 \)).

In our compactifications, this no-go theorem no longer needs to hold, because some of these compactifications have geometric fluxes with schematic potentials

\[ V_f \propto \pm \rho^{-1} \tau^{-2}. \]  

(7.7)

Such a contribution to the potential would weaken (7.4) to

\[ -\rho \frac{\partial V}{\partial \rho} - 3\tau \frac{\partial V}{\partial \tau} = 9V + \sum_p pV_p - 2V_f. \]  

(7.8)

If \( V_f \) turns out to be negative, the above expression would still be at least \( 9V \) just as before, and the no-go theorem expressed in the form (7.6) would still hold. Thus, if geometric fluxes alone are to circumvent this no-go theorem, they can do so at most if they are positive:

\[ V_f > 0 \quad \text{(Necessary condition for evading the no-go theorem).} \]  

(7.9)

In fact, we can immediately find the geometric part of the potential from the Einstein-Hilbert term in the ten-dimensional action:

\[ V_f = -\frac{1}{2} M_p^4 \kappa_{10}^2 e^{2\Phi} \text{Vol}^{-1} R = -\frac{1}{2} M_p^4 \kappa_{10}^2 \tau^{-2} R, \]  

(7.10)

where \( R \) is the scalar curvature of the internal manifold. For cosets/group manifolds \( R \) can be calculated from (4.7). This expression has indeed the expected scaling behaviour since
\( R \propto g^{-1} \propto \rho^{-1} \). It follows that the condition (7.9) for avoiding the no-go theorem can be rephrased as

\[ R < 0. \quad (7.11) \]

Let us display the scalar curvature for some of our coset models:

\[
\begin{align*}
\frac{G_2}{SU(3)} : \quad R &= \frac{10}{k_1}, \\
\frac{Sp(2)}{SU(2) \times U(1)} : \quad R &= \frac{6}{k_1} + \frac{2}{k_2} - \frac{k_2}{2(k_1)^2}, \\
\frac{SU(3)}{U(1) \times U(1)} : \quad R &= 3 \left( \frac{1}{k_1} \right) + \frac{1}{k_2} + \frac{1}{k_3} - \frac{1}{2} \left( \frac{k_1}{k_2k_3} + \frac{k_2}{k_1k_3} + \frac{k_3}{k_1k_2} \right), \\
\frac{SU(3) \times U(1)}{SU(2)} : \quad R &= \frac{1}{\sqrt{1 + \rho^2}} \left( \frac{6}{k_1} - \frac{3\rho k_2}{4(1 + \rho^2)k_1^2} \frac{u_2}{u_1} \right),
\end{align*}
\]

where \( k_i > 0 \) are the Kähler moduli and \( u_i \) the complex structure moduli that enter the expansion of \( J \) and \( \text{Im} \Omega \) in the basis (6.1), (6.4), (6.7) and (6.13), respectively (where we put \( a = 1 \)). We see that for \( \frac{G_2}{SU(3)} \) the curvature is always positive, so inflation is still excluded, however for the other models there are values of the moduli such that \( R < 0 \).

For \( SU(2) \times SU(2) \) we did not display the curvature, because taking generic values of the complex structure and Kähler moduli, its expression is quite complicated and not very enlightening. However, also in that case it is possible to choose the moduli such that \( R < 0 \).

Note that this does not yet guarantee that the \( \epsilon \) parameter is indeed small, it just says that the theorem that requires it to be at least \( 27/13 \) no longer applies. Hence, a logical next step would be to calculate \( \epsilon \) in this region, ideally by taking also all other moduli into account (see the general expression (7.5)) and try to make \( \epsilon \) small or zero. These would be necessary conditions for, respectively, inflation or de Sitter vacua. They are not sufficient however, because for inflation, we would also need the \( \eta \) parameter to be small and further obtain a satisfactory inflationary model which could end in a meta-stable de Sitter vacuum. For a meta-stable de Sitter vacuum, on the other hand, one would also have to check that the matrix of second derivatives only has negative eigenvalues.\(^{16}\)

We remark that (7.4) also played a rôle in the (failed) F-term uplifting attempts by [12], so one might also reconsider that in the present context.

For our coset models, we have completely explicit expressions for the low energy effective theory so we have the necessary tools to address all these questions and we hope to come back to them in future work.

\(^{16}\)The eta-parameter, or, more generally, the matrix of second derivatives of the potential, played the key role in the recent works [57] (see also the older paper [58]), in which difficulties for inflation or meta-stable de Sitter vacua in various string inspired supergravity potentials are discussed. Note that the no-go theorem of [14] has no direct connection to these works, as it makes the much stronger statement that a small epsilon parameter or a de Sitter critical point of the potential do not exist, whereas this existence was assumed in [57].
8. Conclusions

In this paper, we studied a number of type IIA SU(3)-structure compactifications on nilmanifolds and cosets, which are tractable enough to allow for an explicit derivation of the low energy effective theory. In particular, we calculated the mass spectrum of the light scalar modes, using $\mathcal{N} = 1$ supergravity techniques. For the torus and the Iwasawa solution, we have also performed an explicit Kaluza-Klein reduction, which led to the same result, supporting the validity of the effective supergravity approach, with superpotential (5.13) and Kähler potential (5.15), also in the presence of geometric fluxes. Furthermore we have demonstrated that this superpotential and Kähler potential lead to sensible results in type IIB string theory with static SU(2)-structure as well. For the nilmanifold examples we have found that there are always three unstabilized moduli corresponding to axions in the RR sector. On the other hand, in the coset models, except for $SU(2) \times SU(2)$, all moduli are stabilized.

It would be interesting to study the uplifting of these models to de Sitter space-times. This might be accomplished by incorporating a suitable additional uplifting term in the potential along the lines of, e.g., [5]. Although a negative mass squared for a light field in AdS does not necessarily signal an instability, after the uplift all fields should have positive mass squared. Unless the uplifting potential can change the sign of the squared masses, it is thus desirable that they are all positive even before the uplifting. We find that this can be arranged in the coset models $G_2/SU(3)$, $Sp(2)/SU(2) \times U(1)$, and $SU(3)/U(1) \times U(1)$ for suitable values of the orientifold charge.

An alternative approach towards obtaining meta-stable de Sitter vacua could also be to search for non-trivial de Sitter minima in the original flux potential away from the AdS vacuum. In such a case, one would have to re-investigate the spectrum of the light fields and the issue of the Kaluza-Klein decoupling.

We discussed this Kaluza-Klein decoupling in section 2.2 for the original AdS vacua and found that it requires going to the nearly-Calabi Yau limit. For our nilmanifolds, this can be easily arranged by tuning the parameters, while for our coset models it is somewhat harder. Indeed, we found that for $Sp(2)/SU(2) \times U(1)$ and $SU(3)/U(1) \times U(1)$ one has to make a continuation to negative values of the “shape” parameters. Strictly speaking, this can no longer be described as a left-invariant SU(3) structure on a coset anymore, but it can still be described in terms of a twistor bundle over a four-dimensional hyperbolic space. It would be interesting to study these models in more detail, as there are more examples of this type. Another class of vacua may be obtained by quotienting out the internal manifold by a discrete group $\Gamma$, where $\Gamma$ is a subgroup of SU(3). This possibility may be of interest for model-building.

Another promising avenue would be to include space-time filling D-branes supporting the matter and gauge structure of the Standard Model. A lot is already known on model building with intersecting D6-branes, see, e.g., [59, 3] for reviews and many references. In our models it is indeed possible to insert D6-branes that do not break the supersymmetry by having them wrap special Lagrangian cycles: for a discussion in the context of AdS$_4$ compactifications see [60]. We further remark that a superpotential for D-brane moduli,
which fits nicely together with the superpotential (5.13), was given in [61], but it is not complete in that it does not describe the charged fields coming from open strings ending on different D-branes, which are of course exactly the important ones in reproducing the Standard Model.

We have also discussed how geometric fluxes leading to negative scalar curvature circumvent the assumptions that underlie the no-go theorem [14] against modular inflation in type IIA string theory. We found that we can arrange for negative curvature in all the coset models except for \( \frac{G_2}{SU(3)} \). Circumventing this no-go theorem is just a first step towards a successful inflationary model. It would certainly be worthwhile to study this possibility in more detail, perhaps including extra ingredients such as NS5-branes and other types of branes [15, 16].

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A. Type II supergravity

The bosonic content of type II supergravity consists of a metric \( g \), a dilaton \( \Phi \), an NSNS 3-form \( H \) and RR-fields \( F_n \). In the democratic formalism of [44], where the number of RR-fields is doubled, \( n \) runs over 0, 2, 4, 6, 8, 10 in IIA and over 1, 3, 5, 7, 9 in type IIB. We write \( n \) to denote the dimension of the RR-fields; for example \((-1)^n\) stands for +1 in type IIA and \(-1\) in type IIB. After deriving the equations of motion from the action, the redundant RR-fields are to be removed by hand by means of the duality condition:

\[
F_n = (-1)^{\frac{(n-1)(n-2)}{2}} e^{\frac{n+2}{2}\Phi} \star_{10} F_{(10-n)} ,
\]

(A.1)

given here in the Einstein frame. We will often collectively denote the RR-fields, and the corresponding potentials, with polyforms \( F = \sum_n F_n \) and \( C = \sum_n C_{(n-1)} \), so that:

\[
F = d_H C.
\]

In the Einstein frame, the bosonic part of the bulk action reads:

\[
S_{\text{bulk}} = \frac{1}{2\kappa_10^2} \int d^{10}x \sqrt{-g} \left[ R - \frac{1}{2} (\partial \Phi)^2 - \frac{1}{2} e^{-\Phi} H^2 - \frac{1}{4} \sum_n e^{\frac{5+n}{2}\Phi} F_n^2 \right] ,
\]

(A.2)

where for an \( l \)-form \( A \) we define

\[
A^2 = A \cdot A = \frac{1}{l!} A_{M_1...M_l} A_{N_1...N_l} g^{M_1 N_1} \cdots g^{M_l N_l} .
\]

(A.3)

Since (A.1) needs to be imposed by hand this is strictly-speaking only a pseudoaction. Note that the doubling of the RR-fields leads to factors of \( 1/4 \) in their kinetic terms.
The contribution from the calibrated (supersymmetric) sources can be written as:

\[ S_{\text{source}} = \int \langle C, j \rangle - \sum_n e^{\frac{2}{3} \Phi} \int \langle \Psi_n, j \rangle, \tag{A.4} \]

with

\[ \Psi_n = e^A dt \wedge \frac{e^{-\Phi}}{(n-1)!} \epsilon_1^T \gamma_{M_1} \ldots \gamma_{M_{n-1}} \epsilon_2 dX^{M_1} \wedge \ldots \wedge dX^{M_{n-1}}, \tag{A.5} \]

with \( \epsilon_1, \epsilon_2 \) nine-dimensional internal supersymmetry generators. For space-filling sources in compactifications to AdS\(_4\) this becomes [60]

\[ \Psi_n = \text{vol}_4 \wedge e^{4A-\Phi} \text{Im} \Psi_{1E}|_{n=4}, \tag{A.6} \]

with \( \Psi_{1E} \) the pure spinor \( \Psi_1 \) in the Einstein frame.

The dilaton equation of motion and the Einstein equation read

\[
0 = \nabla^2 \Phi + \frac{1}{2} e^{-\Phi} H^2 - \frac{1}{8} \sum_n (5 - n) e^{\frac{2}{3} \Phi} F_n^2 + \frac{\kappa_{10}^2}{2} \sum_n (n - 4) e^{\frac{2}{3} \Phi} \epsilon_n \wedge \epsilon_n \langle \Psi_n, j \rangle, \tag{A.7a} \\
0 = R_{MN} + g_{MN} \left( \frac{1}{8} e^{-\Phi} H^2 + \frac{1}{32} \sum_n (n - 1) e^{\frac{5}{2} - \frac{n}{2} \Phi} F_n^2 \right) - \frac{1}{2} \partial_M \Phi \partial_N \Phi - \frac{1}{2} e^{-\Phi} H_M \cdot H_N - \frac{1}{4} \sum_n e^{\frac{5}{2} - \frac{n}{2} \Phi} F_{nM} \cdot F_{nN} - 2 \kappa_{10}^2 \sum_n e^{\frac{2}{3} \Phi} \epsilon_n \wedge \epsilon_n \left( -\frac{1}{16} n g_{MN} + \frac{1}{2} g_{P(M} d\xi^P \otimes \epsilon_n \right) \Psi_n, j, \right), \tag{A.7b}
\]

where we defined for an \( l \)-form \( A \)

\[
A_M \cdot A_N = \frac{1}{(l-1)!} A_{MM_2 \ldots M_l} A_{NN_2 \ldots N_l} g^{M_2 N_2} \ldots g^{M_l N_l}. \tag{A.8} 
\]

The Bianchi identities and the equations of motion for the RR-fields, including the contribution from the ‘Chern-Simons’ terms of the sources, take the form

\[
0 = dF + H \wedge F + 2 \kappa_{10}^2 j, \tag{A.9a} \\
0 = d \left( e^{\frac{5}{2} - \frac{n}{2} \Phi} \epsilon_n \wedge F_n \right) - e^{\frac{5}{2} - \frac{n}{2} \Phi} H \wedge \epsilon_n \wedge F_{(n+2)} - 2 \kappa_{10}^2 \alpha(j). \tag{A.9b}
\]

Finally, for the equation of motion for \( H \) we have:

\[
0 = d(e^{-\Phi} \epsilon_n \wedge H) - \frac{1}{2} \sum_n e^{\frac{5}{2} - \frac{n}{2} \Phi} \epsilon_n \wedge F_n \wedge F_{(n-2)} + 2 \kappa_{10}^2 \sum_n e^{\frac{2}{3} \Phi} \epsilon_n \wedge \epsilon_n \wedge \epsilon_n \wedge \epsilon_n (j) \bigg|_8. \tag{A.10}
\]

In the above equations we can redefine \( j \) in order to absorb the factor of \( 2 \kappa_{10}^2 \),

\[
(2 \kappa_{10}^2) j \rightarrow j, \tag{A.11}
\]

which we do in this paper.

The equations of motion resulting from \( S_{\text{bulk}} + S_{\text{source}} \) were given in this form (in the string frame) in [45], where it was shown that, under certain mild assumptions, imposing the supersymmetry equations together with the Bianchi identities for the forms, is enough to guarantee that the dilaton and Einstein equations are also satisfied.
B. Structure groups

In this paper we have assumed the following \( \mathcal{N} = 1 \) compactification ansatz for the ten-dimensional supersymmetry generators [28]

\[
\begin{align*}
\epsilon_1 &= \zeta_+ \otimes \eta_+^{(1)} + \zeta_- \otimes \eta_-^{(1)}, \\
\epsilon_2 &= \zeta_+ \otimes \eta_+^{(2)} + \zeta_- \otimes \eta_-^{(2)},
\end{align*}
\]

(B.1)

for IIA/IIB, where \( \zeta_\pm \) are four-dimensional and \( \eta_\pm^{(1,2)} \) six-dimensional Weyl spinors. The Majorana conditions for \( \epsilon_{1,2} \) imply the four- and six-dimensional reality conditions \( (\zeta_+)^* = \zeta_- \) and \( (\eta_\pm^{(1,2)})^* = \eta_\mp^{(1,2)} \). This reduces the structure of the generalized tangent bundle to \( SU(3) \times SU(3) \) [62]. The structure of the tangent bundle itself on the other hand is a subgroup of \( SU(3) \) since there is at least one invariant internal spinor. What subgroup exactly depends on the relation between \( \eta^{(1)} \) and \( \eta^{(2)} \). Combining the terminology of [28] and [63] the following classification can be made:

- strict \( SU(3) \)-structure: \( \eta^{(1)} \) and \( \eta^{(2)} \) are parallel everywhere;
- static \( SU(2) \)-structure: \( \eta^{(1)} \) and \( \eta^{(2)} \) are orthogonal everywhere;
- intermediate \( SU(2) \)-structure: \( \eta^{(1)} \) and \( \eta^{(2)} \) at a fixed angle, but neither a zero angle nor a right angle;
- dynamic \( SU(3) \times SU(3) \)-structure: the angle between \( \eta^{(1)} \) and \( \eta^{(2)} \) varies, possibly becoming a zero angle or a right angle at a special locus.

Since for static and intermediate \( SU(2) \)-structure there are two independent internal spinors the structure of the tangent bundle reduces to \( SU(2) \), while for dynamic \( SU(3) \times SU(3) \)-structure no extra constraints beyond \( SU(3) \) are imposed on the topology of the tangent bundle since the two internal spinors \( \eta^{(1)} \) and \( \eta^{(2)} \) might not be everywhere independent.

In [32] it was realized that in type IIB strict \( SU(3) \) compactifications to \( AdS_4 \) are impossible\(^{17}\). We will review the argument in section B.2, while we will also show that, conversely, in type IIA static \( SU(2) \)-compactifications are impossible (which was previously noted in [64]). But in fact we will show more: intermediate \( SU(2) \)-structure \( AdS_4 \) vacua with left-invariant pure spinors are impossible in both type IIA and type IIB. The way out of this no-go theorem is that in type IIA we must allow \( e^{2A - \Phi} \eta_+^{(2)} \eta_-^{(1)} \) to vary along the internal manifold, while in type IIB we need a genuine dynamic \( SU(3) \times SU(3) \)-structure that changes type to static \( SU(2) \) on a non-zero locus. So the most interesting but also the most complicated case, the dynamic \( SU(3) \times SU(3) \)-structure is still possible, but we will leave this to further work. Note that in [45, 63] examples of constant intermediate \( SU(2) \)-structure on \( Minkowski \) compactifications were provided.

In this paper we will mostly focus on the effective theory around strict \( SU(3) \) vacua in type IIA and also give one example of a T-dual static \( SU(2) \) vacuum in type IIB. So let us discuss the conventions for these cases in some more detail in the next subsections.

\(^{17}\)That is at a pure classical level. Taking non-perturbative corrections into account the authors of [5] indeed constructed an \( AdS_4 \) vacuum with \( SU(3) \)-structure. See also [32] for a discussion.
B.1 SU(3)-structure

A real non-degenerate two-form $J$ and a complex decomposable three-form $\Omega$ completely specify an SU(3)-structure on the six-dimensional manifold $M$ iff:

\begin{align*}
\Omega \wedge J &= 0, \\
\Omega \wedge \Omega^* &= \frac{4i}{3} J^3 \neq 0,
\end{align*}

and the associated metric (B.15) is positive definite. Up to a choice of orientation, the volume normalization can be taken such that

\begin{equation}
\frac{1}{6} J^3 = -\frac{i}{8} \Omega \wedge \Omega^* = \text{vol}_6.
\end{equation}

When the internal supersymmetry generators of (B.1) are proportional,

\begin{equation}
\eta^{(2)}_+ = (b/a) \eta^{(1)}_+,
\end{equation}

with $|\eta^{(1)}|^2 = |a|^2$, $|\eta^{(2)}|^2 = |b|^2$, they define an SU(3)-structure as follows. First let us define a normalized spinor $\eta_+$ such that $\eta^{(1)}_+ = a \eta_+$ and $\eta^{(2)}_+ = b \eta_+$ and moreover we choose the phase of $\eta$ such that $a = b^*$. Note that in compactifications to AdS$_4$ the supersymmetry imposes $|a|^2 = |b|^2$ such that $b/a = e^{i\theta}$ is just a phase. Now we can construct $J$ and $\Omega$ as follows

\begin{equation}
J_{mn} = i \eta^+_+ \gamma_{mn} \eta^+_+, \quad \Omega_{mnp} = \eta^+_+ \gamma_{mnp} \eta^+_+.
\end{equation}

The intrinsic torsion of $M$ decomposes into five modules (torsion classes) $W_1, \ldots, W_5$. These also appear in the SU(3) decomposition of the exterior derivative of $J$, $\Omega$. Intuitively, this is because the intrinsic torsion parameterizes the failure of the manifold to be of special holonomy, which can also be thought of as the deviation from closure of $J$, $\Omega$. More specifically we have:

\begin{align*}
\text{d}J &= \frac{3}{2} \text{Im}(W_1 \Omega^*) + W_4 \wedge J + W_3, \\
\text{d}\Omega &= W_1 J \wedge J + W_2 \wedge J + W_5^* \wedge \Omega,
\end{align*}

where $W_1$ is a scalar, $W_2$ is a primitive (1,1)-form, $W_3$ is a real primitive (1,2) + (2,1)-form, $W_4$ is a real one-form and $W_5$ a complex (1,0)-form. For the vacua of interest to us only the classes $W_1$, $W_2$ are non-vanishing and they are purely imaginary, which we will indicate with a minus superscript. Indeed, we can readily see that eq. (2.5) follows from eq. (B.6) above, upon setting $W_{3,4,5}$ to zero and imposing $W_{1,2} = W_{1,2}^- = i \text{Im} W_{1,2}^-$. Note that by definition $W_2$ is primitive, which means

\begin{equation}
W_2 \wedge J \wedge J = 0.
\end{equation}

One interesting property of a primitive (1,1)-form is

\begin{equation}
* (W_2 \wedge J) = -W_2,
\end{equation}

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which can be shown using $J_{mn} \mathcal{W}_{2mn} = 0$ (which follows from the primitivity) and $J_m^a J_p^a \mathcal{W}_{mnp} = \mathcal{W}_{mnp}$ (which follows from the fact that $\mathcal{W}_2$ is of type (1,1)).

Let us now calculate the part of $d\mathcal{W}_2^-$ proportional to $\Re \Omega$:

$$d\mathcal{W}_2^- = \alpha \Re \Omega + (2, 1) + (1, 2),$$

for some $\alpha$. Taking the exterior derivative of $\Omega \wedge \mathcal{W}_2^- = 0$ and using (B.9) as well as the eqs. (B.2b), (B.6), we arrive at:

$$\mathcal{W}_2^- \wedge \mathcal{W}_2^- \wedge J = \frac{2i}{3} \alpha J^3.$$

We can now use (B.8) to show

$$\mathcal{W}_2^- \wedge \mathcal{W}_2^- \wedge J = 2|\mathcal{W}_2^-|^2 \text{vol}_6,$$

from which we obtain $\alpha = -i|\mathcal{W}_2^-|^2/8$, in accordance with (2.8).

From the SU(3)-structure (B.2b), we can read off the metric as follows [65]. From $\Re \Omega$ alone we can construct an almost complex structure. First we define

$$\tilde{T}_k = -\varepsilon^{lm_1...m_5}(\Re \Omega)_{km_1m_2}(\Re \Omega)_{m_3m_4m_5},$$

where $\varepsilon^{m_1...m_6} = \pm 1$ is the totally antisymmetric symbol in six dimensions, and then properly normalize it

$$\mathcal{I} = \frac{\tilde{T}}{\sqrt{-\text{tr} \frac{1}{6} \tilde{T}^2}},$$

so that $\mathcal{I}^2 = -1$. Note that

$$H(\Re \Omega) = \text{tr} \frac{1}{6} \tilde{T}^2$$

is called the Hitchin functional. The metric can then be constructed from $\mathcal{I}$ and $J$ via:

$$g_{mn} = \mathcal{I}_m \mathcal{I}_n.$$

Finally, let us mention some useful formulæ for $J$ and $\Omega$ as defined in (B.5)

$$\gamma_m \eta_- = -i J_m \gamma_n \eta_-,$$

$$\gamma_{mn} \eta_+ = -i J_{mn} \eta_+ + \frac{1}{2} \Omega_{mnp} \gamma^p \eta_-,$$

$$\gamma_{mnp} \eta_- = 3i J_{[mn} \gamma_{p]} \eta_- - \Omega^*_{mnp} \eta_+.$$  

**B.2 SU(3)×SU(3)-structure and static SU(2)-structure**

It turns out that in order to study static SU(2)-structure, it is most convenient to use the generalized geometry formalism. The supersymmetry generators $\eta^{(1)}$ and $\eta^{(2)}$ from (B.1) are then collected into two spinor bilinears, which using the Clifford map, can be associated with two polyforms of definite degree

$$\Psi_+ = \frac{8}{|a||b|} \eta_+^{(1)} \otimes \eta_+^{(2)}, \quad \Psi_- = \frac{8}{|a||b|} \eta_-^{(1)} \otimes \eta_-^{(2)}.$$  

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It can be shown that these are associated to pure spinors of \( SO(6,6) \) and that they satisfy the normalization
\[
\langle \Psi_+, \Psi^*_+ \rangle = \langle \Psi_-, \Psi^*_- \rangle \neq 0 ,
\]
with the Mukai pairing \( \langle \cdot, \cdot \rangle \) given by
\[
\langle \phi_1, \phi_2 \rangle = \phi_1 \wedge \alpha(\phi_2)\big|^{\text{top}} .
\]
The operator \( \alpha \) acts by inverting the order of indices on forms. The Mukai pairing has the following useful property:
\[
\langle e^b \phi_1, e^b \phi_2 \rangle = \langle \phi_1, \phi_2 \rangle ,
\]
for an arbitrary two-form \( b \). Since there are two compatible invariant pure spinors the structure of the generalized tangent bundle is reduced to \( SU(3) \times SU(3) \). In order to obtain similar equations in IIA and IIB one redefines
\[
\Psi_1 = \Psi_+ , \quad \Psi_2 = \Psi_- ,
\]
with upper/lower sign for IIA/IIB. We collect all the RR-fields of the democratic formalism into one polyform and make the following compactification ansatz
\[
F = \hat{F} + \text{vol}_4 \wedge \tilde{F} ,
\]
with \( \text{vol}_4 \) the four-dimensional (AdS_4) volume form. In fact, in this paper we will drop the hat and hope that it is clear from the context whether we mean the full \( F \) or only the internal part.

With these definitions the supersymmetry conditions (in string frame) take the following concise form in both IIB and IIA [28]
\[
d_H \left( e^{A-\Phi} \text{Im} \Psi_1 \right) = 3 e^{3A-\Phi} \text{Im} (W^* \Psi_2) + e^{4A} \hat{F} ,
\]
\[
d_H \left[ e^{3A-\Phi} \text{Re} (W^* \Psi_2) \right] = 2 |W|^2 e^{2A-\Phi} \text{Re} \Psi_1 ,
\]
\[
d_H \left[ e^{3A-\Phi} \text{Im} (W^* \Psi_2) \right] = 0 ,
\]
with \( |a|^2 = |b|^2 \propto e^A \). From the above, the equations of motion for \( F \) follow as integrability conditions, as well as the following equation:
\[
d_H \left( e^{2A-\Phi} \text{Re} \Psi_1 \right) = 0 .
\]
Here \( W \) is defined in terms of the AdS Killing spinors
\[
\nabla_\mu \zeta_- = \pm \frac{1}{2} W_{\mu} \zeta_+ ,
\]
for IIA/IIB.

These equations should be supplemented with the Bianchi identities for the RR-fluxes (A.9a) where the (localized or smeared) sources \( j \) have to be calibrated
\[
\langle \text{Re} \Psi_1, j \rangle = 0 , \quad \langle \Psi_2, X \cdot j \rangle = 0 , \quad \forall X \in \Gamma(T_M \oplus T^*_M) .
\]
Analogously to the SU(3)-case, an easy way to solve these calibration conditions is to choose
\[ j = -k \Re \Psi_1, \] (B.27)
for some function \( k \), which is positive for net D-brane charge and negative for net orientifold charge. Applying an exterior derivative on (B.23a), taking (B.23b), (A.9a), (B.22) into account, it can be shown that
\[ \pm d_H \{ \alpha [ \ast d_H (e^{3A-\Phi} \Im \Psi_1)] \} = -e^{4A}j - 6|W|^2 e^{A-\Phi} \Re \Psi_1, \] (B.28)
for IIA/IIB.

With the SU(3)-structure ansatz (B.4) we get from (B.17)
\[ \Psi_- = -\Omega, \quad \Psi_+ = e^{-i\theta} e^{iJ}, \] (B.29)
where \( J \) and \( \Omega \) are defined in (B.5). For IIA we arrive at (2.1) and (2.6) after plugging (B.21) into (B.23). For IIB on the other hand, where the above definitions of \( \Psi_1 \) and \( \Psi_2 \) are switched, it is immediately obvious from (B.23b) that there is no SU(3)-structure solution possible. Indeed, the left-hand side is a four-form, which would put the zero- and two-form part of the right-hand side to zero, making (B.18) impossible to be satisfied, unless \( W = 0 \) – implying the vanishing of the AdS\(_4\) curvature. We can go even further and show that in fact no intermediate structure is possible for type IIB unless it is really a dynamic SU(3)\( \times \)SU(3)-structure that changes type to static SU(2) somewhere in the manifold. Indeed, for intermediate structure \( \Psi_1 \) is a pure spinor of type 0, which means that the lowest form in \( \Psi_1 \) is a zero-form. According to Gualtieri [62] the generic form of such a pure spinor is
\[ e^{2A-\Phi} \Psi_1 = ce^{i\omega+b}, \] (B.30)
where \( \omega \) and \( b \) are arbitrary real two-forms. From (B.23b) we find, unless \( W = 0 \),
\[ \Re c = 0, \quad c \omega \text{ exact}. \] (B.31)
It follows that
\[ \Im c \langle e^{2A-\Phi} \Psi_1, e^{2A-\Phi} \Psi_1^\ast \rangle = \frac{8}{3!} (c \omega)^3 \] (B.32)
is exact. At the same time it is proportional to the volume form, which is non-exact. The only way to satisfy (B.18) is to have \( \Im c = 0 \) at least somewhere, which means that the type changes on that locus to static SU(2).

For IIB we are interested in the static SU(2)-structure case for which
\[ \eta^{(2)}_+ = V^i \gamma_i \eta^{(1)}_-. \] (B.33)
It is convenient to define the following SU(2)-structure quantities
\[ \omega_2 = J + 2iV \wedge V^\ast, \quad \Omega_2 = \iota_{V^\ast} \Omega. \] (B.34)
where \( J \) and \( \Omega \) form the SU(3)-structure associated to \( \eta_1 = |a|^{-1} \eta_1^{(1)} \) as in (B.5). We then find for the pure spinors

\[
\Psi_+ = -e^{2V^V \Omega_2^*}, \\
\Psi_- = -2V^V \wedge e^{i\omega_2}.
\]  

(B.35a)

(B.35b)

Plugging this ansatz in (B.23), one finds equations for the SU(2)-structure quantities \( V \), \( \omega_2 \) and \( \Omega_2 \), but it should be less complicated to try to solve these equations directly in terms of the pure spinors.

In the same vein as above it follows from (B.23b) that IIA compactifications to AdS\(_4\) are incompatible with static SU(2)-structure, as already noted in [64]. Indeed, this equation would, unless \( W = 0 \), force \( \text{Re} \, \Psi_1 |_1 = 0 \), making it impossible to satisfy (B.18). We can extend the argument to intermediate structure with left-invariant pure spinors. In this case \( \Psi_2 = \Psi_+ \) starts with a zero-form instead of a two-form. However, because of the assumption of left-invariance the zero-form on the left-hand side of (B.23b) is constant and thus zero upon acting with the exterior derivative. Again we find then \( \text{Re} \, \Psi_1 |_1 = 0 \) and (B.18) is not satisfied. The conclusion is that we can only have intermediate SU(3) \( \times \)SU(3)-structure if \( d(e^{3A - \Phi} \Psi_+ |_0) \neq 0 \).

Nilmanifold 5.1

Let us now apply the above formalism to the solution of section 3.3. We can compute the pure spinors using (B.35). One can also check that

\[
-d_H \{ \alpha [d_H (\text{Im} \, \Psi_+)] \} = -4 \beta^2 \text{Re} \, \Psi_+ ,
\]

(B.36)

which leads to an orientifold source as in (B.27) with

\[
k = -\frac{5}{2} \beta^2.
\]

(B.37)

C. How to dress smeared sources with orientifold involutions

Suppose we are given a form \( j \) representing the Poincaré dual of smeared orientifolds. How do we decide what the orientifold involutions should be? Let us first give an example for a localized orientifold in flat space. If we have an orientifold along the directions \( \Sigma = (x^1, x^2, x^3) \) then the corresponding source is

\[
j = T_{Op} j_\Sigma = -T_{Op} \delta(x^1, x^2, x^3) dx^4 \wedge dx^5 \wedge dx^6,
\]

(C.1)

where \( T_{Op} < 0 \) for an orientifold and \( j \) is the Poincaré dual of \( \Sigma \) satisfying

\[
\int_\Sigma \phi = \int_M \langle \phi, j_\Sigma \rangle = - \int_\Sigma \phi \wedge j_\Sigma ,
\]

(C.2)

for an arbitrary form \( \phi \). \(^{18}\) In this case the orientifold involution is of course

\[O6: \quad x^4 \rightarrow -x^4, \quad x^5 \rightarrow -x^5, \quad x^6 \rightarrow -x^6.\]

(C.3)

\(^{18}\)The definition with the Mukai pairing is the one appropriate for generalizing to D-branes with world-volume gauge flux as explained in [66]. Here it will just give an extra minus sign.
Suppose we now introduce many orientifolds and completely smear them in the directions \((x^1, x^2, x^3)\) obtaining
\[
j = -T_{\text{Op}} c \, dx^4 \wedge dx^5 \wedge dx^6, \tag{C.4}
\]
where \(c\) is a constant representing the orientifold density. We have now lost information about the exact location but we would still like to associate the orientifold involution
\[
O_6 : \ dx^4 \rightarrow -dx^4, \ dx^5 \rightarrow -dx^5, \ dx^6 \rightarrow -dx^6. \tag{C.5}
\]

An important observation is that \(dx^4 \wedge dx^5 \wedge dx^6\) is not just any form, it is a decomposable form, i.e. it can be written as a wedge product of three one-forms. These one-forms span the annihilator space of \(T_{\Sigma}\), the tangent space of \(\Sigma\). So if we are given a smeared orientifold current \(j\) we should write it as a sum of decomposable forms and then associate to each term an orientifold involution as above.

Let us now study more formally how we could write \(j\) as a sum of decomposable forms and whether the decomposition is unique. First, let us introduce a basis of forms \(e^i \in V^*\) that span (locally) \(T_M\). Indeed, for the case of group manifolds we have such a basis, which is even defined globally. For the cosets left-invariant forms in this basis are also globally defined.

Now, let \(V\) be a \(d\)-dimensional vector space and \(V^*\) its dual. A (real/complex) \(p\)-form \(j \in \Lambda^p V^*\) is called simple or decomposable if it can be written as a wedge product of \(p\) one-forms.\(^{19}\) What we are interested in is that there is a one-to-one correspondence between \((d-p)\)-planes (our orientifold planes) and decomposable \(p\)-forms (up to a proportionality factor). This isomorphism is called the Plücker map. A discussion of the criteria for having a simple form can be found in e.g. [67] pp. 209-211. We will use here the criterion based on
\[
j^\perp = \{X \in V : \iota_X j = 0\} \subset V, \tag{C.6}
\]
and
\[
W = \text{Ann}(j^\perp) \subset V^*. \tag{C.7}
\]
In [67] it is shown that \(j\) is simple if and only if \(\dim W = p\). Using this the following alternative criterion is shown:

**Theorem:** A \(p\)-form \(j \in \Lambda^p V^*\) is simple if and only if for every \((p-1)\)-polyvector \(\xi \in \Lambda^{p-1} V\),
\[
\iota_\xi j \wedge j = 0, \tag{C.8}
\]
where \(\iota_\xi j\) is the one-form contraction of \(j\) with \(\xi\).

Now for the special case of three-forms in six dimensions there is another useful theorem due to Hitchin [65].

**Theorem:** Consider a real three-form \(j \in \Lambda^3 V^*\) and calculate its Hitchin functional \(H(j)\) defined in (B.14). Then

\(^{19}\)Note that a (real/complex) form of fixed dimension is a pure spinor if and only if it is simple. In fact, we could regard the notion of pure spinor as a generalization of the notion of decomposable forms to polyforms.
• $H(j) > 0$ if and only if $j = j_1 + j_2$ where $j_1, j_2$ are unique (up to ordering) real decomposable three-forms and $j_1 \wedge j_2 \neq 0$;

• $H(j) < 0$ if and only if $j = \alpha + \bar{\alpha}$ where $\alpha$ is a unique (up to complex conjugation) complex decomposable three-form and $\alpha \wedge \bar{\alpha} \neq 0$.

Now we have two base-independent characterizations of $j$: the Hitchin functional $H(j)$ and $\dim W$. To get a feeling of the relation between both we prove the following

**Theorem:** If $H(j) \neq 0$ then $\dim W = 6$ (but not the other way round!).

Indeed, let us first consider $H(j) > 0$. We use the above decomposition and try to find $X$ such that $\iota_X (j_1 + j_2) = 0$. From this relation follows that $\iota_X (j_1 \wedge j_2) = \iota_X j_1 \wedge j_2 - j_1 \wedge \iota_X j_2 = -\iota_X j_2 \wedge j_2 + j_1 \wedge \iota_X j_1$, which is zero because $j_1$ and $j_2$ are simple and (C.8). On the other hand it must be non-zero since $j_1 \wedge j_2 \propto \text{vol}_6 \neq 0$. It follows that there is no such solution for $X$ and thus $j^\perp$ is empty. Then $\dim W = 6$. Analogously for $H(j) < 0$, but we use now the above theorem for complex simple forms.

Using these two characterizations we can classify the possible $j$ and decompose it in simple terms:

• if $H(j) > 0$ it follows immediately that $j$ is a sum of exactly two real simple terms;

• if $H(j) < 0$ then $j$ is a sum of exactly two (conjugate) complex simple terms and thus of exactly four real simple terms. This will in fact be almost always the case for the orientifold sources in this paper.

• if $H(j) = 0$ we have three cases. Either (C.8) is satisfied (equivalently $\dim W = 3$) and $j$ is simple, either $\dim W = 5$ and then $j$ will be a sum of two simple terms $j_1$ and $j_2$ such that $j_1 \wedge j_2 = 0$, or $\dim W = 6$ and $j$ will be a sum of three simple terms. All this is easy to prove by looking at possible types of sums of two and three simple terms.

An important remark is in order: while the Hitchin theorem states that for $H(j) \neq 0$ the two real/complex forms in the decomposition of $j$ are unique (up to ordering/complex conjugation), the choice of one-forms out of which these forms are made is not unique. In the case of $H(j) < 0$ it is the freedom of choosing a basis of complex one-forms belonging to a complex structure, which is $\text{SL}(3, \mathbb{C})$. As a consequence the choice of the four real forms in which $j$ is decomposed is not unique. Indeed, suppose we choose one basis of complex one-forms and associated $x$ and $y$ coordinates: $e^{x_i} = e^{x_i} - ie^{y_i}$. Then $j$ can be written as the sum of the following four terms:

\[
  j = \text{Re} (e^{x_1} e^{x_2} e^{x_3}) = e^{x_1} e^{x_2} e^{x_3} - e^{x_1} e^{y_2} e^{y_3} - e^{y_1} e^{x_2} e^{y_3} - e^{y_1} e^{y_2} e^{x_3}, \tag{C.9}
\]

which leads to the following orientifold involutions:

\[
\begin{align*}
06 : & \quad e^{x_1} \rightarrow -e^{x_1}, \quad e^{x_2} \rightarrow -e^{x_2}, \quad e^{x_3} \rightarrow -e^{x_3}, \\
06 : & \quad e^{x_1} \rightarrow -e^{x_1}, \quad e^{y_2} \rightarrow -e^{y_2}, \quad e^{y_3} \rightarrow -e^{y_3}, \\
06 : & \quad e^{y_1} \rightarrow -e^{y_1}, \quad e^{x_2} \rightarrow -e^{x_2}, \quad e^{y_3} \rightarrow -e^{y_3}, \\
06 : & \quad e^{y_1} \rightarrow -e^{y_1}, \quad e^{y_2} \rightarrow -e^{y_2}, \quad e^{x_3} \rightarrow -e^{x_3}.
\end{align*} \tag{C.10}
\]
If we perform a \( SL(3, \mathbb{C}) \) transformation, \( j \) takes exactly the same form, but now in the \textit{new} basis. So alternatively we could have chosen four orientifold involutions taking the same form as the old ones, but now in the \textit{new} basis, which is rotated. This means that our choice of orientifold involutions is not unique. We must then further choose them such that the structure constant tensor of the group or coset is even, and \( \Re \Omega \) and \( J \) are odd.

In the case of \( H(j) > 0 \) the argument does not apply because the remaining freedom \( \text{GL}(3, \mathbb{R}) \times \text{GL}(3, \mathbb{R}) \) leaves the two terms of the decomposition \textit{separately} invariant and the choice of orientifold involutions is unique.

**Application to SU(2) \times SU(2)**

Let us now apply the above procedure to the model of section (4.4). Calculating the Hitchin functional \( H(j^6) \) of (4.30) we find that it is negative so that it contains four orientifold involutions. We must now fix the freedom of choosing them such that \( \Re \Omega \) and \( J \) are odd, and the structure constant tensor \( f \) is even. Some reflection should make clear that if \( \Re \Omega \) is to be odd it should be a sum of the same four terms as \( j^6 \), but with different coefficients. In fact, we could reverse the procedure and choose a complex basis \( e^{z^i} \) in which \( \Omega \) and \( J \) take their standard form:

\[
\Omega = e^{z^1 \bar{z}^2 \bar{z}^3}, \quad J = -\frac{i}{2} \sum_i e^{z^i \bar{z}^i}.
\]  

(C.11)

Then \( \Re \Omega \) and \( J \) are automatically odd under the associated orientifold involutions (C.10). However, this should of course also be the orientifold involutions that follow from \( j^6 \). This will be the case if and only if \( j^6 \) has the same terms as \( \Re \Omega \) (but with different coefficients) or equivalently \( j^6 \) should take the form

\[
j^6 = \Re \left( c^0 e^{z^1 \bar{z}^2 \bar{z}^3} + c^{11} e^{z^1 \bar{z}^2 \bar{z}^3} + c^{22} e^{z^1 \bar{z}^2 \bar{z}^3} + c^{33} e^{z^1 \bar{z}^2 \bar{z}^3} \right),
\]  

(C.12)

with all coefficients \( c \) real. To accomplish this we still have the freedom to make a base transformation such that \( \Omega \) and \( J \) invariant, i.e. an SU(3)-transformation. A priori, \( j^6 \) is an arbitrary three-form which transforms under SU(3) as

\[
20 = 1 + \bar{1} + 3 + 3 + 6 + \bar{6}.
\]  

(C.13)

However, we know that \( j^6 \) has to satisfy the calibration conditions (2.11), which remove the \( 3 + \bar{3} \) representation and only leave the form proportional to \( \Re \Omega \) out of \( 1 + \bar{1} \). Here the \( 6 \) is the \((3 \times 3)_S \) i.e. the symmetric product of two fundamental representations of SU(3). It follows that the most general \( j^6 \) satisfying the calibration conditions looks like

\[
j^6 = c_0 \Re \Omega + \Re \left[ c^{ki} g_{(k|l)j} \bar{d} \bar{z}^j \wedge \epsilon_{z^i} \Omega \right]
= c_0 \Re \Omega + \Re \left[ c^{11} e^{z^1 \bar{z}^2 \bar{z}^3} + c^{22} e^{z^1 \bar{z}^2 \bar{z}^3} + c^{33} e^{z^1 \bar{z}^2 \bar{z}^3} + c^{12} \left( e^{z^2 \bar{z}^2 \bar{z}^3} + e^{z^2 \bar{z}^1 \bar{z}^3} \right) + c^{13} \left( e^{z^2 \bar{z}^2 \bar{z}^3} + e^{z^1 \bar{z}^2 \bar{z}^3} \right) + c^{23} \left( e^{z^2 \bar{z}^2 \bar{z}^3} + e^{z^1 \bar{z}^2 \bar{z}^3} \right) \right].
\]  

(C.14)
with \( c_0 \) real and the entries of the coefficient matrix
\[
C = \begin{pmatrix}
c_{11} & c_{12} & c_{13} \\
c_{21} & c_{22} & c_{23} \\
c_{31} & c_{32} & c_{33}
\end{pmatrix},
\] (C.15)
complex. Now we have to find an SU(3)-transformation to put \( j^6 \) in the form (C.12). \( c_0 \) does not transform but is luckily already of the right form, while the coefficient matrix transforms as
\[
C \rightarrow U C U^T.
\] (C.16)
From (C.12) we see that we want to transform \( C \) to a diagonal real matrix. In fact, since the above transformation cannot change the determinant this is only possible if
\[
\det C \in \mathbb{R}.
\] (C.17)
This is a condition we have to add to the calibration conditions. For the \( j^6 \) of (4.30) one can check that it is indeed satisfied and it is possible to find the complex coordinates with the required properties. Also, under the associated orientifold involution the structure constant tensor \( f \) is even as required. Note that alternatively, as we actually did in (4.31), we can also construct a complex basis associated to \( \Omega \) such that \( f \) is even. This then automatically implies that \( j \) is odd and that it is a sum of the same four terms as Re\( \Omega \).

D. Kaluza-Klein reduction: calculational details

We assemble here the details on the calculation of the mass spectrum through Kaluza-Klein reduction.

D.1 Solving the Bianchi identities

Here we will obtain an expression of the fluctuations of the gauge flux in terms of the fluctuations of potentials ensuring that the Bianchi identities are automatically satisfied. The analysis is complicated by the presence of a source.

We assume that the source does not fluctuate since it is associated to smeared orientifolds. For the Bianchi identities of the background and the fluctuation we find then respectively
\[
(d + \hat{H}) \hat{F} = -j, \quad (d + \hat{H} + \delta H)(\hat{F} + \delta \hat{F}) = -j.
\] (D.1a) (D.1b)
The integrability equations read
\[
(d + \hat{H}) j = 0, \quad (d + \hat{H} + \delta H) j = 0,
\] (D.2a) (D.2b)
from which follows
\[
\delta H \wedge j = 0.
\] (D.3)
This implies also
\[(d + \hat{H})(e^{\delta B} \wedge j) = 0,\] (D.4)
so that, subtracting (D.1a), we can define (locally)
\[-(e^{\delta B} - 1) \wedge j = (d + \hat{H})\delta \omega.\] (D.5)

Now, for orientifold sources the left hand side of this equation always vanishes. This follows because the pull-back of \(\delta B\) to the orientifold, \(\delta B|_{\Sigma}\), must be zero, which implies using (C.2):
\[\delta B \wedge j = 0,\] (D.6)
and the same for all powers of \(\delta B\). Then, we can also choose \(\delta \omega = 0\).

The difference between (D.1a) and (D.1b) gives the Bianchi identity for the fluctuations
\[\left(d + \hat{H} + \delta H\right)\delta F + \delta H \wedge \hat{F} = 0,\] (D.7)
which can be rewritten as
\[\left(d + \hat{H}\right)\left(e^{\delta B} \delta F\right) + \delta H \wedge e^{\delta B} \hat{F} = 0.\] (D.8)

One can easily show that (with \(\delta F_0 = 0\)) this Bianchi identity can be satisfied by introducing potentials \(\delta C\) and putting
\[e^{\delta B} \delta F = (d + \hat{H})\delta C - (e^{\delta B} - 1)\hat{F} + \delta \omega,\] (D.9)
where we can set \(\delta \omega = 0\) so that we obtain eq. (5.9).

Expanding this expression we find for the IIA-fluctuations
\[\delta F_0 = 0,\]
\[\delta F_2 = d\delta C_1 - m\delta B,\]
\[\delta F_4 = d\delta C_3 + \hat{H} \wedge \delta C_1 - \delta B \wedge (\hat{F}_2 + \delta F_2) - \frac{1}{2}m(\delta B)^2,\] (D.10)
\[\delta F_6 = d\delta C_5 + \hat{H} \wedge \delta C_3 - \delta B \wedge (\hat{F}_4 + \delta F_4) - \frac{1}{2}(\delta B)^2 \wedge (\hat{F}_2 + \delta F_2) - \frac{1}{3!}m(\delta B)^3,\]
and for the IIB-fluctuations
\[\delta F_1 = d\delta C_0,\]
\[\delta F_3 = d\delta C_2 + \hat{H} \wedge \delta C_0 - \delta B \wedge (\hat{F}_1 + \delta F_1),\] (D.11)
\[\delta F_5 = d\delta C_4 + \hat{H} \wedge \delta C_2 - \delta B \wedge (\hat{F}_3 + \delta F_3) - \frac{1}{2}(\delta B)^2 \wedge (\hat{F}_1 + \delta F_1).\]

For the Kaluza-Klein reduction we will only need the terms linear in the fluctuations while for an analysis of finite fluctuations using the Kähler potential and superpotential we need higher orders too.

Also, in the Kaluza-Klein reduction we will only need fluctuations of the physical fields \(\delta F_2, \delta F_4\) since the higher-form fluxes are removed from the equations of motion using (A.1), while in the superpotential approach, which is formulated in the democratic formalism, we should work with the internal part of \(\delta F_5\) instead of the external part of \(\delta F_4\). As we explain in section D.5 we could actually have done this in the Kaluza-Klein approach too.
D.2 Expansion/Truncation

For the Kaluza-Klein reduction on $T^6$ we will expand the fluctuations of the various fields in the following basis:

\[ \delta B(x, y) = b_{i, \vec{n}}(x) Y_{i, \vec{n}}^{(2)}(y) + b_{1, \vec{n}}(x) Y_{1, \vec{n}}^{(1)}(y) + b_{2}(x) Y_{R, \vec{n}}^{(0)}(y), \]

(D.12a)

\[ \delta \phi(x, y) = \delta \phi^{(1)}(x) Y_{i, \vec{n}}^{(0)}(y), \]

(D.12b)

\[ \delta C^{(1)}(x, y) = c^{(1)}_{i, \vec{n}}(x) Y_{i, \vec{n}}^{(1)}(y) + c_{1}^{(1)}(x) Y_{R, \vec{n}}^{(0)}(y), \]

(D.12c)

\[ \delta C^{(3)}(x, y) = c^{(3)}_{i, \vec{n}}(x) Y_{i, \vec{n}}^{(3)}(y) + c_{1}^{(3)}(x) Y_{R, \vec{n}}^{(0)}(y) + c_{2}^{(3)}(x) Y_{i, \vec{n}}^{(1)}(y) \]

(D.12d)

\[ \delta g(x, y) = h^{i, \vec{n}}(x) X^{(2)}_{i, \vec{n}}(y) + h^{i}_{1}(x) Y_{i, \vec{n}}^{(1)}(y) + h^{2}_{2}(x) Y_{R, \vec{n}}^{(0)}(y). \]

(D.12e)

The functions $Y_{i, \vec{n}}^{(l)}(y)$ are the $l$-eigenforms of the Laplacian operator and are given by

\[ Y_{i, \vec{n}}^{(l)}(y) = Y^{(l)}_{i}(e^{\vec{p}} \vec{y}), \quad \vec{p} = \frac{\vec{n}}{R}, \quad \vec{n} \in \mathbb{Z}^{6}, \]

(D.13)

where the $Y_{i}^{(l)}$ form a basis of harmonic $l$-forms on $T^6$. $X^{(2)}_{i, \vec{n}}$ are symmetric two-tensors

\[ X^{(2)}_{i, \vec{n}}(y) = X_{i}^{(2)}(e^{\vec{p}} \vec{y}), \quad \vec{p} = \frac{\vec{n}}{R}, \quad \vec{n} \in \mathbb{Z}^{6}, \]

(D.14)

Since we will restrict our analysis to the zero modes ($\vec{p} = 0$), we only keep $Y_{i, \vec{n}=0}^{(l)}(y) = Y_{i}^{(l)}$ and $X_{i, \vec{n}=0}^{(2)}(y) = X_{i}^{(2)}$ in the expansions above and derivatives only act on the external fields. For the Iwasawa manifold we will use for the expansion forms $Y_{i}^{(l)}$ left-invariant forms, which will not necessarily be all harmonic. When exterior derivatives act on these forms terms will be generated of the order of the geometric fluxes.

D.3 IIA on $AdS_4 \times T^6$

We would now like to perform a Kaluza-Klein reduction on the $AdS_4 \times T^6$ solution described in section 3.1.

A basis for the harmonic $l$-forms $Y_{i}^{(l)}$ is simply given by all exterior products of the form $dy^{m_1} \wedge \cdots \wedge dy^{m_l} = e^{m_1 \cdots m_l}$, $1 \leq l \leq 6$. Hence:

\[ b_{l} = \binom{6}{l}, \]

(D.15)

where $b_{l}$ denotes the real dimension of the $l$th cohomology group of $T^6$. However, we must then impose the orientifold projection, which means that suitable expansion forms must be even or odd under all the orientifold involutions. For the torus we find from (3.4):

| type         | basis                           | name     |
|--------------|---------------------------------|----------|
| odd 2-form   | $e^{12}, e^{34}, e^{56}$        | $Y_{i}^{(2-)}$ |
| even 3-form  | $e^{135}, e^{146}, e^{236}, e^{245}$ | $Y_{i}^{(3+)}$ |
| odd 3-form   | $e^{136}, e^{145}, e^{235}, e^{246}$ | $Y_{i}^{(3-)}$ |
| even 4-form  | $e^{1234}, e^{1256}, e^{3456}$  | $Y_{i}^{(4+)}$ |
| even symmetric 2-tensor | $e^{1} \otimes e^{1}, e^{2} \otimes e^{2}, \ldots, e^{6} \otimes e^{6}$ | $X_{i}^{(2)}$ |
Under the orientifold projection we find from (2.15) that \( \Phi, g, F_0, C_3 \) are even, while \( B, C_1 \) are odd. This simplifies the expansion (D.12) considerably

\[
\begin{align*}
\delta B(x, y) &= b^i(x)Y_i^{(2-)} , \\
\delta \Phi(x, y) &= \Phi(x), \\
\delta C^{(3)}(x, y) &= c^{(3)i}(x)Y_i^{(3+)} + \epsilon^{(3)}_3(x) , \\
\delta g(x, y) &= h^i(x)X_i^{(2)} + h_2(x) .
\end{align*}
\] (D.16)

Note in particular that the orientifold projection removes all four-dimensional gauge fields, which in fact holds for all type IIA models for which the orientifolds project out all one-forms and even two-forms.

We find then from (D.10) the linear fluctuations of the field strengths

\[
\begin{align*}
\delta F_2 &= -m \delta B , \\
\delta F_4 &= \delta \delta C_3 .
\end{align*}
\] (D.17)

To derive the mass matrix for the four-dimensional fields we proceed as follows. We first compute the variation of all the equations of motion (A.7a),(A.7b),(A.9b) and (A.10) to first order. Remember that we should use (A.1) to remove the redundant RR-fields so that the only RR-fluctuations are the ones above. Next we plug in the background values and the truncated expansion of the fields (D.16). Since we are only considering the zero internal modes we use that for the torus derivatives only act on the external fields.

Let us consider first the equations for the RR-fields and \( H \). It will turn out that these do not mix with the dilaton and the metric. Applying the steps described above we get from the equation of motion for \( H \) the following equation, which has (external, internal) index structure \((0, 2)\):

\[
\begin{align*}
0 &= \Delta (b^iY_i^{(2-)}) - \star (\hat{F}_4 \wedge d\epsilon_3^{(3)}) - m \star (\star \hat{F}_4 \wedge b^iY_i^{(2-)}) + m^2 b^iY_i^{(2-)} .
\end{align*}
\] (D.18)

From the variation of the equation of motion of \( F_4 \) we get a \((0, 3)\)-equation and a \((1, 6)\)-equation

\[
\begin{align*}
0 &= \Delta (c^{(3)i}Y_i^{(3+)}) - \star (\hat{H} \wedge d\epsilon_3^{(3)}) , \\
0 &= d \star d\epsilon_3^{(3)} + db^i \wedge Y_i^{(2-)} \wedge \hat{F}_4 + \hat{H} \wedge d\epsilon_3^{(3)i} \wedge Y_i^{(3+)} ,
\end{align*}
\] (D.19)

and from \( F_2 \) a \((4, 5)\)- and \((3, 6)\)-equation

\[
\begin{align*}
0 &= \hat{H} \wedge \star \left[ h^iX_i^{(2)} \cdot \hat{F}_4 \right] , \\
0 &= \hat{H} \wedge \star (d\epsilon_3^{(3)i} \wedge Y_i^{(3+)} ) ,
\end{align*}
\] (D.20)

where we used in the upper equation the variation of the \(\star\)

\[
(\delta \star)F_l = \left( \frac{1}{2} g^{MN} \delta g_{MN} \right) \star F_l - \star [\delta g \cdot F_l] ,
\] (D.21)
where
\[ [\delta g \cdot F]_{M_1 \cdots M_l} = l \cdot \delta g_{[M_1} A g^{AB} F_{|M_2 \cdots M_l]} . \] (D.22)

The equations (D.20) are automatically satisfied using the orientifold projection. Indeed, the right-hand sides should have contained an even internal five-form respectively six-form under all orientifold involutions, which do not exists, so they must vanish.

Next, we integrate (D.19b) and put the integration constant to zero because it would correspond to changing the background value of \( f \). This procedure corresponds to dualizing \( c_3^{(3)} \) as explained in [33, 29]. We come back to it in section D.5.

To proceed we make a choice of expansion basis for the even three-forms
\[
Y_0^{(3+)} = \text{Im} \Omega, 
\]
\[ Y_i^{(3+)} , \quad i = 1, 2, 3 : \quad 3 \text{ real (2,1)+(1,2) forms} , \] (D.23a)
and the odd two-forms
\[
Y_0^{(2+)} = J, 
\]
\[ Y_i^{(2+)} , \quad i = 1, 2 : \quad 2 \text{ primitive real 2-forms} , \] (D.24a)
where a primitive two-form is defined in (B.7).

In the end we obtain the following result for the eigenvalues \( \tilde{M}^2 = M^2 + 2/3 \Lambda \):

| mass eigenmode | mass (in units \( m^2/25 \)) |
|----------------|-----------------------------|
| \( b^i, \quad i = 1, 2 \) | 10 |
| \( c^i, \quad i = 1, 2, 3 \) | 0 |
| \( b^0 - 4c^{(3)0} \) | 10 |
| \( 3b^0 + c^{(3)0} \) | 88 |

Next we consider the dilaton and Einstein equation. With the same procedure as above, we arrive at the following equations
\[
0 = (\Delta + \frac{67m^2}{25}) \delta \Phi + \frac{7m^2}{25} \sum_{i=1}^{6} h^i , \] (D.25)
and
\[
0 = \Delta h^i + \frac{8m^2}{25} h^i + \frac{7m^2}{50} g_{ii} \delta \Phi + \frac{m^2}{50} g_{ii} \sum_{j=1}^{6} h^j + \frac{2m^2}{5} g_{ii} h^{i-(-1)^i} . \] (D.26)

The result of diagonalizing the mass matrix is

| mass eigenmode | mass (in units \( m^2/25 \)) |
|----------------|-----------------------------|
| \( -h_{z1}z1 + h_{z2}z2 = -h^1 + h^2 + h^3 + h^4 \) | 18 |
| \( -h_{z1}z1 + h_{z2}z2 = -h^1 + h^2 + h^3 + h^4 \) | 18 |
| \( -3 \delta \Phi + 7 \sum h_i \) | 18 |
| \( 7 \delta \Phi + \sum h_i \) | 70 |
| \( \text{Re} h_{z1}z1 = -h^1 + h^2 \) | -2 |
| \( \text{Re} h_{z2}z2 = -h^3 + h^4 \) | -2 |
| \( \text{Re} h_{z3}z3 = -h^5 + h^6 \) | -2 |
The external part of the Einstein equation on the other hand becomes
\[\frac{1}{2}\Delta_L h_{\mu\nu} + \nabla(\mu \nabla^\rho h_{\nu}) = \frac{1}{2} \nabla(\mu \nabla^\rho h_{\nu}) + \frac{3}{25} m^2 h_{\mu\nu} - \frac{3}{20} m^2 g_{\mu\nu} \sum h_i - \frac{21}{100} m^2 g_{\mu\nu} \delta \Phi = 0.\] (D.27)

At this point we have to take into account that so far we worked in the ten-dimensional Einstein frame. From (E.21) we find that the conversion to the four-dimensional Einstein frame is as follows
\[g_{E\mu\nu} = g_{6} \sqrt{g_{\mu\nu}},\] (D.28)
where the constant factor \(c = M^{-2} \kappa^{-2} V_s\) does not matter here, so that
\[c^{-1} h_{E\mu\nu} = \sqrt{g_{6}} h_{\mu\nu} + \frac{1}{2} \sqrt{g_{6}} g_{\mu\nu} \sum h_i.\] (D.29)

Plugging this into (D.27) and using (D.26) we find for \(h_{E\mu\nu}\) exactly equation (5.3c) with \(M^2 = 0\) so that \(h_{E\mu\nu}\) indeed describes a massless graviton.

D.4 IIA on the Iwasawa manifold

For the solution on the Iwasawa manifold of section 3.2 with \(m = 0\) we get the same even and odd forms under the orientifold involution as on the torus (but now in the left-invariant basis appropriate for the Iwasawa). Again \(\Phi, g, F_0, C_3\) are even, while \(B, C_1\) are odd, resulting in the same expansion (D.16) as for the torus. This time we get from (D.10) for the linear fluctuations of the field strengths
\[\delta F_2 = 0,\] (D.30a)
\[\delta F_4 = d\delta C_3 - \delta B \wedge \tilde{F}_2.\] (D.30b)

Expanding the equation of motion for \(H\) around the Iwasawa solution, we obtain
\[0 = \Delta b^i Y_i^{(2-)} + b^i \left(\ast_6 d \ast_6 d Y_i^{(2-)}\right) - c^{(3)i} \ast_6 \left(\ast_6 d Y_i^{(3+)} \wedge \tilde{F}_2\right) + b^i \ast_6 \left[\ast_6 \left(Y_i^{(2-)} \wedge \tilde{F}_2\right) \wedge \tilde{F}_2\right] + f c^{(3)i} \ast_6 d Y_i^{(3+)} - b^i f \ast_6 \left(Y_i^{(2-)} \wedge \tilde{F}_2\right),\] (D.31)
while the equation of motion for \(F_4\) splits in (1,6) and (4,3) index structures
\[0 = d \ast_4 d c_3^{(3)} + \frac{1}{2} f d \left(\delta g^\mu _\mu - \delta g^m _m - \delta \Phi\right),\] (D.32a)
\[0 = \Delta c^{(3)i} Y_i^{(3+)} + c^{(3)i} \left(\ast_6 d \ast_6 d Y_i^{(3+)}\right) + f b^i \ast_6 d Y_i^{(2-)} - b^i \ast_6 d \left(Y_i^{(2-)} \wedge \tilde{F}_2\right),\] (D.32b)

In a similar way as in the torus case, we integrate (D.32a), put the integration constant to zero and plug the result for \(d c_3^{(3)}\) in the other equations.

As expansion forms we take the same three-forms as in eq. (D.23), while for the two-forms we take this time
\[Y_0^{(2-)} = \beta^2 e^{56},\] (D.33a)
\[Y_1^{(2-)} = e^{12} + e^{34},\] (D.33b)
\[Y_2^{(2-)} = e^{12} - e^{34}.\] (D.33c)
Note that this time $Y_{0}^{(3+)}$ and $Y_{0}^{(2-)}$ are not closed. Introducing $m_T$ such that $\beta = \frac{2}{5} e^\Phi m_T$ (this is of course the Romans mass of the T-dual torus solution), we get the following masses:

| mass eigenmode | mass (in units $m^2_T/25$) |
|---------------|-----------------------------|
| $c^i, i = 1, 2, 3$ | 0 |
| $b^0 + b^1$ | 10 |
| $b^2$ | 10 |
| $8c^{(3)0} + 5b^0 + 3b^1$ | 10 |
| $c^{(3)0} - b^0 + 2b^1$ | 88 |

Due to T-duality the mass eigenvalues are the same as for the torus solution.

The equation for the variation of the dilaton around the background reads

$$0 = (\Delta + \frac{27}{25} m^2_T) \delta \phi - \frac{9}{25} m^2_T \sum_{i=5}^{6} h^i + \frac{3}{25} m^2_T \sum_{i=1}^{4} h^i.$$  \hspace{1cm} (D.34)

For the Einstein equation we find:

$$0 = \Delta h^i + \frac{49 m^2_T}{50} h^i + \frac{53 m^2_T}{50} h^i - \frac{11 m^2_T}{5} \sum_{j=5}^{6} h^j - \frac{33 m^2_T}{50} \delta \phi \quad \text{for} \quad i = 5, 6, \hspace{1cm} (D.35a)$$

$$0 = \Delta h^i + \frac{8 m^2_T}{25} h^i + \frac{2 m^2_T}{5} h^i - \frac{3 m^2_T}{10} \sum_{j=5}^{6} h^j + \frac{m^2_T}{10} \sum_{j=1}^{4} h^j + \frac{3 m^2_T}{10} \delta \phi \quad \text{for} \quad i = 1, 2, 3, 4. \hspace{1cm} (D.35b)$$

Here we used that

$$\delta R_{mn} = \frac{1}{2} \Delta_L \delta g_{mn} + \nabla_m (\nabla^n \delta g_{n}) - \frac{1}{2} \nabla_m \nabla_n \delta g_{Q} Q,$$  \hspace{1cm} (D.36)

where $\Delta_L$ is the Lichnerowicz operator defined in (5.4) and all covariant derivatives and contractions are with respect to the background metric. In (D.36) the last two terms are vanishing.

Diagonalizing the mass matrix we find the following eigenmodes:

| mass eigenmode | mass (in units $m^2_T/25$) |
|---------------|-----------------------------|
| $-h_{z,1} + h_{z,2}$ | 18 |
| $11h_{z,1} + 5h_{z,2} = 11(h^1 + h^2) + 5(h^5 + h^6)$ | 18 |
| $5\delta \Phi - 3(h^1 + h^2)$ | 18 |
| $3\delta \Phi - 3(h^5 + h^6) + (h^1 + h^2 + h^3 + h^4)$ | 70 |
| $\Re h_{z,1} = -h^1 + h^2$ | -2 |
| $\Re h_{z,2} = -h^3 + h^4$ | -2 |
| $\Re h_{z,3} = -h^5 + h^6$ | -2 |

Once again, we find the same masses as in the torus example.
D.5 A note on integrating out $d_{c_3}^{(3)}$

Both in the torus and in the Iwasawa analysis we integrated out $d_{c_3}^{(3)}$. In general one gets from the part of the equation of motion of $F_4$ with $(1,6)$ index structure

$$e^\frac{i}{2} \Phi \star_4 d_{c_3}^{(3)} \land \text{vol}_6 = + \frac{1}{2} e^\frac{i}{2} \Phi f (\delta g^{\mu}_m - \delta g^m_{\mu} - \delta \Phi) \land \text{vol}_6$$
$$+ e^{(3)i} \tilde{\mathbf{H}} \land Y^{(3+)}_i - b^i \land Y^{(2-)}_i \land \hat{F}_4 + \delta f ,$$

where the integration constant $\delta f$ corresponds to a variation of the background flux $f$, which we put to zero.

This describes the external part of $F_4$, which equivalently can be described by the internal part of $F_6$. Indeed, from varying

$$F_6 = e^\frac{i}{2} \Phi \star F_4 ,$$

which we got from (A.1), follows

$$\delta F_{6,\text{int}} = \frac{1}{2} e^\frac{i}{2} \Phi f (\delta g^{\mu}_m - \delta g^m_{\mu} - \delta \Phi) \land \text{vol}_6 + e^\frac{i}{2} \Phi \star d_{c_3}^{(3)} ,$$

so that plugging in (D.37) we find

$$\delta F_{6,\text{int}} = e^{(3)i} \tilde{\mathbf{H}} \land Y^{(3+)}_i - b^i \land Y^{(2-)}_i \land \hat{F}_4 .$$

This corresponds to the part of $\delta F_6$ in (D.10) that is first order in the fluctuations. We conclude that instead of introducing $d_{c_3}^{(3)}$, the external part of $F_4$, we might as well have worked with the internal part of $F_6$. That is exactly what we will do in the superpotential analysis.

E. Effective supergravity

The superpotential for $\text{SU}(3) \times \text{SU}(3)$-structure was derived in various ways in [30, 31, 32] (based on [55, 29]). Here we will follow the approach of [32], which calculated the superpotential and the (conformal) Kähler potential in the superconformal formalism of [68].

The bosonic part of the effective four-dimensional superconformal action takes the following form

$$S = \int d^4x \sqrt{-g_4} \left( \frac{1}{2} \mathcal{N} \mathcal{R} + 3 \mathcal{N}_IJ g^{\mu\nu} D_\mu X^I D_\nu X^* J + \frac{1}{3} \mathcal{W}_I \left( \mathcal{N}^{-1} \right)^{IJ} \mathcal{W}^*_J + \cdots \right) ,$$

where the vector multiplet sector, including D-terms, has been omitted. Here the $X^I$ are the $n + 1$ scalars and $D_\mu X^I = \partial_\mu X^I - \frac{1}{3} i A_\mu X^I$, where $A_\mu$ is the gauge field associated to the $\text{U}(1)$-transformations, generated by $\alpha$ (see (E.4)), in the complex Weyl transformation. From dimensional reduction of the ten-dimensional supergravity action the conformal
Kähler potential $\mathcal{N}$ and the superpotential $\mathcal{W}$ were found and read (here we reinstate dimensionful coupling constants)

$$\mathcal{N} = \frac{1}{\kappa_{10}^2} \int_M \text{d}^6y \sqrt{\text{det} h} e^{2A-2\Phi} = \frac{1}{8\kappa_{10}^2} \left( i \int_M e^{-4A} \langle Z, \bar{Z} \rangle \right)^{1/3} \left( i \int_M e^{2A} \langle t, \bar{t} \rangle \right)^{2/3}, \quad (E.2a)$$

$$\mathcal{W} = \frac{1}{4\kappa_{10}^2} \int_M \langle Z, F + i \text{d}_H(\text{Re} T) \rangle. \quad (E.2b)$$

Here $Z$, $\text{Re} T$ and $t$ are defined through

$$Z = -ie^{3A-\Phi} \Psi_2, \quad (E.3a)$$

$$t = e^{-\Phi} \Psi_1, \quad (E.3b)$$

$$\text{Re} T = \text{Im} t = e^{-\Phi} \text{Im} \Psi_1. \quad (E.3c)$$

The dimensionally reduced action is naturally invariant under the following complex Weyl symmetry

$$A \rightarrow A + \sigma, \quad g \rightarrow e^{-2\sigma} g, \quad Z \rightarrow e^{3\sigma+i\alpha} Z, \quad N \rightarrow e^{2\sigma} N. \quad (E.4)$$

Since the scalars $X^I$ transform as

$$X^I \rightarrow e^{\sigma+i\alpha} X^I, \quad (E.5)$$

we find that $Z$ must be homogeneous of degree 3 in the $X^I$. To go to the usual Einstein frame, we must gauge-fix the Weyl symmetry. We first explicitly isolate the unphysical degree of freedom, which is called the conformon, as follows

$$X^I = Y x^I(\phi^i), \quad Z = Y^3 Z(\phi^i) \quad \mathcal{N} = |Y|^2 e^{-K/3}, \quad \mathcal{W} = Y^3 M_P^{-3} \mathcal{W}_E(\phi^i), \quad (E.6)$$

where $Y$ is the conformon, $\phi^i$ are the $n$ scalar degrees of freedom in the Einstein frame and $M_P$ the four-dimensional Planck mass. $K$ and $\mathcal{W}_E$ will turn out to be the Kähler potential and the Einstein-frame superpotential after gauge-fixing. Indeed, in the new coordinates the action (E.1) becomes

$$S = \int \text{d}^4x \sqrt{-g_4} \left[ \frac{1}{2} |Y|^2 e^{-K/3} R - |Y|^2 e^{-K/3} K_{ij} g^{\mu\nu} \partial_\mu \phi^i \partial_\nu \bar{\phi}^j + \cdots ight.$$  

$$-M_P^{-6} |Y|^4 e^{K/3} (K^{ij} D_i D_j \mathcal{W}_E \mathcal{W}_E^* - 3|\mathcal{W}_E|^2) + \cdots \right], \quad (E.7)$$

where for the kinetic term of the scalars we omitted pieces that will vanish after the gauge-fixing.

We then impose the following gauge

$$\mathcal{N} = |Y|^2 e^{-K/3} = M_P^2, \quad (E.8)$$

which obviously gives us the usual Einstein-frame action

$$S = \int \text{d}^4x \sqrt{-g_4} \left( \frac{M_P^2}{2} R - M_P^2 K_{ij} \partial_\mu \phi^i \partial^\mu \bar{\phi}^j - V(\phi, \bar{\phi}) \right), \quad (E.9)$$
and also leads to the standard expression for the potential
\[ V(\phi, \bar{\phi}) = M_F^{-2} e^K \left( K^{ij} D_i W_E D_j W_E^* - 3 |W_E|^2 \right). \]  
(E.10)

The U(1)-symmetry must also be gauged, but for more details on this we refer to \cite{68}.

The Kähler potential reads
\[ K = -\ln i \int_M e^{-4A} (Z, \bar{Z}) - 2 \ln i \int_M e^{2A} (t, \bar{t}) + 3 \ln (8\kappa^2_{10} |Y|^2). \]  
(E.11)

Note that in \cite{69} it is shown that Im\( t \) is a function of Re\( t \) so that \( t \) can be seen as (non-holomorphically) dependent on \( T \). To take this relation properly into account we use the fact that the Kähler potential for the \( t \)-sector may be written as
\[ K_t = -2 \ln 4 \int_M e^{2A} H(\text{Im} t), \]  
(E.12)

where \( H(\text{Im} t) \) is the Hitchin functional \cite{65, 69, 30}. For stable pure spinors of \( SO(6,6) \) it is defined as follows
\[ H(\text{Im} t) = \sqrt{-\frac{1}{12} \mathcal{J}^{\Pi \Sigma} \mathcal{J}^{\Sigma \Pi}}. \]  
(E.13)

In the case of SU(3)-structure \( \text{Im} t = -\text{Im} \Omega \) and the Hitchin functional reduces to (B.14).

Note that if we make an expansion of the warp factor \( A \) in harmonic modes
\[ A = A^0 + \sum_{\bar{n} \neq 0} A_{\bar{n}} \gamma^{(0)}_{\bar{n}}(y) = A^0 + \bar{A}, \]  
(E.15)

the Weyl transformation (E.4) only acts on \( A^0 \) since \( \sigma \) is constant in the internal coordinates (while of course it can depend on the four-dimensional coordinates). Suppose \( A \) and \( \Phi \) are constant over the internal space (so \( \bar{A} = 0 \)). A good choice of \( Y \) in (E.6) would be
\[ Y = e^{A - \Phi/3} M_P, \]  
(E.16)

where the \( M_P \) is introduced for convenience as it allows \( K \) to be dimensionless upon imposing the Einstein gauge (E.8). With this choice we find for the superpotential and the Kähler potential
\[ K = -\ln i \int_M \langle \Psi_2, \bar{\Psi}_2 \rangle - 2 \ln i \int_M \langle t, \bar{t} \rangle + 3 \ln (8\kappa^2_{10} M^2_P), \]  
(E.17a)

\[ W_E = \frac{-i}{4\kappa^2_{10}} \int_M \langle \Psi_2, F + i d_H(\text{Re} T) \rangle. \]  
(E.17b)

Note that another choice \( Y' = f Y \) would amount to a Kähler transformation
\[ W'_E = f^{-3} W_E, \quad K' = K + 3 \ln f + 3 \ln f^*. \]  
(E.18)
From the four-dimensional Einstein-frame action (E.9) we compute the equation of motion for the scalar fields
\[ \Delta \phi^k + M_p^{-2}(\hat{K}^{-1}\hat{M})^k_\ell \phi^\ell = 0, \tag{E.19} \]
where \( \hat{M}_{ij} = \frac{1}{2} \frac{\partial^2 V}{\partial \phi_i \partial \phi_j} \big|_{\text{background}} \) is the mass matrix and \( \hat{K}_{ij} \) is the Kähler metric in real coordinates in the background. Therefore, to compare the results for the masses in the analysis with the superpotential and the Kähler potential with the results from the Kaluza-Klein reduction we need to diagonalize the matrix \( M_p^{-2}\hat{K}^{-1}\hat{M} \). We also have to take into account that the results from the Kaluza-Klein reduction were in the ten-dimensional Einstein frame, while here we get the result in the four-dimensional Einstein frame:
\[ g_s = e^{\Phi} g_{E10}, \]
\[ g_s = M_p^2 N^{-1} g_{E4}, \tag{E.20} \]
and thus
\[ g_{E10} = M_p^2 e^{-\Phi/2} N^{-1} g_{E4} = M_p^2 \kappa_{10}^2 e^{-2A \text{Vol}_E^{-1}} g_{E4}, \tag{E.21} \]
where in the last expression we assumed \( A \) and \( \Phi \) constant over the internal space. The conversion for the mass is
\[ m^2_E = \kappa_{10}^2 M_p^2 e^{-2A \text{Vol}_E^{-1}} m^2_{E10}. \tag{E.22} \]

**F. \( \mathcal{N} = 2 \) for IIA on \( \frac{\text{SU}(3) \times \text{U}(1)}{\text{SU}(2)} \)**

From (4.37) we see that \( 5\hat{c}_1^2 = 4\hat{m}^2 \) is a special point in that we have only two orientifolds, one along 345 and one along 125. As a result there are more odd/even forms:
\[ Y^{(1+)} = e^5, \quad Y^{(1-)} = e^6, \]
\[ Y^{(2+)} = e^{12} + e^{34}, \quad Y^{(2-)} = e^{13} - e^{24}, \quad Y^{(2-)} = e^{14} + e^{23}, \quad Y^{(2-)} = e^{56}, \]
\[ Y^{(3-)} = e^{145} + e^{235}, \quad Y^{(3-)} = e^{136} - e^{246}, \quad Y^{(3-)} = e^{126} + e^{346}, \]
\[ Y^{(3+)} = e^{146} + e^{236}, \quad Y^{(3+)} = e^{136} - e^{246}, \quad Y^{(3+)} = e^{125} + e^{345}, \]
\[ Y^{(5+)} = e^{12345}, \quad Y^{(5-)} = e^{12346}, \tag{F.1} \]
where we did not display the four-forms, which are dual to the two-forms, because we do not need them for the expansion below. However we find that \( Y^{(3+)}_1 \wedge Y^{(2-)}_2 \neq 0 \) and \( Y^{(3+)}_2 \wedge Y^{(2-)}_1 \neq 0 \) so that not all fluctuations expanded in these forms turn out to be consistent. Indeed, suppose we make the following expansion
\[ J_c = J - i\delta B = t^i Y^{(2-)}_i, \]
\[ e^{-\Phi} e^{i\delta B} \text{Im} \Psi + i\delta C = z^1 Y^{(1-)} + z^{1+i} Y^{(3+)}_i + z^5 Y^{(5-)}_i, \tag{F.2} \]
we see first of all that if \( z^1 \neq 0 \) or \( z^5 \neq 0 \) we do not have strict SU(3)-structure anymore but rather intermediate SU(2) and secondly that the compatibility between the two pure spinors is not automatic anymore. Thirdly there is a \( \delta B \) fluctuation that affects both pure
spinors. It is best to absorb $\delta B$ into the pure spinors $\Psi_{1,2}' = \Psi_{1,2} e^{\delta B}$. The generalization of the strict SU(3) compatibility condition $J \wedge \Omega = 0$ is \cite{30}

$$\langle \text{Im} \Psi_1', \mathbf{X} \cdot \Psi_2' \rangle = 0,$$

(F.3)

for arbitrary $\mathbf{X}$. Working this out we find the following constraint

$$-2i \left[ (\text{Re} z^2) t^2 + (\text{Re} z^3) t^1 \right] - \text{Re} z^1 \left[(t^1)^2 + (t^2)^2\right] + \text{Re} z^5 = 0.$$  \hspace{1cm} (F.4)

One way to solve this constraint is to put

$$t^2 = -\rho t^1, \quad \text{Re} z^3 = \rho \text{Re} z^2, \quad z^1 = z^5 = 0,$$

(F.5)

which brings us to the restricted set of fluctuations in (6.13).

However, in this model there can be three extra fluctuations in the NSNS-sector. One physical one, involving only $\text{Im} t_1, \text{Im} t_2$, satisfying to linear order around the background

$$\text{Im} t^1 = \rho \text{Im} t^2$$

(F.6)

and inducing $\delta B_{(2,0)} + \delta B_{(0,2)}$. There are also two spurious ones

$$\delta \Omega = -4(\bar{g} \chi) \wedge J, \quad \delta J = i \chi \Omega + i \bar{\chi} \bar{\Omega},$$

(F.7)

with $\chi = E^5 - \frac{i a}{2 \sqrt{1 + \rho^2}} E^6$, and the other involving $\text{Re} z^1$ and $\text{Re} z^5$ such that

$$-\text{Re} z^1 \left[(t^1)^2 + (t^2)^2\right] + \text{Re} z^5 = 0.$$  \hspace{1cm} (F.8)

Indeed, one can check that these do not affect the metric nor the $B$-field, only the pure spinors defining the structure \cite{30}. As such, one would not expect them to appear in the low energy effective action.

The presence of a spurious fluctuation that leads to leaving the strict SU(3)-structure case and that is still allowed under the orientifold projection, indicates that the theory is in fact $\mathcal{N} = 2$ and therefore outside the scope of this paper. Indeed, suppose $\eta$ was the original internal spinor generating the supersymmetry and satisfying $\sigma^*(\eta_{\pm}) = \eta_{\mp}$ under all orientifolds (for the required transformation behaviour of the internal spinors under supersymmetric orientifolds see \cite{45}), there is now a second one $\eta' = \gamma_5 \eta$ also satisfying $\sigma^*(\eta'_{\pm}) = \eta'_{\mp}$ under all orientifolds. It follows that we can make the $\mathcal{N} = 2$ spinor ansatz

$$\epsilon_1 = \xi_+^{(1)} \otimes \eta_+ + \xi_+^{(2)} \otimes \eta'_{\mp} + (\text{cc}),$$

$$\epsilon_2 = \xi_+^{(1)} \otimes \eta_+ + \xi_+^{(2)} \otimes \eta'_{\mp} + (\text{cc}).$$  \hspace{1cm} (F.9)

Note that this spinor ansatz is different from the usual $\mathcal{N} = 2$ ansatz as found in e.g. \cite{30}. The spurious deformations are not physical, so they are not really in the spectrum. However, their partners in the RR-fields are. In total, the extra sector contains four scalars: one from $\delta B_{(2,0)} + \delta B_{(0,2)}$, from $\delta C_1$, from $\delta C_3$ and from $\delta C_5$. We also have four vectors: one from the metric (along $e^5$), from $C_3$, from $C_5$ and from $B$. This makes up the bosonic content of a massive gravitino multiplet in $\mathcal{N} = 2$. Note that since there is an extra internal spinor, we also have an extra gravitino.
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