THE STRUCTURE OF HALOS: IMPLICATIONS FOR GROUP AND CLUSTER COSMOLOGY

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ABSTRACT

The dark matter halo mass function is a key repository of cosmological information over a wide range of mass scales, from individual galaxies to galaxy clusters. N-body simulations have established that the friends-of-friends (FOF) mass function has a universal form to a surprising level of accuracy (∼ 10%). The high-mass tail of the mass function is exponentially sensitive to the amplitude of the initial density perturbations, the mean matter density parameter, \( \Omega_m \), and to the dark energy controlled late-time evolution of the density field. Observed group and cluster masses, however, are usually stated in terms of a spherical overdensity (SO) mass which does not map simply to the FOF mass. Additionally, the widely used halo models of structure formation—and halo occupancy distribution descriptions of galaxies within halos—are often constructed exploiting the universal form of the FOF mass function. This again raises the question of whether FOF halos can be simply related to the notion of a spherical overdensity mass. By employing results from Monte Carlo realizations of ideal Navarro–Frenk–White (NFW) halos and N-body simulations, we study the relationship between the two definitions of halo mass. We find that the vast majority of halos (80%–85%) in the mass-range \( 10^{12.3}–10^{15.5} \, h^{-1} M_\odot \) indeed allow for an accurate mapping between the two definitions (∼ 5%), but only if the halo concentrations are known. Nonisolated halos fall into two broad classes: those with complex substructure that are poor fits to NFW profiles and those “bridged” by the (isodensity-based) FOF algorithm. A closer investigation of the bridged halos reveals that the fraction of these halos and their satellite mass distribution is cosmology dependent. We provide a preliminary discussion of the theoretical and observational ramifications of these results.

Key words: large-scale structure of universe – methods: N-body simulations

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1. INTRODUCTION

A large number of astronomical and cosmological observations now provide compelling evidence for the existence of dark matter. Although the ultimate nature of the dark matter remains unknown, its large-scale dynamics is completely consistent with that of a self-gravitating collisionless fluid. In an expanding universe, the gravitational instability is the driver of the growth of structure in the dark matter, the final distribution arising from the nonlinear amplification of primordial density fluctuations. The existence of localized, highly overdense clumps of dark matter, termed halos, is an essential feature of nonlinear gravitational collapse in cold dark matter models.

Dark matter halos occupy a central place in the paradigm of structure formation: Gas condensation, resultant star formation, and eventual galaxy formation occur within halos. The distribution of halo masses—the halo mass function—and its time evolution, are sensitive probes of cosmology, particularly so at low redshifts, \( z < 2 \), and high masses. This last feature allows cluster observations to constrain the dark energy content, \( \Omega_{\Lambda} \), and the equation of state parameter, \( w \) (Holder et al. 2001). In addition, phenomenological modeling of the dark matter in terms of the halo model (reviewed in Cooray & Sheth 2002) requires knowledge of the halo mass distribution and density profiles, as does the halo occupancy distribution (HOD) approach to modeling galaxy bias.

Because accurate theoretical results for the mass function (and other halo properties) do not exist, many numerical studies of halos and their properties, and of the mass function, have been carried out over widely separated mass and redshift ranges. Despite the intuitive simplicity and practical importance of the halo paradigm, halo definitions and characterizations have been somewhat ad hoc, mostly because of the lack of an adequate theoretical framework. For the purposes of this work, there are two main ways to define halos, both based on determining the halo boundary. The first one is the friends-of-friends algorithm (FOF; Einasto et al. 1984; Davis et al. 1985), a percolation motivated halo finder, which tracks isodensity contours. Note that there is no notion of halo center in the FOF definition, although the center can be suitably defined after finding the halo. The other definition is based on finding spherical overdensities (SO). This algorithm adopts a certain definition for the halo center (often the gravitational potential minimum of the halo), and then grows spheres around it until the mean volume density reaches a given overdensity, defined with respect to either critical density, \( \rho_c \), or the mean density of matter in the universe, \( \rho_m \). The two halo mass definitions will be explained in more detail in Section 2.

In this paper, we will make use of two crucial results that have been well established by numerical studies. The first is that spherically averaged halo profiles are well described by the two-parameter NFW profile (Navarro et al. 1996, 1997) (this shape is consistent with observational studies of clusters), and second, that a simple “universal” form for the FOF halo mass function (with linking length, \( b = 0.2 \)) holds for standard cold dark matter cosmologies (Jenkins et al. 2001). A detailed understanding of both these numerically established results remains elusive.
The universality of the FOF mass function has been recently verified to the level of $\lesssim 10\%$ accuracy for essentially all observationally relevant redshifts ($z \lesssim 10$) by several simulation efforts (e.g., Heitmann et al. 2006b; Reed et al. 2003, 2007; Lukić et al. 2007). The result is potentially very useful, because at this level of accuracy there is no longer any reason to simulate individual cosmologies, as the universal form already covers the parametric region of interest. However, there is one serious problem: the universal form of the mass function does not hold for the SO mass as defined and used by observers when determining the masses of galaxy groups and clusters (White 2001; Voit 2005). In principle, isodensity-based methods can be used in observations, but require significantly more work than the SO approach.

At this point, one could ask the question whether the SO and FOF masses could be mapped to each other if more information regarding halo properties were available (or one could forsake universality and attack the SO mass function problem directly via simulations, e.g., Evrard et al. 2002; Tinker et al. 2008). The aim here is to proceed along the first path and investigate whether an effective solution to the problem can be found (for an earlier discussion, see White 2002, who noted that FOF and SO masses are correlated, but with a significant scatter). We first show that even for perfect NFW halos, there is no simple direct mapping between FOF and SO masses, because of a significant dependence on the halo concentration. Also, the mapping depends on the number of particles sampling a given halo, something that needs to be taken into account when interpreting results from simulations. However, we establish the useful result that for NFW halos sampled by a given number of particles, a two-parameter map utilizing concentration and particle number indeed connects the two masses (with a small Gaussian scatter, quantified below in Section 3).

The key question is whether these relationships for idealized NFW halos survive when applied to the more realistic case of halos within cosmological $N$-body simulations. We find that this is indeed the case for halos that can be considered to be relatively isolated (a notion to be made more concrete in Section 3), and not possess significant substructure; i.e., approximately $80\%-85\%$ of all halos in the mass range $10^{12.5}$-$10^{15.5} h^{-1} M_{\odot}$ explored by the simulations. This fraction of isolated halos is close to the conclusion of Evrard et al. (2008) who analyzed results from a large suite of simulations. For these halos, the two-parameter map derived above succeeds remarkably well in accurately converting the FOF mass function to the corresponding SO mass function, at the $\sim 5\%$ level—the current level of descriptive accuracy as limited by the robustness of halo definitions and numerical results from simulations (Lukić et al. 2007; Heitmann et al. 2008). We show that the concentration dependence of the FOF–SO mass relation is significant at the current levels of accuracy for the determination of halo masses. Conversion between FOF and SO masses will incur significant error if halo concentration is not considered. To transform between the FOF and the SO mass function, the scatter in concentration must also be considered. Our work has implications for observationally determined mass functions, and for HOD and other methods of deriving mock galaxy catalogs.

An additional point is that, in the $N$-body simulations, there not only exists a simple relationship between the halo concentration and the SO (or FOF) mass with a (relatively) large scatter, but that the scatter can be very well fit by a Gaussian distribution at a given mass. Using this simple concentration–mass relation and its Gaussian variance, one may go directly from the FOF mass function to the SO mass function or vice versa. This procedure solves the mass function mapping problem for the subset of isolated halos that comprise the bulk of the halo population. It does not, however, enable one to transform from the universal FOF mass function to a chosen SO mass function because of the $15\%-20\%$ fraction of FOF halos with irregular morphologies, most of which are “bridged” halos (density peaks connected by high density filaments or ridges). A potential way around this difficulty is to treat explicitly the “multiplicity” of apparently discrete SO halos within FOF halos in the transformation between FOF and SO mass functions. This possibility is under investigation.

Based on our runs for two cosmologies, we have good evidence that the fraction of bridged halos rises as a function of mass, and that this fraction is also “universal,” i.e., more or less independent of the cosmology when written in units of $M/M_*$, where $M_*$ is the characteristic halo mass scale set by matching the rms linear density fluctuation to the threshold density for collapse. We also find that the fraction of halos with major satellites as a function of the satellite mass fraction (with respect to the main halo) is cosmology dependent. This may pave the way for constraining cosmology from clusters of galaxies in a new way, essentially independent of the sampling volume, and therefore with enhanced immunity against selection effects. At the very least, using the major satellite halo fraction should provide a valuable cross-check for cosmological constraints derived from the mass function in the conventional manner.

2. MASS DEFINITIONS

The spherical overdensity and FOF methods are the two main approaches to defining halos and their associated masses in simulations. SO identifies halos by identifying spherical regions with prescribed spherical overdensities $\Delta$:

$$M_{\Delta} = \frac{4\pi}{3} R_{\Delta}^3 \Delta \rho_c,$$

(1)

where $\rho_c$ is the critical density. Overdensities are sometimes stated with respect to the background density: $\rho_b = \Omega_m \rho_c$, here we restrict ourselves to defining them with respect to $\rho_c$. An often-used value for the overdensity is $\Delta = 200$, roughly the theoretically predicted value given by the spherical collapse model, $18\pi^2$, for virialized halos in an Einstein–de Sitter universe. For the currently favored $\Lambda$CDM model ($\Omega_m = 0.7$, $\Omega_{\Lambda} = 0.3$), spherical collapse actually predicts a smaller overdensity at virialization: $\Delta \approx 100$. X-ray observers, on the other hand, prefer higher density contrasts, $\Delta = 500$ or 1000, because structures on those scales are much brighter, and more relaxed compared to the outer regions.

The main drawback of the SO mass definition is that it is somewhat artificial, enforcing spherical symmetry on all objects, while in reality halos often have an irregular structure (e.g., White 2002; Figure 1 of this paper). For some applications, such an approach may well be founded (e.g. X-ray cluster analysis for relaxed clusters), but may not be universally applicable. Furthermore, defining an SO mass can be ambiguous, since for two close density peaks, the corresponding SO spheres might overlap, and one has to decide how to distribute particles between them (or assign them to both, breaking mass conservation).

The FOF algorithm, on the other hand, is not based on the notion of a certain overdensity structure, but defines instead an object bound by some isodensity contour. The mass of a halo
is then simply the sum of all particles inside a given contour. By linking particles which are separated at most by the distance \( \Delta \) (where \( n \) is the number density of particles in the simulation, and \( b \) is the so-called “linking length”), the FOF method, in effect locates an isodensity surface of

\[
\rho_{\text{iso}} \approx k b^{-3} \rho_b, \tag{2}
\]

where \( k \) is a constant of order 2 (Frenk et al. 1988). For \( b = 0.2 \), and the concordance ΛCDM cosmology, this leads to \( \rho_{\text{iso}} = 75 \rho_c \). Given their percolation-centric nature, FOF halos can have complicated shapes and topologies (Figure 1).

3. MASS MAPPING FROM MOCK HALOS

In order to address the relation of FOF and SO masses, we first turn to a controlled test using idealized “mock” halos. These are taken to be spherical dark matter halos with the NFW density profile

\[
\rho(r) = \frac{\rho_s}{r/r_s (1 + r/r_s)^2}, \tag{3}
\]

where \( \rho_s \) and \( r_s \) are the core density and scale radius respectively. Instead of \( \rho_s \) and \( r_s \), it is often convenient to use physically more transparent quantities: the SO mass \( M_s \) and the concentration \( c = r_s/R_s \);

\[
\rho_s = \frac{\Delta \rho_c c^3}{3 \left[ \ln(1 + c) - c/(1 + c) \right]}; \tag{4}
\]

\[
r_s = \left( \frac{3 M_s}{4 \pi \Delta \rho_c} \right)^{1/3}. \tag{5}
\]

The cumulative mass within a radius \( r \) can be calculated as

\[
M(r) = \int_0^r 4\pi r'^2 \frac{\rho_s}{r/r_s (1 + r/r_s)^2}dr = 4\pi \rho_s r_s^3 \left[ \ln(1 + r/r_s) - (r/r_s)/(1 + r/r_s) \right]. \tag{6}
\]

While it is still unclear whether the very inner parts of the halos (~1% of \( R_{200} \)) have density profiles steeper than NFW (e.g., Ghigna et al. 2000; Jing & Suto 2000; Klypin et al. 2001; Navarro et al. 2004; Reed et al. 2005), the inner asymptotic slope is not of concern here, and does not affect our results.

We generate mock NFW halos in the following way: first we fix the SO mass (\( M_s = M_{200} \)) of a halo and choose the number of particles which will reside in it (\( N_{200} \)). Then we populate the halo with particles according to the NFW distribution such that we enforce the desired mass to be \( M_{200} \) within the radius \( R_{200} \). We then extend the NFW distribution further out, adding particles to a “halo tail.” Note that for our mock halo tests, we have kept \( M_{200} \) fixed, and varied \( N_{200} \) and concentration, \( c \). The choice of \( \Delta = 200 \) can easily be changed to some other desired value such as \( \Delta = 500 \) or 1000 as more appropriate for cluster studies. In any case, for a given NFW profile choice, all overdensity masses are immediately fixed, so there is no lack of generality in our specific choice (which corresponds to an approximate notion of the “virial mass;” Navarro et al. 1996, 1997).

Knowing \( M_{200} \) for all the mock halos, we now determine the FOF mass for every halo. Because the particles are randomly sampled inside a halo (following the NFW density profile), one cannot expect that for every realization of a mock halo, the FOF finder will return exactly the same mass. Given a large number of mock halos with the same density profile and statistical independence of the realizations, the central limit theorem predicts a Gaussian distribution for the FOF masses. Indeed, just as expected, a normal distribution gives an excellent description for \( M_{\text{FOF}}/M_{200} \). Thus, one can not only determine to what SO mass a certain \( M_{\text{FOF}} \) corresponds (on average), but can also quantify the systematic deviation of an FOF halo finder through a standard deviation (Figures 2). The Gaussian spread of FOF masses is centered around a mean value that shifts systematically with the number of sampling particles, \( N \), as empirically noted by Warren et al. (2006; Figure 3).

Besides this \( N \)-dependence, we also wish to examine how \( M_{\text{FOF}}/M_s \) depends on the underlying profile. We have found that this dependence leads to another source of bias for FOF masses relative to SO masses. As the concentration of a (fixed SO mass) halo increases, the isodensity contours will move towards smaller radii, making the FOF mass smaller. Note that this effect exists even in the case of infinite mass resolution. In Figure 3, we show average values of \( M_{\text{FOF}} \) over a range of particle numbers and concentrations. It is clear that one cannot accurately match a given \( M_{200} \) to a corresponding \( M_{\text{FOF}} \) without the concentration being specified. Concentration variation from \( c \approx 20 \) (typical for galaxies) to \( c \approx 5 \) (typical for clusters; Bullock et al. 2001; Eke et al. 2001) corresponds to systematic FOF mass shifts of 30%, much larger than can be tolerated by the accuracy to which the FOF mass function can currently be determined numerically (~5%). For any given \( N_{200} \), this concentration dependence follows the functional form

\[
\frac{M_{\text{FOF}}}{M_{200}} = \frac{a_1}{c^2} + \frac{a_2}{c} + a_3, \tag{7}
\]

where the coefficients \( a_1, a_2, a_3 \), depend on \( N_{200} \) only (Table 1).
The number of Monte Carlo samples are 10^6, 10^5, and 10^4 for M concentration, the next logical step is to see whether the mean such a low intrinsic scatter in the mass relationship for a given below. This expectation turns out to be valid, as shown approach based on idealized halos may well provide an adequate we are interested in an averaged quantity, the halo mass, an (Kasun & Evrard 2005; Allgood et al. 2006). Nevertheless, as accretion, and in fact are much better described as ellipsoids it should be noted that actual simulated halos are not expected to actually applies to individual halos in N-body simulations. Here, should be noted that actual simulated halos are not expected to be spherical due to the episodic and anisotropic nature of mass accretion, and in fact are much better described as ellipsoids (Kasun & Evrard 2005; Allgood et al. 2006). Nevertheless, as we are interested in an averaged quantity, the halo mass, an approach based on idealized halos may well provide an adequate description. This expectation turns out to be valid, as shown below.

4. MASS MAPPING IN N-BODY SIMULATIONS

In order to investigate the validity of the mock halo mass relationships, we use results from four cosmological simulations for two flat ΛCDM cosmologies, each simulated with 174 and 512 h^{-1} Mpc boxes. The pre-WMAP, high-σ_8 cosmology has the following parameters: matter density, Ω_m = 0.3; dark energy density, Ω_λ = 0.7; fluctuation amplitude, σ_8 = 1.0; Hubble constant h = 0.7 (in units of 100 km s^{-1} Mpc^{-1}); primordial spectral index, n_s = 1; and the Bardeen et al. (1986) transfer function with γ = Ω_m h. For the WMAP-3 compatible cosmology runs, the parameters are: Ω_m = 0.26, Ω_λ = 0.74, σ_8 = 0.75, h = 0.71, n_s = 0.938, and a transfer function generated using CMBFAST (Seljak & Zaldarriaga 1996). We use the parallel gravity solver GADGET2 (Springel 2005) to follow the evolution of 512^3 dark matter particles starting from a redshift z = 99, high enough to satisfy the initial redshift requirements given in Lukić et al. (2007). The particle masses are 3.3 × 10^9 and 8.3 × 10^{10} h^{-1} M_⊙ for the high-σ_8 run, and 2.8 × 10^9 and 7.2 × 10^{10} h^{-1} M_⊙ for the WMAP-3 cosmology. These masses are small enough to comfortably resolve groups and clusters to the level required for this study (see, e.g., Power et al. 2003; Reed et al. 2005; Neto et al. 2007). The FOF mass functions from these simulations are in very close agreement with the results of Lukić et al. (2007), well within a few percent. By using cosmologies with normalizations that bracket the currently favored cosmology (e.g., Spergel et al. 2007), we are able to show that our results are applicable to any likely cosmology, once (cosmology dependent) halo concentrations are specified.

To carry out a realistic test of the mass relationships, we adopt the following procedure: (1) first run an FOF halo finder on the

Figure 2. Distribution of b = 0.2 FOF masses for NFW halos with concentrations c = 3 (left panel), and c = 10 (right panel), sampled with different particle numbers: 100 (blue), 1000 (green), 10,000 (red). The number of Monte Carlo samples are 10^6, 10^5, and 10^4 for N_{200} = 100, 1000, and 10,000, respectively. The solid curves are Gaussian fits. Note that the two panels have different units along both axes.

(A color version of this figure is available in the online journal.)

Well sampled halos, with N > 1000, are characterized by a small variance in the M_{200}/M_{200} ratio, with a maximum value of σ ~ 0.02–0.03, depending on the concentration. With such a low intrinsic scatter in the mass relationship for a given concentration, the next logical step is to see whether the mean M_{200} (M_{200}, c) relationship obtained from the mock NFW halos actually applies to individual halos in N-body simulations. Here, it should be noted that actual simulated halos are not expected to be spherical due to the episodic and anisotropic nature of mass accretion, and in fact are much better described as ellipsoids (Kasun & Evrard 2005; Allgood et al. 2006). Nevertheless, as we are interested in an averaged quantity, the halo mass, an approach based on idealized halos may well provide an adequate description. This expectation turns out to be valid, as shown below.

| Coeff. | 10^2 | 10^3 | 3 × 10^3 | N_{200} | 6 × 10^3 | 10^4 | 10^5 | 10^6 |
|--------|------|------|---------|---------|---------|------|------|------|
| a_1    | -0.3887 | -0.3063 | -0.2790 | -0.2368 | -0.2210 | -0.1970 | -0.1642 | -0.1374 |
| a_2    | 1.6195  | 1.4130  | 1.3669  | 1.2849  | 1.2459  | 1.2157  | 1.1392  | 1.0900  |
| a_3    | 1.0715  | 1.0313  | 1.0226  | 1.0081  | 1.0008  | 0.9960  | 0.9800  | 0.9714  |

Note. Best-fit coefficients for different N_{200}, as obtained from the mock halo analysis. For all values of N_{200}, the functional form of the fit is given by Equation (7).
Figure 3. Ratio of the ($b = 0.2$) FOF mass to $M_{200}$ for NFW mock halos with different concentrations and particle number, $N$, but the same value of $M_{200}$. Low concentration halos have up to a factor of 2 higher FOF mass than $M_{200}$. For high concentration halos, the ratio of the two mass definitions is closer to unity, the FOF mass always being higher.

(A color version of this figure is available in the online journal.)

Figure 4. Distribution of distances between FOF center of mass, and potential minimum for $512 \, h^{-1}$ Mpc box (red) and $174 \, h^{-1}$ Mpc box (blue), scaled by $R_{200}$.

(A color version of this figure is available in the online journal.)

Figure 5. Same as Figure 4, but for the WMAP-3 cosmology.

(A color version of this figure is available in the online journal.)

final particle distribution, and define halo centers by identifying the local potential minima, for all halos with $N > 1000$.

(2) Construct individual SO0 profiles around these minima, thereby determining $M_{200}$. The halo density is computed in 32 logarithmically equidistant bins, and we fit the NFW profile treating both $r_s$ and $\rho_s$ as free parameters. As a consistency check, we use an alternative approach, where $M_{200}$ is measured directly from the mass within a sphere, and NFW is treated as a one-parameter function (by fixing $\rho_s$ such that the enclosed overdensity is $200\rho_c$). No significant differences were found between the two approaches.

The $N > 1000$ halo particle cut keeps the variance in the mass ratios small (see Figures 2 and 3) and also allows stable calculations of the individual halo concentrations. Details of the procedures followed will be given elsewhere (D. Reed et al. 2009, in preparation). For each FOF halo we find its center of mass from all the particles linked together by the halo finder. On occasion, the FOF finder connects apparently distinct halos (bridging); these halos may well be in some stage of merging. Since it makes little sense to define an SO profile and an associated concentration for very close halos and those undergoing major mergers, we use the distance between the center of mass and the potential minima to exclude such FOF halos. Such an offset parameter is often applied on SO halos to separate relaxed from unrelaxed halos (e.g., D’Onghia & Navarro 2007; Macciò et al. 2007; Neto et al. 2007). In Figures 4 and 5, we show the distribution of that distance ($d$) for all halos with $N > 1000$ from both simulation boxes. While most of the halos appear to be isolated objects where the difference between the two center definitions is due to substructure, there are outliers at high mass, and even objects where the FOF center of mass is more than $R_{200}$ away from the potential minimum.

To proceed further, we first set aside all halos with $d/R_{200} > 0.4$. Although this cut is somewhat arbitrary, the results are relatively insensitive to the particular choice, as discussed below. Furthermore, the mock halo analysis on regular NFW halos shows that, even at low concentrations, one expects approximately $M_{\text{FOF}}/M_{200} \sim 1.5$ (see Figure 3). Larger values therefore are a signal of a potential merger, as was verified directly by confirming with the simulation results. In Figures 6 and 7, where we plot both “isolated” (blue) and “bridged” (red) halos, the strong correlation between our cut, based on the difference between halo mass and potential centers, and
Figure 6. Scatterplot of the ratio of FOF and SO(200) masses from the simulations as a function of the measured concentration for (1) halos passing the criterion \( d/R_{200} < 0.4 \) (blue), where \( d \) is the distance between the center of mass and the potential minima (see discussion in the text), and (2) halos not passing this criterion (red). The solid line shows the mock halo prediction for halos with particle number, \( N_{200} = 10^3 \), which dominate the sample. The dashed line is the relation between \( M_{\text{FOF}} \) and \( M_{200} \) as defined in White (2001).

(A color version of this figure is available in the online journal.)

Figure 7. Same as Figure 6, but for the WMAP-3 cosmology. Note that the x-axis has a different scale.

(A color version of this figure is available in the online journal.)

the high values of \( M_{\text{FOF}}/M_{200} \) (with respect to the mock halo expectation) can be easily verified.

Finally, we compare our halo selection criterion with an SO analysis of FOF halos: for each halo we find the minimum potential particle and \( R_{200} \) around it, and then move to the next particle in the potential hierarchy which resides outside \( R_{200} \) (if inside, we define it as a piece of substructure rather than a ‘satellite halo’ bridged by the FOF procedure), find \( R_{200} \) and \( M_{200} \) for the second halo, and iterate this procedure until all FOF particles are exhausted. When separate SO halos overlap we assign particles in the overlapping region to all SO halos, keeping the overdensity idea straightforward, but breaking mass conservation. Of course, if one goes down to a few particles, then virtually all FOF halos will be resolved into multiple SO objects. But if the threshold of the satellite mass is raised to 20% of the main halo mass, most of the FOF halos appear as a single SO halo. The two methods: \( d/R_{200} > 0.4 \), and \( M_{\text{satellite}}/M_{\text{main}} > 0.2 \) correlate extremely well, agreeing in 85%–90% of all cases (the agreement is worse for larger masses, and better for smaller halo masses). This gives us additional confidence that our cutoff criterion separates isolated from bridged halos. We will return to an analysis of the excluded halos by both of the discussed exclusion criteria in Section 5.

Figures 8 and 9 show the quality of the estimated \( M_{200} \) using the measured \( M_{\text{FOF}} \) and our formula based on NFW profiles. We take into account the dependence of \( M_{\text{FOF}}/M_{200} \) on the number of particles in a halo, as given in Table 1. This effect is of a minor concern here, as we analyze only halos sampled with more than 1000 particles. While for both cosmologies it is possible to successfully apply our formula on isolated halos (Figure 8), for the excluded halos the overdensity mass is virtually always overestimated (Figure 9). This large systematic error, usually more than a factor of 2, may seriously corrupt HOD-based analyses, if conducted on FOF halos.

The halo exclusion cut eliminates only about 15%–20% of all halos, so it is not very statistically significant, though certainly not negligible. For the retained halos, we now apply the \( M_{\text{FOF}}(M_{200}, c) \) relationship determined by the mock halo
results of Figure 3, as encapsulated in the fits specified in Table 1. The results of this halo by halo mass mapping test are shown in Figures 10 and 11 for the mass function, where the measured mass functions are displayed in terms of a ratio to a fitting form for the FOF mass function given by Warren et al. (2006). This ratio is taken only for ease of interpretation, as any other mass function fit would have done just as well. The undernormalization of the FOF mass function relative to the fit is simply due to the exclusion procedure described above. Note that the FOF and SO mass functions, as numerically determined, differ by as much as 20%–40% depending on the mass bin. However, application of the mock halo mass relationship to every individual FOF halo correctly reproduces the SO mass function at the 5% level, the current (numerical) limiting accuracy of mass function determination. The success of this simple mapping idea is a testimony to the accuracy of the NFW description for (spherically averaged) realistic halos in simulations, and is consistent with the overall conclusion of Evrard et al. (2008) that the vast majority of cluster-scale halos are structurally regular.

Using the expression for the cumulative NFW mass (Equation (6)), we can find the mass for any desired overdensity \( \Delta \) in terms of \( M_{200} \); defining \( M_c = M_{\Delta}/M_{200} \), we have

\[
M_c = A(c) \left[ \ln \left( 1 + \sqrt{\frac{200}{\Delta} M_c c} \right) - \sqrt{\frac{200}{\Delta} M_c c} \right],
\]

where \( A(c) \) is a prefactor which depends on \( c \) only:

\[
A(c) = \frac{1}{\ln(1+c) - c/(1+c)}.
\]

Employing this approach one can easily move from one SO mass function to another, and in Figure 11 we show that this mass transformation gives accurate results for halos in simulations. Furthermore, this shows that if one is interested in any overdensity other than 200 (as considered in our mock halo analysis), our best fit for \( M_{FOF}/M_{200} \) (Equation (7) and Table 1) can simply be rescaled for any \( M_{\Delta} \) using Equation (8).

The results shown in Figures 10 and 11 depend only weakly on the cut imposed by a particular value of \( d/R_{200} \). Choosing a value below \( d/R_{200} = 0.4 \) such as 0.3 is more conservative; one loses more halos (another 5%), but the mass function mapping results remain excellent. Increasing the cut threshold to 0.5 adds 5% more halos while the mapping accuracy remains more or less the same. Beyond this point the results slowly degrade, as is to be expected.

With this important result at the level of individual halos in hand, the global mass function can be realized without knowing individual halo concentrations, and independent of cosmology, provided one has a form for the (mean) concentration–mass relation for SO (or FOF) halos as well as the PDF for the scatter in this relation. The latter cannot be ignored since the scatter in the concentration–mass relation is known to be significant (Jing 2000; Bullock et al. 2001; Eke et al. 2001; Macciò et al. 2007; Neto et al. 2007). Particularly in the mass regime typical

\[
Figure 9. Same as Figure 8, but for the excluded halos. A systematic overestimation of \( M_{200} \) is evident.
\]

\[
Figure 10. Measured mass functions normalized to the Warren et al. (2006) fit as an (arbitrary) reference, for the high \( \sigma_8 \) (upper panel) and WMAP-3 (lower panel) cosmologies. Black: FOF halo masses with \( b = 0.2 \) and bridged halos removed as shown in Figure 6. Red: \( M_{200} \) masses measured from the simulation for the same set of halos, and using the same (FOF) halo centers. Blue: the mass function for \( M_{200} \) halos using the idealized mock halo prediction (Figure 3 and Table 1), the measured FOF masses for each halo as mapped to the predicted SO mass. The agreement between measured (red) and predicted (blue) mass functions is excellent, better than 5%.
\]
might be. Nevertheless, our exclusion was designed mostly to eliminate the bridged halos; our results show that the unrelaxed population is apparently subdominant at least in terms of biasing the mass function results. Even so, it is clear that the existence of these types of substructured halos has ramifications for the simple HOD program, although the quantitative impact needs to be studied.

The halos that are bridged by the FOF procedure are typically close neighbors, the majority being partners in the hierarchical process of structure formation via halo merging (Busha et al. 2005). Some of these close neighbors might be “backsplash halos” that have previously been within \( \mathcal{R}_{200} \) (see Gill et al. 2005; Ludlow et al. 2009). In both the high-\( \sigma_8 \) and \( \Lambda \)CDM cosmologies, we find that the fraction of bridged halos has a tendency to increase with increase in mass. This is as expected from the hierarchical merging picture since very massive halos are still forming at the current epoch; this effect is clearly shown in Figure 13. We have checked that the two different-sized boxes (for each cosmology) agree well in the region of overlap, supporting the argument that numerical effects (finite mass and force resolution) are negligible for this consideration. For the two box sizes, the mass resolution differs by a factor of approximately 25, and the force resolution by a factor of 3.

The overall effect can certainly depend on cosmology: the results from the \( \Lambda \)CDM cosmology are clearly separated from the high \( \sigma_8 \) cosmology (Figure 13). Since the structures grow differently in the two different cosmologies (due to different \( \sigma_8 \) and \( \Omega_m \)), we can try to parameterize our exclusion as a function of \( M/M_\ast \), where \( M_\ast \) is the characteristic collapse mass at the current epoch, defined through

\[
\sigma[M_\ast(z)] = 1.686,
\]

where \( \sigma \) is the variance of the linear density fluctuation field \( P(k) \), smoothed by a top-hat filter \( W(k,M) \) on a scale \( M \), and normalized to the present epoch \( z = 0 \) by the growth function \( d(z) \):

\[
\sigma^2(M,z) = \frac{d^2(z)}{2 \pi^2} \int_0^\infty k^2 P(k)W^2(k, M)dk.
\]

As shown in Figure 14, with the mass rescaled in terms of \( M_\ast \), the fraction of bridged halos agrees for the two cosmologies and may very well be “universal.” This intriguing fact indicates, first, that our method of excising bridged halos (the principle, not necessarily the specific choice of \( d/R_{200} > 0.4 \)) is physically well motivated. Second, if the universality is borne out, the bridged halo fraction can be combined with the cosmology independent mock halo analysis, to yield a method for translating the universal FOF mass function to any desired SO mass function. Moreover, these results suggest that the bridged halo fraction can also provide a separate probe of cosmology, being particularly sensitive to the same parameters as the mass function itself (Figure 13).

An additional way of probing the growth of structure in the Universe using clusters, aside from the mass function, would be to measure the fraction of isolated clusters versus those that have (major) satellites. In our simulations, we measure the fraction of multiple SO dark matter halos in the mass range of interest for clusters: \( M_{200} \geq 10^{14} M_\odot/h \) (see also Evrard et al. 2008). If we plot this fraction as a function of \( f \), where \( f \) is defined through \( M_{\text{satellite}} \geq f M_{\text{main}} \) we find again that the two cosmologies considered are clearly separated, as shown in Figure 15. The advantage of this analysis compared to the mass function method

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**Figure 11.** Testing the mapping to masses other than \( M_{200} \) with mass functions shown as in Figure 10. Black: \( M_{200} \) masses measured from the simulation. Red: \( M_{100} \) and \( M_{500} \) masses measured from the simulation using the same halo centers. Blue: idealized NFW predictions for \( M_{100} \) and \( M_{500} \) using the measured \( M_{200} \) mass for each halo. Measured and predicted quantities (red vs. blue) are again in very good agreement.

(A color version of this figure is available in the online journal.)

for clusters, i.e., halo masses above \( \sim 3 \times 10^{14} h^{-1} M_\odot \), the variation in concentration with mass is in fact much smaller than the concentration scatter for halos of similar mass.

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**5. THE BRIDGED HALOS**

We now turn to understanding the FOF halos that cannot be simply mapped as individual NFW profiles. Broadly speaking, we find that these halos are of two types: (1) Halos with density bridges across major substructures, and (2) halos with complex substructure (“unrelaxed”). Halos of the first type are those largely excluded by our halo mass and potential centers-based cut and correspond mostly to the high mass ratio region in Figures 6 and 7. While our cut is very efficient in terms of identifying bridged halos, there is a very small contamination fraction due to chance symmetric bridging which does not lead to significant differences between the mass and potential minima. The second type of halos corresponds largely to the low concentration/low mass ratio region. Representative halo types are shown in Figure 12: typical isolated halo (upper panel), bridged halo (middle panel), and complex substructure (lower panel).

It is clear that the idea of a single concentration or a simple mass ratio \( M_{\text{FOF}}/M_{\text{SO}} \) makes little sense for either the bridged halos or the unrelaxed halos. For the unrelaxed halos, absent a sub-halo analysis, it is not even clear what an appropriate \( M_{\text{SO}} \)
Figure 12. Top panel: typical isolated FOF halo (FOF-linked particles shown as white dots) with NFW concentration, $c = 9.0$, and $M_{\text{FOF}}/M_{\text{200}} = 1.15$ (profile fit to the right). Green dots are particles within $R_{\text{200}}$ of the corresponding SO halo. Middle panel: example of a bridged halo. The SO halo found at the FOF center has concentration $c = 8.1$ (the NFW profile fit is a good fit), however the mass ratio $M_{\text{FOF}}/M_{\text{200}} = 1.8$ is high due to the bridged minor halo in the left upper corner. Bottom panel: halo with major substructure, for which the NFW profile is not a good fit.

(A color version of this figure is available in the online journal.)

is that it does not require measurements in a controlled volume, and will work for a random sample of observed galaxy clusters. Depending on observational possibilities (McMillan et al. 1989; Mohr et al. 1995; Zabludoff & Zaritsky 1995; Jones & Forman 1999; Kolokotronis et al. 2001; Jeltema et al. 2005; Ramella et al. 2007), this might provide a new way of characterizing cosmologies using clusters of galaxies, or at least be a valuable method of cross checking results from mass function constraints.
Figure 13. Distribution of bridged halos as a function of mass for the high $\sigma_8$ and WMAP-3 cosmologies. In both cosmologies, the relative fraction of such halos tends to increase with increasing mass. The shaded regions are Poisson error bars. (A color version of this figure is available in the online journal.)

Figure 14. Possible universality of the bridged halo fraction: the same data as in Figure 13, but with the mass now scaled by $M_\star$. (A color version of this figure is available in the online journal.)

The halo outliers with values of $M_{\text{FOF}}/M_{200} > 1.5$ are also a possible source of systematic bias for certain HOD applications. Given some halo mass bin above the fiducial mass cutoff for a given HOD, a bridged halo would be assigned a central galaxy with the same probability as an isolated halo. The probability of a satellite galaxy in a bridged halo (with the main halo having high mass companion(s)) is likely significantly higher than in an isolated halo. Therefore, applying the same HOD to both halo types would downweight the number of satellite galaxies, the precise amount depending on the mass range considered.

6. CONCLUSIONS AND DISCUSSION

We have presented results from an analysis of idealized NFW halos and N-body simulations with the aim of clarifying the connection between FOF and SO halos, focusing mainly on the issue of halo masses and attempting to account for some of the unavoidable difficulties in simplifying a multiscale problem in terms of primitive halo concepts. We found that a large fraction of FOF halos in N-body simulations (80%–85%) are relatively isolated and well fitted by NFW profiles. This allows them to have SO counterparts, albeit the mass mapping is a two-parameter function $M_{\text{SO}} = M_{\text{SO}}(M_{\text{FOF}}, c)$, inferred from the properties of idealized NFW halos ($c$ is the NFW halo concentration). In principle, this mock halo technique can be trivially extended to $M_\Delta$ with $\Delta$ values more directly useful for cluster analyses (e.g., $\Delta = 500, 800, 1000$), or indeed to any other useful definition of the observable mass.

The rest of the halos, a fraction of 15%–20%, appear to be dominated mainly by bridged halos. These halos consist of apparently localized structures (visually, or according to the SO halo definition) linked via density “ridges” into a common FOF halo, as discussed in Section 4. This degree of bridging is roughly consistent with X-ray observations of clusters, where in approximately 10%–20% of all cases there is a significant second component roughly within $R_{100}$, corresponding to the scale length of a $b = 0.2$ FOF halo (A. Vikhlinin 2007, private communication). We have found that the bridged halo fraction rises as a function of mass, and when rescaled by the collapse mass scale $M_\star$, also appears to be universal. We also find that in the cluster mass regime, the fraction of halos with major satellites as a function of the satellite’s mass fraction is cosmology dependent.
The bridged FOF halo fraction complicates the procedure for transforming the global mass function. Accurate mapping between the global FOF and SO mass function must take into account SO multiplicity within FOF halos due to the bridging (which should be distinguished from the substructure mass function). Fortunately, if the bridged halo fraction is universal, then this problem can be (approximately) solved by one more iteration of the procedure described here. A simple prescription for handling the bridging problem, for example, may be the simultaneous use of two different linking lengths as a way of solving the bridging problem, for example, may be the iteration of the procedure described here. A simple prescription for transforming the global mass function. Accurate mapping between the global FOF and SO mass function must take into account the bridging halo fraction which should be almost free of bridging artifacts to at least the 5% level. This possibility is currently under investigation.

In this work, systematic and statistical uncertainties were held to ~ 5%, which represents the current state of the art in determining the halo mass function. The sensitivity of halo masses to simulation parameters such as force and mass resolution has not yet been satisfactorily controlled below this level. While further improvement is not ruled out, the universality of the FOF mass function is not known to be valid at or better than this level either.

The finite bridged halo fraction points to the existence of some level of bias when applying simple HOD schemes for the distribution of galaxies in halos, due to the existence of (minor/major) halo substructure. In standard HOD methods, halos are often selected, or assumed to be selected, by the FOF algorithm. However, this standard method then assumes a spherically-symmetric (usually NFW) distribution of satellite galaxies within halos, which is possibly at odds with a significant fraction of real halos (see, e.g., Berlind & Weinberg 2002; Tinker et al. 2005). The fraction of problematic, irregular morphology FOF halos is mass-dependent, creating thereby a mass dependent source of error. Furthermore, any concentration dependence of the fraction of bridged FOF halos makes it difficult to parameterize halo properties purely as a function of halo mass, which is standard within HOD methods.

Despite these difficulties, the availability of sufficiently high resolution simulations should yield a completely satisfactory HOD more or less independent of the particular halo definition used (FOF or SO), provided that a realistic satellite distribution is implemented. The point is that, even with such a simulation, a simplified description of halos such as an NFW profile for populating halos with galaxies would certainly fail for a not insignificant fraction of halos, and be a cause of systematic errors.

As an alternative to mapping SO mass functions beginning with the uniform form of the FOF mass function, and utilizing the cosmology-dependent concentration–mass relation and its scatter, one could instead take the more computationally expensive approach of computing SO mass functions from simulations that sample a range of plausible cosmologies (e.g., Tinker et al. 2008). The additional expense of such an approach can be drastically reduced by the use of efficient statistical sampling and interpolation techniques that have been successfully demonstrated for cosmic microwave background temperature anisotropy and for the mass power spectrum (Heitmann et al. 2006a; Habib et al. 2007). This work is currently in progress.

We remain agnostic as to the value of particular choices of halo definitions and masses in cosmological applications. For X-ray observations of relaxed clusters, the SO approach appears to be more natural since one fits directly to a spherically averaged profile as is observational practice. High-resolution views of the gas distribution in clusters (e.g., Jeltema et al. 2005) are hardly consistent with spherical symmetry, however, and the physics of the underlying robustness of the mass-observable relations remains to be fully established. Turning to other applications such as optical group and cluster and subcluster member identification, there may be no option but the use of (modified) FOF techniques. Analogous to our bridged FOF halos, Sunyae–Zel’ dovich observations are likely to suffer from bridging of closely-neighboring clusters. Mock catalogs for ongoing and future cluster observations carried out via the Sunyaev–Zel’dovich effect have been built using FOF definitions for clusters (albeit with shorter linking lengths than $b = 0.2$), as the possible systematics from using spherical halo definitions are not clear (Schulz & White 2003).

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