Measuring $D^0-\bar{D}^0$ Mixing and Relative Strong Phases
at a Charm Factory

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ABSTRACT

We propose a set of measurements at a charm factory to determine the $D^0-\bar{D}^0$ mixing parameters $\Delta M$ and $\Delta \Gamma$ and the strong phase difference $\delta$ between doubly-Cabibbo-suppressed (DCS) and Cabibbo-favored (CF) neutral $D$ decays into $K^-\pi^+$. The method can also be used to measure strong phase differences between other corresponding DCS and CF amplitudes. These phase differences are important for studies of $D^0-\bar{D}^0$ mixing.

The time development of decays of neutral $D$ mesons to doubly-Cabibbo-suppressed (DCS) modes such as $K^+\pi^-$ exhibits interesting behavior. At $t = 0$ the only term in the amplitude is the direct DCS $D^0 \rightarrow K^+\pi^-$ term, but for $t > 0$ a $D^0-\bar{D}^0$ mixing contribution appears. The interference of this term with the DCS contribution involves the lifetime and mass differences of the neutral $D$ mass eigenstates as well as the final-state strong phase difference $\delta$ between the Cabibbo-favored (CF) $\bar{D}^0 \rightarrow K^+\pi^-$ and DCS $D^0 \rightarrow K^+\pi^-$ decay amplitudes.

A recent study of this process by the CLEO Collaboration [1], when combined with direct measurements of the lifetime difference between CP-even and CP-odd neutral $D$ mesons [2, 3, 4, 5, 6], suggests that this strong phase difference may be large [7]. The FOCUS Collaboration has also presented data in agreement with CLEO’s results.

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Several attempts to estimate this phase theoretically [8, 10, 11, 12] involve various degrees of model-dependence. A recent proposal to measure the phase [13] assumes that \( \Delta I = 0 \) and \( \Delta I = 1 \) DCS \( D \to K\pi \) amplitudes into \( I = 1/2 \) final states involve equal strong phases. In the present note we suggest an entirely assumption-free method for determining this phase experimentally. We show that related measurements are capable of determining \( D^0 - \bar{D}^0 \) mixing parameters at the percent level.

A charm factory operating at the \( \psi(3770) \) resonance produces \( D^0\bar{D}^0 \) pairs in a state of definite charge-conjugation eigenvalue \( C = - \). At slightly higher energies one can produce such pairs with \( C = + \) [14]. One may tag one of the neutral \( D \) mesons as a CP eigenstate through its decays into CP eigenmodes, such as \( K_S(\pi^0, \rho^0, \omega, \eta, \eta', \phi), K^+K^- \), and \( \pi^+\pi^- \). (These decays are important in a different context of studying the weak phase \( \gamma \) in \( B \to D^0K \) [15, 16].) The other neutral \( D \) meson must then have opposite CP if \( C(D^0\bar{D}^0) = - \) and the same CP if \( C(D^0\bar{D}^0) = + \) [14]. One measures its decay rate into \( K^-\pi^+ \), which includes an interference between CF and DCS amplitudes. The measured rate thus is given in terms of the already-measured CF and DCS rates and the relative strong phase \( \delta \), which permits a determination of \( \delta \). (This phase plays an important role in a method [17] for measuring \( \gamma \) in \( B \to D^0K \).)

We now discuss details of this method, beginning with conventions and notation and ending with an estimate of the achievable precision. For the majority of our discussion we shall neglect CP violation in neutral \( D \) mixing and decays, which is expected to be very small in the Standard Model. Our master equations for correlated hadronic decays of \( D^0\bar{D}^0 \) pairs, Eqs. (10)–(11) below, apply also to the case of CP violation. Towards the end of our study we will argue that including CP violation beyond the Standard Model has a negligible effect on the proposed measurement of the relative strong phase \( \delta \).

The mass eigenstates in the neutral \( D \) meson system may be defined as

\[
D_1 \equiv p|D^0\rangle + q|\bar{D}^0\rangle ,
\]

\[
D_2 \equiv p|D^0\rangle - q|\bar{D}^0\rangle ,
\]

where \( |p|^2 + |q|^2 = 1 \), with corresponding eigenvalues \( \mu_{1,2} \equiv M_{1,2} - i\Gamma_{1,2}/2 \). Neglecting CP violation in \( D^0 - \bar{D}^0 \) mixing, we adopt the convention [7] in which \( D_1 \) is the CP-odd and \( D_2 \) the CP-even state, choosing further \( p = q = 1/\sqrt{2} \). We define

\[
M \equiv \frac{M_1 + M_2}{2} , \quad \Gamma \equiv \frac{\Gamma_1 + \Gamma_2}{2} , \quad \mu \equiv \frac{\mu_1 + \mu_2}{2} ,
\]

\[
x \equiv \frac{M_2 - M_1}{\Gamma} , \quad y \equiv \frac{\Gamma_2 - \Gamma_1}{2\Gamma} , \quad \Delta \mu \equiv \mu_2 - \mu_1 = \Gamma(x - iy) .
\]

The decay amplitudes are defined as

\[
\langle K^-\pi^+|D^0\rangle \equiv A e^{i\delta_R} , \quad \langle K^-\pi^+|\bar{D}^0\rangle \equiv \bar{A} e^{i\delta_W} , \quad \delta \equiv \delta_R - \delta_W ,
\]

where \( \delta_R \) and \( \delta_W \) refer, respectively, to “right-sign” and “wrong-sign” strong phases in the \( K^-\pi^+ \) system.
With the above-mentioned convention for CP eigenstates, self-consistency requires us to take $D^0 \equiv c\bar{u}$, $\bar{D}^0 \equiv -u\bar{c}$ as in Ref. [18]. The decay $D_1 \rightarrow K^+K^-$ then is forbidden. With this convention, in the SU(3) limit, we would have $\bar{A}/A = -V_{ud}V_{us}^*/V_{us}V_{cs} = +\tan^2\theta_C \simeq 0.05$, where $\theta_C$ is the Cabibbo angle. We thus take, more generally, $\bar{A}/A \equiv r > 0$.

We note that (4) implies $\langle K^+\pi^-|D^0\rangle = -\bar{A}^*e^{i\delta_W}$ which is equal, by a U-spin $s \leftrightarrow d$ substitution and a replacement of CKM elements, to $[V_{cd}V_{us}/V_{ud}V_{cs}]A^*e^{i\delta_R}$. Thus $\delta = 0$ in the SU(3) limit [19, 20].

We begin by considering the process studied by CLEO and FOCUS [5, 8], which does not require a symmetric charm factory. A state which is identified at the time independent determination of $\delta$ in the SU(3) limit [19, 20]. If this is so, the last term in (7) may be inaccessible even though there may exist evidence for the $y$ term. However, most (but not all) estimates of $x$ and $y$ within the Standard Model are considerably less than a percent [21]. If this is so, the last term in (4) may be inaccessible even though there may exist evidence for the $ry'$ term, and we need an independent determination of $\delta$. This may be achieved through experiments at a charm factory, as we now show.
We consider a $D^0\bar{D}^0$ pair produced in a state of definite charge-conjugation eigenvalue $\eta_C = \pm 1$. The initial states $\Psi_{\eta_C}$, namely,

$$\Psi_{\pm} = \frac{1}{\sqrt{2}} [D^0(\hat{p})\bar{D}^0(\hat{-p}) \pm \bar{D}^0(\hat{p})D^0(\hat{-p})] ,$$

evolve in time to

$$\Psi_{\pm}(t, \bar{t}) = \frac{1}{\sqrt{2}} e^{-i\mu(t+\bar{t})} \left\{ \cos[\Delta \mu(t \pm \bar{t})/2] [D^0(\hat{p})\bar{D}^0(\hat{-p}) \pm \bar{D}^0(\hat{p})D^0(\hat{-p})] \right. \\
\left. \pm i \sin[\Delta \mu(t \pm \bar{t})/2] [D^0(\hat{p})D^0(\hat{-p}) \pm \bar{D}^0(\hat{p})\bar{D}^0(\hat{-p})] \right\} ,$$

(9)

Here $t$ refers to the proper time of a state traveling along the $+\hat{p}$ direction, while $\bar{t}$ refers to one traveling along $-\hat{p}$.

We now consider decays of these correlated systems into various final states, searching in particular for interference effects depending on $\delta$. In all cases we integrate with respect to proper time, since vertex separation in a symmetric $e^+e^-$ “charm factory” is likely to be problematic. An early study of correlated $D^0\bar{D}^0$ decays into specific flavor final states, assuming $\delta = 0$, was carried out by Bigi and Sanda [22]. More recently Xing [23] has considered both time-dependent and time-integrated decays into correlated pairs of states, including some effects of a nonzero final state phase difference. However, he has not considered the cases (3$^-$) and (3$^+$) below, from which we propose to measure $\delta$. For completeness, we derive general expressions for time-integrated decay rates into a pair of final states $f_1$ and $f_2$, from $C = -1$ and $C = +1 \, D^0\bar{D}^0$ states, in agreement with [28]:

$$\Gamma^{C=-1}(f_1, f_2) = \frac{1}{2} |A(-)|^2 \left[ \frac{1}{1 - y^2} + \frac{1}{1 + x^2} \right]$$

$$+ \frac{1}{2} |B(-)|^2 \left[ \frac{1}{1 - y^2} - \frac{1}{1 + x^2} \right] , \quad (10)$$

$$\Gamma^{C=+1}(f_1, f_2) = \frac{1}{2} |A(+)|^2 \left[ \frac{1 + y^2}{(1 - y^2)^2} + \frac{1 - x^2}{(1 + x^2)^2} \right]$$

$$+ \frac{1}{2} |B(+)|^2 \left[ \frac{1 + y^2}{(1 - y^2)^2} - \frac{1 - x^2}{(1 + x^2)^2} \right]$$

$$+ 2 \text{Re} \left\{ A^{(+)*} B^{(+)} \left[ \frac{y}{(1 - y^2)^2} + \frac{ix}{(1 + x^2)^2} \right] \right\} , \quad (11)$$

We focus on the CP-even states, corresponding to $y > 0$.
where
\[
A^{(\pm)} = \langle f_1|D^0\rangle\langle f_2|D^0\rangle \pm \langle f_1|D^0\rangle\langle f_2|D^0\rangle , \quad (12)
\]
\[
B^{(\pm)} = \langle f_1|D^0\rangle\langle f_2|D^0\rangle \pm \langle f_1|D^0\rangle\langle f_2|D^0\rangle . \quad (13)
\]

The rate expressions simplify if one of the states (say, \(f_2\)) is a CP eigenstate \(S_\zeta\) with eigenvalue \(\zeta = \pm 1\):
\[
\Gamma^{C=-1}(f_1, S_\zeta) = |A_{S_\zeta}|^2\frac{|\langle f_1|D^0\rangle + \zeta \langle f_1|\overline{D^0}\rangle|^2}{1 - y^2} , \quad (14)
\]
\[
\Gamma^{C=+1}(f_1, S_\zeta) = |A_{S_\zeta}|^2\frac{|\langle f_1|D^0\rangle - \zeta \langle f_1|\overline{D^0}\rangle|^2}{(1 + \zeta y)^2} , \quad (15)
\]
where \(A_{S_\zeta} = \langle S_\zeta|D^0\rangle\), and we have used \(CP|D^0\rangle = -|\overline{D^0}\rangle\).

The expressions for products of amplitudes in Eqs. (12) and (13) can be easily generalized allowing for CP violation:
\[
A^{(\pm)} = \frac{p}{q} A_1 A_2 (\lambda_2 \pm \lambda_1) , \quad (16)
\]
\[
B^{(\pm)} = \frac{p}{q} A_1 A_2 (1 \pm \lambda_2 \lambda_1) , \quad (17)
\]
where
\[
A_i \equiv \langle f_i|D^0\rangle , \quad \tilde{A}_i \equiv \langle f_i|\overline{D^0}\rangle , \quad \lambda_i \equiv \frac{q}{p} \frac{\tilde{A}_i}{A_i} . \quad (18)
\]
Here and everywhere we use a normalization in which \(\Gamma(D^0 \to f_i) = |A_i|^2\).

Keeping terms up to order \(r^2, x^2, y^2\) in the expressions for rates, and assuming CP conservation we list the following results for various cases:

\[C = -1\ D^0\overline{D^0}\ states:\]
\[\begin{align*}
(1^-) \ K^-\pi^+(-\hat{p})K^-\pi^+(-\hat{p}). \\
\Gamma^{(\pm)}(K^-\pi^+, K^-\pi^+) &= \frac{1}{2} A^4 |1 - r^2 e^{-2i\delta}|^2 (x^2 + y^2) \\
&\approx \frac{1}{2} A^4 (x^2 + y^2) , \quad (19)
\end{align*}\]

where \(A\) was defined in Eq. (4). This process serves to measure mixing effects when normalized by the one which follows.

\[\begin{align*}
(2^-) \ K^-\pi^+(-\hat{p})K^+\pi^+(-\hat{p}). \\
\Gamma^{(\pm)}(K^-\pi^+, K^+\pi^-) &= A^4 |1 - r^2 e^{-2i\delta}|^2 [1 - \frac{1}{2} (x^2 - y^2)] \\
&\approx A^4 [1 - 2r^2 \cos 2\delta - \frac{1}{2} (x^2 - y^2)] . \quad (20)
\end{align*}\]
The dominant contribution is proportional to $A^4$, so by comparing this case with the previous one we can learn the combination $x^2 + y^2$ describing mixing. Similar information could be gleaned from a fit to the time-distribution of a single tagged $D^0$ as described above, if $x^2 + y^2$ is sufficiently large, but this may not be the case. The small interference terms in (20) are probably unmeasurable.

\[(3^-) \ K^+\pi^+(\hat{p})S_\zeta(-\hat{p}). \]
\[
\Gamma^{(-)}(K^+\pi^+, S_\zeta) = A^2 A^2_{S_\zeta} |1 + \zeta r e^{-i\delta}|^2 (1 + y^2) \\
\approx A^2 A^2_{S_\zeta} (1 + 2 \zeta r \cos \delta). \tag{21}
\]

Recall $r \approx \tan^2 \theta_C \simeq 0.05$ in the SU(3) limit. By comparing rates with $\zeta = +1$ final states such as $K^+K^-$ and $\zeta = -1$ final states such as $K_S(\rho, \omega, \phi)$ one can measure the ratio $(1 + 2 r \cos \delta)/(1 - 2 r \cos \delta)$ and, given an independent measurement of $r$, one can obtain $\cos \delta$.

\[(4^-) \ K^+\pi^+(\hat{p})\ell^-(-\hat{p}). \]

Using a leptonic $\overline{D^0}$ flavor tag and defining $A_{\ell^-} = \langle \ell^-X|\overline{D^0}\rangle$, one finds
\[
\Gamma^{(-)}(K^+\pi^+, \ell^-) = A^2 A^2_{\ell^-} [1 - \frac{1}{2} (x^2 - y^2)]. \tag{22}
\]

This process serves as a normalization for the one which follows describing the opposite-sign $D^0$ flavor tag.

\[(5^-) \ K^+\pi^+(\hat{p})\ell^+(-\hat{p}). \]
\[
\Gamma^{(-)}(K^+\pi^+, \ell^+) = A^2 A^2_{\ell^+} [r^2 + \frac{1}{2} (x^2 + y^2)] . \tag{23}
\]

where $A_{\ell^+} = \langle \ell^+X|D^0\rangle = A_{\ell^-}$. By comparing this process with the previous one, we obtain $r^2 + (x^2 + y^2)/2$. This may be of interest for mixing parameters if $r, x, y$ are of comparable size but, as mentioned, it is much more likely that $x, y \ll r$ in which case this process can be used to measure $r$.

\[(6^-) \ S_\zeta(\hat{p})\ell^+(-\hat{p}). \]
\[
\Gamma^{(-)}(S_\zeta, \ell^+) = A^2 A^2_{S_\zeta} (1 + y^2). \tag{24}
\]

Strictly speaking, the $y^2$ correction is one order higher in small parameters than we have been keeping, since $A^2_{S_\zeta}$ is already of order $r$. This process serves as a normalization for others.

$C = +1$ $D^0\overline{D^0}$ states:

\[(1^+) \ K^+\pi^+(\hat{p})K^-\pi^+(-\hat{p}). \]
\[
\Gamma^{(+)}(K^+\pi^+, K^-\pi^+) = 4A^4[r^2 + ry' + \frac{3}{8} (x^2 + y^2)]. \tag{25}
\]

This expression gives information similar to that learned from the time-dependence (7) in the decay of a single tagged neutral $D$. 

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\[
(2^+) \quad K^-\pi^+(-\hat{p})K^+\pi^-(-\hat{p}).
\]

\[
\Gamma^{(+)}(K^-\pi^+, K^+\pi^-) = A^4[1 + 2r^2 \cos 2\delta + 4r\tilde{y} - \frac{3}{2}(x^2 - y^2)] ,
\]

where \(\tilde{y} \equiv y \cos \delta + x \sin \delta\). The correction terms are probably unmeasurable, so this process serves as a normalization in comparison with the previous one.

\[
(3^+) \quad K^-\pi^+(-\hat{p})S_\zeta(-\hat{p}).
\]

\[
\Gamma^{(+)}(K^-\pi^+, S_\zeta) = A^2A^2_\zeta |1 - \zeta r e^{-i\delta}|^2(1 - 2\zeta y + 3y^2)
\approx A^2A^2_\zeta (1 - 2\zeta r \cos \delta)(1 - 2\zeta y) .
\]

There thus appears to be ample information to constrain \(\cos \delta\) once \(r\) and \(y\) are well-enough known.

\[
(4^+) \quad K^-\pi^+(-\hat{p})\ell^-(-\hat{p}).
\]

\[
\Gamma^{(+)}(K^-\pi^+, \ell^-) = A^2A^2_\ell [1 + 2r\tilde{y} - \frac{3}{2}(x^2 - y^2)] .
\]

There is a small difference in comparison with the corresponding \(C = -1\) case.

\[
(5^+) \quad K^-\pi^+(-\hat{p})\ell^+(-\hat{p}).
\]

\[
\Gamma^{(+)}(K^-\pi^+, \ell^+) = A^2A^2_\ell [r^2 + 2ry' + \frac{3}{2}(x^2 + y^2)] .
\]

In contrast to the corresponding \(C = -1\) case, this expression involves interference between mixing and DCS decay similar to Eq. \((25)\).

\[
(6^+) \quad S_\zeta(-\hat{p})\ell^+(-\hat{p}).
\]

\[
\Gamma^{(+)}(S_\zeta, \ell^+) = A^2_\zeta A^2_\zeta (1 - 2\zeta y + 3y^2) .
\]

This differs from the \(C = -1\) case by a term of first order in \(y\).

An additional set of cases involves the detection of one flavor eigenstate with direction \(\hat{p}\) and a different flavor eigenstate with direction \(-\hat{p}\). Examples are \(K\pi, K\rho\), and \(K^*\pi\). Processes with opposite-flavor final states [such as \(K^-\pi^+(-\hat{p})K^+\rho^+(-\hat{p})\)] serve for normalization. Processes with same-flavor eigenstates [such as \(K^-\pi^+(-\hat{p})K^+\rho^-(-\hat{p})\)] from \(C = -1\) \(D^0\overline{D}^0\) pairs then give rise to a leading term in the rate proportional to \(|r_i e^{-i\delta_i} - r_j e^{-i\delta_j}|^2\), where \(i\) and \(j\) denote two different channels, \(r_i\) is the ratio of DCS to CF amplitudes in the channel \(i\), and \(\delta_i\) is the strong phase difference between right-sign and wrong-sign decays in that channel. Given three such measurements, and independent determinations of two of the three \(r_i\), one can solve for the three phase differences \(\delta_i - \delta_j\) up to discrete ambiguities. Measurement of the third \(r_i\) reduces those ambiguities by a factor of 2 and provides one constraint. There remains an overall ambiguity in the common sign of all phases. One can thereby check whether the quantities \(\delta_i\) differ from channel to channel. The comparison of strengths of \(K^*\pi\) and \(K\rho\) Dalitz plot bands...
in (DCS) $D \to K\pi\pi$ and (CF) $D \to \bar{K}\pi\pi$ decays already indicate the possibility of this difference [24], since U-spin relations of flavor SU(3) [12] appear to be violated in such cases. This method is unable to provide absolute values of any of the phases $\delta_i$, in contrast to those based on the examples $(3^-)$ and $(3^+)$ above.

At this point let us comment briefly on how our results may be modified in the presence of CP violation. In the Cabibbo-Kobayashi-Maskawa framework CP violation in neutral $D$ mixing and decays, dominated by the first two generations, is very small and can be safely neglected. Extensions of the Standard Model could induce new sources of CP violation. The most likely sizable effect is a possible new CP violating phase, $\phi = \arg(q\bar{A}/pA)$, occurring in the interference between $D^0 - \bar{D}^0$ mixing and decay amplitudes into $K\pi$ or other hadronic states. Dependence on such a phase requires mixing and would affect, for instance, the term linear in $t$ in Eq. (7) [7] and the $ry'$ terms in Eqs. (25) and (24). However, it affects Eq. (21), from which $\cos \delta$ is obtained, only in terms quadratic in $x, y$, which we neglected. This is true also for possible CP violation in mixing, $|q/p| \neq 1$.

Other CP violating effects in Eq. (21) can come from direct CP violation in Cabibbo-favored $D^0 \to K^-\pi^+$ and in singly Cabibbo-suppressed $D^0 \to S_\zeta$ decays. This would introduce in the amplitudes $A$ and $A_{S_\zeta}$ corrections of order $|\langle K^+\pi^-|\bar{D}^0 \rangle/\langle K^+\pi^+|D^0 \rangle| - 1$ and $|\langle S_\zeta|\bar{D}^0 \rangle/\langle S_\zeta|D^0 \rangle| - 1$, respectively. Such effects are expected to be very small in extensions of the Standard Model, where direct CP violation was shown to be negligible even in DCS decays [23]. This can be checked directly by comparing $D^0$ and $\bar{D}^0$ branching ratios into CP-conjugate states. Furthermore, these small effects can be shown to occur only at second order when combining the rate of Eq. (21) with its CP-conjugate.

In order to estimate the total sample of events needed to perform a useful measurement of $\delta$, we note that the rates for the processes of interest are given in terms of the already measured CF and DCS rates and the relative strong phase $\delta$. Let us define an asymmetry

$$A \equiv \frac{\Gamma^-(S_+) - \Gamma^-(S_-)}{\Gamma^-(S_+) + \Gamma^-(S_-)} ,$$

(31)

where $\Gamma^-(S_\pm)$ is a rate for a $C = -1$ $D^0\bar{D}^0$ configuration to decay into a CP-eigenstate $S_\pm$ with direction $-\hat{p}$ and a flavor eigenstate such as $K^-\pi^+$ with direction $+\hat{p}$ [the case $(3^-)$ noted above]. Eq. (21) implies a small asymmetry, $A = 2r \cos \delta$. For a small asymmetry, a general result is that its error $\Delta A$ is approximately $1/\sqrt{N}$, where $N$ is the total number of events tagged with CP-even and CP-odd eigenstates. Thus we have

$$\Delta (\cos \delta) \simeq \frac{1}{2r\sqrt{N}} .$$

(32)

The number $N$ of CP-tagged events decaying to $K^-\pi^+$ is related to the total number of $D^0\bar{D}^0$ pairs $N(D^0\bar{D}^0)$ through $N \approx 0.01N(D^0\bar{D}^0)B(D^0 \to K^-\pi^+) \approx 4 \times 10^{-4}N(D^0\bar{D}^0)$, since the branching-ratio-times-efficiency factor for tagging CP eigenstates is only about 1.1% [13] (the total branching ratio into CP eigenstates is larger than about 5% [24]). With $r = 1.2 \tan^2 \theta \simeq 0.06$ [1, 8], one then has

$$\Delta (\cos \delta) \approx \frac{400}{\sqrt{N(D^0\bar{D}^0)}} .$$

(33)
The cross section for $e^+e^- \rightarrow \psi(3770)$ is about 10 nb at the peak \(^27\), while a foreseen integrated luminosity for a charm factory operating at this energy is about 3 fb\(^{-1}\) \(^28\).

One can thus envision collecting $3 \times 10^7 D\bar{D}$ pairs, of which half are charged and half are neutral. We are entitled to a factor of 2 for considering both $K^-\pi^+$ and $K^+\pi^-$ final states. We thus estimate that one may be able to reach an accuracy of about 0.07 in $\cos \delta$.

The achievable error on $(x^2 + y^2)^{1/2}$ is of the order of a percent. To see this, we note that the case (1\(^-$) mentioned above should yield about $15 \times 10^6 (x^2 + y^2) |B(D^0 \rightarrow K^-\pi^+)|^2 \simeq 2.2 \times 10^4 (x^2 + y^2) (K^+\pi^\pm) (K^+\pi^\mp)$ events under the conditions mentioned above, giving rise to sensitivity to $x^2 + y^2$ at the level of about $10^{-4}$. Modest improvements will be possible by adding other final states.

The parameter $y' = y \cos \delta - x \sin \delta$ is obtained from the $ry'$ terms occuring in the time-dependence (\(^7\)) in the decay of a single tagged neutral $D$, and in the rates (\(^23\)) and (\(^29\)) measured by producing $D^0\bar{D}^0$ pairs with $C = +1$. Since $r > x, y$, the accuracy of measuring $y'$ is expected to be better than of measuring $(x^2 + y^2)^{1/2}$. Once $\cos \delta$ is measured with the above calculated precision, separate measurements of $x$ and $y$ at a corresponding level of sensitivity may be achieved.

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