Utility-Scale Energy Storage in an Imperfectly Competitive Power Sector

Vilma Virasjoki\textsuperscript{a} Afzal S. Siddiqui\textsuperscript{a,b} Fabricio Oliveira\textsuperscript{a}
Ahti Salo\textsuperscript{a}

\textsuperscript{a} Aalto University

\textsuperscript{b} Stockholm University, e-mail address: asiddiq@dsv.su.se

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Introduction
Role of Flexible Generation (https://energy-charts.info/)

- Marginal costs: €30/MWh (lignite), €39/MWh (CCGT), €53/MWh (gas)
- Daily average prices: €22.85/MWh to €43.64/MWh
Is Energy Storage the Answer? (https://www.climatetechwiki.org/technology/jiqweb-ph)
Equilibrium Analysis of Storage

- Schill and Kemfert (2011) consider the use of storage in Germany without transmission constraints
- Sioshansi (2014) demonstrates when storage can reduce social welfare or increase GHG emissions (Sioshansi, 2011)
- Siddiqui et al. (2019) compare welfare impacts of ownership and market power
  - Cournot oligopoly: the merchant invests less than the welfare maximiser does to keep price differences high and benefit from temporal arbitrage (margin trading)
  - Perfect competition: the merchant invests in more capacity than a welfare maximiser does (volumetric trading)
- Nasrolahpour et al. (2016) use a bi-level model to assess storage investment by a merchant
- Dvorkin et al. (2018) consider transmission congestion but not strategic investors and market power at the lower level
Research Objective and Findings

- How are storage capacity and social welfare affected by the type of storage owner?
- Bi-level model for Western Europe with network and VRE: profit-maximising generators with a profit-maximising merchant investor (or a welfare-maximising entity)
- Market structure and spatio-temporal variations affect investment decisions more than the type of investor
  - Perfectly competitive lower level: 300 MWh of storage investment in Belgium and France with welfare transfer from producers to consumers
  - Cournot oligopolistic lower level: 100 MWh of storage investment in Germany and similar welfare transfers (although producers avoid losses)
- Impact of investor type: welfare maximiser never invests less vis-à-vis the merchant under perfect competition, but low storage-investment cost may spur a merchant to adopt more capacity vis-à-vis the welfare maximiser under Cournot oligopoly
Mathematical Formulation
Setup

- Inverse demand at each node, $D_{m,t,n}^{\text{int}} - D_{m,t,n}^{\text{slp}}q_{m,t,n}$
- DC load flow based on network transfer admittance, $H_{\ell,n}$, and susceptance, $B_{n,n'}$, with voltage angles, $v_{m,t,n}$
- Constant marginal costs, $C_{u}^{\text{conv}}$, with capacities, $G_{n,i',u}^{\text{conv}}$
- VRE has capacities $G_{n,i'}^{e}$ with availability factors, $A_{m,t,n}^{e}$
- Leader’s problem
  - Maximise welfare (or, profit if merchant) by investing in storage capacity, $\sum_{y \in Y} z_{n,j,y}R_{d}^{d}$
  - Anticipate the response of followers
- Followers’ problems
  - Power producers: maximise profit from net generation, $g_{m,t,n,i',u}^{\text{conv}} + g_{m,t,n,i'}^{e} + r_{m,t,n,i'}^{\text{out}} - r_{m,t,n,i'}^{\text{in}}$
  - ISO: maximise gross surplus by managing flows, $v_{m,t,n}$, and consumption, $q_{m,t,n}$
  - Merchant: maximise profit from storage operations, $r_{m,t,n,j}^{\text{out}} - r_{m,t,n,j}^{\text{in}}$
Playing Games

Central-Planning Model

Single-Level Optimization Problem
- Storage Investment
- Power Market Operations

Bi-Level Model

Upper-Level Optimization Problem
- Storage Investment

Lower-Level Optimization Problem
- Power Market Operations (ISO)
Mathematical Program with Primal and Dual Constraints (MPPDC)

- Replace lower-level problems (1)–(4), (5)–(10), and (11)–(20), \( \forall i' \in \mathcal{I}' \), by a single-agent quadratic programming (QP) problem using an extended-cost function.
- Replace lower-level QP by:
  - Primal constraints
  - Dual constraints
  - QP strong duality (Dorn, 1960; Huppmann and Egerer, 2015)
- After resolving bilinear terms in strong-duality expression and merchant’s upper-level objective function (23) via binary expansion, we render the bi-level problems as mixed-integer quadratically constrained quadratic programs (MIQCQPs).
- Also, we implement a benchmark central-planning problem that is a simple mixed-integer quadratic program (MIQP).
Numerical Examples
Network Topology
# Generation Technologies’ Marginal Costs (with CO\(_2\) price of 20\(\text{\euro}/\text{t}\)), Ramp Rates, and Emission Rates

| Type        | Marginal cost (\(\text{\euro}/\text{MWh}\)) | Max hourly ramping rate (%) | CO\(_2\) emissions per unit of electricity output (kg/kWh) |
|-------------|-----------------------------------------------|-----------------------------|----------------------------------------------------------|
| u1 (nuclear)| 9                                             | 10                          | 0                                                        |
| u2 (lignite)| 30                                            | 10                          | 0.94                                                     |
| u3 (coal)  | 44                                            | 20                          | 0.83                                                     |
| u4 (CCGT)  | 39                                            | 30                          | 0.37                                                     |
| u5 (gas)   | 53                                            | 30                          | 0.50                                                     |
| u6 (oil)   | 91                                            | 70                          | 0.72                                                     |
| u7 (hydro) | 0                                             | 30                          | 0                                                        |
## Installed Generation Capacity (GW) and Used Availability Percentages for Conventional Technology $u_1$–$u_7$, Solar, and Wind

| Node | Producer | $u_1$ | $u_2$ | $u_3$ | $u_4$ | $u_5$ | $u_6$ | $u_7$ | $S$  | $W$  |
|------|----------|-------|-------|-------|-------|-------|-------|-------|------|------|
| $n_1$ | Uniper   | -     | 0.9   | 3.2   | 2.7   | 0.5   | 1.2   | -     | -    | 0.3  |
|       | RWE      | 2.6   | 9.1   | 2.8   | 2.5   | 1.7   | -     | 0.3   | -    | 0.3  |
|       | EnBW     | 2.7   | 0.9   | 3.0   | 0.4   | -     | 0.4   | 0.2   | -    | 0.3  |
|       | Vattenfall | -    | -     | 2.9   | 0.6   | 0.9   | 0.1   | -     | -    | 0.6  |
|       | FringeD  | 4.2   | 7.4   | 9.3   | 10.9  | 2.2   | 0.4   | 1.3   | 40.1 | 54.6 |
| $n_2$ | EDF      | 63.1  | -     | 4.0   | 1.4   | -     | 7.0   | 15.0  | 0.3  | 1.5  |
|       | FringeF  | -     | -     | -     | 3.8   | 2.4   | -     | 3.6   | 6.5  | 12.3 |
| $n_3$ | Electrabel | 5.9  | -     | -     | 1.7   | 1.4   | -     | -     | -    | 0.5  |
| $n_6$ | EDF (Luminus) | -   | -     | -     | 0.4   | 0.4   | -     | -     | -    | 0.2  |
|       | FringeB  | -     | -     | -     | 1.0   | -     | -     | -     | 3.3  | 2.2  |
| $n_4$ | Electrabel | -   | -     | -     | 2.8   | 0.1   | -     | -     | -    | -    |
| $n_5$ | Essent/RWE | -   | -     | 1.3   | 1.9   | 0.6   | -     | -     | -    | -    |
| $n_7$ | Nuon/Vattenfall | -   | -     | 0.9   | 3.2   | 1.1   | -     | -     | -    | -    |
|       | FringeN  | 0.5   | 2.9   | 3.2   | 0.7   | -     | -     | 2.0   | 4.3  |      |
| %    | Available | 80    | 85    | 84    | 89    | 86    | 86    | 30    | Fig. | Fig. |
### Representative Weeks and Demand/VRE Clusters

| Week, \(m\) (1-52) | Weight, \(W_m\) | Demand profile | Wind profile | Solar profile |
|------------------|-----------------|----------------|--------------|--------------|
| 6                | 7/52            | high           | low          | low          |
| 18               | 12/52           | low            | high         | high         |
| 20               | 23/52           | low            | low          | high         |
| 47               | 10/52           | high           | high         | low          |

**Clusters for weekly demand and wind power generation**

**Clusters for weekly demand and solar PV power generation**
# Welfare Effects of Storage Investment on Social Welfare (SW), Investor Surplus (IS), Producer Surplus (PS), Consumer Surplus (CS), and Merchandising Surplus (MS) under PC

| Cost (€/MWh) | Model                      | SW (k€)     | IS (k€)     | PS (k€)     | CS (k€)     | MS (k€)     | Capacity (GWh) |
|--------------|----------------------------|-------------|-------------|-------------|-------------|-------------|----------------|
|              | No inv. PC                 | 2 201 628.15 | –           | 438 683.35  | 1 749 290.80 | 13 654.01   | –              |
| 65           | 1. CP / 2. SW-PC / 3. M-PC | +2.542      | +1.95       | -86.53      | +88.02      | -0.90       | 0.3            |
|              |                            | +2.541      | +2.23       | -82.50      | +81.90      | +0.92       | 0.2            |
| 50           | 1. CP / 2. SW-PC / 3. M-PC | +7.04       | +6.45       | -86.53      | +88.02      | -0.90       | 0.3            |
|              |                            | +7.04       | +6.45       | -86.53      | +88.02      | -0.90       | 0.3            |
| 35           | 1. CP / 2. SW-PC / 3. M-PC | +13.06      | +11.95      | -101.63     | +103.54     | -0.80       | 0.6            |
|              |                            | +12.83      | +11.98      | -90.93      | +93.13      | -1.34       | 0.5            |
Welfare Effects of Storage Investment on Social Welfare (SW), Investor Surplus (IS), Producer Surplus (PS), Consumer Surplus (CS), and Merchandising Surplus (MS) under PC Cost Model

| Cost (€/MWh) | SW (k€) | IS (k€) | PS (k€) | CS (k€) | MS (k€) | Capacity (GWh) |
|--------------|---------|---------|---------|---------|---------|---------------|
| –            | 2 201 628.15 | – | 438 683.35 | 1 749 290.80 | 13 654.01 | – |
| 65           | +2.542 | +1.95 | -86.53 | +88.02 | -0.90 | 0.3 |
| 50           | +7.04  | +6.45 | -86.53 | +88.02 | -0.90 | 0.3 |
| 35           | +13.06 | +11.95 | -101.63 | +103.54 | -0.80 | 0.6 |

1. CP / 2. SW-PC 3. M-PC  
2. SW-PC+2.542  
3. M-PC+2.23  
4. CP / 2. SW-PC+7.04  
5. M-PC+7.04  
6. CP / 2. SW-PC+13.06  
7. M-PC+12.83
### Welfare Effects of Storage Investment on Social Welfare (SW), Investor Surplus (IS), Producer Surplus (PS), Consumer Surplus (CS), and Merchandising Surplus (MS) under CO

| Cost (€/MWh) | Model | SW (k€)   | IS (k€) | PS (k€) | CS (k€) | MS (k€) | Capacity (GWh) |
|--------------|-------|-----------|---------|---------|---------|---------|----------------|
|              | No inv. CO | 1 923 769.99 | –       | 989 457.19 | 917 432.27 | 16 880.53 | –              |
| 55           | 4. SW-CO | +0.16 | -0.40 | +0.56 | +0.60 | -0.60 | 0.1 |
|              | 5. M-CO | -      | -      | -      | -      | -      | -              |
| 50           | 4. SW-CO | +0.66 | +0.10 | +0.56 | +0.60 | -0.60 | 0.1 |
|              | 5. M-CO | +0.66 | +0.10 | +0.56 | +0.60 | -0.60 | 0.1 |
| 25           | 4. SW-CO | +4.88 | +1.78 | +0.88 | +0.10 | +2.13 | 0.4 |
|              | 5. M-CO | +3.95 | +2.87 | -0.25 | +1.33 | -      | 0.2 |
| 15           | 4. SW-CO | +9.15 | +6.34 | +0.87 | +0.45 | +1.50 | 0.5 |
|              | 5. M-CO | +8.43 | +6.92 | -1.05 | +3.27 | -0.71 | 0.6 |
Welfare Effects of Storage Investment on Social Welfare (SW), Investor Surplus (IS), Producer Surplus (PS), Consumer Surplus (CS), and Merchandising Surplus (MS) under CO

| Cost (€/MWh) | Model   | SW (k€)     | IS (k€) | PS (k€)   | CS (k€) | MS (k€) | Capacity (GWh) |
|--------------|---------|-------------|---------|-----------|---------|---------|----------------|
| –            | No inv. CO | 1 923 769.99 | –       | 989 457.19 | 917 432.27 | 16 880.53 | –              |
| 55           | 4. SW-CO | +0.16       | -0.40   | +0.56     | +0.60   | -0.60   | 0.1            |
|              | 5. M-CO  |            |         |           |         |         |                |
| 50           | 4. SW-CO | +0.66       | +0.10   | +0.56     | +0.60   | -0.60   | 0.1            |
|              | 5. M-CO  | +0.66       | +0.10   | +0.56     | +0.60   | -0.60   |                |
| 25           | 4. SW-CO | +4.88       | +1.78   | +0.88     | +0.10   | +2.13   | 0.4            |
|              | 5. M-CO  | +3.95       | +2.87   | -0.25     | +1.33   | -       | 0.2            |
| 15           | 4. SW-CO | +9.15       | +6.34   | +0.87     | +0.45   | +1.50   | 0.5            |
|              | 5. M-CO  | +8.43       | +6.92   | -1.05     | +3.27   | -0.71   | 0.6            |
### Welfare Effects of Storage Investment on Social Welfare (SW), Investor Surplus (IS), Producer Surplus (PS), Consumer Surplus (CS), and Merchandising Surplus (MS) under CO Cost Model

| Cost (€/MWh) | SW (k€) | IS (k€) | PS (k€) | CS (k€) | MS (k€) | Capacity (GWh) |
|--------------|---------|---------|---------|---------|---------|----------------|
| –            | No inv. CO | 1 923 769.99 | – | 989 457.19 | 917 432.27 | 16 880.53 | – |
| 55           | 4. SW-CO | +0.16 | -0.40 | +0.56 | +0.60 | -0.60 | 0.1 |
|              | 5. M-CO | – | – | – | – | – | – |
| 50           | 4. SW-CO | +0.66 | +0.10 | +0.56 | +0.60 | -0.60 | 0.1 |
|              | 5. M-CO | +0.66 | +0.10 | +0.56 | +0.60 | -0.60 | 0.1 |
| 25           | 4. SW-CO | +4.88 | +1.78 | +0.88 | +0.10 | +2.13 | 0.4 |
|              | 5. M-CO | +3.95 | +2.87 | -0.25 | +1.33 | – | 0.2 |
| 15           | 4. SW-CO | +9.15 | +6.34 | +0.87 | +0.45 | +1.50 | 0.5 |
|              | 5. M-CO | +8.43 | +6.92 | -1.05 | +3.27 | -0.71 | 0.6 |
Optimal Storage Investment Size and Location under PC

Welfare maximiser

Merchant
Optimal Storage Investment Size and Location under CO

Welfare maximiser

Merchant
Conclusions
Summary

- Directly compare the impact of market structure and investor type on storage adoption
  - Market power affects investment more than the investor type
  - PC: higher investment capacity because of higher temporal price differentials, especially in nuclear-dominated Belgium and France
  - CO: higher but smoother prices, which results in storage arbitrage to be sought in Germany due to its high VRE capacity
  - Welfare maximiser generally invests in at least as much capacity as the merchant
  - Exception: low storage-investment cost spurs a merchant to adopt more capacity, i.e., to assume a volumetric strategy, under CO

- Future work: enhance solution methods for large-scale MIQCQP problem instances, represent uncertain VRE output, transmission expansion
Mathematical Appendix
\[
\max_{\Omega^{\text{ISO}}} \sum_{m \in M} \sum_{t \in T} \sum_{n \in N} W_m \left( D^{\text{int}}_{m,t,n} q_{m,t,n} - \frac{1}{2} D^{\text{slp}}_{m,t,n} q_{m,t,n}^2 \right) \\
\text{s.t.} \quad \sum_{n \in N} T_t \ell, n v_{m,t,n} - T_t K_{\ell} \leq 0 \left( \mu_{m,t,\ell} \right), \forall m, t, \ell \\
- \sum_{n \in N} T_t \ell, n v_{m,t,n} - T_t K_{\ell} \leq 0 \left( \mu_{m,t,\ell} \right), \forall m, t, \ell \\
q_{m,t,n} - \sum_{i' \in I'} \sum_{u \in U_{n,i'}} g_{m,t,n,i',u} - \sum_{i' \in I'} \sum_{e \in E} g_{m,t,n,i'} - \sum_{i \in I} r_{m,t,n,i}^\text{out} \\
+ \sum_{i \in I} r_{m,t,n,i}^\text{in} - \sum_{n' \in N} T_t B_{n,n'} v_{m,t,n'} = 0 \left( \theta_{m,t,n} \right), \forall m, t, n
\]

where \( \Omega^{\text{ISO}} \equiv \{ q_{m,t,n} \geq 0, v_{m,t,n} \ \text{u.r.s.} \} \)
Mathematical Appendix

Lower-Level Problems

ISO

\[
\max_{\Omega^{ISO}} \sum_{m \in M} \sum_{t \in T} \sum_{n \in N} W_m \left( D^\text{int}_{m,t,n} q_{m,t,n} - \frac{1}{2} D^\text{slp}_{m,t,n} q^2_{m,t,n} \right)
\]

s.t. \[
\sum_{n \in N} T_t H_{\ell,n} v_{m,t,n} - T_t K_{\ell} \leq 0 \ (\bar{\mu}_{m,t,\ell}), \ \forall m, t, \ell
\]

\[
- \sum_{n \in N} T_t H_{\ell,n} v_{m,t,n} - T_t K_{\ell} \leq 0 \ (\mu_{m,t,\ell}), \ \forall m, t, \ell
\]

\[
q_{m,t,n} - \sum_{i' \in I'} \sum_{u \in U_{n,i'}} g^\text{conv}_{m,t,n,i',u} - \sum_{i' \in I'} \sum_{e \in E} g^e_{m,t,n,i'} - \sum_{i \in I} r^\text{out}_{m,t,n,i}
\]

\[
+ \sum_{i \in I} r^\text{in}_{m,t,n,i} - \sum_{n' \in N} T_t B_{n,n'} v_{m,t,n'} = 0 \ (\theta_{m,t,n}), \ \forall m, t, n
\]

where \( \Omega^{ISO} \equiv \{ q_{m,t,n} \geq 0, v_{m,t,n} \ \text{u.r.s.} \} \)
\[
\max_{\Omega_{\text{ISO}}} \sum_{m \in M} \sum_{t \in T} \sum_{n \in N} W_m \left( D_{\text{int},m,t,n} q_{m,t,n} - \frac{1}{2} D_{\text{slp},m,t,n} q_{m,t,n}^2 \right)
\]
\[\text{s.t.} \quad \sum_{n \in N} T_t H_{\ell,n} v_{m,t,n} - T_t K_{\ell} \leq 0 \ (\bar{\mu}_{m,t,\ell}), \ \forall m, t, \ell \]
\[\quad - \sum_{n \in N} T_t H_{\ell,n} v_{m,t,n} - T_t K_{\ell} \leq 0 \ (\underline{\mu}_{m,t,\ell}), \ \forall m, t, \ell \]
\[\quad q_{m,t,n} - \sum_{i' \in \mathcal{I}', u \in \mathcal{U}_{n,i'}} g_{m,t,n,i',u} - \sum_{i' \in \mathcal{I}', e \in \mathcal{E}} g_{m,t,n,i'} + \sum_{i \in \mathcal{I}} r_{m,t,n,i}^\text{in} - \sum_{n' \in \mathcal{N}} T_t B_{n,n'} v_{m,t,n'} = 0 \ (\theta_{m,t,n}), \ \forall m, t, n \]

where \(\Omega_{\text{ISO}} \equiv \{ q_{m,t,n} \geq 0, v_{m,t,n} \ \text{u.r.s.} \} \)
Storage Operator $j$

\[
\max_{\Omega^j} \sum_{m \in M} \sum_{t \in T} \sum_{n \in N} W_m \left[ \frac{\theta_{m,t,n}}{W_m} (r_{m,t,n,j}^{\text{out}} - r_{m,t,n,j}^{\text{in}}) - C^{\text{sto}} r_{m,t,n,j}^{\text{out}} \right]
\]

\[
\text{s.t.} \quad r_{m,t,n,j}^{\text{sto}} - (1 - E_{j}^{\text{sto}}) T_t r_{m,t-1,n,j}^{\text{sto}} - E_{j}^{\text{in}} r_{m,t,n,j}^{\text{in}} + r_{m,t,n,j}^{\text{out}} = 0 \quad (\lambda_{m,t,n,j}^{\text{bal}}), \quad \forall m, t, n
\]

\[
= 0 \quad (\lambda_{m,t,n,j}^{\text{bal}}), \quad \forall m, t, n
\]

\[
r_{m,t,n,j}^{\text{in}} - T_t R_j^{\text{in}} \sum_{y \in Y} z_{n,j,y} R_y^{d} \leq 0 \quad (\lambda_{m,t,n,j}^{\text{in,p}}), \quad \forall m, t, n
\]

\[
r_{m,t,n,j}^{\text{out}} - T_t R_j^{\text{out}} \sum_{y \in Y} z_{n,j,y} R_y^{d} \leq 0 \quad (\lambda_{m,t,n,j}^{\text{out,p}}), \quad \forall m, t, n
\]

\[
r_{m,t,n,j}^{\text{sto}} - \sum_{y \in Y} z_{n,j,y} R_y^{d} \leq 0 \quad (\lambda_{m,t,n,j}^{\text{ub,p}}), \quad \forall m, t, n
\]

\[
R_{n,j} \sum_{y \in Y} z_{n,j,y} R_y^{d} - r_{m,t,n,j}^{\text{sto}} \leq 0 \quad (\lambda_{m,t,n,j}^{\text{lb,p}}), \quad \forall m, t, n
\]

where $\Omega^j \equiv \{ r_{m,t,n,j}^{\text{out}} \geq 0, r_{m,t,n,j}^{\text{in}} \geq 0, r_{m,t,n,j}^{\text{sto}} \}$
Storage Operator $j$

\[
\max_{\Omega^j} \sum_{m \in M} \sum_{t \in T} \sum_{n \in N} W_m \left[ \frac{\theta_{m,t,n}}{W_m} (r^\text{out}_{m,t,n,j} - r^\text{in}_{m,t,n,j}) - C^\text{sto} r^\text{out}_{m,t,n,j} \right]
\]

s.t.
\[
\begin{align*}
& r^\text{sto}_{m,t,n,j} - (1 - E^\text{sto}_j) T_t r^\text{sto}_{m,t-1,n,j} - E^\text{in}_j r^\text{in}_{m,t,n,j} + r^\text{out}_{m,t,n,j} = 0 \quad (\lambda^\text{bal}_{m,t,n,j}), \ \forall m, t, n \\
& r^\text{in}_{m,t,n,j} - T_t R^\text{in}_j \sum_{y \in Y} z_{n,j,y} R^d_y \leq 0 \quad (\lambda^\text{in,p}_{m,t,n,j}), \ \forall m, t, n \\
& r^\text{out}_{m,t,n,j} - T_t R^\text{out}_j \sum_{y \in Y} z_{n,j,y} R^d_y \leq 0 \quad (\lambda^\text{out,p}_{m,t,n,j}), \ \forall m, t, n \\
& r^\text{sto}_{m,t,n,j} - \sum_{y \in Y} z_{n,j,y} R^d_y \leq 0 \quad (\lambda^\text{ub,p}_{m,t,n,j}), \ \forall m, t, n \\
& \frac{R_{n,j}}{} \sum_{y \in Y} z_{n,j,y} R^d_y - r^\text{sto}_{m,t,n,j} \leq 0 \quad (\lambda^\text{lb,p}_{m,t,n,j}), \ \forall m, t, n
\end{align*}
\]

where $\Omega^j \equiv \{ r^\text{out}_{m,t,n,j} \geq 0, r^\text{in}_{m,t,n,j} \geq 0, r^\text{sto}_{m,t,n,j} \}$
Storage Operator $j$

$$\max_{\Omega^j} \sum_{m \in M} \sum_{t \in T} \sum_{n \in N} W_m \left[ \frac{\theta_{m,t,n}}{W_m} (r_{m,t,n,j}^{\text{out}} - r_{m,t,n,j}^{\text{in}}) - C_{\text{sto}}^{st} r_{m,t,n,j}^{\text{out}} \right]$$

s.t. $$r_{m,t,n,j}^{\text{sto}} - (1 - E_{j}^{\text{sto}}) T_t r_{m,t-1,n,j}^{\text{sto}} - E_{j}^{\text{in}} r_{m,t,n,j}^{\text{in}} + r_{m,t,n,j}^{\text{out}} = 0 \quad (\lambda_{\text{bal}}^{m,t,n,j}), \ \forall m, t, n \quad (5)$$

$$r_{m,t,n,j}^{\text{in}} - T_t R_{j}^{\text{in}} \sum_{y \in Y} z_{n,j,y} \overline{R}_{y}^{d} \leq 0 \quad (\lambda_{\text{in,p}}^{m,t,n,j}), \ \forall m, t, n \quad (6)$$

$$r_{m,t,n,j}^{\text{out}} - T_t R_{j}^{\text{out}} \sum_{y \in Y} z_{n,j,y} \overline{R}_{y}^{d} \leq 0 \quad (\lambda_{\text{out,p}}^{m,t,n,j}), \ \forall m, t, n \quad (7)$$

$$r_{m,t,n,j}^{\text{sto}} - \sum_{y \in Y} z_{n,j,y} \overline{R}_{y}^{d} \leq 0 \quad (\lambda_{\text{ub,p}}^{m,t,n,j}), \ \forall m, t, n \quad (8)$$

$$R_{n,j} \sum_{y \in Y} z_{n,j,y} \overline{R}_{y}^{d} - r_{m,t,n,j}^{\text{sto}} \leq 0 \quad (\lambda_{\text{lb,p}}^{m,t,n,j}), \ \forall m, t, n \quad (9)$$

where $\Omega^j \equiv \{ r_{m,t,n,j}^{\text{out}} \geq 0, r_{m,t,n,j}^{\text{in}} \geq 0, r_{m,t,n,j}^{\text{sto}} \}$
Storage Operator $j$

$$\max_{\Omega^j} \sum_{m \in M} \sum_{t \in T} \sum_{n \in N} W_m \left[ \frac{\theta_{m,t,n}}{W_m} (r_{m,t,n,j}^{out} - r_{m,t,n,j}^{in}) - C_{sto} r_{m,t,n,j}^{out} \right]$$

s.t.  

$$r_{m,t,n,j}^{sto} - (1 - E_{j}^{sto}) T_t r_{m,t-1,n,j}^{sto} - E_{j}^{in} r_{m,t,n,j}^{in} + r_{m,t,n,j}^{out} = 0 \ (\lambda^{bal}_{m,t,n,j}), \ \forall m, t, n $$

$$r_{m,t,n,j}^{in} - T_t R_{j}^{in} \sum_{y \in Y} z_{n,j,y} R_{y}^{d} \leq 0 \ (\lambda^{in,p}_{m,t,n,j}), \ \forall m, t, n $$

$$r_{m,t,n,j}^{out} - T_t R_{j}^{out} \sum_{y \in Y} z_{n,j,y} R_{y}^{d} \leq 0 \ (\lambda^{out,p}_{m,t,n,j}), \ \forall m, t, n $$

$$r_{m,t,n,j}^{sto} - \sum_{y \in Y} z_{n,j,y} R_{y}^{d} \leq 0 \ (\lambda^{ub,p}_{m,t,n,j}), \ \forall m, t, n $$

$$R_{n,j} \sum_{y \in Y} z_{n,j,y} R_{y}^{d} - r_{m,t,n,j}^{sto} \leq 0 \ (\lambda^{lb,p}_{m,t,n,j}), \ \forall m, t, n $$

where $\Omega^j \equiv \{ r_{m,t,n,j}^{out} \geq 0, r_{m,t,n,j}^{in} \geq 0, r_{m,t,n,j}^{sto} \}$
Storage Operator \( j \)

\[
\max_{\Omega^j} \sum_{m \in M} \sum_{t \in T} \sum_{n \in N} W_m \left[ \frac{\theta_{m,t,n}}{W_m} (r_{m,t,n,j}^{\text{out}} - r_{m,t,n,j}^{\text{in}}) - C_{m,t,n,j}^{\text{sto}} r_{m,t,n,j}^{\text{out}} \right]
\]

s.t.

\[
r_{m,t,n,j}^{\text{sto}} - (1 - E_j^{\text{sto}}) T_t r_{m,t-1,n,j}^{\text{sto}} - E_j^{\text{in}} r_{m,t,n,j}^{\text{in}} + r_{m,t,n,j}^{\text{out}} = 0 \quad (\lambda_{m,t,n,j}^{\text{bal}}), \quad \forall m, t, n
\]

\[
r_{m,t,n,j}^{\text{in}} - T_t R_j^{\text{in}} \sum_{y \in Y} z_{n,j,y} R_y^d \leq 0 \quad (\lambda_{m,t,n,j}^{\text{in,p}}), \quad \forall m, t, n
\]

\[
r_{m,t,n,j}^{\text{out}} - T_t R_j^{\text{out}} \sum_{y \in Y} z_{n,j,y} R_y^d \leq 0 \quad (\lambda_{m,t,n,j}^{\text{out,p}}), \quad \forall m, t, n
\]

\[
r_{m,t,n,j}^{\text{sto}} - \sum_{y \in Y} z_{n,j,y} R_y^d \leq 0 \quad (\lambda_{m,t,n,j}^{\text{ub,p}}), \quad \forall m, t, n
\]

\[
R_{n,j} \sum_{y \in Y} z_{n,j,y} R_y^d - r_{m,t,n,j}^{\text{sto}} \leq 0 \quad (\lambda_{m,t,n,j}^{\text{lb,p}}), \quad \forall m, t, n
\]

where \( \Omega^j \equiv \{ r_{m,t,n,j}^{\text{out}} \geq 0, r_{m,t,n,j}^{\text{in}} \geq 0, r_{m,t,n,j}^{\text{sto}} \} \)
Mathematical Appendix

Lower-Level Problems

**Storage Operator** \( j \)

\[
\max_{\Omega^j} \sum_{m \in M} \sum_{t \in T} \sum_{n \in N} W_m \left[ \frac{\theta_{m,t,n}}{W_m} (r_{m,t,n,j}^{\text{out}} - r_{m,t,n,j}^{\text{in}}) - C_{\text{sto}}^{\text{out}} r_{m,t,n,j}^{\text{out}} \right]
\]

s.t.

\[
\begin{align*}
\text{r}^{\text{sto}}_{m,t,n,j} - (1 - E_{j}^{\text{sto}}) T_t \text{r}^{\text{sto}}_{m,t-1,n,j} - E_{j}^{\text{in}} \text{r}^{\text{in}}_{m,t,n,j} + \text{r}^{\text{out}}_{m,t,n,j} &= 0 \ (\lambda_{m,t,n,j}^{\text{bal}}), \ \forall m, t, n \\
\text{r}^{\text{in}}_{m,t,n,j} - T_t R_{j}^{\text{in}} \sum_{y \in \mathcal{Y}} z_{n,j,y} \overline{R}_y &\leq 0 \ (\lambda_{m,t,n,j}^{\text{in,p}}), \ \forall m, t, n \\
\text{r}^{\text{out}}_{m,t,n,j} - T_t R_{j}^{\text{out}} \sum_{y \in \mathcal{Y}} z_{n,j,y} \overline{R}_y &\leq 0 \ (\lambda_{m,t,n,j}^{\text{out,p}}), \ \forall m, t, n \\
\text{r}^{\text{sto}}_{m,t,n,j} - \sum_{y \in \mathcal{Y}} z_{n,j,y} \overline{R}_y &\leq 0 \ (\lambda_{m,t,n,j}^{\text{ub,p}}), \ \forall m, t, n \\
\overline{R}_{n,j} \sum_{y \in \mathcal{Y}} z_{n,j,y} \overline{R}_y - \text{r}^{\text{sto}}_{m,t,n,j} &\leq 0 \ (\lambda_{m,t,n,j}^{\text{lb,p}}), \ \forall m, t, n
\end{align*}
\]

where \( \Omega^j \equiv \{ r_{m,t,n,j}^{\text{out}} \geq 0, r_{m,t,n,j}^{\text{in}} \geq 0, r_{m,t,n,j}^{\text{sto}} \} \)
Firm $i'$

$$\max_{\Omega^{i'}} \sum_{m \in M} \sum_{t \in T} \sum_{n \in N} W_m \left[ (g_{m,t,n,i'}, u + g^e_{m,t,n,i'} + r^{out}_{m,t,n,i'} - r^{in}_{m,t,n,i'}) \times (D^{int}_{m,t,n} - D^{slp}_{m,t,n} q_{m,t,n}) - \sum_{u \in U_{n,i'}} C_u g_{m,t,n,i', u} - C^{sto} r^{out}_{m,t,n,i'} \right]$$  \hspace{1cm} (11)

s.t.  
$$g_{m,t,n,i', u} - T_t \overline{G}_{n,i', u} \leq 0 \left( \beta^{conv}_{m,t,n,i', u} \right), \ \forall m, t, n, u \in U_{n,i'}$$  \hspace{1cm} (12)

$$g_{m,t,n,i', u} - g_{m,t-1,n,i', u} - T_t R^u_{u} \overline{G}_{n,i', u} \leq 0 \left( \beta^{up}_{m,t,n,i', u} \right), \ \forall m, t, n, u \in U_{n,i'}$$  \hspace{1cm} (13)

$$g_{m,t-1,n,i', u} - T_t R^down_{u} \overline{G}_{n,i', u} \leq 0 \left( \beta^{down}_{m,t,n,i', u} \right), \ \forall m, t, n, u \in U_{n,i'}$$  \hspace{1cm} (14)

$$g^e_{m,t,n,i'} - T_t A^e_{m,t,n} \overline{G}^e_{n,i'} = 0 \left( \beta^{e}_{m,t,n,i'} \right), \ \forall m, t, n, e$$  \hspace{1cm} (15)

$$r^{sto}_{m,t,n,i'} - (1 - E^{sto}_i) T_t r^{sto}_{m-1,t,n,i'} - E^{in}_i r^{in}_{m,t,n,i'} + r^{out}_{m,t,n,i'} = 0 \left( \lambda^{bal}_{m,t,n,i'} \right), \ \forall m, t, n$$  \hspace{1cm} (16)

$$r^{in}_{m,t,n,i'} - T_t R^{in}_i \overline{R}_{n,i'} \leq 0 \left( \lambda^{in,p}_{m,t,n,i'} \right), \ \forall m, t, n$$  \hspace{1cm} (17)

$$r^{out}_{m,t,n,i'} - T_t R^{out}_i \overline{R}_{n,i'} \leq 0 \left( \lambda^{out,p}_{m,t,n,i'} \right), \ \forall m, t, n$$  \hspace{1cm} (18)

$$r^{sto}_{m,t,n,i'} - \overline{R}_{n,i'} \leq 0 \left( \lambda^{ub,p}_{m,t,n,i'} \right), \ \forall m, t, n$$  \hspace{1cm} (19)

$$\overline{R}_{n,i'} - r^{sto}_{m,t,n,i'} \leq 0 \left( \lambda^{lb,p}_{m,t,n,i'} \right), \ \forall m, t, n$$  \hspace{1cm} (20)

where $\Omega^{i'} \equiv \{ g_{m,t,n,i', u}^{conv} \geq 0, g^{e}_{m,t,n,i'} \geq 0, r^{out}_{m,t,n,i'} \geq 0, r^{in}_{m,t,n,i'} \geq 0, r^{sto}_{m,t,n,i'} \geq 0 \}$
Firm $i'$

\[
\begin{align*}
\max_{\Omega^{i'}} \quad & \sum_{m \in M} \sum_{t \in T} \sum_{n \in N} W_m \left[ (g_{m,t,n,i}', u + g_e^{m,t,n,i'} + r^{out}_{m,t,n,i'} - r^{in}_{m,t,n,i'}) \right. \\
& \times \left. (D^{\text{int}}_{m,t,n} - D^{\text{slp}}_{m,t,n} q_{m,t,n}) - \sum_{u \in U_{n,i'}} C_u^{\text{conv}} g_{m,t,n,i}', u - C^{\text{sto}} r^{out}_{m,t,n,i'} \right] \\
\text{s.t.} \quad & g^{\text{conv}}_{m,t,n,i}', u - T_t \bar{G}^{\text{conv}}_{n,i}', u \leq 0 (\beta^{\text{conv}}_{m,t,n,i',u}), \forall m, t, n, u \in U_{n,i'} \\
& g^{\text{conv}}_{m,t,n,i}', u - g^{\text{conv}}_{m,t-1,n,i}', u - T_t R^{\text{up}}_u \bar{G}^{\text{conv}}_{n,i}', u \leq 0 (\beta^{\text{up}}_{m,t,n,i',u}), \forall m, t, n, u \in U_{n,i'} \\
& g^{\text{conv}}_{m,t-1,n,i}', u - g^{\text{conv}}_{m,t,n,i}', u - T_t R^{\text{down}}_u \bar{G}^{\text{conv}}_{n,i}', u \leq 0 (\beta^{\text{down}}_{m,t,n,i',u}), \forall m, t, n, u \in U_{n,i'} \\
& g^{e}_{m,t,n,i'} - T_t A^{e}_{m,t,n} \bar{G}^{e}_{n,i'} = 0 (\beta^{e}_{m,t,n,i'}), \forall m, t, n, e \\
& r^{\text{sto}}_{m,t,n,i}' - (1 - E^{\text{sto}}_{i}) T_t r^{\text{sto}}_{m-1,t,n,i'} - E^{\text{in}}_{i} r^{\text{in}}_{m,t,n,i'} + r^{\text{out}}_{m,t,n,i'} = 0 (\lambda^{\text{bal}}_{m,t,n,i'}), \forall m, t, n \\
& r^{\text{in}}_{m,t,n,i'} - T_t R^{\text{in}}_{i} \bar{R}_{n,i'} \leq 0 (\lambda^{\text{in,p}}_{m,t,n,i'}), \forall m, t, n \\
& r^{\text{out}}_{m,t,n,i'} - T_t R^{\text{out}}_{i} \bar{R}_{n,i'} \leq 0 (\lambda^{\text{out,p}}_{m,t,n,i'}), \forall m, t, n \\
& r^{\text{sto}}_{m,t,n,i'} - \bar{R}_{n,i'} \leq 0 (\lambda^{\text{ub,p}}_{m,t,n,i'}), \forall m, t, n \\
& \bar{R}_{n,i'} \bar{R}_{n,i'} - r^{\text{sto}}_{m,t,n,i'} \leq 0 (\lambda^{\text{lb,p}}_{m,t,n,i'}), \forall m, t, n \\
\end{align*}
\]

where $\Omega^{i'} \equiv \{g^{\text{conv}}_{m,t,n,i}', u \geq 0, g^{e}_{m,t,n,i'} \geq 0, r^{\text{out}}_{m,t,n,i'} \geq 0, r^{\text{in}}_{m,t,n,i'} \geq 0, r^{\text{sto}}_{m,t,n,i'} \geq 0\}$
Firm \textit{i}'

\[
\max_{\Omega_{i}'} \sum_{m \in M} \sum_{t \in T} \sum_{n \in N} W_m \left[ (g_{\text{conv}} m,t,n,i',u + g_{e} m,t,n,i' + r_{\text{out}} m,t,n,i' - r_{\text{in}} m,t,n,i') \right.
\]
\[
\times (D_{\text{int}} m,t,n - D_{\text{slp}} m,t,n q_{m,t,n} - \sum_{u \in U_{n,i'}} C_{u} g_{\text{conv}} m,t,n,i',u - C_{\text{sto}} r_{\text{out}} m,t,n,i') \right] \] (11)

s.t.
\[
g_{\text{conv}} m,t,n,i',u - T_t G_{n,i',u} \leq 0 (\beta_{\text{conv}} m,t,n,i',u), \forall m, t, n, u \in U_{n,i'} \] (12)
\[
g_{\text{conv}} m,t,n,i',u - g_{\text{conv}} m,t-1,n,i',u - T_t R_{u}^{\text{up}} G_{n,i',u} \leq 0 (\beta_{\text{up}} m,t,n,i',u), \forall m, t, n, u \in U_{n,i'} \] (13)
\[
g_{\text{conv}} m,t-1,n,i',u - g_{\text{conv}} m,t,n,i',u - T_t R_{u}^{\text{down}} G_{n,i',u} \leq 0 (\beta_{\text{down}} m,t,n,i',u), \forall m, t, n, u \in U_{n,i'} \] (14)
\[
g_{e} m,t,n,i' - T_t A_{m,t,n} G_{e,n,i'} = 0 (\beta_{e} m,t,n,i'), \forall m, t, n, e \] (15)
\[
r_{\text{sto}} m,t,n,i' - (1 - E_{i}^{\text{sto}}) T_t r_{\text{sto}} m,t-1,n,i' - E_{i}^{\text{in}} r_{\text{in}} m,t,n,i' + r_{\text{out}} m,t,n,i' = 0 (\lambda_{\text{bal}} m,t,n,i'), \forall m, t, n \] (16)
\[
r_{\text{in}} m,t,n,i' - T_t R_{i}^{\text{in}} R_{n,i'} \leq 0 (\lambda_{\text{in,p}} m,t,n,i'), \forall m, t, n \] (17)
\[
r_{\text{out}} m,t,n,i' - T_t R_{i}^{\text{out}} R_{n,i'} \leq 0 (\lambda_{\text{out,p}} m,t,n,i'), \forall m, t, n \] (18)
\[
r_{\text{sto}} m,t,n,i' - R_{n,i'} \leq 0 (\lambda_{\text{ub,p}} m,t,n,i'), \forall m, t, n \] (19)
\[
R_{n,i'} - r_{\text{sto}} m,t,n,i' \leq 0 (\lambda_{\text{lb,p}} m,t,n,i'), \forall m, t, n \] (20)

where \( \Omega_{i}' \equiv \{ g_{\text{conv}} m,t,n,i',u \geq 0, g_{e} m,t,n,i' \geq 0, r_{\text{out}} m,t,n,i' \geq 0, r_{\text{in}} m,t,n,i' \geq 0, r_{\text{sto}} m,t,n,i' \geq 0 \} \)
Firm $i'$

$$\begin{align*}
\max_{\Omega^{i'}} \sum_{m \in \mathcal{M}} \sum_{t \in \mathcal{T}} \sum_{n \in \mathcal{N}} W_m \left[ (g_{m,t,n,i'}, u + g_{m,t,n,i'}^e + r_{m,t,n,i'}^\text{out} - r_{m,t,n,i'}^\text{in}) \right. \\
\left. \times (D_{m,t,n}^\text{int} - D_{m,t,n}^\text{slp} q_{m,t,n}) - \sum_{u \in \mathcal{U}_{n,i'}} C_u g_{m,t,n,i',u}^\text{conv} - C_{\text{sto}}^\text{out} r_{m,t,n,i'} \right] \tag{11}
\end{align*}$$

subject to

$$\begin{align*}
g_{m,t,n,i', u}^\text{conv} - T_t G_{n,i'}^\text{conv}, u & \leq 0 (\beta_{m,t,n,i', u}^\text{conv}), \forall m, t, n, u \in \mathcal{U}_{n,i'} \tag{12} \\
 g_{m,t,n,i', u}^\text{conv} - g_{m,t-1,n,i', u}^\text{conv} - T_t R_u G_{n,i'}^\text{conv}, u & \leq 0 (\beta_{m,t,n,i', u}^\text{up}), \forall m, t, n, u \in \mathcal{U}_{n,i'} \tag{13} \\
 g_{m,t-1,n,i', u}^\text{conv} - g_{m,t,n,i', u}^\text{conv} - T_t R_u G_{n,i'}^\text{conv}, u & \leq 0 (\beta_{m,t,n,i', u}^\text{down}), \forall m, t, n, u \in \mathcal{U}_{n,i'} \tag{14} \\
 g_{m,t,n,i'}^e - T_t A_{m,t,n} G_{n,i'}^e, u & = 0 (\beta_{m,t,n,i'}^e), \forall m, t, n, e \tag{15} \\
 r_{m,t,n,i'}^\text{sto} - (1 - E_i^\text{sto}) T_t r_{m,t-1,n,i'}^\text{sto} - E_i^\text{in} r_{m,t,n,i'}^\text{in} + r_{m,t,n,i'}^\text{out} & = 0 (\lambda_{m,t,n,i'}^\text{bal}), \forall m, t, n \tag{16} \\
 r_{m,t,n,i'}^\text{in} - T_t R_i^\text{in} R_{n,i'} & \leq 0 (\lambda_{m,t,n,i'}^\text{in}), \forall m, t, n \tag{17} \\
 r_{m,t,n,i'}^\text{out} - T_t R_i^\text{out} R_{n,i'} & \leq 0 (\lambda_{m,t,n,i'}^\text{out}), \forall m, t, n \tag{18} \\
 r_{m,t,n,i'}^\text{sto} - R_{n,i'} & \leq 0 (\lambda_{m,t,n,i'}^\text{ub}), \forall m, t, n \tag{19} \\
 R_{n,i'} - r_{m,t,n,i'}^\text{sto} & \leq 0 (\lambda_{m,t,n,i'}^\text{lb}), \forall m, t, n \tag{20}
\end{align*}$$

where $\Omega^{i'} \equiv \{ g_{m,t,n,i', u}^\text{conv} \geq 0, g_{m,t,n,i'}^e \geq 0, r_{m,t,n,i'}^\text{out} \geq 0, r_{m,t,n,i'}^\text{in} \geq 0, r_{m,t,n,i'}^\text{sto} \geq 0 \}$
**Firm $i'$**

$$\begin{align*}
\max_{\Omega^i} & \sum_{m \in M} \sum_{t \in T} \sum_{n \in N} W_m \left[ (g_{m,t,n,i',u}^{\text{conv}} + g_{m,t,n,i'}^{e} + r_{m,t,n,i'}^{\text{out}} - r_{m,t,n,i'}^{\text{in}}) \right] \\
& \times (D_{m,t,n}^{\text{int}} - D_{m,t,n}^{\text{slp}} q_{m,t,n}) - \sum_{u \in U_{n,i'}} C_u g_{m,t,n,i',u}^{\text{conv}} - C_{\text{sto}} r_{m,t,n,i'}^{\text{out}} \\
\text{s.t.} & g_{m,t,n,i',u}^{\text{conv}} - T_t G_{n,i'}^{\text{conv}}, u \leq 0 (\beta_{m,t,n,i',u}^{\text{conv}}), \forall m, t, n, u \in U_{n,i'} \\
& g_{m,t,n,i',u}^{\text{conv}} - g_{m,t-1,n,i',u}^{\text{conv}} - T_t R_{u}^{\text{up}} G_{n,i'}^{\text{conv}}, u \leq 0 (\beta_{m,t,n,i',u}^{\text{up}}), \forall m, t, n, u \in U_{n,i'} \\
& g_{m,t-1,n,i',u}^{\text{conv}} - g_{m,t,n,i',u}^{\text{conv}} - T_t R_{u}^{\text{down}} G_{n,i'}^{\text{conv}}, u \leq 0 (\beta_{m,t,n,i',u}^{\text{down}}), \forall m, t, n, u \in U_{n,i'} \\
& g_{m,t,n,i'}^{e} - T_t A_{m,t,n}^{e} G_{n,i'}^{e} = 0 (\beta_{m,t,n,i'}^{e}), \forall m, t, n, e \\
& r_{m,t,n,i'}^{\text{sto}} - (1 - E_{i'}^{\text{sto}}) T_t r_{m,t-1,n,i'}^{\text{sto}} - E_{i'}^{\text{in}} r_{m,t,n,i'}^{\text{in}} + r_{m,t,n,i'}^{\text{out}} = 0 (\lambda_{m,t,n,i'}^{\text{bal}}), \forall m, t, n \\
& r_{m,t,n,i'}^{\text{in}} - T_t R_{i'}^{\text{in}} R_{n,i'}^{\text{in}} \leq 0 (\lambda_{m,t,n,i'}^{\text{in,p}}), \forall m, t, n \\
& r_{m,t,n,i'}^{\text{out}} - T_t R_{i'}^{\text{out}} R_{n,i'}^{\text{out}} \leq 0 (\lambda_{m,t,n,i'}^{\text{out,p}}), \forall m, t, n \\
& r_{m,t,n,i'}^{\text{sto}} - R_{n,i'}^{\text{sto}} R_{n,i'}^{\text{sto}} \leq 0 (\lambda_{m,t,n,i'}^{\text{ub,p}}), \forall m, t, n \\
& R_{n,i'}^{\text{sto}} R_{n,i'}^{\text{sto}} - r_{m,t,n,i'}^{\text{sto}} \leq 0 (\lambda_{m,t,n,i'}^{\text{lb,p}}), \forall m, t, n \\
\end{align*}$$

where $\Omega^{i'} \equiv \{ g_{m,t,n,i',u}^{\text{conv}} \geq 0, g_{m,t,n,i'}^{e} \geq 0, r_{m,t,n,i'}^{\text{out}} \geq 0, r_{m,t,n,i'}^{\text{in}} \geq 0, r_{m,t,n,i'}^{\text{sto}} \geq 0 \}$
Upper-Level Objective Function

\[
\begin{align*}
\text{max} \quad & \sum_{m \in M} \sum_{t \in T} \sum_{n \in N} W_m \left[ \left( D_{m,t,n}^{\text{int}} q_{m,t,n} - \frac{1}{2} D_{m,t,n}^{\text{slp}} q_{m,t,n}^2 \right) ight. \\
& - \sum_{i' \in I'} \sum_{u \in U_{n,i'}} C_{u}^{\text{conv}} g_{m,t,n,i',u}^{\text{conv}} - \sum_{i \in I} C_{m,t,n,i}^{\text{sto}} r_{m,t,n,i}^{\text{out}} \\
& - \sum_{n \in N} \sum_{y \in Y} z_{n,j,y} I R_y^{d} \\
\text{s.t.} \quad & \sum_{y \in Y} z_{n,j,y} = 1, \forall n, \quad z_{n,j,y} \in \{0, 1\}, \forall n, y
\end{align*}
\]

(21)

\[
\begin{align*}
\text{max} \quad & \sum_{m \in M} \sum_{t \in T} \sum_{n \in N} W_m \left[ \frac{\theta_{m,t,n}}{W_m} \left( r_{m,t,n,j}^{\text{out}} - r_{m,t,n,j}^{\text{in}} \right) - C_{m,t,n,j}^{\text{sto}} r_{m,t,n,j}^{\text{out}} \right] \\
& - \sum_{n \in N} \sum_{y \in Y} z_{n,j,y} I R_y^{d} \\
\text{s.t.} \quad & (22)
\end{align*}
\]

(23)
Upper-Level Objective Function

\[
\begin{align*}
\max_{z_{n,j,y}} & \quad \sum_{m \in M} \sum_{t \in T} \sum_{n \in N} W_m \left[ \left( D_{m,t,n}^{\text{int}} q_{m,t,n} - \frac{1}{2} D_{m,t,n}^{\text{slp}} q_{m,t,n}^2 \right) \\
& - \sum_{i' \in I'} \sum_{u \in U_{n,i'}} C_u^{\text{conv}} g_{m,t,n,i',u}^{\text{conv}} - \sum_{i \in I} C_{\text{sto}}^{\text{out}} r_{m,t,n,i}^{\text{out}} \right] \\
- & \sum_{n \in N} \sum_{y \in Y} z_{n,j,y} IR_d^{d} y \\
\text{s.t.} & \quad \sum_{y \in Y} z_{n,j,y} = 1, \forall n, \quad z_{n,j,y} \in \{0, 1\}, \forall n, y 
\end{align*}
\]

\[
\begin{align*}
\max_{z_{n,j,y}} & \quad \sum_{m \in M} \sum_{t \in T} \sum_{n \in N} W_m \left[ \frac{\theta_{m,t,n}}{W_m} (r_{m,t,n,j}^{\text{out}} - r_{m,t,n,j}^{\text{in}}) - C_{\text{sto}}^{\text{out}} r_{m,t,n,j}^{\text{out}} \right] \\
- & \sum_{n \in N} \sum_{y \in Y} z_{n,j,y} IR_d^{d} y \\
\text{s.t.} & \quad (22)
\end{align*}
\]
Upper-Level Objective Function

\[
\max_{z_{n,j,y}} \sum_{m \in \mathcal{M}} \sum_{t \in \mathcal{T}} \sum_{n \in \mathcal{N}} W_m \left[ \left( D_{m,t,n}^\text{int} q_{m,t,n} - \frac{1}{2} D_{m,t,n}^\text{slp} q_{m,t,n}^2 \right) \right. \\
\left. - \sum_{i' \in \mathcal{I}'} \sum_{u \in \mathcal{U}_{n,i'}} C_u^{\text{conv}} g_{m,t,n,i',u} - \sum_{i \in \mathcal{I}} C_{\text{sto}} r_{m,t,n,i}^\text{out} \right] \\
- \sum_{n \in \mathcal{N}} \sum_{y \in \mathcal{Y}} z_{n,j,y} I\bar{R}_y^d
\]  

\text{s.t.} \quad \sum_{y \in \mathcal{Y}} z_{n,j,y} = 1, \forall n, \quad z_{n,j,y} \in \{0, 1\}, \forall n, y \tag{21}

\max_{z_{n,j,y}} \sum_{m \in \mathcal{M}} \sum_{t \in \mathcal{T}} \sum_{n \in \mathcal{N}} W_m \left[ \frac{\theta_{m,t,n}}{W_m} (r_{m,t,n,j}^\text{out} - r_{m,t,n,j}^\text{in}) - C_{\text{sto}} r_{m,t,n,j}^\text{out} \right] \\
- \sum_{n \in \mathcal{N}} \sum_{y \in \mathcal{Y}} z_{n,j,y} I\bar{R}_y^d 
\]  

\text{s.t.} \quad (22)
Upper-Level Objective Function

\[
\begin{align*}
\max_{z_{n,j,y}} \sum_{m \in M} \sum_{t \in T} \sum_{n \in N} W_m \left[ (D_{m,t,n}^\text{int} q_{m,t,n} - \frac{1}{2} D_{m,t,n}^\text{slp} q_{m,t,n}^2 ) ight] \\
- \sum_{i' \in I'} \sum_{u \in U_{n,i'}} C_u^{\text{conv}} g_{m,t,n,i',u}^{\text{conv}} - \sum_{i \in I} C^{\text{sto}} r_{m,t,n,i}^{\text{out}} \\
- \sum_{n \in N} \sum_{y \in Y} z_{n,j,y} IR_d^{d} \end{align*}
\]

\[
\sum_{y \in Y} z_{n,j,y} = 1, \forall n, \quad z_{n,j,y} \in \{0, 1\}, \forall n, y
\]  

\[
\begin{align*}
\max_{z_{n,j,y}} \sum_{m \in M} \sum_{t \in T} \sum_{n \in N} W_m \left[ \frac{\theta_{m,t,n}}{W_m} (r_{m,t,n,j}^{\text{out}} - r_{m,t,n,j}^{\text{in}}) - C^{\text{sto}} r_{m,t,n,j}^{\text{out}} \right] \\
- \sum_{n \in N} \sum_{y \in Y} z_{n,j,y} IR_d^{d} \end{align*}
\]

\[
\text{s.t.} \quad (22)
\]