Towards a dynamical understanding of the non-$D\bar{D}$ decay of $\psi(3770)$

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We investigate the $\psi(3770)$ non-$D\bar{D}$ decays into $VP$, where $V$ and $P$ denote vector and pseudoscalar mesons, respectively, via OZI-rule-evading intermediate meson rescatterings in an effective Lagrangian theory. By identifying the leading meson loop transitions and constraining the model parameters with the available experimental data for $\psi(3770)\rightarrow J/\psi\eta$, $\phi\eta$ and $p\pi$, we succeed in making a quantitative prediction for all $\psi(3770)\rightarrow VP$ with $BR_{VP}$ from 0.41% to 0.64%. It indicates that the OZI-rule-evading long-range interactions are playing a role in $\psi(3770)$ strong decays, and could be a key towards a full understanding of the mysterious $\psi(3770)$ non-$D\bar{D}$ decay mechanism.

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Charmonium state $\psi(3770)$ has a mass just above the open $D\bar{D}$ threshold, which allows it to decay into charmed mesons, i.e. $D\bar{D}$, without the so-called Okubo-Zweig-Iizuka (OZI) rule suppression. This scenario qualitatively explains that the width of the $\psi(3770)$ is about two orders of magnitude larger than those of the $J/\psi$ and $\psi'$ due to the dominant $D\bar{D}$ decay. An interesting and nontrivial question here is whether the $\psi(3770)$ decay is totally saturated by $DD$, or whether there exist significant non-$D\bar{D}$ decay channels. Unfortunately, a definite answer from either experiment or theory is unavailable. CLEO Collaboration measured the exclusive cross sections for $\psi(3770)\rightarrow D\bar{D}$[2,3] and inclusive cross sections for $\psi(3770)\rightarrow$ hadrons[4]. These results lead to $BR_{\psi(3770)\rightarrow D\bar{D}}= (103.0 \pm 1.4^{+5.1}_{-6.8})\%$, of which the lower bound suggests the maximum non-$D\bar{D}$ branching ratio is about 6.8%.

The $D\bar{D}$ production cross sections measured by BES[5] are consistent with CLEO[3]. However, the analyses lead to much larger non-$D\bar{D}$ branching ratios of $\sim 15\%$. Such a significant discrepancy makes the experimental status quite puzzling. Also, the search for exclusive non-$D\bar{D}$ decays has been carried out at both CLEO[6] and BES[7]. In Ref. 8, three non-$D\bar{D}$ hadronic decay branching ratios are listed, i.e. $\psi(3770)\rightarrow J/\psi\pi\pi$, $J/\psi\eta$ and $\phi\eta$, while tens of other channels have only experimental upper limits due to the poor statistics. In the radiative decay channel, $\psi(3770)\rightarrow \gamma\chi_{c0}$ and $\gamma\chi_{c1}$ are listed while an upper limit is given to $\gamma\chi_{c2}$. The sum of those channels, however, is far from clarifying the mysterious situation of the $\psi(3770)$ non-$D\bar{D}$ decays. It hence stimulates intensive experimental and theoretical efforts[4,10,11,12,13,14,15,16] on understanding the nature of $\psi(3770)$ and its strong and radiative transition dynamics.

In this Letter we propose that the dominant $D\bar{D}$ decay is strongly correlated with the non-$D\bar{D}$ ones. We argue that the intermediate $D\bar{D}$ and $D\bar{D}^* + c.c.$ rescatterings, which annihilate the $c\bar{c}$ at relatively large distance by the OZI-rule evading processes, may provide a natural mechanism for quantifying the $\psi(3770)$ non-$D\bar{D}$ decays.

As illustrated in Fig. 1, the $c\bar{c}$ pair first couples to an intermediate meson pair, e.g. $D\bar{D}$, and then these two mesons rescatter into two light mesons via the $c\bar{c}$ annihilation and a light quark pair creation. Qualitatively, with the branching ratio for $\psi(3770)\rightarrow D\bar{D}$ at an order of one, the rescattering process could be suppressed by two or three orders of magnitude. Note that the OZI-evading rescatterings are open to numerous final-state light mesons. It might be possible that a sum of those exclusive final states would account for a sizeable fraction of the $\psi(3770)$ branching ratios.

A natural way of describing the rescattering processes is to expand the amplitude in Fig. 1 via the Mandelstam variables $t \equiv (P_{f1} - p_1)^2$ and $s \equiv (P_{f1} + P_{f2})^2 = M_{\psi(3770)}^2$. At leading order, the $t$-channel is via an additional meson exchange transition, while the $s$-channel can be recognized as the vector meson

Particles

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mixings, e.g. $\psi(2S)$-$\psi(1D)$ mixing \cite{10,11}. The typical transition diagrams are shown in Fig. 2. The intermediate $DD$ rescattering will contribute to the absorptive part of the transition amplitude and is not to be due to the pQCD leading transition via short-range gluon exchanges. This is an explicit indication that long-range interactions can play an important role in such a transition. The intermediate $DD^* + c.c.$ can contribute to the real part of the transition amplitude due to its large coupling to $\psi(3770)$ \cite{17} and the break-down of the local quark-hadron duality \cite{18,19}. By clarifying the above points, we are ready to construct the theory for probing the role played by the intermediate charmed meson loops in $\psi(3770) \to VP$.

The following effective Lagrangians are needed in the evaluation of the $t$ and $s$-channel transitions,

\begin{align}
\mathcal{L}_{\psi DD} &= g_{\psi DD} \{ \bar{D} \partial^\mu \tilde{D} - \partial_\mu D\tilde{D} \} \psi^\mu, \\
\mathcal{L}_{VDD^*} &= -ig_{VDD^*} \epsilon_{\alpha\beta\mu\nu} \partial^\nu \tilde{D}^\beta \tilde{D}^\mu D + H.c., \\
\mathcal{L}_{FDD^*} &= -ig_{FDD^*} \epsilon_{\alpha\beta\mu\nu} \partial^\nu \tilde{D}^\beta \tilde{D}^\mu \mathcal{P} + H.c., \\
\mathcal{L}_{PDD^*} &= g_{PDD^*} \{ \bar{D} \partial_\mu D \mathcal{P} - \partial_\mu \bar{D} \mathcal{P} \} D^{*\mu} + H.c.,
\end{align}

where $\epsilon_{\alpha\beta\mu\nu}$ is the Levi-Civita tensor; $\mathcal{P}$ and $V^\beta$ are the pseudoscalar and vector meson fields, respectively.

The charmed meson couplings to light mesons are obtained in the chiral and heavy quark limits \cite{17},

\begin{align}
g_{DD^* D\pi} &= \frac{2}{f_\pi} g \sqrt{m_D m_{D^*}}, \\
g_{DD^* D\rho} &= g_D \frac{g_{DD\rho}}{m_D}, \\
g_{DD^* D\rho} &= \sqrt{2} \lambda g_\rho, \\
g_{DD\rho} &= g_{DD\rho} = g_{DD\rho} \tilde{M}_D,
\end{align}

where $f_\pi = 132$ MeV is the pion decay constant, and $\tilde{M}_D \equiv \sqrt{m_D m_{D^*}}$ sets a mass scale. The parameters $g_\rho$ respects the relation $g_\rho = m_\rho/f_\pi$ \cite{20}. We take $\lambda = 0.56 \text{ GeV}^{-1}$ and $g = 0.59$ \cite{21,22}.

The coupling $g_{\psi(3770)DD}$ is extracted by,

\begin{align}
\Gamma_{\psi(3770)\to DD} &= \frac{g_{\psi(3770)DD}^2}{6\pi M_D^2}\frac{|\vec{p}|^3}{\psi(3770)},
\end{align}

where $|\vec{p}|$ is the D-meson momentum. The branching ratios for $\psi(3770) \to D^+ D^-$ and $D^0 \bar{D}^0$ are slightly different. They give $g_{\psi(3770)DD^* D^{*\alpha}} = 12.71$ and $g_{\psi(3770)D^{*\alpha} D^{*\beta}} = 12.43$, and reflects the isospin violation due to the mass difference between the $u$ and $d$ quark. Taking into account the consequent kinematic difference, we also have access to isospin violating channels via the meson loops.

For other couplings, we take the SU(3) flavor symmetry as a leading order approximation which leads to $g_{D^* D^0 u\bar{u}} = g_{D^* D^0 d\bar{d}} = g_1$, $g_{D^* D^0 s\bar{s}}$, and $g_{D^* D^0 s\bar{s}} = g_2$. So we have $g_{D^* D^0} = g_1$.
\( \sqrt{2} g_{D^* D q q(0^-)} \), \( g_{D^* D p} = \sqrt{2} g_{D^* D q q(1^-)} \), \( g_{D^* D s} = 0 \), and \( g_{D^* D n n} = 0 \), with \( n \) for \( u \) or \( d \) quark. Similar relations are also implied for \( g_{D^* D^* \pi}, \) and \( g_{D D p}. \)

We adopt coupling constants \( g_{J/\psi D^*} = 3.84 \text{ GeV}^{-1} \) and \( g_{J/\psi D D} = 7.44 \) from Ref. \[23\]. Coupling \( g_{\psi(3770)D^*} \) can be related to \( g_{\psi(3770)D D} \) via \( g_{\psi(3770)D D} = g_{\psi(3770)D D}/M_D \).

The \( \eta-\eta' \) mixing is considered in a standard way,

\[
\eta = \cos \alpha |n\bar{n}\rangle - \sin \alpha |s\bar{s}\rangle,
\eta' = \sin \alpha |n\bar{n}\rangle + \cos \alpha |s\bar{s}\rangle,
\]

where \(|n\bar{n}\rangle \equiv |u\bar{u} + d\bar{d}|/\sqrt{2}\), and the mixing angle \( \alpha_p = \theta_p + \arctan(\sqrt{2}) \) with \( \theta_p \sim -24.6^{\circ} \) or \( \sim -11.5^{\circ} \) for linear or quadratic mass relations, respectively \[8\]. We adopt \( \theta_p = -19.1^{\circ} \) \[21\].

By investigating \( \psi(3770) \rightarrow J/\psi \eta, \phi \eta \) and \( \rho \pi \) simultaneously, we expect to obtain constraints on the theory by which we can then make predictions for other \( VP \) channels. Although these decays are OZI-rule-suppressed processes, their kinematics are slightly different. The production of \( J/\psi \) in \( \psi(3770) \rightarrow J/\psi \eta \) suggests that it is a very soft process. The momentum carried by the final state meson in the \( \psi(3770) \)-rest frame is \( p = 0.359 \text{ GeV} \) which is much less than the masses of both \( \eta \) and \( J/\psi \). Thus, we argue that \( \psi(3770) \rightarrow J/\psi \eta \) is dominated by the intermediate meson loops. Note that the \( t \)-channel loops suffer from divergence \[24\]. We then introduce a cut-off in the loop integrals via a standard dipole form factor,

\[
F(q^2) = \left( \frac{\Lambda^2 - m^2_{ex}}{\Lambda^2 - q^2} \right)^2, \quad (5)
\]

where \( \Lambda \equiv m_{ex} + \alpha \Lambda_{QCD} \), with \( \Lambda_{QCD} = 0.22 \text{ GeV} \); \( m_{ex} \) is the mass of the exchanged meson and \( \alpha \) is a parameter to be determined by experimental data for \( \psi(3770) \rightarrow J/\psi \eta \).

The \( s \)-channel meson loop contributions can be determined via the on-shell approximation. We find that the branching ratio given by the \( \psi' \psi(3770) \)-mixing in \( \psi(3770) \rightarrow J/\psi \eta \) is \( BR = 1.3 \times 10^{-5} \) which is much smaller than the \( t \)-channel, and indicates the dominance of the \( t \)-channel. With \( BR_{J/\psi \eta}^{\text{exp}} = (9.0 \pm 4) \times 10^{-4} \) \[3\], \( \alpha = 1.73 \) can be determined and the exclusive \( t \)-channel contributes \( 8.44 \times 10^{-4} \).

As follows, we fix \( \alpha = 1.73 \) in the form factors as an overall parameter. Two aspects must be taken care of. Firstly, since relatively large momentum transfers are involved in \( \psi(3770) \) decays into light \( VP \), the pQCD leading contribution via SOZI transitions may play a role. This part contributes to the real part of the transition amplitude and will not be dual with the long-range intermediate meson loops as recognized by the absorptive feature of the \( D\bar{D} \) rescattering in the on-shell approximation. Secondly, for light \( VP \) decay channels, their SOZI amplitudes can be related to each other by the flavor-blind assumption \[20\] \[23\] for quark-gluon coupling,

\[
g_S^{\rho \pi} : g_S^{K^{*+}K^-} : g_S^\phi \eta : g_S^{\phi \eta'} : g_S^{\phi \eta} : g_S^{\phi \eta'}
= 1 : 1 : \cos \alpha_p : \sin \alpha_p : (\sin \alpha_p) : \cos \alpha_p ,
\]

with the other isospin channels implied.

The transition amplitude for \( \psi(3770) \rightarrow VP \) can be expressed as

\[
\mathcal{M}_{fi} = \mathcal{M}^L + e^{i\delta} \mathcal{M}^{SOZI} \equiv i(g_L + e^{i\delta} g_S F_S(P_V))
\times \xi_{\alpha \beta \mu \nu} P_\psi^\alpha P_V^\beta F_{\psi \psi \psi \psi} / M_{\psi(3770)}
\]

where the property of antisymmetric tensor is applied to factorize out the effective couplings in the second line and \( \delta \) is the phase angle between the meson loop and SOZI amplitudes. A conventional form factor, \( F_S^2(P_V) \equiv \exp(-P_V^2/8\beta^2) \) with \( \beta = 0.5 \text{ GeV} \) is applied for the SOZI transition with \( P_V \) the final three momentum in the \( \psi(3770) \)-rest frame \[22\] \[23\].

With \( \alpha = 1.73 \) fixed, we can then determine the other two parameters \( g_S \equiv g_S^{\rho \pi} = 0.085 \) and \( \delta = -66^{\circ} \) from experimental data, i.e. \( BR_{\phi \eta} = (3.1 \pm 0.7) \times 10^{-4} \) \[3\] and \( BR_{\rho \pi} < 0.24 \% \) with C.L. of 90\% \[28\]. In Tab. \[1\] theoretical predictions for other \( VP \) decay branching ratios as a maximum rate are
presented. The exclusive results for $t$ and $s$-channel meson loops and SOZI processes are also listed. We also include isospin-violating channels $J/\psi \pi^0$, $\omega \pi^0$, $\rho^0 \eta$, and $\bar{\rho}^0 \eta'$, which can be recognized via the non-exact cancelations between the charged and neutral meson loop amplitudes due to the mass differences between the charged and neutral intermediate mesons. We do not consider $\phi \pi^0$ channel since it involves both OZI doubly disconnected process and isospin violation, thus will be strongly suppressed.

The following points can be learned from Tab. I: (i) Different from the $\psi(2S)$-$\psi(1D)$ mixing scheme discussed in Refs. 10, 11, our $s$-channel $\psi(3770) \to \psi'$ transition element is a complex number. If we neglect the imaginary part due to the widths, we can extract the mixing angle $\phi \simeq 4.57^\circ$ in the convention of 11. We find that the $t$-channel transitions are much more important in $\psi(3770) \to VP$, while the $s$-channel contributions are generally small and even negligible in light $VP$ channels. This is mainly due to the small partial widths for $\psi'$ decays into light $VP$. The only non-negligible $s$-channel is in $\psi(3770) \to J/\psi \eta$, which adds to the $t$-channel constructively. In contrast, the isospin violating channel $J/\psi \pi^0$ experiences a destructive interference between the $t$ and $s$-channel. These results are useful for clarifying the scenario of $\psi(2S)$-$\psi(1D)$ mixing. (ii) The SOZI coupling $g_S$ and phase angle $\delta$ are strongly correlated. Applying the BES data 28, we find that the meson loop and SOZI amplitudes have constructive interferences in $\phi \eta$ and $\phi \eta'$, but have destructive interferences in $\rho \pi$, $K^+ K^- + c.c.$, and $\omega \eta(\eta')$, which are automatically given by the SU(3) flavor symmetry. This is a strong constraint for our model parameters, and a sum over the $VP$ decays gives a rate of $\sim 0.64\%$. By varying $\delta$, but keeping the $\phi \eta$ rate unchanged (i.e. $g_S$ will be changed), we obtain a lower bound for the sum of branching ratios, $\sim 0.41\%$.

It is interesting to see that the intermediate $D$ meson rescatterings indeed account for some deficit for the non-$DD$ decay. In order to clarify this puzzling problem, it is essential to have precise data for $\rho \pi$ and $K^+ K^- + c.c.$ A search for these decays at BES-III 29 is thus strongly recommended. Theoretical investigation of other channels such as $\psi(3770) \to V S, V T$, etc is also needed as a prediction and test of the proposed mechanism.

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**Notes added:** We would also like to mention that upon the submission of this paper, a work based on

### TABLE I: Branching ratios for $\psi(3770) \to VP$ calculated for different mechanisms. The values for $J/\psi \eta$ and $\phi \eta$ are fixed at the central values of the experimental data 8, and the experimental upper limit is taken for $\rho \pi$ 28.

| BR($\times 10^{-3}$) | t-channel | s-channel | SOZI | Total |
|---------------------|-----------|-----------|------|-------|
| $J/\psi \eta$       | 8.44      | 0.13      | -    | 9.0   |
| $J/\psi \pi^0$      | 0.1       | 2.58 $\times$ $10^{-2}$ | -   | 4.4 $\times$ $10^{-2}$ |
| $\rho \pi$          | 34.45     | 7.69 $\times$ $10^{-5}$ | 8.53 | 24.0 |
| $K^{+} K^- + c.c.$  | 10.97     | 6.83 $\times$ $10^{-5}$ | 5.72 | 8.91 |
| $K^{*0} K^0 + c.c.$ | 11.80     | 4.38 $\times$ $10^{-5}$ | 5.72 | 9.90 |
| $\phi \eta$         | 1.25      | 1.13 $\times$ $10^{-3}$ | 1.16 | 3.1  |
| $\phi \eta'$        | 0.87      | 2.53 $\times$ $10^{-3}$ | 1.86 | 3.78 |
| $\omega \eta$       | 6.83      | 9.64 $\times$ $10^{-5}$ | 1.88 | 4.69 |
| $\omega \eta'$      | 0.58      | 2.87 $\times$ $10^{-5}$ | 0.97 | 0.39 |
| $\rho \eta$         | 1.88 $\times$ $10^{-2}$ | 1.77 $\times$ $10^{-5}$ | -   | 1.8 $\times$ $10^{-2}$ |
| $\rho \eta'$        | 1.08 $\times$ $10^{-2}$ | 1.54 $\times$ $10^{-5}$ | -   | 1.0 $\times$ $10^{-2}$ |
| $\omega \pi^0$      | 2.57 $\times$ $10^{-2}$ | 1.82 $\times$ $10^{-3}$ | -   | 2.5 $\times$ $10^{-2}$ |
| Sum                 | 75.34     | 0.16      | 25.84 | 63.87 |
a similar idea was submitted to the arXiv by Liu et al. There, the authors focus on the intermediate $D\bar{D}$ rescattering in an on-shell approximation and investigate its contributions to $J/\psi\eta$, $\rho\pi$ and $J/\psi\pi\pi$. In our case, we calculate all $VP$ channels with full loop integrals and a reasonable estimate of the SOZI processes based on a stringent constraint on the model parameters.

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