Glue Ball Masses and the Chameleon Gauge

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Abstract

We introduce a new numerical technique to compute mass spectra, based on difference method and on a new gauge fixing procedure. We show that the method is very effective by test runs on a $SU(2)$ lattice gauge theory.
Even if computations of the gluonic mass spectrum have reached recently very high levels of precision and reliability [1] looking for more effective physical procedures to measure correlation functions is a very important task. The measurement of connected correlation functions from Monte Carlo procedures is a very time consuming task, and the interesting signal comes from large separation, where the functions we want to determine are exponentially small.

In this note we will continue in exploring the possibility of using continuous updating schemes (Langevin-like, as opposed to the discrete step Metropolis scheme) and to exploit such continuity to define connected correlation functions by means of differences of correlated dynamical processes [2, 3] (for analytic applications of these kinds of methods see for example ref. [4]). Here we will propose a new method which takes care about a crucial ingredient, the gauge invariance of the theory, and that looks far more effective than the pre-existing schemes.

The simple idea of the difference scheme to compute connected correlation functions is the following. In the rest of the paper we will consider the $0^{++}$ glueball mass, defined from the connected correlation function

$$G(t) \equiv \langle E^{(3)}(0)E^{(3)}(t) \rangle - \langle E^{(3)}(0) \rangle \langle E^{(3)}(t) \rangle \simeq \exp\{-m_{0^{++}}t\}, \quad (1)$$

where with the upperscript 3 we denote the sum of the spatial plaquettes on the 3-cube at a given time (0 and $t$ in the previous equation). We will consider a system governed by Wilson Action at inverse temperature $\beta = T^{-1}$, i.e.

$$S_\beta \equiv \beta \sum_{\text{plaquettes}} (1 - \frac{1}{2} \text{Tr } U_P), \quad (2)$$

for $SU(2)$ gauge fields, with the usual definition of the plaquette variables $U_P$.

A possibly effective way to measure connected correlation functions has been suggested [2, 3], and it is based on simulating the dynamics of two copies of the system, one with the original action (2) and one with a modified action. In our case we only modify the Action involving links on the 3d cube at $t = 0$ (we are in 4 euclidian space-time dimensions, and we identify one of these dimensions with the euclidian time). So, for $t \neq 0$ we use the original Wilson Action, while at $t = 0$ we modify the value of the coupling, by setting
\[ \beta \rightarrow \beta + \delta \beta, \]  
with a small \( \delta \beta \). In the limit of \( \delta \beta \rightarrow 0 \) one gets that [3] (we indicate with a tilde expectation values taken over the modified action)

\[ \tilde{E}^{(3)}(t) - E^{(3)}(t) = G(t)\delta \beta + O(\delta \beta^2), \]

where by \( E^{(3)}(t) \) we denote the average of the space-like plaquettes at time \( t \).

In words we consider the difference of the time \( t \) energy of the unperturbed and the perturbed system, in the limit of a small perturbation. This difference, when measured with good statistical precision, gives us a measurement of the connected correlation function \( G \).

The main point of the method is that if we use an appropriate simulation technique \( E \) and \( \tilde{E} \) are correlated, and by exploiting that we can eliminate the most part of the statistical error we would get in a direct measurement of the \( G(t) \). The typical pattern of such a joint simulation is the following. We start from two copies of the same configuration. Their energy distance is zero, since we are out of equilibrium. Now we start the two simulations. The energy distance at a given time \( t \) will hopefully stabilize after a transient period \( (\tau_0, \tau) \), we indicate with \( \tau \) the dynamical fifth time, as opposed to euclidian time \( t \) at the correct distance. Then for large dynamical time \( (\tau > \tau_1) \) the two trajectories in phase space will separate, the statistical noise will dominate the signal and the measurement will be of little use. The measurement window goes from \( \tau_0 \) to \( \tau_1 \), and a good dynamical procedure maximizes its extent.

A discrete dynamics like the Metropolis algorithm does not do the job. Indeed in this case the advantage of the method, which to be well performing exploits the fact that the two trajectories are close in phase space, is lost due to the intrinsic discreteness of the updating procedure. On the contrary a Langevin dynamics can be the basis of our scheme [2, 3]. For the two copies of the system we write

\[ \dot{U} = -\frac{\delta S}{\delta U} + \eta, \]
\[ \dot{V} = -\frac{\delta S}{\delta V} + \eta, \]
where we have denoted by $U$ the unperturbed fields and by $V$ the perturbed ones, and $\eta$ is the same noise for the two systems.

For a gauge model the gauge degrees of freedom create an additional complication. The gauge part of the degree of freedom random walks in phase space, and such a random walk tends to separate the two trajectories. An usual gauge fixing (for example putting to one all time-like gauge variables) does slow down the dynamics making the method impractical [3]. In ref. [3] we have seen that sometimes the phenomenon can be dramatic enough to make any measurement impossible (even for short euclidean time separations).

In ref. [3] we proposed to solve the problem by using some kind of magnetic field, which would insure a smooth, partial fixing of the gauge. Wilson action would be modified by a non-gauge invariant term, selected in such away to have a small effect one gauge-invariant quantities. Even if this method was improving the situation, it did not turned out to be very superior to usual methods. Trajectories in phase space did diverge quite soon, and a slow drift on the system internal energy (that is also modified of a small amount due to the magnetic like term) was difficult to control. Also a non-gauge invariance formulation is definitely not so appealing, and could present a large number of unwanted features.

In this note we propose a new method, and we show that it is indeed very effective. The basic idea is very simple. We fix the gauge where the two configurations are as similar as possible. We call such a gauge the chameleon gauge. This method does indeed keep the gauge part of the two systems as close as possible (reducing the rate of divergence of the two trajectories in phase space) but does not introduce any sizeable slowing down in the dynamics of observable quantities like the energy.

After each full lattice sweep of Langevin update of the link variables $U$ and $V$ we gauge fix the $U$ configuration (obviously it does not change to gauge fix instead the $V$ configuration). We maximize the quantity

$$\sum_{n, \mu} U_\mu(n) V^\dagger_\mu(n), \quad \text{(6)}$$

where $N$ runs over the lattice sites and $\mu$ over the 4 directions. We find the gauge transformation $\{g(n)\}$, with

$$U_\mu(n) \rightarrow g(n) U_\mu(n) g^\dagger(n + \hat{\mu}). \quad \text{(7)}$$
We determine \( \{g(n)\} \) such that
\[
\sum_{n,\mu} g(n) U_\mu(n) g^\dagger(n + \hat{\mu}) V^\dagger_\mu(n) = \text{local maximum}.
\] (8)

The quantity
\[
F \equiv \frac{1}{4V} \sum_{\text{links}} (1 - U_\mu(n)V^\dagger_\mu(n)),
\] (9)
gives a measure of how good a gauge fix we have been able to reach. \( F = 0 \) when the two configurations are identical. A high value of \( F \) signals that a good gauge fix has not been achieved (implying for example that the two gauge field configuration differ because of a physical, non gauge freedom related reason).

Our numerical simulations show that the method is very well performing, and has very pleasant features. The (correct) equilibrium value for the plaquette energy is reached in very short time. That means that all the problems connected to standard gauge fixing (like axial gauge) or to our magnetic field partial gauge fixing have been solved by the chameleon gauge. We show in fig. (1) the plaquette value computed with the magnetic field method, where a slow drift is evident. In fig. (2) we show the new data, where after less than 100 Monte Carlo sweeps the measured value is stable. Correlation functions measured by difference technique on the chameleon gauged configurations are far less noisy than with our old method. In a typical situation we gain a factor of order 5 over the time window we can rely on. It is also quite interesting to note that the breakdown of the correlation functions (that become noisy after a given time \( \tau_1 \)) is always signaled by a sudden growth of the quantity \( F \), i.e. by a collapse of the quality of the gauge fixing we are able to reach.

Let us give a few details about our runs and our numerical results. We simulate an \( SU(2) \) system on a \( 16^4 \) lattice with periodic boundary conditions. We keep one copy of the system at the original \( \beta \) value, while we simulate four parallel systems, where different time slices are set at the inverse square coupling value \( \beta + \delta \beta \). The program size is 44 Mbyte, and on a IBM RISC WS 550E we run 500 steps in close to one day. We have selected \( \beta = 2.5 \), that is close enough to the scaling region, and where our lattice size is large enough to describe infinite volume behavior. Michael and Teper estimate in
Figure 1: The plaquette operator expectation value computed by using the magnetic field method.

16^4 lattice
β=2.5
δβ=0.001
H=0.3
Figure 2: The plaquette operator expectation value computed by using the chameleon gauge fixing method.
ref. [5] that \( m_a \simeq 0.660 \). The result we obtain by using our method is fully compatible with that, and from a global fit to our data we obtain \( m_a \simeq 0.67 \), with an error, including both statistic uncertainty and systematic effects from finite euclidean time distance that we estimate to be smaller than 5%.

In fig. (3) we show the signal we were getting at separation 1 by using our old method (based on the use of the magnetic gauge fixing term), while in fig. (4) we show the result we obtain with our new method. The difference is quite impressive. While signal was becoming noisy at \( \tau \simeq 400 \) and was lost at \( \tau \simeq 800 \) in the old approach, in the chameleon gauge fixed approach we get a stable signal up to time \( \tau > 1200 \).

We believe that there is still much to understand in the physics of the so-called difference methods, both in gauge and in simple spin systems, but we hope that the method we are proposing here is a nice step in the direction of less expensive measurements of mass spectra.

References

[1] See for example the Proceedings of the LAT94 Bielefeld Lattice Conference, Nucl. Phys. B (Proc. Suppl.) 34 (1994) 795, and references therein; see also E. Marinari, M. L. Paciello an B. Taglienti, The String Tension in Gauge Theories, hep-lat/9503027.

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Figure 3: The distance 1 correlation computed by using the magnetic field method.
Figure 4: The distance 1 correlation computed by using the chameleon gauge fixing method.