Experimental study of liquid-liquid interface oscillating in radial hele-shaw cell

Ivan E Karpunin¹, Nikolai V Kozlov² and Viktor G Kozlov¹

¹Laboratory of Vibrational Hydromechanics, Perm State Humanitarian-Pedagogical University, Perm, 614990, Russian Federation
²Laboratory of Hydrodynamic Stability, Institute of Continuous Media Mechanics UrB RAS, Perm, 614013, Russian Federation

E-mail: karpunin_ie@pspu.ru, kozlov.n@icmm.ru, kozlov@pspu.ru

Abstract. The dynamics of the interface between two immiscible liquids with a high viscosity contrast is studied experimentally under steady displacement of interface and periodic variation of the flow rate of the pumped liquid in radial Hele-Shaw cell. Classic Saffman–Taylor instability, which develops when the viscous fluid is monotonously displaced by the inviscid one, is well known. In the present work, the excitation of Saffman–Taylor instability by means of oscillations of the liquid-liquid interface is demonstrated. The interphase boundary performs axisymmetric radial oscillations at small amplitude of oscillations and in the absence of an average pumping. With the growth of the amplitude of radial oscillations the interface instability is excited, which manifests itself in the development of an azimuthally periodic finger structure during a part of the period. “Finger-like” instability is determined by the relative amplitude of the oscillations of the interphase boundary and under the conditions of the performed experiments depends neither on the oscillation frequency nor on the radial size of the interface.

1. Introduction

The study of the dynamics of liquid-liquid interfaces in a flat layer, as a model of a porous medium, is of great fundamental and technological interest. The radial Hele-Shaw cell is commonly used to simulate the motion in a porous medium. When a more viscous fluid is displaced in a radial layer by a less viscous fluid, the Saffman–Taylor instability [1] is observed, which manifests itself in the development of a system of inviscid fingers. The characteristic size and structure of fingers are determined by surface tension [2], viscosity forces [3]. In contrast to the problem of stationary displacement, which has been the subject of many experimental and theoretical studies [3, 4], the influence of oscillations on the dynamics of the liquid-liquid interface has not been so rigorously studied. However, a few related vibrational problems can be found in literature; consider for instance some theoretical [5, 6] and experimental [7] works. It should be noted that theoretical works that studied the liquid-liquid interface in plane cylindrical layer (e.g. [8]) demonstrate that the inertial effects may have a stabilizing action on the interface.
2. Experimental setup and methods

The dynamics and stability of the interface between two immiscible liquids of close densities and different viscosities are investigated in a radial Hele-Shaw cell. Radial displacement of a more viscous liquid by a low-viscosity liquid, leveling of the interface with developed Saffman–Taylor instability, and the effect of harmonic variation in fluid flow rate on the stability of the interface are considered. The cell (Fig. 1) has a plane radial layer, organized by two glass discs 1 with a diameter of 150 mm and 8 mm thick. In the center of one of the glasses there is an opening for feeding a low-viscosity displacing liquid into the layer filled with a viscous liquid. The working liquids are selected in such a way that their densities are close, and their viscosity differs by several orders of magnitude. These are dyed water \( \rho_w = 1.0 \times 10^3 \text{ kg/m}^3 \), \( \eta_w = 1.0 \times 10^{-3} \text{ kg/(m·s)} \) and silicone oil PMS-1000 \( \rho_oil = 0.95 \times 10^3 \text{ kg/m}^3 \), \( \eta_oil = 1.0 \text{ kg/(m·s)} \); the coefficient of interfacial tension is \( \sigma = 27 \text{ N/m} \).

A metallic insertion 2 with a channel of diameter 6 mm for supplying liquid, equipped with a fitting 3, is glued into the opening in the glass. A ball valve 4 serves to fill the cell and remove gas bubbles. Glass discs are glued into aluminum frames 5 so that when the frames are closed, the glass discs are located strictly coaxial and parallel to each other. Between the surfaces of the discs a layer of thickness \( h = 1.70 \pm 0.05 \text{ mm} \) is formed. The construction of the cell allows observing the dynamics of liquids both from the side of a solid glass with a diameter \( D = 140 \text{ mm} \) and from the side of a glass with a fitting, through a window with a radius \( a = 55 \text{ mm} \). Photo and video recording of the dynamics of the interface between liquids was carried out with a Canon 600D camera 6 in transmitted light with a shooting frequency of 50 frames/s. The tightness of the layer is ensured by a rubber o-ring 7; the aluminum frames are tightened with bolts 8. The plane layer formed by the glasses extends along its perimeter into an axisymmetric wide annular cavity designed to equalize the pressure in the liquid along the layer perimeter. A fitting 9 (as the fitting 3) entering the annular channel serves to connect the cuvette to the hydraulic circuit. The flow rate of the fluids pumped through the cell is determined by the law \( Q = Q_0 + Q_0 \cos \Omega t \), where \( Q_0 \) is the constant feed component, \( Q_0 = Q_0 \cos \Omega t \) and \( \Omega = 2\pi f_{vib} \) are an amplitude and a radian frequency of flowrate oscillations.

![Figure 1. Schematic drawing of the experimental cell](image)

A pump of harmonic flow-rate oscillations has a short cylindrical cavity consisting of two chambers separated by a movable membrane (Fig. 2(a)). Oscillatory motion of liquid at the pump outlets is set by oscillations of the membrane. A pump cover 1 is an aluminum disk of diameter 150 mm and thickness 6 mm. Walls of the upper and lower pump chambers 2 and 3 have the shape of rings of height 20 mm and inner and outer diameters 106 mm and 150 mm, respectively. Between the rings, the membrane 4 is squeezed, which is made from a 1 mm-thick sheet of oil-petrol-resistant rubber. The choice of this rubber is based on its elasticity and strength that assure a long period of operation. Chambers are sealed using...
rubber o-rings. In the center of the membrane there are two epoxy-composite disks fastened together, their thickness and diameter equal 2 mm and 90 mm, respectively. A rod is rigidly fastened to the disks and connects them to the platform of a vibrator. In the bottom aluminum cover there is an opening, through which the rod enters the chamber. A fixing ring is used to seal the opening with an additional rubber membrane. The latter is hermetically squeezed on the rod and on the cover assuring the sealing of the bottom chamber and allowing the rod oscillations. The pump is connected to the hydraulic circuit via fittings.

A pump of continuous feed (Fig. 2(b)) assures the constant component of the flow rate . It consists of a cylinder whose inner diameter is 60.0 mm and inside which a piston with rubber sealing rings moves. The piston is mounted on a metal rod of diameter 16.0 mm that makes translational motion inside the cylinder. The tightness of the pump internal volume is provided by two sealing glands installed at the ends of the cylinder. The pump operates by the “push-pull” principle: extruding fluid from one cavity draws it into another. The inlet/outlet fittings connect the pump volume to the hydraulic circuit. The rotation of a stepper motor FL86STH65-2808A with a frequency is transformed into the translational motion of the rod with piston by means of a long nut, fixed on the pump rod, and a coupling fixed to a threaded shaft. The motor and the cylinder are rigidly fixed on a metal plate.

The flow rate of the displacing fluid is determined by the motor rotation speed (with high accuracy: 0.001 rps) and can vary in the range with the precision 0.01 ml/s.

![Figure 2](image)

Figure 2. Schemes of the pump of harmonic flow-rate oscillations (a) and of the pump of continuous feed (b)

The pump of harmonic flow-rate oscillations is rigidly fixed on a stationary platform installed on the body of the electrodynamic vibrator and remains stationary in the laboratory reference frame. The vibrator sets the oscillations of the pump membrane. The displacement amplitude of the vibrator table varies in the range and the frequency – in the range Hz. The pump operation principle is the periodic variation of the volumes of the chambers. The amplitude value of the volume of the pumped liquid in the experiments varies in the range ml. All the elements of the experimental setup are connected using metal-plastic pipes, forming a single hydraulic circuit. The cell is
connected to the circuit through separating flanges with elastic membranes. The construction of the experimental setup makes it possible to simultaneously provide the continuous feed of the liquid into the cell and make it oscillate.

3. Saffman–Taylor instability at steady interface displacement

In the classical formulation, the experiment consists in uniform displacement of a more viscous liquid with a less viscous one. During this process, at a certain threshold displacement rate, the interface is deformed due to the non-uniform rate along the boundary perimeter. In this case, local breakdowns of a low-viscosity liquid into a more viscous one occur at the interface. The pressure gradient between the end of the finger of a less viscous fluid and the outer boundary of a more viscous fluid is greater than, respectively, between the concavity and the outer boundary, therefore, according to Darcy's law, the flow velocity at the end of the finger is greater than in the concavity. This inhomogeneity in the displacement rate and rate of the liquid in non-viscous fingers leads to the development of Saffman–Taylor instability. The experiments show that the uniform displacement of a viscous fluid leads to a loss of stability of the axial symmetry of the interface (Fig. 3). In a series of photographs, it can be seen that as the contact line moves away from the center of the cavity, an inhomogeneity forms at the boundary (ripples in Fig. 3(c)), which, with further displacement, intensifies and leads to the appearance of the Saffman–Taylor instability (Fig. 3(d)) by the mechanism described above.

As the displacement rate increases, the finger-forming effect is enhanced. The fingers are formed already at a small distance from the center of the cuvette (Fig. 4(a)) and then develop independently (Fig. 4(b, c)). The discovered instability refers to the well-studied case of the Saffman–Taylor instability. Note that the wavelength of finger structures in the phase of their appearance is in good agreement with the theoretical and experimental results of other authors [9].
4. Effect of liquid-liquid interface oscillations on its shape

Let us consider the influence of radial oscillations of the interface between liquids on an initially deformed interface with developed Saffman–Taylor instability. Figure 5 (a) shows a picture of the initial state of the liquid-liquid interface, on which the "non-viscous fingers" of colored water in more viscous oil are visible. Under the influence of vibrations, a slow displacement of the base of the fingers occurs, while it is noticeable that the tops of the fingers farther from the center of the cavity are practically not displaced (Fig. 5(b)). When the interface oscillates, the velocity at the base of the fingers, due to the inhomogeneous radial distribution of the velocity, is higher than at the tops of the fingers. An inverse relationship is observed in the propagation velocity compared to the case of uniform displacement of the liquid. Over time, the interface tightens, forming an almost uniform circular contact line. However, it should be noted that drops of more viscous oil can be captured by a less viscous fluid (Fig. 5(c)). At the same time, under the influence of vibrations, over time, a low-viscosity liquid slowly washes away the more viscous oil, gradually leveling the shape of the boundary. The interesting effect is the behavior of a single drop of a low-viscosity liquid formed when the liquid vibrates in the cuvette (Fig. 5(d)). The droplet washed by a more viscous liquid begins to drift towards a less viscous liquid and is eventually absorbed by the main volume of a low-viscosity liquid. The final form of the interphase boundary performing radial oscillations has an almost axisymmetric shape. It can be concluded that radial vibrations of liquids with high viscosity contrast, in the absence of a constant feed of low-viscosity liquid, lead to the alignment of the initially inhomogeneous interface (Fig. 5(f)).

5. Oscillatory finger instability

The dynamics of the interface between two liquids with a high viscosity contrast in a radial Hele-Shaw cell with a harmonic change in the flow rate of the pumped liquid, $Q = Q_0 \sin \Omega t$, was considered in the work [10]. The experiments were carried out with a pair dyed water–silicone oil PMS-1000 at $Q_0 = 0$ ml/s. The dynamics of the interface was studied at a fixed frequency with a change in the vibration amplitude (volume of pumped liquid). At low amplitudes of oscillations (Fig. 6 (a, b)), the interface performs radial oscillations, while the oscillating front of a low-viscosity liquid has an axisymmetric shape. With the increase in the amplitude of oscillations in certain phases of the period, the development of a system of inviscid fingers is observed (Fig. 6(d)).

This instability is formed in the phase of maximum radial displacement of a viscous liquid and completely disappears in the phase of maximum displacement of the boundary in the direction of a low-viscosity liquid (Fig. 6(c)). The oscillations of the interface either with a stable radial boundary or with an unstable one (in the form of a system of inviscid fingers) occurs strictly radially. The oscillations, as in the above-described case of border alignment, have a stabilizing circular effect. The analysis of the data obtained at different $R_0$ and frequencies has shown that the development of fingers occurs in a threshold manner. Instability develops at the same value of the vibration amplitude and does not depend on either the vibration frequency, or the initial size $R_0$. 
Figure 5. Classical Saffman–Taylor instability at \( Q_o = 2.13 \) ml/s, \( V_0 = 0 \) ml and \( t = 0 \) s (a). Stabilization of the interface at \( f_{vib} = 2 \) Hz and \( V_0 = 2.98 \) ml. Panels (b)–(f) correspond to the time moments
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t = (43.6, 129.1, 168.1, 210.1, 236.2) \text{ s}
\]

Figure 6. Interface between two liquids for \( R_0 = 15.3 \) mm at \( f_{vib} = 2 \) Hz: (a, b) \( V_0 = 0.85 \) ml, (c, d) \( V_0 = 1.62 \) ml. Images (a, c) correspond to the phase of maximum contraction of the boundary \( (\Omega = 0) \), images (b, d) – to the phase of maximum radial displacement \( (\Omega = \pi/2) \). A stable contact line of the interface between liquids \( R_{ss} \) is marked by a red solid curve.

6. **Effect of oscillations on liquid-liquid interface at its steady displacement**

Let us dwell upon the case of the combined action of static and oscillating radial pressure gradients, when, in addition to the uniform displacement of a viscous liquid, the interface performs radial high-frequency oscillations. The transformation of a moving interface with time under oscillations (Fig. 7) is qualitatively different from the case of uniform displacement (Fig. 3).
From a comparison with the case of uniform displacement with the same flow rate in the absence of oscillations (Fig. 3), it follows that at small oscillation amplitudes (below the threshold for the development of oscillatory finger instability of the interface) oscillations have a destabilizing effect on the interface (Fig. 7). The developing fingers of a low-viscosity liquid are long. Moreover, their wave number turns out to be significantly higher. The latter is consistent with the fact that in the case of purely oscillatory motion of the boundary, instability manifests itself in the development of fingers with a large wavenumber (Fig. 6). It is worth to note the averaged centering effect of oscillations on the interface (also shown in Fig. 5): despite the development of the fingers, the interface has a pronounced concentric inner border, and the fingers have a similar length (Fig. 7).

7. Conclusion
For the first time, the dynamics of the interface in a radial Hele-Shaw cell under the combined action of oscillating and constant radial pressure gradients is considered. The experiments show that radial oscillations of the interface of liquids with high viscosity contrast (when the high-viscosity liquid is outside) lead to several new effects. Thus, under the influence of oscillations, the interface takes a concentric shape, even if initially it was significantly disturbed. With the increase in the amplitude of the oscillations, a new type of instability appears, when a transient instability in the form of fingers develops at the axisymmetric interface in the phase of radial displacement of a viscous fluid, which completely disappears in the phase of contraction. Finally, oscillations contribute to the development of the classical Saffman–Taylor instability in the case of uniform radial displacement of a viscous fluid; at the same displacement rate, the developing Saffman fingers become longer and thinner under the influence of oscillations.

Acknowledgments
The work was supported by the Government of Perm Region (grant of International Research Group C-26/174.9 and grant for Leading Scientific School C-26/1191) and by the Ministry of Science and Higher Education of the Russian Federation (theme AAAA-A18-118020590106-3 of ICMM UrB RAS).

References
[1] Saffman P G and Taylor G I 1958 Proc. R. Soc. London, Ser. A 245 (1242) 312–329
[2] Park C W and Homsy G M 1985 The Physics of fluids 28 (6) 1583–1585
[3] Bischofberger I, Ramachandran R, Nagel S R 2014 Nat. Commun. 5 5265
[4] Anjos P H A, Dias E O, Miranda J A 2017 Phys. Rev. Fluids 2 084004
[5] Lyubimov D, Lyubimova T and Cherepanov A 2003 Dynamics of Interfaces in Vibration Fields (Physmathlit, Moscow) (in Russian)
[6] Alabuzhev A A 2018 Microgravity Science and Technology 30 25–32
[7] Marfin E A, Garaeva S V and Abdashitov A A 2020 IOP Conf. Ser.: Mater. Sci. Eng. 927 012027
[8] Rabbani S, Abderrahmane H A and Sassi M 2019 Fluids 4(2) 79
[9] Fernandez J, Kurowski P, Limat L and Petitjeans P 2001 Phys. Fluids 13 3120
[10] Kozlov V G, Karpunin I E and Kozlov N V 2020 Physics of Fluids 32 (10) 102102