OPEN FLAVOR CHARMED MESONS

P. C. Vinodkumar,
*Department of Physics, Sardar Patel University,
Vallabh Vidyanagar-388 120, Gujarat, India.

Ajay Kumar Rai,
Department of Applied Sciences and Humanities,
SVNIT, Surat-395 007, Gujarat, India.

*Bhavin Patel,
Jignesh Pandya,
Department of Physics, Veer Narmad South Gujarat University,
Surat-395 007, Gujarat, India.

Abstract

We present here recent results on the investigations of the mass spectrum (S-states and P-states), decay constants, decay widths and life time of the D, D_s, and B_c mesons within the framework of phenomenological potential models. We also present the binding energy and the masses of the di-meson molecular systems with one or more charm meson combinations. Many of the newly found experimental open charm states are identified with the orbital excitations of the conventional open charm mesons while others like X(3872), Y(3930), D_{sJ}(2632, 2700) etc., are identified as molecular like states.
1 Introduction

The study of spectroscopy and the decay properties of the heavy flavour mesonic states provides us useful information about the dynamics of quarks and gluons at the hadronic scale. The remarkable progress at the experimental side, with various high energy machines such as LHC, B-factories, Tevatron, ARGUS collaborations, CLEO etc for the study of hadrons has opened up new challenges in the theoretical understanding of heavy flavour hadrons. In order to understand the structure of the newly observed zoo of open flavour meson resonances\textsuperscript{1} \textsuperscript{2} \textsuperscript{3} \textsuperscript{4} in the energy range of 2-5 GeV, it is necessary to analyze their spectroscopic properties and decay modes based on theoretical models. Many of these states could be the excited charmed mesonic states while for many other states the possibility of multi-quark or molecular like structures are being proposed. Thus, the main objective of the present talk includes the study of spectroscopy and the decay properties of the open flavour charm mesons. We study these open charm states as the excited states of the conventional quark-antiquark systems within the framework of a potential model\textsuperscript{5} \textsuperscript{6}.

We also study, following the molecular interpretation of some of the recently observed meson states, the binding energy and the ground state masses of di-hadronic molecules\textsuperscript{7} \textsuperscript{8}. For the binding energy of the di-hadronic state, we consider a large \( r \rightarrow \infty \) limit of the confined gluon propagator employed in our earlier study on N-N integrations.\textsuperscript{9}

2 Theoretical methodology: A Potential Scheme

For the light-heavy flavour bound system of \( qQ \) or \( \bar{q}Q \) we treat the heavy-quark (Q=c, b) non-relativistically and the light-quark (q = u, d, s) relativistically within the mesonic system. The Hamiltonian for the case be written as\textsuperscript{10}

\[
H = M + \frac{p^2}{2m} + \sqrt{p^2 + m^2} + V(r) + V_{\bar{S}_Q \cdot S_q}(r) + V_{L \cdot S}(r)
\]

(1)

Where \( M \) is the heavy quark mass, \( m \) is the light quark mass, \( p \) is the relative momentum of each quark, \( V(r) \) is the confined part of the quark-antiquark potential, \( V_{\bar{S}_Q \cdot S_q}(r) \) and \( V_{L \cdot S}(r) \) are the spin-spin and spin orbital part of the interaction. Here we consider

\[
V(r) = -\frac{\alpha_c}{r} + Ar^\nu
\]

(2)
where $\alpha_c = \frac{4}{3}\alpha_s$, $\alpha_s$ being the strong running coupling constant, $A$ and $\nu$ are the potential parameters. For computing the hyperfine and spin-orbit splitting, we consider the spin dependent part of the usual OGE potential given by (10)

$$V_{S\bar{Q}Q}(r) = \frac{2}{3} \frac{\alpha_c}{M_Q m_q} \vec{S}_{\bar{Q}} \cdot \vec{S}_Q 4\pi \delta(\vec{r}), \quad V_{L-S}(r) = \frac{\alpha_c}{M_Q m_q} \vec{L} \cdot \vec{S} \frac{4}{r^3}$$

We employ the harmonic oscillator wave function and use the virial theorem, to get the energy expression from the hamiltonian defined by Eqn. (1). Here $\mu$ is the wave function parameter determined using the variational method. The parameters used here are $m_{u/d} = 0.360$ GeV, $m_s = 0.5$ GeV, $m_c = 1.41$ GeV, $m_b = 4.88$ GeV, $\alpha_c = 0.48$ (for open charm meson) and $\alpha_c = 0.36$ (for open beauty-charm meson). The computed $S$ and $P$ wave mass spectrum of $D$, $D_s$ and $B_c$ mesons are tabulated in Table 1 alongwith the experimental and other theoretical results.

3 The decay constants and Lifetime of the open charm mesons

The decay constant of the mesons is an important parameter in the determination of the leptonic, non-leptonic weak decay processes. It is related to the wave function at the origin through Van-Royen-Weisskoff formula. Incorporating a first order QCD correction factor, we compute them using the relation (11)

$$f_P^2 = \frac{12}{M_P} \frac{|\Psi_P(0)|^2}{C^2(\alpha_s)}$$, where

$$C^2(\alpha_s) = 1 - \frac{\alpha_s}{\pi} \left[ 2 - \frac{M_Q - m_q}{M_Q + m_q} \ln \frac{M_Q}{m_Q} \right]$$

where $M_P$ is the ground state mass of the pseudoscalar states.

In the spectator approximation (6) (12) the inclusive widths of $b$ and $c$ quarks decay are given by

$$\Gamma(b \to X) = \frac{9}{192\pi^3} \frac{G_F^2 |V_{Q\bar{Q}}|^2 m_b^5}{M_P^2}, \quad \Gamma(c \to X) = \frac{5}{192\pi^3} \frac{G_F^2 |V_{Q\bar{q}}|^2 m_c^5}{M_P^2}$$

and width of the annihilation channel is computed using the expression given by (6) (12)

$$\Gamma(Anni) = \frac{G_F^2}{8\pi} |V_{Q\bar{q}}|^2 f_P^2 M_P \sum_i m_i^2 \left( 1 - \frac{m_i^2}{M_P^2} \right)^2 C_i$$
Table 1: \( S \)-Wave and \( P \)-Wave Masses (in MeV)

| \( \nu \) | \( 1^1S_0 \) | \( 1^3S_1 \) | \( 1^3P_0 \) | \( 1^3P_1 \) | \( 1^3P_2 \) | \( 2^1S_0 \) | \( 2^3S_1 \) |
|-----|-----|-----|-----|-----|-----|-----|-----|
| D   | 0.5 | 1922 | 1992 | 2195 | 2203 | 2210 | 2218 | 2286 | 2294 |
|     | 1.0 | 1912 | 1993 | 2347 | 2367 | 2390 | 2414 | 2580 | 2639 |
|     | 1.5 | 1905 | 2003 | 2388 | 2435 | 2481 | 2527 | 2599 | 2709 |
|     | Expt.| 1864 | 2006 |       |       |       |       |       |       |
|     |     | 1875 | 2009 |       |       |       |       |       |       |
|     |     | 1915 | 1999 | 2385 | 2417 | 2449 | 2518 | 2653 | 2690 |
|     | Ds  | 0.5 | 2042 | 2089 | 2353 | 2364 | 2375 | 2386 | 2466 | 2476 |
|     |     | 2003 | 2104 | 2512 | 2544 | 2576 | 2608 | 2813 | 2847 |
|     |     | 1937 | 2135 | 2607 | 2678 | 2750 | 2821 | 3149 | 3228 |
|     | Expt.| 1969 | 2112 |       |       |       | 2535 | 2574 |       |
|     |     | 1981 | 2111 | 2508 | 2515 | 2560 | 2569 | 2670 | 2716 |
|     |     | 2009 | 2110 | 2385 | 2417 | 2449 | 2481 | 2778 | 2280 |
|     | Bc  | 1.0 | 6349 | 6373 | 6715 | 6726 | 6738 | 6749 | 6821 | 6855 |
|     |     | 6280 | 6321 | 6727 | 6743 | 6765 | 6783 | 6960 | 6990 |
|     |     | 6356 | 6397 | 6673 |       |       | 6751 | 6888 | 6910 |
|     |     | 6270 | 6332 | 6699 | 6734 | 6749 | 6762 | 6835 | 7072 |

Expt.\(^{3)}\), ALV\(^{12)}\), EFG\(^{13)}\), Pandya\(^{14)}\), Lattice\(^{15)}\), Ebert\(^{16)}\)

where \( C_1 = 1 \) for the \( \tau \nu_\tau \) channel and \( C_1 = 3 |V_{Q\bar{q}}|^2 \) for \( Q\bar{q} \), and \( m_i \) is the mass of the heaviest fermions. Here \( |V_{Q\bar{q}}| \) and \( |V_{QQ}| \) are the respective CKM Matrix, where numerical values are obtained from \(^{3)}\). The total width of the \( Q\bar{q} \) meson decay is the addition of partial widths \( i.e. \Gamma(total) = \Gamma(Q \rightarrow X) + \Gamma(Anni) \).

In the case of the \( B_c \) meson, both the heavy quark, \( b \) and \( c \) under go the decay and the total width is obtained as \( \Gamma_{total}(B_c) = \Gamma(b \rightarrow c) + \Gamma(c \rightarrow X) + \Gamma(Anni) \).

The computed pseudoscalar decay constants with and without the correction factor \( C^2(\alpha_s) \), the total width and lifetime of \( D \), \( D_s \) and \( B_c \) mesons are listed in Table 2 along with other model predictions and experimental values.

4 Di-hadrons as molecular states

The low-lying di-hadronic molecular system consisting of di-meson tetra quark states are treated here by assuming non-relativistic. Hamiltonian given by

\[
H = M + \frac{P^2}{2\mu} + V(R_{12}) + V_{SD}(S_1S_2)
\]  (7)
Table 2: Decay constants ($f_P$) and lifetime of meson.

| System | $\nu$ | $f_P$ (MeV) | $f_P$(cor.) (MeV) | $\Gamma$(total) ($10^{-4}$eV) | $\tau$ (ps) |
|--------|-------|-------------|-------------------|------------------------------|-------------|
| $D$    | 0.5   | 231         | 157               | 6.126                        | 1.074       |
|        | 1.0   | 250         | 170               | 6.142                        | 1.072       |
|        | 1.5   | 276         | 187               | 6.167                        | 1.067       |
| Expt.  |       |             |                   |                              | 1.040±0.007|
| Penin  |       |             |                   |                              | 285±20      |
| Ebert  |       |             |                   |                              | 243±25      |
| $D_s$  | 0.5   | 218         | 156               | 9.148                        | 0.719       |
|        | 1.0   | 321         | 229               | 12.630                       | 0.521       |
|        | 1.5   | 451         | 322               | 18.515                       | 0.356       |
| Expt.  |       |             |                   |                              | 500±0.007   |
| Heister|       |             |                   |                              | 285         |
| $B_c$  | 1.0   | 556         | 13.86             | 0.47                         |             |
| Expt.  |       |             |                   |                              | 0.46±0.18   |

where $M = m_{h_1} + m_{h_2}$, $m_{h_1}$ and $m_{h_2}$ are masses of the hadrons, $\mu$ is the reduced mass, $P$ is the relative momentum of the two hadrons and $V(R_{12})$ is the residual (molecular) interaction potential between the two hadrons given by the asymptotic expression ($r \to \infty$) of the confined one gluon exchange interaction (COGEP) given by [9]

$$V(R_{12}) = -\frac{k_{mol}}{R_{12}} e^{-C R_{12}^2}$$

(8)

where $k_{mol}$ is the residual strength of the strong interaction coupling and $C$ is the effective colour screening parameter of the confined gluons. Using a trial wave function given by

$$\psi(R_{12}) = \left(\frac{4 \Omega^{3/2}}{\sqrt{\pi}}\right)^{1/2} e^{-\Omega R_{12}^2}$$

(9)

By minimizing the expectation value of $H$, the ground state molecule energy is obtained as

$$E(\Omega) = M + \frac{3\Omega}{4\mu} - \frac{4k_{mol}\Omega^{3/2}}{e^2 + 2\Omega} + \frac{8}{9} \frac{\alpha_s}{m_{h_1} m_{h_2}} \vec{S}_1 \cdot \vec{S}_2 |\psi(0)|^2$$

(10)
Table 3: Low-lying masses of Multiquarks as di-hadronic molecule

| Systems | $J^P C$ | $\Omega$ | $\psi$ | BE | Mass | Expt | Others |
|---------|---------|----------|--------|-----|------|-------|--------|
|         | $h_1 - h_2$ | GeV$^2$ | GeV$^3/2$ | GeV | GeV | GeV | GeV |
| $\pi - D$ | 0++ | 0.0186 | 0.0757 | 0.022 | 2.027 | – | – |
| $\pi - D^*$ | 1+ | 0.0188 | 0.0762 | 0.022 | 2.169 | – | – |
| $K - D$ | 0++ | 0.1415 | 0.3465 | 0.015 | 2.344 | $D_s J(2.317)$ | – |
| $K - D^*$ | 1+ | 0.1455 | 0.3539 | 0.016 | 2.485 | $D_s J(2.460)$ | – |
| $\rho - D$ | 1+ | 0.2684 | 0.5602 | 0.033 | 2.603 | – | – |
| $K^* - D$ | 1+ | 0.3265 | 0.6489 | 0.039 | 2.718 | $D_s J(2.700)$ | – |
| $\rho - D^*$ | 1+ | 0.2795 | 0.5775 | 0.235 | 2.845 | – | – |
| $K^* - D^*$ | 1+ | 0.3420 | 0.6718 | 0.158 | 2.976 | – | – |
| $D - D$ | 0++ | 0.3568 | 0.6935 | 0.008 | 3.723 | 3.723 | 19 |
| $D - D^*$ | 1+ | 0.3810 | 0.7285 | 0.006 | 3.878 | X(3.870) | 3.876 | 20 |
| $D^* - D^*$ | 1+ | 0.4081 | 0.7670 | 0.084 | 3.930 | – | – |
| $2++$ | – | – | 0.040 | 4.062 | $\psi(4.040)$ | 3.968 | 21 |

Here, we have added the spin-hyperfine contribution separately. The binding energy of the di-mesons as $BE = |m_{h_1} + m_{h_2} - E|$ and the parameters $k_{mol} = 0.45$ and $c=1.25$ GeV are employed to compute the binding energy (BE) at the charmed sector. The computed masses and binding energies of the di-meson systems are tabulated in Table 3.

5 Conclusion and Discussion:

The properties of open charm mesons vs a vs $D$, $D_s$ and $B_c$ are investigated by us using an effective static quark-antiquark interaction potential of the form $-\frac{\alpha}{r} + Ar^\nu$. We found that the potential form with $\nu = 1.0$ is consistent with the experimental results of the light-heavy flavour mesons. The relativistic treatment of light flavour and non relativistic treatment of heavy
flavour seem to be justifiable in light of the successful prediction of the various properties of light-heavy flavour mesons. In the case of $B_c$-meson study, the non-relativistic treatment for both the heavy quarks yields better result. The $S$-wave and $P$-wave masses, decay constants $f_P$, the decay widths and lifetime of $D$, $D_s$ and $B_c$ mesons are studied within the potential scheme with $0.5 \leq \nu < 2$. The recently observed $D_{s1}(2536)$ and $D_{s1}^*(2857)$ are found to be the $1^3P_1$ and $2^3S_1$ states predicted in our model with $\nu = 1.0$ Other predicted excited states are expected to be identified and observed in future experiments.

The pseudoscalar decay constant $f_P$ predicted without the correction terms $C^2(\alpha_s)$ of Eqn.(4) in our model with potential index $\nu = 1$ is found to be in better agreement with the experimental values of $f_{D^+} = 222.6 \pm 16 MeV$ of CLEO collaboration $^{22}$ and the predicted value of $321 MeV$ for $f_{D_s}$ is within the error bar of the experimental result of $283 \pm 17 \pm 7 \pm 14 MeV$ by BaBar collaboration $^1$. However, the PDG average value for $f_{D_s}$ is $267 \pm 33 MeV$ $^3$. The ratio of $f_{D_s}/f_P$ in our case is 1.34 with the correction factor, while that with out correction factor is 1.28 which is in accordance with the Lattice results of $1.24 \pm 0.01 \pm 0.07$ $^{23}$. The lifetime predictions of $1.07 ps$ for $D$ and $0.52 ps$ for $D_s$ mesons are in good agreement with the respective experimental result of $1.04 \pm 0.007 ps$ of $D^\pm$ and $0.5 \pm 0.007 ps$ with $\nu = 1.0$.

The exotic states such as $X(3872), D_{sJ}(2317, 2460, 2632, 2700$ and $2860), \psi(4040)$ etc are identified as the low lying di-mesonic molecular states at the charm sector as shown in Table 3. Though there exist many attempts, the zoo of open flavour mesonic states continues to pose challenges to both experimental analysis and theoretical predictions.

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