Applications of periodic structures and metamaterials for antenna design

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Submitted: 01/07/2022 Accepted: 06/08/2022 Published online: 31/08/2022

Abstract: Nowadays, wireless telecommunication is playing an increasing role in almost every field. With the spread of IoT and 5G technologies, all electronic devices will communicate over a network and millions of devices can be connected to each other over such network. Due to this, the number of base stations will also increase significantly. As a result, even more antennas will be installed both indoors and outdoors. It follows that the use of smaller and more compact antennas and simpler and more reliable parameter tuning are of great importance to service providers as well as to equipment manufacturers, both in terms of design and antenna selection. Metamaterials are sub-wavelength structures that act as a homogeneous material to create electromagnetic properties that would not be possible with conventional materials. With the help of today's modern design and manufacturing technologies, it is possible to implement and manufacture such structures. In this article, the theoretical background and application possibilities of periodic structures and materials are presented, followed by the tuning of a split ring resonator and its application on a patch antenna.

Keywords: Periodic structure, Metamaterial, Periodic filter, Antenna simulation

I. INTRODUCTION

Periodic structures are naturally occurring phenomena, one of the best examples being the minerals that are made of crystal structures [1]. As such, the crystal structures are made up of many small crystal elements that are identical to each other.

In microwave technology, periodic structures similar to crystal structures have also appeared [1]. These structures can be considered of finite, or even infinite elements, which can be in one-, two-, or three-dimensional shapes. The simplest example is the theoretical model presented in Fig. 1.

II. ANALYSIS OF THE PERIODIC STRUCTURE

The model of the periodic structure shown in Fig. 1 consists of an infinite number of identical elements. Each element has a length $d$, in the center of which is the concentrated parameter $B$ (joint to transmission line impedance) admittance. If a concentrated parameter element is a two-port system, it can be described with a so-called ABCD matrix so that element is divided into three parts, a $d/2$ length line, $jB$, and another $d/2$ of the feed line. The relationship between input and output to $n$-th element is described by the equations (1-5)[2],

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix}, \quad (1)$$

$$A = \cos \theta - \left( \frac{b}{2} \right) \sin \theta, \quad (2)$$

$$B = j \left( \frac{b}{2} \cos \theta + \sin \theta - \frac{b}{2} \right) \quad (3)$$

$$C = j \left( \frac{b}{2} \cos \theta + \sin \theta + \frac{b}{2} \right) \quad (4)$$

Figure 1. One-dimensional periodic structure [1]
\[ D = \cos \theta - \left(\frac{b}{2}\right) \sin \theta \]  

(5)

where \( \theta = kd \) is the electric length of the feed line in the unit element, \( k \) is the wave number equal to the propagation coefficient of the TEM transmission line.

Assume, that the periodic structure is of infinite length, the wave parameters of the \( n \)-th and \( (n + 1) \) cells are identical, except for the phase shift caused by the propagation on the element. The equation in (1) can be reformulated as

\[ \begin{bmatrix} V_n \\ I_n \end{bmatrix} = e^{-\gamma d} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} \]  

(6)

In eq (6) \( \gamma = \alpha + j\beta \), which represents the complex propagation coefficient for the periodic structure. From (1) and (6) the following eigenvalue following simplification (eigenvalue) can be implemented:

\[ \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} e^{-\beta d} & 0 \\ 0 & e^{-\gamma d} \end{bmatrix}. \]  

(7)

After further simplification

\[ \cosh \gamma d = \cos \theta - \left(\frac{b}{2}\right) \sin \theta. \]  

(8)

If \( \alpha = 0 \), then \( \gamma = j\beta \), which leads to unattenuated wave propagation in the periodic structure. If the right side of the equation (8) is greater than one, no wave propagation occurs.

### III. EXAMPLE FOR PERIODIC STRUCTURE

The capacitively loaded periodic structure behaves as follows from the point of view of wave propagation. (see an example in Fig. 2).

![Figure 2. Microstrip periodic structure [2]](image1)

The concentrated parameter representation of the periodic structure in Fig. 2 can be seen in Fig. 3

![Figure 3. Theoretical model of a microstrip periodic structure [2]](image2)

The parameters of the structure are as follows: \( Z_0 = 50\Omega \), \( d = 1\text{cm} \), \( C_0 = 2.6666\text{pF} \). During this example, we define the k-\( \beta \) (Brillouin) diagram, the propagation coefficient, the phase velocity, and the Bloch impedance at \( f = 3\text{GHz} \) [2]. Replacing the dispersion ratio (6), gives the following relationship

\[ \cos \beta d = \cos k_0 d - \left(\frac{C_0 Z_0 c}{2d}\right) k_0 d \sin k_0 d, \]  

(9)

\[ \cos \beta d = \cos k_0 d - 2k_0 d \sin k_0 d. \]  

(10)

If the right side of the eq (10) is between 0 and 1, wave propagation will occur (passband-bandwidth). A Passband is created if it falls between \( \theta \leq k_0d \leq 0.96 \). The next passband is at \( \pi \). Infinite number of passbands with decreased bandwidth may be achieved by increasing the \( k_0d \) value to infinity. At 3 GHz, \( k_0d \) takes the following value

\[ k_0d = 36°. \]  

(11)

It follows that \( \beta d \) is 1.5, so the propagation coefficient is \( \beta = 150 \text{ rad / m} \). The phase velocity is

\[ v_p = \frac{k_0 c}{\beta} = 0.42c. \]  

(12)

An important phenomenon is observed, showing that the phase velocity is much lower than the speed of light. These structures are called Slow-wave structures. From the above, the k-\( \beta \) (Brillouin) diagram can be determined.

Finally, the determination of Bloch impedance, which determines the wave impedance at the input of an element.

\[ B = \frac{b}{2} = \frac{\omega C_0 Z_0}{2} = 1.256, \]  

(13)

\[ \theta = k_0 d = 36°, \]  

(14)

\[ A = \cos \theta - \frac{b}{2} \sin \theta = 0.707, \]  

(15)

\[ B = j \left( \sin \theta + \frac{b}{2} \cos \theta - \frac{b}{2} \right) = j0.3479, \]  

(16)

\[ Z_B = \left( \frac{B Z_0}{\sqrt{A^2 - 1}} \right) = \left( \frac{\left(j0.3479\right)\left(50\right)}{\sqrt{1 - \left(0.707\right)^2}} \right). \]  

(17)

![Figure 3. k-\( \beta \) (Brillouin) diagram [2]](image3)
IV. METAMATERIALS

Left-handed materials [3] are special artificial structures with negative refractive index (Fig. 4). The development of these materials is also emphasized in optical and microwave environments.

\[ n_1 \sin(\theta_1) = n_2 \sin(\theta_2), \]  

(18)

Figure 4. Refraction in a normal and a metamaterial [3]

In nature, substances generally have positive permittivity and permeability. However, there are also substances that have either negative permittivity or permeability. These properties are highly frequency dependent. \( \varepsilon > 0 \) occurs in high frequency ranges, primarily in the optical range and in semiconductors from the THz range to the infrared range [2]. Examples of such materials are gold, silver, and aluminium.

In contrast, \( \mu < 0 \) is available in a lower frequency range, typically in ferro- or antiferromagnetic materials. However, some metamaterials have recently been discovered, such as bismuth at \( \lambda = 60 \) \( \mu \)m. Unfortunately, bismuth cannot be used in practice due to its high resistance and relative rarity.

1. Electromagnetic metamaterials

Double Negative Metamaterials (DNG) [9] have both negative permeability and permittivity properties at the same frequency. This also means that they have a negative refraction index.

Metamaterial structures are often made of two kinds of Single Negative Metamaterials (SNG). SNG materials are closely related to Epsilon Negative Materials (ENG), with negative permittivity and positive permeability and to Mu-Negative Materials (MNG) with positive permittivity and negative permeability. The SNG structures have zero or close to zero positive properties, this is the difference between SNG and ENG/MNG. Two layers of different kinds of SNG can be combined to make an effective DNG material [8].

2. Split ring resonator

One solution is to create a material with the simultaneous use of two different periodic structures with different properties. An example of this is the combination of capacitive load strip and split-ring structures. The wire structure was electrically, while the split-ring structure was magnetically active at a given frequency. Metamaterials can be a big leap in the development of microwave technology. With their help, antennas can be designed more directional, more profitable, and smaller size antennas [6][13].

V. SIMULATION OF A SPLIT RING RESONATOR (SRR)

In this chapter, a simulation of a split ring resonator will be presented. The structure in Fig. 5 was simulated at 12 GHz, its substrate is a 0.25mm thick FR4 epoxy.

The simulation has been performed using Ansys HFSS [5] simulation software in the frequency range of 5 - 15 GHz.

The simulation result in Fig. 6 shows that the periodic structure operates as a DNG material. The structure has negative permeability and permittivity in the same frequency range. According to the results, the structure operates in the required 12 GHz frequency range and also has a quite wideband operation range with about 370 MHz.

Figure 5. SRR element model in HFSS

Figure 6. SRR element simulation results
VI. APPLICATION OF METAMATERIAL STRUCTURES

1. Metamaterial absorbers

Metamaterials can be used as radiofrequency absorbers. Their main advantage over conventional absorbers are suitable for miniaturization, effectiveness, wider frequency range and adaptability [4].

2. Metamaterial lenses

A refraction index that is close to zero is achievable with the use of periodic metamaterial structures. With zero refractive index, the structures behave like perfect lens. These lenses have optical, and microwave applications as well. As an optical device, it can replace glass as metamaterial lenses that are much thinner, lighter, and much more effective. In microwave technology, metamaterial lenses can be used to focus radiofrequency power, resulting in higher-gain antennas. Metamaterials can also be used for parabolic antennas, as their mode of operation is the same, but their size can be drastically reduced [9][10].

3. Metamaterial antennas

Metamaterials are used to increase the efficiency and minimize the size of antennas and antenna systems. Generally, metamaterials are used to increase the gain of conventional antennas using periodic structures. The antenna reflects part of the transmitted power back to the generator. With the use of periodic metamaterial structures, better impedance matching, improved gain, bandwidth, efficiency front-to-back ratio and so on can be achieved [5][11].

The Fig. 7 shows an early experiment to make a more efficient 10 GHz patch antenna. The patch of the antenna is surrounded by split ring resonators.

Figure 7. Experimental metamaterial patch antenna simulation in HFSS

The simulation shows minor improvement in the antenna characteristics, better front-to-back ratio, and wider bandwidth. Fig. 8 shows the difference in the antenna characteristic. The result on the left-hand side was made with the split ring resonators, meanwhile the characteristics shown on the right-hand side just the patch element was simulated. Using metamaterials, the gain of the patch antenna increased by 2.8 dB. The only drawback is that the reflection is 3 dB lower than in case of the regular patch antenna, but the difference can be neglected.

By tuning the split ring resonators, optimizing their placement around the patch element even better results may be achieved. This experiment showed that it is possible to improve the antenna parameters by adding metamaterial structures around the radiating patch. Additionally, the metamaterial structure created out of printed circuit adds no extra cost to the PCB. [7].

Figure 8. Experimental metamaterial patch antenna simulation Results

VII. CONCLUSION

The concept and the theoretical background of metamaterials have been presented. The practical applicability of metamaterials has been shown through an example with the goal of improving antenna characteristics. The results showed that some periodic structures can be used as
metamaterial, as it has a negative refractive index. SRR structures are very promising metamaterials, with further examination and optimization, they can be easily used in practice.

**AUTHOR CONTRIBUTIONS**

**P. Prukner**: Conceptualization, Theoretical analysis, Antenna simulation, Writing, Review and editing

**M. Kuczmann**: Supervision, Review and editing.

**DISCLOSURE STATEMENT**

We have no conflict of interest to declare.

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