Quantum information in Hawking radiation

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In 1974 Steven Hawking showed that black holes emit thermal radiation, which eventually causes them to evaporate. The problem of the fate of information in this process is known as the “black hole information paradox.” It inspired a plethora of theoretical models which, for the most part, assume either a fundamental loss of information or some form of quantum gravity. At variance to the main trends, a conservative approach assuming information retrieval in quantum correlation between Hawking particles was proposed and recently developed within qubit toy-models. Here we leverage modern quantum information to incarnate this idea in a realistic model of quantised radiation. To this end we employ the formalism of quantum Gaussian states together with the continuous-variables version of the quantum marginal problem. Using a rigorous solution to the latter we show that the thermality of all Hawking particles is consistent with a global pure state of the radiation. Surprisingly, we find out that the radiation of an astrophysical black hole can be thermal until the very last particle. In contrast, the thermality of Hawking radiation originating from a microscopic black hole — which is expected to evaporate through several quanta — is not excluded, though there are constraints on modes’ frequencies. Our result support the conservative resolution to the black hole information paradox. Furthermore, it suggests a systematic programme for probing the global state of Hawking radiation.

1 Introduction

When a star runs out of its nuclear fuel the gravitational attraction takes over and the star implodes. If the mass of the stellar remnant exceeds the Tolman–Oppenheimer–Volkoff limit (~ 2.3 solar masses), the gravitational collapse will result in a black hole. According to general relativity, the latter consist of a central spacetime singularity hidden under an event horizon – a surface impenetrable from the inside by any particles or fields. The area of the event horizon continues to grow as the black hole engulfs the surrounding matter. The collapse progresses indefinitely, trapping all infallen information carried by the matter fields in the black hole region.
However, once an event horizon forms, the quantum field theory predicts a steady flux of thermal radiation emitted from the black hole\(^{24}\). Its origin is attributed to the fluctuations of quantum vacuum near the event horizon. These result in spontaneous pair creation, with one of the partners falling into the black hole and the other escaping to infinity. An alternative description of the mechanism involves tunnelling of quantum particles through the event horizon\(^{25}\). In consequence, an ideal Schwarzschild black hole behaves like a black body, while a physical one — spinning and surrounded by matter — as a grey body\(^{26}\). The radiation consists mostly of photons (and, possibly, other free massless elementary particles), as emission of particles of mass \(m\) can only happen when the black body temperature exceeds the threshold \(mc^2/k_B\) for their production\(^{27}\).

The laws of black hole thermodynamics\(^{28}\) allow to explicitly compute the temperature of the Hawking radiation\(^{29}\). For a Schwarzschild black hole \(T_{BH} = \frac{\hbar c^3}{8\pi k_B GM}\) depends only upon the black hole’s mass \(M\) and entails the Planck constant \(\hbar\), the gravitational constant \(G\), Boltzmann constant \(k_B\) and the speed of light in vacuum \(c\). Even for a small black hole of several solar masses the value of \(T_{BH}\) is of the order of \(10^{-8}\) K, much below the temperature of the Cosmic Microwave Background, which equals 2.7 K. In consequence, a newly formed black hole will be acquiring much more energy than it radiates out.

However, according to the standard cosmological model, as the Universe expands the matter will get more and more sparse and the average temperature will drop\(^{30}\). Eventually, the Universe will enter a “dark epoch” when the Hawking radiation dominates. A black hole will gradually loose its mass and, finally, evaporate completely. This process is extremely slow — for stellar-mass black holes it takes roughly \(10^{67}\) years — but becomes more dynamical towards the end since \(T_{BH}\) grows as the mass decreases. A black hole is thus expected to end its life in a very bright explosion\(^{31}\).

It is also possible that primordial black holes were formed from density fluctuations at a very early stage of the Universe’s evolution\(^{31,32}\). Such hypothetical objects could have, depending on the model, masses ranging from the Planck mass \(M_P \approx 2 \cdot 10^{-8}\) kg to several solar masses. Those with an initial mass larger than \(10^{11}\) kg would still be present in the Universe, while the ones of a smaller initial mass would have already evaporated.

### 2 Results

In quantum mechanics an isolated system is modelled by a pure state, i.e. a vector in a Hilbert space. It describes the full knowledge about the system in terms of quantum information. We consider, from the perspective of a distant observer, the isolated system initially consisting of a collapsing star accompanied with all matter which will eventually fall into the forming black hole. The relevant Hilbert space is decomposed into three segments \(\mathcal{H} = \mathcal{H}_M \otimes \mathcal{H}_{BH} \otimes \mathcal{H}_{HR}\) tailored for the infalling matter outside of the event horizon, the matter in the black hole region and the Hawking radiation, respectively. The space \(\mathcal{H}_{HR}\) is infinite-dimensional, as usual in the
Initially, the global pure state of the system is of product form $|\Psi_{in}\rangle = |\psi_0\rangle_M \otimes |0\rangle_{BH} \otimes |0\rangle_{HR}$, where $|0\rangle$ denotes the vacuum. Once the event horizon begins to form the quantum information starts flowing from the M sector to the BH sector. As the quantum systems constituting the infalling matter are generically entangled, the resulting global state will involve entanglement between M and BH: $|\Psi^{(0)}\rangle = \sum_i a_i |\psi_i^{(0)}\rangle_M \otimes |\chi_i^{(0)}\rangle_{BH} \otimes |0\rangle_{HR}$.

When a Hawking particle is emitted at the event horizon, it will be entangled with its partner concealed in the black hole region. The pair forms a pure state $\sum_j b_j |\eta_j\rangle_{BH} \otimes |\phi_j\rangle_{HR}$. Shortly afterwards, the infalling particle interacts with the matter in the black hole region and the outgoing Hawking particle becomes entangled with other quantum systems in BH. The total state will then have the form $|\Psi^{(1)}\rangle = \sum_{i,j} c_{ij} |\psi_i^{(1)}\rangle_M \otimes |\chi_i^{(1)}\rangle_{BH} \otimes |\phi_j^{(1)}\rangle_{HR}$ known as “entangled entanglement”.

At the moment of its emission the second Hawking particle is entangled only with its infalling partner. However, once the latter interacts with the matter in BH the second quantum of radiation gets entangled not only with the BH sector, but also with the first Hawking particle. Consequently, the total state will acquire the form $|\Psi^{(2)}\rangle = \sum_{i,j,k} d_{ijk} |\psi_i^{(2)}\rangle_M \otimes |\chi_i^{(2)}\rangle_{BH} \otimes |\phi_1^{(1)}\rangle_{HR} \otimes |\phi_j^{(1)}\rangle_{HR}$.

Eventually, all of the matter from M will fall in the black hole and the latter will evaporate through the emission of a finite number $N$ of radiation quanta. The final quantum state of the process will then be

$$|\Psi_{out}\rangle = |0\rangle_M \otimes |0\rangle_{BH} \otimes |\Phi\rangle_{HR}, \quad \text{with} \quad |\Phi\rangle = \sum_{k_1,k_2,...,k_N} f_{k_1,k_2,...,k_N} |\phi^1_{k_1}\rangle \otimes |\phi^2_{k_2}\rangle \otimes \cdots \otimes |\phi^N_{k_N}\rangle.$$  

(1)

This global state of the Hawking radiation $|\Phi\rangle$ is pure and contains the full quantum information initially encoded in the state $|\psi_0\rangle$ of matter. It exhibits multi-mode entanglement and is much more complex than the mixed state resulting from the semi-classical analysis. Nevertheless, it can be analysed within the framework of continuous-variables quantum information. Specifically, the question of compatibility between the purity of the global state and thermality of its local reductions can be formulated rigorously using Gaussian states. It is an instance of the quantum marginal problem, which was a significant stumbling block in quantum chemistry, until the solution in the framework of quantum information was given by Klyachko. For infinite-dimensional systems the quantum marginal problem was posed and solved, for an arbitrary number of bosonic modes, by Eisert et al.
The answer is as follows: Suppose that state $|\Phi\rangle$ is a pure Gaussian state of $N$ modes, then the reduced state $\rho_n$ of every mode is a (mixed) Gaussian state. This is consistent if and only if for $n \in \{1, 2, \ldots, N\}$ one has

$$b_n \leq \sum_{m \neq n} b_m, \quad \text{with} \quad b_n = 2 \left( \exp \left( \frac{\hbar \omega_n}{k_B T_n} \right) - 1 \right)^{-1},$$

(2)

where $\omega_n$ is the frequency of the mode and $T_n$ the respective temperature. The conditions (2) would only fail if for one of the modes the value of $b_n$ exceeds the sum of all other $b_n$’s.

The total number of particles emitted by a black hole of mass $M$ during the evaporation process can be estimated as $N \sim \frac{120(2\pi G \xi)}{\pi^3 c^3 M^2}$, where $\xi$ is the Riemann zeta-function and $\zeta(3) \approx 1.2$. Even for the smallest stellar-mass black holes this number is huge $N \sim 10^{77}$. Basing on the standard black body particle-number spectrum one can assume that the Hawking particles are, on the average, emitted with frequencies corresponding to the peak value $\langle \omega \rangle_N(T) = (2 + W(-2e^{-2}))k_B T/h$, where $W$ is the Lambert $W$-function and $W(-2e^{-2}) \approx -0.4$. Alternatively, we could take the peak frequency in the energy spectrum, which gives a similar result $\langle \omega \rangle_E(T) = (3 + W(-3e^{-3}))k_B T/h$, where $W(-3e^{-3}) \approx -0.18$. Consequently, we obtain $\langle b_n \rangle_N \approx 0.51$ and $\langle b_n \rangle_E \approx 0.13$. We shall denote by $\langle b \rangle$ either of these values.

If all Hawking particles are on the average typical, then condition (2) is readily verified. Suppose now that there is a single ‘rogue’ Hawking particle, the frequency $\Omega$ of which is much smaller than the expected peak value in its epoch. This is the worst-case scenario from the perspective of formula (2), because if two or more such rogue particles are emitted, their excessive values of $b_n$ will compensate each other. Formula (2) then gives an explicit bound on the minimal value of $\Omega$:

$$\Omega(T) \gtrsim \frac{2}{\langle b \rangle} \frac{k_B T}{h} \frac{1}{N},$$

(3)

or, equivalently, for the maximal wavelength

$$\Lambda(T) \lesssim \frac{\pi c h}{k_B \langle b \rangle} N = 8\pi^2 \langle b \rangle N R_S(T),$$

(4)

where $R_S(T)$ is the Schwarzschild radius of the black hole in the epoch with temperature $T$. Given that the number $N$ for astrophysical black holes is extremely large $N \gtrsim 10^{77}$, condition (4) is satisfied in every epoch of Hawking radiation. This conclusion is insensitive to the actual value of the prefactor $\langle b \rangle$. It breaks down only when the total number of emitted Hawking particles is small $N \sim 1$. For this to happen, the black hole would need to have an initial mass of the order of Planck mass $M_P$. Even in such case the radiation can still be thermal, but condition (2) then imposes constraints on the frequencies of particles. This is striking, because one would generally not expect the Hawking radiation to be thermal in such a regime.

Furthermore, our analysis based on formula (2) is robust against the modifications of the emission spectra due to the black hole’s intrinsic spin and other physical effects, such as the grey
body factors. Firstly, one can relax the condition on the global state to be a mixed Gaussian state, which induces a more general constraint\(^{21}\)

\[
\sum_{m} b_m \geq \sum_{m} \delta_m, \quad \text{and} \quad b_n \leq \sum_{m \neq n} b_m + \delta_n - \sum_{m \neq n} \delta_m, \tag{5}
\]

where the numbers \(\delta_n \geq 0\) arise from the symplectic spectrum \{1 + \delta_1, \ldots, 1 + \delta_n\} of the global state \(|\Phi\rangle\). Secondly, one could consider a more general class of non-Gaussian states, in which case constraints (5) translate to a consistency conditions between the global and local second moments of the \(N\)-mode quantum state.

Let us stress that the thermal character of the radiation does not exclude correlations between Hawking particles. On the contrary, a typical pure \(N\)-mode Gaussian state will exhibit multi-mode entanglement\(^{18,34}\). For instance, the shared entanglement of a single mode versus the rest of the system can be quantified with the help of the entanglement entropy\(^{18}\)

\[
E(b_n) = \frac{b_n+2}{2} \log_2 \left( \frac{b_n+2}{2} \right) - \frac{b_n}{2} \log_2 \left( \frac{b_n}{2} \right). \tag{6}
\]

Further insights about the global state of Hawking radiation could be gained from modern quantum optics. For instance, condition (2) allows typical modes to be of similar squeezing. This may happen when the ratio \(\omega_n/T_n\) does not vary much with the time step \(n\). Multi-mode squeezed states constitute a well established field of quantum optics and include the so-called Gaussian graph states with the GHZ-type as a prominent example\(^{37}\). From the viewpoint of the multi-mode entanglement structure, the latter exhibits highest symmetry. Therefore, as a first hypothesis which can be tested experimentally one could assume that \(|\Phi\rangle\) is a GHZ-type state. More refined tests should seek possible deviations from it, which in turn would shed light on the entanglement structure of \(|\Phi\rangle\).

Many conceptual models in the literature\(^{11-17}\) posit that the quantum degrees of freedom of a black hole are discrete, which leads to an assumption of \(\mathcal{H}_{BH}\) being finite-dimensional\(^7\). This suggests that realistic models involving quantised radiation may need hybrid discrete-continuous quantum information techniques\(^{40,41}\).

3 Discussion

The application of the continuous-variable quantum marginal problem allowed us to rigorously demonstrate that the purity of the global state of Hawking radiation is compatible with thermal reductions of all individual modes. This is a consequence of the vast realm of pure quantum states available in \(N\)-mode systems with large \(N\). The quantum marginal conditions are thus readily satisfied for Hawking radiation of astrophysical provenance and can accommodate highly irregular modes emitted in the last epoch. On the other hand, for black holes of microscopic origin, with
masses of the order of several Planck masses, thermality induces more stringent constraints on the frequencies of individual particles.

The description of the black hole’s evolution via a global unitary transformation (an \( S \)-matrix) was conceived already in 1980 by Page\(^9\)\(^{10}\). In our picture, the sum of entropies of a subset of modes is an upper bound on the entropy of the global state. Our result hence agrees qualitatively with the Page curve\(^10\), in that the entanglement entropy between the black hole and the Hawking radiation first rises and eventually drops to zero. Whereas the position of the maximum of entanglement entropy, known as the Page time, depends on the details of the process, we emphasise that if the total state remains pure — as assumed — nothing exceptional happens at that moment. Formally, this is analogous to a typical process in nuclear or particle physics when an ensemble of quantum systems decays coherently to an ensemble of a different type.

Our analysis hinges upon the assumption of universal validity of quantum mechanics. The quantum information is then preserved during the black hole formation and evaporation process, because the system’s state follows a unitary evolution. While this assumption, recently dubbed “the central dogma”\(^7\), could and has been questioned\(^6\)\(^43\), there has been no experimental signature of the inadequacy of quantum mechanics, either at the micro-scale\(^44\) or in macroscopic systems\(^45\).

Consequently, we propose that the black hole information paradox can be constructively and quantitatively resolved within a “conservative” approach based solely upon entanglement among Hawking particles. This hypothesis can be tested by probing the global state of Hawking radiation with the help of modern tools from the domain of continuous-variables quantum information.

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