Predictions from Quantum Cosmology

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Abstract

After reviewing the general ideas of quantum cosmology (Wheeler-DeWitt equation, boundary conditions, interpretation of $\psi$), I discuss how these ideas can be tested observationally. Observational predictions differ for different choices of boundary conditions. With tunneling boundary conditions, $\psi$ favors initial states that lead to inflation, while with Hartle-Hawking boundary conditions it does not. This difficulty of the Hartle-Hawking wave function becomes particularly severe if the role of ‘inflatons’ is played by the moduli fields of superstring theories. In models where the constants of Nature can take more than one set of values, $\psi$ can also determine the probability distribution for the constants. This can be done with the aid of the ‘principle of mediocrity’ which asserts that we are a ‘typical’ civilization in the ensemble of universes described by $\psi$. The resulting distribution favors inflation with a very flat potential, thermalization and baryogenesis at electroweak scale, a non-negligible cosmological constant, and density fluctuations seeded either by topological defects, or by quantum fluctuations in models like hybrid inflation (as long as these features are consistent with the allowed values of the constants).

\footnote{Lectures at International School of Astrophysics “D.Chalonge”, Erice, 1995.}
1. Introduction

If the cosmological evolution is followed back in time, we come to the initial singularity where the classical equations of general relativity break down. This led many people to believe that in order to understand what actually happened at the origin of the universe, we should treat the universe quantum-mechanically and describe it by a wave function rather than by a classical spacetime. This quantum approach to cosmology was initiated by DeWitt [1] and Misner [2], and after a somewhat slow start has become very popular in the last decade or so. The picture that has emerged from this line of development [3, 4, 6, 5, 7, 8, 9] is that a small closed universe can spontaneously nucleate out of nothing, where by ‘nothing’ I mean a state with no classical space and time. The cosmological wave function can be used to calculate the probability distribution for the initial configurations of the nucleating universes. Once the universe nucleated, it is expected to go through a period of inflation, which is a rapid (quasi-exponential) expansion driven by the energy of a false vacuum. The vacuum energy is eventually thermalized, inflation ends, and from then on the universe follows the standard hot cosmological scenario. Inflation is a necessary ingredient in this kind of scheme, since it gives the only way to get from the tiny nucleated universe to the large universe we live in today.

Another possible use for quantum cosmology is to determine the probability distribution for the values of the constants of Nature. The constants can vary from one universe to another due to a different choice of the vacuum state, a different compactification scheme in higher-dimensional theories, or to Planck-scale wormhole effects [10]. The cosmological wave function will then be a superposition of terms corresponding to all possible values of the constants.

In these lectures, I would like to review where we stand in this program. The general ideas of quantum cosmology and predictions for the initial state are discussed in Section 2-4, followed by a discussion of predictions for the constants of Nature in Sections 5,6. Due to the time constraints, some important topics will be left out. These include topology-changing processes, third quantization, consistent histories approach, and decoherence.
2. A Simple Model

2.1 ‘THE GREATEST MISTAKE OF MY LIFE’

First I would like to illustrate how the nucleation of a universe can be described in a very simple model. The model is defined by the action

\[ S = \int d^4 x \sqrt{-g} \left( \frac{R}{16\pi G} - \rho_v \right), \]

where \( \rho_v \) is a constant vacuum energy and the universe is assumed to be homogeneous, isotropic, and closed,

\[ ds^2 = \sigma^2 [-dt^2 + a^2(t)d\Omega_3^2]. \]

Here, \( d\Omega_3^2 \) is the metric on a unit three-sphere, and \( \sigma^2 = 2G/3\pi \) is a normalizing factor chosen for later convenience. The scale factor \( a(t) \) satisfies the evolution equation

\[ \dot{a}^2 + 1 - H^2 a^2 = 0, \]

where

\[ H = 4G\rho_v^{1/2}/3. \]

The solution of Eq.\((3)\) is the de Sitter space,

\[ a(t) = H^{-1} \cosh(Ht). \]

The universe contracts at \( t < 0 \), reaches the minimum radius \( a = H^{-1} \) at \( t = 0 \), and re-expands at \( t > 0 \).

This is similar to the behavior of a particle bouncing off a potential barrier, with \( a \) playing the role of particle coordinate. Now, we know that in quantum mechanics particles can not only bounce off, but can also tunnel through potential barriers. This suggests the possibility that the negative-time part of the evolution in \((5)\) may be absent, and that the universe may instead tunnel from \( a = 0 \) directly to \( a = H^{-1} \).

When I suggested this idea in 1982, I made an attempt to estimate the tunneling probability in the semiclassical approximation. To describe the tunneling process, I used the
Figure 1: *A schematic representation of the birth of inflationary universe.*

bounce solution of the Euclidean field equations, which can be obtained by substituting $t = -i\tau$ in Eqs.(2),(3),

$$ds^2 = \sigma^2[d\tau^2 + \dot{a}^2(\tau)d\Omega^2_3],$$

(6)

$$\dot{a}(\tau) = H^{-1}\cos(H\tau).$$

(7)

This metric describes a four-sphere $S^4$ of radius $H^{-1}$. The nucleation of the universe is schematically represented in Fig. 1, where the bounce solution (7) connects to the Lorentzian solution (3) at the turning point $\tau = t = 0$.

For ‘normal’ quantum tunneling, the tunneling probability $P$ is proportional to $\exp(-S_E)$, where $S_E$ is the Euclidean action for the corresponding bounce. In our case,

$$S_E = \int d^4x\sqrt{-g}\left(-\frac{R}{16\pi G} + \rho_v\right) = -2\rho_v\Omega_4\sigma^4H^{-4} = -3/8G^2\rho_v,$$

(8)

where

$$R = 12H^2 = 32\pi G\rho_v$$

(9)
is the scalar curvature, and $\Omega_4 = 4\pi^2/3$ is the volume of a unit four-sphere. Hence, I concluded in Ref. [3] that

$$\mathcal{P} \propto \exp\left(\frac{3}{8G^2\rho}\right).$$  

(10)

Following fashion, I might declare this ‘the greatest mistake of my life’ [11].

2.2 THE TUNNELING WAVE FUNCTION

What I now think is the correct answer is given by $\mathcal{P} \propto \exp(-|S_E|)$. In the case of ‘normal’ quantum tunneling, the Euclidean action is positive-definite, and $|S_E| = S_E$, but for quantum gravity this is no longer so. The reason for using the absolute value of $S_E$ can be understood by considering the tunneling wave function for our problem. To write the corresponding wave equation, we first substitute (2) into (1), and after integrating by parts find the Lagrangian

$$\mathcal{L} = \frac{1}{2}a(1 - \dot{a}^2 - H^2a^2).$$

(11)

The momentum conjugate to $a$ is

$$p_a = -a\dot{a},$$

(12)

and the Hamiltonian is

$$\mathcal{H} = -\frac{1}{2a}(p_a^2 + a^2 - H^2a^4).$$

(13)

The evolution equation (3) implies that

$$\mathcal{H} = 0.$$  

(14)

Quantization of this model amounts to replacing $p_a \rightarrow -i\partial/\partial a$ and imposing the Wheeler-DeWitt equation

$$\mathcal{H}\psi = 0.$$  

(15)

This gives

$$\left[\frac{d^2}{da^2} - U(a)\right]\psi(a) = 0,$$

(16)

where

$$U(a) = a^2(1 - H^2a^2),$$

(17)
and I have ignored the ambiguity in the ordering of non-commuting operators $a$ and $p_a$. (This ambiguity is unimportant in the semiclassical domain which we will be mainly concerned with in these lectures).

Eq. (16) has the form of a one-dimensional Schrodinger equation for a ‘particle’ described by a coordinate $a(t)$, having zero energy, and moving in a potential $U(a)$. The classically allowed region is $a \geq H^{-1}$, and the WKB solutions of (16) in this region are

$$\psi_\pm(a) = [p(a)]^{-1/2} \exp \left[ \pm i \int_{H^{-1}}^{0} p(a') da' \mp i \pi / 4 \right],$$

where $p(a) = [-U(a)]^{1/2}$. The under-barrier, $a < H^{-1}$, solutions are

$$\bar{\psi}_\pm(a) = |p(a)|^{-1/2} \exp \left[ \pm \int_{a}^{H^{-1}} |p(a')| da' \right].$$

For $a \gg H^{-1}$,

$$\hat{p}_a \psi_\pm(a) \approx \pm p(a) \psi_\pm(a),$$

and Eq. (13) tells us that $\psi_-(a)$ and $\psi_+(a)$ describe an expanding and a contracting universe, respectively. In the tunneling picture, it is assumed that the universe originated at small size and then expanded to its present, large size. This means that the component of the wave function describing a universe contracting from infinitely large size should be absent:

$$\psi(a > H^{-1}) = \psi_-(a).$$

The under-barrier wave function is found from the WKB connection formula,

$$\psi(a < H^{-1}) = \bar{\psi}_+(a) - \frac{i}{2} \bar{\psi}_-(a).$$

The growing exponential $\bar{\psi}_-(a)$ and the decreasing exponential $\bar{\psi}_+(a)$ have comparable amplitudes at the nucleation point $a = H^{-1}$, but away from that point the decreasing exponential dominates (see Fig. 2). The ‘nucleation probability’ can be estimated as

$$\mathcal{P} \sim \exp \left( -2 \int_{0}^{H^{-1}} |p(a')| da' \right) = \exp(-|S_E|) = \exp \left( -\frac{3}{8G^2 \rho_v} \right).$$
Figure 2: Tunneling wave function for the de Sitter minisuperspace model. The ‘potential’ $U(a)$ is shown by a solid line and the wave function by a dashed line.

The use of the semiclassical approximation is justified as long as $|S_E| = 3/8G^2\rho_v \gg 1$, or

$$ \rho_v \ll \rho_p, \quad \text{where} \quad \rho_p = G^{-2} = m_p^4 \quad \text{is the Planck density and} \quad m_p \quad \text{is the Planck mass}. $$

Eq.(23) was obtained independently by Linde [5], Zel’dovich and Starobinsky [6], Rubakov [7], and myself [8]. But the story does not end here. Not everybody agrees that my first answer (10) was a mistake. The same expression for $P$ is obtained in the Euclidean approach developed by Hawking and collaborators. We shall return to this on-going debate after discussing the general formalism of quantum cosmology.

3. Wave Function of the Universe

3.1 WHEELER-DE WITT EQUATION

In the general case, the wave function of the universe is defined on superspace, which is the space of all 3-dimensional geometries and matter field configurations,

$$ \psi[h_{ij}(x), \varphi(x)], $$

(24)

where $h_{ij}$ is the 3-metric, and matter fields are represented by a single scalar field $\varphi$. The wave function $\psi$ satisfies the Wheeler-DeWitt (WDW) equation,

$$ \mathcal{H}\psi(h_{ij}, \varphi) = 0, $$

(25)
which can be thought of as representing the fact that the energy of a closed universe is equal to zero. The WDW equation can be symbolically written in the form

$$(\nabla^2 - U)\psi = 0,$$  \hspace{1cm} (26)

which is similar to the Klein-Gordon equation. Here, $\nabla^2$ is the superspace Laplacian, and the functional $U(h_{ij}, \varphi)$ can be called ‘superpotential’. (We shall not need explicit forms of $\nabla^2$ and $U$).

We have no idea how to solve the WDW equation in the general case. Most of what we know about quantum cosmology has been found using minisuperspace models in which the infinite number of degrees of freedom in Eq.(25) is reduced to a few independent variables. The de Sitter model \([10]\) with a single variable is the simplest example of minisuperspace. The minisuperspace approach is justified when the remaining degrees of freedom can be treated as small perturbations. The corresponding wave function can then be calculated perturbatively \([12, 13]\).

Quantum cosmology is based on quantum gravity and shares all of its problems. In addition, it has some extra problems which arise when one tries to quantize a closed universe. The first problem stems from the fact that $\psi$ is independent of time. This can be understood \([1]\) in the sense that the wave function of the universe should describe everything, including the clocks which show time. In other words, time should be defined intrinsically in terms of the geometric or matter variables. However, no general prescription has yet been found that would give a function $t(h_{ij}, \varphi)$ that would be, in some sense, monotonic. A related problem is the definition of probability. Given a wave function $\psi$, how can we calculate probabilities? One can try to use the conserved current \([1, 2]\)

$$J = i(\psi^* \nabla \psi - \psi \nabla \psi^*), \quad \nabla \cdot J = 0.$$  \hspace{1cm} (27)

The conservation is a useful property, since we want probability to be conserved. But one runs into the same problem as with Klein-Gordon equation: the probability defined in this way is not positive-definite. Although we do not know how to solve these problems in general, they can both be solved in the semiclassical domain. In fact, it is possible that this is all we need.
3.2 SEMICLASSICAL UNIVERSES

Let us consider the situation when some of the variables \( \{c\} \) describing the universe behave classically, while the rest of the variables \( \{q\} \) must be treated quantum-mechanically. Then the wave function of the universe can be written as a superposition

\[
\psi = \sum_k A_k(c) e^{iS_k(c)} \chi_k(c, q) \equiv \sum_k \psi_k^{(c)} \chi_k,
\]

(28)

where the classical variables are described by the WKB wave function \( \psi_k^{(c)} = A_k e^{iS_k} \). In the semiclassical regime, \( \nabla S \) is large, and substitution of (28) into the WDW equation (26) yields the Hamilton-Jacobi equation for \( S(c) \),

\[
\nabla S \cdot \nabla S + U = 0.
\]

(29)

The summation in (28) is over different solutions of this equation. Each solution of (28) is a classical action describing a congruence of classical trajectories (which are essentially the gradient curves of \( S \)). Hence, a semiclassical wave function \( \psi_c = A e^{iS} \) describes an ensemble of classical universes evolving along the trajectories of \( S(c) \). A probability distribution for these trajectories can be obtained using the conserved current (27). Since the variables \( c \) behave classically, these probabilities do not change in the course of evolution and can be thought of as probabilities for various initial conditions. The time variable \( t \) can be defined as any monotonic parameter along the trajectories, and it can be shown [1, 14] that in this case the corresponding component of the current \( J \) is non-negative, \( J_t \geq 0 \). Moreover, one finds [13, 16, 12] that the ‘quantum’ wave function \( \chi \) satisfies the usual Schrodinger equation,

\[
i \frac{\partial \chi}{\partial t} = H_\chi \chi
\]

(30)

with an appropriate Hamiltonian \( H_\chi \). Hence, all the familiar physics is recovered in the semiclassical regime.

This semiclassical interpretation of the wave function \( \psi \) is valid to the extent that the WKB approximation for \( \psi_c \) is justified and the interference between different terms in (28) can be neglected. Otherwise, time and probability cannot be defined, suggesting that the
wave function has no meaningful interpretation. In a universe where no object behaves classically (that is, predictably), no clocks can be constructed, no measurements can be made, and there is nothing to interpret.

3.3 BOUNDARY CONDITIONS

As (almost) any differential equation, the WDW equation has an infinite number of solutions. To get a unique solution, one has to specify some boundary conditions in superspace. In ordinary quantum mechanics, the boundary conditions for the wave function are determined by the physical setup external to the system under consideration. In quantum cosmology, there is nothing external to the universe, and it appears that a boundary condition should be added to Eq. (25) as an independent physical law.

Several candidates for this law of boundary conditions have been proposed. Hartle and Hawking [4] suggested that $\psi(h, \varphi)$ should be given by a path integral over compact, Euclidean 4-geometries $g_{\mu \nu}(x, \tau)$ bounded by the 3-geometry $h_{ij}(x)$ with the field configuration $\varphi(x)$:

$$\psi = \int^{(h, \varphi)} dg [d\varphi] \exp[-S_E(g, \varphi)].$$  \hfill (31)

In this path-integral representation, the boundary condition corresponds to specifying the class of histories integrated over in Eq. (31). Compact 4-geometries can be thought of as histories interpolating between a point (‘nothing’) and a finite 3-geometry $h_{ij}$.

Alternatively, I proposed [8, 17] that $\psi(h, \varphi)$ should be obtained by integrating over Lorentzian histories interpolating between a vanishing 3-geometry $\emptyset$ and $(h, \varphi)$ and lying to the past of $(h, \varphi)$:

$$\psi(h, \varphi) = \int^{(h, \varphi)} \emptyset dg [d\varphi] e^{iS}.$$  \hfill (32)

This wave function is closely related to Teitelboim’s causal propagator [18] $K(h_2, \varphi_2|h_1, \varphi_1)$:

$$\psi(h, \varphi) = K(h, \varphi|\emptyset).$$  \hfill (33)

Linde [5] suggested that, instead of the standard Euclidean rotation $t \to -i\tau$, the action $S_E$ in (31) should be obtained by rotating in the opposite sense, $t \to +i\tau$.  

9
Halliwell and Hartle [19] discussed a path integral over complex metrics which are not necessarily purely Lorentzian or purely Euclidean. This encompasses all of the above proposals and opens new possibilities. However, the space of complex metrics is very large, and no obvious choice of integration contour suggests itself as the preferred one.

In addition to these path-integral no-boundary proposals, one candidate law of boundary conditions has been formulated directly as a boundary condition in superspace. This is the so-called tunneling boundary condition [20, 21] which requires that \( \psi \) should include only outgoing waves at boundaries of superspace. It has been argued [22] that, in a wide class of models, this boundary condition is equivalent to the Lorentzian path integral proposal (32).

For the simple de Sitter model of Sec.2, the tunneling wave function \( \psi_T(a) \) is given by Eqs.(21),(22), the Hartle-Hawking wave function is [23]

\[
\psi_H(a^+H^{-1}) = \psi_+(a) - \psi_-(a),
\]

(34)

\[
\psi_H(a^-H^{-1}) = \tilde{\psi}_-(a),
\]

(35)

and the Linde wave function is [3, 24, 25]

\[
\psi_L(a^+H^{-1}) = \frac{1}{2}[\psi_+(a) + \psi_-(a)],
\]

(36)

\[
\psi_L(a^-H^{-1}) = \tilde{\psi}_+(a).
\]

(37)

Unlike the tunneling wave function, both Hartle-Hawking and Linde wave functions include expanding and contracting universe components with equal amplitudes (see Fig. 3).

4. Predictions for the Initial State

4.1 INITIAL VACUUM ENERGY

To see what kind of cosmological predictions we can get from different boundary conditions, I would like to consider a somewhat more realistic model. Instead of a constant vacuum energy \( \rho_v \), I introduce a scalar field \( \varphi \) with a potential \( V(\varphi) \). Since vacuum energy is very small in our part of the universe, \( V(\varphi) \) should have a minimum with \( V \approx 0. \)
The WDW equation for this two-dimensional model can be solved assuming that $V(\varphi)$ is a slowly-varying function and is well below the Planck density,

$$|V'/V| \ll m_p^{-1}, \quad V \ll \rho_p \equiv m_p^4.$$  \hspace{1cm} (38)

A slowly-varying $V(\varphi)$ helps to simplify the equation, but is also necessary for the inflationary scenario. If the condition $V(\varphi) \ll \rho_p$ is violated, then the semiclassical approximation is not valid and higher-order corrections to quantum gravity are important.

After an appropriate rescaling of the scale factor $a$ and the scalar field $\varphi$, the WDW equation can be written as

$$\left[ \frac{\partial^2}{\partial a^2} - \frac{1}{a^2} \frac{\partial^2}{\partial \varphi^2} - U(a, \varphi) \right] \psi(a, \varphi) = 0, \hspace{1cm} (39)$$

where

$$U(a, \varphi) = a^2[1 - a^2V(\varphi)]. \hspace{1cm} (40)$$
With the assumptions (38), one finds [21] that Hartle-Hawking, Linde, and tunneling solutions of this equation are given essentially by the same expressions as for the simple model (16), but with \( \rho \) replaced by \( V(\varphi) \). The only difference is that the wave function is multiplied by a factor \( C(\varphi) \), such that \( \psi(a,\varphi) \) becomes \( \varphi \)-independent in the limit \( a \to 0 \) (with \( |\varphi| < \infty \)).

The initial state of the nucleating universe in this model is characterized by the value of the scalar field \( \varphi \), with the initial value of \( a \) given by \( a = V^{-1/2}(\varphi) \). The probability distribution for \( \varphi \) can be found using the conserved current (27),

\[
\partial_a J^a + \partial_\varphi J^\varphi = 0.
\] (41)

With a proper normalization, the quantity \( \rho(a,\varphi)d\varphi \), where

\[
\rho(a,\varphi) = J^a(a,\varphi) = i(\psi^* \partial_a \psi - \psi \partial_a \psi^*),
\] (42)

can be interpreted as the probability for the scalar field to be between \( \varphi \) and \( \varphi + d\varphi \) when the scale factor is equal to \( a \).

For the tunneling wave function one finds

\[
\rho_T(\varphi) \approx C_T \exp \left( -\frac{3}{8G^2 V(\varphi)} \right),
\] (43)

where \( C_T \) is a normalization constant. This is the same as Eq.(23) with \( \rho_v \) replaced by \( V(\varphi) \). \( \rho \) is independent of \( a \) because \( \varphi \) remains approximately constant along the classical trajectories (with \( a \) playing the role of time). The probability distribution (43) is strongly peaked at the value \( \varphi = \varphi_{\max} \) where \( V(\varphi) \) has a maximum. Thus, the tunneling wave function ‘predicts’ that the universe is most likely to nucleate with the largest possible vacuum energy. This is just the right initial condition for inflation. The high vacuum energy drives the inflationary expansion, while the field \( \varphi \) gradually ‘rolls down’ the potential hill, and ends up at the minimum with \( V(\varphi) \approx 0 \), where we are now.

For Linde’s wave function, evaluation of the current for the expanding-universe component of \( \psi_L \) gives the same probability distribution (43). The Hartle-Hawking wave function
gives a similar distribution, but with a crucial difference in sign,

$$\rho_H(\varphi) = C_H \exp \left( + \frac{3}{8G^2V(\varphi)} \right).$$  \hfill (44)

Note that $\rho_H$ is the same as the nucleation probability (10) found using the instanton method. This is not surprising: the Hartle-Hawking proposal involves the same Euclidean rotation as the one used to obtain the instanton. The distribution (44) is peaked at $V(\varphi) \approx 0$, and thus the Hartle-Hawking wave function appears to predict an empty universe with $V \approx 0$. Such initial condition does not lead to inflation, and is therefore inconsistent with observations.

Hawking and Page [26] have pointed out that things may be not so bad in models of ‘chaotic’ inflation, where $V(\varphi) \to \infty$ at $\varphi \to \infty$ (a typical example is $V(\varphi) \propto \varphi^{2k}$ with $k$ an integer). In such models, $\rho(\varphi \to \infty) \to \text{const}$, the distribution (44) is not normalizable, and the ensemble described by this distribution is dominated by universes with arbitrarily large initial values of $\varphi$. The problem with this argument is that, in order to outweigh the exponentially large values of $\rho_H(\varphi)$ at small $\varphi$, one has to go to extremely large values of $\varphi$, for which the potential $V(\varphi)$ will far exceed the Planck energy density [except, perhaps, for a very special shape of $V(\varphi)$]. The semiclassical approximation, on which the derivation of Eq.(44) was based, cannot be trusted in this regime. For a further discussion of this issue see Refs. [27, 28].

4.2 INITIAL STATE OF THE MODULI

The tunneling vs. Hartle-Hawking debate takes an interesting turn in superstring theories, where the relevant part of the low-energy effective action has the form

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} - K_{AB}(\varphi)g^\mu{}\nu\partial_\mu\varphi^A\partial_\nu\varphi^B - V(\varphi) \right].$$  \hfill (45)

Here, $\varphi^A$ are the moduli fields, $K_{AB}(\varphi)$ is the metric of moduli space, $A, B = 1, 2, ..., n$, and $n$ is the number of moduli. The potential $V(\varphi)$ vanishes to all orders of perturbation theory, but is expected to be generated non-perturbatively, with a characteristic scale well below the Planck mass. It has been argued that moduli are natural candidates for the role of the inflaton in superstring cosmology [29, 30, 31].
As before, we shall restrict ourselves to a closed Robertson-Walker universe with homogeneous moduli fields. Then, after an appropriate rescaling of $t, a, \varphi^A,$ and $V(\varphi)$, the Lagrangian for our model can be written as

$$\mathcal{L} = \frac{1}{2} a(1 - \dot{a}^2) + a^3 \left[ \frac{1}{2} K_{AB}(\varphi) \dot{\varphi}^A \dot{\varphi}^B - V(\varphi) \right]$$

(46)

The momenta conjugate to $a$ and $\varphi^A$ are

$$p_a = -a\dot{a}, \quad p_A = a^3 K_{AB} \dot{\varphi}^B,$$

(47)

and the Hamiltonian is

$$\mathcal{H} = \frac{1}{2a} \left[ -p_a^2 + a^{-2} K^{AB} p_A p_B - U(a, \varphi) \right],$$

(48)

where $U(a, \varphi)$ is given by (41) and $K^{AB}$ is related to $K_{AB}$ by the standard relation $K^{AB} K_{BC} = \delta^A_C$.

The WDW equation is obtained by replacing $p_a \rightarrow -i\partial/\partial a$, $p_A \rightarrow -i\partial/\partial \varphi_A$,

$$\left[ -\frac{\partial^2}{\partial a^2} + \frac{1}{a^2} |K|^{-1/2} \frac{\partial}{\partial \varphi^A} \left( |K|^{1/2} K^{AB} \frac{\partial}{\partial \varphi^B} \right) + U(a, \varphi) \right] \psi = 0,$$

(49)

where $K = \text{det}(K_{AB})$ and the ordering of factors $\varphi^A$ and $\partial/\partial \varphi^A$ has been chosen so that the equation is invariant with respect to reparametrizations of the moduli space, $\varphi^A \rightarrow \tilde{\varphi}^A(\varphi^B)$ [12]. The probability distribution for $\varphi^A$ is then $d\mathcal{P} = \rho(a, \varphi) d^n \varphi$, where

$$\rho = J^a = i |K|^{1/2} (\psi^* \partial_a \psi - \psi \partial_a \psi^*).$$

(50)

For a slowly-varying potential, Eq.(41) is solved in the same way as Eq.(33) for a single scalar field, and one finds

$$\rho(\varphi) = C |K|^{1/2} \exp \left( \pm \frac{3}{8G^2 V(\varphi)} \right),$$

(51)

where the upper sign corresponds to Hartle-Hawking, and the lower sign to the tunneling wave function.
Now, the moduli space is non-compact, and one could expect that the remedy suggested by Hawking and Page [26] to avoid the empty universe problem should work in this case as well. However, Horne and Moore have argued [33] that, despite the existence of non-compact regions, the volume of the moduli space is finite,

\[ \int |K|^{1/2} d^n \varphi < \infty. \]  

(52)

Moreover, it is expected that the moduli potential \( V(\varphi) \) asymptotically vanishes in all non-compact directions. If either of these expectations is correct, then the Hartle-Hawking probability distribution is unavoidably peaked at very low densities, so that the initial states leading to inflation are highly unlikely.

5. Predictions for the Constants of Nature

5.1 VARIABLE CONSTANTS

In theories that allow variation of the constants of Nature, the cosmological wave function is a superposition

\[ \psi = \sum_{\alpha} \psi_{\alpha}(a, \varphi), \]  

(53)

where the subscript \( \alpha \) is a collective symbol for the constants \( \{\alpha_j\} \). In superstring theories, some of the constants parametrize different compactifications of extra dimensions, and different sets of \( \{\alpha_j\} \) correspond to different moduli spaces with their own potentials \( V(\varphi) \). The number of different compactifications is believed to be \( \sim 10^4 \). Hence, the spectrum of \( \{\alpha_j\} \) can be rather dense.

The wave function \( \psi_{\alpha}(a, \varphi) \) gives the amplitude for a universe to nucleate with a set of constants \( \{\alpha_j\} \) and to have the values \( a, \varphi \) for the scale factor and moduli fields. The relative normalization of different components is fixed for both Hartle-Hawking and tunneling wave functions if one uses the path integral formulation of the corresponding boundary conditions, Eqs.(31),(32). The overall normalization is determined by

\[ \sum_{\alpha} \int \rho_{\alpha}(\varphi) d\varphi = 1. \]  

(54)
We can think of the probability distribution $\rho_\alpha(\varphi)$ as describing an ensemble of universes which, following Gell-Mann [34], I will call ‘multiverse’. The probability that a universe arbitrarily picked in this multiverse will have a particular set of $\{\alpha_j\}$ is

$$w_\alpha = \int \rho_\alpha(\varphi) d\varphi.$$  \hfill (55)

Adopting the tunneling boundary condition for $\psi$, we expect

$$w_\alpha \propto \exp \left( -\frac{3}{8G^2(\alpha)V_{\text{max}}(\alpha)} \right),$$  \hfill (56)

where $V_{\text{max}}(\alpha) = \max\{V(\varphi)\}$ for the constants $\{\alpha\}$.

It has been recently suggested [35] that most, if not all, moduli spaces may actually be connected to one another. The potential on this interconnected web of moduli spaces may still have a large number of maxima and minima, with different low-energy physics at each minimum. The constants $\{\alpha_j\}$ then parametrize different minima of $V(\varphi)$, and the probabilities $w_\alpha$ are obtained by integrating $\rho(\varphi)$ over the basin of attraction of the corresponding minimum. We still expect the estimate (56) to apply, with $V_{\text{max}}(\alpha)$ being the highest maximum of $V(\varphi)$ in the basin of attraction of the minimum $\alpha$.

The potential $V(\varphi)$ on the moduli space may have some flat directions. The associated massless fields may not be in conflict with observations if they are very weakly coupled. The values of these fields, which affect the ‘constants’ of low-energy physics, will then be determined by the initial conditions at nucleation and by the following cosmological evolution. Such fields should be included in $\{\alpha_j\}$ as continuous variables parametrizing the constants of Nature [36].

Finally, Coleman [10] has argued that all constants appearing in sub-Planckian physics may become totally undetermined due to Planck-scale wormholes connecting distant regions of spacetime. Then the spectrum of the constants is also continuous, but unlike massless moduli, they cannot vary from one spacetime point to another, but only from one universe to another (disconnected) universe. To simplify the discussion, I will assume a discrete spectrum of the constants.

### 5.2 PRINCIPLE OF MEDIOCRITY
It is quite possible that a randomly picked universe will be unsuitable for life, and therefore the distribution (56) is not adequate for predicting the observed values of the constants. Moreover, the number of civilizations in some of the universes may be much greater than in the others, and this difference should also be taken into account when evaluating the probabilities [40]. The probability distribution of constants for a civilization randomly picked in the multiverse is

\[ P_\alpha = C^{-1} w_\alpha N_\alpha, \]  

(57)

where \( N_\alpha \) is the average number of civilizations in a universe with a set of constants \( \{\alpha_j\} \) and \( C = \sum_\alpha w_\alpha N_\alpha \) is a normalization constant. \( N \) is taken to be the total number of civilizations through the entire history of the universe and is assumed to be finite. The case of eternal inflation, where \( N = \infty \), will be discussed in Section 6.

If we assume that our civilization is a ‘typical’ inhabitant of the multiverse, then we ‘predict’ that the constants of Nature in our universe are somewhere near the maximum of the distribution (57). The assumption of being typical was called ‘the principle of mediocrity’ in Ref. [39]. It is a version of the ‘anthropic principle’ which has been extensively discussed in the literature [42].

The number \( N \) can be expressed as

\[ N_\alpha = V_\alpha \nu_{\text{civ}}(\alpha), \]  

(58)

where \( V_\alpha \) is the volume of the universe at the end of inflation (that is, the 3-volume of the hypersurface that divides the spacetime into inflating and thermalized parts), and \( \nu_{\text{civ}}(\alpha) \) is the average number of civilizations originating per unit thermalized volume.

The definition of probability (57) based on the number of civilizations is somewhat arbitrary. One could, for example, assign a weight to each civilization, depending on its lifetime and/or the number of individuals. We shall deal with this uncertainty by concentrating on stable ‘predictions’ from (57) which are not sensitive to the choice of the definition.

The concept of ‘naturalness’ that is commonly used to assess the plausibility of elementary particle models is based on the assumption that the probability distribution for the constants is nearly flat, \( P_\alpha \approx \text{const.} \) The principle of mediocrity gives a very different perspective on
what is natural and what is not. It predicts that the constants \( \{\alpha_j\} \) are likely to be such that the product

\[
\mathcal{P}_\alpha \propto w_\alpha \nu_{\text{civ}}(\alpha)
\]  (59)

is maximized. The factors in this product have a strong (exponential) dependence on \( \{\alpha_j\} \), and the distribution \( \mathcal{P}_\alpha \) can be strongly peaked in some region of \( \alpha \)-space.

It should be emphasized that predictions of the principle of mediocrity are not guaranteed to be correct. After all, our civilization may be special in some respects. The predictions can be expected to have only statistical accuracy. That is, with a large number of predictions, only few of them are likely to be wrong.

5.3 PREDICTIONS FOR FINITE INFLATION

From Eq.(56), the nucleation probability is maximized when the maximum of the potential approaches the Planck scale, \( V_{\text{max}}(\alpha) \sim \rho_p \). (I assume that \( V(\varphi) \) cannot get much greater than \( \rho_p \)).

The volume factor \( V \) is given by \( V = V_0 Z^3 \), where \( V_0 \sim (GV_{\text{max}})^{-3/2} \) is the initial volume at nucleation and \( Z \) is the expansion factor during inflation. The maximum of \( Z \) is achieved by maximizing the highest value of the potential \( V_{\text{max}} \), where inflation starts, and minimizing the slope of \( V(\varphi) \): the field \( \varphi \) takes longer to roll down for a flatter potential.

The cosmological literature abounds with remarks on the ‘unnaturally’ flat potentials required by inflationary scenarios. With the principle of mediocrity the situation is reversed: flat is natural. Instead of asking why \( V(\varphi) \) is so flat, one should now ask why it is not flatter.

The ‘human factor’ \( \nu_{\text{civ}}(\alpha) \) may impose stringent constraints on the constants \( \{\alpha_j\} \). We do not know what other forms of intelligent life are possible, but the principle of mediocrity favors the hypothesis that our form is the most common in the multiverse. The conditions required for life of our type to exist [the low-energy physics based on the symmetry group \( SU(3) \times SU(2) \times U(1) \), the existence of stars and planets, supernova explosions] may then fix, by order of magnitude, the values of the fine structure constant, and of electron, nucleon, and W-boson masses, as discussed in Ref. [42]. Anthropic considerations also impose a bound on the allowed flatness of the potential \( V(\varphi) \). If it is too flat, then the thermalization
temperature after inflation is too low for baryogenesis. The lowest temperature at which baryogenesis can still occur is set by the electroweak scale, $T_{\text{min}} \sim m_W$. Hence, if other constraints do not interfere, we expect the universe to thermalize at $T \sim m_W$.

Superflat potentials required by the principle of mediocrity typically give rise to density fluctuations which are many orders of magnitude below the strength needed for structure formation. This means that the observed structures must have been seeded by some other mechanism. An alternative mechanism is based on topological defects: strings, global monopoles, and textures, which could be formed at a symmetry breaking phase transition \cite{13}. The required symmetry breaking scale for the defects is $\eta \sim 10^{16}$ GeV. With ‘natural’ (in the traditional sense) values of the couplings, the transition temperature is $T_c \sim \eta$, which is much higher than the thermalization temperature ($T_{\text{th}} \sim m_W$), and no defects are formed after inflation. It is possible for the phase transition to occur during inflation, but the resulting defects are inflated away, unless the transition is sufficiently close to the end of inflation. To arrange this requires some fine-tuning of the constants. However, the alternative is to have thermalization at a much higher temperature and to cut down on the amount of inflation. Since the dependence of the volume factor $V$ on the duration of inflation is exponential, we expect that the gain in the volume will more than compensate for the decrease in ‘$\alpha$-space’ due to the fine-tuning. We note also that in some supersymmetric models the critical temperature of superheavy string formation can ‘naturally’ be as low as $m_W$ \cite{14}.

Another possibility is to use more complicated models of inflation, such as ‘hybrid’ inflation \cite{15}, which involve several scalar fields and can give reasonably large density fluctuations even when the potentials are very flat in some directions in the field space \cite{16}. The amount of fine-tuning required in these models appears to be comparable to that in the case of topological defects.

The symmetry breaking scale $\eta \sim 10^{16}$ GeV for the defects is suggested by observations, but we have not explained why this particular scale has been selected. The value of $\eta$ determines the amplitude of density fluctuations, which in turn determines the time when galaxies form, the galactic density, and the rate of star formation in the galaxies. Since these parameters certainly affect the chances for civilizations to develop, it is quite possible that $\eta$ is significantly constrained by the anthropic factor $\nu_{\text{civ}}(\alpha)$. It would therefore be interesting
to study how structure formation would proceed in a universe with a very different amplitude of density fluctuations (and a very different baryon density). Some steps in this direction have been made in Ref. [47].

If $\nu_{\text{civ}}$ is indeed sharply peaked at some value of $\eta$ and thus fixes the amplitude of density fluctuations and the epoch of active galaxy formation, then an upper bound on the cosmological constant can be obtained by requiring that it does not disrupt galaxy formation until the end of that epoch. An anthropic bound on the cosmological constant has been first discussed by Weinberg [48]. He argued that, since there is evidence for the existence of some quasars and protogalaxies as early as $z \sim 4$, the anthropic principle cannot rule out vacuum energy domination at $z \sim 4$. The matter density at $z = 4$ is greater than the present matter density $\rho_{m0}$ by a factor $(1 + z)^3 = 125$, and he concluded that the anthropic bound on $\rho_v$ cannot be stronger than $\rho_v/\rho_{m0} \lesssim 100$. This falls short of the observational upper bound [49], $|\rho_v/\rho_{m0}|_{\text{obs}} \lesssim 10$, by a factor $\sim 10$ [50].

On the other hand, the principle of mediocrity suggests that we look not for the value of $\rho_v$ that makes galaxy formation barely possible, but for the value that maximizes the amount of matter in galaxies [51]. This amount grew substantially after $z = 4$, and it is quite possible that it increased, say, by a factor $\sim 2$ as late as $z \sim 1$. Requiring that $\rho_v$ does not dominate before $z \sim 1$, we obtain $\rho_v/\rho_{m0} \lesssim 10$. The actual value of $\rho_v$ is likely to be comparable to this upper bound. Negative values of $\rho_v$ are bounded by requiring that our part of the universe does not recollapse while stars are still shining and new civilizations are being formed. This gives a bound comparable to that for positive $\Lambda$ (by absolute value). A more detailed discussion of the bounds on the cosmological constant will be given elsewhere.

Let us now summarize the ‘predictions’ of the principle of mediocrity for the case of finite inflation [39]. The preferred models have very flat inflaton potentials, thermalization and baryogenesis at the electroweak scale, a non-negligible cosmological constant, and density fluctuations seeded either by topological defects, or by quantum fluctuations in models like hybrid inflation (as long as these features are consistent with the spectrum of the constants $\{\alpha_j\}$).

6. Predictions for Eternal Inflation
I have assumed so far that inflation has a finite duration, so that the thermalized volume $\mathcal{V}$ and the number of civilizations $\mathcal{N}$ are both finite. This, however, is not generally the case. The evolution of the inflaton field $\varphi$ is influenced by quantum fluctuations, and as a result thermalization does not occur simultaneously in different parts of the universe. In many models it can be shown that at any time there are parts of the universe that are still inflating [52, 53, 60]. The conclusions of Section 5.3 are directly applicable only if inflation is finite for all the allowed values of the constants $\{\alpha_j\}$. For eternally inflating universes the situation is substantially more complicated. This subject is now under active investigation, and I will have time only for a quick review.

6.1 DISCONNECTED UNIVERSES

Let us first suppose that different sets of constants $\{\alpha_j\}$ correspond to different, disconnected universes. In order to calculate the probabilities (57), we should then be able to compare the thermalization volumes $\mathcal{V}_\alpha$.

In an eternally inflating universe, the thermalization volume $\mathcal{V}$ is infinite and has to be regulated. If one simply cuts it off by including only parts of the volume that thermalized prior to some moment of time $t_c$, with the same value of $t_c$ for all universes, then one finds that the results are extremely sensitive to the choice of the time coordinate $t$. For example, cutoffs at a fixed proper time and at a fixed scale factor $a$ give drastically different results [54]. An alternative procedure [53] is to introduce the cutoff at the time when all but a small fraction $\epsilon$ of the initial (co-moving) volume of the universe has thermalized. The value of $\epsilon$ is taken to be the same for all universes, but the corresponding cutoff times $t_c$ are generally different. The limit $\epsilon \to 0$ is taken after calculating the probability distribution $\mathcal{P}_\alpha$. It was shown in [53] that the resulting distribution is not sensitive to the choice of $t$.

The regularized volume $\mathcal{V}$ can be calculated in terms of the distribution function $\mathcal{P}(\varphi, a)$, which is defined so that $\mathcal{P}(\varphi, a)d\varphi$ gives the fraction of the co-moving volume where the scalar field(s) takes values between $\varphi$ and $\varphi + d\varphi$, with the scale factor $a$ playing the role of a time coordinate. The function $\mathcal{P}(\varphi, a)$ satisfies a ‘diffusion’ equation [52, 54], which I will not reproduce here. The important thing for us to know is that the asymptotic form of $\mathcal{P}$ at
large $a$ is

$$\mathcal{P}(\varphi, a \to \infty) \approx f(\varphi)a^{-\gamma}.$$  

(60)

The positive constant $\gamma$ can be found by solving an eigenvalue problem \[56\], and $d = 3 - \gamma$ has the meaning of the fractal dimension of the inflating region \[57\]. (It can be shown that $\gamma \leq 3$). I will omit the calculations performed in Ref. \[55\] and even the rather lengthy expression for $V$ obtained as a result of those calculations. The essence of the result can be expressed as

$$V \propto \epsilon^{-(3-\gamma)/\gamma}Z^3.$$  

(61)

Here, $Z$ is the expansion factor during the slow-roll phase of inflation, when quantum fluctuations are small.

In the limit $\epsilon \to 0$, non-vanishing probabilities are obtained only for $\{\alpha_j\}$ corresponding to the smallest value of $\gamma$,

$$\gamma(\alpha) = \text{min}.  \quad (62)$$

The eigenvalue $\gamma$ decreases as the potential $V(\varphi)$ becomes flatter \[57, 54\], and thus the condition (62) tends to select maximally flat potentials.

It is possible that the condition (62) selects a unique set of $\{\alpha_j\}$. Then all constants of Nature can, at least in principle, be predicted with 100% certainty. On the other hand, it is conceivable that the minimum of $\gamma$ is strongly degenerate, so that Eq.(62) selects a large subset of all $\{\alpha_j\}$. Then all values of $\alpha$ not in this subset have a vanishing probability, and the probability distribution within the subset is proportional to $w_\alpha Z^3\nu_{\text{civ}}(\alpha)$ [see Eq.(59)]. The probability maximum is then determined by the same considerations as in the case of finite inflation.

It should be emphasized that the conditions of minimizing $\gamma(\alpha)$ and maximizing $w_\alpha Z^3\nu_{\text{civ}}(\alpha)$ are not on an equal footing, with the first of these conditions always taking precedence. Even a tiny decrease in $\gamma$ leads to an infinite increase of the thermalization volume in the limit $\epsilon \to 0$. Suppose, for example, that we have two sets of constants, $\{\alpha_j^{(1)}\}$ and $\{\alpha_j^{(2)}\}$, such that $\gamma(\alpha^{(1)}) < \gamma(\alpha^{(2)})$, but the thermalization temperature for the constants $\alpha^{(1)}$ is too low for baryogenesis, while for $\alpha^{(2)}$ it is sufficiently high. We would still have to conclude that the constants $\alpha^{(1)}$ are infinitely more probable than $\alpha^{(2)}$. In a universe described by the
constants $\alpha^{(1)}$, life can appear only as a result of a huge fluctuation of the baryon density. The probability of such a fluctuation per unit volume is incredibly small, but its smallness is more than compensated for when the volume is increased by an infinite factor.

### 6.2 MULTIPLE VACUA IN A SINGLE UNIVERSE

Let us now consider eternal inflation in a single universe where the potential $V(\varphi)$ has a large number of minima, parametrized by the constants $\{\alpha_j\}$. Thermalization will then occur in different minima in different parts of the universe. The asymptotic form of the distribution function $P(\varphi, a)$ in this case is still given by Eq.(60), but now $\gamma$ has the same value everywhere and is independent of $\{\alpha_j\}$. From Eq.(61), the regularized thermalization volumes are $V_\alpha \propto Z^{3}_\alpha$, and the corresponding probabilities are

$$P_\alpha \propto Z^{3}_\alpha \nu_{civ}(\alpha).$$

(63)

The probability distribution (63) has the same dependence on the slow-roll expansion factor $Z$ and on the anthropic factor $\nu_{civ}$ as we found in the case of finite inflation. The predictions for $\{\alpha_j\}$ are, therefore, also the same (see Section 5.3).

### 6.3 ETERNAL INFLATION AND QUANTUM COSMOLOGY

The ideas of eternal inflation and quantum cosmology have always had a somewhat uneasy coexistence. The picture of an eternally inflating universe, with new islands of thermalization constantly being formed, makes one wonder about the possibility of extending this picture to the infinite past. The universe would then be in a steady state of eternal inflation without a beginning, the problem of the initial singularity would be avoided, and there would be no need for quantum cosmology. However, it has been shown that, under rather general assumptions, inflation cannot be eternal in both future and past directions. Hence, an eternally inflating universe must still have a beginning, and we probably need quantum cosmology to describe it.

On the other hand, the eternal nature of inflation can make the initial state at the nucleation of the universe completely irrelevant. The universe eventually reaches the steady-state regime described by the asymptotic form (60) and stays in this regime thereafter. If
transitions between vacua with different $\{\alpha_j\}$ are in principle possible, then the universe completely forgets its initial conditions. In this case, all one needs from the cosmological wave function is that the probability for eternal inflation to start should be non-zero. Hartle-Hawking and tunneling wave functions are then in equally good agreement with observations, and the probability distribution for $\{\alpha_j\}$ is given by Eq.(63). (This equation was derived without relying on quantum cosmology).

Thus, we see that eternal inflation is quite capable of determining the values of $\{\alpha_j\}$ on its own, without any help from quantum cosmology. The inverse is also true: in models where inflation is finite for all allowed values of $\{\alpha_j\}$, quantum cosmology can determine the probability distribution for the initial states and for the constants of Nature, without any need for eternal inflation [59]. At this time it is hard to tell which of the two approaches is more promising, and both are probably worth pursuing.

Acknowledgements

It is a pleasure to thank the organizers of the course for their warm hospitality. I am also grateful to Arvind Borde, Allen Everett, Cumrun Vafa, Serge Winitzki, and particularly Andrei Linde and Don Page for discussions and comments. This work was supported in part by the National Science Foundation.

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This argument assumes that the probability distribution for $\rho_v$ in the range of interest is nearly flat. It is possible, however, that the ‘fundamental’ variable that has a flat distribution at sub-Planckian scales is the characteristic energy scale $\eta = \rho_v^{1/4}$. Then the discrepancy between the anthropic and observational bounds on $\eta$ is only by a factor $\sim 2$.

More exactly, we look for the values of $\rho_v$ that achieve a balance between fine-tuning and maximizing the amount of matter in galaxies. To make this quantitative, let $w(\rho_v)d\rho_v$ be the probability distribution for $\rho_v$ for the nucleating universes, and let $f(\rho_v)$ be the fraction of baryonic matter that ends up in galaxies for a given value of $\rho_v$. (Here I assume that $\rho_v$ has a continuous spectrum). Then we want to maximize the product $f(\rho_v)w(\rho_v)\rho_v$.

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