On the determination of the spin and disc truncation of accreting black holes using X-ray reflection

A.C. Fabian, M.L. Parker, D.R. Wilkins, J.M. Miller, E. Kara, C.S. Reynolds and T. Dauser

1 Institute of Astronomy, Madingley Road, Cambridge CB3 0HA
2 Department of Astronomy, University of Michigan, Ann Arbor, MI 48109, USA
3 Department of Astronomy, University of Maryland, College Park, MD 20742-2421, USA
4 Dr Karl Remeis-Observatory and Erlangen Centre for Astroparticle Physics, Sternwartstr. 7, D-96049 Bamberg, Germany

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ABSTRACT

We discuss the application of simple relativistically-blurred X-ray reflection models to the determination of the spin and the inner radius of the disc in accreting black holes. Observationally, the nature of the corona is uncertain a priori, but a robust determination of the inner disk radius can be made when the disc emissivity index is tightly constrained. When the inner disc is well illuminated, the black hole spin can also be determined. Using reflection modelling derived from ray tracing, we show that robust determination of disc truncation requires that the location of the coronal source is quasi-static and at a height and radius less than the truncation radius of the disc. Robust spin measurements require that at least part of the corona lies less than about 10 gravitational radii above the black hole in order that the innermost regions, including the innermost stable circular orbit, are well illuminated. The width of the blurring kernel (e.g., the iron line) has a strong dependence on coronal height. These limitations may be particularly applicable at low Eddington fractions (e.g. the low/hard state, and low-luminosity AGN) where the height of the corona may be relatively large, or outflowing, and tied to jet production.

Key words: black hole physics: accretion discs, X-rays: binaries, galaxies

1 INTRODUCTION

X-ray reflection is becoming a common tool in X-ray astronomy, particularly for studying the inner regions around compact objects. It occurs when matter is irradiated by a primary X-ray source, giving rise to backscattering, fluorescence and secondary emission (see Fabian & Ross 2010 for a review). The primary source is here referred to as the corona. We initially assume it to be pointlike, static and on axis, but in practice it may be extended and outflowing, perhaps the base of a jet. The reflection spectrum provides diagnostics for elemental abundances and ionization parameters of the matter as well as its velocity and radius range through doppler and gravitational redshifts. For sources with significant variability, reverberation of the reflection component compared with the primary can reveal the source size and geometry in physical units. Here we are concerned with the use of X-ray reflection to measure the innermost radius of the accretion disc around a black hole. If this is identified with the Innermost Stable Circular Orbit (ISCO) then the spin of the black hole can be determined. Alternatively, the disc may be truncated well before the ISCO in which case the truncation radius may be located.

Considerable effort has been expended recently on tracing light rays in the Kerr metric in order to determine the emissivity profile on the disc plane produced by a source above the disc (Wilkins & Fabian 2012; Dauser et al 2013). The emissivity profile and the radius range over which it is relevant are key factors in constructing the spectral response of the reflection spectrum. Here we discuss limitations to the measurement of spin and disc truncation when the primary source or corona is much more than 10 gravitational radii (10GM/c^2) above the disc. When that occurs, the innermost regions around the ISCO are poorly irradiated so making the reflection spectrum from that part of the disc, which is crucial to spin determination, difficult to measure. This issue has been discussed before (e.g. Vaughan & Fabian 2004; Chiang et al 2012; Dauser et al 2013) but is not widely recognised. The purpose of this paper is to explore it further.

Spin determinations of AGN using X-ray reflection have been ongoing since Dabrowski et al (1997) with recent results including MCG-6-30-15 (a = cJ/GM^2 = 0.989^{+0.009}_{-0.002}, Brenneman...
& Reynolds 2006), 1H0707-495 ($a > 0.98$, Fabian et al 2009); IRAS13224-3809 ($a = 0.989 \pm 0.001$, Fabian et al 2013a) (see recent reviews by Reynolds 2013 and Brenneman 2013 for summaries of more AGN spin results). The high statistical precision of the above three results is due to a combination of low coronal height and high iron abundance, which emphasizes features in the reflection spectrum. X-ray reflection-based spin determinations in black hole binaries (BHB) are reviewed by Miller (2007) following Miller et al (2002, 2004); more recent results include Cyg X-1 ($a = 0.97^{+0.01}_{-0.02}$, Fabian et al 2012; $a = 0.98 \pm 0.01$, Tomskick et al 2014) and GRS 1915+105 ($a = 0.98 \pm 0.01$, Miller et al 2013). Objects with intermediate spin, include SWIFT J2127.4+5654 ($a = 0.6 \pm 0.2$, Miniutti et al 2009), XTE J 1752-223 ($a = 0.52 \pm 0.11$, Reis et al 2013) and Fairall 9 ($a = 0.52^{+0.19}_{-0.15}$, Lohfink et al 2012). Complementary work on BHB using the thermal continuum method (McClintock, Narayan & Steiner 2013) is providing general agreement with X-ray reflection methods, for objects where both methods can be applied (e.g. Steiner et al 2012).

Both methods rely on the identification of $r_{in}$, the innermost disc radius detected, with the ISCO. This systematic uncertainty is being tackled with numerical simulations of discs (Reynolds & Fabian 2008; Shafee et al 2008; Noble et al 2011; Penna et al 2010; Kulkarni et al 2011). The simulations indicate that $r_{in}$, the inner radius of the dense disc (which is relevant to reflection), lies within a pressure scale-height of the ISCO, which corresponds to an uncertainty of $\sim (L/L_{Edd})r_G$. Different methods may give (slightly) different values for $r_{in}$ depending upon their sensitivity to surface density or emission. The disc may be thicker when the thermal continuum method is optimally applied than for the reflection approach. The low ionization often found with the reflection method for the inner disc of AGN (e.g. $\xi < 20$ for 1H0707-495 and IRAS13224-3809) indicates that the disc is dense and thus probably thin. There are currently no published simulations of the ISCO region of thin irradiated discs.

Disc truncation in which the optically-thick, physically-thin, disc terminates beyond the ISCO and the inner region is filled with a hot advection-dominated flow is often invoked as a mechanism for explaining state changes in stellar mass black hole sources (Esin et al 1997; Done et al 2007). In this model a source in the low, hard state is interpreted as having a truncated disc. Spectral evidence for truncation is confused in the literature with some work claiming that it does not occur (e.g. Miller et al 2004, 2006, Reis et al 2008, 2010) and some (Done & Diaz Trigo 2010) that it does. Recent work by Plant et al (2013) on the source GX339-4, which has featured much in this debate, appears to show disc truncation in the low state using a simple relativistic reflection model (see also Petrucci et al 2013). As discussed here in Section 2, this interpretation may be too simplistic. Truncation has also been inferred in some radio-loud AGN, such as 3C120 (Marscher et al 2003; Lohfink et al 2013).

2 DETERMINATION OF THE INNER RADIUS OF THE DISC BY REFLECTION

For the purpose of illustration, we concentrate here on a single emission line, but the principle is the same if the whole reflection spectrum is used. We employ a simple lamp-post geometry for the illumination, which means that the coronal emission source is point-like, lies along the central rotation axis of the disc and emits isotropically. We consider more complex geometries later. We use RELLINE-LP of Dau et al (2013) in the spectral-modelling package XSPEC to model the response of a single emission line from the disc. This model is based on ray-tracing calculations, so accurately represents the strong gravity effects of light bending and gravitational redshift. The disc emissivity profile of the reflected spectrum is implicitly included in RELLINE-LP. It can be explicitly specified using a broken power-law with RELLINE, its convolution kernel RECONV, or any other similar kernel.

The emission line predicted as the source is raised from 3 to 30 gravitational radii ($3 - 30G/(c^2)$) is shown in Fig. 1. The black hole has a high spin of 0.998, and the disc continues down to the ISCO, which is at 1.235$r_g$. When the source is high above the disc, at height $h > 6r_g$, the line does not look very broad. This is because little illumination is reaching the innermost part near the ISCO. As the height is increased further the illumination on the disc approaches the Newtonian pattern which is roughly constant out to radius $r \sim h$, beyond which the emissivity of the disc drops with radius as a power-law of index 3, as plotted in Vaughan & Fabian (2004).

To highlight the effect of illumination on detection of the emission from the ISCO when the source is at a height of $30r_g$, we show in Fig. 2 the line profile for $r = r_{ISCO} = 1.235r_g$ and when the disc is truncated at $30r_g$. The small difference in normalization would in reality be unmeasurable since the absolute value is unknown; the main indication of truncation lies below 5.5 keV, where the line strength is less than 10 per cent of its peak. Truncation would be difficult to detect clearly with current instruments and even a bright source.

The conclusion here is that at least part of the coronal source needs to be situated at a height below at most $10r_g$ in order that clear evidence for truncation beyond the ISCO is obtainable. This

![Figure 1](source_height.png)

**Figure 1.** Model iron line profiles expected from an on-axis point source at heights of 3, 6, 10 and 30$r_g$ above an accretion disc inclined at 30 deg around a rapidly spinning black hole with spin $a = 0.998$. The profiles were obtained using RELLINE-LP of Dau et al 2013.
also applies to spin measurements, which only yield a lower limit to the spin unless the coronal height is small so that the ISCO is well illuminated (Dauser et al. 2013). Any firm recommendation depends on the quality of the data, and source conditions including iron abundance, inclination and the spin itself. (Fig. 1 in Wilkins & Fabian 2011 shows an example of the energy range relevant to various inner disc radii). The above recommendation, $h < 10r_g$, is relevant to current data quality.

Note that similar results apply if the source is even a mildly relativistic outflow, as suggested by Beloborodov (1999; see also Malzac, Beloborodov & Poutanen 2001). Due to special relativistic aberration the emission would be beamed away from the disc, mimicking the appearance of a high source. This could well be relevant to the situation when a jet develops in a source where the coronal emission occurs from the base of that jet (Markoff, Nowak & Wilms 2005). The location of the base of that jet may be quasistationary, yet the emitting matter within the base may flow rapidly upward. This rapid motion is of course what is relevant for the emission pattern and disc irradiation.

Both coronal height and motion could be relevant to jetted sources, such as the low state of black hole binaries, for example GX339-4, and radio-loud AGN such as 3C120. Lohfink et al (2013) find that the inner radius of the disc in 3C120 appears to change following a radio outburst (see also Marscher et al 2002). Possible alternative interpretations are that either coronal height has increased or the coronal material has developed an outward velocity of a few tenths of the speed of light.

When a disc is truncated and the inner radius is well illuminated then the effects of truncation can be seen in the line profile (Fig. 3). In this extreme example the source height is only $3r_g$. Note that the peak of all model lines is normalised to unity and does not show the drop in overall reflected emission which would result from the larger truncation radii in this example.

2.1 Simulations

The height of the X-ray source above the disc is manifest in the emissivity profile of the accretion disc used in the computation of the broadened line profile. If the spin of the black hole is to be obtained by fitting to the profile of the broadened emission line, some assumption is required as to the emissivity profile of the disc in the model that is fit to the data.

To quantify the effect of the assumed emissivity profile, spectra were simulated using a model consisting of a power-law continuum and relativistically-broadened iron Kα line resulting from illumination by an X-ray source at different heights ($3, 5$ and $10r_g$) above the disc. (This extends the simulations shown in Dauser et al 2013.) The emission line was modelled with the RELLINE-LP
model and the fluxes of the continuum and reflected line correspond to those found in the spectrum of the narrow line Seyfert 1 galaxy 1H0707-495 (Fabian et al 2009), but with a photon index $\Gamma = 3$ and disc inclination of 30 deg. A 500ks observation made with XMM Newton was assumed using the \textsc{fakemt} command in \textsc{XSPEC}. In each case, the spin was set to be either 0.998 (Fig. 4, upper) or 0.5 (Fig. 4, lower). We also include examples where an ionized disc model (Ross & Fabian 2005), with ionization parameter $\xi = 2000$, is assumed. There is now a deep smeared edge in the spectrum which provides additional sensitivity and thus tighter constraint on spin.

Just as for real observations, the spin is determined by fitting the profile of the iron K\textalpha line in the fake spectra over the energy range 0.3–10 keV, using either the \textsc{relline-lp} model or \textsc{reconv} convolved with \textsc{replionx} plus power-law continuum, both of which assume the correct emissivity profile for the relevant illumination case.

In Fig. 5 we show results from simulations using \textsc{relline-lp} which uses power-law approximations of the emissivity profile. In this case, when the source is at a greater height above the disc, overestimation of the emissivity index parameter (the power law slope) causes the measured spin to be systematically underestimated (Fig. 4). When the source is high, there exists a significant flattened portion of the emissivity profile at radii between the innermost peak and $r h$ (Wilkins & Fabian 2012). Even assuming the Newtonian emissivity index $q = 3$ causes the redshifted emission from the inner part of the disc to be overestimated (since the overall normalisation of the emission line is driven by the blueshifted peak). The best fit to the observed spectrum therefore requires that the disc be artificially truncated at larger radius to compensate for this extra emission created from the inner part of the disc. This artificial truncation of the accretion disc is interpreted as a lower measurement of the black hole’s spin.

Thus an assumed emissivity profile index of 3 can mean that the spin is underestimated when the illuminating X-ray source is as little as $5r_g$ above the plane of the accretion disc, though it is important to emphasise that this effect systematically underestimates the black hole spin, meaning that conclusions drawn about rapidly spinning black holes remain valid.

### 3 MORE COMPLEX GEOMETRIES

The situation becomes more complicated if the coronal source is extended, as it surely must be. Observations including reflection spectra, reverberation and microlensing (Fabian et al 2012; Kara et al 2013; Morgan et al 2012) indicate that the corona in at least some sources is compact, meaning confined to $r \sim 10 r_g$ or less (Fabian 2012; Reis & Miller 2013). This seems plausible for rapidly spinning sources where most of the disc energy release occurs within that radius. The corona is presumably powered by magnetic fields anchored in the disc so any quasi-stationary corona is likely to have a height $h < 10 - 30 r_g$, with smaller values applying to higher spin.

Even in such cases, the effects of the inevitable strong light bending mean that the source appears anisotropic to the observer. This is the essence of the light-bending model (Miniutti & Fabian 2004) where the intensity of the reflection spectrum changes with source height relatively little whereas the observed primary coronal emission changes markedly.

If the source is extended uniformly in height then the lower regions, which give the least direct emission, will give the most irradiation to the innermost parts of the disc near the ISCO, whereas the opposite applies for the upper parts. If the Comptonization spectrum of the corona changes with height (the electron temperature and optical depth are unlikely to be constant with height) then it is possible that the innermost parts near the ISCO are irradiated by a power-law of different photon index $\Gamma$ to that which characterizes the mean observed power-law spectrum. Similar effects should occur for a radially-extended source.

They do not necessarily make spin difficult to measure but

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3 Half the disc power is emitted within $5r_g$ for a highly spinning black hole and within $\sim 33r_g$ at low spin, Thorne (1974). If $h \sim r$ in radio-quiet objects, then $h \sim 5 + 28(1 - a)$.  
4 On the basis of the emissivity profile, Wilkins & Fabian (2012) show that the corona in 1H0707-495 is extended. There is however significant reflection from the inner disc so the spin measurement is robust.
Reflection on truncation and spin

Figure 5. Model fitting using broken power-law approximations for the emissivity profile (see e.g. Wilkins & Fabian 2012). Maximum spin is assumed with $r_{\text{in}} = r_{\text{ISCO}}$. The inner and outer power-law indexes, $q$, are listed together with the break radius. If only one index is given then a single power-law is assumed. Upper panel: $h = 3r_{g}$, Middle panel: $h = 5r_{g}$, Lower panel: $h = 10r_{g}$. Poor approximations lead to underestimation of spin. (The true spin in the simulation is 0.998.)

4 SYSTEMATICS IN THE X-RAY REFLECTION METHOD

We now outline some of the systematics inherent in the measurement of spin and inner radius using the X-ray reflection method. Beside the level at which the inner radii are illuminated, relevant issues include the ionization, density, elemental abundances, level of surface turbulence and uniformity of the disc, the extent to which the corona is extended and thus whether a correct emissivity model is adopted. The important issue of whether the ISCO and $r_{\text{in}}$ coincide has already been discussed in the Introduction.

The ionization state of the surface of the disc, which determines the ionization parameter $\xi$ in the reflection model (e.g. either reflionx (Ross & Fabian 2005) or xillver (Garcia & Kallman 2010); Garcia et al 2013a), affects the overall shape of the reflection spectrum. If the ionization parameter is high, the (vertically) outermost disc gas is highly ionized and much of the iron line scatters out from below the outer Thomson depth, leading to additional line broadening due to Compton scattering (Ross & Fabian 2007). The ionization state depends upon the illumination level and pattern (i.e. radial dependence) and on the disc density profile. If, as expected, the power for the corona is extracted magnetically from the disc, then it is not obvious that the radial dependence of that extraction should match that of the analytic dissipation relation of a Shakura-Sunyaev (1973) or Novikov-Thorne (1974) disc, nor, therefore, that the density profile is known or can be found other than by observation. Svoboda et al (2012) show some effects produced by varying the ionization parameter across the disc. An extended corona exacerbates the uncertainties in the ionization profile.

Svoboda et al (2009) discuss the effects produced by limb darkening, or brightening, of the emergent reflected radiation. Note that the vertical structure of the disk surface depends on the pressures involved, which in most cases are dominated by magnetic
fields (Blaes 2013), so simple thermal hydrostatic support has little relevance. There are currently no detailed simulations of irradiated discs to inform us of the expected density structure. Commonly used constant density models are an approximation. The vertical density profile of the disc surface affects the outer ionization parameter. A decreased density leads to an increased ionization which can affect the degree of limb darkening.

Disc inclination leads to further uncertainties. In principle it is measurable by the blue wing of the line, which is sensitive to the increased doppler shifts produced by higher inclination. Measurements of spin and inclination are correlated, but in a model dependent manner; for examples see Tomick et al (2014). In addition, there are intrinsic inclination effects on the reflection spectrum, now modelled by RELXILL (Garcia et al 2013b).

Numerical models of the inner regions of thin discs show them to be turbulent (Armitage & Reynolds 2003; Beckwith et al 2008). This could lead to the surface density of the disc being patchy (see e.g. Dexter & Quataert 2012), thus leading to a range of ionization parameter at each radius. There might moreover be additional doppler broadening due to the turbulent motions (Tomick et al 2014). This is presumably subsonic, which means that it should not be important for objects where a low disc ionization, and thus low disc surface temperature, is inferred.

Finally we note that the geometry of the corona is probably not known a priori so a model-independent approach is required for determining the emissivity profile. Singly or doubly broken power-law models are a good guide (Wilkins & Fabian 2012).

5 SELECTION EFFECTS

Observational selection effects mean that the brightest sources in the Sky will tend to have both high spin and inclination, even if the underlying distribution favours no spin or inclination. If the mass accretion rate is independent of black hole spin, then the increase in efficiency of energy release with spin means that the most rapidly spinning black holes will be the most luminous and thus dominate flux-limited samples (Brenneman et al 2011; Reynolds et al 2012). Subject to the uncertainty in the relation between the ISCO and $r_{in}$, excellent estimates of spin and inner disc radius are being made with X-ray reflection using the methods outlined above. The robust measurements of spin from the reflection method which have yielded high spin are perfectly valid and simply indicate that much of the irradiating radiation originates from within $10r_g$. The effects discussed above suggest that less certain results (due to the corona being higher or outflowing, for example) indicate lower limits on spin.

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