Dual Description of Brane World Cosmological Constant with $H_{MNPQ}$

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We present a short review of the recent 5D self-tuning solution of the cosmological constant problem with $1/H^2$ term, and present the dual description of the solution. In the dual description, we show that the presence of the coupling of the dual field($\sigma$) to the brane(which is a bit different from the original theory) maintains the self-tuning property.

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I. INTRODUCTION

The cosmological constant problem is the most severe hierarchy problem or fine-tuning problem known to particle physicists since 1975 [1-3]. It was introduced in the Einstein equation as

$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - 8\pi G V_0 g_{\mu\nu} = 8\pi G T_{\mu\nu}$

(1)

where the term $8\pi G V_0 (V_0$ is the vacuum energy density considered in particle physics.) is the so-called cosmological constant(c.c.) $\Lambda$. One can see that it is quite natural to introduce a constant in the action. Therefore, the constant is of order the mass scale in question. The parameter appearing in gravity is the Planck mass $M = 2.44 \times 10^{18}$ GeV which is astronomically larger than the electroweak scale. Since gravity introduces a large mass $M$, any other parameter in gravity is expected to be of that order, which is a natural expectation. Namely, $V_0$ is expected to be of order $M$. However, the bound on the vacuum energy was known to be $< (0.01 \text{ eV})^4$, which implied a fine-tuning of order $10^{-120}$.

This c.c. problem surfaced as a very serious one when one considered the spontaneous symmetry breaking in particle physics [1]. The minimum position of potential does not matter in particle physics. But in gravity its position determines the c.c. There have been several attempts toward the solution, by Hawking [4], Witten [5], Weinberg [6], Coleman [7], etc, under the name of probabilistic interpretation in Euclidian gravity, boundary of different phases, anthropic solution, wormhole solution, etc.

The self-tuning solution, which attracted a great deal of attention recently, is basically that the equations of motion choose the vanishing cosmological constant solution. For example, if the flat space solution exists, then it is a zero cosmological constant solution. In 4D, a nonzero vacuum energy does not allow the flat space solution. Thus, to obtain the flat space one has to fine-tune $V_0$ to zero very accurately.

In Sec. II, we review the self-tuning ideas. In Sec. III, we summarize the self-tuning solution with $H_{MNPQ}$ discussed in Ref. [8]. In Sec. IV, we present a self-tuning solution in the dual description in terms of $\sigma$. This self-tuning property is shown to be maintained even though we introduce the brane coupling of the dual field $\sigma$. Sec. V is a conclusion.

II. SELF-TUNING IDEAS

We can distinguish the self-tuning solutions into the old version and the new version.

A. Old version

If there exists a solution for the flat space, then the existence itself is called a self-tuning solution, or a solution with the undetermined integration constant(UIC). For a nonzero $\Lambda$ in 4D, a flat space ansatz $ds^2 = dx^2 - dt^2$ does not allow a solution. The de Sitter space(dS, $\Lambda > 0$) or anti de Sitter space(AdS, $\Lambda < 0$) solution is possible. To reach a nearly flat space solution, one needs an extreme fine tuning, which is the c.c. problem.

But suppose that there exists an UIC. Witten used the four index field strength $H_{\mu\nu\rho\sigma}$ to obtain an UIC [6]. The equation of motion of $H$ leads to an UIC, say $c$. Thus, the vacuum energy contains a piece $\sim c^2$. This UIC $c$ can be adjusted so that the final c.c. is zero. Once $c$ is determined, there is no more UIC because $H_{\mu\nu\rho\sigma}$ is not a dynamical field in 4D. When vacuum energy is added later, there is no handle to adjust further. In a sense, it was another way of fine-tuning. However, if there exists a dynamical field allowing an UIC, it is a desired old style self-tuning solution. This old version did not care whether there also exist de Sitter or anti de Sitter solutions. Selection of the flat space out of these solutions is from a principle such as Hawking’s probabilistic choice.
B. New version

In recent years, a more ambitious attempt was proposed, where only the flat space ansatz has the solution \[ (3) \]. (But at present there does not exist a solution in this category.) The Randall-Sundrum II type models \[ (1) \] were constructed in 5D bulk AdS, i.e. the 5D bulk cosmological constant \( \Lambda_b < 0 \), with brane(s) located at the fixed point at \( y = 0 \). The RS II model uses only one brane. At this brane one can introduce a brane tension \( \Lambda_1 \). Thus, the gravity Lagrangian contains two free parameters \( \Lambda_b \) and \( \Lambda_1 \). In Fig. 1, we show this situation schematically, where the extra dimension is the \( y \)-direction. \( x^\mu \) is the 4D index. The 4D flat space ansatz allows a solution for a specific choice of \( k_1 \) (basically \( \Lambda_1 \), \( k_1 = \Lambda_1 / 6M^3 \)) and \( k \) (basically \( \Lambda_b \), \( k = \sqrt{-\Lambda_b / 6M^3} \)): \( k_1 = k \). Therefore, it requires a fine-tuning between parameters as in the 4D case. However, it seems to be an improvement since we reach at the 4D flat space from non-zero cosmological constants and the RS II type models seems to be a good playground to obtain UIC solutions.

The RS II model is an interesting extension of the 4D case. However, it seems to be an improvement since we reach at the 4D flat space from non-zero cosmological constants and the RS II type models seems to be a good playground to obtain UIC solutions.

We will discuss the property of this solution somewhat in detail, to compare with our new solution obtained in the dual picture.

For \( \Lambda = 0 \), there exists a bulk solution satisfying \( A' = \alpha \phi' \),

\[
\phi = \pm \frac{3}{4} \ln \left( \frac{4}{3} y + c \right) + d, \quad \alpha = \pm \frac{1}{3}
\]

where \( c \) and \( d \) are determined without fine-tuning of the parameters. The solution has a singularity at \( y_c = -(3/4)c \) or diverges logarithmically at large \( |y| \). The logarithmically diverging solution does not realize the localization of gravity. If we restrict the space up to the singular point \( y_c \), then at every \( y \) inside the space it is flat. However, the effective 4D theory is the one after integrating out the allowed \( y \) space. Since \( y_c \) is the naked singularity, we do not know how to cut the \( y \) integration near \( y_c \), implying a possibility that the flat space ansatz does not lead to a solution. Depending on how to cut the integral, one may introduce a nonzero c.c. Förste et. al. tried to understand this problem by curing the singularity by putting a brane at \( y_c \) \[ (1) \]. Then, a flat 4D space solution is possible but one needs a fine-tuning. It is easy to understand. If one more brane is introduced, then there is one more tension parameter \( \Lambda_2 \), i.e. in the Lagrangian one adds \( \Lambda_2 \delta(\text{distance from brane}) \). If the space is flat for one specific value of \( \Lambda_2 \), then it must be curved for the other values of \( \Lambda_2 \), since the \( y \) integration gives a c.c. contribution directly from \( \Lambda_2 \).

This example teaches us that the self-tuning solution better should not have a singularity in the whole \( y \) space. Even if we neglect the problem of singularity, the brane interaction Eq. \( (4) \) must have a specific value for \( b \), which is a fine-tuning. Thus, we have not obtained a new self-tuning solution yet.
III. THE SELF-TUNING SOLUTION WITH $1/H^2$

As we have seen in the RS II model, the Einstein-Hilbert action alone does not produce a self-tuning solution. Inclusion of higher order gravity does not improve this situation \[2\]. We need matter field(s) in the bulk. The first try is a massless spin-0 field in the bulk as Ref. \[9\] tried so that it affects the whole region of the bulk. However, it may be better if there appears a symmetry in the spin-0 sector. These are achieved by a three-index antisymmetric tensor field $A_{MNP}$. In 5D the dual of its field strength is interpreted as a scalar. The field strength $H_{MNP}$ is invariant under the gauge transformation $A_{MNP} \rightarrow A_{MNP} + \partial_M \lambda_{NP}$, thus masslessness arises from the symmetry. There will be one $U(1)$ gauge symmetry remaining with one massless pseudoscalar field which is $a$, $\partial_M a = (1/4) \sqrt{-g} \varepsilon_{MNPQR} H^{NPQR}$. But the interactions are important for the solution, as in Ref. \[8\] a bulk solution was found for the specific form of the interaction.

The first guess is the bulk term $-(M/48)H^2$ \[8\] where $H^2 = H_{MNPQ}H^{MNPQ}$. The brane with tension $\Lambda_1$ is located at $y = 0$, and the bulk c.c. is $\Lambda_b$. The ansatze for the solution are

$$\text{Ansatz 1} : \quad ds^2 = \beta(y)^2 \eta_{\mu\nu} dx^\mu dx^\nu + dy^2,$$
$$\text{Ansatz 2} : \quad H_{\mu\nu\rho\sigma} = \epsilon_{\mu\nu\rho\sigma} \frac{\sqrt{-g}}{n(y)}$$

where $\mu, \cdots$ are the 4D indices, and $n(y)$ is a function of $y$ to be determined. It is sufficient to consider (55) and $(\mu\nu)$ components Einstein equations and the $H$ field equation. By setting $M = 1$, we obtain the bulk solution \[8\]

$$\Lambda_b < 0 : \quad \beta(|y|) = \left(\frac{\Lambda_b}{k}\right)^{1/4} (\pm \sinh(4k|y| + c))^{1/4}$$
$$\Lambda_b > 0 : \quad \beta(|y|) = \left(\frac{\Lambda_b}{k}\right)^{1/4} (\sin(4k|y| + c'))^{1/4}$$
$$\Lambda_b = 0 : \quad \beta(|y|) = \sqrt{|4a|y} + c''^{1/4}.$$  (8)

For a localizable (near $y = 0$) metric, there exists a singularity at $y = -c/4k$, etc., except for some cases with $\Lambda_b > 0$. Thus, for these singular cases another brane is necessary to cure the singularity, and we need a fine-tuning as in the case of Kachru et. al. \[9\]. The solution with the bulk de Sitter space without a singularity, the second one in Eq. \[8\], is worth commenting. Such a solution is periodic and depicted in Fig. 2. We can consider only $|y| \leq y_c$, then $\beta' = 0$ at $y = \pm y_c$. The boundary condition at $y = 0$ determines $c' = \cot^{-1}(k_1/k)$ and the boundary condition at $y_c$ determines $y_c$ such that $c' = 4k_y/c_0 - \cot^{-1}(k_2/k)$, so it looks like a UIC. Indeed, Ref. \[13\] claims that such a solution is a self-tuning one. But for $y_c$ to behave like an undetermined integration constant, it should not appear in the equations of motion. Note, however, that $y_c$ is the VEV of the radion $g_{55}$, and hence it cannot be a strictly massless Goldstone boson. If it were massless, it will serve to the long range gravitational interaction and hence give different results from the general relativity predictions in the light bending experiments. Therefore, it should obtain a mass and hence $y_c$ is not a free parameter but fixed. So the boundary condition at $y_c$ is a fine-tuning condition \[8,14\].

A working self-tuning model is obtained with the $1/H^2$ term \[8\].

$$S = \int d^4x \int dy \sqrt{-g} \left(\frac{1}{2} R + \frac{2}{12} \Lambda_b - \Lambda_1 \delta(y)\right).$$  (9)

A. Flat space solution

For the flat space ansatze, we use

$$ds^2 = \beta(y)^2 \eta_{\mu\nu} dx^\mu dx^\nu + dy^2,$$
$$H_{\mu\nu\rho\sigma} = \epsilon_{\mu\nu\rho\sigma} \frac{\sqrt{-g}}{n(y)}, \quad H_{5\mu\nu\rho} = 0.$$  (10)

The $H$ field equation is $\partial_M [\sqrt{-g}H^{MNPQ}/H^4] = \partial_\mu [\sqrt{-g}H^{MNPQ}/H^4] = 0$, and hence fixes $n$ as a function of $y$ only. The two relevant Einstein equations are

$$\text{(55)} : \quad 6 \left(\frac{\beta'}{\beta}\right)^2 = -\Lambda_b - \frac{\beta^8}{A}$$
$$\text{(11)} : \quad 3 \left(\frac{\beta'}{\beta}\right)^2 + 3 \left(\frac{\beta''}{\beta}\right) = -\Lambda_b - \Lambda_1 \delta(y) - 3 \left(\frac{\beta^8}{A}\right)$$

where $\Lambda_b < 0$ and $2n^2 = \beta^8/A$ with $A > 0$. We require the $Z_2$ symmetry, and the bulk equation is easily solved. The boundary condition at $y = 0$ is $(\beta'/\beta)|_{0^+} = -\Lambda_1/6$. Then, we find a solution

$$\beta(|y|) = \frac{1}{\left(\frac{a}{k}\right) \cosh(4k|y| + c)}^{1/4}$$  (12)

where

$$k = \sqrt{-\Lambda_b/6}, \quad a = \sqrt{\frac{1}{6A}}, \quad k_1 = \frac{\Lambda_1}{6}.$$  (13)

The flat solution is shown in Fig. 3.

This solution has the integration constants $a$ and $c$, $a$ is basically the charge of the universe and determines the 4D Planck mass. $c$ is the UIC which is fixed by the boundary condition at $y = 0$, 

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\[ \tanh(c) = \frac{k_1}{k} = \frac{\Lambda_1}{\sqrt{-6\Lambda_b}}. \] (14)

This solution shows that, for any value of \( \Lambda_1 \) in the finite region allowing \( \tanh \), it is possible to have a flat space solution. Even if the observable sector adds some constant to \( \Lambda_1 \), still it is possible to have the flat space solution, just by changing the shape a little bit via \( c \). The change is acceptable since \( H \) is a dynamical field. Note that \( \beta(y) \) is a decreasing function of \( |y| \) in the large \( |y| \) region and it goes to zero exponentially as \( |y| \to \infty \). We propose that this property is needed for a self-tuning solution.

The key points found in our solution are

(i) \( \beta \) has no singularity: Our solution extends to infinity without singularity, and \( \beta' \to 0 \) as \( y \to \infty \).

(ii) 4D Planck mass is finite: Even if the extra dimension is not compact, this theory can describe an effective 4D theory since gravity is localized. Integrating with respect is not compact, this theory can describe an effective solution. Even if the observable sector adds some constant to \( \Lambda \), it is possible to have a flat space solution, so we cannot obtain the solution in closed forms, we cannot show this by integration. However, we have checked this kind of behavior in the RS II model, using the Karch-Randall form \([15]\). Here, we show just that the de Sitter and anti de Sitter space solutions exist, and show the warp factor numerically. In our model, the \( y \) derivative of the metric is

\[ \beta' = \pm (k_1^2 + k^2\beta^2 - a^2\beta^4)^{1/2}, \quad k_\lambda = (\lambda/6)^{1/2} \] (20)

At \( y = y_h \) where \( \beta(y_h) = 0 \), \( \beta'(y_h) \) needs not be zero due to the presence of the nonvanishing \( k_\lambda \). Therefore, there exists a point \( y_h \) where \( \beta' \) is finite. It is the de Sitter space horizon. It takes an infinite amount of time to reach \( y_h \). Also, we can see that it is possible that \( \beta \) can be nonzero where \( \beta' \) is zero for \( k_\lambda^2 < 0 \). It is the anti de Sitter space solution. These de Sitter space and anti de Sitter space solutions are shown in Figs. 4 and 5. For the de Sitter space, we can integrate from \( -y_h \) to \( +y_h \). As in the Karch-Randall example, it should give the 4D c.c. \( \lambda \). The AdS solution does not give a localized gravity.

In the presence of de Sitter and anti de Sitter space solutions, the c.c. problem relies on the old self-tuning solution. Namely, the c.c. is probably zero, following Hawking \([3]\).

Hawking showed that in 4D the probability is maximum, using the Euclidian space action

\[ -S_E = -\frac{1}{2} \int d^4x \sqrt{-\eta} [(R + 2\Lambda) + (1/48)H_{\mu\nu\rho\sigma}H^{\mu\nu\rho\sigma}] \] (21)

where we use the unit \( 8\pi G = 1 \). The Einstein equation and field equations are

\[ R_{\mu\nu} - (1/2)g_{\mu\nu}R = \Lambda g_{\mu\nu} - T_{\mu\nu} \]

\[ \partial_{[\mu} [\sqrt{\eta} H^{\nu\rho\sigma}] = 0. \]
From the field equation, one has $H^{\mu\nu\rho} = (1/\sqrt{g})e^{\mu\nu\rho\sigma}c$ or $H^2 = 4c^2$. Thus, he obtained

$$T^{\mu\nu} = -\Lambda_H g^{\mu\nu}, \Lambda_H = -c^2/2, \quad R = -4\Lambda_{eff}, \Lambda_{eff} = \Lambda + \Lambda_H.$$ (22)

Thus,

$$-S_E = -(1/2) \int d^4x \sqrt{g}(R + 2\Lambda_{eff}) = \text{Volume} \cdot (\Lambda + \Lambda_H) = \frac{3M^4}{\Lambda_{eff}}$$ (23)

which is maximum at $\Lambda_{eff} = 0^+$ which is shown in Fig. 6.

Note that Hawking used equations of motion. Duff, on the other hand, used the action itself to calculate the Euclidian action, and obtained $-S_E = 3M^4(\Lambda - (3/2)c^2)/(\Lambda - (1/2)c^2)^2$ which is minimum at $0^+$ [4]. But the consideration of the surface term in the action would give additional contribution and should give Hawking’s result. The surface term is essential as we have shown in our self-tuning solution.

Thus, the maximum probability occurs when the c.c. is zero. Our self-tuning solution relies on this probabilistic choice of the flat one from the flat, de Sitter and anti de Sitter space solutions.

### IV. SELF-TUNING SOLUTION IN THE DUAL DESCRIPTION

Let us now proceed to consider a dual description of the self-tuning solution with $1/H^2$. When we introduce the $1/H^2$ term in the Lagrangian, the equation of motion for a three form field $A$ with $H=dA$ is

$$d^4y \left( \frac{^*H}{(H^2)^2} \right) = 0$$ (24)

where $^*H$ is the Hodge dual of $H$. Note that we have not introduced a source term for $A_{MNP}$.

The corresponding field strength $H=dA$ should satisfy the Bianchi identity, $dH=0$, locally. In the dual description, the roles of the equation of motion and the Bianchi identity are interchanged from the original one. Therefore, we can take a dual of the field strength $H$ in 5D such that the equation of motion for $A$ is trivially satisfied, as a Bianchi identity in the dual description, as follows:

$$d\sigma = \frac{^*H}{(H^2)^2}$$ (25)

where $\sigma$ is a dual scalar field. Then, we can identify the field strength $H$ in terms of the scalar field $\sigma$ as

$$H_{MNPQ} = \sqrt{-g} MNPQ \frac{\partial R\sigma}{(4!)^{1/3}[(\partial\sigma)^2]^{2/3}}.$$ (26)

As a result, the original Bianchi identity, $dH=0$, becomes in the dual picture

$$\partial_M \left( \sqrt{-g} \frac{\partial M \sigma}{[(\partial\sigma)^2]^{2/3}} \right) = 0.$$ (27)

which will be the equation of motion in the dual picture. In this case, we can add a source term on the right hand side of Eq. (27).

To show the self-tuning solution with $1/H^2$ in the dual picture explicitly, let us insert the dual relation (25) or (26) back to the Lagrangian and make a variation of the action with respect to the dual scalar field. When the surface term for $A$ is included, the dual 5D action is

$$S = \int d^4x dy \sqrt{-g} \left( \frac{1}{2}R - \gamma[(\partial\sigma)^2]^{1/3} - \Lambda_h - \Lambda_1 f(\sigma)\delta(y) \right)$$ (28)

where $\gamma = 6(4!)^{1/3}$ and $f(\sigma)$ is an arbitrary coupling of the dual field to the brane, which breaks the shift symmetry of the dual field at the brane.

Then, the energy-momentum tensor from the dual scalar field is

$$T^\sigma_{MNP} = 2\gamma \left( \frac{1}{3}[(\partial\sigma)^2]^{-2/3}\partial_M\sigma\partial_N\sigma - \frac{1}{2} g_{MN}[(\partial\sigma)^2]^{1/3} \right).$$ (29)

And the equation of motion for the dual scalar field becomes

$$\partial_M \left( \sqrt{-g} \frac{\partial^M \sigma}{[(\partial\sigma)^2]^{2/3}} \right) = \sqrt{-g} (4!)^2 \Lambda_1 \frac{df}{d\sigma}\delta(y)$$ (30)

where the original Bianchi identity (27) is not respected at the brane due to the coupling $f(\sigma)$, i.e.,

$$dH_{5\mu\nu\rho} = \frac{1}{(4!)^{1/3}} \sqrt{-g} (4!) \epsilon_{\mu\nu\rho\sigma\tau} \Lambda_1 \frac{df}{d\sigma}\delta(y).$$ (31)

Note that the self-tuning solution with $1/H^2$ is the case with $df/d\sigma=0$. In the dual picture, a new Bianchi identity

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*The text mode(Roman) characters are the form notation while equation mode(Italic) characters are the component notation.
is $d\sigma = 0$. Thus, the theory we study is different from the original one in that the bulk field $\sigma$ couples to the brane with $f(\sigma)$.

If we take the ansatze for the metric and the scalar field as

$$ds^2 = \beta^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \quad \sigma = \sigma(y),$$

then the relevant Einstein’s equations and the scalar equation are

$$3\left(\frac{\beta'}{\beta}\right)^2 + 3\frac{\beta''}{\beta} = -\Lambda_b - \Lambda_1 f(\sigma)\delta(y) - \gamma|\sigma'|^{2/3},$$

$$6\left(\frac{\beta'}{\beta}\right)^2 = -\Lambda_b - \frac{1}{3}\gamma|\sigma'|^{2/3},$$

$$\left(\frac{\beta^4}{|\sigma'|^{4/3}}\right)' = \beta^4 A_1 \frac{df}{d\sigma}\delta(y).$$

Thus, we find that the solutions of the above equations consistent with the $Z_2$ symmetry become

$$\beta(y) = \left(\frac{k}{a}\right)^{1/4}[\cosh(4k|y| + c)]^{-1/4},$$

$$\sigma'(y) = \pm \left(\frac{18\alpha}{\gamma}\right)^{3/2}\beta^{12}\epsilon(y)$$

where $\epsilon(y)$ is the step function which is +1 for $y > 0$ and -1 for $y < 0$, and

$$k = \sqrt{-\frac{\Lambda_b}{6}}, \quad a = \sqrt{\frac{1}{6\Lambda_1}}.$$ (38)

On the other hand, from Eqs. (33) and (35), the boundary conditions at the brane for the metric and the scalar are given by

$$\left.\frac{\beta'}{\beta}\right|_{y=0^+} = -k_1 f(\sigma(0)), \quad \left.\frac{\sigma'}{|\sigma'|^{4/3}}\right|_{y=0^+} = 3k_1 \frac{df}{d\sigma}(\sigma(0))$$

where

$$k_1 = \frac{\Lambda_1}{6}.$$ (41)

Therefore, there arise two consistency conditions due to the existence of the brane and the scalar coupling:

$$\tanh(c) = \frac{k_1}{k} f(\sigma(0)),$$

$$\pm \sqrt{\frac{\gamma}{2k}}\cosh(c) = 9k_1 \frac{df}{d\sigma}(\sigma(0)).$$ (43)

That is, the condition for the scalar coupling function at the brane becomes

$$\frac{df}{d\sigma}(\sigma(0)) = \pm \sqrt{\frac{\gamma}{162k_1}} \left(1 - \left(\frac{k_1}{k}\right)^2 f^2(\sigma(0))\right)^{-1/2}.$$ (44)

Without the scalar coupling, Eq. (43) is absent and Eq. (42) is the self-tuning solution obtained in terms of the three form field $\mathcal{J}$, and we reproduce the solution in the dual picture.

Even with the scalar coupling, i.e. the original Bianchi identity becoming the equation of motion in the dual picture with the scalar coupling to the brane, the self-tuning property is maintained because $\sigma(0)$ acts as an integration constant of Eq. (47). For a given set of $\Lambda_b, \Lambda_1$, and parameters appearing in $f(\sigma)$, Eq. (44) can be satisfied by choosing an appropriate value of the integration constant $\sigma(0)$. For example, if $f(\sigma) = e^{b\sigma}$ then for some $b$ we can find $\sigma(0)$ as far as Eq. (44) is satisfied. Note that the condition Eq. (44) is given only for the field value at $y = 0$. Of course, there also exist de Sitter and anti de Sitter space solutions. Thus, our self-tuning solution is of the old style.

To see it explicitly, let us consider $f(\sigma)$ as an exponential function

$$f(\sigma) = e^{b\sigma}$$

where we set the overall constant as 1 and $b$ is a number. Then, Eq. (44) becomes

$$b^2 e^{2b\sigma(0)} \left[1 - \frac{k_1^2}{k^2} e^{2b\sigma(0)}\right] = \frac{\gamma}{162(kk_1)^2}$$

which is a quadratic equation of $e^{2b\sigma(0)}$. For a finite range of parameters, it has two solutions, proving the self-tuning property. It is different from the Kachru et al. solution which required a fine-tuning of $b$ as 4/3.

V. CONCLUSION

It is pointed out that a self-tuning solution of the cosmological constant is possible in the RS II type models. We have shown this $1/H^2$ term in the Lagrangian as a review to guide the self-tuning solution [7]. We presented this solution in the dual picture. The kinetic energy term in the dual picture has a peculiar form $((\partial_\sigma \sigma)^2)^{1/3}$. In the dual picture, this self-tuning property is still maintained even with the presence of the dual field coupling to the brane. However, the new type of self-tuning solution is not obtained. The self-tuning relies on the old
idea of choosing the flat one out of numerous possibilities.

The dual picture description is intriguing. As discussed in Sec. IV, the scalar field $\sigma$ has an integration constant $\sigma(0)$. We can choose $\sigma(0)$ such that a flat space solution results. However, for another nearby value of $\sigma(0)$, a de Sitter or anti de Sitter space solution is possible, which is the reason that we could have not obtained a new type of self-tuning solution. The unique flat space solution requires a fine-tuning of $\sigma(0)$, at present. In the future, it will be interesting to see if some principle chose such a fine-tuning of $\sigma(0)$.

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Fig. 1. The RS II model with a brane at $y = 0$.

Fig. 2. The flat space solution with $H^2$

Fig. 3. The flat space solution.

Fig. 4. The de Sitter space solution.

Fig. 5. The anti de Sitter space solution.

Fig. 6. Hawking’s probability

$\exp(\frac{3M^4}{\Lambda})$