Spin structure functions and intrinsic motion of the constituents *

Petr Závada

Institute of Physics, Academy of Sciences of the Czech Republic,
Na Slovance 2, CZ-182 21 Prague 8,
e-mail: zavada@fzu.cz

(January 4, 2002)

The spin structure functions of the system of quasifree fermions on mass shell are studied in a consistently covariant approach. Comparison with the basic formulas following from the quark-parton model reveals the importance of the fermion motion inside the target for the correct evaluation of the spin structure functions. In particular it is shown, that regarding the moment Γ1, both the approaches are equivalent for the static fermions, but differ by the factor 1/3 in the limit of massless fermions (m ≪ p0, in target rest frame). Some other sum rules are discussed as well.

I. INTRODUCTION

Measuring of the nucleon spin structure functions represents an important tool not only for better understanding of the nucleon internal structure in the language of the QCD, but also for better understanding of QCD itself. These functions contain an information, which is a crucial complement to the structure functions obtained in the unpolarized deep inelastic scattering (DIS) experiments.

The polarized experiments are more complex and difficult than the unpolarized ones, nevertheless the last decade has brought remarkable results also for the nucleon spin functions from the experiments at CERN (EMC, SMC) and SLAC (E142, E143, E154, E155). And the new experiments are running (HERMES) or are being under preparation (COMPASS). The data on polarized pp collisions are expected from the collider RHIC. For the present status of the research in structure functions see e.g. [1], the overview [2] and citation therein. The more formal aspects of the polarized DIS are explained in [3].

Also the interpretation and understanding of polarized structure functions seem be more difficult. For an example, until now it is not well understood, why the integral of the proton spin structure function g1 is substantially less, than expected from very natural assumption, that the nucleon spin is generated by the valence quarks. Presently, there is a tendency to explain the missing part of the nucleon spin as a contribution of the gluons. It has been also suggested, that the quark orbital momentum can play some role as well [4] - [6].

The spin in general is a very delicate quantity, which requires correspondingly precise treatment. It has been argued, that for correct evaluation the quark contribution to the nucleon spin it is necessary to take properly into account the intrinsic quark motion [1] - [13]. Necessity of the covariant formulation of the quark - parton model (QPM) for the spin functions has been pointed out in [14]. These requirements are not satisfied in the standard formulation of the QPM, which is currently used for analysis and interpretation of the experimental data.

In this paper we shall attempt to demonstrate the role of the intrinsic motion for the spin structure functions, using very simple model of the system quasifree fermions on mass shell. The basic requirement is consistently covariant formulation of the task for the system of fermions, which are not static, being characterized by some momenta distribution in the frame of their centre of mass. The spin structure functions of such system are obtained in Sec. II and the sum rules following from these functions are shown in Sec. III. In the Sec. IV a comparison with the formulas of the standard QPM is done. The last section is devoted to the short summary.

II. SPIN STRUCTURE FUNCTIONS IN COVARIANT APPROACH

Let us imagine a system of three quasifree charged fermions with the spin 1/2 and mass m, for which the following conditions are satisfied:

*to be published in Phys. Rev. D
1) The distribution of fermion momenta in the frame of their centre of mass is described by some spherically symmetric function $G$,

$$\int G(p_0) d^3p = 3; \quad p_0 = \sqrt{m^2 + p^2}. \quad (1)$$

The free fermion states are described by the spinors

$$\psi_{p,\lambda}(x) = \frac{1}{\sqrt{\Omega}} u(p, \lambda) \exp(-ipx); \quad \int \psi_{p,\lambda}^\dagger(x) \psi_{p,\lambda}(x) d^3x = 1, \quad (2)$$

where $\Omega$ is the normalization volume and

$$u(p, \lambda) = \frac{1}{\sqrt{N}} \left( \frac{\phi_\lambda}{p_0+m} \phi_\lambda \right); \quad N = \frac{2p_0}{p_0+m}, \quad \phi_\lambda^\dagger \phi_\lambda = 1. \quad (3)$$

We assume

$$\frac{1}{2} n_\lambda \phi_\lambda = \lambda \phi_\lambda, \quad \lambda = \pm \frac{1}{2}, \quad (4)$$

which means, that the spin projection of the fermion in its rest frame is $\pm 1/2$ in the given direction $\mathbf{n}; |\mathbf{n}| = 1$.

2) By $G_{\pm 1/2}$ we denote function, which measures probability, that fermion is in the state $\psi_{p, \pm 1/2}$, so that

$$G(p_0) = G_{+1/2}(p_0) + G_{-1/2}(p_0) \quad (5)$$

and we assume

$$\int \Delta G(p_0) d^3p = 1; \quad \Delta G(p_0) \equiv G_{+1/2}(p_0) - G_{-1/2}(p_0). \quad (6)$$

The difference $\Delta G$ consists of the corresponding contributions $\Delta h_j$ from the three fermions:

$$\Delta G(p_0) = \sum_{k=1}^3 \Delta h_k(p_0); \quad \Delta h_k(p_0) \equiv h_{k,+1/2}(p_0) - h_{k,-1/2}(p_0). \quad (7)$$

Later on, we shall need also the distribution

$$H(p_0) \equiv \sum_{k=1}^3 e_k^2 \Delta h_k(p_0), \quad (8)$$

where $e_k$ are the fermion charges.

What is the resulting spin (total angular momentum) related to the whole system? Let us calculate the integral of the matrix elements

$$\langle \mathbf{n} \rangle = \int \int \sum_\lambda G_\lambda(p_0) \left( \psi_{p,\lambda}^\dagger(x) \mathbf{n} \psi_{p,\lambda}(x) \right) d^3x d^3p, \quad (9)$$

where the angular momentum $\mathbf{j}$ consists of the spin and orbital part

$$j_k = \Sigma_k + l_k = \frac{1}{2} \begin{pmatrix} \sigma_k & 0 \\ 0 & \sigma_k \end{pmatrix} - i\varepsilon_{klnm} p_l \frac{\partial}{\partial p_m}. \quad (10)$$

Since the total angular momentum $\mathbf{j}$ is a conserving quantity, which commutes with the term $\mathbf{p}\sigma$, a simple calculation gives

$$\psi_{p,\lambda}^\dagger(x) \mathbf{n} \psi_{p,\lambda}(x) = \frac{1}{\Omega} \left( \lambda + \varepsilon_{klnm} n_k p_l x_m \right). \quad (11)$$

So, after inserting to Eq. (11) and using the assumption (10) one gets
\[ \langle \mathbf{n} | \mathbf{j} \rangle = \frac{1}{\Omega} \int \int_\Omega \left[ (G_{+1/2}(p_0) - G_{-1/2}(p_0)) / 2 + G(p_0) \varepsilon_{klm} n_k p_l x_m \right] d^3x d^3p \]

\[ = \frac{1}{2} \int \Delta G(p_0) d^3p = \frac{1}{2}, \]

since the term \( \varepsilon_{klm} n_k p_l x_m \), due to spheric symmetry, vanishes. One can check, if \( \mathbf{n}', \mathbf{n}'' \) are vectors, which together with \( \mathbf{n} \) generate an orthonormal base in the frame of centre of mass of the three fermions, then a similar calculation gives

\[ \langle \mathbf{n}' | \mathbf{j} \rangle = \langle \mathbf{n}'' | \mathbf{j} \rangle = 0. \]

Obviously, the simplest way is to use the base like:

\[ \mathbf{n} = (0, 0, 1), \quad \mathbf{n}' = (0, 1, 0), \quad \mathbf{n}'' = (1, 0, 0). \]

Since we work with the probabilistic description (in terms of quantum mechanics with the statistical mixture of states) by means of the distributions \( G_\lambda \), as a result we can obtain only the mean values of the total spin projections \( \langle J \rangle = (0, 0, 1/2) \). Nevertheless one could consider a more rigorous (but more complicated) approach, in which the three fermion system is not constructed as the statistical mixture of plane waves, but as the composition of the three pure states \( J = 1/2 \), \( J_z = \pm 1/2 \) with the condition, that the whole system represents a pure state \( J = 1/2 \), \( J_z = 1/2 \). These states are represented by the relativistic spheric waves (spinors), which imply the corresponding probabilistic distributions \( G_\lambda, \Delta G \) and \( H \) have spheric symmetry. In other words, if in our approach we assume the system in a pure state \( J = 1/2 \), then its probabilistic description in terms of the plane waves will be defined by the distributions \( G_\lambda \), which are spherically symmetric. In fact, that is the reason, why we require spheric symmetry, deformed distributions \( G_\lambda \) would contradict the eigenstate \( J = 1/2 \).

Let us point out, in the relativistic case, having one fermion state with definite projection \( \mathbf{n} j \) of the total angular momentum, one cannot separate its orbital and spin part (with exception of the special case when \( \mathbf{n} \parallel \pm \mathbf{p} \)), i.e. account with the fermion orbital momentum is crucial for a consistent calculation of the resulting spin. On the other hand, the similar calculation, in which the orbital part \( \mathbf{l} \) is ignored, gives

\[ \psi_{\lambda,\lambda}'(x) \mathbf{n} \Sigma \psi_{\lambda,\lambda}(x) = \frac{1}{\Omega N} \left( \lambda \phi^\dagger_\lambda \phi_\lambda + \phi^\dagger_\lambda \frac{\mathbf{p} \cdot \mathbf{n} \sigma \cdot \mathbf{p} \sigma}{2(p_0 + m)^2} \phi_\lambda \right) \]

\[ = \frac{1}{\Omega N} \left( \lambda + \phi^\dagger_\lambda \frac{\mathbf{p} \cdot \mathbf{n} \sigma \cdot \mathbf{p} \sigma}{2(p_0 + m)^2} \phi_\lambda \right) \]

\[ = \frac{1}{\Omega N} \left( \lambda - \lambda \frac{\mathbf{p}^2}{(p_0 + m)^2} + \phi^\dagger_\lambda \frac{\mathbf{p} \cdot \mathbf{n} \sigma \cdot \mathbf{p} \sigma}{3(p_0 + m)^2} \phi_\lambda \right). \]

Since

\[ \mathbf{p} \sigma \cdot \mathbf{n} = \sum_{i=1}^3 p_i^2 \sigma_i n_i + \sum_{i \neq j} p_i p_j \sigma_i n_j \]

one can write

\[ \langle \mathbf{n} \Sigma \rangle = \int \int_\Omega \sum_\lambda G_\lambda(p_0) \left( \psi_{\lambda,\lambda}'(x) \mathbf{n} \Sigma \psi_{\lambda,\lambda}(x) \right) d^3x d^3p \]

\[ = \int \sum_\lambda G_\lambda(p_0) \frac{\lambda}{N} \left( 1 - \frac{\mathbf{p}^2}{(p_0 + m)^2} + \frac{2\mathbf{p}^2}{3(p_0 + m)^2} \right) d^3p, \]
where inserting the formula (15), we take into account, that due to spheric symmetry the terms \( p_i p_j \) \((j \neq i)\) vanish and the terms \( p_i^2 \) can be substituted by \( p^2/3 \). The last relation can be further simplified:

\[
\langle n \Sigma \rangle = \frac{1}{2} \int \Delta G(p_0) \left( \frac{1}{3} + \frac{2m}{3p_0} \right) d^3 p \leq \frac{1}{2} \tag{16}
\]

One can observe, that the correspondence with Eq. (12) takes place only for the system of static fermions.

For further consideration, it will be useful to substitute the vector \( n \), representing the direction of the fermion polarization, by the corresponding covariant polarization vector \( w^\sigma(\lambda) \), which satisfies

\[
w^2(\lambda) = -1, \quad w(\lambda) \cdot p = 0 \tag{17}
\]

and

\[
w(\lambda) = \frac{\lambda}{|\lambda|}(0, n); \quad \lambda = \pm \frac{1}{2} \tag{18}
\]
in the fermion rest frame. The explicit representation of the vectors \( w(\lambda) \) will be defined hereinafter.

Now, let us expose this system as a (fixed) target to the beam of polarized electrons (e.g. helicity \( +1/2 \)) coming with the momentum

\[
k = \left( k_0, \sqrt{k_0^2 - m_e^2}, 0, 0 \right) \tag{19}
\]

and let us calculate the form of corresponding differential cross-section. The spin dependent part of the cross-section for interaction with a single fermion in one photon approximation has the form

\[
d\sigma \sim - L^{\alpha\beta(A)}(q, s) T^{(A)}_{\alpha\beta} \tag{20}
\]

The antisymmetric tensor \( L^{\alpha\beta(A)} \), (see e.g. \cite{3}) related to the electron beam reads:

\[
L^{\alpha\beta(A)} = m_e \varepsilon_{\alpha\beta\lambda\sigma} q^{\lambda} w^{\sigma}, \tag{21}
\]

where \( m_e \) is the electron mass, \( s \) denotes its polarization vector

\[
s = \frac{1}{m_e} \left( \sqrt{k_0^2 - m_e^2}, k_0, 0, 0 \right); \quad s^2 = -1, \quad ks = 0 \tag{22}
\]

and \( q = k - k' \) is the photon momentum. The antisymmetric tensor \( T^{\alpha\beta(A)} \) related to the single fermion inside the target has a similar form:

\[
T^{\alpha\beta(A)} = m \varepsilon_{\alpha\beta\lambda\sigma} q^{\lambda} w^{\sigma}(\lambda), \tag{23}
\]

where \( m \) and \( w(\lambda) \) denote the fermion mass and polarization vector. If one assumes, that the electron scattering can be described as the incoherent sum of the interactions with the single plane waves, then the tensor \( T^{\alpha\beta(A)} \) reads

\[
T^{(A)}_{\alpha\beta} = \varepsilon_{\alpha\beta\lambda\sigma} q^{\lambda} m \int H(p_0) w^{\sigma} \delta((p + q)^2 - m^2) \frac{d^3 p}{p_0}, \tag{24}
\]

Here the charge factors and the two possible signs of \( w^\sigma \) are included into the tensor through the distribution \( \delta \). By the symbol \( w^\sigma \) we mean \( w^\sigma(\lambda = +1/2) \). Let us remark, this form of the antisymmetric part of the hadronic tensor is very similar to that used in \cite{3}. Further, we can modify the \( \delta \)–function term:

\[
\delta((p + q)^2 - m^2) d^3 p = \delta(2pq + q^2) d^3 p = \frac{1}{2\xi} \delta(pq) + \frac{q^2}{2\xi} d^3 p, \tag{25}
\]

where \( \xi \) is arbitrary constant, which only rescales the integration variable. Now, let us imagine, that our target is a part of the greater system, which is at rest with respect to the given reference frame and has the mass \( M \), but at the same time the probing electron interact only with the three fermions. If we put

\[
\xi = M q_0 = M \nu, \tag{26}
\]
then in the $\delta-$function one can identify the terms known from the formalism of deep inelastic scattering:

$$-\frac{q^2}{2M\nu} = \frac{Q^2}{2M\nu} = x,$$

which is the Bjorken scaling variable, its value can be directly determined using only initial and final momenta of the scattered electron. This variable is in the $\delta-$function compensated by the ratio $pq/M\nu$, which after boosting the whole target of mass $M$ to the infinite momentum frame approximately represents ratio of dominating momenta components $p'/P'$ of the fermion and the target.

The explicit form of the polarization vector $w$ can be found as follows. First, let us transform the vector $w = (0, n)$ from the fermion rest frame to the target rest frame. After decomposition of the vector $n$ to longitudinal and transversal parts with respect to the momentum fermion $p$, the corresponding Lorentz boost gives

$$(0, n) \rightarrow w = \left( \frac{pn}{m}, n + \frac{pn}{m(m + p_0)} p \right).$$

(28)

Secondly, let us make a Lorentz boost of the whole target with mass $M$ to some another frame, which is defined by the new components of the target momentum

$$(M, 0, 0, 0) \rightarrow P = (P_0, P); \quad P^2 = M^2.$$  

(29)

Next, if we define the covariant vector $S$ by its components in the target rest frame as

$$S = (0, n),$$

(30)

then the polarization vector $w$ can be written in manifestly covariant form

$$w^\sigma = AP^\sigma + BS^\sigma + Cp^\sigma,$$

(31)

where $A, B, C$ are invariant functions (scalars) of the vectors $P, S, p$. These three functions are fixed by two the conditions (17) and by the constraint (28) valid in the target rest frame. A simple calculation gives:

$$A = -\frac{pS}{pP + mM}, \quad B = 1, \quad C = \frac{M}{m}A.$$

(32)

So, we have obtained explicit covariant form of the polarization vector $w$ entering the tensor (24), which can be now in accordance with the relations (25)-(27) rewritten

$$T^{(A)}_{\alpha\beta} = \varepsilon_{\alpha\beta\lambda\sigma} q^\lambda \frac{m}{2Pq} \int H\left(\frac{pP}{M}\right) w^\sigma \delta\left(\frac{pq}{Pq} - x\right) \frac{d^3p}{p_0},$$

(33)

where we use the invariant term $Pq$ instead of $M\nu$ and $H(pP/M)$ instead of $H(p_0)$.

On the other hand, in accordance with the general rule (see e.g. [3]), the antisymmetric tensor $T^{(A)}_{\alpha\beta}$ appearing in the formula for the cross-section (20), has the form

$$T^{(A)}_{\alpha\beta} = \varepsilon_{\alpha\beta\lambda\sigma} q^\lambda \left\{ MS^\sigma G_1 + [(Pq)S^\sigma - (qS)P^\sigma] \frac{G_2}{M} \right\},$$

(34)

where $M, P, S$ represent the target mass, momentum and spin polarization vector, which satisfies

$$S^2 = -1, \quad PS = 0.$$  

(35)

The invariants $G_1$ and $G_2$ are the spin structure functions. In the next we shall identify the parameters $M, P, S$ in Eq. (34) with those in the model described above and simultaneously we shall attempt to determine the spin structure functions corresponding to our target. First of all, we modify the Eq. (34) by the substitution

$$G_S = MG_1 + \frac{Pq}{M}G_2, \quad G_P = \frac{qS}{M}G_2,$$

(36)

which gives
\[ T_{\alpha\beta}^{(A)} = \varepsilon_{\alpha\beta\lambda\sigma} q^\lambda \{ S^\sigma G_S - P^\sigma G_P \}. \]  

Comparison with Eq. (33) gives the equation for the structure functions:

\[ \varepsilon_{\alpha\beta\lambda\sigma} q^\lambda \{ S^\sigma G_S - P^\sigma G_P \} = \varepsilon_{\alpha\beta\lambda\sigma} q^\lambda \frac{m}{2P q} \int H \left( \frac{p P}{M} \right) \left( \frac{pq}{P q} - x \right) \frac{d^3 p}{p_0}. \]  

Because of the antisymmetry of the tensor \( \varepsilon \) and after inserting from the relation (31) it follows that

\[ S^\sigma G_S - P^\sigma G_P = \frac{m}{2P q} \int H \left( \frac{p P}{M} \right) (AP^\sigma + BS^\sigma + C\rho^\sigma) \delta \left( \frac{pq}{P q} - x \right) \frac{d^3 p}{p_0} + Dq^\sigma, \]  

where \( D \) is some scalar function and the functions \( A, B, C \) are given by the relations (24). After contracting with \( P_\sigma, S_\sigma \) and \( q_\sigma \) one gets the equations for unknown functions \( G_S, G_P \) and \( D \):

\[ -M_s G_P = \frac{m}{2P q} \int H \left( \frac{p P}{M} \right) (AM^2 + C \cdot P P) \delta \left( \frac{pq}{P q} - x \right) \frac{d^3 p}{p_0} + D \cdot P q, \]  

\[ -G_S = \frac{m}{2P q} \int H \left( \frac{p P}{M} \right) (-B + C \cdot P S) \delta \left( \frac{pq}{P q} - x \right) \frac{d^3 p}{p_0} + D \cdot q S, \]  

\[ q S \cdot G_S - P q \cdot G_P = \frac{m}{2P q} \int H \left( \frac{p P}{M} \right) (A \cdot P q + B \cdot q S + C \cdot P q) \]  

\[ \times \delta \left( \frac{pq}{P q} - x \right) \frac{d^3 p}{p_0} + D q^2 \]  

and inserting \( G_P, G_S \) from the first two equations to the last one gives the condition for \( D \):

\[ \frac{m}{2P q} \int H \left( \frac{p P}{M} \right) (C \cdot pu) \delta \left( \frac{pq}{P q} - x \right) \frac{d^3 p}{p_0} + D \cdot qu = 0, \]  

where we denote

\[ u \equiv q + (qS) - \frac{(pq)}{M^2} P. \]  

Finally, inserting \( D \) from this equation to Eqs. (40), (41) gives with the use of relations (23) the structure functions

\[ G_P = \frac{m}{2P q} \int H \left( \frac{p P}{M} \right) \frac{p S}{p P + m M} \left[ 1 + \frac{1}{m M} \left( p P - pu \right) \frac{P q}{q S} \right] \delta \left( \frac{pq}{P q} - x \right) \frac{d^3 p}{p_0}, \]  

\[ G_S = \frac{m}{2P q} \int H \left( \frac{p P}{M} \right) \left[ 1 + \frac{p S}{p P + m M} \frac{M}{m} \left( p S - pu \right) \frac{q S}{q S} \right] \delta \left( \frac{pq}{P q} - x \right) \frac{d^3 p}{p_0}. \]  

The spin structure functions in the standard notation \( g_1 = M \cdot P q \cdot G_1, g_2 = (P q)^2 / M \cdot G_2 \) can be now obtained from Eqs. (44):

\[ g_1 = P q \left( G_S - \frac{P q}{q S} G_P \right), \quad g_2 = \frac{(P q)^2}{q S} G_P, \quad g_1 + g_2 = P q G_S, \]  

where the functions \( G_S, G_P \) are given by relations (44), (45). Corresponding integrals, as shown in the Appendix, can be simplified to the form (A14), (A15). Let us remark, resulting functions \( g_1, g_2 \), after inserting from the relations (A14), (A15) into Eq. (40) do not depend on the variable \( q S \) despite the fact, that such terms are present in the starting integrals (44), (45) in a non-trivial way. This is a consequence of spheric symmetry of the distribution \( H \), which as we have suggested, follows from the requirement \( J = 1/2 \).
III. SUM RULES

For next analysis of the obtained structure functions it is convenient to express the integrals (44), (45) in the target rest frame, where \( P = (M, 0, 0, 0) \) and \( S = (0, \mathbf{n}) \). Detailed calculation is done in the Appendix. Now, let us assume \( Q^2 \gg 4M^2x^2 \), then

\[
\frac{|q|}{\nu} = \sqrt{1 + 4M^2x^2/Q^2} \to 1
\]

and using the second relation \( (46) \) and Eq. \( (A7) \) one gets

\[
\Gamma_2 \equiv \int g_2(x)dx = -\pi \int \int H(p_0) \left(p_1 + \frac{p_1^2 - p_T^2/2}{p_0 + m}\right) \delta \left(\frac{p_0 + p_1}{M} - x\right) \frac{p_T dp_1 dp_T}{p_0} dx
\]

(47)

In the last integral, due to spheric symmetry of the distribution \( H \), the terms proportional to \( p_1 \) and \( p_T^2/2 \) vanish, insofar that

\[
\Gamma_2 = 0,
\]

(48)

which is the known Burkhardt-Cottingham sum rule \[15\]. Similarly the third relation \( (46) \) and Eq. \( (A8) \) give

\[
\int (g_1(x) + g_2(x))dx = \pi \int H(p_0) \left(m + \frac{p_T^2}{2(p_0 + m)}\right) \frac{p_T dp_1 dp_T}{p_0}.
\]

(49)

After the substitution

\[
2\pi p_T dp_1 dp_T = d^3p
\]

(50)

and using relation \( (18) \) one gets

\[
\Gamma_1 \equiv \int g_1(x)dx = \frac{1}{2} \int H(p_0) \left(m + \frac{p^2}{3(p_0 + m)}\right) \frac{d^3p}{p_0}.
\]

(51)

Simple modification then gives

\[
\Gamma_1 = \frac{1}{2} \int H(p_0) \left(\frac{1}{3} + \frac{2m}{3p_0}\right) d^3p.
\]

(52)

More detailed analysis of this result will be done in the next section.

The relations \( (A7) \) and \( (A8) \) can be used also for the calculation of the higher momenta. Generally, if \( F \) is a function defined as

\[
F(x) = \int K(p)\delta \left(\frac{p_0 + p_1}{M} - x\right) d^3p,
\]

then

\[
\int x^n F(x)dx = \int \int K(p)x^n \delta \left(\frac{p_0 + p_1}{M} - x\right) d^3pdx
\]

\[
= \int \int K(p) \left(\frac{p_0 + p_1}{M}\right)^n \delta \left(\frac{p_0 + p_1}{M} - x\right) d^3pdx
\]

\[
= \int K(p) \left(\frac{p_0 + p_1}{M}\right)^n d^3p.
\]
Application of this rule to Eqs. (A7) and (A8) gives after the substitution (50) and with the use of the second and third relation (46):

\[
\int xg_2 dx = -\frac{1}{6M} \int H(p_0) \left(p_0 - \frac{m^2}{p_0}\right) d^3p,
\]

(53)

\[
\int x(g_1 + g_2) dx = \frac{1}{6M} \int H(p_0) \left(p_0 + 2m\right) d^3p.
\]

(54)

These equalities imply relation

\[
\int x(g_1 + 2g_2) dx = \frac{1}{6M} \int H(p_0) \left(2m + \frac{m^2}{p_0}\right) d^3p,
\]

(55)

which in the limit of the massless fermions coincides with the Efremov - Leader - Teryaev (ELT) sum rule [13]:

\[
\int x(g_1 + 2g_2) dx = 0.
\]

(56)

**IV. DISCUSSION**

In the previous sections we have studied the properties of the spin structure functions related to the system of quasifree fermions on mass shell. This system can be compared with the naïve QPM, which is with embedded QCD corrections yet the basic tool for the analysis and interpretation of polarized and unpolarized deep inelastic scattering data. What is the difference between our approach and the naïve QPM, if one speaks about the proton spin structure functions? To simplify this discussion, let us assume:

1) Spin contribution from the sea of quark-antiquark pairs and gluons can be neglected. Then the three fermions in our approach correspond to the three proton valence quarks. So, in this simplified scenario, the proton spin is generated only by the valence quarks.

2) In an accordance with the non-relativistic SU(6) approach the spin contribution of individual valence terms is given as

\[
s_u = \frac{4}{3}, \quad s_d = -\frac{1}{3}.
\]

(57)

Let us point out, in the given context the term valence quarks means nothing else, than the three fermions with defined momenta distribution, charge, mass and polarization.

Then according to the naïve SU(6) version of the QPM we have

\[
g_1(x) = \frac{1}{2} \sum e_j^2 \Delta q_j(x) = \frac{1}{2} \left(\frac{2}{3}\right)^2 \frac{2}{3} u_{\text{val}}(x) - \left(\frac{1}{3}\right)^2 \frac{1}{3} d_{\text{val}}(x),
\]

(58)

corresponding to two the quarks with distribution \(u_{\text{val}}(x)\) and the one with distribution \(d_{\text{val}}(x)\), which are normalized as

\[
\frac{1}{2} \int u_{\text{val}}(x) dx = \int d_{\text{val}}(x) dx = 1.
\]

(59)

It follows, that

\[
\Gamma_1 = \int g_1(x) dx = \frac{5}{18} \approx 0.28.
\]

(60)

This number overestimates more than twice the experimental value. Disagreement is generally interpreted as a contradiction with the assumption, that the proton spin is generated only by spins of the valence quarks.

Now let us calculate the \(\Gamma_1\) in our approach. Let us denote momenta distributions of the valence quarks in the target rest frame by symbols \(h_u\) and \(h_d\) with the normalization
\[ \frac{1}{2} \int h_u(p_0) d^3p = \int h_d(p_0) d^3p = 1. \]  

(61)

These distributions are connected with the \( u_{val}(x) \) and \( d_{val}(x) \) defined above by the relation

\[ q_{val}(x) = \int h_q(p_0) \delta \left( \frac{p_0 \nu + p_1 |q|}{M \nu} - x \right) d^3p. \]  

(62)

The charge weighted distribution \( H \), in an \( SU(6) \) picture, reads

\[ H(p_0) = \sum e_j^2 \Delta h_j(p_0) = \left( \frac{2}{3} \right)^2 \frac{2}{3} h_u(p_0) - \left( \frac{1}{3} \right)^2 \frac{1}{3} h_d(p_0). \]  

(63)

Now, for simplicity let us assume the same shape of the distributions for both the flavours:

\[ \frac{1}{2} h_u(p_0) = h_d(p_0) \equiv h(p_0). \]  

(64)

Then it follows

\[ G(p_0) = 3h(p_0), \quad \Delta G(p_0) = h(p_0), \quad H(p_0) = \frac{5}{9} h(p_0) \]  

(65)

and the relations (65) and (62) can be rewritten:

\[ \langle n\Sigma \rangle = \frac{1}{2} \int h(p_0) \left( \frac{1}{3} + \frac{2m}{3p_0} \right) d^3p, \]  

(66)

\[ \Gamma_1 = \frac{5}{18} \int h(p_0) \left( \frac{1}{3} + \frac{2m}{3p_0} \right) d^3p. \]  

(67)

These relations imply:

a) Because the distribution \( h \) has the defined normalization, the corresponding integrals reach their maximum in the limit, when the fermions are static \( (p_0 = m) \). On the other hand in the limit of massless fermions \( (m \ll p_0) \) these integrals represent only one third of their maximal value. In particular, the \( \Gamma_1 \) satisfies:

\[ \frac{5}{18} \geq \Gamma_1 \geq \frac{5}{54}. \]  

(68)

b) Both the integrals are (up to the factor 5/9) equal. It follows, that in the case of non static fermions the \( \Gamma_1 \) "measures" only the contribution from their spins, which is only part of the their angular momenta, see derivation of the relation (16). Fermions with momentum \( p \neq 0 \), which is not parallel to \( \pm n \), necessarily contribute to the total angular momentum also by some orbital part.

Further let us notice, if we denote

\[ \gamma_{ELT} \equiv \int x (g_1 + 2g_2) dx, \]  

(69)

then Eq. (65) and the third relation (55) imply

\[ \frac{2}{3} m \leq \frac{18}{5} \gamma_{ELT} \cdot M \leq m. \]  

(70)

Why these two very simple approaches for description of the target consisting of the three fermions differ so strongly regarding the prediction \( \Gamma_1 \)? The reason is following. The standard formulation of the QPM is closely connected with the preferred reference system - infinite momentum frame (IMF). The basic relations between the distribution and structure functions like

\[ g_1(x) = \frac{1}{2} \sum e_j^2 \Delta q_j(x), \quad F_2(x) = x \sum e_i^2 q_i(x) \]  

(71)

are derived with the use of approximation.
which seems to be plausible in the IMF. Nevertheless, in the covariant formulation this relation is equivalent to the assumption, that the quarks are static with respect to the proton, since the velocities \( p_j/p_0 \) and \( P_j/P_0 \) are the same. In the proton rest frame it means \( p = 0 \). That is why both the approaches are equivalent for the static quarks but differ for the quarks, which have some intrinsic motion inside the proton. In our approach we do not use assumption (72) and as a result if \( p_\alpha \neq xP_\alpha \) we obtain different relations between the distribution and structure functions. In other words, the fact, that the experimental value \( \Gamma_1 \) is substantially under the value predicted by the naïve QPM in standard formulation, can be in our approach interpreted as a direct consequence of the quark intrinsic motion.

Of course, the approach discussed above concerns the simplified scenario of the quasifree fermions on mass shell. Naïve QPM represents only a first approximation for a description of real nucleon, but the consistent accounting for the quark intrinsic motion as suggested in our approach can, in some aspects, improve this approximation considerably.

Nevertheless, in the realistic case of partons inside the nucleon the situation is still much more delicate. The interaction among the quarks and gluons is very strong, partons themselves are mostly in some shortly living virtual states, is it possible to speak about their mass at all? Strictly speaking probably not. The mass in the exact sense is well defined only for free particles, whereas the partons are never free. However one can assume the following. The relations obtained in the previous sections can be used as a good approximation even for the interacting quarks, but provided that the term mass of quasifree parton is substituted by the term parton effective mass. By this term we mean the mass, which a free parton would have to have to interact with the probing photon equally as the real, bounded one. Intuitively, this mass should correlate to \( Q^2 \): a lower \( Q^2 \) roughly means, that the photon "sees" the quark surrounded by some cloud of gluons and quark-antiquark pairs as a one particle - by which this photon is absorbed. And on contrary, the higher \( Q^2 \) should mediate interaction with more "isolated" quark. Moreover, one should accept that the value of the effective mass can even for a fixed \( Q^2 \) fluctuate. Such phenomenological model was suggested in [13], but unfortunately calculation was based on the form of quark polarization vector which is not correct. Despite of that, the general considerations in mentioned paper can be sensible. Corresponding numeric recalculation with the correct input obtained in the present study for the invariants \( A,B,C,D \) [relations (32),(43)] should be done in a separate paper.

\[ p_\alpha = xP_\alpha, \]

\[ (72) \]

V. SUMMARY AND CONCLUSION

In the present paper we have studied the spin structure functions of the system of quasifree fermions on mass shell and with spherically symmetric distribution of their momenta. The main results can be summarized as follows:

1) Using consistently covariant description of this simple system, we have shown how the structure functions depend on the intrinsic motion of the fermions. In particular, we have suggested, that the momenta \( \Gamma_1 \) corresponding to the two extreme scenarios, of the static (massive) fermions and massless fermions, can differ significantly: \( \Gamma_1(m \ll p_0)/\Gamma_1(p_0 \approx m) = 1/3. \)

2) We have shown, what sum rules follow from the obtained spin structure functions. Further we have shown, how these rules are related to some sum rules well known from the QPM phenomenology.

3) We have done a comparison with the corresponding relations for the structure functions following from the standard formulation of the naïve QPM. Both the approaches are basically equivalent for the static quarks. Differences for quarks with intrinsic motion inside the proton are result of the conflict with the assumption \( p_\alpha = xP_\alpha \), which is crucial for derivation of the relations between structure and distribution functions in the standard QPM.

4) The difference between the experimental value \( \Gamma_1 \) for the proton and the corresponding value expected from the naïve QPM, or at least a part of this difference, can be interpreted as a consequence of the quark motion inside the proton.

**Acknowledgement:** I would like to thank Anatoli Efremov and Oleg Teryaev for many useful discussions and valuable comments.

**APPENDIX A: CALCULATION OF THE INTEGRALS RELATED TO \( G_P, G_S \)**

Integrals in the relations \[ \text{(44), (47)} \] expressed in the target rest frame read

\[ G_P = -\frac{m}{2M^2\nu} \int H(p_0) \]

\[ \text{(A1)} \]
× \left( \frac{\mathbf{p} \mathbf{n}}{p_0 + m} \left[ 1 + \frac{1}{m} \left( \frac{p_0 - \frac{\mathbf{p} \mathbf{q} - (\mathbf{n} \mathbf{p})}{\mathbf{q}^2 \sin^2 \omega} \right) \right] \delta \left( \frac{pq}{M \nu} - x \right) \frac{d^3 p}{p_0} \right),

G_S = \frac{m}{2M \nu} \int H(p_0)

× \left[ 1 + \frac{\mathbf{p} \mathbf{n}}{p_0 + m} \left( \frac{\mathbf{p} \mathbf{n} - \frac{\mathbf{p} \mathbf{q} - (\mathbf{n} \mathbf{p})}{\mathbf{q}^2 \sin^2 \omega} \mathbf{q} \mathbf{\cos \omega} \right) \right] \delta \left( \frac{pq}{M \nu} - x \right) \frac{d^3 p}{p_0},

where \( \cos \omega \equiv \frac{\mathbf{q} \mathbf{n}}{\mathbf{|q|}} \). For integration we use the orthonormal system in which

\[
\mathbf{p} = p_1 \mathbf{e}_1 + p_2 \mathbf{e}_2 + p_3 \mathbf{e}_3, \quad \mathbf{e}_1 = -\frac{\mathbf{q}}{\mathbf{|q|}}, \quad \mathbf{e}_2 = \frac{\mathbf{n} - (\mathbf{n} \mathbf{e}_1) \mathbf{e}_1}{\sqrt{1 - (\mathbf{n} \mathbf{e}_1)^2}}, \quad \mathbf{e}_3 = \mathbf{e}_1 \times \mathbf{e}_2,
\]

so one gets

\[
\mathbf{p} \mathbf{q} = -p_1 \mathbf{|q|}, \quad \mathbf{p} \mathbf{n} = -p_1 \cos \omega + p_2 \sin \omega, \quad \cos \omega = \frac{\mathbf{q} \mathbf{n}}{\mathbf{|q|}}.
\]

After the substitution \( p_2 = p_T \cos \varphi, \ p_3 = p_T \sin \varphi \) and taking into account that the terms proportional to \( \cos \varphi \) disappear, the integrals can be rewritten

\[
G_P = \frac{\cos \omega}{2M^2 \nu} \int H(p_0) \left( p_1 + \frac{\nu}{\mathbf{|q|}} \frac{\mathbf{p}^2 - \mathbf{p}^2_T \cos^2 \varphi}{p_0 + m} \right) \delta \left( \frac{p_0 \nu + p_1 \mathbf{|q|}}{M \nu} - x \right) \frac{p_T dp_T dT d\varphi}{p_0},
\]

\[
G_S = \frac{m}{2M \nu} \int H(p_0) \left( 1 + \frac{\mathbf{p}^2_T \cos^2 \varphi}{m (p_0 + m)} \right) \delta \left( \frac{p_0 \nu + p_1 \mathbf{|q|}}{M \nu} - x \right) \frac{p_T dp_T dT d\varphi}{p_0},
\]

where \( p_0 = \sqrt{m^2 + \mathbf{p}^2_T + p_1^2} \). Integration over \( \varphi \) gives

\[
G_P = \frac{\pi \cos \omega}{M^2 \nu} \int H(p_0) \left( p_1 + \frac{\nu}{\mathbf{|q|}} \frac{\mathbf{p}^2 - \mathbf{p}^2_T / 2}{p_0 + m} \right) \delta \left( \frac{p_0 \nu + p_1 \mathbf{|q|}}{M \nu} - x \right) \frac{p_T dp_T}{p_0},
\]

\[
G_S = \frac{\pi m}{M \nu} \int H(p_0) \left( 1 + \frac{\mathbf{p}^2_T / 2}{m (p_0 + m)} \right) \delta \left( \frac{p_0 \nu + p_1 \mathbf{|q|}}{M \nu} - x \right) \frac{p_T dp_T}{p_0}.
\]

Further, using the relation

\[
\frac{\mathbf{|q|} \nu}{\nu} = \sqrt{1 + 4M^2 x^2 / Q^2}
\]

one can check, that the argument of \( \delta - \) function equals zero for

\[
p_1 = \tilde{p}_1 \equiv \frac{M x - m_\pi^2 / M x}{\sqrt{1 + 4m_\pi^2 / Q^2 + 1 + 4M^2 x^2 / Q^2}}, \quad m_\pi^2 \equiv m^2 + p_T^2.
\]

This is the first root of the corresponding quadratic equation, the second one is excluded, since in the effect of the \( \delta - \) function this root is compatible only with negative energy \( p_0 \). The energy corresponding to the root \( \tilde{p}_1 \) is

\[
p_0 = \tilde{p}_0 \equiv M x - \frac{\tilde{p}_1 \mathbf{|q|}}{\nu} = M x - \tilde{p}_1 \sqrt{1 + 4M^2 x^2 / Q^2}.
\]

Then in an accordance with the rule
\[ \delta(f(x))dx = \sum_j \frac{\delta(x - x_j)}{|f'(x_j)|} dx, \quad f(x_j) = 0 \] (A12)

the \(\delta\)-function in the integrals can be rewritten

\[ \delta \left( \frac{p_0 \nu + p_1 |q|}{M \nu} - x \right) dp_1 = \frac{M \delta(p_1 - \tilde{p}_1) dp_1}{\tilde{p}_1/\tilde{p}_0 + \sqrt{1 + 4M^2x^2/Q^2}} \] (A13)

and afterwards the integrals are simplified

\[ G_P = \frac{\pi \cos \omega}{M \nu} \int_0^{p_{T \text{ max}}} H(\tilde{p}_0) \left( \tilde{p}_1 + \frac{\nu \tilde{p}_1^2 - \tilde{p}_1^2/2}{\tilde{p}_0 + m} \right) \frac{p_T dp_T}{\tilde{p}_1 + \tilde{p}_0 \sqrt{1 + 4M^2x^2/Q^2}}. \] (A14)

\[ G_S = \frac{\pi m}{\nu} \int_0^{p_{T \text{ max}}} H(\tilde{p}_0) \left( 1 + \frac{p_T^2/2}{m(\tilde{p}_0 + m)} \right) \frac{p_T dp_T}{\tilde{p}_1 + \tilde{p}_0 \sqrt{1 + 4M^2x^2/Q^2}}. \] (A15)

where \(\tilde{p}_1\) and \(\tilde{p}_0\) depend on \(p_T\) according to Eqs. (A10) and (A11). For the numeric calculation one should know the upper limit \(p_{T \text{ max}}\) for given \(x, Q^2\) and \(\tilde{p}_{0 \text{ max}}\). After inserting \(\tilde{p}_1\) from Eq. (A10) into Eq. (A11) one gets equation for \(m_T^2\):

\[ \frac{\tilde{p}_{0 \text{ max}} - Mx}{\sqrt{1 + 4M^2x^2/Q^2}} = \frac{Mx - m_T^2/Mx}{\sqrt{1 + 4m_T^2/Q^2} + \sqrt{1 + 4M^2x^2/Q^2}}. \] (A16)

Instead of \(m_T^2\) it is useful to solve this equation first for \(y = \sqrt{1 + 4m_T^2/Q^2}\) obtaining the two roots

\[ y_{\pm} = A \pm \frac{\sqrt{A^2 + 4\alpha(\tilde{p}_{0 \text{ max}} + a)}}{2a}, \quad A = \frac{\tilde{p}_{0 \text{ max}} - Mx}{\sqrt{1 + 4M^2x^2/Q^2}}, \quad a = \frac{Q^2}{4Mx}. \] (A17)

Since \(y_- < 0\), this root is excluded. The second root \(y_+\) after some computation implies

\[ m_{T \text{ max}}^2 = Mx(2\tilde{p}_{0 \text{ max}} - Mx) + \frac{(\tilde{p}_{0 \text{ max}} - Mx)^2}{1 + Q^2/4M^2x^2}, \quad p_{T \text{ max}} = \sqrt{m_{T \text{ max}}^2 - m^2}. \] (A18)

In this way we have the recipe how to calculate the integrals related to the structure functions \(G_P, G_S\) corresponding to the distribution \(H(\tilde{p}_0) d^3p\).

[1] Proceedings of the 9th International Workshop on Deep Inelastic Scattering and QCD - DIS2001, Bologna, Italy, 27Apr.-1May, 2001; (in press).
[2] E.W. Hughes, R. Voss, Annu. Rev. Part. Sci. 49, 303 (1999).
[3] M. Anselmino, A. Efremov, E. Leader, Phys. Rep. 261, 1 (1995).
[4] S.J. Brodsky, Dae Sung Hwang, Bo-Qiang Ma and I. Schmidt, Nucl. Phys. B 593, 311 (2001).
[5] Bo-Qiang Ma and Ivan Schmidt, Phys. Rev. D58, 096008, (1998).
[6] Liang Zuo-tang and R. Rittel, Mod. Phys. Lett. A12, 827 (1997).
[7] Bo-Qiang Ma, J. Phys. G: Nucl. Part. Phys. 17, L53 (1991).
[8] Bo-Qiang Ma and Qi-Ren Zhang, Z. Phys. C 58, 479 (1993).
[9] Bo-Qiang Ma, Phys. Lett. B 375, 320 (1996).
[10] Bo-Qiang Ma, I. Schmidt and J. Soffer, Phys. Lett. B441, 461 (1998).
[11] P. Zavada, Phys. Rev. D55, 4290 (1997).
[12] P. Zavada, Phys. Rev. D56, 5834 (1997).
[13] P. Zavada, Acta Physica Slovaca 49, 2, 273 (1999); updated version in e-print hep-ph/9810540 v2.
[14] J.D. Jackson, G.G. Ross and R.D. Roberts, Phys. Lett. B226, 159 (1989).
[15] H. Burkhardt, W.N. Cottingham, Ann. Phys. 56, 453 (1970).
[16] A.V. Efremov, O.V. Teryaev, E. Leader, Phys. Rev. D55, 4307 (1997).