On Intrinsic Geometric Stability of Controller

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\textbf{Abstract}

This work explores the role of the intrinsic fluctuations in finite parameter controller configurations characterizing an ensemble of arbitrary irregular filter circuits. Our analysis illustrates that the parametric intrinsic geometric description exhibits a set of exact pair correction functions and global correlation volume with and without the variation of the mismatch factor. The present consideration shows that the canonical fluctuations can precisely be depicted without any approximation. The intrinsic geometric notion offers a clear picture of the fluctuating controllers, which as the limit of the ensemble averaging reduce to the specified controller. For the constant mismatch factor controllers, the Gaussian fluctuations over equilibrium basis accomplish a well-defined, non-degenerate, flat regular intrinsic Riemannian surface. An explicit computation further demonstrates that the underlying power correlations involve ordinary summations, even if we consider the variable mismatch factor controllers. Our intrinsic geometric framework describes a definite character to the canonical power fluctuations of the controllers and constitutes a stable design strategy for the parameters.

\textbf{Keywords:} Correlation, Fluctuation, Geometry, Controller, Stability

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1 Introduction

Stable design of controllers is one of the most interesting research issues since the proposition of the robust controllers. Such a principle has been extensively successful, which applies in both the domestic and industrial application \[1\]. This follows because of the fact that it is easy in implementation and requires fewer numbers of parameters. Further, the stably designed controllers are the best alternative to the existing controllers. This follows from the fact that the intrinsic geometric design takes an account of the model uncertainties and specifically allows for a clear-cut determination of the controller settings and parameters. Such an observation is supported from the fact that a class of controllers remains insensitive to the time delay and parametric deviations for the first order systems \[1\].

Global Stability phenomenon are the subject matter of \[2\]. Also, the parametric approach robust controllers have been brought out in the picture with the notions of \[3\]. An important filter design taking an account of the load disturbance is proposed \[4\] in order to improve the speed of the tuning response function. The stability of such a class of controllers depends only on the domain of the controller parameters and so the construction of a nominal plant. In addition, although the system can result in control input saturation, but the stability of the designed controller can be maintained to only depend on the parameters of the intrinsically designed controller and that of the plant, as per the outlines of the Ref. \[5\]. Similar direct model reference adaptive controllers \[6, 7, 8\] are explored in diverse applications.

Parametric model controller design is one of the best methods to improve the robust recital of the controller because of the fact that a polynomial approach is involved in the performance improvement of the controller. The parameters concerning the speed of the controller depend on the model parameters and mismatch factor of the low pass filter circuit. This is because the robustness of the controllers and their performance depend on the model parameters \(\{a, b\}\) and the mismatch factor \(f\) of the filter. In the non-linear domain of the above parameters, our intrinsic geometric analysis provides a stable characterization of the controllers. From the viewpoint of the present interest, Ref. \[9\] leads us to provide an appropriate mathematical design and parameterization for the controllers. The parameterized equation controller block diagram is further brought out into the present attention. From the out-set of the above reference \[9\], we can track the desired trajectory and minimize the plant error. The key issues concerning the controllers are the speed response, low pass filter and geometric model. The robust performance of such a controller is based on these parameters \[10, 11\].

A novel approach is thus made possible in the history of controllers via the present investigation. The method as outlined above is very general in its own and it leaves different possible versions to be explored further. One of the key issues is an appropriate design of the controller circuits. To design the parametric internal model, one traditionally assumes that one of the parameters of the controller is in the subset of parametric family of the controller. It is sometime called finite dimensional parametric model. The filtering operations in these controllers are associated with the parameter involved. The distribution of parameters in the parametric model can be taken to be finite dimensional. Furthermore, the model reference parameterizations are taken into an account \[12\] and the associated parameterizable methods are used to obtain the parametric controllers.

Based on the conventional design method, we offer intrinsic stability analysis of the controllers. The linear parametric polynomial approach which improves structures of the bounded parametric controller integral-derivative (id) designs, is explicitly presented. Improvements in the limiting linear parametric controllers are shown. It turns out that the present intrinsic geometric notion is particularly well suited for the practical applications. The mathematical design procedure thus taken can be extended for any controllers. In fact, we have a clear picture of further investigation. This has been a real bestow for stabilization of the conventional controllers. The present analysis of controller can further be used to explore the intrinsic nature of the unmod-eled part of the plant and the associated bounded disturbances arising from the fluctuations of parameters and mismatch factor.

Intrinsic geometric modelings involving equilibrium configurations of the extremal and the non-extremal black holes in string theory \[13, 14, 15, 16, 17, 18, 19, 20\] and M-theory \[21, 22, 23, 24\] possess rich intrinsic geometric structures \[25, 26, 27, 28, 29\]. There has been much well focused attention on the equilibrium perspective of black holes, and thereby explicates the nature of concerned parametric pair correlations and associated stability of the solutions containing a large number of branes and antibranes. Besides several general notions which have earlier been analyzed in the condensed matter physics \[30, 31, 32, 33, 34, 35\], we consider specific controller configurations thus mentioned with equilibrium parameters and analyze possible parametric pair correlation functions and their correlation relations. Basically, the investigation entails an intriguing feature of the underlying fluctuations which are defined in terms of the parameters.

Given a definite covariant intrinsic geometric description of a consistent controller configuration, one can
expose (i) for what conditions the considered configuration is stable?, (ii) how its parametric correlation functions scale in terms of a set of chosen fluctuating circuit parameters? In this process, one can enlist a complete set of non-trivial parametric correlation functions of the controller configurations \[9\]. It may further be envisaged that similar considerations remain valid over the black hole solutions in general relativity \[30, 37, 38, 39\], attractor black holes \[40, 41, 42, 43, 44, 45\] and Legendre transformed finite parameter chemical configurations \[40, 47\], quantum field theory and the associated Hot QCD backgrounds \[48\]. Thus, the differential geometry plays an important role in the thermodynamic study of the controllers.

In this paper, we analyze the stability of the controllers under the fluctuation of the parameters and the mismatch factor. The controller under consideration is depicted in the Fig.[1]. The stability is demonstrated for a suitable design and its parameterizations. From the general parametrization equation of the controller, the stability of block diagram is drawn into attention. To the best of authors’ knowledge, this approach is made possible for the first time towards the intrinsic geometric stability analysis of controllers. The proposed method is very general and different Legendre transformed versions of the present analysis are possible. By employing the standard notion of the intrinsic Riemannian geometry, the rest of the sections are devoted to the local and global stability properties of fluctuating controllers.

2 Fluctuations in the Controllers

The controller circuit of the present interest is depicted in the Fig.[1]. Although the analysis of the present investigation remains for any controllers, nevertheless we illustrate it for a class of controllers, which are of an immediate interest as shown in the Fig.[1]. Here, \(S\) is a complex signal having a modulus and an angle of phase. In the subsequent analysis, we denote a locally constant signal by the corresponding uppercase notation \(S\). Furthermore, we show that the investigation of the stability analysis is valid for the general controllers, and demonstrate that our approach remains consistent with the other existing ones. Given the controller, we consider the intrinsic geometric stability of the underlying low pass filter system. The parametric stability criterion offers a proficient method to determine the parameters of the circuit and thereby to design the controller as per ones requirement.

Ref \[49\] implements the principle for as associated class of controllers. From a close perspective, such an analysis of the controllers takes an account of the model uncertainties. Specifically, it allows a straightforward relation of controller settings with the associated model parameters. Notice further that the first order consideration of the controllers is insensitive to time delay and parameter deviation, and the output is approximately equal to the PI controllers \[1\]. The response of the controller is sluggish, although it does not have important overshoot effects \[30\] and the integral action of such a controller is used to eliminate the offset of the system. To explore the stability of the first order controllers \[51\], we design stable controllers from the perspective of the intrinsic geometry.

Having mentioned the domains of the parameters, we consider the following two specification of the controllers, viz., constant mismatch factor controllers, and variable mismatch factor controllers. For arbitrary \(n^{th}\) order low pass filter, the controllers of the intrinsic interest are

\[
G_\text{con}(a, b, f) := \frac{(S - a)(S + b)}{(1 + fS)^2(S - a)^2 - (S + b)^2}
\]

(1)

Notice further that the aim of the present paper is to expose the power of the intrinsic geometry. In this concern, the controller described by the Eqn.(1) serves only as an example of the present consideration. Subsequently, our analysis as the exposition of the intrinsic geometric investigation remains valid for any smooth controller and thus the above class of the controllers. In order to begin the subsequent intrinsic geometric analysis of fluctuations, we introduce the correlation in the controller arising from the fluctuations of the circuits parameters. Thus, we consider an ensemble of controllers fluctuating over the limiting Gaussian ensemble. In this analysis, we consider that the controllers can have non-zero fluctuations due to the vibrations of frequencies, residual ripple factors in the filter circuits, and possible other practical uses. This follows from the fact that we do not restrict ourselves in the specific domains of filter circuit used in the controller.

Consequently, we allow an ensemble of limiting configurations with finite fluctuations in an arbitrary non linear domain of the parameters and thereby analyze the nature of a class of generic controllers. We also take an account of the variable mismatch factor defining the speed response tuning parameter of the controller. Notice further that the analysis of the present exposition is valid for all range of the parameters of the controllers. Physically, their deviation from the origin signifies a contribution of the non-linear effects of the controller. Specifically, the values \(a = 0\) and \(b = 0\) of the parameter signify a purely linear model controller. Subsequently,
correlation functions reduce to the set of following expressions:

\begin{align*}
  g_{aa} &= \frac{2(S+b)(S-a)(1+fS)n(3(S-a)^2(1+fS)^n + (S+b)^2)}{(1+fS)^n(S-a)^2 - (S-b)^2)^3} \\
  g_{ab} &= \frac{(S-a)^4(1+fS)^n + 6(S-a)^2(S+b)^2(1+fS)^n + (S+b)^4)}{(1+fS)^n(S-a)^2 - (S-b)^2)^3} \\
  g_{bb} &= \frac{2(S+b)(S-a)(3(1+fS)n(S-a)^2 + (S+b)^2)}{(1+fS)^n(S-a)^2 - (S-b)^2)^3}
\end{align*}

(2)

In this framework, we observe that the geometric nature of parametric pair correlations divulges the notion of fluctuating controllers. Thus, the fluctuating controllers may be easily determined in terms of the intrinsic parameters of the underlying circuit configurations. Moreover, it is evident for a given controller that the principle components of the metric tensor signify self pair correlations, which are positive definite functions over a range of the parameters. Physically, this signifies a set of heat capacities against the intrinsic interactions on the configuration \((M_2(R), g)\) of the controller.

It is worth mentioning that the controllers turn out to be well-behaved for the generic values of the parameters. Over the domain of the circuit parameters \(\{a, b\}\), we notice that the Gaussian correlations form stable correlations, if the determinant of the metric tensor

\begin{equation}
  \text{Det}(g) = -\frac{(1+fS)^n(S-a)^2 + (S+b)^2}{(1+fS)^n(S-a)^2 - (S-b)^2}^4
\end{equation}

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remains a positive function on the parametric surface \((M_2(R), g)\). What follows further is that we specialize ourselves for the physical values of the parameters, and subsequently, we analyze the stability for \(a = 0, b = 0\) corresponding to the linear controllers Fig.[2]. Under such a limiting specification of the parameters, the local correlation functions reduce to the following expressions:

\begin{align*}
  g_{aa} &= \frac{6(1+fS)^{2n} + 2(1+fS)^n}{(1+fS)^n - 1)^3}, \quad g_{bb} = \frac{2(3(1+fS)^n + 1)}{(1+fS)^n - 1)^3} \\
  g_{ab} &= \frac{1(1+fS)^{2n} + 6(1+fS)^n + 1}{(1+fS)^n - 1)^3}
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(5)

3 Constant Mismatch Controllers

Let us first describe the intrinsic stability of the controller with a given mismatch factor. The correlations are described by the Hessian matrix of the controller, defined with a set of desired corrections over a chosen model mismatch factor \(f\) under the tuning response function. Following Eqn.(1), the components of the metric tensor defined as \(Hess(G_{con}(a, b))\) reduce to the following expressions:

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  g_{aa} &= \frac{2(S+b)(S-a)(1+fS)n(3(S-a)^2(1+fS)^n + (S+b)^2)}{(1+fS)^n(S-a)^2 - (S-b)^2)^3} \\
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\end{equation}

(5)
Figure 2: Linear controller as the limiting values of the parameters \( a = 0 \) and \( b = 0 \), described with an intrinsic mismatch factor \( f \).

Figure 3: The determinant of the metric tensor plotted as the function of an intrinsic mismatch factor \( f \) with given parameters \( a \) and \( b \), describing the fluctuations in the controllers.

The behavior of the determinant of the metric tensor shows that such controllers become unstable for specific values of the mismatch factor. For \( S = 1 \) and \( n = 1 \), the nature of the determinant of the metric tensor is depicted in the Fig.[3]. It is worth mentioning further that the constant mismatch controllers become highly unstable in the limit of vanishing mismatch factor. Specifically, the system acquires a throat in the regime of \( |f| \leq 0.7 \).

In order to explain the nature of transformation of the parameters \( \{a, b\} \) forming an intrinsic surface, we examine the functional behavior of the associated Christoffel connections. A direct computation yields that the limiting non-trivial Christoffel connections reduce to the following expressions:

\[
\begin{align*}
\Gamma_{aaa} & = \frac{3}{S^3} \frac{(1 + fS)^{3n} + 6(1 + fS)^{2n} + (1 + fS)^n}{((1 + fS)^n - 1)^4} \\
\Gamma_{aab} & = \frac{1}{S^3} \frac{(1 + fS)^{3n} + 14(1 + fS)^{2n} + 9(1 + fS)^n}{((1 + fS)^n - 1)^4} \\
\Gamma_{aba} & = \frac{1}{S^3} \frac{(1 + fS)^{3n} + 14(1 + fS)^{2n} + 9(1 + fS)^n}{((1 + fS)^n - 1)^4} \\
\Gamma_{abb} & = \frac{1}{S^3} \frac{9(1 + fS)^{2n} + 14(1 + fS)^n + 1}{((1 + fS)^n - 1)^4} \\
\Gamma_{bba} & = \frac{1}{S^3} \frac{9(1 + fS)^{2n} + 14(1 + fS)^n + 1}{((1 + fS)^n - 1)^4} \\
\Gamma_{bbb} & = \frac{3}{S^3} \frac{(1 + fS)^{2n} + 6(1 + fS)^n + 1}{((1 + fS)^n - 1)^4}
\end{align*}
\]

The present investigation shows that a typical controller is globally un-correlated over all possible Gaussian
fluctuations of the parameters \( \{a, b\} \). As the matter of fact, the correlation length of underlying nearly equilibrium system is global characterized by the scalar curvature of \((M_2, g)\). This follows from the fact that the scalar curvature, arising from the definition of the Gaussian fluctuations over the parameters \( \{a, b\} \), vanishes identically for the constant mismatch factor controllers.

For the above type of controllers with a chosen mismatch factor, we see that the Riemann Christoffel curvature tensor vanishes identically over the entire \( \{a, b\} \) surface. Thus, the present intrinsic geometric analysis anticipate that a constant mismatch controller is always a non-interacting system over the surface of fluctuating parameters \( \{a, b\} \).

4 Fluctuating Mismatch Controllers

In the present section, we explore the nature of an ensemble of generic controllers generated with a variable mismatch factor. To consider the most general case, we chose the controller as the function of mismatch factor along with the other system parameters. When the mismatch factor is allowed to fluctuate, we exploit the definition of the Hessian function \( H_{\text{ESS}}(G_{\text{con}}(a, b, f)) \). Following Eqn.(1), we see for the variable mismatch factor controllers a set of interesting properties. It follows that the pure pair correlations \( \{g_{aa}, g_{ab}, g_{bb}\} \) between the parameters \( \{a, b\} \) remain the same as depicted in the Eqn.(2) for the constant mismatch factor controllers. The remaining parametric pair correlations, involving the variation of the mismatch factor \( f \), are given by the following set of equations:

\[
\begin{align*}
g_{af} &= \frac{-nS(S + b)(S - a)^2(1 + fS)^{n-1}\{(1 + fS)^n(S - a)^2 + 3(S + b)^2\}}{(1 + fS)^n(S - a)^2 - (S + b)^2)^3} \\
g_{bf} &= \frac{-nS(S - a)^2(1 + fS)^{n-1}\{(1 + fS)^n(S - a)^2 + 3(S + b)^2\}}{(1 + fS)^n(S - a)^2 - (S + b)^2)^3} \\
g_{ff} &= \frac{nS^2(S - a)^2(S + b)^2(1 + fS)^{n-2}\{(n + 1)(1 + fS)^n(S - a)^2 + (n - 1)(S + b)^2\}}{(1 + fS)^n(S - a)^2 - (S + b)^2)^3}
\end{align*}
\]

We see that the fluctuations of the mismatch factor controller comply physically expected conclusions. In particular, the heat capacities, defined as the self-pair correlations, remain positive quantities for well-defined controllers. A straightforward computation further demonstrate the overall nature of the parametric fluctuations. A variable mismatch factor controller is stable under the set of Gaussian fluctuations, if the associated principle minors \( \{p_2, p_3\} \) remain positive functions on the manifold \((M_3, g)\). Subsequently, an explicit computation shows that the stability constraint on the \( ab \)-surface is given by

\[
p_2 := \frac{-m_0 + m_1(1 + fS)^2n + m_2(1 + fS)^4n}{(1 + fS)^n(S - a)^2 - (S + b)^2)^6}
\]

where the polynomials \( m_i(S) \) are given as

\[

m_0(S) &= S^6 + 4S^5 + 2S^4 + 6S^3 + 5a^3S^2 + 70a^4S + 56a^5S^3 + 28a^6S^2 + 85a^7S + b^8 \\
m_1(S) &= -2a^8 + 8(a - b)aS^7 - 4(3b^2 + 3a^2 - 8ab)S^6 + 8(a^3 - b^3 + 6ab - 6a^2b)bS^5 \\
&\quad - 2(a^4 + b^4 + 36a^2b^2 - 16a^3b - 16b^3a)S^4 + 8(6a^3b^2 - 6a^2b^3 - a^3b + b^7a)S^3 \\
&\quad - 3(4a^4b^3 + 6b^6a^3 - 8b^6a^3(a - b)b - 4b^5a^2 - 2b^6a^4)S^2 - 16a^4b^5S + b^8 \\
m_2(S) &= S^8 - 8a^2S^7 + 28a^6S^6 - 56a^7S^5 + 70a^8S^4 - 56a^9S^3 + 28a^{10}S^2 - 8a^{11}S + a^{12}
\]

The stability constraint on the entire configuration is determined by the determinant of the metric tensor

\[
\det(g) = \frac{(S - a)^3(S + b)nS^2}{(1 + fS)^n(S - a)^2 - (S + b)^2)^3} g_1(a, b, f)
\]

Notice that the co-ordinate charts on \((M_3, g)\) are described by the parameters \( \{a, b\} \) and mismatch factor \( f \) of the controller. In Eqn.(10), the determinant of the metric tensor can have a positive value, if the functions \( g_1(a, b, f) \) defined as

\[
g_1(a, b, f) := (a + 1)h_1(S)(1 + fS)^n + (3n - 1)h_2(S)(1 + fS)^2n + (3n + 1)h_3(S)(1 + fS)^3n + (n + 1)h_4(S)(1 + fS)^4n
\]

take a negative value over \((M_3, g)\). In Eqn.(11), it turns out that \( \{h_k(S)\} \) reduce to the following polynomials:

\[

h_1(S) &= S^6 + 20a^3b^3 + 6a^5b + 15S^4b^4 + 15S^4b^2 + 6b^5S + 3b^6 \\
h_2(S) &= S^6 - 2S^6a + 4S^6b - 8S^6ab + 6S^6b^2 + S^6a^2 - 12S^4a^2b - 12S^3ab^2 + 4S^8b^3 + 6S^2a^2b^2 + S^8b^4 - 8S^6ab^5 + 45S^6b^7 - 28ab^6 + a^3b^3 \\
h_3(S) &= S^6 - 4aS^5 + 2S^5b + 6a^2S^4 + S^4b^7 - 8a^3b^3 - 4a^3S^3 + 12a^2S^2b - 4aS^3b^2 + 3Sb^3 + 9S^2b^2 - 6a^3Sb^2 + 2aS^2b + 1a^2b^2 \\
h_4(S) &= S^6 - 6aS^5 + 15a^2S^4 - 20a^3b^3 + 15a^4S^3b - 6S^5a^3 + a^6
\]
It is not difficult to compute an exact expression for the scalar curvature describing the global parametric intrinsic correlations. By defining set of controller functions, we find explicitly that the most general scalar curvature can be presented as

\[
R = -\frac{n}{2D^2} \sum_{k=0}^{5} \frac{\omega_k r_k (1 + f S)^{kn}}{(S + b)(S - a)}
\]  

(13)

where the co-efficients \( \{r_i(a, b)\} \) appearing in the numerator can be written as the following polynomials

\[
r_0 = S^{10} + 10S^9b + 45S^8b^2 + 120S^7b^3 + 210S^6b^4 + 252S^5b^5 + 210b^6S^4 + 45b^7S^3 + 10b^8S + b^{10}
\]

\[
r_1 = -S^{10} + 2(a - 4b)S^9 + (16ab - 28b^2 - a^2)S^8 + (7ab^2 - 7b^3 - a^2b)S^7 + 14(8ab^3 - 54b^4 - 2a^2b^2)S^6 + 28(5ab^4 - 2b^5 - 2a^3b^3)S^5 + 14(8ab^5 - 26b^6 - 5a^2b^4)S^4 + 8(7ab^6 - b^7 - 7a^2b^5)S^3 + (16ab^7 - b^8 - 28ab^6)S^2 + 2ab(b^7 - 4ab^6)S - a^4b^6
\]

\[
r_2 = -S^{10} + (4a - 6b)S^9 + (24ab - 15b^2 - 6a^2)S^8 + (60a^2b - 36a^2b^2 - 20b^3 + 4a^3)S^7 + (80ab^3 - 15b^4 - 24ab^2 - 90a^2b^2 - a^4)S^6 + (60a^3b^2 - 120ab^3b^3 + 60a^4b - 6a^4b - 6b^6)S^5 + (80a^3b^3 - 6^6 - 90a^4b^4 + 24ab^5 - 15a^4b^3)S^4 + (60a^4b^4 + 4ab^6 - 20a^5b^3 - 36a^2b^5)S^3 + (24a^5b^5 - 6a^4b^6 - 15a^6b^4)S^2 + (4a^6b^6 - 6a^4b^7)S - a^6b^6
\]

\[
r_3 = S^{10} + (4b - 6a)S^9 + (6b^2 + 15a^2 - 24ab)S^8 + (4b^3 - 20b^2 + 60a^2b - 36a^2b^2)S^7 + (b^4 - 80a^2b + 90a^2b^2 + 15a^4 - 24ab^3)S^6 + (60a^4b - 120ab^3b^2 + 60a^5b - 6ab^4)S^5 + (a^6 - 24ab^2 - 80a^2b^3 + 90a^4b^2 + 15a^2b^4)S^4 + (4a^2b^5 - 20a^3b^4 + 60a^2b^4 - 36a^4b^2)S^3 + (6a^4b^6 - 24a^5b^5 + 15a^6b^4)S^2 + (4a^4b^8 - 6a^5b^2)S + a^6b^4
\]

\[
r_4 = S^{10} + (2b - 8a)S^9 + (5b^2 - 16ab + 28a^2)S^8 + (56a^2b - 56a^3b^2 - 8ab^2)S^7 + (70a^4 + 28a^2b^2 - 112a^3b)S^6 + (140a^6b - 56a^6b^2 - 56a^7b)S^5 + (28a^8 - 112a^6b + 70a^8b^2)S^4 + (56a^8b - 56a^8b^2 - 8a^7b)S^3 + (a^8 - 16a^7b + 28a^6b^2)S^2 + (2a^8b - 8a^7b)S + a^8b^2
\]

\[
r_5 = S^{10} - 10aS^9 + 45a^2S^8 - 120abS^7 + 210b^2S^6 - 252ab^2S^5 + 210a^3S^4 - 120a^2b^3 + 45a^4S^3 - 10a^5S^2 + a^6S + 10
\]  

(14)

The weights \( \{w_i\} \) occurring in the summation of the numerator of Eqn. [13] are given by the sequence
\[
w_i := \{-6(n - 1), (9n - 1), (10n + 16), (8n - 18), (16n + 18), (n + 1)\}
\]  

(15)

Furthermore, it turns out that \( D(a, b) \) appearing in the denominator of the scalar curvature, Eqn. [13], is expressed as the following function

\[
D = \frac{(1 + f S)^n (2nS^4 + 4n(b - a)S^3 + 2n(b^2 - 4ab + a^2)S^2 + 4nab(a - b)S + 2na^2b^2)}{(1 + f S)^n + (n + 1)S^4 + 4S^3a + 65S^2a^2 - 4Sa^3 + a^4}
\]

(16)

Consequently, we may easily expose the associated important conclusions for the specific considerations of the variable mismatch factor controllers. The global nature of phase transitions can be thus explored over the range of parameters describing the controllers of interest. For the limiting linear controllers corresponding to the values \( a = 0, b = 0 \), the limiting intrinsic scalar curvature simplifies to the following ratio of series

\[
R = \frac{\sum_{k=0}^{5} t_k (1 + f S)^{kn}}{(n + 1)(1 + f S)^{2n} + 2n(1 + f S)^n + n - 1)^2}
\]  

(17)

where \( t_k(n) \) are defined by the following sequence
\[
t_k := \{3n(n - 1), 9, 2(n(n - 1), n(5n + 8), -n(4n - 9), -n(8n + 9), -\frac{1}{2}n(n + 1)\}
\]  

(18)
Figure 4: The curvature scalar plotted as the function of a variable mismatch factor $f$ and $S$ with given parameters $a$ and $b$, describing the fluctuations in the controllers.

Figure 5: The determinant of the metric tensor plotted as the function of a variable mismatch factor $f$ and $S$ with given parameters $a$ and $b$, describing the stability of the controllers.

Figure 6: The determinant of the metric tensor plotted as the function of a variable mismatch factor $f$ with given parameters $a$ and $b$, describing the fluctuations in the linear controllers.
Figure 7: The curvature scalar plotted as the function of a variable mismatch factor $f$ with given parameters $a$ and $b$, describing the fluctuations in the linear controllers.

Eqn. (17) shows that the global interactions exist for the limiting linear values of the variable mismatch factor controllers. This follows from the fact that the coefficients of the scalar curvature Eqn. (17), signifying the global correlation volume of controller, remain non-zero in the linear limit. In the limit of $n = 1$, the Eqn. (17) shows further that the scalar curvature diverges on the $fS$-surface of the root of $(2 + fS)^2$.

The graphical views of the curvature scalar of the variable mismatch factor controllers are depicted in the two and three dimensions. The intrinsic characterization offered in (i) Fig.[4] is over the three dimensions and (ii) Fig.[7] is over the two dimensions. This describes the precise global behavior of the parametric fluctuations over the entire intrinsic manifold $(M_3, g)$ for the limiting linear variable mismatch controllers Fig.[2].

For the limiting linear controllers, it is worth mentioning that the local pair correlations, as the components of the metric tensor, have an expected behavior. The pure pair correlations reduce as the Eqn.(4). The others involving a variation of the mismatch factor reduce to the following equations:

$$
g_{af} = -n \frac{(1 + fS)^{2n-1} + 3(1 + fS)^{n-1}}{(1 + fS)^{n-1} - 1)^3}, \quad g_{sf} = -n \frac{(1 + fS)^{2n-1} + 3(1 + fS)^{n-1}}{(1 + fS)^{n-1} - 1)^3}\n$$

$$
g_{ff} = S^2 n \frac{(1 + fS)^{2n-2(n+1)} + (1 + fS)^{n-2(n+1)}}{(1 + fS)^{n-1} - 1)^3} \tag{19}\n$$

In the case of the limiting linear controllers, we observe further that the determinant of the metric tensor reduces to the following expression:

$$
\text{Det}(g) = -nS^{-2}(1 + fS)^{n-2}((1 + fS)^n - 1)^{-7} \left( (n+1)(1 + fS)^n + (3n+1)(1 + fS)^{2n} + (3n-1)(1 + fS)^n + (n-1) \right) \tag{20}\n$$

It is important to notice that the global stability of the controllers may be determined by observing the sign of the determinant of the metric tensor. For $n = 1$, we find that the determinant of the metric tensor reduces to the following expression:

$$
\text{Det}(g) = - \frac{2S}{3^2} \frac{(1 + fS)^2 + 2(1 + fS)^3 + (1 + fS)^4}{(1 + fS)^2((1 + fS)^2 - 1)^2} \tag{21}\n$$

The behavior of the determinant of the metric tensor shows that the variable mismatch factor controller becomes unstable for the specific values $|f| \leq 1.2$ of the associated mismatch factor. For the general value the $S$, the nature of the determinant of the metric tensor is depicted in the Fig.[5]. For $S = 1$ and $n = 1$, the corresponding surface nature of the determinant of the metric tensor is depicted in the Fig.[6].

In contrast to the constant mismatch factor controllers, we notice in the present section that the determinant of the metric tensor reduces to the cusp form in the regime of $f \to 0$. In this domain of the mismatch factor,
we observe that the variable mismatch factor controllers are relatively less stable than the constant mismatch factor controllers. The corresponding surface behavior of the scalar curvature is depicted in the Fig.[7]. This describes the global phase properties of the variable mismatch factor controllers.

It is shown that the non-zero value of intrinsic scalar curvature further demonstrate the existence of a finite correlation volume. Specifically, the phase stability of typical controllers with a variable mismatch factor can thus be easily determined by analyzing the nature of the scalar curvature in the domain of interest. This has been depicted in the Figs.[4, 7], in which we show the global nature of the parametric correlations.

The further observation of the Fig.[4] shows that the variable mismatch factor controller systems have no phase transitions on the parametric manifold $(M_3, g)$. Subsequently, the global nature of variable mismatch factor controllers is well explicable against the local fluctuations of the model parameters $\{a, b\}$ and mismatch factor $f$ of the controller.

5 Conclusion

The intrinsic geometric design of controllers is offered under the fluctuations of the model parameters and mismatch factor. Such fluctuations are expected to arise due to non-zero heating effects, chemical reactions and possible conventional corruptions associated with the controller under the application. The intrinsic geometric method is used to improve the structure of the bounded parametric controller id thus designed. It is pictorially presented for the limiting polynomial approach corresponding to the limiting linear parametrization. The stability analysis thus introduced is most generic for the fluctuations of the parameter and the mismatch factor the controllers.

The present analysis is well suited for practical applications. The intrinsic geometric design procedure is presented for the controllers with a (i) constant and (ii) variable mismatch factor. In this concern, the Fig.(3) and Fig.(6) show the respective determinants of the metric tensor for the constant and variable mismatch factor controllers. These figures illustrate that the typical instability appears as (i) a throat for the constant mismatch factor controllers and (ii) a cusp for the variable mismatch factor controllers. Subsequently, a straightforward comparison may be made between the stability properties of the constant and the variable mismatch factor controllers. In the first case, it turns out that the associated controllers correspond to a non-interacting system, while in the second case such a controller configuration corresponds to an interacting system. This follows from the fact that the manifold of parameters is flat in the first case, while it becomes curved for the variable mismatch factor controllers.

In the limit of $f \rightarrow 0$, we have shown in the first case that the determinant of the metric tensor acquires a throat, whereas the determinant of the metric tensor of the variable mismatch factor controllers acquires a cusp in this limit. Thus, the present investigation predicts that the controller systems with the constant mismatch factor are relatively more stable and better-behaved than those with the variable mismatch factor. In addition, our model is well suited for the robust controllers. Such controllers are very popular now a days, because of their high performance and low maintenance needs. From the commercial viewpoints, such a robust controller is very lucrative. It is worth mentioning that the use of the intrinsic geometric principle is rapidly growing in robust controller design in recent years.

Based on the definition of the controller, the intrinsic stability analysis remains compatible for parametrically stable designs of the controller and their modern appliances. The present analysis thus provides the intrinsic geometric front to the stability analysis of existing controllers and their possible future generations. It may be also used, in order to model in a suitable fashion the un-modeled part of the plant and the bounded disturbances. Finally, it is envisaged that our analysis offers perspective stability grounds, when applied to the electrical plants. It is expected further that the present investigation would be an important factor in an appropriate design of the safety guards, which can work as the indicators under fluctuations of the parameters, mismatch factor and the other possible components.

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