Interacting Strings in Matrix String Theory

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Abstract: It is here explained how the Green-Schwarz superstring theory arises from Matrix String Theory. This is obtained as the strong YM-coupling limit of the theory expanded around its BPS instantonic configurations, via the identification of the interacting string diagram with the spectral curve of the relevant configuration. Both the GS action and the perturbative weight $g_s^{-\chi}$, where $\chi$ is the Euler characteristic of the world-sheet surface and $g_s$ the string coupling, are obtained.

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1 Introduction

During the last few years it has been developed a new approach to string theory. This new approach arose to embody the dream of a non perturbative formulation of string theory. This theory has been called M-theory \([1]\), where however M, by now, still stands for Moon. In fact, notwithstanding the great activity in this direction, the full structure of M-theory remains largely elusive. The major proposal for its definition has been given in \([2]\) where it has been conjectured to be a quantum mechanical system of \(9 + 1\) bosonic matrices plus a fermionic counterpart which carries the model to have a dimension 16 (nonlinear) supersymmetry. In this formulation the dimension of the matrices is to be taken to infinity to generate extended objects (M-branes) and the resulting theory should describe M-theory in the infinite momentum frame. A possible way to confirm this hypothesis is to understand if it does really reproduce, in the appropriate corners of the moduli space, the known features of perturbative string theories. The matrix string theory program \([3, 4]\) \(^1\) has been formulated in the conjectured neighborhood of the type IIA superstring theory. Here the theory is realized as an \(U(N)\) SYM with \((8,8)\) supersymmetry on a cylinder and the thesis is that its strong coupling limit should describe type IIA with \(g_s \sim g_{YM}^{-1}\).

For what concerns the free theory reproduction, things are not so difficult to realize. The situation is as follows: consider the unique limit

\[
\begin{align*}
\left( \begin{array}{c}
U(N) \\
SYM \\
\mathcal{N} = (8, 8)
\end{array} \right) & \xrightarrow{g_{YM} \to \infty} \left( \begin{array}{c}
(R^8)^N/S_N \\
CFT \\
\mathcal{N} = (8, 8)
\end{array} \right)
\end{align*}
\]

Due to the fact that there is no interacting realization of the \(D = 2, (8, 8)\)-superconformal algebra, the IR CFT is forced to be the free theory twisted by the \(U(N)\)-Weyl group. Moreover, the orbifold sectors of the theory, which are identified with \(S_N\) classes \([g] = (1)^{n_1} \cdot (2)^{n_2} \cdots \cdot (N)^{n_N}\), where \(N = \sum_{a=1}^{N} a n_{a}\), get a natural string interpretation as states composed of \(\sum_a a\) free strings each of length \(n_a\). This length is then identified with the discretized light-cone momentum in appropriate units. In \([4]\) there is also a step forward in the direction of understanding the interacting perturbative string regime. The starting point is the observation that in the CFT the string states (the orbifold sectors) are orthogonal. Therefore, to let strings interact, one should exit the conformal point with some vertex. The conjectured DVV vertex is essentially the Mandelstam string vertex \([5]\) and properly generates the

\[^1\]Let me thank T. Banks and L. Motl for pointing me out reference \([8]\) which was lacking in the previous version of this proceeding report.
superstring perturbative expansion in the light-cone (see also R. Dijkgraaf’s lecture in this volume).

The problem we want to tackle here is how and where to find superstring interaction in the very structure of the SYM theory. We will start with looking for (punctured) Riemann Surfaces in the theory: if they would be found, then these surfaces should be identified with the interpolating world-sheet between different string states. Before entering in any detail, let us try to give some reasonable form in which interpolating surfaces could appear. As we saw above, the relevant string theory should arise already in the light-cone gauge. Therefore, one is led to look for the relative Mandelstam diagrams which are, as it has been fully explained in [6], representable as branched coverings of a cylinder. Let us first review very quickly, as a preliminary point, the basics of how Riemann curves can be represented as branched coverings.

Let \( z \) be a coordinate on a connected set \( \mathcal{A} \subset \mathbb{CP}^1 \) and let \( a_i(z), i = 0, \ldots, N - 1 \), be analytic functions on \( \mathcal{A} \). Let also \( x \in \mathbb{CP}^1 \) be an indeterminate variable. Consider the curve \( \Sigma \) in \( \mathcal{A} \times \mathbb{CP}^1 \) defined by the polynomial equation

\[
P(x) = x^N + \sum_{i=0}^{N-1} a_i(z)x^i = \prod_{k=1}^{N} (x - x_k(z)) = 0
\]

and notice that, for generic \( a_i \), the root functions \( x_k(z) \) are not one-valued functions on \( \mathcal{A} \). In fact they can exchange by continuing along paths encircling points where two (or more) of them coincide. These points are called the branching points of the covering. The covering structure of \( \Sigma \) is given in terms of the copies \( \mathcal{A}_k = \text{Im} (\mathcal{A}, x_k) \) in the following way: on each copy coherently give a cuts system connecting the branching points and possibly the boundary \( \partial \mathcal{A}_k \), then glue them together along the cuts in the way dictated by the exchange in the roots set to get the surface. In the following, \( \mathcal{A} \) will be taken to be \( \mathbb{C} - \{0\} \), which is an infinite cylinder \( \mathcal{C} \).

The following results have been obtained in collaboration with L. Bonora and F. Nesti [8, 10].
2 (4,4) preserving instantons and the strong coupling expansion of the partition function around them

What we are going to show in the rest of the talk is how the above program can be realized if one looks at the instantonic sector of the theory. More precisely, we will find a rich mathematical structure in that sector and perform a full stringy interpretation of the strong coupling expansion of the theory around the generic instantonic configuration. The outcome will be the Green–Schwarz IIA superstring partition function.

2.1 (4,4) instantons

As a first step, let us explain the emergence of the relevant Riemann surfaces from the instanton equations. The bulk action of the theory is

$$S = \frac{1}{\pi} \int_C d^2w \, \text{Tr} \left( D_w X^i D_{\bar{w}} X^i - \frac{1}{4g^2} F_{w\bar{w}}^2 - \frac{g^2}{2} [X^i, X^j]^2 + i(\theta_s D_{\bar{w}} \theta_s + \theta_c D_w \theta_c) + 2ig\theta_s \gamma_l [X^i, \theta_c] \right)$$

where $(X^i, \theta_s, \theta_c)$s are in the adjoint w.r.t. the $U(N)$ gauge group and in the $(8_v, 8_s, 8_c)$ of the $SO(8)$ R-symmetry group respectively. The gauge connection is of course an R-singlet. This action is invariant under the following $\mathcal{N} = (8, 8)$-supersymmetry transformations

$$\delta X^i = \frac{i}{g}(\epsilon_s \gamma^j \theta_c + \epsilon_c \tilde{\gamma}^j \theta_s), \quad \delta A_w = -2\epsilon_s \theta_s, \quad \delta A_{\bar{w}} = -2\epsilon_c \theta_c$$

$$\delta \theta_s = (-\frac{i}{2g^2} F_{w\bar{w}} + \frac{1}{2} [X^i, X^j] \gamma_{ij}) \epsilon_s - \frac{1}{g} D_w X^i \gamma_l \epsilon_c$$

$$\delta \theta_c = (-\frac{i}{2g^2} F_{w\bar{w}} + \frac{1}{2} [X^i, X^j] \tilde{\gamma}_{ij}) \epsilon_c - \frac{1}{g} D_{\bar{w}} X^i \tilde{\gamma}_l \epsilon_s$$

There exists a full class of (4,4)-susy preserving classical configurations [7, 8]: $\theta_s = 0, \theta_c = 0, X^i = 0$ for $i = 3, \ldots, 8$ while $X = X^1 + iX^2$ and the connection $A$ satisfy the Hitchin system [3]

$$F_{w\bar{w}} + ig^2 [X, \bar{X}] = 0, \quad D_w X = 0, \quad D_{\bar{w}} \bar{X} = 0.$$ (1)
This system is known to be integrable in terms of spectral curves. These spectral curves are what we were looking for. To see them explicitly, parametrize with full generality the above fields as

\[ X = Y^{-1}MY \quad \text{and} \quad A_w = -iY^{-1}\partial_wY, \quad Y \in SL(N, \mathbb{C}) \]  

(2)

where \( Y \) and \( M \) are still well defined fields on the cylinder.

As for \( M \), it satisfies \( \partial_w M = 0 \) without any further restriction. Consider now the polynomial

\[ P_X(x) = \text{Det}(x - X) = \text{Det}(x - M) = x^N + \sum_{i=0}^{N-1} x^i a_i, \]

where \( x \) is a complex indeterminate. Since \( \partial_w M = 0 \), we have \( \partial_w a_i = 0 \) which means that the set of functions \( \{a_i\} \) are antianalytic on the cylinder. Therefore the equation

\[ P_X(x) = 0 \]  

(3)

identifies in the \( (w, x) \) space a Riemann surface \( \Sigma \), which is an \( N \)-sheeted branched covering of the cylinder \( \mathcal{C} \). We can choose \( M \) to be in a standard form as

\[ M = \begin{pmatrix}
-a_{N-1} & -a_{N-2} & \ldots & \ldots & -a_0 \\
1 & 0 & \ldots & \ldots & 0 \\
0 & 1 & 0 & \ldots & 0 \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
0 & 0 & \ldots & 1 & 0
\end{pmatrix} \]

Notice that the branched covering structure is completely encoded in \( M \) and is independent on the value of the coupling. This is the rigid part of the \((4,4)\)-preserving instanton under the coupling flow.

As for \( Y \), it contains all the dependence on the value of the coupling and is determined by the following deformed WZNW equation

\[ \partial_w \left( \partial_w \Omega \Omega^{-1} \right) + g^2 \left[ M, \Omega M^+ \Omega^{-1} \right] = 0, \quad \text{where} \quad \Omega \equiv YY^+. \]  

(4)

The calculation we are going to perform is a strong coupling expansion of the partition function. As a necessary step, we need to understand what the fate is of our instantons in this strong coupling limit [10].

As it is immediate from the Hitchin system equations [II], at strong coupling we have

\[ [X, \bar{X}] = 0 \quad \Rightarrow \quad X = U \hat{X} U^+, \quad U \in U(N) \]

\[ \hat{X} = \text{diag}(x_1, \ldots, x_N) \]
where \( x_i \) are the roots of (3). On the other hand we parametrized the solutions as (2), so we have

\[ X = Y_s^{-1} MY_s, \]

where \( Y_s \) is \( Y \) at \( g \sim \infty \).

Therefore, diagonalizing \( M = S \hat{X} S^{-1} \) with \( S_{ij} \equiv (x_j)^{N-i} \), we get \( Y_s = SU^+ \) and \( \Omega_s = Y_s Y_s^+ = SS^+ \) satisfies, coherently with (4), \( \partial_w (\partial_w \Omega_s \Omega_s^{-1}) = 0 \).

Summarizing, at strong coupling the instantonic configuration is

\[ A_w = -iU \partial_w U^+ \quad \text{and} \quad X = U \hat{X} U^+ \quad (5) \]

Notice that along fixed time curves on the cylinder \( \text{Re} \, w = T \) we get

\[ \hat{X} \to P_T \cdot \hat{X} \cdot P_T \quad \text{and} \quad U \to U \cdot P_T \]

with \( P_T \in S_N \) describing the intermediate string state at time \( T \) in the way described at the beginning of the talk. Moreover, the unitary field \( U \) defines a Cartan subalgebra \( t = U t_d U^+ \), where \( t_d \) is the diagonal one. This will be the Cartan subalgebra we will choose to split the fields in the strong coupling limit expansion of the theory.

### 2.2 Expanding the action functional

We are now going and face the problem of performing the strong coupling expansion of the action. Let us write the bulk action of the theory as \[ S = \frac{1}{\pi} \int_C d^2 w \text{Tr} \left( D_w X^I D_w X^I - \frac{g^2}{2} [X^I, X^J]^2 - g^2 [X^I, X] [X^J, \bar{X}] + \right. \]

\[ + D_w X D_w \bar{X} - \frac{1}{4g^2} \left( F_{w\bar{w}} + ig^2 [X, \bar{X}] \right)^2 + i \left( \theta_s^+ D_w \theta_s^- + \theta_c^+ D_w \theta_c^- + i g \theta^T \Gamma_i [X^i, \theta] \right) \]

where \( I = 3, 4, ..., 8 \). To perform the expansion around any given instanton, write any field \( \Phi \) as

\[ \Phi = \Phi^{(b)} + \phi^v + \phi^n \equiv \Phi^{(b)} + \phi^v + \phi^n, \]

\[ \text{notice that the following holds up to boundary terms. These terms are inessential for determining the bulk structure of the theory at strong coupling. Nevertheless, they could become very interesting once one would like to have a full control of the theory at the boundary of the cylinder. In such a refined analysis, one should start from the beginning with a boundary control on the form of the action to start with.} \]
where $\Phi^{(b)}$ is the background value of the field (which is $\Phi$), $\phi^c$ are the fluctuations along the Cartan directions and $\phi^n$ are the fluctuations along the non–Cartan directions.

It is appropriate at this point to fix the gauge of the theory in the following way

$$G_w \bar{G}_{\bar{w}} = D^\circ w \bar{a}_w + D^\circ \bar{w} a_w + ig^2 ([X^\circ, \bar{x}] + [\bar{X}^\circ, x]) + 2ig^2 [X^\circ, x'] = 0,$$

and to apply the Faddeev–Popov procedure by adding to the action $S_{FP} = S_{gf} + S_{ghost} = \frac{1}{4\pi g^2} \int d^2 w \left[ G_w \bar{G}_{\bar{w}} - \frac{1}{2\pi g^2} \int d^2 w \frac{\delta G_w \bar{G}_{\bar{w}}}{\delta c} c \right],$

where $\delta$ represents the gauge transformation with parameter $c$. We get a total action

$$S_{tot.} = S + S_{FP}$$

To extract the leading terms of the action, rescale fields as

$$A_w = A_w^{(b)} + g a^w, \quad X = X^{(b)} + x^w + \frac{1}{g} x_a^w, \quad X^I = x^I_w + \frac{1}{g} x^{Ia}_w,$$

$$\theta = \theta^w + \frac{1}{\sqrt{g}} \theta^n, \quad c = g c^w + \sqrt{g} c^n, \quad \bar{c} = g \bar{c}^w + \frac{1}{\sqrt{g}} \bar{c}^n$$

It is important to notice that these rescalings induce a unit Jacobian in the path integral measure of the non–zero modes, but they may produce a non-trivial factor due to the presence of zero modes. After the above rescalings the action becomes

$$S = S_{sc} + Q_n + o \left( \frac{1}{\sqrt{g}} \right),$$

where

$$S_{sc} = \frac{1}{\pi} \int \mathcal{D}^2 \theta \text{Tr} \left[ D_w^{(b)} x^I \bar{x}^I D_w^{(b)} x^I \bar{x}^I + D_w^{(b)} x^w \bar{a}_w + D_w^{(b)} \bar{a}_w + i(\theta^w, \theta^c) A \left( \frac{\partial^w}{\partial^c} \right) \right]$$

and $Q_n$ is a quadratic term in $\phi^n$.

Let us now show that the integration along non-Cartan directions does not contribute to the effective action. The exact expression for $Q_n$ is

$$Q_n = \frac{1}{\pi} \int d^2 w \text{Tr} \left[ x^a^w q^a x^a + x^I_n q^I x^I_n + a^a w q a^a_w + \bar{c}^a q c^a + i(\theta^a_n, \theta^c_n) A \left( \frac{\partial^a}{\partial^c} \right) \right],$$

where
\[ Q = \text{ad}_{X^{i\circ}} \cdot \text{ad}_{X^{i\circ}} + \text{ad}_{a_t^w} \cdot \text{ad}_{a_t^w} \quad \text{and} \quad A = \begin{pmatrix} i\text{ad}_{a_t^w} & \gamma_i \text{ad}_{X^{i\circ}} \\ \bar{\gamma}_i \text{ad}_{X^{i\circ}} & i\text{ad}_{a_t^w} \end{pmatrix}. \]

Notice that \( Q \) is a purely algebraic quadratic term in the \( \phi^n \) fluctuations which can be easily integrated over without any zero-mode problem contribution to the path-integral measure. The integration over \( a^n \) and \( c^n \) exactly cancels to 1 and also the integration over \( x^n \) and \( \theta^n \) gives again 1 due to supersymmetry (\( AA^\dagger = A^\dagger A = -Q \)). Summing up the net result of integrating over the non-Cartan modes is 1 and, in the strong coupling limit, we are left with the action \( S_{sc} \) over the Cartan modes.

### 2.3 Lifting the action to the world-sheet

Let us now show that \( S_{sc} \) corresponds to the Green–Schwarz superstring action plus a free Maxwell action on the world-sheet identified with the spectral curve of the relevant background instanton. The free Maxwell sector will be integrated out at the end of the story. The result of the integration along this sector will be a nice expected contribution.

Begin rewriting \( S_{sc} \) in a diagonal representation of the background just undoing the \( U \) rotation relative to the background structure (5). The covariant derivative \( D^{(b)}_w \) becomes the simple derivative \( \partial_w \) and the Cartan subalgebra gets rotated to the diagonal one

\[
S_{sc} = \frac{1}{\pi} \int_C d^2 w \text{Tr} \left[ \partial_w x^{i\circ} \partial_w x^{i\circ} + \partial_w x^{z_d} \partial_w x^{z_d} + i(\theta_s^{z_d} \partial_w \theta_s^{z_d} + \theta_c^{z_d} \partial_w \theta_c^{z_d}) + \right.
\]

\[
\left. + \partial_w a_{\bar{w}}^{r_d} \partial_w a_{\bar{w}}^{r_d} + \partial_w c^{r_d} \partial_w c^{r_d} \right]
\]

Since all the matrices are diagonal we can rewrite this action in terms of the diagonal modes \( \phi^{z_d} = \text{diag} \left( \phi_1, \ldots, \phi_N \right) \)

\[
S_{sc} = \frac{1}{\pi} \int_C d^2 w \sum_{n=1}^N \left[ \partial_w x_{(n)}^i \partial_w x_{(n)}^i + i(\theta_{s(n)} \partial_w \theta_{s(n)} + \theta_{c(n)} \partial_w \theta_{c(n)}) + \right.
\]

\[
\left. + \partial_w a_{\bar{w}(n)} \partial_w a_{\bar{w}(n)} + \partial_w c_{(n)} \partial_w c_{(n)} \right]
\]

As anticipated, the individual components \( \phi_{(i)} \) are not well defined fields on the cylinder and to give a meaning to the theory we must understand if they can be considered to be well defined fields on some other space. To do this, observe that since \( \phi^z = U \phi^{z_d} U^\dagger \) is well-defined on the cylinder and since following once \( \text{Re} w = T, \ U \rightarrow U \cdot P_T \) with \( P_T \in S_N \), then, along \( \text{Re} w = T \), we get \( \phi^{z_d} \rightarrow P_T^+ \cdot \phi^{z_d} \cdot P_T \).
What we want to show now is that these are exactly the properties a
set of fields on a cylinder should have to be the representation of a single
local field on a Riemann surface represented as a branched covering of the
cylinder.

If Σ is a branched covering of the cylinder C, then there exists a projection
map \( \pi : \Sigma \to C \) whose local inverse image is N-valued
\[
\pi^{-1} : w \to (x_1(w), \ldots, x_N(w)),
\]
where \( \{x_i(w)\} \) is the set of the roots of its polynomial equation (3). So, let \( \tilde{\psi} \)
be a local complex field on Σ:
\[
\pi \ast \tilde{\psi} = (\psi(1)(w), \ldots, \psi(N)(w))
\]
represents the
field on each copy of the cylinder C composing the covering Σ and the \( \psi(i)(w) \)’s
are related exactly by the \( P_T \) monodromy along the curves \( \text{Re } w = T \).

From the point of view of Σ, the \( w \) coordinate is locally defined via an
abelian differential \( \omega = dw \) with imaginary periods \( [6] \). This generates the
factors needed to keep into account the differential weights of the various
fields.

All this implies that the field \( \phi^{t_d} \) represents a well-defined field on Σ
when rescaled with the appropriate \( \omega \) factor and we can lift the action to the
Riemann surface Σ obtaining
\[
S_{sc} = S_{\text{GS}} + S_{\text{Maxwell}},
\]
where
\[
S_{\text{GS}} = \frac{1}{\pi} \int_{\Sigma} \frac{1}{2} \left( \partial_{\bar{z}} \bar{a} \partial_{\bar{z}} \bar{a} + i(\tilde{\theta} s \partial_{\bar{z}} \tilde{\theta} s + \tilde{\theta} c \partial_{\bar{z}} \tilde{\theta} c) \right)
\]
\[
S_{\text{Maxwell}} = \frac{1}{\pi} \int_{\Sigma} \left( g_{z\bar{z}} \partial_{z} \bar{a} \partial_{\bar{z}} \bar{a} + \partial_{z} \tilde{c} \partial_{\bar{z}} \tilde{c} \right)
\]
and the metric in the Maxwell term is \( g_{z\bar{z}} = \omega_z \omega_{\bar{z}} \) with \( z \) a system of local
coordinates on Σ.

An expected nice present from the Maxwell sector will be soon received.
To get it, let us integrate over this sector. Since the action is quadratic
the integration produces a ratio of determinants, which turns out to be a
constant (there is no dynamics for a massless vector field in two dimensions),
but we have to take account of the zero modes for the fields that have been
rescaled:
\[
\tilde{a}_{\bar{z}} \to g \tilde{a}_{\bar{z}}, \quad \tilde{a}_{\bar{z}} \to g \tilde{a}_{\bar{z}}, \quad \tilde{c} \to g \tilde{c}, \quad \tilde{\bar{c}} \to g \tilde{\bar{c}}.
\]
The Maxwell partition function is then
\[
Z_{\text{Maxwell}}^\Sigma = \int D[\tilde{a}, \tilde{\bar{c}}] \ e^{-S_{\text{Maxwell}}^\Sigma(\tilde{a}, \tilde{\bar{c}})} \propto \frac{\text{Det}'\nabla_c}{\text{Det}'\nabla_a} \propto g^{8-\xi_a}
\]
where $\nabla$ denotes the relevant laplacian, $'$ means that the zero modes have been excluded from the computation of the regularized determinants and $\mathcal{q}$ is the number of these zero modes.

As for the ghost fields, which are scalars, the only zero modes of the $\nabla_c$ operator on $\Sigma$ is the constant. The zero modes of the Maxwell field correspond instead to the harmonic 1-differentials on $\Sigma$. If $\Sigma$ were a closed Riemann surface of genus $h$, their number would be $h$. But $\Sigma$ is a Riemann surface with boundaries $\partial$ and the counting needs a little trick to be performed. Construct the double $\hat{\Sigma}$ of $\Sigma$ ($\hat{\Sigma} \sim \Sigma \times \mathbb{Z}_2$): $\hat{\Sigma}$ has genus $\hat{h} = 2h + b - 1$ and $\hat{\partial} = 0$ and the number of analytic differential on $\Sigma$ that extend to $\hat{\Sigma}$ (analytic Schottky differentials) is $\hat{h} = 2h + b - 1$. Summing up, we have therefore $\mathcal{q}_c - \mathcal{q}_a = 1 - \hat{h} = 2 - 2h - b = \chi_{\Sigma}$ and we get

$$Z_{\Sigma}^{\text{Maxwell}} \propto \left(\frac{1}{g}\right)^{-\chi_{\Sigma}}.$$

3 The string theory interpretation

Let us recollect the various terms to reconstruct the strong coupling limit of the $(8, 8)$ YM partition function:

$$Z_{sc} \sim \int_{\mathcal{M}_{sc}} dm \ (1/g)^{-\chi} \ [\text{Jac}] \int D [\tilde{x}, \tilde{\theta}] e^{-S_{GS}[\tilde{x}, \tilde{\theta}]}$$

(6)

$\mathcal{M}_{sc}$ is the space of instantons at strong coupling: in this regime each instantonic configuration is determined uniquely by a branched covering of the cylinder $\Sigma$, i.e. by a Mandelstam diagram; $dm$ is the field theory induced measure on $\mathcal{M}_{sc}$: the integral in this sector is split as a sum over $h$ and an integral over $\mathcal{M}_{sc}^h$; in the $(1/g)^{-\chi}$ factor, $\chi = 2 - 2h - b$ is the Euler characteristic of $\Sigma$; the [Jac]-obian factor has been produced by the background dependent field splitting we performed (it depends of course on $\Sigma$) and $S_{GS}$ is the Green-Schwarz superstring action on $\Sigma$.

At this point, looking at (6), one is tempted to say that MST in its strong coupling regime reproduces a discretized version of the perturbative type IIA superstring theory in the light-cone with $g_s \propto 1/g$.

Indeed we proved the above statement up to a couple of technical points: prove that $dm \cdot [\text{Jac}]$ generates the right superstring measure on the moduli space and complete the analysis of the theory at the boundary of the cylinder to reconstruct the boundary terms in the Mandelstam light-cone string.

Higher order terms in the expansion of the partition function should then represent non-perturbative contributions to string theory. One should also
be able to include in the analysis D-branes. Up to D-particles, this seems to be done with a careful sight at the $N \rightarrow \infty$ limit. Are there other relevant subleading saddle-points to consider to get the full theory?

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