Investigating Loop Quantum Gravity with EHT Observational Effects of Rotating Black holes

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ABSTRACT

A mathematically consistent rotating black hole model in Loop Quantum Gravity (LQG) is yet lacking. The scarcity of rotating black hole solutions in LQG substantially hampers the development of testing LQG from observations, e.g., from the Event Horizon Telescope (EHT) observations. The EHT observation revealed event horizon-scale images of the supermassive black holes Sgr A* and M87*. The EHT results are consistent with the shadow of a Kerr black hole of general relativity. We present LQG-motivated rotating black hole spacetimes (LMRBH), which are regular everywhere and asymptotically encompass the Kerr black hole as a particular case. LMRBH metric describes a multi-horizon black hole in the sense that it can admit up to three horizons, such that an extremal LMRBH, unlike Kerr black hole, refers to a black hole with angular momentum $a > M$. The metric, depending on the parameters, describes (1) black holes with only one horizon (BH-I), (2) black holes with an event and Cauchy horizons (BH-II), (3) black holes with three horizons (BH-III) or (4) no-horizon spacetime (NH) which, we show, is almost ruled out by the EHT observations. We constrain the LQG parameter with an aid of the EHT shadow observational results of M87* and Sgr A*, respectively, for an inclination angle of 17° and 50°. In particular, the VLTI bound for the Sgr A*, $\delta \in (-0.17, 0.01)$, constrains the parameters $(a, l)$ such that for $0 < l \leq 0.347851M$ ($l \leq 2 \times 10^6$ Km), the allowed range of $a$ is $(0, 1.0307M)$. Together with EHT bounds of Sgr A* and M87* observables, our analysis concludes that the substantial part of BH-I and BH-II parameter space agrees with the EHT results of M87* and Sgr A*. While EHT M87* results totally rules out the BH-III, but not that by Sgr A*.

Keywords: black hole physics - black hole shadow - gravitational lensing: strong

1. INTRODUCTION

That the gravitational collapse of a massive star ($\geq 3.5M$) leads to a spacetime singularity in general relativity is confirmed by an elegant theorem by Hawking and Penrose (Hawking & Penrose 1970; Hawking & Ellis 1973). However, it is widely believed that these singularities result from a classical treatment of spacetime. By its very definition, the existence of a singularity means spacetime fails to exist, signalling a breakdown of the laws of physics. Thus, singularities must be substituted by some other objects in a more unified theory for these laws to exist and will not be present when quantum effects are considered (Wheeler 1963). While we do not yet have any complete quantum gravity theory, permitting us to dig the interior of the black hole and settle it separately, we must turn our attention to regular models motivated by quantum arguments. The first regular black hole solution was proposed by Bardeen (1968). Bardeen (1968) asserts that although there are horizons, there is no curvature singularity. Instead, black hole center develops a de-Sitter-like region, ultimately known as black hole with a regular centre. Thus, its maximal extension is the one of the Reissner–Nordström spacetime but with a regular centre (Barrabes & Frolov 1996; Bronnikov et al. 2003). Thereafter several regular black hole models have been proposed based on Bardeen’s idea, which mimics the demeanour of the Schwarzschild black hole at large distances (Poisson & Israel 1988; Dymnikova 1992; Barrabes & Frolov 1996; Bronnikov et al. 2003; Hayward 2006; Bronnikov & Fabris 2006; Simpson & Visser 2019). Also, there has been a
significant advance in the analysis and application of regular black holes (Ayon-Beato & Garcia 1998; Bronnikov 2001; Hayward 2006; Zaslavskii 2009; Lemos & Zanchin 2011). There is evidence that the loop quantum gravity (LQG) may be competent to fix the inevitable singularities in classical general relativity (Ashtekar et al. 2006, 2007; Vandersloot 2007). Because of the inherent problems in solving the complete system, the emphasis has been on spherically symmetric black holes (Ashtekar & Bojowald 2006; Modesto 2006; Boehmer & Vandersloot 2007; Campiglia et al. 2008; Gambini & Pullin 2008).

The phase space quantization or semiclassical polymerization that maintains aspects of the discreteness of underlying spacetime suggested by the LQG turns out to be a fruitful technique to resolve the singularity issue, and has been used recently to significant effect (Boehmer & Vandersloot 2007; Campiglia et al. 2008; Gambini & Pullin 2008). Different polymerizations can give qualitatively other regularized spacetimes, so it is of great interest to examine a more comprehensive class of models and methods. Peltola and Kunstatter (2009a), motivated by earlier works (Ashtekar & Bojowald 2006; Modesto 2006, 2010, 2008; Boehmer & Vandersloot 2007, 2008; Campiglia et al. 2008; Gambini & Pullin 2008), used the effective field theory and the partially polymerized theory arguments to determine a static and spherically symmetric regular black hole (Peltola & Kunstatter 2009b,a) that is asymptotically flat. One of the most striking features of this quantum-corrected black hole, unlike other regular black holes that have two horizons, is that it has a single horizon and it also encompasses Schwarzschild black hole.

The no-hair theorem embodies the unique qualities of the GR black hole, stating that Kerr (1963a) is the only stationary, axially symmetric, and asymptotically flat vacuum solution to the Einstein Field equations (Israel 1967, 1968; Carter 1971; Hawking 1972; Robinson 1975). No-hair theorem suggests that astrophysical black hole candidates are Kerr black hole, but it still lacks definitive proof, and its exact nature is not yet verified. It opens an arena for investigating properties of black holes that differ from Kerr’s black hole.

Also, the spherical black hole cannot be sampled by astrophysical observations as rotating black holes are typically found in nature. The black hole spin plays a fundamental role in any astrophysical process. Further, the lack of rotating black hole models in LQG hinders the progress of testing LQG theory from observations. It encouraged us to consider rotating or axisymmetric generalization of the spherical metric (Peltola & Kunstatter 2009a) recently obtained by Walia (2022). It is a Kerr-like metric, derived through the revised Newman-Janis algorithm, and hereafter called as the LQG-inspired rotating black hole (LMRBH) that is suitable to test with astrophysical observations. Also, it turns out that when applied to other models in LQG, the revised Newman-Janis works quite well in generating rotating metrics starting with their non-rotating seed metrics (Brahma et al. 2021; Liu et al. 2020; Chen 2022).

We also show that it is feasible, in principle, to constrain the LQG parameter $l$ using the Event Horizon Telescope (EHT) observed shadows of the M87* and Sgr A* black holes. In precise, we find that the effects made by the parameter $l$ on the shadow size are more noteworthy than those on the deviation of circularity of the shadow silhouette. The EHT collaboration released the first horizon-scale image of the M87* black hole in 2019 (Akiyama et al. 2019a,b,c,d,e,f). Using a distance $d = 16.8$ Mpc and estimated mass of M87* $M = (6.5 \pm 0.7) \times 10^9 M_\odot$ (Akiyama et al. 2019a,d,b), the EHT results have bounds on the compact emission region size with angular diameter $\theta_d = 42 \pm 3 \mu$as and circularity deviation $\Delta C \lesssim 0.1$ along with the central flux depression with a factor of $\gtrsim 10$ -identified as the shadow. The observed shadow image of the M87* black hole is compatible with the Kerr black hole as predicted by the GR. However the present uncertainty in the measurement of spin and the relative deviation of quadrupole moments do not eliminate modification of Kerr black holes (Akiyama et al. 2019a,d,b; Cardoso & Pani 2019). In 2022, the EHT collaboration released the shadow results of black hole Sgr A* in the Milky Way depicting shadow angular diameter $d_{sh} = 48.7 \pm 7 \mu$as and thick emission ring of diameter $\theta_d = 51.8 \pm 2.3\mu$as; considering a black hole of mass $M = 4.0^{+1.1}_{-0.6} \times 10^6 M_\odot$ and distance 8$kpc$ from the Earth, the EHT images of Sgr A* are agreeing with the expected appearance of a Kerr black hole (Akiyama et al. 2022a,b,c,d,e,f). Furthermore, when compared with the EHT results for M87*, it exhibits consistency with the predictions of GR pushing across three orders of magnitude in central mass (Akiyama et al. 2022e).

The remainder of the paper is structured as follows. In Sect. 2, we cover the geometric properties of the rotating polymerized black hole and the conformal diagrams. In Sect. 3, we discuss the polymerized black hole shadows. Considering the observer at the equatorial plane, the black hole shadow observables and their applications in estimating the LQG
2. LQG MOTIVATED POLYMERIZED BLACK HOLE

A static and spherically symmetric black hole model in four-dimensional partially polymerized theory reads as (Peltola & Kunstatter 2009b,a)

\[
\begin{align*}
    ds^2 &= \left(\sqrt{1 - \frac{l^2}{y^2} - \frac{2M}{y}}\right) dt^2 - \left(\frac{1 - \frac{l^2}{y^2}}{\sqrt{1 - \frac{l^2}{y^2} - \frac{2M}{y}}}\right) dy^2 \\
    &\quad - y^2 (d\theta^2 + \sin^2 \theta d\phi^2),
\end{align*}
\]

with \( M \) being the black hole mass and \( l \) is the bounce radius. Metric (1) has an unique horizon at \( y_H = \sqrt{4M^2 + l^2} \) that demonstrates a quantum correction as a result of polymerization. In limited proper time, the solution evolves from the horizon at \( y_H \) to the smallest radius \( l \), and then expands in infinite proper time to \( y = \infty \) where the solution is also asymptotically flat. All across, the Ricci and Kretschmann scalars are finite, non-singular and rapidly dissipate away from the black hole. The polymerized black hole metric (1) thus characterizes a globally regular spacetime in a way that allows a sphere of radius \( y = l \) to replace the curvature singularity at \( y = 0 \) and bounce into an infinitely expanding Kantowski-Sachs spacetime. The solution although do violate the classical energy requirements, according to a calculation of the Einstein tensor, but the violations are of order \( l^2/y^4 \) and thus diminish far beyond the bounce radius \( l \) (Kumar Walia 2022). This implies that the Schwarzschild solution which the metric (1) recovers in the limit \( l \to 0 \), is well approximated everywhere far outside the horizon, and that the exterior is equipped with nonzero quantum stress energy that is statistically insignificant for macroscopic black holes \((y_H \gg l)\). Additionally, the radial coordinate \( y \) admits a minimum value of \( y = l \), and the 2-sphere at the center is known as the black bounce, whose geometries are represented by solutions to the Einstein equations with phantom scalar fields (Bronnikov & Fabris 2006; Bronnikov et al. 2007; Bronnikov & Korolyov 2015; Bronnikov 2018; Bronnikov & Walia 2022). The coordinate singularity at \( y = l \) in metric (1), can be explicitly eliminated by coordinate transformation \( y = \sqrt{l^2 + r^2} \) resulting in the much simpler and regular metric which reads

\[
\begin{align*}
    ds^2 &= \left(\frac{r - 2M}{\sqrt{r^2 + l^2}}\right) dt^2 - \frac{1}{\sqrt{r^2 - 2M}} dr^2 \\
    &\quad - (r^2 + l^2)(d\theta^2 + \sin^2 \theta d\phi^2).
\end{align*}
\]

The radial coordinate \( r \) assumes its entire range in this instance, i.e., \( 0 \leq r \leq \infty \). Unlike metric (1), the horizon in this case is independent of \( l \) and is always fixed at \( 2M \). The majority of the regular black holes that are closely related to a potential theory of quantum gravity might be seen as being inspired by quantum gravity rather than deriving from it. As such, they constitute useful phenomenological models but making their physical justification less straightforward (Ashtekar & Lewandowski 2004). However, by treating the quantum geometry corrections as an “effective” matter contribution, Walla (Kumar Walia 2022) has shown that phantom scalar field and the non-linear electrodynamics field together sourced the static and spherically symmetric polymerized black hole (1). Thus enhanced the significance of the polymerized black hole as an interesting solution of the Einstein’s field equations. The metric (2) also reduces to the Schwarzschild black hole in the \( l \to 0 \) limit.

2.1. Rotating black holes

The EHT observations of Sgr A* and M87* black holes shadows have confirmed that these astrophysical black holes are indeed rotating (Akiyama et al. 2019a, 2022f). This serves as our motivation to search for an axisymmetric extension of metric (2), more specifically, a LMRBH, which can be tested using the EHT observations. A direct loop quantization of stationary and axisymmetric black holes spacetime is still an unsolved problem. However, the Newman-Janis algorithm (NJA) introduced a revolutionary way to produce spinning spacetimes from a static, spherically symmetric seed metric without integrating any field equations (Newman & Janis 1965). Additionally, the revised NJA was used to obtain the first-ever rotating black hole solution in the loop quantum gravity (Brahma et al. 2021), and has been subsequently used to produce rotating black holes in modified gravities (Johannsen & Psaltis 2011; Ghosh 2015; Kumar & Ghosh 2018; Kumar et al. 2020a; Kumar & Ghosh 2020a). Starting with a partially polymerized static and spherically symmetric black hole solution (2) and applying the revised NJA algorithm (Azreg-Aînou 2014a,b), the resulting rotating spacetime LMRBH is always expressible in
Figure 1. Parameter space \((a,l)\) for LMRBH metric (3). Acronyms are spelled in the text. The parameters along the red line describe an extremal black hole with degenerate horizons. The blue line is a transition surface, with one positive and two degenerate horizons where as on green line with \(\Delta(0) = 0\), for \(l < 0.601 M\) one has \(\Delta(0) = 0\) and two horizons while for \(l > 0.601 M\), we have \(\Delta(0) = 0\) only. The LMRBH extremal black hole refers to a black hole with \(a > M\) (Kumar et al. 2022).

Boyer-Lindquist form, which is given as (Kumar et al. 2022)

\[
\begin{align*}
    ds^2 &= \left(1 - \frac{2M(r)\sqrt{r^2 + l^2}}{\rho^2}\right) dt^2 - \frac{\rho^2}{\Delta} dr^2 - \rho^2 d\theta^2 \\
    &\quad + 4aM(r)\sqrt{r^2 + l^2}\sin^2 \theta dt d\phi - \frac{A\sin^2 \theta}{\rho^2} d\phi^2 \\
\end{align*}
\]

where

\[
\begin{align*}
    M(r) &= M - \frac{r - \sqrt{r^2 + l^2}}{2} \\
    \rho^2 &= r^2 + l^2 + a^2 \cos^2 \theta, \\
    \Delta &= r^2 + l^2 + a^2 - 2M(r)\sqrt{r^2 + l^2}, \\
    A &= (r^2 + l^2 + a^2)^2 - a^2 \Delta \sin^2 \theta.
\end{align*}
\]

Although the rotating metric, derived from the NJA, does not guarantees to satisfy the LQG field equations, it captures key aspects of LQG, such as the global regularity of spacetime and the presence of a transition surface at the black hole center. Firstly, we note that LMRBH metric (3) encompasses the Kerr spacetimes (Kerr 1963b) in the limit \(l \to 0\), while in the limit \(a \to 0\), the spherical LQG black hole (2) is regained.

Furthermore, when \(a \to 0\), \(M \to 0\), and \(l \to 0\) the metric (3) gives flat spacetime.

The null surface \(\Delta(r) = 0\) - a coordinate singularity of the metric (3) defines the horizons of LMRBH. It turns out that depending on the values of \(a\) and \(l\), \(\Delta(r) = 0\) can admit up to three real roots with one to three positive roots and can also have one negative root. We shall consider only positive roots as they correspond to the horizon. We identified the roots \(r_1, r_2, r_3\) with \(r_3 \leq r_2 \leq r_1\). The largest root, \(r_1\), is always an event horizon, whereas \(r_2\), if it exists, is always a Cauchy horizon. While \(r_3\) is an additional horizon inside the Cauchy horizon and when \(r_3 < 0\), it corresponds to a horizon inside \(r = 0\) surface. The parameter space \((a,l)\) for LMRBH is depicted in Fig. 1. The spacetime structure of LMRBH spacetime intensely depends on the parameter \((M, a, l)\) as shown in cf. Fig. 1. We have identified four regions: BH-I, BH-II, BH-III and NH. The boundaries shown by coloured lines split these regions (see Kumar et al. (2022) for details), and lead to interesting geometrical features:
Figure 2. Plot showing $\Delta(r)$ in the region of parameter space (Fig. 1) (a) BH-I, where $\Delta(r) = 0$ admits only one positive root $r_1$ corresponding to event horizon (Top left), (b) BH-II where $\Delta(r) = 0$ admits two distinct positive roots $r_1$ (event horizon) and $r_2$ (Cauchy horizon), and one negative root $r_3$ corresponding to additional third horizon (Top right). Penrose-Carter diagrams of the parameter space ($M, a, l$) showing the region (a) BH-I (Bottom left) and (b) BH-II (Bottom right) (Kumar et al. 2022).

- **Region BH-I**: In this region of parameter space (gray region in Fig. 1) $\Delta(r) = 0$ has only one positive root corresponding to the single horizon.

- **Region BH-II**: Here (light blue region in Fig. 1), $\Delta(r) = 0$ has two positive roots ($r_2, r_1$) ($r_2 < r_1$) and one negative root ($r_3$). The former case corresponds to the Cauchy horizon ($r_2$) and event horizon ($r_1$). This region is the most physically relevant and is referred to as the *generic black hole*.

- **Region BH-III**: In this region (yellow region in Fig. 1), $\Delta(r) = 0$ admits three positive roots viz $r_3, r_2, r_1$ corresponding to three horizons, viz., inner horizon ($r_3$), Cauchy horizon ($r_2$) and event horizon ($r_1$). They degenerate at black dot (cf. Fig. 1).

- **Region NH**: Here (red region in Fig. 1), $\Delta(r) = 0$ has only one negative root. We refer to it as no-horizon spacetime, which, we show, is almost ruled out by EHT observations.

Thus, unlike Kerr black hole, $a = M$ does not yield an extremal black hole in LMRBH. It turns out that LMRBH can admit one negative root and one to three positive roots. Thus LMRBH, unlike Kerr’s black hole, admits up to three horizons. The case of two distinct positive horizons viz $r_1, r_2$ ($r_1 > r_2$) is referred as *generic black hole* with event horizon ($r_1$) and the Cauchy horizon ($r_2$). For further details see Ref. (Kumar et al. 2022) where we have given horizon structure and conformal diagrams of LMRBH. Thus, we have a LMRBH, which are regular everywhere and asymptotically encompass the Kerr black hole as a particular case. Also LMRBH metric (3) describes a multi-horizon rotating black hole in the sense that it can admit up to three horizons, and that an extremal LMRBH, unlike Kerr black hole, refers to a black hole with angular momentum $a > M$. Besides having properties, viz., asymptotic flatness and regularity,
next, we check the separability of the geodesic equations, which helps to test the LMRBH spacetime with its shadow via EHT observations.

Penrose diagram—The Penrose diagram of the region BH-I (cf. Fig 2 (left) ) is precisely the same as for its spherical counterpart (Peltola & Kunstatter 2009b). It reveals two exterior regions, the black hole and the white hole interior regions, and two quantum corrected interior regions (Peltola & Kunstatter 2009b). The transition surface, \( r = 0 \) is space-like and hidden behind the event horizon. Unlike Kerr black holes, the Penrose diagram of the generic black hole region BH-II has quantum corrected regions (\( r_3 \) to \( r = -\infty \)) where \( t \) and \( r \) swap their roles and replicate infinite times horizontally in both paths. The time-like transition surface substitutes the classical ring singularity occurring in the Kerr black hole at \( r = 0, \theta = \pi/2 \) (cf. Fig 2 (right)). The region BH-III has a similar Penrose diagram and hence is not presented (Kumar et al. 2022).

3. BLACK HOLE SHADOW

The null geodesics describing the photon orbits around the black hole are intriguing because of their observational significance in analysing the gravitational impact of the black holes on the surrounding radiation. Light rays, coming from the background source behind the black hole, with impact parameter greater than the critical value, get strongly deflect around the black hole and reach the observer, whereas those with impact parameter smaller than the critical value fall into the event horizon, resulting in a dark region on the observer’s sky, shadow, encompassed by the bright photon ring (Bardeen 1973; Synge 1966; Luminet 1979; Cunningham & Bardeen 1973). The influential work led by Synge (1966) and Luminet (1979), who furnished the formula to measure the angular radius of the photon captured region around the Schwarzschild black hole by identifying the diverging light deflection angle. Later, Bardeen (1973) in his pioneering work investigated the shadow of the Kerr black hole and showed that the spin would cause a distortion in shadow shape. The photon ring, encompassing the black hole shadow, explicitly depends on the spacetime geometry and thus its shape and size is a potential tool to determine the black hole parameters and to reveal the valuable information regarding the near-horizon field features of gravity. Later a flurry of activities in the analytical/numerical investigations and observational studies of shadows for large varieties of black holes have been reported (de Vries 2000; Shen et al. 2005; Amarilla et al. 2010; Yumoto et al. 2012; Amarilla & Eiroa 2013; Atamurotov et al. 2013; Abdujabbarov et al. 2016, 2015; Cunha & Herdeiro 2018; Mizuno et al. 2018; Shaikh 2019; Mishra et al. 2019; Kumar et al. 2020). Shadows have also been investigated for black hole parameter estimations (Kumar & Ghosh 2020b) and testing theories of gravity (Kramer et al. 2004).

We start with the Hamilton-Jacobi equation to determine the null geodesics followed by photons in the LMRBH spacetime (3) (Carter 1968; Chandrasekhar 1985)

\[
\frac{\partial S}{\partial t} = -\frac{1}{2}g^{\alpha\beta} \frac{\partial S}{\partial x^\alpha} \frac{\partial S}{\partial x^\beta},
\]

where \( \tau \) is the affine parameter along the geodesics, and \( S \) is the Jacobi action. The metric (3) is time translational and rotational invariant, which leads to conserved quantities along null geodesics, namely energy \( E = -p_t \) and axial angular momentum \( L = p_\phi \), where \( p_\mu \) is the photon’s four-momentum. In addition, the Petrov-type D character of metric (3) ensures the existence of Carter’s separable constant. Therefore, the Jacobi action can be written as

\[
S = -\mathcal{E}t + \mathcal{L}\phi + S_r(r) + S_\theta(\theta),
\]

\( S_r(r) \) and \( S_\theta(\theta) \), respectively, are functions only of the \( r \) and \( \theta \) coordinates. Therefore, the four integrals of motions, the Lagrangian, energy \( \mathcal{E} \), axial angular momentum \( \mathcal{L} \) and the Carter constant associated to the latitudinal motion of the photon, are sufficient to determine the geodesics equations of motion in the first-order differential form (Carter 1968; Chandrasekhar 1985), as follows

\[
\sum \frac{dt}{d\tau} = \frac{r^2 + l^2 + a^2}{\Delta} (\mathcal{E}(r^2 + a^2) - a\mathcal{L}) - a(\mathcal{E}\sin^2 \theta - \mathcal{L}),
\]

\[
\sum \frac{dr}{d\tau} = \pm \sqrt{\mathcal{R}(r)},
\]

\[
\sum \frac{d\theta}{d\tau} = \pm \sqrt{\Theta(\theta)},
\]

\[
\sum \frac{d\phi}{d\tau} = \frac{a}{\Delta} (\mathcal{E}(r^2 + l^2 + a^2) - a\mathcal{L}) - \left(a\mathcal{E} - \frac{\mathcal{L}}{\sin^2 \theta}\right),
\]

where \( \mathcal{R}(r) \) and \( \Theta(\theta) \), respectively, pertain to the following radial and polar motion effective potentials:

\[
\mathcal{R}(r) = [a^2 - \Delta \mathcal{K} - (a\mathcal{E} - L)^2] \Delta \mathcal{K} + (a\mathcal{E} - L)^2,
\]

\[
\Theta(\theta) = \mathcal{K} - \left[\frac{\mathcal{L}^2}{\sin^2 \theta} - a^2\mathcal{E}^2\right] \cos^2 \theta.
\]

where the separability constant \( \mathcal{K} \) is related to the Carter constant \( Q \) through \( Q = \mathcal{K} + (a\mathcal{E} - L)^2 \) (Carter...
which, in essence, represents the isometry of metric (3) along the second-order Killing tensor field. The photon’s motion is influenced by the $Q$, but restricted to an equatorial plane when $Q = 0$. Meanwhile, the $\mathcal{L}$ governs the $\phi$-motion. Contrary to the Schwarzschild black hole, where all null circular orbits are planar due to spherical symmetry, i.e., orbits with $\dot{\theta} = 0$, the rotating black hole also has nonplanar orbits because of the effects of frame dragging. The black hole shadow silhouette is outlined by the unstable spherical photons, which can be determined by solving $\dot{r} = \ddot{r} = 0$ from Eqs. (8) and (11). The radii $r_p$ of photon spherical orbits is positive root of the following equations
\begin{equation}
\mathcal{R}|_{r=r_p} = \frac{\partial \mathcal{R}}{\partial r}\bigg|_{r=r_p} = 0, \quad \text{and} \quad \frac{\partial^2 \mathcal{R}}{\partial r^2}\bigg|_{r=r_p} > 0. \quad (13)
\end{equation}

To proceed further, following Chandrasekhar (1985), we can introduce the dimensionless parameters $\xi \equiv \mathcal{L}/\mathcal{E}$, $\eta \equiv K/\mathcal{E}^2$ to reduce the degree of freedom of photons geodesics to one. Solving Eq. (13) for Eq. (11) results in the critical impact parameters as follows (Chandrasekhar 1985)
\begin{align}
\xi_c &= \frac{(a^2 + l^2 + r_p^2)\Delta'(r_p) - 4r_p\Delta(r_p)}{a\Delta(r_p)} \nonumber \quad (14) \\
\eta_c &= \frac{1}{a^2\Delta'(r_p)^2}(16r_p^2\Delta(r_p)(a^2 - \Delta(r_p)) \\
&\quad + (l^2 + r_p^2)\Delta'(r_p)[8r_p\Delta(r_p) - (l^2 + r_p^2)\Delta'(r_p)]
\end{align}

where $'$ stands for the derivative with respect to the radial coordinate. The Eq. (14) in the limit $l \to 0$ reduces to the following
\begin{align}
\xi_{cK} &= \frac{a^2(M + r_p) + r_c(r_p - 3M)}{a(M - r_p)} \\
\eta_{cK} &= \frac{r_p^3}{a^2(M - r_p)^2}[4a^2M - r_p(r_p - 3M)M] \quad (15)
\end{align}

which are exactly same as that for the Kerr black hole (Chandrasekhar 1985). For light rays coming from the bright source, there are three possible trajectories a) capture orbit, b) scatter orbit and c) unstable orbit. The light rays which are plunged into the black hole form the black hole shadow. The shape of the black hole shadow depends on the spin $a$ and observation angle $\theta_0$ with respect to the spin axis. The relationship between the observer’s celestial coordinates, $X$ and $Y$, and two constants, $\xi_c$ and $\eta_c$, is derived using the tetrads components of the four momentum $p^{(a)}$ and geodesic Eqs. (7), (8), (9), and (10) as
\begin{align}
X &= -r_o\frac{p^{(\phi)}}{p^{(\theta)}} = -r_o\frac{\xi_c}{\sqrt{g_{\phi\phi}(\xi - \gamma \xi_c)}} \quad (r_o, \theta_o) \\
Y &= r_o\frac{p^{(\theta)}}{p^{(\phi)}} = \pm r_o\frac{\eta_c}{\sqrt{g_{\phi\phi}(\xi - \gamma \xi_c)}} \quad (r_o, \theta_o)
\end{align}

where
\begin{equation}
\gamma = -\frac{g_{\phi\phi}}{g_{\phi\phi}} \quad (17)
\end{equation}

The parametric plots of Eqs. (18) in the $(X, Y)$ plane cast a variety of black hole shadows for different range of parameters as shown in figures. 3 and 4. For $a = l = 0$, the contour of a Schwarzschild black hole shadow from Eq. (18) takes the form $X^2 + Y^2 = 27M^2$, which ensures that the shadow is perfectly circular. The shadow of non-rotating LQG black holes, when compared with the Schwarzschild black holes are larger and the size of the shadow increases with parameter $l$ (Kumar Walia 2022). We depict the shadow silhouette of LMRBH spacetime for various parameter values in Fig. 4. In fact, the shadow size, for a fixed value of spin $a$, increases and becomes more distorted with increasing $l$. Additionally, we also see a horizontal shift in the shadow along the $X$-axis, with an increase in inclination angle $\theta_0$ and the spin $a$.

4. OBSERVABLES AND BLACK HOLE PARAMETER ESTIMATION

We wish to understand the LMRBH model properly. Hence, using shadow observables, we also estimate parameters associated with LMRBH assuming that the observer is in the equatorial plane. It may be beneficial to determine information about an LMRBH ultimately.

We will employ two independent methods, Hioki-Maeda (2009) and Kumar-Ghosh (2020b), to
characterize the LMRBH shadows using the shadow observables. A prior knowledge of these observables through observations can be used to back estimate the black hole parameters. We assume the observer is in the equatorial plane, i.e., at an inclination angle $\theta_0 = \pi/2$.

**Hioki – Maeda method:** —Hioki and Maeda (2009) proposed two observables, radius $R_s$ and distortion $\delta_s$, to characterise the black hole shadow silhouette. They used a reference perfect circle with center $(X'_c, 0)$ that coincides with the shadow silhouette at three points, $(X_t, Y_t)$, $(X_b, Y_b)$, $(X_r, 0)$, to approximate the shape of the black hole shadow as shown in Fig. 5. The points $(X_t, 0)$, $(X'_t, 0)$, represent the intersections of the

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**Figure 3.** Shadow silhouette of LMRBH for $l = 0.5$ with varying $a$ (left) and for $a = 0.95$ with varying $l$ (right) as seen from the equatorial plane, i.e., inclination angle $\theta_0 = \pi/2$.

**Figure 4.** Shadow silhouette of LMRBH for $a = 0.95$ with varying $l$ at inclination angle (left) $\theta_0 = 17^\circ$ and (right) $\theta_0 = 50^\circ$. 
The shadow observables, radius $R_s$ and distortion $\delta_s = d_{cs}/R_s$, for the apparent shadow shape of LMRBH ($a = 0.99M$, $l = 0.4M$ and $\theta_0 = \pi/2$). The distortion is the difference between the left endpoints of the dotted circle and of the shadow.

$R_s = \frac{(X_t - X_r)^2 + Y_t^2}{2|X_t - X_r|}$,

$\delta_s = \frac{|X'_t - X_t|}{R_s}$. (20)

We determine the black hole parameters by combining two contour plots for these observables, resulting in a one-to-one correspondence between ($a$, $l$) and ($R_s$, $\delta_s$). With the prior knowledge of radius $R_s$ and distortion $\delta_s$, we can accurately calculate the spin parameter $a$ and the parameter $l$ using Fig. 6 and Table 1.

**Table 1.** Table representing the estimated values of parameters $a/M$ and $l/M$ from given shadow observables $R_s$ and $\delta_s$.

| $R_s/M$ | $\delta_s$ | $a/M$   | $l/M$   |
|--------|------------|--------|--------|
| 5.197  | 0.01       | 0.2884 | 0.0437 |
| 5.197  | 0.02       | 0.4038 | 0.0405 |
| 5.21   | 0.01       | 0.2897 | 0.1788 |
| 5.21   | 0.10       | 0.8131 | 0.1684 |
| 5.23   | 0.02       | 0.4129 | 0.2803 |
| 5.23   | 0.20       | 0.9808 | 0.2673 |
| 5.30   | 0.01       | 0.3058 | 0.4934 |
| 5.30   | 0.20       | 1.023  | 0.4866 |
| 5.39   | 0.05       | 0.6787 | 0.6868 |
| 5.39   | 0.16       | 1.038  | 0.6735 |
| 5.39   | 0.20       | 1.078  | 0.6728 |

**Figure 5.** The shadow observables, radius $R_s$ and distortion $\delta_s = d_{cs}/R_s$, for the apparent shadow shape of LMRBH ($a = 0.99M$, $l = 0.4M$ and $\theta_0 = \pi/2$). The distortion is the difference between the left endpoints of the dotted circle and of the shadow.

**Figure 6.** Contour plot of the shadow observables $R_s/M$ (red solid lines) and $\delta_s$ (blue dashed lines). A given lines of $R_s$ and $\delta_s$ intersect at a unique point ($a$, $l$) in the parameter space plane.

**Kumar – Ghosh method:** Kumar and Ghosh 2020b proposed an alternate method to characterize the black hole shadows using the observables, namely, area ($A$) enclosed by a black hole shadow and the shadow oblateness ($D$). The potential advantages of using these observables is that they can be used coordinate independently by different teams analysing the noisy observational data, and they do not require to compare the shadow silhouette with a perfect circle and thus applicable to a general shadow of any shape and size. The observables $A$ is defined by

$$A = 2\int Y(r_p)dX(r_p) = 2\int_{r_p}^{r_{p}} Y(r_p)\frac{dX(r_p)}{dr_p} dr_p,$$

(21)

By defining the dimensionless observable $D$: the ratio of the horizontal and vertical diameters as the oblateness (Takahashi 2004; Grenzebach et al. 2015;...
Tsupko 2017), a degree of distortion (circular asymmetry), a characterisation of the rotating black hole’s shadow can be done. The oblateness can be as follows

\[ D = \frac{X_r - X_t}{Y_t - Y_b}. \]  

(22)

The subscripts \( r, l, t, \) and \( b \), respectively, stand for right, left, top, and bottom of the shadow silhouette. \( D = 1 \) for a spherically symmetric black hole shadow, but \( \sqrt{3}/2 < D < 1 \) for a Kerr shadow (Tsupko 2017). The shadow structure (see Fig. 4) clearly reveals that the parameters \( a \) and \( l \) have a substantial effect on both the shadow area and the oblateness. It is worth noting that a single shadow observable, either \( A \) or \( D \), will lead to degeneracy in estimating more than one black hole parameters. However, a set of shadow observable \((A, D)\) can uniquely determine two parameters. A one-to-one correspondence between the observables \((A, D)\) and parameters \((a, l)\) is depicted in Table 2 and Fig. 7.

5. CONSTRAINTS FROM THE EHT OBSERVATION

In spite of several astrophysical phenomena including accretion flow, jet outflow, gravitational lensing, emission phenomena, etc., the black hole shadow shape acts as a direct diagnostic test of strong field gravity since it is the most obvious manifestation of the background spacetime. The black hole shadow boundary is constructed by the photons which can go closest to the black hole but still manage to escape the black hole gravitational field and reach the observer, as a result shadow bear imprints of the strong-field characteristics (Jaroszynski & Kurpiewski 1997; Falcke et al. 2000), so much so that the black hole shadow observations by the EHT collaboration have opened up an exciting arena to make a precision test of the gravitational theory in the strong and relativistic field regimes. It is worth-noting that the current angular resolution of the EHT is not enough to capture the quantum-gravity effects imprints. The EHT collaboration, a very long baseline interferometry experiment, that measures radio brightness distributions at a wavelength of 1.3 mm on the sky with an unprecedented angular resolution of 20 μas, has captured the horizon-scaled emission image of the Sgr A* and M87* black holes (Akiyama et al. 2022e). Both shadows have some common features; the shadow center have significant brightness depression due to photon capture by the event horizon of the black hole which are enclosed by a bright nearly-circular ring. EHT collaboration put bounds on the size and the shape of this ring. Using the EHT observations of the shadows cast by black holes M87* and Sgr A*, we explore potential constraints on the LMRBH parameters.

5.1. Observational constraints from the EHT results of M87*

The EHT analysis suggested that, based on a priori known estimates for the mass and distance from stellar dynamics, the M87* shadow size is consistent within 17% for a 68% confidence interval of the size predicted.
from the Kerr black hole general relativistic-magneto-hydrodynamics (GRMHD) image (Akiyama et al. 2019a). However, several other studies altogether have not entirely precluded the possibility of non-Kerr black holes (Vincent et al. 2021; Mizuno et al. 2018). Using the M87* shadow angular size, constraints are placed on the second post-Newtonian metric coefficients, which were inaccessible in the earlier weak-field tests at the Solar-scale (Psaltis et al. 2020). Therefore, it is both legitimate and timely to test the viability of the LQRBH using the M87* black hole shadow observations. We determine the constraints on the LMRBH parameters using the deviation from circularity of the black hole shadow \( \Delta C \) and the angular diameter \( \theta_{sh} \), to validate the suitability of LMRBH as a candidate for the M87* black hole.

Let the shadow boundary is described by polar coordinates \((R(\varphi), \varphi)\) and shadow has centre at \((X_c, Y_c)\) with \(X_c = (X_r - X_l)/2\) and \(Y_c = 0\). To assess the shadow deviation from a perfect circle, we define the dimensionless circularity deviation observable in terms of root-mean-square deviation from average shadow radius as (Johannsen & Psaltis 2010; Johannsen 2013; Kumar et al. 2020b)

\[
\Delta C = \frac{1}{\bar{R}} \sqrt{\frac{1}{2\pi} \int_0^{2\pi} \left( R(\varphi) - \bar{R} \right)^2 d\varphi},
\]

where \( \bar{R} \) is the shadow average radius defined as (Johannsen & Psaltis 2010)

\[
\bar{R} = \frac{2}{\pi} \int_0^{2\pi} R(\varphi) d\varphi.
\]

with \( \varphi \equiv \tan^{-1}[Y/(X - X_C)] \) being the subtended polar angle and \( R(\varphi) = \sqrt{(X - X_C)^2 + (Y - Y_C)^2} \) being the radial distance from the centre \((X_c, Y_c)\) of the shadow to any point \((X, Y)\) on the boundary. The variables connected with black holes, such as their mass \( M \), spin parameter \( a \), \( l \), or inclination angle \( \theta_0 \), influence celestial coordinates, and as a result, also the observables \( \bar{R} \) and \( \Delta C \). To determine the black hole parameter values that are most likely to be observed, we compare our theoretical prediction to the EHT observation and use their result \( \Delta C < 0.1 \) to impose constraints on the LMRBH parameter space while accounting for the inclination angle with respect to the observational line of sight to be \( 17^\circ \) (Craig Walker et al. 2018).

Next, the shadow angular diameter (Kumar & Ghosh 2020a; Kumar et al. 2021) is given by

\[
\theta_{sh} = \frac{2}{d} \sqrt{\frac{A}{\pi}},
\]

where \( A \) is the black hole shadow area, defined as in Eq. 21, and \( d \) is the distance of M87* from the Earth. In Fig. 8, we demonstrate the LMRBH shadow angular diameter for \( \theta_0 = 17^\circ \) as a function of \( a \) and \( l \). The M87* black hole shadow angular diameter within the 1\( \sigma \) bound, \( 39\text{\,mas} \leq \theta_{sh} \leq 45\text{\,mas} \), place bound \( 0.0 \leq a \leq 0.52757M \), whereas all values of \( l \) are allowed. For \( a > 0.52757M \), there is an upper limit on the value of \( l \) which depends on the value of \( a \) (cf. fig. 8). Thus, in this constrained parameter space, the M87* can be LMRBH spacetime. Since there are number of possible parameter points \((a, l)\) within the confined parameter space, the LMRBH’s compatibility with the M87* observations demonstrates that they can be excellent candidates for astrophysical black holes. Circularity deviation is shown in Fig. 10 for \( \theta_0 = 17^\circ \). The M87* bound \( \Delta C \leq 0.10 \) is satisfied by the LMRBH for the entire parameter space. This is because, the rotating black hole shadows are nearly circular for small inclination angle.
5.2. Observational constraints from the EHT results of Sgr A*  

Based on the 2017 VLBI observing campaign at 1.3 mm wavelength, the EHT collaboration has published the shadow data for the Sgr A* black hole (Akiyama et al. 2022a,b,c,d,e,f). Sgr A* black hole shadow images have advantages to test the nature of astrophysical black hole (i) Sgr A* probes a $10^6$ order of higher curvature than the M87* (ii) independent prior estimates for mass to distance ratio are used for Sgr A*. The shadow images were created using a variety of imaging and modelling techniques, and they are astonishingly similar in features. We will use the EHT bounds on the two observables, shadow angular diameter $\theta_{sh}$ and Schwarzschild shadow deviation $\delta$, to set constraints on the LMRBH parameters. We determine $\theta_{sh}$, which, in addition to other black hole parameters, relies on the mass $M$, the distance $d$ of the black hole, LQG parameter $l$ and inclination $\theta_0$ as depicted in Figure 9. We show that $41.7 \mu\text{as} \leq \theta_{sh} \leq 55.7 \mu\text{as}$, i.e., within the 1$\sigma$ region of the SgrA* shadow angular diameter, is satisfied for entire parameter space in case of LMRBH. Moreover, EHT used three independent imaging algorithms, namely, eht-imaging, SIMLI, DIFMAP, to determine the Sgr A* shadow’s average diameter $46.9\mu\text{as} \leq \theta_{sh} \leq 50 \mu\text{as}$. This range of angular diameter strongly constrains the parameters.
0.356355M ≤ a ≤ a_c and 0 ≤ l ≤ l_u at θ_o = 50°. a_c are the critical values of parameter a represented by the red line in Fig. 9 and l_u is the maximum value of l which depends on a. For these constrained parameter range, the LMRBH shadows are consistent with the shadow of Sgr A*.

The Schwarzschild shadow deviation (δ), assesses the disparity between the shadow angular diameter of LMRBH, θ_sh, and the Schwarzschild shadow diameter, θ_sh, Sch, and is given by Akiyama et al. (2022e,f)

\[
δ = \frac{θ_{sh}}{θ_{sh, Sch}} - 1. \tag{26}
\]

For Kerr black hole with \( a \leq M \), the Schwarzschild shadow deviation lies in \(-0.075 \leq δ \leq 0 \) as inclination varies from 0 to \( \pi/2 \). EHT used the two separate priors for the Sgr A* angular size from the Keck and Very Large Telescope Interferometer (VLTI) observations to estimate the bounds on the fraction deviation observable \( δ \) (Akiyama et al. 2022e,b)

\[
δ_{Sgr} = \begin{cases} 
-0.08^{+0.09}_{-0.09} & \text{VLTI} \\
-0.04^{+0.09}_{0.10} & \text{Keck}
\end{cases} \tag{27}
\]

Thus modeling Sgr A* as LMRBH, the entire parameter space in case of LMRBH is satisfied for inferred bound Keck (-0.14,0.05). On the other hand the VLTI bound (-0.17,0.01), constrains the parameters \((a,l)\) such that for \( 0 < l < 0.347851M (l \leq 2 \times 10^{8} \text{Km}) \), the allowed range of \( a \) is \((0,1.0307M)\). If \( a > 1.0307M \), there is an upper bound on the value of \( l \) which is not specific and depends on the value of ‘a’, for e.g., at \( a = 0.5094M \), \( l \leq 0.4864M \) and at \( a = 0.8399M \), \( l \leq 0.6758M \) (cf. Fig 11).

6. CONCLUSION

The scarcity of rotating black hole models in the LQG restricts the progress of testing LQG from observations, like EHT observation of M87* and SgrA*. Furthermore, because of the rapidly growing astronomical observations of rotating black holes, starting from a nonrotating LQG seed metric (2) and using a modified NJA generating technique we got a rotating LMRBH in an earlier work (Kumar et al. 2022). The LMRBH metric possesses a Kerr-like form and has several exciting properties (Kumar et al. 2022). It is nonsingular everywhere and reduces to the Kerr solution when the quantum effects are switched off \((l = 0)\). Depending on values of parameters \((M, a, l)\), the LMRBH can describe an NH, a generic regular black hole with Cauchy and Event horizons (BH-II), or a black hole with one or three horizons (BH-I or BH-III).

Here, we are interested in shadow and analyzed the LMRBH to find that the Hamilton-Jacobi equations are separable, resulting in null geodesics equations in the first-order differential form. In Fig. 4, we depict the shadow cast by the LMRBH in the parameter space to find black hole shadow size increases monotonically, and the shape gets more distorted with an increasing \( l \). For given shadow observables, we have estimated parameters associated with LMRBH by two popular methods, assuming that the observer is in the equatorial plane. We show that by using these shadow observables one can ultimately determine information about an LMRBH.

The supermassive black holes, M87* and Sgr A*, at the centre of the Milky Way and Messier 87, are a superior and realistic laboratory for testing the strong-field predictions of GR. The EHT collaboration released the first horizon-scale images of both M87* and Sgr A*, and the EHT results are consistent with the prediction on the Kerr metric, and there is no evidence for any breaches of the theory of GR. Here, we used the EHT observation of the black hole shadow in M87* and Sgr A* to place constraints on deviation parameters associated with LMRBH.

To place constraints on the deviation parameter \( l \), we employ the angular size and asymmetry results of the EHT for the M87* black hole and the EHT bounds on the Sgr A* shadow angular diameter and Schwarzschild shadow deviation from the Sgr A* results. The allowed range of M87* shadow angular diameter within 1σ region, i.e., \( 39\mu as \leq θ_{sh} \leq 45\mu as \) is possible for the entire range of \( l \) and \( 0.0 \leq a \leq 0.52757M \). However, if \( a > 0.52757M \), then there is an upper limit on \( l \) which depends on \( a \). Likewise, we show that the Sgr A* shadow angular diameter within 1σ credible region, \( 41.7\mu as \leq θ_{sh} \leq 55.7\mu as \), is satisfied for the entire parameter space in the case of LMRBH. But, the EHT observation also employed three different techniques to determine that the shadow’s average measured angular diameter is within the range \( θ_{sh} \in (46.9, 50)\mu as \) which strongly constrains the parameters, such that \( 0.356355M \leq a \leq a_c \) and \( 0 < l < l_u \) at \( θ_o = 50° \)is allowed. Here, \( a_c \) and \( l_u \) are, respectively, the critical values of parameter \( a \) represented by the red line in Fig. 9 and maximum value of \( l \). Further, modelling Sgr A* as LMRBH, the entire parameter space with LMRBH is obeyed for inferred Keck bound (-0.14,0.05). The VLTI bound (-0.17,0.01), constrains the parameters \((a,l)\) such that for \( 0 \leq l \leq 0.347851M \), the allowed range of \( a \) is \((0,1.0307M)\). For \( a > 1.0307M \), the maximum value of \( l \) depends on ‘a’. The shadow circularity deviation \( \Delta C \) bound for M87*
black hole allows the entire parameter space for LMRBH due to low inclination angle, whereas the $\Delta C$ bound for Sgr A* is not available.

The main restriction of our proposition is that the LMRBH metric does not result from a direct loop quantization of the Kerr spacetime. However, we note that the existence of the region BH-I have similar features of nonrotating LQG black holes (1) (cf Penrose Diagram 2) and also LMRBH provide singularity resolution of Kerr black holes. Thus, it is pragmatic to expect that LMRBH can capture some aspects of the effective regular spacetime description of LQG.

7. ACKNOWLEDGMENTS

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