Developing learning trajectories with the RME of phytagorean theorem

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Abstract - There are three main parts of the hypothetical learning trajectory (HLT), are (1) learning objectives, (2) learning activities, (3) student understanding and problem solving strategies. The researcher tried to develop a context that helped students in finding the concept of the phytagorean theorem using the RME approach. In this study, the type of research used by researchers was design research developed by Gravemeijer and Cobb. There are three phases in design research according to Gravemeijer and Cobb, namely preliminary design, experimental design, and retrospective analysis. In this paper, researchers describe limited to the first phase of design research.

1. Introduction
In everyday life, we are not free from problems. Not all problems are mathematical problems, but mathematics has an important role in answering these daily problems. This is in accordance with what was revealed [1] regarding the importance of learning mathematics because mathematics is a means (1) clear and logical thinking, (2) to solve everyday life problems, (3) recognize patterns relationship patterns and generalization of experience, (4) developing creativity, (5) to increase awareness of cultural developments. According to Holmes [2] individuals who are skilled at solving problems will be able to race against their needs, become more productive workers, and understand complex issues related to global society. In other words, individuals who have good problem solving skills will be able to face problems in daily life both in the world of education and in the world of work. From the aspect of the education curriculum, problem solving becomes one of the goals in learning mathematics in schools that is training ways of thinking and reasoning in drawing conclusions, developing the ability to solve problems and developing the ability to convey information or communicate ideas through oral, written, picture, graph, map, diagrams.

Based on interview with teacher started that students tend to memorize formulas so that when the problems forms change students have difficulty completing them so the teacher needs a lot of time to reexplain the concept of phytagoras. In this study, researchers gave initial tests to students who had studied pythagoras material with a total of 17 students. The aim is to find out how far students understand. The problem given to students is Mr. Tono is a bus driver. He drove a bus from city A to the north towards city B with a distance of 60km. After arriving in city B, the bus stopped for a while and then continued eastward towards city C with a distance of 80km. If Mr. Tono wants to go back to city A directly from city C, calculate the distance that will be taken by the bus! Some of the settlement strategies carried out by students are as follows:

1. One student has not been able to determine the direction of the track and the distance traveled.

Following are examples of student work:
Students do not accurately represent the problem given. If according to the direction of the wind, the north direction intended from city A to city B is the arrow upwards (↑), then eastward from city B towards city C is the arrow to the right (→), while the representation of students is shows north but eastward from city A to city B and south from city B to city C. The next step students finish by writing $\frac{1}{2} \times 80 \times 60$. The operation is an operation in calculating the area of a triangle. Students assume that the base side is BC = 80km and the AB side = 60 km is high. When viewed from the representation of the image, the base of the triangle is AB and the hypotenuse is BC. Then students simplify the number 80 with $\frac{1}{2}$ to 40 so that the AC path length = 2400 km is obtained. From the results of the settlement it can be concluded that students have not understood well the concept of the side of a triangle and the pythagorean theorem. In addition, the settlement process is not yet right because it is not in accordance with the purpose of the question.

2. Two students are not right in expressing symbols. The following are examples of student work.

$$AC^2 = AB^2 + BC^2$$

$$= 60^2 + 80^2$$

$$= 3600 + 6400$$

$$= 10000 \sqrt{10000}$$

$$= 1000 \text{ km}$$

Furthermore, $$AC^2 = \sqrt{10000}$$ so that AC = 100 km. In this process it can be concluded that students have difficulty expressing their ideas in symbolic form.
3. Eight students don’t correctly write the formula for pythagoras. The following are examples of student work.

\[ AC^2 = \sqrt{BC^2 + AB^2} \]

Students represent the problem by drawing a trajectory in the form of a right triangle, but students have not precisely determined the direction of the trajectory of the city A towards city B. If seen from the results of the representation, the direction of trajectory formed from city A to city B is south. This is not in accordance with the purpose of the question. In solving the problem, students calculate the AC distance using the pythagoras theorem. In the first step, students write the pythagoras formula \( AC^2 = \sqrt{BC^2 + AB^2} \). Writing the formula is not right because students mistakenly give the second power to the AC on the left hand side, while on the right hand side the student writes the root of the number \( BC^2 \) and \( AB^2 \). Then students write the appropriate values for \( BC \) and \( AB \). In the fourth step of completion the students write \( AC^2 = 10.000 \) and get \( AC = 100 \). When students classify the right segment in the form of roots and get an AC value of 100, students actually understand the concept of pythagoras only students do not pay attention to their mistakes in writing the formula of pythagoras.

4. Six students can determine the direction of the track and the distance traveled. Following is one example of student answers.

Students represent a problem by drawing a path in the form of a right triangle. From the representation results, students can determine the correct path direction in accordance with the description of the question, students clarify the direction of the track by writing up arrows (↑) and symbol U for north direction and arrows to the right (→) and letter T to declare eastward. In
solving problems, students calculate the AC path length using the Pythagoras theorem. Students can write the Pythagoras formula correctly, namely $AC^2 = AB^2 + BC^2$ and do the completion operation correctly.

From these data it can be concluded that students still experience obstacles in understanding Pythagorean theorem. Most students have difficulty expressing their ideas in the form of symbols, difficulty representing the direction of the path, it is not appropriate to make mathematical models using the Pythagorean theorem, besides that the concepts in the settlement process are not well understood.

Although research related to student learning on Pythagorean theorem is quite a lot like that of Siregar [3] but research related to student learning regarding the discovery of the Pythagorean concept is still relatively small. This is also one of the reasons that encouraged researchers to design learning tools using the RME model on Pythagorean theorem.

1.1. Hypothetical Learning Trajectory
Hypothetical Learning Trajectory (HLT) is a hypothesis or prediction of how students thinking and understanding develop in a learning activity. HLT consists of three main components, namely (1) learning objectives, (2) learning activities and devices or media used in the learning process, (3) conjecture (presumption/anticipation) of the learning process on how to understand students understanding and strategies that arise and develop when learning activities are carried out in class, Gravemeijer [4].

1.2. Realistic Mathematics Education
Realistic Mathematics Education (RME) is a mathematical learning approach developed since 1971 by a group of mathematicians from the Freudenthal Institute, Utrecht University in the Netherlands. This approach is based on the assumption of Hans Freudenthal (1991) that mathematics is a human activity. Freudenthal does not place mathematics as a finished product, but rather a form of activity or process in constructing mathematical concepts. In this approach, Freudenthal [4] believes that students should not be seen as passive receivers of ready-made mathematics. According to him education must direct students to the use of various situations and opportunities to rediscover mathematics in their own way. Treffers [4] classifies two types of mathematical namely vertical and horizontal mathematical, which are described by Gravemeijer (1994) as the process of reinvention.

![Figure 1. Horizontal and vertical mathematics, Gravemeijer [3]](image)

In horizontal mathematical, students start from contextual questions, try to describe them with their own language and symbols, then solve the problem. Next students complete using their own ways that might be different from others [4] According to De Lange [7] the horizontal mathematical process begins by identifying mathematical concepts based on regularities and relations found through visualization and schematic problems. Whereas vertical mathematical is a form of formalization...
process, where mathematical models made by students previously become the foundation in developing more formal mathematical concepts [7]. In vertical mathematics, we also start from contextual questions, but in the long run we can compile certain procedures that can be used to solve similar questions directly without using context help [4]

2. Research Method
The instrument that will be developed by the researcher is a learning device (Hypothetical Learning Trajectory) that can be used to teach phytagoras material with the RME model to student of grade VIII junior high school. The type of research in research is design research according to Gravemeijer & Cobb [6]. In this study, the results of the development of HLT will be discussed in the first phase, namely:
1. Reviewing the competencies students have learned about material square roots, rhythms, triangles and rectangles.
2. Develop learning trajectories with the possibility of students' answers to the material Pythagoras uses realistic mathematics education.

3. Result and Discussion
The researcher reviews the basic competencies that students have learned previously related to phytagoras based on the 2013 curriculum revision. The results are as follows:
1. Class 1:
   a. Get to know the space and get up flat by using various concrete objects.
   b. Classify building space and build flat by using various concrete objects.
2. Class 2:
   a. Explain line segments by using concrete models to build flat and construct spaces.
   b. Explain building flat and build space based on its characteristics.
   c. Identify line segments using concrete objects to build flat and build space.
   d. Classifying wake up flat and build space based on its characteristics.
3. Class 3:
   a. Analyzing a variety of flat builds based on the properties possessed.
   b. Grouping various flat shapes based on their properties.
4. Class 4:
   a. Explain and determine the circumference and area of a square, a rectangle and a triangle.
   b. Resolve problems related to circumference and area of a square, rectangle and triangle.
5. Class 5:
   a. Explain and carry out the removal and withdrawal of chopped root numbers.
   b. Resolve problems related to the removal and withdrawal of chopped root numbers.
6. Class 7
   a. Identify the properties of flat building and use it to determine circumference and area.
   b. Resolve real problems related to the application of triangular and quadrilateral properties.

3.1. RME based learning
1.) Indicator of the teaching and learning process
   a) Students can represent / model a problem in daily life related to phytagoras.
   b) Students can find the concept of the phytagorean theorem.
   c) Students can describe the phytagorean theorem using their own sentences.
2.) Teacher and student activities
   a) Activities to build social norms in the class. The teacher gives the social norm that will be set in the class, namely:
      1) Students must maintain calm during learning.
      2) If there are students who want to ask questions, express opinions, or answer questions from both the teacher and other students, you should raise your hand first.
3) If there are students who submit opinions or questions then other students listen.
4) When the teacher asks the student again does not mean the answer is wrong, but the teacher only wants to know the students’ understanding.

b) Exploration of problems
1) The teacher asks students to form discussion groups heterogeneously where each group consists of 4 students.
2) The teacher gives contextual problems to students as follows:
   A taint will paint a building that is 4m high. for that he uses a ladder and props it against the wall of the building. The painters put the foot of the stairs on the floor which is 3m from the wall. What is the length of the ladder used by the painter?
3) Students are given the opportunity to discuss in groups. This is an attempt to issue the interactivity characteristics of RME.
4) If students are not right in stating the symbol,
   \[ t^2 = d^2 + l^2 \]
   \[ t^2 = 4^2 + 3^2 \]
   \[ t^2 = 16 + 9 \]
   \[ t^2 = 25 \]
   \[ t = 5m \]
   The teacher gives a support in the form of a question that is, try to look back at the results of your settlement. Try to investigate again, is \(25\sqrt{25} = 5\)? If students find fault, the teacher again explains that writing menuliskan25 should be in the next step, then in the left section becomes \(t\).

c) Class discussion
1) If students finish discussing and solving problems, the teacher asks one of the groups to present the results of the resolution. The expected answers are as follows:
   Students represent problems in the form of pictures and suppose the length of the stairs with the letter \(t\), the building wall with the letter \(d\), and the floor with the letter \(l\). Then students complete by applying the phytagoras theorem.
   \[ t^2 = d^2 + l^2 \]
   \[ t^2 = 4^2 + 3^2 \]
   \[ t^2 = 16 + 9 \]
   \[ t = \sqrt{25} \]
   \[ t = 5m \]
   2) Other students are asked to respond to the results of their friend's presentation.
   3) The teacher asks other students who have different resolution strategies to convey ideas.
   4) Other students are asked to respond to what is presented by the group presenting the results.
   5) The teacher leads the course of class discussion until students get the length of the ladder used by painters.

d) Exploration of the problem
1) The teacher provides contextual problems to students. The contextual problem is as follows:
   There are four points, namely A, B, C, and D as shown. The position of these points forms a square.
   ![Diagram of points A, B, C, and D forming a square]
   A child will run from point C to point A. Make the fastest track or the shortest possible child and give your reason!
a) Students are given the opportunity to discuss in groups. This is an attempt to issue the interactivity characteristics of RME.

b) students represents the path from point C to point A through point B as follows.

\[ AC = 2m + 2m = 4m \]

The teacher gives a support in the form of a question which is trying to prove mathematically that the track is the fastest track. If students have found the length of the track, the teacher asks again, is there another path that is faster than that? Next the students try to find the fastest track that is selected.

c) If students represent the path from point C to point A as follows:

\[ \text{The reason is, suppose the child runs from point C to the north is } 2m, \text{ then westward } 2m, \text{ then he moves northward } 4m \text{ and westward } 4m. \text{ then the path length that the child traverses from point C to point A is } 12m. \]

So the teacher gives a support which is to try to think of another method that states the shortest path from point C to point A.

e) Class Discuss

1) The teacher asks one of the groups to present the results of the resolution. The expected answer is as follows:

Students represent the direction of the path from point C directly to point A.

\[ \text{The reason is that Points A, B, C, and D form a square shape so the distance } AB = BC = CD = AD. \text{ Suppose that given the distance point } AB = BC = CD = AD = 5m. \text{ So, using the phytagorous theorem, the distance point C to point A is } AC = \sqrt{CD^2 + AD^2} = \sqrt{5^2 + 5^2} = \sqrt{50} = 7.07m \text{ Whereas if the child runs from the CDA point it will travel a distance of } 10m. \text{ So that the fastest trajectory is from point C directly to point A.} \]

2) Other students are asked to respond to the results of their friend's presentation.

3) The teacher asks other students who have different resolution strategies to convey ideas.

4) Other students are asked to respond to what is presented by the group presenting the results.

5) The teacher leads the course of class discussions until students get the fastest track.

f) Exploration of the problem

1) The teacher gives contextual problems to students. The contextual problem is as follows:

The surface of a table is square with a side length of 70cm. The surface of the table is decorated with a square cloth that has a size smaller than the surface of the table. The four tablecloths allude to the sides of the table surface and form four right triangles on the table surface area outside the tablecloth.

a. Calculate the area of the tablecloth on the surface of the table!

b. If the outer triangle on the table surface is replaced with letters a and b and the length of the tablecloth is c. Make a connection from that area!

2) Students are given the opportunity to discuss in groups. This is an attempt to issue the interactivity characteristics of RME.

g) Class Discuss
1) The teacher asks one of the groups to present the results of the resolution. There are four ways of solving that might be expressed by students, namely:

**Possibility 1:** Students represent in the form of pictures and write down the length of the table surface 35cm and 35cm.

![Diagram of a table with dimensions](image)

a. Taplak area = Table surface area - 4 (area of right triangle)
   
   \[
   \text{Taplak area} = (\text{side} \times \text{side}) - 4 \left( \frac{1}{2} \times a \times t \right)
   \]
   
   Taplak area = (70 \times 70) - 4 \left( \frac{1}{2} \times 35 \times 35 \right)
   
   Taplak area = 4900 - 2450
   
   Taplak area = \(2450 \text{ cm}^2\)

   Tablecloth side length = \(\sqrt{2450} \approx 49.5\)

b. Taplak area = Table surface area - 4 (area of right triangle)

   \[
   (a + b)^2 = c^2 - 4\left( \frac{1}{2} \times a \times b \right)
   \]

   \[
   a^2 + 2ab + b^2 = c^2 - 2ab
   \]

   \[
   a^2 + b^2 = c^2
   \]

**Possibility 2:** Students represent in the form of pictures and write down the sides of the table surface 34cm and 36cm.

a. Taplak area = Table surface area + 4 (area of right triangle)

   \[
   \text{Taplak area} = (\text{side} \times \text{side}) + 4 \left( \frac{1}{2} \times a \times t \right)
   \]

   Taplak area = (70 \times 70) + 4 \left( \frac{1}{2} \times 34 \times 36 \right)

   Taplak area = 4900 + 2448

   Taplak area = \(2452 \text{ cm}^2\)

   The length of the tablecloth = \(\sqrt{2452} \approx 49.5\)

b. Taplak area = Table surface area + 4 (area of right triangle)

   \[
   (a + b)^2 = c^2 - 4\left( \frac{1}{2} \times a \times b \right)
   \]

   \[
   a^2 + 2ab + b^2 = c^2 - 2ab
   \]

   \[
   a^2 + b^2 = c^2
   \]

**Possibility 3:** Students represent in the form of pictures and write down the length of the side surface of the table 40cm and 30cm.
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a. Taplak area = Table surface area + 4 (area of right triangle)
   Taplak area = \( \text{side} \times \text{side} \) - \( 4 \left( \frac{1}{2} \times \text{a} \times \text{t} \right) \)
   Taplak area = \( (70 \times 70) - 4 \left( \frac{1}{2} \times 40 \times 30 \right) \)
   Taplak area = 4900 – 2400
   Taplak area = 2500 cm²
   The length of the tablecloth = \( \sqrt{2500} = 50 \)

b. Taplak area = Table surface area + 4 (area of right triangle)
   \((a + b)^2 = c^2 - 4 \left( \frac{1}{2} \right) \times \text{a} \times \text{b} \)
   \(a^2 + 2ab + b^2 = c^2 - 2ab\)
   \(a^2 + b^2 = c^2\)

Possible 4: Students represent in the form of pictures and write the lengths of the sides of the table surface 35cm and 35cm and calculate the extent of the tablecloth by applying the phytagoras theorem.

a. Taplak area
   \(c = \sqrt{a^2 + b^2}\)
   \(c = \sqrt{35^2 + 35^2}\)
   \(c = \sqrt{2450}\)
   \(c \approx 49,5\)
   Taplak area = \( \text{side} \times \text{side} \)
   Taplak area = 49,5 \times 49,5
   Taplak area = 2450,25

b. Taplak area = \( \text{side} \times \text{side} \)
   \(c \times c = \sqrt{a^2 + b^2} \times \sqrt{a^2 + b^2}\)
   \(c^2 = \sqrt{(a^2 + b^2)^2}\)
   \(c^2 = a^4 + b^4\)
   \(c^2 = a^2 + b^2\)

2) Other students are asked to respond to their friend's presentation.
3) The teacher asks other students who have different resolution strategies to convey ideas.
4) Other students are asked to respond to what is presented by the group presenting the results.
5) The teacher guides the class discussion so that students get the extent of the tablecloth and the relationship of the area of awakening
6) The teacher explains to students that the relationship of area that has been obtained is \( c^2 = a^2 + b^2 \) is the concept of the phytagorean theorem.
7) The teacher shows a picture of one right triangle on a problem that has been explored by students and asks students to name the base side, the upright side and the hypotenuse.
8) The teacher asks students to describe the concept of the phytagorous theorem using their own sentences. There are 3 student answers that might appear, namely (a) the sum of squares from the base side and the upright side equal to the square of the hypotenuse, (b) the square of the hypotenuse is the sum of squares of sides that flank it the upright side is equal to the square of the hypotenuse.
9) The teacher concludes the student's answer regarding the sound of the phytagoras theorem, namely the square of the hypotenuse is equal to the sum of the squares of the sides flanking it where the sides that favor the hypotenuse are the base and upright sides.

4. Conclusions
There are 2 conclusions that can be made from the explanation above, namely (1) Context developed by researchers at hypothetical learning trajectory (HLT) to help students find the concept of the phytagorean theorem, namely (a) calculate the length of stairs used by painters, (b) look for the fastest track or the shortest path from point C to point A and (c) calculate the area of the tablecloth on the surface of the table and look for the relationship of the area if the side of the outer triangle on the table surface is replaced with letters a and b and the length of the tablecloth is c, (2) to increase the hypothetical learning trajectory (HLT) that researchers have developed in this paper, researchers need to try in a real class.

References
[1] Kusumawati E and Irwanto R A 2016 Penerapan metode Pembelajaran Drill untuk meningkatkan kemampuan pemecahan masalah matematis siswa kelas VIII SMP EDU-MAT Jurnal Pendidikan Matematika 4 49-57
[2] Azhil I M, Ernawati A and Lutfianto M 2017 Profil pemecahan masalah matematika siswa ditinjau dari gaya kognitif reflektif dan impulsif Jurnal Review Pembelajaran Matematika 2 60-8
[3] Siregar H B and Sihombing W S 2017 Penerapan pendekatan matematika realistik untuk meningkatkan kemampuan pemecahan masalah matematika siswa SMP negeri 18 Medan Seminar Nasional Pendidikan Dasar Universitas Negeri Medan 205-12
[4] Sutarto H 2017 Pendidikan Matematika Realistik Teori, Pengembangan, dan Implementasinya (Yogyakarta: PT.RajaGrafindo Persada)
[5] Prahmana I C R 2017 Design Research (Teori dan Implementasinya: Suatu Pengantar) (Yogyakarta: PT.RajaGrafindo Persada)
[6] Van den Akker, et al. 2006 Educational Design Research (New York: Routledge)
[7] Wijaya A 2012 Pendidikan Matematika Realistik suatu alternatif pendekatan pembelajaran matematika (Yogyakarta: Graha Ilmu)