Multicore runup simulation by under water avalanche using two-layer 1D shallow water equations

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Abstract. The increasing of layers in shallow water equations (SWE) produces more dynamic model than the one-layer SWE model. The two-layer 1D SWE model has different density for each layer. This model becomes more dynamic and natural, for instance in the ocean, the density of water will decreasing from the bottom to the surface. Here, the source-centered hydro-static reconstruction (SCHR) numerical scheme will be used to approximate the solution of two-layer 1D SWE model, since this scheme is proved to satisfy the mathematical properties for shallow water equation. Additionally in this paper, the algorithm of SCHR is adapted to the multicore architecture. The simulation of runup by under water avalanche is elaborated here. The results show that the runup is depend on the ratio of density of each layers. Moreover by using grid sizes \( N_x = 8000 \), the speedup and efficiency by 2 threads are obtained 1.74779 times and 87.3896 % respectively. Nevertheless, by 4 threads the speedup and efficiency are obtained 2.93132 times and 73.2830 % respectively by similar number of grid sizes \( N_x = 8000 \).

1. Introduction

Simulating fluid flow in estuaries, lake, river, coastal area, etc, can be done by shallow water equations (SWE). The type of this system equations is a hyperbolic type of partial differential equation. The equation is also known as Saint-Venant equations which is derived from the Navier-stokes equations for describing fluid motion [1]. Commonly, the original shallow water equations is described in one layer fluid surface. Moreover, recently the two- and multi-layer shallow water equations are developed for giving the model more dynamic and natural. Here the model consists of various density for each multiple layers.

In two-layer form of SWE, for \( i \in \mathcal{L} = \{1, 2\} \), the equations can be rewritten in compact form as follows

\[
\partial_t U_i + \partial_x F(U_i) = -Q(U)\partial_x S_i, \tag{1}
\]
where,

\[ U_i = \begin{pmatrix} h_i \\ h_i u_i \end{pmatrix}, \quad F(U_i) = \begin{pmatrix} h_i u_i \\ h_i u_i^2 + \frac{g}{2} h_i^2 \end{pmatrix}, \quad (2) \]

\[ Q(U_i) = \begin{pmatrix} 0 \\ g h_i \end{pmatrix}, \quad S_i = \begin{pmatrix} 0 \\ \sum_{k<i} \rho_k h_k + \sum_{k>i} \rho_k h_k + Z + f_{fric} \end{pmatrix}, \quad (3) \]

where \( h(x,t) \) denotes the water height, \( u(x,t) \) the average velocity, \( g \) the gravitational force, and \( Z(x) \) the bottom elevation. The subscripts 1 and 2 are used to describe the first and second layers of water respectively. Moreover, the space and time are shown as \( x \) and \( t \) respectively. Figure 1 shows the configuration of primitive variables of (1-3).

Generally, the numerical methods for approximating one-layer SWE should have following properties\[2, 3, 4, 5\]:

- preserving the positivity of water height \( (h > 0)\),
- gratifying well-balanced,
- satisfying discrete entropy balance, and
- satiating the dry/vacuum condition \( (h = 0)\).

In other side, the numerical scheme of two-layer SWE also should be satisfied previous mathematical properties for each layers. Bouchut, et. al.,\[2\], introduce the source-centered hydrostatic reconstruction (SCHR) scheme which is a robust scheme for approximating the mass and momentum conservation of two-layer SWE. Moreover, the SCHR also is proved to satisfied the previous basic properties of SWE (see \[2\] for more detail).

The aims of this paper are given as follows:

- to elaborate two-layer model with SCHR scheme for describing runup by under water avalanche phenomena and,
- to analyze the computational time of SCHR scheme for multiple cores architecture.

Here, the original algorithm of SCHR scheme will be modified such that the algorithm in accordance with the parallel architecture.
2. Source-centered Hydrostatic Reconstruction (SCHR) Scheme

Follow the paper of Bouchut [2], the source-centered hydrostatic reconstruction (SCHR) scheme is a modified original hydrostatic reconstruction (HR) scheme [3, 4, 6] for two-layer model of SWE due to the problem of momentum conservation. In [7], two schemes using original HR are called splitting and sum scheme. The problem using splitting method with original HR produces wrong solutions due to the total momentum did not conserve. Even, this scheme is developed into a variant of splitting which is called sum scheme still produces wrong solutions.

The detail of numerical scheme of SCHR can be found in [2], however the brief summary of this scheme will be written here. Let start with the discrete domain of one-dimensional space \( \Omega = [0 : L], \ L \in \mathbb{R} \) and time \( T = [0, \infty) \) as follows

\[
\Delta x = \frac{L}{N_x}, \quad N_x \in \mathbb{Z}^+, \quad x_j = j \times \Delta x, \quad j \in \mathcal{M} = \{0, 1, \ldots, N_x\},
\]

\[
t^n = n \times \Delta t, \quad n \in \mathcal{T} = \{0, 1, \ldots\},
\]

where here the uniform of discrete space \( \Delta x \) is considered. The time step \( \Delta t \) will be given depend on the stability condition (15). Therefore, the variable \( U^n_j \) denotes the the value of variable \( U \) at point \( x_j \) and current time \( t^n \).

The finite volume numerical scheme using numerical fluxes SCHR is given as

\[
\frac{U_{i,j}^{n+1} - U_{i,j}^n}{\Delta t} + \frac{\mathcal{F}_l(U_{i,j}^n, U_{i,j+1}^n, b_{i,j}, b_{i,j+1}^n) - \mathcal{F}_r(U_{i,j-1}^n, U_{i,j}^n, b_{i,j-1}^n, b_{i,j}^n)}{\Delta x} = 0,
\]

\( i \in \mathcal{L}, \quad j \in \mathcal{M}, \quad n \in \mathcal{T}, \tag{4} \)

where

\[
b_i = \sum_{k<i} \frac{\rho_k}{\rho_i} h_k + \sum_{k>i} h_k + Z. \tag{5}\]

Moreover the numerical fluxes \( \mathcal{F}_{l/r}(u_l, u_r, b_l, b_r) \) are defined as

\[
\mathcal{F}_l = \mathcal{F}^{HR}_{l} + \mathcal{P}_l, \tag{6}
\]

\[
\mathcal{F}_r = \mathcal{F}^{HR}_{r} + \mathcal{P}_r, \tag{7}
\]

where \( \mathcal{F}^{HR}_{l/r} \) denote the numerical fluxes of the original hydrostatic reconstruction scheme which can be found in [4] and \( \mathcal{P}_{l/r} = (\mathcal{P}^0, \mathcal{P}^1_{l/r}) \).

Under the Proposition 1 in the paper of Bouchut [2], in order to satisfy the semi-discrete entropy condition, thus \( \mathcal{P}^0 \) and \( \mathcal{P}^1_{l/r} \) with \((\cdot)_+ = \max(\cdot, 0)\) and \((\cdot)_- = \min(\cdot, 0)\) are defined as follows

\[
\mathcal{P}^0 = \frac{\kappa ((1 + \theta)u_l + (1 - \theta)u_r)}{2} \tag{8}
\]

\[
\mathcal{P}^1_l = \frac{\kappa(1 + \theta)(h_r - h_l + \Delta b)g}{2} + u_l(\mathcal{P}^0)_+ + u_r(\mathcal{P}^0)_- \tag{9}
\]

\[
\mathcal{P}^1_r = -\left(\frac{\kappa(1 - \theta)(h_r - h_l + \Delta b)g}{2}\right) + u_l(\mathcal{P}^0)_+ + u_r(\mathcal{P}^0)_- \tag{10}
\]
with $\theta$ and $\kappa$ are the parameters for hydrostatic solver which are given as follows

$$\theta = \min \left( 1, \frac{(u_l)_+}{\sqrt{gh_l}} \right) - \max \left( 1, \frac{(-u_r)_+}{\sqrt{gh_r}} \right),$$ (11)

$$\kappa = \min(\bar{\kappa}, 2.5 \times \min(h_l, h_r)), (12)$$

$$\bar{\kappa} = \frac{\Delta b}{2} \begin{cases} 
\frac{(\Delta b - h_l)h_r}{2(hr + \Delta b - h_l)} & \text{if } \Delta b > h_l, \\
0 & \text{if } -h_l \leq \Delta b \leq h_l, \\
\frac{-(h_r + \Delta b)h_l}{2h_l - 2(hr + \Delta b)} & \text{if } \Delta b < -h_r, 
\end{cases}$$ (13)

$$\Delta b = \begin{cases} 
\min(\Delta b, h_l) & \text{if } \Delta b \geq 0, \\
\max(\Delta b, -h_r) & \text{otherwise}. 
\end{cases}$$ (14)

The stability condition of the SCHR scheme is presented as

$$\Delta t \leq \frac{\Delta x}{\lambda(U_{i,j}^n, U_{i,j+1}^n, b_{i,j+1}^n - b_{i,j}^n)}$$ (15)

where $\lambda$ is the wave speed which is defined in [2] in detail.

3. Numerical Algorithms using Multicore Architecture

In this paper, the multicore architecture algorithm for computing the numerical scheme of two-layer will be given. Figure 2 shows the serial and parallel parts of main algorithm. In terms of memory, parallel architecture is divided into two parts, shared memory and distributed memory [8, 9, 10]. Shared memory is used in this simulation to run the parallel algorithm using OpenMP (Open-Multi-Processing) platform. Here, OpenMP is used because it is straightforward and simple to implement to the parallel algorithm [11, 12, 13, 14, 15].

Here is the explanation of parallelization algorithm process:

- **Initialization process**: all variables at $t = 0$ are given.
- **Compute mass**: when time less than final time, mass equation is computed using (4). It will obtain new water depth $h$.
- **Compute momentum**: after mass equation is computed, momentum equation will be computed to deliver new discharge $hu$ using (4).
- **Determine boundary condition**: in this case, two treatments are used to determine the boundary. On left boundary, Dirichlet is used for the velocity and Von-Neumann is used for the water depth. Meanwhile, Von-Neumann is used for both velocity and water depth on the right boundary [16, 17].
- **Update mass and momentum**: all variables are updated.

4. Numerical Simulation and OpenMP Performance

In this section, the numerical simulation of runup due to under water avalanche will be elaborated. Moreover, the parallel performance using OpenMP also will be discussed in this section.
4.1. Simulation of runup by under water avalanche
Here, the configuration of avalanche simulation is illustrated in Figure 3. The detail of this configuration is given as follows

\[ Z(x) = \begin{cases} 0 & \text{if } x < 70, \\ \alpha x + c & \text{otherwise,} \end{cases} \] \quad (16)

\[ \eta_1(x,0) = \max(30, Z(x)), \] \quad (17)

\[ \eta_2(x,0) = \begin{cases} 20 & \text{if } 30 \leq x \leq 60, \\ 0 & \text{otherwise}, \end{cases} \] \quad (18)

\[ u_1(x,0) = u_2(x,0) = 0, \] \quad (19)

where \( \alpha = 1 \) and \( c = -70 \).

In this simulation, the domain is given in \( \Omega = [0 : 500] \) and various ratio of density are given. According to (19), the initial velocity for first and second layer is set to be zero. Thus, the steady condition at initial time is described in Figure 3.

The results by various ratio of density are given in Figure 4. Figure 4(a) is shown the runup profile in three various final time using ratio \( \rho_1/\rho_2 = 0.5 \). Runup profile at \( t = 5.40 \), \( t = 10.4 \) and \( t = 14.2 \) are described by blue straight line, red line points, and green dash line respectively. It shown that the first runup reached \( x = 108.5 \) and wetting the bottom \( (Z) \) with water depth \( h = 0.3592 \) at \( t = 5.40 \). The second runup is observed only reach \( x = 102 \) with water depth \( h = 0.1676 \). Moreover, the third runup is observed reach \( x = 104.5 \) with depth \( h = 0.4423 \).

Here, the first runup is shown has the highest value, meanwhile the second runup is shown has the lowest value. Indeed the impact of first sediment slides (layer 2) on bottom causes the
Figure 3. The scenario of wave propagation simulation in enclosed bay. A slope bottom is given with 45 degrees from \( x = 70 \) until \( x = 500 \). The spatial domain of this simulation is \( \Omega = [0 : 500] \) m. The avalanche will be simulated using second layer (dash line), thus it will generating the wave which is described by first layer (straight line).

energy and velocity of first layer increasing (see Figure 4(b)). When the sediment avalanche moves to the left side, the velocity becomes negative then it makes the second runup loss of energy. However, due to the moves of sediment back again to the right side, then the third runup is pushed the water higher than the second runup.

Similar profiles are observed for ratio of density \( \rho_1/\rho_2 = 0.25 \) in Figure 4(c) and \( \rho_1/\rho_2 = 0.125 \) in Figure 4(e). However using small ratio of density, the runup for three final time are obtained increasing. For instance, the first runup using ratio of density \( \rho_1/\rho_2 = 0.25 \) and \( \rho_1/\rho_2 = 0.125 \) are conducted at \( x = 113.5 \) and \( x = 116 \) respectively. The distance of first and second runup for ratio of density \( \rho_1/\rho_2 = 0.5 \) is 2.00208 meters. Meanwhile, The distance of first and second runup for ratio of density \( \rho_1/\rho_2 = 0.125 \) is 6.0077 meters.

4.2. Parallel performance
Here, the parallel performance using OpenMP platform for simulating the runup due to avalanche is given using the following specifications of computer (Table 1).

Table 1. The specifications of computer which is used for measuring the OpenMP performances.

| Computer Specifications |
|-------------------------|
| Processor | Intel(R) Core(TM) i5-2500 |
| Num of cores (threads) | 4 |
| Clock (MHz) | 2600.00 |
| Memory DDR3 (GB) | 8 |
| Operating System | Ubuntu 16.04.1 LTS |

In Table 2, the parallel performances for using two and four threads are conducted from the average of 10 times experiments. It is shown that the CPU time for each of simulations is accelerated well by OpenMP. By two threads in parallel algorithm, serial time is obtained around 1.65 times the parallel time. Meanwhile, the parallel time is 2.81 times faster than the
Figure 4. The runup profile of the first layer (on the left side) and sediment profile of the second layer (on the right side). (a) The runup profile of first layer using ratio $\rho_1/\rho_2 = 0.5$. (b) The sediment profile of second layer using ratio $\rho_1/\rho_2 = 0.5$. (c) The runup profile of first layer using ratio $\rho_1/\rho_2 = 0.25$. (d) The sediment profile of second layer using ratio $\rho_1/\rho_2 = 0.25$. (e) The runup profile of first layer using ratio $\rho_1/\rho_2 = 0.125$. (f) The sediment profile of second layer using ratio $\rho_1/\rho_2 = 0.125$. 
Table 2. The CPU time, speedup and efficiency of the program.

| $N_x$ | Serial time | Parallel time | Speedup | Efficiency (%) | Serial time | Parallel time | Speedup | Efficiency (%) |
|-------|-------------|---------------|---------|----------------|-------------|---------------|---------|----------------|
| 500   | 17.9467     | 10.9958       | 1.63214 | 81.6071        | 7.00126     | 2.56335       | 64.0838 |
| 1000  | 35.3949     | 21.0549       | 1.68108 | 84.0538        | 12.4394     | 2.84539       | 71.1347 |
| 2000  | 70.1040     | 41.6844       | 1.68243 | 84.1213        | 24.3071     | 2.88410       | 72.1024 |
| 4000  | 139.959     | 83.0917       | 1.68439 | 84.2196        | 48.1345     | 2.90767       | 72.6916 |
| 8000  | 278.771     | 159.499       | 1.74779 | 87.3896        | 95.1009     | 2.93132       | 73.2830 |

In contrast with the CPU time, the performance of efficiency using two threads is observed better than using four threads. For instance in Table 2, using $N_x = 2000$, the efficiency for two and four threads are conducted 84.1213% and 72.1024% respectively.

Profile of speedup and efficiency can be seen in Figure 5. Figure 5(a) is shown the speedup profile for both experiments, two and four threads. Overall, the simulations using four threads have higher speedup than using two threads. Meanwhile in Figure 5(b), the efficiency profile is clearly shown in contrast with the speedup profile. The simulations using four threads have lower efficiency percentage than using two threads.

5. Conclusion
Runup simulation by under water avalanche using two-layer 1D SWE and SCHR method is successfully simulated. The SCHR method is used to approximate the solution of two-layer 1D SWE model. Three simulations have been done with various ratio of density $\rho_1/\rho_2$ to investigate the runup and sediment movement. From the results, the runup and sediment movement are...
observed depend on the ratio of density $\rho_1/\rho_2$. By lighter ratio, the highest runup is obtained and vice versa. Moreover, the experiments of parallel computing in order to analyze the parallel performances using OpenMP are given. Two experiments are elaborated, using 2 and 4 threads. The results of parallel performance are shown satisfying by the observation of acceleration in CPU time for computing. Using a large number of grid sizes $N_x = 8000$, the speedup by 2 and 4 threads are obtained 1.74779 and 2.93132 times respectively. However, the efficiency 87.3896 % and 73.2830 % are observed by using 2 and 4 threads respectively in similar number of grid sizes.

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