A flank modification method for spiral bevel gears based on mismatch topography adjustment

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Abstract
In order to improve the meshing performance or correct contact area of spiral bevel gears, a new method of tooth surface topography modification is proposed in this paper. Firstly, the construction method of tooth surface mismatch topography is studied by means of calculating the deviations between the pinion initial and fully conjugated tooth surface derived from the mating gear. Secondly, by means of describing tooth surface mismatch topography with second order surface approximately, the factors of tooth surface mismatch topography along five directions are calculated, and the mismatch degree of tooth surface is assessed. Subsequently, by adjusting the factors of tooth surface mismatch topography, the pinion target tooth surface is established, and the deviations between the pinion initial and target tooth surface are calculated. The pinion machine settings are corrected by establishing the modification mathematical model between the tooth surface deviations and machine settings of pinion. Finally, the method of tooth surface mismatch modification is executed using a numerical example of a pairs of spiral bevel gears, and the simulation result verifies the effectiveness of the proposed method.

Keywords: Spiral bevel gear, Tooth surface numerical calculation, Tooth surface mismatch adjustment, Machine settings modification, Meshing analysis

1. Introduction

Spiral bevel gears have wide application in automobile axle transmission because of its stable transmission, high load capacity and low rolling noise. With the people higher demands to the automobile quality, the higher and higher requirement for the meshing performance of spiral bevel gears become an inevitable trend. In order to reduce vibration and lower noise, the tooth surface modification technology as a valid method is widely applied into the design and manufacture process of spiral bevel gears. Many researchers have made lots of studies for the tooth surface modification method of spiral bevel gears.

The traditional modification methods mainly include changing the motion relation in the process of gear generating or modifying the blade profile shape. In the aspects of motion modification, Stadtfeld and Gaiser (2000) used the universal motion concept to introduce a tooth surface modification based on fourth-order kinematical motion. Wang and Fong (2006) studied a tooth surface modification method with the modified radial motion by predetermining fourth-order motion curve. Zhang et al., (2007) studied the modified roll motion method to modify the tooth lengthwise curvature of spiral bevel gears based on CNC machine. Liu et al., (2011) studied the tooth surface modification method with modified roll motion for the grinding process of spiral bevel gears, subsequently. For the blade profile modification, Nie et al., (2012) studied a modification method with arc blade to modify the tooth profile of cycloid bevel gears. Zhang et al., (2016) analyzed the influence law of arc blade on tooth meshing performance for duplex helical method of spiral bevel gears.

Compared with the traditional modification methods, the tooth surface topography modification technology can...
modify tooth surface through the whole topography structure, and the modification process can be represented by the graph easily, and the modification process becomes more intuitive and flexible, therefore, it becomes the current research hot topic. Shih and Fong (2007) studied the flank modification methodology based on ease-off topography for face-hobbing hypoid gears, on this basis Shih (2010) proposed a novel ease-off topography modification method which can pre-control the meshing performance. Su et al., (2013) studied a higher-order polynomial function of TE and present the advantages of it over second-order and forth-order polynomial functions of TE in terms of load sharing, bending, tensile, and contact stresses. Su and He (2014) studied the high precision tooth surface modification method of spiral bevel gears, which can modify the pinion tooth surface by predetermining the overlap ratio, transmission error curve with parabolic shape and contact elliptic parameters, Cao et al., (2015) studied the ease-off flank topography design method for aviation spiral bevel gears with higher-order transmission errors, Du et al., (2015) studied the active tooth surface design method for double-cut cycloid bevel gears, which can modify the pinion tooth surface by predetermining the meshing performance parameters based on the fully conjugate pinion tooth surface, and Du and Fang (2016) proposed the active tooth surface design methodology to be used in the phase of trial-manufacture and only one time of correction of the pinion real tooth surface.

The above researches are all based on the fully conjugated pinion tooth surface to redesign the pinion target tooth surface by pro-controlling the meshing performance parameters, however, these approaches are not convenient for modifying the existed pinion tooth surface, meanwhile the modification process is not very intuitive. Therefore, a new tooth surface topography modification method for spiral bevel gears is proposed in this paper, which can modify contact area by adjusting the current mismatch relation between the pinion and gear. The method is more intuitive and effective, so it is easy to be applied into the production practice.

The whole modification process mainly includes the following basic steps: (1) Establishing the mathematical model for the generating process of spiral bevel gears and deriving the theoretical equations of tooth surface. (2) Establishing the current tooth surface mismatch relation, calculating the factors of tooth surface mismatch topography and assessing the current mismatch degree of tooth surface. (3) Adjusting the factors of tooth surface mismatch topography, building the pinion target tooth surface and calculating the deviations between the pinion initial and target tooth surface. (4) Establishing the mathematical model of tooth surface deviations correction, building the sensitivity matrix with respect to the machine settings of pinion and calculating the machine settings modifications using linear regression method. (5) Developing the software of tooth surface mismatch modification and building the visual modification process between the tooth surface mismatch topography and tooth contact analysis (TCA) simulation.

2. Derivation of tooth surface equations
2.1 Mathematical model for cutter head

In the process of spiral bevel gears cutting, the cutter head rotates around its axis, and the blade projection pass through the center of cutter head. According to the structure of cutter head and the motion relation, the mathematical model of cutter head is built, as shown in Fig.1. Coordinate system $S_t(X, Y, Z)$ is rigidly connected to the cutter head, rotating together with the cutter head. $S_a(X, Y, Z)$ is auxiliary coordinate system that describes the initial position of cutter head. Assuming that the cutter head rotates in the clockwise direction, the variable $\theta$ represents the rotation angle of the cutter head, here, $j=1,2$ designates the pinion and gear, respectively, below is the same. $P_t$ and $P_a$ are the blade top point of inner and outer blade, and the corresponding cutter radius are represented by variables $r_{ta}$ and $r_{ia}$, respectively. $G_t$ and $G_a$ are the random point of inner and outer blade, and the corresponding parameters are represented by variable parameters $u_{ta}$ and $u_{ia}$, respectively. $\alpha_{ta}$ and $\alpha_{ia}$ are the profile angle of inner and outer blade, assuming that the inner profile angle $\alpha_{ta}$ takes the positive value and the outer profile angle $\alpha_{ia}$ takes the negative value.

According to the relative position relation of coordinate systems in Fig.1, the position vector of cutter blade in the coordinate system $S_a(X, Y, Z)$ can be represented by the following equation:

$$r_k(u_{ta},\theta)=[(r_{ta}+u_{ta}\sin\alpha_{ta})\cos\theta]
\begin{array}{c}
(r_{ta}+u_{ta}\sin\alpha_{ta})\sin\theta \\
n_{ta}\cos\alpha_{ta}
\end{array}1$$

(1)

where the subscript $k=1,A$ represents the inner and outer blade, respectively, below is the same.

The unit normal vector of cutter blade is represented by the following equation:
2.2 Mathematical model for gear generating

During gear generating process, the tooth surface of imaginary generating gear is formed by the locus of cutter blades, and the work gear tooth surface is formed by the tooth surface of imaginary generating gear because of the conjugate motion relation between the imaginary generating gear and work gear. According to the relative position and motion relation between the cutter head and work gear, the basic mathematical model of gear generating is built, as shown in Fig.2. Coordinate system $S(X_m,Y_m,Z_m)$ is rigidly connected to the machine, and it lies in the blade top plane. Coordinate system $S(X_p,Y_p,Z_p)$ is rigidly connected to the cradle structure, rotating around the cradle axis in the counterclockwise direction in the process of gear generating, and the variable $\phi_j$ is the cradle rotation angle. Coordinate system $S(X_w,Y_w,Z_w)$ is rigidly connected to the work gear, rotating around the gear axis in the counterclockwise direction in the process of gear generating, and the variable $\phi_w$ is the work gear rotation angle. Coordinate systems $S(X_q,Y_q,Z_q)$ and $S(X_p,Y_p,Z_p)$ are the auxiliary coordinate systems that describe the relative position and motion relation between the imaginary generating gear and work gear. $q_j$ is the basic cradle angle, $s_r$ is the radial setting, $\sigma_{m0}$ is the machine root angle, $E_m$ is the vertical offset, $X_b$ is the sliding base feed setting, $X_g$ is the increment of machine center to back, $R_b$ is the roll ratio. In the process of gear generating, the angle $\phi_j$ and $\phi_w$ satisfy the relation $\phi_j = R_b\phi_w$.

![Coordinate systems for the cutter head](image1)

**Fig. 1** Coordinate systems for the cutter head

![Coordinate systems between the imaginary generating gear and work gear](image2)

**Fig. 2** Coordinate systems between the imaginary generating gear and work gear
By transforming matrices from $S_o$ to $S_r$, the equation of work gear tooth surface can be represented as follows:

$$
r_w(u_{ij}, \theta_j) = M_{w0}M_{w1}M_{w2}M_{r0}r_w(u_{ij}, \theta_j)
$$

Here,

$$
M_w = M_{w0}M_{w1}M_{w2}M_{r0} = \begin{bmatrix}
1 & 0 & 0 & 0 & \cos \delta_{ij} & 0 & \sin \delta_{ij} & -X_{ij} & 1 & 0 & 0 & 0 \\
0 & \cos \phi_i & \sin \phi_i & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & E_{ij} \\
0 & -\sin \phi_i & \cos \phi_i & 0 & 0 & 0 & \cos \delta_{ij} & 0 & 0 & 1 & -X_{ij} & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1
\end{bmatrix}
$$

(4)

The unit normal vector of work gear tooth surface can be represented by the following equation:

$$
n_{wj}(u_{ij}, \theta_j) = \frac{\partial r_w}{\partial \theta_j} \times \frac{\partial r_w}{\partial u_{ij}}
$$

(5)

In the process of gear generating the meshing equation can be represented as follows:

$$
\Phi(u_{ij}, \theta_j, \phi_i) = n_{wj}(\psi^{(w)}) = \begin{bmatrix}
\frac{\partial r_w}{\partial \theta_j} & \frac{\partial r_w}{\partial u_{ij}} & \frac{\partial r_w}{\partial \phi_i} & 0
\end{bmatrix} \frac{\partial r_w}{\partial \phi_i} = 0
$$

(6)

where $\psi^{(w)}$ is the relative velocity between the imaginary generating gear and work gear in coordinate system $S_r$. The relation $\phi_i = \phi_i(u_{ij}, \theta_j)$ can be derived according to the Eq. (6), and substituting it into Eqs. (3) and (5), respectively, the equation of work gear tooth surface $r_w(u_{ij}, \theta_j)$ and the unit normal vector $n_{wj}(u_{ij}, \theta_j)$ can be obtained, which only include two variable parameters.

3. Establishing of tooth surface mismatch topography

3.1 Derivation of pinion base tooth surface

Figure 3 shows the assembly position relation between the pinion and gear. Coordinate system $S_o(X_h, Y_h, Z_h)$ is rigidly connected to the rolling test machine which is still. Coordinate system $S_i(X_i, Y_i, Z_i)$ is rigidly connected to the gear, rotating together with the gear, the rotation angle is represented by variable $\phi_i$. Coordinate system $S_j(X_j, Y_j, Z_j)$ is rigidly connected to the pinion, rotating together with the pinion, the rotation angle is represented by variable $\phi$. The origins $O_2$ and $O_1$ are the design crossing points of the gear and pinion, respectively. $\Sigma$ is the shaft angle, $E$ is the offset distance between the pinion and gear.

Fig. 3 Assembly coordinate systems between the pinion and gear
According to the Eqs. (3) and (5) we can acquire the gear tooth surface equation \( r_k(u, \theta_k) \) and the unit normal vector \( n_k(u, \theta_k) \), by transforming matrices \( M_{k2} \) and \( L_{k2} \) from \( S_2 \) to \( S_1 \), we can acquire the following equations:

\[
\begin{align*}
    r_k &= M_{k2} r_2 \\
    n_k &= L_{k2} n_2 \\
\end{align*}
\]  

(7)

When the pinion and gear are the fully conjugate meshing, the rotation angles \( \phi_2 \) and \( \Phi_2 \) must satisfy the equation relation \( \phi_2/\Phi_2 = z_1/z_2 \), here \( z_1 \) and \( z_2 \) are the tooth number of pinion and gear, respectively.

During the meshing process between the pinion and gear, the meshing equation in the coordinate system \( S_1 \) is written as follows:

\[
\Phi = f(u_i, \theta_i, \theta_k) = n_i \cdot v^{(i)}(u_i, \theta_k) = 0
\]  

(8)

where \( v^{(i)}(u_i, \theta_k) \) is the relative velocity between the pinion and gear in coordinate system \( S_1 \).

According to the meshing Eq. (8) we can obtain the equation \( \Phi = \phi(u_i, \theta_i) \), so by means of the transformation matrices \( M_{ik} \) and \( L_{ik} \) from \( S_1 \) to \( S_i \), the position vector and unit normal vector of the fully conjugated pinion can be represented as follows:

\[
\begin{align*}
    r^{-1}(u_i, \theta_i) &= M_{ik} r_i(u_i, \theta_i) \\
    n^{-1}(u_i, \theta_i) &= L_{ik} n_i(u_i, \theta_i) \\
\end{align*}
\]  

(9)

From Eq. (9) we obtain the fully conjugated pinion tooth surface derived from the mating gear, here we call the fully conjugated pinion tooth surface as the pinion base tooth surface, represented by vector \( C_{r} \).

3.2 Numerical calculation of tooth surface

At present the ease-off topography is often used to reflect the tooth surface mismatch relation between the pinion and the mating gear (Kolivand et al., 2009; Artoni et al., 2013), and the ease-off values are obtained by calculating the difference of topographical grid points, in this paper the proposed tooth surface mismatch topography is equal to the ease-off topography. Before calculating the ease-off values, the tooth surface needs to be discretized and numerically calculated. The calculation steps mainly include: (a) Dividing grids in the rotary projection plane of tooth surface, and calculating the data of grid points;(b) Building the numerical equation according to the corresponding relation of grid points between the rotary projection plane and the tooth surface;(c) Solving the equation with numerical algorithm to acquire the data of tooth surface points.

Figure 4 shows the grids dividing in the rotary projection plane of tooth surface, where \( A_0, A_1, A_2, A_3 \) are the four edge point of tooth surface in the projection plane. The origins \( O_z \) is the design crossing point. In the coordinate system \( S_z(X_z, Y_z, Z_z) \), the axis \( X_z \) pass through the gear axis.

![Fig. 4 Grids dividing of tooth surface](image)

Assuming that the coordinate of any grid point \( P_i (i = 1\ldots5, j = 1\ldots9) \) is represented as \((X_{r_i}, R_{r_i})\) in the projection plane, and its position vector is represented as \( r_i = (x_i(u, \theta), y_i(u, \theta), z_i(u, \theta)) \), according to the corresponding relation of grid points between the rotary projection plane and the tooth surface, we can obtain the following equations:
where \( i = 5 \) and \( j = 9 \) designate the row number and the column number of grid points, respectively. Solving the Eq. (10) with Newton-Raphson algorithm, we can obtain the variable parameters \((u_x, \theta)\), substituting them into the equations of tooth surface, the data of tooth surface grid points can be acquired.

### 3.3 Calculation of tooth surface mismatch values

The pinion initial tooth surface can be obtained from equation (3), and is represented by vector \( r_{GM}^{(G)} \). The deviations between the pinion initial and base tooth surface are the tooth surface mismatch values. In order to calculate the deviations easily, the pinion initial tooth surface needs to be rotated around its axis to make the middle grid point \( M \) of the initial tooth surface coincide with the middle grid point \( M \) of the base tooth surface. \( \Delta \theta \) is the rotation angle. Figure 5 shows the position relation between the pinion initial and base tooth surface.

![Fig. 5 Position relation between the pinion initial and base tooth surface](image)

Assuming that the coordinate of any point in the pinion initial tooth surface is represented as \((x_i, y_i, z_i)\), it will become coordinate \((x', y', z')\) after rotating angle \( \Delta \theta \), so we can obtain the following matrix equations:

\[
\begin{pmatrix}
    x_i \\
    y_i \\
    z_i \\
    1
\end{pmatrix} =
\begin{pmatrix}
    1 & 0 & 0 & 0 \\
    0 & \cos \Delta \theta & -\sin \Delta \theta & 0 \\
    0 & \sin \Delta \theta & \cos \Delta \theta & 0 \\
    0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
    x' \\
    y' \\
    z' \\
    1
\end{pmatrix}
\]

(11)

Figure 6 shows the position relation between the pinion rotated initial and base tooth surface. The middle grid point of the pinion initial tooth surface coincides with that of the pinion base tooth surface after rotating. \( M \), is any point in the pinion base tooth surface, and its coordinate is \((x_M, y_M, z_M)\). Making the vertical line to the pinion rotated initial tooth surface trough point \( M \), the crossing point is point \( M' \), and the connect line between point \( M \) and \( M' \) coincides with the unit normal vector \( n_{\|} \). \( \Delta e = |M_M'| \) is the tooth surface deviation between the pinion rotated initial and base tooth surface.

![Fig. 6 Deviation between the pinion rotated initial and base tooth surface](image)

The coordinate of point \( M \), in the pinion initial tooth surface can be represented by the position vector \( r_{Mi} = (z_{Mi}(u_z, \theta), y_{Mi}(u_y, \theta), z_{Mi}(u_z, \theta)) \) after rotating, and the unit normal vector can be represented as

\[
\begin{pmatrix}
    x_i \\
    y_i \\
    z_i \\
    1
\end{pmatrix} =
\begin{pmatrix}
    1 & 0 & 0 & 0 \\
    0 & \cos \Delta \theta & -\sin \Delta \theta & 0 \\
    0 & \sin \Delta \theta & \cos \Delta \theta & 0 \\
    0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
    x' \\
    y' \\
    z' \\
    1
\end{pmatrix}
\]

(11)
According to the vector relation in Fig. 6, we can obtain the following equations:

\[
\begin{align*}
\nu_{x}(u, \theta) + \Delta \nu_{x}(u, \theta) &= x_{u_{1}} \\
\nu_{y}(u, \theta) + \Delta \nu_{y}(u, \theta) &= y_{u_{1}} \\
\nu_{z}(u, \theta) + \Delta \nu_{z}(u, \theta) &= z_{u_{1}}
\end{align*}
\]  

(12)

Solving the Eq. (12) with Newton-Raphson algorithm, we can obtain the variable parameters \((u, \theta)\) and tooth surface deviation \(\Delta \varepsilon\). Substituting all grid points coordinates into the Eq. (12), the deviations \(\Delta \varepsilon_{i} (i = 1 \sim 45)\) in all grid points can be acquired, and we call the deviations \(\Delta \varepsilon_{i} (i = 1 \sim 45)\) as the tooth surface mismatch values.

4. Assessment of tooth surface mismatch degree

Because the tooth surface mismatch values reflect the deviations between the pinion initial and base tooth surface, their nature is also the tooth surface deviations. Figure 7 shows the topography of tooth surface deviations.

![Fig. 7 Topography of tooth surface deviations](image)

Generally speaking, the tooth surface deviations can be approximately represented with second-order surface by the following equation:

\[
\Delta \varepsilon = \Delta \varepsilon + d\beta X + d\alpha Y + LBX^2 + HBY^2 + dvXY
\]  

(13)

where \((X, Y)\) is the coordinate of grid point along tooth length direction and tooth height direction in coordinate system \(S(X, Y)\), which origin is the middle grid point of tooth surface. \(\Delta \varepsilon\) represents the tooth thickness deviation, here \(\Delta \varepsilon = 0\), because the backlash have been eliminated by rotating the tooth surface. \(d\beta\) represents the factor of spiral angle difference, \(d\alpha\) represents the factor of pressure angle difference, \(LB\) represents the factor of tooth lengthwise crowding, \(HB\) represents the factor of tooth profile crowing, \(dv\) represents the factor of tooth longitudinal twist. The five factors reflect the mismatch degree of tooth surface along the five directions.

Assuming that the tooth surface deviations in forty five grid points are known, the Eq. (13) can be represented by the following matrix equation:

\[
\begin{bmatrix}
\Delta \varepsilon_{1} \\
\Delta \varepsilon_{2} \\
\Delta \varepsilon_{3} \\
\vdots \\
\Delta \varepsilon_{45}
\end{bmatrix}
= 
\begin{bmatrix}
X_1 & Y_1 & X_1^2 & Y_1^2 & X_1Y_1 \\
X_2 & Y_2 & X_2^2 & Y_2^2 & X_2Y_2 \\
X_3 & Y_3 & X_3^2 & Y_3^2 & X_3Y_3 \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
X_{45} & Y_{45} & X_{45}^2 & Y_{45}^2 & X_{45}Y_{45}
\end{bmatrix}
\begin{bmatrix}
d\beta \\
d\alpha \\
LB \\
HB \\
dv
\end{bmatrix}
\]  

(14)

From Fig. 4 we can obtain the coordinate \((X_{uj}, Y_{uj})\) in coordinate system \(S_{j}(X, Y)\), transforming it into the coordinate system \(S(X, Y)\), we can obtain the coordinate \((X_{i}, Y_{i}) (i = 1 \sim 45)\) of each grid point. Because the number of unknown parameters is less than the number of equations, the Eq. (14) is overdetermined. The factors \((d\beta, d\alpha, LB, HB, dv)\) can be solved by means of the least squares method.

5. Adjusting tooth surface mismatch relation and modifying machine settings

5.1 Adjustment of the tooth surface mismatch relation

The current tooth surface mismatch relation between the pinion and gear initial tooth surface has been built. Because the tooth surface mismatch relation reflects the meshing state between the pinion and gear tooth surface, the improvement of current meshing performance can be achieved by adjusting the tooth surface mismatch relation. The
calculation steps mainly include: (a) Solving the five factors of the current tooth surface mismatch topography; (b) Adjusting the five factors to produce the new tooth surface mismatch topography, and calculating the deviations between the pinion initial and target tooth surface produced by adjusting; (c) Modifying the machine settings of pinion to eliminate the deviations.

Figure 8 shows the difference relation between the pinion initial and target tooth surface. The mismatch values of current tooth surface mismatch topography is represented as $\Delta \varepsilon_i (i = 1 - 45)$, and the mismatch values of target tooth surface mismatch topography is represented as $\Delta \varepsilon_i (i = 1 - 45)$, thus the deviations of the topographical grid points between the pinion initial and target tooth surface are represented as $\Delta \delta_i = \Delta \varepsilon_i - \Delta \varepsilon_i$.

![Fig. 8 Difference relation between the pinion initial and target tooth surface](image)

**5.2 Modification of the pinion machine settings**

The pinion initial tooth surface is represented by the position vector $r^{(i)}(u_1, \theta, \psi_i)$ and the unit normal vector $n^{(i)}(u_1, \theta, \psi_i)$ which includes the tooth surface parameters $(u_1, \theta)$ and the machine settings $\psi_i (q = 1 - n)$, here, $n$ is the number of machine settings. The pinion target tooth surface is represented as $r^{(i)}$, and the difference vector between the pinion initial and target tooth surface can be represented by the following equation:

$$r = r^{(i)} - r^{(i)}(u_1, \theta, \psi_i) \quad (15)$$

The normal variation of difference vector can be represented by the following equation:

$$\delta r \cdot n^{(i)} = - \left( \frac{\partial r^{(i)} \cdot n^{(i)}}{\partial \psi_1} \times \delta \psi_1 + \frac{\partial r^{(i)} \cdot n^{(i)}}{\partial \psi_2} \times \delta \psi_2 + \ldots + \frac{\partial r^{(i)} \cdot n^{(i)}}{\partial \psi_n} \times \delta \psi_n \right) \quad (16)$$

where $\Delta \delta = \delta r \cdot n^{(i)}$ represents the tooth surface deviations, assuming that the tooth surface deviations of the forty five grid points are known, the Eq. (16) can be represented by the following matrix form:

$$\begin{bmatrix} \Delta \delta_1 \\ \Delta \delta_2 \\ \Delta \delta_3 \\ \vdots \\ \Delta \delta_{45} \end{bmatrix} = \begin{bmatrix} \eta_{11} & \eta_{12} & \eta_{13} & \ldots & \eta_{1n} \\ \eta_{21} & \eta_{22} & \eta_{23} & \ldots & \eta_{2n} \\ \eta_{31} & \eta_{32} & \eta_{33} & \ldots & \eta_{3n} \\ \vdots & \vdots & \vdots & \ldots & \vdots \\ \eta_{45} & \eta_{45} & \eta_{45} & \ldots & \eta_{45} \end{bmatrix} \begin{bmatrix} \Delta \psi_1 \\ \Delta \psi_2 \\ \Delta \psi_3 \\ \vdots \\ \Delta \psi_n \end{bmatrix} \quad (17)$$

Equation (17) can be simplified by the following form:

$$\{ \Delta \delta \} = \{ S \} \{ \Delta \psi \} \quad (i = 1 - 45, q = 1 - n) \quad (18)$$

where $\{ \Delta \delta \} (i = 1 - 45)$ represents the tooth surface deviations of the topographical grid points, $S$ is the sensitivity matrix with respect to the machine settings, which component is $\eta_{iq} (q = 1 - n)$ approximately represented by the difference equation $\eta_{iq} = \Delta \delta_i / \Delta \psi_q \cdot \{ \Delta \psi \}$ represents the modifications of the machine settings.

In order to avoid ill conditional matrix causing the numerical divergence, the Eq. (18) is often solved with singular value decomposition method (Shih and Fong, 2008; Tang et al, 2012), the solved result can be represented by the following equation:

$$\{ \Delta \psi \} = \{ S \}^{-1} \{ S \}^\top \{ \Delta \delta \} \quad (19)$$

6. Flow chart of tooth surface mismatch modification

In order to illustrate the process of tooth surface mismatch modification clearly, Figure 9 gives the flow chart of modification method. In the modification process of flow chart, if the initial meshing result can’t meet our expectations, we will adjust the five factors of the initial tooth surface mismatch topography to modify the pinion tooth surface until the final meshing performance meet the requirement. The whole modification process may be executed repeatedly, but it can be accomplished by use of the developed software of tooth surface mismatch topography modification (see Fig.10). The software makes the modification process become intuitive and effective by establishing the visual graph mapping between the tooth surface mismatch topography and TCA results.

![Flow chart of tooth surface modification](image)

**Fig. 9 Flow chart of tooth surface modification**

![Developed software of tooth surface mismatch topography modification](image)

**Fig. 10 Developed software of tooth surface mismatch topography modification**
7. Numerical example

In order to illustrate the effectiveness of tooth surface mismatch modification methodology, the numerical example applies the method to modify a pairs of face-milling spiral bevel gear which are widely applied to drive axle of the truck. The pinion and the gear are all manufactured by the generating method. The basic geometrical parameters and the machine settings for the numerical example are shown in Tables 1 and 2, respectively.

Figure 11 shows the initial simulation results corresponding to the initial pinion and gear tooth surface. Table 3 gives the five factors of the initial tooth surface mismatch topography.

From Fig.11 we can see that the initial contact area and transmission error in unloaded condition are fine, however, in the actual working condition, the automobile bench test showed that the loaded contact pattern exceeded the tooth surface boundary, meanwhile the edge contact occurred on the top and root of gear tooth surface because of heavy load and drive axle housing deformation. The real loaded meshing state caused big transmission noise, therefore, we need make modification for the initial contact state. In order to avoid the loaded contact pattern exceeding the tooth surface boundary, the unloaded contact pattern length should be shorten and the bias in trend should be weaken. In order to eliminate the edge contact, the profile crowning should be increased, thereby the amplitude of unloaded transmission error need be increased. So our modification goal is to shorten the length of contact area, increase the transmission error and low the bias in trend of contact area.

| Items                      | Pinion | Gear  |
|----------------------------|--------|-------|
| Number of teeth            | 26     | 33    |
| Spiral hand                | LH     | RH    |
| Shaft angle /°              |        | 90    |
| Transverse module/mm       | 7.57   |       |
| Mean spiral angle /°        | 35     |       |
| Pressure angle /°           | 20     |       |
| Face width/mm              | 46     |       |
| Whole tooth height /mm      | 13.535 |       |
| Addendum height/mm         | 6.094  | 6.094 |
| Outer pitch diameter /mm   | 196.82 | 249.81|
| Pitch angle /°              | 38.234 | 51.766|
| Face angle /°               | 40.913 | 54.445|
| Root angle /°               | 35.554 | 49.087|

| Items                              | Pinion(Concave) | Pinion(Convex) | Gear  |
|------------------------------------|-----------------|----------------|-------|
| Cutter radius/mm                   | 112.07          | 116.234        | 114.3 |
| Profile angle /°                   | -18             | 22             | 22/-18|
| Radial setting/mm                  | 112.737         | 118.739        | 117.084|
| Cradle angle /°                    | 53.355          | 53.839         | 53.097|
| Machine root angle /°              | 35.554          | 35.554         | 49.087|
| Vertical offset/mm                 | -3.325          | 0.8958         | 0     |
| Increment of machine center to back/mm | -0.811        | -0.383         | 0     |
| Sliding base feed setting/mm       | 0.471           | 0.223          | 0     |
| Roll ratio                         | 1.591399        | 1.603634       | 1.271696|
Generally speaking, the factors of tooth surface mismatch topography have direct influence on the meshing performance. For example, $d\beta$ can produce main influence on the position of contact area in tooth lengthwise direction, $da$ can produce main influence on the position of contact area in tooth profile direction, $LB$ can produce main influence on the length of contact ellipse, $HB$ can produce main influence on the amplitude of the transmission error. $dv$ can produce main influence on the bias degree of contact area. In order to achieve our modification goal, according to the above influence law we decide to make the adjustment for the factors $LB$, $HB$ and $dv$, the factors after adjustment are shown in Table 4.

### Table 4 Factors after adjustment

|       | $d\beta$ | $da$     | $LB$     | $HB$     | $dv$    |
|-------|----------|----------|----------|----------|---------|
| Drive side | 0        | -0.00088 | 0.000108 | 0.000769 | 0.0003778 |
| Coast side | 0.000265 | 0.000075 | 0.0001312 | 0.000615 | -0.0003174 |

Table 3 Factors of initial tooth surface mismatch topography

|       | $d\beta$ | $da$     | $LB$     | $HB$     | $dv$    |
|-------|----------|----------|----------|----------|---------|
| Drive side | 0        | -0.00088 | 0.000108 | 0.000769 | 0.0003778 |
| Coast side | 0.000265 | 0.000075 | 0.0001312 | 0.000615 | -0.0003174 |
After adjusting the factors, the deviations between the pinion initial and target tooth surface are shown in Fig.12. In order to improve the correction accuracy, selecting the machine settings of pinion \((S_1, q_1, c_{m1}, X_{m1}, c_{g1}, M_{\delta}, R_{ni})\) as the modification parameters, the tooth surface deviations for pinion concave and convex are modified, respectively, and the modifications are shown in Table 5.

![Fig. 12 Deviations between the pinion initial and target tooth surface (\(\mu m\))](image)

(a) Tooth surface deviations for pinion concave  (b) Tooth surface deviations for pinion convex

Table 5 Modifications of the machine settings of pinion

| Items                          | Pinion(Concave) | Pinion(Convex) |
|-------------------------------|-----------------|----------------|
| Radial setting/mm             | -2.334          | 4.871          |
| Cradle angle /°               | -1.482          | -1.918         |
| Machine root angle /°         | -0.169          | 0.732          |
| Vertical offset/mm            | -1.438          | 5.576          |
| Increment of machine center to back/mm | -2.644          | 3.173          |
| Sliding base feed setting/mm  | -0.815          | -7.977         |
| Roll ratio                    | -0.04           | 0.071          |

After modifying the machine settings of pinion, the deviations between the pinion modified and target tooth surface are shown in Fig.13.

![Fig. 13 Deviations between the pinion modified and target tooth surface (\(\mu m\))](image)

(a) Tooth surface deviations for pinion concave  (b) Tooth surface deviations for pinion convex

According to the deviations shown in Fig.13, we can acquire the conclusion that the modified pinion tooth surface is nearly in accordance with the pinion target tooth surface because of the very small deviations between the pinion modified and target tooth surface. Figure 14 shows simulation results corresponding to the modified pinion and gear tooth surface.

Contrasting the simulation results shown in Figs.11 and 14, we can get the conclusions that after modifying the tooth surface mismatch topography, the bias in trend of contact area weakens, the amplitude of transmission error increases and the length of major axis of contact ellipse becomes small (see Fig.15). The simulation results are nearly in accordance with our expectations. This indicates that the meshing performance can be improved effectively by adjusting the factors of tooth surface mismatch topography to modify the pinion tooth surface.
Fig. 14 Simulation results for the modified pinion tooth surface

(a) Mismatch topography for drive side (μm)

(b) Mismatch topography for coast side (μm)

(c) Contact area for gear convex

(d) Contact area for gear concave

(e) Transmission error for drive side

(f) Transmission error for coast side

Fig. 15 Contrast of contact ellipse major axis length between the initial and modified contact area
8. Conclusions

From the above investigations the following conclusions can be drawn:

(1) A new tooth surface topography modification method for face-milling spiral bevel gears has been proposed, on the basis of building the current tooth surface mismatch relation, adjusting the factors of tooth surface mismatch topography to modify the pinion tooth surface, the meshing performance of tooth surface can be improved.

(2) The paper introduces the whole flow chart of tooth surface mismatch modification, and develops the software of tooth surface mismatch modification. The modification process becomes intuitive and flexible by establishing the dynamic visualization relation between tooth surface topography and TCA simulation.

(3) The applied numerical example verifies the effectiveness of the proposed method, and the modification method can be extended to other gear types.

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