Sea Quark Chemistry and Correlations in the Initial Conditions of Heavy Ion Collisions

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Abstract. QCD dynamics are directly studied in two different but intersecting contexts: high-temperature QCD in heavy-ion collisions and zero-temperature hadronic structure in electron-hadron collisions. The approaches to high-energy nuclear physics in these two contexts are very different, however, and bringing the two fields into dialogue can reveal new opportunities. In this paper I highlight one interesting intersection between the two: leveraging the sea quark chemistry of the proton to study charge transport in top-energy heavy-ion collisions.

1. ICCING: Initial Conserved Charges in Nuclear Geometry

The equations of motion for hydrodynamics solve the gross conservation laws for the energy-momentum tensor $T^{\mu\nu}$ and the conserved currents $N^{\mu}_i$ where $i = B, S, Q$ denotes the conserved charges baryon number, strangeness, and electric charge, respectively:

$$\partial_\mu T^{\mu\nu} = 0$$
$$\partial_\mu N^{\mu}_i = 0$$

with initial conditions $T^{\mu\nu}_0, N^{\mu}_i$ specified at at a fixed-proper-time hypersurface $\tau_0$. A range of initial condition models exist for $T^{\mu\nu}_0$ with varying levels of theoretical input, from models with only nucleonic degrees of freedom [1] to those which incorporate calculations or models of nucleonic substructure [2, 3]. All models must specify the energy density $T^{00}$, while some models also incorporate the initial momentum flux $T^{0i}$ or the initial shear tensor $T^{ij}$. At top collider energies, baryon stopping is suppressed and the QGP carries no net charge: $\mu_B = \mu_S = \mu_Q = 0$; for this reason, the initial charge currents $N^{\mu}_i$ are usually set to zero.

However, while the total charge of the QGP at top collider energies may be zero, that does not mean that the charge is homogeneously zero, locally. Consider the initial state from the point of view of perturbative QCD. At small $x$ (corresponding to high energy), the dominant degrees of freedom are gluons, as seen in the parton distribution functions extracted from HERA (Fig. 1). The steep decrease in the valence quark contributions $xv_u, xv_d$ at small $x$ is equivalent to the suppression of baryon stopping in this regime. Since gluons carry zero baryon number, strangeness, and electric charge, an initial state energy profile $T^{\mu\nu}_0$ with no charge currents $N^{\mu}_i = 0$ can be naturally interpreted as composed of gluons. This is indeed the correct leading-order picture of the initial state.

But as can already be seen at the level of the PDFs, there are substantial corrections to the initial state from sea quark fluctuations, which do carry the conserved charges B, S, Q. These $q\bar{q}$
Figure 1. Parton distribution functions (PDFs) extracted from HERA showing the comparable growth rates of gluons and sea quarks at small $x$. This figure reproduced following Creative Commons Attribution License guidelines from Fig. 19(b) of Ref. [4].

Figure 2. Left: the ICCING algorithm as it begins to resample an event, selecting blobs of energy and giving them the chance to split into $q\bar{q}$ pairs. Right: a different event after being fully resampled by the ICCING algorithm, resulting in a reconstructed energy density as well as new distributions of the three conserved charges. Note that some artifacts of the energy redistribution can be seen in the modified energy density.

Pairs carry no net charges, but because the quark and antiquark are displaced from one another, they create regions of local positive and negative charge. So even at top collider energies where $\mu_B = \mu_S = \mu_Q = 0$, there can still be nontrivial local fluctuations which constitute a nonzero $N_{i0}$. 
This idea is the key concept behind the Monte Carlo event generator ICCING [5, 6] (Initial Conserved Charges in Nuclear Geometry), as illustrated in Fig. 2. Intended as an additional layer of simulation on top of an existing initial condition for the energy density $\epsilon_0 = T_0^{00}$, ICCING uses input on the differential probability for gluons to split into quark-antiquark pairs to resample the initial condition. By redistributing charge and energy in space, ICCING produces an initial condition for the conserved charge densities $\rho_i = N_i^0$ out of the initial energy density $\epsilon_0$.

![Figure 3. Differential $q\bar{q}$ splitting probability as a function of the distance $r_T$ between them.](image)

In this plot we use the down quark mass $m = 4.8$ MeV, representative values of the splitting fraction $\alpha = 0.3$ and saturation scale $Q_s = 1.5$ GeV, and we compare the model shown in (3) (GBW) with another alternative (MV). See Ref. [6] for details.

As input, ICCING takes information on the differential probability distribution to produce a quark-antiquark pair with transverse separation $r_T$ and fractional energy $\alpha$ carried by the quark. In a simple dilute-dense calculation [7] in the Color Glass Condensate (CGC) framework (see e.g. Ref. [8] and references therein), one obtains the distribution

$$
\frac{dP}{d\alpha d\alpha} = \frac{\alpha_s}{4\pi} m^2 r_T \left[ 1 - \exp \left( -\frac{1}{3} \alpha^2 + (1 - \alpha)^2 \right) r_T^2 Q_s^2 (\vec{u}_\perp) \right]
\left[ (\alpha^2 + (1 - \alpha)^2) K_1^2 (m r_T) + K_0^2 (m r_T) \right]
$$

(3)

shown in Fig. 3. We note that the $r_T$ distribution is controlled by the Bessel functions in the last line; these terms correspond precisely to the vacuum DGLAP splitting kernels [9, 10, 11] (light-front wave functions) in coordinate space. In this calculation, all the CGC effects are contained in the prefactor $[1 - \exp(\cdots)]$, which leads to a slight dependence of the flavor chemistry (the total probability to produce $q\bar{q}$ pairs of a given flavor) on the saturation scale $Q_s$ (see Fig. 4).

The energy/charge redistribution algorithm is illustrated in Figs. 5 and 6. A random point from the input energy grid is chosen as the seed for a “gluon,” which in this model is a circular blob of energy. First the total probabilities to split into the various quark flavors are sampled to determine the outcome of the splitting. If the outcome is that the gluon does not split into a $q\bar{q}$ pair, then its energy is pasted into the output grid as shown in Fig. 5. If instead the gluon splits into quarks of a given flavor, then a second sampling of the differential probability distribution (see Fig. 3) is performed to determine the distance $r_T$ and energy fraction $\alpha$ of the splitting. Then “quark” and “antiquark” blobs of energy and charge are deposited at the
Figure 4. The flavor chemistry of the initial state in the calculation (3). Plotted are the ratios of quark to gluon multiplicities (normalized by the strong coupling $\alpha_s$) as a function of the saturation scale $Q_s$.

Figure 5. Illustration of how the ICCING algorithm transfers energy when the gluon does not split into a $q\bar{q}$ pair. The energy is deducted from the input grid (left) and deposited in the output grid (right) as shown. The energy transfer is done point by point and proportionately to the total enclosed energy. As a result, the transferred energy retains the underlying geometric structure of the original energy density, as seen in both the input energy grid after subtraction and the output energy grid after deposition. The gluon radius here has been greatly increased to clearly show these details.

displaced positions. In this way, the algorithm redistributes the energy and separates a region of zero charge density into two displaced regions of separate positive and negative charge.

The final output of the ICCING algorithm is a fully resampled event as shown in the right
Figure 6. Illustration of how the ICCING algorithm transfers energy when the gluon does split into a $q\bar{q}$ pair. The energy is deducted from the input grid (left plot) proportionately, preserving the underlying geometry in the input grid. But it is deposited in Gaussian blobs for the $q\bar{q}$ pair, which are displaced relative to the original gluon position. This modifies the energy density distribution (center plot) and also leads to a net displacement of positive and negative charge (here baryon density, right plot). Note that the energy is in general shared unequally between the quark and antiquark; in this case, the quark carried about 75% of the original gluon energy, as visible in the center plot. Here both the radii and overall $q\bar{q}$ displacement have been greatly increased to clearly show these details.

Figure 7. Histograms of the eccentricities for positive and negative baryon number, strangeness, and electric charge. For comparison, the distribution of the energy density eccentricities is shown.

These features are seen quantitatively in the eccentricity histograms shown in Fig. 7. The eccentricities $\varepsilon_{n,m}$ are measures which quantify the shape of the initial state distributions of.
some quantity $f$:

$$
\varepsilon_{n,m} = \frac{\left| \int d^{2}r \, r^{m} \, e^{i n \phi} \, f(\vec{r}) \right|}{\left| \int d^{2}r \, r^{m} \, f(\vec{r}) \right|},
$$

with $\varepsilon_{2} \equiv \varepsilon_{2,2}$ and $\varepsilon_{3} \equiv \varepsilon_{3,3}$ for brevity. In Fig. 7 we plot the eccentricities for the energy density $\varepsilon$ as well as for the conserved charges $B, S, Q$. For the charge densities we separately calculate the eccentricities of the positive regions $\rho_{i} > 0$ and negative regions $\rho_{i} < 0$ of charge.

As can be immediately seen from Fig. 7, the strange quark distribution is far more eccentric in all measures than the bulk geometry. Additionally, one readily confirms the picture from Fig. 2 that the $\rho_{B}, \rho_{Q}$ geometries mirror the energy density $\varepsilon$.

A fully-developed framework to study the charge transport properties of the QGP at top collider energies would truly open the door to “multi-messenger femtoscopy”: the use of multiple, independent degrees of freedom to reveal different slices of the initial state geometry and subsequent transport.

2. PDF Chemistry in ICCING

While the particular probabilities used in (3) are specific to the model chosen, it is clear that the mass threshold effect responsible for the different behavior of the strangeness distribution is much more general. In fact, the spatial distribution in (3) is dominated by the vacuum-like DGLAP kernels which are fundamental ingredients in QCD. For this reason, it seems natural to relax the CGC input to ICCING and consider other ways to specify the quark flavor chemistry and the quark-antiquark spatial correlations.

This seems like a natural opportunity to introduce the quark PDFs to fix the partonic flavor chemistry. The simplest thing to do is to would be to use an extraction of the various quark and gluon PDFs to set the flavor chemistry for the appropriate kinematics of the collision. For the spatial correlations, one can simply use the vacuum DGLAP kernels directly, rather than ones modified by CGC effects as in (3). The subtlety is in what scales one should reasonably evaluate the PDFs at to be relevant for a heavy-ion collision.

While the longitudinal momentum fraction $x$ is fairly straightforward to specify based on the energy of the collision, the resolution scale $Q^{2}$ is far more subtle. On one hand, the $Q^{2}$ evolution of the PDFs is very slow (logarithmic) so the chemistry dependence of the PDFs on $Q^{2}$ is weak. But the inability to set the hard resolution scale $Q^{2}$ reflects a deeper mismatch between a perturbative point of view and the reality of heavy ion collisions. The absence of a clear scale for $Q^{2}$ is an indication that there is no external scale in the initial condition which justifies a perturbative treatment. Put differently, a justification for thinking of the initial state in terms of perturbative “gluons” is hard to come by. By this measure, the CGC framework for gluon saturation stands alone. Because the high gluon density in a boosted nucleus gives rise to a semi-hard saturation scale $Q_{s}$, it provides a rare justification for the use of perturbative degrees of freedom in the initial conditions of heavy-ion collisions.

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References

[1] J. S. Moreland, J. E. Bernhard and S. A. Bass, Alternative ansatz to wounded nucleon and binary collision scaling in high-energy nuclear collisions, Phys. Rev. C92 (2015) 011901, [1412.4708].
[2] B. Schenke, P. Tribedy and R. Venugopalan, Fluctuating Glasma initial conditions and flow in heavy ion collisions, *Phys. Rev. Lett.* **108** (2012) 252301, [1202.6646].

[3] J. Moreland, J. E. Bernhard, W. Ke and S. A. Bass, Flow in small and large quark-gluon plasma droplets: the role of nucleon substructure, *Nucl. Phys. A* **967** (2017) 361–364, [1704.04486].

[4] H1 and ZEUS Collaboration collaboration, F. Aaron et al., Combined Measurement and QCD Analysis of the Inclusive e+e− p Scattering Cross Sections at HERA, *JHEP* **1001** (2010) 109, [0911.0884].

[5] M. Martinez, M. D. Sievert, D. E. Wertepny and J. Noronha-Hostler, Initial state fluctuations of QCD conserved charges in heavy-ion collisions, 1911.10272.

[6] M. Martinez, M. D. Sievert, D. E. Wertepny and J. Noronha-Hostler, Toward Initial Conditions of Conserved Charges Part II: The ICCING Monte Carlo Algorithm, 1911.12454.

[7] M. Martinez, M. D. Sievert and D. E. Wertepny, Toward Initial Conditions of Conserved Charges Part I: Spatial Correlations of Quarks and Antiquarks, *JHEP* **07** (2018) 003, [1801.08986].

[8] Y. V. Kovchegov and E. Levin, *Quantum Chromodynamics at High Energy*. Cambridge University Press, 2012.

[9] V. N. Gribov and L. N. Lipatov, Deep inelastic e p scattering in perturbation theory, *Sov. J. Nucl. Phys.* **15** (1972) 438–450.

[10] G. Altarelli and G. Parisi, Asymptotic Freedom in Parton Language, *Nucl. Phys.* **B126** (1977) 298.

[11] Y. L. Dokshitzer, Calculation of the Structure Functions for Deep Inelastic Scattering and e+e− Annihilation by Perturbation Theory in Quantum Chromodynamics, *Sov. Phys. JETP* **46** (1977) 641–653.