The Mathematical Model for a Prediction of Residual Discharge Time

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Abstract. The mathematical model is established by using the sampling data of the discharge curve of different batteries and the recording data of different attenuation state discharge of the same cell. Analysis of the same batch of new battery at different discharge current intensity test complete discharge curve of sampling data, based on quadratic function model, through matlab simulation, get the current strength of the corresponding function. In order to find the residual discharge time of the battery and the higher accuracy of MRE, select the relation between the time and voltage of the exponential function. Finally, in the model improvement, the turning point of the discharge curve is the segmentation point, and the discharge curve is fitted with the.

1. Introduction
Lead–acid batteries are widely used as power sources in industry, military and daily life. During the lead–acid battery discharge at a constant current intensity, the voltage decreases monotonously with the discharge time until the rated minimum protection voltage (generally 9 V). This article is based on 2016 college students' mathematical contest in modeling problem C measured 6532 discharge data (hereinafter referred to as attachment 1 and attachment 2) [1], the application of data fitting methods of lead–acid battery discharge curve, respectively to batch cell under different current strength and the same battery in different attenuation state residual discharge time is forecasted, explores the same battery discharge the rest of the time and the strength of the current relationship.

2. Prediction of discharge function and residual time of the same battery at different current intensity

2.1. The elementary function represents the discharge curve
Firstly, preprocess the measured discharge data, and use matlab to draw the scatter diagram of voltage changes with the time when current is 20A, 30A ...100A respectively (as shown in figure 1):
Figure 1. The relation diagram of voltage changes with time under different current intensity.

You can see from the diagram, no matter what the current intensity is, the voltage of the battery drops rapidly in the initial stages of discharge and rises within a short time. This is because at the moment when the battery changes from charging state to discharging state, the charge near the battery electrode plates released rapidly, while the charge far away from the electrode plate needs to be transported to the vicinity of the electrode plate gradually, and then it can be released. This process leads to a big trough in the terminal voltage of the battery [2]. After the cell terminal voltage changes over this period of time, a ‘turning point’ will be formed. The curve after the ‘turning point’ will show a monotonic drop in voltage until the end of the discharge curve. We can treat the data points at the beginning of a short period of time as singular points, and remove this part of data at the time of modeling. In order to obtain the accurate discharge curve function, the rate of charge of voltage at each observation time is calculated, that is, the observed value of the derivative of the voltage function, and the corresponding scatter diagram (as shown in figure 2) is made:

Figure 2. Diagram of voltage change rate over time.

After observation, the diagram shows approximately a straight line, and we use linear significance test \( r = \frac{\sum (t_i - \bar{t})(u_i - \bar{u})}{\sqrt{\sum (t_i - \bar{t})^2 (u_i - \bar{u})^2}} \) [3], and get \( r = 0.823857 \), therefore, we determine the rate of
change of voltage is fitted with the linear relation of time, and the original function, namely discharge curve function, is a quadratic function. MATLAB is used to fit the curve of voltage and time under 9 current intensities.

2.2. Average relative error test

According to the function table 2, the inverse function can be obtained $t = f^{-1}(v) = \frac{-b - \sqrt{b^2 - 4ac - d}}{2a}$, and then the discharge time values corresponding to each voltage under different current intensities can be calculated.

The definition of MRE average relative error given in annex1 is that 231 voltage sample points are extracted from $U_m$ (9v) at the maximum interval not exceeding 0.005v, and the average relative error between the discharge time calculated by the model corresponding to the voltage value of these sample points and the sampling discharge time is MRE.

Through excel screening, voltage sample points that meet the conditions can be obtained with current intensity of 20A, 30A, 40A, 50A, 60A, 70A and 80A. However, when current intensity is 90A and 100A, there are less than 231 sample points that meet the maximum interval of 0.0005v, and the value of MRE can be used as reference.

Then according to the calculation formula of average relative error $MRE = \frac{1}{n} \sum \frac{|Observation\ value - calculated\ value|}{Observation\ value}$ [4], and get the average relative error of each current's discharge time as shown in table 1:

| i  | MRE       |
|----|-----------|
| 20 | 0.01965705 |
| 30 | 0.03052824 |
| 40 | 0.02706103 |
| 50 | 0.0224981  |
| 60 | 0.03614013 |
| 70 | 0.067527814|
| 80 | 0.069882304|
| 90 | 0.119575957|
| 100| 0.09756359 |

Current between 20A to 60A, the average relative error is small (less than 5%), with the increase of current (70A-100A), the average relative error increases, its reason is that with the increase of the intensity of discharge current, the voltage decreases faster. The fitting accuracy of the model is somewhat weakened. In general, the average relative error is al less than 0.1, resulting in a higher accuracy of the model.

2.3. The remaining discharge time of the battery

With the current intensity of 30A, 40A, 50A, 60A and 70A, when the measured voltage is 9.8v, we establish the elementary function relation corresponding to each current, and use the method of $2.2 t = \frac{-b - \sqrt{b^2 - 4ac - d}}{2a}$, and use excel to calculate the difference between $t_{v=9}$ and $t_{v=9.8}$, then the residual discharge is obtained, the residual discharge time under various current intensities is shown in table 2
Table 2. Residual discharge time at different current intensity.

| i(A) | U=9.8v residual discharge time(min) |
|------|-----------------------------------|
| 30   | 955.6034301                      |
| 40   | 657.5889348                      |
| 50   | 517.55743                        |
| 60   | 420.2225793                      |
| 70   | 350.5968161                      |

3. Model and test of relationship between discharge function and current intensity

3.1. Mathematical model of discharge curve at any constant current intensity discharge

As can be seen from table 1, the discharge curves under 9 kinds of current intensities are quadratic function relations, but the quadratic function term coefficients are different, and the relationship between the coefficients of the quadratic formula and the current intensity can be obtained. Taking the coefficients a, b, c of the quadratic function as dependent variables and the current intensity i as independent variables, we conducted data fitting, and obtained the relational expressions of a, b, c and i respectively. Using the elementary function relation obtained in table 1, the scatter diagram is made for the coefficients of the quadratic term and the current intensity. According to the scatter diagram of the over coefficient and the matlab fitting toolbox analysis, the coefficients a, b, c and current intensity i are suitable for the quadratic fitting, cubic fitting and linear fitting respectively.

The result of a, b, c fitting function relation is:

\[
a = -4.897 \times 10^{-10} \times i^2 + 1.821 \times 10^{-8} \times i - 2.497 \times 10^{-7}
\]
\[
b = -2.347 \times 10^{-9} \times i^3 + 4.261 \times 10^{-7} \times i^2 - 2.449 \times 10^{-5} \times i + 2.587 \times 10^{-4}
\]
\[
c = -4.017 \times 10^{-3} \times i + 10.68
\]

3.2. MRE evaluation model accuracy

According to the functional expressions of above equations a, b, and c, the function of the discharge curve of any constant voltage can be obtained. At this time, the voltage is a binary function of current intensity and time, that is:

\[
u = (-4.897 \times 10^{-10} \times i^2 + 1.821 \times 10^{-8} \times i - 2.497 \times 10^{-7}) \times t^2
\]
\[
+ (-2.347 \times 10^{-9} \times i^3 + 4.261 \times 10^{-7} \times i^2 - 2.449 \times 10^{-5} \times i + 2.587 \times 10^{-4}) \times t - 4.017 \times 10^{-3} \times i + 10.68
\]

3.3. MRE evaluation model accuracy

The mathematical model of discharge curve is obtained through 2.1, substitute the current intensity i = 20a, 30A...100A and the relationship between discharge voltage and time is retrieved under these 9 current intensities. The discharge time is calculated by using the excel software and the known voltage, and the average relative error is also calculated (Table 3):

| i   | MRE    |
|-----|--------|
| 20  | 0.03558071 |
| 30  | 0.019305  |
| 40  | 0.023389  |
| 50  | 0.026201  |
| 60  | 0.03078058 |
| 70  | 0.02036667 |
| 80  | 0.083960596 |
| 90  | 0.07747544 |
| 100 | 0.09493028 |
By comparing the average relative error (table 4) and the accuracy grade table (table 4) [5], it can be seen that when the current is 20A to 70A, the average relative error is less than 0.05, and the accuracy grade is level 2. When the current is 80A to 100A, the average relative error is less than 0.10, and the accuracy grade is 3. In general, the model has high precision and can be applied in practice.

| Accuracy grade | Grade 1 | Grade 2 | Grade 3 | Grade 4 |
|----------------|---------|---------|---------|---------|
| MRE            | 0.01    | 0.05    | 0.10    | 0.20    |

3.4. Discharge curve at current intensity of 55A
According to the mathematical model calculated in 2.1, the generation of I = 55 will get a function of voltage and time, 

\[ u = -7.2949 \times 10^{-7} \times t^2 - 1.8978 \times 10^{-4} \times t + 10.4591 \]

MATLAB is used to draw the graph of the function (as shown in figure 3).

![Figure 3. Relationship between voltage and discharge time at i =55.](image)

4. Conclusion

4.1. Advantages of the model
The scatter diagram made by MATLAB is helpful for us to analyze the function and preliminarily obtain the discharge curve model. This method can be popularized in many engineering problems.

The model can basically solve the relationship between voltage and time at constant current intensity discharge.

The mean relative error (MRE) was applied to improve the model step by step while modeling and testing.

The model can not only accurately reflect the dynamic process of battery discharge, but also is not too complicated and practical.

4.2. Disadvantages of the model
The accuracy of the model decreases with increasing current intensity.

4.3. Model improvement
In order to solve the problem that the accuracy of the model decreases when the current intensity is gradually larger, we improved the model by changing the fitting function from a simple elementary function to a piecewise function, and took the current intensity equal to 20A as an example to illustrate the improvement process of the model.
According to the known data, find the change rate of voltage when the current is 20A, that is, the slope k1 of the tangent line of the voltage curve. Looking at the graph of the slope k1, it is found that the slope does not change much for a long time, but after reaching a "turning point", the slope decreases rapidly. The piecewise function is used to fit these two sections.

After analysis, the turning point of slope K1 is at t=3362 min, so we use it as the piecewise point of the piecewise function. The discharge curves of the battery at t<3362 min and t>3362 min were obtained by using MATLAB toolbox for fitting (as shown in figure 4):

![Figure 4. The discharge curves of the battery at t<3362 min and t>3362 min.](image)

At this time, the piecewise function expression of voltage and time is:

\[ u = \begin{cases} 
-2.003 \times 10^{-1} \times t^2 - 0.0002834 \times t + 10.59 & 22 \leq t \leq 3362 \\
-8.766 \times 10^{-6} \times t^2 + 0.02892 \times t - 14.34 & 3362 < t \leq 3764 
\end{cases} \]

Using MRE to evaluate the accuracy of this piecewise function, the average relative error is 0.00446409, which is far less than the model 1.1 average relative error of current intensity of 20A, which is 0.01965705. Therefore, the accuracy of this model is greatly improved.

References
[1] Organizing committee of national college students mathematical modeling contest. Title C of national college students mathematical modeling contest of higher education society cup in
2016. http://www.shumo.com/home/html/3384.html

[2] Baidu library, Study on discharge characteristics of valve-controlled lead-acid battery, 2009-04-04. http://wenku.baidu.com/view/0f467bc2d5bbfd0a79567387.html

[3] Er-xun Yu, Ming-fu Luo, Shao-zong Wang. Mathematical statistics. China Forestry Publishing House 1986-5.

[4] Baidu library, Relative error, 2016-09-09. http://baike.baidu.com/link?url=kNEPTbr-YsEhLQscEwAW9vez3Q8562tWw_eZ1K1VZo59JcSfocm4qL3a8otQWUzi3nt3S7hy5aw-7RKAUm-3K

[5] Shu-Jun Zhang, Jian-Zhong Zhao, Hui-Wu Zhang, Guang-Bing Ding. The forecast method of ordnance maintenance equipment consumption based on improved grey prediction model. Sichuan journal of military engineering (J), 2011-05.