Stability Analysis of a Single-Input/Two-Output, Variable Loop Transmission System for Wind Turbine Control

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Abstract.
Large wind turbine control performance is restricted due to a host of intrinsic feedback limitations including low frequency structural modes and slow blade pitch rate. A variable loop transmission system applied to a multiple output controller smoothly transitions the limited feedback available between channels in real-time to enhance performance. This work is focused on the stability analysis of a variable loop transmission system applied to a 1.5 MW wind turbine. Simulation results illustrate interesting challenges to compensator design in the form of more sophisticated loop shaping required to satisfy conditions of absolute stability.

1. Introduction
The US Department of Energy suggests wind power capacity will increase by roughly an order of magnitude (305 GW) by 2030 [1]. An economy through size is indicated for future deployments and thus wind turbines will be larger and more flexible than contemporary variants. This presents a dual detriment in the form of greater structural loads and reduced feedback available with which to mitigate these forces. The latter disadvantage is an amalgam of limitations to automatic control exacerbated by increased turbine size: low frequency modes that restrict bandwidth, a sufficiently wide interval of frequency separation for stable, multiple output control, and slow blade pitch actuation [2]. This work is focused on single-input/two-output (SITO) control of large turbines to provide simultaneous structural control and rotor rate regulation for application in region 3 (wind is faster than rated speed).

Wind turbine Region 3 rate regulation and load mitigation control has been extensively researched. Model-based gain selection strategies for PID Region 3 compensators have been developed [3, 4]. Control of hybrid renewable energy systems are reported in [5]. Researchers at the National Renewable Energy Center (NREL) have designed and implemented advanced control systems for rotor rate regulation and load mitigation applied to the Controls Advanced Research Turbine 2 (CART2), a two-bladed 600 kW Westinghouse wind turbine, for comparison to fixed gain low order controllers. Independent blade pitch control strategies to mitigate the effect of asymmetric wind variations across the rotor disk implemented on CART2 are reported in [6, 7]. Combined collective and individual blade pitch control for load mitigation is investigated in [8]. $H_{inf}$ control is used for speed and tower damping [9]. An optimal tuning procedure for model
predictive control for wind turbines is investigated in [10]. In [11], a Linear Quadratic Gaussian design with novel refinements is used to account for CART2 actuator delay. State-space control with disturbance accommodation is investigated in [12, 13, 14]. Full-state feedback is used for tower damping and torsion mode control for NREL’s three-bladed CART3 turbine in [15].

Fixed loop transmission, multiple output control systems are limited in performance to what feedback the designer applies over each output frequency interval. Variable loop transmission (VLT) is an algorithm that smoothly reshapes the SITO loop transmission in closed loop so that the available feedback is transitioned between the two output paths as a function of a performance parameter [16]. For the wind turbine plant with collective blade pitch actuation, loop gain is shifted between the low frequency transmission of the rotor rate (primary output) control and the bandpass transmission in an interval subsuming a structure mode(s) (secondary output). VLT has the effect of concentrating limited feedback at frequencies were it is most needed with ancillary benefits that include greater frequency separation in output channels and reduced blade pitch rate demands due to reductions in structure loop (high frequency) feedback in intervals of time where rotor rate error is large.

The determination of stability of a general class of variable loop transmission systems is of paramount importance and is the focus of this work. As a preliminary step, the absolute stability is assessed over a sector for which nonlinear VLT algorithms and nonlinearities associated with system limits are expected to be confined. A VLT system is designed and compared to fixed transmission SITO, and proportional-integral (PI) single-input/single-output (SISO) control using NREL’s FAST simulation of a 1.5 MW wind turbine [17]. Multivariable circle criterion analysis reveals intriguing sensitivities to loop gain that require compensator modifications to ensure absolute stability.

1.1. Control Terminology

Rational function $T(s)$ of the Laplace variable $s$ is the loop transmission (alternatively return ratio) of a feedback loop. Frequency $\omega_b$, where $|T(j\omega_b)| = 1$, is the control bandwidth (alternatively 0 dB crossover frequency). $F(s) = 1 + T(s)$ is the return difference; $|F(s)|$ is the feedback. $|F(s)| > 1$, $|F(s)| < 1$ and $|T(s)| \ll 1$ define negative, positive and negligible feedback, respectively [18]. $|F(s)| \gg 1$ defines large feedback. These definitions indicate the effect of feedback on the logarithmic response of the closed loop system to disturbances. Nonminimum phase is the phase lag not found using the Bode phase/gain relationship [19]. When comparing two systems, the system with greatest feedback in a particular frequency band will be superior in the rejection of disturbance in that band. $G(s) \in \mathbb{R}^{n \times m}$ is an $n \times m$ matrix of rational functions.

2. Compensator Design

The system is single-input (collective blade pitch), two-output (rotor rate and tower fore/aft acceleration); the block diagram of is shown in figure 1. The primary goal is the regulation of rotor rotational velocity (a Region 3 application); the secondary goal is tower acceleration attenuation at the mode frequency of 18 rad/s. The compensator designs are accomplished without benefit of linearized models at specific wind speeds ($P_r(s)$ or $P_t(s)$), instead the FAST model of the 1.5 MW turbine with all DOFs active and a turbulent wind profile is used for on-line designs. The rotor controller $C_r(s)$ is PI and is found using Ziegler-Nichols [20]. The acceleration compensator $C_a(s)$ is bandpass to stably enhance the loop gain at the 18 rad/s tower mode. The compensators are as follows.
\[ C_r(s) = 0.105 \frac{s + 0.72}{s} \]  

\[ C_t(s) = \frac{(s + 6.28)^2}{(s^2 + 80s + 2500)(s^2 + 37.7s + 1421)} \]

In a linear condition (i.e. there is sufficiently small variance about a wind speed at which the turbine dynamics are linearized using the NREL FAST model), the SITO system may be characterized by a single loop transmission function identified by opening the loop at the actuator and finding the return ratio. The dynamics of the wind turbine are sensitive to wind speed; linear stability analysis is required using plant transfer functions \( P_r(s) \) and \( P_t(s) \) calculated for wind speeds spanning the expected operating conditions. Figure 2 shows a family of loop transmission Nichols plots for wind speeds from 14 to 22 m/s. It is noted there is sufficient frequency separation of negative feedback between the rotor rate (the low pass response) and the acceleration (bandpass response) channels [18]. It is noted that the acceleration feedback is limited to 10 dB and at lower wind speeds is only slightly more than 1. This limited performance is due to plant limitations including high frequency poles and blade pitch actuators limited in rate. The fixed-transmission system using (1) and (2) is stable by the Nyquist Stability Criterion at each of these wind speeds.

Several features of the wind turbine conspire to limit available feedback for wind turbine control including multiple, low frequency modes, nonminimum phase zeros, low blade rate limits, and the need for adequate frequency separation in the channels of a SITO system [2],[21]. A variable loop transmission strategy is presented that shifts this limited feedback between the two SITO channels to improve performance.

3. Variable Loop Transmission System

Figure 3 shows the SITO system enhanced with a system that shifts loop gain between the rotor rate and fore/aft acceleration channels via systems \( \psi_1 \) and \( \psi_2 \). While many variable transmission algorithms are feasible, variable gains explicit in rotor rate error are found for this work as this
is the principle performance variable. The algorithms and parameters established are found heuristically and are presented to show efficacy of the concept (formal methods of design for more sophisticated variable loop transmission systems will be developed in future investigations).

\begin{align}
    f_r(t) &= 0.25 + \frac{3}{8}|e_r(t)| \\
    f_t(t) &= 1 - \frac{1}{20}|e_r(t)|
\end{align}

where \(e_r(t)\) is the rotor rate error. Define \(f_{rp}(t) = f_r(t)\) if \(f_r(t) < 1\) otherwise \(f_{rp}(t) = 1\) and \(f_{tp}(t) = f_t(t)\) if \(f_t(t) > 0.5\) otherwise \(f_{tp}(t) = 0.5\). These signals are passed through unit gain, one pole filters to slow the response of the variable loop transmission system. \(K_r(t) = f_{rp}(t) \times 10e^{-10t}\) and \(K_t(t) = f_{tp}(t) \times 0.5e^{-0.5t}\) are variable gains. These variable gains have negative relationships to rotor error. When the rotor error increases, the gain in the rotor regulator channel of the SITO loop is increased while the gain in the acceleration channel is reduced. This improves the frequency separation between control channels viz-a-viz a system that shifts the loop transmission across all frequencies, and reduces the actuator effort at high frequency. When the rotor rate error is small, feedback is shifted from the rotor rate to the acceleration channel to improve load mitigation.

4. Stability Analysis

It is desired to establish stability conditions sufficiently conservative and flexible to allow for a wide range of variable loop transmission algorithms and a large suite of potential nonlinearities in the feedback channels. Multivariable Circle Criterion analysis is performed to this end [22].

Consider a minimal state space realization \(\{A, B, C\}\) of the stable, strictly proper matrix \(T(s)\) (i.e. \(\dot{x}(t) = Ax(t) + Bu(t), y(t) = Cx(t) \in \mathbb{R}^m, u \in \mathbb{R}^n, A \in \mathbb{R}^{n \times n}\) Hurwitz, \((A, B)\) controllable, \((A, C)\) observable). This system is in feedback connection with a system of \(m\) decoupled nonlinearities \(u = -\Psi(t, y) = \left[ \psi_1(t, y_1) \psi_2(t, y_2) \ldots \psi_m(t, y_m) \right]^T\). Nonlinearity \(\psi_i(t, y_i) : [0, \infty) \times \mathbb{R} \rightarrow \mathbb{R}\) satisfies the sector condition if

\[\psi_i(t, y_i)\psi_i(t, y_i) - K_i y_i) \leq 0, \forall y_i \in \Gamma_i\]

where \(K_i > 0\) and interval \(\Gamma_i\) contains the origin [22]. \(\psi_i(\cdot)\) that satisfies equation (5) is said to belong to sector \([0, K_i]\) over finite domain \(\Gamma_i\) (globally if \(\Gamma_i = \mathbb{R}\)). If all decoupled \(m\) nonlinearities satisfy (5), \(\Psi\) satisfies the sector condition for \(K = diag(K_1, K_2, ..., K_m)\) over domain \(\Gamma\) (globally if \(\Gamma \in \mathbb{R}^m\)). If the origin \(x = 0\) of this feedback system is asymptotically stable for all nonlinearities in the sector \([0, K]\), the system is said to be absolutely stable over this sector.
\[
Z(s) = I + KT(s)
\]
is strictly positive real, then the feedback system is absolutely stable by the \textit{Multivariable Circle Criterion} \cite{22}.

Proper, square rational transfer matrix \(Z(s)\) is strictly positive real if

(i) All poles of \(Z(s)\) have negative real parts,
(ii) \(Z(j\omega) + Z^T(-j\omega) > 0, \forall \omega \in \mathbb{R}\), and
(iii) \(Z(\infty) + Z^T(\infty) > 0\).

4.1. VLT Stability Analysis

Figure 3 shows a block diagram of the VLT controller as a 2x2 system feedback connected to systems \(\Psi_1(t, y_1)\) and \(\Psi_2(t, y_2)\), where the \(y_1\) is rotor rate and \(y_2\) is tower acceleration, respectively. The systems are equivalently expressed as a feedback connection of the linear system in the forward path \([ P_r(s) \quad P_t(s) ]^T [ C_r(s) \quad C_t(s) ] = T(s)\) and the nonlinear system described by \(2 \times 2\) diagonal matrix \(\Psi_1\) on the diagonal). For this analysis both channel sectors are chosen to be \([0 \quad 1]\) as the variable loop transmission system functions defined in section 3 (i.e. \(\Psi_1(t, y_1) = K_r(t)y_1(t),\ \Psi_2(t, y_2) = K_t(t)y_2(t)\)) and most limiting-type nonlinearities (e.g. saturation) are subsumed by this sector. Thus, \(K = I_{2x2}\). The feedback system is absolutely stable by the Multivariable Circle Criterion if \(Z(s) = I + KT(s)\) is strictly positive real \cite{22}.

Figure 4 shows the minimum eigenvalues of \(Z(j\omega) + Z^T(-j\omega)\) for the system of figure 3 evaluated at wind speeds 14 – 22 m/s. \(Z(s)\) fails positive realness in the neighbourhood of 0.1 rad/s. This is an interesting result in that there are no nearby modes of the structure and the feedback is still negative at this frequency as seen in figure 2; the turbine dynamics vary substantially with wind speed at these frequencies and present a threat to absolute stability.

4.2. Stability Analysis of the Modified VLT

The rotor rate compensator is modified to reduce feedback in the neighborhood of 0.1 rad/s, a tradeoff of performance in exchange for satisfying the Circle Criterion.

\[
Cr_m(s) = 2.08 \frac{(s + 0.72)(s + 0.1)(s + 0.002)}{s(s + 20)(s + 0.3)(s + 0.001)}
\] (6)

Figure 5 shows the Nichols plots for SITO loop transmission at 21 m/s wind speed using rotor rate compensators of equations (1) and (6), illustrating the stability of the modified
controller and its modulus attenuation in the neighborhood of 0.1 rad/s. Figure 6 shows the minimum eigenvalue plots for the modified feedback system described by $Z_2(s) = I + K \begin{bmatrix} P_r(s) & P_t(s) \end{bmatrix}^T \begin{bmatrix} C_{rm}(s) & C_t(s) \end{bmatrix}$; the positive real condition is satisfied. Thus, the variable loop transmission system with the modified rotor rate compensator is absolutely stable at each wind speed considered over the sector $[0 \ 1]$ for both output channels.

Figure 5. SITO loop transmission functions at 21 m/s wind speed: blue, rotor rate compensator of (1); red, rotor rate compensator of (6).

Figure 6. Minimum eigenvalues of $Z_2(j\omega) + Z_2^T(-j\omega)$.

5. Performance
The performance of the two versions of the VLT on the 1.5 MW turbine is compared to SISO PI (rotor rate control only using $C_r(s)$) and fixed SITO control using $C_r(s)$ and 0.5$C_t(s)$. The 6 dB reduction in fixed SITO acceleration transmission viz-a-viz the VLT is a consequence of relatively poor performance and high blade rate demands. The simulations are performed using NREL’s FAST model of the 1.5 MW turbine with all DOFs turned on and with a turbulent wind profile shown in figure 7. The mean is 16 m/s with sharp changes in wind speed starting at 17.5 s. Regulation operations are maintained for the entire 40 second event.

Rotor rate and tower fore/aft acceleration standard deviations are used to quantify and compare performance. Blade pitch rate standard deviation is also reported to illustrate required control effort as it is assumed that slow actuation will be a limiting feature of this type of control; the FAST model has no saturation in blade rate, and a particular limit was not assumed in this work. Figures 8 and 9 show rotor rate and tower fore/aft acceleration from 10-25 seconds which subsumes the challenging wind speed change event. Table 1 shows the performance parameters. The SITO controllers provide modest reductions in fore/aft acceleration commensurate with the limited available feedback. The unmodified VLT controller provides better performance than the fixed SITO system with 4 deg/s rms less blade pitch rate. The compromise of decreased loop gain for absolute stability results in the expected reduced performance most significant in the rotor rate response where feedback was removed.
6. Conclusions and Future Work
Absolute stability analysis of a variable loop transmission system for two output wind turbine control is presented. The satisfaction of absolute stability guarantees convergence to the origin with the implementation of appropriate variable loop transmission systems and in the presence of plant nonlinearities confined to a sector. It is shown that additional considerations to compensator design for VLT are required to satisfy absolute stability. The approach used in this work is enhanced loop shaping to reduce negative feedback over frequencies where conditions
of absolute stability fail. Future work includes a formalization of VLT design for wind turbine applications with more sophisticated, nonlinear systems considered. The VLT stability analysis presented will be expanded to consider sector conditions for nonlinear responses to changes in wind speed and other factors.

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