Streak-less wall-bounded turbulence

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Abstract

The effect of damping the longest streaks in wall-bounded turbulence is explored using numerical experiments. It is found that long streaks are not required for the self-sustenance of the bursting process, which is relatively little affected by their absence. The only perturbations of the streamwise velocity required to sustain turbulence are shown to be those associated with the bursts themselves. This suggests that long streaks are byproducts, rather than active parts of the process.

1 Introduction

Elongated streaks of the streamwise velocity have long been recognised as important features of wall-bounded turbulence (Kline et al., 1967), and it was soon proposed that their instability is the cause of the intermittent ‘bursting’ that was later shown to be important in the cross-stream transfer of momentum and in the generation of turbulent drag (Lu & Willmarth, 1973). While their relevance was initially hypothetical, detailed mechanisms were eventually suggested (Swearingen & Blackwelder, 1987), and more concrete evidence appeared. For example, Jiménez & Moin (1991) showed that a minimal self-sustaining unit of plane Poiseuille turbulence consists of an infinitely long streak and a pair of shorter quasi-streamwise vortices, whose intensity evolves in counter-phase with the streak. The kinematics of this unit was made clearer in plane Couette flow by Hamilton et al. (1995), and eventually became codified in a turbulence regeneration cycle in which the vortices generate the streaks by deforming the mean velocity profile, and the streaks create the vortices by some sort of instability (Jiménez, 1994; Hamilton et al., 1995). Flores & Jiménez (2010) extended minimal simulations to the logarithmic layer and, by analogy with the viscous-layer results, concluded that a similar regeneration cycle is active across the shear-dominated part of the channel, with the streamwise vortices substituted by more generic bursts of the cross-stream velocities, as in figure 1(a). Although seldom made explicitly, it was usually implied that the arrows in this figure represent time. Reduced dynamical models by Waleffe (1997) and others also typically include infinite streaks and shorter vortices, and Jiménez & Pinelli (1999), by modifying different terms of the evolution equations in turbulent channels at low Reynolds numbers, showed that both components are required. Damping the streaks leads to the decay of the vortices, and vice versa, and the result in both cases is laminarisation of the flow. Exact travelling waves and other exact solutions that mimic the near-wall structures also include an infinitely long streak of the streamwise velocity and quasi-streamwise vortices (Nagata, 1990; Kawahara et al., 2012).
Although no linear instability of the mean velocity profile is known for wall-bounded turbulence (Reynolds & Tiederman 1967), the different branches of the classical regeneration cycle can be modelled by transient, non-modal, linear amplification processes. The bursts create the streaks by linear advection of the mean profile, and the bursts are amplified by the shear, through a tilting process in which eddies at different distance from the wall overtake each other (Orr 1907; Jiménez 2013). The optimum transient growth of linearised perturbations of the mean velocity profile results in streaks and bursts compatible with those observed in experiments (Butler & Farrell 1993; del Alamo & Jiménez 2006, Jiménez 2013), and (Vaughan et al. 2015) have presented plausible self-sustaining models of wall turbulence based on time-dependent infinitely long streaks. All these mechanisms are subcritical. Bursts only grow if they are triggered, and streaks are only created if bursts exist. No linear triggering mechanism is known.

Bursts and streaks have been extensively studied in turbulence simulations, both in the viscous and in the logarithmic layer. Down- and up-drafts (sweeps and ejections) are statistically found in pairs consistent with the short quasi-streamwise rollers that substitute the vortices mentioned above, and sit at the interface between longer high- and low-velocity streaks (Lozano-Durán et al. 2012). Their lifetimes are controlled by the shear of the mean velocity profile (Lozano-Durán & Jiménez 2014b) and, when their evolution is conditionally averaged with respect to the maximum intensity of the burst, the progressive tilting of the Orr mechanism is clearly seen (Encinar & Jiménez 2020). Jiménez (2018) summarises these observations and their relation with theory.

Considering the above evidence, it may be surprising that the rest of this paper is dedicated to showing that long streaks are not essential features of wall-bounded turbulence, and that the only required perturbations of the streamwise velocity are short ones, associated with the bursts themselves.

In fact, the evidence for a close connection between bursts and streaks is not strong. Their dimensions are fairly different. If we denote the streamwise, wall-normal, and spanwise directions by $x, y$ and $z$, respectively, and the corresponding velocity components by $u, v$ and $w$, the three-dimensional correlations of the transverse velocities, $v$ and $w$, are approximately isotropic in the cross plane, $\Delta_z/y \approx 1.5$, and only moderately elongated streamwise, $\Delta_x/y \approx 3$ (Sillero et al. 2014). The correlation of the streamwise velocity is more elongated, with a spanwise aspect ratio similar to that of the transverse velocities.
Table 1: Parameters of the large-box simulations. The size of the doubly periodic computational box is \( L_x \times L_z = (8\pi \times 4\pi)h \), and \( U_b h / \nu = 10^4 \), where \( U_b \) is the bulk velocity. All simulations use the same collocation grid \((1536 \times 257 \times 1536)\) in \( x, y, z \). The friction Reynolds number is measured at the end of each simulation, and \( \lambda_{z,f} \) is the longest undamped wavelength. Wall units for D30 are based on D20.

The present paper revisits the experiments of Jiménez & Pinelli (1999), which can now be performed at higher Reynolds numbers and in larger boxes and analysed in the context of the new structural information gained since then. The numerical details are given in §2, results are presented in §3, and are discussed and summarised in §4.

2 Numerical experiments

We analyse simulations of pressure-driven incompressible turbulent channels between flat plates separated by a distance \( 2h \), in doubly periodic computational boxes whose streamwise and spanwise periods
are \(L_x\) and \(L_z\), respectively. The code is Fourier-spectral in \((x,z)\) and Tchebychev-spectral in \(y\), and follows closely [Kim et al. (1987)]. More details are found in del Álamo & Jiménez (2003), whose simulation at \(Re_x \equiv h^+ \approx 550\) is used here as a reference. The code integrates evolution equations for \(\nabla^2 v\) and for the wall-normal vorticity, \(\omega_y\), from where other variables are obtained using continuity. The volume flux per unit span, \(2hU_0\), is kept constant. To control the spanwise inhomogeneity of the streamwise velocity, the variable to be modified is \(\omega_y\), which is approximately equal to \(\partial_x u\) for long and narrow features. Following Jiménez & Pinelli (1999), this is implemented by explicitly zeroing at each time step all the long harmonics of the Fourier expansion \(\omega_y(x, y, z) = \sum \sum \tilde{\omega}_y(k_x, y, k_z) \exp[i(k_x x + k_z z)]\), so that

\[
\tilde{\omega}_y(k_x, y, k_z) \to 0, \quad \text{for all } k_z, \text{ and } k_x < 2\pi/\lambda_{xf}.
\]

This includes the infinitely long harmonics with \(k_z = 0\). A few simulations were run in which the damping is only applied above or below a given distance from the wall, \(y_f\),

\[
\tilde{\omega}_y(k_x, y, k_z) \to \tilde{\omega}_y(k_x, y, k_z) F(y), \quad \text{for all } k_z, \text{ and } k_x < 2\pi/\lambda_{xf},
\]

where

\[
F(y) = \frac{1}{2} \left[ 1 + \tanh \frac{y - y_f}{\sigma_f} \tanh \frac{2 - y - y_f}{\sigma_f} \right], \quad y \in (0, 2).
\]

The plus sign damps the vorticity below \(y_f\), and the minus sign damps it above that level. In our simulations, \(y_f = 0.2\) and \(\sigma_f = 0.05\). No modification is applied to the \(\nabla^2 v\) equation. Other numerical parameters are summarised in table 1 and some wavelength limits are overlaid on the spectra in figure 1(b,c).

The damping of the vorticity is equivalent to a body force acting in the plane parallel to the wall, but because of homogeneity, and because it acts on velocity derivatives rather than on the velocities themselves, its average cancels over time. The total stress profile of all the statistics discussed below is linear.

3 Results

The temporal evolution of the \(L_2\) norm of two velocity components of the damped simulations is given in figure 2 normalised with the equilibrium values of an undamped reference simulation at the same nominal Reynolds number [del Álamo & Jiménez (2003), F00]. The damped cases initially decay, but later recover and stabilise at a lower friction Reynolds number than the reference. Case D30
laminarises completely, and does not recover. Figure 3 shows wall-parallel sections of the streamwise velocity in the buffer \((y^+ = 15)\) and outer \((y/h \approx 0.35)\) regions of two surviving damped cases, D10 and D20, and for the reference case.

Case D20 contains a mix of laminar and turbulent patches, reminiscent of transition, which persists for as long as the case was run. Case D10, which filters only wavelengths longer than \(\lambda_x \approx 2.5h\), stays turbulent, but with a very different organisation from the nominal case, whose streaky structure is substituted by a rhomboidal pattern. This organisation is easy to rationalise, because the damping ensures that the streamwise average of the velocity fluctuations vanishes over streamwise distances longer than \(\lambda_{xf}\), so that a positive fluctuation has to be followed by a negative one. But the most interesting aspect of figure 3(c,d) is that the spanwise wavelength of the velocity is approximately the same as in the nominal case, so that the angle of the rhombuses increases as this wavelength increases with the distance from the wall. It is intriguing that a weaker, but similar, rhomboidal pattern with an angle close to 45 degrees is observed in the spanwise velocity of natural flows (Sillero et al., 2014). If the above explanation applied to it, it would imply that \(w\) tends to cancel over streamwise distances of the order of its spanwise scale. This is consistent with the lack of long tails in its spectrum (Jiménez, 2018).
Figure 3. Wall-parallel sections of the streamwise velocity. The flow is from left to right, and all normalisations use the friction velocity from del ´Alamo & Jiménez (2003). Each panel contains (a) D10. (b, d, f) D20. (a,b,d,e) $k_z k_z E$. (a,d) Averaged over the buffer layer, $y^+ \leq 60$, as in figure 1(b). (b,e) Averaged over the outer layer, $y/h \in (0.4, 0.7)$, as in figure 1(c). (c,f) $k_z E_{\bullet \bullet}(k_z, y)/\Phi_\bullet^2(y)$, versus wavelength and distance from the wall.

Figure 4: Mean and fluctuation velocity profiles of the filtered simulation in table 1. Colours as in figure 2. Dashed black line is F00. (a) Mean streamwise velocity, $U^+$. (b) $u'^+$. (c) $v'^+$. (d-f) $k_z k_z E$. (a) Averaged over the buffer layer, $y^+ \leq 60$, as in figure 1(b). (b) Averaged over the outer layer, $y/h \in (0.4, 0.7)$, as in figure 1(c). (c,f) $k_z E_{\bullet \bullet}(k_z, y)/\Phi_\bullet^2(y)$, versus wavelength and distance from the wall.

Figure 5: Premultiplied energy spectrum. The dashed contours are F00. The solid ones are damped cases. The damped region is in grey, and contours contain 75% of the spectral mass. Black lines, $\Phi_{uu}$; red ones, $\Phi_{vv}$. (a-c) D10. (d-f) D20. (a, b, d, e) $k_z k_z E$. (a,d) Averaged over the buffer layer, $y^+ \leq 60$, as in figure 1(b). (b,e) Averaged over the outer layer, $y/h \in (0.4, 0.7)$, as in figure 1(c). (c,f) $k_z E_{\bullet \bullet}(k_z, y)/\Phi_\bullet^2(y)$, versus wavelength and distance from the wall.

and may be linked to the streak meandering discussed by Hutchins & Marusic (2007). It also has the right dimensions to be related to the bursts described in the introduction.

The mean and fluctuation velocity profiles for the damped simulations are shown in figure 4, compared
Table 2: Parameters of the small-box simulations. The size of the doubly periodic computational box is \( L_x \times L_z = (\pi h/2 \times \pi h/4) \), and \( U_b h/\nu = 19340 \), where \( U_b \) is the bulk velocity. All simulations use the same collocation grid (192 \( \times \) 385 \( \times \) 192 in \( x,y,z \)). The friction Reynolds number is averaged over the last half of each simulation, and \( \lambda_{xf} \) is the longest undamped wavelength. Wall units for D950-2 are based on D950-1.

| Case       | \( Re_\tau \) | \( \Delta x^+ \) | \( \Delta z^+ \) | \( \Delta y_{max}^+ \) | \( L_x^+ \) | \( L_z^+ \) | \( \lambda_{xf}/h \) | Result   |
|------------|----------------|-----------------|-----------------|-----------------|--------|--------|-----------------|---------|
| F950       | 949            | 7.8             | 3.9             | 7.8             | 1490   | 745    | \( \infty \)   | Jiménez [2013] |
| D950-0     | 830            | 6.8             | 3.4             | 6.8             | 1300   | 651    | 1.57            | Turbulent |
| D950-1     | 555            | 4.5             | 2.3             | 4.5             | 870    | 435    | 0.78            | Turbulent |
| D950-2     | 241            | 4.5             | 2.3             | 4.5             | 870    | 435    | 0.52            | Laminar  |

with the reference case. In all cases except the transitional D20, the streamwise fluctuations are weaker than the reference case, while the wall-normal ones are stronger. This is characteristic of controlled flows, because the relative magnitude of the two components has to adjust itself to generate the tangential Reynolds stress used to define wall units. It is clear from the mean velocity profiles in figure 4(a) that D20 is basically laminar above \( y^+ \approx 200 \), with no trace of a logarithmic layer, but the behaviour of D10 is interesting. When the whole channel is damped, as in D10, the mean velocity can be described as a logarithm with a low von Kármán constant, reflecting the missing Reynolds stresses. The same is true when only the outer layer is filtered, as in D10out, whose red dashed line is difficult to distinguish from the solid line of D10. However, when only the inner layer is filtered, as in the black solid line of D10in, the mean velocity and the fluctuation profile of \( u' \) tend to recover their turbulent behaviour in the undamped layer above \( y/h = 0.3 \) \( (y^+ \approx 150) \). This supports the idea that the large scales of the outer part of the channel are not driven by the wall.

The premultiplied spectra in figure 1(b,c) suggest that the behaviour of the flow depends on how much of the \( v \) spectrum is damped. Case D10 barely modifies \( \Phi_{vv} \), either in the buffer or in the outer layer, but D20 damps a large fraction of \( v \), especially in the outer layer where figure 4 shows that turbulence has died. The streamwise velocity spectrum, \( \Phi_{uu} \), is heavily damped in both cases, but it does not appear to have a strong effect. This is confirmed by figure 5 which compares spectra of damped and undamped simulations. In figure 5(a-c), \( \Phi_{uu} \) is heavily truncated in D10, but the remaining part agrees well with a truncated version of the undamped spectrum. Up to a point, the same is true of \( \Phi_{vv} \), most likely because its truncation is minor. The truncation of \( \Phi_{vv} \) in the near-wall region of D20 is also moderate, but all the spectra are heavily distorted in the outer layer of D20 in figure 5(e), where half of \( \Phi_{vv} \) is damped. In fact, although the contours in these figures are chosen to represent a fixed fraction of the fluctuation energy (75%), figure 4 shows that the spectra in figure 5(e) correspond to weak residual fluctuations, which are not turbulent. Another interpretation of these spectra, especially clear from the (\( \lambda_x,y \)) projections in figure 5(c,f), is that turbulence requires that the fluctuations of \( u \) be longer than those of \( v \), but only by a small factor of two or three. The very large difference between the length of \( u \) and \( v \) (and \( w \)) in nominal turbulence does not appear to be required for its survival.

To explore the dependence on the Reynolds number, and since the previous results suggest that features longer than \( \lambda_x \approx h \) do not have a strong effect on the dynamics of the structures in the core of the spectrum, two sets of simulations were run in the relatively small boxes used by Jiménez [2013] to study the logarithmic layer (case F950 in table 2). According to Flores & Jiménez [2010], turbulence in these boxes should be healthy below \( y = 0.3 L_z \approx 0.24 h \). The first set of simulations, detailed in table 2, are damped ones in which the first few wavenumbers are zeroed. The second set are undamped versions of F950 in which the streamwise length of the box is progressively shrunk until
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Case $Re$, $\Delta x$, $\Delta z$, $\Delta y$, $L_{\text{max}}$, $x$, $L$, $z$, $\lambda_x f/h$, $\lambda_x f$.

Result

F950 949 7.8 3.9 7.8 1490 745 Turbulent

As the damping becomes more severe, the friction Reynolds number decreases with respect to its undamped value, and

Figure 6: (a) Decay of the friction Reynolds number with the streamwise wavelength. Red triangles and blue circles are cases in tables 1 and 2, respectively, plotted versus $\lambda_x f^+$; solid squares are undamped channels with the same parameters as F950 in table 2, but with shorter boxes, plotted against $L_+^x$. The arrows mark cases that laminarise, scaled in the wall units of the closest surviving simulation. (b) Temporal evolution of the band-averaged $v$ fluctuation intensity, normalised with its temporal mean, and offset for clarity. Top line is F950, and bottom one is D950-0. Symbols mark the snapshots in figure (c) Dashed lines are $C(v'_b, v'_b^{\text{c}})$, and solid ones at $C(v'_b, u'_b)$. Black is F950; red is D950-0.

turbulence is no longer sustained. They are the solid symbols in figure (a).

The open symbols in figure 6(a) summarise the behaviour of the damped boxes, compared with the corresponding undamped simulations. As the damping becomes more severe, the friction Reynolds number decreases with respect to its undamped value, and the flow laminarises for $\lambda_x f^+ \lesssim 400$. We have already mentioned that Jiménez & Pinelli (1999) were able to sustain turbulence for $\lambda_x f^+ \gtrsim 600$, but not for shorter wavelengths. This is the same limit found by Jiménez & Moin (1991) for the $L_+^x$ of minimal channels at $L_+^x \approx 100$, $Re_x \approx 180$, suggesting that the flow only laminarises when the box is too short even for the smallest structures in the buffer layer. This is confirmed by the collapse of figure 6 in wall units. On the other hand, the behaviour of damped and undamped boxes is different. Figure 6(a) shows very little degradation of the $Re_x$ of the solid symbols representing undamped simulations, until they laminarise for very short $L_x$. The limit of short and wide channels has been extensively explored by Tob & Itano (2005) and coworkers, who also find little degradation for boxes longer than $L_+^x \approx 260$ at $Re_x = 137$. Figure 6(a) confirms that a similar limit holds at higher Reynolds number.

It may be significant that the undamped limit is shorter than the damped one by a factor of about two, which would confirm the similar suggestion from the spectra in figure 5.

An advantage of small boxes is that the uppermost band of wall distances for which turbulence remains healthy is essentially minimal for the energy-containing eddies, and their temporal evolution can be studied by tracking band-averaged integral quantities. For the rest of this section we will use fluctuation intensities averaged over $y/h \in (0.15 - 0.3)$, denoted by a ‘$b$’ subscript.

Figure 6(b) displays the temporal behaviour of $v'_b$ in cases F950 and D950-0 of table 2, which only differ by the zeroing in the latter of $k_x = 0$. Both quantities burst intermittently with an approximately similar period, and with an amplitude that is only slightly larger in the undamped case. The temporal evolution can be quantified by the temporal correlation, which is defined for any two variables $a$ and $b$ as,

$$ C(a, b; t) = \frac{\langle a(s)b(s + t) \rangle_s}{\langle a^2 \rangle_s \langle b^2 \rangle_s^{1/2}}, $$

where

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Figure 7: Wall-parallel views of small-box simulations. Colours are distance from the wall of the \( U^+ = 18 \) isosurface. Flow is from the bottom upwards, and lengths are in wall units. (a-c) F950. Time between consecutive frames, \( u_\tau \Delta t/h \approx 0.52 \). (d-f) D950-0. Time between consecutive frames, \( u_\tau \Delta t/h \approx 0.30 \).

where the average \( \langle \rangle_s \) is taken over time and over the two sides of the channel.

Figure 6(c) shows correlations of \( u'_b \) and \( v'_b \). The correlation \( C(v'_b, v'_b) \) is symmetric and agrees closely for the two flows. It was shown by Jiménez [2013] to represent well an Orr [1907] burst. Such bursts also typically involve \( u'_b \), but the correlation \( C(v'_b, u'_b) \) is not necessarily symmetric, and depends on the location of the burst with respect to pre-existing streaks. In agreement with Jiménez [2013], \( C(v'_b, u'_b) \) is tilted towards negative times in F950, implying that the best linear estimator,

\[
u'_b = [C(v'_b, u'_b)/C(v'_b, v'_b)] v'_b,
\]

decreases as the burst progresses, and suggesting, somewhat surprisingly, that bursts tend to damp the streaks in which they appear. On the other hand, the correlation in the damped simulation is approximately symmetrical, implying that there are no pre-existing streaks on average, and that no residual streak is left by the burst.

Snapshots of flow fields in small boxes are presented in figure 7. The three leftmost panels are the undamped case F950, and the three rightmost ones are D950-0. The figures map the distance from the wall of the \( U^+ = 18 \) isosurface, which has been chosen because its average position is \( y^+ \approx 100 \) in both flows. In practice, it ranges over \( y^+ \in (15 - 200) \), and the colour code is uniform among panels. The three snapshots in each flow are chosen to span a ‘burst’ of \( v'_b \), with its peak at the central panel. Red areas are low-velocity regions, and it is interesting that the maps of D950-0 show the same rhomboidal pattern as the large-box damped flows in figure 3(c-f). The burst in figure 7(c) forms when the symmetry of the rhomboids is broken and one orientation dominates over the other. It is tempting to see a similar process in the undamped flow fields, where the breakdown takes places as the tail of the streak ‘catches up’ with itself, but it should be remembered that these snapshots cover a spatially periodic domain, and that their behaviour may partly be an artefact of periodicity.

4 Discussion and conclusions

This note can be considered one more step in the simplification of the models for wall turbulence. Our evidence suggests that the long correlations of the streamwise velocity are by-products rather than
drivers of the dynamics of the wall. It can even be interpreted as that streaks are not required for the maintenance of wall-bounded turbulence, since there is little that can be interpreted as a streak in the flows in figures 3(c-f) or 7(d-f). The sketch of the self-sustaining process in figure 1(a) could then be substituted by the one-loop process in figure 8 opening several questions. The first one is how bursts are restarted, because, as was the case of figure 1(a), figure 8 portraits a transient process. The hope is that the new system might be easier to model than the classical one, because one of the components of the latter is missing. The second question is what is the role of streaks in the maintenance of turbulence. It is unlikely that they have none, because they contain a substantial fraction of the kinetic energy and of the Reynolds stresses. We have shown that they are not an essential component of the bursting cycle, but they may still participate in some other equally important process. Lastly, the question of how the wall gets its long streaks remains open.

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