The stability of a compressed resiliently supported rod of variable stiffness

Zverev V V, Liubavskaia I V, Meshcheryakova E V and Sotnikova M V
Lipetsk State Technical University, Moskovskaya st. 30, Lipetsk, 398055, Russia
i.sotnikova@yandex.ru

Abstract. Solution of the problem of the stability of a compressed resiliently supported rod is considered in the article. Elastic fixing models the spatial work of neighboring structural elements (frames). The asymmetric rod of variable stiffness is considered in work. The problem is connected with the resistance of a compressed resiliently supported rod of variable stiffness with the reduction in center.

1. Introduction
Currently, the frame constructions made of cold-formed steel profiles, which are combined with connecting strips, are widespread in the field of industrial construction. Constructive solutions of the junction nodes of certain elements (ridge and cornice nodes) of frame constructions lead to a sharp stepwise change in stiffness along the length of beams and columns. Loss of stability is the most common cause of loss of sustaining capacity of frame construction made of steel roll-formed profiles. As the analysis of the research has shown, [1-3] the actual work of the cane segment made of steel roll-formed profiles with a connection gusset plate has not been studied enough.

In frame constructions made of cold-formed steel profiles the junction nodes of certain elements have increased stiffness. The theoretical and experimental researches [4, 5] have shown that in such constructions real work of the conjunction node of post-and-lintel is similar to elastic restraint. Introducing a spring into the design scheme, the node is actually pliable, and it causes the involvement of the neighboring elements in work.

2. Problem setup
In Figure 1, the rod consisting of three parts is presented. The outermost parts of the length $l_1$ and $l_3$ have stiffness $EI_1$, and the central part of the length $l_2$ have stiffness $EI_2$, where $l_2 < l_1$.

3. Mathematical model
To compile the differential equation of the curved axis for each of the three parts, the $x_1$ coordinate will be counted from the end of the rod $O$, and $x_2, x_3$ will be counted from the junction point of the sections. Let us compose a differential equation of the curved axis for each part. The equations are as follows:
Figure 1. The design scheme of the compressed rigid rod from the one side and the resiliently supported rod from the other side.

\[
\begin{align*}
EI_1 \frac{d^2 v_1}{dx_1^2} &= P(\delta - v_1) - c_{np} \delta (l-x_1) - d_{np} \phi = 0, \\
EI_2 \frac{d^2 v_2}{dx_2^2} &= P(\delta - v_2) - c_{np} \delta (l-l_1-x_2) - d_{np} \phi = 0, \\
EI_1 \frac{d^2 v_3}{dx_3^2} &= P(\delta - v_3) - c_{np} \delta (l-l_1-l_2-x_3) - d_{np} \phi = 0.
\end{align*}
\]

In system (1), the following notation is used: \( EI_1, EI_2 \) are the rod stiffness, \( P \) is a normal force; \( \delta \) is the movement of the rod in place of elastic fastening; \( v_1, v_2, v_3 \) are the deflection of the splitting rod sections 1, 2, 3; \( c_{np}, d_{np} \) are the spring characteristics, \( \phi \) is the angle of deflection of the rod in the place of elastic fastening.

Let us introduce the following notation \( \frac{P}{EI_1} = k_1^2, \frac{P}{EI_2} = k_2^2. \)

Now we can write the integrals of the equations (1):

\[
\begin{align*}
v_1 &= A \sin k_1 x_1 + B \cos k_1 x_1 - \frac{\delta}{P} (c_{np} (l-x_1) - P) - \frac{d_{np} \phi}{P}, \\
v_2 &= C \sin k_2 x_2 + D \cos k_2 x_2 - \frac{\delta}{P} (c_{np} (l-l_1-x_2) - P) - \frac{d_{np} \phi}{P}, \\
v_3 &= E \sin k_3 x_3 + F \cos k_3 x_3 - \frac{\delta}{P} (c_{np} (l-l_1-l_2-x_3) - P) - \frac{d_{np} \phi}{P}.
\end{align*}
\]

We put the boundary conditions at the ends and at the junction points:

\( v_1 = 0, \frac{dv_1}{dx_1} = 0 \) where \( x_1 = 0, \)

\( \frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}, v_1 = v_2 \) where \( x_1 = l_1 \) and \( x_2 = 0, \)
\[ v_3 = v_2, \quad \frac{dv_3}{dx_2} = \frac{dv_1}{dx_3} \] \quad \text{where} \quad x_2 = l_2 \quad \text{and} \quad x_3 = 0

\[ \frac{dv_3}{dx_3} = \varphi \quad \text{where} \quad x_3 = l_3 \quad \text{and} \quad v_3 = \delta. \]

Taking the boundary conditions into account, we get the following inhomogeneous system of linear algebraic equations:

\[
\begin{aligned}
    Ak_1 + \delta \frac{C_{np}}{P} &= 0, \\
    B - \delta \left( \frac{C_{np}}{P} l - 1 \right) - \frac{d_{np} \varphi}{P} &= 0, \\
    Ak_1 \cos k_1 l_1 - Bk_1 \sin k_1 l_1 &= Ck_2, \\
    A \sin k_1 l_1 + B \cos k_1 l_1 &= D, \\
    C \sin k_2 l_2 + D \cos k_2 l_2 &= F, \\
    Ck_2 \cos k_2 l_2 - Dk_2 \sin k_2 l_2 &= Ek_1, \\
    E \sin k_1 l_3 + F \cos k_1 l_3 - \frac{d_{np} \varphi}{P} &= 0, \\
    Ek_1 \cos k_1 l_3 - Fk_1 \sin k_1 l_3 + \delta \frac{C_{np}}{P} &= \varphi.
\end{aligned}
\tag{3}
\]

The analysis of the equations (3) gives an opportunity to go to the stability loss criterion of the system given that is the transcendental equation of the connection between section parameters, critical force and a bending moment [6]. The least root of the equation determines the coefficient value of the effective length \( \mu \).

\[
\begin{aligned}
    &k_1 \cdot \left[ \cos k_1 l_1 \cdot k_2 \cdot (\cos k_2 l_2 \cdot \left( \left( \frac{C_{np}}{P} l - 1 \right) \cdot \frac{d_{np}}{P} k_1 \sin k_1 l_3 + \cos k_1 l_3 - \cos k_1 l_3 \cdot C_{np} \cdot d_{np} \right) - k_2 \cdot \sin k_1 l_3 \cdot \left( \frac{d_{np}}{P} k_1 \cdot \cos k_1 l_3 - \sin k_1 l_3 - \frac{C_{np}}{P} \cdot \sin k_1 l_3 + \frac{C_{np}}{P} \right) + \sin k_2 l_2 \cdot k_2 \cdot \sin k_1 l_3 \cdot \left( \frac{C_{np}}{P} l - 1 \right) \cdot \left( k_1 \cdot \sin k_1 l_3 - \cos k_1 l_3 - \frac{d_{np}}{P} \cdot \cos k_1 l_3 \cdot \frac{C_{np}}{P} \right) + k_2 \cdot \cos k_2 l_2 \cdot \left( \left( \frac{C_{np}}{P} l - 1 \right) \cdot \frac{d_{np}}{P} \cdot \cos k_1 l_3 - \sin k_1 l_3 - \frac{d_{np}}{P} \cdot \sin k_1 l_3 + \frac{d_{np}}{P} \cdot \frac{C_{np}}{P^2} \right) \right] + \\
    &\quad \left( - (k_1 \cdot \cos k_1 l_3 \cdot k_2 \cdot \cos k_2 l_2 \cdot \sin k_1 l_3 + k_2 \cdot \sin k_1 l_3 \cdot \left( \cos k_2 l_2 \cdot \left( k_1 \cdot \cos k_1 l_3 + k_2 \cdot \sin k_1 l_3 \cdot \frac{d_{np}}{P} \right) - k_2 \cdot \sin k_2 l_2 \cdot (k_1 \cdot \cos k_1 l_3 \cdot \frac{d_{np}}{P} - \sin k_1 l_3) \right) - \right. \\
    &\quad \left. + \frac{C_{np}}{P} \cdot \left( \frac{d_{np}}{P} \cdot \cos^2 k_1 l_3 \cdot k_2 \cdot \left( \sin k_1 l_3 \cdot \left( k_1 \cdot \sin^2 k_2 l_2 + k_2 \cdot \cos^2 k_2 l_2 \right) + \frac{d_{np}}{P} ight) - \frac{d_{np}}{P} \cdot \cos k_1 l_3 \cdot k_2 \cdot \left( \sin^2 k_2 l_2 - \cos^2 k_2 l_2 \right) \cdot \left( \sin^2 k_2 l_2 - \cos^2 k_2 l_2 \right) \right) \right] = 0
\end{aligned}
\]
Since \( l_2 > l_1 \), the critical load can be calculated as

\[
P_{cr} = K \frac{EI_2}{l^2},
\]

where the coefficient \( K \) is connected with the effective length coefficient \( \mu \) as \( K = \frac{\pi^2}{\mu^2}. \)

4. Experimental research

To compare the theoretical data with real construction work, in-place testing has been carried out. The construction tested work as part of a block of four frames. At the end of the frame, there is a linkage block with vertical column links and horizontal links on the covering (Figure 2).

The elements of the frames are paired profiles of the C-shaped section combined with joint spacers. The profiles were made of coiled galvanized steel according to the TU 1122-181-02494680-99 by “Eksergiya” company. The bearing frame profiles along the axis 3 (Figure 2) are made of C345 steel, the rest of the elements of the tested block were made of C235 steel. As the covering, a profiled metal sheet H75-1000-0,7 is laid on the lintels. The step of the lintels is 2.25 mm. Lintel supporting is carried out in one level with the girder with the help of bolts of normal accuracy M10. The sections of lintels are pared profiles PGS 150C with thickness of 1.5 mm, jointed by connecting gaskets.

![Figure 2. Composition of a block of four frames with ties the tested frame.](image)

Girder section is paired profiles PGS 200C with thickness of 2 mm jointed by connecting gaskets. According to the project, the girder section is strengthened with the attachment of the bent channel to the upper shelves.

The connection of girder elements among themselves in a ridge and with the support pillar in a cornice knot is carried out using gusset plates with thickness of 6 mm with the help of bolts of normal accuracy M10. The support pillar section is paired profiles PGS 200C with thickness of 2 mm, jointed by connecting gaskets.

The bracing is made of round bar steel with the diameter of 30 mm. Steel is C235. The connection of bracing with the cornice gusset plate is carried out using one-sided overlay with thickness of 6 mm. The attachment of the overlay to the gusset plate is carried out using four bolts M20.

The ties BC-1, GS-1 (Figure 2) are carried out using the profiles PGS 150C with thickness of 1.5 mm. The assembly view of the constructions tested is given in Figure 3.
Figure 3. The assembly view of the tested construction.

For this task, where the construction working in a block of 4 frames, the coefficient of effective length is \( \mu = 1.76 \). The comparison of the theoretical and experimental data (Figure 4) shows a good coincidence within 8-11%.

Figure 4. Comparison of theoretical and experimental data.

5. Conclusions
According to the results of the study the following conclusion can be made. When calculating the stability of frame structures from steel bent profiles, the change in stiffness along the length of the rods and elastic coupling with girder has to be considered.

References
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