QED corrections with partial angular integration to fermion pair production in $e^+e^-$ annihilation

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Abstract
Analytic formulae are derived for the complete photon energy spectrum due to QED corrections to fermion pair production in case of a limited angular acceptance for the final state fermions. After a numerical integration over the energy of non-observed photons, this corresponds to typical experimental conditions at LEP/SLC.

1 Introduction

One of the main tasks of the $e^+e^-$ storage rings LEP and SLC is a verification of the standard electroweak theory with unprecedented precision. An important reaction in this context is fermion pair production:

$$e^+ + e^- \rightarrow (\gamma, Z) \rightarrow f^+ + f^- + n\gamma, \quad f \neq e. \quad (1)$$

Within the standard theory, the corresponding cross sections can be predicted with a precision well below 1% either with (semi-) analytic formulae or Monte-Carlo simulation. For the analysis of experimental data, the use of (semi-)analytic formulae is preferred due to their fast performance and well understood accuracy. A disadvantage is the small flexibility concerning the choice of experimental cuts.

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In a series of papers, we derived analytic formulae taking into account QED corrections to the total cross section and the integrated forward-backward asymmetry as well as to the differential cross-section (see \cite{4} and references quoted therein). The demand of experimentalists for fast and elegant algorithms motivated us to derive in addition analytic formulae for the photon energy spectrum in presence of an angular acceptance cut for the final state fermions. Such formulae allow a description of the quite realistic situation of fermion pair production including photon bremsstrahlung (with soft photon exponentiation) with cuts on the photon energy and on the fermion scattering angle using only one numeric integration. These new formulae are the subject of the present article.

We denote with $\sigma_T(-c_1, c_2)$ and $A_{FB}(-c_1, c_2)$ the total cross section and forward-backward asymmetry, both integrated over the angular range $\cos \theta \in (-c_1, c_2)$:

$$\sigma_T(-c_1, c_2) = \sigma_T(c_2) - \sigma_T(-c_1), \quad (2)$$

$$A_{FB}(-c_1, c_2) = \sigma_T(-c_1, c_2)^{-1}[\sigma_{FB}(c_2) + \sigma_{FB}(-c_1)]. \quad (3)$$

The building blocks $\sigma_A(c), A = T, FB$ have to be determined by an explicit integration over the angular distribution:

$$\sigma_A(c) = \sum_{m,n=0,1} \sum_{a=e,i,f} Re[\sigma^a_{s,s^0}(s, s; m, n) R^a_A(c; m, n)], \quad (4)$$

$$R^a_A(c; m, n) = d_A^{-1} \int_0^\Delta dv R^a_A(c, v; m, n) \frac{\sigma^a_{s,s^0}(s, s'; m, n)}{\sigma^a_{s,s^0}(s; m, n)}, \quad (5)$$

$$d_T = \frac{4}{3}, \quad d_{FB} = 1, \quad s' = (1 - v)s. \quad (6)$$

Here, $\sigma^a_{s,s^0}(s, s'; m, n)$ are normalisation factors containing the Born resonance function and all the couplings. Indices $m, n$ are for photon ($m = 0$) and $Z$ boson ($m = 1$) exchange. The remaining (numerical) integration in (5) is to be performed over the photon energy with cut-off $\Delta \in (0, 1)$. The problem to be solved is the determination of the angular dependent functions,

$$R^a_A(c, v; m, n) = \int_0^c dcos\theta r^a_A(cos\theta, v; m, n). \quad (7)$$

The general notations have been taken over from \cite{4}, where also the functions $r^a_A(cos\theta, v; m, n)$ may be read off.

The following chapters are devoted to an explicit representation of rather compact formulae for the QED corrections (7) originating from initial ($a = e$) and final ($a = f$) state radiation and their interference ($a = i$).
2 Initial State Radiation

The major part of radiative corrections originates from the initial state. Therefore, an inclusion of higher order contributions is necessary. An adequate accuracy for LEP/SLC physics may be achieved if one combines the exact to order $O(\alpha)$ cross sections with a soft photon exponentiation procedure \[1\text{ ]}. We split the QED corrections into two parts containing soft and hard bremsstrahlung, correspondingly. For initial state radiation, they both are independent of the kind of exchanged vector boson:

\[
R_A^e(c, v; m, n) = d_A(c)[1 + \tilde{S}(\beta_e)]\beta_e v^{\beta_e - 1} + \bar{H}_A^e(c, v),
\]

\[
d_T(c) = d_T^{-1}(c + c^3/3), \quad d_{FB}(c) = c^2,
\]

\[
\tilde{S}(\beta_e) = \frac{3}{4} \beta_e + \frac{\alpha}{\pi} Q_e^2 \left( \frac{\pi^2}{3} - \frac{1}{2} \right),
\]

\[
\bar{H}_A^e(c, v) = \frac{\alpha}{\pi} Q_e^2 d_A^{-1} h_A^e(c, v)/v - \frac{\beta_e}{v} d_A(c), \quad A = T, FB,
\]

\[
\beta_e = \frac{2\alpha}{\pi} Q_e^2 (L_e - 1), \quad L_e = \ln \frac{s}{m_e^2}, \quad Q_e = -1.
\]

The second term in (11) is the remnant of soft photon exponentiation and makes the hard photon correction infra-red finite. The hard radiator functions are different for $\sigma_T$ and $A_{FB}$. They depend on the reduced photon energy $v$ and the range of observed scattering angles $c$:

\[
h_T^e(c, v) = \frac{4}{5} r_2 \left[ \ln \frac{\gamma_+}{\gamma_+^3} (c_+^3 - z^3 c_+^3) - \ln \frac{\gamma_-}{\gamma_-^3} (c_-^3 - z^3 c_+^3) \right] + \frac{2c}{\gamma_+^2 \gamma_+^2} \left\{ zr_2 [(L_e - 1) - \ln z] \left[ \frac{2}{3} z^2 (1 - c_+ c_-) + v^2 c_+ c_- (z + r_2 c_+ c_-) \right] + 2v^2 (c_+ c_-)^2 \left( r_2 z - r_4 + \frac{4}{3} \frac{z^2}{c_+ c_-} - \frac{22}{3} z^2 \right) + 2v^4 (c_+ c_-)^3 \left( \frac{5}{3} r_2 z - 2z^2 - \frac{1}{3} r_4 \right) \right\},
\]

\[
h_{FB}^e(c, v) = 8z \frac{r_2}{r_1^2} \ln \frac{r_1}{2} + \frac{2z}{\gamma_-^2 \gamma_+^2} \left\{ [(L_e - 1) - \ln z] \left[ 2r_2 \frac{z^2}{r_1} \right] - r_2^2 c_+ c_- + 4r_2 c_+ c_- \frac{v^2}{r_1^2} (z + r_2 c_+ c_-) \right\} - 4zr_2 c_+ c_- \left( \ln \frac{\gamma_+}{\gamma_+^2} + \ln \frac{\gamma_-}{\gamma_-^2} \right).
\]

The following abbreviations are used:

\[
r_n = 1 + z^n, \quad v = 1 - z,
\]

\[
c_\pm = \frac{1}{2} (1 \pm c), \quad \gamma_+ = c_+ + zc_-, \quad \gamma_- = c_- + zc_+.
\]

For an unrestricted angular acceptance, $c \to 1$, the above formulae agree with those derived earlier for the uncut convolution kernels $\sigma_T$ \[3\] and $A_{FB}$ \[4\].

3
3 Initial-Final State Interference Radiation

Under usually realised experimental conditions, the $O(\alpha)$ initial-final interference bremsstrahlung is nearly negligible in the $Z$ resonance region. This is not true if a tight cut on the photon energy is applied or in case of scattering angles near $\cos \theta \approx 1$. After an angular integration, the second type of singularity is removed completely. Away from the peak, the interference contributions become comparable to the other non-leading radiative corrections.

It is sufficient to present the diagonal interference corrections. The non-diagonal terms obey the following simple relation:

\[
R^i_A(c, v; m, n) = \frac{1}{2}[R^i_A(c, v; m, m) + R^i_A(c, v; n, n)]^*, \quad A = T, FB. \tag{17}
\]

The initial-final state interference contributions are composed of soft and hard bremsstrahlung parts $S^i_A(c, \epsilon)$, $h^i_A(c, v)$ and of $\gamma\gamma$ and $\gamma Z$ box diagrams $B_A(c; m, n)$:

\[
R^i_A(c, v; m, n) = \frac{a}{\pi} Q_e Q_f \{ \delta(v) [S^i_A(c, \epsilon) + B_A(c; m, n)] + \theta(v - \epsilon) h^i_A(c, v) \sigma^0_A(s, s'; m, n) \}, \tag{18}
\]

where $\bar{A} = T, FB$ if $A = FB, T$. The box contributions are the only ones with an explicit dependence on the kind of exchanged vector boson; they are written in (anti-)symmetrised form:

\[
B_{T, FB}(c; m, n) = b_{T, FB}(c; m, n) \pm b_{T, FB}(-c; m, n). \tag{19}
\]

The corrections $R^i_T(c, v; m, n)$ to the total cross section $\sigma_T(c)$ contain the following terms:

\[
S^i_T(c, \epsilon) = -4 \ln \epsilon [ \left( c^2 - 1 \right) \ln \frac{c_+}{c_-} + 2c + 2(c^2 - 1)[Li_2(c_+) - Li_2(c_-)] - (c^2 - 1) \ln(c_+ c_-) \ln \frac{c_+}{c_-} - 4c \ln(c_+ c_-) - 4 \ln \frac{c_+}{c_-} + 8c, \tag{20}
\]

\[
b_T(c; 0, 0) = \frac{1}{2} \left( c^2 - 1 \right) \ln^2 c_+ + 2c \ln c_+ - (c^2 - 3) \ln c_+ - 3c - i[\pi(c^2 - 1) \ln c_+], \tag{21}
\]

\[
b_T(c; n, n) = -2 \ln(1 - \frac{1}{R}) \{2cR(1 - R) \ln c_+ - 2cR(c^2 + 1) c_+ - cR(R + 1) \} + 4R(R - 1)c Li_2(1 - \frac{1}{R}) - 2c \ln R + (c^2 - 1) \ln^2 c_+ + 4c \ln c_+ + 2[2R(c^2 - 1) - c^2 + 3] \ln c_+ - 4cR(R - 1) Li_2(1 - \frac{c_+}{R}) - 2[2R^2 - R(c^2 + 1)] Li_2(1 - \frac{c_+}{R}) + 2Rc - 6c, \quad n \neq 0, \tag{22}
\]

\[
R = \frac{1}{s} [M_Z^2 - iM_Z \Gamma_Z(s)], \tag{23}
\]

\[
h^i_T(c, v) = 4c_+ c_- \left[ \frac{r_1 r_2}{v} \ln \frac{c_+}{c_-} + \left( \frac{c_+^2}{c_+ c_-} - z - v^2 \right) \ln \frac{\gamma_+}{\gamma_-} \right] + 4cz \left( -\frac{r_1}{v} + \ln z \right). \tag{24}
\]
The interference corrections $R_{FB}(c,v;m,n)$ to the numerator of the forward-backward asymmetry, $\sigma_{FB}(c)$, contain the following terms:

\[
\frac{3}{4} S_{FB}(c, \epsilon) = \ln \epsilon \left[ -8 \ln 2 - 4 \ln(c_+ c_-) - 3\left( \frac{c^3}{3} + c \ln \frac{c_+}{c_-} - c^2 \right) \right] \\
+ \frac{3}{2} \left( \frac{c^3}{3} + c \right) [L_i(c_+) - L_i(c_-)] + 2[L_i(c_+) + L_i(c_-)] - \frac{3}{4} \left( \frac{c^3}{3} + c \right) \ln(c_+ c_-) \ln \frac{c_+}{c_-} \\
- (\ln^2 c_+ + \ln^2 c_-) + 4 \ln^2 2 + \ln 2 - \frac{1}{2} \ln(c_+ c_-)(c^2 - 1) + \frac{c^2}{2} - 2L_i(1),
\]

(25)

\[
b_{FB}(c;0,0) = -\frac{1}{2}(c^2 - 1) \ln^2 c_+ + (c^2 - 3) \ln c_+ - 2c \ln c_+ - \frac{1}{2} (\ln^2 2 + 6 \ln 2 + c^2) \\
- i\pi \left[ \frac{5}{3} \ln 2 + (c^2 + \frac{5}{3}) \ln c_+ + 2\left( \frac{c^3}{3} + c \right) \ln c_+ - \frac{2}{3} c^2 \right],
\]

(26)

\[
b_{FB}(c;n,n) = \left( \frac{c^3}{3} + c \right) \ln^2 c_+ + \frac{4}{3} \ln^2 c_+ \\
+ \ln(1 - \frac{1}{R}) \left\{ (4R^2 - 2R + \frac{10}{3}) \ln 2 + [4R^2 - 2R(c^2 + 1) + 2c^2 + \frac{10}{3}] \ln c_+ \\
+ 4[cR(R - 1) + (\frac{c^3}{3} + c) \ln c_+ + c^2 (-R^2 + 3R - \frac{4}{3})] \right\} \\
+ c^2 \left( \frac{4}{3} R - \frac{5}{3} \right) \ln R + \left( \frac{16}{3} R^3 - 4R^2 + 2R - \frac{2}{3} \right) L_i \left( \frac{1}{1 - \frac{1}{2R}} \right) \\
+ 2c^2(R - 1)L_i \left( \frac{1}{1 - \frac{1}{R}} \right) - \frac{4}{3} \ln^2 2 + \left( \frac{8}{3} R^2 + \frac{8}{3} - R - 6 \right) \ln 2 \\
+ \left[ \frac{8}{3} R^2 + R(-\frac{4}{3} c^2 + \frac{8}{3}) + 2(c^2 - 3) \right] \ln c_+ + \left( \frac{8}{3} R^2 + \frac{4}{3} - R - 4 \right) c \ln c_+ \\
+ 2 \left[ \frac{8}{3} R^3 + 2R^2 - R(c^2 + 1) + c^2 + \frac{1}{3} \right] L_i \left( \frac{1}{1 - \frac{c_+}{R}} \right) \\
+ 2 \left[ 2c(R^2 - R) + \left( \frac{c^3}{3} + c \right) \right] L_i \left( \frac{1}{1 - \frac{c_+}{R}} \right) + \left( \frac{2}{3} R - 1 \right) c^2, \quad n \neq 0,
\]

(27)

\[
h^i_{FB}(c,v) = \frac{2c^2z}{\gamma_+ \gamma_-} \left( \frac{1}{3} \nu^2 - \frac{2}{3v} + \frac{2z}{r_1} + \frac{1}{3} c^2 vr_1 \right) \\
- 2c^2 z v \ln z - 4 \frac{r_1}{v} c_+ c_- r_2 \ln(c_+ c_-) \\
- \frac{16}{3} \frac{r_3}{v} (c_+ \ln c_- + c_- \ln c_+) + \frac{2}{3} \left( 2z - 5r_2 \right) \frac{r_1}{v} \ln r_1 \\
- 4 \ln \gamma_- \left[ 2c c_- - \gamma_- (v + \frac{4}{3} \gamma_- - 4c_- + \gamma_-) \right] \\
+ 4 \ln \gamma_+ \left[ 2c c_+ + \gamma_+ (v + \frac{4}{3} \gamma_+ - 4c_+ + \gamma_+) \right].
\]

(28)

Again, for an unrestricted angular acceptance the above formulae yield those derived earlier.
4 Final State Radiation

The final state radiator functions for the angular distribution, \( r_A^f(\cos \theta, v; m, n) \), as used in (6) for the definition of \( R_A^f(c, v; m, n) \) are described in detail in chapter 5.1 of [4]. Their angular dependence to order \( O(\alpha) \) is the same as that of the Born cross section. Consequently, an integration is trivial. For applications which deserve highest precision, a common exponentiation of initial and final state radiation corrections is recommended (see also [11, 12]). An inspection of the formulae in chapter 5.3 of [4] shows that the only nontrivial angular dependence is due to the initial state hard radiator function. The result of their integration may be found in section 2 of the present article. Further, there exists a minor angular variation in the non-leading part of the final state factor which is proportional to \[ \frac{3-4}{(1 + \cos^2 \theta)} \]. We have shown numerically that this term can be neglected completely for all applications.

As a result, a description of common soft photon exponentiation of initial and final state bremsstrahlung is obtained by a direct combination of the treatment in [4] with the integrated expressions for the initial state hard photon corrections (13), (14).

5 Discussion

The formulae of this article may be used both in a model-independent way with input parameters chosen to be e.g. \( M_Z, \Gamma_Z, \Gamma_e, \Gamma_f \), or within the standard theory using e.g. \( \alpha, G_\mu, M_Z \) [13, 14]. The corresponding Fortran codes ZBIZON and ZFITTER are described in [16] and have been applied recently to data obtained by LEP experiments (e.g. [17, 18, 19, 20]). In a unique way, one can perform a semi-analytical calculation of either observables which are integrated over a wide angular range or of pseudo-differential distributions with bins filled using differences of angular slices. As typical examples, we show in Figs. 1 and 2 some predictions for such distributions of \( \sigma_T \) and \( A_{FB} \).

To summarise, we think that the analytic calculation of QED corrections with partial angular integration could prove to be a powerful tool for the phenomenological analysis of fermion pair production.

\footnote{Besides the calculational chain based on the formulae of this article, these codes contain also a branch with different choice of cuts [15].}
References

[1] F.A. Berends et al., Z Line Shape, in: Z Physics at LEP 1, CERN report CERN 89-08 (1989), vol. 1, p. 89.

[2] M. Böhm, W. Hollik et al., Forward-Backward Asymmetries, in: Z Physics at LEP 1, CERN report CERN 89-08 (1989), vol. 1, p. 203.

[3] R. Kleiss et al., Monte Carlos for Electroweak Physics, in: Z Physics at LEP 1, CERN report CERN 89-08 (1989), vol. 3, p. 1.

[4] D. Bardin et al., Zeuthen prepr. PHE 89-19(1989), to appear in Nucl. Phys. B.

[5] G. Bonneau, F. Martin, Nucl. Phys. B27(1971)381.

[6] M. Greco, G. Pancherivi, Y. Srivastava, Nucl. Phys. B101(1975)11; ibid, B171(1980)118; E: Nucl. Phys. B197(1982)543.

[7] D. Bardin et al., Phys. Letters B229(1989)405.

[8] F.A. Berends, R. Kleiss, S. Jadach, Nucl. Phys. B202(1982)63; Comput. Phys. Commun. 29(1983)185.

[9] D. Bardin, M. Bilenky, O. Fedorenko, T. Riemann, JINR Dubna prepr. E2-88-324(1988).

[10] S. Jadach, Z. Was, Phys. Letters B219(1989)103.

[11] O. Nicrosini, L. Trentadue, Phys. Letters 196B(1987)551; Z. Physik C39(1988)479.

[12] M. Greco, O. Nicrosini, Phys. Letters B240(1990)219.

[13] D. Bardin et al., Z. Physik C44(1989)493; Comput. Phys. Commun. 59(1990)303.

[14] D. Bardin, W. Hollik, T. Riemann, München prepr. MPI-PAE/PTh 32/90(1989), subm. to Z. Physik C.

[15] M. Bilenky, A. Sazonov, JINR Dubna prepr. E2-89-792(1989).

[16] D. Bardin et al., DELPHI note 89-71phys52; Zeuthen prepr. PHE 90-18 (1990), in preparation.

[17] B. Adeva et al. (L3 collab.), L3 prepr. 008 (1990), L3 prepr. 009 (1990), subm. to Phys. Letters B.

[18] D. Decamp et al. (Aleph collab.), prepr. CERN-PPE/90-104(1990), subm. to Z. Physik C.

[19] P. Abreu et al. (Delphi collab.), prepr. CERN-PPE/90-119(1990), contrib. to 25th Int. Conf. on High Energy Physics, Singapore, Aug. 1990.

[20] T. Mori (Opal collab.), talk at 25th Int. Conf. on High Energy Physics, Singapore, Aug. 2, 1990.
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Fig. 1.
The total cross section \( \sigma_T(c_1, c_2)/\Delta c \), \( \Delta c = c_2 - c_1 \), as function of the scattering angle and of the photon energy cut-off \( \Delta = 2E_\gamma/\sqrt{s} \); \( M_H = 100 \text{ GeV} \), \( m_t = 100 \text{ GeV} \).

Fig. 2.
Cumulative forward-backward asymmetry \( A_{FB}(-c, c) \) as a function of \( c \) and of the photon energy cut-off; parameters as in fig. 1.