A Novel Compact Wideband Microstrip Wilkinson Power Divider

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Abstract In this paper, a novel compact wideband Wilkinson Power Divider (WPD) is proposed and is analyzed by using the even- and odd-mode theory. The even-mode equivalent circuit of the WPD is designed to have both a wide stopband and a wide passband. The passband is then used as the wide operating frequency band of the WPD with low return loss. The even-mode circuit parameters are determined by using the derived formulas and an optimization algorithm. The remaining circuit parameters of the WPD are obtained from the analysis of the odd-mode circuit of the WPD. Finally, a microstrip WPD is designed with a center frequency of 3.0 GHz and an operating frequency band covering 1.5 ~ 4.5 GHz (FBW=100%). The microstrip WPD is compact, occupying an area of only $0.20\lambda_g \times 0.13\lambda_g$, and its measured performance satisfies the design specifications quite well.

key words: Wilkinson power divider (WPD), miniaturization, wideband, even-mode, odd-mode.
Classification: Microwave and millimeter wave devices, circuits, and hardware

1. Introduction

The Wilkinson power divider (WPD) is one of the key components in microwave circuits and is widely used in communication systems because of its many advantages like low-insertion loss, large isolation, and simple structure [1, 2]. The conventional one-section two-way WPD consists of a pair of quarter-wavelength ($\lambda/4$) transmission lines and one absorbing resistor between its two output ports [1], but it has a narrow operating frequency band. To deal with this problem, the WPD with multi-sections of $\lambda/4$ transmission lines and multiple absorbing resistors are developed [3], which can realize equal-ripple characteristics in its wide operating frequency band and low VSWR. However, the occupied circuit area becomes large due to the multi-sections of $\lambda/4$ transmission lines.

To reduce the circuit size, WPDs using coupled transmission lines were developed to replace the conventional $\lambda/4$ transmission lines [4]-[8], or by adding other coupling structures and lumped capacitors [9]-[13], but the operating frequency band obtained was close to that of a conventional WPD. To broaden the operating frequency band, [14]-[19] improved the works by using multi-sections of coupled lines and absorbing resistors. On the other hand, more complicated WPDs are presented by using open-ended stubs [20], slot lines [21], [22], strip lines [23], and defected ground structures (DGS) [24], [25] to reduce the lengths of transmission lines in the conventional WPD. But they led to a limited bandwidth and the DGS on the background plane of the substrate made the design and fabrication of the circuit more difficult. In [26] and [27], capacitive loadings were used which can reduce the conventional $\lambda/4$ transmission lines to between $\lambda/5$ and $\lambda/12$. But the design shown a poor in-band response with an obvious bandwidth reduction at high frequencies due to the capacitive loading and the significantly increased characteristic impedance of the transmission line. In [28]-[30], WPDs using both coupled lines and short-circuited stubs were developed which shown very wide operating frequency bands, but the multi-section structures were relatively large, and extra via-holes were needed which made both the simulation and fabrication of the circuits more complicated.

In this paper, a novel wideband WPD is proposed by using a compact ring structure. The WPD is analyzed by employing the even- and odd-mode method, from which, design formulas for determining all the circuit parameters of the WPD are derived. Then, an optimization algorithm is developed for obtaining all the circuit parameters. A microstrip WPD using the low-price FR4 substrate is designed with a center frequency of 3.0 GHz and an operating frequency band covering 1.5 ~ 4.5 GHz, i.e., a fractional bandwidth (FBW) of 100%. The measured responses of the WPD are provided, with comparisons with the simulated results. The microstrip WPD is very small, occupying an area of only $0.20\lambda_g \times 0.13\lambda_g$, where $\lambda_g$ is the microstrip wavelength at the center frequency of the WPD. The structure of the proposed WPD is simple. Only one chip resistor is used, and no coupling lines, via-holes, and defected ground structures (DGS) are needed. It is easy for low-cost fabrication.
2. Circuit configuration and design method

Fig. 1 shows the circuit configuration of the proposed WPD. It consists of a transmission line ring and two output transmission lines connected by an absorbing resistor \( R_c \). In the figure, \( Z_i \) \((i=1, 2, 3)\) and \( Z_{st} \) are the characteristic impedances of the transmission lines, \( \theta \) and \( 2\theta \) indicate their electrical lengths, and \( Z_s \) and \( Z_l \) are the source and load impedances. The circuit has a symmetrical structure, so it can be analyzed by employing the even- and odd-mode method [3]. In the case of the even-mode, the symmetrical plane is an open-circuit, and half of the odd-mode equivalent circuit is shown in Fig. 2(b). Both the even-mode circuit and the odd-mode circuit are two port circuits and can be analyzed readily by using the transmission line theory, from which we can derive all the circuit parameters, including the characteristic impedances. The circuit has a symmetrical structure, so it readily by:

\[
[M] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = [M_1] \cdot [M_2] \cdot [M_{st}] \cdot [M_3]
\]

where \([M_1] = \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix}\) is the ABCD matrix of the open stub given by

\[
[M_{st}] = \begin{bmatrix} A_{st} & B_{st} \\ C_{st} & D_{st} \end{bmatrix}
\]

Then the transmission coefficient \( S_{21} \) between the source impedance \( 2Z_s \) and load impedance \( Z_l \) can be obtained by:

\[
|S_{21}(\theta)| = 1/\sqrt{1 + |(AZ_s + B - 2CZ_l - DZ_s)/2\sqrt{2Z_lZ_s}|^2}
\]

In order to realize a wide stopband with Chebyshev equal-ripple passband characteristics, the following function is choosing as the target function

\[
|S_{21}^{\text{er}}(\theta)| = 1/\sqrt{1 + |\varepsilon F_4(\theta)|^2}
\]

where

\[
F_4(\theta) = \frac{-1}{\cos \theta} \left[ T_4 \left( \frac{\sin \theta}{\sin \theta_c} \right) + aT_3 \left( \frac{\sin \theta}{\sin \theta_c} \right) \right]
\]

with

\[
a = \left( \frac{1}{\sin^2 \theta_c} - 1 - \frac{1}{\sin \theta_c} \right) \cdot \sin \theta
\]

and \( T_n(\alpha) \) is the \( n \)-th order Chebyshev polynomial of the first kind (here we have a fourth-order Chebyshev polynomial), and \( \theta \) is the electrical length at the lower equal-ripple edge-frequency of the passband of the circuit, and \( \varepsilon \) is determined by the value of the required passband ripple.

Next, an object function below is defined with the circuit parameters \( Z_i \) \((i=1, 2, 3)\) and \( Z_{st} \) as the optimization variables.

\[
F(Z_1, Z_2, Z_3, Z_{st}) = \sum_{i=1}^{n} |S_{21}^{\text{er}}(f_i) - S_{21}(f_i)|^2
\]

where

\[
f_i = f_c + (i - 1) \cdot \Delta f
\]

and \( f_i (i=1, 2, \ldots, n) \) are the sampling frequencies in the equal-ripple band of the circuit, and \( n \) is the number of sampling points used in the simulation.

In the design, a center frequency \( f_c \), and fractional bandwidth \( FBW \) of the stopband of this even-mode circuit are appropriately chosen so that its lower side passband can cover the operating frequency band of the WPD. The
electrical length $\theta$ of the transmission lines is $\pi/2$ (a quarter-wavelength) at the center frequency $f_{cs}$ of the stopband. An optimization program is coded to minimize the value of the above object function in (8), from which, the optimal values of all the characteristic impedances $Z_i$ ($i=1, 2, 3$) of the transmission lines and $Z_{st}$ of the open stub are determined.

2.2 Odd-mode analysis
In the odd-mode circuit shown in Fig. 2(b), all the characteristic impedances $Z_i$ ($i=1, 2, 3$) and $Z_{st}$ of the transmission lines have been determined through the above-described even-mode analysis, and the electrical length of the transmission lines is a quarter-wavelength at the center frequency $f_{cs}$ of the stopband, i.e., $\theta = \theta_{cs} = \pi/2 @ f_{cs}$.

The remaining unknown circuit parameter is the absorbing resistor $R_s$, which can be determined through the impedance matching analysis of the circuit. By referring to Fig. 2(b), we get the following formulas:

\[
\begin{align*}
Z_{in1} &= jZ_s \tan \theta \\
Z_{in2} &= Z_s \frac{j(Z_s+Z_2) \tan \theta}{Z_s+Z_2} \\
Z_{in3} &= \frac{Z_{in2}+jZ_2 \tan \theta}{Z_{in2}+jZ_2 \tan \theta} \\
Z_{in4} &= Z_3 \frac{jZ_{in3}+jZ_3 \tan \theta}{Z_{in3}+jZ_3 \tan \theta} \\
Z_{in} &= \frac{Z_{in4}+R_s/2}{Z_{in4}+R_s/2}
\end{align*}
\]

with
\[
\theta = \theta_{cs} \cdot f / f_{cs} = \pi/2 \cdot f / f_{cs}
\]

The final impedance-matching condition is
\[
Z_{in5} = Z_s
\]

From this equation, we can solve the absorbing resistor $R_s$ of the WPD.

3. Design example of a wideband microstrip WPD
Based on the above description, we design a compact wideband microstrip WPD with the following specifications:

(i) Center frequency: $f_c = 3.0$ GHz
(ii) Operating frequency band: 1.5 ~ 4.5 GHz ($FBW=100\%$)
(iii) VSWR < 2.0, i.e., $|S_{11}| < 10.0$ dB ($i=1, 2, 3$)
(iv) Maximum insertion loss, i.e., Max ($|S_{11}|, |S_{12}|) < 4.0$ dB
(v) Isolation between the output ports $|S_{23}|=|S_{32}| > 10.0$ dB

The design of the microstrip WPD is done in three steps described below:

(1) Step-1: Design of the even-mode circuit
The even-mode circuit in Fig. 2(a) of the WPD is designed, from which we determine the characteristic impedances $Z_i$ ($i=1, 2, 3$) of the transmission lines and $Z_{st}$ of the open stub. In this step, the center frequency of the stopband of this circuit is chosen as $f_{cs} = 8.0$ GHz, a fourth-order Chebyshev polynomial in Eq. (5) and (6) is used, and the ripple factor in Eq. (5) is chosen as $e = 1.0807$ so that the maximum ripple level of the return loss is $15.0$ dB in the equal-ripple passband of the WPD which covers the operating frequency band 1.5~4.5 GHz. Also, in the design, $Z_s = Z_L = 50.0$ Ω is assumed.

Using the parameters listed in Table I, the frequency response of the even-mode circuit is calculated using the circuit simulator ADS and is shown in Fig. 3. It is seen that the frequency range from 1.5 GHz to 4.5 GHz, shown by the light-grey region in the figure, the return loss $S_{11}$ is less than the required 15.0 dB.

(2) Step-2: Determination of $R_s$ from the odd-mode circuit
By using the impedances given in Table I, the equation (10) ~ (15) is solved to determine the absorbing resistor $R_s$. The calculation yields $R_s = 106.5$ Ω.

(3) Step-3: Design of the microstrip WPD
After all the circuit parameters of the WPD in Fig. 1, including the characteristic impedances $Z_i$ ($i=1, 2, 3$) and $Z_{st}$, the electrical length $\theta$ and the absorbing resistor $R_s$, are obtained following Step-1 and Step-2 described above, we designed the microstrip WPD using the well-known FR4 substrate with a relative dielectric constant $\varepsilon_r = 4.5$, a loss tangent $\tan \delta = 0.015$, and a thickness $t = 1.0$ mm. An electromagnetic (EM) simulator, Sonnet em, is used, and the finally obtained circuit configuration with geometrical parameters is shown in Fig. 4.

To verify the design, the simulated frequency responses of the WPD using the circuit simulator ADS and the EM simulator Sonnet em are compared in Fig. 5(a)-(d). In the EM simulation, all the losses, including the conductor loss, the dielectric loss, and the radiation loss of the circuit are considered.

Fig. 5(a) and 5(b) show the return loss $S_{11}$ and $S_{22}$ ($=S_{32}$) at Port 1 and Port 2 (Port 3), respectively. Although there are
some discrepancies between and circuit and EM simulation results, all the return loss at the three ports are less than the required 10.0 dB over the wide operating frequency band 1.5 ~ 4.5 GHz. The EM simulation result of $S_{11}$ shows only one reflection zero, compared with two in the circuit simulation result, which may be due to the extra reflections introduced by the many right angels and T-junctions in the microstrip circuit of the WPD. The effect of right angels and T-junctions are not considered in the circuit simulation.

Fig. 5(c) shows the power division $S_{21}(=S_{31})$ at Port 2 and 3. Over the whole operating frequency band 1.5 ~ 4.5 GHz, the output power is quite stable with a fluctuation less than 0.5 dB. Also, the discrepancy between the circuit and EM simulation is less than 0.5 dB. Fig. 5(d) show the isolation $S_{32}(=S_{23})$ between the two output Ports 2 and 3. Over the whole operating frequency band 1.5 ~ 4.5 GHz, the isolation is larger than 10 dB, satisfying the design requirement.

4. Measured performance of the microstrip WPD

The designed microstrip WPD is fabricated, and its photograph is shown in Fig. 6. The circuit occupies an area of about $0.20\lambda_g \times 0.13\lambda_g$, where $\lambda_g$ is the wavelength at the center frequency 3.0 GHz of the WPD. The circuit is tested by using an Anritsu MS46122A vector network analyzer. Fig. 7(a)-(d) show the measured performance of the WPD which is compared with the EM simulation result.
Fig. 6 Photograph of the fabricated microstrip WPD

Fig. 7(a) and 7(b) show the return loss $S_{11}$, $S_{22}$, and $S_{33}$, respectively. Due to the fabrication precision and soldering of the connectors and absorbing resistor, there are some discrepancies between the measured and EM simulation results. However, over the whole operating frequency band 1.5 ~ 4.5 GHz, all the measured and simulated return losses at the three ports are less than 10 dB, which means that the corresponding VSWRs at the three ports are all less than 2.0, satisfying the design specifications.

Fig. 7(c) show the power division $S_{21}$ and $S_{31}$, respectively. Over the whole operating frequency band 1.5 ~ 4.5 GHz, the discrepancy between the measured $S_{21}$ and $S_{31}$ is less than 0.1 dB. The difference between the measured and EM simulation is less than 0.25 dB.

Fig. 7(d) show the isolation $S_{23} = S_{32}$ between the two output ports 2 and 3. Over the operating frequency band 1.5 ~ 4.5 GHz, both the measured and simulated isolations are larger than 10 dB except a few points near 1.5 GHz.

5. Conclusion

In this paper, a novel compact wideband WPD is proposed, and its design method is developed. Based on the derived formulas, all the circuit parameters are determined with the assistance of a self-coded optimization program. The designed and fabricated microstrip WPD at 3.0 GHz occupies an area of only $0.20 \lambda_g \times 0.13 \lambda_g$. Within the operating frequency band 1.5 ~ 4.5 GHz (FBW=$100\%$), the maximum insertion loss is 3.8 dB, the VSWRs at all three ports are less than 2.0, the deviation of the power division at the two output ports is less than 0.1 dB, and the isolation between the two output ports is larger than 10.0 dB. The configuration of the WPD is simple with moderate geometrical dimensions (the smallest strip-width is 0.25 mm) and is thereby easy for low-cost chemical etching. Only one chip-resistor is used, and no coupling lines, via-holes, and defected ground structures (DGS) are needed.

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