Fault location method of active distribution network based on graph theory and matrix algorithm

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Abstract. In view of the large scale distributed power distribution network, distribution network from the traditional single main transformer power supply system becomes more complex broken power supply network, the trend of the distribution network flow and network frame produced change, failure fault feature information of great change, the traditional distribution network fault location method can not accurately obtain the location of the fault zone, the low accuracy of fault location, the error bigger problem a new fault location method of active distribution network combining graph theory and matrix was proposed. According to the knowledge of graph theory, the distribution network is simplified to topology diagram and described in the form of matrix, and then the fault judgment matrix is obtained through a series of matrix addition operation, and the type of fault interval can be identified accurately and quickly by combining the fault criterion table. The simulation results show that this method is simple in principle, fast and effective in criterion, suitable for the flexible operation mode of active distribution network structure, can accurately and quickly determine the fault segment, and can meet the requirements of complex distribution network fault location.

1. Introduction

In recent years, with the development and utilization of new energy, the scale of distributed power supply, mainly photovoltaic power generation and wind power, connected to the distribution network is increasing, which also brings many new problems. Distribution network from the traditional single main transformer power supply system becomes more complex broken power supply network, the trend of the distribution network flow and network frame produced change, dramatic change failure fault characteristic information, such as the failure of short-circuit current source, direction and size all can change, causing the original distribution network fault location scheme and relay protection technology are no longer applicable[1]. And with the rapid development of economy and the improvement of residents' pursuit of life, users have higher and higher requirements for power supply reliability of distribution network. In order to solve the problem caused by the massive distributed power supply connected to the distribution network, CIGRE C6.11 project team proposed the concept of Active Distribution Network (ADN) in 2008. It aims to solve the compatibility of power grid and the application of large-scale intermittent renewable energy, improve the utilization rate of green energy and optimize the primary energy structure[2]. Therefore, it is of great significance to study fault location technology of active distribution network under the new situation.

At present, distribution network fault location based on feeder terminal (FTU) is widely used in active distribution network fault location methods at home and abroad. This method is mainly divided into two research directions: intelligent algorithm and matrix algorithm: One is the fault location algorithm based on artificial intelligence, such as artificial neural network, expert system, genetic
algorithm, fuzzy theory, etc. Due to the long calculation time, this kind of algorithm is difficult to meet the requirements of real-time online processing, and is generally used less[3]. The other is the matrix algorithm based on graph theory, which is widely used because of its small amount of calculation and simplicity. The topology of ADN with distributed energy is flexible and changeable, so the traditional fault segment determination algorithm can not be applied completely. Therefore, the improved matrix algorithm has more advantages in computing speed and efficiency. Based on the above, this paper proposes a new matrix algorithm based on graph theory. The algorithm is simple in calculation, avoids complex matrix multiplication operation, and does not need to redefine the direction when the network structure changes. It can quickly and accurately locate faults and terminal faults between multiple power nodes and achieve fault isolation.

2. Fault location method of active distribution network

2.1 Graph theory and network structure simplification

Distribution network is a connected network composed of switches, distribution feeders and distribution transformers. The structure of active distribution network with distributed energy can be simplified and further studied by graph theory in discrete mathematics.

A figure consisting of several different vertices with edges connecting some of them is called a graph. Among the elements of the graph, the nodes of the active distribution network are taken as the vertices in the graph, and the branches of the active distribution network are taken as the edges in the graph. If two vertices can be connected by an edge, the two vertices are said to have adjacent relations. Usually figure can be expressed as $G = (V, E)$, $V$ is the set of all its vertices $V = \{v_1, v_2, \cdots, v_n\}$, $E$ is a collection of all its edges to $E = \{e_1, e_2, \cdots, e_m\}$.

Active distribution network is usually divided into radial network and ring network according to the structure characteristics. Based on the above graph theory knowledge, distribution network composition can be simplified, load nodes are regarded as vertices, distribution feeders and distribution transformers between two adjacent vertices are regarded as edges, and the distribution network topology can be described by undirected graph, and then the topology description of distribution network in accordance with graph theory knowledge can be obtained. On the basis of the undirected graph, the power direction of the main power supply of the network is selected as the reference forward direction, and the network is described as a directed graph. Then, the directed dynamic half-angle adjacency matrix is obtained, which can reflect the structure state of the network, the direction relation between nodes and the electrical connection relation.

2.2 Fault information coding method and matrix principle

$\beta_i (i = 1, 2, 3, \ldots, D)$ represents the fault state information of the section in article I, and the diagonal element represents the state of this section, and its encoding mode is shown in Formula (1), where $D$ is the total number of sections contained in the system.

$$\beta_i = \begin{cases} 1 & \text{Sector } i \text{ is normal} \\ 0 & \text{Sector } i \text{ out of service} \end{cases}$$ (1)

According to graph theory, A connected graph with $n$ nodes, $b$ branches and no self-loop can be represented by an association matrix $A$ of order $n \times b$. The rows and columns of matrix $A$ correspond to nodes and branches in the network respectively, and the elements of matrix $A$ satisfy $A_{ij}$.

$$A_{ij} = \begin{cases} 0, & \text{Node } i \text{ is not connected to branch } j \\ 1, & \text{Node } i \text{ is connected to branch } j \end{cases}$$ (2)

The operation of row vector $A_i$ and vector $\beta$ of matrix $A$ gives the electrical correlation matrix $E$, which describes the electrical connection relation between nodes.

$$A = [\alpha_1^T, \alpha_2^T, \cdots, \alpha_m^T]^T$$

$$E = [(\alpha_1 \cap \beta)^T, (\alpha_2 \cap \beta)^T, \cdots, (\alpha_m \cap \beta)^T]^T$$ (4)
Adjacency matrix $R$ describes the unicom relation between network nodes, and its row and column numbers correspond to corresponding node numbers, satisfying:

$$R_{ij} = \begin{cases} 
0, & \text{Node } i \text{ is not connected to node } j \\
1, & \text{Node } i \text{ is connected to node } j 
\end{cases} \quad (5)$$

Matrix $R$ changes with the status of network branches, and electrical correlation matrix $E$ describes the electrical connection relationship between nodes and network branches. The adjacency matrix $R$ can be obtained by the operation of Equation (6) on matrix $E$. See reference [4] for details.

$$R = (EE^T) \cap \bar{I} \quad (6)$$

When the fault occurs in the active distribution network, the data acquisition and Monitoring control system (SCADA) obtains the switch fault current information uploaded by the feeder terminal unit (FTU) installed on the node switch, and the master station collects and generates the n-dimensional diagonal matrix $F$, which is defined as follows:

$$f_{ii} = \begin{cases} 
1, & \text{The } I_f \text{ direction is consistent with the specified positive direction} \\
0, & I_f = 0 \\
-1, & \text{The direction of } I_f \text{ is opposite to the specified direction}
\end{cases} \quad (7)$$

Finally, the fault judgment matrix $J$ is obtained by the operation $J = R + F$ between the directed dynamic adjacency matrix $R$ and the fault information matrix $F$.

2.3 Common Branch Node types

The following are five common branch node types in active distribution networks. As shown in Figure 1, they are power outlet node, terminal load node, 2 branch node, 3 branch node failure, and 4 branch node respectively.

![Figure 1. Common branch node types in distribution network](image)

2.4 Fault criteria

Due to the different types of branch nodes in the network, the corresponding fault characteristics are different, so the fault criterion corresponding to one and one is proposed [5]. After the fault occurs, the fault judgment matrix $J$ is obtained by calculating the directed dynamic adjacency matrix $R$ and the fault information matrix $F$ in the protected host, that is, $J = R + F$. After the fault judgment matrix $J$ is generated, fault location can be realized according to the six criteria in Table 1 below.

| Fault Type          | Fault Feature                                                                                                                                  | Fault criterion                                                                 |
|---------------------|-----------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------|
| The power export    | The system only detected fault current supplied by DG which was opposite to the reference forward direction                                    | The matrix $J$ has only elements -1 and 0 on the diagonal                           |
| Two node range      | The $i$th row of the matrix $J$ contains one and only one of the other elements except $J_{ii}$                                               | $J_{ii} = 1$, $J_{ij} = 1(i \neq j)$ and $J_{jj} = 0$ or $J_{ij} = -1$, make $J_{ii} \& J_{jj} = 0$ |
3. Example analysis and simulation

3.1 Directed dynamic matrix acquisition

The following takes a 14 node active distribution network as an example to explain the principle of fault location algorithm adopted in this paper. The distribution network structure is shown in Figure 2. In Figure 2, there are 14 nodes and 13 branches. DG1 and DG2 are access nodes 5 and 14 respectively, FTU is installed on nodes 1 to 13, \( S_1 \) is the main power supply. Based on the above explanation, Figure 2 can be further simplified into an undirected graph, as shown in Figure 3.

![Figure 2. active distribution network of 14 nodes](image1)

![Figure 3. Undirected diagram of 14-node active distribution network](image2)

The branch vector representing the state of this section is as follows

\[
F = [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1] \quad (8)
\]

Figure 3 shows the connected graph without isolated nodes and self-loops, corresponding to which A 14-row and 13-column association matrix \( A \) can be generated. Substitute the matrix \( A \) and the matrix \( F \) into Equation (4), the electrical correlation matrix \( E_{14\times13} \) is obtained, whose form is the same as that of matrix \( A \).

\[
A = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} \quad (9)
\]

\[
E = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} \quad (10)
\]
The adjacency matrix $R_{13 \times 13}$ obtained from Equation (6) is as follows. In addition, the adjacency matrix $R$ is an undirected symmetric Boolean matrix. By selecting the power direction of main power supply $S_1$ as the reference positive direction, the node relation expressed in adjacency matrix $R$ is simplified to the forward connection relation, and the element value of the negative connection relation is set to 0 to obtain the directed dynamic half-angle matrix. Therefore, matrix $R$ is a simplified adjacency matrix $R$ after digitizing to omit the lower half angle zero element for convenience.

\[
R = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

(11)

\[
R = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

(12)

3.2 Fault section determination

The fault diagram of active distribution network is shown in Figure 4, K1, K2, K3, K4 and K5, respectively, indicating that the fault occurs at the exit, two-node, three-node, four-node interval and terminal.

According to the network fault diagram of 14-node active distribution network, the corresponding fault information matrix of K1, K2, K3, K4 and K5 faults.

| Fault location | Fault information matrix |
|----------------|-------------------------|
| K1             | $F_1 = \begin{bmatrix} -1 & -1 & -1 & -1 \\
                        -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$ |
| K2             | $F_2 = \begin{bmatrix} 1 & -1 & -1 & -1 \\
                        -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$ |
| K3             | $F_3 = \begin{bmatrix} 1 & 1 & -1 & -1 \\
                        -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$ |
| K4             | $F_4 = \begin{bmatrix} 1 & 1 & 1 & -1 \\
                        -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$ |
| K5             | $F_5 = \begin{bmatrix} 1 & 1 & -1 & -1 \\
                        -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$ |
After the fault occurs, the fault judgment matrix $J$ is obtained by calculating the directed dynamic adjacency matrix $R$ and the fault information matrix $F$ in the protected host, that is, $J = R + F$. According to the above information, the fault determination matrices $J_1, J_2, J_3, J_4, J_5$ are obtained by the operation of fault information matrices $F_1, F_2, F_3, F_4, F_5$ with the reduced adjacency matrix $R$. The fault location results of the active distribution network of the 14 nodes can be obtained according to the fault determination matrices and fault criteria.

### Table 3. Fault location results

| Fault Type      | Fault criterion                                                                 | Locating result                                      |
|-----------------|---------------------------------------------------------------------------------|-------------------------------------------------------|
| The power export| The fault information matrix $J_1$ contains only elements -1 and 0 on the diagonal | The main power outlet is faulty.                      |
| Two node range  | $J_{11} = 1$, $J_{12} = 1$ and $J_{22} = -1$, there is $J_{11} \& J_{22} = 0$ | The fault occurs between nodes 1 and 2                |
| Three node range| $J_{22} = J_{23} = J_{26} = 1$, and $J_{33} = -1$, $J_{66} = 0$, there is $J_{22} \& [J_{33} \odot J_{66}] = 0$ | Nodes 2, 3, and 6 are faulty.                          |
| Four node range | $J_{33} = J_{34} = J_{39} = J_{3,10} = 1$, and $J_{44} = -1, J_{99} = 0, J_{10,10} = -1$, there is $J_{33} \& [J_{44} \odot J_{99} \odot J_{10,10}] = 0$ | Nodes 3, 4, 9, and 10 are faulty.                       |
| Line end interval| In matrix $J_5$, all elements in row 8 are 0 except $J_{88} = 1$ | The cable terminal of node 8 is faulty.                |

### 4. Conclusion

This paper introduces a new method for fault location of active distribution network based on graph theory and matrix algorithm, which can accurately identify power outlet, line end and multi-branch interval faults. For the complex distribution network including distributed power supply, due to its complex and changeable architecture, the power direction of main power supply $S_{1}$ in the distribution network is defined as the only positive direction, so the adjacency moment $R$ can be automatically updated only by using real-time power flow direction. At the same time, the innovative fault judgment matrix can be generated only through matrix addition operation, which not only greatly reduces the complexity of matrix operation but also shortens the time of fault judgment compared with the tedious multiplication operation. Simulation results show that the algorithm is simple in principle, fast and effective in criterion, and can meet the requirements of accuracy and rapidity in fault location of complex distribution network. In this paper, there is a lack of research on the authenticity of alarm information. In the future, the distorted alarm information will be screened and extracted.

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