Enhanced Singular Spectrum Decomposition and Its Application to Rolling Bearing Fault Diagnosis

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ABSTRACT
Singular spectrum analysis (SSA) has proven to be a powerful technique for processing non-stationary signals and has been widely used in the fault diagnosis of rolling bearings. Based on the SSA, an adaptive signal decomposition algorithm called singular spectrum decomposition (SSD) was developed. The SSD realizes the adaptive selection of two critical parameters of SSA (i.e., embedding dimension selection and principal components grouping) by concentrating on the frequency components of the signal. Despite that SSD makes the SSA techniques more automated and has shown its potentials in detecting bearing faults, it may fail to separate the fault bearing signals whose frequencies are not outstanding among the frequency components of the signal. Hence, this paper presents an enhanced SSD (ESSD) approach to better detect bearing faults by introducing the differentiation and integration operators into SSD. Specifically, the raw vibration signal is first differentiated to highlight the fault signal components. Then, the new signal retrieved through the differentiation process is subjected to SSD to yield a number of singular spectrum components (SSCs). Finally, each SSC is integrated to obtain the enhanced SSC (ESSC). The simulation analysis indicates that the ESSD improves the anti-interference capability of the SSD. The ESSD provides more pleasant results in an experimental bearing fault signals’ analysis compared with the original SSD, variational mode decomposition (VMD), and Kurtogram algorithms, which illustrates the superiority of the ESSD for detecting bearing faults.

INDEX TERMS
Enhanced singular spectrum decomposition, differentiation operator, integration operator, rolling bearing, fault diagnosis.

I. INTRODUCTION
Rolling bearing is among the most critical and defect-prone components of rotating machinery. Its failures can degrade the performance and efficiency of rotating machinery and even lead to serious safety [1]. Therefore, scholars have studied many fault diagnosis techniques based on vibration signal [2], acoustic emission signal [3], oil slag [4], etc., to ensure that bearing faults can be diagnosed timely and accurately. Among these fault diagnosis techniques, the ones based on vibration analysis are the most highly respected owing to their high efficiency and convenience [5].

The core of the vibration analysis techniques is to employ practical signal processing approaches to extract the signatures of the fault impulse signal generated by bearing defect. However, the vibration signal of the defective bearing is always the mixture of fault impulse signal, harmonic interferences and background noise. Hence, the separation of fault impulse signal is a critical part of bearing fault diagnosis. To reach this goal, some signal de-noising and deconvolution algorithms have been introduced for enhancing the fault signal of bearing [6], [7]. Despite the signal de-noising algorithms have good performances in noise suppression, they are difficult to exclude the harmonic interferences. At the same time, the weak pulses of the fault signal are easily smoothed by signal de-noising algorithms. The idea of the deconvolution method is to use a designed FIR filter to make the output signal reach the predetermined target value. Maximum correlated kurtosis deconvolution (MCKD) [8] and multipoint optimal minimum entropy deconvolution adjusted
(MOMEDA) [9] are two recently invented deconvolution algorithms, which have the excellent talent of incipient fault detection. However, the target period and filter length need to be set artificially in their application [10]. With the occurrence of the local defect, the high-frequency resonance band will appear in the bearing fault signal. Kurtogram [11] and its modified versions such as improved Kurtogram [12] and Autogram [13] can adaptively locate the resonant frequency band of the signal, thereby demodulating the fault information. However, the Kurtogram-like approaches tend to yield incorrect results when the signal has accidental impulses.

Signal decomposition can retrieve the potential components with different frequency bands from the original data, which facilitates it as an important tool for separating the fault signals of bearing. In recent years, the signal decomposition method has been continuously updated, and it has become an indispensable technical solution for diagnosing bearing faults. Wavelet transform (WT) and empirical mode decomposition (EMD) are two of the most primitive signal decomposition methods [14], [15]. The decomposition efficiency of WT relies on the choice of basis function and may lead to the energy leakage related to the fault signal. Some well-designed WT methods such as over-complete rational dilation WT (ORDWT) [16] and tunable Q-factor WT (TQWT) [17] show stronger bearing fault detection capabilities. However, parameter selection and more calculation time are needed. The decomposition process of EMD is based on the local features of signals. When dealing with complex bearing signals, EMD often has modal aliasing problems [18]. In recent years, scholars have attempted to introduce more efficient signal decomposition methods such as ensemble EMD (EEMD) [19], complementary EEMD (CEEMD) [20], adaptive local iterative filtering (ALIF) [21], empirical WT (EWT) [22], time-varying filtering based EMD (TVF-EMD) [23], swarm decomposition (SWD) [24] and variational mode decomposition (VMD) [25] into bearing fault diagnosis. These methods greatly promote the development of bearing fault diagnosis technology. However, they are all parametric algorithms that require a reasonable combination of parameters. This flaw makes them less suitable for the online fault diagnosis of bearings.

Singular spectrum analysis (SSA) is a mighty signal processing technique, which is capable of analyzing the multi-dimensional signal and can realize the decomposition of one-dimensional signals [26]. SSA has exhibited excellent characteristics in noise reduction [27] and fault feature separation [28] of bearing fault vibration signals. When using SSA for signal decomposition, it is inevitable to artificially select the embedding dimension to establish the trajectory matrix and select the useful principal component for signal reconstruction. The choice of these two parameters has a decisive influence on the decomposition results. A novel signal decomposition algorithm called singular spectrum decomposition (SSD) was developed by Bonizzi \textit{et al.} [29]. This algorithm can adaptively determine the two critical parameters of SSA by focusing on the frequency contents of the signal. SSD overcomes the modal aliasing flaw of EMD and is not susceptible to noise. It has been proven to be more capable of detecting the faults of rotating machinery compared to EMD [30], [31]. In our study, it is found that the weak fault signal, whose frequency is not the dominant frequency in the bearing signal, is difficult to be recovered by SSD. This situation is not uncommon for rolling bearings because the weak fault signals are easily overwhelmed by harmonics and noise. The differentiation operator can improve the signal-to-interferences ratio (SIR) of the signal, and the integration operator can inhibit the noise [32]. An enhanced SSD (ESSD) algorithm is presented in this paper to promote the detection capability of weak fault features of bearing by introducing the differentiation and integration operators into SSD. The proposed algorithm and three comparison algorithms (i.e., SSD, VMD and Kurtogram) are applied to simulation data and two experimental data to evaluate the superiority of the proposed algorithm in detecting weak signatures of bearings.

The rest of this paper is organized as follows. Section II describes the principle of SSD and the effects of signal differentiation and signal integration. Section III describes the proposed ESSD algorithm. Sections IV and V implement the simulated and experimental verifications, respectively. A few conclusions are concluded in Section VI.

II. THEORETICAL BACKGROUND

A. SINGULAR SPECTRUM DECOMPOSITION

For a multi-component signal with $N$ data points, $s(n)$, its sub-components can be iteratively separated using the SSD algorithm until the iteration stopping condition is met. Each iteration can be specifically actualized as follows [29]:

\textit{Step 1 (Embedding):} Assuming the embedding dimension is set to $P$ ($1 < P < N$), $P$ lagged vectors, $S_i = \{s(i), \ldots, s(N), s(1), \ldots, s(i-1)\}$, where $i = 1, 2, \ldots, N - P + 1$, can be produced via the embedding process. These lagged vectors construct a $P \times N$ trajectory matrix, $S = [S_1, S_2, \ldots, S_P]^T$. The construction of the trajectory matrix for the series $s(n) = \{1, 2, 3, 4, 5\}$, is used as an instance to better explain the embedding process of SSD. Equation (1) shows the corresponding trajectory matrix for $P = 3$.

$$S = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \\ 3 & 4 & 5 & 1 & 2 \end{bmatrix}$$

The embedding dimension is a key parameter in the SSA technique. An improvement of SSD over SSA is to propose a scheme for the adaptive selection of this parameter. Specifically, the power spectral density (PSD) of the residual signal at iteration $j$ is firstly computed as: $r_j(n) = s(n) - \sum_{k=1}^{j-1} r_k(n)(r_0(n) = s(n))$. Then, detect the primary frequency with the largest amplitude in the PSD, i.e., $f_{\text{max}}$. For $j = 1$, if $f_{\text{max}}/f_s < 0.001$ ($f_s$ denotes the sampling frequency), that means the signal has a sizeable trend, $P$ is then chosen as $N/3$. For $j > 1$, $P$ is set to $1.2 \times (f_s/f_{\text{max}})$. 


Step 2 (Decomposition): Apply singular value decomposition (SVD) to the trajectory matrix $S$, we get:

$$S = U \Lambda V^T$$

$$= [u_1, u_2, \cdots, u_P] \begin{bmatrix}
\delta_1 & 0 & 0 & 0 \\
0 & \delta_2 & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \delta_p
\end{bmatrix} \begin{bmatrix}
v_1^T \\
v_2^T \\
\vdots \\
v_p^T
\end{bmatrix}$$

$$= \delta_1 u_1 v_1^T + \delta_2 u_2 v_2^T + \cdots + \delta_p u_p v_p^T$$

where $U \in R^{P \times P}$ and $u_k \in R^{P \times 1}$ is its $k$-th column vector, $V \in R^{N \times N}$ and $v_k \in R^{N \times 1}$ denotes its $k$-th column vector, $\Lambda \in R^{P \times N}$ represents the singular value matrix and $\delta_k$ is its $k$-th singular value.

Let $S_k = \delta_k u_k v_k^T$, the trajectory matrix $S$ can be converted to:

$$S = S_1 + S_2 + \cdots + S_P$$

The trajectory matrix is described as the sum of $P$ sub-matrices through SVD. Each sub-matrix is seen as a principal component.

Step 3 (Grouping): How to group the principal components to reconstruct a specific signal is another issue of SSA, and this is automatically actualized in SSD. More explicitly, the frequency band $[f_{\text{min}} - f_a, f_{\text{max}} + f_a]$ in the PSD of $r_f(n)$ is regarded as the target frequency band, $f_a$ is the bandwidth and it can be calculated by the Gaussian interpolation of the PSD of $r_f(n)$. The $Q (Q < P)$ principal components, whose left eigenvectors have an outstanding frequency in the target frequency band, are chosen to reconstruct a new matrix.

Step 4 (Reconstruction): A specific component signal can be acquired through the diagonal averaging of the reconstructed matrix. To implement the diagonal averaging, some elements of the reconstructed should be moved. For example, the elements of the matrix shown in Equation (1) should be rearranged as:

$$X = \begin{bmatrix}
1 & 2 & 3 & 4 & 5 \\
2 & 3 & 4 & 5 \\
3 & 4 & 5
\end{bmatrix}$$

The recovered component is then subtracted from $s(n)$. The above procedures are applied to the residual to extract the next component. Iteratively, the SSD decomposition can be implemented. An iteration stopping criteria based on the energy of the residual is used in SSD. Specific details of SSD can refer to [29].

From the principle of SSD, we can find that the selection of the key parameters of SSD, i.e., embedding dimension and principal components grouping, are based on the dominant frequency of the signal to be decomposed at each iteration. The impact fault signal is the target signal we want to separate from the rolling bearing vibration signal. However, the impact fault signal may not exhibit a prominent frequency due to the effects of the influences. In this case, SSD will fail to separate the weak impact fault signal. Only by breaking this obstacle can SSD better detect bearing failures.

B. SIGNAL DIFFERENTIATION

This section introduces the effects of differentiation on bearing vibration signals. Given a discrete signal $y(n)$, its differentiation can be calculated as [32]:

$$D(y(n)) = (y(n) - y(n-1))/\Delta t, \quad \Delta t = 1/f_s$$

where $f_s$ is the sampling frequency.

As described in previous studies, the differentiation process can contribute to amplifying the energy of high frequency impact signals [33] and boosting signal-to-interferences ratio (SIR) [32]. The researches conducted in [32] are introduced to clearly illustrate the above functions of differentiation on the signal.

The model of the shock fault signal of bearing can be simplified as [32]:

$$s(t) = Ae^{-\alpha t} \cos(\omega_r t + \phi)$$

where $A$ represents the amplitude of $s(t)$, $\alpha$ is the coefficient of resonance damping, $\omega_r$ denotes the resonant frequency aroused by $s(t)$ and $\phi$ is the initial phase.

By applying differentiation to the signal in Equation (6), we get:

$$D(s(t)) = -A\omega_r e^{-\alpha t} \sin(\omega_r t + \phi) - A\alpha e^{-\alpha t} \cos(\omega_r t + \phi)$$

Let $\gamma = A\sqrt{\omega_r^2 + \alpha^2}, \sin\xi = A\omega_r/\gamma, \cos\xi = A\alpha/\gamma$ and $\phi_\xi = \phi - \xi$, the derivative of $s(t)$ can be transformed into:

$$D(s(t)) = -\gamma e^{-\alpha t} \cos(\omega_r t + \phi_\xi)$$

Equation (9) is used to describe the multiple interferences:

$$i(t) = \sum_{m=1}^{M} P_m \cos \omega_{lm} t$$

where $P_m$ and $\omega_{lm}$ are the amplitude and frequency of the $m$-th interference, respectively. It is assumed that all $\omega_{lm}$ are smaller than the $\omega_r$ with practical reason.

The derivative of $i(t)$ can be calculated as:

$$D(i(t)) = - \sum_{m=1}^{M} \cos \omega_{lm} P_m \sin \omega_{lm} t$$

The definition of SIR is given as [32]:

$$SIR = \frac{(1/T) \int_0^T s^2(t) dt}{(1/T) \int_0^T i^2(t) dt}$$

where $s(t)$ and $i(t)$ represent the defective bearing signal and interferences, respectively.
According to Equations (6), (9) and (11), the SIR of the signal before differentiation can be deduced as:

\[
SIR_{\text{original}} = \frac{\left(\frac{1}{T_p}\right) \int_0^{T_p} A^2 e^{-2\alpha t} \cos^2(\omega_r t + \phi) dt}{\left(\frac{1}{T}\right) \sum_{k=1}^K P_k^2 \cos^2(\omega_{rk} t) dt} \frac{A^2}{2T_p \alpha^2 [\sum_{k=1}^K (P_k)^2] + \frac{A^2 (\alpha \cos 2\phi - \omega_r \sin 2\phi)}{2T_p \left[ \sum_{k=1}^K (\omega_{rk} P_k)^2 \right]}}
\]

(12)

where \(T_p\) is the period of the fault signal and \(T_k\) is the period of the \(k\)-th interference component.

According to Equations (7), (10) and (11), the SIR of the signal after differentiation can be deduced as:

\[
SIR_{\text{differentiation}} = \frac{\left(\frac{1}{T_p}\right) \int_0^{T_p} A^2 (\omega_r^2 + \alpha^2)e^{-2\alpha t} \cos^2(\omega_r t + \phi) dt}{\left(\frac{1}{T}\right) \sum_{k=1}^K P_k^2 \cos^2(\omega_{rk} t) dt} \frac{A^2}{2T_p \alpha^2 [\sum_{k=1}^K (\omega_{rk} P_k)^2] + \frac{A^2 (\alpha \cos 2\phi - \omega_r \sin 2\phi)}{2T_p \left[ \sum_{k=1}^K (\omega_{rk} P_k)^2 \right]}}
\]

(13)

Combing Equations (12) and (13), it can be deduced [32]:

\[
\frac{SIR_{\text{differentiation}}}{SIR_{\text{original}}} \approx \frac{(\omega_r^2 + \alpha^2) \left[ \sum_{k=1}^K (P_k)^2 \right]}{\sum_{k=1}^K (\omega_{rk} P_k)^2} \geq 1
\]

(14)

From Equation (14), we can conclude that the value of SIR increases after the differentiation process, which means the fault signal is highlighted. In the same way as the definition of signal-to-noise ratio (SNR), the following equation is adopted to measure the SIR of the signal more conveniently:

\[
SIR_e = 10 \log_{10} \frac{\sum_{i} s(t_i)^2}{\sum_{i} t(t_i)^2}
\]

(15)

More details about the effects of signal differentiation can refer to [32].

C. SIGNAL INTEGRATION

Given a discrete signal \(y(n)\), its integration can be calculated using the trapezoidal rule:

\[
I(y(n)) = \Delta t[y(n) + y(n - \Delta t)]/2
\]

(16)

where \(\Delta t\) denotes the step factor.

The effects of signal integration have been studied in [32]. It proves that the integration operator can improve the SNR of the signal.

III. ENHANCED SINGULAR SPECTRUM DECOMPOSITION BASED ON DIFFERENTIATION AND INTEGRATION

By introducing signal differentiation and integration, a non-parametric signal decomposition algorithm termed enhanced singular spectrum decomposition (ESSD), is put forward in this work to improve the fault detection capability from interferences. Fig. 1 gives the flowchart of ESSD, which includes three main procedures:

1. Differentiate the raw vibration data of rolling bearings.
2. Decompose the differential signal with SSD to get a number of SSCs.
3. Apply integration to each SSC to obtain the enhanced singular spectrum component (ESSC).

IV. SIMULATION ANALYSIS

A composite signal, which is composed of the fault signal, vibration interference signal and noise, is employed to test the capability of the proposed ESSD algorithm in detecting the weak fault signal of rolling bearing. According to [34], [35], the fault signal is generated using the following equation:

\[
s(t) = \sum_{k=1}^K A_k e^{-\alpha (t - k T_b - \tau_i)} \times \cos \omega_r (t - k T_b - \tau_i + \theta_k) \sigma(t - k T_b - \tau_i)
\]

(17)

where \(A_k\) denotes the magnitude of the \(k\)-th fault impulse, \(T_b\) is the interval between two adjacent fault impulses, \(\alpha\) represents the damping coefficient, \(\omega_r\) denotes the induced resonance frequency, \(\tau_i\) is a random number within the range \([0.01 T_b, 0.02 T_b]\), \(\theta_k\) is the phase and \(\sigma(t)\) represents the step function. Table 1 shows the specific settings of these parameters. We can deduce the fault characteristic frequency as \(f_b = 1/T_b = 100\text{Hz}\).

According to Equation (9), the vibration interference signal is constructed by four harmonics (\(\omega_{Im} = 8, 35, 430, 650\text{Hz}\)). The mixed noise signal is yielded by employing the ‘awgn’ matlab function. The SIR and SNR of the composite signal are set as \(-30\text{dB}\) and \(-5\text{dB}\), respectively.
FIGURE 2. Simulation signal: (a) the fault signal; (b) the composite signal; (c) the frequency spectrum of (b); (d) the enveloping result of (b).

Figs. 2(a) and (b) show the fault signal and the composite signal, respectively. The trend of the signal in Fig. 2 (b) is like an oscillating sinusoidal signal and the impulses shown in the fault signal cannot be located. Fig. 2 (c) displays the frequency spectra of the composite signal. Only the interference frequencies stand out in this figure, and the high-frequency resonance band is invisible. This indicates that the frequency of the fault signal is not the dominant frequency. The enveloping result displayed in Fig. 2 (d) fails to exhibit a spectra line at $f_b$ either.

The SSD algorithm is applied to the composite signal and five SSCs are separated, as shown in Fig. 3(a). The waveforms of these SSCs are similar to the harmonic signals. Fig. 3(b) depicts the frequency spectrums corresponding to the SSCs. Frequencies of 430, 650, 35 and 8 Hz are respectively presented in the spectrums of SSC$_2$ $\sim$ SSC$_5$, which infers that the SSD algorithm separates the four vibration interferences successfully. However, none of the five SSCs shows the impulsive features of the fault signal. The SSD algorithm fails to retrieve the fault signal as the frequency of the fault signal is not a prominent frequency in the signal.

The composite signal is then analyzed by the proposed ESSD algorithm. To emphasize the role of the differentiation, the results of the signal after differentiation are firstly shown in Fig. 4. Fig. 4(a) presents the differential signal. The trend of the differential signal is no longer an oscillating sinusoidal signal. Compared to the spectrum of the original composite signal, only two interference frequencies of 430 and 650 Hz are reserved while the other two interference frequencies disappear in the spectrum of the differential signal as plotted in Fig. 4 (b). Moreover, a high-frequency band appears in this spectrum. As known to us, the resonance frequency band that contains fault signatures locates in the high-frequency components. This highlights the function of the differentiation operator for enhancing the high-frequency components. Fig. 4 (c) depicts the enveloping result of the differential signal. Clearly, the interference frequencies of Fig. 4 (c) decrease compared to Fig. 2 (d). However, $f_b$ cannot be identified in this spectrum either.

The differential signal is further decomposed by SSD, and the decomposition components are subjected to the
integration to perform the ESSD algorithm. Five ESSCs are retrieved as plotted in Fig. 5(a). The second decomposed component, i.e., ESSC₂, reveals the fault impulses. Fig. 5(b) describes the spectrum of the ESSCs. A resonance band appears in the spectrum of ESSC₂. Moreover, the two interference frequencies of 8 and 35 Hz cannot be found in these spectrums. The second decomposed component shown in Fig. 5(c) is selected for enveloping analysis and the result is given in Fig. 5(d), which obviously exhibits \( f_b \) and its 2X-7X harmonics. The proposed ESSD algorithm detects the fault signatures successfully.

VMD is considered to be the most advanced signal decomposition algorithm at present and has shown the powerful capability of extracting fault signatures of bearings [25]. Hence, it is used for comparing analysis to demonstrate the superiority of the proposed ESSD algorithm. Two key parameters, i.e., the penalty factor \( \alpha \) and mode number \( K \), should be well considered in advance for VMD to achieve pleasant results. According to the recommendation of [36], [37], \( \alpha \) is set to 500 in this paper. The mode number is determined according to the number of the decomposition components obtained by ESSD. Hence, \( K \) is set to 5 in the simulation analysis. In order to ensure the fairness of the comparison, differentiation and integration are also involved in the VMD algorithm. Fig. 6(a) shows the decomposition components of VMD. The third component contains the fault signatures and it is separately displayed in Fig. 6(b). We can find heavier noise in this signal compared to Fig. 5(c). Its enveloping result shown in Fig. 6(c) has peaks at \( f_b \) and its 2X-4X harmonics. The proposed ESSD algorithm extracts more harmonics of \( f_b \) compared to VMD. The simulation analysis indicates the proposed ESSD method has a stronger ability to detect weak fault signals and outperforms the VMD algorithm.
B. Pang et al.: ESSD and Its Application to Rolling Bearing Fault Diagnosis

The Kurtogram approach is adopted to analyze the composite signal to further illustrate the advantage of the ESSD algorithm. Fig. 7(a) represents the Kurtogram and Fig. 7(b) displays the filtered signal obtained based on the Kurtogram. The shock features of the signal depicted in Fig. 7(b) are not as distinct as those of the signal shown in Fig. 5(c). Fig. 7(c) depicts the enveloping result of the filtered signal. Only the 1X-4X harmonics of the fault characteristic frequency are outstanding in this spectrum. The fault features extracted by the Kurtogram method are not as evident as those obtained by the ESSD algorithm.

The fault feature detection capabilities of the ESSD, VMD and Kurtogram methods are evaluated quantitatively by using the characteristic frequency intensity coefficient (CFIC) defined in the following equation [18]:

$$CFIC = \frac{\sum_{l=1}^{P} A_{f_l}}{\sum_{i=1}^{M} A_{f_i}}$$  \hspace{1cm} (18)

where $A_{f_l}$ is the amplitude of the $l$-th harmonic of the fault characteristic frequency $f_c$, $P$ represents the total number of the harmonics of $f_c$, $A_{f_i}$ denotes the amplitude of the frequency $f_i$, and $M$ is the number of the spectral lines.

The CFIC indicator reflects the richness of fault signature information in the frequency spectrum. A bigger CFIC value indicates a more satisfactory result. Fig. 5 (d), Fig. 6 (c) and Fig. 7(c) correspond to the enveloping results of the ESSD, VMD and Kurtogram, respectively. Their CFIC values are calculated with $f_c = 100$ Hz and the results are shown in Table 2. The CFIC value of the ESSD algorithm is the largest, which implies the ESSD algorithm has the best performance.

V. EXPERIMENTAL ANALYSIS

Two experimental data of rolling bearings, including an inner race fault of a cylindrical roller bearing and compound faults (which contain an inner race fault and an outer race fault) of a ball bearing, are used for validation in this section. Similarly, the proposed ESSD algorithm is compared with the original SSD, VMD and Kurtogram methods to prove its superiority. Both experiments are performed on the QPZZ
TABLE 3. Information of the cylindrical roller bearing.

| Bearing type | Geometric parameters | Theoretical fault characteristic frequency |
|--------------|----------------------|--------------------------------------------|
| N205         | Roller diameter | 7.5 mm | Pitch diameter | 38.5 mm | Number of rollers | 12 | Contact angle | 0° | Fault characteristic frequency of inner race ($f_i$) | 172 Hz | Fault characteristic frequency of outer race ($f_o$) | 116 Hz | Fault characteristic frequency of rollers ($f_r$) | 118 Hz | Fault characteristic frequency of cage ($f_c$) | 10 Hz |

TABLE 4. Information of the ball bearing.

| Bearing type | Geometric parameters | Theoretical fault characteristic frequency |
|--------------|----------------------|--------------------------------------------|
| LYC6205F    | Roller diameter | 7.94 mm | Pitch diameter | 38.5 mm | Number of rollers | 9 | Contact angle | 0° | Fault characteristic frequency of inner race ($f_i$) | 133 Hz | Fault characteristic frequency of outer race ($f_o$) | 88 Hz | Fault characteristic frequency of rollers ($f_r$) | 115 Hz | Fault characteristic frequency of cage ($f_c$) | 10 Hz |

FIGURE 8. Structure diagram of QPZZ test bench.

Test bench. The rotational frequencies of two experiments for the cylindrical roller bearing and ball bearing are 24 Hz and 24.5 Hz, respectively. The signal sampling frequencies of both experiments are 12800 Hz. This test bench can perform fault tests on different types of bearings with the same inner and outer diameters. Fig. 8 represents its structure diagram. From this, we can see that a motor is the power source, which drives the shaft to rotate through a belt drive. The defective bearings in the two experiments are mounted at the far right end of the shaft, as shown in Fig. 8. Tables 3 and 4 describe the parameters and theoretical fault characteristic frequencies of the two experimental bearings.

A. INNER RACE FAULT DETECTION OF CYLINDRICAL ROLLER BEARING

Fig. 9(a) shows the experimental setup of the cylindrical roller bearing. As depicted in Fig. 9(a), the eddy current sensors and PCB accelerometers are adopted to measure the vibration data of the test stand. The faulty bearing is displayed in Fig. 9(b). To illustrate the capability of the proposed ESSD algorithm to resist interference signals, the signal collected by the eddy current sensor, as shown in Fig. 10(a) is employed for analysis. Fig. 10(b) represents the enveloping result of the analyzed signal. This spectrum is obscured by the rotational frequency ($f_r$) and its harmonics while the fault characteristic frequency of inner race ($f_i$) is submerged. The signal is processed by the original SSD algorithm and the result is represented in Fig. 11. All the components retrieved by SSD are like harmonics, and none of them has impulses.

Fig. 12 displays the analysis results of the inner race fault signal by using the proposed ESSD algorithm. From the decomposition results as plotted in Fig. 12(a), we can determine the second decomposed component is the fault impulse signal, whose waveform is separately shown in Fig. 12(b). Distinct fault features, including the 1X-5X harmonics of $f_i$ and modulation frequency bands, are exhibited in Fig. 12(c), which represents the enveloping result of the second
The proposed algorithm implements the fault impulse signal separation ideally. The fault signal in this experiment is then analyzed by the VMD algorithm. The differentiation and integration procedures are also added to the VMD method to ensure an equal comparison. Fig. 13(a) represents its decomposition components. Among these decomposition components, the second one shown in Fig. 13(b) alone is identified as the fault signal. The fault signal separated by VMD is similar to that obtained by ESSD. However, its envelope spectrum as plotted in Fig. 13(c) only shows the 1X-3X harmonics of \( f_i \). The detection results of the proposed ESSD algorithm is a little better compared to VMD.

The Kurtogram approach is applied to the fault signal displayed in Fig. 10(a) to detect the fault features. The frequency band with the largest kurtosis in the Kurtogram as shown in Fig. 14(a) is employed to obtain the filtered signal. Fig. 14(b) and (c) represent the waveform and the enveloping result of the filtered signal, respectively. Three harmonics of \( f_i \) can be located from Fig. 14(c). However, the second and third harmonics of \( f_i \) are not as outstanding as their sidebands. The Kurtogram approach has a weaker performance compared with the proposed ESSD algorithm.

Furthermore, the performances of the ESSD, VMD and Kurtogram methods are compared by employing the CFIC indicator. Table 5 shows the CFIC of Fig. 12(c), Fig. 13(c) and Fig. 14(c). The CFIC of the ESSD algorithm is the largest. Hence, the performance of the ESSD algorithm is determined as the best.

**B. COMPOUND FAULT DETECTION OF BALL BEARING**

Under actual working conditions, a certain period of the evolution process of bearing faults often manifests as a compound fault in which a plurality of fault components are coupled, and the fault levels of different fault components are different. How to separate the weaker fault component is a difficulty in bearing fault diagnosis. A compound fault detection case is given to more fully reveal the weak fault detection capability of the proposed algorithm. The compound fault experiment is completed on the test stand shown...
The test bearing with an inner race defect and an outer race defect is shown in Fig. 15(a). Two PCB accelerometers mounted on the bearing housing as displayed in Fig. 15(c) are adopted for measuring the vibration signals. The compound fault signal is shown in Fig. 16(a). Fig. 16(b) represents the enveloping result, which shows significant outer race fault characteristics at the outer fault characteristic frequency \( f_o \) and its five harmonics. However, the amplitude of the inner race fault characteristic frequency \( f_i \) is smaller than that of the rotational frequency \( f_r \). The fault features corresponding to the inner race are hard to be recognized. If the fault features can be separated, it will help to determine the compound fault more accurately.

The original SSD method is employed to detect the compound fault. Fig. 17(a) displays its decomposition components and their associated envelope spectrums are plotted in Fig. 17(b). Among these spectrums, the first two spectrums are dominated by the outer race fault features. Despite the spectra peak of \( f_i \) is recognized in the spectrum of SCC1, it is still feeble. The domain frequencies of the third and the fourth spectrums are composed of \( f_i \) and its harmonics, which corresponds to the vibration information of the shaft. However, the SSD approach fails to detect obvious fault features of the inner race fault.

Fig. 18(a) represents the components of the compound fault signal after the ESSD decomposition. Their corresponding enveloping results are shown in Fig. 18(b). Obviously, the fault signatures of the inner race fault (the 1X-7X frequency doubling of \( f_i \)) are exhibited in the envelope spectrum of ESSC1, which indicates the inner race fault signal is successfully retrieved by ESSD. At the same time, we can find that \( f_o \) and its 2X-9X harmonics dominate the envelope spectrums of ESSC2 and ESSC3, which illustrates that the fault information of the outer race fault is distributed in two different frequency bands. The related information about the shaft vibration can be visible in the envelope spectrums of ESSC4 and ESSC5. The proposed ESSD algorithm realizes the detection of the weak fault features of inner race fault.
Fig. 19(a) shows the decomposition results of the compound signal by VMD, which also involves the differentiation and integration operators. Fig. 19(b) describes the corresponding envelope spectrums, which indicates the first two decomposition components mainly reflect the fault features of the outer race fault and the last three ones represent...
the fault signatures of inner race fault. The VMD method extracts up to five times the frequency of $f_i$ and six times the frequency of $f_o$. Hence, the fault feature signals separated by ESSD have more abundant fault information.
Fig. 20 represents the analysis results of the compound fault signal by using the Kurtogram approach. Fig. 20(a) and (b) display the Kurtogram and the filtered signal, respectively. Fig. 20(c) shows the enveloping result of Fig. 20(b). Five harmonics of \( f_i \) can be recognized from Fig. 20(c). However, the fault features representing the outer race fault are invisible. The optimal frequency band of the Kurtogram only carries the fault information of the inner race fault. The proposed ESSD algorithm outperforms the Kurtogram in extracting more comprehensive fault features.

Similarly, the CFIC indicator is employed to quantify the performances of the ESSD, VMD and Kurtogram methods. The CFIC indicator of the outer race fault named CFIC\(_o\) and CFIC of the outer race fault termed CFIC\(_o\) are obtained by using \( f_i \) and \( f_o \) as the fault characteristic frequency, respectively. Table 6 shows the CFIC values of the three comparison methods. The CFIC\(_i\) and CFIC\(_o\) corresponding to the ESSD algorithm are the largest. Hence, the ESSD algorithm outperforms the other two methods.

**VI. CONCLUSION**

In order to overcome the deficiency of SSD in detecting weak faults of rolling bearings, a modified SSD termed ESSD is developed in this work. In the process of ESSD, the differentiation operator is firstly applied to the vibration signal of rolling bearings to improve the SIR of the signal and enhance the high-frequency components in which the fault features are always concentrated. The output signal of the differentiation is then decomposed by SSD. Finally, the integration operator is applied to the decomposition components to retrieve the ESSCs. The proposed ESSD algorithm is tested by one simulation and two experimental cases of rolling bearings, and it is compared with the original SSD, VMD and Kurtogram algorithms. In these analysis cases, the original SSD algorithm cannot separate the weak fault signals as their frequencies are not the dominant frequencies in the signal. The proposed ESSD algorithm breaks through this limitation. Similar and even better fault feature detection results are obtained using the ESSD algorithm compared to the VMD and Kurtogram algorithms. A distinct advantage of the proposed ESSD algorithm over VMD is that it does not need to set any parameters artificially. The proposed ESSD algorithm expands the application scope of SSD to detect bearing faults.

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