Emergence of small numbers in complex systems 
and the origin of the electroweak scale

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In sufficiently complex models with many parameters that are unknown or undetermined from 
first principles, a small coupling or mass can naturally arise even if it is not protected by a symmetry 
or a result of some dynamics. For the naturalness criterion, we advocate specifying all the model 
parameters with one significant figure. This automatically avoids outcomes of a given observable 
that would require special highly-tuned choices for parameters. The obtained range of outcomes is 
an attribute of a given model resulting from its specific characteristics and complexity. Using these 
criteria in the minimal supersymmetric model, we demonstrate that the electroweak scale up to 3 
orders of magnitude below superpartner masses naturally occurs.

Introduction. In particle physics, it is usually argued 
that a parameter, written in units of maximal energy 
at which a given model is valid, can naturally be signifi-
cantly smaller than unity if it is protected by a sym-
metry from quantum corrections due to various inter-
actions [11] [2]. When applied to the Higgs boson mass, 
which is related to the electroweak (EW) scale, this im-
mediately implies the existence of a new physics not far 
above the EW scale, since there is no symmetry in the 
standard model (SM) protecting the Higgs mass from 
radiative corrections [3–6]. The naturalness problem re-
lated to the Higgs boson mass is one of the most studied 
problems in particle physics and the main motivation for 
physics beyond the standard model.

Realizing that supersymmetry (SUSY) can provide the 
needed symmetry for any scalar mass [7], together with 
the intriguing picture of unification of strong and elec-
trouwk forces [8] and the possibility to build simple re-
alistic models [9] made supersymmetric extensions of the 
SM the most promising candidates for new physics. In 
these models, the EW scale is a result of quantum correc-
tions and is calculable. These quantum corrections are 
comparable to masses of superpartners of SM particles 
and thus a generic prediction of these models, based on 
the naturalness argument, is that superpartners are at or 
very near the EW scale [10].

So far negative results of searches for superpartners, 
with some limits exceeding an order of magnitude above 
the EW scale, cast a shadow on this framework (similar 
cconcerns apply to other proposals for physics beyond the 
SM). Moreover, the measured value of the Higgs boson 
mass, taking aside naturalness considerations, indirectly 
points to superpartners two orders of magnitude above 
the EW scale [11]. Since the relevant parameters are 
masses squared, this would mean that the EW scale cor-
responds to a number which is 4 orders of magnitude 
smaller than expected. It is commonly argued that such 
an outcome, apparently contradicting naturalness prin-
ciple, would require fine tuning of model parameters at 
the level of 1 part in $10^4$ and thus it is very unlikely [10].

In this letter we argue that in sufficiently complex mod-
els with many parameters that are unknown or undeter-
dined from first principles, a small coupling or mass can 
naturally arise even if it is not protected by a symmetry 
or a result of some dynamics. Rather than relying on 
various probabilistic arguments or measures of sensitiv-
ity, the naturalness criterion adopted in this letter advoc-
ates specifying all the model parameters with one sig-
nificant figure. This automatically avoids outcomes of a 
given observable that would require special highly-tuned 
choices for parameters. The obtained range of outcomes is 
an attribute of a given model resulting from its spe-
cific characteristics and complexity. Within that range any outcome is indistinguishable with respect to the way 
model parameters are specified and thus is considered 
natural. Using these criteria, we will demonstrate that 
in the minimal supersymmetric model the electroweak scale up to 3 orders of magnitude below superpartner 
masess naturally occurs.

Besides complexity, an important characteristic of the 
EW symmetry breaking in supersymmetric models is 
that the EW scale is typically a result of the cancella-
tion of comparable contributions. In a toy model that 
closely mimics features of electroweak symmetry break-
ing, we will show that in such situations, even if all the 
model parameters are specified with just one significant 
figure, the outcomes several orders of magnitude smaller 
compared to dominant contributions arise with probabil-
ities comparable to the most likely outcome.

Toy model. Let us consider an observable $X$, which is, 
up to much smaller contributions from other parameters 
in the model, given by the difference of two parameters 
$A$ and $B$ of comparable size:

$$X \simeq A - B. \quad (1)$$

We can always choose appropriate units so that $A$ and 
$B$ are random real numbers close to 1 and, in order to
be specific, let us allow them to vary by 50% in both directions, between 0.5 and 1.5. The distribution of \(X\) is then an almost symmetric triangle peaked at 0, similar to that in Fig. 1(a) (which is slightly shifted to the right due to perturbations). For unequal intervals of \(A\) and \(B\) or their central values the distribution could be less peaked, more broad and shifted to the left or right. Nevertheless, as far as there is a non-negligible overlap in intervals over which \(A\) and \(B\) are allowed to vary, an outcome in the vicinity of zero will have a comparable probability to the most probable outcome.

If \(A\) and \(B\) were integers in overlapping ranges, there would be no problem associated with \(X\) being a small number. It would be just one out of many possible comparably likely outcomes. Being integers supplies a natural bin size in which the outcomes are plotted and compared. For real numbers, an arbitrarily small outcome of \(X\) is possible. However it would require specifying \(A\) and \(B\) with many significant figures and having almost identical values. This, without any symmetry or dynamical reason, would seem like a huge coincidence and it is the essence of the fine tuning problem.

For example, if we specify the departure of \(A\) and \(B\) from 1 by one significant figure, the outcomes of \(X\) will be distributed in intervals of 0.1, indicated by highlighted lines in Fig. 1(a). A mismatch of the central values of \(A\) and \(B\) would shift the 0 outcome, but since the outcomes appear in 0.1 intervals, an \(X\) as small as 0.1 with either sign will naturally appear with probability comparable to the most likely outcome. A significantly smaller \(X\) would however require specifying departures of \(A\) and \(B\) from 1 by more than one significant figure.

However, in many physical systems there are more than two parameters contributing to a given observable. Let us consider a parameter \(C\) contributing to \(X\) at an order of magnitude smaller level than \(A\) and \(B\). The exact value of \(C\) is not important since it only shifts the \(A - B\) distribution slightly to the left or right and thus we can simply assume it is equal to 0.1. Let us again allow the parameter \(C\) vary in the interval ±50% around the central value, similarly to what we allowed for parameters \(A\) and \(B\), and specify the departure from the central value with one significant figure. Adding \(C\) is then, up to a constant shift, equivalent to adding random numbers: 0.1 + \(\{0, \pm 0.01, \pm 0.02, \ldots, \pm 0.05\}\). Let us define such a set of random numbers as \(R[0.1]\). In such a notation, our \(A\) and \(B\) specified with one significant figure are \(R[1]\). Varying \(C\) in steps corresponding to specifying it with one significant figure results in emergence of new outcomes of \(X\) in intervals of 0.01, or 0.1C. These new outcomes are indicated by highlighted lines in Fig. 1(b).

If there are more parameters contributing at smaller levels, even if specified by just one significant figure, new outcomes will appear in intervals ±0.1 of the corresponding parameter. The distribution of \(X\) resulting from adding three such perturbations, \(R[0.1] + R[0.01] + R[0.001]\), is shown in different intervals in Fig. 1(a)-(d). In this specific example, outcomes as small as \(10^{-4}\) appear and they are still about 10 times more likely than outcomes of order 1. No parameter needed to be specified with more than one significant figure, no tuning was required.

In order to obtain, without fine tuning, outcomes of \(X\) orders of magnitude smaller than contributions from \(A\) or \(B\), it is necessary that a given model is sufficiently complex with at least several more parameters contributing to it. More importantly, contributions of these parameters have to be distributed in a way that the space of possible outcomes is continuously covered without any gaps. If, for example, there was a new parameter contributing to \(X\) at 10^{-8} level, it would not change our previous conclusions. This new contribution would only split the existing outcomes into new outcomes tightly clustered around those shown in Fig. 1(d). Because of the arbitrary shift from all the parameters of the model, from digits that were not specified, the naturally occurring outcomes would still not be smaller than \(10^{-4}\). It is the largest gap in the distribution that determines the smallest naturally occurring outcome from the minimally specified model parameters. It is easy to see that our toy model in Fig. 1 is optimal for achieving the smallest outcome.

The obtained range of outcomes is a characteristic of a given model. While probabilities of individual outcomes
depend on intervals over which parameters are allowed to vary and prior distributions of model parameters, the range itself is independent on these assumptions. For example, it is very easy to change the central values of the parameters in the toy model in Fig. 1 so that the distribution peaks at any other value. Nevertheless, as far as intervals of $A$ and $B$ overlap, outcomes of $X$ as small as $10^{-4}$ are inevitable. However, no matter what assumptions we make, outcomes of $X$ smaller than $10^{-4}$ cannot be guaranteed without specifying model parameters with more than one digit. Such outcomes would disappear after an arbitrary shift and thus they are purely accidental.

Armed with the intuition from the toy model we can find the range of hierarchy between the electroweak scale and the scale of superpartners that corresponds to specifying parameters in the minimal supersymmetric model with one significant figure. As in our toy model, the electroweak scale is a result of the cancellation of comparable numbers. In addition, the complexity of these models is enormous, with more than 100 parameters contributing to the determination of the EW scale.

**EW symmetry breaking.** In the standard model, EW symmetry breaking is the result of a negative quartic term and a positive quartic term in the Higgs potential. The minimum of the potential, corresponding to the vacuum expectation value of the Higgs field, is away from zero and determines the scale of electroweak interactions and masses of $Z$, $W$ and Higgs bosons.

One of the most attractive features of supersymmetric extensions of the SM is the fact that the negative mass squared term, triggering EW symmetry breaking, is obtained by radiative corrections in vast ranges of parameters of various models. The relevant mass squared parameter is a combination of the supersymmetric Higgs mass parameter, $\mu$, and the soft SUSY breaking mass squared parameter, $m_{H_u}^2$:

$$|\mu|^2 + m_{H_u,0}^2 + \Delta m_{H_u}^2,$$

where we intentionally split the $m_{H_u}^2$ into the value generated by a SUSY breaking scenario at a given scale, $m_{H_u,0}^2$, and the radiative correction from the renormalization group (RG) running to the electroweak scale, $\Delta m_{H_u}^2$. The $|\mu|^2$ part could also be split in a similar way, but radiative corrections to this term are not dramatic and thus we use the EW scale value directly. The negative of the combination of parameters in Eq. (2) is directly related to the $Z$ boson mass or the Higgs mass and thus it should be close to $(100 \text{ GeV})^2$.

The power of radiative corrections is illustrated in Fig. 2 (a)–(c) where the RG evolution of $m_{H_u}^2$ is shown together with the most contributing other parameters, the stop masses and the gluino mass for three representative cases. Although all superpartner masses can be free parameters, simple scenarios with just few parameters are commonly assumed. Here we assume a universal scalar mass, $m_0$, a universal gaugino mass, $M_{1/2}$, and consider three cases: $M_{1/2} = 0, m_0 = 0$ and $M_{1/2} = m_0$. We also define $M_{SUSY} = \max(M_{1/2}, m_0)$ and the RG evolution in Fig. 2 is plotted in this unit starting at the grand unification scale (it is another simple and common assumption although it is not necessary for our discussion). In all three cases, and thus for any case in between, the $m_{H_u}^2$ parameter turns negative.

Remarkable feature of the RG flow is that $\Delta m_{H_u}^2$ eventually reaches $-M_{SUSY}^2$, but it doesn’t run to much larger negative values. More specifically, for the case of $m_0$ domination, the $\Delta m_{H_u}^2 \simeq -M_{SUSY}^2$ after about 12 orders of magnitude of the RG flow. Similarly, in the case of gaugino domination, the same situation is achieved after 11 orders of magnitude of the RG flow and finally for $M_{1/2} = m_0$ we need about 9 orders of magnitude. In further RG evolution $\Delta m_{H_u}^2$ does not exceed $-1.2M_{SUSY}^2$, $-3M_{SUSY}^2$, and $-4M_{SUSY}^2$ respectively, even if evolved all the way to the EW scale which can be seen in Fig. 2 (d). Note, however, that stop masses no longer contribute to $\Delta m_{H_u}^2$ at energies below their masses. For stop masses which can result in the measured value of the Higgs boson mass without significant additional contributions, $5 – 20$ TeV, indicated by shaded region in Fig. 2 (d), the $\Delta m_{H_u}^2$ ranges between $-1M_{SUSY}^2$ and $-2.5M_{SUSY}^2$.

The value of the $\mu$ term in Eq. (2) or its origin, a-
though subject to intense research and usually the center of attention in naturalness considerations, is not crucial for our discussion. We assume $\mu \lesssim M_\text{SUSY}$ and sufficiently small not to prevent electroweak symmetry breaking. With $|\mu|^2 + m_{H_u,0}^2 \simeq M_\text{SUSY}^2$, either because $m_{H_u,0}^2 \simeq M_\text{SUSY}^2$ or $\mu \simeq M_\text{SUSY}$ or both, we can see that the negative of the mass squared combination in Eq. (2) that sets the EW scale squared, $M_{EW}^2$, is the result of cancellation of two comparable contributions as in our toy model: $X = -M_{EW}^2$, $A = |\mu|^2 + m_{H_u,0}^2 \simeq M_\text{SUSY}^2$, and $B = -\Delta m_{H_u}^2 \simeq M_\text{SUSY}^2$.

Although written in a simple way, the mass squared parameter in Eq. (2) depends on every single soft SUSY breaking scalar and gaugino mass, every gauge coupling and every Yukawa coupling in the model because they all contribute in the RG evolution to $\Delta m_{H_u}^2$. Assuming all soft masses comparable (not necessarily equal) in order $M_\text{SUSY}$ at the grand unification scale, from two loop RG equations [12] we find that the approximate individual contributions to $\Delta m_{H_u}^2$ in units of $M_\text{SUSY}^2$ are as follows: -2.6 from gluino, -0.3 from each scalar top and $H_u$, 0.2 from the SU(2) gaugino, 0.05 from the $U(1)_Y$ gaugino, $-4 \times 10^{-3}$ from the trace of masses squared of all SU(2) doublet scalars, $-4 \times 10^{-4}$ from the trace of masses squared of all scalars carrying hypercharge, $10^{-3} - 10^{-5}$ from each scalar bottom and $H_d$ (for the ratio of vacuum expectation values of two Higgs doublets, tan $\beta$, in the range tan $\beta = 50 - 5$), and $-4 \times 10^{-6}$ from each scalar charm. After scalar charm contributions, there is a gap of about 5 orders of magnitude. Each scalar up contributes at $3 \times 10^{-11}$ and the contribution from each scalar strange is about $10^{-8}$ of the contribution from scalar bottom.

In the list above, we did not include contributions from neutrino Yukawa couplings and scalar neutrinos, since these couplings are not known and are highly model dependent, and thus also from scalar charged leptons since contributions of these depend on the neutrino sector. In addition, we omitted the hypercharge weighted trace of all scalar masses squared which does not contribute if scalar masses are equal. Furthermore, we assumed diagonal Yukawa and scalar mass matrices, and neglected contributions from soft trilinear couplings that, if flavor diagonal, would contribute at comparable levels to scalar masses.

We have found that contributions of various parameters to $\Delta m_{H_u}^2$ in first 6 orders of magnitude below $M_{SUSY}^2$ are very dense, with 13 parameters contributing, followed by a gap to below $10^{-10}$ level. As in the toy model, having many new parameters contributing in 6 orders of magnitude below $M_{SUSY}^2$ without gaps means that, specifying with one significant figure the way they are varied around central values, the outcomes for the electroweak scale squared as small as $10^{-6}$ $M_{SUSY}^2$, or $M_{EW} \approx 10^{-3} M_{SUSY}$, occur with probabilities comparable to the most likely outcome.

As already discussed for the toy model, probabilities of individual outcomes do depend on the choice of intervals and prior distributions of model parameters, but the range of outcomes, $10^{-3} M_{SUSY} \lesssim M_{EW} \lesssim M_{SUSY}$, is a characteristic of the minimal supersymmetric model with all the mass parameters of the same order at sufficiently high scale and specified with one digit. The range of hierarchy between the EW scale and $M_{SUSY}$ characteristic to constrained versions of the MSSM will be explored in Ref. [13].

**Discussion.** Conclusions drawn in this letter are in a sharp contrast with commonly accepted views. In our analysis, the EW scale two orders of magnitude below superpartner masses, as suggested by the Higgs boson mass, is well within the range of comparably likely outcomes, $10^{-3} M_{SUSY} \lesssim M_{EW} \lesssim M_{SUSY}$, that do not need specifying any of the parameters with more than one significant figure. On the other hand, it is commonly argued that the EW scale resulting from 10 TeV superpartners, requires fine tuning of model parameters at the level of 1 part in $10^4$. This is usually estimated using various probabilistic arguments or Barbieri-Giudice measure, $|\partial \ln M_{EW}/\partial \ln p_i|$, where $p_i$ are model parameters and the role of $M_{EW}$ is played by the Z boson mass, the Higgs mass or the vacuum expectation value in various versions [13][10].

This quantity, being a derivative, clearly represents the sensitivity of the EW scale to model parameters. However, as we showed, this sensitivity does not necessarily indicate the need for special choices of parameters. In our toy model, the outcome $X \approx 10^{-4}$ corresponds to $|\partial \ln X/\partial \ln A| \approx 10^4$, possibly suggesting fine tuning needed at the level of 1 part in $10^4$ just like for the EW symmetry breaking with 10 TeV superpartners. If there were just two model parameters, $A$ and $B$, this would indeed be the level of fine tuning required: for any $A$, the $B$ would have to be specified with 4 significant figures. However, with more parameters contributing at smaller levels without significant gaps, none of the parameters needed to be specified with more than 1 significant figure. The Barbieri-Giudice measure does not take into account the complexity of a given model and indicates the same for a model with two parameters as for a model with many parameters.

The naturalness criterion adopted in this letter automatically avoids outcomes of a given observable that would require special highly-tuned choices for parameters. The obtained range of outcomes is an attribute of a given model resulting from its specific characteristics and complexity. Within that range any outcome, even the smallest one, is indistinguishable with respect to the way model parameters are specified. If the obtained range is large, the small probability of an individual outcome is a reflection of the complexity of a given model rather than a sign of unnaturalness.

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