Modelling of the interstellar dust distribution under the influence of the interstellar magnetic field

Egor Godenko¹,²,³, Vladislav Izmodenov²,³,¹

¹ Ishlinsky Institute for Problems in Mechanics of RAS, ² Space Research Institute of RAS, ³ Moscow Center for Fundamental and Applied Mathematics, Lomonosov Moscow State University

E-mail: eg24@yandex.ru

Abstract. Due to the relative motion of the Sun, the interstellar dust (ISD) particles can penetrate into the heliosphere. Three main forces influence the ISD motion in the heliosphere and near its boundaries: the solar gravitational force, the radiation pressure force and the electromagnetic force. Our general goal is to create a model of the ISD distribution in the heliosphere and, particularly, find regions of the increased ISD number density. In our previous works ([13], [3]) we investigated the peculiarities of the ISD distribution in the heliosphere/astrospheres and the effects of velocity dispersion on these peculiarities. Here we study the influence of the interstellar magnetic field on the ISD distribution in the outer heliospheric interface. For this purpose, we also calculate the electric charge of the ISD particles, which depends mainly on the currents of surrounding plasma ions and electrons, secondary emitted electrons and photoelectrons. We demonstrate the filtration of the ISD particles in the vicinity of the heliopause: small ISD particles align along the lines of the interstellar magnetic field and don’t penetrate into the heliosphere and, on the contrary, the interstellar magnetic field almost doesn’t affect the motion of big ISD particles. We also show that regions of the increased ISD number density appear in the vicinity of the heliopause and discuss mechanisms of their origin.

1. Introduction

One of the most important components of the interstellar medium (ISM) is the interstellar dust (ISD) particles. Because of the relative motion of the Sun in the ISM, the ISD particles can penetrate into the heliosphere. The ISD grains are solid grains of relatively uncertain shape. It is usually supposed that their chemical composition is carbon or astronomical silicate. Due to the interaction with plasma particles and photons from the Sun and interstellar background, the ISD grains have nonzero electric charge. This charge makes them sensitive to electromagnetic fields, and, in particular, to the interstellar magnetic field.

Modelling of the ISD distribution in the heliosphere was performed by many authors. Some of them studied the distribution in the inner heliosphere (inside the region limited by the Termination Shock). In the series of works [17], [18] they developed a model created by [12] and, consequentely, presented a relatively advanced model of the ISD distribution in the inner heliosphere. Their model takes account of the effects related to the motion of the heliospheric current sheet and they also use relevant numerical values of the physical parameters included in the model. They performed detailed computations for the ISD particles of different sizes, studied various configurations of the determining parameters and, as a result, applied this
model to interpret experimental measurements obtained by the Ulysses spacecraft ([11], [21], [19]). However, they found some features in data, which they couldn’t fully explain using their model. They supposed, that these features are associated with the influence of the heliospheric interface on the motion of the ISD particles ([20]), and, particularly, with the filtration of dust grains at the heliospheric boundaries.

Modelling of the ISD distribution in the outer heliosphere was performed by [16], [1], [16]. [16] presented a relatively advanced model of the ISD distribution in the outer heliosphere. For computations they used a heliosphere model from [14] and applied a state-of-the-art model of the ISD charging described in [22]. They performed computations for the ISD particles of different sizes and showed the filtration of small ISD particles at the heliospheric boundaries. They also demonstrated maps of the ISD number density distribution in the different coordinate planes of the ecliptic reference frame and presented the density enhancements of the ISD grains. [1] executed computations in the same manner as it was done in [16], but they applied another heliosphere model ([5]) and used a simplified model for the charging of ISD (more precisely, they used simplified formulae for the photoelectron emission and the secondary electron emission currents). However, they performed detailed computations of the ISD distribution for the ISD particles of different sizes, also demonstrated the regions of the ISD number density accumulations in the heliosphere and near its boundaries, and attempted to explain the physical mechanism of the origin of these number density features. They also tried to quantitatively estimate the filtration effect using a specially included characteristic: the normalized average number density of the dust particles inside the heliopause.

Our general goal is to create a model of the ISD density distribution in the heliosphere. In our previous works ([13], [3]) we explored the number density singularities of the ISD particles in the inner heliosphere: their origin and the influence of the velocity dispersion on them. In this work we focus on the effects related to the interstellar magnetic field. We model the trajectories of the ISD particles under the influence of the interstellar magnetic field, calculate the number density of these particles and demonstrate the number density enhancements in the vicinity of the heliopause. Comparing with previous modelers, we apply a modified model of the ISD charging (based on [9]) and attempt to explain the reasons for the forming of the mentioned number density singularities. The structure of the paper is the following. In Section 2 we describe main concepts of the model. In Section 3 we provide the results of modelling of the ISD trajectories and the ISD number density distributions. Section 4 concludes the paper.

2. Description of the model

2.1. Coordinate systems

In our model we use two Cartesian coordinate systems. All computations for the ISD distributions and ISD trajectories we perform in the coordinate system related to the ecliptic plane (see Figure 1). The Sun is located at the origin of coordinates $S$, axis $Sx$ corresponds to the solar rotation axis, axis $Sz$ lies in the ecliptic plane so that the plane $Sxz$ contains the velocity vector of the undisturbed ISM $v_{ISM}$, axis $Sy$ completes the right-handed system of coordinates. In order to calculate the electromagnetic force we use the distributions of plasma parameters, such as bulk velocity and magnetic field. In this paper we apply a heliosphere model from [6]. The distributions of plasma parameters are given in the special coordinate system related to the BV-plane, i.e. the plane, which contains plasma velocity vector and magnetic field vector in the undisturbed ISM. The description of this coordinate system can be found in [5] (see Section 2.3 there). For more detailed understanding of the mutual location between these two coordinate systems see Figure 2.
2.2. Mathematical formulation of the problem

We use a kinetic approach for the description of the ISD distribution. This approach is based on the ISD velocity distribution function \( f_d(t, \mathbf{r}, \mathbf{v}) \).

The kinetic equation for \( f_d(t, \mathbf{r}, \mathbf{v}) \) is:

\[
\frac{\partial f_d}{\partial t} + \mathbf{v} \cdot \frac{\partial f_d}{\partial \mathbf{r}} + \mathbf{F} \cdot \frac{\partial f_d}{\partial \mathbf{v}} = 0,
\]

where \( \mathbf{F} \) is the net force acting on the dust particles. We have zero on the right side \( (1) \), because in the heliosphere elastic collisions between dust grains may be neglected \( (4) \), which could be the source for the inflow and outflow of particles. In this article we consider the stationary model of the interaction between solar wind and interstellar medium \( (6) \), and therefore the solution of the kinetic equation is also stationary, i.e. \( \frac{\partial f_d}{\partial t} = 0 \):

\[
\mathbf{v} \cdot \frac{\partial f_d}{\partial \mathbf{r}} + \mathbf{F} \cdot \frac{\partial f_d}{\partial \mathbf{v}} = 0,
\]

(2)

In order to obtain the solution of \( (2) \) we should formulate the boundary conditions. In this paper we mainly study the influence of the interstellar magnetic field on the ISD distribution, that is why we formulate a boundary condition in the undisturbed ISM. We suppose the flow of the ISD particles is the uniform flow in the undisturbed ISM with average velocity \( \mathbf{v}_{ISM} \) and zero thermal velocity: in other words, we assume that all ISD particles have identical velocity equal to \( \mathbf{v}_{ISM} \). It turns out that at the distance of 1500 a.u. from the Sun in the upwind direction the ISM plasma is almost undisturbed and the boundary condition can be formulated on the plane \( z = 1500 \) a.u. as singular distribution:

\[
f_d(\mathbf{r}, \mathbf{v})|_{ISM} = \delta(\mathbf{v} - \mathbf{v}_{ISM}).
\]

(3)

To complete the correct mathematical formulation of the problem we should set the boundary condition in velocity space:

\[
f_d(\mathbf{r}, \mathbf{v})|_{v \to \infty} = 0.
\]

(4)
2.3. Force analysis
The analysis of forces acting on the ISD particles in the heliosphere was done by [8]. We are mainly interested in studying the electromagnetic force $F_{el}$ influence on the ISD distribution in the ISM. However, we also take into consideration gravitational force $F_{grav}$ and radiation pressure force $F_{rad}$, but they don’t produce any significant effect on the distribution because they are proportional to $r^{-2}$, where $r$ is the distance to the Sun. Coulomb and collisional drag force $F_{drag}$ are not important in the considered region ([4]).

The expression for the net force acting on particles (see details e.g. in [3]):

$$F = (\beta - 1) \frac{GM_S}{r^2} e_r + \frac{q}{c_0 m} ((v - v_p) \times B),$$

where $G$ is the gravitational constant, $M_S$ is the Sun mass, $\beta$ is the special dimensionless parameter equal to the ratio between absolute values of gravitational force and radiation pressure force (see e.g. [8]). $v_p$ and $B$ are the plasma velocity and magnetic field correspondingly, $c_0$ is the speed of light, $q$ and $m$ are the charge of particle and its mass.

2.4. Calculation of charge
In this paper, for simplicity, we consider spherical particles, because a real shape of the ISD particles is extremely uncertain. For spherical particles (in Gaussian system) $q = U \cdot a$, $m = \frac{4}{3} \rho \pi a^3$, where $U$ is the charge surface potential, $a$ is the radius of the particle, $\rho$ is the mass density. Since parameter $\frac{q}{m} \propto \frac{1}{a^2}$, smaller particles are more affected by the electromagnetic force. In order to calculate the charge of the particle one should consider the following ordinary differential equation:

$$\frac{dq}{dt} = \sum J_k,$$

where $J_k$ are the currents corresponding to different physical processes influencing charge of the particles. Here we consider sticking of plasma ions and electrons, secondary electron emission due to bombardments of plasma particles and photoelectron emission. [9] showed that the characteristic time for particles to reach the equilibrium surface potential is less than the characteristic time of particle motion in the heliosphere. Thus, $\frac{dq}{dt} = 0$ and that is why we can solve the algebraic equation:

$$\sum J_k = 0$$

instead of equation [6]. On the left side of (7) currents $J_k$ depend on the charge potential, plasma parameters, distance to the Sun. Since plasma parameters in the stationary case are the functions of $r$, in general, we can present all these currents as $J_k = J_k(U, r)$ and, therefore, numerically solving (we use the combination of Bisection method and Secant method) the algebraic equation (7), it is possible to receive the expression for charge potential:

$$U = U(r).$$

The expressions for $J_k$ in (7) are based on the paper [9] (see Appendix A for details). Figure 2 demonstrates the distributions of charge potential along the upwind direction for different sizes of the ISD grains. It should be noted that smaller particles have bigger values of charge potential, which is in agreement with the results of previous modelers (see e.g. Figure 2 in [16]), although we have some qualitative differences in the inner heliosphere. It may be related to the fact that they consider irradiance of high-energy photons (in addition to the usual UV-irradiance), and consequently the rate of the photoelectron emission in their charge model increases. In Figure
Figure 2. The distributions of charge potential along the upwind direction (axis $S_{21}$) for the ISD grains of different sizes. It is visible that in the inner part of the heliospheric interface (between HP and TS) the value of charge potential increases as a result of the growing rates of the secondary electron emission. Smaller ISD particles have bigger values of charge potential in the whole domain. **TS** - Termination Shock, **HP** - Heliopause, **BS** - Bow Shock.

We can also see that the assumptions in our previous works ([13], [3]) about the constant value of charge potential in the region limited by TS are roughly confirmed in the results of our modelling.

2.5. Numerical method

For the solving of the equation [2], a Monte-Carlo method is used. The whole computational domain is divided into cube cells $5 \text{ au} \times 5 \text{ au} \times 5 \text{ au}$. At the starting plane $z = 1500 \text{ au}$ initial coordinates of the ISD particles are randomly generated corresponding to the uniform spatial distribution. Trajectories of the ISD grains are integrated using 4th order Runge-Kutta method with the net force from (5). For the calculation of the net force we are given the distributions of plasma parameters from [6]. During particle motion we record the time periods which a particle resides in each computational domain cell (time period equals to 0, if a particle doesn’t cross the corresponding cell). Then, we average these time periods over the whole set of simulated particles and calculate the velocity distribution function and the number density of the ISD grains (for more details see [3]).

For the computations we use the following numerical values of the parameters: $v_{ISM} = 26.4 \text{ km/s}$, $M_S = 2 \cdot 10^{30} \text{ kg}$, $\rho_d = 3300 \text{ kg/m}^3$, $c_0 = 3 \cdot 10^8 \text{ m/s}$. All numerical values used for the calculation of the charge potential can be found in [9] and in the corresponding links in the Appendix A.

3. Results

[16], [1] demonstrated the filtration of the ISD particles at the heliospheric interface region. In our model we also show this effect. In Figure 3 maps of the ISD number density distribution are presented for the different values of the ISD particle radius $a$: 20 nm, 50 nm, 100 nm, 200 nm, 500 nm, 1000 nm. For the smallest particles ($a = 20 \text{ nm}$) we can see the effect of full filtration.
Figure 3. Maps of the number density distribution in the planes $X = 0$ (top panel) and $Y = 0$ (bottom panel) for the ISD particles of different sizes (20 nm, 50 nm, 100 nm, 200 nm, 500 nm, 1000 nm). The smallest ISD particles don’t penetrate into the heliosphere due to the effects of the interstellar magnetic field, and, on the contrary, the biggest particles do it almost freely. The number of particles for each computation $N = 25000000$. Relative statistical error is limited by 5% at each point out of the heliosphere, where the ISD particles appear.

at the heliopause: no one particle penetrates into the heliosphere. The reason is the fact, that these particles have a small enough gyroradius and that is why they align along the lines of the interstellar magnetic field which flow around the heliopause (left panel of Figure 4). In front of the heliopause there is an increase of the magnetic field strength (see distribution for $B_y$ component on the right panel of Figure 4), which serves as a source for the deflection of the ISD particles. Due to the gyro rotation of small ISD particles, wave structures also appear in the number density and velocity component distributions (see pictures for $\alpha = 20$ nm in Figures 3 and 6). This effect for astrospheres was discussed in [7]. Since the value of determining parameter $q_m$ for the smaller particles is greater as well as the value of charge potential, the smaller particles are more affected by the interstellar magnetic field, and, as a result, the number density at the nose part of the heliopause and the number of peaks in wave structure increase. For this
Figure 4. The left panel presents examples of the ISD grains trajectories. Small ISD particles flow around the heliopause. The radius of particles $a = 20 \text{ nm}$. On the right panel the distribution in the plane $Y = 0$ of the $B_y$ magnetic field component from the heliosphere model [6] is shown. The increase of the absolute value of this component in front of the heliopause serves as a source for the filtration of small ISD particles.

Figure 5. Distribution of the ISD number density along the line ($X = 0, Y = 0$). The number of peaks in wave structure and the ISD number density at the nose part of the heliopause increase for smaller particles. The resolution of the grid along the $z$-axis direction is $0.1 \text{ au}$. The radius of particles $a = 5 \text{ nm}$. The number of particles $N = 1000000$. Relative statistical error is limited by 2-3 % at each point.

purpose, we performed a test computation (Figure 5) for the particles of size $a = 5 \text{ nm}$ using a computational grid with higher resolution (but because of the limitations in memory we recorded results only in grid cells located along the line $X = 0, Y = 0$). The reasons for the ISD density accumulations in front of the heliopause for small particles are clearly visible in Figure 6. The absolute value of the main velocity component $v_z$ decreases (even down to zero) and so the ISD
particles spend more time moving across the cells in this region.

In the case of $a = 50 \text{ nm}$, a minor part of the ISD particles enter the heliosphere just in the ecliptic plane ($X = 0$) and just from one of the flanks. In this case the gyroradii is too small for almost all particles to penetrate into the heliosphere, and at the same time too big for them to create noticeable wave structures in the number density distribution. Also it should be noted, that here the filtration mechanism has significant asymmetry: on the plane $X = 0$ it can be seen, that the exclusion zone (black colour region) shifts from the heliopause. This asymmetry is supposed to relate to the asymmetry in the interstellar magnetic field distribution in our heliosphere model, which becomes significant only for the particles with medium values of the determining parameter $\frac{q_m}{m}$.

Then, for $a = 100 \text{ nm}$ and $a = 200 \text{ nm}$ we observe almost the same situation: such ISD particles penetrate almost freely into the heliosphere in the ecliptic plane and its surroundings, but near the poles they still experience the filtration effect. For these particles the asymmetry of the exclusion zone practically disappears. From the pictures for these sizes on the bottom panel ($Y = 0$) of Figure 3 one can see, that the scale of the region at the heliopause, from which the ISD particles can penetrate into the heliosphere, increases with particle radius. In particular, in the picture for $a = 200 \text{ nm}$ approximately the whole nose part of the heliopause is permeable for the ISD particles. After the crossing of the heliopause, particles are focused around the ecliptic plane due to the influence of the heliospheric magnetic field (in our heliosphere model the heliospheric current sheet is supposed to be in the focusing phase). Such strong focusing (almost immediately after the crossing of the heliopause) relates to the fact, that in the inner heliospheric interface the values of charge potential are large compared to the values in the outer heliospheric interface (see in Figure 2 region between TS and HP).

The biggest ISD particles ($a = 500 \text{ nm}$ and $a = 1000 \text{ nm}$) almost don’t experience the influence of the interstellar magnetic field: one can see that in Figures 3, 6 for these sizes the maps of distributions have almost monotonic colour out of the heliosphere. All features of the number density and velocity component distributions are located inside the heliosphere, but it is beyond the scope of this paper.

4. Conclusions

In this paper we demonstrated the results of the modelling of the ISD motion and distribution in the outer heliospheric interface. We built a kinetic model based on the kinetic equation and velocity distribution function. The solution of the kinetic equation was computed using a Monte-Carlo method. The model of the ISD particles charging was implemented as well. In this model we took account of currents of the impinging plasma ions and electrons, the secondary emitted electrons and the photoelectrons.

We showed that small enough ISD particles don’t penetrate into the heliosphere due to the interaction with the interstellar magnetic field. They flow around the heliopause and make a gyro rotation around the lines of this field. Due to the gyro rotation, wave structures in the number density appear. They also form the number density accumulations in front of the heliopause according to their deceleration in this region.

For the ISD particles of medium sizes, part of the particles can penetrate into the heliosphere through the nose part of the heliopause. At the poles of the heliopause the filtration effect remains. In the inner heliospheric interface these particles immediately focus in the vicinity of the ecliptic plane and that is why, for these sizes, there are some regions in the heliosphere without dust grains. This focusing appears due to the large values of the charge potential in this region. The reason for this growth of the charge potential is an increased current rate of the secondary emitted electrons.

The biggest ISD particles move almost freely across the heliospheric interface. It relates to the small values of the determining parameters of the electromagnetic force in this case. For
Figure 6. Maps of the $z$-axis velocity component distributions in the planes $X = 0$ (top panel) and $Y = 0$ (bottom panel) for the ISD particles of different sizes (20 nm, 50 nm, 100 nm, 200 nm, 500 nm, 1000 nm). One of the reasons for the dust particle accumulation in the vicinity of the heliopause is deceleration of the ISD flow. The number of particles for each computation $N = 2500000$. Relative statistical error is limited by 5% at each point out of the heliosphere, where the ISD particles appear.

de these particles, all features in distributions are located in the inner heliosphere.

In future we plan to develop our model, add to it the effects related to the inner heliosphere and consider the influence of the interstellar magnetic field on the ISD distributions inside the heliosphere.

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Appendix A. Formulae for currents.
Here we follow Section 3 from [9] with some remarks and annotations. As it was mentioned above, we take account of three main physical processes responsible for the charging of the ISD particles: sticking of plasma particles, secondary electron emission, photoelectron emission.
Appendix A.1. Sticking of plasma particles.
The current rate due to the sticking of plasma particles can be expressed by the number of collisions between the ISD particle and surrounding plasma particles, and this number can be written using a general integral form with velocity distribution function. Thus, the corresponding current rate is:

\[
J_{\text{imp},j} = Z_j e \int_0^{\infty} \int_0^{\pi} \int_0^{\infty} \sigma_j(v) f_j(v, \vartheta) v^3 \sin \vartheta dv d\vartheta d\varphi,
\]

where \( j \) runs two values: protons and electrons. All definitions and formulae for the values which compose the integrative function can be found in [9] (formulae (2) - (13) in that paper). In our calculations we use formula (7) from that paper, which corresponds to the integration over the Maxwellian distribution of plasma particles. This formula also takes account of the absolute value of the relative velocity between the ISD grain and plasma.

Appendix A.2. Secondary electron emission.
For the current of secondary electrons caused by impinging plasma particles the general form is similar to the one for the current of impinging plasma particles (A.1), but contains also the secondary electron yield (see e.g. formulae (21) and (25) in [9]) and supplementary integral multiplier related to the fact that not all emitted electrons are able to escape the ISD grain (in the case of positively charged ISD grain, emitted electrons must have enough energy to overcome the attraction of the grain and escape its surface):

\[
J_{\text{sec},j} = Z_j e \int_0^{\infty} \int_0^{\pi} \int_0^{\infty} \sigma_j(v) f_j(v, \vartheta) v^3 \sin \vartheta d\vartheta d\varphi \times \int_{\epsilon_{\text{min}}}^{\epsilon_{\text{max}}} \rho_j(\epsilon) d\epsilon,
\]

where again \( j \) runs two values and the value \( \delta_j(E_0) \) is the secondary electron yield, i.e. the number of secondary electrons produced per one collision with plasma particles. Here we fully follow the description in [9]. Also it should be noted that in this case we can’t receive analytical formula, as it was done for the currents of impinging plasma particles due to the special form of the expressions for the secondary electron yield. But we can still simplify this triple integral to the simpler one-dimensional integral using the analytical integration over \( \theta \) and \( \varphi \) variables. This transfer from triple integral to one-dimensional integral allows us to increase the computational efficiency of the charge potential calculation.

Appendix A.3. Photoelectron emission.
For the current of photoelectrons we use the expression (28) from [9]:

\[
J_{\text{ph}} = e \int_{W} d(h\nu)C_{\text{abs}}(h\nu)F(h\nu)Y(h\nu) \int_{\epsilon_{\text{min}}}^{\epsilon_{\text{max}}} d\epsilon \rho_{\text{ph}}(\epsilon).
\]

The calculation of absorption cross section \( C_{\text{abs}}(h\nu) \) is performed by means of Mie-theory (e.g. see Chapter 4 in [2]). One can find complex refractive indices used in the calculation of \( C_{\text{abs}}(h\nu) \), for example, in the following link: https://www.astro.princeton.edu/draine/dust/dust.diel.html subsection "Original Astronomical Silicate". The photon flux \( F(h\nu) \) is the sum of solar flux (in our computations we use data from https://lasp.colorado.edu/lisird/data/fism_p_ssi_earth/) and photon flux from the interstellar background (here we use data from [15], solid curve in Figure 1). It should be noted, that the solar photon flux is inversely proportional to squared heliodistance and data from the link
represents measurements at 1 au. For the photoelectric yield $Y(h\nu)$ we use formulae from [10] (see Appendix A from that paper). After the analytical integration over energy $\epsilon$ this integral is also converted to one-dimensional integral, that again increases efficiency of computations.