TS-CHIEF: A Scalable and Accurate Forest Algorithm for Time Series Classification

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Abstract Time Series Classification (TSC) has seen enormous progress over the last two decades. HIVE-COTE (Hierarchical Vote Collective of Transformation-based Ensembles) is the current state of the art in terms of classification accuracy. HIVE-COTE recognizes that time series are a specific data type for which the traditional attribute-value representation, used predominantly in machine learning, fails to provide a relevant representation. HIVE-COTE combines multiple types of classifiers: each extracting information about a specific aspect of a time series, be it in the time domain, frequency domain or summarization of intervals within the series. However, HIVE-COTE (and its predecessor, FLAT-COTE) is often infeasible to run on even modest amounts of data. For instance, training HIVE-COTE on a dataset with only 1,500 time series can require 8 days of CPU time. It has polynomial runtime w.r.t training set size, so this problem compounds as data quantity increases. We propose a novel TSC algorithm, TS-CHIEF, which is highly competitive to HIVE-COTE in accuracy, but requires only a fraction of the runtime. TS-CHIEF constructs an ensemble classifier that integrates the most effective embeddings of time series that research has developed in the last decade. It uses tree-structured classifiers to do so efficiently. We assess TS-CHIEF on 85 datasets of the UCR archive, where it achieves state-of-the-art accuracy with scalability and efficiency. We demonstrate that TS-CHIEF can be trained on 130k time series in 2 days, a data quantity that is beyond the reach of any TSC algorithm with comparable accuracy.

Keywords time series, classification, metrics, bag of words, transformation, forest, scalable
1 Introduction

Since March 2017, the European Space Agency’s Sentinel-2 programme has been capturing a full high-resolution picture of the Earth, every 5 days, and at no cost to the end-users [12]. Analysing the dynamics of each ‘pixel’ over time has proven to be invaluable for climate monitoring, including land-use mapping [19], fire modelling [32], flood modelling [43] and agricultural applications such as disease outbreak monitoring and yield prediction [3]. Creating such a map requires observing the evolution of each of these quadrillions of geographic areas over a given period (e.g., one year) for which a set of time series is extracted to represent the evolution of the observed variables over time. These quadrillions of time series can each then be classified into land cover categories such as ‘corn crop’, ‘eucalyptus forest’ or ‘urban’ [36].

Further applications of time series classification (TSC) that require training from large volumes of data include human activity recognition [48], classification of medical data from Electrocardiograms (ECG) [47], electric device identification from power consumption patterns [29], and many more [9, 8]. The diversity of such applications are evident from the commonly used University of California Riverside (UCR) archive of time series datasets [9, 8].

A number of recent TSC algorithms [31, 41, 39] have tackled this issue of ever increasing data volumes, achieving greater efficiency and scalability than typical TSC algorithms. However, none has been competitive in accuracy to the state-of-the-art HIVE-COTE algorithm. In contrast, our novel TSC algorithm, TS-CHIEF, achieves scalability without sacrificing accuracy. It is orders of magnitude faster than both FLAT-COTE and HIVE-COTE while attaining accuracy that ranks at least as well as them when assessed on the benchmark UCR archive, as illustrated in Figure 1.

Figure 2 shows an experiment that demonstrates the scalability of TS-CHIEF using the Satellite Image Time Series (SITS) dataset [45]. It is 900x faster than HIVE-COTE for 1,500 time series (13 min versus 8 days).

Fig. 1: Critical difference diagram showing the average ranks on error of leading TSC algorithms (described in Section 2) across 85 datasets from the benchmark UCR archive [9]. The lower the rank (further to the right) the lower the error of an algorithm relative to the others on average. While being scalable and efficient, our method TS-CHIEF ranks marginally better on accuracy than the current state-of-the-art method HIVE-COTE.
Fig. 2: Training time in logarithmic-scale for TS-CHIEF versus HIVE-COTE with increasing training size using the Satellite Image Time Series dataset [45]. Even for 1,500 time series, TS-CHIEF is more than 900 times faster than the current state of the art HIVE-COTE. The parameter $k$ indicates the number of trees used in the TS-CHIEF forest. The choice of parameters will be discussed in Section 4.

Moreover, the relative speedup grows with data quantity: at 132k instances TS-CHIEF is 46,000x faster. For a training size that took TS-CHIEF 2 days, we estimated 234 years for HIVE-COTE.

The rest of the paper is organized as follows: Section 2 discusses related work. Section 3 presents our algorithm TS-CHIEF. In Section 4, we compare TS-CHIEF against state-of-the-art TSC classifiers and investigate the scalability of TS-CHIEF and finally, Section 5 draws conclusions.

2 Related Work

Time Series Classification (TSC) aims to predict a discrete label $y \in \{1, \ldots, c\}$ for an unlabeled time series, where $c$ is the number of classes in the TSC task. Although our work could be extended to time series with varying lengths and multi-variate time series, we focus here on univariate time series of fixed lengths. A univariate time series $T$ of length $\ell$ is an ordered sequence of $\ell$ observations of a variable over time, where $T = \langle x_1, \ldots, x_\ell \rangle$, with $x_i \in \mathbb{R}$. We use $D$ to represent a training time series dataset and $n$ to represent the number of time series in $D$. 
2.1 Similarity-based techniques

These algorithms usually use 1-Nearest Neighbour (1-NN) with elastic similarity measures. Elastic measures are designed to compensate for local distortions, miss-alignments or warpings in time series that might be due to stretched or shrunk subsections within the time series.

The classic benchmark for TSC has been 1-NN using Dynamic Time Warping (DTW), with cross validated warping window size \[11\]. The warping window is a parameter that controls the \textit{elasticity} of the similarity measure. A zero window size is equivalent to the Euclidean distance, while a larger warping window size allows points from one series to match points from the other series over longer time frames.

Commonly used similarity measures include variations of DTW such as Derivative DTW (DDTW) \[23,13\], Weighted DTW (WDTW) \[20\], Weighted DDTW (WDDTW) \[20\], and measures based on edit distance such as Longest Common Subsequence (LCSS) \[18\], Move-Split-Merge (MSM) \[44\], Edit Distance with Real Penalty (ERP)\[7\] and Time Warp Edit distance TWE \[33\]. Most of these measures have additional parameters that can be tuned. Details of these measures can be found in \[29,2\].

Ensembles formed using multiple 1-NN classifiers with a diversity of similarity measures have proved to be significantly more accurate than 1-NN with any single measure \[29\]. Such ensembles help to reduce the variance of the model and thus help to improve the overall classification accuracy. For example, Elastic Ensemble (EE) combines 11 1-NN algorithms, each using a different one of 11 elastic measures \[29\]. For each measure, the parameters are optimized with respect to accuracy using cross-validation \[29,2\]. Though EE is a relatively accurate classifier \[2\], it is slow to train due to high computational cost of the leave-one-out cross-validation used to tune its parameters – \(O(n^2 \cdot \ell^3)\). Furthermore, since EE is an ensemble of 1-NN models, the classification time for each time series is also high – \(O(n \cdot \ell^2)\).

Our recent contribution, Proximity Forest (PF), is more scalable and accurate than EE \[31\]. It builds an ensemble of classification trees, where data at each node are split based on similarity to a representative time series from each class. This contrasts with the standard attribute-value splitting methods used in decision trees. Degree of similarity is computed by selecting at random one measure among the 11 used in EE. The parameters of the measures are also selected at random. Proximity Forest is highly scalable owing to the use of a divide and conquer strategy, and stochastic parameter selection in place of computationally expensive parameter tuning.

2.2 Interval-based techniques

These algorithms select a set of intervals from the whole series and apply transformations to these intervals to generate a new feature vector. The new feature vector is then used to train a traditional machine learning algorithm,
commonly a Random Forest. For instance, Time Series Forest (TSF) applies three time domain transformations – mean, standard deviation and slope – to each of a set of randomly chosen intervals, and then trains a decision tree using this new data representation. The operation is repeated to learn an ensemble of decision trees, similar to a Random Forest (RF) model, on different randomly chosen intervals. Other notable interval-based algorithms are Time Series Bag of Features (TSBF) [3], Learned Pattern Similarity (LPS) [4], and the recently introduced Random Interval Spectral Ensemble (RISE) [30].

RISE computes four different transformations for each random interval selected: Autocorrelation Function (ACF), Partial Autocorrelation Function (PACF), and Autoregressive model (AR) which extracts features in time domain, and Power Spectrum (PS) which extracts features in the frequency domain [30, 1]. Coefficients of these functions are used to form a new transformed feature vector. After these transformations have been computed, a Random Forest is trained on the new vectors, which is then used for classification. The algorithm presented in this paper have components inspired by RISE, therefore, further details are presented later (see Section 3.2.3).

2.3 Shapelet-based techniques

Rather than extracting intervals, where the location of sub-sequences are important, shapelet-based algorithms seek to identify sub-sequences that allow discrimination between classes irrespective of where they occur in a sequence [50]. Ideally, a good shapelet candidate should be a sub-sequence similar to time series from the same class, and dissimilar to time series from other classes. Similarity is usually computed using the minimum Euclidean distance of shapelet to all sub-sequences of same length from another series.

The original version of the shapelet algorithm [50, 34], enumerates all possible sub-sequences among the training set to find the “best” possible shapelets. It uses Information Gain criteria to assess how well a given shapelet candidate can split the data. The “best” shapelet candidate and a distance threshold is used as a decision criterion at the node of a binary decision tree. The search for the “best” shapelet is then recursively repeated until obtaining pure leaves. Despite some optimizations proposed in the paper, it is still a very slow algorithm with training complexity of $O(n^2 \cdot \ell^4)$.

Much of the research about shapelets have been focused on ways of speeding up the shapelet discovery phase. Instead of enumerating all possible shapelet candidates, researchers have tried to come up with ways of quickly identifying possible “good” shapelets. These include Fast Shapelets (FS) [37] and Learned Shapelets (LS) [16]. Fast Shapelet proposed to use an approximation technique called Symbolic Aggregate Approximation (SAX) [27] to shorten the time series during the shapelet discovery process in order to speed up by reducing the number of shapelet candidates. Learned Shapelets (LS) attempted to “learn” the shapelets rather than enumerate all possible candidates. Fast Shapelets algorithm is faster than LS, but it is less accurate [2].
Another notable shapelet algorithm is Shapelet Transform (ST) [17]. In ST, the ‘best’ \( k \) shapelets are first extracted based on their ability to separate classes using a quality measure such as Information Gain, and then the distance of each of the “best” \( k \) shapelets to each of the samples in the training set is computed [17, 6, 24]. The distance from \( k \) shapelets to each time series forms a matrix of distances which defines a new transformation of the dataset. This transformed dataset is finally used to train an ensemble of eight traditional classification algorithms including Logistic Regression, SVM, and Random Forest. Although very accurate, ST also has a high training-time complexity of \( O(n^2 \cdot \ell^4) \) [17, 30].

One algorithm that speeds up the shapelet-based techniques is Generalized Random Shapelet Forest (GRSF) [21]. GRSF selects a set of random shapelets at each node of a decision tree and performs the shapelet transformation at the node level of the decision tree. GRSF is fast, because it is tree-based, and uses random selection of shapelets, instead of enumerating all shapelets. GRSF experiments were carried out on a subset of the 85 UCR datasets where the values of the hyperparameters – the number of randomly selected shapelets as well as the lower and upper shapelet lengths – are optimized by using a grid search.

2.4 Dictionary-based techniques

Dictionary-based algorithms transform time series data into bag of words [12, 38, 25]. Dictionary based algorithms are good at handling noisy data and finding discriminatory information in data with recurring patterns [38]. Usually, an approximation method is first applied to reduce the length of the series [22, 27, 40], and then a quantization method is used to discretize the values, and thus to form words [38, 25]. Each time series is then represented by a histogram that counts the word frequencies. 1-NN with a similarity measure, that compares the similarity between histograms, can then be used to train a classification model. Notable dictionary based algorithms are Bag of Patterns (BoP) [25], Symbolic Aggregate Approximation-Vector Space Model (SAX-VSM) [12], Bag-of-SFA-Symbols (BOSS) [38], BOSS in Vector Space (BOSS-VS) [38], and Word eXtrAction for time SEries cLassification (WEASEL) [41].

To compute an approximation of a series, BOP and SAX-VSM use a method called Symbolic Aggregate Approximation (SAX) [27]. SAX uses Piecewise Aggregate Approximation (PAA) [22] which concatenates the means of consecutive segments of the series and uses quantiles of the normal distribution as breakpoints to discretize or quantize the series to form a word representation. By contrast, BOSS, BOSS-VS, and WEASEL use a method called Symbolic Fourier Approximation (SFA) [40] to compute the approximated series. SFA applies Discrete Fourier Transformation (DFT) on the series and uses the coefficients of DFT to form a short approximation, representing the frequencies in the series. This approximation is then discretized using a
data-adaptive quantization method called Multiple Coefficient Binning (MCB) \[40\] [38].

The most accurate algorithm in this category is Bag-of-SFA-Symbols (BOSS), which is an ensemble of dictionary-based 1-NN models [38]. Due to this accuracy, our algorithm incorporates elements of this approach. Further details of the BOSS algorithm will be presented in Section 3. BOSS has a training time complexity of $O(n^2 \cdot \ell^2)$ [38]. Variations of BOSS such as BOSS-VS [39] and WEASEL [41] were developed to be more scalable, but significantly sacrifice accuracy [31].

2.5 Combinations of Transformations

Two leading algorithms that combine multiple transformations are Flat Collective of Transformation-Based Ensembles (FLAT-COTE) [11] and the more recent variant Hierarchical Vote COTE (HIVE-COTE) [30]. FLAT-COTE is a meta-ensemble of 35 different classifiers that use different time series classification methods such as similarity-based, shapelet-based, and interval-based techniques. In particular, it includes other ensembles such as EE and ST. The label of a time series is determined by applying weighted majority voting, where the weighting of each constituent depends on the training leave-one-out cross-validation (LOO CV) accuracy. HIVE-COTE works similarly, but it includes new algorithms, BOSS and RISE, and changes the weighted majority voting to make it balance between each type of constituent modules. These modifications result in a major gain in accuracy, and it is currently considered as the state of the art in TSC for accuracy. However, both variants of COTE have high training complexity, lower bounded by the slow cross-validation used by EE – $O(n^2 \cdot \ell^3)$ – and exhaustive shapelet enumeration used by ST – $O(n^2 \cdot \ell^4)$.

2.6 Deep Learning

Deep learning is interesting for time series both because of the structuring dimension offered by time (deep learning has been particularly good for images and videos) and for its linear scalability with training size. Most related research has focused on developing specific architectures based mainly on Convolutional Neural Networks (CNNs) [49, 13], coupled with data augmentation, which is required to make it possible for them to reach high accuracy on the relatively small training set sizes present in the UCR archive [26, 13]. While these approaches are computationally efficient, the two leading algorithms, Fully Connected Network (FCN) [49] and Residual Neural Network (ResNet) [49], are still less accurate than FLAT-COTE and HIVE-COTE [13].
This section introduces our novel algorithm TS-CHIEF, which stands for Time Series Combination of Heterogeneous and Integrated Embeddings Forest. TS-CHIEF is an ensemble algorithm that makes the most of the scalability of tree classifiers coupled with the accuracy brought by decades of research into specialized techniques for time series classification. Traditional attribute-value decision trees form a tree by recursively splitting the data w.r.t the value of a selected attribute. These techniques (and ensembles thereof) do not in general perform well when applied directly to time series data [2]. As they treat the value at each time step as a distinct attribute, they are unable to exploit the information in the series order. In contrast, TS-CHIEF utilizes splitting criteria that are specifically developed for time series classification.

Our starting point for TS-CHIEF is the Proximity Forest (PF) algorithm [31], which builds an ensemble of classification trees with ' splits' using the proximity of a given time series $T$ to a set of reference time series: if $T$ is closer to the first reference time series, then it goes to the first branch, if it is closer to the second reference time series, then it goes to the second branch, and so on. PF integrates 11 time series measures for evaluating similarity. At each node a set of reference series are selected, one per class, together with a similarity measure and its parameterization. These selections are made stochastically. PF attains accuracies that are comparable to BOSS and ST (see Figure 1). TS-CHIEF complements PF’s splitters with dictionary-based and interval-based splitters, which we describe below. Our algorithmic contributions are three-fold:

1. We take the ideas that underlie the best dictionary-based method, BOSS, and develop a tree splitter based thereon.
2. We take the ideas behind the best interval-based method, RISE, and develop a tree splitter based thereon.
3. We develop techniques to integrate these two novel splitters together with those introduced by PF, such that any of the 3 types might be used at any node of the tree.

TS-CHIEF is an ensemble method: we thus paid particular attention to maximizing the diversity between the learners in its design. We do this by creating a very large space of possible splitting criteria. This diversity for diversity sake would be unreasonable if the objective was to create a single standalone classifier. By contrast, by ensembling, this diversity can be expected to reduce the covariance term of ensemble theory [46]. If ensemble member classifiers are too similar to one another, their collective decision will differ little from that of a single member.

3.1 General Principles

During the training phase, TS-CHIEF builds a forest of $k$ trees. The general principles of decision trees remain: tree construction starts from the root node
and recursively builds the sub-trees, and at each node the data are split into branches using a splitting function. Where TS-CHIEF differs is in the use of time-series-specific splitting functions. The details of these splitting functions will be discussed in Section 3.2. In short, we use different types of splitters either using time series similarity measures, dictionary-based or interval-based representations. At each node, we generate a set of candidate splits and select the best one using the weighted Gini index, i.e. the split that maximises the purity of the created branches (similar to a classic decision tree). We describe the top-level algorithm in Algorithm 1; note that this algorithm is very typical of decision trees and that all the time-series-specific features are in the way we generate candidate splits, as shown in Algorithms 2 to 3.

3.2 Splitting Functions

As mentioned earlier, we choose splitting functions based on similarity measures, dictionary representations and interval-based transformations. This is motivated by the components of HIVE-COTE, namely EE (similarity-based), BOSS (dictionary-based) and RISE (interval-based). We do not include ST (shapelets) because of their high computational complexity. We also omit TSF because it has accuracy ranked lower than 1-NN DTW [2]. We next describe how we generate each of these types of splitting function.

3.2.1 Similarity-based

This splitting function uses the method of Proximity Forest [31], which splits the data based on the similarity of each time series to a set of reference time series. At training time, for each candidate splitter, a random measure $\delta_M$, that is randomly parameterized, is selected, as well as a set $\delta_E$ of random reference time series, one from each class (see [31, Algorithm 2]). Note that if TS-CHIEF is trained with only the similarity-based splitter enabled (i.e. $C_b = C_r = 0$), then it is exactly Proximity Forest.

When splitting the data at training time and at classification time, the similarity of a query instance $Q$ to each reference time series $e_c$ in $\delta_E$ is evaluated using the selected measure $\delta_M$. $Q$ is passed down the branch corresponding to the $e_c$ to which $Q$ is closest.

3.2.2 Dictionary-based

This type of split functions also uses a similarity-based splitting mechanism, except that it works on a set of time series that have been transformed using the BOSS transformation [38, Algorithm 1], and that it uses a variant of the Euclidean distance ([38 Definition 4]) to measure similarity between transformed time series.

The BOSS transformation is used to convert the time series dataset into a bag-of-word model. We start by describing the BOSS transformation. To
Algorithm 1: build_tree($D, C_e, C_b, C_r$)

**Input:** $D$: a time series dataset
**Input:** $C_e$: no. of similarity-based candidates
**Input:** $C_b$: no. of dictionary-based candidates
**Input:** $C_r$: no. of interval-based candidates
**Output:** $T$: a TS-CHIEF Tree

1. if is_pure($D$) then
   2. return create_leaf($D$)
3. // Create tree represented by its root node
4. $T \leftarrow$ create_node()
5. $S \leftarrow \emptyset$ // the set of candidate splitters
6. for $i = 1$ to $C_e$ do
   7. // See [31, Algorithm 2] for a detailed description of
      generate_similarity_split which returns the distance measure $M$ and a
      set of exemplars $E$
   8. $(M, E) \leftarrow$ generate_similarity_split($D$)
   9. $\delta \leftarrow (M, E)$
   10. Add splitter $\delta$ to $S$
11. end
12. for $i = 1$ to $C_b$ do
   13. $\delta \leftarrow$ generate_dictionary_split($D$)
   14. Add splitter $\delta$ to $S$
15. end
16. $\delta^* \leftarrow$ select_interval_split($D, C_r$)
17. Add splitter $\delta^*$ to $S$
18. $T_B \leftarrow \emptyset$
19. // Partition the data using $\delta^*$ and recurse
20. if $\delta^*$ is interval-based then
   21. $D^\leq \leftarrow \{d \in D \mid \text{get_att_val}(\delta^*, \langle d_{s+1}, \ldots, d_{s+m} \rangle) \leq \delta^*_v\}$
   22. $T_{\text{left}} \leftarrow$ build_tree($D^\leq, C_e, C_b, C_r$)
   23. Add branch $T_{\text{left}}$ to $T_B$
   24. $D^> \leftarrow \{d \in D \mid \text{get_att_val}(\delta^*, \langle d_{s+1}, \ldots, d_{s+m} \rangle) > \delta^*_v\}$
   25. $T_{\text{right}} \leftarrow$ build_tree($D^>, C_e, C_b, C_r$)
   26. Add branch $T_{\text{right}}$ to $T_B$
   27. else
   28. if $\delta^*$ is similarity-based then
      29. $\text{dist} \leftarrow \delta^*_\lambda$
   30. else if $\delta^*$ is dictionary-based then
      31. $\text{dist} \leftarrow BOSS,\text{dist} // \text{see [38, Definition 4]}$
   32. foreach $e \in \delta^*_E$ do
      33. $D^+ \leftarrow \{d \in D \mid \text{dist}(d, e) = \min_{x \in E}(\text{dist}(d, x))\}$
      34. $t \leftarrow$ build_tree($D^+, C_e, C_b, C_r$)
      35. Add branch $(e, t)$ to $T_B$
   36. end
37. end
38. return $T$
Algorithm 2: \texttt{generate\_dictionary\_split(D)}

\textbf{Input:} $D$: a time series dataset
\textbf{Output:} $\delta$: a trained splitting function

1. $\mathcal{T} \leftarrow \text{get\_random\_BOSS\_transformation()}$ \texttt{\// see [33]}
2. \texttt{// Select BOSS histograms of reference time series $E$}
3. $E \leftarrow \emptyset$

4. \texttt{foreach class $c$ present in $D$ do}
5. \hspace{1em} // $D_c$ is the data for class $c$
6. \hspace{1em} $D_c \leftarrow \{d \in D \mid \text{class}(d) = c\}$
7. \hspace{1em} $e \leftarrow \text{sample a reference histogram uniformly at random from } D_c$
8. \hspace{1em} Add $\mathcal{T}(e)$ to $E$ \texttt{// $\mathcal{T}(e)$ is a BOSS histogram}
9. \texttt{end}

10. $\delta \leftarrow (\mathcal{T}, E)$
11. \texttt{return } $\delta$

Algorithm 3: \texttt{select\_interval\_split(D, $C_r$)}

\textbf{Input:} $D$: a time series dataset
\textbf{Input:} $C_r$: no. of interval-based candidates
\textbf{Output:} $\delta^*$: a trained interval splitting function

1. $m_{\text{min}} \leftarrow 16$ \texttt{// minimum length of intervals}
2. $C_r^* \leftarrow \lfloor C_r / A \rfloor$ \texttt{// no. of splitters per transform}
3. $R \leftarrow \lceil C_r^* / m_{\text{min}} \rceil$ \texttt{// no. of intervals to compute}
4. $S \leftarrow \emptyset$ \texttt{// candidate interval-based splitters}

5. \texttt{for } $i = 1$ \texttt{to } $R$ \texttt{do}
6. \hspace{1em} \texttt{// Select a random interval of length } $m$ \texttt{starting at } $s$
7. \hspace{1em} $(s, m) \leftarrow \text{get\_random\_interval}(s, m_{\text{min}})$
8. \hspace{1em} \texttt{// Add a splitter for each transformation}
9. \hspace{1em} \texttt{foreach } $\lambda$ \texttt{in } $\{\text{ACF, PACF, AR, PS} \}$ \texttt{do}
10. \hspace{2em} \texttt{// Apply } $\lambda$ \texttt{to each time series}
11. \hspace{2em} $D_T \leftarrow \emptyset$
12. \hspace{2em} \texttt{foreach } $d$ \texttt{in } $D$ \texttt{do}
13. \hspace{3em} \texttt{// Create } $d_T$, \texttt{a vector of } $m$ \texttt{attribute-values obtained by}
14. \hspace{3em} \texttt{applying } $\lambda$ \texttt{to the interval}
15. \hspace{3em} $d_T \leftarrow \lambda(d_{s}, \cdots d_{s+m-1})$
16. \hspace{3em} Add $d_T$ to $D_T$
17. \hspace{1em} end
18. \hspace{1em} \texttt{// Calculate no. of attributes to evaluate so that } $C_r$ \texttt{attributes are evaluated in total}
19. \hspace{1em} $A \leftarrow \lfloor C_r^* / R \rfloor$
20. \hspace{1em} \texttt{// Select at random } $A$ \texttt{attributes in } $D_T$
21. \hspace{1em} $\tilde{T} \leftarrow \text{get\_random\_attributes}(D_T, A)$
22. \hspace{1em} \texttt{foreach attribute } $a$ \texttt{in } $\tilde{T}$ \texttt{do}
23. \hspace{2em} \texttt{v} \texttt{\leftarrow find\_best\_threshold}(a)
24. \hspace{2em} Add $((s, m), \lambda(a, v))$ to $S$
25. \hspace{1em} end
26. \texttt{end}
27. \texttt{// Select the best splitter}
28. $\delta^* \leftarrow \arg \max_{\delta \in S} \text{Gini}(\delta)$
29. \texttt{return } $\delta^*$
compute a BOSS transformation of a single time series, first, a window of fixed length $w$ is slid over the time series, while converting each window to a Symbolic Fourier Approximation (SFA) word of length $f$ \cite{35,38}. SFA is a two-step procedure: 1) it applies a low pass filter – using only the low frequency coefficients of the Discrete Fourier Transformation (DFT) –, 2) it converts each window (subseries) into a word using a data adaptive quantization method called Multiple Coefficient Binning (MCB). MCB defines a matrix of discretization levels for an alphabet size $\alpha$ (default is $\alpha = 4$) and a word length $f$. This leads to $\alpha^f$ possible words. There is also a normalization parameter. If it is equal to true, the window is $z$-normalised before applying the DFT. SFA words are then counted to form a word frequency histogram that is used to compare two time series. BOSS uses a bespoke Euclidean distance, namely $BOSS_{\text{dist}}$, to compare the similarity between the histograms \cite{35}.

We now turn to explaining how we use BOSS transformations to build our forest. Since BOSS has four different hyperparameters, many possible BOSS transformations of a time series can be generated. Before we start training the trees, $t$ BOSS transformations (histograms for all time series) of the dataset are pre-computed based on $t$ randomly selected sets of BOSS parameters. Similar to the values used in BOSS, the four parameters are selected uniformly at random from the following ranges: the window length $w \in \{10 \cdots \ell\}$, SFA word length $f \in \{6,8,10,12,14,16\}$, the normalization parameter $\text{norm} \in \{\text{true, false}\}$, and $\alpha = 4$.

At training time (Algorithm 2), for each candidate splitter $\delta$, a random BOSS transformation $\delta_T$ is chosen, as well as a set $\delta_E$ of random reference time series from each class for which the transformation $\delta_T$ has been applied. Each training time series is then passed down the branch of the reference series for which the BOSS distance between histogram of the series and the reference time series is highest. We then generate several such splitters and choose the best one according to the Gini index.

At classification time, when a query time series $Q$ arrives at a node with a dictionary-based splitter, we start by calculating its transformation into a word histogram (the transformation $\delta_T$ selected at training). We then compare this histogram to each reference time series in $\delta_E$, and $Q$ is passed down the branch corresponding to the reference time series to which $Q$ is closest.

3.2.3 Interval-based

This type of splitting function is designed to work in a similar fashion to the RISE component used in the HIVE-COTE. Recall that RISE is an interval-based algorithm that uses four transformations (ACF, PACF, AR - in time domain and PS - in frequency domain) to convert a set of random intervals to a feature vector. Once the feature vectors have been generated, RISE uses a classic attribute-value splitting mechanism to train a forest of binary decision trees (similar to Random Forest). Intervals are selected per tree in HIVE-COTE, whereas TS-CHIEF selects the random intervals per candidate split at the node level. This is done to increase the diversity of the forest.
Each candidate splitter \(\delta\) is defined by a pair \((\delta_s, \delta_m)\) that represent the interval start and its length respectively, a function \(\delta_\lambda\) (one of ACF, PACF, AR or PS) which is applied to the interval and a pair \((\delta_a, \delta_v)\) that indicates the attribute \(\delta_a\) and threshold value \(\delta_v\) on which to split. The values of \((\delta_s, \delta_m)\) are randomly selected to get a valid interval of a minimum length \(m_{min} = 16\). The values of the pair \((\delta_a, \delta_v)\) are optimized such that the Gini index is maximized when the data are split on the attribute \(\delta_a\) for a threshold value \(\delta_v\). Algorithm 3 describes the process to select the best similarity-based splitter among \(C_r\) candidates.

When splitting the data at training time and at classification time, \(\delta_\lambda\) is applied to the interval of query instance \(Q\) defined by \(\delta_s\) and \(\delta_m\), obtaining the attribute vector \(Q_\lambda\). If \(\text{get\_att\_val}(Q_\lambda, \delta_a) \leq \delta_v\) (the value of attribute \(\delta_a\) of \(Q_\lambda\) is less than the threshold value), \(Q\) is passed down the left branch. Otherwise it is passed down the right. Contrary to the similarity- and dictionary-based splitting functions, only binary splits are built by the interval-based splitting functions (Lines 22 to 26 in Algorithm 1).

### 3.3 Classification

For each tree, a query time series \(Q\) is passed down the hierarchy from the root to the leaves. The branch taken at each node depends on the splitting function selected at the node. Once \(Q\) reaches the leaf, it is labelled with the class with which the training instances that reached that leaf were classified. Recall that the tree is repeatedly split until pure, so all training instances that reach a leaf will have the same class. This process is presented in the Algorithm 4. Finally, a majority vote by the \(k\) trees is used to label \(Q\).

### 3.4 Complexity

**Training time complexity** Proximity Forest, on which TS-CHIEF builds, has average training time complexity that is quasi-linear with the quantity of training data, \(O(k \cdot n \log(n) \cdot C_e \cdot c \cdot \ell^2)\) for \(k\) trees, \(n\) training time series of length \(\ell\), \(C_e\) similarity-based candidate splits, and \(c\) classes [31]. The term \(k\) comes from the number of trees to train and \(\log(n)\) from the average depth of the trees. In the worst case, tree depth may be \(n\), however, on average, tree depth can be expected to be \(\log(n)\). The term \(n \cdot C_e \cdot c \cdot \ell^2\) represents the order of time required to select the best of \(C_e\) candidate splits and partition the data thereon, based on the similarity of \(n\) training instances to \(c\) reference time series at the node using a random similarity measure. The slowest of the similarity measures used (WDTW) is bounded by \(O(\ell^2)\).

The addition of the dictionary-based splitter adds a new initialization step and a new selection step to the PF algorithm. The initialization part pre-computes \(t\) BOSS transformations for \(n\) time series. Since the cost of BOSS transforming one time series is \(O(\ell)\) [38, Section 6], the complexity of the
Algorithm 4: classification($Q, T$)

Input: $Q$: Query Time Series
Input: $T$: TS-CHIEF Tree
Output: a class label $c$

1. if is_leaf($T$) then
   2. return majority class of $T$
3. if $T$ is similarity-based then
   4. $(e, T^*) \leftarrow \arg\min_{(e', T') \in T_B} \delta_A(Q, e')$
5. else if $T$ is dictionary-based then
   6. $(e, T^*) \leftarrow \arg\min_{(e', T') \in T_B} \text{BOSS}_\text{dist}(\delta_T(Q), e')$
7. else if $T$ is interval-based then
   8. $Q_\lambda \leftarrow \delta_\lambda(Q_{\delta_\lambda}, \ldots, Q_{\delta_\lambda + \delta_m - 1})$
   9. // compare the $\lambda$th attribute value from $Q_{\lambda}$ to the split value
10. if get_att_val($Q_{\lambda}, \delta_v$) $\leq \delta_v$ then
    11. $T^* \leftarrow T_{\text{left}}$
    12. else
    13. $T^* \leftarrow T_{\text{right}}$
14. // recursive call on subtree $T^*$
15. return classification($Q, T^*$)

initialization part is $O(t \cdot n \cdot \ell)$. The Euclidean-based BOSS distance has a complexity of $O(\ell)$ [38, Definition 4] and must be applied to every example at the node for each of the $C_b$ (dictionary-based candidate splits), resulting in order $O(C_b \cdot c \cdot n \cdot \ell)$ complexity for generating and evaluating dictionary splitters at each node of each tree.

The interval-based splitting functions are attribute-value splitters; we detail the complexity for training a node receiving $n'$ time series. Each interval is transformed using 4 different functions (ACF, PACF, AR and PS), which takes at most $O(\ell^2)$ time [30, Table 4], leading to $O(r \cdot n' \cdot \ell^2)$ for $r$ intervals taken where $r$ is proportional to $C_r$. For each of the $C_r$ candidate splits the data is then sorted and scanned through to find the best split – $O(C_r \cdot n \log(n))$. Put together, this adds $O(C_r \cdot n \cdot \ell^2 + C_r \cdot n \log(n))$ complexity to the split selection stage. Note that $\ell$ in this term represents an upper bound on the length of random intervals selected. The expected length of random interval is $1/3$ of $\ell$.

Overall, TS-CHIEF has quasi-linear average complexity:

$$O \left( \frac{t \cdot n \cdot \ell}{\text{initialization}} + \frac{k \cdot \log(n)}{\text{avg. depth for } k \text{ trees}} \left[ \frac{C_c \cdot c \cdot n \cdot \ell^2 + C_b \cdot c \cdot n \cdot \ell}{\text{similarity}} + \frac{C_r \cdot n \cdot \ell^2 + C_r \cdot n \log(n)}{\text{dictionary}} \right] \right).$$

In the Appendix, we have included an experiment to measure the fraction of training time taken by each splitter type over 85 UCR datasets [8]. As expected, the dominant term in the training complexity is the term representing the similarity-based splitter. In practice, our experiments show that the similarity-based splitter takes about 80% of the training time (See Figure [8].)
Classification time complexity Each time series is simply passed down \( k \) trees, traversing an average of \( \log(n) \) nodes. Moreover, the complexity at each node is dominated by the similarity-based splitters. Overall, this is thus a \( O(k \cdot \log(n) \cdot c \cdot \ell^2) \) average case classification time complexity.

Memory complexity The memory complexity is linear with the quantity of data. We would need to store one copy of \( n \) time series of length \( \ell \) – this is \( O(n \cdot \ell) \). In the worst case there are as many nodes in each of the \( k \) trees as there are time series and at each node, and we store one exemplar time series for each of the \( c \) classes, \( O(k \cdot n \cdot c) \). We pre-store all \( t \) dictionary-based transformations, \( O(t \cdot n \cdot \ell) \). Overall, this is \( O(n \cdot \ell + k \cdot n \cdot c + t \cdot n \cdot \ell) \).

4 Experiments

We start by evaluating the accuracy of TS-CHIEF on the UCR archive, and then assess its scalability on a large time series dataset. In essence, we show that TS-CHIEF can reach the same level of accuracy as HIVE-COTE but with much greater speed, thanks to TS-CHIEF’s quasi-linear complexity. We finish this section by an assessment of the contribution of each type of splitter in TS-CHIEF.

Throughout the experiments, unless mentioned otherwise, we use the following parameter values for TS-CHIEF: \( t = 1000 \) dictionary-based (BOSS) transformations, \( k = 500 \) trees in the forest. When training each node, we concurrently assess the following number of candidates: 5 similarity-based splitters, 100 dictionary-based splitters and 100 interval-based splitters. Ideally, we would also want to raise the number of candidates for the similarity-based splitter, but this has a significant impact on training time (due to passing the instances down the branches measures in \( O(\ell^2) \)) with marginal improvement in accuracy [31]. Note that we have not done any tuning of these numbers of candidates of each type.

4.1 Accuracy on the UCR Archive

We evaluate TS-CHIEF on the UCR archive [8], as is the de facto standard in TSC research [2]. We use the 2015 version with 85 datasets, because the very recent update adding further datasets is still in beta [9]. All 85 datasets are fixed length univariate time series that have been \( z \)-normalized. We use the standard train/test split available at [http://www.timeseriesclassification.com](http://www.timeseriesclassification.com). To compare multiple algorithms over the 85 datasets, we use critical difference diagrams, as is standard in machine learning research [10]. For each dataset, the classifiers are ranked on accuracy and the critical difference diagram displays the average rank for each algorithm, as well as which differences between them are statistically significant at level \( \alpha = 0.05 \).

We first compare TS-CHIEF to the 3 time series classifiers identified by [2] as the most accurate on the UCR archive (FLAT-COTE, ST and BOSS), as
well as the de facto standard 1-NN DTW, deep learning method ResNet and
the more recent HIVE-COTE (the current most accurate on the URC archive)
and PF (the inspiration for TS-CHIEF). We use results reported at the
http://www.timeseriesclassification.com website for these algorithms, except
for TS-CHIEF, PF (our result [31]) and the deep learning ResNet method for
which we obtained the results from Fawaz et. al’s review of Deep Learning
methods for TSC [13].

Figure 1 (on page 2) displays average ranks and critical differences between
the 8 algorithms. The critical difference of 1.14 is indicated by the width of the
horizontal line below the label “CD”: an algorithm’s rank has to be at least
1.14 greater than another for it to be considered significantly better. The thick
line represents clusters of algorithms that are within one CD of each other.

Figure 1 gives the main result of this paper in terms of accuracy: TS-
CHIEF ranks the highest with an average rank of 2.92, with HIVE-COTE
almost on par with it at 2.93. FLAT-COTE comes next with an average rank of
3.78 that is not statistically distinguishable from TS-CHIEF or HIVE-COTE.
TS-CHIEF is ranked significantly higher than 1-NN DTW, BOSS, Proximity
Forest (PF), Residual Neural Network (ResNet) and Shapelet Transform (ST).

To further examine the accuracy of TS-CHIEF against both COTE algo-
rithms, Figure 3 presents a scatter plot of pairwise accuracy. Each point rep-
resents a UCR dataset. TS-CHIEF wins above the diagonal line. TS-CHIEF
wins 41 times against HIVE-COTE (blue squares), loses 35 times and ties on
9 datasets. Compared to FLAT-COTE (red circles), TS-CHIEF wins 44 times,
and loses 33 times, with 8 ties. With respect to the benchmark UCR archive,
TS-CHIEF is thus the most accurate TSC to date, outperforming both COTE
algorithms. It is interesting to see that TS-CHIEF nonetheless gives results
that are quite different to both COTE algorithms, with a few datasets for
which the difference in accuracy is quite large.

Although we were not able to compare running time with either of the
COTE algorithms because of their very high running time, even on the UCR
archive, we give here a few indications of runtime for TS-CHIEF. The experi-
ment was carried out using an AMD Opteron CPU (1.8 GHz) with 64 GB
RAM, with 16 CPU threads.

Average training and testing times were respectively of about 3 hours and
27 min per dataset, but with quite a large difference between datasets. TS-
CHIEF was trained on 69 datasets in less than 1 hour each and less than
one day was sufficient to train TS-CHIEF on all but 10 datasets. It however
took about 10 days to complete training on all the datasets, mostly
due to the HandOutlines dataset which took more than 4 days to complete.
Our experiments confirmed our theoretical developments about complexity:
TS-CHIEF was largely unaffected by dataset size with the largest dataset
ElectricDevices trained in 2h24min and tested in 9min. HandOutlines is
the dataset with the longest series and in the top-10 in terms of training size,
which shows that the quadratic complexity with the length has still a non-
negligible influence on training time. The next section details scalability w.r.t
length and size.
4.2 Scalability

TS-CHIEF is designed to be both accurate and highly scalable. Section 3.4 showed that the complexity of TS-CHIEF scales quasi-linearly w.r.t number of training instances $n$ and quadratically w.r.t length of the time series $\ell$. To assess how this plays out in practice, we carried out two experiments to evaluate the runtime of TS-CHIEF when 1) the number of training instances increases, and 2) the time series length increases. We compare TS-CHIEF to the HIVE-COTE algorithm which previously held the title of most accurate on the UCR archive. We performed these experiments with 100 trees and repeated 10 times. As the accuracy on the UCR archive has been evaluated for 500 trees (Section 4.1), we also estimated the timing for 500 trees (5 times slower). The experiments used a single run of each algorithm using 1 CPU on a machine with an Intel(R) Xeon(R) CPU E5-2680 v3 @ 2.50GHz processor with 200 GB of RAM.

4.2.1 Increasing training set size

First, we assessed the scalability of TS-CHIEF w.r.t training set size. We used a Satellite Image Time Series (SITS) dataset composed of 1 million time series of length 46, with 24 classes. We evaluated the accuracy and the total runtime as a function of the number of training time series, starting from a subsample of 58, and logarithmically increasing up to 131,879 (a sufficient quantity to clearly define the trend).

Figures 2 (on page 3) and 4 show the training time and the accuracy, respectively, as a function of the training set size for TS-CHIEF (in olive)
and HIVE-COTE (in red). Figure 2 shows that TS-CHIEF trains in time that is quasi-linear w.r.t the number of training examples, rather than the quadratic time for HIVE-COTE. For about 1,500 training time series, HIVE-COTE requires about 8 days to train, while TS-CHIEF was able to train in about 13 minutes. This is thus a 900x speed-up.

Figure 4 shows that TS-CHIEF has similar accuracy to HIVE-COTE for any given number of training time series. However, TS-CHIEF achieves 67% accuracy within 2 days by learning from about 132k time series. By fitting a quadratic curve through HIVE-COTE training time, we estimate that it will require 234 years for HIVE-COTE to learn from 132k time series. This is a speed-up of 46,000 times over HIVE-COTE. Furthermore, to train all one million time series in the SITS dataset, we estimated that it would take 13,550 years to train HIVE-COTE, while TS-CHIEF is estimated to take 44 days. This is a speed-up of 90,000 times over HIVE-COTE for 1M time series.

Moreover, Figure 4 indicates that HIVE-COTE can only achieve 60% after 2 days of training, i.e. a decrease of 7.9% compared to TS-CHIEF. In practice, the execution time of TS-CHIEF thus scales very close to its theoretical average complexity (Section 3.4) by scaling quasi-linearly with the training set size.

![Fig. 4: Accuracy as a function of training set size for SITS dataset.](image)

### 4.2.2 Increasing length

Second, we assessed the scalability of TS-CHIEF w.r.t to the length \( \ell \) of the time series. We use here InlineSkate, a UCR dataset composed of 100 time series and 550 test time series of original length 1882. We resampled the length from 32 to 2048 by using an exponential scale with base 2.
Figure 5 displays the training time for both TS-CHIEF (in olive) and HIVE-COTE (in red) as a function of the length of the time series. TS-CHIEF can learn from 100 time series of length 2,048 in about 4 hours, while HIVE-COTE requires more than 3 days. This is a 24x speed up. It also mirrors the theoretical training complexity of TS-CHIEF in $O(\ell^2)$, and HIVE-COTE in $O(\ell^4)$ w.r.t length of the time series.

4.3 Contribution of Splitting Functions

We also conducted ablation experiments to assess the contribution of each type of splitting function: similarity-based, dictionary-based and interval-based. For this purpose, we assess each variant of TS-CHIEF created by disabling one of the functions or a pair of the functions. We performed these experiments with 100 trees and repeated 10 times.

Figure 6 displays six scatter-plots comparing the accuracy of TS-CHIEF using all splitting functions to that of the six ablation configurations. The vertical axes indicate the accuracy of TS-CHIEF with all split functions enabled. The first row compares TS-CHIEF to variants with a single splitting function disabled. The second row compares TS-CHIEF to variants with only a single splitting function enabled. Please note that the use of only the similarity-based splitting function (first column, second row) corresponds to the PF algorithm [31]. Each point indicates one of the 85 UCR datasets. Points above the diagonal dashed line indicate that TS-CHIEF with all three splitting functions has higher accuracy than the alternative.

Fig. 5: Training time as a function of the series length $\ell$ for a one UCR dataset.
Fig. 6: Pairwise comparison of accuracy with one or two types of split functions removed versus TS-CHIEF (where all three types of split functions were used), to determine the contribution of each type of split function to the overall accuracy.

Fig. 7: Critical difference diagram showing the contribution of each type of split function.

The scatter plots on the bottom row indicate that, individually, the dictionary-based splitter contributes most to the accuracy with 18 wins, 59 losses and 8 ties relative to TS-CHIEF. We can also observe that the magnitudes of its losses tend to be smaller. Conversely, the interval-based splitter contributes least to the accuracy, with losses of the greatest magnitude relative to TS-CHIEF. However, it still achieves lower error on 17 datasets, demonstrating that there are some datasets for which the interval-based approach performs well.
In addition, the similarity-based splitter in conjunction with the dictionary-based splitter (that is, the variant with interval-based disabled) is closest to the accuracy of TS-CHIEF, with 26 wins against TS-CHIEF, 42 losses and 8 ties.

Figure 7 shows a critical difference diagram summarizing the the relative accuracy of all combinations of the splitting functions. This confirms our observations from the graphs in Figure 6. The combination of all three types of splitters has the highest average rank. Next come the pairs of splitters, with all pairs out-ranking the single splitters, albeit marginally for the pair that excludes the dictionary splitter.

5 Conclusions

We have introduced TS-CHIEF, which is a scalable and highly accurate algorithm for TSC. We have shown that TS-CHIEF makes the most of the quasi-linear scalability of trees, together with the last decade of research into deriving accurate representations of time series. Our experiments carried out on 85 datasets show that our algorithm reaches state-of-the-art accuracy that rivals HIVE-COTE, an algorithm which cannot be used in many applications because of its computational complexity. We showed that on an application for land-cover mapping, TS-CHIEF is able to learn a model from 130,000 time series in 2 days, whereas it takes HIVE-COTE 8 days to learn from only 1,500 time series – a quantity of data from which TS-CHIEF learns in 13 minutes. TS-CHIEF offers a general framework for time series classification. We believe that researchers will find it easy to integrate novel transformations and similarity measures and apply them at scale.

Supplementary material

To ensure reproducibility, a multi-threaded version of this algorithm implemented in Java and the experimental results have been made available in the github repository https://github.com/dotnet54/TS-CHIEF.

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### Appendix

Table 1: Accuracy of leading TSC classifiers on 85 UCR datasets. The classifiers are 1-Nearest Neighbour with DTW (labelled DTW), BOSS, PF (Proximity Forest), ST (Shapelet Transform), Residual Neural Network (ResNet), FLAT-COTE (FCT), HIVE-COTE (HCT), and TS-CHIEF (CHIEF). The last two rows show the number of wins and average ranking of accuracy (Refer to Figure 1).

| Dataset     | DTW   | BOSS  | PF    | ResNet | ST    | FCT   | HCT   | CHIEF |
|-------------|-------|-------|-------|--------|-------|-------|-------|-------|
| Adiac       | 60.87 | 76.47 | 73.40 | 82.89  | 78.26 | 79.03 | 81.07 | 79.80 |
| ArrHead     | 80.00 | 83.43 | 87.54 |        |       |       |       |       |
| Beef        | 66.67 | 80.00 | 72.00 |        |       |       |       |       |
| BeeFly      | 65.00 | 90.00 | 87.50 |        |       |       |       |       |
| BirdChi     | 70.00 | 95.00 | 86.50 |        |       |       |       |       |
| Car         | 76.67 | 83.33 | 84.67 | 92.50  | 91.67 | 90.00 | 86.67 | 85.00 |
| CBF         | 99.44 | 99.78 | 99.33 |        |       |       |       |       |
| ChConc      | 65.00 | 66.09 | 63.39 | 84.36  | 69.97 | 72.71 | 71.20 | 72.06 |
| CinCETCGT   | 93.04 | 88.70 | 93.45 | 82.61  | 95.43 | 99.49 | 99.64 | 95.00 |
| Coffee      | 100.00| 100.00| 100.00| 100.00 | 96.43 | 100.00| 100.00| 100.00|
| Comp        | 62.40 | 75.60 | 64.44 | 81.48  | 73.60 | 74.00 | 76.00 | 71.20 |
| CricketX    | 77.95 | 73.50 | 80.21 | 79.13  | 77.18 | 80.77 | 82.31 | 79.74 |
| CricketY    | 75.64 | 75.38 | 79.38 |        | 77.95 | 82.56 | 84.87 | 80.26 |
| CricketZ    | 73.59 | 74.62 | 80.10 | 81.15  | 78.72 | 81.54 | 83.08 | 85.59 |
| DiSh2Red    | 93.46 | 93.14 | 96.57 | 93.13  | 92.48 | 92.81 | 94.12 | 97.71 |
| DiPhoAG     | 62.50 | 74.82 | 73.09 | 71.65  | 76.98 | 74.82 | 76.26 | 74.10 |
| DiPhoGC     | 72.46 | 72.83 | 79.28 |        | 77.10 | 77.54 | 76.09 | 77.17 |
| DiPhTW      | 63.31 | 67.63 | 65.97 | 66.47  | 66.19 | 69.78 | 68.35 | 68.35 |
| Earthquake  | 72.66 | 74.82 | 75.40 | 71.15  | 74.10 | 74.82 | 74.82 | 74.82 |
| ECG200      | 88.00 | 87.00 | 90.90 | 87.40  | 83.00 | 88.00 | 85.00 | 86.00 |
| ECG5000     | 92.51 | 94.13 | 93.65 | 93.42  | 94.38 | 94.60 | 94.02 | 94.58 |
| ECG5D       | 79.67 | 100.00| 84.92 | 97.48  | 98.37 | 99.88 | 100.00| 100.00|
| ElectDev    | 63.08 | 79.92 | 70.60 | 72.91  | 74.70 | 71.33 | 77.03 | 75.24 |
| FaceAll     | 80.77 | 78.17 | 89.38 | 83.88  | 77.87 | 91.78 | 80.30 | 84.26 |
| FaceFour    | 89.77 | 100.00| 97.39 | 95.45  | 85.23 | 89.77 | 95.45 | 100.00|
| FacesUCR    | 90.78 | 95.71 | 94.59 | 95.47  | 90.59 | 94.24 | 96.29 | 96.49 |
| 50Words     | 76.48 | 70.55 | 83.14 | 73.96  | 70.55 | 79.78 | 80.99 | 84.62 |
| Fish        | 83.43 | 98.86 | 93.49 | 97.94  | 98.86 | 98.29 | 98.86 | 99.43 |
| FordA       | 66.52 | 92.95 | 85.46 | 92.05  | 97.12 | 95.68 | 94.44 | 94.70 |
| FordB       | 59.88 | 71.11 | 71.49 | 91.49  | 80.74 | 87.37 | 82.35 | 83.21 |
| GunPoint    | 91.33 | 100.00| 99.73 | 99.1   | 100.00| 100.00| 100.00| 100.00|
| Ham         | 60.00 | 66.67 | 66.00 | 75.71  | 68.57 | 64.76 | 66.67 | 71.43 |
| HandOut     | 87.84 | 90.27 | 92.14 | 91.11  | 93.24 | 91.89 | 93.24 | 92.97 |
| Haptics     | 41.56 | 46.10 | 44.45 | 51.88  | 52.27 | 52.27 | 51.95 | 51.62 |
| Herring     | 53.13 | 54.69 | 57.97 | 61.88  | 67.19 | 62.50 | 68.75 | 57.81 |
| InSlSkate   | 38.73 | 51.64 | 54.18 | 37.31  | 37.27 | 49.45 | 50.00 | 53.64 |
| InSwSd      | 57.37 | 52.32 | 61.87 | 50.65  | 62.68 | 65.25 | 65.51 | 64.65 |
| ItPwDem     | 95.53 | 90.86 | 96.71 | 96.30  | 94.75 | 96.11 | 96.31 | 97.18 |
| LbKtApp     | 79.47 | 76.53 | 78.19 | 89.97  | 85.87 | 84.53 | 86.40 | 78.93 |
| Light2      | 86.89 | 83.61 | 86.56 | 77.05  | 73.77 | 86.89 | 81.97 | 77.05 |
| Light7      | 71.23 | 68.49 | 82.19 | 84.52  | 72.60 | 80.82 | 73.97 | 75.34 |
| Mallat      | 91.43 | 95.82 | 95.76 | 97.16  | 96.42 | 95.39 | 96.20 | 97.74 |
| Meat        | 93.33 | 90.00 | 93.33 | 96.83  | 85.00 | 91.67 | 93.33 | 90.00 |
| MdImg       | 74.74 | 71.84 | 75.82 | 77.03  | 66.97 | 75.79 | 77.76 | 79.74 |

Continued on next page
| Dataset   | DTW  | BOSS | FF   | ResNet | ST   | FCT  | HCT  | CHIEF |
|-----------|------|------|------|--------|------|------|------|-------|
| MdPhOAG   | 54.95| 53.55| 56.23| 90.88  | 64.29| 63.64| 59.74| 59.09 |
| MdPhOC    | 76.63| 78.01| 83.64| 80.89  | 79.38| 80.41| 83.16| 85.22 |
| MdPhTW    | 50.65| 54.55| 52.92| 48.44  | 51.95| 57.14| 57.14| 55.84 |
| MtStrain  | 86.58| 87.86| 90.24| 92.76  | 89.70| 93.69| 93.29| 94.41 |
| NoECGT1   | 82.90| 83.82| 90.66| 94.54  | 94.96| 93.13| 93.03| 90.74 |
| NoECGT2   | 87.02| 90.08| 93.99| 94.61  | 95.11| 94.55| 94.45| 94.45 |
| OliveOil  | 86.67| 86.67| 86.67| 83.00  | 90.00| 90.00| 90.00| 92.00 |
| OSULeaf   | 59.92| 95.45| 82.73| 97.85  | 96.69| 96.69| 97.93| 98.76 |
| PhalanxOC | 76.11| 77.16| 82.35| 83.90  | 76.34| 77.04| 80.65| 84.85 |
| Phoneme   | 22.68| 26.48| 32.01| 33.43  | 32.07| 34.92| 38.24| 36.08 |
| Plane     | 100.0| 100.0| 100.0| 100.0  | 100.0| 100.0| 100.0| 100.0 |
| PrxPhOAG  | 78.54| 83.41| 84.63| 85.32  | 84.39| 85.37| 85.85| 84.88 |
| PrxPhOC   | 79.04| 84.88| 87.32| 93.13  | 88.32| 86.94| 87.97| 89.69 |
| PrxPhTW   | 76.10| 80.00| 77.90| 78.05  | 80.49| 78.05| 81.46| 81.46 |
| RefDev    | 44.00| 49.87| 53.23| 52.53  | 58.13| 54.67| 55.73| 53.87 |
| ScrType   | 41.07| 46.40| 45.52| 62.16  | 52.00| 54.67| 58.93| 50.40 |
| ShipSim   | 69.44| 100.0| 77.61| 77.94  | 95.56| 96.11| 100.0| 100.0 |
| ShipAll   | 80.17| 90.83| 88.58| 92.13  | 84.17| 89.17| 90.50| 93.00 |
| SKitApp   | 67.20| 72.53| 74.43| 78.61  | 79.20| 77.60| 85.33| 81.60 |
| SonyRS1   | 69.55| 63.23| 84.58| 95.81  | 84.36| 84.53| 76.54| 82.70 |
| SonyRS2   | 85.94| 85.94| 89.63| 97.78  | 93.39| 95.17| 92.76| 92.86 |
| StarCurv  | 89.83| 97.78| 98.13| 97.18  | 97.85| 97.96| 98.15| 98.20 |
| Strwbe    | 94.59| 97.57| 96.84| 98.05  | 96.22| 95.14| 97.03| 96.76 |
| SwdLeaf   | 84.64| 92.16| 94.66| 95.63  | 92.80| 95.52| 95.36| 96.64 |
| Symbols   | 93.77| 96.68| 96.16| 90.64  | 88.24| 96.38| 97.39| 97.99 |
| SynCtl    | 98.33| 96.67| 99.53| 99.83  | 98.33| 100.0| 99.67| 100.0 |
| ToeSeg1   | 75.00| 93.86| 92.46| 96.27  | 96.49| 97.37| 98.25| 96.93 |
| ToeSeg2   | 90.77| 96.15| 86.23| 90.62  | 90.77| 91.54| 95.38| 95.38 |
| Trace     | 99.00| 100.0| 100.0| 100.0  | 100.0| 100.0| 100.0| 100.0 |
| 2LeadECG  | 80.83| 98.07| 98.86| 100.0  | 99.74| 99.30| 99.65| 99.65 |
| 2Pttrns   | 99.85| 99.30| 99.96| 99.99  | 95.50| 100.0| 100.0| 100.0 |
| UWAAll    | 96.23| 95.89| 97.23| 85.95  | 94.22| 96.43| 96.85| 96.87 |
| UWAx      | 77.44| 76.21| 82.86| 78.05  | 80.29| 82.19| 83.98| 84.17 |
| UWAy      | 70.18| 68.51| 76.15| 67.01  | 73.03| 75.85| 76.55| 77.16 |
| UWAz      | 67.50| 69.49| 76.40| 75.01  | 74.85| 75.04| 78.31| 77.97 |
| Wafer     | 99.59| 99.48| 99.55| 99.86  | 100.0| 99.98| 99.94| 99.99 |
| Wine      | 61.11| 74.07| 56.85| 74.44  | 79.63| 64.81| 77.78| 88.89 |
| WordSyn   | 74.92| 63.79| 77.87| 62.24  | 57.05| 75.71| 73.82| 78.68 |
| Worms     | 53.25| 55.84| 71.82| 79.09  | 74.03| 62.34| 55.84| 79.22 |
| Worms2C   | 58.44| 83.12| 78.44| 74.68  | 83.12| 80.52| 77.92| 81.82 |
| Yoga      | 84.30| 91.83| 87.86| 87.02  | 81.77| 87.67| 91.77| 84.83 |
| Avg_Rank  | 6.99 | 5.41 | 4.81 | 4.34   | 4.82 | 3.78 | 2.93 | 2.92  |
| Wins      | 3    | 12   | 9    | 18     | 14   | 12   | 24   | 33    |

Table 1 – continued from previous page
In this experiment, we selected the hyper-parameters as follows: number of similarity-based splitters $C_s = 5$ (in blue), number of dictionary-based splitters $C_d = 100$ (in orange), and the number of interval-based splitters $C_i = 100$ (in green). We ran this experiment with $k = 10$ trees to evaluate the fraction of training time used by each splitter type.