Response relation in Pb-Pb and p-Pb collisions at 5.02 TeV

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We carry out simulations using A Multi-Phase Transport (AMPT) model to describe the collective flow in Pb-Pb and p-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV, respectively. Based on event-by-event AMPT simulations, we study the response relation between $v_2$ and $v_2$. We found that the contribution of non-flow effects of response relation can be neglected in Pb-Pb systems but can not be neglected in p-Pb systems. Without non-flow effects, the results of hydrodynamic response where in Pb-Pb collisions and p-Pb collisions are very similarly. Furthermore, we study the differential response relation both in Pb-Pb collisions and p-Pb collisions. The differential figures show that the response relation as a function of the transverse momentum, pseudorapidity and hadron cascade time, respectively.

I. INTRODUCTION

One remarkable achievement in high-energy heavy-ion experiments is the creation of a fluid-like quark-gluon system—the Quark-Gluon Plasma (QGP). The Collective flow behavior is found in the azimuthal correlations of particles emitted in relativistic nucleus-nucleus collisions at the BNL RHIC [1] and at the CERN LHC [2–4]. These correlations, which are long-range in pseudorapidity, suggest the formation of a strongly interacting QGP that exhibits nearly ideal hydrodynamic behavior [5]. In recent years, similar long-range collective azimuthal correlations have also been observed in events with high final-state particle multiplicity in proton-proton [6, 7], proton-nucleus [8–12], and lighter nucleus-nucleus collisions [13], where it naturally raises the question that whether a fluid-like QGP is created in these much smaller systems [14].

Anisotropic flow, which quantifies the degree of anisotropic azimuthal correlations of final state particles, is found to be very sensitive both to the event-by-event fluctuating initial geometry of the overlap region of colliding nucleons and to the transport properties and its equation of state [5, 15, 16]. One character of the azimuthal anisotropy of the generated particle spectrum with respect to the probability distribution of particles in momentum space is the harmonic flow $v_n$ [17], as

\[ V_n = v_n e^{i n \Psi_n} \equiv \int \frac{d\phi}{2\pi} e^{i n \phi} f(\phi_p). \]  

where the magnitude $v_n$, and phase $\Psi_n$, are fluctuate on event-by-event basis. And the initial eccentricity $\mathcal{E}_n$, which is defined with respect to the initial-state energy density profile $\rho(\vec{x}_\perp, \tau_0)$ as [18, 19]

\[ \mathcal{E}_n = \varepsilon_n e^{i n \Phi_n} \equiv -\frac{\int d^2 \vec{x}_\perp \rho(\vec{x}_\perp, \tau_0) r^n e^{i n \phi}}{\int d^2 \vec{x}_\perp r^n \rho(\vec{x}_\perp, \tau_0)} . \]  

On hydrodynamic figure, one possibility is that fluctuation-driven asymmetries in the initial-state nucleon locations within the overlap region are transferred to the final-state particle distributions through the hydrodynamic evolution of an hot density expanding plasma [20, 21]. Since $\mathcal{E}_n$ is defined, harmonic flow $V_n$ can be expanded with respect to $\mathcal{E}_n$, and a series of response relation can be obtained. A recent review on these response relation can be found in Ref. [22]. It has also been show that the statistics of fluctuation-driven initial-state anisotropies in proton-proton, proton-nucleus and nucleus-nucleus collisions is to a large extent universal [20, 21, 23].

However, the origin of the strong collectivity in small collision systems is still under debate (see Ref. [24] for a review of experimental and theoretical status). It has been argued that the system size is too small and life-time is too short for the matter in small system to hydrodynamize and approach local isotropization [25]. Quantitative hydrodynamic predictions of azimuthal correlations in pp and p-Pb systems at low-multiplicities still have large uncertainties, mainly due to the limited knowledge of initial-state fluctuations of energy density deposition at sub-nucleonic scales, and the results are seriously contaminated by non-collective correlations (non-flow effects) out of the hydrodynamic picture [26, 27].

One of the challenges in the study of azimuthal correlations for small collision systems is how to distinguish the long-range ridge from significantly non-flow correlations involving only a few particles, such as jets, resonance decays, and Bose-Einstein correlations between identical particles [28]. In order to suppress and explore these non-flow correlations and possible collective correlation signals in the low-multiplicity region in small systems, subevent cumulant techniques with a rapidity gaps among particles were proposed [12, 29–31].

Though the non-flow contribution in cumulant calculations had strongly suppressed by the subevent techniques, but the role of non-flow response in the small system is not yet studied. The main purpose of this paper is to address a particular picture on response theory in p-Pb systems: To extract the response relation and to compare with Pb-Pb systems.

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II. A MULTI-PHASE TRANSPORT (AMPT) MODEL

The AMPT model is a Monte Carlo transport model for high-energy heavy ion collisions [32]. In AMPT model, the initial-state particle distributions are given by the Heavy Ion Jet Interaction Generator (HIJING) model [33]. For the current study, string melting is considered so that the produced hadrons from HIJING model are further converted into valence quarks and antiquarks. Right before parton scatterings, we record the generated energy density profile of the system \(\rho(x, \tau_0)\), as the initial state of medium evolution. Initial state eccentricities of each event are then calculated with respect to Eq. (2). Parton scatterings, and accordingly the space-time evolution of QGP, are determined via ZPC parton cascade model [34]. In AMPT model, quarks and antiquarks combine to form hadrons via a spatial coalescence model when scatterings stop. Hadronic phase of the system evolves according to a relativistic transport model until hadrons freeze-out. More thorough discussions of the AMPT model can be found in Ref. [32].

In this paper, in addition to parameters that control the Lund string fragmentation, \(a = 0.5\) and \(b = 0.9\) GeV\(^{-2}\), \(\alpha_s = 0.33\) and \(\mu = 3.2\) fm\(^{-1}\) are found in Ref. [19]. This paper is organized as follows: In Section II we briefly describe the AMPT model which used in the present simulations. The numerical results of flow and non-flow response for \(v_2\) is presented in Section III, where we emphasize the non-flow effects in small system. Finally, we will summarize the main results in Section IV. Throughout this paper, our results and analyses are mostly obtained based on AMPT simulations with respect to Pb-Pb and p-Pb collisions at \(\sqrt{s_{NN}} = 5.02\) TeV. We use natural unit \(k_B = c = \hbar = 1\).

III. RESPONSE RELATION IN AMPT

A. Flow response relation

In recent experiments at the LHC energies, more sophisticated measurements of elliptic flow have been carried out, revealing the fluctuating nature of \(v_2\) (cf. Ref. [36]). To estimate the fluctuation effect of \(v_2\), AMPT results had been completed for the ratio of \(v_2\{6\}/v_2\{4\}\) and ratio \(v_2\{8\}/v_2\{4\}\) as a function of centrality percentile on Pb-Pb collisions at \(\sqrt{s_{NN}} = 2.76\) TeV and 5.02 TeV, respectively [19]. The information of \(v_2\) fluctuations can be as well captured by skewness. Given \(v_2\{2\}, v_2\{4\}\) and \(v_2\{6\}\), one may estimate the standardized skewness as [35]

\[
\gamma_{\text{exp}} = -6\sqrt{2}v_2\{4\}^2 \frac{v_2\{4\} - v_2\{6\}}{[v_2\{2\}^2 - v_2\{4\}^2]^{3/2}}. \quad (3)
\]

Fig. 1 depicts the estimated standardized skewness of \(v_2\) fluctuations from our AMPT simulations, as colored bands. Note that, the width of the band is determined by statistical errors via a jackknife resampling. AMPT results from Ref. [19] are agree well with the recent experimental data (red points) for Pb-Pb collisions. One observes a negative value of the skewness, with its magnitude increases as centrality percentile grows. Similar result of p-Pb collisions is also shown in the figure. Note that the negative skewness of \(v_2\) fluctuations is understood as a consequence of the negative skewness of \(\varepsilon_2\), due to the combined effect of an upper bound \(\varepsilon_2 < 1\) and a non-zero mean of ellipticity in the reaction plane.

The compatibility of the fluctuation of \(v_2\) and the fluctuation of \(\varepsilon_2\) implies that a simple response relation where between the \(V_2\) and \(\varepsilon_2\) has been studied [19, 37],

\[
V_2 = \kappa_2 \varepsilon_2 + \kappa_2' \varepsilon_2^2 \varepsilon_2 + \delta_2. \quad (4)
\]

Owing to the condition of rotational symmetry, the leading order term is a linear response proportional to \(\varepsilon_2\), with \(\kappa_2\) the linear response coefficient determined by medium dynamical expansion. The next leading order contribution is of cubic order, and is dominantly determined by \(O(\varepsilon_2^3)\). By minimizing the effect of additional event-by-event fluctuations \(\delta_2\), one solves \(\kappa_2\) and \(\kappa_2'\), which is called the unsuppress non-flow effects (USNF) [19],

\[
\kappa_2 = \text{Re} \left( \frac{\langle \varepsilon_2^6 \rangle (V_2 \varepsilon_2^*) - \langle \varepsilon_2^4 \rangle (V_2 \varepsilon_2^2 \varepsilon_2^*)}{\langle \varepsilon_2^2 \rangle (\varepsilon_2^2) - \langle \varepsilon_2^2 \rangle^2} \right), \quad (5a)
\]

\[
\kappa_2' = \text{Re} \left( -\frac{\langle \varepsilon_2^4 \rangle (V_2 \varepsilon_2^2) + \langle \varepsilon_2^2 \rangle (V_2 \varepsilon_2^4 \varepsilon_2^2)}{\langle \varepsilon_2^2 \rangle (\varepsilon_2^2) - \langle \varepsilon_2^2 \rangle^2} \right). \quad (5b)
\]
where bracket \( \langle \ldots \rangle \) indicates the particles are average over events.

Although hydrodynamic response of Pb-Pb collisions are established in the AMPT modelings [19]. In particular, one notices that non-flow effects result in additional event-by-event fluctuations, influencing the response relation which is not characterized in small system by AMPT. Based on AMPT model [32] simulations, we re-examine the large and the small system response relation between \( V_2 \) and \( \varepsilon_2 \), respectively. In the present setup of AMPT simulations, for the Pb-Pb collisions on 40-50% centrality class and the p-Pb collisions at \( b= 0-3 \) fm, we focus on the collisional energy at \( \sqrt{s_{NN}} = 5.02 \) TeV. We generate approximately \( 10^4 \) events for Pb-Pb collisions and \( 3 \times 10^5 \) events for p-Pb collisions in the AMPT simulations. To avoid the large fluctuations of the few-particles correlation, we focus on the charged-particle multiplicity which is larger than 50. A scatter plot of \( v_2 \) versus \( \varepsilon_2 \) is obtained and shown in Fig. 2. We take \( \varepsilon_3 \) from 0.0 to 0.80, with size as \( \Delta \varepsilon_3=0.2 \) in each bin. Fig. 2 shows an event-by-event \( v_2 \) from AMPT simulations for Pb-Pb and p-Pb collisions, as a function of \( \varepsilon_2 \). Each point in Fig. 2 corresponds to one collision event. It is worth mentioning that, the statistical uncertainty of \( v_2 \) in each event due to finite multiplicity is not included, which would in principle lead to a smearing along \( v_2 \) in Fig. 2. In the up panel of Fig. 2, the distribution of scatter points are concentrated in the middle values of \( \varepsilon_3 \), due to the fluctuation-driven asymmetries initial states. It found that a response relation between magnitudes derived from Eq. (4), describing well the trend of Pb-Pb systems,

\[
v_2 = \kappa_2 \varepsilon_2 + \kappa'_2 \varepsilon_2^3.
\]  

Given these solved values of \( \kappa_2 \) and \( \kappa'_2 \) according to Eq. (5), Eq. (6) is plotted in the up panel of Fig. 2 as the white dashed line. It should be emphasized that the white dashed line is not fitting the scattering points, but determined with respect to the solved values of \( \kappa_2 \) and \( \kappa'_2 \) according to Eq. (5). By Eq. (6), similar points distribution of p-Pb collisions are shown in the down panel of Fig. 2. One can see that, a larger fluctuation in p-Pb systems appears compared with that in Pb-Pb systems. Note that the evolution of expanding plasma from fluctuation-driven asymmetries initial-state to the final-state particle distributions is hydrodynamically in the Pb-Pb systems, but non-hydrodynamic effects (named non-flow effects) can not to be neglected in the p-Pb systems.
B. Non-flow response relation

To subtract non-flow effects in the harmonic flow, one may either take a pseudo-rapidity gap or rely on multi-particle cumulants. Both methods have been applied extensively in experiments [38–40]. Therefore, based on the response relation in Eq. (4), we find from the two-particle and four-particle correlations, which is called the suppress non-flow effects (SNF) [19],

\[ v_2\{2,|\Delta \eta|\} = \kappa_2 \varepsilon_2\{2\} \left[ 1 + \frac{\kappa_2'}{\kappa_2} \left( \frac{\varepsilon_2^4}{\varepsilon_2^2} \right) \right], \quad (7a) \]

\[ v_2\{4\} = \kappa_2 \varepsilon_2\{4\} \left[ 1 + \frac{\kappa_2'}{\kappa_2} \frac{2\langle \varepsilon_2^2 \rangle^2 - \langle \varepsilon_2^6 \rangle}{2\langle \varepsilon_2^2 \rangle^2 - \langle \varepsilon_2^4 \rangle} \right]. \quad (7b) \]

In writing Eq. (7), we have assumed that a pseudo-rapidity gap is sufficient to take out the non-flow contribution, i.e., \( \delta_2 \), in \( v_2\{2\} \). In practice, a pseudo-rapidity gap \(|\Delta \eta| > 1\) is taken into account in our AMPT simulations for \( v_2\{2,|\Delta \eta|\} \). Similarly, \( \delta_2 \) does not appear in \( v_2\{4\} \). Equation (7) then allows us to solve \( \kappa_2 \) and \( \kappa_2' \), without non-flow effects, as

\[ \kappa_2 = \frac{v_2\{2,|\Delta \eta|\} \cdot B - v_2\{4\} \cdot A}{\varepsilon_2\{2\} \cdot B - \varepsilon_2\{4\} \cdot A}, \quad (8a) \]

\[ \kappa_2' = \frac{-(v_2\{2,|\Delta \eta|\} \varepsilon_2\{4\} - v_2\{4\} \varepsilon_2\{4\})}{\varepsilon_2\{2\} \cdot B - \varepsilon_2\{4\} \cdot A}. \quad (8b) \]

where

\[ A = \langle \frac{\varepsilon_2^4}{\varepsilon_2^2} \rangle \cdot \varepsilon_2\{2\}, \quad B = \frac{2\langle \varepsilon_2^2 \rangle^2 - \langle \varepsilon_2^6 \rangle}{2\langle \varepsilon_2^2 \rangle^2 - \langle \varepsilon_2^4 \rangle} \cdot \varepsilon_2\{4\}. \]

We had been studied the response coefficient which relates elliptic flow \( v_2 \) and initial ellipticity \( \varepsilon_2 \) for Pb-Pb collisions at \( \sqrt{s_{NN}} = 2.76 \) TeV via AMPT simulations [19]. Although the strategy is very similar to hydrodynamic simulations, with the response coefficient obtained through minimizing event-by-event fluctuations, AMPT simulations contain non-flow effects. These non-flow effects are beyond pure hydrodynamic calculations, including, e.g., short-ranged correlations in the deconfined medium from particle scatterings, hence they are non-hydrodynamic. Nonetheless, considering the fact that response relations are hydrodynamic and are dominated by the evolution of long-wavelength modes of the medium system in Pb-Pb collisions, one would expect that the response relation barely dependent on the non-flow effects.

In order to better to understand the relationship between the strong suppression of non-flow effects and hydrodynamical medium in p-Pb system, differential response relation are introduced in Fig. 3 and Fig. 4, respectively. To test those simulations, we fix parameters where \( \varepsilon_3 \in [0.2, 0.6] \), for Pb-Pb collisions on 40-50% centrality and p-Pb collisions on b=0-3 fm, respectively.

Fig. 3 shows that the response relation \( \kappa_2 \) and \( \kappa_2' \) as a function of the transverse momentum, from AMPT simulations of Pb-Pb (40-50%) and p-Pb (b = 0-3 fm) collisions at \( \sqrt{s_{NN}} = 5.02 \) TeV, respectively. The results of \( \kappa_2 \) and \( \kappa_2' \) are calculated according to the USNF and the SNF method, respectively. (a) for Pb-Pb systems; (b) for p-Pb systems.
state to the final-state particle distributions is significantly affected by the non-flow effects. On the other hand, initial fluctuation-driven asymmetries in the viscosity deconfined medium from few-particle scatterings which enhance non-hydrodynamic correlation. By using the SNF method, similarly results of the hydrodynamic viscosity deconfined medium from few-particle scatterings hand, initial fluctuation-driven asymmetries in the viscosity fluctuations affect the results over the full simulated pseudorapidity range.

In Pb-Pb collisions, the collective flow mainly develops in the QGP phase \[44\] and further accumulated in the hadronic stage \[45\]. To determine the fluid-like evolution of small system, simulation on the response relation for different hadron evolution time are adopted. The \( \kappa_2 \) and \( \kappa'_2 \) as a function of the hadron evolution time, from AMPT simulations of p-Pb (b = 0-3 fm) collisions at \( \sqrt{s_{NN}} = 5.02 \) TeV, are shown in Fig. 5, respectively. One can see that, \( \kappa_2 \) is weakly dependence on the mechanism of hadrons cascade, and \( \kappa'_2 \) is increases with the hadron cascade time increasing. The non-zero response relation by SNF method implies that the contribution of flow is dominated by the stage of medium expansions. The narrowing of the non-flow effects of hadrons decay, which plays a minor role in the response relation.

IV. SUMMARY

In this work, we have carried out AMPT simulations for the Pb-Pb collisions and p-Pb collisions at the LHC energy \( \sqrt{s_{NN}} = 5.02 \) TeV. In the event-by-event AMPT simulations, we found that the contribution of non-flow effects on the response relation in Pb-Pb systems are different from in p-Pb systems. There had confirmed the fact that elliptic flow \( v_2 \) in large system is indeed a consequence of medium response to initial state \( \varepsilon_2 \). Even though event-by-event fluctuations are non-hydrodynamic, as they are not sensitive to dissipative effect of the hot density medium system, they do not affect the response relation. However, these non-hydrodynamic fluctuations will induce a large dissipative effect on small medium system. As a consequence, the response relation

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FIG. 4. (Color online) Response coefficient \( \kappa_2 \) and \( \kappa'_2 \) as a function of the pseudorapidity, for Pb-Pb(40-50 %) and p-Pb(b = 0-3 fm) collisions at \( \sqrt{s_{NN}} = 5.02 \) TeV, respectively. The results of \( \kappa_2 \) and \( \kappa'_2 \) are calculated according to the SNF method.

FIG. 5. (Color online) Response coefficient \( \kappa_2 \) and \( \kappa'_2 \) as a function of the hadron evolution time, for p-Pb collisions at \( \sqrt{s_{NN}} = 5.02 \) TeV. The results of \( \kappa_2 \) and \( \kappa'_2 \) are calculated according to the USNF and the SNF method, respectively.
are significantly affected by these dissipative effects.
Furthermore, our results show that the response relation as a function of transverse momentum, pseudorapidity and hadron evolution time, respectively.
Together, the contribution of non-flow effects of response relation can be neglected in Pb-Pb systems but cannot be neglected in p-Pb systems. Without non-flow effects, the results of hydrodynamic response where in Pb-Pb collisions and p-Pb collisions are very similarly.

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