Simulation of non-destructive inspections and acoustic emission measurements involving guided waves

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Abstract. In a structure that guides elastic waves, a discontinuity (defect, shape variation) causes scattering (reflection, partial extinction or mode conversion). Two modal formulations have been developed to link separate models dealing with the calculation of the modal decomposition, with the generation and reception of guided waves (GW), with their scattering. The first concerns pulse-echo configurations (involving a single transducer), the other concerns pitch-catch configurations (two transducers involved). A new finite element (FE) method has been developed to compute the scattering by an arbitrary discontinuity, based on the modal decomposition of the field. Perfectly transparent boundary conditions (Dirichlet-to-Neuman boundaries) are developed, allowing the FE computation zone to be reduced to a minimum. A specific variational problem including these boundary conditions was obtained and solved using FE tools. By combining the modal formulations, the new FE scheme and tools for GW radiation, propagation and reception based on the Semi-Analytical Finite Element (SAFE) method, a new simulation tool has been developed. It can address almost arbitrary configurations of GW nondestructive testing. Moreover, a source inside the FE computation zone can be defined so that configurations of testing by acoustic emission can also be simulated. Examples of use of this tool are shown, some dealing with junctions of complex geometry between two guides, other with surface or bulk sources of acoustic emission.

1. Introduction
The simulation of a complete inspection (a configuration of nondestructive testing) by elastic guided waves requires specific models to describe \textit{i}) the radiation and reception of guided waves by a transducer, \textit{ii}) their propagation and \textit{iii}) their scattering by a flaw or by a variation of the guide shape (a guide discontinuity subsequently). Two theoretical formulations have been proposed \cite{1} allowing these phenomena to be computed separately. They result in two overall formulae linking independent models of the various phenomena; the first (resp. second) concerns pulse-echo (resp. pitch-catch) configurations. These formulations are based on a modal description of wave phenomena in the guide. In \cite{1}, the method has been derived to deal with waveguides of arbitrary cross section, but the model for computing scattering could only take into account guide discontinuities in the form of planar defects normal to the guide axis.
The object of the present work is to show how the method has been extended to arbitrary discontinuities of the guide. The difficulty is therefore concentrated on the computation of the scattering matrix (such a matrix appears in the two formulae) that links incident modes to either transmitted or reflected modes. An original finite element formulation (FE) has been proposed in [2]; transparent boundary conditions based on modal expansions were derived and used on each side of the discontinuity. This leads to computation zones which can be of very small extent, all the smaller since the number of evanescent modes taken into account is large; this constitutes the main advantage of the method in terms of performances.

The present paper aims at presenting how the various models and formulations can be used together to predict results of nondestructive examination using guided waves. It is organized as follows. First, the various theoretical tools making it possible to predict such results are reviewed: some general results concerning guided modes are briefly recalled, the main point being the biorthogonality relation extensively used subsequently, then the overall modal formulations are described; the FE formulation for computing the scattering of guided waves by an arbitrary scatterer is also presented. Then, examples of results for various configurations are given for demonstrating the versatility of the tools developed. Since source terms can also be included inside or at the surface of the FE computation zone as second members of the system of equations to be solved, the same tools can also be used to model Acoustic Emission testing configurations where transducers are passive; it implies to also model acoustic emission by a defect under stress, which is done here in a very simplified manner. The question of transducer diffraction effects is not addressed here. It is solved in a number of publications for transducers acting at the guiding surface; for transducers acting from the guide section, an original account of typical effects and of the modeling approach to compute them may be found in [3].

2. Theory
2.1. Some results concerning guided waves of specific interest

We consider an isotropic elastic waveguide of section $S$ in the $x_S = (x_1, x_2)$ plane and of axis $x_3$ with a stress-free boundary. The density $\rho$ and stiffness coefficients may depend on $x_S$. The propagation in $\Omega$ is modelled by the following classical equations (where $\mathbf{u}$ denotes the displacement field and $\omega > 0$ the angular frequency):

$$
\begin{align*}
- \text{div} \sigma(\mathbf{u}) + \omega^2 \rho \mathbf{u} &= 0 \quad \text{in } \Omega, \\
\sigma(\mathbf{u}) \cdot \mathbf{v} &= 0 \quad \text{on } \partial \Omega,
\end{align*}
$$

where $\sigma(\mathbf{u})$, the stress tensor, is related to the strain tensor $\varepsilon(\mathbf{u}) = (\nabla \mathbf{u} + \nabla^T \mathbf{u}) / 2$ by Hooke’s law and $\mathbf{v}$ denotes the outward unitary normal to $\partial \Omega$. It is assumed in equation (1) that there is no source terms in $\Omega$ or on $\partial \Omega$. Because the guide may be described as an arbitrary section $S$ translated along an axis ($x_3$ axis), new notations are more suitable: we denote by $\mathbf{u}_S$, $\mathbf{t}_S$, $\sigma_S$, and $\varepsilon_S$ the transverse part of the displacement field $\mathbf{u}$, the normal stress $\sigma(\mathbf{u}) \cdot e_3$, the stress tensor, and the strain tensor, respectively:

$$
\begin{align*}
\mathbf{u}_S &= \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, \\
\mathbf{t}_S &= \begin{pmatrix} \sigma_{31} \\ \sigma_{32} \end{pmatrix}, \\
\sigma_S &= \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}, \\
\varepsilon_S &= \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{21} & \varepsilon_{22} \end{pmatrix},
\end{align*}
$$

Introducing now the following vectors

$$
\begin{align*}
\mathbf{X} &= \begin{pmatrix} \mathbf{t}_S \\ \mathbf{u}_3 \end{pmatrix}, \\
\mathbf{Y} &= \begin{pmatrix} \mathbf{u}_S \\ \mathbf{t}_3 \end{pmatrix},
\end{align*}
$$

it is possible to re-write the elastodynamic system as an evolution problem (with respect to the coordinate $x_3$ of the waveguide) expressed for the new variables $\mathbf{X}$ and $\mathbf{Y}$ (by doing so, we extend the formulation proposed in [4, 5] to the general 3D case). On gets
where $F$ and $G$ are defined as follows, assuming subsequently that the elastic propagation medium is isotropic ($\lambda$ and $\mu$ denote the Lamé’s coefficients). The two operators $F$ and $G$ write

$$
F_Y = \begin{pmatrix}
-\text{div}_s \sigma_s(Y) - \alpha^2 \rho u_s \\
-\alpha \text{div}_s u_s - \alpha \tau_s / \lambda
\end{pmatrix},
G_X = \begin{pmatrix}
t_s / \mu - \nabla_s u_s \\
\text{div}_s t_s + \alpha^2 \rho u_s
\end{pmatrix},
$$

(5)

with $\sigma_s(Y) = (\delta \text{div}_s u_s - \alpha \tau_s) I + 2 \mu \varepsilon_s(u_s)$, and where $\delta = 2\lambda\mu / (\lambda + 2\mu)$ and $\alpha = \lambda / (\lambda + 2\mu)$. Since the vector normal to $\partial \Omega$, $\nu = (n_s, 0)$, is independent of the axial ($x_3$) coordinate, the boundary condition can be written equivalently in terms of two uncoupled boundary conditions in $X$ and $Y$:

$$
\sigma(u) \cdot \nu = 0 \text{ on } \partial \Omega \iff \begin{cases}
\sigma_s(Y) \cdot n_s = 0 \\
t_s \cdot n_s = X_1 n_1 + X_2 n_2 = 0
\end{cases} \text{ on } \partial S.
$$

(6)

Now, a solution of equation (4) can be sought in the form

$$
\begin{pmatrix}
X(x_s) \\
Y(x_s)
\end{pmatrix} e^{i\beta x_3}, \beta \in \mathbb{C},
$$

(7)

assuming $x_3$ separable of others coordinates $x_s$. This leads to the eigen-value problem that writes

$$
i \beta \begin{pmatrix}
X \\
Y
\end{pmatrix} = \begin{pmatrix}
0 & F \\
G & 0
\end{pmatrix} \begin{pmatrix}
X \\
Y
\end{pmatrix}.
$$

(8)

A so-called biorthogonality relation can be derived between two solutions $(\beta, X, Y)$ and $(\bar{\beta}, \bar{X}, \bar{Y})$, as demonstrated in due details in [2]

$$
\left(FGX|\bar{Y}\right)_S = \left(X|G\bar{Y}\right)_S \iff (\beta \bar{\beta} - \beta^2)(X|\bar{Y})_S = 0.
$$

(9)

In equation (9), the notation $\left(\begin{array}{c} X \\ Y \end{array}\right)_S$ stands for

$$
\left(X|Y\right)_S = \int_S (X_1 Y_1 + X_2 Y_2 + X_3 Y_3) dS = \int_S (t_s \cdot u_s + t_u u_s) dS.
$$

(10)

Equation (9) implies that the eigen-solution $(\beta, X, Y)$ is “orthogonal” to any other mode $(\bar{\beta}, \bar{X}, \bar{Y})$ except if $\beta = \pm \bar{\beta}$. Let $(\beta_n, X_n, Y_n)$ for $n \in \mathbb{N}$ denote the eigen-elements corresponding to right going modes, thus, $(\beta_n, X_n, Y_n) = (-\beta_n, -X_n, Y_n)$ for $n \in \mathbb{N}$ being the family of left going modes, according to Auld’s labeling convention [6]. One gets after suitable normalization:

$$
\left(X_m|Y_n\right)_S = \delta_{mn}.
$$

(11)

2.2. Modal formulations for GW nondestructive testing configurations

As it has been mentioned in the introduction of the present paper, two overall formulations to model NDT experiments using guided waves have been derived in [1] for the cases of a single transducer working in the transmit-receive mode and that of two separate transducers for the radiation and the reception. In these models, it is assumed that a waveguide is locally nonuniform, due to the presence of a guide discontinuity. The guide discontinuity is a finite volume $\Omega_d$ of the guide bounded by two surfaces $\Sigma$ and $\Sigma^+$ as shown by Figure 1. The first configuration results in an expression that is a function of reflection coefficients while the second is expressed as a function of transmission coefficients. Respective expressions for the amplitude received write
\[
\delta \Gamma_1 = \frac{-i\omega}{P} \sum_{n \in \mathbb{N}} \sum_{m \in \mathbb{N}} A'_n A'_m R_{mn} e^{i(\beta_n + \beta_m)\xi'_{ij}}, \quad \delta \Gamma_2 = \frac{-i\omega}{P} \sum_{n \in \mathbb{N}} \sum_{m \in \mathbb{N}} A'_n A'_m T_{mn} e^{i\beta_n \xi'_{ij}} e^{i\beta_m (L-x_3)},
\]
(12)

where \(L\) denotes the position of the receiver along the \(x_3\)-axis, the emitter being located at \(x_3 = 0\). \(x_3^\pm\) are the coordinates of the two boundaries \(\Sigma^\pm\). \(A'_n\) and \(A'_m\) are the amplitude of the various modes emitted (the sensitivity to the modes received) by the transmitting (receiving) transducer. \(T_{mn}\) and \(R_{nm}\) coefficients denote respectively the corresponding coefficients of reflection from the guide discontinuity and transmission through it, that is, the amplitude of the \(n^{th}\) mode reflected or transmitted for the \(m^{th}\) incident mode. In (12), it is assumed that the right waveguide in which waves are transmitted is the same as the left one in which the incident waves were propagating. Considering two different guides on each side of \(\Omega_d\) is however straightforward.

In what follows, a FE method is obtained to compute these coefficients, using the various formulae recalled in section 2.1.

2.3. A modal FE scheme for computing the scattering coefficients

The aim here is to derive a FE method based on the modal decomposition of elastic waves in the guide so that the scattering coefficients appearing in equations (12) are readily computed. The interest of doing so is that a finite element method can handle an arbitrary local configuration of scattering in \(\Omega_d\) between the two boundaries \(\Sigma^-\) and \(\Sigma^+\) at which the modal solutions for the uniform waveguide are properly defined. This implies that a solution is sought to define boundary conditions on \(\Sigma^-\) and \(\Sigma^+\) such that these surfaces are transparent for waves diffracted in \(\Omega_d\).

For a right-going mode incident on \(\Sigma^-\) given by at \(x = (x_s, x_3)\)

\[
u^{inc}(x) = u^\pm(x_s)e^{i\beta_n(x_3-x_3)},
\]
(13)

the diffracted field \(u^{diff}(x)\) [with \(u^{diff}(x) = u^{tot}(x) - u^{inc}(x)\)] is sought in the form of a sum of outgoing waves using the modal decomposition in the uniform waveguide, as

\[
u^{diff}(x) = \sum_{n \in \mathbb{N}} A^{diff \pm}_n u^{\pm n}(x_s)e^{\pm i\beta_n(x_3-x_3)},
\]
(14)

where \(A^{diff \pm}_n\) denotes the amplitude of the \(n^{th}\) mode of the diffracted field on \(\Sigma^\pm\). Then introducing the mixed variables \(X\) and \(Y\) for the diffracted field, these variables can be similarly expanded thanks to the modal decomposition. One gets:
Using the biorthogonality relation (11), it is possible to extract the amplitude $A_n^{\text{diff}}$ of the $n^{\text{th}}$ mode as

$$A_n^{\text{diff}} = \left( X_n^{\text{diff}} | Y_{\Sigma}^{\text{inc}} \right)_S$$

Introducing (16) into (15), one can express two coupling operators (similar to a DtN operator, for Dirichlet-to-Neuman) allowing mapping $X_{\Sigma}^{\text{diff}}$ on $Y_{\Sigma}^{\text{diff}}$ for the first or vice-versa for the second, as:

$$\begin{pmatrix} X_{\Sigma}^{\text{diff}} \\ Y_{\Sigma}^{\text{diff}} \end{pmatrix} = \sum_{n \in \mathbb{N}} A_n^{\text{diff}} \begin{pmatrix} X_n^{\text{diff}}(x_n) \\ Y_n^{\text{diff}}(x_n) \end{pmatrix} e^{i \beta (x_n-x_i)}.$$

Now, using one of these so-called boundary maps, one can formulate the diffraction problem in the bounded domain $\Omega_d$, where the boundary conditions on the artificial sections $\Sigma^\pm$ will be expressed by means of the chosen coupling operator. They correspond to an outgoing wave condition for the computational domain $\Omega_d$. This original condition as expressed as a modal formula is of the same nature as what is usually called a transparent boundary condition for bulk elastic waves.

The aim is to propose a variational formulation in displacement. To introduce the coupling operator – equation (17) – $X_Y$ which are mixed variables in displacement and stress and unknown at present stage, a further and last ingredient must be introduced. This is done in the form of a Lagrange multiplier corresponding to either $t_3$ or $t_3$ depending on the choice made for the boundary map ($X$ to $Y$ or $Y$ to $X$, respectively). Now, $X$ or $Y$ is known as a function of $u$ and of the Lagrange’s multiplier. Therefore, the work of external stresses on the artificial boundaries can be expressed within the variational formulation.

Scattering coefficients can now be defined in terms of reflection and transmission coefficients as

$$T_{nm} = \left( Y^{\text{tot}} | X_n \right)_n / \left( Y^{\text{inc}} | X_n \right)_n = (A_n^{\text{diff}} + \left( Y^{\text{inc}} | X_n \right)_n) / \left( Y^{\text{inc}} | X_n \right)_n,$$

$$R_{nm} = \left( Y^{\text{tot}} - Y^{\text{inc}} | X_n \right)_n / \left( Y^{\text{inc}} | X_n \right)_n = A_n^{\text{diff}} / \left( Y^{\text{inc}} | X_n \right)_n,$$

by choosing the $Y$ to $X$ boundary operator, and where

$$A_n^{\text{diff}} = \left( t_3^{\Sigma^+} X_n^{\text{diff}} \right)_n + \left( u_3^{\Sigma^+} X_n^{\text{diff}} \right)_n = \left( Y^{\text{diff}} | X_n \right)_n.$$

For the tangential stress components, this leads to

$$t^{\text{tot}} \frac{s^\Sigma}{s^\Sigma} - \delta_\Sigma t^{\text{inc}} \frac{s^\Sigma}{s^\Sigma} = \sum_{n \in \mathbb{N}} (A_n^{\text{diff}} + \delta_\Sigma A_n^{\text{inc}}) t_3^{\Sigma^+}.$$

with

$$\delta_\Sigma = \begin{cases} 0, & \text{for } \Sigma = \Sigma^+ \\ 1, & \text{for } \Sigma = \Sigma^- \end{cases} \text{ and } A_n^{\text{inc}} = \left( Y^{\text{inc}} | X_n \right)_n.$$

The third component of the displacement, required as a consequence of the choice of the $Y$ to $X$ boundary operator, is used to complete the formulation as we introduced the $t_3$ Lagrange multiplier. It similarly writes

$$u^{\text{tot}} \frac{s^\Sigma}{s^\Sigma} - \delta_\Sigma u^{\text{inc}} \frac{s^\Sigma}{s^\Sigma} = \sum_{n \in \mathbb{N}} (A_n^{\text{diff}} + \delta_\Sigma A_n^{\text{inc}}) u_3^{\Sigma^+}.$$

Equations (21, 23) are used to form the final (mixed) variational formulation where the unknowns are the displacement $u$ and the $t_3$ stress component. Readers interested in full details on the FE
formulation are referred to [2]. At this stage, the scattering by an arbitrary guide discontinuity can be predicted for an arbitrary incident field. The numerical scheme can only deal with a finite number of modes. All the propagative modes (a finite set) must be accounted for. The infinite sets of inhomogeneous or evanescent modes are truncated after they have been ordered; amplitude of these modes is a decreasing function of the distance to the scatterer that generates them. Thus, the larger their number accounted for in the formulation, the smaller the computation zone surrounding the scatterer or guide discontinuity. This has been numerically studied and shown in [7]. In our implementation, GW are computed using the Semi-Analytical Finite Element (SAFE) method, a general method based on the early work of Dong and Nelson [8] (infinite layer method as called by these authors) generalised by many authors and still under development to deal with complex cases.

3. Results for GW-NDT configurations

3.1. Introduction

In this first section giving and discussing numerical results, two configurations of NDT by guided waves are considered. The first is an example of application of industrial interest: it concerns the reflection and transmission of guided modes from and through the bonding of two identical plates. The second example deals with a part made of two plates of dissimilar thickness linked by a part of linearly varying section in between. This does not correspond to a specific industrial application; but this exemplifies simulation capabilities of our theoretical model and associated computation code to deal with complex configurations. The propagation of guided waves in several guides of different nature (geometry, material etc.) that are linked together can be predicted. In both cases, the parts considered are supposed to be sound in a first computation; in a second computation, the presence of a flaw (debonding, crack) is assumed. In these results, transducer diffraction effects (either in radiation or in reception) are not taken into account. Thanks to the overall formulations proposed in [1], such effects are easily accounted for by simple but specific amplitude coefficients $A'$ and $A''$.

3.2. Scattering by a bonded region

Figure 2 displays the first configuration considered. Two semi-infinite 2-mm-thick plates made of Aluminium (with bulk wave velocities $c_L = 6180 \text{ m.s}^{-1}$, $c_T = 3040 \text{ m.s}^{-1}$ and density $\rho = 2700 \text{ kg.m}^{-3}$) are bonded, either perfectly (first computation) or partially debonded (second).

![Figure 2](image_url)

**Figure 2.** Configurations: two 2-mm-thick plates are bonded (6-mm-overlap). This junction is undamaged in a first computation, partly debonded in a second. The grey region is meshed for the FE computation.

The bonded region is treated as a guide discontinuity, meshed by 27512 triangular elements for the computation of the local FE solution. The same modal solution computed by SAFE (though the classical Lamb wave solution could have been also efficiently computed) is used for both semi-infinite plates. In the frequency bandwidth 0.825 – 1.15 MHz considered, only three modes are propagative. Figure 3 shows the total field in the bond at the frequency of 0.825 MHz, assuming an A0 incident mode in the upper plate. Two components of the particle displacement are shown (2D computations in this example). The transparent boundary operator is applied on $\Sigma^{-}$ by taking into account 41 modes.
Figure 3. Total fields of particle displacement (x- and y- components in left and right columns, respectively) in the sound bonded region (top) and in the debonded region (bottom); an A0 incident mode is considered, at an excitation frequency of 0.825 MHz.

In these results, the various components in the two cases are not displayed at the same color scale; they should not be quantitatively compared (of course, quantitative results were obtained in the computations). The only interest in presenting them qualitatively here (with a color map maximizing the range between the smallest negative value and the largest positive value) is to show how the incident A0 mode is strongly scattered in the presence of the flaw. Full quantitative results are shown by figure 4 where reflection and transmission coefficients (amplitude of reflected and transmitted modes relatively to that of the incident) are given as functions of frequency in the absence or in the presence of the flaw, again assuming an A0 incident mode in the left plate.

Figure 4. Reflection from (left) and transmission through (right) the bonded zone of the various modes as functions of frequency (in MHz), relatively to an A0 mode incident on the bonded zone, in the absence (top) or presence (bottom) of partial debonding.

Note that by summing the square of the coefficients, one easily checks energy conservation relatively to that carried by the incident mode. Resonances and anti-resonances appear, depending on
the mode considered. Such results demonstrate that the prediction of coefficient behaviours cannot be intuitively sought but requires a proper numerical scheme, considering how their variation is complex in this limited bandwidth. Such results together with proper parametric studies (influence of the size, position and nature of the debonding) can be very helpful to design for a given part, the best configuration to maximise the sensitivity of the NDT method to defects sought.

3.3. Scattering by a variation of guide section

Figure 5 displays the second configuration. Two semi-infinite plates of different thicknesses (10- and 20-mm, respectively) are linked by a part of linearly varying thickness. This time, the constitutive material is steel (bulk wave velocities $c_L = 5960 \text{ m.s}^{-1}, c_T = 3260 \text{ m.s}^{-1}$ and density $\rho = 7930 \text{ kg.m}^{-3}$).

![Figure 5](image)

**Figure 5.** Configurations considered: a thin (10-mm) plate and a thick (20-mm) semi-infinite plate are linked by a part of linearly varying section. In the first computation, this part is sound whereas it is cracked in the second. The grey region is meshed for the FE computation.

The part of varying section is supposed to be undamaged in a first computation and contains a 4-mm-long surface-breaking crack normal to the outer surface in a second one. The intermediate region is again treated as a guide discontinuity; it is meshed by 8688 triangular elements for the computation of the local FE solution using the formulation described in the previous section. The same solution computed by SAFE is used for both semi-infinite plates; since they are of different thickness, the solution is computed over a bandwidth that includes the actual bandwidth of the modelled experience (0.165 – 0.265 MHz) for the incident modes in the thin plate and that (0.33 – 0.53 MHz, frequencies doubled) for dealing with the thick plate which is twice thicker than the first. Corresponding dispersion curves (phase and group velocities) are shown in figure 6.

![Figure 6](image)

**Figure 6.** Phase (left) and group (right) velocities [mm.µs] in the thin plate of the propagative modes as functions of the frequency for a 10-mm-thick plate.

Figure 7 shows the total field in the part of varying section at the frequency of 0.165 MHz,
assuming a S0 incident mode in the thin plate. Two components of the particle displacement are shown (2D computations were also made in this example). In the bandwidth considered, three (six) modes are propagative in the thin (thick) plate. The Dirichlet-to-Neuman transparent boundary operator on $\Sigma^\pm$ is applied by taking into account 41 modes. As in the previous case, color amplitude scales are not identical in the various field representations. Due to symmetry breaking in the presence of the crack and the associated abrupt change in cross-sectional area, images show that the total field is strongly affected in such a case as compared to the flawless case.

Full quantitative results are shown by figure 8 where reflection and transmission coefficients (relatively to that of the incident mode) are given in the absence or presence of the flaw. A S0 incident mode in the thin plate is assumed. The various scattering coefficients are given as functions of the frequency, for the three modes A0, A1 and S0 in the thin plate and the six modes (same as in the thin plate plus S1, S2 and A2) in the thick plate.

When the flaw is absent (top results in figure 8), the symmetry of the configuration together with the symmetric incident mode considered leads to null-valued reflection and transmission coefficients for antisymmetric modes, as expected. Again, it is easily checked that the energy conservation criteria are fulfilled by squaring then summing the various coefficients.

In the presence of the surface-breaking crack (bottom results in figure 8), the configuration loses its symmetry: antisymmetric modes are generated by the complex scattering phenomena occurring in the part of varying section that includes the crack. The behaviour is far more complex and requires a proper numerical scheme to be computed.
4. Early results for NDT by Acoustic Emission

4.1. Introduction

Acoustic Emission (AE) techniques of NDT are used in many industrial domains. Internal defects in pieces submitted to external stress can generate elastic waves. The use of several receivers to catch some of the energy released in the structure as elastic waves allows not only detecting but also locating defects by means of triangulation algorithms. Such algorithms require however the knowledge of the velocity of propagation of the various waves measured. In typical structures for which AE methods are used, waves propagate as guided waves (pressure vessels, pipes etc.), in the frequency bandwidth of the receivers typically used; therefore, the determination of the wavespeed to input in the algorithms for processing a given indication may be a difficult task. In practice, calibration tests are specifically made, one standard source for this being the brittle fracture of a pencil lead at the surface of the piece under test.

Simulation of AE experiments requires the simulation of guided wave propagation in a structure together with that of the source of acoustic emission and that of the reception process by transducers. The simulation tools presented herein, though they were developed more specifically to deal with active NDT using guided waves, can help simulating AE experiments. For this, a necessary step is that consisting in simulating the amplitude of the various wave modes emitted by a given source in the structure. Interestingly, the FE scheme presented in this paper can be modified to do so. The fact is that the FE element code dealing with dynamic elasticity can handle source terms that appear as second members in the system of equations. Basically, the system of equation (1) is now replaced by

\[
\begin{align*}
\text{div} \sigma(u) - \omega \rho u &= f \quad \text{in } \Omega, \\
\sigma(u) \cdot \nu &= 0 \quad \text{on } \partial \Omega,
\end{align*}
\]

where \( f \) denotes the equivalent force of the acoustic emission source which can concern source points located either inside the piece or at its surface.
Here, we want to demonstrate, in the very simple example of a plate, differences one can observe depending on the nature and location of the source, therefore, to demonstrate the usefulness of a proper simulation tool to predict which wave modes are potentially measured in an experiment. Note that in this example, the bulk AE source is not realistic. However, more realistic sources can easily be introduced in the FE computation; specific work is in progress to model realistic AE sources in the frame of simulation tools presented in this paper.

4.2. Numerical results

Two sources are considered as depicted by figure 9. The first is a set of three point forces acting at or close to the centre of a 20-mm-thick steel plate directed along the propagation axis. The second is a point source of normal force due to a pencil lead break acting on one of the guiding surface.

![Figure 9. Two sources of acoustic emission are considered: a) an internal source of horizontal force, an external source (lead break).](image)

2D computations have been made. A 2D region of the 20-mm-thick plate (20 x 20 mm²) in which the two sources act has been meshed by 24728 elements, though a smaller zone could have been considered. It is assumed that receivers are sensitive in a narrow bandwidth centred on the frequency 0.818 MHz. This rather high frequency was chosen in this example so that many (17) propagative modes coexist. In the common practice, lower frequencies are used for such a thick plate just to avoid this. Knowledge of higher modes radiation could be used for more accurate source characterization; this is not a conclusion of present work but may constitute an objective for future researches. Dirichlet-to-Neuman transparent boundary conditions were applied on a set of 41 modes. Tables 1 and 2 give the amplitude obtained for the various modes (normalized in each case relatively to the highest amplitude value). Table 1 (2) concerns (anti)symmetric modal decomposition. The behaviour is very different in the two cases. A remarkable fact is that anti-symmetric modes are almost not radiated by the lead break. The relative amplitudes of A0 and S0 modes in the internal source case are also remarkably equal. More importantly, as far as application to acoustic emission testing is concerned, it must be noticed that modes of highest amplitude are different in the two cases. Since they propagate at different wavespeeds, if the second case be used for calibrating the experience, the wavespeed deduced being used in triangulation algorithms, errors could easily occur in source location.

Table 1. Relative amplitudes of modes (S-modes) emitted by the two sources of acoustic emission considered in the plate, for a receiver working at the frequency of 0.818 MHz.

| modes   | S0  | S1  | S2  | S3  | S4  | S5  | S6  | S7  | S8  |
|---------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| internal| 1.00| 0.25| 0.37| 0.48| 0.13| 0.19| 0.41| 0.62| 0.17|
| lead break| 0.00| 0.26| 0.44| 0.52| 0.55| 0.52| 0.82| 0.06| 1.00|
Table 2. Relative amplitudes of modes (A-modes) emitted by the two sources of acoustic emission considered in the plate, for a receiver working at the frequency of 0.818 MHz.

| modes        | A0 | A1 | A2 | A3 | A4 | A5 | A6 | A7 |
|--------------|----|----|----|----|----|----|----|----|
| internal     | 1.00 | 0.14 | 0.32 | 0.43 | 0.48 | 0.22 | 0.09 | 0.40 |
| lead break   | 0.00 | 0.00 | 0.01 | 0.03 | 0.04 | 0.03 | 0.02 | 0.05 |

5. Conclusion
A numerical method has been developed for simulating NDT experiments based on elastic guided waves. This method relies on a modal formulation [1] to deal with the radiation by an ultrasonic transducer, the guided propagation, the scattering by a defect and the reception by the same transducer (pulse-echo configurations) or another one (transmission configurations), each stage or phenomenon being computed separately. The modal formalism is particularly interesting to easily interpret typical complicated waveforms observed in experiments; furthermore, number of interesting results can be obtained by means of simple post-processing, thus limiting computational efforts in multi-parametric studies. In practice, eigenmodes are computed by means of the well-established SAFE method for numerical efficiency and versatility.

A specific Finite Element model has been derived to compute in the smallest possible zone the scattering of an incident field by an arbitrary defect or generally speaking by a discontinuity of the guide; it combines exact transparent boundary conditions and the account of the modal decomposition of an arbitrary field thanks to biorthogonality relations [2]. Further, this FE scheme can admit a second member term allowing one to account for source terms inside the FE computation zone; it is thus possible to simulate also an Acoustic Emission experiment involving guided wave propagation. These developments result in a versatile and numerically efficient method that can address complicated configurations typical of those encountered in practice. At present, the numerical tool developed is used to deal with various configurations of interest for NDT applications.

6. References
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