Original Article

Development and heat transfer analysis of thermoelectric self-powered fuel-fired residential boiler

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Abstract
Solid-state thermoelectric (TE) devices offer many interesting features compared with other methods of generation, such as no moving parts, high reliability, low maintenance, and straightforward integration with other heating equipment. In this study, a thermoelectric assembly was integrated into a residential heating boiler to convert a portion of combustion heat to electricity while meeting space and/or water heating needs. A prototype was developed, in which recently developed thermoelectric modules were incorporated into a gas-fired boiler. The electricity generated by the thermoelectric assembly would be sufficient to power the electrical auxiliary components of the heating system. In this way, the heating system could operate entirely on fuel combustion and provide the consumer with heating system reliability and a reduction in power consumption. A model for the thermoelectric conversion system was established and the thermal resistance analysis was carried out to show the influence of heat transfer coefficients and other parameters on power output and efficiency, and thus improve the system design. In this study, the single-module integrated assembly can produce a 22.5 W power output. However, each module of the four-/eight-module integrated assembly can only generate a power output of 7.85/9.175 W on average. The performance of a single module in a multimodule assembly is worse than that of a single module assembly. The number of modules placed on the furnace wall is not as many as possible but has an optimal value. In this experimental environment, the assembly of eight modules is the optimal choice. Furthermore, it is found that enhancing combustion-side heat transfer capacity is an effective way to promote system performance. The power output of the assembly composed of four modules will increase by 70% from 7.85 to 13.5 W due to the installation of the fin on the hot end.

Keywords
boiler, self-powered, thermal resistance analysis, thermoelectric generation
1 INTRODUCTION

Solid-state thermoelectric (TE) power generation devices offer many interesting features compared with other methods of power generation, such as no moving parts, high reliability, low maintenance, and straightforward integration with other heating equipment. However, subject to the existing thermoelectric materials, the thermal-to-electric energy conversion efficiency of the thermoelectric power generator (TEG) is relatively low. Fortunately, on some occasions, with economical heat sources, especially waste heat recovery, TEG technology has its special advantages and broad application prospects.

TEGs are sometimes used only for electricity production, but most of the current projects concerning TEGs are used in combined heat and power (CHP) systems. Recently, many researchers have developed a number of TEG applications integrated with other thermal equipment so as to pursue a higher energy utilization efficiency of the whole thermal system. Qiu and Hayden have conducted a study on thermoelectric self-powered heating equipment. Experimental results show good applicability of thermoelectric power generation to fuel-fired heating equipment. Huang presents a regenerative concept for TE-based waste heat recovery systems. Montecucco develops an application of TEGs to a solid-fuel stove to concurrently charge a lead–acid battery and transfer heat to the water for heating or household use. In Zhao’s work, performance optimization for a combustion-based microthermoelectric generator with a two-stage thermoelectric module has been conducted. Rasfuldi develops a self-heat recuperative heat circulation processing with a TE device. A few laboratories have conducted research into the installation of TE modules (Bi$_2$Te$_3$) on cookers or stoves.

What is the actual performance of such applications, and what problems exist in their operation? Undoubtedly, both the two questions are worthy of in-depth discussion. Nuwayhid et al. have studied the possibility of using a proportion of the heat from wood stoves to provide a continuous electric power supply. Their TE generator used commercially available, low-cost power generator modules. The cold side of the TE modules was cooled naturally with the surrounding air. The test results showed that the output power per module decreased when the number of TE modules in the TE generator increased. However, the authors did not further analyze the heat transfer process of the system but explored the effect of heat transfer enhancement from the experimental point of view instead. They tried to improve heat transfer, making a TE generator use heat pipes for the heat sink.

In addition to experimental projects, the practical use of TEGs is the focus of attention. Najjar proposed a multipurpose furnace with a 12-module TEG as a heat exchanger. The temperature difference was less than 45K, and the maximum power output of 12 modules was less than 17 W, but the TEG and its expanded heat dissipation surface improved the heat exchange with room air. More than 80% of the energy generated in the burner is converted into space heating through TEG fins, thereby improving the energy efficiency of the entire system. Lettsarkinon studied the possibility of self-powering a rice husk gasifier to use it as a cooking stove. To enhance the heat dissipation of the cold end, a fan needs to be arranged, for which it is necessary to provide electrical energy for the fan. The first experiment indicated that the system needs to be improved because the power generated by the prototype is not enough to power the fan.

Obviously, to achieve large-scale improvements in system performance to meet self-powered demand, a more powerful cooling method (forced air or even water cooling) is required to create and maintain large temperature differences. Even in the conversion of thermal energy to electrical energy involving new energy and new materials, the above technologies are equally critical. Kraemer et al. reported experimental measurements of concentrating solar thermoelectric generators (STEGs) with a peak efficiency of 9.6% at an optically concentrated normal solar irradiance of 211 kW/m$^2$, and a system efficiency of 7.4% after considering optical concentration losses. The performance improvement is achieved by the use of segmented thermoelectric legs, a high-temperature spectrally selective solar absorber enabling stable vacuum operation with absorber temperatures up to 600°C, and combining optical and thermal concentration. Of course, it is very important to maintain the effective heat dissipation of the cold end, so as to keep a large temperature difference. It can be seen that the performance improvement of the TEG devices is also limited by engineering factors such as the high-temperature tolerance of materials.

To this end, some researchers have done further studies based on some optimization issues to realize the practical use of TEG devices. Cheng et al. have carried out a performance comparison of single- and multi-stage onboard thermoelectric generators and stage number optimization at a large temperature difference. A TEG model with variable stage numbers considering the flow and heat transfer process of the heat source and the cold source has been developed to compare the performances of single- and multi-stage TEGs at large temperature differences. The results indicate that at a constant stage height, the thermoelectric performance with
different stage numbers is strongly influenced by the geometry factor. In a word, the multistage TEGs have significant advantages over single-stage TEG at the large temperature differences (over 500K).

In addition to the self-powered cogeneration (or CHP) system that this paper focuses on, it should be noted that the TEGs have been used in the recycling of waste heat from automobile exhaust. This field is also a very important research direction for the real application of TEG devices, and it has recently become a research hotspot. Zhao presents the characteristics analysis of an exhaust thermoelectric generator system with heat transfer fluid circulation. Lu conducts an experiment on thermal uniformity and pressure drop of exhaust heat exchanger for automotive thermoelectric generator. In addition to modeling analysis and performance testing, the energy efficiency analysis of the entire vehicle system is also a research hotspot. Kim carries out an experimental study of the energy utilization effectiveness of thermoelectric generators on diesel engines, and an assessment of the energy recovery potential of a thermoelectric generator system for passenger vehicles under various drive cycles. Massaguer finishes a fuel economy analysis under a WLTP cycle on a mid-size vehicle equipped with a thermoelectric energy recovery system. The results show that, with the incorporation of the TEG presented, a fuel consumption savings and CO₂ emissions reduction of 0.53% can be achieved. Zhu studies the related structural and thermal properties of vehicle exhaust thermoelectric power generation and lithium–ion battery energy storage.

Whether it is research on self-powered CHP systems or on automobile exhaust recovery TEGs, the studies mentioned above are either separate theoretical analyses or separate prototype tests. There are few studies conducted by integrating practical applications and detailed thermal process analysis based on modeling. The novelty of this study is to combine modeling analysis and experimental testing. On the one hand, a thermoelectric self-powered fuel-fired residential boiler is developed; on the other hand, a detailed heat transfer analysis based on this prototype is carried out. The research in this paper not only pays attention to the scientific nature of the research method, and fully reveals the inherent heat transfer mechanism that affects the performance of the system, but also pays attention to the real application of the TEGs. Based on prototype construction and testing, the problems of TEG devices in practical applications are explored.

To construct a realistic thermoelectric self-powered heating system and investigate its operation characteristics, in this study, a thermoelectric generator assembly was integrated into a residential heating boiler to convert a portion of combustion heat to electricity while meeting space and/or water heating needs. A prototype system was developed, in which recently developed thermoelectric modules were incorporated into a gas-fired residential boiler. The structure of this thermoelectric generation system was introduced and the power performance was measured. The electricity generated by the thermoelectric generator assembly would be sufficient to power the electrical auxiliary components of the heating system. In this way, the heating system could operate entirely on fuel combustion and provide the consumers with heating system reliability and a reduction in electric power consumption.

The study found that the phenomenon of “diminishing marginal utility” mentioned in Nuwayhid et al.17 reappeared. The law of diminishing marginal utility is an important concept that comes from economics. Its basic meaning is that when other conditions remain unchanged, if an input element continuously increases in equal amounts, after increasing to a certain output value, the increment of the product provided will decrease, that is, the marginal output of the variable element will decrease. In this study, it represents such a phenomenon, that the performance of a single module in a multistage assembly is worse than that of a single module assembly, and as the number of modules increases, this effect becomes more and more obvious. This shows that no matter how large the temperature difference is and how much power is generated, and no matter whether it is air cooling or water cooling, there is such a phenomenon, which is considered to be an optimization problem that should be worthy of further study. Based on the prototype system developed above, a mathematical model for the thermoelectric energy conversion system was established and simulations were carried out to show the influence of heat transfer coefficients and other parameters on electric power output and efficiency. The simulation results are in good agreement with the testing results, including the phenomenon of “diminishing marginal utility,” which reveals the bottleneck of the performance improvement of the system from the perspective of the heat transfer mechanism.

In general, the work of this paper provides both practical and theoretical guidance for the development and optimization of thermoelectric self-powered systems. More refined system design and thermal system analysis of the subprocess will be gradually carried out in the follow-up work. The author will consider incorporating energy analysis, exergy analysis, and entransy analysis into the design and operation optimization of thermoelectric power generation systems.
2 | EXPERIMENTAL SETUP

In this study, a natural gas-fired residential heating boiler (Product Serial No. Baxi Luna 3 Comfort 310 Fi) is employed. As shown in Figure 1, the furnace wall of the boiler is easy to disassemble and suitable for installing a flat TEG assembly to conduct energy conversion.

Figure 2 is the schematic diagram of the integration of a thermoelectric generator assembly with the gas-fired residential boiler. To convert a portion of combustion heat to electricity while meeting space and/or water heating needs, as shown in Figure 2, the dual furnace wall is replaced with two TEG assemblies. Each TEG assembly consists of several recently developed thermoelectric modules which are tightly sandwiched between a thermal conductive finned aluminum block (hot-side) and a flow channel (cold-side). The finned aluminum block is fixed inside the furnace to enhance combustion heat transfer. And the flow channel through which the cooling water flows acts as a heat sink of the TEG assembly.

To obtain the maximum possible power output, two types of high-power thermoelectric modules are used. One is the TEG1-12611-6.0 module which could generate a power output of up to 14 W when the temperature difference is up to 200°C; the other is the HZ-20 module that could generate a 20 W power output when a temperature gradient of 200°C exists. The detailed specification of the modules such as TE material, element size, number of the couple, and electrode material is included in the official documents which can be downloaded from the official website of the respective manufacturer, that is, TECTEG MFR. and Hi-Z. For the TEG1-12611-6.0 module, the dimension is 56 mm × 56 mm; the hot end can withstand a maximum temperature of 300°C; the matched load resistance is 1.2 ohms. For the HZ-20 module, the hot end can withstand a maximum temperature of 250°C; the dimension of the TE leg is 4.98 mm × 4.98 mm × 3 mm and the number of legs is 142. The Seebeck coefficient of a TE pair (P-N junction) is about 0.000375 VK⁻¹ at an average temperature of 400K. As shown in Figure 3, two pieces of ceramic substrates are integrated into a TEG1-12611-6.0 module (hot- and cold-side), but each HZ-20 module needs to be covered with two insulation wafers manually.
3 | TESTING AND DISCUSSION

3.1 | Performance test of a single module

Considering the different actual performances of the two modules above, we conduct a test for every single module at first. Figure 4 is the picture of the TEG assembly designed for a single module. The single TE module is sandwiched between a thermal conductive finned aluminum block (hot-side) and a flow channel (cold-side) using bolts. Because the assembly of a single TEG module is very difficult to install and fix, to fasten the thermal conductive finned aluminum block (hot-side), the TEG module, and the flow channel (cold-side), a thermal conductive finned aluminum block of the same size as the hot-side is installed on the outside of the flow channel. In this way, the above configuration can be fastened with bolts. Such a sandwich configuration is clearly shown in Figure 4. Both surfaces of the module are coated with thermal grease to reduce contact thermal resistance. The single-module integrated heating and generation system is shown in Figure 5.

The TEG assembly output is recorded when the system is stable after start-up. The major measurement parameter in the experiment is DC voltage, and current and power can be obtained by calculation. The standard uncertainty of the DC voltage measuring instrument within the experimental range is 0.00027 V. Open circuit voltage of the TEG1-12611-6.0 assembly is 10.4 V. The power output is 22.5 W when the load resistance matches the internal resistance of the module (1.2 ohms). And open-circuit voltage of the HZ-20 assembly is 5.6 V. The power output is 19.6 W when the load resistance matches the internal resistance of the module (0.4 ohms).

Although the rated power output of the HZ-20 module is larger than that of the TEG1-12611-6.0 module, it is interesting that the result is the opposite. The thermal resistance introduced by the manual mounting of the two insulation wafers is likely to cause the power output reduction of the HZ-20 module. For better output, we adopt the TEG1-12611-6.0 module to conduct further testing.

3.2 | Multimodule testing

As shown in Figure 6, a TEG assembly which consists of four TEG1-12611-6.0 modules is developed and integrated into the residential heating boiler. In this case, to compare testing results, the hot-side aluminum block is a
flat plate without fins. The four TE modules are connected electrically in series, but thermally in parallel, and then connected to a 0–10 ohms variable resistor as a load to form a power circuit.

The TEG assembly output is recorded and shown in Figure 7 when the system is stable. The open circuit voltage is 25 V. And the power output reaches 31.4 W when the load resistance is 5 ohms. It should be noted that the maximum power output occurs when the load resistance is slightly larger than the TEG internal resistance (4.8 ohms).

On the basis of a series of experiments above, as shown in Figure 8, a TEG assembly which consists of eight TEG1-12611-6.0 modules is developed. In this case, every two TE modules share a finned aluminum plate. Thus, there are four finned aluminum plates attached to the hot-side surface of the modules in this assembly. And the eight TE modules are connected electrically in series, but of course thermally in parallel. The eight-module integrated heating and generation system is shown in Figure 9.

Based on a 0–10 ohms variable resistor connected in series with the modules as a circuit load, as shown in Figure 10, the power output characteristics of the eight-module integrated TEG assembly are recorded when the system is stable. The open circuit voltage is 53 V. The power output reaches 73.4 W when the load resistance is 10 ohms. And the maximum power output occurs when the load resistance is larger than the TEG internal resistance (9.6 ohms).

### 3.3 Discussion

In this study, the single-module integrated assembly can produce a 22.5 W power output. However, each module of the four-/eight-module integrated assembly can only generate a power output of 7.85 \((31.4 \div 4)\) W/9.175 \((73.4 \div 8)\) W on average. That is, the performance of a single module in a multimodule assembly is worse than that of a single module assembly, which is the phenomenon of diminishing marginal utility. From the mathematical theory, the concept of calculus, especially the derivative, can be used to explain the law of diminishing marginal utility. However, our focus in this study is not to study the complex mathematical theory of
this phenomenon but to reveal the guiding significance of this phenomenon to engineering practice through physical analysis. Obviously, under the same heat source, the heat absorbed and used by only placing one module on the furnace wall is maximized (but not optimal, because the power output of a single module has a limit, in this case, 22.5 W). As the number of modules placed on the furnace wall increases, multiple modules share the same heat source, so the amount of heat allocated to each module is getting less and less. Analyzed quantitatively, the power output (9.175 W) of a single module in the eight-module integrated assembly is just 40% of the power output (22.5 W) of the single-module integrated assembly. Based on this, according to a simple linear analysis, a rough forecast of the number of modules placed on the furnace wall and its power output can be obtained, which is shown in Table 1.

According to the above rough forecast, the number of modules placed on the furnace wall is not as many as possible but has an optimal value. Here, the total power output of the assembly composed of 16 modules is lower than that of the assembly composed of eight modules. Obviously, in this experimental environment, the assembly of eight modules is the relatively optimal choice. Further, it is found that enhancing the heat transfer at the hot end of the module is critical to improving TEG performance. The power output of the assembly composed of four modules will increase by 70% from 7.85 to 13.5 W due to the installation of the fin on the hot end.

It should be noted that if we install another TEG assembly consisting of eight modules to the other furnace wall and further improve the heat transfer, as shown in Figure 2, the 16-module integrated heating and generation system could produce $2 \times 73.4 = 146.8$ W or more power output. The electricity generated by the thermoelectric generator assembly would be sufficient to power the electrical auxiliary components of the heating system. Thus, the heating system could operate entirely on fuel combustion.

According to the combustion theory, as long as the flame can be generated, the furnace temperature is not too low, which is enough to ensure the heat input of the hot end of the TEG component. Considering the TEG component only needs to use a small amount of heat transferred from the wall of the furnace (This can be verified by subsequent thermal resistance analysis), although the test is carried out under the condition of the stable load of the boiler, the self-powered heating system can operate stably even if the boiler load is greatly changed. In fact, it is found that the temperature difference between the inlet and outlet of the cooling water is small, indicating that the cooling capacity of the cold end of the TEG component is sufficient to ensure the stability of the self-powered heating system.

To clarify the operational relationship between the gross power output and the auxiliary load under the variation of the main residential boiler load, TEG performance tests under variable load conditions of the boiler are carried out. The test results show that, in the normal working range of the boiler (the firepower has a range from large to small), the deviation of the maximum power output (under matching load) is 10%. In other words, under the above device capable of outputting 73.4 W (under maximum boiler firepower), the maximum output power will not be less than 66 W (under minimum boiler firepower). A deviation of 10% can ensure the normal operation of the auxiliary load and realize true “self-powered.” The reason for this result is that there is always a lot of heat (from the flame) on the furnace wall that can be absorbed, and TEG can only convert less than 10% of it into electricity. The TEG efficiency is higher when the boiler thermal load is large, and the TEG efficiency is slightly lower when the thermal load is low. Therefore, the maximum power output of TEG fluctuates at 10% within the normal operating range of the boiler.

### Table 1 Number of modules placed on the furnace wall and its power output

| Number of modules | 1   | 2   | 4   | 8   | 16  |
|-------------------|-----|-----|-----|-----|-----|
| Average power output of a single module (W) | 22.5<sup>a</sup> | 18  | 13.5 (7.85<sup>b</sup>) | 9.175<sup>c</sup> | 4.5  |
| Power output of the whole modules (W) | 22.5<sup>a</sup> | 36  | 54  (31.4<sup>b</sup>) | 73.4<sup>a</sup> | 72  |

<sup>a</sup>Test value.

<sup>b</sup>Test value without fins installed on the hot end.

<sup>c</sup>Test value.
To construct a realistic thermoelectric self-powered heating system, a thermoelectric generator assembly was integrated into a residential heating boiler to convert a portion of combustion heat to electricity while meeting space and/or water heating needs. Although it is only a demonstration project, in addition to performance, its cost is still an important consideration for the system to be successfully implemented and promoted. In this case, the most expensive cost is the thermoelectric power generation module. TEG1-12611-6.0 employed in the assembly is the most cost-effective TEG module in the same applicable temperature range. According to the official quotation, the module is priced at about $40 per piece. For this self-powered heating system with 16 modules (output power 146.8 W), the cost of the TEG module is about $640. Assuming that the price of 1 kWh of electricity is $0.15, which is a normal electricity price in the United States, the cost of the installation of $640 requires the TEG system to generate 4267 (640/0.15) kWh of electricity to recover. For the TEG system, it takes 6.8 (1000/146.8) h to generate 1 kWh of electricity, which needs to work for 29016 (4267 × 6.8) h to recover the cost. Assuming that the system operates continuously 24 h a day (uninterrupted heating and power supply), the cost recovery period of the thermoelectric self-powered heating system is 3.3 (29016/24/365) years. Taking into account that the actual cost will be higher and the actual daily operating time will be less, the cost recovery period may be several years, which is unfavorable for the application and promotion of the TEG technology. But in general, the cost recovery cycle can be expected, and it will always become shorter and shorter with technological progress and economic development. First of all, we hope that the cost can be greatly reduced, which accelerates the application and promotion of the TEG technology. Second, in cold areas, boilers are required to operate for long-time meeting space and/or water heating needs all year round. The thermoelectric self-powered heating system is relatively friendly to this application scenario, and the cost recovery period of which is the shortest. Finally, for application scenarios where it is inconvenient to connect to the grid in remote areas or the cost of grid connection is extremely high, the thermoelectric self-powered heating system has its unique advantages, which are their greatest value and cannot be measured by cost alone.

4 | MODELING AND ANALYSIS

4.1 | Mathematic model

To investigate the detailed operation characteristics of the thermoelectric energy conversion system and thus improve the system design, a mathematic model based on thermal resistance analysis was established and simulations were carried out to show the influence of heat transfer coefficients and other parameters on electric power output and efficiency.

This study at first is based on the generalized thermoelectric energy balance equations:

\[
Q_h = n [\alpha IT_h + K (T_h - T_c) - 0.5I^2R],
\]

\[
Q_c = n [\alpha IT_c + K (T_h - T_c) + 0.5I^2R],
\]

where \( Q_h \) and \( Q_c \) represent the heat absorbed by the generator from the high-temperature reservoir and released to the low-temperature reservoir per unit time, respectively. \( n \) is the number of TE couples of the generator, \( I \) is the electric current in the generator circuit, and \( T_h \) and \( T_c \) represent the hot- and cold-side temperature of the TE couples. In addition, \( \alpha_p \) and \( \alpha_n \) are the Seebeck coefficients of the p- and n-type semiconductor legs, respectively, and \( \alpha = \alpha_p - \alpha_n \). \( K \) and \( R \) are the thermal conductance \((WK^{-1})\) and the electric resistance of the single thermoelectric couple, respectively.

For a detailed derivation of the generalized thermoelectric energy balance equations, please refer to Xiao et al.\(^{38}\) This literature details the entire process of modeling analysis and the rationality of some of these assumptions about the model. It should be noted that the work of this paper does not use the modified equations derived in the literature to consider the effect of natural convection heat dissipation. However, if the TEGs are quite large, this effect must be considered.

The thermal resistance model is based on the most basic concept of heat transfer. It is assumed in this study that there is no heat loss during the heat transfer process and the contact thermal resistance between the two contact heat transfer surfaces is negligible. Although this assumption has an impact on the accuracy of the model, we will later introduce a factor to correct the deviation caused by the assumption (compared with the experimental test results). On account of a series of thermal resistance existing between the heat reservoirs and the generator (thermoelectric couples), by employing heat transfer and thermal resistance analysis, the heat flux of the multicouple TEG can be expressed as follows:

\[
Q_h = \frac{T_b - T_h}{R_h},
\]

\[
Q_c = \frac{T_c - T_l}{R_c},
\]

where \( T_b \) and \( T_l \) represent the combustion temperature of the boiler and the cooling fluid temperature. \( R_h \) and \( R_c \)
are the total thermal resistance \((\text{KW}^{-1})\) between the hot-/cold-side heat reservoirs and thermoelectric couples, respectively.

As shown in Figure 11, either \(R_h\) or \(R_c\) consists of three thermal resistances in series, which can thus be expressed as:

\[
R_h = R_b + R_{Ah} + R_{waf},
\]

\[
R_c = R_{waf} + R_{Alc} + R_f,
\]

where \(R_b\) and \(R_f\) refer to the combustion-side and fluid cooling-side thermal resistances, respectively. \(R_{waf}\) is the thermal resistance of the thermally conductive ceramic substrate (wafer). And \(R_{Ah}\) and \(R_{Alc}\) represent the thermal resistances of the thermally conductive aluminum block (including fins) on the hot side and the aluminum flow channel plate on the cold side, respectively.

The heat transfer path of the entire system is visually and clearly presented in Figure 11. First, the furnace combustion flame transfers heat to the thermally conductive aluminum block by radiation and convection; then, it is transferred to the hot end thermal conductive ceramic substrate, the thermoelectric material, the cold end thermal conductive ceramic substrate, and the aluminum flow channel plate in turn by heat conduction; Finally, the heat is carried away by the cooling water through convective heat transfer. In the process of heat transfer, due to the existence of thermal resistance, the temperature of each heat transfer node decreases in turn, but the heat transfer amount remains the same in each node (assuming that the heat loss of each node is negligible).

The following are the detailed expressions of the above thermal resistance terms:

\[
R_b = \frac{1}{A_h h_b},
\]

where \(h_b\) and \(h_f\) refer to heat transfer coefficients \((\text{Wm}^{-2}\text{K}^{-1})\) at the combustion-side and fluid cooling-side, respectively. Note \(h_b\) includes both effects of convection and radiation. \(\delta_{Ah}\), \(\delta_{Alc}\), and \(\delta_{waf}\) represent the thickness of the aluminum block base on the hot side and the aluminum flow channel plate on the cold side, and the thickness of the ceramic substrate (wafer), respectively. \(\lambda_{Al}\) and \(\lambda_{waf}\) are thermal conductivity \((\text{Wm}^{-1}\text{K}^{-1})\) of aluminum and the ceramic substrate (wafer). \(A_b\) and \(A_f\) refer to heat transfer areas of the combustion-side surface of the aluminum block (including fins) and the fluid cooling-side surface of the aluminum flow channel plate, respectively. And \(A_{Ah}\), \(A_{Alc}\), and \(A_{waf}\) are heat transfer surface areas of the aluminum block base, the aluminum flow channel plate, and the ceramic substrate (wafer), respectively.

In addition, when the system circuit is closed, the electric current \(I\) is determined by the following formula:

\[
I = \frac{n\alpha (T_h - T_c)}{nR + R_L},
\]

where \(R_L\) represents a load resistance that is connected in series with the TEG circuit.

Consequently, we can obtain the expressions of power output \(P_{out}\) and system efficiency \(\eta\):

\[
P_{out} = \beta I^2 R_L,
\]

\[
\eta = \frac{P_{out}}{Q_h},
\]

where \(\beta\) is a power correction factor \((0 < \beta < 1)\) which reflects the fact that the TEG system model established above is an one-dimensional model. When the multi-element TEG works in the actual conditions, the one-dimensional model could not consider the different working conditions that each part of the TEG bears. And usually, this model would overestimate the system performance. In this paper, the value of the factor is not arbitrarily chosen but is determined by the comparison of
simulation analysis results and experimental results. As will be seen in the following discussion, the results of the simulation analysis are basically consistent with the experimental results in the qualitative trend, but there are some deviations in the quantitative. The value of the factor is determined by the following method: First, according to the test data, the performance curve of power output versus load resistance is fitted and partially predicted; Then, based on the above mathematical model, the simulation performance curve of power output versus load resistance without correction factor is made; Finally, using a similar idea of least squares method, under the condition that the overall trend of the simulation curve remains unchanged, the objective function is optimally solved, that is, the value of the power correction factor is determined to minimize the sum of squares of the difference between the experimental data and the simulated data within the simulation range.

4.2 | Thermal resistance analysis

Based on the above TEG system model, we can carry out simulations and thermal resistance analysis employing an iterative algorithm to show the influence of heat transfer coefficients and other parameters on power output and system efficiency. Considering that the mathematical models employed are all composed of algebraic equations, that is, the generalized thermoelectric energy balance equations, the thermal resistance model, and related circuit equations, it is most suitable to solve these models by using an iterative method. The equations are collated and entered into Matlab software, and some of the commands for solving algebraic equations can be used to iteratively calculate the results. When calculating the solution, pay attention to the coupling relationship between the electric current value and the temperature values. The performance parameters, including power output and system efficiency, can be easily calculated by first iterating over the above parameters.

Figure 12 shows the simulation results on system performance in the case of utilizing eight TEG1-12611-6.0 modules. To better compare the simulation analysis results against the experimental results, the data of Figure 10 is integrated into Figure 12. It is found that the simulation results are reasonable compared with the actual conditions. Although there are some acceptable deviations in quantitative between the two results, the results of the simulation analysis can fully and qualitatively reflect the characteristics of the actual device. For the calculation of efficiency, it should be pointed out that, on the one hand, because the test is carried out under high-temperature conditions, the system efficiency is higher than that under medium- and low-temperature conditions. On the other hand, due to the simplification of the model (one-dimensional model), the system efficiency calculated here will be slightly higher than the actual operating system efficiency.

Furthermore, there is an interesting feature revealed by simulation, also mentioned in the experimental study above, that the maximum power output occurs when the load resistance is larger than the TEG internal resistance rather than when the load resistance matches the internal resistance. And, the operating state of the maximum efficiency for the TEG is near (not equal) to that of the maximum power output. According to the circuit theory, the load resistance at the maximum power output should be equal to the internal resistance, as shown in Figure 13A. Although it is an equivalent circuit system in form, the TEG system has different characteristics from the conventional circuit system due to the influence of thermoelectric materials and heat transfer characteristics. On one hand, the physical properties of thermoelectric materials, including resistivity, vary with temperature to a greater extent than ordinary metals. On the other hand, and more importantly, as Chen and Wu analyzed, it is the external and internal irreversible heat transfer processes considered in the above modeling that bring about this interesting feature. In fact, expressing and understanding the existence of internal irreversibility in an intuitive way is equivalent to adding an internal resistance inside the TEG system in the circuit, as shown in Figure 13B. Anyway, to make a thermoelectric generator operate optimally, it is necessary to consider the optimal matching of load resistance. For a detailed study about the effect of heat transfer on
the performance of thermoelectric generators, works on finite time thermodynamics and Chen et al.\textsuperscript{40} are recommended. It is found that the optimum matching resistance value of the TEG system should be between the resistance value at the maximum power output and the resistance value at the maximum efficiency.

On the basis of simulation, system thermal resistance analysis is conducted. Table 2 shows the simulation value of each thermal resistance. For the calculation process of a single thermal resistance, refer to the detailed expression of each thermal resistance introduced in the previous section. The relevant physical properties and geometric parameters (measured values from the experimental device) are as follows when calculating the thermal resistance values:

- $h_b = 10 \text{ Wm}^{-2}\text{K}^{-1}$, $h_f = 1000 \text{ Wm}^{-2}\text{K}^{-1}$,
- $\lambda_{Al} = 200 \text{ Wm}^{-1}\text{K}^{-1}$, $\lambda_{waf} = 36 \text{ Wm}^{-1}\text{K}^{-1}$,
- $\delta_{Al} = 0.01 \text{ m}$, $\delta_{waf} = 0.001 \text{ m}$, $A_b = 0.255 \text{ m}^2$, $A_{Alh} = 0.0426 \text{ m}^2$, $A_{waf} = 0.025088 \text{ m}^2$, $A_{Alc} = 0.045 \text{ m}^2$, $A_f = 0.0405 \text{ m}^2$.

Obviously, the combustion-side and fluid cooling-side thermal resistance, that is, $R_b$ and $R_f$, are the dominant thermal resistances in the hot- and cold-side heat transfer processes of the TEG system, respectively. According to Expressions (6) and (10), to reduce the two thermal resistances and thus improve the system performance effectively, we need to enhance both the hot-/cold-side heat transfer processes, that is, increasing both heat transfer coefficients $h_b$ and $h_f$. Though expanding both heat transfer areas $A_b$ and $A_f$ also seem to be an effective way, there is not enough space to expand and it will affect the compactness of the system.

As shown in Figures 14 and 15, the effects of the combustion-/fluid cooling-side heat transfer coefficients on system performance (power output and system efficiency) are simulated, respectively. It is found that, for this setup, enhancing fluid cooling-side heat transfer capacity from the rated condition 1000 Wm\textsuperscript{-2}K\textsuperscript{-1}, can only promote system performance to a small extent. However, enhancing combustion-side heat transfer capacity can promote system performance effectively to a large extent. It is clear that the system performance is more sensitive to combustion-side heat transfer capacity.

The theoretical analysis does guide the optimization of the experimental study, which is mainly reflected in two aspects: On the one hand, theoretical analysis shows that the key to improving the performance of the system is to enhance the heat transfer at the hot end, that is, the combustion side. Because the heat transfer capacity of the cold end has a limit within the experimental range, it is necessary to expand the heat absorption area of the hot end (using fins to expand the surface), which is a cost-effective means of improving system performance. On the other hand, when building a power output circuit, it is necessary to adjust the load resistance value to be slightly larger than the internal matching resistance value of the TEG to maximize its output power. However, the mathematical model and theoretical analysis of this study are not detailed and in-depth enough, especially in the quantitative aspect, and further research is needed. It is believed that more valuable results can be achieved in the future using

\begin{table}[h]
\centering
\caption{Simulation value of each thermal resistance for the TEG system}
\begin{tabular}{|c|c|}
\hline
\textbf{Thermal resistance} & \textbf{Simulation value (K/W)} \\
\hline
$R_b$ & 0.3922 \\
$R_{Alh}$ & 0.0012 \\
$R_{waf}$ & 0.0011 \\
$R_{Alc}$ & 0.0006 \\
$R_f$ & 0.0247 \\
$R_b = R_b + R_{Alh} + R_{waf}$ & 0.3945 \\
$R_c = R_{waf} + R_{Alc} + R_f$ & 0.0264 \\
\hline
\end{tabular}
\end{table}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure13.png}
\caption{System equivalent circuit diagram under ideal (A) and actual (B) conditions. TEG, thermoelectric power generator.}
\end{figure}
theoretical analysis to guide experimental research, product development, and optimization.

5 | CONCLUSION

In this study, a thermoelectric generator assembly that consists of several recently developed thermoelectric modules was developed and integrated into a gas-fired residential boiler to convert a portion of combustion heat to electricity while meeting space and/or water heating needs. The electricity generated by the thermoelectric generator assembly would be sufficient to power the electrical auxiliary components of the heating system. In this way, the heating system could operate entirely on fuel combustion and provide the consumers with heating system reliability and a reduction in electric power consumption.

In addition, a mathematic model based on thermal resistance analysis was established and simulations were carried out to show the influence of heat transfer coefficients and other parameters on power output and system efficiency. It is found that the combustion-side and fluid cooling-side thermal resistance, that is, \( R_b \) and \( R_f \), are the dominant thermal resistances in the hot- and cold-side heat transfer processes of the TEG system, respectively.

In this study, the single-module integrated assembly can produce a 22.5 W power output. However, each module of the four-/eight-module integrated assembly can only generate a power output of 7.85/9.175 W on average. The performance of a single module in a multimodule assembly is worse than that of a single module assembly. The number of modules placed on the furnace wall is not as many as possible but has an optimal value. In this experimental environment, the assembly of eight modules is the optimal choice. Further, it is also found that the system performance is sensitive to combustion-side heat transfer coefficient \( h_b \). Therefore, enhancing combustion-side heat transfer capacity is an effective way to promote system performance. The power output of the assembly composed of 4 modules will increase by 70% from 7.85 to 13.5 W due to the installation of the fin on the hot end.

By the way, the combustion-side heat transfer coefficient \( h_b \) is just a macroscopic coefficient that reflects the effects of both convection and radiation. The enhancement of combustion-side heat transfer is a more complex technical issue that we plan to carry out in future work.

NOMENCLATURE

- \( A \) area (m²)
- \( h \) heat transfer coefficient (Wm⁻²K⁻¹)
- \( I \) electric current (A)
- \( K \) thermal conductance (WK⁻¹)
- \( n \) number
- \( P \) power (W)
- \( Q \) heat transfer rate (W)
- \( R \) electric resistance (Ω), thermal resistance (KW⁻¹)
- \( T \) temperature (K)

GREEK LETTERS

- \( \alpha \) Seebeck coefficient (VK⁻¹)
- \( \beta \) correction factor
- \( \delta \) thickness (m)
- \( \eta \) efficiency
- \( \lambda \) thermal conductivity (Wm⁻¹K⁻¹)
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