The Casimir frictional drag force between a SiO$_2$ tip and a graphene-covered SiO$_2$ substrate

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The possibility of the mechanical detection of the Casimir friction using non-contact force microscope is discussed. On a SiO$_2$ tip situated above a graphene-covered SiO$_2$ substrate will act the frictional drag force mediated by a fluctuating electromagnetic field produced by a current in the graphene sheet. This friction force will produce the bending of the cantilever, which can be measured by state-of-art non-contact force microscope. Both the thermal and quantum contributions to the Casimir frictional drag force can be studied using this experimental setup. This result paves the ways for the mechanical detection of the Casimir friction and for the application of the frictional drag effect in micro- and nano- electromechanical devices (MEMS and NEMS).

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I. INTRODUCTION

All media are surrounded by a fluctuating electromagnetic field due to the thermal and quantum fluctuations of the current and charge densities inside them. These electromagnetic fluctuations are the cornerstone of the Casimir physics which includes the Casimir – van der Waals forces, the Casimir friction with its limiting case - quantum friction, and the near-field radiative heat transfer. The thermal and quantum fluctuation of the current density in one body induces the current density in other body; the interaction between these current densities is the origin of the Casimir interaction. When two bodies are in relative motion, the induced current will lag slightly behind the fluctuating current inducing it, and this is the origin of the Casimir friction. At present the Casimir friction is attracting a lot of attention due to the fact that it is one of the mechanisms of noncontact friction between bodies in the absence of direct contact. The noncontact friction determines the ultimate limit to which the friction force can be reduced and, consequently, also the force fluctuations because they are linked to friction via the fluctuation-dissipation theorem. The force fluctuations (and hence friction) are important for the ultrasensitive force detection.

The Casimir friction has been studied in the configurations plate-plate and neutral particle-plate. While the predictions of the theory for the Casimir forces were verified in many experiments, the detection of the Casimir friction is still challenging problem for experimentalists. However, the frictional drag between quantum wells and graphene sheets, and the current-voltage dependence of nonsuspended graphene on the surface of the polar dielectric SiO$_2$, were accurately described using the theory of the Casimir friction.

The frictional drag effect is in driving an electric current in one metallic layer and registration of the effect of the frictional drag of the electrons in a second (parallel) metallic layer (Fig. 1). Such experiments were predicted by Pogrebinskii and Price and were performed for 2D quantum wells and graphene sheets. In these experiments, two quantum wells are separated by a dielectric layer thick enough to prevent electrons from tunneling across it but allowing interlayer interaction between them. A current of density $J_2 = n_2 e v$ is driven through layer 2 (where $n_2$ is the carrier concentration per unit area in the second layer), see Fig. 1. Due to the proximity of the layers, the interlayer interactions will induce a current in layer 1 due to a friction stress $\sigma$ acting on the electrons in layer 1 from layer 2. To linear order in the drift velocity $\sigma = \gamma v$ where $\gamma$ is the friction coefficient. If layer 1 is an open circuit, an electric field $E_1$ will develop in the layer whose influence cancels the frictional stress $\sigma$ between the layers. Thus the frictional stress $\sigma = \gamma v$ must equal the induced stress $n_1 e E_1$ so that

$$\gamma = n_1 e E_1 / v = n_2 n_1 e^2 E_1 / J_2 = n_1 n_2 e^2 \rho_D,$$

where the drag resistivity $\rho_D = E_1 / J_2 = \gamma / n_1 n_2 e^2$ is defined as the ratio of the induced electric field in the first layer to the driving current density in the second layer. The transresistivity is often interpreted in terms of a drag rate which, in analogy with the Drude model, is defined by $\tau_D^{-1} = \rho_D n_2 e^2 / m^* = \gamma / n_1 m^*$. Frictional drag between graphene sheets was measured recently in Refs. 26, 27. This study has fueled the recent theoretical investigations of frictional drag between graphene sheets mediated by a fluctuating Coulomb field (see Ref. 46 for recent review).

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Most of this work focused on interlayer Coulomb interaction, the most obvious coupling mechanism and the one considered in the original theoretical papers.\textsuperscript{22}, though the contributions due to an exchange of phonons between the layers have also been considered.\textsuperscript{39,47–50} In the Fermi-liquid regime, $k_BT \ll \epsilon_F$, and in the limit of strong screening, $k_T F d \gg 1$, the drag resistivity for both the 2D-quantum wells and the graphene sheets is given by\textsuperscript{39,47–50}:

$$\rho_D = \frac{\gamma}{(ne)^2} = \frac{\hbar}{e^2} \frac{\pi \zeta(3)}{32} \left( \frac{k_B T}{\epsilon_F} \right)^2 \frac{1}{(k_F d)^2} \left( \frac{1}{k_T F d} \right)^2,$$

where for the 2D-quantum wells $k_T F = 2a_0^{-1}/\varepsilon$ is the single-layer Tomas-Fermi screening wavevector, $a_0 = \hbar^2/m^*e^2$, and for the graphene sheets $k_T F = 4e^2\epsilon_F/\varepsilon h v_F$, $m^*$ is the electron effective mass, $k_F$ is the Fermi wave number, $\epsilon_F$ is the Fermi energy, $v_F$ is the Fermi velocity, $d$ is the separation between 2D layers, $\varepsilon$ is the dielectric constant for the surrounding medium.

The close connection of the Casimir friction with frictional drag effect is illustrated in Fig. 2. Application of the theory of the Casimir friction to these experiment is based on the assumption that the Fermi-liquid theory is valid for electrons in graphene. In this case the graphene with the drift motion of electrons with the drift velocity $v_{Drift}$ produces the same fluctuating electromagnetic field and has the same reflection amplitudes as graphene moving with velocity $v = v_{Drift}$. The hydrodynamic model for electrons in a medium was used in Ref.\textsuperscript{39} to calculate the fluctuating electromagnetic field produced by the medium in the presence of the drift motion of the electrons. A theory of the Casimir friction was used for the description of the frictional drag effect in the 2D quantum wells in Ref.\textsuperscript{39} and in the graphene double layer in\textsuperscript{22}. Eq. (2) was reproduced using such approach. However, the theory of the Casimir friction, in contrast to the theory of the Coulomb drag, includes the contribution not only from the fluctuating Coulomb field due to the charge fluctuations but also the contribution from the electromagnetic field produced by the transverse current density fluctuations.

In one experiment (see\textsuperscript{22}) the drift velocity $v_{Drift} \sim 10^2$ m/s. According to the theory of the Casimir friction\textsuperscript{34–36}, at such velocities the thermal fluctuation give the dominant contribution to the friction, and the theoretical predictions are in agreement with experiment. However, at large drift velocities the Casimir friction is dominated by the contribution from quantum fluctuations which are not included in the theory of the Coulomb drag. Using the theory of the Casimir friction in Ref.\textsuperscript{22} it was shown that the current-electric field dependence of nonsuspended graphene on the SiO$_2$ substrate can be explained by quantum friction between electrons in graphene and surface phonon polaritons in SiO$_2$. 

FIG. 1: Scheme of experiment for observation of the drag effect.

FIG. 2: Two ways of studying of the Casimir friction. Left: a metallic block sliding relative to the metallic substrate with the velocity $v$. An electronic fricitional stress will act on the block (and on the substrate). Right: the shear stress $\sigma$ can be measured if instead of sliding the upper block, a voltage $U_2$ is applied to the block resulting in a drift motion of the conduction electrons (velocity $v$). The resulting frictional stress $\sigma$ on the substrate electrons will generate a voltage difference $U_1$ (proportional to $\sigma$) as indicated in the figure. Both approaches are equivalent if the conduction electrons are in the Fermi-liquid state and it is possible to neglect scattering of the free carries by lattice.
In the frictional drag experiment the electric field induced by the the Casimir friction force is measured. For the graphene sheet situated nearby the polar dielectric substrate the Casimir friction force between the charge free carries in graphene and the surface phonon polaritons in dielectric gives rise to the change of the resistivity of graphene which also can be measured. So far the Casimir friction was detected only using the electrical effects, which it produces. Thus, the frictional drag effect can only be observed between the two 2D conducting structures and the electrical transport in graphene can only be measured for nonsuspended graphene when the heat conductance between graphene and underlying dielectric is high.

In this paper the possibility of the mechanical detection of the Casimir friction using non-contact atomic force microscope (AFM) is considered. This topic we also discussed recently in Ref.\textsuperscript{52}. The schemes for the experimental setups are shown on Fig.\textsuperscript{3}. On Fig.\textsuperscript{3a} a SiO\textsubscript{2} tip and a SiO\textsubscript{2} substrate have clean surfaces (DD structure). On Fig.\textsuperscript{3b} a SiO\textsubscript{2} substrate is covered by graphene and a SiO\textsubscript{2} tip has clean surface (DGD structure), and on Fig.\textsuperscript{3c} both surfaces of the tip and the dielectric are covered by graphene (DGGD structure).

This paper is organised as follows. In Sec.\textsuperscript{II} the Casimir friction is considered in the plate-plate configuration for the SiO\textsubscript{2} plates and the graphene-covered SiO\textsubscript{2} plates. To linear order in the relative velocity \(v\) the friction stress between plates \(f = \gamma v\) where the friction coefficient \(\gamma\) was calculated analytically in the resonant and off-resonant approximations. In Sec.\textsuperscript{III} the Casimir friction in the tip-plate configurations is calculated using the proximity force approximation. In Sec.\textsuperscript{IV} the Casimir frictional drag force acting on the tip due to the drift motion of the electrons in the graphene sheet on the SiO\textsubscript{2} plate is calculated. The summary of the obtained results is given in Sec.\textsuperscript{V}.

\section{Casimir Friction in the Plate-Plate Configuration}

According to the theory of the Casimir friction, the frictional stress between two plates in the parallel relative motion with the velocity \(v\) along the \(\hat{x}\)-axis and with the \(\hat{z}\)-axis normal to the plate is given by the \(xz\)-component of the Maxwell stress tensor: 
\[
\begin{align*}
\sigma_{xz} &= f_T + f_0, \quad \text{where at} \quad d \ll \lambda_T = \hbar c/\kappa B T \quad \text{and} \quad v \ll c \quad \text{the contributions from thermal} \ (f_T) \quad \text{and quantum} \ (f_0) \quad \text{fluctuations are given by} \ \text{eqs.}\textsuperscript{4,7,10,35},
\end{align*}
\]

\[
\begin{align*}
\sigma_{xz} &= \frac{\hbar}{\pi^3} \int_0^\infty dq_y \int_0^\infty dq_x q_x e^{-2q d} \left\{ \int_0^\infty d\omega \left( \frac{\text{Im}R_1(\omega)\text{Im}R_2(\omega^+)}{1 - e^{-2q d} R_1(\omega) R_2(\omega^+)} \right)^2 \times [n_1(\omega) - n_2(\omega^+)] + (1 \leftrightarrow 2) \right\} \\
&\quad - \int_0^{q_x v} d\omega \left( \frac{\text{Im}R_1(\omega)\text{Im}R_2(\omega^-)}{1 - e^{-2q d} R_1(\omega) R_2(\omega^-)} \right)^2 n_1(\omega) + (1 \leftrightarrow 2) \} \\
&= -\frac{\hbar}{2\pi^3} \int_0^\infty dq_y \int_0^\infty dq_x q_x e^{-2q d} \int_0^{q_x v} d\omega \left( \frac{\text{Im}R_1(\omega)\text{Im}R_2(\omega^-)}{1 - e^{-2q d} R_1(\omega) R_2(\omega^-)} \right)^2 + (1 \leftrightarrow 2). \quad (3)
\end{align*}
\]

\[
\begin{align*}
f_0 &= -\frac{\hbar}{2\pi^3} \int_0^\infty dq_y \int_0^\infty dq_x q_x e^{-2q d} \int_0^{q_x v} d\omega \left( \frac{\text{Im}R_1(\omega)\text{Im}R_2(\omega^-)}{1 - e^{-2q d} R_1(\omega) R_2(\omega^-)} \right)^2 + (1 \leftrightarrow 2). \quad (4)
\end{align*}
\]
where $\omega^\pm = \omega \pm q_z v$, $R_{ip}$ is the reflection amplitude for the $p$-polarized electromagnetic wave for the plate $i$, $n_i(\omega) = \exp(\omega/k_B T_i) - 1)^{-1}$. The symbol $(1 \leftrightarrow 2)$ denotes the terms that are obtained from the preceding terms by permutation of 1 and 2. The reflection amplitude for the dielectric for $d < c/(\omega_0|\epsilon_d|)$

$$R_d = \frac{\epsilon_d - 1}{\epsilon_d + 1},$$  

(5)

where $\epsilon_d$ and $\omega_0$ are the dielectric function and the characteristic frequency for dielectric. The dielectric function of amorphous SiO$_2$ can be described using an oscillator model

$$\epsilon(\omega) = \epsilon_\infty + \sum_{j=1}^2 \frac{\sigma_j}{\omega_0^2 - \omega^2 - i\omega\gamma_j},$$

(6)

where parameters $\omega_0,j$, $\gamma_j$ and $\sigma_j$ were obtained by fitting the actual $\epsilon$ for SiO$_2$ to the above equation, and are given by $\epsilon_\infty = 2.0014$, $\sigma_1 = 4.4767 \times 10^{12} \text{s}^{-1}$, $\omega_{0,1} = 8.6732 \times 10^{13} \text{s}^{-1}$, $\gamma_1 = 3.3026 \times 10^{12} \text{s}^{-1}$, $\sigma_2 = 2.3584 \times 10^{28} \text{s}^{-2}$, $\omega_{0,2} = 2.0219 \times 10^{14} \text{s}^{-1}$, and $\gamma_2 = 8.3983 \times 10^{12} \text{s}^{-1}$. For a graphene-covered SiO$_2$ substrate the reflection amplitude $R_{dg}$ can be expressed through the reflection amplitudes for the clean substrate surface $R_d$ given by Eq. (5) and for isolated graphene given by

$$R_g = (\varepsilon_g - 1)/\varepsilon_g,$$

(7)

where the dielectric function of graphene

$$\varepsilon_g = 1 + \frac{2\pi i q \sigma_I}{\omega},$$

(8)

where $\sigma_I$ is the longitudinal conductivity of the sheet. To get the expression for $R_{dg}$ we assume that the graphene layer is located at $z = 0$ and the dielectric at $z < -a$. The electric field can be written in the form

$$E(q,\omega,z) = \begin{cases} R_{dp} \hat{n}_p^+ e^{-qz} + \hat{n}_p^- e^{qz}, & \text{for } x > 0 \\ v_p \hat{n}_p^- e^{-qz} + w_p \hat{n}_p^+ e^{qz}, & \text{for } -a < z < 0 \end{cases},$$

(9)

where $\hat{n}_p^\pm = (\mp i q, a)$, $a$ is the separation between graphene and the dielectric. From continuity of the tangential component of the electric field at $z = 0$ follows: $1 - R_{dg} = w_p - v_p$. The second boundary condition at $z = 0$ follows from the requirement that at $-a < z < 0$ the amplitude of the outgoing wave is equal to the amplitude of the reflected wave plus the amplitude of the transmitted wave: $w_p = R_g v_p + 1 - R_g$, and at $z = -a$ the amplitude of the outgoing wave is equal to the amplitude of the reflected wave: $v_p = e^{-2q a} R_{dw} R_p$. From these boundary conditions for $qa \ll 1$ follows

$$R_{dg} = 1 - \frac{(1 - R_d)(1 - R_g)}{1 - R_d R_g} = \frac{\epsilon_d - 1 + 2(\varepsilon_g - 1)}{\epsilon_d + 1 + 2(\varepsilon_g - 1)}.$$  

(10)

In the study below we used the dielectric function of graphene, which was calculated within the random-phase approximation (RPA). The dielectric function is an analytical function in the upper half-space of the complex $\omega$-plane:

$$\varepsilon_0(\omega, q) = 1 + \frac{4k_F e^2}{\hbar v_F q} - \frac{e^2 q}{2\hbar\sqrt{\omega^2 - v_F^2 q^2}} \left( G\left(\frac{\omega + 2v_F k_F}{v_F q}\right) - G\left(\frac{\omega - 2v_F k_F}{v_F q}\right) - i\pi \right),$$

(11)

where

$$G(x) = x\sqrt{x^2 - 1} - \ln(x + \sqrt{x^2 - 1}),$$

(12)

where the Fermi wave vector $k_F = (\pi n)^{1/2}$, $n$ is the concentration of charge carriers, the Fermi energy $\epsilon_F = \hbar v_F k_F$, $v_F \approx 10^6$ m/s is the Fermi velocity.

To linear order in the velocity $v$ the friction force $f = \gamma v$ where at $T_1 = T_2 = T$, the friction coefficient

$$\gamma = \frac{\hbar^2}{8\pi^2 k_B T} \int_0^\infty \frac{d\omega}{\sinh^2 \left( \frac{\hbar \omega}{2kB T} \right)} \int_0^\infty dq q^3 e^{-2qd} \frac{\text{Im} R_{1p} \text{Im} R_{2p}}{|1 - e^{-2qd} R_{1p} R_{2p}|^2},$$

(13)
The reflection amplitude for a dielectric given by Eq. 5 has the resonance at the condition $\epsilon'_d(\omega_s) = -1$ where $\epsilon'_d$ is the real part of the dielectric function $\epsilon_d$. This condition determines the frequency $\omega_s$ of the surface phonon polariton mode. Close to the resonance the reflection amplitude can be written in the form:\[16,56\]

$$ R_d \approx -\frac{\omega_a}{\omega - \omega_s + i\eta}. \tag{14} $$

Close to the resonance the transmission coefficient in Eq. 13 for two identical dielectrics can be written in the form

$$ t = \frac{(\text{Im} R_d e^{-qd})^2}{|1 - e^{-2qd} R_d R_d'|^2} \approx \frac{(\omega_a \eta e^{-qd})^2}{(|\omega - \omega_+|^2 + \eta^2)(|\omega - \omega_-|^2 + \eta^2)} \tag{15} $$

where

$$ \omega_\pm = \omega_s \pm \omega_a e^{-qd}. $$

Using Eq. 15 in Eq. 13 gives the resonant contribution to the friction coefficient

$$ \gamma_{\text{res}} \approx \frac{\hbar^2 \eta}{4\pi k_B T \sinh^2 \left( \frac{\omega_a}{k_B T} \right)} \int_0^\infty dq q^3 \frac{(B e^{-qd})^2}{(B e^{-qd})^2 + 1} = \frac{\hbar^2 \eta}{16\pi k_B T \sinh^2 \left( \frac{\omega_a}{k_B T} \right)} \int_0^q dq q^3 $$

$$ = \frac{\hbar^2 \eta \omega_c^4}{16\pi k_B T \sinh^2 \left( \frac{\omega_a}{k_B T} \right)} \tag{16} $$

where it was assumed that $B = \omega_a / \eta > 1$, $\omega_s \gg \omega_a \exp(-qd)$, $q_c = \ln B / d$. At small frequencies far from the resonance ($\omega \ll \omega_s$) $t \approx (\omega \exp(-qd) / \omega^*)^2$ and the off-resonant contribution to the friction coefficient

$$ \gamma_{\text{off res}} \approx \frac{\hbar}{16d^2} \left( \frac{k_B T}{\hbar \omega^*} \right)^2. \tag{17} $$

For SiO$_2$ $\omega_s = 9.6 \cdot 10^{13} \text{s}^{-1}$, $\omega_s = 4.5 \cdot 10^{12} \text{s}^{-1}$, $\omega_s = 1.7 \cdot 10^{12} \text{s}^{-1}$, $\omega_s = 2.3 \cdot 10^{16} \text{s}^{-1}$. With these parameters Eqs. 10 and 17 at $T = 300$K and $d = 1 \text{nm}$ give $\omega_{\text{res}} = 3.5 \cdot 10^{-2} \text{kg s}^{-3} \text{m}^{-2}$ and $\omega_{\text{off res}} = 1.8 \cdot 10^{-5} \text{kg s}^{-3} \text{m}^{-2}$.

Another resonance is possible in the condition of the anomalous Doppler effect when $q_c v \approx 2 \omega_s d / \omega_a \eta$. At this resonant condition, taking into account that $R(-\omega) = R^*(\omega)$, at $q_c = 0$ the denominators in the integrands in Eqs. 5 and 14 contain the factor

$$ 1 - |R(\omega_s)|^2 e^{-\frac{4qd}{\omega_s}}. \tag{18} $$

At the resonance $|R(\omega_s)|$ can be larger than unity thus the denominator is equal to zero at

$$ v_c = \frac{2\omega_s d}{\ln |R(\omega_s)|} \tag{19} $$

what means that above the threshold velocity at $v \gg v_c$ the friction force for two identical plates is divergent. The origin of this divergence is related with the creation above the threshold velocity $v_c$ of the non-dissipative collective eigenmode for two identical plates even in the case when the surface phonon polariton modes for the isolated surfaces are dissipative. Above the threshold velocity $v_c$ the amplitude of this mode increases infinitely with time which gives rise to the divergence of the dissipation and friction\[59\].

Close to the resonance at $\omega' = q_x v - \omega = \omega_s$ the critical velocity $v_c = 2\omega_s d / \ln(\omega_a / \eta)$ and the reflection amplitude can be written in the form

$$ R_d(\omega - q_x v) \approx -\frac{\omega_a}{\omega - \omega_s + i\eta}. \tag{20} $$

Close to the resonance at $\omega \approx \omega' \approx \omega_s$ the transmission coefficient in Eq. 13 for two identical dielectrics can be written in the form

$$ t = \frac{\text{Im} R_d(\omega) \text{Im} R_d(\omega - q_x v) e^{-2qd}}{|1 - e^{-2qd} R_d(\omega) R_d(\omega - q_x v)|^2} \approx \frac{(\omega_a e^{-qd})^2}{(\omega - \omega')^2 + [\omega(\omega - \omega_s) + \eta^2 - \omega^2 \exp(-2qd)]^2}. \tag{21} $$

Using Eq. 21 in Eq. 14 gives the resonant contribution to the friction coefficient from $\omega \approx \omega_s \approx q_x v - \omega_s$

$$ f_{\text{friction}} \approx -\frac{\hbar \eta^2 \ln^2(\omega_a / \eta)}{2 \pi \omega_s d^3} \ln \frac{v_c - v}{v_c}. \tag{22} $$

The divergent factor $\ln(v_c - v) / v_c$ in Eq. 22 can be also written in the form $\ln(d - d_0) / d_0$ where $d_0 = v \ln(\omega_a / \eta) / (2\omega_s)$. Thus at the given velocity $v$ the friction force diverges at $d < d_0$. 

III. CASIMIR FRICTION IN THE TIP-PLATE CONFIGURATION

An atomic force microscope tip with the radius of curvature $R \gg d$, at a distance $d$ above a flat sample surface, can be approximated by a sphere with radius $R$. In this case the friction force between the tip and the plane surface can be estimated using the approximate method of Derjaguin, later called the proximity force approximation (PFA). According to this method, the friction force in the gap between two smooth curved surfaces at short separation can be calculated approximately as a sum of forces between pairs of small parallel plates corresponding to the curved geometry of the gap. Specifically, the sphere-plane friction force is given by

$$ F = 2\pi \int_0^R d\rho \rho f(z(\rho)), $$

where $z(\rho) = d + R - \sqrt{R^2 - \rho^2}$ denotes the tip-surface distance as a function of the distance $\rho$ from the tip symmetry axis, and the friction force per unit area $f(z(\rho))$ is determined in the plate-plate configuration. This scheme was proposed in Refs. 60, 62 for the calculation of the conservative van der Waals interaction; in this case the error is not larger than 5-10% in an atomic force application, and 25% in the worst case situation. We assume that the same scheme is also valid for the calculation of the Casimir friction. However, as it was discussed in Sec. III for two identical plates the Casimir friction diverges at the velocities above the Cherenkov threshold velocity. However, for different plates the friction force can be finite even above the threshold velocity. For the SiO$_2$ the threshold velocity $v_c \approx 2 \times 10^9$ m/s. Thus in the present study the numerical calculations are performed at the velocities below the threshold velocity when one can assume that the PFA gives sufficiently accurate estimation of the Casimir friction including the quantum correction. During the last few years, the most general method available for calculating both Casimir force and radiative heat transfer between many bodies of arbitrary shapes, materials, temperatures, and separations was obtained which expresses the Casimir force and radiative heat transfer in terms of the scattering matrices of individual bodies. Specifically, the numerically exact solution for the near-field radiative heat transfer between a sphere and an infinite plane was first performed using the scattering matrix approach. In principle the same approach can be used for the calculation of the Casimir friction. We assume that the tip has a paraboloid shape given in cylindrical coordinates $(z, \rho)$ by the formula: $z = d + \rho^2/2R$, where $d$ is the distance between the tip and the flat surface. If

$$ f = \frac{C}{(d + \rho^2/2R)^n}, $$

we get

$$ F = \frac{2\pi R}{n-1} \frac{C}{d^{n-1}} = \frac{2\pi Rd}{n-1} f(d) = A_{\text{eff}} S(d), $$

where $A_{\text{eff}} = 2\pi Rd/(n-1)$ is the effective surface area. In a more general case one must use numerical integration to obtain the friction force.

The friction coefficient in the tip-plane configuration $\Gamma$ can be obtained from Eq. 13 using the proximity force approximation. In an experiment $\Gamma$ is determined by measuring the quality factor of the cantilever vibration parallel to the substrate surface. At present only can be detected the friction coefficient in the range $10^{-12} - 10^{-13}$ kg/s. Fig. 3 shows the dependence of the friction coefficient on the separation between a tip and a substrate surface for the different configurations. The red, green and blue curves represent the results for the configurations (a) SiO$_2$-SiO$_2$ (DD) (Fig.4a), (b) SiO$_2$+graphene-SiO$_2$ (DGD) (Fig.4b), and (c) SiO$_2$+graphene - SiO$_2$+graphene (DGGD) (Fig.4c), respectively. The friction coefficient in these configurations is below $10^{-10}$ kg/s thus it can not be tested by the modern experimental setup. However, it has been predicted in Ref. 56, that for the some configurations involving adsorbates the Casimir friction coefficient can be large enough to be measured by state-of-art non-contact force microscope.

During the cantilever vibration the velocity of the AFM tip does not exceed 1 m/s. However, the Casimir friction force can be strongly enhanced at the large relative sliding velocity. This friction force can be detected if it produces sufficiently large bending of the cantilever. Fig. 5 shows the dependence of the friction force, acting on the tip with the radius of the curvature $R = 1\mu$m, on the relative velocity $v$ between the tip and substrate for the SiO$_2$-SiO$_2$ configuration (see Fig.3a) at the separation $d = 1$ nm and for the different temperatures. The friction force $F = F_0 + F_T$ where $F_0$ is the contribution from quantum fluctuations which exist even at $T = 0$K (this friction is denoted as quantum friction), $F_{\text{friction}}(T = 0K) = F_0$ and $F_T$ is the contribution from the thermal fluctuations which exist only at finite temperature. The thermal contribution dominates for $v < k_B T d/h$ and quantum contribution dominates for $v > k_B T d/h$. On Fig. $F > 10^{-12}$ N at $v > 10^5$ m/s. In the modern experiment the spring constant of the cantilever are between $k_0 = 30$ and $k_0 = 50\mu$N/m. The friction force $\approx 10^{-12}$ N will produce the displacement of the tip of the order $10^2$ nm which can be easily detected. However, at present there is no experimental setup...
FIG. 4: The dependence of the friction coefficient $\Gamma$ for the spherical tip with the radius of the curvature $R = 1 \mu m$ on the separation $d$ between the tip and the substrate for the different configurations at $T = 300K$. The red, green and blue curves represent the results for the configurations (a) SiO$_2$-SiO$_2$ (Fig. 3a), (b) SiO$_2$+graphene-SiO$_2$ (Fig. 3b), and (c) SiO$_2$+graphene-SiO$_2$+graphene (Fig. 3c), respectively. The charge carriers concentration for graphene $n = 10^{16} m^{-2}$.

FIG. 5: The dependence of the different contributions to the friction force on the relative sliding velocity $v$ between a tip and a substrate for the SiO$_2$-SiO$_2$ configuration (see Fig. 3a). The blue and red lines show the thermal contributions at $T = 600K$ and $T = 300K$, respectively. The green line shows the quantum contribution ($T = 0K$). The radius of the curvature of the tip $R = 1 \mu m$. The separation between the tip and substrate $d = 1nm$.

with the relative sliding velocity between the tip and substrate $\sim 10^5 m/s$. Fig. 3 shows the friction force for the SiO$_2$+graphene-SiO$_2$ configuration (see Fig. 3b) which is one order of the magnitude smaller the friction force for the SiO$_2$-SiO$_2$ configuration (see Fig. 5).

IV. CASAIMIR FRICTION DRAG BETWEEN A TIP AND A GRAPHENE COVERED SiO$_2$ PLATE

An alternative method for the detection of the Casimir friction is possible for the SiO$_2$+graphene - SiO$_2$ configuration (see Fig. 3b). For this configuration inducing current in a graphene sheet with the drift velocity of the free charge carriers $v_{\text{Drift}}$ will produce the fluctuating electromagnetic field which is similar to the electromagnetic field due to the mechanical motion of the sheet with the velocity $v = v_{\text{Drift}}$ $^{4,34-36}$. Due to the high mobility of the charge carriers in graphene, in a high electric field electrons (or holes) can move with very high velocities (up to $10^6 m/s$). The drift motion of charge carries in graphene will result in a modification of dielectric properties of graphene due to the Doppler effect $^6$. The reflection amplitude for the graphene sheet with induced current density is determined by the reflection amplitude $R'_{g}$ in the reference frame co-moving with the drift motion of the charge carriers in graphene: $R'_{g} = R_{g}(\omega^-)$, where $R_{g}(\omega)$ is the reflection amplitude amplitude in the rest reference frame of the graphene sheet without current, $\omega^- = \omega - q_{x}v$, $v = v_{\text{Drift}}$.

In the vacuum gap between two plates in the configuration SiO$_2$+graphene - SiO$_2$ the electric field $\mathbf{E}(q, \omega, z)$ can
be written in the form\cite{10}

\[ \mathbf{E}(\mathbf{q}, \omega, z) = v_p \hat{n}_p^+ e^{-qz} + w_p \hat{n}_p^- e^{qz}, \]  

(26)

where \( \hat{n}_p^\pm = (\mp i\mathbf{q}, q) \),

\[ v_p = \frac{E_{dg}^{f'} + R_{dg}' E_d' e^{-q d}}{1 - e^{-2q d} R_{dg}' R_d}, \quad w_p = \frac{R_d E_{dg}^{f'} e^{-2q d} + E_d' e^{-q d}}{1 - e^{-2q d} R_{dg}' R_d}. \]  

(27)

where \( E_{dg}^{f'} \) and \( E_d \) are the amplitudes of the fluctuating electric fields created on the surfaces of plates by the charge density fluctuations inside the SiO\(_2\) + graphene and SiO\(_2\) plates, respectively, and where, in the presence of the drift motion of the free charge carriers in a graphene sheet with the drift velocity \( v_{\text{Drift}} \), the reflection amplitude \( R_{dg}' \) is given by Eq. (10) with \( R_g \) replaced on \( R_g' = R_g(\omega^-) \)

\[ R_{dg}' = 1 - \frac{(1 - R_d)(1 - R_{dg})}{1 - R_d R_{dg}} = \frac{\varepsilon'_d - 1 + 2(\varepsilon'_g - 1)}{\varepsilon'_d + 1 + 2(\varepsilon'_g - 1)}, \]  

(28)

where \( \varepsilon'_g = \varepsilon_g(\omega^-) \). To get the expression for \( E_{dg}^{f'} \) resulting from the interference of the electric fields created by the SiO\(_2\) plate and the graphene sheet, we assume that the graphene layer is located at \( z = 0 \) and the dielectric at \( z < -a \). The electric field can be written in the form

\[ \mathbf{E}(\mathbf{q}, \omega, z) = \begin{cases} E_{dp}^{f'} \hat{n}_p^+ e^{-qz}, & \text{for } x > 0 \\ v_p \hat{n}_p^+ e^{-qz} + w_p \hat{n}_p^- e^{qz}, & \text{for } -a < z < 0 \end{cases}, \]  

(29)

where \( \hat{n}_p^\pm = (\mp i\mathbf{q}, q) \), \( a \) is the separation between graphene and the dielectric. From continuity of the tangential component of the electric field at \( z = 0 \) follows: \( E_{dg}^{f'} = v_p - w_p \). The second boundary condition at \( z = 0 \) follows from the requirement that at \( -a < z < 0 \) the amplitude of the outgoing wave is equal to the amplitude of the reflected wave plus the amplitude of the wave emitted by the graphene sheet due to the charge fluctuations inside the sheet: \( w_p = R_g v_p - E_d^{f'} \), and at \( z = -a \) the amplitude of the outgoing wave is equal to the amplitude of the reflected wave plus the amplitude of the wave emitted by the SiO\(_2\) plate: \( v_p = e^{-2q d} R_d w_p + E_d^{f'} \). From these boundary conditions for \( qa \ll 1 \) follows

\[ E_{dg}^{f'} = \frac{E_d^{f'} (1 - R_d') + E_g^{f'} (1 - R_d)}{1 - R_d R_{dg}'} = \frac{E_d^{f'} (\varepsilon_d + 1) + 2 E_d^{f'} \varepsilon_g'}{\varepsilon_d + 1 + 2(\varepsilon'_g - 1)}, \]  

(30)

where \( E_d^{f'} \) and \( E_g^{f'} \) are the electric fields created by the charge density fluctuations in the SiO\(_2\) plate and in the graphene sheet with the drift motion of the charge carriers, respectively. According to the general theory of the fluctuating electromagnetic field\cite{11} the spectral density of the fluctuations for the electric field:

\[ < |E_f|^2 >_\omega = \frac{2\hbar}{q} \left( n_i(\omega) + \frac{1}{2} \right) \text{Im} R_i, \]  

(31)
FIG. 7: The same as on Fig. [6] but for the dependence of the thermal contributions to the friction force on the drift velocity \( v_{\text{Drift}} \) of the charge carriers in graphene. The quantum friction (not shown) is negligible in the range of the velocities shown on the figure.

where \(< \ldots >\) denote statistical average over the random field, \( i = d, g \). The frictional stress \( f_x \) acting on the surface of the SiO\(_2\) plate is determined by \( xz\)-component of Maxwell’s stress tensor \( \sigma_{ij} \), calculated at \( z = +0 \):

\[
f_x = \sigma_{xz} = \frac{1}{8\pi} \int_{-\infty}^{+\infty} d\omega \langle (E_z E_x^*) + \langle E_z E_x \rangle \rangle_{z=+0} = 2\text{Im} \int_{0}^{\infty} \frac{d\omega}{2\pi} \int \frac{d^2q}{(2\pi)^2} \frac{q_x}{q} \langle w^*_p v_p \rangle,
\]

where the symbol \( \langle \ldots \rangle \) denotes statistical averaging on the random fields \( E_d \) and \( E_g \). Using Eqs. (26)-(30) in Eq. (32), after averaging over the random electric fields in Eq. (32) using Eq. (31), we get

\[
f_x = \frac{\hbar}{2\pi^3} \int_{0}^{\infty} d\omega \int d^2q \frac{\text{Im} R_d}{1 - e^{-2g_d \text{Im} R_d \text{Im} R'_d} + 2|\text{Im} R_d| \text{Re} \text{Im} R'_d} \left( \text{Im} R'_d [n_g(\omega) - n_d(\omega)] \right.
\]

\[
+ \frac{2\text{Im} \epsilon_d}{|\epsilon_d + 1 + 2(\epsilon'_d - 1)|^2} [n_g(\omega) - n_g(\omega^-)] \right) \right)
\]

In Eq. (33) the factors \( n_g \) and \( n_d \) are calculated at the temperatures \( T_g \) and \( T_d \) for the SiO\(_2\)+graphene and SiO\(_2\) plates, respectively. The contribution to the friction from the quantum fluctuations can be obtained from Eq. (33) at \( T_g = T_d = 0 \text{ K} \)

\[
f_x(T = 0 \text{ K}) = -\frac{2\hbar}{\pi^3} \int d^2q \int_{0}^{q_{x,v}} d\omega \frac{\text{Im} R_d \text{Im} \epsilon'_d}{1 - e^{-2g_d \text{Im} R_d \text{Im} R'_d} + 2|\text{Im} R_d| \text{Re} \text{Im} R'_d} \left( \text{Im} R'_d [n_g(\omega) - n_g(\omega^-)] \right.
\]

Fig.7 shows the dependence of the frictional drag force acting on the SiO\(_2\) tip in the SiO\(_2\)+graphene-SiO\(_2\) configuration on the drift velocity \( v_{\text{Drift}} \) of electrons in the graphene sheet. For \( v_{\text{Drift}} > 10^6 \text{m/s} \) the friction force is above \( 10^{-12} \text{N} \) and can be measured by state-of-art non-contact force microscope. It is important to note that in the contrast to the SiO\(_2\)-SiO\(_2\) configuration, where the bending of the cantilever due to the Casimir friction can only be detected for very large relative sliding velocity between a tip and a substrate, in the SiO\(_2\)+graphene-SiO\(_2\) configuration the friction of the same order can be obtained inducing the current density in the graphene sheet by high electric field what can be easily obtained using the modern experimental setup.

V. SUMMARY

A current in the graphene sheet produces a fluctuating electromagnetic field which is similar to the field produced by the moving sheet. In a high electric field electrons in non suspended graphene on the SiO\(_2\) substrate can move with sufficiently large drift velocity (above \( 10^6 \text{m/s} \)) to produce the frictional drag force acting on a tip which can be measured by state-of-art non-contact force microscope. Both the thermal and quantum contributions to the Casimir
friction can be detected using this experimental setup. These results pave the ways for the mechanical detection of the Casimir friction and for the application of the frictional drag effect in micro- and nano-electromechanical devices (MEMS and NEMS).

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