Group Method of Data Handling as a Tool to Determine Vertical Displacements

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Abstract. Surveying measurements carried out to determine displacements and deformations of existing civil structures and their surroundings provide information making it possible to represent their geometry in space and any changes it undergoes over time. Data acquired through geodetic monitoring can be modelled using artificial neural networks, capable of learning (adaptability) and quick operation and providing the possibility of visualisation by means of computer simulation. Neural networks, however, require specification of an optimal architecture by the user, as a result of which any resulting solutions are flawed by a difficult to identify error of method. Therefore, this article proposes an alternative approach in the form of the Group Method of Data Handling (GMDH) based on evolutionary algorithms. The article presents the fundamental assumptions for the GMDH and the principles of development and training of static neural networks with multiple inputs and one output. The GMDH network was used to develop a geometric model of vertical displacements determined on the basis of periodic measurements taken on civil structures.

1. Introduction
The structure of artificial neural networks is based on that of the human neuron, therefore they are ideal to solve problems which require learning, associating facts, classifying elements and recognising patterns. Artificial neural networks solve the problems without knowing mathematical relationships and connections between input and output data [1] [3] [10]. However, neural networks – despite a number of advantages such as adaptability, the capability of generalization of acquired knowledge, parallel computing, the capability of modelling non-linear correlations and resistant to disturbances occurring in datasets – are flawed by an error of method which is difficult to identify and eliminate and which results from the network architecture assumed by the user. In order to minimize the influence of the error on the effects of neural network training evolutionary algorithms can be used, in which the training comprises the topology of connections between neurons [2]. A polynomial GMDH (Group Method of Data Handling) network is an example of such an approach. The network creates its architecture through adaptation, on the basis of a training dataset [6] [11]. The structure of such a network comprises partial models based on polynomials developed by [5] [2], which enable the achievement of the most satisfactory end result of network training within the assumed error criterion.

GMDH algorithms are applied to solve a number of problems related to, for example, short- and long-term forecasting, approximation of multidimensional processes, classification, pattern
recognition or equipment and process diagnostics. In this study, the GMDH is used to predict vertical displacements of a civil structure on which regular measurements were taken in 2012.

2. GMDH algorithm

Multi-layer artificial neural networks, whose structure (the number of layers and neurons in each layer) is defined in an arbitrary way, are most often applied in practice [8]. As a result, they comprise a hardly avoidable error of method which can be minimized if an approach based on evolutionary algorithms is taken, whereby the training process can be combined with the determination of an optimal structure for the neural network.

Evolutionary algorithms provide the possibility of finding solutions to optimization problems, using a solution selection process based on the assumed objective function (measure of adaptation) to successive iterations. This kind of approach enables the formation of new sets of solutions which become better and better adapted to a given environment with each subsequent iteration. The Group Method of Data Handling (GMDH) is an example of a method which belongs to the class of evolutionary algorithms. It consists in the replacement of a whole neural network model by a hierarchical structure made of polynomial partial models. A GMDH network is made up of a number \(m\) of single neurons (the structure of a single neuron is shown in figure 1) which process the input signal – vector \(x\) – and turn it into an output signal, \(y\). The signal is processed when at least two input signals are stimuli, according to the following relation:

\[
y = f(x) = f(x_1, x_2, \ldots, x_m),
\]

where \(f\) is the transfer function.

The transfer function must not be too complex, as this would extend the time required for training and would prevent an accurate assessment of the training error. Therefore, although the GMDH algorithm permits the application of various forms of the transfer function, the function is most often merely an approximation of the \(n\)th-degree Kolmogorov-Gabor polynomial, defined as [4]:

\[
y = a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_i x_j + \cdots,
\]

where \(a_0, a_i, a_{ij}\) are polynomial parameters. Assuming that the polynomial has a degree of \(n=2\), the transfer function is defined as:

\[
y = a_0 + a_1 x_1 + a_2 x_2 + a_{11} x_1^2 + a_{22} x_2^2 + a_{12} x_1 x_2.
\]

![Figure 1. Structure of a single GMDH neuron (own work)](image)
selection methods. Such a course of action makes the network structure closer to the optimum with each subsequent solution.

A schematic representation of a GMDH network synthesis is shown in figure 2. It is an iterative process, in which the first iteration involves the formation of an input layer described by the transfer function (1), taking into account all possible combinations of the input signals. The transfer function describes input neurons and contains the parameters of the Ivakhnenko polynomial, individually optimized for each neuron using, for example, the least squares method.

![Schematic representation of a GMDH network synthesis](image)

**Figure 2.** Synthesis of a GMDH type network (own work)

The selection performed as the second stage of the synthesis eliminates the neurons for which the processing error exceeded the assumed criterion, $\varepsilon$. Previous studies [9] demonstrated that the best effects when processing surveying measurement results are obtained by means of the optimal population method used to perform the selection of neurons. This method was used here, as well.

Having selected the neurons (figure 3), a new layer has to be created and then incorporated in the network, and in a subsequent iteration output signals from the previous layer are used as input data.

![Selection of neurons in the input layer](image)

**Figure 3.** Selection of neurons in the input layer (for 1 neuron in the first layer (1)) (own work based on [2])

The process of selection and learning is continued in subsequent iterations until the optimal structure of the network is achieved, satisfying the optimality criterion $Q_{opt}$ [7]. In this study, the optimality criterion was assumed as follows [2]:

$$Q_{opt}$$
where: \( p \) – data set size, \( y \) – known value of input signal, \( \tilde{y} \) – estimated value of input signal.

An error of the processing of a neuron determined on the basis of the assumed criterion is the reason for removing the neuron from the network or including it in another layer. The process of synthesis of a polynomial GMDH network is verified by identifying errors for the data which was not used in the learning process (a testing set).

![Figure 4. Sketch of the location of control network points (own work)](image)

3. Numerical example
The numerical analysis was based on the results of displacement measurements taken on a complex of buildings where cracks had been observed, indicating differential settlement. A network of control
points was set up on the civil structure. The scope of field work and desktop analysis covered measurements taken using precision levelling and analysis of the accuracy of the measurements. Figure 4 shows the location of the control points. The control and measurement network comprised 40 control points:

- 24 points fixed on external walls of the buildings which were directly threatened by subsidence,
- 8 points fixed inside a link structure joining the threatened buildings,
- 8 reference points on surrounding buildings.

The measurements were taken using a Zeiss Ni 007 precise automatic level on a wooden tripod and a 1.75 m invar levelling staff. The measurements were validated on site. The deviations did not exceed the permitted tolerances. Upon network approximation, the obtained values of mean error of the control point height, $m_{ht}$, ranged from $\pm 0.03$ mm to $\pm 0.07$ mm, and the mean measurement error, $m_{\Delta h}$, ranged from $\pm 0.02$ mm to $\pm 0.05$ mm. A prediction of vertical displacements was undertaken using the GMDH neural networks on the basis of the results of periodic measurements of the control network established on the analysed civil structure. In order to accomplish the task and perform network training, an input vector with the local coordinates of the control points was introduced at the start along with the values of vertical displacement obtained from previous periods of measurement. An analysis of the calculations revealed that the mean error of prediction amounted to $\pm 0.03$ mm. Table 1 contains a summary of the displacement values obtained through conventional measurement (precise levelling) and through the application of the GMDH algorithm, and figure 5 shows their graphic representation.

Table 1. Summary of displacement values obtained through conventional measurement and using the GMDH algorithm

| CP No. | Precise levelling [mm] | GMDH [mm] | Difference [mm] |
|-------|------------------------|-----------|----------------|
| 1     | 0.16                   | 0.12      | -0.04          |
| 2     | -0.01                  | 0.03      | 0.04           |
| 3     | 0.07                   | 0.11      | 0.04           |
| 4     | 0.13                   | 0.15      | 0.02           |
| 5     | 0.09                   | 0.11      | 0.02           |
| 6     | 0.17                   | 0.15      | -0.02          |
| 7     | 0.10                   | 0.12      | 0.02           |
| 8     | -0.03                  | -0.07     | -0.04          |
| 9     | -0.16                  | -0.17     | -0.01          |
| 10    | -0.18                  | -0.21     | -0.03          |
| 11    | -0.01                  | 0.01      | 0.02           |
| 12    | -0.01                  | 0.02      | 0.03           |
| 13    | -0.02                  | -0.04     | -0.02          |
| 14    | 0.00                   | 0.02      | 0.02           |
| 15    | 0.07                   | 0.11      | 0.04           |
| 16    | 0.02                   | -0.01     | -0.03          |

| CP No. | Precise levelling [mm] | GMDH [mm] | Difference [mm] |
|-------|------------------------|-----------|----------------|
| 17    | 0.12                   | 0.14      | 0.02           |
| 18    | 0.01                   | 0.05      | 0.04           |
| 19    | -0.20                  | -0.22     | -0.02          |
| 20    | 0.02                   | 0.05      | 0.03           |
| 21    | 0.21                   | 0.24      | 0.03           |
| 22    | 0.07                   | 0.05      | -0.02          |
| 23    | 0.06                   | 0.04      | -0.02          |
| 24    | 0.20                   | 0.15      | -0.05          |
| 25    | 0.12                   | 0.09      | -0.03          |
| 26    | 0.30                   | 0.32      | 0.02           |
| 27    | 0.18                   | 0.24      | 0.06           |
| 28    | 0.30                   | 0.33      | 0.03           |
| 29    | 0.16                   | 0.19      | 0.03           |
| 30    | 0.20                   | 0.18      | -0.02          |
| 31    | 0.20                   | 0.24      | 0.04           |
| 32    | 0.14                   | 0.16      | 0.02           |
Figure 5. Values of vertical displacement obtained through measurement and through the application of the GMDH algorithm for particular controlled points (own work).

As shown, the maximum difference between the values of displacement obtained using both methods is 0.06 mm and was recorded for CP No. 30. The results of the application of the two methods are visualized in geometric network operation models in figure 6.

Figure 6. Geometric displacement model: a) measurement, b) prediction (own work)
4. Conclusions
This article describes the possibility of application of a GMDH algorithm to determine vertical displacements of a network of control points established on a civil structure. The presented example involves the Group Method of Data Handling which enables the construction of an optimal structure of a neural network during the training process, thus improving the efficiency of its operation. An approach like this may be used even if only small and limited datasets are available, which is often the case in surveying measurements. The values of displacement obtained using conventional measurements and approximation were compared with those determined using the GMDH algorithm.

The results revealed that the prediction error did not exceed ±0.03 mm, and the maximum difference between the values of vertical displacement obtained using the two methods was 0.06 mm. This proves that the results of calculations based on conventional methods are equivalent to the results obtained through the application of a GMDH algorithm.

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