Weak charges quantization in $SU(3)_c \otimes SU(n)_L \otimes U(1)_Y$ gauge models

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received 29 September 2022; accepted in final form 23 November 2022
published online 28 December 2022

Abstract – After proving, in a previous paper, that the electric charge quantization occurs as a natural consequence in renormalizable $SU(3)_c \otimes SU(n)_L \otimes U(1)_Y$ gauge models, we take here a step further within the same paradigm in order to obtain the precise weak charges quantization. To this end a viable boson mass spectrum is obtained first, once a proper parametrization in the Higgs sector is taken into consideration. Hence, by diagonalizing the neutral bosons mass matrix, the quantized neutral weak charge operators are obtained. The Standard Model phenomenology is not affected at all, as its scale ($v_{SM} = 246$ GeV) is decoupled from the higher scale ($V \sim 10$ TeV) specific to our generalized electro-weak unification.

Introduction. – In a recent paper [1] the author worked out in some detail the particular method (conceived by Cotăescu [2]) for treating the generalized gauge models of the type $SU(3)_c \otimes SU(n)_L \otimes U(1)_Y$. This kind of models is supposed to play a decisive role for some of the yet unsolved challenges in particle physics. For several decades now, such gauge models (for the particular cases $n = 3, 4$) have been regarded in the literature as plausible extensions of the well-established Standard Model (SM) [3–5]. The so called 3-3-1 models [6–41] or 3-4-1 models [42–66] are still extensivly investigated in their own right, since promising results in addressing the tiny neutrino masses, flavor physics and CP-phase violation issue, dark matter candidates or $g–2$ muon discrepancy have already been inferred. Meanwhile, some papers [67–69] dared to propose even 3-5-1 models in order to successfully face the available observational data [70]. However, among many shortcomings the theory of the SM contains, one can list, for instance, the lack of predictions regarding the pattern of the electric charge quantization observed in nature and the number of precisely 3 fermion generations. Both these undisputed observational facts can be part of the outcome in each of the particular extensions mentioned above. In ref. [1] it was elegantly proved, for the most general case $SU(3)_c \otimes SU(n)_L \otimes U(1)_Y$ (with $n$ arbitrary), that the renormalization requirement involving the cancellation of the triangle anomalies leads precisely to the true electric charge quantization and to the necessity for 3 fermion generations (no more, no less). These consequences occur regardless of the values $n$ can take, as long as $SU(3)_c$ remains the gauge group of the vector QCD.

What about the “weak charges”, once the predicted electric charges are quantized? Here we deal with this topic, which – we prove straightforwardly in the following – arises as a consequence of the Higgs sector’s parametrization, by simply assuming and exploiting the prescriptions of the general method [2]. Obviously, the natural decoupling of the higher scale of the generalized model ($V \sim 10$ TeV) from the SM’s scale ($v_{SM} = 246$ GeV) must occur as an undisputed feature of our approach, since the avoidance of any alteration to the SM phenomenology is to be sought. A smart choice of the parameters in the Higgs sector ensures unequivocally that outcome at the end of the proceedings. The resulting values for the weak couplings precisely match the SM ones for the known quarks and leptons, while the predicted values for the exotic quarks and leptons indeed open up a rich phenomenological path to be worked out in future works.

This letter contains 5 sections. In the next section the model’s sectors are briefly reviewed, while the boson mass spectrum is obtained in the third section, as a consequence of a suitable parametrization in the Higgs sector. The fourth section is properly providing us with the quantized weak charges and is followed by the last section reserved for some comments.

$SU(n)_L \otimes U(1)_Y$ electro-weak sector. –

Fermion sector. – The particle content of the 3–n–1 model under investigation here is displayed below. As [1]
instructs us, there must be precisely three left-handed generations of leptons and quarks, occurring in the following distinct multiplets \((n\text{-plets})\):

\[
L_{iL} = \begin{pmatrix} N'_i \\ N_i \\ \nu_i \\ e_i \end{pmatrix}_L, \quad Q_{1L} = \begin{pmatrix} U' \\ U \\ d \\ u \end{pmatrix}_L, \quad Q_{2L,3L} = \begin{pmatrix} D_{2,3} \\ D_{2,3} \\ u_{2,3} \end{pmatrix}_L
\]

with \(i = 1, 2, 3\). A right-handed singlet partner corresponds to each left-handed fermion displayed in multiplets above.

The irreducible representations with respect to the model’s gauge group \(SU(3)_c \otimes SU(n)_L \otimes U(1)_Y\) are [1]

\[
L_{iL} \simeq \begin{pmatrix} 1, n, -1/n \end{pmatrix}, \quad e_{iR} \simeq \begin{pmatrix} 1, 1, -1 \end{pmatrix},
\]

\[
\nu_{iR}, N_{iR}, N'_{iR}\ldots \simeq \begin{pmatrix} 3, 1, 0 \end{pmatrix},
\]

\[
Q_{1L} \simeq \begin{pmatrix} 3, n, 2n-3 \end{pmatrix}/3, \quad Q_{kL} \simeq \begin{pmatrix} 3, n^*, 3-n \end{pmatrix}/3,
\]

\[
u_{iR}, u_{kR}, U_{iR}, U'_{iR}\ldots \simeq \begin{pmatrix} 3, 1, 2 \end{pmatrix}/3,
\]

\[
d_{kR}, d_{kR}, D_{kR}, D'_{kR}\ldots \simeq \begin{pmatrix} 3, 1, -1/3 \end{pmatrix},
\]

with \(k = 2, 3\).

These assignments are not arbitrary at all, but inferred (see sect. 3.2 in [1]) by simply demanding two distinct features to be observed: i) the renormalizability criterion —ensured by the cancellation of the axial anomalies—and ii) the algebraic constraint on the gauge bosons whose electric charges are allowed to be only \(\pm e \) or 0. The last condition restricts the electric charges of all fermions in the model to the following values: \(\pm e, \pm 2e/3, \pm e/3\) or 0, and nothing else (i.e., no exotic electric charges).

**Gauge sector.** Now let us turn our attention to the electro-weak interactions in the model at hand. They are mediated by vector bosons defined by the adjoint representation algebra of the (still unbroken) semi-simple gauge group \(SU(n)_L \otimes U(1)_Y\) of the electro-weak part of the model.

That is,

\[
A_\mu = \begin{pmatrix} D_\mu \ldots Y_{\mu}^{00} \ Y_{\mu}^0 \ Y_{\mu}^0 \ Y_{\mu}^0 \ Y_{\mu}^0 \\ \vdots \vdots \vdots \vdots \vdots \\ Y_{\mu}^{00} \ldots D_{\mu}^{-4} \ X_{\mu}^{00} \ X_{\mu}^0 \ Y_{\mu}^{+} \\ Y_{\mu}^{00} \ldots X_{\mu}^{00} \ D_{\mu}^{-3} \ X_{\mu}^0 \ Y_{\mu}^{+} \\ Y_{\mu}^{00} \ldots X_{\mu}^{00} \ X_{\mu}^{00} \ D_{\mu}^{-2} \ W_{\mu}^{+} \\ \vdots \vdots \vdots \vdots \vdots \\ Y_{\mu}^{-} \ Y_{\mu}^{-} \ W_{\mu}^{-} \ D_{\mu}^{-1} \\ \end{pmatrix},
\]

with the diagonal entries (corresponding to the Cartan subalgebra of the \(su(n)_L \otimes u(1)_Y\) algebra):

\[
D_\mu^1 = \frac{1}{2} A_\mu^3 + \frac{1}{2\sqrt{3}} A_\mu^8 + \ldots + Y B_\mu^0, \\
D_\mu^2 = -\frac{1}{2} A_\mu^3 + \frac{1}{2\sqrt{3}} A_\mu^8 + \ldots + Y B_\mu^0, \\
D_\mu^3 = -\frac{1}{\sqrt{3}} A_\mu^8 + \frac{1}{2\sqrt{6}} A_\mu^{15} + \ldots + Y B_\mu^0, \\
\ldots \\
D_\mu^{-1} = -\frac{\sqrt{n-1}}{\sqrt{2n}} A_\mu^{2n-1} + Y B_\mu^0.
\]

The off-diagonal entries are cast as \(B_\mu^{\alpha\beta} = \frac{1}{\sqrt{2}} (A_\mu^\alpha \pm iA_\mu^\beta)\) with \(\alpha, \beta = 1, 2, \ldots, n, \alpha \neq \beta\). Evidently, the off-diagonal entries account either for charged bosons (if \(\alpha = n = \beta\)), or for neutral bosons (if simultaneously \(\alpha \neq n \) and \(\beta = n\)). That means there are gauge bosons exhibiting only \(\pm e\) charges, as we stated above.

**Scalar sector.** The spontaneous symmetry breaking (SSB) is achieved by means of an appropriate scalar sector consisting of the following \(n\) scalar multiplets [1]:

\[
\phi^{(k)} = \begin{pmatrix} \phi^{(k)}_1 \\ \phi^{(k)}_2 \\ \vdots \\ \phi^{(k)}_n \end{pmatrix} \sim \begin{pmatrix} 1, n, -1/n \end{pmatrix}, \quad k = 1, \ldots, n-1,
\]

\[
\phi^{(n)} = \begin{pmatrix} \phi^{(n)}_1 \\ \phi^{(n)}_2 \\ \vdots \\ \phi^{(n)}_n \end{pmatrix} \sim \begin{pmatrix} 1, n, n-1/n \end{pmatrix},
\]

developing each of them its own vacuum expectation value (VEV), in the manner \(\langle \phi^{(i)} \rangle = \eta_i V\), due to a set of real parameters \(\eta_i \in (0, 1)\) once a unique overall scale \(V\) in the model is assumed. These parameters are organized in a \(n \times n\) diagonal matrix \((\eta)\) whose trace must obey (following the prescriptions of the general method [2]) the constraint \(\text{Tr} (\eta)^2 = 1\). More explicitly, \(\eta_1^2 + \eta_2^2 + \ldots + \eta_n^2 = 1\), so that the relation among all \(n\) VEVs \(\langle \phi^{(1)} \rangle^2 + \langle \phi^{(2)} \rangle^2 + \ldots + \langle \phi^{(n)} \rangle^2 = V^2\) holds. A convenient parameter matrix could be, without loss of generality, the following:

\[
\eta^2 = \text{Diag} \left( \frac{1-a}{n-2}, \frac{1-a}{n-2}, \ldots, \frac{1-a}{n-2}, \frac{a-b}{2}, \frac{a+b}{2} \right),
\]

which, evidently, fulfills the trace requirement. Moreover, it will split the VEVs, if one considers tuning \(a, b \to 0\) (very small), so that \(\langle \phi^{(1)} \rangle, \langle \phi^{(2)} \rangle, \ldots, \langle \phi^{(n-2)} \rangle \sim V\) and \(\langle \phi^{(n-1)} \rangle, \langle \phi^{(n)} \rangle \sim v_{SM}\). This play an important role in keeping the SM phenomenology decoupled from the high scale of this model \((v_{SM} \ll V)\).
\[ \frac{2m^2}{n-1} \begin{pmatrix} 1 & \left( \frac{n^2 - 4n + 2}{2} a - \frac{n - 2}{2} b \right) \\ \frac{1}{\sqrt{n(n-2)\cos \theta}} \left(1 - \frac{n - 2}{2} b\right) & \frac{1}{n\cos^2 \theta} \left[1 + \frac{n(n-2)}{2} a + \frac{n(n-2)}{2} b\right] \end{pmatrix} \]  

(11)

\[ M^2_{2 \times 2}(Z, Z') = \frac{2m^2}{n-1} \begin{pmatrix} 1 + a \frac{n^2 - 4n + 2}{2} b & \frac{1}{\sqrt{n(n-2)\cos \theta}} \left[1 - \frac{n - 2}{2} b\right] \\ \frac{1}{\sqrt{n(n-2)\cos \theta}} \left[1 - \frac{n - 2}{2} b\right] & \frac{1}{n\cos^2 \theta} \left[1 + a \frac{n(n-2)}{2} (1 - 2s_W^2)\right] \end{pmatrix} \]  

(13)

\[ \omega = \frac{1}{\sqrt{2(n-1)c_W}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\sqrt{-n-2} & \sqrt{-n-2} \\ 0 & -\sqrt{-n-2} & \sqrt{-n-2} \end{pmatrix} \]  

(14)

\[ M^2(Z_1) \begin{pmatrix} M^2(Z_1) & 0 & 0 \\ 0 & M^2(Z_{n-3}) & 0 \\ 0 & 0 & M^2_{2 \times 2}(Z, Z') \end{pmatrix} \]  

(10)

Boson mass spectrum. — With the above parameter choice and the natural tuning \( a, b \to 0 \), one gets — once the SSB is achieved (according to the prescriptions of the general method [2]) — the mass matrix for the Hermitian bosons mediating the weak interactions,

\[ M^2 = \begin{pmatrix} M^2(Z_1) & 0 & 0 \\ 0 & M^2(Z_{n-3}) & 0 \\ 0 & 0 & M^2_{2 \times 2}(Z, Z') \end{pmatrix} \]  

(10)

As expected, \( n - 3 \) heavier bosons are completely decoupled from \( Z \) and \( Z' \). The latter ones mix with one another. Therefore, only \( Z' \) could, in principle, interfere with the SM phenomenology supplied by \( Z \). Hence, the \( 2 \times 2 \) mass matrix \( M^2_{2 \times 2}(Z, Z') \) goes actually through the diagonalization procedure. Here \( \theta \) represents the rotation angle of a generalized Weinberg transformation (see sect. 5 in ref. [2]) that singles out the massless electromagnetic direction from other directions in the parameter space. It is connected to the SM Weinberg angle \( \theta_W \) in this way: \( \sin \theta = \sqrt{\frac{2m^2 - n}{n}} \sin \theta_W \), as was proved in [1].

At this stage one must enforce the SM mass relation (while making use of the notation \( m^2 = \frac{1}{2} g^2 V^2 \)). That is, \( M^2(Z) = m^2 a/\cos^2 \theta_W \) (with the PDG [70] estimation \( M(Z) \simeq 91.2 \text{ GeV} \)) is promoted to the status of eigenvalue of the matrix in eq. (11). In our parametrization [1], obviously \( M(W^\pm) = m \sqrt{a} \) (with \( M(W^\pm) \simeq 80.4 \text{ GeV} \) [70]).

Hence, after a few manipulations, one obtains a nice constraint on the two free parameters \( a \) and \( b \), namely

\[ (a \tan^2 \theta_W + b)^2 = 0. \]  

(12)

Under these circumstances, the mass matrix to be diagonalized becomes the following one-parameter matrix:

\[ \text{see eq. (13) above} \]

The diagonalization of the mass matrix in eq. (10) is now performed by simply employing the following \( SO(n - 1) \) matrix:

\[ \text{see eq. (14) above} \]

in the manner

\[ \omega M^2 \omega^T = \text{Diag} [M^2(Z_1), M^2(Z_2), \ldots, M^2(Z_{n-3}), M^2(Z), M^2(Z')] \]  

(15)

The eigenvalues, specific to our 3-n-1 model, read

\[ M^2(Z') = \frac{4m^2(1 - s_W^2)}{(n-2)(n-1)s_W^2} \times \left[ 1 + a \frac{n(n-4)(1 - 2s_W^2) - 4s_W^4}{4(1 - s_W^2)^2} \right] \]  

(16)

and respectively

\[ M^2(Z_1) = M^2(Z_2) = \ldots = \]  

\[ M^2(Z_{n-3}) = m^2 \left( \frac{2}{n-2} \right) (1 - a). \]  

(17)

The above expressions (16), (17) yield masses much heavier than \( Z \) mass, if one considers the parameter \( a \) very small, say its order of magnitude is \( O(10^{-3}) \) or smaller. Such a tuning is meant to ensure an overall breaking scale \( V \) around 10 TeV or higher, according to \( v_{SM} = \sqrt{3} V \) (see [1]).
Weak charges quantization. – Having established the diagonalization matrix $\omega$, all the weak charge operators $Q^\rho(Z_i)$ can be immediately inferred, according to the prescriptions of the general method [2], in the following way:

$$Q^\rho(Z_i) = g \left[ D_k^\rho - \nu_k (D^\rho_{\nu} (1 - \cos \theta) - \nu_k g Y^\rho \sin \theta) \right] \omega^k_i,$$

(18)

where $\nu_k$ are associated to the Hermitian diagonal generators of the gauge group. As was proved in ref. [1], the avoidance of the exotic electric charges implies the selection $\nu_{n^2 - 1} = 1$ accompanied simultaneously by the vanishing condition for all the other versors $\nu_3 = \nu_8 = \nu_{15} = \ldots = 0$. That means, in fact, these very versors properly discriminate among the various models based on the same gauge group. (A different versors choice leads to different versions of the model that could include exotic electric charges, that we avoid here.)

Thus, the neutral charge operators become, in our particular $3$-$n$-$1$ model (up to a $e/s_w c_w$ factor),

$$Q^0(Z) = -\sqrt{\frac{n - 2}{2(n - 1)}} T^0_{n^2 - 2n} + \sqrt{\frac{n - 2(n - 1)s_w^2}{2(n - 1)}}$$

$$\times \left( T^0_{n^2 - 1} \frac{n - 2(n - 1)s_w^2}{n} \right)$$

$$-\frac{2(n - 1)}{n - 2(n - 1)s_w^2 Y^\rho},$$

(19)

$$Q^\rho(Z) = -\sqrt{\frac{n - 2(n - 1)s_w^2}{2(n - 1)}} T^\rho_{n^2 - 2n} - \sqrt{\frac{n - 2}{2(n - 1)}}$$

$$\times \left( T^\rho_{n^2 - 1} \frac{n - 2(n - 1)s_w^2}{n} \right)$$

$$-\frac{2(n - 1)}{n - 2(n - 1)s_w^2 Y^\rho}. \quad (20)$$

The decoupled heavier bosons’ couplings straightforwardly yield

$$Q^\rho(Z_1) = g T^\rho_{1} = \frac{e}{2s_w} \text{Diag}(1, -1, 0, \ldots, 0, 0),$$

(21)

$$Q^\rho(Z_2) = g T^\rho_{4} = \frac{e}{2\sqrt{3} s_w} \text{Diag}(1, 1, -2, \ldots, 0, 0),$$

(22)

$$Q^\rho(Z_{-3}) = g T^\rho_{n^2 - 3} = \frac{e}{\sqrt{2(n - 2)(n - 3)s_w}}$$

$$\times \text{Diag}(1, 1, \ldots, 3 - n, 0, 0),$$

(23)

where we made use of the notations $s_w = \sin \theta_W$ and $c_w = \cos \theta_W$ along with the identification $e = g \sin \theta_W$ (once we assumed that the coupling $g$ of the $SU(n)_L$ in our model is identical to $g$ of the $SU(2)_L$ in the SM).

Now, with some simple algebraic computations, the neutral charges are inferred for all the irreducible representations in our model. For the sake of simplicity, we will express these charges (as usual) in $e/2s_w c_w$ units, in order to instantly compare them to the well-known SM predicted values.

The resulting couplings with the SM neutral vector boson $Z$ are

$$Q^{(n - \frac{3}{2})}(Z) = \begin{pmatrix}
0 \\
\cdot \\
0 \\
1 - 2s_w^2
\end{pmatrix},$$

(24)

for the lepton sector, and

$$Q^{(n - \frac{2n - 3}{2})}(Z) = \begin{pmatrix}
\frac{4s_w^2}{3} \\
\cdot \\
\frac{4s_w^2}{3} \\
1 - \frac{4s_w^2}{3}
\end{pmatrix},$$

(25)

for the quark sector, respectively. One can immediately notice that $Z$ exhibits vector couplings with new fermions (other than SM ones).

In the case of the new $Z'$ neutral vector boson, the couplings are—up to a supplementary factor $\frac{n - 2}{\sqrt{n - 2(n - 1)s_w^2}}$—the following:

$$Q^{(n - \frac{3}{2})}(Z') = \begin{pmatrix}
-\frac{2s_w^2}{n - 2} \\
\cdot \\
-\frac{2s_w^2}{n - 2} \\
1 - \frac{2s_w^2}{n - 2}
\end{pmatrix},$$

(27)
for the lepton sector, and

\[
Q^{(n, \frac{2n-3}{3n})} (Z') = \begin{pmatrix}
-\frac{2[3+(1-2n)s_W]}{3(n-2)} \\
\vdots \\
-\frac{2[3+(1-2n)s_W]}{3(n-2)} \\
-1 + \frac{4s_W^2}{3} \\
-1 + \frac{4s_W^2}{3}
\end{pmatrix}
\] (28)

for the quark sector, respectively.

All the neutral charges of the SM fermions are summarized in table 1 of the Supplementary Material. It is now something of an evidence that the couplings connecting SM fermions to the neutral SM vector boson (Z) are utterly recovered, meaning that the SM is not altered at all at tree level. At the same time, \(Z_1, \ldots, Z_{n-3}\) are completely decoupled and exhibit no interactions with the SM fermions. That means those bosons are specific only to our generalized model and have nothing to do with the established SM phenomenology. However, there is \(Z'\) that could eventually influence the SM phenomenology. Therefore, when it comes to effects produced by this very boson, our investigation in such models must continue in future works by estimating the loop corrections that are supposed to provide us with certain restrictions regarding the parameters space.

Further on, the \(Z_2\) couplings are inferred as

\[
Q^{(n, \frac{2n-3}{3n})} (Z_2) = c_W \begin{pmatrix}
1 \\
-1 \\
0 \\
\vdots \\
0
\end{pmatrix}
\] (31)

\[
Q^{(n, \frac{2n-3}{3n})} (Z_1) = c_W \begin{pmatrix}
1 \\
-1 \\
0 \\
\vdots \\
0
\end{pmatrix}
\] (32)

for the lepton sector, respectively.

The couplings for the last heavy boson, namely \(Z_{n-3}\), read

\[
Q^{(n, \frac{2n-3}{3n})} (Z_{n-3}) = c_W \begin{pmatrix}
1 \\
1 \\
\vdots \\
1 \\
\sqrt{2c_W} \sqrt{\frac{1}{(n-2)(n-3)}} \\
\sqrt{2c_W} \sqrt{\frac{1}{(n-2)(n-3)}}
\end{pmatrix}
\] (33)

for the quark sector, respectively.

For the heavier \(Z_1, Z_2, \ldots, Z_{n-3}\) neutral bosons the computation is actually simpler. Their couplings are given by their associated diagonal generators in the \(su(n) \otimes u(1)\) algebra and nothing else. That is, only \(T_3\) accounts for \(Z_1\), only \(T_3\) accounts for \(Z_2\) couplings, and so forth, with no admixture at all. The resulting values are summarized in table 2 of the SI, once the explicit expressions are given below.

Under these circumstances, the couplings for \(Z_1\) yield

\[
Q^{(n, \frac{2n-3}{3n})} (Z_1) = c_W \begin{pmatrix}
1 \\
-1 \\
0 \\
\vdots \\
0
\end{pmatrix}
\] (30)

for the lepton sector, and

\[
Q^{(n, \frac{2n-3}{3n})} (Z_2) = c_W \begin{pmatrix}
1 \\
1 \\
\vdots \\
1 \\
\sqrt{2c_W} \sqrt{\frac{1}{(n-2)(n-3)}} \\
\sqrt{2c_W} \sqrt{\frac{1}{(n-2)(n-3)}}
\end{pmatrix}
\] (34)

for the quark sector, respectively.

The couplings for the last heavy boson, namely \(Z_{n-3}\), read

\[
Q^{(n, \frac{2n-3}{3n})} (Z_{n-3}) = \begin{pmatrix}
1 \\
1 \\
\vdots \\
1 \\
\frac{\sqrt{2c_W}}{\sqrt{(n-2)(n-3)}} \\
\frac{\sqrt{2c_W}}{\sqrt{(n-2)(n-3)}}
\end{pmatrix}
\] (35)

for the quark sector, respectively.

\[
Q^{(n, \frac{2n-3}{3n})} (Z_n) = \begin{pmatrix}
1 \\
1 \\
\vdots \\
1 \\
\frac{\sqrt{2c_W}}{\sqrt{(n-2)(n-3)}} \\
\frac{\sqrt{2c_W}}{\sqrt{(n-2)(n-3)}}
\end{pmatrix}
\] (36)
for the lepton sector, and

\[ Q^{(n, \frac{2n-3}{3})}(Z_{n-3}) = \frac{\sqrt{2}c_W}{\sqrt{(n-2)(n-3)}} \begin{pmatrix} 1 & 1 & \cdot & \cdot & \cdot & \cdot & 3-n & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & n-3 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & 0 \end{pmatrix} , \] (37)

\[ Q^{(n^* : -\frac{n-3}{3})}(Z_{n-3}) = \frac{\sqrt{2}c_W}{\sqrt{(n-2)(n-3)}} \begin{pmatrix} -1 & -1 & \cdot & \cdot & \cdot & \cdot & n-3 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & 0 \end{pmatrix} , \] (38)

for the quark sector, respectively.

Let us turn now to the fermion singlets. Their weak charges are computed in the simplest way, since one takes into account only the $U(1)$ hypercharges $Y$ and none of the $T$ generators of the $SU(n)$. Hence, for the right-handed singlets, there are no interactions at all with the heavier $Z_1, Z_2, \ldots, Z_{n-3}$. At the same time, as expected, all the neutral right-handed fermions have no weak interactions, regardless of the SM or non-SM bosons.

For the sake of completeness we display below the weak interactions of the right-handed representations explicitly. In the case of charged leptons one obtains the following couplings:

\[ Q^{(1, -1)}(Z) = 2s_W^2, \quad Q^{(1, -1)}(Z') = -\frac{2\sqrt{n-2}s_W^2}{\sqrt{n-2}(n-1)s_W^2}, \] (39)

while all kinds of right-handed neutrinos are sterile,

\[ Q^{(1, 0)}(Z) = 0, \quad Q^{(1, 0)}(Z') = 0, \] (40)

as expected.

In the quark sector things change a little bit, all the right-handed up-type quarks couplings yield

\[ Q^{(1, 2/3)}(Z) = -\frac{4s_W^2}{3}, \quad Q^{(1, 2/3)}(Z') = 0, \] (41)

while all the right-handed down-type quarks interact weakly in the manner

\[ Q^{(1, -1/3)}(Z) = \frac{2s_W^2}{3}, \quad Q^{(1, -1/3)}(Z') = -\frac{2\sqrt{n-2}s_W^2}{3\sqrt{n-2}(n-1)s_W^2}. \] (42)

Summary. – The starting point of our discussion here has been the quantized electric charge operator in $SU(3)_c \otimes SU(n)_L \otimes U(1)_Y$ gauge models, proved in ref. [1] to match the exact electric charge pattern observed in nature. That was achieved based only on renormalization criteria imposed to the chiral anomalies, with no supplemental hypothesis or computational artifacts. Here we enlarged our approach and inferred the quantization of all the weak charge operators in the model, as a mere consequence of the same renormalizable paradigm. Moreover, by combining eqs. (19), (20) with the generalized Gell-Mann–Nishijima formula (eq. (13) in ref. [1]) in order to get rid of the would-be hypercharges $Y$, one obtains (up to a factor $\frac{e}{s_W^2c_W}$) the following expressions:

\[ Q^p(Z) = -T_{n^2-2n}^p \left[ \frac{n-2}{2(n-1)} + T_{n^2-1}^p \frac{\sqrt{n}}{\sqrt{2(n-1)}} \right] - s_W^2 Q_{em}^p, \] (43)

\[ Q^p(Z') = -T_{n^2-2n}^p \left[ \frac{n-2(n-1)s_W^2}{2(n-1)} \right] - T_{n^2-1}^p \frac{\sqrt{n}(n-2)}{2(2(n-1)s_W^2)} + Q_{em}^{p} \frac{n-2}{n-2(n-1)s_W^2}, \] (44)

for $Z$ and $Z'$ couplings, while the rest of the $n-3$ weak interactions can be picked up from eqs. (22)–(23). It goes without saying that the weak charges are quantized, once the electric charge is quantized and they are strictly related for each irreducible representation $\rho$, in the manner displayed above.

This is a remarkable outcome even though, for the moment, our discussion regards the tree level only. Any quantum correction remains to be worked out, once a specific process is taken into consideration. As one can easily observe from the results derived in the present work, the SM fermions preserve their couplings predicted by the SM. Moreover, they have no couplings at all with the new bosons $Z_1, Z_2, \ldots, Z_{n-3}$, but only with $Z'$ ($\leq 5.1$ TeV according to [70]). The influence $Z'$ could exert on SM phenomenology must be investigated in a future work. If worked out properly such corrections could enforce restrictions on the overall scale of the model and other parameters as well (for instance, the appropriate Yukawa couplings in the neutrino sector or quark sector). At the same time, a nice feature of our generalized gauge model resides in the fact that the SM boson $Z$ has vector interactions with all heavier fermions (other than SM fermions), while $Z'$ makes no distinction on the electric charge basis when it comes to the left-handed fermions in the same SM doublet. A plethora of suitable candidates for the cold Dark Matter can be found in such models, since a lot of neutral fermions, neutral scalars and even neutral vector bosons are not interacting with ordinary matter particles—as the new scale $V$ is actually decoupled from
Consequently, the electro-weak sector could be based on more than three, and the asymptotic freedom could restrict the $SU(3)\otimes SU(n)_L\otimes U(1)_Y$ gauge models. For the time being, this could seem only a hypothetical speculation, but the history of science proved many times that it is not wise to discard even the weirdest hypothesis. What if the QCD one day will be reassessed on a larger gauge group than $SU(3)$? Then the number of colors could be more than three, and the asymptotic freedom could restrict the number of quark flavors to a much larger number than 16, as results now from the renormalization group procedure. Consequently, the electro-weak sector could be based on larger $SU(n)_L\otimes U(1)_Y$ group and our approach would become a very useful tool.

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