The influence of operational and environmental loads on the process of assessing damages in beams

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Abstract. Damage detection methods based on vibration analysis make use of the modal parameter changes. Natural frequencies are the features that can be acquired most simply and inexpensively. But this parameter is influenced by environmental conditions, e.g. temperature and operational loads as additional masses or axial loads induced by restraint displacements. The effect of these factors is not completely known, but in the numerous actual research it is considered that they affect negatively the damage assessment process. This is justified by the small frequency changes occurring due to damage, which can be masked by the frequency shifts due to external loads. The paper intends to clarify the effect of external loads on the natural frequencies of beams and truss elements, and to show in which manner the damage detection process is affected by these loads. The finite element analysis, performed on diverse structures for a large range of temperature values, has shown that the temperature itself has a very limited effect on the frequency changes. Thus, axial forces resulted due to obstructed displacements can influence more substantially the frequency changes. These facts are demonstrated by experimental and theoretical studies. Finally, we succeed to adapt a prior contrived relation providing the frequency changes due to damage in order to fit the case of known external loads. Whereas a new baseline for damage detection was found, considering the effect of temperature and external loads, this process can be performed without other complication.

1. Introduction

In the scientific literature different models for obtaining modal parameters are presented, that rely on beam slenderness [1-3], cross-section variation due to constructive reasons [4] or damage [5-8]. Among them, those that take into account the operational loads are much fewer [9]. The temperature effect on natural frequent changes of the healthy and damaged structures is shown in [10-11].

In previous research a method for detecting the damage position was developed; it bases on an original mathematical relation that allows the calculation of natural frequencies of the damaged beam [12-16]. In studies presented in this paper we researched the effect that the temperature changes have on the frequency changes and, as a result, we succeed to develop a damage detection method which takes into account these changes of temperature.
2. Buckling of beams

In vibration analysis performed to detect damages in beams under changing conditions (temperature variations for instance) we must take into account the compressive loads that may occur. In reality, in practical cases, embedding at both ends of the beams is avoided. If at one end a relative displacement is allowed, for instance hinge with friction, then the change in temperature will have a lower influence, depending on the compression loads that occurs due to friction. Thus, in this paper the case of clamped-clamped beam is analyzed.

For a beam of length $L$, width $b$ and height $h$, mass density $\rho$ at an initial temperature $T_0$, clamped at both ends, if the temperature changes with $\Delta T$, an internal load $P$ occurs, that is:

$$ P(T) = \alpha \cdot E \cdot A \cdot (T - T_0) = \alpha \cdot E \cdot A \cdot \Delta T $$

Here $A$ is the cross-section area, $E$ is the Young’s modulus and $\alpha$ is the thermal expansion coefficient. When the first critical temperature $T_{cr-1}$ is reached, the first critical buckling load $P_{cr-1}$ appears and the structure will suffer a vertical displacement, as can be seen in figures 1 and 2.

![Figure 1. Double-clamped beam in initial state and subjected to buckling (first mode)](image)

![Figure 2. Beam in buckling mode one, after passing the critical load](image)

The equation of the deformed beam, due to buckling, is given by:

$$ EI \frac{d^4 v}{dx^4} + P \frac{d^2 v}{dx^2} = \frac{d^4 v}{dx^4} + \varsigma \frac{d^2 v}{dx^2} = 0 $$

where $v$ is the lateral displacement and $\varsigma$ is the buckling eigenvalue.

The solution of this equation is of the form [17]:

$$ v(x) = c_1 \sin(\varsigma x) + c_2 \cos(\varsigma x) + c_3 x + c_4 $$

with $c_1 \ldots c_4$ coefficients to be derived for the specific boundary conditions. For the double clamped beam we obtain the characteristic equation:

$$ \varsigma \sin \varsigma + 2 \cos \varsigma - 2 = 0 $$

with the first six buckling eigenvalues presented in table 1

| Symmetric buckling modes | Eigenvalues | Asymmetric buckling modes | Eigenvalues |
|--------------------------|-------------|---------------------------|-------------|
| 1                        | 6.2831853071 | 2                         | 8.9868189158 |
| 3                        | 12.566370614 | 4                         | 15.450503673 |
| 5                        | 18.849555921 | 6                         | 21.808243318 |
The mode shape value, for any location $x$ along the beam, is given by the following function:

$$v_i(x) = c \left[ 1 - \cos \left( \frac{\xi_i}{L} x \right) \right] - \left[ \sin \left( \frac{\xi_i}{L} x \right) - \frac{\xi_i}{L} \sin \xi_i \frac{x}{L} \right] \frac{1 - \cos \xi_i}{\sin \xi_i}$$  \hspace{1cm} (5)

The critical loads are obtained from:

$$P_{cr-i} = \frac{\xi_i^2 E I}{L^2}$$  \hspace{1cm} (6)

so, the critical temperatures are:

$$T_{cr-i} = T_0 + \Delta T_{cr} = T_0 + \frac{P_{cr-i}}{\alpha E A}$$  \hspace{1cm} (7)

These temperatures are critical because at these points the frequency becomes zero and then, from here on, the apparent stiffness increases. If we have negative temperatures, $P$ decreases and the frequency increase. This frequency is derived as:

$$f_i = \frac{\lambda_i^2}{2\pi L^2} \sqrt{\frac{EI}{\rho A} - \frac{P}{\rho A} \left( \frac{L^2}{\xi_i^2} \right)}$$  \hspace{1cm} (8)

where $\lambda_i$ is the dimensionless wave number of bending vibration mode $i$.

We made an analytical calculation and determined the critical load values and the critical temperatures for a clamped steel beam with the length $L = 1200$ mm, width $b = 50$ mm and the thickness $h = 5$ mm and the yield strength of 530 N/mm$^2$, elastic modulus $E = 2 \cdot 10^5$ N/mm$^2$, Poisson’s ratio $\mu = 0.3$, mass density $\rho = 7850 \cdot 10^{-9}$ kg/mm$^3$ and the thermal coefficient $\alpha = 1.25 \cdot 10^{-5}$ $^\circ$C$. The results of calculation are given in table 2.

### 3. FEM analysis

For finite element analysis was used SolidWorks simulation software with which was performed an analysis over a clamped steel beams using the features stated in the previous paragraph. In Thermal analysis module we perform a thermal analysis for temperatures in the range of 0...100 °C, with reference temperature of 24.85°C. Next, we test whether the critical temperature obtained from the analysis in Buckling module corresponds to the one analytically obtained.

**Table 2. Critical loads and temperatures for the analyzed double-clamped beam**

| Mode | Critical load | Critical temperature |
|------|---------------|----------------------|
|      | Analytic [N]  | FEM [N]              |
| 1    | 2855.79       | 2857.47              |
| 2    | 5842.22       | 5846.48              |
| 3    | 11423.1       | 11440.6              |
| 4    | 17268.3       | 17301.3              |
| 5    | 25702.1       | 25776.3              |
| 6    | 34403.9       | 34523.2              |
|      | Analytic [°C] | FEM [°C]             |
| 1    | 29.8          | 29.8                 |
| 2    | 35.01         | 35.01                |
| 3    | 44.71         | 44.74                |
| 4    | 54.88         | 54.94                |
| 5    | 69.54         | 69.68                |
| 6    | 84.68         | 84.89                |

Analyzing the data in table 2 is seen that the critical buckling load values and the critical temperature values are very similar both for analytical and FEM analysis.
In figure 3 we graphically represented the buckling shapes for both, symmetric and asymmetric modes analytically obtained and in figure 4 the buckling shapes for the same modes obtained from the simulation program.

Comparing figures 3 and 4 is observed similarity between the shapes of vibration modes analytically obtained and those obtained from finite element analysis.

Further analysis will be done using the Modal analysis tool in which we will consider temperatures between 0 and 100 °C, ranging from 5 to 5 °C in the interval 0…24.85 °C and then, for temperatures higher than the reference temperature, will go with a step of 1 °C.
The frequencies obtained by modal analysis for the first six weak-axis transversal vibration modes are presented in figure 5, presenting good similarity with that presented in [18].

In figure 5 we observed that the frequency decreases with increasing temperature until reaching the buckling in that mode. After this point, the frequency increases due to the important contribution of the apparent stiffness increase.

Figure 6 presents the frequency values calculated analytically for the vibration mode three in the state before buckling and with continuous line the natural frequency values obtained from simulation program for temperatures between 25 °C and 45 °C.

![Figure 6: Frequency decrease due to compression for vibration mode three](image)

It is important to mention that the vibration mode shapes deviate from the classical look due to thermal loads and expansion.

4. Testing the damage location indicator for buckled beams

Analyzing the frequency distribution depending on the position of the damage, and depending on the depth of the damage, we conclude that the reduction of stiffness in a transverse segment does not automatically changes frequencies. With other words, damages located in two different positions produce different effects on their frequencies for a given mode of vibration. On the other hand, if we place it in the same position the damage produces different effects for different vibration modes. All the above observations help us to understand the phenomena occurring and provide the basis for developing a mathematical model and an algorithm by which it is possible to identify the position of the damage which occurs in a beam.

The natural frequency of the beam with transverse open is [12]

\[ f_{i,D}(x_D, a) = f_{i,U} \left(1 - \gamma(a)\right) (\phi''(x_D))^2 \]  

where \( f_{i,D} \) is the damaged beam frequency, \( f_{i,U} \) is the frequency for the undamaged beam, \( \gamma(a) \) reflect the damage severity, \( i = 1..n \) the mode number and \( n \) the number of the weak-axes bending vibration modes considered. The normalized mode shape curvature at location \( x_D \) is denoted \( \phi''(x_D) \).

If we have a set of frequencies measurements for an undamaged beam, we can calculate the relative changes in frequencies using the following equation:

\[ \delta f_i(x,a) = \frac{f_{i,U} - f_{i,D}(x,a)}{f_{i,U}} \]  

(10)
If these relative changes in frequencies will be normalized by dividing each value with the maximum of the sequence

\[
\Psi_i = \frac{\delta f_i}{\max(\delta f_i)} , \Psi_2 = \frac{\delta f_2}{\max(\delta f_2)} , \ldots , \Psi_n = \frac{\delta f_n}{\max(\delta f_n)} , \quad i = 1..n
\]  

we get a sequence of sub unitary positive numbers, only one element having the value 1, that is:

\[
\Psi : \{ \Psi_1, \Psi_2, \ldots, \Psi_n \}
\]

On the other hand, introducing equation (9) in equation (10) results:

\[
\delta f_i (x,a) = \frac{\dot{f}_i(x,a)}{f_i(x,a)} = \gamma(a) \cdot (\overline{\varphi}(x))^2
\]  

By normalization we obtain a sequence which is independent of severity:

\[
\Phi_i = \frac{\delta f_i}{\max(\delta f_i)} = \frac{\gamma(a) (\overline{\varphi}(x))^2}{\max \{\gamma(a) (\overline{\varphi}(x))^2\}} = \frac{\gamma(a) (\overline{\varphi}(x))^2}{\max \{(\overline{\varphi}(x))^2\}}
\]

\[
\Phi_i = \frac{\gamma(a) (\overline{\varphi}(x))^2}{\max \{(\overline{\varphi}(x))^2\}}
\]  

If we want to generalize, we consider \( k \) positions of the damage on the beam. For each position \( x_1, x_2, \ldots, x_k \) on the beam we determine the squared of the mode shape curvature at that point \( (\overline{\varphi}(x_j))^2 \).

Normalizing these values for each position \( x_1, x_2, \ldots, x_k \) considered of the beam, by dividing with the maximum values on a line we obtain the influence coefficients obtained defect position:

\[
\Phi_1(x_1) = \frac{(\overline{\varphi}(x_1))^2}{\max \{(\overline{\varphi}(x_1))^2\}} , \Phi_2(x_1) = \frac{(\overline{\varphi}(x_2))^2}{\max \{(\overline{\varphi}(x_1))^2\}} , \ldots , \Phi_\ast(x_1) = \frac{(\overline{\varphi}(x_\ast))^2}{\max \{(\overline{\varphi}(x_\ast))^2\}}
\]

\[
\Phi_1(x_2) = \frac{(\overline{\varphi}(x_1))^2}{\max \{(\overline{\varphi}(x_2))^2\}} , \Phi_2(x_2) = \frac{(\overline{\varphi}(x_2))^2}{\max \{(\overline{\varphi}(x_2))^2\}} , \ldots , \Phi_\ast(x_2) = \frac{(\overline{\varphi}(x_\ast))^2}{\max \{(\overline{\varphi}(x_\ast))^2\}}
\]

\[
\Phi_1(x_k) = \frac{(\overline{\varphi}(x_1))^2}{\max \{(\overline{\varphi}(x_k))^2\}} , \Phi_2(x_k) = \frac{(\overline{\varphi}(x_2))^2}{\max \{(\overline{\varphi}(x_k))^2\}} , \ldots , \Phi_\ast(x_k) = \frac{(\overline{\varphi}(x_\ast))^2}{\max \{(\overline{\varphi}(x_k))^2\}}
\]

So, for the position \( x_j \) on the beam, we obtain the damage location indicator DLI as

\[
\Phi_j : \{ \Phi_1(x_j), \Phi_2(x_j), \ldots, \Phi_\ast(x_j) \}
\]

Considering \( k \) positions equidistantly distributed along the beam, i.e. \( j = 1..k \) we produce a matrix which has \( k \) rows and \( n \) columns. Each matrix row, or set \( \Phi_j \), uniquely characterize a certain location on the beam [12]. The sequence \( \Psi \), as a single row matrix, is compared successively with each row of
the matrix derived using equation (16). The value of distance \( x_j \), for which the elements of row \( j \) of the matrix \( \Phi \) are closest to the values of the matrix \( \Psi \), indicate the damage location.

In this study we want to see if we can use the damage location indicators (DLI) without considering the influence of temperature. For this, we consider the double-clamped beam presented in paragraph 2, with a transverse damage with a width of 0.1 mm, situated at a distance of 400 mm to one end and reducing the cross-sectional area by 50%. The analysis was made for the temperature of 0 °C, 10 °C, 15 °C, 20 °C, 26 °C having a reference temperature by 24.85 °C. Using the frequency values obtained the graphs in figure 6 have been traced.

![Damage Location Indicators](image)

**Figure 6.** Damage location indicators for damage located at 400 mm

Analyzing the results, it can be seen that there are quite large deviations of frequency values obtained at different temperatures. From this, we can conclude that either we have to analyze the frequencies at the same temperature, or we have to find a formula to compensate the temperature change. This is possible by introducing the explicit relation of the buckling load given in equation (1) in the frequency equation (8). It results

\[
f_i(T) = \frac{\lambda_i^2}{2\pi L^2} \sqrt{\frac{EI}{\rho A} - \frac{P(T)}{\rho A} \left( \frac{L}{L_i^2} \right)} = \frac{\lambda_i^2}{2\pi L^2} \sqrt{\frac{EI}{\rho A} - \frac{\alpha EA \cdot \Delta T}{\rho A} \left( \frac{L}{L_i^2} \right)}
\]

(17)

For practical applications it is sufficient to study the frequency until the first buckling mode occurs.

5. Conclusion
The paper presents the influence of axial loads induced by temperature on the damage detection process. It is shown that, if in the structure monitoring the vibration measurements are made at different temperatures, the initially acquired frequencies of the healthy clamped-clamped beam cannot be considered a reference as they are. This happens because the frequencies of different modes are affected in different ways by the same temperature change. A solution of the problem was tested and presented in this paper: the adjustment of frequencies by considering the loads induced by temperature changes. Thus, a new reference is contrived and the Damage Location Indicators introduced by the authors can be used to assess the damage position. This approach is valid also if compression or tensile loads are applied on the beam; in this case the induced stress has to be evaluated and introduced in the frequency relation in order to make the monitored frequencies compare. Since in real cases the boundaries are not perfectly fixed, but small displacements are possible, it is preferable to apply this second approach also if temperature changes produce the internal loads.
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