Numerical investigation of the opposing mixed convection in an inclined flat channel using turbulence transition models

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Abstract. Many researchers analyzed laminar and turbulent mixed convection in channel flows, but despite wide investigations there are still many cases which are not well understood and difficult to predict. Investigations of heat transfer in the laminar-turbulent (transition) region under the effect of buoyancy (mixed convection) are rather limited. In this paper we present the results on numerical investigation of the local opposing mixed convection heat transfer in the laminar-turbulent (transition) region in an inclined flat channel (inclination angle $\phi = 60^\circ$ from horizontal position). Numerical two dimensional steady state simulations were performed using Ansys Fluent code in air flow. Sensitivity analysis was performed using laminar and different transitional (kkl-$\omega$, SST and Re stress-$\omega$) models. Modelling results demonstrate that vortices exist in the channel. This makes velocity profiles asymmetrical. The results of numerical modelling have been compared with the heat transfer results of experiments performed at the same conditions.

1. Introduction

The interaction of the natural and forced convection (mixed convection) in channels of various cross-sections and orientations are important for nuclear power technology, heat exchangers, electronic cooling systems, solar energy systems, etc. Due to the importance of the problem to engineering applications, a lot of researchers concentrated their attention on the turbulent mixed convection heat transfer investigations in vertical circular tubes [1 - 3]. Wide investigations on this problem (turbulent flow in vertical channels) were performed at the Lithuanian Energy Institute [4, 5] also. Different studies showed that compared to the forced convection, heat transfer is higher in case of the opposing mixed convection (upward oriented flow due to natural convection and downward oriented flow due to forced flow). It was revealed that in case of the aiding mixed convection flows (upward oriented flow due to natural convection and upward oriented flow due to forced flow), the effectiveness of heat transfer could be seriously impaired as a result of buoyancy forces modifying the production of turbulence and laminarizing the flow. However, if higher buoyancy parameters are applied, heat transfer recovers and becomes even higher than forced convection heat transfer.

Investigations of heat transfer in the laminar-turbulent transition region under the effect of buoyancy (mixed convection) are rather limited. The flow character in pipes in this region was investigated in [6, 7]. In order to visualize the flow, paint was injected in the central part of the pipe. It was reported, that the shape of the paint thread in aiding mixed convection flows with the loss of flow...
stability was similar to the shape of sinusoid, and the pulsation of the temperature on the pipe’s wall took its place. With the increase of the buoyancy parameter, the amplitude of the paint thread sinusoid became bigger, while eventually the paint thread intermitted at the end. The initial instability of the flow depended not only on the buoyancy parameter, but also on the length of the channel (L/d). In the case of the opposing mixed convection flows, the instability of the flow was demonstrated by the appearance of the asymmetry of the paint thread just before the heating. If the buoyancy parameter was increased significantly, the flow began pulsating. It was noticed that an asymmetric flow was formed inside the channel. It was also noticed in [1, 8], that the existence of the inflection points in velocity profiles and especially the possibility of the appearance of the reversed flow stimulated the loss of stability of laminar flow. The loss of stability and the transition to turbulent flow took place when \( Re = Re_{cr} < 2.3 \times 10^3 \). With the increase of \( Grq/Re \), critical Reynolds number decreased.

The structure of the flow in case of the opposing mixed convection flows was studied in a vertical tube in [9] with moving thermocouples. After these investigations it was concluded that at the beginning of the heating, vortices appear near the walls, which causes the fluctuation of wall temperature. As the influence of the buoyancy forces becomes stronger, the flow inside the channel becomes turbulent. Turbulent flow structure was investigated experimentally in a vertical annular channel \((d_1/d_2 = 0.54)\) in [10]. A range of Reynolds number from \(1.5 \times 10^3\) to \(2 \times 10^4\) was covered. Grashof number was varied from \(2 \times 10^5\) to \(1.5 \times 10^6\) in the case of water flowing upwards and downwards (height \(-5\) m, heated part \(-3\) m (there are unheated sections also), the outer tube has internal diameter of \(d_2 = 140\) mm, the inner tube has external diameter of \(d_1 = 76\) mm and wall thickness \(-1.6\) mm.). The inner surface was uniformly heated and outer one was adiabatic. The two-component laser Doppler anemometry system was used to measure radial profile of velocity and turbulence quantities. Results for a Reynolds number of \(2 \times 10^4\) showed that the mean flow is actually reversed near the heated surface in the downward flow case and is very turbulent. Velocity fluctuations are larger than those for isothermal flow at a Reynolds number of \(8.5 \times 10^3\). The Reynolds shear stress is very high and its maximum value is larger than that for isothermal flow at a Reynolds number of \(8.5 \times 10^3\). Regrettfully the spectrum of the velocity pulsations was not measured that could provide some information about the possible vortex flow structure. Flow instability in a vertical tube in case of the aiding mixed convection was analyzed in [11]. Measurements of temperature and velocity fluctuations in airflow for \(Re = 1000, 1300\) and 1600 were performed. It was determined that flow lost its stability under \(Grq/Re > 1500 (Grq/Re^2 > 0.7)\).

In [12] the results on experimental investigation of the opposing mixed convection in a vertical flat channel with symmetrical heating in the laminar-turbulent transition region were presented. Numerical two-dimensional simulations were performed for the same conditions as in experiments using the Fluent code. The performed experimental investigations showed that for the higher than ambient air pressure in some Re region heat transfer rate was more intensive than for the turbulent flow. The numerical investigations indicated that as the influence of the buoyancy became stronger, the vortical flows appeared at the wall of the channel, which caused the intensification of the heat transfer. The correlations for determination of the critical Re number and heat transfer in vortex flow region were suggested.

In this paper the results on numerical investigations of the local opposing mixed convection heat transfer in the laminar-turbulent transition region in the inclined flat channel (inclination angle \(\varphi = 60^\circ\) from horizontal position) are presented. Numerical two dimensional simulations have been performed for the same channel and for the same conditions as in experiments using Ansys Fluent code. Simulations have been performed at air pressure \(p = 0.2\) MPa with symmetrical heating for Reynolds numbers \(Re_n = 2.3 \times 10^4, 2.8 \times 10^4, 4 \times 10^4\) and Grashof numbers \(Grq_{pin} = 3.9 \times 10^6, 4.5 \times 10^6, 4.1 \times 10^8\) respectively.
2. Numerical method
In this paper the results on two dimensional numerical modelling of opposing mixed convection in an inclined flat channel (height – 0.0408 m, length – 6 m, hydrodynamic unheated length – 2.5 m, heated (calorimeter) length 3.5 m) with two-sided symmetrical heating \( q_{w1} = q_{w2} = \text{const} \) are presented for the steady state flow conditions in airflow. The modelling has been performed using laminar and turbulence transition models: kkl-\( \omega \), SST and Reynolds stress \( \omega \) model.

The modelling has been carried out using Ansys Fluent code. It is a contemporary computational fluid dynamics code, which is used for modelling the fluid flow and heat transfer in complex two-dimensional or three-dimensional systems [13]. This code solves the main flow and energy equations. In this case a control volume based technique is used which is based on division of the domain into discrete control volumes using a computational grid (which at the same time describes the channel geometry).

The steady state conservation of mass (continuity), conservation of the x and y momentum and energy equations were solved in the two dimensional problem. In case of modeling the laminar flow (laminar model), in Reynolds equations turbulent viscosity \( \mu_t = 0 \), and in energy equation turbulent thermal conductivity \( \mu_t/Pr_t = 0 \).

The transition to turbulence is three-dimensional and transient but in this case the transition is significantly extended due to formation of vortices in the flow [12]. So, it seems that even steady state two-dimensional laminar modelling can provide some useful information about formation of the vortices.

Three turbulence transition models: Re stress-\( \omega \) [14], SST (Menter-Langtry model) [15] and kkl-\( \omega \) [16] were also used in these investigations. The transition SST model is based on the coupling of the SST k-\( \omega \) transport equations [17] with two other transport equations, one for the intermittency and one for the transition onset criteria, in terms of momentum-thickness Reynolds number. The transition kkl-\( \omega \) model is considered to be a three-equation eddy-viscosity type model, which includes transport equations for turbulent kinetic energy, laminar kinetic energy, and the inverse turbulent time scale. The low-Re stress-\( \omega \) model is a stress-transport model that is based on the k-\( \omega \) equations [14] and LRR model [18].

Boundary conditions are:
- At the inlet to the hydrodynamic unheated part longitudinal air velocity \( u_x \) is equal to inlet velocity \( u_m \), transversal velocity \( u_y = 0 \). Air enthalpy \( i \) at the inlet to the calorimeter is equal to the inlet air enthalpy \( i_m \).
- On the walls (\( y = 0 \) and \( y = h \)) longitudinal \( u_x \) and transversal \( u_y \) velocities are equal to 0. Heat flux on the calorimeter walls are \( q_{w1} = q_{w2} = \text{const} \).

In this work, a grid of \( N_x \times N_z = 80 \times 7500 \) was used (Figure 1), based on grid sensitivity analysis [19]. The first node of the used grid was close to the wall at \( y^+ \leq 0.2 \). Not less than 10 nodes were in the viscous sublayer (\( 0 < y^+ < 5 \)) and no less than 15 nodes were in the buffer layer (\( 5 < y^+ < 30 \)).

3. Results
The velocity profiles and flow structure at \( \text{Re}_m = 2.3 \cdot 10^3 \) for various transitional models: kkl-\( \omega \), SST, Re stress-\( \omega \) and laminar model are illustrated in Figure 2. It can be seen that in case of laminar model (Figure 2a) the vortices exist near channel walls. Circular flow (in the heat transfer stabilized region the chequerwise circular flow) takes place at the walls. As a result, the velocity profiles become
distorted and the flow in the centre of the channel finally gains sinusoid character. Velocity profiles are chaotic, always asymmetric, and the backward flow (negative \( u_x \)) in the analyzed cross-sections (except \( x/D_h = 1.9 \) and 22.3; \( D_h \) – hydraulic diameter of the channel) near the bottom (right) wall occupies much larger part of the cross-section than that near the upper (left) one.

**Figure 1.** Computational domain and partial view of the grid.

We can see that in case of kkl-\( \omega \) model (Figure 2b) the flow in the central part of the channel is always downward oriented for all \( x/D_h \) (the direction of forced flow), but velocity profiles at some \( x/D_h \) (\( x/D_h = 1.9, 3.9 \)) are with vortical flow. The flow structure shows vortical flow near both walls (Figure 2b). At \( x/D_h = 1.9 \) (Figure 2b, curve 1) the vortical flow near the upper (left) wall occupies a larger part of the cross-section than that near the bottom (right) one, and the maximum velocity is at \( y/h \approx 0.55 \). At \( x/D_h = 3.9 \) (Figure 2b, curve 2) the vortical flow near the bottom wall occupies a larger part of the cross-section than that near the upper one, but the maximum velocity is at \( y/h \approx 0.5 \). In the remaining part of the channel the flow is symmetric and near both walls it is also downwards oriented.

In case of SST model (Figure 2c) we observe very similar situation as it was with kkl-\( \omega \) model. The flow in the central part of the channel is also always downward oriented for all \( x/D_h \), and velocity profiles are asymmetric at some \( x/D_h \) (\( x/D_h = 1.9, 3.9 \)) as well. But in general it can be stated that velocity profiles are less fulfilled and gains more parabolic shape comparing with kkl-\( \omega \) model case. In the very beginning of the heated part of the channel also vortical flow exists at the walls.

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Almost the same situation as in laminar model case we observe with Re stress-\( \omega \) model (Figure 2d). Velocity profiles are chaotic, always asymmetric, and the vortical flow in analyzed cross-sections (except \( x/D_h = 1.9 \) and 22.3) near the bottom (right) wall occupies much larger part of the cross-section than that near the upper (left) one.

The analysis of the velocity profiles and flow structure in case of the same transitional models and laminar model, for higher Re number (\( Re_m = 4 \cdot 10^5 \)) was performed also. Modelling results showed that in general for such Re number vortical flow becomes less expressed. In case of laminar model velocity profiles are asymmetric in the whole heated part of the channel, but the asymmetry is not such drastic as in case of lower Re number. Vortical flow remains in the whole channel.
Figure 2. The dynamics of velocity profiles and flow structure for $Re_{in} = 2.3 \cdot 10^3$, $Gr_{in} = 3.9 \cdot 10^8$ using different models: a) laminar, b) kkl-$\omega$, c) SST, d) Re stress-$\omega$. 1 - $x/D_h = -2.2 - 0$ (the end of hydrodynamic stabilization region and beginning of heating); 2 - $x/D_h = 1.9 - 3.9$; 3 - $x/D_h = 39.2 - 42$. 
Using kkl-ω model (in case of $Re_{in} = 4 \cdot 10^3$) large vortical structures are formed at the very beginning of the heated part of the channel and this affects velocity profiles with $x/D_h = 1.9$ and $3.9$ are very asymmetric. Later on the vortical flow intermits and in the heat transfer stabilized region vortices are not well expressed. SST model shows very similar results as in case for smaller $Re$ number with the vortical flows at the very beginning of the heated part of the channel. And finally $Re$ stress-ω model shows quite big asymmetry of velocity profiles for the smallest $x/D_h$ ($x/D_h = 1.9, 3.9$) but in the heat transfer stabilized region vortices are not well expressed.

So after analysis of the flow structure it can be stated that in case of laminar model for smaller $Re$ number and for higher $Re$ number shows that the vortices are at the beginning of the heating section and the scale of vortices is diminishing along the channel but in heat transfer stabilized region vortical flow remains. $Re$ stress-ω model gives similar results for smaller $Re$ number, but in case of higher $Re$ number vortical flow intermits already at the beginning of the heated part of the channel. Another two turbulence transition models (kkl-ω and SST) show clear vortical structures only at the very beginning of the heated part of the channel.

Let's analyze first of all heat transfer from upper wall. Variation of heat transfer along the channel for different $Re$ in case of different models is presented in Figure 3a, c, e. Flow structure in case of laminar model (Figure 2a) shows that intensive vortices are at the beginning of the heating section and the scale of vortices is diminishing along the channel. Therefore at the beginning of the heating section heat transfer is more intensive comparing to heat transfer in stabilized region (Figure 3a). The difference between modelling results and experimental data is very large at small $x/D_h$ and diminishing along the channel. The smallest difference is at highest $x/D_h$, but nevertheless modelling results remain higher than experimental data. This indicates that laminar model gives too intensive vortical flow. Similar situation is observed for higher $Re$ numbers (Figure 3c, e).

In case of the $Re$ stress-ω model for $Re_{in} = 2.3 \cdot 10^3$ and $2.8 \cdot 10^3$ (Figure 3a, c) the situation is similar as in laminar model case. But for $Re_{in} = 4 \cdot 10^3$ (Figure 3e) this model gives the best correlation with experimental data.

Modelling results in case of other two turbulence transition models (kkl-ω, SST) correlate rather well with experimental data for $Re_{in} = 2.3 \cdot 10^3$ and $Re_{in} = 2.8 \cdot 10^3$.

As it can be seen from Figure 3 for smaller $Re$ numbers $Re$ stress-ω model gives Nusselt profiles significantly far from the two other models in the case of a stable density stratification (Figure 3a, c). Apparently this is because $Re$ stress-ω is a turbulent model with a transition option while kkl-ω and SST models are developed as transition flow models. Whereas for the highest $Re$ ($Re_{in} = 4 \cdot 10^3$) even in the case of a stable density stratification the $Re$ stress-ω model results correlate rather well with experimental data.

Figure 3 demonstrates clear difference between intensity of heat transfer from upper (stable air density stratification) and bottom (unstable air density stratification) walls. This difference is decreasing with increasing of $x/D_h$, but for all $x/D_h$ heat transfer from bottom wall is higher than from upper one.

Analyzing heat transfer variation from bottom wall we can observe that in all cases modeling results correlate better with experimental data comparing with upper wall results. For all analyzed $Re$ numbers at $x/D_h > 5$ the best correlation with experimental data is in case of the laminar model when intensive vortical structures are noticed along the channel.
Figure 3. Variation of heat transfer (Nu) along the channel for different Re in case of different models: 1 – laminar model, 2 – kkl-ω model, 3 – SST model, 4 – Re stress-ω model; 5 – experiments. a, c, e - upper wall; b, d, f - bottom wall.

Conclusions
Analysis of the numerical modelling data on opposing mixed convection in transition region in an inclined flat channel (inclination angle $\varphi = 60^\circ$ from horizontal position) with symmetrical heating leads to the following conclusions:

1. Modelling results demonstrated that in case of the laminar model intensive vortical structures could be noticed along the channel. In case of the Re stress-ω model such situation is noticed only for $Re_{in} = 2.3 \cdot 10^3$. For higher Re numbers clear vortical structures are only at the beginning of the heating section which are intermitting with increasing of $x/D_h$, as in the case of kkl-ω and SST models for all Re numbers analyzed.
2. For upper wall the best correlation with experimental data is in case of the kkl-ω and SST models for Re_in = 2.3 \cdot 10^3 and Re_in = 2.8 \cdot 10^3, and in case of the Re stress-ω model for Re_in = 4 \cdot 10^3. In all these cases clear vortical flow is noticed only at the beginning of the heated part of the channel and it intermits with increasing of x/Dh.

3. For bottom wall for all analyzed Re numbers at x/Dh > 5 the best correlation with experimental data is revealed in case of the laminar model when intensive vortical structures are noticed along the channel.

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