Classical strings in $AdS_4 \times \mathbb{CP}^3$ with three angular momenta

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Abstract

In this paper, rotating strings in three directions of $AdS_4 \times \mathbb{CP}^3$ geometry are studied; its divergent energy limit, and conserved charges are also determined. An interpretation of these configurations as either giant magnons or spiky strings is discussed.

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I. INTRODUCTION

Semi-classical quantization [1] is one possible way to test the AdS/CFT correspondence, as there is currently no full quantization of string theory. The energy spectrum of a specific string solution must be identical to an anomalous dimension spectrum of an also specific dilation operator of a dual gauge theory. More recently, a string state dual to an excitation of an infinite spin chain gauge operator has been proposed [2]. This solution is called giant magnon and, for a string restricted to the $\mathbb{R} \times S^2$ subspace of $AdS_5 \times S^5$, its dispersion relation has the characteristic expression of a magnon in a spin chain

\[ E - J = 2T \left| \sin \frac{p}{2} \right|, \]

where $E$, $J$, and $T$ are respectively the energy, the angular momentum, and the tension of the string. $p$ is identified from the angle between the extremes of the giant magnon and $E, J \to \infty$. A solution related to the giant magnons, the spiky string [3, 4], also has divergent energy and obeys the dispersion relation

\[ E - T \Delta \phi = 2T \left( \frac{\pi}{2} - \theta \right), \]

where $T$ is the string tension, $\Delta \phi \to \infty$ is the deficit angle and $\theta$ is the coordinate where the string is peaked.

String configurations that obey the prescription of a giant magnon were pursued in $AdS_5 \times S^5$, e.g. [3, 5, 6], but also in other backgrounds where the gauge/string correspondence was stated, for example, Lunin and Maldacena [6, 10], Maldacena and Núñez [11], and $AdS_4 \times \mathbb{C}P^3$ [12–25]. In $AdS_4 \times \mathbb{C}P^3$ [26], giant magnons with one [12] and two [13] angular momenta were found. The solution with three angular momenta has not yet been found, although it has been predicted [14].

With this work we propose candidates for giant magnons and spiky strings with three angular momenta in $AdS_4 \times \mathbb{C}P^3$. We call them candidates because the dispersion relations of these configurations are similar to those found by [13, 27], although they do not match (1) and (2) precisely. However, the prescriptions can be reached in a one dimensional limit of our solutions.

This text is organized as follows: in section (II) we describe the motion of strings $\mathbb{R} \times \mathbb{C}P^3$ using a specific ansatz, and we calculate their conserved charges; in section (III) the divergent
energy solutions and their possible giant magnon and spiky string forms are constructed. The reasons why these structures do not exactly have the expected giant magnon dispersion relation are also discussed in this section. Section (IV) is a brief conclusion of our findings.

II. THE CLASSICAL STRING

We start with the complete $AdS_4 \times \mathbb{CP}^3$ metric

$$ds^2 = \frac{R^2}{4} \left( - \cosh^2 \rho dt^2 + d\rho^2 + \sinh^2 \rho d\Omega_2^2 \right) +$$

$$+ R^2 \left[ \frac{d\xi^2}{\sin^2 \xi} + \cos^2 \xi \left( d\psi + \frac{1}{2} \cos \theta_1 d\phi_1 - \frac{1}{2} \cos \theta_2 d\phi_2 \right)^2 + \right.$$

$$+ \frac{1}{4} \cos^2 \xi \left( d\theta_1^2 + \sin \theta_1^2 d\phi_1^2 \right) + \frac{1}{4} \sin^2 \xi \left( d\theta_2^2 + \sin \theta_2^2 d\phi_2^2 \right) \right], \quad (3)$$

where $R^2 = 4\pi \sqrt{2\lambda}$ and the ranges of the variables are $\xi \in [0, \pi/2]$, $\psi \in [-2\pi, 2\pi]$, $\theta_{i=1,2} \in [0, \pi]$ and $\phi_{i=1,2} \in [0, 2\pi]$. The first term of (3) corresponds to the $AdS_4$ space and the other to the $\mathbb{CP}^3$ space.

We are seeking string solutions in the subspace where $\rho = 0$, namely $\mathbb{R} \times \mathbb{CP}^3$. Additionally, the $\theta_{i=1,2}$ will be held constant. In this case, the change of variables $\psi \rightarrow (\psi + \frac{1}{2} \cos \theta_1 \phi_1 - \frac{1}{2} \cos \theta_2 \phi_2)$, and $\phi_i \rightarrow \sin \theta_i \phi_i$ is equivalent to make $\theta_i = \pi/2$ in (3). So, we are going to consider the ansatz

$$t = \kappa \tau, \quad \theta_{i=1,2} = \frac{\pi}{2}, \quad \xi = \xi(y), \quad \psi = \omega \tau + P(y), \quad \phi_{i=1,2} = \omega_i \tau + f_i(y) \quad (4)$$

where $y = \alpha \sigma + \beta \tau$, and $\alpha$, $\beta$, $\kappa$, $\omega$, and $\omega_i$ are constants. The Polyakov action that will be used to (4) is

$$S = T \int d\sigma d\tau \frac{1}{4} \left( - \partial_a t \partial^a t + 4 \partial_0 \xi \partial^a \xi + 4 \cos^2 \xi \sin^2 \xi \partial_a \psi \partial^a \psi + \right.$$

$$+ \cos^2 \xi \partial_a \phi_1 \partial^a \phi_1 + \sin^2 \xi \partial_a \phi_2 \partial^a \phi_2 \right), \quad (5)$$

where $T = \sqrt{2\lambda}$ is the string tension, $a = \{0, 1\}$, $\partial_0 = \partial_\tau$, and $\partial_1 = \partial_\sigma$. The equations of
motion are

\[ \partial_a \partial^a t = 0 \]  
\[ 8\partial_a \partial^a \xi - 4 \left( \sin^2 \xi \cos^2 \xi \right) \partial_a \psi \partial^a \psi - (\cos \xi) \partial_a \phi_1 \partial^a \phi_1 - (\sin \xi) \partial_a \phi_2 \partial^a \phi_2 = 0 \]  
\[ \partial_a \left( \sin^2 \xi \cos^2 \xi \partial^a \psi \right) = 0 \]  
\[ \partial_a \left( \cos^2 \xi \partial^a \phi_1 \right) = 0 \]  
\[ \partial_a \left( \sin^2 \xi \partial^a \phi_2 \right) = 0. \]

The \( \xi \) subscript in (7) indicates a derivative with respect to the variable. The conserved charges of the action (5) are

\[ E = \frac{T}{2} \int_0^{2\pi} d\sigma i \]  
\[ J_\psi = 2T \int_0^{2\pi} d\sigma \cos^2 \xi \sin^2 \xi \dot{\psi} \]  
\[ J_1 = \frac{T}{2} \int_0^{2\pi} d\sigma \cos^2 \xi \dot{\phi}_1 \]  
\[ J_2 = \frac{T}{2} \int_0^{2\pi} d\sigma \sin^2 \xi \dot{\phi}_2. \]

Using (4) we integrate (8), (9), and (10) with respect to \( y \) to obtain

\[ f_1 y = \frac{A_1}{\cos^2 \xi} + \frac{\beta \omega_1}{\alpha^2 - \beta^2} \]  
\[ f_2 y = \frac{A_2}{\sin^2 \xi} + \frac{\beta \omega_2}{\alpha^2 - \beta^2} \]  
\[ P_y = \frac{A_\psi}{\sin^2 \xi \cos^2 \xi} + \frac{\omega \beta}{\alpha^2 - \beta^2}, \]

where \( A_1, A_2 \) and \( A_\psi \) are integration constants, and \( f_{i y} \) and \( P_y \) are first derivatives with respect to \( y \). The Virasoro constraints

\[ g_{\mu\nu} \partial_\tau X^\mu \partial_\sigma X^\nu = 0 \quad \text{and} \quad g_{\mu\nu} \left( \partial_\tau X^\mu \partial_\tau X^\nu + \partial_\sigma X^\mu \partial_\sigma X^\nu \right) = 0, \]

and the ansatz (11) give

\[ 4\omega A_\psi + \omega_1 A_1 + \omega_2 A_2 + \frac{\kappa^2 \beta}{\alpha^2 - \beta^2} = 0. \]
To work out the sum of the Virasoro constraints, we use the variable $x = \cos 2\xi$, the choice

$$\omega_1 = \omega_2 = \nu \quad \text{and} \quad A_1 = A_2 = A,$$

and (17) to get

$$\frac{dx}{\sqrt{(x^2 - x_-^2)(x_+^2 - x^2)}} = \pm \frac{\omega \alpha}{\alpha^2 - \beta^2} dy,$$

where

$$x_\pm^2 = \frac{1}{2 \omega^2 \nu \alpha^2} \left\{ \nu \left[ \alpha^2 (\nu^2 + 2 \omega^2) - (\alpha^2 + \beta^2) \kappa^2 \right] \pm \right.$$  

\[ \pm \left[ -64 \alpha^2 \omega^2 (\alpha^2 - \beta^2)^2 (\omega^2 + \nu^2) A_\psi^2 - 32 \omega^3 \alpha^2 \beta \kappa^2 (\alpha^2 - \beta^2) A_\psi - 
-4 \omega^2 \alpha^2 \beta^2 \kappa^4 + \nu^2 \left[ (\alpha^2 + \beta^2)^2 - \nu^2 \alpha^2 \right] \right]^{\frac{1}{2}} \right\}.

Using (15) and (19) we can rewrite the conserved charges (11), (12), (13), and (14). For appropriate integration limits we have

$$E = \frac{T \kappa \alpha^2 - \beta^2}{\omega \alpha^2} \int_{x_i}^{x_f} \frac{dx}{\sqrt{(x_+^2 - x^2)(x^2 - x_-^2))}}$$

$$J_{i=1,2} = \left( \beta A_i + \frac{1}{2} \omega_i \alpha^2 \frac{E}{\kappa} \pm \frac{T \omega_i}{4 \omega} \int_{x_i}^{x_f} \frac{x dx}{\sqrt{(x_+^2 - x^2)(x^2 - x_-^2))}} \right)$$

$$J_\psi = \left( 4 A_\psi \beta + \frac{\omega \alpha^2}{\alpha^2 - \beta^2} \right) \frac{E}{\kappa} - \frac{T}{2} \int_{x_i}^{x_f} \frac{x^2 dx}{\sqrt{(x_+^2 - x^2)(x^2 - x_-^2))}},$$

where the upper sign in (22) corresponds to $i = 1$ and the bottom sign to $i = 2$; this notation is used in the whole paper. We also calculate the deficit angles using the general formula $\delta \varphi = \int d\varphi$ and the results are

$$\Delta \phi_{i=1,2} = \left( 2 A_i + \frac{\beta \omega_i}{\alpha^2 - \beta^2} \right) \frac{E}{\kappa T} \pm 2 A_i \frac{\alpha^2 - \beta^2}{\alpha \omega} \int_{x_i}^{x_f} \frac{x}{1 \pm x \sqrt{(x_+^2 - x^2)(x^2 - x_-^2))}} \frac{dx}{(x^2 - x_-^2))}$$

$$\Delta \psi = \left( 4 A_\psi + \frac{\beta \omega}{\alpha^2 - \beta^2} \right) \frac{E}{\kappa T} + 4 A_\psi \frac{\alpha^2 - \beta^2}{\alpha \omega} \int_{x_i}^{x_f} \frac{x^2}{1 - x^2 \sqrt{(x_+^2 - x^2)(x^2 - x_-^2))}} \frac{dx}{(x^2 - x_-^2))}.$$

String energy (21) can be written in terms of an elliptic integral of first kind. From (19) we get

$$\int_{x_1}^{x_f} \frac{dx}{\sqrt{(x_+^2 - x^2)(x^2 - x_-^2))}} = g F(\varphi, k).$$
The parameters $g$, $\varphi$ and $k$ depend on $x_1$, $x_2$, $x_+$, and $x_-$. There are some possibilities of integration according to the choice of the integration limits. Excluding the non-physical cases of $x_+ < 0$ and $x_+ > 1$, the results for the general formula (26) are arranged in Table I.

| Case | $x_\pm$ | Range of $x$ | $g^{-1}$ | $\sin^2 \varphi$ | $k^2$ |
|------|---------|-------------|---------|-----------------|-------|
| 1.   | $x_+^2 > 1$ and $x_-^2 < 0$ | $[0, 1]$ | $\sqrt{x_+^2 + x_-^2}$ | $x_+^2 + |x_-|^2$ | $x_+^2 + |x_-|^2$ |
| 2.   | $x_+^2 > 1$ and $x_-^2 \in [0, 1]$ | $[x_-, 1]$ | $x_+$ | $\frac{x_+^2}{x_+^2 - x_-^2}$ | $1 - \frac{x_+^2}{x_+^2}$ |
| 3.   | $x_+^2 \in [0, 1]$ and $x_-^2 < 0$ | $[0, x_+]$ | $\sqrt{x_+^2 + x_-^2}$ | $1$ | $\frac{x_+^2}{x_+^2 + |x_-|^2}$ |
| 4.   | $x_+^2 \in [0, 1]$ | $[x_-, x_+]$ | $x_+$ | $1$ | $1 - \frac{x_+^2}{x_+^2}$ |

Case (1) corresponds to a circular string and the others to folded strings, all along the $\xi$ coordinate. We also see that every case has divergent energy on the $x_- = 0$, where $\sin \varphi = k = 1$. In this limit we calculate the integrals necessary for the deficit angles and for the momenta,

$$
\int_0^{x_+} \frac{dx}{(1-x)\sqrt{x_+^2 - x^2}} = \frac{\pi - 2\xi_+}{\sin 2\xi_+}, \quad \int_0^{x_+} \frac{dx}{(1+x)\sqrt{x_+^2 - x^2}} = \frac{2\xi_+}{\sin 2\xi_+},
$$

$$
\int_0^{x_+} \frac{x dx}{(1-x^2)\sqrt{x_+^2 - x^2}} = \frac{\pi - 2\xi_+}{\sin 2\xi_+}, \quad \int_0^{x_+} \frac{x dx}{\sqrt{x_+^2 - x^2}} = \cos 2\xi_+ \quad \text{and}
$$

$$
\int_0^{x_+} \frac{dx}{\sqrt{x_+^2 - x^2}} = \frac{\pi}{2},
$$

where we used the definition $x = \cos 2\xi$. $x_- = 0$ also implies that

$$
x_-^2 = 2 + \frac{\nu^2}{\omega^2} - \left(1 + \frac{\beta^2}{\alpha^2}\right)\frac{\kappa^2}{\omega^2} \quad (28)
$$

$$
A_\psi = \frac{1}{4(\omega^2 + \nu^2)(\alpha^2 - \beta^2)} \left(-\beta \omega \kappa^2 \pm \nu \sqrt{(\kappa^2 - \nu^2 - \omega^2)(\alpha^2 (\omega^2 + \nu^2) - \kappa^2 \beta^2)}\right) \quad (29)
$$

$A_\psi$ must be real, and so we obtain the following relations among the other parameters

$$
\frac{\kappa^2}{\omega^2 + \nu^2} \in \left[1, \frac{\alpha^2}{\beta^2}\right] \quad \text{if} \quad \alpha^2 > \beta^2, \quad \text{and} \quad \frac{\kappa^2}{\omega^2 + \nu^2} \in \left[\frac{\alpha^2}{\beta^2}, 1\right] \quad \text{if} \quad \beta^2 > \alpha^2.
$$

(30)
If $\beta^2 > \alpha^2$, then the velocity of the pulse is greater than the velocity of the light, and so the string has a tachyonic character. Thus, we have characterized all the possible motions that a string performs using (4) as ansatz. Now we consider the particular cases where the energy, the momenta, and the deficit angle are divergent.

III. PARTICULAR CASES

A. Divergent Momenta and Finite Deficit Angles

Looking at (24) and (25) we can infer that by choosing

$$A_\psi = -\frac{1}{4} \frac{\omega \beta}{\alpha^2 - \beta^2} \quad \text{and} \quad A = -\frac{1}{2} \frac{\nu \beta}{\alpha^2 - \beta^2},$$

we get finite deficit angles. Accordingly, the momenta are

$$J_i = \frac{1}{2} \frac{\nu}{\kappa} E \mp \frac{T \pi \nu}{4 \omega} \quad \text{and} \quad J_\psi = \frac{\omega}{\kappa} E - T \cos \xi_+$$

and the deficit angles are,

$$\Delta \phi_1 = 2 \frac{\beta \nu}{\alpha \omega} \frac{2 \xi_+}{\sin 2 \xi_+}, \quad \Delta \phi_2 = 2 \frac{\beta \nu}{\alpha \omega} \frac{2 \xi_+ - \pi}{\sin 2 \xi_+} \quad \text{and} \quad \Delta \psi = \frac{\beta}{\alpha} \frac{2 \xi_+ - \pi}{\sin 2 \xi_+}.$$

Looking at the difference between the Virasoro constraints (17) we also obtain a relation among the constant parameters

$$\kappa^2 = \omega^2 + \nu^2,$$

which is consistent with (29) and implies that

$$\cos^2 2 \xi_+ = x_+^2 = 1 - \frac{\beta^2}{\alpha^2} \left(1 + \frac{\nu^2}{\omega^2}\right),$$

The choice $\nu = 0$ leads to

$$J_i = \Delta \phi_i = 0, \quad \sin 2 \xi_+ = \frac{\beta}{\alpha}, \quad \Delta \psi = 2 \xi_+ - \frac{\pi}{2},$$

and we get the dispersion relation of a giant magnon in the coordinate $\psi$

$$E - J_\psi = -T \sin \Delta \psi.$$

This is expected, because the choice $\nu = 0$ restricts the movement of the string to an $\mathbb{R} \times S^2$ sector. On the other hand, $\nu \neq 0$ keeps the string with three angular momenta. Defining
\( J = J_1 + J_2 \) and \( j = J_1 - J_2 \), we write the dispersion relation

\[
\sqrt{E^2 - J^2} - J_\psi = -T \sin \left( \sqrt{1 + \frac{4j^2}{T^2 \pi^2}} \Delta \psi \right), \tag{38}
\]

where

\[
\frac{\nu}{\omega} = \frac{\Delta \phi_1 + \Delta \phi_2}{4 \Delta \psi} = -\frac{2j}{T \pi} \tag{39}
\]

was used. The question which arises is whether the dispersion relation (38) describes a genuine giant magnon with three angular momenta, and we hypothesize that it does. Dispersion relations like (38) have already been used to describe a giant magnon with two angular momenta in \([13, 27]\), and the novelty in the above relation is the square root multiplying \( \Delta \psi \) in the argument of the sine. However, imposing the additional constraint

\[
\sqrt{1 + \frac{4j^2}{T^2 \pi^2}} = 1 + \frac{n\pi}{\Delta \psi}, \quad \text{with} \quad n \in \mathbb{N} \tag{40}
\]

results in the expected relation, and of course \( n = 0 \) returns (37). This interesting constraint between \( j \) and \( \Delta \psi \) seems to quantize its relation in order to produce a giant magnon. Nevertheless, if this quantizing constraint is not imposed, one hypothesis can immediately be raised: as the sine argument is always greater than \( \Delta \psi \), it is possible for the deficit angle to be a lower bound, and so deficit angles greater than \( \Delta \psi \) could represent a giant magnon. Of course, these possibilities need a careful study on the gauge side of the duality, and we hope this result could be used to clarify this point in the future.

**B. Finite Momenta and Divergent Deficit Angles**

To this case the choice

\[
A = -\frac{1}{2 \beta} \frac{\nu \alpha^2}{\alpha^2 - \beta^2}, \quad A_\psi = -\frac{1}{4 \beta} \frac{\omega \alpha^2}{\alpha^2 - \beta^2}, \tag{41}
\]

in (29) and (28) implies that

\[
\alpha^2 \left( \omega^2 + \nu^2 \right) - \kappa^2 \beta^2 = 0 \quad \text{and} \quad x_+^2 = 1 - \frac{\alpha^2}{\beta^2} \left( 1 + \frac{\nu^2}{\omega^2} \right), \tag{42}
\]

and the momenta are

\[
J_i = \pm \frac{T}{4} \frac{\nu}{\omega} \quad \text{and} \quad J_\psi = -T x_+. \tag{43}
\]
From these relations, we can write
\[ \frac{\beta^2}{\alpha^2} = \frac{T^2 + 4j^2}{T^2 - j^2}, \tag{44} \]
which shows the dependence between the velocity of the pulse on the string and the angular momenta on the coordinates. From (42) we see that \( \beta^2 > \alpha^2 \), and so this string is tachyonic.

A tachyon like this also appears in the case of a spiky string with two angular momenta in this background \[13\]. Now, we write the deficit angles
\[
\Delta \phi_1 = \frac{2 \alpha \nu}{\beta \omega} \sin 2\xi_+ - \frac{\nu}{\sqrt{\omega^2 + \nu^2}} \frac{E}{T},
\tag{45}
\]
\[
\Delta \phi_2 = \frac{2 \alpha \nu}{\beta \omega} \sin 2\xi_+ - \frac{\nu}{\sqrt{\omega^2 + \nu^2}} \frac{E}{T},
\tag{46}
\]
\[
\Delta \psi = \frac{\alpha}{\beta} \frac{2\xi_+ - \pi}{2} \sin 2\xi_+ - \frac{\omega}{\sqrt{\omega^2 + \nu^2}} \frac{E}{T}.
\tag{47}
\]
\( \nu = 0 \) generates the relation expected for a spiky string in one dimension
\[
E + T \Delta \psi = T \left( 2\xi_+ - \frac{\pi}{2} \right), \tag{48}
\]
which allows us to write the dispersion relation
\[
E + \sqrt{T^2 + 4j^2} \left[ \Delta \psi + \frac{T}{8j} (\Delta \phi_2 - \Delta \phi_1) \right] = T 2\xi_+, \tag{49}
\]
which gives the spiky string with one angular momentum \[18\] in \( j \to 0 \). The case is similar to the case of the giant magnon of the previous item: we have a dispersion relation that is not exactly the expected one but with a well-known one dimensional limit. As in the former case, we use this evidence to claim that \[49\] represents a spiky string, and we envisage that further studies on the gauge sector of the correspondence will confirm it.

C. Finite \( J_\psi \) and Divergent \( J_i \)

We can also study cases that do not have both the divergent momenta or both the finite momenta. If we have \( J_\psi \)finite and \( J_i \) divergent, the choice
\[
A_\psi = -\frac{1}{4\beta} \frac{\omega \alpha^2}{\alpha^2 - \beta^2}, \quad A = \frac{1}{2} \frac{\nu \beta}{\alpha^2 - \beta^2}, \tag{50}
\]
and \[17\], \[29\], and \[28\] give
\[
\kappa^2 = \frac{\alpha^2}{\beta^2} \omega^2 + \nu^2 \quad \text{and} \quad x_+^2 = 1 - \frac{\alpha^2}{\beta^2} - \frac{\beta^2 \nu^2}{\alpha^2 \omega^2}, \tag{51}
\]
which implies in $\beta^2 > \alpha^2$, and $\omega^2 > \nu^2$, which means that the configuration is tachyonic, as was found in the preceding case. The momenta in this case are

$$J_i = \frac{1}{2\sqrt{1 + \frac{\alpha^2 \omega^2}{\beta^2 \nu^2}}} E \pm \frac{T \pi \nu}{4 \omega} \quad \text{and} \quad J_\psi = -Tx_+. \quad (52)$$

The deficit angles read

$$\Delta \phi_1 = \frac{\beta \nu}{\alpha \omega} \frac{4 \xi_+}{\sin 2 \xi_+}, \quad \Delta \phi_2 = \Delta \phi_1 - \frac{\beta \nu}{\alpha \omega} \frac{2\pi}{\sin 2 \xi_+}, \quad \text{and}$$

$$\Delta \psi = \frac{1}{2 \nu} \Delta \phi_2 - \frac{E}{T \sqrt{1 + \frac{\beta^2 \omega^2}{\alpha^2 \nu^2}}}. \quad (53)$$

A giant magnon is not possible in this case, because it would need a $\omega \to 0$. On the other hand, looking at the deficit angles, we write

$$\sqrt{E^2 - J_i^2} + T \Delta \psi = -\frac{T^2 \pi \Delta \phi_1 + \Delta \phi_2}{4 j}, \quad (54)$$

which is not exactly an expected relation for a spiky string, but it has the same structure as the spiky string with two angular momenta found by Ryang [13]. The right hand side of (54) is given in terms of deficit angles in the $\phi_i$ directions, which is unexpected, as the string peaks along the $\xi$ direction. Yet, choice $\nu = 0$ leads to

$$\frac{\kappa^2}{\omega^2} = \frac{\alpha^2}{\beta^2} = \sin^2 2 \xi_+ \quad (55)$$

and consequently we can write the relation

$$E + T \Delta \psi = T \left(4 \xi_+ - \pi\right), \quad (56)$$

which shows that there is a one-dimensional spiky string limit in the configuration. Indeed (54) is telling us that there is a constraint among the deficit angles in the $\phi_i$ directions and the peak in the $\xi$ direction. This interpretation conforms with the former cases, where new constraints were found, and a well-behaved one-dimensional limit can be reached.

D. Finite $J_i$ and Divergent $J_\psi$

In this case the integration constants of the problem are

$$A = -\frac{1}{2 \beta} \frac{\nu \alpha^2}{\alpha^2 - \beta^2}, \quad A_\psi = -\frac{1}{4} \frac{\omega \beta}{\alpha^2 - \beta^2} \quad (57)$$
which, using (29) and (28), imply that
\[ \kappa^2 = \omega^2 + \frac{\alpha^2}{\beta^2} \nu^2 \quad \text{and} \quad x_+^2 = 1 - \frac{\beta^2}{\alpha^2} - \frac{\alpha^2 \nu^2}{\beta^2 \omega^2} \] (58)
which means that \( \alpha^2 > \beta^2 \), and \( \omega^2 > \nu^2 \). Accordingly, we calculate the momenta
\[ J_i = \pm \frac{\nu T \pi}{\omega} \frac{1}{4}, \quad \text{and} \quad J_\psi = \frac{1}{\sqrt{1 + \frac{\alpha^2 \nu^2}{\beta^2 \omega^2}}} E - T x_+, \] (59)
and the deficit angles
\[ \Delta \phi_1 = \frac{\alpha \nu}{\beta \omega \sin 2\xi_+} - \frac{1}{2T} \frac{E}{\sqrt{1 + \frac{\beta^2 \omega^2}{\alpha^2 \nu^2}}} \Delta \phi_2 = \Delta \phi_1 - \frac{\alpha \nu}{\beta \omega \sin 2\xi_+} \frac{2 \pi}{\Delta \phi_2} \] (60)
\[ \Delta \psi = \frac{\beta \frac{\pi}{2} - 2\xi_+}{\alpha \sin 2\xi_+} \quad \text{and} \quad \Delta \phi_1 - \Delta \phi_2 = \frac{\alpha^2 \beta}{\beta^2 \pi^2 2\xi_+} \frac{\Delta \psi}{\Delta \phi_2} \] (61)
This is probably the most unusual of the four cases studied here. Now we have a \( \nu = 0 \) limit where a giant magnon is found. On the other hand, if \( \nu \neq 0 \) there is a spiky string-like dispersion relation involving the \( \phi_i \) angles. Our interpretation is that we have the giant magnon and the spiky string coexisting and with a coupled dispersion relation, namely
\[ \frac{E}{T} - \sqrt{1 + \frac{\beta^2 T^2 \pi^2}{\alpha^2 4 \beta^2}} \frac{\Delta \phi_1 + \Delta \phi_2}{2} = \frac{1}{2} \alpha \beta \sqrt{1 + \frac{\alpha^2 4 j^2}{\beta^2 T^2 \pi^2}} \Delta \psi \] (62)
\[ \frac{E}{\sqrt{1 + \frac{\alpha^2 4 j^2}{\beta^2 T^2 \pi^2}}} - J_\psi = T \cos 2\xi_+ \] (63)
This is a new kind of structure and, of course, it requires further studies for a better understanding.

IV. CONCLUSION

In this article divergent energy classical strings with three angular momenta were studied. There is evidence that these strings actually correspond to the until-now-undiscovered spiky string and giant magnon configurations, although this evidence is not conclusive. On the other hand, these structures do not obey the usual prescriptions which are known for an \( \mathbb{R} \times S^2 \) subspace or even to strings moving in higher dimensional spherically symmetrical spaces. As the \( \mathbb{R} \times \mathbb{C}P^3 \) is something different from \( \mathbb{R} \times S^n \) we expect different dispersion
relations, at least while there are not, to our knowledge, gauge studies that confirm or deny this hypothesis.

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