**Hi 21 cm observation and mass models of the extremely thin galaxy FGC 1440**

K. Aditya\(^1\) *, Peter Kamphuis\(^2\), Arunima Banerjee\(^1\) †, Sviatoslav Borisov\(^3,4,5\), Aleksandr Mosenkov \(^6\), Aleksandra Antipova \(^5\), Dmitry Makarov \(^5\)

\(^1\)Department of Physics, Indian Institute of Science Education and Research (IISER) Tirupati, Tirupati - 517507, India
\(^2\)Ruhr Universität Bochum, Astronomisches Institut, Universitätsstrasse 150, D-44801 Bochum, Germany
\(^3\)Department of Astronomy, University of Geneva, Chemin Pegasi 51, 1290 Versoix, Switzerland
\(^4\)Sternberg Astronomical Institute, M.V. Lomonosov Moscow State University, Universitetsky prospect 13, Moscow, 119234 Russia
\(^5\)Special Astrophysical Observatory, Russian Academy of Sciences, Nizhnii Arkhyz, 369167 Russia
\(^6\)Department of Physics and Astronomy, N283 ESC, Brigham Young University, Provo, UT 84602, USA

**ABSTRACT**

We present observations and models of the kinematics and distribution of neutral hydrogen (Hi) in the superthin galaxy FGC 1440 with an optical axial ratio \(a/b = 20.4\). Using the Giant Meterwave Radio telescope (GMRT), we imaged the galaxy with a spectral resolution of 1.7 km s\(^{-1}\) and a spatial resolution of 15\(\prime\)'9 \(\times\) 13\(\prime\)'5. We find that FGC 1440 has an asymptotic rotational velocity of 141.8 km s\(^{-1}\). The structure of the Hi disc in FGC 1440 is that of a typical thin disc warped along the line of sight, but we can not rule out the presence of a central thick Hi disc. We find that the dark matter halo in FGC 1440 could be modeled by a pseudo-isothermal (PIS) profile with \(R_c/R_d < 2\), where \(R_c\) is the core radius of the PIS halo and \(R_d\) the exponential stellar disc scale length. We note that in spite of the unusually large axial ratio of FGC 1440, the ratio of the rotational velocity to stellar vertical velocity dispersion, \(v_{\text{rot}}/\sigma_z \approx 5 - 8\), which is comparable to other superthins. Interestingly, unlike previously studied superthin galaxies which are outliers in the \(\log_{10}(j_\text{\textmacron}) - \log_{10}(M_\star)\) relation for ordinary bulgeless disc galaxies, FGC 1440 is found to comply with the same. The values of \(j\) for the stars, gas and the baryons in FGC 1440 are consistent with those of normal spiral galaxies with similar mass.

**Key words:** galaxies: individual, galaxies: FGC 1440, galaxies: kinematics and dynamics, galaxies: structure, method: data analysis

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**1 INTRODUCTION**

Superthin galaxies are subset of edge-on low-surface brightness (\(\mu_B(0) > 22.7 \text{mag arcsec}^{-2}\)) (Bothun et al. 1997; McGaugh 1996) disc galaxies with an axial ratio \(a/b > 10\) (Karachentsev et al. 2003). These late type disc structures observed at high inclination (Matthews et al. 1999; Dalcanton & Bernstein 2000; Dalcanton & Shectman 1996) are some of least evolved disc structure in the Universe (Vorontsov-Vel’yanov 1974; Kautsch 2009; Uson & Matthews 2003) characterized by ratio of high gas mass to blue luminosity, \(M_{\text{HI}}/L_B \approx 1M_\odot L_\odot^{-1}\) (Goad & Roberts 1981; Uson & Matthews 2003), and low star formation rates \(\sim 0.01 - 0.05 M_\odot\text{yr}^{-1}\) (Narayanan & Banerjee 2021; Wyder et al. 2009).

The Hi distribution offers interesting clues to the physical processes regulating the structure, dynamics, and evolution of galaxies. Recent large Hi surveys, THINGS (The Hi Nearby Galaxy Survey) (Walter et al. 2008), LITTLE THINGS (Local Irregulars That Trace Luminosity Extremes, The Hi Nearby Galaxy Survey) (Hunter et al. 2012) have focused on mapping the Hi distribution in the nearby spiral and dwarf galaxies, and explain the role of Hi in regulating the stability/instability and star formation in these galaxies (Leroy et al. 2008; Bigiel et al. 2008). The high-resolution rotation curves derived in these studies have been used for constraining the dark matter mass in these galaxies (De Blok et al. 2008; Oh et al. 2015). The observed rotation curves in conjugation with the stellar photometry,
see for example; SPARC (Spitzer Photometry and Accurate Rotation Curves) (Lelli et al. 2016), and also (Rubin et al. 1980, 1985; De Blok et al. 2001b; Banerjee & Bapat 2017; Kurapati et al. 2018b) have been used to test fundamental scaling relations between the total acceleration and the acceleration due to baryons (Li et al. 2018; Ghari et al. 2019; Li et al. 2019), checking the consistency of A-CDM model (Keller & Wadsley 2017) and testing alternate theories of gravity (Gentile et al. 2011; Naik et al. 2019; Chan & Hui 2018). High resolution H\textsc{i} observations may help in better understanding and characterizing instabilities like warps, bars and spiral arms, etc. in gaseous discs of the galaxies (Phookun et al. 1993), also for indirect characterization using Tremaine-Weinberg method, see for example (Banerjee et al. 2013; Patra & Jog 2019). This may also help in uncovering previously unseen gaseous companions, see, for example, detection of satellite galaxy close to NGC 973 in HEROES (HERschel Observations of Edge-on Spirals) survey (Alaert et al. 2015). The observation of H\textsc{i} halos see for example HALOGAS (Hydrogen Accretion in LOCal GALaxies) survey (Heald et al. 2011) in galaxies provides us with important pointers for understanding the accretion mechanism for replenishing the gas needed for star formation (Voigtlander et al. 2013; Zschaechner et al. 2012; Kamphuis 2008), also see EDGE(Extragalactic Database for Galaxy Evolution survey) - CALIFA (Calar Alto Legacy Integral Field Area) survey (Barrera-Ballesteros et al. 2021; Bolatto et al. 2017).

Measurement of the H\textsc{i} distribution along with the H\textsc{i} dispersion and the radial variation of the scaleheight can be used to model the shape of the dark matter (Olling 1995, 1996). The method was applied to investigate the shape of dark matter halo of eight edge-on gas-rich galaxies with $a/b > 10$ by O’Brien et al. (2010); Peters et al. (2017). Dark matter halo plays an essential role in regulating the structure of the stellar disc; previous studies by Banerjee & Jog (2013); Banerjee et al. (2010) have shown that a compact dark matter halo plays a vital role in regulating the super-thin structure and in stabilizing the galactic disc against axis-symmetric instability (Garg & Banerjee 2017; van der Kruit et al. 2001; Ghosh & Jog 2014). Since these galaxies are rich in neutral hydrogen, it contributes significantly to the total potential and plays an important role in regulating the vertical structure of both the stellar and the H\textsc{i} disc (Narayan & Jog 2002). Similarly, the role of H\textsc{i} gas on the stability of galaxy disc was investigated by Jog (1996); Romeo & Falstad (2013). Recent studies have pointed out that the low value of the observed stellar scaleheight is a direct outcome of very low vertical stellar velocity dispersion and that these superthin galaxies are highly stable despite low values of dispersion (Aditya & Banerjee 2021).

Of all the well known superthin galaxies extensively studied in literature till now for the structure of neutral hydrogen, stellar dynamics and properties of dark matter haloes ; UGC 7321 (Uson & Matthews 2003; Matthews & Wood 2003; Matthews et al. 1999; Banerjee et al. 2010; Sarkar & Jog 2019a; Aditya & Banerjee 2021; Komanduri et al. 2020; Pohlen et al. 2003) has the highest axial ratio $a/b$ equal to 15.4, followed by FGC 1540 (Kurapati et al. 2018b) which has $a/b$ equal to 12.25. Other superthin galaxies IC 2233 (Matthews & Uson 2008, 2007; Gallagher & Hudson 1976) and IC 5249 (Abe et al. 1999; van der Kruit et al. 2001; Byun 1998; Yock et al. 1999) have an $a/b$ equal to 8.9 and 10.4 respectively.

In this paper, we report H\textsc{i} 21 cm observation of FGC 1440, which has an axial ratio equal to 20.36 ($B$-band) and is among the flattest known galaxies. To our knowledge, no previous studies have mapped the distribution of the neutral hydrogen studied shape of dark matter halo in such extremely thin galaxies; $a/b \geq 20$. We derive the basic structural and kinematic properties of FGC 1440 by modeling the three-dimensional distribution of the neutral hydrogen in FGC 1440. Comparing different model datacubes, we derive limits on the scaleheight, velocity dispersion, and inclination. We finally use the total rotation curve in conjugation with the stellar photometry to derive constraints on the shape of dark matter density in FGC 1440. Further, using the stellar and the H\textsc{i} surface density, along with the dark matter mass models, we solve the two-component Jeans equation for modeling the vertical stellar dispersion as a function of radius. We use the observed stellar scaleheight and the limits on the H\textsc{i} velocity dispersion and the H\textsc{i} scaleheight as constraints on the two-component Jeans equation (Narayan & Jog 2002; Banerjee et al. 2010; Sarkar & Jog 2020, 2019b; Patra 2020b,a, 2018).

The paper is organized as follows; in §2, we introduce the target FGC 1440, in §3 we discuss the data reduction method, in §4 and §5, we present the analysis of the H\textsc{i} data cube and the results from the modeling of the three-dimensional structure of neutral hydrogen. In §6, we describe the optical photometry of FGC 1440. In §7 and §8, we present the results from mass models and constrain the vertical velocity dispersion using the observed scaleheight in conjugation with the best-fit mass model using the two-component Jeans equation. We finally present the associated discussion in §9 and conclude in §10.

### 2 TARGET: FGC 1440

FGC 1440 is an edge on disc galaxy included in the ultra flat galaxy catalogue (Karachentseva et al. 2016). Ultra flat galaxies are defined by very large axial ratio ($a/b$)$_R > 10$, where $a/b$ is the ratio of the major axis to minor axis. The major and minor axes of FGC 1440 (Karachentsev et al. 1993, 2003) in $B$-band is $a_B \times b_B = 2.24^\prime \times 0.11^\prime$, which gives an ($a/b$)$_B = 20.36$. FGC 1440 is an edge-on ($i = 90^\circ$) late type Sd spiral galaxy (De Vaucouleurs et al. 1991), located at a distance of 59.6 Mpc (Kourkchi et al. 2020). In a study Hoffman et al. (1989) report an H\textsc{i} diameter 3.8$^\prime$ and further indicate that the ratio of dynamical mass to blue luminosity$(M_{HI}/L_B)$ is equal to 11.9 $M_\odot/L_\odot$, lower limit on the ratio of H\textsc{i} mass to blue luminosity being $M_{HI}/L_B > 0.86 M_\odot/L_\odot$. The H\textsc{i} properties of FGC 1440 have also been delineated in the ALFALFA H\textsc{i} source catalogue, Haynes et al. (2018) report H\textsc{i} flux $F = 9.53 \pm 0.09$ Jy km s$^{-1}$, H\textsc{i} mass $log(M_{HI}) = 10.08 \pm 0.18 M_\odot$, and $W_{50} = 298 \pm 2$ km s$^{-1}$. In their detailed study on the optical photometry of 47 late-type galaxies Dalcanzzi & Bernstein (2000, 2002) have observed that FGC 1440 has an extremely small bulge in the center, but they conclude that it might not be a kinematic bulge but rather an edge-on orientation of pseudo-bulge. Further, from their study of vertical color gradients Dal-
canton et al. (2004) point out that the FGC 1440 might also host concentrated dust lanes. The studies investigating the kinematics of the thick disc in Yoachim & Dalcanton (2008) find that FGC 1440 does not show signatures of a thick disc component, based on their measurement of the off-plane rotation curve, which they find is very similar to the mid-plane rotation curve. We have summarized the basic properties of FGC 1440 in Table 1.

### 3 OBSERVATIONS AND DATA REDUCTION

We observed FGC 1440 with the Giant Meterwave Radio Telescope (GMRT) on August 19, 2019, for 7 hours (including the overheads) with 26 working antennas. The target FGC 1440 was observed for 5.5 hours in 11 scans of 30 minutes each, interspersed by 11 observations of phase calibrator 1150-003 of five minutes each. The flux calibrator 3C286 was observed at the beginning and the end of the observation for a total of 30 minutes. The observation of the central frequency 1402.5 MHz was done in GSB mode with 512 channels and a spectral resolution of 8.14 kHz (1.71 km s\(^{-1}\)) and a total bandwidth of 4.14 MHz. Details of the observation are summarized in Table 2.

#### 3.1 Flagging and Calibration

We perform the data reduction of our GMRT observation using Common Astronomy Software Application (CASA) (McMullin et al. 2007). We begin by flagging the known bad antennae E04, E05, E06, and S02, which were offline during the observation. We then visually inspect the data set and flag the corrupted data. After flagging, we follow the usual procedures for cross-calibration. After cross-calibrating and splitting the ‘target’ from the measurement set, we average the visibilities in time to locate the channels containing the spectral line and then flag those channels to create again a ‘continuum – only’ measurement set which we will use for self-calibration.

#### 3.2 Imaging the spectral line

We make our zeroth dirty image using CASA task TCLEAN and manually mask the emissions. We further do about four rounds of ‘phase – only’ self-calibration and three rounds of ‘amplitude – phase’ self-calibration, lowering the cleaning threshold in each round. We don’t find any improvement in the rms value with further self-calibration and post continuum imaging. We do not detect any continuum emission from the center or around the galaxy FGC 1440. We further apply the final amplitude-phase self-calibration table to the ‘target – only’ measurement set containing both the spectral lines and continuum emissions before performing continuum subtraction using CASA task UVCORSUB with zeroth order interpolation excluding the spectral channels. We create a data cube with the task tclean and clean the emission within a mask made by the Source Finding Application (SoFiA, Serra et al. (2015)) down to a level of 0.5σ. We iterate this process until the SoFiA mask is stable. We have experimented with different weighting schemes in TCLEAN, and we find that briggs weighting scheme with robustness parameter equal to zero and a uvtaper of 10k gives us the best compromise between resolution and sensitivity. We finally perform Hanning Smoothing on the cube and find that the final resolution of the data cube is 15.9″ × 13.5″, and the rms noise is 1.01 mJy/beam compared to the expected theoretical noise is 1mJy/beam.

#### 4 ANALYSIS

##### 4.1 Global H\(\text{I}\) profile

In Figure 1, we present the global H\(\text{I}\) profile of the galaxy FGC 1440. We note that our integrated H\(\text{I}\) flux 9.6 \(Jy\) km s\(^{-1}\) is comparable to that obtained in single dish observation by Haynes et al. (2018). We fit the observed profile using busy-function (Westmeier et al. 2014) to derive the parameters corresponding to the profile. We find that the velocity widths 20% and the 50% of the peak maximum are 304.9
disc as two halves and fits 9 data cube as an input and automatically estimates flux. emission extends out of line. flux density.  

\[ I \propto \text{beamwidth} \]

5.3 Manual TRM Models

In an attempt to break the degeneracy between the parameters, by constructing a Flat–Disc model, we perform several rounds of manual fitting with TiRiFiC followed by visual inspection of channel maps, PV diagrams at various offsets, and the moment maps to estimate the parameters describing the Flat–Disc model. The method of iterative visual inspection to routinely carried out to fine-tune the parameter and arrive at the final model data cube see for example Allaert et al. (2015) and also Zschaechner et al. (2012), Gentile et al. (2013), Kamphuis et al. (2013).

Using the model data cube output by FAT as the base model, we iteratively construct model data cubes, each time changing the following parameters; inclination (i), dispersion (\( \sigma \)), and the scaleheight\( (h_z) \) in the model output by FAT. In
the Flat–Disc model, we fix the center’s values (RA, Dec), systemic velocity, surface brightness, and position angle to the values derived by FAT. We make different model data cubes by varying the parameters \(i, \sigma, h\) either one by one, in pairs or all together at the same time. We compare each model with the data through a visual-inspection, wherein we compare the model and data channel-wise, comparing 3D models of data and the model data cubes using volume rendering software astroslicer (Punzo 2017), and comparing the datacube by taking slices at various offsets along the major axis (as it preserves the 3D structure of the cube) to arrive at the secondary base model called the ‘Flat–Disc’ model. In Figure 6, we show the Moment 0 and the Moment 1 map derived for the ‘flat-disc model’ superposed on the data.

In order to show the effect of varying the inclination \(i\),
Figure 3. In the top panel we have plotted the Moment0 map the contours are at \[ 2.5, 5.0, 10, 15, 22 \] \times 40 \text{ mJy beam}^{-1} \text{ kms}^{-1}. In the bottom panel, we have shown the Moment 1 map, and the contours start at 4000 kms^{-1} increasing at 35 km s^{-1}.

Figure 4. In the above figure we have plotted the rotation velocity and the surface brightness profile obtained from the tilted ring modeling. The surface brightness is fitted independently for the approaching and the receding side.

Table 4. Parameters describing 'FAT' and 'Flat-Disc' models.

| Parameter       | FAT-Model | Flat-Disc-Model |
|-----------------|-----------|-----------------|
| \(X_0^a\)       | +187.2    | +187.2          |
| \(Y_0^b\)       | +4.29     | +4.29           |
| \(i^c\)         | 85°       | 88.5°           |
| \(V_{sys}^d\)   | 4179.45   | 4179.45         |
| \(PA^e\)        | 53.5°     | 53.5°           |
| Surface Brightness \(f\) | fig 4 | fig 4 |
| Dispersion \(g\) | 6.6 km s^{-1} | 15 km s^{-1} |
| Rotation velocity \(h\) | fig 4 | fig 4 |
| \(h_z^i\)       | 6.10 '' (1.8 kpc) | 0.45 '' (0.13 kpc) |

\(a\) Right ascension  
\(b\) Declination  
\(c\) Inclination  
\(d\) Systemic velocity  
\(e\) Position angle  
\(f\) Surface brightness as a function of radius  
\(g\) Velocity dispersion  
\(h\) Rotation velocity as a function of radius  
\(i\) Scaleheight of the H\(_\text{i}\) disc

dispersion (\(\sigma\)) and the scaleheight (\(h_z\)), we plot the minor axis PV diagrams (Figure 7 to 11) at different offsets, varying each of the parameter \((i, \sigma, h_z)\) one by one, while keeping the values of the other two-parameters to that of the Flat−Disc model (Table. 4). For example, in the second row of the Figure 6, we show the effect of variation of the inclination and how it compares with the data. We keep the values of the dispersion and scaleheight equal to that of the Flat−Disc model and vary the values of the inclination. Similarly, in the third row, we keep the values of inclination and scaleheight fixed to that of Flat−Disc and vary only the dispersion.

Also, we consider the models in which we vary the values of the inclination, dispersion, and scaleheight as a function of radius, keeping the value of other parameters to be same as that of the Flat−Disc model:

- Radially inclination \((i(R))\): The inner rings are kept at an inclination equal to 90° and the outer rings at 85°.
• Radially varying dispersion $\sigma(R)$: Dispersion varies from 20 km s$^{-1}$ for the inner rings to 5 km s$^{-1}$ at the outer rings in steps of 5 km s$^{-1}$.

• Radially varying scaleheight $h_z(R)$: The inner rings are kept at an $h_z = 0.45''$, the central rings are kept at $h_z = 1.97''$, and the outer rings at $h_z = 5.3''$.

Inclination ($\theta$). For studying the effect of inclination, we fix the values of all the parameters to that of the 'flat disc model' and vary just the inclination to different values. From the minor axis PV diagrams (Figure 7 to Figure 11), the first immediate observation is that we can rule out the models with an $i < 85^\circ$, for example, the PV plot at an offset equal to 0, Figure 7, the inner model contours at 9$\times$rms are not extended sufficiently to describe the emissions and, further models with lower inclination only increase this discrepancy. Comparing the PV diagrams at different offsets, we find that the inclination is restricted in the range $85^\circ < i < 90^\circ$. We note that the inclination value estimated by FAT $i = 85^\circ$ is the lower limit for the inclination of FGC 1440. Models with an inclination lower than $85^\circ$ do not describe the data accurately.

Dispersion ($\sigma$). The value of dispersion is set to 15 km s$^{-1}$, whereas FAT fits dispersion profile; 6.63 km s$^{-1}$. By comparing the PV diagrams (figure 7 to 11) we find that the data is not very sensitive to the change in dispersion as the models with dispersion 5 km s$^{-1}$to 15 km s$^{-1}$show little variation at the level of data. However, we find that the models with dispersion greater than 15 km s$^{-1}$clearly start to deviate from the data. We find that the value of the dispersion describing the data is restricted in the range 5 km s$^{-1}$ < $\sigma_\text{rms}$ < 15 km s$^{-1}$.

Scaleheight ($h_z$). In Figure 6 - 9, we observe that the models with higher scaleheight are spatially more extended as compared to models with lower scaleheight, which is more extended along the velocity axis but not spatially. Further, we observe that, in the case of models with higher scaleheight, the model contours do not follow the data contours in the inner regions, i.e., the contours at 9$\times$rms. The models with lower scaleheight do follow the data contours in the inner regions of the PV diagrams, but it is not easy to distinguish between the models with $h_z = 0.45''$ and $h_z = 1.97''$. We further construct a model with radially varying scaleheight, based on the observation that the inner rings at lower scaleheight give a better description of the data contours in the innermost region of the PV diagram, and the outer rings kept at the higher scaleheight will match the spatially extended data contours in the outer region of the PV diagram. We find the model with radially varying scaleheight is barely distinguishable from models with $h_z = 0.45''$ or $h_z = 1.97''$. To further investigate the vertical structure, in Figure 12, we plot the normalized vertical density profile extracted at various slices from the moment 0 maps and overlaid the major axis FWHM of the synthesized beam. We observe that the synthesized beam is comparable to the vertical density profile extracted from the data, thereby indicating that the disc’s thickness is barely resolved in these observations. From comparing PV diagrams, we find that the upper limit on scaleheight is 5.3''.

By comparing the model and data in the minor axis PV diagrams Figure 7 - 11, following above discussion, we find the lower and upper limits for the values for the inclination ($85^\circ \leq i \leq 88.5^\circ$), dispersion (5 km s$^{-1}$ $\leq \sigma \leq$ 15 km s$^{-1}$) and the scaleheight ($h_z \leq 5.3''$).

5.4 Thickness of the H$^i$ disc
Is it a Flare, thick disc or a line of sight warp?
In Figure 2, from the channel maps, we observe that the emissions from the channels close to the systemic velocity extend above the plane as compared to the emissions from the end channels, possibly indicating the presence of a thick H$^i$ disc or a line of sight warp. We can possibly rule out a flaring disc. The flare would be circular and hence every where along the line of sight. The flare would be better noticeable in the outer channel because the sight line through the outer parts of the disc would be longer and hence the flare is harder to identify in the center. In order to investigate the origin of the observed thickness of the H$^i$ disc in the FGC 1440, we make the Moment 0 map considering only the starting channels (4023 km s$^{-1}$ - 4100 km s$^{-1}$), the central channels close to systemic velocity (4100 - 4242 km s$^{-1}$) and the end channels (4242 - 4325 km s$^{-1}$). We then compare the data contours for the said velocity ranges with the model with radially varying scaleheight, inclination, and the flat disc model. We find that the data contours (see Figure 13) are as thin as the model contours in high-velocity channels, indicating that possibly the H$^i$ disc in FGC 1440 does not show flaring behavior. Further absence of flaring is supported by the observation that instead of the end channels at higher velocities, channels close to the central velocity contribute to the thickness of the H$^i$ disc. Thus, a thick H$^i$ disc at the center with thickness tapering off radially leaves us only with two options; that either we are observing a line of sight warp or a thick H$^i$ disc at the center.

In order to further investigate this effect we consider the following models 1) A model with radically varying inclinations and 2) A two-component model with a lagging thick disc ($\frac{dv}{dz} = -10$ km s$^{-1}$ kpc$^{-1}$) and ($h_z = 5.3''$) and a thin disc ($h_z = 0.45''$). The value of scaleheight of the thick disc in the two-component model is equal to the upper limit on the scaleheight obtained for the one component model. We then iteratively compare models with data through visual-inspection for different values of $\frac{dv}{dz}$, and find that $\frac{dv}{dz} = -10$ km s$^{-1}$ kpc$^{-1}$ better reproduces the data contours. From Figure 13 and 14, we find that overall models with radially varying inclination and the two-component model are very similar; it is barely possible to distinguish if the observed central thickness is the result of radially varying inclination or if the galaxy host central thick H$^i$ disc. Further, in Figure 12, we compare the synthesized major axis beam with the density profile, indicating that we possibly can not resolve the thick disc in our observations, although we can not rule out the presence of a central thick H$^i$ disc. We adopt a model with a line of sight warp as it is simpler model than the two-component model.

6 OPTICAL PHOTOMETRY
In this section, we present the photometric analysis of FGC 1440 in SDSS g, r band, and UKIDSS K band. As a first step, we mask all the surrounding objects and the galaxy
Aditya et al.

Figure 6. In the top panel we have plotted the moment 0 (top) and the moment 1 (bottom) map the contours are at [2.5, 5.0, 10, 15, 18, 22]×40 mJy beam⁻¹ km s⁻¹, and the contours for moment 1 map start at 4000 km s⁻¹ increasing at 35 km s⁻¹ respectively. The contours corresponding to the data are rendered in black and the contours Flat – Disc – Model are in shown in red color.

Figure 7. Position velocity maps parallel to the the minor-axis comparing the tilted rings model to data at offset equal to 0 by varying the model parameters. The contours at [1.5, 3, 6, 9]×1.01 Jy/beam.

itself and estimate the positions and the positional angles (PA) using SExtractor (Bertin & Arnouts 1996). We subtract the background by fitting it with a two-dimensional second-order polynomial and then further rotate the entire frame by the PA. Next, we eliminate all the objects lying close to the galaxy by replacing them with the regions symmetric to the galaxy’s mid-plane. We then further integrate the light in a rectangular box to estimate the total magnitude. The size of the box was chosen in such a way that the growth curve becomes flat near the borders of the box. The estimated value of the total magnitude in the g, r, and K bands are 15.43, 14.72, and 11.83. We derive the structural parameters of the galaxy using GalFit (Peng et al. 2011). The intensity profile is \( I \sim \frac{1}{R/R_d K_1(R/R_d) \sech^2(z/h_z)} \), where \( R_d \) is the disc scale radius, \( h_z \) is the disc scaleheight, and \( K_1 \) is the modified Bessel function of the second kind. The parameters derived using optical photometry are summarized in Table 5. The data, the galfit model, and their normalized difference are shown in shown in appendix A.
Figure 8. Same as figure 6 but at offset = 20.

Figure 9. Same as figure 6 but at offset equal to -20.
Figure 10. Same as figure 6 but at an offset equal to 40.0.

Figure 11. Same as figure 6 but at offset equal to -40.
7 MASS MODELING

In this section we will present our analysis and results from mass models of FGC 1440. By decomposing the total rotation curve of the galaxy into baryonic (stars+HI) and dark matter components we will determine the contribution of each mass - component to the total rotation curve ($V_{\text{Total}}$) (see Figure 4). The total rotation curve of the galaxy is obtained by adding in quadrature the circular velocity profiles produced by each component separately.

$$V_{\text{Total}}^2 = \gamma V_*^2 + V_{\text{gas}}^2 + V_{\text{DM}}^2$$  \hspace{1cm} (1)

where $\gamma$ is the mass to light ratio (M/L), $V_*$, $V_{\text{gas}}$, and $V_{\text{DM}}$ are the circular velocity profiles due to the stars, gas and dark matter components respectively.

7.1 Neutral gas distribution and rotation curve

We model the gas disc as thin concentric rings and use GIPSY (Van der Hulst et al. 1992) task ROTMOD to derive $V_{\text{gas}}$. We use the surface densities as a function of radius obtained from the tilted ring model (panel 2 Figure 4) as the input parameter in ROTMOD. The gas surface density is scaled by a factor of 1.4 to account for helium and other metals.

7.2 Stellar distribution and rotation curve

For ascertaining the contribution of the stars to the observed rotation curve, we model the stellar distribution in the optical g band and the near-infrared (NIR) UKIDSS K band. We derive mass to light ratio ($\gamma^*$) ratio following the empirical relations in Bell et al. (2003) and Bell & de Jong (2001).
Based on stellar population synthesis models. The scaling between the color magnitude and the mass to light ratio ($\gamma^*$) is given by:

$$\gamma^* = 10^{(a_1 + b_1 \times \text{Color})} \tag{2}$$

In the above equation $\gamma^*$ is the mass to light ratio, $a_1$ and $b_1$ are the intercept and slope of the $\log_{10}(\gamma^*)$ versus color calibration obtained by Bell & de Jong (2001) using stellar population synthesis models. We compare the mass models derived using 'diet' Salpeter IMF with the mass models derived using Kroupa IMF (Kroupa 2001). The $\gamma^*$ ratios assuming Kroupa IMF are derived by subtracting 0.15 dex from the constant term $a_1$ (table 5). We use the input parameters described in table 5 to derive the stellar rotation curve using the GIPSY task ROTMOD.

### 7.3 H$\alpha$ Rotation curve

For mass modeling, we derive the hybrid rotation curve, wherein the inner region of the total rotation curve is composed of points from the H$\alpha$ rotation curve, and the H I 21 cm data is used to define the outer points (De Blok et al. 2001a). We have taken the optical rotation for FGC 1440 from Yoachim & Dalcanton (2008). In order to derive a smooth curve passing through the raw optical rotation curve, we fit a simplified multi-parameter function as in Yoachim & Dalcanton (2005); Courteau (1997),

$$V(r) = V_0 + \frac{V_c}{(1 + x^2)^{\frac{1}{2}}} \tag{3}$$

where $V_0$ is the recession velocity of the galactic center, $V_c$ is the asymptotic rotation velocity (the flat part), $x$ is defined...
Table 6. Best fit values describing the Multi-parameter model fitted to raw optical curve.

| $V_{ao}^{(a)}$ | $V_c^{(b)}$ | $i_0^{(c)}$ | $r_i^{(d)}$ | $\gamma^{(e)}$ |
|---------------|-------------|--------------|-------------|--------------|
| km s$^{-1}$   | km s$^{-1}$ | kpc          | kpc         |              |
| 4253.5        | 149.4 ± 4.2 | 1.4 ± 0.04   | 12.3 ± 0.6  | 1.3 ± 0.1    |

(a): Recession velocity at the galaxy center.
(b): Asymptotic rotation velocity at the flat part.
(c): Center of the galaxy.
(d): Point of transition between the rising and the flat part.
(e): Degree of sharpness of transition.

as $x = r_i/R - r_0$, where $r_i$ is the transition section between the rising and the flat part of the rotation curve and $r_0$ and $\gamma$ define the center of the galaxy and degree of sharpness of transition respectively. The best-fitting parameters obtained by fitting Equation 3 to the raw rotation curve are detailed in Table 6. In Figure 15 we show the smooth optical rotation curve and the raw data points.

7.4 Dark matter models

In this work, we parameterize the dark matter distribution using observationally motivated pseudo isothermal (PIS) dark matter halo (Begeman et al. 1991; Fuchs et al. 1998) dominated by a constant density core and, also the Navarro-Frenk-White (NFW) Navarro et al. (1997) dark matter halo profile derived from the cold dark matter (CDM) simulations.

7.4.1 Pseudo-Isothermal halo model

The density profile of the observationally motivated cored spherical pseudo-isothermal (PIS) dark matter halo is,

$$\rho(r) = \frac{\rho_0}{1 + (\frac{r}{R_c})^2}$$

(4)

where $\rho_0$ is the central density of the halo and $R_c$ is the core radius. The rotation curve of the PIS halo is given by

$$V(R) = \sqrt{\left(\frac{4\pi G\rho_0 R_c^2}{1 - \left(\frac{R}{R_c}\right) \arctan\left(\frac{R}{R_c}\right)}\right)^2}$$

(5)

where $V_{\infty} = \sqrt{4\pi G\rho_0 R_c^2}$ is the asymptotic rotation velocity of the dark matter halo. The inner density distribution for PIS halo i.e $R_c \geq R$ is dominated by a constant density core.

7.4.2 NFW halo model

The density distribution of the cuspy NFW dark matter halo derived from the CDM simulations is

$$\rho(R) = \frac{\delta_c}{\rho_{crit}} \left(\frac{R}{R_c}\right)^{1 + \frac{3}{2}}$$

(6)

where $\delta_c$ is the density of the universe at the time of the collapse of the dark matter halo, $R_c$ is the characteristic scale radius, and $\rho_{crit} = 3H^2/8\pi G$ is the critical density of the universe. The inner density of the NFW in the inner radii $R \leq R_c \rho \neq r^{-1}$, corresponding to steep and cuspy density distribution. The rotation curve due to the NFW density distribution is,

$$V(r) = V_{200} \sqrt{\frac{\ln(1 + cx) - cx/(1 + cx)}{\ln(1 + c) - c/(1 + c)}}$$

(7)

where, $x = \frac{R}{R_{200}}$, $R_{200}$ is the radius at which the mean density of the dark matter halo is 200 times the critical density. $V_{200}$ is the rotation velocity at $R_{200}$. The concentration parameter is defined as $c = \frac{R_{200}}{R_c}$.

7.5 Modified Newtonian gravity (MOND)

Apart from the standard dark matter model described in the above section, we use our rotation curve data to test if just the baryonic matter suffices to explain the observed rotation curve in the context of the Modified Newtonian dynamics (MOND) paradigm. The net rotation curve in MOND is given by (Milgrom 1983)

$$V(r) = \sqrt{\frac{1}{\sqrt{2}} V_{gas} + \gamma V_2^2} \left[1 + \left(\frac{2R_a}{V_{gas}^2 + \gamma V_2^2}^2\right)^{-\alpha}\right]$$

(8)

where $a$ is acceleration and $\gamma^*$ is the mass to light ratio.

7.6 Fitting Method

We define the likelihood function as $\exp(-\chi^2)$, where $\chi^2$ is given by,

$$\chi^2 = \sum_R \left(\frac{V_{obs}(R) - V_T(R)}{V_{err}^2}\right)^2$$

(9)

where $V_{obs}$ is the observed rotation curve (see top panel Figure 4), $V_T$ is the total rotation curve (see Equation 1) obtained by adding in quadrature the baryonic and the dark matter components and $V_{err}$ is the error on the observed rotation curve. For optimizing the likelihood function, we use the publicly available python package LMFIT (Newville et al. 2016). The residuals and the corresponding reduced chi-square values $\chi^2_{red}$ are shown in Figures 15 and 16. The H$\alpha$ rotation curve derived for FGC 1440 using 3D tilted ring modeling (section 5.2) has few points in the inner region of the galaxy, and the slope of the dark matter density critically depends on the shape of the rotation curve in the inner region i.e. a steeply rising or slowly rising rotation curve. In order to overcome this problem we use the H$\alpha$ rotation curve in the inner region and the points from the H$\alpha$ rotation curve in the outer region, where the H$\alpha$ data is not available. Although the shape of the H$\alpha$ rotation curve is well defined, in order to account for the scatter between the points, we derive a smooth representation of the H$\alpha$ rotation curve (§7.3). Further, to derive the hybrid rotation curve consisting of the points from the smooth H$\alpha$ rotation curve in the inner region and the H$\alpha$ rotation curve in the outer region, we use B-spline method from python package scipy (Virtanen et al. 2020) and create a smooth spline approximation of the data. Further, in order to gauge the effectiveness of using the hybrid rotation curve for estimating the dark matter parameters and mass models, we also derive
the mass models constrained by the HI rotation curve only, the results of which are shown in Appendix-B. We find that the results using only the HI rotation curve and those using the hybrid rotation curve are comparable. We assume uniform error bars equal to 10 km s$^{-1}$ for the data points defining the observed rotation curve as these are typical conservative error estimates, see for example (De Naray et al. 2008; McGaugh et al. 2001), also McGaugh et al. (2001) show that the halo parameters are robust against precise definition of the error-bars.

In case when the dark matter distribution is parameterized using the pseudo-isothermal dark matter halo, the free parameters are $\rho_0$ and $R_c$, and mass to light ratio $\gamma^*$ along with $\rho_0$ and $R_c$; in case of mass, models derived keeping the mass to light ratio as a free parameter. When the dark matter density is parameterized using the NFW halo, the free parameters are $c$ and $R_{200}$, and $\gamma^*$ is the free parameter along with $c$ and $R_{200}$ in case of mass models in which the mass to light ratio is kept as a free parameter.

### 7.7 Results from mass modeling

This section presents the mass models constructed using the SDSS optical g-band and NIR K-band rotation curve in conjunction the derived HI and the total rotation curve as detailed in the above sections. For each of the photometric band, we fit both the NFW and PIS dark matter halo profile and discuss the following cases:

- **The constant $\gamma^*$** We derive $\gamma^*$ using population synthesis models as described by Bell & de Jong (2001).
  - a) diet - Salpeter IMF which gives highest disc mass for a given photometric band (De Blok et al. 2008) and
  - b) Kroupa IMF which produces lower disc mass Kroupa (2001) as compared to ’diet - Salpeter IMF’.

- **Free $\gamma^*$** In this model, the $\gamma^*$ is kept as a free parameter along with the parameters corresponding to the dark matter density profile.

- **Maximum Disc** We scale the stellar rotation curve by scaling the $\gamma^*$ such that the observed rotation curve in the inner region is entirely due to the stellar component. The maximum disc model sets the lower limits on the dark matter density.

- **Minimum Disc** We set the contribution of the gaseous disc and stars to zero and attribute the observed rotation curve to be entirely due to the underlying dark matter distribution. The minimum disc model sets the upper limit on the dark matter density.

In Figure 16, we present mass models constructed using the stellar rotation curve derived using g-band Photometry and the HI rotation curve. We find that the reduced chi square($\chi^2_{\text{red}}$) for the cored pseudo-isothermal dark matter halo is lower than the cuspy NFW dark matter halo, only in the case of the model derived using ‘diet’ - Salpeter IMF both the halos give similar ($\chi^2_{\text{red}}$) values. For mass models using PIS halo, we find that the Kroupa IMF gives lower reduced ($\chi^2_{\text{red}}$) as compared to the ‘diet’ - Salpeter IMF, indicating that the mass distribution is possibly dominated by dark matter. In mass models for PIS halo with $\gamma^*$ as a free parameter, $\gamma^*$ values tend to values closer to that derived using the ‘diet’ - Salpeter IMF, in case of the mass models with NFW halo and $\gamma^*$ kept as a free parameter, we find that the $\gamma^*$ tends to values higher than that derived using the stellar population synthesis models. In the case of the maximum disc models, we have scaled the stellar rotation curve by a factor of 14 to maximize the contribution of the stellar disc and set the baryonic to zero in the case of the minimum disc models.

Similarly, in Figure 17, we present the results from mass models using the stellar rotation curve in K-band. The $\chi^2_{\text{red}}$ for the mass model with PIS dark matter halo is systematically lower than that corresponding to the NFW dark matter halo. The $\chi^2_{\text{red}}$ for the Kroupa IMF is lower than that for ‘diet’ - Salpeter IMF. Whereas in the case of mass models with $\gamma^*$ as a free parameter, $\gamma^*$ is even lower than that derived using Kroupa IMF in the case of both PIS halo and NFW halo, possibly indicating the mass models prefer lighter IMF in K-band. We scale the stellar rotation curve by a factor of 1.1 to derive the maximum disc model and set the baryonic contribution to zero for the minimum disc models.

The definition of compact dark matter halo follows from the ratio of the core radius to the disc scalelength $R_c/R_d < 2$ (Banerjee & Bapat 2017). In Table 7, we have indicated $R_c/R_d$ for each model, in all the cases other than the maximum disc case in K-band $R_c/R_d$ is less than 2. We also note that the definition of a compact halo is not independent of the choice of IMF, as models which prefer maximum disc in a given photometric band have a larger core radius and less compact dark matter halo.

For models with constant IMF, we find that the concentration parameter ranges between 2.8 (diet-Salpeter K-band) and 5.5 (Kroupa IMF in g-band). Similar to the compactness parameter, the IMF models preferring higher disc masses have smaller concentration parameters. The scaling between asymptotic rotation velocity $V_{\text{max}}$ and concentration parameter given in Bottema & Pestana (2015) $c_{\text{exp}} = 55.5 (V_{\text{max}}/[\text{km s}^{-1}])^{-0.2933}$, gives us $c = 12.97$. We note that the value of concentration parameters derived in this work are much lower than the value predicted by the scaling. Similarly the the scaling between $V_{\text{max}}$ and $R_{200}$, where $R_{200} = 0.0127(V_{\text{max}})^{1.37}c_{\text{exp}}$ gives $R_{200} = 146kpc$ which is closer to the values of $R_{200}$ obtained in case of maximum disc models. We also derive mass models using HI rotation curve derived using the tilted ring models, see Appendix-B, we find that the reduced $\chi^2_{\text{red}}$ for the PIS halo is smaller than that obtained for the NFW halo.

We further compare our results with theoretical predictions between the ratio of $V_{\text{max}}/V_{200}$ and concentration parameter from Dutton & Maccio (2014). The relation between $V_{\text{max}}/V_{200}$ and concentration parameter $c$ is defined as $V_{\text{max}}/V_{200} = 0.216c_{200}/f(c_{200})$, where $f(c_{200}) = \ln(1 + c) - c/(1 + c)$. Using the the values of concentration parameter from Table 7, we find that the value of $V_{\text{max}}/V_{200}$ is close to 1.

We test for the correlation between the dark matter core radius and the disc scalelength using the relation given in Donato et al. (2004) $\log(R_c) = (1.05 \pm 0.11)\log(R_d) + (0.33 \pm 0.04)$. With K-band disc scalelength $R_d = 2.58kpc$ we get a core radius equal to 5.78 kpc, which is closer to the diet-Salpeter and the maximum disc case. Similarly the g-band scalelength gives core radius equal to 10.72 kpc.
Table 7. Dark matter density parameters derived from mass-modeling using the optical g-band and NIR K band photometry using the hybrid (HI + H2) rotation curve.

| Model               | $c^{(a)}$ | $R_{200}^{(b)}$ (kpc) | $\gamma^{(c)}$ | $\chi_{\text{red}}^{(d)}$ | $\gamma^{(e)}$ | $R_{\gamma}^{(f)}$ (kpc) | $\chi_{\text{red}}^{(g)}$ |
|---------------------|----------|----------------------|----------------|-------------------------|----------------|----------------------------|-------------------------|
| g-band              | NFW profile |                     |                 |                         |                |                            |                         |
| 'diet' Salpeter     | 5.06 ± 0.5 | 84.1 ± 0.3           | 3.8            | 0.1                      | 2.3 ± 0.01     | 3.8                        | 0.5                     |
| Kroupa IMF          | 5.5 ± 0.06 | 84.4 ± 0.3           | 2.7            | 0.1                      | 2.2 ± 0.01     | 2.7                        | 0.5                     |
| Free $\gamma^*$     | 3.8 ± 0.1  | 85.0 ± 0.3           | 6.4 ± 0.2      | 2.3                      | 50.8 ± 0.61    | 3.6 ± 0.05                 | 0.5                     |
| Maximum Disc        | 0.06 ± 0.04 | 218.4 ± 9.6          | 14             | 0.07                     | 1.4 ± 0.05     | 19.2 ± 0.8                 | 14                      |
| Minimum Disc        | 5.7 ± 0.06 | 96.1 ± 0.3           | 0              | 2.0                      | 71.9 ± 0.43    | 2.4 ± 0.00                 | 0.5                     |

K-Band

| Model               | $d^{(a)}$ | $\gamma^{(e)}$ | $\chi_{\text{red}}^{(g)}$ |
|---------------------|----------|----------------|-------------------------|
| 'diet' Salpeter     | 0.42 × 10^{-10} | 12.7 ± 0.1 | 0.2                     |
| Kroupa IMF          | 1.2 × 10^{-10} | 4.1 ± 0.1 | 1.8                     |
| Free $\gamma^*$     | 0.85 × 10^{-10} | 1.0      | 0.5                     |
| Maximum Disc        | 1.2 × 10^{-10} | 0.6 ± 0.01 | 0.8                     |

MOND

| Model               | $d^{(a)}$ | $\gamma^{(e)}$ | $\chi_{\text{red}}^{(g)}$ |
|---------------------|----------|----------------|-------------------------|
| g-band              | 1.2 × 10^{-10} | 4.1 ± 0.1 | 1.8                     |
| K-Band              | 0.85 × 10^{-10} | 1.0      | 0.5                     |

(a): Concentration parameter of the NFW profile
(b): Radius at which the mean density equal to 200 times the critical density.
(c): Mass to light ratio derived using population synthesis models or estimated as a free parameter.
(d): Ratio of the asymptotic velocity to the velocity at $V_{200}$.
(e): Reduced chi-square value corresponding to the fit.
(f): The central dark matter density of the PIS dark matter halo model.
(g): The core radius of the PIS dark matter halo model.
(h): Mass to light ratio derived using population synthesis models or estimated as a free parameter.
(i): Ratio of the core radius and the disc scale length.
(j): Reduced chi square value corresponding to the fit.
(k): Acceleration per length in MOND.
(l): Estimated Mass to light ratio in MOND.
(m): Reduced chi square corresponding to the fit.

We compare the parameters $V_{200} = \sqrt{4\pi G \rho_0 R_s^2}$ for the PIS halo and $R_s = R_{200}/c$ of FGC 1440 with the that of other superhumps in the literature as $R_s$ and $V_{200}$ constitute single parameter encompassing both the fitting parameters. In the study of three superhumps Banerjee & Bapat (2017) find $V_{200}$ equal to 110 km s$^{-1}$, 112 km s$^{-1}$ and 99 km s$^{-1}$ for UGC7321, IC5249 and IC2233 respectively. In another study, Kurapati et al. (2018b) find $V_{200}$ equal to 82.7 km s$^{-1}$ for FGC 1540. We find $V_{200} = 135$ for FGC 1440. Banerjee & Bapat (2017) find $R_s$ equal to 8.55 and 22.6 for UGC7321 and IC5249, Kurapati et al. (2018b) find $R_s$ equal to 5.25 for FGC 1540. For FGC 1440, we find $R_s$ equal to 24.3.

7.7.1 Mass models MOND

In the last panels of figures 16 and 17, we have shown the mass models derived using MOND. Keeping both $a$ and $\gamma$ as a free parameter we find that the acceleration parameter $a = 0.42 \times 10^{-10} \text{ms}^{-2}$ and $\gamma^* = 0.97$. We note that in the case when both the $a$ and $\gamma^*$ are kept as free parameters, $\gamma^*$ tends to values maximizing the disc mass, i.e., they are closer to the maximum disc case of dark matter models. We also try the case in which we fix the value of $a = 1.2 \times 10^{-10} \text{ms}^{-2}$ and only vary $\gamma^*$, we find that the value of $\gamma^*$ is equal to 4.14 and 0.65 in g-band and K-band respectively, and is closer to the values derived using population synthesis models.

8 VERTICAL STRUCTURE OF FGC 1440.

We model the galaxy disc as a co-planar, co-axial gravitationally coupled star + gas system under the influence of the force field of external dark matter halo. Solving the two-component Jeans equation using the methods outlined in Aditya & Banerjee (2021) and Komanduri et al. (2020) we derive the stellar vertical velocity dispersion ($\sigma_z$) as function of radius constrained by the observed stellar scaleheight. We use the best fitting mass model, i.e., the pseudo-isothermal (PIS) profile with stellar surface density scaled by Kroupa...
IMF, along with the observed HI surface density and the HI dispersion as the input parameters. From the 3-D models of the HI data cube, we find that the HI dispersion is constrained in the range $5 \text{ km s}^{-1} < \sigma_{HI} < 15 \text{ km s}^{-1}$, so we model the stellar vertical dispersion fixing the HI dispersion at $5 \text{ km s}^{-1}$, $10 \text{ km s}^{-1}$, $15 \text{ km s}^{-1}$ at all radius. The stellar dispersion is modeled as an exponential function $\sigma_z(R) = \sigma_0 e^{-\alpha R}$, where $\sigma_0$ is the central dispersion and the $\alpha$ is the steepness parameter. The values of the $\sigma_0$ and $\alpha$ derived using the mass models in g-band, and K-band are summarized in Table 8. We find that the central value of the stellar dispersion is not sensitive to the variation of the $\sigma_{HI}$ in the allowed range, but the steepness parameter is. The steepness parameter ($\alpha$) varies between $3.2 - 4.2$ in g-band and $4.2 - 6.3$ in K-band for $5 \text{ km s}^{-1} < \sigma_{HI} < 15 \text{ km s}^{-1}$. For the different values of the $\sigma_{HI}$, we find that the modeled stellar scaleheight agrees with the observed scaleheight up to $3R_d$. For the values of $\sigma_{HI} = 5 \text{ km s}^{-1}$ we find that that the the model predicted HI scaleheight (panel 3 in Figure 17) is constrained between the limits obtained from tilted ring modeling ($0.45'' < h_z < 5.3''$).
Table 8. Values of vertical stellar dispersion.

| Parameter | g-Band $\sigma_0$ ($\text{km s}^{-1}$) | g-Band $\alpha$ | K-Band $\sigma_0$ ($\text{km s}^{-1}$) | K-Band $\alpha$ |
|-----------|-------------------------------|----------------|---------------------------------|----------------|
| $\sigma_{HI}=5 \text{ km s}^{-1}$ | 29.0 | 4.2 | 18.6 | 6.3 |
| $\sigma_{HI}=10 \text{ km s}^{-1}$ | 29.7 | 3.6 | 19.1 | 4.8 |
| $\sigma_{HI}=15 \text{ km s}^{-1}$ | 29.9 | 3.2 | 20.1 | 4.2 |

9 DISCUSSION

- In this Section we discuss the main dynamical properties of FGC 1440, and compare them with those determined in the literature for other superthin galaxies.

  - Disc Heating

    It has been shown that bars, spiral arms, and globular clusters play an important role in disc heating. Bars and spiral arms heat the disc in the radial direction whereas globular clusters heat the disc isotropically in both the radial and vertical direction (Aumer et al. 2016; Jenkins...
Figure 18. The plots show the vertical velocity dispersion $\sigma_z$, the modeled stellar ($h_z^{\text{mod}}$) and H\textsc{i} scaleheights ($h_z^{\text{gas}}$), and the stability parameter ($Q$). The top panel in blue ink depicts the results for g-band and lower panel in red, the results in K-band. The vertical dashed line marks the $3R_s$ in g-band and K-band respectively. The horizontal black line in panel depicting $h_z^{\text{mod}}$ marks the observed scaleheight. The horizontal line in plot showing $h_z^{\text{gas}}$ marks the upper and the lower limit on the H\textsc{i} scaleheight derived using the tilted ring modeling.

& Binney 1990; Grand et al. 2016; Saha 2014). Further, Banerjee & Jog (2013) showed that a compact dark matter halo regulates the distribution of the stars in the vertical direction and gives rise to superthin disc structure. Aditya & Banerjee (2021) show for a sample of superthin galaxies that the absolute values of the vertical stellar velocity dispersion is small as compared to the Milky way. And upon comparing the the ratio of vertical velocity dispersion to the total rotation velocity ($\dot{\Omega}$), they find that the value of $\frac{V_{\text{rot}}}{\dot{\Omega}}$ is comparable to stars in the thin disc of Milky way, indicating that superthin galaxies are dynamically cold systems. Thus, it is imperative to compare the value of $\frac{V_{\text{rot}}}{\dot{\Omega}}$ for FGC 1440 with previously studied superthin galaxies to quantify the effect of the disc heating. Using the values of vertical velocity dispersion constrained using optical photometry; we find that the ratio $\frac{V_{\text{rot}}}{\dot{\Omega}}$ for FGC 1440 is comparable to the sample of superthin galaxies studied in Aditya & Banerjee (2021) equal to 5.0, except for UGC 7321 which has $\frac{V_{\text{rot}}}{\dot{\Omega}} = 10$. Similarly using the values of the velocity dispersion constrained with the g-band and K-band data we find that value of $\frac{V_{\text{rot}}}{\dot{\Omega}}$ = 8 for FGC 1440 compared to 10 for other superthin galaxies, except for IC2233 which has $\frac{V_{\text{rot}}}{\dot{\Omega}} = 14$ and UGC00711 which has $\frac{V_{\text{rot}}}{\dot{\Omega}} = 5$.

Disc dynamical stability

The disc dynamical stability $Q$ as quantified by Toomre (1964) $Q = \frac{\kappa\Sigma}{\sigma}$, where Toomre stability criterion 'Q' is a subtle balance between the epicyclic frequency of the self-gravitating matter $\kappa$, the radial velocity dispersion $\sigma$ and the surface density $\Sigma$. The epicyclic frequency $\kappa$ at a radius $R$ is defined as $\kappa^2(R) = \sqrt{\frac{G(M+\Sigma)}{R}} + 4\Omega(R)^2$, where $\Omega$ is the angular frequency defined as $\Omega(R)^2 = \frac{1}{R}\frac{d\Phi_{\text{Total}}}{dR} = \frac{V_{\text{rot}}^2}{R}$, $\Phi_{\text{Total}}$ is the total gravitational potential and $V_{\text{rot}}$ is the total rotation velocity.

Garg & Banerjee (2017) show that dark matter plays a decisive role in regulating the stability of the low surface brightness galaxies and that in the absence of dark matter, the galaxy disc would be susceptible to axisymmetric instabilities. Further, in Aditya & Banerjee (2021) it has been shown for a sample of superthin galaxies that the median values of stability for the star + gas system is higher than the typical spiral galaxies studied by Romeo & Mogotsi (2017). This section compares the value of the stability of the star + gas disc of FGC 1440 with that of previously studied superthin galaxies. Using the two-component stability parameter derived in Romeo & Wiegert (2011), we compute the dynamical stability of the star - gas system for FGC 1440.

The two-component disc stability parameter $Q_{RW}$ appraising the stability of the composite star + gas disc is given by

$$
\frac{1}{Q_{RW}} = \begin{cases} 
\frac{W_g}{T_gQ_g} + \frac{W_s}{T_sQ_s} & \text{if } T_sQ_s > T_gQ_g \\
\frac{W_s}{T_sQ_s} + \frac{W_g}{T_gQ_g} & \text{if } T_sQ_s < T_gQ_g 
\end{cases}
$$

(10)

where the weight function $W$ is given by

$$
W = \frac{2\sigma_s\sigma_g}{\sigma_s^2 + \sigma_g^2}
$$

(11)

The thickness correction is defined as:

$$
T \approx 0.8 + 0.7 \frac{\sigma_s}{\sigma_R}
$$

(12)

In the above equation $\sigma_s$ and $\sigma_g$ are the velocity dispersion of the stars and gas respectively. $Q_s$ and $Q_g$ are the Toomre Q of the stellar disc and gas disc respectively. A value of $Q_{RW} > 1$ indicates that the composite stars + gas disc is stable against axisymmetric perturbations.

We calculate the disc dynamical stability parameter ($Q$) (panel 4 in Figure 18) against local, axisymmetric perturbations of the galaxy as a function of the radius using the two-component stability parameter derived in Romeo 2011, where $\kappa\Sigma/\sigma$ is the Toomre stability criterion 'Q' is a subtle balance between the epicyclic frequency of the self-gravitating matter $\kappa$, the radial velocity dispersion $\sigma$ and the surface density $\Sigma$. The epicyclic frequency $\kappa$ at a radius $R$ is defined as $\kappa^2(R) = \sqrt{\frac{G(M+\Sigma)}{R}} + 4\Omega(R)^2$, where $\Omega$ is the angular frequency defined as $\Omega(R)^2 = \frac{1}{R}\frac{d\Phi_{\text{Total}}}{dR} = \frac{V_{\text{rot}}^2}{R}$, $\Phi_{\text{Total}}$ is the total gravitational potential and $V_{\text{rot}}$ is the total rotation velocity.

Garg & Banerjee (2017) show that dark matter plays a decisive role in regulating the stability of the low surface brightness galaxies and that in the absence of dark matter, the galaxy disc would be susceptible to axisymmetric instabilities. Further, in Aditya & Banerjee (2021) it has been shown for a sample of superthin galaxies that the median values of stability for the star + gas system is higher than the typical spiral galaxies studied by Romeo & Mogotsi (2017). This section compares the value of the stability of the star + gas disc of FGC 1440 with that of previously studied superthin galaxies. Using the two-component stability parameter derived in Romeo & Wiegert (2011), we compute the dynamical stability of the star + gas system for FGC 1440.

The two-component disc stability parameter $Q_{RW}$ appraising the stability of the composite star + gas disc is given by

$$
\frac{1}{Q_{RW}} = \begin{cases} 
\frac{W_g}{T_gQ_g} + \frac{W_s}{T_sQ_s} & \text{if } T_sQ_s > T_gQ_g \\
\frac{W_s}{T_sQ_s} + \frac{W_g}{T_gQ_g} & \text{if } T_sQ_s < T_gQ_g 
\end{cases}
$$

(10)

where the weight function $W$ is given by

$$
W = \frac{2\sigma_s\sigma_g}{\sigma_s^2 + \sigma_g^2}
$$

(11)

The thickness correction is defined as:

$$
T \approx 0.8 + 0.7 \frac{\sigma_s}{\sigma_R}
$$

(12)

In the above equation $\sigma_s$ and $\sigma_g$ are the velocity dispersion of the stars and gas respectively. $Q_s$ and $Q_g$ are the Toomre Q of the stellar disc and gas disc respectively. A value of $Q_{RW} > 1$ indicates that the composite stars + gas disc is stable against axisymmetric perturbations.

We calculate the disc dynamical stability parameter ($Q$) (panel 4 in Figure 18) against local, axisymmetric perturbations of the galaxy as a function of the radius using the two-component stability parameter derived in Romeo & Wiegert (2011).
& Wiegert (2011). We compute the stability parameter using value of HI dispersion; $\sigma_{HI} = 5 \text{ km s}^{-1}$. Similarly, in K-band the minimum value of $Q$ is equal to 1.1 $\sigma_{HI} = 10 \text{ km s}^{-1}$ for $R < 3 R_d$. Moreover, we find that the specific angular momentum of FGC 1440 is lower than the median value of 5.5 (Aditya & Banerjee 2021) for previously studied sample of superthin galaxies.

10 CONCLUSION

We have presented the analysis and results pertaining to HI imaging of ultra-flat galaxy FGC 1440 observed using GMRT.

- We fit the busyness function to the HI spectrum and find that the velocity widths 20% and the 50% of the peak maximum are 295 km s$^{-1}$ and 306 km s$^{-1}$, respectively. We find that the total flux density is 10.46 Jy km s$^{-1}$.

- From our preliminary analysis of the HI data cube, we find that the moment 0 and moment 1 maps show that the HI disc is slightly warped on the north-eastern side.

- We use our final HI data to construct tilted rings model of the HI emission and derive the kinematic parameters using TiRiFic (Józsa et al. 2012) and FAT (Ramphuls et al. 2015). We find that a model with radially varying inclination equal to 90° in the inner rings and 85° for the outer rings gives a better description of the data. The best fit value of the position angle is 53.6°. We find that FGC 1440 has a slowly rising rotation curve with an asymptotic rotation velocity equal to 141.8 km s$^{-1}$.

- By manually comparing the PV diagrams at different offsets, we find that the HI velocity dispersion lies in between $5 \text{ km s}^{-1} < \sigma < 15 \text{ km s}^{-1}$ and the measurement of the scaleheight is limited by the resolution of the synthesized beam. $h_z < 5.3''$.

- By comparing the data with models with a varying inclination and scaleheight, we find that FGC 1440 possibly hosts a thin HI disc warped along the line of sight.

- We use the total rotation velocity along with the stellar photometry to derive the mass models in optical g-band and NIR UKIDSS K-band. We find that the mass models derived using cored pseudo-isothermal dark matter halo in conjunction with stellar rotation curves derived using Kroupa initial mass function give better fits to the observed rotation curve.

- We also derive mass models in modified Newtonian paradigm, we find that models in which both acceleration and the $\gamma^*$ are kept as free parameters, the values of acceleration are lower than $1.2 \times 10^{10} \text{ ms}^{-2}$ and the mass to light ratios tend to values maximizing the disc mass. Whereas as models in which a is fixed is $1.2 \times 10^{10} \text{ ms}^{-2}$, $\gamma^*$ tends to values predicted by stellar population synthesis models.

- Using the observed stellar scaleheight as we constrain the vertical velocity dispersion in g-band and K-band. We find that the value of central dispersion is equal to 29.0 km s$^{-1}$ in g-Band and 18.6 km s$^{-1}$ in K-band. We note the values of vertical dispersion are comparable to the values of dispersion quoted in Aditya & Banerjee (2021).

- Using the two-component stability parameter proposed by Romeo & Wiegert (2011), we calculate the stability factor for FGC 1440, we find that Q-value is greater than 1 for $R < 3 R_d$, indicating that the galaxy is stable against axis-symmetric instabilities. The value of Q for FGC 1440 is lower than the median value of Q for superthin galaxies, equal 5.5 (Aditya & Banerjee 2021) and is closer to the median value of Q for calculated for a sample of spiral galaxies equal to $2.2 \pm 0.6$ by (Romeo & Mogotsi 2017).
in spite of the large axial ratios the values of $\frac{V_{\text{rot}}}{\sigma}$ = 5 in g-band and $\frac{V_{\text{rot}}}{\sigma}$ = 8 in K-band are comparable to the superthin galaxies previously studied in the literature.

- Inspecting the Fall relation, for ordinary disc galaxies, we find that FGC 1440 follows the regression line for the $\log_{10}(j_*) - \log_{10}(M_\star)$, $\log_{10}(j_b) - \log_{10}(M_b)$ and $\log_{10}(j_g) - \log_{10}(M_g)$ relations. The values of $j$ for the stars, gas and the baryons in FGC 1440 are consistent with those of normal spiral galaxies with similar mass.

11 DATA AVAILABILITY

The data from this study are available upon request.

12 ACKNOWLEDGEMENT

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Figure 20. The image shows the data (panel 1), the Galfit model (panel 2) and the normalized difference between the data and the model (panel 3) for optical photometry of FGC 1440 using the SDSS g-band image.

Figure 21. The image shows the data (panel 1), the Galfit model (panel 2) and the normalized difference between the data and the model (panel 3) for optical photometry of FGC 1440 using the SDSS r-band image.

Figure 22. The image shows the data (panel 1), the Galfit model (panel 2) and the normalized difference between the data and the model (panel 3) for optical photometry of FGC 1440 using the UKIDSS K-band image.
Figure 23. We present the mass-model of the galaxy FGC 1440 derived using SDSS g-band photometry. The mass models are constrained using the HI 21 cm data.
Figure 24. We present the mass-model of the galaxy FGC 1440 derived using UKIDSS K-band photometry. The mass models are constrained using the H$^121$ cm data.
Table 9. Dark matter density parameters derived from mass-modeling using the optical g-band and NIR K band photometry using just the HI rotation curve.

| Model            | $c^{(a)}$ | $R_{200}^{(b)}$ (kpc) | $\gamma^{(c)}$ | $\frac{v_{\text{max}}}{V_{200}}^{(d)}$ | $\chi^{2}_{\text{red}}^{(e)}$ | $\rho_0^{(f)} \times 10^{-3}$ $M_\odot$/pc$^3$ | $R_\gamma^{(i)}$ (kpc) | $\gamma^{(k)}$ | $\frac{R_\gamma}{R_{200}}^{(l)}$ | $\chi^{2}_{\text{red}}^{(m)}$ |
|------------------|-----------|-----------------------|----------------|--------------------------------------|-------------------------------|----------------------------------|----------------------|----------------|----------------------------------|-------------------------------|
|                  | g-band    | NFW profile           | PIS profile    |                                      |                               |                                   |                      |                |                                  |                                |
| 'diet' Salpeter  | 4.65 ± 1.18 | 87.15 ± 7.2           | 3.8            | 2.23                                 | 1.02                          | 45.92 ± 12.7                    | 2.66 ± 0.42          | 3.8              | 0.60                             | 0.26                           |
| Kroupa IMF       | 5.10 ± 1.27 | 87.4 ± 7.07           | 2.6            | 2.22                                 | 1.17                          | 54.78 ± 16.4                    | 2.48 ± 0.42          | 2.6              | 0.56                             | 0.35                           |
| Free $\gamma^*$  | 1.06 ± 1.43 | 103.5 ± 35.8          | 12.19 ± 2.84   | 1.87                                 | 0.52                          | 13.08 ± 8.65                    | 4.76 ± 1.60          | 9.26 ± 2.10 | 1.07                             | 0.11                           |
| Maximum Disc     | 0.05 ± 0.74 | 221.36 ± 179.9        | 14             | 0.87                                 | 0.55                          | 2.7 ± 0.86                      | 11.19 ± 2.7          | 14               | 2.53                             | 0.19                           |
| Minimum Disc     | 5.4 ± 1.16  | 98.7 ± 7.23           | 0              | 1.97                                 | 1.3                           | 65.81 ± 13.04                   | 2.56 ± 0.29          | 0                | 0.57                             | 0.23                           |
|                  | K-Band    |                        |                |                                      |                               |                                   |                      |                  |                                  |                                |
| 'diet' Salpeter  | 3.13 ± 1.3  | 94.3 ± 13.95          | 0.85           | 2.06                                 | 1.69                          | 18.69 ± 8.63                    | 4.15 ± 1.22          | 0.85             | 1.60                             | 0.79                           |
| Kroupa IMF       | 3.94 ± 1.32 | 91.38 ± 10.51         | 0.6            | 2.12                                 | 1.55                          | 29.035 ± 11.81                  | 3.42 ± 0.84          | 0.6              | 1.3                              | 0.62                           |
| Free $\gamma^*$  | 4.72 ± 2.73 | 90.71 ± 10.37         | 0.37 ± 0.63    | 2.14                                 | 1.81                          | 46.7 ± 34.2                     | 2.79 ± 1.04          | 0.32 ± 0.36 | 1.08                             | 0.67                           |
| Maximum Disc     | 2.10 ± 1.14 | 117.56 ± 26.3         | 1.1            | 1.65                                 | 1.9                           | 12.16 ± 6.46                    | 5.43 ± 1.83          | 1.1              | 2.10                             | 1.06                           |
| Minimum Disc     | 5.41 ± 1.61 | 98.7 ± 7.23           | 0              | 1.97                                 | 1.3                           | 65.81 ± 13.04                   | 2.56 ± 0.29          | 0                | 0.99                             | 0.23                           |

| MOND             | $\frac{d^{(i)}}{m^2 s^{-2}}$ | $\gamma^{(j)}$ | $\chi^{2}_{\text{red}}^{(m)}$ |
|------------------|--------------------------------|----------------|-------------------------------|
| g-Band           | $0.4 \times 10^{-10}$         | 13.89 ± 1.64   | 0.34                           |
| g - Band$^{\text{fixed}}$ | $1.2 \times 10^{-10}$         | 2.61           |                               |
| K-Band           | $0.8 \times 10^{-10}$         | 1.12 ± 0.4     | 1.96                           |
| K - Band$^{\text{fixed}}$ | $1.2 \times 10^{-10}$         | 0.69 ± 0.18    | 2.33                           |

(a): Concentration parameter of the NFW profile
(b): Radius at which the mean density equals 200 times the critical density.
(c): Mass to light ratio derived using population synthesis models or estimated as a free parameter.
(d): Ratio of the asymptotic velocity to the velocity at $R_{200}$, where $V_{200} = 0.73 R_{200}$ (Navarro et al. 1997)
(e): Reduced chi-square value corresponding to the fit.
(f): The central dark matter density of the PIS dark matter halo model
(g): The core radius of the PIS dark matter halo model
(h): Mass to light ratio derived using population synthesis models or estimated as a free parameter.
(i): Ratio of the core radius and the disk scalelength.
(j): Reduced chi square value corresponding to the fit.
(k): Acceleration per length in MOND.
(l): Estimated Mass to light ratio in MOND.
(m): Reduced chi square corresponding to the fit.