Renormalization group equations and infrared quasi fixed point behaviors of non-universal soft terms in MSSM

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Abstract

The renormalization group equations (RGEs) of non-universal soft supersymmetric breaking terms with CP violating phases are analyzed in this paper. We obtain the analytic solutions of RGEs by directly solving the RGEs themselves. Compared with the method of spurion expansion our approach proves to be simple and succinct, and easy to extend to the case of complex parameters. With the analytical forms of the solutions we obtained the infrared quasi fixed point behavior of soft terms are analyzed and it turns out to support the notion in scenarios with CP violating phases.
1 Introduction

As an excellent candidate to embed the standard model(SM) in a more fundamental theory, the minimal supersymmetric standard model(MSSM) has many good features. It ingeniously tackles the abominable gauge hierarchy problem from which ordinary unification theories suffer [1], provides a mechanism that supergravity models all share which breaks the electroweak(EW) symmetry dynamically via radiative corrections, naturally provides the lightest supersymmetric particle(LSP) as a candidate of the dark matter in astrophysics and cosmology[2], etc. In spite of these theoretical virtues, however, the MSSM suffers from great uncertainty which arises from the large number of free parameters describing the soft supersymmetry (SUSY) breaking. One may greatly reduce the parameter space and make the theory much more predictive via adopting the universality conditions at high energy scale, e.g., one may choose the minimal supergravity model(MSUGRA). Furthermore, with the aid of the concept of infrared quasi fixed point(IRQFP), further reduction of parameter space has been observed in many works[3, 4, 5, 6]. It reveals the screening effect for soft breaking parameters implied entirely by the renormalization group equations(RGEs) and the large top Yukawa coupling, and with this one may rely on two[5] or even one[6] free soft SUSY breaking terms, i.e., $M_0$ and $M_{1/2}$ or only $M_{1/2}$. In these cases one can work in models of highly predictive power leaving some of the universality boundary assumptions. This IRQFP analysis was extended to the large $\tan\beta$ scenario[7] in which the whole set of Yukawa coupling of the third generation has to be considered. The result shows that the theory also exhibit the IRQFP for large $\tan\beta$ and allows one to predict SUSY mass spectra as functions only of $M_{1/2}$, but the analysis turns out to be much more complicated.

While hindered in this situation, a crucial observation was made in Ref. [8] which showed the intrinsic connection between the solutions for the RGEs of soft terms and those of the gauge coupling constants and Yukawa couplings. It tells that one can get the solutions of RGEs for the soft SUSY breaking terms via substituting the modified expressions for gauge and Yukawa couplings in the solutions of the RGEs for gauge and Yukawa couplings [8, 9, 10].

$$\alpha_i \Rightarrow \tilde{\alpha}_i = \alpha_i (1 + M_i \eta + \bar{M}_i \bar{\eta} + 2 M_i \bar{M}_i \eta \bar{\eta}), \quad \eta = \theta^2, \quad \bar{\eta} = \bar{\theta}^2,$$

and expanding over the Grassmannian parameters $\theta$ and $\bar{\theta}$, which is exactly the spurion field description of soft SUSY breaking [11]. The technique was immediately used to probe the implication of RGEs for the soft SUSY breaking terms in large $\tan\beta$ scenario, but one faced with the problem of finding good analytical forms for Yukawa couplings. This problem was solved in Ref. [12] in which an integral form for Yukawa coupling evolution was given which was shown to be a convergent scheme for numerical calculations and a convenient and concise form for qualitative analysis. In view of this solution, the general analytical forms for the solutions of the soft SUSY breaking terms were obtained in this complicated case via the spurion field expansion and the IRQFP feature was analyzed for non-universal boundaries of soft terms[13].

Although the solution for soft SUSY breaking terms look concise and simple, the procedure described above, especially the actual calculation of spurion expansion is still complicated due to the a little bit complicated iterative solutions for Yukawa couplings [12]. So in Ref. [13] the author restrict their attention to real trilinear couplings and real gaugino masses. However, in a lot of works on superstring theory and M-theory phenomenology non-universal complex
soft terms appear naturally \[14\], which provide new sources of CP violation. And the SUSY CP violating phases may not be constrained to be small due to the cancellation mechanism of SUSY contributions to electron and neutron electric dipole moments (EDME and EDMN), and one has extensively discussed the phenomenological implications of large phases of $\mu$, gaugino masses and trilinear terms while satisfying the EDME and EDMN constraints \[13\]. Therefore, it is important to extend the study of non-universality of soft terms to the complex parameter case. In this paper we will show that the convenient iterative forms \[12, 13\] of the solutions of RGEs can be obtained very succinctly in the approach of directly solving RGEs and extend to the complex non-universal soft terms. Our results show that the real and imaginary parts of trilinear couplings have infrared quasi fixed points respectively if the initial values of Yukawa couplings are large enough (at least $Y_k^0 > \alpha^0$).

The paper is organized as follows. In section two we will show briefly how one can directly solve the RGEs and arrive at the forms in Ref. \[13\], and extend the analysis to more general case, complex parameter case. In section three we will analyze the IRQFP in the case with SUSY CP violating phases and discuss the phenomenological implications. Finally, conclusions are drawn in section four.

## 2 Solution of the RGEs

The one loop RGEs for gauge and Yukawa coupling and soft breaking terms can be written as

\[
\begin{align*}
\dot{\alpha}_i &= -b_i \alpha_i^2, \\
\dot{Y}_k &= Y_k (\sum_i c_{ki} \alpha_i - \sum_l a_{kl} Y_l), \\
\dot{M}_i &= -b_i \alpha_i M_i, \\
\dot{A}_k &= -\sum_i c_{ki} \alpha_i M_i - \sum_l a_{kl} Y_l A_l, \\
\dot{\Sigma}_k &= 2 \sum_i c_{ki} \alpha_i |M_i|^2 - \sum_l a_{kl} Y_l (\Sigma_l + |A_l|^2),
\end{align*}
\]

where $\alpha_i = \frac{y_i^2}{16\pi^2}, i=1,2,3, Y_i = \frac{y_t^2}{16\pi^2}, l=t,b,\tau, \cdots \equiv d/dt, \ t = \log M^2_{GUT}/Q^2$ and

\[
\begin{align*}
\Sigma_t &= \tilde{m}^2_{Q3} + \tilde{m}^2_{U3} + m^2_{H2}, \\
\Sigma_b &= \tilde{m}^2_{Q3} + \tilde{m}^2_{D3} + m^2_{H1}, \\
\Sigma_{\tau} &= \tilde{m}^2_{L3} + \tilde{m}^2_{E3} + m^2_{H1}, \\
b_i &= \{33/5, 1, -3\}, \\
c_{ti} &= \{13/15, 3, 16/3\}, \ c_{bi} = \{7/15, 3, 16/3\}, \ c_{\tau i} = \{9/5, 3, 0\}, \\
a_{tl} &= \{6, 1, 0\}, \ a_{bl} = \{1, 6, 1\}, \ a_{\tau l} = \{0, 3, 4\}.
\end{align*}
\]

The solutions for $\alpha_i$’s and $M_i$’s can be easily obtained to be:

\[
\alpha_i = \frac{\alpha_i^0}{1 + b_i \alpha_i^0 t}, \quad M_i = \frac{M_i^0}{1 + b_i \alpha_i^0 t}.
\]
For $Y_k$’s, solutions turn out to be

$$Y_k = \frac{Y_0^k u_k}{1 + a_{kk} Y_0^k \int_0^t u_k}$$

(7)

where $u_k$’s are defined iteratively as

$$u_k = E_k \prod_{l \neq k} (1 + a_{kl} Y_l^0 \int_0^t u_l)^{-a_{kl}/\alpha_l},$$

(8)

with

$$E_k = \prod_{i=1}^3 (1 + b_i \alpha_i^0 t)^{c_{ki}/b_i}.$$  

(9)

To find the analytic solutions of the renormalization group equations for $A_k$’s and $\Sigma_k$’s, we rewrite them as the form of the Bernoulli equation. Consider the trilinear coupling $A_k$ first. We can rewrite Eq. (4) to be

$$\frac{dA_k}{dt} = -\frac{de_k}{dt} - a_{kk} Y_k A_k,$$

(10)

with $e_k$ defined by

$$\frac{de_k}{dt} = \sum_i c_{k_i} \alpha_i M_i + \sum_{l \neq k} a_{kl} Y_l A_l,$$

$$e_0^k = e_k(t)|_{t=0} = 0.$$

(11)

Eq. (10) can then be solved in the standard way. The solution is

$$A_k = -e_k + \frac{A_0^k + a_{kk} Y_0^k \int_0^t u_k e_k}{1 + a_{kk} Y_0^k \int_0^t u_k}$$

(12)

where $A_0^k = A_k|_{t=0}$. In order to solve Eq. (11), we rewrite Eq. (10) as

$$\frac{d(A_k + e_k)}{dt} = -a_{kk} Y_k A_k$$

(13)

which means that

$$\int_0^t Y_k A_k = -\frac{1}{a_{kk}} (A_k + e_k - A_0^k) = Y_0^k A_0^k \frac{\int_0^t u_k - \int_0^t u_k e_k}{1 + a_{kk} Y_0^k \int_0^t u_k}.$$  

(14)

Integrating Eq. (11) over $t$ and inserting Eq. (14) in it, we find the iterative integral equations for $e_k$’s

$$e_k = t \sum_i c_{k_i} \alpha_i^0 M_i^0 + \sum_{l \neq k} a_{kl} \frac{A_l^0 \int_0^t u_l - \int_0^t u_l e_l}{1/Y_l^0 + a_{ll} \int_0^t u_l}.$$  

(15)
For $\Sigma_k$'s, the situation is similar but a little more complicated. From Eq. (13) we can write
\[
\frac{d|A_k + e_k|^2}{dt} = -a_{kk}Y_kA_k(A_k + e_k)^* - a_{kk}Y_kA_k^*(A_k + e_k),
\]
or
\[
\frac{d(|A_k|^2 + A_k^*e_k + A_ke_k^*)}{dt} = -\frac{d|e_k|^2}{dt} - a_{kk}Y_k|A_k|^2 - a_{kk}Y_k(|A_k|^2 + A_k^*e_k + A_ke_k^*).
\]
Define
\[
\tilde{\Sigma}_k = \Sigma_k - |A_k|^2 - A_k^*e_k - A_ke_k^*,
\]
then we find from Eqs. (5) and (16) that
\[
\frac{d\tilde{\Sigma}_k}{dt} = \frac{d\xi_k}{dt} - a_{kk}Y_k\tilde{\Sigma}_k
\]
which is the same as Eq. (10) with $A_k$ and $e_k$ substituted by $\tilde{\Sigma}_k$ and $\xi_k$ respectively. In Eq. (18) $\xi_k$'s are defined by
\[
\frac{d\xi_k}{dt} = 2\sum c_{ki}\alpha_i |M_i|^2 - \sum_{i \neq k} a_{kl}Y_l(\Sigma_l + |A_l|^2) + \frac{d|e_k|^2}{dt},
\]
\[
\xi_k^0 = \xi_k(t)|_{t=0} = 0.
\]
Then from Eqs. (17) and (18) we get
\[
\Sigma_k = \xi_k + |A_k|^2 + A_k^*e_k + A_ke_k^* - \frac{|A_k|^2 - \Sigma_k^0 + a_{kk}Y_k^0 \int_0^t u_k\xi_k}{1 + a_{kk}Y_k^0 \int_0^t u_k}.
\]
The iterative equations for $\xi_k$'s are derived in a way similar to that for $e_k$ by noting that Eq. (18) can be rewritten as
\[
\frac{d\Sigma_k}{dt} = \frac{d\xi_k}{dt} - \frac{d|e_k|^2}{dt} - a_{kk}Y_k(\Sigma_k + |A_k|^2),
\]
which leads to
\[
\int_0^t Y_k(\Sigma_k + |A_k|^2) = -\frac{1}{a_{kk}}(\Sigma_k - \Sigma_k^0 - \xi_k + |e_k|^2).
\]
From the above equation and Eq. (19), we find
\[
\xi_k = t^2|\sum c_{ki}\alpha_i M_i^0|^2 + 2t \sum c_{ki}\alpha_i |M_i^0|^2 - t^2 \sum c_{ki}\alpha_i^2 |M_i^0|^2
\]
\[
+ t \sum c_{ki}\alpha_i M_i^0 \sum_{l \neq k} a_{kl} A_{l0}^* \int_{0}^{t} u_l - \int_{0}^{t} u_l e_{l} + t \sum c_{ki}\alpha_i M_i^0 \sum_{l \neq k} a_{kl} \frac{A_{l0}^* \int_{0}^{t} u_l - \int_{0}^{t} u_l e_{l}}{1/Y_l^0 + a_{ll} \int_{0}^{t} u_l}
\]
\[
+ \sum_{l \neq k} \frac{A_{l0}^* \int_{0}^{t} u_l - \int_{0}^{t} u_l e_{l}}{1/Y_l^0 + a_{ll} \int_{0}^{t} u_l}^2 + \sum_{l \neq k} a_{kl} \frac{A_{l0}^* \int_{0}^{t} u_l - \int_{0}^{t} u_l e_{l}}{1/Y_l^0 + a_{ll} \int_{0}^{t} u_l} \bigg| A_{l0}^* \int_{0}^{t} u_l - \int_{0}^{t} u_l e_{l} \bigg|^2
\]
\[
- \sum_{l \neq k} a_{kl} \frac{(|A_{l0}^*|^2 + \Sigma_k^0) \int_{0}^{t} u_l - A_{l0}^* \int_{0}^{t} u_l e_{l} + A_{l0}^* \int_{0}^{t} u_l e_{l} + \int_{0}^{t} u_l \xi_l}{1/Y_l^0 + a_{ll} \int_{0}^{t} u_l}.
\]
The Eqs. (12), (15), (20) and (21) are our main results. When $M_i$ and $A_l$ are real they reduce to the results given in Ref. [3], as it should be. The Eqs. (12) and (15) are the same as those in the case of real $A_l$’s and $M_i$’s so that it follows that $A_l$ is independent of its initial value when the initial values of all three Yukawa couplings are large enough ($Y_l^0$ larger than $\alpha^0$ ). That is, the real and imagine parts of $A_l$ divided by $M_3$ have the IQFPs, respectively. By inspecting Eqs. (20) and (21), one is led to that $\Sigma_k/M_3 \tau$ possesses an IQFP if the initial values of all three Yukawa couplings are large enough. In conclusion Yukawa couplings $Y_l$ and soft terms $A_l$ and $\Sigma_l$ have IQFP behaviors in the complex non-universal $A_l$ and $M_i$ case when the initial values of Yukawa couplings, $Y_l^0$’s, are large enough.

It is worth to note that having the analytic solutions of RGEs, it is easy to find the RG invariants. For example, if we define

$$E_k' = \prod_{i=1}^{3} \alpha_i^{c_{ki}/b_i},$$

$$u'_k = E_k^0 u_k = u_k^0 u_k,$$

with $E_k^0 = E_k(t = 0)$ and $u_k^0 = u_k'(t = 0)$, then from Eqs. (7), (12) and (20) we obtain the RG invariants

$$F_{1k} = \frac{u'_k}{Y_k} - a_{kk} \int_0^t u'_k,$$

$$F_{2k} = \left( A_k + e_k - a_{kk} \frac{Y_k}{u'_k} \int_0^t u'_k e_k \right) \left( 1 - a_{kk} \frac{Y_k}{u'_k} \int_0^t u'_k \right)^{-1},$$

$$F_{3k} = \left( \Sigma_k - \xi_k - |A_k|^2 - A_k e_k - A_k e_k^* + A_{kk} \frac{Y_k}{u'_k} \int_0^t u'_k e_k \right) \left( 1 - a_{kk} \frac{Y_k}{u'_k} \int_0^t u'_k \right)^{-1},$$

where $k = t, b, \tau$, which extend the results in Ref. [10] to the case with multi Yukawa couplings.

### 3 Numerical Study of Dependences of Soft Terms on Their Initial Values

In order to see the dependences of soft terms on their initial values one can write

$$A_k(t) = \sum_i c_i^k(t) A_i^0 + \sum_i d_i^k(t) M_i^0, \quad (22)$$

$$\Sigma_k(t) = \sum_{\alpha} f_{\alpha}^k(t)(m_{\alpha})^2 + \sum_{\nu} g_{\nu}^k(t) A_i^{0*} A_{\nu}^{0} + \sum_{i,l} h_{il}^{k}(t)(M_i^{0} A_i^{0*} + c.c.) + \sum_{i,j} k_{ij}^k(t) M_i^0 M_j^{0*}, \quad (23)$$

where the superscript 0 of $A_{\nu}^0$, $M_i^0$ etc. means that they are the initial values, i.e., the values at unification scale, the asterisk, *, means the complex conjugate, $g_{\nu_i}^k = g_{\nu_i}^k$ and $k_{ij}^k = k_{ij}^k$. In Eq. (23) $\alpha$ runs over all the third generation scalar quarks and Higgs scalars. The coefficients in the equation depend on the initial values of Yukawa and gauge couplings and , of course, are determined by the solutions of RGEs, (12), (15), (20) and (21).
The IFQFPs exist for any values of $\tan\beta$ if the initial values of all three Yukawa couplings, $Y_k^0$'s, are large enough (say, larger than the gauge coupling at GUT scale), as shown in the last section. However, that whether the condition can be satisfied is in fact dependent of $\tan\beta$ masses, then run them to we want to make the theory realistic, i.e., to impose the requirement that the third generation quark masses are given by experiments. Therefore, for numerical calculations, we choose the procedures as follows. First, run gauge couplings from $M_Z$ scale up to find the scale where gauge couplings unify with the SUSY threshold effects taken at $M_{SUSY} = 400$ Gev. The values of gauge couplings at $M_Z$ are taken directly from Ref. \[7\]. For three Yukawa couplings, we take corresponding current masses of the third generation quarks from Ref. \[7\] and calculate their pole masses, then run them to $M_Z$ scale to find the Yukawa couplings at that scale with $\tan\beta$ fixed (hence the SUSY corrections to the pole mass of top are not included in our calculations). The values of Yukawa couplings at GUT scale, $Y_k^0$'s, are found by running their energy scale value up by using Eqs. (7,8,9) and again the SUSY threshold are taken into account at $M_{SUSY}$. The second step is to run the gauge couplings, Yukawa couplings, gaugino masses, trilinear soft SUSY breaking terms and $\Sigma_l(l = t, b$ and $\tau)$ down to the low scale (e.g., the $M_{SUSY}$ scale) with the aid of the integral equations (6-9), (12), (15), (20), (21). The masses of the sfermions of the third and the Higgs bosons can be get via $\Sigma_l(l = t, b$ and $\tau)$ as shown in Ref. \[13\]. With such a procedure the dependences on Yukawa couplings of the coefficients in Eqs. (22 \[23\]) are translated into the dependence on $\tan\beta$. We show below an example of numerical results at $t_S = \log(M_{GUT}^2/M_{SUSY}^3) = 63.28$ for $\tan\beta = 58$:

$$A_t(t_S) = 0.19118 A_t^0 - 0.04580 A_t^0 + 0.01059 A_t^0 - 0.02728 M_1^0 - 0.20187 M_2^0 - 1.45580 M_3^0,$$

$$A_b(t_S) = -0.04019 A_b^0 + 0.06052 A_b^0 - 0.03687 A_b^0 - 0.00168 M_1^0 - 0.14498 M_2^0 - 1.32411 M_3^0,$$

$$A_\tau(t_S) = 0.03346 A_\tau^0 - 0.20839 A_\tau^0 + 0.28668 A_\tau^0 - 0.08353 M_1^0 - 0.22527 M_2^0 + 0.47013 M_3^0,$$

$$\Sigma_l(t_S) = -0.04580 (m_{D_3}^0)^2 - 0.03520 (m_{H_1}^0)^2 + 0.19118 (m_{H_2}^0)^2 + 0.01059 (m_{L_3}^0)^2 + 0.01059 (m_{E_3}^0)^2 + 0.14538 (m_{Q_3}^0)^2 + 0.19118 (m_{U_3}^0)^2 - 0.15170 |A_t^0|^2 + 0.00140 |A_b^0|^2 + 0.00227 |A_\tau^0|^2 + 0.02344 (A_t A_t^{*0} + A_b A_b^{*0} A_{\tau}^{*0}) + 0.00841 (M_1 A_t^{*0} + M_1^{*0} A_t) + 0.00194 (M_1 A_t^{*0} + M_1^{*0} A_t) + 0.00000 (M_2 A_t^{*0} + M_2^{*0} A_t) + 0.04673 (A_t A_t^{*0} + A_t A_t^{*0} A_{\tau}^{*0}) + 0.00865 (M_2 A_t^{*0} + M_2^{*0} A_t) + 0.00072 (M_2 A_t^{*0} + M_2^{*0} A_t) + 0.18528 (M_3 A_t^{*0} + M_3^{*0} A_t) - 0.03551 (M_3 A_t^{*0} + M_3^{*0} A_t) + 0.00515 (M_3 A_t^{*0} + M_3^{*0} A_t) - 0.00418 (M_1 M_2^{*0} + M_1^{*0} M_2) - 0.01949 (M_1 M_3^{*0} + M_1^{*0} M_3) - 0.13369 (M_2 M_3^{*0} + M_2^{*0} M_3) + 0.03411 |M_1|^2 + 0.33267 |M_2|^2 + 4.96704 |M_3|^2,$$

$$\Sigma_b(t_S) = 0.06052 (m_{D_3}^0)^2 + 0.02364 (m_{H_1}^0)^2 - 0.04019 (m_{H_2}^0)^2 - 0.03687 (m_{L_3}^0)^2 - 0.03687 (m_{E_3}^0)^2 + 0.02032 (m_{Q_3}^0)^2 - 0.04019 (m_{U_3}^0)^2 + 0.00420 |A_t^0|^2 - 0.03908 |A_b^0|^2 - 0.00028 |A_\tau^0|^2 + 0.00441 (A_t A_t^{*0} + A_t A_t^{*0} A_{\tau}^{*0}) + 0.00225 (A_t A_t^{*0} + A_t A_t^{*0} A_{\tau}^{*0}) + 0.00859 (A_t A_t^{*0} + A_t A_t^{*0} A_{\tau}^{*0}) + 0.00017 (M_1 A_t^{*0} + M_1^{*0} A_t) + 0.00028 (M_1 A_t^{*0} + M_1^{*0} A_t) + 0.00089 (M_1 A_t^{*0} + M_1^{*0} A_t) - 0.00243 (M_2 A_t^{*0} + M_2^{*0} A_t) + 0.00974 (M_2 A_t^{*0} + M_2^{*0} A_t) - 0.00246 (M_2 A_t^{*0} + M_2^{*0} A_t) - 0.01353 (M_3 A_t^{*0} + M_3^{*0} A_t) + 0.05688 (M_3 A_t^{*0} + M_3^{*0} A_t) + 0.03167 (M_3 A_t^{*0} + M_3^{*0} A_t) - 0.00146 (M_1 M_2^{*0} + M_1^{*0} M_2) - 0.00279 (M_1 M_3^{*0} + M_1^{*0} M_3) - 0.09627 (M_2 M_3^{*0} + M_2^{*0} M_3) - 0.000098 |M_1|^2 + 0.23446 |M_2|^2 + 4.64976 |M_3|^2,$$

$$\Sigma_\tau(t_S) = -0.20839 (m_{D_3}^0)^2 + 0.07828 (m_{H_1}^0)^2 + 0.03346 (m_{H_2}^0)^2 + 0.28668 (m_{L_3}^0)^2 + 0.28668 (m_{E_3}^0)^2.$$
\[-0.17493 (\tilde{m}_{Q3})^2 + 0.03346 (\tilde{m}_{t3})^2 + 0.00725 |A^{0}_{t}|^2 - 0.01117 |A^{0}_{b}|^2 - 0.18803 |A^{0}_{\tau}|^2
\]
\[+0.00192 (A^{0}_{t^{0}} A^{0}_{b} + A^{0}_{t^{0}} A^{0}_{\tau}) - 0.00469 (A^{0}_{t^{0}} A^{0*}_{b} + A^{0*}_{t^{0}} A^{0}_{b}) + 0.07397 (A^{0}_{b^{0}} A^{0}_{b^{0}} + A^{0}_{\tau^{0}} A^{0}_{\tau^{0}})
\]
\[+0.00035 (M^{0}_{t^{0}} A^{0}_{t^{0}} + M^{0*}_{t^{0}} A^{0*}_{t^{0}}) - 0.00932 (M^{0}_{b^{0}} A^{0}_{b^{0}} + M^{0*}_{b^{0}} A^{0*}_{b^{0}}) + 0.01609 (M^{0}_{\tau^{0}} A^{0}_{\tau^{0}} + M^{0*}_{\tau^{0}} A^{0*}_{\tau^{0}})
\]
\[-0.00196 (M^{0}_{t^{0}} A^{0}_{t^{0}} + M^{0*}_{t^{0}} A^{0*}_{t^{0}}) - 0.01776 (M^{0}_{b^{0}} A^{0}_{b^{0}} + M^{0*}_{b^{0}} A^{0*}_{b^{0}}) + 0.03236 (M^{0}_{\tau^{0}} A^{0}_{\tau^{0}} + M^{0*}_{\tau^{0}} A^{0*}_{\tau^{0}})
\]
\[-0.02048 (M^{0}_{A^{0}} A^{0}_{A^{0}} + M^{0*}_{A^{0}} A^{0*}_{A^{0}}) + 0.04148 (M^{0}_{A^{0}} A^{0*}_{A^{0}} + M^{0*}_{A^{0}} A^{0*}_{A^{0}}) - 0.04521 (M^{0}_{A^{0}} A^{0}_{A^{0}} + M^{0*}_{A^{0}} A^{0*}_{A^{0}})
\]
\[-0.00595 (M^{0}_{M^{0}} A^{0}_{M^{0}} + M^{0*}_{M^{0}} A^{0*}_{M^{0}}) + 0.00716 (M^{0}_{M^{0}} A^{0}_{M^{0}} + M^{0*}_{M^{0}} A^{0*}_{M^{0}}) - 0.01505 (M^{0}_{M^{0}} A^{0}_{M^{0}} + M^{0*}_{M^{0}} A^{0*}_{M^{0}})
\]
\[+0.10639 |M^{0}_{0}|^2 + 0.38267 |M^{0}_{0}|^2 - 1.72886 |M^{0}_{0}|^2. \quad (24)
\]

One can see from Eq. (24) that for $A_t$, $A_b$ and $\Sigma_t$, $\Sigma_b$ there are very large coefficients appearing before $M^0_3$ or $|M^0_3|^2$ which arise from the large gauge coupling of SU(3) group in the RGs, Eqs. (4) and (5), and hence make them inevitably greatly depend on the gluino mass. If we consider the scenario with the mass spectrum below 1 TeV, we find that the effects of other high energy boundary values on the low energy spectrum are negligible as illustrated in Eq. (24) and actually it is gluino mass which mainly govern the SUSY mass spectrum (note that $m_0$ is required due to the constraint from cosmology [18]) so that non-universality has a little influence on low energy mass spectrum. This is the screening effects induced by the large Yukawa couplings of top, bottom and tau for which a detailed discussion in the real parameter case has been given in Ref. [13]. For $A_t$ and $\Sigma_t$, the screening effects are weak since here $M^0_3$ is not as important as for $A_t$, $A_b$, $\Sigma_t$ and $\Sigma_b$ and the coefficient of $M^0_3$ can compete with that of $M^0_3$. In particular, for $A_t$, the coefficient of $A^0_t$ is the same order as that of $M^0_3$. That is, we are in the vicinity of the IRQFP in the example. Note that a crucial difference between our discussion and the one in Ref. [13] is that we use the Yukawa couplings at $m_Z$ scale induced from their corresponding current masses of quarks, while in Ref. [13] Yukawa couplings at the GUT scale much larger than the unified gauge couplings are assumed when the IRQFP behavior is discussed for large $\tan \beta$. Another difference is that we take the SUSY threshold at $M_{SUSY}$ whereas in Ref. [13] SUSY threshold effects was not considered. As a consequence we find that the value of $\tan \beta$ for which the IFQFP is reached depend weakly on the choice of $M_{SUSY}$. For example the theory exhibits IRQFP behavior at $\tan \beta \approx 59$ for $M_{SUSY} = 400$ GeV and at $\tan \beta \approx 60$ for $M_{SUSY} = 800$ GeV.

In Fig. 1(a) we illustrate how the dependences of $A_t$, $A_b$ and $A_\tau$ on their initial values, i.e., the coefficient $c^0_t(t_s)$ (1=t,b,\tau) in Eq. (22), change with $\tan \beta$. One may find in the figure that the coefficient $c^0_t(t_s)$ increases rapidly for $\tan \beta$ less 5 and decrease slowly afterwards. This is because the top Yukawa coupling, $y_t$, behaves as $\propto \frac{1}{\sin \beta}$ and for somewhat large $\tan \beta$ it remains almost unchanged. So, according to the procedure described above, when $\tan \beta$ increases $Y^0_t$ cannot reach the value large enough to make $A_t$ independent of $A^0_t$. On the other hand, the coefficients $c^0_b$ and $c^0_{\tau}$ decrease considerably as $\tan \beta$ increases in the whole range and drop sharply to zero when $\tan \beta$ approaches 59, which is exactly the case as expected since their corresponding Yukawa couplings behave as $\propto \frac{1}{\cos \beta}$. When $\tan \beta$ reaches some large value ($\approx 59$), $Y^0_{b,\tau}$ become large enough to make the dependences of $A_{b,\tau}$ on their initial values, $A^0_{b,\tau}$, almost disappear, i.e., IRQFP is reached. In Fig. 1(b-d) we plot the the dependences of $\Sigma_t(t_s)$, $\Sigma_b(t_s)$ and $\Sigma_{\tau}(t_s)$ on the initial values of their corresponding three parameters in the definitions of $\Sigma_t$, $\Sigma_b$ and $\Sigma_{\tau}$, i.e., $f^k(t_s)$, $k=t,b,\tau$ in Eq. (23). One can find the similar behavior of the dependences on initial conditions when changing $\tan \beta$. It is clear from Fig. 1 that except for $A_t$ and $\Sigma_t$, IRQFP behavior becomes more and more evident as $\tan \beta$ approaches 59, i.e., the
point where Landau pole appear as we run bottom and tau Yukawa couplings up.

In Fig. 2 we plot $\rho_l = A_l / M_3^l (l = t, b$ and $\tau)$ versus $\alpha_s$ for $\tan \beta = 58.7$, running from the GUT scale down to $M_{SUSY}$ scale as indicated by the axes of the strong coupling $\alpha_s$. One can clearly see the IRQFP behavior of $\rho_l (l = t, b$ and $\tau)$ from the figure. In summary, the numerical results confirm that despite the CP violating phases introduced, the model still exhibit the IRQFP behavior for soft terms $A_l$ and $\Sigma_l$, as pointed out in section two.

Now we come to a position to address the effects of CP violating phases on sparticle spectrum. For the effects of phases of gaugino masses, $M_1^0$ and $M_2^0$, on the mass spectrum (because of R-symmetry, we take $M_3^0$ real for convenience in the following discussion), one can see from Eq. (24) that the only way for the phases of $M_1^0$ and $M_2^0$ to be important is that their magnitudes are much larger than $M_3^0$ (say, at least one order of magnitude), which means a very heavy bino or chargino mass and so is not a preferable scenario. For the phases of trilinear terms $A_t^0$, $A_b^0$ and $A_\tau^0$, naively one may think that it is possible for them to play important role in the mass spectrum through the interference terms with $M_3^0$ as illustrated in Eq. (24). However, notice that because the coefficients of the $A_l^0 M_3^0$ terms are much smaller than that of $|M_3|^2$ term (e.g., for $\Sigma_t$, the former are only 1/20 of the latter) we have to choose $|A_l^0| (l = t, b$ and $\tau)$ much larger than $M_3^0$ (say, at least one order of magnitude) in order to make the effects of their phases considerable so that one is led to far from IRQFP. Moreover, since trilinear soft SUSY breaking terms appear in the nondiagonal terms of squark or slepton mass matrices, the large $A_l$ induced by very large $A_l^0$, together with terms proportional to $\mu \tan \beta$, may result in such light stau that stau becomes LSP. Therefore, we can infer from the above discussion that the initial values of trilinear soft SUSY breaking terms cannot be much larger than the magnitudes of $M_l^0$. Thus we can conclude that in physically interesting regions of parameter space where the notion of IRQFP can be applied, CP violating phases of gaugino masses and soft trilinear SUSY breaking terms are not important for determining the mass spectrum through their roles played in RGEs.

4 Conclusion

In conclusion, we have shown in this paper another way to find the solutions of the RGEs of the soft SUSY breaking terms in the third generation sector of MSSM by directly solving the RGEs themselves. Compared with the method of the spurion expansion, our approach proves to be a simple and convenient way especially for models with complex soft trilinear SUSY breaking terms and gaugino masses. The results in the scenario with complex trilinear terms and gaugino masses turn out to support the notion of IRQFP. It follows that non-universality of trilinear couplings $A_l$ and gaugino masses have no influence on sparticle spectrum at the IRQFP. We have studied the effects on mass spectrum of CP violating phases of trilinear terms and the non-universal gaugino masses and find that the effects on the mass spectrum are limited to be small in the physically interesting region of parameter space where the notion of IRQFP can be applied and physical requirements such as LSP being neutral particle are imposed.

Acknowledgements

The work was partly supported by National Natural Science Foundation of China.
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Figure Caption

Fig. 1(a) lines labelled by 1, 2 and 3 correspond respectively to coefficient of $A_t^0$ in $A_t(t_S)$, i.e., $c_t^i(t_S)$ in (22), coefficient of $A_b^0$ in $A_b(t_S)$, $c_b^i(t_S)$, and coefficient of $A_\tau^0$ in $A_\tau(t_S)$, $c_\tau^i(t_S)$, as functions of $\tan\beta$. In 1(b) lines 1 and 2 refer respectively to coefficients of $(\tilde{m}^0_{Q3})^2$ and $(\tilde{m}^0_{U3})^2$ in $\Sigma_t(t_S)$. In 1(c) lines labeled by 1, 2 and 3 are respectively coefficients of $(\tilde{m}^0_{Q3})^2$, $(m^0_{H1})^2$ and $(\tilde{m}^0_{D3})^2$ in $\Sigma_b(t_S)$. In 1(d) lines 1 and 2 correspond respectively to coefficients of $(m^0_{H1})^2$ and $(\tilde{m}^0_{L3})^2$ in $\Sigma_\tau(t_S)$.

Fig. 2 Three dimensional plots of $\rho_l = A_l/M_3$ ($l = t, b$ and $\tau$) as functions of $\alpha_s$ for $\tan\beta = 58.7$. The other parameters are chosen as $M_1^0 = M_2^0 = 500$ GeV, $M_3^0 = 400$ GeV and $M_0 = 800$ GeV. Initial values of $|\rho_l|$'s are chosen as 1 or 2 and the phases of $\rho_l$'s are taken as $i \times \pi/4 (i = 0, ..., 7)$. 
Fig. 1
Fig. 2