Application of the Point-Descriptor-Precedence representation for micro-scale traffic analysis at a non-signalized T-junction

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ABSTRACT

An intersection of two or more roads poses a risk for potential conflicts among vehicles. Often the reasons triggering such conflicts are not clear, as they might be too subtle for the human eye. The environment also plays a part in understanding where, when, and why a particular vehicle interaction has occurred in a certain way. Therefore, it is of paramount importance to dive deeper into the vehicle interaction at a micro-scale within the embedded geographical environment, particularly at the intersections. This would in turn assist in evaluating the association of vehicle interactions with conflict risks and near-miss accidents. Moreover, detection of such micro traffic interactions could also be used to improve the complexity of the already established transport infrastructure. Conversely, traffic at intersections has been explored mainly for flow estimation, capacity and width measurements, and traffic congestion, etc., whereas the detection of micro-scale traffic interactions at intersections remains relatively under-explored. In this paper, we present a novel approach to retrieve and represent micro-scale traffic movement interactions at a non-signalized T-junction by extending a recently introduced qualitative spatiotemporal Point-Descriptor-Precedence (PDP) representation. We study how the PDP representation offers a fine solution to study the interaction of traffic flows at intersections. This permits tracking the micro-movement of vehicles in much finer detail, which is used later to retrieve movement patterns from a motion dataset. Unlike conventional approaches, we start our approach with the actual movements before modeling the static intersection environment. Additionally, with the aid of illustrative examples, we discuss how the length, width, and speed of the vehicles can be exploited in our approach to detect specific patterns more accurately. Additionally, we address the potential benefits of our approach for traffic safety assessment and how it can be extended to a network of intersections using different transport modes.

1. Introduction

As an intersection is the junction of three or more roads, it is considered as an accident-prone zone and one of the hazardous sites of a road network (Huang 2015). Regardless of the improved design, innovation of vehicles, and sophisticated Intelligent Transportation Systems technology, there is an ever-growing demand for improved traffic management and safety protocols at intersections (Dirnbach et al. 2020). Traffic authorities and policy-makers are in high need of methods to analyze movements at intersections for identifying potential dangerous maneuvers. Needless to say that micro-traffic analysis is important to gain more insight into factors affecting traffic safety and how the intersection design can be improved for future mobility needs, including those of autonomous vehicles (Xin et al. 2011).

From a micro-analysis perspective, intersections have been mainly analyzed for some specific inter-vehicle crash types, e.g., rear-end, crossing, and merging (Huguenin, Torday, and Dumont 2005; Wolfrum 2018). Some studies also considered lane changing, frontal, run-off, and hit pedestrian accidents (Stone, Chae, and Pillalamarri 2002; Mak and Sicking 2003; Dijkstra et al. 2010). Different attempts have been made to extract traffic movement patterns by using quantitative approaches as in (Zhang and Wang 2019; Sarkar et al. 2020), but due to insufficient prior knowledge, these approaches are suitable under specific situations. With the technological advances, many micro-simulation tools such as PARAMICS (Fritzsche and Ag 1994), AIMSUM (Huguenin, Torday, and Dumont 2005), VISSIM (Dedes et al. 2011) and SUMO (Siligui et al. 2021) can simulate an intersection environment for the analysis of a wide range of traffic activities. However, typically these models have been developed to look at the capacity of the transport system and its performance in terms of delay, travel time, etc., but not for movement pattern detection that can lead to safety analysis (Young et al. 2014). Mostly, the safety-related data is aggregated into summary statistics (such as the total
number of crashes/conflicts), which constitutes a major simplification of the real risk levels that change over time. Therefore, these models would need further refinements to be able to accurately detect movement patterns and address different types of conflicts on parts of a road network (Mahmud et al. 2019). Besides, most of these approaches consider vehicles as moving points, whereas each vehicle has its own dimensions. These dimensions have an impact on the severity of accidents. To the best of our knowledge, no micro-analysis qualitative spatiotemporal representation model has ever considered the dimensions of the vehicles to differentiate between movement patterns.

Generally, the detection of traffic movement patterns is not only based on the analyses of motion data (vehicle’s trajectories), but also on the embedded geographical context such as the environment, mode of transportation, or any other domain-specific information (Gschwend and Laube 2012; Keler, Krisp, and Ding 2017). Modeling the geographical context is important to understand the variations in traffic movement patterns and route planning, which is why the existing analysis approaches typically start with representing the road environment. The way one models and represents the road environment has a fundamental impact on the outcomes of the study (Marshall et al. 2018). An inspection of existing studies shows that the dominant representation of a road network is a graph with intersections as vertices and roads as edges (Winter and Unbacher 2002; Batty & Rana 2004; Porta, Crucitti, and Latora 2006b). Alternative approaches represent intersections as edges and roads as vertices (Tomko, Winter, and Claramunt 2008; Delling et al. 2011; Gil 2014; Figueiredo 2015; Porta, Crucitti, and Latora 2006a). Steadman (2004) represented an intersection as a conflation of a street network model and a graph representation. Cova et al. (2008); Scheider and Kuhn (2010) modeled intersections as planar graphs to capture their structure and connectivity. Some of the conventions made by Scheider and Kuhn (2010), such as merging and diverging bifurcations, are also used in our approach to represent intersections. Additionally, Ticha et al. (2018) showed that many other approaches rely on an oversimplified representation of a road network, while in reality, intersections are complex. Hence, from a micro-analysis perspective, it is important to perceive intersections as considerably more complex than just vertices or edges.

To the best of our knowledge, no approach exists for traffic analysis at intersections that can simultaneously account for the different flows and the complexity of the intersection environment at micro-scale. In this work, such an approach is being developed by relying on the Point-Descriptor-Precedence (PDP) representation (Qayyum et al. 2021), which allows to qualitatively capture moving objects as precedences among points. The proposed approach consists of three steps, which are explained by means of an exemplary non-signalized T-junction. Though Rodegerdts et al. (2004) described a detailed list of the most common three-way (e.g. trumpet interchange and semi-directional T-interchange) and four-way intersection types (e.g. quadrant roadway intersection, stacked interchange, and continuous flow intersection), we believe that the T-intersection is less complex than other crossroad types and can be helpful in demonstrating the basics of our proposed approach. This would in turn lay the foundation for applying the approach to other intersection types effortlessly.

In the first step of the proposed approach, the main traffic flows at the T-junction are identified, together with the bifurcation points where different flows split or merge. The result of this step is a flow table that includes all sub-flows between different roads of the T-junction. This step is crucial since the movements are of utmost importance for micro-traffic analysis. In the second step, the T-junction environment is incorporated by defining static and dynamic entry and leave points. This way, the analysis is limited to the crucial area between the entry and leave points of the T-junction, avoiding excessive calculations. In the third step, the PDP representation is used to define the relations between all points generated in the previous two steps. The resulting PDP representations are entered in every cell of the flow table, which was created in the first step. Hence, the end-result is a tabular PDP representation of the T-junction, which captures all important flows. Our approach is flexible since as many points can be added as needed for the analysis and hence, suited for a lot of applications.

The flexibility and relevance of the approach are demonstrated in the second part of this paper, by applying it for movement pattern recognition on a case study intersection in four different ways. Vehicles are subsequently represented by different points to capture the centroid, length, size, and speed. Hence, we also pay attention to the dimensions of the vehicles in the analysis of movement patterns. Our approach is used to successively retrieve the exact movement patterns in a traffic flow dataset for each of the four under consideration cases making it suitable for micro-scale traffic analysis. It can be used to detect and recognize specific (dangerous) movements for a variety of applications ranging from traffic cameras to autonomous vehicles.

The rest of the paper is structured as follows. Section 2 provides a quick overview of the PDP representation and its relevance in micro-scale traffic analysis, in particular at intersections. Section 3 introduces the approach to describe the traffic
movements at a T-junction by using the PDP representation. Section 4 presents the analyses based on the proposed approach. Lastly, Section 5 discusses further extensions of the approach, directions for future research, and conclusions.

2. The Point-Descriptor-Precedence (PDP) representation

Our proposed approach is based on the Point-Descriptor-Precedence (PDP) representation, which was introduced by Qayyum et al. (2021). The PDP representation is a qualitative point representation of (moving) objects with respect to a directed line by means of distance descriptors as depicted in Figure 1. Conventional approaches like the Point Calculus (Vilain and Kautz 1986; Balbiani and Condotta 2002) describe the distance relations between points on a straight directed line only, whereas the PDP representation can also represent movements on a curved directed line. Directed lines are important since they represent the direction of the movement of interest, which can be straight (e.g. in the case of highways), curved (e.g. movements at intersections), circular (e.g. movements at roundabouts), or any other shape. Since the direction of movement can change during movement, the PDP representation uses “distance descriptors” to describe the ordering between the points via a system of relational symbols (\(<\), \(=\), \(>\)). For instance in Figure 1, the (moving) points are shown across the straight and curved directed lines by using the precedence of two distance descriptors \(d_1\) and \(d_2\), where \(d_1\) describes the distance of the points traveled along the directed line and \(d_2\) is the signed orthogonal distance of the points to the directed line measured positive on the left, and negative on the right with respect to the direction. Hence, the PDP representation can represent movements along a curved directed line, which makes it interesting for the representation and analysis of traffic at intersections.

Figure 1 further illustrates how the same point configuration can result in different precedences for straight and curved directed lines. This is one of the main strengths of the PDP representation as it can differentiate between the same movements in different directions at a micro-scale level.

3. Approach

In this section, the proposed approach to represent movements at the T-junction at micro-scale is introduced. From a micro-analysis perspective, it is important to perceive intersections as being much more complex than just vertices or edges. The aim is to represent intersections in such a way to allow for a careful analysis of the traffic movement patterns by relying on the PDP representation. In the upcoming sections, the proposed approach will be explained gradually. First, in Subsection 3.1, movements at the T-junction are broadly defined as flows and recorded in a flow table. Next, in Subsection 3.2, the environmental variables are defined to characterize the fixed surroundings of the flows. The movements and the environment are then represented together in the form of a PDP representation in Subsection 3.3. Finally, this PDP representation is used to describe the movement of the vehicles at micro-scale in Subsection 3.4.

![Figure 1. Simple illustration of PDP representation using straight and curved directed lines (Qayyum et al. 2021).](image-url)
3.1. Scanning for the movements

Figure 2 represents a simple non-signalized T-junction \( I_1 \) comprising three roads \( R_1 \), \( R_2 \), and \( R_3 \), where each road consists of two lanes. In terms of an absolute frame of reference, the left lanes in \( R_1 \) and \( R_3 \) are termed as slow lanes (shown as \( a \)) for the vehicles traveling at slower speed, whereas the right lanes are termed as fast lanes (shown as \( b \)) for faster moving traffic. For illustration purposes and simplicity, we use the basic three-leg intersection (T-junction) where one road meets another one at right angles (or close to a right angle). To set the scale, we assume that the width of a road here is 8 m and that there is also a stop line in \( R_2 \). The collective movement of traffic, which is repeated several times at \( I_1 \) is shown as flows (\( F \)) in red. These flows might, for example, be obtained by detecting the trajectories of all the vehicles entering/exitng the T-junction through video recognition followed by a spatiotemporal clustering. Another way to obtain these vehicle trajectories is to ask domain specialists for details about a certain intersection. Note that the more flows there are at the T-junction, the more complex the behavior is, and thus the more detail is required to understand the movements. Our approach is capable to do so since movement (i.e. flows) is the core of our approach. In some studies, these flows are interpreted as the possible allowed routes at the T-junction (Cova et al. 2008). However, routes tend to appear as a generic term in the literature, whereas flows seem closer to the movements that are represented here.

At our simple T-junction \( I_1 \), there are three detected flows: \( F_{1a-2} \) going from \( R_1 \) to \( R_2 \), \( F_{1a-3} \) going from \( R_1 \) to \( R_3 \), and \( F_{2-3} \) going from \( R_2 \) to \( R_3 \). A flow always consists of one or more sub-flows. \( F_{1a-3} \) consists of two sub-flows: \( F_{1a-3a} \) going from the slow lane \( a \) of \( R_1 \) to the slow lane \( a \) of \( R_3 \) and \( F_{1a-3b} \) going from the fast lane \( b \) of \( R_1 \) to the fast lane \( b \) of \( R_3 \). \( F_{1a-2} \) consists of \( F_{1a-2a} \) and \( F_{1a-2b} \). \( F_{2-3} \) consists of \( F_{2-3a} \). Of course, there might be outliers where one drives differently from the flows shown. For instance, there might be vehicles driving from the slow lane \( a \) of \( R_1 \) to the fast lane \( b \) of \( R_1 \) and then coming back in the slow lane \( a \) of \( R_1 \) again or vice versa. However, for brevity, we have not presented the outlier flows here. Nonetheless, the possible outliers can also be handled if desired.

All the flows appearing at the T-junction are summarized in a flow table \( F \) (see Figure 2). The flow table \( F \) gives the number of sub-flows (\( subF \)) between all roads \( R \).

There often exist certain diverging or converging bifurcations (\( B \) points) (Scheider and Kuhn 2010) at the T-junction where flow(s) split or merge. Hence, it is important to incorporate such bifurcations when dealing with the representation and analysis of movements at micro-scale. That is why these common road network features are implemented in our study. Figure 2 represents the two bifurcations (\( B \) points) for T-junction \( I_1 \): the diverging bifurcation \( B_{1a-2,3a} \) where the two flows \( F_{1a-2} \) and \( F_{1a-3a} \) split, and the converging bifurcation \( B_{1a,2-3a} \) where the two flows \( F_{1a-3a} \) and \( F_{2-3a} \) merge.

3.2. Incorporating the environment

In the second step, the aim is to incorporate the T-junction environment into the analysis up to the extent necessary (Figure 3): the beginning and the end of the T-junction, where the traffic enters or leaves. The first step is to align the T-junction \( I_1 \) with the alignment lines (\( A \)) shown in blue in Figure 3(a). The \( A \) lines are drawn here at a distance \( d \) of 5 m from the stop line in \( R_2 \) (taken as a reference), but \( d \) might also be linked to the speed of the vehicles, or their direction indicators, or where two lanes diverge into three lanes. The exact location of the \( A \) lines is not that important since we use a qualitative representation.

The next step is to incorporate the relevant static and dynamic entry \( E \) and leave \( L \) points of the T-junction environment (Figure 3(b)). The static
points are used for marking the boundaries of the intersection environment, whereas the dynamic points are used to define the entry/leave of the traffic movements. We refer to these points as A points since they are located at the A lines. Some studies have named these points as source and sink nodes (Scheider and Kuhn 2010), but these terms are generally used for a comprehensive road network. We intend to distinguish the intersection from the rest of the road network and introduce terminology related to the intersection only. For instance, Figure 3(b) shows the static E and L points in blue, and the dynamic points in yellow. \( R_1 \) \( E_0 \), \( R_1 E_1 \) and \( R_1 E_2 \) are the static entry points of lanes at the A line of \( R_1 \). Likewise, \( R_3 L_0 \), \( R_3 L_1 \), and \( R_3 L_2 \) are the static leave points of lanes at the A line of \( R_3 \). Similarly, \( R_1 E_{1a-2} \) is the dynamic entry point of \( F_{1a-2} \) and \( R_3 L_{1a-2} \) is the dynamic leave point of \( F_{1a-2} \), etc. Without this context, one might argue that the \( E \) and \( L \) points are the same and lie at the same location, so why go for different labellings and notations? In fact, using these extra labellings allows to clearly distinguish the static points from the dynamic points, which is beneficial for further analysis. The static points are labeled in numeric form, such as \( E_1 \), \( E_2 \), \( E_3 \), . . . , and the dynamic points are labeled according to the flows, such as \( E_{1a-2} \), \( L_{1a-2} \), . . . and so on.

An important aspect of this representation is that it allows to consider (only) the most relevant aspects of the T-junction environment when capturing movements. For instance, in Figure 3(b), the points where the roads meet need not to be considered since such points have already been defined as B points on the flows.

3.3. Combining movements with the environment

In the third step, the movements described in Section 3.1 are combined with the environment points illustrated in Section 3.2 using the PDP representation depicted in Figure 1. Figure 4 shows how this combination has been done. The flows in the flow table \( F \) are considered one by one and the direction of the flow becomes the directed line for which the PDP representation is built. All points (static and dynamic) surrounding the respective flow are ordered using the small set of relational symbols (\(<\),\(=\),\(>\)) for two distance descriptors \( d_1 \) and \( d_2 \).

Consider, for instance, flow \( F_{1a-2} \) going from \( R_1 \) to \( R_2 \), which is a curved directed line. First, we need to check which static/dynamic points are going to be represented. To do so, a green buffer zone \( Z \) is created around this flow (see Figure 4). The points that need to be considered for representing this flow are then \( R_1 E_0 \), \( R_1 E_{1a-2} \), \( R_1 E_1 \), \( B_{1a-2,3a} \), \( R_3 L_0 \), \( R_3 L_{1a-2} \), and \( R_3 L_1 \) as these are located within \( Z \). The next step is to check which distance descriptors can be used to describe the distance of these points with respect to the directed line. We used two distance descriptors: \( d_1 \) describing the distances between the points on the directed line and \( d_2 \) describing the signed orthogonal distance of the points to the directed line. Finally, the precedence of these points is derived based on the two descriptors.
used and recorded accordingly (see Figure 4a). The same is repeated for all other flows. Hence, the movements (flows) together with the environment points are transformed into the respective PDP representations shown in Figure 4.

It is important to create relations between different flows  \( F \) (both merging/splitting) and different directions visible at the T-junction. Therefore, equal point relations are defined by observing which points on the flows are identical in that specific direction. For instance, at  \( R_1, E_{1a-2} = E_{1a-3a} \). Similarly at  \( R_2, E_1 = L_1 \) and at  \( R_3, L_{3a-3a} = L_{2a-3a} \). We create these relations for the diagonal entries of  \( P \) as shown in Table 1.

Finally, following the entries of the flow table  \( F \), the PDP representation of each flow is combined together in a tabular form as  \( P \), which is shown in Table 1. This is the representation of a simple T-junction, but any other type of intersection could also be represented using the same approach.

### 3.4. Adding vehicles for the analysis

Now that the PDP representation  \( P \) of the T-junction  \( I_1 \) is established, it is quite straightforward to import all vehicles appearing at  \( I_1 \) into the PDP representation  \( P \) for the analysis purposes. For instance, by considering a top view, suppose that a vehicle  \( a \) is driving in the flow  \( F_{1a-3a} \) with a speed of 50 km/h and is logged for 9 timestamps at a temporal resolution of 0.2 s (Figure 5).

This movement of  \( a \) can be represented and analyzed through  \( P \) as depicted in Table 0. The positions of  \( a \) at timestamps  \( t_1-t_9 \) are represented as  \( |t_1-a|t_9 \) in Figure 5.

**Table 1.** The PDP representation  \( P \) of the T-junction  \( I_1 \).

|  \( h \)  |  \( r_1 \) |  \( r_2 \) |  \( r_3 \) |
|-------|---------|---------|---------|
| \( h_1 \) | 90 \( E_{1a-2} = E_{1a-3a} \) |  \( d_1^\prime \) |  \( d_1 \) |
| \( h_2 \) |  / |  \( E_1 = L_1 \) |  \( d_1 \) |
| \( h_3 \) |  / |  / |  / |
and Table 2. Representing the micro-movement of $k$ for all nine timestamps together is just one way. An alternative approach is to represent the micro-movement of the vehicles for separate timestamps, which would result in generating $P$ for every timestamp. In the next section, we show how the analysis can be done by generating $P$ for individual timestamps.

### 4. Retrieving a movement pattern

Extracting patterns from spatiotemporal data could be very useful for understanding human mobility, designing intelligent transportation systems, and traffic management for urban mobility. That is why the PDP representation $P$ of the T-junction has been designed in Section 3 to support the retrieval of movement patterns or for performing a similarity analysis. In this section, we will use this PDP representation $P$ of the T-junction to retrieve a movement pattern from a motion dataset. To keep things simple, we will focus on retrieving the exact matches of a given reference movement pattern, i.e., a tuple of vehicles following the same sequence of the reference movement pattern.

#### 4.1. Dataset

To illustrate the retrieval of movement patterns, we created a dataset of 46 vehicles moving at a similar T-junction $I_1$ as shown in Figure 2(a) with a temporal resolution of 0.1 s. The duration of the video is 64 s. The vehicles vary in length, width (e.g., cars versus trucks), and speed to mimic a real-world traffic scenario at the T-junction. From now onwards, we will refer to it as "target dataset".

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**Table 2. Importing the movement of vehicle $k$ in $P$ at 9 timestamps.**

| $I_1$ | $E_{1a-2} = E_{1a-3a}$ | $d_1$ | $R_1$ |
|-------|------------------------|-------|-------|
| $R_1$ | $E_{1a-2} = E_{1a-3a}$ | $d_1$ | $R_1$ |
| $R_2$ | / | $E_1 = L_1$ | $d_2$ | $R_2$ |
| $R_3$ | / | / | $d_3$ | $R_3$ |

**Figure 5. Movement of vehicle $k$ for 9 timestamps at $I_1$.**
4.2. Reference movement pattern with one point

By taking the centroid “c” of each vehicle, we created a reference movement pattern of four vehicles $V_k, V_l, V_m$ and $V_n$, comprising six timestamps ($t'_{1-6}$) as depicted in Figure 6. We used $t'_{i}$ to differentiate between the timestamps of the reference movement pattern and the target dataset. At $t'_{1}$, $V_k$ and $V_n$ are moving from $R_1$ to $R_3$, vehicle $V_l$ is moving from $R_2$ to $R_3$ and $V_m$ is moving from $R_1$ to $R_2$. From $t'_{2}$ to $t'_{4}$, vehicle $V_l$ awaits for $V_k$ and $V_n$ to pass and from $t'_{5}$ to $t'_{6}$, $V_l$ and $V_m$ complete their moves. We call this the “one-point reference movement pattern” since this pattern has been constructed using one point (i.e. the centroid) only. Using the approach described in Section 3, the PDP representations ($P^{ref}$) of the one-point reference movement pattern for six timestamps is given in Appendix A (Tables A1–A6).

4.3. Retrieving the exact matches for the one-point reference movement pattern

Since the one-point reference movement pattern is already created, the goal is now to identify its exact matches in our target dataset. As there were four vehicles in the reference movement pattern, we first generated a set $T$ consisting of all 4-tuples of the 46 vehicles of the target dataset. Then, using the approach described in Section 3, we created the PDP representations ($P^{tar}$) of each target tuple in $T$ along the entire

![Figure 6. The one-point reference movement pattern by taking the centroid per vehicle.](image-url)
duration of the target dataset. As a next step, the $P_{\text{tar}}$ of each tuple in $T$ is compared with the $P_{\text{ref}}$ of the reference movement pattern. If both PDP representations match exactly irrespective of the temporal length, then the tuple is taken as a match of the one-point reference movement pattern. In this way, we found these six exact matches: Match-I ($V_2, V_4, V_5, V_6$), Match-II ($V_{10}, V_{11}, V_{12}, V_{13}$), Match-III ($V_{24}, V_{25}, V_{26}, V_{27}$), Match-IV ($V_{31}, V_{32}, V_{33}, V_{34}$), Match-V ($V_{36}, V_{37}, V_{38}, V_{39}$), and Match-VI ($V_{43}, V_{44}, V_{45}, V_{46}$). A static representation of these six matches is shown in Figures A1–A3.

Each of these matches matched with the reference movement pattern during a specific interval irrespective of its temporal length. This is because we have used the event-based approach where the time at which the event occurred is recorded for the analysis purposes. For instance, in Figure 7, Match-I ($V_2, V_4, V_5, V_6$) matched with the $P_{\text{ref}}$ of the one-point reference movement pattern from $t_{26}$–$t_{47}$. Note that these matches are found by checking whether the centroids of the vehicles lie inside the buffer zone $Z$ in accordance with the one-point reference movement pattern.

4.4. Does the length of a vehicle matter? analysis with two points

A vehicle is an object with its own dimensions. These dimensions vary according to vehicle type and have a major impact on the probability and severity of crashes. That is why an important question is whether the length of the vehicles affects the process of finding the exact matches. We tried to answer this question by making a two-point reference movement pattern using the front-end ($f$) and the back-end ($b$) of the vehicles as shown in Figure 8(a). In this way, we captured the length of the vehicles and sought for the exact matches of this two-point reference movement pattern in the target dataset. The goal is to check if the six matches found for the one-point reference movement pattern remain the same for the two-point reference movement pattern or whether considering two points results in different matches than considering a single point. A complete static representation of the two-point reference movement pattern is given in Figure A4.

Certainly with two points, the representation $P_{\text{ref}}$ of the two-point reference movement pattern differs significantly from that of the one-point reference pattern.
Resultantly, the matches for the two-point reference pattern were found to be only two out of six: \((V_{31}, V_{32}, V_{33}, V_{34})\) and \((V_{36}, V_{37}, V_{38}, V_{39})\) shown in Figure A2 and A3 respectively.

Figure 9 explains that due to the length of the vehicles, there are only two exact matches for the two-point reference movement pattern. Consider the remaining four tuples \((V_{2}, V_{4}, V_{5}, V_{6})\), \((V_{10}, V_{11}, V_{12}, V_{13})\), \((V_{24}, V_{25}, V_{26}, V_{27})\), and \((V_{44}, V_{45}, V_{46})\) given in Appendix A, which are the other exact matches for the one-point reference movement pattern. At \(t_{26}\) in Figure 9(b), \(V_{4}\) did not match with \(V_{5}\) of the two-point reference movement pattern shown in Figure 9(a) and also \(V_{1}\) appeared in the buffer zone \(Z\). Hence, the PDP representation \(P_{1}^{\text{tar}}\) of the tuple \((V_{3}, V_{4}, V_{5}, V_{6})\) did not match the representation \(P_{1}^{\text{ref}} | t'_{1}\). Likewise, \((V_{10}, V_{11}, V_{12}, V_{13})\) was not found as an exact match because at \(t_{122}\), \(V_{10}\) and \(V_{10}^{b}\) differed from \(V_{k}\) and \(V_{k}^{b}\) of the two-point reference pattern (Figure 9(c)). Similarly, at \(t_{342}\), the back-end of \(V_{24}\) \((V_{24}^{b})\) appeared before the branching point \(B_{1}^{a} \rightarrow 2, 3, 4\) whereas it should have coincided with \(B_{1}^{a} \rightarrow 2, 3, 4\) just like \(V_{k}\) at \(P_{1}^{\text{ref}} | t'_{1}\) (Figure 9(d)). The same goes for the tuple

Figure 8. Static representation of the reference movement pattern at \(t'_{1}\) with two points (front-end \(f\) and back-end \(b\)) (a), four points (front-left \(fl\), back-left \(bl\), front-right \(fr\), and back-right \(br\)) (b), and two points (centroid \(c\) and a safe following distance point \(s\)) (c).

Figure 9. The two-point reference movement pattern at \(t'_{1}\) shown in (a). Tuple \((V_{2}, V_{4}, V_{5}, V_{6})\) at \(t_{26}\) shown in (b), tuple \((V_{10}, V_{11}, V_{12}, V_{13})\) shown at \(t_{122}\) in (c), tuple \((V_{24}, V_{25}, V_{26}, V_{27})\) at \(t_{342}\) shown in (d), and tuple \((V_{44}, V_{45}, V_{46})\) at \(t_{608}\) shown in (e). All these tuples do not coincide with \(t'_{1}\) of the two-point reference movement pattern shown in (a).
(V_{43}, V_{44}, V_{45}, V_{46}) in Figure 9(e) at t_{608}. As a result, subtle differences appeared between the PDP representations of these vehicles as compared with those of the two-point reference movement pattern as shown in Tables A7–A10. This shows that the length of the vehicles is important in the process of extracting exact movement patterns out of a given spatiotemporal motion dataset. The length of the vehicles must be included in the analysis to gain more insights into the traffic movements since different lengths of the vehicles might result in different traffic maneuvers.

4.5. Does the width along with the length of a vehicle matter? analysis with four points

Another related question is whether the width of the vehicles matters in analyzing movement patterns. This could be answered by taking four points per vehicle in the reference movement pattern (Figure 8(b)), i.e. the front-left fl, the front-right fr, the back-left bl, and the back-right br end-points. These points capture the width along with the length of the vehicles. A glimpse of the static four-point reference movement pattern at t'_{1} is given in Figure 8(b), whereas the complete static representation of the same pattern can be found in Figure A5. Following the approach described in Section 3, the only exact match of the four-point reference movement pattern was found to be (V_{36}, V_{37}, V_{38}, V_{39}). This time again, the PDP representations \(I_{\text{pro}}\) of the four-point reference movement pattern differ from the PDP representations \(I_{\text{tar}}\) of other tuples.

The other match (V_{31}, V_{32}, V_{33}, V_{34}) for the two-point reference movement pattern was not detected as a match for the four-point reference movement pattern due to the width of V_{33} (see Figure 10(b)). At t_{468}, the front-right and the back-right end-points of V_{33} (V_{33r,1}) did not match with those of V_{m} (V_{m,r,1}) at t'_{5} of the four-point reference movement pattern in Figure 10(a). This resulted in a conflict between the PDP representations \(I\) of both reference and target tuples, as shown in Table A11. The slight variations in \(I\) could result in a different movement pattern, which shows the strength of PDP representation for the micro-scale analysis. This proves that the width of the vehicles matters in the retrieval of movement patterns from a motion dataset. Hence, the presented approach can differentiate between the subtle differences in \(I\) caused by varying the physical dimensions of the vehicles.

4.6. Does the speed of a vehicle matter?

Vehicles moving at different speeds can have a variety of motion interaction, but does the speed of the vehicles affect the process of pattern retrieval as well? To represent the speed of the vehicles, we considered the following two points per vehicle: the centroid \(c\) and a safe following distance \(s\). Usually, the higher the speed of the vehicle, the farther is its \(s\). Utilizing these two points, we reconfigured our reference movement pattern as shown in Figure 8(c) and searched the target dataset for finding its exact matches, since the vehicles in our target dataset also had different speed profiles. The complete static representation for this two-point reference movement pattern is given in Figure A6.

Repeating the same approach, we found only one match for our two-point reference movement pattern, namely (V_{36}, V_{37}, V_{38}, V_{39}), which is also a match for the four-point reference movement pattern. This tuple

![Four-point reference movement pattern at t'_{5}](a)

![Tuple (V_{31}, V_{32}, V_{33}, V_{34}) at t_{468}](b)

**Figure 10.** The four points of V_{33} shown in (b) do not match with those of V_{m} at t'_{5} of the four-point reference movement pattern shown in (a).
matched the four-point and two-point reference movement pattern in the same dimensions and speed profiles. The rest of the tuples could not meet the speed profile of the two-point reference movement pattern constructed here and thus, signify the importance of speed in the process of retrieving movement patterns.

5. Discussion

Analyzing movement patterns could help to improve safety regulations, especially for road intersections by detecting undesirable or even dangerous constellations of moving entities such as traffic jams, anomalous driving behaviors, or speed crashes. In fact, the environment also plays an important role in analyzing these constellations. For instance, obstacles, limited visibility due to vegetation, a mix of different traffic participants (cyclists, pedestrians, car drivers), or a busy situation on a street due to an accident. Which conditions lead to unsafe driving maneuvers are not always clear, also not to a human driver, as the causes might be very subtle for the expert eyes. Therefore, it is important to investigate how such situations can be detected at micro-scale, and then search for their triggers.

Considering the importance of movement patterns, we presented an approach to represent the T-junction by combining movements with the intersection environment in the form of a tabular PDP representation \( P \). We have developed the approach based on the PDP representation for a simple three-leg or T-T-junction, which can be applied to multiple intersection types for traffic analysis. The proposed approach can also be used to represent the intermediate intersections to model the whole traffic network. This might be the next conceptual step, which involves labeling different intersections \( I \), roads \( R \), and streets \( S \). For instance, in a network \( N \) of intersections \( I \), the roads connecting to a particular intersection might be labeled as \( I_1 R_1, I_2 R_1, I_1 R_2 = S, R_2 \) and so on. Similarly, each intersection can be represented by a flow table \( F \) (e.g. \( I_1 F, I_2 F, etc. \)). Lane linings at complex intersections can also be taken into account according to the presented approach. Each lining-end is a point. Besides, similar intersections have similar PDP representations, apart from the overall orientation.

It is worthwhile mentioning here that adding context such as different transport modes, different types of lines, traffic lights, signs, etc. is possible in the presented approach at the cost of added complexity. For example, other transport modes can be incorporated into the approach by considering \((E_0)^b\) for bikes, \((L_1)^t\) for trams, and \((F_{1a-3a})^c\) for cars. Moreover, differentiating between “overtaking line” \( O \) and “full-line” \( F \) could be done by adding extra symbols, e.g. \( I_1 R_1 E_1^O \) and \( I_1 R_1 E_1^F \).

In Section 4, we discussed how the proposed approach can be used to retrieve a particular movement pattern from a given motion dataset. This is important to develop analysis tools that help to improve the level of traffic management and safety at intersections. Additionally, we considered the dimensions of the vehicles such as length, width, and speed as points in our analysis since these dimensions turn out to have a major impact on the probability of dangerous traffic maneuvers. The importance of considering length becomes evident when introducing the so-called eco-combies or supertrucks on the roads; and how such vehicles would maneuver in cities as compared to other smaller vehicles (cars, vans, etc.). The longer vehicles may induce unsafe situations for the other road users. The width of the vehicles becomes equally important with the so-called special transport, which requires designated routes; or in case of emergency when two lanes become three lanes with the rescue lane in the middle. These dimensions can also be combined with speed. The more points per vehicle, the more refined the analysis obtains, and the more complex the representation becomes. Furthermore, the existing traffic design guidelines like HBS in Germany, HCM in the US, and Austroads in Australia can incorporate our approach in their microsimulation models to simulate the attributes of vehicles, such as speed, length, and safety distance to the vehicle in front when traveling, to advise on design plans, congestion resolution, or real-time risk assessment of autonomous driving at various road junctions. This could be interesting for autonomous and automated driving testing, especially in a heterogeneous traffic environment where different vehicle types share the same road.

The presented analysis for the exact matches in Section 4 is based on the idea to introduce a conceptual approach for traffic analysis at a simple non-signaled T-junction. The practical implementation of this approach certainly requires an in-depth analysis and needs to be further investigated. It is important to validate the presented concepts in practice in order to detect the implementation issues with real-world traffic datasets. Instead of the exact matches, similarities could also be calculated via the proposed approach since, in real life, the dimensions and speed profiles of the vehicles vary significantly. Therefore, considering flexibility in the pattern matching might be important.

The tabular PDP representation \( P \) of the T-junction represents traffic movements by considering all flows between the roads. This is important if the objective is to analyze the whole intersection. However, if the movement pattern is related to a specific flow, then it is possible to consider only that respective flow movement (or a part of it) represented in the specific columns of \( P \) and ignore the rest. Of course, this mainly depends upon the type of application and what is required to be analyzed. Since our reference movement pattern was
constructed by considering all flows at the T-junction $I_1$, it was necessary to consider all flows while searching for the exact matches. Nevertheless, if our reference movement pattern was based on three vehicles moving in some specific flows, e.g. $V_1$ moving from $R_1$ to $R_3$, $V_1$ moving from $R_2$ to $R_3$ and $V_m$ moving from $R_1$ to $R_3$, then we could have considered the slow part of the flow $F_1\rightarrow 3$ only, i.e. $F_{1a\rightarrow 3a}$ instead of taking both parts of the flow ($F_{1a\rightarrow 3a}$ and $F_{1b\rightarrow 3b}$) for finding the exact matches.

6. Conclusions

In this work, we have presented a novel approach to represent and retrieve micro-scale traffic movement interactions at a non-signalized T-junction by extending the qualitative spatiotemporal Point-Descriptor-Precedence (PDP) representation. First, we represented the traffic motion as flows ($F$) in the curved and straight directions at the T-junction. Next, we represented the micro-movement of individual vehicles along these flows in the form of a PDP representation $\mathcal{P}$. Then, we compared these PDP representations to retrieve movement patterns from a motion dataset. Finally, with the aid of illustrative examples, we discussed that the length, width, and speed of the vehicles can be exploited in our approach to detect unsafe or subtle patterns more accurately. By comparing the PDP representations of the T-junction at different timestamps, similarities in the movement patterns can be detected. This way, hazardous situations can be perceived and monitored by traffic analysts to facilitate safer traffic flows and reduce traffic accident costs for society. Since our approach provides an in-depth micro-scale traffic analysis, we believe that, when used in microsimulation models, it can assist in evaluating the association of vehicle interactions with conflict risks and near-miss accidents.

Data availability statement

The data and codes that support the findings of this study are available at https://dx.doi.org/10.17632/vhng7jdvfb.2.

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References

Balbiani, P., and J.-F. Condotta. 2002. “Spatial Reasoning about Points in a Multidimensional Setting.” Applied Intelligence 17 (3): 221–238. doi:10.1023/A:1020079114666.

Batty, M., and S. Rana. 2004. “The Automatic Definition and Generation of Axial Lines and Axial Maps.” Environment and Planning B: Planning and Design 31 (4): 615–640. doi:10.1068/b2985.

Cova, T.J., H.J. Miller, K. Beard, A.U. Frank, and M. F. Goodchild, Eds. 2008. Road Networks and Their Incomplete Representation by Network Data Models. Berlin: Heidelberg.

Dedes, G., D. Grejner-Brzezinska, D. Guenther, G. Heyding, K. Mouskos, B. Park, and C. Toth. 2011. “Integrated GNSS/INU, Vehicle Dynamics, and Microscopic Traffic Flow Simulator for Automotive Safety.” In 14th International Conference on Intelligent Transportation Systems (ITSC), Washington, D.C., October 5–7, pp 513–519.

Delling, D., A. Goldberg, T. Pajor, and R. Werneck. 2011. “Customizable Route Planning.” In Panos M. Pardalos, Steffen Rebennack. 10th International Symposium on Experimental Algorithms (SEA’11) Lecture Notes in Computer Science. Greece: Springer Verlag. https://www.microsoft.com/en-us/research/publication/customizable-route-planning/

Dijkstra, A., P. Marchesini, F. Bijleveld, V. Kars, H. Drolenga, and M. van Maarseveen. 2010. “Do Calculated Conflicts in Microsimulation Model Predict Number of Crashes?” Transportation Research Record 2147 (1): 105–112. doi:10.3141/2147-13.

Dörnbach, I., T. Kubiatko, E. Kolla, J. Ondruš, and Ž. Šarić. 2020. “Methodology Designed to Evaluate Accidents at Intersection Crossings with respect to Forensic Purposes and Transport Sustainability.” Sustainability 12 (5): 1972. https://www.mdpi.com/2071-1050/12/5/1972.

Figueiredo, L. 2015. “A Unified Graph Model for Line and Segment Maps.” In 10th International Space Syntax Symposium, 46. London.

Fritzsch, H.-T., and D.-B. Ag. 1994. “A Model for Traffic Simulation.” Traffic Engineering+Control 35: 317–321.

Gil, J. 2014. “Analyzing the Configuration of Multimodal Urban Networks.” Geographical Analysis 46 (4): 368–391. doi:10.1111/gean.12062.

Gschwend, C., and P. Laube. 2012. “Challenges of Context-aware Movement Analysis – Lessons Learned about Crucial Data Requirements and Pre-processing.” In GIS Research UK 20th Annual Conference, 241–246. 1 vol. Lancaster, UK: GIS Research UK.

Huang, H. 2015. “Anomalous Behavior Detection in Single-trajectory Data.” International Journal of Geographical Information Science 29 (12): 2075–2094. doi:10.1080/13658816.2015.1063640.

Huguenin, F., A. Torday, and A.-G. Dumont. 2005. “Evaluation of Traffic Safety Using Microsimulation.” In 5th Swiss Transport Research Conference, 1–19. Monte Verità, Ascona, Switzerland.

Keler, A., J.M. Krisp, and L. Ding. 2017. “Detecting Vehicle Traffic Patterns in Urban Environments Using Taxi Trajectory Intersection Points.” Geo-Spatial Information Science 20 (4): 333–344. doi:10.1007/s11067-018-9427-9.

Mahmud, S.M.S., L. Ferreira, M.S. Hoque, and A. Tavassoli. 2019. “Micro-simulation Modelling for Traffic Safety: A Review and Potential Application to Heterogeneous Traffic Environment.” IATSS Research 43 (1): 27–36. http://www.sciencedirect.com/science/article/pii/S0386111217302133.

Mak, K.K., and D.L. Sicking. 2003. Roadside Safety Analysis Program (RSAP). Transportation Research Board.

Marshall, S., J. Gil, K. Kropf, M. Tomko, and L. Figueiredo. 2018. “Street Network Studies: From Networks to Models and Their Representations.” Networks and Spatial Economics 18 (3): 1–15. doi:10.1007/s11057-018-9427-9.

Porta, S., P. Crucitti, and V. Latora. 2006a. “The Network Analysis of Urban Streets: A Dual Approach.” Physica A: Statistical Mechanics and Its Applications 369 (2): 853–866. http://www.sciencedirect.com/science/article/pii/S0378437106001282.

Porta, S., P. Crucitti, and V. Latora. 2006b. “The Network Analysis of Urban Streets: A Primal Approach.” Environment and Planning B: Planning & Design 33 (5): 705–725. doi:10.1068/b32045.

Qayyum, A., B.D. Baets, M.S. Baig, F. Witlox, G.D. Tré, and N.V. de Weghe. 2021. “The Point-Descriptor-Precedence Representation for Point Configurations and Movements.” International Journal of Geographical Information Science 35 (7): 1374–1391. doi:10.1080/13658816.2020.1864378.

Rodegerdts, L.A., B.L. Nevers, B. Robinson, J. Ringert, P. Koonce, J. Bansen, T. Nguyen, et al. 2004. “Signalized Intersections: Informational Guide.” Technical Report United States. Federal Highway Administration.

Sarkar, N.C., A. Bhaskar, Z. Zheng, and M.P. Miska. 2020. “Microscopic Modelling of Area-based Heterogeneous Traffic Flow: Area Selection and Vehicle Movement.” Transportation Research Part C: Emerging Technologies 111: 373–396. https://www.sciencedirect.com/science/article/pii/S0968090X18313810.

Scheider, S., and W. Kuhn. 2010. “Affordance-based Categorization of Road Network Data Using a Grounded Theory of Channel Networks.” International Journal of Geographical Information Science 24 (8): 1249–1267. doi:10.1080/13658810903514198.

Silgu, M.A., I.G. Erdagi, G. Göksu, and H.B. Celikoglu. 2021. “Combined Control of Freeway Traffic Involving Cooperative Adaptive Cruise Controlled and Human Driven Vehicles Using Feedback Control through SUMO.” IEEE Transactions on Intelligent Transportation Systems. doi:10.1109/TITS.2021.3098640.

Steadman, P. 2004. “Developments in Space Syntax.” Environment and Planning B: Planning and Design 31 (4): 483–486. doi:10.1068/b3104ed.

Stone, J.R., K. Chae, and S. Pillalamarri. 2002. “The Effects of Roundabouts on Pedestrian Safety.” Technical Report Southeastern Transportation Center, University of Tennessee Knoxville.
Ticha, B., A. Hamza, F. Nabil, Dominique, and A. Quilliot. 2018. "Vehicle Routing Problems with Road-network Information: State of the Art." Networks 72 (3): 393–406. https://onlinelibrary.wiley.com/doi/abs/10.1002/net.21808

Tomko, M., S. Winter, and C. Claramunt. 2008. "Experiential Hierarchies of Streets." Computers, Environment and Urban Systems 32 (1): 41–52. http://www.sciencedirect.com/science/article/pii/S0198971507000221

Vilain, M.B., and H.A. Kautz 1986. “Constraint Propagation Algorithms for Temporal Reasoning.” In In AAAI'86, Philadelphia, August 11–15. pp 377–382. 86. vols. Philadelphia Pennsylvania.

Winter, S., and A. Unbacher. 2002. "Modeling Costs of Turns in Route Planning." Geoinformatica 6 (4): 345–361. doi:10.1023/A:1020853410145.

Wolfgram, J. 2018. "A Safety and Emissions Analysis Of Continuous Flow Intersections." Master’s thesis University of Massachusetts Amherst.

Xin, L., D. Yang, Y. Chen, and Z. Li 2011. "Traffic Flow Characteristic Analysis at Intersections from Multi-layer Spectral Clustering of Motion Patterns Using Raw Vehicle Trajectory." In 14th International IEEE Conference on Intelligent Transportation Systems (ITSC). Washington, DC, USA, 513–519.

Young, W., A. Sobhani, M.G. Lenné, and M. Sarvi. 2014. "Simulation of Safety: A Review of the State of the Art in Road Safety Simulation Modelling." Accident Analysis and Prevention 66: 89–103. http://www.sciencedirect.com/science/article/pii/S0001457514000128

Zhang, W., and W. Wang. 2019. "Learning V2V Interactive Driving Patterns at Signalized Intersections." Transportation Research Part C: Emerging Technologies 108: 151–166. https://www.sciencedirect.com/science/article/pii/S0968090X18313329
### Appendix A

Table A1. $x_0 \rightarrow x_0'$ of the one-point reference movement pattern.

| $t_1$ | $\text{R}_1$ | $\text{R}_2$ | $\text{R}_3$ |
|------|-------------|-------------|-------------|
| $E_{10-2} = E_{10-3a}$ | $d_1$: $R_1E_0 = R_1E_{10-2} = R_1E_1 < E_{10-2,3a}$ | $d_1$: $R_3E_0 = R_3E_{10-3a} = R_3E_1 = R_3E_{10-3b} = R_3E_2 < E_{10-2,3a}$ | $E_{10-2a} < E_{10-2b} < E_{10-2c}$ |
|      | $m_{1}|t'_1| <$ | $m_{1}|t'_1| <$ | $E_{10-2a} < E_{10-2b} < E_{10-2c}$ |
|      | $E_{10-2,3a} <$ | $E_{10-2,3a} <$ | $E_{10-2a} < E_{10-2b} < E_{10-2c}$ |
|      | $k_0|t'_1| <$ | $k_0|t'_1| <$ | $E_{10-2a} < E_{10-2b} < E_{10-2c}$ |
|      | $R_2L_0 = R_2L_{10-2} = R_2L_1$ | $R_3L_0 = R_3L_{10-3a} = R_3L_1 = R_3L_{10-3b} = R_3L_2 < E_{10-2a} < E_{10-2b} < E_{10-2c}$ |
|      | $d_2$: $R_1E_0 < R_1L_0 < R_1E_{10-2} = R_1L_{10-2} = m_1|t'_1| < R_1E_1 < R_1L_1 < R_1E_{10-3a} = m_1|t'_1| < k_0|t'_1| < E_{10-2a} < E_{10-2b} < E_{10-2c}$ |

### Table A2. $x_{10} \rightarrow x'_{10}$ of the one-point reference movement pattern.

| $t_1$ | $\text{R}_1$ | $\text{R}_2$ | $\text{R}_3$ |
|------|-------------|-------------|-------------|
| $E_{10-2} = E_{10-3a}$ | $d_1$: $R_1E_0 = R_1E_{10-2} = R_1E_1 < E_{10-2,3a}$ | $d_1$: $R_3E_0 = R_3E_{10-3a} = R_3E_1 = R_3E_{10-3b} = R_3E_2 < E_{10-2,3a}$ | $E_{10-2a} < E_{10-2b} < E_{10-2c}$ |
|      | $m_{1}|t'_1| <$ | $m_{1}|t'_1| <$ | $E_{10-2a} < E_{10-2b} < E_{10-2c}$ |
|      | $E_{10-2,3a} <$ | $E_{10-2,3a} <$ | $E_{10-2a} < E_{10-2b} < E_{10-2c}$ |
|      | $k_0|t'_1| <$ | $k_0|t'_1| <$ | $E_{10-2a} < E_{10-2b} < E_{10-2c}$ |
|      | $R_2L_0 = R_2L_{10-2} = R_2L_1$ | $R_3L_0 = R_3L_{10-3a} = R_3L_1 = R_3L_{10-3b} = R_3L_2 < E_{10-2a} < E_{10-2b} < E_{10-2c}$ |
|      | $d_2$: $R_1E_0 < R_1L_0 < R_1E_{10-2} = R_1L_{10-2} = m_1|t'_1| < R_1E_1 < R_1L_1 < R_1E_{10-3a} = m_1|t'_1| < k_0|t'_1| < E_{10-2a} < E_{10-2b} < E_{10-2c}$ |

| $\text{R}_1$ | $\text{R}_2$ | $\text{R}_3$ |
|-------------|-------------|-------------|
| $E_1 = L_1$ | $d_2$: $R_1E_0 < R_1L_0 < R_1E_{10-2} = R_1L_{10-2} = m_1|t'_1| < R_1E_1 < R_1L_1 < R_1E_{10-3a} = m_1|t'_1| < k_0|t'_1| < E_{10-2a} < E_{10-2b} < E_{10-2c}$ |
| $E_1 = L_1$ | $R_3L_0 = R_3L_{10-3a} = R_3L_1 = R_3L_{10-3b} = R_3L_2 < E_{10-2a} < E_{10-2b} < E_{10-2c}$ |
| $R_2$ | $E_1 = L_1$ | $R_3L_0 = R_3L_{10-3a} = R_3L_1 = R_3L_{10-3b} = R_3L_2 < E_{10-2a} < E_{10-2b} < E_{10-2c}$ |
| $E_1 = L_1$ | $R_3E_0 < R_3E_{10-3a} = R_3E_1 < l_0|t'_1| < k_0|t'_1| < E_{10-2a} < E_{10-2b} < E_{10-2c}$ |

| $\text{R}_3$ | $\text{R}_2$ | $\text{R}_3$ |
|-------------|-------------|-------------|
| $L_{0-3a} = L_{20-3a}$ | $R_3L_0 = R_3L_{10-3a} = R_3L_1 = R_3L_{10-3b} = R_3L_2 < E_{10-2a} < E_{10-2b} < E_{10-2c}$ |

Note: $E_{10-2a}$, $E_{10-2b}$, and $E_{10-2c}$ represent different points in the reference movement pattern. $R_1$, $R_2$, and $R_3$ denote the reference segments.
Table A3. $z_{ref}$ for $\psi_3'$ of the one-point reference movement pattern.

| $l_i$ | $R_1$ | $R_2$ | $R_3$ | $R_4$ |
|------|-------|-------|-------|-------|
| $R_1$ | $E_{1a-2} = E_{1a-3a}$ | $d_1$: | $d_1$: | $d_1$: |
|      | $R_1E_0 = R_1E_{1a-2} = R_1E_1$ | $R_2E_0 = R_2E_{1a-3a} = R_2E_1 = R_2E_{1b-3b} = R_2E_2$ | $m_1|\psi_3'\rangle <$ | $m_1|\psi_3'\rangle <$ |
|      | $B_{1a-2.3a} <$ | $B_{1b-2.3b} <$ | $B_{1a-2.3a} <$ | $B_{1a-2.3a} <$ |
|      | $R_1L_0 = R_1L_{1a-2} = R_1L_1$ | $k_1|\psi_3'\rangle <$ | $k_1|\psi_3'\rangle <$ | $k_1|\psi_3'\rangle <$ |
|      | $d_1$: | $d_1$: | $d_2$: | $d_1$: |
|      | $R_1E_0 <$ | $R_2E_0 = R_2L_0 <$ | $R_2E_0 = R_2L_0 <$ | $R_3E_0 = R_3L_0 <$ |
|      | $R_2L_0 <$ | $R_1E_{1a-3a} = R_1L_{1a-3a} = m_1|\psi_3'\rangle <$ | $R_1E_{1a-3a} = R_1L_{1a-3a} = m_1|\psi_3'\rangle <$ | $R_1E_{1a-3a} = R_1L_{1a-3a} = m_1|\psi_3'\rangle <$ |
|      | $R_1L_1 <$ | $R_2E_{1a-3a} = R_2L_{1a-3a} = n_1|\psi_3'\rangle <$ | $R_2E_{1a-3a} = R_2L_{1a-3a} = n_1|\psi_3'\rangle <$ | $R_2E_{1a-3a} = R_2L_{1a-3a} = n_1|\psi_3'\rangle <$ |
|      | $R_1E_1$ | $R_3E_1 = R_3L_1 <$ | $R_3E_1 = R_3L_1 <$ | $R_3E_1 = R_3L_1 <$ |
| $R_2$ | / | $E_1 = L_1$ | $E_1 = L_1$ | $E_1 = L_1$ |
| $R_3$ | / | / | $E_{1a-3a} = L_{2a-3a}$ | / |

Table A4. $z_{ref}$ for $\psi_3'$ of the one-point reference movement pattern.

| $l_i$ | $R_1$ | $R_2$ | $R_3$ | $R_4$ |
|------|-------|-------|-------|-------|
| $R_1$ | $E_{1a-2} = E_{1a-3a}$ | $d_1$: | $d_1$: | $d_1$: |
|      | $R_1E_0 = R_1E_{1a-2} = R_1E_1$ | $R_2E_0 = R_2E_{1a-3a} = R_2E_1 = R_2E_{1b-3b} = R_2E_2$ | $m_1|\psi_4'\rangle <$ | $m_1|\psi_4'\rangle <$ |
|      | $B_{1a-2.3a} <$ | $B_{1b-2.3b} <$ | $B_{1a-2.3a} <$ | $B_{1a-2.3a} <$ |
|      | $R_1L_0 = R_1L_{1a-2} = R_1L_1$ | $k_1|\psi_4'\rangle <$ | $k_1|\psi_4'\rangle <$ | $k_1|\psi_4'\rangle <$ |
|      | $d_1$: | $d_1$: | $d_2$: | $d_1$: |
|      | $R_1E_0 <$ | $R_2E_0 = R_2L_0 <$ | $R_2E_0 = R_2L_0 <$ | $R_3E_0 = R_3L_0 <$ |
|      | $R_2L_0 <$ | $R_1E_{1a-3a} = R_1L_{1a-3a} = m_1|\psi_4'\rangle <$ | $R_1E_{1a-3a} = R_1L_{1a-3a} = m_1|\psi_4'\rangle <$ | $R_1E_{1a-3a} = R_1L_{1a-3a} = m_1|\psi_4'\rangle <$ |
|      | $R_1L_1 <$ | $R_2E_{1a-3a} = R_2L_{1a-3a} = n_1|\psi_4'\rangle <$ | $R_2E_{1a-3a} = R_2L_{1a-3a} = n_1|\psi_4'\rangle <$ | $R_2E_{1a-3a} = R_2L_{1a-3a} = n_1|\psi_4'\rangle <$ |
|      | $R_1E_1$ | $R_3E_1 = R_3L_1 <$ | $R_3E_1 = R_3L_1 <$ | $R_3E_1 = R_3L_1 <$ |
| $R_2$ | / | $E_1 = L_1$ | $E_1 = L_1$ | $E_1 = L_1$ |
| $R_3$ | / | / | $E_{1a-3a} = L_{2a-3a}$ | / |
Table A5. $z_{-\epsilon \alpha} | \tau_1^e$ of the one-point reference movement pattern.

| $t_1$ | $R_1$ | $R_2$ | $R_3$ | $R_4$ |
|-------|-------|-------|-------|-------|
| $R_1$ | $E_{10-2} = E_{10-3a}$ | $d_1$: $R_1E_2 = R_1E_{14-2} = R_1E_1 < B_{10-2,3a} < m_1|\tau'_s < R_1L_4 = R_1L_{10-2} = R_1L_1$ | $d_2$: $R_2E_2 = R_2E_{14-2} = R_2E_1 = R_2E_{10-2} = R_2E_2 < B_{10-2,3a} < l_2|\tau'_s < R_2L_4 = R_2L_{10-3a} = R_2L_1 = R_2L_{10-3a} = R_2L_2$ | $R_3$ | $L_{10-3a} = L_{2a-3a}$ |
| $R_2$ | $/ | E_1 = L_1 | / | / |
| $R_3$ | $/ | / | / | / |

Table A6. $z_{-\epsilon \alpha} | \tau_1^e$ of the one-point reference movement pattern.

| $l_1$ | $R_1$ | $R_2$ | $R_3$ | $R_4$ |
|-------|-------|-------|-------|-------|
| $R_1$ | $E_{10-2} = E_{10-3a}$ | $d_1$: $R_1E_2 = R_1E_{14-2} = R_1E_1 < B_{10-2,3a} < m_1|\tau'_s < R_1L_4 = R_1L_{10-2} = R_1L_1$ | $d_2$: $R_2E_2 = R_2E_{14-2} = R_2E_1 = R_2E_{10-2} = R_2E_2 < B_{10-2,3a} < l_2|\tau'_s < R_2L_4 = R_2L_{10-3a} = R_2L_1 = R_2L_{10-3a} = R_2L_2$ | $R_3$ | $L_{10-3a} = L_{2a-3a}$ |
| $R_2$ | $/ | E_1 = L_1 | / | / |
| $R_3$ | $/ | / | / | / |
### Table A7. Differences between $\pi^{def}$ of the tuple $(V_{24}, V_{25}, V_{26}, V_{27})$ and $\pi^{def}$ of two-point reference movement pattern shown in red.  

| $h_1$ | $R_1$ | $d_1$ | $R_2$ | $d_2$ | $R_3$ | $d_3$ | $R_4$ |
|---|---|---|---|---|---|---|---|
| $R_1$ | $E_{10-2} = E_{10-30}$ | $R_{10-2} = R_{10-30} = R_{1} < $ | $E_{1} = L_1$ | $R_{L_1} = R_{l_{10-30}} = R_{L_1} < $ | | | |
| $R_2$ | / | / | / | / | / | / | |
| $R_3$ | / | / | / | / | / | / | |

- $B_{18-23a} < R_{10a-2} = R_{18-23a} = R_{10a-2} = R_{18-23a} = 26b | t_{142} = 26b | t_{142} = 26b | t_{142} < $ 
- $R_{10a-2} = R_{18a-2} = 25b | t_{142} < $ 
- $R_{10a-2} = R_{18a-2} = 25b | t_{142} < $ 
- $B_{18-23a} < R_{10a-2} = R_{18a-2} = 25b | t_{142} < $ 
- $R_{10a-2} = R_{18a-2} = 25b | t_{142} < $ 
- $B_{18-23a} < R_{10a-2} = R_{18a-2} = 25b | t_{142} < $ 
- $24b | t_{142} = 24b | t_{142} = 24b | t_{142} < $ 
- $R_{10a-2} = R_{18a-2} = 25b | t_{142} < $
Table A3. Differences between $\varphi_{V_{0}}(\varphi_{A_{0}})$ of the tuple $(V_{2}, V_{3}, V_{4})$ and $\varphi_{V_{0}}(\varphi_{A_{0}})$ of the two point reference movement pattern shown in red.

| $h$ | $R_{1}$ | $R_{2}$ |
|-----|---------|---------|
| $E_{L_{0}}$ | $E_{F_{0}}$ | $E_{F_{0}}$ |
| $E_{L_{0}}$ | $E_{F_{0}}$ | $E_{F_{0}}$ |
| $E_{L_{0}}$ | $E_{F_{0}}$ | $E_{F_{0}}$ |
| $E_{L_{0}}$ | $E_{F_{0}}$ | $E_{F_{0}}$ |
| $E_{L_{0}}$ | $E_{F_{0}}$ | $E_{F_{0}}$ |
| $E_{L_{0}}$ | $E_{F_{0}}$ | $E_{F_{0}}$ |
| $E_{L_{0}}$ | $E_{F_{0}}$ | $E_{F_{0}}$ |
| $E_{L_{0}}$ | $E_{F_{0}}$ | $E_{F_{0}}$ |
| $E_{L_{0}}$ | $E_{F_{0}}$ | $E_{F_{0}}$ |

\[ E_{L_{0}} = R_{1} , E_{F_{0}} = R_{2} \]
Table A9. Differences between $\theta^\text{ref}_{1,26}$ of the tuple $(V_2, V_4, V_5, V_6)$ and $\theta^\text{ref}_1$ of the two-point reference movement pattern shown in red. 1.

| $l_1$ | $R_1$ | $E_{l_0-2} = E_{l_0-30}$ | $R_2$ | $E_1 = L_1$ | $R_3$ |
|-------|-------|------------------------|-------|--------------|-------|
| $R_1$ | $E_{l_0-2} = E_{l_0-30}$ | $d_{l_1}$ |
| $R_1$ | $E_{l_0-2} = E_{l_0-30}$ | $d_{l_1}$ |
| $R_1$ | $E_{l_0-2} = E_{l_0-30}$ | $d_{l_1}$ |
| $R_1$ | $E_{l_0-2} = E_{l_0-30}$ | $d_{l_1}$ |
| $R_1$ | $E_{l_0-2} = E_{l_0-30}$ | $d_{l_1}$ |
Table A10. Differences between $\delta^\text{diff}\{t_{122}\}$ of the tuple $(V_{10}, V_{11}, V_{12}, V_{13})$ and $\delta^\text{ref}\{t'_{12}\}$ of the two-point reference movement pattern shown in red.

| $t_0$ | $R_1$ | $R_2$ | $R_1$ | $R_2$ | $R_1$ |
|-------|-------|-------|-------|-------|-------|
| $R_1$ | $E_{10-2} = E_{10-3a}$ | $d_1$: | $R_1E_3 = R_1E_{10-3a} = R_1E_1 = R_1E_{10-2b} = R_1E_2$ | $12_0|122 <$ | $R_1E_3 = R_1E_{10-3a} = R_1E_1 = R_1E_{10-2b} = R_1E_2$ | $13_0|122 <$ |
| | | $d_1$: | $R_1E_3 = R_1E_{10-3a} = R_1E_1 = R_1E_{10-2b} = R_1E_2$ | $120|122 <$ | $R_1E_3 = R_1E_{10-3a} = R_1E_1 = R_1E_{10-2b} = R_1E_2$ | $121|122 <$ |
| | | | $R_1E_3 = R_1E_{10-3a} = R_1E_1 = R_1E_{10-2b} = R_1E_2$ | $10_0|122 <$ | $R_1E_3 = R_1E_{10-3a} = R_1E_1 = R_1E_{10-2b} = R_1E_2$ | $10_1|122 <$ |
| | | | $R_1E_3 = R_1E_{10-3a} = R_1E_1 = R_1E_{10-2b} = R_1E_2$ | $R_1L_0 = R_1L_{10-2} = R_1L_1$ | $R_1L_0 = R_1L_{10-2} = R_1L_1$ |
| $R_2$ | / | $E_1 = l_1$ | / | / | / |
Table A11. Differences between $\tilde{\mu}_a |f_{468}$ of the tuple $(V_{31}, V_{32}, V_{33}, V_{34})$ and $\tilde{\mu}_a |t'_0$ of the four-point reference movement pattern shown in red. I.

| $r_1$ | $r_2$ | $r_3$ |
|-------|-------|-------|
| $E_{1a} = E_{1b} = E_{2a}$ | $E_{1c} = E_{1d}$ | $E_{1e} = E_{1f}$ |
| $R_1$ | $R_2$ | $R_3$ |
| $d_1$ | $d_2$ | $d_3$ |

Figure A1. Match-I $(V_{2}, V_{4}, V_{5}, V_{6})$ and Match-II $(V_{10}, V_{11}, V_{12}, V_{13})$ for the one-point reference movement pattern.
Figure A2. Match-III ($V_{24}, V_{25}, V_{26}, V_{27}$) and Match-IV ($V_{31}, V_{32}, V_{33}, V_{34}$) for the one-point reference movement pattern.

Figure A3. Match-V ($V_{36}, V_{37}, V_{38}, V_{39}$) and Match-VI ($V_{43}, V_{44}, V_{45}, V_{46}$) for the one-point reference movement pattern.
Figure A4. Two-point (front-end f and back-end b) reference movement pattern.

Figure A5. Four-point (front-left fl, front-right fr, back-left bl, and the back-right br) reference movement pattern.

Figure A6. Reference movement pattern with two points: centroid c and safe-following distance s for speed analysis.