The response of primordial abundances to a general modification of $G_N$ and/or of the early universe expansion rate

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Abstract

We discuss the effects of a possible time variation of the Newton constant $G_N$ on light elements production in Big Bang Nucleosynthesis (BBN). We provide analytical estimates for the dependence of primordial abundances on the value of the Newton constant during BBN. The accuracy of these estimates is then tested by numerical methods. Moreover, we determine numerically the response of each element to an arbitrary time-dependent modification of the early universe expansion rate. Finally, we determine the bounds on possible variations of $G_N$ which can be obtained from the comparison of theoretical predictions and observational data.

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1 Introduction

The idea that fundamental constants may vary with time dates back to Dirac [1]. Even if, at present, there is no robust experimental evidence in favor of this possibility, this idea continues to be widely discussed, since many extensions of the standard theories (e.g. superstring theories, scalar-tensor theories of gravitation, etc.) predict a variation of "fundamental constants" with time.

On a pure theoretical level, the space-time dependence of the fundamental parameters\(^1\) is forbidden by the Strong Equivalence Principle (SEP), and in particular by the statement of Local Position Invariance (LPI) (see e.g. [2]). On the other hand, the weakest form of the Equivalence Principle, the so-called Einstein Equivalence Principle (EEP) – which is the essence of the geometrical theory of gravitation – refers (in its LPI) only to non-gravitational physics and, thus, allows the Newton constant \(G_N\) to be time-dependent. This is what happens, e.g., in the Brans-Dicke, or more generally in the (multi-) scalar-tensor theories of gravity.

The above argument suggests that \(G_N\) has a special role in the subject of time variation of the fundamental parameters. A dependence of \(G_N\) on time is a symptom of the violation of the SEP, but not necessarily of the EEP, whereas the non-constancy of the other constants, like the electroweak or strong coupling constants, necessarily represents a violation of the equivalence principle in both its forms.

In this paper, we discuss the effects of time variations of \(G_N\) on the light element production in Big Bang Nucleosynthesis (BBN), completing and extending the results of previous analysis on the subject (see e.g. [3, 4, 5]). BBN is evidently a good probe of a possible time variation of \(G_N\), since it is the earliest event in the history of the universe for which we can obtain solid and well-testable predictions. Even a weak (or very peculiar) time dependence, which gives no observable effects in high accuracy experiments performed at the present epoch, could give sizable effects when translated over cosmological time scales. BBN, however, is a complex phenomenon, since each element responds in its own way to a modification of \(G_N\). We devote particular attention to this point, introducing suitable response functions which relate the abundance of each element to an arbitrary time-dependent modification of the early universe expansion rate.

The plan of the paper is the following: In the next section, we discuss the role of \(G_N\) in BBN and we derive, analytically, the dependence of the primordial abundances on the value of the Newton constant at the key epochs. In section 3, we calculate numerically the response functions, emphasizing that different elements are sensitive to the value of the Newton constant at slightly different times. In section 4, we discuss the bounds on \(G_N\) variations that can be obtained from the comparison of theoretical predictions with the observational data for light elements primordial abundances. We summarize our results in section 5.

2 The role of \(G_N\) in BBN

The production of light elements (namely \(^2\text{H}, ^3\text{He}, ^4\text{He}\) and \(^7\text{Li}\)) in BBN is the result of the efficiency of weak reactions (\(p + e \leftrightarrow n + \nu_e\) and related processes) and nuclear reactions (which build light nuclei from neutrons and protons) in the expanding universe. The value of the gravitational constant determines the expansion rate of the universe and thus, in

\(^1\)In this context, the term "fundamental parameters" is more appropriate than "fundamental constants".
turn, the relevant time scales for the above processes. As a consequence, if we assume that the gravitational constant at time of BBN is different from its present value, this translates into a variation of light element abundances with respect to standard BBN predictions.

The above argument clarifies in simple terms the role of $G_N$ in BBN. In order to be more quantitative, one needs, as a first step, to identify the key epochs for light element production with respect to possible $G_N$ variations. As we shall see, the relevant periods are those during which the weak reaction rates and/or the nuclear reaction rates are not vanishing or exceeding the universe expansion rate. Essentially they are the weak-interaction “freeze-out” epoch (which occurs at the temperature $T_f \sim 0.8$ MeV) and the “deuterium bottleneck” epoch (which corresponds to $T_d \sim 0.08$ MeV) \[9,10,11\]. In the following, we use $G_{N,f}$ and $G_{N,d}$ to indicate the value of the Newton constant during these periods, while we use $G_{N,0}$ to indicate the present value.

In order to estimate the dependence of the various elemental abundances on $G_{N,f}$ and $G_{N,d}$, it is necessary to quickly review the basic physical mechanisms responsible for light element production. When $T \gg T_f$, the rate of weak processes which interchange neutrons and protons, $\Gamma_W \sim G_N^2 T^5$, is large with respect to the expansion rate of the universe: 

$$H = 1.66 \sqrt{g_* G_N^2 T^2},$$  

where $g_*$ counts the total number of relativistic degrees of freedom. As a consequence, neutrons and protons are in chemical equilibrium and the neutron abundance $X_n = n_n/n_B$, defined as the ratio of neutron to baryon densities, is simply given by $X_n(T) = [1 + \exp(\Delta m/T)]^{-1}$, where $\Delta m \simeq 1.29$ MeV is the neutron-proton mass difference.

For $T \leq T_f$ the weak reaction rate drops below the Hubble expansion rate, the neutron abundance freezes out at the equilibrium value $X_n(T_f)$ and it then evolves only due to the neutron decay: $X_n(t) \simeq X_n(T_f) \exp(-t/\tau)$, where $\tau = 885.7$ s is the neutron lifetime. The “freeze-out” temperature is basically determined by the condition $\Gamma_W(T_f)/H(T_f) \simeq 1$ and, clearly, depends on the value of the gravitational constant. One obtains: 

$$T_f = 0.784 \left( \frac{G_{N,f}}{G_{N,0}} \right)^{1/6} \text{MeV},$$  

where $G_{N,f}$ is the value of the Newton constant during the freeze-out, i.e. when the temperature of the universe is $0.2 \leq T \leq 2$ MeV (see next section). The larger is $G_{N,f}$, the earlier is the freeze-out of the neutron abundance, at an higher value, and the larger is the $^4$He abundance produced in BBN.

Light element production in BBN occurs through a sequence of two body reactions, such as $p(n, \gamma)^2$H, $^2$H$(d, n)^3$He, $^2$H$(d, p)^3$H, $^3$He$(p, \gamma)^4$He, etc. Deuterium has to be produced in appreciable quantity before the other reactions can proceed at all. Nucleosynthesis effectively begins when the rate of deuterium processing through $^2$H$(d, n)^3$He and $^2$H$(d, p)^3$H reactions becomes comparable with the expansion rate of the universe. By imposing this condition one finds the “deuterium bottleneck” temperature which, in standard BBN, is given by: $T_d = 0.08(1 + 0.16 \log(\eta/10^{-10}))$ MeV \[7\].

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\[2\] Here and in the following, we use a natural system of units in which $\hbar = c = k_B = 1$.

\[3\] Strictly speaking, equation 11 is derived in the context of General Relativity in which $G_N$ is constant. In any extension of the standard theory, one has additional terms related to time derivatives of $G_N$. In this paper, we make the usual assumption that $G_N$ is slowly varying (with respect to the early universe expansion rate) which implies that these extra terms are negligible.

\[4\] See \[8,10\] for the precise numerical calculation of the total weak rate $\Gamma_W$. 

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2
After nucleosynthesis has started, light nuclei ($^2$H, $^3$He, $^4$He and $^7$Li) are quickly produced. The $^4$He abundance is basically determined by the total number of neutrons that survive till the onset of nucleosynthesis, since nearly all available neutrons are finally captured in $^4$He nuclei. The synthesized elemental abundances of $^2$H, $^3$He and $^7$Li are, instead, the result of the complex interplay of the various nuclear reactions efficient during and after the d-bottleneck. The value of the Newton constant during this period, $G_{N,d}$, clearly plays a relevant role both for $^4$He and other elements production.

The situation with $^4$He is particularly simple. The primordial $^4$He mass fraction is approximatively given by

$$Y_4 \sim 2X_n(t_d) \simeq 2X_n(T_f) \exp(-t_d/\tau),$$

where $t_d$ is the age of the universe at the d-bottleneck. Neglecting the weak dependence of the temperature $T_d$ on $G_{N,d}$, one has:

$$t_d \sim 206 \left( \frac{G_{N,d}}{G_{N,0}} \right)^{-1/2} (1 - 0.32 \log(\eta/10^{-10})) \text{ sec},$$

where $G_{N,d}$ is the value of the Newton constant when the temperature of the universe is $0.02 \leq T \leq 0.2$ Mev (see next section), i.e. when $^4$He and the other elements are effectively synthesized. By using this formula and considering eq. (2), one is able to estimate the dependence of $Y_4$ from $G_{N,f}$, $G_{N,d}$ and $\eta$. One obtains:

$$\delta Y_4 \equiv \frac{\Delta Y_4}{Y_4} = 0.23 \delta G_{N,f} + 0.09 \delta G_{N,d} + 0.07 \log(\eta/\eta_{CMB})$$

where $\delta G_{N,i}$ (with $i = f, d$) represents the fractional variation of the Newton constant at a given epoch with respect to its present value:

$$\delta G_{N,i} = \frac{G_{N,i} - G_{N,0}}{G_{N,0}},$$

and $\log(\eta/\eta_{CMB})$ is the logarithmic variation of the baryon to photon ratio with respect to the value $\eta_{CMB} = 6.14 \cdot 10^{-10}$ presently favored by cosmic microwave background (CMB) and deuterium data.

The situation with $^2$H, $^3$He and $^7$Li is slightly more complicated. In principle, one has to integrate the rate equations, which can be written formally as:

$$\frac{dY_i}{dt} \propto \eta n_\gamma \sum_{+, -} Y \times Y \times \langle \sigma v \rangle_T,$$

where $Y_i$ indicate the abundance of a given element, the sum runs over the relevant source (+) and sink (−) terms, and $\langle \sigma v \rangle_T$ are the thermally-averaged reaction rates. In order to estimate the role of $G_N$, one can simply note that, since the temperature of the universe evolves as $dT/dt \propto -T^3\sqrt{G_N}$, the above equation can be rewritten as:

$$\frac{dY_i}{dT} \propto -\frac{\eta}{G_{N,d}^{1/2} T^3} \sum_{+, -} Y \times Y \times \langle \sigma v \rangle_T,$$

In writing eqs. (6) and (7), we implicitly assumed that only two-baryon reactions are relevant (neglecting reactions such as $p + e \rightarrow n + \nu$ and related processes or $d + \gamma \rightarrow n + p$). This is reasonable after deuterium bottleneck and provides a good framework to discuss the abundances of $^2$H, $^3$He and $^7$Li, whose abundance is essentially established after the d-bottleneck. On the contrary, this is clearly not adequate to discuss the synthesis of $^4$He.
which shows that $^2\text{H}$, $^3\text{He}$ and $^7\text{Li}$ depend on $\eta$ and $G_N$ essentially through the combination $\eta/G_N^{1/2}$. This suggests that the synthesized elemental abundances of $\text{D}/\text{H}$, $^3\text{He}/\text{H}$, and $^7\text{Li}/\text{H}$ (indicated, in the following, with $Y_2$, $Y_3$, and $Y_7$ respectively) for an arbitrary value of $G_{N,d}$ can be related to the standard case ($G_{N,d} = G_{N,0}$) through an appropriate rescaling in $\eta$:

$$Y_i(\eta, G_{N,d}) = Y_i \left( \eta(G_{N,0}/G_{N,d})^{1/2}, G_{N,0} \right). \quad (8)$$

The previous equation, linearized, implies that:

$$\delta Y_i = \frac{\Delta Y_i}{Y_i} = \gamma_i(\eta_0, G_{N,0}) \left( \Delta \log(\eta) - 0.22 \delta G_{N,d} \right) \quad (9)$$

where $\eta_0$ is an arbitrary pivot point, $\gamma_i = (1/Y_i) \partial Y_i/\partial \log \eta$ and we have assumed that $\Delta \log(\eta) = \log(\eta/\eta_0)$ and $\delta G_{N,d}$ are small. We remark that, according to eqs. (8), fixed values of the abundances correspond in the plane $(\log \eta, \delta G_{N,d})$ to straight parallel lines (with slope $\sim 1/0.22$).

The above argument is accurate enough to describe $^2\text{H}$ and $^3\text{He}$ abundances. However, it can be slightly improved in order to predict correctly the behavior of $^7\text{Li}$. In order to produce $^7\text{Li}$, one has to use $^4\text{He}$ as a target, whose abundance is strongly dependent on the value of the Newton constant. This clearly introduce an extra dependence on $G_{N,f}$ and $G_{N,d}$. One expects an extra factor in eq. (5) proportional to $Y_4$ which, linearized, gives:

$$\delta Y_7 = (0.23 \delta G_{N,f} + 0.09 \delta G_{N,d}) + \gamma_7(\eta_0) \left( \Delta \log(\eta) - 0.22 \delta G_{N,d} \right) \quad (10)$$

where the first terms in the r.h.s are obtained from eq. (6) and we neglected the weak dependence of $Y_4$ on the baryon to photon ratio $\eta$.

The analytical results discussed above allow us to understand the relevance of the various physical mechanisms in light elements production and to obtain simple quantitative relations (eq. (11) for $^4\text{He}$, eq. (12) for $^2\text{H}$ and $^3\text{He}$ and eq. (13) for $^7\text{Li}$) between the various elemental abundances and the parameters $\eta$, $\delta G_{N,f}$ and $\delta G_{N,d}$. In order to check their validity, we compare them with the results of numerical calculations. As a first step, we consider the case of a constant variation of $G_N$ during the entire period relevant for BBN (i.e. $\delta G_{N,f} = \delta G_{N,d} \equiv \delta G_N$). In this assumption, a linear fit to the numerical result gives for the $^4\text{He}$ abundance:

$$\delta Y_4 \simeq 0.35 \delta G_N + 0.09 \log(\eta/\eta_{\text{CMB}}) \quad (11)$$

with an accuracy at the level of 2% or better in the range $\delta G_N = 0.75 - 1.25$ and $\eta = 2 \times 10^{-10} - 10^{-9}$, in reasonable agreement with estimate (9). For $^2\text{H}$ and $^3\text{He}$, expanding around $\eta_0 = \eta_{\text{CMB}}$, one obtains:

$$\delta Y_2 \simeq \gamma_2(\eta_{\text{CMB}})(\log(\eta/\eta_{\text{CMB}}) - 0.25 \delta G_N) \quad (12)$$

$$\delta Y_3 \simeq \gamma_3(\eta_{\text{CMB}})(\log(\eta/\eta_{\text{CMB}}) - 0.24 \delta G_N) \quad (13)$$

in good agreement with predictions (9), with $\gamma_2(\eta_{\text{CMB}}) = -3.7$ and $\gamma_3(\eta_{\text{CMB}}) = -1.3$. In addition, the $^7\text{Li}$ behavior can be described by:

$$\delta Y_7 \simeq 0.32 \delta G_N + \gamma_7(\eta_{\text{CMB}})(\log(\eta/\eta_{\text{CMB}}) - 0.22 \delta G_N) \quad (14)$$

as predicted by eq. (10), with $\gamma_7(\eta_{\text{CMB}}) = 4.8$.\footnote{We remark that, for $^2\text{H}$, $^3\text{He}$ and $^7\text{Li}$, expanding around an arbitrary value $\eta_0$ in the range $\eta_0 = 3 \times 10^{-10} - 10^{-9}$ (and using the proper values $\gamma_i(\eta_0)$) one obtains the same results as those described by relations (12), (13) and (14), with essentially the same numerical coefficients.}
3 Response functions

As underlined in the previous section, the production of each element responds in its own way to a variation $\delta G_N$ of the Newton constant. For example, a change of $G_N$ at the time of weak interaction freeze-out would have important consequences on the observed helium abundance, giving instead negligible corrections to that of deuterium. So far we have implicitly assumed that $G_N(t)$ stays constant (at a value not necessarily equal to the present one) during BBN, or that it takes two different values at the two key epochs, marked as $T_f$ and $T_d$. A more general analysis, that can account for a time dependence of $G_N(t)$ along the all BBN period, requires the introduction of suitable functions, which describe the response of each elemental abundance to an arbitrary time-dependent modification of the early universe expansion rate.

We have determined numerically the response functions: $\rho_i(\eta, T)$, which are defined by:

$$\delta Y_i(\eta, \delta H(T)) = 2 \int \rho_i(\eta, T) \delta H(T) \frac{dT}{T}, \quad (15)$$

where $i = 2, 3, 4$ and $7$ and $\delta H(T)$ is the fractional variation (assumed to be small) of the expansion rate of the universe at the temperature $T$ with respect to its standard value. We remark that, in the assumption that $G_N(t)$ is slowly varying, one has that $\delta G_N = 2 \delta H(T)$ from eq.(1). The above equation can then be simply rewritten as

$$\delta Y_i(\eta, \delta G_N(T)) = \int \rho_i(\eta, T) \delta G_N(T) \frac{dT}{T}, \quad (16)$$

which shows that $\rho_i(\eta, T)$ is basically the functional derivative of $\ln Y_i(\eta, \delta G_N(T))$ with respect to $\delta G_N(T)$.

The response functions allows us to identify unambiguously the key epochs for the production of the various elements and to emphasize that different elements are sensitive to the value of the Newton constant at slightly different times. Our results, calculated for $\eta = \eta_{\text{CMB}}$, are shown in Fig. 1. As expected, the functions $\rho_i(\eta, T)$ have two peaks corresponding to the weak interaction freeze-out and to the epoch, just after the deuterium bottleneck, during which the various elements are effectively synthesized. The width of the two peaks reflects the fact that the above processes are not instantaneous. One can essentially identify the range $\Delta T_f = (0.2 - 2)\text{MeV}$ with the weak interaction freeze-out epoch and $\Delta T_d = (0.02 - 0.2)\text{MeV}$ with the various elements synthesis period. The behavior of the functions $\rho_i(\eta, T)$ also allows to give a more quantitative meaning to the parameters $G_{N,f}$ and $G_{N,d}$, which have to be intended, evidently, as the average values of the Newton constant during the periods $\Delta T_f$ and $\Delta T_d$, respectively.

The total area under the curves in fig. 1 (integrated in $\ln T$) gives the numerical coefficient $\delta Y_i/\delta G_N$, which are obtained in eqs.(11-14) in the assumption of constant $G_N$ variations. It is interesting, however, to consider separately the early time and the late time behavior of the functions $\rho_i(\eta, T)$ in order to have a feeling of the relative importance of the different epochs in the various elements production. In this spirit, we have calculated the numerical values:

$$\alpha_i = \int_{0.02\text{MeV}}^{2\text{MeV}} \rho_i(\eta, T) \frac{dT}{T}, \quad \beta_i = \int_{0.02\text{MeV}}^{0.2\text{MeV}} \rho_i(\eta, T) \frac{dT}{T}, \quad (17)$$
The functions $\rho_i(\eta, T)$ describe the effect of an arbitrary time-dependent modification of the early universe expansion rate on the various elemental abundances (see text for details).

for $i = 2, 3, 4$ and 7, which have to be compared with the coefficients $\delta Y_i/\delta G_{N,f}$ and $\delta Y_i/\delta G_{N,d}$ estimated in eqs.(10). For $\eta = \eta_{\text{CMB}}$, one obtains $\alpha_2 = 0.12$ and $\beta_2 = 0.80$ for deuterium, $\alpha_3 = 0.04$ and $\beta_3 = 0.29$ for helium-3, $\alpha_4 = 0.22$ and $\beta_4 = 0.12$ for helium-4 and $\alpha_7 = 0.14$ and $\beta_7 = -0.83$ for lithium-7, in good agreement with our predicted values.

One sees that $^2\text{H}$, $^3\text{He}$ and $^7\text{Li}$ abundances are essentially determined by the expansion rate of the universe during and after the d-bottleneck. As a consequence, the bounds obtained from these elements are basically bounds on $\delta G_{N,d}$. Helium-4, instead, is mainly sensitive to the value of the Newton constant during the weak interaction freeze-out. However, the “response” function $\varrho_4(T, \eta)$ is rather broad, showing that $^4\text{He}$ is sensitive to a rather long period, $0.05 \leq T \leq 2$ MeV, of the early universe evolution. In terms, of $G_{N,f}$ and $G_{N,d}$, the bounds obtained from $^4\text{He}$ observational data can be considered limits on the combination $0.65\delta G_{N,f} + 0.35\delta G_{N,d}$.

Finally, we remark that the response functions $\varrho_i(\eta, T)$ have a quite general meaning and may be easily applied to discuss any non-standard schemes (e.g. new light - stable or decaying - particles, non vanishing muon or tau neutrino chemical potentials, etc.) whose main effect is to modify the early universe expansion rate.

4 The BBN bound on $\delta G_N$

By comparing theoretical predictions with observational data for light element primordial abundances one is able, in principle, to obtain bounds for $\delta G_{N,f}$ and $\delta G_{N,d}$. However, comparison of theoretical results with observational data is not straightforward because the
data are subject to poorly known systematic errors and evolutionary effects (see [14] for a review). The present situation can be summarized as it follows:

i) Recent determinations of deuterium in quasar absorption line systems (QAS) report values of D/H in the range D/H \sim 2 - 4 \times 10^{-5}. However, the dispersion among the different determinations is not consistent with errors in the single measurements. We will use, in the following, the value D/H = 2.78^{+0.44}_{-0.38} \times 10^{-5} given in [13], which is the weighted mean of most recent deuterium determinations (see [13] for detailed discussion and references).

ii) Independent determinations of $^4\text{He}$ primordial abundance have statistical errors at the level of 1 - 2% but differ among each others by about \sim 5%. In particular, by using independent data sets, Olive et al. [15, 16] have obtained $Y_4 = 0.234 \pm 0.003$, while Izotov et al. [17, 18] have found $Y_4 = 0.244 \pm 0.002$. We will use the “average” value $Y_4 = 0.238$, quoted in [10], with the error estimate $\Delta Y_4 = 0.005$, which is obtained from the dispersion of the various $Y_4$ determinations.

iii) The $^7\text{Li}$ and $^3\text{He}$ primordial abundances are not known, at present, with a level of uncertainty comparable to the other elements. The reported values for the primordial abundances (see [20] for $^7\text{Li}$ and [21] for $^3\text{He}$) are, evidently, important as a confirmation of the BBN paradigm, but are presently not very effective in constraining possible non-standard BBN scenarios. For this reason we will not include these elements in our analysis.

In Fig. 2 we discuss the bounds on $\delta G_N$ and $\eta$ that can be obtained by comparing theoretical predictions with observational data for primordial $^2\text{H}$ and $^4\text{He}$. Theoretical calculations are made in the assumption of a constant variation, $\delta G_N$, of the Newton constant in the period relevant for BBN. This simple assumption is motivated by the fact that the present observational situation does not allow to determine the evolution of $G_N(t)$ during BBN. However, when considering a theoretical framework in which a specific time-dependence for $G_N(t)$ is predicted, one has to keep in mind that $^2\text{H}$ and $^4\text{He}$ respond differently to a non-constant modification of the early universe expansion rate.

The results shown in Fig. 2 are obtained by defining a $\chi^2(\eta, \delta G_N)$ as prescribed in [11] which takes into account both observational and theoretical errors in the various elemental abundances. The best fit points in the plane ($\log \eta, \delta G_N$) are obtained by minimizing the $\chi^2$. The three confidence level (C.L.) curves (solid, dashed, and dotted) are defined by $\chi^2 - \chi^2_{\text{min}} = 2.3, 6.2, 11.8$, corresponding to 68.3%, 95.4% and 99.7% C.L. for two degrees of freedom ($\eta$ and $\delta G_N$), i.e. to the probability intervals designated as 1, 2, and 3 standard deviation limits. The upper panels are obtained by considering BBN alone, while the lower panels show the bounds which can be obtained by combining BBN data with the measurement of the baryon to photon ratio from CMB and LSS. This is done by adding the contribution:

$$\chi^2_{\text{CMB}}(\eta) = \frac{(\eta - \eta_{\text{CMB}})^2}{\sigma^2_{\text{CMB}}}$$

(19)

to the BBN chi-square, where $\eta_{\text{CMB}} = 6.14 \cdot 10^{-10}$ is the baryon-to-photo ratio determined by CMB and LSS data and $\sigma_{\text{CMB}} = 0.25 \cdot 10^{-10}$ is the error in this determination [22]. In this case, the C.L. curves are determined by the condition $\chi^2 - \chi^2_{\text{min}} = 1, 4, 9$, corresponding to 1, 2, and 3 standard deviations for one degree of freedom ($\delta G_N$), since $\eta$ is considered as a measured quantity $^7$.

The results displayed in Fig. 2 allow to obtain the following conclusions:

$^7$Technically speaking, one should define $\chi^2_{\text{tot}}(\delta G_N) = \min[\chi^2(\eta, \delta G_N) + \chi^2_{\text{CMB}}(\eta)]$ which depends only on the parameter $\delta G_N$. For graphical reasons and to facilitate the comparison with the bounds obtained only from BBN, we presented the results in the two dimensional plane $(\eta, \delta G_N)$.
i) The $^2\text{H}$ and $^4\text{He}$ data (panels a and b) select bands in the plane $(\log(\eta), \delta G_N)$, which can be easily interpreted in terms of the analytical (eqs. (31) and (31)) and numerical (eqs. (11) and (12)) relations discussed in the previous section. In order to have a bound on $\delta G_N$ one has to combine the $^2\text{H}$ and $^4\text{He}$ observational results (panel c) (in the assumption that $\delta G_N(t)$ stays nearly constant during BBN) and/or to consider the independent information on $\eta$ given by CMB+LSS (lower panels).

ii) If we combine $^2\text{H}$ and CMB+LSS data (panel d), we obtain $\delta G_N = 0.09^{+0.22}_{-0.19}$, in agreement with \cite{3}. The quoted bound is consistent with the standard assumption that $G_N$ has not varied during the evolution of the universe and is, at present stage, the most robust piece of information on the value of the gravitational constant in the early universe. We recall that this bound essentially applies to the value of the Newton constant during and after the d-bottleneck (i.e. when $0.02\text{MeV} \leq T \leq 0.2\text{MeV}$) or, equivalently (in the previous sections notations), to the parameter $\delta G_{N,d}$.

iii) If we combine the $^4\text{He}$ and CMB+LSS data (panel e), we obtain $\delta G_N = -0.11 \pm 0.05$, which shows that $^4\text{He}$ observational data favor a reduction of $G_N$ in the early universe, even if errors are large enough to allow for the standard value $\delta G_N = 0$. This result is emphasized (reduced) if we consider the “low” (“high”) helium value $Y_4 = 0.234 \pm 0.003$ given in \cite{15, 16} ($Y_4 = 0.244 \pm 0.002$ given in \cite{17, 18}), which results in $\delta G_N = -0.15 \pm 0.03$ ($\delta G_N = -0.05 \pm 0.02$) and is also obtained in panels c) and f) where $^2\text{H}$ observational data are also included. If confirmed (and strengthened) by future data, this could be an important indication of non-standard effects in BBN. We remark, however, that the present situation is quite delicate. The uncertainty in the quoted bound, $\delta G_N = -0.11 \pm 0.05$, is completely dominated by (not well known) systematic errors in $^4\text{He}$ measurements. It is thus advisable to wait for a better comprehension of these errors, before a final result can be obtained.

iv) In panel f we show the bound, $\delta G_N = -0.09 \pm 0.05$, that is obtained by considering $^2\text{H}$, $^4\text{He}$ and CMB+LSS data. We see that the fit is dominated by helium-4 and CMB+LSS observational data, and that the information provided by deuterium only marginally reduces the error bar with respect to the previous case. We remark that the above result is obtained in the assumption that $\delta G_N$ is nearly constant during BBN. In principle, one could fit the data by considering $\delta G_{N,f}$ and $\delta G_{N,d}$ as independent parameters. In a future perspective, this could be interesting as a possible test for variations of $G_N$ during BBN (or equivalent schemes). In the present experimental situation this appears too ambitious. The value of $\chi^2_{\text{min}} = 1.0$ for the best-fit point indicate, in fact, that the quality of the fit is good and that there is no real evidence, at present, in favor of theoretical schemes which predict non-constant modification of the early universe expansion rate.

5 Summary and conclusions

We summarize the main points of this letter and provide some perspective:

i) We have discussed the role of $G_N$ in BBN and we have derived analytically the dependence of light element primordial abundances on the values $(G_{N,f}$ and $G_{N,d})$ of the Newton constant at the key epochs: the weak interaction freeze-out epoch and the epoch, just after the deuterium bottleneck, during which the elements are effectively synthesized.

ii) The production of each element responds in its own way to a variation $\delta G_N$ of the Newton constant. A general study, that can account for a time dependence of $G_N(T)$ during the BBN period, requires the introduction of suitable functions which describe the response of
Figure 2: The bounds on $\delta G_N$ which can be obtained from $^2$H and $^4$He observational data. The upper panels are obtained considering BBN alone. The lower panels are obtained by combining BBN data with the measurement of the baryon to photon ratio from CMB and LSS (see text for details).

Each element to an arbitrary time-dependent modification of the early universe expansion rate. We have numerically calculated these response functions, obtaining a good agreement with the analytical estimates.

iii) We have emphasized that different elements are sensitive to the value of the Newton constant at slightly different times. The $^2$H, $^3$He and $^7$Li abundances are essentially determined by the expansion rate of the universe close to the d-bottleneck. Helium-4 is, instead, mainly sensitive to the value of the Newton constant during the weak interaction freeze-out. In a future perspective, this could be interesting as a possible test for variations of $G_N$ during BBN (or equivalent schemes).

iii) We have discussed the observational bounds on the possible variations of the gravitational constant in the early universe. Our best limit, $\delta G_N = 0.09^{+0.22}_{-0.19}$, is obtained by combining $^2$H observational results with the measurements of the baryon to photon ratio obtained from CMB and LSS data. This limit refers to the value of $G_N$ when the temperature of the universe is $0.02 \leq T \leq 0.2$ MeV (i.e. during and immediately after the d-bottleneck epoch) and is consistent with the standard assumption that $G_N$ has not varied during the evolution of the universe.

iv) To conclude, we remark that the results obtained for $\delta G_N$ may be easily applied to a more general context in which other constants are allowed to vary and/or new light particles are included. The limits on $\delta G_{N,f}$ and $\delta G_{N,d}$ are, indeed, essentially obtained by comparing the expansion rate of the Universe with the weak reaction rate (in the case of $\delta G_{N,f}$) or with the light element nuclear reaction rates (in the case of $\delta G_{N,d}$). One immediately understands, then, that the bound on $G_{N,f}$ is basically a bound on $g_*(T_f) G_{N,f}/G_F^4$, where $G_F$ is the Fermi constant and $g_*(T_f)$ is the number of relativistic degrees of freedom at the weak interaction freeze-out, while the bound on $G_{N,d}$ is essentially a bound on $g_*(T_d) G_{N,d}$, where $g_*(T_d)$ is the number of relativistic degrees of freedom at the d-bottleneck.
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