Coexistence of p-wave Cooper pairing and ferromagnetism

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A two-band model for coexistence of p-wave superconductivity with localized ferromagnetism is studied using the equation of motion approach. It shows that ferromagnetic and superconducting states enhance each other but in a different way from that of the one-band model. The Curie temperature is not only determined by the exchange interactions between localized spins, but also can be increased with the coupling between electrons and spins, and with the p-wave Cooper-pairing interaction. These results are complementary to those of the one-band model, which suggest that the Curie temperature is unlikely to ever be below the superconducting transition temperature.

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I. INTRODUCTION

Motivated by the recent discoveries of the ferromagnetic superconductors, e.g., UGe$_2$ and URhGe$_2$, much attention has been drawn to understanding the underlying physics of the coexistence of superconductivity and ferromagnetism. Early investigations supposed that electrons form conventional s-wave Cooper pairs. At present, the scenario of spin-triplet p-wave pairing is generally accepted.

Nevidomskyy proposed a microscopic model of the coexistence of a p-wave spin-triplet superconductivity with weakly itinerant ferromagnetism. Supposing that the coexisting state is a uniform Meissner state, he explained the enhancement of superconductivity by the established ferromagnetism. On the other hand, Jian et al. studied the feedback effect of superconductivity upon ferromagnetism. Due to the interplay between the ferromagnetic order and p-wave Cooper pairing, it was suggested that the Curie temperature is unlikely to ever be below the superconducting transition temperature once the ferromagnetism is established. These results are to some extent consistent with the observed phase diagram of UGe$_2$ and with the theoretical discussion of Walker and Samokhin.

It is now well-believed that the electrons involved in both the ferromagnetic (FM) and superconducting (SC) orders are within the same itinerant band. Nevertheless, it helps for arriving at a complete understanding of ferromagnetic superconductivity to investigate the coexistence of superconductivity and localized magnetic order. Suhl proposed a mechanism of simultaneous appearance of ferromagnetism and superconductivity based on interactions between electrons mediated by localized spins. More recently, Singh discussed a model consisting of a pairing interaction and a term describing the scattering of Cooper pairs by localized electrons.

Here we investigate the interplay between FM and SC orders based on a two-band model. A band of itinerant electrons which can exhibit the A-phase p-wave Cooper pairing is coupled to a lattice of localized spins with FM couplings. The model is treated using equations of motion truncated at the lowest nontrivial order. It is shown that ferromagnetism and superconductivity affect on each other in a different way from that of the one-band model.

II. THE MODEL

We use the Heisenberg model on a simple cubic lattice to describe localized spins,

$$ H_S = -J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j. $$

(1)

The superconducting electrons are described by the BCS Hamiltonian,

$$ H_e = \sum_{k\sigma}(\varepsilon_k - \mu)a_{k\sigma}^\dagger a_{k\sigma} + \frac{1}{2\nu} \sum_{kk'\sigma\sigma'} V_{kk'} a_{k\sigma}^\dagger a_{k'\sigma}^\dagger a_{-k'\sigma'} a_{-k\sigma}. $$

(2)

There exists an exchange interaction between localized spins and itinerant electrons,

$$ H_I = -g \sum_j \{ S_j^z (n_{j\uparrow} - n_{j\downarrow}) + S_j^+ a_j^\dagger a_{j\uparrow} + S_j^- a_j^\dagger a_{j\downarrow} \}. $$

(3)

Here $S_j$ represents the spin operator at site $i$. $J$ is the exchange integral between localized spins, and $J > 0$ for the FM state. $a_{k\sigma}^\dagger(a_{k\sigma})$ is the creation (annihilation) operator of electrons. We discuss the case of ferromagnetic coupling between electrons and spins, so the coupling strength $g$ is positive. The pairing potential is assumed to have the p-wave type, $V_{kk'} = -V \mathbf{k} \cdot \mathbf{k'}$, and here we choose the SC order parameters to have the A-phase symmetry, $\Delta_{\pm j}(k) = (\hat{\mathbf{k}}_2 + i\hat{k}_3)\Delta_{\pm j}$. The Hamiltonian is dealt with using the Green’s function method within the mean-field theory framework. The Green’s functions are defined as follows,

$$ \langle S_k^+(t); S_k^- \rangle = -i\Theta(t)\langle \{ S_k^+(t), S_k^- \} \rangle, $$

(4)

$$ \langle a_{k\sigma}^\dagger(t); a_{k\sigma} \rangle = -i\Theta(t)\langle [a_{k\sigma}^\dagger(t), a_{k\sigma}] \rangle, $$

(5)

$$ \langle a_{k\sigma}^\dagger(t); a_{-k\sigma}^\dagger \rangle = -i\Theta(t)\langle [a_{k\sigma}^\dagger(t), a_{-k\sigma}^\dagger] \rangle. $$

(6)
Using the equations of motion approach and truncation technique, we get Green’s functions at the lowest nontrivial order and derive the self-consistent equations,

\[
M = \left[ 2 + \frac{1}{2\pi^3} \int_{-\pi}^{\pi} dk_x \int_{-\pi}^{\pi} dk_y \int_{-\pi}^{\pi} dk_z \left( 2\mathcal{M}(3 - \cos k_x - \cos k_y - \cos k_z) + \Delta \right) - 1 \right]^{-1},
\]

(7)

\[
n = \frac{3}{16} \int_{0}^{\infty} d\varepsilon \int_{0}^{\pi} d\theta \sqrt{\varepsilon} \sin \theta \times \left( 2 - \frac{\omega_1 \tanh \left( \frac{\pi}{2T} \right)}{\eta} - \frac{\omega_2 \tanh \left( \frac{\pi}{2T} \right)}{b} \right),
\]

(8)

\[
m = \frac{3}{16} \int_{0}^{\infty} d\varepsilon \int_{0}^{\pi} d\theta \sqrt{\varepsilon} \sin \theta \times \left( \frac{\omega_1 \tanh \left( \frac{\pi}{2T} \right)}{\eta} - \frac{\omega_2 \tanh \left( \frac{\pi}{2T} \right)}{b} \right),
\]

(9)

\[
1 = \frac{3\sqrt{\eta + \Delta_+ + \omega_c}}{32} \int_{\eta - \Delta_+ - \omega_c}^{\eta + \Delta_+ + \omega_c} d\varepsilon \int_{0}^{\pi} d\theta \sqrt{\varepsilon} \sin^3 \theta \left( \frac{T}{2T} \right),
\]

(10)

\[
1 = \frac{3\sqrt{\eta - \Delta_- - \omega_c}}{32} \int_{\eta - \Delta_- - \omega_c}^{\eta + \Delta_- + \omega_c} d\varepsilon \int_{0}^{\pi} d\theta \sqrt{\varepsilon} \sin^3 \theta \left( \frac{T}{2T} \right),
\]

(11)

where we define \( M = \langle S^z \rangle \), and \( \bar{a} = \sqrt{\omega_1^2 + \Delta_+^2 \sin^2 \theta} \), \( \bar{b} = \sqrt{\omega_2^2 + \Delta_-^2 \sin^2 \theta} \), \( \bar{\omega}_1 = \bar{\omega} - \bar{\mu} - \bar{\eta} \), \( \bar{\omega}_2 = \bar{\omega} - \bar{\mu} + \bar{\eta} \). Here all the parameters are nondimensionalized, for example, \( \bar{\eta} = \eta / \epsilon_f \), \( \bar{V} = V / \epsilon_f \). Other dimensionless parameters, \( \bar{\tau}, \bar{m}, \bar{\Delta}_\pm \), and \( \bar{\omega}_c \) are rescaled analogously. The rescaled factor \( \epsilon_f = \frac{\hbar}{2m}(3\pi^2 n)^{\frac{1}{3}} \) and \( n = 1 \) at half-filling. The dimensionless temperature is defined as \( T = k_B T / \epsilon_f \). The energy cutoff \( \omega_c = 0.01 \tau_F \) is chosen to be consistent with the one-band model, where \( \tau_F \) is the dimensionless Fermi energy.

### III. RESULTS AND DISCUSSION

We first calculate the \( T = 0 \) properties. Figure 1 plots the p-wave SC order parameters, \( \Delta_\pm \), magnetization density of itinerant electrons, \( m = \langle n_+ \rangle - \langle n_- \rangle \), and magnetization of localized spins, \( M = \langle S^z \rangle \). \( m \) is improved as the exchange interaction \( V \) increases. Correspondingly, the SC gap \( \Delta_+ \) is strengthened, while \( \Delta_- \) is weakened.

Figure 2 shows the variation of \( m \) and \( \Delta_\pm \) with the p-wave interaction strength \( V \). Apparently, \( m \) rises as \( V \) increases for each given value of \( \bar{\eta} \), indicating that p-wave Cooper pairing can also enhance the ferromagnetism. As shown in the inset, with increasing \( \bar{V}, \Delta_+ \) rises accordingly, while \( \Delta_- \) initially rises and then decreases. These results are consistent with the one-band model.

Figure 3 illustrates all the order parameters at finite temperatures. It seems that the FM transition temperature \( T_{FM} \) is mainly determined by the exchange interaction between localized spins, \( J \), when the coupling between localized spins and itinerant electrons, \( \bar{\eta} \), is small. So the FM state can vanish earlier than the SC state as \( J \) is weak enough. Once \( m \) and \( M \) disappear, \( \Delta_+ \), \( \Delta_- \) become equal. \( T_{FM} \) rises as \( \bar{\eta} \) increases and it tends to \( T_{SC} \) as \( \bar{\eta} \) goes up to about 0.52. And then \( T_{FM} \) surpasses \( T_{SC} \) at larger \( \bar{\eta} \) values. These results are different from those of the one-band model for which \( T_{FM} \) is unlikely below \( T_{SC} \) once the ferromagnetism is established.
Figure 3 also indicates that the magnetization displays an inflexion at the SC transition temperature.

IV. CONCLUSION

In conclusion, we study a two-band model describing coexistence of p-wave superconductivity with localized ferromagnetism. It is shown that ferromagnetism can be enhanced by the p-wave Cooper pairing, as suggested in a one-band model previously. However, the Curie temperature in this model is not only determined by the exchange interaction between localized spins, also by the coupling between localized spins and itinerant electrons. The Curie temperature can be lower than the superconducting transition temperature.

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FIG. 3: Plots of all order parameters, $\overline{\Delta}$, $m$, and $M$, as functions of $T$ with $V = 100$, $J = 0.01$, at different $g$ values. Inset: Enlargement of the region $0.125 < T < 0.13$.

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1 Saxena, S. S., Agarwal, P., Ahilan, K., Grosche, F. M., Haselwimmer, R. K. W., Steiner, M. J., Pugh, E., Walker, I. R., Julian, S. R., Monthoux, P., Lonzarich, G. G., Huxley, A., Sheikin, I., Braithwaite, D., and Flouquet, J., Nature (London) 406, 587 (2000).
2 Aoki, D., Huxley, A., Ressouche, E., Braithwaite, D., Flouquet, J., Brison, J. P., Lhotel, E. and Paulsen, C., Nature (London) 413, 613 (2001).
3 Karchev, N. I., Blagoev, K. B., Bedell, K. S. and Littlewood, P. B., Phys. Rev. Lett. 86, 846 (2001).
4 Suhl, H., Phys. Rev. Lett. 87, 167007 (2001).
5 Walker, M. B., and Samokhin, K.V., Phys. Rev. Lett. 88, 207001 (2002).
6 Kirkpatrick, T. R., Belitz, D., Phys. Rev. Lett. 92, 037001 (2004).
7 Nevidomskyy, A. H., Phys. Rev. Lett. 94, 097003 (2005).
8 Jian, X. L., Zhang, J. C., Gu Q. and Klemm, R. A., Phys. Rev. B 80, 224514 (2009).
9 Singh, P., J. Supercond. Nov. Magn. 24, 945-949 (2011).