Identification and Assessment of Zones with Interfacial Partially Healed Cracks Using Lamb Waves Dispersion

A V Dudchenko

1LLC “PIK-PROJECT”, Rastorguevskij pereulok, d. 3, korp. 16, Moscow, 123557, Russia

E-mail: aleks_dud@mail.ru

Abstract. Requirements of modern technics and technology resulted in development of composite materials that provides higher target properties than individual materials. Then, self-healing technology was proposed to increase the durability of such materials. This technology can be applied to various material types, for instance to cementitious one like concrete and mortar as well as multilayer laminar composites. In the both cases one of the most important issue is to estimate the degree of healing as well as the change in the material mechanical and physical properties like strength, stiffness, permeability, etc. during healing process. Various methods are possible to implement for such assessment including non-destructive ways of evaluation. In the present work ultrasonic diagnostics involving Lamb waves is considered and evaluated to show its applicability for practical testing of multilayer composites. The method is based on Six-dimensional Cauchy formalism developed earlier.

1. Introduction
In modern civil, mechanical, electronic and other branches of engineering the need of lightweight and durable materials has arisen due to development of technics and technology in the last century. As a result, composite materials have been developing to fulfill the requirements of modern engineering. These materials can be a composition of several layers with different mechanical properties e.g. laminar composites or it can be a matrix with inclusions of other materials such as reinforced concrete. In composition these materials demonstrate better required properties rather than individually.

The next development of such materials is connected with the technology which is called self-healing. It includes various methods that can be used for crack closure as well as improving mechanical and permeability properties [1-3]. This can be performed by using various additions including bacteria [4,5], microcapsules with special organic or inorganic agents [6,7], various admixtures [8] etc. to initial material composition. As a result, this provides crack closure and improvement in material properties, thus, prolonging its utilization.

At the same time, one of the most important issues for such materials is to estimate healing degree and improvement in material properties. In [9,10] review of experimental methods to characterize self-healing performance of cementitious materials and polymeric composites is conducted to show different approaches of mechanical and physical property characterization. In work [11], Ahn et al. focus on ultrasonic methods of self-healing characterization and shows the applicability of such methods to evaluate “healed” properties of cementitious materials.
The present study is focused on theoretical analysis of the method of non-destructive testing of self-healing multilayer composites. The approach is based on Lamb-wave dispersion properties. The proposed method of theoretical analysis is developed earlier by Kuznetsov in [12] and consists in calculating and analysis of dispersion curves for multilayer plates by using six-dimensional Cauchy formalism. As a result, the calculated dispersion curves can be analyzed and the patterns corresponding to various healing degrees can be identified. Avershieva et al. in [13] show that analytical solution for Lamb wave velocities calculated using Cauchy formalism is in a good agreement with numerical simulation using finite element method in Abaqus software. Apart from that, many sources confirm applicability of Lamb wave for non-destructive testing e.g. [14-16]. Therefore, it is possible to perform healing assessment through comparing experimentally obtained dispersion curves with the ones obtained theoretically for various stages of internal damage. In this work, change of dispersion curve character for three-layer plate with inner rigid layer after appearance of an interfacial crack and at various stages of self-healing process is calculated and analyzed to show the main patterns characterizing each stage.

2. Lamb waves in diagnostics of internal defects

2.1. Methods of self-healing evaluation

Self-healing process can be estimated through direct measurements or experimental procedures. The most obvious way is to measure crack width through various types of cameras [17-19] and microscopy [20-26]. The experiments in the works [17-26] showed the positive effect of self-healing for surface crack closure. Internal volumes of the cracks can be estimated through tomography [27-31], electrical methods [32-34], etc. One of the key issues for these approaches is sample preparation to detect crack borders accurately. This may include polishing, cleaning with compressed air, drying, etc. Eventually, it is possible to measure crack width in various points, average crack width and crack volume to estimate crack closure during healing process.

Another group of methods of self-healing evaluation includes test of liquid [35-40] and gas-permeability [41-42], sorptivity [43-46], etc. These properties can be measured at different stages of self-healing and related to durability properties of materials. Water permeability properties relates to change in water pressure with time or the volume of water that goes through a sample. Similarly to crack measurements, this method represents change in the specimen continuity due to self-healing, thus, indirectly characterizing crack closure. Correlation between this method, crack closure and ultrasonic pulse velocity tests is studied by Van Tittelboom et al. in [47] showing good agreement. Similar approach is adopted for gases. The results for gas permeability in [41,42] showed a significant change in that property during healing process.

On the other hand, the methods mentioned above do not provide direct measurement of mechanical characteristics like stiffness and residual strength which are the key parameters for structural engineers. As a result, mechanical properties from the measurements can be found using correlation between durability or crack parameters with Young’s modulus and strength.

Direct methods of mechanical properties evaluation include direct mechanical tests that damage specimens and non-destructive ultrasonic wave based methods. Direct mechanical tests may include compression tests [48,49], tensile tests [31,50], splitting tests [51], flexural tests [52-55]. Non-destructive evaluation of mechanical properties is based on acoustic wave propagation and measurements of various wave properties in different points of a sample. Ultrasonic pulse velocity measurement is well-established and extensively used test method [56] showing its effectiveness in internal defect detections. Additionally, it can be used to assess self-healing [53,55,57-58]. However, these approaches are not free from drawbacks because of reinforcement bars, voids, temperature and moisture change that can significantly affect the results making them doubtful. Apart from UPV methods it is possible Rayleigh wave based approaches [59] that also showed its applicability. Lamb waves are frequently used for material testing and the common approach are quite well established [14-16]. Application of nondestructive testing of self-healing composite involving Lamb waves is
performed in [61-63] using array of fiber Bragg grating sensors. The possibility to use Lamb waves to evaluate and detect internal defects is shown.

2.2. Constitutive equations.
Although several types of surface waves can travel in a layered elastic media including Lamb waves, Rayleigh waves, Stoneley and Love waves, Lamb and Love waves can be considered as the most appropriate to study the properties of individual layers because of dispersion properties (the dependency of the phase velocity on the oscillation frequency). Thus, they allow obtaining mechanical characteristics of individual layers. Lamb waves are characterized by elliptical polarization in the plane formed by the normal to the wave front and surface of the layer as well they can propagate in separate layers. In the following text a brief description of the method that is used to calculate dispersion curves for Lamb waves is provided.

Three-dimensional formalism for analysis of wave propagation in an anisotropic half-space was initially developed in [64] for Rayleigh waves. Afterwards, the approach was modified to study Lamb waves travelling in anisotropic plates, e.g. [65-70]. Further development of the formalism was Stroh six-dimensional formalism [71] that was also initially applied to study Rayleigh waves propagating on a free surface of an anisotropic half-space. Then, in [72,73] this approach was used to analyze Lamb waves propagating in anisotropic plates.

Cauchy six-dimensional formalism, that is used in the present work, was initially developed in [74] for analysis of Lamb waves propagating in anisotropic media. In the following text a brief description is provided.

Equation of motion for continuous general anisotropic media has the following form:

$$A(\partial_x, \partial_t)u = \text{div}_x C \cdot \nabla_x u - \rho \ddot{u} = 0$$

where $C$ is an elasticity tensor, $u$ is the vector of displacements and $x$ is a coordinate vector. Equation for Lamb wave travelling in anisotropic elastic media can be written using the following vector function:

$$v(x^n) = \partial_{x^n} \cdot f(x^n).$$

This allows modifying the equation of motion for an anisotropic media (Eq.(1)) to normal Cauchy form:

$$\partial_{x^n} \left( \begin{array}{c} f \\ v \end{array} \right) = G \cdot \left( \begin{array}{c} f \\ v \end{array} \right)$$

In Eq. (3) $G$ is fundamental matrix that can be defined as:

$$G = \begin{pmatrix} 0 & I \\ -A_1 \cdot A_3 & -A_1 \cdot (A_2 + A'_2) \end{pmatrix}.$$ 

Matrixes $A_1, A_2, A_3$ in Eq. (4) are calculated as:

$$A_1 = v \cdot C \cdot v, \quad A_2 = v \cdot C \cdot n$$

$$A_3 = n \cdot C \cdot n - \rho c^2 I.$$ 

Using Eq. (3) allows obtaining solution of Eq. (2) in exponential form:

$$\left( \begin{array}{c} f(x^n) \\ v(x^n) \end{array} \right) = \exp(x^n G) \cdot \tilde{C}_6.$$
In Eq. (6) $C_6$ is a six-dimensional vector of unknown coefficients that can be calculated from boundary conditions up to a multiplier. In the considered approach the boundary conditions can be formulated in terms of $A_1, A_2$ as:

$$t_v(x^\prime) = (A_1 \partial_x + A_2) \cdot f(x^\prime),$$

Using this equation for the surface traction vector and combining it with Eq. (2) and Eq. (6) gives surface tractions as well as displacements on the both plate surfaces:

$$
\begin{bmatrix}
  f(\pm irh) \\
  t_v(\pm irh)
\end{bmatrix} = Z \cdot \exp(\pm irhG) \cdot \hat{C}_6,
$$

where $Z = \begin{pmatrix} I & 0 \\ A_2 & A_1 \end{pmatrix}$. Unknown coefficients vector can be excluded by using the relation between the displacements and traction on the one side and with the corresponding vectors on the other boundary producing the equation:

$$
\begin{bmatrix}
  f(-irh) \\
  t_v(-irh)
\end{bmatrix} = T \cdot \begin{bmatrix}
  f(+irh) \\
  t_v(+irh)
\end{bmatrix}
$$

where $T = Z \cdot \exp(-2irhG) \cdot Z^{-1}$. Matrix $T$ can be called the transfer matrix not depending on the used boundary conditions. In more details, the presented approach and its use for calculating dispersion curves in particular problems can be observed in the works [12,74-75].

3. Calculation for model problem

3.1. Problem formulation

Evaluation of the presented method is performed using three-layer model with rigid inner layer (Figure 1). The layer in the middle of the plate is supposed to be 10 times stiffer than the outer ones. Ideal mechanical contact between the layers is considered for initial state of the plate. The surfaces of the plate are free. The parameters of the layers are taken as follows:

$$E_{1,3} = 1.0, \quad E_2 = 10.0, \quad \rho_{1,3} = 1.0, \quad \rho_2 = 2.0, \quad \nu_{1,2,3} = 0.0.$$  

Crack is simulated by adding a thin layer between the rigid and soft ones (the thickness of the layer 100 times less than the thickness of the soft and rigid ones) simulating delamination of the plate. The stiffness and density of the crack layer is varied ($E_{cr} = 10^{-5} \div 0.9, \quad \rho_{cr} = 0.001 \div 0.9, \nu_{cr} = 0.0$) to model self-healing process. As a result, dispersion curves are calculated for various states of the plate including: (1) initial state without a crack; (2) damaged state ($E_{cr} = 10^{-5}, \quad \rho_{cr} = 0.001, \nu_{cr} = 0.0$), (3) self-healing stage 1 ($E_{cr} = 0.001, \quad \rho_{cr} = 0.01, \nu_{cr} = 0.0$) (2) self-healing stage 2 ($E_{cr} = 0.1, \quad \rho_{cr} = 0.1, \nu_{cr} = 0.0$). Then, based on the obtained dispersion curves the principal difference for each stage are identified.
Figure 1. Model problem. Three-layer plate with rigid internal layer. Initial state (top scheme).

3.2. Calculation results
Dispersion curves for initial state of the plate are shown in figure 2. Their change due to damage is shown in figure 3. It can be seen that the curve character totally changes with appearance of interfacial crack for each mode and becomes virtually horizontal except for some phase velocity ranges.

Dispersion curves change due to self-healing can be seen in figures 4 and 5 where loss of horizontal character of the curves can be observed due to stiffness recovery at various crack layer properties. Eventually when the recovered stiffness of the crack layer becomes close to the stiffness of the neighbor layers dispersion curve character becomes close to the initial. This allows detecting interfacial cracks and estimating damage degree of the internal layer.

Figure 2. Dispersion curve for the plate in undamaged state.

Figure 3. Dispersion curves for an interfacial crack related to unhealed crack \( E_{cr} = 1 \times 10^{-5} \).
Figure 4. Dispersion curves for an interfacial crack related to unhealed crack \((E_{cr} = 1 \times 10^{-3})\).

Figure 5. Dispersion curves for an interfacial crack related to unhealed crack \((E_{cr} = 0.1)\).

4. Conclusion

Non-destructive test method based on Six-dimensional Cauchy formalism is adopted for analysis of three-layer composite plate with inner rigid layer. The appearance of a crack is modeled using additional thin layer with low Young’s modulus and density.

As a result, it is obtained that it is possible to detect interlayer cracks and evaluate healed degree for this layer by building dispersion curves. Apart from that, comparing theoretically calculated dispersive curves with the experimentally obtained ones allows restoring mechanical properties of the healed layers.

As perspectives of this work, calculation and assessment of dispersion curves for the plates composed of larger number of layers will be performed. This will be used to analyze and determine common patterns in dispersion curves for multilayer plates including identification of crack location.

5. References

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