Autonomous Mobile Robot Navigation Using Harmonic Potential Field

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Abstract. Mobile robot navigation has been an area of robotics which has gained massive attention among the researchers of robotics community. Path planning and obstacle avoidance are the key aspects of mobile robot navigation. This paper presents harmonic potential field based navigation algorithm for mobile robots. Harmonic potential field method overcomes the issue of local minima which was a major bottleneck in the case of artificial potential field method. The harmonic potential field is calculated using harmonic functions and Dirichlet boundary conditions are used for the obstacles, goal and initial position. The simulation results shows that the proposed method is able to overcome the local minima issue and navigate successfully from initial position to the goal without colliding into obstacles in static environment.

Keywords : Autonomous mobile robot, harmonic potential field, obstacle avoidance, path planning

1. Introduction

Mobile robot navigation has been an area of robotics which has been researched extensively during the past three decades. This has led to development of different algorithms focusing on solving the issues related to mobile robot navigation namely path planning and obstacle avoidance. Artificial potential field, Vector field histogram method, bug algorithms etc., are some of the most commonly used schemes in mobile robot navigation. Artificial potential field method for mobile robot navigation proposed by J Borenstein et, al \cite{1} has been one of the most used navigation scheme. The core principle of this scheme is based on attractive and repulsive potential field concepts. The goal point is assumed to be negatively charged and rest other elements in the working environment of the robot like the obstacles and the initial point of the robot are assumed to be positively charged. The magnitude of the attractive or repulsive potential field is calculated based on the distance between the robot and the goal or the obstacles respectively. The attractive feature of the artificial potential field method is its simplicity and elegance with respect to mathematical analysis. The main bottleneck of the artificial potential field methods lies in its inability to counter the issue of local minima in the working environment. Local minima is a condition where in the magnitude of the attractive potential field and repulsive potential field is equal. This situation leads to the robot getting stuck in a position and being not able to compute its next move. In order to overcome the issue of artificial potential field, Conolly et al \cite{2} proposed harmonic potential field method. Harmonic potential filed method is a novel artificial function based on harmonic functions, which overcomes the limitations of potential field methods. Harmonic functions are solution to Laplaces equations. The most
important property of harmonic functions is that they are free from local minima[3-5]. The core idea of this method lies in creation of only one minimum in the working environment i.e., the global minimum which is represented by the goal. If the goal is represented by a global minimum and no other minimum exists in the environment then the robot will arrive at the goal location always. Harmonic potential field algorithm has gained a lot of interest in mobile robot navigation due to its distinct feature of avoiding local minima and it also produces a smooth path for mobile robot navigation [6]. Unlike potential field methods, harmonic potential field method not only performs well in static environment but also works efficiently in dynamic environment and can cope the rapid changes in the environment. Some of the research articles related to the use of harmonic potential field for mobile robot navigation are mentioned as follows, Kalavsky et al [7] have explained the mechanism to generate potential field using harmonic functions and to formulate an algorithm for computing a safe path for mobile robot in the environment with created potential field. Keymeulen et al [8] have presented the effect of boundary conditions for the obstacles. The application of potential field method for autonomous ground vehicles has been presented in [9-11]. This paper presents a path planning of a differential wheel drive mobile robot using harmonic potential field approach, Dirichlet boundary conditions are used to specify the boundary values for the obstacles. Simulation is carried out to evaluate the proposed method, Matlab software has been used for setting the boundary conditions and generating the environment for implementation of harmonic potential field method. The simulation results shows the effectiveness of the proposed approach.

The organization of the paper is as follows. Kinematics modelling of the robot is discussed in section 2. The description of harmonic potential field is discussed in section 3. Simulation results are discussed in section 4. Discussion and conclusion are presented in section 5.

2. Kinematic Modeling of Wheeled Mobile Robot

The geometrical description of a mobile robot is shown in fig.1. Assuming that the mobile robot wheels are non-deforming, and moves without slip on the surface, two wheels are independently driven by two actuators to achieve the motion and orientation, the kinematic model of the wheeled mobile robot is given by following equation eq.1

\[
\begin{pmatrix}
\dot{x} \\
\dot{y} \\
\dot{\theta}
\end{pmatrix} =
\begin{pmatrix}
V \cos \theta \\
V \sin \theta \\
\omega
\end{pmatrix}
\]  

(1)

Figure 1. A two wheeled non-holonomic mobile robot

3. Harmonic potential field

Harmonic Potential filed method is a novel method based on harmonic functions, it overcomes the limitations of Potential field methods. The most important trait of harmonic potential field method is that they are free from local minima. Harmonic potential field method uses harmonic functions and boundary value conditions to overcome the local minima problem. To build an
artificial potential, we use harmonic function. A harmonic function should satisfy Laplace’s equation, it should not have local extrema in a space free from singularities, it should have second order derivatives. Solution of Laplace equation is also known as the mobile robot velocity potential:

\[
\begin{align*}
V = -\nabla \Psi_{i,j} &= \left( \frac{\partial \Psi_{i,j}}{\partial x} \right) \\
&= -\left( \frac{\partial \Psi_{i,j}}{\partial x} \right)
\end{align*}
\] (2)

A harmonic function should also satisfy principle of superposition and principle of maxima and minima. These principles indicate that the harmonic function has its extremes only on the boundary, so it does not have local maxima/minima inside the boundary. Hence, it is convenient for us to define boundary conditions for boundary of all obstacles and boundary of goal. In this research Dirichlet boundary conditions have been used. The Dirichlet boundary condition states that the boundary of all obstacles will be assigned with the maximum value in the region and the boundary of goal position has the minimum value in the region. By defining the boundary conditions in this format the potential field is harmonic field with only global minimum represented by goal position.

3.1. Finite Difference Approximation Method

The finite difference approximation has been used in the proposed research to find the numerical solution of Laplace equation. Finite element method and finite volume method are some of the other methods used to find the numerical solutions to Laplace equations. The main advantage of using finite difference approximation method lies in its simplicity to implement and can be used to obtain numerical solution of partial differential equation (PDE) [12]. By using segmentation of environment, we can obtain potential distribution of the environment as shown in fig.2.

![Figure 2. Sub-region of potential distribution in the environment](image)

In order to solve finite difference approximation for the derivative \( \Psi_{xx} \), we use the following equations:

\[
\begin{align*}
\Psi_{i-1,j} &= \Psi_{i,j} - T_x \Delta x_A + \frac{1}{2} \Psi_{xx} \Delta x_A^2 \\
\Psi_{i+1,j} &= \Psi_{i,j} - T_x \Delta x_B + \frac{1}{2} \Psi_{xx} \Delta x_B^2
\end{align*}
\] (3)

Subtracting equation eq.3 and eq.4 and solving for \( \Psi_{xx} = \frac{\partial^2 \Psi}{\partial x^2} \), results in
\[ \Psi_x = \frac{\Psi_{i+1,j} - \Psi_{i-1,j}}{\Delta x_A + \Delta x_B} \]  

Adding equations eq.3 and eq.4 and solving for \( \Psi_x \), after replacing the expression for \( \Psi_x \), from eq.5, results in the following expression:

\[ \Psi_{xx} = \frac{2(\Psi_{i-1,j} + \Psi_{i+1,j} - 2\Psi_{i,j})}{\Delta x_A + \Delta x_B} + \frac{\left(\Psi_{i+1,j} - \Psi_{i-1,j}\right)(\Delta x_A - \Delta x_B)}{(\Delta x_A + \Delta x_B)} \]  

To simplify the expression we introduce the following definitions:

\[ \alpha_x = \Delta x_A - \Delta x_B, \quad \beta_x = \Delta x_A + \Delta x_B, \quad r_x = \frac{\alpha_x}{\beta_x}, \quad \gamma_x^2 = \Delta x_A^2 + \Delta x_B^2, \]

Thus

\[ \Psi_{xx} = \frac{2(\Psi_{i-1,j} + \Psi_{i+1,j} - 2\Psi_{i,j} + \alpha_x (\Psi_{i+1,j} - \Psi_{i-1,j})))}{\gamma_x^2} \]

Similarly, we can obtain the following equation \( \Psi_{yy} \) for the following derivatives in y:

\[ \Psi_{yy} = \frac{2(\Psi_{i,j-1} + \Psi_{i,j+1} - 2\Psi_{i,j} + \alpha_y (\Psi_{i,j+1} - \Psi_{i,j-1})))}{\gamma_y^2} \]

If we now replace the results of eq.8 and eq.9 into the Laplace equation \( \Psi_{xx} + \Psi_{yy} = 0 \), results in the following finite-difference approximation:

\[ \frac{2(\Psi_{i-1,j} + \Psi_{i+1,j} - 2\Psi_{i,j} + \alpha_x (\Psi_{i+1,j} - \Psi_{i-1,j})))}{\gamma_x^2} + \frac{2(\Psi_{i,j-1} + \Psi_{i,j+1} - 2\Psi_{i,j} + \alpha_y (\Psi_{i,j+1} - \Psi_{i,j-1})))}{\gamma_y^2} = 0 \]

An explicit solution for the value of the unknown \( \Psi_{i,j} \) at the center of the computational cell can be obtained from above equation:

\[ \Psi_{i,j} = \frac{\gamma_x^2(\Psi_{i-1,j} + \Psi_{i+1,j} + r_x (\Psi_{i+1,j} - \Psi_{i-1,j}))}{2(\gamma_x^2 + \gamma_y^2)} + \frac{\gamma_y^2(\Psi_{i,j-1} + \Psi_{i,j+1} + r_y (\Psi_{i,j+1} - \Psi_{i,j-1}))}{2(\gamma_x^2 + \gamma_y^2)} \]  

If we consider a rectangular region where the increments in both x and y are uniform which means

\[ \Delta x_A = \Delta x_B = \Delta x, \quad r_x = \alpha_x = 0 \]
\[ \Delta y_A = \Delta y_B = \Delta y, \quad r_y = \alpha_y = 0 \]
\[ \beta_x = 2\Delta x, \quad \gamma_x^2 = 2\Delta x^2 \]
\[ \beta_y = 2\Delta y, \quad \gamma_y^2 = 2\Delta y^2 \]

and eq.10 simplifies to

\[ \Psi_{i,j} = \frac{\Psi_{i-1,j} + \Psi_{i+1,j} + \Psi_{i,j-1} + \Psi_{i,j+1}}{4} \]
4. Simulation

The simulation environment is modelled with static obstacles (walls), the mobile robot is required to traverse through these obstacles from initial position to reach the goal position (see fig.10). The area of the environment is 100×100 metres. Fig.[5] and fig.[6] illustrate the path which the robot followed to reach the target and displays the result of the proposed method in this paper. It can be observed that the mobile robot could safely avoid obstacles and successfully navigate towards the goal position.

![Simulation environment](image1)

**Figure 3.** Simulation environment

Harmonic potential field method has the ability to generate path for any given initial position and any given target point of the mobile robot. The vector field generated is shown in fig.4 below.

![Potential field](image2)

**Figure 4.** Potential field

In order to evaluate the proposed path planning scheme, two scenarios have been chosen. In the first scenario, the initial position and target position of the robot are assigned to be (90,90) and(2,20) respectively. The simulation result for the first scenario is shown in fig.5.
Figure 5. Mobile robot navigation: Scenario-1

The simulation results of the second scenario are shown in fig.6, in this scenario the initial position is considered to be (90,10) and goal position is considered to be (4,80). The simulation results for the second scenario is shown in fig.6.

Figure 6. Mobile robot navigation: Scenario-2

5. Discussion
This paper presents a harmonic potential field based navigation scheme for mobile robots. The potential fields are generated using the Laplace equation, Dirichlet's conditions are used to specify the boundary conditions for the obstacles, initial and goal positions. Finite difference approximation method is used to find the numerical solution of Laplace equation. The proposed method has been evaluated using MATLAB simulation. Two types of simulation scenarios with different initial and goal points have been considered. It can be observed from the simulation results that the streamlines generated by the harmonic potential field method is smooth and it enables the robot to completely avoid the obstacles and efficiently reach the goal position in static environment.

6. Conclusion
The mobile robot navigation in static environment based on Harmonic potential field method has been discussed in this paper. Two scenarios with different initial position and goal position
were considered and mobile robot navigation based on harmonic potential field was simulated in matlab environment. The simulation results show that harmonic potential field can be used to efficiently navigate the mobile robot in static environment.

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