Simiplified textures of $M_D$ in the seesaw model for the trimaximal neutrino mixing

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Abstract

In this paper, in the basis of $M_R$ being diagonal, we survey the simplified textures of $M_D$ for realizing the trimaximal neutrino mixing and their consequences for the low-energy neutrino observables and leptogenesis. We first formulate the general texture of $M_D$ for realizing the trimaximal mixing, and then, following the Occam’s razor principle, examine if its parameters can be further reduced, giving more simplified textures of it. Our analysis is restricted to the simple but instructive scenario that there is only one phase parameter $\phi$ responsible for the CP violating effects at low energies and leptogenesis. Our attention is paid to the simplified textures of $M_D$ where there are some vanishing elements (a sign of Abelian flavor symmetries) or equalities among non-vanishing elements (a sign of non-Abelian flavor symmetries). On the basis of these simplified textures, we further examine if $\phi$ can also take a particular value (a sign of CP symmetries). The consequences of the phenomenologically-viable simplified textures for the low-energy neutrino observables and leptogenesis are studied.

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1 Introduction

The observation of the phenomenon of neutrino oscillations indicates that neutrinos are massive and the lepton flavors are mixed [1]. In the literature, the most popular and natural way of generating the tiny but non-zero neutrino masses is the type-I seesaw mechanism, in which three right-handed neutrinos $N_I$ (for $I = 1, 2, 3$) are added into the SM [2]. The right-handed neutrinos not only can have the usual Yukawa couplings with the left-handed neutrinos (which lead to the Dirac neutrino mass matrix $M_D$ after the electroweak symmetry breaking) but also are allowed to have the Majorana masses $M_I$ (eigenvalues of their Majorana mass matrix $M_R$). The essence of the seesaw mechanism is that the right-handed neutrino masses are much larger than the electroweak scale so that we are led to a highly-suppressed Majorana mass matrix for the light neutrinos

$$M_\nu = M_D M_R^{-1} M_D^T .$$

Then, in the basis where the flavor eigenstates of three charged leptons are identical with their mass eigenstates, the neutrino mixing matrix $U$ [3] coincides with the unitary matrix for diagonalizing $M_\nu$:

$$U^\dagger M_\nu U^* = \text{Diag}(m_1, m_2, m_3) ,$$

where $m_i$ are three light neutrino masses. In the standard form, $U$ is parameterized in terms of three mixing angles $\theta_{ij}$ (for $ij = 12, 13, 23$), one Dirac CP phase $\delta$ and two Majorana CP phases $\rho$ and $\sigma$ as

$$U = \begin{pmatrix}
 c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\
 -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\
 s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13}
\end{pmatrix} \begin{pmatrix}
 e^{i\rho} \\
 e^{i\sigma} \\
 1
\end{pmatrix} ,$$

where the abbreviations $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$ have been used.

Neutrino oscillations are sensitive to the following six parameters: three mixing angles, two independent neutrino mass squared differences $\Delta m^2_{ij} \equiv m_i^2 - m_j^2$ (for $ij = 21, 31$), and $\delta$. Several groups have performed global analyses of the existing neutrino oscillation data to extract the values of these parameters [4] [5]. For definiteness, we will use the results in Ref. [4] (which are reproduced in Table 1 here) as reference values in the following numerical calculations. Note that the sign of $\Delta m^2_{31}$ remains undetermined, allowing for two possible neutrino mass orderings: the normal ordering (NO) $m_1 < m_2 < m_3$ and inverted ordering (IO) $m_3 < m_1 < m_2$. On the contrary, neutrino oscillations have nothing to do with the absolute values of neutrino masses and Majorana CP phases. In order to extract or constrain their values, one has to resort to some non-oscillatory experiments such as the neutrino-less double beta decay experiments [6]. However, such experiments have not yet placed any lower constraint on the lightest neutrino mass, nor any constraint on the Majorana CP phases.

From Table 1 one can see that $\theta_{12}$ and $\theta_{23}$ are close to some special values: $\sin^2 \theta_{12} \sim 1/3$ and $\sin^2 \theta_{23} \sim 1/2$. Before the value of $\theta_{13}$ was pinned down in 2012, the conjecture that it might be vanishingly small was very popular. For the extreme case of sin $\theta_{12} = 1/\sqrt{3}$, sin $\theta_{23} = 1/\sqrt{2}$ and $\theta_{13} = 0$ (referred to as the tribimaximal (TBM) mixing [7]), the neutrino mixing matrix can be described by a few simple numbers and their square roots

$$U_{\text{TBM}} = \frac{1}{\sqrt{6}} \begin{pmatrix}
 2 & \sqrt{2} & 0 \\
 -1 & \sqrt{2} & \sqrt{3} \\
 1 & -\sqrt{2} & \sqrt{3}
\end{pmatrix} .$$

Such a particular mixing may be suggestive of an underlying flavor symmetry in the lepton sector. And many flavor symmetries have been employed to realize it [8]. However, the observation of a relatively
and their consequences for the neutrino mixing parameters, among which the following expressions of $\theta$:

- $0.17$—$0.19$ and $0.36$—$0.33$

At the 3σ level, $s_{12}$ and $s_{13}$ ranges of $0.317$—$0.319$ (0.340—0.342) and $0.33$—$0.59\pi$ (0.31π—1.00π) for the TM1 (TM2) mixing, which are in good agreement with the experimental results. Due to its simple structure and phenomenologically-appealing consequences, the
trimaximal mixing has attracted a lot of attention (in particular after the observation of a relatively large \(\theta_{13}\) \([10]\).

In this paper, in the basis of \(M_R\) being diagonal, we survey the simplified textures of \(M_D\) for realizing the trimaximal mixing and study their consequences for the low-energy neutrino observables and leptogenesis. The rest part of this paper is organized as follows. In the next section, we first formulate the general texture of \(M_D\) for realizing the trimaximal mixing. Then, following the Occam’s razor principle, we examine if its parameters can be further reduced, giving more simplified textures of it. The consequences of the phenomenologically-viable simplified textures for the low-energy neutrino observables and leptogenesis will be studied. The studies for the TM1 and TM2 mixings will be given in sections 3 and 4, respectively. Finally, the last section summarizes our main results.

## 2 General texture of \(M_D\) for the trimaximal mixing

In this section, in the basis of \(M_R = \text{diag}(M_1, M_2, M_3)\) being diagonal, we formulate the general texture of \(M_D\) for realizing the trimaximal mixing. It is convenient to parameterize \(M_D\) as

\[
M_D = \begin{pmatrix}
\sqrt{a_1} M_1 & \sqrt{b_1} M_2 & \sqrt{c_1} M_3 \\
\sqrt{a_2} M_1 & \sqrt{b_2} M_2 & \sqrt{c_2} M_3 \\
\sqrt{a_3} M_1 & \sqrt{b_3} M_2 & \sqrt{c_3} M_3
\end{pmatrix},
\]

where \(a_i, b_i\) and \(c_i\) are complex parameters. The seesaw formula immediately yields an \(M_\nu\) as

\[
M_\nu = \begin{pmatrix}
a_1^2 + b_1^2 + c_1^2 & a_1 a_2 + b_1 b_2 + c_1 c_2 & a_1 a_3 + b_1 b_3 + c_1 c_3 \\
a_1 a_2 + b_1 b_2 + c_1 c_2 & a_2^2 + b_2^2 + c_2^2 & a_2 a_3 + b_2 b_3 + c_2 c_3 \\
a_1 a_3 + b_1 b_3 + c_1 c_3 & a_2 a_3 + b_2 b_3 + c_2 c_3 & a_3^2 + b_3^2 + c_3^2
\end{pmatrix}.
\]

Given that the trimaximal mixing matrix can be expressed as in Eq. (5), one can formulate the general texture of \(M_D\) for realizing the trimaximal mixing by first transforming \(M_\nu\) to \(M_\nu' \equiv U_{\text{TBM}}^\dagger M_\nu U_{\text{TBM}}^*\), and then deriving the constraints on the texture of \(M_D\) from the requirement that \(M_\nu'\) should be such that it can be diagonalized by \(U_{23}\) or \(U_{13}\) \([11]\). Here \(M_\nu'\) appears as

\[
M_\nu' = \begin{pmatrix}
\frac{A_1^2}{6} + \frac{B_1^2}{6} + \frac{C_1^2}{6} & \frac{A_1 A_2}{3\sqrt{2}} + \frac{B_1 B_2}{3\sqrt{2}} + \frac{C_1 C_2}{3\sqrt{2}} & \frac{A_1 A_3}{3\sqrt{2}} + \frac{B_1 B_3}{3\sqrt{2}} + \frac{C_1 C_3}{3\sqrt{2}} \\
\frac{A_1 A_2}{3\sqrt{2}} + \frac{B_1 B_2}{3\sqrt{2}} + \frac{C_1 C_2}{3\sqrt{2}} & \frac{A_2^2}{3} + \frac{B_2^2}{3} + \frac{C_2^2}{3} & \frac{A_2 A_3}{\sqrt{6}} + \frac{B_2 B_3}{\sqrt{6}} + \frac{C_2 C_3}{\sqrt{6}} \\
\frac{A_1 A_3}{3\sqrt{2}} + \frac{B_1 B_3}{3\sqrt{2}} + \frac{C_1 C_3}{3\sqrt{2}} & \frac{A_2 A_3}{\sqrt{6}} + \frac{B_2 B_3}{\sqrt{6}} + \frac{C_2 C_3}{\sqrt{6}} & \frac{A_3^2}{2} + \frac{B_3^2}{2} + \frac{C_3^2}{2}
\end{pmatrix},
\]

with \n\begin{align*}
A_1 &= 2a_1 - a_2 - a_3 , & A_2 &= a_1 + a_2 - a_3 , & A_3 &= a_2 + a_3 , \\
B_1 &= 2b_1 - b_2 + b_3 , & B_2 &= b_1 + b_2 - b_3 , & B_3 &= b_2 + b_3 , \\
C_1 &= 2c_1 - c_2 + c_3 , & C_2 &= c_1 + c_2 - c_3 , & C_3 &= c_2 + c_3 . \end{align*} \n
With the help of this result, we make the following observations. (1) In the case of \(A_2 = A_3 = B_1 = C_1 = 0\), for which \(M_D\) takes a form as

\[
M_D = \begin{pmatrix}
2a_3 \sqrt{M_1} & \frac{b_2 - b_3}{2} \sqrt{M_2} & \frac{c_2 - c_3}{2} \sqrt{M_3} \\
-a_3 \sqrt{M_1} & b_2 \sqrt{M_2} & c_2 \sqrt{M_3} \\
a_3 \sqrt{M_1} & b_3 \sqrt{M_2} & c_3 \sqrt{M_3}
\end{pmatrix},
\]

(12)
$M'_\nu$ becomes

$$
M'_\nu = \begin{pmatrix}
6a_3^2 & 0 & 0 \\
0 & 3(b_2 - b_3)^2 / 4 + 3(c_2 - c_3)^2 / 4 & 3(b_2^2 - b_3^2) / 2\sqrt{6} + 3(c_2^2 - c_3^2) / 2\sqrt{6} \\
0 & 3(b_2^2 - b_3^2) / 2\sqrt{6} + 3(c_2^2 - c_3^2) / 2\sqrt{6} & (b_2 + b_3)^2 / 2 + (c_2 + c_3)^2 / 2
\end{pmatrix},
$$

(13)

which can be diagonalized by $U_{23}$. In the whole, $M_\nu$ can be diagonalized by a successive action of $U_{\text{TBM}}$ and $U_{23}$, thus realizing the TM1 mixing. (2) In the case of $A_2 = B_1 = B_3 = C_2 = 0$, for which $M_D$ takes a form as

$$
M_D = \begin{pmatrix}
(a_3 - a_2)\sqrt{M_1} & b_3 \sqrt{M_2} & (c_3 - c_2) \sqrt{M_3} \\
a_2 \sqrt{M_1} & b_3 \sqrt{M_2} & c_2 \sqrt{M_3} \\
a_3 \sqrt{M_1} & -b_3 \sqrt{M_2} & c_3 \sqrt{M_3}
\end{pmatrix},
$$

(14)

$M'_\nu$ turns out to be

$$
M'_\nu = \begin{pmatrix}
\frac{3(a_2 - a_3)^2}{2} + \frac{3(c_2 - c_3)^2}{2} & 0 & -\frac{\sqrt{3}(a_2^2 - a_3^2)}{2} - \frac{\sqrt{3}(c_2^2 - c_3^2)}{2} \\
0 & \frac{3b_3^2}{2} & 0 \\
-\frac{\sqrt{3}(a_2^2 - a_3^2)}{2} - \frac{\sqrt{3}(c_2^2 - c_3^2)}{2} & 0 & \frac{(a_2 + a_3)^2}{2} + \frac{(c_2 + c_3)^2}{2}
\end{pmatrix},
$$

(15)

which can be diagonalized by $U_{13}$. In the whole, $M_\nu$ can be diagonalized by a successive action of $U_{\text{TBM}}$ and $U_{13}$, thus realizing the TM2 mixing. To summarize, the textures of $M_D$ in Eqs. (12) (Eq. 14) can realize the TM1 and TM2 mixings, respectively. Apparently, the first (second) column of $M_D$ in Eq. (12) (Eq. 14) is proportional to the first (second) column of $U_{\text{TBM}}$ while the other two columns are orthogonal to it. It should be noted that three columns of $M_D$ are actually interchangeable, because they are on an equal footing in contributing to $M_\nu$ (see Eq. 9).

The above conclusion can be understood in a more transparent way with the help of the QR parametrization of $M_D$. In such a parametrization, a generic $M_D$ in Eq. (8) is decomposed as $M_D = U_L \Delta U^T$. Here $U_L$ is a unitary matrix as

$$
U_L = \begin{pmatrix}
a_1 / a & \frac{b_1 - a^* b}{a^2 a_1} & \frac{c_1 - a^* c}{a^2 a_1} - \frac{a^2 b^* c - b^* a a^* c}{a^2 b^2 - |a^* b|^2} \left( b_1 - \frac{a^* b}{a^2 a_1} \right) \\
\frac{b_2 - a^* b}{a^2 a_2} / a & \frac{c_2 - a^* c}{a^2 a_2} - \frac{a^2 b^* c - b^* a a^* c}{a^2 b^2 - |a^* b|^2} \left( b_2 - \frac{a^* b}{a^2 a_2} \right) \\
\frac{b_3 - a^* b}{a^2 a_3} / a & \frac{c_3 - a^* c}{a^2 a_3} - \frac{a^2 b^* c - b^* a a^* c}{a^2 b^2 - |a^* b|^2} \left( b_3 - \frac{a^* b}{a^2 a_3} \right)
\end{pmatrix},
$$

(16)

with $a^2 \equiv |a_1|^2 + |a_2|^2 + |a_3|^2$ and $a^* b \equiv a_1^* b_1 + a_2^* b_2 + a_3^* b_3$ (and so on). On the other hand, $\Delta$ is a
triangular matrix as

$$\Delta = \begin{pmatrix} a \sqrt{M_1} & \frac{a^*b}{a} \sqrt{M_2} & \frac{a^*c}{a} \sqrt{M_3} \\ 0 & \sqrt{b^2 - \frac{|a^*b|^2}{a^2}} \sqrt{M_2} & \frac{a^2 b^*c - b^*a a^*c}{a \sqrt{a^2 b^2 - |a^*b|^2}} \sqrt{M_3} \\ 0 & 0 & \sqrt{c^2 - \frac{|a^*c|^2}{a^2} - \frac{|a^2 b^*c - b^*a a^*c|^2}{a^2 (a^2 b^2 - |a^*b|^2)}} \sqrt{M_3} \end{pmatrix}.$$  \quad (17)

In this case, \( M_\nu \) appears as \( M_\nu = U_L \Delta M_R^{-1} \Delta^T U_R^T \). And the neutrino mixing matrix can be obtained as \( U = U_L U_R \) with \( U_R \) being the unitary matrix for the diagonalization \( U_R \Delta M_R^{-1} \Delta^T U_R^* = \text{diag}(m_1, m_2, m_3) \). With the help of this result, it is direct to verify the above conclusion: if \( U_R = U_{23} \) or \( U_{13} \), which will be the case for \( a^*b = a^*c = 0 \) or \( a^*b = b^*c = 0 \), then \( U \) will retain the first or second column of \( U_L \) (which is in turn proportional to the corresponding column of \( M_D \)). Therefore, in order to achieve a TM1 or TM2 mixing, \( M_D \) can be arranged in such a way that one column of it is proportional to the first or second column of \( U_{\text{TBM}} \) while the other two columns are orthogonal to it. Of course, such a guiding principle can also be employed to formulate the general texture of \( M_D \) for realizing a neutrino mixing matrix with one column being of some desired pattern.

### 3 Simplified textures of \( M_D \) for the TM1 mixing

In this section, following the Occam’s razor principle, we examine if the parameters of the general texture of \( M_D \) for realizing the TM1 mixing (that given in Eq. (12)) can be further reduced, giving more simplified textures of it. The consequences of the phenomenologically-viable simplified textures for the low-energy neutrino observables and leptogenesis will be studied.

Our analysis will be restricted to the simple but instructive scenario that three elements in the same column of \( M_D \) share a common phase, which is often the case in flavor-symmetry models \cite{FlavorSymmetry}.

Given that an overall rephasing of \( M_D \) is of no physical meaning, the second column of \( M_D \) is taken to be real without loss of generality. Furthermore, considering that the first-column phase (i.e., \( \arg(a_3) \)) only contributes to \( \rho \) in a trivial way (see Eq. (13)), the first column of \( M_D \) is also taken to be real. (For \( \arg(a_3) \neq 0 \), one just needs to make a simple replacement \( \rho \rightarrow \rho + \arg(a_3) \) for our results.) We are therefore left with only one phase parameter, the third-column phase, which will be responsible for both the CP-violating effects at low energies and leptogenesis. It is useful to note that this phase works with a period of \( \pi \) in determining the low-energy neutrino observables (see Eq. (13)) and the CP asymmetries responsible for leptogenesis (as will be seen from Eq. (33)). To summarize, under our setup, \( a_3, b_2 \) and \( b_3 \) are real parameters while \( c_2 \) and \( c_3 \) are complex parameters with a common phase. Accordingly, for convenience of the following discussions, \( M_D \) in Eq. (12) is reexpressed as

$$M_D = \begin{pmatrix} 2l \sqrt{M_1} & mx \sqrt{M_2} & ne^{i\phi}y \sqrt{M_3} \\ -l \sqrt{M_1} & m(1+x) \sqrt{M_2} & ne^{i\phi}(1+y) \sqrt{M_3} \\ l \sqrt{M_1} & m(1-x) \sqrt{M_2} & ne^{i\phi}(1-y) \sqrt{M_3} \end{pmatrix},$$  \quad (18)

with \( l, m, n, x \) and \( y \) being real parameters and \( \phi \) the only phase parameter. Correspondingly, \( M_\nu' \) in Eq. (13) becomes

$$M_\nu' = \begin{pmatrix} 6l^2 & 0 & 0 \\ 0 & 3m^2 x^2 + 3n^2 y^2 e^{2i\phi} & \sqrt{6}m^2 x + \sqrt{6}n^2 y e^{2i\phi} \\ 0 & \sqrt{6}m^2 x + \sqrt{6}n^2 y e^{2i\phi} & 2m^2 + 2n^2 e^{2i\phi} \end{pmatrix}.$$  \quad (19)
the following relations for $\pi$:

$$-1 -1/2 0 1/2 1$$

| pattern | $(-1,0,2)^T$ | $(-1,1,3)^T$ | $(0,1,1)^T$ | $(1,3,1)^T$ | $(1,2,0)^T$ |

Table 2: For the TM1 mixing, the particular values of $x$ ($y$) and the corresponding column patterns.

Our analysis will be further restricted to the following simplified textures of $M_D$, which will be instructive for the model-building exercises: (1) there are some vanishing elements (a sign of Abelian flavor symmetries \[13\]); (2) there are some equalities among the non-vanishing elements (a sign of non-Abelian flavor symmetries \[8\]). It is direct to see that such textures of $M_D$ correspond to some particular values of $x$ and $y$: (1) the value $-1$, $0$ or $1$ of $x$ ($y$) corresponds to a column pattern as $(-1,0,2)^T$, $(0,1,1)^T$ or $(1,2,0)^T$ which has one vanishing element; (2) the value $-1/2$, $0$ or $1/2$ of $x$ ($y$) corresponds to a column pattern as $(-1,1,3)^T$, $(0,1,1)^T$ or $(1,3,1)^T$ where there is an equality between two elements. To summarize, the particular values of $x$ and $y$ that are phenomenologically appealing include $-1$, $-1/2$, $0$, $1/2$ and $1$, the column patterns corresponding to which are listed in Table 2. On the basis of particular $(x, y)$ combinations, we will further examine if $\phi$ can also take a particular value such as $\pi/3$ (a sign of CP symmetries).

Before proceeding, let us enumerate the formulas useful for our numerical calculations. For $M_D$ in Eq. (18), the resulting neutrino mixing matrix can be obtained as $U_{\text{TM1}} = U_{\text{TBM}} U_{23}$. Here $U_{23}$ is the unitary matrix for diagonalizing $M'_\nu$ in Eq. (19), whose parameters $\theta$ and $\phi$ can be calculated as

$$\tan 2\theta = \frac{2 |M'_{22}M'_{23} + M'_{23}M'_{33}|}{|M'_{33}|^2 - |M'_{22}|^2}, \quad \varphi = \arg (M'_{22}M'_{23}+M'_{23}M'_{33}),$$

with $M'_{ij}$ denoting the $ij$th element of $M'_\nu$. From this equation one can see that the equality between $x$ and $y$ is denied, which would otherwise lead to the unacceptable $\varphi = 0$. Then, the three mixing angles and $\delta$ can be extracted from $U_{\text{TM1}}$ according to the formulas in Eqs. (6, 7). On the other hand, the resulting neutrino mass eigenvalues are given by

$$m_1 e^{2i\alpha} = M'_{11} = l^2,$$

$$m_2 e^{2i\beta} = M'_{22} \cos^2 \theta + M'_{33} \sin^2 \theta e^{-2i\varphi} - M'_{23} \sin 2\theta e^{-i\varphi},$$

$$m_3 e^{2i\gamma} = M'_{33} \cos^2 \theta + M'_{22} \sin^2 \theta e^{2i\varphi} + M'_{23} \sin 2\theta e^{i\varphi},$$

from which $\rho$ and $\sigma$ can be obtained as

$$\rho = \varphi - \delta + \alpha - \gamma,$$

$$\sigma = \varphi - \delta + \beta - \gamma.$$

It is easy to see that, under the transformation $\phi \to -\phi$, the results for the CP phases undergo a sign reversal while those for the neutrino masses and mixing angles keep invariant.

3.1 Phenomenologically-viable simplified textures

Now, we confront $M_D$ against the experimental results to examine if $x$ and $y$ can take some particular values. Let us first perform the study in the NO case, for which the left panel of Fig. 1 shows the values of $y$ versus $x$ that can be phenomenologically viable within the $3\sigma$ level. The results are obtained in a way as follows: for randomly selected values of $x$ and $y$ in the range of $-2$ to $2$, $\phi$ in the range of $0$ to $\pi$ and $m_1$ in the range of 0.001 eV to 0.1 eV, the values of $l$, $m$ and $n$ are determined by virtue of the following relations for $M'_\nu$ in Eq. (19):

$$m_1 m_2 m_3 = |\text{Det}(M'_{\nu})|,$$

$$m_1^2 + m_2^2 + m_3^2 = \text{Tr}(M'^T_{\nu} M'_{\nu}),$$

$$m_1 = l^2,$$

$$m_2 = 2$$

$$m_3 = 3.$$
where \( m_2 = \sqrt{m_1^2 + \Delta m_{21}^2} \) and \( m_3 = \sqrt{m_1^2 + \Delta m_{31}^2} \) with \( \Delta m_{21}^2 \) and \( \Delta m_{31}^2 \) taking random values in their 3\( \sigma \) ranges. Then, we check if the resulting values of \( \theta \) and \( \varphi \) (calculated as in Eq. (20)) can give some values of \( s_{13}^2 \) and \( s_{23}^2 \) (calculated as in Eq. (6)) in their 3\( \sigma \) ranges. (Meanwhile, the values of \( \theta_{12} \) and \( \delta \) are determined as in Eq. (7).) If yes, then these values of \( x \) and \( y \) will be recorded. A repetition of the above procedure for enough times yields the results in Fig. 1 For the present, we have not taken into account the experimental constraint on \( \delta \), but will do so in the following \( \chi^2 \) calculations.

It is apparent that the results exhibit a symmetry with respect to the interchange \( x \leftrightarrow y \). This can be understood as follows: after a successive action of \( x \leftrightarrow y, m \leftrightarrow n, \phi \rightarrow -\phi \) and \( M'_{\nu} \rightarrow M'_{\nu}e^{2i\phi} \), \( M'_{\nu} \) in Eq. (19) keeps invariant except for the replacement \( l^2 \rightarrow l^2e^{2i\phi} \). This means that the results of \( (x, y) = (y_0, x_0) \) can be obtained from those of \( (x, y) = (x_0, y_0) \) by making the replacements \( \phi \rightarrow -\phi \) and \( \rho \rightarrow \rho + \phi \), where \( x_0 \) and \( y_0 \) are any given values of \( x \) and \( y \). For this reason, we will just consider the \( x < y \) cases. Furthermore, there is a connection between the results of \( (x, y) = (x_0, y_0) \) and those of \( (x, y) = (-y_0, -x_0) \): after a successive action of \( x \leftrightarrow -y, m \leftrightarrow n \) and \( M'_\nu \rightarrow M'_\nu e^{-2i\phi} \), \( M'_\nu \) in Eq. (19) becomes

\[
M'_\nu = \begin{pmatrix} 6l^2e^{-2i\phi} & 0 & 0 \\ 0 & 3m^2x^2 + 3n^2y^2e^{-2i\phi} & -(\sqrt{6m^2x} + \sqrt{6n^2y}e^{-2i\phi}) \\ 0 & -\left(\sqrt{6m^2x} + \sqrt{6n^2y}e^{-2i\phi}\right) & 2m^2 + 2n^2e^{-2i\phi} \end{pmatrix}.
\] (24)

From Eqs. (6, 7, 20, 22) it is deduced that the results of \( (x, y) = (-y_0, -x_0) \) can be obtained from those of \( (x, y) = (x_0, y_0) \) by making the replacements \( \rho \rightarrow -(\rho + \phi) \), \( \sigma \rightarrow -\sigma \), \( \delta \rightarrow \pi - \delta \) and \( \Delta s_{23}^2 \rightarrow -\Delta s_{23}^2 \) (for \( \Delta s_{23}^2 \equiv s_{23}^2 - 1/2 \)).

Let us consider the possibility that both \( x \) and \( y \) take some particular values, for which all the three columns of \( M_D \) take some simple but instructive patterns. It is found that \( (x, y) = (-1, -1/2), (-1, 0), (-1, 1/2), (-1/2, 0), (-1/2, 1), (0, 1/2), (0, 1) \) and \( (1/2, 1) \) can be phenomenologically viable within the 3\( \sigma \) level. Note that the last four cases are the \( x \leftrightarrow -y \) counterparts of the first four cases, so their results can be related by the replacement rules below Eq. (24). For the first (last) four cases, Fig. 2 (Fig. 3) shows the parameter spaces of \( \phi \) and the predictions for \( \delta, \rho \) and \( \sigma \) versus \( m_1 \), which are summarized in Table 3 Some comments about these results are given as follows. (1) The predictions for \( \delta \) are around \( \pm \pi/2 \), in good agreement with the experimental preference for \( \delta \sim -\pi/2 \). (2) The \( (x, y) = (-1/2, 0) \) and \( (0, 1/2) \) combinations, which can be phenomenologically viable for a negligibly
Figure 2: For the TM1 mixing and NO case, the parameter spaces of $\phi$ versus $m_1$ for $(x,y) = (-1,-1/2), (-1,0), (-1,1/2)$ and $(-1/2,0)$ to be phenomenologically viable within the $3\sigma$ level, and the predictions for $|\delta|, \rho$ and $\sigma$.

Figure 3: For the TM1 mixing and NO case, the parameter spaces of $\phi$ versus $m_1$ for $(x,y) = (-1/2,1), (0,1/2), (0,1)$ and $(1/2,1)$ to be phenomenologically viable within the $3\sigma$ level, and the predictions for $|\delta|, \rho$ and $\sigma$.
can be phenomenologically viable within the 3σ level. (3) It is obvious that φ = π/2 would render $M'_\nu$ in Eq. (19) real, which in turn leads to trivial δ, ρ, and σ. Nevertheless, for $(x, y) = (-1, -1/2)$ and $(1/2, 1)$, φ ≃ π/2 leads to $|\delta| \approx \pi/2$.

A careful analysis reveals that this is because there occurs a large accidental cancellation for the real part of $M_{23}$, making its imaginary part (which is controlled by the small deviation of φ from π/2 and should have been subdominant) dominant, which subsequently leads to a nearly maximal $|\varphi|$ and thus $|\delta|$.

Then, we further examine if φ can also take a particular value (out of ±π/6, ±π/4, ±π/3), on the basis of particular $(x, y)$ combinations. Table 4 lists the particular $(x, y, \phi)$ combinations that can be phenomenologically viable within the 3σ level and their predictions for the low-energy neutrino observables at $\chi^2_{\text{min}}$. Here the $\chi^2$ function is defined as

$$\chi^2 = \sum_i \left( \frac{O_i - \bar{O}_i}{\sigma_i} \right)^2 ,$$

(25)

where the sum is over three mixing angles, two neutrino mass squared differences and δ, and $O_i$, $\bar{O}_i$ and $\sigma_i$ denote their predicted values, best-fit values and 1σ errors, respectively. Note that the experimental constraint on δ has now been taken into account. For $(x, y) = (-1, -1/2)$ and $(1/2, 1)$, although φ is not allowed to exactly take π/2, we have also listed their results in Table 4 which might be instructive for the model-building exercises.

As for the IO case, the right panel of Fig. 1 shows the values of y versus x that can be phenomenologically viable within the 3σ level. It turns out that only the particular combination $(x, y) = (-1, -1/2)$ can be phenomenologically viable within the 3σ level, for which φ is not allowed to exactly take π/2 but is very close to it.

### 3.2 Consequences for leptogenesis

Let us proceed to study the consequences of the above phenomenologically-viable particular $(x, y, \phi)$ combinations for leptogenesis. As is known, the seesaw model via the leptogenesis mechanism offers an appealing explanation for the baryon asymmetry of the Universe [16]

$$Y_B \equiv \frac{n_B - n_{\bar{B}}}{s} = (8.67 \pm 0.15) \times 10^{-11} ,$$

(26)
$\chi^2_{\min}$, and the values of $M_2$ ($M_3$) for leptogenesis to be viable in the $M_2 < M_3$ ($M_3 < M_2$) case. The units of $m_l$, $|\Delta m^2_{21}|$, and $|\Delta m^2_{31}|$ are eV, $10^{-5}$ eV$^2$, and $10^{-3}$ GeV, respectively.

with $n_B$ ($n_{\bar{B}}$) being the baryon (anti-baryon) number density and $s$ the entropy density. This mechanism proceeds in a way as follows [17, 18]: a lepton asymmetry $Y_L \equiv (n_L - n_{\bar{L}})/s$ is firstly generated during the decays of right-handed neutrinos, and then partially converted into the baryon asymmetry through the sphaleron process [19]. According to the temperature where leptogenesis takes place (approximately the mass of the right-handed neutrino responsible for leptogenesis), there are three distinct leptogenesis regimes [20]. (1) Unflavored regime: in the temperature range above $10^{12}$ GeV, the charged-lepton Yukawa ($y_\alpha$) interactions have not yet entered thermal equilibrium, so the three lepton flavors are indistinguishable and thus to be treated in a universal way. (2) Two-flavor regime: in the temperature range $10^9$—$10^{12}$ GeV, the $y_\tau$-related interactions enter thermal equilibrium, making the $\tau$ flavor distinguishable from the other two flavors which remain indistinguishable. In this regime, the $\tau$ flavor and a superposition of the $e$ and $\mu$ flavors should be treated separately. (3) Three-flavor regime: in the temperature range below $10^9$ GeV where the $y_\mu$-related interactions also enter thermal equilibrium, all the three flavors are distinguishable and should be treated separately.

Here we consider the scenario of hierarchical right-handed neutrino masses, for which the contribution to leptogenesis mainly comes from the lightest right-handed neutrino $N_I$. In the unflavored regime, the final baryon asymmetry can be calculated as

$$Y_B = -cr\varepsilon_I\kappa(\bar{m}_I),$$

where $c = 28/79$ is the conversion efficiency from the lepton asymmetry to the baryon asymmetry through the sphaleron process [22], and $r \approx 3.9 \times 10^{-3}$ is the ratio of the equilibrium number density of $N_I$ to the entropy density at the temperature above $M_I$. $\varepsilon_I$ is the total CP asymmetry for the decays of $N_I$

$$\varepsilon_I = \frac{1}{8\pi(M_D^2M_D^\dagger)_{II}v^2} \sum_{J \neq I} \text{Im} \left[ (M_D^I)^2_{JI} \right] F \left( \frac{M^2_J}{M^2_I} \right),$$

For some exceptional scenarios, see Refs. [21].
which is a sum of the flavored CP asymmetries \[ [17] [23]

\[
\varepsilon_{I\alpha} \equiv \frac{\Gamma(N_I \to L_{\alpha} + H) - \Gamma(N_I \to \bar{L}_{\alpha} + \bar{H})}{\sum_\alpha [\Gamma(N_I \to L_{\alpha} + H) + \Gamma(N_I \to \bar{L}_{\alpha} + \bar{H})]}
\]

\[
= \frac{1}{8\pi(M_D^I M_D^I)_{IJ} v^2} \sum_{j \neq I} \left\{ \text{Im} \left[ (M_D^I)^\dagger (M_D^I)_{\alpha j} (M_D^I)^\dagger M_D^I \right] F \left( \frac{M_D^J}{M_I^J} \right) \right. \\
\left. + \text{Im} \left[ (M_D^I)^\dagger (M_D^I)_{\alpha j} (M_D^I)^\dagger M_D^I \right] G \left( \frac{M_D^J}{M_I^J} \right) \right\}
\]

(29)

with \( v = 174 \text{ GeV} \) being the Higgs vacuum expectation value, \( F(x) = \sqrt{x} \left( (2 - x)/(1 - x) + (1 + x) \ln[x/(1 + x)] \right) \) and \( G(x) = 1/(1 - x) \). Finally, \( \kappa(\tilde{m}_I) \) is an efficiency factor accounting for the washout effects due to the inverse-decay and lepton-number-violating scattering processes. Its value is determined by the washout mass parameter \( \tilde{m}_I \), which is a sum of the flavored washout mass parameters

\[
\tilde{m}_{I\alpha} = \frac{|(M_D^I)^\dagger|^2}{M_I^J}.
\]

(30)

In our numerical calculations, the following empirical fit formula \[ [24] \] will be employed to calculate the values of this efficiency factor

\[
\frac{1}{\kappa(x)} \simeq \frac{3.3 \times 10^{-3} \text{ eV}}{x} + \left( \frac{x}{5.5 \times 10^{-4} \text{ eV}} \right)^{1.16}.
\]

(31)

In the two-flavor regime, the final baryon asymmetry receives two contributions from \( \varepsilon_{I_\tau} \) and \( \varepsilon_{I_\gamma} = \varepsilon_{I_e} + \varepsilon_{I_\mu} \) which are subject to different washout effects controlled by \( \tilde{m}_{I_\tau} \) and \( \tilde{m}_{I_\gamma} = \tilde{m}_{I_e} + \tilde{m}_{I_\mu} \) \[ [20] \]

\[
Y_B \simeq -c r \left[ \varepsilon_{I_\tau} \kappa \left( \frac{390}{589} \tilde{m}_{I_\tau} \right) + \varepsilon_{I_\gamma} \kappa \left( \frac{417}{589} \tilde{m}_{I_\gamma} \right) \right].
\]

(32)

Due to the special form of \( M_D \) in Eq. (18), which leads to \( (M_D^I M_D^I)_{12} = (M_D^I M_D^I)_{13} = 0, N_1 \) is not relevant for leptogenesis. One is thus left with the following two possibilities: (1) for \( M_2 < M_3 \), the contribution to leptogenesis mainly comes from \( N_2 \). The CP asymmetries for the decays of \( N_2 \) are explicitly given by

\[
\varepsilon_{2\tau} = \frac{M_3 n^2 (1 - x)(1 - y)(2 + 3xy)}{8\pi v^2 (2 + 3x^2)} F \left( \frac{M_2^J}{M_2}\right) \sin 2\phi,
\]

\[
\varepsilon_2 = \frac{M_3 n^2 (2 + 3xy)^2}{8\pi v^2(2 + 3x^2)} F \left( \frac{M_3^J}{M_2}\right) \sin 2\phi,
\]

\[
\varepsilon_{2\gamma} = \varepsilon_2 - \varepsilon_{2\tau}.
\]

(33)

As mentioned above, \( \phi \) works with a period of \( \pi \) in determining the CP asymmetries for leptogenesis, so \( \phi = \pi/2 \) would prohibit a viable leptogenesis. For the phenomenologically-viable particular \( (x, y, \phi) \) combinations listed in Table 4, the values of \( M_2 \) for leptogenesis to be viable are calculated (given in the last column of Table 4). In obtaining these results, we have taken \( M_3 = 3M_2 \) as a benchmark value. (In fact, the dependence of the generated baryon asymmetry on the concrete ratio of \( M_3 \) to \( M_2 \) is mild provided they are hierarchical.) The results show that leptogenesis works successfully in the unflavored regime for \( (x, y, \phi) = (-1, -1/2, 0.497\pi), (-1, 1/2, 1/6\pi) \) and \( (1/2, 1, 0.484) \) and in the two-flavor regime for the other \( (x, y, \phi) \) combinations in the NO case. (2) For \( M_3 < M_2 \), the contribution to leptogenesis mainly comes from \( N_3 \). The explicit expressions of the CP asymmetries
for the decays of $N_3$ can be obtained from Eq. (33) by making the replacements $x \leftrightarrow y$, $M_2 \leftrightarrow M_3$ and $\phi \rightarrow -\phi$. In obtaining the value of $M_3$ for leptogenesis to be viable, we have taken $M_2 = 3M_3$ as a benchmark value instead. It is found that leptogenesis can only work successfully (in the unflavored regime) for $(x, y, \phi) = (-1, -1/2, 0.504\pi)$ in the IO case.

4 Simplified textures of $M_D$ for the TM2 mixing

In this section, we perform a parallel study for the TM2 mixing. Namely, we examine if the parameters of the general texture of $M_D$ for realizing the TM2 mixing (that given in Eq. (14)) can be further reduced, giving more simplified textures of it, and study the consequences of the phenomenologically-viable simplified textures for the low-energy neutrino observables and leptogenesis. As will be seen, the results for the TM2 mixing have a lot in common with those for the TM1 mixing.

As before, the first and second columns of $M_D$ are taken to be real while the third-column elements to share a common phase. Accordingly, for convenience of the following discussions, $M_D$ in Eq. (14) is reexpressed as

$$M_D = \begin{pmatrix} 2lx\sqrt{M_1} & m\sqrt{M_2} & 2ne^{i\phi}y\sqrt{M_3} \\ l(1-x)\sqrt{M_1} & m\sqrt{M_2} & ne^{i\phi}(1-y)\sqrt{M_3} \\ l(1+x)\sqrt{M_1} & -m\sqrt{M_2} & ne^{i\phi}(1+y)\sqrt{M_3} \end{pmatrix}.$$  \hspace{1cm} (34)

Correspondingly, $M'_\nu$ in Eq. (15) becomes

$$M'_\nu = \begin{pmatrix} 6l^2x^2 + 6n^2y^2e^{2i\phi} & 0 & 2\sqrt{3l^2x} + 2\sqrt{3n^2y}e^{2i\phi} \\ 0 & 3m^2 & 0 \\ 2\sqrt{3l^2x} + 2\sqrt{3n^2y}e^{2i\phi} & 0 & 2l^2 + 2n^2e^{2i\phi} \end{pmatrix}.$$  \hspace{1cm} (35)

Now the particular values of $x$ and $y$ which are phenomenologically viable include $-1$, $-1/2$, 0, 1/2 and 1, the column patterns corresponding to which are listed in Table 5.

For $M_D$ in Eq. (34), the resulting neutrino mixing matrix can be obtained as $U_{\text{TM2}} = U_{\text{TBM}}U_{13}$. Here $U_{13}$ is the unitary matrix for diagonalizing $M'_\nu$ in Eq. (35), whose parameters $\theta$ and $\phi$ can be calculated as

$$\tan 2\theta = \frac{2|M_{11}'M_{13}'' + M_{13}'M_{33}''|}{|M_{33}''|^2 - |M_{11}'|^2}, \quad \phi = \arg \left( M_{11}'M_{13}' + M_{13}'M_{33}' \right).$$  \hspace{1cm} (36)
Table 5: For the TM2 mixing, the particular values of $x$ ($y$) and the corresponding column patterns.

| $x$ ($y$) | $-1$ | $-1/3$ | 0 | $1/3$ | 1 |
|-----------|------|--------|---|-------|---|
| pattern   | $(-1, 1, 0)^T$ | $(-1, 2, 1)^T$ | $(0, 1, 1)^T$ | $(1, 1, 2)^T$ | $(1, 0, 1)^T$ |

Figure 5: For the TM2 mixing and NO case, the parameter spaces of $\phi$ versus $m_1$ for $(x, y) = (-1, -1/3), (-1, 0)$ and $(-1/3, 0)$ to be phenomenologically viable within the 3$\sigma$ level, and the predictions for $|\delta|, \rho$ and $\sigma$.

From this equation one can see that the equality between $x$ and $y$ is denied, which would otherwise lead to the unacceptable $\varphi = 0$. Then, the three mixing angles and $\delta$ can be extracted from $U_{\text{TM2}}$ according to the formulas in Eqs. (6, 7). On the other hand, the resulting neutrino mass eigenvalues are given by

$$m_1 e^{2i\alpha} = M'_{11} \cos^2 \theta + M'_{33} \sin^2 \theta e^{-2i\varphi} - M'_{13} \sin 2\theta e^{-i\varphi},$$

$$m_2 e^{2i\beta} = M'_{22} = 3m^2,$$

$$m_3 e^{2i\gamma} = M'_{33} \cos^2 \theta + M'_{11} \sin^2 \theta e^{2i\varphi} + M'_{13} \sin 2\theta e^{i\varphi},$$

from which $\rho$ and $\sigma$ can also be calculated as in Eq. (22). One can also see that, under the transformation $\phi \rightarrow -\phi$, the results for the CP phases undergo a sign reversal while those for the neutrino masses and mixing angles keep invariant.

It is analogously deduced that the results of $(x, y) = (y_0, x_0)$ can be obtained from those of $(x, y) = (x_0, y_0)$ by making the replacements $\phi \rightarrow -\phi$ and $\sigma \rightarrow \sigma + \phi$, so we will just consider the $x < y$ cases. Furthermore, the results of $(x, y) = (-y_0, -x_0)$ can be obtained from those of $(x, y) = (x_0, y_0)$ by making the replacements $\rho \rightarrow -\rho$, $\sigma \rightarrow -(\sigma + \phi)$, $\delta \rightarrow \pi - \delta$ and $\Delta s^2_{23} \rightarrow -\Delta s^2_{23}$.

Now let us consider the possibility that both $x$ and $y$ take some particular values. Fig. 4 shows the values of $y$ versus $x$ that can be phenomenologically viable within the 3$\sigma$ level in the NO (left).
and IO (right) cases. The results are obtained in the same way as for the TM1 mixing, except that now the values of $l$, $m$ and $n$ are determined by virtue of the following relations for $M'_\nu$ in Eq. (35):

$$m_1 m_2 m_3 = |\text{Det}(M'_\nu)|, \quad m_1^2 + m_2^2 + m_3^2 = \text{Tr}(M'_{\nu}^\dagger M'_{\nu}), \quad m_2 = 3m_1^2. \quad (38)$$

In the NO case, $(x, y) = (-1, -1/3), (-1, 0), (-1/3, 0), (0, 1/3), (0, 1)$ and $(1/3, 1)$ can be phenomenologically viable within the $3\sigma$ level. Note that the last three cases are the $x \leftrightarrow -y$ counterparts of the first three cases, so their results can be related by the aforementioned replacement rules. For the first (last) three cases, Fig. 5 (Fig. 6) shows the parameter spaces of $\phi$ and the predictions for $\delta, \rho$ and $\sigma$ versus $m_1$, which are summarized in Table 6. Some comments about these results are given as follows.

(1) The allowed ranges of these parameters are significantly larger than those for the TM1 mixing. This can be attributed to the results below Eq. (6). (2) None of these cases can be phenomenologically viable for a negligibly small $m_1$ and thus accommodated in the minimal seesaw framework. (3) For $(x, y) = (-1, -1/3)$ and $(1/3, 1)$, due to a large accidental cancellation, $\phi \simeq \pi/2$ has chance to induce $|\delta| \sim \pi/2$. As for the IO case, only $(x, y) = (-1, -1/3)$ and $(1/3, 1)$ can be phenomenologically viable within the $3\sigma$ level.

Then, we further examine if $\phi$ can also take a particular value, on the basis of particular $(x, y)$ combinations. Table 7 lists the phenomenologically-viable particular $(x, y, \phi)$ combinations and their predictions for the low-energy neutrino observables at $\chi^2_{\text{min}}$. Note that for $(x, y) = (-1, -1/3)$ and $(1/3, 1)$, $\phi$ is not allowed to exactly take $\pi/2$ but is very close to it in both the NO and IO cases.

Finally, we study the consequences of the phenomenologically-viable particular $(x, y, \phi)$ combinations for leptogenesis. Due to the special form of $M_D$ in Eq. (14), which leads to $(M_D^\dagger M_D)_{12} = (M_D^\dagger M_D)_{23} = 0$, $N_2$ is not relevant for leptogenesis. One is thus left with the following two possibilities:
Table 6: For the TM2 mixing, the parameter spaces of $\phi$ for the phenomenologically-viable particular $(x, y)$ combinations, and the predictions for $m_1$, $\delta$, $\rho$ and $\sigma$.

| $x$ | $y$ | $\phi/\pi$ | $\Delta m_{21}^2$ | $\Delta m_{31}^2$ | $s_{12}^2$ | $s_{13}^2$ | $s_{23}^2$ | $\delta/\pi$ | $\rho/\pi$ | $\sigma/\pi$ | $M_1$ (eV) |
|-----|-----|------------|-----------------|-----------------|-----------|-----------|-----------|-----------|-----------|-----------|-------------|
| NO  | $-1$| $-1/3$     | $\pm(0.484-0.499)$ | $>0.022$        | $\pm(0.54-0.99)$ | $\mp(0.484-0.499)$ | $\mp(0.486-0.499)$ |
|     | $-1$| 0          | $\pm(0.00-0.41)$  | 0.011—0.024     | $\mp(0.31-0.99)$ | $\pm(0.00-0.38)$ | $\mp(0.00-0.40)$ |
|     | $-1/3$| 0          | $\pm(0.01-0.40)$  | 0.003—0.008     | $\mp(0.31-0.99)$ | $\pm(0.00-0.32)$ | $\mp(0.00-0.36)$ |
|     | 0   | $1/3$      | $\pm(0.21-0.50)$  | 0.004—0.009     | $\mp(0.31-0.99)$ | $\pm(0.12-0.50)$ | $\mp(0.00-0.05)$ |
|     | 1   | $1/3$      | $\pm(0.484-0.496)$ | $>0.040$        | $\pm(0.31-0.46)$ | $\pm(0.486-0.496)$ | $\pm(0.000-0.002)$ |
| IO  | $-1$| $-1/3$     | $\pm(0.486-0.496)$ | $>0.040$        | $\pm(0.31-0.46)$ | $\pm(0.486-0.496)$ | $\pm(0.487-0.497)$ |
|     | $1/3$| 1          | $\pm(0.484-0.499)$ | $>0.022$        | $\pm(0.54-0.99)$ | $\pm(0.484-0.499)$ | $\pm(0.000-0.002)$ |

(1) for $M_1 < M_3$, the contribution to leptogenesis mainly comes from $N_1$. The CP asymmetries for the decays of $N_1$ are explicitly given by

$$
\varepsilon_{1\tau} = \frac{M_3 n^2 (1 + x) (1 + y) (1 + 3 xy)}{8 \pi v^2 (1 + 3 x^2)} F \left( \frac{M_3^2}{M_1^2} \right) \sin 2\phi ,
$$

$$
\varepsilon_1 = \frac{M_3 n^2 (1 + 3 xy)^2}{8 \pi v^2 (1 + 3 x^2)} F \left( \frac{M_3^2}{M_1^2} \right) \sin 2\phi ,
$$

$$
\varepsilon_{1\gamma} = \varepsilon_1 - \varepsilon_{1\tau} .
$$

(39)

In obtaining the values of $M_1$ for leptogenesis to be viable (given in the last column of Table 7), we have taken $M_3 = 3 M_1$ as a benchmark value. The results show that leptogenesis works in the unflavored regime for $(x, y, \phi) = (1/3, 1, 0.496\pi)$ and in the two-flavor regime for the other $(x, y, \phi)$ combinations in the NO case. (2) For $M_3 < M_1$, the contribution to leptogenesis mainly comes from $N_3$. The explicit expressions of the CP asymmetries for the decays of $N_3$ can be obtained from Eq. (39) by making the
replacements $x \leftrightarrow y$, $M_1 \leftrightarrow M_3$ and $\phi \rightarrow -\phi$. In obtaining the values of $M_3$ for leptogenesis to be viable, we have taken $M_1 = 3M_3$ as a benchmark value instead. It is found that leptogenesis can only work successfully for the phenomenologically-viable particular $(x, y, \phi)$ combinations in the IO case.

5 Summary

In summary, due to its simple structure and phenomenologically-appealing consequences, the trimaximal mixing has attracted a lot of attention. In this paper, in the basis of $M_R$ being diagonal, we survey the simplified textures of $M_D$ for realizing this mixing and their consequences for the low-energy neutrino observables and leptogenesis.

We first formulate the general texture of $M_D$ for realizing the trimaximal mixing (see Eqs. (12, 14)), and then, following the Occam’s razor principle, examine if its parameters can be further reduced, giving more simplified textures of it. Our analysis is restricted to the simple but instructive scenario that three elements in the same column of $M_D$ share a common phase. Furthermore, for the TM1 (TM2) mixing, only the phase difference between the second and third (first and third) columns is responsible for $\delta$ and leptogenesis, while the phase of the first (second) column only contributes to $\rho$ ($\sigma$) in a trivial way. Therefore, without loss of generality, our analysis is further restricted to the scenario that there is only one phase parameter $\phi$ (i.e., the third-column phase), for which $M_D$ can be conveniently reexpressed as in Eqs. (18, 34).

It is immediate to note that the equality between $x$ and $y$ is denied, which would otherwise lead to the unacceptable $\delta = 0$. For the TM1 (TM2) mixing, the results of $(x, y) = (y_0, x_0)$ can be obtained from those of $(x, y) = (x_0, y_0)$ by making the replacements $\phi \rightarrow -\phi$ and $\rho \rightarrow \rho + \phi$ ($\sigma \rightarrow \sigma + \phi$). So we just consider the $x < y$ cases. Furthermore, the results of $(x, y) = (-y_0, -x_0)$ can be obtained from those of $(x, y) = (x_0, y_0)$ by making the replacements $\rho \rightarrow -(\rho + \phi)$ ($\sigma \rightarrow -(\sigma + \phi)$), $\sigma \rightarrow -\sigma$ ($\rho \rightarrow -\rho$), $\delta \rightarrow \pi - \delta$ and $\Delta s^2_{23} \rightarrow -\Delta s^2_{23}$.

Our attention is paid to the simplified textures of $M_D$ where there are some vanishing elements (a sign of Abelian flavor symmetries) or equalities among non-vanishing elements (a sign of non-Abelian flavor symmetries). Such textures of $M_D$ correspond to some particular values of $x$ and $y$ (see Tables 2, 5). The phenomenologically-viable particular $(x, y)$ combinations and the allowed ranges of $m_1$, $\phi$, $\delta$, $\rho$ and $\sigma$ are listed in Tables 3, 6. On the basis of these particular $(x, y)$ combinations, we further examine if $\phi$ can also take a particular value (a sign of CP symmetries). The phenomenologically-viable particular $(x, y, \phi)$ combinations and their predictions for the low-energy neutrino observables at $\chi^2_{\min}$ are listed in Tables 4, 7. Finally, the consequences of these particular $(x, y, \phi)$ combinations for leptogenesis are studied. Due to the special form of $M_D$, for the TM1 (TM2) mixing, only $N_2$ and $N_3$ ($N_1$ and $N_3$) are relevant for leptogenesis. The values of $M_2$ or $M_3$ ($M_1$ or $M_3$) for leptogenesis to be viable in the $M_2 < M_3$ or $M_3 < M_2$ ($M_1 < M_3$ or $M_3 < M_1$) case are calculated.

The simplified textures of $M_D$ for realizing the trimaximal mixing studied in this paper constitute some benchmark flavor models testable for the future measurements. If any one of them happens to be close to the truth, then it will need more detailed studies (e.g., a realistic flavor-symmetry model realization, the impacts of renormalization group running effects).

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