Intensity interferometry for ultralight bosonic dark matter detection

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Ultralight bosonic dark matter (UBDM) can be described by a classical wave-like field oscillating near the Compton frequency of the bosons. If a measurement scheme for the direct detection of UBDM interactions is sensitive to a signature quadratic in the field, then there is a near-zero-frequency (dc) component of the signal. This opens the possibility of a detection scheme analogous to Hanbury Brown and Twiss intensity interferometry. Assuming that the UBDM is virialized in the galactic gravitational potential, the random velocities produce slight deviations from the Compton frequency. These result in stochastic fluctuations of the intensity on a time scale determined by the spread in kinetic energies. In order to mitigate ubiquitous local low-frequency noise, a network of sensors can be used to search for the stochastic intensity fluctuations by measuring cross-correlation between the sensors. This method is inherently broadband, since a large range of Compton frequencies will yield near-zero-frequency components within the sensor bandwidth that can be searched for simultaneously. Measurements with existing sensor networks have sufficient sensitivity to search experimentally unexplored parameter space.

A wide variety of evidence suggests the existence of dark matter, an invisible substance that constitutes a large fraction of the matter in the Universe [1–3]. Despite decades of research, its microscopic nature remains unknown. A promising hypothesis is that dark matter predominantly consists of ultralight bosons with masses \( m \approx 1 \text{ eV}/c^2 \), such as axions [4–6], axion-like particles (ALPs) [7, 8], or hidden photons [9–11]. Ultralight bosons can couple to Standard Model (SM) particles through a variety of “portals” [12, 13], which have been used to search for ultralight bosonic dark matter (UBDM) — see, for example, Refs. [14–31].

Numerous experiments looking for UBDM are based on resonant systems, which require accurate tuning to the unknown Compton frequency, \( \omega_c = m_c c^2/h \), in order to be sensitive to UBDM. Therefore, a time-consuming scan of tunable parameters must be performed to search a wide range of masses. However, if a search is based on the measurement of the intensity of the UBDM, part of the signal is down-converted to near-zero frequency [32, 33], regardless of the particular Compton frequency of the UBDM.

In the commonly considered simplest version of the standard halo model (SHM) for dark matter [34–36], the net UBDM field results from the superposition of numerous virialized bosons [37]. Such a model assumes minimal self-interactions [38] and ignores possible non-virialized dark matter streams [39] and composite dark matter structures such as boson stars [40] or topological defects [41]. The velocity dispersion of the dark matter particles in the neighborhood of our solar system is \( v_0 \approx 10^{-3}c \). Consequently, the oscillation frequencies associated with the virialized bosons are Doppler-shifted. This generates a fractional shift from \( \omega_c \) and a spread of frequencies of \( v_0^2/(2c^2) \approx 5 \times 10^{-7} \). Because of the random distribution of frequencies of the UBDM, the amplitude of the net UBDM field stochastically fluctuates, as discussed in detail in Refs. [42–49]. We emphasize that this is an essential feature of the UBDM field in the SHM. The characteristic time scale, \( \tau_{\omega_c} \approx 2h/(m_c v_0^2) \), and length scale, \( \lambda_c \approx h/(m_c v_0) \), of the fluctuations depend on the mass, \( m_c \), and velocity dispersion, \( v_0 \), of the bosons [37]. In comparison to a direct measurement of the UBDM field, a measurement of the UBDM field intensity produces a frequency down-conversion of the UBDM signal to near-dc. The spectral linewidth of this
near-dc signal is \(\sim 10^6\) times smaller than \(\omega_c\). Looking for this near-dc feature allows sensors with a limited bandwidth to probe UBDM with masses \(\sim 10^6\) times larger than searching for direct field oscillations at \(\omega_c\).

The frequency down-conversion discussed above occurs naturally when considering quadratic portals where the interaction with SM particles is proportional to the square of the UBDM field [32, 33, 41, 50–53]. A spin-0 field \(\varphi(r, t)\) can interact with SM fermions and electromagnetic fields according to the phenomenological quadratic scalar Lagrangian [32]

\[
\mathcal{L}_s = \hbar c \left( \pm m_f c^2 \frac{\partial \bar{\psi}_f \gamma_5 \psi_f}{\Lambda_f^2} \right) \varphi^2(r, t),
\]

where \(\Lambda_f\) and \(\Lambda_\gamma\) parametrize the couplings to fermions and photons, respectively, where \(\pm\) indicates the sign of the coupling, \(m_f\) is the fermion mass, \(\psi_f\) is the fermion field, and \(F_{\mu\nu}\) is the Faraday tensor.

The effects of the interactions described by Eq. (1) can be understood in terms of redefinitions of the effective fermion masses and the fine-structure constant \(\alpha\) [32, 50]:

\[
m_f^{(\text{eff})}(r, t) = m_f \left( 1 \mp \frac{\hbar c\varphi^2(r, t)}{\Lambda_f^2} \right),
\]

\[
\alpha^{(\text{eff})}(r, t) = \alpha \left( 1 \mp \frac{\hbar c\varphi^2(r, t)}{\Lambda_\gamma^2} \right).
\]

Equation (6) features a structure similar to that of the Zeeman Hamiltonian, \(\mathcal{H}_Z = \gamma S \cdot B\), where \(\gamma\) is the gyromagnetic ratio and \(B\) is a magnetic field (with the above noted difference that \(\mathcal{H}_Z\) is CP-even while \(\mathcal{H}_\varphi\) is CP-odd). Therefore \(\nabla \varphi^2(r, t)\) couples to a fermion spin \(S\) in a manner similar to a magnetic field [41], playing the role of a “pseudo-magnetic” field. Thus, such pseudoscalar fields can be searched for in the spin dynamics of electrons or nuclei. While a variety of sensors could be used, here we focus on atomic (nuclear) magnetometers since these are intrinsically sensitive to Zeeman shifts [76–82].

The effects described by Eqs. (2), (3), and (6) are related to \(\varphi^2\), which is in turn proportional to the UBDM “intensity” and exhibit near-dc stochastic amplitude fluctuations. Although measuring low-frequency signals is technically challenging due to multiple sources of low-frequency noise, an array of independent, geographically distributed sensors will tend to have uncorrelated noise. In contrast, the slowly-changing UBDM intensity will lead to a common-mode fluctuating signal present in all detectors within a coherence length of one another, which will appear in correlations between the sensors. We advocate for the use of networks of sensors to search for these stochastic fluctuations. There are existing and proposed dark matter searches sensitive to these interactions using networks consisting of a variety of sensor types such as atomic clocks [50, 53–56, 83], atomic magnetometers [41, 76–80, 84–86], gravimeters [87–89], laser interferometers [52, 57, 58, 60–62, 90], and atom interferometers [59]. The methodology described below is analogous to Hanbury Brown and Twiss intensity interferometry [91], and can be used to dramatically expand the range of UBDM Compton frequencies that particular sensors can probe [92].

Stochastic properties of the UBDM field. — The properties of the UBDM field can be derived using the framework described in Refs. [37, 42–49]. In brief, assuming that UBDM does not interact with itself, each individual
particle can be treated as an independent wave. In this scenario, the UBDM field is well described by a superposition of these individual waves.

Assuming that the local dark-matter energy density $\rho_{dm}$ is solely in the form of UBDM, such field can be modeled as the superposition of $N$ oscillators

\[ \varphi(r, t) \approx \sum_{n=1}^{N} \frac{\varphi_0}{\sqrt{N}} \cos(\omega_n t - \mathbf{k}_n \cdot \mathbf{r} + \theta_n), \]  

where the oscillation amplitude is given by [73]

\[ \varphi_0 = \frac{\hbar}{m_c c} \sqrt{2 \rho_{dm}}, \]  

such that the average energy density in the UBDM field comprises the totality of the local dark matter. Here, $\mathbf{k}_n = m_c \mathbf{v}_n / \hbar$ is the wave vector corresponding to $\mathbf{v}_n$, the velocity of the $n^{th}$ oscillator in the laboratory frame. The phases $\theta_n$ are randomly distributed between 0 and $2\pi$.

The oscillation frequency, $\omega_n$, is determined mostly by the Compton frequency, $\omega_c$, of the underlying ultralight boson. The kinetic energy correction to the rest energy introduces small deviations from $\omega_c$, so that

\[ \omega_n \approx \omega_c \left(1 + \frac{v_n^2}{2c^2}\right), \]  

for $v_n \ll c$. Therefore, the distribution of $\omega_n$ (and $\mathbf{k}_n$) is determined by the velocity distribution as observed in the laboratory frame. According to the standard halo model, $\mathbf{v}_n$ follows a displaced Maxwell-Boltzmann distribution defined as

\[ f_{lab}(v) \approx \frac{1}{\pi^{3/2} v_0^3} \exp \left(-\frac{(v - v_{lab})^2}{v_0^2}\right), \]  

where the velocity of the lab frame is $v_{lab} = |v_{lab}| \hat{z}$, and we have ignored the escape-velocity cut-off.

Sufficiently strong self-interactions could introduce effects in the coherence properties of the UBDM field not considered in this work. However, since these interactions are expected to be relatively weak [38, 44], they are commonly neglected in studies of direct detection searches [37, 48, 93], as in this work. Here, our interest lies in quadratic interactions with the field, proportional to $\varphi^2(r, t)$ or $\nabla \varphi^2(r, t)$. These quantities have two terms: one near-dc component and one fast oscillating component at $\approx 2\omega_c$. We consider sensors with a limited bandwidth $\Delta \omega \ll \omega_c$, such that the fast oscillating terms can be ignored. Therefore, only the near-dc components (denoted by the subscript $s$) of $\varphi^2$ and $\nabla \varphi^2$ are considered. These are written as

\[ \varphi_s^2(r, t) = \frac{\varphi_0^2}{2N} \sum_{n,m=1}^{N} \cos(\omega_{nm} t - \mathbf{k}_{nm} \cdot \mathbf{r} + \theta_{nm}), \]  

\[ \nabla \varphi_s^2(r, t) = \frac{\varphi_0^2}{2N} \sum_{n,m=1}^{N} \mathbf{k}_{nm} \sin(\omega_{nm} t - \mathbf{k}_{nm} \cdot \mathbf{r} + \theta_{nm}), \]  

where $\omega_{nm} = \omega_n - \omega_m$, $\mathbf{k}_{nm} = \mathbf{k}_n - \mathbf{k}_m$, and $\theta_{nm} = \theta_n - \theta_m$. Since the sensors are assumed to be within the same coherence patch (such that $\Delta \mathbf{k} \cdot \Delta \mathbf{r} \approx 0$, where $\Delta \mathbf{k}$ is the characteristic spread of values in $\mathbf{k}_{nm}$ and $\Delta \mathbf{r}$ is the difference in the position vectors for the pair of sensors at $\mathbf{r}_n$ and $\mathbf{r}_m$), the $\mathbf{r}$ dependence can be neglected and we can evaluate the expressions at $\mathbf{r} = 0$ in the following calculations [94].

The signal measured with each sensor would have a small UBDM-related component $\kappa \xi(t)$, where $\kappa$ accounts for the coupling of the sensor to the UBDM field, and $\xi(t)$ is either $\varphi_s^2$ (scalar interaction) or $\mathbf{m} \cdot \nabla \varphi_s^2$ (pseudoscalar gradient interaction, where $\mathbf{m}$ represents the sensitive direction of the sensor). The correlations between measurable signals produced in different sensors by a UBDM field can be quantified using the degree of first-order coherence $g^{(1)}(\tau)$ for different delay times $\tau$.

\[ g^{(1)}(\tau) = \langle \xi(t) \xi(t + \tau) \rangle_t / \langle \xi^2 \rangle_t, \]  

where $\langle \cdots \rangle_t$ denotes the time average.

To illustrate the stochastic properties of $\nabla \varphi_s^2(0, t)$ and $\varphi_s^2(0, t)$, their time evolution was simulated. Plots of $g^{(1)}(\tau)$ for $\varphi_s^2(t)$ and for projections of $\nabla \varphi_s^2(t)$ onto parallel and perpendicular directions with respect to $v_{lab}$ can be seen in Fig. 1. In order to numerically calculate the near-dc components of $\varphi^2$ and $\nabla \varphi^2$, and avoid the double summation in Eq. (12), it is convenient to introduce the field in complex notation

\[ \varphi_c(0, t) = \frac{\varphi_0}{\sqrt{N}} \sum_{n=1}^{N} \exp[i(\omega_n t + \theta_n)]. \]  

Then the near-dc component of the field squared can be calculated using

\[ \varphi_s^2 = \frac{1}{2} \varphi_c \varphi_c^*. \]  

This yields a real number related to the average value of $\varphi^2$ over a cycle of the oscillation.
Similarly, $\nabla \varphi^2_s(r, t)$ is numerically evaluated by applying the chain rule

$$\nabla \varphi^2_s(r, t)\bigg|_{t=0} = \frac{i}{2} \varphi \frac{\varphi_0}{\sqrt{N}} \sum_{n=1}^{N} k_n \exp [-i(\omega_n t + \theta_n)]$$

$$-\frac{i}{2} \varphi \frac{\varphi_0}{\sqrt{N}} \sum_{n=1}^{N} k_n \exp [i(\omega_n t + \theta_n)].$$

(16)

Note that by first evaluating Eq. (14) to obtain $\varphi(r, t)$ and then evaluating Eqs. (15) and (16) to find $\nabla \varphi^2_s$, we only evaluate sums with $O(N)$ terms for $N$ oscillators. This is in contrast to the equivalent expressions presented in Eqs. (11) and (12), which have double sums with $O(N^2)$ terms. This makes numerical calculations using Eqs. (15) and (16) considerably faster for large $N$.

The individual wave vectors $k_n$ and frequencies $\omega_n$ are calculated from the velocities $v_n$. These velocities are drawn from the displaced Maxwell-Boltzmann distribution defined in Eq. (10). In our simulations, we take the isotropic velocity dispersion of the local UBDM to be determined by the characteristic virial velocity $v_0 \approx 220 \text{ km/s}$, and $v_{\text{lab}}$ to be dominated by the motion of the Sun in the galactic frame, $|v_{\text{lab}}| \approx 233 \text{ km/s}$. The phases $\theta_n$ are drawn from a uniform distribution spanning from 0 to $2\pi$.

The simulations consider $N = 10^3$ oscillators evolving during $20 \tau_\varphi$ with a time resolution of 0.05 $\tau_\varphi$, where $\tau_\varphi$ is calculated using Eq. (18). The number of oscillators used to model the UBDM field reflects the quantity that can be comfortably simulated with our available hardware. By repeating the simulation hundreds of times, we observe that the results converge. Additional checks confirmed that the spectral properties of the simulations matched theoretical predictions. For example, an analytical solution for $g^{(1)}(\tau)$ in the limit $|v_{\text{lab}}| > v_0$ can be found in Appendix D, which is shown to agree with our simulations. The temporal resolution and the duration of the simulated field evolution were chosen considering plausible values for an experimental search.

After generating $\varphi^2_s$ and $\nabla \varphi^2_s$, $g^{(1)}(\tau)$ is calculated using Eq. (13). Note that the mean value of the field is subtracted so $g^{(1)}(\tau) \rightarrow 0$ for $\tau \gg \tau_\varphi$ (see Appendix B).

The coherence time $\tau_\varphi$ is the characteristic time after which the correlation in the UBDM field is lost. We define the coherence time as the power-equivalent width of $g^{(1)}(\tau)$ [97, 98],

$$\tau_\varphi = \int_{-\infty}^{+\infty} |g^{(1)}(\tau)|^2 d\tau,$$

(17)

which describes a characteristic temporal width of $g^{(1)}(\tau)$. We used this expression to quantify the coherence time in our simulations.$^2$

As a useful benchmark to compare our results with, we have used the coherence time $\tau_\varphi$ of the field assuming an exact Lorentzian lineshape (with a full width at half maximum of $\omega_n v_0^2/c^2$) [37, 49],

$$\tau_\varphi \approx \frac{2\hbar}{m_\varphi v_0^2}.$$

(18)

Because the actual spectral lineshape describing $\varphi$ is non-Lorentzian [49], the coherence time of the field $\varphi$ derived from the simulations differs from $\tau_\varphi$. For the considered $v_{\text{lab}}$, simulations show a coherence time of $\approx 1.12(1)\tau_\varphi$.

The coherence time for $\varphi^2_s$ is approximately half of that for the field $\varphi$. This is a result of the field $\varphi^2_s$ being a sum over terms depending on the difference of frequencies $\omega_{nm}$, as opposed to $\varphi$, which only contains terms depending on $\omega_n$. The probability distribution of $\omega_{nm}$ can be calculated as the convolution of the distribution of $\omega_n$ with itself. This results in a distribution for $\omega_{nm}$ that is broader. Consequently, the coherence time is shorter since $g^{(1)}(\tau)$ is given by the Fourier transform of the power spectral density of $\varphi^2_s$ (proportional to the $\omega_{nm}$ distribution), according to the Wiener–Khinchin theorem. The gradient coupling features even shorter coherence times due to the $k_{nm}$ factor weighting the contribution of the oscillating terms. Larger $\omega_{nm}$ tends to correspond to larger $k_{nm}$: this effectively broadens the power spectral density of $\nabla \varphi^2_s$, leading to a shorter coherence time. This is also the reason why parallel and perpendicular components of the gradient have different coherence times, as discussed below.

![FIG. 1: Degree of first-order coherence as a function of delay time $\tau$ for $\varphi^2_s$ and different projections of the gradient $\nabla \varphi^2_s$ relative to $\varphi^2_s$. Each curve is the result of 100 averages simulating $10^9$ particles, where the mean was subtracted. The approximate values of the coherence times are given in colors matching their respective plot traces.](image)

$^2$ For the simulations presented here, the coherence time was obtained by integrating Eq.(17) numerically. The integration was done over the time interval $[0, 5\tau_\varphi]$, and multiplying by two in order to account for the negative segment of the range.
The coherence time is also related to the mass of the UBDM particle as can be seen in Eq. (18). The relationship between the UBDM mass and the coherence time exhibited by $\varphi^2$ and $\nabla \varphi^2$ is shown in Fig. 2. The coherence times for $\varphi^2$ and $\nabla \varphi^2$ are proportional to the coherence time of $\varphi$, $\tau_\varphi$ as defined in Eq. (18). In the case of detection, this could be used to estimate the mass of the UBDM particles.

A possible method to look for UBDM is to use multi-sensor intensity interferometry to measure the cross-correlation between time-series data from different sensors. When pairs of geographically distributed sensors, a correlated global background field will produce a nonzero cross-correlation $g_{AB}^{(1)}(\tau)$ between sensors $A$ and $B$ proportional to $g^{(1)}(\tau)$, as discussed in Appendix B. Note that uncorrelated noise in the sensors will reduce the expected value of $g_{AB}^{(1)}(0)$ in the presence of a UBDM signal, making it smaller than the maximum value of one (see Appendix B). In the case of the gradient coupling, a relative misalignment of the sensitive axes of the sensors also leads to a reduction in the value of $g_{AB}^{(1)}(0)$ (see Appendix C). In order to distinguish a correlated signal from uncorrelated noise, $g_{AB}^{(1)}(0)$ can be compared with $g_{AB}^{(1)}(\tau \gg \tau_\varphi)$.

Accessible UBDM parameter space. — Existing sensor networks have sufficient sensitivity to probe experimentally unexplored parameter space describing UBDM by searching for correlated stochastic fluctuations using intensity interferometry. Atomic magnetometers can search for ALP fields by detecting Zeeman shifts caused by the interaction described in Eq. (6). In analogy with the Zeeman Hamiltonian, the gradient of the square of the ALP field acts as a “pseudo-magnetic field” $\mathbf{B}_q$ given by

$$\mathbf{B}_q \approx \mp \frac{2\hbar^2 c^2}{g_F \mu_B f_q^2} \nabla \varphi^2(r, t),$$

(19)

where $g_F$ is the Landé factor, $\mu_B$ is the Bohr magneton (or, for nuclear-spin-based magnetometers, the nuclear magneton). The projection of the near-dc component of $\mathbf{B}_q$ along the sensitive axis of a magnetometer defined by the unit vector $\mathbf{m}$ can be estimated in a manner similar to that discussed in, for example, Ref. [49], by evaluating the sum in Eq. (12), yielding a characteristic magnitude (with an average value of zero) of

$$\mathbf{m} \cdot \mathbf{B}_q \sim \frac{\hbar^2 \rho_{\text{dm}} v_0}{g_F \mu_B m_\varphi f_q^2}.$$

(20)

In the derivation of the above equation we assumed that frequency and wave-vector are uncorrelated. The accuracy of this approximation is discussed in Appendix A. Nonetheless, Eq. (20) is suitable for a rough estimate of the sensitivity of a magnetometer network to such UBDM, given that $|v_{\text{lab}}| \approx v_0$.

The sensitivity of a sensor network depending on the number of sensors ($N_m$), the UBDM field coherence time ($\tau_\varphi$), and total acquisition time ($T$) is discussed in Appendix B. Combining Eqs. (20) and (B6), an estimate for the UBDM coupling constant to which a magnetometer network would be sensitive is given by

$$f_q^2 \lesssim \frac{\hbar^2 \rho_{\text{dm}} v_0}{g_F \mu_B m_\varphi \delta B (\tau_\varphi T)^{1/4} \sqrt{N_m}}.$$

(21)
the fine-structure constant $\alpha$ is given by

$$\frac{\delta \nu(t)}{\nu} = \kappa_\alpha \frac{\delta \alpha(t)}{\alpha},$$

(22)

where $\nu$ is the clock frequency and $\kappa_\alpha$ is a dimensionless sensitivity coefficient that depends on the type of clock: $\kappa_\alpha \approx 2$ for most current optical atomic clocks, but note that there are exceptions, such as the proposed clock with $\kappa_\alpha \approx -15$ described in Ref. [104], and the possibility of future clocks based on highly charged ions [105] or a Th nuclear transition [106] that could have orders of magnitude larger values of $\kappa_\alpha$.

It is important to note that the sensitivity of intensity interferometry can be significantly impacted by back-action of the surrounding matter density on the scalar field as pointed out in Refs. [33, 103] and also discussed in Refs. [51, 107, 108]. The accessible range of parameter space for current optical clocks near the surface of the Earth is well within the regime where such back-action effects are significant (above the long-dashed blue line in Fig. 4, see Appendix E). However, for the range of boson masses and coupling constants considered in the present work, it turns out that for a space-based network of sensors [109] the screening effects can be largely neglected [103], and so for simplicity we consider such a space-based network in our sensitivity estimates.

Assuming the effect described by Eq. (3) and a scalar field that makes up the entirety of the dark matter density, the amplitude of the fractional frequency variation is given by

$$\frac{\delta \nu}{\nu} \approx \kappa_\alpha \frac{2 \hbar \rho_{dm}}{m_s^2 c^2 \Lambda^2},$$

(23)

Optical clock networks, with $N_c$ independent clocks, can achieve a fractional frequency uncertainty [83, 102]

$$\frac{\delta \nu}{\nu} \approx 3 \times 10^{-16} \left(\frac{\tau_c T}{N_c} \right)^{1/4},$$

(24)

which translates to the sensitivity to the quadratic scalar coupling constant shown in Fig. 4. Appendix F offers a heuristic argument for the significant sensitivity difference between atomic clock and magnetometer networks to the respective coupling parameters $\Lambda_q$ and $f_q$.

**Conclusion.** — In summary, we propose a new method to search for UBDM by using intensity interferometry with sensor networks. We show that when the sensors measure signals quadratic in the UBDM field, there is a near-de component of the signal that enables finite-bandwidth sensors to search for UBDM with Compton frequencies many orders of magnitude larger than possible if a traditional search for signals oscillating at the Compton frequency is carried out. Here we have focused on quadratic UBDM interactions and sensors with a linear response; however, the results are also valid for linear interactions and sensors that respond quadratically to the field (square-law detectors). The method of intensity

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**Figure 4:** Estimated parameter space describing UBDM fields that can be probed by an optical clock network such as those described in Refs. [83, 102] (dotted line, light purple shaded region) in $\approx 100$ days of searching for correlated stochastic fluctuations using $N_c = 10$ clocks, not accounting for (anti)screening from back-action [33, 103], which can play a significant role near Earth’s surface above the long-dashed blue line as indicated by the blue arrow. Clocks are sensitive to the interactions described by Eq. (1); $\Lambda_q$ parameterizes the strength of the coupling of the ultralight bosons to photons. The vertical dashed red line marks the Compton frequency and mass for which the ultralight boson’s coherence length equals the Earth’s diameter. The dark green shaded area represents astrophysical bounds on such quadratic scalar interactions between ultralight bosons and photons from stellar cooling and observations of supernova 1987a [51, 106]; the light green shaded region represents bounds from Big Bang nucleosynthesis (BBN) [32].

Figure 3 shows sensitivity estimates for the Global Network of Optical Magnetometers for Exotic physics searches ( GNOME) [76, 77, 80, 82] based on alkali vapor magnetometers with $\delta B \approx 100 \text{fT}/\sqrt{\text{Hz}}$ and the Advanced GNOME network based on noble gas magnetometers with $\delta B \approx 1 \text{fT}/\sqrt{\text{Hz}}$, assuming $T = 100$ days and $N_m = 10$. Note that for $\tau_c \gg 24$ hours, the signal amplitude is partially modulated at the frequency of Earth’s rotation since the signals are $\propto \hat{m} \cdot \hat{B}_q$ and $\hat{m}$ rotates with the Earth while $\hat{B}_q$ does not, which can in principle enable the detection of UBDM with coherence times much longer than a day. Notable is the extent to which GNOME and Advanced GNOME can probe UBDM with Compton frequencies far beyond the nominal sensor bandwidths.

Optical atomic clocks are an example of a sensor that can search for scalar fields through the apparent variation of fundamental constants as described in Eqs. (2) and (3), due to, for example, the variation of the fine-structure constant $\alpha$ and relativistic effects (see Ref. [13] and references therein). For example, the fractional frequency variation in an atomic clock due to variation of
interferometry is intrinsically broadband, with the potential to search for UBDM with particle masses ranging over many orders of magnitude without having to probe individual narrow frequency bands. UBDM searches with intensity interferometry using existing sensor networks can probe unexplored parameter space.

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Appendix A: The effect of velocity of the laboratory frame

The velocity offset due to the movement of the laboratory frame, here equivalent to the galactic velocity of Earth $v_{\text{lab}} = v_{\text{lab}}^x$, breaks the isotropy of the field. However, this anisotropy is not directly observed when measuring the average intensity because the quadratic interactions depend on the difference of wave-vectors and frequencies as can be seen in Eqs. (11) and (12). Yet, the coherence time of the field is different for directions parallel and perpendicular to $v_{\text{lab}}$ and depends on its magnitude.

![Figure 5](image_url)

**FIG. 5:** Coherence time, as defined in Eq. (17), as a function of $v_{\text{lab}}$ for the different cases of the UBDM field $\varphi$ studied.

To understand the influence of $v_{\text{lab}}$ on the coherence time it is instructive to consider the limiting cases: $v_{\text{lab}} = 0$ and $|v_{\text{lab}}| \gg v_0$. Decomposing $\vec{k}$ into parallel and perpendicular components to $v_{\text{lab}}$, yields the following relationships:

$$k_{nm}^\parallel = \frac{m v_{nm}^\parallel}{\hbar}, \quad \text{and} \quad k_{nm}^\perp = \frac{m v_{nm}^\perp}{\hbar}. \tag{A1}$$

The velocity difference $v_{nm} = v_n - v_m$ and the superscripts $\parallel$ and $\perp$ indicate whether the component is parallel or perpendicular to $v_{\text{lab}}$. One can also write the difference of frequencies in terms of the galactic-rest-frame velocity components as

$$\frac{2\varphi^2}{\omega_c} \omega_{nm} = (v_n^\parallel + v_m^\parallel) \cdot v_{nm}^\parallel + (v_n^\perp + v_m^\perp) \cdot v_{nm}^\perp + 2v_{\text{lab}} \cdot v_{nm}^\parallel. \tag{A2}$$

For $v_{\text{lab}} \ll v_0$ there is almost no anisotropy and therefore parallel and perpendicular components of $\nabla \varphi^2(r, t)$ have the same coherence time. However, as can be seen in Eqs. (A1) and (A2), $\omega_{nm}$ and $k_{nm}$ have different dependence on $v_{nm}$. For $k_{nm}$ the relationship is linear, while for $\omega_{nm}$ it is non-linear and depends on the product of the sum and the difference of velocities. This implies a nonlinear correlation between $\omega_{nm}$ and $k_{nm}$ as can be seen in Fig. 5. Larger $v_{nm}$ correlates with larger $\omega_{nm}$ and $k_{nm}$. Since the gradient coupling is weighted by $k_{nm}$ [see Eq. (12)], the larger $\omega_{nm}$ have more weight in the sum. This broadens the frequency spectrum and shortens the coherence time of the gradient with respect to the field squared as can be seen in Fig. 5.

For large $v_{\text{lab}}^2$, the behavior of $\nabla \varphi^2$ resembles that of $\varphi^2$ (see Fig. 5) because the perpendicular terms of the velocity $v_{nm}^\perp$ contribute little to $\omega_{nm}$. Therefore, $k_{nm}^\perp$ and $\omega_{nm}$ are effectively independent and Eq. (12) resembles Eq. (11) if multiplied by an appropriate scaling factor.

In contrast, the coherence time of $\nabla \varphi^2$ reduces faster than that of $\varphi^2$ as can be seen in Fig. 6. The reason for this is that for large $v_{\text{lab}}$, $\omega_{nm}$ becomes approximately $\propto k_{nm}^\parallel$ and this proportionality scales with $v_{\text{lab}}^2$. This increases the weight in the sum of terms with large $\omega_{nm}$ [in Eq. (12)], leading to a broader power spectrum and consequently, a shorter coherence time.

In the derivation of Eq. (20) in the main text we assumed that the average spread of values of $\vec{m} \cdot k_{nm}$ is $\approx m_\varphi v_\varphi / \hbar$. This estimate implicitly assumes that $\omega_{nm}$ and $k_{nm}$ are uncorrelated, and thus the accuracy of this approximation varies with $v_{\text{lab}}$. Nonetheless, Eq. (20) is suitable for a rough estimate of the sensitivity of a magnetometer network to such UBDM, given that $|v_{\text{lab}}| \approx v_0$.

![Figure 6](image_url)

**FIG. 6:** Visualization of the dependence of $\omega_{nm}$ on $v_{nm}^\perp$ and $v_{nm}^\parallel$. The Monte Carlo simulation was performed drawing $10^7$ velocities from the distribution in Eq. (10). The number of instances in which $v_{nm}^\perp$ or $v_{nm}^\parallel$ results in a $\omega_{nm}$ within a 20 km/s $\times 4.6 \times 10^{-7}$ s$^{-1}$ bin is represented in color. The black contour-lines indicate when the number of counts is $10^2$, $10^3$ or $10^4$ and serve as a guide to illustrate the shape of the distributions.
Appendix B: Search with correlated sensor networks

The stochastic properties of \( \varphi^2 \) and \( \nabla \varphi^2 \) can be used to search for UBDM. However, magnetometers and optical clocks are subjected to systematic noise sources in the laboratory, which feature some degree of temporal self-coherence similar to the UBDM field. For this reason, we focus our search on the cross-correlation of independent sensors, as their noise can be assumed to be mostly uncorrelated.

Prosaic natural phenomena may also limit the sensitivity of a sensor network. Known natural phenomena that could generate noise with long-range correlations include, for example, time-dependent electromagnetic fields associated with resonances of the conducting Earth-ionosphere cavity [84] (such as the Schumann resonances [110]) and vibrational noise due to free oscillations of the Earth excited by large earthquakes [87] (such as the “breathing” mode of the Earth at \( \approx 800 \, \mu \text{Hz} \) [111, 112]). Auxiliary measurements with other instruments may be able to rule out such systematic effects. For example, GNOME uses unshielded magnetometers to monitor the magnetic environment near the shielded dark matter sensors to veto signals from anomalously large local magnetic field excursions [77]. Additionally, as showed in Appendix A, the coherence time of the UBDM has a dependence on the measurement axis relative to \( \mathbf{v}_{\text{lab}} \). The characteristic daily and annual modulation of the coherence time due to Earth motion can be used to confirm a UBDM signal. The possibility of the data containing non-dark-matter long-range correlated signals is not considered in this analysis.

In order to illustrate a possible method that could be used to search for UBDM fields using intensity interferometry, let us consider a measured time-series \( \tilde{S}_{A,B}(t) \) lasting several \( \tau_p \) from two different sensors, \( A \) and \( B \). These time-series have their mean subtracted, such that \( \tilde{S}_{A,B} = \tilde{S}_{A,B} - (\tilde{S}_{A,B})_t \). The measurement in each sensor would have a small UBDM-related component \( \kappa s \), and a noise term \( \mathcal{N}(t) \), \( \tilde{S}_{A,B} = \kappa s A,B s_{A,B}(t) + \mathcal{N}_{A,B}(t) \). The factor \( \kappa s \) accounts for coupling of the sensor to the UBDM field, and \( s_{A,B}(t) \) is either \( \varphi^2 \) (scalar interaction) or \( \tilde{m} \cdot \nabla \varphi^2 \) (pseudoscalar gradient interaction).

For a particular time series the degree of first-order coherence of these signals can be calculated as

\[
g^{(1)}_{AB}(\tau) = \frac{\langle S_A(t)S_B(t+\tau) \rangle_t}{\sqrt{\langle S_A^2 \rangle_t \langle S_B^2 \rangle_t}}. \tag{B1}
\]

While the signals \( s_A(t) \) and \( s_B(t) \) are correlated, the corresponding noise contributions, \( N_A(t) \) and \( N_B(t) \), are not. As a consequence, one can suppress the cross-terms \( \langle N_A N_B \rangle_t, \langle N_A s_B \rangle_t, \) and \( \langle N_K s_A \rangle_t \) with sufficient amount of averaging (determined by the signal-to-noise ratios of sensors \( A \) and \( B \)). Then, Eq. (B1) reduces to

\[
g^{(1)}_{AB}(\tau) = \frac{\kappa \langle s_A(t)s_B(t+\tau) \rangle_t}{\sqrt{\langle S_A^2 \rangle_t \langle S_B^2 \rangle_t}}. \tag{B2}
\]

Strictly speaking, there will be a difference between \( s_A(t) \) and \( s_B(t) \) because of the spatial dependence of \( \varphi^2 \) \((\nabla \varphi^2)\) that arises from the \( \mathbf{k} \cdot \mathbf{r} \) term in Eq. (7). However, for the considered UBDM mass range, the coherence length is much larger than the distance between the sensors. Therefore, \( \varphi^2 \) \((\nabla \varphi^2)\) will approximately be the same for both sensors, \( s_A(t) \approx s(t) \); and so \( g^{(1)}(\tau) \) is proportional to the autocorrelation of the slow-varying component of the UBDM field squared (or its gradient):

\[
g^{(1)}_{AB}(\tau) \approx \frac{\kappa_K B}{\sqrt{\langle S_A^2 \rangle_t \langle S_B^2 \rangle_t}} \langle s(t)s(t+\tau) \rangle_t. \tag{B3}
\]

As an example, evaluating Eq. (B3) at \( \tau = 0 \) yields

\[
g^{(1)}_{AB}(0) \approx \frac{\kappa_K B \sigma_A \sigma_B}{\sqrt{\langle S_A^2 \rangle_t \langle S_B^2 \rangle_t}} \langle s^2 \rangle_t. \tag{B4}
\]

The quantity \( q^{(1)}(0) \) can be used as an estimator for the slow-varying component (of the gradient) of the field squared; it can be used to determine whether or not a UBDM signal is present in the data. For the scalar interaction, \( \langle s^2 \rangle_t = \langle \varphi^2 \rangle t \) and it can be shown that for the gradient interaction \( \langle \mathbf{g} \cdot \nabla \varphi^2 \rangle t = \langle \cos \varphi (\mathbf{g} \cdot \nabla \varphi^2) \rangle t \), where \( \varphi \) is the angle between the sensitive axes of the two sensors, \( \mathbf{m}_A \) and \( \mathbf{m}_B \) (see Appendix C). Assuming Gaussian noise with standard deviation \( \sigma \), \( \mathcal{N} \sim \mathcal{G}(0, \sigma_\nu^2) \), that dominates over the UBDM-signal component, \( \kappa s \ll \sigma \), Eq. (B4) can be simplified to:

\[
g^{(1)}_{AB}(0) \approx \frac{\kappa_K B \sigma_A \sigma_B}{\sigma_A \sigma_B} \langle s^2 \rangle_t. \tag{B5}
\]

The presence of a common correlated UBDM signal between two sensors will lead to a nonzero \( g^{(1)}(0) \). This method is inherently broadband, as a large range of UBDM masses could lead to \( g^{(1)}(0) > 0 \).

In order to estimate the sensitivity to a nonzero \( g^{(1)}(0) \), we assume \( N_m \) identical sensors having the same directional sensitivity and dominated by Gaussian noise with the same variance (of course, this is not the typical case for real networks, these feature different directional sensitivities — see Appendix C — and are affected by 1/\( f \) noise). In the cases of interest, the coherence length of the UBDM is much larger than the spacing between the sensors, so the UBDM signal will be identical in all sensors. To understand the scaling of the sensitivity with \( N_m, \tau_{\nu} \), and total acquisition time \( T \), suppose that we divide the network into two distinct groups each with \( N_m/2 \) sensors and average the data in each group. Furthermore, suppose that the time-series data are binned in \( \tau_{\nu} \)-long segments and time-averaged in each bin. Based on this approach, in the absence of a correlated UBDM signal, each group of sensors will exhibit a bin-to-bin variance in the measured magnetic field of approximately \( \delta B / \sqrt{\tau_{\nu} N_m/2} \sim \Delta B / \sqrt{\tau_{\nu} N_m} \), where \( \delta B \) is the sensitivity of a single magnetometer in units of magnetic field strength times the square root of time. For a total acquisition time \( T \), there will be \( N_b = T/\tau_{\nu} \) bins. The
expected residual from noise in the cross-correlation between the two groups provides an estimate of the network resolution, which in turn gives the minimum detectable pseudo-magnetic field squared
\[
\langle B_g(\text{min})^2 \rangle_t \approx \frac{\delta B^2}{N_m \tau_c} \frac{1}{\sqrt{N_b}} \approx \frac{\delta B^2}{N_m \tau_c T}. 
\]
B6

An in-depth discussion of the $T$ and $N_m$ scaling for a similar case can be found in Ref. [37].

This scheme does not rely on the boson having a Compton wave resonant with the experimental set-up. Such resonant searches require the interrogation of numerous narrow frequency bands which is a time-consuming process [14–18, 23–27, 30]. In the case of detection in a search for correlated stochastic fluctuations, the mass range of the UBDM particle could be narrowed by analyzing the coherence time, as illustrated in Fig. 2.

Appendix C: Angle dependence of the correlation between magnetometers

We are interested in understanding the coherence between two signals, $s_A$ and $s_B$, that are the result of the gradient coupling. In particular, for $j \in \{A, B\}$, let $\hat{m}_j$ be the sensitive axis of magnetometer $j$, so $s_j = \hat{m}_j \cdot \nabla \varphi^2$. The calculation of the coherence requires one to understand $\langle s_A s_B \rangle_t$, but each $s_A$ and $s_B$ depends on the relative angle between the respective sensitive axis and the field gradient. These angles are not known and change over time as the field evolves.

Note that $\nabla \varphi^2$ is a sum of terms $\propto v_{nm}$. Additionally, since the field gradient experiences the same bias velocity at the location of both sensors, the relative velocities $v_{nm}$ have no bias. The angle dependence between the field gradient and the sensitive axes can be accounted for under some basic assumptions. In particular, assume that $v_{nm}$ is ergodic in direction — that is, over time, $\hat{v}_{nm}$ will take all values over the sphere $S^2$. Ergodicity implies that $\langle s_A s_B \rangle_t = \langle s_A s_B \rangle_{S^2}$.

As another simplifying assumption, let the relative speed $v_{nm}$ be independent of the direction $\hat{v}_{nm}$ so
\[
\langle (\hat{m}_A \cdot v_{nm})(\hat{m}_B \cdot v_{nm}) \rangle_t = \bar{v}^2 \langle (\hat{m}_A \cdot \hat{v}_{nm})(\hat{m}_B \cdot \hat{v}_{nm}) \rangle_t,
\]
for $\bar{v}^2 = \langle \|v_{nm}\|^2 \rangle_t$.

The correlation between magnetometers can be calculated explicitly with the above assumptions. This will still depend on the relative angle $\vartheta_{AB}$ between the sensitive axes $\hat{m}_A$ and $\hat{m}_B$. Let us define the axes such that $\hat{m}_B$ points along the $z$-axis and $\hat{m}_A$ lies in the $xz$-plane in the $+x$-direction. The gradient direction $\hat{v}$ can be arbitrary, described by the azimuthal angle $\phi$ and polar angle $\theta$ in this coordinate system. Thus
\[
\hat{m}_A = (\sin \vartheta_{AB}, 0, \cos \vartheta_{AB}) ,
\hat{m}_B = (0, 0, 1) ,
\hat{v}_{nm} = (\cos \phi \sin \theta, \sin \phi \sin \theta, \cos \theta) .
\]

The following calculation can be made explicitly:
\[
\langle s_A s_B \rangle_t \approx \langle s_A s_B \rangle_{S^2} = \frac{\bar{v}^2}{4\pi} \int_0^{2\pi} d\phi \int_0^\pi d\theta (\sin \vartheta_{AB} \cos \phi \sin \theta + \cos \vartheta_{AB} \cos \theta) \cos \theta \sin \theta = \frac{\bar{v}^2}{3} \cos \vartheta_{AB} ,
\]
where the $\sin \vartheta_{AB}$ term averages out over the sphere over both the azimuthal and polar directions.

The results of simulations shown in Fig. 7 illustrate this dependence. An interesting feature is that the dispersion in $g^{(1)}(0)$ also depends on the relative angle between the magnetometers. This dispersion in $g^{(1)}(0)$ is not due to noise, as noise was not added to the simulated magnetometer data, but rather results from the UBDM fields themselves. If the sensitive axes of the sensors are not parallel, dispersion in $g^{(1)}(0)$ is introduced by the uncorrelated components of the UBDM field measured; because the orthogonal components of $\nabla \varphi^2$ (e.g., along $\hat{x}, \hat{y}, \hat{z}$) are uncorrelated, measuring different orientations produces a spread in $g^{(1)}(0)$. This characteristic feature may be a useful signature that could be used for validating a UBDM signal.

Appendix D: Analytic solution for $g^{(1)}(\tau)$

In this Appendix, an analytic expression for the degree of coherence is derived. Let us consider a signal given by the sum of sinusoidal functions,
\[
S(t) = \sum_n a_n \cos (\omega_n t + \phi_n) .
\]
D1
For simplicity, we consider the case in which the average signal is zero, $\omega_n \neq 0$, and the frequencies are all different, $\omega_n \neq \omega_m$ for $m \neq n$. Considering the two-point correlation function defined as,

$$C_2(\tau) = \langle S(t)S(t+\tau) \rangle_t,$$

then $g^{(1)}(\tau) = C_2(\tau)/C_2(0)$.

When averaging over the product of the two signal functions, the oscillating components average out. As a result, the only components that contribute to the coherence are from products of the same sinusoidal terms. The resulting two-point correlation function is then

$$C_2(\tau) = \frac{1}{2} \sum_n a_n^2 \cos(\omega_n \tau).$$  

For this study, we consider the case in which there are $N \gg 1$ sinusoidal components in the signal. The frequencies $\omega_n$ and amplitudes $a_n$ are sampled from some probability distribution. Here, this distribution is derived from a distribution of particle velocities $P(v)dv$ given by Eq. (10).

We approximate the sum in Eq. (D3) as an integral in which $a_n^2$ is replaced with the differential $a^2(x)P(x)dx$,

$$C_2(\tau) = \frac{1}{2} \int dx a^2(x)P(x)\cos(\omega(x)\tau).$$  

Here, for both the $\varphi^2$- and $\nabla \varphi^2$-coupling, the probability distribution function is parameterized by $x = (k_1,k_2)$, for $k_i = (m_c/\hbar)v_i$ described by a Gaussian distribution of velocities offset by the laboratory reference frame. It is useful to consider the average $\bar{k} = (k_1+k_2)/2$ and difference $\Delta = k_1-k_2$. In these variables, $d^2k_1d^2k_2 = d^2k d^2\Delta$ and the difference in frequencies $\omega_{12} = (\hbar/m_c)\bar{k} \cdot \Delta$. For either coupling, the contribution from $O(2\nu_c)$ frequency components and the constant offset will be neglected. Also, let $\sigma_\theta = (m_c/\hbar)\sigma_\nu (\nu_c^2 = 2\sigma_\nu^2)$ describe the width of the $k$-vector distribution and $k_{lab} = (m_c/\hbar)v_{lab}$ describe the offset of the distribution due to the laboratory reference frame.

For a signal given by $\varphi^2$, the amplitude of a sinusoidal component is constant while the frequency is given by the difference $\omega(k_1) - \omega(k_2)$. The integral in Eq. (D4) can be approximated for $k_{lab} \gg \sigma_k$. Under this approximation, $\bar{k} \approx k_{lab}$ (i.e., one can neglect the uncertainty in the average $\bar{k}$), which reduces the six-dimensional integral to a three-dimensional integral over $\Delta$. The result is the degree of first-order coherence (see Fig. 8)

$$g^{(1)}(\tau) \approx \exp \left[ -\left(\frac{m_c v_{lab}\sigma_\nu}{\hbar}\right)^2 \tau^2 \right].$$

Note that we consider a lab-frame for which $v_{lab} \approx v_k$, so this is only a rough approximation. Heuristically, one would expect that the coherence is overestimated, because this approximation neglects a component of the frequency dispersion.

A signal given by coupling to the gradient $\nabla \varphi^2$ will have amplitudes of different sinusoidal components $\sim \hat{n} \cdot \Delta$. Here, we denote the angle between $\hat{n}$ and $v_{lab}$ with $\theta$. Using the approximation $k_{lab} \gg \sigma_k$ as before, the coherence is (see Fig. 8)

$$g^{(1)}(\tau) \approx \frac{\sin^2(\theta) + 2\pi \left[1 - 2 \left(\frac{m_c v_{lab}\sigma_\nu}{\hbar}\right)^2 \cos^2(\theta)\right] \cos^2(\theta)}{\sin^2(\theta) + 2\pi \cos(\theta) \frac{m_c v_{lab}\sigma_\nu}{\hbar}^2 \tau^2} \times \exp \left[ -\left(\frac{m_c v_{lab}\sigma_\nu}{\hbar}\right)^2 \tau^2 \right].$$

Observe that the coherence becomes negative for $\theta \neq \pi/2$ when

$$\tau > \frac{\hbar}{m_c v_{lab}\sigma_\nu} \sqrt{\frac{1}{2} + \tan^2(\theta) / 2\pi}.$$

This is a result of the fact that for such time shifts $S(t)$ and $S(t+\tau)$ are out-of-phase.
Appendix E: Back-action effects

As noted in the main text, a consideration particular to searches for ultralight scalar fields with quadratic couplings to SM particles and fields is the possible back-action of mass density on the scalar field, which can either reduce or enhance the field amplitude [33, 51, 103, 107, 108]. This back-action arises due to the fact that the bare potential for the scalar field, namely $m^2_{\phi} \phi^2 / (2h^2)$, is modified by the presence of the Lagrangian in Eq. (1).

The quadratic terms in the interaction Lagrangian endow the scalar field with an effective mass $m_{\text{eff}}$ in the presence of matter; this effect is analogous to the screening of magnetic fields caused by the Meissner effect (where the photon gains an effective mass inside a superconductor). Depending on the sign of the interaction, back-action can lead to either screening or antiscreening [33].

For the interaction between $\phi$ and the electromagnetic field leading to variation of $a$ considered in Eq. (1), in the presence of matter the scalar field acquires an effective mass given by

$$m_{\text{eff}}^2 \approx m_{\phi}^2 \pm \frac{2\hbar^3}{c} \rho_\gamma,$$  \hspace{1cm} (E1)

where $\rho_\gamma \approx -E_{\mu\nu}/4$ is the Coulomb energy density of a nonrelativistic nucleus averaged over the characteristic volume outside of which the UBDM field changes appreciably, which in our considered case is orders of magnitude larger than the size of a nucleus and thus depends on the average local matter density. Based on the numerical estimates of Ref. [103], for the Earth’s interior $\rho_\gamma \approx 6 \times 10^{21}$ GeV/cm$^3$ and for Earth’s atmosphere $\rho_\gamma \approx 6 \times 10^{17}$ GeV/cm$^3$. The (anti)screening effect due to back-action can be important if

$$m_{\phi}^2 \Lambda_\gamma^2 \lesssim \frac{2\hbar^3}{c} \rho_\gamma.$$  \hspace{1cm} (E2)

Based on the above expression (E2), for values of $m_{\phi}\Lambda_\gamma$ above the long-dashed blue line in Fig. 4 (anti)screening effects can be important and cover the entire range of parameter space accessible with a network of current optical atomic clocks. Here $\rho_\gamma$ for Earth’s interior is used since this is the relevant value for the long-wavelength interactions considered. Thus sensitivity to positive-signed quadratic scalar interactions [as in Eq. (E1)] is significantly diminished near Earth’s surface. On the other hand, a space-based clock [109, 113] (or atom interferometer [114]) network could largely avoid such screening effects due to the significantly lower average mass density in the interplanetary medium. In the case of negative-signed quadratic scalar interactions [as in Eq. (E1)], there is antiscreening and the field amplitude can be significantly enhanced [33]. However, further analysis is required to calculate the form of the signal from stochastically fluctuating UBDM in this scenario. Finally, we note that constraints on coupling constants based on screening and antiscreening of scalar fields can be derived from experiments testing the equivalence principle [33].

Appendix F: Heuristic argument for the difference in sensitivity between magnetometers and clocks

As can be seen in Figs. 3 and 4, the sensitivity of an optical clock network to the coupling parameters $1/\Lambda_\gamma$ or $1/\Lambda_{\gamma e}$ far exceeds that of magnetometer networks to the equivalent coupling parameter $1/f_q$. Here we offer a heuristic explanation for this difference between the two cases. In the case of the scalar interaction [described by Eq. (1)] probed by clocks, the UBDM field modulates the electromagnetic binding energy of the atom ($\sim e^2/(2a_B) \approx \alpha^2 m_e c^2/2$, where $e$ is the electron charge magnitude, $a_B$ is the Bohr radius, and $m_e$ is the electron mass). This effect leads to an energy shift approximately given by

$$\Delta E_{\text{clock}} \approx \left(\frac{\hbar c^2}{\Lambda_{\gamma e}}\right) \times \left(\frac{1}{2} \alpha^2 m_e c^2\right),$$  \hspace{1cm} (F1)

where the first factor in parentheses describes the effective coupling constant associated with the UBDM field (proportional to the UBDM-induced fractional energy shift) and the second factor in the parentheses describes the scale of the electromagnetic binding energy of the atom. This manifestation of the scalar coupling is seen from the fact that it results in an effective modulation of the fine-structure constant on ad/or electron mass $m_e$ as described by Eqs. (2) and (3). In the case of the pseudoscalar interaction Eq. (6) probed by magnetometers, it is the energy associated with the field gradient ($\sim m_{\phi}c v_0$) that is modulated, leading to an energy shift approximately given by [see Eq. (6)]

$$\Delta E_{\text{mag}} \approx \left(\frac{\hbar c^2}{f_q}\right) \times \left(\frac{m_{\phi}^2 c^2 v_0}{c}\right).$$  \hspace{1cm} (F2)

By comparing Eqs. (F1) and (F2), it is evident that the scale of the fractional energy modulation given by the first numerator in parentheses is similar in the two cases. Taking the ratio between the two energy shifts yields

$$\frac{\Delta E_{\text{clock}}}{\Delta E_{\text{mag}}} \approx \frac{f_q^2}{\Lambda_{\gamma e}^2} \frac{\alpha^2 c m_e}{2 v_0 m_{\phi}},$$  \hspace{1cm} (F3)

and thus the ratio of the squares of the coupling constants determined by given energy shift measurements is

$$\frac{\Lambda_{\gamma e}^2}{f_q^2} \approx \frac{\Delta E_{\text{mag}}}{\Delta E_{\text{clock}} \cdot 0.1 \Delta E_{\text{mag}}} \frac{m_e}{m_{\phi}}.$$  \hspace{1cm} (F4)

For $m_{\phi}c^2 \approx 10^{-14}$ eV, $\Delta E_{\text{mag}} \approx 10^{-19}$ eV (corresponding to a magnetic field measurement at the level of a few pT for a nuclear spin), $\Delta E_{\text{clock}} \approx 10^{-16}$ eV (corresponding to a fractional uncertainty in a measurement of an...
optical transition at the $10^{-16}$ level) and $c/\nu_0 \sim 10^3$, we see that $\Lambda^2/f_A^2 \sim 10^{15}$. This estimate demonstrates that the source of the enhanced sensitivity of clocks is from the large $m_e/m_\varphi$ mass ratio, expected for UBDM ($m_\varphi \ll 1 \text{ eV}/c^2$).