Combined system for indeterminate non-affine plant with control delay on the set of functioning states

E. L. Eremin¹, L. V. Nikifororva¹, E. A. Shelenok²

¹Amur State University, Blagoveshchensk, Russia; ²Pacific National University, Khabarovsk, Russia
cidshell@mail.ru

Abstract. In the present paper authors consider the synthesis of combined nonlinear control law for non-affine control plant with delay in the input variable with step-by-step changing of its dynamics. The control plant operates in conditions of a priori parametric and structural uncertainties in the presence of external noise and measurement of the controlled variable only. The structure of the control system includes the predictor-compensator and the filter-corrector.

1. Introduction
For the modern theory of automatic control, the development of regulators based on analytical synthesis methods for non-affine control systems with various types of delays and stepwise changing dynamics is one of the urgent tasks. This circumstance is due to the fact that many of such plants are typical for energy, mechanical, and chemical industries. Every control system for such plants includes the set of switched subsystems and some switching law that determines activity of the subsystem at certain period of time [1 – 4].

In this paper, we obtain the combined nonlinear control algorithm for non-affine plant with a known delay in the input variable. The proposed regulator takes into account the switching of parameters of unstable plant, changes in external perturbation and changes in the relative degree of the plant, i.e. the considered control plant is a priori structurally indefinite.

2. Mathematical model of the control system
We discuss the control plant that functioning in the conditions of priory structurally and parametrically uncertainties and its dynamics is described by composite system of differential equations at Q time intervals \( 0 = t_0 < t_1 < \cdots < t_Q \):

\[
\frac{dx^{(q)}(t)}{dt} = A^{(q)} x^{(q)}(t) + B(u^{(q)}(t - h))F^{(q)}(u^{(q)}(t - h)) + \phi^{(q)}(t),
\]

\[
y^{(q)}(t) = (L^{(q)})^T x^{(q)}(t),
\]

where \( q = 1, \ldots, Q \); \( (x^{(q)}(t))^T = (x_1^{(q)}(t), x_2^{(q)}(t), \ldots, x_n^{(q)}(t)) \) is the composite state vector of the plant; \( A^{(q)} \) are some matrices in the Frobenius form; \( B^T = [0, \ldots, 0, 1] \) is the state vector of size; \( u^{(q)}(t) \in R \) is the composite control signal; \( h \) is a known control delay; \( F^{(q)}(t) \) are smooth nonlinear functions; \( \phi^{(q)}(t) \) are permanent external perturbations; \( L^{(q)} \) are constant vectors of \( m_y \) size, \( y^{(q)}(t) \in R \) is the composite measurable output signal of the plant.

For the considered control plant (1) the following conditions are fulfilled:
1. The state variables are not accessible for direct measurements; for direct measurement only the composite output of the plant $y^{(q)}(t)$ is accessible and and trajectories of composite systems of equations (1) are joined:

$$x^{(q+1)}(t_0) = x^{(q)}(t_0), q = 1, 2, \ldots, Q - 1.$$

2. A priori parametric uncertainty of the plant (1) is due to the coefficients of the matrices $A^{(q)}$ and the vectors $L^{(q)}$ are unknown numbers belonging to known bounded numerical set.

3. Unknown functions $F^{(q)}(u^{(q)}(t-h))$ and $\varphi^{(q)}(t)$ satisfy the following estimates:

$$0 \leq F^{(q)}(u^{(q)}(t-h)) \leq F_{0q} = \text{const} > 0, \ |\varphi^{(q)}(t)| \leq \varphi_{qy} = \text{const}, \ \forall \ t \geq 0;$$

where $F_{0q}, \varphi_{qy}$ are unknown numbers.

4. A priori structural indefinite is due to the size of the vectors $L^{(q)}$ at the different time intervals may change since $m_q$ may change at the interval $[m_0, n - 1]$ where $m_0 > 0$, $m_0 = \min m_q$. Then the relative order of the plant (1) $r^{(q)}$ may change like: $1 \leq r^{(q)} \leq (n - m_0)$.

To the output of the plant (1) we connect the filter-corrector with following dynamics:

$$y^{(q)}(s) = \left[\frac{T_0s + 1}{T_0s + 1}\right]^{(n-m_0)} y^{(q)}(s), \quad (2)$$

where $y^{(q)}(s)$ is the output signal of the filter-corrector; $T_0, T_0 = \text{const} > 0$ are time constants; $T_0$ has a rather small value $[5, 6]$.

The serial connection of the plant (1) and the filter-corrector (2) can be represent as serial connection of the modified control plant (MCP) and the block of structural perturbation (BSP) as follows

$$y^{(q)}(s) = W^{(q)}_{MCP}(s) \cdot W^{(q)}_{BSP}(s) a^{(q)}(s) = \frac{L^{(q)}(s)}{a^{(q)}(s)} \cdot \frac{1}{(T_0s + 1)^{(n-m_0)} a^{(q)}(s)}, \quad (3)$$

where $a^{(q)}(s) = a^{(q)}(s) \cdot (T_0s + 1)^{(m_q - m_0)}$, deg $a^{(q)} = n + m_q - m_0$, $a^{(q)}(s)$ is denominator of the plant (1) transfer function; $L^{(q)}(s) = L^{(q)}(s)(T_0s + 1)^{(n-m_0)}$, deg $L^{(q)} = m_q + n - m_0 - 1$, $L^{(q)}$ is numerator of the plant (1) transfer function, $a^{(q)}(s) = u^{(q)}(t-h) F^{(q)}(u^{(q)}(t-h)) + \varphi^{(q)}(t)$.

In this case the relative order of the MCP ($\rho_{MCP}^{(q)}$) will be equals to one ($\rho_{MCP}^{(q)} = 1$) at any period of the system functioning.

We exclude from the model (3) the BSP following $[5 - 8]$. Then in extended state space, the MCP can be written as follows:

$$\frac{d\tilde{x}^{(q)}(t)}{dt} = \tilde{A}^{(q)} \tilde{x}^{(q)}(t) + \tilde{B}(u^{(q)}(t-h) F^{(q)}(u^{(q)}(t-h)) + \varphi^{(q)}(t)), \quad (4)$$

where $(\tilde{x}^{(q)}(t))^T = (\tilde{x}^{(q)}(t), \ldots, \tilde{x}^{(q)}_{(m_q + n - m_0)}(t))^T, \ \tilde{y}^{(q)}(t) \in R$ is the composite output.

It is supposed that the required dynamics of the plant is determined with the help of the first-order aperiodic link. We can extend the state space of the such reference model using the approach of $[9]$:

$$\tilde{y}^{(q)}_{M}(s) = \frac{a_{M0}}{s + a_{M0}} L^{(q)} \tilde{r}(s), \quad (5)$$

$$\frac{d\tilde{x}^{(q)}_{M}(t)}{dt} = \tilde{A}^{(q)} \tilde{x}^{(q)}_{M}(t) + \tilde{B}_{M} r(t), \quad \tilde{y}^{(q)}_{M}(t) = (E^{(q)})^T \tilde{x}^{(q)}_{M}(t), \quad (6)$$

where $a_{M0} = \text{const} > 0, r(t) \in R$ is the command signal; $(\tilde{x}^{(q)}_{M}(t))^T = (\tilde{x}^{(q)}_{M1}(t), \ldots, \tilde{x}^{(q)}_{M(m_q + n - m_0)}(t))^T$ is the reference state vector; $\tilde{y}^{(q)}_{M}(t) \in R$ is required output.
For compensation in the system the control delay we connect in parallel to plant (1) the predictor-corrector [10, 11] that dynamics we determine by following mathematical model

\[ \frac{d\tilde{x}^q_k(t)}{dt} = \tilde{A}_M^{(q)} \tilde{x}^q_k(t) + \tilde{B}_M u^q(t) - u^q(t-h), \]

\[ \tilde{y}^q_k(t) = (\tilde{L}^{(q)})^T \tilde{x}^q_k(t), \]  

where \( (x^q_M)^T = (x^q_M(t), ..., x^q_M(t)) \) is the composite state vector of the predictor-compensator; \( \tilde{A}_M^{(q)} \) are Hurwitz state matrices in the Frobenius form of \( (n \times n) \) size; \( \tilde{B}_M \) is constant vector of \( n \) size; \( \tilde{y}^q_k(t) \in R \) is the composite output.

3. Problem statement

For the non-affine plant (1), operating under conditions of a priori parametric and structural uncertainties, it is required to synthesize the explicit form of the control law \( u^q(t) \) of the control system (1), (2), (6), (7) so that for any initial conditions \( x^{(q)}(0) \) and bounded external perturbations \( \phi^{(q)}(t) \) it will be fulfilled following control goal:

\[ \lim_{t \to \infty} [\tilde{y}_M^{(q)}(t) - y^{(q)}(t)] \leq \sigma_0 = \text{const} > 0, \]

where \( q = 1, ..., Q; \sigma_0 \) is a rather small number.

4. Nonlinear control algorithm

The explicit form of the control law \( u^q(t) \) in this paper was determined on the basis of the hyperstability criterion, following which were considered the composite signal

\[ e^{(q)}(t) = \tilde{x}_M^{(q)}(t) - (\tilde{x}_M(t) + \tilde{x}_k^{(q)}(t)) \]

and the equivalent mathematical description

\[ \frac{de^{(q)}(t)}{dt} = \tilde{A}_M^{(q)} e^{(q)}(t) + \tilde{B}_M \mu^{(q)}(t), \]

\[ \nu^{(q)}(t) = \tilde{y}_M^{(q)}(t) - \tilde{y}^{(q)}(t) - \tilde{y}_k^{(q)}(t), \]

\[ \mu^{(q)}(t) = -\left[ u^{(q)}(t) - r(t) - \tilde{x}_0^{(q)}(t) + \left( a_{M0}^{-1} F^{(q)}(u^{(q)}(t-h)) - 1\right) u^{(q)}(t-h) + a_{M0}^{-1} \phi(t) \right]. \]

Requirement of hyperstability criterion with respect to a strict positive definiteness of the real frequency response of linear part of equivalent system (9) is that it will be fair the inequality:

\[ \text{Re}\left\{ (\tilde{E}^{(q)})^T (s E - \tilde{A}_M^{(q)})^{-1} \tilde{B}_M \right\} > 0, \ \forall \omega \geq 0. \]

This inequality at each time interval always be fair since the transfer function of the linear stationary part of the system (9) with respect to (5) matoes to transfer function of the first-order aperiodic link:

\[ W^{(q)}(s) = \left( \tilde{E}^{(q)} \right)^T \left( s E - \tilde{A}_M^{(q)} \right)^{-1} \tilde{B}_M = \frac{a_{M0}}{s + a_{M0}}. \]

For the nonlinear non-stationary part of the system (9) following the criterion of hyperstability must be satisfied following integral inequality:

\[ \eta^{(q)}(0, t) = -\int_0^t \mu^{(q)}(\mathcal{G}) \nu^{(q)}(\mathcal{G}) d\mathcal{G} \geq -\left( \eta_0^{(q)} \right)^2, \]

\[ \eta_0^{(q)} = \text{const}, \ \forall t > 0. \]

By setting the explicit form of the control action, the requirement for the validity of integral inequality (10) was fulfilled, and the synthesized control law was given the explicit form as follows
\[ u(t) = r(t) + h_1 \tilde{v}^{(q)}(t) t \int_0^t \ddot{v}^{(q)}(\theta) \nu^{(q)}(\theta) d\theta + h_2 \left( \ddot{v}^{(q)}(t) + h_1 \right) \nu^{(q)}(\theta) d\theta + + h_3 \nu^{(q)}(t-h) \int_0^t u^{(q)}(\theta-h) \nu^{(q)}(\theta) d\theta + h_4 \left( u^{(q)}(t-h) \right)^2 \nu^{(q)}(t), \]

where \( h_i = \text{const} > 0, i = 1, \ldots, 5 \) are parameters of the control law the values of that selected with the help of system simulation.

If we choose the small value of the parameter \( T^* \), then the obtained combined control law (11) guarantees \( L \)-dissipativity of the system (1), (2), (6), (7), (11), and hence the fulfillment of the control goal (8).

5. Example of the system simulation

Let us to consider the non-affine plant (1) during the functioning of that at some time intervals it is appear structural and parametric switchings:

1. At time interval \( 0 \leq t < t_1 = 120 \) (c):
   \( a^{(1)}(s) = s^3 + s^2 + 5s - 0.3 \);
   \( L^{(1)}(s) = 2s + 1 \);
   \( F^{(1)}(u^{(1)}(t-1)) = 0.3\arctan^2(1 + |u^{(1)}(t-1)|) \);
   \( q^{(1)}(0) = -0.3\sin^2(0.075t) \);
   \( (x^{(1)}(0))^T = (0.5, 0.5, 0.5) \).

2. At time interval \( t_1 = 120 \leq t < t_2 = 270 \) (c):
   \( a^{(2)}(s) = s^3 + 1.7s^2 + 2s - 0.2 \);
   \( L^{(2)}(s) = 0.6s^2 + 2s + 0.1 \);
   \( F^{(2)}(u^{(2)}(t-1)) = 0.5/(1 + \arctan(1 + |u^{(2)}(t-1)|)) \);
   \( q^{(2)}(t) = 0.1 - 0.15\arctan(0.05t) \);
   \( x^{(2)}(t_1) = x^{(1)}(t_1) \).

3. At time interval \( t_2 = 270 \leq t < 500 \) (c):
   \( a^{(3)}(s) = s^3 + 3s^2 + 4s - 0.1 \);
   \( L^{(3)}(s) = s + 2 \);
   \( F^{(3)}(u^{(3)}(t-1)) = 0.2|u^{(3)}(t-1)|^{3/4} + 2/(1 + |u^{(3)}(t-1)|^{1/4}) \);
   \( q^{(3)}(t) = 0.15\cos(0.1t) \);
   \( x^{(3)}(t_2) = x^{(2)}(t_2) \).

The maximum degree of the considered plant is equals to three and the maximum relative order is equals to two.

Transfer function of the filter corrector has the following form:
\[ W(s) = \frac{0.1s + 1}{0.0075s + 1} \]

The command signal and parameter of the predictor-corrector are:
\( r(t) = 0.4\sin(0.04t) - 0.3\sin(0.08t) - 0.4\sin(0.1t) \);
\( a_{\text{M}} = 1 \).

In the course of simulation values of parameters of the combined control law (11) were chosen like \( h_1 = 200, h_2 = 1000, h_3 = 2000, h_4 = 1000, h_5 = 5, h_6 = 2 \).

Simulation results are depicted on Fig. 1.

6. Conclusion

With the help of hyperstability criterion in this paper it is synthesized the combined control law for one class of non-affine plants with control delay, stepwise variable dynamics that functioning under parametric and structural uncertainties and the influence of external uncontrollable permanent disturbances.
In the course of modeling, the value of the control error in the steady state did not exceed 2%. This circumstance indicates a good enough quality of the obtained control system operation.

Acknowledgement
The work was supported by Russian Foundation for Basic Research (project 20-08-00712).

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