Clustering without replication: approximation and inapproximability

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Abstract—We consider the problem of clustering the nodes of directed acyclic graphs when the replication of logic is not allowed. We show that the problem is NP-hard even when the vertices of the DAG are unweighted and the cluster capacity is 2. Moreover, when the vertices of the DAG are weighted, the problem does not admit a \((2-\epsilon)\)-approximation algorithm for each \(\epsilon > 0\), unless \(P=NP\). On the positive side, we show that in case the vertices of the DAG are unweighted and \(M=2\), the problem admits a 2-approximation algorithm. Finally, we present some cases when the problem can be solved in polynomial time.

Index Terms—Clustering without replication, NP-completeness, approximation, inapproximability.

I. INTRODUCTION

In general setting, a combinatorial network is a directed acyclic graph (or DAG), that is, the underlying network can be represented as a directed graph \(G = (V, E)\) with vertex set \(V\) and edge-set \(E\) having no directed cycles. In such networks the elements of \(E\) are directed edges or are arcs, and each vertex from \(V\) will represent a gate, each arc \((u, v)\) will represent an interconnection between gates \(u\) and \(v\). A cluster is defined as a subset of vertices of \(V\).

Fanin and fanout of a vertex are the number of arcs of \(G\) that enter and leave the vertex, respectively. An input or source is a vertex with fanin 0, and the output or sink is a vertex with fanout 0. As the example from figure 1 shows, a network may have more than one source and sink.

![Fig. 1. DAG representing a combinatorial network. It contains two sources and two sinks.](image)

Let \(I\) and \(O\) be the set of inputs and outputs of \(G\), respectively. In this example \(I = \{a, b\}\) and \(O = \{e, f\}\). Moreover \(C_1 = \{a, c, d\}\) and \(C_2 = \{b, e, f\}\) represent a pair of disjoint clusters in the network.

The gates and their interconnections in the DAG represent the implementations of one or more boolean functions. A realization of a boolean function is the assignment of the gates to chips. Observe that due to fabrication and capacity constraints, it is not possible in the general case to realize all the gates in the same chip. Consequently, the nodes of the DAG need to be partitioned into clusters, with each cluster representing a chip in the realization of the overall design.

The gates in the circuit have a delay associated with them, which represents the propagation time...
for a signal from its input to its output. An edge of the DAG can suffer one of two types of delay, viz. (1) if the edge connects two nodes in the same cluster, then it suffers an intra-modular delay $d$, (2) if the edge connects two nodes in different clusters, then it suffers an inter-modular delay $D$.

Traditionally, the inter-modular delay tends to dominate in all delay calculations, since $D > d$. The delay of the circuit is the longest delay on any path between a source and a sink.

Technology and design paradigms impose a number of constraints on the clustering of the circuit. A clustering is feasible if all clusters obey the imposed constraints. Constraints can be either monotone or non-monotone. In [5] the following definition is given:

Definition 1.1: A constraint is said to be monotone if and only if any connected subset of nodes in a feasible cluster is also feasible.

Typical constraints include capacity, that is, the weight of a cluster is bounded by $M$, a fixed constant (which is an example of a monotone constraint) and pin-limitations (which is an example of a non-monotone constraint), that is the total degree of a cluster is bounded by $P$ a fixed constant.

A clustering partitions the vertex set into a number of subsets, which may or may not be disjoint (depending whether or not replication is allowed). The edges of the underlying graph that connect a node in one cluster to a node in another cluster are called cut-edges.

A clustering algorithm tries to achieve one or both of the following goals, subject to a combination of one or more constraints.

1) Delay minimization through the circuit [8], [7], [1], [5]
2) Minimize the total number of cut-edges. [3], [4], [6], [9], [10], [2]

The rest of the paper is organized as follows. In section [II] we give the formal statement of the problem. The related work is surveyed in section [III] In section [IV] we present a hardness result for the clustering problem, then in section [V] we observe that this result already implies an inapproximability result for the problem. In section [VI] we present an approximation algorithm for solving the clustering problem, when the vertices of the DAG are unweighted and the cluster capacity is 2. In section [VII] we present some cases when the problem can be solved exactly. Finally, we conclude the paper in section [VIII] where we present a summary of the main results that are obtained in the paper.

II. Statement of the Problem

In the paper we will study a clustering problem under a delay model, where replication of the logic is not allowed. The delay model is the following.

1) Associated with every gate $v$ of the network there is a delay $\delta(v)$ and a weight $w(v)$.
2) The delay encountered on an interconnection between two gates in a cluster is $d$.
3) An interconnection between gates in different clusters suffers a delay of $D > d$.

The weight of a cluster is the sum of the weights of the nodes in the cluster. The delay of the network is the maximum delay suffered on a path from a source to any of the sinks.

The precise mathematical formulation of the problem can now be specified as follows: given a network $G = (V, E)$, with weights on the nodes, partition nodes into clusters so that

1) The weight of the cluster is bounded by $W$.
2) The delay of the network is minimized.

In the figure below we consider a simple example of a clustering of a network where replication of the logic is not allowed.

In this example, weights and delays are equal to one, the upper bound for the weight of the cluster is $M = 2$, $d = 1$, $D = 2$. It can be easily seen that the partition $\Sigma = \{\{s, a\}, \{b, d\}, \{c, t\}\}$ forms a feasible clustering whose weight is 8. Moreover, it can be checked that this clustering is optimal.

III. Related Work

In [5] a polynomial-time optimal solution to the clustering problem is given assuming the unit delay model. In this model, there is no delay that is associated with any gate or with any edge linking two gates within a cluster. A delay of one unit
allowed. Below we prove this.

Theorem 4.1: The problem of finding the optimal clustering in case replication of the logic is not allowed is **NP-hard**.

**Proof:** We will present a reduction from the partition problem. Recall that in an instance of this problem, we are given a set $S = \{a_1, a_2, \ldots, a_n\}$, and the goal is to check whether there is a set $S_1 \subset S$, such that $\sum_{i \in S_1} a_i = \sum_{j \in S - S_1} a_j$. Observe that without loss of generality, we can assume that $B = \sum_{i \in S} a_i$ is even, or else the problem is trivial.

We now construct an instance of Exact Clustering as shown in the Figure 3. There is a source $s$ connected to a sink $t$ through $n$ nodes labeled $a_1$ through $a_n$. None of the $a_i$ nodes are connected to each other. Each of the $a_i$ nodes has a weight $a_i$ and $s$ and $t$ have weights equal to $\frac{B}{2}$. We set the inter-cluster delay to be 1 i.e. an edge has a weight 1 if it connects nodes in different clusters and it has a weight of 0 if it connects nodes in the same cluster. All nodes are given a delay of 0.

The packing size is set to $B$. The description of the reduction is completed.

To complete the proof of the theorem, let us make the following observation: if there exists a partition of $S$ into $S_1$ and $S - S_1$ such that $\sum_{i \in S_1} a_i = \sum_{j \in S - S_1} a_j = \frac{B}{2}$, then we can group the nodes corresponding to the elements in $S_1$ with $s$ and the remaining nodes with $t$ in $G$. The packing constraint is met and the maximum delay from $s$ to $t$ is 1. Likewise, if we are able to pack the nodes in $G$ into clusters such that the maximum delay from $s$ to $t$ is 1, then we observe that:

1) Every node must be packed with either $s$ or $t$, or else the maximum delay will have to be 2 going through that node,

2) $w(s) + w(s_i) + w(t) + w(t_i) = 2 \cdot B$, where, $w(s)$ is the weight of node $s$, $w(s_i)$ is the sum of the weights of nodes packed with $s$ and likewise for $t$,

3) Since $w(s) + w(s_i), w(t) + w(t_i) \leq B$, we have that $w(s_i), w(t_i) \leq \frac{B}{2}$, which implies that $w(s_i) = \frac{B}{2}$ and we have obtained the desired partition.

**IV. COMPUTATIONAL COMPLEXITY OF THE CLUSTERING PROBLEM**

It turns out that it is **NP-hard** to find an optimal clustering in case replication of the logic is not
We have also focused on the unweighted case of the problem, which is the case when the weights of vertices of the input DAG are 1. We also assumed that the vertices have no delays, and that the cluster capacity $M = 2$. For this case, observe that the following simple lemma holds:

**Lemma 4.1:** There is always an optimal clustering of the vertices of DAG $G$, such that any two vertices lying in the same cluster are adjacent, and these edges form a maximal matching in the underlying simple graph of $G$.

Recall that a matching of a graph is called maximal, if it does not lie in a larger matching.

For this case we were able to strengthen Theorem 4.1 by proving Theorem 4.2. The proof of the latter requires the following:

**Lemma 4.2:** For any maximal matching of $d$-edges of the DAG in Figure IV, there exists no path with delay $4D$, and there is at least one path of delay $(d + 3D)$.

**Proof:**

First, observe that either arc $ab$, or arc $ac$ must have delay $D$ (it may be the case that they both have a delay of $D$, but of course they cannot both have a delay of $d$ since they are both incident from vertex $a$). So, without loss of generality, suppose arc $ac$ has a delay of $D$.

Likewise, observe that either arc $hj$, or arc $ij$ must have delay $D$ (it may be the case that they both have a delay of $D$, but of course they cannot both have a delay of $d$ since they are both incident to vertex $j$). So, without loss of generality, suppose arc $ij$ has a delay of $D$.

Now, observe that either arc $cf$, or arc $cg$ must have delay $D$, but not both since it would violate the fact that we have chosen a maximal matching. So, suppose $cf$ has delay $D$, then $cg$ must have delay $d$. This also means that $gi$ must have delay $D$, and $fi$ must have delay $d$ since we have chosen a maximal matching.

Notice that if $ab$ and $hj$ both have delays, then an argument similar to the previous one holds for the internal arcs in the upper half of the DAG, and all paths have delay $d + 3D$, with no path having delay $4D$.

If $ab$ and $hj$ both have delay $d$, then the internal arcs in the upper half of the DAG would all have delays of $D$, so that the delay along the paths starting at vertex $a$ and ending at vertex $j$ would be $2d + 2D$.

If $ab$ or $hj$ (and not both) arcs have a delay of $d$, then exactly one of the internal arcs in the upper half of the DAG must have a delay of $d$ (the one with an endpoint not incident with an arc having delay $d$). In this case, one $(a, j)$-path in the upper half of the DAG has delay $2d + 2D$, while the other $(a, j)$-path has delay $d + 3D$.

Finally, suppose $ac$ and $hj$ both have a delay of $D$, and $ab$ and $ij$ both have a delay of $d$. Then, exactly one of the internal arcs in both the upper and lower halves of the DAG must have a delay of...
(namely, the arcs that do not have vertices b or i as endpoints). In this case, an $(a, j)$-path in both the upper and lower halves of the DAG have delay $2d + 2D$, while the other $(a, j)$-path in the upper and lower halves have delay $d + 3D$. The argument is similar if arcs ab and ij both have delay $D$, when arcs ac and hj both have delay $d$.

In every case, there is at least one $d + 3D$-path, and no $4D$-paths. This means that the maximum delay of the DAG is $d + 3D$.

**Theorem 4.2:** The clustering problem is NP-hard even when $M = 2$ and the maximum degree of each vertex in the underlying simple graph of the input DAG $G$ is at most 3.

**Proof:** The reduction, from 3-SAT, is formulated as follows:

**3-SAT:** Given a 3-CNF formula $K(x_1, \ldots, x_n) = D_1 \land \cdots \land D_s$, the goal is to check whether the boolean variables, $x_1, \cdots, x_n$, have a truth assignment such that all clauses are satisfied. A clause is said to be satisfied under the truth assignment if its value is true.

Figure 5 shows the DAG which corresponds to a literal which is a boolean variable. Notice that the underlying simple graph of this DAG contains two perfect matchings. If this DAG is clustered with respect to perfect matchings of $G$, then it can be shown that the maximum delay is either $2 \cdot d + D$ or $2 \cdot D + d$. Let us interpret the clustering as follows: if horizontal edges of the 4-cycle are taken, then the value of the variable should be taken as true; and if the vertical edges are taken, then the value of the variable should be taken as false. The variable is satisfied if and only if the corresponding DAG is clustered in such a way that the delay is at most $2 \cdot d + D$ (this is directly verifiable). Thus, this DAG simulates a boolean variable.

Figure 6 shows the DAG which corresponds to a literal which is the negation of a boolean variable. Notice that the underlying simple graph of this DAG also contains two perfect matchings. If this DAG is clustered with respect to the perfect matchings of $G$, then it can be shown that the maximum delay is either $2 \cdot d + D$ or $2 \cdot D + d$. Let us interpret the clustering as follows: if horizontal edges of the 4-cycle are taken, then the value of the variable should be taken as true; and if the vertical edges are taken, then the value of the variable should be taken as false. The negated variable is satisfied if and only if the corresponding DAG is clustered in such a way that the delay is at most $2 \cdot d + D$ (this is directly verifiable). Thus, this DAG simulates a negated boolean variable.

Now, suppose we are given a disjunction of literals. For simplicity’s sake, assume the disjunction is one consisting of three boolean variables denoted by $x$, $y$ and $z$. The DAG corresponding to such a disjunction can be found in Figure 7. It can be shown that the underlying simple graph of this DAG contains a perfect matching. Let us interpret the clustering as follows: if horizontal edges of the 4-cycle are taken, then the value of the variable should be taken as true; and if the vertical edges are taken, then the value of the variable should be taken as false. The negated variable is satisfied if and only if the corresponding DAG is clustered in such a way that the delay is at most $2 \cdot d + D$ (this is directly verifiable). Thus, this DAG simulates a negated boolean variable.
Fig. 7. The DAG corresponding to a disjunction \( x \lor y \lor z \).

cycle are taken, then the value of the corresponding variable should be taken as true; and if the vertical edges are taken, then the value of the corresponding variable should be taken as false. The disjunction is satisfied if and only if the corresponding DAG is clustered in such a way that the delay is at most \( 8 \cdot d + 13 \cdot D \) (this is directly verifiable). Thus, this DAG simulates a disjunction of three boolean variables. One main observation is that if the first arc of a 2-path leading out of, or the second arc of a 2-path leading into a 4-cycle has delay \( d \), then the delay can always be swapped so that the other arc in the 2-path has delay \( d \) instead. This swapping technique is also supported by the lemma, since the max delay of the DAG that helps to bridge the 4-cycles will never be impacted (i.e., the max delay is always \( d + 3D \)). So, notice that one can always take a perfect matching and not some arbitrary matching.

Finally, assume we are given a 3-CNF formula \( K(x_1, \ldots, x_n) = D_1 \land \cdots \land D_s \). Suppose that DAGs corresponding to each of the clauses are already constructed. For each of the variables \( x_i \) \((1 \leq i \leq n)\), take the four-cycles corresponding to them, and join them cyclically, as shown in Figure 8. The resulting DAG is the one corresponding to the original conjunctive normal form \( K(x_1, \ldots, x_n) = D_1 \land \cdots \land D_s \). Observe that the number of vertices of the resulting DAG is \( 38 \cdot s \), hence the described reduction is linear. It is a matter of direct verification that \( K(x_1, \ldots, x_n) = D_1 \land \cdots \land D_s \) is satisfiable, if and only if the resulting DAG has a clustering which corresponds to a maximum matching where two \( d \) edges from each 4-cycle are taken, and whose delay is at most \( 8 \cdot d + 13 \cdot D \).

Fig. 8. The four-cycles corresponding to the same variable are joined cyclically.

V. AN INAPPROXIMABILITY RESULT OVER THE CLUSTERING PROBLEM

In this section we show that there is no approximation algorithm that can achieve a delay which is less than twice the optimal.

Theorem 5.1: For each \( \epsilon > 0 \) the problem of finding the optimal clustering in case replication of the vertices is not allowed does not admit a \((2 - \epsilon)\)-approximation algorithm unless \( P=NP \).

**Proof:** Consider the reduction from partition problem described in the proof of theorem 4.1. Observe that in any approximate solution of the clustering problem, there must exist at least one node which is not packed with either \( s \) or \( t \). This means that \( s \) and \( t \) do not belong to the same cluster. Hence the approximation algorithm can be used to solve the partition problem exactly.

\[ \square \]
VI. POSITIVE APPROXIMATION RESULT

Theorem 6.1: The clustering problem when $M = 2$ admits a 2-approximation algorithm.

Algorithm VI.1: A 2-approximation algorithm for the clustering problem

1: Input: a DAG $G$;
2: Output: a clustering of vertices of $G$;
3: Take a longest path $P$ in DAG;
4: Declare the central edge $e$ of $P$ as a $d$-edge.
5: The edges adjacent to $e$ are declared as $D$-edges.
6: Remove the edge $e$ together with edges incident to it.
7: Continue this process till all edges of $G$ are exhausted.

Proof: For a path $P$, let $l(P)$ be the length of $P$, that is, the number of edges of $P$. Moreover, let $l = \max_P l(P)$, that is, $l$ denotes the length of a longest path of $G$.

We proved the following lower bound for $OPT$ - the delay of the optimal clustering of $G$ when $M = 2$.

$$OPT \geq \left\lceil \frac{l(P)}{2} \right\rceil \cdot d + \left\lfloor \frac{l(P)}{2} \right\rfloor \cdot D.$$ 

Here $P$ is any path, so particularly this bound implies that:

$$OPT \geq \left\lceil \frac{l}{2} \right\rceil \cdot d + \left\lfloor \frac{l}{2} \right\rfloor \cdot D.$$ 

Now, let us estimate $ALG$ - the delay of the clustering found of the algorithm. Let $P$ be an optimal path with respect to this clustering. We will consider three cases.

Case 1: $l(P) < l$. Then one has the following chain of inequalities:

$$ALG \leq (l - 1) \cdot D$$
$$\leq 2 \cdot \left( \left\lfloor \frac{l}{2} \right\rfloor \cdot d + \left\lfloor \frac{l}{2} \right\rfloor \cdot D \right)$$
$$\leq 2 \cdot OPT.$$ 

Case 2: $l(P) = l$ and $P$ contains at least one $d$-edge. Then one has the following chain of inequalities:

$$ALG \leq d + (l - 1) \cdot D$$
$$\leq 2 \cdot \left( \left\lceil \frac{l}{2} \right\rceil \cdot d + \left\lfloor \frac{l}{2} \right\rfloor \cdot D \right)$$
$$\leq 2 \cdot OPT.$$ 

Case 3: $l(P) = l$ and all edges of $P$ are $D$-edges. We will consider two sub-cases of this case.

Sub-case (3a): $l$ is even. Then one has the following chain of inequalities:

$$ALG \leq l \cdot D$$
$$\leq 2 \cdot \left( \left\lceil \frac{l}{2} \right\rceil \cdot d + \left\lfloor \frac{l}{2} \right\rfloor \cdot D \right)$$
$$\leq 2 \cdot OPT.$$ 

Sub-case (3b): $l$ is odd. Assume that $l = 2 \cdot k + 1$. Let us show that in this case, one has:

$$OPT \geq \left\lceil \frac{l}{2} \right\rceil \cdot d + \left\lfloor \frac{l}{2} \right\rfloor \cdot D.$$ 

Clearly, we need to rule out the case, when

$$OPT = \left\lceil \frac{l}{2} \right\rceil \cdot d + \left\lfloor \frac{l}{2} \right\rfloor \cdot D = (k + 1) \cdot d + k \cdot D,$$

which is equivalent to saying that in any optimal path, which is of odd length $l$, the edges with odd number of indices, are $d$-edges. This particularly means that all central edges of longest paths are $d$-edges, hence it is impossible to have adjacent central edges of longest paths unless they are the same.

Since we are in case 3, all edges of $P$ are $D$-edges, particularly the central, $(k + 1)$-edge $e = (u, v)$ of $P$. Now, since this edge is also $D$-edge, it means that there is an edge $e'$ adjacent to $e$, so that the algorithm has declared $e'$ as a $d$-edge. Clearly, this means that $e'$ is a central edge of some other path $P'$ of length $l$ contradicting our conclusion
made above.

We are ready to complete the consideration of this case as well. Observe that one has the following chain of inequalities:

\[
ALG \leq l \cdot D \\
\leq 2 \cdot (\lceil \frac{l}{2} \rceil \cdot d + \lfloor \frac{l}{2} \rfloor \cdot D) \\
\leq 2 \cdot OPT.
\]

The proof of the theorem is completed.

Again, there is an example of a DAG, for which the algorithm achieves the bound 2.

![Fig. 9. The DAG achieving the bound 2](image)

Observe that in this example, \(OPT = 2 \cdot d + D\). However, the algorithm picks up the central edge first, then the delay of the resulting clustering would be \(ALG = 2 \cdot D + d\), which shows that the bound 2 cannot be improved for this algorithm. Finally observe that one can construct infinite sequence of DAGs, such that the algorithm achieves the bound 2 (simply take vertex disjoint copies of three-paths).

VII. EXACT RESULT

Theorem 7.1: There is a polynomial-time algorithm for solving the clustering problem for \(M = 2\), when the underlying simple graph of the input DAG \(G\) is a tree.

**Proof:** The problem is easy to solve when \(|V(T)| \leq 3\). Now, by induction assume that we can solve the problem for all trees \(T'\) with \(|V(T')| < |V(T)|\), and let us consider the tree \(T\) with \(|V(T)| \geq 4\).

Let \(V(0)\) and \(V(1)\) be the set of vertices of degree one in \(T\) and \(T - V(0)\), respectively. Since \(|V(T)| \geq 4\), we have that \(T - V(0) \neq \emptyset\). Let \(x\) be a vertex with \(x \in T - V(0)\). Observe that since \(x\) is pendant in \(T - V(0)\), we have that \(x\) has all its neighbours in \(V(0)\), except may be one.

We will consider three cases.

Case 1: \(d_T(x) \geq 4\). Then \(x\) is incident to at least three pendant vertices in \(V(0)\), hence at least two of them, say \(y_1\) and \(y_2\), are either a source or a sink. Since \(M = 2\), we cannot put \(x\), \(y_1\) and \(y_2\) in the same cluster, hence in any optimal clustering either \(y_1\) or \(y_2\) lies in a cluster that \(x\) does not lie. Taking into account that \(y_1\) and \(y_2\), are either a source or a sink at the same time, there is no difference which of them should be put in the different cluster. Hence we can remove the vertex \(y_2\) from \(T\), cluster the resulting DAG, optimally, and put \(y_2\) back as an isolated cluster.

This will result into an optimal clustering of \(G\) by the argumentation given above.

Case 2: \(d_T(x) = 3\). In this case, \(x\) is adjacent to two pendant vertices \(y_1\) and \(y_2\). If both of them are a source or both of them are a sink, then we can come up with this exactly by the same argumentation given in case 1, hence without loss of generality, we can assume that \(y_1\) is a source, and \(y_2\) is a sink. Moreover, we can assume that the other edge \(e\) (the edge, not incident to \(y_1\) and \(y_2\)), that is incident to \(x\), is oriented so that it goes out of \(x\). The other case can be handled similarly.

Observe that \(OPT \geq d + D\). This follows from the existence of a path of length two connecting \(y_1\) and \(y_2\). Now, we remove \(y_1\) and \(y_2\), and by induction we get an optimal clustering of the resulting DAG.

Now, if the delay of \(e\) is \(D\), then we put \(x\) and \(y_1\) in the same cluster, and \(y_2\) as an isolated cluster. Observe that the resulting clustering of \(G\) is optimal. On the other hand, if the delay of \(e\) is \(d\), then we remove \(x\) from its old cluster, and put \(x\) and \(y_1\) in the same cluster, and \(y_2\) as an isolated cluster. Observe that the resulting clustering of \(G\) is again optimal.

Case 3: \(d_T(x) = 2\). Let \(y\) be the only pendant neighbor of \(x\). Now, if \(x\) is a source or a sink, then we can remove \(y\), cluster the resulting DAG optimally by induction, and add \(y\) as an isolated cluster. Clearly, we are going to get an optimal clustering of \(G\).

Thus we can assume that \(x\) is neither a source nor a sink. Observe that without loss of generality we can assume that \(y\) is a source. The other case
can be handled similarly. Now, we remove \( x \) and \( y \) from \( G \), and apply the induction hypothesis to the resulting DAG. By induction we have an optimal clustering of the smaller DAG. Now, we add \( x \) and \( y \) as a new cluster. Clearly, we are going to get an optimal clustering of \( G \).

The proof of the theorem is completed.

VIII. Conclusion

In this paper, we considered the problem of clustering the nodes of DAG when the replication of the logic is not allowed. We showed that the problem is \textbf{NP-hard}. Our reduction was from the well-known partition problem. The strategy developed for the proof allows us to prove that the problem does not admit a \((2 - \epsilon)\)-approximation algorithm for each \( \epsilon > 0 \) unless \( \text{P}=\text{NP} \). We also showed that the problem remains \textbf{NP-hard} even when the vertices of the DAG are unweighted, their degree in the underlying simple graph of the DAG is at most three and the cluster capacity \( M = 2 \). On the positive side, we have designed a 2-approximation algorithm for the unweighted case of the problem when \( M = 2 \). Finally, we have shown that there is a polynomial algorithm for the problem when the underlying simple graph of the DAG is a tree.

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