Probing 10 μK stability and residual drifts in the cross-polarized dual-mode stabilization of single-crystal ultrahigh-Q optical resonators

Jinkang Lim1, Wei Liang2, Anatoliy A. Savchenkov2, Andrey B. Matsko2, Lute Maleki2 and Chee Wei Wong1

Abstract
The thermal stability of monolithic optical microresonators is essential for many mesoscopic photonic applications such as ultrastable laser oscillators, photonic microwave clocks, and precision navigation and sensing. Their fundamental performance is largely bounded by thermal instability. Sensitive thermal monitoring can be achieved by utilizing cross-polarized dual-mode beat frequency metrology, determined by the polarization-dependent thermorefractivity of a single-crystal microresonator, wherein the heterodyne radio-frequency beat pins down the optical mode volume temperature for precision stabilization. Here, we investigate the correlation between the dual-mode beat frequency and the resonator temperature with time and the associated spectral noise of the dual-mode beat frequency in a single-crystal ultrahigh-Q MgF2 resonator to illustrate that dual-mode frequency metrology can potentially be utilized for resonator temperature stabilization reaching the fundamental thermal noise limit in a realistic system. We show a resonator long-term temperature stability of 8.53 μK after stabilization and unveil various sources that hinder the stability from reaching sub-μK in the current system, an important step towards compact precision navigation, sensing, and frequency reference architectures.

Introduction
High-precision clocks and oscillators are cornerstones for global navigation to provide accurate timing and distance information, via satellite-to-satellite or satellite-to-ground communications, for the synchronization of electronic device communications and for quantum information processing using atomic qubits1–3. Likewise, precision time–frequency transfer in timing links is helpful for synchronization and data exchange in coherent arrays, gravitation sensing, and relativity measurements4. Current remote free-space links relying on radio-frequency communications have limited data transfer rates largely due to the available spectral bands, and often require large antenna sizes and high power consumptions due to diffraction, which naturally demands higher frequency carriers. However, the oscillators based on electronics attain their spectral purity from the high-quality factor of the resonator circuit, which usually degrades with increasing frequency. This design, therefore, potentially increases not only the size and the power consumption but also the complexity of the signal generation subsystems. As a result, free-space low-noise optical carrier networks provide an alternative with compact footprints for high-precision timing synchronization and high data rate links5–7.

Optical microresonator devices are a potential platform to provide both high spectral purity and stable clock operation for future timing networks. High-quality factor
transitions14. Single-crystal whispering-gallery resonators
tors disciplined to radio-frequency atomic hyper
logous architectures of quartz-crystal microwave oscilla-
also be disciplined to optical atomic transitions in ana-
resonators18modes with different thermal sensitivities21. In our
stabilization of quartz oscillators using two mechanical
–
approximation. To overcome the detection and control
resonator mode. The different thermorefractive coef
frequency, is subsequently locked to an ordinary polarized
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K levels with a precisely measurable cross-
Radiation of the microresonators towards advanced applications in
and heat diffusion with an associated time delay in the
radius by a factor of 2.407 compared to the one used in
extraradial deformation caused by the variation in tem-
caused by the resonator temperature gradient, and
however, the resonator temperature stabilization using the dual-mode
theoretical model, implying that the stringent correlation between the resonator temperature
and the cross-polarized dual-mode beat frequency could
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stringent correlation between the resonator temperature
and the cross-polarized dual-mode beat frequency could no longer hold true in time. This discrepancy raises the
question of whether the thermodynamically limited
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success stability is required for achieving the
thermodynamically limited noise level corresponding to a

For sub-millimeter-scale resonators, theoretical models
Q
predict that sub-μK stability is required for achieving the
thermodynamically limited noise level at a, resonance frequency stability of 10−13 or less57. Such sub-
monitoring cannot be achieved with conventional
resonators. For instance, the achievable bias stability of
microresonators experience large thermal noise induced by the
medium, potentially hindering the ultimate performance of the microresonators towards advanced applications in
ultrastable laser oscillators and photonic microwave
Clocks. Temperature-induced aging could also be detri-
mental for sensitive optical sensors using the micro-
resonators. In fact, the achievable bias stability of the
microresonator resonant gyroscopic sensor is typically
larger than 3° h−1 16 and is bounded by the resonator
thermal stability.

For sub-millimeter-scale resonators, theoretical models
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thermodynamically limited noise level at a resonance frequency stability of 10−13 or less57. Such sub-
monitoring cannot be achieved with conventional
thermistors. For example, the achievable bias stability of
resonators is generally approximated by using the
Steinhart–Hart thermistor third-order
approximation. To overcome the detection and control
limit, a cross-polarized dual-mode temperature stabiliza-
method has been suggested in birefringent crystalline
resonators18–20. The approach draws from the frequency
stabilization of quartz oscillators using two mechanical
modes with different thermal sensitivities21. In our
approach and prior studies, the laser frequency is locked to
an extraordinary polarized resonator mode, and its
radio-frequency modulated sideband, or a second laser
frequency, is subsequently locked to an ordinary polarized
resonator mode. The different thermorefractive coef-
cients of the orthogonally polarized modes allow the
thermal sensitivity of the resonator mode to be detected
down to sub-μK levels with a precisely measurable cross-
polarized dual-mode radio-frequency beat, which facil-
states the precise thermal stabilization of the resonator.

Previous experiments implementing this approach have
reported the microresonators that are temperature-
stabilized with 10- to 100-nK precision, which is cali-
bra ted by the in-loop cross-polarized dual-mode beat
frequency measurement. For the sub-μK resonator tem-
perature, the laser frequency instability locked to each
resonator should reach less than 10−13 even at 1000 s
integration time based on the theoretical thermodynamic
noise models22. However, the frequency instability
reported in the experiments shows a substantial dis-
crepancy from the theoretical model, implying that the
stringent correlation between the resonator temperature
and the cross-polarized dual-mode beat frequency could
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no longer hold true in time. This discrepancy raises the
question of whether the thermodynamically limited
resonator temperature stabilization using the dual-mode
beat stabilization method is fundamentally insufficient for
the realistic millimeter-scale ultrahigh-Q resonator or the
discrepancy stems from extraneous noise that needs to be
identified and compensated.

First, one can consider that this mismatch arises from
the temperature nonuniformity or temperature gradient
caused by the temperature difference between the reso-
nator and its enclosure or between the substrate and the
resonator, leading to the imperfect control of the reso-
nator thermal expansion. Second, the waveguide effects
and the modal interactions experienced by optical modes
can possibly modify the refractive indices and the mode
volumes of the resonator. Third, the cross-polarization
metrology probes predominantly the optical mode
volume temperature with the optical mode distribution at
the resonator rim, while the resonator volume tempera-
ture—and hence the volumetric size fluctuation—may be
less correlated. In addition, the finite heat diffusion time-
scale results in a time lag (before thermal equilibrium) in
the resonator stabilization. The first question was studied
by Baumgartel et al.19. Varying the temperature difference
between the resonator and its enclosure introduces an
extraradial deformation caused by the variation in tem-
perature gradient along the radial direction, and the
deformation increases with the size of the resonator. In
this work, we implement a thin cylindrical ultrahigh-Q
magnesium difluoride (MgF2) resonator with a smaller
radius by a factor of 2.407 compared to the one used in
ref. 19 to alleviate the impact of the radial deformation
caused by the resonator temperature gradient, and
investigate the others. Light guiding effects such as
refractive index modification, mode extinction, and modal
area expansion are further supported by finite-element-
method (FEM) numerical simulations on potential effects
on the dual-mode temperature stabilization. Moreover,
we study the impact of circulating laser-induced heating
and heat diffusion with an associated time delay in the
resonator.

To interrogate the resonator mode volume temperature,
two low-phase-noise continuous wave (cw) lasers are
locked to a transverse magnetic (TM) mode and a transverse electric (TE) mode of the whispering-gallery resonator, and the dual-mode beat frequency is measured by heterodyning the two stabilized lasers at a photodetector. We measure and demonstrate the time-dependent frequency correlation of the dual-mode beat frequency with the resonator temperature, represented by the resonant frequency drift due to the thermal expansion, by simultaneously counting the dual-mode beat frequency and a polarization-mode stabilized laser frequency. The degree of correlation between the open-loop dual-mode beat frequency and the resonance frequency approaches the theoretical bounding limit, where the averaged resonator volume temperature change is equal to the optical mode volume temperature change, with increasing integration time. By locking the dual-mode beat frequency to a radio-frequency clock with a passive suppression of the extraneous noise, we achieve a 51.78× enhancement in long-term frequency instability down to 14.55 kHz at 1000 s integration time, corresponding to a temperature instability of 8.53 µK and a linear drift of 0.54 kHz min⁻¹.

**Results**

**Theoretical model of dual-mode temperature stabilization**

The resonator is made from a z-cut magnesium difluoride (MgF₂) single crystal. It is a thin disk-shaped resonator with a radius (R) of 1.35 mm, which is sizably smaller than that of our prior study to mitigate the temperature gradient and shorten the heat diffusion time to achieve fast thermal equilibrium. A ring-down time of 3.63 µs is measured, corresponding to Q of 2.1×10⁶, as illustrated in Fig. 1a. MgF₂ is a birefringent crystal with n₁ = 1.382 and n₀ = 1.37, where (o) represents the ordinary polarized light or polarized with a TM mode and (e) represents the extraordinary polarized light or polarized with a TE mode, for the z-cut resonator. The resonator frequency in the whispering-gallery resonator has a linear function of temperature dependence that can be quantified using the thermal expansion coefficient, describing the resonator expansion perpendicular to the crystal axis (α₁,(o,e)), and thermorefractive coefficients (α_n,(o,e)). The expansion along the symmetry axis has a much weaker impact on the mode frequency and can be neglected. At room temperature, these coefficients are α₁,(o) = (1/R)(dR/dT) = 8.9 × 10⁻⁶, α₁,(e) = (1/n₁)(dn₁/dT) = 0.64 × 10⁻⁶, and α_n,(o,e) = (1/n₀)(dn₀/dT) = 0.23 × 10⁻⁶. ¹²

The thermal dependence of the TM and TE mode frequency difference, f_TM – TE = f₁ – f₂ (i.e., heterodyne beat note), can be determined by Δf_TM – TE = T/m (α₁,(o,e) – α_n,(o,e)), where v₀ is the optical carrier frequency. To derive this expression, we assume that the modes occupy the same mode volume and are nearly degenerate so that an increase in resonator size does not change their relative frequency. For an optical carrier wavelength of 1565.5 nm, we characterize the dual-mode beat frequency thermal fluctuation (i.e., Δf_TM – TE) in our MgF₂ resonator, which is 78.71ΔT_m MHz based on the difference between the TM and TE mode thermorefractive coefficients, where ΔT_m is the temperature change in the whispering-gallery mode channel. On the other hand, the frequency shift induced by thermal expansion is described by v₀α₁,(o,e) for the ordinary polarized mode (or the electric field of the mode is perpendicular to the crystal axis) in the resonator. The coefficient α₁,(o,e) is typically greater than α_n,(o,e). The value of the frequency fluctuation of the ordinary polarized mode is estimated to be 1.705ΔT_m GHz based on the thermal expansion coefficient, where ΔT_m is the resonator volume temperature change. We note that the temperature fluctuations are quantified with precision frequency metrology. The ratio of the dual-mode beat frequency variation and a single mode frequency variation due to thermal expansion becomes a factor of 21.66 when the mode volume temperature variation (ΔT_m) is equal to the resonator volume temperature variation (ΔT). This correlation can be utilized for stabilizing the resonator temperature by locking the radio-frequency dual-mode beat to a low-noise radio-frequency clock with proper feedback actuation.

**Integration of waveguide effects on the resonator temperature stabilization**

To detail the impacts of waveguide modal effects on the resonator temperature stabilization, we perform the FEM modeling of the MgF₂ resonator with a 12.5 μm radius of curvature and a 1.35 mm bending radius. The simulations clearly show the well-confined TM and TE modes at our experimental laser frequencies. The purities of these two modes are high enough, and their extinction ratios from the orthogonal modes are 35 dB (Supplementary Information S1). The eigenfrequencies of the TE and TM modes are determined to be f_TM = 191.4484274346256 THz and f_TE = 191.45602707701247 THz, respectively. The separation of these frequencies (Δf_TM – TE) is 7.599 GHz, which is close to the experimentally observed value of 5.953 GHz. The deviation is attributed to slight differences in the waveguide geometry between the numerical model and the fabricated device, as well as in minute material constant variations.

We calculate the effective refractive indices including the waveguide modal effect for both TM and TE resonant modes in the resonator. The FEM simulation shows that the modal effect modifies the refractive indices and the resulting refractive indices decrease by a factor of 5 x 10⁻³ for both modes, as illustrated in Fig. 1c. This is small because the modal areas for both confined modes are fairly large (modal area of 75.52 μm² for the TM mode and 74.56 μm² for the TE mode) and the bending radius is also large.
The refractive index change at the different temperature \((T + \Delta T)\) is computed by using temperature-dependent refractive index equations:

\[
\Delta n_e \approx \Delta T \left[0.09797 - 5.57293 \times 10^{-4}T \right] \times 10^{-5} \quad \text{and} \quad \Delta n_o \approx \Delta T \left[0.04183 - 5.63233 \times 10^{-4}T \right] \times 10^{-5}
\]

from ref. 17. The thermorefractive index changes due to the temperature change are small—we use a \(\Delta T = 10\) K to visualize the modification. Regardless of the temperature (even with \(\Delta T = 10\) K), the refractive index modification due to the waveguide modal effect is equally a factor of \(5 \times 10^{-3}\) for each mode, implying that the difference in refractive indices \(\alpha_{n,o} - \alpha_{n,o}\) can thus be safely considered the same for our dual-mode temperature stabilization. We also simulate the possible anisotropic thermal expansion of the mode cross-sections. For 1 K of the temperature change, the resonator deformation is small enough, as illustrated in Supplementary Information S2, and the distinct mode area change is not observed. If we consider that our resonator mode volume temperature can be stabilized to sub-\(\mu\)K, the modal size fluctuations can therefore be effectively neglected for stabilization.

**Correlation measurements between the dual-mode frequency and the resonator temperature**

We encapsulate the microresonator in a compact aluminum oven with temperature control and photodetector units in a small form factor, as shown in Fig. 1b. The resonator temperature is prestabilized at 295.7 K with mK precision over hours via the proportional–integral–differential (PID) control of a thermoelectric cooler (TEC) in a resonator mount. The resonator is then placed in a compact vacuum chamber, and the chamber is evacuated to maintain a high vacuum level (8 \(\times\) 10\(^{-6}\) torr) to eliminate convective heat transfer and environmental perturbations such as pressure and humidity. For instance, strong acoustic noise peaks are eliminated from the dual-mode beat phase noise, as illustrated in Supplementary Information S3. Figure 2 shows the schematic experimental measurement set-up. We utilize two cw lasers to interrogate a TM mode and a TE mode, respectively. The lasers that we used for stabilization to the resonator have a linewidth of 3 kHz at 100 ms integration time. The laser helps to interrogate ultrahigh-Q resonances without unwanted heating of the resonator so that the

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**Fig. 1 Integrated thin-disk MgF\(_2\) ultrahigh-Q whispering-gallery resonator and FEM simulations of the MgF\(_2\) whispering-gallery resonator modes.**

**a** Ring-down measurement of the resonator shows a cavity lifetime of 3.63 \(\mu\)s corresponding to \(Q = 2.1 \times 10^9\). Inset: image of the thin-disk whispering-gallery resonator. **b** Integrated package. The resonator is placed in a compact aluminum oven with a PID controlled TEC and the laser light is delivered into the resonator via an optical fiber. Both TM and TE laser signals are measured at a single photodetector. Optical resonance mode frequencies for TM and TE are located at \(f_{TM} = 191.45602707701247\) THz and \(f_{TE} = 191.4484272436256\) THz. **c** The frequency difference between the two orthogonal modes \((f_{TM} - f_{TE})\) is evaluated to be 7.599 GHz, close to the experimentally observed value. The FEM simulation shows modal areas and effective index changes due to the whispering-gallery resonator guiding effect for individual modes. The waveguide effect lowers the refractive indices by a factor of \(5 \times 10^{-3}\) for both polarization modes, numerically examined across a \(\Delta T\) of 10 K.

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**Table 1**

| Temperature | Refractive index | Modified index |
|-------------|-----------------|----------------|
| 295.7 K     | 1.382           | 1.374416       |
| 1.370       | 1.362363        |
| 3.63 \(\mu\)s | 1.382008225     | 1.374424       |
| 1.370002595 | 1.362366        |
| 7.599 GHz   | 1.380625        | 1.374424       |
| 1.370002595 | 1.362366        |
Fig. 2 Orthogonally polarized dual-mode temperature stabilization set-up. The two cw lasers are coupled into the ultrahigh-Q MgF2 resonator in a vacuum chamber with two orthogonally polarized modes. Each laser is locked to the resonance via a Pound–Drever–Hall (PDH) lock scheme. The inset (right bottom) shows the PDH error signals for the TM and TE modes interrogated by the two lasers, respectively. The dual-mode beat frequency is locked to a radio-frequency clock via intensity modulation with an acousto-optic modulator (AOM). Both the TM laser frequency and the dual-mode beat frequency are simultaneously counted and their phase noises are recorded. FBC fiber-optic beam combiner, FBS fiber-optic beam splitter, EOM electro-optic modulator, PBC polarization beam combiner, PD photodetector.

short-term scale temperature stability is not limited by the laser. The orthogonally polarized laser beams are combined with a fiber-optic polarization beam combiner and evanescently coupled into the whispering-gallery resonator via a prism.

Each laser frequency is locked to the resonance mode via the Pound–Drever–Hall (PDH) technique. The optical power used to interrogate the resonator is 20 μW for each resonance mode. We first find the resonance frequency of a TM mode and then interrogate a TE mode near the TM mode, offering the best PDH error signals. A single photodetector is implemented, and the two PDH error signals are separated by two different electro-optic modulation frequencies. The frequency difference in TM and TE modes is deterministically measured at 5.953 GHz. This dual-mode beat frequency is divided by 64 with a prescaler for stabilization to a 93.015 MHz radio-frequency clock. We then examine the Allan deviation (AD) of the TM laser frequency and the dual-mode beat frequency to measure the degree of correlation in time. The stability of the TM laser frequency is measured by downconverting the optical frequency via the heterodyne beating of the TM laser against a commercial laser. The stability of the TM laser frequency is measured by the heterodyne beating of it against an ultrastable Fabry–Pérot mirror cavity laser reference possessing 1 Hz linewidth and ≈1 Hz s⁻¹ drift. The dual-mode beat frequency is locked to a radio-frequency clock via intensity modulation with an acousto-optic modulator (AOM). Both the TM laser frequency and the dual-mode beat frequency are simultaneously counted and their phase noises are recorded.

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FBC fiber-optic beam combiner, FBS fiber-optic beam splitter, EOM electro-optic modulator, PBC polarization beam combiner, PD photodetector.

frequency drift over time. The long-term stability of 750 kHz at 1000 s integration time implies less than mK resonator temperature stability dominated by the thermal expansion coefficient. We extract the correlation between Δf_{TM} and Δf_{TM–TE} along the integration time (τ) by measuring their ratio as expressed by: β(τ) = Δf_{TM}(τ)/Δf_{TM–TE}(τ).

Figure 3c shows that β(τ) approaches the theoretically estimated value of 21.66 when ΔT_R = ΔT_m with increasing integration time, which indicates that the resonator volume temperature equilibrates to the mode temperature via heat diffusion and time averaging. This is supported by a heat transfer simulation with circulating laser power as a heat source (details in Supplementary Information S4). The simulation shows a heat gradient along the radius due to the light absorption by the resonator in Fig. 3d. The average temperatures near the mode volume and the entire resonator volume are probed individually and their difference is calculated in time, as illustrated in Fig. 3e. The difference between the mode volume temperature and the resonator volume temperature becomes constant after 30 s, which could be the characteristic time required to reach heat equilibrium. The longer time averaging helps β(τ) to be closer to the theoretical value. The deviation of β(τ) from the theoretically estimated value of 21.66 is attributed to the atmospheric temperature variation triggering the TEC control to maintain the enclosure temperature, which can be an extra heat source for the resonator.
Dual-mode temperature stabilization and residual drifts

We stabilize the dual-mode beat frequency by controlling the intensity of the TE laser into the resonator with an acousto-optic modulator (AOM). The inset in Fig. 4a shows the frequency measurement of the in-loop dual-mode beat carrier locked to a radio-frequency reference.
was applied, we found strong 60 Hz harmonic peaks in the 300 Hz offset frequency. When the higher feedback gain beats the frequency stabilization loop suppresses noise up to its single-side-band (SSB) phase noise. The dual-mode stabilized dual-mode beat frequency spectrum (inset) and mode beat frequency is locked. Figure 5a illustrates the noise of the radio-frequency clock to which the dual-frequency and analyzing the frequency noise of the dual-mode beat frequency spectrum at 5.953 GHz.

We investigate this frequency drift by measuring and analyzing the frequency noise of the dual-mode beat frequency and find that the drift could be attributed to the noise of the radio-frequency clock to which the dual-mode beat frequency is locked. Figure 5a illustrates the stabilized dual-mode beat frequency spectrum (inset) and its single-side-band (SSB) phase noise. The dual-mode beat frequency stabilization loop suppresses noise up to 300 Hz offset frequency. When the higher feedback gain was applied, we found strong 60 Hz harmonic peaks in the error signal, which limits the available feedback gain. However, we found that the feedback bandwidth and gain could not be the fundamental limiting factors for the long-term temperature stability of the resonator. In principle, time averaging with more than the characteristic time, which is required to reach the resonator heat equilibrium (30 s), can help to rule out fast frequency fluctuations. The dual-mode beat frequency noise starts to converge to the reference clock noise below 1 Hz offset frequency. The locked dual-mode beat frequency SSB phase noise has a decaying slope of $f^{-1}$ while the radio-frequency reference SSB phase noise has a decaying slope of $f^{-4.6}$ near the carrier frequency, which implies that the two curves could have a crossover so that the achievable dual-mode beat frequency stability can be bounded by the radio-frequency clock noise beyond the crossover point, which potentially introduces the frequency drift in our current system. To verify this assumption, we deliberately reduce proportional feedback gain without losing frequency locking and measure the stability again. Although the stabilities at $\tau < 100$ s are rather worse, the frequency drift is significantly suppressed, as illustrated in Fig. 5b. The drifts estimated with a linear function fitting show 11.8 and 0.54 kHz min$^{-1}$ at the two different gains. The

Fig. 5 Single-sideband (SSB) phase noise of the dual-mode beat frequency, Allan deviation, and their enhancement at different proportional feedback gains. a SSB phase noise measurements of the dual-mode beat frequency at 5.953 GHz. Inset: the stabilized dual-mode beat frequency spectrum at 5.953 GHz. b Measured TM laser frequency in time with the two different feedback gains. The frequency drifts estimated with a linear function fitting show two examples at 11.8 kHz min$^{-1}$ (navy) and 0.54 kHz min$^{-1}$ (green), respectively. The red line is the unlocked (open-loop) TM laser frequency for comparison. c TM laser frequency Allan deviations with different feedback gains. By lowering the proportional gain, the drift is significantly suppressed, illustrating that the radio-frequency reference noise could be responsible for the frequency drift. d The lower proportional gain makes the enhancement slower along the integration time, but the enhancement roll-off is avoided with the higher gain. The enhancement factor at 1000 s integration time is 51.78
AD at 1000 s integration time is improved by an order-of-magnitude compared to that with the higher gain and the resonator temperature remains to be approximately 10 μK from 1 to 1000 s integration time. The TM laser frequency instability is improved approximately by a factor of 51.78–14.55 kHz at 1000 s integration time compared to the instability (753.8 kHz) before stabilizing the dual-mode mode beat frequency as illustrated in Fig. 5d.

Residual intensity noise contributions
We investigate the impact of the coupled laser intensity fluctuation resulting in the fluctuation of the absorbed circulating optical power via the amplitude fluctuation of the intracavity laser field. Since we implement the TE laser intensity modulation feedback, the intensity fluctuation of the TE laser into the resonator is under control, but the intrinsic relative intensity noise (RIN) or the coupled power change of the TM laser could trigger the intracavity laser field fluctuation. To quantify the impact of RIN, we measure the resonance frequency shift induced by the laser intensity modulation as illustrated in Supplementary Information S5. The coupled laser power is modulated by an AOM with a 1 Hz top-hat function and the modulated laser power (0.75%) into the resonator and the corresponding TM laser frequency shift are measured. We determine the resonance frequency shift on the coupling power modulation—2.66 kHz for 1% coupled power change corresponding to 200 nW. The integrated RIN of the lasers used in this measurement is ≈10 ppm from 10 Hz to 10 MHz offset frequencies. By extrapolating RIN to the carrier frequency with 20 dB/decade, we estimate that the impact of RIN could be one part in 10^3 up to 1 Hz offset frequency. Hence, the RIN-associated power fluctuation can introduce a TM laser frequency fluctuation of 266 Hz, which is small compared to our currently measured frequency instability, but the laser RIN or the coupling power fluctuation could be a contributing bound when operating at the thermodynamic noise limit.

Discussion
We implement the cross-polarized dual-mode temperature stabilization for a birefringent high-Q whispering-gallery resonator and improve the long-term stability by ≈51× at 1000 s integration time. We achieve 10 μK resonator temperature instabilities even up to 1000 s integration time, enabling this compact optical resonator module to serve as a high-performance frequency reference in potential metrology, timing, and frequency transfer applications. The numerical simulations reveal the possible thermorefractive coefficient and mode volume modification due to the waveguide effect and the heat diffusion process in the MgF₂ resonator. The characteristic time for the resonator temperature equilibrium is deduced and matched with the experimental result. Further, more accurate models consider the impact of TEC control because the ambient temperature variations can trigger the TEC to maintain the enclosure temperature, leading to a change in the resonator temperature. Improved system thermal isolation from the surrounding environment may be needed. Although the current long-term frequency or temperature stability seems to be restricted by the low Fourier frequency noise of the radio-frequency clock used for the dual-mode beat frequency stabilization, further investigations and understandings on noise are desirable for operating the resonator stability at fundamental thermodynamic noise limits.

Materials and methods
FEM optical and thermal modeling and simulations for the MgF₂ resonator
The thin cylindrical MgF₂ resonator is modeled using 2D-axis symmetry in COMSOL Multiphysics with a 1.35 mm bending radius and a 12.5 μm radius of curvature. The guided modes in the resonator are calculated by solving Maxwell’s electromagnetic wave equations in the frequency domain, which provides the eigenfrequencies of the modes. By performing mode analysis near the optical carrier frequency, the family of TE and TM modes associated with the eigenfrequencies are found. We probe the effective modal areas and the effective mode indices, and study their resonator temperature dependence by performing simulations at different resonator temperatures. For the resonator heat diffusion modeling and simulation, the time-dependent heat transfer equation solver in COMSOL is implemented, where a user-defined heat source, the resonator absorption of circulating laser power in the modal area, is used. The size of the heat source is estimated by the modal area calculated by the mode analysis study. The physical constants and the optical parameters used in the simulation are listed in Supplementary Information S4.

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Author contributions
J.L., C.W.W., A.B.M., and L.M. designed the experiment, and J.L. developed the experimental set-up, performed the experiment, and analyzed the measurements. W.L. and A.A.S. developed the optical resonator along with the package assembly. J.L. performed the FEM numerical simulations and analyzed them with A.B.M. and C.W.W. All authors contributed to the manuscript preparation.

Conflict of interest
The authors declare that they have no conflict of interest.

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