Dynamics of two-component electromagnetic and acoustic extremely short pulses

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Abstract
The distinctive features of passing the two-component extremely short pulses through the nonlinear media are discussed. The equations considered describe the propagation in the two-level anisotropic medium of the electromagnetic pulses consisting of ordinary and extraordinary components and an evolution of the transverse-longitudinal acoustic pulses in a crystal containing the paramagnetic impurities with effective spin $S = 1/2$. The solutions decreasing exponentially and algebraically are studied.

Keywords: nonlinear coherent optics, extremely short pulse, self-induced transparency, optical anisotropy, nonlinear acoustics

1 Introduction

In recent years, an important subject of the theoretical studies became the coherent phenomena in the anisotropic media. This is caused by the significant progress in the technologies of growing the semiconductor crystals and producing the low-dimensional quantum structures. Unlike to the case of isotropic media, the parity of the stationary states of the quantum particles located in the anisotropic medium is not well defined. As a result, the diagonal elements of the matrix of the dipole moment operator and their difference usually called the permanent dipole moment (PDM) of the transition may not vanish.

The dynamics of one-component extremely short electromagnetic pulses in the presence of PDM was considered in works\textsuperscript{1−5}. The stationary pulse solutions decreasing exponentially and algebraically of the full Maxwell–Bloch equations for the two-level medium were found\textsuperscript{4}. It was shown that these solutions possess an asymmetry on the pulse polarity. The complete integrability of the corresponding system of the reduced Maxwell–Bloch equations with the help of the inverse scattering transformation method\textsuperscript{6−8} was revealed in paper\textsuperscript{2}. The multi-soliton solutions were constructed on fixed background, which was selected in such a manner that the problem under consideration is reduced to isotropic case. The effects of a pump on dynamics of these pulses were discussed in\textsuperscript{3}. Also, the numerical study of the pulse formation in the medium possessing PDM was performed\textsuperscript{5}. An existence of solitary stable bipolar signal with nonzero time area was discovered. The asymmetry on the polarity manifested itself there in that the sign of the time area is determined by the sign of PDM.

The propagation through optically uniaxial medium of the two-component electromagnetic pulses consisting of short-wavelength ordinary and long-wavelength extraordinary components was investigated in\textsuperscript{9−12}. As opposite to the case of one-component pulses mentioned above, an application of the slowly varying envelope approximation does not eliminate the influence of PDM in this problem. It was shown that, under conditions

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of a strong interaction between the components, the regimes of the pulse propagation may differ from the self-induced transparency. Various regimes of the optical transparency were classified with respect to the pulse velocity and the degree of the resonant medium excitation. The two-component pulse solutions possess also the PDM-induced asymmetry: signs of the pulse extraordinary component and PDM are opposite. It is remarkable that the equations distinguishing from ones studied in\textsuperscript{9}−\textsuperscript{12} by notations only describe the two-component (transverse-longitudinal) acoustic pulse propagation in the low-temperature paramagnetic crystal\textsuperscript{13,14}.

To investigate the propagation of the two-component pulses in optically uniaxial medium, we apply here the approximation of the unidirectional propagation instead of the slowly varying envelope approximation. The two-component system of reduced Maxwell–Bloch equations is derived and, as in the case treated in works\textsuperscript{9}−\textsuperscript{12}, its acoustic analogue is found. The properties of exponentially and algebraically decreasing pulse solutions of these equations are discussed.

2 Two-component system of the Maxwell–Bloch equations for an anisotropic medium and its acoustic analogue

Consider the optically uniaxial medium containing the two-level quantum particles. Let the medium optical axes coincide with $z$ axes of the Cartesian coordinate system. Suppose that the plane electromagnetic pulse propagates through the medium in the positive direction of $y$ axes. Then, ordinary $E_o$ and extraordinary $E_e$ components of the electric field of the pulse are parallel to $x$ and $z$ axes, respectively. The Maxwell equations yield in the case we study the next system for the electric field components:

\begin{align}
\frac{\partial^2 E_o}{\partial y^2} - \frac{n_o^2}{c^2} \frac{\partial^2 E_o}{\partial t^2} &= \frac{4\pi}{c^2} \frac{\partial^2 P_o}{\partial t^2}, \\
\frac{\partial^2 E_e}{\partial y^2} - \frac{n_e^2}{c^2} \frac{\partial^2 E_e}{\partial t^2} &= \frac{4\pi}{c^2} \frac{\partial^2 P_e}{\partial t^2},
\end{align}

where $P_o$ and $P_e$ are the ordinary and extraordinary components of the polarization, which is induced by the two-level particles; $n_o$ and $n_e$ are ordinary and extraordinary refractive indices, respectively; $c$ is the speed of light in free space.

To describe the evolution of the state of the quantum particle, we make use the von Neumann equation on density matrix $\hat{\rho}$:

\begin{equation}
\iota \hbar \frac{\partial \hat{\rho}}{\partial t} = [\hat{H}, \hat{\rho}],
\end{equation}

Here Hamiltonian $\hat{H}$ of the two-level particle is defined as given

\begin{equation}
\hat{H} = \text{diag}(0, \hbar \omega_0) - \hat{d}_x E_o - \hat{d}_z E_e,
\end{equation}

where $\omega_0$ is the resonant frequency of quantum transition $|1\rangle \rightarrow |2\rangle$; $\hbar$ is the Plank’s constant; $\hat{d}_x$ and $\hat{d}_z$ are the matrices of the projection of the dipole moment operator on $x$ and $z$ axes, respectively.

In the geometry chosen, the expressions for the polarization components are

\begin{align}
P_o &= N \text{ Tr} (\hat{\rho} \hat{d}_x), \\
P_e &= N \text{ Tr} (\hat{\rho} \hat{d}_z)
\end{align}

where $N$ is the concentration of the quantum particles. Since the medium is axially symmetric, the matrices of the dipole moment have the following form:

\begin{equation}
\hat{d}_x = \begin{pmatrix} 0 & d_{12} \\ d_{12} & 0 \end{pmatrix},
\end{equation}
\[ \dot{z} = \begin{pmatrix} D_{11} & 0 \\ 0 & D_{22} \end{pmatrix}. \] (8)

Then Hamiltonian (4) reads as
\[ H = \begin{pmatrix} -D_{11}E_e & -d_{12}E_o \\ -d_{12}E_o & \hbar \omega_0 - D_{22}E_e \end{pmatrix}, \] (9)

and equation (3) gives the next system for the density matrix elements:

\[ \frac{\partial \rho_{11}}{\partial t} = \frac{\partial \rho_{22}}{\partial t} = i \frac{d_{12}}{\hbar} E_e (\rho_{21} - \rho_{12}), \] (10)

\[ \frac{\partial \rho_{12}}{\partial t} = i \left( \omega_0 + \frac{D}{\hbar} E_e \right) \rho_{12} + i \frac{d_{12}}{\hbar} E_o (\rho_{22} - \rho_{11}), \] (11)

where \( D = D_{11} - D_{22} \) is PDM of the transition. Formulas (1), (2) and (5)–(8) imply

\[ \frac{\partial^2 E_o}{\partial y^2} = \frac{n_o^2 \hbar^2}{c^2} \frac{\partial^2 E_o}{\partial t^2} = 4\pi N \frac{n_o}{c} d_{12} \frac{\partial^2}{\partial t^2} (\rho_{12} + \rho_{21}), \] (12)

\[ \frac{\partial^2 E_e}{\partial y^2} = \frac{n_e^2 \hbar^2}{c^2} \frac{\partial^2 E_e}{\partial t^2} = 4\pi N \frac{n_e}{c} \frac{\partial^2}{\partial t^2} (D_{11}\rho_{11} + D_{22}\rho_{22}). \] (13)

Let us suppose that concentration of the quantum particles is small \( \left( \frac{\pi d_{12}^2 N}{\hbar \omega_0} \ll 1 \right) \). The order of derivatives in the equations for the electric field components can be reduced in this case. Indeed, by applying the approximation of the unidirectional propagation\(^{15} \) to equations (12), (13) and excluding the derivatives of the elements of the density matrix with the help of formulas (10) and (11), we obtain:

\[ \frac{\partial E_o}{\partial y} + \frac{n_o d_{12}}{c} \frac{\partial E_o}{\partial t} = -i \frac{2\pi N}{n_o c} \frac{d_{12}}{\hbar} \left( \omega_0 + \frac{D}{\hbar} E_e \right) (\rho_{12} - \rho_{21}), \] (14)

\[ \frac{\partial E_e}{\partial y} + \frac{n_e d_{12}}{c} \frac{\partial E_e}{\partial t} = i \frac{2\pi N}{n_e c} \frac{d_{12}}{\hbar} D (\rho_{12} - \rho_{21}). \] (15)

It is convenient for the subsequent consideration to rewrite equations (10), (11) and (14), (15) in dimensionless form

\[ \frac{\partial u}{\partial \xi} = i(1 + v)(\sigma^* - \sigma), \] (16)

\[ \frac{\partial v}{\partial \xi} + \delta_n \frac{\partial v}{\partial \tau} = ik \frac{n_o}{n_e} u (\sigma - \sigma^*), \] (17)

\[ \frac{\partial \sigma_3}{\partial \tau} = i u (\sigma - \sigma^*), \] (18)

\[ \frac{\partial \sigma}{\partial \tau} = i(1 + v) \sigma + 2iu \sigma_3, \] (19)

where

\[
\begin{align*}
\tau &= \omega_0 \left(t - \frac{n_o}{c} y\right), \quad \xi = \frac{2\pi N d_{12}^2}{\hbar n_o c} y, \\
u &= \frac{d_{12}}{\hbar \omega_0} E_o, \quad v = \frac{D}{\hbar \omega_0} E_e, \\
\sigma_3 &= \frac{\rho_{22} - \rho_{11}}{2}, \quad \sigma = \rho_{12}, \\
k &= \frac{D}{d_{12}}, \quad \delta_n = \frac{n_o \hbar \omega_0}{2\pi N d_{12}^2} (n_e - n_o).
\end{align*}
\]
Obviously, system (16)–(19) coincides with the reduced Maxwell–Bloch equations for isotropic medium if \(k = v = 0\). One can see that the electric field components fulfill different functions here. The ordinary component causes the quantum transitions, while the extraordinary one shifts its frequency. Also, the same functions were executed by these components in works\(^9\)–\(^{12}\), where the system, which follows equations (16)–(19) in the slowly varying envelope approximation, was considered.

It is remarkable that the system of equations equivalent to (16)–(19) appeared in the acoustics. Namely, the propagation of the transverse-longitudinal acoustic pulses in a crystal containing paramagnetic impurities with effective spin \(S = 1/2\) in a direction parallel to the external magnetic field is described by next equations (see\(^{16,17}\)):

\[
\frac{\partial E}{\partial \chi} = -bUS_y, \tag{21}
\]
\[
\frac{\partial U}{\partial \chi} + \delta_v \frac{\partial U}{\partial \eta} = bES_y, \tag{22}
\]
\[
\frac{\partial S_y}{\partial \eta} = bUS_x - ES_z, \tag{23}
\]
\[
\frac{\partial S_x}{\partial \eta} = -bUS_y, \tag{24}
\]
\[
\frac{\partial S_z}{\partial \eta} = ES_y, \tag{25}
\]

where

\[
\delta_v = (v_1 - v_2) \frac{v_1 n_0 g^2}{\hbar \omega_B n f_3}.
\]

\(E\) and \(U\) are dimensionless variables defining transverse and longitudinal components of the acoustic pulse, respectively; \(S_x, S_y\) and \(S_z\) are expressed through the elements of the density matrix of the paramagnetic impurities; \(\chi\) and \(\eta\) are dimensionless spatial and retarded time variables; constant \(b\) is defined through the coupling constants of the spin-phonon interaction; \(v_1\) and \(v_2\) are linear velocities of the transverse and longitudinal acoustic waves, respectively; \(\omega_B\) is the frequency of the Zeeman splitting of the Kramers’s doublets; \(n\) is the concentration of the paramagnetic impurities; \(n_0\) is the mean crystal density; \(g\) is the component of the Lande tensor; \(f_3\) is the coupling constant of the spin-phonon interaction.

The variables of both systems are connected by relations:

\[
E = ku, \quad U = \sqrt{\frac{n_c}{n_0}} (1 + v), \tag{26}
\]
\[
S_x = -k(\sigma + \sigma^*), \quad S_y = ik(\sigma - \sigma^*), \quad S_z = 2k\sigma_3, \tag{27}
\]
\[
\chi = \frac{2}{k} \xi, \quad \eta = \frac{2}{k} \tau, \tag{28}
\]
\[
b = \sqrt{\frac{n_0}{n_c}} \frac{k}{2}, \quad \delta_v = \delta_n. \tag{29}
\]

Complete integrability of equations (21)–(24) in the frameworks of the inverse scattering transformation method\(^6\)–\(^8\) in the case, when the linear velocities of transverse and longitudinal acoustic waves are equal, was established in paper\(^{17}\). Consequently, system (16)–(19) is also integrable if \(\delta_n = 0\).
3 Simplest solutions

Let us begin with system (16)–(19). At first, we suppose that the refractive indices of the medium are equal: \( n_e = n_o \). As it was noted at the end of previous section, system we deal with is integrable by inverse scattering transformation method in this case, and its multisoliton solutions can be obtained by means of the algebraic methods of the soliton theory. The Darboux transformation technique can be used for constructing these solutions, for instance. Since the one-soliton solution is stationary, it can be found by direct integration of equations (16–19). After straightforward calculations, one reveals that there can exist two families of the one-soliton solutions. These families are distinguished by the domains on the spectral parameter plane, in which the points of the discrete spectrum of the solution spectral data lie. Solution of the first family has next form:

\[
\begin{align*}
    u &= \frac{\sqrt{A} \sinh \theta}{A \sinh^2 \theta + K}, \\
    v &= -\frac{2K}{A \sinh^2 \theta + K}, \\
    \sigma &= -\frac{2T \sqrt{A} \sigma_0 (T \sinh \theta + i \cosh \theta)}{(1 + T^2)(A \sinh^2 \theta + K)}, \\
    \sigma_3 &= \sigma_0 \left( 1 - \frac{2T^2}{(1 + T^2)(A \sinh^2 \theta + K)} \right),
\end{align*}
\]

where

\[
K = \frac{k^2}{4}, \quad A = K(1 + T^2) - T^2,
\]

\[
\theta = \frac{\tau}{T} + \frac{4\sigma_0 T}{1 + T^2} \xi,
\]

\( \sigma_0 \) is an initial population of the quantum level (\( |\sigma_0| \leq 1/2 \)). Free parameter of this solution is real constant \( T \). It is assumed here that parameter \( T \) takes the values such that condition \( A > 0 \) holds. This condition is true with arbitrary \( T \), if \( K \geq 1 \). For \( K < 1 \), parameter \( T \) has to satisfy constraint

\[
T^2 \leq \frac{K}{1 - K}.
\]

In the case \( A < 0 \), the one-soliton solution is written as follows

\[
\begin{align*}
    u &= \frac{\sqrt{-A} \cosh \theta}{A \cosh^2 \theta - K}, \\
    v &= \frac{2K}{A \cosh^2 \theta - K}, \\
    \sigma &= -\frac{2T \sqrt{-A} \sigma_0 (T \cosh \theta + i \sinh \theta)}{(1 + T^2)(A \cosh^2 \theta - K)}, \\
    \sigma_3 &= \sigma_0 \left( 1 + \frac{2T^2}{(1 + T^2)(A \cosh^2 \theta - K)} \right).
\end{align*}
\]

This solution belongs to the second family of the one-soliton solutions. It can exist only if \( K < 1 \). Also, the parameter \( T \) value is such that

\[
T^2 > \frac{K}{1 - K}.
\]

The well-known one-soliton solution of the reduced Maxwell–Bloch equations\(^{15} \) is obtained from formulas (34)–(37) if we put \( k = 0 \).
It is seen from equations \ref{20}, \ref{31} and \ref{35} that there exists asymmetry on the polarity of the extraordinary component of the pulse electric field: signs of $E_e$ and PDM are opposite. Similar asymmetry was found in\textsuperscript{9–12} for two-component pulses, in which the ordinary component has slowly varying envelope. The number of peaks of variable $u$ of the second family solution depends on parameter $T$. If

$$T^2 \leq \frac{2K}{1 - K}$$

it has two peaks, while at

$$T^2 > \frac{2K}{1 - K}$$

only single peak exists. One can show that a degree of the medium excitation grows with increase of $|T|$ for the first one-soliton solution family and, on the contrary, with decrease of $|T|$ for second one. The full inversion of the quantum level population can take place only if $K \leq 1$. The curves of variables $u$ and $\sigma_3$ of the first and second families of one-soliton solution are presented on Figs 1, 2 and Fig. 3, respectively. The distance between the peaks of $u$ depends on parameter $T$ and can take an arbitrary value if $K < 1$ (see curves on Figs 2a and 3a). If $K > 1$, there exists an upper limit of this distance that corresponds to Fig. 6 below (compare with Fig. 1a). The strongest excitation of the quantum particles occurs in the center of the pulses in all cases.

Consider different limiting cases of the one-soliton solutions. If $K < 1$, we can tend $A$ to zero. Then, after appropriate shift alone $\tau$ (or $\xi$) axes, we obtain from formulas \ref{30}–\ref{33} or \ref{34}–\ref{37} following expressions:

$$u = \frac{T \exp \theta}{T^2 + K \exp 2\theta},$$

$$v = \frac{-2K \exp 2\theta}{T^2 + K \exp 2\theta},$$

$$\sigma = -\frac{2T^2 \sigma_0 (T - i \exp \theta)}{(1 + T^2)(T^2 + K \exp 2\theta)},$$

$$\sigma_3 = \sigma_0 \left(1 - \frac{2T^2 \exp 2\theta}{(1 + T^2)(T^2 + K \exp 2\theta)}\right).$$

It is necessary to put $T = k/\sqrt{4 - K^2}$ in this solution in accordance with the limiting procedure. If $|T| \to \infty$,
Fig. 2. Profiles of $u$ and $\sigma_3$ of the first one-soliton family with $K = 0.8$, $\sigma_0 = -0.5$, $T = 1.4$ (solid curves) and $T = 1.9999$ (dotted curves).

Fig. 3. Profiles of $u$ and $\sigma_3$ of the second one-soliton family with $K = 0.8$, $\sigma_0 = -0.5$, $T = 2.001$ (solid curves) and $T = 4$ (dotted lines).

then formulas (30)–(33) lead to algebraic solution:

$$
u = \frac{\sqrt{K-1}(\tau + 4\sigma_0 \xi)}{(K-1)(\tau + 4\sigma_0 \xi)^2 + K},$$ (42)

$$v = -\frac{2K}{(K-1)(\tau + 4\sigma_0 \xi)^2 + K},$$ (43)

$$\sigma = \frac{-2\sigma_0 \sqrt{K-1}(\tau + 4\sigma_0 \xi + i)}{(K-1)(\tau + 4\sigma_0 \xi)^2 + K},$$ (44)

$$\sigma_3 = \sigma_0 \left(1 - \frac{2}{(K-1)(\tau + 4\sigma_0 \xi)^2 + K}\right).$$ (45)

This limit is allowed in the case $K > 1$ only.

Dependence of variables $u$ and $\sigma_3$ of solutions (38)–(41) and (42)–(45) on $\tau$ are presented in Figs 4, 5 and 6, respectively. Comparing with curves on Figs 2a and 3a, we can say for the variable $u$ case that the first limiting procedure ($A \to 0$) corresponds to moving one of its peaks into infinity. The pulse, whose velocity exceeds the
light velocity, is plotted in Fig. 5. Note that variable $v$ tends to $-2$ in the limit $\tau \to -\infty$. For this reason, the energy levels trade their places owing to the Stark effect.

Let us now consider the case of unequal refractive indices ($n_e \neq n_o$). If we define coefficient $K$ as

$$K = \frac{n_o k^2}{4 n_e} \left( 1 + \frac{1 + T^2}{4 \sigma_0 T^2 \delta_n} \right)^{-1}$$

in the formulas (30)–(33) and (34)–(37), then they give solutions of system (16)–(19) with $\delta_n \neq 0$. Since coefficient $K$ characterizing the anisotropy of the medium depends now on the parameter of the pulse, we can say that the anisotropy becomes effective. It is seen that this coefficient is unbounded if

$$T^2 = -\frac{1}{1 + 4 \sigma_0 / \delta_n}$$

and it can change the sign. Similar results were obtained in works\textsuperscript{9–12}, where the approximation of the slowly varying envelope was applied for the description of the dynamics of the two-component pulses.

Obviously, the limiting cases studied above can be also modified to satisfy equations (16)–(19) with $n_e \neq n_o$.  

Fig. 4. Profiles of $u$ (solid curve) and $\sigma_3$ (dotted curve) in the case $A = 0$ with $K = 0.8$, $T = 2.0$, $\sigma_0 = -0.5$.

Fig. 5. Profiles of $u$ (solid curve), $v$ (thick curve) and $\sigma_3$ (dotted line) in the case $A = 0$ with $K = 0.8$, $T = -2.0$, $\sigma_0 = 0.5$.
Fig. 6. Profiles of \( u \) (solid curve) and \( \sigma_3 \) (dotted line) of the algebraic solution with \( K = 1.15, \sigma_0 = -0.5 \).

In particular, one has to put

\[
K = \frac{n_0 k^2}{4 n_e} \left( 1 + \frac{\delta_n}{4 \sigma_0} \right)^{-1},
\]

in formulas (42)–(45). Finally, coefficient \( K \) should be determined by relation (46) in equations (38)–(41), while parameter \( T \) should be equal to

\[
T = \pm \sqrt{\frac{\alpha}{4 - \alpha}},
\]

where

\[
\alpha = \frac{n_0}{n_e} k^2 - \frac{\delta_u}{\sigma_0}.
\]

The solutions of system (21)–(25) can be found using its equivalence to system (16)–(19) (see relations (26)–(29)). In particular, formulas (30)–(33) and (34)–(37) give two families of the pulse solutions of the acoustic system:

\[
E = \frac{2 \sqrt{B} \sinh \Theta}{B \sinh^2 \Theta + 1},
\]

\[
U = 1 - \frac{2}{B \sinh^2 \Theta + 1},
\]

\[
S_x = \frac{2b \sqrt{B} \hat{T}^2 S_0 \sinh \Theta}{(1 + b^2 \hat{T}^2)(B \sinh^2 \Theta + 1)},
\]

\[
S_y = \frac{2 \sqrt{B} \hat{T} S_0 \cosh \Theta}{(1 + b^2 \hat{T}^2)(B \sinh^2 \Theta + 1)},
\]

\[
S_z = S_0 \left( 1 - \frac{2 \hat{T}^2}{(1 + b^2 \hat{T}^2)(B \sinh^2 \Theta + 1)} \right),
\]

and

\[
E = \frac{2 \sqrt{-B} \cosh \Theta}{B \cosh^2 \Theta - 1},
\]

\[
U = 1 + \frac{2}{B \cosh^2 \Theta - 1}.
\]
\begin{align*}
  S_x &= \frac{2b\sqrt{-B\hat{T}^2}S_0\cosh\Theta}{(1+b^2\hat{T}^2)(B\cosh^2\Theta - 1)}, \\
  S_y &= \frac{2\sqrt{-B\hat{T}}S_0\sinh\Theta}{(1+b^2\hat{T}^2)(B\cosh^2\Theta - 1)}, \\
  S_z &= S_0 \left( 1 + \frac{2\hat{T}^2}{(1+b^2\hat{T}^2)(B\cosh^2\Theta - 1)} \right),
\end{align*}

where

\[ B = 1 + (b^2 - 1)\hat{T}^2, \quad \Theta = \frac{\eta}{\hat{T}} + \frac{b\hat{T}S_0}{1+b^2\hat{T}^2}\chi, \]

\( S_0 \) is an initial population of the quantum level, real constant \( \hat{T} \) is free parameter of these solutions. Coefficient \( B \) has to be positive for the former solution and negative for latter one. For the sake of simplicity, we suppose here that \( \delta_v = 0 \). The main properties of the pulse solutions presented are, obviously, the same as for the system (10–19) solutions. The appropriate modification of \( U \) and \( \hat{b} \) in these formulas allows one to obtain the solutions of system (21–25) with \( \delta_v \neq 0 \).

\section{4 Conclusion}

The two-component system of the reduced Maxwell–Bloch equations for anisotropic medium and its acoustic analogue have been considered in this report. As it was done in works\textsuperscript{11,12}, the linear velocities of both the components are not supposed to be equal. Two families of the one-parameter extremely short pulse solutions and their limiting cases, including one that leads to algebraically decreasing two-component pulse, are studied. The ordinary component of the pulse solutions found can have one peak like in the isotropic case or, also, two peaks, whose signs can be identical or opposite. If PDM is small (\(|D/2d| < 1\)), then the distance between the peaks can be unbounded. Nevertheless, the strongest degree of the medium excitation is always achieved in the center of the pulses. The asymmetry on the polarity of the signal takes place here similarly to that in the cases of one-component pulses\textsuperscript{4,5} and two-component pulses, in which the ordinary component has higher-frequency filling\textsuperscript{9–12}. Namely, the signs of the extraordinary component and PDM are opposite.

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