Asteroseismic radii of dwarfs: New accuracy constraints from Gaia DR2 parallaxes

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ABSTRACT
Precise stellar masses and radii can be determined using asteroseismology, but their accuracy must be tested against independent estimates. Using radii derived from Gaia DR2 parallaxes, we test the accuracy of asteroseismic radii for a sample of 93 dwarfs based on both individual frequency fitting and the seismic scaling relations. Radii from frequency fitting are about 1 per cent smaller than Gaia radii on average; however, this difference may be explained by a negative bias of 30 µas in the Gaia parallaxes. This indicates that the radii derived from frequency fitting are accurate to within 1 per cent. The scaling relations are found to overestimate radii by more than 5 per cent, compared to the Gaia radii, at the highest temperatures. We demonstrate that this offset is reduced to 3 per cent after applying corrections based on model frequencies to the scaling relation for ∆ν, but only when the model frequencies are corrected for the surface effect. With corrections to ∆ν, the scaling relation gives radii accurate to about 2–3 per cent for dwarfs in the temperature range 5400–6700 K. The remaining offset at the highest temperatures may indicate the need for a correction to the scaling relation for νmax.

Key words: asteroseismology – stars: fundamental parameters

1 INTRODUCTION
During the last few decades, asteroseismology of solar-like oscillators has emerged as a powerful tool for determining precise stellar parameters, not least thanks to the high-precision photometry from space-based telescopes (see Chaplin & Miglio (2013) for a review). In the best-case scenario, the frequencies of the individual stellar oscillations can be identified in the power spectrum and fitted to stellar evolutionary models in order to determine precise fundamental parameters like the radius, mass, age, etc. (e.g. Silva Aguirre et al. 2015, 2017). However, for the large majority of targets observed by the space missions, the signal-to-noise ratio is not high enough to reliably identify the individual frequencies. Still, two average parameters of the power spectrum can be determined: the mean frequency separation, ∆ν, and the frequency of maximum power, νmax. These parameters scale with the stellar mean density and surface gravity, and allow for estimates of the stellar mass and radius through a set of scaling relations. The scaling relations have been applied to estimate stellar parameters for more than 500 main sequence and subgiant stars (Chaplin et al. 2014) and thousands of red giants (e.g., Pinsonneault et al. 2018; Silva Aguirre et al. 2018) observed by the Kepler spacecraft.

In recent years, much effort has been devoted to testing the accuracy of the stellar parameters derived from asteroseismic analyses. For the scaling relations, such investigations have shown that the seismic radii of dwarfs are accurate to within about 5 per cent, based e.g. on comparisons with radii from interferometry (Huber et al. 2012; White et al. 2013). Following the first data release from the Gaia mission (Gaia DR1; Gaia Collaboration et al. 2016a,b), a number of studies were carried out comparing distances (or parallaxes) derived from seismic radii from scaling relations to those from the Tycho-Gaia astrometric solution (Lindegren et al. 2016). These studies generally found good agreement between seismic and Gaia parallaxes for dwarf stars, confirming once again that the scaling relations provide radii accurate to within 5 per cent (De Ridder et al. 2016; Huber et al. 2017). However, seismic parallaxes derived from the radii of dwarfs based on modelling of individual frequencies have been found to be systematically overestimated by about 3 per cent compared to the Gaia DR1 parallaxes (Silva Aguirre et al. 2017; Sahlholdt et al. 2018). Due to the possibility of systematic errors in the Gaia DR1 parallaxes, it was not clear whether this was due to a 3 per cent
systematic in the seismic radii or the Gaia parallaxes (or a combination of the two).

With the recent second data release from the Gaia mission (Gaia DR2; Gaia Collaboration et al. 2018; Lindegren et al. 2018) the precision of the parallaxes of Kepl er stars has improved significantly and the level of systematic errors has decreased. In this paper we use parallaxes from Gaia DR2, to put improved constraints on the accuracy of seismic radii of dwarf stars.

2 SAMPLE AND METHODS

We consider a sample of dwarf stars with individual frequencies, as well as $\Delta \nu$ and $v_{\text{max}}$, determined from Kepler data. The sample consists of the 33 stars of 'simple' type from the Kages sample (Davies et al. 2016) and the 66 stars of the LEGACY sample (Lund et al. 2017). These two studies had 4 stars in common making the total sample size 95 stars. Two of these were not included in Gaia DR2 which leaves us with 93 dwarfs with individual frequencies and Gaia parallaxes. The typical parallax precision for these stars has improved by a factor of 10 from 300 $\mu$as in Gaia DR1 to 30 $\mu$as in Gaia DR2.

We have collected spectroscopic metallicities, [Fe/H], for the sample from Buchhave & Latham (2015). Some of the LEGACY stars were not included in that study, for these we use the same [Fe/H] as in the original study of the LEGACY stars (Silva Aguirre et al. 2017). For effective temperatures, $T_{\text{eff}}$, and angular diameters, $\theta$, we adopt the values based on the infrared flux method (IRFM; calibration by Casagrande et al. 2010) derived in our previous study of this sample (Sahlholdt et al. 2018). The IRFM angular diameters are derived from the fundamental relations between bolometric flux, effective temperature, and angular diameter (see Casagrande et al. 2006); hence, they correspond to limb-darkening corrected angular diameters obtained from interferometry. They are, however, subject to uncertainties related to bolometric corrections, reddening, and the zero-point of the temperature scale. Throughout our analysis we assume that there is no significant bias in the angular diameters.

2.1 Seismic radii

We test two sets of seismic radii: one calculated by fitting stellar models to the observed frequencies, and the other calculated directly from the seismic scaling relations. For the first set we use stellar models calculated using the Garching Stellar Evolution Code (GARSTEC; Weiss & Schlattl 2008) with theoretical oscillation frequencies from the Aarhus Adiabatic Oscillation Package (adipla; Christensen-Dalsgaard 2008). When fitting to the frequencies, we use the ratios of frequency differences described by Roxburgh & Vorontsov (2003). The advantage is that these ratios are insensitive to the stellar surface layers, so the systematic overestimation of the highest model frequencies, known as the surface effect, becomes irrelevant. For simplicity, we refer to this as fitting to individual frequencies. We fit the stellar models to the individual frequencies, $T_{\text{eff}}$, and [Fe/H] using the latest version of the BAyesian STellar Algorithm (BASTA; Silva Aguirre et al. 2015). The stellar models are the same as those described in Sahlholdt et al. (2018, section 3.1).

For the second set of seismic radii we use the scaling relations for $\Delta \nu$ and $v_{\text{max}}$ which are given by (Ulrich 1986; Brown et al. 1991)

$$\frac{\Delta \nu}{\Delta \nu_\odot} \simeq \left( \frac{M}{M_\odot} \right)^{1/2} \left( \frac{R}{R_\odot} \right)^{-1/2},$$

$$\frac{v_{\text{max}}}{v_{\text{max,}\odot}} \simeq \left( \frac{M}{M_\odot} \right)^{2} \left( \frac{R}{R_\odot} \right)^{2} \left( \frac{T_{\text{eff}}}{T_{\text{eff,}\odot}} \right),$$

Combining these relations, the radius is given by

$$\frac{R}{R_\odot} \simeq \left( \frac{v_{\text{max}}}{v_{\text{max,}\odot}} \right) \left( \frac{\Delta \nu}{\Delta \nu_\odot} \right)^{2} \left( \frac{T_{\text{eff}}}{T_{\text{eff,}\odot}} \right).$$

For the solar values we adopt $v_{\text{max,}\odot} = 3090$ $\mu$Hz, $\Delta \nu_\odot = 135.1$ $\mu$Hz (Huber et al. 2011), and $T_{\text{eff,}\odot} = 5777$ K.

2.2 Gaia radii

To calculate a set of radii based on the Gaia parallaxes, $\sigma$, we use them in combination with the IRFM angular diameters. The stellar radius and angular diameter can be used to calculate the distance and thereby the parallax:

$$d = \frac{2R}{\theta} \Rightarrow \sigma = \frac{1}{d} = \frac{\theta}{C \times 2R},$$

where $C$ is the factor which converts the distance into parsec. Then the radius is given by

$$R = \frac{\theta}{C \times 2\sigma}.$$ 

In Equation 4 we assume that the inverse of the parallax is a good estimate of the distance which is not necessarily the case when the parallax is very uncertain (see e.g. Bailer-Jones 2015). However, for all but two of the stars in our sample, the parallax uncertainties are below 2 per cent (the other two have uncertainties of 3 and 5 per cent), so we do not expect this assumption to bias our radii to any significant degree.

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1 Most of our analysis has been carried out in a Jupyter notebook which is publicly available on GitHub: https://github.com/csahlholdt/testing_ast_radii.
In Figure 1 we show the uncertainties on the \textit{Gaia} radii and the seismic radii derived from individual frequency fitting. The seismic radii all have uncertainties below 0.05 $R_\odot$ or about 2 per cent. Overall, the \textit{Gaia} radii are more uncertain, and the majority of them are precise to within 5 per cent. The precision of the \textit{Gaia} radii is limited by the angular diameters which have uncertainties of about 3–5 per cent. This includes uncertainties in the photometry, reddening, temperature, metallicity, and surface gravity, and these terms are now the limiting factors, rather than the parallax, in the radius precision for these stars.

### 3 RESULTS AND DISCUSSION

In Figure 2, the two sets of seismic radii are compared with the \textit{Gaia} radii for the full sample of 93 stars. A number of outliers are clearly visible; we have marked in red the stars for which the difference between the \textit{Gaia} radius and the seismic radius based on individual frequency fitting is greater than 3\sigma. They are all outliers in the sense that the \textit{Gaia} radius is too large which may be explained by contaminated photometry leading to overestimated angular diameters. In all results discussed in the following, these nine outliers have been removed leaving 84 stars.

By fitting a straight line to the data using orthogonal distance regression, we find that the slope is consistent with one for radii based on individual frequencies. However, there is a small but significant average offset of $(R_{\text{seismic}} - R_{\text{Gaia}}) = -0.015 \pm 0.005 R_\odot$, suggesting that the seismic radii are underestimated. If we make the comparison in terms of parallaxes instead, by calculating seismic parallaxes using Equation 4, we find a mean difference of $(\sigma_{\text{Gaia}} - \sigma_{\text{seismic}}) = -35 \pm 16$ mas with seismic parallaxes based on the individual frequency analysis. This offset is similar in direction and magnitude to those found based on seismic parallaxes of red giants in the \textit{Kepler} field (Zinn et al. 2018) and parallaxes of eclipsing binaries (Stassun & Torres 2018). Our offset is also in agreement with the possible negative bias in the \textit{Gaia} DR2 parallaxes of about 30\mu as based on the parallaxes of quasars (Lindgren et al. 2018).

This means that the offset between seismic and \textit{Gaia} radii in the left-hand panels of Figure 2 may be entirely explained by a negative bias in the \textit{Gaia} parallaxes leading to overestimated \textit{Gaia} radii. For the comparison based on scaling relations, we find a slope larger than one and an average offset of $(R_{\text{seismic}} - R_{\text{Gaia}}) = 0.014 \pm 0.005 R_\odot$ making the seismic radii slightly overestimated compared to the \textit{Gaia} radii. They may be slightly more overestimated than this number suggests if the \textit{Gaia} radii are in fact underestimated as discussed above.

In Figure 3 the ratios of seismic to \textit{Gaia} radii are shown as a function of both $T_{\text{eff}}$ and [Fe/H]. The overestimation of the seismic radii based on scaling relations, and the slope in Figure 2, is mainly due to an offset at the highest temperatures in the sample where the mean offset reaches about 7 per cent. For radii based on individual frequencies, the offset is more or less constant as a function of both $T_{\text{eff}}$ and [Fe/H] with a mean ratio of $(R_{\text{seismic}}/R_{\text{Gaia}}) = 0.988 \pm 0.004$. Again, this offset is consistent with a negative bias in the \textit{Gaia} parallaxes of about 30\mu as, indicating that frequency fitting gives radii accurate to within 1 per cent. Furthermore, this confirms that the offset of 3 per cent between seismic and \textit{Gaia} parallaxes found previously (Silva Aguirre et al. 2017; Sahlholdt et al. 2018) was mainly caused by a bias in the \textit{Gaia} DR1 parallaxes for these particular stars.

The fact that the offset in the radii from the scaling relation depends on the temperature can be explained by biases in the scaling relations. Comparisons have been made for stellar models between $\Delta \nu$ from the scaling relation (Equation 1) and $\Delta \nu$ from theoretical oscillations frequencies of the models (e.g. White et al. 2011). They show that the size of the deviation between the two is mainly a function of temperature. To investigate whether a correction to the scaling relation for $\Delta \nu$ can explain the temperature trend we see, we have derived correction factors for $\Delta \nu$, $f_{\Delta \nu}$. For each star we take the best-fitting model from the individual frequency fits and calculate the large frequency separation based on the model frequencies following White et al. (2011); this value is referred to as $\Delta \nu_{\text{fit}}$. Using the mass, radius, and $T_{\text{eff}}$ of the same model, we also calculate $\Delta \nu_{\text{cal}}$ using Equation 1 and define $f_{\Delta \nu} = \Delta \nu_{\text{fit}}/\Delta \nu_{\text{cal}}$. We find that our corrections agree qualitatively with what others have found when comparing $\Delta \nu_{\text{fit}}$ and $\Delta \nu_{\text{cal}}$ for stellar models (White et al. 2011; Sharma et al. 2016; Serenelli et al. 2017).

Due to the previously mentioned surface effect which causes the high-order model frequencies to be overestimated, $\Delta \nu_{\text{fit}}$ is also overestimated if this effect is not taken into account. Therefore, we calculate a second set of $\Delta \nu_{\text{fit}}$ and $f_{\Delta \nu}$ after applying the Ball & Gizon (2014) correction for the surface effect to the model frequencies.

Figure 4a shows the ratios of seismic to \textit{Gaia} radii with the uncorrected scaling relations (which is the data behind the bins in the upper panel of Figure 3), and Figure 4b,c show the same after applying the correction factor by dividing $\Delta \nu$ by $f_{\Delta \nu}$ in Equation 3. After correction of $\Delta \nu$ based on $\Delta \nu_{\text{fit}}$ without correcting for the surface effect, the temperature trend is reduced but only by increasing the radii of stars at intermediate temperatures which leaves a significant mean offset. In fact, the agreement between the seismic and \textit{Gaia} radii is significantly worse for stars of intermediate temperatures after applying this correction. For these stars, the good agreement is restored when we take into account the surface effect as seen in Figure 4c, which also brings the offset down to about 3 per cent at the highest temperatures.

Overall, these results show that theoretically motivated corrections to $\Delta \nu$ improve the accuracy of the seismic radii of dwarfs from scaling relations, but it is important to take the surface effect into account. After correction including the surface effect, the radii from the scaling relations agree with the \textit{Gaia} radii within 2–3 per cent across the temperature range considered here. In this work we have calculated a correction for the surface effect for each individual star. This is possible since we have the observed frequencies for all of them, but this is not generally the case when applying the scaling relations to larger samples of stars. However, we find that the impact of the surface effect on the correction factors, $f_{\Delta \nu}$, is very close to being a constant offset. Therefore, it is a good approximation to simply apply the surface correction obtained for the Sun to all dwarfs in the temperature range 5400–6700 K. This can be done by scaling all values of $f_{\Delta \nu}$ with a common factor such that the value is 1.0 for a solar model.

Although the temperature trend is reduced by the $\Delta \nu$
correction, it is not completely gone in Figure 4c. The remaining temperature trend could be due to inaccuracies in the scaling relation for $r_{\text{max}}$ which we have not attempted to include here.

4 CONCLUSIONS

We have computed seismic radii based on individual frequency modelling and the scaling relations for 93 dwarf stars observed by Kepler and included in Gaia DR2. By combining the Gaia parallaxes with IRFM angular diameters, we have computed Gaia radii for the stars and used them to test the accuracy of the seismic radii. After removing nine 3$\sigma$ outliers, we find only small, although statistically significant, differences between seismic and Gaia radii. The mean radius differences, in the sense seismic minus Gaia, are $-0.015 R_\odot$ and $0.014 R_\odot$ for individual frequencies and scaling relations, respectively.

The offset of the radii based on individual frequencies is equivalent to an offset in parallax of $\langle \sigma_{\text{Gaia}} - \sigma_{\text{seismic}} \rangle = -35 \pm 16 \, \mu\text{as}$. Since this is consistent with the possible negative bias in the Gaia DR2 parallaxes, the bias may be in the Gaia radii rather than the seismic ones. With this in mind, the radii from individual frequencies are likely accurate to a level of 1 per cent or better.

We find that the scaling relations overestimate the stellar radii mainly at high temperatures, reaching values of over 5 per cent larger than the Gaia radii. By introducing theoretically motivated correction factors to $\Delta r$ based on individual frequencies of stellar models, we are able to reduce this offset. However, the corrections actually worsen the accuracy of the scaling relations unless we correct the model frequencies for the surface effect. After including the surface effect, our corrections bring the seismic radii into average agreement with the Gaia radii, and the average offset at the highest temperatures is brought down to 3 per cent. Taking into account the possible negative bias in the Gaia parallaxes, we conclude that the corrected scaling relation for the radius is accurate to within about 2–3 per cent for dwarfs.

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Figure 3. Mean ratios of seismic to Gaia radii binned by the effective temperature (top) and the metallicity (bottom). The filled circles show the comparison with seismic radii based on modelling individual frequencies and the open circles are based on the scaling relations.

Figure 4. Ratios of seismic to Gaia radii as a function of temperature, with seismic radii based on the scaling relations. The black dashed line marks unity, and the grey dashed lines mark a difference of 3 per cent to guide the eye. The open circles are mean values binned by temperature. a) No corrections to the scaling relation (i.e. Equation 3). b) $\Delta \nu$ corrected using $\Delta \nu$ from the models of our best fits to individual frequencies. c) Like b) but with $\Delta \nu_{\text{hr}}$ corrected for the surface effect. See the text for more details on the $\Delta \nu_{\text{hr}}$ correction.

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