Strong $WW$ interactions at the LHC

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Abstract

We present a brief pedagogical introduction to the Effective Electroweak Chiral Lagrangians, which provide a model independent description of the WW interactions in the strong regime. When it is complemented with some unitarization or a dispersive approach, this formalism allows the study of the general strong scenario expected at the LHC, including resonances.

1 Introduction

As is well known, the Standard Model (SM), which is a $SU(3)_C \times SU(2)_L \times U(1)_Y$ quantum gauge theory, is able to describe all our knowledge about the strong and electroweak interactions, even at the high level of precision reached at LEP (see for instance [9]). The SM can be divided in three sectors: The first one is the matter content (quarks and leptons), whose elementary particles

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interact between themselves by mediating bosons that belong to the second sector. These fields are the eight gluons associated to the $SU(3)_{C}$ group of the strong interactions as well as the $W^{+}, W^{-}$ and $Z$ bosons together with the photon, which are associated to the $SU(2)_{L} \times U(1)_{Y}$ group of the electroweak interactions. Finally, there is the so-called Symmetry Breaking Sector (SBS). It is responsible for the spontaneous symmetry breaking of the electroweak gauge group $SU(2)_{L} \times U(1)$ down to the electromagnetic group $U(1)_{em}$. Through the Higgs mechanism it provides masses for the $W^{+}, W^{-}$ and $Z$ bosons while leaving the photon massless. In addition, the SBS is also connected with the matter sector through Yukawa couplings which give rise to the quark and lepton masses, quark mixing (Cabbibo-Kowayashi-Maskawa matrix) and eventually to CP violation because of the complex phase in this matrix.

Now we arrive to the first important remark of this lecture. In contrast with the matter and the gauge sectors, the SBS is very poorly known from the experimental point of view. In fact, several different theoretical scenarios have been widely discussed in the literature. Generically they can be grouped in three kinds, the Minimal Standard Model (MSM), the Minimal Supersymmetric Standard Model (MSSM) and QCD-like theories. Let us briefly review the main features and problems of these scenarios.

### 1.1 Minimal Standard Model

It contains the minimum ingredients to explain the present data. However, it does not shed much light on possible new physics effects and it does not address several problems, among others:

- The Higgs potential is introduced *ad hoc*. It is not a gauge interaction as the rest of the the known forces in Nature such as the strong, the electroweak or even gravity. The origin and nature of this Higgs field remains a mystery.

- Keeping the mass of this scalar field at scales close to the electroweak symmetry breaking (100 GeV to 1 TeV) requires a very fine tuning, since radiative corrections tend to make its mass of the order of the next new physics scale. This is known as the naturalness problem.

- The electron and top masses fall five orders of magnitude apart. The problem of why the masses present such a hierarchy is not addressed.
There are hints in the literature suggesting that the simple realization of
the Higgs sector in the MSM could indeed be a trivial (non-interacting)
quantum field theory.

Nowadays the existence of the Higgs is taken for granted by many people as
it was the case for the top quark. However the Higgs is not the top. By
this we mean that the SM model would have not been consistent without the
top quark (it becomes an anomalous gauge theory), whereas the Higgs boson
is not a theoretical need. It is possible to postulate different versions of the
SM differing in the SBS which are theoretically consistent. Indeed, in most
of the physical systems that present an spontaneous symmetry breaking (like
chiral symmetry breaking in QCD or, in solid state physics, the Cooper pair
formation, magnetization, etc.), there is nothing analogous to a fundamental
Higgs field.

Nevertheless, for its simplicity, this model is very useful to describe the data,
without additional assumptions.

1.2 Minimal Supersymmetric Standard Model

In this model, an additional symmetry relating fermions and bosons is intro-
duced. As a consequence the Higgs potential is related to the gauge couplings
and the scalar particles appear in a natural way. The advantage of this new
symmetry is that for each fermion loop there is a corresponding boson loop with
similar couplings and masses but opposite sign, thus avoiding the naturalness
problem. However:

- Nature is not supersymmetric. “Soft” breaking terms must be added by
  hand in order to break spontaneously the $SU(2)_L \times U(1)_Y$ gauge sym-
  metry, without spoiling too much the cancellations needed to solve the
  naturalness problem. Those terms break supersymmetry explicitly.

- The values of the parameters in those soft breaking terms (more than
  a hundred) are unknown, and they severely limit the predictive power of
  these models. The origin of those soft breaking terms are the origin of
  further speculation.

- Probably the most robust (soft breaking parameter independent) pre-
  diction is that a Higgs should appear below around 120 GeV. Thus this
particle could have been produced at LEP. So far nothing has been found, but there is still a small room for discovery. However, if nothing of this kind is found at the next generation of colliders the low-energy supersymmetric scenarios would be in serious trouble.

1.3 QCD-like scenario

These models mimic the spontaneous chiral symmetry breaking of QCD and are generically known as Technicolor (TC). The Higgs does not exist as a fundamental field although some other composite fields with different quantum numbers play a similar role. However

- There is no completely consistent and universally accepted Technicolor model.
- Predictions are very vague due to the strong nature of the interactions.
- The simplest versions, like a direct rescaling of QCD, are ruled out by the LEP data or by the appearance of undesired flavor changing neutral currents.

Therefore we arrive to the second main remark of this lecture: *it possible that the SBS of the SM has nothing to do with our current theoretical expectations.*

At this point one could ask which are the main experimental constraints on the SBS or, in other words, what we really know about this sector. The main pieces of our knowledge are the following [3]:

1. First of all there must be a physical system coupled to the SM displaying a spontaneously symmetry breaking pattern from a *global* $G$ group to a subgroup $H$. This symmetry breaking triggers the Higgs mechanism that breaks the electroweak *gauge* group $SU(2)_L \times U(1)_Y$ down to the electromagnetic group $U(1)_{em}$. Thus we have $SU(2)_L \times U(1)_Y \in G$ and $U(1)_{em} \in H$.

2. Since we need three would-be Goldstone bosons in order to give masses to the $W^+, W^-$ and the $Z$ gauge bosons, we have $\dim G - \dim H = 3$.

3. Experimentally we know that the $\rho$ parameter (which measures the relative strength of the charged and neutral weak currents) is very close to one - apart from some radiative corrections proportional to the hypercharge coupling $g'$ squared. Probably the most natural explanation for
this fact is to assume that the unbroken $H$ group of the SBS contains the so-called custodial group $SU(2)_{L+R}$ (as it happens in the MSM). Any other assumption leads to some fine tuning.

With these conditions on $G$ and $H$ it is very easy to show that the only possible solution is $G = SU(2)_L \times SU(2)_R$ and $H = SU(2)_{L+R}$.

4. Finally, from the muon mean life it is possible to obtain the dimensional parameter $v \simeq 250$ GeV which sets the scale of the SBS dynamics in the SM.

At this point it is reasonable to think whether it is possible to build a model independent description of the SBS. As we will see this can be done by using the Effective Electroweak Chiral Lagrangian (EChL), which is based on a similar formalism used in low-energy hadron physics, namely, Chiral Perturbation Theory (ChPT) \cite{5}. As we will see, this approach is especially useful when the SBS is strongly interacting.

## 2 The Electroweak Chiral Lagrangian

The EChL provides a phenomenological description of the Goldstone boson dynamics associated to the symmetry breaking of $SU(2)_L \times SU(2)_R$ down to $SU(2)_{L+R}$. As far as we are not introducing any other field, it has to be realized nonlinearly. That will limit the applicability of the approach up to the energies where the other relevant degrees of freedom show up. In the case of strong dynamics, we expect these other modes to appear at energies much higher than $v \simeq 250$ GeV and the formalism will be very useful. In contrast, for theories with, for instance, a light Higgs (as in the MSSM), there is no applicability region for this formalism, but in that case we will have additional information to disentangle the SBS physics when measuring these light modes.

Therefore, we will be assuming a strong SBS. For simplicity, let us then switch off momentarily the gauge fields, whose interactions with the SBS are comparatively weak. In such case, no other degrees of freedom are present at low energies except the Goldstone bosons $\omega^a(x)$, which will be gathered in the $SU(2)_{L+R}$ matrix

$$U(x) = \exp \left( \frac{i \omega^a(x) \sigma^a}{v} \right),$$

where the $\sigma^a$ are the Pauli matrices.
A low energy expansion of the amplitudes is nothing but a derivative expansion of the Lagrangian. Then, the simplest $G$-invariant Lagrangian relevant at low energies (with two derivatives) can be written as

$$L_2 = \frac{v^2}{4} \text{tr} \partial_\mu U \partial^\mu U^\dagger.$$  \hspace{1cm} (2)

From this Lagrangian it is possible to obtain the exact behavior of the elastic low-energy scattering amplitude for the Goldstone bosons. Indeed, using the SU(2) and crossing symmetries, any amplitude can be obtained from that of $\omega^+\omega^- \rightarrow \omega^0\omega^0$, which is given by

$$A(s, t, u) = \frac{s}{v^2} + O\left(\frac{s^2}{v^4}\right).$$  \hspace{1cm} (3)

Of course, the Goldstone bosons are not directly observable, since through the Higgs mechanism they are “eaten” by the $W^\pm$ and $Z$ longitudinal components, that we will denote, generically, by $W_L$. Indeed, the so-called Equivalence Theorem \cite{6} (ET) relates the Goldstone bosons amplitudes with the corresponding longitudinal components of the electroweak gauge bosons for the MSM, as follows

$$A(W_L^a W_L^b \rightarrow W_L^c W_L^d) \simeq A(\omega^a \omega^b \rightarrow \omega^c \omega^d) + O\left(\frac{M_W}{\sqrt{s}}\right).$$  \hspace{1cm} (4)

This result is a consequence of the Slavnov-Taylor identities coming from the $SU(2)_L \times U(1)_Y$ gauge symmetry. The $O(M_W/\sqrt{s})$ corrections can be understood by noting that the Goldstone bosons are massless in contrast with the gauge bosons, whose mass is $O(100 \text{ GeV})$. Note that the ET is a high energy limit, whereas the EChL is a low energy limit. Indeed, for the EChL the formulation of the Equivalence Theorem is not so simple \cite{7} but we will not discuss the details here since, later we will unitarize the amplitudes of the effective lagrangian and in such case the above formulation is valid (the interested reader can find a complete account of this issue in \cite{6} and \cite{8}). At this point the following comments are in order:

- First we see that the low energy dynamics of the Goldstone bosons is dictated by symmetry and the scale $v$ only. In this sense, it is universal, i.e. independent of the details of the SBS. The amplitudes obtained from eq.(2) are called the Low Energy Theorems.

- The amplitudes above grow with the energy. Thus, if we assume that no other particles modify this behavior at low energies, they give rise to
strong interactions for the Goldstone bosons as well as for the longitudinal components of the electroweak gauge bosons, according to the Equivalence Theorem.

- However, the growth of this amplitudes is in conflict with unitarity around $O(1\text{TeV})$ energies.

In conclusion, provided there are no other light modes in the SBS, we expect strongly interacting $W_L'$s. From unitarity constraints we also expect new physics at the TeV scale, possibly in the form of resonances.

## 3 Beyond the Low Energy Theorems

In order to switch on the gauge fields in the low energy EChL we change the derivatives in eq.(2) into the appropriate covariant derivatives, $D_\mu$, containing the electroweak gauge fields. That is

$$L_2 = \frac{v^2}{4} \text{tr} D_\mu U (D^\mu U)^\dagger.$$  \hspace{1cm} (5)

In addition we can introduce the next to leading order (four derivative) terms in the EChL

$$L_4 = L_1 \left( \text{tr} D_\mu U D_\nu U^\dagger \right)^2 + L_2 \left( \text{tr} D_\mu U D_\nu U^\dagger \right)^2 + \ldots$$  \hspace{1cm} (6)

where we have only displayed those terms that give rise to $O(s^2/v^4)$ contributions to the Goldstone boson elastic scattering amplitude. These $O(s^2/v^4)$ terms depend on several $L_i$ constants, which parameterize our ignorance on the SBS. For special values we recover some particular models. For example, the MSM with a 1 TeV Higgs corresponds to $L_1 = 0.007$ and $L_2 = -0.0022$ whereas the simplest TC model with three technicolors has $L_1 = -0.001$ and $L_2 = 0.001$. In addition, under renormalization these parameters can absorb the divergences appearing in the one-loop contributions to the amplitudes coming from $L_2$, which are $O(s^2/v^4)$. From precision test of the SM it is possible to set bounds on some of these parameters. However, these bounds are too weak for $L_1$ and $L_2$, which are expected to lie in the $10^{-3}$ to $10^{-2}$ range. That precision could only be reached after a few years of LHC running at full luminosity.

7
4 Unitarization and dispersion Relations

Customarily, the longitudinal gauge boson amplitudes are given in a basis of states of definite angular momentum, $J$, and the “weak isospin”, $I$, associated to the SU(2) group. These “partial waves”, $t_{IJ}$, are also obtained as an expansion of the form

$$t_{IJ}(s) = t_{IJ}^{(2)}(s) + t_{IJ}^{(4)}(s) + O(s^3),$$

(7)

where the superscript refers to the corresponding energy (momentum) power. As we have already seen, they grow with the energy and violate unitarity around 1 TeV. In this basis, the elastic unitarity constraint can be easily written for physical values of $s$; it reads

$$\text{Im} t_{IJ}(s) = |t_{IJ}(s)|^2 \Rightarrow \text{Im} \frac{1}{t_{IJ}(s)} = -1,$$

(8)

which is basically the Optical Theorem. Although the results obtained from the Chiral Lagrangian break unitarity, they are nevertheless unitary in the perturbative sense

$$\text{Im} t_{IJ}^{(4)}(s) = |t_{IJ}^{(2)}(s)|^2 \Rightarrow \frac{\text{Im} t_{IJ}^{(4)}(s)}{|t_{IJ}^{(2)}(s)|^2} = -1.$$

(9)

It is however possible to obtain unitary amplitudes from the effective Lagrangian. Note that from eq.(8) we know exactly the imaginary part of the inverse of the amplitude. As a consequence, any unitary amplitude will satisfy

$$\frac{1}{t_{IJ}(s)} = \text{Re} \frac{1}{t_{IJ}(s)} - i \Rightarrow t_{IJ}(s) = \frac{1}{\text{Re} t_{IJ}^{-1}(s) - i}. $$

(10)

That is, we only have to approximate the real part of the inverse of the amplitude $t_{IJ}^{-1}$, by means of eq.(7). Formally: $\text{Re} t_{IJ}^{-1} = (t_{IJ}^{(2)})^{-1}[1 - \text{Re} t_{IJ}^{(4)}/t_{IJ}^{(2)} + ...]$. Finally, using eq.(7) we can write

$$t_{IJ}(s) = \frac{t_{IJ}^{(2)}}{1 - t_{IJ}^{(4)}/t_{IJ}^{(2)}}$$

(11)

which is known as the Inverse Amplitude Method (IAM). It can be derived alternatively, by writing a two subtracted dispersion relation for the inverse amplitude. Using some extra hypothesis and approximations it is possible to solve the dispersion relation for $t_{IJ}(s)$ to find the same result.

This partial wave is strictly unitary and has the proper analytical structure with the appropriate cuts. In addition it is able to reproduce poles which can be interpreted as resonances generated dynamically. Note also that, by expanding this amplitude in power of s, we recover the chiral low-energy expansion.
4.1 The Inverse Amplitude Method at work

The IAM method has been successfully applied in a completely different physical context: the low-energy hadron dynamics [10]. As it is well known, QCD is the proper theory to describe strong interactions, but it cannot be applied directly at low energies due to the breaking of standard perturbation theory. However in the limit where the three lightest quarks are massless, the QCD Lagrangian possesses a global symmetry (chiral symmetry) which rotates right quarks or left quarks between themselves. The symmetry group is $SU(3)_L \times SU(3)_R$ and for different reasons it is known that it is spontaneously broken to the $SU(3)_{L+R}$ group. The corresponding Goldstone bosons are identified with the pseudoscalar mesons $\pi^0, \pi^\pm, K^0, \bar{K}^0$ and $\eta$ and their relative low physical masses compared with the typical hadronic scale of 1 GeV, can be considered as a perturbation effect due to the very small, but non-zero, quark masses. Note that the symmetry pattern is very close to that of the SBS (it would be the same if we just considered two quarks).

As we did before we can gather the $\omega^a$ mesons fields in an $SU(3)$ matrix as $U(x) = \exp(i \omega^a \lambda^a / F)$, where $\lambda^a$ are the Gell-Man matrices and $F$ is basically the pion decay constant. Once more we can describe the low energy hadron dynamics in terms of a chiral Lagrangian. This approach is known as Chiral Perturbation Theory (ChPT)[13]. At the lowest order this Lagrangian is given by:

$$\mathcal{L}_2 = \frac{F^2}{4} \text{tr} \partial_\mu U \partial^\mu U^\dagger. \quad (12)$$

which reproduces the well know current algebra results in a very simple way. At the next order (four derivatives) one has additional terms whose precise form is not relevant here, although some of them have the same structure of those in eq.(1). As a matter of fact, the formalism that we have presented for the SBS is inspired in the massless limit of SU(2) ChPT, although rescaled from $F \simeq 93 \text{ MeV}$ up to $v \simeq 250 \text{ GeV}$. The main difference is the existence of real data on meson physics, from which it is possible to determine the values of the $\mathcal{L}_4$ ChPT Lagrangian parameters, whereas they are undetermined for the SBS.

The amplitudes can now be obtained as a truncated series in powers of the momentum $p^2$ over $4\pi F \simeq 1 \text{ GeV}$. This formalism is only suitable at low-energies up to about 500 MeV. We should not extrapolate them naively to higher energies since they would severely violate unitarity and they would not
Figure 1: IAM fit to the phase shifts for $\pi\pi \rightarrow \pi\pi$ and $\pi K \rightarrow \pi K$. For data references see\textsuperscript{5}.

reproduce resonances.

However, we can use the IAM to extend the applicability of the effective Lagrangian approach. In Fig.1 we show an example of the results obtained with the IAM when applied to $\pi\pi \rightarrow \pi\pi$ (analogous to the Goldstone boson $\omega\omega \rightarrow \omega\omega$) and $\pi K \rightarrow \pi K$ scattering. Note that now the data is reproduced up to approximately 1 GeV. In addition, resonances like the $\sigma$, $\rho$ and $K^*$ are correctly reproduced with an associated pole in the second Riemann sheet.

Starting from the corresponding effective Lagrangians, the IAM has also been applied very successfully to other processes with coupled channels\textsuperscript{11} or even nucleons\textsuperscript{12}, reproducing correctly many other resonances. Thus we arrive to the conclusion that the IAM greatly improves the range of applicability of the effective Lagrangians and, moreover, it is able to reproduce resonances in the channels where they are present.

5 Resonances in the SBS

Let us then apply the IAM to the SBS. In this case we do not have experimental information yet, and therefore we do not know the values of the $O(p^4)$
parameters, which are model dependent. Nevertheless, we expect them to lie between $10^{-2}$ and $10^{-3}$ if the SBS is strongly interacting.

For elastic $W_LW_L$ scattering only two $O(p^4)$ parameters appear in the amplitudes, namely, $L_1$ and $L_2$. By changing their values we can therefore reproduce the behavior of $W_LW_L$ scattering in any strongly interacting model. In Fig.3 we show the phase shifts $\delta_{ij}$ that we expect in three different models. The first one corresponds to the SM with 1 TeV Higgs and, consistently, we see a resonance in the scalar isoscalar channel (a “Higgs”). The second set of values mimics a simple TC model with three technicolor and thus presents features very similar to ChPT (compare with the $\pi\pi$ curves in Fig.1), mainly, a vector resonance (a “techni-$\rho$”). The last model is chosen to show two behaviors that deserve further comments. First, it could happen that a resonance becomes so broad that it may be hard to identify as a resonance, in such case we say there is a “saturation” of unitarity. (The situation with the $\sigma$ particle in QCD is of this kind). Second, we have to remember that in the effective Lagrangian approach we only have a finite theory order by order in energy, but it is not renormalizable in the strict sense. Thus, the set of possible consistent fundamental theories is “smaller” than that of effective theories. By that we mean that there could be a choice of parameters which are not the low-energy limit of any fundamental theory. That may seem obvious if we take an absurd value for some parameters, like $L_1 = 10^6$. But it could also occur for values that look “reasonable”. In such case, however, we would find inconsistencies in the effective theory. That happens indeed for the last model in Fig.3, which yields a pole in the first Riemann sheet of the $I = 2$ channel, which should not be present in a renormalizable quantum field theory. That can be used as a criterion to exclude a set of parameters. There are several arguments that support this interpretation[14].

We are now in conditions to study the general resonance spectrum of the strongly interacting SBS. We only have to vary the values of $L_1$ and $L_2$ in their expected ranges, and identify what resonances appear below 3 TeV (an estimate of the LHC $W_LW_L$ scattering reach). In Fig.5 we present the results of this approach[14], which deserves some comments:

- The presence of an scalar resonance is represented by the areas that contain an “H”, whereas vector resonances are represented by a “$\rho$”. Saturation effects are labeled by “$S_I$” for each $I = 0, 1, 2$ channel.
Figure 2: Phase shifts expected for different choices of electroweak chiral parameters.
• For illustrative purposes, we have signaled the pairs of parameters that mimic some simple scenarios. The black triangles stand at the position of a QCD-like model with 5 or 3 technicolors. The black dots correspond to the SM with a Higgs whose tree level mass is 800, 1000 or 1200 GeV.

• Note that there are scenarios where we could find two resonances in two different channels, or a resonance in one channel and a saturation behavior in another, or two saturation effects.

• The black area is the part of parameter space which is excluded by the appearance of poles in the first Riemann sheet, it suggests that we cannot find heavy resonances in the \( I = 2 \) channel (doubly charged Higgses). This difficulty has also been found when trying to construct models with such particles: there is no model where they are heavier than \( \simeq 375 \) GeV \cite{15}, a bound obtained from a renormalization group analysis. In this lecture we are assuming that no such “light” particles are present. In such case, from the figure it seems that either nothing at all or a “saturation” behavior is possible in that channel.

• Finally, there is a small shaded region where no resonance or saturation effect would be clearly visible. In this region it also seems very hard to obtain a measurement of just the chiral parameters and probably we would only get some bounds on their values \cite{16}. In such case, not even with the IAM we could get any information of other more massive resonances that may lie ahead.

6 Summary

The main conclusions from the discussion below are the following:

• There is not any fundamental reason for the Higgs (Standard or Supersymmetric) to exist. We should therefore keep an open mind to alternative scenarios.

• However unitarity requires new physics to appear before below 1 TeV.

• This new physics could be new particles in the best of the worlds or even a completely new and unexpected physics.
Figure 3: Resonance spectrum of the strong SBS in the $L_1, L_2$ plane. The black area is excluded. On the white areas, we have represented broad resonances or saturation effects in the $I$ channel by $S_I$; Higgs-like narrow resonances by $H$ and $\rho$-like narrow resonances by $\rho$. In the grey area there is no saturation of unitarity, nor resonances, below 3 TeV. The black dots represent the MSM with $M_H = 800, 1000, 1200$ GeV and the triangles a QCD-like model with 3 or 5 technicolors.

- In the worst case we will have an enhancement of the $WW$ production. It will be difficult to observe at the LHC but not impossible. A lot of work should be done in this direction and chiral Lagrangians, supplemented with the inverse amplitude method, can provide a model independent approach to new phenomena like the strong $W_LW_L$ scattering and the resonances that may appear over the LHC energy range.

- In any case we have to wait for the LHC with an open mind. Nature will tell us.

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