Control of Cricket System Using LQR Controller Optimized by Particle Swarm Optimization

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Abstract. The cricket system, as a strongly coupled, nonlinear and multivariable two dimensional cue system, is a typical representative of unstable and underactuated system. In this paper, the system is analyzed by modeling based on the cricket experimental platform, and while designing the LQR controller, the particle swarm algorithm is introduced to optimize the controller parameters to determine the optimal weight matrix Q in order to solve the difficult problem of controller parameter rectification. The simulation results show that the PSO-LQR can restore the balance of the system in a shorter period of time with enhanced stability compared to a single LQR controller, which exhibits good control performance.

1. Introduction
The cricket system is a typical nonlinear, multivariable and unconstrained control system, which drives the ball movement by controlling the tilt angle of the plate surface. It has become a typical experimental control platform in the study of control theory such as system modeling, decoupling analysis, and path planning⁴. At present, domestic and international researchers have achieved a series of results in fuzzy control, sliding membrane control, and neural network control combined with optimization algorithms.

Aiming at the control accuracy of the cricket system, Changzheng Wang proposed using particle swarm optimization (PSO) algorithm and genetic algorithm to improve the RBF-PID control theory on the basis of the PID control algorithm and the RBF (Radial Basis Function) neural network research, to achieve the control effect of the system to reach a stable state with low vibration frequency in a short time⁵. Yuyun Xu used the advantages of strong robustness and simple implementation of synovial controller to control the cricket system and introduced the RBF neural network to weaken the vibration introduced in the synovial control, to realized the adaptive synovial control of the cricket system⁶.

In the process of LQR controller design, the selection of weight Q will directly affect the response speed of system and time to reach steady state, and the performance of the system have close relations. Usually, set the weighted Q need strong knowledge of experience, it is not easy to get. Aiming at this problem, particle swarm optimization algorithm is adopted to improve the optimization operation, selecting the optimal matrix Q as controller parameters. Lagrange equation was used to model the cricket system, and LQR controller and PSO-LQR controller were used to control the system. Finally, the validity of LQR parameters of particle swarm optimization was verified by the simulation curve and the results were analyzed.
2. Cricket System
The cricket system is mainly composed of ball, connecting rod, servo motor, power supply, reducer, etc. The structure is shown in figure 1. The control objective: the plate surface with two degrees of freedom is controlled by the motor, so that the ball rolling freely can be stabilized at a specific position. Due to the complex structure of the system, the interaction between the plate surface and the ball, and the strong coupling force, it is difficult to analyze. So the Lagrange theory is used to model the system.

![Cricket system model](image1)

Figure 1. Cricket system model.

Referring to the cricket system model, the coordinate system is established with the supporting point of the cricket plane as the origin. As shown in figure 2.

![Cricket coordinate system](image2)

Figure 2. Cricket coordinate system.

As a control device with complex structure and typical multi-variables, the cricket system is difficult to write accurate mathematical expressions. Therefore, without affecting the actual properties, some interference factors are ignored to establish the model\[^4\], and assume the following:

- Not considering the friction factor in the system.
- The ball is always in contact with the cricket surface and does not slide during the movement.
- Do not consider the size and angle of the cricket plane.
- The plate is symmetrical in the x- and y-axis directions with respect to the support points.

The specific parameters and meanings used in the modeling process are shown in table 1.

| Parameter | Meaning | Parameter | Meaning |
|-----------|---------|-----------|---------|
| \( m \)   | Ball quality | \( w_x, w_y \) | Angular velocity of x and y of ball rotation |
| \( r_b \) | Radius of the sphere | \( \Omega \) | Angular velocity of plate rotation |
| \( x, y \) | Displacement of x and y of the ball | \( I_b \) | The moment of inertia of the ball |
| \( \alpha, \beta \) | Angle of x and y of the plate | \( I_p \) | Moment of inertia of the plate |
| \( r \)   | Ball position | \( \tau_x, \tau_y \) | The x and y torques of the flat plate |

Table 1. Cricket system modeling parameters.

Based on the above cricket system, Lagrange equation is used to analyze the system, and the nonlinear equations can be written as follows.

\[
x: \left( m + \frac{I_p}{r_b} \right) \ddot{x} - m \left( x\dot{\alpha}^2 + y\dot{\beta}\dot{\beta} \right) + mg \sin \alpha = 0 \tag{1}
\]

\[
y: \left( m + \frac{I_p}{r_b} \right) \ddot{y} - m \left( y\dot{\beta}^2 + x\dot{\alpha}\dot{\alpha} \right) + mg \sin \beta = 0 \tag{2}
\]
Equations (1) and (2) describe the motion of the ball, the tilt angle and angular velocity of the plate. Equations (3) and (4) describe the relationship between ball and plate motion and external forces. According to the analysis, the actual control volume of the system is the angle, so the position of the motor will not be affected. Therefore, equations (3) and (4) are ignored. Linearization of the nonlinear model can be obtained as follows.

\[
\alpha : (I_b + I_p + mx^2)\ddot{\alpha} + m(x y \ddot{\beta} + xy \dot{\beta} + x y \dot{\delta} + 2x \dot{\alpha}) + mgx \cos \alpha = \tau_x
\]  

\[
\beta : (I_b + I_p + my^2)\ddot{\beta} + m(x y \alpha + xy \dot{\alpha} + xy \dot{\beta} + 2y \dot{\beta}) + mgy \cos \beta = \tau_y
\]  

Equations (3) and (4) describe the relationship between ball and plate motion and external forces.

According to the analysis, the actual control volume of the system is the angle, so the position of the motor will not be affected. Therefore, equations (3) and (4) are ignored. Linearization of the nonlinear model can be obtained as follows.

\[
x : \left( m + \frac{I_b}{r_b^2} \right) \ddot{x} + mg \alpha = 0
\]  

\[
y : \left( m + \frac{I_b}{r_b^2} \right) \ddot{y} + mg \beta = 0
\]

The state space expression for the cricket system is:

\[
\begin{align*}
x(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t)
\end{align*}
\]

The linearization process is a decoupling process, that is, the x direction is decoupled from the y direction. So only the x axis can be analyzed. If \( I_b = \frac{2}{5}mr_b^2 \), the system state matrices A and B in the x direction can be obtained as: \( A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \).

3. LQR Controller Design

LQR (linear quadratic regulator) is available optimal control law in the process of linear state feedback, and it is easy to form a closed-loop optimal control. The essence is to ensure the minimum error through a small amount of control. The ultimate goal of the controller design is to find the optimal feedback matrix \( K \) according to the quadratic performance index \( J \) to obtain the optimal control \( u \). When the system deviates from the equilibrium point, the control \( u \) is applied to make the system return to the equilibrium point and minimize the value of \( J \).

The quadratic performance index function \( J \) and the optimal control \( u \) are respectively shown in equation (8) and equation (9)\(^5\).

\[
J = \frac{1}{2} \int_0^T \left[ x^T(t)Q(t)x(t) + u^T(t)R(t)u(t) \right] dt + \frac{1}{2} x^T(t_f)P(t_f)x(t_f)
\]  

\[
u = -Kx
\]

As in equations (8) and (9), \( Q \) is the semi-positive definite real symmetric constant matrix; \( R \) is the positive definite real symmetric constant matrix; \( P \) is a semi-positive constant matrix; \( K \) is the optimal feedback gain matrix.

The former term of integration represents the penalty of dynamic tracking error. The larger the value of the weighted coefficient, the faster the decay of \( x \) will be; The integral term represents the sum of the energy consumption of the system. The larger the weight matrix, the slower the decay of \( x \) will be.

The LQR controller simulation model is built by simulation is shown in figure 3. Set \( R = 2 \) and \( Q \) matrix is selected by trial and error method. The control performance is observed through simulation results. And the appropriate \( Q \) matrix is determined by adjusting the parameters.
Figure 3. LQR controller simulation model.

The analysis shows that the cricket system has the same structure in both directions and can be divided into two motion processes. In this paper only the four state variables of the x-axis are analyzed, so Q is a matrix of $4 \times 4$. It can be expressed as $Q = \begin{bmatrix} 300 & 0 & 0 & 0; 0 & 4 & 0 & 0; 0 & 0 & 1000 & 0; 0 & 0 & 0 & 10 \end{bmatrix}$.

The feedback matrix is obtained by lqr (A, B, Q, R): $K = \begin{bmatrix} -12.2474 & -12.6925 & 45.5128 & 9.7993 \end{bmatrix}$.

4. Particle Swarm Optimization LQR Controller

4.1. Particle Swarm Optimization Algorithm Description

Particle swarm optimization (PSO) is an optimization algorithm that modifies parameters by simulating foraging behavior of birds. Each particle represents a potential solution of the space. The particle is mainly updated by two extremes: individual extreme and global extreme. The individual extreme is shared with the whole particle swarm, and the optimal value in the individual extreme is found as the current global optimal solution. The velocity and position of particles are updated by equation (10) and equation (11). The position represents the direction of particle movement, and the velocity represents the speed of particle movement\(^{[6]}\).

$$v_{ij}(t+1) = w v_{ij}(t) + c_1 r_1 (t) (p_{ij}(t) - x_{ij}(t)) + c_2 r_2 (t) (p_{gj}(t) - x_{ij}(t))$$  \hspace{1cm} (10)

$$x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1)$$  \hspace{1cm} (11)

As in equations (10) and (11), $v_{ij}$ represents the $i(1,2,...,m)$ particles velocity in $j(1,2,...,D)$ dimensional space, $x_{ij}$ represents the position of the particle, $p_{ij}$ represents the individual extreme value of the particle, $p_{gj}$ represents the global extreme value of the particle, $r_1$ and $r_2$ are random numbers, $w$ is the inertia weight, $c_1$ and $c_2$ are acceleration constants. The flow chart of PSO algorithm is shown in figure 4.

Figure 4. Particle swarm optimization process.
4.2. Particle Swarm Optimization Method for LQR Controller

When LQR parameters are optimized by particle swarm optimization algorithm, the setting of basic parameters of particles will directly affect the final optimization effect in the algorithm. After consulting literature and experimental debugging, the particle swarm parameters are set as follows:\(^7\):
The inertia weight is 0.8; The group size is 40; The learning factor is 2; The maximum velocity of the particles are 1.

By setting the parameters of the particle swarm, the optimal matrix \(Q\) is sought and the optimal feedback matrix \(K\) is calculated. Set the \(Q = \begin{bmatrix} q_1 & 0 & 0 & 0 \\ & q_2 & 0 & 0 \\ & & q_3 & 0 & 0 \\ & & & q_4 & 0 \end{bmatrix}\).

The performance index function can be expressed as:

\[
J = \int_0^\infty \left( q_1 x_1^2 + q_2 x_2^2 + q_3 x_3^2 + q_4 x_4^2 + R u^2 \right) dt
\]

As in equation (12), the \(q\) corresponds to the weight coefficients of ball displacement, ball velocity, plate tilt angle and angular velocity respectively. The specific process of using the particle swarm algorithm to optimize the parameters of the controller: By running the PSO.m file to assign the particle to the weight matrix \(Q\), calling Simulink to run the cricket system, and the output adaptation value, then return to the PSO.m file to update the speed and position of the particles, so iterative. When the conditions are satisfied, the program stops and outputs the optimal weight matrix, as shown in figure 5.

The weight matrix \(Q\) is obtained by programming optimization of the controller parameters: \(Q = \begin{bmatrix} 210 & 4 & 2300 & 10 \end{bmatrix}\).

The feedback matrix is obtained by \(lqr(A, B, Q, R)\): \(K = [-10.2470, -12.6064, 53.6527, 10.5974]\).

5. Cricket Simulation Analysis

In this section, the model of cricket system was established in simulink environment. The control performance of LQR controller and PSO-LQR controller is compared by simulation results. Figure 6 shows the change curve of the fitness function. The fitness value gradually converges to the stable with the increase of the number of iterations of the particle swarm.

Figure 5. PSO optimized LQR structure diagram.

The weight matrix \(Q\) is obtained by programming optimization of the controller parameters: \(Q = \begin{bmatrix} 210 & 4 & 2300 & 10 \end{bmatrix}\).

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Figure 6. Convergence curve of fitness value.
Figure 7 shows the response curves of the two controllers to the step signal before and after the optimization, which are the control output curve, the motion position curve of the ball, the tilt angle of the plate and the change curve of the angular velocity, respectively. As can be seen from (a) and (b), the optimized controller can shorten the transition time, quickly return to the desired position, the system responds faster, and the oscillation amplitude of the curve is smaller. As can be seen from (c) and (d), the overshoot of the optimized curve is reduced and the system adjustment time is shortened. In general, the control effect of using PSO algorithm to optimize LQR controller is better than a single LQR controller, which can achieve better control effect.

![Figure 7](image)

**Figure 7.** Comparison of response curves before and after optimization.

6. Conclusion

In order to solve the problem that the parameter selection of LQR controller is empirical, this paper designs a weight matrix optimization procedure based on particle swarm algorithm by Matlab, and completes the weight matrix search process by combining Simulink. From the simulation results, the introduction of the particle swarm algorithm can quickly and effectively determine the optimal weight matrix of the controller, and the control performance of the cricket system is also significantly improved to some extent. Not only shorten the time for the system to recover to a stable state, but also improve the vibration frequency. This method can realize online rectification of controller parameters, which greatly improves work efficiency. At the same time, the combination of intelligent algorithms and control algorithms is very helpful in improving system performance and has certain application value.

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