Analytic Expressions for the Inner-rim Structure of Passively Heated Protoplanetary Disks

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Abstract

We analytically derive the expressions for the structure of the inner region of protoplanetary disks based on the results from the recent hydrodynamical simulations. The inner part of a disk can be divided into four regions: a dust-free region with a gas temperature in the optically thin limit, an optically thin dust halo, an optically thick dust condensation front, and the classical, optically thick region, in order from the innermost to the outermost. We derive the dust-to-gas mass ratio profile in the dust halo using the fact that partial dust condensation regulates the temperature relative to the dust evaporation temperature. Beyond the dust halo, there is an optically thick condensation front where all the available silicate gas condenses out. The curvature of the condensation surface is determined by the condition that the surface temperature must be nearly equal to the characteristic temperature $\sim 1200$ K. We derive the midplane temperature in the outer two regions using the two-layer approximation, with the additional heating by the condensation front for the outermost region. As a result, the overall temperature profile is step-like, with steep gradients at the borders between the outer three regions. The borders might act as planet traps where the inward migration of planets due to gravitational interaction with the gas disk stops. The temperature at the border between the two outermost regions coincides with the temperature needed to activate magnetorotational instability, suggesting that the inner edge of the dead zone must lie at this border. The radius of the dead zone inner edge predicted from our solution is $\sim 2$–$3$ times larger than that expected from the classical optically thick temperature.

Key words: protoplanetary disks

1. Introduction

The inner region of protoplanetary disks is the birthplace of rocky planetesimals and planets. One preferential site of rocky planetesimal formation is the inner edge of the so-called dead zone (e.g., Kretke et al. 2009). The dead zone is the location where magnetorotational instability (MRI; Balbus & Hawley 1998) is suppressed because of poor gas ionization (Gammie 1996). In the inner region of protoplanetary disks, the dead zone is likely to have an inner edge where the gas temperature $T$ reaches $\sim 1000$ K, above which the thermal ionization of the gas is effective enough to activate MRI (Gammie 1996; Desch & Turner 2015). Across the dead zone inner edge, the turbulent viscosity arising from MRI steeply decreases from inside out, resulting in a local maximum in the radial profile of the gas pressure (e.g., Dzyurkevich et al. 2010; Flock et al. 2016, 2017). The pressure maximum traps solid particles (Whipple 1972; Adachi et al. 1976) and may thereby facilitate dust growth (Brauer et al. 2008; Testi et al. 2014) and in situ planet formation (Tan et al. 2015). The dead zone edge may even trap planets by halting their inward migration (Masset et al. 2006). Because the location of the dead zone inner edge is determined by the gas temperature, understanding the temperature structure is important for understanding rocky planetesimal formation as well as the orbital architecture of inner planets.

The temperature structure of the innermost regions of protoplanetary disks has been extensively studied in the context of their observational appearance (see Dullemond & Monnier 2010 for review, and also Dullemond et al. 2001; Natta et al. 2001; Isella & Natta 2005). Recently, Flock et al. (2016) performed radiation hydrodynamical calculations of the inner-rim structures of disks around Herbig Ae stars, taking into account the effects of starlight and viscous heating, evaporation and condensation of silicate grains, and gas opacity. They found that the inner disk consists of four distinct zones with different temperature profiles. According to their results, the dead zone inner edge occurs at the border between the two outermost regions, where the temperature drops steeply across $1000$ K. Thus, this complex temperature structure potentially influences where rocky planetesimals and planets preferentially form. However, there has been no simple analytic model that reproduces the entire structure of the temperature profile. In addition, because Flock et al. (2016) performed simulations only for the disks around Herbig Ae stars, it has been unclear how the inner-rim structures depend on stellar parameters.

In this paper, we analytically derive the temperature and dust-to-gas mass ratio profile for the inner region of disks and reveal how these structures depend on the central star parameters based on the results from Flock et al. (2016). Our analysis and the analytic solutions are shown in Section 2. We discuss the implications for the planetesimal and planet formation and the disk observation in Section 3. A summary is in Section 4.

2. Analytic Solutions for the Inner-rim Structure

We derive analytic expressions for the inner-rim structure of protoplanetary disks based on the recent numerical simulations conducted by Flock et al. (2016). They found that the inner region of protoplanetary disks can be divided into four regions (regions A, B, C, and D) in terms of the temperature profile. Figure 1 schematically shows the inner region of
The dust-to-gas mass ratio, \( \rho_{\text{d}} \), because given by in this region.

\[ \rho_{\text{d}} = \frac{\rho_{\text{d}}^0}{1 + \frac{v}{c_{\text{d}}}} \]

where \( v \) is the gas velocity and \( c_{\text{d}} \) is the speed of sound in the gas. Inside the Kelvin–Helmholtz layer, the dust-to-gas mass ratio is determined in each region.

2.1. Region A: An Optically Thin Dust-free Region

Region A is the innermost region where the temperature is above the evaporation temperature of dust. This region is therefore free of dust and is optically thin to the starlight. In an optically thin region, the temperature profile is generally given by

\[ T = \left( \frac{R_*}{2r} \right)^{1/4} T_*, \quad (1) \]

where \( r \) is the distance from the central star, and \( R_* \) and \( T_* \) are the stellar radius and temperature, respectively. The dimensionless quantity \( \epsilon \) is the ratio between the emission and absorption efficiencies of the disk gas, including the contribution from dust,

\[ \epsilon \equiv \frac{\kappa_{\text{g}} + f_{\text{d}2g} \kappa_{\text{d}}(T)}{\kappa_{\text{g}} + f_{\text{d}2g} \kappa_{\text{d}}(T_*)}, \quad (2) \]

where \( f_{\text{d}2g} \) is the dust-to-gas mass ratio, \( \kappa_{\text{g}} \) is the Planck mean gas opacity (here assumed to be independent of the temperature), and \( \kappa_{\text{d}}(T_*) \) and \( \kappa_{\text{d}}(T) \) are the Planck mean dust opacities at the stellar and disk temperatures, respectively. Flock et al. (2016) adopted \( \kappa_{\text{g}} = 10^{-4} \text{ cm}^2 \text{ g}^{-1} \), \( \kappa_{\text{d}}(T_*) = 2100 \text{ cm}^2 \text{ g}^{-1} \), and \( \kappa_{\text{d}}(T) = 700 \text{ cm}^2 \text{ g}^{-1} \).

For region A, \( \epsilon \approx 1 \) because \( f_{\text{d}2g} \approx 0 \). Therefore, the temperature profile in region A, \( T_A \), is given by

\[ T_A = \left( \frac{R_*}{2r} \right)^{1/4} T_* \quad (3) \]

Flock et al. (2016) showed that Equation (3) accurately reproduces the temperature profile in region A (see the top panel of their Figure 1). The real gas temperature might be more difficult to understand because it depends on gas opacity (Dullemond & Monnier 2010; Hirose 2015), which is assumed to be independent of wavelength in this paper.

2.2. Region B: An Optically Thin Dust Halo Region

Region B is the location where dust starts to condense but is still optically thin. This region, which Flock et al. (2016) called the dust halo, has a uniform temperature that is nearly equal to the evaporation temperature \( T_{\text{ev}} \) (see Figure 1 of Flock et al. 2016). These features can be understood by noting that the condensing dust acts as a thermostat: if \( T \) falls below \( T_{\text{ev}} \), dust starts to condense, but the condensed dust pushes the temperature back up because the emission-to-absorption ratio \( \epsilon \) of the dust is lower than that of the gas. Thus, dust evaporation and condensation regulate the temperature in this region to \( \sim T_{\text{ev}} \). Strictly speaking, the temperature of region B obtained by Flock et al. (2016) is higher than \( T_{\text{ev}} \) by about 100 K. However, this is due to their numerical treatment of the dust evaporation, where the dust-to-gas ratio is given by a smoothed step function of \( T \) with a smoothing width of 100 K.

\[ T \sim T_{\text{ev}} \quad \text{in region B can be used to predict the spatial distribution of the dust-to-gas mass ratio} \quad f_{\text{d}2g} \quad \text{in this region. Substituting Equation (2) into Equation (1) and solving the equation with respect to} \quad f_{\text{d}2g} \quad \text{we obtain} \]

\[ f_{\text{d}2g} = \frac{\kappa_{\text{g}}(T) R_*^2 T_*^4 - R_B^2 T_B^4}{\kappa_{\text{d}}(T_*) R_*^2 T_*^4 - 4 \kappa_{\text{d}}(T_*) r^2 T_B^4}, \quad (4) \]

where \( T_B \) stands for the temperature in region B. Assuming that \( T_B \) is constant, Equation (4) determines \( f_{\text{d}2g} \) as a function of \( r \). Figure 2 compares Equation (4) with the radial profile of \( f_{\text{d}2g} \) at the midplane taken from the radiation hydrostatic disk model S100 of Flock et al. (2016; see the bottom panel of their Figure 1. The stellar temperature, radius, mass, and luminosity are set to \( T_0 = 10,000 \text{ K}, \quad R_* = 2.5 R_\odot = 0.0116 \text{ au}, \quad M_* = 2.5 M_\odot \) and \( L_* = 56 L_\odot \), respectively. See also their Table 1 for details). Equation (4) perfectly reproduces the results of Flock et al. (2016) when \( T_B \) is set to \( T_{\text{ev}} + 100 \text{ K} \approx 1470 \text{ K} \), where we have used that \( T_{\text{ev}} \) in region B is \( \approx 1370 \text{ K} \) in their calculation.

As we can see from Figure 2, the inner and outer edges of region B correspond to the locations where \( f_{\text{d}2g} \) given by Equation (4) goes to zero and infinity, respectively. The radii of
solid angle to the solid angle 

\[ \frac{4\pi}{3} \] 

is 

\[ s = 1 \] 

radiation hydrostatic disk model S100 of Flock et al. (2016) with the highest resolution, respectively. We have expressed the inner radius of region B as 

\[ R_{AB} = \frac{1}{2} \left( \frac{T_*}{10^4 \text{ K}} \right)^2 \left( \frac{T_\text{B}}{1470 \text{ K}} \right)^{-2} \left( \frac{R_*}{2.5 R_\odot} \right) \text{ au} \] 

(5)

and

\[ R_{BC} = \frac{1}{2} \left( \frac{\kappa_\text{d}(T_*)}{\kappa_\text{d}(T_\text{B})} \right)^{1/2} \left( \frac{T_*}{10^4 \text{ K}} \right)^{2} \left( \frac{R_*}{2.5 R_\odot} \right) \text{ au}, \] 

(6)

respectively. We have expressed the inner radius of region B as 

\[ R_{AB} \] 

because it also stands for the outer radius of the dust-free zone (region A). Similarly, \( R_{BC} \) also stands for the inner radius of the optically thick inner-rim (region C). We note that our expression for \( R_{BC} \) is equivalent to that of Monnier & Millan-Gabet (2002).

Equations (5) and (6) give the orbital radii of the boundaries of region B if \( T_\text{B}(\approx T_{ev}) \) is given. In fact, \( T_{ev} \) weakly depends on the gas density, and hence on \( r \). Therefore, if one wants to determine \( R_{AB} \) and \( R_{BC} \) precisely, one has to solve Equations (5) and (6) simultaneously with the equation for \( T_{ev} \) as a function of \( r \). This is demonstrated in the Appendix.

As we showed, a dust halo naturally appears. The dust halo is a main contributor of the near-infrared emission in the spectral energy distribution of disks around Herbig Ae/Be stars (e.g., Vinković et al. 2006).

2.3. Region C: An Optically Thick Dust Condensation Front

At \( r \approx R_{BC} \), dust grains near the midplane fully condense (see Figure 2), and the visual optical depth \( \tau_{\text{v}} \) measured along the line directly from the star exceeds unity at the midplane. Further out, stellar irradiation is absorbed at a height where \( \tau_{\text{v}} = 1 \), and the midplane temperature is determined by the reprocessed, infrared radiation from dust grains lying at the \( \tau_{\text{v}} = 1 \) surface. Figure 3 schematically illustrates the surface structure of regions C and D. As we show below, region C can be regarded as the zone where the \( \tau_{\text{v}} = 1 \) surface coincides with the condensation front; the surface structure is approximately determined by the condition that the surface temperature \( T_{C} \) must be equal to the characteristic temperature \( \sim 1200 \text{ K} \).

The radial extent of region C is of particular interest because the outer edge of this region sets the inner edge of the dead zone (Flock et al. 2016, 2017). We will see in Section 2.4 that the border of regions C and D is related to how the height of the \( \tau_{\text{v}} = 1 \) surface, \( z_\text{c} \), depends on disk radius. Here, we try to estimate this height by considering the energy balance between stellar irradiation on the \( \tau_{\text{v}} = 1 \) surface and the thermal emission from the surface,

\[ \frac{L_*}{4\pi R^2} \sin \theta = \left( \frac{C_\text{bw}}{4} \right) \varepsilon_\text{SB} T_{C}^4, \] 

(7)

where \( R \) is the radial distance on the midplane, \( \varepsilon_\text{SB} \) is the Stefan–Boltzmann constant, and \( \theta \) is the angle between the starlight and the disk surface (the so-called grazing angle). \( C_\text{bw} \) is a backwarming factor (e.g., Dullemond et al. 2001), which is defined as the ratio of a full \( 4\pi \) solid angle to the solid angle subtended by the empty sky seen by particles on the surface. The backwarming is heating by surrounding dust particles. If a dust particle radiates into the sky area covered by nearby dust particles, a similar amount of energy is returned from surrounding particles (Kama et al. 2009). We assume \( C_\text{bw} = 4 \), which means that the \( \tau_{\text{v}} = 1 \) surface can be approximated as a flat wall. In region C, \( \varepsilon = 1/3 \) because dust totally condenses. The grazing angle \( \theta \) is related to the surface height \( z_\text{c}(R) \) as (Chiang & Goldreich 1997; Tanaka et al. 2005)

\[ \theta = \arcsin \left( \frac{4R_*}{3\pi R} \right) + \arctan \left( \frac{z_\text{c} d \ln z_\text{c}}{K d \ln R} \right) - \arctan \left( \frac{z_\text{c}}{R} \right) \approx \frac{4R_*}{3\pi R} + \frac{z_\text{c} d \ln z_\text{c}}{K d \ln R} - \frac{z_\text{c}}{R^2}, \] 

(8)

where the second expression assumes \( \theta \ll 1 \). As mentioned earlier, the \( \tau_{\text{v}} = 1 \) surface in region C coincides with the condensation front, and the surface temperature \( T_{C} \) is approximately equal to 1200 K according to the simulations by Flock et al. (2016; see their Figure 2). Substituting Equation (8) and \( T_{C} \approx \text{constant} \) into Equation (7) and solving the differential equation with respect to \( z_\text{c} \), we obtain the
surface height in region C as

$$z_{*,C} = \frac{1}{6R^2_x} \left( \frac{T_{*,C}}{T_*} \right)^4 \left( R^3 - RR_0^3 \right) + \frac{4R_x}{3 \pi R_0} (R - R_0),$$  

(9)

where $R_0$ is the radius at which $z_{*,C} = 0$. Figure 4 shows the location of the $\tau = 1$ plane obtained by Equation (9) and that from 3D simulations by Flock et al. (2016). In Figure 4, we set $R_0 = 0.4$ au, which explains Flock et al. (2016) better than $R_0 = R_{BC} \approx 0.46$ au. Our analytic solution (Equation (9)) reproduces the shape of the condensation front obtained by Flock et al. (2016).

In order to obtain the midplane temperature in region C, $T_{\text{mid,C}}$, we assume that half of the infrared emission from the surface layer comes into the interior. Then we have

$$T_{\text{mid,C}} = 2^{-1/4}T_{*,C} \approx 1009 \text{ K}.$$  

(10)

2.4. Region D: An Optically Thick Region

Region D is the outermost, coldest region where dust condenses at all heights. Therefore, the temperature structure of this region is essentially the same as that of the classical two-layer disk model by Chiang & Goldreich (1997) and Kusaka et al. (1970). Specifically, the surface temperature profile in this region follows Equations (7) with (8), $\epsilon = 1/3$, and $C_{bw} = 4$, i.e.,

$$T^4_{*,D} = 3 \left( \frac{R}{R_*} \right)^{-2} \left( \frac{4R_x}{3 \pi R} \right) \left( \frac{d \ln z_{*,D}}{d \ln R} - 1 \right) \left( \frac{z_{*,D}}{R} \right)^2,$$

(11)

where $z_{*,D}$ is the surface height in region D. Assuming that $z_{*,D}$ is proportional to the gas scale height $h_g = c_s/\Omega_K$, where $c_s$ is the sound speed and $\Omega_K$ is Keplerian angular velocity, we obtain (Kusaka et al. 1970)

$$T^4_{*,D} = T^4_1 + T^4_2,$$

(12)

where

$$T_1 = \left( \frac{4}{\pi} \right)^{1/4} \left( \frac{R}{R_*} \right)^{-3/4} T_*$$

$$= 33 \left( \frac{R}{1 \text{ au}} \right)^{-3/4} \left( \frac{R_*}{2.5R_{\odot}} \right)^{3/4} \left( \frac{T_*}{10^4 \text{ K}} \right) K,$$

(13)

and

$$T_2 = \left( \frac{6}{7} \right)^{2/7} \left( \frac{z_{*,D}}{h_g} \right)^{2/7} \left( \frac{R}{R_*} \right)^{-3/7} \left( \frac{k_B T_* R_*}{m G M_*} \right)^{1/7} T_*$$

$$= 413 \left( \frac{z_{*,D}}{h_g} \right)^{2/7} \left( \frac{R}{1 \text{ au}} \right)^{-3/7} \left( \frac{R_*}{2.5R_{\odot}} \right)^{4/7} \times \left( \frac{M_*}{2.5M_{\odot}} \right)^{-1/7} \left( \frac{T_*}{10^4 \text{ K}} \right)^{8/7},$$

(14)

where $k_B$ is the Boltzmann constant, $G$ is the gravitational constant, and $m = 4 \times 10^{-24}$ g is the mean molecular mass of the disk gas. As in region C, the midplane temperature of region D is given by

$$T_{\text{mid,D}} = 2^{-1/4}T_{*,D}.$$  

(15)

The border between regions C and D can be defined as the position where the $\tau = 1$ surfaces in the two regions intersect. Equating $z_{*,C}$ given by Equation (9) with $z_{*,D} = (z_{*,D}/h_g) h_g = (z_{*,D}) \sqrt{k_B T_{\text{mid,D}} R^3/m G M_*}$, we obtain the transcendental equation for the radius $R_{CD}$ of the border,

$$\frac{1}{6R^2_x} \left( \frac{T_{*,C}}{T_*} \right)^4 \left( \frac{R_{CD}^2 - R^2_{BC}}{R_{CD}^2 - R^2_{BC}} \right) + \frac{4R_x}{3 \pi R_{BC} R_{CD}} (R_{CD} - R_{BC})$$

$$= \left( \frac{z_{*,D}}{h_g} \right) \sqrt{k_B T_{\text{mid,C,D},R_{CD}} R_{CD}} m G M_*,$$

(16)

where the subscript CD stands for the value at $R = R_{CD}$ and we have used $R_0 \approx R_{BC}$ (see Section 2.3). In order to derive a closed-form expression for $R_{CD}$, we simplify Equation (16) as follows. First, we neglect the second term on the left side of Equation (16) because for T-Tauri and Herbig stars the inner-rim radius is much larger than the stellar radius. Second, we replace the factor $T_{\text{mid,C,D}}^{1/2} = (2^{-1} T_{*,C,D}^{1/8})$ on the right side of Equation (16) with $(T_{*,C,D}^{1/8})$, where $T_{*,C,D}$ is the value of $T_2$ at $R = R_{CD}$, because $T_1 \sim T_2$ at $R \sim 0.1-1$ au (see Equations (13) and (14)). Third, we neglect the $R_{CD}$-dependence of the right side, by substituting $R_{CD} = 2R_{BC}$ on the right side only, because the dependence is generally weak ($\sqrt{R_{CD} T_{\text{mid,C,D}}}$
If we use Equation (6) to eliminate $R_{BC}$ from $\Gamma$, we obtain

$$\Gamma = 3 \left( \frac{z_{*,D}/h_g}{4.8} \right)^{8/7} \left( \frac{L_*}{56L_\odot} \right)^{-2/7} \left( \frac{M_*}{2.5M_\odot} \right)^{-1/2},$$

(19)

where we set $\kappa_d(T_*)/\kappa_d(T_B) = 1/3$ and $T_B = 1470$ K. The resultant equations (Equations (17) and (19)) justify the validity of the approximation of $R_{CD} = 2R_{BC}$. Our simplifications introduce an error of less than 10% in the estimate of $R_{CD}$, if we solve Equation (16) exactly, we obtain $R_{CD} = 0.85$ au with $T_* = 10,000$ K, $R_* = 2.5R_\odot$, $M_* = 2.5M_\odot$, and $z_{*,D}/h_g = 4.8$, while Equation (17) leads to $R_{CD} = 0.92$ au. As seen in Figure 4, the ratio $z_{*,D}/h_g$ is a function of the distance from the central star; the ratio $z_{*,D}/h_g$ is about 3.6 at the outer part and 4.8 at the inner part of region D, due to the extra heating by the hot rim surface discussed below. Therefore, when we estimate the location of $R_{CD}$, it is better to use the higher value for $z_{*,D}/h_g$ ($\approx 4.8$). On the other hand, when we estimate the temperature in region D, it is better to use the lower value for $z_{*,D}/h_g$ ($\approx 3.6$).

\[ R_{CD}^{1/8} \text{ for } T_{mid,CD} \approx 2^{-1/4} T_{1,CD}, \text{ and } \sqrt{R_{CD}^{T_{mid,CD}}} \propto R_{CD}^{2/7} \text{ for } T_{mid,CD} \approx 2^{-1/4} T_{L,CD}. \]

\[ R_{CD} = \sqrt{1 + \Gamma R_{BC}}, \]

\[ \Gamma = 3 \left( \frac{R_{BC}}{0.46 \text{ au}} \right)^{-12/7} \left( \frac{z_{*,D}/h_g}{4.8} \right)^{8/7} \left( \frac{T_*}{10^4 \text{ K}} \right)^{32/7} \times \left( \frac{M_*}{2.5M_\odot} \right)^{-1/2} \left( \frac{R_*}{2.5R_\odot} \right)^{16/7}. \]

\[ \text{If we use Equation (6) to eliminate } R_{BC} \text{ from } \Gamma, \text{ we obtain} \]

\[ \Gamma = 3 \left( \frac{z_{*,D}/h_g}{4.8} \right)^{8/7} \left( \frac{L_*}{56L_\odot} \right)^{-2/7} \left( \frac{M_*}{2.5M_\odot} \right)^{-1/2}, \]

\[ \text{where we set } \kappa_d(T_*)/\kappa_d(T_B) = 1/3 \text{ and } T_B = 1470 \text{ K. The resultant equations (Equations (17) and (19)) justify the validity of the approximation of } R_{CD} = 2R_{BC}. \text{ Our simplifications introduce an error of less than 10% in the estimate of } R_{CD}; \text{ if we solve Equation (16) exactly, we obtain } R_{CD} = 0.85 \text{ au with } T_* = 10,000 \text{ K, } R_* = 2.5R_\odot, M_* = 2.5M_\odot, \text{ and } z_{*,D}/h_g = 4.8, \text{ while Equation (17) leads to } R_{CD} = 0.92 \text{ au. As seen in Figure 4, the ratio } z_{*,D}/h_g \text{ is a function of the distance from the central star; the ratio } z_{*,D}/h_g \text{ is about 3.6 at the outer part and 4.8 at the inner part of region D, due to the extra heating by the hot rim surface discussed below. Therefore, when we estimate the location of } R_{CD}, \text{ it is better to use the higher value for } z_{*,D}/h_g \text{ (} \approx 4.8\text{). On the other hand, when we estimate the temperature in region D, it is better to use the lower value for } z_{*,D}/h_g \text{ (} \approx 3.6\text{).} \]
\( \tau_b = 1 \) surface in region D and should be determined by the optical depth along the ray emitted by each position on the \( \tau_b = 1 \) surface in region C. If one needs a more accurate temperature structure, detailed radiative transfer simulations would be needed.

Figure 7 shows the analytic temperature profile refined with Equation (21). We set \( h = 0.05R_{\text{CD}} \), which is a bit larger than the typical gas scale height and \( R_{\text{CD}} = 0.86 \) au, which explains Flock et al. (2016) better than the value estimated from Equation (17) (0.83 au with \( z_{D,D}/h_y = 3.6 \)).

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### 3. Discussion

#### 3.1. The Position of the Dead Zone Inner Edge

One of the goals of this work is to determine the position of the dead zone inner edge, \( R_{\text{DBIB}} \). Our results show that at the boundary between regions C and D, the temperature steeply decreases from \( \sim 1000 \) K to \( \sim 600 \) K. According to Desch & Turner (2015), the position of the dead zone inner edge should be located where \( T \sim 1000 \) K. Therefore, the boundary between regions C and D is thought to be the dead zone inner edge. From Equation (17), \( R_{\text{DBIB}} \) can be estimated as 0.09 au, with \( T_b = 5000 \) K and \( M_b = 1M_\odot \), and 0.8 au with \( T_b = 10000 \) K and \( M_b = 2.5M_\odot \), with \( z_{D,D}/h_y = 4.8 \), \( \kappa_4(T_b)/\kappa_4(T_b) = 1/3 \), and \( T_b = 1470 \) K, while the classical optically thick temperature profile (Equation 15) leads to \( R_{\text{DBIB}} \approx 0.1 \) au, with \( T_b = 5000 \) K and \( M_b = 1M_\odot \), and \( R_{\text{DBIB}} \approx 0.3 \) au with \( T_b = 10000 \) K and \( M_b = 2.5M_\odot \). Therefore, the location of the dead zone inner edge estimated from our model is 2–3 times farther out than that estimated from the classical optically thick temperature profile. This is because the temperature in region C is much higher than that estimated from the classical model due to the stellar irradiation on the condensation front with a high incident angle.

#### 3.2. The Effect of Viscous Heating

We have focused on passively irradiated disks where the effect of viscous heating on the temperature structure is negligibly small. In the opposite case where viscous heating dominates over stellar irradiation, the midplane temperature \( T_{\text{mid,vis}} \) is given by

\[
T_{\text{mid,vis}} = \left(1 + \frac{3\gamma_{\text{IR}}}{4}\right)^{1/4} T_{\text{vis}},
\]

where \( T_{\text{vis}} \) is the infrared optical depth at the midplane, and

\[
T_{\text{vis}} = \frac{3GM\dot{M}}{8\pi\Omega_{\text{SH}}R^3} \approx 107 \left(\frac{R}{1 \text{ au}}\right)^{-3/4} \left(\frac{M_\odot}{2.5M_\odot}\right)^{1/4} \left(\frac{\dot{M}}{10^{-8}M_\odot \text{ yr}^{-1}}\right)^{1/4} \text{ K}
\]

is the surface temperature determined by the mass accretion rate \( \dot{M} = 3\gamma_{\text{SH}}L/\nu \) (e.g., Shakura & Sunyaev 1973), where \( \nu \) is the turbulent viscosity.

To see when viscous heating can be neglected, we compare Equation (22) with (15), which is the minimum estimate for the midplane temperature of a passively irradiated disk (see Figure 7). Combining Equations (15) and (22), together with \( \gamma_{\text{IR}} = (1/2)\kappa_4\Sigma_d = (1/2)\kappa_4\Sigma_4 \) (where \( \Sigma_4 \) and \( \Sigma_d \) are the surface densities of gas and dust), we find that \( T_{\text{mid,D}} > T_{\text{mid,vis}} \) if

\[
0.8 \left(\frac{L_\odot}{56L_\odot}\right)^{-2/7} \left(\frac{M_\odot}{2.5M_\odot}\right)^{31/70} \left(\frac{\kappa_4}{700 \text{ cm}^2 \text{ g}^{-1}}\right)^{1/5} \left(\frac{f_{\text{d2g}}}{10^{-2}}\right)^{1/5} \times \left(\frac{\dot{M}}{10^{-8}M_\odot \text{ yr}^{-1}}\right)^{2/5} \left(\frac{\alpha}{10^{-5}}\right)^{-1/5} \left(\frac{R}{1 \text{ au}}\right)^{-33/70} < 1.
\]

Here, we have used the \( \alpha \)-prescription for the turbulent viscosity (Shakura & Sunyaev 1973), i.e., \( \nu = \alpha c_s h_y \). We have also assumed \( \gamma_{\text{IR}} \gg 1 \), \( z_{D,D}/h_y = 3.6 \), and \( T_{\text{D,D}} = 2T_\odot^2 \) for simplicity. Equation (24) suggests that in a disk around a Herbig Ae star of \( L_\odot = 56L_\odot \) and \( M_\odot = 2.5M_\odot \), and \( \dot{M} = 10^{-8}M_\odot \text{ yr}^{-1} \), stellar irradiation dominates over viscous heating if \( f_{\text{d2g}} \lesssim 10^{-2} \), in agreement with the result of the hydrodynamical simulation by Flock et al. (2016; see their Figure 8). Even in an inner part (\~{}0.1 au) of a disk around a T-Tauri star of \( L_\odot = 5L_\odot \) and \( M_\odot = 2M_\odot \), stellar irradiation dominates if \( f_{\text{d2g}} \lesssim 10^{-3} \) and \( M \lesssim 6 \times 10^{-10}M_\odot \text{ yr}^{-1} \). Therefore, our temperature model is applicable to weakly accreting T-Tauri disks where tiny, opacity-dominating dust is depleted (through, e.g., dust coagulation).

#### 3.3. Migration Trap

A steep temperature drop as seen in the borders of regions B, C, and D is known to act as a planet trap where the strong corotation torque halts the inward migration of planets due to the negative Lindblad torque from the gas disk (e.g., Paardekooper et al. 2010). In addition, if the boundary between regions C and D sets the dead zone inner edge as discussed in Section 3.1, the positive surface density gradient arising from
the change in turbulent viscosity also prevents planets from inward migration (Masset et al. 2006).

It is meaningful to compare the locations of the borders to the orbital distribution of observed planets in order to reveal whether the borders really act as the migration trap. Our solutions show that the positions of the boundaries between regions B, C, and D are roughly proportional to $L_{\text{sh}}^{1/2}$, though the position of the boundary between regions C and D has the additional factor $\sqrt{1 + \Gamma}$, which slightly depends on the stellar parameters. If we assume $L_{\text{sh}} \propto M_{\text{sh}}^{3/2}$ (Mulders et al. 2015), these boundaries are scaled as $M_{\text{sh}}^{3/4}$. Mulders et al. (2015) investigated Kepler Objects of Interest (KOIs) and showed that the distance from the star where the planet occurrence rate drops scales with semimajor axis as $M_{\text{sh}}^{1/3}$, which is different from our estimate. However, the spectral range is not enough for a detailed comparison because the majority of the host stars for KOIs are F, G, and K stars. In order to discuss how the boundaries really act as a migration trap, observations of planets around A and B stars are needed. Mulders et al. (2015) also showed that many KOIs orbit at $\sim 0.1$ au, which is similar to the value of $R_{\text{lim}}$ estimated from our model.

3.4. Instabilities at the Inner Region of Protoplanetary Disks

Steep temperature gradients at the boundaries between regions B, C, and D would induce various kinds of instabilities. One possible instability is the subcritical baroclinic instability, which is the instability powered by radial buoyancy due to the unstable entropy gradient (Klahr & Bodenheimer 2003; Lyra 2014). If we assume the power-law density and temperature profile as $\rho \propto R^{-\beta_r}$ and $T \propto R^{-\beta_l}$, respectively, the condition for the subcritical baroclinic instability is given by (Klahr & Bodenheimer 2003)

$$\beta_T - (\gamma_{\text{ad}} - 1)\beta_p > 0,$$

(25)

where $\gamma_{\text{ad}}$ is the adiabatic index. This condition would be satisfied at the boundaries between regions B, C, and D, where $\beta_T$ is large. The boundary between regions C and D might also have a positive surface density gradient ($\beta_p < 0$) if the dead zone edge inner edge lies there. The positive density gradient would further enhance the instability on the boundary. However, the subcritical baroclinic instability has not been observed in Flock et al. (2016). This is because the spatial resolution of Flock et al. (2016) is not enough to resolve the instability. Lyra (2014) showed that for resolutions $> 32$ cells/h$_{\text{g}}$ the subcritical instability converges, while Flock et al. (2016) used around 15 cells/h$_{\text{g}}$. Also, Flock et al. (2016) calculated for around 20 local orbits at 2 au, which is not enough for the subcritical instability to operate. The growth timescale for the instability requires many hundreds of local orbits (Lyra 2014).

The Rossby wave instability could also be induced at the boundary between regions C and D due to the steep surface density gradient produced by the dead zone edge (Lyra & Mac Low 2012). In fact, the magnetohydrodynamical simulation by Flock et al. (2017) shows that the boundary between regions C and D produces a vortex, indicating that the Rossby wave instability should operate there.

If such instabilities operate on these boundaries, vortices caused by the instabilities would accumulate dust (e.g., Barge & Sommeria 1995), which in turn could lead to rocky planetesimal formation via the streaming instability (e.g., Youdin & Goodman 2005). We will explore this possibility in future work.

4. Summary

We have analytically derived the temperature and dust-to-gas mass ratio profile for the inner region of protoplanetary disks based on the results from the recent hydrodynamical simulations conducted by Flock et al. (2016). The temperature profile for the inner region of protoplanetary disks can be divided into four regions. The innermost region is dust-free and optically thin, with the temperature determined by the gas opacity (Equation (3)). As the temperature goes down and approaches the dust evaporation temperature, silicate dust starts to condense, producing an optically thin dust halo with a nearly constant temperature regulated by partial dust condensation. We have derived the dust-to-gas mass ratio profile in the dust halo using the fact that partial dust condensation regulates the temperature to the dust evaporation temperature (Equation (4)). Beyond the dust halo, there is an optically thick condensation front where all the available silicate gas condenses out. The curvature of the condensation surface is simply determined by the condition that the surface temperature must be nearly equal to the characteristic temperature $\sim 1200$ K (Equation (9)). The temperature profile for the outermost region is essentially the same as the classical optically thick temperature profile (e.g., Kusaka et al. 1970). We have derived the midplane temperature in the outer two regions using the two-layer approximation, with additional heating by the condensation front for the outermost region. As a result, the overall temperature profile follows a step-like profile with steep temperature gradients at the borders between the outer three regions. The borders might act as planet traps where the inward migration of planets due to gravitational interaction with the gas disk stops. The temperature at the border between the two outermost regions coincides with the temperature needed to activate magnetorotational instability, suggesting that the inner edge of the dead zone must lie at this border. The radius of the dead zone inner edge predicted from our temperature profile is $\sim 2$–3 times larger than that expected from the classical optically thick temperature.

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Appendix

Appendix Effect of the Evaporation Temperature

Dust evaporation temperature $T_{\text{ev}}$ is modeled as a function of the gas density $\rho$ as (Isella & Natta 2005)

$$T_{\text{ev}} = 2000\left(\frac{\rho}{1 \text{ g cm}^{-3}}\right)^{0.0195} \text{ K.}$$

(26)
Using the relation \( \rho = \frac{\Sigma_g}{\sqrt{\pi} h_0} \) and substituting Equation (26) as \( T_B \), Equations (5) and (6) can be rewritten as

\[
R_{AB} = \left[ 0.079 \left( \frac{R_e}{0.01 \text{ au}} \right) \left( \frac{T_e}{5000 \text{ K}} \right)^2 \right] \times \left( \frac{\Sigma_0}{100 \text{ g cm}^{-2}} \right)^{-0.0386} \left( \frac{M_*}{M_\odot} \right)^{-0.0193} \text{ au} \tag{27}
\]

and

\[
R_{BC} = \left( \frac{\kappa_d(T_e)}{\kappa_d(T)} \right)^{-\gamma/2} R_{AB} \tag{28}
\]

respectively. Here, \( \Sigma_0 \) is the surface density at 1 au. The index \( \gamma \) is related to the index of the radial surface density profile, \( \Sigma_0 \propto r^{-\beta} \), as \( \gamma = 2.0195/(1.9025 - 0.078\beta) \). For \( \beta = 1 \), \( \gamma \approx 1.107 \). Equation (27) is similar to the expression derived by (Kama et al. 2009, their Equation (A.4)), although the \( \beta \) dependence is different. If the disk is massive (\( \Sigma_0 \gg 1000 \text{ g cm}^{-2} \)), the boundary between regions B and C (i.e., the radial position for \( \tau_e = 1 \)) moves closer to the star compared to \( R_{BC} \) written by Equations (6) and (28) because of the large optical depth. In this case, in order to determine the radial position of \( \tau_e = 1 \), we have to calculate the optical depth using Equation (4).

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