Observing Dark Matter via the Gyromagnetic Faraday Effect

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If dark matter consists of cold, neutral particles with a non-zero magnetic moment, then, in the presence of an external magnetic field, a measurable gyromagnetic Faraday effect becomes possible. This enables direct constraints on the nature and distribution of such dark matter through detailed measurements of the polarization and temperature of the cosmic microwave background radiation.

Introduction. The existence of dark matter (DM) was first inferred in 1933 from Zwicky’s observations of extragalactic nebulae [1]. In recent years, our ability to assay its abundance has sharpened considerably, and a concordance of disparate observations reveal that DM comprises some twenty-three percent of the energy density of the universe, with a precision of a few percent [2]. Yet, despite this progress, the fundamental nature of DM remains unclear. One cannot say whether DM consists of a single species of particle, or of many, or even if it consists of stable, elementary particles at all. Dark matter could comprise aggregates of some kind, or be mimicked, in part, by a modification of gravity at large distances [3, 4, 5]. We do know that light, massive neutrinos cannot explain the galactic rotation curves [6], yet, despite this progress, the fundamental nature of DM remains unclear.

In recent years, our ability to assay its abundance has sharpened considerably, and a concordance of disparate observations reveal that DM comprises some twenty-three percent of the energy density of the universe, with a precision of a few percent [2]. Yet, despite this progress, the fundamental nature of DM remains unclear. One cannot say whether DM consists of a single species of particle, or of many, or even if it consists of stable, elementary particles at all. Dark matter could comprise aggregates of some kind, or be mimicked, in part, by a modification of gravity at large distances [3, 4, 5].

We consider the possibility that DM consists of neutral objects, which need not be elementary particles, of mass M with non-zero magnetic moments. The empirical limits on this possibility vary with the particle’s mass [5, 6] and can be evaded if the particle is composite.

Although our scenario naturally permits the dark constituents to be mutually interacting [10], it does differ significantly from usual ideas. For example, models of electroweak symmetry breaking with an additional discrete symmetry can yield viable DM candidates. In models with supersymmetry, the DM candidate — the “lightest supersymmetric particle” — is a Majorana particle, and its static magnetic moment is identically zero. Thus if the effect we discuss is observed, it demonstrates that the wave number for states with right- (+) or left-handed (−) circular polarization, then the rotation angle is given by φ = (k+ − k−)l/2, where l is the length of transmission through the medium. If the medium contains free electric charges, this is the Faraday effect known for light travelling through the electrons and magnetic fields of the warm interstellar medium (ISM) [10]. A Faraday effect can also occur in a magnetizable medium which is electrically neutral [12, 13]. We term these the gyroelectric (GE) and gyromagnetic (GM) Faraday effects [19], respectively. We study the GM Faraday effect associated with cold DM carrying a non-zero magnetic moment. We begin by comparing the Faraday effects in the ISM, for which the GE effect is familiar, before turning to a discussion of their impact on the cosmic-microwave background (CMB) polarization and the constraints such measurements can yield on models of DM.

Faraday Effects in the ISM. The ISM contains free electrons and external magnetic fields; it is GE and gives rise to a Faraday effect. We consider an external magnetic field \( \mathbf{H}_0 \) in the \( \hat{x} \)-direction with circularly polarized electromagnetic waves propagating parallel to it. In this case, an electron with charge \( -e \) and mass \( m \) suffers a displacement \( s \) via the Lorentz force

\[
\mathbf{m}s = -e(\mathbf{E} + \mathbf{s} \times \mathbf{H}_0),
\]

where \( \mathbf{H}_{\text{tot}} = \mathbf{H}_0 + \mathbf{H} \). The electric field, e.g., associated with the wave is \( \mathbf{E}(x,t) = E_0 e_\pm \exp(i k_{\pm} x - i \omega t) \), where \( e_\pm \equiv \hat{y} \pm i \hat{z} \). We define the polarization state with positive helicity, \( e_+ \), to be right-handed. Assuming \( |\mathbf{H}_0| \gg |\mathbf{H}| \), the steady-state solution for \( \mathbf{s} \) yields, for a medium of electrons with number density \( n_e \), the polarization \( \mathbf{P} = -n_e e_0 \mathbf{s} \) and the electric susceptibility \( \chi_e \), recalling \( \mathbf{P}_\pm = e_0 \chi_{e\pm} \mathbf{E}_\pm \). We thus determine the permittivity \( \epsilon_{\pm} \):

\[
\frac{\epsilon_{\pm}}{\epsilon_0} = 1 + \chi_{e\pm} = 1 - \frac{\omega_P^2}{\omega(\omega \mp \omega_H)},
\]

where the plasma frequency \( \omega_P \) is given by \( \omega_P^2 = n_e e_0^2 / m \) and \( \omega_H = eH_0 / m \). With \( k_\pm = (\omega / c) \sqrt{\epsilon_{\pm} / \epsilon_0} \),
and with $\omega \gg \omega_H, \omega_H$, we have $\phi = -\omega_H^2/2c\omega^2$ to leading order in $\omega$. Generalizing this to variable electron densities and magnetic fields along the line of sight yields

$$\phi = -\frac{e^3}{2c\omega^2\epsilon_0 m^2} \int_0^l \! dx \, n_e(x) H_0(x),$$

where $x = 0$ marks the location of the source. The $\omega$ dependence makes knowledge of the intrinsic source polarization unnecessary; one measures the position angle of linear polarization, in a fixed reference frame, as a function of $\omega$, so that the line integral of $n_e(x) H_0(x)$ can be inferred \cite{20, 21}. A pulsed radio source also permits the measurement of the frequency dependence of the ar
tival time, to yield the line integral of $n_e(x)$ \cite{20}, so that the average magnetic field along the line of sight can also be determined.

If the electrons can be aligned to yield a magnetization, the ISM can be regarded as GM as well. We shall treat the GE and GM effects independently. Applying a magnetic field in a GM medium induces a magnetization $M_{\text{tot}}$, i.e., a net magnetic moment/volume, where $M_{\text{tot}} = \hat{\mathbf{x}} M_0 + \mathbf{M}$ and $M_0$ results from $H_0$ alone. The resulting magnetization obeys

$$\dot{\mathbf{M}}_{\text{tot}} = \gamma \mathbf{M}_{\text{tot}} \times \mathbf{H}_{\text{tot}},$$

where $\gamma$ is the gyromagnetic ratio of the magnetic-moment-carrying particle. If the constituents possess an electric dipole moment as well, an additional term appears in Eq. \cite{1, 22}. We assume $|\mathbf{H}_0| \gg |\mathbf{H}|, |M_0| \gg |\mathbf{M}|$, and the conventions of the GE case to determine the steady-state solution, which, neglecting the $\mathbf{M} \times \mathbf{H}$ term, is

$$M_\pm = \pm \frac{\chi_0 \omega_H}{\omega \pm \omega_H} H_\pm \equiv \chi_\pm H_\pm,$$

where $\chi_0 \equiv M_0/H_0$ and $\omega_H \equiv \gamma H_0$. We recall the magnetic susceptibility $\chi_m$ obeys $\mathbf{M} = \chi_m \mathbf{H}$, so that

$$\frac{\mu_+}{\mu_0} = 1 + \chi_m \pm = 1 \pm \frac{\chi_0 \omega_H}{\omega \pm \omega_H},$$

where $k_\pm = (\omega/c) \sqrt{\mu_+/\mu_0}$. Noting $\omega_H/\omega \ll 1$ and working to leading order in this quantity, one has $k_{\text{diff}} = k_+ - k_-$, which controls $\phi$, with

$$k_{\text{diff}} = \frac{\chi_0 \omega_H}{c} + \frac{\chi_0 \omega_H}{c \omega} + \frac{\chi_0 \omega_H}{2c \omega^2} + \ldots$$

The magnetization induced by $H_0$ on a system of spin-1/2 particles each with magnetic moment $\mu$ in equilibrium at temperature $T$ is \cite{23}

$$M_0 = n_e \mu \tanh \left( \frac{\mu H_0}{k_B T} \right) = n_e \left( \frac{\mu^2 H_0}{k_B T} \right),$$

where the corrections to the last equality are negligible in the ISM, though diverse environmental conditions do exist. The magnetic field $H_0$ is no larger than a few $\mu G$ and its cold patches are no colder than a few $100 K$ \cite{20}. We can thus neglect non-leading powers in $\chi_0$. We separate the rotation angle $\phi$ into frequency-independent and frequency-dependent pieces, so that $\phi = \phi_0 + \phi_\omega$, to yield

$$\phi_0 = \frac{\mu^2 \gamma}{2ck_B T} \int_0^l \! dx \, n_e(x) H_0(x),$$

$$\phi_\omega = \frac{\mu^2 \gamma}{2c \omega^2 k_B T} \int_0^l \! dx \, n_e(x) H_0^2(x)/T(x),$$

where $\gamma = g \mu_B / \hbar, \mu_B \equiv e/2m$, and $g$ is the usual Landé factor. The appearance of higher powers in $H_0$ in $\phi_\omega$ makes it, as well as the time delay, negligible in comparison to $\phi_0$ in the ISM. If we neglect any $T$ variation along the line of sight, then the frequency-independent GM and GE effects share a common integral. We can then compare

$$\phi_0 = \frac{\mu^2 \gamma}{2ck_B T} \int_0^l \! dx \, n_e(x) H_0(x)$$

by computing

$$|\hat{\chi}| \equiv \frac{\gamma \mu^2}{k_B T} = \frac{|g| \mu^2 \mu_B}{k_B T} \sim \frac{2 \mu_B^3}{h k_B T} \sim 4.6 \cdot 10^{-19} \left[ \frac{300 K}{T} \right] \text{cm}^3 \text{G}^{-1}$$

and

$$\chi \equiv \frac{e^3}{\omega^2 \epsilon_0 m^2} = \frac{\alpha}{\pi} \frac{h}{mc} \frac{e}{m} \lambda^2 \sim 1.6 \cdot 10^{-6} \left[ \frac{\lambda}{1 \text{cm}} \right]^2 \text{cm}^3 \text{G}^{-1},$$

where $|g| \sim 2, \mu_B \sim 5.79 \cdot 10^{-9} \text{eV/G}, k_B T \sim 1/38.7 \text{eV}$ for $T \sim 300 K$, $1 \text{eV} \sim 4.03 \cdot 10^{-11} \text{G}^2 \text{cm}^3$, $\alpha \sim 1/137$, $e/m \sim 1.76 \cdot 10^7 \text{rad/Gs}$, and $h/mc \sim 3.86 \cdot 10^{-11} \text{cm}$. Recent surveys have used wavelengths in the $\lambda = 6$ and 20 cm bands \cite{20, 24}, and most Faraday rotation accrues in the warm ISM, for which $T \sim 5000 K$. We thus find the GM effect to be negligible for radio sources. We note $\phi_\omega$ is smaller than $\phi_0$ by a factor of $\gamma^2 H_0^2/\omega^2 \sim 9 \cdot 10^{-21} \lambda/(1 \text{cm})^2$, using $H_0 \sim 10^{-6} \text{G}$.

**Faraday Effects on the CMB Polarization.** Our study of the GM Faraday effect shows $\phi_0$ to be the most important numerically, though its frequency independence means we must employ sources of known polarization to determine it. To realize this, we turn to the CMB radiation, for the scalar gravitational perturbations which dominate the temperature fluctuations in inflationary cosmologies give rise to $E$-mode, or gradient-type, polarization exclusively \cite{25, 26}. The Faraday effects provide a mechanism by which $B$-mode, or curl-type, polarization can be produced from an initial state of $E$-mode polarization; ultimately, we wish to interpret the $B$-mode polarization as a constraint on DM with a
magnetic moment. A variety of sources of $B$-mode polarization exist, however, and it is important to separate the possibilities. Let us enumerate some of them explicitly. Primordial tensor or vector gravitational perturbations in the CMB can give rise to $B$-mode polarization [23, 26], and $B$-mode polarization can be generated from primordial $E$-mode polarization via gravitational lensing [27]. Magnetic fields can also imprint $B$-mode polarization. Primordial magnetic fields can do this both through the perturbations they engender [28], as well as through the GE Faraday rotation they mediate [29]. Magnetic fields in galactic clusters [24] can also give rise to GE Faraday rotation [30], impacting the CMB polarization at small angular scales [25, 26, 31]. The GM Faraday effect is distinguished by its $\omega^{-2}$ frequency dependence; the $B$-polarizations engendered by gravitational lensing and radiation are frequency-independent.

The GM Faraday effect can operate if the medium has a magnetization; this can occur if a non-zero magnetic field exists while the DM is still in thermal equilibrium in the early Universe. These conditions suffice to define the effective path length $\tilde{\ell}$, so that $\phi_0 = \mu_0^2 \gamma H^\text{prim}_o n_o \tilde{\ell} / 2ck_B T_o$, where $H^\text{prim}_o$, $n_o$, and $T_o$ are the primordial magnetic field, DM number density, and temperature, all scaled to the present epoch. We solve for $H(z)$, the Hubble constant at a redshift of $z$, using the Friedmann equation in a flat $\Lambda$CDM cosmology with a matter energy density of $\Omega_M = 0.27$ and with $H_0 = 71 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [34]. For a spin-1/2 DM particle of mass $M$ we define the magnetic moment $\mu \equiv k_M$, so that $k$ is the Pauli moment, as well as the gyromagnetic ratio $\gamma = 2k_M/h$ with $\mu_M = e/2M$. DM has been established in the recombination era [2], so that we compute the angle $\phi_0$ engendered by CMB photons propagating from $z \approx 1100$ to the present. To estimate the present-day DM temperature we consider galactic DM and use the gravitational infall velocity, assuming a Maxwell-Boltzmann distribution in galactic DM velocities, to determine that the root-mean-square velocity obeys $v_{\text{rms}} / c = \sqrt{3k_B T / M c^2}$. Thus

$$\phi_0 \sim 3.6 \cdot 10^{-18} \frac{\text{cm}^3}{\mu \text{G Mpc}} \left( \frac{\mu}{\mu_B} \right)^3 \left( \frac{M}{M_z} \right)^2 \left( \frac{v_{\text{rms}}}{c} \right)^{-2} \times n_o [\text{ cm}^{-3}] H^\text{prim}_o [\mu G][\text{ Mpc}],$$

where $n_o \sim 2.17 \cdot 10^{-3} \text{ cm}^{-3}$ noting $n_o \equiv \rho_{\text{dm}} / m_e$ and $\rho_{\text{dm}} \sim 1.98 \cdot 10^{-30} \text{ g cm}^{-3}$ [34], $v_{\text{rms}} \sim 200 \text{ km/s}$, and $\tilde{\ell} \sim 1.3 \cdot 10^{10} \text{ Mpc}$. We can consider light cold dark matter because its annihilation cross section is mediated by its magnetic moment [33]. Some observational evidence suggests that $M$ is of MeV scale [36]. Using the bound $H^\text{prim}_o \lesssim 10^{-3} \mu G$, for primordial magnetic fields coherent across the present horizon [37], we find a bound of $|k| \lesssim 0.8$ if $M = m/10$ and if $\phi_0$ can be determined to $\phi_0 \sim 10^{-2}$ rad. Precision electroweak measurements also constrain the magnetic moment [9]. The quantity $\Delta^r$ represents the radiative corrections to the relationship between the fine-structure constant $\alpha$, the Fermi constant $G_F$, and the $W^\pm$ and $Z$ masses, $M_W$ and $M_Z$ [38]; the difference between the empirically determined value of $\Delta^r$ and that computed in the Standard Model provides a window $\Delta^\text{new}$ to which a DM particle can contribute. Thus we find from the vacuum polarization correction to the photon self-energy, with $a \equiv (M_Z/M)^2 \gg 1$,

$$\Delta^\text{DM} \sim -\kappa^2 \frac{\alpha}{4\pi} \left( \frac{a}{6} \log a - \frac{a}{9} + O(1) \right) \left( 1 - \frac{M_Z^2}{M_c^2} \right)^{-4},$$

where we include a form factor at each vertex with a compositeness scale of $M_c$. With $\Delta^\text{new} < 0.0010$ at 95% CL [39], we find with $M = m/10$ that $|k| < 3.4 \cdot 10^{-7}$ if $M_c \to \infty$, which relaxes to $|k| < 1.5$ if $M_c = 2 \text{ GeV}$, e.g. We thus conclude that a useful constraint on $k$ from a Faraday rotation measurement is possible.

**Summary.** A Faraday effect also exists for light transiting a dark medium of electrically neutral particles with non-zero magnetic moments in an external magnetic field. We have shown that this possibility can serve as a new source of $B$-mode polarization in the CMB and...
that it can be disentangled from other sources. Thus a non-zero effect due to such DM can be identified, if it exists, with the implication that supersymmetric models do not provide an exclusive solution to the DM problem.

The GM Faraday effect can be used to probe the nature and distribution of DM, to realize a picture of our Universe shaped by what we observe, rather than by what we believe to be so.

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