Analysis of Water Temperature of Lake Water Source Heat Pump and Calculation of Lake Water Area

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Abstract. In order to solve the problem of heat pollution of lake water source heat pump and standardize the design of lake water source heat pump, this paper takes Yuehu water source heat pump of Hunan University of Science and Technology as the research object, establishes the water temperature model based on the heat balance equation, uses MATLAB software to solve numerically, obtains the water temperature model of lake water and the water temperature model of lake water source heat pump. According to the stipulation of <Surface Water Environmental Quality Standard>, the maximum temperature rise of Yuehu Lake is 0.4929, and the designed Lake area should be no less than 43000 m².

1. Introduction
Lake water source heat pump is an effective energy saving technology by heat exchange with lake water to convert low energy source in lake water into high energy source. If the scale of the heat pump system is very large and the corresponding water capacity is small, the heat will be discharged into the lake day after day in the high temperature season, which may cause the surface water temperature to rise and even lead to heat pollution when serious [2]; Water thermal pollution will not only affect the living things in the water environment, but also reduce the performance of the heat pump system.

For the study of water temperature model, the MIT model is modified and perfected by the China Institute of Water Conservancy and Hydropower, and the "Lake temperature No. 1" model is proposed to predict the temperature field of the lake water [3], and Zhang Xiane uses the two-dimensional model to simulate the water temperature distribution of the reservoir [4]. Chen Xiao established the mathematical model of the closed lake water source heat pump system, and analyzed the influence of the depth and area of the closed water body on the temperature of the water inlet of the unit under the condition of refrigeration and heating [5]. Zhao Jian established the surface water source heat pump water temperature model, and analyzed the cooling capacity of water body [6]; Liu Yikun used lumped parameter model to establish water temperature model of lake water source heat pump, and made use of simplified lumped parameters The maximum cooling capacity of lake water was calculated by numerical model [7], and the water body heat pollution assessment method based on analysis was put forward by Hu Qiuming [8].

According to the Standard for Environmental quality of Surface Water, the variation of water temperature caused by man should be limited to the following: the maximum temperature rise per week ≤ 1 °C, and the maximum temperature drop ≤ 2 °C per week [9]. The water temperature model is...
established by MATLAB to analyze the water temperature of the lake water source heat pump and calculate the minimum water body area to avoid heat pollution.

2. Engineering survey
This project uses the artificial lake moon lake of Hunan University of Science and Technology to form the water source heat pump air conditioning system of lake water. The application area of air conditioning is about 20 000 m² and the total building area is about 24 000 m². The total cooling load is 2483 kw and the total heat load is 1875 kW. The water area of Yuehu Lake is about 51000 m², and the average depth of lake water is 3.5 m. The operating time of water source heat pump is 8 am-18:00. The winter and summer seasons of Xiangtan account for 67% of the whole year, the cold period in summer is July to September, and the heating period in winter is December 1. The effect since operation is remarkable.

3. Natural water temperature model
3.1. Water temperature model
For the water body using the surface water source heat pump system, the heat exchange between the surface water and the outside world mainly includes: solar shortwave radiation, atmospheric long wave radiation, evaporation of water body, convection heat transfer with air, Convection heat transfer with riverbed and heat transfer from heat pump unit [10]. In establishing the water temperature model, the following assumptions are made:

1) Ignore the natural flow of lake water, that is, ignore the natural inflow and outflow of lake water.
2) The shore is adiabatic from the lake bottom and does not exchange heat with the lake water.
3) The lake water is considered as a whole, regardless of vertical and horizontal heat transfer.
4) The reflectivity of water to solar radiation is fixed at 0.1.

According to the first law of thermodynamics, the mathematical model of water temperature is established as follows:

\[
\rho c_p V \frac{dT_w}{dt} = Q_z = Q_{in} - Q_{out}
\]

(1)

Where \( T_w \) is represented for the water temperature(°C), t is the time(s), \( \rho \) is the density of water(kg/m³), \( c_p \) is the specify heat of water(J/(kg·°C)), \( V \) is the water volume(m³), \( Q_z \) is the total heat flow(W), \( Q_{in} \) is the heat flow into the water(W), and \( Q_{out} \) is the outflow heat from the water(W).

(1) The total heat flow \( Q_z \):

\[
Q_z = Q_{sn} + Q_{an} - Q_{br} - Q_e - Q_i
\]

(2)

Where \( Q_{sn} \) is the solar short-wave radiant heat(W), \( Q_{an} \) is the atmospheric long-wave radiant heat of the atmosphere(W), \( Q_{br} \) is the long wave radiant heat of water body(W), \( Q_e \) is the heat exchange rate of evaporation of water body(W), and \( Q_i \) is the convection heat transfer between water surface and air(W).

(2) The solar short-wave radiant heat \( Q_{sn} \):

\[
Q_{sn} = Q_s \times (1 - r_s) \times A \times c_s
\]

(3)
Where $r^s$ is the reflectivity (take the value of 0.1), $A$ is the area of the water body ($m^2$), $c^s$ is the occlusion coefficient of surface vegetation, (when there is no vegetation, $c^s=1$, otherwise $c^s$ is range from 0 to 1).

(3) The atmospheric long-wave radiant heat $Q_{an}$

$$Q_{an} = (1-r_a) \times \sigma \times \varepsilon_a \times (273 + T_a)^4 \times A$$

Where $r_a$ is the long wave reflectivity (take the value of 0.03), $\sigma$ is the stefan boltzmann constant, the value of $5.67 \times 10^{-8} W/(m^2\cdot K^4)$. $T_a$ is the temperature of air above water($^\circ C$), $\varepsilon_a$ is the emissivity of the atmosphere on cloudy days.

(4) The long wave radiant heat of water $Q_{br}$

$$Q_{br} = \sigma \times \varepsilon_w \times (273 + T_w)^4 \times A$$

Where $T_w$ is the average temperature of water ($^\circ C$), $\varepsilon_w$ is the long wave emissivity of the water surface, which is set to 0.97.

(5) The convection heat transfer between water surface and air $Q_c$

$$Q_c = 0.47 f_w \times (T_w - T_a) \times A$$

Where $f_w$ is the function of wind speed ($W/m^2 \cdot mmHg$).

$f_w = 9.2 + 0.46 W_z$

$W_z$ Is represented the wind speed (m/s).

(6) Evaporation of water $Q_e$

$$Q_e = f_w \times (e_s - e_a) \times A$$

Where $e_s$ is represents the saturated vapor pressure of air near the surface of the water, $e_a$ is represents the air vapor pressure over the water surface.

(7) Exhaust heat of heat pump unit $Q_{hp}$

$$Q_{hp} = Q_h \times (cop + 1)/cop$$

Where $Q_h$ is represents the cold (thermal) load of a building, $cop$ is performance coefficient of heat pump unit, summer takes +, winter takes -.

4. Water temperature solution

4.1. Lake water temperature model

In this paper, the lake water is regarded as a whole and the water temperature is uniformly distributed without considering the heat discharge of the lake water source heat pump. $t_j$ Indicates the current lake
temperature, $t_{j+1}$ indicating the temperature of the lake at the next moment. The water temperature expression of lake water at $j+1$ is:

$$t_{j+1} = t_j + \Delta t_j, \quad \Delta t_j$$

And, $\Delta t_j$ represents the change in temperature within the time step $h$, $\Delta t_j = A \times \Delta Q \times h / \rho c_p V$. Referring to the Euler method, the initial lake water temperature $T_w$ at 1 am on July 15 is given. The lake water temperature from 1 o'clock to 24:00 on July 15 is calculated time by hour and the variation of lake water temperature for one week is obtained. The temperature of the lake is shown in figure 1.

4.2. Water temperature Model of Lake Water Source Heat pump

Runge-Kutta method is a high-precision single-step algorithm which is widely used in engineering. The classical fourth-order Runge-Kutta method takes the weighted mean of slope at four points as the approximate value of the mean slope and constitutes a series of fourth-order Runge-Kutta. It has fourth order precision.

Because the (1) equation is a nonlinear function about water temperature, the water temperature model of the lake water source heat pump is solved by the classical fourth order Runge-Kutta method, and the water temperature model of the rest period is solved by the Euler method. The program diagram is shown in figure 3.

START

Enter weather parameters, given initial temperature $T_w, dt=600$

$J=1, \Delta t=0$

$m=1$

Calculation $Q, \Delta t=\Delta t+Q*dt/(\rho*C*v)$

$m=6$

$J=J+1$

$T_w(J+1)=T_w(J)+\Delta t$

$J>168$

Output water temperature $T_w$

END

Fig. 1 Lake water temperature program diagram
Enter weather parameters, given initial temperature $T_w$, $dt=3600$

$J=1, h=1$

Rem($J$,24)$>7$ & Rem($J$,24)$<19$

Calculation $Q$, $\Delta t = Q^2/\left( P \cdot C \cdot \gamma \right)$

$J=J+1$

$T_w(J+1) = T_w(J) + \Delta t$

Rem($J$,24)$>7$ & Rem($J$,24)$<19$

$K_1 = f(t_1, T_w)$
$K_2 = f(t_1+h/2, T_w+h*K_1/2)$
$K_3 = f(t_1+h/2, T_w+h*K_2/2)$
$K_4 = f(t_1+h, T_w+h*K_3/2)$

$T_w = T_w + h*(K_1+2K_2+2K_3+K_4)/6$

J$>168$

Output water temperature $T_w$

END

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**Fig. 2** Program Diagram of Water temperature of Lake Water Source Heat pump

Four points as the approximate value of the mean slope and constitutes a series of fourth-order Runge-Kutta. It has fourth order precision.

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### 4.3. Results and discussion

It can be seen from fig. 4 that the solar shortwave radiation of the water surface decreases during the day due to the cover of vegetation on the water surface, which results in a decrease in the rise of water temperature and does not tend to be balanced. The effect of vegetation occlusion coefficient on natural water temperature is also different. The closer the $C_s$ is to 1 (no vegetation on the surface), the more the water temperature tends to be balanced. So for the lake water source heat pump water source, it is particularly important to clear the water barrier.
During a day lake water area

Table. 1 Maximum circumference temperature rise under different lake water area

| Lake area A(m²) | Week maximum temperature rise(℃) |
|-----------------|----------------------------------|
| 51000           | 0.4929                           |
| 45000           | 0.8656                           |
| 43000           | 1.0124                           |
| 40000           | 1.2593                           |
| 30000           | 2.9967                           |

In this paper, the effect of water area on the water temperature of lake water source heat pump is studied, and the water depth is kept constant at 3.5 meters, and the lake water area is changed. From figure 4, we can see that the larger the area of lake water, the closer the water temperature curve of water source heat pump is to the normal temperature curve of lake water. When the area of lake water
is 51000m$^2$, the maximum temperature rise is 0.4929 ℃, less than 1 ℃. In line with the National Surface Water Environmental quality Standards.

Table 1 shows that the larger the lake area, the smaller the maximum temperature rise around the lake. The larger the lake area, the larger the heat capacity of the lake, which can reduce the heat pollution caused by water source heat pump. For the Yuehu water source heat pump project of Hunan University of Science and Technology, the designed lake area should not be less than 43000m$^2$.

5. conclusion

In this paper, based on the heat balance equation, the water temperature model of the Yuehu Lake heat pump and the water temperature of the lake water source heat pump are established based on the research object of Yuehu water heat pump in Hunan University of Science and Technology. In accordance with the provisions of the Environmental quality Standard for Surface Water, the following conclusions are drawn:

(1) The sheltering coefficient of water surface vegetation has an important influence on the change of lake water temperature. The closer it is to 1 (that is, there is no vegetation on the surface of the water), the more the temperature changes of the lake tend to be balanced.

(2) By calculation, the water source heat pump project of Yuehu Lake, Hunan University of Science and Technology, the maximum temperature rise around the lake is 0.4929 ℃, less than 1 ℃, which accords with the regulation of the National Surface Water Environmental quality Standard. The design area of lake water should not be less than 43000m$^2$.

References

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