Polarization conversion in resonant magneto-optic gratings

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\textit{New Journal of Physics} 8 (2006) 205
Received 4 May 2006
Published 22 September 2006
Online at http://www.njp.org/
doi:10.1088/1367-2630/8/9/205

Abstract. Strong polarization conversion effects, including large Kerr rotation and ellipticity, are predicted in zero-order reflection of light from a Bi : YIG grating embedded in a dielectric material. The field localization that gives rise to the enhanced magneto-optic effects is due to the excitation of leaky guided waves, which propagate in (and around) the high-index Bi : YIG layer in directions perpendicular to the external static magnetic field as well as the incident, transmitted and reflected optical fields. The resonant interaction with the incident field then results in narrow-band reflection spectra. Large magneto-optic polarization conversion effects result from the splitting of the degeneracy of right- and left-circularly polarized eigenmode resonances with highly dispersive phase spectra near the resonance peak.

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1. Introduction

Large Faraday and Kerr rotations obtained in photonic bandgap (PGP) structures with magneto-optic (MO) defect layers have received considerable attention in recent years. Both one-dimensional multilayer film stacks [1]–[6] and two-dimensional PGP structures [7, 8] composed of MO and dielectric materials have been shown to give rise to greatly enhanced polarization conversion effects per unit thickness of the MO material. In these distributed-feedback systems, light effectively traverses back and forth through the Bi:YIG layer many times, and the rotation is enhanced in each pass. Thus, even though the total thickness of the structured region is only tens of wavelengths (for example, less than 20 \( \mu \text{m} \)) and the Bi:YIG layer thickness is just a few wavelengths (even less than 1 \( \mu \text{m} \)), rotations of tens of degrees are possible.

In this paper, we consider an even more compact system, in which only a single layer with a thickness of a fraction of a wavelength can produce polarization conversion effects of the same magnitude as predicted in previously studied PGP structures. The field enhancement in the Bi:YIG layer is obtained, instead of distributed feedback, by means of the guided-mode resonance effect: a grating excites a leaky guided wave, which is continuously coupled in and out of the high-index layer and produces a sharp reflection peak around the resonance wavelength [9, 10]. In this structure, the field is enhanced in the modulated Bi:YIG layer due to the guided modes, which propagate in the \( xy \) plane perpendicular to the external magnetic field \( \vec{B}_{\text{ex}} \) (oriented in the \( z \)-direction), and thus Faraday or Kerr rotations might not be expected at first sight. However, in view of the geometrical-optics interpretation of guided-mode propagation, rays travel inside the waveguide along zig-zag paths by total internal reflection. Since such rays possess also wave vector components in \( \pm z \)-directions, this interpretation suggests multiple passes through the Bi:YIG layer. Thus enhanced Kerr rotation and other polarization conversion effects around the resonance wavelength may be anticipated.

It should be pointed out that polarization conversion ability is a general property of resonance-domain gratings illuminated at conical mountings; see, e.g., [11]–[14]. Guided-mode resonance filters are an exception in this respect. However, the effects to be discussed here are specifically due to the MO layer and no polarization conversion is observable at normal incidence without the external magnetic field.

The paper is organized as follows. In section 2, we first present the geometrical configuration of the system considered; it is one of the many possible implementations of a guided-mode resonance filter [9, 10]. Then appropriate values for the simulation parameters are determined and the method of analysis, in which no approximations are made, is discussed. In section 3, we give numerical simulation results to illustrate the main optical effects uncovered by the analysis if the structure is illuminated by a monochromatic plane wave. Some specific features of the Kerr (and Faraday) rotation and ellipticity spectra are studied in more detail in section 4 to show that useful optical functions can be realized by resonant magneto-optic gratings. We also point out a somewhat simplified, approximate method of analysis and design of such gratings. Moreover, the effects of the finite size of the incident beam and the finite spectral width of the illuminating field are discussed briefly in section 5. Finally, conclusions are drawn in section 6.
Figure 1. Cross section of a magneto-optic guided-mode resonance filter with illustration of characteristic parameters $D$, $d$, $t$, $h$, $n$ and $\epsilon$.

2. Geometrical configuration and method of analysis

The geometry to be considered in this paper is illustrated in figure 1. The structure consists of square pillars of size $D \times D$, arranged in a Cartesian array of period $d \times d$ in the $xy$-plane. The pillars of height $h$ and the underlying film of thickness $t$ are assumed to be made of a magneto-optic material (Bi : YIG is considered in this work) and buried in a dielectric of refractive index $n$. The fabrication of this structure follows the same lines as the fabrication of a conventional filter of the same type, the only difference being that the magneto-optic layer must be grown using some known technique such as that described in [15] and a suitable etchant is used to form the grating structure.

The structure of figure 1 is illuminated by a normally incident, monochromatic electromagnetic plane wave $\vec{E}^{\text{inc}}$ with wave vector $\vec{k}^{\text{inc}}$ (these assumptions are relaxed in section 5). We are interested in the zero-order reflected and transmitted waves $\vec{E}^{\text{refl}}$ and $\vec{E}^{\text{trans}}$ with wave vectors $\vec{k}^{\text{refl}}$ and $\vec{k}^{\text{trans}}$, which are the only propagating waves outside the structure if the period $d$ is chosen small enough compared to the wavelength $\lambda$ of the illuminating wave. The coupling of energy between $\vec{E}^{\text{inc}}$, $\vec{E}^{\text{refl}}$ and $\vec{E}^{\text{trans}}$ takes place via guided modes that are confined in and near the Bi : YIG layer, producing strongly wavelength-dependent effects, in particular narrow reflection peaks close to the guided-mode excitation wavelengths.

In the presence of an external static magnetic field $\vec{B}^{\text{ex}}$, the Bi : YIG grating layer acts as an anisotropic dielectric material, and this will be seen to affect the properties of the structure quite dramatically. The permittivity tensor takes the form

$$\epsilon(\lambda) = \begin{pmatrix} \epsilon_1 & -i\epsilon_2 & 0 \\ i\epsilon_2 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3 \end{pmatrix},$$

where $\epsilon_1$, $\epsilon_2$ and $\epsilon_3$ are, in general, complex-valued wavelength-dependent quantities and the value of $\epsilon_2$ depends on $B = |\vec{B}^{\text{ex}}|$. The relative-permittivity data for Bi : YIG are computed using the model of [16], assuming that all diagonal elements are equal (because $\epsilon_3 \approx \epsilon_1$ in room temperature) and that $\epsilon_2$ has reached its saturation value.
The real and imaginary parts of \( \epsilon_1 \) and \( \epsilon_2 \) given by Doormann et al.’s model [16] are plotted in figure 2. It is seen that the imaginary part of \( \epsilon_1 \) is large for wavelengths smaller than 550 nm, which renders the material rather useless in resonant optical structures because it leads to enhanced absorption. This forces us to leave much of the interesting part of the \( \epsilon_2 \) curves out of consideration, including the point at \( \lambda = 421 \) nm where \( \Im \{ \epsilon_2 \} = 0 \) but \( \Re \{ \epsilon_2 \} \approx -0.2 \). Thus, to minimize absorption losses, we consider in this paper only wavelengths longer than 600 nm. Even here Doormann et al.’s model predicts small imaginary parts for \( \epsilon_1 \) and \( \epsilon_2 \). However, since the model is approximate and does not consider the electronic band structure of Bi : YIG, these are ignored and we assume \( \Im \{ \epsilon_1 \} = \Im \{ \epsilon_2 \} = 0 \) (i.e., the material is lossless) from now on.

Considering first the interaction of plane waves with a uniform MO material, the Kerr rotation and ellipticity experienced by a linearly polarized wave are related to the reflection coefficients of the two eigenmodes of the MO material, namely the right-circularly polarized (RCP) and left-circularly polarized (LCP) modes, which experience different refractive indices in the material due to circular birefringence:

\[
 n_\pm = (\epsilon_1 \pm \epsilon_2)^{1/2},
\]

where the \( \pm \) signs refer to the RCP and LCP modes, respectively. Denoting the complex reflection coefficients by

\[
 r_\pm = a_\pm \exp(i\phi_\pm),
\]

the Kerr rotation angle (the angle between the long axis of the polarization ellipse and the incident polarization direction) is given by

\[
 \theta = \frac{1}{2} (\phi_+ - \phi_-)
\]

Figure 2. Permittivity spectra used in this work. Real and imaginary parts of the diagonal elements \( \epsilon_1 \) (left-hand scale) and the off-diagonal elements \( \epsilon_2 \) (right-hand scale) of the relative permittivity tensor, assuming Bismuth concentration \( x = 1.26 \). Real parts are shown by solid lines and imaginary parts by dashed lines; the arrows indicate whether left or right scales should be considered.
Table 1. Structural parameters used in figures 3–5.

| n   | d (nm) | D/d | t (nm) | h (nm) |
|-----|--------|-----|--------|--------|
| 1.8 | 300    | 0.5 | 320    | 150    |
| 2.0 | 300    | 0.5 | 275    | 115    |
| 2.5 | 250    | 0.5 | 320    | 160    |

and the Kerr ellipticity $\chi$ is obtained from

$$\tan \chi = \frac{a_+ - a_-}{a_+ + a_-}.$$  \hspace{1cm} (5)

These parameters, as well as the amplitude reflection coefficient, determine the elliptically polarized reflected plane wave completely. Clearly the ellipticity vanishes and a linearly polarized output wave is produced if $a_+ = a_-$. 

Let us then consider the structure of figure 1, assuming that $d < \lambda / n$, which implies that the zero-order wave is the only reflected wave produced by the structure (similarly, only the zero-order transmitted wave is produced). Because of the symmetry of the grating, the RCP and LCP modes are eigenmodes of the entire structure, i.e., illuminating the element with RCP and LCP waves produces RCP and LCP reflected waves, respectively. Thus, in the rest of this paper, $r_+$ and $r_-$ denote the zero-order Rayleigh coefficients of the structure of figure 1 associated with normally incident RCP and LCP plane waves, respectively.

The complex coefficients $r_+$ and $r_-$, as well as the parameters of the reflected wave produced by a linearly polarized input wave, are determined using the rigorous Fourier modal method for crossed gratings made of anisotropic materials [17]. The air–dielectric boundaries are ignored in this analysis for simplicity, assuming for example that both interfaces are antireflection-coated.

3. Simulation results

Figures 3–5 illustrate the general trends observed in the numerical simulations, which show that the grating parameters $d$, $h$, $t$ and $D/d$ mainly influence the position of the resonance (as they do in case of non-magnetic guided-mode resonance filters) and that the magneto-optically induced effects depend most dramatically on the index contrast of the structure, i.e., the ratio of $n$ to $\sqrt{\epsilon_1}$, which has the largest effect in the width of the resonance peak in conventional resonance filters. Thus, in the figures, three values of $n$ are considered, namely $n = 1.8$ in figure 3, $n = 2.0$ in figure 4 and $n = 2.5$ in figure 5. The grating parameters, listed in table 1, are chosen such that the resonance takes place close to the He–Ne laser wavelength $\lambda = 632.8$ nm (these parameters can be modified slightly to tune the resonance position in the neighbourhood of this wavelength). Good fits in Doormann et al’s model [16] are obtained from formulas

$$\Re\{\epsilon_1\} \approx 12.69 \exp (-0.001033\lambda)$$ \hspace{1cm} (6)

and

$$\Re\{\epsilon_2\} \approx 1.765 \exp (-0.006934\lambda)$$ \hspace{1cm} (7)
Figure 3. Reflectivities $R$ (a), phases $\phi_{\pm}$ of RCP and LCP waves and the Kerr rotation angle $\theta$ (b) and amplitudes $a_{\pm}$ of the RCP and LCP waves and the Kerr ellipticity $\chi$ (c), in the case $n = 1.8$. Dashed lines refer to RCP, dotted lines to LCP and solid lines to linearly polarized illumination.

(where $\lambda$ is expressed in nanometres) in the region $620 \text{ nm} < \lambda < 640 \text{ nm}$, which is of main interest here. The dispersion of $n$ is neglected in the considered region.

In figures 3–5, the zero-order reflectivities $R_{\pm} = |r_{\pm}|^2$ for RCP and LCP illumination, as well as the reflectivity $R$ for linearly polarized illumination, are shown in panel (a) by dashed, dotted and solid lines, respectively. In panel (b) of figures 3–5, we show (left scale) the phases of the reflected RCP and LCP waves (the phase of the incident wave is zero) as well as (right scale) the Kerr rotation angle. Finally, in panel (c) of figures 3–5, the amplitudes $a_{\pm}$ of the RCP and LCP waves are shown (left scale) along with (right scale) the tangent of the Kerr ellipticity, $\tan \chi$. The numerical calculations agree with equations (4) and (5) well, and the relation $R = \frac{1}{2} (R_+ + R_-)$ is also valid as for a uniform MO material.

Several interesting features become clear by inspection of figures 3–5. Firstly, the magneto-optic effect is responsible for splitting the degenerate resonance peak of a non-magnetic guided-mode resonance filter into two peaks, which are spectrally separated and related directly to the RCP and LCP eigenmodes of the structure: with pure RCP and LCP illumination a 100\% reflectivity is obtained at the respective resonance peak. These RCP and LCP peaks shift downwards and upwards in the wavelength scale, respectively, from the resonance position obtained when $\varepsilon_2 = 0$. In case of linearly polarized illumination (which of course can be decomposed into RCP and LCP waves), the reflectivity at the positions of the RCP and LCP resonances is the average (as expected) of the two eigenmode-induced reflectivities and therefore always less than 100\%.
While the structural parameters given in table 1 determine the spectral position of the resonance peak, the index contrast is mainly responsible for the widths of the RCP and LCP resonance lines. Thus, two well-separated reflection peaks with $R \approx 50\%$ are obtained for linearly polarized light when the index contrast is large, but by decreasing it (for a given value of $\epsilon$) one can effectively merge the two peaks. At the same time, the spectra of $\phi_+^\pm$ and $\phi_-^\pm$ become identical with the phase spectrum obtained when $\epsilon_2 = 0$. Consequently, the maximum polarization rotation angle $\theta$, which occurs at the middle point of the RCP and LCP resonances, is reduced. On the other hand, with well-separated peaks, the reflectivity at the centre position is practically zero. Hence, if we want to obtain a large rotation without introducing Kerr ellipticity, and achieving high reflectance at the same time, the index contrast is the main parameter to be optimized. Additionally, the ellipticity takes its maximum values at the eigenmode resonance positions, where the rotation is also considerable.

4. Design examples

Let us now consider some useful optical functions that can be achieved with magneto-optic guided-mode resonance filters. The most obvious application is to use the filter as a magneto-optically switchable polarization rotator as suggested by the results given in figures 3–5. At the middle point of the RCP and LCP resonances, a linearly polarized incident wave remains linearly polarized upon reflection, but with $\vec{E}$ rotated by an angle $\theta$. This is the resonance peak position of the structure when $\vec{B} \parallel = 0$, so a pure polarization rotation by an angle $\theta$ takes place when

**Figure 4.** Same as figure 3 but for $n = 2$. 
the magnetic field is applied. With proper choices of grating parameters one can obtain, e.g., a 90° polarization rotation at a specified wavelength. One set of appropriate parameters for the design wavelength $\lambda = 632.8$ nm are given in table 2, design A. The efficiency of this design is $R = 24\%$ and the spectral characteristics of the structure are illustrated in figure 6.

Thus far, we have considered only Kerr effects in magneto-optic guided-mode resonance filters, but polarization conversion occurs also in the transmitted zero order. Because there are no absorption losses in the structure, energy is split between the zero reflected and transmitted orders. In the case of design A in table 2, $T = 76\%$ and the polarization vector of the transmitted beam is rotated by $-25.7°$ as seen from figure 6(b).

As a second example, we consider polarization beam splitters, which transform an incident linearly polarized wave into a reflected LCP (RCP) and a transmitted RCP (LCP) wave, each with an efficiency of 50%. Such designs are obtained at the locations of the eigenmode resonance peaks. The LCP reflection peak is tuned to $\lambda = 632.8$ nm using the parameters of design B in table 2, while the RCP peak is tuned to $\lambda = 632.8$ nm with the parameters of design C. The spectral characteristic of designs B and C are illustrated in figures 7 and 8, respectively.
To end this section, we note that there is an approximate method of analysis and design of magneto-optic guided-mode resonance filters, which does not require a full analysis by rigorous diffraction theory of gratings made of anisotropic materials [17] used in this paper, but only rigorous analysis of the case $\vec{B}_{\text{ex}} = \epsilon_2 = 0$ accompanied by certain simple manipulations. As pointed out above, the introduction of the external magnetic field splits the degeneracy of the Kerr resonance peak into two (RCP and LCP) peaks, which in first approximation have the same line shape and line width as the original peak, but are shifted towards shorter and longer wavelengths, respectively, by an amount $\Delta \lambda$. Mathematically, if we denote the amplitude of the reflectance spectrum in the case $\vec{B}_{\text{ex}} = 0$ by $a(\lambda)$, the RCP and LCP amplitude reflectance spectra take the forms

$$a_{\pm}(\lambda) \approx a(\lambda \pm \Delta \lambda).$$

(8)

Similarly, the phase spectrum of the reflected wave, $\phi(\lambda)$, splits into two identical but wavelength-shifted spectra

$$\phi_{\pm}(\lambda) \approx \phi(\lambda \pm \Delta \lambda).$$

(9)

Then, in view of equations (4) and (5), the Kerr ellipticity and rotation are given by

$$\theta(\Delta \lambda) = \frac{1}{2}[\phi(\lambda + \Delta \lambda) - \phi(\lambda - \Delta \lambda)].$$

(10)
Figure 7. Same as figure 6, but for design B in table 2.

and

\[
\tan \chi(\Delta \lambda) = \frac{a(\lambda + \Delta \lambda) - a(\lambda - \Delta \lambda)}{a(\lambda + \Delta \lambda) + a(\lambda - \Delta \lambda)},
\]

respectively. The shift \(\Delta \lambda\) can be well approximated by applying the standard Fourier modal method for isotropic materials to the structure of figure 1 assuming, in view of equation (2), that the refractive index of the grating region is changed from \(\sqrt{\epsilon_1}\) to \(\sqrt{\epsilon_1 + \epsilon_2}\) and observing the resulting shift in the resonance peak position. A similar procedure is applicable to the transmitted field as well.

5. Discussion

We have so far assumed that the illumination beam is a normally incident, uniform-amplitude, monochromatic plane wave. However, since the resonances produced by guided-mode filters are sharp and the polarization conversion properties vary strongly within the resonance line, it is necessary to pay some attention to the effect of the finite extent (and divergence) as well as the finite spectral bandwidth of the incident beam.

Let us assume a Gaussian beam with half-width \(w\) and an angular spectrum with divergence \(\phi_w = \lambda / \pi w\) incident on design A of table 2. The beam can be decomposed into plane waves incident on the structure at different angles, and a small change (a few milliradians, typical of an unexpanded gas laser beam) of the angle of incidence \(\phi\) shifts the
resonance peak centre position \( \tilde{\lambda} \) as well as the amplitude and phase spectra according to the approximate formula

\[
\tilde{\lambda} \approx -1.751 \times 10^{-3} \phi^2 - 3.274 \times 10^{-4} \phi + 632.8,
\]  

where the fit is made for \( 0 < \phi < 5.2 \) mrad and \( \tilde{\lambda} \) is expressed in nanometres.\(^2\) The result is that non-normal plane wave components of the incident field are not purely linearly polarized. If we take a Kerr ellipticity value of \( \chi < 5.7^\circ \) (or \( \tan \chi < 0.1 \)) as the criterion for acceptable deviation from linear polarization, we find an angular tolerance of \( \phi < 2.8 \) mrad, which transforms into \( w > 75 \) \( \mu \)m. For beam-waist radii smaller than this, the polarization state in the far-field diffraction pattern becomes appreciably direction-dependent.

Considering next incident fields with a finite bandwidth, we must first make a difference between stationary fields and pulses. In case of stationary fields, different spectral components are uncorrelated and the dispersion of polarization rotation and ellipticity causes partial depolarization of reflected and transmitted beams, i.e., the degree of polarization is reduced if a wavelength-integrating detector is used. In the case of pulses emitted by lasers, different frequency components in the spectrum are usually fully correlated. Thus the degree of polarization is not reduced even though the different frequency components are still differently polarized. A more detailed and quantitative discussion of these aspects is beyond the scope of this paper and will be provided elsewhere, but it is easy to estimate the maximum spectral

\(^2\) A change of the incident angle changes also all other features of the spectra in a complex manner, but the shift is the one of primary concern in the present context.

Figure 8. Same as figure 6, but for design C in table 2.
bandwidth for which a wavelength-integrating detector will show pure polarization conversion effects: this follows directly from our criterion of polarization purity. In the case considered above, we chose $|\lambda - 632.8 \text{ nm}| < 0.015 \text{ nm}$ to obtain $\tan \chi < 0.1$ (or $\chi < 5.7^\circ$). This condition is easily satisfied, e.g., by single-longitudinal-mode continuous-wave lasers, but implies a minimum duration of $\sim 0.1 \text{ ps}$ for a coherent pulse.

6. Conclusions

We have shown that substantial polarization conversion effects, including $90^\circ$ Kerr rotation, can take place in diffraction of light by magneto-optic guided-mode resonance filters. The effects were explained by the lifting of the degeneracy of the right- and left-circularly polarized modes propagating in the structure due to anisotropy caused by an externally applied static magnetic field, which leads to a splitting of the resonance peak and the associated phase spectrum into two. Large polarization rotation effects are due to the rapid change in the phase spectrum around the original reference peak. Some useful device applications were also pointed out.

Acknowledgments

This work was supported by the Academy of Finland (contract 207523) and the Network of Excellence in Micro-optics (NEMO; http://www.micro-optics.org). Many stimulating discussions with Lifeng Li are greatly appreciated.

References

[1] Inoue M and Fujii T 1997 J. Appl. Phys. 81 5659–61
[2] Inoue M, Arai K, Fujii T and Abe M 1998 J. Appl. Phys. 83 6768–70
[3] Sakaguchi S and Sugimoto N 1999 J. Lightwave Technol. 17 1087–92
[4] Steel M J, Levy M and Osgood R M 2000 IEEE Photon. Technol. Lett. 12 1171–73
[5] Steel M J, Levy M and Osgood R M 2000 J. Lightwave Technol. 18 1297–308
[6] Steel M J, Levy M and Osgood R M 2000 J. Lightwave Technol. 18 1289–96
[7] Jalali A A and Friberg A T 2005 Opt. Lett. 30 1213–15
[8] Jalali A A and Friberg A T 2005 Opt. Commun. 253 145–50
[9] Golubenko G A, Svakhin A S, Tischenko A V and Sychugov V A 1985 Sov. J. Quantum Electron. 15 886–87
[10] Magnusson R and Wang S S 1992 Appl. Phys. Lett. 61 1022–24
[11] Elston S J, Bryan-Brown G P and Sambles J R 1991 Phys. Rev. B 44 6393–400
[12] Bryan-Brown G P, Sambles J R and Hutley M C 1990 J. Mod. Opt. 37 1227–32
[13] Kats A V and Spevak I S 2002 Phys. Rev. B 65 195406
[14] Bristow A D, Astratov V N, Shimada R, Culshaw I S, Skolnick M S, Whittaker D M, Tahraoui A and Krauss T F 2002 IEEE J. Quantum Electron. 38 880–84
[15] Kahl S and Grishin A M 2004 J. Magn. Magn. Mater. 287 244–55
[16] Doormann V, Krumme J-P and Lenz H 1990 J. Appl. Phys. 68 3544–53
[17] Li L 2003 J. Opt. A Pure Appl. Opt. 5 345–55