Alternative to mathematical models of gas production: numerical experiment

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Abstract. Computational experiment for comparison of quasi-one dimensional and two dimensional mathematical statements of the conjugate problem of heat exchange between gas producing well and the rocks has been carried out. The problem consists of solution of differential equations describing non-isothermal gas flow in the well and the heat conduction equation for surrounding rocks. The dependence of the coefficient of heat exchange of the gas with the hydrate layer on the time-varying flow area is taken into account in a quasi-stationary mathematical model of hydrates formation (dissociation) in gas well. The results of computational experiment correspond to two cases of production of wet and dry gas at the Otradninsky gas-condensate field.

1. Introduction

In the extraction and transportation of natural gas in the Northern regions, such natural factors as low climatic temperature and the presence of a thick layers of permafrost largely determine the technological modes of gas production. This is caused by the fact that under certain thermodynamic conditions, natural gas, when combined with water, forms solid crystalline compounds, namely gas hydrates, which can be formed in both bottom-hole zone and the wells. The formation of hydrates in the bottom-hole zone leads to a reduction in well productivity, whereas their formation in the wells can lead to a complete cessation of gas extraction. Such emergency situations can have dire consequences. Currently the only means of combating this unwanted phenomenon is the injection of methanol or other hydrate inhibitors into wells. This measure is ineffective, since the methanol is removed from the wells along with the produced gas, and, in addition, it significantly increases the cost of gas production.

Modern mathematical models of gas production consist of gas dynamic equations (tube hydraulics) as well as heat conduction equation for surrounding rocks. In Arctic regions their upper part is permafrost. So for appropriate description of gas production and control of possible accidents (well head damage because of ground thawing, hydrate plugs) one needs to solve the conjugated Stefan problem. The set of differential equations should be added with corresponding initial and boundary conditions where the most important ones are conditions of 1) heat exchange between rocks close to wellhead and atmospheric air and 2) heat exchange between flowing gas and surrounding rocks. The model should be more complicated in the case of hydrate formation in a well.

Up till now the second of heat exchange conditions was stated using approximation based on the fact that convective heat transfer in gas well has much shorter characteristic time than conductive one in rocks. Then only radial components will remain in the equation of heat conduction for each of well
sections while their connection is implemented through boundary condition at outside surface of a well [1, 2]. The paper is devoted to numerical experiment to test validity of this approximation for real cases of gas production.

2. Problem statement
The authors use quasi-stationary mathematical model of real gas flow in tubes [1, 2] and mathematical model of hydrate formation [1 – 6] based on the generalized Stefan problem where phase transition temperature depends on gas flow pressure. The model consists of two ordinary differential equations.

\[
\frac{dp}{dx} = -\rho_g g \sin \varphi - \frac{\sqrt{\pi} \psi M^2}{4 \rho_S S^2 S_0^{1.5}},
\]

\[
\frac{dT}{dx} - \varepsilon \frac{dp}{dx} = \frac{\pi D \alpha}{c_p M} (T_e - T) - \frac{g}{c_p} \sin \varphi,
\]

where \( c_p \) – specific heat capacity of gas, \( D \) – inner tube diameter, \( g \) – gravity acceleration, \( M = \rho_g \nu S S_0 \) – constant mass flow rate, \( p \) – pressure, \( S \) – dimensionless cross-section, \( S_0 \) – initial cross-section, \( T \) – gas temperature, \( T_e \) – temperature of rocks around well, \( \nu \) – gas flow velocity, \( x \) – distance along tube axe, \( \alpha \) – total heat transfer coefficient, \( \rho_g \) – gas density, \( \varphi \) – angle of tube inclination to horizon, \( \psi \) – hydraulic resistance, \( \varepsilon \) – throttle coefficient.

Gas equation of state is written in the form

\[
\rho_g = \frac{p}{ZRT}, \quad Z = Z(p, T), \quad \varepsilon = \frac{RT^2}{c_p M \left( \frac{\partial Z}{\partial T} \right)_p},
\]

where \( Z \) – real gas compressibility, empirical function of reduced pressure and temperature; \( R \) – gas constant; for real gas compressibility the Latonov-Gurevich formulae \( Z = (0.17376 \ln T + 0.73)^{a} + 0.1p \) is used [7].

Dynamics of well cross-section \( S \) can be found from the dimensionless equation [6]:

\[
\frac{dS}{d\tau} = b_2 \frac{T_e - T_h(p)}{1 - b_2 \ln S} - b_1 \sqrt{S(T_e(p) - T)} ,
\]

where \( b_1 = \alpha_1 D_0 / \lambda_h \), \( b_2 = \alpha_2 D_0 / 4 \lambda_h \), \( \alpha_1 \) – coefficient of heat transfer between gas and hydrate layer; \( \alpha_2 \) – coefficient of heat transfer between hydrate and rocks, \( D_0 \) – initial tube diameter, \( \tau = \frac{\lambda_h T_e}{\rho_h q_h D_0^2} \) – dimensionless time, \( \lambda_h \), \( \rho_h \) – heat conduction and density of hydrate, \( q_h \) – specific latent heat of hydrate formation, \( t \) – time, \( T_h(p) = a \ln p + b \) – equilibrium temperature of hydrate formation. Coefficients \( a \) and \( b \) can be found from a curve built according to the \( D \). Sloan method [8], properties of mixture components being known. Critical temperature and pressure are determined by the W.B. Kay relations [9]: \( p_c = \sum y_i p_{c_i}, \quad T_c = \sum y_i T_{c_i} \); where \( y_i \) – volume content and \( p_{c_i}, T_{c_i} \) – critical pressure and temperature of \( i \)-th component of natural gas. Coefficient \( \alpha_i \) depends on tube cross-section, which in turn is a function of time. In the sections of well tube where hydrate formation (dissociation) takes place, it should be calculated by the following relation:
where \( \lambda_g , \eta_g \) – coefficients of heat conduction and viscosity of gas, \( \text{Pr} \) – the Prandtl number. In so doing rock temperature \( T_e \) should be substituted with equilibrium temperature of hydrate formation \( T_h \).

Initial conditions for the equations (1), (2) and (4) are:

\[
p(0) = p_0 , \quad T(0) = T_0 , \quad S(0) = \text{const}.
\]

Equations (2) and (4) include rock temperature \( T_e \), which are found from the solution of heat conduction equation

\[
\frac{\partial T}{\partial t} = \alpha \left( T_e - T \right), \quad r = r_b .
\]

At conditional radius the heat flow is zero:

\[
\frac{\partial T_e}{\partial r} = 0 , \quad r = r_b .
\]

At well bottom hole rock temperature is constant and equals reservoir temperature \( T_{w0} \):

\[
T_e = T_{w0} , \quad x = 0 .
\]

At daylight surface condition of convective heat transfer with atmosphere includes radiation heat inflow [10, 11]:

\[
\frac{\partial T_e}{\partial x} = \alpha \left( T_e - T_a + \frac{Q_a (1 - A)}{\alpha_a} \right) , \quad x = L .
\]

where \( T_a \) – air temperature, \( Q_a \) – total solar radiation, \( A \) – albedo of the Earth surface; \( \alpha_a = 15.12 \sqrt{u_a} \) – coefficient of heat transfer between soil surface and atmosphere, dependent on wind velocity \( u_a \) [12]; \( \alpha_a = (1/\alpha_a + \delta_m/\lambda_m)^{-1} \) – total heat transfer coefficient taking into account thickness of snow layer \( \delta_m \) with heat conductivity \( \lambda_m = 1.163 \cdot (0.03 + 0.303 \rho_m - 0.177 \rho_m^2 + 2.25 \rho_m^3) \), where \( \rho_m \) – snow density (g/cm\(^3\)) [10]. To describe cyclic variations of \( Q_a , A , \nu_a , \delta_m , \rho_m \), their month averaged values [12] were approximated by linear functions.

Initial rocks temperature:

\[
T_e = \begin{cases} T_{w0} - \Gamma_1 x , & 0 < x < L - H \\ T_{w0} - \Gamma_2 x , & L - H < x < L \end{cases}
\]
where $T_{bo}$ – temperature at bottom hole, $T_{ph}$ = 273.15 K – temperature at lower permafrost boundary, 
$\Gamma_1$ and $\Gamma_2$ – geothermal gradient of thawed and frozen rocks, $L$ – well depth, $H$ – permafrost thickness.

Algorithm of numerical solution of the problem (1) – (12) looks in the following way:
I. Choose geometrical and physical characteristics and state conditions (6) and (12).
II. Initial value of cross-section is given, solve equations (1) – (3) to find pressure $p(x)$ and temperature of gas $T(x)$.
III. Making a time-step and looking at $x$ as a parameter, from equations (4) and (6) find a new value of cross-section.
IV. Solve the problem (7) – (12) to find rocks temperature. Since smoothed coefficients in the equation (7) are temperature dependent, the finite-difference problem will be non-linear, simple iterations method in combination with a sweeping algorithm should be used.
Tasks II – IV are repeated at each time step.

3. Calculation results

Given data correspond to Otradninsky gas-condensate field in Sakha (Yakutia) Republic [13]: $a=6.635\, K$, $b=182951\, K$, $c_p=2300\, J/(kg\cdot K)$, $D_0=0.146\, m$, $H=680\, m$, $L=2480\, m$, $\rho_g=188.35\cdot10^5\, Pa$, $\rho_c=44.71\cdot10^5\, Pa$, $q_u=510000\, J/kg$, $q_{ph}=334400\, J/kg$, $R=438.3\, J/(kg\cdot K)$, $T_0=286.35\, K$, $T_c=195.376\, K$, $T_{bo}=286.48\, K$, $T_{ph}=273.15\, K$; $\alpha=5.82\, W/(m^2\cdot K)$, $\varphi=90^\circ$, $\Gamma_1=0.0074\, K/m$, $\Gamma_2=0.0073\, K/m$, $\eta_g=1.3\cdot10^{-5}\, Pa\cdot s$, $\lambda_g=0.0307\, W/(m\cdot K)$, $\lambda_n=1.88\, W/(m\cdot K)$, $\rho_n=920\, kg/m^3$, $\psi=0.02$.

The formulae for atmospheric temperature:

$$T_a=\frac{T_{sa}-T_{ma}}{2}\cdot\sin\left(\frac{\pi(30(m-1)+t+75)}{180}\right)+\frac{T_{sa}+T_{ma}}{2},$$

where $T_{sa}$, $T_{ma}$ – minimal and maximal air temperature according to [14], $t$ – time, days, $m$ – month number, starting at well start-up.

At first, consider a gas producing well without adding inhibitors which make hydrate formation and even well plugging possible. Results of calculations are shown at Figure 1 – Figure 3.

![Figure 1](image1.png)

**Figure 1.** Temperature ($a$) and pressure ($b$) distribution along a well: 1 – 5 min., 2 – 5 hours, 3 – equilibrium temperature, 4 – temperature at outer well surface.
At Figure 1 curves 1 and 2 for both models are indistinguishable, so they are marked with the same figures. It is worth to notice that hydrate plugs only the upper part of a well, which is entirely in permafrost rocks (compare curves 1 and 2). Another important feature of the process is very short time of complete plugging (about 5 hours). The calculations lead to the following conclusion: if hydrates plug producing well then both models give identical results for all technological parameters (temperature (Figure 1a) and pressure (Figure 1b) distribution along a well, dynamics of free cross-section (Figure 2) and phase transition boundary in rocks (Figure 3). It is due to a very short period of thermal interaction between gas flow and surrounding rocks (see, for example, a very small thawing zone at Figure 3).

**Figure 2.** Dynamics of free cross-section distribution in well: 1 – quasi-one dimensional model, 2 – two dimensional model.

**Figure 3.** Dynamics of phase transition boundary in lower part of permafrost rocks: 1 – quasi-one dimensional model, 2 – two dimensional model.
Second case corresponds to injection of inhibitor into well. The results of calculation can be seen at Figure 4 – Figure 5. Figure 4 shows that both models give equal temperature and pressure distributions along gas well (see curves 1 and 2 at Figure 4a, Figure 4b). However, phase transition boundary value for quasi-one dimensional model exceeds that for two-dimensional model (compare curves 1 and 2 at Figure 5). However, the difference can be seen only near the lower part of permafrost rocks, about 60 m which is shown at Fig. 5. Still, it can be important while one estimates the danger of well head damage due to permafrost thawing.

Figure 4. Temperature (a) and pressure (b) distribution along a well after a year of gas producing:
1 – quasi-one dimensional model, 2 – two dimensional model,
3 – equilibrium temperature, 4 – temperature at outer well surface.

Figure 5. Phase boundary position along permafrost zone:
1 – quasi-one dimensional model, 2 – two dimensional model.
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