Efficient Partial Snapshot Implementations

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Abstract

A snapshot object is a concurrent data structure that has numerous applications in concurrent programming. Snapshots can be used to record the state of the system, so they can provide solutions to problems where an action should be taken when the global state of the system satisfies some conditions. A snapshot object consists of $m$ components, each storing a value from a given set. Processes can read/modify the state of the object by performing $UPDATE$ and $SCAN$ operations. An $UPDATE$ operation gives processes the ability to change the value of a component, while the $SCAN$ operation returns a “consistent” view of all the components. In most literature, two variants (in terms of the number active scanners) of snapshot objects are studied. The first one is the single-scanner snapshot object, where at most one $SCAN$ operation is performed at any given time (whilst supporting many concurrent $UPDATE$ operations). The second one is the multi-scanner snapshot object that can support multi concurrent $SCAN$ operations at any given time.

In this work, we propose the $\lambda$-scanner snapshot, a variation of the snapshot object, which supports any fixed amount of $0 < \lambda \leq n$ different $SCAN$ operations being active at any given time. Whenever $\lambda$ is equal to the number of processes $n$ in the system, the $\lambda$-scanner object implements a multi-scanner object, while in case that $\lambda$ is equal to 1, the $\lambda$-scanner object implements a single-scanner object. We present the $\lambda$-Snap snapshot object, a wait-free $\lambda$-scanner snapshot implementation that has a step complexity of $O(\lambda)$ for $UPDATE$ operations and $O(\lambda m)$ for $SCAN$ operations. The space complexity of $\lambda$-Snap is $O(\lambda m)$. $\lambda$-Snap provides a trade-off between the step/space complexity and the maximum number of $SCAN$ operations that the system can afford to be active on any given point in time. The low space complexity that our implementations provide makes them more appealing in real system applications. Moreover, we provide a slightly modified version of the $\lambda$-Snap implementation, which is called partial $\lambda$-Snap, that is able to support dynamic partial scan operations. In such an object, processes can execute modified $SCAN$ operations called $PARTIAL\_SCAN$ that could obtain a part of the snapshot object avoiding to read the whole set of components.

In this work, we first provide a simple single-scanner version of $\lambda$-Snap, which is called $1$-Snap. We provide $1$-Snap just for presentation purposes, since it is simpler than $\lambda$-Snap. The $UPDATE$ in $1$-Snap has a step complexity of $O(1)$, while the $SCAN$ has a step complexity of $O(m)$. This implementation uses $O(m)$ CAS registers.
1 Introduction

We inarguably live in an era where almost any activity is supported either by smart devices or potent servers, relying on multi-core CPUs. As this new equipment promises to perform more services per time unit, executing increasingly complex jobs, any application that does not use the many cores that are provided by the hardware is gradually becoming obsolete.

At the heart of exploiting the potential that multiple cores provide, are concurrent data structures, since they are essential building blocks of concurrent algorithms. The design of concurrent data structures, such as lists [23], queues [17, 22], stacks [6, 22], and even trees [7, 11] is a thoroughly explored topic. Compared to sequential data structures, the concurrent ones can simultaneously be accessed and/or modified by more than one process. Ideally, we would like to have the best concurrent implementation, in terms of space and step complexity, of any given data structure. However, this cannot always be the case since the design of those data structures is a complex task.

In this work, we present a snapshot object, a concurrent object that consists of components which can be read and modified by any process. Concurrent snapshot objects are used in numerous applications in order to provide a coherent “view” of the memory of a system. They are also used to design and validate various concurrent algorithms such as the construction of concurrent timestamps [13], approximate agreement [5], etc, and the ideas at their core can be further developed in order to implement more complex data structures [2]. Applications of snapshots also appear in sensor networks where snapshot implementations can be used to provide a consistent view of the state of the various sensors of the network. Under certain circumstances, snapshots can even be used to simulate concurrent graphs, as seen e.g. in [20]. The graph data structure is widely used by many applications, such as the representation of transport networks [1], video-game design [8], automated design of digital circuits [19], making the study of snapshot objects pertinent even to these areas.

There are many different implementations of snapshot objects based on the progress guarantee that they provide. However, in order to be fault tolerant against process failure, a concurrent object has to have strong progress guarantees, such as wait-freedom, i.e. the progress guarantee which ensures that an operation invoked by any process that does not fail, returns a result after it executes a finite number of steps. We provide two wait-free algorithms that implement a snapshot object, namely an algorithm for a single-scanner snapshot object, i.e. a snapshot object where only one process is allowed to read the values of the components, although any process may modify the values of components; and an algorithm for a \( \lambda \)-scanner snapshot object, where up to \( \lambda \) predefined processes may read the components of the object, while any process may change the value of any component. Note that \( \lambda \) should be lower than or equal to \( n \), i.e. the number of processes in the system. In case the value of \( \lambda \) is equal to \( n \), we obtain a general multi-scanner snapshot object. Our \( \lambda \)-scanner implementation allows us to study trade-offs, since the increase of the value of \( \lambda \) leads to a linear increase of the space and step complexity. Our algorithms can be modified to obtain partial snapshot implementations (see Sections 3.1 and 4.1), where processes execute modified SCAN operations that can obtain the values of just a subset of the snapshot components.

In terms of shared registers, our algorithm \( \lambda - \text{Snap} \) has a low space complexity of \( O(\lambda m) \), where \( m \) is the number of the components of the snapshot object. This does not come with major compromises in terms of step complexity, since the step complexity of an UPDATE operation is \( O(\lambda) \), while that of a SCAN operation is \( O(\lambda m) \). The registers we use are of unbounded size, although the only unbounded value that they store is a sequence number. This is a common practice from many state-of-the-art implementations [12, 24]. The atomic primitive the registers need to support is CAS (Compare And Swap), although we present a version of the
algorithm using LL/SC registers in order to be more comprehensive and easier to prove correct. An LL/SC register can be constructed by CAS registers using known constructions [18,23].

The rest of this work is organized as follows. Section 1.1 provides a brief comparison of our work with other state-of-the-art algorithms that solve similar problems. Section 2 exposes the theoretical framework we use. Section 3 presents 1 − Snap, our wait-free implementation of a single-scanner snapshot object, and Section 4 presents λ − Snap, our wait-free λ-scanner implementation. Section 5 contains a concluding discussion.

1.1 Related work

Most of current multi-scanner snapshot implementations that use registers of relatively small size either have step complexity that is linear to the number of processes \( n \) [4,14] or the space complexity is linear to the number of \( n \) [3,4,16,20]. The only exception is the multi-scanner snapshot implementation presented by Fatourou and Kallimanis in [10]. However, this snapshot implementation uses unrealistically large registers, since it requires registers that contain a vector of \( m \) values as well as a sequence number. The step complexity of \( \lambda - \text{Snap} \) is \( O(\lambda m) \) for SCAN and \( O(\lambda) \) for UPDATE, while it uses \( O(\lambda m) \) LL/SC registers. In cases where \( \lambda \) is a relatively small constant, the number of registers used can be reduced almost to \( O(m) \), while the step complexity of SCAN is almost linear to \( m \) and the step complexity of UPDATE is almost constant. Compared to current single-scanner snapshot implementations [4,10,12,15,21,24], \( \lambda - \text{Snap} \) offers the capability to have more than one SCAN operation at each point of time by slightly worsening the step complexity. In the worst case where the value of \( \lambda \) is equal to \( n \), \( \lambda - \text{Snap} \) provides an implementation of a multi-scanner snapshot object that uses a smaller amount of registers compared to the implementations in [4,15,16,21,24]. To the best of our knowledge, \( \lambda - \text{Snap} \) provides the first trade-off between the number of active scanners and the step/space complexity.

We now compare \( \lambda - \text{Snap} \) snapshot with other multi-scanner algorithms. In Table 1 we present the basic characteristics of each snapshot implementation that is reviewed in this section. Riany et al. have presented in [23] an implementation of snapshot objects that uses \( O(n^2) \) registers and achieves \( O(n) \) and \( O(1) \) step complexity for SCAN and UPDATE operations respectively. Attiya, Herlihy & Rachman present in [4] a snapshot object that has \( O(n \log^2 n) \) step complexity for both SCAN and UPDATE operations, while it uses dynamic Test&Set registers.

Fatourou and Kallimanis [10] present a multi-scanner implementation with \( O(m) \) for step complexity SCAN operations and \( O(1) \) step complexity for UPDATE operations. In contrast to \( \lambda - \text{Snap} \), this snapshot implementation requires registers that contain a vector of \( m \) values as well as a sequence number. Moreover, the multi-scanner snapshot implementation of [10] does not support partial snapshots.

Kallimanis and Kanellou [20] present a wait-free implementation of a graph object. This implementation can be slightly modified to simulate a snapshot object, which supports partial SCAN operations. This algorithm manages to implement UPDATE and SCAN operations with step complexity of \( O(k) \), where \( k \) is the number of active processes in a given execution. It also maintains a low space complexity of \( O(n + m) \) but the registers used are of unbounded size. In essence, the algorithm needs registers that can contain \( O(n) \) integer values, where half of these values are unbounded.

Imbs and Raynal [14] provide two implementations of a partial snapshot object. The first implementation uses simpler registers than the registers used in the second implementation, but it has a higher space complexity. Thus, we concentrate on the second implementation that achieves a step complexity of \( O(nr) \) for SCAN and \( O(r_in) \) for UPDATE, where \( r_i \) is a value
that is relative to the helping mechanism the UPDATE operations provide. This implementation uses \(O(n)\) Read/Write (abbr. RW) and LL/SC registers. Finally, the implementation of Imbs and Raynal provides a new helping mechanism by implementing the “write first, help later” technique in their work.

Attiya, Guerraoui and Ruppert [3] provide a partial snapshot algorithm that uses \(O(m + n)\) CAS registers. The UPDATE operations of this implementation have a step complexity of \(O(r^2)\). The step complexity of SCAN is \(O(\overline{C}_S^{2}r_{max})\), where \(\overline{C}_S\) is the number of active SCAN operations, whose execution interval overlaps with the execution interval of \(S\), and \(r_{max}\) is the maximum number of components that any SCAN operation may read in any given execution.

We now compare \(\lambda - \text{Snap}\) and \(1 - \text{Snap}\) snapshot with other single-scanner algorithms. Recall that \(\lambda - \text{Snap}\) gives the ability to a snapshot object to have more than one snapshot with other single-scanner algorithms.

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In [10] and [12], Fatourou and Kallimanis provide a single-scanner snapshot implementation, which is called \(T - \text{Op}\), that achieves \(O(1)\) step complexity for UPDATE and \(O(m)\) for SCAN. By applying some trivial modifications to \(T - \text{Op}\), a partial snapshot implementation with \(O(r)\) step complexity for SCAN and \(O(1)\) for UPDATE could be derived. In contrast to \(1 - \text{Snap}\), \(T - \text{Op}\) uses an unbounded number of registers. Moreover, the RT and RT – \text{Op} snapshot implementations presented in [10] [12] do not support partial SCAN operations. In [16], Jayanti presents a single-scanner snapshot algorithm with \(O(1)\) step complexity for UPDATE and \(O(m)\) for SCAN, while it uses \(O(m)\) LL/SC & RW registers. The algorithm of [16] could be easily modified to support partial SCAN operations without having any negative impact on step and space complexity. Therefore, \(1 - \text{Snap}\) and \(\lambda - \text{Snap}\) for \(\lambda = 1\) match the step complexity of implementations presented in [10] [12] [16], which is \(O(m)\) for SCAN and \(O(1)\) for UPDATE. Denote that the single-scanner implementations of [10] [12] use RW registers, while \(1 - \text{Snap}\) and \(\lambda - \text{Snap}\) use LL/SC registers. The partial versions of \(1 - \text{Snap}\) and \(\lambda - \text{Snap}\) for \(\lambda = 1\) have step complexity of SCAN that is reduced to \(O(r)\), where \(r\) is the amount of components the SCAN operation wants to read.

Kirousis et al. [21] provide a single scanner implementation that uses an unbounded number of registers and has unbounded time complexity for SCAN. A register recycling technique is applied to this snapshot implementation resulting a snapshot implementation with \(O(mn)\) step complexity for SCAN and \(O(1)\) for UPDATE. Riany, et al. [24] present an algorithm a single-scanner implementation, which is a simplified variant of the algorithm presented in [21].

| Implementation | Partial | Regs type  | Regs number | SCAN | UPDATE |
|----------------|---------|------------|-------------|------|--------|
| A-Snap         |         | CAS & RW   | \(O(\lambda m)\) | \(O(\lambda m)\) | \(O(\lambda)\) |
| partial \(\lambda\)-Snap | ✓       | CAS & RW   | \(O(\lambda m)\) | \(O(\lambda)\) | \(O(\lambda)\) |
| Attiya, et. al. [1] | ✓       | dynamic Test&Set | unbounded | \(O(n \log^2 n)\) | \(O(n \log^2 n)\) |
| Fatourou & Kallimanis [10] | ✓       | CAS & RW   | \(O(m)\) | \(O(m)\) | \(O(1)\) |
| Jayanti [16] | ✓       | CAS or LL/SC & RW | \(O(mn^2)\) | \(O(m)\) | \(O(m)\) |
| Jayanti [14] | ✓       | CAS or LL/SC & RW | \(O(mn^2)\) | \(O(m)\) | \(O(m)\) |
| Riany et al. [24] | ✓       | CAS or LL/SC & Fetch&Inc & RW | \(O(n^2)\) | \(O(n)\) | \(O(1)\) |
| Kallimanis & Kanellou [20] | ✓       | CAS or LL/SC & RW | \(O(n + m)\) | \(O(k)\) | \(O(k)\) |
| D. Imbs & M. Raynal [14] | ✓       | LL/SC & RW | \(O(n)\) | \(O(nr)\) | \(O(r,n)\) |
| Attiya, Guerraoui & Ruppert [3] | ✓       | CAS & RW   | \(O(n + m)\) | \(O(r^2)\) | \(O((\overline{C}_S)^2 r_{max})\) |

Table 1: Known multi-scanner snapshot implementations
snapshot implementation achieves $O(1)$ step complexity for UPDATE and $O(n)$ for SCAN. By applying some trivial modifications, a partial snapshot implementation could be derived. However, the snapshot implementation of [24] is a single-updater snapshot object, since it does not allow more than one processes to update the same component at each point of time.

In [9, 12], Fatourou and Kallimanis provide the Checkmarking algorithm that achieves $O(m^2)$ step complexity for both SCAN and UPDATE, while it uses $O(m)$ RW registers. This implementation does not support partial SCAN operations.

## 2 Model

We consider a system consisting of $n$ uniquely distinguishable processes modeled as sequential state machines, where processes may fail by crashing. The processes are asynchronous and communicate through shared base objects. A base object stores a value, and it provides a set of primitives, through which the object’s value can be accessed and/or modified.

1. A Read – Write register $R$ (RWregister), is a shared object that stores a value from a set and that supports the primitives: (i) Write($R, v$) that writes the value $v$ in $R$, and returns true, and (ii) Read($R$) that returns the value stored in $R$.

2. An LL/SC register $R$ is a shared object that stores a value from a set and supports the primitives: (i) LL($R$) which returns the value of $R$, and (ii) SC($R, v$) which can be executed by a process $p$ only after the execution of an LL($R$) by the same process. An $SC(R, v)$ writes the value $v$ in $R$ only if the state of $R$ hasn’t changed since $p$ executed the last $LL(R)$, in which case the operation returns true; it returns false otherwise.

3. An LL/SC – Write register $R$ is a shared object that stores a value from a set. It supports the same primitives as an LL/SC register $R$ and in addition, the primitive Write($R, v$) that writes the value $v$ in $R$, and returns true.

A shared object is a data structure that can be accessed and/or modified by processes in the system. Each shared object provides a set of operations. Any process can access and/or modify the shared object by invoking operations that are supported by it. An implementation of a shared object uses base objects to store the state of the shared object and provides a set of algorithms that use the base objects to implement each operation of the shared object. An operation consists of an invocation by some process and terminates by returning a response to the process that invoked it. Similar to each base object, each process also has an internal state. A configuration $C$ of the system is a vector that contains the state of each of the $n$ processes and the value of each of the base objects at some point in time. In an initial configuration, the processes are in an initial state and the base objects hold an initial value. We denote an initial configuration by $C_0$. A step taken by a process consists either of a primitive to some base object or the response to that primitive. Operation invocations and responses are also considered steps. Each step is executed atomically.

| Implementation | Partial | Regs type | Regs number | SCAN | UPDATE |
|----------------|---------|-----------|-------------|------|--------|
| 1 – Snap       | ✓       | LL/SC & SW RW | $O(m)$ | $O(m)$ | $O(1)$ |
| 1 – Snap (partial) | ✓ | LL/SC & SW RW | $O(m)$ | $O(r)$ | $O(1)$ |
| Checkmarking   | ✓       | LL/SC & SW RW | $O(m)$ | $O(m^2)$ | $O(m^2)$ |
| T – Opt (modified) | ✓ | RW | Unbounded | $O(m)$ | $O(1)$ |
| RW            | ✓       | RW | $O(mn)$ | $O(n)$ | $O(1)$ |
| RT            | ✓       | RW | $O(mn)$ | $O(m)$ | $O(1)$ |
| Riany et al. [24] | ✓ | RW | $O(mn)$ | $O(mn)$ | $O(1)$ |
| Jayanti [16]  | ✓       | LL/SC & RW | $O(m)$ | $O(m)$ | $O(1)$ |

Table 2: Known single-scanner snapshot implementations
An execution $a$ is a (possibly infinite) sequence $C_o, e_1, C_1, e_2, C_2, \ldots$, alternating between configurations and steps, starting from some initial configuration $C_o$, where each $C_k$, $k > 0$, results from applying step $e_k$ to configuration $C_{k-1}$. If $C$ is a configuration that is present in $a$ we write $C \in a$. An execution interval of a given execution $a$ is a subsequence of $a$ which starts with some configuration $C_k$ and ends with some configuration $C_l$ (where $0 \leq k < l$). An execution interval of an operation $op$ is an execution interval with its first configuration being the one right after the step where $op$ was invoked and last being the one right after the step where $op$ responded.

Given an execution $a$, we say that a configuration $C_k$ precedes $C_l$ if $k < l$. Similarly, we say that step $e_k$ precedes step $e_l$ if $k < l$. We say that a configuration $C_k$ precedes the step $e_l$ in $a$, if $k < l$. On the other hand, we say that the step $e_l$ precedes $C_k$ in $a$ if $l \leq k$. We furthermore say that $op$ precedes $op'$ if the step where $op$ responds precedes the step where $op'$ is invoked. Given two execution intervals $I, I'$ of $a$, we say that $I$ precedes $I'$ if any configuration $C$ contained in $I$ precedes any configuration $C'$ contained in $I'$.

An operation $op$ is called concurrent with an operation $op'$ in execution $a$ if there is at least one configuration $C \in a$, such that both $op$ and $op'$ are active in $C$. An execution $a$ is called sequential if in any given $C \in a$ there is at most one active $op$. An execution $a$ that is not sequential is called concurrent. Executions $a$ and $a'$ are equivalent if they contain the same operations and only those operations are invoked in both of them by the same process, which in turn have the same responses in $a$ and $a'$.

An execution $a$ is linearizable if it is possible to assign a linearization point, inside the execution interval of each operation $op$ in $a$, so that the response of $op$ in $a$ is the same as its response would be in the equivalent sequential execution that would result from performing the operations in $a$ sequentially, following the order of their linearization points. An implementation of a shared object is linearizable if all executions it produces are linearizable. An implementation $IM$ of a shared object $O$ is wait–free if any operation $op$, of a process that does not crash in $a$, responds after a finite amount of steps. The maximum number of those steps is called step complexity of $op$.

A snapshot $S$ is a shared object that consists of $m$ components, each taking values from a set, that provides the following two primitives: (i) $SCAN(i)$ which returns a vector of size $m$, containing the values of $m$ components of the object, and (ii) $UPDATE(i, v)$ which writes the non NULL value $v$ on the $i$–th component of the object. A partial snapshot $S$ is a shared object that consists of $m$ distinct components denoted by $c_0, c_1, \ldots, c_{m-1}$, each taking values from a set, that provides the following two primitives: (i) $PARTIAL\_SCAN(A)$ which, given a set $A$ that contains integer values ranging from 0 to $m-1$, returns for each $i \in A$ the value of the component $c_i$, and (ii) $UPDATE(i, v)$ which writes the non NULL value $v$ on $c_i$. A snapshot implementation is single–scanner if in any execution $a$ produced by the implementation there is no $C \in a$ in which there are more than one active $SCAN$ operations. Similarly, a snapshot implementation is $\lambda$–scanner if in any execution $a$ produced by the implementation there is no $C \in a$ in which there are more than $\lambda$ active $SCAN$ operations.

### 3 1-Snap

In this section, we present the 1–Snap snapshot object (see Listings [13]).

In 1–Snap, only a single, predefined process is allowed to invoke $SCAN$ operations, while all processes can invoke $UPDATE$ operations on any component of the snapshot object. 1–Snap uses shared integer variable $seq$, with initial value 0, in order to provide sequence numbers to operations. Each applied operation gets a sequence number by reading the value of $seq$. An operation $op$ that is applied with a smaller sequence number than that of another operation $op'$
Listing 1: Data Structures of 1-Snap.

```c
struct valuestruct {
    val value;
    int seq;
    val proposed value;
};

struct pre valuestruct {
    val value;
    int seq;
};

shared int seq;
shared value struct values [0...m-1]= [NULL,NULL,NULL,...,<NULL,NULL,NULL>];
shared pre value struct pre values [0...m-1]= [NULL,NULL,...,<NULL,NULL>];
private int view [0...m-1]= [NULL,NULL,...,NULL,NULL];
```

is considered to be applied before op'. Since only SCAN operations can increase the value of seq by one and since in any given configuration there is only one active SCAN operation in our implementation, the seq register can safely be a RW register.

1 − Snap uses shared vector values, consisting of m structs, to represent the components of the snapshot object. Each struct of values is stored in an LL/SC register and any process can execute LL and SC operations on each of them. The i − th component of the snapshot object is stored in the i − th struct of the values data structure, this struct is denoted values [i] and its type is valuestruct. Each of those structs contains the following three fields: (1) a val variable called value which stores the value of the i − th component of the snapshot object that is simulated by 1 − Snap, (2) an integer variable called seq, which stores the sequence number of the last UPDATE operation that has been applied to the i − th component of the snapshot. This is also referred to as the sequence number of the i − th component, and (3) a val variable called proposed value which stores the value that the announced UPDATE operation wants to apply on the i − th component of the snapshot.

This means that each component of the snapshot object can store two values, namely its current value and the proposed value. The process that executes the SCAN operations uses a unique vector pre values, which consists of m structs that are stored in an LL/SC register and any process can execute LL and SC operations on them. The i − th struct of pre values, pre values[i], contains a previous value of the i − th component and a sequence number of the component of the snapshot object. This sequence number is always smaller than that of the SCAN executed by process p. Since we apply a helping mechanism, any UPDATE and SCAN operation can read and modify the components of this data structure regardless of their process id.

A SCAN operation increases the value of seq by one and uses this increased value as its sequence number (line 54). UPDATE operations that have been applied with a greater or equal sequence number than that of this SCAN, are not “visible” by it (recall that operations are considered to be applied in increasing order of their assigned sequence number). Afterwards, for each component of the snapshot object (lines 55 14), the SCAN performs the following steps: (1) It tries to copy the value of this component to pre values data structure if the sequence number of the component is lower than the sequence number of the corresponding SCAN (lines 53 60). (2) It tries to apply an announced UPDATE to this component of the snapshot object (lines 61 66). (3) Finally, SCAN returns its copy of the snapshot object (line 45).

An UPDATE operation U on the j − th component executed by process p first tries to announce the new value that it wants to store on the j − th component of the snapshot. This is
Listing 2: UPDATE and SCAN implementations of 1-Snap.

```c
14  void UPDATE(int j, value val)
15  {
16      int i;
17      struct value struct up_val, cur_val;
18      for (i=0; i<2; i++)
19          cur_val=LL(values[j]);
20          up_val=cur_val;
21          if (cur_val.proposed_val==value)
22              ApplyUpdate(j);
23          if (SC(values[j], up_val))
24            break;
25          ApplyUpdate(j);
26      }
27  }
28
29  pointer SCAN()
30  {
31      int j;
32      struct value struct v1;
33      struct prev_val struct v2;
34      seq=seq+1;
35      for (j=0; j<m; j++)
36          ApplyUpdate(j);
37      v1=values[j];
38      v2=prev_values[j];
39      if (v1.seq<seq)
40          view[j]=v1.value;
41      else{
42          view[j]=v2.value;
43      }
44      return view[0..m-1];
45  }
```

achieved by trying to write on the proposed_val field of the j – th component (lines 18 – 22). Afterwards, U tries to copy the value of the j – th component of the snapshot to prev_values data structure if needed (lines 52 - 60). Then it tries to update the value of the j – th component of the snapshot using a local copy of seq as its sequence number (lines 61 - 67). If the announcement was successful, then the UPDATE operation ends its execution after the aforementioned last step. Otherwise, it repeats all previous steps for one last time. Doing so will make sure that an UPDATE operation (may or may not be the same as U) on the j – th component of the snapshot object is applied and furthermore linearized inside the execution interval of U.

3.1 A partial version of 1-Snap

The 1 – snap snapshot implementation can be trivially modified in order to implement a partial snapshot object (see Listing 4). In order to do that, a new function Read is introduced. This function is invoked by PARTIAL_SCAN operations in order to read the values of the components indicated by A, which a subset of the components of the snapshot object. For each component c_j that is contained in A, the PARTIAL_SCAN operation tries to help an UPDATE operation that wants to update the value of c_j by invoking the ApplyUpdate. Afterwards, it reads the value of c_j by invoking the Read function.
Listing 3: ApplyUpdate implementation of 1-Snap.

```c
void ApplyUpdate(int j) {
    struct value struct cur_value;
    struct pre_value struct cur_pre_value, proposed_pre_value;
    cur_value = LL(values[j]);
    cur_seq = seq;
    for (t = 0; t < 2; t++) {
        cur_pre_value = LL(pre_values[j]);
        cur_value = values[j];
        if (cur_value.seq < seq) {
            proposed_pre_value.seq = cur_value.seq;
            proposed_pre_value.value = cur_value.value;
            SC(pre_values[j], proposed_pre_value);
        }
    }
    if (cur_value.proposed_value != NULL) {
        cur_value.value = cur_value.proposed_value;
        cur_value.seq = cur_seq;
        cur_value.proposed_value = NULL;
        SC(values[j], cur_value);
    }
}
```

Listing 4: Partial version of 1-Snap.

```c
void PARTIAL_SCAN(A) {
    seq = seq + 1;
    for each j in A{
        ApplyUpdate(j);
        Read(j);
    }
    for Read(j){
        struct value struct v1;
        struct pre_value struct v2;
        v1 = values[j];
        v2 = pre_values[j];
        if (v1.seq < seq) {
            view[j] = v1.value;
        } else {
            view[j] = v2.value;
        }
    }
    return view[j];
}
```

3.2 Step and space complexity of 1-Snap

The step complexity of any operation of 1 − Snap is measured by the number of accesses that are executed in shared registers, inside its execution interval.

We start with the worst-case analysis of ApplyUpdate.

1. In lines 48-51 only an LL operation is performed at line 50 and a read of shared variable seq (line 51).
2. Lines 52-60 contain a loop that is executed at maximum two times. In each iteration of this loop, there are executed at maximum two LL/SC operations (the LL of line 53 and the SC of line 58) and one read of line 54.
3. Lines 61-66 contain just a single SC operation (line 65).

Thus, ApplyUpdate executes $O(1)$ shared memory accesses.
We now proceed with the worst-case analysis of the step complexity of any \textit{UPDATE}. The loop of lines 17-28 can be executed two times at maximum and contains an \textit{LL} (line 18), an \textit{SC} (line 22) and two invocations of \textit{ApplyUpdate} (lines 23 and 27). We previously proved that any \textit{ApplyUpdate} executes $O(1)$ shared memory accesses. It follows that any \textit{UPDATE} operation executes $O(1)$ shared memory accesses.

Finally, the worst-case analysis of the step complexity of any \textit{SCAN} is as follows.

1. A write operation on the shared value $\text{seq}$ is executed on line 34.
2. Lines 35-44 contain a loop that is executed exactly $m$ times. In each iteration of the loop an invocation of \textit{ApplyUpdate} is executed (line 36) and two read operations (lines 37 and 38) are performed.

It follows that any \textit{SCAN} operation executes $O(m)$ shared memory accesses.

Both partial $1 - \text{Snap}$ and non-partial $1 - \text{Snap}$ provide the same step complexity to \textit{UPDATE} operations of $O(1)$ and have the same space complexity of $O(m)$. However, partial $1 - \text{Snap}$ provides a step complexity of $O(r)$ to \textit{SCAN} operations, where $r$ is the number of elements contained in $A$. In contrast, the step complexity that non-partial $1 - \text{Snap}$ provides to \textit{SCAN} operation is $O(m)$, higher than that of the partial version, since $r \leq m$.

The space complexity of $1 - \text{Snap}$ algorithm is measured through counting the number of shared registers that are needed for its implementation. The implementation of $1 - \text{Snap}$ deploys three different shared objects:

1. A shared integer variable called $\text{seq}$ which is stored in a multi-read/write register.
2. A shared table called $\text{values}$ that is consisted of $m$ \textit{LL/SC} registers.
3. A shared table called $\text{pre\_values}$ that is consisted of $m$ \textit{LL/SC} registers.

Thus, our implementation deploys $2m$ \textit{LL/SC} unbounded registers and 1 \textit{RW} register. It follows that the space complexity of our algorithm is $O(m)$.

The implementation of $1 - \text{Snap}$ presented in this work uses \textit{LL/SC} registers of unbounded size (one sequence number and two integer values). Although registers should be unbounded it can be proven that they need to have a size of $O(\log(s))$, where $s$ is the maximum number of \textit{SCANS} in a given execution. Thus, in executions that the maximum number of \textit{SCAN} operation is not too big, $1 - \text{Snap}$ may use bounded registers.

\textbf{Theorem 3.1.} $1 - \text{Snap}$ is a wait-free linearizable concurrent single-scanner snapshot implementation that uses $O(m)$ registers, and it provides $O(1)$ step complexity to \textit{UPDATE} operations and $O(m)$ to \textit{SCAN} operations.

4 \textbf{$\lambda$-Snap}

In this section, we present the $\lambda - \text{Snap}$ snapshot object (see Listings 5-7).

In $\lambda - \text{Snap}$, only a predefined set of $1 \leq \lambda \leq n$ processes are allowed to invoke \textit{SCAN} operations, while all processes can perform \textit{UPDATE} operations on any component. Each applied operation gets a sequence number by reading the shared register $\text{seq}$. Sequence numbers assigned both to \textit{SCAN} and \textit{UPDATE} operations. More specifically, \textit{SCAN} operations get a sequence number during the beginning of their execution, while \textit{UPDATE} operations get an actual sequence number at the point they successfully update the component with their value. We often refer to that as the sequence number of the operation. A role of the sequence number is that an operation $\text{op}$ with a smaller sequence number than that of another operation $\text{op}'$ is considered to be applied before $\text{op}'$. Also, a sequence number predetermines which \textit{UPDATE} operations are visible to a \textit{SCAN} operation. More specifically, \textit{UPDATE} operations that have been applied with a greater or equal sequence number than that of the sequence number of a \textit{SCAN} operation, are not visible from this \textit{SCAN}.
For assigning sequence numbers to SCAN and UPDATE operations, λ−Snap employs a shared LL/SC register seq (line 10), which takes integer values. Only SCAN operations are able to increase the value of seq by one (lines 36–46). In contrast to 1−Snap, SCAN operations in λ−Snap get sequence numbers in more complex way (lines 35-47). More specifically, SCAN operations use a consensus-like protocol in order to increase the seq (using LL/SC instructions) and get a new sequence number. In contrast to 1−Snap, more than one SCAN operations may get the same sequence number. However, for all SCAN operations that get the same sequence number, the following hold: (1) they are performed by different processes, (2) the increment of the seq register using LL/SC instructions takes place inside their execution interval, and (3) all these SCAN operations are eventually linearized at the same point of the increment of register seq. Note that an UPDATE operation U, which has been applied with a sequence number greater or equal to the sequence number of some SCAN S operation, is not visible to S. Since U is not visible to S, U is linearized after S. In order to ensure that both LL/SC instructions take place in the execution interval of a SCAN operation and try helping themselves and other SCAN operations, the consensus-like protocol is executed 3 times (lines 36–46).

Each process p that is able to execute SCAN operations, owns a shared array of m registers, which is called pre_values (line 16). This array of registers stores a previous value and the sequence number for each component that wants to read. As a first step, each SCAN operation tries to increase the value of seq by executing the consensus-like protocol of lines 36 – 46. Afterwards, for each component of the snapshot object a SCAN operation does the following steps: (1) it tries to copy the value of this component to every pre_values[p] data structure that is used by SCAN operations (lines 66 – 73 of ApplyUpdate), (2) if the sequence number of the component is lower than that of the sequence number of the corresponding SCAN (line 70), it tries to apply an announced UPDATE to this component of the snapshot object (line 73), and (3) it returns its copy of the snapshot object (line 59).

We now concentrate on describing UPDATE operations. Each component of the snapshot object stores two values. The first one is the current value of the component (i.e. the value field of value_struct at line 2) and the second one is the proposed value (i.e. the proposed_value

Listing 5: Data structures of λ-Snap.

```c
struct value Struct {
  val value;
  val proposed_value;
  int seq;
};

struct pre_value Struct {
  val value;
  int seq;
};

struct scan_struct {
  int seq;
  boolean write_enable;
};

shared int seq;
shared value_struct values [0...m-1]=[<NULL, NULL, NULL>, ..., <NULL, NULL, NULL>];
shared pre_value_struct pre_values [0...λ-1][0...m-1]=[<NULL, NULL>, ..., <NULL, NULL>];
shared scan_struct s_table [0...λ-1]=[<NULL, 0>, <NULL, 0>, ..., <NULL, 0>];
private int view [0...m-1]=[NULL, NULL, ..., NULL, NULL];
```
Listing 6: \textit{UPDATE} and \textit{SCAN} implementations of \(\lambda\)-Snap.

```c
19 void UPDATE(int j, value value){
20 struct value struct up_value, cur_value;
21 for (i=0; i<2; i++){
22 cur_value=LL(values[j]);
23 up_value=cur_value;
24 if (cur_value.proposed_value==NULL){
25 if (SC(values[j], up_value)){
26 ApplyUpdate(j);
27 break;
28 }
29 ApplyUpdate(j);
30 }
31 }
32
33 pointer SCAN(){
34 s_table[p_id]={1,seq};
35 for (i=0;i<3;i++){
36 cur_seq=LL(seq);
37 for (j=0;j<A;j++){
38 cur=s_table=LL(s_table[j]);
39 if (cur.s_table.seq<seq+2 && cur.s_table.write_enable==1){
40 cur.s_table.seq=seq+2;
41 SC(s_table[j],cur.s_table);
42 }
43 }
44 SC(seq,cur_seq+1);
45 }
46 for (j=0;j<m;j++){
47 ApplyUpdate(j);
48 vl=values[j];
49 v2=pre_values[p_id][j];
50 if (vl.seq<s_table[p_id].seq){
51 view[j]=vl.value;
52 } else {
53 view[j]=v2.value;
54 }
55 }
56 return view[0..m-1];
57 }
```

field of \textit{value} \textit{struct}, simpler said this is the value that an \textit{UPDATE} currently wants to write on the component. An \textit{UPDATE} operation \textit{U} on \textit{j}-th component executed by some process \textit{p}, it first tries to propose the new value that it wants to store on the \textit{j}-th component of the snapshot. This is achieved by trying to write on the \textit{proposed\_value} of the \textit{j}-th component of the snapshot object (lines 22 – 26). Afterwards, it tries to copy the current value of the \textit{j}-th component of the snapshot to every \textit{pre\_values}[\textit{p}] register (one for each scanner) if needed (lines 66 – 76). Then it tries to \textit{UPDATE} the value of the \textit{j}-th component of the snapshot using a local copy of \textit{seq} as its sequence number (line 71). If the proposal of the new value was successful, then the \textit{UPDATE} operation ends its execution (line 26). Otherwise, it repeats all previous steps for one last time. Doing so will make sure that an \textit{UPDATE} operation (may or may not be the same as \textit{U}) on the \textit{j}-th component of the snapshot object is applied and furthermore linearized inside the execution interval of \textit{U}. By writing a sequence number with its value, an \textit{UPDATE} operation \textit{U} that has been applied with a sequence number less or equal to the sequence number of some \textit{SCAN S} operation is visible to \textit{S}.

In \(\lambda\)-\textit{Snap}, we employ a helping mechanism where \textit{UPDATE} and \textit{SCAN} operations
try to help UPDATE operations that are slow or stalled (lines 77 – 82). More specifically, an UPDATE operation on some component $j$ helps at most 2 UPDATE operations on the $j$-th component (see lines 77 – 82). On the other hand, a SCAN operation helps at most 2 UPDATE operations per component that it reads. Thus, the non partial version of $\lambda$-Snap helps at most 2$m$ UPDATE operations (in the case of the partial version of $\lambda$-Snap, a SCAN operation helps at most $\lambda r$ UPDATE operations, where $r$ is the number of components that wants to read).

### 4.1 A partial version of $\lambda$-Snap

We now present a slightly modified version of $\lambda$-Snap (see Listing 8) that implements a partial snapshot object. The data structures used in this modified version of $\lambda$-Snap remain exactly the same, as shown in Listing 5. Furthermore, the pseudocode of UPDATE and ApplyUpdate function remain the same as shown in Listings 6 and 7. A new function is introduced called Read (Listing 8). This function is invoked by PARTIALSCAN operations in order to read the values of the snapshot object.

The only modification in this version of $\lambda$-Snap is that the PARTIAL_SCAN operations do not read every component of the snapshot object, they only read the components of set $A$. For each component $j$ that is contained in $A$ (the set of components that a SCAN wants to read), the PARTIAL_SCAN operation tries to help UPDATE operations on the $j$-th component by invoking the ApplyUpdate function (lines 15 – 18). Afterwards, it reads the value of the $j$-th component by invoking the Read function.

Both partial $\lambda$-Snap and non-partial $\lambda$-Snap have the same step complexity of UPDATE operations, and the same space complexity. Although, $\lambda$-Snap provides a step complexity to SCAN operations of $O(\lambda r)$ where, $r$ is the number of components that the PARTIAL_SCAN operation reads.
4.2 Step and space complexity of λ-Snap

The step complexity of an operation of λ – Snap is measured by the number of operations that are executed in shared registers, inside its execution interval.

We start with the worst-case analysis of ApplyUpdate.

1. In lines 62-65 only an LL operation is performed at line 68 and a read of shared variable seq (line 69).

2. In lines 66-76 contain a loop that is executed exactly λ times. In any iteration of this loop the loop of lines 67-75 is executed exactly two times. In any iteration of the later loop, four shared register operations are executed at maximum. An LL at line 68, two read operations (line 69 and 70) and an SC operation at line 73. Thus, the loop of lines 67-75 executes at maximum eight shared register operations. Furthermore, the loop of lines 67-75 is a nested loop of that of lines 66-76, so it is executed exactly λ times. It follows that the loop of lines 66-76 executes at maximum \(8\lambda\) shared register operations.

3. Lines 77-82 contain just a single SC operation (line 81).

It follows that ApplyUpdate executes at maximum \(3 + 6\lambda\) shared memory accesses. Thus, ApplyUpdate has a step complexity of \(O(\lambda)\).

We now proceed with the worst-case analysis of the step complexity of any UPDATE. The loop of lines 21-32 can be executed two times at maximum and contains an LL (line 22), an SC (line 26) and two invocations of ApplyUpdate (lines 27 and 31). We previously proved that any

Listing 8: UPDATE and SCAN implementations for the partial version of λ-Snap.

```c
1  #include <stdio.h>
2  #include <stdlib.h>
3  #include <string.h>
4  #include <pthread.h>
5
6  #define MAX_SEQ 10
7
8  struct value_struct {
9      int value;
10     int seq;
11  }
12
13  struct p_value_struct {
14      int value;
15     int seq;
16  }
17
18  #define MAX_TABLE 10
19
20  enum Write_enable {
21      OFF, ON
22  }
23
24  int main() {
25      int i;
26      int j;
27
28      p_value_struct values[MAX_TABLE];
29      p_value_struct pre_values[MAX_TABLE];
30
31      for (i=0; i<MAX_TABLE; i++) {
32          values[i].seq = i;
33          values[i].value = i;
34          pre_values[i].seq = i;
35          pre_values[i].value = 0;
36      }
37
38      pthread_t thread1, thread2;
39
40      pthread_create(&thread1, NULL, update, NULL);
41      pthread_create(&thread2, NULL, update, NULL);
42
43      pthread_join(thread1, NULL);
44      pthread_join(thread2, NULL);
45
46      return 0;
47  }
```
ApplyUpdate executes $O(\lambda)$ shared memory accesses. It follows that any UPDATE operation executes $O(\lambda)$ shared memory accesses.

We can finally proceed with the worst-case analysis of step complexity of any SCAN.

1. A write operation on the shared table $s\table$ is executed in line 35.

2. Lines 36-47 contain a loop that is executed exactly three times. In each iteration of that loop, an LL is executed at line 37 and an SC at line 46. Furthermore, the loop of lines 38-45 is executed, and exactly $\lambda$ iterations of it are performed. In any iteration of loop of lines 38-45 at maximum three shared memory accesses are performed (an LL at line 39, a read of the shared seq variable at line 40 and an SC at line 43). It follows that the loop of lines 38-45 executes $O(\lambda)$ shared memory accesses. Since the loop of lines 36-47 is executed exactly three times it executes $O(\lambda)$ shared memory accesses.

3. Lines 49-58 contain a loop that is executed exactly $m$ times. In each iteration of that loop an ApplyUpdate is invoked (line 50) and two read operations are performed (lines 51, 52). Since ApplyUpdate executes $O(\lambda)$ shared memory accesses and at lines 49-58 is invoked exactly $m$ times it follows that lines 49-58 execute $O(\lambda m)$ shared memory accesses.

It follows that any SCAN operation executes $O(\lambda m)$ shared memory accesses.

The space complexity of $\lambda - Snap$ algorithm is measured through counting the number of shared registers that are needed for its implementation. The implementation of $\lambda - Snap$ deploys four different shared objects:

1. A shared LL/SC register called seq.

2. A shared array called values that is consisted of $m$ LL/SC registers.

3. A shared array called pre-values that is consisted of $\lambda m$ LL/SC registers.

4. A shared array called s\table that is consisted of $\lambda$ LL/SC write registers.

Thus, our implementation deploys $1 + m + \lambda m + \lambda$ LL/SC write registers. It follows that the space complexity of our algorithm is $O(\lambda m)$.

**Theorem 4.1.** $\lambda - Snap$ is a wait-free linearizable concurrent $\lambda$-scanner snapshot implementation that uses $O(\lambda m)$ registers, and it provides $O(\lambda)$ step complexity to UPDATE operations and $O(\lambda m)$ to SCAN operations.

## 5 Discussion

This work proposes the $\lambda - Snap$ snapshot object and its implementations, providing a solution to the single-scanner snapshot problem and the multi-scanner snapshot problem simultaneously. If $\lambda$ is equal to 1, then our algorithm simulates a single-scanner snapshot object, while if $\lambda$ is equal to the maximum number of processes, then it simulates a multi-scanner snapshot object. To the best of our knowledge, there is no publication that provides a solution to the snapshot problem that can support a preset amount of SCAN operation that may run concurrently.

$1 - Snap$ solves the single-scanner flavor of snapshot problem. Although, in our algorithm, we only allow one process with a certain id to invoke SCAN operations, this is a restriction that can be easily lifted. The system can support invocations of SCAN operations by any process, although only one process can be active in any given configuration of the execution. In this case, our algorithm would be correct only in executions that no more than one SCAN is active in any given configuration of the execution.
A λ—Snap snapshot can efficiently be applied in systems where only a preset amount of processes may want to execute SCAN operations. Especially in systems that the amount of processes that may want to invoke a SCAN operation is small enough, our algorithm has almost the same performance as a single-scanner snapshot object. An example of such a system may be a sensor network, where many sensors are communicating with a small amount of monitor devices. In this case, sensors essentially perform UPDATE operations while monitor devices may invoke SCAN operations.

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