Ultra-low-frequency gravitational waves from individual supermassive black hole binaries as standard sirens

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Ultra-low-frequency gravitational waves (GWs) generated by individual inspiraling supermassive black hole binaries (SMBHBs) in the centers of galaxies may be detected by pulsar timing arrays (PTAs) in the future. These GW signals encoding absolute cosmic distances can serve as bright and dark sirens, having potential to be developed into a precise cosmological probe. Here we show that an SKA-era PTA consisting of 100 millisecond pulsars may observe about 25 bright sirens and 41 dark sirens during a 10-year observation. The bright sirens, together with the CMB data, have comparable capabilities to current mainstream data for measuring the equation of state of dark energy. The dark sirens could make the measurement precision of the Hubble constant close to that of current distance-ladder observation. Our results indicate that ultra-low-frequency GWs from individual SMBHBs are of great significance in exploring the nature of dark energy and measuring the Hubble constant.
Gravitational waves (GWs) are ripples in the fabric of spacetime, produced when large masses accelerate. The detection of GW150914, the first GW event of binary black hole coalescence, has marked the beginning of the era of GW astronomy. The luminosity distances of GW sources, encoded in the amplitudes of GW waveforms, can be inferred from GW measurements, usually referred to as “standard sirens”. The standard sirens with electromagnetic (EM) counterparts can be used as “bright sirens” to directly constrain cosmological parameters via the distance-redshift relation. For the standard sirens without EM counterparts, one can use GW signals to find their potential host galaxies in galaxy catalogs. A statistical analysis of these galaxies’ redshifts together with the GW signals can also provide constraints on cosmological parameters and such GW data are usually called “dark sirens”.

Typical sources of standard sirens are compact binary coalescences, including stellar-mass compact binaries and supermassive black hole binaries (SMBHBs). Stellar-mass compact binaries, such as binary neutron stars (BNSs) and stellar-mass binary black holes (SBBHs), can be detected by ground-based GW detectors in the frequency band between $O(10) - O(10^3)$ Hz. BNS coalescences are expected to have EM counterparts and have been experimentally confirmed by the GW170817 event that is the only available bright siren till now, providing a $\sim 14\%$ measurement for the Hubble constant $H_0$. SBBH coalescences are commonly thought to have no EM counterparts but they can serve as dark sirens. 47 such GW sources from the Third LIGO-Virgo-KAGRA Gravitational-Wave Transient Catalog provide a $\sim 19\%$ measurement of the Hubble constant with the dark siren method. In the future, the third-generation ground-based GW detectors (the Einstein Telescope and the Cosmic Explorer) enable ones to acquire numerous available standard
sirens of stellar-mass compact binaries\textsuperscript{[11].}

Low-frequency GWs emitted by SMBHBs with masses of $10^4 \text{–} 10^8 M_\odot$ can be detected in the mHz frequency band by the planned space-borne GW observatories, e.g., the Laser Interferometer Space Antenna\textsuperscript{12}, Taiji\textsuperscript{13}, and TianQin\textsuperscript{14}. These SMBHBs may produce EM emissions due to their surrounding gas-rich environments and external magnetic fields\textsuperscript{15,16}, and therefore they are also expected to serve as bright sirens\textsuperscript{17–21}. Recent studies show that such SMBHBs can also serve as dark sirens and provide precise measurements for $H_0$\textsuperscript{22,23}.

Ultra-low-frequency GWs emitted by SMBHBs with masses of $10^8 \text{–} 10^{10} M_\odot$ are expected to be detected in the nHz frequency band by the natural galactic-scale detector comprised of an array of millisecond pulsars (MSPs), usually referred to as “pulsar timing array” (PTA). When GWs pass between pulsars and the Earth, the paths of the pulsar signals change, thus affecting the times of arrival (ToAs) of radio pulses. Nanohertz GWs from individual inspiraling SMBHBs could be detected by monitoring the spatially correlated fluctuations of ToAs induced by GWs. With the concept proposed decades ago, there are three major PTA projects, namely, the Parkes Pulsar Timing Array\textsuperscript{24}, the European Pulsar Timing Array\textsuperscript{25}, and the North American Nanohertz Observatory for Gravitational Waves\textsuperscript{26}. They have also been combined to form the International Pulsar Timing Array\textsuperscript{27} aimed at significantly enhancing sensitivities. So far, most of the efforts have been devoted to detecting the stochastic gravitational wave background (SGWB)\textsuperscript{28,30}. Although challenging, the detections of individual SMBHBs will have immense scientific return. The capability of detecting individual SMBHBs using PTAs has been investigated in Refs.\textsuperscript{31–33}. With the participation of more
advanced radio telescopes such as the Five-hundred-meter Aperture Spherical Telescope (FAST) in China and the planned Square Kilometre Array (SKA), there is a great possibility that GWs produced by individual SMBHBs (other than SGWBs) could be detected by SKA-era PTAs.

Recently, it was proposed in Ref. that inspiraling SMBHBs to be detected by PTAs may also be used as bright sirens. The luminosity distances of currently available SMBHB candidates detected by EM observations with known redshifts may be measured by the PTA GW observations, then the distance-redshift relation can be used to constrain cosmological parameters. In Ref., a preliminary study on constraining dark energy parameters was performed, in which only the equation-of-state (EoS) parameters of dark energy are set free but other cosmological parameters are all fixed. Obviously, such a treatment cannot reveal how well the PTA nanohertz GW observations could constrain cosmological parameters. Actually, the most prominent advantage of GW bright sirens in cosmological parameter estimations is that they can break the degeneracies between cosmological parameters. The capabilities of the bright sirens from ground-based detectors and space-borne observatories of breaking the parameter degeneracies have been widely discussed (see Ref. for a recent review), but the relevant studies on the standard sirens from PTA observations are still absent. Here the first question to be answered is what role the ultra-low-frequency GW bright sirens can play in breaking the degeneracies between cosmological parameters.

Although the SMBHB bright sirens from the PTA observations are thought to be useful in measuring cosmological parameters, they also have limitations, because the SMBHB candidates
with known redshifts may not really be SMBHBs and the actual detected SMBHBs may not be the members of these candidates. Therefore, it is important to find a way to measure cosmological parameters when the SMBHB bright sirens are not available. We propose that SMBHBs detected by PTAs may serve as dark sirens. Dark sirens require suitable galaxy catalogs to provide potential host galaxies of SMBHBs. Since the redshifts of the existing galaxy catalogs are relatively low, only SMBHBs in the local Universe might be used as dark sirens. Along this line, the second question we wish to answer is whether SMBHBs in the local Universe can be used as ultra-low-frequency GW dark sirens to precisely measure cosmological parameters.

In this work, we analyze the ability of SKA-era PTAs to detect the existing SMBHB candidates and the local-Universe SMBHBs by simulating the timing residuals of pulsar signals, and then use the mock GW bright-siren and dark-siren data to perform cosmological parameter estimations. The system of units in which $G = c = 1$ is adopted in this paper.

**Results**

The number of available MSPs, $N_p$, in the SKA-era PTAs is still uncertain, and therefore we select 100, 200, and 500 MSPs within 3 kpc from the Earth, obtained from the Australia Telescope National Facility (ATNF) pulsar catalog, to construct PTAs. The root mean square (rms) of timing residual, $\sigma_t$, reflecting the stability of the pulsar and the quality of the ToA data, consists of red noise and white noise. Since the GW strain induced by an individual source in the frequency domain appears as a single peak on the PTA-detection time scale, which is essentially different
from frequency-dependent SGWB\textsuperscript{32}, the red noise mainly affects the detection of SGWB while it is less critical on the detections of individual sources, especially at relatively high frequency. For simplicity, we ignore the influence of the red noise in this work. The white noise mainly includes jitter noise and radiometer noise. The jitter noise will dominate for most bright pulsars and the total white noises are around $10 \sim 50$ ns\textsuperscript{45}. Considering that FAST and SKA could make the noise lower, we expect that $\sigma_t$ could reach $\sim 20$ ns for SKA-era PTAs. We consider two cases of $\sigma_t = 20$ ns and $\sigma_t = 100$ ns for comparison. Here we assume that the GW spectrum induced by SGWB can be well measured in the forthcoming years and the GW signals from individual SMBHBs can be resolved from SGWB\textsuperscript{46}, therefore we do not consider SGWB in this work. We assume that the ToA data are obtained via monitoring the pulses from MSPs with the typical cadence of two weeks and the observation span is 10 years\textsuperscript{37}.

We analyze the ability of SKA-era PTAs to detect SMBHBs by simulating the timing residuals (see Methods). The detection curves of SKA-era PTAs, averaged over the sky locations of the GW sources, are plotted in Fig.1 by using the hasasia package\textsuperscript{47,48}. The solid dots without black borders represent 154 SMBHB candidates and the solid dots with black borders represent 84 SMBHBs simulated from the 2 Micron All Sky Survey (2MASS)\textsuperscript{49} Extended Source Catalog\textsuperscript{50}. As $N_p$ increases and $\sigma_t$ decreases, the more sensitive detection curves enable ones to detect more SMBHBs. The dotted curves ($\sigma_t = 20$ ns) are obviously lower than the solid curves ($\sigma_t = 100$ ns), indicating that the rms timing residual has a more dominating effect than the number of MSPs on the detections of SMBHBs.
To simulate GW bright-siren data, we adopt 154 currently available SMBHB candidates, mainly obtained via the observations of periodic variations in their light curves from the Catalina Real-time Transient Survey and the Palomar Transient Factory. These methods are appropriate for SMBHBs in the inspiral phase. Actually, SMBHBs in the merger phase are likely to emit dual jets that may be detected by future telescopes, such as the Vera C. Rubin Observatory (formerly known as LSST) and the European Extremely Large Telescope. These EM signals can also be used as EM counterparts to provide redshifts. According to the analysis in Refs., in a 5-year observation, dozens of SMBHBs (10⁴ – 10⁸ M☉) with the dual-jet EM counterparts could be observed by space-borne observatories in the mHz band. Usually, SMBHBs in the PTA band will inspiral for a long time and we need to wait hundreds of years for the merger phase. Therefore, it is more difficult to detect the merger-phase EM signals for PTA-band SMBHBs.

The relative errors of the luminosity distances (∆d_L/d_L) of the mock SMBHB bright and dark sirens as a function of signal-to-noise ratio (SNR), ρ, are shown in Fig. 2. The corresponding numbers of detected bright and dark sirens (ρ > 10) are shown in Table 1. In the case of N_p = 100, the number of detected bright sirens increases from 14 (σ_t = 100 ns) to 25 (σ_t = 20 ns) and the number of detected dark sirens increases from 13 (σ_t = 100 ns) to 41 (σ_t = 20 ns). Although the number of MSPs can also affect the detection number of SMBHBs, its effect is not as obvious as σ_t. For example, in the case of σ_t = 100 ns, the number of detected bright sirens increases from 14 (N_p = 100) to 15 (N_p = 500) and the number of detected dark sirens increases from 13 (N_p = 100) to 27 (N_p = 500). This indicates that the rms timing residual is the most important factor in reducing the errors of luminosity distances. Our results show that about 100 MSPs are
sufficient for detecting individual SMBHBs, if the timing measurement could reach high-enough precision.

Assuming different $N_p$ and $\sigma_t$, we simulate six sets of bright-siren data that contain $d_L$, $\Delta d_L$, and the redshift $z$ of the SMBHB candidates (see Methods). We use these bright-siren data to constrain the $\Lambda$CDM and $w$CDM models, respectively. The constraint results of the $\Lambda$CDM model solely from the bright-siren data are listed in Table 1. We define the constraint precision of the parameter $\xi$ as $\varepsilon(\xi) = \sigma(\xi)/\xi$ with $\sigma(\xi)$ representing the marginalized absolute error. In the case of $\sigma_t = 100$ ns, as $N_p$ increases from 100 to 500, $\varepsilon(H_0)$ decreases from 2.1% to 1.8%. In the case of $N_p = 100$, as $\sigma_t$ decreases from 100 ns to 20 ns, $\varepsilon(H_0)$ decreases from 2.1% to 1.5%. We note that reducing $\sigma_t$ is more effective than increasing $N_p$ on improving the constraining capability of bright sirens. If $\sigma_t$ could reach 20 ns, 100 MSPs are sufficient to make the measurement precision of $H_0$ comparable to that of the current cosmic distance-ladder observation.

In the $w$CDM model, the cosmic microwave background (CMB) data cannot provide tight constraints on the EoS parameter of dark energy ($w$), because CMB encodes the information of the early Universe, while dark energy becomes dominant in the late Universe. Nevertheless, Fig. 3 shows that the CMB data and the bright-siren data (simply referred to as the PTA data) have distinct degeneracy orientations in the $w$-$H_0$ plane, indicating that although the PTA data alone cannot constrain $w$ well either, it can provide tight constraints on $H_0$, thus breaking the degeneracy between the parameters $w$ and $H_0$. Extended Data Table 1 shows that, in the case of $N_p = 100$ and $\sigma_t = 20$ ns, the combination of the CMB and PTA data gives the relative error $\varepsilon(w) = 4.7\%$, which
is roughly comparable with the result of Planck 2018 TT,TE,EE+lowE+lensing+SNe+BAO \cite{63}. The results suggest that the SMBHB bright sirens will be a useful probe to explore the nature of dark energy.

To simulate GW dark-siren data, we consider 5119 galaxies in the 2MASS catalog as SMBHBs’ possible host galaxies. We simulate 84 SMBHBs according to the probability of a galaxy hosting an SMBHB in the PTA band (see Methods). Furthermore, we simulate the GW signals emitted by the 84 SMBHBs and consider those with $\rho > 10$ as detected SMBHBs. Table 1 shows the numbers of detected SMBHBs in different cases. For these SMBHBs, we determine their localization volumes by the Fisher matrix (see Methods). The redshifts of the galaxies within the localization volumes can be utilized to infer the posterior distribution of cosmological parameters. Since the dark sirens are simulated in the local Universe in which the $d_L-z$ relation is weakly dependent on cosmological models, these data cannot constrain $w$ well. Therefore, we only calculate the posterior distribution of $H_0$ via the Bayesian analysis method (see Methods).

The results of the SMBHB dark sirens are shown in Fig. 4 and Table 1. In the case of $\sigma_t = 100$ ns, increasing $N_p$ from 100 to 500 can significantly improve the measurement of $H_0$. The 1$\sigma$ errors of $H_0$ with $\sigma_t = 20$ ns are obviously smaller than those with $\sigma_t = 100$ ns. It is worth noting that even with only 100 MSPs ($N_p = 100$ and $\sigma_t = 20$ ns), the precision of $H_0$ could reach $\sim 1.8\%$. Compared with the SMBHB bright sirens, the SMBHB dark sirens have a similar ability in measuring $H_0$. This indicates that even if it is difficult to detect EM counterparts of SMBHBs in the future, dark sirens could solely provide precise measurement of $H_0$. The bright and dark sirens
have potential to complement each other to provide precise measurements for both \(w\) and \(H_0\).

**Discussion**

In this work, we assume that the cadence of monitoring the pulses from MSPs is two weeks\(^{37}\) and consider \(N_p = 100, 200, 500\) respectively. Actually, observing 500 MSPs is not achievable with this cadence due to the time required for each observation, and therefore the case of \(N_p = 500\) is just used as an extreme case for comparison. To show the effect of the cadence, we consider another case with the cadence of one month instead of two weeks. In this case, 23 bright sirens could be observed during 10 years when \(N_p = 100\) and \(\sigma_t = 20\) ns, and the measurement precision of \(H_0\) reach 1.66\%, similar with the result obtained with the cadence of two weeks \([\varepsilon(H_0) = 1.47\%]\), indicating that even if the observation time is reduced by a factor of 2, SMBHB standard-siren data could still maintain tight constraints on \(H_0\).

In the analysis of dark sirens, all SMBHBs are simulated at \(z < 0.05\) based on the 2MASS catalog. In the future, the Stage IV space-based telescopes, such as the China Space Station Telescope (CSST)\(^{64}\), the Vera C. Rubin Observatory, and the Euclid space mission\(^{65}\), could provide galaxy catalogs at higher redshift. According to our preliminary estimation, CSST is expected to provide a complete galaxy catalog up to \(z \sim 0.3\) at which \(O(10^3) - O(10^4)\) SMBHBs could be observed by PTAs. Although the measurements on \(H_0\) are mainly contributed by the local- Universe SMBHBs considered in this work, larger numbers of SMBHB dark sirens may help to measure other cosmological parameters, such as the EoS parameter of dark energy.
The chirp mass, $M_c$, depends on the mass ratio between the two black holes forming an SMBHB. Therefore, the mass ratio affects not only SNRs of GWs but also the probability of the existence of an SMBHB in a galaxy. This two effects both affect the constraint precision of cosmological parameters. We define $q = m_1/m_2$ as the mass ratio with $q \in (0, 1]$. The results in the main text is based on the assumption that $q$ is randomly chosen between $[0.25, 1]$ with a log-normal distribution. We discuss the effect of $q$ in detail in Methods and show the constraint results of $H_0$ with different $q$ values in Extended Data Table 2.

Compared with the GW standard sirens in other frequency bands, the ultra-low-frequency GW standard sirens have some advantages. (i) The masses of the GW sources are at the top of the mass range of SMBHBs, leading to higher SNRs. Fig. 2 shows that the highest SNR could reach ~ 700. Such high SNRs are helpful to accurately localize GW sources and thus contribute to the precise measurements of $H_0$. (ii) Unlike BNSs that are thought to emit EM signals only in the merger phase, SMBHBs could produce observable EM signals in the inspiral phase, i.e., the characteristic signals of the SMBHB candidates. The inspiral-phase EM signals not only provide redshifts for bright sirens but also provide the early alerts for GW detections, which can help us to choose MSPs at suitable sky positions to obtain the best sensitivity in the direction of the GW source. (iii) When an SMBHB evolves to the late stage, the GW frequency may fall in the frequency band of space-borne GW detectors. Although most PTA-band SMBHBs inspiral for a long time, in a few cases, for example, an SMBHB with $M \sim 10^9 M_\odot$, $z \lesssim 1$, and $f_0 \sim 10^{-7}$ is expected to enter the merger phase after 17 years. Once such cases are discovered, the joint observation in the mHz and nHz frequency bands can be realized, which is helpful to localize GW
sources and explore the various physical properties of SMBHBs.

We conclude that ultra-low-frequency GWs emitted by individual SMBHBs can serve as both bright and dark sirens and have promising potential in two aspects. (i) The bright-siren data could effectively break the cosmological-parameter degeneracy inherent in the CMB data. The bright-siren data combined with the CMB data have a comparable capability to the mainstream observational data (Planck 2018 TT,TE,EE+lowE+lensing+SNe+BAO) for measuring $w$. (ii) The dark sirens in the local Universe have high SNRs and could be well localized, making the measurement precision of $H_0$ close to that of the current distance-ladder observation. The bright and dark sirens can complement each other to measure both $w$ and $H_0$ precisely. Ultra-low-frequency GWs detected by SKA-era PTAs could be developed into a precise late-Universe probe to explore the nature of dark energy and measure the Hubble constant.
Figure 1 | Detection curves of SKA-era PTAs with a 10-year observation time span. The solid and dotted lines represent the cases of $\sigma_t = 100$ ns and $\sigma_t = 20$ ns, respectively. The data points represent the GW strain amplitudes ($h_0$) when $f = f_0$, with $h_0 = 2[M_c (1 + z)]^{5/3} (\pi f)^{2/3} d_L^{-1/37}$ and $f_0$ the GW frequency at the time of the first observation. The solid dots without black borders represent 154 SMBHB candidates and the solid dots with black borders represent 84 SMBHBs simulated from the 2MASS catalog.
Figure 2 | Measurement precision of luminosity distance ($\Delta d_L/d_L$) as a function of SNR ($\rho$). The red stars and the blue dots represent the detected SMBHBs with $\rho > 10$, used as the bright and dark sirens, respectively. The impacts of $N_p$ and $\sigma_t$ on the SMBHB detections can be explicitly seen.
Figure 3 | 2D marginalized contours (68.3% and 95.4% confidence level) in the $w-H_0$ plane for the $w$CDM model by using the CMB, PTA, and CMB+PTA data. Here the PTA data refer to the mock GW bright-siren data.
Figure 4 | 1D posterior distribution of $H_0$ inferred from the mock GW dark-siren data. The dotted and solid lines represent the cases of $\sigma_t = 100$ ns and $\sigma_t = 20$ ns, respectively. The errors of $H_0$ become smaller as $N_p$ increases and $\sigma_t$ decreases.
Table 1 | Relative errors of $H_0$ in the $\Lambda$CDM model. The GW bright-siren data are simulated based on the 154 SMBHB candidates and the GW dark-siren data are simulated based on the 5119 galaxies in the 2MASS catalog. $N_s$ is the number of detected SMBHBs ($\rho > 10$) and $\varepsilon(H_0)$ is the relative error of $H_0$. 

| $N_p$ | $\sigma_t$ (ns) | $N_s$ | $\sigma(H_0)$ | $\varepsilon(H_0)$ | $N_s$ | $\sigma(H_0)$ | $\varepsilon(H_0)$ |
|-----|----------------|------|---------------|----------------|------|---------------|----------------|
| 100 | 100           | 14   | 1.4           | 0.0209         | 13   | 3.458         | 0.0500         |
| 200 | 100           | 14   | 1.3           | 0.0193         | 19   | 2.350         | 0.0339         |
| 500 | 100           | 15   | 1.2           | 0.0183         | 27   | 1.824         | 0.0270         |
| 100 | 20            | 25   | 1.0           | 0.0147         | 41   | 1.248         | 0.0184         |
| 200 | 20            | 40   | 1.0           | 0.0151         | 49   | 1.066         | 0.0159         |
| 500 | 20            | 53   | 0.95          | 0.0141         | 56   | 0.895         | 0.0131         |
Methods

The detection of individual SMBHBs. GW signals are detected in the timing residuals of MSPs by removing model-predicted ToAs from the observational ToA data. The timing residuals induced by a single GW source, measured at time \( t \) on the Earth, can be written as

\[
s(t, \hat{\Omega}_s, \hat{\Omega}_p) = F_+(\hat{\Omega}_s, \hat{\Omega}_p) \Delta A_+(t) + F_\times(\hat{\Omega}_s, \hat{\Omega}_p) \Delta A_\times(t),
\]

with \( F_+ (\hat{\Omega}_s, \hat{\Omega}_p) \) the geometric factors, equivalent to the antenna pattern functions of laser interferometric GW detections. \( \hat{\Omega}_s \) and \( \hat{\Omega}_p \) are the unit vectors pointing from the GW source and the pulsar to the observer, respectively, determined by the sky positions of the GW source \((\alpha_s, \beta_s)\) and the pulsar \((\alpha_p, \beta_p)\). Extended Data Fig. 1 shows the sky positions of the selected 500 MSPs used in this work. \( \Delta A_{+\times}(t) = A_{+\times}(t) - A_{+\times}(t_p) \) is the difference between the Earth term \( A_{+\times}(t) \) and the pulsar term \( A_{+\times}(t_p) \), with \( t_p \) the time at which GW passes the MSP.

\( \Delta A_{+\times}(t) \) encodes the GW strain amplitude \( h(t) \). We assume that SMBHBs inspiral in circular orbits, and then \( h(t) \) can be written as

\[
h(t) = 2 \left( \frac{G M_c}{c^4} \right)^{5/3} \frac{[\pi f(t)]^{2/3}}{d_L}.
\]

Here, \( M_c = M_c(1 + z) \) represents the redshifted chirp mass and \( M_c \) is the chirp mass defined as \( M_c = \eta^{3/5} M \). \( M = m_1 + m_2 \) is the total mass of the binary system with component masses \( m_1 \) and \( m_2 \), and \( \eta = m_1 m_2/(m_1 + m_2)^2 \) is the symmetric mass ratio. \( d_L \) represents the luminosity distance of the GW source. The GW frequency \( f(t) \) is given by

\[
f(t) = f_0^{-8/3} - \frac{256}{5} \pi^{8/3} \left( \frac{G M_c}{c^3} \right)^{5/3} t^{-3/8},
\]
where \( f_0 = 2f_{\text{orb}} \) is the GW frequency at the time of the first observation. Here \( f_{\text{orb}} = (2\pi T)^{-1} \) is the orbit frequency and \( T \) is the orbital periods of SMBHBs. When simulating the GW signals of bright sirens, we calculate \( f_0 \) using the orbital periods of the 154 SMBHB candidates\(^{38,51,57}\).

When simulating the GW signals of dark sirens, we calculate \( f_0 \) using Eq. (11).

The SNR (\( \rho \)) of the GW signal detected by a PTA is given by

\[
\rho^2 = \sum_{i=1}^{N_p} \sum_{n=1}^{N} \left[ \frac{s_i(t_n)}{\sigma_{t,i}} \right]^2,
\]

where \( N \) is the total number of data points for each MSP, \( N_p \) is the number of MSPs, \( s_i(t_n) \) is the timing residual induced by the GW signal in the \( i \)-th MSP at time \( t_n \) [see Eq. (1)], and \( \sigma_{t,i} \) is the rms of the timing residual of the \( i \)-th MSP.

Fisher information matrix is adopted to estimate the parameters of GW sources. For a PTA including \( N_p \) independent MSPs, the Fisher matrix \( F \) is expressed as

\[
F_{ab} = \sum_{i=1}^{N_p} \sum_{n=1}^{N} \frac{\partial s_i(t_n)}{\sigma_{t,i}} \frac{\partial s_i(t_n)}{\sigma_{t,i}} \frac{\partial \theta_a}{\partial \theta_b},
\]

where \( \theta \) denotes the free parameters to be estimated. The instrumental error of the parameter \( \theta_a \) is estimated as \( \Delta \theta_a = \sqrt{(F^{-1})_{aa}} \). Here, nine parameters are taken into account in the Fisher matrix, including eight parameters of a GW source and the pulsar distance, i.e., \( M_c, \alpha_s, \beta_s, \iota, \psi, \phi_0, f_0, d_L, \) and \( d_p \). The inclination angle \( \iota \) is randomly chosen between \([0, \pi]\). The polarization angle \( \psi \) and the initial phase \( \phi_0 \) of SMBHBs are randomly chosen between \([0, 2\pi]\).

In addition to the instrumental error (\( \Delta \theta_{\text{inst}} \)) estimated by the Fisher matrix, the total error of
$d_L$ should also include the weak lensing error ($\Delta d_L^{\text{lens}}$)\(^{69}\) and the peculiar velocity error ($\Delta d_L^{\text{pv}}$)\(^{70}\).

\[
\Delta d_L = \sqrt{(\Delta d_L^{\text{inst}})^2 + (\Delta d_L^{\text{lens}})^2 + (\Delta d_L^{\text{pv}})^2},
\]

with

\[
\Delta d_L^{\text{lens}}(z) = d_L(z) \times 0.066 \left( \frac{1 - (1 + z)^{-0.25}}{0.25} \right)^{1.8},
\]

\[
\Delta d_L^{\text{pv}}(z) = d_L(z) \times \left[ 1 + \frac{c(1 + z)^2}{H(z)d_L(z)} \right] \frac{\sqrt{\langle v^2 \rangle}}{c},
\]

where $H(z)$ is the Hubble parameter and $\sqrt{\langle v^2 \rangle}$ is the peculiar velocity of the GW source with $\sqrt{\langle v^2 \rangle} = 500 \text{ km s}^{-1}\(^{71}\)$. Since $\Delta d_L^{\text{lens}}$ is relatively small at $z < 0.1\(^{72}\)$, we consider this error only when simulating the bright-siren data and ignore it when simulating the dark-siren data. When simulating the bright-siren data, we use Eq. (8) to calculate the peculiar velocity error of $d_L$ and add it to the total error of $d_L$; when simulating the dark-siren data, we consider the peculiar-velocity effect in the error of $z$ instead of in the error of $d_L$ [see Eq. (15)].

**SMBHB bright sirens.** When we simulate the GW bright-siren data, we adopt 154 currently available SMBHB candidates obtained from various characteristic signatures. The redshifts of these SMBHB candidates are taken from Refs.\(^{38,51-57}\). Extended Data Fig. 2 shows these SMBHB candidates in the $z$-$M$ plane. We use their redshifts to calculate their luminosity distances based on the $\Lambda$CDM model in which the cosmological parameters are set to the Planck 2018 results.

We use Eqs. (1)–(3) to simulate the GW signals emitted by these SMBHB candidates. Fig. 1 shows the strain amplitudes (when $f = f_0$) of the GW signals. Here we only plot the strain amplitudes when $f = f_0$, because a simple calculation using Eq. (3) shows that the variation of
the GW frequency of an inspiraling SMBHB with $M = 10^9 M_\odot$ and $f_0 = 10^{-7}$ Hz in a 10-year observational time span is $4.36 \times 10^{-9}$ Hz, and this variation is so minuscule that the amplitude of the GW strain undergoes only negligible changes over the given time span.

We use Eq. (4) to calculate SNRs of the SMBHB candidates. We use $N_s$ to represent the number of SMBHBs with $\rho > 10$, shown in Table [1] For these SMBHBs, we use the Fisher matrix to estimate the errors of luminosity distances. $d_L$, $\Delta d_L$, and $z$ compose the bright-siren data and can be used to constrain cosmological parameters via the $d_L$-$z$ relation. Two representative sets of the GW bright-siren data are shown in Extended Data Fig. [3]. The numbers of detected bright sirens in the case of $\sigma_t = 20$ ns are much larger than those in the case of $\sigma_t = 100$ ns for the same number of MSPs. Improved SNRs of the GW events by decreasing $\sigma_t$ reduce the measurement errors of luminosity distances.

Here we consider the base $\Lambda$CDM model ($w = -1$) and the $w$CDM model ($w = \text{constant}$). The $d_L$-$z$ relation can be written as

$$d_L = \frac{(1 + z)}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_m(1 + z')^3 + (1 - \Omega_m)(1 + z')^{3(1+w)}}},$$  

where $\Omega_m$ represents the current matter density parameter. The constraint results of the base $\Lambda$CDM model are shown in Extended Data Fig. [4] and listed in Table [1]. The constraint results of the $w$CDM model are shown in Fig. [3] and listed in Extended Data Table [1].

Smaller $q$ could decrease the chirp mass of SMBHBs and thus decrease SNRs. According to our calculation, two SMBHBs ($z = 0.05$ and $M = 10^9 M_\odot$) with $q = 1$ and $q = 0.1$ have SNR
= 19.52 and SNR = 6.45, respectively, when a PTA with \(N_p = 100\) and \(\sigma_t = 20\) ns is considered.

We set the range of \(q\) to \([q_{\text{min}}, 1]\) according to the log-normal distribution. The main results in this paper are based on \(q_{\text{min}} = 0.25\). To show the effect of \(q\) more explicitly, we show the constraint results of \(H_0\) with different \(q_{\text{min}}\) values in Extended Data Table 2. We consider four cases, i.e., \(q_{\text{min}} = 1, 0.25, 0.1\) and 0.01, where \(q_{\text{min}} = 1\) indicates that \(q\) is fixed at 1. It is shown that, for the bright-siren data, the values of \(q_{\text{min}}\) have negligible effects on constraining \(H_0\).

**SMBHB dark sirens.** For dark sirens, we consider 5119 galaxies in the 2MASS catalog as SMBHBs’ possible host galaxies. These galaxies are in the local Universe \((z < 0.05)\) and the 2MASS catalog can be considered complete in this redshift range\(^{67}\). The mass distribution of these galaxies is between \(10^{11} - 10^{12} M_\odot\).\(^{67,74,75}\) We estimate the masses of SMBHBs in these galaxies according to the \(M\)-\(M_{\text{buldge}}\) relationship, with \(M_{\text{buldge}}\) the bulge mass of a galaxy\(^{73}\).

The probability that a galaxy hosts an SMBHB in the PTA band, \(p_j\), can be written as

\[
p_j = \frac{t_{c,j}}{T_{\text{life}}} \int_{0.25}^{1} d\mu_\star \frac{dN}{dt} (M_\star, \mu_\star, z') T_{\text{life}}.
\]

Eq. (10) shows that \(p_j\) depends on \(t_{c,j}\), and \(t_{c,j}\) is related to \(M_\star\) that is determined by \(q\). Therefore, \(q\) could affect the merger probability of SMBHBs and further
affect the number of mock SMBHBs. As shown in Extent Data Table 2, smaller values of $q_{\text{min}}$ increase the numbers of both mock and detected SMBHBs. As $q_{\text{min}}$ decreases from 1 to 0.01, the number of mock SMBHBs increases from 54 to 197. The main results of this paper are based on the assumption of $q_{\text{min}} = 0.25$. Under this assumption, we find that there are approximately 84 SMBHBs in the total 5119 galaxies. Then we randomly select 84 galaxies from the total galaxies as SMBHBs’ host galaxies according to the probability distribution $p_j$.

Extended Data Fig. 5 shows the 84 SMBHBs in the $z-M$ plane. Fig. 1 shows the strain amplitudes (when $f = f_0$) of the GW signals emitted by the 84 SMBHBs, with $f_0$ calculated by the following formula,

$$f_0 = \pi^{-1} \left( \frac{GM_c}{c^3} \right)^{-5/8} \left( \frac{256}{5} t_c \right)^{-3/8},$$

(11)

where $t_c$ is taken from a uniform distribution in [100 yr, 26 Myr].

Using the GW signal, we can localize the GW source within a localization volume. Fig. 2 shows the relative errors of luminosity distances, $\Delta d_L/d_L$, as a function of SNR. By matching the localization volume to the galaxy catalog, we can find the host galaxy of the GW source. The localization volume usually contains more than one galaxy, and thus we need to consider these galaxies’ redshifts in a statistical way. The Bayesian analysis is a statistical method commonly used for dark sirens.

In the Bayesian method, the posterior distribution of $H_0$ can be written as

$$p(H_0|D_{\text{GW}}, D_{\text{EM}}) \propto p(D_{\text{GW}}, D_{\text{EM}}|H_0)p(H_0),$$

(12)
where $D_{GW}$ and $D_{EM}$ represent the GW and EM data, respectively. $p(H_0)$ represents the prior probability of $H_0$, assumed to be uniformly distributed in the interval [50, 80] km s$^{-1}$ Mpc$^{-1}$. For a single GW event, the likelihood term, $p(D_{GW}, D_{EM}|H_0)$, can be written as

$$p(D_{GW}, D_{EM}|H_0) = \int p(D_{GW}|d_L(z, H_0), \alpha, \beta)p(D_{EM}|z, \alpha, \beta)p_0(z, \alpha, \beta)d\gamma(H_0).$$

(13)

$p(D_{GW}|d_L(z, H_0), \alpha, \beta)$ in Eq. (13) is the likelihood of the GW data, expressed as

$$p(D_{GW}|d_L(z, H_0), \alpha, \beta) \propto e^{-\chi^2/2},$$

(14)

with $\chi^2 = (x - x_{gw})^T C^{-1} (x - x_{gw})$. Here $x = (d_L(z, H_0), \alpha, \beta)$ represents an arbitrary three-dimensional (3D) position in the sky. $x_{gw} = (d_{L,s}, \alpha_s, \beta_s)$ represents the 3D position of the GW source, that is, the 3D position of the true host galaxy in our simulation. We calculate $d_{L,s}$ with the galaxies’ redshifts ($z_s$) by assuming the $\Lambda$CDM model and setting the cosmological parameters to the Planck 2018 results. $C$ is the $3 \times 3$ covariance matrix only relevant to $(d_L, \alpha, \beta)$, obtained from the Fisher matrix. We use $\chi^2 = 11.34$ (corresponding to 99% confidence) to determine the boundary of GW source’s localization volume. If the position of a galaxy satisfies $\chi^2 < 11.34$, we consider this galaxy to be within the localization volume and regard it as a potential host galaxy of the GW source. We define $N_{in}$ to describe the number of potential host galaxies. Generally, a small value of $N_{in}$ means a strong ability to localize the GW source. Extended Data Fig. 6 shows the numbers of SMBHBs satisfying $N_{in} < 10$. It can be seen that more SMBHBs satisfy $N_{in} < 10$ as $N_p$ increases and $\sigma_t$ decreases. Here $N_{in}$ is calculated by fixing $H_0 = 67.36$ km s$^{-1}$ Mpc$^{-1}$. 

24
\( p(D_{\text{EM}}|z, \alpha, \beta) \) in Eq. (13) is the likelihood of the EM data and is given by

\[
p(D_{\text{EM}}|z, \alpha, \beta) = \frac{1}{N_{\text{in}}} \sum_{j=1}^{N_{\text{in}}} \mathcal{N}(z_j, \sigma_{z,j}) \delta(\alpha - \alpha_j) \delta(\beta - \beta_j),
\]

where \( \mathcal{N}(z_j, \sigma_{z,j}) \) represents a Gaussian distribution centered at \( z_j \), with a standard deviation \( \sigma_{z,j} \) arising from the peculiar velocity of the \( j \)-th galaxy and we take \( \sigma_z = (1 + z) \frac{\sqrt{\langle v^2 \rangle}}{c} \). Under the assumption of \( \sqrt{\langle v^2 \rangle} = 500 \text{ km s}^{-1} \), \( \sigma_z \) is around 0.0017 in the redshift range [0, 0.05]. Here we assign an equal weight to each galaxy in the localization volume for simplicity. A more rigorous way is to assign different weights to different galaxies, for example, replacing \( 1/N_{\text{in}} \) with \( \omega_j \) that is the weight of the \( j \)-th galaxy, proportional to the stellar or star-forming luminosity.

\( p_0(z, \alpha, \beta) \) in Eq. (13) represents the prior distribution of galaxies in the Universe, which is set to be uniform in the comoving volume, and it is expressed as

\[
p_0(z, \alpha, \beta) \propto \frac{d^2 V_C}{dz d\Omega} \propto \frac{d_C^2(z)}{H(z)},
\]

with \( V_C \) the comoving volume and \( d_C \) the comoving distance.

The normalization term \( \gamma(H_0) \) in Eq. (13) can be written as

\[
\gamma(H_0) = \int D_{\text{det}}^{GW}(d_L(z, H_0), \alpha, \beta) D_{\text{det}}^{EM}(z, \alpha, \beta) p_0(z, \alpha, \beta) dz d\alpha d\beta
\]

where \( D_{\text{det}}^{GW}(d_L(z, H_0), \alpha, \beta) \) and \( D_{\text{det}}^{EM}(z, \alpha, \beta) \) represent the GW selection effect and the EM selection effect, respectively, reflecting that only the events exceeding the detection threshold can be taken into account. The GW selection effect can be expressed as

\[
D_{\text{det}}^{GW}(d_L(z, H_0), \alpha, \beta) = \int_{d_{\text{GW}}>d_{GW}^0} p(D_{\text{GW}}|d_L(z, H_0), \alpha, \beta) dD_{\text{GW}},
\]
with
\[ p(D_{GW}|d_L(z, H_0), \alpha, \beta) = \begin{cases} 1, & \text{if } \rho > \rho_{th}, \\ 0, & \text{if } \rho < \rho_{th}, \end{cases} \] (19)

where \( \rho_{th} = 10 \) is the threshold of SNR. Since the 2MASS galaxy catalog is complete in our considered redshift range \( z < 0.05 \), all the potential host galaxies will be contained in this galaxy catalog. Thus, the EM selection effect can be expressed as
\[ D_{EM}^E(z) = \int_{D_{EM} > D_{EM}^{th}} p(D_{EM}|z, \alpha, \beta) dD_{EM} = \mathcal{H}(z_{max} - z), \] (20)

where \( \mathcal{H} \) is the Heaviside step function and \( z_{max} \) is set to 0.05 in our analysis.

Using Eqs. (15)–(20), we can calculate the likelihood of a single GW event. The total likelihood of SMBHB events can be written as
\[ p(D_{GW}, D_{EM}|H_0) = \prod_{k=1}^{N_{SMBHB}} p(D_{GW,k}, D_{EM,k}|H_0), \] (21)

where \( N_{SMBHB} \) is the total number of SMBHB events and \( k \) represents the \( k \)-th GW event.

Extended Data Table 2 shows the constraint results from the mock GW dark-siren data with different \( q_{min} \) values. Unlike the bright-siren data giving almost identical results, the dark-siren data constrain \( H_0 \) tighter as \( q \) decreases. The reason is that when we simulate the dark-siren data, \( q \) affects not only SNRs of GWs but also the number of mock SMBHBs, and the effect of the number of mock SMBHBs is more dominated than the effect of SNRs for the dark-siren method.
Data Availability  The data that support the results in this paper are available from the corresponding author upon reasonable request.

Code Availability  The code hasasia is publicly available at https://hasasia.readthedocs.io, and the code to simulate the SMBHB catalog is publicly available at https://github.com/ChiaraMingarelli/nanohertz_GW.

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Competing Interests  The authors declare no competing interests.
Extended Data Figure 1 | Positions of the selected MSPs on the sky. We select 500 pulsars within 3 kpc from the Earth obtained from the ATNF pulsar catalog[44].
Extended Data Figure 2 | Distribution of 154 SMBHB candidates in the $z$-$M$ plane, taken from Refs.\textsuperscript{38,51-57}. These SMBHB candidates are used in the analysis of bright sirens.
Extended Data Figure 3 | GW bright-siren and dark-siren data simulated from the 154 SMBHB candidates and 84 mock SMBHBs, respectively. Upper and lower panels correspond to the bright and dark sirens, respectively. The redshifts of dark sirens shown in lower panel are the redshifts of mock SMBHBs' host galaxies. The luminosity distances $d_L$ are calculated based on the $\Lambda$CDM model in which the fiducial values of cosmological parameters are set to be the Planck 2018 results. The error bars of the data points ($\Delta d_L$) in the figure are obtained from the Fisher matrix. The central values of the data points are randomly chosen in the range of $[d_L - \Delta d_L, d_L + \Delta d_L]$. The data points with $\Delta d_L/d_L > 1$ are not displayed in the figure.
Extended Data Figure 4 | 2D marginalized contours (68.3% and 95.4% confidence level) in the $\Omega_m$-$H_0$ plane for the $\Lambda$CDM model by using the PTA data. Here the PTA data refer to the mock GW bright-siren data.
Extended Data Figure 5 | Distribution of 84 SMBHBs simulated from the 2MASS catalog in the $z$-$M$ plane. We use these SMBHBs in the analysis of dark sirens.
Extended Data Figure 6 | Numbers of SMBHBs with $N_{in} < 10$ in the analysis of dark sirens. $N_s$ is the number of detected SMBHBs ($\rho > 10$). Here $N_{in}$ is calculated by fixing $H_0 = 67.36$ km s$^{-1}$ Mpc$^{-1}$. As $N_p$ increases and $\sigma_t$ decreases, more SMBHBs satisfy $N_{in} < 10$. 
### Extended Data Table 1 | Relative errors of the cosmological parameters in the \( \omega \)CDM model using the CMB, PTA, and CMB+PTA data. \( N_s \) is the number of detected SMBHB \( (\rho > 10) \). Here the PTA data refer to the mock GW bright-siren data.

| Data        | \( N_p \) | \( \sigma_t \)(ns) | \( \sigma(\Omega_m) \) | \( \varepsilon(\Omega_m) \) | \( \sigma(H_0) \) | \( \varepsilon(H_0) \) | \( \sigma(w) \) | \( \varepsilon(w) \) |
|-------------|-----------|---------------------|------------------------|------------------------|-----------------|-----------------|-----------------|-----------------|
| CMB         | –         | –                   | 0.054                  | 0.179                  | 6.0             | 0.858           | 0.20            | 0.185           |
| PTA         | 100       | 100                 | 0.083                  | 0.407                  | 3.9             | 0.056           | 0.50            | 0.424           |
|             | 500       | 100                 | 0.090                  | 0.292                  | 3.8             | 0.057           | 0.68            | 0.548           |
|             | 100       | 20                  | 0.053                  | 0.157                  | 3.6             | 0.053           | 0.63            | 0.460           |
| CMB+PTA     | 100       | 100                 | 0.014                  | 0.042                  | 1.4             | 0.020           | 0.053           | 0.055           |
|             | 500       | 100                 | 0.011                  | 0.033                  | 1.1             | 0.017           | 0.042           | 0.044           |
|             | 100       | 20                  | 0.010                  | 0.031                  | 1.1             | 0.017           | 0.045           | 0.047           |

\( q_{\min} \) | \( N_s \) | \( \sigma(H_0) \) | \( \varepsilon(H_0) \) | \( N_{\text{mock}} \) | \( N_s \) | \( \sigma(H_0) \) | \( \varepsilon(H_0) \) |  
|-------------|-----------|---------------------|------------------------|------------------------|-----------------|-----------------|-----------------|-----------
| 1           | 32        | 0.99                | 0.0147                 | 54                     | 34              | 1.41            | 0.0210         |
| 0.25        | 25        | 1.00                | 0.0147                 | 84                     | 41              | 1.25            | 0.0184         |
| 0.1         | 22        | 1.00                | 0.0147                 | 134                    | 65              | 1.21            | 0.0178         |
| 0.01        | 21        | 1.10                | 0.0165                 | 197                    | 87              | 1.15            | 0.0171         |

### Extended Data Table 2 | Relative errors of \( H_0 \) in the \( \Lambda \)CDM model with different \( q_{\min} \). \( N_{\text{mock}} \) and \( N_s \) are the numbers of mock SMBHBs and detected SMBHBs \( (\rho > 10) \), respectively, and \( \varepsilon(H_0) \) is the relative error of \( H_0 \). Here we set \( N_p = 100 \) and \( \sigma_t = 20 \) ns.