Inflationary quantum cosmology for FRLW supersymmetric models

N.E. Martínez-Pérez, C. Ramírez
Benemérita Universidad Autónoma de Puebla, Facultad de Ciencias Físico Matemáticas, P.O. Box 165, 72000 Puebla, México.

V. Vázquez-Báez
Benemérita Universidad Autónoma de Puebla, Facultad de Ingeniería, 72000 Puebla, México.

We consider inflationary scenarios in quantum cosmology for FRLW supersymmetric models with a scalar field, whose Wheeler-DeWitt equation has analytic solutions. An evolution for the scale factor is obtained from mean trajectories in superspace, which can be computed from the mean value of the scale factor with the conditional probability amplitude, with the scalar taken as time, following a previous work of the authors. We analyse several superpotentials, for which the resulting evolutions correspond to consistent inflationary scenarios.

I. INTRODUCTION

The knowledge of the convergence of the matter of the observable universe in the past, the high degree of homogeneity and isotropy of the microwave radiation background, the relative concentrations of light elements, and the large scale matter structure, allow to infer that the observable universe originates from a homogeneous early phase, beginning presumably around the Planck scale, just after a less understood quantum spacetime phase. This knowledge has lead to a classical description of the early universe by a spacetime with FRLW metric, with one function of time, the scale factor, and a time dependent scalar matter density, see e.g. [1]. This highly symmetric phase is embedded in spacetime, hence it allows for inhomogeneous perturbations of the metric and scalar fields. The quantization of these perturbations leads to quantum fluctuations, whose evolution during inflation explains the seeds of structure of the universe [2]. Thus, even though there is not a quantum theory of gravity as a predecessor of the homogeneous phase, the generation of structure seeds at the ending of the homogeneous phase is given by means of a semiclassical quantum gravity. On the other side, due to its simplicity as a mechanical system, the quantization of the homogeneous phase can be given in the framework of quantum mechanics. Thus, a quantum treatment seems to be natural for the homogeneous phase. Quantum cosmology [3] gives a canonical quantization of cosmology, and the hamiltonian constraint, generator of time reparametrizations, is implemented as a time independent Schrödinger equation, the Wheeler-DeWitt equation. Additionally, the solution of the WdW equation, the so called wave function of the universe, has to be supplemented by a way to obtain probabilities, considering the singular character of the universe. Further, as the hamiltonian operator acting on the wave function vanishes, this is a timeless theory. However, time must be reinstated in some way for the corresponding classical theory [4–7].

One way to give a time has been shown in [8], where a scalar field is fixed to be time [9], and a time dependent effective wave function is defined by the conditional probability of measuring a value of the scale factor for a given value of the scalar field. This wave function allows to compute time dependent mean values. In this work supersymmetric quantum cosmology was considered, which allows to obtain an analytic solution for the WdW equation.

The fact that the elementary constituents of matter are fermionic, besides the bosonic intermediaries of interactions and the particles that induce symmetry breaking, in a universe subject to exact Poincaré symmetry and interactions restricted by exact or approximated symmetries, has lead to the search of unified theories, which relate in a nontrivial way fermions and bosons. It is well known [10], that supersymmetry is such a theory, which has opened the way to supergravity, the supersymmetric theory of gravity, and to string theory. On the other side, the elementary particles with half-integer intrinsic angular momentum are described by quantum mechanics. Thus, supersymmetric theories must be quantum theories, and there is the belief that these theories can be a step in the way to a quantum theory of general relativity. Thus, supersymmetric quantum cosmology [11–13] is a relevant option for the study of cosmology. Supersymmetry can be formulated by the extension of spacetime translations to translations in a Grassmann, extended spacetime, which includes fermionic coordinates, called superspace [1]. The fields on this supersymmetry-superspace are calculated superfields and supergravity can be formulated as a general relativity theory on a supermanifold [10–13]. There are several formulations for supersymmetric extensions of homogeneous cosmological models [12–13]. One class of such formulations comes from dimensional reduction of four or higher dimensional supergravity theories, by considering...
only time dependent fields, and integrating the space coordinates \[15\]. The other class is obtained by supersymmetric extensions of homogeneous models, invariant under time general reparametrizations, to supersymmetric theories \[16\], or to theories invariant under general reparametrizations on a superspace of time, with anticommutative coordinates besides time \[17, 18\]. In \[19\], following the WKB method, classical equations of motion are obtained and analyzed, for the two relevant spinorial components of the wave function. Supersymmetric quantum cosmologies have spinorial wave functions, whose components satisfy a system of first degree homogeneous differential equations. In the simple cases these equations have exact solutions \[17 \[18] \[20\], and the integration constants can be assimilated in the normalization, i.e. no initial conditions are required, although the state will depend on the model.

In this work we consider the quantum cosmology of \[8\] to explore inflationary scenarios. In sections II and III we review the models, given in superfield formalism. In section IV we review the quantization of the models from \[8\], the supersymmetric Wheeler-DeWitt equations have an analytic solution, which depends on the scale factor and the superpotential. In section V we make a discussion of the problem of time. The identification of high probability paths in configuration space, to which mean trajectories correspond, leads to the identification of the scalar field as time. Thus, following \[8\], a time dependent, effective wave function, can be given. This effective wave function allows to compute mean values of the scale factor, which give a classical evolution. This scale factor is inversely proportional to the two relevant spinorial components of the wave function. Supersymmetric quantum cosmologies have spinorial wave functions, see e.g. \[10, 14\].

The large scale observable universe has been studied in general relativity by the FRLW metric with scalar fields. This is a quite general setting that could follow from a fundamental theory, and can account for inflation, primordial matter generation and structure formation, and dark energy. We consider the most studied model, the simplest one, i.e. no initial conditions are required, although the state will depend on the model.

II. SUPERSYMMETRIC FRLW MODEL WITH A SCALAR FIELD

The large scale observable universe has been studied in general relativity by the FRLW metric with scalar fields. This is a quite general setting that could follow from a fundamental theory, and can account for inflation, primordial matter generation and structure formation, and dark energy. We consider the most studied model, the simplest one, i.e. no initial conditions are required, although the state will depend on the model.

\[ I = \frac{1}{\kappa^2} \int \left\{ -\frac{3}{c^2} N^{-1} a \dot{a}^2 + 3 N k a - N a^3 \Lambda + \kappa^2 a^3 \left[ \frac{1}{2c^2} N^{-1} \dot{\phi}^2 - NV (\phi) \right] \right\} dt. \] (1)

This Lagrangian is invariant under general time reparametrizations. From this action follow the Friedmann equations and the conservation equation for a perfect fluid described by the scalar field \(\phi(t)\). i.e. in natural units and comoving gauge \(\frac{\dot{a}^2}{a^2} - \frac{\Lambda}{3} + \kappa^2 \rho + \frac{\dot{a}^2}{a^2} - \frac{\Lambda}{3} + \kappa^2 \rho = -\kappa^2 \dot{p} + \rho + \frac{3}{a} (p + \rho) = 0\), with \(\rho = \frac{1}{2} \dot{\phi}^2 + V(\phi)\) the energy density, and \(p = \frac{1}{2} \dot{\phi}^2 - V(\phi)\) the pressure for the perfect fluid \(\phi(t)\). The momenta are \(\pi_a = -\frac{6}{c^2 a^2} N^{-1} a \dot{a}\) and \(\pi_\phi = -\frac{1}{c^2} N^{-1} a^3 \dot{\phi}\).

The Hamiltonian is \(H = NH_0\), where \(H_0\) is the hamiltonian constraint, which generates time reparametrizations.

A. Supersymmetric cosmology

Supersymmetric cosmology can be obtained from one dimensional supergravity \[17\]. Here we shortly review the derivation of the supersymmetric Wheeler-DeWitt equation following \[8 \[18\]. In these works, we have formulated it as general relativity on supersymmetry-superspace, \(t \rightarrow z^M = (t, \Theta, \bar{\Theta})\), where \(\Theta\) and \(\bar{\Theta}\) are anticommuting coordinates, the so called “new” \(\Theta\)-variables \[10\]. Hence, under \(z^M \rightarrow z'^M = z^M + \epsilon^M(z)\), the superfields, see e.g. \[10 \[14\], transform as \(\delta \epsilon \Phi(z) = -\epsilon^M(z) \partial_M \Phi(z)\), and their covariant derivatives are \(\nabla_M \Phi = \nabla^A_M(z) \partial_M \Phi\). \(\nabla^A_M(z)\) is the superspace vielbein, whose superdeterminant gives the invariant superdensity \(\mathcal{E} = Sdet \nabla^A_M \delta \epsilon = (-1)^m \partial_M (\epsilon^M \mathcal{E})\). For the supersymmetric extension of the FRLW metric, in the covariant Wess-Zumino gauge \[14\], we have \(\mathcal{E} = -N - \frac{1}{2} (\Theta \bar{\psi} + \bar{\Theta} \psi)\) \[18\].

In this formulation, to the scale factor and the scalar field correspond real scalar superfields \[10 \[17\]

\[ A (t, \Theta, \bar{\Theta}) = a(t) + \Theta \lambda(t) - \bar{\Theta} \bar{\lambda}(t) + \Theta \Theta B (t); \]
\[ \Phi (t, \Theta, \bar{\Theta}) = \phi(t) + \Theta \eta(t) - \bar{\Theta} \bar{\eta}(t) + \Theta \bar{\Theta} G(t). \]

Here scalar has the usual meaning, of invariant under time reparametrizations. Note that under time reparametrizations, all the components of these superfields, i.e. \(a(t), \lambda(t), \bar{\lambda}(t), B(t)\), and \(\phi(t), \eta(t), \bar{\eta}(t), G(t)\), are scalar.
The supersymmetric extension of the action \( I \), for \( k = 0, 1 \), is \( I = I_G + I_M \), where \( I_G \) is the supergravity action, and \( I_M \) is the matter term [8, 17, 21]

\[
I_G = \frac{3}{\kappa^2} \int \mathcal{E} \left( A \nabla^2 \phi A - \sqrt{k} \mathcal{A}^2 \right) d\Theta d\bar{\Theta},
\]

\[
I_M = \int \mathcal{E} \mathcal{A}^2 \left[ -\frac{1}{2} \nabla^2 \Phi \nabla^2 \Phi + W(\Phi) \right] d\Theta d\bar{\Theta},
\]

where \( W \) is the superpotential.

### III. COMPONENT FORMULATION

The component action follow from (4) and (5), after performing the Grassmann integrals, and solving the equations of motion of the auxiliary fields \( B \) and \( G \), see [8]. Then, after the redefinitions \( \lambda \rightarrow a^{1/2} \lambda, \bar{\lambda} \rightarrow a^{1/2} \bar{\lambda}, \eta \rightarrow a^{3/2} \eta \), and \( \bar{\eta} \rightarrow a^{3/2} \bar{\eta} \)

\[
L_{Tot} = \frac{3a}{2c} \phi^2 + \frac{3}{2} N \lambda^2 - 3 \sqrt{a} k \left( \psi \lambda - \bar{\psi} \bar{\lambda} \right) - \frac{3a}{2c} N \phi^2 - 3 N \phi^2 - \frac{1}{2} N \phi^2 + \frac{3}{2} N \phi^2 \bar{\phi} \lambda + N \left( -\frac{3}{2a} \right) \bar{\lambda} + N \left( -\frac{3}{2a} \right) \phi^2 \lambda - N \left( -\frac{3}{2a} \right) \phi^2 \bar{\phi} \bar{\lambda} + N \left( -\frac{3}{2a} \right) \phi^2 \bar{\phi} \bar{\lambda},
\]

where \( W \equiv W(\phi) \), \( W' \equiv \partial_\phi W(\phi) \), and \( W'' \equiv \partial_\phi^2 W(\phi) \). The Hamiltonian is \( \mathcal{H} = \mathcal{N} \mathcal{H}_0 + \frac{1}{2} \psi S - \frac{1}{2} \bar{\psi} \bar{S} \), where \( \mathcal{H}_0 \) is the hamiltonian constraint and \( \mathcal{S} \) and \( \bar{\mathcal{S}} \) are the supersymmetric constraints [8]. The canonical Dirac Brackets are \( \{a, \pi_a\} = \{\phi, \pi_\phi\} = 1, \{\lambda, \bar{\lambda}\} = \frac{\sqrt{a}}{k}, \{\eta, \bar{\eta}\} = -c \), where \( \{,\} \) are fermionic Dirac brackets. The constraints satisfy

\[
\{S, \bar{S}\} = -2 \mathcal{H}_0, \quad \{\mathcal{H}_0, S\} = \{\mathcal{H}_0, \bar{S}\} = 0.
\]

Regarding time reparametrizations, the three constraints \( \mathcal{H}_0, S, \mathcal{S} \) are scalar.

The scalar potential in the hamiltonian \( \mathcal{H}_0 \) is [8]

\[
V_S = \frac{3 \sqrt{a}}{2} W - \frac{3 \kappa^2}{4} W^2 + \frac{1}{2} W''.
\]

Note that for \( k = 0 \), the sign of the superpotential does not matter for the scalar potential.

### IV. QUANTIZATION

Homogeneous cosmology is a mechanical system, hence it can be quantized by the methods and with the interpretation of conventional quantum mechanics. However, there are several well known problems. The first one is that this is a system that cannot be observed from the outside, hence it is not possible to perform repeated measurements in identical, observer shaped, conditions [23]. Nevertheless, observables like the scale factor, can be measured by a system that cannot be observed from the outside, hence it is not possible to perform repeated measurements.

Quantum mechanics in the Schrödinger picture tells us that the observables are represented by time independent operators, whose eigenvalues are the allowed values of these observables. The fact that the theory does not give a time evolution is only a consequence of the invariance under time reparametrizations. We have argued that time is an internal property that can be determined by the choice of a clock [5]. On the other side, the observed universe is classical [23], hence its description is given by mean values of the quantum operators. We further discuss the time problem in section [V].
A. Supersymmetric Wheeler-DeWitt equations

For the derivation of the supersymmetric Wheeler-DeWitt equations, we follow [3]. We choose a slightly different operator ordering for fermions, which gives simpler solutions. For consistency it is required that the Hamiltonian operator is hermitian, hence the supercharges must satisfy $\bar{S} = S^\dagger$ and $S = S^\dagger$. The non-zero (anti)commutators are

$$[a,\pi_a] = [\phi,\pi_\phi] = i\hbar, \quad \{\lambda,\bar{\lambda}\} = \frac{4\pi}{3}\ell_p^2, \quad \{\eta,\bar{\eta}\} = -\hbar c,$$

where $\ell_p^2 = \frac{hc}{8\pi}$ is the Planck length. For the quantization, we redefine the fermionic degrees of freedom as $\lambda = \sqrt{\hbar c}\kappa\alpha$, $\bar{\lambda} = \sqrt{\hbar c}\kappa\beta$, $\eta = \sqrt{\hbar c}\beta$, and $\bar{\eta} = \sqrt{\hbar c}\beta$. Hence the anticommutators are

$$\{\alpha,\bar{\alpha}\} = 1, \quad \{\beta,\bar{\beta}\} = -1.$$

as well as $\alpha^2 = \beta^2 = \bar{\alpha}^2 = \bar{\beta}^2 = 0$. The bosonic momenta are represented by derivatives, $\alpha$ and $\beta$ are annihilation operators, and $\bar{\alpha}$ and $\bar{\beta}$ are creation operators. We fix the ordering ambiguities by Weyl ordering, which for fermions is antisymmetric. Hence

$$\begin{align*}
\frac{1}{\sqrt{\hbar c}}S &= \frac{c\kappa}{2\sqrt{6}} \left( a^{-\frac{3}{2}}\pi_a + \pi_a a^{-\frac{1}{2}} \right) \alpha + ca^{-\frac{3}{2}}\pi_\phi\beta + \frac{3\kappa}{\sqrt{6}}a^2 W\alpha + ia^2 W\beta - i\frac{\sqrt{3}}{\kappa}a^2 \alpha - i\frac{\sqrt{2}}{\kappa}\hbar\kappa a^{-\frac{3}{2}}\alpha[\beta,\bar{\beta}], \\
\frac{1}{\sqrt{\hbar c}}\bar{S} &= \frac{c\kappa}{2\sqrt{6}} \left( a^{-\frac{3}{2}}\pi_a + \pi_a a^{-\frac{1}{2}} \right) \bar{\alpha} + ca^{-\frac{3}{2}}\pi_\phi\bar{\beta} - \frac{3\kappa}{\sqrt{6}}a^2 W\bar{\alpha} - ia^2 W\bar{\beta} + i\frac{\sqrt{3}}{\kappa}a^2 \bar{\alpha} + i\frac{\sqrt{2}}{\kappa}\hbar\kappa a^{-\frac{3}{2}}\alpha[\beta,\bar{\beta}].
\end{align*}$$

The anticommutator $\{S,\bar{S}\} = -2\hbar cH_0$ gives the quantum Hamiltonian

$$H_0 = -\frac{\hbar^2 c^2}{24} \left( a^{-1}\pi_a^2 + \pi_a^2 a^2 \right) + \frac{\hbar^2 c^2}{2} a^{-3} \pi_\phi^2 - \frac{3\hbar c^2}{2\sqrt{2}} \hbar\kappa a^{-3} \pi_\phi (\alpha\beta + \bar{\alpha}\bar{\beta}) - \frac{3\hbar c^2}{\kappa^2} a - \frac{\hbar c\kappa}{4} \hbar\kappa a^{-1}\alpha, \bar{\alpha} + \frac{3\hbar c\kappa}{4} \hbar\kappa a^{-1}\beta, \bar{\beta}
$$

$$- \frac{3\kappa}{4} a^3 W^2 + 3\sqrt{5}\kappa a^2 W + \frac{1}{2} a^3 W^2 + \frac{3}{8} \hbar\kappa^2 W[\alpha,\bar{\alpha}] - \frac{3}{8} \hbar\kappa^2 W[\beta,\bar{\beta}] + \frac{\sqrt{3}}{2\sqrt{2}} \hbar\kappa W'(\alpha\bar{\beta} - \bar{\alpha}\beta) + \frac{1}{2} \hbar\kappa W''[\beta,\bar{\beta}]
$$

$$+ \frac{3}{16} (\hbar\kappa)^2 a^{-3} (\bar{\alpha}\alpha\bar{\beta} + \alpha\bar{\alpha}\beta).$$

The Hilbert space is generated from the vacuum state $|1\rangle$, which satisfies $\alpha|1\rangle = \beta|1\rangle = 0$. Hence, there are four orthogonal states

$$|1\rangle, \quad |2\rangle = |\alpha\rangle, \quad |3\rangle = |\beta\rangle \quad \text{and} \quad |4\rangle = |\bar{\alpha}\bar{\beta}\rangle,$$

which have norms $\langle 2|2\rangle = \langle 1|1\rangle$, $\langle 3|3\rangle = -\langle 1|1\rangle$ and $\langle 4|4\rangle = -\langle 1|1\rangle$. Hence, a general state will have the form

$$|\Psi\rangle = \psi_1(a,\phi)|1\rangle + \psi_2(a,\phi)|2\rangle + \psi_3(a,\phi)|3\rangle + \psi_4(a,\phi)|4\rangle.$$

Therefore, from the constraint equation $S|\Psi\rangle = 0$, we get

$$a \left( \partial_a - \frac{3}{\hbar c} a^2 W + \frac{6\sqrt{5}}{\hbar c\kappa^2} a + \frac{1}{2} a^{-1} \right) \psi_2 - \frac{\sqrt{6}}{\kappa} (\partial_\phi - a^3 W') \psi_3 = 0,$$

$$\left( \partial_a - \frac{3}{\hbar c} a^2 W + \frac{6\sqrt{5}}{\hbar c\kappa^2} a - a^{-1} \right) \psi_4 = 0 \quad \text{and} \quad \left( \partial_\phi - \frac{1}{\hbar c} a^3 W' \right) \psi_4 = 0,$$

while from $\bar{S}\Psi = 0$

$$a \left( \partial_a + \frac{3}{\hbar c} a^2 W - \frac{6\sqrt{5}}{\hbar c\kappa^2} a + \frac{1}{2} a^{-1} \right) \psi_3 - \frac{\sqrt{6}}{\kappa} (\partial_\phi + a^3 W') \psi_2 = 0,$$

$$\left( \partial_a + \frac{3}{\hbar c} a^2 W - \frac{6\sqrt{5}}{\hbar c\kappa^2} a - a^{-1} \right) \psi_1 = 0, \quad \text{and} \quad \left( \partial_\phi + \frac{1}{\hbar c} a^3 W' \right) \psi_1 = 0.$$
B. Solutions

As the Wheeler-DeWitt equation is second order, its solutions require boundary conditions. However, in the
supersymmetric theory the equations are first order \(16\)–\(19\), and have unique solutions, which can be fixed by
consistency and normalization. The equations for \(\psi_1\) and \(\psi_4\) can be straightforwardly solved yielding the, up to
constant factors, unique solutions \(17\)

\[
\psi_1(a, \phi) = a \exp \left[ -\frac{1}{\hbar c} \left( a^3 W(\phi) - \frac{3\sqrt{\kappa} a^2}{\kappa^2} \right) \right], \quad (20)
\]

\[
\psi_4(a, \phi) = a \exp \left[ \frac{1}{\hbar c} \left( a^3 W(\phi) - \frac{3\sqrt{\kappa} a^2}{\kappa^2} \right) \right]. \quad (21)
\]

For the equations \(16\) and \(18\), the solutions for \(W(\phi) = 0\) and \(k = 0\) are \(\psi_2(a, \phi) = e^{\frac{1}{2} [f_+ (a^\kappa \phi) + f_- (a^{-\kappa} \phi)]}\) and \(\psi_3(a, \phi) = e^{\frac{1}{2} [f_+ (a^\kappa \phi) - f_- (a^{-\kappa} \phi)]}\), where \(f_\pm\) are arbitrary functions. These solutions are not defined
at \(a = 0\), unless they are trivial. Thus \(8\), we choose the solutions \(|\Psi\rangle = C_1 \psi_1(a, \phi) |1\rangle + C_4 \psi_4(a, \phi) |4\rangle\), where the
factors are arbitrary constants. The norm of this state is

\[
\langle \Psi | \Psi \rangle = \left[ |C_1|^2 \int |\psi_1(a, \phi)|^2 d\phi - |C_4|^2 \int |\psi_4(a, \phi)|^2 d\phi \right] |1\rangle |1\rangle. \quad (22)
\]

Classically \(a \geq 0\), and could be a problem for quantization, see e.g. \(6\). It could require an infinite wall. However,
the solutions \(20\) and \(21\) already vanish at \(a = 0\). For a positive superpotential, \(\psi_1\) has a bell form and tends to
zero as \(a\) increases, see figure \(1\). In this case the solution \(\psi_2\) must be set the trivial one. Oppositely, for a negative
superpotential, \(\psi_1\) tends to zero as \(a\) increases, and \(\psi_1\) must be discarded. For \(\phi \to \pm \infty\), the behavior of \(20\) and
\(21\) depends on the form of the superpotential. Therefore, we consider only positive or negative superpotentials, and
we choose \(|1\rangle |1\rangle = 1\) for positive superpotentials, and \(|1\rangle |1\rangle = -1\) for negative superpotentials. Hence

\[
|\Psi\rangle = C_1 \psi_1(a, \phi) |1\rangle, \quad \text{if} \ W(\phi) > 0, \quad (23)
\]

\[
|\Psi\rangle = C_4 \psi_4(a, \phi) |4\rangle, \quad \text{if} \ W(\phi) < 0. \quad (24)
\]

From the expansion \(15\), and \(14\), we see that these states correspond to scalars. By construction, these states
are invariant under supersymmetry transformations. Usually, a wave function corresponds to a localized particle,
with well defined position probabilities, and probability conservation. These conditions also guarantee hermiticity of
operators. On the other side, free particles cannot be localized in a finite volume, and their wave functions do not
vanish at infinity, but can be compared. If we restrict the superpotential to be an even function of \(\phi\), then any of
the two states \(23\) or \(24\) is an even function of \(\phi\), and the operators \(\pi_\phi\) and \(H_0\) are self-adjoint, even if the wave
function does not vanish at \(\phi \to \pm \infty\). In the following we will consider only such superpotentials. The self-adjoint
Hamiltonian constraint is consistent with the lack of evolution, in the Heisenberg picture

\[
\langle \Psi | \frac{d \Psi}{dt} | \Psi \rangle = \frac{i}{\hbar} \langle \Psi | [H_0, a] | \Psi \rangle = 0. \quad (25)
\]

In the following, unless otherwise stated, we will consider \(k = 0\). In this case we can write \(23\) and \(24\) as

\[
\Psi(a, \phi) = Ca \exp \left[ -\frac{1}{\hbar c} a^3 W(\phi) \right]. \quad (26)
\]

\[
\Psi(a, \phi) = C_4 a \exp \left[ \frac{1}{\hbar c} a^3 W(\phi) \right]. \quad (27)
\]

In the appendix we give the expressions for \(k = 1\).

V. TIME

The hamiltonian constraint corresponds to the invariance of the theory under time reparametrizations. This invar-
ance allows the use of arbitrary clocks, with times related by monotonic functions. Locally, space-time is homogeneous,
the geometry is flat and the laws of classical mechanics take very simple form for very simple clocks, given by harmonic
oscillators. Also elementary phenomena have simple descriptions in terms of harmonic oscillators, as follows from the
Schrödinger equation. In particular light propagation, which allows to compare local clocks with remote clocks, and sets limits to causality. Time direction cannot be changed by a change of clock. The question of the arbitrariness of the laws of physics, e.g. due to the freedom in the choice of clock, is handled by general relativity by the fundamental ansatz of independence of the coordinate system. Hence it should be possible to define the state of the universe by a set of observables independent of space-time coordinates, the superspace\textsuperscript{2}.

On the other side, quantum mechanics assigns to observables real spectra, and probabilities for their occurrence. Spectra are represented by operators acting on the linear space of probability amplitudes. A time dependence of the wave function is generated by the Hamiltonian operator. However, in a real occurring state, time requires that there is a certain energy indeterminacy. Hence, the energy spectrum must have at least two values, and time dependent mean values arise by interference among different energy states. Thus, if the Hamiltonian is a constraint and vanishes, time is undetermined. Otherwise, if there are different energy states, there must be transitions among them, hence there is an environment. If we mean by universe everything, there is no environment. However, we could consider a scalar component of the universe as time; this component must be present in the theory.

We consider the conventional interpretation of quantum mechanics for the solutions of the Wheeler-DeWitt equation, i.e. the square module of the wave function is the probability density of measuring a certain three-geometry. Thus, the wave function must give probabilities for all possible three-geometries. Further, invariance under time reparametrizations ensures that the superspaces (in the sense of geometrodynamics) corresponding to all space slices are equivalent. Hence, this wave function describes, in quantum mechanical sense, the space geometries of the whole space-time.

Furthermore, if the model has enough freedom, it should be possible to identify mean trajectories in superspace along regions around maxima of the wave function, and it should be possible to parametrize these trajectories following the idea of Misner’s supertime \cite{22}. For perturbative configurations, these trajectories should be around classical ones \cite{23}. Strictly speaking, measurements should give random values around these mean values. Moreover, this time, besides its arbitrariness due to the reparametrization freedom, would correspond itself to a sort of mean value.

In our case, the configuration space is given by the scale factor and the scalar field. If the probability density has crests, we could speak of “trajectories” in the \((a,\phi)\) plane. From (26) we see, that if we keep \(\phi\) constant, the wave function (26) has a bell form, with the maximum at \(a_{\text{max}}(\phi) = \left(\frac{c\hbar}{3W(\phi)}\right)^{1/3}\), and \(\psi_{\text{max}}(\phi) = \psi(a_{\text{max}}, \phi) = e^{-1/3}a_{\text{max}}(\phi)\).

To illustrate it, we set \(W(\phi) = \phi^{-1}(e^{-\phi^2} + 1)\), with the probability density given in figure 2, and if we take the

![Graph](https://via.placeholder.com/150)

**FIG. 1.** Profile of \(\psi(a, \phi)\), for \(\phi\) constant.

\textsuperscript{2} Note that this superspace is not related to the superspace of supersymmetry, and in the following we will mean superspace as the space of three-configurations of general relativity.
domain of the scalar field $\phi \in [0, \infty)$, then the most probable value for the universe would be somewhere at $\phi \to \infty$; hence, the system is not localized. Therefore, if the universe is at some value $\phi_0$, it will be less probable to find it for $\phi < \phi_0$, and more probable to find it for $\phi > \phi_0$, around $a_{\text{max}}(\phi)$. The previous behaviour suggests us that the field $\phi$ can take the role of time [6, 8], as far as it is causal. This corresponds to the comoving gauge, where the scalar field is constant on the spacial slices.

Hence we set $\phi \to t$, with a probability amplitude [8]

$$
\Psi(a, t) = \frac{1}{\sqrt{\int_0^\infty da |\psi(a,t)|^2}} \psi(a, t) = \sqrt{\frac{6|W(t)|}{\hbar c}} a \exp \left[ -\frac{a^3|W(t)|}{\hbar c} \right], \quad (28)
$$

normalized as $\int_0^\infty da |\Psi(a,t)|^2 = 1$. The resulting probability density [8]

$$
|\Psi(a,t)|^2 = \frac{|\psi(a,t)|^2}{\int_0^\infty da |\psi(a,t)|^2}, \quad (29)
$$

is the conditional probability of the universe being at $a$ for a given $\phi = t$. It satisfies the conservation equation

$$
\frac{d}{dt} |\Psi(a,t)|^2 + \frac{1}{3} \frac{\partial}{\partial a} \left[ a^3 |\psi(t)|^2 \right] |\Psi(a,t)|^2 = 0. \quad (28)
$$

Further, as the observed universe is classical, what we can give a meaning is to mean values, following Ehrenfest theorem. Thus, under the preceding ansatz, the classical setup arises from mean values with the probability amplitude (28). We get a time dependent scale factor

$$
a(t) = \int_0^\infty a |\Psi(a,t)|^2 da = \Gamma(4/3) \left[ \frac{\hbar c}{2|W(t)|} \right]^{1/3}. \quad (30)
$$

For the standard deviations $\Delta a$ and $\Delta \pi_a$ we get

$$
(\Delta a)^2 = \left[ \Gamma(5/3) - \Gamma(4/3)^2 \right] \left[ \frac{\hbar c}{2|W|} \right]^{2/3}, \quad (31)
$$

and

$$
(\Delta \pi_a)^2 = \frac{\hbar^2}{2} \Gamma(1/3) \left[ \frac{2|W|}{\hbar c} \right]^{2/3}, \quad (32)
$$

from which follows the uncertainty relation

$$
\Delta a \Delta \pi_a = \sqrt{\Gamma(1/3) \left[ \Gamma(5/3) - \Gamma(4/3)^2 \right]} \approx 0.53 \hbar. \quad (33)
$$

The results for this section can be given also analytically for $k = 1$, see Appendix. They involve Hypergeometric, AiryBi, and AiryBi' functions, which have exponential behaviour, and their numerical evaluation is troublesome. As we are here interested on qualitative features, we will restrict ourselves to $k = 0$. In this case, considering that the sign of the superpotential does not have consequences for the wave function, nor for the scalar potential, we will choose the superpotential as positive definite in the following.
VI. INFLATIONARY SCENARIOS

In this section we will consider several superpotentials that lead to cosmological scenarios. The scalar field has real values, from $-\infty$ to $\infty$, and as mentioned in section [\ref{superpotentials}], we require that the superpotential is a symmetric function and always positive. To describe the origin of the universe, it is convenient to choose time beginning at $t = 0$, since we are interested in the expansion of the universe, we will use a new time variable $\tau = t$.

From the time interval where the acceleration is positive, and a consistent Hubble horizon. As we will see, a very small $\lambda N \sim 60$ e-folds, evaluated considering the beginning and exit of inflation from the time interval where acceleration is positive, and a consistent Hubble horizon. As we will see, a very small $\lambda$ is required for enough e-folds, and we set $\lambda = e^{-\nu}$. Regarding other parameters, as the tensor-scalar ratio, an estimate considering an effective FRLW cosmology with one scalar, leads to inconsistent results, a more precise evaluation is needed.

We will consider superpotentials depending on the initial value of the scale potential. First $a(0) = 0$, hence $\lim_{t \to 0} W(t) = \infty$, and the wave function (28) will vanish at $t = 0$, $\Psi(a, 0) = 0$, similar to figure [1]. Hence, the universe will arise for $t > 0$ from nothing. Otherwise, for finite $W(0)$, we have $a(0) = \Gamma(4/3) \left[ \frac{\hbar c}{2W(0)} \right]^{1/3}$. We can obtain superpotentials of the first sort by $W(\phi) \to \phi^{-2\alpha} W(\phi)$, with $\alpha$ positive. From (30) we get

$$\dot{a}(t) = \frac{a(t)}{9} \left[ \frac{4W^2(t)}{W^2(t)} - 3 \frac{W(t)}{W'(t)} \right].$$

(34)

Hence the acceleration is positive if $W^2(t) > \frac{3}{4} W(t) W'(t)$. We consider inflation starting when the scale factor acceleration becomes positive, at $t = t_i$, until the exit when it becomes zero again $\ddot{a}(t_e) = 0$, with $N = \ln \frac{\alpha(t_e)}{\alpha(t_i)} = \frac{1}{3} \ln \frac{W(t_i)}{W(t_e)}$ e-folds. Note that in this way, as the scale factor is scalar, $N$ is an independent of the time parametrization.

An indicative of the feasibility of inflation, is the horizon

$$(aH)^{-1} = -\frac{3 \times 2^{1/3} W(t)^{4/3}}{\Gamma(4/3)(c\hbar)^{1/3} W(t)}. \quad (35)$$

In order to have an effective model in more familiar terms, we can substitute the scale factor in the Friedmann equations to obtain the energy density and the pressure for $k = 0, 1$, as well as a time dependent potential, $V(t) = \rho(t) - p(t)/c^2$; this potential does not coincide with the scalar potential [3].

For simplicity, in the following we will consider an adimensional time $t \to \tau = t/t_p$, in Planck time unities, $t_p = \sqrt{\hbar G/c^5}$ is the Planck time. However, the dot will continue to denote time derivatives, $\dot{t} \equiv \frac{dt}{dt}$. A. Superpotentials

Here, we will consider two types of superpotentials, which show the main features:

Gaussian superpotentials

$$W(\tau) = \frac{c^4 M_p^3}{\hbar^2} \tau^{-2\alpha} \left( e^{-\tau^2} + \lambda \right), \quad (36)$$

and Step superpotentials

$$W(\tau) = \frac{c^4 M_p^3}{\hbar^2} \tau^{-2\alpha} \left( \frac{1}{\tau^2 + \lambda} + 1 \right), \quad (37)$$

where $M_p = \sqrt{\frac{\hbar G}{c^3}}$ is the Planck mass, $\alpha$ positive integer, and $\lambda$ positive constant. We consider representative cases which reproduce an inflationary period with $N \sim 60$ e-folds, evaluated considering the beginning and exit of inflation from the time interval where acceleration is positive, and a consistent Hubble horizon. As we will see, a very small $\lambda$ is required for enough e-folds, and we set $\lambda = e^{-\nu}$. Regarding other parameters, as the tensor-scalar ratio, an estimate considering an effective FRLW cosmology with one scalar, leads to inconsistent results, a more precise evaluation is needed.

Initially, for these superpotentials, the behaviour is as follows: for $\alpha = 0$ the scale factor has a value $a(0) > 0$ with a very small positive acceleration, and for $\alpha \geq 1$, $a(0) = 0$ with a vanishing acceleration. In order to see if there is a consistent inflation, we evaluate the values of the scale factor at $\tau = 0$, and the corresponding acceleration. If $a(0) > 0$, then inflation begins at the time $\tau = \tau_i$, when the acceleration becomes positive, until it becomes negative, at $\tau = \tau_f$. If the acceleration is positive already at $\tau = 0$, inflation begins right away. Further, if $a(0) = 0$ with
positive acceleration from the beginning, then at the exit there are infinite e-folds, unless we discard an initial full quantum period. Otherwise, if \( a(0) = 0 \) and \( \dot{a}(0) < 0 \), then inflation will begin as soon as the acceleration becomes positive, at \( \tau_i \), and finish when the acceleration becomes negative again, at \( \tau_f \), with \( N = \ln \frac{a(\tau_f)}{a(\tau_i)} \). In the following we will consider natural unities \( c = 1 \) and \( h = 1 \), i.e. \( M_P = 1/\sqrt{G} \), and the Planck and time length \( l_P = t_P = M_P^{-1} \).

### B. Gaussian superpotentials

The scale factor for (36) is

\[
a(\tau) = \frac{\Gamma(4/3)\tau^{2\alpha/3}l_P}{2^{1/3}(e^{-\tau^2} + \lambda)^{1/3}}.
\]

Therefore, for \( \alpha = 0 \), neglecting \( \lambda \) with respect to 1, we have, figure 3

\[
a_0 = a(0) = \frac{\Gamma(4/3)l_P}{2^{1/3}(1 + \lambda)^{1/3}} \approx \frac{\Gamma(4/3)l_P}{2^{1/3}},
\]

with a positive acceleration, figure 4

\[
\ddot{a}(0) \approx \frac{2^{2/3}\Gamma(4/3)l_P}{3}.
\]

On the other side, for higher times, the acceleration vanishes around the value at which the superpotential (36) becomes nearly constant, at \( e^{-\tau^2} \approx \lambda \), i.e. \( \tau \approx \sqrt{\nu} \), see figure 5 and the scale factor becomes

\[
a_f \approx \frac{\Gamma(4/3)l_P}{2^{1/3}\lambda^{1/3}},
\]

Thus, for \( \alpha = 0 \), the e-fold number satisfies

\[
N \lesssim \ln \frac{a_f}{a_0} = -\frac{1}{3} \ln \lambda = \nu/3.
\]

Hence, a value \( \lambda \sim 10^{-79} \) is necessary for \( \sim 60 \) e-folds. From (39), we obtain that \( a(0) \sim 0.7l_P \), with an initial velocity \( \dot{a}(0) = 0 \). The initial acceleration is \( \dot{a}(0) \sim 0.5M_P \), hence inflation begins right away, until the acceleration slows and becomes negative at \( \tau_f \approx 13.4 \), see figure 5. We get \( N = \ln \frac{a(\tau_f)}{a(\tau_i)} \approx 60.1 \), consistently with (42). The Hubble parameter is plotted in figure 6 and the horizon in figure 7.

For \( \alpha > 0 \), \( a(0) = 0 \), and at early times, \( \tau \ll 1 \), the scale factor can be approximated by

\[
a(\tau) \approx \frac{\Gamma(4/3)l_P}{2^{1/3}} \tau^{2\alpha/3} e^{\frac{\nu}{3}},
\]

and \( \ddot{a}(\tau) \approx \frac{1}{2} \Gamma(4/3)2^{2/3}\alpha(2\alpha - 3)\tau^{2\alpha/3 - 2}M_P \). For \( \alpha = 0, 1, 2, 3 \) the acceleration is shown, for early times in figure 6 and for late times in figure 7. Thus we have:

For \( \alpha = 1 \), \( \dot{a}(0) = -\infty \), then grows and for \( \lambda \sim 10^{-76} \), the acceleration becomes positive at \( \tau_i \sim 0.4 \), with \( N = \ln \frac{a(\tau_f)}{a(\tau_i)} \sim 60.2 \).

For \( \alpha = 2 \), \( \dot{a}(0) = \infty \), decreases almost to zero for \( 0 < \tau \lesssim 0.4 \), and then grows again. Hence as \( a(0) = 0 \), \( N \) is unbounded. However, the initial acceleration is very quickly decreasing, see figure 6 and we could discard this initial period. In this case, if we consider inflation starting at \( \tau \sim 0.4 \), we get \( N \sim 60.2 \).

For \( \alpha = 3 \), \( \dot{a}(0) \simeq 1.4M_P \), and we have a situation similar to \( \alpha = 2 \). If we consider inflation starting at \( \tau \sim 0.5 \), we get \( N \sim 60.5 \).

In figure 7 we show the plots of the scale factor for the previous cases in logarithmic scale. Even if the previous results are alike, the event horizons, plotted in figure 7 are quite different.

With the previous results, we can compute the effective matter density and pressure for a perfect fluid. It turns out that the resulting potential differs from the scalar potential following from the hamiltonian (8). Moreover, for the gaussian potential and \( k = 0 \), in the analysed cases, the fluid has phantom energy. The effective potentials for \( k = 0 \) are given in figure 8.

As we can see from figures 7 and 12, Hubble’s radius decreases during a time interval consistent with that of inflation solving the sphericity problem. Hubble’s radius must go down during inflation so that we have less structures causally connected for those times and, subsequently, at late times Hubble’s radius grows and more structures are causally reconnected again [24]. We must note that different combinations for the values of \( \alpha \) and \( \lambda \) give similar behaviours but some correspond to a slightly bigger radius and thus to a longer inflationary period, allowing us to tune the duration of inflation according to data. The minimum, along with an adequate neighbourhood, in Hubble’s radius for each curve could be interpreted as a reheating epoch once some matter is introduced in a more realistic model.
C. Step superpotentials

For the step superpotentials (37), we have

\[ a(\tau) = \frac{\Gamma(4/3)}{2^{1/3}} l_P \tau^{2\alpha/3} \left( (\tau^2 + \lambda)^{-1} + 1 \right)^{1/3} \]  

(43)

As well as for gaussian superpotentials, for \( \alpha = 0 \) the scale factor initial value is (39), and for large time values, \( \tau \gg 1 \), the scale factor tends to (41), figure 9. Hence, the \( e \)-folds satisfy as well (42). The situation is similar as for the gaussian superpotential, the initial velocity and acceleration are zero, \( \dot{a}(0) = 0 \), \( \ddot{a}(0) = 0 \), and inflation begins at \( \tau \approx 0.51 \), when the acceleration becomes negative. For \( N \sim 60 \) a value \( \lambda \sim 10^{-80} \) is required. In figures 10, 11, 12, and 13, the acceleration, horizon, and Hubble parameter are plotted.

For \( \alpha = 1 \) the initial acceleration is minus infinite, and becomes positive at \( \tau \sim 10^{-14} \), and further negative at \( \tau \sim 3.8 \), see figures 10 and 11. The 60 \( e \)-folds are reached for \( \lambda \sim 10^{-53} \). The horizon and the Hubble parameter are plotted in figures 12 and 13. Note that the time scales are different than for previous case.

VII. DISCUSSION

Quantum cosmology gives a canonical quantization of general relativity in the Schrödinger picture. The operators give the spectra of the observables. The Hamiltonian constraint leads to a zero energy time independent Schrödinger equation, the Wheeler-DeWitt equation. There is no time dependent Schrödinger equation to generate a time dependence for the wave function. Rather, the wave function must give the probability amplitudes for all possible space configurations, at any time. On the other side, the observed universe is classical and has time, and classical physics should follow from quantum physics, hence a time evolution should follow from the quantum description. Further, the Wheeler-DeWitt equation is a second order scalar equation, hence it requires boundary conditions for the wave function, which should ensure also normalization. In the supersymmetric case there is a system of first order equations, whose solutions are spinorial wave functions, see e.g. [12] [13]. A particular meaning of the components of these wave functions has not been given; it would quickly lead to a Machian discussion. Frequently, there are only two nonvanishing components, see e.g. [20], given by real exponentials with opposite exponent sign. Here we have
considered a homogeneous theory with a minimally coupled scalar field. It is well known, that such a model describes surprisingly good the inflationary era. In this case, the solution of the supersymmetric Wheeler-DeWitt equation (16)-(19) has four components, two of them have analytic expressions (20),(21), and only one is clearly consistent, normalizable. As the equations are homogeneous, and (20),(21) decouple, the other components can be taken to be trivial. The form of the solution (26), suggests the ansatz that, in a certain gauge, time can be given by the scalar field, and trajectories for the observables are mean values on an effective wave function that gives conditional probabilities given a value of the scalar (30). Thus, for the scale factor we get an evolution $a(t)$, and we can perform time reparametrizations, considering that it is scalar $a'(t') = a(t)$. In general, the resulting classical evolution does not correspond to a perturbative setting. It remains the question of how far can we get classical trajectories from quantum states in this way, in particular for late times. In this formulation, we have considered several inflationary scenarios, with superpotentials (36) and (37). These superpotentials have a factor $\tau^{-2\alpha}$ which, for $\alpha \neq 0$, gives a wave function that vanishes for $\tau = 0$, and produces a path of increasing probabilities, which could be seen as a time direction. The inflation exit is implemented in the superpotentials through a constant $\lambda \sim e^{-3\mathcal{N}}$, where $\mathcal{N}$ are the required $e$-folds. For each type of superpotential we considered several cases, plotted in the graphics 5-10. From these results we can expect that this sort of models could be explored for a realistic scenario, perhaps with more scalar fields, as in [18]. Moreover, inhomogeneous perturbations should be considered, to compute the evolution of the fluctuations. An interesting perspective presents the study of this formalism for dark energy.

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FIG. 7. Comoving Hubble radius \((aH)^{-1}\) for gaussian superpotentials, the graphics are logarithmic for clarity.

FIG. 8. Effective potentials from Friedmann equations, for \(k = 0\), and \(\alpha = 0, 1, 2, 3\) the potential becomes \(\infty\) at \(\tau = 0\).

VIII. APPENDIX

In this appendix we give, for \(k = 1\), the expressions for the normalization factor for the wave function (20) for \(k = 1\), and the time dependent scale factor (30). The normalization factor of

\[
\psi(a, \phi) = |C|a \exp \left[ \frac{1}{\hbar c} \left( -a^3 |W(\phi)| + \frac{3\sqrt{k}a^2}{\kappa^2} \right) \right],
\]

is given by

\[
|C|^{-2} = \frac{\hbar c}{6} \int_{-\infty}^{\infty} \frac{1}{|W(\phi)|} \left[ 32F_2 \left( \frac{1}{2}, 1; \frac{1}{3}, \frac{2}{3}; \frac{8}{\hbar c k^6 W^2(\phi)} \right) + 4\pi \left( \frac{1}{\hbar c k^6 W(\phi)^2} \right)^{1/3} e^{-\frac{4}{\hbar c k^6 W^2(\phi)}} \right] d\phi,
\]

where \(Bi\) is the Airy function of second kind. Further, for the denominator of (28)

\[
\int_{0}^{\infty} da |\psi(a, \phi)|^2 = \frac{\hbar c}{18W(\phi)} \times \left\{ 3 \cdot 2F_2 \left( \frac{1}{2}, 1; \frac{1}{3}, \frac{2}{3}; \frac{8}{\hbar c k^6 W^2(\phi)} \right) + 4 \times 6^{1/3} \pi \left( \frac{1}{\hbar c k^6 W(\phi)^2} \right)^{2/3} e^{-\frac{4}{\hbar c k^6 W^2(\phi)}} \right\}
\]

\[
\times \left[ 6^{1/3} Bi \left( \left( \frac{6}{\hbar c k^6 W(\phi)^2} \right)^{2/3} \right) - \left( \frac{6}{\hbar c k^6 W(\phi)^2} \right)^{1/3} Bi' \left( \left( \frac{6}{\hbar c k^6 W(\phi)^2} \right)^{2/3} \right) \right].
\]
from which follows

\[
a(t) = \left\{ \begin{array}{l}
9 \sqrt{3} \kappa^4 (c \hbar)^{2/3} W(t)^{4/3} \quad 2 F_2 \left( \begin{array}{c}
\frac{1}{2}, \frac{2}{3} \\
\frac{1}{2}, \frac{1}{3}
\end{array} ; \frac{2}{3} \right) \frac{8}{c \kappa^6 \hbar W(t)^2} e^{-\frac{c \kappa^6 \hbar W(t)^2}{2}} \\
+ 2^{2/3} \pi \left[ 24 + c \kappa^6 \hbar W(t)^2 \right] \text{Bi} \left( \left[ \frac{6}{c \kappa^6 \hbar W(t)^2} \right]^{2/3} \right) \\
+ 8 \sqrt{2} 2^{2/3} \pi \kappa^2 \sqrt{c \kappa^6 \hbar W(t)^2} \text{Bi}' \left( \left[ \frac{6}{c \kappa^6 \hbar W(t)^2} \right]^{2/3} \right) \end{array} \right} \\
+ \left\{ \begin{array}{l}
3 \sqrt{3} \kappa^6 (c \hbar)^{2/3} W(t)^{7/3} \quad 2 F_2 \left( \begin{array}{c}
\frac{1}{2}, \frac{1}{3} \\
\frac{1}{2}, \frac{1}{3}
\end{array} ; \frac{2}{3} \right) \frac{8}{c \kappa^6 \hbar W(t)^2} e^{-\frac{c \kappa^6 \hbar W(t)^2}{2}} \\
+ 4 \sqrt{2} 2^{2/3} \pi \kappa^4 \sqrt{c \kappa^6 \hbar W(t)^2} \left( \left[ \frac{6}{c \kappa^6 \hbar W(t)^2} \right]^{2/3} \right) \\
+ 12 2^{2/3} \pi \kappa^2 W(t) \text{Bi} \left( \left[ \frac{6}{c \kappa^6 \hbar W(t)^2} \right]^{2/3} \right) \end{array} \right. 
\right\}
\]

Due to the exponential behaviour of the hypergeometric and Airy functions, which appear in the numerator and denominator in these expressions, it is difficult to handle them numerically.

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FIG. 11. Acceleration $\ddot{a}(\tau)$ for step superpotentials.

FIG. 12. Comoving Hubble radius $(aH)^{-1}$ for step superpotentials.

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FIG. 13. Hubble parameter $H(\tau)$ for step superpotentials.