Research Article

Design of a Performance-Adaptive 1-Parameter Tuning PID Controller

Yoichiro Ashida, Shin Wakitani, Toru Yamamoto

1Department of Electrical Engineering and Computer Science, National Institute of Technology Matsue College, 14-4 Nishikamacho, Matsue city, Shimane, Japan
2Department of System Cybernetics, Graduate School of Engineering, Hiroshima University, 1-4-1 Kagamiyama, Higashihiroshima city, Hiroshima, Japan

ARTICLE INFO

ABSTRACT

Many self-tuning Proportional–Integral–Derivative (PID) controllers have been proposed. However, these methods are rarely employed to actual systems because of low reliability of an on-line PID parameter estimator. In this work, a performance adaptive controller that has two stages of PID parameters tuning mechanisms is proposed. One of the tuners is a 1-parameter tuner using the recursive least squares. The other tunes PID parameters using ordinary least squares. The effectiveness of the proposed method is evaluated by an experimental example.

1. INTRODUCTION

Most of the controllers in industries have fixed control parameters. However, the controllers cannot obtain good control performances against time-variant systems. To tackle this problem, self-tuning controllers [1,2] have been proposed. In these controllers, control parameters are tuned adaptively and can maintain a good control performance to time-variant systems. However, it is difficult to use these methods to actual systems because of low reliability of an on-line estimator. In addition, it is not preferred to tune many control parameters simultaneously in terms of the safeness of a closed-loop system.

Especially in process industries, Proportional–Integral–Derivative (PID) controllers [3] have been widely used. The reasons why PID controllers have been widely applied include that: (1) PID controllers have simple structure, and (2) physical meanings of the control actions are clear. Therefore, some self-tuning PID control schemes [2] have been proposed.

In this work, a performance-adaptive controller that has two stages of PID parameters tuning mechanisms is proposed. One of the tuners is a 1-parameter tuner which calculates only the proportional gain using the recursive least squares. The proportional gain depends only on a static gain of the controlled object, thus the reliability of an identification of the proportional gain is higher than the other gains. The other is a PID parameters tuner using ordinary least squares. When the 1-parameter tuner cannot maintain a good control performance, the PID parameters tuner determines new PID parameters to maintain a desired control performance. The effectiveness of the proposed method is evaluated by an experimental example.

2. THE TUNING METHOD OF PID PARAMETERS

The PID parameters tuning method is described at this section. A velocity-type PID controller in discrete time domain is given as follows:

\[
\Delta u(k) = k \left\{ \frac{T_p}{T_i} e(k) - \Delta y(k) - \frac{T_d}{T_i} \Delta^2 y(k) \right\},
\]

where \(k\), \(T_p\), and \(T_d\) are the proportional gain, the integral time and the derivative time, respectively. \(\Delta\) denotes the differencing operator defined by \(\Delta := 1 - z^{-1}\), where \(z^{-1}\) is the shift operator which means \(z^{-1} y(k) = y(k-1)\). In addition, \(u(k)\) is the input signal and \(y(k)\) is the output signal. Furthermore, \(T_s\) denotes the sampling time, and \(e(k)\) denotes the output error signal defined by \(e(k) := r(k) - y(k)\) where \(r(k)\) denotes the reference signal. Equation (1) can be rewritten as

\[
\Delta u(k) = K_p e(k) - K_i \Delta y(k) - K_d \Delta^2 y(k),
\]

where \(K_p\), \(K_i\), and \(K_d\) are proportional, integral, and derivative gains, respectively. By assuming \(K_i \neq 0\), Equation (3) can be obtained by rewriting Equation (2).

\[
r(k) = \frac{1}{K_i} \Delta u(k) + \frac{K_p + K_i + K_d}{K_i} y(k) - \frac{K_p + 2K_i}{K_i} y(k-1) + \frac{K_d}{K_i} y(k-2).
\]
Generalized output \( \Phi(k) \) is defined as follows:

\[
\Phi(k) := a_i \Delta u(t) + a_i \{ y(k) - y(k-2) \} + a_i \{ y(k-1) - y(k-2) \} + y(k-2). 
\] (4)

Where coefficients \( a_i, a_s, \) and \( a_d \) are defined as follows:

\[
\begin{align*}
  a_i := & \frac{1}{K_i} \\
  a_s := & \frac{K_p + K_i + K_d}{K_s} \\
  a_d := & \frac{K_p + 2K_d}{K_s}
\end{align*}
\] (5)

From (3) to (5), the following relationship can be obtained:

\[
r(k) = \Phi(k). 
\] (6)

In the proposed method, it is desired to make the system output \( y(k) \) track the reference model output \( y_r(k) \) which is defined by:

\[
y_r(x) = G_m(z^{-1})r(k), \quad G_m(z^{-1}) := \frac{z^{-d+i\sigma}}{P(z^{-1})},
\] (7)

where \( d \) denotes the time-delay of the controlled object, and \( d \) is assumed to be known. \( P(z^{-1}) \) is given as follows:

\[
P(z^{-1}) = 1 + p_1 z^{-1} + p_2 z^{-2}. 
\] (9)

Where \( p_1 \) and \( p_2 \) are determined by following equations:

\[
\begin{align*}
  p_1 = & \exp \left( -\frac{\rho}{2\mu} \right) \cos \left( \frac{\sqrt{4\mu - 1} - \rho}{2\mu} \right) \\
  p_2 = & \exp \left( -\frac{\rho}{\mu} \right) \\
  \rho = & \frac{T_s}{\sigma} \\
  \mu = & 0.25(1 - \delta) + 0.51\delta
\end{align*}
\] (10)

\( \sigma \) and \( \delta \) denote the rise-time and the damping coefficient, respectively, and \( \mu \) is adjusted by \( \delta \). When \( \delta = 0 \), \( G_m(z^{-1}) \) indicates the Binomial model, and when \( \delta = 1 \), \( G_m(z^{-1}) \) indicates the Butterworth model. In practice, the value of \( \delta \) is considered to be designed between 0 and 2, and by using bigger \( \delta \), the response becomes oscillating.

The evaluation function \( J \) is defined as follows:

\[
J = \sum_{j=0}^{N} \varepsilon(j)^2 ,
\] (11)

where \( N \) denotes the number of data and \( \varepsilon(k) \) is defined as follows:

\[
\varepsilon(k) := y(k) - G_m(z^{-1})\phi(k).
\] (12)

By minimizing the evaluation function \( J \), the following relationship can be obtained:

\[
G_m(z^{-1})\phi(k) \rightarrow y(k).
\] (13)

From Equation (4), parameters to be optimized are \( a_i, (i = 1, 2, 3) \). When the minimization has been finished, it leads the following relationship:

\[
y(k) \rightarrow G_m(z^{-1})\phi(k),
\] (14)

Therefore, the reference response is realized by using the optimized \( a_i \). To apply a PID controller, PID parameters are derived from \( a_i \) as follows:

\[
k_1 = \frac{2a_1 + a_2 - 2}{a_1} \\
T_1 = (2a_1 + a_2 - 2)T, \\
T_s = \frac{1 - a_1 - a_2}{2a_1 + a_2 - 3}T.
\] (15)

### 3. DESIGN OF A PERFORMANCE DRIVEN PID CONTROLLER

#### 3.1. 1-Parameter Tuning

Both 1-parameter tuning and retuning of all PID parameters are realized by using the PID tuning method described at the previous section. In the 1-parameter tuning, only overall gain of the PID controller \( k_1 \) is tuned. From Equation (15), \( k_1 \) is rewritten as follows:

\[
k_1 = \frac{T_1}{a_1T_s}. 
\] (16)

Furthermore, \( T_1 \) and \( T_s \) are constant values, and \( a_1 \) and \( a_2 \) are also constant values. Therefore, 1-parameter tuning can be realized by tuning only \( a_1 \). In this paper, a recursive least squares method [4] is applied to minimize \( \varepsilon(k) \). Estimated parameter vector \( \hat{\theta}(k) \) and data vector \( \psi(k) \) are defined as follows:

\[
\hat{\theta}(k) := \hat{a}_1(k), \quad \psi(k) := G_m(z^{-1})\Delta u(k),
\] (17)

\[
\hat{\theta}(k) = \hat{\theta}(k - 1) + K(k)e(k)
\] (19)

\[
K(k) = \frac{P(k - 1)\psi(k)}{\omega(k) + \psi^T(k)P(k - 1)\psi(k)},
\] (21)

\[
P(k) = \frac{1}{\omega(k)}[P(k - 1) - K(k)\psi(k)\psi^T(k)P(k - 1)].
\] (22)

#### 3.2. Control Performance Evaluation

When 1-parameter tuning cannot maintain good control performance, all PID parameters should be tuned. In this subsection, the evaluation criterion of control performance is described. A reference
signal is assumed to be the step-type signal. The criterion of control performance is defined as follows:

\[ e_{\text{G}_k}(k) = \frac{1}{k - k_0 + 1} \sum_{i=k_0}^{k} |G_m(z^{-1})r(i) - y(i)|, \]

(23)

where \( k_0 \) denotes a recent step when a reference signal is changed. \( e_{\text{G}_k}(k) \) denotes an average of errors between controlled output and reference model output. All PID parameters are tuned when \( e_{\text{G}_k}(k) \) satisfies the following condition:

\[ e_{\text{G}_k}(k) > \beta |\Delta r(k_0)|, \]

(24)

where \( \alpha \) is a user-specified parameter which determines a threshold. \( |\Delta r(k_0)| \) is introduced because \( e_{\text{G}_k}(k) \) have proportional relationship to the \( |\Delta r(k_0)| \).

### 3.3. A Method of PID Parameters Tuning

In order to tune all PID parameters, the PID tuning method described at Section 2 are used. The optimized parameters are \( a_1, a_2, \) and \( a_3 \). In addition, least squares method is used to minimize \( e(k) \). In the ordinary least squares, the data between \( k_0 \) and \( k \) steps are used.

### 4. AN EXPERIMENTAL RESULT

The usefulness of the proposed self-tuning method is experimentally evaluated on a pilot-scale temperature control system. Figure 1 shows an appearance of the controlled object, and the schematic figure is illustrated in Figure 2. In this experimental system, two heaters are secured on a steel plate. These heaters work synchronously, corresponding to the input signal from the computer. One thermo-couple is also prepared, and the measured temperature of the steel is sent to the computer as the system output signal. The control objective is to regulate the temperature of the steel to the desired reference signal by manipulating the power of the heater. In this experimental result, the sampling time \( T_s \) is 1 s and the reference signal is changed alternately from 70 (°C) in the temperature and 100 (°C).

To simulate heater deterioration, the relationship between controlled input \( u(k) \) and the voltage of the power conditioner \( v(k) \) is varied as follows:

\[
\begin{align*}
    v(k) &= 1 + \frac{4}{100} u(k) \quad (k \leq 200) \\
    v(k) &= 1 + \frac{4 - 3(k - 200)}{100 \cdot 600} u(k) \quad (200 < k \leq 800) \\
    v(k) &= 1 + \frac{1}{100} u(k) \quad (k < 800)
\end{align*}
\]

(25)

The control result by using the proposed performance-adaptive controller is shown as Figure 3, and the trajectories of PID parameters corresponding to Figure 3 is shown in Figure 4. In this result, the initial PID parameters were determined by using the PID tuning method described at Section 2 as follows:

\[ k_c(0) = 0.70, T_i(0) = 28.9, T_d(0) = 4.67. \]

(26)

Parameters of the proposed method were set as follows: \( \alpha = 1, \beta = 0.2, d = 8, w(k) = 0.995, \sigma = 30, \) and \( \delta = 0. \) From Equation (26), initial estimated parameter was determined as \( \theta(0) = 41.3. \)

Figure 3 indicates that the control performance gradually deteriorated after 200 steps. Figure 4 shows that only \( k_c \) was tuned before

---

**Figure 1** Appearance of the experimental temperature control system.

**Figure 2** Schematic figure of the experimental temperature control system.

**Figure 3** Experimental control result by using the proposed control method.
863 steps, and all PID parameters are tuned at 863 steps. The proportional gain $k_c$ became larger on the 1-parameter tuning period. Considering that the system gain became smaller on the period as Equation (25), the variation of the $k_c$ is correct. In addition, after the tuning of the all parameters, the trajectory of the control output is likely the same as the reference trajectory.

5. CONCLUSION

In this paper, a performance-adaptive 1-parameter tuning PID controller is proposed, and effectiveness of the proposed method is evaluated by the pilot-scale temperature control system. The proposed performance adaptive controller has two stages of PID parameters tuning mechanisms. The features of the proposed method are summarized as follows:

- The proposed method is the 1-parameter tuning method, and only $k_c$ is tuned.
- All PID parameters are retuned only when control performance deteriorates.
- Both the 1-parameter tuning and the retuning method of all PID parameters are based on the same PID tuning scheme.

From these characteristics, the proposed method can be easily understood. However, the effectiveness of the 1-parameter tuning in this paper does not be proved analytically. This is a future work.

CONFLICTS OF INTEREST

There is no conflicts of interest.

REFERENCES

[1] D.W. Clarke, P.J. Gawthrop, Self-tuning control, Proc. IEE 126 (1979), 633–640.
[2] Y. Ashida, S. Wakitani, T. Yamamoto, Design of an implicit self-tuning PID controller based on the generalized output, IFAC-PapersOnLine 50 (2017), 13946–13951.
[3] K.J. Åström, T. Hägglund, Advanced PID Control, International Society of Automation, North Carolina, 2006.
[4] E.B.M. Costa, G.L.O. Serra, Self-tuning robust fuzzy controller design based on multi-objective particle swarm optimization adaptation mechanism, J. Dyn. Syst. Meas. Control 139 (2017), 12.
[5] G.C. Goodwin, K.S. Sin, Adaptive filtering prediction and control, Prentice-Hall, New Jersey, 1984.
He received his B.Eng. and M.Eng. degrees from the University of Tokushima, Japan, in 1984 and 1987, respectively, and the D.Eng. degree from Osaka University, Japan, in 1994. He is currently a Professor with the Department of System Cybernetics, Graduate School of Engineering, Hiroshima University, Japan. He was a Visiting Researcher with the Department of Mathematical Engineering and Information Physics, University of Tokyo, Japan, in 1991, and he was an Overseas Research Fellow of the Japan Society for Promotion of Science (JSPS) with the University of Alberta for six months in 2006. His current research interests are in the area of self-tuning and learning control, data-driven control, and their implementation for industrial systems. He was a General Chair of ADCONIP 2014 and SICE Annual Conference 2019.