TELEPEN SOUTH project:
Measurement of the Earth gravitomagnetic field
in a terrestrial laboratory

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Abstract. We will expose a preliminary study on the feasibility of an experiment leading to a direct measurement of the gravitomagnetic field generated by the rotational motion of the Earth. This measurement would be achieved by means of an appropriate coupling of a TELEscope and a Foucault PENdulum in a laboratory on ground, preferably at the SOUTH pole. An experiment of this kind was firstly proposed by Braginski, Polnarev and Thorne, 18 years ago, but it was never re-analyzed.

1 Introduction

The search for measurable effects of a gravitational field due to the angular momentum of the source, within the framework of General Relativity (GR), continues. In the weak and slow motion approximation of GR, the gravitomagnetic part of the gravitational potential gives rise to the Lense-Thirring effect [1]. The actual detection of this effect is entrusted both to Earth satellites experiments and to Earth based laboratory experiments. So far, the only positive indirect result concerns an experiment of the first kind, the precession of the nodes of the orbit of the LAGEOS satellite [2].

On the other hand, in the next years the space mission Gravity Probe B (GPB) is planned to fly, carrying gyroscopes which should verify the Lense-Thirring precession effect directly [3],[4]. Moreover, different possibilities connected both with the clock effect and the gravitational Sagnac effect have been considered [5],[6].

Recently, after the completion of this work, a Earth based laboratory experiment to test directly the quadratic terms in the angular momentum of a gravitational potential, has been proposed by Tartaglia (see [7]). This proposal deserves further study. However, in what follows, I will remind a different Earth based laboratory experiment to test directly the Lense-Thirring effect, which was firstly proposed by Braginski, Polnarev and Thorne, 18 years ago [8], but never reconsidered or re-analyzed.
2 Gravitomagnetic Maxwell-like equations

First at all, we give a fast review of the well-known linear and slow motion approximation of GR. Our starting point will be the Einstein field equations:

$$R_{ab} - \frac{1}{2} g_{ab} R = -\frac{8\pi G}{c^4} T_{ab}. \quad (1)$$

If the gravitational field is weak, then the metric tensor can be approximated by

$$g_{ab} \simeq \eta_{ab} + h_{ab}, \quad (2)$$

where $$\eta_{ab}$$ is the flat Minkowski spacetime metric. Now define the gravitational potentials as

$$h_{ab} = h_{ab} - \frac{1}{2} \eta_{ab} \eta. \quad (3)$$

The analogy with Maxwell-Lorentz electrodynamics can be made explicit by writing the linear gravitational equations in terms of first-order derivatives of the gravitational potential, i.e., acceleration fields. With this aim in view, we first introduce the object

$$G^{abc} = \frac{1}{4} (\overline{h}^{ab,c} - \overline{h}^{ac,b} + \eta^{ab} \overline{h}^{cd,d} - \eta^{ac} \overline{h}^{bd,d}) . \quad (4)$$

and impose the four harmonic de Donder gauge conditions:

$$\overline{h}^{ab,b} = 0. \quad (5)$$

From (4) and (5) reads:

$$G^{abc} = \frac{1}{4} (\overline{h}^{ab,c} - \overline{h}^{ac,b}), \quad (6)$$

and retaining only linear terms, one obtains the weak field equations in terms of the object $$G^{abc}$$, in which only first-order derivatives of the gravitational potential occur:

$$\frac{\partial G^{abc}}{\partial x^c} = -\frac{4\pi G}{c^4} T_{ab}. \quad (7)$$

After defining the gravitoelectric Newtonian scalar potential $$\Phi$$ and the gravitomagnetic vector potential $$a$$ as

$$\Phi := -\frac{c^2 \overline{h}^{00}}{4} \quad (8)$$

$$a^i := \frac{c^2 \overline{h}^{0i}}{4}, \quad a = (a^1, a^2, a^3), \quad (9)$$

let us introduce new symbols and substitute equations (8) and (9) into equation (3), to get the gravitoelectric Newtonian field $$g$$ as

$$g = -\nabla \Phi - \frac{1}{c} \frac{\partial a}{\partial t}, \quad (10)$$
where
\[ g^i = c^2 G^{00i} = -\frac{\partial \Phi}{\partial x^i} - \frac{1}{c} \frac{\partial a^i}{\partial t}, \]
(11)
and
\[ G^{00i} = \frac{1}{4} \left( \theta^{00,i} - \theta^{0i,0} \right), \]
(12)
and to get the gravitomagnetic field \( b \) as
\[ b = \nabla \wedge a, \quad c^2 G^{0ij} = a^i,j - a^j,i, \]
(13)
Now, performing the first order slow motion approximation for the energy momentum tensor we neglect quadratic terms in velocity, i.e., neglect the stress part of the energy-momentum tensor. Thus, the energy-momentum tensor will only have the components
\[ T^{00} = \rho c^2 \]
(14)
and
\[ T^{0i} = \rho c v^i. \]
(15)
Thus, when the first order effects of the motion of the sources are taken into account, one arrives at the following gravitomagnetic (Maxwell-like) equations [9]:
\[ \nabla g = -4\pi G \rho, \]
(16)
\[ \nabla b = 0, \]
(17)
\[ \nabla \wedge g = -\frac{1}{c} \frac{\partial b}{\partial t}, \]
(18)
\[ \nabla \wedge b = -\frac{4\pi G}{c} \rho v + \frac{1}{c} \frac{\partial g}{\partial t}. \]
(19)
For a weak stationary gravity field, from the geodesic equation
\[ \frac{d^2 x^\alpha}{d\tau^2} + \Gamma^\alpha_{\beta\gamma} \frac{dx^\beta}{d\tau} \frac{dx^\gamma}{d\tau} = 0, \]
(20)
one obtains the Lorentz-like force law, reads
\[ \frac{du}{dt} = g + \frac{4}{c} u \wedge b, \]
(21)
where \( u \) is the velocity of the test particle.

3 Lense-Thirring precession on a spin

For a weak stationary field the gravitomagnetic potential is
\[ a = -\frac{1}{2} \frac{G}{c} \frac{J \wedge r}{r^3}, \]
(22)
and the gravitomagnetic field

\[ \mathbf{b} = \nabla \wedge \mathbf{a} = -\frac{1}{2} \frac{G}{c} \frac{3 \mathbf{n} (\mathbf{J} \cdot \mathbf{n}) - \mathbf{J}}{r^3} \]  

(23)

where \( \mathbf{J} \) is the intrinsic angular momentum of the source and \( \mathbf{n} \) is the unit position vector. These equations are analogous to the magnetostatic ones, replacing the magnetic dipole moment by minus half the angular momentum.

For an arbitrary accelerated observer in a gravitational field with 4-velocity \( u^a = dx^a/d\tau \) and 4-acceleration \( a^a = Du^a/d\tau \), the equation of motion for the torque-free point-like 4-spin vector is given by the Fermi-Walker transport law

\[ \frac{dS^a}{d\tau} + \Gamma^a_{b\ c} u^b S^c = u^a a_d S^d, \]

(24)

where the 4-spin vector \( S^a \) is constrained by the condition

\[ u_a S^a = 0, \]

(25)

which assures that the length of the 3-spin vector \( \mathbf{S} \) does not change as measured by an observer comoving with the spinning particle. The total precession of the 3-spin vector with respect to an asymptotic inertial frame given by a “fixed star” trained on by a telescope, whose associated tetrad realizes Frenet-Serret transport, is given by the equation

\[ \frac{d\mathbf{S}}{dt} = \Omega \wedge \mathbf{S}. \]

(26)

The general expression for the spin precession rate in the Lense-Thirring metric (Schiff formula) contains three terms

\[ \Omega = \Omega_{\text{Th}} + \Omega_{\text{geo}} + \Omega_{\text{LT}}, \]

(27)

where

\[ \Omega_{\text{Th}} = \frac{1}{2c^2} \mathbf{a} \wedge \mathbf{u}, \]

(28)

and

\[ \Omega_{\text{geo}} = \frac{3}{2} \frac{G}{c^2} \frac{M}{r^2} \mathbf{n} \wedge \mathbf{u} = \frac{3}{2} \frac{G}{c^2} \mathbf{u} \wedge \mathbf{g}. \]

(29)

Only the Thomas precession \( \Omega_{\text{Th}} \) would be present for accelerated motion in a flat Minkowski spacetime. Both geodetic \( \Omega_{\text{geo}} \) and Thomas \( \Omega_{\text{Th}} \) precessions are present for accelerated motion in Schwarzschild geometry, but only the geodetic \( \Omega_{\text{geo}} \) remains, in the case of free fall (\( \mathbf{a} = 0 \)) motion. Moreover, the geodetic de Sitter-Fokker precession \( \Omega_{\text{geo}} \), due to the mass \( M \), is in the same sense as the orbital motion.

The additional Lense-Thirring gravitomagnetic precession effect \( \Omega_{\text{LT}} \), due to the angular momentum \( \mathbf{J} \) of the source, is manifested in a Kerr spacetime.
or in its weak field and slow motion approximation (the Lense-Thirring metric \([11]\)), in which the Lense-Thirring precession rate \(\Omega_{LT}\) is

\[
\Omega_{LT} = -\frac{2}{c^2} b = \frac{G}{c^2} \frac{3n(J \cdot n) - J}{r^3}
\]  

(30)

3.1 Particular cases of the Schiff formula

(1) Free fall gyroscopes in Earth’s orbit (GP-B experiment).
As \(a = 0\) and \(u \neq 0\), then \(\Omega_{Th} = 0\) and only two terms survive,

\[
\Omega = \Omega_{geo} + \Omega_{LT}
\]  

(31)

This formula is used in the GP-B gyroscope (Stanford) experiment to obtain the precession, due to stationary gravitomagnetic \(b\) field generated by the rotation of the Earth mass, with respect to an asymptotic inertial frame given by the “fixed star” TM Pegasus.

(2) Gyroscope at rest on Earth (except at the poles).
In this case, \(a = -g\), hence

\[
\Omega = \Omega_{Th} + \Omega_{geo} + \Omega_{LT}
\]  

(32)

with \(\Omega_{Th} + \Omega_{geo} = \frac{2}{c^2} u \wedge g\).

(3) Gyroscope or Foucault pendulum at rest on Earth at a pole (South).
As \(u = 0\), then only the Lense-Thirring term remains

\[
\Omega = \Omega_{LT}.
\]  

(33)

Hence, this last is the clean experiment because one has the GM precession only, without competing geodetic or Thomas effects. Moreover, the magnitude of \(\Omega_{LT}\) is five times larger in this last case \((220 \text{ mas/year})\) than in the GP-B experiment \((42 \text{ mas/year})\).

4 TELEPENSOUTH experimental apparatus

The experimental apparatus would consist of a Foucault pendulum and an astrometric telescope in a underground vacuum chamber. To avoid the classical Foucault effect, due to Earth’s rotation, it is necessary to operate just in the South pole.

Furthermore, another reason for using a Foucault pendulum instead of a gyroscope is due to the required sensitivity compared to the Earth rotation rate: \(\Omega_{LT} = \frac{d\Phi}{dt} \approx 5.10^{-10} \omega_{\oplus}\). The telescope must have its optical axis locked to the azimuth of a “fixed” star, which is an approximate asymptotic inertial frame.

The pendulum swinging fiber is used as a light pipe and the mass as a lens to focus a swinging light beam onto an optical system, which monitors the angle \(\Phi\) between the principal axis of the pendulum and the telescope and the ellipticity of the swing \(\epsilon\).
4.1 Sources of error of the experiment

For the sake of completeness, we give here a summary of the sources of error of the experiment, which has been mainly extracted from the original work [8]. For a two-month experiment, 10% of accuracy requires a precision of $\delta \Phi = 4 \text{ mas}$.

(A) For the pendulum, two different kinds of error:

(1) Velocity dependent forces: Gravitomagnetic (to measure), magnetic, anisotropic frictional damping, Pippard precession.

Effect: Change of the principal axis direction $\Phi$, no change in $\epsilon$.

(2) Position dependent forces orthogonal to the principal axis of the pendulum: frequency anisotropy, seismic displacements of the support.

Effect: First order change in $\epsilon$, second order in $\Phi$.

(B) For the telescope, the errors come from:

(1) Atmospheric refraction.

(2) Distortion.

(3) Tilts of the mirror.

4.2 Control of the sources of error of the pendulum

(1) Velocity dependent forces:

(a) Against magnetic forces: Coat the mass and the fiber with metal.

(b) Against anisotropic frictional damping: pendulum support held fixed relative to the Earth. Conclusion: Sapphire fiber with diameter $d = 0.1 \text{ mm}$ and mass, $M = 100 \text{ gr}$.

(c) Pippard precession: due to the spin of the pendulum mass, if the support is fixed relative to the Earth. It is $10^3$ times larger than the GM effect. Remedy: subtract it from the data, mass must be long, thin and dense, e.g., tungsten.

(2) Position dependent forces:

(a) Frequency anisotropy: due to the ellipticity $\epsilon$ and large amplitude $A = 5 \text{ cm}$. Cure: Length of the fiber, $l = 2 \text{ m}$, and gravitational and electrostatic pulls of large masses in parallel plates placed on each side of the pendulum or use of a pendulum without fiber, with the magnetically levitated mass sliding over a superconducting surface.

(b) Seismic noise. Remedy: sapphire fiber.

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