Enhanced spectral profile in the study of Doppler-broadened Rydberg ensembles

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Combination of the electromagnetically-induced-transparency (EIT) effect and Rydberg-state atoms has attracted great attention recently due to its potential application in the photon-photon interaction or qubit operation. In this work, we studied the Rydberg-EIT spectra with room-temperature $^{87}$Rb atoms. Spectroscopic data under various experimental parameters all showed that the contrast of EIT transparency as a function of the probe field intensity is initially enhanced, reaches a maximum value and then decays gradually. The contrast of spectral profile at the optimum probe field intensity is enhanced by 2–4 times as compared with that at weakest intensity. Moreover, the signal-to-noise ratio of the spectrum can potentially be improved by 1 to 2 orders of magnitude. We provided a theoretical model to explain this behavior and clarified its underlying mechanism. Our work overcomes the obstacle of weak signal in the Rydberg-EIT spectrum caused by an apparent relaxation rate of the Rydberg polariton and weak coupling transition strength, and provides the useful knowledge for the Rydberg-EIT study.

Rydberg atom has become a popular research topic in recent decades, especially in the context of quantum information science, thanks to its physical properties. The weak dipole transition between ground and highly-excited Rydberg states prolongs the lifetime of Rydberg atoms. The large polarizability of Rydberg atoms gives rise to strong long-range interactions. It would couple the nearby atoms strongly through the immense dipole-dipole interaction. The strong interaction between Rydberg atoms leads to a blockade effect, implying a double excitation for a distance smaller than the blockade radius is strongly suppressed. With the unique features of above, Rydberg atom is a good candidate for the demonstration of novel quantum devices, such as single-photon transistors as well as quantum phase gate, single-photon sources, and quantum simulator.

Electromagnetically-induced-transparency (EIT) spectrum provides the direct nondissipative optical detections of Rydberg energy levels, atom-atom interaction, and wall-atom interaction in a thin cell. An additional microwave field can break the symmetry of Rydberg-EIT interference, making it a good way to precisely determine the electric field of the microwave. Besides the EIT spectra, the quantum information carried by photons can be dynamically encoded in Rydberg polaritons, allowing for storage, control, and retrieval of quantum states. To perform the above mentioned studies with Rydberg-state atoms, it is necessary to lock laser frequencies to a two-photon transition frequency. The EIT spectrum provides a convenient way to stabilize the laser frequencies based on a high contrast EIT peak. The EIT peak height increased with the probe field intensity has been experimentally observed and theoretically analyzed in a $\Lambda$-type open transition EIT system. Here we report that there exists an optimum probe intensity, which makes EIT peak height reach its maximum value, in a $\Xi$-type cycling Rydberg EIT transition.

We carried out our study in a vapor cell which is filled with the admixture of $^{87}$Rb and $^{85}$Rb atoms at the room temperature of about 300 K. In this paper, we present the investigation of Rydberg-state EIT spectra based on $^{87}$Rb atoms. The EIT peak height, i.e. the difference between the probe transmission at the EIT peak and that in the absence of the EIT effect, has been enhanced by 2–4 times at the optimum probe intensity as compared with that at weakest probe intensity. Remarkably, the optimum intensity is influenced very little by the light polarization, the principal quantum number of Rydberg state $n$, and $nS$ or $nD$ Rydberg states. We will provide a theoretical model for the observed behavior of the EIT peak height as a function of the probe intensity. In addition, when one...
under perturbation limit of the probe field, i.e. the Rabi frequency of probe field \( \Omega_p \) is much weaker than that of the coupling field \( \Omega_c \). As shown in Fig. 1(a), the hyperfine levels \( |3\overline{D}_{5/2}\rangle \) and \( |3\overline{D}_{3/2}\rangle \) of the ground state, an intermediate excited state, and a Rydberg state, as shown in Fig. 1(a). The probe field couples the ground state \( |5S_{1/2}, F = 2\rangle \equiv |g\rangle \) and the intermediate state \( |5P_{3/2}, F = 3\rangle \equiv |e\rangle \), while the coupling field drives the transition of \( |e\rangle \) to Rydberg state \( |r\rangle \). Further details of our setup can be found in the section of Setup and Measurement in Methods. For each EIT spectrum measurement, we kept the probe field frequency resonant to the ground state \( |5S_{1/2}, F = 2\rangle \) or a lower Rydberg state (with fixed coupling intensity of 18 W/cm²), respectively. Both light fields had the same polarization \( \sigma_+ \) in (b); and the probe field was \( \sigma_- \) polarized and coupling field was \( \sigma_+ \) polarized in (c). The values of NEPH are 0.014, 0.10, and 0.022 from left to right peaks in (b,c).

### Results and Discussion

#### Experimental Observation.

Rydberg EIT has a cascade-type (\( \Xi \)-type) level structure, which consists of a ground state, an intermediate excited state, and a Rydberg state, as shown in Fig. 1(a). The probe field couples the ground state \( |5S_{1/2}, F = 2\rangle \equiv |g\rangle \) and the intermediate state \( |5P_{3/2}, F = 3\rangle \equiv |e\rangle \), while the coupling field drives the transition of \( |e\rangle \) to Rydberg state \( |r\rangle \). Further details of our setup can be found in the section of Setup and Measurement in Methods. For each EIT spectrum measurement, we kept the probe field frequency resonant to \( |g\rangle \)-\( |e\rangle \) transition, while the frequency of the coupling field was swept across the \( |e\rangle \)-\( |r\rangle \) transition. The probe field has the maximum transmission when the two light fields are resonant to the Rydberg state due to the quantum interference of EIT effect. Figure 1(b,c) show the typical Rydberg-state EIT-type spectra. Far from resonance, atoms in the excited state are less likely pumped to the Rydberg state. It can be viewed as a two-level transition, resulting in a high absorption. Here, we define the normalized EIT peak height (NEPH) as the transmission difference between the transmissions of EIT peak and baseline. The intensities of the probe and coupling fields were 0.029 and 18 W/cm², respectively. Both light fields had the same polarization \( \sigma_+ \) in (b); and the probe field was \( \sigma_- \) polarized and coupling field was \( \sigma_+ \) polarized in (c). The values of NEPH are 0.014, 0.10, and 0.022 from left to right peaks in (b,c).

To systematically study the NEPH, we varied the intensities of the probe and coupling fields, the Rydberg states, and the light polarization configurations for Rydberg S-state and D-state orbitals. As shown in Fig. 2, a stronger \( \Omega_c \) (with fixed Rydberg state of \( |38D_{3/2}\rangle \)) or a lower Rydberg state (with fixed coupling intensity of 18 W/cm²) leads to a greater NEPH. Note that the intensity of the coupling or probe field specified in this article is the intensity at the center of the Gaussian beam profile of the laser field. The phenomenon can be explained by the expression of EIT transmission \( T \sim \exp[-2\alpha\gamma\Gamma/\Omega_c^2] \) under perturbation limit of the probe field, i.e. the Rabi frequency of probe field \( \Omega_p \) is much weaker than that of the coupling field \( \Omega_c \). Here the relaxation rate \( \gamma \) represents the decay of coherence between the ground and Rydberg states, which includes two-photon detuning effect (caused by the atomic motion), atomic collisions, coupling with external fields, laser linewidth, and nature lifetime of Rydberg state. \( \alpha \) is the optical density of the medium, and \( \Gamma \) is the spontaneous decay rate of the intermediate state. The coupling Rabi frequency \( \Omega_c \) is proportional to a power law \( n^{-3\alpha} \), where \( n \) is the effective principal quantum number of Rydberg state (see Method). The coupling strength between a higher Rydberg state and the intermediate state is weaker, resulting in a smaller \( \Omega_c \). The expression shows that a stronger \( \Omega_c \), leads to a higher transmission of the probe field, which can qualitatively describe the data.

Besides, the polarization configurations of the laser fields were adjusted as \( \sigma_+ - \sigma_- \), \( \sigma_+ + \sigma_- \), \( \sigma_+ \), \( \sigma_- \), \( \sigma_\parallel \), or \( \sigma_\perp \). For \( |nD_{5/2}\rangle \) Rydberg state, the best polarization configuration is \( \sigma_\parallel \) (Fig. 3(a)). The hyperfine levels \( F = 2, 3, 4 \) of \( |nD_{5/2}\rangle \) state are not resolvable so that all transition channels among the Hyperfine states and their Zeeman sub-levels need to be taken into account. The Zeeman state
transition $|F' = 3, m_F' = 3\rangle$ to $|F'' = 4, m_F'' = 4\rangle$ is a cycling transition, leading to a larger effective $\Omega_c$. On the contrary, when the coupling field couples to $|nS_{1/2}\rangle$ Rydberg state, the polarization of $\sigma_+ - \sigma_-$ is the best configuration shown in Fig. 3(b). For $|nS_{1/2}\rangle$ state, only the excitation to $F'' = 2$ hyperfine state is allowed. For example, in the polarization configuration of $\sigma_+ - \sigma_-$, there are some probe excitations do not couple with the coupling field and hence, the EIT transmission is reduced. Therefore, in order to get the best EIT contrast for a given $|nS\rangle$ or $|nD\rangle$ state, one should well select the best light polarization.

For each light polarization configuration or each Rydberg state, $n$, as well as the orbital, $S$ or $D$, of Rydberg state, the Rabi frequencies of light shall be modified by averaging Clebsch-Gordan coefficients (CGC) among different hyperfine and Zeeman states. We define $\Omega_{p,0.1}$ as the Rabi frequency of the probe field with intensity of 0.1 W/cm$^2$ and $\Omega_{p,1}$ as $2\pi \times 22.7$ MHz for each polarization of $\sigma_+$ or $\sigma_-$ or $\pi$. The Rabi frequencies used in the measurements are summarized in Table 1. A detailed description of the derivation of Rabi frequencies can be found in Method. From the measurements of Figs 2 and 3 and the analysis of Rabi frequencies, a greater NEPH corresponding to a larger $\Omega_c$ is quantitatively proved.

Moreover, all measured results reveal a universal phenomenon, that as we enlarged the probe field intensity, NEPH initially increased, then reached a maximum (around intensity of 0.03 W/cm$^2$), and finally decreased. The optimum probe power is about 20 times stronger than the weakest power used in the measurement so that the $S/N$ of NEPH can be enhanced. In most of EIT-relevant studies, people typically applied a weak probe intensity or Rabi frequency ($\Omega < \Omega_c$) because EIT theories work well under the perturbation limit. To our knowledge, this rising phenomenon of NEPH is not presented in any article so far and promotes a further study in this paper. After the optimization of the probe intensity, the best NEPH is 2–4 times larger, and therefore, the overall $S/N$ can have 1–2 order of magnitude enhancement. Qualitatively, such enhancement or the behavior of rising NEPH is universal, which is irrelevant to the laser polarization, the coupling intensity, the Rydberg $S$-state or $D$-state orbitals, and the principal quantum number $n$ of Rydberg state. The physics mechanism and criterion or experimental condition of the behavior will be discussed in the remaining part of the Results and Discussion.

Figure 2. NEPH as a function of the intensity of the input probe field at (a) different intensities of the coupling field for the transition to Rydberg state of $|38D_{5/2}\rangle$ and (b) different principal quantum numbers of Rydberg states for the coupling intensity of 18 W/cm$^2$. Both measurements have the same polarization configuration of $\sigma_+ - \sigma_-$. The coupling intensities in units of W/cm$^2$ in (a) and selected Rydberg states in (b) are shown in the legends. Solid lines are the curves to guide the eye. The typical errors (standard deviation) are smaller than the symbol size, which do not reveal in the following figures.

Figure 3. NEPH as a function of the intensity of the input probe field at different polarization configurations and Rydberg states, shown in the legends. The coupling intensity was 18 W/cm$^2$ in the both measurements. Solid lines are the curves to guide the eye.
Theoretical Model. We present the theoretical simulation by solving the optical Bloch equations (OBEs) and Maxwell–Schrödinger equation (MSE). According to the EIT theoretical studies, the spectrum or the dynamic behaviors in Doppler-broadened media can be simulated by a simple model used for Doppler-free media with the modifications of parameters, such as decoherence rate \( \gamma \), optical depth \( \alpha \), and Rabi frequencies \( \Omega_i \). Hence, the following equations are widely used in the EIT-relevant studies,

\[
\begin{align*}
\partial_t \rho_{gg} &= \frac{i}{2} \left( \Omega_p^* \rho_{eg} - \Omega_p \rho_{eg}^* \right) + \frac{\Gamma_e}{2} \rho_{gg}, \\
\partial_t \rho_{er} &= \frac{i}{2} \left( \Omega_p^* \rho_{eg} + \Omega_r \rho_{er} - \Omega_r \rho_{er}^* \right) - \left( \frac{\Gamma_e + \Gamma_r}{2} + i \Delta \right) \rho_{er}, \\
\partial_t \rho_{eg} &= \frac{i}{2} \left( \Omega_r^* \rho_{eg} - \Omega_r \rho_{eg}^* \right) - \left( \gamma - i \Delta \right) \rho_{eg}, \\
\partial_t \rho_{gg} &= \frac{i}{2} \left( \Omega_r^* \rho_{eg} - \Omega_r \rho_{eg}^* \right) + \Gamma_r \rho_{gg}, \\
\partial_t \rho_{er} &= \frac{i}{2} \left( \Omega_p^* \rho_{eg} - \Omega_p \rho_{eg}^* \right) - \Gamma_r \rho_{er}, \\
\Gamma_e &= \frac{\alpha \Gamma_p}{2L} \rho_{gg}.
\end{align*}
\]

Here \( \rho_{ij} \) is an element of the density-matrix operator of the three-level system, \( \Delta \) is the detuning of the coupling field, \( \gamma \) is the decay rate of the coherence between the Rydberg and ground states, \( \Gamma_e \) is the spontaneous decay rate of the intermediate-level excited state and \( \Gamma_r \) is that of the Rydberg state, and \( L \) is the length of the medium.

We will describe how the predictions from above theoretical model can be consistent with the data. We first determine the optical density of the medium \( \alpha \) and the ratio of \( \Omega_i \) to \( \Gamma_i \) by numerically fitting the data of the probe transmission at the coupling frequency detuned far away from the two-photon resonance versus the intensity of the input probe, as shown in Fig. 4(a). The transmission at small values of \( \Omega_i \) determines the value of optical density mainly. As \( \Omega_i \) gets larger, the power broadening effect, indicated by \( \Omega_p^2 / \Gamma_i^2 \), influences the increment of the transmission. Hence, the best fit gives the optical depth \( \alpha = 0.45 \) and \( \Omega_i = 0.95 \). Next, we fix \( \Gamma_r = 2 \pi \times 5 \text{kHz} \), which is the spontaneous decay rate of 38D23, in the calculation of the NEPH predictions (Fig. 4(b)). The ratios of \( \Omega_i \) to \( \Gamma_r \), \( \gamma \), and \( \Gamma_i \) were resolved by the fitting because both of \( \Omega_i \) and \( \gamma \) can affect the EIT peak height. This can be realized from the EIT peak transmission \( T \sim \text{Exp} \left( -2 \pi \text{Rabi} / \Gamma_i \right) \) under perturbation limit of the probe field. In addition, as all the frequency-related quantities, \( \Omega_i, \Omega_p, \gamma, \Gamma_i \), etc., are normalized to or divided by \( \Gamma_i \), the calculation result becomes invariant with respect to \( \Gamma_i \). Therefore, the NEPH can determine \( \Omega_i / \Gamma_i \) (or \( \gamma / \Gamma_i \)) but not \( \Omega_i \) and \( \Gamma_i \) (or \( \gamma \) and \( \Gamma_i \)) individually. The actual value of \( \Gamma_i \) in units of MHz was derived by matching the theoretical EIT linewidths to the experimental ones as illustrated in Fig. 4(c). The three kinds of data shown in Fig. 4(a–c) can uniquely determine a set of \( \alpha = 0.45 \) and \( \Gamma_i = 2 \pi \times (17.1, 4.5, 3.1, 18) \text{MHz} \).

The best fit of EIT spectra gives \( \Gamma_i = 2 \pi \times 18 \text{MHz} \), indicating that only a small fraction of room-temperature atoms interacts with the probe field. Note that the actual angular natural linewidth of the intermediate state \( |e \rangle \) is \( \Gamma_e = 2 \pi \times 6 \text{MHz} \). In each measurement of the EIT-like spectrum, we kept the probe field frequency fixed at the resonant frequency of the transition from \( |5S_{1/2}, F = 2 \rangle \) to \( |5P_{3/2}, F' = 3 \rangle \), and swept the coupling field frequency. Atoms with high velocities \( v \) do not interact with the probe field at all due to the Doppler shift \( kv \) being large, where \( k \) is the wave vector of the probe light. Because the EIT scheme is formed by the cascade- or ladder-type
transitions, no interaction with the probe field also means no influence from the coupling field. Only the atoms with low velocities can interact with the probe field and, consequently, also be influenced by the coupling field. In the lab frame, these atoms with low but non-zero velocities possess different resonance frequencies of the transition. A distribution of resonance frequencies, arising from the atoms with various velocities distributed around \( v = 0 \), can equivalently be seen as the broadening of the excited-state linewidth in the theoretical model treating all atoms with \( v = 0 \). Therefore, \( \Gamma_c \), the ratio of the EIT linewidth is predominately determined by the coupling field intensity and optical density of the medium at the weak probe field regime. Employing room-temperature or hot atomic media, one typically applies a strong coupling field to diminish the influence of non-negligible decoherence rate \( \gamma \) on the EIT transmission. For a strong probe field, the power broadening effect occurs, resulting in the spectral width becomes broader. The linewidth at the optimum probe intensity, shown in Fig. 4(c), is only twice broader than that at weak intensity. As discussed before, the overall S/N of EIT peak can have 1–2 order of magnitude enhancement. The EIT dispersion, i.e. the slope of a frequency-modulated EIT spectrum, becomes much larger. If the probe field is further stronger, the increment of EIT linewidth and the decrement of NEPH becomes fast. The EIT spectrum doesn’t benefit the Rydberg studies relied on the optical frequency lock.

**NEPH Behavior.** We now discuss the universal phenomena, that NEPH increases initially with weak \( \Omega_p \) regime and then decreases at strong \( \Omega_p \) regime. The theoretical model for Doppler-free atoms with effective parameters can phenomenologically describe the Doppler-broadened atomic medium well. Hence, in the following discussion, we will utilize this model to explain the observed behavior of NEPH. For simplicity, we start from the analysis of the absorption cross sections, which is equal to \( \sigma_{\text{abs}} \approx \text{Im}[\rho_{p}\Gamma_{d}/\Omega_a^2] \), where \( \sigma_{\text{abs}} \) is the resonant absorption cross section under perturbation limit of the probe field. We define \( \sigma_{\text{TL}} \) as the absorption cross section of the baseline in a spectrum, which results from the resonant transition in the two-level system. \( \sigma_{\text{EIT}} \) is defined as the absorption cross section of the EIT peak. Based on the steady-state solution of the optical-Bloch equations, we derive \( \sigma_{\text{TL}} \) and \( \sigma_{\text{EIT}} \) under the assumptions of \( \Gamma_a \ll \Gamma_c \) and \( \Gamma_a, \Gamma_c \ll \Omega_a^2 \) as

\[
\frac{\sigma_{\text{TL}}}{\sigma_0} = \frac{\Gamma^2}{2\Omega_p^2 + \Gamma_c^2}, \quad \frac{\sigma_{\text{EIT}}}{\sigma_0} = \frac{\Gamma^2}{\Omega_a^2(2\Omega_p^2 + \Gamma_c^2) + \Gamma_c^2(6\Omega_p^2 + \Gamma_c^2 + 2\Omega_a^2)}. \tag{2}
\]

The analysis of absorption cross section as well as the probe transmission will be discussed in two regimes: weak and strong \( \Omega_p \) regimes. In the weak \( \Omega_p \) regime, we further assume \( \Omega_p^2 \ll \Gamma_a^2, \Omega_c^2 \) and \( \Gamma_a, \Gamma_c \ll \Gamma_a, \Omega_a^2 \). The absorption cross sections of two-level and Rydberg-EIT systems can be expanded as

\[
\frac{\sigma_{\text{TL}}}{\sigma_0} \approx 1 - \frac{2\Omega_p^2}{\Gamma_c^2}, \quad \frac{\sigma_{\text{EIT}}}{\sigma_0} \approx \frac{2\gamma\Gamma_c}{2\gamma + \Gamma_c} - \frac{2\Omega_p^2(4\gamma + \Gamma_c)}{(\Omega_a^2 + 2\gamma + \Gamma_c)^2}. \quad \Delta \sigma = \sigma_{\text{TL}} - \sigma_{\text{EIT}} = A_1 + \frac{2B_1\Omega_p^2}{\Gamma_c^2}, \tag{3}
\]

where

\[
B_1 = \frac{\Gamma_c^2(2\gamma + \Gamma_c) - \Omega_a^2(4\gamma + \Gamma_c + \Omega_p^2)}{(\Omega_p^2 + 2\gamma + \Gamma_c)^2}.
\]

If \( B_1 \) is positive, \( \Delta \sigma \) as well as NEPH increases with increasing \( \Omega_p \). In the physical explanation, on one hand, the transmission of the probe field under the two-level transition slightly decreases as increasing \( \Omega_p \) due to the power broadening effect. On the other hand, in the EIT transition, a highly-excited Rydberg state can be treated as a meta-stable state because of the slow spontaneous decay rate. A strong \( \Omega_p \) results in more populations to be driven.

**Figure 4.** (a) Baseline transmission, i.e. the probe transmission at the coupling field frequency detuned far away from the two-photon resonance, (b) NEPH, and (c) linewidth, i.e. full width at half maximum, of the EIT profile versus the intensity of the input probe field. Black squares are the experimental data of the Rydberg state of \([38D_{3,2}]\), and red lines are the theoretical predictions. The coupling intensity was 18 W/cm², and two light fields were both \( \sigma_+ \)-polarized in the experimental measurement. We set \( \Gamma_c = 2\pi \times 5 \text{ kHz} \) in the theoretical calculation. To fit the data well, \( \alpha = 0.45 \) and \( (\Omega_{p,0.1}^a, \gamma_p, \Gamma_c) = 2\pi \times (17.1, 4.5, 3.1, 18) \text{ MHz} \). We first determined \( \alpha \) and the ratio of \( \Omega_{p,0.1}^a \) to \( \Gamma_c \) by the fitting of (a). Then, the ratios of \( \Omega_p \) and \( \gamma \) to \( \Gamma_c \) were resolved by the fitting of (b). Finally, we settled \( \Gamma_c \) by the fitting of (e).
into the excited state |e⟩ and Rydberg state |r⟩. According to EIT theory, the population ratio of |r⟩ to |g⟩ is determined by the ratio of Ω_p^2 to Ω_c^2 and, thus, the population ratio becomes larger as Ω_p^2 increases under fixed Ω_c^2. The less population in the ground state leads to a higher transmittance of the probe field. With a weak Ω_c, the rising transmittance for EIT transition would be faster than that for two-level transition. The initially-rising phenomenon in the data of NEP versus the probe intensity can occur under B_1 > 0.

We now intend to explain the decreasing behavior at stronger Ω_p regime. Here, we assume Ω_p^2 >> Ω_c^2, σ_{TL}, σ_{EIT} and σ_{EIT} are presented as

\[
\frac{\sigma_{TL}}{\sigma_{EIT}} \approx \frac{\Omega_p^2}{\Omega_c^2}, \quad \frac{\sigma_{EIT}}{\sigma_{EIT}} \approx \frac{\Omega_p^2}{\Omega_c^2}(\Omega_c^2 + \Gamma_p^2 + \Gamma_{pe}^2), \quad \Delta \sigma = \sigma_{TL} - \sigma_{EIT} = \frac{A_2 \Omega_p^2}{\Omega_c^2},
\]

where

\[
A_2 = \frac{(2\gamma + \Gamma_r^2)\Omega_c^2}{(6\gamma + \Gamma_p^2)\Omega_c^2 + 2\Gamma_{pe}^2\Omega_c^2}.
\]

A_2 is always larger than zero, implying that Δσ as well as NEP decreases with increasing Ω_p. At a large Ω_p, the EIT-like peak transmission becomes saturated and has a little room for improvement, while the baseline transmission can still be significantly increased. Consequently, increment of the probe Rabi frequency makes the baseline transmission approach to the EIT peak transmission, implying the NEP decreases to zero. The above argument is supported by Δσ decreasing monotonically with Ω_p as shown by Eq. (4). Since NEP initially increases at small Ω_p, and finally decreases at large Ω_p there must exist an optimum probe Rabi frequency, Ω_{opt}, which makes NEP reach its maximum value.

To observe the phenomenon that NEP as a function of the probe intensity initially increases, then reaches a maximum, and eventually decreases to zero, B_1 must be positive. The condition \( \Omega_p^2 \leq (6\gamma^2 + 2\gamma - 2\Gamma_r^2) \) makes B_1 > 0. We numerically solve the OBEs of the atomic density-matrix operator and the MSE of the light pulse, i.e. Eqs (1a) to (1g), to predict the EIT peak height. For a given set of Ω_c and γ_c, we search for the optimal probe Rabi frequency, \( \Omega_{opt} \), as shown in Fig. 5(a). The solid line sets the boundary in the parameter space between regions with zero and positive \( \Omega_{opt} \). The experimental condition of the measurements in Fig. 4 were far from the border line, shown as \( \gamma_c \) in the contour plot. Note that for a sufficient small optical density, e.g. OD < 0.1, the border line (blue line in Fig. 5(b)) well fits the above-mentioned condition. As the OD gets larger, the border line locates at the stronger Ω_c, as shown in Fig. 5(b). The increasing NEP phenomenon is within reach for a sufficient weak Ω_c. Generally, the enhancement of EIT contrast or the increasing phenomenon of NEP can be observed in the Λ-EIT or Rydberg-EIT systems with Doppler-broadened atoms or with cold atoms under an apparent decoherence rate.

Conclusion

We systematically investigated the best contrast of EIT peak in different polarization configurations of light fields, coupling field intensities, and Rydberg states. From the measurements and the analysis of Rabi frequencies, a greater NEP corresponding to a larger Ω_c is quantitative proved. In addition, a universal phenomenon was experimentally observed that the EIT contrast increasing with the probe intensity and then decreasing for further stronger light field. The EIT contrast is enhanced by 2–4 times at the optimum probe field intensity as compared with that at weak intensity. Meanwhile, the signal-to-noise ratio of the profile can be improved by 1 to 2 orders of magnitude. The enhancement can be achieved in most of the Λ-EIT or Rydberg-EIT experiments using Doppler-broadened atoms or using cold atoms under an apparent decoherence rate. Without considering the Doppler effect caused by the atomic motion, the thermal Rydberg-EIT spectrum can be predicted by using the simulation for a Doppler-free medium with the modification of parameters. The theoretical model can well fit the
data and simulate the rising behavior of NEPH. Our studies provide a better way of locking the upper transition frequency through a high contrast Rydberg-EIT spectrum and advances the Rydberg-atom-relevant studies.

Methods
Setup and Measurements. The probe field couples $|g\rangle$ and $|e\rangle$ states with wavelength of 780 nm, generated from an external cavity diode laser. The coupling field drives atoms from $|e\rangle$ to $|f\rangle$ states with wavelength of 480 nm, produced from a frequency doubled diode laser system (Toptica TA SHG pro). These two beams are sent into a counter-propagation direction to diminish the Doppler effect due to the atomic motion. We applied a dichroic mirror (DM) to separate the mixed probe field from the coupling beam after the vapor cell. The full width at $e^{-2}$ maximum of the probe and coupling beams were 1.4 mm and 2.2 mm, respectively. The vapor cell is filled with the admixture of $^{87}$Rb and $^{85}$Rb atoms at the temperature of 300 K.

Deviation of Table 1. Considering the equivalent CGC$^{32,33}$, the Rabi frequency is derived as a function of laser intensity,

$$\Omega_p = \left(a_p^*\right)^I \sqrt{I_p} = \frac{e a_p}{\hbar} C_{SD}(a_p)^{I_p} \sqrt{\frac{2I_p}{C_{SD}^2}} n^{-\frac{3}{2}},$$

where $(a_p) = \sum P_{C_p} \sqrt{P_{C_p}}$ and $(a_p) = \sum P_{C_p} \sqrt{P_{C_p}}/a_p$. $I_p$ and $I_c$ are the laser intensities of probe and coupling fields, $I_{sat}$ is the saturation intensity of the probe field transition. $n^*$ is the effective principal quantum number of Rydberg state which replaces the true $n$ with the relation of $n^* = n - \delta(n, l, J)$. The quantum defects $\delta(n, l, J)$ of $^{85}$Rb atoms were measured by Mack et al.$^{34}$, and for $n > 20$ are around 1.34(1) and 3.13 for $|nD_{3/2,3/2}\rangle$ and $|nS_{1/2}\rangle$ states, respectively. $C_{S} = 4.5$ and $C_{P} = 8.5$ for S and D orbitals of Rydberg state. In the expressions of $(a_p)$ and $(a_c)$, $P_i$ represents the population in the $i$-th Zeeman ground state, $C_{S}$ and $C_{P}$ are the CGCs of the probe and coupling transitions for each subsystem.

The polarization configurations of the laser fields were adjusted as $\sigma_+ - \sigma_-, \text{lin} \perp \text{lin}, \text{lin} || \text{lin}$, and $\sigma_- - \sigma_+$. By half- or quarter-wave plates. To derive the effective $\Omega_p$, we consider all the transition channels among Zeeman states from $|S_{1/2, F = 2}\rangle$ to $|S_{1/2, F = 3}\rangle$. The transition of $|S_{1/2, F = 2}\rangle$ or $|S_{1/2, F = 3}\rangle$ polarizations of the probe field all gives $\left(a_p^*\right) = \sqrt{7}/15$. The coupling field drives atoms from $|S_{1/2, F = 2}\rangle$ to $|D_{3/2, F = 2, 3, 4}\rangle$ or $|S_{1/2, F = 2}\rangle$. Note that the $D$ state hyperfine levels are not resolvable, and hence all transition channels need to be taken into account. The values of $\Omega_p$ are summarized in Table 1.

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Author Contributions
I.A.Y. conceived the study and designed the experiment. Y.-W.C. measured the spectra supervised by M.-S.C. B.-H.W., Y.-H.C., and J.-C.Y. studied the theory and analyzed the data supervised by I.A.Y. B.-H.W. and Y.-H.C. made the figures and table. The manuscript was written by Y.-H.C. with the help from B.-H.W., I.A.Y., and M.-S.C.

Additional Information
Competing Interests: The authors declare that they have no competing interests.

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