Dissecting the microphysics behind the metallicity-dependence of massive stars radii

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ABSTRACT
Understanding the radii of massive stars throughout their evolution is important to answering numerous questions about stellar physics, from binary interactions on the main sequence to the pre-supernova radii. One important factor determining a star’s radius is the fraction of its mass in elements heavier than Helium (metallicity, Z). However, the metallicity enters stellar evolution through several distinct microphysical processes, and which dominates can change throughout stellar evolution and with the overall magnitude of Z. We perform a series of numerical experiments with 15M⊙ MESA models computed doubling separately the metallicity entering the radiative opacity, the equation of state, and the nuclear reaction network to isolate the impact of each on stellar radii. We explore separately models centered around two metallicity values: one near solar Z = 0.02 and another sub-solar Z ~ 10^{-3}, and consider several key epochs from the end of the main sequence to core carbon depletion. We find that the metallicity entering the opacity dominates at most epochs for the solar metallicity models, contributing to on average ~ 60 – 90% of the total change in stellar radius. Nuclear reactions have a larger impact (~ 50 – 70%) during most epochs in the sub-solar Z models. The methodology introduced here can be employed more generally to propagate known microphysics errors into uncertainties on macrophysical observables including stellar radii.

Key words: stars: massive – stars: evolution

1 INTRODUCTION
Stars are the most numerous observable objects in galaxies, and much of astrophysics is pinned to the detailed understanding of stellar physics. In particular, many applications depend on precise knowledge of stellar radii.

Physically, stars extend into their winds without a well defined end-point (e.g., Parker 1958), and observationally their radius is a wavelength-dependent quantity because of the wavelength dependence of the optical depth \( \tau \). Nevertheless, the bulk of a star’s mass is contained within its photosphere\(^1\).

The value of the photospheric radius \( R \) enters in the global dynamical timescale of the star \( \tau_{\text{dyn}} \sim 1/\sqrt{G\rho} \sim \sqrt{R^3/GM} \), where \( G \) is the gravitational constant, \( \bar{\rho} \) is the average stellar density, expressed as \( \bar{\rho} \sim M/R^3 \), and \( M \) is the stellar mass. Therefore, the interpretation of dynamical phenomena, such as pulsations, is sensitive to the precise value of the radius. The value of \( R \) also enters directly in the surface gravity of a star, thus in the interpretation of observed spectra. Whether or not a star will interact with binary companion(s) depends on the extent of its maximum radius (e.g., Sana et al. 2012). If binary interactions occur, the rate of radial change with mass \( \zeta = d \log R/dM \) contributes to determining the dynamical stability of the mass transfer process (e.g., Soberman et al. 1997, Vigna-Gómez et al. 2020). Knowledge of the stellar radius is often a limiting factor in the determination of the masses and radii in eclipsing planetary systems (e.g., Johnson et al. 2017). In the case of massive \( M \geq 8 M_\odot \) stars, their radial extent at the end of their nuclear burning evolution sets the delay between their final core collapse and shock breakout, or, in other words, the delay between a possible neutrino and gravitational-wave signal (e.g., Ott 2009) and the first photons from a (hypothetical) supernova explosion (e.g., Gill et al. 2022). Tidal interactions in stellar binaries are extremely sensitive to stellar radius, \( \propto R^\xi \) where \( \xi \approx 8 \) (e.g., Zahn 1977). Finally, stellar radii are also important in determining the maximum luminosity achieved by individual stars, and thus by a given stellar population, with cosmological implications (e.g., Jang & Lee 2017).

In stellar evolution models of single stars, the radius depends sensitively on many “macrophysical” and “microphysical” effects described by parametric algorithms. These can impact stellar radii in different ways (e.g., Farrell et al. 2021a), and include among the “macrophysical” parameters those regulating mixing in the stellar interior (\( \zeta_{\text{MLT}}, \) overshooting, etc., see for example Dessart et al. 2013), wind (e.g., Renzo et al. 2017) and other mass loss mechanisms (e.g., Quataert & Shiode 2012; Fuller 2017; Fuller & Ro 2018; Morozova et al. 2018), rotation (e.g., Heger & Langer 2000), and the primordial initial composition.

The primordial composition is particularly important, because of the existence of observational tests comparing stellar populations in M31, various parts of the Galaxy, the Magellanic Clouds, and other dwarf galaxies (Bellazzini et al. 2001; Aloisi et al. 2007; Annibali

\(^1\) The photosphere is also wavelength-dependent. Here, we follow the standard assumption of defining the photosphere of a star where the optical depth is \( \tau = 2/3 \) using the Rosseland mean opacity to calculate \( \tau \).
et al. 2018; Wang et al. 2020; Martins et al. 2021). Moreover, the kind of binary interactions and the outcome of massive binary evolution, especially for rare paths leading to the formation of gravitational wave sources, are sensitive to the initial composition (e.g., Klencki et al. 2021a,b). The composition is usually summarized in terms of fraction of mass made of helium (Y) and elements heavier than helium, so-called metallicity Z, assuming a known scaling of the relative proportion of each metal (e.g., Asplund et al. 2009; Grasha et al. 2021).

The composition also enters predominantly in determining the “microphysical” influence on the stellar radii. These can roughly be divided in three categories relating to the radiative opacity (the “microphysical” influence on the stellar radii. These can roughly be divided in three categories relating to the radiative opacity, the nuclear burning or the EOS). These can roughly be divided in three categories relating to the radiative opacity, the nuclear burning or the EOS. These can roughly be divided in three categories relating to the radiative opacity, the nuclear burning or the EOS. These can roughly be divided in three categories relating to the radiative opacity, the nuclear burning or the EOS. These can roughly be divided in three categories relating to the radiative opacity, the nuclear burning or the EOS. These can roughly be divided in three categories relating to the radiative opacity, the nuclear burning or the EOS. These can roughly be divided in three categories relating to the radiative opacity, the nuclear burning or the EOS. These can roughly be divided in three categories relating to the radiative opacity, the nuclear burning or the EOS. These can roughly be divided in three categories relating to the radiative opacity, the nuclear burning or the EOS. These can roughly be divided in three categories relating to the radiative opacity, the nuclear burning or the EOS.

More specifically, the radiative opacity governs the energy transport in radiative regions, such as the envelopes of main sequence (MS) massive stars. Indirectly, it also governs the temperature gradient and thus the onset of convection, like in the core of MS massive stars. Finally, the radiative opacity is key in the driving of massive stars wind mass loss. The dependence of k on composition comes dominantly from the bound-bound and bound-free transitions which are the main source of opacity in the envelope (versely electron-scattering dominates in the fully-ionized stellar core, Kippenhahn & Weigert 1994).

In massive MS stars with non-zero Z, the main source of nuclear energy generation is the CNO cycle (e.g., Bethe 1939). In this cycle, carbon (C), nitrogen (N), and oxygen (O) catalyze the conversion of hydrogen (H) into helium (He), and the efficiency of the process depends on the total mass fraction of these three elements. The subsequent evolution is less directly dependent on the chemical composition, as the mass fractions in the core are determined by the ashes of preceding nuclear burning phases. Thus the role of the composition in the MS nuclear burning governs the response of the energy generation in the stellar core to surface energy losses via photons (and later neutrinos, e.g., Fraley 1968).

Finally, the EOS determines the relation between thermodynamical variables of the stellar gas, such as the pressure as a function of density and temperature $P = P(\rho, T, (X_i))$, where $(X_i)$ represents set of mass fractions for every isotope in the star. In massive stars the pressure is primarily due to non-degenerate gas, with a contribution from radiation pressure the importance of which increases with stellar mass. Typically, the total pressure is given by an ideal gas component, $P_{\text{gas}} = \rho T / \mu$, where the composition enters through the mean molecular weight $\mu$, plus a radiation pressure component independent of composition. Since the hydrostatic equilibrium of a star is ensured by the balance between gravity and the pressure gradient, the dependence of the composition on $\mu$ impacts the radius indirectly.

Here, we isolate the metallicity-dependent effects of each microphysical ingredient on the stellar radii. We perform a series of controlled numerical experiments in which the metallicity used in each microphysical ingredient can be varied separately. This allows us to create unphysical stars with, for example, a different value of metallicity for the radiative opacity and the nuclear burning or the EOS. Our goal is to determine which microphysical input dominates at each stage of evolution for different metallicities by comparing these models.

This paper is organized as follows. We describe our MESA stellar models and the numerical scheme we employ to dissect the microphysics-dependence of stellar radii in section 2. After defining a quantitative measure of the fractional radius change due to each combination of microphysics input in section 3.1, we report our findings for select physical epochs of stellar evolution in the rest of section 3. In Sec. 4, we compare our results to previous studies investigating stellar radii from different approaches, and explore the relative impact of particular choices made in our experiments (e.g., the assumed helium and hydrogen abundances; Sec. 4.3). Our conclusions are summarized in Sec. 5. In Appendix A we quantify the main sources of numerical error which enter our radius estimates.

2 STELLAR MODELS

We compute stellar structure and evolution models using Modules for Experiments in Stellar Astrophysics (MESA, version 12778; Paxton et al. 2011, 2013, 2015, 2018, 2019). Each model consists of the evolution of a 15M$_\odot$ star from the pre-main sequence to carbon depletion, defined as when the central carbon mass fraction reaches $X_c(12\text{C}) \leq 10^{-8}$.

Table 1 lists all our models, and our input files and numerical results are available at https://doi.org/10.5281/zenodo.6621643 or https://github.com/cx2204/stellar-radius.

2.1 Fiducial Models (Standard Microphysics)

We first construct fiducial models, which employ standard microphysics available in MESA, for two pairs of metallicity values: Z = $(10^{-3}, 2 \times 10^{-3})$ and $(0.02, 0.04)$ (Models 1, 9, 10, and 18 in Table 1, respectively). Our motivation is to separately explore stellar radii around a characteristic ‘high’ metallicity value near solar (Z ≈ Z$\odot$ ≈ 0.02) and a characteristic ‘low’ metallicity (Z ≈ 10$^{-3}$), insofar as the dominant physical processes at low- and high-metallicity may differ substantially.

For a 15M$_\odot$ star, the EOS module in MESA uses the OPAL tables (for Z ≤ 0.04; Rogers & Nayfonov 2002) and HELM (Timmes & Swesty 2000). MESA uses tabulated opacities to construct stellar structures as functions of density and metallicity (Lederer & Aringer 2009; Marigo & Aringer 2009). The radiative opacities are primarily from OPAL (Iglesias & Rogers 1993, 1996), with the low-temperature data from Ferguson et al. (2005) and high-temperature, Compton-scattering-dominated regime from Buchler & Yueh (1976). The electron conduction opacities are from Cassisi et al. (2007). The nuclear reaction rates used in MESA come from NACRE (Angulo et al. 1999) and JINA REACLIB (Cyburt et al. 2010), plus additional tabulated weak reaction rates (Fuller et al. 1985; Oda et al. 1994; Langanke & Martínez-Pinedo 2000). Screening is included via the prescription of Chugunov et al. (2007). The nuclear networks we use is basic.net, which includes the species $^{12}\text{C}$, $^{14}\text{N}$, $^{16}\text{O}$, $^{20}\text{Ne}$, $^{24}\text{Mg}$, and is extended to co_burn.net and approx21.net, a 21-isotope network in later evolutionary phases. This approach is sufficient to capture the bulk of the energy generation in the star until carbon depletion (e.g., Farmer et al. 2016), but does not capture the details of nucleosynthesis. Nuclear neutrino loss rates are accounted for in the nuclear reaction rates; thermal neutrino loss rates are from Itoh et al. (1996).

The initial metal isotope fractions are assumed to scale with the solar abundances from Grevesse & Sauval (1998). With the goal of performing a “controlled” numerical experiment, we want to isolate the effects of changing metallicity on stellar radii from those which occur due to the associated changes in hydrogen and helium abundances. We fix the hydrogen mass fraction $X = 0.75$ in all of the models in this paper, with the helium mass fraction then following from $Y = 1 - X - Z$. This choice differs from the usual assumption of...
changing both $X$ and $Y$ simultaneously with $Z$ (e.g., Pols et al. 1998, see also Sec. 4.3).

In addition to the microphysical processes whose effects are explored in this paper (namely, photon opacities responsible for energy transport, equation of state and nuclear reaction rates), the metallicity in the stellar atmosphere impacts the mass-loss rate of the star $M$ (most importantly, through radiation pressure on the atomic lines of iron-group elements; Castor et al. 1975). It will prove convenient to define the “wind metallicity” $Z_{\text{wind}}$ as that which enters $M$. All of our models employ the mass-loss prescription of Vink et al. (2001) and de Jager et al. (1988) for high and low temperature phase, respectively (Glebbeek et al. 2009), with metallicity-scaling $M \propto Z_{\text{wind}}^{0.85}$ (Vink & De Koter 2005). We fix the value of $Z_{\text{wind}} = 0.02$ and $10^{-3}$ in all of the high-Z and low-Z models, respectively, rather than varying them directly with the overall metallicity of the model (see Farmer et al. 2019 for a similar approach in a different context). We explore the quantitative impact of a varying wind metallicity as well in Sec 4.3.

The treatment of convection also influences the stellar radius (e.g., Dessart et al. 2013). We determine convectively unstable regions using the Schwarzschild criterion and adopt mixing length theory (Böhm-Vitense 1958) with $\alpha_{\text{MLT}} = 2.0$ (Cox & Giuli 1968). Schootemeijer et al. (2019) showed that the radial gradient of the (composition-dependent) mean molecular weight affects the post-main sequence expansion (when using the Ledoux criterion) and could lead to larger stellar radii. However, the hydrodynamics of convective boundary mixing is still an active topic of research (e.g., Anders et al. 2022a,b), and which instability criterion is physically more appropriate when modeling stars should not depend on the metallicity-dependence of the microphysics we focus on here. We adopt a step-function overshooting for the boundaries of any convective region (Paxton et al. 2011), with free parameters $(\Omega,f,f_\text{e}) = (0.345,0.01)$ (Brott et al. 2011).

Figure 1 shows evolutionary tracks of stellar luminosity $L$ versus stellar radius $R$ for our fiducial models, starting at the beginning of the MS. Different symbols along the tracks denote key physical stages of interest discussed below. Models with higher metallicity are less luminous at any given epoch (or, equivalently, possess larger radii at fixed luminosity). This trend is well established in the literature (e.g. Garnett 2002; Zahid et al. 2011) but its origin in terms of the relative contributions of different microphysical input has to our knowledge not been elucidated in a systematic way.

### 2.2 Models with Altered Microphysics Input

We focus on understanding what factors determine the radius differences between the $Z = 10^{-3}$ and $2 \times 10^{-3}$ models, and between the $Z = 0.02$ and $Z = 0.04$ models, respectively. We do this by changing separately the metallicity values which enter in the opacity, EOS, and nuclear reactions in MESA with customized routines (see, e.g., Walmswell et al. 2015 for a related approach). The metallicity values for each microphysical input are denoted in Table 1 by the symbols $Z_{\mu}$ for the metallicity in the opacity calculations, $Z_{\text{e}}$ for the metallicity value in the EOS calculations, and $Z_{\epsilon}$ for the metallicity in the nuclear energy generation calculations, respectively. Models 2-9 and models 11-17 cover all different permutations of the values of $\{Z_{\mu},Z_{\text{e}},Z_{\epsilon}\}$ away from the fiducial high-metallicity models (1 and 9) and low-metallicity models (10 and 18), respectively.

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**Table 1.** Summary of the MESA models used in our study. Columns give the different metallicity values used in each numerical experiment to dissect the effects of individual microphysics on stellar radii. See Sec. 2.2 for the notation. The rows with empty entries are the fiducial models, for which all metallicity values are the same as in Column 1.
To implement the changes in $Z_k$, we implement a routine that mimics the default opacity treatment in MESA with a separate input for metallicity. Within this routine, we rescale the total metallicity and the abundances of the metal isotopes in our simulations – $^{12}\text{C}$, $^{14}\text{N}$, $^{16}\text{O}$, $^{20}\text{Ne}$, $^{24}\text{Mg}$, which are used to interpolate opacities from the tabulated values in MESA. We choose the initial mass fractions of each isotope coming from the higher metallicity fiducial model in each category, which follows the solar abundances pattern, effectively setting $Z_k = Z$. However, as the star evolves, in the core the fraction of metals eventually exceeds $2Z$ due to nuclear burning. Our models allow the elemental abundances to change from their initial (possibly modified) values self-consistently in later evolutionary stages, following the nuclear reaction networks in MESA.

Similarly, we create separate EOS routines that mimic the default behavior but setting a floor $Z_\mu = 2Z$ in mean molecular weight. In this case too, as the composition of the star evolves, $Z_\mu$ is allowed to follow. As described in the previous section, all of our models assume an initial hydrogen mass fraction $X = 0.75$ and helium mass fraction $Y = 1 - X - Z$.

The nuclear reaction rates also depend on the stellar composition, because they are proportional to (a power of) the density of isotopes involved in each reaction. To account for the change from $Z_k = Z$ to $Z_k = 2Z$, we double all the reaction rates in the nuclear networks involving metal isotopes. These consist of 20 reactions out of the total 30 used in the largest (21-isotope) nuclear network we adopt. We keep the rate factors constant throughout the entire stellar evolution, but these changes are most relevant on the MS, when our simulation employ basi.c.net, where the primordial metal isotopes enter the CNO cycle. After the main sequence the star synthesizes its basic.net.

3 Results

In this section, we systematically analyze the stellar radius of each model in Table 1 at five notable epochs of stellar evolution (highlighted by markers for the fiducial models of Fig. 1): Terminal age main sequence (Sec. 3.2), corresponding to the maximum radius during core hydrogen burning; Hertzsprung Gap (Sec. 3.3), where the rate of change of radius (dlog(R)/dt) is largest; Helium core burning (Sec. 3.4), where the central helium mass fraction first reaches 90%; Beginning of Hayashi track (Sec. 3.5), where half of the envelope mass is convective; and Late Hayashi track (Sec. 3.6), where the stellar luminosity reaches $L = 10^5L_\odot$.

Figure 2 shows the luminosity-radius evolution for each model in Table 1. The left panels show the ‘high’ metallicity models ($Z \sim 10^{-2}$) while the right panels show the ‘low’ metallicity models ($Z \sim 10^{-3}$). The top rows compare the fiducial models to those in which the primordial metallicity ($Z_k$) is varied. The bottom row shows models with various combinations of microphysics changes. The top left panel shows the fiducial model of metallicity $Z = 0.04$ fiducial model. By comparison, doubling the metallicity in the EOS or nuclear physics has a less pronounced effect, producing stellar tracks more similar to the fiducial $Z$ model. We also notice that the $Z_{\mu \kappa \epsilon}$ model (for which all three microphysics metallicities are set to the $2Z$ value), does not perfectly match the fiducial $2Z$ model; we return to the origin of this discrepancy later in this section.

By contrast, in the low metallicity ($Z \sim 10^{-3}$) models, similar trends are more challenging to discern from the figure alone because the absolute difference in the radii between models are more modest, due to the smaller absolute variation in metallicity ($\Delta Z = 2 \times 10^{-3}$ vs. $\Delta Z = 0.02$ in the high metallicity case). Nevertheless, the offset of the symbols on the horizontal axis, particularly in the ‘mixed’ models shown in the lower right plot, demonstrate the metallicity-sensitivity of the radii at key stages of stellar evolution at low metallicity. The next section describes a more quantitative method to dissect the metallicity dependencies of the radius.

3.1 Radius Matrix

Before proceeding to quantify our results for the partitioning of microphysical effects on stellar radii, we first introduce the notation used to present and interpret our findings. We define

$$\Delta_{2Z} = \frac{R_{2Z} - R_Z}{R_Z},$$

where $R_Z$ ($R_{2Z}$) is the radius of the fiducial model of metallicity $Z$ ($2Z$). Thus, $\Delta_{2Z}$ is the fractional change in the stellar radius for the fiducial model of metallicity $2Z$ (models #9, 18) relative to the radius in the fiducial model of metallicity $Z$ (models #1, 10) at each key epoch of stellar evolution (denoted by symbols in Fig. 1). Our aim is to quantitatively break down $\Delta_{2Z}$ into the contributions from different microphysics.

Similarly, we define

$$\Delta_{\{k, \mu, \epsilon\}} = \frac{R(Z, Z_k, Z_\mu, Z_\epsilon) - R_Z}{R_Z},$$

as the fractional change in the stellar radius $R(Z, Z_k, Z_\mu, Z_\epsilon)$ of a given model with enhanced metallicities $Z_k$, $Z_\mu$, and/or $Z_\epsilon = 2Z$ (models #2-8 and #11-17), again relative to the fiducial metallicity $Z$ (models #1, 10). For example, $\Delta_{\mu \kappa \epsilon}$ is the result of Eq. 2 for the model in which $Z_\mu = Z_\kappa = 2Z$ but $Z_\epsilon = Z$, while $\Delta_{\mu \kappa \mu \kappa}$ is the result of increasing all three microphysical metallicities simultaneously ($Z_k = Z_\mu = Z_\kappa = Z_\epsilon = Z = 2Z$; models #8, #17). We calculate $\Delta$ values separately for the pairs of models at high ($Z \sim 10^{-2}$) and low metallicities ($Z \sim 10^{-3}$).

We arrange our results in $3 \times 3$ matrices with entries $\Delta_{\{k, \mu, \epsilon\}}$ in percentage units, of the following form:

$$\mu \kappa \epsilon \begin{bmatrix} \Delta_{\mu} & \Delta_{\mu \kappa} & \Delta_{\mu \epsilon} \\ \Delta_{\kappa \mu} & \Delta_{\kappa \mu \kappa} & \Delta_{\kappa \mu \epsilon} \\ \Delta_{\epsilon \mu \kappa} & \Delta_{\epsilon \mu \kappa \epsilon} & \Delta_{\epsilon \mu \kappa \epsilon} \end{bmatrix} \text{Tr}(\Delta) = \Delta_{\mu} + \Delta_{\kappa} + \Delta_{\epsilon}$$

The matrix is symmetric by construction, and we report its trace denoted as Tr($\Delta$) = $\Delta_{\mu} + \Delta_{\kappa} + \Delta_{\epsilon}$. We generate one matrix for each of the key epochs of stellar evolution shown with symbols in Fig. 1-2 (and subsequent figures), and discuss them separately in the sections to follow.

Naively, one should expect $\Delta_{2Z}$ to equal $\Delta_{\mu \kappa \epsilon}$, because activating all microphysics inputs to their $2Z$ value should be equivalent to doubling the metallicity of the fiducial model. However, as shown in Fig. 2, this expectation is not typically realized in practice, as a result of numerical inaccuracies in our experiments. Appendix A addresses
partially for physical rather than numerical reasons. Stellar evolution is a highly non-linear problem (e.g., Kippenhahn et al. 2012), so it should not be surprising that non-linear effects can arise. For example, a different choice of opacity influences the stellar evolution by different fractional amounts depending on the metallicity entering the EOS or nuclear reaction rates.

3.2 Main Sequence

Stars spend the majority of their lifetime on the MS, corresponding to $\approx 1.5 \sim 1.9 \times 10^7$ years in our fiducial $15 M_\odot$ models, roughly independent of metallicity. We first consider the impact of changing $(Z_\mu, Z_\kappa, Z_\epsilon)$ on the stellar radius at the end of the MS. We define the terminal age main sequence (TAMS) as a central hydrogen mass-fraction $X_c(H) < 10^{-3}$, which correspond to the maximum radial extent of stars before core hydrogen exhaustion (cf. Fig. 1 and 2).

The radii matrices (Eq. 2) for the high and low metallicity models are given, respectively, by:

**High Z**

| $\Delta$ (%) | $\mu$ | $\kappa$ | $\epsilon$ | $\Delta_{\mu\kappa\epsilon}$ |
|--------------|-------|----------|------------|-----------------|
| $\mu$        | 0.6(0.07) | 14.9(0.1) | 7.8(0.07)  | 24.5(0.1)       |
| $\kappa$     | 14.1(0.07) | 23.2(0.07) | 6.9(0.0)   | 21.6(0.1)       |
| $\epsilon$   | -      | -        | -          | 24.3(0.0)       |

**Low Z**

The Z-dependent microphysics of stellar radii

Figure 2. Luminosity-radius tracks for all models in Table 1, in both high metallicity $Z \approx 10^{-2}$ (left panels) and low metallicity $Z \approx 10^{-3}$ (right panels) models. The four fiducial models are superimposed with black and dark red solid lines, while the dashed lines show models where the metallicity for one (top panels) or more (bottom panels) microphysical input has been increased by a factor of 2 ($Z \rightarrow 2Z$). Key physical epochs are marked with the same symbols as in Fig. 1.
Thus, doubling $Z_C$ (CNO cycle (times the amount of fuel available at that point on the MS) is approximately fixed. Assuming the star is in thermal equilibrium, i.e. $L = L_{\text{nuc}} \sim \varepsilon_{\text{CNO}} (T_e)$, where $L_{\text{nuc}}$ is the luminosity from nuclear burning, which is proportional to the energy generation rate from the CNO cycle (times the amount of fuel available at that point on the MS). Taking $L = \mu$ constant and hence $\varepsilon_{\text{CNO}} = \text{constant}$ then implies $T_e \propto Z_e^{1/\alpha}$ and hence from the virial theorem $R \propto T_e^{-1} \propto Z_e^{1/\alpha}$. Thus, doubling $Z_e$, should act to increase the radius by a factor $\sim 2^{1/20} \sim 1.4$. This roughly agrees with our numerical results, $\Delta R \approx 7\%$ and $5\%$ in the high- and low-Z models, respectively.

Finally, consider the effect of the EOS-metallicity. Gas pressure dominates in our stellar models: the mass averaged gas to total pressure ratios on the MS are $(\rho) = \rho_{\text{gas}}/\rho_{\text{tot}} \approx 0.81$ and 0.85 for $Z = 10^{-3}$ and $Z = 0.02$ fiducial models, respectively. Metallicity enters the EOS primarily through the dependence $P_{\text{gas}} \propto \rho T_e/\mu$ on the mean molecular weight, $\mu$. For fully ionized gas, the latter can be written

$$\mu \approx \frac{1}{2X} + \frac{3}{4} \frac{Y}{Z} + \frac{1}{2} \frac{Z}{Z} \mu = \frac{1}{\frac{3}{4} - \frac{3}{4} X - \frac{1}{4} Z \mu}$$

where in the final equality we have used $X + Y + Z \mu = 1$ to express everything in terms of the hydrogen mass fraction, $X$, which is fixed in our models. An increase in the EOS-metallicity $Z_M$ by an amount $\delta Z_M$ thus increases the value of $\mu$, such that for small changes $\delta \mu/\mu \approx -1/(4) \mu \delta Z_M = 0.158 \mu \delta Z_M$ (where we take $\mu \approx 0.6$). From the virial theorem $R \propto \mu/T_e$, such that for the approximately fixed central temperature $T_e$ set by nuclear energy generation (see above), we expect $R$ to increase with increasing $\mu$, as $\delta R \propto R \propto 0.158 \mu \delta Z_M$. These expected changes are roughly borne out by our numerical results, $\Delta R \approx 0.6\%$ and $0.02\%$, for $\delta Z_M = 0.02$ and $\delta Z_M = 10^{-3}$, respectively, to within a factor of $\sim 2$.

### 3.3 Hertzsprung Gap

Most interacting massive binaries experience mass transfer when the donor star crosses the “Hertzsprung Gap” (HG), as this is the phase of largest radial expansion (van den Heuvel 1969; Sana et al. 2012; Renzo et al. 2019); see Fig. 3 for the evolution of stellar radius with age for our fiducial models. The time a star takes to cross the Hertzsprung Gap is therefore important in determining the timescale for these mass transfer episodes (although ultimately the dynamical stability of mass transfer is determined by the reactions of the stellar radii and the orbital separation to the changes in the stellar masses, not radii; e.g., Soberman et al. 1997; Vigna-Gómez et al. 2020). If the donor is intrinsically evolving “fast” (e.g., on a thermal timescale), this forces an at least equally “fast” mass transfer timescale in a binary. Conversely, if the donor star is evolving “slowly” (e.g., on a nuclear timescale, Klencki et al. 2021b), then other process (e.g., the structural reaction of the secondary or the secular evolution of the orbit) can determine the binary mass-transfer timescale.

These reasons motivate us to explore the metallicity effects on stellar radii during the epoch of fastest radial expansion, i.e. when $d \log_{10} R/dt$ is maximum (green dots in Figs. 2, 3). As illustrated by the matrices below, doubling each microphysics metallicity value acts to reduce the stellar radius at this stage (negative $\Delta$ values), except for the EOS-metallicity at high-Z and nuclear-metallicity at low-Z. Opacity plays the largest role in changing the radius, with $\Delta \mu \sim 11\%$ and $\sim 34\%$ in the high-Z and low-Z models, respectively.
Unfortunately, however, the uncertainties in the low-Z model results are similar in magnitude to the matrix entries (see Appendix A), so no firm conclusions can be drawn in this case.

### 3.4 Helium core burning

After a brief phase of contraction at H exhaustion (causing the so-called “Henyey hook”), massive stars rapidly ignite core He burning; this is the last long-lived phase of evolution, about ~10% of the total lifetime. In an observed population of stars, this is typically the most evolved phase for which a statistically significant number of stars can be assembled. Our 15 $M_\odot$ models take about 10$^5$ yrs to evolve from a central $^4$He mass fraction of $X_c(^4\text{He}) \sim 1$ to ~0.9, and 10$^6$ yrs to reach He-depletion ($X_c(^4\text{He}) = 0.01$ in the center).

Whether a star in this phase appears as a blue supergiant (BSG) with a small radius or a red supergiant (RSG) with a large radius is not fully understood. The answer depends sensitively on the previous evolution (e.g., Woosley 1988; Kippenhahn et al. 2012; Farrell et al. 2021a, and references therein), specifically the assumptions made to model convective boundary mixing and semiconvection (e.g., Langer et al. 1989; Brott et al. 2011; Schootemeijer et al. 2019), wind mass loss and rotation (e.g., Renzo et al. 2017; Farrell et al. 2021b; Sabahhit et al. 2021). Observations of massive stars in the Galaxy, Large and Small Magellanic clouds suggest a metallicity dependence of the BSG/RSG ratio (Klencki et al. 2020).

To place this phase into context, Figure 4 shows L-R tracks for several models calculated under fiducial model assumptions (Sec. 2.1), for a wide range of metallicities, $Z = 10^{-4}$, $10^{-3}$, 0.01, 0.02, 0.03, 0.04. We color the tracks according to the core $^4$He mass fraction. He ignition occurs with a drastic decrease in $X_c(^4\text{He})$, i.e. the quickest transition in colors in Fig. 4. Our 15 $M_\odot$ low metallicity stars ($Z = 10^{-4}$, $10^{-3}$) burn He early in the Herzspring Gap, corresponding to a high effective temperature of a BSG, while in the higher metallicity stars, He ignition occurs at a lower effective temperature during the RSG phase.

The matrices below compare stellar radii at the epoch when $X_c(^4\text{He}) = 0.9$ (red diamonds in Fig. 4); however, because the radius changes by $\leq 5\%$ throughout the He core burning phase, our results are not sensitive to this precise definition.

#### High Z

| $\Delta(\%)$ | $\mu$  | $\kappa$ | $\epsilon$ | $\Delta_{\mu\kappa\epsilon}$ | $\Delta_{Z}$ |
|-----------|--------|----------|-----------|-----------------------------|-------------|
| $\mu$ | [-7.4(-6.4)] | 6.5(-3.9) | 7.2(-6.4) | $-0.2(-3.9)$ | $11.7(-15.3)$ |
| $\kappa$ | - | 6.6(-8.9) | -2.9(-8.9) | | $12.6(0.0)$ |
| $\epsilon$ | - | - | 12.6(0.0) | | $4.7(0.9)$ |

#### Low Z

| $\Delta(\%)$ | $\mu$  | $\kappa$ | $\epsilon$ | $\Delta_{\mu\kappa\epsilon}$ | $\Delta_{Z}$ |
|-----------|--------|----------|-----------|-----------------------------|-------------|
| $\mu$ | [-10(13)] | -14(-21) | -4.7(13.4) | $33(-21)$ | $8.3(-11.2)$ |
| $\kappa$ | - | -7.1(-25) | -4.0(-25) | | $25.6(0.0)$ |
| $\epsilon$ | - | - | - | | $51(-5.7)$ |

![Figure 4](image_url)  
**Figure 4.** Luminosity-radius tracks showing the helium core burning phase for a range of metallicities – $Z = 10^{-4}$, $10^{-3}$, 0.01, 0.02, 0.03 and 0.04 from top left to bottom right, calculated under the fiducial model assumptions. The color of each track represents the mass fraction of $^4$He in the core. Gray points represent the radius of helium ignition, defined as $X_c(^4\text{He}) = 90\%$.

We see that the nuclear-metallicity has the highest impact on stellar radius in both the high-Z and low-Z cases, at $\Delta_\epsilon \approx 18\%$ and $\Delta_\epsilon \approx 26\%$, respectively, followed by EOS-metallicity and then the opacity. Unfortunately the uncertainties on the matrix entries are comparable to or larger than their physical values, particularly in the low-Z models (see Appendix A2).

The large differences between $\Delta_{\mu\kappa\epsilon}$ and $\Delta(\mu)$ in both the high- and low-Z models implicate the presence of non-linear interactions between the impact of the three microphysical effects. There is also a smaller but still significant difference between $\Delta_{\mu\kappa\epsilon}$ and $\Delta_{Z}$, which is noticeably larger in the high-Z case. This discrepancy may be related to the difference in the initial helium mass fraction of the 2Z versus $Z_c = Z_\mu = Z_\epsilon = 2Z$ models (Sec. 4.3).

#### 3.5 Beginning of Hayashi Track (Half Convective Envelope)

A key element determining the stability of mass transfer – and the outcome of common envelope events – in a binary system is the density stratification of the donor star’s envelope. Efficient convection in RSG envelopes enforces a flat entropy profile; if mass is removed from the surface, the envelope receives expanding, leading more likely to unstable mass transfer (although thin radiative surface layers can greatly stabilize the mass transfer, Pavlovskii et al. 2017; Marchant et al. 2021).

As stars evolve redward on the HR diagram, they develop a convective envelope which grows inward. Therefore, the fraction of envelope mass that is convective ($f_{\text{conv,env}}$) grows at this stage. Below, we compare the model stellar radii at the point when the convective mass of the envelope has grown to half of the total envelope mass, or $f_{\text{conv,env}} = 0.5$ (purple crosses in Figure 2). We obtain the envelope mass by subtracting the mass of the He core from the total mass of the star, where the helium core boundary is defined at the outermost location where the hydrogen mass fraction is $\leq 0.01$ and the helium mass fraction is $\geq 0.1$. The $f_{\text{conv,env}} = 0.5$ epoch occurs before the He
core burning phase (Sec. 3.4) in the high-Z models and after in the low-Z models.

### High Z

| Δ (%) | μ | κ | ε | Δμεκ = -1.8(1.3) |
|-------|---|---|---|------------------|
| 0.2(3.9) | -1.8(1.3) | 1.9(3.9) |

### Low Z

| Δ (%) | μ | κ | ε | Δμεκ = -1.8(1.3) |
|-------|---|---|---|------------------|
| -0.3(0.4) | -0.2(-0.7) | 0.03(0.4) |

We see that the opacity-metallicity has the largest effect on radii at this phase in the high-Z models, nuclear-metallicity is a close second, and the EOS-metallicity is the least important, within reasonable degrees of uncertainties. However the results in the low-Z case are again obscured by large numerical uncertainties (Appendix A1).

### 3.6 Late Hayashi Track

Neglecting possible late, dynamical mass loss events (e.g., Quataert & Shiode 2012; Shiode & Quataert 2014; Khazov et al. 2016; Fuller 2017), our 15 M☉ models spend their final evolutionary stages on the Hayashi track, at temperatures 3200 K ≤ Teff ≤ 4000 K. Since the envelope structure is essentially frozen during the final stages of stellar evolution due to high neutrino luminosity of the core (Fraley 1968), its radius will be similar to that at the time of core collapse and (potential) supernova explosion (however, see Quataert & Shiode 2012). The final stellar radius has important implications for the early-time light curves of supernovae (e.g., Nakar & Sari 2010; Piro & Nakar 2013; Morozova et al. 2018; Goldberg & Bildsten 2020; Gill et al. 2022).

Comparing our models on the Hayashi track at the point they reach a common luminosity, \( L = 10^3 L_☉ \), we obtain the following matrices:

### High Z

| Δ (%) | μ | κ | ε | Δμεκ = 9.2(0.3) |
|-------|---|---|---|------------------|
| -0.11(0.06) | 12.7(0.3) | 1.2(0.03) |

### Low Z

| Δ (%) | μ | κ | ε | Δμεκ = 6.3(-0.8) |
|-------|---|---|---|------------------|
| -0.11(-0.06) | 5.7(-0.8) | -0.6(-0.06) |

In both high-Z and low-Z models, the opacity-metallicity has the biggest effect on the stellar radius. To quantitatively understand this behavior, we can approximate the star as being almost fully convective. Since convection in the bulk of RSG envelopes is efficient, the entropy profile will be constant and \( P \propto ρ^{5/3} \). We further assume an ideal gas EOS, \( P = P_{gas} = ρ/T/μ_p \), following (Hayashi 1961).

These approximations are well-justified for our models: the fraction by radius of the envelope which is still radiative at \( L = 10^3 L_☉ \) is less than 0.1% in the high Z models and ≤ 8% for the low-Z models. Even at this late stage of stellar evolution, gas pressure dominates over radiation pressure in our 15 M☉ models (\( ⟨\delta E⟩ \approx P_{gas}/P_{tot} ≥ 0.7 \)). The central pressure and temperature for a \( γ = 5/3 \) polytrope are given by, \( P_c = 0.77GM^2/R^4 \) and \( kT_c = 0.54GM(μ)m_p/R \), where \( (μ) \) is the average mean molecular weight.

The photosphere occurs roughly where the average photon mean-free path \( 1/kP \) equals the atmospheric scale-height, \( H \equiv c_{s,ph}^2/g \), where \( g = GM/R^2 \) is the surface gravity and \( c_{s,ph} \) the sound speed at the photosphere. The dominant opacity source near the photosphere is the bound-free absorption by \( H^- \) ions, which we approximate as (e.g., Kippenhahn et al. 2012)

\[
\kappa_{H^-} = 2.5 \times 10^{-31} \left( \frac{Z_α}{Z_☉} \right) \frac{1}{P_{ph}} \rho_{ph}^2 \epsilon_{eff} \ cm^2 g^{-1}.
\]  

where the linear dependence with \( Z_α \) assures that the electrons at these low temperatures are from singly-ionized alkali metals rather than hydrogen or helium. Combining the above relations, we obtain

\[
R \propto L^{39/102} M^{-14/51} \langle μ ⟩^{-26/51} Z_α^{-8/51}.
\]  

Equation 5 predicts that doubling \( Z_α \) at fixed \( \langle M, R, (μ) \rangle \) should increase \( R \) by \( ~11\% \), roughly consistent with the high–Z matrix entries above, e.g. \( Δ_μ ≈ 12.5\% \). The agreement for the low–Z models is not as good (though still within a factor \( ≤ 2 \)), possibly due to the star not being fully-convective or a break-down of the assumption that alkali metals supply the electrons which contribute to the \( H^- \) opacity.

The much weaker dependence of stellar radius on \( Z_μ \) can be understood because the mean molecular weight is dominated by hydrogen and helium in the envelope. For a fully ionized stellar interior one would predict \( δ_μ/μ \sim 0.9δZ_μ \) (Eq. 3 and surrounding discussion), such that from Eq. 5 one predicts \( δR/R \sim -(26/51)(0.9δZ_μ) \sim -0.5δZ_μ \); this is consistent with sign, but not the magnitude, of the \( Δ_μ \) entry for the high-Z models. However, the stellar envelope will not be completely ionized given the lower temperature of the giant star envelope and hence a more detailed consideration of ionization-state-dependence of \( (μ) \) is needed to make a quantitative prediction. Finally, the weak dependence on \( Z_α \) we observe is also expected, given that the properties of the nuclear energy source do not enter to first order in setting the radius of a fully convective star.

## 4 DISCUSSION

The previous sections have quantified the relative importance of different microphysical processes on the metallicity-dependence of massive star radii. This section describes some implications, applications, and caveats of our results.

### 4.1 Microphysics Error Propagation

One application of the general technique developed in this paper is as a tool to propagate theoretical or experimental uncertainties in the microphysics inputs into corresponding theoretical uncertainties in modelled stellar radii. We briefly discuss the implications of our findings in this regard for each microphysics input.

**Opacity.** We have found that the metallicity-dependence of the opacity has the largest impact on stellar radii during most evolutionary epochs. This implies that theoretical estimates of stellar radii are particularly sensitive to uncertainties in opacity. Our stellar models employ the OPAL opacity tables (see Sec. 2 for details); however, the knowledge of atomic physics in stellar environment is still evolving. As an example of the potential magnitude of such changes, the updated OPAL tables in Iglesias & Rogers (1996) yielded up to a
20% increase in opacity versus the previous baseline. Furthermore, the wavelength-dependent opacity of iron was recently measured by Bailey et al. (2015) to be ~75% higher than in the OP and OPAL tables for solar conditions; this has a significant impact on the location of the $\beta$ Cep pulsational instability strip on the Hertzsprung-Russell diagram (Moravveji 2016). If more generally representative of opacities for solar conditions; this has a significant impact on the location of stellar radii of a MS star is comparable to that expected in moving from Milky-Way (Z $\sim Z_{\odot}$) to SMC-like (Z $\sim 0.1Z_{\odot}$) metallicities.

**Equation of State.** Unlike opacity, uncertainties in the OPAL equation of state used in our numerical experiments in the regimes of temperature and density relevant to the evolution of a 15$M_{\odot}$ star are typically small (~ 10%; Rogers & Nayfonov 2002). Furthermore, our experiments indicate that the EOS-metallicity has the weakest impact on stellar radii. We therefore conclude that the EOS is not a major contributor to the uncertainty in massive star radii.

**Nuclear Reactions.** Our results show that in the early evolutionary stages of low-Z stars, nuclear reactions dominate the metallicity-dependence of the stellar radius. However, the experimental uncertainties in the nuclear reactions rates at a given temperature and density in MESA are typically ~ 10% (Sallaska et al. 2013; Fields et al. 2018) for the CNO reactions (though we note the bottle neck reaction $^{14}$N(p, y)$^{15}$O of the cycle has undergone rate changes as large as a factor of ~ 2; e.g., Forni cola et al. 2003; Imbriani et al. 2004). However, because the metallicity-sensitivity of stellar radii at low-metallicity is low in an absolute sense, nuclear reaction uncertainties should not be a major source of error.

**4.2 Observational Implications**

The Introduction enumerated several applications which depend on stellar radii and being able to quantify their accuracy. Here, we expand this discussion guided by the results of our numerical experiments.

**Timing of Shock Breakout.** The 15 $M_{\odot}$ single non-rotating stellar progenitors explored in our models reach core-collapse as RSG with large radii (cf. Fig. 1–4). Assuming such stars undergo successful explosions, they are likely to produce Type IIp supernovae (SNe). Core collapse SNe are sources of thermal neutrino emission (e.g., Burrows & Lattimer 1986) and of gravitational waves (GW; e.g., Ott 2009), within seconds of core bounce. The sensitivity of GW searches is sensitive to the time-window of the data-stream being searched, particularly for a weak signal. Absent a coincident neutrino detection, electromagnetic observations of the SN shock breaking out of the stellar surface can provide an estimate of the core collapse time (e.g., Goldberg & Bildsten 2020; Gill et al. 2022); however, the accuracy of the obtained time delay relies on knowledge of the stellar radius.

**Supernova Light Curves.** The initial rise-time and luminosity of core collapse SNe light curves depends sensitively on the stellar radius at the time of explosion and any pre-explosion mass loss (e.g., Nakar & Sari 2010; Piro & Nakar 2013; Valenti et al. 2016; Morozova et al. 2017; Morozova et al. 2018). Our results show that during the final evolutionary stages, the radius uncertainties are dominated by Z-dependent opacity, being ~6% and 12% in subsolar and solar case, respectively. Other microphysics have < 1% effects in stellar radii, negligible compared to the uncertainties in progenitor evolution (e.g., O’Connor & Ott 2011; Sukhbold et al. 2018; Laplace et al. 2021; Renzo & Götberg 2021) and macrophysical effects (e.g., Woosley et al. 2002; Langer 2012; Renzo et al. 2017; Davies & Beasor 2018; Zapartas et al. 2021a,b). However, the microphysics effects explored in this paper may be swamped by the three-dimensional effects not captured by a 1D stellar evolution model (e.g., Goldberg et al. 2022).

**Other Applications.** Although this work focuses on massive stars, our general technique could also be applied to assess uncertainties on lower-mass stellar radii. They are relevant in the context of inferring stellar properties through eclipsing star (Maiz-Apellaniz et al. 2004; Sota et al. 2008; Handler et al. 2012; Williams et al. 2013; Cazorla et al. 2017; Pozo Nuñez et al. 2019; Trigueros Páez et al. 2021; Johnston et al. 2021) and transiting exoplanets observations (e.g., Henry et al. 2000; Charbonneau et al. 2000; Mandel & Agol 2002; Seager & Mallen-Ornelas 2003). The observed radius of a companion star/exoplanet obtained through these methods rely on the assumptions in radius of the primary/host star (e.g., Johnson et al. 2017). Our method of analysis could be used to break down the radius uncertainty budget in any of these situations, or for other applications involving stars of different masses.

**4.3 Other Effects on Stellar Radii**

The initial hydrogen mass fraction $X$ and stellar wind-metallicity $Z_{\text{wind}}$ were held fixed in all of the models presented thus far (one exception being the value of $X$ in the fiducial ZZ model), in order to isolate effects that arise from the metallicity-dependent microphysics from those related to the associated change in the hydrogen and helium abundances, or the details of the wind mass-loss prescription. This section attempts to quantify the impact of these specific choices on stellar radii relative to the microphysics-effects of interest. We conclude by describing some technical caveats associated with our calculations that also contribute to radii uncertainties.

**Wind Metallicity $Z_{\text{wind}}$.** Figure 5 shows how the luminosity-radius evolution of the $Z = 0.04$ fiducial models change for different assumptions about the wind metallicity. As $Z_{\text{wind}}$ increases from 0 to 0.04, the star loses mass at a higher rate, causing a reduction in its luminosity. The impact of $Z_{\text{wind}}$ on the stellar radius at the key epochs of interest are summarized in Table 2.

In the high-Z models, increasing the value of $Z_{\text{wind}}$ from 0 to 0.04 results in only a small change in the radius ($\Delta Z \lesssim 5\%$), except during the Hayashi epochs when the change is as large as $\Delta Z \sim 10\%$. For comparison, the largest single microphysics-dependent change during the Hayashi phase was $\Delta X \approx 8 \sim 12\%$. Thus, the impact of the metallicity dependence of microphysics on stellar radii is at most comparable to the impact of the metallicity dependence of the mass-loss prescription ($M \propto Z^{0.85}$, Vink & De Koter 2005), at least as can be encapsulated through order-unity changes in the parameter $Z_{\text{wind}}$.

Conversely, the low-Z models can exhibit larger variations by changing $Z_{\text{wind}}$ compared to changing the metallicity of microphysical ingredients. Increasing $Z_{\text{wind}}$ from 0 to 0.04 causes $\Delta Z_{\text{wind}}$ to change by as much as ~ 50 – 70%, however the large variations occur during HG and He core burning phases, where the numerical uncertainties are also large (Appendix A). Besides these epochs, on the MS and the Hayashi track, the $Z_{\text{wind}}$-dependence is smaller than the dominant microphysics effects. Namely, $\Delta X = 5\%$ on the MS, compared to ~ 0.3 – 1% changes due to $Z_{\text{wind}}$, and $\Delta X = 1 \sim 6\%$ on the Hayashi track, compared to $\lesssim 1\%$ due to $Z_{\text{wind}}$.

In addition to changes in stellar radii that arise from changing $Z_{\text{wind}}$, different choices about the wind prescription itself have an impact. Renzo et al. (2017) explored the uncertainties in stellar radii
that arise from imposing different wind mass-loss prescriptions. They find that using different wind parameters results in a ∼7% change in stellar radius in the RSG phase, for example.

Hydrogen and Helium Abundances. Figure 6 compares the effects of different choices for the initial hydrogen (X) or helium (Y) mass fractions on the luminosity-radius evolution of the high-Z fiducial models (Z = 0.02 and Z = 0.04). We show models in which X or Y are held fixed with varying Z, as well as those where X, Y, and Z vary together following Pols et al. (1998). Table 3 summarizes the fractional change in the stellar radii ΔZ in these choices at the key epochs of evolution.

The effect of changing Z illustrated in Figure 6 broadly agrees with those found by Farrell et al. (2021a) through a different numerical experiment. Specifically, fixing Y changes the value of ΔZ from the X-fixed case by ∼1−7% and ≤ 20% in high-Z and low-Z models, respectively (Table 3); these variations are smaller than the dominating effect of microphysics at any given epoch, except at the HG and He core burning where large numerical uncertainties persist. Varying X and Y together following Pols et al. (1998) changes ΔZ by an even larger amount compared to the X-fixed case, since now neither the hydrogen nor helium content remain the same as in the fiducial Z models.

The different value of Y which characterize our {Z, ZH, Ze} = 2Z models versus that of the fiducial 2Z model to which they are compared, introduces another source of uncertainty in attributing radius changes solely to individual Z-dependent microphysical effects, in addition to those described in Appendix A.

Limitations of 1D modeling. The post-MS evolution of massive stars is notoriously sensitive to the composition and entropy profiles between the H-depleted core and the envelope (e.g., Walmswell et al. 2015; Schootemeijer et al. 2019). These aspects are determined by processes hard to model in 1D, such as the recession of the convective boundary during the MS, semiconvective and rotational mixing above the core during and after H-core burning (e.g., Langer et al. 1989; A. Maeder & Meynet 2000; Schootemeijer & Langer 2018; Schootemeijer et al. 2019; Klencki et al. 2020, 2021a), the mass-loss history (e.g., Belczynski & Taam 2008; Renzo et al. 2017), and the possible accretion episodes (e.g., Renzo & Göbbert 2021), all of which can have different metallicity dependencies. For stars more massive than the 15 Msolar models that we considered here, another possibly metallicity-dependent, macrophysical ingredient enters in the radius determination: the treatment of energy transport in radiatively inefficient super-Eddington convective layers (e.g., Paxton et al. 2018).

Table 2. Percentage change, ΔZ, in the radius of the 2Z fiducial model relative to the fiducial Z model as a result of adopting the wind metallicity Zwind denoted in each column. In all other input, the high- and low-Z models assume Z = 0.04 and Z = 2 × 10−3, respectively. *Wind metallicity used in the 2Z fiducial models in Table 1.

| Zwind          | High-Z | Low-Z |
|---------------|--------|-------|
| MS            | 27.0   | 0.04  |
| HG            | -7.6   | -8.8  |
| He Burning    | 3.6    | 5.9   |
| Early Hayashi | -2.4   | -5.1  |
| End Hayashi   | 13.6   | 6.8   |

Table 3. Percentage fraction change, ΔZ, in the radius of the 2Z fiducial model relative to the fiducial Z model as a result of varying the initial hydrogen X and helium Y mass fractions at different epochs of stellar evolution as marked. From left to right, the columns assume initial composition values (X, Y, Z) as follows: (0.64, 0.32, 0.04), (0.73, 0.23, 0.04), (0.75, 0.21, 0.04), (0.75, 0.244, 2 × 10−3), (0.749, 0.249, 2 × 10−3), and (0.75, 0.248, 2 × 10−3). In all other input, the high- and low-Z models assume Z = 0.04 and Z = 2 × 10−3, respectively. *Composition used in the 2Z fiducial models introduced in Section 2.

| Composition | High-Z | Low-Z |
|------------|--------|-------|
| MS         | 1.9    | 9.4   |
| HG         | 0.1    | 9.7   |
| He Burning | 9.4    | 10.0  |
| Early Hayashi | 2.3 | 0.03  |
| Late Hayashi | 8.0  | 6.9   |

Figure 5. Luminosity-radius tracks for three different choices of the metallicity entering the stellar wind prescription − Zwind = 0.04 (dashed red), 0.02 (black; fiducial assumption of all high-Z models in Table 1) and 0 (dot-dashed purple, no mass loss). All models assume X = 0.75; Y = 0.21.

Figure 6. Dependence of the luminosity-radius evolution in the high-Z models on the choice of initial hydrogen X and helium Y mass-fractions. The fiducial Z = 0.02 model, for which X = 0.75 and Y = 0.23, is shown with a black curve. For the Z = 0.04 case, we compare three different choices: X = 0.64, Y = 0.32, i.e. following the default scaling in MESA (Pols et al. 1998) (dotted maroon line); Y = 0.23 is fixed and X = 1 − Y − Z = 0.75 decreases (red dashed line); X is fixed at 0.75 and Y = 1 − X − Z = 0.21 decreases (orange dot-dashed line). This last option (X fixed) corresponds to that adopted in our 2Z models in Table 1.
The detailed treatment of such regime, which is an intrinsically three-dimensional problem because of the interplay between turbulence seeded with the iron opacity bump and the helium opacity bump (Jiang et al. 2018), determines whether more massive star inflate at the end of their main sequence (Sanyal et al. 2015, 2017) or instead trigger eruptive mass loss events. However, our 15 $M_\odot$ models are sufficiently low-luminosity to not enter this regime common for $M \gtrsim 25 M_\odot$.

**Comparison with observational uncertainty.** Nevertheless, we are being extremely demanding with the numerical resolution adopted in our models. Appendix A2 shows that the largest uncertainties that arise from varying the spatial and time resolution parameters occur on the Hertzsprung Gap and has magnitude of $\lesssim 10\%$. In reality, the observational errors on the stellar radii of massive stars are much larger compared to the variations we see here. For example, the closest O-type MS star, $\zeta$ Ophiuchi ($\sim 20 M_\odot$), has a radius uncertainty of $\sim 10 - 20\%$ (see, e.g., Villamariz & Herrero 2005; Gordon et al. 2018; Renzo & Götberg 2021; Shepard et al. 2022, and references therein), while the radius uncertainty of the nearest RSG $\alpha$ Orionis (Betelgeuse, $\sim 15 - 20 M_\odot$) is up to $\sim 30\%$ (e.g., Chatzopoulos et al. 2020, and references therein).

**5 CONCLUSIONS**

We have constructed MESA stellar evolution models of a 15$M_\odot$ star to dissect the effects that metallicity-dependent opacity, mean molecular weight and nuclear physics on stellar radii at key points in the star’s life. We explore models centered around a solar ($Z = 0.02$) and subsolar ($Z = 10^{-3}$) metallicity value, which characterize the isolated and aggregate impacts of the three $Z$-dependent microphysics.

As a visual demonstration of the fractional radius-change matrices presented in Sec. 3, we summarize our findings in Fig. 7. We show the partitioning of microphysics with bar plots for each epoch in the high-$Z$ and low-$Z$ case. The heights and error bars are defined as:

$$\text{(Height of bar)} = \frac{\Delta_i}{\Delta_{2Z}} \times 100\%$$

$$\text{(Error bar)} = \frac{\delta_i}{\Delta_{2Z}} \times 100\%$$

$i \in \{\mu, \kappa, \epsilon\}$ corresponds to each diagonal value in the matrix. The error bars are obtained from the null matrices (‘$\delta$’s’) given in Appendix A1, which represent the uncertainties that arise from our implementation method. Our main findings are summarized as follows:

- In both high-$Z$ and low-$Z$ cases, the $Z$-dependent effect of EOS is the weakest on radius across all five epochs.
- Our numerical results on the MS and at late Hayashi track broadly agree with the corresponding analytical solutions, under reasonable assumptions.
- Our choice to fix stellar wind metallicity to $Z_{\text{wind}} = Z$ in high-$Z$ and low-$Z$ models and to fix $X = 0.75$ in all models, does not change the resulting partitioning of the microphysics. The different assumptions about $Z_{\text{wind}}$ and the composition ($X$ and $Y$) influence the $2Z$ fiducial models, although the influence is small compared to that of the microphysics during most epochs.
- Our results highlight the importance of quantifying uncertainties in stellar evolution models, particularly when conducting numerical experiments where the measurable effects can be subtle. In this work, we evaluate the uncertainties resulting from (i) our specific implementations of different microphysical metallicities in MESA, (ii) the adopted time and spatial resolution. We conclude that our implementation leads to slightly larger uncertainties than numerical resolutions.

Overall our findings support the promise of controlled numerical experimenting with stellar evolution codes to elucidate the dominant physical processes at work. Future work could include the application of our method to stars of other masses, in order to systematically dissect the effects of different microphysics on macrophysical observables, such as stellar age, luminosity, rotation, stellar mass-loss, etc.. The matrix formalism we have introduced provides a convenient framework to present and interpret these results.
APPENDIX A: NUMERICAL UNCERTAINTIES

Here we evaluate the numerical and algorithmic uncertainties associated with the method introduced in Sec. 2. These uncertainties arise from: (1) our method of implementing Z-dependent microphysics into MESA; (2) numerical resolution, both in time and in mass-coordinate. The amplitudes of these two effects are summarized and compared in Fig. 4.1.

A1 Microphysics Implementation

We first explore the impact of the artificial modifications we have made to the microphysics input to MESA on the precision of our results for stellar radii. As a sanity check, we rerun all the models in Table 1 using our customized routines, except now fixing the input metallicity Z at its original value \( Z_\mu = Z_k = Z_\epsilon = Z \) rather than doubling it \( (Z_\mu = Z_k = Z_\epsilon = 2Z) \) as before. Models #1 and #9 are now assigned the same metallicity \( (Z = 0.02) \), as are models #10 and #18 \( (Z = 10^{-3}) \); however, the altered wind-metallicity is only activated in models #9 and #18, both at \( Z_{\text{wind}} = Z \). We refer to the new series of radius matrices generated by comparing the \( Z_\mu = Z_k = Z_\epsilon = Z \) and fiducial Z models as “null” because, given a perfectly controlled experiment, their entries should all be strictly zero. We denote the elements of the null matrices by \( \delta \)'s to distinguish them from the normal matrix elements \( \Delta \)’s introduced in §3.1.

The expectation \( \delta \to 0 \) will not be realized in practice if our alterations to the microphysics input of MESA (§2.2) do not exactly reproduce the default assumptions present in the fiducial Z model. The magnitude of the residual \( \delta \) values can therefore be used to estimate the minimum uncertainty on the \( \Delta \) values presented in §3 (in addition to the effects of resolution explored in Appendix A2).

The null matrices corresponding to each phase of stellar evolution are given below. The value of \( \delta \epsilon \) is always zero; this is because our alterations to the nuclear reactions input involve simply changing the reaction rate factors, which is a built-in option already present in MESA. Also note that \( \delta Z \) is small compared to the other \( \delta \)'s in all cases; this matrix entry compares two fiducial models with the same Z but different \( Z_{\text{wind}} \). The fact that \( \delta Z \) is small (strictly zero in the case of MS and late Hayashi track) shows that our method of artificially fixing the value \( Z_{\text{wind}} \) in our numerical experiments does not itself impart a significant uncertainty to \( \Delta \) (however, see discussion in Sec. 4.3).

In general the other \( \delta \) matrix elements are non-zero. The radii at the five key evolutionary points discussed in Sec 3 are marked as symbols in Fig. A1. The spread in radii between models is depicted graphically by the colored bands, enabling an easy comparison of the uncertainty that arise due to our microphysics implementation (top panels) and due to the numerical resolution (bottom panels; Sec. A2).

In broad terms, the implied uncertainties in the high-metallicity \( Z = 0.02 \) model sequence (top panel left of Fig. A1) are sufficiently small so as to not dominate the physical radii changes we are trying to measure (the \( \delta \)'s are much smaller than the \( \Delta \)'s), except during the He core burning and early Hayashi phases. However, in the \( Z = 10^{-3} \) models (top right panel of Fig. A1), the algorithmic uncertainties become large also in the Hertzsprung Gap, and are generally larger at all epochs than for the high-Z models.

Below now present and describe the null matrices for each stage of stellar evolution in greater detail.

| Main Sequence | High Z |
|---------------|--------|
| \( \delta \) (%) | \( \mu \) | \( \kappa \) | \( \epsilon \) |
| \( \delta Z \) | \( \delta_{\mu\kappa\epsilon} = 0.127 \) |
| --- | --- | --- | --- |
| \( \delta_{\mu\kappa\epsilon} = 0.127 \) | 0.072 | 0.127 | 0.072 |
| \( \delta_{\mu\kappa\epsilon} = 0.074 \) | - | 0.074 | 0.074 |
| \( \delta_{\mu\kappa\epsilon} = 0.074 \) | - | - | 0.0 |
| \( \delta_{\mu\kappa\epsilon} = 0.074 \) | - | - | 0.0 |
| \( \delta_{\mu\kappa\epsilon} = 0.074 \) | - | - | 0.0 |
Figure A1. Evolution of stellar luminosity with radius from a grid of “null” models used to assess the numerical uncertainties in our results due to (1) our implementation of the metallicity-dependent microphysics in MESA (top panels; Sec. A1); and (2) temporal and mass-grid resolution (bottom panels; Sec. A2). We show separately the high-
\( Z \) \((Z = 0.02; \text{left panels})\) and low-
\( Z \) \((Z = 10^{-3}; \text{right panels})\) models. The numerical implementation models were run with \( Z_{\text{wind}} = Z \) for all different combinations of \( \kappa, \mu, \epsilon \). For the grid resolution runs, the time resolution parameter “vct” was varied between \( 10^{-3} \) to \( 10^{-4} \) and mass-coordinate resolution \( d_s \) from 1.0 to 0.5. The five key evolutionary epochs are marked with the same symbols as those in Fig. 2. The spread of the points in radius-coordinate in each epoch indicates the degree of uncertainty – highlighted with colored bands. Comparing the left and right panels across each row, we see that the uncertainties in the high-
\( Z \) models are small compared to those in the low-
\( Z \) case. A comparison between the top and bottom panels in each column shows that the uncertainty introduced by our implementation is typically comparable or larger than that arising from our choice of numerical resolution.

On the MS, the null matrices are small \( \delta \) relative to the \( \Delta \) values in both high-
\( Z \) and low-
\( Z \) models. Our results on the MS (§3.2) are the most robust of all the epochs we have explored.

### Hertzsprung Gap

#### Low Z

| \( \delta \) (%) \( \mu \) \( \kappa \) \( \epsilon \) | \( \mu \) \( \kappa \) \( \epsilon \) | \( \mu \) \( \kappa \) \( \epsilon \) |
|---|---|---|
| 0.026 | 0.017 | 0.026 |

### High Z

| \( \delta \) (%) \( \mu \) \( \kappa \) \( \epsilon \) | \( \mu \) \( \kappa \) \( \epsilon \) | \( \mu \) \( \kappa \) \( \epsilon \) |
|---|---|---|
| 9.9 | 4.1 | 9.9 |

### Resolution

#### Low Z

| \( \delta \) (%) \( \mu \) \( \kappa \) \( \epsilon \) | \( \mu \) \( \kappa \) \( \epsilon \) | \( \mu \) \( \kappa \) \( \epsilon \) |
|---|---|---|
| 6.4 | -42 | 6.4 |

### Helium Core burning

#### High Z

| \( \delta \) (%) \( \mu \) \( \kappa \) \( \epsilon \) | \( \mu \) \( \kappa \) \( \epsilon \) | \( \mu \) \( \kappa \) \( \epsilon \) |
|---|---|---|
| -6.4 | -3.9 | -6.4 |

In the Hertzsprung Gap, the null matrix entries are sufficiently small in the high-
\( Z \) models relative to the \( \Delta \)'s, that our conclusion from Sec. 3.3 that metallicity entering the opacity has the greatest impact on stellar radii at this stage still holds well. By contrast, in the low-
\( Z \) models the \( \delta \) entries are large compared to the corresponding \( \Delta \) values, precluding definitive conclusions about the dominant microphysical effect at this epoch.
Similarly, at He core burning the uncertainties implied by the null-matrix values are too large compared to the Δ values presented in Sec. 3.4. In the high-Z models, δκ and δκε are large compared to Δκ and Δκε, challenging our conclusion that nuclear reactions are the dominant effect on stellar radii at this epoch (even though Δκ ≫ Δκ). In the low-Z models, the δ-implied uncertainties are also too large, again preventing us from drawing meaningful conclusions.

### Early Hayashi track

**High Z**

| δ (%) | μ | κ | ε |  \( \delta_{\mu\kappa\epsilon} = 1.3 \) |
|-------|---|---|---|----------------------------------|
| μ     | 3.9 | 1.3 | 3.9 |
| κ     | -0.3 | -0.3 |
| ε     | -0.0 | -0.0 |

**Low Z**

| δ (%) | μ | κ | ε |  \( \delta_{\mu\kappa\epsilon} = -0.73 \) |
|-------|---|---|---|-------------------------------|
| μ     | 0.35 | -0.73 | 0.35 |
| κ     | -1.2 | -1.2 |
| ε     | -0.0 | -0.0 |

### Late Hayashi track

**High Z**

| δ (%) | μ | κ | ε |  \( \delta_{\mu\kappa\epsilon} = 0.25 \) |
|-------|---|---|---|------------------|
| μ     | 0.03 | 0.25 | 0.03 |
| κ     | -0.48 | -0.48 |
| ε     | -0.0 | -0.0 |

**Low Z**

| δ (%) | μ | κ | ε |  \( \delta_{\mu\kappa\epsilon} = -0.84 \) |
|-------|---|---|---|------------------|
| μ     | -0.06 | -0.84 | -0.06 |
| κ     | -0.92 | -0.92 |
| ε     | -0.0 | -0.0 |

The late Hayashi track matrices exhibit the second best accuracy (lowest δ values), next to those of the MS, in both high-Z and low-Z model sequences. The uncertainties are sufficiently small compared to all of the Δ values (§3.6) that our conclusions regarding this phase should also be robust.

### A2 Numerical Resolution

Our numerical implementation not only modifies the physics but also numerical details of the MESA models wherever we change the microphysics, including, for example, the precision of the initial abundances that we manually enter for the opacity routine. Such change could result in different resolution requirements in each model. We estimate the uncertainties from choices of numerical resolutions below, and compare to those from our implementation method.

The bottom panels of Fig. A1 show Zfiducial models in which the temporal or spatial resolution have been changed from their default assumptions. Specifically, we vary the two resolutions separately by changing mesh_delta_coeff (dx = 1.0 and 0.5) and varcontrol_target (vct=10^-3 and 10^-4), respectively, where smaller values impose more stringent limits on the mesh and timestep. As a result, the maximum timesteps (dt) decreases from 10^3 to 10^4 years and the spatial resolution is roughly double the default setting in MESA.

Comparing the width of the radii bands at each epoch in Fig. 2, the uncertainties associated with the choice of resolution in the low-Z models are larger than those in high-Z models. As a conservative choice, we use the highest set of resolution, mesh_delta_coeff = 0.5 and varcontrol_target = 1.0d-4, in all of our models in this paper. However if we compare the bottom and the top panels in Fig. A1, we see that the uncertainty associated with algorithmic implementation of microphysics typically introduces a larger uncertainty than the choice of resolution.

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### DATA AVAILABILITY

The input files, modified MESA routines, and output files are available on https://doi.org/10.5281/zenodo.6621643.

### REFERENCES

A. Maeder Meynet G., 2000, The Evolution of Rotating Stars
Alosi A., Clementini G., Tosi M., Annibali F., Contreras R., Fiorentino G., Mack J., 2007, ApJ, pp 183–186
Anders E. H., Jermyn A. S., Lecoanet D., Brown B. P., 2022a, ApJ, 926, 169
Anders E. H., Jermyn A. S., Lecoanet D., Fraser A. E., Cresswell I. G., Joyce M., Fuentes J. R., 2022b, ApJ, 928, L10
Angulo C., et al., 1999, Nuclear Physics A, 656, 3
Annibali F., et al., 2018, Astronomical Journal, 146
Asplund M., Grevesse N., Sauval A. J., Scott P., 2009, ARA&A, 47, 481
Bailey J. E., et al., 2015, Nature, 517, 56
Belczynski K., Taam R. E., 2008, ApJ, 685, 400
Bellazzini M., Ferraro F. R., Pancino E., 2001, Monthly Notices of the Royal Astronomical Society, 327, 15
Bethe H. A., 1939, Physical Review, 55, 434
Böhmi-Vitense E., 1958, Z. Astrophys., 46, 108
Brott I., et al., 2011, Astronomy and Astrophysics, 530, 1
Buchler J. R., Yueh W. R., 1976, ApJ, 210, 440
Burrows A., Lattimer J. M., 1986, ApJ, 307, 178

2 For the time resolution, this is not the recommended approach in more recent MESA versions, see the documentation.
The Z-dependent microphysics of stellar radii 15

Kippenhahn R., Weigert A., Weiss A., 2012, Stellar Structure and Evolution, doi:10.1007/978-3-642-30304-3.

Klencki J., Nelemans G., Istrate A., Pols O., 2020, Massive Donors in Interacting Binaries: Impact of Metallicity (arXiv: 2004.00628)

Klencki J., Istrate A. G., Nelemans G., Pols O., 2021a, Partial-Envelope Stripping and Nuclear-Timescale Mass Transfer from Evolved Supergiants at Low Metallicity (arXiv: 2111.10271)

Klencki J., Istrate A. G., Nelemans G., Pols O., 2021b, arXiv e-prints, p. arXiv:2111.10271

Klencki J., Nelemans G., Istrate A. G., Chruslinska M., 2021c, A&A, 645, A54

Langanke K., Martínez-Pinedo G., 2000, Nuclear Physics A, 673, 481

Langer N., 2012, Annual Review of Astronomy and Astrophysics, 50, 107

Langer N., El Eid M. F., Baraffe I., 1989, A&A, 224, L17

Laplace E., Justham S., Renzo M., Göttberg Y., Farmer R., Vartanyan D., De Mink S. E., 2021, Astronomy and Astrophysics, 656

Lederer M. T., Aringer B., 2009, A&A, 494, 403

Maiz-Apellaniz J., Walborn N. R., Galue H. A., Wei L. H., 2004, A Galactic O Star Catalog, doi:10.1086/381380

Mandel K., Agol E., 2002, The Astrophysical Journal, 580, L171

Marchant P., Pappas K. M. W., Gallegos-Garcia M., Berry C. P. L., Taam R. E., Kalogera V., Podsidiawlski P., 2021, A&A, 650, A107

Marigo P., Aringer B., 2009, A&A, 508, 1539

Martins F., Chantereau W., Charbonnel C., 2021, Astronomy and Astrophysics, 650, 1

Mehta A. K., Buonanno A., Gair J., Miller M. C., Farag E., DeBoer R. J., Wiescher M., Timmes F. X., 2022, The Astrophysical Journal, 924, 39

Moravec E., 2016, Monthly Notices of the Royal Astronomical Society: Letters, 455, L67

Morozova V., Piro A. L., Valenti S., 2017, The Astrophysical Journal, 838, 28

Morozova V., Piro A. L., Valenti S., 2018, ApJ, 858, 15

Nakar E., Sari R., 2010, Astrophysical Journal, 725, 904

O’Connor E., Ott C. D., 2011, Astrophysical Journal, 730

Ota D., Hino M., Muto K., Takahara M., Sato K., 1994, Atomic Data and Nuclear Data Tables, 56, 231

Ott C. D., 2009, Classical and Quantum Gravity, 26, 063001

Parker E. N., 1958, ApJ, 128, 664

Pavlovskii K., Ivanova N., Belczynski K., Van K. X., 2017, MNRAS, 465, 2092

Paxton B., Bildsten L., Dotter A., Herwig F., Lesaffre P., Timmes F., 2011, Astrophysical Journal, Supplement Series, 192

Paxton B., et al., 2013, Astrophysical Journal, Supplement Series, 208

Paxton B., et al., 2015, Astrophysical Journal, Supplement Series, 220

Paxton B., et al., 2018, The Astrophysical Journal Supplement Series, 234, 34

Paxton B., et al., 2019, The Astrophysical Journal Supplement Series, 243, 10

Piro A. L., Nakar E., 2013, Astrophysical Journal, 769

Pols O. R., Schröder K.-P., Hurley J. R., Tout C. A., Eggleton P. P., 1998, MNRAS, 298, 525

Pozio N., Minozz F., Chiri C., Barr Domínguez A., Fein C., Hackstein M., Pietrzyński G., Murphy M., 2019, Monthly Notices of the Royal Astronomical Society, 490, 5147

Quataert E., Shioide J., 2012, MNRAS, 423, L92

Renzo M., Götberg Y., 2021, The Astrophysical Journal, 923, 277

Renzo M., Ott C. D., Shore S. N., de Mink S. E., 2017, A&A, 603, A118

Renzo M., et al., 2019, A&A, 624, A66

Rogers F. J., Nayfonov A., 2002, The Astrophysical Journal, 576, 1064

Sabahit G. N., Vink J. S., Higgins E. R., Sander A. A. C., 2021, MNRAS, 506, 4473

Sallaska A. L., Iliadis C., Champagne A. E., Goriely S., Starfield S., Timmes F. X., 2013, Astrophysical Journal, Supplement Series, 207

Sana H., et al., 2012, Science, 337, 444

Sanyal D., Grassitelli L., Langer N., Bestenlehner J. M., 2015, A&A, 580, A20

Sanyal D., Langer N., Szécsi D., -C Yoon S., Grassitelli L., 2017, A&A, 597, A71

MNRAS 000, 1–10 (2022)
Schootemeijer A., Langer N., 2018, *Astronomy & Astrophysics*, 611, A75
Schootemeijer A., Langer N., Grin N. J., Wang C., 2019, *A&A*, 625, A132
Seager S., Mallen-Ornelas G., 2003, *The Astrophysical Journal*, 585, 1038
Shepard K., Gies D. R., Kaper L., De Koter A., 2022, arXiv e-prints, p. arXiv:2204.07473
Shiode J. H., Quataert E., 2014, *ApJ*, 780, 96
Soberman G. E., Phinney E. S., van den Heuvel E. P. J., 1997, *A&A*, 327, 620
Sota A., Apellániz J. M., Walborn N. R., Shida R. Y., 2008, Revista Mexicana de Astronomía y Astrofísica: Serie de Conferencias, 33, 56
Stothers R. B., Chin C.-W., 1993, *ApJ*, 408, L85
Sukhbold T., Woosley S. E., Heger A., 2018, *The Astrophysical Journal*, 860, 93
Timmes F. X., Swesty F. D., 2000, *The Astrophysical Journal Supplement Series*, 126, 501
Trigueros Páez E., Barbó R. H., Negueruela I., Maíz Apellániz J., Simón-Díaz S., Holgado G., 2021, *Astronomy & Astrophysics*, 655, A4
Valenti S., et al., 2016, *Monthly Notices of the Royal Astronomical Society*, 459, 3939
Vigna-Gómez A., MacLeod M., Neijssel C. J., Broekgaarden F. S., Justham S., Howitt G., de Mink S. E., Mand el I., 2020, arXiv e-prints, p. arXiv:2001.09829
Villamariz M. R., Herrero A., 2005, *A&A*, 442, 263
Vink J. S., De Koter A., 2005, a&a, 442, 587
Vink J. S., de Koter A., Lamers H. J. G. L. M., 2001, *Astronomy and Astrophysics*, 369, 574–
Walmswell J. J., Tout C. A., Eldridge J. J., 2015, *MNRAS*, 447, 2951
Wang C., Langer N., Schootemeijer A., Castro N., Adscheid S., Marchant P., Hastings B., 2020, *The Astrophysical Journal*, 888, L12
Williams S. J., Gies D. R., Hillwig T. C., McSwain M. V., Huang W., 2013, *Astronomical Journal*, 145
Woosley S. E., 1988, *ApJ*, 330, 218
Woosley S. E., Heger A., Weaver T. A., 2002, *Reviews of Modern Physics*, 74, 1015
Zahid H. J., Kewley L. J., Bresolin F., 2011, *Astrophysical Journal*, 730
Zahn J. P., 1977, *A&A*, 57, 383
Zapartas E., De Mink S. E., Justham S., Smith N., Renzo M., De Koter A., 2021a, *Astronomy and Astrophysics*, 645, 1
Zapartas E., et al., 2021b, *Astronomy and Astrophysics*, 656, 1
de Jager C., Nieuwenhuijzen H., van der Hucht K. A., 1988, *A&A*, 72, 259
van den Heuvel E. P. J., 1969, *AJ*, 74, 1095

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