I-CONVERGENT TRIPLE DIFFERENCE SEQUENCE SPACES USING SEQUENCE OF MODULUS FUNCTION

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Abstract. The main objective of this paper is to introduce classes of I-convergent triple difference sequence spaces, $c^3_0(\Delta, F)$, $c^3(\Delta, F)$, $\ell^3(\Delta, F)$, $M^3_1(\Delta, F)$ and $M^3_0(\Delta, F)$, by using sequence of modulii function $F = (f_{pqr})$. We also study some algebraic and topological properties of these new sequence spaces.

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1. Introduction

A triple sequence (real or complex) is a function $x : \mathbb{N} \times \mathbb{N} \times \mathbb{N} \to \mathbb{R}(\mathbb{C})$, where $\mathbb{N}$, $\mathbb{R}$ and $\mathbb{C}$ are the set of natural numbers, real numbers, and complex numbers respectively. We denote by $\omega''''$ the class of all complex triple sequence $(x_{pqr})$, where $p, q, r \in \mathbb{N}$. Then under the coordinate wise addition and scalar multiplication $\omega''''$ is a linear space. A triple sequence can be represented by a matrix, in case of double sequences we write in the form of a square. In case of triple sequence it will be in the form of a box in three dimensions.

The different types of notions of triple sequences and their statistical convergence were introduced and investigated initially by Sahiner et. al [16]. Later Debnath et.al [1,2], Esi et.al [3,4,5], and many others authors have studied it further and obtained various results. Kizmaz [11] introduced the notion of difference sequence spaces, he defined the difference sequence spaces $\ell_{\infty}(\Delta)$, $c(\Delta)$ and $c_0(\Delta)$ as follows.

$$Z(\Delta) = \{ x = (x_k) \in \omega: (\Delta x_k) \in Z \}$$

for $Z = \ell_{\infty}$, $c$ and $c_0$

Where $\Delta x = (\Delta x_k) = (x_k - x_{k+1})$ and $\Delta^0 x_k = x_k$ for all $k \in \mathbb{N}$

The difference operator on triple sequence is defined as

$$\Delta x_{mnk} = x_{mnk} - x_{(m+1)nk} - x_{m(n+1)k} - x_{mn(k+1)} + x_{(m+1)(n+1)k}$$

$$+ x_{(m+1)(n+k+1)} + x_{m(n+1)(k+1)} - x_{(m+1)(n+1)(k+1)}$$

and $\Delta^0_{mnk} = (x_{mnk})$. 

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Statistical convergence was introduced by Fast [6] and later on it was studied by Fridy [7-8] from the sequence space point of view and linked it with summability theory. The notion of statistical convergent double sequence was introduced by Mursaleen and Edely [14].

$I$-convergence is a generalization of the statistical convergence. Kostyrko et. al. [12] introduced the notion of $I$-convergence of real sequence and studied its several properties. Later Jalal [9-10], Salat et.al [15] and many other researchers contributed in its study. Tripathy and Goswami [19] extended this concept in probabilistic normed space using triple difference sequences of real numbers. Sahiner and Tripathy [17] studied $I$-related properties in triple sequence spaces and showed some interesting results. Tripathy[18] extended the concept in $I$-convergent double sequence and later Kumar [13] obtained some results on $I$-convergent double sequence.

In this paper we have defined $I$-convergent triple difference sequence spaces, $c_{0}^{3I}(\Delta, F)$, $c_{1}^{3I}(\Delta, F)$, $\ell_{\infty}^{3I}(\Delta, F)$, $M_{1}^{3I}(\Delta, F)$ and $M_{0}^{3I}(\Delta, F)$, by using sequence of modulii function $F = (f_{pqr})$ and also studied some algebraic and topological properties of these new sequence spaces.

2. Definitions and preliminaries

**Definition 2.1.** Let $X \neq \emptyset$. A class $I \subset 2^{X}$ (Power set of $X$) is said to be an ideal in $X$ if the following conditions holds good:

(i) $I$ is additive that is if $A, B \in I$ then $A \cup B \in I$;

(ii) $I$ is hereditary that is if $A \in I$, and $B \subset A$ then $B \in I$.

$I$ is called non-trivial ideal if $X \notin I$

**Definition 2.2** (16). A triple sequence $(x_{pqr})$ is said to be convergent to $L$ in Pringsheim’s sense if for every $\epsilon > 0$, there exists $N \in \mathbb{N}$ such that

$$|x_{pqr} - L| < \epsilon \text{ whenever } p \geq N, q \geq N, r \geq N$$

and write as $\lim_{p,p,r \to \infty} x_{pqr} = L$.

**Note:** A triple sequence is convergent in Pringsheim’s sense may not be bounded [16].

**Example** Consider the sequence $(x_{pqr})$ defined by

$$x_{pqr} = \begin{cases} p + q & \text{for all } p = q \text{ and } r = 1 \\ \frac{1}{p^2qr} & \text{otherwise} \end{cases}$$

Then $x_{pqr} \to 0$ in Pringsheim’s sense but is unbounded.

**Definition 2.3.** A triple sequence $(x_{pqr})$ is said to be $I$-convergence to a number $L$ if for every $\epsilon > 0$, 

$$\{(p, q, r) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : |x_{pqr} - L| \geq \epsilon\} \in I.$$ 

In this case we write $I - \lim x_{pqr} = L$.

**Definition 2.4.** A triple sequence $(x_{pqr})$ is said to be $I$-null if $L = 0$. In this case we write $I - \lim x_{pqr} = 0$. 

Definition 2.5 (16). A triple sequence \((x_{pqr})\) is said to be Cauchy sequence if for every \(\epsilon > 0\), there exists \(N \in \mathbb{N}\) such that
\[
|x_{pqr} - x_{lmn}| < \epsilon \quad \text{whenever} \quad p \geq l \geq N, q \geq m \geq N, r \geq n \geq N
\]

Definition 2.6. A triple sequence \((x_{pqr})\) is said to be \(I\)–Cauchy sequence if for every \(\epsilon > 0\), there exists \(N \in \mathbb{N}\) such that
\[
\{(p, q, r) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : |x_{pqr} - a_{lmn}| \geq \epsilon\} \in I
\]
whenever \(p \geq l \geq N, q \geq m \geq N, r \geq n \geq N\)

Definition 2.7 (16). A triple sequence \((x_{pqr})\) is said to be bounded if there exists \(M > 0\), such that
\[
|x_{pqr}| < M \quad \text{for all} \quad p, q, r \in \mathbb{N}
\]

Definition 2.8. A triple sequence \((x_{pqr})\) is said to be \(I\)–bounded if there exists \(M > 0\), such that
\[
\{(p, q, r) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : |x_{pqr}| \geq M\} \in I \quad \text{for all} \quad p, q, r \in \mathbb{N}
\]

Definition 2.9. A triple sequence space \(E\) is said to be solid if \((\alpha_{pqr} x_{pqr}) \in E\) whenever \((x_{pqr}) \in E\) and for all sequences \((\alpha_{pqr})\) of scalars with \(|\alpha_{pqr}| \leq 1\), for all \(p, q, r \in \mathbb{N}\).

Definition 2.10. Let \(E\) be a triple sequence space and \(x = (x_{pqr}) \in E\). Define the set \(S(x)\) as
\[
S(x) = \{(x_{\pi(pqr)}): \pi \text{ is a permutations of } \mathbb{N}\}
\]
If \(S(x) \subseteq E\) for all \(x \in E\), then \(E\) is said to be symmetric.

Definition 2.11. A triple sequence space \(E\) is said to be convergence free if \((y_{pqr}) \in E\) whenever \((x_{pqr}) \in E\) and \(x_{pqr} = 0\) implies \(y_{pqr} = 0\) for all \(p, q, r \in \mathbb{N}\).

Definition 2.12. A triple sequence space \(E\) is said to be sequence algebra if \(x \cdot y \in E\), whenever \(x = (x_{pqr}) \in E\) and \(y = (y_{pqr}) \in E\), that is product of any two sequences is also in the space.

Definition 2.13. A function \(f : [0, \infty) \to [0, \infty)\) is called a modulus function if it satisfies the following conditions

(i) \(f(x) = 0\) if and only if \(x = 0\).

(ii) \(f(x + y) \leq f(x) + f(y)\) for all \(x \geq 0\) and \(y \geq 0\).

(iii) \(f\) is increasing.

(iv) \(f\) is continuous from the right at 0.

Since \(|f(x) - f(y)| \leq f(|x - y|)\), it follows from condition (4) that \(f\) is continuous on \([0, \infty)\). Furthermore, from condition (2) we have \(f(nx) \leq nf(x)\), for all \(n \in \mathbb{N}\), and so
\[
f(x) = f(nx\left(\frac{1}{n}\right)) \leq nf\left(\frac{x}{n}\right)
\]
Hence \(\frac{1}{n}f(x) \leq f\left(\frac{x}{n}\right)\) for all \(n \in \mathbb{N}\).
We now define the following sequence spaces
\[ c_0^3(\Delta, F) = \left\{ x \in \omega'' : I - \lim f_{pqr}(|\Delta x_{pqr}|) = 0 \right\} \]
\[ c_2^3(\Delta, F) = \left\{ x \in \omega'' : I - \lim f_{pqr}(|\Delta x_{pqr} - b|) = 0, \text{ for some } b \right\} \]
\[ \ell_\infty^3(\Delta, F) = \left\{ x \in \omega'' : \sup_{pqr \in \mathbb{N}} f_{pqr}(|\Delta x_{pqr}|) = 0 \right\} \]
\[ M_1^3(\Delta, F) = c_0^3(\Delta, F) \cap \ell_\infty^3(\Delta, F) \]
\[ M_2^3(\Delta, F) = c_2^3(\Delta, F) \cap \ell_\infty^3(\Delta, F) \]

3. Algebraic and Topological Properties of the new Sequence spaces

**Theorem 3.1.** The triple difference sequence spaces $c_0^3(\Delta, F)$, $c_2^3(\Delta, F)$, $\ell_\infty^3(\Delta, F)$, $M_1^3(\Delta, F)$ and $M_2^3(\Delta, F)$ all are linear for the sequence of modulii $F = (f_{pqr})$.

**Proof.** We shall prove it for the sequence space $c_2^3(\Delta, F)$, for the other spaces, it can be established similarly.

Let $x = (x_{pqr}), y = (y_{pqr}) \in c_2^3(\Delta, F)$ and $\alpha, \beta \in \mathbb{R}$ such that $|\alpha| \leq 1$ and $|\beta| \leq 1$, then
\[ I - \lim f_{pqr}(|\Delta x_{pqr} - b_1|) = 0, \text{ for some } b_1 \in \mathbb{C} \]
\[ I - \lim f_{pqr}(|\Delta y_{pqr} - b_2|) = 0, \text{ for some } b_2 \in \mathbb{C} \]

Now for a given $\epsilon > 0$ we set
\[ X_1 = \left\{ (p, q, r) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : f_{pqr}(|\Delta x_{pqr} - b_1|) > \frac{\epsilon}{2} \right\} \subset I \quad (2.1) \]
\[ X_2 = \left\{ (p, q, r) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : f_{pqr}(|\Delta y_{pqr} - b_2|) > \frac{\epsilon}{2} \right\} \subset I \quad (2.2) \]

Since $f_{pqr}$ is a modulus function, so it is non-decreasing and convex, hence we get
\[
\begin{align*}
f_{pqr}(|(\alpha \Delta x_{pqr} + \beta \Delta y_{pqr}) - (\alpha b_1 + \beta b_2)|) &= f_{pqr}(|(\alpha \Delta x_{pqr} - \alpha b_1) + (\beta \Delta y_{pqr} - \beta b_2)|) \\
&\leq f_{pqr}(|\alpha| |\Delta x_{pqr} - b_1|) + f_{pqr}(|\beta| |\Delta y_{pqr} - b_2|) \\
&= |\alpha| f_{pqr}(|\Delta x_{pqr} - b_1|) + |\beta| f_{pqr}(|\Delta y_{pqr} - b_2|) \\
&\leq f_{pqr}(|\Delta x_{pqr} - b_1|) + f_{pqr}(|\Delta y_{pqr} - b_2|)
\end{align*}
\]

From (2.1) and (2.2) we can write
\[
\{(p, q, r) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : f_{pqr}(|(\alpha \Delta x_{pqr} + \beta \Delta y_{pqr}) - (\alpha b_1 + \beta b_2)|) > \epsilon \} \subset X_1 \cup X_2
\]

Thus $\alpha x + \beta y \in c_2^3(\Delta, F)$

This completes the proof. \qed
Theorem 3.2. The triple difference sequence \( x = (x_{pq}) \in M_1^3(\Delta, F) \) is \( I \)-convergent if and only if for every \( \epsilon > 0 \) there exists \( I, J, K \in \mathbb{N} \) such that

\[
\{(p, q, r) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : f_{pq}(|\Delta x_{pq} - \Delta x_{I, J, K}|) \leq \epsilon \} \subseteq M_1^3(\Delta, F)
\]

Proof. Let \( b = I - \lim \Delta x \). Then we have

\[
A_\epsilon = \left\{(p, q, r) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : f_{pq}(|\Delta x_{pq} - b|) \leq \frac{\epsilon}{2} \right\} \subseteq M_1^3(\Delta, F)
\]

Next fix \( I, J, K \in A_\epsilon \) then we have

\[
|\Delta x_{pq} - \Delta x_{I, J, K}| \leq |\Delta x_{pq} - b| + |b - \Delta x_{I, J, K}| \leq \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon
\]

Thus

\[
\{(p, q, r) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : f_{pq}(|\Delta x_{pq} - \Delta x_{I, J, K}|) \leq \epsilon \} \subseteq M_1^3(\Delta, F)
\]

Conversely suppose that

\[
\{(p, q, r) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : f_{pq}(|\Delta x_{pq} - \Delta x_{I, J, K}|) \leq \epsilon \} \subseteq M_1^3(\Delta, F)
\]

we get \( \{(p, q, r) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : f_{pq}(|\Delta x_{pq} - \Delta x_{I, J, K}|) \leq \epsilon \} \subseteq M_1^3(\Delta, F) \), for all \( \epsilon > 0 \).

Then given \( \epsilon > 0 \) we can find the set

\[
B_\epsilon = \{(p, q, r) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : \Delta x_{pq} \in [\Delta x_{I, J, K} - \epsilon, \Delta x_{I, J, K} + \epsilon] \} \subseteq M_1^3(\Delta, F)
\]

Let \( J_\epsilon = [\Delta x_{I, J, K} - \epsilon, \Delta x_{I, J, K} + \epsilon] \) if \( \epsilon > 0 \) is fixed then \( B_\epsilon \subseteq M_1^3(\Delta, F) \) as well as \( B_\frac{\epsilon}{2} \subseteq M_1^3(\Delta, F) \).

Hence \( B_\epsilon \cap B_\frac{\epsilon}{2} \subseteq M_1^3(\Delta, F) \)

Which gives \( J = J_\epsilon \cap J_\frac{\epsilon}{2} \neq \emptyset \) that is \( \{(p, q, r) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : \Delta x_{pq} \in N \} \subseteq M_1^3(\Delta, F) \)

Which implies \( \text{diam } J \leq \text{diam } J_\epsilon \)

where the \( \text{diam} \) of \( J \) denotes the length of interval \( J \).

Now by the principal of induction a sequence of closed interval can be found

\[
J_\epsilon = I_0 \supseteq I_1 \supseteq I_2 \supseteq \cdots \supseteq I_s \supseteq \cdots
\]

with the help of the property that \( \text{diam } I_s \leq \frac{1}{2} \text{diam } I_{s-1} \), for \( s = 1, 2, \cdots \) and

\[
\{(p, q, r) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : \Delta x_{pq} \in I_{pq} \} \quad \text{for} \quad (p, q, r) = (1, 2, 3, \cdots)
\]

Then there exists a \( \xi \in \cap I_s \) where \( s \in \mathbb{N} \) such that \( \xi = I - \lim \Delta x \)

So that \( f_{pq}(\xi) = I - \lim f_{pq}(\Delta x) \) therefore \( b = I - \lim f_{pq}(\Delta x) \).

Hence the proof is complete. \( \square \)

Theorem 3.3. The \( F = (f_{pq}) \) be a sequence of modulus functions then the inclusions \( c_0^3(\Delta, F) \subseteq c_0^3(\Delta, F) \subseteq \ell_\infty^3(\Delta, F) \) holds.

Proof. The inclusion \( c_0^3(\Delta, F) \subseteq c_0^3(\Delta, F) \) is obvious.

We prove \( c_0^3(\Delta, F) \subseteq \ell_\infty^3(\Delta, F) \), let \( x = (x_{pq}) \in c_0^3(\Delta, F) \) then there exists \( b \in \mathbb{C} \) such that \( I - \lim f_{pq}(|\Delta x_{pq} - b|) = 0 \),

Which gives \( f_{pq}(|\Delta x_{pq}|) \leq f_{pq}(|\Delta x_{pq} - b|) + f_{pq}(|b|) \)
On taking supremum over \( p, q \) and \( r \) on both sides gives
\[
x = (x_{pqr}) \in \ell^3_{\infty I}(\Delta, F)
\]
Hence the inclusion \( c^3_{0I}(\Delta, F) \subset c^3_1(\Delta, F) \subset \ell^3_{\infty I}(\Delta, F) \) holds. \( \square \)

**Theorem 3.4.** The triple difference sequence spaces \( c^3_{0I}(\Delta, F) \) and \( M^3_{0I}(\Delta, F) \) are solid.

**Proof.** We prove the result for \( c^3_{0I}(\Delta, F) \).
Consider \( x = (x_{pqr}) \in c^3_{0I}(\Delta, F) \), then
\[
I - \lim_{p,q,r} f_{pqr}(|\Delta x_{pqr}|) = 0
\]
Consider a sequence of scalar \( (\alpha_{pqr}) \) such that \( |\alpha_{pqr}| \leq 1 \) for all \( p, q, r \in \mathbb{N} \).
Then we have
\[
I - \lim_{p,q,r} f_{pqr}(|\Delta \alpha_{pqr}(x_{pqr})|) \leq I - |\alpha_{pqr}| \lim_{p,q,r} f_{pqr}(|\Delta x_{pqr}|)
\]
\[
\leq I - \lim_{p,q,r} f_{pqr}(|\Delta x_{pqr}|)
\]
\[
= 0
\]
Hence \( I - \lim_{p,q,r} f_{pqr}(|\Delta \alpha_{pqr} x_{pqr}|) = 0 \) for all \( p, q, r \in \mathbb{N} \).
Which gives \( (\alpha_{pqr} x_{pqr}) \in c^3_{0I}(\Delta, F) \).
Hence the sequence space \( c^3_{0I}(\Delta, F) \) is solid.
The result for \( M^3_{0I}(\Delta, F) \) can be similarly proved. \( \square \)

**Theorem 3.5.** The triple difference sequence spaces \( c^3_{0I}(\Delta, F) \), \( c^3_1(\Delta, F) \), \( \ell^3_{\infty I}(\Delta, F) \), \( M^3_1(\Delta, F) \) and \( M^3_{0I}(\Delta, F) \) are sequence algebras.

**Proof.** We prove the result for \( c^3_{0I}(\Delta, F) \).
Let \( x = (x_{pqr}), y = (y_{pqr}) \in c^3_{0I}(\Delta, F) \).
Then we have
\[
I - \lim_{p,q,r} f_{pqr}(|\Delta x_{pqr}|) = 0 \quad \text{and} \quad I - \lim_{p,q,r} f_{pqr}(|\Delta y_{pqr}|) = 0
\]
and
\[
I - \lim_{p,q,r} f_{pqr}(|\Delta (x_{pqr} \cdot y_{pqr})|) = 0 \quad \text{as}
\]
\[
\Delta (x_{pqr} \cdot y_{pqr}) = x_{pqr} \cdot y_{pqr} - x_{(p+1)qr} \cdot y_{(p+1)qr} - x_{p(q+1)r} \cdot y_{p(q+1)r} - x_{pq(r+1)} \cdot y_{pq(r+1)} + x_{(p+1)(q+1)r} \cdot y_{(p+1)(q+1)r} + x_{(p+1)qr} \cdot y_{(p+1)qr} - x_{(p+1)(q+1)(r+1)} \cdot y_{(p+1)(q+1)(r+1)}
\]
It implies that \( x \cdot y \in c^3_{0I}(\Delta, F) \).
Hence the proof.
The result can be proved for the spaces \( c^3_{1I}(\Delta, F) \), \( \ell^3_{\infty I}(\Delta, F) \), \( M^3_1(\Delta, F) \) and \( M^3_{0I}(\Delta, F) \) in the same way. \( \square \)

**Theorem 3.6.** In general the sequence spaces \( c^3_{0I}(\Delta, F) \), \( c^3_1(\Delta, F) \) and \( \ell^3_{\infty I}(\Delta, F) \) are not convergence free.

**Proof.** We prove the result for the sequence space \( c^3_1(\Delta, F) \) using an example.

**Example 3.7.** Let \( I = I_f \) define the triple sequence \( x = (x_{pqr}) \) as
\[
x_{pqr} = \begin{cases} 
0 & \text{if } p = q = r \\
1 & \text{otherwise}
\end{cases}
\]
Then if \( f_{pqr}(x) = x_{pqr} \) \( \forall p, q, r \in \mathbb{N} \), we have \( x = (x_{pqr}) \in c^3_I(\Delta, F) \).

Now define the sequence \( y = y_{pqr} \) as

\[
y_{pqr} = \begin{cases} 
0 & \text{if } r \text{ is odd, and } p, q \in \mathbb{N} \\
lmn & \text{otherwise}
\end{cases}
\]

Then for \( f_{pqr}(x) = x_{pqr} \) \( \forall p, q, r \in \mathbb{N} \), it is clear that \( y = (y_{pqr}) \not\in c^3_I(\Delta, F) \).

Hence the sequence spaces \( c^3_I(\Delta, F) \) is not convergence free.

The space \( c^3_I(\Delta, F) \) and \( \ell^3_\infty(\Delta, F) \) are not convergence free in general can be proved in the same fashion. \( \square \)

**Theorem 3.8.** In general the triple difference sequences \( c^3_{0f}(\Delta, F) \) and \( c^3_I(\Delta, F) \) are not symmetric if \( I \) is neither maximal nor \( I = I_f \).

**Proof.** We prove the result for the sequence space \( c^3_{0f}(\Delta, F) \) using an example.

**Example 3.9.** Define the triple sequence \( x = (x_{pqr}) \) as

\[
x_{pqr} = \begin{cases} 
0 & \text{if } r = 1, \text{ for all } p, q \in \mathbb{N} \\
1 & \text{otherwise}
\end{cases}
\]

Then if \( f_{pqr}(x) = x_{pqr} \) \( \forall p, q, r \in \mathbb{N} \), we have \( x = (x_{pqr}) \in c^3_{0f}(\Delta, F) \).

Now if \( x_{\pi(pqr)} \) be a rearrangement of \( x = (x_{pqr}) \) defined as

\[
x_{\pi(pqr)} = \begin{cases} 
1 & \text{for } p, q, r \text{ even } \\
0 & \text{otherwise}
\end{cases}
\]

Then \( \{x_{\pi(p,q,r)}\} \not\in c^3_{0f}(\Delta, F) \) as \( \Delta x_{\pi(pqr)} = 1 \)

Hence the sequence spaces \( c^3_{0f}(\Delta, F) \) is not symmetric in general.

The space \( c^3_I(\Delta, F) \) is not symmetric in general can be proved in the same fashion. \( \square \)

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