Simulation of electromagnetic parameters in a laboratory installation

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Abstract. The paper devoted to the numerical simulation of electromagnetic and heat transfer processes in laboratory installation with two concentric cylindrical electrodes and liquid metal. The electromagnetic and temperature fields of laboratory installation are presented. The results obtained for the laboratory installation can be used in the metallurgical industry for the simulation and optimization of electrometallurgical furnaces.

1. Introduction
Among the various magnetohydrodynamic effects the electrovortex flows, which arise as a result of the interaction of an inhomogeneous electric current density $\vec{j}$ with its magnetic field $\vec{B}$ have a large practical application [1].

An example of such electrovortex flow is a device placed in a magnetic field, which consists of a cylindrical vessel and two electrodes (cylindrical and ring).

2. The method description and first laboratory results
In laboratory conditions, the vortex flow arises as a physical effect when an electric current flow through an electrically conductive liquid medium placed in gradients of magnetic field and temperature [2, 3].

When the current is connected, the vortex flow of molten tin is observed in the direction shown by the arrow in Figure 1a. A deep funnel forms in the center. With increasing current, the velocity of the vortex flow increases. When the direction of the magnetic field changes, the direction of the vortex motion of the molten tin changes to the opposite. A significant temperature gradient and the formation of a solid phase of the metal in the center of the vessel are also observed [2-4]. To simulate this phenomenon, we present the studied setup in the form of a model. The geometric parameters of this model are shown in Figure 1b (dimensions are given in millimeters).
3. Physics of processes

The inhomogeneous current density in space, which is obtained by passing a current through a melt placed in a cylindrical vessel, will cause a vortex flow that is unstable in space. However, it is assumed that under the influence of an external magnetic field, the stability of the motion of the conductive melt should increase. This assumption follows from the non-uniform distribution of the current density in the melt, and the formation of its magnetic field, which, according to the Lenz principle, seeks to reduce the external magnetic flux. Consequently, the magnetic field seeks to suppress the existing instability or to prevent new disturbances. The field tends to suppress mainly the components of the vortices, in this case, the axis is perpendicular to the field, and does not exert any influence on disturbances is parallel to the magnetic field lines. It is also possible to reorient part of the vortices to a position in which the field does not affect them. Turbulence in a magnetic field transforms into a system of two-dimensional perturbations, on which the field does not act at all and which are scattered only by viscous forces. A similar situation is characterized by the so-called two-dimensional turbulence in the flow. It is also assumed that the two-dimensional turbulence of the magnetohydrodynamic flow will allow the implementation of the magnetic field generation mode. The application of a magnetic field to a turbulent flow leads to change in all magnetohydrodynamic flow parameters. With an increase in the Hartmann number, i.e. when a very strong magnetic field is applied to the flux, even more complex transformations in the flux are possible.

4. The mathematical formulation of the problem

The installation has axial symmetry therefore electromagnetic processes will also have axial symmetry. The installation in Figure 1 has molten metal 2 – tin, which has the following parameters: density $\rho = 6980 \text{ kg/m}^3$, specific conductivity $\sigma = 8.69 \times 10^6 \text{ (S/m)}$, kinematic viscosity $\nu = 3.1 \times 10^{-7} \text{ m}^2/\text{s}$, melt temperature $T = 232 \text{ °C}$ [5].

The electrodes 4, 5 in Figure 1 are copper with electrical conductivity $\sigma = 58.1 \times 10^6 \text{ (S/m)}$, their surfaces are considered equipotential. The current supplied to the unit is $I = 40 \text{ A}$. The internal
electrode is the anode. Induction of the external axial magnetic field \( B_z = 0.01 \) T, the field is directed vertically upward.

The following assumptions are made in the work
- the process is stationary and axisymmetric;
- the physical characteristics of the medium (conductivity, viscosity and thermal conductivity coefficients, etc.) are assumed to be homogeneous and isotropic and independent of temperature and pressure;
- chemical reactions are not taken into account;
- the medium is considered non-magnetic;
- the Earth’s magnetic field is not taken into account.

The processes in laboratory installation can be described by the following governing equations [6-9] the Maxwell’s equations

\[
\nabla \times \vec{B} = \mu_0 \vec{j}, \nabla \cdot \vec{B} = 0, \tag{1}
\]

\[
\nabla \times \vec{E} = 0, \nabla \cdot \vec{E} = \frac{\rho_e}{\varepsilon_0}, \tag{2}
\]

Ohm’s law

\[
\vec{j} = \sigma (\vec{E} + \vec{u} \times \vec{B}), \tag{3}
\]

and charge conservation law

\[
\nabla \cdot \vec{j} = 0, \tag{4}
\]

where \( \vec{j} \) – current density, \( \rho_e \) – charge density, \( \vec{B} \) – magnetic induction intensity vector, \( \vec{E} \) – electrical field intensity, \( \sigma \) – specific conductance, \( \mu_0 \) – permeability of free space, \( \varepsilon_0 \) – the permittivity of free space, \( \vec{u} \) – liquid velocity.

The heat transfer processes can be described by following equation

\[
\rho C_p \vec{u} \cdot \nabla T = \nabla \cdot \left( (a + a_T) \nabla T \right) + \frac{j^2}{\sigma}, \tag{5}
\]

where \( \rho \) – density, \( C_p \) – specific heat, \( T \) – temperature, \( a \) – heat conduction coefficient, \( a_T \) – turbulent heat conduction coefficient, \( j^2 / \sigma \) – Joule heat source.

The hydrodynamic processes can be described by the Navier–Stokes equations

\[
\rho \vec{u} \cdot \nabla \vec{u} = \nabla \cdot (\rho \vec{u} (\nabla \vec{u} + (\nabla \vec{u})^T) - (2/3)(\nabla \cdot \vec{u}) \vec{I}) + \rho \vec{g} + \vec{j} \times \vec{B}, \tag{6}
\]

and the continuity equation

\[
\nabla \cdot (\rho \vec{u}) = 0, \tag{7}
\]

where \( \rho \) – pressure, \( \vec{g} \) – gravitation, \( \theta = \eta / \rho \) – coefficient of kinematics viscosity, \( \eta \) – coefficient of dynamic viscosity, \( \rho \) – liquid density, \( \vec{I} \) – identity operator for points on the boundary.

5. Numerical simulation

The problem under consideration does not have an analytical solution due to the complexity of the ongoing processes. Therefore, the problem was solved numerically by the finite element method.

A two-dimensional electromagnetic model problem is calculated. The solution technique with varying boundary conditions at the boundaries of the computational domain is debugged. The influence of the dimensions of the computational domain on the influence of the ongoing processes is considered. The optimal computational grid and type of analysis are selected.

As the initial conditions, we set in one case the values of the electric potential supplied to the electrodes and the current value in another. The influence of the dimensions of the computational domain was also studied. It is established that the dimensions do not significantly affect the result. Therefore, the laboratory facility itself, without regard to the environment, was taken as the calculation area.

The calculations were carried out on various grids with different numbers and shapes of finite elements. The optimal computational grid was chosen. Its elements have the shape of a triangle. The computational domain was divided unevenly, depending on the gradient of the electromagnetic
parameters near the electrodes, the grid consists of small elements, in the remaining areas where the gradients of the magnitudes are not so significant the finite elements are large. This partition allows to get the most accurate results without the high cost of the simulation time. The convergence of the results in different parts of the simulation area was investigated. Convergence is already achieved with a grid containing about 4000 finite elements.

As a result of the numerical simulation, the dependencies of the electromagnetic values were obtained and presented in Figure 2. A plot of the current density $\mathbf{j}$ showed non-uniform distribution. The maximum values of the current density are localized at a distance around anode radius and rapidly decrease at the periphery, as a result of the interaction of a nonuniformly distributed current density $\mathbf{j}$ with magnetic induction $\mathbf{B}$ an electrovortex flow arises.

![Figure 2. The vector and contour field of current density.](image)

From the dependence of the Joule heat, the highest intensity of heat generation localized at the contact point between the cylindrical electrode and liquid metal as it is shown in Figure 3.

![Figure 3. The contour field of Joule heat.](image)

The change of Joule heats at a distance of one radius from the electrode rapidly changes and the dependency presented in Figure 4.

6. Conclusions
To describe the electromagnetic processes in a cylindrical vessel with a cylindrical and ring electrode an electromagnetic two-dimensional problem was solved in an axisymmetric formulation. The physical and mathematical model of the processes in laboratory installation was described. The optimal computational grid was selected. The distributions of electromagnetic parameters are presented, which confirm the hypothesis that electrovortex flows are the result of an uneven distribution of current density and magnetic field.
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