Fluid-Structure Interaction Approach to Single Particle in a Square Microchannel

Xiang Li and Ying Lin
School of Mechanical Engineering, Shanghai Institute of Technology, Shanghai, 201418, China
Email: liny@sit.edu.cn

Abstract. Inertial microfluidic technique has been widely applied on particle/cell manipulation and detection. To understand the physical principle of this technique more detailed, the interaction of fluid and particle was studied through the Fluid-Structure Interaction (FSI) method. The equilibrium positions of finite-size particles with different diameters were simulated at moderate Reynolds numbers. The flow structure around two typical particles was analysed. The vortex in the front of the particle retards particle’s translation leading to the lag velocity increasing. Finally, the rotation velocity and the rotational-induced force analysed quantitatively to demonstrate that particle’s self-rotation significantly promotes its inertial migration.

Keywords. Inertial microfluidics, FSI, Rotation-induced force.

1. Introduction
In recent years, inertial microfluidic technique was widely applied on particle/cell manipulation and detection, such as cell focusing in flow cytometry [1], real-time detection of airborne bacteria [2], and monitoring chemical species in the environment [3], and so on. This technique originated from a phenomenon observed by Segre and Silberberg in 1961 [4], that particles would migrate laterally to the annulus equilibrium positions in a macroscale cylindrical pipe. To explore the mechanism of inertial migration in microscales, many researchers conducted theoretical investigations by the point-like-particle hypothesis [5-7]. They found the net lift force, \( F_L \approx \rho U^3 a^4/\eta^2 \) [5], dominated the inertial migration, where \( U \) is the flow velocity and \( H \) is the channel dimension. However, as using the \( F_L \) to focus randomly distributed particles in rectangular section channels, Di Carlo et al. [8] unexpectedly observed that particles ordered longitudinally along the channel wall, which could not be explained by theoretical results. Furthermore, Miura et al. [9] observed the particle equilibrium position moved monotonously towards the channel center with an increasing \( Re \) through experiments, which was contrary to the antecedent studies. And, Amini et al. [10] discovered the presence of particle would unavoidably disturb the fluid around it which was neglected before. These stimulated indepth researches and, more quantitively analysis need to understand the inertial effect deeply. Thus, numerical methods, such as Direct Numerical Simulation (DNS) [11], Immersed Boundary Method (IBM) [11], Lattice Boltzmann Method (LBM) [12] and Fluid-Structure Interaction methods (FSI), were applied for the available quantitative motion parameters of the fluid and particles. Among these methods, the FSI can handle boundaries of large displacements through remeshing which leads the simulation work similarly to experiment status. Meanwhile, the FSI can monitor particle’s rotational velocity and calculate the force exerted on the particle [13-15], which exhibit its power to investigate the interaction of a particle and fluid field.
In this paper, the interaction of fluid and particle with different diameters was researched at moderate Reynolds numbers through the FSI method. The equilibrium positions of different particles were investigated at different Reynolds numbers in square microchannel. Then, the flow structures around two typical particles were presented and analysed to understand the particle’s active property in the flow. Finally, the rotation velocity of equilibrium particles was discussed and then, preliminarily analysed the rotation-induced lift force exerted on particles.

2. Method and Model
In order to study the fluid and particle interaction and further the particle focusing characteristics, the Fluid-Structure Interaction (FSI) module in the commercial computational fluid dynamics (CFD) software COMSOL Multiphysics (version 5.5, COMSOL Inc., Stockholm, Sweden) was applied. In this module, the fluid flow is described by the Navier-Stokes equations, which provides a solution for the velocity field $\vec{u}_{fluid}$ and the total force exerted on the particle boundary:

$$\vec{f} = \vec{n} \cdot [-pI + (\mu(\nabla \vec{u}_{fluid} + (\nabla \vec{u}_{fluid})^T) - \frac{2}{3} \mu(\nabla \cdot \vec{u}_{fluid})I)]$$

(1)

where $\vec{n}$ is the normal vector to particle surface, $p$ the pressure, $I$ the identity matrix, and $\rho$ and $\mu$ the density and viscosity of the fluid, respectively.

The 3D geometry and meshes were modeled, as shown in figure 1. The coordinate system locates at the center of channel’s inlet (the left plane). In all the models, the length $L$ was defined 12-20 times of the particle diameter to cover the flow field affected by the particle [16]. The particle was released freely in the plane of $y=0$ and, its initial position in the cross section was shown in figure 1(a). The residual error that controlled the internal time steps was $5 \times 10^{-4}$, which lead the time step to be about $1 \times 10^{-6}$ s. And, the remeshing occurred when the mesh quality became less than the limit 0.2 (defined by the software). The mesh independence tests were conducted for all working conditions. The density of the particles was $1.05 \times 10^3$ kg·m$^{-3}$ as the same value of general experimental particle. The density and viscosity of water was $1 \times 10^3$ kg·m$^{-3}$ and $1 \times 10^{-3}$ kg·m$^{-1}$·s$^{-1}$, respectively. The validity of the FSI method was testified through comparing the equilibrium position between present simulation and precedent experiment [17]. The maximum deviation of 8.2% was obtained, and the present method considered reliable.

![Figure 1](image-url)
3. Results and Discussion

3.1. Equilibrium Positions of Different a/Hs at Different Re

The free particle would migrate to a fixed position near the center of channel face in the cross section named the channel face equilibrium (CFE) position. The equilibrium position $z_{eq}$ was defined by the distance between the particle center and the channel axis. The nondimensional expression $z_{eq}/h$ ($h=H/2$) was depicted as a function of $Re$ (figure 2). By comparing the data at the same $Re$, it is clear that the particle with a small $a/H$ obtains a large $z_{eq}/h$ value. This means small particles close to channel wall and large particles to channel center relatively.

The wall-induced lift force, $F_{LW}=C_{LW} \rho U^2 a^6/H^4$, which drives the particle to the channel center increases faster than the shear-induced lift force, $F_{LS}=C_{LS} \rho U^2 a^3/H$, which drives particle migrate to the channel wall, where $C_{LW}$ is the wall-lift coefficient and $C_{LS}$ the shear-lift coefficient.

Furthermore, the particle of $a/H=0.1$ would migrate to the channel wall slightly with $Re$ risen, which is agreeable with previous investigations [17]. In Poiseuille flow, $F_{LS}$ and $F_{LW}$ govern the inertial migration of particles of the small $a/H$. As increasing $Re$, the $C_{LW}$ keeps constant while the $C_{LS}$ rises [18]. $F_{LS}$ dominates the particle migration and drives particle to the channel wall. When $a/H\geq0.2$, the equilibrium position vibrates in a tiny scale as $Re$ increased. That is not coincident with the trend displayed by the particle of $a/H=0.1$. The equilibrium position of the large particle would not be simply determined by Reynolds number. It reveals that the large particle does not act as a passive component in the flow only. As a positive component, the particle would disturb the flow field around it and, interfere its lateral motion. This would be discussed in following analysis.

![Figure 2](image_url)

**Figure 2.** The nondimensional equilibrium position as a function of the Reynolds number.

3.2. Flow Structure around Particle

Two typical conditions of the migration of 5µm and 15µm particles at $Re=80$ were selected to present the flow characteristic. The absolute velocity contours of fluid around the equilibrium particles on the plane of the symmetry ($y=0$) were plotted in figure 3. The streamlines representing the relative velocity of the fluid to particle were illustrated on the same plane. For the 5µm particle, three vortices are formed to occupy about one eighth of the selected area as shown in the figure 3(a). The streamlines are not disturbed in a large space near the channel center (over half of the selected area). For the 15µm particle, four vortices nearly occupy half of the selected area as shown in figure 3(b).
Figure 3. The absolute velocity contour as the background and the streamlines representing the relative velocity of fluid to particle of two typical conditions: (a) $a=5 \, \mu m, \, Re=80$, (b) $a=15 \, \mu m, \, Re=80$.

The vortex at the downstream of the particle engenders hydrodynamic resistance force on the particle. This leads to the decrease of the actual translational velocity of the particle $U_p$. It was compared with the fluid velocity of Poiseuille flow at the particle center $U_f$ to obtain the lag velocity ($\Delta U=U_f-U_p$). The lag velocity of the two selected particles was plotted in figure 4 as a function of Re. Obviously, two groups of data from $a/H=0.1$ and $a/H=0.3$ show negative and positive results, respectively. For $a/H=0.1$, the negative values of $\Delta U$ mean that the fluid moves slower than the particle. The traction exerted on a small particle is stronger than the blockage of the downstream fluid at moderate $Re$. With the increase of $Re$, the particle hinders the fluid flow to some extent. Then the vortex at the downstream grows up, thus the particle slowing down. For $a/H=0.3$, the fluid runs too fast to make the particle lag behind. At this circumstance, the vortex augments due to the constraint of the particle as shown in figure 3(b), and strongly impedes the particle move ahead. The large $Re$ enhances the effect of vortices. Then it results in the increase of the lag velocity in the condition of $a/H=0.3$.

Figure 4. The lag velocity as a function of $Re$.

3.3. Particle’s Rotational Velocity

The lateral migration would be promoted by the particle rotation which was ignored in most previous reports. As well known, when the velocity gradient exists between the mainstream and particle surface, the particle will be subjected by shear and then rotate. The rotation velocity Omega ($\Omega$) was depicted as a function of $a/H$ at different Reynolds numbers in figure 5. At all Reynolds numbers, the $\Omega$ is diminished with the $a/H$ increasing. The trend was strongly enhanced at high $Re$. To further discuss
the contribution of the particle rotation, the particle rotation induced lift force, \( F_{LR} = \frac{1}{8} \pi a^3 \rho_f (\vec{u}_f - \vec{u}_p) \times \vec{u} \) [19], was introduced and calculated through all the results of simulations, where \( \vec{u}_f \) is the fluid velocity and \( \vec{u}_p \) the particle velocity. The net lift force \( F_L \) was also calculated, and the data of two forces were listed in Table 1. The \( F_{LR} \) combines the \( F_{LS} \) and \( F_{LW} \) into the net lift force \( F_L \) which governs the lateral migration. It was found that the direction of the forces is different between \( a/H = 0.1 \) and \( a/H \geq 0.2 \). It can be explained by the lag velocity discussed above. Moreover, the direction of \( F_{LR} \) and \( F_L \) is identical at a given \( a/H \), meaning that the positive contribution of the \( F_{LR} \) to the migration. In addition, the data of the \( F_{LR} \) and \( F_L \) are both on the scale of \( 10^{-11} \) N at \( a/H = 0.1 \). During the range of \( a/H \geq 0.2 \), the \( F_{LR} \) and the \( F_L \) amplify enormously. The \( F_{LR} \) even exceeds the \( F_L \) more than one order of magnitude. It indicates that the particle rotation has the extreme influence on the lateral migration of a large particle.

![Figure 5. Rotation velocity of particles with different \( a/H \).](image)

### Table 1

| \( a/H \) | \( F_L \) (\( \times 10^{-11} \) N) | \( F_{LR} \) (\( \times 10^{-11} \) N) | \( F_L \) (\( \times 10^{-10} \) N) | \( F_{LR} \) (\( \times 10^{-10} \) N) | \( F_L \) (\( \times 10^{-9} \) N) |
|---|---|---|---|---|---|
| 0.1 | -3.6 | -2.1 | 3.89 | 57.6 | 1.99 |
| 0.2 | -3.6 | -2.1 | 3.89 | 57.6 | 1.99 |
| 0.3 | -3.6 | -2.1 | 3.89 | 57.6 | 1.99 |
| 0.4 | -3.6 | -2.1 | 3.89 | 57.6 | 1.99 |
| 0.5 | -3.6 | -2.1 | 3.89 | 57.6 | 1.99 |

### 4. Conclusion

The interaction of fluid and particle was studied at moderate Reynolds numbers through the FSI method in this paper. The equilibrium positions of different particles (0.1 \( \leq a/H \leq 0.5 \)) were investigated at different Reynolds numbers (20 \( \leq Re \leq 80 \)). Then, flow structure around two typical particles (\( a/H = 0.1 \) and \( a/H = 0.3 \)) was analysed. The vortex in the front of the particle retards particle’s translation leading to the lag velocity increasing. Finally, the particle rotation and the rotation-induced force were discussed quantitatively. The rotation-induced force tremendously promotes large particle’s inertial migration. In the present research, the particle in a fixed position was selected to analyse which was limited by the computer resources. In the future, more attention needs to be paid to the process of the overall inertial migration to explore the kinetics of inertial microfluidics.

### Acknowledgments

This study was financially supported by Foundation of Scientific and Technological Talents Development for Young and Middle-aged Teacher of Shanghai Institute of Technology (No. ZQ2019-9).
References
[1] Hur S C, Tse H T and Di Carlo D 2010 Lab Chip 10 274-80
[2] Choi J, Hong S C, Kim W and Jung J H 2017 ACS Sens 2 513-21
[3] Marle L and Greenway G M 2005 TrAC Trends in Analytical Chemistry 24 795-802
[4] Segre G and Silberberg A 1961 Nature 189 209
[5] Asmolov E S 1999 Journal Of Fluid Mechanics 381 63-87
[6] Matas J P, Morris J F and Guazzelli E 2004 Journal of Fluid Mechanics 515 171
[7] Schonberg J A and Hinch E 1989 Journal of Fluid Mechanics 203 517-24
[8] Di Carlo D, Irimia D, Tompkins R G and Toner M 2007 Proc. Natl. Acad. Sci. U. S. A. 104 18892-7
[9] Miura K, Itano T and Sugihara-Seki M 2014 Journal of Fluid Mechanics 749 320-30
[10] Amini H, Sollier E, Weaver W M and Di Carlo D 2012 Proc. Natl. Acad. Sci. U. S. A. 109 11593-8
[11] Esipov D V, Chirkov D V, Kuranakov D S and Lapin V N 2020 Journal of Fluids Engineering 142
[12] Jiang D, Tang W, Xiang N and Ni Z 2016 RSC Advances 6 57647-57
[13] Tian F B, Dai H, Luo H, Doyle J F and Rousseau B 2014 Journal of Computational Physics 258 451-69
[14] Pedrol E, Massons J, Díaz F and Aguiló M 2018 Fluids 3
[15] He Y, Bayly A E and Hassanzour A 2018 Powder Technology 325 620-31
[16] Yuan C, Pan Z and Wu H 2018 Microfluidics and Nanofluidics 22 102
[17] Di Carlo D, Edd J F, Humphry K J, Stone H A and Toner M 2009 Phys. Rev. Lett. 102 094503
[18] Shi P Y and Rzehak R. 2020. Chemical Engineering Science 211
[19] Zhang J, Yan S, Yuan D, Alici G, Nguyen NT, Ebrahimi W M and Li W 2016 Lab Chip 16 10-34