Scherk-Schwarz twist in 5D conformal SUGRA

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Abstract

We reinterpret the Scherk-Schwarz (SS) boundary condition for $SU(2)_R$ in a compactified five-dimensional (5D) Poincaré supergravity in terms of the twisted $SU(2)_U$ gauge fixing in 5D conformal supergravity. In such translation, only the compensator hypermultiplet is relevant to the SS twist, and various properties of the SS mechanism can be easily understood. Especially we show the equivalence between the SS twist and boundary constant superpotentials at the full supergravity level.

1 Introduction

The Scherk-Schwarz (SS) mechanism [1] of SUSY breaking has been revisited as a phenomenologically interesting candidate for the physics beyond the standard model [2]. The simplest setup in such context was constructed within the framework of five-dimensional (5D) supergravity compactified on an orbifold $S^1/Z_2$. Here we reinterpret the SS boundary condition for $SU(2)_R$ in the compactified 5D Poincaré supergravity as the twisted $SU(2)_U$ gauge fixing in the 5D conformal supergravity $^{3}$. In such an interpretation, only the compensator hypermultiplet is relevant to the SS twist.

2 SU(2)$_U$ gauge fixing and Scherk-Schwarz twist

2.1 Twisted SU(2)$_U$ gauge fixing

In the derivation of 5D Poincaré supergravity from the 5D conformal supergravity using the hypermultiplet compensator $^{4}$, the $SU(2)_R$ symmetry is defined as the diagonal subgroup of the direct product of the original $SU(2)_U$ gauge symmetry among the superconformal symmetries and $SU(2)_C$ which rotates the compensator hyperscalars ($A^1_i, A^2_i$) ($i = 1, 2$ is an $SU(2)_U$-index),

$$SU(2)_U \times SU(2)_C \rightarrow SU(2)_R ,$$

through the $SU(2)_U$-gauge fixing $A^a_i \propto \delta^a_i$. The dilatation- and $SU(2)_U$-gauge fixings completely fix the quaternionic compensator hyperscalars as

$$A^a_i \equiv \delta^a_i \sqrt{1 + A^a_j A^j_i} .$$

However, if we consider a torus compactification of the fifth dimension with the radius $R$, we have inequivalent classes of the $SU(2)_U$ gauge fixing which are parameterized by a twist vector $\vec{\omega} = (\omega_1, \omega_2, \omega_3)$ as

$$A^a_i \equiv (e^{i\vec{\omega} \cdot \vec{\sigma} f(y)})^a_i \sqrt{1 + A^a_j A^j_i} ,$$

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where \( y \) is the coordinate of the fifth dimension and \( f(y) \) is a function satisfying \( f(y + 2\pi R) = f(y) + 2\pi \). Nonzero \( \omega_{1,2} \) correspond to the SS twist parameter.\(^4\) In other words, from Eq. (1), the SS boundary condition for all the fields with \( SU(2)_R \) index in the Poincaré supergravity is simply (equivalently) given by the twisted gauge fixing condition for \( SU(2)_U \) in the framework of the conformal supergravity [7].

Next we derive SUSY breaking terms induced by the SS twist in the periodic basis. By the following field redefinition, the compensator fixing condition (3) reduces to the normal one (2).

\[
\mathcal{A}^a_i \rightarrow U^a_b(y)\mathcal{A}^b_i, \quad \zeta^a \rightarrow U^a_b(y)\zeta^b, \quad (4)
\]

where \( \zeta^a \) is the compensator hyperinos and

\[
U^a_b(y) \equiv \left(e^{-i\vec{\omega} \cdot \vec{\sigma}} f(y)\right)^a_b.
\]

As compensation for it, we have additional \( \vec{\omega} \) dependent terms which arise from the \( y \)-derivatives of the compensator fields in the action. They are given by

\[
e^{-1}\mathcal{L}_\omega = f'(y)(i\vec{\omega} \cdot \vec{\sigma})_{ij} \left\{2i\vec{\bar{\omega}} \cdot \gamma^4 \mathcal{A}_m^i \mathcal{A}_n^j(1 + \mathcal{A}_k^k)\mathcal{A}_k^a(1 + \mathcal{A}_k^k)\right\} - 2(f'(y)|\vec{\omega}|)^2(\mathcal{A}_i^i\mathcal{A}_i^a + (\mathcal{A}_i^i\mathcal{A}_i^a)^2),
\]

after integrating out the auxiliary fields. This contains the mass terms of the gravitinos \( \psi_m^i \), the gauginos \( \Omega^I_i \) and the physical hyperscalars \( \mathcal{A}_i^a \).

### 2.2 Singular gauge fixing and boundary interpretation

An explicit function form of \( f(y) \) in Eq. (3) does not affect the physical consequence because \( f(y) \) is just a gauge fixing parameter. We usually choose \( f(y) = y/R \) which gives the simplest description. However in this section, motivated by the argument of generalized symmetry breaking in Ref. [8], we choose it as

\[
f(y) = \pi \left(\text{sgn}(y) - \text{sgn}(-n\pi R)\right), \quad (5)
\]

where \( \text{sgn}(y) \) is the sign-function. Namely,

\[
f'(y) = \pi \sum_n \delta(y - n\pi R).
\]

For this \( f(y) \), \( \mathcal{L}_\omega \) totally becomes boundary terms. In fact, we can show that the SS twist reproduces the same action as the untwisted case with the constant superpotentials \( W \) at both the orbifold boundaries, provided that

\[
W = \pi(\omega_1 + i\omega_1). \quad (6)
\]

From this correspondence between the SS twist and the constant superpotentials, we confirm that SUSY breaking caused by the SS twist is not explicit because the boundary constant superpotential is \( N = 1 \) invariant. We remark that this correspondence has been easily found at the full supergravity level, not in the effective theory, thanks to our simplified interpretation of the SS twist.

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\(^4\) The consistency with the orbifold projection requires that \( \omega_3 = 0 \).
2.3 Scherk-Schwarz twist and AdS$_5$ geometry

It was suggested in Ref. [9] that the SS twist yields an inconsistency in the supergravity on AdS$_5$ geometry. In this section, we see how this fact can be seen within our framework of the twisted SU(2)$_U$ fixing. It is known that the gauging of U(1)$_R$ symmetry by the graviphoton is necessary to realize the AdS$_5$ geometry keeping SUSY [10, 11, 12]. In fact, the negative cosmological constant is proportional to the U(1)$_R$ gauge coupling in such a case. Thus the covariant derivatives of the compensator hyperscalars are

$$D_\mu A^a_i = \partial_\mu A^a_i - (g_R W^R_\mu)^a_b A^b_i + \cdots,$$

where

$$(g_R W^R_\mu)^a_b \equiv g_R W^R_\mu (\vec{q} \cdot i\vec{\sigma})^a_b = \begin{cases} |\vec{q}| g_R W^R_\mu (i\sigma_1 \sin \theta_R + i\sigma_2 \cos \theta_R)^a_b & (g_R : Z_2\text{-even}) \\ |\vec{q}| g_R W^R_\mu (i\sigma_3)^a_b & (g_R : Z_2\text{-odd}) \end{cases}.$$ 

Since $A^a_i$ are fixed to constants at the leading order for the gravitational coupling $5$ (see Eq. (2)), the nonvanishing mass term for the graviphoton comes out from the kinetic terms for $A^a_i$ if the commutator $[\vec{q} \cdot \vec{\sigma}, \vec{\omega} \cdot \vec{\sigma}]$ does not vanish. Such a mass term breaks the unitarity of the theory because the graviphoton is the gauge field in this case. Therefore the following condition must be satisfied for the theory to be consistent.

$$[\vec{q} \cdot \vec{\sigma}, \vec{\omega} \cdot \vec{\sigma}] = 0.$$ 

To obtain the GP-FLP [11] (BKVP [12]) model for a supersymmetric warped brane world, we need to gauge the U(1)$_R$ symmetry by the graviphoton with Z$_2$-odd gauge coupling$6$ $g_R$, i.e. $\vec{q} = |\vec{q}|(0, 0, 1)$. From the condition (8), the only possible twist vector in this case is $\vec{\omega} = 0$. Namely the SS twist is impossible in this case.

On the other hand, in the ABN model [10] in which the U(1)$_R$ symmetry is gauged by the graviphoton with Z$_2$-even gauge coupling$6$ $g_R$, i.e. $\vec{q} = |\vec{q}|(\sin \theta_R, \cos \theta_R, 0)$, the possible twist vector is $\vec{\omega} = |\vec{\omega}|(\sin \theta_R, \cos \theta_R, 0)$. So we find a possibility to have the SS twist in the warped spacetime. However it was pointed out that ABN model is not derived from the known off-shell formulations with the linear multiplet [13] or the hypermultiplet compensator [6]. We can not find the Killing spinor on this background in those off-shell formulations for ABN model even without the SS twist. This is still an open question.

3 Conclusion

By noticing that the twisted SU(2)$_R$ boundary condition in $S^1/Z_2$-compactified 5D Poincaré supergravity is equivalent to the twisted SU(2)$_U$ gauge fixing in 5D conformal supergravity, we have reexamined the SS twist boundary condition in the latter terminology. In this case, only the compensator hypermultiplet is relevant to the SS twist, and various properties of the SS mechanism can be easily understood. We reproduced the 5D Poincaré supergravity with the SS twist from the 5D conformal supergravity with the twisted SU(2)$_U$ gauge fixing. Thanks to this interpretation, we can explicitly show the Wilson line interpretation of the SS twist [14], the correspondence between the SS twist and the boundary constant superpotentials [8], and the quantum inconsistency of the twist in the AdS$_5$ background [9] at the full supergravity level.

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5 We have taken the unit of $M_5 = 1$, where $M_5$ is the 5D Planck mass.
6 The Z$_2$-odd gauge coupling can be realized in the supergravity through the four-form mechanism [12].
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