Weinberg Angle and Integer Electric Charges of Quarks

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Abstract

Orthogonality between $\gamma$ and $Z$ boson involves the Weinberg angle and a scheme for assignments of electric charge and weak isospin to leptons and quarks coupling to $\gamma$ and $Z$. The Han-Nambu scheme, with integer electric charges for quarks, satisfies $\gamma Z$ orthogonality with $\sin^2(\theta_W) = 0.25$ in leading order. Experimental results for photon-photon fusion into $c\bar{c}$ and $b\bar{b}$ pairs provide further support for assigning integer electric charges to quarks.

1 Introduction

Whether quarks have fractional or integer electric charges is not an issue of present particle physics but has been discussed extensively in the past, e.g. [1] - [5].

The present Standard Model [6] of particle physics (see [7] and [8]) assumes that quarks have fractional charges. In units of $e$ they are $+2/3$ for up, charm and top quarks and $-1/3$ for down, strange and bottom quarks independent of their color, as proposed by Gell-Mann [9] and Zweig [10] in the framework of flavor-$SU(3)$ and later adopted by Fritzsch, Gell-Mann and Leutwyler in the framework of color-$SU(3)$ and QuantumChromoDynamics [11].

As an alternative scheme, Han and Nambu [1] had proposed that quarks have integer charges, but different for the three colors. Here, the fractional charge is the result of the superposition of the three quarks to a color singlet. Table 1 shows the electric charges $Q_{fc}$ according to this scheme for one generation of quarks, consisting of three up- and three down-like quarks with different strong color charges.

Table 1: Electric charges $Q_{fc}$ according to the Han-Nambu scheme

| Quark | Strong Charge | Electric Charge |
|-------|---------------|-----------------|
| Up    | +1/3          | +1/3            |
| Strange | +1/3         | +1/3            |
| Bottom | +1/3          | +1/3            |
| Down  | -2/3          | -1/3            |
| Strange | -2/3         | -1/3            |
| Bottom | -2/3          | -1/3            |

Obviously, for an up quark in a color singlet state $|up\rangle$ with the short notation:

$|up\rangle = (|up(Q = +1)_{red}\rangle + |up(Q = +1)_{blue}\rangle + |up(Q = 0)_{green}\rangle)/\sqrt{3}$

the average charge $\langle Q \rangle$ is $+2/3$ and, correspondingly, for the down quark, $-1/3$.

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Table 1: Electric charge $Q_{fc}$ in units of $e$, the absolute value of the electron charge, for the two quark flavors $f = \text{up}$ and $\text{down}$, and three color charges $c = \text{red}$, blue, and green, according to the Han-Nambu scheme.

| Flavor | red | blue | green |
|--------|-----|------|-------|
| up     | +1  | +1   | 0     |
| down   | 0   | 0    | -1    |

It has been realized, also decades ago [4], that as long as quarks are confined in color singlet states the direct coupling of photons to quarks measures the average charge and cannot distinguish between the two alternatives, fractional charges according to the Standard Model or a superposition of integer charged quarks according to Han-Nambu.

The strongest experimental evidence for fractional (average) electric quark charges comes from $e^+e^-$ annihilations into hadrons. The standard description is based on the Feynman diagram, see Fig.1a, where the virtual photon results from the $e^+e^-$ annihilation. Firstly, the virtual photon $\gamma$ decays into a quark and antiquark and subsequently this pair fragments into hadrons. The ratio of the cross sections $\sigma(e^+e^- \rightarrow \text{hadrons})$ and $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$: $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ displays the emergence of new quarks as a function of the virtual photon invariant mass, starting from thresholds dictated by the effective mass of the quark-antiquark pair, and provides a direct measure of the electric charges of the quarks and the number ($N_c = 3$) of different colors. The ratio is described by the formula $R = \sum_f(\sum_c Q_{fc}/\sqrt{3})^2$, where the indices $f$ and $c$ indicate the quark flavor and color charge and the sum extends over all quark flavors which can be produced at the given virtual photon invariant mass.

Consider the production of a quark-antiquark pair $f\bar{f}$ of flavor $f$ with electric charge(s) $Q_{fc}$ in the Standard Model or the Han-Nambu scheme. The transition amplitude $T_f$ is proportional to the coupling of the photon to the effective electric quark charge, the quark being in the color singlet state, i.e.

$$T_f \propto (Q_{f\text{red}} + Q_{f\text{green}} + Q_{f\text{blue}})/\sqrt{3} = 3\langle Q_f \rangle/\sqrt{3}$$

This (partial) amplitude contributes to the cross section ratio $R$ with $(3\langle Q_f \rangle/\sqrt{3})^2 = N_c\langle Q_f \rangle^2$. The experimental data are in perfect agreement with the assumption of the Standard Model up to the highest measured virtual photon masses. But they are also in agreement with the Han-Nambu scheme as long as quarks are confined to color singlet states.

Another (lost) testing ground for the distinction between fractional and integer quark charges is Deep Inelastic Scattering (DIS) of leptons. The exchanged virtual photon, now spacelike with imaginary invariant mass, couples in first order to quarks in color singlet states. (This part of DIS is as well described by diagram Fig.1a). Thus, DIS as well as $e^+e^-$ annihilation cannot distinguish between the Standard Model and the Han-Nambu scheme. The proof that the (average) electric charge is fractional for DIS can be derived from the ratio of $F_2$ structure functions measured in charged electron and muon DIS (photon exchange) on average nucleons over that measured in neutrino DIS (W boson exchange). This ratio amounts to about $5/18 = 1/2(\langle Q_{\text{up}} \rangle^2 + \langle Q_{\text{down}} \rangle^2)$. 


Estimating the Weinberg angle

In order to distinguish between integer and fractional charges of quarks, transition amplitudes are required where the square of the electric charges of quarks enter. This has been emphasized by Witten already in 1977 [12].

First consider the diagram shown in Fig.1b. It displays the lowest order coupling between Z boson and photon $\gamma$ through the internal loop of a fermion $f$. The Standard model postulates orthogonality between $\gamma$ and $Z$.

\[ T_{\gamma Z} = \langle Z | \gamma \rangle = 0 \]

The amplitude $T_{\gamma Z}$ implies the summation over all fermions, quarks and leptons. The product of coupling constants of each fermion to $\gamma$ and $Z$ enter. The coupling to $Z$ depends on electric charge and third component $I_3$ of the weak isospin of the fermion. The left-handed fermions have, according to the Standard Model, $I_3 = +1/2$ (neutrinos and up quarks), $I_3 = -1/2$ (charged leptons and down quarks). Right-handed charged leptons, up and down quarks have $I_3 = 0$. Thus, for a given internal fermion $f$ (with color charge $c$, in case of quarks), the coupling $C_{\gamma Z fc}$ between photon and Z boson is, according to the Standard Model, omitting a common factor $1/(\sin(\theta_W)\cos(\theta_W))$ :

\[ C_{\gamma Z fc} = Q_{fc} \cdot [I_{3f} - Q_{fc} \cdot \sin^2(\theta_W)] \]

with the Weinberg angle $\theta_W$.

Let us assume, all fermions are massless. And restrict the consideration to the first family of known fermions, since the additional families are repetitions of the first
with respect to the quantum numbers relevant for the coupling to $\gamma$ and $Z$. Summing over all fermions, leptons and quarks, in order to have orthogonality (1) simplifies to requiring that the sum of the couplings $C_{\gamma Z f c}$ vanishes. There are 14 family members which contribute: the left and right handed electron, $e_L, e_R$ and quarks $u_R, u_L, d_R, d_L$, with the three colors $c = \text{red}, \text{blue}, \text{green}$. Summing over color and handiness, the partial contributions $C_{\gamma Z f}$ of electron, up and down quarks to the orthogonality condition are obtained. They are of course identical for the electron, in both schemes, Standard Model (SM) and Han-Nambu (HN):

$$C_{\gamma Z e} = \frac{1 - 4 \sin^2(\theta_W)}{2}.$$  

For the quarks, the results differ in both schemes. The SM obtains:

$$C_{\gamma Z u} = \frac{1 - (8/3) \sin^2(\theta_W)}{2} \quad C_{\gamma Z d} = \frac{1 - (4/3) \sin^2(\theta_W)}{2}$$

For the HN scheme one obtains:

$$C_{\gamma Z u} = 2 \left(1 - 4 \sin^2(\theta_W)\right) \quad C_{\gamma Z d} = \frac{1 - 4 \sin^2(\theta_W)}{2}$$

Requiring the internal loops of quarks to be color singlets, all the quark contributions have to be multiplied with a factor $1/\sqrt{N_c} = 1/\sqrt{3}$ for both schemes.

As a final step, the contributions of electron and the two quark flavours are added up to extract the Weinberg angle from the orthogonality relation (1) (under the assumption of massless fermions):

$$T_{\gamma Z} \propto \sum_f C_{\gamma Z f} = 0$$

The SM scheme leads to $\sin^2(\theta_W) = 3/8 = 0.37$ or $\sin^2(\theta_W) = 0.35$, whereas the latter value is obtained applying the factor $1/\sqrt{N_c}$ to the quark amplitudes.

According to the HN scheme, the multiplet of one family with integer charges yields $\sin^2(\theta_W) = 1/4 = 0.25$, independent of the color singlet assumption. The latter value is much closer to the best present value of the Weinberg angle determined from many experimental results and extrapolated to the $Z$ mass $M_Z$ in the modified minimal subtraction scheme:

$$\sin^2(\theta_W(M_Z)) = 0.23116 \pm 0.00012$$

In the above derivations the mass of all fermions has been neglected. To calculate $T_{\gamma Z}$, each coupling factor $C_{\gamma Z f}$ has to be multiplied with a weight ($\propto 1/m_f^2$), which depends on the mass $m_f$ of the fermion in the internal loop of the diagram Fig. 1b.

However, under the assumptions of the HN scheme, all three charged fermions of one generation contribute to the sum with a product containing the same factor $\left(1 - 4 \sin^2(\theta_W)\right)$. Hence, lepton, up and down quark, separately satisfy the orthogonality condition, independent of their mass, for a Weinberg angle $\sin^2(\theta_W) = 1/4$. Even more generally: Every single colored fermion with non-zero integer electric charge and with the same assignment rules for the signs of $I_3$ and $Q$ as those valid for charged fermions in the Standard Model: equal signs of $I_3$ and $Q$ for the left handed, separately satisfies the orthogonality (1).

Adopting the HN scheme, orthogonality is satisfied for a Weinberg angle with $\sin^2(\theta_W) = 1/4$, since the $Z$ couples to a purely axial and $\gamma$ to a purely vector fermion current. In the above derivation the internal loop with a $W$ gauge boson instead of fermion has been neglected. This is only justified by the empirical fact that the mass of $W$ is large compared to the masses of fermions of the first generation.

In the absence of a better prediction for the Weinberg angle based on the SM scheme this may be considered as support for the HN scheme, i.e. integer electric charges of
quarks.

3 Two-photon physics

Two photon interactions have been extensively studied at the Large Electron Positron Collider LEP. Of particular interest in the present context are processes where two photons, emitted from the scattered electron and positron, fuse to create quark-antiquark pairs, see Fig. 1c for the diagram describing the amplitude. Obviously, this amplitude is proportional to

\[ T \propto (Q_{f_{\text{red}}}^2 + Q_{f_{\text{green}}}^2 + Q_{f_{\text{blue}}}^2) / \sqrt{3} \]

The corresponding ratio \( R_2 \) of the cross section for photon-photon fusion into a specific quark-antiquark pair \( ff \) over that for the fusion into \( \mu^+\mu^- \), again neglecting QCD radiative corrections and quark mass dependences far above threshold is

\[ R_2 = 3\langle Q_f^2 \rangle^2 \]

With the SM assumption of fractional electric quark charges \( R_2 \) equals \( 3(4/9)^2 = 16/27 \) for up quarks and \( 1/27 \) for down quarks. The HN scheme yields \( 3(2/3)^2 = 4/3 \) for the fusion into up quarks and \( R_2 = 1/3 \) for down quarks. The differences (a factor \( 9/4 \) for up quarks and \( 9 \) for down quark, for the ratio of the HN over SM cross section) are striking, in particular for the down-like quarks.

At LEP, the production of charm-anticharm and bottom-antibottom by photon-photon fusion has been studied, see [13]. It has been observed that indeed the measured cross section for \( b\bar{b} \) production is significantly larger than the expected (SM) cross section. Ferreira [14] has proposed to adopt the theoretical calculations for the production by the direct (unresolved) photon-photon fusion to charm and bottom quarks and to multiply them with the factors \( 9/4 \) and \( 9 \), respectively. Good agreement between prediction and experiment is achieved for this calculation.

This is considered as a further support for the Han-Nambu scheme.

4 Conclusion

The assumption that quarks have integer electric charge according to the Han-Nambu scheme largely simplifies the orthogonality condition between photon and Z boson, in leading order. It allows a straightforward derivation of the Weinberg angle, based on all fermions of the three known families. Surprisingly, the result is almost independent of the mass sector of the Standard Model. The obtained value of \( \sin^2(\theta_W) = 1/4 \) deviates only by 8% from the quoted present best theory-corrected experimental value.

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