Addition of fraction in swimming context

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Abstract. This study aimed to produce learning trajectory that can help students in learning fractions by using swimming context. The study involved 37 fourth grade students with different capabilities in Elementary School IBA, South Sumatra, Indonesia. This study used an instructional theory called Indonesian version of Realistic Mathematics Education (PMRI). This research used design research method with three stages: preliminary design, the design experiment, and retrospective analysis. Several techniques used for collecting data including a video recording of students interaction in the group discussion, students’ work, and interviewing the students. To conclude, the swimming context could stimulate students’ informal knowledge about the meaning of fractions in which it can be used in the additional learning either the same denominator or different denominator.

1. Introduction
Building the understanding of fractions for elementary school students is not easy to do. Learning fractions in Indonesia tend to focus on procedures only. The problem is a variety of fractions meaning is one of the difficulty causes in the learning fractions [1]. The students should be given the widest possible opportunity to explore the meaning of fractions before students study the relationship between fractions and operations on fractions [2]. Another problem faced by students is they got difficulty in adding the fractions, especially with different denominator fractions. The difficulty in conventional teaching is caused by the hazy understanding of fractions itself. Students do not understand the actual meaning of fractions, but they are forced to continue the learning process by operating fractions. Also, students tend to follow the way given by teacher mechanistically. The teacher shows a method, with some examples, which pupils then apply to similar [3].

Mathematics has always been one of the most significant issues in an individual’s life [4]. Children need mathematics when they graduate from school or when they start working [4]. Mathematical competence is increasingly recognized as crucial for study in a wide range of science disciplines [5]. Learning should be more meaningful so that students are more active and understanding of the subjects taught. Understanding how mathematical conceptions are constructed can help in thinking about teaching with the aim of encouraging learning [6]. Therefore, it is necessary a promising approach be applied, that is Realistic Mathematics Education, in Indonesia [7]. PMRI selected because in line with Curriculum 2013 in which its objectives is to provide knowledge to the students completely and not fragmented. The curriculum 2013 emphasizes the involvement of the students to find the lesson concepts through the teachers’ role as a facilitator. PMRI provide opportunities for
teachers and students to interact between the teacher with students and students with students. With the interaction, the learning atmosphere in the classroom is expected to be more conducive to achieving the learning objectives. PMRI is an approach that starts from the real things for the students, and emphasize the skills of discussing the process with classmates so that at last their discovery that they can use to solve problems either individually or in groups [8]. The situation of students’ lives is not only what is the real extent of the students' views but also all imaginable of students, affordable by their imagination [9, 10]. Mathematics must be close to children and be relevant to everyday life situation, therefore the contextual situation that is relevant and familiar to students need to be elaborated within mathematics learning [11]. Learning starts from a real thing so that students can get involved in the learning process significantly. In PMRI, the teaching is built on informal knowledge of students; it is important to give students the opportunity to explore some of the situations of daily lives where the fractions play a role [9].

PMRI requires a context which is close to children to help students in understanding the lesson [8]. Freudenthal stated education should lead students to use a variety of situations and opportunities to reinvent mathematics in their way [12, 13]. Mathematical concepts arise from the process of mathematization-starting from accomplishment which related to the context (context-link solution), students gradually developed it as the device and comprehend the mathematics to a more formal level [13]. Therefore, in this study using real context that can be imagined by the students to stimulate informal knowledge to the formal knowledge. The swimming context was chosen because it can represent fractions using measurements. The measurement concerns the identification of a length which uses the length as a part in determining the length [14]. The shape of the pool is one model that allows representing parts of the whole.

From the above discussion, the aim of this paper is to develop a theory of learning to assist students to understand the concept of the adding fractions using the swimming context.

2. Method

2.1. The subject of the research
The study involved 37 fourth grade students of IBA Palembang, South Sumatra, Indonesia. Subjects were selected randomly with heterogeneous capabilities that will be divided into groups with the heterogeneous capabilities. Those students had not studied about fractions yet before. Six students participated in the pilot experiment to make some adjustments to the Hypothetical Learning Trajectory (HLT). Then, thirty-one students participated in the teaching experiment.

2.2. The design of the research
This study used research design as the proper approach to achieving the objectives of the research. Research design consisted of three stages, namely the preliminary design, the design of experiment (pilot experiment and teaching experiment), and retrospective analysis [15].

In the perspective of design research, the objective of the first stage was to formulate the local instructional theory (LIT) which can be elaborated and perfected while performing experiments [16, 17]. At the preliminary stage, researcher examined the literature about fractions, the concept of supporting fractions material (meaning of fractions, the addition of fractions with the same denominator, and the addition of fractions with the different denominator), the fractions learning model, the swimming context, the fractions learning in the curriculum, PMRI approach, and design research which is used as research methods. After that, the researcher discussed with the teacher who will teach at the teaching experiment stage to learn more about the condition and capabilities of students. After receiving information from the teacher, researcher designed HLT as an image of fractions learning sequence using swimming context with PMRI approach.
Students are able to determine the distance that have been taken by athletes in the fractions form

Students are able to add the fractions using

Students are able to add the fractions using

**Figure 1. HLT**

Swimming was chosen because it can be represented through Athlete’s Distance Measurement theory while swimming in the certain time can be determined the parts. In this activity, students also used fractions bar as a model for studying fractions. Through the bar, students used a smaller field to find the names of fractions for the parts that were provided [14].

After that, the prototype which had been designed at the preliminary design stage was tested at the experiment stage. The purpose of teaching experiment is to test and improve LIT which is developed at an early stage and to develop an understanding of how it works [16]. The teaching experiment has purposed to collect data to answer the research questions [17]. The design experiment stage was conducted in two cycles, pilot experiment, and teaching experiment.

**Figure 2. A cumulative cyclic process in design research [16, 17]**

The first cycle was conducted as an experiment design; this stage involved six students with each of the two high-ability students, two medium-ability students, and two low-ability students. The purpose of this pilot experiment was to adjust the content and sequence of activities that have been arranged and to increase the learning activities in order to gain a better design in the next cycle [17], the second cycle was conducted as the actual experiment activity, in this cycle, involving 31 students with heterogeneous capabilities, and the activities were divided into groups consisting of students with heterogeneous capabilities. A reflection on the learning process at the end of each lesson concerning the strong and weak points of the subjects was also done [17].

2.3. Data collection and data analysis

Some activities for data collection such as interviews, video observation, and students’ answer sheets. Before conducting the learning, researcher and teacher discussed learning which will be carried out. The role of the researcher in this learning activities was to ask the students some additional questions,
observe the learning activities, coordinate activities, and to make changes to activities which are required to provide relevant information on the research [18, 19].

During the experiments, learning activities thoroughly and students’ discussion activity was recorded. In the process of discussion, the teacher while interviewing the members of the group asked about how far their understanding toward the problems on the activity sheet. After the teaching experiment, we collected the students’ answers on the answer sheet. This data was analyzed to obtain insights from the learning process of students and to investigate how far the learning objectives were achieved.

Data analysis was conducted in a retrospective analysis stage. At this stage, HLT which is used as a guideline in analyzing all the data on the experimental stage compared with the actual learning in the classroom. The conclusion from this analysis is used as an answer to the problem formulation [17]. The main result is not a worked design, but the reasons of how it works [15, 16, 17].

3. Results and discussion

3.1. Activity 1: Understanding the meaning of fraction

The problems are given in activity 1 related to elements of fractions. At the beginning of learning, the teacher gave apperception by asking "Have you swum?", "Tell your experience!", "How do you think of a swimming athlete can be declared victorious in the race?". This apperception activity made students enthusiastic in answering the teacher’s questions. This activity also aimed to inform the context that will be used during the learning process.

Next, the teacher gave instruction to the students to sit in groups and gave LAS 1 to students, and they started working in groups. Students discussed and did the activity and the teacher as a facilitator and to observe students’ activities. After finishing the discussion and working LAS 1, the students presented the results of their answers. Problems presented in LAS 1 were part of athletes that have been taken while swimming. It can be seen in the following figure.

![Figure 3. Problem in activity 1](image)

On the first question, it was asked the length of swimming pool. Then, on the second question, it was asked the distance of athletes. After that, the students determined which parts which are reached by athletes. From those answers, students' understanding of the meaning of fractions began to be built. Students were able to know that the part that has been taken by athletes was one of the fractions meanings. At last, the students concluded that the distance which is reached by the athletes was the numerator, while the length of the pool was the denominator of the fraction. The results of the students' answers can be seen in Figure 4.

To clarify the students’ answers, here are following transcripts of conversations with students.

Teacher : How far the distance that has taken by the athlete?
Student : Half of the pool
Teacher : Why did you say that?
Student : Let's see. The end of the pool here, and here is the beginning of the pool (pointing 0 and 50)
Teacher : What is the meaning of 25?
Student : 25 is the location where he arrives
Teacher : Oh ..., what about the 50's?
Student : It is the whole pool length
Teacher: Did you know in the fractions, what is the meaning of the number 25?
Student: Numerator, and here is the denominator (pointing to the number 50)

| Translation: |
|--------------|
| 1. From the picture above, how long the pool? (in meters) |
| 2. How much distance taken by athlete? (in meters) |
| 3. How much part of the pool has been taken by athlete? |
| 4. Fill the top box is a section taken by athlete and the bottom box is the length of the pool! |

**Conclusion:**
The form that you write is faction. Fractions are numbers that can be expressed by \( \frac{a}{b} \).

**Teacher:** Did you know in the fractions, what is the meaning of the number 25?
**Student:** Numerator, and here is the denominator (pointing to the number 50)

**Figure 4.** The student’s answer in activity 1

The conversation above describes how the student begins to understand the situation of this problem. The student concluded that the distance of that athlete reach the swimming pool is the part that has been taken, while the length of the pool is the whole should be taken. So that when answering the question "how many parts have been taken by the athlete?" He was able to answer half the pool. The answer was the expected answer; the students knew that the meaning of parts in question was about the form of fractions. Then, from that problem, he was led to know the elements of fractions; those are the numerator and denominator. By answering the last question, he was able to answer that the distance enumerated by the athlete was the numerator and the distance of the whole pool was the denominator. From this, it was concluded that the students’ understanding of the real meaning of fractions had shown good results.

3.2. Activity 2: Adding the fractions with the same denominator
In this activity, students were asked to resolve the problem where an athlete reached distance in certain seconds, and then the two distances added together. In this activity, students were asked to use the fraction bar in the process of finding the result of the fractions addition. Students’ activity started from shading the bar based on the distance covered for 20 seconds; then students shaded more another fraction bar based on the distance in the 30 seconds. Furthermore, the students cut one fraction bar by its shading, after that the students patched the shading which has been cut to the top of the intact bar. From the bar, it was got the addition results of the parts that have been taken by athletes. Students’ answer can be seen in Figure 5.

**Translation:**

**What can you conclude?**

Meli has been take \( \frac{1}{5} \) parts of the pool, then she swim again as far as \( \frac{2}{5} \) parts of the pool. So, the result is \( \frac{3}{5} \).

**Figure 5.** The Student’s answer using fraction bar
In Figure 5, students can shade properly in the parts which have been determined on the questions. Then, by the bar helping, students added those parts and moved it to the answers’ column. Here is the following conversation transcript of the students’ answers.

Teacher : How much does yellow represent?
Student 1 : \( \frac{1}{5} \)
Teacher : How many regions will be shaded?
Student 1 : 1
Teacher : Of how many regions?
Student 1,2 : 5 regions
Teacher : And how about the blue one?
Student 1,2,3 : \( \frac{3}{5} \)
Teacher : Then what should we do?
Student 2 : There! (Student attached Bar A and B)
Teacher : So how much is that?
Student 1 : \( \frac{4}{5} \) (Students shaded the results boxes)
Teacher : So how much is the sum?
Student 1,2 : \( \frac{4}{5} \)
Teacher : What can you conclude?
Student 1 : \( \frac{1}{5} + \frac{3}{5} = \frac{4}{5} \)
Teacher : When the bottoms have the same number, you can directly add them up.
Student 1 : Yes, you can.

The conversation above illustrates activities in one of the groups in adding the same denominator fractions using a fraction bar media. We can see how students develop their understanding of the concept of fractions. Fraction bar which is used likened to a pool, and then the students shading the first distance reached by athlete in the 20 seconds, then shading the second distance reached by athlete in 30 seconds. From this bar shading activity, understanding of the meaning of fractions is required. Students knew that athletes have to take what parts of the pool, which will both parts should be added together. Seeing the conversations and results of students’ answers, fraction bar was very helpful to solve the problem. Fraction bar represented trajectory of swimming pool in the smaller units, so that it can be folded or torn to fractions addition activities. Students were able to answer questions on the problem of the same denominator well.

The next problem in this activity was the students were asked to add fractions with the same denominator by using the number line. The number line had been in the column, the students only determined the exact point of the swim athlete’s distance. The distance was determined by the students should be added using the number line. On these problems, students should add up within two athletes who were swimming. Students determined the distance of the first athlete on the number line, and then it is added with the second athlete’s distance. So that students can draw the conclusion that the sum of both these athletes can be seen from the number line that has been drawn by the students. Here are the students’ answers shown in Figure 6.
Translation:
1. How the overall distance that has been taken by Indah and Silvy in 20 seconds? (in meters)
2. If expressed as a fraction, how many parts have been taken by Indah and Silvy?

**Figure 6. The Student’s answer using fraction bar number line**

Figure 6, students correctly determined the point that represented the athlete’s distance. From the students’ answers, it can be seen that students have understood how to add fractions with the same denominator by using the number line.

**3.3. Activity 3: Adding the fractions with different denominator**

In this activity, students were asked to solve problems where athletes from Japan No. 5 and Korea No. 8 reached distance at the certain second, then the two distances added together. The problem can be seen from Figure 7.

**Figure 7. Problem in Activity 3**

In this activity, students were asked to use the fraction bar in the process of finding the result of the addition of fractions. The way using fraction bar on this activity was equivalent to the fraction bar on activity 2. Students were given fraction bar with different parts, and each student shading fraction bar by a predetermined part. Then, the students cut one of fraction bar and patched it to the intact fraction bar, after that the students calculated the amount of the existing parts in the fraction bar. Pictures of students’ activity can be seen in Figure 8.
1. Bar for athlete number 5
2. Bar for athlete number 8
3. Bar for the addition of athlete number 5 and 8’s travelled
4. What can you conclude? $\frac{3}{4}$

Figure 8. The student’s answer using fraction bar

In Figure 8, students could add fractions with the same denominators. Students added up the two fractions by using a bar that has been given. Then, they moved the results on the activity sheet that has been provided. This is in accordance with HLT that has been designed by researcher. For more details can be seen in the following transcript.

Teacher: How did you point the athlete position in number 5?
Student: Measured, half of the pool.
Teacher: How do know it’s half of the pool?
Student: Because it’s in the middle.
Teacher: Is there any other way? How about using the bar?
Student: We fold it.
Teacher: How do you show a half when you fold it?
Student: It was 2, when you fold them, it becomes $\frac{1}{2}$
Teacher: Now for number 8, which bar will you use?
Student: This one, the athlete is still in the $\frac{1}{4}$ of the box. So you fold it, if it’s collapsible, fold it again, and again. Done.
Teacher: Oh .. so you folded it, then how much is left?
Student: 1 (He cuts the bar)
Teacher: The bar you cut, how much does it show?
Student: $\frac{1}{2}$, then you attach them together.
Teacher: Then how much is it?
Student: $\frac{3}{4}$

In the next activity, the students resolved different problems but still used the swimming context. Students were required to add the two athletes’ distance. The two athletes’ distance was known by using part of the pool that has been taken. However, in this question, the pool part used fractions with different denominator. Then, students solved the problem using a number line. Figure 9 shows the student’s work. He correctly added the fractions using a number lines.
Also, one of the groups have understood that equating the denominator can be done by finding the Least Common Multiple from both denominators. Students have known that the addition of fractions with different denominator cannot be done directly, but it must equalize the denominator first.

This learning was designed to produce the learning trajectory in the learning of addition of fractions material by using swimming context. There were three learning activities of addition of fractions on the learning trajectory design that have been designed and implemented previously. The first activity helped students in understanding the actual meaning of fractions through the swimming context. The second activity helped students in adding fractions with the same denominator through problems of adding distance from the specified swimming athletes. The third activity helped students in adding fractions with different denominator through the problems of adding distance of the two swimming athletes. All of these three activities conducted during the three sessions of learning.

Several previous studies have also been getting maximum results in learning fractions using the PMRI approach [21]. Other results indicated by the research that showed that the results of the use of learning media in the form of fractional card can support students’ understanding in comprehending the students’ fractions from informal stage to formal stage [22].

In develops teaching materials based PMRI approach on summation material fractions also showed the good results, namely learning process of students using teaching materials of the addition of fractions PMRI approach based extremely guide students to develop ideas and foster the creativity in solving problems [1]. From the results of the researches on the fractions material that use PMRI, show that the PMRI approach is good enough to be applied in either the fractions material or other materials [23].

4. Conclusion
Using the results and discussions, it can be concluded that the activities in the student worksheet has been designed to help students understand and resolve problems in an addition of fraction intuitive informally to the formal problem-solving. The things in the learning trajectory in this study to assist students in learning the real meaning of fractions using swimming context and experience in learning meaningfully and fun given that the activity using fraction bars which can help students add fractions.

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