Spectrally recycling space-time wave packets

Layton A. Hall1 and Ayman F. Abouraddy1
1 CREOL, The College of Optics & Photonics, University of Central Florida, Orlando, FL 32816, USA

‘Space-time’ (ST) wave packets are propagation-invariant pulsed optical beams that travel rigidly in linear media without diffraction or dispersion at a potentially arbitrary group velocity. These unique characteristics are a result of spatio-temporal spectral correlations introduced into the field; specifically, each spatial frequency is associated with a single temporal frequency (or wavelength). Consequently, the spatial and temporal bandwidths of ST wave packets are correlated, so that exploiting an optical source with a large temporal bandwidth or achieving an ultralow group velocity necessitate an exorbitantly large numerical aperture. Here we show that ‘spectral recycling’ can help overcome these challenges. ‘Recycling’ or ‘reusing’ each spatial frequency by associating it with multiple distinct but widely separated temporal frequencies allows one to circumvent the proportionality between the spatial and temporal bandwidths of ST wave packets, which has been one of their permanent characteristics since their inception. We demonstrate experimentally that the propagation invariance, maximum propagation distance, and group velocity of ST wave packets are unaffected by spectral recycling. We also synthesize a ST wave packet with group velocity c/14.3 (c is the speed of light in vacuum) with a reasonable numerical aperture made possible by spectral recycling.

I. INTRODUCTION

Since Brittingham discovered in 1983 a propagation-invariant (diffraction-free and dispersion-free) pulsed optical beam [1], a host of other wave packets that share this feature have been identified [2][10]. Such pulsed beams have been recently dubbed generically ‘space-time’ (ST) wave packets, because their propagation invariance is undergirded by tight spectral associations between the spatial and temporal degrees of freedom [11–13]. Specifically, each spatial frequency underlying the spatial profile of a ST wave packet is assigned to a single temporal frequency (or wavelength) underlying its temporal profile [19][21]. Nevertheless, exploiting the unique characteristics predicted theoretically for such propagation-invariant wave packets (such as arbitrary group velocities in free space [22][29]) has been hampered in practice by the lack of a precise methodology for spatio-temporal wave-packet synthesis [9]. Recently, a new strategy based on spatio-temporal spectral-phase modulation [30][31] is now helping realize the full potential of ST wave packets. This technique has enabled demonstrations of arbitrary group velocities in free space [32][33], transparent dielectrics [34][35], planar waveguides [36], and plasmonic interfaces [37], accelerating wave packets [38], non-accelerating Airy wave packets [39], self-healing [40], novel ST Talbot effects [41], and even realizations using incoherent light [42][43].

Nevertheless, a fundamental constraint is imposed on ST wave packets because the tight spatio-temporal spectral correlations entail a proportionality between the spatial and temporal bandwidths. Such a proportionality is absent from traditional pulsed optical beams in which the spatial and temporal degrees of freedom are more-or-less independent; that is, the spatial beam profile and the temporal pulse linewidth can be manipulated independently. In general, ultrashort optical pulses need not require large numerical apertures. For ST wave packets, on the other hand, the proportionality between the spatial and temporal bandwidths entails that exploiting a large temporal bandwidth will exact a high price in terms of system design: a large spatial bandwidth – and thus high numerical aperture – is necessary. In general, the temporal bandwidth $\Delta \omega$ and the spatial bandwidth $\Delta k_x$ (in one dimension) are related through $\Delta \omega \propto (\Delta k_x)^2$ [11][39]. This proportionality is particularly exacerbating for ultralow ST wave packets (group velocity $\tilde{v} \ll c$, where $c$ is the speed of light in vacuum) [35]. Indeed, this bandwidth restriction has been a permanent feature of ST wave packets since their introduction into optical physics. Although progress has been made in exploiting larger temporal bandwidths by relying on lithographically inscribed phase plates [44][45], the limit on lithographic spatial resolution is nevertheless quickly reached with increased bandwidths. Consequently, this constraint sets upper limits for the practically utilizable light-source bandwidth and the minimally achievable group velocity in free space.

Here we show that the proportionality between spatial and temporal bandwidths that is intrinsic to all propagation-invariant ST wave packets can be circumvented without negative impact on any of their unique characteristics, thus paving the way to reducing the numerical aperture necessary for accommodating large temporal bandwidths and enabling the synthesis of ultralow pulses in free space. Our strategy relies on spectral ‘recycling’; that is, each spatial frequency is recycled (or reused) $N$ times in association with distinct but widely separated temporal frequencies, rather than used only once as is the case in typical ST wave packets. Consequently, the temporal spectrum can be increased – in principle, indefinitely – while retaining a fixed spatial bandwidth. In this approach, the full temporal bandwidth $\Delta \omega$ is divided into $N$ sub-bandwidths $\Delta \omega_n$, $n = 1, 2, \cdots, N$, each associated with a fixed spatial bandwidth that is reduced by a factor $\sim \sqrt{N}$ with respect...
II. THEORY OF SPECTRAL RECYCLING

We start our analysis from a traditional optical wave packet \( E(x, z; t) = e^{i(k_o x - \omega_o t)}\psi(x, z; t) \) and express its envelope \( \psi(x, z; t) \) in terms of an angular spectrum:

\[
\psi(x, z; t) = \int dk_x d\Omega \tilde{\psi}(k_x, \Omega) e^{i k_x x e^{i(k_o x - \omega_o t)} e^{-i\Omega t}}, \tag{1}
\]

where \( \tilde{\psi}(k_x, \Omega) \) is the spatio-temporal spectrum, \( \Omega = \omega - \omega_o \) is the temporal frequency with respect to the carrier frequency \( \omega_o \), \( k_0 = \frac{\omega_o}{c} \), \( k_x \) (the spatial frequency) is the transverse component and \( k_z \) is the longitudinal component of the wave vector along the transverse \( x \) and axial \( z \) coordinates, respectively. For simplicity, we consider the field to be uniform along \( y \) \((k_y = 0)\). The support domain of this pulsed beam is a two-dimensional patch on the surface of the light-cone \( k_x^2 + k_z^2 = (\frac{\omega}{c})^2 \) \([38, 31]\). In contrast, the spatio-temporal spectrum of a propagation-invariant ST wave packet [Fig. 1(a)] lies at the intersection of the light-cone with the spatial plane

\[
\Omega = (k_z - k_o) c \tan \theta, \tag{2}
\]

where the spectral tilt angle \( \theta \) is measured with respect to the \( k_z \)-axis \([30, 46]\); see Fig. 1(a)-i). Consequently, each spatial frequency \( k_x \) is associated with a single temporal frequency \( \Omega \), \( \tilde{\psi}(k_x, \Omega) \to \psi(k_x, \Omega) \delta(\Omega - \Omega(k_x)) \), where \( \Omega(k_x) \) is the conic section at the intersection of the spatial plane with the light-cone [Fig. 1(a)-ii]. The envelope of the ST wave packet takes the form:

\[
\psi(x, z; t) = \int dk_x \tilde{\psi}(k_x) e^{i k_x x e^{-i\Omega t}} e^{i(k_o x - \omega_o t)}, \tag{3}
\]

which corresponds to a propagation-invariant wave packet that propagates rigidly in free space at a group velocity \( \bar{v} = c \tan \theta \) \([22, 32]\). Therefore, tuning \( \theta \) changes \( \bar{v} \) in free space above or below \( c \) (without violating relativistic causality \([33, 40, 19]\)). According to this construction, higher \( \omega \) is associated with lower \( k_x \) when \( \theta < 45^\circ \), and with higher \( k_x \) otherwise. Because the spectral projection onto the \((k_z, \frac{\omega}{c})\)-plane in Fig. 1(a)-iii is a straight line, the wave packet propagates at a well-defined group velocity \( \bar{v} = c \tan \theta \) without group velocity dispersion (GVD) \([59]\).

Increased deviation of the spectral projection onto the \((k_z, \frac{\omega}{c})\)-plane away from the light-line \( k_z = \frac{\omega}{c} \) [Fig. 1(a)-iii] is associated with higher \( k_x \) [Fig. 1(a)-ii].

Finally, the spatio-temporal intensity profile \( I(x, z; t) \) at a fixed axial plane \( z \) is shown in Fig. 1(a)-iv, which exhibits a characteristic X-shape \([2, 33]\). The pulse width \( \Delta \tau \) at the beam center, defined as the width of \( I(0, z; t) \) [Fig. 1(a)-iv], is the inverse of the temporal bandwidth \( \Delta \omega \). The beam width \( \Delta x \) at the pulse center, defined as the width of \( I(z, 0; t) \), is the inverse of the spatial bandwidth \( \Delta k_x \).

It is useful to consider the narrowband paraxial limit \((\Delta k_x \ll k_o \text{ and } \Delta \omega \ll \omega_o)\), whereupon the spectral projection of the conic section onto the \((k_x, \frac{\omega}{c})\)-plane can be approximated by a parabola \( \frac{\Omega(k_x)}{\omega_o} = \frac{1}{1 - \cot \theta/2k_o^2} \). As such, the spatial and temporal bandwidths are related through

\[
\Delta \omega/\omega_o = \frac{1}{1 - \cot \theta/2k_o^2} \left( \frac{\Delta k_x}{k_o} \right)^2, \tag{4}
\]

i.e., \( \Delta \omega \propto (\Delta k_x)^2 \). The proportionality constant between \( \Delta \omega \) and \( (\Delta k_x)^2 \) depends sensitively on the spectral tilt angle \( \theta \). When \( \theta \to 45^\circ \) and \( \bar{v} \to c \), the proportionality constant is large, so that only a small spatial bandwidth \( \Delta k_x \) is required to accommodate a given temporal bandwidth \( \Delta \omega \). However, ultraslow ST wave packets \( \bar{v} \ll c \) require \( \theta \to 0^\circ \), in which case the proportionality constant is extremely small and a larger \( \Delta k_x \) is required for the same \( \Delta \omega \). For example, with respect to a ST wave packet having \( \theta = 44^\circ \) \((\bar{v} = 0.97c)\), reducing the spectral tilt angle to \( \theta = 4^\circ \) \((\bar{v} = 0.07c)\) requires increasing \( \Delta k_x \) by a factor of \( \approx 19 \) for the same \( \Delta \omega \).

We are now in a position to elucidate our proposed concept of spectral recycling. Spectral recycling aims to reduce the separation between the spectral projection of the ST wave packet onto the \((k_x, \frac{\omega}{c})\)-plane and the light-line, which reduces the maximum \( k_x \) and hence maintains a low numerical aperture despite increase in the temporal bandwidth. To reduce the spatial bandwidth \( \Delta k_x \) of the ST wave packet associated with a fixed temporal bandwidth \( \Delta \omega \), we ‘reuse’ the spatial frequencies, or ‘recycle’ them. That is, instead of each \( k_x \) being assigned to a unique spatial frequency \( \omega \), each \( k_x \) is associated with \( N \) distinct and widely separated temporal frequencies: \( \omega_1, \omega_2, \ldots, \omega_N \). Consider a non-recycled ST wave packet of temporal bandwidth \( \Delta \omega \) starting at \( \omega_o \) with an associated spatial bandwidth \( \Delta k_x \). Its spectrally recycled counterpart has the same total temporal bandwidth \( \Delta \omega \), but is segmented into \( N \) sub-bands \( \Delta \omega_n \) starting at distinct frequencies \( \omega_{o+n}, n = 1, 2, \ldots, N \), which are selected such that the associated spatial bandwidths \( \Delta k_x \) are all equal \( (\Delta k_x) = \Delta k_x' < \Delta k_x \). That is, the spatial bandwidth of the spectrally recycled ST
A ST wave packet is less than that of its non-recycled counterpart.

This concept is illustrated via two examples depicted in Fig. 1(a,b,c). In Fig. 1(b), we show a spectrally recycled ST wave packet with \( N = 2 \) having the same temporal bandwidth as that of the non-recycled ST wave packet in Fig. 1(a). The spatio-temporal spectrum results from the intersection of two spectral planes \( \omega - \omega_{o,n} = (k_z - k_{o,n})c \tan \theta \) with the light-cone [Fig. 1(b)-i], \( k_{o,n} = \omega_{o,n}/c \), \( n = 1, 2 \), such that the sub-bandwidths are staggered with no spectral gaps [Fig. 1(b)-ii]. The spatial spectra associated with these two sub-bandwidths \( k_{z} \) being now associated with two temporal frequencies \( \omega \) (one belonging to each sub-bandwidth) – rather than to one \( \omega \) as in the non-recycled ST wave packet. Consequently, the spatial bandwidth of the spectrally recycled ST wave packet \( \Delta k'_{z} \) is smaller than that of its non-recycled counterpart despite having the same temporal bandwidth \( \Delta \omega \). A further example illustrates the concept of spectral recycling for the case \( N = 4 \) as shown in Fig. 1(c). The sub-bandwidths \( \Delta \omega_{n} \) and the associated fixed spatial bandwidth \( \Delta x'_{k} \) are smaller that those for \( N = 1 \), and each \( k_z \) is recycled by associating it with 4 distinct temporal frequencies \( \omega \).

In general, holding the recycled spatial spectrum \( \Delta k'_{z} \) fixed necessitates selecting unequal sub-bandwidths \( \Delta \omega_{n} \) because \( \Delta \omega_{n} \) is inversely proportional to \( \omega_{o,n} \) for fixed \( \Delta k'_{z} \) as seen in Eq. 4. Consequently, the separations between the distinct temporal frequencies \( \omega_{1}, \omega_{2}, ..., \omega_{N} \) associated with a particular spatial frequency \( k_z \) are also unequal. Within the paraxial approximation used in Eq. 4 we have \( \omega_{n} - \omega_{n-1} = (\omega_{o,n} - \omega_{o,n-1})(1 - \Omega_{n-1}(k_z)/\omega_{o,n}) \). Only for small bandwidths is \( \omega_{n} - \omega_{n-1} = \omega_{o,n} - \omega_{o,n-1} \). Indeed, in the narrow bandwidth limit \( \Delta \omega \ll \omega_{o} \), the sub-bandwidths are approximately equal \( \Delta \omega_{n} \approx \frac{\Delta \omega}{N} \), and \( \Delta k'_{z} \approx \frac{\Delta k_{z}}{\sqrt{N}} \), \( \omega_{o,n} \approx \omega_{o} \pm (n - 1)\frac{\Delta \omega}{N} \). The negative sign corresponds to \( \theta < 45^\circ \), whereupon \( \omega_{o,N} = \omega_{o} \), and the positive sign used otherwise, where-
Upon \( \omega_{0,1} = \omega_{0} \); examples of the latter case are depicted in Fig. 1(b,c).

It is clear from comparing Fig. 1(a)-iv with Fig. 1(c)-iv that spectral recycling results in added structure to the spatio-temporal intensity profile. To gain insight into these changes, we write the spectrally recycled ST wave packet as

\[
E_{\text{rec}}(x, z; t) = \sum_{n=1}^{N} e^{i(k_{0,n} z - \omega_{0,n} t)} \psi_n(x, z; t),
\]

where \( \psi_n(x, z; t) \) is the envelope associated with the \( n \)th sub-bandwidth,

\[
\psi_n(x, z; t) = \int_{\Delta \omega_n} dk_x \tilde{\psi}_n(k_x) e^{ik_x x} e^{-i(\omega(k_x) - \omega_{0,n})(t - z/\tilde{v})},
\]

Note that \( \psi_n(x, z; t) = \psi_n(x, 0; t - z/\tilde{v}) \), so that the envelope associated with each sub-bandwidth also propagates rigidly at the same group velocity \( \tilde{v} = c \tan \theta \) as its non-recycled counterpart associated with the full temporal bandwidth \( \Delta \omega \).

A particularly simple case is that of a narrowband wave packet whereupon \( \Delta \omega_n \approx \frac{\lambda}{\Delta \lambda} \), and the associated envelopes \( \psi_n(x, z; t) \) are thus almost identical, \( \psi_n(x, z; t) \approx \psi_1(x, z; y) \) for all \( n \), in which case:

\[
I_{\text{rec}}(x, z; t) \propto I_1(x, z; t) I_1(z - ct),
\]

where \( I_1(x, z; t) = |\psi_1(x, z; t)|^2 \) and the form factor \( I_1(z - ct) \) is the familiar term resulting from a phasor sum,

\[
I_1(z - ct) = \frac{\sin^2 \left( \frac{\Delta \omega}{2} N \left( \frac{\tilde{v}}{c} - 1 \right) \right)}{\sin^2 \left( \frac{\Delta \omega}{2} \left( \frac{\tilde{v}}{c} - 1 \right) \right)}.
\]

The spectrally recycled ST wave packet is therefore the product of two envelopes: the first is the ST wave packet envelope \( I_1(x, z; t) \) that corresponds to a single sub-bandwidth and the second is the form factor \( I_1(z - ct) \) that is independent of \( x \). Temporal walk-off occurs between \( I_1(x, z; t) \) that travels at \( \tilde{v} \) and \( I_1(z - ct) \) that travels at \( c \).

To elucidate the structure of spectrally recycled ST wave packets, we plot first in Fig. 2(a) the spatio-temporal profiles of a non-recycled ST wave packet \( I(x, z; t) \) of temporal bandwidth \( \Delta \omega \), and the envelope \( I_1(x, z; t) \) for a single sub-bandwidth \( \Delta \omega_1 \approx \frac{\lambda}{4N} \) in Fig. 2(b), whose pulsewidths at the beam center are \( \Delta \tau_1 \sim \frac{\Delta \omega_1}{4} \) and \( \Delta \tau = \frac{\pi}{\Delta \omega_1} = N \Delta \tau_1 \), respectively. Similarly, the beam widths at the pulse center are \( \Delta x_1 \sim \frac{2L}{\Delta \omega_1} \) and \( \Delta x \sim \frac{2L}{\Delta \omega} \sim \sqrt{N} \Delta x_1 < \Delta x \). This is expected because of the smaller spatial and temporal bandwidths associated with \( I_1(x, z; t) \) in comparison to those associated with \( I(x, z; t) \).

Including the \( N \) sub-bandwidths in the spectrally recycled ST wave packet results in multiplication of \( I_1(x, z; t) \) by the form factor \( I_1(z - ct) \) plotted in Fig. 2(c), resulting in the spatio-temporal intensity profile \( I_{\text{rec}}(x, z; t) \) shown in Fig. 2(d), where the periodic structure of \( I_1(z - ct) \) modulates \( I_{\text{rec}}(x, z; t) \). The pulsewidth at the beam center of the spectrally recycled wave packet is \( \Delta \tau_{\text{rec}} \sim \frac{4 \pi}{N \Delta \omega} = \Delta \tau \), similar to that of the non-recycled ST wave packet \( I(x, z; t) \). Nevertheless, because \( I_1(z - ct) \) is independent of \( x \), the beam width at the pulse center remains larger than that for \( I(x, z; t) \), \( \Delta x_{\text{rec}} = \Delta x_1 < \Delta x \); indeed, \( \Delta x_1 \sim \sqrt{N} \Delta x \). We have thus retained the narrow temporal width associated with the full temporal bandwidth, while increasing the beam transverse width and thus reducing the associated numerical aperture. Increasing \( N \) increases the separation \( T = N \frac{\Delta \omega}{2} \) between the discrete features in the spatio-temporal profile, while decreasing the width of these features.

We can draw several conclusions from this theoretical analysis. First, spectral recycling does not affect the critical properties of a ST wave packet, including its propagation invariance and its group velocity \( \tilde{v} \). Second, the temporal linewidth of the pulse at the beam center of the spectrally recycled ST wave packet is similar to that of its non-recycled counterpart, while maintaining a larger
beam width at the pulse center. This indicates that the proportionality between the spatial and temporal bandwidths has been modified without changing the spectral tilt angle $\theta$. We now proceed to verify experimentally the concept of spectrally recycling ST wave packets and to validate these theoretical predictions concerning their properties.

III. EXPERIMENTAL CONFIGURATION

The setup utilized for our proof-of-principle demonstration of spectral recycling is similar to that used previously for synthesizing ST wave packets [30, 33, 46]. Beginning with $\sim 100$-fs pulses from a mode-locked Ti:sapphire laser (Tsunami; Spectra Physics) of central wavelength $\sim 800$ nm, the pulse spectrum is spread via a diffraction grating and collimated by a cylindrical lens before impinging on a two-dimensional, reflective, phase-only spatial light modulator (SLM; Hamamatsu X10468-02). The SLM imparts a phase distribution to the wave front designed to associate each wavelength $\lambda$ with a prescribed spatial frequency $k_x$ in order to satisfy the constraint in Eq. 2. The retro-reflected field is reconstituted into a ST wave packet at the diffraction grating. We record the spatio-temporal spectrum $|\tilde{\psi}(k_x, \lambda)|^2$, the spatio-temporal profile of the ST wave packets $I(x, z; \tau)$ at different axial planes $z$, which allows us to assess the group velocity $\tilde{v}$, in addition to the axial evolution of the time-averaged intensity $I(x, z)$.

We consider three ST wave packets having spectral tilt angles $\theta = 30^\circ$ (subluminal $\tilde{v} = 0.58c$), $\theta = 70^\circ$ (superluminal, $\tilde{v} = 2.75c$) and $\theta = 110^\circ$ (negative-$\tilde{v}, \tilde{v} = -2.75c$). In each case we carry out the measurements for different extents of spectral recycling: $N = 1$ (non-recycled, $\Delta\lambda = 2$ nm), $N = 2$ ($\Delta\lambda_n \approx 1$ nm, $n = 1, 2$), and $N = 4$ ($\Delta\lambda_n \approx 0.5$ nm, $n = 1, 2, 3, 4$).

IV. MEASUREMENTS RESULTS

The spatio-temporal spectra for all 9 ST wave packets are plotted in Fig. 3. In each case, we observe that the measured spatio-temporal spectrum $|\tilde{\psi}(k_x, \lambda)|^2$ is divided into $N$ segments all centered at $k_x = 0$. Note the switch in sign of the curvature of the spectral projections onto the $(k_x, \frac{\lambda}{c})$-plane from the subluminal case ($\theta = 30^\circ$) to the superluminal cases ($\theta = 70^\circ$ and $110^\circ$). The impact of spectral recycling is particularly clear when examining the spectral projection onto the $(k_z, \frac{\omega}{c})$-plane, as shown in Fig. 4. Here we see clearly that the spectral projection for the non-recycled ST wave packet extends further away from the light-line (corresponding to larger values of associated $k_x$) than in its spectrally recycled counterparts. Spectral recycling divides this projection into $N$ sections, and the segments are shifted accordingly along $k_x$ such that the sub-bandwidths are staggered, leaving no spectral gaps. The maximum deviation away from the light-line is reduced as $N$ increases, indicating a reduction in the spatial bandwidth and thus also the numerical aperture. We plot the spatio-temporal profiles $I(x, z; \tau)$ at fixed $z$ for all these cases in Fig. 3. The non-recycled ST
The theoretical curve, indicating that spectral recycling does not affect \( \bar{v} \).

We finally demonstrate that circumventing the usual proportionality between the spatial and temporal bandwidths, \( \Delta k_x \) and \( \Delta \omega \), respectively, through spectral recycling, facilitates for the synthesis of ultraslow ST wave packets. Previously, the slowest subluminal ST we prepared corresponded to \( \theta = 10^\circ \). Here, making use of a temporal bandwidth of \( \Delta \lambda \approx 2 \text{ nm} \) \( \approx c/14.3 \). For a spatial bandwidth \( \Delta k_x \approx 0.26 k_o \), which is prohibitively difficult to produce. Instead, through spectral recycling with \( N = 16 \), the spatial bandwidth required is reduced to \( \Delta k_x \approx 0.064 k_o \), which is that corresponding to a non-recycled ST wave packet having \( \bar{v} = c \) and the same full temporal bandwidth \( \Delta \lambda \approx 2 \text{ nm} \). In Fig. 3(a) we plot the measured spectral projection onto the \((k_x, \lambda)\) planes, which show clearly the spectrally recycled structure of the spatio-temporal spectrum. We plot in Fig. 3(b) the spatio-temporal intensity profile \( I_{rec}(x, z; t) \).

V. DISCUSSION

A final characteristic of the spectrally recycled ST wave packets to consider is their time-averaged intensity \( I(x, z) = \int dt I(x, z; t) \), that is captured by a slow detector such as a CCD camera. Because the form-factor \( I_s(z - ct) \) is independent of \( x \), the time-averaged intensity of the spectrally recycled ST wave packet \( I_{rec}(x, z) \) is diffraction-free with width \( \Delta x_{rec} \) (corresponding to the spatial bandwidth \( \Delta k_x^{rec} \)) of a single sub-bandwidth.

The question remains whether spectral recycling affects the maximum propagation distance \( L_{max} \) of a ST wave packet, defined as the propagation distance after

\[ \Delta = 4 \frac{\omega}{c}. \]
VI. CONCLUSIONS

In conclusion, we have proposed and demonstrated a technique we have called ‘spectral recycling’ that helps circumvent the proportionality between spatial and temporal bandwidths intrinsic to ST wave packets. In contrast to traditional pulsed optical beams where the spatial and temporal degrees of freedom are more-or-less independent, each spatial frequency \( k_x \) in a ST wave packet is typically associated with a single temporal frequency \( \omega \) (or wavelength). The spectral recycling strategy ‘reuses’ or ‘recycles’ each spatial frequency \( k_x \) by associating it with \( N \) distinct and well-separated temporal frequencies \( \omega_1, \omega_2, \ldots, \omega_N \) to maintain a fixed spatial bandwidth in-
dependently of an increasing temporal bandwidth. We confirmed experimentally that spectral recycling even up to $N = 16$ does not affect the unique characteristics of a ST wave packet: its propagation invariance, its group velocity $\bar{v}$, and its maximum propagation distance $L_{\text{max}}$ all remain intact. This strategy facilitates the synthesis of ultraslow ST wave packets in free space without exorbitant increase in the required numerical aperture. Indeed, we demonstrated here $\bar{v} = c/14.3$ with the same numerical aperture associated with $\bar{v} = c/2$. It is expected that spectral recycling will enable the utilization of broadband optical sources in the synthesis of ST wave packets, while maintaining a low numerical aperture. This will be particularly useful in application in which a broadband ST wave packet is required, such as in omni-resonant coupling to planar cavities, nonlinear optics, and laser-plasma interactions. Finally, it would also be interesting to examine whether spectral recycling can benefit another recently developed class of optical wave packets having controllable group velocity known as a ‘flying focus’.

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