The optimization of the lateral motion control system of an unmanned aerial vehicle

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Abstract. The article is devoted to the possibility of constructing an aircraft lateral motion control system only using the rudder; a mathematical model of such a system is obtained based on the structuring of an unmanned aerial vehicle model. The design of optimal lateral motion control of a UAV is carried out using the linear-quadratic optimization method in the complex domain for single-control systems. As an optimization criterion, a quadratic criterion in the form of restrictions on the control energy is used. Examples of system modeling are given.

1. Introduction

The unmanned aerial vehicle (UAV) in landing mode, especially in difficult conditions (above the sea surface), is very sensitive to control inputs, so special attention is paid to the modeling and design of its control systems. In this case, the task of automatic control of the lateral movement of UAV is very important [1-3]. The linear and non-linear mathematics models of UAV are used in the time and the complex domain, [4]. The deep mutual relations between the channels of the onboard control systems that affect the flight of the UAV lead to the complication of mathematical models that describe the dynamic processes of the UAV movement. In the synthesis of UAV (the aircraft type) control laws, the spatial motion of UAV is often separated using various approaches [5, 6]. A very clear representation of the system is in the Laplace domain, and therefore, its representation in the form of a structural diagram.

The robust and adaptive approaches to the construction of UAV control [7, 8] in particular, lateral movement, as well as modal control methods [6] are widely known. It is of considerable interest in the development of UAV control algorithms, including lateral movement, obtaining regulators that implement the optimal laws, which is carried out using linear-quadratic optimization (LQO) methods both in the time and in the complex domain. This task does not lose its relevance, especially concerning UAV, given its high maneuverability and high-speed characteristics.

2. The mathematical model of the lateral movement of the UAV

We consider a linearized model of the lateral movement of the UAV in the form:

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t)$$  \hspace{1cm} (1)

where \( x = (v, \omega_y, \omega_z, \gamma, \psi, z)^T \); \( v = -V_z = -V \sin \beta = -V \beta \) – side glide speed; \( u(t) \) – the control vector; \( u = (\delta_a, \delta_n)^T \), \( \delta_a \) – the aileron deflection angle; \( \delta_n \) – the rudder angle; \( \gamma \) – the roll angle; \( \beta \) – the glide.
angle; \( \psi \) – the yaw angle; \( \omega_x \) – the angular velocity of rotation around the longitudinal axis (the roll angular velocity); \( \omega_y \) – the horizontal angular rotation speed (the yaw rate); \( V \) – the UAV speed.

When structuring the mathematical model, the factors of the second order of smallness are discarded, i.e. is accepted:

\[
0 = a_{11} = a_{22} = a_{33} = b_{12} = b_{21} = b_{32} = 0.
\]

Applying the Laplace transform to simplified equations, we obtain:

\[
\begin{align*}
(s - a_{11})\nu(s) & = a_{12}\omega_x(s) + a_{14}\gamma(s), \\
(s - a_{22})\omega_y(s) & = a_{21}\nu(s) + b_{22}\delta_H(s), \\
(s - a_{33})\omega_z(s) & = a_{31}\nu(s) + b_{31}\delta_H(s), \\
ss\gamma(s) & = \omega_x(s),/ss\psi(s) = \omega_y(s), ss\zeta(s) = V(s) + a_{63}\psi(s).
\end{align*}
\]

Adding the relationships of control laws that reflect the idea of the nested tracking loops,

\[
\begin{align*}
\delta_j(s) & = W_{p_j}\nu\gamma_j(s), \quad \varepsilon_j(s) = \gamma_j(s) - \gamma(s), \quad \gamma_j(s) = W_{p_j}\gamma_j(s)\varepsilon_j(s), \\
\delta_n(s) & = W_{p_\psi}\nu\psi_j(s), \quad \varepsilon_\psi(s) = \psi_j(s) - \psi(s), \quad \psi_j(s) = W_{p_\psi}\psi_j(s)\varepsilon_\psi(s).
\end{align*}
\]

we get the system model in the form of the following structural diagram:

In the case of small values of \( a_{11} \) and \( a_{31} \), we accept \( a_{11} = 0, a_{31} = 0 \). Then \( \lambda(s) \equiv 0 \) and the influence of yaw on the roll is excluded, and the lateral coordinate can be controlled only with the help of the roll. The yaw loop performs the functions of a stabilization loop (maintaining \( \psi = 0 \), i.e., suppressing disturbances in angular motion).

In the general case, the exclusion of the mutual influence of the circuits can be achieved by introducing the compensating relationships in the laws of control

\[
\begin{align*}
\delta_j(s) & = W_{p_j}\nu\gamma_j(s) + \left[-s\lambda(s)\psi(s)\right], \quad \delta_n(s) = W_{p_\psi}\nu\psi_j(s) + \left[-\mu(s)\gamma(s)\right].
\end{align*}
\]

In the case when \( a_{11} \neq 0 \), the lateral force has a significant effect, so it is possible to consider the control of the lateral coordinate using the yaw control. Then the roll loop from the lateral displacement control can be excluded and perform only the functions of the roll stabilization loop (the process of maintaining \( \gamma = 0 \)).

3. The statement of the problem and the basic theoretical principles

We will consider the problem of synthesis of regulators, i.e. transfer functions \( W_{p_\nu}(s) \) indicated in Figure 1, by the method of linear-quadratic optimization (LQO). After eliminating the mutual influence of the contours (due to the accepted assumptions or due to the introduction of compensating
relationships), it is possible to use the methods of one-dimensional optimization for each channel in two stages - optimization of the internal and then the external circuits.

For a one-dimensional system (Figure 2) there are the relations

$$\Phi(s) = \frac{W_p(s)W_o(s)}{1+W_p(s)W_o(s)}; \quad W_p(s) = \Phi(s)\frac{W_o(s)(1-\Phi)}{W_o(s)(1-\Phi)}$$

Figure 2. The structural diagram of a one-dimensional system

We introduce the optimality criteria:

$$I_{21} = \int_0^\infty \varepsilon^2(t)dt \to \min; \quad I_{22} = \int_0^\infty \delta^2(t)dt \leq C, \quad C > 0 \tag{7}$$

where the parameter C determines the restrictions on the energy of the control.

Given the relation (6), the transfer function of the controller can be determined by the transfer function of the closed system $\Phi(s)$, which is optimal in the sense of criterion (7) and belongs to the class of fractional rational functions with the left poles. We assume that the input action also belongs to this class.

Following figure 2, we write the expression for the error and deviation of the control in the complex area:

$$\varepsilon(s) = [1-\Phi(s)]u(s); \quad \delta(s) = \frac{\Phi(s)}{W_o(s)}u(s). \tag{8}$$

Let a stable system be considered and $\varepsilon(\infty) = 0$.

Following (8), one can write the following expressions for the integral quality criteria:

$$I_{21}\{\Phi\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[ 1 - \Phi(s) \right] \left[ 1 - \Phi(-s) \right] u(s)u(-s)ds; \tag{9}$$

$$I_{22}\{\Phi\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\Phi(s)\Phi(-s)}{W_o(s)W_o(-s)} u(s)u(-s)ds. \tag{10}$$

To find the optimal transfer function $\Phi(s)$, we use the following known position. Let $\mathfrak{J}$ means the class of fractional rational functions with left poles. Let:

1. $\Phi(s, \lambda)$ - the solution to the problem of unconditional extremum:

$$\Phi(s, \lambda) = \arg \min_{\Phi \in \mathfrak{J}} I_{21}\{\Phi\} + \lambda I_{22}\{\Phi\}; \quad \lambda \in R^+.$$  

2. $\forall \hat{C} \leq C \quad (\hat{C} > 0) \quad \exists \lambda : I_{22}\{\Phi(s, \lambda)\} \leq \hat{C}.$
3. The area of valid parameter values is determined \( \lambda \in R^+: I_{21}\{\Phi(s, \lambda)\} \leq C \) and 
\[ \lambda_0 = \arg \min_{\lambda \in \Lambda} [I_{21}\{\Phi(s, \lambda)\}] \] 

Then \( \Phi(s) = \Phi(s, \lambda_0) \) – the optimum transfer function.

Let \( R(s^2) = 1 + \frac{\lambda}{W_0(s)W_0(-s)} \) and \( R(s) = Y^+(s)Y^-(s) \), where \( Y^+(s), Y^-(s) \) – the rational functions, with zeros and poles in the left and right half-planes, respectively.

Then, to solve the problem of unconditional extremum \( \Phi(s, \lambda) \), we have

\[ Y^+(s)u(s)\Phi(s, \lambda) = \begin{bmatrix} \frac{u(s)}{Y^-(s)} \end{bmatrix} ; u(s) \in \mathbb{S}, \]

where the "+" sign means the separation result, i.e. separation of the fractional rational function with the left poles.

The factorization \( R(s^2) = \frac{A(s^2)}{B(s^2)} \) is performed to obtain \( Y^+(s) \) and \( Y^-(s) \) in the general case, by calculating the roots of the polynomials \( A(x), B(x), x = s^2 \) with subsequent representations

\[ A(x) = a_n(x - x_1^a) \cdots (x - x_n^a) ; B(x) = b_m(x - x_1^b) \cdots (x - x_m^b), \quad (11) \]

\[ x - x_i = (s^2 - x_i) = (s + \sqrt{x_i})(s - \sqrt{x_i}), \quad \text{Re} \sqrt{x_i} > 0. \quad (12) \]

The separation of a fractional rational function is performed using its decomposition into simple fractions.

The solution to the whole problem, namely, the optimal transfer function of a closed system will be the function \( \Phi(s) = \Phi(s, \lambda_0) \). To determine it based on the well-known methods for calculating the integrals

\[ I_2 = \frac{1}{2\pi} \int_{-j\infty}^{+j\infty} \frac{c(s^2)}{q(s)q(-s)} ds \quad (13) \]

the construction of dependencies of optimality criteria and \( I_{21}(\lambda), I_{22}(\lambda) \).

4. The simulation results

For the UAV model in landing configuration (indicated in dimensionless quantities)

\[
A = \begin{bmatrix}
-0.089 & -2.21 & 0.328 & 0.319 & 0 & 0 \\
0.076 & -0.217 & -0.166 & 0 & 0 & 0 \\
-0.602 & 0.327 & -0.975 & 0 & 0 & 0 \\
0 & 0.15 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 2.21 & 0
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
0 & -0.0327 \\
0.0264 & 0.151 \\
0.227 & -0.0636 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\]

Correspondences \( I_{21}(\lambda) \) and \( I_{22}(\lambda) \) with a single step effect are of the form:
Given the value of the restriction parameter $C$, the dependence $I_{22}(\lambda)$ determines the value of $\lambda_0$, the left boundary value of the set $\Lambda$.

The procedure for the synthesis of optimal contours at given levels of stepwise influences and the constraints of the integral quadratic deviations of the controls includes, in the general case, the calculation of the roots of polynomials, factorization and separation of rational functions, the calculation of integral quadratic functional for different $\lambda \in R$, the calculation of the parameters of the optimal transfer functions and ends with modeling of optimal processes. As a result, tables of form 1, 2 can be calculated (the data given are calculated at inputs $l(t)$).

**Table 1.** The maximum value of the deviation of the ailerons and the angle of heel at various restriction parameters and $C_\delta$ и $C_\gamma$

| $C_\gamma$ (deg) | $C_\delta$ | $C_\delta$ | $C_\gamma$ (deg) | $C_\delta$ | $C_\gamma$ (deg) |
|-----------------|----------|----------|-----------------|----------|-----------------|
| $\delta_{\max}$ | $\gamma_{\max}$ | $\delta_{\max}$ | $\gamma_{\max}$ | $\delta_{\max}$ | $\gamma_{\max}$ |
| 3               | 0.6      | 0.8      | 1.26            | 1.9      | 3               | 0.09      | 0.11      | 0.15      | 0.19      |
| 6               | 1.1      | 1.68     | 2.6             | 4        | 6               | 0.12      | 0.17      | 0.23      | 0.27      |
| 9               | 1.6      | 2.4      | 3.85            | 5.95     | 9               | 0.15      | 0.2       | 0.26      | 0.35      |
| 12              | 2.1      | 3.2      | 5               | 7.8      | 12              | 0.17      | 0.22      | 0.3       | 0.4       |

**Figure 3.** Dependencies $I_{21}(\lambda)$ and $I_{22}(\lambda)$ for the roll control

**Figure 4.** Dependencies $I_{21}(\lambda)$ and $I_{22}(\lambda)$ for the yaw control
Table 2. The maximum value of the deviation of the rudder and yaw angle at various parameters of restriction and $C_\delta$ и $C_{\psi^*}$.

| $C_{\psi^*}$ | 15 | 20 | 30 | 40 |
|--------------|----|----|----|----|
| $C_\delta$   | 5  | 0.64| 0.77| 1  | 1.19|
|              | 10 | 1.7 | 2   | 2.6| 3.1 |
|              | 15 | 3.9 | 4.7 | 6  | 7.2 |
|              | 20 | 5.5 | 6.6 | 8.6| 10  |

| $\psi_{\max}$ (deg) |
|---------------------|
| $\psi_{\max}$ (deg) |
| 15                  |
| 20                  |
| 30                  |
| 40                  |

The totality of such tables for different levels of exposure allows you to choose the parameters of the optimal controllers for the specified permissible deviations of the controls.

5. Conclusion
An approach to developing an optimal UAV lateral motion control system using only the rudder has been proposed in the paper. A mathematical model of such a system is obtained based on the structuring the initial UAV model. A linear-quadratic optimization method in the complex domain for systems with single control input was used as the optimization method. After eliminating the mutual influence of the yaw and roll control loops, it is possible to use one-dimensional optimization methods for each loop. The results indicate that optimization of the aircraft lateral motion control system using the rudder method using the LQO method of the integrated area for systems with one control body gives quite acceptable results and confirms the possibility of building a control system of the UAV lateral movement with separate control channels.

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