Color-singlet and color-octet \( J/\psi \) production in top quark rare decays

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Abstract

\( J/\psi \) production in top quark rare decays is investigated under the framework of NRQCD factorization formalism. Various production channels are studied, and we find that the contributions from the color-singlet quark fragmentation and the color-octet gluon fragmentation are both over 3 orders larger than that from the leading order color-singlet process. The numerical results show that the branching ratio \( B(t \to c + J/\psi + X) \) is about \( 10^{-14} \) in the SM, and \( 10^{-10} \) in the MSSM.

PACS number(s): 14.65.Ha, 14.40.Gx
Top quark has been discovered by CDF and D0 collaborations at the Fermilab Tevatron \[^{[1]}\] and its measured mass is around 175 GeV. Because it has a mass of the order of the Fermi scale, the top quark may be strongly associated with the electroweak symmetry breaking (ESB) sector. So, the studies of top quark (including the production cross section and the decay widths etc.) may put forward our knowledge about the ESB mechanism. Furthermore, the fact that both $R_b$ and $A_b$ remain approximately $2\sigma$ away from the SM expectations \[^{[2]}\] gives us some hints of the probability that the third generation might couple to some new physics (may be associated with the ESB mechanism). All these distinguishing features about the top quark and the third generation require us to measure the top quark properties precisely.

The flavor changing neutral current (FCNC) rare decays of heavy quarks have long been a subject of intense theoretical and experimental study. The top quark rare decays have also been investigated in literatures \[^{[3-5]}\]. Because in the SM the branching ratios of the rare decay modes are shown to be very small to be observable, $(B(t \to c\gamma) \sim 10^{-12}, B(t \to cZ) \sim 10^{-12} - 10^{-13}$ and $B(t \to cg) \sim 10^{-10})$. The FCNC decay modes make an excellent probe for models beyond the SM. The Two-Higgs-Doublet models give an enhancement to these decay rates by $3 \sim 4$ orders of magnitude \[^{[3]}\]. The minimal supersymmetric SM (MSSM) also gives the same orders enhancement \[^{[4]}\]. Recently, some studies of searching for top quark rare decays at hadron colliders have also been performed \[^{[5]}\].

In searching for rare decay $t \to cg$, a $c$ quark jet and a gluon jet must be identified, which makes it more difficult than the other two modes because of the larger background in the hadron collisions. Otherwise, if the $c$ quark or the gluon hadronizes into heavy quarkonium, searching for heavy quarkonium in top quark rare decays would further identify the top quark effective coupling vertex $tcg$. For this purpose, in this paper we calculate the $J/\psi$ production rates in top quark rare decays. $J/\psi$ has an extremely clean signature through its leptonic decays into $e^+e^-$ or $\mu^+\mu^-$, and this makes it very valuable as trigger for top quark rare decays at high energy hadron colliders. Furthermore, on the $J/\psi$ production sector, top quark rare decays provide another facility to study different production mechanisms. There are $c$ quark and gluon in the decay $t \to cg$, so we can study the $J/\psi$ production via quark fragmentation and gluon fragmentation in top quark rare decays.

In the past few years, a new factorization formalism of heavy quarkonium production in
high energy collisions has been developed by Bodwin, Braaten and Lepage in the context of NRQCD \cite{Bodwin-et-al}. In this approach, the production process is factorized into short and long distance parts, while the latter is associated with the nonperturbative matrix elements of four-fermion operators. Addition to the conventional color-singlet mechanism of heavy quarkonium production, this factorization formalism provides a new mechanism called color-octet mechanism, in which the heavy-quark and antiquark pair is produced at short distance in a color-octet configuration and subsequently evolves nonperturbatively into physical quarkonium state. After including this new production mechanism, one might explain the \( \psi' \) (J/\( \psi \)) surplus measured by the CDF group at the Tevatron \cite{CDF}. In the past few years, the applications of the NRQCD factorization formalism to J/\( \psi \)(\( \psi' \)) production at various experimental facilities have been performed \cite{applications}.

In top quark decays, J/\( \psi \) mainly comes from \( t \to W^+ b \) followed by \( b \) decays \( b \to J/\psi X \) as well as from direct production \( t \to bW^+ g^* \) followed by \( g^* \to J/\psi + X \) \cite{top_quark}. However, the J/\( \psi \) from \( b \) decays can be distinguished by using the so-called Second Vertex Detector, \( b \)-tagging, and the directly produced J/\( \psi \) can be isolated by studying its energy spectrum because it has a very soft energy spectrum \cite{CMS}. As shown in the following calculations, the J/\( \psi \) from top quark rare decays mostly come from parton fragmentation processes in which they have hard energy spectrum. So, in top quark rare decays, J/\( \psi \) is easily detected and distinguished, and it therefore can be used to observe the top quark rare decay mode \( t \to cg \).

According to the NRQCD factorization formalism, the inclusive production of J/\( \psi \) in top quark rare decays has the following factorized form,

\[
\Gamma(t \to c + J/\psi + X) = \sum_n \hat{\Gamma}(t \to c + (c\bar{c})[n] + X) \langle O_n^{J/\psi} \rangle. \tag{1}
\]

Here, \( \hat{\Gamma}_n \) is the short-distance coefficient for producing a \( c\bar{c} \) pair in a configuration denoted by \( n \) (including the angular momentum \( 2S+1L_J \) and the color index 1 or 8). \( \langle O_n^{J/\psi} \rangle \) is the long-distance nonperturbative matrix element demonstrating the probability of the \( c\bar{c} \) pair in \( n \) configuration evolving into a physical charmonium state J/\( \psi \). \( \hat{\Gamma}_n \) can be calculated perturbatively as an expansion in coupling constant \( \alpha_s \). Whereas, \( \langle O_n^{J/\psi} \rangle \) is a nonperturbative parameter, and practically is determined by fitting to the experimental data. The matrix element \( \langle O_n^{J/\psi} \rangle \) can be a color-singlet matrix element or a color-octet matrix element due to the color index number of the \( c\bar{c} \) pair being 1 or 8. The color-singlet matrix elements
are related to the nonrelativistic radial wave function or its derivatives at the origin, but
the color-octet ones have not the corresponding relations and can only be extracted from
experiment now. For $J/\psi$ production, fortunately the element $\langle O_{8}^{J/\psi}(3S_{1}) \rangle$ has been well
determined from large $p_T$ prompt $J/\psi$ production at the Tevatron [4].

The leading order color-singlet contributions to $J/\psi$ production in top quark rare decays
come from the process displayed in Fig.1(a). In the Feynman diagrams, the tcg vertex
(represented by a black fat dot in Fig.1 and Fig.2) is an effective vertex. In the SM, this
effective vertex can be calculated by loop calculations and the final result has the following
form [3],

$$M_i^\mu = V_{ci}V_{ti}^*(a_1p^\mu + a_2q_2^\mu + a_3\gamma^\mu)L + (b_1p^\mu + b_2q_2^\mu + b_3\gamma^\mu)R,$$

for an internal quark $i$. Here $V_{ij}$ is KM matrix element, $L = (1 - \gamma_5)/2$, and $R = (1 + \gamma_5)/2$.
$q_2$, $q_1$, and $p$ are moment of the top quark, $c$ quark and gluon associated with the tcg vertex
respectively. The explicit expressions of the six form factors $a_i$ and $b_i$ can be found in Ref.
[3]. Adopting this effective tcg vertex in the SM, the decay rates for the leading order color-
singlet process can be calculated straightforward. The result is lengthy, and it is too tedious
to write it here. If we take the limit $m_c \ll m_t$, we will get

$$\Gamma^{(LO)}(t \to c + J/\psi) = \frac{2\alpha_s}{243} \frac{\langle O_{1}^{J/\psi}(3S_{1}) \rangle}{m_tm_c} (a_3 b_3^* + b_3^* a_3).$$

In the numerical calculations, we take the input parameters as [3]

$$m_t = 175 GeV, \quad M_Z = 91.2 GeV, \quad M_W = 80.10 GeV,$$

$$\alpha_{em} = 1/128.8, \quad \alpha_s(m_t) = 0.10, \quad \sin^2\theta_w = 0.23,$$

$$m_u = 10 MeV, \quad m_s = 150 MeV, \quad m_c = 1.5 GeV, \quad m_b = 5.0 Gev.$$

So, the branching ratio for this process is

$$B^{(LO)}(t \to c + J/\psi) = \frac{\Gamma^{(LO)}(t \to c + J/\psi)}{\Gamma(t \to bW^+)} = 5.4 \times 10^{-19},$$

where, the color-singlet matrix element value $\langle O_{1}^{J/\psi}(3S_{1}) \rangle = 1.2 GeV^3$ [10] has been used.

This result is dramatic small, and it is about 8 orders smaller than the branching ratio of
the inclusive top quark rare decay $t \to cg$ which is about $10^{-10}$ [3]. This is mainly because the gluon propagator has a suppression factor of order $O(1/m_t^2)$ due to the large momentum transfer in this process.

Actually, the dominant color-singlet contributions come from the so-called fragmentation processes shown in Fig.1(b) and Fig.1(c) for quark and gluon fragmentation respectively. In Fig.1(b), on-shelled $c$ quark (gluon in Fig.1(c)) is produced from top quark rare decays and then fragmentating into the $J/\psi$. The contributions from the fragmentation processes mostly come from the region of phase space in which the $\psi - c$ ($\psi - gg$ in Fig.1(c)) system has large momentum of order $m_t$ and small invariant mass of order $m_c^2$ [10] [11]. In the fragmentation approximation, the decay rates are given by, for the quark fragmentation [11],

$$
\Gamma(t \to J/\psi cg) = \Gamma(t \to gc) \times P_{c \to J/\psi} \\
= \Gamma(t \to gc) \times \frac{16\alpha_s^2(2m_c)}{243} \frac{\left< O_1^{J/\psi}(3S_1) \right>}{m_c^3} \left( \frac{1189}{30} - 57\ln2 \right)
$$

$$
= \Gamma(t \to gc) \times 1.2 \times 10^{-4}, \quad (8)
$$

and for gluon fragmentation [11],

$$
\Gamma(t \to J/\psi cgg) = \Gamma(t \to cg) \times P_{g \to J/\psi}^{(1)} \\
= \Gamma(t \to cg) \times 8.28 \times 10^{-4} \frac{\alpha_s^3(2m_c)}{m_c^3} \left< O_1^{J/\psi}(3S_1) \right> \\
= \Gamma(t \to gc) \times 3.2 \times 10^{-6}. \quad (9)
$$

Adding all these contributions Eqs.(3), (8) and (9) together, we get the total color-singlet contributions to $J/\psi$ production in top quark rare decays, and the branching ratio is about

$$
B_{\text{singlet}}(t \to c + J/\psi + X) = 1.1 \times 10^{-14}. \quad (10)
$$

The dominant color-octet contributions come from the color-octet gluon fragmentation process shown in Fig.2. According to the velocity scaling rules, the color-octet process is suppressed by a factor of order $v^4$, but as shown in Fig.2, the color-octet process has $O(1/\alpha_s^2)$ enhancement compared with the color-singlet process in Fig.1(c). In the fragmentation approximation, the decay rates for the color-octet gluon fragmentation process is given by [10]
\[ \Gamma(t \rightarrow J/\psi c + X) = \Gamma(t \rightarrow cg) \times P^{(g \rightarrow J/\psi)}_{9 \rightarrow J/\psi} \]
\[ = \Gamma(t \rightarrow cg) \times 1.31 \times 10^{-1} \frac{\alpha_s(2m_c)}{m_c^3} \left\langle 8_J^{J/\psi}(3S_1) \right\rangle \]
\[ = \Gamma(t \rightarrow gc) \times 1.5 \times 10^{-4}, \]
(11)

and the branching ratio is

\[ B_{octet}(t \rightarrow c + J/\psi + X) = 1.5 \times 10^{-14}, \] (12)

where the color-octet matrix element \( \langle 8_J^{J/\psi}(3S_1) \rangle = 1.5 \times 10^{-2} GeV^3 \) [11].

After including the color-singlet and color-octet contributions, the total decay rates for \( J/\psi \) production in top quark rare decays is

\[ B_{SM}(t \rightarrow c + J/\psi + X) = 2.6 \times 10^{-14}. \] (13)

From the above results, we can see that the \( J/\psi \) production in top quark rare decays is dominated by the color-singlet quark fragmentation and the color-octet gluon fragmentation, and the contributions from these two processes are both 4 orders larger than those from the leading order color-singlet process. As shown in Ref. [12], the energy spectrum of \( J/\psi \) from color-singlet quark fragmentation is hard. The \( J/\psi \)s from color-octet gluon fragmentation also have large energies because the \( t \rightarrow c + J/\psi \) decay in Fig.2 is a two body decay process and the energies of the produced \( J/\psi \)s in this process are fixed around one half of the top quark mass. That means the \( J/\psi \)s produced in top quark rare decays have much larger energies than those from direct production [3]. So, the signals of the \( J/\psi \) produced in top quark rare decays are easily to be detected and distinguished.

In the MSSM, the SUSY QCD correction would provide an enhancement to the top quark rare decays [4]. The effective \( tcg \) vertex is given in Ref. [4], for an internal squark \( \alpha \),

\[ M^\alpha_\mu = -i \frac{\alpha_s}{2\pi} K_{\alpha t} K_{\alpha c} (\gamma_\mu p_L V^\alpha + \frac{p_\mu}{m_t} P_R T^\alpha), \] (14)

where \( P = q_1 + q_2 \), and \( K_{\alpha q} \) is the supersymmetric version of KM matrix. The explicit expressions of the form factors \( V \) and \( T \) can be found in Ref. [4]. In the MSSM, the mass eigenstates of squark are obtained by mixing the left- and right-handed squark with the mixing angle \( \theta \). Here, we only consider the unmixing case (\( \theta = 0 \)) in which the left-handed squark is a mass eigenstate. Our calculations can be easily extended to the mixing case.
The mass splitting of the scale top quark and the scale charm quark comes into account, which is taken to be $m_\tilde{c} = 0.9m_\tilde{t}$. If all scale quark masses would be the same, the decay rates would be identical to zero.

With this effective $tcg$ vertex, we can calculate the $J/\psi$ production rates in top quark rare decays by using the same way discussed above. The decay rates for the leading order color-singlet process Fig.1(a) is

$$\Gamma_{SUSY}^{(LO)}(t \to c + J/\psi) = \frac{\alpha_s^2 \epsilon^2}{486\pi^2 (m_\tilde{t}^2 - m_\tilde{c}^2)^2 m_\tilde{c} m_\tilde{t}^3} \left[ -36m_\tilde{c}^6 T^2 + 24m_\tilde{c}^4 m_\tilde{t}^2 T^2 - 24m_\tilde{c}^4 m_\tilde{t}^2 T V 
- 27m_\tilde{c}^4 m_\tilde{t}^2 V^2 + 12m_\tilde{c}^2 m_\tilde{t}^4 T^2 + 24m_\tilde{c}^2 m_\tilde{t}^4 TV + 2m_\tilde{c}^4 m_\tilde{t}^2 V^2 + m_\tilde{c}^6 V^2 \right], \quad (15)$$

where $V = V^i - V^\tilde{c}$ and $T = T^i - T^\tilde{c}$. Here $\epsilon$ is a small number appearing in the SUSY-KM matrix, which is taken as $\epsilon^2 = \frac{1}{4}$. For the quark and gluon fragmentation (including color-singlet and color-octet contributions), the branching ratio is

$$B_{SUSY}^{(frag)}(t \to c + J/\psi + X) = B_{SUSY}(t \to cg)[P_{c \to J/\psi} + P_{g \to J/\psi}^{(1)} + P_{g \to J/\psi}^{(8)}] \quad (16)$$

All the above results are summarized in Fig.3 and Fig.4. In Fig.3, we demonstrate the dependence of $J/\psi$ production branching ratio on the scale top quark mass $m_\tilde{t}$ in the case that $m_\tilde{g} = 150GeV$. The dotted line represents the contribution from leading order color-singlet process, the dashed line from color-singlet quark and gluon fragmentation processes, and the dotted-dashed line from color-octet gluon fragmentation process. In Fig.4, we plot the branching ratio as a function of the gluino mass $m_\tilde{g}$ in the case that $m_\tilde{t} = 200GeV$. The curves in Fig.4 represent the same things as those in Fig.3. From these two figures, we can see that similar to that in the SM the $J/\psi$ production is dominated by the partons fragmentations (including the color-singlet and color-octet contributions) in the MSSM. Except for some lower stop mass region ($< 150GeV$) the branching ratio of $B(t \to c + J/\psi + X)$ is not sensitive to the stop quark mass and the gluino mass. Compared with the results obtained in the SM Eqs.([13]), it is shown that the SUSY QCD contributions can enhance the $J/\psi$ production rates in top quark rare decays by over 4 orders, and the branching ratio of $B(t \to c + J/\psi + X)$ is now up to about $10^{-10}$. But it is still too small to be observable.

In conclusion, in this paper we have calculated $J/\psi$ production rates in top quark rare decays. We find that both in the SM and in the MSSM the contributions from partons fragmentations (color-singlet quark fragmentation and color-octet gluon fragmentation) are
over 3 orders larger than that from leading order color-singlet process. However, the total production rates of \( J/\psi \) in top quark rare decays are too small to observable, \( i.e., \ 10^{-14} \) in the SM and \( 10^{-10} \) in the MSSM. So, the new physics effects would be crucial important to this decay mode, such as some anomalous top quark interactions. We hope that the searching for \( J/\psi \)'s signals in top quark rare decays will be hold at the top quark factories such as the LHC and NLC.

**Acknowledgements**

This work was supported in part by the National Natural Science Foundation of China, the State Education Commission of China and the State Commission of Science and Technology of China.
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Figure Captions

Fig.1. Color-singlet processes in $J/\psi$ production in top quark rare decays. (a) The leading order process; (b) quark fragmentation process; (c) gluon fragmentation process.

Fig.2. Color-octet gluon fragmentation process in $J/\psi$ production in top quark rare decays.

Fig.3. The branching ratio $B(t \to c + J/\psi + X)$ as a function of the scale top quark mass in the MSSM. The dotted line represents the contribution from the leading order color-singlet process, the dashed line from the color-singlet quark and gluon fragmentation processes, and the dotted-dashed line from the color-octet gluon fragmentation process.

Fig.4. The branching ratio $B(t \to c + J/\psi + X)$ as a function of the gluino mass in the MSSM. The meanings of the curves are as the same as those in Fig.3.
Fig. 1.
Fig. 2
Fig. 3

$B(t \rightarrow c + J/\psi + X)$

Stop Mass (GeV)

Logarithmic scale for $B(t \rightarrow c + J/\psi + X)$ and linear scale for Stop Mass (GeV).
Fig. 4

$B(t \rightarrow c + J/\psi + X)$

Gluino Mass (GeV)