Evidence for the suppressed decay \( B^- \rightarrow DK^-, D \rightarrow K^+\pi^-\pi^0 \)

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(The Belle Collaboration)

1 University of the Basque Country UPV/EHU, Bilbao
2 Beijing University, Beijing 100191
3 University of Bonn, 53115 Bonn
4 Budker Institute of Nuclear Physics SB RAS and Novosibirsk State University, Novosibirsk 630090
5 Faculty of Mathematics and Physics, Charles University, Prague 121 16 Prague
6 University of Cincinnati, Cincinnati, Ohio 45221
7 Deutsches Elektronen-Synchrotron, 22607 Hamburg
8 Department of Physics, National Taiwan University, Taipei 2409
9 Justus-Liebig-Universität Gießen, 35392 Gießen
10 Hanyang University, Seoul 133-791
11 University of Hawaii, Honolulu, Hawaii 96822
12 High Energy Accelerator Research Organization (KEK), Tsukuba 305-0801
13 Hiroshima Institute of Technology, Hiroshima 731-5193
14 Ikerbasque, 48011 Bilbao
15 Indian Institute of Technology Guwahati, Assam 781039
16 Indian Institute of Technology Madras, Chennai 600036
17 Institute of High Energy Physics, Vienna 1050
18 Institute for High Energy Physics, Protvino 142281
19 INFN – Sezione di Torino, 10125 Torino
20 Institute for Theoretical and Experimental Physics, Moscow 117218
21 J. Stefan Institute, 1000 Ljubljana
22 Kanagawa University, Yokohama 221-8686
23 Institut für Experimentelle Kernphysik, Karlsruher Institut für Technologie, 76131 Karlsruhe
24 Korea Institute of Science and Technology Information, Daejeon 305-806
25 Korea University, Seoul 136-713
26 Kyungpook National University, Daegu 702-701
27 University of Maribor, 2000 Maribor
28 Max-Planck-Institut für Physik, 80805 München
29 School of Physics, University of Melbourne, Victoria 3010
30 Moscow Physical Engineering Institute, Moscow 115409
31 Moscow Institute of Physics and Technology, Moscow Region 141700
We report a study of the suppressed decay $B^- \to DK^-$, $D \to K^+\pi^-\pi^0$, where $D$ denotes either a $D^0$ or a $\bar{D}^0$ meson. The decay is sensitive to the CP-violating parameter $\phi_3$. Using a data sample of $772 \times 10^6 BB$ pairs collected at the $\Upsilon(4S)$ resonance with the Belle detector, we measure the ratio of branching fractions of the above suppressed decay to the favored decay $B^- \to DK^0$, $D \to K^+\pi^-\pi^0$. Our result is $R_{DK} = [1.98 \pm 0.62(\text{stat.}) \pm 0.24(\text{syst.})] \times 10^{-2}$, which indicates the first evidence of the signal for this suppressed decay with a significance of 3.2 standard deviations. We measure the direct CP asymmetry between the suppressed $B^-$ and $B^+$ decays to be $A_{DK} = 0.41 \pm 0.30(\text{stat.}) \pm 0.05(\text{syst.})$. We also report measurements for the analogous quantities $R_{Ds}$ and $A_{Ds}$ for the decay $B^- \to D\pi^-$, $D \to K^+\pi^-\pi^0$.

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Several hadronic weak decays related by the combined charge-conjugation and parity (CP) transformations exhibit different behavior. Such violation of CP symmetry is described by the Standard Model of particle physics via an irreducible complex phase in the $3 \times 3$ Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [1], which has elements $V_{qq'}$, with $q = u, c, t$ and $q' = d, s, b$. The unitarity triangle (UT) is used to represent the amount of CP violation parameterized by the CKM matrix. The UT angle $\phi_3 = \gamma \equiv \arg (-V_{cd}V^*_{ub}/V_{cd}V^*_{cb})$ is less precisely measured compared to the other two angles $\phi_1(\equiv \beta)$ and $\phi_2(\equiv \alpha)$. The particular importance of improving the determination of $\phi_3$ lies in the fact that it is the only CP-violating parameter that describes the UT that can be measured solely in tree-level processes. As a result, such measurements provide a benchmark to search for new physics contributions in loop-dominated processes that would otherwise constrain the UT.

Various methods to determine $\phi_3$ in the tree decay $B^- \to DK^-$, where $D$ is a $D^0$ or $\bar{D}^0$ decaying to a common final state [2], have been proposed [3-5]. In this paper, we focus on the Atwood-Dunietz-Soni (ADS) method [4] using the decay $B^- \to DK^-$ followed by $D \to K^+\pi^-\pi^0$. Several ADS measurements have been made using $D \to K^+\pi^-\pi^0$ [6-10]. However, given a significantly larger branching fraction for $D^0 \to K^+\pi^-\pi^0$
the second is the direct decay and the favored $B^+\to D^0 K^-$ decay followed by the doubly Cabibbo-suppressed (DCS) $D^0 \to K^+\pi^-\pi^0$ decay interferes with the suppressed $B^-\to D_s^- K^-$ decay followed by the Cabibbo-favored (CF) $D_s^0 \to K^+\pi^-\pi^0$ decay. The interference between the two amplitudes can lead to a large direct CP asymmetry between the suppressed decays. We use $B^-\to D\pi^-$ as a control channel because of the kinematic similarity to $B^-\to DK^-$ and its much larger branching fraction.

One observable measured is the ratio of the suppressed to favored branching fractions

$$R_{DK} = \frac{B([K^+\pi^-\pi^0]_D K^-)}{B([K^-\pi^+\pi^0]_D K^+)} \frac{B([K^-\pi^+\pi^0]_D K^-)}{B([K^+\pi^-\pi^0]_D K^+)}$$

The definition of $R_{DK}$ is such that its value is bounded between zero and one. Sensitivity to $\phi_3$ through measurements of $R_{DK}$ and $A_{DK}$ is maximal when $R_{DK}$ is unity. The measured value of $R_{DK}$ is 0.84 ± 0.07 [13], which means that these observables are suitable to obtain information about $\phi_3$. The previous measurement of this channel [13] has constrained $R_{DK}$ to be less than $2.1 \times 10^{-2}$ at the 90% confidence level; no limit on $A_{DK}$ is presented.

The observables for the $B^- \to D\pi^-$ mode are $R_{D\pi}$ and $A_{D\pi}$. They can be defined using Eqs. (1) and (2) with the following substitutions: $K^+ \to \pi$ for the $B$ daughter, $r_B \to r_{D\pi}$, and $\delta_B \to \delta_{D\pi}$. Here, $r_{D\pi}$ and $\delta_{D\pi}$ are the absolute ratio and strong-phase difference between the suppressed and favored $B^- \to D\pi^-$ decay amplitudes. The sensitivity to $\phi_3$ is reduced in this mode because $r_{D\pi}$ is approximately an order of magnitude smaller than $r_B$.

There have been no previous measurements of $R_{D\pi}$ and $A_{D\pi}$. In Ref. [15] it has been shown that the corrections due to $D$-mixing on $R_{D\pi}$ and $A_{D\pi}$ are potentially large; therefore, such corrections would need to be taken into account if these measurements are used in the determination of $\phi_3$.

Our measurement uses a data sample of 772 × 10^6 $B\bar{B}$ pairs, collected with the Belle detector [16] located at the KEKB asymmetric-energy $e^+e^-$ (3.5 GeV) collider [17] operating near the Υ(4S) resonance. The principal detector elements used in this analysis are a silicon vertex detector, a 50-layer central drift chamber (CDC), an array of aerogel threshold Cherenkov counters (ACC), a barrel-like arrangement of time-of-flight scintillation counters (TOF), and an electromagnetic calorimeter comprised of CsI(Tl) crystals located inside a superconducting solenoid coil that provides a 1.5 T magnetic field.

We reconstruct Π^0 candidates from photon pairs that have a momentum greater than 400 MeV/c in the $e^+e^-$ center-of-mass (CM) frame and an invariant mass between 120 and 145 MeV/c^2, which corresponds to approximately ±3.2σ in resolution around the nominal $\pi^0$ mass [11]. Each photon candidate is required to have an energy greater than 50 MeV. We apply a mass-constrained fit to the $\pi^0$ candidate to improve its momentum resolution.

Neutral $D$ meson candidates are reconstructed from a pair of oppositely charged tracks and a $\pi^0$ candidate. Each track must have a distance of closest approach to the interaction point of less than 0.2 cm in the plane transverse to the positron beam direction and less than 1.5 cm along the positron beam axis. We also define $L_K$ ($L_{\pi}$), the likelihood of a track being a kaon (pion), based on particle identification (PID) information [18] from the ACC and the TOF, combined with specific ionization measured in the CDC. We apply likelihood-ratio requirements of $L(K/\pi) = L_{K/\pi} > 0.6$ for a kaon candidate and $L(K/\pi) < 0.4$ for a pion candidate. The efficiency to identify a kaon (pion) is approximately 83% (88%) averaged over momentum and the probability of misidentifying a pion (kaon) as a kaon (pion) is approximately 8% (7%).

The invariant mass of $K\pi\pi^0$ candidates is required to satisfy 1.804 GeV/c^2 < $M_{K\pi\pi^0} < 1.885$ GeV/c^2, which corresponds to approximately ±2.5σ in resolution around the nominal $D$ mass [11]. To improve the four momentum resolution of the daughters, we apply a $D$-mass constrained fit.

A B meson candidate is reconstructed by combining
the $D$ candidate with a charged hadron. The same set of $L(K/\pi)$ requirements is applied for the prompt track that used for $D$ meson reconstruction. The signal is identified with the beam-energy-constrained mass \( M_{bc} = c^{-2} \sqrt{E_{beam}^2 - |p_B|^2c^2} \) and the energy difference \( \Delta E = E_B - E_{beam} \), where \( E_{beam} \) is the beam energy and \( p_B \) is the momentum (energy) of the $B$ meson candidates in the CM frame. For $B \to DK$ decays, $M_{bc}$ peaks at the nominal mass of the $B$ meson [11] and $\Delta E$ peaks at zero. We select candidates in the ranges $5.27 \text{ GeV}/c^2 < M_{bc} < 5.29 \text{ GeV}/c^2$ and $-0.1 \text{ GeV} < \Delta E < 0.2 \text{ GeV}$.

To suppress background coming from the $D^{*\pm} \to D \pi^\pm$ decays in $e^+e^- \to \ell\ell$, we use the mass difference between the $D^{*\pm}$ and $D$ candidates ($\Delta M$). We reconstruct $D^{*\pm}$ candidates from the $D$ meson used for $B$ reconstruction and a $\pi^\pm$ candidate not used in the $B$ reconstruction. No PID requirement is applied to the $\pi^\pm$ because of its low momentum when coming from the $D^{*\pm}$ decay. After requiring $\Delta M > 0.15 \text{ GeV}/c^2$, we remove 99% of $D^{*\pm}$ backgrounds and 17% of all $\ell\ell$ backgrounds. The relative loss of signal efficiency is 3.4%.

A possible source of peaking background is the favored $B^- \to [K^-\pi^+\pi^0, J\psi^0 \pi^-] dh^-$ ($h = K$ or $\pi$) decay, which can contribute to the signal region of the respective suppressed decay, due to misidentification of both the $K^-$ and $\pi^+$ mesons in the $D$ decay. To reject this background, we veto events satisfying $1.804 \text{ GeV}/c^2 < M_{K\pi^+\pi^-} < 1.885 \text{ GeV}/c^2$ when the mass assignments of the $K^-$ and $\pi^+$ are exchanged. This criterion reduces the background to a negligible level with a relative loss of signal efficiency of around 17%. About 6% of events have multiple $B$ candidates; the candidate with $M_{K\pi^+\pi^-}$ and $M_{bc}$ most consistent with the corresponding nominal values is retained for further analysis.

The dominant remaining background for both the favored and the suppressed $Dh$ decays comes from $e^+e^- \to q\bar{q}$ ($q = u, d, s,$ or $c$) continuum events. The daughters from $B\bar{B}$ events tend to emerge isotropically in the CM frame whereas the particles from continuum events are collimated into back-to-back jets. We exploit this difference in event topology by using a neural network [7, 19] to combine shape variables that describe the particle distribution with other properties of the event that differentiate between $q\bar{q}$ and $B\bar{B}$ events.

The neural network utilizes the following nine input variables: 1) the likelihood ratio of the Fisher discriminant formed from 17 modified Fox-Wolfram moments [20]; 2) the absolute value of the cosine of the angle in the CM frame between the thrust axis of the $B$ decay and that of the remaining particles in the event; 3) the vertex separation between the $B$ candidate and the remaining charged tracks along the beam direction; 4) the cosine of the angle between the direction of the $K$ candidate from the $D$ decay and the direction opposite the flight of the $B$ candidate measured in the $D$ rest frame; 5) the absolute value of the $B$ flavor tagging dilution factor [21]; 6) the cosine of the angle between the $B$ flight direction and the beam axis in the CM frame; 7) the cosine of the angle between the $D$ and $\Upsilon(4S)$ directions in the rest frame of the $B$; 8) the product of the charge of the $B$ candidate and the sum of the charges of all kaons not used for the reconstruction of the $B$ candidate; and 9) the difference between the sum of the charges of particles in the $D$ hemisphere and the sum of charges in the opposite hemisphere, excluding the particles used in the $B$ meson reconstruction.

The neural network output $C_N$ is in the range $-1$ to 1, where events at $C_N = 1$ ($-1$) are signal (continuum) like. The training and optimization of the neural network are carried out with signal and $qq$ Monte Carlo (MC) samples after event-selection requirements are imposed. We require $C_N > -0.6$, which rejects 70% of the $qq$ continuum background and only 3% of the signal. The selection efficiency after all criteria have been applied is 10.9% (11.2%) for $B \to DK$ ($B \to D\pi$) decays.

The $C_N$ distribution peaks strongly at $|C_N| \sim 1$ and is therefore difficult to model with a simple analytic function. Therefore, to improve this modeling, we transform $C_N$ to a new variable $C'_N$:

$$C'_N = \log \left( \frac{C_N - C_{N,min}}{C_{N,max} - C_N} \right) .$$

Here, $C_{N,min} = -0.6$ and $C_{N,max} = 1$ are the minimum and maximum values of $C_N$ for the events used for the signal extraction. The distribution of $C'_N$ can be modeled by Gaussian or asymmetric Gaussian functions.

We extract the signal yield using an unbinned extended maximum likelihood fit to $\Delta E$ and $C'_N$ distributions. We perform separate fits to the suppressed and favored $B \to DK$ ($B \to D\pi$) modes. The total PDF for each component is formed by multiplying the individual PDFs for $\Delta E$ and $C'_N$, as they have negligible correlation. The $\Delta E$ and $C'_N$ PDF for each fit component are described as follows.

For signal, the $\Delta E$ distribution is parameterized by a sum of two Gaussian functions of common mean. The $C'_N$ distribution is parameterized by the sum of a symmetric Gaussian and an asymmetric Gaussian having different means. The PDF shape parameters used in the fit to the suppressed mode are fixed to the values obtained from the fit to the favored mode.

For $B \to DK$ decays, there is a background from $B \to D\pi$ decays where the $\pi$ daughter of the $B$ is misidentified as a $K$. This background peaks in $\Delta E$ at around 45 MeV and is modeled by the sum of a symmetric Gaussian and an asymmetric Gaussian. The distribution of $C'_N$ is the same as for the signal, so the same PDF is used. For the fit to the suppressed $DK$ data, the $D\pi$ background yield is fixed to that measured in the suppressed $D\pi$ signal fit multiplied by the misidentification rate; this procedure reduces the statistical uncertainty on the signal yield.
For this background component, all other PDF shape parameters in the suppressed mode are fixed to those measured in the fit to the favored mode.

The $B\bar{B}$ background in the favored $Dh$ modes has two components. The first is from $B^- \to D^* h^-$ and $B^- \to D_{0}^- h^-$ events and peaks at $\Delta E < -0.1$ GeV, so an upper tail is observed within the fit range. The second component is combinatorial. The peaking and combinatorial components are modeled by an exponential and first-order polynomial, respectively. The suppressed $Dh$ has a much smaller peaking $B\bar{B}$ background contribution than the favored mode, so an exponential function is used to model the whole peaking and combinatorial background. The $C'_{NB}$ distribution for the $B\bar{B}$ background is parameterized by a Gaussian function, which is determined separately for suppressed and favored modes from the $B\bar{B}$ MC sample.

The $\Delta E$ and $C'_{NB}$ distributions for the $q\bar{q}$ continuum background are parametrized by a first-order polynomial and a sum of two Gaussian functions of common mean, respectively. The parameters for $C'_{NB}$ are determined using the $M_{bc}$ sideband, given by $5.20 \text{ GeV/}c^2 < M_{bc} < 5.24 \text{ GeV/}c^2$, for all modes. For the suppressed mode, the mean of one of the Gaussians is left free in the fit to data; this minimizes the cross feed between the $q\bar{q}$ and combinatorial $B\bar{B}$ backgrounds.

The projections of the fits for the suppressed and favored $Dh$ modes are shown in Figs. 1 and 2, respectively. Suppressed $DK$ and $D\pi$ signal peaks are visible. The values of $R_{DK}$ are determined using the signal yields and efficiencies given in Table 4.

$$R_{DK} = [1.98 \pm 0.62\text{ (stat.)} \pm 0.24\text{ (syst.)}] \times 10^{-2}, \quad (6)$$

$$R_{D\pi} = [1.89 \pm 0.54\text{ (stat.)}^{+0.22}_{-0.25}\text{ (syst.)}] \times 10^{-3}. \quad (7)$$

The systematic uncertainties associated with $R_{DK}$ and $R_{D\pi}$ are listed in Table II and estimated as follows. The uncertainties due to fixed PDF shape parameters that are obtained from data are estimated by varying each fixed parameter by $\pm 1\sigma$. The uncertainty due to the $B\bar{B}C'_{NB}$ PDF is estimated by varying the mean and width of the Gaussian by the maximum differences observed between data and MC for the $C'_{NB}$ PDF from favored signal. Possible bias related to the fit is checked with 10000 simulated experiments. No bias is observed, and the systematic uncertainty due to possible bias is taken to be the error on the mean residual. A small bias is observed in the yields of $B\bar{B}$ and $q\bar{q}$ backgrounds in the suppressed $B \to DK$ mode simulations. This is due to an imperfect modeling of the continuum $C'_{NB}$ distribution in the signal region by the fits to the $M_{bc}$ sideband. The impact of this bias on the signal yield is estimated using simulated experiments to be at most 3%.

Charmless $B^- \to K^- K^+ \pi^- \pi^0$ decay could result in an irreducible peaking background to the signal. The size of this background is bounded by fits to the sidebands of the reconstructed $D$ mass: $1.45 \text{ GeV/}c^2 < M_D < 1.80 \text{ GeV/}c^2$ and $1.90 \text{ GeV/}c^2 < M_D < 2.25 \text{ GeV/}c^2$. We apply the same fitting method used in the signal extraction to the sideband sample to obtain an expected yield of $-9 \pm 7$ and $-11 \pm 8$ events for suppressed $DK$ and $D\pi$, respectively. Since the yields are consistent with zero, we include the uncertainty on the obtained yield as a systematic uncertainty. This is the dominant source of systematic uncertainty on the measurement of $R_{DK}$.

There are also uncertainties on the efficiency coming from the limited statistics of the MC sample and the calibration of the PID efficiency for potential data-MC differences. The uncertainty due to fixing the $B \to D\pi$ yield in the fit to the suppressed $B \to DK$ sample is found to be negligible.

The signal significance is calculated as $S = \sqrt{2\ln(L_0/L_{\text{max}})}$, where $L_{\text{max}}$ is the maximum likelihood and $L_0$ is the likelihood when the signal yield is constrained to be zero. In order to include systematic uncertainty in the significance, we convolve the fit likelihood with a Gaussian whose width is equal to the systematic
We measure $A_{Dh}$ in a separate fit to the suppressed candidates, including the charge of the kaon or pion from the $B$ decay as an additional observable and $A_{Dh}$ as a new free parameter. Since asymmetries associated with $B\bar{B}$ and $q\bar{q}$ parameters are expected to be negligible, they are fixed to zero in the $A_{Dh}$ fit. The measured values are:

$$A_{DK} = 0.41 \pm 0.30(\text{stat.}) \pm 0.05(\text{syst.}),$$

$$A_{D\pi} = 0.16 \pm 0.27(\text{stat.}) \pm 0.04(\text{syst.}).$$

The $\Delta E$ projections for signal $Dh^-$ and $Dh^+$ are shown in Fig. 3. The systematic uncertainties (see Table I).

In summary, for the mode $B^- \rightarrow Dh^-$, $D \rightarrow K^{+}\pi^{-}\pi^{0}$ ($h = K, \pi$), we report the measurements $R_{Dh}$ and $A_{Dh}$, using $772 \times 10^6 B\bar{B}$ pairs collected by the Belle detector.

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TABLE II: Summary of the systematic uncertainties for $R_{Dh}$ and $A_{Dh}$. Negligible contributions are denoted by “−”.

| Source | $R_{DK}$(%) | $R_{Ds}$(%) | $A_{DK}$ | $A_{Ds}$ |
|--------|-------------|-------------|----------|----------|
| ΔE and $C_{NB}$ PDFs | +9.9 | +9.4 | +0.03 | +0.02 |
| Fit bias | –7.1 | –10.3 | –0.02 | –0.03 |
| Due to $B\bar{B}$ and $q\bar{q}$ bias | +0.1 | +0.4 | – | – |
| Peaking background | ±3.0 | – | – | – |
| Efficiency | ±9.5 | ±8.2 | ±0.04 | ±0.01 |
| Detector asymmetry | ±0.1 | ±0.1 | – | – |
| Total | +11.9 | +11.7 | ±0.05 | ±0.04 |

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