\( \gamma^*, Z^* \) production in polarised \( p-p \) scattering as a probe of the proton spin structure

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Abstract

We present the results of a detailed study of the large transverse momentum Drell-Yan process, \( pp \rightarrow (\gamma^*, Z^*)X \rightarrow l^+l^-X \) at collider energies, with either one or both protons polarised, allowing the study of single- and double-spin asymmetries respectively. We show how these asymmetries obtained from angular distributions of the leptons in the \( \gamma^* \) (or \( Z^* \)) rest frame, can be used to get information on the polarised parton distributions. Numerical results for the asymmetries and the cross-sections are presented, and the sensitivity of the asymmetries to the initial parton distributions indicates that these can be used as effective probes of the spin structure of the proton.

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1. Introduction

Over the last two decades, deep-inelastic scattering experiments have given us important clues to the structure of the nucleon. Many of the predictions of the QCD-improved parton model have now been tested in these experiments, and we have a reasonable understanding of the structure of the nucleon in terms of its constituents—quarks and gluons. This understanding, however, is far from complete. There are several aspects of hadronic phenomenology that do not yield to a complete description in terms of the QCD-improved parton model. One striking example of this is the description of the spin structure of the nucleon.

The limited amount of experimental information on the spin structure of the nucleon that is available is from polarised deep-inelastic scattering experiments—scattering of polarised leptons off polarised nucleon targets. Over five years ago, the EMC collaboration first published its results [1] obtained from polarised muon-proton scattering experiments. For the polarised structure function $g_1^p(x, Q^2)$, which the EMC experiment measured in the range $0.015 \leq x \leq 0.7$, the first moment gives

$$\Gamma_1^p \equiv \int_0^1 g_1^p(x, Q^2)dx = 0.128 \pm 0.032.$$  (1)

This integral was obtained by assuming a smooth extrapolation based on Regge behaviour for the unmeasured low-$x$ region $x \leq 0.015$. In the parton model, we have the relation

$$g_1^p(x) = \frac{1}{2} \sum_i e_i^2 \Delta q^i(x).$$  (2)

In the above equation, $\Delta q^f(x) = (q^+_f(x) - q^-_f(x))$, where $q^\pm_f$ refer to densities of quarks with $\pm$ helicity in a proton with helicity $\frac{1}{2}$, i.e. the $\Delta q^i$’s are twice the contribution to the nucleon’s spin of a quark of flavour $f$. Data [2] from hyperon decays determine two independent linear combinations of the first moments of $\Delta q^f$’s. Using the information in Eq. [2] with that available from hyperon decays, it is possible to determine the first moments of $\Delta q^f$’s ($i = u, d, s$) separately. This yields for the integral of twice the sum of the spins of the quarks [3]

$$\Delta \Sigma \equiv \int \Delta \Sigma dx = \int (\Delta u + \Delta d + \Delta s)dx = 0.12 \pm 0.17.$$  (3)

This is the famous EMC result that the total spin carried by the quarks is small and is actually compatible with zero.

More recently, data from the SMC muon-deuteron scattering experiment [4] and from the SLAC muon-$^3$He scattering experiment [5] have become available. Taken at face value, the SLAC results seem to be in disagreement with the EMC and SMC results. However, a careful analysis [6] of all the uncertainties show that the values of $\Delta \Sigma$ obtained from the three sets of data are not incompatible with each other. A global average yields

$$\Delta \Sigma = 0.22 \pm 0.10.$$  (4)
This value is not very different from the value in Eq. 3; and the global average also gives a rather small value for $\Delta \Sigma$. In the naive parton model, the theoretical problem is to understand the smallness of this quantity which could arise from a large, negative strange quark polarisation. However, in QCD, there is an alternative interpretation of the data on $\Gamma^0_p$. Via the axial anomaly, there arises a gluonic contribution to $g_1^p(x)$ and to $\Gamma^0_p$ so that the conclusion that $\Delta \Sigma$ is small can be avoided, if the gluonic contribution is large enough, i.e. if the polarised gluon distribution function is non-negligible.

It thus becomes important to have other independent information on the polarised quark and gluon densities. In this paper, we study the large transverse momentum Drell-Yan process, taking into account $Z$-interference, as a probe of the gluon polarisation in the nucleon. We use a range of polarised parton distributions, fitted to the data from the EMC experiment. We study this process for the energies ($\sqrt{s} \sim 500$ GeV) planned at the proposed polarised $p-p$ experiment at the Relativistic Heavy Ion Collider (RHIC) in Brookhaven. A detailed account of the proposal can be found in Ref. 4. At RHIC, experiments with both incident protons polarised and those with only one of the incident protons polarised, are envisaged. The process that we are interested in is $pp \rightarrow l^+l^- + X$, where the lepton pair has large transverse momentum and where either one or both initial protons are polarised, allowing the study of single-spin and double-spin asymmetries, respectively. For this process, we study the angular distributions of the leptons, and demonstrate how this is sensitive to the polarised parton distributions. At the high energies under consideration, virtual photons and virtual $Z$’s contribute to lepton pair production. The parity-non-conserving $Z$ vertex is crucial to our considerations because this results in a non-zero single-spin asymmetry.

Some of the results discussed in this paper, namely those relating to the simplest asymmetries of the cross-sections, were published in an earlier letter. In this paper, we discuss in detail the theoretical framework for obtaining the angular distributions, and present complete expressions for the additional asymmetries associated with the multipole parameters. Numerical results beyond that presented in Ref. 4, are also included in the present paper. The remainder of this paper is organised as follows: in Sec. 2, we begin with a discussion of the explanation of the EMC experiment in terms of the axial anomaly and a large gluon polarisation and emphasise the need to have independent experimental information on the magnitude and the shape of the polarised gluon distributions. After briefly discussing the proposals to measure the gluon polarisation that exist in the literature, we discuss the conditions under which a non-vanishing single-spin asymmetry can be obtained. In Sec. 3, we discuss the process $pp \rightarrow l^+l^- + X$ and give the general framework for obtaining the angular distributions of the leptons in the rest frame of the vector boson ($\gamma^*$ or $Z^*$), and in Sec. 4 we present the numerical results for the asymmetries and the cross-sections. We state our conclusions in Sec. 5. In Appendix A, we list the complete expressions for the density matrices and the multipole parameters, and give the expressions for the decay parameters in Appendix B.
2. Gluon polarisation and the single-spin asymmetry

We begin this section with a brief description of the anomaly explanation \[8\] of polarised deep-inelastic scattering experiments. In the naive parton model, the flavour-singlet part of the first moment of the polarised structure functions is given by

$$\Gamma_{1|\text{singlet}}^p = \langle e^2 \rangle \Delta \Sigma. \tag{5}$$

However, in the QCD-improved parton model, this relation is no longer true: there is an additional piece due to the interaction of the photon with a gluon via a quark loop. Naively, this contribution would be expected to be small, as it involves an extra factor of $\alpha_s$ as compared with the lowest-order photon-quark diagram. But the contribution is not small because the longitudinally polarised gluon distribution evolves in such a way as to compensate exactly the logarithmic decrease with $Q^2$ of $\alpha_s$. For the first moment of the singlet part of $g_1^p$, the gluon contribution is given by the triangle axial anomaly, so the singlet part of the measured polarised structure function includes a gluon contribution, which is given by the following relation

$$\Gamma_{1|\text{singlet}}^p = \langle e^2 \rangle \Delta \Sigma - \frac{\alpha_s}{2\pi} N_f \Delta g, \tag{6}$$

where $N_f$ is the number of quark flavours that circulate in the loop. The empirical smallness of $\Gamma_{1|\text{singlet}}^p$ is explained in this picture by postulating a cancellation between the $\Delta \Sigma$ and the $\Delta g$ contributions; in order that this cancellation occurs one requires a large gluon polarisation.

It is important to realise that the anomaly explanation of the EMC experiment does not provide any estimate of $\Delta g$, neither do we have any other theoretical reason to believe that it must be large. An independent experimental determination of this quantity is crucial, and will, in fact, help us understand the spin structure of the nucleon. There have been several suggestions in the literature \[11\] on how the gluon polarisation may be experimentally determined. Most of these are aimed at experiments where the beam and the target are both longitudinally polarised – the asymmetries that can be studied in this case are the double-spin asymmetries. From the experimental point of view, however, it would be much simpler if only one of the initial particles were polarised. There exist in the literature few suggestions on how the single-spin asymmetries thus measured may be used to determine the polarised parton densities.

The most direct way of measuring a single-spin asymmetry is to measure the polarisation of a final-state particle, and the measured asymmetry would then simply correlate the final polarisation to the initial polarisation. The measurement of the polarisation of a final-state photon in direct photon production \[12\] or of a final-state lepton in Drell-Yan dilepton production \[13\] have, therefore, been suggested. However, it is not an easy task to measure the polarisation of a final-state particle at high energies, and, hence, it becomes necessary to look for other ways of obtaining a finite single-spin asymmetry.
One way is to measure at least two final-state momenta and couple them through their cross-product to the spin of the initial particle. The cross-product and the spin are both axial vectors and yield a scalar correlation; this correlation, however, is $T$-odd and can, therefore, be induced only through loop corrections. A detailed study of one-loop corrections to polarised Drell-Yan has been made [14], in order to estimate the size of the asymmetry arising from the imaginary part of the amplitudes. The only other way of obtaining a non-vanishing single-spin asymmetry is to consider parity-violating processes like $W$ production [15, 16]. This asymmetry obtains because the initial spin couples with the axial part of the $W$-fermion vertex. In this paper, we consider another parity-violating process: large $p_T$ Drell-Yan with $\gamma^*-Z^*$ interference, and study both single- and double-spin asymmetries. This we do by studying the angular distributions of the lepton pair in the vector boson rest-frame.

3. Large-$p_T$ Drell-Yan with $\gamma^*-Z^*$ interference

The subprocesses that contribute to the Drell-Yan process at large $p_T$ are $q\bar{q}$ annihilation and $qg$ Compton scattering:

\[
\begin{align*}
q + \bar{q} & \rightarrow \gamma^*, Z^* + g, \\
q(\bar{q}) + g & \rightarrow \gamma^*, Z^* + q(\bar{q}),
\end{align*}
\]

followed by $\gamma^*, Z^* \rightarrow l^+l^-$. The Compton subprocess dominates over the annihilation subprocess in the case of $p\bar{p}$ scattering. Consequently, this process is sensitive to gluon distributions in the $p\bar{p}$ case. We consider polarised $p\bar{p}$ collisions (with the first proton labelled $A$ and the second labelled $B$). The corresponding partons from protons $A$ and $B$ are labelled $a$ and $b$ and these carry momentum fractions $x_1$ and $x_2$, respectively. We will consider the case where both $A$ and $B$ are polarised and also the case where $A$ is polarised and $B$ is not.

It is useful to think of the Drell-Yan process in terms of a production process where the photon or the $Z$ is produced, and a “decay” process for the boson into an $l^+l^-$ pair. We analyse the decay angular distributions of the lepton in the rest frame of the $\gamma^*$ (or the $Z^*$) with the $z$-axis taken to be the direction of the momentum of the $\gamma^*$ or the $Z^*$ (in the c.m. frame). In this frame, $\theta_l$ and $\phi_l$ are the polar and azimuthal angles of the lepton.

We can factorise the Feynman amplitude for the Drell-Yan process $pp \rightarrow l^+l^-X$ into two parts: one part of the amplitude specifying the production of the virtual vector boson ($\gamma^*$ or $Z^*$), and the other describing the decay of this boson into a pair of leptons. Thus, we may write (with $l^-$, $l^+$ standing for the helicities of the lepton and the antilepton, respectively)

\[
M_{l^+l^-} = \frac{1}{M^2} \bigg[ M_{\lambda\mu;\rho,\sigma}(\gamma)A_{l^+l^-}(\gamma) + \chi(M)M_{\lambda\mu;\rho,\sigma}(Z)A_{l^+l^-}(Z) \bigg],
\]

\[8\]
where the $M_{\lambda\mu\rho\sigma}(\gamma, Z)$ are the production amplitudes and $A^{\mu\nu}_{\lambda}(\gamma, Z)$ the decay amplitudes for the $\gamma^*$ and the $Z^*$, and $\lambda, \mu, \rho, \sigma$ are the helicities of the vector-meson, the final parton and the initial partons $b$ and $a$, respectively; $M$ is the invariant mass of the lepton pair and the function $\chi(M)$ is given as

$$\chi(M) = \frac{M^2}{M^2 - M_Z^2 + iM\Gamma_Z},$$

where $M_Z$ and $\Gamma_Z$ are the mass and the width of the $Z$, respectively. Squaring the amplitude in Eq. (8) and summing over all helicities except that of the vector boson, we arrive at the following expression:

$$|M|^2(\theta_l, \phi_l) = \sum_{l^+l^-} \frac{1}{M^2} \left[ \rho_{\lambda\lambda'} A^{\mu\nu}_{\lambda}(\gamma) A^{\mu\nu}_{\lambda'}(\gamma) + |\chi|^2 \rho_{\lambda\lambda'}^Z A^{\mu\nu}_{\lambda}(Z) A^{\mu\nu}_{\lambda'}(Z) + 2\text{Re}[\chi^* \rho_{\lambda\lambda'}^Z A^{\mu\nu}_{\lambda}(Z) A^{\mu\nu}_{\lambda'}(\gamma)] \right],$$

where the unnormalised density matrices $\rho_{\lambda\lambda'}$ are defined as

$$\rho_{\lambda\lambda'} = \sum_{\mu\rho\sigma} f^A_{\sigma}(x_1) f^B_{\rho}(x_2) M_{\lambda\mu\rho\sigma}(\gamma) M_{\lambda'\mu\rho\sigma}^*(\gamma),$$

etc. The factors $f^A_{\sigma}(x_1), f^B_{\rho}(x_2)$ are the appropriate density functions for partons of helicity $\rho$. We write the Feynman amplitude for the decay as

$$A^{\mu\nu}_{\lambda} = M_{\alpha} e^{i\phi_{\alpha}} d_{\alpha\lambda}(\theta_l) \quad \text{(no sum on } \alpha),$$

where $\alpha = l^- - l^+ = \pm 1$ only, because for fast leptons only the helicities ($+ -$) or ($- +$) are allowed. The $M_{\alpha}$'s are essentially standard model coupling constants. Then expanding the density matrices in terms of multipole parameters, and integrating over $\phi_l$ yields

$$|M|^2(\theta_l) = \frac{\sqrt{7}}{M^4} \sum_{l=0}^{2} \left[ C_l^\gamma \text{d}_{l0}^\gamma(\gamma) + |\chi|^2 C_l^Z \text{d}_{l0}^Z(Z) + 2\text{Re}[\chi^* C_l^Z \text{d}_{l0}^Z(\gamma Z)] \right] Y_{l0}(\theta_l),$$

where $Y_{l0}$ are the spherical harmonics and the $C_l$ are decay parameters \cite{17}, and we have

$$\text{d}_{l0} = \sum_{\lambda} \langle 1\lambda|1\lambda; l0\rangle \rho_{\lambda\lambda},$$

$$C_l = \sqrt{3} \sum_{\alpha} \langle l0|1\alpha; 1\alpha - 1\rangle M_{\alpha}^2.$$  

In the above equation $M_{\alpha}^2$ stands for $|M_{\alpha}(\gamma)|^2$ and $|M_{\alpha}(Z)|^2$ for the pure $\gamma$ and $Z$ terms and $M_{\alpha}(\gamma)M_{\alpha}^*(Z)$ for the interference terms.

Finally Eq. (13) can be rewritten in the form

$$|M|^2(\theta_l) = \frac{4e^2\sqrt{7}}{M^2} \sum_{l=0}^{2} D_l Y_{l0}(\theta_l),$$
where the $D_l$’s are:

$$
D_0 = t_0^0(\gamma) + (v^2 + a^2)|\chi|^2 t_0^0(Z) + 2v(\text{Re} \chi^*) t_0^0(\gamma Z),
$$

$$
D_1 = -\sqrt{6}a[v|\chi|^2 t_0^1(Z) + (\text{Re} \chi^*) t_0^1(\gamma Z)],
$$

$$
D_2 = \frac{1}{\sqrt{2}}[t_0^2(\gamma) + (v^2 + a^2)|\chi|^2 t_0^2(Z) + 2v(\text{Re} \chi^*) t_0^2(\gamma Z)].
$$

Here $v$ and $a$ are the vector and the axial couplings of the $Z$ to the leptons. The expressions for the multipole parameters for the annihilation and the Compton subprocesses, which involve the parton distributions and the couplings, are presented in Appendix A. The decay parameters $C_l$ are discussed in Appendix B.

4. Numerical results

For the Drell-Yan process $AB \rightarrow l^+l^-CX$, ($C$ is an associated quark- or gluon-initiated jet) where the pair has transverse momentum $p_T$ and rapidity $y_1$, the differential cross-section to produce the lepton at angle $\theta_l$ (in the reference frame described in the previous section) is then given as

$$
\frac{d\sigma_{AB \rightarrow l^+l^-CX}}{dp_T dy_1 d\tau d\cos \theta_l} = \int dx_1 \sqrt{\pi} F \sum_{l=0}^{2} D_l Y_{l0}(\theta_l),
$$

where $\tau = M^2/s$, with $s$ the c.m. energy, and $x_1$ the momentum fraction of proton $A$ carried by parton $a$. The factor $\mathcal{F}$ is given by

$$
\mathcal{F} = \frac{\alpha x_T}{48\pi^2 \sqrt{5} M^2 x_1 x_2 \left[ x_1 - \frac{1}{2} \sqrt{x_T^2 + 4\tau e^{y_1}} \right]},
$$

where $x_T = 2p_T/\sqrt{s}$.

For each setting of the spins of the colliding protons the differential cross-section in $\theta_l$ is controlled by the independent parameters $D_{0,1,2}$, each of which contains information about the parton distributions. To isolate information on the polarised distributions, it is necessary to form asymmetries, either by reversing the spin direction of one of the protons with the other being unpolarised (single-spin asymmetries), or reversing the spin direction of one of the protons, the other being polarised (double-spin asymmetries).

For each spin setting, the $D_j$’s can be projected out from a knowledge of the angular distribution :

$$
D_l = \frac{2\sqrt{\pi}}{\mathcal{F}} \int \frac{d\sigma_{AB \rightarrow l^+l^-CX}}{dp_T dy_1 d\tau d\cos \theta_l} Y_{l0}(\theta_l) \sin \theta_l d\theta_l,
$$

Clearly $D_0$ is the simplest, being essentially the cross-section to produce an $l^+l^-$ pair with given $p_T$, $y_1$ and $\tau$. 

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The full set of asymmetries that one can, in principle, measure is the following:

\[ A_s = \frac{D_0^+ - D_0^-}{D_0^+ + D_0^-} = \frac{d\sigma_+ - d\sigma_-}{d\sigma_+ + d\sigma_-}, \]

\[ A_d = \frac{D_0^{++} - D_0^{--}}{D_0^{++} + D_0^{--}} = \frac{d\sigma_{++} - d\sigma_{--}}{d\sigma_{++} + d\sigma_{--}}. \]  

(20)

where the labels +, − refer to the helicities of the protons A and B, and for \( j = 1, 2, \)

\[ A^j_s = \frac{D_j^+ - D_j^-}{D_j^+ + D_j^-}, \]

\[ A^j_d = \frac{D_j^{++} - D_j^{--}}{D_j^{++} + D_j^{--}}. \]  

(21)

In what follows, we shall present results for all these asymmetries. We shall assume that the jet recoiling from the lepton pair is not detected and so we must integrate over \( x_1. \)

We illustrate our results for the case \( y_1 = 0 \) and fixed values of \( M, \) and show how the asymmetries vary with \( p_T. \) We take \( \sqrt{s} = 500 \text{ GeV} \) to correspond to RHIC energies.

In order to study the sensitivity of our asymmetries to the polarised parton distributions we utilise the following range of models:

Set I : \( \Delta g \) large, \( \Delta s = 0, \)

Set II : \( \Delta g \) and \( \Delta s \) both moderately large,

Set III : \( \Delta g = 0, \Delta s \) large,

which were described in Ref. [18] and which all fit the EMC data on \( g_1^p(x). \) For the unpolarised distributions we use Owens’ Set 1.1 distributions [19].

In Fig. 1, we show the calculated asymmetries for \( M = 10 \) and 50 GeV and for \( M = M_Z \) and with \( \theta_l \) integrated over. The figures in the top row are predictions for single-spin asymmetries, the bottom row for double-spin asymmetries. In each figure, the three curves correspond to the three different sets of polarised densities used. We see that for small dilepton masses the single-spin asymmetries are very small, but that they are appreciable for large dilepton masses. This is because the single-spin asymmetry arises from the parity-violating nature of the Z-fermion coupling, and hence is large only when the effects of Z-interference are important. The double-spin asymmetries are large over the whole range of dilepton masses. In Fig. 1, there is a strong dependence upon the choice of polarised parton distributions and the asymmetries, if measured with reasonable accuracy, will be very helpful in teaching us to what extent the various partons in a proton are polarised. In Fig. 2 we have plotted the cross-section as a function of \( p_T \) for the three values of dilepton mass given above. In Figs. 3 and 4 we have plotted the asymmetries \( A_{s,d}^j (j = 1, 2). \) Since the structure of the contributions to \( D_2 \) is very similar to those of \( D_0 \) contribution, the corresponding asymmetries also look very similar in shape. The single-spin asymmetries \( A_{s,d}^{1,2} \) are large only close to \( M = M_Z. \)
5. Conclusions

Data on the polarised structure function $g_1^p(x)$ in deep-inelastic scattering, from the EMC, SMC and SLAC experiments, if interpreted in the naive parton model, suggest that only a small fraction of the proton’s spin arises from the spin of its quark constituents. One explanation has the contribution of the valence quarks cancelled by an unexpectedly large strange quark contribution. Another approach argues that the naive parton model formulae are invalidated by the axial anomaly and explains the data in terms of a large gluon contribution to the proton’s spin. Independent information on the magnitude of the strange quark and gluon polarised distributions is thus of great importance and needed urgently. In this paper we have studied both the single- and the double-spin asymmetries of the parameters which describe the angular distribution of the lepton in the large transverse momentum Drell-Yan process at RHIC energies ($\sqrt{s} = 500$ GeV). At these high energies, $Z^*$’s can contribute significantly to the Drell-Yan process, inducing a parity-violating single-spin asymmetry. This asymmetry may be measured in experiments where only one of the initial particles is polarised.

We have discussed the framework in which we compute the angular distributions of the leptons in the vector-boson rest-frame, and have presented the complete results for the multipole parameters, which describe the decay angular distributions of the lepton. Using these angular distributions, we have constructed single- and double-spin asymmetries, which we then studied numerically using three sets of polarised parton distributions that are consistent with the deep-inelastic scattering data. We find that the single-spin asymmetries are measurably large only at large dilepton masses, i.e. close to the $Z$ peak. The double-spin asymmetries are large even at smaller dilepton masses. Both the single-spin and the double-spin asymmetries are sensitive to the polarised gluon distribution and may be used as probes of the spin structure of the proton.
Appendix A

In this appendix, we present the expressions for the density matrices and the multipole parameters, for the annihilation and the Compton processes.

The couplings of the $\gamma^*$ to both quarks and leptons is given by the interaction term $-ieQ_f \gamma^\mu$, while the coupling of the $Z^*$ to the fermions is given by the term $ie\gamma^\mu(v_j - a_j \gamma_5)$, where as usual

$$v_j = \frac{I_3^L - 2Q_j \sin^2\theta_W}{2\sin\theta_W \cos\theta_W},$$

$$a_j = \frac{I_3^L}{2\sin\theta_W \cos\theta_W}$$

In the above equations, $I_3$ is the third component of the weak isospin, $\theta_W$ the Weinberg angle and $Q_f$, $Q_j$ are the charges in units of $e$. In what follows, we use $v$ and $a$ to denote the $Z^*$ couplings to leptons and $v_f$ and $a_f$ to quarks of flavour $f$. The results are presented in terms of the variables $\hat{\tau}$ and $\hat{\theta}$, where $\hat{\theta}$ is the subprocess scattering angle, and

$$\hat{\tau} = \frac{\tau}{x_1 x_2}.$$  \hspace{1cm} (23)

The unnormalised density matrices for $Z^*$ production via the annihilation process $q\bar{q} \rightarrow Z^* g$ are given as:

$$\rho_{11}(Z) = K_0 (1 + \hat{\tau}^2) \sum_f \left\{ (\bar{q}_f(x_1)q_f(x_2) - \Delta \bar{q}_f(x_1) \Delta q_f(x_2)) \left[ (v_f^2 + a_f^2) \left( \cot^2 \frac{\hat{\theta}}{2} + \tan^2 \frac{\hat{\theta}}{2} \right) + 2v_f a_f \left( \cot^2 \frac{\hat{\theta}}{2} - \tan^2 \frac{\hat{\theta}}{2} \right) \right] + (\Delta \bar{q}_f(x_1)q_f(x_2) - \bar{q}_f(x_1) \Delta q_f(x_2)) \left[ (v_f^2 + a_f^2) \left( \cot^2 \frac{\hat{\theta}}{2} - \tan^2 \frac{\hat{\theta}}{2} \right) + 2v_f a_f \left( \cot^2 \frac{\hat{\theta}}{2} + \tan^2 \frac{\hat{\theta}}{2} \right) \right] \right\},$$

$$\rho_{00}(Z) = K_0 8\hat{\tau} \sum_f \left\{ (\bar{q}_f(x_1)q_f(x_2) - \Delta \bar{q}_f(x_1) \Delta q_f(x_2)) (v_f^2 + a_f^2) + (\Delta \bar{q}_f(x_1)q_f(x_2) - \bar{q}_f(x_1) \Delta q_f(x_2)) (2v_f a_f) \right\},$$

$$\rho_{1-1}(Z) = K_0 4\hat{\tau} \sum_f \left\{ (\bar{q}_f(x_1)q_f(x_2) - \Delta \bar{q}_f(x_1) \Delta q_f(x_2)) (v_f^2 + a_f^2) + (\Delta \bar{q}_f(x_1)q_f(x_2) - \bar{q}_f(x_1) \Delta q_f(x_2)) (2v_f a_f) \right\},$$

$$\rho_{10}(Z) = K_0 \sqrt{2}(1 + \hat{\tau}) \sum_f \left\{ (\bar{q}_f(x_1)q_f(x_2) - \Delta \bar{q}_f(x_1) \Delta q_f(x_2)) \left[ (v_f^2 + a_f^2) \left( \tan \frac{\hat{\theta}}{2} - \cot \frac{\hat{\theta}}{2} \right) - 2v_f a_f \left( \tan \frac{\hat{\theta}}{2} + \cot \frac{\hat{\theta}}{2} \right) \right] + (\Delta \bar{q}_f(x_1)q_f(x_2) - \bar{q}_f(x_1) \Delta q_f(x_2)) \times \right\}.$$
\[
\rho_{0-1}(Z) = K_a \sqrt{2\tau} (1 + \hat{\tau}) \sum_f \left\{ \left( \bar{q}_f(x_1)q_f(x_2) - \Delta \bar{q}_f(x_1)\Delta q_f(x_2) \right) \left( v_f^2 + a_f^2 \right) \left( \cot \frac{\hat{\theta}}{2} - \tan \frac{\hat{\theta}}{2} \right) \right\},
\]

\[
\rho_{-1-1}(Z) = K_a (1 + \hat{\tau})^2 \sum_f \left\{ \left( \bar{q}_f(x_1)q_f(x_2) - \Delta \bar{q}_f(x_1)\Delta q_f(x_2) \right) \left( v_f^2 + a_f^2 \right) \left( \tan \frac{\hat{\theta}}{2} + \cot \frac{\hat{\theta}}{2} \right) \right\},
\]

where the sum runs over all flavours. The overall factor \( K_a \) is given by

\[
K_a = \frac{4}{9 \left( 1 - \hat{\tau} \right)^2}.
\]

The unnormalised density matrices for \( Z^* \) production via the Compton process \( qg \rightarrow Z^*q \) are given as:

\[
\rho_{11}(Z) = K_c \sum_f \left\{ G(x_1)q_f(x_2) \left[ (v_f - a_f)^2 \left( 1 - \hat{\tau} \right)^2 \sec^2 \frac{\hat{\theta}}{2} + \hat{\tau}^2 \cos^2 \frac{\hat{\theta}}{2} \tan^4 \frac{\hat{\theta}}{2} \right] + (v_f + a_f)^2 \cos^2 \frac{\hat{\theta}}{2} \right\},
\]

\[
+ G(x_1)\Delta q_f(x_2) \left[ (v_f - a_f)^2 \left( 1 - \hat{\tau} \right)^2 \sec^2 \frac{\hat{\theta}}{2} + \hat{\tau}^2 \cos^2 \frac{\hat{\theta}}{2} \tan^4 \frac{\hat{\theta}}{2} \right] - (v_f + a_f)^2 \cos^2 \frac{\hat{\theta}}{2} \right\},
\]

\[
+ \Delta G(x_1)q_f(x_2) \left[ (v_f - a_f)^2 \left( 1 - \hat{\tau} \right)^2 \sec^2 \frac{\hat{\theta}}{2} - \hat{\tau}^2 \cos^2 \frac{\hat{\theta}}{2} \tan^4 \frac{\hat{\theta}}{2} \right] + (v_f + a_f)^2 \cos^2 \frac{\hat{\theta}}{2} \right\},
\]

\[
+ \Delta G(x_1)\Delta q_f(x_2) \left[ (v_f - a_f)^2 \left( 1 - \hat{\tau} \right)^2 \sec^2 \frac{\hat{\theta}}{2} - \hat{\tau}^2 \cos^2 \frac{\hat{\theta}}{2} \tan^4 \frac{\hat{\theta}}{2} \right] - (v_f + a_f)^2 \cos^2 \frac{\hat{\theta}}{2} \right\},
\]

\[
\rho_{00}(Z) = K_c \hat{\tau} \sin^2 \frac{\hat{\theta}}{2} \sum_f \left\{ (G(x_1)q_f(x_2) - \Delta G(x_1)\Delta q_f(x_2)) (v_f^2 + a_f^2) 
\right\},
\]

\[
- 4v_f a_f (G(x_1)\Delta q_f(x_2) - \Delta G(x_1)q_f(x_2)) \right\},
\]

\[
\rho_{1-1}(Z) = K_c \hat{\tau} \sin^2 \frac{\hat{\theta}}{2} \sum_f \left\{ (G(x_1)q_f(x_2) + \Delta G(x_1)\Delta q_f(x_2)) (v_f^2 + a_f^2) 
\right\}.
\]
+4\nu_{af}(G(x_1)\Delta q_f(x_2) - \Delta G(x_1)q_f(x_2))\right\},

\rho_{10}(Z) = K_c\sqrt{2}\tau\cos\hat{\theta}\sin\hat{\theta}\frac{\hat{\theta}}{2}\sum_f\left\{(-G(x_1)q_f(x_2) + \Delta G(x_1)\Delta q_f(x_2))\left[(v_f - a_f)^2\hat{\tau}\tan^2\hat{\theta}\frac{\hat{\theta}}{2} + (v_f + a_f)^2\right] + (\Delta G(x_1)q_f(x_2) - G(x_1)\Delta q_f(x_2))\left[(v_f - a_f)^2\hat{\tau}\tan^2\hat{\theta}\frac{\hat{\theta}}{2} - (v_f + a_f)^2\right]\right\},

\rho_{0-1}(Z) = K_c\sqrt{2}\tau\cos\hat{\theta}\sin\hat{\theta}\frac{\hat{\theta}}{2}\sum_f\left\{(G(x_1)q_f(x_2) - \Delta G(x_1)\Delta q_f(x_2))\left[(v_f - a_f)^2 + (v_f + a_f)^2\hat{\tau}\tan^2\hat{\theta}\frac{\hat{\theta}}{2}\right] + (\Delta G(x_1)q_f(x_2) - \Delta G(x_1)\Delta q_f(x_2))\left[(v_f - a_f)^2 - (v_f + a_f)^2\hat{\tau}\tan^2\hat{\theta}\frac{\hat{\theta}}{2}\right]\right\},

\rho_{-1-1}(Z) = K_c\sum_f\left\{G(x_1)q_f(x_2)\left[(v_f + a_f)^2\left(1 - \hat{\tau}\right)^2\sec^2\hat{\theta}\frac{\hat{\theta}}{2} + \hat{\tau}\cos^2\hat{\theta}\frac{\hat{\theta}}{2}\tan^2\hat{\theta}\frac{\hat{\theta}}{2}\right] + (v_f - a_f)^2\cos^2\hat{\theta}\frac{\hat{\theta}}{2}\right\} + G(x_1)\Delta q_f(x_2)\left[+(v_f + a_f)^2\left(1 - \hat{\tau}\right)^2\sec^2\hat{\theta}\frac{\hat{\theta}}{2} - \hat{\tau}\cos^2\hat{\theta}\frac{\hat{\theta}}{2}\tan^2\hat{\theta}\frac{\hat{\theta}}{2}\right] - (v_f - a_f)^2\cos^2\hat{\theta}\frac{\hat{\theta}}{2}\right\} + \Delta G(x_1)\Delta q_f(x_2)\left[+(v_f + a_f)^2\left(1 - \hat{\tau}\right)^2\sec^2\hat{\theta}\frac{\hat{\theta}}{2} - \hat{\tau}\cos^2\hat{\theta}\frac{\hat{\theta}}{2}\tan^2\hat{\theta}\frac{\hat{\theta}}{2}\right] - (v_f - a_f)^2\cos^2\hat{\theta}\frac{\hat{\theta}}{2}\right\},

where, again the sum is over all flavours, and the overall factor $K_c$ is given as

$$K_c = \frac{1}{6\left(1 - \hat{\tau}\right)}.$$  \hspace{1cm} (27)

We can construct the unnormalised multipole parameters using the expressions for the density matrices given above. For the annihilation process, we obtain

$$t_0^0(Z) = 2K_a\sum_f\left\{(\bar{q}_f(x_1)q_f(x_2) - \Delta \bar{q}_f(x_1)\Delta q_f(x_2))(v_f^2 + a_f^2) + (\Delta \bar{q}_f(x_1)q_f(x_2) - \bar{q}_f(x_1)\Delta q_f(x_2))\right\}2v_f a_f\left[\left(1 + \hat{\tau}\right)\left(\tan^2\hat{\theta}\frac{\hat{\theta}}{2} + \cot^2\hat{\theta}\frac{\hat{\theta}}{2}\right) + 4\hat{\tau}\right],$$

$$t_1^0(Z) = \sqrt{2}K_a\sum_f\left\{(\bar{q}_f(x_1)q_f(x_2) - \Delta \bar{q}_f(x_1)\Delta q_f(x_2))2v_f a_f + (\Delta \bar{q}_f(x_1)q_f(x_2) - \bar{q}_f(x_1)\Delta q_f(x_2))\right\}(v_f^2 + a_f^2)\left[\left(1 + \hat{\tau}\right)\left(\cot^2\hat{\theta}\frac{\hat{\theta}}{2} - \tan^2\hat{\theta}\frac{\hat{\theta}}{2}\right)\right],$$

$$t_0^2(Z) = 2\sqrt{\frac{1}{10}}K_a\sum_f\left\{(\bar{q}_f(x_1)q_f(x_2) - \Delta \bar{q}_f(x_1)\Delta q_f(x_2))(v_f^2 + a_f^2) + \right.$$
\[
(\Delta q_f(x_1)q_f(x_2) - \bar{q}_f(x_1)\Delta q_f(x_2))2v_f a_f \right) \left[ (1 + \hat{\tau}^2) \left( \tan^2\frac{\hat{\theta}}{2} + \cot^2\frac{\hat{\theta}}{2} \right) - 8\hat{\tau} \right],
\]

and for the Compton process, we get

\[
t^0_0(Z) = K_e \sum_f 2(v_f^2 + a_f^2) \left\{ G(x_1)q_f(x_2) \left[ (1 - \hat{\tau})^2 \sec^2\frac{\hat{\theta}}{2} \cos^2\frac{\hat{\theta}}{2} \left( 1 + \hat{\tau}^2 \tan^2\frac{\hat{\theta}}{2} \right) + 2\hat{\tau} \sin^2\frac{\hat{\theta}}{2} \right] 
+ \Delta G(x_1)\Delta q_f(x_2) \left[ (1 - \hat{\tau})^2 \sec^2\frac{\hat{\theta}}{2} - \cos^2\frac{\hat{\theta}}{2} \left( 1 + \hat{\tau}^2 \tan^2\frac{\hat{\theta}}{2} \right) - 2\hat{\tau} \sin^2\frac{\hat{\theta}}{2} \right] \right\}
- 4v_f a_f \left\{ G(x_1)\Delta q_f(x_2) \left[ (1 - \hat{\tau})^2 \sec^2\frac{\hat{\theta}}{2} + \cos^2\frac{\hat{\theta}}{2} \left( \hat{\tau}^2 \tan^4\frac{\hat{\theta}}{2} - 1 \right) \right] 
+ \Delta G(x_1)q_f(x_2) \left[ (1 - \hat{\tau})^2 \sec^2\frac{\hat{\theta}}{2} + \cos^2\frac{\hat{\theta}}{2} \left( \hat{\tau}^2 \tan^4\frac{\hat{\theta}}{2} - 1 \right) \right] \right\}
+ 2(v_f^2 + a_f^2) \left\{ G(x_1)\Delta q_f(x_2) \left[ (1 - \hat{\tau})^2 \sec^2\frac{\hat{\theta}}{2} - \cos^2\frac{\hat{\theta}}{2} \left( \hat{\tau}^2 \tan^4\frac{\hat{\theta}}{2} - 1 \right) \right] 
+ \Delta G(x_1)q_f(x_2) \left[ (1 - \hat{\tau})^2 \sec^2\frac{\hat{\theta}}{2} - \cos^2\frac{\hat{\theta}}{2} \left( \hat{\tau}^2 \tan^4\frac{\hat{\theta}}{2} - 1 \right) \right] \right\},
\]

\[
t^2_0(Z) = \sqrt{\frac{1}{10}} K_e \sum_f 2(v_f^2 + a_f^2) \left\{ G(x_1)q_f(x_2) \left[ (1 - \hat{\tau})^2 \sec^2\frac{\hat{\theta}}{2} + \cos^2\frac{\hat{\theta}}{2} \left( 1 + \hat{\tau}^2 \tan^4\frac{\hat{\theta}}{2} \right) - 4\hat{\tau} \sin^2\frac{\hat{\theta}}{2} \right] 
+ \Delta G(x_1)\Delta q_f(x_2) \left[ (1 - \hat{\tau})^2 \sec^2\frac{\hat{\theta}}{2} - \cos^2\frac{\hat{\theta}}{2} \left( 1 + \hat{\tau}^2 \tan^4\frac{\hat{\theta}}{2} \right) + 4\hat{\tau} \sin^2\frac{\hat{\theta}}{2} \right] \right\}
- 4v_f a_f \left\{ G(x_1)\Delta q_f(x_2) \left[ (1 - \hat{\tau})^2 \sec^2\frac{\hat{\theta}}{2} + \cos^2\frac{\hat{\theta}}{2} \left( 1 + \hat{\tau}^2 \tan^4\frac{\hat{\theta}}{2} \right) - 4\hat{\tau} \sin^2\frac{\hat{\theta}}{2} \right] 
+ \Delta G(x_1)q_f(x_2) \left[ (1 - \hat{\tau})^2 \sec^2\frac{\hat{\theta}}{2} - \cos^2\frac{\hat{\theta}}{2} \left( 1 + \hat{\tau}^2 \tan^4\frac{\hat{\theta}}{2} \right) + 4\hat{\tau} \sin^2\frac{\hat{\theta}}{2} \right] \right\}.\]

The multipole parameters given are only for quark-gluon Compton scattering. The corresponding multipole parameters for antiquark-gluon Compton scattering are given by

\[ a_f \rightarrow -a_f \]

in Eq. 29.

So far, we have listed only the $Z^*$ production density matrices and multipole parameters. We need to have similar expressions for $\gamma^*$ productions and the expressions for the
interference terms. These expressions can be easily obtained from the above expressions for \(Z^*\) by simple replacements of the couplings, which we summarise in the following rules:

1. To obtain the \(\gamma^*\) multipole parameters, the replacements \(a_f \rightarrow 0, v_f \rightarrow Q_f\), should be made in the expressions for the \(Z^*\) production multipole parameters.

2. To obtain the multipole parameters corresponding to the interference terms, the replacements \(v_f^2 + a_f^2 \rightarrow -2Q_f v_f, 2v_f a_f \rightarrow -Q_f a_f\), should be made in the expressions for the \(Z^*\) production multipole parameters.

Having listed all the multipole parameters, we are now in a position to write down the complete expressions for the \(D_j\)'s defined in Eq. 16. To present these in a compact form, we define the following combinations of the couplings

\[
\alpha_f = Q_f^2 - 2v_f Q_f \Re \chi^* + (v^2 + a^2)(v_f^2 + a_f^2)|\chi|^2, \\
\beta_f = 2a_f v_f [(v^2 + a^2)|\chi|^2 - v Q_f \Re \chi^*], \\
\gamma_f = a a_f [2v v_f |\chi|^2 - Q_f \Re \chi^*], \\
\delta_f = v v_f^2 |\chi|^2 - v_f Q_f \Re \chi^*.
\]

and the functions,

\[
R_{\pm}(\hat{\tau}, \hat{\theta}) = (1 - \hat{\tau})^2 \sec^2 \hat{\theta} \pm \left[ \cos \hat{\theta} + \hat{\tau} \sin \frac{\hat{\theta}}{2} \right] \tan \frac{\hat{\theta}}{2}, \\
S_{\pm}(\hat{\tau}, \hat{\theta}) = (1 - \hat{\tau})^2 \sec^2 \hat{\theta} \pm \cos^2 \hat{\theta} \left[ \hat{\tau}^2 \tan^4 \frac{\hat{\theta}}{2} - 1 \right], \\
T_{\pm}(\hat{\tau}, \hat{\theta}) = R_{\pm}(\hat{\tau}, \hat{\theta}) \mp 6\hat{\tau} \sin^2 \frac{\hat{\theta}}{2}, \\
U(\hat{\tau}, \hat{\theta}) = (1 + \hat{\tau}^2) \left[ \tan^2 \frac{\hat{\theta}}{2} + \cot^2 \frac{\hat{\theta}}{2} \right] + 4\hat{\tau}, \\
V(\hat{\tau}, \hat{\theta}) = (1 + \hat{\tau}^2) \left[ \tan^2 \frac{\hat{\theta}}{2} - \cot^2 \frac{\hat{\theta}}{2} \right].
\]

We may now write each \(D_j\) as

\[
D_j = \sum_f [D_j^{\hat{\theta}} + D_j^{\hat{\phi}} + D_j^{\hat{\phi}G}] + [\hat{\theta} \rightarrow \pi - \hat{\theta}],
\]

where the sum is over flavours, and by \([\hat{\theta} \rightarrow \pi - \hat{\theta}]\) is intended to imply the following symmetrization

\[
f_A(x_1)g_B(x_2)F(\hat{\theta}) \rightarrow g_A(x_1)f_B(x_2)F(\pi - \hat{\theta}),
\]

where \(f, g\) are parton densities pre-multiplying some function \(F\) of \(\hat{\theta}\).
Then we have, for the annihilation subprocess,

\[ D_{0}^{q\bar{q}} = 2K_{a}U(\hat{\tau}, \hat{\theta}) \left\{ (\bar{q}_{f}(x_{1})q_{f}(x_{2}) - \Delta \bar{q}_{f}(x_{1})\Delta q_{f}(x_{2}))\alpha_{f} + (\Delta \bar{q}_{f}(x_{1})q_{f}(x_{2}) - \bar{q}_{f}(x_{1})\Delta q_{f}(x_{2}))\beta_{f} \right\}, \]

\[ D_{1}^{q\bar{q}} = 2\sqrt{3}K_{a}V(\hat{\tau}, \hat{\theta}) \left\{ (\bar{q}_{f}(x_{1})q_{f}(x_{2}) - \Delta \bar{q}_{f}(x_{1})\Delta q_{f}(x_{2}))\gamma_{f} + (\Delta \bar{q}_{f}(x_{1})q_{f}(x_{2}) - \bar{q}_{f}(x_{1})\Delta q_{f}(x_{2}))\delta_{f} \right\}, \]

\[ D_{2}^{q\bar{q}} = \frac{1}{5}K_{a}[U(\hat{\tau}, \hat{\theta}) - 12\hat{\tau}] \left\{ (\bar{q}_{f}(x_{1})q_{f}(x_{2}) - \Delta \bar{q}_{f}(x_{1})\Delta q_{f}(x_{2}))\alpha_{f} + (\Delta \bar{q}_{f}(x_{1})q_{f}(x_{2}) - \bar{q}_{f}(x_{1})\Delta q_{f}(x_{2}))\beta_{f} \right\}, \] (35)

and for the Compton subprocess,

\[ D_{0}^{G} = 2K_{c} \left\{ [G(x_{1})q_{f}(x_{2})R_{+}(\hat{\tau}, \hat{\theta}) + \Delta G(x_{1})\Delta q_{f}(x_{2})R_{-}(\hat{\tau}, \hat{\theta})]\alpha_{f} \right. \]
\[ - [G(x_{1})\Delta q_{f}(x_{2})R_{+}(\hat{\tau}, \hat{\theta}) + \Delta G(x_{1})q_{f}(x_{2})R_{-}(\hat{\tau}, \hat{\theta})]\beta_{f} \}, \]

\[ D_{1}^{G} = 2\sqrt{3}K_{c} \left\{ [G(x_{1})q_{f}(x_{2})S_{+}(\hat{\tau}, \hat{\theta}) + \Delta G(x_{1})\Delta q_{f}(x_{2})S_{-}(\hat{\tau}, \hat{\theta})]\gamma_{f} \right. \]
\[ - [G(x_{1})\Delta q_{f}(x_{2})S_{+}(\hat{\tau}, \hat{\theta}) + \Delta G(x_{1})q_{f}(x_{2})S_{-}(\hat{\tau}, \hat{\theta})]\delta_{f} \}, \]

\[ D_{2}^{G} = \frac{1}{5}K_{c} \left\{ [G(x_{1})q_{f}(x_{2})T_{+}(\hat{\tau}, \hat{\theta}) + \Delta G(x_{1})\Delta q_{f}(x_{2})T_{-}(\hat{\tau}, \hat{\theta})]\alpha_{f} \right. \]
\[ - [G(x_{1})\Delta q_{f}(x_{2})T_{+}(\hat{\tau}, \hat{\theta}) + \Delta G(x_{1})q_{f}(x_{2})T_{-}(\hat{\tau}, \hat{\theta})]\beta_{f} \}. \] (36)

\( D_{j}^{G} \)'s are obtained from \( D_{j}^{G} \)'s by making the replacements \( \beta_{f} \rightarrow -\beta_{f} \) and \( \gamma_{f} \rightarrow -\gamma_{f} \).

**Appendix B**

In this appendix, we discuss the decay process \( V \rightarrow l^{+}l^{-} \), where \( V \) denotes \( \gamma^{*} \) or \( Z^{*} \). We first list the Feynman amplitudes \( A_{\lambda}^{l^{+}l^{-}} \), where \( l^{+}, l^{-} \) refer to the helicities of the antilepton and the lepton, respectively, and \( \lambda \) to the helicity of the vector-boson. Since the vector-boson is massive (of mass \( M \)), we compute these amplitudes in the vector-boson rest frame. In this frame, we take the polar and azimuthal angles of the lepton to be \( \theta_{l} \) and \( \phi_{l} \), respectively.

For fast leptons, because of the vector and the axial-vector couplings, only the helicities \((+, -)\) or \((-+, +)\) can occur and the Feynman amplitude has the form

\[ A_{\lambda}^{l^{+}l^{-}} = M_{\alpha}e^{i\phi_{l}^{\lambda}}d_{\lambda\alpha}^{1}(\theta_{l}) \] (37)

where \( \alpha = l^{-} - l^{+} = \pm 1 \).
Upon calculating the Feynman amplitudes, we find that
\[ M_\alpha = \sqrt{2}i M \beta(\alpha) \] \hspace{1cm} (38)
where \( \beta(\alpha) \) depends only on the electroweak coupling constants. For the decay of the photon one finds
\[ \beta^{(\gamma)}(1) = ie \] \hspace{1cm} (39)
and, by parity invariance, we get
\[ \beta^{(\gamma)}(-1) = \beta^{(\gamma)}(1) \] \hspace{1cm} (40)

Then, from Eq. 14, the decay parameters \( C_l \) are given by
\[ C_l^{(\gamma)}(00; 00) = \sqrt{3}(2M^2) \sum_\alpha |\beta^{(\gamma)}(\alpha)|^2 \langle 0|1\alpha; 1 - \alpha \rangle; \] \hspace{1cm} (41)
which gives
\[ C_0^{(\gamma)}(00; 00) = 4e^2 M^2, \]
\[ C_1^{(\gamma)}(00; 00) = 0, \]
\[ C_2^{(\gamma)}(00; 00) = 2\sqrt{2}e^2 M^2. \] \hspace{1cm} (42)

For the \( Z^* \), \( \beta^{(Z)}(1) = ie(v - a) \) and \( \beta^{(Z)}(-1) = ie(v + a) \), and so we get
\[ C_0^{(Z)}(00; 00) = (v^2 + a^2)C_0^{(\gamma)}(00; 00), \]
\[ C_1^{(Z)}(00; 00) = -4\sqrt{6}M^2 e^2 va, \]
\[ C_2^{(Z)}(00; 00) = (v^2 + a^2)C_2^{(\gamma)}(00; 00). \] \hspace{1cm} (43)

Finally, for the interference term, we obtain
\[ C_0^{(\gamma Z)}(00; 00) = vC_0^{(\gamma)}(00; 00), \]
\[ C_1^{(\gamma Z)}(00; 00) = -2\sqrt{6}M^2 e^2 a, \]
\[ C_2^{(\gamma Z)}(00; 00) = vC_2^{(\gamma)}(00; 00). \] \hspace{1cm} (44)
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Figure captions

Fig. 1 Asymmetries for the Drell-Yan process at $\sqrt{s} = 500$ GeV, as a function of $p_T$. The asymmetries shown in this figure are constructed out of the cross sections differential in $p_T$, $y_1$, and $\hat{\tau}$ (the jet rapidities have been integrated over), with $\hat{\tau}$ fixed at $M/\sqrt{s}$ and $y_1 = 0$. The cross sections are integrated over the lepton angle $\theta_l$. The dilepton mass $M$ is shown on top of the figures. The full, dashed and dotted lines correspond respectively to Set I, II and III polarised densities, described in the text. The upper row of plots are for the single-spin asymmetries $A_s$, the lower for the double-spin asymmetries $A_d$. Note that the scale for the first two plots is much smaller than for the rest.

Fig. 2 Cross-sections for the Drell-Yan process at $\sqrt{s} = 500$ GeV, as a function of $p_T$. The caption $d\sigma/d\Gamma$ refers to $d\sigma/dp_Tdy_1d\hat{\tau}$, with $\hat{\tau}$ fixed at $M/\sqrt{s}$ and $y_1 = 0$. The cross sections are integrated over the lepton angle $\theta_l$. The full, dashed and dotted lines correspond respectively to $M=10$ GeV, $M=50$ GeV and $M = M_Z$, where $M$ is the dilepton mass. The cross sections are in nb/GeV.

Fig. 3 Same as in Fig. 1, but for the asymmetries $A_s^1$ and $A_d^1$ defined in Eq. [21].

Fig. 4 Same as in Fig. 1, but for the asymmetries $A_s^2$ and $A_d^2$ defined in Eq. [21].