A simple elastoplastic dynamic constitutive model of saturated structural soft clay

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Abstract. The dynamic constitutive relation of soil is widely applied to the ground design of marine structure and infrastructure in offshore geotechnical engineering. Increasing evidence from marine geotechnical design has shown that the traditional dynamic constitutive model has uncertain theories, many parameters, and complexity. Therefore, it is urgent to overcome these limitations and develop a simple and practical elastoplastic dynamic constitutive relation for the saturated structural soft clay. A simple dynamic constitutive relation with 5 parameters was proposed in this study, which can reflect the strain accumulation, hysteretic characteristics and cyclic degradation of the stress-strain for structural soft clay. First, the Drucker-Prager model was selected as the bounding surface, hardening modulus equation was established by the hardening law, interpolation function and structural strength coefficient. Then, the incremental stress-strain relationship was derived based on the incremental theory. The UMAT subroutine was compiled by FORTRAN language. Finally, the cyclic triaxial test and numerical simulation were carried out, and the stress-strain relationship was analyzed. The research results showed that the test results were in good agreement with the prediction results of the model.

1. Introduction

With the proposal of the national strategy of "marine potestatem" proposed by the central government of China, the development of offshore energy was booming. Many offshore structures were built on clay seabed, such as suction caisson, gravity foundation, and pile foundation. The offshore structures were subjected to long-term wave loading, which often led to large deformation or instability of soft clay ground. The offshore structures could not work normally and many engineering accidents were arising when engineering designers neglected soft clay property and complex dynamic loading effect. Moreover, the design of the offshore ground was different from that of onshore ground. The offshore ground bears cyclic loading for a period of a long time so that the stress-strain relationship of soil captured nonlinear, strain accumulation, and hysteretic characteristics. At present, the static or quasi-static model was adopted, which was not consistent with the actual situation [1]. Although some complex dynamic models could suitably reflect the stress-strain characteristics of soft clay, they contained more than ten parameters, which was not convenient for engineers to apply in the engineering practice [2]. In addition, it was generally believed that structural soft clay often presented...
cyclic degradation under cyclic loading. However, most of the dynamic constitutive models do not consider the structural characteristics of the soil. The structural characteristics were the inherent property of the undisturbed soil, the remolded soil had no structural characteristics [3]. Most of the constitutive models were established on the basis of remolded soil property, such as the Cambridge clay model [4], bounding surface model [5], etc. Increasing evidence from engineering design had demonstrated that the settlement of soil was overestimated and underestimated, which depended on whether engineers considered the structural behaviors of soil [6]. Therefore, knowledge about a simple and applicable elastoplastic dynamic constitutive relation of structural clay was essential for offshore ground design.

To solve the above problems, a simple elastoplastic dynamic constitutive model of saturated structural soft clay was provided based on the bounding surface theory in this paper. Bounding surface equation, hardening law and stress-strain relation were expressed. And the structural strength coefficient was defined to reflect the structural behaviors of soft clay, which introduced in the model of elastoplastic modulus. Furthermore, the elastoplastic modulus was obtained through the interpolating function between the initial modulus and the bounding modulus. Two kinds of hardening parameters of structural strength coefficient and hardening modulus were presented in this research. The validity of the model was verified by comparing the experimental results with the model predicted results. The model had the advantages of fewer model parameters and can reasonably present nonlinear, strain accumulation, hysteresis characteristics and cyclic degradation of saturated structural soft clay, which provided a reference for geotechnical engineering design.

2. Establishment of dynamic constitutive model

2.1. Bounding Surface Equation

Bounding surface equation characterized the ultimate failure of the soil element. With the increase of the stress, and the loading surface reached the state of strength of the soils, which indicated that the soil would be a failure. Drucker-Prager, describing the effect of hydrostatic pressure on yield and failure, was chosen to establish the bounding surface equation $f$, as shown in equation (1):

$$f = ad_k + \sqrt{J_2} - k = 0$$  

(1)

where $\alpha$ and $k$ were the parameter of materials, $J_1$ was the first stress tensor invariant, $J_2$ was the second deviatoric stress tensor invariant. To simplify the calculation, equation (1) was also expressed by equation (2) as follows:

$$\alpha(\sigma_x + \sigma_y + \sigma_z) + \frac{1}{\sqrt{6}}[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2] + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2) - k = 0$$  

(2)

where $\sigma_x$, $\sigma_y$, $\sigma_z$ were the normal stress of plane which was perpendicular to the axis $x$, $y$, $z$, respectively. Soil stress state respectively was $\sigma_x=\sigma_y$, $\tau_{xy}=0$, $\tau_{yz}=\tau_{xz}$ with respect to the in-situ state. Hence, equation (3) was obtained by equation (2)

$$\alpha(2\sigma_x + \sigma_z) + \frac{1}{\sqrt{3}}(\sigma_x - \sigma_z)^2 + 12\tau_{yz}^2 - k = 0$$  

(3)

To simplify the problem, the influence of hydrostatic pressure on plastic deformation was not considered, namely, $\alpha=0$. Therefore, equation (4) was defined as the bounding surface equation according to equation (3)

$$(\sigma_y - \sigma_x)^2 + 36\tau_{yz}^2 = 3k^2$$  

(4)

Equation (4) demonstrated that the bounding surface was the circle, and the radius was $\sqrt{3}k$ when the $Y$-axis represented $\sigma_y$, $\sigma_x$, and $X$-axis represented $6\tau_{yz}$. Combined with the Mohor-Coulomb criterion, the radius parameter $k$ was obtained as follows:
It was noted that $k$ was determined by the cyclic triaxial test which could confirm dynamic cohesive force, $c_d$, and dynamic friction angle, $\phi_d$, in the given cyclic numbers.

### 2.2. Hardening law

In the triaxial stress state, the shear stress $\tau_{yz}=0$, the point always remained on the Y axis, which was plotted in Figure 1. For example, the track O-A-B in the stress-strain relation corresponded to the track O-A-B in the stress space. Noted that the elastoplastic modulus of stress-strain relation was maximum in the initial loading, which was expressed by $G_{\text{max}}$ equal to the elastic shear modulus $G_0$. The track B-C of the stress-strain relation corresponded to the track B-C of the stress space in the unloading phase. Similarly, the track D-E, E-F, G-H of the stress-strain relation corresponded to the track D-E, E-F, G-H of the stress space. However, when the point reached to the bounding surface, the soil element was a failure.

**Figure 1.** Hardening law.

To take the maximum elastoplastic modulus, $G_{\text{max}}$, and the structural characteristics into account, the evolution of elastoplastic modulus was defined in the loading and unloading phase, as shown in Figure 2. The structural strength coefficient $m$ was defined as equation (6), which was designed to illustrate the difference between the static strength and the dynamic strength, caused by the structural property of clay.

$$m = \frac{m_0}{m_c}$$

where $m_c$ was the undrained strength of soft clay under the static loading, $m_0$ was the undrained strength of soft clay under the dynamic loading. Dynamic strength was less than static strength for long period wave loading [7], $0<m<1$.

Figure 2 showed $O$ was the initial loading point, $B$ and $E$ were the stress reversal point, $A$ and $C$ were loading point and unloading point, respectively. $\delta$ was the distance between the loading point or unloading point and bounding surface. $\delta_0$ was the distance between the initial loading point and bounding surface or the distance between the stress reversal point and the bounding surface. It was assumed that the plastic modulus was equal to the maximum value $G_0$ at initial loading when the stress was reversed. Therefore, the modulus considering the structural strength coefficients could be expressed as equation (7). When $\delta$ approached $\delta_0$, it was demonstrated that the current stress point was on the bounding surface, the plastic modulus was equal to $\zeta m G_0$. On the contrary, when $\delta$ approached 0, it was illustrated that the current stress was at the reverse loading point or the initial loading point, the plastic modulus was equal to $\zeta G_0$. The range of the plastic modulus was $\zeta m G_0 \sim \zeta G_0$. 

$$k = \sqrt{J_2} = \frac{1}{\sqrt{2}} = \frac{2\sqrt{3}c_d \cos \phi_d}{3 - \sin \phi_d}$$ (5)
where $\zeta$ was related to cyclic numbers, which could control the modulus degradation.

\[ G = \zeta \left( G_m + (G_n - mG_m)(1 - \frac{c}{c_0}) \right) \quad (7) \]

2.3. Stress-strain Relation Based on Incremental Theory

The elastoplastic theory contained two methods of plasticity total theory and plasticity incremental theory. However, the conditions of the establishment of constitutive relations were extremely harsh based on the plasticity total theory, it was not applicable to the soil. Hence, the relation between the stress increment and strain increment was popularly established based on the plasticity incremental theory, which could be expressed as follows:

\[ d\varepsilon_{ij} = d\varepsilon_{ij}^e + d\varepsilon_{ij}^p \quad (8) \]

where $d\varepsilon_{ij}$ was total strain increment tensor, $d\varepsilon_{ij}^e$ was the elastic strain increment tensor, $d\varepsilon_{ij}^p$ was the plastic strain increment tensor. The elastic deformation was only caused by deviatoric stress in the elastic deformation stage. The elastic strain could be expressed as equation (9)

\[ d\varepsilon_{ij}^e = d\varepsilon_{ij}^e = \frac{ds_{ij}}{2G} \quad (9) \]

where $d\varepsilon_{ij}^e$ was the deviatoric elastic strain increment tensor, $ds_{ij}$ was the deviatoric elastic stress increment tensor, $G$ was shear modulus.

According to the Drucker-Prager yield criterion, and associated flow rule was taken into consideration, which explained that plastic potential function was equal to yield function. Therefore, $\frac{\partial f}{\partial \sigma_{ij}}$ was obtained.

\[ \frac{\partial f}{\partial \sigma_{ij}} = a\sigma_{ij} + \frac{s_{ij}}{2\sqrt{J_2}} \quad (10) \]

where $f$ was the plastic potential function, $\sigma_{ij}$ was the stress tensor.

It was assumed that the effects of volume stress and strain were not taken into account, the equation (10) could be simplified into equation (11).

\[ \frac{\partial f}{\partial \sigma_{ij}} = \frac{s_{ij}}{2\sqrt{J_2}} \frac{s_{ij}}{s_{ij}} ds_{ij} \quad (11) \]

According to the plasticity incremental theory, the plastic strain increment was expressed as follows:
The vibration frequency of 0.5Hz was selected. The total strain increment was shown as follows:

\[
de_{ij} = \frac{ds_{yy}}{2G} + d\lambda \left( \frac{\partial f}{\partial \sigma_{yy}} \right)_{s} + \frac{ds_{zz}}{2J_{2}} + \frac{s_{yy}}{2J_{2}} ds_{zz}
\]

Considering the stress state in the triaxial test, the deviatoric elastic stress tensor was shown as follows:

\[
s_{ij} = \begin{bmatrix}
\sigma_{x} - \frac{2\sigma_{x} + \sigma_{z}}{3} & 0 & 0 \\
0 & \sigma_{y} - \frac{2\sigma_{y} + \sigma_{z}}{3} & 0 \\
0 & 0 & \sigma_{z} - \frac{2\sigma_{z} + \sigma_{x}}{3}
\end{bmatrix}
\]

Because the dynamic loading was applied under the undrained conditions, the volume strain was zero. Therefore, the deviatoric elastic strain in the X, Y, Z direction was obtained as follows:

\[
\begin{align*}
\Delta e_{x} &= \Delta e_{y} = \Delta e_{z} \\
\Delta e_{c} &= \Delta e_{x} = \Delta e_{y} = \Delta e_{z}
\end{align*}
\]

Equation (15) was taken into equation (14), the strain increment was obtained as follows:

\[
\Delta e_{c} = 2\left( \frac{1}{2G} + d\lambda \right) d(\sigma_{c} - \sigma_{c})
\]

where, \( d\lambda + 1/2G \) was equal to \( 1/(G+G_{0}) \), and \( d\lambda \) was obtained as follows:

\[
d\lambda = \frac{1}{G + G_{0}} - \frac{1}{2G}
\]

Therefore, the elastoplastic matrix \( D_{ep} \) could be obtained, which laid the foundation for developing the UMAT program.

\[
D_{ep} = [D] - \frac{[D]}{(0.5G + d\lambda)} \left[ \frac{\partial f}{\partial \sigma_{ij}} \right]_{(s)} \left[ \frac{\partial f}{\partial \sigma_{ij}} \right]_{(s)}^{T}
\]

where \([D]_{s}\) was the elastic matrix.

3. Cyclic triaxial test and numerical simulation establishment

To verify the correctness of the model, a cyclic triaxial testing apparatus (British Geotechnical Digital Systems (GDS) Instruments, Ltd., Unit 32 Murrell Green Business Park, London, United Kingdom) was used to simulate the low-frequency characteristics of wave loading. The cyclic loading device was controlled by an advanced servo motor, which could effectively filter the low-frequency noise signals. Krica et al., (2013) [8] and Feng et al. (2020) [9] proposed that the vibration frequency of 0.5Hz was more suitable to simulate the wave load with low-frequency characteristics. Therefore, the sinusoidal loading and vibration frequency (1/T) of 0.5Hz were selected. Three groups of cyclic triaxial tests (three samples) were carried out to consider the influence of different cyclic stress ratios (CSRs) on relation between stress and strain of soil. The vibration amplitude (A) was respectively 10kPa, 20kPa and 30kPa, the corresponding CSRs were 0.1, 0.2 and 0.3. It is noted that there are uncertainties and
controversies about the failure criteria of soft clay. Chen et al. (2005) [10] and Zheng et al. (2013) [11] employed the accumulative plastic strain of 1%~5% as the soil failure criterion. Lei et al. (2009) [12] demonstrated that the accumulative plastic strain of 1.27%~2.13% was more applicable to evaluate the failure criterion of structural soft clay. Combined with the wave loading characteristics with low frequency and low cyclic stress level, the accumulative plastic strain of 2% was selected in this paper to evaluate the end time of cyclic triaxial test.

Based on the triaxial sample size of 39.1mm × 80mm, the numerical software ABAQUS was used to establish the model, as shown in Figure 3. The displacement boundary conditions in the X, Y and Z directions of the bottom constraint were set. The boundary conditions of the confining pressure of 100kPa were set around the cylinder and on the top surface. C3D8P element was selected and 525 meshes were divided. The initial void ratio of 1.62 and the anisotropic consolidation stress of \( \sigma_{11}=200kPa, \sigma_{22}=\sigma_{33}=100kPa \) were predefined on the elements.

![Figure 3. Establishment of numerical model.](image)

Because the accuracy of implicit algorithm was higher than that of explicit algorithm, an improved Euler implicit integration algorithm with error control was employed to achieve UMAT subroutine in this paper. In this method, the strain increment \([\Delta e]\) was decomposed into several sub-step strain increments. The length of each sub-step could be controlled by the iterative sub-step control variable \(\Delta s\), and the value range of \(\Delta s\) was \([0,1]\). The accuracy of the result was controlled by artificially setting the error SSTOL. SSTOL < 10\(^{-4}\) met the calculation accuracy and convergence. The key to FORTRAN programming was to form the elastic-plastic stiffness matrix \(DDSD\).

In addition, radius parameter \(k\), structural strength coefficient \(m\), initial shear modulus \(G_0\), degradation parameter \(\zeta\) and Poisson's ratio \(\mu\) should be determined by experimental test and empirical equation, as shown in Table 1. \(G_0\) was calculated by the compression modulus \(E\) value of 2.3 MPa determined by a compression test \((G_0=0.5E/(1+\mu))\). It was noted that \(\zeta\) was constantly adjusted with the increase of vibration number to capture the modulus degradation.

| Parameter | Value   | \(k\) | \(m\) | \(G_0\) MPa | \(\zeta\) | \(\mu\) |
|-----------|---------|-------|-------|-------------|----------|--------|
|           |         | 8.17  | 0.73  | 0.8         | 1.0-0.54lgN | 0.432  |

4. Model validation

Figure 4 showed the test results were in good agreement with the model prediction results. With the increase of CSRs, the hysteresis loop width increased. The larger the number of cycles or the smaller the CSR presented, the smaller the hysteresis loop width performed. When the CSR was 0.1, the hysteretic loop width was the thinnest. With the increase of vibration numbers, the hysteretic curve became denser and presented elastic deformation. When the CSR was 0.3, the deformation of the test results suddenly increased in the initial stage. This showed that under the condition of a high-stress level, the accumulated plastic deformation of soft clay was concentrated several cycles of loading in the initial phase. With the increase of vibration numbers, the soil showed a stable accumulative deformation trend. In addition, the hysteresis loop slope showed a minor declining trend with the increase of cyclic number, which can demonstrate the dynamic modulus presented slight cyclic degradation. To illustrate the cyclic degradation, the structural strength coefficient was adapted to
control the hysteresis loop slope in the constitutive model. Through comparative analysis, it was found that the hysteresis loop slope predicted by the constitutive model was in good consistency with that measured by the cyclic triaxial test.

(a) Test result for CSR=0.1

(b) Model prediction result for CSR=0.1

(c) Test result for CSR=0.2

(d) Model prediction result for CSR=0.2

(e) Test result for CSR=0.3

(f) Model prediction result for CSR=0.3

Figure 4. Comparative analysis of model prediction results and test results.

5. Conclusion
Combined with the Drucker-Prager model and triaxial stress state, the circular bounding surface equation was established. The structural strength coefficient was introduced in elastoplastic modulus to characterize the cyclic degradation behaviors based on the hardening law. The model consisted of 5 parameters with understandable physical meaning, and numerical software ABAQUS was employed to design UMAT compiled by FORTRAN language. The test results and model prediction results were compared and analyzed. The model prediction results and test data had good consistency, which verified the correctness of the model. The specific conclusions were as follows:
The Drucker-Prager model contributed to a simplified circular bounding surface equation. By defining the relationship between the current stress point and the boundary point, the interpolation function of the hardening modulus was constructed, and the structural coefficient was introduced into the hardening modulus to control the slope of the hysteretic curve to reasonably express the cyclic
degradation characteristics. The UMAT subroutine was developed by FORTRAN programming and an improved Euler implicit integration algorithm with error control was studied, and the iterative error control was SSTOL < 10^{-4}. Combined with the numerical simulation and the laboratory undrained cyclic triaxial test, the hysteretic curve was analyzed. It was found that the test results were in good agreement with the model prediction results. With the increase of the cyclic stress ratio, the width of the hysteresis loop increased, and the hysteresis loop presented a slight cyclic degradation with the increase of the cyclic numbers.

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