Geopotential measurements with synchronously linked optical lattice clocks

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According to Einstein’s theory of relativity, the passage of time changes in a gravitational field1,2. On Earth, raising a clock by 1 cm increases its apparent tick rate by 1.1 parts in 1018, allowing chronometric levelling3 through comparison of optical clocks4–6. Here, we demonstrate such geopotential measurements by determining the height difference of master and slave clocks separated by 15 km with an uncertainty of 5 cm. A subharmonic of the master clock laser is delivered through a telecom fibre6 to synchronously operate7 the distant clocks. Clocks operated under such phase coherence reject clock laser noise and facilitate proposals for linking clocks8,9 and interferometers10. Taken over half a year, 11 measurements determine the fractional frequency difference between the two clocks to be 1,652.9(5.9) × 10−18, consistent with an independent measurement by levelling and gravimetry11. Our system demonstrates a building block for an internet of clocks, which may constitute ‘quantum benchmarks’, serving as height references with dynamic responses.

Stable disseminations of clock signals between metrology laboratories at the level of 10−18 are being actively pursued with a future redefinition of the second in sight12. Optical frequency links13 using telecom fibres allow stable, better clocks than achieved with satellite links14, and are therefore used for frequency comparisons that require accuracies below 10−16 (refs 15,16). Fibre links17–19 over hundreds of kilometres are being developed worldwide and two optical lattice clocks that are separated by 700 km have been compared to 5 × 10−17 (ref. 20), limited only by the clocks’ uncertainties. Such fibre-linked optical clocks enable chronometric levelling20 by measuring the geopotential differences Δϕ between the clocks via the gravitational redshift21, Δν/ν0 = Δϕ/c2, where ν0 is the clock frequency and c is the speed of light. We envision a future internet of clocks that consists of local clock networks as shown in Fig. 1a, where some of the slave clocks may be transportable21 to allow local clock networks as shown in Fig. 1a, where some of the slave clocks may be transportable21 to allow

optical lattice clocks that interrogate hundreds of atoms or more, the Dick effect noise22, which originates from the frequency noise of the clock laser, overwhelms the quantum projection noise and limits the clock instability23. In this regard, reducing the frequency noise of the laser is the primary approach to improving the clock instability. However, the stabilities of mobile clock lasers24 that are used in transportable clocks23 are often inferior to state-of-the-art laboratory lasers that permit longer25 and colder26 cavities with better shielding from acoustic and seismic noise.

We employ three cryogenic Sr optical lattice clocks2 that are equipped with master and slave clock lasers as depicted in Fig. 1b. The master clocks at RIKEN are interrogated by a master laser with an instability of 3 × 10−16 at 1 s. The slave laser at The University of Tokyo (UTokyo) has an independent instability of 2 × 10−15 at 1 s that is compatible with the performance of typical mobile lasers24. The three clocks, RIKEN1, RIKEN2 and UT, operate on the 1S0 – 1P0 transition of 87Sr at ν0 ≈ 429.2 THz (698 nm), and are connected by a 30-km-long telecom fibre. The master clock delivers a subharmonic of the clock laser ν0/2 ≈ 214.6 THz (1.397 nm) to the slave clock through the stabilized fibre link5 without employing frequency combs. Our direct clock link significantly simplifies the set-up compared with reported clock links15,16,20 and improves the long-term reliability.

The cryogenic clocks5 suppress the uncertainty due to the black-body radiation shift to below 1 × 10−18 by surrounding the lattice-trapped atoms with copper-enclosures that are coated for high emissivity and stabilized at temperatures of about 100 K. After spin-polarizing the atoms, clock laser pulses with a duration of Tτ = 200–400 ms interrogate N ≈ 500–1,200 atoms every Tc = 1.5 s, where Tτ is the interrogation time and Tc is the cycle time. The excitation probabilities of the clock transitions, as displayed in Fig. 2a–c, are used to steer the clock laser frequencies by applying fRIKEN1,2 and fUT to frequency shifters (FS1, FS2 and FS3 in Fig. 1b). Phase-locking the slave laser to the master laser reduces the scatter of the excitation probabilities of UT, as shown in Fig. 2b. Moreover, by operating the master and slave clocks synchronously by referencing timing signals delivered using the same optical fibre, the excitation probabilities correlate as shown in Fig. 2c (see Methods).

In this phase-locked and synchronous mode of operation, as the clock laser is common to both of the clocks, the frequency difference between the clocks is directly obtained from numerical frequency data that is applied to synthesizers (DDS1 and DDS3 in Fig. 1b) as ΔνUT-RIKEN(tj) = fUT(tj + τd) − fRIKEN(tj) for the jth measurement sequence at tj = jTc, where τd ≈ 0.16 ms is the transit time of
Three cryogenic Sr optical lattice clocks, two (RIKEN1 and RIKEN2) at RIKEN and one (UT) at UTokyo, act as master and slave clocks, forming a local clock network. At RIKEN, a master laser is stabilized to a 40-cm-long reference cavity. Its subharmonic ($\nu_{C,RIKEN} = 429\text{ THz}$) is sent to UTokyo via a 30-km-long telecom fibre equipped with a fibre noise cancellation (FNC) system. $\nu_{C,RIKEN}/2$ denotes the transferred laser contaminated with the residual fibre noise. A slave laser $\nu_{C,UT}$ is pre-stabilized to a 7.5-cm-long cavity. Using $\nu_{beat} = \nu_{C,UT}/2 - \nu_{C,RIKEN}/2$, the slave laser is offset-locked to the master laser with a frequency shifter (FS4). The steering frequencies $f_{RIKEN,2}$ and $f_{UT}$ are generated by direct digital synthesizers (DDS1, DDS2, DDS3). The master clock sends a trigger signal to synchronously interrogate the slave clock using a 1.55-μm laser coupled to the same fibre through wavelength-division multiplexers (WDM). PD, photodiode. Frequency noise of the master and slave lasers. After stabilizing to the master laser, the frequency noise of $\nu_{C,UT}$ (black dashed line) is suppressed to the blue solid line, which is the square root of the squared sum of the noise of the master laser $\nu_{C,RIKEN}$ (red dashed line) and the fibre (orange dashed line). The green dashed line shows the residual fibre noise assuming the free-running fibre noise $h_s \approx 1\text{ Hz}^2\text{ Hz}^{-1}\text{ km}^{-1}$ (ref. 27). The yellow shaded area shows the non-common frequency noise (see Methods). Purple circles (plotted against the right y axis) show the sensitivity to the Dick effect, which has a sharp cutoff at $f_s = 1/T_c$.

Figure 1 | Schematic of the experimental setup. a, Local clock networks, each consisting of synchronously interrogated master and slave clocks, are linked by long-distance fibres to create an internet of clocks. b, Three cryogenic Sr optical lattice clocks, two (RIKEN1 and RIKEN2) at RIKEN and one (UT) at UTokyo, act as master and slave clocks, forming a local clock network. At RIKEN, a master laser is stabilized to a 40-cm-long reference cavity. Its subharmonic ($\nu_{C,RIKEN}/2 = 215\text{ THz}$) is sent to UTokyo via a 30-km-long telecom fibre equipped with a fibre noise cancellation (FNC) system. $\nu_{C,RIKEN}/2$ denotes the transferred laser contaminated with the residual fibre noise. A slave laser $\nu_{C,UT}$ is pre-stabilized to a 7.5-cm-long cavity. Using $\nu_{beat} = \nu_{C,UT}/2 - \nu_{C,RIKEN}/2$, the slave laser is offset-locked to the master laser with a frequency shifter (FS4). The steering frequencies $f_{RIKEN,2}$ and $f_{UT}$ are generated by direct digital synthesizers (DDS1, DDS2, DDS3). The master clock sends a trigger signal to synchronously interrogate the slave clock using a 1.55-μm laser coupled to the same fibre through wavelength-division multiplexers (WDM). PD, photodiode. Frequency noise of the master and slave lasers. After stabilizing to the master laser, the frequency noise of $\nu_{C,UT}$ (black dashed line) is suppressed to the blue solid line, which is the square root of the squared sum of the noise of the master laser $\nu_{C,RIKEN}$ (red dashed line) and the fibre (orange dashed line). The green dashed line shows the residual fibre noise assuming the free-running fibre noise $h_s \approx 1\text{ Hz}^2\text{ Hz}^{-1}\text{ km}^{-1}$ (ref. 27). The yellow shaded area shows the non-common frequency noise (see Methods). Purple circles (plotted against the right y axis) show the sensitivity to the Dick effect, which has a sharp cutoff at $f_s = 1/T_c$.
case, the overlapping Allan deviation improves to $6 \times 10^{-16} \, (\tau/s)^{1/2}$ (red circles). Dashed lines with corresponding colours show the Dick effect limited instability due to the residual fibre noise $\sqrt{S_v^{\text{free}}(f)}$ (orange dashed line in Fig. 1c) measured at $v \approx 429$ THz (see Methods). This frequency noise is consistent with the delay-suppressed fibre noise given by $S_v^{\text{free}}(f) = (2\pi f)^2 a h L$ for our free-running fibre noise $h(\nu_f) \approx 70 \, \text{Hz}^2 \, \text{Hz}^{-1} \, \text{km}^{-1}$ (ref. 6), where we assume $a \sim 1$ and a fractional frequency instability is unchanged in the harmonic generation, that is, $S_v^{\text{free}}(f) = 4S_v^{\text{free}}(f(i))$. Such synchronous comparisons should work more effectively for quiet fibres with $h(\nu_f) \approx 1 \, \text{Hz}^2 \, \text{Hz}^{-1} \, \text{km}^{-1}$ as reported in the German link27, which (for the same 30 km fibre length) would improve the Dick effect limited instability to $6 \times 10^{-17} \, (\tau/s)^{1/2}$, as shown by a red line in Fig. 2d.

Blue circles in Fig. 3a show 11 measurements performed over 6 months. The mean value weighted by the statistical uncertainty on each measurement determines the frequency difference to be $\Delta_{\text{UT–RIKEN}}/\nu_0 = -1,652.9(5.9) \times 10^{-18}$ as indicated by the blue shaded region, where the total uncertainty includes statistical (calculated by the standard error) and systematic uncertainties of $1.5 \times 10^{-18}$ and $5.7 \times 10^{-18}$, respectively. Table 1 summarizes the corrections and uncertainties for the slave clock $\nu_{\text{UT}}$, the master clock $\nu_{\text{RIKEN1}}$, and the frequency difference $\Delta_{\text{UT–RIKEN}} = (\nu_{\text{UT}} - \nu_{\text{RIKEN1}})$. The observed gravitational redshift determines the geopotential difference to be $\Delta h_{\text{UT–RIKEN}} = (\Delta_{\text{UT–RIKEN}}/\nu_0) c^2 = -148.55(53) \, \text{m}^2 \, \text{s}^{-2}$, which agrees with the geodetically determined value of $\Delta h_{\text{UT–RIKEN}} = -148.14(6) \, \text{m}^2 \, \text{s}^{-2}$ (red area) measured with spirit-levelling and gravimetry by the Geospatial Information Authority of Japan in February 2014 (see Methods).

The complete data set for the last measurement in Fig. 3a is displayed in Fig. 3b, which monitors the frequency difference for three days with an uptime of 73%. The two clocks at RIKEN agree with $\Delta \nu_{\text{RIKEN2–RIKEN1}}/\nu_0 = 1.0 \pm (1.3)_{\text{stat}} \pm (3.2)_{\text{syst}} \times 10^{-18}$ (blue dots), demonstrating the reproducibility of these clocks. On the other hand, the master–slave comparison shows a gravitational
redshift $\Delta \nu_{\text{UT-RIKEN}}/\nu_0 = [-1,653.3 \pm (4.8)_{\text{stat}} \pm (5.7)_{\text{sys}}] \times 10^{-18}$ (red dots). The relevant overlapping Allan deviation (shown by empty blue circles in Fig. 2d) reaches $6.4 \times 10^{-19}$ after $6 \text{ h}$, corresponding to a height uncertainty of 5.8 cm.

Figure 4 shows the calculated tidal perturbation $\phi_{\text{tidal}}(t)$ on the geopotential due to astronomical tides and ocean tidal loading for 15 days, including the days of the measurements (see Methods). The tidal perturbation (black line) is composed predominantly of semi-diurnal and diurnal variations. The root-mean-square amplitudes of the perturbation are retained to about 70% (red line) and about 92% (blue line), for 6 h and 3 h averaging, respectively, suggesting that 6 h averaging is the characteristic timescale over which clocks reveal the tidal perturbation. Although this tidal perturbation amounts to approximately $6 \times 10^{-17}$ in terms of the proper time passage (Fig. 4a), it is common for clocks at UTokyo and RIKEN; it cancels to approximately $2 \times 10^{-19}$ in the clock comparison shown in Fig. 4b. The clock shift therefore directly measures the height difference $\Delta h=(\Delta \nu_{\text{UT-RIKEN}}/\nu_0)(c^2/g) = -1,516(5) \text{ cm}$ with $g = 9.798 \text{ m s}^{-2}$ the gravitational acceleration. Assuming a clock positioned at the tidal observation station at Akune, separated

| Table 1 | Uncertainty budgets for clocks. |
|----------------------------------|-------------------------------|
| **Contributor**                  | **$^{87}$Sr (UT)** | **$^{87}$Sr (RIKEN)** | **$^{87}$Sr (UT) - $^{87}$Sr (RIKEN)** |
| Quadratic Zeeman shift           | 109.0             | 0.9                     | 117.1             | 0.9       | -8.2 | 0.5 |
| Blackbody radiation shift*       | 79.1              | 0.9                     | 54.2              | 0.9       | 24.9 | 1.2 |
| Lattice light shift              | 3.5               | 5.0                     | 3.5               | 3.4       | 0.0  | 4.4 |
| Clock light shift                | 0.047             | 0.023                   | 0.047             | 0.023     | 0.0  | 0.014 |
| First-order Doppler shift        | 0                 | 0.5                     | 0                 | 0.5       | 0.0  | 0.07 |
| AOM chirp and switching          | 1.3               | 3.9                     | 0.7               | 2.5       | 0.6  | 0.5 |
| Servo error                      | 1.1               | 5.2                     | 0.4               | 1.9       | 0.7  | 3.3 |
| Systematic total                 | 194.0             | 8.3                     | 176.0             | 4.8       | 18.0 | 5.7 |

The systematic corrections and uncertainties for the UT clock ($\Delta \nu_{\text{UT}}$), one of the RIKEN clocks ($\Delta \nu_{\text{RIKEN}}$) and for the fractional frequency difference $\Delta \nu_{\text{UT-RIKEN}}/\nu_0 = (\Delta \nu_{\text{UT}} - \Delta \nu_{\text{RIKEN}})/\nu_0$ are shown. The uncertainties of $\Delta \nu_{\text{UT-RIKEN}}$ originating from the quadratic Zeeman shift, lattice shift, clock light shift, servo error, and density shift are smaller than the square root of quadratic sum of the individual clocks’ uncertainties due to the partial cancellation of the effects. Representative values for the three-day-long campaign (Fig. 3b) are given. Acousto-optic modulator (AOM) is used for frequency shifters FS1 (RIKEN) and FS3 (UT). The cryogenic chamber for UT (RIKEN) operates at a temperature of 105 K (95 K). See Methods.
from RIKEN by 970 km and 9.4° of longitude, the tidal potential difference would introduce peak-to-peak frequency variations Δ\(\Phi_{\text{tidal}}(t)/c^2\) that exceed 1 × 10^{-17}, as shown in Fig. 4c, which is within the reach of the clocks' uncertainties.

In summary, we have demonstrated centimetre-level chronometric levelling using synchronously operated cryogenic optical lattice clocks as a building block for an internet of clocks that is applicable to geosciences and fundamental physics. The observed instability of 6 \times 10^{-16} (\tau/s)^{1/2} with the master–slave configuration will be reduced further by improving the stability of the master laser, which would allow an extended interrogation time, thereby reducing the impact of the residual noise of the fibre link. On the other hand, the systematic uncertainty of the clock, which is mainly limited by the lattice-induced light shift, can be reduced to a low level of 10^{-19} by employing an operational magic frequency\(^{28}\) leading to millimetre-level chronometric levelling. Quantum benchmarks that consist of such networked clocks will improve the realization of a long-range height reference frame over a distance of 1,000 km by an order of magnitude as the clocks are periodically interrogated atomic resonator. IEEE Trans. Ultrason. Ferroelectr. Freq. Control 45, 887–894 (1998).

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Acknowledgements

This work is partially supported by the Japan Society for the Promotion of Science through its Funding Program for World-Leading Innovative R&D on Science and Technology Program and by the Photon Frontier Network Program of the Ministry of Education, Culture, Sports, Science and Technology, Japan. The authors thank M. Das for the development of cryogenic clocks, N. Nemitz and K. Gibble for careful reading of the manuscript and for useful comments, and M. Musha for the loan of a phase-locked loop circuit for the clock laser.

Author contributions

T.T., M.T. and I.U. operated the clocks at UTokyo and RIKEN and analysed data. T.A. and A.Y. were responsible for the fibre link and M.T. and N.O. for the clock lasers. Y.K. evaluated the tidal perturbation. B.M. supervised the spirit-levelling measurements. Y.K., B.M., H.M. and H.K. discussed the geodetic applications of a quantum benchmark. T.T., Y.K. and H.K. wrote the manuscript. H.K. planned and supervised the experiments. All authors discussed the results and commented on the manuscript.

Additional information

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Competing financial interests

The authors declare no competing financial interests.
Methods

Definition of height and its geodetic determination. The commonly used notion of ‘height above sea level’, or orthometric height in geodetic terms, is defined as the vertical distance to the geoid—the equipotential surface that best fits the global mean sea level. Here, the geopotential of the Earth is the sum of its gravitational potential and the potential of its centrifugal force due to its rotation. Orthometric height cannot be determined by direct measurement, so it is defined as the negative of the differential geopotential with respect to the geoid by the local gravitational acceleration. Conventionally, the height difference between two points on the Earth is precisely determined by geodetic techniques. Typically, a combination of spirit-leveling and gravimetry determines height differences quite accurately over short distances (1 m/mm per km but with a precision of 1 mm over a distance of 10 km), but its measurement accuracy deteriorates, predominantly in proportion to the square root of the observation distances. Moreover, the measurements can be time-consuming. For example, it took approximately 10 years to complete the first-order levelling network of the whole of Japan, inevitably giving it a poor time resolution. On the other hand, ellipsoidal height is geometrically defined as the normal distance to a reference rotational ellipsoid that best fits global mean sea level. It can be measured with space geodetic positioning, such as Global Positioning System (GPS). The ellipsoidal height difference measured by GPS offers a better time resolution (up to 1 Hz) than the orthometric height, but suffers from uncertainties introduced by the atmospheric delay in radio transmission, requiring over 24 h of averaging time to achieve an uncertainty of 1 cm (ref. 31).

Correction of clock shifts. The UT clock is nearly identical to the RIKEN clocks and the difference measured by GPS offers a better time resolution (up to 1 Hz) but suffers from uncertainties introduced by the atmospheric delay in radio transmission, requiring over 24 h of averaging time to achieve an uncertainty of 1 cm (ref. 31).

Laser stabilization. Each optical lattice consists of two polarization counterpropagating laser beams. The lattice frequencies of the RIKEN and UT clocks are stabilized to 368,554,490(5) MHz and 368,554,490(3) MHz, respectively. The axial trap frequencies of atoms in the one-dimensional (1D) lattice are set to 60 kHz for all the clocks, that is, RIKEN1, RIKEN2, and UT. The lattice light shift, including the multipolar and hyperpolarizability effects, is estimated to be 3.5(3.4) × 10⁻¹⁸ using RIKEN1 and RIKEN2. For the UT clock, the uncertainty caused by the fluctuation of the lattice frequency of 3 MHz is added to this value. For the frequency comparison budget, where common effects are added out, the uncertainty is estimated to be 4.4 × 10⁻¹⁸, which is the square root of quadratic sum of the uncertainties arising from the travelling wave contamination and from the lattice frequency fluctuation.

Servo error. We evaluate the servo error by calculating the mean value and instability of the error signals of each measurement, which are obtained from the difference of the excitation fractions for the right and left shoulders of the Rabi spectrum. Because of the common effects, the uncertainty is estimated to be 3.3 × 10⁻¹⁸ by taking the atom number difference $N_{\text{master}} - N_{\text{slave}} = 750$.

Transfer of a master laser stability and rejection of the Dick effect. The master laser at RIKEN is stabilized to a 40-cm-long reference cavity with an instability that is estimated to be $3 \times 10^{-18}$ at 1 s based on the analysis of the clock excitation probability density with the help of RIKEN2, whereas the slave laser at UTokyo is stabilized to a 7.5-cm-long cavity with an instability of $2 \times 10^{-18}$ at 1 s measured through the beat with the transferred master laser. These instabilities infer a frequency noise of $\nu_{\text{RLRIKEN}}$ and $\nu_{\text{UT}}$ as shown by the red and black dashed lines in Fig. 1c. When the RIKEN and UT clocks are independently interrogated by the respective clock laser, the instability of the clock comparison is estimated to be $6 \times 10^{-15}/\sqrt{\text{Hz}}$, as shown by the black circles in Fig. 2d (inset). An external-cavity diode laser at 1,397 nm, whose second harmonic is phase-locked to $\nu_{\text{RLRIKEN}}$, is used to deliver the clock signal to UTokyo. This subharmonic generation unit is denoted $m/2$ in Fig. 1b. After activating the fibre noise cancellation, the residual fibre noise is shown by a dashed orange line in Fig. 1c. The beat note $f_{\text{beat}} = \nu_{\text{RLRIKEN}} - \nu_{\text{UT}}$ is phase-locked to the master laser $\nu_{\text{RLRIKEN}}$ containing the residual fibre noise with a frequency shifter (FS4). The feedback bandwidth is carefully chosen to minimize the contamination of the residual fibre noise.

The power spectral density (PSD) of the frequency noise of the slave clock laser after phase-locking to the master laser is given by

\[
S_{\text{UT}}(f) = \left| \frac{G(f) + 1}{1 + G(f)} \right|^2 S_{\text{UT}}(f) + \left| \frac{1}{1 + G(f)} \right|^2 S_{\text{SLAVE}}(f)
\]

where $S_{\text{RLRIKEN}}(f)$, $S_{\text{SLAVE}}(f)$, and $S_{\text{UT}}(f)$ are the PSDs of the master laser $\nu_{\text{RLRIKEN}}$, the slave laser $\nu_{\text{SLAVE}}$, and the uncompensated fibre noise measured at $\nu = 429$ Hz, $G(f)$ is the open-loop gain. The slave laser is tightly locked to the transferred laser light for $\lesssim 3$ Hz, where the Dick effect gives the dominant contribution. For higher frequencies, $f > 15$ Hz, the servo gain is reduced to zero not to add excess fibre noise beyond $S_{\text{SLAVE}}(f)$. The blue line in Fig. 1c shows the frequency noise $S_{\text{SLAVE}}(f)$ after the phase lock, which tracks the noise characteristic of $\nu_{\text{RLRIKEN}}$ below 1 Hz. This clock–laser stability transfer improves the comparison instability to $1 \times 10^{-18}/\sqrt{\text{Hz}}$ as shown by only the circles in Fig. 2d (inset). Here we operate the two distant clocks asynchronously by applying the timing offset of 750 ms that is longer than the interrogation time $T_1 = 400$ ms. By triggering a laser with a timing uncertainty of less than $T_1$, we synchronize the clock sequence in both sites. This allows the laser frequency noise for both the clocks to correlate in time, except for the non-common fibre noise (see the discussion below). Consequently, the Dick effect due to the clock lasers is rejected, improving the frequency instability to $6 \times 10^{-19}/\sqrt{\text{Hz}}$ as shown by the red circles in Fig. 2d (inset).

Instability of the frequency comparison. The instability of the synchronous frequency comparison is limited by the non-common frequency noise $S_{\text{NC}}(f)$

\[
S_{\text{NC}}(f) = \left| \frac{G(f) + 1}{1 + G(f)} \right|^2 S_{\text{SLAVE}}(f) + \left| \frac{1}{1 + G(f)} \right|^2 S_{\text{SLAVE}}(f)
\]

where $g_{\text{c}}$ is the 1-cycle average of the sensitivity function $g(t)$, and $g_{\text{c}}$ and $g_{\text{c}}$ are the nth sine and cosine components of the fourier series expansion of $g(t)$ for a cycle $T_1$, respectively. Purple circles in Fig. 1c shows $\nu_{\text{RLRIKEN}}^2 + (\nu_{\text{RLRIKEN}}^2)$ for $T_1 = 1.5 s$ and the interrogation time $T_1 = 400$ ms. Because of a sharp cut-off of $g_{\text{c}}^2 + g_{\text{c}}^2$ that decreases with $f > 1 T_1$, the frequency noise $S_{\text{SLAVE}}(f)$ with $f > 1 T_1$ barely contributes to the Dick effect limit shown as in Fig. 1c. The Dick effect limit, therefore, may be further improved by extending $T_1$, as observed in Fig. 2d, as it reduces the impact of residual fibre noise. Our calculation (red dashed line) that assumes a linear atomic response to frequency deviations predicts reduced instability than that observed in experiments with $T_1 = 400$ ms. This is because the actual atomic response becomes nonlinear and less sensitive (than assumed in the calculation) as the frequency deviations approach $1/T_1$. For our master laser with an instability of $3 \times 10^{-16}$, this nonlinear response prevents improving instability when extending the interrogation times to $T_1 = 400$ ms.

While the instability of the synchronous clock comparison improves with $T_1$, in practice, the continuous operation of the clocks becomes more difficult due to a narrower locking range. For the three-day-long measurement in Fig. 3b, we therefore apply a 300-ms-long interrogation time to allow operation of the clocks for up to a half day without any interruption of the frequency lock.

Fibre noise cancellation with an extended phase pulling range. The clock laser is transferred to UTokyo using a commercial 30-km-long telecom fibre, installed alongside a subway line. We find that the vibrations induced by the train operation cause frequency deviations as large as $\delta f \sim 40$ kHz with a modulation frequency of about $\sim 200$ Hz that occurs every 5 min for a duration of 15–20 s. To cope with this frequency noise, we lock our master laser to a wavelength-controlled SAW Oscillator (VCSO) with a frequency pulling range of 100 kHz and a digital phase and frequency detector (DPFD) with a 6-bit counter in our fibre noise cancellation system. Because of the time-delay-limited feedback bandwidth of $1/(4\pi\tau) \approx 1.6$ kHz, the frequency deviation $\delta f \sim 40$ kHz at $\nu_{\text{RLRIKEN}} \sim 200$ Hz is suppressed down to 3 kHz. This means to a residual fibre noise of 30 rad, which is temporarily stored in the 6-bit DPFD with a phase detection range of 64 rad to prevent cycle slips. This permits a robust clock link, even when the subway is in operation.

Fibre noise extension and reduction of the fibre. Typical fibre running fibre noise is reported to be $\delta f \sim 1$ and $4$ Hz/Hz$^{-1}$ km$^{-1}$ for Germany[9] and Boulder[10], while that of Toshiba during the daytime amounts to $70$ Hz/Hz$^{-1}$ km$^{-1}$. When further extending the fibre length, the installation of (n-1) repeaters by dividing the total fibre length by n will be beneficial, as they allow the unsuppressed fibre noise to be reduced and improve the link instability as $\sigma_{\delta f}(\tau) = \sigma_{\delta f}(n)/n$ (refs 18, 34). Although the transmission loss of the fibre at 1,397 nm is, in general, slightly larger (0.25 dB km$^{-1}$) than that at 1,550 nm, this is not a major limiting factor for a fibre length of up to
100 km. Use of OH-free fibres\(^{35}\) will significantly reduce the transmission loss characteristic to this wavelength.

**Tidal perturbation.** In combination with revolution of the Earth, the differential gravitational attractions from extraterrrestrial bodies (predominantly the Moon and the Sun) with respect to Earth’s centre of mass, deform the solid Earth and shift the ocean water masses. These phenomena are known as the solid Earth tides and the ocean tides, respectively, or collectively as the astronomical tides. In addition, the redistributed ocean masses due to the ocean tides affect the ocean bottom as loads and induce additional deformation of the Earth, which further perturbs the geopotential. This phenomenon is called the ocean tide loading (OTL). Figure 4 shows the tidal perturbation on the geopotential due to the astronomical tides and the OTL, which are calculated using the tide4n (ref. 36) and SPOTL (ref. 37) software packages, respectively, with minor modifications. The former uses the tidal potential harmonic development\(^{38}\) and contains tidal constituents of periods longer than diurnal but zero-frequency, which result in a DC offset in time series of tidal perturbation. The latter uses DTU10 (ref. 39) for a global ocean tide model and NAO99b (ref. 40) for a regionally detailed model around Japan.

The uncertainty of the computed solid Earth tides is evaluated to be within 0.2% in terms of gravity by comparing observed gravity variations at four different locations in the US\(^{41}\). The uncertainty of the ocean tide models is evaluated by comparing model-derived tidal heights with independent observations at coastal tidal stations: the uncertainty of DTU10 is evaluated to be 10.3% for 57 stations worldwide\(^{42}\) and that of NAO99b is evaluated to be about 5% around Japan based on the fact that it shows improvement by a factor of two over the global ocean tide models for 80 stations in Japan\(^{43}\).

**Traditional geodetic measurements of geopotential difference by spirit levelling and gravimetry.** Geopotential differences along a levelling survey route on the ground can be determined by summing up the consecutively measured vertical height differences between two ends of each piecewise measurement section at which levelling rods are set up, multiplied by the gravity values there. To complete a survey between the optical lattice clocks at UTokyo and RIKEN, levelling campaigns were carried out at clock sites and combined with a regional levelling survey made by the Tokyo Metropolitan Government, whose routes include benchmarks distributed along the Nankai–Suruga trough in southwest Japan, the Philippine Sea Plate is subducting beneath the Eurasian plate. The interplate coupling between these plates causes land subsidence on the side of the overriding plate. The rate of subsidence amounts to 8 mm yr\(^{-1}\), equivalent to a geopotential change rate of 0.08 m\(^2\) s\(^{-1}\) yr\(^{-1}\) or a fractional frequency shift by 8 \times 10^{-19}\) yr\(^{-1}\) at Cape Omaezaki, located 200 km southwest of Tokyo\(^{44}\). If quantum benchmarks are operated on the long term (a few years or more), it may be possible to detect signals and their changes regarding the interplate coupling status, which is valuable information in understanding how large earthquakes occur\(^{45}\).

At present these changes are often monitored by space geodetic techniques such as GPS or interferometric synthetic aperture radar (InSAR). Geopotential changes monitored with quantum benchmarks will complement these methods because they are not affected by atmospheric disturbances like space geodetic measurements, and because when combined with gravimetric measurements\(^{39}\) they are helpful in resolving density changes in the Earth’s crust that are not related to the height changes. Most importantly, quantum benchmarks are optimally suited to monitor such changes stably and accurately in the long term by probing geopotential changes based on atomic clocks without any instrumental drift, which is guaranteed by the stability of fundamental constants.

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