Fantappiè’s group as an extension of special relativity on $\epsilon^{(\infty)}$ Cantorian space-time

G. Iovane, P. Giordano
Dipartimento di Ingegneria dell’Informazione e Matematica Applicata, Università di Salerno, Italy.

E. Laserra
Dipartimento di Matematica e Informatica
Università di Salerno, Italy.

31.03.2004

Abstract

In this paper we will analyze the Fantappiè group and its properties in connection with Cantorian space-time. Our attention will be focused on the possibility of extending special relativity. The cosmological consequences of such extension appear relevant, since thanks to the Fantappiè group, the model of the Big Bang and that of stationary state become compatible. In particular, if we abandon the idea of the existence of only one time gauge, since we do not see the whole Universe but only a projection, the two models become compatible. In the end we will see the effects of the projective fractal geometry also on the galactic and extra-galactic dynamics.

1 Introduction

It is known that invariance under Lorentz transformations is a fundamental principle underlying both relativity and quantum field theory. Recently it has been suggested that global Lorentz invariance is an approximation of nature, that can be broken at high-energy physics. Also in the detection of ultrahigh energy cosmic rays and TeV photon spectra some anomalies are found [1, 2, 3]. Many authors showed that vacuum fluctuations and quantum gravity effects introduce stochastic perturbations in the space-time geometry at Plank energy scale [4, 5, 11, 2, 6, 7, 8, 9, 10]. In [11] G. Iovane showed the relevant

\[ \text{iovane@diima.unisa.it} \]
consequences of a Stochastic Self-Similar and Fractal Universe. Starting from an universal scaling law, the author showed its agreement with the well–known Random Walk equation or Brownian motion relation that was used by Eddington [12], [13]. Consequently, he arrived at a self-similar Universe. It appears that the Universe has a memory of its quantum origin as suggested by R.Penrose with respect to quasi-crystal [14]. Particularly, the model was related to Penrose tiling and thus to ε(∞) theory (Cantorian space-time theory) as proposed by El Naschie [15], [16] as well as with Connes Noncommutative Geometry [17]. In [18] the authors presented a descriptive model of segregated universe, then considered a dynamical model to explain the results and to give the evolution of the structures.

It is already well known that cosmology can be analyzed thanks to different theories which are not always compatible with each other. Actually the major part of the theories of gravitation are obtained by modifying the Einstein-Hilbert action, adding scalar fields or curvature invariants in the form φ^2R, R^2, R^\mu\nuR^\mu\nu or R□R (19), [20], [21]. If we apply Einstein’s equations to the whole Universe, we find the relativistic cosmology, in which the cosmological principle can be postulated and a model of constant spatial curvature obtained. We have to pay close attention to general relativity, where, inevitably, the application of Einstein’s equations to cosmological problems requires an extreme extrapolation of their validity to very far regions of space-time. Therefore we can look for solutions which present a cosmological interest, as long as we take into account, for this type of problem, such equations could be little more than a good model. As it is known, minor changes to the equations, while exhibiting all the classical verifications, produce completely different cosmologically–interesting solutions [22], [23]. It is useful to observe that even though cosmology accepts general relativity as a definitive theory of gravitation, there are still some uncertain aspects due to a baffling pluralism. Possible universes are numerous and differ from each other substantially. Moreover some astronomers, such as Arp and Hoyle, believe that to connect the red shift to the recession is an error because it is known there are other mechanisms which produce the red shift. Arp affirms that some astronomical objects appear to be gravitationally interacting among themselves, and so they should be spatially near. Instead their red shift indicates very different velocities of recession. In addition there are some objects which appear to be older than the Universe and, for this reason, Arp has proposed to return to a variation of the old stationary model in which there isn’t an origin of time [24], [25], [26]. According to this theory, formulated by Bondi, Gold and Hoyle, the Universe has always been as we see it today. Concerning the theory of the Big-Bang, the primary difficulty is the presence of the initial singularity which brings up the problem of behavior of matter when it is reduced to no dimensions with infinite density and temperature. Many difficulties of the Big Bang standard model can be overcome by inflationary cosmology [27], [28], [29]. It is actually difficult to find a theory that explains what Big-Bang really looked like. Probably, many think that all this will be resolved when we are able to formulate a quantum theory of gravitation. As it is known, Hawking and Hartle have looked for a way, based on quantum mechanics, to explain how time could
have spontaneously begun in correspondence with Big–Bang [30]. The idea is that time could have been imaginary, similar to space, near Big–Bang [31]. That is, in proximity to Big-Bang, it would be more exact to speak of 4-dimensional space instead of space–time. The hypothesis that the Universe had its origin in a singularity of infinite compression can be graphically represented by a cone with its point at the base of the diagram. As it is well known, in the quantum cosmology of Hartle and Hawking the point of the cone is substituted by a half–sphere with a radius equal to the length of Planck, $10^{-33} cm$. In the upper part of this half–sphere the cone widens in the usual way representing the standard development of the Universe in expansion. The transition from imaginary time to real time is gradual and cannot be proven all at once. The conclusion is that according to this approach, there is no origin of the Universe even if time is limited in the past. In [32] Mohamed El Naschie also consider the imaginary time and seek a formal definition of nowness. Starting from this result De Felice et al, consider Lorentz transformations and complex space-time functions [33].

In this paper we want to show that, even if Einstein’s ideas triggered a revolutionary process in our comprehension of space and time, the relativistic space–time could not be sufficient to completely explain the physical Universe and our perception of it. Expanding the idea of Hartle and Hawking to the whole space–time manifold, we find ourselves in a model of the Universe in which geometry is linked to the group that Fantappiè obtained by generalizing Poincarè’s relativistic group. Thus, according to the official interpretation of quantum mechanics, the observer plays a fundamental role in the description of the atomic world. Analogously, also if in a different way, we think that in the description of the universe, the observer becomes more deeply involved in respect to what is generally believed. For this reason we have made a distinction between space–time, external to the observer and associated with gravitation, and the internal space–time, associated with the other interactions. This last one is regulated by Fantappiè’s transformations. A relevant point is that this space-time could be not only Euclidean or curved one, but also a Cantorian space-time. In this way, accepting the new laws of space–time distortion, one can formulate an elegant cosmology from the point of view of the theory of groups, and be able to unify concepts apparently not conciliatory as in the following: the stationary and the expanding Universes, the curved and the flat Universes, the finite and the infinite ages of the Universe, and finite and infinite space.

The paper is organized as follows: we firstly discuss the coordinate lines of imaginary time in Sect.2; Sect.3 presents hyperbolic geometry and special relativity; Sect.4 is devoted to Fantappiè’s group; in Sect.5 we consider Nuclear electro–weak space–time and gravitational space–time, while in Sect.6 conclusions are drawn.
2 Coordinate lines of imaginary time

Let us start by considering the following problem. Even if, notwithstanding relativity, we are able to define the notion of universal cosmic time, how can we be sure that the cosmic clock has always ticked in the same way from the beginning of time? There is no logical need, nor are there physical theories, which answer this question. The first to explore the possibility of the existence of more time gauges were Milne and Dirac who, however, were not able to find a mathematics to support their ideas. Let us remember that, in the construction of cosmological models, it is not possible to deduce the contour conditions around the outside of the Universe. We can choose many different conditions, but we need to calculate their consequences to see if they agree with the observations. Hartle and Hawking eliminated the problem of contour conditions because their Universe has no frontier \cite{30,31}. The main difficulty is to understand in what way real time emerges continuously from imaginary time. To overcome this difficulty, we develop the hypothesis that imaginary time is the fundamental structure of the entire space–time manifold, while real time does not emerge in a past age because of some unknown physical mechanism. However, it is relevant to stress that actually we find a quantum and relativistic imprinting as showed in \cite{11}. It simply originates from our senses; it is simply what we are able to perceive and measure. Along the coordinate lines of imaginary time all the events are placed. These events, past, present and future, simply exist in the Universe of imaginary time. The entirety of space–time is represented by a 4–dimensional pseudo-hypersphere\footnote{Pseudo means in the context of Cantorian support, that is an hypersphere with stochastic self-similar fluctuations on the surface.}, which exists in its entirety and is immutable. By sectioning the sphere with planes orthogonal to the coordinate lines of imaginary time the model represents a stationary Universe of cyclic imaginary time. By transforming imaginary time into real time, we obtain the passage from a space–time pseudo-hypersphere of imaginary time, to a pseudo-hyperboloid of real time. Therefore, we obtain a space-time manifold of hyperbolic structure. Among all the possible structures for space–time, the hyperbolic is the most natural, since, as we will see in the next paragraph, Lobacevskij–Bolyai’s geometry is formally analogous to special relativity.

3 Hyperbolic geometry and special relativity

Let $K$ be an inertial frame of reference and consider the hyperbole of equation \footnote{For sake of simplicity we are considering a 2–dimensional relativistic space–time.} $\frac{x^4}{2} - \frac{x^1}{2} = R^2$. The two branches of the hyperbole approach the universe lines described by the light rays.asintotically. Let $K'$ be another inertial frame of reference moving with respect to $K$, and let $P$ be the space–time point which is the intersection of the upper branch of the hyperbole with the axis $x^4$ represented by $x^4 = \beta x^1$. 

\[
\frac{\beta}{R} - \frac{1}{R} = \frac{x^1}{R} = \beta x^4.
\]
The coordinates of this point, obtained by combining the equations of the two curves, are

\[
\begin{align*}
\begin{cases}
    x^1 = \frac{\beta R}{\sqrt{1 - \beta^2}}, \\
    x^4 = \frac{R}{\sqrt{1 - \beta^2}}.
\end{cases}
\end{align*}
\]

(2)

By comparing Lorentz's transformations \[22\]

\[
\begin{align*}
\begin{cases}
    x'^1 = x^1 - \beta x^4 \sqrt{1 - \beta^2}, \\
    x'^4 = x^4 - \beta x^1 \sqrt{1 - \beta^2},
\end{cases}
\end{align*}
\]

(3)

one sees that the coordinates of the point represent time $R/c$ and zero length in the primed frame of reference. For each relative velocity of $K'$ with respect to $K$, and so for each inclination of the axes, the intersection of time axis with this hyperbole will give time $R/c$. Therefore, the hyperboles represent, in Minkowski’s space–time, the locus of the points equidistant from the origin and therefore they are the analogous of the circumferences in the euclidian plane. By adding a spatial dimension, one sees that the two–sheeted hyperboloids are the analogous of the spheres and similarly, by adding other spatial dimensions, we can construct the hyperspheres of relativistic geometry. These spheres in the pseudo–euclidian spaces have been amply studied by mathematicians and it has been demonstrated that their intrinsic geometry is Lobacevski’s hyperbolic geometry. Precisely, they are surfaces of negative constant curvature $K = -1/R^2$.

We see from this that hyperbolic geometry, in respect to euclidian geometry and elliptic geometry, has a particularly important role in space–time, and from the point of view of group theories, is very similar to the theory of relativity where, in space–time, relativity and hyperbolic geometry share Poincarè’s symmetry group.

4 Fantappiè’s group

Fantappiè noted that general relativity follows an extraneous approach to the tradition of mathematical physics in that it does not follow the group structure of physics. This is different from classical mechanics and special relativity.

Let us remember that Galileo’s group is the main group of classical physics and is formed by the composition of the following transformations:

a) Spatial Rotations - characterized by three parameters

\[
x'_\mu = a_{\mu\nu} x_\nu , \quad t' = t,
\]

(4)

where $[a_{\mu\nu}]$ is an orthogonal matrix whose determinant is $+1$.

b) Inertial Movements - characterized by the three components of velocity,

\[
x'_\mu = x_\mu + v_\mu t , \quad t' = t.
\]

(5)
c) Spatial translations - characterized by three parameters,
\[ x'_{\mu} = x_{\mu} + a_{\mu}, \quad t' = t. \] (6)
d) Temporal translations - characterized by only one parameter,
\[ x'_{\mu} = x_{\mu}, \quad t' = t + t_0. \] (7)

Therefore Galileo’s group has order 10 and expresses Galileo’s well-known relativity principle. Moving on to relativistic physics, spatial rotations and inertial movements become fused in a unique operation, the rotations of a euclidian space \( M_4 \), characterized by 6 parameters,
\[ x'_i = a_{ik}x_k, \] (8)
where \( |a_{ik}| = 1, x_1 = x, x_2 = y, x_3 = z, x_4 = ict \).

These transformations, called Lorentz’s special transformations, form Lorentz’s proper group and joining the reflections, form Lorentz’s extended group. Then we need to add the translations of \( M_4 \)
\[ x'_i = x_i + a_i, \] (9)
characterized by 4 parameters, which comprise spatial and temporal translations. By composing the transformations of these two groups, we obtain Lorentz’s general transformations which form Poincaré’s group of 10 parameters
\[ x'_i = a_{ik}x_k + a_i. \] (10)

Poincaré’s group mathematically translates Einstein’s relativity principle. When \( c \to \infty \) so that \( \frac{v}{c} \ll 1 \), Minkowski’s space–time reduces to that of Newton’s and Poincaré’s group reduces to Galileo’s group.

Fantappiè went on in this direction and tried to understand if Poincaré’s group could be the limit of a more general group, in the same manner as Galileo’s group is the limit of Poincaré’s group. In [34] He wrote a new group of transformations, which had as limit Poincaré’s group and He was able also to demonstrate that his group was not able to be the limit of any continuous group of 10 parameters. That is, by limiting to groups of 10 parameters and to 4-dimensional spaces, what happened with Galileo’s and Poincaré’s groups cannot be repeated. For this reason this group is called the final group.

Fantappiè, moving from a space–time with hyperbolic structure, showed that, through a flat projective representation, one could obtain a space–time which generalizes Minkowski’s space–time.

Let us remember that to have a flat representation of hyperbolic geometry, we fix a circle in the plane, with center \( O \) and radius \( r \), called the absolute of Cayley-Klein. Relative to this we get the following definitions:
point ⇒ point inside the circle;
straight line ⇒ chord of the circle (without extremes);
plane ⇒ region of points inside the circle;
movements ⇒ projections on the plane that transform
the region of the internal points in itself;
congruent figures ⇒ figures which can be transformed from
one to the other through a projection.

Let us introduce a system of orthogonal coordinates with origin in the center
of the circle. It is not possible to represent the distance of two points $A(x, y)$
and $B(x', y')$ in the form $\sqrt{(x - x')^2 + (y - y')^2}$ because it is not invariant for
the projective transformations. An expression of the coordinates of $A$ and $B$, which
remains invariant for all the projective transformations which leave the
limit circle fixed, is the anharmonic ratio of the four points $A, B, M, N$

$$(ABMN) = (AM/BM) : (BN/AN), \quad (11)$$

where $M$ and $N$ are the extremes of chord $AB$.

We assume as distance

$$\text{dist}(AB) = k \log(ABMN). \quad (12)$$

In this way one sees that every point of the hyperbolic plane, no matter how they
are moved, always remain at infinite distance from the points of the absolute. So
the hyperbolic plane is finite and limited if we make the measures in a euclidian
sense. On the contrary, if the measures are not euclidian, the hyperbolic plane
is infinite and unlimited.

Fantappiè chooses, as the absolute quadric, the hypersphere

$$x_E^2 + y_E^2 + z_E^2 - c^2 t_E^2 + R^2 = 0, \quad (13)$$

which, in the 2-dimensional case becomes the circumference

$$x_E^2 - c^2 t_E^2 + R^2 = 0. \quad (13')$$

He showed that Minkowski’s space–time can be considered as a limit case of
the projected space–time when $R \to \infty$. Therefore Poincaré’s group proves
the limit of the group of motions of new space–time in itself. To determine
the transformations of the new group, we observe that the space–time motions
are represented by the projections that transform the absolute circumference
in itself. Such absolute, with the introduction of imaginary time, can be written as

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 + R^2 = 0 \quad (14)$$
and, by considering the homogeneous coordinates \( \mathbf{x}_A (A = 1, 2, \ldots, 5) \) so defined\(^3\)

\[ x_k = R \mathbf{x}_k / \mathbf{x}_5, \]

it becomes

\[ x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 = 0. \]

(16)

It follows that the motions we are searching for are those which leave the prior quadratic form invariable and are the orthogonal substitutions on the five variables \( \mathbf{x}_A \). These transformations form the group of 5-dimensional rotations and are three types (moving to the non-homogeneous coordinates):

a) \textit{Time translations}: considering two observers standing still in the same place, but separated by a great distance in time, that is, the same observer in two different moments

\[
\begin{align*}
\mathbf{x}' &= \begin{pmatrix} x' \\ y' \\ z' \\ t' \\ 1 \end{pmatrix} = \begin{pmatrix} x \sqrt{1 - \eta^2} \\ y \\ z \\ t - \frac{\eta t}{R/c} \\ 1 \end{pmatrix}, \\
\mathbf{t}' &= \begin{pmatrix} t' \\ 1 \end{pmatrix} = \begin{pmatrix} t - T \\ 1 \end{pmatrix},
\end{align*}
\]

with \( \eta = \frac{T}{R/c} \) where \( T \) is the parameter of time translation. It follows that for \( t = \pm R/c \), one has \( x' = 0 \).

These transformations for \( R \to \infty \) are reduced to the classic time translations

\[ x' = x, \quad t' = t - T \]

(18)

b) \textit{Spatial Translations}: considering two observers at the same time and standing still compared to each other, but separated by great distance in space (for example along the \( x \) axis)

\[
\begin{align*}
x' &= \frac{x - S}{1 + \alpha x/R}, \\
y' &= \frac{y}{1 + \alpha x/R}, \\
z' &= \frac{z}{1 + \alpha x/R}, \\
t' &= \frac{t}{1 + \alpha x/R},
\end{align*}
\]

(19)

\(^3\) Let us remember that if \((x_0, y_0, z_0)\) are the cartesian orthogonal coordinates of a point \( P \) in ordinary space, one defines the four homogeneous coordinates by the relations

\[ x_0 = \frac{x_1}{x_4}, \quad y_0 = \frac{x_2}{x_4}, \quad z_0 = \frac{x_3}{x_4}. \]

If \( Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + L = 0 \) is the equation of a quadric in cartesian orthogonal coordinates, then in homogeneous coordinates we have

\[ Ax_1^2 + Bx_2^2 + Cx_3^2 + Dx_1x_2 + Ex_1x_3 + Fx_2x_3 + Gx_1x_4 + Hx_2x_4 + Ix_3x_4 + Lx_4^2 = 0. \]
with $\alpha = \frac{S}{R}$ and where $S$ is the parameter of translation along the $x$ axis. At the relativistic limit, that is for $R \to \infty$, equations (19) reduce to
\begin{equation}
    x' = x - S, \quad y' = y, \quad z' = z, \quad t' = t. \tag{20}
\end{equation}

c) Pullings: considering two observers that initially coincide, and one moving rectilinearly and uniformly to the other, with velocity parallel to the $x$ axis
\begin{equation}
    \begin{aligned}
    x' &= \frac{x-Vt}{\sqrt{1-\beta^2}}, \\
    y' &= y, \\
    z' &= z, \\
    t' &= t - \frac{Vx/c^2}{\sqrt{1-\beta^2}}. 
    \end{aligned} \tag{21}
\end{equation}

Let us synthesize the geometric structure of Fantappiè’s group in the following scheme:

\begin{align*}
\text{GALILEO’S GROUP} & \Rightarrow \text{Rotations } S_3 \\
& \Rightarrow \text{Pullings} \\
& \Rightarrow \text{Translations } S_3 \\
& \Rightarrow \text{Time Translations} \\
\text{POINCARE’S GROUP} & \Rightarrow \text{Rotations } S_4 \\
& \Rightarrow \text{Translations } S_4 \\
\text{FANTAPPIE’S GROUP} & \Rightarrow \text{Rotations } S_5
\end{align*}

5 Nuclear electro–weak space–time and gravitational space–time

Some gravitational phenomena show us that in a curved space and in some conditions, we cannot localize an object in its effective position (this is the case of the gravitational lenses). Instead the source is seen by the observer in the direction of the tangent to the light rays in the point where we are, and as a curved space appearing as if it were flat. It could seem likely that something like this can also happen for time. It could seem that the entire space–time manifold, in which every observer can watch the phenomena, is only a flat representation of the space–time tangent to curved manifold. Applying the flat representation of hyperbolic geometry, the group of motions of the space–time manifold in itself is represented by Fantappiè’s group. At this point, the idea is that there is a difference between the projected space–time ($x_E, t_E$) that every observer can see and is associated with atomic processes and with light frequencies, and the non–projected space–time ($x_G, t_G$) associated with gravitational phenomena. The
projected space–time coordinates are regulated by Fantappiè’s transformations and are on a Cantorian support. Let us determine, therefore, the relations that link projected and non–projected coordinates together. Let us consider the hyperbolic model with real time and let us make a flat representation of it by choosing as absolute the hypersphere

\[ c^2 t_E^2 - x_E^2 - y_E^2 - z_E^2 - R^2 = 0, \] (22)

which, to simplify, in the 2–dimensional case becomes the circumference

\[ c^2 t_E^2 - x_E^2 = R^2. \] \(22'\)

Let us calculate the time distance of two points, A(0) and B\(t_E\), placed on the \(t_E\) axis. The time axis meets the absolute at two points C\(R/c\) and D\(−R/c\) and so

\[ (ABCD) = \frac{AC \cdot DB}{CB \cdot AD} = \frac{C - A \cdot B - D}{B - C \cdot D - A} = \frac{R/c \cdot t_E + R/c}{t_E - R/c \cdot -R/c} = \frac{R + ct_E}{R - ct_E}. \] (23)

The measure of the time interval between the two events on the \(t_E\)-axis is

\[ t_G = \frac{R}{c} \log \frac{R + ct_E}{R - ct_E}, \] (24)

by posing \(k = \frac{R}{c}\).

So we conclude that time \(t_E\), linked to the non–gravitational interactions, is slower than gravitational time. Taking into account the observed data (current radius of the Universe, speed of light, etc.) one can easily verify that we have to consider intervals of many thousands of years in such a way that the two temporal scales differ by only a second. Going back instead to a past cosmologic epoch, the differences increase and when electro–magnetic time nears \(−t_{EU} = R/c\), which for us coincides with the beginning of time, gravitational time extends into the infinite past. That is, in gravitational time, the Universe is infinitely old. Obviously every observer sees only a part of the manifold and that is what we call the past. So, as the background cosmic radiation demonstrates, there was an initial instant in respect to the electro–magnetic waves, while the Universe is eternal in the gravitational scale. By considering two points placed on the \(x_E\)-axis, A(0) and B\(x_E\), we will have, instead, \(P(-iR)\) and \(Q(iR)\), where their spatial distance will be

\[ x_G = \frac{R}{2i} \log \frac{iR - x_E}{x_E + iR} = \frac{R}{2i} \log \frac{-R - ix_E}{ix_E - R} = \frac{R}{2i} \log \frac{R + ix_E}{R - ix_E} = R \arctg \frac{x_E}{R}, \] (25)

by posing \(k = \frac{R}{2i}\).

In other words, non–projected space is smaller than projected one.

From the prior relations the two inverse formulas follow:

\[
\begin{align*}
  t_E & = \frac{R}{2i} \tgh \frac{ix_E}{R}, \\
  x_E & = R \tgh \frac{x_G}{R}.
\end{align*}
\] (26)
In conclusion, therefore, in cosmologic times, the astronomic clocks slowly lose their synchronization with the atomic clocks.

In gravitational time Universe space is finite and time is infinite; in a projected Universe space is infinite and time is finite.

This physical interpretation of Fantappiè’s transformations implies that the electromagnetic age of the Universe is constant. The temporal translations demonstrate

\[ t_E = \frac{t_{E1} + t_{E2}}{1 + t_{E1} t_{E2}/t_{EU}^2}. \]  

(27)

This relation is equal in form to relativistic law of the composition of velocities. Therefore, as such, the speed of light is the same for each observer in whichever motion, and as being finite, cannot be exceeded. In the same way the electromagnetic age of the Universe is the same for each observer, whichever its space–time position. Every observer will see the same Universe globally, not only from every point of space, but also in any era. This is not different from the perfect cosmological principle postulated by the authors of the stationary model, who however, had to hypothesize the creation of new matter from nothing in order to verify it. In this treatment the perfect cosmological principle can be obtained as a consequence of Fantappiè’s group.

Differentiating Fantappiè’s temporal translations, we obtain

\[ dx_E' = \frac{\sqrt{1 - \eta^2}}{1 + \eta t_E/t_{EU}} dx_E - \frac{x_E \sqrt{1 - \eta^2}}{(1 + \eta t_E/t_{EU})^2 t_{EU}} dt_E, \]  

(28)

\[ dt_E' = \frac{1 + \eta t_E/t_{EU} - (t_E + T_{EU})\eta/t_{EU}}{(1 + \eta t_E/t_{EU})^2} dt_E = \frac{1 - \eta^2}{(1 + \eta t_E/t_{EU})^2} dt_E, \]  

(29)

and so

\[ V_E' \sqrt{1 - \eta^2} = V_E(1 + \eta t_E/t_{EU}) - x_E \eta/t_{EU}, \]  

(30)

which furnishes the link between the velocities of a point measured in two instants of electromagnetic time separated by the interval \( T_{EU} \).

In particular, if the interval of time is \( T_{EU} = t_{EU} = R/c \), since \( \eta = 1 \), we obtain,

\[ V_E = \frac{x_E}{t_E + t_{EU}} = H(t_E) x_E. \]  

(31)

At the current time, that is \( t_E = 0 \), we have

\[ H = 1/t_{EU} = c/R. \]  

(32)

Therefore our physical interpretation of Fantappiè’s transformations says that every observer will see an expanding Universe with escape velocity proportional to the distance, and this agrees with Hubble’s law.
6 Physical Remarks and Conclusions

We know that in order, for a body in rotation on itself, to be dynamically stable there must be a condition of equilibrium between gravitational force, which depends on its mass, and centrifugal force, which depends on its velocity of rotation. If the body rotates faster than a certain maximum velocity, it will disintegrate because of its own centrifugal force. Observations of stellar objects with highly elevated velocities of rotations, actually show periods of rotation of the order of a few milliseconds. To withstand the centrifugal force, they should have a density of the order of $10^{14} \text{gr/cm}^3$. Such high density is equal only to that of an atomic nucleus, so for this reason astrophysicists think these objects must be neutron stars, but it could be another answer: they could stay on a Cantorian space-time and our view is just a projection.

There are galaxies which rotate faster than the theoretical maximum velocity and beyond this the velocity of the stars along the arms do not seem to decrease in a keplerian way. To justify the equilibrium one hypothesizes the existence of dark matter which increases the mass. However, the study of space–time through Fantappiè’s group acknowledges a new law of time dilatation that we associated with the two different time scales. That is, it seen through electromagnetic time, processes such as the rotation of celestial bodies were strongly accelerated in the past, while the behavior of light and atomic processes remained invariable. Therefore, even in this context, the application of Fantappiè’s transformations agrees with the observations. It is obvious that, for small space–time scales, the results are not different for $t_G$ in respect to $t_E$.

The results of our paper are not conclusive and just put into evidence that the cosmological problems and Fantappiè’s physical–mathematical observations, based on the theory of groups, can be framed in the same interpretive scheme. It is particularly interesting to observe that projective geometry on Cantorian space-time behaves as natural geometry in cosmology. In addition, we have shown how Hartle and Hawking’s ideas of imaginary time, extended to the entire Cantorian Universe, allow us to see the model of Big Bang and the stationary model, as two different projections of the same reality. In particular, the two models appear to be connected to two different time scales; the latter linked to space–time geometry, and therefore to gravitational interaction; the prior is linked to the other fundamental interactions which measure time for each observer.

Acknowledgements

The authors wish to thank E.Benedetto for comments and discussions.

References

[1] C.Bech, Physica A, 305, 2009, 2002.

[2] G.Amelino-Camelia and T.Piran, Phys.Rev.D 64, 36005, 2001.
[3] G. Amelino-Camelia, Int. J. Mod. Phys. D 11, 35, 2002.

[4] R. Aloisio, et al, Phys. Rev. D 62, 53010, 2000.

[5] O. Bertolami and C. S. Carvalho, Phys. Rev. D 61, 103002, 2002.

[6] S. Coleman and S. L. Glashow, Phys. Rev. D 59, 116008, 1999.

[7] M. S. El Naschie, Chaos, Complex vacuum fluctuation as a chaotic "limit" set of any Kleinian group transformation and the mass spectrum of high energy particle physics via spontaneous self-organization, Solitons, & Fractals, 17, 4, 631, 2003.

[8] M. S. El Naschie, VAK, vacuum fluctuation and the mass spectrum of high energy particle physics, Chaos, Solitons, & Fractals, 17, 4, 797, 2003.

[9] M. S. El Naschie, The VAK of vacuum fluctuation, Spontaneous self-organization and complexity theory interpretation of high energy particle physics and the mass spectrum, Chaos, Solitons, & Fractals, 18, 2, 401, 2003.

[10] E. Goldfain, Derivation of the fine structure constant using fractional dynamics, Chaos, Solitons, & Fractals, 17, 5, 811, 2003.

[11] G. Iovane, Varying G, accelerating Universe, and other relevant consequences of a Stochastic Self-Similar and Fractal Universe, Chaos, Solitons, & Fractals, 20, 4, 657, 2004.

[12] M. S. El Naschie, On the unification of the fundamental forces and complex time in the $\varepsilon(\infty)$ space, Chaos, Solitons, & Fractals, 11, 1149-1162, 2000.

[13] B. J. Sidharth, Chaos Solitons Fractals 11, 2155, 2000; Chaos, Solitons, & Fractals 12, 795, 2001.

[14] R. Penrose, The Emperor’s New Mind, Oxford University Press, 1989.

[15] M. S. El Naschie, On the uncertainty of Cantorian geometry and the two slit experiment, Chaos, Solitons, & Fractals, 9, 3, 517-529, 1998.

[16] M. S. El Naschie, Penrose universe and Cantorian spacetime as a model for noncommutative quantum geometry, Chaos, Solitons, & Fractals, 9, 931-933, 1998.

[17] A. Connes, Noncommutative Geometry, Academic Press, New York, 1994.

[18] G. Iovane, E. Laserra and P. Giordano, Dust Fractal Universe with spatial pseudo-spherical symmetry for a possible description of the actual segregated Universe as a consequence of its primordial fluctuations, in press on Chaos, Solitons, & Fractals, 2004.

[19] S. Weinberg, Gravitation and Cosmology, Wiley, New York, 1972.
[20] A.A.Starobinsky, Phys.Lett. B 91, 99, 1980.
[21] D.H.Eckhardt, Phys.Rev.D 48, 3762, 1993.
[22] R.Ruffini, H.Ohanian, Gravitation and spacetime, Norton & Company, 1994.
[23] E.W.Kolb, M.S.Turner, The Early Universe, Addison-Wesley, 1990.
[24] H.Arp, Origins of Quasars and Galaxy Clusters, XXXVI Recontres de Moriond, 2001, astro-ph/0105325
[25] H.Arp, Redshifts of New Galaxies, 194th IAU Symp. on ”Activity in galaxies and related phenomena”, held in Byurakan, Armenia, August 17-21, 1998, Eds. Y.Terzian, E.Khachikian, and D.Weedman, PASP Conf. Series, astro-ph/9812144
[26] H.Arp, Association of X-ray Quasars with Active Galaxies, IAU183, Kyoto 18-22 August 1997, astro-ph/9712164
[27] A.D.Linde, Inflation and quantum cosmology, Accademic Press, Boston, 1990.
[28] S.Hawking, E.Israel, Three hundred years of gravitation, Cambridge University Press, 1987.
[29] J.V.Narlikar, T.Padmenabhan, Ann.Rev.Astron.Astrophys. 29, 325, 1991.
[30] S.Hawking, R.Penrose, The nature of space and time, Princeton University Press, 1996.
[31] S.Hawking, The quantum state of the universe, Nucl. Phys. B, 239, 1984.
[32] M.S. El Naschie, On the nature of complex time, diffusion and two-slit experiment, Chaos, Solitons, & Fractals, 5, 1031, 1995.
[33] F.de Felice et al., Lorentz transformations and complex space-time functions, Chaos, Solitons, & Fractals, 21, 573, 2004.
[34] L.Fantappie', Su una nuova teoria di relatività finale, Rend. Lincei. Vol. 17, 5, 1954.
[35] P.Schneider,S.Ehlers, E.E.Falco, Gravitational lenses, Springer-Verlay, 1992.