Inflationary perturbation spectra at next-to-leading slow-roll order in effective field theory of inflation

Guang-Hua Ding\textsuperscript{1,a}, Jin Qiao\textsuperscript{1,b}, Qiang Wu\textsuperscript{1,c}, Tao Zhu\textsuperscript{1,d}, Anzhong Wang\textsuperscript{1,2,e}

1 Institute for Theoretical Physics and Cosmology, Zhejiang University of Technology, Hangzhou 310032, China
2 GCAP-CASPER, Physics Department, Baylor University, Waco, TX 76798-7316, USA

Abstract The effective field theory (EFT) of inflation provides an essential picture to explore the effects of the unknown high energy physics in the single scalar field inflation models. For a generic EFT of inflation, possible high energy corrections to simple slow-roll inflation can modify both the propagating speed and dispersion relations of the cosmological scalar and tensor perturbations. With the arrival of the era of precision cosmology, it is expected that these high energy corrections become more important and have to be taken into account in the analysis with future precise observational data. In this paper we study the observational predictions of the EFT of inflation by using the third-order uniform asymptotic approximation method. We calculate explicitly the primordial power spectra, spectral indices, running of the spectral indices for both scalar and tensor perturbations, and the ratio between tensor and scalar spectra. These expressions are all written in terms of the Hubble flow parameters and the flow of four new slow-roll parameters and expanded up to the next-to-leading order in the slow-roll expansions so they represent the most accurate results obtained so far in the literature. The flow of the four new slow-roll parameters, which arise from the four new operators introduced in the action of the EFT of inflation, can affect the primordial perturbation spectra at the leading-order and the corresponding spectral indices at the next-to-leading order.

1 Introduction

Inflation is the leading paradigm for the hot Big Bang origin of the Universe [1–3] (see Ref. [4] for an updated review). The most remarkable features of the inflationary scenario is that it provides an elegant mechanism for generating structures in the Universe and the spectrum of cosmic microwave background (CMB) anisotropies, which are fully consistent with the cosmological observations with a spectacular precision [5–8]. In general, there are a lot of approaches to realize inflation that originate from very different background physics. The recent precise observational data set supports the key predictions of the standard single-field inflationary models. In this model, a single scalar degrees of freedom with a self-interacting potential produces a slow-roll phase during which the energy density of the matter field remains nearly constant and the space-time behaves like a quasi-de Sitter spacetime.

Instead of constructing inflation models with scalar degrees of freedom from different fundamental theories, the EFT provides a general framework for describing the most generic single scalar field theory and the associated fluctuations on a quasi de-Sitter background [9,10]. This framework is based on the decomposition of space and time with a preferred time foliation defined by the scalar field, which provides a clock that breaks time but preserves spatial diffeomorphism invariance. Similar idea has been used in the construction of the Horava-Lifshitz (HL) theory of quantum gravity [11–15], in which the symmetry of the theory is broken from the general covariance down to the foliation-preserving diffeomorphisms. With this symmetry, the action of the theory has to be constructed only in terms of three dimensional spatial diffeomorphism invariants. This allows us to characterize all the possible high energy corrections to simple slow-roll inflation and their impacts on the primordial perturbation spectra. Such considerations have attracted a lot of attention and the observational effects of high-order operators on inflationary perturbation spectra have been extensively studied both in the framework of the EFT of inflation and the inflation in HL theory [16–19] (see [20] for an updated review). With such thoughts, an extension of the EFT of inflation by
adding high dimension operators has also been constructed
and the corresponding primordial perturbation spectra have
also been explored in refs. [21–23].

In the EFT of inflation, possible high energy corrections
to simple slow-roll inflation can affect the propagations of
both the cosmological scalar and tensor perturbations in two
aspects. One is that it may lead to a time-dependent propa-
gating sound speeds associated with the scalar and tensor
perturbation modes, and another is that it could modify the
conventional linear dispersion relation of the perturbation
modes to nonlinear ones. Both the time-dependent propa-
gating sound speeds and the modified dispersion relation can
make important corrections in the primordial scalar and ten-
sor perturbation spectra. In most of previous works, how-
ever, in order to estimate the spectra, the time-dependent
speed has been assumed to be constant and the operators
which produces nonlinear dispersion relations are ignored
[9,24–26]. One of the main reasons for such treatment is that
the considerations of the time variation of the sound speed
and the nonlinear dispersion relations make it very difficult
to calculate the corresponding power spectra and spectral
indices.

However, to match with the accuracy of the current and
forthcoming observations, as pointed out in [27–30], con-
sideration of the contributions from the time variation of
the sound speed and the modified dispersion relation are
highly demanded. To achieve this goal, one has to calcu-
late the primordial perturbation spectra up to the second-
order in the slow-roll expansions. Recently, we have devel-
oped a powerful method, the uniform asymptotic approxi-
mation method [31–33], to calculate precisely the primor-
dial perturbation spectra for a lot of inflation models. The
robustness of this method has been verified for calculating
primordial spectra in k-inflation [28,34,35], and inflation
with nonlinear dispersion relations [32,33,36], inflation in
loop quantum cosmology [37–39], and inflation with
Gauss-Bonnet corrections [27] (For an alternative approach
by using Green’s function method, see [40,41] and its
recent application to the scalar-tensor theories in differ-
ent frames [42].) We note here that this method was first
applied to inflationary cosmology in the framework of gen-
eral relativity (GR) in [43–45], and then we have devel-
oped it, so it can be applied to more general cases and
high accuracy [32,36]. The accurate results of some infla-
tonary models derived from this method have also been used
to the study of the adiabatic regularisation of the pri-
mordial power spectra [46,47]. It is worth noting that this
approximation has also been applied to study the parametric
resonance during reheating [48] and the quantization
conditions in quantum mechanics [49]. The main pur-
pose of the present paper is to use this powerful method
to derive the inflationary observables at the second-order in
slow-roll inflation in the framework of the EFT of infla-
tion. With the general expressions of power spectra and
spectral indices obtained in [33,34] in the uniform asym-
pototic approximation, we calculate explicitly these quanti-
ties for both scalar and tensor perturbations of EFT of
inflation. Then, ratio between the tensor and scalar spec-
tra is also calculated up to the second-order in the slow-
roll expansions. These observables represent a significant
improvement over the previous results obtained so far in the
literature.

We organize the rest of the paper as follows. In Sect. 2, we
present a brief review of the EFT of inflation and the corre-
sponding equations of motion for both cosmological scalar
and tensor perturbations. In Sect. 3, we give the most general
formulas of the primordial perturbation spectra in the uni-
form asymptotic approximation. Then, in Sect. 3, with these
general expressions we calculate explicitly the power spec-
tra, spectral indices, and running of the spectral indices of
both scalar and tensor perturbations of the slow-roll inflation
in the EFT of inflation. Our main conclusions and discussions
are presented in Sect. 4.

2 The effective field theory of inflation

In this section, we present a brief introduction of the EFT
of inflation [9,10]. In general, the EFT provides a frame-
work for describing the most generic single scalar field the-
ory on a quasi de-Sitter background. With this framework,
it is shown that the action of the EFT of inflation around a
flat Friedmann-Lemaître-Robertson-Walker (FLRW) back-
ground reads

\[ S_{\text{eft}} = M_{Pl}^2 \int d^4x \sqrt{-g} \left\{ \frac{R}{2} + \dot{H} g^{00} - (3H^2 + \dot{H}) 
+ \frac{M_{Pl}^2}{2} (g^{00} + 1)^2 - \frac{M_{Pl}^3}{2} \left( g^{00} + 1 \right) \delta K_{\mu}^{\mu} 
- \frac{M_{Pl}^2}{2M_{Pl}^3} (\delta K_{\mu}^{\mu})^2 - \frac{M_{Pl}^2}{2M_{Pl}^3} \delta K_{\nu}^{\nu} \delta K_{\mu}^{\mu} \right\}. \] (2.1)

where

\[ K_{\mu \nu} = (g_{\mu \sigma} + n_{\mu} n_{\sigma}) \nabla^\sigma n_{\nu} \]

denotes the extrinsic curvature at constant time hypersur-
faces with \( n_{\nu} \) being the unit normal vector. \( \delta K_{\mu \nu} \) denotes
the perturbation of \( K_{\mu \nu} \) about the flat FLRW background.
For the calculation of the power spectra for both the scalar
and tensor perturbation later, we only consider operators up
to second order fluctuation in metric, which is sufficient to
extract the information of two-point function from scalar and
tensor modes. Therefore, we truncate the EFT action to (2.1)
in the above by cutting terms, for example, with \( n > 2 \) in
\( \hat{O}_{(i)} = \sum_{n=2}^{\infty} \frac{M_{Pl}^2}{n!} (1 + g^{00})^n. \)
We would like to mention that, in writing the above action, one requires the perturbation in inflaton field $\phi$ to be zero, i.e., $\delta \phi = 0$. This is achieved by considering the following linear transformations under time diffeomorphism,

$$
\delta \bar{t} = t + \xi(t, x^i), \quad \delta \phi = \delta \bar{\phi} + \xi^{0}(t, x^i) \phi_0(t),
$$

(2.2)

which leads to a particular gauge (unitary gauge) with $\xi^{0}(t, x^i) = -\delta \phi / \phi_0$ where there is no inflaton perturbation. Obviously the action (2.1) that respects the unitary gauge has no time diffeomorphism invariance. In order to restore the time diffeomorphism invariance, one can introduce a Goldstone mode $\pi(t, x^i)$ and require it transforms as $\pi(t, x^i) \rightarrow \pi(t, x^i) - \xi^{0}(t, x^i)$. By introducing the Goldstone mode, the perturbation of inflaton field is not required to be zero and it relates to $\pi(t, x^i)$ via $\delta \phi = \phi_0 \pi$.

It is worth noting that the standard single scalar slow-roll inflation can be simply recovered by setting $M_2, M_1, M_2, M_3$ to zero. As mentioned in [9], the terms with $M_2, M_1, M_2, M_3$ encodes the possible effects of high energy physics on the simple slow-roll model of inflation under the current accurate observation. As we will see later, these new terms can affect both the propagating speed and dispersion relations of the cosmological scalar and tensor perturbations. Therefore, studying their effects on the primordial perturbation spectra can provide a way to probe the possible high energy effects in the observational data.

2.1 Scalar perturbations

As mentioned above, the Goldstone mode $\pi(t, x^i)$ can be introduced to restore the time diffeomorphism invariance of action and it also describes the scalar perturbations around the flat FLRW background. Thus, in order to study the scalar perturbations, one can transform the action in (2.1) in unitary gauge to $\pi$-gauge by evaluating the action explicitly for $\pi$. Proceeding this procedure [9], one obtains

$$
S_{\text{eff}}^\pi = \int d^4x a^3 \left[ A_0 (\pi'^2 + 3H^2\epsilon_1 \pi^2) + A_1 \frac{(\partial_i \pi)^2}{a^2} + A_2 \frac{(\partial^2 \pi)^2}{a^4} \right],
$$

(2.3)

where

$$
A_0 = -M_{P}^2 \dot{H} + 2M_1^2 - 3\tilde{M}_1^2 H - \frac{9}{2} H^2 \tilde{M}_2^2 - \frac{3}{2} H^2 \tilde{M}_3^2,
$$

(2.4)

$$
A_1 = M_{P}^2 \dot{H} + \frac{1}{2} \tilde{M}_1^2 H + \frac{3}{2} H^2 \tilde{M}_2^2 + \frac{1}{2} \tilde{M}_3^2 H^2,
$$

(2.5)

$$
A_2 = -\frac{1}{2} \left( \tilde{M}_2^2 + \tilde{M}_3^2 \right).
$$

(2.6)

Here two remarks about the above action are in order. First, the above action for scalar perturbations is valid in the weak decoupling limit. In this way, we ignore most of the terms arising from mixing between gravity and Goldstone Boson, except the leading-order mixing term $M_{P}^2 \dot{H} \pi \delta \phi^{00}$. This term gives rise to a mass term $3a^3 H^2 \epsilon_1 \pi^2$ in the action for the scalar perturbations. Second, in order to match the precision of the current and future observations, both the scalar spectral index $n_s$ and tensor-to-scalar ratio $r$ are required be accurate up to the second-order in the slow-roll expansion. In this way, in order to simplify our calculations, we only interest in terms in the action which contribute to $n_s$ and $r$ at the first two orders in the slow-roll expansion. Other terms which lead to contributions beyond this order are all neglected in the above action.

Changing the variable $\pi$ to $u = \dot{\pi} \pi$ we cast the action into the form

$$
S_{\text{eff}}^\pi = \int d^4x a^3 \left[ u'' + \left( \frac{z''}{z} + 3a^2 H^2 \epsilon_1 \right) u^2 + \frac{A_1}{A_0} (\partial_i u)^2 + \frac{A_2 (\partial^2 u)^2}{a^2} \right],
$$

(2.7)

where a prime denotes the derivative with respect to the conformal time $\eta$ and

$$
z = a \sqrt{A_0}.
$$

(2.8)

Then variation of the action with respect to the Fourier modes $u_k(\eta)$ of $u$ leads to the equation of motion,

$$
u''_k + \left( \omega_k^2(\eta) - \frac{z''}{z} - 3a^2 H^2 \epsilon_1 \right) u_k = 0,
$$

(2.9)

where

$$
\omega_k^2 = \frac{A_1}{A_0},
$$

(2.10)

and

$$
\omega_k^2(\eta) = c_s^2 k^2 \left( 1 - \frac{A_2}{A_0 c_s^2} \frac{k^2}{a^2} \right).
$$

(2.11)

It is easy to see that in the EFT of inflation, both the effective sound speed and the nonlinear terms in the dispersion relation receive contributions from the operators with $M_2, M_1, M_2, M_3$. If one ignores $M_2$ and $M_3$ terms, the nonlinear dispersion relation reduces to the usual linear one. In Refs. [9, 24–26], the nonlinear term ($A_2$ term in (2.11)) has been dropped and the time-dependent sound speed $c_s$ has been assumed to be constant during inflation. In this paper, we are going to consider effects of both the nonlinear term and the time variation of the sound speed in the primordial power spectra.
2.2 Tensor perturbations

For tensor perturbations, the perturbed spacetime is set as
\[ g_{ij} = a^2 (\delta_{ij} + h_{ij}), \]
(2.12)
where \( h_{ij} \) represents the transverse and traceless tensor perturbations, which satisfies
\[ h_i^j = 0 = \partial_i^j h_{ij}. \]
(2.13)

Then expanding the total action \( S_{\text{eff}} \) up to the second-order gives
\[ S_h^2 = \frac{M_{\text{Pl}}^2}{8} \int dt d^3 x a^3 \left( c_i^{-2} \partial_i h_{ij} \partial_j h^{ij} - a^{-2} \partial_i h_{ij} \partial^j h^{ij} \right), \]
(2.14)
where the effective sound speed \( c_i \) for tensor perturbations is given by
\[ c_i^2 = \left( 1 - \frac{\bar{M}_3^2}{M_{\text{Pl}}^2} \right)^{-1}. \]
(2.15)
In order to avoid superluminal propagation for tensor modes, one has to require
\[ \bar{M}_3^2 < 0. \]
(2.16)

Variation of the action with respect to \( h_{ij} \) leads to the equation of motion
\[ \frac{d^2 \mu_k^{(i)}(\eta)}{d\eta^2} + \left( c_i^2 k^2 - \frac{a^{\prime\prime}}{a} \right) \mu_k^{(i)}(\eta) = 0, \]
(2.17)
where \( d\eta = dt/a \) is the conformal time and \( \mu_k^{(i)}(\eta) = a M_{\text{Pl}} h_k/\sqrt{2} \) with \( h_k \) denoting the Fourier modes of the two helicities of the tensor perturbations.

It is obvious to observe that the new operators introduced in the action (2.1) of EFT of inflation can affect sound speed of the tensor modes. As pointed out in [50, 51], this effective sound speed can be redefined to unity by a disformal transformation. Under this disformal transformation, the new effects in the sound speed shift to the effective time-dependent mass term \( a^{\prime\prime}/a \rightarrow \tilde{a}^{\prime\prime}/\tilde{a} \) in (2.17) such that the primordial tensor spectrum keeps invariant. With this property, one can either do the calculation with \( c_i \neq 1 \) or with \( c_i = 1 \) by using the disformal transformation. In this paper, we adopt the former case.

2.3 Slow-roll parameters

In order to consider the slow-roll inflation, we first need to impose the following slow-roll conditions,
\[ \left| \frac{\dot{H}}{H^2} \right|, \left| \frac{\ddot{H}}{H H} \right| \ll 1. \]
(2.18)

With these conditions, it is convenient to introduce a set of the slow-roll parameters, the Hubble flow parameters, which are defined by
\[ \epsilon_1 = -\frac{\dot{H}}{H^2}, \quad \epsilon_{n+1} = \frac{d \ln \epsilon_n}{d \ln a}. \]
(2.19)

In the EFT of inflation, in general one also requires all the coefficients \( (M_2^1, M_1^1, M_2^2, M_3^2) \) of the new operators in the action (2.1) satisfy the approximation conditions:
\[ \left| \frac{M_2^1}{M_{\text{Pl}}^2 H^2 \epsilon_1} \right|, \left| \frac{M_1^1}{M_{\text{Pl}}^2 H \epsilon_1} \right|, \left| \frac{M_2^2}{M_{\text{Pl}}^2 \epsilon_1} \right|, \left| \frac{M_3^2}{M_{\text{Pl}}^2 \epsilon_1} \right| \ll 1. \]
(2.20)

In the slow-roll approximation, we also need all the above quantities are slow-varying. With this assumption, we can introduce four new sets of slow-roll parameters, which are defined by
\[ \delta_1 = \frac{M_2^4}{M_{\text{Pl}}^2 H^2 \epsilon_1}, \quad \delta_{n+1} = \frac{d \ln \delta_n}{d \ln a}. \]
(2.21)
\[ \sigma_1 = \frac{M_1^3}{M_{\text{Pl}}^2 H \epsilon_1}, \quad \sigma_{n+1} = \frac{d \ln \sigma_n}{d \ln a}. \]
(2.22)
\[ \xi_1 = \frac{M_2^2}{M_{\text{Pl}}^2 \epsilon_1}, \quad \xi_{n+1} = \frac{d \ln \xi_n}{d \ln a}. \]
(2.23)
\[ \kappa_1 = \frac{M_3^2}{M_{\text{Pl}}^2 \epsilon_1}, \quad \kappa_{n+1} = \frac{d \ln \kappa_n}{d \ln a}. \]
(2.24)

Then using these slow-roll parameters, up to the second-order, the effective sound speed for scalar perturbations and the modified dispersion relation \( \omega_k^2(\eta) \) can be expressed as
\[ \omega_k^2(\eta) \simeq c_s^2 k^2 + b(\eta) k^4 \eta^2, \]
(2.26)
where \( b(\eta) \) is a small and slow-varying quantity which reads
\[ b(\eta) \simeq \frac{\xi_1 + \kappa_1}{2} - \xi_1 \epsilon_1 - \kappa_{1} \epsilon_1 - b_0 \xi_1 - b_1 \kappa_1 
+ \frac{3 \xi_1 \kappa_1}{2} + \frac{3 \xi_1 \sigma_1}{2} + \frac{9 \xi_1^2}{4} + \frac{3 \kappa_1 \sigma_1}{2} - \frac{3 \kappa_1^2}{4}. \]
(2.27)
3 Scalar and tensor perturbation spectra in the uniform asymptotic approximation

3.1 General formulas of primordial spectra in the uniform asymptotic approximation

In this subsection, we present a very brief introduction of the general formulas of primordial perturbations with a slowly-varying speed and parameter $b(\eta)$.

In the uniform asymptotic approximation, we first write Eqs. (2.9) and (2.17) in the standard form [32,52]

$$\frac{d^2 \mu(y)}{dy^2} = (\lambda^2 \hat{g}(y) + q(y)) \mu(y),$$  \hspace{1cm} (3.1)

where we introduce a new variable $y = -k \eta$, $\mu(y) = \mu_\lambda(y)$ and $\mu_b(y)$ corresponding to scalar and tensor perturbations respectively; and

$$\lambda^2 \hat{g}(y) + q(y) = \frac{v^2(\eta) - 1/4}{y^2} - c^2(\eta) - b^2(\eta)y^2.$$  \hspace{1cm} (3.2)

Here $c(\eta) = c_{s,t}(\eta)$ is the effective sound speed for scalar and tensor perturbation modes respectively. For scalar perturbation, we have [24,26]

$$v^2_s(\eta) = \eta^2 \frac{z''(\eta)}{z(\eta)} + \frac{1}{4} + 3a^2 H^2 \epsilon_1,$$  \hspace{1cm} (3.3)

and for tensor modes,

$$v^2_t(\eta) = \eta^2 \frac{a''(\eta)}{a(\eta)} + \frac{1}{4} \quad \text{and} \quad b(\eta) = 0.$$  \hspace{1cm} (3.4)

Note that in the above equation, $\lambda$ is supposed to be a large parameter and used to label the different orders of the approximation. As we will see later in the general formulas of power spectrum in Eq. (3.8), $\lambda$ with different powers denote different approximate orders in the uniform asymptotic approximation. In the final calculation we can set $\lambda = 1$ for simplification. Now in order to construct the approximate solutions of the above equation by using the uniform asymptotic approximation, one needs to choose [32]

$$q(y) = -\frac{1}{4y^2},$$  \hspace{1cm} (3.5)

to ensure the convergence of the errors of the approximate solutions. Then, we have

$$\lambda^2 \hat{g}(y) = \frac{v^2(\eta)}{y^2} - c^2(\eta) - b(\eta)y^2.$$  \hspace{1cm} (3.6)

In order to get a healthy UV limit, for scalar perturbation we have to impose the condition $b(\eta) > 0$.

Considering that $b(\eta)$ is a small quantity, it is obvious to see that the function $\lambda^2 \hat{g}(y)$ has a single turning point, which can be expressed as

$$y^2_{\lambda}(\eta_0) = \frac{-c^2(\eta_0) + \sqrt{c^4(\eta_0) + 4b(\eta_0)\bar{v}^2(\eta_0)}}{2b(\eta_0)}.$$  \hspace{1cm} (3.7)

Then following [34], the general formula of the power spectrum reads

$$\Delta^2(k) = \frac{k^3 H^2}{4\pi^2} \left| \frac{u(y)}{z(\eta)} \right|^2_{y \to 0^+} = \frac{k^2}{8\pi^2} \frac{-4\eta}{z^2(\eta) \nu(\eta)} \exp \left( 2\lambda \int_{y_0}^{\eta_0} \sqrt{\hat{g}(y') dy'} \right) \times \left[ 1 + \mathcal{H}(+\infty) \frac{\bar{v}}{\lambda} + \frac{\mathcal{H}^2(+\infty)}{2\lambda^2} + O \left( \frac{1}{\lambda^3} \right) \right].$$  \hspace{1cm} (3.8)

We would like to mention that, in the square bracket of the above equation, the parameter $\lambda$ with different powers denote different approximate orders in the uniform asymptotic approximation, which shows clearly the above formula is at the third-order approximation. In this formulas, the integral of $\sqrt{\hat{g}}$ is given by Eqs. (A.8) with $L_0$ and $L_1$ being given by Eqs. (A.11) and (A.13) and the error control function is given by (A.15) with $H_0$ and $H_1$ being given by Eqs. (A.16) and (A.17).

Now we turn to consider the corresponding spectral indices. In order to do this, we first specify the $k$-dependence of $\bar{v}_0(\eta_0)$, $\bar{v}_1(\eta_0)$ through $\eta_0 = \eta_0(k)$. From the relation $-k\eta_0 = \bar{v}_0$, we have

$$\frac{d \ln(-\eta_0)}{d \ln k} = -1 + \frac{d \ln \bar{v}_0}{d \ln(-\eta_0)} \frac{d \ln(-\eta_0)}{d \ln k},$$  \hspace{1cm} (3.9)

which leads to

$$\frac{d \ln(-\eta_0)}{d \ln k} \simeq -1 - \frac{d \ln \bar{v}_0}{d \ln(-\eta_0)}.$$  \hspace{1cm} (3.10)

Then using this relation, the spectral index is given by

$$n_s - 1, \quad n_t \equiv \frac{d \ln \Delta^2(t)}{d \ln k} \simeq 3 - 2\bar{v}_0 - \frac{2\bar{v}_1 \bar{v}_0}{3c_0^3} + \frac{2c_1 \bar{v}_0}{c_0^2} + \frac{2\bar{v}_1 \bar{v}_0^3}{3c_0^5} - 2\bar{v}_1 \ln 2 + \frac{\bar{v}_1}{6\bar{v}_0^2}.$$  \hspace{1cm} (3.11)

Similarly, we find that the running of the spectral index $\alpha \equiv dn/d\ln k$ is given by

$$\alpha_{s,t}(k) \simeq 2\bar{v}_1 + \frac{2\bar{v}_2 \bar{v}_0}{3c_0^3} - \frac{2c_2 \bar{v}_0}{c_0^2} - \frac{2\bar{v}_2 \bar{v}_0^3}{3c_0^5} + 2\bar{v}_2 \ln 2 - \frac{\bar{v}_2}{6\bar{v}_0^2}.$$  \hspace{1cm} (3.12)

In the above, we present all the formulas (Eqs. (3.8), (3.11), and (3.12)) that can be directly used to calculate the...
primordial perturbation spectra from different inflation models. Note that these formulas are easy to use because they only depend on the quantities $H(\eta), (c_0, c_1, c_2), (v_0, v_1, v_2)$ evaluated at the turning point. These quantities can be easily calculated from Eqs. (2.25, 3.3) for scalar perturbations and Eqs. (2.15, 3.4) for tensor perturbations. In the following subsections, we apply these formulas to calculate the slow-roll power spectra for both scalar and tensor perturbations in the EFT of inflation.

3.2 Scalar spectrum

We first consider the scalar perturbations. As we already pointed out in the introduction, in order to match the accuracy of forthcoming observations, we need to calculate the spectral indices up to the next-leading order (second-order) in the expansions of the slow-roll approximation. For this purpose, we only need to consider the quantities ($\tilde{c}_6, c_{\mathcal{R}1}$) and ($v_{\mathcal{R}0}, v_{\mathcal{R}1}, v_{\mathcal{R}2}$) up to the second-order in the slow-roll expansions.

For the slow-varying sound speed $c_{\mathcal{R}}$, from Eq. (2.25) we find

$$
\tilde{c}_{\mathcal{R}0} \approx 1 - \tilde{\delta}_1 + \frac{3\tilde{\delta}_1}{2} + \frac{\tilde{\epsilon}_1}{2} + \frac{5\tilde{\delta}_1}{4} - 6\tilde{\delta}_1\tilde{\epsilon}_1 - 2\bar{\delta}_1\bar{\epsilon}_1 \\
- \frac{17\tilde{\epsilon}_1\bar{\epsilon}_1}{4} + \frac{3\tilde{\delta}_1^2}{2} + \frac{15\tilde{\delta}_1\bar{\epsilon}_1}{4} + \frac{33\tilde{\epsilon}_1\bar{\epsilon}_1}{4} \\
+ \frac{45\tilde{\delta}_1^2}{8} + \frac{11\tilde{\epsilon}_1\bar{\epsilon}_1}{4} + \frac{5\tilde{\delta}_1^2}{8} + \frac{95\tilde{\epsilon}_1^2}{32}. 
$$

(3.13)

For $v_{\mathcal{R}}$, from Eqs. (A.2) and (3.3) we find

$$
v_{\mathcal{R}0} \approx \frac{3}{2} + \tilde{\epsilon}_1 + \frac{\tilde{\epsilon}_2}{2} + \frac{\tilde{\epsilon}_1^2}{6} + \frac{\tilde{\epsilon}_2\tilde{\epsilon}_3}{6} \\
+ \tilde{\delta}_1\tilde{\delta}_2 - \frac{9\tilde{\epsilon}_1\tilde{\epsilon}_2}{4} - \frac{3\tilde{\delta}_1\tilde{\epsilon}_2}{4} - \frac{3\tilde{\delta}_1\tilde{\epsilon}_2}{2},
$$

(3.14)

$$
v_{\mathcal{R}1} \approx -\bar{\epsilon}_1\bar{\epsilon}_2 - \frac{\tilde{\epsilon}_2\tilde{\epsilon}_3}{2},
$$

(3.15)

and

$$
v_{\mathcal{R}2} \equiv \frac{d^2v_{\mathcal{R}}}{d\ln^2(-\eta)} = \mathcal{O}(\tilde{\epsilon}_i^3).
$$

(3.16)

Then, using the above expansions, the power spectrum for the curvature perturbation $\mathcal{R}$ can be calculated via Eq. (3.8).

After tedious calculations we obtain,

$$
\Delta^2_{\mathcal{R}}(k) = \tilde{\Delta}_{\mathcal{R}} \left[ 1 - (2 - 2\tilde{D}_p)\tilde{\epsilon}_1 - \tilde{D}_p\tilde{\epsilon}_2 + \tilde{\delta}_1 + \left( \frac{\tilde{D}_p - 9}{8} \right)\bar{\epsilon}_1 \\
- \frac{3\tilde{\delta}_1}{4} + \left( \frac{\tilde{D}_p - 9}{8} \right)\bar{\epsilon}_1 \\
+ \left( \frac{\tilde{D}_p^2 - \tilde{D}_p + \tilde{\Delta}_1 + 2\tilde{\Delta}_2 + \frac{7\pi^2}{12} - 8 \right)\tilde{\epsilon}_1\tilde{\epsilon}_2 \\
+ \left( \frac{\tilde{D}_p^2}{2} + \frac{\tilde{\Delta}_1}{4} + \frac{\pi^2}{8} - \frac{3}{2} \right)\tilde{\epsilon}_2^2 + \left( \frac{-\tilde{D}_p^2}{2} + \tilde{\Delta}_2 + \frac{\pi^2}{24} \right)\tilde{\epsilon}_2^3 \\
+ \left( 2\tilde{D}_p + 2\tilde{D}_p + \tilde{\Delta}_1 + \frac{\pi^2}{2} - 5 \right)\tilde{\epsilon}_1^2 + \frac{9\tilde{\delta}_1\bar{\epsilon}_1}{4} \\
+ \left( -\frac{3\tilde{D}_p}{4} - \frac{5}{2} \right)\tilde{\delta}_1\bar{\epsilon}_2 \\
+ \left( -2\tilde{D}_a\tilde{D}_p - \frac{9031\tilde{D}_a}{2715} + \frac{9\tilde{D}_p}{4} + \frac{9}{4} \right)\tilde{\xi}_1\tilde{\epsilon}_1 \\
- \frac{\tilde{\delta}_1^2}{2} + (2\tilde{\delta}_1 - 2)\tilde{\delta}_1\tilde{\epsilon}_1 + \left( \frac{3\tilde{D}_a}{8} + \frac{9}{8} \right)\tilde{\delta}_1\bar{\epsilon}_1 \\
+ \left( \tilde{D}_a\tilde{D}_p - \frac{449\tilde{D}_a}{724} - \frac{9\tilde{D}_p}{8} - \frac{4405}{2896} \right)\tilde{\kappa}_1\tilde{\kappa}_2 \\
+ \frac{3}{4}\tilde{D}_p\tilde{\delta}_1\tilde{\epsilon}_2 - \frac{57\tilde{\delta}_1^2}{32} + (\tilde{D}_p + 2)\tilde{\delta}_1\tilde{\delta}_2 - \tilde{D}_p\tilde{\delta}_1\tilde{\epsilon}_2 \\
+ \left( 3\tilde{D}_a - \frac{15}{8} \right)\tilde{\delta}_1\tilde{\epsilon}_1 \\
+ \left( -\frac{\tilde{D}_a\tilde{D}_p + 1829\tilde{D}_a}{5430} + \frac{9\tilde{D}_p}{8} - \frac{9}{8} \right)\tilde{\xi}_1\tilde{\epsilon}_2 \\
+ \left( -2\tilde{D}_a\tilde{D}_p - \frac{9031\tilde{D}_a}{2715} + \frac{9\tilde{D}_p}{4} + \frac{9}{4} \right)\tilde{\kappa}_1\tilde{\kappa}_1 \\
+ \left( \frac{39}{32} - \frac{11\tilde{D}_a}{4} \right)\tilde{\kappa}_1\tilde{\epsilon}_1 \\
+ \left( -\frac{\tilde{D}_a\tilde{D}_p + 1829\tilde{D}_a}{5430} + \frac{9\tilde{D}_p}{8} - \frac{9}{8} \right)\tilde{\kappa}_1\tilde{\kappa}_2 \\
+ \left( -\frac{11\tilde{D}_a}{4} - \frac{81}{32} \right)\tilde{\kappa}_1\tilde{\epsilon}_1 + \left( \frac{297}{640} - \frac{69\tilde{D}_a}{10} \right)\tilde{\xi}_1^2 \\
+ \left( \frac{3\tilde{D}_p}{2} + \frac{3}{2} \right)\tilde{\delta}_1\tilde{\epsilon}_1 \\
+ \left( \frac{2457}{320} - \frac{79\tilde{D}_a}{5} \right)\tilde{\xi}_1\tilde{\kappa}_1 + \left( \frac{3657}{640} - \frac{89\tilde{D}_a}{10} \right)\tilde{\kappa}_1^2 \\
+ \left( \tilde{D}_a\tilde{D}_p - \frac{449\tilde{D}_a}{724} - \frac{9\tilde{D}_p}{8} - \frac{10197}{2896} \right)\tilde{\xi}_1\tilde{\xi}_2 \right].
$$

(3.17)

where $\tilde{\Delta}_{s} \equiv \frac{188\tilde{H}^2}{724\pi^5(1 + \frac{2}{3}H)}\tilde{\xi}_1\tilde{\xi}_2\mathcal{M}_p^6$, $\tilde{D}_p \equiv \frac{67}{18\pi^2} - \ln 2$, $\tilde{\Delta}_{s} \equiv \frac{90}{18\pi^2}$, $\tilde{\Delta}_1 \equiv \frac{183606}{52761} - \frac{\pi^2}{2}$, and $\tilde{\Delta}_{s} \equiv \frac{8926}{88909979}$. Note that a letter with an over bar denotes quantity evaluated at the turning point $\bar{\eta}_0$. From the scalar spectrum (3.17), we observe that the effects of the four new operators affect the spectrum at the leading-order in the slow-roll expansion. We would like to mention that when we wrote down the action (2.3), we have neglected all the terms those do not contribute to the scalar power spectrum at the leading-order. But they may lead to contributions in the scalar spectrum at the second-
order which are not included in the above expression. This is because their contributions only affect the spectral index $n_s$ and tensor-to-scalar ratio $r$ at the third-order and our intention here would be to obtain results which are accurate up to the second-order in the spectral index $n_s$ and $r$ for matching the accuracy of future observational data.

Then with the scalar power spectrum given above, the scalar spectral index is,

$$n_s - 1 \simeq -2\tilde{e}_1 - \tilde{e}_2 - 2\tilde{e}_1^2 + 3\tilde{\delta}_1\tilde{\delta}_2 - (2\tilde{D}_n + 3)\tilde{e}_1\tilde{e}_2$$

and the running of the scalar spectral index is expressed as

$$\alpha_s \simeq -2\tilde{e}_1\tilde{e}_2 - \tilde{e}_2\tilde{e}_3 - 6\tilde{e}_1^2\tilde{e}_2 - (2\tilde{D}_n + 3)\tilde{e}_1\tilde{e}_2^2 - \tilde{D}_n\tilde{e}_2\tilde{e}_3^2$$

The second-order in the spectral index

$$\tilde{e}_3 = \frac{5\tilde{\xi}_1\tilde{\xi}_2}{8} - \frac{5\tilde{\kappa}_1\tilde{\kappa}_2}{8} - 3\tilde{\sigma}_1\tilde{\sigma}_2,$$

(3.18)

For scalar spectral index, the new effects denoted by the four new sets of the slow-roll parameters appear at the next-to-leading order, while for the running of the index, they only contribute to the third-order of the slow-roll approximation.

3.3 Tensor spectrum

Now we consider the tensor spectrum. First we need to derive the expressions of $v_{t0}$, $v_{t1}$, $v_{t2}$, and $c_{t0}$, $c_{t1}$, $c_{t2}$. Repeating similar calculations for scalar perturbations, we obtain

$$\tilde{v}_{t0} = \frac{3}{2} + \tilde{\xi}_1 + \tilde{\xi}_1^2 + \frac{4\tilde{e}_1\tilde{e}_2}{3} + O(\tilde{e}_3^3),$$

(3.20)

$$\tilde{v}_{t1} = \frac{d\tilde{v}_t}{d\ln(\eta)} = -\tilde{e}_1\tilde{e}_2 + O(\tilde{e}_3^3),$$

(3.21)

$$\tilde{v}_{t2} = \frac{d^2\tilde{v}_t}{d\ln(\eta)^2} = O(\tilde{e}_3^3),$$

(3.22)

and

$$\tilde{c}_{t0} = 1 + \tilde{\xi}_1\tilde{e}_1 + O(\tilde{e}_3^3),$$

(3.23)

$$\tilde{c}_{t1} = \frac{dc_{t1}}{d\ln(\eta)} = O(\tilde{e}_3^3).$$

(3.24)

Then, the power spectrum for the tensor perturbation $h_k$ reads

$$\Delta^2_s(k) = \tilde{A}_t \left[ 1 - \left( 2\tilde{D}_p - 2\right) \tilde{e}_1 + \left( 2\tilde{D}_p^2 + 2\tilde{D}_p + \tilde{\Delta}_1 + \frac{\pi^2}{2} - 5 \right) \tilde{e}_1^2 + \left( -\tilde{D}_p^2 - 2\tilde{D}_p + 2\tilde{\Delta}_2 + \frac{\pi^2}{12} - 2 \right) \tilde{e}_1\tilde{e}_2 - \frac{\tilde{e}_1\tilde{\kappa}_1}{2} \right],$$

(3.25)

where $\tilde{A}_t = \frac{1811}{36e_\star^2}$. Also the tensor spectral index and its running are given by

$$n_t \simeq -2\tilde{e}_1 - 2\tilde{e}_2^2 - (\tilde{D}_n + 1)2\tilde{e}_1\tilde{e}_2,$$

(3.26)

and

$$\alpha_t \simeq -2\tilde{e}_1\tilde{e}_2 - 6\tilde{e}_1^2\tilde{e}_2 - (\tilde{D}_n + 1)2\tilde{e}_1\tilde{e}_2^2 - 2(\tilde{D}_n + 1)2\tilde{e}_1\tilde{e}_2^2.$$

(3.27)

We observe that the new effects (represented by $\tilde{\kappa}_1$) can only affect the tensor spectrum at the next-to-leading order. In this case, the tensor spectral index and its running are keeping the same as the standard slow-roll inflation.

3.4 Expressions at horizon crossing

In the last two subsections, all the results are expressed in terms of quantities that are evaluated at the turning point. However, usually those expressions were expressed in terms of the slow-roll parameters which are evaluated at the time $\eta_\star$ when scalar or tensor perturbation modes cross the horizon, i.e., $a(\eta_\star)H(\eta_\star) = c_s(\eta_\star)k$ for scalar perturbations and $a(\eta_\star)H(\eta_\star) = c_t(\eta_\star)k$ for tensor perturbations. Consider modes with the same wave number $k$, it is easy to see that the scalar and tensor modes left the horizon at different times if $c_s(\eta) \neq c_t(\eta)$. When $c_s(\eta) > c_t(\eta)$, the scalar mode leaves the horizon later than the tensor mode, and for $c_s(\eta) < c_t(\eta)$, the scalar mode leaves the horizon before the tensor one.

As we have two different horizon crossing times, it is reasonable to rewrite all our results in terms of quantities evaluated at the later time, i.e., we should evaluate all expressions at scalar horizon crossing time $a(\eta_\star)H(\eta_\star) = c_s(\eta_\star)k$ for $c_s(\eta_\star) > c_t(\eta_\star)$ and at tensor-mode horizon crossing $a(\eta_\star)H(\eta_\star) = c_t(\eta_\star)k$ for $c_s(\eta_\star) < c_t(\eta_\star)$.

3.4.1 $c_s(\eta_\star) > c_t(\eta_\star)$

Then, we shall re-write all the expressions in terms of quantities evaluated at the time when the scalar-mode horizon crossing $c_s(\eta_\star) > c_t(\eta_\star)$. Skipping all the tedious calculations, we find that scalar spectrum can be written in the form

$$\Delta^2_s(k) = A_s^\star \left[ 1 - (2D^*_\star + 2)\epsilon^\star_1 
- D^*_\star \epsilon^\star_2 - \left( \frac{D^*_\star}{2} \right) \epsilon^\star_4 
- D^*_\star \epsilon^\star_2 - \left( \frac{D^*_\star}{8} \right) \epsilon^\star_4
+ \left( \frac{D^*_\star}{2} + 2D^*_\star + \Delta^*_\star + \frac{\pi^2}{2} - 5 \right) \epsilon^\star_4
\right].$$

(3.25)
\[
+ \left( \frac{D^2_p}{2} + \frac{\Delta^*_p}{4} + \frac{\pi^2}{8} - 1 \right) \epsilon_{s2}^2 \\
+ \left( \frac{D^2_p - D^*_p + \Delta^*_t + 2\Delta^*_s + \frac{7\pi^2}{12} - 7}{2} \right) \epsilon_{s1}\epsilon_{s2} \\
- \left( 3D^*_a - \frac{15}{8} \delta_{s1} \kappa_{s1} + \frac{3D^*_p}{2} - \frac{15}{4} \right) \epsilon_{s1}\epsilon_{s2} \\
- \left( D^*_a D^*_p + \frac{53117D^*_p}{17376} - \frac{9D^*_p}{8} \right) \xi_{s1}\epsilon_{s2} \\
- \frac{2509}{320} D^*_s \xi_{s1} \sigma_{s1} \\
+ \left( \frac{3D^*_a + 9}{8} \delta_{s1} \xi_{s1} \right) \\
+ \left( \frac{3D^*_p}{2} + \frac{1}{2} \right) \sigma_{s1} \epsilon_{s2} \\
+ \left( D^*_a D^*_p - \frac{223033D^*_a}{28960} - \frac{9D^*_p}{8} \right) \xi_{s1}\epsilon_{s2} \\
+ \frac{9\sigma_{s1}\epsilon_{s1}}{4} - \frac{38187D^*_a \xi_{s1}^2}{6400} \\
+ \left( \frac{3D^*_a}{4} + \frac{5}{2} \right) \sigma_{s1} \sigma_{s2} \\
- \left( \frac{D^*_a D^*_p}{4} + \frac{46213D^*_p}{43440} - \frac{9D^*_p}{4} \right) \xi_{s1}\epsilon_{s1} - \frac{\delta_{s1}^2}{2} \\
- \left( D^*_a D^*_p + \frac{46213D^*_a}{43440} - \frac{9D^*_p}{4} \right) \kappa_{s1}\epsilon_{s2} \\
- \left( D^*_a D^*_p + \frac{46213D^*_a}{43440} - \frac{9D^*_p}{4} \right) \kappa_{s1}\epsilon_{s2} - \frac{57\sigma_{s1}^2}{32} \\
- \left( 2D^*_p + 2 \right) \delta_{s1}\epsilon_{s1} \\
- \frac{D^*_a \delta_{s1}\epsilon_{s2} - \frac{1147D^*_a \xi_{s1} \kappa_{s1}}{3200} - \frac{287}{960} D^*_a \kappa_{s1} \sigma_{s1}}{3200} \\
+ \frac{49759D^*_a \epsilon_{s1}^2}{19200} + (D^*_p + 2) \epsilon_{s1}\delta_{s2} \right]. \tag{3.28}
\]

where the subscript "\(^*\)" denotes evaluation at the horizon crossing, \(A_i^* = \frac{181H_i^2}{12^i - \pi^2 \epsilon_{s1}}\), \(D^*_p = \frac{67}{181} - \ln 3\), \(D^*_s = \frac{90}{181}\), \(\Delta^*_t = \frac{485206}{98283} - \frac{\pi^2}{2}\), and \(\Delta^*_s = \frac{9269}{58990}\). For the scalar spectral index, one obtains

\[
n_s - 1 \simeq -2\epsilon_{s1} - \epsilon_{s2} - 2\epsilon_{s1}^2 + \delta_{s1}\delta_{s2} \\
- \left( 2D^*_n + 3 \right) \epsilon_{s1}\epsilon_{s2} - D^*_n \epsilon_{s2}^2 + \frac{5\kappa_{s1} \kappa_{s2}}{8} - \frac{3\kappa_{s1} \kappa_{s2}}{4} \sigma_{s1} \sigma_{s2}. \tag{3.29}
\]

The running of the scalar spectral index reads

\[
\alpha_s \simeq -2\epsilon_{s1}\epsilon_{s2} - \epsilon_{s1}\epsilon_{s2} - 3\epsilon_{s1}^2 \epsilon_{s2} - 6\epsilon_{s1}^2 \epsilon_{s2} + \delta_{s1}\delta_{s2}\delta_{s3} \\
- 4\epsilon_{s1}\epsilon_{s2} \epsilon_{s3} + \delta_{s1}\delta_{s2}^2 - 2D^*_n \epsilon_{s2}^2 - \frac{3}{4} \sigma_{s1} \sigma_{s2} \sigma_{s3} \\
- D^*_n \epsilon_{s2}^2 - D^*_n \epsilon_{s2} \epsilon_{s3}^2 - \frac{5}{8} \kappa_{s1} \kappa_{s2} \epsilon_{s2} - \frac{5}{8} \kappa_{s1} \kappa_{s2} \kappa_{s3} \\
- \frac{5}{8} \kappa_{s1} \kappa_{s2} \kappa_{s3} - 2D^*_n \epsilon_{s1}\epsilon_{s2}\epsilon_{s3} - \frac{5}{8} \kappa_{s1} \kappa_{s2}^2 - \frac{3}{4} \sigma_{s1} \sigma_{s2}^2. \tag{3.30}
\]

Similar to the scalar perturbations, now let us turn to consider the tensor perturbations, which yield

\[
\Delta^2_t(k) = -2 \left( 1 + D^*_p \right) \epsilon_{s1} \\
+ \left( 2D^*_p + 2D^*_p - \Delta^*_t + \frac{\pi^2}{2} - 5 \right) \epsilon_{s1}^2 \\
- \left( -D^*_p - 2D^*_p + \frac{\pi^2}{12} - 5 + 2\Delta^*_t \right) \epsilon_{s1}\epsilon_{s2} \\
- 2\delta_{s1}\epsilon_{s1} + 3\xi_{s1}\epsilon_{s1} + \frac{\kappa_{s1} \epsilon_{s1}}{2} + 2\sigma_{s1} \epsilon_{s1}. \tag{3.31}
\]

For the tensor spectral index, we find

\[
n_t \simeq -2\epsilon_{s1} - \left( 2D^*_n + 2 \right) \epsilon_{s1}\epsilon_{s2} - 2\epsilon_{s1}^2. \tag{3.32}
\]

Then, the running of the tensor spectral index reads

\[
\alpha_t \simeq -2\epsilon_{s1}\epsilon_{s2} - 6\epsilon_{s1}^2 \epsilon_{s2} - \left( 2D^*_n + 1 \right) \epsilon_{s1}\epsilon_{s2}^2 \\
- 2 \left( 2D^*_n + 1 \right) \epsilon_{s1}\epsilon_{s2}\epsilon_{s3}. \tag{3.33}
\]

Finally with both scalar and tensor spectra given above, we can evaluate the tensor-to-scalar ratio at the horizon crossing time \(n_s\), and find that

\[
r \simeq 16\epsilon_{s1} \left[ 1 + D^*_p \epsilon_{s2} - \delta_{s1} \right] \\
+ \left( \frac{9}{8} - D^*_n \right) \kappa_{s1} + \left( \frac{9}{8} - D^*_a \right) \xi_{s1} \\
+ \frac{3\sigma_{s1}}{4} + \left( D^*_p - \Delta^*_t + \frac{\pi^2}{2} + 5 \right) \epsilon_{s1} \epsilon_{s2} \\
+ \left( \frac{D^*_p}{4} - 2\Delta^*_t + \frac{\pi^2}{24} \right) \epsilon_{s2} \epsilon_{s3} \\
+ \left( \frac{D^*_p}{2} - 2\Delta^*_t + \frac{\pi^2}{8} + 1 \right) \epsilon_{s2}^2 \\
- \left( D^*_p + 2 \right) \delta_{s1}\delta_{s2} \\
+ \left( \frac{3}{4} D^*_a + \frac{1}{2} \right) \sigma_{s1}\epsilon_{s2} + \frac{421}{192} D^*_a \kappa_{s1} \sigma_{s1} \\
+ \frac{3\sigma_{s1}^2}{2} + \left( \frac{9}{8} D^*_p - D^*_p \right) D^*_a + \frac{223033D^*_p^2}{14400} \xi_{s1}\epsilon_{s2} \\
+ \left( \frac{9}{8} D^*_p - D^*_n \right) D^*_a + \frac{53117D^*_p^2}{8640} \xi_{s1}\epsilon_{s2} \\
- 2\delta_{s1}\epsilon_{s1} - \left( D^*_a + \frac{27}{8} \right) \delta_{s1}\xi_{s1}. \tag{3.34}
\]
For $c_s(\eta_*) < c_t(\eta_*)$, as the scalar mode leaves the horizon before the tensor mode does, we shall rewrite all the expressions in terms of quantities evaluated at the time when the tensor mode leaves the Hubble horizon $a(\eta_*)H(\eta_*) = c_t(\eta_*)k$. Skipping all the tedious calculations, we find that the scalar spectrum can be written in the form

$$
\Delta_s^2(k) = A_s^* \left[ 1 - \left( 2D_p^* + 2 \right) \epsilon s_1 \right. \\
- D_p^* \epsilon s_2 - \left( D_a^* - \frac{9}{8} \right) \xi s_1 - \left( D_a^* - \frac{9}{8} \right) \kappa s_1 \\
+ \delta s_1 - \frac{3 \sigma s_1}{4} - \left( \frac{D_p^2}{2} - \Delta^*_a - \frac{\pi^2}{24} \right) \epsilon s_2 \epsilon s_3 \\
+ \left( \frac{5 \pi}{2} - \frac{7 \pi^2}{12} \right) \epsilon s_2 \epsilon s_3 \\
+ \left( D_a^2 - D_a^* + \Delta^*_a + 2 \Delta^*_s + \frac{7 \pi^2}{12} - 7 \right) \epsilon s_1 \epsilon s_2 \\
- \left( 3D_a^* - \frac{15}{8} \right) \delta s_1 \kappa s_1 + \left( \frac{3 D_p^2}{4} - \frac{5}{4} \right) \sigma s_1 \epsilon s_2 \\
- \left( D_a^2 D_p^* - \frac{178591 D_p^2}{14400} \right) - \frac{9 D_p^*}{8} \right] \epsilon s_1 \epsilon s_2 \\
- \frac{2509}{320} D_a^* \xi s_1 \sigma s_1 \\
+ \left( 3 D_a^* + \frac{9}{8} \right) \delta s_1 \xi s_1 + \left( \frac{3 D_p^*}{2} - 1 \right) \sigma s_1 \epsilon s_1
$$

For the scalar spectral index, one obtains

$$
n_s - 1 \approx -2 \epsilon s_1 - \epsilon s_2 - 2 \epsilon s_1^2 + \delta s_1 \delta s_2 \\
- \left( 2 D_a^* + 3 \right) \epsilon s_1 \epsilon s_2 - D_a^* \epsilon s_1 \epsilon s_2 \epsilon s_3 \\
- \frac{5 \xi s_1 \xi s_2}{8} - \frac{5 \kappa s_1 \kappa s_2}{8} - \frac{3 \sigma s_1 \sigma s_2}{4}.
$$

The running of the scalar spectral index reads

$$
\alpha_s \approx -2 \epsilon s_1 \epsilon s_2 - \epsilon s_2 \epsilon s_3 - 3 \epsilon s_1 \epsilon s_2^2 - 6 \epsilon s_1 \epsilon s_2 + \delta s_1 \delta s_2 \delta s_3 \\
- 4 \epsilon s_1 \epsilon s_1 \epsilon s_2 \epsilon s_3 + \delta s_1 \delta s_2^2 - 2 D_n^* \epsilon s_1 \epsilon s_2^2 - \frac{3}{4} \sigma s_1 \sigma s_2 \sigma s_3 \\
- D_n^* \epsilon s_1 \epsilon s_2^2 - D_n^* \epsilon s_1 \epsilon s_2 \epsilon s_3 - \frac{5}{8} \xi s_1 \xi s_2^2 - \frac{5}{8} \kappa s_1 \kappa s_2 \kappa s_3 \\
- \frac{5}{8} \xi s_1 \xi s_2 \xi s_3 - 2 D_n^* \epsilon s_1 \epsilon s_2 \epsilon s_3 - \frac{5}{8} \kappa s_1 \kappa s_2 \kappa s_3 - \frac{3}{4} \sigma s_1 \sigma s_2 \sigma s_3.
$$

Similar to the scalar perturbations, now let us turn to consider the tensor perturbations, which yield

$$
\Delta_t^2(k) = -2 \left( 1 + D_p^* \right) \epsilon s_1 \\
+ \left( 2 D_p^2 + 2 D_p^* - \Delta^*_t + \frac{\pi^2}{2} - 5 \right) \epsilon s_1 \\
- \left( D_p^2 - 2 D_p^* + \Delta^*_t + 2 \Delta^*_s + \frac{7 \pi^2}{12} - 7 \right) \epsilon s_1 \epsilon s_2 \\
- \left( D_a^2 + \frac{9}{8} \right) \delta s_1 \xi s_1 + \left( \frac{3 D_p^*}{2} - 1 \right) \sigma s_1 \epsilon s_1
$$
For the tensor spectral index, we find
\[ n_T \simeq -2 \epsilon_{s_1} - (2 D^*_n + 2) \epsilon_{s_1} \epsilon_{s_2} - 2 \epsilon_{s_1}^2. \] (3.39)

Then, the running of the tensor spectral index reads
\[ \alpha_T \simeq -2 \epsilon_{s_1} \epsilon_{s_2} - 6 \epsilon_{s_1}^2 \epsilon_{s_2} - 2 \left( D^*_n + 1 \right) \epsilon_{s_1} \epsilon_{s_2}^2 - 2 \left( 2 D^*_n + 1 \right) \epsilon_{s_1} \epsilon_{s_2} \epsilon_{s_3}. \] (3.40)

Finally with both scalar and tensor spectra given above, we can evaluate the tensor-to-scalar ratio at the horizon crossing time \( \eta_h \), and find that
\[ r \simeq 16 \epsilon_{s_1} \left[ 1 + D^*_p \epsilon_{s_2} - \delta_{s_1} \right] \]
\[ + \left( \frac{9}{8} - D^*_a \right) \kappa_{s_1} + \left( \frac{3}{4} - D^*_a \right) \xi_{s_1} + 3 \sigma_{s_1} \frac{3}{4} \]
\[ + \left( D^*_n - \Delta \right) \kappa_{s_1} \epsilon_{s_2} - 2 \left( D^*_n + 1 \right) \epsilon_{s_1} \epsilon_{s_2} \]
\[ + \left( D^*_p - \Delta \right) \kappa_{s_1} \epsilon_{s_2} \]
\[ + \left( D^*_p - \Delta \right) \frac{3}{4} \epsilon_{s_2} \]
\[ + \left( D^*_p + 2 \right) \delta_{s_1} \delta_{s_2} + \left( \frac{3}{4} - D^*_p + \frac{5}{4} \right) \sigma_{s_1} \epsilon_{s_2} - \frac{15 \delta_{s_1}}{4} \sigma_{s_1} \]
\[ - \left( D^*_p + 2 \right) \delta_{s_1} \delta_{s_2} + \left( \frac{3}{4} - D^*_p + \frac{5}{4} \right) \sigma_{s_1} \epsilon_{s_2} - \frac{15 \delta_{s_1}}{4} \sigma_{s_1} \]
\[ + \left( \frac{9}{8} D^*_p - D^*_n D^*_a + \frac{223033 D^*_a^2}{14400} \right) \frac{\kappa_{s_1} \xi_{s_2}}{\delta_{s_1}} \]
\[ + \left( \frac{9}{8} D^*_p - D^*_n D^*_a + \frac{175891 D^*_a^2}{14400} \right) \frac{\kappa_{s_1} \xi_{s_2}}{\delta_{s_1}} \]
\[ + \left( \frac{3}{4} - D^*_p + \frac{5}{4} \right) \sigma_{s_1} \epsilon_{s_2} + \frac{38_2}{2} \frac{\kappa_{s_1} \epsilon_{s_2}}{\delta_{s_1}} \]
\[ + \left( \frac{9}{8} D^*_p - D^*_n D^*_a + \frac{191789 D^*_a^2}{25920} \right) \frac{\kappa_{s_1} \epsilon_{s_2}}{\delta_{s_1}} \]
\[ + \frac{15481}{800} D^*_a^2 \epsilon_{s_2} \]
\[ + \frac{75 \sigma_{s_2}^2}{32} + \frac{1125697 D^*_a^2 \xi_{s_1} \kappa_{s_1}}{288000} \]
\[ + \left( \frac{3}{4} - D^*_p + \frac{5}{4} \right) \sigma_{s_1} \epsilon_{s_2} + \frac{38_2}{2} \frac{\kappa_{s_1} \epsilon_{s_2}}{\delta_{s_1}} \]
\[ + \left( \frac{5 \sigma_{s_2}^2}{64800} + \frac{27 \sigma_{s_2}^2}{8} \right) \delta_{s_1} \xi_{s_1} \]
\[ + \left( \frac{9}{8} D^*_p - D^*_n D^*_a + \frac{1058843 D^*_a^2}{129600} \right) \frac{\kappa_{s_1} \epsilon_{s_2}}{\delta_{s_1}} \]
\[ + \frac{421}{192} D^*_a \kappa_{s_1} \sigma_{s_1} - \delta_{s_1} \epsilon_{s_2}. \] (3.41)

### 3.5 Comparison with results from generalized slow-roll approach

In this subsection, we would like to provide a roughly comparison between our results and those derived by using the generalized slow-roll approach in the unified EFT of inflation [55] at the leading slow-roll order. For this purpose, we can map the coefficients \( C_N, C_{KK}, C_{NK}, \) and \( \tilde{C}_{KK} \) in Eq. (16) in [55] with the four new coefficients \( M_2, M_1, M_2, \) and \( M_3 \) as
\[ C_N + \frac{1}{2} C_{NN} \rightarrow -\dot{H} M^2_{Pl} + 2 M^4, \] (3.42)
\[ C_{KK} \rightarrow -\dot{M}^3, \] (3.43)
\[ C_{KK} \rightarrow -\dot{M}^2_2 - M^2_{Pl}, \] (3.44)
\[ C_{R} \rightarrow \frac{1}{2} M^2_{Pl}. \] (3.46)

In [55], the authors only consider the second-order spatial derivative terms in the action and make the fourth spatial derivative terms vanishing by imposing the condition
\[ C_{KK} = -\tilde{C}_{KK}, \] (3.47)

which corresponds to \( \tilde{M}^2_2 = -\tilde{M}^2 \) in the notation of this paper. One of essential differences of our calculation in this paper is that we have included the contributions of the fourth spatial derivative terms in the calculation of the power spectra.

The scalar power spectrum in [55] (Eq. (85)) is calculated at the leading-order in the optimized slow-roll approximation. In order to compare with our results at least at the first-order in the slow-roll expansion, we have to recalculate their power spectra by including the \( p = 1 \) order in Eq. (65) in [55]. The scalar power spectrum can be calculated as (c.f. Eq. (65) in [55])
\[ \Delta^2 \chi \simeq G(\ln x_f) + p_1(\ln x_f) G'(\ln x_f), \] (3.48)

where
\[ G(\ln x_f) \simeq \ln \left( \frac{H^2}{8 \pi^2 \xi_{s_1} \xi_{s_2}H_{s_1}} \right) - \frac{10}{3} \epsilon_H - \frac{2}{3} \delta_1 \]
\[ - \frac{7}{3} \sigma_{s_1} - \frac{1}{3} \xi_{s_1} - \frac{8}{3} \sigma_{s_2}, \] (3.49)
\[ G'(\ln x_f) \simeq 4 \epsilon_H + 2 \delta_1 + \sigma_{s_1} + \xi_{s_1} + \frac{2}{3} \delta_2 \]
\[ + \frac{7}{3} \sigma_{s_2} + \frac{1}{3} \xi_{s_2}, \] (3.50)
\[ p_1(\ln x_f) = -\ln 2 - y_E - \ln x_f, \] (3.51)

with \( y_E \) being the Euler-Mascheroni constant. In the above expressions, \( c_3, b_3, \delta_1, \epsilon_H, \sigma_{s_1}, \sigma_{s_2}, \xi_{s_1}, \xi_{s_2}, \delta_2, x_1, \) and \( x_f \) are all defined in [55]. In these quantities, \( \sigma_{s_1}, \sigma_{s_2}, \delta_2, \xi_{s_1}, \) and \( \xi_{s_2} \) represent second-order contributions in the notation of
our paper since they are defined as the derivatives of the slow-roll quantities $c_s, \delta_1, b_s$. Therefore we do not consider these terms when we compare the power spectrum at the leading-order. Then we can transform the above power spectrum in our notation, which is

$$\Delta^2(k) = \frac{H^2}{8\pi^2} \left[ 1 + 2(\alpha_\epsilon - 1)\epsilon_1 + \alpha_\epsilon \epsilon_2 - \frac{3}{4} \sigma_1 + \delta_1 \right].$$

(3.52)

where $\alpha_\epsilon \equiv 2 - \ln 2 - \gamma_E \simeq 0.729637$ and the power spectrum is evaluated at the Horizon crossing point $x_f = 1$. By comparing the coefficients of each term with those in (3.35) and considering the assumption $\zeta_1 = -\kappa_1$ if $M_2^2 = -\tilde{M}_2^2$ in [55], we observe the two expressions are almost the same. The overall amplitude has a relative difference $\lesssim 0.15\%$, and the coefficients of $\epsilon_1$ and $\epsilon_2$ have relative differences 0.44\% and 0.16\%. The coefficients of $\sigma_1$ and $\delta_1$ are exactly the same in the two expressions.

4 Conclusion and outlook

The uniform asymptotic approximation method provides a powerful, systematically improvable, and error-controlled approach to construct accurate analytical solutions of linear perturbations. In this paper, by applying the third-order uniform asymptotic approximations, we have obtained explicitly the analytical expressions of power spectra, spectral indices, and running of spectral indices for both scalar and tensor perturbations in the EFT of inflation with the slow-roll approximation.

Comparing to the standard slow-roll inflation, the EFT of inflation introduces four new operators which can modify both the effective sound speed and the linear dispersion relation. In order to calculate the effects of these four new operators in the primordial spectra, we defined four new slow-roll parameters as in (2.25). All the final expressions are written in terms of both the Hubble flow parameters defined in (2.19) and the flow of four new slow-roll parameters and expanded up to the next-to-leading order in the slow-roll expansions so they represent the most accurate results obtained so far in the literature. We observe that the four new operators introduced in the action of the EFT of inflation can affect the primordial perturbation spectra at the leading-order and the corresponding spectral indices at the next-to-leading order. The running of the indices are keeping the same with that in the standard slow-roll inflation up to the next-to-leading order.

The next-to-leading order corrections to the scalar index $n_s$ and the tensor-to-scalar ratio $r$ in the slow-roll approximation are very important and useful in the future analysis with forthcoming observational data. As pointed out in [53,54], the future CMB measurements can achieve the errors of the measurements on both $n_s$ and $r$ are $\lesssim 10^{-3}$. This implies, if the contributions of the new effects (for examples, $\delta_1, \delta_2$, $\zeta_1, \zeta_2$, etc) at the magnitude of $O(10^{-3})$, then they cannot be ignored in the analysis with future experiments. Therefore, it would be interesting to constrain these new effects by using the more precise forthcoming observational data in the future. We expect that such constraints could help us to understand the physics of the early Universe.

Acknowledgements G. H. D., J. Q., Q. W., and T. Z. are supported by National Natural Science Foundation of China under the Grants no. 11675143, the Zhejiang Provincial Natural Science Foundation of China under Grant no. LY20A050002, and the Fundamental Research Funds for the Provincial Universities of Zhejiang in China under Grants No. RF-A2019015. A. W. is supported by National Natural Science Foundation of China with the Grants nos. 11675145.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors’ comment: This is a theoretical work, so we have not use any data.]

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Appendix A: Integrals of $\sqrt{g}$ and the error control function

Here we present the calculation of the integral of $\sqrt{g}$ and the error control function $\mathcal{E}(+\infty)$ in the general expression of the power spectra (3.8). Let us first consider the integral of $\sqrt{g}$, which in general reads

$$\int_{\bar{y}}^{\tilde{y}_0} \sqrt{g(y') dy'} = \int_{\bar{y}}^{\tilde{y}_0} \sqrt{\frac{v^2(\eta)}{y^2} - c^2(\eta) - b(\eta)y^2 dy'}.$$  

(A.1)

Here we note that $v(\eta)$, $c(\eta)$, and $b(\eta)$ are all slow-varying quantities. In order to evaluate the integral with these time-dependent quantities, we consider expanding them about the turning point $y = \bar{y}_0$ as

$$v(\eta) = \bar{v}_0 + \bar{v}_1\ln \frac{y}{\bar{y}_0} + O\left( \ln^2 \frac{y}{\bar{y}_0} \right),$$

(A.2)

$$c(\eta) = \bar{c}_0 + \bar{c}_1\ln \frac{y}{\bar{y}_0} + O\left( \ln^2 \frac{y}{\bar{y}_0} \right),$$

(A.3)

$$b(\eta) = \bar{b}_0 + \bar{b}_1\ln \frac{y}{\bar{y}_0} + O\left( \ln^2 \frac{y}{\bar{y}_0} \right),$$

(A.4)

where

$$v(\eta) = \frac{dv(\eta)}{d\ln(-\eta)}.$$

(A.5)
where
\[
\lim_{\nu \to 0} \mathcal{H}_0(\xi) \simeq \frac{c_0^2 + 8\nu v_0^2}{6c_0 v_0^2 + 24\nu v_0^3}, \tag{A.16}
\]
\[
\lim_{\nu \to 0} \mathcal{H}_1(\xi) \simeq \frac{c_1}{6c_0 v_0^2} - \frac{23 + 12 \ln 2}{72v_0^2} \nu v_0 + \frac{(1 + 4 \ln 2) v_0}{6c_0} b_1. \tag{A.17}
\]

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