Four-dimensional Simulation of the Hot Electroweak Phase Transition with the SU(2) Gauge-Higgs Model

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We study the finite-temperature phase transition of the four-dimensional SU(2) gauge-Higgs model for intermediate values of the Higgs boson mass in the range $50 \lesssim m_H \lesssim 100\text{GeV}$ on a lattice with the temporal lattice size $N_t = 2$. The order of the transition is systematically examined using finite size scaling methods. Behavior of the interface tension and the latent heat for an increasing Higgs boson mass is also investigated.

1. INTRODUCTION

The possibility that the baryon number asymmetry of the Universe has been generated in the course of the electroweak phase transition has led to recent lattice investigations of the SU(2) gauge-Higgs model by several groups[1]. Their studies have shown that a first-order phase transition takes place at a finite temperature in this system for a light Higgs boson, which, however, becomes rapidly weak as the Higgs boson mass increases. A central question, relevant for the baryon number asymmetry problem, is whether the first-order transition survives with a sufficient strength for realistically large Higgs boson mass, experimentally bounded by $M_H \geq 64\text{GeV}$[2]. To answer this question many studies have been done within the perturbatively reduced three-dimensional model[1]. On the other hand, only a few studies with the original four-dimensional model exist in this region of Higgs boson mass[1]. In this article, we report results of our simulation of the four-dimensional model, aiming at a systematic finite-size scaling analysis of the order of the transition for the Higgs boson mass in the range $50 \lesssim m_H \lesssim 100\text{GeV}$.

2. SIMULATION

We employ the standard action given by

$$S = \sum_n \left\{ \sum_{\mu > \nu} \frac{\beta}{2} \text{Tr} U_{n,\mu \nu} + \sum_{\mu} \kappa \text{Tr} (\Phi_n^\dagger U_{n,\mu} \Phi_{n+\mu}) \right\} - \rho_n^2 - \lambda (\rho_n^2 - 1)^2, \tag{1}$$

with the complex $2 \times 2$ matrix $\Phi$ decomposed as $\Phi_n = \rho_n \alpha_n$, $\rho_n \geq 0$; $\alpha_n \in \text{SU}(2)$. All of our simulations are made for the temporal extent $N_t = 2$. We set the gauge coupling $\beta = 1/g^2 = 8$, and make simulations for 6 values of the scalar self-coupling $\lambda$ as listed in Table 1. Also listed in the table are the zero-temperature Higgs boson mass $M_H$ at the transition point of $N_t = 2$ lattice, estimated by interpolating available data for $M_H$[3–5] as a function of $\lambda$. For each value of $\lambda$ runs are made on an $N_s^3 \times 2$ lattice with $N_s = 8, 12, 16, 24, 32$, and in addition with $N_s = 40$ for $\lambda = 0.001, 0.0017235, 0.003 (M_H = 67, 85, 102\text{GeV})$. Gauge and scalar fields are updated with a combination of the heat bath[6] and overrelaxation[7] algorithms in the ratio reported to be the fastest in ref. [4]. For each parameter point we make $10^5$ iterations of the combined updates.

3. FINITE-SIZE SCALING ANALYSIS

Let us define the angular part of the spatial component of the hopping term in the action by

$$\Lambda_s = \frac{1}{3 N_s^3 N_t} \sum_n \sum_{j=1}^3 \left[ \frac{1}{2} \text{Tr} (\alpha_n^\dagger U_{n,j} \alpha_{n+j}) \right]. \tag{2}$$

In Fig. 1 we show the volume dependence of the maximum height of the susceptibility defined by

$$\chi_\Lambda_s \equiv V \left( \langle \Lambda_s^2 \rangle - \langle \Lambda_s \rangle^2 \right), \tag{3}$$

with $V = N_s^3$, which is calculated by the standard reweighting technique. For $47 \leq M_H \leq 102\text{GeV}$,
Table 1  
Choice of scalar self coupling and corresponding zero temperature Higgs boson mass.

| λ  | 0.0005 | 0.000625 | 0.00075 | 0.001  | 0.0017235 | 0.003  |
|----|--------|-----------|----------|--------|------------|--------|
| $M_H$(GeV) | 47  | 53  | 58  | 67  | 85  | 102 |

Figure 1. Maximum height of the susceptibility of $\Lambda_s$ as a function of the volume. Lines are guides for eyes.

At $M_H = 67$GeV the maximum value increases linearly toward larger volumes, which is expected for the case of a first-order transition. In contrast, we observe a very flat volume dependence for $85 \leq M_H \leq 102$GeV, albeit the maximum value is increasing slowly in the range of volume used here.

In Fig. 2 we plot the valley depth of the Binder cumulant defined by

$$B_{\Lambda_s} \equiv 1 - \frac{1}{3} \frac{\langle \Lambda_s^4 \rangle}{\langle \Lambda_s^2 \rangle^2}. \tag{4}$$

as a function of inverse volume. Lines are linear fits to the largest 3 volumes for each $M_H$.

For $47 \leq M_H \leq 67$GeV the value extrapolated to the infinite volume clearly deviates from $2/3$, providing additional evidence for a first-order transition. For $85 \leq M_H \leq 102$GeV the deviation decreases by an order of magnitude, although still finite within the error.

These results clearly show that the transition is of first order for $47 \leq M_H \leq 67$GeV. It is also clear that the transition, if first order, is a very weak one at $M_H = 85$GeV and 102GeV. It is quite possible that the transition has turned into a crossover for this range of $M_H$. Data for larger volumes are needed, however, for a conclusive analysis on this point.

4. LATENT HEAT AND INTERFACE TENSION

The Higgs boson mass dependence of the latent heat $\Delta \epsilon$ provides a physical measure of the weakening of the first-order transition as $M_H$ increases. Here we calculate $\Delta \epsilon$ for $M_H = 47, 53, 58, 67$GeV using the Clausius-Clapeyron equation.

$$\Delta \epsilon \simeq -M_H^2 \kappa \Delta \langle \rho^2 \rangle. \tag{5}$$

where the gap $\Delta \langle \rho^2 \rangle$ is estimated from the run on the largest volume for each $M_H$. The result is shown in Fig. 3.

Interface tension provides another indicator of the strength of the first-order transition. Let $P_{\text{max}}$ and $P_{\text{min}}$ be the peak and valley height of the distribution of $\Lambda_s$, reweighted such that the two peaks have an equal height. Define

$$\hat{\sigma}_V \equiv -\left( N_t^2 / 2 N_s^2 \right) \ln(P_{\text{min}} / P_{\text{max}}).$$

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Figure 3. Latent heat as a function of Higgs boson mass. Open symbol is from ref. [4] and is shifted slightly left for visualization.

cubic lattices, finite-size formula for the true interface tension $\sigma$ is given by

$$\hat{\sigma}_V(N_s, N_t) = \frac{\sigma}{T_c^3} \left[ c - \frac{1}{4} \ln N_s + \frac{1}{2} \ln 3 \right],$$

(6)

where $c$ is a constant independent of $N_s$. Making a two parameter fit of $\hat{\sigma}_V$ obtained for the largest three volumes for $M_H = 47, 53, 58\text{GeV}$ or two volumes for $M_H = 67\text{GeV}$ we find $\sigma/T_c^3$ shown in Fig. 4.

Both the latent heat and the interface tension rapidly decrease with an increasing Higgs boson mass and seem to vanish around $M_H \sim 80\text{GeV}$.

5. SUMMARY

Our finite-size scaling study establishes a first-order transition for $47 \leq M_H \leq 67\text{GeV}$. For larger Higgs boson masses a rapid weakening of the transition makes it difficult to draw a definitive conclusion on the order within the range of lattice volumes employed in our simulation. However, combining finite-size data with results for the latent heat and the interface tension, our four-dimensional study suggests that the first-order transition terminates around $M_H \sim 80\text{GeV}$ in the $N_t = 2\text{ SU}(2)$ gauge-Higgs model. This is consistent with the result of a recent finite-size scaling study carried out in the dimensionally reduced three-dimensional model[10].

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