Magnetic-interference patterns in Josephson junctions with $d + is$ symmetry

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Abstract

The magnetic interference pattern and the spontaneous flux in unconventional Josephson junctions of superconductors with $d + is$ symmetry are calculated for different reduced junction lengths and the relative factor of the $d$ and $s$ wave components. This is a time reversal broken symmetry state. We study the stability of the fractional vortex and antivortex which are spontaneously formed and examine their evolution as we change the length and the relative factor of $d$ and $s$ wave components. The asymmetry in the field modulated diffraction pattern exists for lengths as long as $L = 10\lambda_J$. 
I. INTRODUCTION

In the past several years one of the main questions in the research activity on high-$T_c$ superconductors has been the identification of the order parameter symmetry [1–5]. The most possible scenario is that the pairing state is an admixture of a dominant $d$-wave with some small $s$-wave component. This fact is a direct consequence of the orthorhombic distortion of the systems which makes both the $d$-wave and $s$-wave indistinguishable (they transform according to the identity representation of the group). There is a basic difference in the physics if one takes into account the phase difference between the two parts of the order parameter. The mixing due to orthorhombicity predicts a $d+s$ or equivalently $d−s$ order parameter. This has been analyzed within the Ginzburg-Landau framework, valid close to $T_c$ [6]. Experimental observation of this possibility has been clearly realized in photoemission experiments [7] and the $c$-axis tunneling [8].

In addition to the above work, calculations based in BCS weak-coupling theory [9,10] predict that a mixed symmetry is realized in a certain range of interaction. This state has the time-reversal symmetry $\mathcal{T}$ broken. This symmetry is realized in bulk calculations only as a consequence of the absence of any orthorhombic distortion (the Fermi surface is either circular or tetragonal in the particular examples) which favors a phase difference of $\pi/2$ between the two components as opposed to $\pi$ in the presence of it.

The situation becomes more complicated if we consider surface effects. The observation of fractional vortices on the grain boundary in YBa$_2$Cu$_3$O$_7$ by Kirtley et al. [11], may indicate a possible violation of the time-reversal symmetry near grain boundary (because the boundary breaks the bulk orthorhombic symmetry). Therefore it is interesting to study more this symmetry in the case of interfaces.

In the present paper we study the static properties of one dimensional junction which contains a twin boundary where the pair transfer integral between the two superconductors has an extra relative phase in each twin. The maximum current $I_c$ that a junction can carry versus the external magnetic field $H$ in direction parallel to the plane of the junction is calculated by solving numerically the Sine-Gordon equation. The stability of fractional vortices $f_v$ or antivortices $f_{av}$ which are spontaneously formed as a consequence of the symmetry, is examined in the absence of current and magnetic field for different lengths and relative phases.

In the $\mathcal{T}$-violated state the magnetic interference pattern as has been obtained by Zhu et al. [15] in the short junction limit is assymmetric. They conclude that for a long junction the magnetically modulated critical current is basically identical to the conventional 0-0 junction due to the formation of the spontaneous vortex near the center of the junction. Our exact numerical calculations, show that there is a “dip” near the center of the diffraction patterns even for junctions as long as 10$\lambda_J$.

The rest of the paper is organized as following. In section II we discuss the Josephson effect for a mixed wave symmetry. In section III we present the results for the magnetic flux and the interference pattern. Finally, summary and discussions are presented in the last section.
We discuss the Josephson coupling at the interface between two superconductors (A and B) both with a two component order parameter \( n^A, n^B \). We can think of the interface as a Josephson junction, so the Josephson current phase relation is

\[
J = \sum_{i,j=1}^{2} J_{cij} \sin(\phi_i^B - \phi_j^A),
\]

where \( J_{cij} \) is the coupling between the components \( n^B_i \) on side B and with \( n^A_j \) on side A \( [n_j^\mu = |n_j^\mu| \exp(\iota \phi_j^\mu)] \). We consider some special cases.

(i) For \( d \)-wave symmetry one component of the order parameter vanishes at the interface \( (n^B_2 = 0) \). The Josephson current density becomes \( J = |J_{c11}| \sin(\phi + \pi) \) with \( J_{c11} < 0 \).

(ii) For \( d + s \)-wave we are restricted to the case where \( \phi_1^A - \phi_2^A = \phi_1^B - \phi_2^B = \pi \) is fixed on both sides of the interface. The current density \( J \) depends only on one phase difference through the interface, say \( \phi = \phi_1^B - \phi_1^A \)

\[
J(\phi) = |\tilde{J}_c| \sin(\phi + \theta)
\]

\[
\tilde{J}_c = J_{c11} + J_{c22} - J_{c12} - J_{c21}
\]

with \( \theta = 0 \) for \( \tilde{J}_c > 0 \) and \( \theta = \pi \) for \( \tilde{J}_c < 0 \).

(iii) For \( d + is \)-wave case the intrinsic phase difference within each superconductor A and B can be assumed to be \( \phi_1^A - \phi_2^A = \phi_1^B - \phi_2^B = \pi/2 \). The current density \( J \) is

\[
J(\phi) = \tilde{J}_c \sin(\phi + \theta)
\]

with

\[
\tilde{J}_c = \sqrt{(J_{c11} + J_{c22})^2 + (J_{c12} - J_{c21})^2},
\]

\[
\tan(\theta) = \frac{J_{c21} - J_{c12}}{J_{c11} + J_{c22}}.
\]

We consider two superconducting sheets A and C which overlap for a distance \( L \) with the superconducting sheet B, in the x-direction, (as shown in Fig 1). All three superconducting sheets have dominant \( d \)-wave symmetry, with a small \( s \)-wave component. The angles between the crystalline a axis of each superconductor A, B, C with the junction interface are defined as \( \theta_1, \theta_2, \theta_3 \). We describe the entire junction with width \( w \) small compared to \( \lambda_J \) in the y direction, of length \( L \) in the x direction, in external magnetic field \( H \) in the y direction. The intrinsic phase difference \( \theta(x) \) is \( \phi_{c1} \) in \( 0 < x < L/2 \) and \( \phi_{c2} \) in \( L/2 < x < L \). The superconducting phase difference \( \phi \) across the junction is then the solution of the Sine-Gordon equation

\[
\frac{d^2 \phi(x)}{dx^2} = \frac{1}{\lambda_J^2} \sin[\phi(x) + \theta(x)],
\]
with the inline boundary condition
\[
\frac{d\phi}{dx} \bigg|_{x=0,L} = \pm \frac{I}{2} + H
\]  
(8)

The Josephson penetration depth is given by
\[
\lambda_J = \sqrt{\frac{\hbar c^2}{8\pi edJ_c}}
\]
where \(d\) is the sum of the penetration depths in two superconductors plus the thickness of the insulator layer. We also assume that \(J_c\) is constant within each segment of the interface.

To check the stability we consider small perturbations \(u(x, t) = v(x) e^{st}\) on the static solution \(\phi(x)\), and linearize the time-dependent Sine-Gordon equation to obtain:
\[
\frac{d^2v}{dx^2} + \cos[\phi(x) + \theta(x)]v(x) = \lambda v(x),
\]  
(9)
under the boundary conditions \(\frac{dv(x)}{dx} \bigg|_{x=0,L} = 0\), where \(\lambda = -s^2\). It is seen that if the eigenvalue equation has a negative eigenvalue the static solution \(\phi(x)\) is unstable.

We can also compute the free energy of the solution for zero current and external magnetic field
\[
F = \frac{\hbar J_c w}{2e} \int_0^L \left[ 1 - \cos [\phi(x) + \theta(x)] + \frac{\lambda^2}{4} \left( \frac{\partial\phi}{\partial x} \right)^2 \right] dx,
\]  
(10)
Note that the no vortex solution \(\phi = 0\) everywhere is not a solution of this problem.

When \(\phi_{c1} = \phi_{c2} = 0\) we have the conventional s-wave junction. In case \(\phi_{c1} = 0\), \(\phi_{c2} = \pi\) we have the \(d\)-wave or \(d + s\)-wave junction. The above cases have time reversal symmetry (\(T\)-conservation). When \(\phi_{c1}, \phi_{c2}\) are slightly different from 0 and \(\pi\), we have the \(d + is\)-wave pairing, which is a broken time reversal symmetry state (\(T\)-violation). In this work, the particular parameters we use are \(\phi_{c1} = 0.01\pi\), \(\phi_{c2} = 1.08\pi\), and the pairing state is \(d + is\).

III. SPONTANEOUS MAGNETIC FLUX AND INTERFERENCE PATTERNS FOR THE \(T\)-VIOLEATING PAIRING STATE

In Fig. 2 we plot the maximum current for a symmetric \(0 - \pi\) junction as a function of the magnetic flux \(\Phi\) (in units of \(\Phi_0 = \frac{\hbar c}{2e}\)) for different junction lengths: (a) \(L = 10\), (b) \(L = 4\), (c) \(L = 2\), (d) \(L = 1\) (\(\lambda J = 1\)). The circles and squares in this figure correspond to the fractional vortex (\(f_v\)) and antivortex (\(f_{av}\)) branch. For most of the range of existence of \(f_v\) (\(f_{av}\)) the magnetic flux is positive (negative) while there is a small region where it turns into antivortex (vortex). Similar calculation has been done [13], [14] who considered the \(f_v\), since in this case the plot is symmetric in \(H\). As we can see, there is a "dip" at \(\Phi = 0\), for lengths as long as \(L = 10\). In Fig. 3 we present our calculations for the \(T\)-violation case where \(\phi_{c1} = 0.01\pi\) and \(\phi_{c2} = 1.08\pi\). We also plot for \(L = 1\) the analytical result (solid line) of Zhu et al. [13]. In contrast to the pure \(d\)-wave case, for small lengths this pattern is
asymmetric and the “dip” in the maximum current does not occur at $\Phi = 0$, but at a finite $\Phi$ value. This behavior also exists for lengths as long as $L = 10$.

If we plot $I_c$ vs $H$ (and not $\Phi$) then the two branches in Fig. 3, will be almost coincident and one might draw the conclusion that the behavior for a long junction is the same independent of the symmetry. The proper quantity to consider though is the total magnetic flux which includes both the contribution from the external field and the induced self field. It should be remarked that for an $s$-wave junction the relation between $\Phi$ and $H$ is linear for small $H$ so that the plot of $I_c$ vs $H$ or $\Phi$ does not show any differences for small $H$. For higher $H$ however the overlapping branches (for long $L$) are unfolded. In the case of a different symmetry even the small $H$ form can change due to the existence of spontaneous magnetization. In this case, if we consider the zero current solutions, and vary the magnetic field, both $f_v$, $f_{av}$ are stable, whereas for $L = 4$, $L = 2$, $L = 1$ the stable regions in the magnetic field are separated by the unstable ones, and this behavior persists in both the $f_v$, $f_{av}$ cases. Also in the short junction limit (i.e. $L = 1$) these two branches coincide at the maximum current.

Figure 4 addresses the question of spontaneous flux generation in junctions with broken time reversal symmetry ($\mathcal{T}$-violation) as a function of the reduced length ($L$) and the relative factor of $s$ and $d$ components. The long dashed line is the result of [14] which compares with our numerical result (solid line). Both cases have $\phi_{c,1} = 0.01\pi, \phi_{c,2} = 1.08\pi$. We have also used two other values for $\phi_{c,2}$ i.e. $0.9\pi$ (dotted line) and $0.8\pi$ (dashed line). We conclude that as we decrease the value of $\phi_{c,2}$ the fractional vortex $f_v$ tends out to be a $2\pi$ vortex, whereas the fractional antivortex gradually loses its flux content.

In Fig. 5a we have plotted the magnetic flux $\Phi$ (solid line, $\Phi_0 = 1$) versus the value $\phi_{c,2}$ for $L = 10$ and $H = 0$. We see that as we decrease $\phi_{c,2}$ the magnetic flux increases linearly for the $f_v$ branch from $\Phi \approx 0.45$, for $\phi_{c,2} = 1.08\pi$ to $\Phi \approx 1$ for $\phi_{c,2} = 0$. But this last point is unstable, as seen by the stability analysis from which the lowest eigenvalue is also displayed (light line). This is expected since it goes to a point in the unstable (1, 2) branch of the usual $s$-type junction [16]. The $f_{av}$ branch increases linearly its flux as we decrease $\phi_{c,2}$ and goes to the stable (0, 1) branch. Here we follow the notation of [17]. This linear dependence of $\Phi$ from $\phi_{c,2}$ can also be seen in the analytical result of [15] for large lengths, where the approximation they made is valid. On the other hand as we increase $\phi_{c,2}$ from $1.08\pi$ to $2\pi$, the $f_v$ branch decreases its flux and goes to the stable (0, 1) branch, while the $f_{av}$ branch goes to the unstable $(-2, -1)$ branch. When we change $\phi_{c,1}$ and keep $\phi_{c,2} = 0$, from 0 to $2\pi$, the (0, 1) branch goes to $f_v$ and then to the unstable (1, 2), while the unstable $(-2, -1)$ branch goes to $f_{av}$ and then to the stable (0, 1). The situation is a little bit different for small lengths as can be seen from Fig. 5b where $L = 1, H = 0$. Here $f_{av}$ branch is unstable for $\phi_{c,2} = 1.08\pi$ and by decreasing $\phi_{c,2}$ it gets stabilized, but $f_v$ branch is stable for $\phi_{c,2} = 1.08\pi$ and then it becomes unstable. We can see that from Fig. 6 where we plot the ratio $F/F_0$ of the free energy of the state with some spontaneous flux to the state with no flux. This ratio becomes larger than one as we decrease the $\phi_{c,2}$, for the $f_v$ branch, for small lengths. On the other hand, when $F/F_0 < 1$ the no flux state is metastable and the final state will be the one with spontaneous flux. Notice that the magnetic flux remains almost constant - almost zero - which can be expected since we are in the short junction limit where self currents are neglected.
IV. CONCLUSIONS

We have studied the static properties of a one dimensional junction with $d + is$ order parameter symmetry. The magnetic interference pattern is asymmetric, and there exist a “dip” near $\Phi = 0$ for lengths as long as $10\lambda_f$. The diffraction pattern of a junction can give us information about the pairing symmetry, at least where junctions are formed.

We have followed the evolution of spontaneously formed vortex and antivortex solutions for different mixing between the $s$ and $d$ components of the order parameter. We have shown that for small lengths the fractional vortex becomes unstable as we decrease the extra phase of the pair transfer integral in the right part of the junction. We conclude that when a mixing state symmetry is realized, the fractional vortex and antivortex solutions evolve differently and this characterizes the $d + is$-wave pairing. We expect these findings to hold even if a bulk $d + s$ state evolves continuously as a function of distance from the interface to a $d + is$ one, as long as the there is a well defined area close to the interface where the time reversal symmetry is not conserved and the junction is formed.
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FIGUREs

FIG. 1. The one dimensional junction geometry. The dashed line marks the twin boundary.

FIG. 2. Critical current $I_c$ versus the magnetic flux $\Phi$ (in units of $\Phi_0$) for a symmetric $0 - \pi$ junction, for different junction lengths: (a) $L = 10$, (b) $L = 4$, (c) $L = 2$, (d) $L = 1$.

FIG. 3. Critical current $I_c$ versus the magnetic flux $\Phi$ for a junction with $d + is$ symmetry, $\phi_{c1} = 0.01\pi$, $\phi_{c2} = 1.08\pi$, for different junction lengths: (a) $L = 10$, (b) $L = 4$, (c) $L = 2$, (d) $L = 1$.

FIG. 4. The spontaneous magnetic flux $\Phi$ as a function of the reduced junction length $L$, for different values of the intrinsic phase $\phi_{c2}$ in the right part $L/2 < x < L$ of the junction and $\phi_{c1} = 0.01\pi$.

FIG. 5. The evolution of the fractional vortex $f_v$ and antivortex $f_{av}$ as a function of $\phi_{c2}$, for two different lengths (a) $L = 10$, (b) $L = 1$. The stability of these branches is also denoted by the lowest eigenvalue $\lambda_1$ of the linearized eigenvalue problem. Note that the double arrow connects the flux with its stability curve.

FIG. 6. The ratio of the free energy $F/F_0$ as a function of the reduced junction length $L$, for different values of $\phi_{c2}$: (a) $f_v$, (b) $f_{av}$.
$\phi_{c2} = 1.08\pi$

$\phi_{c2} = 0.9\pi$

$\phi_{c2} = 0.8\pi$

ref. [15]
\( \phi_c = 1.08\pi \)

\( \phi_c = 0.9\pi \)

\( \phi_c = 0.8\pi \)

ref. [15]