RELATIONSHIP BETWEEN THERMAL TIDES AND RADIUS EXCESS

ARISTOTLE SOCRATES
Institute for Advanced Study, Einstein Drive, Princeton, NJ 08540

ABSTRACT

Close-in extrasolar gas giants – the hot Jupiters – display departures in radius above the zero-temperature solution, the radius excess, that are anomalously high. The radius excess of hot Jupiters follows a relatively close relation with thermal tidal tidal torques and holds for \( \sim 4 - 5 \) orders of magnitude in a characteristic thermal tidal power in such a way that is consistent with basic theoretical expectations. The relation suggests that thermal tidal torques determine the global thermodynamic and spin state of the hot Jupiters. On empirical grounds, it is shown that theories of hot Jupiter inflation that invoke a constant fraction of the stellar flux to be deposited at great depth are, essentially, falsified.

Subject headings: planets: hot Jupiters

1. INTRODUCTION

The radii of hot Jupiters are anomalously larger than their zero temperature radii, \( R_0 \) (Bodenheimer et al. 2003; Arras & Bildsten 2007). The powerful stellar radiation field serves as an ample source of energy which may inflate hot Jupiters to their current radii (Guillot & Showman 2002). The major theoretical challenge is in finding a mechanism that can transfer a small amount \((\lesssim 1\%)\) of the stellar insolation to great depth. Current ideas include thermal tides (Arras & Socrates 2009a,b; 2010), delayed gravitational contraction (Ibigu et al. 2010) and ohmic dissipation due to magnetic activity on the heavily irradiated surface (Batygin & Stevenson 2010).

It seems that there is an relation between observed radius excess \( \Delta \) defined as

\[
\Delta = \frac{R_p - R_0}{R_p},
\]

where \( R_p \) is the measured planet radius, and a characteristic thermal tidal power \( L_{\text{thT}} \) defined as

\[
L_{\text{thT}} = \frac{\sigma_{\text{SB}} n^2 T_{eq}^3 R_p^2}{c_p}
\]

where \( \sigma_{\text{SB}} \) is the Stefan-Boltzmann constant, \( c_p \) is the specific heat, \( n \) is the mean motion and \( T_{eq} \) is its equilibrium surface temperature.

The radius excess \( \Delta \) is determined by a balance of heating and cooling. In the thermal tide scenario, the thermal tide torque is balanced by the gravitational torque. While the thermal tide torque is non-dissipative, the gravitational tide torque necessarily is the result of dissipation. Torque balance in steady state then implies continuous heating as a result of gravitational tidal dissipation. It follows that the planet radius is determined by a balance of this steady state heating with cooling at the photosphere. The ultimate source of energy is the stellar radiation field, which performs work by moving material across the tidal potential, near the surface of the irradiated planet (Arras & Socrates 2009a; 2010).

The plan of this paper is as follows: In 2 we motivate the need for thermal tidal torques in hot Jupiters, compute their characteristic values for both equilibrium and dynamical thermal tides as well as the the characteristic thermal tidal power, all in terms of physical quantities that can be observationally inferred. The thermal tide - radius excess relation is presented in 3 A brief discussion is contained in 4. A brief summary is given in 5.

2. THEORY OF THERMAL TIDES IN HOT JUPITERS

The radiation field of a star forces the atmospheres of planets in orbit. The response of a planetary atmosphere to this forcing is referred to as the “thermal tide.” Planets that spin asynchronously experience a sharp change in thermal forcing during twilight hours and consequently, a broad spectrum of tidal harmonics are excited. For circular orbits, the semi-diurnal \((l = m = 2)\) harmonic of the forcing leads to a thermal tidal bulge that can couple to the leading order term of the gravitational tidal potential. The atmosphere’s finite thermal inertia leads to a lag in the response with respect to the forcing, which allows the tidal potential to exert a net torque on the thermal tide during the heating cycle. Since the thermal tidal bulge peaks in the morning and after sunset, but before midnight, the acceleration increases the planet’s rotational energy.

The picture described above has a long history. On the basis of the measured magnitude and phase of Earth’s thermal tide, Thompson (Lord Kelvin) computed Earth’s thermal tidal torque and found it to be approximately equal to one tenth the dissipative lunar tidal torque (Thompson 1882; see also Munk & Macdonald 1960). Due to its heavy atmosphere and correspondingly large thermal tides, Gold & Soter (1969) hypothesized that the current spin state of Venus results from a balance of thermal and gravitational tidal torques so that its current state of spin can be maintained over the age of the solar system.

In what follows, we motivate the importance of thermal tides and thermal tidal torques in hot Jupiters, within the context of understanding their inflated radii.

2.1. tidal power as a source of dissipation

The characteristic value for the maximum instantaneous tidal power for a uniform spherical body of radius

\[
L_{\text{thT}} = \frac{\sigma_{\text{SB}} n^2 T_{eq}^3 R_p^2}{c_p}
\]
at a heliocentric distance $D$ inflated hot Jupiters is of order the power required to maintain the radii of heavily in-
spired in the Universe required to produce inflated hot Jupiters in steady-state. Only a small fraction of the total available tidal power is
then eq. 1 may be written as ing frequency in eq. 1 is taken to be the mean motion. If the planet is
the spin frequency and mean motion. If the planet is
forced, the forcing frequency is given by the difference of
the spin and mean motion. If the planet is
 ordinarily inferred from the Jupiter-Io interaction (Goldreich & Soter 2009a; 2010).

A fundamental underlying assumption of this picture is that dissipation of the gravitational tide takes place at
great depth, where the pressure scale height is compara-
table to the radius of the planet.

Also note that, in the absence of thermal tidal toques,
gravitational tidal torques are likely to synchronize
the planet spin extremely fast if the relative strength of tidal
dissipation in hot Jupiters is comparable to what is com-
monly inferred from the Jupiter-Io interaction (Goldreich
& Soter 1966; Socrates et al. 2012).

2.2. the role of thermal tides

Stellar irradiation creates a tidal bulge that leads max-
imum forcing at noon, where the phase shift results from
thermal inertia. The tidal gravitational field of the star
forces the thermal tide, accelerating the planet away
from a state of synchronous spin. As the rate of spin
increases, the amount of energy absorbed per cycle de-
creases and consequently, so does the magnitude of the
thermal tidal quadrupole. The thermal tide torque event-
ually comes into balance with the usual dissipative grav-
itational tidal spin-down torque. In steady-state, the
planet’s spin is asynchronous and therefore, gravitational
tidal power is continuously dissipated. The ultimate
source of energy is the starlight of the primary, which
performs work by moving material across the tidal po-
tential (Arras & Socrates 2009a; 2010).

2.3. spin equilibrium and steady tidal power: Gold-Soter
approximation

Balance between the gravitational tidal torque and the
thermal tidal torque determines the planet’s spin rate and
therefore, determines the tidal forcing frequency $\omega$.
For a circular or nearly circular orbit, torque balance is
obtained by equating the respective quadrupoles induced
by thermal and gravitational forcing. The portion of the
gravitationally excited quadrupole responsible for dissi-
pation and secular evolution is given by

$$q_{grav} = \frac{n^2 R_p^5}{Q J_1 G^3} \omega$$

where the above expression for the tidal quality factor
$Q$ reflects the frequency dependence of the constant lag
model of Hut (1981). Here $Q J_1$ and $\omega_1$ is the tidal
quality factor ($\approx 10^5$; Goldreich & Soter 1966) and
forcing frequency ($\approx 10$ hours) of the Jupiter-Io interac-
tion. Note that we are, equivalently, setting the lag time
$\tau \approx 0.1$ s, consistent with the resonant configuration of
the Galilean satellites (Socrates et al. 2012; Leconte et
al. 2010; see also Socrates & Katz 2012).

The Gold-Soter quadrupole due to thermal forcing is
approximatively given by

$$q_{th}^3 = \frac{\Delta M R_p^2}{\tau_{th} \omega} = \frac{\sigma_{SN} R_p^4 T_3^3}{c_p \omega}$$

where $\Delta M$ and $\tau_{th}$ are the mass and thermal relaxation
time, respectively, of the absorbing layer and $t_{th}$. In
terms of physical quantities, the the thermal time may
be written as

$$t_{th} = \frac{c_p}{\sigma_{SN} R_p^4 T_3^3} \Delta M.$$
By equating eq. 4 with eq. 5 the equilibrium forcing frequency becomes
\[ \omega_{GS} = \sqrt{\frac{\sigma_{sb} Q_1 \omega_j T_{eq}^3}{c_p n^2 R_p}} \]
\[ \approx \frac{2\pi}{2 \text{ days}} T_{2000}^{3/2} P_4^{1/2} R_{10}^{1/2} Q_5^{1/2} \]  
(7)
where \( T_{2000} \), \( P_4 \), \( R_{10} \), \( Q_5 \) are the equilibrium temperature in units 2000 K, the orbital period normalized to four days, dimensionless planet radius normalized to 10\(^{10}\) cm and the tidal quality factor of the Jupiter-Io interaction, normalized to 10\(^5\).

In the equilibrium – Gold-Soter – approximation, the tidal dissipation rate resulting from the balance of thermal and gravitational tidal torques, which we refer to as the thermal tidal power \( L_{thT}^{GS} \), is given by
\[ L_{thT}^{GS} = \omega_{GS}^2 q_{grav} = \omega_{GS}^2 \frac{n^4 R_p^5}{G Q_1 \omega_j} \]
\[ \approx \frac{\sigma_{in}}{c_p} n^2 T_{eq}^3 R_p^4 \]
\[ = 1.5 \times 10^{28} P_4^{-2} T_{2000}^{3} R_{10}^{4} \text{ erg/s.} \]  
(8)

2.4. spin equilibrium and steady tidal power: dynamical thermal tides

Consider the computed thermal tide quadrupole from Arras & Socrates (2010; see their figure 5) for the case with outgoing radiation boundary condition. We imagine an evolutionary scenario in which the planet was born spinning rapidly and was then placed in its current orbit via some migration mechanism. As it spins down, the forcing frequency approaches the g-mode “bump” in the thermal tidal quadrupole, which is responsible for arresting any further spin evolution. The first spin up feature of the g-mode bump is located near the cutoff frequency \( \omega_0 \)
\[ \omega_0 = \frac{H}{R_p} N \]  
(9)
where \( H \) is scale height of the isothermal layer that lies above the convection zone and \( N \) is the Brunt-Vaisalla frequency, the characteristic rate at which the buoyancy force restores motion.

It is worth noting that the cutoff frequency \( \omega_0 \) is close to the mean motion \( n \) of hot Jupiters. If \( \omega_0 \ll n \), the quadrupole moments associated with low radial order gravity waves would be small and therefore, so would the quadrupole of the thermally-forced response. From this perspective, it is somewhat of a co-incidence that dynamical thermal tides in the hot Jupiters can lead to significant spin-up tidal torques.

The spin-up quadrupole due to the first feature in the g-mode bump may be expressed as
\[ q_{th}^g = q_{th}^{gs}(\omega = \omega_0) f(\omega, \omega_0) \]  
(10)
where \( f \) can be approximated by a sharply-peaked Gaussian of the form
\[ f(\omega, \omega_0) \approx \beta e^{-(\omega - \omega_0)^2/T^2} \]  
(11)
where \( \beta \approx 0.1 \) quantifies the amount by which gravity waves near the cut-off frequency do not perfectly overlap with both thermal and gravitational forcing. The width of the Gaussian is narrow so that \( T \ll \omega_0 \).

Spin equilibrium is determined by setting \( q_{th}^g = q_{grav} \), which gives
\[ e^{-(\omega - \omega_0)^2/T^2} \approx \frac{c_p}{\sigma_{gb} G \beta T_{eq}^3 Q_j \omega_j} \]  
(12)
The right hand side must be less than unity in order for torque balance to be satisfied. For the physical parameters that describe the hot Jupiter population, torque balance is only possible for \( Q_j \gtrsim 10^5 \). If there is some level of sub-surface reflection the excited gravity waves are quantized and their response is therefore, characterized by a Lorentzian profile near resonance, implying that the induced quadrupolar response can be much larger. Consequently, values of \( Q_j \approx 10^4 \) or \( \tau \approx 1 \text{ s} \) are then possible for a steady-state spin equilibrium (Arras & Socrates 2010).

Due to the sharpness of the Gaussian, the forcing frequency in spin equilibrium, given by eq. (12) is pinned to a value that is close to the cutoff frequency \( \omega \approx \omega_0 \).
\[ \omega \approx \omega_0. \]  
(13)
By following the steps in eq. 5, a similar expression for thermal tidal power, \( L_{thT}^{dyn} \) is derived due to the presence of dynamical tides
\[ L_{thT}^{dyn} \approx 1.2 \times 10^{28} \omega_2^2 P_4^{-4} R_{10}^5 Q_5^{-1} \text{ erg/s} \]  
(14)
where \( \omega_2 \) and \( Q_5 \) are the semi-diurnal forcing frequency normalized to \( (2\pi/2 \text{ days}) \) and the tidal quality factor of the Jupiter-Io interaction, normalized to \( 10^5 \).

The thermal tidal dissipation rate, eq. 14 resulting from the dynamical excitation of gravity waves, is similar in form and magnitude to that resulting from the Gold-Soter approximation, given by eq. 5. From here on, when comparing with data, we employ the Gold-Soter approximation since (i) it is simple (ii) in terms of spin equilibrium and persistent tidal luminosity, it roughly agrees with the more realistic dynamical tide results of Arras & Socrates (2010) (iii) the theory of dynamical thermal tides is currently in a state of development.

3. THERMAL TIDE-RADIUS EXCESS RELATION

The fractional radius anomaly \( \Delta \), given by
\[ \Delta \equiv 1 - R_0/R_p, \]  
(15)
where \( R_0 \) is a fiducial radius that is close the zero temperature solution depends upon planet mass and composition. The value for \( R_0 \) is obtained by using the online table provided by Baraffe et al. (2008) by choosing \( Z = 0.02 \) with no irradiation and their largest computed age of 7 Gyr.

The primary reason for utilizing the models of Baraffe et al. for the values of \( R_0 \), rather than the actual zero temperature or isothermal radius is that they are publicly available. Their solutions serve as a fiducial value of low planetary entropy that depends on planet mass. It follows that any observed departure in planet radius \( R_p \) above \( R_0 \) necessitates a source of thermal energy from within the planet.

The sample of hot Jupiters is taken from the Exoplanet Data Explorer, located on the world wide web. Only hot Jupiters with measured mass, radius, effective temperature and orbit that have measurements i.e., a sizable
fraction of the total sample, are considered. Furthermore, the sample is limited to hot Jupiters with masses $M > 0.5M_J$. All of the planets considered have both transit and radial velocity measurements, for a total of 63 objects.

In figure 1, we plot radius anomaly $\Delta$ against the dissipation rate given by eq. (5). There is a clear relationship. Any level of deep internal energy generation increases the central entropy, lifts the degeneracy and expands the planetary radius. Figure 1 indicates that the radius anomaly, or departure from the zero temperature radius, increases with an increasing internal dissipation rate that is correlated to the characteristic thermal tidal dissipation rate $L_{\text{thT}}$. That is, figure 1 lends strength to the hypothesis that thermal tidal torques power the core luminosities of hot Jupiters.

Figure 1 implies that radius excess $\Delta$, rather than planet radius $R_p$ is more sensitive in ascertaining the actual internal luminosity of a given planet and is therefore, a more useful parameter. The reason for this is straightforward: the mass-radius relation of cold spheres (e.g., Zapolsky & Salpeter 1969) indicates that – for a fixed composition – radius is approximately independent of mass. Thus, the gravitational binding energy and approximately, the Fermi energy, vary by nearly two orders of magnitude. As a result, the energy and power requirements to lift the degeneracy and inflate the radius are far more severe for massive objects. The relatively small scatter in figure 1 in comparison to similar figures that utilize planet radius (e.g., figure 1 of Demory & Seager 2011; figure 1 of Laughlin et al. 2011; figure 1 of Fortney et al. 2011) is partially due to this fact.

4. DISCUSSION

Figure 1 is important irrespective of whether or not it supports the thermal tide model of Arras & Socrates. It indicates that the internal luminosity of the hot Jupiters varies by $\sim 4 - 5$ orders of magnitude. Such a conclusion can be indirectly inferred from e.g., recent work by Spiegel & Burrows (2013), independent of the actual source of thermal power at great depth.

With the exception of the thermal tide scenario advocated here, nearly every plausible model of steady-state hot Jupiter inflation invokes or presumes some fixed fraction of absorbed stellar energy to be redistributed to produce the observed trend in radius excess $\Delta$, the internal luminosity must vary by $\sim 4 - 5$ orders of magnitude for the population of inflated hot Jupiters. This empirical requirement lends support to any tidal scenario that inflates the hot Jupiters steadily over the age of the Galaxy and therefore, lends support to the steady-state thermal tide scenario of Demory & Socrates (2009a; 2010). Furthermore, the empirical requirement that a large spread of inferred internal luminosity is required to explain the trend in radius excess $\Delta$ from figure 1 falsifies theories of hot Jupiter inflation that invoke, presume or calculate that a fixed fraction of the absorbed stellar power is transferred to great depths.

Both equilibrium and dynamical thermal tides approximately yield comparable values for equilibrium tidal power and planetary spin rate. The role of thermal tides can be viewed as allowing for the steady dissipation of tidal energy that is continuously replenished by the stellar radiation field as it performs work by moving matter across the tidal potential (Arras & Socrates 2009a; 2010).

I thank P. Arras for extensive conversations and for assistance. Subo Dong, Andy Gould and Scott Tremaine provided helpful suggestions. AS acknowledges support from a John N. Bahcall Fellowship at the Institute for Advanced Study, Princeton.

REFERENCES

Applegate, J. H., & Shaham, J. 1994, ApJ, 436, 312
Arras, P., & Bildsten, L. 2006, ApJ, 650, 394
Arras, P., & Socrates, A. 2009, arXiv:0901.0735
Arras, P., & Socrates, A. 2009, arXiv:0912.2318
Arras, P., & Socrates, A. 2010, ApJ, 714, 4
Baraffe, I., Chabrier, G., & Barman, T. 2008, A&A, 482, 315
Batygin, K., & Stevenson, D. J. 2010, ApJ, 714, L238
Bodenheimer, P., Laughlin, G., & Lin, D. N. C. 2003, ApJ, 592, 555
Demory, B.-O., & Seager, S. 2011, ApJS, 197, 12
Fortney, J. J., Demory, B.-O., Désert, J.-M., et al. 2011, ApJS, 197, 9
Fruchter, A. S., Stinebring, D. R., & Taylor, J. H. 1988, Nature, 333, 237
Gold, T., & Soter, S. 1969, Icarus, 11, 356
Goldreich, P., & Soter, S. 1966, Icarus, 5, 375
Guillot, T., & Showman, A. P. 2002, A&A, 385, 156
Hut, P. 1981, A&A, 99, 126
Ibgui, L., Spiegel, D. S., & Burrows, A. 2011, ApJ, 727, 75
Laughlin, G., Crismani, M., & Adams, F. C. 2011, ApJ, 729, L7
Leconte, J., Chabrier, G., Baraffe, I., & Levrard, B. 2010, A&A, 516, A64
Munk, W. H., & MacDonald, G. J. F. 1960, Cambridge [Eng.] University Press, 1960.
Spiegel, D. S., & Burrows, A. 2013, arXiv:1303.0293
Socrates, A., Katz, B., & Dong, S. 2012, arXiv:1209.5724
Socrates, A., & Katz, B. 2012, arXiv:1209.5723
Thompson, W. 1882, Proc. Roy. Soc. Edinburgh 11, 369
Youdin, A. N., & Mitchell, J. L. 2010, ApJ, 721, 1113
Zapolsky, H. S., & Salpeter, E. E. 1969, ApJ, 158, 809
This figure "framework.png" is available in "png" format from:

http://arxiv.org/ps/1304.4121v1