Critical Speed Analysis of Rotor System of Sealing Machine

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Abstract: The sealing machine is the key part of the copper pen tip processing machine, and its working stability will have a direct impact on the processing quality of the pen tip. Taking copper pen tip desktop processing equipment for an example, to improve the quality of pen tip, the modal analysis was carried out on the system by large-scale finite element analysis software ANSYS. The modal parameters of the rotor system were calculated, and through the Campbell diagram, the critical speeds of the system were obtained. Finally, the critical speed under different structural parameters is obtained. By analyzing data, the critical speed of the system is negatively correlated with the bearing spacing, and positively correlated with the diameter of the long shaft section, and independent of the bearing stiffness. The analysis results provide a theoretical basis for the optimization design of structural parameters of the system and the design of the maximum working speed, which also has certain reference value for the research and design of copper pen tip desktop processing equipment in China.

1. Introduction
Since the introduction of the copper pen industry to China, it has always been the main industry in China's pen industry. In recent years, new pen types such as gel pens and fountain pens have been developed, which have seized a large number of ballpoint pen markets. However, copper pens still occupy a place in the low-end market due to their price advantages, and the demand has been increasing [1]. The production and manufacturing of copper pen tips are mainly distributed in Jiangsu and Zhejiang, Yiwu is the most, and most of the factories are family workshops, using relatively old equipment based on stand-alone machine tools, which are backward. However, the processing accuracy of this old model is far from reaching the requirements, the qualification rate and the production efficiency are low [2]. The internal quality of a ballpoint pen is affected by many indicators, such as comfort, aesthetics, ink type, durability, and pen tip quality, but the most important thing is the pen tip quality. If the quality of the pen tip is not high, it is prone to writing interruptions, ink leakage, uneven ink discharge, etc. which affect the normal use of the ballpoint pen [3]. If this situation cannot be improved, our country will lose a large amount of the world ballpoint pen market under the impact of the pen manufacturing industry in other countries.

The quality of the sealing is directly related to the stability of the sealing machine. In order to improve the stability of the sealing machine, the critical speed should be kept away from the working speed during the design to obtain higher processing quality [4]. There are many calculation methods for critical speed, such as transfer matrix method, finite element method, modal synthesis method, etc. [5-7]. This paper uses ANSYS analysis software to calculate the critical speed of shaft of the sealing machine, and analyzes the influence of the main structural parameters of the shaft on the critical speed. The research of rotational speed provides a reference for the design of copper pen tip processing equipment in our country.
2. Modal analysis of the critical speed of the rotor system

This paper uses the modal module of Workbench19.0 software to analyze and calculate the shaft of the sealing machine. The shaft is made of steel, and its elastic modulus and mass density are $2\times10^{11}$ N·m$^{-3}$ and $7.85\times10^{3}$ kg·m$^{-3}$. The analysis steps are: creating a geometric model, model simplification, analysis type selection, material loading, discretization, constraints and loads, solving, post-processing [8].

In this article, the locker rotor of the copper pen tip desktop processing equipment adopts the vertical axis drive mode. The overall structure of the machine is shown in Fig.1, and the rotor structure is shown in Fig.2, including belt wheel and cutter structure. The mass of the wheel is about 0.1kg, the speed is 2400r/min, and the length of it is 340mm. The entire rotor system contains two bearings in total, and the installation distance between them is 250mm.

Fig.1 Structure drawing of the sealing machine. 1. pulley; 2. shaft; 3. Connecting frame; 4. Outer support sleeve; 5. Internal support sleeve; 6. Bearing; 7. spring; 8. cutting tool.

Fig.2 Two-dimensional structure diagram of the rotor system

2.1. Basis of modal analysis of rotor system

In actual production, the damping of the mechanical system is very small, so the influence of damping on the natural frequency of the system and the corresponding mode shape can be ignored in the calculation. The finite element method is used to discretely analyze the vibration structure of the rotor. The eigenvalues and eigenvectors of the system can be solved by an approximate method [9]. The differential equation of a multi-degree-of-freedom machine is:

$$ M\ddot{x}(t) + Kx(t) = 0 $$

In the formula:
- $M$——the overall quality matrix of the system;
- $K$——the overall stiffness matrix of the system;
- $\ddot{x}(t)$——finite element nodal acceleration;
- $x(t)$——nodal displacement of finite element.

Assuming that the displacement in the formula is in the form of simple harmonic function, and then according to the boundary conditions, we can get:

$$ \det[K - \omega^2M] = 0 $$

Calculate the eigenvalue $\omega_i$ and eigenvector $\varphi_i$ in the formula respectively, and each pair of eigenvalue and eigenvectors forms a vibration form. According to formula (3), the natural frequency and mode shape of the system can be obtained.

$$ f_i = \frac{\omega_i}{2\pi} $$

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In the formula:
\[ f_i \] —— the i-th natural frequency of the rotor.

2.2. Model simplification and import
The rotor model of the sealing machine is modeled by SolidWorks2018. For easy analysis, the rotor is simplified as follows: (a) Optimize the rotor sections of the rotor into equal-diameter rotor sections with constant diameter, that is, ignore the fine structures such as fillets, chamfers, and threads, and simplify the tool part to a section with constant diameter. (b) Each bearing is simplified into a spring damping unit. (c) Ignore the load on the rotor and the influence of speed on the stiffness of the bearing. (d) Simplify the pulley into a turntable based on the principle of quality equivalence. Importing it into the Workbench, as shown in Fig.3.

![Fig.3 3-D diagram of the rotor system after import](image)

2.3. Material setting and meshing
Discretize the model and follow-up operations on the main interface. Hexahedral meshing has high calculation accuracy and fast speed, but for some complex models, the difficulty of hexahedral meshing will consume a lot of time, which is not worth. In addition, tetrahedral meshes are also widely used in meshing, but the accuracy is lower and the speed is slower. However, because the model is simplified, the slight accuracy gap is within the error tolerance, and with the development of computer hardware, computing speed is no longer a limitation. Therefore, this analysis uses a tetrahedral mesh (SOLID187 10-Node) to divide the model, and the mesh size used is 3mm. The more detailed the mesh is, the more precise the analysis result is and the longer time the calculation takes.

2.4. Imposed constraints and speed setting
A spring-damped bearing unit needs to be set at the bearing of the rotating rotor. The bearing unit 214 (COMBI214) can be used for simulation in ANSYS. Because the stiffness of the bearing is related to the preload of the contact area of the ball and the inner and outer raceways, bearing installation method, part tolerances and bearing operating temperature, the stiffness of the bearing is difficult to measure. The classic stiffness coefficient of the bearing measured according to the experiment is 1e7~1e8N/m, the value in this calculation is 1e7N/m. In order to prevent the rotor from moving in the axial direction, it is necessary to restrict the distal displacement of the entire rotor.

In order to obtain the critical speed, it is also necessary to perform analysis settings. First, set the number of maximum modal end to 6th order, that is, solve the first 6th order modes of the rotor system, and then set Damp to "yes", and open the Coriolis Effect and Campbell Diagram, and then set the number of points for solving the critical speed to 6. Finally, add six speed points (0, 2500, 5000, 7500, 10000, 12500 rpm) to the rotating rotor. This speed range only represents the critical speed search range. If the calculation result fails to find the critical speed or the accuracy is insufficient, the search value should be changed to obtain a more accurate result.

2.5. Finite element solution and post-processing
After the constraints and speed are set, the results can be solved. After the solution is completed, add the required results into the solution module. What needs to be added in this solution are the modal diagrams and Campbell diagrams of each order. After the solution is completed, the results can be
directly generated, which is convenient and quick.

The mode diagram is shown in Fig. 4. The mode diagram only shows a deformed state when the rotor reaches the resonance frequency. The value displayed in the software only represents a relative quantity, and it is not equal to the actual value.
Fig.4 first to sixth mode figure. a) First mode figure; b) Second mode figure; c) Third mode figure; d) Fourth mode figure; e) Fifth mode figure; f) Sixth mode figure

It can be seen from the displacement cloud diagram that the vibration shapes of the first and second, third and fourth, fifth and sixth modes are respectively the same, and the vibration direction is vertical. This is because the rotor model is a symmetrical model, which makes the characteristic equation have two identical solutions, that is, their natural frequencies are the same in pairs. Through the Campbell diagram, the critical speed of each order of the shaft system can be obtained. The specific values are shown in Tab.1.

| Natural frequency (Hz) | 183.90 | 183.93 | 388.96 | 389.23 | 756.74 | 756.89 |
|------------------------|--------|--------|--------|--------|--------|--------|
| Critical speed (rpm)   | 10099  | 12087  | 21176  | 28814  | 44612  | 47191  |

3. Influencing factors of the vibration of the rotor system

There are many factors that affect the vibration of the rotor system, which are mainly divided into external factors and internal factors. The external factors mainly include: external excitation force, the influence of the bearing's own characteristics, and the influence of operation adjustment. And the internal factors mainly include: the uneven mass distribution of the rotor system, the structure of the rotor, and the bearing arrangement plan. This section mainly studies the impact of bearing support spacing, bearing support stiffness and rotor diameter on the critical speed of the rotor system. The critical speed of the rotor system under different conditions is calculated separately, and then the influence of various factors on the vibration of the rotor system is analyzed. When analyzing individual factors, the other two factors remain constant.

3.1. The impact of bearing support spacing on critical speed

The bearing arrangement scheme of the sealing machine rotor adopts one end fixed and the other end floating support. According to the Campbell diagram, the critical speed and natural frequency under the corresponding conditions of each scheme can be calculated. The results are shown in Tab.2.
The relationship between bearing spacing and critical speed is shown in Fig.5. It can be clearly seen from the figure that as the bearing spacing increases, the critical speed decreases. This is because the increase of the bearing spacing reduces the bending stiffness of the rotor, as a result, the natural frequency of the rotor system is reduced. It can be seen from Tab.3 that from Scheme 1 to Scheme 5, the bearing spacing has increased by about 19.05%, while the critical speed has been reduced by about 14.56%, indicating that the bearing spacing has a greater impact on the critical speed and needs to be considered during design. The design of the sealing machine should be based on a compact design, which can not only reduce the size of the equipment, but also increase the critical speed of the system, which is beneficial to reduce the vibration of the rotor system.

Fig.5 Relation diagram of critical speed and bearing spacing

3.2. The influence of support stiffness on critical speed

In order to analyze the influence of bearing stiffness on the rotor system, this section will study the rotor vibration with bearing stiffness from 1e6N/m to 1e8N/m. According to the Campbell diagram, the critical speed under each bearing stiffness coefficient can be obtained, and the results are shown in Tab.3.

The relationship between the support stiffness and the critical speed is shown in Fig.6. It can be seen from the figure that when the support stiffness is small, the first-order critical speed of the rotor system increases significantly with the increase of the support stiffness. When the support stiffness is greater
than 1e7N/m, as the support stiffness increases, the natural frequency of the rotor system and the critical speed increment become smooth. The support stiffness increased from 1e7 to 1e8, increasing by an order of magnitude, but the critical speed increases by less than 0.1%. Because when the support stiffness reached a certain value, it was close to the rigid support \([10]\). It can be seen that when the support stiffness changes within the support stiffness range of the ball bearing, there is basically no effect on the critical speed of the rotor system. Even if the stiffness of the bearing changes due to other factors, it will not affect the vibration of the rotor system, so this helps the working speed to avoid the critical speed, thus it can ensure the stable operation of the sealing machine.

![Fig.6 Relation diagram of critical speed and bearing stiffness](image)

3.3. The influence of rotor diameter on critical speed

The diameter of the rotor section explored in this article refers to the diameter of the long rotor section between the bearings. When calculating different schemes, the diameter of the rotor section changes, and the diameters of other rotor sections will also change. Since the other rotor sections are all short rotor sections, their diameters will not have a big influence on the calculation results. According to the Campbell diagram, the critical speed under each rotor diameter can be obtained, and the results are shown in Tab.4.

| Scheme | Diameter (mm) | First-order natural frequency (Hz) | First-order critical speed (r/min) | Support stiffness (N/m) | bearing support spacing (mm) |
|--------|---------------|-----------------------------------|----------------------------------|------------------------|-----------------------------|
| 1      | 6             | 120.24                            | 6444                             | 1e7                    | 250                         |
| 2      | 7             | 153.12                            | 8358                             | 1e7                    | 250                         |
| 3      | 8             | 183.90                            | 10099                            | 1e7                    | 250                         |
| 4      | 9             | 216.22                            | 11891                            | 1e7                    | 250                         |
| 5      | 10            | 246.94                            | 13568                            | 1e7                    | 250                         |

It can be seen from Fig.7 that with the increase of the rotor diameter, the first-order critical speed of the rotor system increases, the rotor diameter increases by 66.7%, and the critical speed increases by 110.6%, which indicates that the critical speed is more sensitive to rotor diameter changes. This is because the stiffness and quality of the rotor have opposite effects on the natural frequency. The greater the stiffness of the rotor, the higher the natural frequency, and the greater its mass, the lower the natural frequency. In this frequency band, the diameter of the rotor section has a greater impact on the stiffness than the mass. Increasing the diameter of the rotor section appropriately can increase the maximum working speed of the rotor and improve the operating stability of the system. However, in order to reduce the weight of the system, the diameter of the rotor section should not be too large.
4. Conclusion

The sealing machine is the key part of the copper pen tip desktop processing equipment. In this paper, the rotor system of the machine is simplified, and the modal analysis of the rotor system is carried out by using ANSYS analysis software, and the vibration diagrams of each order are obtained, and the critical speed is obtained by the Campbell diagram, and the results show that: (a) The working speed of the sealing machine satisfies the requirements of the critical speed, and there is room for improvement. (b) The size of the rotor diameter has the greatest impact on the rotor system, and choosing a suitable rotor diameter is important. (c) The bearing spacing has minor impact on the rotating rotor system. Reducing the bearing spacing appropriately is also beneficial to increase the maximum operating speed. (d) After selecting a suitable bearing, the change of bearing stiffness has almost no effect on the vibration of the rotor system, which is conducive to the smooth operation of the rotor system. The research in this article provides a certain reference for the design of Chinese pen-making machinery.

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