Graphical shapes of the 2nd type singularities of a 3-RRR planar mechanism

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Abstract. This paper intends to discuss about singularity curves of 2nd type inside the workspace of a 3 RRR planar parallel mechanism used as robot structure. In order to attain this goal we will use certain variation of the links dimensional parameters. This characterization of the mechanism singularities located inside mechanism workspace depends on the dimensional parameters and can be useful in mechanism designing accorded to some functional particularities in the sense that it can help in avoiding singular configurations.

1. Introduction

Due to their evident advantages and opportunities, mechanisms of this category have been largely treated in technical literature [1-40], and wide applied in practical purposes. As is well known, the singularities represent points located inside workspace or limiting workspace, where mechanism movements become uncontrolled or even impossible (the mechanism self-locks). The pose of mechanism (or a structural group) when a singularity occurs is called singular configuration of mechanism or structural group. According to scientific literature [2], singularities limiting mechanism workspace are named of first degree and those located inside mechanism workspace were named of second degree.

2. Singularities of the 3 RRR planar mechanism

In order to illustrate our research we used the example of a 3 DoF 3-RRR planar parallel mechanism with the three actuated joints located on the fixed platform, equally distanced from each other, having in addition equal lengths for proximal and distal links and joints of the mobile platform being also equally distanced from each other. These all particularizations have been done in order to simplify our analysis and not disturbing results nor reducing the method generality.

The problem of singularities in the 3 RRR planar mechanism is detailed treated in technical scientific world [1-8, 11-23]. So, we will use for our purpose, the classical procedure of its determination and study.

Let consider a 3 DoF planar parallel mechanism of a 3-RRR as it was defined in literature [3] (figure 1). Related to this mechanism we do following notations: $O_1A_1 = l_1$ - proximal links length; $A_iB_i = l_2$ - distal links length; $L$, $b$ - sides of the two platforms triangle; $O_1O_2O_3$ - fixed platform; $B_1B_2B_3$ - mobile platform; $Ox$, $Mx_M$, $y_M$ - fixed and mobile systems; $M$ - end effector characteristic point; $q = [x, y, \varphi]$ - output data matrix; $\theta = [\theta_1, \theta_2, \theta_3]$ - input data matrix; $\varphi = const.$.

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platform orientation angle; \( \psi_i = \angle x'M'B_i \), angles depicting position of \( M \) point on the mobile platform.

![Figure 1. 3-RRR planar mechanism - calculus scheme.](image)

To determine the singularities of this mechanism we used an implicit vector function \( \theta = [\theta_1, \theta_2, \theta_3] \) formed by three dimensional function of a three dimensional variable \( q = [x, y, \phi] \):

\[
F(\theta, q) = 0.
\] (1)

Differentiating this relation with respect to time, a formula between input and output velocities were obtained [2]:

\[
J_q \cdot \dot{q} + J_\theta \cdot \dot{\theta} = 0.
\] (2)

The two Jacobian matrices \( J_q \) and \( J_\theta \) depict the two type of singularities: \( J_q \) - related to second type, representing singularities inside workspace and \( J_\theta \) related to first type, representing workspace boundaries which define the workspace shape. In this paper we will deal with singularities of second type only, i.e. located inside workspace boundaries. In order to perform this, Jacobian matrix \( J_q \) will be expressed by following formula, from mathematics:

\[
J_q = \begin{bmatrix}
\frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} & \frac{\partial F_1}{\partial \phi} \\
\frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} & \frac{\partial F_2}{\partial \phi} \\
\frac{\partial F_3}{\partial x} & \frac{\partial F_3}{\partial y} & \frac{\partial F_3}{\partial \phi}
\end{bmatrix}.
\] (3)

Accomplishing the calculations in this matrix, it may be written \( \Delta J_q \), the Jacobian matrix determinant. Taking \( \Delta J_q = 0 \) we will able to discuss about the situations when singularities of this type occur. The idea of this paper is based on the fact that for a given mechanism assembled in a given (from the eight possible), manner – each point of workspace may be touched in an one way only, i.e. given \( q = [x, y, \phi] \) implies one \( \theta = [\theta_1, \theta_2, \theta_3] \) only, thus the problem being determined.
3. The singularity curves tracing aided configuration determinant

We will use to evaluate the configuration determinant \( \Delta J_q \) we shall calculate it in \( Ox \) plane of movement taking into account \( x, y \) variable in this plane and orientation of the mobile platform \( \phi \) - as being constant. So \( \Delta J_q \) can be shown as a surface in \( Ox \) plane, for each angle \( \phi \).

![Image of singularity curves tracing](image1.png)

(a) Workspaces and singularity curves intersecting \( \Delta J_q \) surface with \( Ox \) plane.

![Image of workspaces](image2.png)

(b) Workspaces by changing viewpoint of \( \Delta J_q \) surface.

![Image of \( \Delta J_q \) surfaces](image3.png)

(c) \( \Delta J_q \) surfaces. \( L=3; l1=1; l2=1.5; b=0.5; \phi=\pi/6 \); (left); \( L=3; l1=1; l2=1.5; b=0.5; \phi=\pi/2 \); (right).

**Figure 2.** Singularities shapes for \( L=3 \), \( \phi=\pi/6 \) and \( \phi=\pi/2 \), \( d=[1,1,1] \).
a) Workspaces and singularity curves intersecting $\Delta J_q$ surface with $Oxy$ plane.

b) Workspaces by changing viewpoint of $\Delta J_q$ surface.

c) $\Delta J_q$ surfaces. $L=2; l_1=1; l_2=1.5; b=0.5; \phi=\pi/6$ (left); $L=2; l_1=1; l_2=1.5; b=0.5; \phi=\pi/2$ (right).

Figure 3. Singularities shapes for $L=2$, $\phi=\pi/6$ and $\phi=\pi/2$, $d=[1,1,1]$. 
a) Workspaces and singularity curves intersecting $\Delta J_q$ surface with $Oxy$ plane.

b) Workspaces by changing viewpoint of $\Delta J_q$ surface.

c) $\Delta J_q$ surfaces. $L=2;\ l_1=1;\ l_2=1.5;\ b=0.5;\ \phi=\pi/6$, (left); $L=2;\ l_1=1;\ l_2=1.5;\ b=0.5;\ \phi=\pi/2$ (right).

**Figure 4.** Singularities shapes for $L=2$, $\phi=\pi/6$ and $\phi=\pi/2$, $d=[-1,1]$. 
4. Discussions

In figures 2, 3, 4 it presents some results obtained aided configuration determinant $\Delta J_q$, imposing certain values of constructive parameters $L$, $l_1$, $l_2$, $b$, two orientation angles of mobile platform $\varphi$ and two of eight ($2^3 = 8$) possible assembling modes of proximal and distal links $(l_1$, $l_2)$. For each of these situations, $\Delta J_q$ is 3D plotted, as a surface $\Delta J_q(x, y)$ with $\varphi = \text{const.}$, $x$ and $y$ being the coordinates of the plane where the mechanism operates. To constructive parameter were attributed arbitrarily shown values, taking care the mechanism to be able to work and averages of calculates values not be exaggerate, so that, these values are small. We must specify that values signifying lengths do not represent measurement units; they are abstract ones, chosen only by purpose to facilitate our research.

Regarding the $\Delta J_q$ 3D representations, it can see they define the mechanism workspace if it intersects obtained surfaces with $Oxy$ plane (figure 1). The obtained curves in $Oxy$ plane can be also regarded as solutions of the $\Delta J_q(x, y) = 0$ equation or a 2D graphical representation of an implicit done function $\Delta J_q(x, y) = 0$. Therefore, these planar graphical representations describe rigorously (theoretically at less) the mechanism workspace and located inside it curves of $2^{nd}$ singularity type. These curves divide the mechanism workspace in several regions (different coloured in figures 2, 3, 4), where the mechanism can run properly. However, passing through demarcation (singularity) curves, the mechanism comes out from the properly running domain. It can self-blocks, the movement can become indeterminate or energetically inefficient (unfavourable force transmission index). The singularity curves of $2^{nd}$ type and formed by it regions are shown in figures 2, 3 and 4, in the top and middle sides of them. In the bottom side of the figures 2, 3 and 4 are shown 3D plotted $\Delta J_q$ for a constant orientation of the mobile platform $\varphi$. Images seen in the top and middle sides of figures 2, 3 and 4 represent intersection between $\Delta J_q$ surface and $Oxy$ plane obtained using two different graphical procedures: level curves representation of the surface (in the top) and changing the view point (in middle). These 2D and 3D representations were made varying parameter $L$, orientation of the mobile platform $\varphi$, and assembling mode.

Regarding presented images from figures 2, 3, 4, it can be observed some characteristic traces of the obtained curves:
- These curves divide $Oxy$ plane in zones with special implications for studied mechanism: ones do not take part from mechanism workspace, they represent points that cannot be attained by end-effector when mechanism runs, others represent distinct workspace parts, separated by singularity curves. Inside these zones mechanism work properly but it cannot pass across the demarcation lines without danger of self-blocking or indeterminate movements. These zones are different coloured in figures 2, 3, 4.
- $\Delta J_q = 0$ depicts singularities of $2^{nd}$ type but here are included the singularities of $1^{st}$ type too. It is explained because mathematical expressions of $\Delta J_q$ and $\Delta J_\theta$ contain a square root from a same term.
- Varying dimensional parameters of mechanism $(L$, $l_1$, $l_2$, $b)$, orientation of mobile platform $\varphi$ or assembling mode, it obtains a large variety of workspace shapes and singularity curves shapes too.

5. Conclusions

In this paper, a series of past researches performed by authors is continued, following only the aspect of singularity curve shapes inside the mechanism workspace. We consider this procedure (of tracing singularity curves of $2^{nd}$ type inside mechanism workspace), as being a real aid in mechanism designing process. So, considering as done a mechanism of this type we can easily verify if the end effector trajectory enters in an acceptable domain, avoiding singularity zones. It can observe that by a small modification in exposed input parameters is possible to obtain great effects in effective mechanism workspace.
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