Dyson–Schwinger calculations of meson form factors

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Abstract

The ladder-rainbow truncation of the set of Dyson–Schwinger equations is used to study the pion and kaon electromagnetic form factors and the $\gamma^* \pi^0 \gamma$ transition form factor in impulse approximation. With model parameters previously fixed by the pseudoscalar meson masses and decay constants, the obtained form factors are in good agreement with the data.

1 The Dyson–Schwinger Equations

Our aim is to obtain the hadron spectrum and observables such as coupling constants and form factors from the underlying theory, QCD. The set of Dyson–Schwinger equations [DSEs] form a useful tool for this purpose [1]. In rainbow-ladder truncation, they have been successfully applied to calculate the masses and decay constants of light pseudoscalar and vector mesons [2,3]. The dressed-quark propagator, as obtained from its DSE, together with the Bethe–Salpeter amplitude and the $qq\gamma$ vertex as obtained from the homogeneous and inhomogeneous Bethe–Salpeter equations [BSE] respectively, form the necessary elements for form factor calculations in impulse approximation [4,5].

The DSE for the renormalized quark propagator in Euclidean space is

$$S(p)^{-1} = i Z_2 \not p + Z_4 m(\mu) + Z_1 \int \frac{d^4 q}{(2\pi)^4} g^2 D_{\mu\nu}(k) \frac{\lambda^a}{2} \gamma_\mu S(q) \Gamma^a_\nu(q; p), \quad (1)$$

where $D_{\mu\nu}(k)$ is the dressed-gluon propagator, $\Gamma^a_\nu(q; p)$ the dressed-quark-gluon vertex, and $k = p - q$. The most general solution of Eq. (1) has the form $S(p)^{-1} = i \not p A(p^2) + B(p^2)$ and is renormalized at spacelike $\mu^2$ according to $A(\mu^2) = 1$ and $B(\mu^2) = m(\mu)$ with $m(\mu)$ the current quark mass.

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1 Supported by NSF grant No. PHY97-22429 and computer resources from NERSC.
The DSE for the $qq\gamma$ vertex $\Gamma_{\mu}(p_+, p_-)$ is the inhomogeneous BSE

$$\Gamma_{\mu}(p_+, p_-) = Z_2 \gamma_{\mu} + \int \frac{d^4q}{(2\pi)^4} K(p, q; Q) S(q_+) \Gamma_{\mu}(q_+, q_-) S(q_-),$$

(2)

where $p_{\pm} = p \pm \frac{1}{2}Q$ are the incoming and outgoing quark momenta, and similarly for $q_{\pm}$. The kernel $K$ is the renormalized, amputated $\bar{q}q$ scattering kernel that is irreducible with respect to a pair of $\bar{q}q$ lines. Solutions of the homogeneous version of Eq. (2) define vector meson bound states at timelike photon momenta $Q^2 = -m_v^2$. It follows that $\Gamma_{\mu}(p_+, p_-)$ has poles at those locations. Pseudoscalar solutions $\Gamma_{ps}(p_+, p_-; Q)$ of the homogeneous BSE define bound states such as pions and kaons. Together with the canonical normalization condition for $q\bar{q}$ bound states, Eqs. (1) and (2) completely determine all elements needed for form factor calculations in impulse approximation.

To solve the BSE, we use a ladder truncation, with an effective quark-antiquark interaction that reduces to the perturbative running coupling at large momenta [3]. In conjunction with the rainbow truncation for the quark DSE, the ladder truncation preserves both the vector Ward–Takahashi identity [WTI] for the quark-photon vertex and the axial-vector WTI. This ensures the existence of massless pseudoscalar mesons connected with dynamical chiral symmetry breaking [2]. In combination with impulse approximation, it also guarantees current conservation [5]. Our model preserves the one-loop renormalization group behavior of QCD and reproduces perturbative results in the ultraviolet region. As long as the interaction is sufficiently strong in the infrared region, it leads to chiral symmetry breaking and confinement. The model gives a good description of the $\pi$, $\rho$, $K$, $K^*$ and $\phi$ masses and electroweak decay constants [3], with only two parameters in the effective interaction fitted to $f_\pi$ and the condensate, and realistic current quark masses fitted to $m_\pi$ and $m_K$.

2 Results for meson electromagnetic form factors

Meson form factors in impulse approximation are described by two diagrams, with the photon coupled to the quark and to the antiquark respectively. We can define a form factor for each of these diagrams, e.g.

$$2 P_{\nu} F_{ab\hat{b}}(Q^2) = N_c \int \frac{d^4q}{(2\pi)^4} \text{Tr}[S^a(q) \Gamma_{ps}^{ab}(q, q_; P_-) \times S^b(q_+) i\Gamma^{\hat{b}}_\nu(q_+, q_-) S^{\hat{b}}(q_-) \Gamma_{ps}^{\hat{a}}(q; -P_+) ],$$

(3)

where $q = k + \frac{1}{2}P$, $q_\pm = k - \frac{1}{2}P \pm \frac{1}{2}Q$, $P_\pm = P \pm \frac{1}{2}Q$. We work in the isospin symmetry limit, so for the pion form factor we have $F_\pi(Q^2) = F_{u\bar{u}u}(Q^2)$. The
charged and neutral kaon form factors are given by $F_{K^+} = \frac{2}{3}F_{usu} + \frac{1}{3}F_{u\bar{s}\bar{s}}$ and $F_{K^0} = -\frac{1}{3}F_{d\bar{d}d} + \frac{1}{3}F_{d\bar{s}\bar{s}}$ respectively. Our results for $Q^2F_{\pi}, Q^2F_{K^+}$, and $Q^2F_{K^0}$ are shown in Fig. 1, together with the corresponding charge radii.

The impulse approximation for the $\gamma^*\pi\gamma$ vertex with $\gamma^*$ momentum $Q$ is

$$\Lambda_{\mu\nu}(P, Q) = i\frac{\alpha}{\pi f_\pi} \epsilon_{\mu\nu\alpha\beta} P_\alpha Q_\beta g_{\pi\gamma\gamma} F_{\gamma^*\pi\gamma}(Q^2)$$

$$= \frac{N_c}{3} \int \frac{d^4q}{(2\pi)^4} \text{Tr} [S(q) i\Gamma_\nu(q, q') S(q') i\Gamma_\mu(q', q'') S(q'') \Gamma_\pi(q'', q; P)] .$$

where the momenta follow from momentum conservation. In the chiral limit, the value at $Q^2 = 0$, corresponding to the decay $\pi^0 \rightarrow \gamma\gamma$, is given by the axial anomaly and its value $g_{\pi\gamma\gamma}^0 = \frac{1}{2}$ is a direct consequence of only gauge invariance and chiral symmetry; this value is reproduced by our calculations and corresponds well with the experimental width of 7.7 eV. In Fig. 2 we show our results with realistic quark masses, normalized to the experimental $g_{\pi\gamma\gamma}$.

All our form factor results are remarkably close to the data, without any readjustment of the parameters. Up to about $Q^2 = 3\text{GeV}^2$, they can be fitted quite well by monopoles. Asymptotically, our calculated form factors behave like $Q^2F(Q^2) \rightarrow c$ up to logarithmic corrections. However, numerical limitations
Fig. 2. The $\gamma^* \pi \gamma$ form factor, with data from CLEO and CELLO [8].

prevent us from accurately determining these constants. Around $Q^2 = 3 \text{ GeV}^2$, our result for $Q^2 F_\pi$ is well above the pQCD result $16\pi f_\pi^2 \alpha_s(Q^2)$ [9], and clearly not yet asymptotic; $F_{\gamma^* \pi \gamma}$ lies in between monopoles fitted to the Brodsky–Lepage asymptotic limit, $8\pi f_\pi^2$ [10], and to the DSE limit, $\frac{16}{3}\pi f_\pi^2$ [11].

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