Inverse Kinematic Modeling of a 6-PSS Compliant Parallel Platform for Optoelectronic Packaging

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Abstract. This paper presents an inverse kinematic analysis of a 6-Prismatic-Spherical-Spherical (6-PSS) compliant parallel platform for optoelectronic packaging. The platform is driven by the piezoelectric motors, which combines the advantages of parallel platforms and wide-range flexure hinges. Since the rotation center of the wide-range flexible hinges constantly varies with the motion of the platform, it causes much error for positioning accuracy of the platform in applications. In this paper, an inverse kinematic solution based on elastokinematic analysis is proposed, in which the deformation of the wide-range flexure hinges is considered. And finally, several different cases based on the elastokinematic model are given, the results prove this method effective.

1. Introduction

With the emergence of new optoelectronic devices with high-capacity multi-channel, it is necessary to improve the alignment accuracy of optoelectronic packaging system [1]. The six degrees of freedom (6-DOF) motion platforms are a basic component in the field of optoelectronic packaging, and that are used to adjust the position and attitude of the optical fibers [2]. To achieve high-precision coupling of optoelectronic packaging, the 6-DOF motion platforms have not only a few millimetres stroke to meet the adjustment, but also the high precision.

Compared with the traditional serial platforms, the parallel platforms have the advantages of high stiffness, high bearing capacity and high accuracy. The flexible hinges are a new type of joints, and they have many superiorities such as the zero backlash, lack of friction compare with conventional mechanism joints [3, 4, 5]. Meanwhile, the wide-range flexure hinges are utilized for a larger stroke [6, 7]. So the compliant parallel platforms with the wide-range flexure hinges are more suitable for optoelectronic packaging system. However, the center of rotation of the wide-range flexure hinges constantly varies with the motion of the platform, and the difficulty in obtaining an accurate kinematic model, affect the positioning accuracy of the platform in applications [8].

To overcome this problem, many scholars have adopted the pseudo-rigid-body model concept to obtain a more accurate kinematic model of the compliant mechanisms [9]. Yao et al derived and calibrated a kinematic model of a parallel-kinematic micropositioning XY stage [10]. Shi et al studied
a kinematic-model-based calibration of a hexapod nano-positioner, and which is used to get more accurate results for the spatial [11]. Dong et al established a kinematics model via analyzing the stiffness model of the whole 6-PSS platform based on FEA method [6]. Ryu et al described a computer based inverse kinematic model of a coupled flexure hinge mechanism and presented the experimental verification for the model [12]. Li et al proposed a constraint-force-based approach to calculate the deformation of a decoupled XYZ compliant parallel mechanism under an intermediate motion range by using the nonlinear closed-form spatial beam model [13]. Wang et al established the load-displacement model of the three-segmented compliant limb and derived the inverse kinematics model of a 6-PSS compliant parallel platform in closed-form [14]. Rouhani et al developed a method on the basis of elastokinematic analysis of a microhexapod manipulator, in which the elastic deformation of the flexure hinges was considered for the kinematic analysis [15].

In this paper, we develop an elastokinematic analysis based inverse kinematics solution of a 6-PSS compliant parallel platform with wide-range flexure hinges for optoelectronic packaging. The rest of the paper is organized as follows: Section 2 describes the structure of the proposed 6-PSS compliant parallel platform. In section 3, the inverse kinematics solution based on elastokinematic analysis is deduced. In section 4, several cases based on the proposed method are given. Finally, the paper is concluded in Section 5.

2. System description of the 6-PSS compliant parallel platform
As shown in Fig.1 (a), the 6-PSS compliant parallel platform is comprised of a moving platform, a fixed platform, six driving components and six identical kinematic limbs connecting the moving platform and the fixed platform via wide-range flexure hinges. Each kinematic limb consists of wide-range flexure hinges at both ends and a rigid rod in the middle. As shown in Fig.1 (b), the wide-range flexure hinges can be considered as a slender and spatial beam structure, they can rotate around three axes as a passive joint (S). They can not only achieve high-accuracy motion, but also a wide range motion for parallel platforms. The driving components as active joint (P), and the piezoelectric motors are utilized as actuator with incomparable superiority in high-precision, large driving force, small size and infinite travel. Here, beryllium bronze and duralumin are chosen as the material of the wide-range flexure hinges and the others.

![Figure 1. (a). Basic configuration of the 6-PSS compliant parallel platform. (b). A description of the wide-range flexure hinge.](image)

3. Inverse kinematic modeling
Considering the elastic deformation of the wide-range flexure hinges and the displacement of rotation center, an inverse kinematics solution of a 6-PSS compliant parallel platform are developed based on elastokinematic analysis.

A model with coordinate systems for a 6-PSS compliant parallel platform is shown in Fig.2. A reference frame $B-xyz$ is attached to the fixed platform at the center $B$. A local coordinate system $P-xyz$ is attached to a moving platform at the center $P$. For the $i^{th}$ kinematics chain, a local coordinate system
$P_i$-xyz is attached to the moving platform at the connection point between the platform and the $i^{th}$ upper flexure hinges, and always parallel to $P$-xyz. The other three local coordinate systems are attached to the end of the upper flexure hinge denoted by $u_i$-xyz, the rigid rod denoted by $r_i$-xyz and the lower flexure hinge denoted by $l_i$-xyz separately. The $z$ axis of these coordinate systems is oriented along the geometric axis. A local coordinate frame $s_i$-xyz is attached to the center of gravity at the slider, the $z$ axis along the only moving direction of the slider.

![Figure 2. A model with coordinate systems for a 6-PSS compliant parallel platform.](image)

A deformation unit in space as shown in Fig.3, within the elastic limit range, when the load is applied on a certain point, the relationship between load and displacement of nodes can be established in the local coordinate system as follows:

$$d = C \cdot F$$

(1)

Where $d$ is nodal deformation vector ($6 \times 1$), $F$ is the nodal load vector ($6 \times 1$) and $C$ is compliance matrix ($6 \times 6$) of the deformation unit. Specifically, $C$ can be obtained in ref [16].

For the $i^{th}$ kinematics chain of the 6-PSS compliant parallel platform, the compliance model in the local coordinate system for the deformation unit can be derived, as follows:

$$d_{ui} = u_i C_u \cdot F_{ui}$$

(2)

$$d_{ri} = r_i C_r \cdot F_{ri}$$

(3)
\[ d_i = \dot{i}C_u \cdot F_i \]  
\[ d_s = \dot{s}C_s \cdot F_s \]  

Where \( \dot{i}C_u \), \( \dot{i}C_r \), \( \dot{i}C_l \), and \( \dot{s}C_s \) are the compliance matrices of the upper flexure hinge, the rigid rod, the lower flexure hinge, and the slider in the local coordinate frame. Note that the slider has only one degree of freedom as a variable defined by input value. We assume that \( \dot{s}C_s \) as a compliance matrix \((6 \times 6)\), in which a large number is assigned to the element related to the degree of freedom, the other elements of the matrix are set to zero.

The displacement for the upper flexure hinge, the rigid rod, the lower flexure hinge and the slider in the local coordinate system can be transformed to the displacement of the local coordinate system \( P_i-xyz \) at \( P_i \) for the \( i^{th} \) kinematics chain, respectively, as follows:

\[ d_{u,i}^P = \frac{\dot{r}_i}{\dot{u}_i} J \cdot d_{u,i} \]  
\[ d_{r,i}^P = \frac{\dot{r}_i}{\dot{r}_i} J \cdot d_{r,i} \]  
\[ d_{l,i}^P = \frac{\dot{r}_i}{\dot{l}_i} J \cdot d_{l,i} \]  
\[ d_{s,i}^P = \frac{\dot{r}_i}{\dot{s}_i} J \cdot d_{s,i} \]  

Where \( d_{u,i}^P \), \( d_{r,i}^P \), \( d_{l,i}^P \), and \( d_{s,i}^P \) are the nodal displacement of \( P_i \) in the coordinate frame \( P_r-xyz \) that is caused by the displacement of the upper flexure hinge, the rigid rod, the lower flexure hinge and the slider in the local coordinate system. \( \frac{\dot{r}_i}{\dot{u}_i} J \), \( \frac{\dot{r}_i}{\dot{r}_i} J \), \( \frac{\dot{r}_i}{\dot{l}_i} J \), and \( \frac{\dot{r}_i}{\dot{s}_i} J \) represent the transformation matrix from \( u_i-xyz \), \( r_i-xyz \), \( l_i-xyz \), and \( s_i-xyz \) frame to \( P_r-xyz \) coordinate system. Let \( J \) be the equivalent transformation matrix from the frame \( j \) to \( i \), and which can be obtained in ref [17].

The nodal displacement of \( P_i \) in the coordinate frame \( P_r-xyz \) can be derived as a linear combination of \( d_{u,i}^P \), \( d_{r,i}^P \), \( d_{l,i}^P \), and \( d_{s,i}^P \):

\[ d_P = d_{u,i}^P + d_{r,i}^P + d_{l,i}^P + d_{s,i}^P \]  

Let expect output \( d_p = \begin{bmatrix} x & y & z & \theta_x & \theta_y & \theta_z \end{bmatrix}^T \) as the displacement vector of the center point of moving platform in the \( P-xyz \) coordinate system. The displacement of \( P_i \) in the local coordinate system \( P_l-xyz \) can be expressed as follows:

\[ d_{P,i} = \frac{\dot{r}_i}{\dot{r}_i} J \cdot d_P \]  

Where \( \frac{\dot{r}_i}{\dot{r}_i} J \) represents the transformation matrix from \( P-xyz \) coordinates system to \( P_r-xyz \) coordinates system.

When a force acts on the center of the moving platform in a local coordinate system \( P_l-xyz \). For the \( i^{th} \) kinematics chain in \( P_l \)-frame, this force can be converted into \( F_{P,i} \) at \( P_i \), so the force is applied to
the upper flexure hinge denoted by $F_u$, the rigid rod denoted by $F_r$, the lower flexure hinge denoted by $F_l$ and the slider denoted by $F_s$ in the local coordinate system that can be derived as follows:

$$ F_u = \frac{u_i}{p_i} J \cdot F_p $$  \hspace{1cm} (12)

$$ F_r = \frac{r_i}{p_i} J \cdot F_p $$  \hspace{1cm} (13)

$$ F_l = \frac{l_i}{p_i} J \cdot F_p $$  \hspace{1cm} (14)

$$ F_s = \frac{s_i}{p_i} J \cdot F_p $$  \hspace{1cm} (15)

Substituting Eqs. (12) - (15) into Eqs. (2) - (5), and then in Eq. (10) yields:

$$ d_p = \left( \frac{p_i}{u_i} J \cdot u C_u \cdot \frac{u_i}{p_i} J + \frac{p_i}{r_i} J \cdot r C_r \cdot \frac{r_i}{p_i} J + \frac{p_i}{l_i} J \cdot l C_l \cdot \frac{l_i}{p_i} J + \frac{p_i}{s_i} J \cdot s C_s \cdot \frac{s_i}{p_i} J \right) \cdot F_p $$  \hspace{1cm} (16)

Therefore, the above equation can be written into:

$$ F_p = \left( \frac{p_i}{u_i} J \cdot u C_u \cdot \frac{u_i}{p_i} J + \frac{p_i}{r_i} J \cdot r C_r \cdot \frac{r_i}{p_i} J + \frac{p_i}{l_i} J \cdot l C_l \cdot \frac{l_i}{p_i} J + \frac{p_i}{s_i} J \cdot s C_s \cdot \frac{s_i}{p_i} J \right)^{-1} \cdot d_p $$  \hspace{1cm} (17)

Substituting $F_p$ from Eq. (17) into Eq. (15) yields:

$$ F_s = \frac{s_i}{p_i} J \cdot \frac{s_i}{p_i} J \cdot \left( \frac{p_i}{u_i} J \cdot u C_u \cdot \frac{u_i}{p_i} J + \frac{p_i}{r_i} J \cdot r C_r \cdot \frac{r_i}{p_i} J + \frac{p_i}{l_i} J \cdot l C_l \cdot \frac{l_i}{p_i} J + \frac{p_i}{s_i} J \cdot s C_s \cdot \frac{s_i}{p_i} J \right)^{-1} \cdot d_p $$  \hspace{1cm} (18)

Combining (5) and (18), we can get the following equation:

$$ d_s = \frac{s_i}{p_i} J \cdot \left( \frac{p_i}{u_i} J \cdot u C_u \cdot \frac{u_i}{p_i} J + \frac{p_i}{r_i} J \cdot r C_r \cdot \frac{r_i}{p_i} J + \frac{p_i}{l_i} J \cdot l C_l \cdot \frac{l_i}{p_i} J + \frac{p_i}{s_i} J \cdot s C_s \cdot \frac{s_i}{p_i} J \right)^{-1} \cdot d_p $$  \hspace{1cm} (19)

Using (11) and the above equation, calculating $d_s$ results in:

$$ d_s = \left[ \frac{s_i}{p_i} J \cdot \left( \frac{p_i}{u_i} J \cdot u C_u \cdot \frac{u_i}{p_i} J + \frac{p_i}{r_i} J \cdot r C_r \cdot \frac{r_i}{p_i} J + \frac{p_i}{l_i} J \cdot l C_l \cdot \frac{l_i}{p_i} J + \frac{p_i}{s_i} J \cdot s C_s \cdot \frac{s_i}{p_i} J \right)^{-1} \cdot \frac{p_i}{s_i} J \right] \cdot d_p $$  \hspace{1cm} (20)

Therefore, the six linear inputs $d_s$ of a 6-PSS parallel platform can be obtained by solving Eq. (20) in the presence of expect output $d_p$ of the moving platform.

4. Case results

Based on the above elastokinematic analysis, an inverse kinematic solution program by means of MATLAB for a 6-PSS compliant parallel platform is developed. We assume that the moving platform has a expect output $(x, y, z, \theta_x, \theta_y, \theta_z)$. Then the input displacements of the six motors $(d_u, d_r, d_l, d_s, d_s, d_s)$ will be calculated by above program. The main geometric and structure parameters are listed in Table 1. The inverse kinematics solution results are shown in Table 2.
Table 1. The structure parameters of the 6-PSS compliant parallel platform.

| Item                                      | Value |
|-------------------------------------------|-------|
| Diameter of moving platform (mm)          | 50    |
| Diameter of fixed platform (mm)           | 180   |
| Distribution angle of upper flexure hinge α (°) | 30    |
| Distribution angle of lower flexure hinge β (°) | 60    |
| Length of rigid rod (mm)                  | 74    |
| Length of wide-range flexure hinge (mm)   | 13    |
| Diameter of rigid rod (mm)                | 10    |
| Diameter of wide-range flexure hinge (mm) | 1     |
| Density of rigid rod (Kg/m³)              | 2700  |
| Density of wide-range flexure hinge (Kg/m³) | 8100  |
| Modulus of elasticity of rigid rod (Gpa)   | 70    |
| Modulus of elasticity of wide-range flexure hinge (Gpa) | 130    |

Table 2. The inverse kinematics solution results.

| Output / (mm, mm, mm, °, °, °) | Input / (mm, mm, mm, mm, mm, mm) |
|--------------------------------|----------------------------------|
| (6, 0, 0, 0, 0, 0)             | (5.9999, 0.5819, 6.0000, 6.0000, -0.5819, 5.9999) |
| (0, 6, 0, 0, 0, 0)             | (4.2856, 5.9999, -2.7290, 2.7290, 5.9999, -4.2856) |
| (0, 0, 6, 0, 0, 0)             | (-8.1883, -6.6944, 7.3200, 7.3200, 6.6944, -8.1883) |
| (5, 5, 0, 0, 0, 0)             | (8.5713, 5.4849, 2.7258, 7.2742, 4.5151, 1.4287) |
| (0, 0, 3.5, 3.5, 3.5)          | (1.9822, -2.1532, 1.2853, 0.3036, -1.1696, 0.9981) |

5. Conclusion
In this paper, we present a 6-PSS compliant parallel platform for optoelectronic packaging. The platform is driven by the piezoelectric motors, which combines the advantages of parallel platforms and wide-range flexure hinges. Considering that the elastic deformation of the wide-range flexure hinges and the displacement of rotation center, an inverse kinematic solution based on elastokinematic analysis is developed. An inverse kinematic solution program based on elastokinematic analysis is given, several different cases results show that the elastokinematic model is efficient for modeling compliant parallel platforms with wide-range flexure hinges.

Acknowledgments
This work was supported in part by the Fundamental Research Funds for the Central Universities of Central South University under Grant 2018zzts470, in part by the National Key Research and Development Program of China under Grant 2017YFB1104800, in part by the National Natural Science Foundation of China under Grant 51575534.

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