Symmetry and Uncountability of Computation

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Abstract

This paper talks about the complexity of computation by Turing Machine. I take attention to the relation of symmetry and order structure of the data, and I think about the limitation of computation time. First, I make general problem named “testing problem”. And I get some condition of the NP complete by using testing problem. Second, I make two problem “orderly problem” and “chaotic problem”. Orderly problem have some order structure. And DTM can limit some possible symbol effectively by using symmetry of each symbol. But chaotic problem must treat some symbol as a set of symbol, so DTM cannot limit some possible symbol. Orderly problem is P complete, and chaotic problem is NP complete. Finally, I clear the computation time of orderly problem and chaotic problem. And $P \neq NP$. 

1 Introduction

This paper talks about the complexity of computation by Turing Machine. I take attention to the relation of symmetry and order structure of the data, and I think about the limitation of computation time.

First, I make general problem named “testing problem”. And I get some condition of the NP complete by using testing problem.

Second, I make two problem “orderly problem” and “chaotic problem”. Orderly problem have some order structure. And DTM can limit some possible symbol effectively by using symmetry of each symbol. But chaotic problem must treat some symbol as a set of symbol, so DTM cannot limit some possible symbol. Orderly problem is P complete, and chaotic problem is NP complete. Finally, I clear the computation time of orderly problem and chaotic problem. And $P \neq NP$.

2 Make “Testing problem”

I make the general problem that have DTM and NTM problem in it.

“Verifier” is the TM that can decide a problem. Verifier can go to halting (accepting or rejecting) configuration finally. “Verify data” is the data that verifier compute. “Verifying problem” is the problem that verifier decide. “Specific
symbol” is the symbol that make up verify data. “Verify problem” is the problem that verifier can compute.

“Checker” is the TM that can use “Generic symbol”. “Generic symbol” is the special symbol that mean one of the specific symbol, and one of the generic symbol that mean fewer specific symbol. “Check data” is the data that have generic symbol. Checker reject the check data that verifier reject all verify data that change check data’s generic symbol to any specific symbol, and checker accept other check data. That is to say, checker accept the check data that verifier accept one or more verify data that change check data’s generic symbol to any specific symbol. “Checking problem” is the problem that checker can decide.

“Selected symbol” is the specific symbol (or the generic symbol that mean fewer specific symbol) that can change from the generic symbol in check data.

Checker is the Verifier that have transition function to use specific symbol. If check data don’t have specific data, checker compute like verifier. And checker decide same (accepting or rejecting) configuration.

“Testing problem” is verifying problem and checking problem. “Tester” is verifier and checker. “Test data” is verify data and check data.

“Computation result” is halting (accepting or rejecting) situation that tester compute test data. And “remaining configuration” is the configuration to clear the situation that tester do not decide accept or reject.

It’s some relation between testing problem and verifying problem.

Theorem 1. If verifying problem is P, testing problem is NP-complete.

Proof. It’s easy to proof that testing problem is in NP.

If a test data is acceptable, some verify data is acceptable that match any specific symbol with the testing data. The verifying problem is P that verify the verifying data, so testing problem is in NP.

To proof testing problem is NP-hard, I clear that SAT can reduce to testing problem with polynomial time.

Those test data have SAT’s logical formula as specific symbol, and SAT’s truth value as general symbol. The test data have polynomial length of logacal formula because truth value is shorter than logical fomula. And It’s can polynomial time to get variable from logacal fomula. So SAT can reduce to testing problem with polynomial time.

So, if verifying problem is P, testing problem is NP-complete.

3 Symmetry and order of the problem

To talk about complexity of problem, I make two testing problem.

3.1 Orderly problem

First, I make the problem that can separate generic symbol each other.
Definition 2. "Orderly problem" is the testing problem that all test data have at least one of the selected symbol fix the computation result, and tester can get selected symbol with polynomial time. "Focus symbol" is the generic symbol, "Symmetry symbol" is the selected symbol that the computation result fix. And "Simplify symbol" is the selected symbol that keep same computation result as focus symbol.

Orderly problem’s test data have at least one of selected symbol that’s computation result is symmetry. Simplify symbol have same computation result that other generic symbol is any selected symbol. And to use this symmetry, tester can change from generic symbol to selected symbol orderly. If the computation result is reject, tester change the generic symbol to other generic symbol that do not have the selected symbol in it with keeping computation result. If the computation result is accept, tester change the generic symbol to selected symbol with keeping computation result. That is to say, Focus symbol have free part from other generic symbol at point of view of the computation result.

If Tester can decide accept/reject/remind when focus symbol change any selected symbol, tester can decide simplify symbol in polynomial time. Because selected symbol count is limited, tester can decide accept/reject/remind of all selected symbol in polynomial time.

And, test data that change from focus symbol to simplify symbol is keep condition of orderly problem. So I can say that;

Theorem 3. Generic symbol in test data of orderly problem have at least one of the order structure.

Proof. Because of 2, test data of orderly problem have at least one of the focus symbol, and tester can change from focus symbol to simplify symbol. And the test data that change simplify symbol also have focus symbol and simplify symbol. And this can do till all generic symbol change to specific symbol. So, all generic symbol have some order to change specific symbol.

Theorem 4. If verifying problem is P, orderly problem is P-complete.

Proof. First, I clear orderly problem is in P. Because of 2, tester of orderly problem can change from every focus symbol to simplify symbol in polynomial time. And tester can change from test data to verify data in polynomial time (because test data have shorter length generic symbol than the test data length). And tester can decide computation result of verify data in polynomial time, so tester can compute all in polynomial time.

To proof orderly problem is P-hard, I clear that HornSAT can reduce to orderly problem with log space.

Like 1 tester can make from HornSAT to test data. Tester need only logical formula’s current position to get variable. So HornSAT can reduce to testing problem with log space.

I clear that Test data of HornSAT have orderly problem’s condition. HornSAT’s variable have some order. Tester can put variable value that is necessary to accept test data. Finally tester can decide computation result. This is same
condition HornSAT and Test data of HornSAT, tester can change from generic symbol as variable to specific symbol as value. So test data of HornSAT is orderly problem’s test data.

So, if verifying problem is $P$, orderly problem is $P$-complete. 

### 3.2 Chaotic problem

Second, I make the problem that is necessary to compute generic symbol as a large body.

**Definition 5.** "Chaotic problem" is the testing problem that tester cannot decide computation result by changing any one generic symbol to selected symbol. If tester decide computation result, tester must decide many generic symbol to selected symbol. And such generic symbol is not limited. “Extended symbol” is the generic symbol above. “Unit symbol” is each generic symbol that make extended symbol. “Extended symbol length” is count of extended symbol. Extended symbol that change from partly or all generic symbol to selected symbol is Extended symbol. Extended symbol is out of count.

“Universal problem” is the co-problem of chaotic problem.

Chaotic problem do not have good condition to decide focus symbol and simplify symbol, so tester must decide another generic symbol to decide any generic symbol. Tester must compute some generic symbol as a large mass, cannot compute like orderly problem. Tester must compute extended symbol as a increasong generic symbol or specific symbol, so tester must compute increasong symbol that come exponent size of extended symbol length. And each unit symbol that make extended symbol is symmetry for tester. If tester break the symmetry by using some unit symbol first, chaotic problem keep symmetry when all test data get together.

By the way, if extended symbol length is limited, tester can compute the problem as orderly problem because tester can compute the extended symbol change to new generic symbol or specific symbol.

Chaotic problem is the set of testing problem cleaning the orderly problem. If orderly problem is testing problem, chaotic problem is empty. If testing problem have the problem that is not in orderly problem, the problem is chaotic problem.

**Theorem 6.** If verifying problem is $P$, chaotic problem is not empty.

**Proof.** To proof this theorem, I use reduction to absurdity. I suppose that chaotic problem is empty if verifying problem is $P$. So, all test data have focus symbol and symmetry symbol, and the testing data that change from focus symbol to symmetry symbol have same computation result.

But testing problem is NP-complete having SAT, so test data do not have computation result symmetry each test data like orderly problem. For example, SAT have any logical formula that make given truth table. So, SAT have any logical formula that result is true and false if a part of variable fix the value. Testing data from SAT do not have focus symbol and symmetry symbol, and testing data is not orderly problem.
That is to say, this assumption conflicts with the testing problem condition. So, if verifying problem is P, chaotic problem is not empty.

4 Difference between Orderly problem and Chaotic problem

I clear the difference of complexity of chaotic problem and orderly problem.

Theorem 7. If verifying problem is P, chaotic problem is not in P.

Proof. To proof this theorem, I use reduction to absurdity. I suppose that chaotic problem is in P if verifying problem is P.

Some testing data A and set of testing data that change testing data A of a generic symbol to a selected symbol. Tester can get all testing data's computation result in polynomial time. And tester can get all testing data that have same computation result with testing data A. So, tester can get testing data with same computation result testing data A and change from a generic symbol to selected symbol. Testing data A have condition of focus symbol and simplify symbol. And testing data A is any testing data of chaotic problems.

But, if testing data A is chaotic problem's data, this generic symbol and selected symbol is not independent other generic symbol. So tester can only change from generic symbol to selected symbol with other generic symbol, and can not change only one generic symbol without changing other generic symbol. Because unit symbol that make Extended symbol is symmetry to tester, if tester use some unit symbol first, chaotic problem keep symmetry when all test data get together. So tester must use all extended symbol as free symbol each other. Extended symbol count is exponent size of extended symbol length, so tester can not use extended symbol as generic symbol and specific symbol same polynomial time.

If this generic symbol can change selected symbol at any extended symbol, the generic symbol is free from extended symbol. And if all test data have such generic symbol, testing problem do not have extended data. So, this assumption conflicts with the chaotic problem condition.

So, if verifying problem is P, chaotic problem is not in P.

If the verifying problem is P, the orderly problem is P-complete and the chaotic problem is NP-complete. So, I clear $P \neq NP$.

Theorem 8. $P \neq NP$

5 More detail about difference between orderly problem and chaotic problem

I talk about chaotic problem's computation time in more detail.

To clear the limit of the orderly problem, I make special orderly problem.
**Definition 9.** "Saturated orderly problem" is the orderly problem that do not have all the data that make some generic symbol and some specific symbol, and the count of data is not limited.

**Theorem 10.** *Saturated orderly problem is not empty.*

*Proof.* To proof this theorem, I use the testing problem from CNFSAT.

Same as [1] I make testing problem from CNFSAT. At this testing problem, logical formula is specific symbol and true value is generic symbol.

Now, to change generic symbol to specific symbol, I change CNFSAT to HornSAT. I can change CNFSAT to HornSAT by putting false to some variable. The variable is over positive literal of each clause. And I can do same operation to testing problem from CNFSAT. The testing problem that do same operation is like HornSAT and fill orderly problem condition.

But, the testing problem like HornSAT have the variables that force false, so testing problem do not have all data of generic symbol and specific symbol. And the count of structure is not limited.

So, The orderly problem like this is saturated orderly problem.

Saturated orderly problem is the limit of orderly problem. To clear difference between chaotic problem and orderly problem, I make new problem.

**Definition 11.** "Saturated chaotic problem" is the testing problem that’s verifying problem is saturated orderly problem.

**Theorem 12.** *Saturated chaotic problem is not empty.*

*Proof.* The testing problem that’s verifying problem is saturated orderly problem. The saturated orderly problem’s generic symbol is the testing problem’s specific symbol. And It is same computation result that change the saturated orderly problem’s generic symbol to testing problem’s generic symbol.

But, saturated orderly problem do not have all data of generic symbol and specific symbol because of [9]. So, if I group all test data that match a part of data, some group have only the test data that is different specific symbol part only. Saturated orderly problem do not have test data that change the part of the specific symbol to generic symbol.

So, the testing problem that’s verifying problem is saturated orderly problem have the data that is not in the saturated orderly problem. And testing problem is not saturated orderly problem.

Saturated chaotic problem is complexer than saturated orderly problem. Tester compute saturated chaotic problem in exponent time.

**Theorem 13.** If *Saturated orderly problem’s verifying problem is P*, tester compute saturated chaotic problem in exponent time of extended symbol size.

*Proof.* I make the test data that have the generic symbol. Saturated orderly problem do not have the generic symbol, and saturated chaotic problem have the generic symbol. So, it is not orderly problem that can computate the generic
symbol without changing to specific symbol. And the verifying problem that change the generic symbol to specific symbol is P-complete because the saturated orderly problem’s verifying problem is P. Count of the extended symbol that maked the generic symbol is amount exponent size, so the computation time of the saturated chaotic problem is exponent size of extended symbol size.  

Now, if saturated orderly problem’s verifying problem is P, saturated orderly problem is P-complete and saturated chaotic problem is NP-complete. And because of 9 and 13 saturated chaotic problem’s extended symbol size is not limited, and computation time is exponent size of extended symbol size. So, I clear $P \neq NP$.

**Theorem 14.** $P \neq NP$