Note on (D6,D8) Bound State, Massive Duality and Non-commutativity *

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Abstract: In this paper we study half-supersymmetric (D6,D8) bound state brane configuration of massive type IIA supergravity. We show this bound state can also be generated by using massive T-duality rules of type II strings in $D = 9$ and starting from D7-branes. We write down corresponding Killing spinors and find that these backgrounds indeed preserve 16 supersymmetries like any other $D_p$-brane bound state with $B_{\mu\nu}$ field. We also make a point on the massive nature of $B_{\mu\nu}$ field in this background. The Seiberg-Witten limits to obtain non-commutative Yang-Mills theories in $D = 9$ are also discussed, but the full understanding of such gauge theories remains unanswered.

Keywords: strings, compactifications, dualities.

*This work is in part supported by: AvH – the Alexander von Humboldt foundation.
1. Introduction

The idea of non-commutativity in string theory [1] has led to new insights into the understanding of AdS/CFT conjecture [2]. It has been understood that in the presence of NS-NS $B_{\mu\nu}$ field open strings behave in such a way that low energy field theory on the anti-de Sitter (AdS) boundary could be described by a non-commutative super-Yang-Mills (NCYM) theory. The Seiberg-Witten map [1]

$$G_{\mu\nu} + \frac{\theta_{\mu\nu}}{2\pi\alpha'} = \frac{1}{g_{\mu\nu} + 2\pi\alpha'B_{\mu\nu}}$$

(1.1)

defines for us open string metric, $G_{\mu\nu}$, and noncommutativity parameter, $\theta_{\mu\nu}$, in terms of closed string metric, $g_{\mu\nu}$, and background, $B$, field. The open string coupling is given by

$$G_o^2 = g_s^2 \frac{\det(g + B)}{\det(g)}.$$  

(1.2)

The Yang-Mills coupling is defined through $g_{YM}^2 = G_o$. 

Note that the above map works so long as the closed string backgrounds are constant which is the case for all Dp-branes with $2 \leq p \leq 6$. For $p > 6$ branes the backgrounds are not asymptotically constant or flat and the holographic picture is less clear. We focus in this paper on D8-branes which have only one transverse direction. We would like to study the dual Yang-Mills theories for $N$ D8-branes, with or without $B$ field. It is not clear if the above Seiberg-Witten relations hold good when the supergravity backgrounds are not constant as is the case with D8-branes. Previously, $D0 - D8$ system with a $B$-field have been studied in [3], see also [4]. Interestingly, in a recent paper [5], Hashimoto and Sethi have studied holography for time-dependent backgrounds assuming that backgrounds are sufficiently locally constant (see also [6]). Results have been interesting and the application of above Seiberg-Witten relations reproduces the desired results.

We will employ the similar idea for the D8-branes here and we assume that the string backgrounds, though not constant, but are locally constant so that Seiberg-Witten maps could be applicable. Under this assumption we basically study the decoupling limits of the supergravity backgrounds and make some observations about the maximally supersymmetric NCYM theories on the nine-dimensional boundaries of AdS$_{10}$ regions. We find that under the decoupling limits the closed strings indeed get decoupled.

The paper is organized in the following way. In section-2 we aim to reconstruct a (D6,D8) bound state with $B$-field by using massive duality relations in nine dimensions [7]. We also obtain the Killing spinors and also discuss the nontrivial massive nature of the $B$-field in this background. In section-3 we study the decoupling limits and discuss the nature of boundary conformal field theories (CFT) in nine spacetime dimensions. We also construct (D4,D6,D8) bound state in section-4. The conclusions are given in section-5.

2. The (D6,D8) bound state

Recently, the following background configuration was obtained in [8] as a solution of massive type IIA supergravity theory [9]

$$ds^2_{10} = H^{\frac{1}{2}} \left\{ H^{-1}(-dt^2 + dx_1^2 + \cdots + dx_6^2) + H'^{-1}(dy_1^2 + dy_2^2) + dz^2 \right\},$$

$$e^{2\phi} = g_s^2 H^{-\frac{3}{2}} H'^{-1}, \quad dA_{(1)} = -\frac{m \sin \theta}{g_s} dy_1 \wedge dy_2,$$

$$B_{y_1y_2} = \tan \theta (1 - H'^{-1})$$

where $H = 1 + m|z|$ and $H' = 1 + \cos^2 \theta (H - 1)$. Here $m = m_0 g_s / \cos \theta$ and we reserve $m_0$ to denote the mass parameter (cosmological constant) of the massive type IIA supergravity [9]. The parameter $g_s$ represents string coupling. This configuration has nontrivial $B$-field along with a constant flux of gauge fields and is interpreted as
a bound state of D6 and D8 branes. Note the value of \( B = \tan \theta (1 - H'^{-1}) \) which is usually the case with all \((D(p-2),Dp)\) bound states for \(2 \leq p \leq 6\), see [10–12]. It is therefore interesting to study the non-commutative Yang-Mills decoupling limits [1] for \((D6,D8)\) bound state (2.1), which we will do in the next section.

The bound state (2.1) was obtained by exploiting the massive T-duality symmetries in \( D = 8 \), see [8] for details. However, we shall show next that above bound state can also be constructed by using massive T-duality rules in nine dimensions [7]. These nine-dimensional massive T-duality rules were constructed in order to relate massive type IIA backgrounds with type IIB backgrounds in nine dimensions.

### 2.1 D7-brane and massive duality in \( D = 9 \)

In the case of asymptotically flat branes, in order to construct \((D(p-2),Dp)\) bound states with nontrivial \( B_{\mu \nu} \) field there is a well known procedure described in [13].

According to this method, we need to start with parallel \( D(p-1) \)-branes delocalised along one transverse direction, \( y \) (say), and subsequently make a rotation in a plane involving the isometry direction \( y \) and a spatial direction parallel to the branes. A subsequent application of T-duality along one of the rotated coordinates generates a solution with \( B \)-field. We shall be adopting this method to obtain \((D6,D8)\) brane bound state.

We start with the delocalised D7-branes in type IIB string theory given in [7],

\[
\begin{align*}
    ds^2 &= H^{\frac{1}{2}} \left\{ H^{-1}(-dt^2 + dx_1^2 + \cdots + dx_7^2) + dz^2 + dy^2 \right\}, \\
    e^{2\phi(b)} &= H^{-2}, \quad \chi(0) = m \ y, 
\end{align*}
\]

with the harmonic function \( H(z) = 1 + m|z| \). (We have set \( g_s = 1 \) in this section).

Note that the harmonic function is linear in \( z \), like in a domain-wall, and is continuous at \( z = 0 \) where the brane is localized. However, \( \partial_z H \) is discontinuous at that point. This discontinuity is related to the tension of delocalised D7-branes (or D8-branes after duality). Following [13] our next step would be to make the rotation in \((y, x_7)\) plane,

\[
\begin{pmatrix}
    y \\
    x_7
\end{pmatrix} = \begin{pmatrix}
    \cos \theta & -\sin \theta \\
    \sin \theta & \cos \theta
\end{pmatrix} \begin{pmatrix}
    y_1 \\
    y_2
\end{pmatrix}
\]

\[ (2.3) \]

In these new coordinates the solution (2.2) becomes

\[
\begin{align*}
    ds^2 &= H^{\frac{1}{2}} \left\{ H^{-1}(-dt^2 + \sum_{i=1}^{6} dx_i^2) + \left( \frac{\sin^2 \theta}{H} + \cos^2 \theta \right) dy_1^2 \\
    &\quad + \left( \frac{\cos^2 \theta}{H} + \sin^2 \theta \right) dy_2^2 + \sin 2\theta (H^{-1} - 1) dy_1 dy_2 + dz^2 \right\}, \\
    e^{2\phi(b)} &= H^{-2}, \quad \chi(0) = m \ (\cos \theta y_1 - \sin \theta y_2). 
\end{align*}
\]

\[ (2.4) \]

\[ \text{1Dp/D(p-2) brane bound states have also been worked out in [14].} \]
Now we would like to make T-duality along $y_1$ direction. One will note that neither $y_1$ nor $y_2$ is an isometry direction in the usual sense because the type IIB axion $\chi$ depends linearly on both of them. However, since the field strength $d\chi$ is constant we can make use of generalized (massive) T-duality rules constructed in [7]. Let us identify the direction $y_1$ to be along the circle. This will fix the mass $m_0$ of massive type IIA theory to be given by $m \cos \theta$. Using the duality relations in [7]

$$e^{2\phi(a)} = e^{2\phi(b)}/g_{y_1 y_1}^{(b)}, \quad g_{y_1 y_1}^{(a)} = 1/g_{y_1 y_1}^{(b)}$$

$$B_{y_1 \mu}^{(a)} = -g_{y_1 \mu}^{(b)}/g_{y_1 y_1}^{(b)}, \quad A_{y_1} = -\chi + m_0 y_1$$

we obtain correspondingly a massive type IIA background

$$ds_{10}^2 = H^{1/2} \left\{ H^{-1}(-dt^2 + dx_i^2) + H'^{-1}(dy_1^2 + dy_2^2) + dz^2 \right\},$$

$$e^{2\phi(a)} = H^{-3/2}H'^{-1}, \quad dA_{(1)} = -m \sin \theta dy_1 \wedge dy_2,$$

$$B_{y_1 y_2}^{(a)} = \tan \theta (1 - H'^{-1}),$$

(2.6)

with $H = 1 + m|z|$ and $H' = 1 + \cos^2 \theta (H - 1)$. This is precisely the configuration written in eq.(2.1). Thus we have shown that in two different ways, one as we employed in [8] and the second which we have described in this section, we lead to the same end result. This is nothing but proves the compatibility of the Scherk-Schwarz reductions of massive type IIA supergravity on $T^1$ [7] and on $T^2$ [8] with constant background RR-fluxes.

### 2.2 Supersymmetry

It is presumed that massive T-duality preserves the supersymmetries of the background configurations in the same way as the ordinary T-duality does. Based on this hypothesis we did claim in [8] that (D6,D8) solution preserves 16 supersymmetries since it had been obtained through an SL(2,R) rotation of the D8-brane solution [8]. Let us clarify on the aspects of supersymmetry, we know that massive type IIA does not have any maximally supersymmetric ground state instead the theory admits D8-branes which are half supersymmetric. On the other hand type IIB supergravity does admit maximally supersymmetric Minkowskian ground state and also $1/2$-supersymmetric brane configurations including the D7-branes above. Under the $T^1$ compactification these 1/2-susy backgrounds are mapped from IIB side to the massive IIA side and vice versa [7]. Note that supersymmetries do match on the both sides. Thus from this argument also (D6,D8) bound state obtained from D7-branes in last subsection must have 1/2 supersymmetries. So we would like to make an explicit check of the supersymmetries of the (D6,D8) background in question and provide explicit solution for the Killing spinors.

Let us first write down most general $SL(2,R)$ covariant set of (D6,D8) solutions as

$$ds_{10}^2 = H^{1/2} \left\{ H^{-1}(-dt^2 + dx_i^2) + H'^{-1}(dy_1^2 + dy_2^2) + dz^2 \right\},$$
\[ e^{2\phi} = H^{-\frac{3}{2}}H'^{-1}, \quad dA_{(1)} = -b \, m \, dy_1 \wedge dy_2, \quad B_{y_1y_2} = \frac{1}{d}(b + cH'^{-1}) \quad (2.7) \]

where the harmonic functions are given by

\[ H = 1 + m|z|, \quad H' = c^2 + d^2H, \quad m = m_0/d \quad (2.8) \]

and \( m_0 \) is the mass parameter of the massive type IIA supergravity. The real parameters \( a, b, c, d \) describe an \( SL(2, R) \) matrix \( \begin{pmatrix} a & b \\ c & d \end{pmatrix} \). For the particular choice \( \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} \) the solution (2.7) reduces to the D8-brane and for the case \( \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \) it reduces to the background in (2.1).

The supersymmetric variations of dilatino and gravitino can be obtained from [9] which for our case are (in Einstein metric)

\[ \delta \lambda \equiv -\frac{1}{2\sqrt{2}}\partial_\mu \phi \Gamma^\mu \epsilon - \frac{5}{8\sqrt{2}}m_0 e^{\frac{5}{4}\phi} \epsilon + \frac{3}{16\sqrt{2}}e^{\frac{3}{4}\phi} F_{\mu \nu} \Gamma^{\mu \nu} \Gamma_{11} \epsilon + \frac{1}{24\sqrt{2}}e^{-\frac{1}{2}\phi} H_{\mu \nu \lambda} \Gamma^{\mu \nu \lambda} \Gamma_{11} \epsilon \]

\[ \delta \psi_\mu \equiv D_\mu \epsilon - \frac{1}{32} m_0 e^{\frac{3}{4}\phi} \Gamma_\mu \epsilon - \frac{1}{64} e^{\frac{3}{4}\phi} F_{\nu \lambda} (\Gamma_\mu^{\nu} - 14\delta_\mu^{\nu} \Gamma^{\nu \lambda}) \Gamma_{11} \epsilon + \frac{1}{48} e^{-\frac{1}{2}\phi} H_{\mu \nu \lambda} (\Gamma^{\mu \nu \lambda} - 9\delta_\mu^{\nu} \Gamma^{\nu \lambda}) \Gamma_{11} \epsilon \quad (2.9) \]

where \( F_{(2)} = dA_{(1)} + m_0 B_{(2)} \) and \( H_{(3)} = dB \) and \( \Gamma_{11} \) is the chirality operator in ten dimensions. The Killing spinors are those solutions for which these variations vanish. The dilatino equation \( \delta \lambda = 0 \) for the background (2.7) simplifies to

\[ [(c \gamma_z \gamma_{y_1} \gamma_{y_2} \Gamma_{11} + (H')^{\frac{1}{2}}) - d \, H^{\frac{1}{2}} \gamma_z] \epsilon = 0 \quad (2.10) \]

where all small \( \gamma \) matrices are constant 10-dimensional gamma matrices and their indices are raised and lowered with the tangent space metric. To find a solution of (2.10) let us make an ansatz

\[ \epsilon = f \epsilon_0^+ + g \epsilon_0^- \quad (2.11) \]

where \( \epsilon_0^\pm = \frac{1 + \gamma}{2} \epsilon_0 \) with \( \gamma = \gamma_z \gamma_{y_1} \gamma_{y_2} \Gamma_{11} \). The constant spinors \( \epsilon_0 \) satisfy the condition \( \gamma_z \epsilon_0 = \epsilon_0 \). Note that this projects out 16 spinors out of 32 constant spinors and thus eventually breaks half of supersymmetries. Substituting the ansatz (2.11) in (2.10) gives us the following relation between \( f \) and \( g \),

\[ f = \frac{d\sqrt{H}}{c + \sqrt{H'}} g \quad (2.12) \]

in terms of which \( \epsilon = g \left( \frac{d\sqrt{H}}{c + \sqrt{H'}} \epsilon_0^+ + \epsilon_0^- \right) \). The overall function \( g \) can be determined by gravitino variations. Consider the equation \( \delta \psi_z = 0 \), this implies that \( g \) must satisfy

\[ \partial_z g - \frac{m}{32} \left( \frac{1}{H} + \frac{6c^2}{H H'} - \frac{8c}{H \sqrt{H'}} \right) g = 0 \quad (2.13) \]
Now taking $g$ to be of the form $g \equiv H^p H^q (c + \sqrt{H'})^r$ and substituting it in the eq. (2.13) we find that $p, q, r$ have a unique solution

$$p = -\frac{1}{32}, \quad q = -\frac{3}{16}, \quad r = \frac{1}{2}.$$  

Thus the Killing spinors for the (D6,D8) background are

$$\epsilon = H^{-\frac{1}{2}} H^{\frac{14}{16}} \left[ \left(1 - \frac{c}{\sqrt{H'}}\right)^{\frac{1}{2}} \epsilon_0^+ + \left(1 + \frac{c}{\sqrt{H'}}\right)^{\frac{1}{2}} \epsilon_0^- \right].$$  

(2.14)

It could be checked that all other Killing equations are satisfied by this solution. When $c = 0$ eq.(2.7) becomes a D8-brane background and eq.(2.14) also reduces to standard Killing spinors for these branes. In summary, we have proved that the (D6,D8) bound state preserves 16 supercharges same as D8-branes. Thus the action of massive duality rotations on the backgrounds do not break supersymmetries of the backgrounds. It also indirectly means that massive type II theories obtained through generalized Scherk-Schwarz reduction of type II supergravities compactified on $T^2$ are maximal supergravity theories.

2.3 Massive $B$-field

Let us briefly discuss the massive nature of the 2-rank tensor field $B$ in the background (2.1). As it can be seen from the 2-form field strength $F_{(2)} = dA + m_0 B$ and the 3-form field strength $H_{(3)} = dB$ that there is a (stueckelberg) gauge invariance through which $B$-field can eat one-form $A$ and become massive. Let us define $B' = B + \frac{1}{m_0} dA$ and replace everywhere in the action $F_{(2)} \equiv m_0 B'$ while $H = dB'$. Under this gauge fixing background (2.1) can be reexpressed as

$$ds_{10}^2 = H^{1/2} \left\{ H^{-1} (-dt^2 + dx_i^2) + H'^{-1} (dy_1^2 + dy_2^2) + dz^2 \right\},$$

$$e^{2\phi} = g_s^2 H^{-\frac{5}{2}} H'^{-1}, \quad B' = -\frac{\tan \theta}{H'} dy_1 \wedge dy_2,$$

with $H = 1 + m|z|$, $H' = 1 + \cos^2 \theta (H - 1)$ and $m = m_0 g_s / \cos \theta$. Here $B'$ field is explicitly massive with mass being $m_0$.

Nevertheless background in (2.13) is half-supersymmetric.

The scalar curvature for above background metric is

$$R = -\frac{14m'^2 + 2mm' (5 + 19m'|z|) + m^2 (21 + 52m'|z| + 45m'^2|z|^2)}{4H^{\frac{5}{2}} H'^2}$$

(2.16)

where we have defined $m' = m \cos^2 \theta$. This result will be used in the next section. When $\theta = 0$, $m'$ becomes equal to $m$ and the expression in eq.(2.16) reduces to the curvature for pure D8-brane background.

\footnote{Although it is difficult to define a mass in the domain-wall (curved) backgrounds. Here mass means that field has $m$ term in the action, see the Appendix.}
So far we chose to keep plus sign in the harmonic function of the type $H = 1 \pm m|z|$ although solutions exist with both the signs. It has been found in [15] that for D8-branes with +ve tension $^3$, a lower sign in the harmonic function $H = 1 \pm m|z|$ is favored. If we use a negative sign in the harmonic function $H$ in (2.13), it follows that as we go far away from the 8-branes not only string coupling but also B-field diverges. Thus such D8-brane configurations cannot be defined independently and far away from the orientifold 8-planes [15]. Near the O8-planes the above geometry has to take into account the back reaction from the orientifolds also and the geometry will be appropriately modified. In terms of string quantities $g_s$ and $\alpha'$, the type IIA mass parameter $m_0$ can suitably be expressed as $m_0 = \frac{c_8 N}{\sqrt{\alpha'}}$, with $c_8$ being an appropriate combinatoric factor and $N$ the number of D8-branes.

### 3. Non-commutative Field Theories

We are now ready to formulate a discussion in the field theory direction. It is well known fact that in the Seiberg-Witten limit ($\alpha' \to 0$) the closed string backgrounds describe holographic dual picture of the boundary conformal field theories (CFTs) in various brane pictures. Precisely, a CFT defined on the boundary of an anti-de Sitter (AdS) spacetime is holographic dual to the gravity (string) theory in the bulk which constitutes the AdS space [2]. Near horizon limits of various brane solutions in (M-)string theories give rise to AdS spacetimes. We would like to see whether the same picture of AdS/CFT emerges in the case of D8-branes also. Since D8-branes are not asymptotically flat we have to be careful.

#### 3.1 No $B$ field

We consider O8-D8 combination so we are eventually in type $I'$ picture [15]. We will shall first consider the case without $B$-field. Let us consider $N$ ($N < 8$) positive tension D8-branes situated at one of the orientifold plane. Including the backreaction of the O8-plane the background geometry for $N$ D8-branes can be written as (i.e. with an effective mass parameter $m_0 = \frac{c_8 (8-N)}{\sqrt{\alpha'}}$)

\[
ds^2_{10} = H^{\frac{1}{2}} \left\{ H^{-1} (dt^2 + dx_1^2 + \cdots + dx_8^2) + dz^2 \right\},
\]
\[e^{2\phi} = g_s^2 H^{-\frac{1}{2}}, \quad H = 1 + \frac{c_8 (8-N) g_s}{\sqrt{\alpha'}} |z|.
\] (3.1)

Thus so long as $N < 8$ we can consider the following decoupling limit, in analogy with other D$p$-branes [17],

\[
\alpha' \to 0, \quad |z| \to \alpha' u, \quad g_s \to \tilde{g}(\alpha')^{-\frac{1}{2}}, \quad g_{YM}^2 \tilde{N} = fixed,
\] (3.2)

$^3$Tension of D$p$-brane is defined as $T_p \sim \frac{1}{g_s (\alpha')^\frac{p+1}{2}}$.
where various parameters $\tilde{g}(= g_{YM}^2), \tilde{N}$ and the energy scale $u$ (the expectation value of the Higgs) are kept fixed. We are using notation $\tilde{N} = c_8(8 - N)$ in order to distinguish it from $N$, the number of parallel D8-branes. Note that D8 background is not asymptotically flat nevertheless we shall implement above scaling limit. Under this limit $H \sim \tilde{g}\tilde{N}u/\alpha'$ and (3.1) becomes

$$ds^2 \sim \alpha' \sqrt{\tilde{g}\tilde{N}u^5} \left( -dt^2 + dx_1^2 + \cdots + dx_8^2 + \frac{du^2}{u^2} \right),$$

$$e^{2\phi} \sim \frac{1}{N^2 g_{\text{eff}}},$$

where effective super-Yang-Mills coupling at the scale $u$ is defined as $g_{\text{eff}}^2 = \tilde{g}\tilde{N}u^5$. It can be seen that the expression within angular brackets on the r.h.s. of (3.3) is a space-time filling AdS$_{10}$ geometry. Therefore we can discuss holographic field theory on the nine-dimensional boundary of AdS spacetime in this decoupling limit. The background does not have transverse isometries, so the boundary field theory in nine dimensions would be $\mathcal{N} = 1$ super-Yang-Mills theory with a gauge group $SO(2N)$ for all $N < 8$. There is no R-symmetry in the gauge theory because the D8 background has only one transverse direction. From eq.(2.16) we find that the curvature scalar in string units is given by

$$-\alpha'R = \frac{21}{4} \sqrt{\frac{1}{\tilde{g}\tilde{N}u^5}} = \frac{21}{4} \frac{1}{g_{\text{eff}}}. $$

Thus in the IR region, $u^5 \ll \frac{1}{\tilde{g}\tilde{N}}$, where $g_{\text{eff}}^2 \ll 1$, super Yang-Mills description holds good. But in this region the curvature and string coupling are both large and the supergravity is not a valid description. While in the UV region, $u^5 \gg \frac{1}{\tilde{g}\tilde{N}}$, curvature and string coupling are small and low energy sugra is a valid description. It is useful since in UV region $g_{\text{eff}} \gg 1$ and the field theory breaks down at some point. Since field theories in $D > 4$ show bad UV behaviour, it is useful that supergravity can make sense out there. However the ten-dimensional Newton’s constant $G_N^{10} \sim g_{\text{eff}}^2 \alpha'^4$ goes as $1/\alpha'$ in the decoupling limit and thus blows up. Which is some what contrary to what one expects in the decoupling limits. We will see next that Newton’s constant indeed vanishes if $B$-field is present.

### 3.2 $B \neq 0$

Now we go over to the case of D8-branes where $B$-field is present. The background in discussion here is given in eq.(2.15). Again the background is not asymptotically flat but we will insist that the background variations are small enough locally so that we

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4 The decoupled background in (3.3), which is conformally AdS$_{10}$ type, is nevertheless a solution of massive type IIA supergravity with an effective mass parameter $m_0 = \tilde{N}/\sqrt{\alpha'}$, for any value of $\alpha'$. 

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can implement the decoupling limit. In this case the decoupling limits are slightly modified as

\[
\begin{align*}
\alpha' &\to 0, \quad g_s \to \tilde{g}(\alpha')^{2/3}, \quad g_{YM}^2 \tilde{N} = \text{fixed}, \\
|z| &\to \alpha'u, \quad \cos \theta \to \frac{\alpha'}{b}, \quad (y_1, y_2) \to \frac{\alpha'}{b}(\tilde{y}_1, \tilde{y}_2).
\end{align*}
\] (3.4)

Various parameters \(\tilde{g}, u, b, \tilde{N}\) are held fixed and \(b\) is the noncommutativity parameter. Under these limits the harmonic functions in (2.15) become (with \(m_0 = \frac{c_8(8-N)}{\sqrt{\alpha'}}\))

\[
H \sim \frac{\tilde{g}\tilde{N}bu}{\alpha'^2}, \quad H' = 1 + \frac{\tilde{g}\tilde{N}u}{b}
\] (3.5)

and

\[
ds^2 \sim \alpha' \sqrt{\tilde{g}b\tilde{N}u^5} \left[ \frac{1}{\tilde{g}b\tilde{N}u^3} \left( -dt^2 + \sum_{i=1}^{6} dx_i^2 + (1 + \frac{b}{\tilde{g}b\tilde{N}u})^{-1}(d\tilde{y}_1^2 + d\tilde{y}_2^2) \right) + \frac{du^2}{u^2} \right],
\]

\[
e^{2\phi} \sim \frac{\tilde{g}^2}{(\tilde{g}b\tilde{N}u)^{1/2}} H', \quad B' = \frac{\alpha'}{b}(1 + \frac{\tilde{g}\tilde{N}u}{b})^{-1}d\tilde{y}_1d\tilde{y}_2
\] (3.6)

where effective gauge coupling of nine-dimensional NCYM at the scale \(u\) is defined as \(g_{eff}^2 = \tilde{g}b\tilde{N}u^5\). Note that the ten-dimensional Newton’s constant \(G_N^{10}\) is of the order of \(\alpha'\) and vanishes in the limit \(\alpha' \to 0\). This is a sign that closed strings indeed get decoupled in the \(\alpha' \to 0\) limit when \(B\)-field is present. Note from (3.6) that after the decoupling limit massive \(B'\) field precisely behaves as in 4-dimensional NCYM theories [10–12].

The decoupling limits (3.4) and the decoupled geometry (3.6) do indicate that there is a dual NCYM theory, but where does this NCYM live? In the type-I’ theory there are two orientifold fixed planes, one at \(z = 0\) and other at \(z = \pi\), and 16 D8-branes are sandwiched between these two fixed points. At the fixed points the NS-NS \(B\)-field vanishes, as it can be seen from eq.(2.1) also. Therefore, there cannot be any noncommutativity if we place \(N\) D8-branes right at the fixed point \(z = 0\) and rest \((16-N)\) at the other fixed point. However if we place \(N\) D8-branes at some finite distance away, say at \(z = z_0\), there is a nonvanishing \(B\)-field background there.\(^5\) Note that such a \(z\)-dependent \(B\)-field along the world-volume directions of the D-brane is not projected out under orientifolding in type I’ theory and nor the constant RR 2-form ’flux’ in (2.1) [16].\(^6\) Under the scaling limit (3.4), \(z_0 \to \alpha'\lambda\) and \((z - z_0) \to \alpha'u\), where \(\lambda\) would act like a IR cutoff in the YM theory and it will measure the separation between O8-plane and \(N\) D8-branes.\(^7\) While dealing

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\(^5\)The harmonic function \(H(z)\) in (2.13) and (2.15) would become \(H = 1 + \frac{c}{\alpha'}z_0 + \frac{c_8(8-N)}{\sqrt{\alpha'}}(z - z_0)\) in the region \(z_0 < z < \pi\) since cosmological constant \(m_0\) jumps between the branes.

\(^6\)I thank Ashoke for discussion on these aspects.

\(^7\)The actual expression for \(g_{eff}^2 \sim \tilde{g}b(1 + \frac{8}{(8-N)\alpha})u^5\). Effectively speaking \(\lambda\) would become a UV cutoff for the case when \(N > 8\).
with the decoupling limits (3.4), we have ignored $\lambda$ assuming that it is infinitesimal. The resultant background written in (3.6) should be seen from that perspective. These D8-branes under the decoupling limit will be described by a non-commutative Yang-Mills with gauge group $SU(N)$ with $N < 8$.

Now, it is easy to see from eq.(3.6) that only when $u \gg b_{\tilde{g}}$ (i.e. $b^2 u^4 \ll g_{\text{eff}}^2$), the geometry in (3.6) becomes conformally AdS$_{10}$ and the whole background reduces to that in (3.3). Thus we have a commutative phase in YM theory in the UV region. But in the UV region effective gauge coupling is large, so field theory is not quite well defined. However, the dual supergravity description holds good since string coupling and the curvature

$$e^{2\phi_{UV}} \sim \frac{1}{N^2 g_{\text{eff}}}, \quad -\alpha'R_{UV} = \frac{45}{4} \frac{1}{g_{\text{eff}}}$$

both are small in UV region.

Let us go to the IR region where $u \ll \frac{b}{\tilde{g}N}$ (i.e. $b^2 u^4 \gg g_{\text{eff}}^2$). We have a non-commutative phase where the coordinates $\tilde{y}_1, \tilde{y}_2$ are noncommutative; i.e. $[\tilde{y}_1, \tilde{y}_2] \sim b$. In IR region string coupling and curvature are given by

$$e^{2\phi_{IR}} \sim \frac{g_{\text{eff}}}{N^2 b^2 u^4}, \quad -\alpha'R_{IR} = \frac{21}{4} \frac{1}{g_{\text{eff}}}.$$ (3.8)

In the IR region where $b^2 u^4 \gg g_{\text{eff}}^2 \gg 1$ (i.e. $\frac{1}{(\tilde{g}N)^2} \ll u \ll \frac{b}{\tilde{g}N}$) the string coupling and curvature are small and sugra description holds good. This region can be approached if parameters are chosen such that $b^2 \gg \tilde{g}N$. Since $g_{\text{eff}}^2 \gg 1$ the NCYM is strongly coupled. Further towards the lower IR region $u \ll \frac{1}{(\tilde{g}N)^2}$ and into deep IR region, both the string quantities are large, but the field theory description is perturbatively well defined due to the weak gauge coupling, $g_{\text{eff}}^2 \ll 1$.

In the strong string coupling region type IIA brane systems are well described only in an appropriate M-theory picture. Note that (D6,D8) background is a solution of Romans’ theory which has no straightforward M-theory relationship, see [8,19,20]. We shall describe next a possible way to go to M-theory side based on the approach in [8].

### 3.3 (M5,KK) bound state

The M-theory background can be obtained by mapping (D6,D8) solution first to (D4,D6) solution of type IIA supergravity in the following way. We start with (D6,D8) bound state (2.1) and compactify two coordinates, $x_5, x_6$, on a $T^2$. Then we follow it up with an SL(2,R) rotation \(
\begin{pmatrix}
0 & -1 \\
1 & 0
\end{pmatrix}
\) . Up-lifting the rotated 8-dimensional configuration back to ten dimensions (using the rules described in [8]) would give us...
following (D4,D6) configuration of type IIA

\[
ds_{10}^2 = H^{\frac{\beta}{2}} \left\{ H^{-1}(-dt^2 + \sum_{i=1}^{4} dx_i^2) + H'^{-1}(dy_1^2 + dy_2^2) + (dz^2 + dx_5^2 + dx_6^2) \right\},
\]

\[e^{2\phi} = g_s^2 H^{-\frac{\beta}{2}} H'^{-1}, \quad dC_{(3)} = -\frac{m \sin \theta}{g_s} dy_1 \wedge dy_2 \wedge dx_5 \wedge dx_6,
\]

\[dA_{(1)} = \frac{m \cos \theta}{g_s} dx_5 \wedge dx_6, \quad B_{(2)} = \tan \theta (1 - H'^{-1}) dy_1 \wedge dy_2 \]

with harmonic functions \( H = 1 + m|z|, \quad H' = 1 + \cos^2 \theta (H - 1) \). The parameter \( m \) will be appropriately related to the relevant stringy quantities. This type IIA configuration is delocalized (smeared) over transverse \( x_5, x_6 \) \( T^2 \)-plane. We can now easily lift this type IIA background to M-theory solution

\[
ds_{11}^2 = e^{4\phi/3}(dx_{11} + A_{(1)})^2 + e^{-2\phi/3} ds_{10}^2
\]

\[= \left( \frac{H}{H'} \right)^{\frac{2}{3}} \left\{ -dt^2 + \sum_{i=1}^{4} dx_i^2 \right\} \left( H')^{-1}[dx_{11} + \frac{m}{2} \cos \theta (x_5 dx_6 - x_6 dx_5)]^2 \right\}
\]

\[+ \left( \frac{H}{H'} \right)^{\frac{2}{3}} (dy_1^2 + dy_2^2 + H'(dz^2 + dx_5^2 + dx_6^2)),
\]

\[G_{(4)} \equiv dC_{(3)} = -m \sin \theta dy_1 \wedge dy_2 \wedge dx_5 \wedge dx_6 + \frac{m \cos \theta \sin \theta}{H^2} dz \wedge dx_{11} \wedge dy_1 \wedge dy_2 \]

with \( H = 1 + m|z|, \quad H' = 1 + \cos^2 \theta (H - 1) \), where \( m \) is now related to the M-theory quantities as \( m \sim \frac{N_5}{a_x a_y} \). This solution represents a bound state system of M5-brane and Kaluza-Klein (Taub-NUT) monopoles and is smeared over two transverse \( T^2 \)-s. Coordinates \( t, x_1, \ldots, x_4, x_{11} \) are along M5-branes while \( C_{\mu \nu \lambda} \)-field is along \( x_{11}, y_1, y_2 \) which is responsible for having Taub-NUT (TN) charges in this background. When \( \theta = 0 \) in (3.10) the background reduces to \( TN \times Mink_7 \) [8]. If we set \( \theta = \pi / 2 \) then solution reduces to pure M5-branes with G-flux over \( T^2 \times T^2 \).

It should be clear that solution (3.10) represents an equivalent M-theory background for (D6,D8) solution with \( B \)-field. It is rather appropriate to discuss decoupling limits of this solution when string coupling becomes large. Corresponding scaling limits for (3.10) when \( \alpha' \to 0 \) can be determined and these are

\[
\left\frac{|z|}{\alpha'} = u = \text{fixed}, \quad l_p \to (\alpha')^{1/3}, \quad R_{11} = N_5 = \text{Fixed}
\]

\[\cos \theta \to \frac{\alpha'}{b}, \quad a_x \to \alpha'^2 a_x, \quad a_y \to \alpha'^2 a_y \]

(3.11)

\[8\]The radius, \( R_{11}, \) of the circle coordinate \( x_{11} \) is related to the string coupling as \( R_{11} = e^{2\phi/3} l_p \) and 11-dimensional Planck length as \( l_5^2 = g_s^{2/3} \alpha' \). \( N_5 \) is the number of M5-branes , \( a_x \) and \( a_y \) are related to the sizes of the two transverse \( T^2 \)-s, \( x_5, x_6 \) and \( y_1, y_2 \) respectively.
with \( \tilde{a}_x \) and \( \tilde{a}_y \) are fixed area parameters. Note that the areas of transverse \( T^2 \)-s also shrink to zero under this scaling. It can be checked that the background (3.10) indeed gets decoupled in the limit (3.11). So in the IR region where the size of eleventh dimension measured in Planck units \( R_{11}(u)/l_p = e^{2\phi/3} \) becomes large it is useful to study above decoupling limits where \( l_p \to 0 \). The corresponding boundary field theory would be a non-local 6D (0,2) SCFT on a circle [17]. The nonlocality arises due to the presence of Taub-NUT charges in the M5-brane solutions.\(^9\) Let us note down the curvature of the 11-dimensional spacetime measured in the Planck units in the IR region (using eq. (3.8))

\[
 l_p^2 R \sim e^{2\phi/3}(\alpha' R) \approx \left( \frac{1}{N^2 g_s^2 b^2 u^4} \right)^{1/3}.
\]

(3.12)

The eleven-dimensional curvature measured in Planck units is still large when \( u \to 0 \). Therefore this low energy supergravity description will not be reliable as corresponding (M5,KK) backgrounds would receive higher curvature corrections. But as we saw NCYM and the CFT theories in this region are weakly coupled and can make a good description.

4. (D4,D6,D8) bound state

It is desirable to obtain D8-branes with \( B \)-field of higher rank. To obtain such solutions we can apply the same method described in section-5 of [8] which led to the construction of (D6,D8) solution. We start with (D6,D8) bound state (2.1) and compactify two coordinates, \( x_5, x_6 \), on \( T^2 \). Then follow it up with an SL(2,R) rotation \((\begin{pmatrix} \cos \psi & \sin \psi \\ -\sin \psi & \cos \psi \end{pmatrix})\). Up-lifting the rotated 8-dimensional configuration to ten dimensions (using the rules described in [8]) would give us following new configuration of massive type IIA supergravity,

\[
 ds_{10}^2 = H^{1/2} \left\{ H^{-1}(-dt^2 + \sum_{i=1}^{4} dx_i^2) + f^{-1}(dx_5^2 + dx_6^2) + H'^{-1}(dy_1^2 + dy_2^2) + dz^2 \right\},
\]

\[
 e^{2\phi} = g_s^2 H^{-1/2} f^{-1} H'^{-1}, \quad dC(3) = \frac{m \sin \psi \sin \theta}{g_s} dy_1 \wedge dy_2 \wedge dx_5 \wedge dx_6,
\]

\[
 dA_{(1)} = -\frac{m \cos \psi \sin \theta}{g_s} dy_1 \wedge dy_2 - \frac{m \cos \theta \sin \psi}{g_s} dx_5 \wedge dx_6,
\]

\[
 B_{(2)} = \tan \theta (1 - H'^{-1}) dy_1 \wedge dy_2 + \tan \psi (1 - f^{-1}) dx_5 \wedge dx_6
\]

(4.1)

with harmonic functions \( H = 1 + m|z| \), \( H' = 1 + \cos^2 \theta (H - 1) \) and \( f = 1 + \cos^2 \psi (H - 1) \). Here parameter \( m = \frac{m_0 g_s}{\cos \psi \cos \theta} \) and as usual \( m_0 \) denotes the mass.

\(^9\)See [18] for nonlocal 6-dimensional field theories.
(cosmological constant) of the massive type IIA supergravity. This solution has sixteen supersymmetries and can be described as a bound state of D4, D6 and D8-branes as corresponding magnetic charges are present in this solution. Note that the $B$-field in the above solution has rank four while in the (D6,D8) solution it had rank two only. One may also describe NCYM decoupling limits for this bound state as well, similar to the case of (D6,D8) solution, but we simply do not attempt it here.

5. Summary

In this paper we have shown that the (D6,D8) bound state [8] with $B$ field can also be obtained by using T-duality map between massive-type-IIA supergravity and type-IIB supergravity in $D = 9$ [7]. We have also explicitly written down the Killing spinors which are preserved by this bound state configuration. We find that though $B$-field is explicitly massive the (D6,D8) background preserves 16 supersymmetries.

We have then studied Yang-Mills decoupling limits and have discussed the behaviour of field theories at various energy scales. We are surprised to note that these 9-dimensional super-Yang-Mills theories with maximal supersymmetries are non-commutative in the IR region while they become commutative in UV region. This is quite opposite to what we observe in the case of NCYMs in four dimensions where non-commutativity appears only in the UV region and it disappears as we go to IR region and the theories become ordinary super-Yang-Mills. On one hand this may not surprise us so much as we know that noncommutative field theories any way show UV/IR mixing.

Thus the appearance of non-commutativity as we go to IR region is some what very peculiar feature of the nine-dimensional NCYMs presented here. We could not understand this unusual behaviour of the $D = 9$ NCYMs, nevertheless we are able to expose this property simply by studying the decoupling limits involving D8-branes with $B$-field. From section-3 we note that there is a decreasing jump in the spacetime curvature as we move from the IR region to UV region of dual NCYM theory. Since $g^{UV}_{eff} \gg g^{IR}_{eff}$, the AdS curvature is more in the IR region as compared to the UV region. This would mean the NCYM theory flows from higher curvature (weak gauge coupling) IR region to a smaller curvature (strong gauge coupling) UV point. It is not unusual to have such a flow, the gauge theories already in five dimensions flow to strong coupling ($g_{YM} = \infty$) UV fixed point [21] where gauge symmetry enhancement takes place. There the symmetries are enhanced to exceptional groups $E_{N+1}$. These gauge groups could be any $E_8$, $E_7$, $E_6$, $E_5 = Spin(10)$, $E_4 = SU(5)$, $E_3 = SU(3) \times SU(2)$, $E_2 = SU(2) \times U(1)$ and $E_1 = SU(2)$ depending upon the number, $N$, of D8-branes present at the orientifold. Therefore in UV region, the 9D NCYMs must flow to these enhanced symmetry fixed points where commutativity is also restored.

Finally, we note that in a recent paper [5] it has been observed, that for non-constant (but slowly varying) closed string backgrounds, $g_{\mu\nu}$, $B_{\mu\nu}$, the Seiberg-
Witten relations give rise to open string metric $G$ and noncommutativity parameter $\theta$ which are spacetime dependent. For constant $g$ and $B$ the open string metric and noncommutativity parameter are however constant as well as the Yang-Mills coupling. We have not tried to obtain these open string quantities for D8-branes with $B$ field. In our case both $g$ and $B$, however, depend on the holographic coordinate $z$ itself. We do expect, in general, open string metric and non-commutativity parameter also to be dependent on $z$ (i.e. $u$). Lastly, since $z$ is a coordinate transverse to the brane directions the Moyal star product $f \ast g$ should be well defined locally (at any given position $z = z_0$ of the boundary). It will also be associative. The $z$-dependence is probably an indication of the fact that nine-dimensional NCYMs are nonrenormalizable and heavily cut-off dependent.

Acknowledgments

I would like to thank D. Ghosal, R. Gopakumar, D. Jatkar and A. Sen for helpful discussions. This work commenced when I was a member of the theory group at Fachbereich Physik, Martin-Luther-Universität, Halle (Germany) for which I would like to thank Jan Louis for great hospitality and for providing enriching environment for research.

A. Romans’ type IIA supergravity

The 10-dimensional type IIA supergravity, which describes the low energy limit of type IIA superstrings, contains in the massless bosonic spectrum the graviton $g_{MN}$, the dilaton $\phi$, NS-NS two-form $B_{(2)}$, a R-R one-form $A_{(1)}$ and a R-R three-form $C_{(3)}$. The fermionic sector consists of two gravitini and two Majorana $\frac{1}{2}$-spinors. The Romans’ supergravity theory [9] is a generalization of the type IIA supergravity to include a mass term for the NS-NS $B$-field without disturbing the supersymmetry content of the theory. The bosonic action for Romans’ theory in the string frame can be written as (after some rescalings)\(^{10}\)

\[
S = \int \left[ e^{-2\phi} \left\{ R \ast 1 + 4d\phi \ast d\phi - \frac{1}{2} H_{(3)} \ast H_{(3)} \right\} - \frac{1}{2} F_{(2)} \ast F_{(2)} - \frac{1}{2} F_{(4)} \ast F_{(4)} - \frac{m_0^2}{2} \ast 1 \\
+ \frac{1}{2} dC_{(3)} dC_{(3)} B_{(2)} + \frac{1}{2} dC_{(3)} dA_{(1)} B_{(2)}^2 + \frac{1}{3!} dA_{(1)} dA_{(1)} B_{(2)}^3 + \frac{1}{3!} m_0 dC_{(3)} B_{(2)}^3 \\
+ \frac{1}{8} m_0 dA_{(1)} B_{(2)}^4 + \frac{1}{40} m_0^2 B_{(2)}^5 \right],
\]

\(^{10}\)Our conventions are same as in [20] where every product of forms is understood to be a wedge product. We denote a $p$-form with a lower index like $(p).
where \( m_0 \) is the mass parameter. The field strengths in the action \((A.1)\) are given by

\[
H^{(3)} = dB^{(2)} , \quad F^{(2)} = dA^{(1)} + m_0 B^{(2)} , \quad F^{(4)} = dC^{(3)} + B^{(2)} dA^{(1)} + \frac{m_0}{2} B^{(2)} .
\]

(A.2)

Note that potentials \( A \) and \( C \) appear only through their derivatives in the action \((A.1)\) and thus obey the standard \( p \)-form gauge invariance \( A^{(p)} \rightarrow A^{(p)} + d\lambda^{(p-1)} \).

The two-form \( B \) on the other hand also appears without derivatives but nevertheless the ‘Stueckelberg’ gauge transformation

\[
\delta A = -m_0 \lambda^{(1)} , \quad \delta B = d\lambda^{(1)} , \quad \delta C = -\lambda^{(1)} dA
\]

leaves the action invariant.

Now, if we define, \( dA + m_0 B = m_0 B' \), \( C'_3 = C_3 - \frac{1}{2m_0} A dA \) then \( H = dB' \), \( F^{(4)} = dC' + \frac{m_0}{2} B'B' \). The above action reduces to

\[
S = \int \left[ e^{-2\phi} \left\{ R \ast 1 + 4 d\phi \ast d\phi - \frac{1}{2} H^{(3)} \ast H^{(3)} \right\} - \frac{1}{2} m_0^2 B' \ast B' - \frac{1}{2} F^{(4)} \ast F^{(4)} - \frac{m_0^2}{2} \right] + \frac{1}{2} dC'_3 dC'_3 B'_3 + \frac{1}{3!} m_0 dC'_3 (B'_3) B'_3 + \frac{1}{40} m_0^2 (B'_3)^5 \right) ,
\]

(A.4)

For the kind of backgrounds in \((2.13)\) for which \( B' \wedge B' = 0 \), \( C' = 0 \) above action reduces to (with fermionic backgrounds vanishing)

\[
S = \int \left[ e^{-2\phi} \left\{ R \ast 1 + 4 d\phi \ast d\phi - \frac{1}{2} H^{(3)} \ast H^{(3)} \right\} - \frac{1}{2} m_0^2 B' \ast B' - \frac{m_0^2}{2} \right] \]

(A.5)

which involves an explicit mass term for \( B' \) field and a cosmological constant term.
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