Diffractive $J/\psi$ production as a probe of the gluon component in the Pomeron

Feng Yuan

Department of Physics, Peking University, Beijing 100871, People’s Republic of China

Kuang-Ta Chao

China Center of Advanced Science and Technology (World Laboratory), Beijing 100080, People’s Republic of China

and Department of Physics, Peking University, Beijing 100871, People’s Republic of China

Abstract

Presented here is a study of the large $p_T$ $J/\psi$ production in hard diffractive process by the pomeron exchange at the Fermilab Tevatron. We find that this process ($p\bar{p} \rightarrow p + J/\psi + X$) can be used to probe the gluon content of the pomeron and to measure the gluon fraction of the pomeron. And the diffractive direct $J/\psi$ production can also provide another crucial test for the color-octet fragmentation mechanism. Using the renormalised pomeron flux factor $D \approx 1/9$, the single diffractive $J/\psi$ production cross section at large $p_T$ ($\geq 8\text{GeV}$) is found to be of order of 0.01nb, and the ratio of the single diffractive to the non-diffractive $J/\psi$ production is $0.65 \pm 0.15\%$ for the gluon fraction $f_g = 0.7 \pm 0.2$.

PACS number(s): 12.40.Nn, 13.85.Ni, 14.40.Gx
In the past few years there has been a renaissance of interest in diffractive scattering. Diffractive processes in hadron collisions are well described by the Regge theory in terms of the pomeron (P) exchange [1,2]. The pomeron carries quantum numbers of the vacuum, so it is a colorless entity in QCD language, which may lead to the “rapidity gap” events in experiments. However, the nature of pomeron and its reaction with hadrons remain a mystery. In [3], the hard diffractive scattering processes have been suggested to resolve the quark and gluon content in the pomeron. That is to say, the pomeron has partonic structure, just as hadrons and nuclei. Therefore, various processes may be considered to probe the partonic structure of the pomeron at high energy colliders [3–6].

On the experimental side, the UA8 collaboration at CERN Sp¯pS collider have studied diffractive dijet production at $\sqrt{s} = 630$ GeV [7], which indicate a dominant hard partonic structure of the pomeron. The H1 and ZEUS have studied the diffractive deep inelastic scattering (DDIS) and dijet photoproduction in ep collision at $\sqrt{s} = 300$ GeV [8]. From these measurements, ZEUS determined that the gluon fraction of the pomeron $f_g$ is in the range of $0.3 < f_g < 0.8$, and H1 determined the quark fraction of the pomeron $f_q \approx 0.2$. The partonic structure of the pomeron is also studied recently by the CDF collaboration at the Fermilab Tevatron through the diffractive W production [9] and dijet production [10], which give further evidence for the hard partonic structure of the pomeron. The combination of these two measurements determined the gluon fraction of the pomeron to be $0.7 \pm 0.4$.

In this paper, we will discuss another diffractive process, the single diffractive (SD) J/ψ production at large $p_T$, (shown in Fig.1),

$$p + \bar{p} \rightarrow p + J/\psi + X.$$  

By the following calculations, we will show that the SD J/ψ production is sensitive to the gluon fraction of the pomeron. So, the measurement of diffractive J/ψ production at the Fermilab Tevatron would provide a probe of the gluon distribution in the pomeron. Importantly, at hadron colliders the J/ψ production is of special significance because it has extremely clean signature through its leptonic decay modes. Furthermore, the SD J/ψ
production is also interesting to the study of heavy quarkonium production mechanism, which is another hot topic in the past few years.

For a long time, it was believed that the heavy quarkonium production at large $p_T$ dominantly comes from the leading order color-singlet processes [11]. But, as pointed out by Braaten and Yuan [12], the fragmentation contributions may dominate over those from leading-order processes at sufficiently large $p_T$, although the fragmentation processes are of higher order in strong coupling constant $\alpha_s$. However, the measurements of large $p_T$ charmonia production from the CDF at the Tevatron show a large excess of direct production (excluding the contribution from $b$ decays and the feeddown from $\chi_c$) both for $J/\psi$ and $\psi'$ [13] [14]. The experimental measurement is a factor of $30 \sim 50$ larger than the theoretical prediction of the Color-Singlet Model even if including the fragmentation contributions. Motivated by this “surplus” problem, a new mechanism for heavy quarkonium production at large $p_T$ in hadronic collisions, named as Color-Octet gluon fragmentation has been proposed [15], which is based on the factorisation formalism of non-relativistic quantum chromodynamics (NRQCD) [16]. In the past few years, applications of the NRQCD factorisation formalism to $J/\psi(\psi')$ production at various experimental facilities have been studied [17].

According to NRQCD factorisation formalism, the gluon fragmentation to $J/\psi$ production can be factorized as

$$D_{g \rightarrow J/\psi}(z, \mu^2) = \sum_n d_{g \rightarrow n}(z, \mu^2) \langle O_{n}^{J/\psi} \rangle,$$

where $z$ is the longitudinal momentum fraction carried by the produced $J/\psi$ in gluon fragmentation, $\mu = 2m_c$ is the fragmentation scale. $d_{g \rightarrow n}$ represent the short-distance coefficients associated with the perturbative subprocesses in which a $c\bar{c}$ pair is produced in a configuration denoted by $n$ (angular momentum $2S+1L_J$ and color index 1 or 8). $\langle O_{n}^{J/\psi} \rangle$ are the long distance nonperturbative matrix elements demonstrating the probability of a $c\bar{c}$ pair evolving into the physical state $J/\psi$. The short-distance coefficients $d_{g \rightarrow n}$ can be obtained from perturbative calculations in powers of coupling constant $\alpha_s$. $\langle O_{n}^{J/\psi} \rangle$ consist of two kinds of matrix elements, i.e., the color-singlet and color-octet matrix elements (according
to the color index is 1 or 8).

For \( J/\psi \) production in gluon fragmentation, the color-octet matrix element \( \langle O_{8}^{J/\psi}(3S_{1}) \rangle \) is smaller than the color-singlet matrix element \( \langle O_{1}^{J/\psi}(3S_{1}) \rangle \) by a factor of order \( v^{4} \) according to the NRQCD velocity scaling rules. However, the short-distance coefficient for the color-octet term in Eq.(2) is larger than that for the color-singlet term by a factor of order \( 1/\alpha_{s}^{2} \). Numerical results show that color-octet contributions are 50 times larger than color-singlet contributions [15]. In the following calculations, we neglect the color-singlet term in gluon fragmentation in Eq.(3), and only consider the color-octet gluon fragmentation. The leading-order color-octet gluon fragmentation to \( J/\psi \) production gives [15]

\[
D_{g\rightarrow J/\psi}(z,\mu^{2}) = \frac{\pi\alpha_{s}(2m_{c})}{24} \frac{\langle O_{8}^{J/\psi}(3S_{1}) \rangle}{m_{c}^{2}} \delta(1-z). \tag{3}
\]

In our calculations, the effects of the evolution of gluon fragmentation function with scale \( \mu^{2} \) are neglected, which may introduce some error. However, as argued in [18], including evolution would not necessarily be an improvement, since naive Altarelli-Parisi equations do not respect the phase-space constraint \( D_{g\rightarrow J/\psi}(z,\mu^{2}) = 0 \) for \( z < m_{J/\psi}^{2}/\mu^{2} \) [19].

As shown in Fig.1 and Fig.2, the SD process \( p\bar{p} \rightarrow p + J/\psi + X \) by pomeron exchange consists of three steps. First, a pomeron is emitted from the proton with a small squared momentum transfer, \( t = (p_{i} - p_{f})^{2} \), where \( p_{i} \) and \( p_{f} \) are the momenta of the initial and the final states of the proton respectively. Second, partons interaction between the pomeron and the antiproton takes place in the large momentum transfer processes (see Fig.2). In the third step, \( J/\psi \) is produced via the fragmentation processes. Because the fragmentation contributions dominantly come from the color-octet gluon fragmentation process, in our calculations we calculate the \( gg(q\bar{q}) \rightarrow gg, q(\bar{q})g \rightarrow q(\bar{q})g \) processes and followed by the gluon fragmentation Eq.(3).

Using the pomeron factorisation formalism [3,4], we write the SD \( J/\psi \) production cross section as

\[
\frac{d\sigma(p\bar{p} \rightarrow p + J/\psi + X)}{d\xi dt} = f_{\bar{p}p}(\xi,t)d\sigma(IP\bar{p} \rightarrow J/\psi + X), \tag{4}
\]
where $\xi$ is the momentum fraction of the proton carried by the pomeron. $f_{P/p}$ is the pomeron “flux” factor,

$$f_{P/p}(\xi, t) = \frac{d^2\sigma_{SD}/d\xi dt}{\sigma_T^{pp}(s', t)} = \frac{\beta^2(t)}{16\pi} \xi^{1-2\alpha(t)} F^2(t)$$

$$= K \xi^{1-2\alpha(t)} F^2(t). \quad (5)$$

Following [22], the parameters are chosen as

$$K = 0.73 GeV^{-2}, \quad \alpha(t) = 1 + 0.115 + 0.26(\text{GeV}^{-2})t, \quad F^2(t) = e^{4.6t}. \quad (6)$$

In our calculations, we use the renormalized flux factor $D \cdot f(\xi, t)$ [22], which may preserve the shapes of $M^2$ and $t$ distributions in SD and predict the experimentally observed SD cross section at all energies. Here $D$ is defined as

$$D = \text{Min}(1, 1/N), \quad (7)$$

with

$$N = \int_{\xi_{min}}^{\xi_{max}} d\xi \int_{t=0}^{\infty} f_{P/p}(\xi, t), \quad (8)$$

where, $\xi_{min} = M_0^2/s$ with $M_0^2 = 1.5 GeV^2$ (effective threshold) and $\xi_{max} = 0.1$ (coherence limit). For the SD process at the Tevatron ($\sqrt{s} = 1800 GeV$), the renormalized factor $D \approx 1/9$. As a conservative estimate, we will take this as a tentative value for $D$ in the following calculations. (However, we should keep in mind that this flux factor $D$ has not been well determined experimentally. If the precise value can be obtained in the future, our results will change accordingly.)

On the parton structure functions of the pomeron, we assume the hard form [6,9,10],

$$\beta G(\beta) = (f_q + f_g)[6\beta(1 - \beta)], \quad (9)$$

where $\beta$ is the momentum fraction of the pomeron carried by the quarks and gluons. $f_q$ and $f_g$ are the quark and gluon fractions of the pomeron respectively. The momentum sum rule constrains $f_q + f_g = 1$. We neglect any $Q^2$ evolution in the above parton densities of the pomeron [6].
With these partons densities in the pomeron and the parton distribution functions in the antiproton, \(d\sigma(P\bar{P} \rightarrow J/\psi + X)\) can be calculated by employing the usual way in the parton model calculations in hadronic collisions. We use the MRS(A) parton distribution functions [21] to generate the production cross section, and set the renomalization scale and the factorization scale both equal to the transverse momentum of the fragmenting gluon \(\mu = p_T(g) \approx p_T(\psi)\). In gluon fragmentation, the input parameters are taken to be

\[m_c = 1.5 GeV, \quad \alpha_s(2m_c) = 0.26, \quad \langle O_S^{J/\psi}(3S_1) \rangle = 0.0106 GeV^3.\]  

The value of the color-octet matrix element \(\langle O_S^{J/\psi}(3S_1) \rangle\) follows the fitted value in [20] by comparing the theoretical prediction to the experimental data at the Tevatron. In the total cross section, we also include the contributions from the \(\chi_c\) and \(\psi'\) feeddowns through gluon fragmentation \(g \rightarrow \chi_c\) and \(g \rightarrow \psi'\) followed by \(\chi_c \rightarrow \psi \gamma\) and \(\psi' \rightarrow \psi X\). The feeddown contributions give the same \(p_T\) distribution of \(J/\psi\) and contribute about one third of the total prompt \(J/\psi\) production cross section (see Ref. [14]). The leptonic decay branching ratio \(Br(J/\psi \rightarrow \mu^+\mu^-) = 0.0597\) is also multiplied in the cross section. The above described procedure (including the fragmentation approximation) can reproduce the large \(p_T\) \(J/\psi\) production in the central region (i.e., \(|\eta(J/\psi)| < 0.6\)) at the Tevatron [14,15].

Because the \(q\bar{q}\) annihilation process only contributes a small portion to the total cross section of large \(p_T\) \(J/\psi\) production, the SD \(J/\psi\) production is insensitive to the quark flavor number of the pomeron. In our calculations we only consider two-quark flavors. A pseudorapidity cut of \(2.0 < \eta(\psi) < 4.0\) was also performed on the produced \(J/\psi\). The diffractive variable \(\xi\) and \(t\) are integrated over the range of \(0 < \xi < 0.1\) and \(|t| < 1 GeV^2\).

In Fig. 3, We show the cross section of SD \(J/\psi\) production as a function of the minimum transfer momentum of the produced \(J/\psi\). Because the \(gg\) process is the dominant process in the production of \(J/\psi\), the cross section is sensitive to the gluon component in the pomeron (i.e., the gluon fraction \(f_g\)). In this figure, we plot three curves correspondence to three different values of the gluon fraction \(f_g\). The solid line represents the cross section \(\sigma(pp \rightarrow p + J/\psi + X) \times Br(J/\psi \rightarrow \mu^+\mu^-)\) for the gluon fraction set to be \(f_g = 0.7\) (which is
determined by the experiments at the Tevatron [10]). The dotted line is for $f_g = 0$ ($f_q = 1$) and the dashed line for $f_g = 1$ ($f_q = 0$). These curves show that the SD $J/\psi$ production cross section may reach the level of order of 0.01nb (for $p_T(\text{min}) = 8\, GeV$), and therefore is observable at the Fermilab Tevatron at present.

In the above calculations, we use the widely used parametrization of the gluon distribution in the pomeron, i.e., the hard form Eq. (9). However, the precise form of the gluon density of the pomeron is unknown at present and this will affect the $p_T$ distribution of the SD $J/\psi$ production. Different parametrizations will give rise to different spectra. If the gluon in the pomeron is soft, e.g., the gluon distribution in the pomeron behaves like,

$$\beta G(\beta) = 6(1 - \beta)^5,$$

the spectra would be different. In Fig.4, we show the $p_T$ distributions of the cross section of the SD $J/\psi$ production in both the hard gluon and soft gluon cases. The result shown in Fig.4 is consistent with the expectation that the softer gluon will favor $J/\psi$ production with smaller $p_T$, while the harder gluon will favor larger $p_T$. The differential cross sections at large $p_T$ for these two cases are different, but their differences are not so critical to distinguish between them. So, in the following discussion, we mainly limit us to the hard gluon parametrization of the pomeron, but we will also mention the result for the soft gluon parameterization.

As a comparison, we also calculate the non-diffractive (ND) forward $J/\psi$ production ($p + \bar{p} \rightarrow J/\psi + X$) in the same kinematic region, i.e., $2.0 < \eta(\psi) < 4.0$. The forward region $J/\psi$ production is also interesting to the study of the $J/\psi$ production mechanism, because the relative contributions of different mechanisms may vary with $J/\psi$ rapidity. Furthermore, the comparison of forward and central region $J/\psi$ production can provide a consistent test of the “surplus” problem of $\psi'$ and $J/\psi$ found at the Tevatron [14]. In Fig.5, we plot the ND forward $J/\psi$ production cross section as a function of $p_T(\text{min})$. This theoretical prediction can be used to compare with the experimental data, and may provide a further test for the color-octet gluon fragmentation production mechanism. (The D0 collaboration at the
Fermilab Tevatron have reported the forward $J/\psi$ production data [24]. However, these data do not exclude the contributions from $b$ decays. We hope that the prompt $J/\psi$ production data in the forward region may be obtained in the near future.

In Fig.6, we plot the ratio $R(\psi) = \sigma_{SD}/\sigma_{ND}$ as a function of $f_g$ (solid line), where $\sigma_{SD}$ is the cross section for SD $J/\psi$ production, and $\sigma_{ND}$ is for the ND $J/\psi$ production in the same kinematic region. The kinematic constraints for the produced $J/\psi$ are the same for these two processes, i.e., $p_T(\text{min}) = 8\,\text{GeV}$ and $2.0 < \eta(\psi) < 4.0$. The ratio $R(\psi)$ increase from 0.1% as $f_g = 0$ to 0.9% as $f_g = 1.0$. For $f_g = 0.7 \pm 0.2$, the ratio $R(\psi)$ will be $0.65 \pm 0.15\%$. We must note that $R(\psi)$ is independent of the choice of the color-octet matrix element $\langle O_{8}^{J/\psi(3S_1)} \rangle$ because its dependence is cancelled in the ratio $\sigma_{SD}/\sigma_{ND}$. So, measuring this ratio $R(\psi)$ can determine the gluon fraction $f_g$ precisely, provided that the color-octet gluon fragmentation is the dominant mechanism for the $J/\psi$ production at large $p_T$.

One more thing that must be noted in the above calculations of the ratio $R(\psi) = \sigma_{SD}/\sigma_{ND}$ is the approximation of neglecting the fragmentation function smearing. If the $p_T$ distributions of the SD and ND $J/\psi$ production are much different, the smearing effects will influence the ration $R(\psi)$ and the extraction of $f_g$ from this ratio. To see these effects, we calculate the ratio $R(\psi) = \sigma_{SD}/\sigma_{ND}$ for different values of $p_T(\text{min})$, which is shown in Table I. From this table, we can see that the ratio $R(\psi)$ is almost a constant (with a fluctuation less than 10%) as a function of $p_T(\text{min})$ for both the hard quark and hard gluon cases. This implies that the ratio $R(\psi)$ is not sensitive to the smearing, and we may neglect the smearing effects in the calculations of the ratio $R(\psi)$ and the extraction of the gluon fraction $f_g$.

In Table I, we also give the result for the soft gluon parametrization. The ratio $R(\psi)$ for the soft gluon is larger than the hard gluon by a factor $\leq 2$ at $p_T(\text{min}) \geq 10\,\text{GeV}$.

Experimentally, the non-diffractive background to the diffractive $J/\psi$ production must be dropped out to obtain useful information of the above calculations. Theoretically, the SD $J/\psi$ production events can be distinguished from those non-diffractive events by performing the Rapid Gap (RG) analysis. However, the acceptance of the RG will affect this
analysis. Here, we adopt the existing results of the background estimate obtained by the CDF diffractive Dijet experiment, where they give the non-diffractive background to the SD events to be 20% \cite{10}. By the same reason, we expect that the non-diffractive background to the SD $J/\psi$ production is about 20%.

Finally, we discuss the theoretical uncertainty coming from the choice of the factor $D$ of Eq.(7). The factor $D$ represents the momentum fraction of the pomeron carried by the hard partons with the standard pomeron flux. In our calculations, we use the renormalized factor, which is about $1/9$ at the Tevatron energy region. But the value cited here is not unique, because it may change with different choices of the parameters such as $M_0$ and $\xi_{\text{max}}$ in Eq.(4). The momentum fraction $D$ can be measured in the diffractive processes at various collider faculties. At the Tevatron, the CDF have determined the fraction $D$ to be $0.18 \pm 0.04$ \cite{10}, which is well below the range $0.4 < D < 1.6$ reported by ZEUS. If we adopt the CDF measurement, there must be difference in the above calculations of the ratio $R$, which is also shown in Fig.6. The shaded region in Fig.6 represent the range of the ratio $R$ calculated as a function of $f_g$ by using the fraction $D = 0.18 \pm 0.04$.

As discussed in previous studies \cite{15,20}, color-octet mechanism is crucially important to direct $J/\psi$ production (excluding the contributions from $b$ decays and the $\chi_c$ and $\psi'$ feed-downs) at large $p_T$, here the color-octet mechanism is also crucially important to the direct $J/\psi$ production in the diffraction region. If only considering the color-singlet contributions (mainly coming from gluon fragmentation) the SD direct $J/\psi$ production rate will be smaller than the curves shown in Fig.3 by a factor of 50. This will make the measurement of the SD direct $J/\psi$ production very difficult at present luminosity at the Tevatron. That is to say, the SD direct $J/\psi$ production can also be regarded as another important test for the color-octet mechanism.

As a final remark, we note that our proposal, the diffractive $J/\psi$ production, may also be used to extract the gluon fraction in photoproduction at the HERA. There are more diffractive events at the HERA than that at the Tevatron. So, more interesting results may be obtained. The work along this way is in progress \cite{23}.
In conclusion, in this paper we show that the diffractive $J/\psi$ production at large $p_T$ is sensitive to the gluon fraction of the pomeron $f_g$. The measurement of this process at the Tevatron would provide a determination of $f_g$. We have also discussed the uncertainties caused by the renormalised factor $D$ and the gluon parameterizations of the pomeron. These uncertainties, however, can be reduced by combining other experimental measurements such as dijet diffractive production and $W$ diffractive production. We believe that with the proposed SD $J/\psi$ production presented in this paper, we will get a better understanding for the property of the pomeron. And also, the SD direct $J/\psi$ production will provide another crucial test for the color-octet production mechanism.

Acknowledgments

We would like to thank Prof. H.A. Peng for his reading of the manuscript and his comments. We also thank Dr. J.S. Xu for interesting discussions, and especially we thank Prof. H.Y. Zhou for providing us with the computer program and many enthusiastic discussions. This work was supported in part by the National Natural Science Foundation of China, the State Education Commission of China, and the State Commission of Science and Technology of China.
REFERENCES

[1] P.D.B. Collins, *An introduction to Regge theory and high energy physics*, Cambridge University Press, Cambridge (1977).

[2] K. Goulianos, Phys. Reports 101, 169 (1985).

[3] G. Ingelman and P. Schlein, Phys. Lett. B 152, 256 (1985).

[4] E.L. Berger, J.C. Collins, D.E. Soper and G. Sterman, Phys. Lett. B 286, 704 (1987).

[5] H. Fritzsch and K.H. Streng, Phys. Lett. B 164, 391 (1985).

[6] P. Bruni and G. Ingelman, Phys. Lett. B 311, 317 (1993).

[7] R. Bonino, et al., Phys. Lett. B 211, 239 (1988); A. Brandt et al., Phys. Lett. B 297, 417 (1992).

[8] T. Ahmed et al., Phys. Lett. B 348, 681 (1995); M. Drerrick et al., Z. Phys. C 68, 569 (1995); M. Drerrick et al., Phys. Lett. B 356, 129 (1995).

[9] F. Abe et al., Phys. Rev. Lett. 78, 2698 (1997).

[10] F. Abe et al., Phys. Rev. Lett. 79, 2636 (1997).

[11] see G. A. Schuler, Report No. CERN-TH 7170/94.

[12] E. Braaten and T. C. Yuan, Phys. Rev. Lett. 69, 3704 (1992).

[13] CDF collaboration, F. Abe et al., Phys. Rev. Lett. 69, 3704 (1992); Phys. Rev. Lett. 71, 2537 (1993); K. Byrum, FERMILAB-CONF-94/136-E.

[14] CDF collaboration, F. Abe et al., Phys. Rev. Lett. 79, 572 (1997); Phys. Rev. Lett. 79, 578 (1997).

[15] E. Braaten and S. Fleming, Phys. Rev. Lett. 74, 3327 (1995); M. Cacciari, M. Greco, M.L. Mangano and A. Petrelli, Phys. Lett. B 356, 553 (1995); E. Braaten and T.C. Yuan, Phys. Rev. D 52, 6627 (1995).
[16] G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. D 51, 1125 (1995).

[17] For a recent review to see E. Braaten, S. Fleming, and T. C. Yuan, Annu. Rev. Nucl. Part. Sci. 46, 197 (1996).

[18] V. Barger et al, Phys. Lett. B 371, 111 (1996).

[19] E. Braaten et al., Phys. Lett. B 333, 548 (1994).

[20] M. Beneke and M. Krämer, Phys. Rev. D 55, 5269 (1997).

[21] A. D. Martin, R. G. Roberts and W. J. Stirling, Phys. Rev. D 50, 6734 (1994).

[22] K. Goulianos, Phys. Lett. B 358, 379 (1995).

[23] F. Yuan and K.T. Chao, work in progress.

[24] D. K. Fein, (for the D0 Collaboration), Report No. FERMILAB-CONF-97/008-E.
| $p_T(\text{min})(GeV)$ | 5.0 | 6.0 | 7.0 | 8.0 | 9.0 | 10.0 | 11.0 | 12.0 | 13.0 |
|-------------------------|-----|-----|-----|-----|-----|------|------|------|------|
| $R(\psi)$ for hard quark (%) | 0.17 | 0.16 | 0.15 | 0.14 | 0.14 | 0.14 | 0.14 | 0.14 | 0.14 |
| $R(\psi)$ for hard gluon (%) | 0.98 | 0.93 | 0.90 | 0.86 | 0.84 | 0.85 | 0.86 | 0.88 | 0.87 |

TABLE I. The ratio $R(\psi) = \sigma_{SD}/\sigma_{ND}$ as a function of $p_T(\text{min})$. 
Fig. 1 Sketch diagram for the SD $J/\psi$ production by Pomeron exchange at the Tevatron.
Fig. 2 A typical diagram for the color-octet gluon fragmentation $J/\psi$ production in $IP\bar{p} \rightarrow J/\psi + X$ process.
Fig. 3  The SD $J/\psi$ production cross section as a function of the minimum $P_T(\psi)$ for $2.0<\eta(\psi)<4.0$. 
Fig. 4 The differential cross section over $P_T$ for the SD $J/\psi$ as a function of $P_T$ for the hard gluon (dashed line) and soft gluon (dotted line) parametrizations in the pomeron.
Fig. 5 The ND forward $J/\psi$ production cross section for $2.0 < \eta(\psi) < 4.0$. 
Fig. 6. The ratio of $R(\psi) = \sigma_{SD}/\sigma_{ND}$ vs the gluon fraction of the pomeron $f_g$. The solid line corresponds to the momentum fraction $D$ used as the renormalized factor as in Eq.(7). The shaded region represents the range of the ratio $R$ limited by the measured fraction $D=0.18\pm0.04$ by the CDF.