Oscillation modeling of viscoelastic elements of thin-walled structures

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Abstract. The paper presents the results of an oscillation process study of thin-walled structures viscoelastic elements, taking into account the static pressure drop. When studying the oscillations of thin-walled structure elements in a gas flow, a model in the form of a cylindrical panel was used. To describe the viscoelastic properties, the hereditary Boltzmann-Volterra theory of viscoelasticity was applied. When realizing the physicomechanical properties of the object material, the systems of integro-differential equations (IDE) in partial derivatives with corresponding initial and boundary conditions are the mathematical model of the problems under consideration. The obtained nonlinear partial differential equations using the Bubnov-Galerkin method were reduced to the solution of nonlinear ordinary differential equations with constant or variable coefficients with respect to the time function. The integration of the equations obtained using the polynomial approximation of deflections was carried out numerically. Based on this method, an algorithm for the numerical solution of the problem was developed fit for all viscoelastic elements of thin-walled structures of panel type.
1. Introduction

Modeling the oscillatory processes of thin-walled structures under geometrically nonlinear strains is relevant both from theoretical and practical points of view.

The development of this direction is necessary for the rational design of modern structures, which should be based on reliable calculation methods that allow a sufficiently accurate prediction of thin-walled elements behavior under various force actions.

In [1], the dynamics and stability of an elastic structural element in the form of a plate-strip were studied streamlined by the supersonic flow of an ideal gas. Based on the constructed functional, sufficient conditions for the asymptotic stability of partial differential equation solutions, describing the dynamics of the plate, were obtained.

In [2], the influence of the surface viscoelasticity on the effective properties of the plate or shell material was studied. Using the expansion of the Gurtin–Murdoch model and the conformity principle of linear viscoelasticity, the equations of motion for strained plates and shells were obtained.

The effect of viscoelastic parameters and impact velocity on the dynamic characteristics of viscoelastic shells was studied in [3]. Based on the results of the experiment, a model of a viscoelastic material was applied and numerical simulation of the effect of thin-walled spherical shells on a plate was conducted. The effect of viscoelastic parameters and impact velocity on dynamic characteristics was studied. Numerical results are in good agreement with the experimental results.

In [4], orthotropic thin-walled beams subjected to long-term loads were investigated. A finite element model has been developed to analyze the linear viscoelastic behavior of thin-walled beams made of composite materials subject to loads. It was shown that the time points are uniformly distributed on a time scale.

Several analytical and numerical methods have been proposed to predict the behavior of thin-walled structures [5, 6, 7, 8, 9, 10, 15, 16, 17, 18, 19, 20, 21].

The aim of this study is to develop the methods for numerical simulation of nonlinear strain and stability of viscoelastic elements of thin-walled structures in a gas flow.

2. Methods

Consider a viscoelastic cylindrical panel of thicknesses $h$, with sides $a, b$, and radius $R$, streamlined on one side by a gas flow along the generatrix.
The non-linear equations of motion of cylindrical panels, taking into account the static pressure drop, have the form:

\[ \frac{D}{h} \left(1 - R^2\right) V^4W = L(W, \Phi) + \frac{\Delta p}{h} + \frac{1}{R} \frac{\partial^2 \Phi}{\partial x^2} - \rho \frac{\partial^2 W}{\partial t^2} + \]  
\[ + \frac{B}{h} \frac{\partial W}{\partial t} + \frac{BV}{h} \frac{\partial W}{\partial x} - B_1 V^2 \left[ \frac{\partial W}{\partial x}\right]^2, \]
\[ \frac{1}{E} \nabla^4 \Phi = -\left(1 - R^2\right) \left\{ \frac{1}{2} L(W, W) + \frac{1}{R} \frac{\partial^2 W}{\partial x^2} \right\} \]

(1)

Where \( \Delta p \) is the static pressure drop constant in time between the cavity and external flow.

The expression approximating the deflection is chosen as follows

\[ W(x, y, t) = \sum_{n=1}^{N} \sum_{m=1}^{L} W_{nm} \sin \frac{n\pi x}{a} \sin \frac{m\pi y}{b} \]

(2)

where \( W_{nm} = W_{nm}(t) \) is the sought for functions of time.

Applying the Bubnov–Galerkin procedure, a system of nonlinear integro-differential equations is obtained in which, after introducing dimensionless parameters

\[ \frac{W}{h}, \frac{V_x}{a}, \frac{V_y}{a}, \frac{R(t)}{L}, \frac{\Delta p a^4}{Dh} \]

while maintaining the previous notation, we have

\[ \ddot{W} + \dot{\dot{W}} \theta^2 \left[ \left( \frac{k}{\lambda} \right)^2 + \frac{12}{\pi^2} \left( \frac{k}{\lambda} \right)^2 E_\theta \right] \times (1 - R^2) W - \]
\[ - \frac{48k}{\pi^2} \left( \frac{1 - \mu^2}{\lambda} \right) \mathcal{P}^2 \sum_{i,j} K_{i,j} \left( 1 - R^2 \right) W_{nm} W_{ij} + \]
\[ + \frac{12k^2}{\pi^2} \left( \frac{1 - \mu^2}{\lambda} \right) \mathcal{P}^2 \sum_{i,j} a_{i,j} \left( 1 - R^2 \right) W_{ij} W_{ij} - \]
\[ - \frac{12k^2}{\pi^2} \mathcal{P}^2 \sum_{i,j} F_{i,j} \left( 1 - R^2 \right) W_{ij} + MW_{ij} - \]
\[ - 2BM \sum_{n=1}^{N} \sum_{m=1}^{L} \mathcal{P}^2 \sum_{n,i} \sum_{m,j} \Gamma_{i,j} W_{nm} W_{ij} - \mathcal{P}^2 \frac{16k^2}{\pi^2} \frac{\Delta p a^4}{Dh} = 0; \]

\[ W_{ij}(0) = W_{0ij}, \dot{W}_{ij}(0) = W_{1ij}; k = \sqrt{N}, l = \sqrt{L}. \]
The integration of the obtained IDE system with the Koltunov–Rzhanitsin kernel was performed by a numerical method based on the use of quadrature formulas [8, 9, 11, 12, 13, 14].

3. Results and Discussion
Calculation results for viscoelastic cylindrical panels are shown in Table 1 and are reflected in the graphs in Figures 1–6.

As seen from Table 1, an account for static pressure drop reduces the critical flutter velocity. The critical flutter velocity of an elastic cylindrical panel at \( \Delta p = 500 \) decreases by about 9%, compared to the case without \( \Delta p \). With increasing parameter \( \Delta p \), the difference increases. The influence of parameters \( \alpha, \beta, \lambda, a/h \) and \( \beta_i \) on the critical flutter velocity is also studied.

Figures 1 and 2 show the influence of rheological parameters \( A, \alpha \). With increasing parameter \( A \), the amplitude of oscillations decays rapidly. Figure 2 corresponds to the cases \( \alpha = 0.2 \) (curve – 1); \( \alpha = 0.5 \) (curve – 2); \( \alpha = 0.7 \) (curve-3). At \( \alpha = 0.2 \), it leads to a motion with rapidly increasing amplitudes. With an increase in \( \alpha \), the nature of oscillations changes and the amplitude of oscillations decays with time.

Figure 3 shows the influence of the aspect ratio on the oscillatory process. At a value of \( \lambda = 2.2 \), the oscillation amplitude increases, and the panel becomes unstable at velocity \( \lambda = 450 \text{ m/s} \). At values of \( \lambda \) greater than 2.2, the oscillations of finite amplitude appear.

Figures 3 and 5 show some numerical results obtained by formula (3), to study the role of parameters \( a/h \) and \( \beta_i \) on the oscillatory process of the panel.

| \( A \) | \( \alpha \) | \( \beta \) | \( \beta_i \) | \( \lambda \) | \( \Delta p \) | \( h/a \) | \( V_{cr} \) |
|---|---|---|---|---|---|---|---|
| 0.0 | 0.25 | 0.05 | 6 | 3 | 0 | 1/400 | 939 |
| | | | | | 500 | | 862.5 |
| | | | | | | 0.1 | 675 |
| 0.05 | 0.35 | 0.05 | 6 | 3 | 500 | 1/400 | 831 |
| 0.5 | | | | | | | 854.5 |
| 0.05 | 0.25 | 0.01 | 6 | 3 | 500 | 1/400 | 798.5 |
| | | | | | | | 797.5 |
| | | 0.1 | 12 | | | | 821 |
| 0.05 | 0.25 | 0.05 | 18 | 3 | 500 | 1/400 | 1016 |
| | | | 24 | | | | 1292 |
| | | | | | | 2 | 402 |
| 0.05 | 0.25 | 0.05 | 6 | 2.5 | 500 | 1/400 | 581 |
At sub-critical velocity $V = 450 \text{ m/s}$, an increase in parameters $a/h$ and $\beta_1$ leads to a decrease in the amplitude of oscillations of viscoelastic panel midpoint.

The effect of static pressure drop on the amplitude and frequency of the viscoelastic system oscillations was studied. Figure 6 shows the curves for various values of the dimensionless parameter $\Delta p$. Analyzing these curves we can say that with an increase in static pressure drop, the oscillation amplitude has a damping form.

![Graph showing oscillation amplitude and frequency](image)

**Figure 1.** $A = 0(1); 0.05(2); 0.1(3); \Delta p = 200; \alpha = 0.25; \lambda = 3; 
\beta = 0.05; \beta_1 = 6; a/h = 400; V = 400 \text{ m/s}$
Figure 2. \( \alpha = 0.2(1); 0.5(2); 0.7(3); \Delta p = 200; A = 0.1; \lambda = 3; \beta = 0.05; \beta_i = 2; a / h = 400; V = 450 \text{ m/s} \)

Figure 3. \( \lambda = 2,2(1); 2,5(2); 3,5(3); \Delta p = 200; A = 0.1; \alpha = 0.25; \beta = 0.05; \beta_i = 6; a / h = 400; V = 450 \text{ m/s} \)
Figure 4. $\beta_i = 6(1); 18(2); 24(3); \Delta p = 200; \ A = 0.05;\
\alpha = 0.25; \beta = 0.05; \lambda = 3; \ a/h = 400; \ V = 450\ m/s$

Figure 5. $a/h = 300 (1) 450 (2); 500 (3); \Delta p = 200; \ A = 0.05;\
\alpha = 0.25; \beta = 0.05; \beta_i = 6; \lambda = 3; \ V = 450\ m/s$
Conclusion

The problems of oscillations of viscoelastic cylindrical panels moving in a gas flow are investigated. A mathematical model is constructed using the Karman theory and A. A. Ilyushin aerodynamic theory.

The partial differential equation system with corresponding initial and boundary conditions was considered when studying the physical-mechanical properties of the object. The obtained nonlinear partial IDEs using the Bubnov-Galerkin method under-considered boundary conditions were reduced to solving the systems of nonlinear ordinary IDEs with constant or variable coefficients with respect to the time function. The integration of the equations obtained using the polynomial approximation of deflections was carried out numerically. Based on this method, an algorithm for the numerical solution of the problem was developed fit for all viscoelastic elements of thin-walled structures of panel type.

References

[1] Ankilov A V Vel’misov P A 2011 Investigation of dynamics and stability of elastic element of structures in supersonic flow Bulletin of SSTU Mathematics and mechanics 3 (57) pp 59-67
[2] Altenbach H Eremeyev V A Morozov N F 2012 Surface viscoelasticity and effective properties of thin-walled structures at the nanoscale International Journal of Engineering Science 59 pp 83-89 doi.org/10.1016/j.ijengsci.2012.03.004
[3] Zhang X W Tao Z Zhang Q M 2014 Dynamic behaviors of viscoelastic thin-walled spherical shells impact onto a rigid plate Lat Am j solids struct 11(14) dx.doi.org/10.1590/S1679-78252014001400009

[4] Bottoni M Mazzotti C Savoi M 2008 A finite element model for linear viscoelastic behaviour of protruded thin-walled beams under general loadings International Journal of Solids and Structures 45 pp 770–793

[5] Mirsaidov M 2012 Using linear hereditary theory of viscoelasticity in dynamic calculation of earth structures Bases Foundations and Soil Mechanics 6 pp 30-34

[6] Mirsaidov M Sultanov T 2013 Use of linear hereditary theory of viscoelasticity for dynamic analysis of earth structures Soil Mechanics & Foundation Engineering 49(6) pp 250-256 DOI: 10.1007/s11204-013-9198-8

[7] Badalov F B Khudayarov B A Abdukarimov A 2007 Effect of the hereditary kernel on the solution of linear and nonlinear dynamic problems of hereditary deformable systems Journal of Machinery Manufacture and Reliability 36 pp 328-335 doi.org/10.3103/S1052618807040048

[8] Badalov F B 1987 Methods for Solving Integral and Integro-differential Equations of the Hereditary Theory of Viscoelasticity (Tashkent Mehmkhat)

[9] Badalov F B Eshmatov Kh Yusupov M 1987 Some Methods of Solution of the Systems of Integro-differential Equations in Problems of Viscoelasticity Applied Mathematics and Mechanics 51(5) pp 867-87

[10] Khudayarov B A Turayev F Zh 2019 Mathematical Simulation of Nonlinear Oscillations of Viscoelastic Pipelines Conveying Fluid Applied Mathematical Modelling 66 pp 662-679 doi.org/10.1016/j.apm.2018.10.008

[11] Khudayarov B A Komilova Kh M Turaev F Zh 2020 Dynamic analysis of the suspended composite pipelines conveying pulsating fluid Journal of Natural Gas Science and Engineering 75 103148 doi.org/10.1016/j.jngse.2020.103148

[12] Khudayarov B A Komilova Kh M Turaev F Zh 2019 The effect of two-parameter Pasternak foundations on the oscillations of composite pipelines conveying gas-containing fluids. International Journal of Pressure Vessels and Piping 176 103946 doi.org/10.1016/j.ijpvp.2019.103946

[13] Khudayarov BA Turaev F Zh 2019 Nonlinear supersonic flutter for the viscoelastic orthotropic cylindrical shells in supersonic
flow Aerospace Science and Technology 84 pp 120-130 doi: 10.1016/j.ast.2018.08.044

[14] Khudayarov B Turaev F Kucharov O 2019 Computer simulation of oscillatory processes of viscoelastic elements of thin-walled structures in a gas flow E3S Web of Conferences 97 06008 doi.org/10.1051/e3sconf/20199706008

[15] Abdikarimov R Khodzhaev D Vatin N 2018 To Calculation of Rectangular Plates on Periodic Oscillations MATEC Web of Conferences 245 01003 EECE-2018 doi.org/10.1051/matecconf/201824501003

[16] Khodzhaev D Abdikarimov R Vatin N 2018 Nonlinear oscillations of a viscoelastic cylindrical panel with concentrated masses MATEC Web of Conferences 245 01001 EECE-2018 doi.org/10.1051/matecconf/201824501001

[18] Khudoyazarov Kh Kh Khalmuradov RvI Nisnonov U A 2018 Nonlinear vibrations ribbed circular plate under influence of pulse loading Int J of Advanced Research in Science, Engineering and Technology 5(3) pp 5289-5296

[19] Khudoyazarov Kh Khudoyberdiyev Z 2018 Symmetrical vibrations of a three-layered elastic plate Int J of Advanced Research in Science Engineering and Technology 5(10) pp 7117-7121 DOI: 10.5862/MCE.53.8

[20] Usarov M K 2015 Buckling of orthotropic plates with bimoments Magazine of Civil Engineering 53(1) pp 80–90

[21] Toshmatov E Usarov M Ayubov G Usarov D 2019 Dynamic methods of spatial calculation of structures based on a plate model E3S Web of Conferences 97 04072 https://doi.org/10.1051/e3sconf/20199704072

[22] Khudoyazarov Kh 2005 Transversal vibrations of thick and thin cylindrical shells interacting with deformable medium Shell Structures Theory and Applications-London Taylor & Francis Group pp 343-347