Simulation of the Optical Vortices Generation by the Silver Spiral Zone Plates

E S Kozlova\textsuperscript{1,2}

\textsuperscript{1}Image Processing Systems Institute - Branch of the Federal Scientific Research Centre "Crystallography and Photonics" of Russian Academy of Sciences, 151 Molodogvardeyskaya st., Samara 443001, Russia
\textsuperscript{2}Samara National Research University, 34 Moskovskoe Shosse, Samara 443086, Russia
kozlova.elena.s@gmail.com

Abstract. The formation of an optical vortex by a silver spiral zone plate was presented by using the finite-difference time-domain method implemented in FullWVE (RSoft). Both the left and right circularly polarized Gaussian beams at the wavelength of 532 nm have been used as an incident light. Spiral zone plate with the topological charge of 2 \( \mu \text{m} \), the diameter of 8 \( \mu \text{m} \) and the focal length of 532 nm made in 100 nm silver film produces an optical vortex with the maximum intensity of 55 a.u. at the focal plane.

1. Introduction
It has been 25 years since the orbital angular momentum (OAM) of a light field was introduced to optics [1]. But the vortex laser beams, which carry the OAM, have not lost their relevance to this day. There are many papers devoted to investigation of optical vortices [2-4]. The high interest in such beams is explained by a large number of different applications in which they can be used. Optical vortices, whose cross section light field is rotating around the optical axis while propagation in free space, are widely used in microscopy while measuring the localization and orientation of individual molecules [5], in the sounding of the atmosphere and in wireless communication systems [6], in underwater information transfer systems [7], as well as in magnetic field sensing [8].

Special optical elements have been used to produce vortex laser beams [9-18]. Investigation and experimental demonstration of a three-dimensional volumetric optical vortices generation based on light-matter interaction with a high-efficiency dielectric metasurface is presented in [9]. Several optical vortices with topological charges 2, 4, 6 are formed in different orders of diffraction by nanoplates made of amorphous silicon (the refractive index \( n = 3.9231 + i 0.1306 \) for the wavelength of 780 nm) with the size of \( 410 \times 175 \times 466 \) nm and period of 600 nm. A series of Pancharatnam-Berry phase elements are designed in [10] to produce the perfect vortex and vector beams. Vectorial beams on the wavelength of 633 nm with the topological charges of 1, 2, 3 and the light rings whose diameter does not depend on the topological charge are formed by a spiral zone plate (SZP), an axicon, and a zone plate (ZP). The size of the elements was 6 mm and the focal length of the lenses was 200 mm. Using an FDTD-aided numerical simulation it is shown that optical vortex can be sharply focused by a spiral metalens made of amorphous silicon, with unit topological charge and unit numerical aperture (NA = 1) [11]. In [12] a diffractive element base on the phase modification of a radially shifted zone was presented to generate an optical vortex. The role of the so-called shifting parameter was also
studied and, as a result, showed that it acts as a managing parameter for the vortex size. All results were examined experimentally and a consistency between the simulation and experimental results was observed. In [13] fractional and de-centered phase spiral zone plates (SZPs) are proposed for anisotropic edge enhancement using a femtosecond laser. It is shown in [14] that by using an SZP with a fractional topological charge and controlling the starting orientation the symmetry of the focusing process can be broken down to give orientation-selective anisotropic vortex foci. Numerical results show that its binary structure gives additional high-order foci on the optical axis and the intensities in the foci can be controlled by properly choosing the fractional topological charge. In [15] spirally phase-shifted zone plate with multi-purpose possibilities in generating the variety of spiral beams and optical vortices with manageable topological charge was theoretically and experimentally investigated. It is demonstrated that the number of spiral arms, as well as spiral turns, can be manipulated by changing shifting parameter of SZP. It is also shown that two vortices with different charges are formed at given distances before and after the focal plane, so the charge of the generated vortex before the focus is gradually transformed to that of the generated vortex after the focus. In [16] the spatiotemporal characteristics of ultrashort optical vortices, involving the evolutions of intensity, phase, OAM, and energy current, are theoretically and numerically studied in the femtosecond regime. The ultrashort vortex pulses are generated by using a spiral multi-pinhole plate and measured by employing a Mach–Zehnder interferometer system. In [17] the concept of spiral zone plates is extended to a specific single optical element, named single-focus spiral zone plate (SFSZP), for the capability of single-focus phase singularity, and describe how to integrate the spiral zone plates’ diffractive structures with a radially sinusoidal transmittance function in the zone area.

In this paper, we consider the formation of optical vortices by using an amplitude ZP fabricated in thin silver films deposited on silica glass. All simulation is carried out by frequency-dependent finite-difference time domain method ((FD)²TD-method) implemented in the FullWAVE software package (RSoft). The wavelength of the radiation was 532 nm. The left and the right circular polarization of the incident light were used for simulation.

2. Light vortex formed by the phase spiral zone plate

It is well known, that SZP can be used to produce and focus optical vortex. Transmittance function of SZP can be written as:

\[ T(r, \theta) = \exp \left[ im \theta + ik \left( \sqrt{f^2 + r^2} - f \right) \right], \]

where \( r \) and \( \theta \) are polar coordinates, \( k \) is wave number, \( f \) is focal length. Fig. 1 shows the binary template of considered SZP and an optical scheme of numerical simulation. The radius of the element is 4 \( \mu \)m and the focal length is equal to the wavelength of incident light \( \lambda = 532 \) nm. The topological charge of SZP \( m = 2 \).

![Figure 1](image1.png)

**Figure 1.** The template of SZP in a transverse (a) and a longitudinal (b) plane. The green line shows the incident intensity distribution.

The circularly polarized Gaussian beam with the wavelength of 532 nm and a waste of 4 nm was chosen as an incident light. Both the left (CL) and right (CR) circular polarizations were studied. MATLAB was used to calculate the spatial distribution of incident field with the necessary
polarization. The 2D projection of incident light intensity in relation to the SZP design is also presented in Fig 1(b).

Phase SZP made of silica glass with the relief height \( h = \lambda = 532 \) nm was considered as a standard for comparison. For silica glass, the Sellmeier's permittivity model was used [18]:

\[
\varepsilon(\lambda) = \varepsilon_{\infty} + \sum_{m} \Delta \varepsilon_{m} \frac{\lambda^2}{\lambda^2 - \lambda_m^2 - i\lambda \eta_m},
\]

where \( \lambda \) is a wavelength; \( \varepsilon_{\infty}(x,z) \) is the permittivity in the limit of infinite frequency; \( \Delta \varepsilon_{m} \) is the resonance strength; \( \lambda_{m} \) is the resonant wavelength; \( \eta_{m} \) is the Sellmeier damping factor. The parameters for silica glass are taken from [18].

While analyzing simulation results, the following parameters of optical vortices are measured: the maximum intensity \( I_{\text{max}} \), the full transverse width of focal ring FWHM. Hereinafter, the averaging of the calculated field intensity for 30 periods was done.

Simulation of light focusing by the proposed SZP was carried out by (FD)²TD-method realized in the FullWAVE package (RSoft). The computation was conducted at a 20 nm and 10 nm step along transverse and longitudinal coordinates. Temporal step \( c \Delta t \) was chosen equal to 5 nm according to Courant condition. These steps ensure the convergence of the numerical method, and a further reduction of steps does not lead to significant changes (the standard deviation 0.001%). Simulation results for the phase SZP are presented in Fig 2-6.

![Figure 2. The average intensity distribution of optical vortex formed by phase SZP from Gaussian beam with the circular left (a) and right (b) polarization.](image)

Fig. 2 shows an average intensity of light at the focal plane at 532 nm after SZP relief. It can be seen from Fig.2 that SZP forms a ring of light, which is wider for CL polarization. To show the presence of optical vortices the simultaneous intensity distribution is shown at Fig. 3.

![Figure 3. Intensity distribution of optical vortex formed by phase SZP from Gaussian beam with circular left (a-b) and right (c-d) polarization at \( cT = 40 \mu m \) (a,c) and \( cT = 39.9 \mu m \).](image)

Analysis of time picture for CR polarized light shows the presence of two peaks which rotate clockwise in relation to the optical axis. At the same time, CL polarized light formed six peaks which
are very close to each other and merge into a ring, which also rotate clockwise in relation to the optical axis.

Moreover, to investigate the obtained results in more details the amplitude and the phase for each electric field component were calculated by using MATLAB script. Fig. 4-6 show the distribution of the amplitude and phase of the electric field vector components in the focal plane at the time $cT = 19.82 \, \mu m$.

![Figure 4](image1.png)

**Figure 4.** Amplitude (a,c) and phase (c,d) distribution of the $E_x$ component for optical vortex formed by phase SZP from Gaussian beam with CL (a-b) and CR (c-d) polarization.

![Figure 5](image2.png)

**Figure 5.** Amplitude (a,c) and phase (c,d) distribution of the $E_y$ component for optical vortex formed by phase SZP from Gaussian beam with CL (a-b) and CR (c-d) polarization.

![Figure 6](image3.png)

**Figure 6.** Amplitude (a,c) and phase (c,d) distribution of the $E_z$ component for optical vortex formed by phase SZP from Gaussian beam with CL (a-b) and CR (c-d) polarization.

From Fig. 4-6 can be seen, the SZP formed vortex beam with complex topological charge: for transverse components ($E_x$ and $E_y$) it is $m=2$ while for longitudinal component ($E_z$) it is $m=3$ for CL polarization and $m=1$ for CR polarization. The difference in the topological charges of the result optical vortices can be explained by the fact that the direction of the SZP twist and the oscillations of the incident light coincide in the case of CR polarization, and they are opposite to each other in the case of CL polarization [11]. The results of analysis shows the presence of phase singularities which produces vortex field with complex structure.
3. Light vortex formed by silver spiral zone plate

Both phase and amplitude ZP (or SZP) are based on geometrical optics to produce tight focusing. It is shown that phase and amplitude ZP can produce similar light fields [19]. The main difference between the obtained distribution is only in the values of maximum intensity. However, the appropriate height of metal relief should be chosen to obtain these results. In [20] it was shown that silver ZP with height of 100 nm gives the most appropriate focal spot.

In this paper several numerical simulations also have been carried out to choose optimal height of the relief for silver SZP. The parameter for optimization was the maximum intensity. Results of numerical simulation are presented in Table 1.

Table 1. Simulation results for silver SZP.

| h, nm | \( I_{\text{max}} \), a.u. |
|-------|----------------------|
|       | CL                  | CR                  |
| 20    | 33.87               | 36.59               |
| 100   | 45.76               | 57.41               |
| 200   | 41.34               | 51.73               |

Table 1 shows that the height of 20 nm is quite small and sufficiently transparent for the incident light and could not produce focus. At the same time the height of 200 nm is quite large and leads to a losses in the energy of the light. It can be seen from Table 1 that SZP with the height of relief \( h = 100 \) nm allows getting maximum intensity in the focal plane and it is only two times smaller than for the case of phase SZP. The results in Table 1 are consistent with the results obtained in [20]. So in this paper the height of silver SPZ relief also was chosen equal to 100 nm to achieve the same results as for phase SZP. It should be also noticed that fabrication of such SZP is easier than for the phase one because of the relief depth.

The Drude-Lorentz' model is used to describe the permittivity of silver [21]:

\[
\hat{\varepsilon}(x, z, \omega) = \varepsilon_{\infty}(x, z) + \frac{\omega_p^2}{\omega^2 - i \omega \gamma} - \sum_{m} \frac{A_m \omega_m^2}{\omega_m^2 - \omega^2 - 2i \omega \delta_m + \omega_m^2},
\]

where \( \omega \) is a frequency; \( \omega_p \) is the plasma frequency; \( \nu \) is the collision frequency; \( A_m \) is the resonance strength; \( \delta_m \) is the damping factor; \( \omega_m \) is the resonant frequency. The parameters for silver are taken from [20]. Fig. 7 shows results of (FD)\(^2\)-TD-aided simulation for amplitude SZP with the height of relief \( h = 100 \) nm.

Figure 7. The average intensity distribution of optical vortex formed by silver SZP from Gaussian beam with the circular left (a) and right (b) polarization.
The simultaneous intensity distribution is shown at Fig. 8.

**Figure 8.** Intensity distribution of optical vortex formed by silver SZP from Gaussian beam with circular left (a-b) and right (c-d) polarization at \( cT = 40 \) μm (a,c) and \( cT = 39.9 \) μm.

The same study of time picture shows that CR polarized light also formed two peaks. However, CL polarized Gaussian beam gives more separate six peaks in the case of silver SZP (Fig. 8). Fig. 9-11 show the distribution of the amplitude and phase of the electric field vector components in the focal plane at the time \( cT = 19.82 \) μm.

**Figure 9.** Amplitude (a,c) and phase (c,d) distribution of the \( E_x \) component for optical vortex formed by silver SZP from Gaussian beam with CL (a-b) and CR (c-d) polarization.

**Figure 10.** Amplitude (a,c) and phase (c,d) distribution of the \( E_y \) component for optical vortex formed by silver SZP from Gaussian beam with CL (a-b) and CR (c-d) polarization.

**Figure 11.** Amplitude (a,c) and phase (c,d) distribution of the \( E_z \) component for optical vortex formed by silver SZP from Gaussian beam with CL (a-b) and CR (c-d) polarization.
Fig 9-11 shows the same results as in the case of phase SZP for each component of the electric field, formed in the focal plane by silver SZP, which can prove that silver SZP can easily substitute phase SZP. It should be also noticed that, however, the plasmonic effects are not investigated in this paper, it can be seen that they have not a big influence on vortex generation as in a case of ordinary ZP [11].

4. Conclusion

In this paper, we considered the formation of optical vortices by using the amplitude ZP fabricated in thin silver films deposited on silica glass. All simulation is carried out by finite-difference time-domain method (FDTD-method) implemented in the FullWAVE software package (RSoft). MATLAB was used for calculation of amplitude and phase distribution of light field components. The wavelength of the radiation was 532 nm. The left and the right circular polarizations of the incident light were used in simulation. It is shown that the silver SZP with the height of relief of 100 nm can form an optical vortex with the maximum intensity, which is only two times smaller than that in the case of phase SZP.

Acknowledgments

This work was supported by the Russian Science Foundation (grant No. 18-19-00595) in part of simulation of optical vortices generation by phase SZP, by the Russian Foundation of Basic Research (18-07-01380) in part of simulation of optical vortices generation by silver SZP from right circular polarization and by the Ministry of Science and Higher Education within the State assignment FSRC «Crystallography and Photonics» RAS in part of simulation of optical vortices generation by silver SZP from left circular polarization.

References

[1] Padgett M J, 2017 Opt Express 25 11265-74
[2] Cheng K, Lu G, Zhong X, 2017 Opt. Commun. 396 58-65
[3] Kotlyar V V, Kovalev A A, 2017 Computer Optics 41 609-615
[4] Mafakheri E, Tavabi A H, Lu P, Balboni R, Venturi F, Menozzi C, Gazzadi G C, Frabboni S, Sit A, Dunin-Borkowski R E, Karimi E, 2017 App. Phys. Lett. 110 093113
[5] Backlund M P, Lew M D, Backer A S, Sahl S J, Grover G, Agrawal A, Piestun R, Moerner W E, 2013 Proc. of SPIE, 8590 85900
[6] Lavery M P J, Peuntinger C, Gunthner K, Banzer P, Elser D, Boyd R W, Padgett M J, Marquardt C, Leuchs G, 2017 Sci. Adv., 3 e1700552
[7] Morgan K S, Miller J K, Cochennour B M, Li W, Li Y, Watkins R J, Johnson E G 2016 J. Opt., 18 104004
[8] Yu S, Pung F, Liu H, Li X, Yang J, Wang T, 2017 Appl. Phys. Lett. 111 091107
[9] Huang L, Song X, Reineke B, Li T, Li X, Liu J, Zhang S, Wang Y, Zentgraf T, 2017 ACS Photonics 4 338-46
[10] Liu Y, Ke Y, Zhou J, Liu Y, Luo H, Wen S, Fan D, 2017 Sci. Rep. 7 44096
[11] Kotlyar V V, Nalimov A G, 2017 Computer Optics 41 645-54
[12] Sabatyan A, Behjat Z, 2017 Opt. Quant. Electron., 49 371
[13] Zhou Y, Feng S, Nie S, Ma J, and Yuan C, 2017 App. Optics 56 2641-8
[14] Wei L, Gao Y, Wen X, Zhao Z, Cao L, and Gu Y, 2013 J. Opt. Soc. Am. A. 30 233-7
[15] Rafighdoost J and Sabatyan A, 2017 J. Opt. Soc. Am. B. 3 608-12
[16] Ma L, Zhang P, Li Z, Liu C, Li X, Zhang Y, Zhang R, and Cheng C, 2017 Opt. Express, 25 29864-73
[17] Liang Y, Wang E, Hua Y, Xie C, and Ye T, 2017 Opt. Lett., 42 2663 - 6
[18] Couairon A, Sudrie L, Franco M, Prade B, and Mysyrowicz A, Phys. Rev. B. 2005 71 125435-
41

[19] Kotlyar VV, Stafeev SS, Nalimov AG, Kotlyar MV, O’Faolain L, and Kozlova ES, 2017 Opt. Express, 25, 19662-71
[20] Kozlova ES, Kotlyar VV, Nalimov AG, Stafeev SS, Kotlyar MV, O’Faolain L, 2017 Proceedings of International Conference on Transparent Optical Networks 2017, 8025096
[21] Vial A, Laroche T, Dridi M, and Le Cunff L, Appl. Phys. A. 2011 103 849-53
[22] Rakic A D, Djurisic A B, Elazar J M, and Majewski M L, 1998 App. Opt. 37 5271-83