Numerical Analysis of RC beam with High Strength Steel Reinforcement using CDP model

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Abstract. The article is based on preliminary research on use of high strength steel for RC elements. A reinforced concrete beam with a span length of 5700 mm has been subjected to four-point bending test. The bottom reinforcement of the beam was made of SAS670/800 high strength steel. This type of steel has no specific yield point. The yield limit declared by the manufacturer is specified as 670 MPa and the ultimate strength is equal to 800 MPa. The aim of the research was to establish a numerical model that would render properly the real beam behaviour. The model was created in the ABAQUS software using the CDP (Concrete Damage Plasticity) material model for concrete. The paper analyses the influence of several parameters on the results of numerical computations. The author's aim is to define values of the dilation angle that should be implemented in the model. Generally, the value of this parameter is established a priori, without any justification. The analysis reveals that in a situation when the dilation angle is greater than or equal to 25 deg, the numerical model predicts properly the damage obtained in the experiment. For lower values of the dilation angle, the failure mode is not consistent with the actual beam behaviour in the experiment. Another issue is whether and how the results from the model are influenced by the choice of specified concrete tensile behaviour. In the CDP model, this property may be set in three ways: by giving the stress-displacement relation, stress-cracking strain relation or determining the fracture energy. All three alternatives are analysed and compared. Another aspect described in the paper, is the difference in load application through force control and displacement control as well as the influence of the amount of shear reinforcement. In order to analyse mesh size sensitivity, the results obtained for models with various mesh densities were compared.

1. Introduction

Buildings are being designed more and more boldly, therefore there is a need to use materials with higher strength parameters. In reinforced concrete structures, we can use both high strength concrete and high strength steel. One of them is SAS670/800 steel with the yield limit of 670 MPa and the ultimate strength equal to 800 MPa. This type of steel is used as reinforcement of columns, e.g. at the Silesian Museum, the Spinnaker Office Tower and the Warsaw HUB. The question arises whether it would be possible to extend the use of this steel also to beam elements. The paper presents a description of a four-point bending test of a beam with SAS670/800 steel reinforcement. The aim of research is to establish a numerical model that would render properly the real beam behaviour. The model was created in the ABAQUS software using the CDP (Concrete Damage Plasticity) material model for concrete.
2. Experimental programme
The test of simply supported RC beam subjected to four-point bending was carried out in the laboratory of the Institute of Building Materials and Structures at Cracow University of Technology. The geometry and the reinforcement of the beam are presented in figure 1. The beam had a rectangular cross-section, 300 mm in width, 600 mm in depth, a total length of 6000 mm and a span of 5700 mm. Two 20 mm steel bars were used as a top reinforcement. The shear reinforcement was made of 8 mm steel stirrups placed at 100 mm intervals. Both, the stirrups and the top reinforcement were made of steel grade B500SP. In the pure bending zone, there was no stirrups. The bottom reinforcement consisted of four 25 mm bars of SAS670/800 steel. The effective depth of the beam was equal to 545 mm.

![Figure 1. Geometry and reinforcement of the beam (dimensions in mm)](image)

The beam was made of C30/37 concrete. The mechanical properties of concrete are presented in table 1. They were evaluated from cylinder specimens. B500SP and SAS 670/800 steels were also subjected to testing. The table 2 presents the average material properties, obtained from five samples from each steel grade and implemented to the ABAQUS model.

|                  | f_c [MPa] | f_t [MPa] | E_s [GPa] | Poisson's ratio |
|------------------|-----------|-----------|-----------|----------------|
| Table 1. Material properties of concrete | 39.66     | 1.813     | 24.68     | 0.2            |

| Reinforcement steel | f_y [MPa] | f_t [MPa] | E_s [GPa] | Poisson's ratio |
|---------------------|-----------|-----------|-----------|----------------|
| B500SP              | 500       | 500       | 200       | 0.3            |
| SAS 670/800         | 540       | 854       | 191       | 0.3            |

The beam was tested under static four-point bending. The load points were located in 1/3 of span length from both sides of the beam. The load was applied in force control mode at a speed of 25 kN/min. During the test, deflections, compression and tension strains were measured in the pure bending zone. Constant collecting data system delivered by HBMs was utilised. The entire applied load was divided into 9 load steps (figure 2). The beam failed suddenly due to concrete crushing and buckling of the top reinforcement, which is illustrated in figure 3. Prior to the failure yielding of the bottom steel was observed. The ultimate load was equal to 775 kN.
3. Numerical simulation

A 3D numerical model of the RC from experiment was created with ABAQUS software. The analysis was carried out mostly through load control technique (additionally, the influence of the displacement control technique for two different values of dilation angle was studied – point 4.3). The details of the numerical model are presented in the further part of the article.

3.1. Discretization of the beam

The numerical model of the beam consisted of two types of finite elements. For concrete, it was the C3D8R - 8-node linear brick with reduced integration and hourglass control, and for reinforcement the T3D2 - 2-node linear displacement [1]. Approximate size of FE was 25 mm. In order to reduce the calculation time, only the quarter of the beam was modelled. The model consisted of 22,258 nodes and 18,296 elements. Figure 4 shows the discretization of the beam. The interaction between concrete and reinforcement was achieved by using the embedded region constraint.

3.2. Materials models

The top and shear reinforcement steel (grade B500SP) was assumed as a linear elastic-plastic material without isotropic hardening. The SAS670/800 steel was assumed as a linear-plastic material with isotropic hardening. Figure 5 shows the average stress - strain response of the SAS670/800 steel obtained from tests. Plastic behaviour of SAS670/800 steel defined in ABAQUS model is presented in figure 6.

The concrete model was defined using the CDP (Concrete Damage Plasticity) material. This model was described theoretically by Lubliner et al. [2] and then enhanced by Lee and Fenves [3]. The CDP model is a continuum, plasticity-based, damage model for concrete. It assumes that the two main failure mechanisms are tensile cracking and compressive crushing of concrete [1]. The compression stress – strain response of concrete was calculated in accordance with [4]. The tension stress –
displacement response of concrete was obtained from the experiment. Tensile behaviour of concrete in the CDP model may be defined in three ways: by displacement, strain or fracture energy. All three options are compared later in the article. In addition to compressive and tensile behaviour, other CDP parameters were assumed as in the table 3.

![Figure 5](image1.png)  
**Figure 5.** Stress - strain response of steel SAS 670/800 from tests.

![Figure 6](image2.png)  
**Figure 6.** Plastic behaviour of steel SAS 670/800 defined in the ABAQUS model.

![Figure 7](image3.png)  
**Figure 7.** Compression stress - inelastic strain response of concrete defined in the ABAQUS model.

![Figure 8](image4.png)  
**Figure 8.** Tension stress - displacement response of concrete from direct tension test on cylinder specimens.

**Table 3.** CDP parameters for ABAQUS material definition of concrete

| Dilation angle [deg] | Eccentricity [-] | fb0/fc0 [-] | K [-] | Viscosity parameter [sec] |
|----------------------|------------------|-------------|-------|---------------------------|
| 35                   | 0.1              | 1.16        | 0.667 | 0.0001                    |
Figure 9. Experimental (Exp) and numerical (FEM) load - midspan deflection curves

Figure 9 shows high correlation between load - midspan deflection obtained from test and from FEM. The FEM response is slightly less stiff in phase II. The experimental beam had failed suddenly under a load equal to 775 kN. This sudden failure is not reflected in the FEM computations. In figure 9, the forces $P_y = 559$ kN and $P_1 = 838$ kN (obtained from an analytical calculations) are marked with a dashed line. These forces correspond to stresses in the bottom reinforcement steel equal to 540 MPa and 854 MPa, respectively. Two black points in figure 9 indicate the moments in which the same stress values (540 MPa, 854 MPa) are obtained in the bottom reinforcement in FEM analysis.

4. Parametric analyses
This section presents parametric analyses carried out on the FEM model. It indicates the influence of the applied method for modelling tensile behaviour of concrete on the CDP model, the dilation angle, a load application through force control and displacement control, the amount of shear reinforcement and FE size.

4.1. Tensile behaviour of concrete in CDP model
There are three possibilities of defining concrete tensile behaviour in the CDP model. All three were taken into consideration in this analysis.

The first alternative is to define the concrete tensile behaviour by determining the stress-displacement relationship. Two beam models apply this strategy. First model, named Exp, was constructed based on experimental tests of concrete samples subjected to a direct tension test (cf. figure 8). The second one, named MC2010, was adopted in accordance with the Model Code 2010 [5] (bilinear model). Next two models were defined by providing cracking strain. One of them was created by dividing the displacement from the Exp model by the FE dimension (25 mm), which is justifiable for plain concrete elements. Another model was defined on the basis of observation of other tests for RC elements assuming that the tension stiffening effect ceases for strain value equal to 2‰ and after it only residual strength equal to 0.10 $f_t$ remains. In the fifth and the last model the fracture energy (GFI) was declared as input variable. Detailed data are provided in table 5. The models are presented in the form of diagrams in figure 10.
### Table 4. Tensile behaviours of concrete tested in CDP model

|          | Displacement | Displacement | Strain | Strain | Tension stiffening | GFI     |
|----------|--------------|--------------|--------|--------|--------------------|---------|
|          | Exp          | MC2010       | Exp    |         |                    |         |
| yield    | stress       | displacement | yield  | stress  | cracking strain    | yield   | stress  | cracking strain | yield | stress  | fracture energy |
| MPa      | MPa          | mm           | MPa    | mm      | -                  | MPa     | -       | -                | MPa   | -       | N/m       |
| 1.81     | 0            | 1.81         | 0      | 1.81    | 0                  | 1.81    | 0       | 1.81             | 110   |
| 1.00     | 0.03         | 0.36         | 0.06   | 1.00    | 0.0011             | 0.18    | 0.002   |
| 0.60     | 0.06         | 0.00         | 0.30   | 0.60    | 0.0022             | 0.18    | 1.00    |
| 0.30     | 0.11         |              | 0.30   | 0.18    | 0.0043             |         |         |
| 0.13     | 0.22         |              | 0.13   | 0.13    | 0.0087             |         |         |

**Figure 10.** Tensile behaviour in CDP model defined by a) displacement, b) strain

**Figure 11.** The results of change in tensile behaviour in CDP model
Figure 11 shows the load-deflection curves obtained through FEM. All five concrete tensile behaviour models defined above gave almost identical results. It may be assumed that the selection of the tensile behaviour model does not affect the beam load capacity. Looking at figure 12 illustrating PEEQT maps (equivalent plastic strain in tension) at load equal to 775 kN, it is obvious that the choice of the concrete tensile behaviour affects the spacing of cracks as well as the volume of the plastic deformation. Significantly lower plastic deformations values were obtained for the DISPLACEMENT MC2010 and the GFI models.

4.2. Dilation angle
One of the parameters that must be defined in the CDP material is the dilation angle. The angle of dilation affects the amount of plastic volume deformation. The selection of a higher value increases the stiffness of the concrete. In works using the CDP model, the value of the dilation angle is usually assumed within the range from 5° to 42° [6, 7, 8, 9, 10, 11]. Determining the dilation angle is essential for obtaining the correct results of the FEM. In this analysis, numerical computations were performed for the following values of the dilation angle: 5°, 15°, 25°, 35°, 45°. The load - midspan deflection curves for the above values are shown in figure 13.
The analysis showed that the dilation angle strongly affects both the deflection and the load-bearing capacity of the beam. The smaller values of the dilation angle, the greater change in the beam behaviour. The analysis revealed that in situation when the dilation angle is greater than or equal to 25 deg, the numerical model predicts correctly the damage mode obtained in the experiment. For lower values of the dilation angle, the failure mode is not consistent with the actual beam behaviour in the experiment. The obtained results are consistent with the numerical observations presented in [6].

4.3. Load or displacement control technique

Figure 14 shows comparison of models with the load and displacement control technique. The models have been varied also with respect to the value of the dilation angle. The results from the models in which control was carried out by applying load are marked in black. The results of the models with displacement control are marked in blue. The difference between these two models is negligible for the dilation angle equal to 35 deg. At a low angle of dilation, the curves diverge at a force level of around 487 kN. It is the maximum force that can be obtained in a model with displacement control and it corresponds to its maximum stress in concrete 28 MPa and in steel 468 MPa. After exceeding the force of 487 kN, the stress in both materials decreases. In the model controlled by force and with a dilatation angle equal to 5 deg, the maximum achievable stresses in concrete are 34 MPa, still less than the declared compressive strength of concrete equal to 39.66 MPa. The maximum stress in steel in this model is 693 MPa (u = 200 mm). The small angle of dilatancy does not allow achieving full compressive strength of the concrete.

Figure 13. The results of change in the dilation angle in the CDP model

Figure 14. The results of change in force controlling technique (GFI)
4.4. Shear reinforcement

To check whether the amount of shear reinforcement has an effect on the results obtained for the beam, a numerical model, in which the stirrup diameter was equal to 12 mm instead of 8 mm (as in the experimental beam), was designed. This change caused more than doubling of the shear reinforcement area (spacing did not change). Figure 15 shows that even such big increase of the stirrup surface area did not affect the bearing capacity of the beam. The stirrups have been appropriately designed so that the beam was not destroyed by the exhausting shear capacity.

![Figure 15. The results of change in stirrups diameter (DISPLACEMENTExp, $\psi = 35^\circ$)](image)

4.5. FE size

In order to analyze mesh size sensitivity, the results obtained for models with various FE densities were compared. For models with the concrete tensile behaviour, defined by the displacement, differences are relatively small. Figure 17 shows that the adoption of the coarser mesh (FE = 50 mm) resulted in less rigid behaviour of the beam and in slight reduction of load-bearing capacity. In models with concrete tensile behaviour declared by STRAINExp, differences in load - midspan deflection curves, are imperceptible (figure 18). This confirms the advice contained in the Abaqus Manual [1], which recommends the use of the STRAIN option for modelling reinforced concrete structures.

![Figure 16. The results of change in FE size (DISPLACEMENTExp, $\psi = 35^\circ$)](image)
5. Conclusions
The failure in the experiment was caused by bending - concrete crushing and steel yielding occurred. The numerical model replicates it well, if the assumed dilation angle is not less than 25 degrees. For smaller angles of dilation, the failure mode is not consistent with the reality.

For small dilation angles, there are significant differences in the load-displacement paths between force and displacement control technique - a phenomenon difficult to explain. There are probably alternative equilibrium paths.

Various descriptions of concrete tensile behaviour have only little effect on the force-displacement path, but they lead to different distribution of cracks and to different values of plastic deformations.

The influence of shearing (see results for different strengths of stirrups) is irrelevant.

There are negligible differences depending on the adopted FE densities when the concrete tensile behaviour is defined by STRAIN option.

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