Second-order curves in architectural and building design

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Abstract. Straight lines, circles and second-order curves are widely used in architectural and building design as the main form-making elements. Straight lines are used in the design of gridwork building structures and engineering facilities. Smooth curves composed of the sections of the second-order curves are used to form complex curvilinear wave surfaces. The grid shells of smooth wave curves have become one of the main means for the form-making of avant-garde buildings, including skyscrapers and high-tech buildings. The article deals with the architectural design of the modern airport terminal developed by Samoo Architects & Engineers Korean company. For the geometric analysis we used an engineering method for designing compound lines formed by the arcs of the second-order curves. It is shown that second-order curves allow us to approximate the curvilinear section of the support profile on the terminal facade. For an unambiguous determination of the support profile, it is sufficient to fix eight points of the profile with tangents indicated in the three of them. The terminal roof surface is formed by the movement of a smooth wave curve of a variable shape along horizontal, parallel to each other, rectilinear guides. The use of second-order curves for the approximation of the undulating roof grade enables to considerably simplify the mathematical description of the roof surface. Approximation algorithms are practically realized by means of computer graphics and specialized software tools.

1. Introduction
The properties of the surface are largely determined by the properties of the lines forming this surface. The properties of the lines, in their turn, are determined by the type of the mathematical functions that describe these lines. In architectural and building design, the simplest lines – straight lines, circles, second-order curves are predominantly used [1-4]. [5] formulates the requirements posed to the methods of designing and setting surfaces. "The method should be constructive, that is, enabling to depict the surface and show techniques of building any points and lines on it. The surface must be mathematically described using the equations that allow the computer to make appropriate calculations. The simpler the constructive and analytical way of setting surfaces, the easier it is to solve various technical problems." These requirements are best satisfied by second-order surfaces and kinematic surfaces formed by the movement of the simplest algebraic lines - straight lines and second-order curves [6-11].

In modern CAD, the main tool for modeling curved lines is NURBS curves embedded in the international CAD/CAM standards. Numerical methods become the main way of designing surfaces. In this case, surface modeling is reduced to its approximation by a given set of reference lines and points, based on a set order of smoothness of the designed surface. In particular, there are many known solutions of the problem of "skinning" the surface on a given spatial contour using various
bicubicspline approximation options (Koonsurface, Bezier surface, NURBS surface, etc.) and a subsequent tabulation of the points of the resulting surface [12-15].

Nevertheless, the tasks of form-making do not always require numerical methods for their solution. Numerical methods have their drawbacks, one of which is the impossibility of obtaining an analytical surface description. In some cases, traditional geometric methods of designing surfaces may be preferable to computational methods. In architectural and building design, the practical value of the kinematic form-making method remains, with the use of second-order curves (SOC) as the main form-making element, due to the simplicity and good previous level of study of these curves.

2. Engineering method for designing composite lines formed by SOC arcs

There is a one-dimensional array of points Ai with the indicated tangents in these points (Figure 1). It is required to draw a smooth curve formed by the arcs of the second-order curves through these points. The shape of the designed curve changes with the change in the position of the control points B₁, B₂, B₃…. The arcs of the second-order curves pass through the points Ai, Bi. In the abutting points Ai, these arcs have common tangents. The first section A₁A₂ is a hyperbola arc, the second section A₂A₃ is a part of the ellipse, the third section is a parabola arc. A special software tool [16] is used for precise drawing of the second-order curves. It does not only draw the arc of the second-order curve, but geometrically precisely determines the center and major axes of this curve. The axis, vertex and focus F are automatically determined for the parabola arc.

![Figure 1. Figure with short caption (caption centred). Smooth compound line formed by the arcs of the second-order curves.](image)

3. Architectural shapes formed by the movement of the second-order curves

In progressive architecture of the 21st century, grid cells - load-bearing building frames made of metal, composite materials, and wood have been widely used. Steel grid shells formed by the arcs of the second-order curves were first used in architecture by Russian engineer and architect V.G. Shukhov (Figure 2). Gridwork shells have become widely used during the last two decades. It was due to the introduction of computer graphics in the practice of geometric modeling.

![Figure 2. Building of a grid shell of V.G. Shukhov’s structure at Vyksa Steel Works, Vyksa, 1897.](image)

Grid shells have become one of the main means of form-making of avant-garde buildings, including skyscrapers and high-tech buildings. For example, modern Chinese architecture uses the shapes formed by smooth wave curves or second-order curves. New Century multifunctional complex (Chengdu, China) is officially recognized to be the largest monolithic building in the world. Its area is
1.6 million square meters. The length of the building is 500 meters, the width is 400 meters, the height is 100 meters (Figure 3). The facade of the building is limited by smooth curved lines from above (Figure 4). The engineering method of designing composite lines formed by the arcs of the second-order curves can be used for a mathematical description of these curves.

The shape of Sheraton hotel in Huzhou, China, which is one of the ten most expensive hotels in the world, is defined by two ellipses with different eccentricities. The building has the shape of a horseshoe (Figure 5).

The terminal project was developed by Samoo Architects & Engineers Korean company. The project uses a kinematic method of surface formation. The facade of the building is formed by the movement of a smooth curve simulating the sea wave profile. A curved line of a variable shape slides along ten rectilinear horizontal guide lines. The guide lines are parallel to each other (Figure 6). The roof of the building is also formed by the kinematic method: a smooth wave curve moves along the horizontal rectilinear guide lines (Figure 7).
Let us show that the profiles of the supports on the terminal façade are formed by the second-order curves. We choose one of the supports and mark characteristic points 1, 2... 8 thereon. The support is completed by rectilinear section 7-8. The directions of the curvature vectors of the wave-like profile change in inflection points 3, 5, 7. Inflection points 3, 5, 7 divide the curvilinear part of the support profile into sections 1-2-3, 3-4-5 and 5-6-7. In initial point 1 and in inflection points 3, 5, 7 the tangents t₁, t₃, t₅, t₇ to the theoretical support profile are indicated. The tangent t₇ in point 7 coincides with straight line section 7-8.

Using the engineering method of designing smooth compound curves, we obtain an undulating grade consisting of the arcs of the second-order curves. Points 1-2-3 and tangents t₁, t₃ define an ellipse arc. Points 3, 4, 5 and tangents t₃, t₅ define the first hyperbola arc. Points 5, 6, 7 and tangents t₅, t₇ define the second hyperbola arc (Figure 8,a).

Thus, to build a profile of the curved support of the terminal facade it is enough to specify eight points 1, 2, ..., 8 and three tangents t₁, t₃, t₅. The tangent t₇ in point 7 coincides with rectilinear section 7-8 of the support (Figure 8,b). The wave curve is approximated by the arcs of the ellipse and the two hyperbolas. Another approximation variant is based on the use of the cross-points of longitudinal load-bearing members of the frame with the vertical supports.

5. Geometrical analysis of the terminal roof
The roof surface is formed by the movement of a smooth curve of the line, called the "generator," along the horizontal straight lines parallel to each other, called "guides". The curve generator is a NURBS-curve of a variable shape, and cannot be described by an algebraic equation [17,18].
The engineering analysis method based on the use of second-order curves allows us to approximate the generator curve by a section of a straight line and three arcs of the second-order curves with a variable eccentricity (Figure 9). Let us choose one of the generators and mark characteristic points 1, 2, ..., 9 thereon. We indicate the tangents $t_2$, $t_8$ in the initial and final points of the curved profile. Sections 1-2 and 8-9 have a rectilinear shape. The tangent $t_8$ coincides with the direction of section 1-2. The tangent $t_8$ coincides with the direction of the section 8-9. Inflection points 4, 6 with the tangents $t_4$, $t_6$ indicated in these points divide the curved profile into three sections 2-3-4, 4-5-6, 6-7-8.

Using the engineering method of designing smooth compound curves, we obtain an undulating grade consisting of the three arcs of the second-order curves. Points 1-2-3 and tangents $t_2$, $t_4$ define the ellipse arc. Points 4, 5, 6 and tangents $t_4$, $t_6$ define the hyperbola arc. Points 6, 7, 8 and tangents $t_6$, $t_8$ define the arc of the second ellipse (Figure 10).

Thus, to build one generator of the terminal roof profile it is enough to specify nine points 1, 2, ..., 9 and three tangents $t_2$, $t_4$, $t_6$. The initial and final sections of profile 1-2 and 8-9 have a rectilinear shape. The wave curve is approximated by the arcs of two ellipses and one hyperbola.

It should be noted that the photographic image of the terminal roof (Figure 10) contains perspective distortions of the actual sizes. Nevertheless, the nature of the profile is not distorted, since the perspective image of the second-order curve remains a second-order curve [19, 20].

6. Conclusion

It is shown that compound non-rectilinear shapes of modern architecture can be obtained not only using NURBS-curves and other standard means of computer graphics, but also using smooth compound curves consisting of the arcs of the second-order curves. The engineering method of designing composite lines formed by the SOC arcs allows us to approximate complex non-rectilinear shapes and to obtain their algebraic equations.

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**Acknowledgments**

The work was supported by Act 211 Government of the Russian Federation, contract no. 02.A03.21.0011.