Microparticle charging in dry air plasma created by an external ionization source

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Abstract. In the present paper the dust particle charging is studied in a dry air plasma created by an external ionization source. The ionization rate is changed in the range $10^{14} - 10^{20} \text{ cm}^{-3} \text{s}^{-1}$. It is found that the main positive ion of the plasma is $O_2^+$ and the main negative ones are $O_2^-$ and $O^-_4$. The point sink model based on the diffusion-drift approach shows that the screening potential distribution around a dust particle is a superposition of four Debye-like exponentials with four different spatial scales. The first scale almost coincides with the Debye radius. The second one is the distance, passed by positive and negative plasma components due to ambipolar diffusion in their recombination time. The third one is defined by the negative ion conversion and diffusion. The fourth scale is described by the electron attachment, recombination and diffusion at low gas ionization rates and by the recombination and diffusion of negative diatomic ions at high ionization rates. It is also shown that the electron flux defines the microparticle charge at high ionization rates, whereas the electron number density is much less than the ion one.

1. The main ions in dry air plasma
The ion components of the plasma are obtained by the analysis of ion-molecular reactions from [1] and the processes caused by an electron beam:

$$O_2 + e_b \rightarrow O_2^+ + e + e_b, \quad O_2 + e_b \rightarrow O^+ + O + e + e_b,$$
$$N_2 + e_b \rightarrow N_2^+ + e + e_b, \quad N_2 + e_b \rightarrow N^+ + N + e + e_b.$$

The resulting densities are presented in table 1. Calculations reveal that the main positive ion type is $O_4^+$ and the main negative ion types are $O_2^-$ and $O^-_4$.

2. The microparticle charging in plasma of an electronegative gas
The microparticle charging is described by the hydrodynamic equation system:

$$\frac{\partial n_e}{\partial t} + \text{div} \, j_e = Q_{\text{ion}} - \beta_{ei} n_e n_i - \alpha n_e,$$
$$\frac{\partial n_i}{\partial t} + \text{div} \, j_i = Q_{\text{ion}} - \beta_{ei} n_e n_i - \beta_{i2} n_i n_2 - \beta_{i4} n_i n_4,$$
$$\frac{\partial n_2}{\partial t} + \text{div} \, j_2 = \alpha n_e - \nu_{24} n_2 + \nu_{42} n_4 - \beta_{i2} n_i n_2,$$
$$\frac{\partial n_4}{\partial t} + \text{div} \, j_4 = \nu_{24} n_2 - \nu_{42} n_4 - \beta_{i4} n_i n_4. \tag{1}$$

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Table 1. The steady-state electron and ion number densities at three different gas ionization rates.

| $Q_{\text{ion}}$ | $10^{14}$ cm$^{-3}$s$^{-1}$ | $10^{16}$ cm$^{-3}$s$^{-1}$ | $10^{18}$ cm$^{-3}$s$^{-1}$ |
|------------------|-----------------------------|-----------------------------|-----------------------------|
| $n_e$            | $2.077 \times 10^6$         | $2.058 \times 10^8$         | $1.929 \times 10^{10}$     |
| $O_2^-$          | $1.856 \times 10^{10}$      | $1.841 \times 10^{11}$      | $1.720 \times 10^{12}$     |
| $O_4^-$          | $1.054 \times 10^{10}$      | $1.045 \times 10^{11}$      | $9.735 \times 10^{11}$     |
| $O_2^+ \cdot O_2$| $2.515 \times 10^{10}$      | $2.506 \times 10^{11}$      | $2.401 \times 10^{12}$     |
| $NO_2^+$         | $1.132 \times 10^{10}$      | $1.140 \times 10^{10}$      | $1.212 \times 10^{11}$     |
| $O_2^+$          | $1.138 \times 10^8$         | $1.168 \times 10^9$         | $1.451 \times 10^{10}$     |
| $O_2^+ \cdot N_2$| $3.055 \times 10^{12}$      | $3.135 \times 10^7$         | $3.888 \times 10^{18}$     |
| $NO^+$           | $2.446 \times 10^5$         | $2.316 \times 10^7$         | $1.577 \times 10^9$        |
| $NO^+ \cdot N_2$ | $5.464 \times 10^4$         | $5.175 \times 10^6$         | $3.521 \times 10^8$        |
| $N_2^+$          | $4.952 \times 10^4$         | $4.451 \times 10^6$         | $4.948 \times 10^8$        |
| $N_4^+$          | $3.406 \times 10^4$         | $3.406 \times 10^6$         | $3.400 \times 10^8$        |
| $N_4^+$          | $3.095 \times 10^3$         | $3.095 \times 10^5$         | $3.094 \times 10^7$        |
| $N_4^+$          | $8.356 \times 10^2$         | $8.355 \times 10^4$         | $8.347 \times 10^6$        |
| $O^+$            | $2.287 \times 10^2$         | $2.287 \times 10^4$         | $2.286 \times 10^6$        |

supplemented by the Poisson equation:

$$\Delta \phi = -4\pi e \left( n_i - n_2 - n_4 - n_e \right).$$

Here $j_\sigma = -\text{sign} (e_\sigma) \mu_\sigma \kappa_\sigma \nabla \phi - D_\sigma \nabla n_\sigma$ are the fluxes of the corresponding particle type ($\sigma = e, i, 2, 4$); $e_\sigma$ is the charge of the $\sigma$-particle, $e$ is the elementary charge ($e_i = e, e_e = e_2 = e_4 = -e$); $n_\sigma$ is the number density of electrons ($\sigma = e$), of positive ions O$^+$ ($\sigma = i$), of negative ions O$^-$ and O$^{-}_4$; $\mu_\sigma$ and $D_\sigma$ are the mobility and diffusion coefficients respectively; $\beta_\sigma$ is the recombination coefficient of electrons and ions O$^+_1$; $\beta_{i2}$ and $\beta_{i4}$ are the recombination coefficients of ions O$^+_1$ with O$^-_1$ and O$^{-}_4$ respectively; $\alpha$ is the attachment coefficient; $\nu_{24}$ and $\nu_{42}$ are the coefficients of conversion of the two-atom ions into the four-atom ones and vice versa.

The system of equations (1) and (2) is solved by linearization and three-dimensional Fourier transform. The screening potential around a charged particle is found to be a superposition of four exponentials:

$$\phi (r) = \frac{e q}{r} \sum_{j=1}^{4} C_j \exp (-k_{shj}r).$$

Calculations reveal that complex constants appear in the limited region of $Q_{\text{ion}}$ values. In this case the potential takes the form:

$$\phi (r) = \frac{e q}{r} \left\{ C_1 \exp (-k_{sh1}r) + C_2 \exp (-k_{sh2}r) + \exp (-k_3 r) \left[ Q_3 \cos (k_4 r) + Q_4 \sin (k_4 r) \right] \right\},$$

where $k_3 = \frac{1}{2} \left( k_{sh3} + k_{sh4} \right)$, $k_4 = \frac{1}{2} \left( k_{sh4} - k_{sh3} \right)$, $Q_3 = \frac{1}{2} \left( C_3 + C_4 \right)$, $Q_4 = \frac{1}{2} \left( C_4 - C_3 \right)$.

3. The comparison of the numerical and analytical calculations

The physical meaning of the screening constants is established within the analytical solution of particular cases of the system of equations (1) and (2). It is revealed, that the first constant is
Figure 1. The comparison of the numerical and analytical calculation of the screening constants. The real parts of the screening constants are found by numerical calculations: $1 - k_{sh1}, 2 - k_{sh2}, 3 - k_{sh3}, 4 - k_{sh4}$; symbols represent their approximate values: $\circ - k_D, \triangle - k_s, \diamond - k_{con}, \square - k_{e2}$.

Figure 2. The dependance of screening constants on $Q_{ion}$ in the crossing region of 3rd and 4th constants. $1 - k_{sh3}, 2 - k_{sh4}, 3 - k_s, 4 - k_{e2}, 5 -$ imaginary part of the screening constants $\kappa_4$.

the inverse Debye length:

$$k^2_D = k^2_{D_e} + k^2_{D_i} + k^2_{D2} + k^2_{D4},$$

where $k^2_{D\sigma} = 4\pi e^2n_{\sigma0}/T_{\sigma}$ and $n_{\sigma0}$ is the undisturbed number density of plasma components. The second one is the inverse length passed by positive and negative ions and electrons due to ambipolar diffusion in the characteristic recombination time:

$$k^2_s \approx \beta_{ei}n_{e0}(D_1^{-1} + D_2^{-1}) + \beta_{ii2}n_{20}(D_1^{-1} + D_2^{-1}) + \beta_{ii4}n_{40}(D_1^{-1} + D_4^{-1}).$$

The third one is the inverse diffusion length of negative ions in their conversion time into each other:

$$k_{con} \approx \nu_{24}D_2^{-1} + \nu_{42}D_4^{-1}.$$  

The fourth constant is defined by the electron attachment and recombination of electrons and diatomic oxygen ions:

$$k^2_{e2} \approx (\alpha + \beta_{ei}n_{i0})D_2^{-1} + \beta_{ii2}n_{20}D_2^{-1}.$$  

The comparison of the numerical and analytical results of the screening constants is shown in figure 1.

Figure 1 shows the curve crossing in two regions of gas ionization rate: $1.8 - 3.4 \times 10^{10}$ and $(0.4 - 1.1) \times 10^{14}$ cm$^{-3}$s$^{-1}$. It means that two of four screening constants have coincident real parts. This coincidence yields the complex constants. The imaginary part $\kappa_4$ of constants $k_{sh3}$ and $k_{sh4}$ is shown in figure 2 as the function of the air ionization rate.

According to the work [2] the electron and ion fluxes are approximately expressed in terms of their undisturbed number densities $n_{\sigma0}$ far from microparticle as follows:

$$J_{\sigma0} = -\frac{\beta_{L\sigma}n_{\sigma0}z_{\sigma}q}{1 - \exp(z_{\sigma}e^2q/T_{\sigma}r_0)}, \quad (5)$$
Figure 3. The plasma component fluxes on a microparticle versus air ionization rate $Q_{\text{ion}}$: 1 – electrons, 2 – positive ions $O_2^+$, 3 and 4 – negative ions $O_2^-$ and $O_4^-$, respectively. Solid curves are numerical calculations, dotted ones are analytical estimations (5).

Figure 4. The reduced potential distribution $\Psi = \phi(r)/eq \times r (1 + k_{sh4}r_0)e^{k_{sh4}(r-r_0)}$ around a dust particle at $Q_{\text{ion}} = 10^{13} \text{cm}^{-3}\text{s}^{-1}$ (curve 1), $10^{14} \text{cm}^{-3}\text{s}^{-1}$ (curve 2) and $10^{15} \text{cm}^{-3}\text{s}^{-1}$ (curve 3). The solid lines correspond to the numerical calculations, the dotted ones with symbols correspond to the sum of (7) with $k_{sh4}$ and the Debye exponentials with $k_{sh2}$ and $k_{sh3}$ (for 1 and 3), and the sum of (8), (9) and the Debye exponential with $k_{sh2}$ (for 2).

where $\beta_{L\sigma} = 4\pi e\mu_\sigma$ is the coefficient of Langevin recombination of $\sigma$-type particles on slow dust particles with charge $q = -z_\sigma$; $z_\sigma = 1$ for positive ions $\sigma = i$ and $z_\sigma = -1$ for $\sigma = e, 2, 4$. The comparison of analytical estimation (5) with numerical calculations is shown in figure 3.

Figure 3 shows that the equation (5) gives underestimated values due to assumption of flux uniformity that is not valid because of attachment, recombination and conversion processes. Nevertheless equation (5) gives proper qualitative character of dependencies. Although the electron number density is much less than the number density of oxygen negative ions (see table 1), the electron flux dominates in microparticle charging at gas ionization rates higher than $10^{14} \text{cm}^{-3}\text{s}^{-1}$ due to high electron mobility.

4. The potential distribution around microparticle

For the verification of the asymptotic theory equations (1) and (2) are solved numerically using finite-difference method with the following boundary conditions:

$$n_\sigma|_{r=r_0} = 0; \quad n_\sigma|_{r=a_d} = n_\sigma 0; \quad E|_{r=r_0} = \frac{q}{r_0^2}; \quad E|_{r=a_d} = 0; \quad \phi|_{r=a_d} = 0. \quad (6)$$

Here $n_d$ is the dust particle number density, $a_d = (4\pi n_d/3)^{-1/3}$ is the Wigner–Seitz cell radius. The solution of the equation (2) in a finite cell with boundary conditions (6) takes the form [3]:

$$\frac{q}{r} [B_1 \exp(-k_{sh4}r) + B_2 \exp(k_{sh4}r)] + B_3 \quad (7)$$
in case of real screening constants and

\[
\frac{q}{r} [G_1 \exp(-\kappa_3 r) + G_2 \exp(\kappa_3 r)] \cos(\kappa_4 r) + G_3,
\]

\[
\frac{q}{r} [K_1 \exp(-\kappa_3 r) + K_2 \exp(\kappa_3 r)] \sin(\kappa_4 r) + K_3,
\]

in case of complex ones. The coefficients \( B_i, G_i, K_i \) (\( i = 1–3 \)) are found from the boundary conditions for the potential and the electric field strength (6).

The comparison of the potential analytical and numerical calculations at three different air ionization rates is shown in figure 4. This figure reveals that the expressions (7)–(9) are in good agreement with the numerical calculations. Note that numerical calculations identify properly only two smallest constants \( k_{sh3} \) and \( k_{sh4} \), and the accuracy of the third-smallest constant \( k_{sh2} \) definition is rather low although this constant becomes apparent at short distances (the potential growth at \( r < 0.01 \) cm), where strong nonlinearity takes place as well as in the \( k_{sh1} \) appearance region. Thus the values of \( k_{sh1} \) and \( k_{sh2} \) obtained within the linear theory are almost physically meaningless.

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