Speeding up adiabatic state conversion in optomechanical systems

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Abstract
Adiabatic state conversion in optomechanical systems requires the evolution process to be very slow, which limits its application in quantum information processing. In this paper, we explicitly show that speeding up adiabatic state conversion in optomechanical systems can be implemented. For the case that each cavity is driven far from the red-detuned mechanical sideband, the high-efficiency population transfer between cavity modes is achieved via an additional Hamiltonian $\hat{H}_{\text{ad}}(t)$, whereas the population of the mechanical resonator can be effectively inhibited. In the meanwhile, in the case of each cavity driven near the red-detuned mechanical sideband, the population transfers can be realized by varying the modified optomechanical couplings without an additional Hamiltonian. Numerical simulation demonstrates that the transfer is robust to the system decays.

Keywords: quantum information, optomechanical systems, speeding up adiabatic state conversion

(Some figures may appear in colour only in the online journal)

1. Introduction

Optomechanical systems, which are composed of an optical (or microwave) cavity and a mechanical resonator, explore the interaction between light and mechanical motion. This system has shown the advantages in quantum information processing [1]. Nontrivial quantum phenomena have been studied in optomechanical systems, such as near-ground-state cooling [2–4], realization of strong coupling [5, 6], and generation of macroscopic quantum superposition [7].

Quantum state conversion in optomechanical systems has been studied [8, 9]. In these schemes, the fidelity is limited due to cavity damping, thermal noise in the mechanical mode, and accuracy of the pump pulses. In order to overcome the effect of thermal noise on the transfer fidelity, the adiabatic quantum state transfer in optomechanical systems was proposed [10–12]. During this process, quantum states were preserved in a mechanical dark mode with negligible excitation to the mechanical mode.

However, the adiabatic state conversion requires that the process is very slow, and thus the decoherence effect on the system should be considered. How to perform fast and high-fidelity quantum state transfer in an optomechanical system is an open question. There are three kinds of approaches, Lewis–Riesenfeld invariant-based inverse engineering [13–18], transitionless quantum driving [19–27] and dressed states [28–30], are widely applied to speed up adiabatic state conversion.

In this paper, we explicitly show that speeding up adiabatic state conversion in optomechanical systems is achieved in both cases. When each cavity is driven far from the red-detuned mechanical sideband, we apply transitionless quantum driving to realize fast population transfer in optomechanical systems. The quantum states population between different cavity modes can be converted by varying the effective optomechanical couplings nonadiabatically, while the population of the mechanical resonator is effectively inhibited. When each cavity is driven near the...
red-detuned mechanical sideband, we can also realize fast population transfer by the design of the coupling coefficients. We find that the damping of the mechanical mode is effectively inhibited when two optical cavities have equal damping rates.

2. Model

We study an optomechanical system, which is composed of two cavity modes and one mechanical mode coupling via optomechanical forces. After executing the standard linearization procedure, the effective Hamiltonian for the proposed coupled system can be expressed under the rotating-wave approximation in the case of \( \hbar = 1 \) [10, 31]:

\[
H_0 = \sum_{i=1,2} \Delta_i a_i^\dagger a_i + g_i(t)(a_i^\dagger b_i + a_i b_i^\dagger) + \omega_m b_i^\dagger b_i,
\]

where \( a_i^\dagger, a_i \) \((i = 1, 2)\) and \( b_i^\dagger, b_i \) are the creation (annihilation) operators for the \( i \)th cavity mode and the mechanical mode, respectively. \( \Delta_i \) is the detuning between the cavity mode and external driving, \( g_i(t) \) is the effective linear coupling strength and \( \omega_m \) is the mechanical frequency.

We assume the detuning between the cavity mode and external driving are equal, i.e. \( \Delta_1 = \Delta_2 = \Delta \). We perform the transformation \( V(t) = \exp(i\Delta t\sum_i a_i^\dagger a_i + b_i^\dagger b_i) \) on this coupled system. In the rotating frame defined by \( V(t) \), the transformed Hamiltonian becomes

\[
H_f = V^\dagger(t)H_0V(t) - iV^\dagger(t)V(t) = \sum_{i=1,2} g_i(t)(a_i^\dagger b_i + a_i b_i^\dagger) + \delta b_i^\dagger b_i,
\]

where \( \delta = \omega_m + \Delta \) is the detuning.

We define the basis vectors \( |v_1\rangle \equiv [1, 0, 0]^T \), \( |v_2\rangle \equiv [0, 1, 0]^T \) and \( |v_3\rangle \equiv [0, 0, 1]^T \), corresponding to states with one excitation of the optical cavity mode \( a_1 \), the mechanical mode \( b \), and the optical cavity mode \( a_2 \), respectively. Under the subspace formed by \( \{ |v_1\rangle, |v_2\rangle, |v_3\rangle \} \), the system Hamiltonian becomes

\[
H_{\text{int}} = \begin{pmatrix}
0 & g_1(t) & \delta \\
g_1(t) & 0 & g_2(t) \\
\delta & g_2(t) & 0
\end{pmatrix}.
\]

3. Adiabatic state conversion

The eigenvalues of the Hamiltonian (3) are given by \( E_1 = 0 \), \( E_2 = g_0 \cot \varphi \) and \( E_3 = -g_0 \tan \varphi \) with the \( g_0 = \sqrt{\Delta^2 + \delta^2} \) and \( \cot \varphi = g_0/(\sqrt{(\delta/2)^2 + g_0^2} - \delta/2) \). Their corresponding eigenstates are given by

\[
|\lambda_1(t)\rangle = \cos \theta |v_1\rangle - \sin \theta |v_2\rangle ,
|\lambda_2(t)\rangle = \sin \theta \sin \varphi |v_1\rangle + \cos \varphi |v_2\rangle + \cos \theta \sin \varphi |v_3\rangle ,
|\lambda_3(t)\rangle = \sin \theta \cos \varphi |v_1\rangle - \sin \varphi |v_2\rangle + \cos \theta \cos \varphi |v_3\rangle ,
\]

where the angle \( \theta \) is defined by \( \tan \theta = g_1/g_2 \). The eigenstate \( |\lambda_1(t)\rangle \) expresses a mechanical dark mode that only involves the cavity modes. When the adiabatic condition [\( d\theta/dt \ll |g_0| \)] is satisfied, the population transfer of the cavity modes is realized by the dark state. The quantum state to be transferred is initially stored in the cavity mode \( a_1 \). Then, \( g_1(t) \) is adiabatically decreasing and \( g_2(t) \) is adiabatically increasing. The population transfer from cavity mode \( a_1 \) to mode \( a_2 \) is achieved [10, 11].

If the \( g_1(t) \) and \( g_2(t) \) are not adiabatic at all, the population cannot be completely transferred from cavity mode \( a_1 \) to mode \( a_2 \). As an example, we choose the time-dependence coupling strengths, which are expressed by [10]:

\[
g_1(t) = g_0 \sin(t),
g_2(t) = -g_0 \cos(t).
\]

Figure 1(a) shows the relation of coupling coefficients \( g_1(t) \) and \( g_2(t) \) with time \( t \). We set the detuning \( \delta = 2\pi \times 10 \text{ MHz} \) and the coupling strength \( g_0 = 2\pi \times 1 \text{ MHz} \) to guarantee large detuning \( \delta \gg |g_1, g_2| \). Under the subspace

![Figure 1](http://example.com/figure1.png)

**Figure 1.** (a) Evolution of coupling coefficients \( g_1 \)(dashed-dotted green line) and \( g_2 \)(solid blue line). (b) The quantum states populations of the cavity mode \( a_1 \)(dashed blue line), cavity mode \( a_2 \)(dashed-dotted red line), and mechanical mode \( b \)(solid green line).
\{ |v_1\rangle, |v_2\rangle, |v_3\rangle \}, we numerically solve the Schrödinger equation of the Hamiltonian (3), i.e. \( \frac{\partial}{\partial t} \psi(t) = \mathcal{H}_{\text{int}} \psi(t) \), where \( \psi(t) = \sum_{i=1}^3 D_i(t) |v_i\rangle \) with normalization coefficients \( \sum_i |D_i(t)|^2 = 1 \). We introduce the symbols \( P_1 = |D_1(t)|^2 \), \( P_2 = |D_2(t)|^2 \) and \( P_3 = |D_3(t)|^2 \), which represent the populations of the basis vectors \( |v_1\rangle \), \( |v_2\rangle \) and \( |v_3\rangle \), respectively. The quantum information transfer can be depicted by the population’s change of quantum states. Figure 1(b) shows the quantum states populations of the system with time \( t \). From figure 1(b), we can see that the population \( P_2 \) of the cavity mode \( a_2 \) cannot reach 1. That is, if the information is encoded in cavity mode \( a_1 \) initially, it is not transferred to mode \( a_2 \) completely. Therefore, we should consider how to improve the population \( P_2 \) and realize perfect quantum information transfer.

4. Shortcut to adiabatic passage

4.1. Each cavity is driven far from the red-detuned mechanical sideband

In the case that each cavity is driven far from the red-detuned mechanical sideband, i.e. \( \delta \gg \{g_1, g_2\} \). We discuss how to implement the fast converting quantum states via transitionless quantum driving. The theory of transitionless quantum driving provides a recipe for finding a suitable supplementary Hamiltonian (counterdiabatic term) which drives the eigenstates of the original Hamiltonian with no transitions in a desired time [20]. In order to implement the fast converting quantum states by transitionless quantum driving, we need to construct a supplementary Hamiltonian \( \mathcal{H}_d(t) \). This supplementary Hamiltonian steers the dynamics along the instantaneous eigenstates \( |\lambda_m(t)\rangle \) without transitions among them and without phase factors, and it is described by \( \mathcal{H}_d(t) = i\sum_m \partial_t |\lambda_m(t)\rangle \langle \lambda_m(t)| \) which takes the form [20, 32]

\[
\mathcal{H}_d(t) = \begin{pmatrix}
0 & 0 & i\bar{\delta}
0 & 0 & 0
-i\bar{\delta} & 0 & 0
\end{pmatrix},
\tag{6}
\]

where \( \bar{\delta} = (g_1 g_2 - g_1 g_2)/g_0^2 \). So, the total Hamiltonian is

\[
\mathcal{H}(t) = \mathcal{H}_{\text{int}} + \mathcal{H}_d(t)
\]

\[
= \begin{pmatrix}
g_1(t) & 0 & 0
g_2(t) & \delta & g_2(t)
g_3(t) & 0 & 0
\end{pmatrix}
+ \begin{pmatrix}
0 & 0 & i\delta
0 & 0 & 0
-i\delta & 0 & 0
\end{pmatrix}.
\tag{7}
\]

Under the subspace formed by \( \{ |v_1\rangle, |v_2\rangle, |v_3\rangle \} \), the total Hamiltonian \( \mathcal{H}(t) \) corresponding to the Schrödinger equation can be numerically solved. Here, the parameters are same as figure 1(b). Figure 2 shows the evolution of the population of the system by the total Hamiltonian \( \mathcal{H}(t) \). From figure (2), we can see that the perfect population transfer from the cavity mode \( a_1 \) to \( a_2 \) can be achieved, while the population of the mechanical resonator is effectively inhibited. Therefore, this scheme can be applied to implement a quantum information transfer between different cavity modes. Under the large detuning conditions \( \delta \gg \{g_1, g_2\} \), there is no energy exchange between the mechanical mode and the cavity modes. The Hamiltonian (6) indicates that there should be a direct transition between cavities \( a_1 \) and \( a_2 \), and the transitions between the cavity modes and the mechanical mode are forbidden.

The transitionless quantum driving provides a fast and robust approach to quantum state transfer. However, ‘there ain’t no such thing as a free lunch.’ Recently, a family of energetic cost functionals of the transitionless quantum driving was introduced in [33]. The cost of the implementation of fast rotation gates was analyzed [34]. The relationship between the quantum speed limit and energetic cost in shortcuts to adiabaticity was studied [35]. The energetic cost of the transitionless quantum driving can be written in general as [33–36]

\[
C = \frac{1}{T} \int_0^T ||\mathcal{H}(t)|| dt,
\tag{8}
\]

with the Frobenius norm \( ||A|| = \sqrt{\text{Tr}(A^\dagger A)} \), where \( A \) is an operator. Due to the \( \text{Tr}(\{\mathcal{H}_{\text{int}}(t), \mathcal{H}_d(t)\}) = 0 \), we obtain

\[
C = \frac{1}{T} \int_0^T \sqrt{\text{Tr}(\mathcal{H}_{\text{int}}^2(t) + \mathcal{H}_d^2(t))} dt
\]

\[
= \frac{1}{T} \int_0^T \sum_m [E_m^2 + \mu_m(t)] dt,
\tag{9}
\]

where \( E_m \) are the energies of the adiabatic Hamiltonian and \( \mu_m(t) = \langle \partial_t |\lambda_m(t)\rangle |\partial_t |\lambda_m(t)\rangle - |\langle \lambda_m(t)|\partial_t |\lambda_m(t)\rangle |^2 \). The instantaneous cost is given by

\[
\partial_t C = \sqrt{g_0^2 \left( \tan^2 \phi + \cot^2 \phi \right) + 2\bar{\delta}^2},
\tag{10}
\]

where the first term is the energy cost of the adiabatic process; the last term is the energy cost due to the transitionless quantum driving. Equation (9) shows an increase in the instantaneous cost to speed up adiabatic evolutions compared to their adiabatic counterparts. That is to say, the faster evolutions cost more energy than slower dynamics. Figure 3 illustrates the behavior of \( \partial_t C/g_0 \) versus the \( g_0 \) for different \( \bar{\delta} \). It is apparent that the \( \partial_t C/g_0 \) is inversely proportional to the increasing \( g_0 \). The cost of speeding up adiabatic process recovers the cost of the adiabatic process with increasing \( g_0 \).
The expressions \( g_m \) and \( g_c \) take equations (4) and (5), respectively. The Hamiltonian (11) corresponding to the Schrödinger equation can be numerically solved. Figure 4 shows that the evolution of populations of the system with the \( g_1(t) \) and \( g_2(t) \) are nonadiabatical. It is obvious the information is not completely transferred from cavity mode \( a_1 \) to mode \( a_2 \). Therefore, we should consider how to improve the population \( P_2 \) while reduce the \( P_m \). We can implement the population transfer from cavity mode \( a_1 \) to \( a_2 \) by the design of the coupling coefficients without an additional Hamiltonian. The total Hamiltonian \( \hat{\mathcal{H}} \) can be rewritten as

\[
\hat{\mathcal{H}} = g_1 \hat{\Xi}_1 + g_2 \hat{\Xi}_2 + \dot{\theta} \hat{\Xi}_3,
\]

where the matrices are

\[
\hat{\Xi}_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \hat{\Xi}_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \hat{\Xi}_3 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \end{pmatrix},
\]

which satisfy the commutation relations \([\hat{\Xi}_1, \hat{\Xi}_2] = i \hat{\Xi}_3, [\hat{\Xi}_2, \hat{\Xi}_3] = i \hat{\Xi}_1\) and \([\hat{\Xi}_3, \hat{\Xi}_1] = i \hat{\Xi}_2\). To get rid of the counterdiabatic term, we introduce the unitary transformation \( U(t) = \exp[-i\phi(t)\hat{\Xi}_2]\), so that the total dynamics matrix becomes

\[
\hat{\mathcal{H}} = U^\dagger \hat{\mathcal{H}} U = U^\dagger (U^\dagger U) U = \begin{pmatrix} g_1 & g_2 & \Omega_d \end{pmatrix}_d \begin{pmatrix} g_1 & g_2 & \Omega_d \end{pmatrix},
\]

where the parameters are

\[
g_1 = g_1 \cos \phi(t) + \dot{\theta} \sin \phi(t), \quad g_2 = g_2 - \dot{\theta} \phi(t), \quad \Omega_d = \dot{\theta} \cos \phi(t) - g_1 \sin \phi(t).
\]

If \( \Omega_d = 0 \), we obtain \( \phi(t) = \arctan(\theta/g_1) \). So, the modified coupling coefficients are

\[
g_1 = \sqrt{g_1^2 + \dot{\theta}^2}, \quad g_2 = g_2 - \dot{\theta} \phi(t),
\]

where \( \phi(t) = \frac{\theta_0 - \theta(t)}{g_1} \). Figure 5(a) shows the modified coupling coefficients \( g_1 \) and \( g_2 \) with evolution time. If the \( g_1(t) \) and \( g_2(t) \) of the Hamiltonian (11) are replaced by \( g_1 \) and \( g_2 \). Then, we obtain a new Hamiltonian

\[
\hat{\mathcal{H}}_{\text{new}} = \begin{pmatrix} 0 & g_1 & 0 \\ g_1 & 0 & g_2 \\ 0 & g_2 & 0 \end{pmatrix}.
\]

Through solving Hamiltonian (19) corresponding to the Schrödinger equation, we plot the population transfer of quantum states by using \( g_1 \) and \( g_2 \) in figure 5(b). The maximal population of the mechanical resonator is \( P_m = 0.0383 \). By comparison to figure 4, the population \( P_2 \) of cavity mode \( a_2 \) is improved and the mechanical resonator population \( P_m \) is reduced. Therefore, to modify the coupling coefficients is an effective method to implement the fast converting quantum states from cavity mode \( a_1 \) to \( a_2 \).

We also considered the damping of the system; \( \gamma \) and \( \kappa_i \) can denote the damping rate of the mechanical resonator and cavity \( i \), respectively. We employ the quantum trajectory method [37], the evolution of the system is governed by the

![Figure 3](image-url)  
Figure 3. The \( \partial C/g_0 \) versus the \( g_0 \) for different \( \theta \) with the \( \tan \varphi = 1 \): \( \theta = 0 \) (solid black line), \( \theta = 0.05 \times 2 \pi \text{ MHz} \) (dashed blue line), and \( \theta = 0.1 \times 2 \pi \text{ MHz} \) (dashed-dotted red line).

![Figure 4](image-url)  
Figure 4. Simulation of the quantum state populations of the cavity mode \( a_1 \) (dashed blue line), cavity mode \( a_2 \) (dashed-dotted red line), and mechanical mode \( b \) (solid black line). The following physical parameters have been used: \( g_1(t) = g_0 \sin(t) \), \( g_2(t) = -g_0 \cos(t) \) and \( g_0 = 2 \pi \times 1 \text{ MHz} \).
adiabatic state conversion is achieved by equations (17) and (18). The influence of system damping on the population number of cavity mode $a_2$ is shown in Figure 6. The population reduces with increasing of the damping rates. It can be seen that the population is robust to the damping of mechanical resonator and sensitive to the damping of the cavity modes. The effective quantum information transfer can be realized by the design coupling coefficients with low damping rates.

This is an non-Hermitian Hamiltonian. The speeding up of the adiabatic state conversion is achieved by equations (17) and (18). The influence of system damping on the population number of cavity mode $a_2$ is shown in Figure 6. The population reduces with increasing of the damping rates. It can be seen that the population is robust to the damping of mechanical resonator and sensitive to the damping of the cavity modes. The effective quantum information transfer can be realized by the design coupling coefficients with low damping rates.

conditional Hamiltonian

$$
\mathcal{H}_c = \begin{pmatrix}
-\frac{i\kappa_1}{2} & g_1 & 0 \\
g_1 & -\frac{i\gamma}{2} & g_2 \\
0 & g_2 & -\frac{i\kappa_2}{2}
\end{pmatrix}.
$$

(20)

5. System-environment coupling

To describe the system-environment coupling, the noise operators $a_{in}^{(i)}(t)$ for the $i$th cavity mode and $b_m(t)$ for the mechanical mode are introduced. We choose the noise correlations $\langle a_{in}^{(i)}(t)a_{in}^{(i)\dagger}(t')\rangle = \delta(t - t')$ and $\langle b_m(t)b_m^{\dagger}(t')\rangle = (n_{th} + 1)\delta(t - t')$ with $n_{th}$ being the thermal phonon number of the mechanical mode [10]. When each cavity is driven near the red-detuned mechanical sideband, i.e. $\omega_m = -\Delta$, the Langevin equation in the interaction picture can be written as

$$
i\frac{d}{dt}A(t) = \mathcal{N}(t)A(t) + i\sqrt{K}A_m(t),
$$

(21)

where the operator $A_m(t) = [a_{in}^{(1)}(t), b_m(t), a_{in}^{(2)}(t)]^T$

$$
\mathcal{N}(t) = \begin{pmatrix}
-\frac{i\kappa_1}{2} & g_1 & 0 \\
g_1 & -\frac{i\gamma}{2} & g_2 \\
0 & g_2 & -\frac{i\kappa_2}{2}
\end{pmatrix},
$$

and the diagonal matrix $K = \text{diag}(\kappa_1, \gamma, \kappa_2)$. The system operators can be derived as [11, 38]

$$
A(t) = U(t)e^{-i\int_0^t dt'\mathcal{E}(t')U^{-1}(0)\mathcal{A}(0)}
+ \int_0^t dt'U(t)e^{-i\int_0^{t'} dt''\mathcal{E}(t'')}U^{-1}(t')\sqrt{K}A_m(t').
$$

(22)

Here $\mathcal{E}(t) = \text{diag}(E_1, E_2, E_3)$ and $U(t) = [\lambda_1, \lambda_2, \lambda_3]$, with $E_i$ being the eigenvalues and $\lambda_i$ the eigenstates of the Hamiltonian (10). After the time $\tau$, we derive

$$
a_2(\tau) = e^{f(0,\tau)}a_1(0) + \int_0^\tau dt'e^{-f(t',\tau)}\beta_1(t'),
$$

(23)

where

$$
f(\tau) = \int_0^\tau dt\left[ \frac{g_1^2(t')}{{2g_0^2}} + \frac{g_2^2(t')}{{2g_0^2}} \right].
$$

(24)
Figure 7. Fidelity $F$ for $\alpha = 0.1$ (dashed red line), $\alpha = 0.2$ (dashed-dotted black line) and $\alpha = 0.3$ (solid blue line).

and

$$\beta(t) = \frac{g_1}{\sqrt{\gamma_1}} a_1^{(1)}(t) + \frac{g_2}{\sqrt{\gamma_2}} a_2^{(2)}(t) + \frac{i(\kappa_1 - \kappa_2)}{2\gamma_0} g_2 g_2 \sqrt{\gamma_2} b(t). \quad (25)$$

If coupling coefficients $g_1$ and $g_2$ are replaced by the $g_{1t}$ and $g_{2t}$, we can implement the speeding up of the adiabatic state conversion. From equation (24), we can see that the damping of the mechanical mode is inhibited for two cavity modes which have equal damping rates, i.e. $\kappa_1 = \kappa_2$.

The fidelity of the state conversion can be defined as

$$F = \left| \sqrt{\frac{\rho_n}{\rho_0}} \right|^2,$$  \hspace{1cm} (26)

where $\rho_n$ is the initial density matrix in cavity mode $a_1$ and $\rho_0$ is the final density matrix in cavity mode $a_2$.

For the coherent state $|\alpha\rangle$, the fidelity is [10]

$$F \approx (1 - f_1(2n_{th} + 1)2n_{th})[1 - f(0, \tau)|\alpha|^2], \quad (27)$$

where $f_1 = \int_{0}^{\gamma_0} dt e^{-2(\gamma/2)t} \left[ \frac{(\kappa_1 - \kappa_2)^2 \gamma_0^2}{2\gamma} \right] \gamma n_{th}$.

When $\kappa_1 = \kappa_2 = \kappa$, the fidelity becomes

$$F \approx 1 - f(0, \tau)|\alpha|^2. \quad (28)$$

Figure 7 shows the relation among the fidelity and the $\kappa\tau$ with the different $\alpha$. The fidelity decreases linearly with the cavity damping rates. Also, the fidelity decreases with increasing $\alpha$. We can see that the high-fidelity quantum states transfer between cavity modes $a_1$ and $a_2$ can be implemented with low damping rates and a small $\alpha$.

6. Discussion and conclusion

The values of the experimental parameters are given to show the feasibility of our scheme. Many studies in optomechanical systems have made great advancements during the past few years. Based on reported literature, the coherent conversion of a microwave photon to a mechanical state has been thoroughly studied and implemented into [40]. The frequency of the microwave resonator is $\omega_m = 2\pi \times 7.5 \text{ GHz}$ and the frequency of the drumhead mode of the membrane is $\omega_m = 2\pi \times 10.5 \text{ MHz}$ with a linewidth $\gamma = 2\pi \times 35 \text{ Hz}$ [40]. Laser cooling of a nanomechanical oscillator into the quantum ground state has been achieved [3]. The mechanical mode has the resonance frequency $\omega_m = 2\pi \times 3.68 \text{ GHz}$ with an intrinsic damping rate $\gamma = 2\pi \times 35 \text{ kHz}$ [3]. The optomechanical coupling strength $g_t = 2\pi \times 910 \text{ kHz}$ [3]. The cavity mode has the resonance frequency $\omega_c = 2\pi \times 195 \text{ THz}$ [3]. So, the condition $g_t \gg \gamma$ is satisfied.

In addition, the expression of the effective linear coupling strength is $g = G_c \sqrt{\gamma_c}$, where $G_c$ is the strength of the single-photon coupling and $n_i$ is the photon number inside the cavity [38]. This strength can be adjusted from the weak coupling limit to the strong coupling limit by varying the driving power [38].

In summary, we have proposed two approaches to speed up the adiabatic state conversion in an optomechanical system. The effective population transfer between cavity mode $a_1$ and $a_2$ can be implemented. The energetic cost and decay of the system were analyzed. Also, this scheme has potential application in generating entanglement and realizing quantum logic gates.

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