Equivalent isentropic expansion efficiency of real fluid subject to concurrent pressure drop and heat transfer

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Abstract. Concurrent pressure drop and cooling of a super-critical or sub-cooled liquid stream can have the same effect as adiabatic expansion even though there is no work extraction. A practical implementation is as straightforward as counter-flow heat exchange with a colder fluid. The concurrent pressure drop need not be continuous with respect to the heat exchange, but may occur in a step-wise manner, between heat exchange. Two aspects of this effect of pressure drop with heat transfer are examined: a thermodynamic and a practical, process equivalent isentropic expansion efficiency. This real fluid phenomenon is useful to understand in applications where work extraction is either not practical or has not been developed. A super-critical helium supply, often around 3 bar and 4.5 K, being ultimately used as a superfluid (usually around 1.8 to 2.1 K) to cool a Niobium superconducting radio frequency cavity or a superconducting magnet is one such particular application. This paper examines the thermodynamic nature of this phenomenon.

1. Introduction
Superconducting devices used in modern accelerators are often required to be cooled using Helium II, operating around 1.8 to 2.1 K (16 to 41 mbar). In this paper, for the purposes of discussion, we will regard the Helium II to be at close to 2 K, or around 31 mbar. Contrary to popular terminology, the Helium II is not ‘produced’ by an isenthalpic process (i.e., Joule-Thompson throttling), any more than light is ‘produced’ by a light switch. The Helium II is produced by the refrigeration process (involving compression, heat exchange, and work extraction), and is physically manifested only within the device being cooled (e.g., superconducting radio frequency (SRF) Niobium cavity, superconducting magnet), which we shall refer to as the ‘load’. Due to the superfluid nature of Helium II, its presence in the refrigeration process, ‘outside’ of the device where it is being used (i.e., the ‘load’), would incur additional exergy loss. Figure 1 depicts the basic elements in such a refrigeration process. Except for the 1.8 K helium refrigerator at the Stanford Linear Accelerator (SLAC) [1], there are no installations known to the authors that implemented work extraction below 4.5 K, which is the nominal temperature for the supply stream from the refrigeration system used to cool such devices. There is no quantitative measure or data provided in [1] regarding the performance of the expander, which was presumably the same type of expander discussed in [2], nor were any particulars regarding its operation given. Referring to figure 1, the 4.5 K cold box supplies positive pressure helium at an enthalpy (typically) close to saturated liquid at 1.25 bar. Small refrigerators will often supply saturated liquid. However, for reasons typically attributed to thermo-hydraulic stability, large refrigerators will normally supply helium to the load as a supercritical fluid; e.g., around 3 bar and 4.5 K. Between the temperatures of the supply stream...
from the 4.5 K cold box and the sub-atmospheric load (e.g., 2 K), there is only heat in-leak (from ambient), heat exchange between the positive pressure supply stream and the sub-atmospheric stream returning from the load, and isenthalpic expansion.

As discussed in [3, 4], if a significant pressure drop occurs in the supply stream of the 4.5 K to 2 K heat exchanger in figure 1, the enthalpy flux to the load will increase. For a nominal supply condition to the heat exchanger of 3 bar 4.5 K and a load pressure of 31 mbar (2.00 K), this corresponds to a load enthalpy flux increase of about 3.3% per 1 bar pressure drop in the supply stream across the heat exchanger. In fact, this pressure drop need not occur within the heat exchanger, but may occur in between successive heat exchanger sections. This has practical advantages if typical heat exchanger designs for this application are used, but requires a higher NTU’s (and a greater net thermal rating, (UA)), as compared to simultaneous supply stream pressure drop and heat transfer with the sub-atmospheric load return stream. The total pressure drop in the supply stream, concurrent (i.e., simultaneously or successively) with heat exchange to the sub-atmospheric stream returning from the load, which brings the outlet pressure below approx. 0.2 bar has been shown to be impractical and not optimum [5], and was earlier identified as such [3, 4].

A performance increase has also been realized with the counter-flow of super-critical helium (below 8 K) and saturated vapor at around 1.25 bar in the well-known Model 1400 and 1600 Linde (formerly, Koch) helium liquefiers.

Figure 1. Basic elements in a 2-K helium refrigeration process

2. ‘Pseudo’ Property – Local Equivalent Efficiency

Figure 2 is a pressure-enthalpy diagram of helium near the two-phase region, but is representative of typical pure single component fluid, except for the Helium-II transition. Note that the pressure (y-) axis is a logarithmic scale, which is typical for such diagrams. Also, for an ideal gas with a constant specific heat, the isotherms would be parallel to the y-axis and isentropes would be logarithmic, with the entropy decreasing as the pressure increases for a constant enthalpy. However, we are interested in the region shown, which is quite far from an ideal gas behavior, except for the sub-atmospheric vapor.

Consider the supply stream out of the 4.5 K to 2 K heat exchanger at a given temperature, e.g. 2.2 K. This is a common specification, keeping the supply stream just above the lambda transition to Helium-II (2.1768 K at 50.418 mbar). For a given range of pressures, at this initial temperature, there are an infinite number of ‘paths’. This range of pressures could be thought of as corresponding to the range of
pressures drops possible in the design (or intermediate throttling) of the aforementioned heat exchanger. Of the infinite number of paths, we are interested in two of these; isothermal and isentropic. Specifically, we are interested in the isentropic and polytropic (‘small stage’) efficiency between \((p_1, T_1)\) and \(p_2\). If the fundamental equation for the fluid is known, the former is readily obtainable, since it is simply an evaluation of state point properties; i.e., \(h_1 = h(p_1, T_1)\), \(s_1 = s(p_1, T_1)\), \(h_{2s} = h(p_2, s_1)\), \(h_{2T} = h(p_2, T_1)\), \(\Delta h_s = h_1 - h_{2s}\), \(\Delta h_T = h_1 - h_{2T}\), \(\eta_s = \Delta h_T / \Delta h_s\).

Making further inspection of figure 2, we consider three cases to examine the isentropes and isotherms over a range of pressures (i.e., \(p_1\) to \(p_2\)). These are shown in figure 3, keeping in mind the implied horizontal axis is enthalpy and the implied vertical axis is the logarithm of pressure. Note that we are not considering the superfluid region, which has a negative volume expansivity and would be similar to ‘case A’ in figure 3, except the indicated isentropes and isotherms would be swapped. It is apparent that the isentropic efficiency for case A is between 0 and 1, for case B it is zero, and for case C it is less than zero (negative). The points ‘1’, ‘2s’, and ‘2’ are shown for each of these three cases on figure 2; e.g., \(A[2s]\) is point ‘2s’ for case A. Note that points \(A[2s]\) and \(A[2]\) appear nearly coincident.

Using Maxwell’s relations we know that,

\[
\left(\frac{\partial h}{\partial p}\right)_{s} = \nu \\
\left(\frac{\partial h}{\partial p}\right)_{T} = \nu(1 - T \cdot \beta)
\]

Where, \(\nu\) is the specific volume \([m^3/kg]\), and \(\beta\) is the volume expansivity \([K^{-1}]\). These have a readily geometric meaning in figure 3 (except that the vertical-pressure axis not be linear, rather than logarithmic), as the isentrope and isotherm slope, respectively. Defining the ‘local’ isentropic efficiency with respect to the ‘local’ isotherm,

\[
\eta_s = \left(\frac{\partial h}{\partial p}\right)_{T} \left/ \left(\frac{\partial h}{\partial p}\right)_{s}\right. = 1 - T \cdot \beta
\]

We can see from figure 3, for case A, \(\eta_s\) is positive and non-zero, but less than unity (i.e., \(0 < \eta_s < 1\). For cases B and C, \(\eta_s\) can vary from positive, but less than unity, to a large negative value (i.e., \(\eta_s < 1\)). And for cases B and C, at some point, \(\eta_s\) is zero. From the above equation, it is apparent that for case A, the volume expansivity is, \(0 < \beta < T^{-1}\). If \(\beta \geq T^{-1}\), then \(\eta_s \leq 0\), which applies for cases B and C. Note that in figure 2, the dashed line, approximately parallel to the “TCL” (transposed critical line), indicates the locus of \(\eta_s = 0\).

The overall isentropic efficiency is integrated between points ‘1’ to ‘2’ as,

\[
\Delta h_s = \int_{p_1}^{p_2} \nu \cdot dp \\
\Delta h_T = \int_{p_1}^{p_2} \nu(1 - T \cdot \beta) \cdot dp \\
\bar{\eta}_s = \frac{\Delta h_T}{\Delta h_s}
\]

Where, \(dp\) indicates integration along constant \(s = s_1\), and \(dp_T\) indicates integration along a constant isotherm \(T\). But this isn’t necessary; just state point at 1 and pressure at point 2, is needed to evaluate the overall isentropic efficiency, as discussed previously.

The polytropic efficiency is defined as,

\[
\eta_p = \frac{1}{v} \left(\frac{dh}{dp}\right)_{s}
\]

For an isothermal path,

\[
\eta_{p(T)} = \frac{1}{v} \left(\frac{dh}{dp}\right)_{T}
\]

And using the Maxwell relation for \(\nu\), we can clearly see that at the state point \((p, T)\),
Where, $\eta$ is the ‘local’ equivalent efficiency. We will designate this as a pseudo-thermodynamic (intensive) state property, in that it appears to be path independent. But its meaning is inherently path dependent, since it is the ‘local’ equivalent efficiency along an isothermal path as compared to an isentropic one.

The overall polytropic efficiency, $\bar{\eta}_{p(T)}$, is integrated between points ‘1’ to ‘2’ as,

$$\Delta h_{p(T)} = \int_{p_1}^{p_2} v \cdot dp_T$$

$$\bar{\eta}_{p(T)} = \frac{\Delta h_{p(T)}}{\Delta h_{p(T)}}$$

Where, $\Delta h_{p(T)}$ is as previously defined. In general, the integrals for $\Delta h_{s}$ and $\Delta h_{p(T)}$ will not be equal. However, for the region associated with case A, these are close, and so will the overall isentropic and polytropic efficiencies.

To provide a sense of these numbers for helium, table 1 presents results for representative cases A, B, and C, referring to the indicated points in figure 2.

Figure 2. Pressure-enthalpy diagram for helium near the two-phase region
Figure 3. Three cases for isentropes and isotherms on a $p-h$ diagram

| Table 1. Results for representative cases A, B, and C |
|-------------------------------------------------------|
| **Case A** | **Case B** | **Case C** |
| Pt. 1 | Pt. 2 | Pt. 2s | Pt. 1 | Pt. 2 | Pt. 2 | Pt. 1 | Pt. 2s | Pt. 2 |
| $T$ [K] | 2.200 | 2.200 | 5.000 | 5.000 | 5.600 | 5.600 | 5.600 | 5.600 | 5.600 |
| $p$ [bar] | 3.000 | 0.200 | 8.449 | 1.960 | 3.000 | 3.000 | 3.000 | 3.000 | 3.000 |
| $h$ [J/g] | 5.024 | 3.146 | 3.188 | 15.85 | 11.00 | 15.85 | 15.85 | 15.85 | 15.85 |
| $s$ [J/g-K] | 1.618 | 1.618 | 3.627 | 4.649 | 3.995 | 3.995 | 5.957 | 5.957 | 5.957 |
| $v$ [l/kg] | 6.596 | 6.828 | 7.155 | 7.337 | 7.985 | 14.20 | 8.185 | 8.185 | 8.185 |
| $\beta^3$ [K] | 100.6 | 161.9 | 97.03 | 11.71 | 97.03 | 11.71 | 0.8857 | 0.4712 | 0.4712 |
| $(\partial h/\partial p)_T$ [J/kg] | 6.451 | 6.673 | 4.101 | -45.80 | 3.498 | -154.6 | 4.750 | 4.750 | 4.750 |
| $\eta$ [-] | 97.8% | 97.7% | 57.3% | -465% | 57.3% | -465% | 97.8% | -94.3% | -94.3% |
| $\Delta p$ [bar] | 2.800 | 6.489 | 7.000 | 5.983 | 4.850 | 5.905 | 97.8% | 97.8% | 97.8% |
| $\Delta h_T$ [J/g] | 1.836 | 0.000 | 5.109 | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| $\eta_T$ [-] | 97.8% | 97.8% | 97.8% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| $p_{2,min}$ [bar] | 0.053 | 1.960 | N/A | N/A | N/A | N/A | N/A | N/A | N/A |

3. Equivalent Process Comparison
The previous section discussed the pseudo-property, termed as the equivalent isentropic efficiency. However, now we would like to examine this from a different perspective. As previously discussed, we know incurring a significant pressure drop in the supply stream of a heat exchanger concurrent with flow returning from the load increases the enthalpy flux to the load. From a capacity perspective, this process seems to be functionally similar to one that incorporates an expander (providing work extraction), but has no (significant) pressure drop in the supply stream concurrent with heat exchange. Figure 4 presents these two basic processes, which we would like to compare. That is, for the same load ($q_L$, see figure 4), what is the equivalent (wet) expander efficiency ($\eta_{ex}$, see figure 4) if there is no supply stream pressure drop concurrent with heat exchange, as compared to a process that does have a significant supply stream pressure drop ($\Delta p_T$, see figure 4) concurrent with heat exchange, but has no
work extraction? The supply stream condition $(p_{h,1}, T_{h,1})$ at the heat exchanger inlet and the load return (liquid ‘bath’) pressure $p_{l,3}$ are the same for both processes. Additionally, heat in-leak (to either supply or return stream) and the return stream pressure drop is neglected. The “x-axis” case is the process on the left hand side of figure 4, which has no work extraction and non-zero supply ($h$) stream pressure drop. The “y-axis” case is the process on the right hand side of figure 4, which has non-zero work extraction and zero supply ($h$) stream pressure drop. Figure 5 and table 2 present results for helium with a supply condition $(p_{h,1}, T_{h,1})$ of (3 bar, 4.5 K), a load return (liquid ‘bath’) pressure $p_{l,3}$ of 31 mbar, and a cold-end heat exchanger stream temperature difference $(\Delta T_{h,l,2})$ of 0.2 K.

As can be seen from these, a process that has a supply stream pressure drop of 2.8 bar with concurrent heat exchange with the returning sub-atmospheric load, using a Joule-Thompson (JT; ‘throttling’) valve from 0.2 bar to 0.031 bar, is equivalent (in a capacity sense) to one that has no supply stream pressure drop and uses an expander (EX, in Figure 4) operating at 89.5% isentropic efficiency. It is unlikely that a practical expander, if one were to be designed and built, operating under such conditions would have such a high efficiency. Keep in mind that the former process has been shown to be practically obtainable [5]. We might be tempted to quickly conclude that there is no reason to bother developing an expander. However, we can also observe from figure 5 and table 2 that the temperature of the sub-atmospheric stream returning to the refrigeration system $(T_{l,1})$ increases as the supply stream pressure drop increases. In the particular case mentioned, this temperature increases by 10.8% (3.225 K to 3.573 K). So, that even though the mass flow would decrease by 8.4% (49.98 (g/s)/kW to 45.77 (g/s)/kW) for the same load $(q_L)$, the total input power to the refrigeration system would decrease less than the mass flow decrease. Or conversely, for the same total input power to the refrigeration system, the load increase would be less than the enthalpy flux increase of 9.2% (20.01 W/(g/s) to 21.85 W/(g/s)). In both cases this has been estimated for the particular process cycle used by [6] and [7] to be approximately 60% [5]. That is, the total refrigeration system input power reduction for the same load is estimated to be 5.0%. Or conversely, the load increase for the same total refrigeration input power is estimated to be 6.1%. Interestingly, the simplified ‘Carnot-Step’ analysis [8, 5] predicted reasonably close results to the full process cycle model, at a considerable computational (time and complexity) reduction.

![Figure 4. Equivalent process comparison](image-url)
Figure 5. Representative results of equivalent process comparison

Table 2. Representative results

| \( \Delta p_{\text{HX}} \) [bar] | \( \eta_{\text{is}} \) [%] | \( 1-x \) [%] | \( \Delta h_{\text{HX}} \) [J/g] | \( T_{\text{LFT}} \) [K] |
|-----------------|-----------------|---------------|-----------------|---------|
| 0.0             | 0.0%            | 85.5%         | 20.01           | 3.225   |
| 0.2             | 6.3%            | 86.1%         | 20.14           | 3.249   |
| 0.4             | 12.6%           | 86.6%         | 20.27           | 3.274   |
| 0.6             | 18.9%           | 87.2%         | 20.40           | 3.298   |
| 0.8             | 25.3%           | 87.7%         | 20.53           | 3.323   |
| 1.0             | 31.6%           | 88.3%         | 20.66           | 3.348   |
| 1.2             | 38.0%           | 88.9%         | 20.79           | 3.372   |
| 1.4             | 44.4%           | 89.4%         | 20.93           | 3.397   |
| 1.6             | 50.8%           | 90.0%         | 21.06           | 3.422   |
| 1.8             | 57.2%           | 90.5%         | 21.19           | 3.447   |
| 2.0             | 63.6%           | 91.1%         | 21.32           | 3.472   |
| 2.2             | 70.1%           | 91.7%         | 21.45           | 3.497   |
| 2.4             | 76.5%           | 92.2%         | 21.59           | 3.522   |
| 2.6             | 83.0%           | 92.8%         | 21.72           | 3.548   |
| 2.8             | 89.5%           | 93.4%         | 21.85           | 3.573   |
| 2.9             | 92.8%           | 93.7%         | 21.92           | 3.586   |
| 2.947           | 94.3%           | 93.8%         | 21.95           | 3.592   |
4. Conclusion
We have introduced the pseudo-thermodynamic property of the equivalent efficiency and discussed its meaning in context to a process that involves a significant pressure drop in a supply stream with concurrent heat exchange with a lower pressure vapor return. A practical equivalent efficiency has also been discussed and the results for a typical helium application presented. These results show that there is considerable benefit in performance, simplicity, and reliability to be obtained by implementing the aforementioned process, as opposed to the implementation of a process that uses a yet-to-be-developed expander.

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