Minimum Distortion Variance Concatenated Block Codes for Embedded Source Transmission

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Abstract—Some state-of-art multimedia source encoders produce embedded source bit streams that upon the reliable reception of only a fraction of the total bit stream, the decoder is able to reconstruct the source up to a basic quality. Reliable reception of later source bits gradually improve the reconstruction quality. Examples include scalable extensions of H.264/AVC and progressive image coders such as JPEG2000. To provide an efficient protection for embedded source bit streams, a concatenated block coding scheme using a minimum mean distortion criterion was considered in the past. Although, the original design was shown to achieve better mean distortion characteristics than previous studies, the proposed coding structure was leading to dramatic quality fluctuations. In this paper, a modification of the original design is first presented and then the second order statistics of the distortion is taken into account in the optimization. More specifically, an extension scheme is proposed using a minimum distortion variance optimization criterion. This robust system design is tested for an image transmission scenario. Numerical results show that the proposed extension achieves significantly lower variance than the original design, while showing similar mean distortion performance using both convolutional codes and low density parity check codes.

I. INTRODUCTION

Multimedia transmission for heterogeneous receivers is a challenging problem due to the unpredictable nature of the communication channels. Recent advances in multimedia compression technology are to account for an adaptation for the time-varying and band limited nature of wireless channels. Progressive source coding is an attractive solution for the transmission problems posed by multimedia streaming over such channels. The bit stream is generally said to be embedded if the removal of the end parts of the source bit stream enables adaptations to end user preferences according to varying terminal and network conditions. For example, the scalable extension of H.264 AVC [1] allows reconstruction of the video at various bit rates using partial bit streams (layers) at the expense of some loss of coding efficiency compared to the single layer counterpart [2]. Also, the bit streams produced by SPIHT [3], JPEG2000 [4] or the MPEG-4 fine grain scalable (FGS) coding [5] standards are embedded and provide a bitwise fine granularity in which the bit stream can be truncated at any point for source decoding. However, embedded source coders provide progressiveness at the expense of possessing some features that make them vulnerable to channel bit errors. For example, it is common to these source coders that the usefulness of correctly received bits depends on the reliable reception of the previous bits. Therefore, an efficient unequal error protection (UEP) scheme is needed for the reliable transmission of such multimedia data. Conventionally, less redundancy is added for each layer with decreasing importance for decoding to allow a graceful degradation of the source at the receiver [6].

Transmission of progressive sources over error prone wireless channels is a well investigated topic. Studies include various cross-layer protection strategies for multimedia streaming over wireless lossy networks [7] and adaptive selections of application layer forward error correction (FEC) coding and deployment for embedded bit streams [3], [8]. For the latter, joint source–channel coding (JSCC) is the most popular. JSCC is extensively used in the literature, in which an appropriate channel code is used to protect the bit stream to optimize some criterion such as minimization of mean distortion or maximization of average useful source rate [9].

In a broadcast transmission scenario, each member of the network is expected to receive at least a decent average multimedia quality in order to meet the fair service guarantee. Excessive quality fluctuations among the users of the same network can be avoided by minimizing the variance of the distortion at the terminal of each user [10]. The main contribution of this study is to consider an efficient coding scheme in a broadcast scenario and introduce major modifications to the original design of [12] for improved distortion variance characteristics.

The concatenated block coded embedded bit streams are shown to give superior performance over conventional coding paradigms while providing flexible and low complexity transmission features over multi-hop networks [12]. There are two assumptions about the previous coding structure that will not fit in a broadcast transmission scenario. First of all, in the original coding scheme, some of the information block sizes (optimized for minimum distortion) might be very large. Typically, the optimal number of encoding stages ($M^*$) are reported to be four or five for the bit budget constraints and raw channel bit error rates considered. This means that there are five or six reconstruction levels at the receiver. This may not be desirable, for example, from an image transmission...
perspective, because the user will only be able to see at most six different quality versions of the transmitted image with possible quality variations in between. Furthermore, it often leads to user dissatisfaction.

Alternatively, each information block in the transmission system can be chopped into smaller chunks to allow a larger number of reconstruction possibilities at the receiver. Due to the embedded nature of the bitstream, this can provide two advantages: (1) one can obtain better mean distortion characteristics and (2) having more reconstruction levels leads to user satisfaction and increases the overall service quality. In other words, the image quality is not expected to vary dramatically because of the availability of larger set of reconstruction levels at the receiver. However, having larger number of chunks in the system means more redundancy allocation for error detection. Given the available bit budget constraint, this will eventually leave less room for source and channel coding bits. Thus, the paper is intended to carry out the optimization needed to resolve this trade-off.

Secondly, the original optimization criterion was to minimize the average distortion of the reconstructed source. Although this criterion could be sufficient in a point-to-point communication scenario, it is rarely found in a broadcast transmission scenario. In order to maintain a decent average source quality among the network users, the second order statistics of the source distortion has to be taken into account. A way to approach this problem is presented in this paper; we consider the minimum distortion variance problem subject to a predetermined average source quality. This way, a reasonable mean source distortion can be obtained while guaranteeing the minimum deviation from the mean performance.

The remainder of this paper is organized as follows: In Section II, the background information about concatenated block codes for embedded source bit streams is explained in detail. In Section III, the proposed extension framework is presented and associated optimization criteria as well as the optimization problems are introduced. Some of the numerical results are given in Section IV. Finally, a brief summary and conclusions follow in Section V.

II. CONCATENATED BLOCK CODING FOR EMBEDDED BIT STREAM TRANSMISSION

Concatenated block codes are considered in [12] for embedded bit stream transmission over error-prone memoryless channels. The proposed $M$-codeword scheme is shown in Fig. 1 and can use any discrete code set $C$. We give a brief description of the original coding structure before giving the details of the extension scheme.

We first describe the coding structure for convolutional codes. The first stage of the encoder is the concatenation of $b_1$ source bits (i.e., source block $I_1$) with two bytes of CRC$^1$, $N_c = 16$ bits based on $b_1$ bits for error detection. If the convolutional codes are selected, they can still be treated as block codes by appending $m$ zero tailing bits to end the trellis at the all zero state. Therefore, $|P_1| = b_1 + N_c + m$ bits constitute the first payload $P_1$. Later, $P_1$ is encoded using the channel code rate $r_1 \in C$ to produce the codeword $c_1$. This ends the first stage of encoding. In the next stage, $c_1$ is concatenated with the second information block $I_2$ (of size $|I_2| = b_2$), $N_c$ and $m$ bits to produce the second payload $P_2$ of size $|P_2| = \left(\lfloor (b_1 + N_c + m)/r_1 \rfloor + b_2 + N_c + m\right)$. The next encoding stage, $N_c$ CRC bits are derived based only on those $b_2$ bits. After the interleaving, the bits in $\pi(P_2)$ are encoded using code rate $r_2 \in C$ to produce codeword $c_2$ where $\pi(x)$ denotes the random block interleaving function that chooses a permutation table randomly according to a uniform distribution, and permutes the entries of $x$ bitwise based on this table.$^2$

This recursive encoding process continues until we encode the last codeword $c_M$. Lastly, the codeword $c_M$ is transmitted over the binary symmetric channel (BSC) channel. Since the errors out of a maximum likelihood sequence estimator (MLSE) are generally bursty, and some of the block codes show poor

\footnotetext[1]{Here, a CRC polynomial is judiciously chosen to minimize undetected-error probability and the same CRC polynomial is used for all information blocks. The selected CRC polynomial is $X^{10} + X^{12} + X^5 + X$. Note that depending on the channel code used, for example low density parity check codes, CRC bits may not even be needed.}

\footnotetext[2]{We choose the size of the random permutation table to be equal to the length of the payload size in each encoding stage except the first.}
Fig. 2: Encoding scheme using concatenated block coding and chopped information blocks. $P_I$: parity bits for codeword $I$.

In order to increase the number of reconstruction levels at the decoder, we have chopped the information blocks into smaller chunks of equal size (e.g., using convolutional codes). Each information chunk is constrained to be an integer multiple of $\nu$ bits each. Let us assume that we are able to collect $r_i \nu$ channel bits per source sample (e.g., pixels). We denote the available code rate by $C = \{r_1, r_2, \ldots, r_J\}$. Let us have $M$ encoding stages having $m_i, 1 \leq i \leq M$, chunks in the $i$-th coding stage. For $i > M$, we define $m_i \equiv 1$ for completeness. We use concatenated block coding mechanism to encode the information chunks to produce the coded bit stream. A code allocation policy $\pi$ allocates the channel code $c^{(i)} \in C$ to be used in the $i$-th stage of the algorithm. Note that the number of packets in each information block depends on the channel coding rate $\pi$ and therefore denoted as $m_i(\pi)$ hereafter. The size of the outermost codeword length is given by

$$
N_s = \left( \left( \sum_{i=1}^{M} \left( \frac{m_1(\pi)}{C_1} + \frac{m_2(\pi)}{C_2} + \ldots \right) \right) \frac{1}{C_\pi} \right) - 1
$$

where $N_s$ is the number of source samples.

Assumption 1: For a tractable analysis, we assume perfect error detection.

For a given channel, let the probability of decoding failure (for example, CRC code flags a failure) for the chunk $i$ of the information block $z$ (where $\sum_{j=1}^{z} m_j(\pi) < i \leq \sum_{j=1}^{z+1} m_j(\pi)$), which is protected by the sequence of channel codes $c_{\pi}(z), c_{\pi}(z+1), \ldots, c_{\pi}(M) \in C$, be $P_e(c_{\pi}(z:M))$. For $z > M$, we define $P_e(c_{\pi}(z:M)) \equiv 1$. Let the operational rate-distortion function of the source encoder be $D(R)$ where $R$ is the source rate in bits per source sample.
\[
D_\pi(n) = \sum_{j=1}^{M+1} \sum_{i=0}^{m_j(\pi)-1} D^n \left( \sum_{t=1}^{j-1} m_t(\pi) + i \right) \frac{k}{N_s} \cdot P_e(e_\pi^{(j,M)}) \left(1 - P_e(e_\pi^{(j,M)})\right)^{j-1} \prod_{s=1}^{j} \left(1 - P_e(e_\pi^{(s,M)})\right)^{m_s(\pi)}
\]

\[D_\pi(n) = \sum_{j=1}^{M+1} \sum_{i=0}^{m_j(\pi)-1} D^n \left( \sum_{t=1}^{j-1} m_t(\pi) + i \right) \frac{k}{N_s} \cdot P_e(e_\pi^{(j,M)}) \left(1 - P_e(e_\pi^{(j,M)})\right)^{j-1} \prod_{s=1}^{j} \left(1 - P_e(e_\pi^{(s,M)})\right)^{m_s(\pi)}
\]

Assumption 2: For the algorithm design purposes, we use a similar approximation in [12] that decoder failure rate is independent for each coded information chunk. This approximation is shown to be good when convolutional and LDPC codes are used with long enough interleavers [12]. In general, our code set \(C\) can be chosen from any code family with a bit processing method (such as interleaving) as long as this assumption closely approximates the code block error performance.

**Lemma 1:** Using Assumption 2, \(n\)-th moment of the distortion at the receiver using the policy \(\pi\), \(D_\pi(n)\) is given by Equation (3).

**Proof:** Let \(X\) be a random variable that takes on the distortion level \(d\) with probability \(p_d \triangleq P(r(X = d))\). Consider the probability of truncating the chunk stream after reliably receiving the \(i\)th chunk of the \(j\)th information block. This corresponds to the source decoder that reconstructs the source up to a distortion level \(d_{j,i} = D \left( \sum_{t=1}^{j-1} m_t(\pi) + i \right) \frac{k}{N_s} \), while the number of correctly decoded chunks is \(\sum_{t=1}^{j-1} m_t(\pi) + i\). Therefore,

\[p_{d_{j,i}} = P \left( X = D \left( \sum_{t=1}^{j-1} m_t(\pi) + i \right) \frac{k}{N_s} \right) = P_e(e_\pi^{(j,M)}) \left(1 - P_e(e_\pi^{(j,M)})\right)^{j-1} \prod_{s=1}^{j} \left(1 - P_e(e_\pi^{(s,M)})\right)^{m_s(\pi)}
\]

Thus using Assumption 2, the \(n\)-th moment of distortion is simply given as follows,

\[D_\pi(n) = \mathbb{E}[X^n] = \sum_{j=1}^{M+1} \sum_{i=0}^{m_j(\pi)} D^n_{j,i} \times p_{d_{j,i}}
\]

Finally, note that \(j = 1, \ldots, M\) and \(i = 0, \ldots, m_j-1(\pi)\) covers all the possibilities except the event that we receive all the chunks correct. This is fixed by letting \(j = M + 1\) and \(m_{M+1}(\pi) = 1\).

**C. Optimization Problems**

Next, we present the optimization problems considered in this study. We start with the original optimization problem i.e., Minimization of Mean Distortion, then we give the Constrained Minimization of Distortion Variance problem for the proposed extension. Finally, we consider Minimum Second Moment of Distortion as an alternative solution for the latter.

**Problem 1: (Minimization of Mean Distortion)**

\[
\min_{\pi, \xi, \nu} D_\pi(1) \text{ such that } r_{tr} = \frac{1}{N_s} \sum_{i=1}^{M} \frac{m_i(\pi)\nu}{\prod_{j=1}^{M} r^{(j)}} \leq B
\]

where \(\xi = \{b_1, \ldots, b_M\}\) and \(B\) is some threshold transmission rate in bits per source sample. As mentioned in the introduction section, we are interested in the minimization of the distortion variance subject to an average source quality constraint. This problem can be formulated as follows

**Problem 2: (Constrained Minimization of Distortion Variance)**

\[
\min_{\pi, \xi, \nu} \sigma^2_\pi \text{ such that } r_{tr} = \frac{1}{N_s} \sum_{i=1}^{M} \frac{m_i(\pi)\nu}{\prod_{j=1}^{M} r^{(j)}} \leq B, \quad D_\pi(1) \leq \gamma_D
\]

where \(\sigma^2_\pi = \sum_{j=1}^{M+1} \sum_{i=0}^{m_j(\pi)-1} (d_{j,i} - D_\pi)^2 \times p_{d_{j,i}} = D_\pi(2) - D_\pi(1)^2\)

and \(\gamma_D\) is some mean distortion constraint on the average performance of the extension system.

**Problem 2** is relatively a harder problem than **Problem 1** because now the each term of the sum in Equation (5) depends on the average distortion, which in turn depends on the parameters of the system subject to optimization. This problem can be simplified by the following observation.

Note that we have \(D_\pi(2) \geq D_\pi^2(1)\) because by definition, the variance cannot be negative. This means that the maximum value of \(D_\pi(1)\) is upper bounded and when the equality holds \((D_\pi(2) = D_\pi^2(1))\), the variance is minimized. On the contrary, if we allow lower \(D_\pi(1)\) in order to obtain a better mean distortion, we will get a positive variance \((\sigma^2_\pi > 0)\). Thus, it is reasonable to assume that \(\sigma^2_\pi\) is a non-increasing function of \(D_\pi(1)\) using the policy \(\pi\). In light of this assumption, we will set \(D_\pi(1) = \gamma_D\) to end up with an easier problem to solve:

**Problem 3: (Minimization of the Second Moment of Distortion)**

\[
\min_{\pi} \quad \sum_{j=1}^{M+1} \sum_{i=0}^{m_j(\pi)-1} (d_{j,i} - D_\pi)^2 \times p_{d_{j,i}} = D_\pi(2) - D_\pi(1)^2 \quad \text{subject to } r_{tr} = \frac{1}{N_s} \sum_{i=1}^{M} \frac{m_i(\pi)\nu}{\prod_{j=1}^{M} r^{(j)}} \leq B
\]

This problem gives the optimal solution of **Problem 2** given that it achieves the minimum when the mean distortion hits the boundary of the constraint set. We solve aforementioned optimization problems using numerical optimization tools. We employ a constrained exhaustive search to find the optimal code allocation policy of the system.

**IV. NUMERICAL RESULTS**

We consider both the original as well as the extension schemes with two different optimization criteria. In general, we have four different possible combinations:

- **ConMinAve:** Concatenated coding with minimum average distortion optimization criterion. Let the minimum distortion be denoted as \(d^*\) at the optimum.
- **ConMinVar:** Concatenated coding with minimum distortion variance optimization criterion.
In our simulation results we have found an optimum approximate solution. In other words, in our simulations results we have that the mean average distortion performance in the system. Thus, in solving Problem 3, we allow a margin of 0.05 with $M = 2$.

- **ConChopMinAve**: Extension scheme with minimum average distortion optimization criterion.
- **ConChopMinVar**: Extension scheme with minimum distortion variance optimization criterion subject to a minimum distortion constraint $\gamma_D \leq d^*$. We do not consider the system ConMinVar, simply because we intend to show how the “chopping” method can be instrumental to improve the performance of the original concatenated coding design. In addition, an increase in the distortion variance performance is expected, as we allow worse mean average distortion performance in the system.

Also, since we constrain the information packet size to be equal to multiples of $v$ and in our simulation results, we have discrete number of code rates in the code set $C$, it is not always possible to meet the average distortion constraint with equality i.e., $\gamma_D = d^*$. Thus, in solving Problem 3, we allow a margin of $\zeta$ in order to find the best approximate solution. In other words, in our simulation results we have $|\gamma_D - d^*| < \zeta$.

We use 512 x 512 monochromatic images Lena and Goldhill with SPIHT and JPEG2000 progressive image coders. In the first simulation, we set $v = 850$ bits, $M = 2$ and use rate compatible punctured convolutional (RCPC) codes with memory 6 [14]. We simulate all three systems and report average distortion and distortion variance performances as functions of the transmission rate in bits per pixel (bpp) when $\epsilon_0 = 0.05$. In all the simulation results using RCPC codes, $\zeta \approx 0$ and $\gamma_D \leq d^*$. As can be seen, chopping the information blocks into smaller size chunks helps decrease the mean distortion and distortion variance in almost all the transmission rates of interest. In addition, allowing some performance degradation in mean distortion, we can obtain much better distortion variance characteristics.

In the second simulation, we set $r_{tr} = 0.5$ bpp and $M = 2$ and vary $v$ to see the effect of variable chunk size on the overall performance. First of all, smaller chunk size does not necessarily mean better performance as the number of redundant CRC bits increase and consume the available bit budget. Consider the system ConChopMinVar. We have seen in the previous simulation that chopping helps to improve the mean system performance. Thus, for a given $M$, we can find an optimum chunk size that will minimize the distortion variance given that it satisfies a mean distortion constraint. In Fig. 4 we note that as we move from left to right on the abscissa, the number of reconstruction levels increase i.e., the block size decreases, number of blocks increases and number of redundancy used for error detection in the bit budget increases. Also, we observe that as we sacrifice some mean distortion performance, we obtain a decrease in distortion variance. This numerical example shows the validity of Assumption 2 about the relationship between the mean distortion and the distortion variance. They are observed to be inversely related.

In Fig. 4 we observe that the minimum variance is achieved when the block size hits 340 bits while satisfying the desired mean distortion constraint $d^* = 41.79$ with equality. At the optimum, ConMinAve has only 3 reconstruction levels (since $M = 2$), while ConChopMinVar has 126 different reconstruction levels. ConMinAve has a variance of 22.65 and shown as a horizontal line for comparison. The variance of ConChopMinVar shows a jump after achieving the optimum at a variance of 9.53 (almost 6 percent decrease from that of ConMinAve). This is because we have more chunks and therefore more reconstruction levels, CRC bits become dominant in the system. In order to satisfy the mean distortion constraint, the optimization mechanism changes the optimum channel code rates from $(4/5, 4/9)$ to $(8/9, 4/11)$. Having more powerful protection now decreases the mean distortion value while causing an increase in the total variance. Thus, ConChopMinVar has 52 less distortion variance compared to ConMinAve while both systems have almost the same mean distortion characteristics. Table II presents a set of performance results using different images, transmission rates at various raw channel BERs. As can be observed, dramatic improvements on
the variance characteristics of the original design are possible using the extension system.

Finally in Table I we provide some of the simulation results using rate compatible LDPC codes [15]. We observe that $\gamma_D \approx d^*$ (i.e., $\max \zeta = 0.7$) can be achieved using LDPC codes. However, we can obtain dramatic improvements in variance performance at the expense of little loss in expected distortion performance of the original design. Table I presents a set of performance results using different images, transmission rates at various raw channel BERs considered in [8] and [12]. As can be observed, similar performance gains are possible. For example at a transmission rate $r_{tr} = 0.505$bpp and $\epsilon_0 = 0.05$, the ConMinAve chooses (4/5, 2/3) as the two optimal code rates with three levels of reconstruction since $M = 2$. In the extension scheme ConChopMinVar, choosing $\nu = 2000$bits and using the same optimal code rate pair, we obtained 44 different levels of reconstruction. The latter design gives almost the same image quality ($\sim 35.3$dB) with a dramatic improvement in the variance, i.e., around 96.8% decrease in variance compared to the that of ConMinAve.

V. CONCLUSIONS

We have considered minimum variance concatenated block encoding scheme for progressive source transmissions. A non-trivial extension of the original design is introduced with better reconstruction properties at the receiver and more importantly better distortion variance characteristics at a given average reconstruction quality. We have considered three different optimization problems and simplified the variance distortion minimization problem. Simulation results show that dramatic improvements can be obtained with the extension system compared to the original coding scheme.

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