Computational and experimental study of mixed mode loading of the cracked semi-circular disc under bending

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Abstract. Numerical and experimental works are performed on the mixed-mode I/II brittle fracture by two types of edge cracked semicircular bend specimens. The first type is edge cracked semi-circular bend (SBC) specimens with vertical and inclined notches. The second type of specimens is semi-circular samples in which the contribution of mode I and mode II components varies by changing the bottom loading support. In the test numerical program fracture tests were conducted at crack inclination angles of 0, 10, 45, 49, 50, 80 degrees. The stress intensity factors for Mode I and Mode II loading and T-stresses are obtained numerically in FEM package SIMULIA Abaqus/CAE. The results of FEM modeling are compared with the experimental results obtained by the photoelasticity method. The experimental approach based on the photoelasticity method allows us to observe the von Mises equivalent stress distribution in the whole specimen and confirm the FEM studies performed. The comparison shows that in the specimens the pure mode II loading can’t be realized. Analysis of numerical studies and the isochromatic fringe patterns allows us to conclude that in the semicircular disc the mixed mode loadings are realized for all angles from 0 to 85 degrees. The pure mode II or sliding fracture can’t be created by so-called mode II loading in disc-type specimens SCB, because there are opening displacements observed for all the specimens investigated numerically and experimentally.

1. Introduction

The fracture mechanics field still contains lots of fracture problems and phenomena that need to be solved [1]. So far, especially the mode I mechanism of failure has been extensively investigated. Nevertheless, many failures occur, especially in their first stage, under mixed mode loading conditions [1].

Nowadays various experimental techniques to determine mixed fracture toughness develop continually together with those mixed-mode fracture criteria [2]. To better serve the engineering practice and achieve consistency in the laboratory measurements four methods to measure the mode I fracture toughness: the chevron-notched bend (CB) specimen, the short rod (SR) specimen, the cracked chevron-notched Brazilian disc (CCNBD) specimen and the semi-circular bend (SCB) specimen have been suggested. The centrally cracked circular disc (CCCD) specimen and the edge cracked semicircular bend (SCB) specimen are two frequently used disc type specimens for the determination of mixed fracture toughness of brittle material, such as rocks. In addition, in order to overcome the shortcomings of using the classical SCB specimen, two improved semi-circular bend
specimens have been proposed. There are several serious reasons to pay particular attention to the SCB specimen [2]. The major advantages in using the specimen to determine the mixed-mode fracture resistance of rock are that it can be easily obtained from rock cores, has a simple geometry and only requires common loading configuration. Therefore, the SCB specimen is more cost-effective, reliable and versatile compared to other specimens. In recent years, several numerical methods capable of modeling crack initiation and propagation have been presented with the development of advanced numerical techniques and constitutive modeling [2].

The semi-circular bend specimen subjected to three-point bending has received much attention in recent years for measuring the mixed mode I/II fracture resistance of rocks [3]. In [3] the experimental results reported in literature and obtained from fracture tests using the semi-circular bend specimen are revisited for several different rocks including marble, sandstone, limestone, and mudstone. It is shown that a two-term expression for the near-crack-tip stresses together with a criterion based on a fixed critical tangential stress under mixed mode loading provide very good estimates for the experimental results reported for mixed mode I/II fracture in the investigated rocks.

The generalized maximum tangential stress (GMTS) criterion proposed in the past for brittle fracture under mixed mode I/II loading is extended to mixed mode I/II/III [4]. The theoretical predictions of the extended criterion were validated by some experimental results reported in literature but only for in-plane loading. Then, the effects of T-stress on the fracture resistance and also on the in-plane and out-of-plane fracture angles are explored in mixed modes I/III and II/III. It is shown that the results of extended criterion depend on the T-stress, the Poisson’s ratio and also on whether the crack problem is plane stress or plane strain. It is shown [4] that despite the considerable influence of T-stress on the fracture behavior in pure mode III, its effect is lower than that of pure mode II.

Brittle fracture of asphalt binder is studied in [5] at low temperatures and under mixed mode crack tip deformation. A set of fracture tests are conducted [5] by two new laboratory specimens made of a type of asphalt binder. The suggested specimens can provide pure mode I (opening), mixed mode I/II, and pure mode II (sliding) crack propagation conditions. The crack propagation angles and fracture toughness values were measured from experiments under pure mode I, pure mode II and mixed mode I/II conditions. The experimental results were then evaluated using different fracture criteria: strain energy density (SED), maximum tangential stress (MTS), and maximum tangential strain (MTSN). The role of the first non-singular term of the elastic stress and strain field (generally called T-term) on the predictions provided by each criterion is discussed in [5]. It was found that both specimens are significantly affected by the T-term, and adding T-term significantly improves the predictions provided by each criterion. It was shown that for the mixed mode conditions, the strain-based criteria provide better predictions for the fracture toughness than stress-based and energy-based criteria. The results of this paper will help researchers to understand the mechanism of mixed mode crack propagation in asphalt binders, and as a result in pavement structures, at low temperatures.

The aim of [6] is to determine the coefficients of singular, T-stress and third terms of crack tip stress field for two disk-type specimens under wide range combinations of mixed-mode I/II loading. The first specimen is a centrally cracked circular disk subjected to diametral load (CCCD sample) and the second one is a semi-circular disk under three-point bending (SCB sample). Using finite element method, the stress intensity factors \( K_I \) and \( K_{II} \) are calculated for two samples and then the state of pure mode II loading is expressed for each specimen. As an important parameter to predict the fracture toughness of quasi-brittle materials taking into account the geometry effect, the sign and magnitude of term \( A_j \) are determined for both samples under pure mode I condition. It is found that the sign of \( A_j \) is negative for any configurations of SCB sample under pure mode I while its sign for CCCD can be negative or positive corresponding to the ratio of \( a/R \).

In [7] it is shown that the pure mode II or sliding fracture can not be created by so-called mode II loading in disc-type specimens including CCBD and SCB without the confinements, because there are opening displacements observed for all the specimens. The present paper is aimed at numerical and experimental study of a semi-circular disk under three-point bending (SCB sample) to verify the possibility of Mode II loading conditions at certain angle of the inclined notch.
2. Computational modelling. Stress intensity factors and T-stress

The semi-circular disk of radius $R$ containing an edge crack of length $a$ subjected to three-point bending is considered. In this specimen, the state of mode mixity can be controlled by changing the orientation of crack relative to the applied load (Figure 1, right). Pure mode I loading takes place in all types of SCB sample when the vertical crack is along the direction of applied load and the distances of bottom supports are symmetry (Figure 1, left).

Prior to using the SCB specimen several crack parameters such as $K_i, i = I, II$ and $T$ -stress for the SCB specimen can be written [8,9] as

$$K_i = \frac{P}{2Rt} \sqrt{\pi a Y_r(\frac{a}{R}, \frac{S}{R}, \alpha)} \quad i = I, II, \quad T = \frac{P}{2Rt} T^r(\frac{a}{R}, \frac{S}{R}, \alpha).$$

(1)

where $P$ is the applied load, $R$ is the radius of the disc, $a$ is the notch length, $2S$ is the loading span in the SCB test, $\alpha$ is the crack inclination angle, $t$ is the specimen thickness. Finite element model for the SCB test is shown in Figure 2. Figure 2 shows the full model and zoomed view of the mesh surrounding the crack tip. The singular elements have been used to model the square root stress singularity in the vicinity of the crack tip. Contour integral method available in ABAQUS software is employed to determine SIF. Eight contour integrals are used to extract the SIF values. The output for contour integral is obtained using a sweep mesh. A mesh insensitivity study is done to decide on the parameters such as the element size near the crack tip, the global element size, number of elements around the crack tip and the number of contour integrals to be used in the analysis. At least 36 elements surrounding the crack tip are required for reliable SIF evaluation as seen in Figure 2. Fifteen contour integrals are sufficient to determine SIF values as shown in Figure 2.

The results of finite element modelling is presented in Table 1 and Figure 3 where stress intensity factors $K_I, K_{II}$ and T-stresses for different values of the angle $\alpha$ are given. Table 1 clearly shows the presence of Mode I for all values of the angle $\alpha$. Thus, one can conclude that in the SBC specimen the pure Mode II can not be realized.
Table 1. Stress intensity factors and T-stress in the SCB disk with the inclined crack for different values of the angle $\alpha$.

| $\alpha$ | $K_1 / \text{mm}^{1/2}$ | $K_\alpha / \text{mm}^{1/2}$ | $T / \text{mm}^2$ |
|----------|--------------------------|-------------------------------|-----------------|
| 0        | 66.68                    | 0                             | -0.90           |
| 10       | 64.74                    | -7.50                         | -0.30           |
| 20       | 58.89                    | -13.43                        | 0.93            |
| 30       | 50.58                    | -17.40                        | 2.62            |
| 40       | 41.17                    | -19.03                        | 4.23            |
| 45       | 36.49                    | -19.08                        | 4.93            |
| 49       | 32.78                    | -18.78                        | 5.40            |
| 50       | 31.90                    | -18.66                        | 5.52            |
| 60       | 23.35                    | -16.75                        | 6.42            |
| 70       | 15.74                    | -13.61                        | 7.02            |
| 80       | 8.81                     | -9.19                         | 7.39            |
| 85       | 5.53                     | -6.16                         | 7.45            |

Figure 3. Dependence of dimensionless functions $Y_1$, $Y_{II}$, $T^*$ on the angle $\alpha$.

To verify the finite element modelling based on the contour integral method in SIMULIA Abaqus the XFEM approach has been used either. The typical mesh used in the analysis is presented in Figure 4 where one can see the crack trajectory.

Figure 4. Distribution of the Mises equivalent stress in the semi-disc with the inclined notch at the angle $49^\circ$.

Together with the finite element analysis the photoelasticity experiment has been performed. Figures 5 and 6 show isochromatic images in the SBC specimen under two loads. There is a very close agreement between the distributions of the von Mises equivalent stresses calculated using experimental (Figure 5 and 6) and finite element methods (Figures 4).
Figure 5. Isochromatic fringe pattern in the semi-disc with the inclined notch at the angle 49° under $P = 100kG$.

Figure 6. Isochromatic fringe pattern in the semi-disc with the inclined notch at the angle 49° under $P = 230kG$.

Figure 7 and 8 show the crack path obtained numerically by XFEM technology and observed experimentally. Finite element study clearly shows that the crack grows to the point of the load application and the experiment study confirms the crack path.

The second type of the SBC is the semi-disc with the vertical crack and different distances between crack location and supports (see Figure 9). By changing the distance between the crack location and supports one can vary the type of the mixed mode loading and the values of mixity parameter \[9-16\]. The stress intensity factors and T-stress can be obtained by the formulae

\[
K_i = \frac{P}{2Rt} \sqrt{\pi a Y_i (a / R, S_i / R, s)} \quad i = I, II
\]

\[
T = \frac{P}{2Rt} T' (a / R, S_i / R, s),
\]

where $s = S_2 / S_1$. The results of FEM modelling are presented in Table 2.

Figure 7. Crack path in the semi-disk - numerical solution performed in Simulia Abaqus. The results are obtained by the extended finite element method (XFEM).
Figure 8. Experimental crack path in semicircular bend specimen.

Figure 9. Scheme of the specimen’s geometry and the applied load.

Table 2. Stress intensity factors and T-stresses for different values of the distances between crack location and supports.

| $S_z$ | $\kappa_1$, H/mm$^{3/2}$ | $\kappa_2$, H/mm$^{3/2}$ | $T$, H/mm$^2$ |
|------|-----------------|-----------------|------------|
| 40   | 58.13           | 0.00            | -0.87      |
| 35   | 53.28           | -0.95           | -0.92      |
| 30   | 39.42           | -1.67           | -0.79      |
| 25   | 33.82           | -2.73           | -0.83      |
| 20   | 27.23           | -4.25           | -1.00      |
| 15   | 15.21           | -5.33           | -1.24      |
| 10   | 7.49            | -8.31           | -2.33      |
| 8    | 5.66            | -13.12          | -4.02      |
| 6    | 0.84            | -8.79           | -2.92      |
| 5    | 0.50            | -5.68           | -1.96      |
| 1    | 0.14            | -0.34           | -0.13      |

Figure 10. Dependence of the dimensionless functions $Y_1, Y_2$ and $T^*$ on the distance $S_z$. 
The results of FEM study is shown in Figure 11 where one can see the von Mises stress distributions for different values of the distance $S_2$ are given.

![Figure 11](image-url)

**Figure 11.** Distribution of the von Mises equivalent stress in the semi-circular disc with the notch for different distances $S_2$: a – $S_2 = 40$ mm; b – $S_2 = 35$ mm; c – $S_2 = 25$ mm; d – $S_2 = 20$ mm; e – $S_2 = 5$ mm; f – $S_2 = 1$ mm.

In the photoelasticity experiments the test specimens shown in Figure 12 are considered.

![Figure 12](image-url)

**Figure 12.** Scheme of the specimen’s geometry and the applied load for the semi-disc with the vertical notch (a), the second type of the semi-disc with different distances between fulcrums (b) and (c).

The isochromatics observed for the vertical notch in the semidisk under different loads are shown in Figures 13 -15. All the isochromatic images observed show the differences between the principal stresses corresponding to the mixed mode loadings. Thus, the loading corresponding to pure mode II is not observed.
3. Conclusions

The fracture properties, $K_I$, $K_{II}$ and T-stress expressions for the semi-disc with vertical and inclined crack were constructed by means of finite element calculation. Experimental verification by using the photoelasticity method is proposed. Numerical and experimental works are performed on the mixed-mode I/II brittle fracture by two types of edge cracked semicircular bend specimens. The first type is edge cracked semi-circular bend (SBC) specimens with vertical and inclined notches. The second type of specimens is semi-circular samples in which the contribution of mode I and mode II components varies by changing the bottom loading support. In the test numerical program fracture tests were conducted at crack inclination angles of $0^\circ$,$10^\circ$ $- 45^\circ$,$49^\circ$,$50^\circ$ $- 80^\circ$. The stress intensity factors for Mode I and Mode II loading and T-stresses are obtained numerically in FEM package SIMULIA Abaqus/CAE. The results of FEM modeling are compared with the experimental results obtained by the photoelasticity method. The experimental approach based on the photoelasticity method allows us to observe the von Mises equivalent stress distribution in the whole specimen and confirm the FEM studies performed. The comparison shows that in the specimens the pure mode II loading can’t be
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