Comparison between Theory and Some Recent Experiments on Quantum Dephasing

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We report on a quantitative comparison between our theory of quantum dephasing at low temperatures and some recent experimental results [D. Natelson et al., Phys. Rev. Lett. 86, 1821 (2001); A.B. Gougam et al., J. Low Temp. Phys. 118, 447 (2000); F. Pierre et al., cond-mat/0012038].

Experiments [1] demonstrated low temperature saturation of the electron decoherence time $\tau_\varphi$ in mesoscopic conductors. This result attracted a lot of attention and triggered theoretical debates on a fundamental issue of quantum dephasing at zero temperature. More recently new experiments in various mesoscopic systems were performed by several groups. The results of these studies are still waiting for a careful analysis.

In this note we will restrict our attention to the results reported by two groups [2] and [3,4]. Natelson et al. [2] found that their data for $\tau_\varphi$ are inconsistent with the geometry dependence predicted in a standard theory [3]. For wider samples they observed saturation of $\tau_\varphi$ at low temperatures. The Saclay–Michigan collaboration detected no saturation in silver [3] and pure gold [4] samples down to 45÷50 mK, but observed clear saturation in copper at temperatures one order of magnitude higher. It is the purpose of this note to analyze if the results [3,4] can be explained within our theory of quantum dephasing [5].

In order to do so let us briefly recollect our key results. The electron decoherence time $\tau_\varphi$ in diffusive conductors was predicted [6] to be finite at any temperature including $T = 0$ due to electron-electron interactions. In simple terms our result can be expressed as follows

$$\frac{1}{\tau_\varphi(T)} = \frac{1}{\tau_\varphi(0)} + \frac{1}{\tau(T)},$$  \hspace{1cm} (1)

where $\tau_\varphi(0)$ is the electron dephasing time at zero temperature and $\tau(T)$ is basically the dephasing time calculated in [6] (note, however, that $\tau(T)$ depends on the full dephasing time $\tau_\varphi(T)$, so the relation (1) should be understood as the equation for $\tau_\varphi(T)$). The value $\tau_\varphi(0)$ is predominantly determined by high-frequency modes of the electromagnetic field which mediates interaction between electrons in disordered conductors. One has [6]

$$\frac{1}{\tau_\varphi(0)} \propto \omega_c^{d/2},$$  \hspace{1cm} (2)

where $\omega_c$ is the high-frequency cutoff and $d$ is the effective dimension. It is important to emphasize that $\omega_c$ cannot exceed $1/\tau_e$, where $\tau_e = l/v_F$ is the electron elastic scattering time. This is because at frequencies above $1/\tau_e$ the electron motion is ballistic and the system is noiseless at such scales. Therefore no contribution from $\omega > 1/\tau_e$ to the noise correlator (which determines $\tau_\varphi$) can occur.

On the other hand, for $\omega \ll 1/\tau_e$ the electron motion is diffusive and the noise correlator is finite decaying to zero as $\omega$ approaches $1/\tau_e$. Thus we choose [6]

$$\omega_c = a_d/\tau_e,$$  \hspace{1cm} (3)

where $a_d$ is some numerical factor $a_d \leq 1$. In [6] we have set $a_d = 1$. This was sufficient for the order-of-magnitude estimate for $\tau_\varphi(0)$. Note, however, that since this sharp cutoff procedure slightly overestimates the contribution of frequencies close to $\omega_c$, the actual value of $\tau_\varphi(0)$ is expected to be bigger than that of Ref. [6] by a numerical factor of order one.

It is also worth mentioning that, provided the elastic mean free path $l$ is smaller than the thickness $t$ and the width $w$ of the sample, $l \ll w, t$, our 3d expression for the dephasing rate

$$\frac{1}{\tau_\varphi(3d)} = \frac{e^2 \rho}{3\pi^2 \sqrt{2} D} \left( \frac{a_3}{\tau_e} \right)^{3/2},$$  \hspace{1cm} (4)

should be used even if the sample is quasi-1d (quasi-2d) in a standard sense, i.e. if both $L_\varphi = \sqrt{D/\tau_\varphi}$ and $L_T = \sqrt{D/T}$ exceed $t$ and $w$ (or $t$). Here $\rho$ and $D$ are respectively the resistivity and the diffusion coefficient. Only if $l \gtrsim w, t$ (or $l \gtrsim t$) our 1d (2d) expressions should be applied. The reason for that was discussed after eq. (81) of the second Ref. [6]. In short, eq. (4) is relevant since $\tau_\varphi(0)$ is dominated by high frequencies at which the electron diffusion is always 3-dimensional provided $l \ll w, t$. Of course, eq. (4) crosses over to 1d (or 2d) results for $l \sim w \sim t$ (or $l \sim t$), as is seen from the following relations (presented at $a_d = 1$)

$$\frac{\tau_\varphi(1d)}{\tau_\varphi(0)} = \frac{tw}{2\pi^2}, \quad \frac{\tau_\varphi(2d)}{\tau_\varphi(0)} = \frac{4}{\pi} \frac{t}{\sqrt{6} l}.$$  \hspace{1cm} (5)

We are now prepared to compare experimental results [2,4] with our theoretical predictions. We will address both the maximum values of the decoherence time $\tau_\varphi^{max}$ reached in these experiments at the lowest measurement temperatures and the observed temperature dependencies of the dephasing time depending on the sample parameters.
I. MAXIMUM DEPHASING TIMES

For all the samples reported in Refs. 2 [4] the mean free path \(l\) deduced from the values of \(D\) was 2 to 5 times shorter than both \(t\) and \(w\) for quasi-1d samples and \(t\) for four quasi-2d samples of Ref. 2. Therefore, even though the condition \(l \ll w, t\) was not very well satisfied in these experiments, it would be more appropriate to use eq. (1) for our comparison. It is also easy to observe from eqs. (1) that, since \(l\) is not drastically smaller than \(w\) and \(t\), our 1d and 2d expressions will yield the same order of magnitude estimates for \(\tau_{\max}^{\phi}\) for all the samples in question (also see below).

The maximum dephasing times were reported in Ref. 2 for ten (six quasi-1d and four quasi-2d) samples fabricated from the same material (AuPd) with practically the same resistivity. For all ten samples the maximum dephasing times were found to be nearly universal \(\tau_{\max}^{\phi} \approx (0.8 \pm 2) \times 10^{-11}\) sek independently of the sample geometry. At \(T \approx 80\) mK these values yield \(T \tau_{\phi} \approx 0.09 \div 0.22\). Since both the material and its properties were the same an important test for our theory is to fit the data [2] for \(\tau_{\max}^{\phi}\) to eq. (1) which only depends on the material parameters and not on geometry.

In experiments [3] the maximum dephasing times were found to be 2 to 3 orders of magnitude bigger as compared to those in Ref. 2. Furthermore, \(\tau_{\max}^{\phi}\) for the silver sample [2] was about an order of magnitude higher than that for the copper sample [3] with similar (although not identical) parameters. Observing this difference the authors [3] suggested that the low temperature saturation of \(\tau_{\phi}\) in disordered metal wires cannot be universal and can be material dependent.

For our comparison we will use the data for 8 samples [2] with nominally identical resistivity \(\rho\approx 24\ \mu\Omega\) cm and \(D \approx 1.5 \times 10^{-3}\) m²/sek (samples C to F and H to K), 2 samples [3] (Ag and Cu) and one sample (AuMSU) [4]. Since all the maximum dephasing times [2] were nearly the same, for the sake of brevity we only quote an average value for \(\tau_{\max}^{\phi}\) obtained by averaging over samples C to F and H to K. In order to estimate \(\tau_{\phi0}\) for Ag and Cu samples we used the parameters given in the Table 1 of Ref. 3. Experimentally observed values of \(\tau_{\max}^{\phi}\) and our theoretical predictions for \(\tau_{\phi0}\) (eq. (4)) with \(a_3 = 1\) are summarized in the following Table:

| sample | \(\tau_{\max}^{\phi}\) (sek) | \(\tau_{\phi0}\) (sek) |
|--------|----------------|---------------------|
| C to F and H to K [2] (averaged) | \(1.3 \times 10^{-11}\) | \(0.3 \times 10^{-11}\) |
| Ag [3] | \(10 \times 10^{-9}\) | \(2 \times 10^{-9}\) |
| Cu [3] | \(1.8 \times 10^{-9}\) | \(0.3 \times 10^{-9}\) |
| AuMSU [4] | \(8 \times 10^{-9}\) | \(4 \times 10^{-9}\) |

These results demonstrate that for all the above samples our formula (4) with \(a_3 = 1\) gives correct order-of-magnitude estimates for \(\tau_{\max}^{\phi}\) even though these times differ by 2 to 3 orders of magnitude depending on the sample and experiment. Already this agreement is fairly good since in weak localization measurements \(\tau_{\phi}\) is defined up to a numerical prefactor. Furthermore, in accordance with our expectations, in all cases our theoretical estimates involving the cutoff parameter \(a_3 = 1\) are smaller than the experimental values by a numerical factor ranging from 2 to 6. This implies that by choosing the cutoff parameter in the range \(a_3 \approx 0.3 \div 0.6\) one can exactly reproduce all the above experimental values for \(\tau_{\max}^{\phi}\) from the same formula (4) which only depends on the material parameters, like \(\rho\) and \(D\).

One can also eliminate the ambiguity related to the cutoff parameter \(a_d\) in (4) by calculating the ratio between maximum dephasing times for different samples. Making use of the experimental results [2] [4] and our theoretical estimates based on eq. (4) one finds:

| sample | experiment | theory (eq. (4)) |
|--------|------------|----------------|
| \(\tau_{\phi0}^{Ag}/\tau_{\phi0}^{Cu}\) | \(1.3 \times 10^{-3}\) | \(1.5 \times 10^{-3}\) |
| \(\tau_{\phi0}^{Ag}/\tau_{\phi0}^{AuMSU}\) | \(0.7 \times 10^{-2}\) | \(1 \times 10^{-2}\) |
| \(\tau_{\phi0}^{AuPd}/\tau_{\phi0}^{AuMSU}\) | \(1.6 \times 10^{-3}\) | \(0.8 \times 10^{-3}\) |

The agreement appears to be even much better than one could possibly expect given all experimental and theoretical uncertainties in determination of \(\tau_{\phi}\). We emphasize that our theoretical predictions presented in both above Tables involve no fit parameters. Previously we already reported on a similar agreement with the results [4]. All these observations strongly favour the conclusion about universality of the low temperature saturation of \(\tau_{\phi}\) in disordered conductors.

To complete this issue we mention that also our 1d and 2d expressions for the maximum dephasing time [2] give correct order-of-magnitude estimates for \(\tau_{\phi}\) for the above samples. For instance, for 2d samples [2] the second eq. (1) yields (for \(a_2 = 1\)) \(\tau_{\phi0}^{(2d)} \sim 0.6 \times 10^{-11}\) sek. This estimate is only 2 ÷ 3 times smaller than the experimental values. Again, although such an accuracy is already more than sufficient, the remaining difference can easily be removed simply by choosing \(a_2 \approx 0.3 \div 0.5\).

For comparison, the authors [2] estimated the value of \(\tau_{\phi}\) which follows from the theory [3] for their 2d samples as \(\tau_{\phi} \sim 6 \times 10^{-9}\) sek at \(T = 1K\). At the lowest measurement temperature \(T \approx 80\) mK this estimate translates into \(\tau_{\phi}\) in the range \(10^{-7}\) sek which is \(3.5 \div 4\) orders of magnitude bigger than the values measured in Ref. 2.

II. TEMPERATURE DEPENDENCE

We now analyze the temperature dependence of \(\tau_{\phi}\) detected in experiments [2] [4]. For quasi-1d samples eq. (4) takes the form [8]

\[
\frac{1}{\tau_{\phi}} = \frac{1}{\tau_{\phi0}} + \frac{\rho T}{R_0tw} \sqrt{2D\tau_{\phi}},
\]
where \( R_q = \pi/2e^2 \approx 6453 \ \Omega \). An essential difference between this result and that of Ref. [3] is that in our case the temperature-independent contribution to the dephasing rate \( 1/\tau_{c0} \) is not equal to zero. If one neglects this contribution, eq. (6) would immediately yield the standard result [3] \( 1/\tau_{cAAK} \propto T^{2/3} \). Similarly, in quasi-2d samples one gets [3] \( 1/\tau_{cAAK} \propto T \ln T \). In order to test these predictions it is appropriate to present experimental data for \( \tau_c(T) \) on a log-log plot, exactly as it was done in Refs. [2,4]. From Ref. [5] one expects (i) to observe straight lines with the slope becoming steeper as one crosses over from quasi-1d to quasi-2d systems and (ii) for two quasi-1d samples with the same \( \rho \) and \( D \) but with different cross-sections \( T \nu \) the straight lines corresponding to \( T^{-2/3} \)-dependence of \( \tau_c \) should be parallel.

None of these two features was observed in the experiments [2]. Just on the contrary, the opposite trend was detected: In all cases the slope of the curves decreased with increasing sample width \( w \) (see Fig. 3 of Ref. [3]). The curves for quasi-1d samples with different \( w \) were not parallel (actually the data points for the sample F on Fig. 3 of [2] clearly saturate and can hardly be fitted to a straight line at all) and, on top of that, the \( T \)-dependence of the decoherence rate for quasi-2d samples – instead of being linear in \( T \) – was very weak already at \( T \sim 1K \).

Now let us see if these features can be accounted for within our theory. Since the low temperature behavior of \( \tau_c \) predicted in Ref. [3] is different from a power-law one it appears to be more useful to replot the same data on a linear scale. Then, according to eq. (6) at sufficiently low \( T \) one expects to observe a linear dependence of the decoherence rate on \( T \) shifted upwards by the value \( 1/\tau_{c0} \). For the same parameters \( \rho \) and \( D \) (and the same \( \tau_{c0} \)) the slope \( d(1/\tau_c)/dT \) for quasi-1d samples should depend only on the cross section \( T \nu \). In particular, since \( w \) for the sample F was reported in [3] to be 4 times larger than for the samples A to E, one expects \( d(1/\tau_c)/dT \) to be 4 times smaller for the sample F.

The experimental data for the samples A, B and F [3] are presented in Fig. 1. We observe that (a) the dephasing rate \( 1/\tau_c \) increases linearly with \( T \), (b) at \( T \to 0 \) these linear dependencies extrapolate to a nonzero (and practically the same) value \( 1/\tau_{c0} \) for all three samples and (c) the slope \( d(1/\tau_c)/dT \) is indeed about 4 times smaller for the sample F as compared to the samples A and B. All these features are fully consistent with eq. (6). The magnitude of the slope is several \( (~4 \div 6) \) times larger than predicted by eq. (6). The origin of this difference is not clear to us at the moment, but possibly it can be due to non-uniformity of the wires. The data for quasi-2d samples I and K [4] also show the trend qualitatively consistent with our eq. (6). Also in that case the temperature effect seems to be more pronounced than it is predicted within our theory.

We conclude that our theory [4] accounts for all the main experimental observations of Ref. [3].

Now we will address the results of Refs. [2,4]. In Fig 3 of [3] the authors presented their data for Au, Ag and Cu samples with similar parameters. Both the magnitude and the temperature dependence of \( \tau_c \) differ drastically for these three samples. This observation led the authors [3] to conclude that the behavior of \( \tau_c \) and, in particular, its saturation at low \( T \) may be material dependent.
Later it was demonstrated by the same group \cite{4} that unusually low values of \( \tau_\varphi \) in gold \cite{3} were most likely due to high concentration of magnetic impurities. A pure gold sample (\textsc{AuMSU}) was fabricated and similar behavior of \( \tau_\varphi \) was observed \cite{4} as previously for \( \text{Ag} \) sample \cite{3}: \( \tau_\varphi \) did not saturate down to \( T \sim \) 45 \( \div \) 50 mK on a log-log plot. At the same time it was confirmed \cite{4} that also very pure \( \text{Cu} \) samples showed saturation similarly to an earlier \( \text{Cu} \) sample \cite{3}.

The material dependence of \( \tau_\varphi \) cannot be ruled out in general. However, with minimum efforts one can demonstrate that no material dependence needs to be assumed in order to quantitatively explain seemingly different behavior of \( \text{Ag} \) (no saturation down to \( T \sim \) 49 mK) and \( \text{Cu} \) (clear saturation already at \( T \gtrsim \) 700 mK) samples \cite{3}.

Let us rescale eq. (3) as
\[
1/\tilde{\tau}_\varphi = A + BT^\varphi \sqrt{\tilde{\tau}_\varphi},
\]
where we introduced dimensionless variables
\[
\tilde{\tau}_\varphi = \frac{\tau_\varphi}{\tau_{\varphi 0}}, \quad \tilde{T} = \frac{T}{T_0}, \quad T_0 = \frac{R_q tw}{\rho \tau_{\varphi 0}^{3/2} \sqrt{2D}}.
\]

Here \( A = B = 1 \) both for \( \text{Cu} \) and \( \text{Ag} \) samples provided one performs the scaling to the values \( \tau_{\varphi 0} \) and \( T_0 \) calculated for the respective sample. The temperature \( T_0 \) sets the scale at which thermal and quantum contributions to the dephasing rate become comparable. Making use of the parameter values for \( \text{Cu} \) and \( \text{Ag} \) samples one finds
\[
T_0^{\text{Cu}} / T_0^{\text{Ag}} \sim 20.
\]

Note, that this estimate is not sensitive to various ambiguities related, e.g., to the cutoff parameter \( a_d \). Eq. (3) just follows from eqs. (4), (6) for the values of \( \rho, D, t \) and \( w \) presented in the Table 1 of Ref. [3]. The estimate (9) remains unchanged if, instead of our eq. (4), one uses experimental values for \( \tau_{\varphi \text{max}} \) \cite{3}. Hence, the temperature range where quantum effects set in can easily vary by one order of magnitude or more for different samples even if their macroscopic parameters are similar \cite{4}.

What remains is to make both log-log and linear scale plots of eq. (3), see Fig. 2. In these plots we scaled both curves to the parameters of one (\( \text{Cu} \)) sample, i.e. \( A_{\text{Cu}} = B_{\text{Cu}} = 1, A_{\text{Ag}} = \tau_{\varphi \text{Cu}}^{\text{Cu}} / \tau_{\varphi \text{Ag}}^{\text{Ag}} \approx 0.2 \) and \( B_{\text{Ag}} = A_{\text{Ag}}^{1/2} T_0^{\text{Cu}} / T_0^{\text{Ag}} \approx 9.5 \). Making use of the values for \( \tau_{\varphi \text{max}} \) \cite{3} we find \( T_0^{\text{Cu}} \approx 730 \) mK and \( T_0^{\text{Ag}} \approx 40 \) mK.

These numbers as well as the temperature dependence of \( \tau_\varphi \) are in the agreement with experimental findings \cite{3}. E.g., on the log-log plot we observe that, while \( \text{Cu} \) sample shows clear saturation around \( \tilde{T} \sim 1 \) (i.e. \( T \sim 700 \) mK), practically no signs of saturation are visible for \( \text{Ag} \) sample even at temperatures one order of magnitude lower. Similar analysis can be performed, e.g., for \( \text{Au} \) \cite{1} and \( \text{AuMSU} \) \cite{4} samples (cf. right panel of Fig. 7 in Ref. 4). In that case one can estimate \( T_0^{\text{Au}} / T_0^{\text{AuMSU}} \approx 6 \).
It is obvious from eq. (6) that by a proper choice of the sample parameters one can tune the effective temperature range at which saturation feature occurs on the log-log plot. E.g. thinner samples with identical other parameters will show no saturation down to lower temperatures. The lack of saturation for such samples even at relatively low T would, of course, not imply that the electron dephasing rate is zero at T = 0.

Let us replot the “non-saturating” data [3,4] on a linear scale. In Fig. 3 we collected the data points for different samples studied in different experiments (Au1, Au2, Au6 of Ref. [1], Ag of Ref. [3] and AuMSU of Ref. [4]). All these samples show essentially the same behavior at low temperatures. Both the slope $d(1/\tau_{\phi})/dT$ and the values $\tau_{\phi0}$ slightly differ from sample to sample, but these differences are unimportant and can easily be accounted for by different values of $D$ and $t_{w}$ as well as by measurement uncertainties. Much more importantly, at low temperatures $1/\tau_{\phi}$ depends linearly on $T$ and clearly extrapolates to a nonzero value for all the samples presented in Fig. 3. This is exactly what one expects from our eq. (6). Also the similarity with the results [6] (our Fig. 1) is quite obvious.

In summary, we demonstrated that the experimental results [2,4] are in a quantitative agreement with our theory [4] which predicts the low temperature saturation of the electron dephasing time in diffusive conductors. No material dependence of $\tau_{\phi}$ needs to be assumed in order to explain the data [3,4] for the samples Ag, Cu and AuMSU.

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[6] D.S. Golubev and A.D. Zaikin, Phys. Rev. Lett. 81, 1074 (1998); Phys. Rev. B 59, 9195 (1999).
[7] This cutoff procedure is correct for sufficiently good metals which we only consider here. In systems like, e.g., granular or strongly disordered polycrystalline materials $\omega_{c}$ (as well as other details) can be different.
[8] Eq. (6) gives correct but slightly oversimplified dependence of $\tau_{\phi}$ on $T$ in the intermediate temperature regime where thermal and quantum contributions are comparable. This inaccuracy (related to the way of definition of $\tau_{\phi}$) is, however, totally unimportant for our conclusions.
[9] From the Table 1 of Ref. [3] we extracted the following numbers: $\rho_{Cu}/\rho_{Ag} \approx 1.17$, $D_{Cu}/D_{Ag} \approx 0.58$, $(t_{w})_{Cu}/(t_{w})_{Ag} \approx 1.7$. 

FIG. 3. The data for $\tau_{\phi}(T)$ for several samples from Refs. [1,3,4]. Dashed and solid lines are the best fits for Ag sample [3] respectively to theories [5] and [6].