Diffraction of light by interfering liquid surface waves

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Abstract

Interfering liquid surface waves are generated by electrically driven vertical oscillations of two or more equispaced pins immersed in a liquid (water). The corresponding intensity distribution, resulting from diffraction of monochromatic light by the reflection phase grating formed on the liquid surface, is calculated theoretically and found to tally with experiments. The curious features of the diffraction pattern and its relation to the interference of waves on the liquid surface are used to measure the amplitude and wavelength of the resultant surface wave along the line joining the two sources of oscillation. Finally, a sample diffraction pattern obtained by optically probing surface regions where interference produces a lattice–like structure is demonstrated and qualitatively explained.

PACS numbers: 42.25.Fx (Diffraction and scattering), 68.03.Kn (Dynamics (capillary waves))
Surface waves are ubiquitous in nature. Also known as the Rayleigh wave, they appear on solid as well as liquid surfaces resulting in diverse phenomena with useful applications. In particular, it is well established that surface acoustic waves (henceforth SAW) are of prime importance in solid state technology [1]. Investigations on SAW in liquids has been relatively less. The traditional ‘ripple tank’ method of measuring the surface tension of a liquid is one of the earliest studies [2]. Recent advances along related directions (thermally activated liquid surfaces, spatial damping of liquid SAW, light scattering/photon correlation spectroscopy of liquid SAW, to name a few) have been carried out and reported in [3, 4, 5, 6, 7]. The basic principle of light diffraction by liquid SAW remains the same as in solids, though their quantitative features, the generation mechanism surely differ. Ripples on liquid surfaces, caused by external vibrations (usually electrically/thermally driven) act as an effective dynamical phase grating for incident radiation. The complex aperture function of the phase grating is determined from the phase shift of the reflected wave due to variation (sinusoidal) of the surface height [8]. A Fourier transform of the aperture function gives the light field strength on the screen from which the intensity can be obtained. Features of the intensity distribution provide information on the liquid SAW as well as properties of the liquid itself. Since liquid properties can be measured by other methods too, one is usually more interested in finding out the nature of the SAW using external non-destructive probes (such as light). Recently, Miao et al [9] have studied diffraction of light from circular ripples on water and experimentally established the correlation between the diffraction pattern and the SAW amplitude. Using the features of the intensity pattern, they have also measured the SAW wavelength. In this letter, we focus our attention on diffraction by interfering SAW, a topic which does not seem to have been dealt with much in the past. In other words, we generate superposed waves on the liquid surface by electrically driven vertical oscillations of slightly immersed metallic pins. The resulting interference pattern on the liquid surface diffracts incident light. Our aim is to first investigate how the superposed character of the wave pattern on the liquid surface can be seen in the diffraction intensity.

A schematic diagram of the experimental set up is shown in Fig. 1. Water (about 1 cm deep) is kept in a petri-dish of diameter 18.5 cm. A pin is glued to the diaphragm of a small loud speaker, which is driven by a low frequency signal generator. A sinusoidal signal of frequency $\Omega$ (220 Hz in our experiment) is used to make the pin vibrate, which acts as a SAW exciter. Best results are obtained when the pin is just below the liquid surface. If the pin
FIG. 1: Schematic diagram of the experimental setup to study SAW on liquid, A: Laser, B: Frequency generator, C: Screen, D: Loudspeaker, E: Microscope, F: Exciter

is immersed more, splashing and other associated effects can occur. To create superposed waves on the surface we have glued two pins on the diaphragm of the same loudspeaker. The separation between the pins is measured by a travelling microscope. The two pins oscillate simultaneously, in phase, on the water surface and create individual circular waves about their location which, in turn, interfere to produce the well-known two-source interference pattern, shown in Fig. 2. A 5 mW He-Ne laser of wavelength $\lambda$ (632.8 nm) and of beam-diameter 1.5 mm is directed to fall on the water surface, where the superposed SAW is formed. The angle of illumination (i.e. the angle at which the laser beam falls on the liquid surface), $i$, in our experiment is 77 degrees. It is known that at frequencies of the order of 220 Hz the SAW decay length is much greater than the size of the illuminated spot [10]. Due to Fraunhofer diffraction of light by the SAW phase grating, spots are observed on a screen, placed at a reasonably large distance (3.15 meters in our case) from the water surface. The images of the diffraction pattern are taken using a digital camera. The spurious noise in the images is removed by using the ‘Photoshop’ software. The intensity of the diffraction spots have been measured by a photodiode detector. The linear relation between the photodiode current and the intensity of diffracted light has been checked.

The circular waves generated on the surface of the liquid due to more than one source of disturbance (oscillation) can be approximated as a linear superposition of individual sinusoidal waves. We can write the individual waves as $\psi_1 = h \sin (\Omega t - kr_1)$ and $\psi_2 = h \sin (\Omega t - kr_2)$, where we assume that the two oscillations have the same frequency $\Omega$, wavenumber $k$ and amplitude $h$. $r_1$ and $r_2$ are the distances of the oscillation sources from the point of observation. In addition, we would be interested in focusing light at a point
FIG. 2: Line drawing of interference between two circular waves. Dotted, dashed and dashed-dotted
lines are explained in the text

along the line joining the two sources (solid line in Fig. 2). A sum of these two waves
produces a resultant wave profile (along the line joining the sources):

$$\psi = 2h \cos \frac{kd}{2} \sin (\Omega t - kr)$$  \hspace{1cm} (1)

where we have assumed \(r_2 - r_1 = d\), \(r_2 + r_1 = 2r\). In general, for a superposition of \(N\) such
oscillations we expect the factor \(2 \cos \frac{kd}{2}\) to be replaced by \(\frac{\sin Nkd}{\sin \frac{kd}{2}}\) (reminiscent of the factor
in the diffraction formula for \(N\) slits in the theory of Fraunhofer diffraction)\(^{[11]}\). Thus,
the resultant phase modulation function for the reflected light from the superposed liquid
surface waves turns out to be \(^{[8]}\):

$$\phi(x) = \frac{2\pi}{\lambda} \left[ (4h \cos \frac{kd}{2} \cos i) \sin \left( \Omega t - \frac{kx}{\cos i} \right) \right].$$ \hspace{1cm} (2)

We can easily rewrite this formula with an equivalent ‘\(h\)’ to make it look like the phase
modulation by a single wave. This will yield the replacement \(h' = 2h \cos \frac{kd}{2}\). In general
for \(N\) sources of oscillation : \(h' = h \frac{\sin (Nkd/2)}{\sin (kd/2)}\). The light field strength \((E)\) is the Fourier
transform of the object (aperture) function given by \(\exp (j\phi)\). The intensity is obtained by
considering \(EE^{*}\) and yields \(^{[12]}\):

$$I(x') = \sum_n J_n^2 \left( 4\pi h' \cos i / \lambda \right) \delta \left( \frac{x'}{\lambda z} - \frac{n}{\Lambda \cos i} \right).$$ \hspace{1cm} (3)

where \(z\) is the horizontal distance between the location of the laser spot on the liquid surface
FIG. 3: Theoretical plot of intensity vs. SAW amplitude for zero order (solid line), first order (dashed line) and second order (dotted line) diffraction spots using eq. 3. Experimentally measured intensity for these spots are shown by black filled, grey filled and open circles, respectively. The linearity in variation of resultant SAW amplitude in water and applied voltage in signal generator (also amplitude imparted by the exciter) is shown in the inset.

and the screen, SAW wavelength \( \Lambda = \frac{2\pi}{k} \). \( x' \) is the position variable on the observation plane.

The intensity distribution, given in eq. 3, demonstrates that at the location of zeros of the Bessel function one would get vanishing intensity of the different orders at different SAW amplitude. For instance, for \( n = 0 \), and \( x' = 0 \), the intensity will vary as \( J_0^2 \). Thus, it will vanish at the value of \( h \) for which \( J_0^2 \) has a value zero. Interestingly, for the superposed waves the value of \( h \) for which the intensity zero will be observed is a factor \( \frac{1}{\cos \frac{kd}{2}} \) (for \( N = 2 \)) smaller than the value for a single wave. We find that the intensity zero at zeroth order will appear nearly at half the \( h \) value obtained for the single wave case when \( \left| \cos \frac{kd}{2} \right| \to 1 \). In our experiment, since \( h \) is proportional to the voltage, we should be able to see this reduction in \( h \) through the reduction in the applied voltage of the frequency generator. For more than two sources of oscillation placed close by, we would expect a reduction by a factor of \( N \) (since \( \left| \sin \frac{Nkd}{2} / \sin \frac{kd}{2} \right| \) tends to \( N \)).

Fig. 3 shows a plot of the intensity as a function of the SAW height \( h \). The black filled, grey filled and open circles are experimentally measured data points for zero, first and second order diffraction spots, respectively; which verifies the well-established theoretical curve, obtained from eq. 3. In the inset of this figure we have shown the relation between the applied voltage in the function generator (amplitude of oscillation imparted to the pin)
FIG. 4: Diffraction pattern of SAW on water surface

and the amplitude of the resultant SAW. A linear correlation is seen though we note that the SAW amplitude is approximately 100 times smaller than the oscillation amplitude of the exciter. We mention that we have obtained this correlation by first calculating the SAW amplitude from eq. 3 and then relating it with the applied voltage for the same intensity on the diffraction pattern. We have measured the imparted amplitude of vibration of the pin (in air) for different values of the applied voltage, using a travelling microscope.

Fig. 4(a) shows the Fraunhofer diffraction patterns of monochromatic light by SAW when a single pin has been used as SAW exciter. As the loudspeaker amplitude (applied voltage) is increased slowly, at a certain amplitude of vibration the central spot vanishes (Fig. 4(b)). We observe the reappearance (disappearance) of the central spot (first order spot) when the amplitude is increased further (Fig. 4(c)). For a single pin this vanishing of the central and first order diffraction spots takes place for applied voltages 4.5 V and 7.4 V, respectively. Exactly similar diffraction patterns appear for two superposed waves (with light incident along the line joining the sources) for applied voltages 2.4 V and 3.9 V, respectively. In other words, for two pins zero order diffraction spot vanishes for nearly half of the applied voltage compared to that in single pin, as we expect from the above theoretical understanding. Similarly, for three pins the zero order spot vanishes when the applied voltage is 1.7 V.

The displacement ($x'$) of the nth order spot from the centre of the diffraction pattern is $\frac{n\lambda_z}{\Lambda \cos \theta}$. Measuring $x'$ for single pin, we determined the SAW wavelength in our experiment to be 2.1 mm. Moreover, with more than one pin (say two) for certain special values of $kd/2$ two interesting possibilities emerge, which, in turn can be used to check the wavelength of the superposed liquid surface wave along the line joining the sources.
**Complete destructive interference:** It is easy to note that if \( d = \frac{mA}{N} \), where \( m \) is an integer but *not* equal to or *not* a multiple of \( N \) the factor \( \sin \frac{Nkd}{2} / \sin \frac{kd}{2} \) vanishes. If \( N = 2 \), this factor vanishes for \( d = \frac{A}{2}, \frac{3A}{2}, \frac{5A}{2} \).... Similarly, for other values of \( N \). This in turn implies that for these values of \( d \) the argument of the Bessel function would vanish. But \( J_0(0) \) is non-zero only for \( J_0 \), which implies that we see only a central spot at maximum (100 %) diffraction efficiency. This special feature, essentially arising out of complete destructive interference can very easily check the value of the wavelength of the liquid surface waves. Keeping in mind that the change in separation between the two pins \((N=2)\) does not change the SAW wavelength estimated above, this should correspond to \( d = \frac{A}{2} = 1.05 \text{ mm}, \frac{3A}{2} = 3.15 \text{ mm} \) and \( \frac{5A}{2} = 5.25 \text{ mm} \). In our experiment, when the separations between the two pins are 1.0 mm, 3.0 mm and 5.2 mm except the central order, the intensities of the other diffraction spots go to zero irrespective of SAW amplitude.

**No change in diffraction pattern with changing \( N \) and \( h \):** A second feature emerges when the diffraction pattern remains unchanged even when we increase the number of oscillation sources (pins). This will happen when the factor \( \sin \frac{Nkd}{2} / \sin \frac{kd}{2} \) takes on the value one, or when \( d = \frac{mA}{N+1} \), where \( m \) is an integer but *not* equal to or *not* a multiple of \( N + 1 \). In particular, for \( N = 2 \) we have \( d = \frac{A}{3}, \frac{2A}{3}, \frac{4A}{3} \).... For \( A=2.1 \text{ mm} \), this observation must match the values of \( d = \frac{A}{3} = 0.7 \text{ mm}, \frac{2A}{3} = 1.4 \text{ mm}, \frac{4A}{3} = 2.8 \text{ mm} \) and \( \frac{5A}{2} = 3.5 \text{ mm} \). In our experiment, when the separation between the two pins are 1.4 mm, 2.7 mm and 3.4 mm we observe identical diffraction patterns as shown in Fig.4. The zero order diffraction spot vanishes for the same applied voltage 4.5 V in each case. With identical conditions, when we lift one of the two pins, we get the vanishing of the central spot at the same voltage. We could not set the separation between the pins to values smaller than 1 mm. This restricted us from checking this phenomenon for lower values of \( d \). Therefore, the wavelength measured by explicitly working with the argument of the delta function in the intensity expression, can now be checked using superposed liquid surface waves.

The dispersion relation \( [\Omega^2 = \frac{\alpha k^3}{\rho}, \text{ where } \alpha \text{ and } \rho \text{ are the surface tension and the density of the liquid, respectively}] \) also provides a third (theoretical) check on the value of the SAW wavelength. Using this relation we have calculated the SAW wavelength to be 2.1 mm. Finally, using the values obtained above one can find out the value of the phase velocity \( [v = \sqrt{\frac{2\pi \alpha}{\Lambda \rho}}] \) of the liquid surface waves along the line joining the two sources \( [13] \). We have calculated \( v_p = 46 \text{ cm/sec} \).
It is certainly true that all our quantitative results are for the region along the line joining the sources. At other oblique points of incidence (of laser light on the surface), or more importantly in the region where the interference pattern shows interesting features, we have not been able to find the exact analytical expression for the intensity distribution of diffracted light. Fig. 4(d) shows the diffraction intensity from the liquid surface when the light is incident on a region near the dashed-dotted line in Fig. 2. The interference pattern on the liquid surface in this region shows a ‘lattice–like structure’ (more precisely a phase lattice). This structure is indeed reflected in the corresponding diffraction pattern shown in Fig. 4(d). Further theoretical and experimental understanding of diffraction of light from this region is left for future investigation.

To conclude, we have shown how the diffraction of light by interfering liquid surface waves can help us quantify the nature of the superposed waves and also find their wavelength along specific directions (here, the line joining the oscillating pins). The change in the values of the liquid SAW height (amplitude) with the number of oscillating sources is a measure of the “interference” effect on the liquid surface. It is possible, therefore, to comment on the number of oscillating sources by systematically observing the changes in the diffraction pattern as a function of height (voltage). Additionally, using the characteristics of the superposed SAW and the resulting light diffraction features we have been able to provide a way of crosschecking the SAW wavelength. Finally, we have been able to quantify how much wave amplitude is generated in the liquid if the pins are vibrated at some definite amplitude of the exciter. This of course is liquid specific and is an entirely empirical consequence.

AR and TKB thank DST for financial support. The authors thank G. P. Sastry for useful discussions and C. S. Kumar for his help with the photographs.

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