Piles and lateral loads: comparison of calculation methods

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ABSTRACT
Introduction. Calculation and analysis of pile resistance to loads remains to be a relevant problem in geoengineering. The design of pile foundations is currently performed using diverse analytical, empirical and numerical methods. However, the reliability of these methods remains to be a topic of interest among researchers and designers. This research paper analyses methods used for calculating the lateral-load capacity of piles in comparison with field-test data.

Materials and methods. The paper dwells upon the development of reliable analytical expressions based on mathematical models of the pile-soil interaction. Main existing mathematical models of the soil environment, including the Mohr – Coulomb elastic ideal plastic model and the hardening soil model (HSM) were analysed. A particular attention was paid to a variety of factors affecting the pile-soil interaction, such as natural factors, pile types, pile sinking depth and technology, configurations of loads, as well as time-changed processes. A comparison of methods for calculating the lateral-load capacity of piles was conducted. To that end, calculations using the Mohr – Coulomb model and the local elastic strain theory (still required by building codes) were performed. High-level solid elements were used to develop and compute a finite-element pile-in-soil model in a spatial setting. Another model on the basis of parametric pile elements was designed using the MIDAS software.

Results. It is established that the use of numerical calculation methods for evaluating the capacity and movements of pile foundations provides results comparable to those of field tests. These methods demonstrate a higher reliability compared to standardized analytical techniques.

Conclusions. The reliability of numerical calculations of pile resistance to lateral impact is shown to be sufficiently high, thus being feasible for use in geoengineering. The use of these methods should be based on advanced non-linear soil models, such as HS, CamClay, etc.

KEYWORDS: pile foundation, ultimate capacity, finite element method, lateral loading, Mohr – Coulomb model

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INTRODUCTION

The creation of strong foundations has been the topic of continued research attention in the field of construction industry. In this respect, deep foundations, including those on piles, have found wide application. The construction of such foundations frequently involves the development of a pile system that is capable of sustaining significant lateral impact.

Methods used at the beginning of the 20th century for lateral-load resistance calculations were developed for sheet piles under the assumption of an absolutely rigid rod that would rotate when exposed to a lateral load, in which case the soil would be displaced in the upper zone [1–3]. Soil resistance was then calculated using the classical theory of the ultimate stress state of the soil [4]. However, numerous experiments have proven these methods to be unreliable [5, 6].

Subsequent studies proposed to calculate pile parameters assuming that each pile was a beam on an elastic basis; these calculations relied on the Winkler – Fuss hypothesis [7, 8]. This approach is based on the differential equation of a deflection curve:

$$E I \frac{d^4 x}{dy^4} = -k x,$$

where $E$ is the elastic modulus, MPa; $I$ is the moment of inertia, cm$^4$; $k$ is the subgrade reaction modulus (SRM).

This approach has gained popularity due to simplified analytical calculations even compared to the general elastic strain theory, while being acceptably reliable provided that the SRM is accurate enough. Various modifications of this approach have been proposed, mainly those aiming to adjust the SRM for depth. This method is still widely applied in Russian design practices and is recommended by the Russian Building Code 22.13330$^1$.

However, the method of calculating lateral loads on the basis of the local elastic strain theory has a number of significant disadvantages, including the following:

- the method neglects the strain emerging at points in the soil immediately adjacent to the load application area but lying in a different plane;
- SRM values cannot be experimentally obtained for a particular construction site [9] and, subsequently, are frequently derived from standardized tables. Such an approach cannot be considered correct, because standardized values cannot represent a broad spectrum of physical and mechanical soil properties, the variety of pile-soil interaction mechanisms across a wide range of technological, geometric, force-related and other factors;
- various empirical modifications that attempt to model a quasi-linear function of the SRM versus depth are somewhat artificial by nature and have not thus far been confirmed experimentally;
- the method ignores a number of boundary conditions that affect the actual non-linear character of changes in the stress and strain of the anisotropic soil environment.

As a result, the restricted mathematical capacity of this methodology has pre-determined the failure of numerous attempts to improve it by introducing various empirical adjustment factors.

In the view of the abovementioned, the development of reliable analytical expressions based on a mathematical model of pile – soil interaction remains to be

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$^1$ SP 22.13330. Foundations of Buildings and Structures. Updated revision of SNiP 2.02.01-83*
of importance. This apparently requires a more accurate calculation of soil properties. In order to describe the behaviour of soils, use should be made of models that consider the mechanical properties of the soil as functions of its physical properties under the effect of diverse loads, as well as the plastic and rheological properties. Such models should rely on the instruments of continuum mechanics and be realized in software packages, including those based on finite elements. Another prospective approach seems to be the application of discrete mechanics instruments within the framework of a microstructural approach.

MATERIALS AND METHODS

In order to analyse the performance of piles under lateral loads, we used numerical methods based on such fundamental models describing the soil behaviour, as

- the Mohr – Coulomb elastic ideal plastic model;
- the hardening soil model (HSM).

Mohr – Coulomb elastic ideal plastic model

This is the most widely used model in today’s engineering [10] due to a relative simplicity of obtaining the input information, which can always be derived from geotechnical reports:

\[ E \] is the elastic modulus, MPa;
\[ \nu \] is Poisson’s ratio;
\[ \phi \] is the internal friction angle, deg.;
\[ C \] is specific cohesion, kPa;
\[ \psi \] is the angle of dilation, deg.

This model demonstrates the linear nature of destruction and comprises two strength components, i.e. the specific cohesion \( C \) and the internal friction angle \( \phi \), thus describing the tangent stress (shear strength) as a function of the acting normal stresses. In general, the model is represented as the slope \( \sigma \) of the destruction line to the stress axis (the X-axis) at the angle \( \phi \) (2) (Fig. 1).

\[
\tau = \sigma \cdot \frac{C}{\cos \phi} + C,
\]

\( \phi \)

Hardening Soil Model (HSM)

This is a hyperbolic model that takes into account soil hardening induced by shear or isotropic loading [11]. Shear-induced hardening (frictional hardening) is the criterion that differentiates HSM from the Mohr – Coulomb model, i.e. implying that the destruction area is not constant but rather a function of plastic (shear) strain, see Figs. 2 and 3.

The main features of this model are as follows:

- the stiffness of the soil \( m \) depends on the stresses present therein;
- consideration of the plastic shaping strain induced by deviator stresses, which depend on the elastic modulus given deviator loading \( E_{\text{dev}}^{\text{md}} \);
- consideration of the plastic linear strain induced by volumetric stresses, which depend on the odometric elastic modulus \( E_{\text{dev}}^{\text{md}} \);
- a mechanism of elastic behaviour under unloading or reloading, which depends on the unloading modulus \( E_{\text{ur}}^{\text{md}} \);
- a soil destruction mechanism determined by the Mohr – Coulomb model parameters (\( \phi, c, \psi \)).

Factors affecting the pile – soil interaction

The pile–soil interaction is a complex process affected by multiple various factors [4, 12, 3, 13]:

- natural factors, including the background history of the soil (affecting, in particular, its density), the current and predicted state of the soil (including its stress-strain state), the complexity of geotechnical elements, the composition and structure of surrounding soils, as well as their physical and mechanical properties;
- type of piles, their physical and geometric parameters;
- pile sinking depth;
- pile sinking technology;
- configuration of pile-sustained loads;
- processes changing overtime, etc.

Fig. 1. Mohr – Coulomb graphical model
The pile type and design play an important role in the pile–soil interaction. Thus, a number of experiments were carried out in the UK in 1969\(^2\) with the purpose of analysing the pile-soil interaction. The experiments used 5.6-meter-long driven piles made of Ø168 mm steel pipes. The piles were sunk in solid clay soils. The tests were being performed during a year after the piles had been immersed. It was found that setting the piles by driving created a technological gap near the top of the pile, at the wellhead. This gap could be as deep as 8x pile diameter. Further observations of the soil in the gap over the year revealed that the gap had not been diminished by rheological soil recovery processes. According to measurements, the pile–soil adhesion was weak at depths from 8 to 14 pile diameters. At greater depth (>16d), the soil adhesion peaked, exceeding the non-drained shear strength by 20 % or greater (Fig. 4). Therefore, under similar conditions, the resistance to any lateral load on the soil surface will be mainly determined by the pile material.

The obtained experimental data also showed that the lateral-load capacity of piles was also affected by the over-consolidation ratio (OCR) and the pile shaft stiffness (flexibility). The over-consolidation ratio (OCR) is herein a quantitative measure that reflects the specific of formation and the age of dispersed soils. This parameter directly affects the effective lateral stress in the soil, thus determining the lateral stress, which can be expressed as a function of the lateral standby pressure coefficient \(K_p\) [14, 15].

According to [16, 17], the soil strain in sandy or clay soils shows different values, which fact also points to the importance of natural (geotechnical) factors. In sands, displacing a pile causes the soil to deposit on its rear face and shift forward, then in different directions from the front face. In cohesive soils, loading causes soil consolidation; in the limiting state, there emerges a cavity, while the vertical wall near the rear face remains [5].

Another important factor is the configuration of loads having an impact of the pile. Apparently, load-induced pile displacement will be affected by the direction of the load with respect to the main pile axes, by the repetitive character of such a loading, by the ratio and intensity of different loads, as well as by many other factors. Thus, the intensity of the load affects the pile resistance, altering the proportion of friction forces on the pile surface. It was shown that the proportion of friction forces reached 36 % of the total pile resistance in some experiments [5].

\(^2\) Research Association of civil engineers. URL: https://www.ciria.org
Comparison of methods for calculating the lateral-load capacity of piles

The high labour intensity of manual calculations when using non-linear mathematical models of the soil behaviour determined the development of simplified empirical methods. Advancements in the digital industry and the widespread use of numerical computations in engineering have enabled researchers to use and develop complex continuous and discrete mechanics models [18]. The current level of computational resources allows unreliable simplified empirical methods based on the Winkler-Fuss hypothesis either to be entirely abandoned or to be limited to first-approximation estimates.

Computer analysis in geoengineering facilitates consideration of various factors affecting the output while being time-effective. Such calculations are devoid of numerous empirical coefficients set forth in regulatory documents. The reliability of numerical calculations is not determined by the pile stiffness, sinking depth, etc. Numerical calculations can be adjusted to a variety of factors: pile impact configuration, physical and geometric nonlinearity of above-ground structures or foundations, rheological properties, etc.

State-of-the-art finite-element software packages (Plaxis, Midas, RS3, etc.) rely on parametric pile models that offer certain user advantages: a faster and less labour-intensive pre-processor modelling; simplified geometric modelling; a higher probability of problem convergence; wider opportunities of the post-processing stage, providing data not only on the stress-strain state but also on the values and distributions of internal forces, etc. It seems important that both the lateral-surface capacity $f_i$ and the base capacity $R_i$ values as found by the currently standardized methodology, Code 24.13330.2011\(^1\), can be used as the input information.

Despite the aforementioned advantages for engineers, parametric modelling of pile foundations has a number of disadvantages. Thus, these approaches are based on a simplified mathematical function of the element behaviour, with the volumetric solid element being replaced with a unidimensional one. In other words, the distribution of forces and stress-strain calculations take an approximate form [13, 19]. Essentially, algorithms first compute the parametric element behaviour function, which is set (and hidden) by the software developer and normally linear, and only then (separately) describe how the element interacts with the finite element of the surrounding mesh representing the soil. Due to such double calculations, algorithms consecutively generate and calculate two matrices of stiffness, each with different values, e.g. the stiffness set forth in the SP or found experimentally, and the stiffness of the adjacent soil. Multiple studies have shown that such an approach produces significantly distorted results.

Thus, reliable calculations require the use of high-level (16, 32, or more nodes) volumetric solid finite elements in a spatial problem statement using mathematical soil behaviour models. Reliability can be improved by modelling the pile-shaft interaction with the adjacent soil, i.e. the “friction effect”, e.g. by adding interface elements [14].

Input information for computational modelling

In order to compare different calculation methods, we performed a number of numerical calculations using

\(^1\) SP 24.13330 Pile Foundations. Updated revision of SNiP 20.02.03-85.
the Mohr-Coulomb model and the hardening soil model (HSM), as well as those based on the local elastic strain theory so far required by building codes (SP 22.13330). The comparison basis was formed by the results of experiments, in which a bored and cast-in-place pile (25 meters long, 800 mm in diameter) was exposed to lateral loads at a high-rise construction site.

Experiments aimed at assessing lateral loads used a single-beam metal stand rested against two piles. The experimental pile was loaded by a DG-200 hydraulic jack. The lateral force was recorded by a gauge placed in the hydraulic system in front of the jack. Pile displacement was measured using Maksimov’s deflection meters attached to an independent reference system.

Fig. 5. Diagram of experimental pile loading
Table 1. Physical and mechanical properties of the soils

| GTU No | Geotechnical unit (GTU)                           | Yield, $I_1$ | Porosity factor, $e$ | Young’s modulus $E$, MPa | Specific cohesion $C$, kPa | Internal friction angle $\phi$, deg | Soil density $\rho$, g/cm$^3$ |
|--------|--------------------------------------------------|--------------|----------------------|---------------------------|-----------------------------|---------------------------------|-----------------------------|
| 37     | Dusty loessial solid loam                        | 0.04         | 0.70                 | 20                        | 30                          | 25                              | 1.70                        |
| 39     | Dusty loessial solid loam with gravel            | –0.10        | 0.70                 | 15                        | 22                          | 20                              | 1.87                        |
| 40     | Heavy brown solid loam with gravel               | –0.10        | 0.65                 | 22                        | 25                          | 12                              | 1.85                        |
| 55     | Moist heavy brown solid loam with gravel         | –0.20        | 0.67                 | 15                        | 33                          | 25                              | 1.85                        |
| 57     | Solid dusty brown clay with gravel               | –0.20        | 0.52                 | 24                        | 35                          | 29                              | 1.85                        |

Fig. 6. Geology site profile
The pile was loaded in increment, with the first and last two increments being 12.5 and 6.25 tf, respectively. The soil strain was deemed stabilized if the displacement rate did not exceed 0.1 mm over an hour of observation. Piles in the test reached a lateral load of $P = 50.0$ tf, while the stabilized displacement totalled $\Delta H = 16.80$ mm.

The value of ultimate pile resistance was taken at the pile head displacement $\Delta H = 10$ mm and reached 37.0 tf as shown in the test curve (Fig. 5).

Fig. 7. Representative finite element model

Fig. 8. Results of numerical calculations (MIDAS) of pile deflection
Table 1 presents the calculated physical and mechanical soil properties at the construction site.

Fig. 6 shows the geology profile of the lateral-load pile testing site.

High-level solid elements were used to develop and compute a finite-element pile-in-soil model in a spatial setting, see Fig. 7.

It should be noted that we also developed and computed a model using parametric pile elements. However, the obtained results are not presented in the current paper, because they deviated significantly from other calculations.

### RESULTS

Fig. 8 displays the results of finite-element modelling.

The graphs in Fig. 9 compare the results obtained by: the analytical method (SP 22.13330), finite-element analysis using the Mohr – Coulomb (MC) model and the hardening soil model (HS) against the experimental field-test results (XPR). The accuracy factor was found from the ratio $K = \frac{R_{\text{XPR}}}{R_M}$, where $R_M$ are the results of mathematical calculations, $R_{\text{XPR}}$ are the experimental results. The accuracy factors are given in Table 2.

![Figure 9](image.png)

**Fig. 9.** Comparative analysis results

| Method                        | $K$   |
|-------------------------------|-------|
| Analytical method per SP 22.13330 | 0.5075 |
| Mohr – Coulomb model         | 0.7716 |
| Hardening soil model         | 2.1518 |
CONCLUSIONS

Our research has produced the following findings:

1. The actual pile–soil interaction mechanism is affected by multiple factors \cite{20, 21}. The existing methods of pile foundation calculations are based on a limited set of boundary conditions, resulting in diverse estimates of pile resistance to loads. The reliability of such methods depends on the pile type, as well as on the geotechnical conditions. To date, no single versatile method has been developed that could take into account the complexity of the pile–soil interaction under various loads \cite{22, 23};

2. The use of numerical calculations for assessing the capacity and movements of pile foundations provides results comparable to those of field tests. However, the reliability of numerical calculations is significantly dependent on the choice of a soil model \cite{24};

3. A high reliability of numerical calculations makes them feasible for assessing the capacity and movements of pile foundations. The choice of a calculation model should take into account the mathematical formulation of the pile–soil interaction, the geotechnical conditions, the availability of the input information, design conditions, etc.

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