Wave parameter sensitivity in structural dependent reliability of jack-up

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Abstract. The response treatment of structural systems subject to stochastic loading requires the mathematical modeling of the input load, propagation such as input through the systems, and the treatment of uncertainty inherent in the output using probabilistic methods. For jack-up structures, the input load from the wave is modeled using the significant wave height and wave period which are statistical averages obtained from long term studies. The variability in the determination of these wave parameters leads to further uncertainty in the response output. This study, therefore, investigated the sensitivity of these wave parameters to the time-dependent reliability of jack-up. The wave was modeled using Karhunen-Loeve representation, and the propagation of uncertainty was performed on a simplified jack-up model using Duhamel convolution integral. The response analysis was performed using VanMarckes time-dependent solution method. The results of jack-up survival probability obtained for different failure criteria in deck displacement show that the probability is more sensitive to the wave period than the wave significant height.

1. Introduction
Acceptance and or rejection assessments of jack-up unit is conducted whenever the unit is offered for use in a given offshore location [1]. This is to determine whether or not, and the unit can withstand the most severe environmental condition that is expected to occur at least once within a given period known as return period. Due to the developments in computational studies, virtual simulations enable these assessments to be made efficiently by developing mathematical models describing the random wave process associated with this return period. The structural performance due to these waves can then be studied, and the decision can be made on the safety of jack-up during its operation in the given location.

Due to natural variabilities in the input waves, the output from the structural performance becomes uncertain, and the use of probability theories can quantify these uncertainties. The use of probability theories in jack-up safety assessment is accomplished by modeling the wave using the two-parameter spectrum models describing the locations of its use and the determination of the survival or failure probabilities. These two parameters, the significant wave height and wave period, are obtained from statistical studies from short and long term observations in the given locations. Consequently, the variability of these statistical parameters results in the variability of the eventual probabilities obtained.
from their use. This variability in the output of the probability studies due to input variation is studied using sensitivity analysis, which measures the importance of the input variables.

Several studies were reported on the jack-up time-independent reliability analysis using extreme events and time-dependent analyses using the distribution of first time to failure [2-5]. However, no studies were made to determine the sensitivity of these input variables used in modeling the wave. This study, therefore, aims to present the sensitivity of wave significant height and wave period in the determination of the jack-up survival probabilities.

2. Methodology

In this study, the sensitivity of wave parameters to the time-dependent survival probabilities of jack-up was investigated. The wave was simulated using the two-parameter JONSWAP wave energy density spectrum by varying the significant wave height and wave periods, as given in Table 1. For each of the wave parameters, the wave was simulated, and the structural response was dynamically evaluated to obtain the response. Time-dependent reliability analysis was then performed to determine the survival probability of the jack-up for three different failure criteria in deck displacement. Table 1 shows the wave parameters selected for use in modeling the wave profile. The wave parameters represent the environmental conditions for different return period that is normally used in the determination of the design wave for use in offshore structural designs.

Table 1. Parameters for simulating the wave.

| Hs (m) | Ts (s) |
|-------|-------|
| 8.98  | 9.35  |
| 10.60 | 10.16 |
| 10.9  | 7.39  |
| 12.0  | 10.81 |
| 14.4  | 11.8  |

The ocean wave modeling was done using KLE expansion method in which the eigenfunctions of prolate spheroidal wave functions were developed and used as the orthogonal basis functions. The KLE expansion of the wave is given as [6-9];

\[ \zeta(t_k) = \sum_{n=1}^{n_{KL}} \alpha_n f_n^{KL}(t_k) \quad -T < t < T \quad (1) \]

Where the function \( f_n^{KL} \) is the nth component of the K-L vector at each time step of the expansion and \( \alpha_n \) are independent identically distributed random variables with zero mean and standard deviation given by the eigenvalues \( \kappa_n \) from the solution of the Fredholm integral equation of the K-L expansion.

The propagation of uncertainty was performed by solving the Duhamel integral equation using a simple computer algorithm. The properties of the simplified jack-up model used were obtained from [10]. From the integral, the response of the structure in discrete time steps is given as [11];

\[ R(t) = q(t)^T z \quad (2) \]

Where \( z \) is the vector whose dimension was given as KL, the number of terms that were kept in the KLE representation of the wave, and it represents the Gaussian random variables associated with each term of the expansion. \( q(t) \) is a vector also with dimension nKL whose components were given as;
\[ q_s(t) = \sum_{l=1}^{k} F_s(t) h(t_l - t_s) d\tau \quad s=1 \ldots n_{sl} \] (3)

Time-dependent reliability for analysis method for structural systems subject to stochastic loading was used in the determination of the probability that the response obtained remains below the allowable threshold value. In this study, the VanMarckes and integral equation methods were used. For each of the wave parameters, the wave was modeled and the response obtained in terms of maximum deck displacement is used.

3. Results and discussions

The two-parameter wave spectral density models used in the wave simulations were described using the wave significant height \( H_s \) and zero-crossing period \( T_z \). These parameters vary according to a given offshore site as well as a given return period used in the selection of the design wave. Consequently, the sensitivity of the parameters to the time-dependent survival probability of jack-up was investigated. Three different criteria of the deck displacements (\( b=0.4m, b=1.6m, \) and \( b=3.0m \)) were selected, representing lower, medium and higher threshold levels.

Figure 1. The sensitivity of wave parameters to probability for \( b=0.4m \).

Figure 1 shows the plot of the jack-up survival probability at different values of the wave parameters for the threshold value of 0.4 m. From the results, both the VanMarckes approximation and the single crossing methods shows high sensitivity to the wave peak period than the wave significant height. This was seen from the plot as the values of the probabilities were skewed more towards the wave period axis. The single crossing methods at this threshold level gives higher survival probabilities than the VanMarckes, due to the accuracy of the method at lower threshold levels.

Figure 2. The sensitivity of wave parameters to probability for \( b=1.6m \)
Figure 2 shows the plot of the jack-up survival probability at different values of the wave parameters for the threshold value of 1.6m. From the results, both the VanMarckes approximation and the single crossing methods show high sensitivity to the wave peak period than the wave significant height. However, at this threshold level, the sensitivity to the wave period is reduced as the plot moved towards the center, indicating an increase in the sensitivity of the wave significant height at medium threshold levels.

Figure 3 shows the plot of the jack-up survival probability at different values of the wave parameters for the threshold value of 3m. Similarly, from the results, both the VanMarckes approximation and the single crossing methods show high sensitivity to the wave peak period than the wave significant height. At high threshold levels, the sensitivity to the wave period was seen to be similar to the medium threshold levels.

4. Conclusion
This study presented the sensitivity of wave parameters to the time-dependent survival probability of jack-up. Wave parameters used in the offshore structural design were selected and varied. For each of the wave parameter sets, the wave was simulated using KLE expansion method with eigenfunctions of Prolate Spheroidal Wave Functions. The structural analysis was performed using Duhamel convolution integral by selecting and solving a simplified jack-up model from the literature. Two different time-dependent reliability analysis methods were used to determine the survival probability of the jack-up due to the different waves. In conclusion, the time-dependent survival probability is found to be more sensitive to the wave period than the wave significant height.

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