Experimental verification and comparison of mode shape-based damage detection methods

M Radzięński\textsuperscript{1}, M Krawczuk\textsuperscript{1,2}

\textsuperscript{1} Technical University of Gdansk, Faculty of Electrical and Control Engineering, Narutowicza 11/12, 80-952 Gdansk, Poland

\textsuperscript{2} Institute of Fluid Flow Machinery, Polish Academy of Sciences, ul. Fiszera 14, 80-952 Gdansk, Poland

Email: Maciej.Radzienski@gmail.com

Abstract. This paper presents experimental verification and comparison of damage detection methods based on changes in mode shapes such as: mode shape curvature (MSC), modal assurance criterion (MAC), strain energy (SE), modified Laplacian operator (MLO), generalized fractal dimension (GFD) and Wavelet Transform (WT). The object of the investigation is to determine benefits and drawbacks of the aforementioned methods and to develop data preprocessing algorithms for increasing damage assessment effectiveness by using signal processing techniques such as interpolation and extrapolation of measured points. Noise reduction algorithms based on moving average, median filter, and wavelet decomposition are also tested. The experiments were performed on an aluminium plate with riveted stiffeners. Damage was introduced in a form of damaged rivets and a saw cut in the angle bar. Measurements were made using a non-contact Scanning Laser Doppler Vibrometer (SLDV) at 101 points in two rows, distributed over the structure height and positioned along two reinforcing ribs.

1. Introduction

In the last two decades non-destructive examination (NDE) and structural health monitoring (SHM) has received a considerable amount of interest in the literature. Among them vibration analysis for damage detection was the most propagate due to its implementation simplicity \cite{1, 2}. Development of Laser Scanning Doppler Vibrometer (SLDV) devices makes those techniques are even more attractive because of non-contact, high resolution vibration measurements in short time periods.

Vibration-based damage detection methods make use of the dynamic responses of a structure, such as natural frequencies and mode shapes. Occurrence of damage leads to changes in structural dynamics. These changes may be used for damage localization and its severity assessment.

Recently, many studies were made proposing different vibration-based damage detection methods. However, in the majority of cases those methods were tested on numerical results. Additionally, it is hard to evaluate the utility of a specific method without comparison of results obtained from the same set of data.

Thus, the purpose of the present study is to determine benefits and drawbacks of the most popular and promising methods, relying on direct comparison of them. Furthermore, it is shown that digital signal processing of the measured data may enhance results in damage localization.
2. Damage identification methods

In this section, six different damage detection methods are presented. The first three of them (2.1-2.3) are based on comparison between two sets of measurements. The first is a reference state and the second is a current state of the structure. Another group of damage detection methods, described in sections 2.4-2.6, make use of only current state data for damage detection and localization. However, using additional reference data may improve effectiveness.

2.1. Mode shape curvature (MSC) and damage index (DI)

Following [3, 4, 5] Pandey in 1991 proposed the use of the second derivative of the mode shape as a tool for damage detection. It has been reported that damage of the structure causes changes in mode shape curvature and that it can be a good indicator for detecting and localizing the damage.

This method is founded on determining the absolute difference of mode shape curvature (MSC) or mode shape curvature squares (MSCS) for the undamaged and damaged object as follows:

\[
MSC_i = \sum_{j=1}^{N_m} \left| \phi''_{i,j} - \phi''_{d,i,j} \right|, \quad MSCS_i = \sum_{j=1}^{N_m} \left| \phi''_{i,j} - \phi''_{d,i,j} \right|^2, \tag{1}
\]

where \(\phi''\) and \(\phi''_d\) are the mode shape second derivatives of undamaged and damage states respectively and \(N_m\) is the number of measured modes and \(i\) is measurement point index.

Stubbs and Kim proposed damage index (DI), which is also based on changes in mode shapes curvatures and it is defined as [4, 5]:

\[
DI_i = \sum_j \left( \frac{\left( \phi''_{i,j} \right)^2 + \sum_{k=1}^{N_m} \left( \phi''_{i,k,j} \right)^2 \cdot \sum_{k=1}^{N_m} \left( \phi''_{d,i,k,j} \right)^2 \right)}{\left( \phi''_{d,i,j} \right)^2 + \sum_{k=1}^{N_m} \left( \phi''_{d,i,k,j} \right)^2 \cdot \sum_{k=1}^{N_m} \left( \phi''_{i,k,j} \right)^2} \cdot \tag{2}
\]

2.2. Modal assurance criterion (MAC)

One of the methods for localizing damage in the structure consists in using the modal assurance criterion (MAC) and co-ordinate modal assurance criterion (COMAC), which were presented by Allemang and Brown (1982) and Lieven and Ewins (1988), respectively. These indexes indicate the correlation between two sets of mode shapes. The MAC is a global index of damage detection and gives information about which modes have been changed the most. The COMAC compares mode shapes in a point-wise manner giving the possibility of locating the damage [5, 6]:

\[
MAC_{j,k} = \left( \frac{\sum_{i=1}^{N_p} \phi_{i,j} \phi_{i,k}^d}{\left( \sum_{i=1}^{N_p} \phi_{i,j}^2 \right)^{1/2} \left( \sum_{i=1}^{N_p} \phi_{i,k}^d^2 \right)^{1/2}} \right)^2, \quad COMAC_i = \left( \frac{\sum_{j=1}^{N_m} \phi_{i,j} \phi_{i,j}^d}{\left( \sum_{j=1}^{N_m} \phi_{i,j}^2 \right)^{1/2} \left( \sum_{j=1}^{N_m} \phi_{i,j}^d^2 \right)^{1/2}} \right)^2 \tag{3}
\]

where: \(\phi_{i,j}\) and \(\phi_{i,k}^d\) stands for a value of mode shape \(j\) and \(k\) at point \(i\), in the undamaged and damage object respectively. \(N_p\) is a number of points, \(N_m\) is a number of modes.

The diagonal value of the MAC matrix varies from 0 to 1 and indicates correlation between the \(j'\)th mode shape of undamaged structure and \(k'\)th damage structure, where 0 is for absence of correlation and 1 for complete correlation.

2.3. Strain energy damage index (SEDI)

Cornwell [6] suggested that a structure can be divided into small areas. If there is a damage in one of them, changes in flexural rigidity for a specific area may be used to localize damage.
For a beam-like structure the strain energy damage index is given by:

\[
\beta_i = \frac{\sum_{j=1}^{N_i} \left( \frac{\partial^2 \phi_j}{\partial x^2} \right)^2 \ dx}{\sum_{j=1}^{N_i} \left( \frac{\partial^2 \phi_j}{\partial x^2} \right)^2 \ dx},
\]

(4)

2.4. Modified Laplacian operator (MLO)

This method is based on localizing changes in slope of a mode shape caused by the damage. Because experimental mode shapes are a series of measured points distributed in space, therefore the second difference of this signal can be estimated by Laplacian difference equation. The one dimensional Laplacian \( \Delta_i \) of discrete mode shape \( \phi_i \) is given by:

\[
\Delta_i = \phi_{i-1} - 2\phi_i + \phi_{i+1}.
\]

(5)

Ratcliffe [4] introduced a modified Laplacian operator to enhance the results for one dimensional structures and Qiao et al. [7] extended this for two dimensional structures. This method is based on calculating the difference between Laplacian and a cubic fit polynomial to the Laplacian. A cubic function is determined for every point of the Laplacian, using only two points on either side of the considered element for coefficients calculations.

2.5. Generalized fractal dimension (GFD)

The fractal dimension (FD) was originally proposed by Mandelbrot [8]. In 1988 Katz’s defined estimation of FD for curvature using a sequence of points. However, this method of estimating FD could give false peaks in higher mode shapes in maximum and minimum points of its first derivative. To overcome this problem Wang and Qiao [2, 9] proposed using scale factor in the FD algorithm. They call this method generalized fractal dimension (GFD) and it is defined as follows:

\[
GFD_M(x) = \frac{\log(n) + \log(ds(x_i, M)/Ls(x_i, M))}{\log(n)},
\]

(6)

\[
Ls(x_i, M) = \sum_{j=1}^{M} \sqrt{(y(x_{i,j}) - y(x_{i,j-1}))^2 + s^2(x_{i,j} - x_{i,j-1})^2},
\]

(7)

\[
ds(x_i, M) = \max_{i,j,M} \sqrt{(y(x_{i,j}) - y(x_i))^2 + s^2(x_{i,j} - x_i)^2}.
\]

(8)

2.6. Wavelet transform (WT)

A wavelet is a function used to decompose a signal \( f(x) \) into series components \( \psi_{x,s}(x) \) derived from a mother wavelet \( \psi(x) \) by scaling and translating, as given by:

\[
\psi_{x,s}(x) = \frac{1}{\sqrt{s}} \psi\left(\frac{x-u}{s}\right),
\]

(9)

Considering the mode shape \( \phi \) as a spatial one-dimensional signal, the continuous wavelet transform (CWT) can be obtained as [10, 11]:

\[
W\phi(u,s) = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} \phi(x)(x-u/s) \ dx.
\]

(10)

The wavelet transform has the ability to find singularities in considered functions. Therefore, an abrupt change or peak in wavelet coefficient can be used as an indicator of the damage location.
3. Experimental setup and results

Figure 1 shows the experimental set-up used in measuring mode shapes and frequency response functions (FRFs) of the aluminium plate by a Polytec PSV-400 Scanning Laser Doppler Vibrometer (SLDV). An electromechanical shaker (GWV B100) was used for vibration excitation.

In the experiment an “FFT” acquisition was firstly performed to obtain FRFs using a periodic chirp excitation. This provides natural frequency determination by finding local maxima in an averaged FRF graph. Those frequencies were used successively as a single-frequency excitation, namely “FAST SCAN”, for measuring corresponding mode shapes. The noise level of measured mode shapes is mostly determined by utilizing frequency bandwidth \( B_w \) and the signal level. The noise level is proportional to \( \sqrt{B_w} \) but the time of acquisition is inversely proportional to \( B_w \), so it should be considered to find a middle ground between acquisition time and precision [12]. The signal can be maximized by coating the measured surface with light back-scattering paint, such as bright acrylic paint used in this research. In addition a scanning head should be positioned away from the examined structure in laser visibility maximum if possible.

![Figure 1. Experimental set-up for measuring aluminium vibrations using a PSV-400 scanning laser vibrometer.](image)

In order to compare presented damage detection methods experimentally, a 70cm x 100cm x 0.4cm aluminum plate with two riveted stiffeners was tested. The test structure was fixed to the ceiling by two sets of 1mm steel cables connected to the bracing, which is an approximate of four free boundary conditions. Stiffeners were equal-sided 70cm x 4cm x 0.4cm aluminum angles and they were jointed to the plate by two rows of 27 aluminum rivets in each of them (totally 108 rivets in the whole structure). Two damage scenarios were tested. At first, 6 and 12 rivets were cut in one or both stiffeners. Next, the damage was made by a saw cutting the orthogonal leg of the right angle in the middle of its length by 25% and 50% depth. Measurements were made in two rows between rivets along the plate height in 101 points each side. The specimen schematic drawing and a photograph are shown in figure 2.

![Figure 2. Aluminum plate: (a) schematic drawing, (b) real object.](image)
3.1. Signal processing

Despite conditioning measurements for minimal noise level, there could still be some disturbances that may bring inaccurate or even false damage localizations. In order to overcome this problem, different noise reduction methods were tested based on moving average, median filter and wavelet decomposition.

Without de-noising the signal, only two methods could localize the damage, namely GFD and WT. This proves that those methods are less noise sensitive than others. Median and moving average filtering are able to reduce noise levels. However, it does not improve the effectiveness of the damage detection methods. Moreover, after filtering only WT was able to indicate damage position. De-noising process based on wavelet decomposition was the only one which benefits in more accurate damage detection in all described methods. For this purpose `wden' MATLAB® function was used with level of decomposition \( L=2 \) for detail coefficients obtaining by wavelet ‘sym8’ and the threshold selection rule set for heuristic without rescaling. After de-noising, mode shapes were interpolated using a cubic spline algorithm to increase number of measured points three times and that gives more accurate damage localization. The last operation in data preprocessing was extrapolating the signal by a few points on each side. This operation helps to minimize the negative boundary effect in some of the damage localization methods. After calculating specific damage index, additional points on edges were cut off.

An example of the mode shape before and after data preprocessing is shown in figure 3. In spite of some subtle changes, the results of damage localization are completely different, which is presented in figure 4, exemplified by damage index calculated for 50% saw cut case, using 8 mode shapes. Using raw data leads to very jagged output function with the maximum value in 0.4 of structure length. The same index computed for preprocessed data gives clearly information about damage localized in the middle of the plate stiffener, which corresponds to the real state.

3.2. Results

Because of limited space, results from only one damage scenario will be presented. Figure 5 presents FRFs of examined plate and its changes caused by the damage occurrence. Natural frequencies read out from FRFs are shown in table 1. All results from relative damage detection methods, for the case of 50% deep saw cut in the middle of the right stiffener are shown in figure 6.

| \( f_{\text{reference}} \) [Hz] | \( f_{50\%\text{cut}} \) [Hz] | \( \Delta f_{50\%\text{cut}} \) [%] | \( f_{50\%\text{cut}} \) [Hz] | \( \Delta f_{50\%\text{cut}} \) [%] |
|-----------------------------|-----------------------------|-------------------------------|-----------------------------|-------------------------------|
| 247.66                      | 247.50                      | 0.06                          | 247.19                      | 0.19                          |
| 318.75                      | 318.59                      | 0.05                          | 316.72                      | 0.64                          |
| 566.09                      | 565.47                      | 0.11                          | 564.22                      | 0.33                          |
| 657.34                      | 656.41                      | 0.14                          | 654.22                      | 0.48                          |
| 994.69                      | 994.06                      | 0.06                          | 992.81                      | 0.19                          |
| 1177.81                     | 1176.25                     | 0.13                          | 1173.75                     | 0.34                          |
| 1406.56                     | 1405.00                     | 0.11                          | 1404.69                     | 0.13                          |
| 1581.88                     | 1581.88                     | 0.00                          | 1581.88                     | 0.00                          |
Figure 5. Frequency response function.

Figure 6. Damage detection relative indexes: (a) mode shape curvature, (b) mode shape curvature squares, (c) damage index, (d) strain energy damage index, (e) modal assurance criterion, (f) co-ordinate modal assurance criterion.

All indexes besides MAC and SEDI were statistically normalized using z-score transformation. This provides easier result analysis and reliable comparison.

Figure 7 shows results obtained from three absolute damage detection methods described in section 2. Additionally, the same methods were used as relative indexes through calculation difference of results for damaged and undamaged plates mode shapes. They are presented in figure 8. All indicators from figure 7 and 8 were obtained by summing indexes estimated for 8 mode shapes separately.
GFD were calculated with scale $s = 100$ and sliding window size of $M = 6$.
For WT ‘gaus4’ mother wavelet was used with scales $s$ from 1 to 50. For this method interpolation was changed from 3 for each point to 10, and extrapolation point number was established as a half of maximum scale parameter on both sides in order to minimize the boundary effect.

![Figure 7](image1.png) **Figure 7.** Damage detection absolute indexes: (a) modified Laplacian operator, (b) generalized fractal dimension, (c) wavelet transform.

![Figure 8](image2.png) **Figure 8.** Damage detection absolute indexes used as relative indexes: (a) modified Laplacian operator, (b) generalized fractal dimension, (c) wavelet transform.

### 3.3. Discussion

All relative damage detection methods were able to correctly pinpoint the damage in the middle of the measured distance. However, COMAC and SEDI are not giving as sharp peaks in the crack position as other methods.

It is worth noticing that an increasing number of measured points does not necessarily provide better results, due to noise propagation in mode shapes second derivatives. This effect was described in detail by Sazonov and Klinkhachorn [13]. However, when de-noise techniques are used, they may take advantage of high spatial measurements.

As far as the absolute damage detection methods are concerned, both GFD and WT clearly identify the right position of damage. Nevertheless, MLO was not producing correct results. Calculation difference between indices for reference and current state leads to the increased effectiveness of those methods. Thus GFD difference computing can remove three peaks near the edges of the structure leaving only one correctly localized damage position. In case of using WT as a relative index, it also brings in more explicit information about damage location.
Similar results of presented damage detection methods for a steal cantilever beam with a notch were obtain by Radzieński et al. [14].

4. Conclusions
The presented work is devoted to comparing damage detection techniques based on vibration analysis. Six various methods are verified and compared on real measured data which lead to the following conclusions and suggestions:

- both measured structure surface preparation and proper positioning of the scanning head are important for providing high quality measurements,
- digital signal processing may hardly enhance the effectiveness of most damage detection methods,
- in relative damage detection methods the bests results are received from MSC and DI computing,
- WT seems to be the most effective, noise independent and versatile damage detection method,
- In case of WT and GFD based methods, primary state of structure data may be used for increasing damage assessment effectiveness if possible,
- Further studies on experimental verification of damage detection methods for multiple damages and for 2D signal measurement should be conducted.

References

[1] Doebling S W, Farrar C R, Prime M B 1998 A summary review of vibration-based damage identification methods Shock and Vibration Digest 20 91-105
[2] Wang J, Qiao P Z 2007 Improved Damage Detection for Beam-type Structures using a Uniform Load Surface Structural Health Monitoring 6(2) 99-112
[3] Pandey A K, Biswas M, Samman M M 1991 Damage detection from changes in curvature mode shapes Journal of Sound and Vibration 145 321-332
[4] Ratcliffe C P 1997 Damage detection using a modified Laplacian operator on mode shape data Journal of Sound and Vibration 204 505-517
[5] Allemang R J, Brown D L 1982 A correlation coefficient for modal vector analysis Proc. 1st Int. Modal Anal. Conf. 1, 110–116.
[6] Lieven N A J, Ewins D J 1988 Spatial correlation of mode shapes the coordinate modal assurance criterion (COMAC) Proc. 6th Int. Modal Anal. Conf. 1 690–695
[7] Qiao P Z, Lu K, Lestari W 2008 A Combined Static/Dynamic Technique for Damage Detection of Laminated Composite Plates Experimental Mechanics 48 17–35
[8] Mandelbrot B B 1967 How long is the coast of Britain? Statistical self-similarity and fractional dimension Science 156 636-638
[9] Wang J, Qiao P Z 2008 On irregularity-based damage detection method for cracked beams International Journal of Solids and Structures 45 688–704
[10] Rucka M, Wilde K 2006 Application of continuous wavelet transform in vibration based damage detection method for beams and plates Journal of Sound and Vibration 297 536-550
[11] Morlier J, Bos F,Castera P 2006 Diagnosis of a portal frame using advanced signal processing of laser vibrometer data, Journal of Sound and Vibration 297 420–431
[12] Jin S, Pai P F 2000 Locating Structural Defects Using Operational Deflection Shapes, Journal of Intelligent Material Systems and Structures 11 613-632
[13] Sazonov E , Klinkhachorn P 2005 Optimal spatial sampling interval for damage detection by curvature or strain energy mode shapes, Journal of Sound and Vibration 285 783–801
[14] Radzieński M, Krawczuk M, Ostachowich W 2009 Experimental verification and comparison of mode shape-based damage detection methods, Proceedings of the 8th International Conference on Damage Assessment of Structures (DAMAS2009)