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Abstract

Entanglement, one of the most intriguing aspects of quantum mechanics, marks itself into different features of quantum states. For this reason different criteria can be used for verifying entanglement. In this paper we review some of the entanglement criteria casted for continuous variable states and link them to peculiar aspects of the original debate on the famous Einstein–Podolsky–Rosen (EPR) paradox. We also provide a useful expression for valuating Bell-type non-locality on Gaussian states. We also present the experimental measurement of a particular realization of the Bell operator over continuous variable entangled states produced by a sub-threshold type-II optical parametric oscillators (OPOs).

Keywords: entanglement, Bell operator, EPR paradox

(Some figures may appear in colour only in the online journal)

1. Introduction

Since the first reply by Schrödinger [1] to the famous EPR paper [2] the word ‘entanglement’ has been primarily used for indicating a class of quantum states that shows non-local features. Discussing the dynamical properties of a composite system made of two subsystems that, after mutual interaction, move away one from the other Einstein Podolsky and Rosen concluded that quantum mechanics was not complete and that some more local (hidden) dynamical variables would have been necessary for a correct description of the physical reality. Essentially, they pointed to two quantum-mechanical aspects that they found counter-intuitive. The first was the possible ambiguity of the wave function so that ‘… as a consequence of two different measurements performed upon the first system, the second system may be left in states with two different wave functions …’. The second aspect was later revealed by Einstein himself as a *spooky action at distance* ‘… since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system …’. We now know that they were wrong and that quantum mechanics gives a complete representation of these strange phenomena.

Up to the late 1950s the debate on entanglement was mostly confined to the fundamental aspect of quantum mechanics and the word itself hadn’t any particular operational meaning. In 1957 a paper by Bohm and Aharonov [3] moved the focus from the original *Gedankenexperiment* toward more feasible and intuitive physical implementations and, in particular, to spin-like systems. This paved the way to a more complete theoretical analysis of the hidden variables scenario that leads to

5 In Bell's original paper only one inequality is discussed, namely equation (15) in [4] that was, five years later, translated into a set of experimentally verifiable inequalities, since then known as CHSH type inequalities [5].
the famous Bell inequalities\(^5\) \([4]\) for dichotomic quantum variables. This made spin-like systems the preferred candidates for proving the failure of any hidden variables hypotheses. Single photons have been widely used in several experimental tests of the Bell inequalities (see for a review \([6]\)). Very recently, a novel experiment made the photon the first physical system for which each of the main loopholes has been closed \([7]\).

On the other hand, the original formulation of the EPR paradox was based on continuous variable systems. In 1986 Reid and Walls proposed the first translation of Bell inequalities into the language of continuous variables (CVs) \([8]\) and in 1992 the first experimental realization of an EPR-like system appeared \([9]\). Since then, a few more attempts have been carried out for translating the Bell argument into the language of CVs \([10]\). Among them the one proposed by Banaszek and Wódkiewicz \([11]\) considers the relation between the Wigner function of the state and non-locality.

Entanglement, in its original formulation, states the existence of global states of a composite system which cannot be written as a product of states of individual subsystems. While this definition set an unequivocal border between separable and entangled states, entanglement gives rise to different features of quantum systems and can be seen under different perspectives \([12]\). On the one hand, mathematically defining entanglement as a property of the composite system wave function makes it intrinsically related to pure states \([13]\). On the other hand, we all know that experimentally accessible states are mixed, so that feasible entanglement tests have to be related to density matrices rather than wave functions \([14]\).

In this paper we discuss the operational meanings of different criteria usually employed for assessing CV entanglement. In particular, we will link each of them to different facets of the original entanglement debate. We will apply them to entangled Gaussian states (GSs) \([15, 16]\) produced by a type-II sub-threshold frequency degenerate OPO. By experimentally analyzing the properties of experimentally generated CV entangled states we will express all these criteria in terms of the covariance matrix (CM) elements. We also provide a novel and useful relation that describes a physical state iff \(\det(\sigma) = 1\). Also, at the time of birth inside the non-linear crystal, the bipartite state we analyze shows \(c_1 = -c_2 = c\). From the symplectic invariants it is possible to provide criteria for distinguishing among physical and non-physical CMs. \(\sigma\) describes a physical state if\(6\)

\[
I_1 + I_2 + 2I_4 \leq 4I_4 + \frac{1}{4}.
\]

We also note that a pure GS is a minimum uncertainty state and that the CM relative to a pure state necessarily has \(\det(\sigma) = I_4 = 1/16\) so that for a pure state

\[
W(K) = \frac{\exp\left\{ -\frac{1}{2}K^T\sigma^{-1}K \right\}}{2\pi\sqrt{\det(\sigma)}}
\]

where \(K \equiv (X_{\alpha,a}, X_{\alpha,b}e^{i\phi}, X_{b,a}, X_{b,b}e^{i\phi})\) is the vector of a set of orthogonal quadratures, for modes \(\alpha\) and \(b\) respectively (being \(\hat{X}_{\alpha,b} \equiv \hat{X}_\alpha e^{i\phi}\)). We would remind the reader that the pair \(X_0 = \hat{X}, \) and \(\hat{X}_{\alpha,\beta} = \hat{Y} ([\hat{X}_b, \hat{Y}_a] = i)\) associated to a single e.m. mode is the analogue to the position/momentum pair for a mechanical oscillator. This makes the optical mode a good candidate for replicating EPR states in their original fashion. All the features of GSs are embedded in the second order momenta of the joint quadrature distribution, namely the CM

\[
\sigma \equiv \begin{pmatrix} \sigma_0 & \sigma_1 \\ \sigma_1^T & \sigma_2 \end{pmatrix}
\]

by means of local symplectic transformations\(^3\) where \(n, m, c_1\) and \(c_2\) are determined by four local symplectic invariants \(I_1 \equiv \det(\sigma) = \tilde{n}^2, I_2 \equiv \det(\beta) = m^2, I_3 \equiv \det(\gamma) = c_1c_2, I_4 \equiv \det(\tau) = (nm - c_1^2)(nm - c_2^2)\). In fact, a sub-threshold type-II OPO, due to the symmetry of its Hamiltonian, can only produce states whose CM is a standard form \([18]\). Hereafter, whenever we refer to CMs we will mean such a standard form. Also, at the time of birth inside the non-linear crystal, the bipartite state we analyze shows \(n = m = c_1 = c_2 = c\).

A continuous-variable bipartite GS is a two-mode state, on the Hilbert space \(\mathcal{H} = \mathcal{H}_a \otimes \mathcal{H}_b\), whose characteristic function or, equivalently, Wigner function in phase space is Gaussian:
\[ c = \sqrt{n^2 - 1/4} \]  
while, for mixed symmetric states, \( c < \sqrt{n^2 - 1/4} \). In general, for a bipartite GS, the purity reads
\[ \mu(\sigma) = \frac{1}{4\sqrt{\text{Det}[\sigma]}}. \]

3. Entanglement criteria

A quantitative measure of entanglement for a mixed state is, so far, an unsolved issue. This is probably due to the different operational implications that different levels of quantum correlation open. At the same time, there exist different necessary and/or sufficient conditions for assessing whether a given state is entangled or not. These criteria are easily translated into experimental tests for entanglement. Here we aim to look at the different criteria and connect them, logically, to the debate on the original EPR paper [2].

3.1. Unseparability criteria: PHS and Duan

The first criterion was developed by considering the definition of an entangled state, which is a state of a composite system whose wave function cannot be given as the product of sub-systems’ wave functions. This definition, in the case of mixed states, can be extended to the density formalism following Werner[14]. In the bipartite case a density matrix represents a separable state if it can be written as a convex combination of the tensor product of density operators relative to the two sub-systems
\[ \rho = \sum_{j} \rho_{j} \otimes \rho_{2}, \]

where \( \sum \rho_{j} = 1 \) while \( \rho_{j} = 1, 2 \) are the density matrices of subsystems 1 and 2. The criterion can be cast by considering that if one performs a partial transposition (i.e. a transposition of the density matrix with respect to only one of the two Hilbert subspaces) \( \rho \) transforms into \( \rho_{PT} \) that, for a state written in the form given in equation (7) will still represent a physical state of the composite system. Conversely, if the state is unseparable, the transformed density operator \( \rho_{PT} \) would no longer have a physical counterpart. This criterion is sometime referred to as the \( \text{ptt} \) criterion (positivity under partial transposition) or PHS from the names of the people that proposed it for discrete (Peres [19] and Horodecki [20]) and continuous variables (Simon [21]). Translated into the CM language a bipartite GS is separable iff
\[ n^2 + m^2 + 2\text{loc}\leq4(nm-c^2)(nm-c^2) \leq \frac{1}{4}, \]
and it is entangled otherwise. We also note that the PHS criterion is invariant under symplectic transformations and that for pure states the inequality is saturated.

The PHS criterion set, then, has its roots in the fact that two systems that have interacted cannot, even if they split apart after the interaction, be described independently.

A second criterion, the Duan one [17], has been derived by considering that, in the presence of an entangled state, the Heisenberg uncertainty principle, written for the joint system and a pair of EPR-like operators, also has to take into account the inherent quantum correlation. The Duan criterion, a necessary condition for entanglement, for a CM in the usual standard form, reads:
\[ \sqrt{(2n-1)(2m-1)} - (c_1 - c_2) < 0. \]

Based on the calculation of the total variance of a pair of EPR-type operators, it relies on the fact that the inherent correlation reduces the total variance that, in separable states, is greater than the sum of the standard quantum limit applied to the single subsystem. In the case of frequency degenerate type-II OPOs, this results in the squeezing of the modes obtained by letting the two entangled companions interfere [22].

3.2. The EPR ‘Reid’ criterion

A stronger bound can be found by considering the original EPR Gedankenexperiment where the paradox was found in the possibility of determining the state of a far-away system by measuring its entangled companion. For this reason this criterion is usually indicated as the EPR criterion and was first introduced by Reid in 1989 [23], in the very early days of quantum information. It describes the ability to infer the expectation value of an observable on a sub-system by performing a suitable measurement on the second sub-system. This criterion sets only a sufficient condition for assessing entanglement and is a stricter condition based on the strength of quantum correlation. It can be easily given in terms of CM elements:
\[ n^2 \left(1 - \frac{c^2}{nm}\right) \left(1 - \frac{c^2}{nn}\right) < \frac{1}{4}. \]

The above criterion is, in general, asymmetric under the exchange of the two sub-systems. Thus, it may result ambiguous if only one of the two inequality, obtained by exchanging the sub-systems, is satisfied. This asymmetry has been recently used for assessing the steering capability in a quantum system: the state of a faraway system can be steered to collapse in a particular quantum state by a suitable measurement on its entangled companion [24]. On the contrary, for balanced systems \( m = n \) as in our case, the inequality is symmetric and asymmetric violations are not allowed.

4. Bell-like inequality (non-locality) in phase space

Generally, if we want to evaluate the non-locality of a state through a CHSH inequality [5], the operative form of the Bell one, we should build a Bell operator representing a combination of dichotomic (true–false) measurements. Then, if the expectation value of such a Bell operator violates the corresponding inequality, the system is not considered local, otherwise it would admit a classical description in terms of hidden variables. A parity operator is dichotomic. It can be constructed, on the photon number, for assigning +1 or –1 depending on whether an even or an odd number of photons has been registered. In [11, 25] a connection between the Wigner function of
the state and the joint measurement of the parity operator performed on the bipartite quantum state has been demonstrated.

Here we want to give a useful expression that relates such a measurement to the CM of a generic GS. We consider the Bell operator in the form given in equation (7) of [11]. The Bell-type function $B$ is then, given by the linear combination of four expectation values

$$B = \langle \mathcal{W}(0, 0) \rangle + \langle \mathcal{W}(\sqrt{I}, 0) \rangle + \langle \mathcal{W}(0, -\sqrt{I}) \rangle - \langle \mathcal{W}(\sqrt{I}, -\sqrt{I}) \rangle,$$

(11)

where

$$\langle \mathcal{W}(a_1, a_2) \rangle = \frac{n^2}{4} W(a_1, a_2);$$

(12)

with $W(a_1, a_2)$ is the Wigner function of the state calculated in $(a_1, a_2)$ and $a_b$ are complex amplitudes (as is $\sqrt{I}$ in equation (11)). Local theories, admitting a description in terms of local hidden variables, set the bound

$$|B| \leq 2.$$

(13)

On the one hand, any Bell inequality concerns the analysis of joint probabilities measured at space-time-separated locations. So, to actually perform a Bell measure, we should need to make repeated simultaneous measurements at different space-time-separated locations, stochastically changing the detector settings (in this case the amounts of displacement). Then by statistical analysis we could conclude as to whether or not the CHSH type inequality has been violated.

On the other hand, equation (12) shows that the knowledge of the Wigner function, i.e. the full reconstruction of the quantum state, gives an insight into the local/non-local character of the state. Without running into delicate questions we wish to show that, being a GS fully described by the CM, a rather simple object, it is possible to, a posteriori, evaluate $B$ on the state so as to assess whether or not it is Bell correlated without the need for reconstructing the whole Wigner function. This paves the way to a useful experimental procedure for discriminating among different levels of quantum correlations.

4.1 Bipartite GS case

Now we consider the bipartite GS generated by a type-II OPO described by the covariance matrix (3) $\sigma$, with $n = m$ and $c_1 = -c_2 = c$.

It can easily be found that the quantity (11) becomes

$$B(I, n, c) = \frac{1 + 2 \exp \left\{ \frac{-n + c + c}{\sqrt{n^2 - c^2} - 2I} \right\} - \exp \left\{ \frac{-n + c + c}{\sqrt{n^2 - c^2} + 2I} \right\}}{4(n^2 - c^2)}.$$

(14)

The Bell function $B(I, n, c)$ depends on the state properties $(n, c)$ and on a free parameter $(I)$. To look for the maximum violation for a given state we need to look for the value of the displacement amplitude $I$ that nullifies the derivative $\frac{\partial B(I, n, c)}{\partial I} = 0$. The maximum is, then, obtained for

$$I(n, c) = \frac{n^2 - c^2}{2n} \ln \left( \frac{n + c}{n} \right).$$

(15)

Figure 1. Region plot of $\mathcal{B}$ as a function of purity $\mu_s$ and the correlation coefficient $C_{ab}$. For the different meanings of the plot regions, see text.

This gives the expectation value of the maximum value of the Bell operator $\mathcal{B}$ as a function of the GS parameters. Since it is possible to experimentally retrieve the CM of such a state [22], this formula can be used to perform an a posteriori test on the non-local property of the state.

It is also possible to relate $\mathcal{B}$ to the purity of the single sub-system $\mu_s \equiv \mu_a = \mu_b = 1/(2n)$. Having in mind the interconnection between entanglement and the purity of the constituent sub-systems [26], we have:

$$\mathcal{B}(\mu_s, C_{ab}) = \mu_s^2 \left[ 1 + (1 + 2C_{ab}) \left( 1 + C_{ab} \right) - \frac{1 + C_{ab}}{1 + 2C_{ab}} \right]$$

(16)

where

$$C_{ab} \equiv \frac{\langle \Delta X_a X_b \rangle}{\sqrt{\langle \Delta X_a^2 \rangle \langle \Delta X_b^2 \rangle}} = \frac{c}{n}$$

(18)

is the correlation coefficient whose limit for a pure state is $C_{ab} = 1/1/(4n^2) = 1 - \mu_s^2$.

It can also be proved that, when the GS is pure, $\mathcal{B}(\mu_s) = C_{ab}$ can be considered a witness to entanglement: any entangled state violates the Bell inequality and vice versa.

4.2 Purity, entanglement and non-locality

For mixed states the above equivalence does not hold. Given a mixed system, one has $\mu_s^2 < 1 - C_{ab}$. For a given correlation...
Figure 2. Schematic representation of the experimental setup. The generation stage is a type-II OPO operating below the oscillation threshold. At the OPO output a neutral adsorber imitates the transmission over a real channel. The state is reconstructed by exploiting data collected by a single homodyne detector.

Figure 3. Experimental evolution of Bell's function versus $T$, the transmittivity of a variable absorber mimicking a realistic transmission channel.

A bipartite state, described by the $CM$ equation (3), subjected to the action of a passive Gaussian channel, undergoes a transformation such that [27]:

$$n_T = \frac{1 - T}{2} + Tn_1,$$

$$c_T = Tc_1,$$  \hspace{1cm} (20)

so that $n_1 < n_T$ indicates entangled states that do not violate the Bell inequality and for $n_T < n_1 < n_2$ the states are entangled and do violate the Bell inequality. For $n_2 > n_2$ the relative CM would not be physical.

We can calculate the evolution of the Bell function $\tilde{B}(n_1, c_1)$ (16) starting from an initially pure state, described by the $CM$ elements $n_1$ and $c_1$ and analyzing $\tilde{B}(n_T, c_T)$ as a function of the coefficient of transmissivity $T (0 < T < 1)$. So Bell's function $\tilde{B}$ becomes a function $\tilde{B}_T$ depending on the initial (pure) state and transmissivity $T$ of the channel. The relative expression, which is indeed rather long and complicated, will be used for evaluating the correspondence among experimental results and theory in the next section.

5. Gaussian noise does not break the entanglement, but it does break Bell's non-locality

In this Section we want to analyze the behavior of Bell's non-locality when subjected to passive Gaussian noise. We will see that when a pure ($c = \sqrt{2n^2 - 1/4}$) entangled and Bell's non-local (i.e. $B_T > 2$) state evolves through a Gaussian channel, it retains its entanglement, but loses its Bell's non-locality. This means that although, at the time of its birth, the state is pure, so that it violates Bell's inequality and breaks the Duan bound, decoherence highlights the different nature of the two markers; the GS becomes local (according to Bell), i.e. it would admit a description in terms of local hidden variables, although it remains entangled.

So Bell's function $\tilde{B}$ becomes a function $\tilde{B}_T$ depending on the initial (pure) state and transmissivity $T$ of the channel. The relative expression, which is indeed rather long and complicated, will be used for evaluating the correspondence among experimental results and theory in the next section.

6. Experimental results

In a recent paper (see [18]) we analyzed how different entanglement and quantum signatures evolve under decoherence. In this paper we wish to include the experimental analysis of the non-local character under decoherence. We have to

$$\mu_D = 1 - C_{ab},$$

$$\mu_B = \left[\frac{2(1-C_{ab}^2)}{1 + (1 + 2C_{ab})(1 + C_{ab})^{3/2}}\right]^{1/2},$$

$$\mu_P = [1 - C_{ab}]^{1/2},$$

so that $\mu_s < \mu_D$ denotes separable states, $\mu_D < \mu_s < \mu_B$ indicates entangled states that do not violate the Bell inequality and for $\mu_B < \mu_s < \mu_P$ the states are entangled and do violate the Bell inequality. For $\mu_s > \mu_P$ the relative CM would not be physical. In this way it is possible to distinguish three (physical) regions in the plane $(\mu_s, C_{ab})$ (see figure 1).

Region (I) Separable states compatible with the local hidden variables theory ($\tilde{B}(\mu_s, C_{ab}) < 2$).

Region (II) Entangled states compatible with the local hidden variables theory ($\tilde{B}(\mu_s, C_{ab}) < 2$).

Region (III) Entangled states not compatible with the local hidden variables theory ($\tilde{B}(\mu_s, C_{ab}) > 2$).

It is clear that there aren't separable GSs that violate Bell's inequality. Instead, a state compatible with a local theory (i.e. compatible with a theory in hidden variables) can also be entangled. This confirms the existence of different forms of quantum correlations and non-locality.

5. Gaussian noise does not break the entanglement, but it does break Bell's non-locality

In this Section we want to analyze the behavior of Bell's non-locality when subjected to passive Gaussian noise. We will see that when a pure ($c = \sqrt{2n^2 - 1/4}$), entangled and Bell's non-local (i.e. $B_T > 2$) state evolves through a Gaussian channel, it retains its entanglement, but loses its Bell's non-locality. This means that although, at the time of its birth, the state is pure, so that it violates Bell's inequality and breaks the Duan bound, decoherence highlights the different nature of the two markers; the GS becomes local (according to Bell), i.e. it would admit a description in terms of local hidden variables, although it remains entangled.
stress that, in view of the restricted region \( T > 90\% \) where one might have expected it, we have not observed any Bell inequality violation. This is essentially due to the maximum overall transmission we can get from the OPO cavity to the homodyne detector (63%). Moreover, we stress that this is rather an a posteriori check of the non-local character of the state than a Bell measure.

The analyzed state is the one outing a sub-threshold type-II OPO [28]. The block scheme of the experiment is given in figure 2. The full covariance matrix is retrieved by a single homodyne detector [29] following the procedure described in detail in [30, 31].

In figure 3 we have plotted the experimental value obtained for \( B_T \). The continuous line represents the theoretical expectations for the pure ancestor state.

As can be seen the experimental data are in good agreement with the expected evolution.

7. Conclusions

Different bounds have been, so far, discussed in the literature for discriminating continuous variable separable and entangled states. Each of them looks at slightly different facets of the EPR paradox. In this paper, they are presented in connection with the original EPR arguments.

For the first time, we have expressed by a useful and direct formula a Bell-type inequality, written for CVs, in terms of the covariance matrix of a GS. We have discussed its relation with the purity of the entangled sub-systems and have analyzed, also experimentally, its behavior under decoherence. In so doing we have proved, experimentally, that, also in a CV regime, there exists mixed entangled states that do not violate the Bell inequality, while, for pure states, any entangled state is Bell non-local.

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