New Model and Numerical Test of a High Temperature Pairing Mechanism in Stripes

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We introduce a new model and mechanism of high temperature pairing in stripes. We propose a way to unambiguously test it by numerical simulations. For example, the implementation of our mechanism in a 6-leg t-J ladder model has the effect of making the spin gap of the doped 6-leg ladder be about 4 times bigger than the spin gap of the same ladder at half filling.

The cuprate superconductors manifest a remarkable variety of unusual and unexplained phenomena. Let us focus on superconductivity. As highlighted in their name, the high temperature superconductors are distinguished by a high pairing energy scale $\Delta_s \approx 35 \text{meV}$. The most significant missing element in our understanding of superconductivity in the cuprates is the microscopic pairing mechanism. Known microscopic characteristics, such as the strength of spin exchange interactions, $J \approx 1500K$, restrict candidate mechanisms.

Therefore, from a theoretical point of view we ask what models can, in principle, produce such high pairing energies over the range of doping as observed in the cuprates? Our aim in this paper is to provide such a model based on stripes. We also propose a straightforward numerical test of our proposed mechanism. The model, and the main effect which is in operation, is distinct from previous proposals.

The paper is organized as follows: First, we present our model and propose a numerical test to verify or negate the operation of the core effect at work (we call it an "environment spin decoupling effect"). Then, we use a two-chain model to illustrate the environment spin decoupling effect using tractable analytical methods. We conclude with a discussion of the implications of our model within the general phenomenology of stripes and superconductivity in the cuprates.

STRIPE MODEL AND NUMERICAL TEST

Experimentally, the spin gap in cuprate superconductors is nearly independent of doping $\delta$ in the range $0.06 < \delta < 0.15$ (with a magnitude $\Delta_s \equiv 2\Delta(0) \approx 800K \sim J/2$). In the same doping range, the distance between stripes changes by a factor two. Therefore, any mechanism of pairing via stripes must be independent of the distance between stripes. The difficulty of achieving such a mechanism can be appreciated by thinking of stripes as a periodic arrangement of ladders.

The mechanism of pairing in stripes is supposed to operate on each stripe independently. Therefore, the same mechanism should manifest itself in a single ladder model where the number of legs of the ladder is determined by the stripe period. Since the stripe period change from 4 to 8 lattice spacings between doping $\delta = 0.12$ and $\delta = 0.06$, it implies that a mechanism of pairing in stripes should be such that the pairing of mobile fermions in the corresponding 4-leg and 8-leg ladder models be the same!! Indeed, the model and mechanism which we propose is aiming to achieve exactly that.

In order to have a sharp goal in mind, we pose the following theoretical question: Is it possible to have a spin gap in an even-leg doped ladder which is several times bigger than the spin gap of the same undoped ladder? Numerical investigation seem to indicate that the answer is negative on uniform ladders. In this paper, we investigate non-uniform ladder models.

One idea in this direction is to make the bond interactions inhomogeneous. Arrigoni & Kivelson considered a 4-leg Hubbard ladder model where the bare hopping interaction was dimerized transverse to the ladder (while maintaining the same uniform hopping interactions along all the legs). They showed that indeed the maximum spin gap of the doped ladder may increase substantially.

We introduce an inhomogeneity in the interactions but instead in the on-site chemical potential. e.g., for t-J type ladders. The model is characterized by three parameters $\{t, J, \mu\}$. In Fig. 2 we draw a 3-leg and 6-leg ladder versions of our models. To begin with, we shall discuss the ladder model from a purely technical perspective and defer the discussion of its association with the stripe state to the concluding section of this paper. The simplest anisotropic ladder model consists of a chemical potential shift $\mu$ in two neighboring legs of an N-leg lad-
The above proposed numerical experiments can quantitatively test our proposed model, yet, in the following section we would like to develop the reader’s analytical intuition about the competition and renormalization of interactions in the model. Instead of directly thinking of the dynamic decoupling of a ladder system into two incoherent sub-systems, let us think of the reverse route: Consider two 1D systems - one is a doped 2-leg Hubbard ladder and the second is an undoped or lightly doped N-leg Hubbard ladder. Turning on hopping interaction $t_\perp$ between the two ladders, there is a competition between the intra-ladder interaction which maintains the spin gap (and pairing) on the 2-leg ladder and inter-ladder interactions which form coherence across the whole coupled system. Similar issues of confinement/deconfinement have risen in the context of coupling equivalent ladder systems where the tuning parameter is inter-ladder interactions (e.g., $t_\perp \neq t$ or $J_\perp \neq J$). In contrast, our model starts with uniform bare interactions ($t_\perp = t$, $J_\perp = J$) while inhomogeneity of doping induced by tuning an inhomogeneous chemical potential. We argue that similar confinement/deconfinement competition of intra- and inter-ladder interactions is at play. We would like to demonstrates this competition of interactions and the core effect where a spin gap and pairing is established on a sub-part of the system and it’s spin degrees of freedom decouple from the rest of the system which may remain gapless. Such an effect is unique to models of coupled inequivalent 1D systems. Below we examine the simplest model of such type.

Consider a model of two inequivalent 1D chains (or two inequivalent 1D bands): Band-A has a Fermi wave number $k_A$ and attractive interactions $-U_A$. Band-B has a different Fermi wave number $k_B$ and repulsive interactions $+U_B$. The filling fractions are such that $k_A < k_B \leq \frac{\pi}{2}$. Hence, in the absence of inter-band interactions, the groundstate of band-A is a Luther-Emery (LE) liquid characterized by a spin gap (due to the attractive $-U_A$ interactions) while band-B has gapless spin excitations. If band-B is half filled, $k_B = \frac{\pi}{2}$, then it is equivalent to a Heisenberg spin chain, or otherwise if $k_B < \frac{\pi}{2}$ then band-B is a Luttinger liquid. Qualitatively, band-Å and band-B correspond to the doped 2-legs and the remaining $N-2$ legs of the inhomogeneous ladder model. (The fact that in a 2-leg ladder the spin gap is due
to repulsive interactions while in this two-chain model the spin gap in band-A is due to attractive interactions is not of conceptual significance).

It is straightforward to determine that the only marginally relevant inter-band interactions (with respect to the non-interacting fixed point) are inter-band spin exchange $M\hat{s}_A(x) \cdot \hat{s}_B(x)$ and Josephson coupling (i.e., inter-band pair hopping) $J\left[ P_{A}^{\dagger} (x) P_{B} (x) + h.c. \right]$ (where the j-band pairing operator $P_{j} (x) = \psi_{j,\uparrow}^{\dagger} \psi_{j,\downarrow}^{\dagger}$). Note, in the case that band-B is half filled then the inter-band Josephson coupling is perturbatively irrelevant. We shall discuss both $k_B < \frac{\pi}{2}$ and $k_B = \frac{\pi}{2}$ cases.

The Hamiltonian density which describe our one dimensional (1D) two-band model is

$$H (x) = H_0^A (x) + U_B \rho^B (x) \rho^B (x) + H_0^B (x) - U_A \rho^A (x) \rho^A (x) + M\hat{s}_A (x) \cdot \hat{s}_B (x) + J\left[ P_{A}^{\dagger} (x) P_{B} (x) + h.c. \right]$$

$$\rho^A (x) = \psi_{A,\uparrow}^{\dagger} (x) \psi_{A,\downarrow}^{\dagger} (x), \quad \hat{s}_A (x) = \psi_{A,\alpha}^{\dagger} (x) \sigma_{\alpha\beta} \psi_{A,\beta} (x).$$

Additional marginally irrelevant interactions (e.g., inter-band charge density interactions) which do not interest us here have been dropped.

The three interaction $\{-U_A, M, J\}$ are all marginally relevant in the initial stages of renormalization. Yet, the first crucial observation that we make is that the inter-band attractive interaction $-U_A$ and the inter-band exchange interaction $M$ are competing. i.e., $-U_A$ and $M$ control distinct fixed points for which in the final stages of renormalization either $M \to 0$ or $U_A \to 0$ respectively. The fixed points and renormalization group flows are drawn in Fig-4. These results are derived using the bosonization method as summarized below.

The fixed points of our model can be inferred by observation from the bosonized representation of the model [3]: Using the bosonized form of chiral (left and right moving) fermion fields $\psi_{L/R,\sigma}^{\dagger} (x) = F_{\sigma} \sqrt{\frac{\theta_{\sigma}}{2\pi}} : e^{i\sqrt{\pi\theta_{\sigma}} (x) \pm \phi_{\sigma} (x)} :$, where $\theta_{\sigma} (x) = 2 \int_{-\infty}^{x} dx' \Pi_{\sigma} (x')$, and $[\Pi_{\sigma} (x'), \phi_{\tau} (x)] = -i\delta (x' - x), \sigma = \uparrow, \downarrow$. The anticommuting Klein factors, $\{F_{\sigma}, F_{\tau}\} = \delta_{\sigma,\tau}$, are needed for the proper anticommutation of fermions with different spin. (Normal ordering eliminates the short-distance regularization factor $1/\sqrt{a}$ that is common in the literature). In the following, all operators are implicitly assumed to be normal ordered. Also, we absorb the infra-red cut-off $\mu$ factors into the interaction coefficients. As commonly done, we re-express the operators in terms of bosonic spin fields $\phi_{\sigma} (x) = \frac{1}{\sqrt{2}} [\phi_{\uparrow} + \phi_{\downarrow}]$, and charge fields $\phi_{\sigma} (x) = \frac{1}{\sqrt{2}} [\phi_{\uparrow} - \phi_{\downarrow}]$, and correspondingly defined momenta $\Pi_{\sigma}$ and $\Pi_{c}$. e.g., pair creation $P_{\uparrow} = \frac{1}{\sqrt{2}} e^{i\sqrt{2\pi\theta_{\sigma}} (x) \pm \phi_{\sigma} (x)}$. The resulting form of the model Hamiltonian density is

$$H (x) = H_0^A (x) - \frac{U_A}{\pi^2} \cos \left( \sqrt{8\pi\theta_{\sigma}^A} \right)$$

$$+ H_0^B (x) + \frac{U_B}{\pi^2} \cos \left( \sqrt{8\pi\theta_{\sigma}^B} \right)$$

$$+ \frac{M}{\pi^2} e^{i\sqrt{2\pi\theta_{\sigma}^A} \mp \theta_{\pi}^A} \cos \left( \sqrt{2\pi\theta_{\sigma}^A} \right) \cos \left( \sqrt{2\pi\theta_{\pi}^A} \right)$$

$$+ \frac{J}{\pi^2} e^{i\sqrt{2\pi\theta_{\sigma}^B} \mp \theta_{\pi}^B} \cos \left( \sqrt{2\pi\theta_{\sigma}^B} \right) \cos \left( \sqrt{2\pi\theta_{\pi}^B} \right)$$

$$+ h.c. + \{\text{marginally irrelevant interactions}\}$$

where

$$H_{0j}^j (x) = \frac{1}{2} \left[ K_j^c (\Pi_j^c)^2 + \frac{1}{K_j^c} (\partial_x \phi_j^c)^2 \right]$$

The difference between attractive and repulsive intra-band interaction is expressed by the Luttinger parameters $K_j^c > 1$ and $K_j^e > 1$ for the charge sector. It is useful to define $\theta_j^c = \frac{\sqrt{4\pi\theta_{\sigma}^c}}{\sqrt{2\pi}} \left( \theta_j^B \pm \theta_j^A \right)$ and $\theta_j^e = \frac{1}{\sqrt{2}} \left( \theta_j^B \pm \theta_j^A \right)$. For the purpose of delineating the potential spin gap fixed points, one needs to keep track only of interaction terms which may develop a non-zero expectation value. Since by general theorems it always holds that $\langle \cos \left( \sqrt{4\pi\theta_{\sigma}^c} \right) \cos \left( \sqrt{4\pi\theta_{\sigma}^e} \right) \rangle = 0$, then we need to keep only the cos $\langle \sqrt{4\pi\theta_{\sigma}^c} \rangle$ cos $\langle \sqrt{4\pi\theta_{\sigma}^e} \rangle$ part of the exchange interaction. Also, since $K_j^c > 1$ then the $U_B$ interaction is irrelevant and can also be dropped. Hence we arrive at

$$H = H_0^B + H_0^A (x) - \frac{U_A}{\pi^2} \cos \left( \sqrt{4\pi} \left( \phi_{\sigma}^+ - \phi_{\sigma}^- \right) \right)$$

$$+ \frac{2M}{\pi^2} \cos \left( \sqrt{4\pi\theta_{\sigma}^c} \right) \cos \left( \sqrt{4\pi\theta_{\sigma}^e} \right)$$

$$+ \frac{2J}{\pi^2} \cos \left( \sqrt{4\pi\theta_{\sigma}^c} \right) \left[ \cos \left( \sqrt{4\pi\phi_{\sigma}^c} \right) + \cos \left( \sqrt{4\pi\phi_{\sigma}^e} \right) \right]$$

$$+ \{\text{irrelevant and marginally irrelevant interaction}\}$$

The competition between $M$ and $U_A$ is inferred from the fact that at a fixed point where the inter-band interaction $M$ is relevant then $\langle \cos \left( \sqrt{4\pi\theta_{\sigma}^c} \right) \rangle \neq 0$ and...
\[ \langle \cos(\sqrt{\pi}\phi_{\xi}^+) \rangle = 0 \] and hence the intra-band \( U_A \) interaction is irrelevant. The resulting phase diagram depicted in Fig.3 shows the fixed points and renormalization group (RG) flows. In an expanded following publication we shall elaborate on the complete RG equations and estimates of the spin gap magnitude renormalization.

As discussed by [3], a fixed point is specified by the gapless order parameters. The enumeration of gapless modes is straightforward by observation. Only the \( \phi_{\xi}^+ \) charge mode remains gapless, as expected from an over all incommensurately filled system. To distinguish between fixed points with the same number of gapless modes, it suffice here to focus on the pairing order parameters

\[
\begin{align*}
\Delta_e & = \frac{1}{2} \bigl[ (R_{B\uparrow}L_{B\downarrow} + L_{B\uparrow}R_{B\downarrow}) + (R_{A\uparrow}L_{A\downarrow} + L_{A\uparrow}R_{A\downarrow}) \bigr] \\
& \approx e^{-i\sqrt{\pi}\phi_{\xi}^+} \left[ e^{-i\sqrt{\pi}\phi_{\xi}^-} \cos(\phi_{\xi}^+) \cos(\phi_{\xi}^-) \right] \\
\Delta_o & = \frac{1}{2} \bigl[ (R_{B\uparrow}L_{B\downarrow} + L_{B\uparrow}R_{B\downarrow}) - (R_{A\uparrow}L_{A\downarrow} + L_{A\uparrow}R_{A\downarrow}) \bigr] \\
& \approx e^{-i\sqrt{\pi}\phi_{\xi}^+} \left[ e^{-i\sqrt{\pi}\phi_{\xi}^-} \sin(\sqrt{\pi}\phi_{\xi}^+) \sin(\sqrt{\pi}\phi_{\xi}^-) \right] \\
\Delta_{te} & = \frac{1}{2} \bigl[ (R_{B\uparrow}L_{B\downarrow} + L_{B\uparrow}R_{B\downarrow}) + (R_{A\uparrow}L_{B\downarrow} + L_{A\uparrow}R_{B\downarrow}) \bigr] \\
& \approx e^{-i\sqrt{\pi}\phi_{\xi}^+} \left[ e^{+i\sqrt{\pi}\phi_{\xi}^-} \sin(\sqrt{\pi}\phi_{\xi}^+) \sin(\sqrt{\pi}\phi_{\xi}^-) \right] \\
\Delta_{to} & = \frac{1}{2} \bigl[ (R_{B\uparrow}L_{B\downarrow} + L_{B\uparrow}R_{B\downarrow}) - (R_{A\uparrow}L_{B\downarrow} + L_{A\uparrow}R_{B\downarrow}) \bigr] \\
& \approx e^{-i\sqrt{\pi}\phi_{\xi}^+} \left[ e^{+i\sqrt{\pi}\phi_{\xi}^-} \cos(\sqrt{\pi}\phi_{\xi}^+) \cos(\sqrt{\pi}\phi_{\xi}^-) \right]
\end{align*}
\]

The subscripts \( e/o \) indicate even/odd parity under exchange of band indices.

Table I delineate the gapless modes at each fixed point.

| \( k_B < \frac{\pi}{2} \) | FP-I | FP-II | FP-III | FP-IV |
|-----------------|------|-------|--------|-------|
| Dominant interaction | \( M - U_A \) | \( -U_A, J \) |
| Gapless modes | \( C2S2 \) | \( C1S0 \) | \( C2S1 \) | \( C1S0 \) |
| \( \Delta_e \) | \( \checkmark \) | 0 | \( \checkmark \) | 0 |
| \( \Delta_o \) | \( \checkmark \) | 0 | \( \checkmark \) | \( \checkmark \) |
| \( \Delta_{te} \) | \( \checkmark \) | \( \checkmark \) | 0 | 0 |
| \( \Delta_{to} \) | \( \checkmark \) | 0 | \( \checkmark \) | 0 |

If band-B is half filled, \( k_B = \frac{\pi}{2} \), then it is equivalent to a Heisenberg spin chain [3] and the fixed points are modified since band-B does not support gapless charge modes and the Josephson coupling \( J \) is perturbatively irrelevant. Hence, modes \( \Delta_{te} \) and \( \Delta_{to} \) are eliminated. For an elaborate discussion of FP-II in this case see [3].

**DISCUSSION**

From a general theoretical perspective, our modified ladder model has the following special properties: (a) The spin gap of the doped ladder model is higher than of the half filled ladder. (b) The predominant flow towards fixed point III (of increased pairing interactions in part of the ladder system while diminishing spin exchange coupling with the rest of the ladder system) leads at intermediate temperatures to the establishment of a spin gap on a sub-part of the ladder (the two hole rich legs), where the temperature scale and the magnitude of the spin gap are characterized by those of a doped 2-leg ladder, and are independent of the number of legs of the full ladder. (c) Consequently, at intermediate temperatures the ladder separates into two subsystems of which the spin dynamics is respectively decoupled. (d) At low temperatures, global coherence is established by the proximity effect (Josephson coupling) as expressed by the flow towards fixed point IV in the phase diagram.

We here argue that the model captures the core physics of the stripe structure in doped antiferromagnets. While the mechanism for the creation of stripes is not generally agreed upon, the end result is manifestly a state of local charge inhomogeneity of elongated domains of alternating hole rich and hole poor strips, i.e., alternating strips with different effective local chemical potential. Therefore, a repeated arrangement of our inhomogeneous ladder model is a model representation of the stripe structure (where the width of the ladder corresponds to the charge period of the stripe). Such a coupled ladder model has the following properties: At intermediate temperatures a spin gap is established independently on each hole rich stripe, while the hole poor strips may remain gapless. Residual interaction may lead to coupling between the hole poor regions across the spin gapped hole rich stripe, but at intermediate temperatures such coupling may just as well be too weak and leave the hole poor regions decoupled and thus manifest only local AFM correlations. At low temperature, global coherence is established by coherent pair tunneling, leading to a superconducting state.

These characteristics of our model realize the goals of the general framework first presented in [3]. Yet, in terms of the pairing mechanism itself, the core effect presented here is orthogonal to that in [3]. The pairing mechanism proposed in [3] had the pairing arising out of the coherent interaction between hole rich and hole poor regions. In contrast, the core mechanism in our new model is a decoupling of the dynamics of the hole rich and hole poor regions. The proximity effect introduced by [3] comes into operation in our model only at low temperatures to establish global coherence (generating the flow towards fixed point IV). Elsewhere we shall expand on the significant advantages (compared with previous stripe based proposals) of the model and mechanism introduced here to address phenomenological properties of pairing and superconductivity in the cuprates.

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