1. Introduction

The study of propagating light beams in liquid crystals, i.e., soft-matter encompassing optical birefringence, nonlocality, anisotropy and all-optical, as well as electro-optic, magneto-optic and thermo-optic responses, has been the subject of extensive experimental and theoretical investigations. These studies encompass light beams in liquid crystals, with applications including imaging, modulation, signal processing, display architectures, lasers, sensors and so on [1]. Owing to such wide-ranging importance, comprehensive papers are available in the literature on the optics and photonics of liquid crystals with reference to solitary waves, random lasing, topological and spin–orbit interactions of light, to cite just a few [2–5]. This Special Issue collates articles of theoretical and applied relevance to liquid crystals in the nematic phase, including one- and two-dimensional waveguides, bulk and periodic geometries and electro-optic and opto-optical phenomena. A brief summary is provided below.

1.1. Guided Waves and Integrated Optics

D’Alessandro and Asquini take up the important issue of tunable optical circuits [6]. Among the applications of liquid crystals in their nematic mesophase, the propagation of guided light waves in confined structures is one of the most promising and, therefore, most investigated. D’Alessandro et al. present an overview of recent achievements in this area, from materials to models and devices in various waveguide configurations, employing substrates such as silicon, glass and photo-dimerized monolayers (PDML) [6]. The authors include all-optical switching and tunable filtering, routers and attenuators, reporting performances competitive with similar integrated optical devices in other materials.

1.2. Self-Localized Wavepackets

The article by Liang et al. [7] concerns the nonlinear optics of reorientational nematic liquid crystals, in particular, light self-localization in unconfined samples. A theoretical review of the nonlocality-controlled transition between self-focusing and defocusing is provided, with a discussion of modulational instability and solitary waves [7]. The authors explore features of the model even beyond available regimes in materials currently known, presenting novel mathematical solutions.

1.3. Nonlinear Propagation in Discrete Arrays

A relatively unexplored area of the nonlinear optics of soft-matter is light propagation in waveguide arrays based on a liquid crystal substrate. The article by P. Panayotaros [8] is a comprehensive theoretical treatment detailing how a Wannier function basis associated with the periodic Schrödinger operator can describe stable light beams propagating in discrete arrays of waveguides, the latter encompassing nonlinearity in the presence of nonlocality, such as in nematic liquid crystals.
1.4. Light Undular Bores

In line with the previous mathematical study of the nonlinear optics of liquid crystals, Baqer et al. [9] investigate dispersive shock waves in the nematic phase of the medium. Such shock waves, also termed undular bores, are the dispersive equivalent of shock waves in compressible flow. Undular bores in nematic liquid crystals are resonant, and six regimes are identified in this paper, each associated with a range of input beam powers. The authors specifically address the nonlocal, nonlinear response based on Whitham modulation theory. The modulation theory solutions are verified using full numerical solutions.

1.5. Ring Pattern Formation

Clerc et al. [10] investigate the amplified coupling of two coherent light beams in dye-doped liquid crystals. The authors experimentally study how light is able to induce ring patterns through the photo-isomerization of specific samples with photosensitive dopants. This process is modeled by a Swift–Hohenberg-type equation. An analysis of this model shows that the rings arise as a trans-critical bifurcation of the isotropic liquid phase. The article characterizes the bifurcation diagram of this complex opto-topological system.

1.6. Tunable Thermoplasmonic Heating

Palermo and coauthors [11] experimentally show that plasmonic heating delivered by a light-illuminated layer of nanoparticles can be adjusted based on the polarization-dependent refractive index change of nematic liquid crystals. A hybrid system consisting of gold nanoparticles immobilized on a glass substrate and layered with photo-aligned liquid crystals is developed, with the photo-aligning material providing molecular reorientation and tunability.

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