Neutral currents and tests of three-neutrino unitarity in long-baseline experiments

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New Journal of Physics 6 (2004) 135
Received 13 July 2004
Published 20 October 2004
Online at http://www.njp.org/
doi:10.1088/1367-2630/6/1/135

Abstract. We examine a strategy for using neutral current measurements in long-baseline neutrino oscillation experiments to put limits on the existence of more than three light, active neutrinos. We determine the relative contributions of statistics, cross-section uncertainties, event misidentification and other systematic errors to the overall uncertainty of these measurements. As specific case studies, we make simulations of beams and detectors that are like the K2K, T2K and MINOS experiments. We find that the neutral current cross-section uncertainty and contamination of the neutral current signal by charge current events allow a sensitivity for determining the presence of sterile neutinos at the 0.10–0.15 level in probability.
1. Introduction

In recent years, a series of exciting experimental results have shown that neutrinos have finite masses and mixings. For a recent review of the status see [1]. Solar neutrino and atmospheric neutrino results indicate that all three known neutrino flavours (e, μ, τ) participate in neutrino mixing and, hence, neutrino oscillations. Consequently, the standard framework to describe the experimental results and analyse neutrino oscillation data is that of three-flavour mixing in which the three flavour eigenstates are related to three mass eigenstates by a 3 × 3 mixing matrix [2]. The positive signal for \( \bar{\nu}_\mu \rightarrow \bar{\nu}_e \) oscillations from the LSND experiment [3] challenges the three-flavour mixing paradigm [4]. However, the neutrino oscillation interpretation of the LSND observations is yet to be confirmed. Independent of whether or not the LSND results are confirmed by MiniBooNE [5], the three-flavour mixing framework deserves further experimental scrutiny in the coming years. Much of the focus on future experiments so far has been directed to the determination of the three-mixing angles and the CP-violating phase with long-baseline oscillation experiments\(^4\) and reactor experiments\(^5\). Of interest in this paper is the measurement of the neutral current, which could allow tests of the unitarity of the 3 × 3 mixing matrix and thus indirectly probe the existence of sterile neutrinos.

\(^4\) Representative long-baseline studies are given in [6].

\(^5\) Representative reactor oscillation studies are given in [7].
In a three-flavour neutrino model, the sum of the oscillation probabilities

\[ \sum_{y=e,\mu,\tau} P(\nu_x \rightarrow \nu_y) \]

is unity. If there are more than three light neutrinos, we know from measurements of the invisible width of Z [8] that the additional neutrinos must be sterile. If additional light neutrinos mix with the three known flavours we can expect a non-zero oscillation probability to sterile neutrinos, \( P(\nu_x \rightarrow \nu_s) \neq 0 \). To test the three-flavour neutrino-mixing paradigm, it is important to search for a sterile neutrino component within the neutrino flux from natural and manmade sources. Since sterile neutrinos have no strong or electroweak interactions, they cannot be detected directly. However, neutral current (NC) measurements allow \( \sum_{y=e,\mu,\tau} P(\nu_x \rightarrow \nu_y) \) to be determined which, by probability conservation, is equal to \( 1 - P(\nu_x \rightarrow \nu_s) \). Therefore, in principle, a NC measurement alone is sufficient to determine \( P(\nu_x \rightarrow \nu_s) \). However, in a realistic detector misidentifications of CC and NC events, together with systematic uncertainties on the relevant neutrino interaction cross-sections, complicate the analysis.

In this paper, we study the use of NC measurements to determine limits on the sterile neutrino content in long-baseline neutrino oscillation experiments. First we consider the sensitivity to the sterile content that might be obtained in a K2K- [9], T2K- [10] and MINOS-like [11] experiment with a ‘perfect’ detector and ‘perfect’ beam if there are no systematic uncertainties. We then consider the impact on the sensitivity of event misidentification and systematic uncertainties. Our study is based on a simple simulation of the long-baseline neutrino beams, neutrino interactions [12], and detector responses. We present our results versus event rates and the size of the cross-section uncertainty in order to show the dependence on these quantities.

2. Using NC data to determine sterile content

2.1. Formalism

The present and proposed long-baseline neutrino oscillation experiments exploit conventional neutrino beams that are produced by the decay of charged pions in a long channel. This produces a beam which is initially almost entirely \( \nu_\mu \). Kaons and muons decaying in the channel introduce a small (typically \( \sim 1\% \)) \( \nu_e \) component in the neutrino beam. As the neutrino beam travels towards a distant detector its flavour content will evolve. In our analysis, we will consider three active neutrinos \( (\nu_e, \nu_\mu, \nu_\tau) \) and one sterile neutrino \( (\nu_s) \), with oscillation probabilities \( P(\nu_\mu \rightarrow \nu_x) \equiv P_{\mu x}, x = e, \mu, \tau, s \). We begin by considering an oscillation experiment that has an initially pure \( \nu_\mu \) beam with well-known neutrino spectrum and flux, and a detector with perfect identification of the produced events. Then the event rates at the far detector will be

\[ N_{NC} = N^0_\mu (1 - P_{\mu s}) \sigma_{NC}/\sigma_\mu, \]

\[ N_\mu = N^0_\mu P_{\mu \mu}, \]

\[ N_e = N^0_\mu P_{\mu e} \sigma_e/\sigma_\mu, \]

\[ N_\tau = N^0_\mu P_{\mu \tau} \sigma_\tau/\sigma_\mu, \]
where \( N^0_\mu \) is the predicted number of \( \nu_\mu \) CC interactions in the detector in the absence of oscillations and the \( \sigma_x \) denote the interaction cross-sections \((\sigma_e, \sigma_\mu, \sigma_\tau)\) for \((\nu_e, \nu_\mu, \nu_\tau)\) CC interactions and \(\sigma_{NC}\) for \(\nu_x\) NC interactions.

In an ideal experiment, \( N_{NC} \) determines \( P_{\mu S} \) and \( N_{CC} \) determines \( P_{\mu\mu} \). In practice, the presence of \( \nu_e \) and \( \nu_\tau \) CC events complicates the analysis if these events are not distinguished from NC events. Under that circumstance, the NC events provide a measure of \( 1 - P_{\mu S} + \epsilon_e P_{\mu e} + \epsilon_\tau P_{\mu\tau} \), where the factors \( \epsilon_e \) and \( \epsilon_\tau \) reflect the contaminations.

Probability conservation \((P_{\mu e} + P_{\mu\mu} + P_{\mu\tau} + P_{\mu S} = 1)\) can be used to eliminate \( P_{\mu\tau} \) or \( P_{\mu e} \), but not both. If the beam energy is below the threshold for \( \tau \) production or the probability \( P_{\mu e} \) is small and can be neglected, then \( P_{\mu S} \) can still be determined. However, for a realistic detector with particle misidentifications and/or a \( \nu_e \) component at the far detector that cannot be neglected, the problem of determining \( P_{\mu S} \) can be complex but still solvable, as we shall discuss.

In general, let the probability that an event of type \( x \) \((NC, \nu_e CC, \nu_\mu CC or \nu_\tau CC)\) be identified in the detector as an event of type \( y \) be given by \( \xi_{xy} \), where \( x, y = (NC, e, \mu, \tau) \) (note that \( \xi_{xx} \) is the efficiency for detecting an event of type \( x \)). If \( N^0_\mu \) is the predicted number of \( \nu_\mu \) CC interactions in the detector in the absence of oscillations, then after including oscillations, detector efficiencies and mis-identifications, and integrating over the energy dependence, the number of measured events of type \( y \) will be

\[
N_y = \frac{N^0_\mu}{\sigma_\mu} \left[ (1 - P_{\mu S})\sigma_{NC}\xi_{NC,y} + \sum_{x=e,\mu,\tau} P_{\mu x}\sigma_x\xi_{xy} \right],
\]

where the interaction cross-sections for \( \nu_e \) CC, \( \nu_\mu \) CC, \( \nu_\tau \) CC and NC events are given by \( \sigma_e, \sigma_\mu, \sigma_\tau \) and \( \sigma_{NC} \), respectively.

In (5), we do not include explicitly a term that accounts for the \( \nu_e \) contamination of the beam. In practice, this contamination could be added to \( P_{\mu e} \) and subtracted from \( P_{\mu\mu} \). The contamination is expected to be of order 1% or less, which is small compared to the misidentification factors, as we subsequently will show. Therefore we neglect this small correction.

2.2. Ignore \( \nu_e \)’s

We consider first the situation in which the \( \nu_e \) component in the beam at the far detector is so small that \( \nu_e \) CC interactions can be neglected. In this case, we let \( P_{\mu e} \to 0 \) (it is known to be small, at the 5% level or less from the CHOOZ experiment [13]). It is convenient to define the following two ratios:

\[
R_{NC} \equiv \frac{N_{NC}}{\xi_{NC}\xi_{NC} N^0_\mu} = (1 - P_{\mu S}) + f_{\mu,NC} P_{\mu\mu} + f_{\tau,NC} P_{\mu\tau},
\]

\[
R_\mu \equiv \frac{N_\mu}{\xi_{\mu}\xi_{\mu} N^0_\mu} = f_{NC,\mu} (1 - P_{\mu S}) + P_{\mu\mu} + f_{\tau,\mu} P_{\mu\tau},
\]

where

\[
N^0_x = \sigma_x N^0_\mu / \sigma_\mu
\]

and

\[
f_{x,y} = \xi_{xy} \sigma_x / \xi_{yx} \sigma_y
\]
is a normalized misidentification factor that gives the ratio of the number of events of type \( x \) identified as type \( y \) to the number of events of type \( y \) that are identified as type \( y \). Measuring \( R_{NC} \) and \( R_\mu \) is sufficient for deducing \( P_{\mu s} \) (and \( P_{\mu \mu} \)). The analysis depends on whether or not we are above the \( \nu_\tau \) CC interaction threshold, i.e., whether or not there are \( \nu_\tau \) CC events produced in the detector.

2.2.1. Below \( \tau \) threshold. For neutrino energies below the \( \tau \) threshold, \( \sigma _\tau = 0 \) and \( f_{\tau,j} = 0 \). In this case we can invert (6) and (7) to obtain

\[
P_{\mu \mu} = \frac{R_\mu - R_{NC} f_{NC,\mu}}{1 - f_{\mu,NC} f_{NC,\mu}},
\]

(10)

\[
P_{\mu s} = 1 - \frac{R_{NC} - R_\mu f_{\mu,NC}}{1 - f_{\mu,NC} f_{NC,\mu}}.
\]

(11)

Adding uncertainties in quadrature we get

\[
\delta P_{\mu \mu} = \sqrt{\left( \delta R_\mu \right)^2 + f_{\mu,NC}^2 \left( \delta R_{NC} \right)^2} \frac{1}{1 - f_{\mu,NC} f_{NC,\mu}},
\]

(12)

\[
\delta P_{\mu s} = \sqrt{\left( \delta R_{NC} \right)^2 + f_{\mu,NC}^2 \left( \delta R_\mu \right)^2} \frac{1}{1 - f_{\mu,NC} f_{NC,\mu}},
\]

(13)

where in the limit of Gaussian statistical uncertainties

\[
\delta R_j = R_j \sqrt{\frac{1}{N_j} + \epsilon_j^2}
\]

(14)

and

\[
\epsilon_j \equiv \delta N_j^0 / N_j^0.
\]

(15)

The first term in each \( \delta R_j \) is the usual statistical uncertainty, the second comes from the normalization uncertainty (flux and cross-section).

Note that the normalized misidentification factors \( f_{\mu,NC} \) and \( f_{NC,\mu} \) will be sensitive to the neutrino energy spectrum and the detector technology, and therefore must be evaluated for each experimental set-up. Most of the misidentification terms are suppressed by \( f^2 \); if \( f \leq 0.1 \) then \( f^2 \leq 0.01 \). If the experimental set-up is such that we can ignore all terms of order \( f^2 \) (see table 1), we have

\[
\frac{P_{\mu s}}{\delta P_{\mu s}} \approx \frac{1 - R_{NC} + f_{\mu,NC} R_\mu}{\delta R_{NC}},
\]

(16)

which measures the significance of the deviation of \( P_{\mu s} \) from zero.

In (16) we have not included the effects of uncertainties of the \( f \) factors. By considering subsamples in our Monte Carlo calculations of the \( f_{x,y} \), we estimate the uncertainties from the Monte Carlo to be about 2\% for the most significant \( f \) factors. This has a small effect since
Table 1. Signal efficiencies ($\zeta_{jj}$) and normalized misidentification factors ($f_{i,j}$) in selected long-baseline experiments.

| Experiment | $j$ (channel) | Signal | $\zeta_{jj}$ | $f_{NC,j}$ | $f_{\mu,j}$ | $f_{\tau,j}$ | $f_{e,j}$ |
|------------|--------------|--------|-------------|------------|------------|------------|--------|
| K2K-like   | NC           | Two $e$-like, no $\mu$-like | 0.391      | -          | 0.068      | 0.052      | -      |
| (basic)    | $\mu$       | One $\mu$-like, no $e$-like | 0.520      | 0.087      | -          | 0.0007     | -      |
|            | $e$         | One $e$-like, no $\mu$-like | 0.497      | 0.003      | 0.0004     | -          | -      |
| K2K-like   | NC           | Even $e$-like, no $\mu$-like | 0.437      | -          | 0.078      | 0.060      | -      |
| (aggressive)| $\mu$      | Even $e$-like, $\geq 1$ $\mu$-like | 0.989      | 0.086      | -          | 0.0003     | -      |
|            | $e$         | Odd $e$-like | 0.993      | 0.002      | 0.005      | -          | -      |
| K2K-like   | NC           | Even $e$-like, no $\mu$-like | 0.494      | -          | 0.081      | 0.011      | -      |
| (Gaussian  | $\mu$       | Even $e$-like, $\geq 1$ $\mu$-like | 0.994      | 0.073      | -          | 0.0007     | -      |
| beam)      | $e$         | Odd $e$-like | 0.999      | 0.0003     | 0.0004     | -          | -      |
| T2K-like   | NC           | Even $e$-like, no $\mu$-like | 0.420      | -          | 0.25       | 0.006      | -      |
| (Gaussian  | $\mu$       | Even $e$-like, $\geq 1$ $\mu$-like | 0.988      | 0.036      | -          | 0.0014     | -      |
| beam)      | $e$         | Odd $e$-like | 0.944      | 0.00002    | 0.00001    | -          | -      |
| MINOS-like | NC           | No $\mu > 1$ GeV | 1.000      | -          | 0.903      | -          | 0.429  |
| $P_{\mu\mu} = 0$ | $\mu$ | Any $\mu > 1$ GeV | 0.749      | 0          | -          | -          | 0      |
| MINOS-like | NC           | No $e, \mu$, or $\gamma$ | 0.520      | -          | 1.067      | 0.005      | 0.347  |
| $P_{\mu\mu} \neq 0$ | $\mu$ | Any $\mu > 1$ GeV | 0.749      | 0          | -          | 0          | 0      |
|            | $e$         | No $\mu > 1$ GeV, $\geq 1$ $e$ or $\gamma$ | 0.999      | 0.125      | 0.090      | -          | 0.064  |

The systematic uncertainty of the NC cross-section (i.e., $\epsilon_{NC}$) is expected to be at least three times larger; this is especially true when the uncertainties are added in quadrature. If there is a systematic uncertainty for an $f$ that is not small compared to $\epsilon_{NC}$, in practice it can be incorporated into $\epsilon_{NC}$, weighted by the value of $f$ itself.

For a perfect detector that can identify each event correctly, $f_{x,y} = \delta_{xy}$. In this limit $P_{\mu\mu} = 1 - R_{NC}$ and

$$\frac{P_{\mu\mu}}{\delta P_{\mu\mu}} \sim \frac{P_{\mu\mu}}{\sqrt{(1 - P_{\mu\mu})(1/\zeta_{NCNC}N_{NC}^0) + (1 - P_{\mu\mu})^2\epsilon_{NC}^2}}. \quad (17)$$

This ratio depends only on $P_{\mu\mu}$, the experimental statistics and the systematic uncertainty on the NC measurement. Thus, (17) defines the maximum sensitivity that is in principle achievable for a given $N_{NC}^0$ and $\epsilon_{NC}$.

2.2.2. Above $\tau$ threshold. If the neutrino energy is above the $\tau$ threshold and there is not a clean signature for $\nu_\tau$ CC events, we can still deduce $P_{\mu\mu}$ and $P_{\mu\tau}$ by using the identity $P_{\mu\mu} + P_{\mu\tau} + P_{\mu\tau} = 1$ to eliminate $P_{\mu\tau}$ in (6) and (7) (we are still assuming $P_{\mu\mu} = 0$), which gives

$$P_{\mu\mu} = \frac{R_{\mu}(1 + f_{\tau,NC}) + R_{NC}(f_{NC,\mu} + f_{\tau,\mu})}{1 + f_{\tau,NC} - f_{\tau,\mu} + f_{\tau,NC}f_{NC,\mu} - f_{\mu,NC}(f_{NC,\mu} + f_{\tau,\mu})}, \quad (18)$$

$$P_{\mu\tau} = 1 - \frac{R_{NC}(1 - f_{\tau,\mu}) - R_{\mu}(f_{\mu,NC} - f_{\tau,NC})}{1 + f_{\tau,NC} - f_{\tau,\mu} + f_{\tau,NC}f_{NC,\mu} - f_{\mu,NC}(f_{NC,\mu} + f_{\tau,\mu})}. \quad (19)$$
If no other process contaminates the $\nu_{\mu}$ CC events (i.e., $f_{j,\mu} = 0$ as appears to be the case for a MINOS-like experiment; see section 3), then

$$\frac{P_{\mu\mu}}{\delta P_{\mu\mu}} \simeq \frac{1 + f_{e,NC} - R_{NC} + R_{\mu}(f_{\mu,NC} - f_{\tau,NC})}{\sqrt{\delta R_{NC}^2 + (f_{\mu,NC} - f_{\tau,NC})^2(\delta R_{\mu})^2}}. \quad (20)$$

For a perfect detector, $P_{\mu\mu}/\delta P_{\mu\mu}$ is again given by (17).

2.3. Do not ignore $\nu_e$'s

If the $\nu_e$ CC interaction rate in the far detector is not negligible (which could be the case if $\sin^2 2\theta_{13}$ is near its upper bound and we want to push the uncertainty in the measurement of $P_{\mu\mu}$ down to the few per cent level), then we need three measurements to be able to solve for all of the probabilities. The potential measurables are

$$R_{NC} \equiv \frac{N_{NC}}{\xi_{NC}N_0^{NC}} = (1 - P_{\mu\mu}) + f_{\mu,NC}P_{\mu\mu} + f_{e,NC}P_{\mu e} + f_{\tau,NC}P_{\mu\tau}, \quad (21)$$

$$R_{\mu} \equiv \frac{N_{\mu}}{\xi_{\mu}N_0^{\mu}} = f_{NC,\mu}(1 - P_{\mu\mu}) + P_{\mu\mu} + f_{e,\mu}P_{\mu e} + f_{\tau,\mu}P_{\mu\tau}, \quad (22)$$

$$R_e \equiv \frac{N_{e}}{\xi_{ee}N_0^{e}} = f_{NC,e}(1 - P_{\mu\mu}) + f_{\mu,e}P_{\mu\mu} + P_{\mu e} + f_{\tau,e}P_{\mu\tau}, \quad (23)$$

and, if we are above the $\nu_{\tau}$ CC interaction threshold,

$$R_\tau \equiv \frac{N_{\tau}}{\xi_{\tau}N_0^{\tau}} = f_{NC,\tau}(1 - P_{\mu\mu}) + f_{\mu,\tau}P_{\mu\mu} + f_{e,\tau}P_{\mu e} + P_{\mu\tau}. \quad (24)$$

2.3.1. Below $\tau$ threshold. Below the $\nu_{\tau}$ CC threshold energy, the three measurements must be $R_{\mu}$, $R_{NC}$ and $R_e$. Then $f_{\tau,j} = 0$, the $P_{\mu\tau}$ terms drop out, and we can invert equations (21)–(23) to obtain

$$P_{\mu\mu} = \frac{R_{\mu}(1 - f_{e,\mu}f_{NC,e}) - R_{NC}(f_{NC,\mu} - f_{NC,e}f_{e,\mu}) - R_e(f_{e,\mu} - f_{NC,\mu}f_{NC,e})}{1 - f}, \quad (25)$$

$$P_{\mu e} = \frac{R_e(1 - f_{e,\mu}f_{NC,\mu}) - R_{NC}(f_{NC,e} - f_{NC,\mu}f_{\mu,\mu}) - R_{\mu}(f_{\mu,e} - f_{NC,\mu}f_{NC,e})}{1 - f}, \quad (26)$$

$$P_{\mu\tau} = 1 - \frac{R_{NC}(1 - f_{e,\mu}f_{NC,e}) - R_{\mu}(f_{\mu,\mu} - f_{\mu,e}f_{e,\mu}) - R_e(f_{e,\mu} - f_{NC,\mu}f_{NC,e})}{1 - f}, \quad (27)$$

where $f \equiv f_{\mu,NC}f_{NC,\mu} + f_{e,NC}f_{NC,e} + f_{\mu,e}f_{e,\mu} - f_{\mu,NC}f_{NC,e}f_{e,\mu} - f_{NC,\mu}f_{\mu,e}f_{\tau,NC}$. The calculation of the $\delta P$’s is straightforward; each $R$ term has a statistical and systematic uncertainty given by (14).

Note that for an idealized detector in which no other processes significantly contaminate $\nu_e$ CC events (i.e., $f_{j,e} \simeq 0$) and $\nu_e$ CC events do not contaminate $\nu_{\mu}$ CC events (i.e., $f_{e,\mu} \simeq 0$), then $P_{\mu e} = R_e$. Since $P_{\mu e}$ is small (of order 0.1 or less, as indicated by current oscillation limits), eliminating terms of order $f^2$ and $fP_{\mu e}$ in this case will recover the situation where we ignored $\nu_e$ (i.e., (16)).
2.3.2. Above $\tau$ threshold, no $\tau$ measurement. For energies above the $\nu_\tau$ CC interaction threshold the $P_{\mu\tau}$ terms do not drop out of equations (21)–(23). If we do not have the means to measure $\nu_\tau$ CC events but can measure $\nu_e$ CC events, then we can use probability conservation to eliminate $P_{\mu\tau}$, giving

$$R_{NC} = (1 - P_{\mu\mu})(1 + f_{e,NC}) + P_{\mu\mu}(f_{\mu,NC} - f_{\tau,NC}) + P_{\mu e}(f_{e,NC} - f_{\tau,NC}), \quad (28)$$

$$R_{\mu} = (1 - P_{\mu\mu})(f_{NC,\mu} + f_{\tau,\mu}) + P_{\mu\mu}(1 - f_{\tau,\mu}) + P_{\mu e}(f_{e,\mu} - f_{\tau,\mu}), \quad (29)$$

$$R_{e} = (1 - P_{\mu\mu})(f_{NC,e} + f_{\tau,e}) + P_{\mu\mu}(f_{\mu,e} - f_{\tau,e}) + P_{\mu e}(1 - f_{\tau,e}). \quad (30)$$

The general solution for the probabilities is somewhat messy, but if we assume that no other processes contaminate the $\nu_\mu$ CC signal (i.e., $f_{j,\mu} \approx 0$) and the $\nu_e$ CC events do not contaminate the other signals ($f_{e,j} \approx 0$) (see section 3), then $P_{\mu\mu} = R_{\mu}$ and we can invert (28) and (30) to obtain

$$P_{\mu\mu} = 1 - \frac{R_{NC}(1 - f_{\tau,e}) + R_{e}f_{\tau,NC} + R_{\mu}[f_{NC,\mu}(1 - f_{\mu,e}) - f_{\mu,NC}(1 - f_{\tau,e})]}{1 + f_{\tau,NC}(1 + f_{NC,e}) - f_{\tau,e}}. \quad (31)$$

The calculation of $\delta P_{\mu\mu}$ is straightforward.

If no other processes contaminate the $\nu_e$ CC signal (i.e., $f_{j,e} \approx 0$), then $P_{\mu e} = R_{e}$ and we obtain

$$\frac{P_{\mu\mu}}{\delta P_{\mu\mu}} = \frac{1 + f_{\tau,NC} - R_{NC} + R_{\mu}(f_{\mu,NC} - f_{\tau,NC}) - R_{e}f_{\tau,NC}}{\sqrt{(\delta R_{NC})^2 + (f_{\mu,NC} - f_{\tau,NC})^2(\delta R_{\mu})^2 + f_{\tau,NC}^2(\delta R_{e})^2}}. \quad (32)$$

2.3.3. Above $\tau$ threshold with a $\tau$ measurement. If $R_{e}$ is also measured, in addition to $R_{e}$, then there are four measurements ($R_{\mu}$, $R_{NC}$, $R_{e}$ and $R_{\tau}$), but there are only three independent quantities (since $P_{\mu\mu} + P_{\mu e} + P_{\mu\tau} + P_{\mu s} = 1$). One possible approach would be to assume that $P_{\mu s}$ is independent of the other probabilities and use these four measurements to test probability conservation. We do not pursue this option here. Instead, we use probability conservation to eliminate one of the probabilities and use three of the four measurements to determine $P_{\mu s}$ (the fourth measurement could be used to check probability conservation afterwards). Since $P(\nu_\mu \rightarrow \nu_e)$ is most likely much larger than $P(\nu_\mu \rightarrow \nu_\tau)$ in the $L/E$ regime we are considering, we use $R_{\tau}$ as the third measurement (along with $R_{\mu}$ and $R_{NC}$). Then the appropriate formulae for the measurables $R_{NC}$, $R_{\mu}$ and $R_{\tau}$ can be found by the interchange $\tau \leftrightarrow e$ in (28)–(30).

3. Detector simulations

We wish to explore how well in principle a neutrino three-flavour unitarity test can be performed with a given muon-neutrino beam as a function of dataset size, and study which systematic uncertainties are likely to be important, and their impact.

We consider first a ‘perfect’ experiment in which the sensitivity of the unitarity test is determined only by the statistical uncertainties, calculated using a parametrization of the known
beam flux and spectrum, together with a simulation of neutrino interactions in the detector. An event simulation is used to determine the relevant detection efficiencies and misidentification factors. We use the NEUGEN Monte Carlo code [12] to simulate neutrino interactions in the detector. Events are classified as $\nu_\mu$ CC, $\nu_e$ CC or NC. In practice, the requirements used to identify events of a given type will depend upon the detector technology. For example, for a water Cherenkov detector in our simple analysis, we will define a $\nu_e$ CC event candidate as an event with an electron candidate above threshold. An electron candidate is either a real electron or a $\pi^0$ with an energy exceeding 1 GeV (in which case the two daughter photons from the high-energy $\pi^0$ produce Cherenkov rings that overlap in the detector and cannot be distinguished from a single electromagnetically showering particle). A NC event candidate would be an event containing a $\pi^0$ candidate but no muon candidate, where a $\pi^0$ candidate has two $e$-like rings above threshold (which come from a $\pi^0$ with energy less than 1 GeV). The definition of CC and NC events can of course be varied, and then tuned to give favourable values for the signal efficiencies and misidentification factors. Examples are shown in table 1.

3.1. K2K- and T2K-like experiments

To identify the most important systematic uncertainties, it is useful to compare the sensitivity of our ‘perfect experiment’ with that of a realistic experiment. We begin with the K2K experiment. K2K uses a beam from the KEK laboratory in Japan. The neutrinos in the KEK beam have a mean energy of 1.3 GeV [9], and the neutrinos travel 250 km to the Super-K water Cherenkov detector. A new experiment T2K is being planned that will exploit a more intense neutrino source that is presently under construction at Tokai, Japan. T2K will also use the Super-K detector, but with a slightly longer baseline (300 km) and narrow-band beam with an axis displaced slightly from pointing directly at the far detector (an ‘off-axis’ beam). The real experimental sensitivities of the K2K and T2K experiments can only be determined by the experimental collaborations. In the following, we use the NEUGEN Monte Carlo program to simulate neutrino interactions together with a simple model for the response of a Super-K-like detector. Although this is inadequate to predict precisely the real K2K and T2K sensitivities, it does enable us to identify the dominant sources of systematic uncertainties, and hence explore how the experimental results will depend upon the sizes of these systematics. We use the following parametrization of a Super-K-like detector response:

(a) A threshold of 197 MeV $c^{-1}$ for the detection and measurement of muons [14], and 100 MeV $c^{-1}$ for electrons and $\pi^0$s. These thresholds approximate those used for the atmospheric neutrino analysis of Super-K [14, 15].

(b) Energy resolutions given by [15]

$$\frac{\Delta E_{\text{rms}}}{E} = 0.005 + \frac{0.025}{\sqrt{E(\text{GeV})}}$$

(33)

for electrons and $\pi^0$s and

$$\frac{\Delta p_{\text{rms}}}{p} = 0.03$$

(34)

for charged pions and muons.

In addition, we use a parametrization of the spectra for the K2K and T2K neutrino beams.

In our analysis, we will use only simulated events with visible energy greater than 0.1 GeV. For our ‘basic’ signals we define a $\nu_\mu$ CC event candidate as an event with a single muon-like
ring, a $\nu_e$ CC event candidate as an event with a single $e$-like ring, and a NC event candidate as an event with two $e$-like rings, which are assumed to be two photons from a single $\pi^0$ decay. Given these definitions, the detector efficiencies and misidentification factors determined from our simulations are listed in table 1. As shown in the table, the efficiencies $\xi_{ij}$ are of order one-half, and there is no significant contamination of one signal by another due to misidentification. Also shown are the results of a more aggressive signal definition, where a simulated event with an odd number of $e$-like rings is labelled as a $\nu_e$ CC event candidate, and the remaining events (those with an even number of $e$-like rings) are labelled as $\nu_\mu$ CC event candidates if they have one or more $\mu$-like rings or NC if they do not. In this more aggressive scenario no events are discarded, i.e., all events were used for one of the targeted signals. Although some of the misidentification factors are slightly larger for the aggressive scenario, overall they are not greatly changed, while there is a significant improvement in the efficiencies for the CC events.

To investigate whether our analysis is sensitive to the assumed details of the neutrino spectrum, we have repeated the calculation of efficiencies and misidentification factors for a K2K-like experiment with a beam that has the same average energy and beam spread as the KEK beam, but with a Gaussian energy spectrum (no long high-energy tail). For the Gaussian beam, the misidentification factors involving $\nu_e$ were greatly reduced (since backgrounds from the high-energy tail are now suppressed), but $f_{\mu,NC}$ and $f_{NC,\mu}$ were only slightly affected. Since $f_{\mu,NC}$ is the dominant $f$ factor for a K2K-like experiment, we conclude that our results are not very sensitive to the detailed beam spectrum we assume.

We now consider a T2K-like experiment, where we have used a beam spectrum that corresponds to a detector $2^\circ$ off-axis. The resulting misidentification factors for a T2K-like experiment are shown in table 1. All of the misidentification factors are reduced except for $f_{\mu,NC}$, which is now 0.25. Therefore, in both the K2K- and T2K-like experiments, the most important contamination is $\nu_\mu$ CC events being misidentified as NC events.

3.2. A MINOS-like experiment

The MINOS experiment is a long-baseline oscillation experiment that will use a neutrino beam from the Fermilab Main Injector and an iron-scintillator sampling calorimeter 730 km away in Minnesota. MINOS is expected to begin data taking early in 2005 with the so-called Low Energy NuMI horn configuration. With a beam energy that is about a factor of three higher than the KEK beam, and a detector that is very different from the water Cherenkov detector used by K2K and T2K, the efficiencies and misidentification factors for MINOS will be very different than those for the experiments in Japan. To compute the numbers given in table 1 we have used a parametrization of the NuMI neutrino beam spectrum for the Low Energy horn configuration, the NEUGEN Monte Carlo Program to simulate neutrino interactions in an iron detector, and a simple parametrization of the response of a MINOS-like detector. In particular we assume:

(a) An energy threshold of 50 MeV for the detection and measurement of electrons, and charged and neutral pions, and a threshold of 1 GeV for the identification and measurement of muons. Note that the MINOS detector is expected to be able to determine the charge and measure the momenta of muons from 0.5 to 100 GeV $c^{-1}$, and to distinguish $\nu_\mu$ CC events from NC events if the muons have momenta exceeding about 1 GeV $c^{-1}$ [16]. Our final results are insensitive to the exact values chosen for the energy thresholds; the most significant changes occur in $f_{e,NC}$, which doubles in size when the thresholds are increased to 100 MeV. However,
$f_{\nu, NC}$ is very small and consequently does not significantly affect our results. The significant $f$ factors change at most by 5%.

(b) Energy resolutions given by

$$\frac{\Delta E_{rms}}{E} = \frac{0.23}{\sqrt{E(\text{GeV})}}$$  \[35\]

for electrons and $\pi^0$'s

$$\frac{\Delta E_{rms}}{E} = \frac{0.55}{\sqrt{E(\text{GeV})}}$$  \[36\]

for charged pions and

$$\frac{\Delta p_{rms}}{p} = 0.05$$  \[37\]

for muons. Note that in practice the muon energy resolution for the MINOS experiment is expected to be somewhat better (worse) than described by (37) if the muon ranges out (does not range out) in the detector. We found that $\Delta p_{rms}/p$ values as high as 0.10 do not appreciably change our results.

As shown in the table, for a MINOS-like experiment there is a very large contamination of the NC channel by $\nu_\mu$ CC events, and misidentification of $\nu_\tau$ CC events as NC events is also significant. The efficiency for identifying NC events is about one-half, similar to the K2K-like and T2K-like experiments.\(^6\)

The size of $f_{\mu, NC}$ may be understood as follows: $f$ depends not only on the misidentification fraction, but also on the relative cross-sections and efficiency of the signal. For $f_{\mu, NC}$ in the case where electron neutrinos are being ignored, the probability of a $\mu$ event being misidentified as a NC event is about 0.25 and the $\mu$ cross-section is about 3.5 times larger than the NC cross-section, which leads to $f_{\mu, NC} \simeq 0.9$.

We have not considered the effect of $\tau$ decays into muons or electrons, each of which occurs with branching fraction of about 18%. If all of the $\tau$ decays to muons were identified as muons, then $f_{\tau, \mu}$ would be at most about 0.08; this has only a small effect because $f_{\tau, \mu}$ is proportional to $\sigma/\sigma_\mu \sim 0.3$. In practice, many muons from $\tau$ decays would not pass the muon minimum energy cut, and $f_{\tau, \mu}$ would be even smaller. Since $f_{\tau, \mu}$ is much smaller than $f_{\mu, NC}$ and $f_{\tau, NC}$, we do not consider the effects of $\tau$ decays.

4. Results

4.1. A perfect detector

We first find the sensitivity of the NC unitarity test for a perfect detector, i.e., a detector that can categorize each event correctly as CC muon or NC, with no misidentification and 100%
Figure 1. Assuming Gaussian statistical uncertainties, the $3\sigma$ sensitivity for measuring $P_{\mu s}$ versus $N^0_{\mu}$ for fixed values of the NC systematic error for a perfect detector (dotted curves, using equation (17), the K2K-like experiment with our basic signal definition (solid), the T2K-like experiment (dashed), and the MINOS-like experiment (dash-dotted). The number of NC events without oscillations is $N_{NC}^0 = 0.156 N_{\mu}^0$. The systematic uncertainties $\delta N^0_{\mu}/N_{\mu}^0$ and $\delta N^0_{e}/N_{e}^0$ are assumed to be 2%, except when $\epsilon_{NC} \equiv \delta N_{NC}^0/N_{NC}^0 = 0$, in which case they are 0. The arrows indicate the approximate statistical sensitivities expected for the K2K, T2K and MINOS experiments.

4.2. More realistic K2K- and T2K-like experiments

Next we find the NC sensitivity for the K2K-like detector described in section 3.1 for the case $P_{\mu e} \simeq 0$. We generated 400 000 neutrino events using the NEUGEN simulator, from which the normalized misidentification factors $f_{x,y}$ were calculated. For a given set of probabilities $P_{x,y}$, the values of $R_{NC}$ and $R_{\mu}$ were calculated, and the corresponding measured value of $P_{\mu s}$ was determined from (11). The uncertainty on $P_{\mu s}$ was calculated using (14), assuming the
uncertainties $\delta R_{NC}$ and $\delta R_{e}$ are uncorrelated and add in quadrature. The 3$\sigma$ sensitivity for $P_{\mu s}$ is shown in figure 1 (solid curves) for various values of $\epsilon_{NC}$ for the case $P_{\mu s} = 1 - P_{\mu\mu}$ (all $\nu_\mu$ oscillating to $\nu_s$; we will consider cases with nonzero $P_{\mu\tau}$ later). For both low and high statistics the K2K-like 3$\sigma$ sensitivity can be approximated by

$$p_{\mu s}^{\text{min}} \simeq \frac{3(1 + f_{\mu,NC})(\delta R_{NC}/R_{NC})}{1 + 3(1 + f_{\mu,NC})(\delta R_{NC}/R_{NC})}, \quad (38)$$

which can be derived from (16), where factors quadratic in the $f_{x,y}$ are ignored. Since $f_{\mu,NC} \simeq 0.08$ for our K2K-like experiment, the NC sensitivity is at most about 1.08, worse than that of the perfect detector for large numbers of events where the statistical uncertainty becomes negligible compared to the systematic uncertainty. At low statistics the efficiency becomes important and the K2K-like performance will be more than 1.08, worse than a perfect detector.

The K2K-like curves in figure 1 are plotted for the simple K2K signals in table 1. The corresponding curves for the more aggressive K2K-like signals are very similar to the simple case; the improved efficiencies are partially compensated for by the slightly higher value of $f_{\mu,NC}$. Thus the result is fairly insensitive to the exact signal criteria used.

We next consider the effects of nonzero $P_{\mu\tau}$. If we assume $P_{\mu\tau} = 1 - P_{\mu s}$ (i.e., $P_{\mu\mu} = P_{\mu e} = 0$), the curves are very close to those of the perfect detector, since the dominant misidentification term $f_{\mu,NC}$ does not contribute to $R_{NC}$ when $P_{\mu\mu} = 0$. If both $P_{\mu\mu}$ and $P_{\mu\tau}$ are both nonzero (with $P_{\mu e} \simeq 0$), the results will lie somewhere between the curves for K2K-like and the perfect detector.

Finally, we consider nonzero $P_{\mu e}$, in which case $R_e$ must also be measured and $P_{\mu s}$ is determined using (27). As discussed in section 2.3.1 if $P_{\mu e}$ is of order 0.1 or less (as indicated by oscillation bounds such as from the CHOOZ reactor), and if the misidentification factors are also of order 0.1 or less, then this case reduces to that where the $v_e$ are ignored. We have verified this numerically for the K2K-like misidentification factors in table 1.

In summary, the sensitivity of the K2K-like detector to the NC signal is only slightly worse than that of a perfect detector, with the dominant loss of sensitivity coming from the misidentification of CC muon events as NC. For comparison, in figure 1, we have also shown sensitivity curves for the T2K-like experiment with a Gaussian beam spread. Since $f_{\mu,NC} = 0.25$ in this case, the sensitivity is about a factor of 1.25/1.08 = 1.16 worse than for K2K.

4.3. The MINOS-like detector

For the MINOS-like case, we generated 320 000 neutrino events using the NEUGEN simulator, and calculated the corresponding misidentification factors. The 3$\sigma$ sensitivity for $P_{\mu s}$ was calculated as described above for the case $P_{\mu s} = 1 - P_{\mu\mu}$; the results are shown in figure 1. At low statistics, the MINOS-like experiment does better than the K2K- and T2K-like experiments because of the higher NC efficiency, but at high statistics it does worse because of the larger misidentification factors.

4.4. Exclusion limit when $P_{\mu s} = 0$

If a 3$\sigma$ signal for $P_{\mu s}$ is not observed, then an exclusion limit (upper bound) for $P_{\mu s}$ can then be obtained. The 90$\%$ CL exclusion limit for $P_{\mu s}$ is shown in figure 2 for a perfect detector (dotted curves), K2K-like with basic signals (solid curves), and T2K-like with basic signals (dashed curves).
Figure 2. Assuming Gaussian statistical uncertainties, the 90% CL exclusion limit for $P_{\mu s}$ versus $N_\mu^0$ for fixed values of the NC systematic error for a perfect detector (dotted curves), the K2K-like experiment with basic signals (solid), the T2K-like experiment with basic signals (dashed), and the MINO-like experiment (dash-dotted). Other assumptions are the same as in figure 1.

curves). To model realistic oscillation probabilities we have assumed a three-neutrino model assuming the parameters $\delta m_{31}^2 = 2.0 \times 10^{-3} \text{ eV}^2$, $\sin^2 2\theta_{23} = 1.0$, and $\sin^2 2\theta_{13} = 0.1$. At high statistics the relative values of the exclusion limits are approximately proportional to $(1 + f_{\mu, NC})$, similar to the $3\sigma$ sensitivity levels calculated previously. As was the case for the $3\sigma$ sensitivity, the MINOS-like detector does better than the K2K- and T2K-like detectors at low statistics, due to the higher NC efficiency, but not as well at high statistics due to larger misidentification factors.

5. Summary

At low statistics ($\lesssim 1000$ events), experiments with a larger NC efficiency, such as our MINOS-like example, tend to have better sensitivity to the sterile oscillation probability $P_{\mu s}$. At high statistics, the sensitivity in the cases we considered is primarily limited by the systematic uncertainty in the NC rate, $\epsilon_{NC}$, and the contamination of the NC signal from CC $\mu$ events, $f_{\mu, NC}$ (and NC contamination from CC $\tau$ events, $f_{\tau, NC}$, above $\tau$ threshold). The best anticipated $\epsilon_{NC}$ is of order a few per cent, so the best $3\sigma$ sensitivity and 90% CL exclusion limits that can be expected for the sterile oscillation probability will be of order 0.10–0.15 (0.2–0.3 for the oscillation amplitude).
The lowest contamination rates are realized for the K2K- and T2K-like cases. Very fine-grain detectors, such as a liquid argon TPC, will be subject to much less event misidentification than the K2K- or MINOS-like detectors. However, even when event misidentification is eliminated, at most an 8% improvement is possible over the K2K-like detectors. Therefore, significant improvements in these sterile probability sensitivities or limits can only be achieved by lowering the uncertainty in NC cross-sections or improving the event selection criteria, both of which could prove to be challenging but very worthwhile.

Acknowledgments

We thank D Harris for a critical reading of the paper and H Gallagher for assistance with NEUGEN. VB thanks the Aspen Center for Physics for hospitality during the completion of this paper. This research was supported in part by the US Department of Energy under Grant nos. DE-FG02-95ER40896, DE-AC02-76CH03000 and DE-FG02-01ER41155, and in part by the Wisconsin Alumni Research Foundation.

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