Estimates of flavoured scalar production in $B$ - decays

Victor Chernyak
Budker Institute of Nuclear Physics, Novosibirsk, 630090, Russia

Abstract

Estimates are presented for the branching ratios of several two-particle $B$-meson decays into flavoured scalar mesons.

It seems that there are no estimates of $B$-meson decays into scalar mesons. The purpose of this short note is to present such estimates. As will be shown, some two-particle decays of $B$-mesons into scalar mesons have sufficiently large branchings to be of current interest.

1. Having in mind $B^- \to K_o(1430)\pi^-$ and $B^- \to K\pi^-$ decays, the main contributions in the factorization approximation which looks reliable for these decays, and in the standard notation, come from two terms in the effective Hamiltonian:

$$H_{eff} = \frac{G_F}{\sqrt{2}} (-V_{tb}V_{ts}^*) \left\{ a_4 \left[ \bar{s} \gamma_\mu (1 + \gamma_5) d \right] \otimes \left[ \bar{d} \gamma_\mu (1 + \gamma_5) b \right] - 2 a_6 \left[ \bar{s} (1 - \gamma_5) d \right] \otimes \left[ \bar{d} (1 + \gamma_5) b \right] \right\}. \quad (1)$$

The effective coefficients $a_i$ in (1) can be expressed either explicitly through the original coefficients $C_i(\mu)$ of the effective Hamiltonian \cite{1}, plus perturbative one loop corrections: $C_i(\mu) \to C_i^{eff}$; this cancels the main scale dependence of $C_i(\mu)$, see e.g. \cite{2} and references therein. Or they can be calculated in "the QCD improved factorization approximation" \cite{3}, which is not much different.
In what follows we will not need their explicit form, but only the typical value of the ratio $a_6/a_4$. \footnote{The coefficients $a_i$ change, of course, when going from $K$ to $K_o(1430)$, as they depend on the form of the meson wave functions. In our estimates below we suppose these differences can be safely neglected for $a_4$, $a_6$ and $a_1$. Using the explicit expressions for $a_i$ from \cite{3}, which include $O(\alpha_s)$ corrections, and the model wave functions of $K$ and $K_o(1430)$, we have estimated that the differences in the values of $a_{1,4,6}$ for $K$ and $K_o(1430)$ are indeed reasonably small.}

The matrix elements are defined as:

$$\langle \bar{K}(q)|\bar{s}\gamma_\mu\gamma_5d|0\rangle = -i f_k q_\mu, \quad \langle \bar{K}(q)|\bar{s}i\gamma_5d|0\rangle = f_k \mu_k, \quad \mu_k = M_k^2/\bar{m}_s,$$

$$\langle K_o(1430)(q)|\bar{s}\gamma_\mu d|0\rangle = f_o q_\mu, \quad \langle K_o(1430)(q)|\bar{s}d|0\rangle = f_o \mu_o, \quad \mu_o = M_o^2/\bar{m}_s,$$

$$\langle \pi^- (p_2)|\bar{d}\gamma_\mu b|B^-(p_1)\rangle = (p_1 + p_2)_\mu f_+(q^2) + q_\mu f_-(q^2),$$

$$M_b \langle \pi^- (p_2)|\bar{d}b|B^-(p_1)\rangle \simeq (M_B^2 - q^2) f_+^B(q^2) \simeq M_B^2 f_+(0). \quad (2)$$

Therefore, the decay amplitudes look as:

$$T(B^- \to \bar{K}\pi^-) \simeq i \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* a_4 f_k M_B^2 f_+^{B\pi}(0) \left( 1 + \frac{2M^2_k}{\bar{m}_s M_B} a_6 \right),$$

$$T(B^- \to K_o\pi^-) \simeq \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^*(-a_4) f_o M_B^2 f_+^{B\pi}(0) \left( 1 - \frac{2M^2_o}{\bar{m}_s M_B} a_6 \right). \quad (3)$$

Now, about parameters entering (3). To avoid main uncertainties it is reasonable to take the ratio $Br(K_o\pi)/Br(K\pi)$. So, it will be sufficient to know ($f_k \simeq 160$ MeV):

$$\frac{a_6}{a_4} \simeq 1.2-1.3, \quad \bar{m}_s \simeq m_s(\mu = 2.5$ GeV $) \simeq 110$ MeV, \quad $M_B^{(\text{eff})} \simeq 4.8$ GeV. \quad (4)$$

The main new parameter is the coupling $f_o$. It can be estimated from the form factor ($\Delta M^2 \equiv M_k^2 - M_\pi^2 \simeq 0.22$ GeV$^2$):

$$\langle K^-(p_2)|\bar{s}\gamma_\mu d\pi^-(p_1)\rangle = (p_1 + p_2 - (\Delta M^2/q^2) q_\mu F_+(q^2) + (\Delta M^2/q^2) q_\mu F_0(q^2),$$

$$m_s \langle K^-|\bar{s}d|\pi^-\rangle \equiv d(q^2) = \Delta M^2 F_0(q^2), \quad F_0(q^2 = 0) \simeq 1. \quad (5)$$
Saturating the dispersion relation for \(d(q^2)\) by two lowest resonances, \(K_o(1430)\) and \(K_o(1950)\), one obtains:

\[
0.22 \text{GeV}^2 \simeq (f_o g_o + f' g'),
\]

where \(f_i\) are couplings of resonances with the scalar current (see eq.(2) above, the coupling \(f'\) of \(K_o(1950)\) with the current \(m_s(\bar{s}d)\) is defined in the same way as those of \(K_o(1430)\); \(f_i = O(m_s)\) at \(m_s \to 0\)), and \(g_i\) are their couplings to the \((K^- \pi^+)\)-pair. These last can be found from their known decays to \(K\pi\): \(g_o \simeq 3.8\text{GeV},\) \(g' \simeq 2.7\text{GeV}\). Besides, there are estimates [4] of the ratio of couplings \(f'/f_o\) which look as:

\[
f'_o f_o = -\gamma \frac{M_o^2}{M'^2} = -0.5\gamma, \quad \gamma = (0.5 \pm 0.3).
\]

Therefore, one obtains:

\[
f_o = (70 \pm 10)\text{MeV}.
\] (6)

Collecting all the above given numbers, we have:

\[
Br \frac{(B^- \to \bar{K}_o(1430)\pi^-)}{(B^- \to K\pi^-)} \simeq 14 \frac{f_o^2}{f^2_k} \simeq 2.7
\]

(for central values of parameters in (4) and (6)).

So, if \(Br(B^- \to \bar{K}\pi^-) \simeq 16 \cdot 10^{-6}\), then \(Br(B^- \to \bar{K}_o(1430)\pi^-)\) will be \(\simeq 43 \cdot 10^{-6}\).

It is interesting not only that \(Br(B^- \to \bar{K}_o(1430)\pi^-)\) is large by itself, but that it receives the dominant contribution from the term \(\sim a_6\) which is a power correction, \(O(\Lambda_{QCD}/M_b)\), in the formal limit \(M_b \to \infty\).

2. Let us consider the decay \(\bar{B}^o \to a^+_o(1450)K^-\). The corresponding form factor \(F^B_{a^+}\) is defined as:

\[
\langle a^+_o(p_2) | \bar{u}\gamma_\lambda \gamma_5 b | \bar{B}^o(p_1) \rangle = -i \left[ (p_1 + p_2)_\lambda F^B_{a^+} + q_\lambda F^B_{-a^+} \right],
\]

\[
M_b \langle a^+_o | \bar{u}\gamma_5 b | \bar{B}^o \rangle \simeq i M^2_B F^B_{a^+}(0).
\] (7)
Such form factors, at not too large $q^2$, can be found by the method proposed in [5] (which is known now as "the light-cone sum rules"). One considers the correlator:

$$K = \int dx e^{ixq} \left< a_0^+(1450)(p) \right| T\{ \bar{u}(x)\gamma_5 b(x),\bar{b}(0)\gamma_5 d(0)\} |0\rangle,$$

and proceeding as in [5] obtains the sum rule ($\Delta = (1 - M_b^2/S_o) \simeq 0.3$):

$$F_{Ba}^+(0) \simeq \frac{M_b^3 \bar{\lambda}^2}{f_B M_B^4} \int_0^\Delta dx \frac{\phi_s(x)}{1 - x} \exp \left\{ \frac{M_B^2 - M_k^2}{1 - x} \right\}, \quad (8)$$

where the wave function $\phi_s(x)$ of $a_0^+(1450)$ is defined as:

$$\langle a_0^+(1450)(p) |\bar{u}(0) d(z)|0\rangle = \lambda^2 \int_0^1 dx e^{ixp} \phi_s(x), \quad \int_0^1 dx \phi_s(x) = 1.$$

The coupling $\bar{\lambda}^2 = \lambda^2(\mu \simeq 1.5 \text{ GeV})$ is related by SU(3) to the matrix element $\langle K^+(1430)(q)|s \bar{d}|0\rangle$ in (2), and so: $\bar{\lambda}^2 \simeq 1.15 \text{ GeV}^2$. For other quantities entering (8) we use: $\phi_s(x) \simeq \phi_s^{asy}(x) = 1, f_B \simeq f_\pi \simeq 130 \text{ MeV}$. One obtains then from (8):

$$F_{Ba}^+(0) \simeq 0.46. \quad (9)$$

Somewhat surprisingly, this transition form factor turns out to be $\simeq 1.5$ times larger than the corresponding $B \to \pi$ form factor: $f_{Ba}^+(0) \simeq 0.30$. Finally, this is due to strong coupling of scalar mesons to the scalar current.

Proceeding now in the same way as above, one obtains the decay amplitude:

$$T(\bar{B}^o \to a_0^+(1450)K^-) \simeq \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* a_4 f_k M_B^2 F_{Ba}^+(0) \left[ 1 - \frac{a_6}{a_4} \frac{2M_k^2}{m_s M_b} \right]. \quad (10)$$

The two terms in square brackets in (10) nearly cancel each other. So, we conclude that $Br(\bar{B}^o \to a_0^+(1450)K^-)$ is very small, in spite of the large form factor.

\[2\] The leading twist wave function of $a_0$ also contributes to the sum rules. Estimates show that this contribution is positive and small. We neglect it.
3. Let us consider now the decay $\bar{B}^o \to a_o^+(1450)\pi^-$. Proceeding as before, one obtains the decay amplitude (it follows from the above that the penguin contribution is negligible):

$$T(\bar{B}^o \to a_o^+(1450)\pi^-) \simeq \frac{G_F}{\sqrt{2}} V_{ub} V^*_{ud} (-a_1) f_\pi M_B^2 F_{Ba}^B (0).$$

(11)

One has then from (11) and (9):

$$Br(\bar{B}^o \to a_o^+(1450)\pi^-) \simeq 20 \cdot 10^{-6}; \quad Br(\bar{B}^o \to a_o^+(1450)\rho^-) \simeq 38 \cdot 10^{-6}.$$

It is seen that these branchings are sufficiently large to be observable.

4. Finally, let us consider production of two scalar mesons, $B^- \to \bar{K}_o(1430)a_o^-(1450)$. Proceeding as before, one obtains from (1) the decay amplitude:

$$T(B^- \to \bar{K}_o(1430)a_o^-(1450)) \simeq i \frac{G_F}{\sqrt{2}} V_{ub} V^*_{ts} a_4 f_\pi M_B^2 F_{Ba}^B (0) \left(1 + \frac{2M_o^2}{m_s M_b a_4}\right).$$

Normalizing by $B^- \to \bar{K}\pi^-$ as before, and using (6), (9) one obtains:

$$Br(B^- \to \bar{K}_o(1430)a_o^-(1450)) \simeq Br(\bar{B}^o \to K_o^- (1430)a_o^+(1450)) \simeq 8.8 Br(B^- \to \bar{K}\pi^-).$$

Therefore, for $Br(B^- \to \bar{K}^o\pi^-) \simeq 16 \cdot 10^{-6}$, these branchings (as well as their charge conjugates) will be $\simeq 140 \cdot 10^{-6}$. It is seen that, in a sense, they are very large.

5. We do not consider here the neutral decay modes like, for instance, $B^o \to J/\Psi K_o(1430)$. Because the main factorizable contributions cancel each other here to large extent, the non-factorizable contributions become of great importance, and these are under poor control for such decays at present. One can expect only that, because the transition form factor $B \to K_o(1430)$ is considerably larger than those of $B \to K$, this mode can hardly be much smaller than $B \to J/\Psi K$.

---

3 $Br(B^o \to a_o^-\pi^+)$ is highly suppressed, and so this mode is self-tagging.

4 The role of the tree $b \to u (\bar{u}s)$ transition is very small here in comparison with the large contribution $\sim a_6$. 

5
We did not consider also flavourless scalars $f_o(980)$, $f_o(1370)$, $f_o(1500)$, etc. There are two reasons for this. First, their nature and quark-gluon composition are not well understood at present and, it seems, are complicated. The main reason, however, is that we expect their production in the $B \to f_i K$ decays can be highly enhanced by the same mechanism which enhances $B \to \eta' K$, and there is no clear understanding of this mechanism up to now.

Acknowledgements

I am indebted to A.E. Bondar who insisted on writing this note. I also thank him for very useful discussions.

References

[1] G. Buchalla, A.J. Buras and M.E. Lautenbacher, Rev. Mod. Phys. 68 (1996) 1125

[2] A. Ali, hep-ph/9812434

[3] M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, Phys. Rev. Lett. 83 (1999) 1914; hep-ph/0007256

[4] M. Jamin and M. Münz, Z. Phys. C66 (1995) 633

[5] V.L. Chernyak and I.R. Zhitnitsky, Preprint INP-88-65, 1988, Novosibirsk; Nucl. Phys. B345 (1990) 137