Coalition Formability Semantics with Conflict-Eliminable Sets of Arguments

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Abstract. We consider abstract-argumentation-theoretic coalition formability in this work. Taking a model from political alliance among political parties, we will contemplate profitability, and then formability, of a coalition. As is commonly understood, a group forms a coalition with another group for a greater good, the goodness measured against some criteria. As is also commonly understood, however, a coalition may deliver benefits to a group X at the sacrifice of something that X was able to do before coalition formation, which X may be no longer able to do under the coalition. Use of the typical conflict-free sets of arguments is not very fitting for accommodating this aspect of coalition, which prompts us to turn to a weaker notion, conflict-eliminability, as a property that a set of arguments should primarily satisfy. We require numerical quantification of attack strengths as well as of argument strengths for its characterisation. We will first analyse semantics of profitability of a given conflict-eliminable set forming a coalition with another conflict-eliminable set, and will then provide four coalition formability semantics, each of which formalises certain utility postulate(s) taking the coalition profitability into account.

1 Introduction

Coalition formation among agents is an important topic in many domains including economics, political science, and computer science. Two groups of agents, by teaming up together, could achieve a task which they cannot on their own. Exploring abstract argumentation theory for finding an apt characterisation of coalition formability looks specially rewarding, and there are already a few papers in the literature looking at this subject matter: with preference-based argumentation frameworks and task allocations [1]; with a cooperative goal generation and fulfilling [2] respecting the property of reciprocity, i.e. agents give to the coalition they are in and benefit from it; and with dialogue games and pay-offs [3]. Shared by them is the theme of identifying a group of individual agents who optimise benefits/social welfare to themselves by being in the group. The optimal group thus formed is free of internal conflicts, and the participating agents do not have to give up anything by being a part. We consider these types of coalitions supportive.

The sort of coalition formation we have in mind, on the other hand, is one that may be found in political alliance among political parties. Such alliance is motivated if, for example, political parties want to reach the voting threshold of passing certain bills. A political-alliance-like coalition exhibits the following unique characteristics:

More organisational than individual It is not possible to freely move agents across multiple political parties such as to connect those having close interests together for formation of optimal political parties. A participant to a political party is often expected to stay in the party during the parliament term. The assumption of self-interested agents as studied in [21] does not fit very well here. There are repercussions to profitability of a coalition, too, in that it is primarily for a party’s, or parties’, benefits than the participants’ benefits that a political alliance is formed.

Partial internal conflicts Agents in a political party should support the party’s agendas and policies. Hence they do not defeat each other about them, broadly spoken. It is common, however, that there are smaller factions within a political party arguing against one another over details. This often entails that some participating agents cannot proclaim their opinions on certain policies as the party’s opinions. Put another way, some individual opinions may be toned down for the party.

Asymmetry in attacks to and from a coalition For a political alliance to retain any credibility for the arguments it expresses, it must argue only by the conflict-free, i.e. self-contradiction-free, portion of the arguments of the party’s participants. But the other political parties not in the alliance are unhindered by the personal circumstance of the alliance. If a political party A is in alliance with another political party B, then an external party C can argue against any argument of the individual participants in A or in B as an argument against the coalition.

Better larger than smaller In [1], the rate of defections is associated to the number of agents in a coalition, thus a smaller set preferred. That does not carry over here: if one single political party or one single political alliance dominates the parliament, it has total freedom in policy making, which is clearly desirable. While not primarily on abstract argumentation, there is a work [4] on alternating-temporal logic incorporating the framework of [1]. In the logic, a larger set is better.

In this work, we contemplate abstract-argumentation-theoretic characterisations of profitability, and then formability, of a coalition of this kind. More specifically we consider the following questions: (1) suppose a set of arguments that may contain partial internal conflicts (as an abstract representation of a political party) and suppose also rational criteria of profitability, with which other (disjoint) sets of arguments that may also contain partial internal conflicts (i.e. representations of other political parties) can it profit by forming a coalition?; and (2) suppose the profitability relation, suppose some rational principles to judge the goodness of a coalition, and suppose such a set of arguments, which other (disjoint) such sets of arguments can it actually form a coalition?

The following are particularly interesting technicalities in the derivation of these semantics. First and foremost, the above-described coalition formability is not simply about whether the re-
sulting coalition is acceptable, which could be handled by adapting the standard acceptability semantics in abstract argumentation theory, but about whether a set of arguments potentially with partial internal conflicts forms a coalition with another similar set. The former concerns the state of the resulting set, while the latter must be parametrised by coalition profitabilities of both sets. Secondly, because of the presence of partial internal conflicts and of the asymmetry in attacks to and from a coalition, we have to: (1) obviously need a weaker notion than conflict-freeness as a property that a set of arguments should primarily satisfy - conflict-eliminability as we term it, which permits members of a set to attack other members of the same set so long as none of them is completely defeated; (2) obtain intrinsic arguments of a conflict-eliminable set, which are the arguments that would remain if any partial internal conflicts within the set were resolved away; and (3) use the intrinsic arguments to determine which external arguments are being attacked by the conflict-eliminable set, while still keeping the original arguments of the set in order to determine if it is being attacked by external arguments (see Asymmetry in attacks to and from a coalition above). Here again, our task is not just whether some set of arguments satisfies conflict-eliminability, and we must consider if any attacks are strong enough to defeat an argument, and, in case an argument is attacked but not defeated, how much it would be weakened/compromised by the attacks. To cope with these, we attach argument capacity, a numerical value, to each argument, and an attack strength, again a numerical value, to each attack. The idea is: a set of arguments is conflict-eliminable just when none of the members of the set attack an argument of the same set with a greater numerical value than the argument’s capacity. As the argument capacities and attack strengths are both numerical, it is easy to derive its intrinsic arguments and their attacks on external arguments.

1.1 Related work

We are not aware of other works in the literature of abstract argumentation theory dealing with this kind of political-alliance-like coalition formation. Also, to the best of our knowledge, the previous approaches proposed in the abstract argumentation literature are not self-sufficient for dealing with the two above-mentioned technicalities. We have already mentioned the key works on coalition formation [1, 2, 21]. They apply Dung’s acceptability semantics [13] for characterising acceptability of a coalition that is individual-benefit-oriented, that is, conflict-free, and that generally prefers a smaller set. In Section 4 of [1] and in [7], a coalition is associated to a set of tasks/goals, and conflicts between coalitions are measured such as by competition which occurs when two coalitions share the same tasks/goals. We, however, focus on political-alliance-like coalition profitability/formability semantics with conflict-eliminable sets of arguments. We do not consider the meta-knowledge of tasks or goals. Instead, we measure profitibility of a coalition for a conflict-eliminable set of arguments by: (1) the size of the coalition; (2) whether the number of attackers to the conflict-eliminable set of arguments increases or decreases in the coalition; and (3) how defended the coalition is from external arguments. Coalition formability is relativised to profitabilities of two conflict-eliminable sets. A rather different perspective of coalition formation: calculation of probabilistic likelihood of a coalition formation capable of achieving some task, given prior probability of agents’ joining in a coalition and of preventing other members from joining in the coalition, was highlighted in probabilistic argumentation frameworks [19]. Such quantitative judgement is out of the scope of this work.

We mention other works relevant to ours.

Attack-tolerant abstract argumentation Characterisation of acceptability semantics for a non-conflict-free set of arguments is gaining attention. Conflict-tolerant semantics [9] relaxes conflict-freeness by using four values (accept, reject, no opinion, and mixed feeling) for labelling arguments for para-consistent abstract argumentation. However, this approach does not incorporate numerical values, which makes it difficult to reason about the strength of attacks. Weighted argument systems [15] attach numerical values to attack relations. There is also a system-wide numerical value called the inconsistency budget. In their systems, conflicts in a set of arguments are quantified as the sum of numerical values given to the attack relations appearing in the set. If the sum does not exceed the given inconsistency budget, then the set is considered para-conflict-free. Their acceptability semantics is relative to the global inconsistency budget. In our setting, having numerical attack strengths alone is not sufficient, as we must know intrinsic arguments of a conflict-eliminable set, which relies both on attack strengths and on argument strengths. We do not use any global and uniform budget. Further, substantively we do not tolerate any inconsistency: intrinsic arguments must be conflict-free. Social abstract argumentation frameworks [18], to which an equational approach by Gabbay and Rodrigues [16] also relates, attach numerical values to arguments in the form of for votes and against votes. They allow for fine-grained para-consistency. We could potentially adapt their approach for characterisation of our conflict-eliminability. However, their numerical attack characterisations by votes are quite specific. We choose more abstract, axiomatic characterisations. In the literature, axiomatic approaches have been considered to for instance ensure logical consistency of an argumentation framework [20]. Classifications of attack relations by axioms they satisfy have been also done [17]. The axiomatic approaches help regulate an abstract argumentation system from a general standpoint.

Dynamic abstract argumentation From one perspective, our framework possesses a certain kind of dynamic nature. So far dynamic changes have been considered within the literature of abstract argumentation to: assume structural argumentation (e.g. Assumption-based argumentation [14] and modify non-falsifiable facts [22]; add a new argument [19]; revise attack relations [12]; revise an argumentation framework by encoding it into propositional logic [11]; and revise an argumentation framework with an argumentation framework [4]). Given an argumentation framework, these works calculate a revised argumentation framework. That is, they derive a post-state from a pre-state given some input. In our setting, however, we require interactions between the initial set of arguments (the pre-state) and intrinsic arguments of a conflict-eliminable set (post-states) due to the asymmetry in attacks to and from a coalition. The pre-state/post-state coordinations are, as far as we are able to fathom, not dealt with in the above-mentioned studies. Meanwhile, one of the works on coalition formation mentioned earlier, namely [7], allows for coalitional and non-coalitional

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3 This should not be confused. We are not meaning that some arguments would disappear as the result. The internal conflicts are assumed non-defeating. We rather mean that some arguments may be weakened of their potency. See Partial internal conflicts. What would remain are then those arguments unaffected by the partial internal conflicts and those weakened arguments.

4 There are also analysis on modularity of acceptable arguments by adding or removing an argument and/or an attack [5, 6].
views of agents. From a non-coalitional view of agents, a number of coalitional views may be derived. Still, what they consider are conflicts among coalitional views (for goal fulfilment), which is a problem possessing a different nature.

1.2 The remaining sections

In the rest, we will: recall Nielsen-Parsons’ argumentation frameworks [20] that generalise Dung’s ones with group attacks (Section 2); introduce our argumentation frameworks for conflict-eliminable sets of arguments (Section 3); and develop semantics for profitability, and then formibility, of coalition formation, at the same time presenting theoretical results (Section 4), before drawing conclusions. We omit trivial proofs.

2 Preliminaries

While Dung’s argumentation frameworks [13] are the most important in the abstract argumentation literature, Nielsen-Parsons’ generalised versions with group attacks are probably closer to our own. Let us recall the key definitions of their frameworks.

An argument is an abstract entity, and the class of all arguments is \( A \). An argumentation framework is a tuple \((A, G)\) where \( A \subseteq \mathbb{N} \) and \( G : (2^{A}) \setminus \emptyset \times A \). A set \( A_1 \subseteq A \) is said to attack an argument \( a \in A \) if and only if, or simply if, \((A', a) \in G\) for some \( A' \subseteq A \). We say that \((A', a) \in G\) is minimal iff for every \( A'' \subseteq A'\), \((A'', a) \notin G\). A set \( A_1 \subseteq A \) is conflict-free iff there exists no \( a \in A_1 \) such that \( A_1 \) attacks \( a \). A set \( A_1 \subseteq A \) is admissible iff for any \( A_2 \subseteq A' \), \( (A', a) \notin G\) for \( A_2 \subseteq A \) is minimal, then \( A_1 \) attacks some \( a \in A_2 \). A set \( A_1 \subseteq A \) accepts \( a \in A \) iff \((A', a) \in G\) for \( A_2 \subseteq A \) is minimal, then \( A_1 \) attacks some \( a \in A_2 \). A set \( A_1 \subseteq A \) is admissible iff \( A_1 \) accepts all its members. A set \( A_1 \subseteq A \) is a preferred set (extension) iff \( A_1 \) is admissible and there exists no \( A_1 \subseteq A_2 \subseteq A \), such that \( A_2 \) is admissible. There are other notions such as complete sets (extensions), stable sets (extensions), and the grounded set (extension). An interested reader will find more information in [13,20].

The value being 0 would mean that \( S \) is attacked on \( s \). The value being positive means that \( S \) is attacking \( s \).

3 Argumentation Frameworks for Conflict-Eliminable Sets of Arguments

Let \( \mathbb{N} \) be the class of natural numbers including 0, and let \( S \) be \( A \times \mathbb{N} \). We refer to any element of \( S \) by \( s \) with or without a subscript. Each of them represents an argument. Not just any set of arguments will we be interested in, however.

Definition 1 (Coherent sets of arguments) Let \( S_1 \) be a subset of \( S \). We say that \( S_1 \) is coherent iff \( S_1 \) satisfies the following conditions.

1. \( S_1 \) is a finite subset of \( S \).
2. For any \((a,n) \in S_1\), it holds that \( n > 0 \).
3. For any \((a,n) \in S_1\), there is no \( m \neq n \) such that \((a,m) \in S_1\).

An argumentation framework usually satisfies the first condition. To explain the second condition, we mention that each argument has argument capacity. For \((a,n) \in S\), \( a \) is its identifier, and \( n \) is its capacity. An argument having argument capacity of 0 basically means that the argument has no utility. The third property is to ensure that each argument identity is used at most by one argument in a chosen subset of \( S \). From here on, by \( S \) with or without a subscript we denote a coherent set of arguments.

Also let \( R \) be a partial function \( 2^S \times S \rightarrow \mathbb{N} \) such that it satisfies the following conditions (or axioms). Informally \( R(S,s) \) represents the attack strength of \( S \)'s attack on \( s \). In the below, \( 'R' \) is defined for \((S,s)\), \( 'R' \) is synonymous to \( 'R(S,s)' \) is defined.

1. \( R \) is undefined for \((\emptyset, s)\) for any \( s \in S \) [Coherence].
2. For any \( S_1 \subseteq S \subseteq S \), \( S_1 \) is a finite subset of \( S \subseteq S \), and for any \( s \in S \), if \( R \) is defined for \((S_1, s)\), then \( R \) is defined for any \((S_2, s)\) for \( \emptyset \subset S_2 \subseteq S_1 \), [Quasi-closure by subset relation].
3. For any \( S_1, S_2 \subseteq S \subseteq S \), and for any \( s \in S \), if \( R \) is defined both for \((S_1, s)\), and for \((S_2, s)\), then \( R \) is defined also for \((S_1 \cup S_2, s)\) [Closure by set union].
4. For any \( S_1 \subseteq S \subseteq S \) and for any \( s \in S \) such that \( R \) is defined for \((S_1, s)\), it holds that \( R(S_1, s) > 0 \) [Attack with a positive strength].
5. For any \((a,n), (a,m) \in S \) such that \( n < m \), the following holds true: if \( R(S_1, s) \) for some \( s \in S \) and for some \( S_1 \subseteq S \) such that \((a,n) \in S_1 \) is defined, then \( R(S_2, s) \) for \( S_2 = (S_1 \setminus (a,n)) \cup (a,m) \) is defined and is such that \( R(S_1, s) \leq R(S_2, s) \) [Attack relation monotonicity (source)].
6. For any \( S_1, S_2 \subseteq S \subseteq S \), and for any \( s \in S \), if \( R \) is defined for \((S_1, s)\), \((S_2, s)\) and \((S_1 \cap S_2, s)\), then \( R(S_1 \cap S_2, s) \leq R(S_1, s) \) for both \( i = 1 \) and \( i = 2 \) [Attack strength monotonicity (source)].
7. For any \((a,n), (a,m) \in S \) such that \( n < m \), it holds that if \( R \) is defined for \((S_1, (a,n))\) or some \( S_1 \subseteq S \) such that \( S_1 \cap \bigcup_{l \in \mathbb{N}} \{(a,l)\} = \emptyset \), then it is defined for \((S_1, (a,m))\), and, moreover, \( R(S_1, (a,n)) \leq R(S_1, (a,m)) \) [Attack relation and strength monotonicity (target)].
8. For any \( S_1 \subseteq S \subseteq S \) and for any \( s \in S \), \( R \) is undefined for \((S_1, s)\) if \( s \in S_1 \) [No self attacks].

[Coherence] ensures that an attack must come from some argument(s). [Quasi-closure by subset relation] ensures that there is a group attack from a set of arguments on an argument just because each member of the set is attacking the argument. This can be contrasted with the group attacks in Nielsen-Parsons’ argumentation frameworks. But it must be noted, as we are to mention shortly, that there is an attack of an argument \((a_1, n_1)\) on another argument \((a_2, n_2)\) does not mean that \((a_1, n_1)\) defeats \((a_2, n_2)\) in our framework. [Closure by set union] is the reverse of [Quasi-closure by subset relation]. The purpose of [Attack with a positive strength] is as follows: we mentioned earlier that \( R(S_1, s) \) is the strength of attack by \( S_1 \) on \( s \), measured in \( \mathbb{N} \). The value being positive means that \( S_1 \) is indeed attacking \( s \). The value being 0 would mean that \( S_1 \) is not attacking \( s \). The purpose of an attack relation in abstract argumentation frameworks is to know which arguments attack which arguments. Consequently, we only consider positive values for \( R \). [Attack relation monotonicity] expresses the following reasonable property: an attack may be occurring from some \( S_1 \subseteq S \subseteq S \) on some \( s \in S \); now, increase the attack capacity of just one argument \( s_1 \in S_1 \), keeping all else equal; then the attack which occurred before the capacity increase should still occur. [Attack strength monotonicity] expresses the property that if an attack occurs from a set of arguments on an argument with some strength, then any superset does not decrease the attack strength. To explain [Attack relation and strength monotonicity], let us say that a set of arguments is attacking an argument with certain argument capacity. That intuitively means that the set intends to tone down the argument. Now, if the argument capacity of the argument increases, the set still intends to tone down the argument just as strongly or even more strongly, but not less strongly, for there are more materials in the argument that the set could attack. This is the direct reading of the condition. In the technical development to follow, the converse reading will be more useful: if the argument ca-
capacity of the argument on the other hand decreases, the set may no longer intend to tone it down further. Finally, if no self attacks prevent self-contradictory arguments from being present.

Additivity of attack strengths is not postulated for $R$: if an argument $s_1$ is attacked by $s_2$ and $s_3$ such that $R(\{s_2, s_1\}) = n_1$ and such that $R(\{s_3, s_1\}) = n_2$, it is not necessary that $R(\{s_2, s_3, s_1\}) = n_1 + n_2$. The additivity holds good when each attack can be assumed independent, but may not in other cases. A generalised version of [Attack relation monotonicity] holds good. Let $\pi$ be a projection function that takes a natural number and an ordered set $\text{List}$ and that outputs a set member, such that $\pi(n, \text{List}) = \{n\}$, the $n$-th member of $\text{List}$. It is undefined if $n$ is larger than the size of the ordered set.

**Proposition 1 (Generalised attack relation monotonicity)** Let $S_1 \subseteq S$ and $s \in S$ be such that $R(S_1, s)$ is defined. Then if $S_2 \subseteq S$ is such that: (1) $\bigcup_{s_1 \in \text{List}} \pi(1, s_1) = \bigcup_{s_2 \in S_2} \pi(1, s_2)$; and that (2) $(a, n) \in S_1$ materially implies $(a, m) \in S_2$ for $n \leq m$, then $R(S_2, s)$ is defined.

**Proof.** By induction on the number of arguments $s_1 \in S_1$ and $s_2 \in S_2$ for which $\pi(1, s_1) = \pi(2, s_2)$ and $\pi(2, s_1) < \pi(2, s_2)$. Here and everywhere, we may make use of and (having the semantics of classic logic conjunction), distinguishing ‘and’ in natural contexts for greater clarity. The base case is vacuous. Use [Attack relation monotonicity] for inductive cases. □

Our argumentation framework is $(S, R)$ for some coherent set of arguments $S$ and for some $R$.

### 3.1 Attacks
We distinguish complete attacks (defeats) from partial attacks (attacks).

**Definition 2 (Attacks and defeats)** We say that $S_1 \subseteq S$ attacks $s \in S$ iff there exists $S_2 \subseteq S_1$ such that $R$ is defined for $(S_2, s)$. We say that $S_1 \subseteq S$ defeats $s \in S$ iff $S_1$ attacks $s$ and $R(S_1, s) \geq \pi(2, s)$.

Informally, an attack defeats its target when the attack strength surpasses the target’s argument capacity.

**Definition 3 (Maximum attack strengths)** We define $V^{\max}(S_1, s)$ to be: 0 if $S_1$ does not attack $s$; otherwise, $R(S_2, s)$ for some $S_2 \subseteq S_1$ such that: (1) $R$ is defined for $(S_2, s)$; and such that (2) if $R$ is defined for $(S_x, s)$ for $S_x \subseteq S_1$, then $R(S_2, s) \leq R(S_x, s)$.

### 3.2 Conflict-eliminable sets

**Definition 4 (Conflict-eliminable sets of arguments)** We say that $S_1 \subseteq S$ is conflict-eliminable iff there exists no $s \in S_1$ such that $S_1$ defeats $s$.

Conflict-eliminability is a weaker notion of the usual conflict-freeness, i.e. the property that there exists no $s \in S_1$ such that $S_1$ attacks $s$. For the rationale behind obtaining this definition and using it as a primitive entity in our argumentation framework, let us consider one mock example of a peer review which, just as political alliance we saw does, exhibits partial internal conflicts:

1. (Authors) We have addressed a gap in the literature about LMN in this paper. Our approach is entirely new.
2. (A reviewer) My overall impression is that the paper contains sufficiently new and interesting materials to be warranted acceptance. Nonetheless, their claim that the approach considered in the paper is entirely new is misleading. There are for instance papers by XYZ et al. to solve problems ABC. I suggest that they include the references.
3. (The handling editor) I accept the paper on the condition that the authors modify the part mentioned by the reviewer.

Now, if not wholly, the reviewer is still attacking the authors. Despite that, the decisions by the reviewer as well as by the handling editor are overall favourable ones to them. They could take the expert opinion of the reviewer into account, weaken their claim; and then their paper will be accepted. In this example, (1) that the authors have addressed some problem about LMN, (2) that a similar method was used by XYZ et al. to solve problems ABC, and (3) that the paper is accepted, are basically intrinsic arguments of the three arguments given. The authors’ contribution is not completely defeated by the reviewer, and the modification is a necessary compromise for the acceptance of their argument (that is, their paper). Intrinsic arguments are the arguments that result from resolving, in a conflict-eliminable set, internal non-defeating attacks.

**Definition 5 (Intrinsic arguments)** Let $\alpha : 2^S \rightarrow 2^S$ be such that it is defined for $S_1 \subseteq S$ iff $S_1$ is conflict-eliminable. If $\alpha$ is defined for $S_1 \subseteq S$, then we define that $\alpha(S_1) = \{\pi(1, s), n) \mid s \in S_1 \text{ and } n = \pi(2, s) - V^{\max}(S_1, s)\}$. We say that $\alpha(S_1)$ are intrinsic arguments of $S_1$.

**Proposition 2 (Well-definedness)** For any $S_1 \subseteq S$, if $\alpha$ is defined for $S_1$, then every member of $\alpha(S_1)$ is a member of $S$: in particular, there exist $a \in A$ and no $n \in N$ such that $(a, -n) \in \alpha(S_1)$.

Intrinsic arguments of a conflict-eliminable set must be substantively conflict-free.

**Definition 6 (The view of intrinsic arguments)** Let $\text{Del}_I(S, S_1)$ be $\{\langle S_1, s \rangle \mid s \in S_1 \text{ and } S_\text{fin}_1 \subseteq S_x \text{ and } R(S_1, s) \text{ is defined},\}$. which is the set of attack relations within $S_1$. Now, let $S_1$ be a subset of $S$. If $\alpha$ is defined for $S_1$, then we say that $(S_1, S_1) \cup \alpha(S_1), R(\text{Del}_I(S_1, S_1))$ is the view that $S_1$ has about $S$, or simply $S_1$’s view of $S$. We denote $S_1$’s view of $S$ by $\text{View}_I(S, S_1)$.

**Proposition 3** Let $S_1$ be a subset of $S$. If $\alpha(S_1)$ is defined, then $\alpha(S_1)$ is conflict-free in $\text{View}_I(S, S_1)$.

### 3.3 Coalition attacks, c-admissible sets and c-preferred sets

A coalition may attack external arguments only by its intrinsic arguments.

**Definition 7 (C-attacks and c-defeats)** We say that $S_1 \subseteq S$ c-attacks $s \in S$ iff $\alpha$ is defined for $S_1$ and there exists some $S_2 \subseteq \alpha(S_1)$ such that $\pi(2, \text{View}_I(S_1, S_1))$ is defined for $(S_2, s)$. We say that $S_1$ c-defeats $s \in S$ iff $S_1$ c-attacks $s$ and $\pi(2, \text{View}_I(S_1, S_1))(\alpha(S_1), s) \geq \pi(2, s)$.

There are no self c.attacks; Cf. Proposition 5.

**Proposition 4** The following are equivalent.

1. $S_1 \subseteq S$ c-attacks $s \in S$.
2. $S_1 \subseteq S$ c-attacks $s \in \pi(1, \text{View}_I(S, S_1))$.
Also, the following are equivalent.

1. \( S_1 \subseteq S \) c-defeats \( s \in S \).
2. \( S_1 \subseteq S \) c-defeats \( s \in \pi(1, \text{View}_R(S, S_1)) \).

Proof. By definition, \( \pi(2, \text{View}_R(S, S_1))(S_x, s) \) is undefined for any \( S_x \subseteq S \cup \alpha(S_1) \) and for any \( s \in S_1 \cup \alpha(S_1) \). Hence if \( S_1 \subseteq S \) c-attacks \( s \in S \), then \( s \not\in S_1 \) and \( s \not\in \alpha(S_1) \). Meanwhile, \( (S \setminus S_1) = (\pi(1, \text{View}_R(S, S_1))\setminus \alpha(S_1)) \). □

We now define the notions of c-admissible and c-preferred sets, which are analogous to admissible and preferred sets we touched upon in Section 2, but which are for conflict-eliminable sets. One part in the definition could appear difficult at first. We italicise the part, and elaborate it later.

**Definition 8 (C-admissible/c-preferred sets)** We say that \( S_1 \subseteq S \) is c-admissible iff \( \alpha \) is defined for \( S_1 \) and if \( S_2 \subseteq \pi(1, \text{View}_R(S, S_1)) \) attacks \( s \in S_1 \) and if \( S_x \subseteq S_2 \) is such that \( R(S_x, s) \) is defined, then there exists some \( S_x \subseteq \alpha(S_1) \) such that \( S_x \) c-defeats some \( s_x \in S_x \). We say that \( S_1 \subseteq S \) is c-preferred iff \( S_1 \) is c-admissible and there exists no \( S_1 \subseteq S_0 \subseteq S \) such that \( S_0 \) is c-admissible.

We explain why in the italicised part in the above definition it is \( S_2 \subseteq \pi(1, \text{View}_R(S, S_1)) \) and not \( S_2 \subseteq S \); and why it is \( s \in S_1 \) and not \( \alpha(S_1) \). The notion of c-admissibility is intuitively the same as that of admissibility in Section 2: subsets of \( S \) are attacking a conflict-eliminable set \( S_1 \); and so \( S_1 \) is not admissible unless some member of \( S_1 \) defeats all the attacking subsets of \( S_2 \). Now, because we are presuming that \( S_1 \) is conflict-eliminable and not necessarily conflict-free, \( S_2 \subseteq S \) would include any partial conflict in \( S_1 \) as an attack. However, c-admissibility, which is the admissibility of a conflict-eliminable set in the view of the set, should not be defined to defeat it. This explains why \( S_x \subseteq \pi(1, \text{View}_R(S, S_1)) \) which compiles away all those purely internal partial conflicts. On the other hand, the reason that it should be \( s \in S_1 \) and not \( \alpha(S_1) \) is because of the asymmetry in attacks to and from a conflict-eliminable set. Recall Asymmetry in attacks to and from a coalition in Section 1.

### 3.4 Reduction to Nielsen-Parsons' argumentation frameworks

**Theorem 1 (Restricted \((S, R)\) as Nielsen-Parsons')** Let \( R^1 \) be such that it satisfies all but [quasi-closure by subset relation]. Let \( S \) be such that for any \( S \subseteq S \) and for any \( s \in S \), if \( R^1 \) is defined for \((S_1, s)\), then \( S \) defeats \( s \). Then \((S, R^1)\) is Nielsen-Parsons’ argumentation framework, and the following all hold good.

1. Any conflict-eliminable set in \( S \) is a conflict-free set.
2. If \( \alpha \) is defined for \((S_1, s)\), then \( \alpha(S_1) = S \) and \( \pi(1, \text{View}_R(S, S_1)) = S \).
3. A c-attack by \( S_1 \subseteq S \) on \( s \in S \) is an attack by \( S_1 \) on \( s \).
4. A c-attack is a c-defeat.
5. A c-admissible set is an admissible set, and a c-preferred set is a preferred set.

**Proof.**

1. By definition, if \( R^1 \) is defined for \((S_1, s)\), then \( S \) defeats \( s \). Hence it is necessary that a conflict-eliminable set be a conflict-free set.
2. By 1., it is vacuous that \( \alpha(S_1) = S \). Further, Del\((S, S_1)\) = \( \emptyset \), and so \( \pi(1, \text{View}_R(S, S_1)) = S \).

3. By 2., both are trivial.
4. By definition of \((S, R^1)\).
5. By 2., both are trivial.

With these, it is straightforward to see that \((S, R^1)\) is Nielsen-Parsons’ argumentation framework. □

**4 Semantics for coalition profitability and formability**

It is of interest to learn meaningful conflict-eliminable sets of arguments. In this section we show semantic characterisations of coalition formability out of conflict-eliminable sets. Since coalition formation presupposes at least two groups of arguments, what we are characterising is not whether a set of arguments is admissible and how good an admissible set over other admissible sets is, but whether a conflict-eliminable set can form a coalition with another conflict-eliminable set, and how good a coalition over other coalitions is. We presume some conflict-eliminable set at front, and will talk of coalition formability semantics relative to the conflict-eliminable set. We will first discuss profitability relation of a coalition, will show its theoretical properties, and will then present four coalition formability semantics each of which formalises certain utility postulate(s) taking the profitability into account. We assume the following notations.

**Definition 9 (One-directional attacks)** Let \( S_1 \subseteq S \) be such that \( \alpha(S_1) \) is defined. We say that \( S_1 \) is one-directionally attacked iff there exists \( S_2 \subseteq \pi(1, \text{View}_R(S, S_1)) \) such that \( S_2 \) attacks \( s \in S_1 \) and \( S_1 \) does not c-attack any \( s \in S_2 \).

**Definition 10 (States of a conflict-eliminable set)** Let \( S \subseteq S \) be binary relation such that \((S_1, S_2) \subseteq \pi \), written also \( S_1 \preceq S_2 \), iff \( \alpha \) is defined both for \( S_1 \) and \( S_2 \) and any of the three conditions below is satisfied:

1. \( S_2 \) is c-admissible.
2. \( S_1 \) is one-directionally attacked.
3. neither \( S_1 \) nor \( S_2 \) is c-admissible or one-directionally attacked.

Informally if \( S_x \subseteq S \) is c-admissible, then it is fully defended from external attacks and is good. If \( S_x \) is one-directionally attacked, then that means that \( S_x \) does not have any answer to external attacks, which is bad. Any conflict-eliminable set that does not belong to either of them is better than being one-directionally attacked but is worse than being c-admissible. Consequently, if \( S_1 \preceq S_2 \), then \( S_2 \) is in a better state or in at least as good a state as \( S_1 \).

**Definition 11 (Coalition permission)** We say that coalition is permitted between \( S_1 \subseteq S \) and \( S_2 \subseteq S \) iff \( S_1 \cap S_2 = \emptyset \) and \( \alpha \) is defined for \( S_1 \cup S_2 \).

The following results tell that this definition of coalition permission is not underspecified.

**Lemma 1 (Defeats are unresolvable)** Let \( S_1 \) and \( S_2 \) be such that \( S_1 \subseteq S \) and \( s \in S_1 \). If \( S_1 \) defeats \( s \), then for all \( S_2 \) such that \( S_1 \subseteq S_2 \subseteq S \) it holds that \( S_2 \) defeats \( s \).

**Proof.** By [Attack strength monotonicity] of \( R \). □

**Proposition 5 (Coalition and conflict-eliminability)** If coalition is permitted between \( S_1 \) and \( S_2 \), then \( \alpha \) is necessarily defined for \( S_1 \) and \( S_2 \).
Proof. Suppose otherwise, then by conflict-eliminability of a set, there must be an argument in \( S_i \), \( i \in \{1, 2\} \), such that \( S_i \) defeats \( s \in S_i \). The required result obtains via Lemma \ref{lem:1}.

We make one notion formally explicit for convenience, and then define coalition profitability.

**Definition 12 (Attackers)** Let \( \text{Attacker} : 2^S \to 2^S \) be such that \( \text{Attacker}(S_1) = \{ s \in S \mid \text{there exists some } s_1 \in S_1 \text{ such that } s \text{ attacks } s_1 \} \). We say that \( \text{Attacker}(S_1) \) is the set of attackers to \( S_1 \).

**Definition 13 (Coalition profitability)** Let \( \mathcal{S} \subseteq 2^S \times 2^S \) be such that \( S_1 \subseteq S_2 \) (or \( S_1, S_2 \in \mathcal{S} \)) satisfies three axioms below.

1. \( S_1 \subseteq S_2 \) (larger set).
2. \( S_1 \models S_2 \) (better state).
3. \( \{ s \in \text{Attacker}(S_1) \mid S_1 \text{ does not c-defeat } s \text{ and } s \notin S_1 \} \geq \{ s \in \text{Attacker}(S_1) \mid S_2 \text{ does not c-defeat } s \text{ and } s \notin S_2 \} \) (fewer attackers).

Due to (better state), \( S_1 \sqsubseteq S_2 \) implies that \( \alpha \) is defined both for \( S_1 \) and \( S_2 \). By (larger set), a set that contains more arguments is a better set. By (better state), a set that is in a better state is a better set. Finally, by (fewer attackers), a set that is attacked by a smaller number of arguments is a better set. The \( S_1 \) in \( \text{Attacker}(S_1) \) on the second line is not a typo: this criterion is for measuring the profits of coalition formation for \( S_1 \).

**Proposition 6** A \((S, R)\) can be chosen in such a way that a pair of \( S_1 \sqsubseteq S \) and \( S_2 \sqsubseteq S \) do not satisfy more than one axioms.

Proof. (larger set)

Let \( S = \{(a_1, 2), (a_2, 2), s_3\} \), and let \( R \) be such that it is defined only for any combination that matches the attack arrows in the two drawings above. Define that \( R(\{(a_1, 2), (a_2, 2)\}) = R(\{(a_1, 2), (a_2, 2)\}) = R(\{(s_3), (a_1, 2)\}) = R(\{(s_3), (a_2, 2)\}) = (a_1, 2) = 1 \), and that \( R(\{(a_1, 2), s_3\}) \geq \pi(2, s_3) \) (among others implicit by the conditions of \( R \)). Then the left and right drawings represent \((S, R)\) and respectively View\(_R\)(\(S, \{(a_1, 2), (a_2, 2)\}) \). Now, let \( S_1 = \{(a_1, 2)\} \) and let \( S_2 = \{(a_2, 2)\} \). Then clearly \( S_1 \sqsubseteq S_2 \). However, it does not satisfy (better state): \( S_1 \) is neither c-admissible nor one-directionally attacked but \( S_2 \) is one-directionally attacked. It does not satisfy (fewer attackers), either: \( \text{Attacker}(S_1) = \{ s_3 \} \), \( S_1 \) c-defeats \( s_3 \), and \( S_2 \) does not c-defect none.

(better state)

Let \( S = \{s_1, s_2, s_3\} \), and let \( R \) be such that it is defined only mutually for \( s_1 \) and \( s_2 \) as shown above, and such that \( R(\{s_1\}, s_2) = \pi(2, s_2) \) and \( R(\{s_2\}, s_1) \geq \pi(2, s_1) \). Now, let \( S_1 = \{s_1\} \) and let \( S_2 = \{s_2\} \). Then \( S_1 \sqsubseteq S_2 \) because \( S_2 \) is c-admissible. However, clearly it is not the case that \( S_1 \sqsubseteq S_2 \). Also, \( \text{Attacker}(S_1) = \{ s_2 \} \), \( S_1 \) c-defeats \( s_2 \), but \( S_2 \) does not c-defeat it.

**Theorem 2 (Profitable coalition for subsets of a c-admissible set)**

Let \( S_1 \subseteq S \) be such that \( \alpha(S_1) = 1 \), and let \( S_2 \subseteq S \) be a c-admissible set. If \( S_1 \sqsubseteq S_2 \), then the following all hold good.

1. \( \alpha \) is defined for \( S_2 = S \setminus S_1 \).
2. Coalition is permitted between \( S_1 \) and \( S_2 \).
3. \( S_1 \sqsubseteq S_2 \).

Proof. 1. Suppose otherwise, then \( S_2 \) would defeat at least one \( s \in S_2 \) such that \( S_2 \) would defeat \( s \). By Lemma \ref{lem:2}, \( S_2 \) would then defeat \( s \) for \( S_2 \subseteq S_3 \). But \( S_3 \) being c-admissible, does not defeat \( s \).

2. \( \alpha \) is defined for \( S_2 \) by definition; and \( S_1 \cap S_2 = \emptyset \), also by definition.

3. \( S_1 \subseteq S_2 \) is a c-admissible set; and \( \{ s \in \text{Attacker}(S_1) \mid S_2 \text{ does not c-defeat } s \text{ and } s \notin S_2 \} = 0 \).

**Theorem 3 (Existence theorem)**

If, for any \( S_1 \subseteq S \), there exists some \( S_2 \subseteq S \) such that coalition is permitted between \( S_1 \) and \( S_2 \) and \( S_1 \sqsubseteq S_2 \) is c-admissible, then there exists some \( S_3 \subseteq S \) such that coalition is permitted between \( S_1 \) and \( S_3 \) and \( S_1 \sqsubseteq S_3 \) and \( S_3 \sqsubseteq S \) and \( S_3 \) is c-preferred and \( S_2 \subseteq S_3 \).

Proof. As \( S \) is of a finite size, it is straightforward to see that if \( S_1 \sqsubseteq S_2 \) is c-admissible, then there must exist some \( S_3 \) such that it is c-preferred and that \( S_1 \sqsubseteq S_2 \sqsubseteq S_3 \). We show that the choice of \( S \setminus S_1 \) for \( S_3 \) ensures the requirement to be satisfied. By Theorem \ref{thm:4} coalition is permitted between \( S_1 \) and \( S_3 \) and \( S_1 \sqsubseteq S_3 \). By assumption, \( S_1 \sqsubseteq S_3 = S_3 \) is c-preferred and \( S_2 \subseteq S_3 \).

**Theorem 4 (A c-preferred set as a mutually maximal coalition)**

Let \( S_1 \subseteq S \) be such that \( \alpha(S_1) = 1 \), and let \( \text{Prof}(S_1) \) be the set of all c-preferred sets that contain \( S_1 \) as their subset. If \( \text{Prof}(S_1) \neq \emptyset \), then the following holds good: for any \( S_2 \in \text{Prof}(S_1) \), \( S_1 \sqsubseteq S_2 \) and \( (S_1 \setminus S_2) \subseteq S_2 \) and there exists no \( S_2 \subseteq S_2 \subseteq S \) such that \( S_1 \sqsubseteq S_2 \) or such that \( (S_1 \setminus S_2) \subseteq S_2 \).

Proof. By Theorem \ref{thm:4} and by definition of a set being c-preferable.

In general, though, mutual profitability is not guaranteed.

**Theorem 5 (Asymmetry of profitability)** There exists an argumentation framework \((S, R)\) such that there are disjoint subsets: \( S_1 \) and \( S_2 \) of \( S \) such that \( S_1 \sqsubseteq S_1 \sqcup S_2 \), but such that it is not the case that \( S_2 \sqsubseteq S_1 \sqcup S_2 \).
Let $S$ be $\{s_1, (a_2, 2), s_3, s_4, s_5\}$, and let $R$ be such that it is defined only for any combination that matches the attack arrows in the drawings above, and such that:

$R(\{s_1\}, (a_2, 2)) = 1$; 
$R(\{s_1, (a_2, 1)\}, s_3) \geq \pi(2, s_3)$; 
$R(\{s_1\}, s_5) < \pi(2, s_5)$; 
$R(\{a_2\}, (a_2, 2)) = 1$; 
$R(\{a_2\} \cup \{s_1\}, (a_2, 1)) \geq \pi(2, s_5)$; 
$R(\{(a_2, 1), s_3\}) \geq \pi(2, s_5)$; 
$R(\{(a_2, 1), s_3\} \cup \{s_1\}, (a_2, 1)) \geq 2$; and 
$R(\{s_5\}, (a_2, 1)) \geq 2$.

Moreover, $S$ does not satisfy what we may at first expect hold good.

**Definition 14 (Continuation property of $\preceq$)** Let $S_1 \subseteq S$ be such that $a(S_1)$ is defined, and let $\Max(S_1)$ be the set of all $S_2 \subseteq S$ such that $S_1 \preceq S_2$, and such that if $S_2 \subseteq S_1 \subseteq S$, then it is not the case that $S_1 \preceq S_2$. Then we say that $S_1$ is weakly continuous for $S_2$ if there exists some $S_1 \in \Max(S_1)$ such that, for any $S_2 \subseteq S_1$, if coalition is permitted between $S_1$ and $S_2 \setminus S_1$, then $S_1 \not\preceq S_2$. We say that $S_1$ is continuous for $S_2$ if it is weakly continuous for $S_2$.

**Theorem 6 (Coalition profitability discontinuity theorem)**
There exist $S_1, S_2, S_3 \subseteq S$ such that: (1) $a(S_1)$ is defined for $S_1$ and $S_2$; (2) $S_2 \not\subset S_1$; (3) $S_3 \subseteq S_2$, but such that $S_1 \not\preceq S_2$ does not hold. Moreover, $\Max(a(S_1))$ would not change this result.

**Proof.** It suffices to consider the case where $\Max(a(S_1)) = \Pref(S_1)$.

Let $S$ be $\{(a_1, 2), (a_2, 2), s_3, s_4, s_5\}$, and let $R$ be such that it is defined only for any combination that matches the attack arrows in the drawings above, and such that:

$R(\{(a_1, 2), s_3\}) \geq \pi(2, s_3)$; 
$R(\{(a_2, 2), s_3\}) \geq \pi(2, s_3)$; 
$R(\{(a_1, 2), (a_2, 2)\}) = 1$; 
$R(\{(a_2, 2), (a_1, 2)\}) = 1$; 
$R(\{(a_2, 1), s_3\}) \geq \pi(2, s_3)$; 
$R(\{(a_2, 1), s_3\} \cup \{s_1\}, (a_2, 1)) \geq 2$; and 
$\Max(a(S_1)) \subseteq S_2 \subseteq S_1 \cup S_2$, and $S_2 = \{(a_1, 2), (a_2, 2), s_3\}$.

Further, $S_2$ is c-preferable. However, it is not the case that $S_2 \preceq S$. In fact, it is also not the case that $S_2 \preceq S_2$, since $S_2$ is one-directionally attacked (see the right drawing for the attacks in $\View_R(S, \{(a_1, 2), (a_2, 2)\})$, whereas $S_1$ and $S_2$ are neither c-admissible nor one-directionally attacked (see the left drawing). \hfill $\Box$

Coalition profitability continuation property satisfies in certain special cases, however.

**Theorem 7 (Coalition profitability continuation theorem)**
Let $S_1 \subseteq S$ be a c-preferred set. Then $\preceq$ is weakly continuous for any $S_1 \subseteq S$ iff any disjoint pair $S_1, S_2$ of subsets of $S$ satisfy $S_2 \not\subset S_1 \cup S_2$.

**Proof.** Suppose, by way of showing contradiction, there are three disjoint subsets of $S$: $S_1, S_2$ and $S_3$, such that $S_1 \subseteq S_2 \subseteq S_1 \cup S_3$ and $S_1 \cup S_3$. By assumption we have $S_1 \subseteq S_1 \cup S_3$, contradiction.

**Only if:** By definition of weak continuation property of $\preceq$. \hfill $\Box$

### 4.2 Coalition formability semantics

We use the profitability relation to express our coalition formability semantics. We set forth three rational utility postulates:

I. Coalition is good when it is profitable at least to one party.

II. Coalition is good when it is profitable to both parties.

III. Coalition is good when maximal potential future profits are expected from it.

Of these, the first two can be understood immediately with the profitability relation. Our interpretation for the last postulate is as follows. Suppose a party, some conflict-eliminable set $S_1 \subseteq S$ in our context, considers coalition formation with another conflict-eliminable set $S_2$. We know that $S_2$ is some subset of $S_1 \subseteq S \setminus S_1$. Before $S_1$ forms a coalition with $S_2$, we have $\Max(S_1)$ as the set of maximal coalitions possible for $S_1$. Once the coalition is formed, we have $\Max(S_1 \cup S_2)$ as the set of maximal coalitions possible for the coalition. Here clearly $\Max(S_1 \cup S_2) \subseteq \Max(S_1)$. What this means is that a particular choice of $S_2$ blocks any possibilities in $\Max(S_1) \setminus \Max(S_1 \cup S_2)$; they become unrealisable from $S_1 \cup S_2$. Hence $S_1$ has an incentive not to form a coalition with such a $S_2$ if all the members of $\Max(S_1 \cup S_2)$ are strictly and comparatively less profitable than some member of $\Max(S_1)$. We reflect this intuition.

**Definition 15 (Maximal profitability relation)**
Let $\leq, \leq_b, \leq_f$ be $2^S \times 2^S$ such that they satisfy all the following:

1. $S_1 \leq S_2$ iff $|S_1| \leq |S_2|$.
2. $S_1 \leq_b S_2$ iff $S_2$ is at least as good (by better state) as $S_1$.
3. $S_1 \leq_f S_2$ iff $S_2$ is at least as good (by weaker attackers) as $S_1$.

We write $S_1 \preceq \preceq_b S_2$ for each $\beta \in \{b, f\}$ just when it is the case that $S_1 \preceq S_2$ and it is not the case that $S_1 \preceq \beta S_2$. Then we define $\preceq_m: 2^S \times 2^S$ to be such that if $S_1 \preceq_m S_2$, then both of the following conditions satisfy:

1. $S_1 \preceq S_2$.
2. Some $S_3 \in \Max(S_2)$ is such that, for all $S_6 \in \Max(S_1)$, if $S_6 \preceq \beta S_2$ for some $\beta \in \{b, f\}$, then there exists $\gamma \in \{\{b, f\} \setminus \beta\}$ such that $S_6 \preceq \gamma S_3$.

Intuitively, if $S_1 \preceq_m S_2$, then at least one set in $\Max(S_1)$ maximal in the three criteria: the set size, the state quality and the number of external attackers, is reachable from $S_2$. The maximality here is judged...
strictly better in $\mathcal{S}_m$ than $\{s_1, (a_2, 2), s_6\}$; and (2) for $\{s_1, s_6\}$, $\{s_6, s_4, s_5, s_6\}$ is strictly better in $\mathcal{S}_m$ than $\{s_1, (a_2, 2), s_6\}$. Similarly $\{s_6\}$ is excluded. Finally, in this particular example, both $\{s_1, (a_2, 2), s_7\}$ and $\{(a_2, 2), (a_3, 2), s_7\}$ are c-preferred, and, moreover, we have: $\{(a_3, 2)\} \subseteq \mathcal{S}_m \subseteq \{(a_2, 2), (a_3, 2), s_7\}$; $\{s_7\} \subseteq \mathcal{S}_m \subseteq \{s_1, (a_2, 2), s_7\}$; and $\{s_7\} \subseteq \mathcal{S}_m \subseteq \{(a_2, 2), (a_3, 2), s_7\}$. By these, together with the subsumption of $\mathcal{S}((a_2, 2))$ in $\mathcal{W}((a_2, 2))$ (Theorem 8), the last equality for $\mathcal{S}((a_2, 2))$ follows.

5 Conclusion

We proposed abstract-argumentation-theoretic coalition profitability and formability semantics for conflict-eliminable sets. Further, we showed that theorems obtained from these semantics. We also showed how our framework can be reduced to Nielsen-Parsons’ argumentation framework. Our work has a connection to several important subfields of abstract argumentation theory such as postulate-based abstract argumentation, attack-tolerant abstract argumentation and dynamic abstract argumentation. It is our hope that this study will further aid in linking the rich knowledge that is being accumulated in the literature.

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