Investigation of the Relativity Errors in Global Positioning System

Parichehr Mesri Alamdari*
Department of Geography, Payame Noor University, Iran; mesri@mailfa.com

Abstract
Objectives: To study the relativity errors in global positioning systems which is very important for computing the position of objects in Earth. Methods: In this research paper by using the physics formalism, relativity errors have arisen, and we estimate the extent of the errors. Findings: The results show that in addition to these errors, the need for precise positioning in GPS, dispersion of the chromatic in the ionospheric layer, refraction in the troposphere layer and the effects of Earth’s rotation should be considered. Also, the gravitational frequency changes the relative quantity. Application: According to the GPS applications in military science, agribusiness, and sea investigation, the results of article can enhance the better accuracy of GPS systems.

Keywords: Gravitational Frequency, Relativity, Satellite, Troposphere Layer, GPS

1. Introduction
GPS or Global Positioning System is a system of circling satellites that send exact subtle elements of their position in space back to earth. The signals are gotten by GPS collectors, for example, route gadgets and are utilized to Figure the correct position, speed and time at the vehicle’s area (Figure 1). GPS is outstanding for its military uses and was first created by the US to help in its worldwide insight endeavors at the stature of the Cold War1–3.

As far back as the mid-1980s, notwithstanding, the GPS has been openly accessed to anybody with a GPS beneficiary. Aircrafts, shipping organizations, trucking firms, and drivers wherever utilize the GPS framework to track vehicles, take over the best course to get them from A to B in the most limited conceivable time. The first GPS framework was produced in the 1960s to enable ships in the US Navy to explore the seas all the more precisely4. The principal framework had five satellites and enabled boats to check their area once consistently. Today, convenient Navigation gadgets can give drivers their exact area to inside a couple meters, which is sufficiently precise to explore roadways. Military applications have significantly higher accuracy so that an area can be pinpointed inside a couple of centimeters. The US NAVSTAR Global Positioning System (GPS) is the main completely operational Global Navigation Satellite System (GNSS) as of now furnishing situating information with worldwide scope. The European Union is presently building up its own GPS known as the Galileo situating framework, which will be operational by 2013. China has a neighborhood framework it might grow all inclusive, while Russia is as of now reestablishing its GLONASS framework. GPS satellite route to decide the time and area of the question, or at the end of the day route framework that utilizations satellites to quantify the position5–8.

The GPS applications in military science, agribusiness, and sea investigation there. GPS rule is extremely straightforward. The satellite sends electromagnetic waves; the beneficiary gets these waves on Earth. The relative separation between the satellite and the collector in light of data gotten from the recipient Figures the waves. This measure

*Author for correspondence
is known as measuring improbable, on the grounds that the estimation mistakes that happen because of different components. So are measured must be amended.

2. Theory

Assume that the satellite coordinates with respect to the ground on points \((x_i,y_i,z_i)\) located at the time \(t_i\) sends radio signals. Recipient due to land in location coordinates \((x, y, z)\), and the radio signal received at time \(t\). The distance between the satellite and the receiver will be as follows:

Radio signals sent by the satellites contain information on where and when the satellite is. Theoretically, if we ignore measurement errors can be determined by solving the equation 1 recipient’s location. The equation 1 should be reviewed, as should the types of errors that arise due to various factors, should be considered. Errors that arise due to special relativity

\[
\chi = \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} = c(t - t_i) \tag{1}
\]

In Equation 1, \((x_i,y_i,z_i)\) can be obtained according to radio signals, while \(t_i\) measured by receiving the instantaneous signal from the satellite. Time is measured by the satellite radio signal, is measured according to this information. Time and quantitatively different. This slight difference is caused by the special theory of relativity and should be considered. Errors that arise due to special relativity

\[
\Delta = \frac{(t - t_i)}{\sqrt{1 - \frac{v_i^2}{c^2}}} \tag{2}
\]

\[
\Delta \text{ is the time interval according to the satellite clock time while on the ground, according to the Earth Hour. If the satellite radio signal when the clocks to be synchronized with each other, with respect to the time when the radio signal reaches the receiver, will be as follows:}
\]

\[
t_i = \frac{t_i'}{\sqrt{1 - \frac{v_i^2}{c^2}}} \tag{3}
\]

According to equation 1, we have:

\[
(t - t_i) = (t - t_i') - t'\frac{1 - \sqrt{1 - \beta^2}}{\sqrt{1 - \beta^2}} \tag{4}
\]

In Equation 4, is. We rewrite the equation 4 as follows:

\[
\chi = \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} + c t' \frac{1 - \sqrt{1 - \beta^2}}{\sqrt{1 - \beta^2}} \tag{5}
\]

Due to the location of the recipient (5) can be calculated. Dispute by special relativity. Since the \(\beta^2\) is very little, the speed of satellite around the Earth is approximately 4,000 and the speed of light is \(3 \times 10^8 \text{ m/s}\). Therefore, the difference will be in the following locations: \(\beta^2 = 9 \times 10^{-11}\). No matter how wide the gap is less accurate positioning system. In the most advanced military GPS location difference in the range of a few centimeters. So clearly these differences cannot be ignored. Differences that arise due to general relativity: According to general relativity can be a force applied by the earth’s curvature of space-time around Earth and can be easily justified. The curvature can be given according to the metric scale. Since the metric scale space for the satellite and the receiver are not the same, the difference measures the distance between the satellite and the recipient arise. Change the frequency difference of gravity according to the theory of general relativity, gravitational field around the Earth called the Schwarzschild field. Schwarzschild metric scale in spherical coordinates is as follows:

\[
dt^2 = -c^2(1 - \frac{2Gm}{c^2r})dt^2 + (1 - \frac{2Gm}{c^2r})^{-1}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \tag{6}
\]

In \(d\tau = \sqrt{-g_{00}}dt\) that \(d\tau\) inherent time. At the same time, inherent in the calculation is evaluated. Time recip-
ient inherent in space 1, and space 2, and the relationship between the two is as follows: \( \Delta \tau = \sqrt{\frac{2Gm}{c^2r_1}} \), that \( \Delta \tau = \sqrt{\frac{2Gm}{c^2r_2}} \) is the time scale of the Schwarzschild metric, \( G \) is the gravitational constant, \( m \) the mass of the Earth, \( c \) speed of light, and \( r \) is the distance between the Earth and the object. If the satellite receiver in the 1 and 2 in space and time according to the time period according to the satellite and receiver clock is \( \Delta \tau \). The relationship between these two-time periods will be as follows: \( \Delta \tau = \sqrt{\frac{2Gm}{c^2r_1}} \Delta \tau ' \)

Since the satellites relative velocity \( V \) (according to the coordinates of the earth) moves: \( \Delta \tau = \frac{\Delta \tau '}{c} \) So the equation 9 to equation becomes:

\[
\Delta \tau = \sqrt{\frac{1 - \frac{2Gm}{c^2r_1}}{\frac{1}{c^2} - \frac{v^2}{c^2}}} \Delta \tau ' \quad \text{(7)}
\]

\( r_1 \) and \( r_2 \) satellite locations and recipient are not. If the recipient is on the ground and the satellite at a height \( h \) above the ground, then \( r_1 = R+h \) and \( r_2 = R \). Equation 11 can be written as follows:

\[
V = \sqrt{\frac{1 - \frac{2Gm}{c^2r_1}}{\frac{1}{c^2} - \frac{v^2}{c^2}}} \quad \text{(8)}
\]

\[
\Delta V = |V - v| = \sqrt{\frac{1 - \frac{2Gm}{c^2r_1}}{\frac{1}{c^2} - \frac{v^2}{c^2}}} - v' \quad \text{(9)}
\]

### 3. Results

Gravitational frequency change relative quantity is calculated as follows:

\[
\Delta V = |V - v| = \sqrt{\frac{1 - \frac{2Gm}{c^2r_1}}{\frac{1}{c^2} - \frac{v^2}{c^2}}} - v' \quad \text{(10)}
\]

Since the equations \( \frac{v^2 2Gm}{c^2r^2} \) are extremely small. According to Taylor series expansion and we are the second to the next, regardless of the series. So, we have:

\[
E \approx \left( 1 - \frac{Gm}{c^2r_2} \right) \left( 1 - \frac{v^2}{2c^2} \right) - \frac{Gm}{c^2r_1} \frac{v^2}{2c^2} \quad \text{(11)}
\]

Here is the thing that a diagram would look like contrasting them for a 2% unconventionality (\( e=0.02 \)) and increasing by \( c \) to change over to position mistake (Figure 2). As can be seen, the forecast of relativity gives a most extreme of 13.73m, while the expectation because of added substance speed gives a greatest of 6.86m. So doubtless added substance speeds can foresee a large portion of the deliberate blunder. Yet, since the beneficiaries are customized to oblige for twofold that sum, does that mean the added substance speed thought isn't right and relativity is right.

![Figure 2. Prediction of errors in GPS.](image)

### 4. Conclusion

Satellite altitude of approximately 20200 km above the Earth, Earth 1024 × 976/5 kg, the radius of 4371 km, the speed of the satellite around the Earth, the speed of light and gravity are constant. Under these conditions, we have: \( E \approx 4.5 \times 10^{-10} \). So, it can be concluded that this difference is not negligible.

### 5. References

1. Ahamed SF, Rao GS, Venkatesh SS. A novel approach to acquire GPS signal in the presence of CWI using DWT. Indian Journal of Science and Technology. 2016 Dec; 9(51):1–5. DOI: 10.17485/ijst/2016/v9i51/107822

2. Kumari R, Mukhopadhyay M. A FPGA software based GPS receiver implementation with signal blocker through simulink. Indian Journal of Science and Technology. 2016 Jul; 9(25):1–5. Crossref
3. Amin MS, Reaz MBI, Nasir SS, Bhuiyan MAS. Low cost GPS/IMU integrated accident detection and location system. Indian Journal of Science and Technology. 2016 Mar; 9(10):1–9. Crossref

4. Kumar V. Effect of environmental parameters on GSM and GPS. Indian Journal of Science and Technology. 2014 Aug; 7(8):1183–8.

5. Kumar JP, Rarotra N, Maheswari U. Design and Implementation of Kalman Filter for GPS receivers. Indian Journal of Science and Technology. 2015 Oct; 8(25):1–5. DOI: 10.17485/ijst/2015/v8i25/80158

6. Daniel TO, Hussein MN, Abdulla R. Localization using GPS coordinates in IPv6 addresses of wireless sensor network nodes. Indian Journal of Science and Technology. 2016 Mar; 9(10):1–8. Crossref

7. Ibrayev A, Ibrayeva A, Akhmedov D. Development of methods and algorithms for improving accuracy of integrated INS/GPS systems for vehicles. Indian Journal of Science and Technology. 2016 Nov; 9(44):1–11. Crossref

8. Zakaria S, Razak NA, Baharudin ZA. Gravitational wave & relativity impact electronic communication & engineering. In MATEC Web of Conferences. 2017; 97:1–5. Crossref