RELATIVISTIC QUANTUM KINETIC EQUATION OF THE VLASOV TYPE 
FOR SYSTEMS WITH INTERNAL DEGREES OF FREEDOM

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Abstract

We present an approach to derive a relativistic kinetic equation of the Vlasov type. Our approach is especially reliable for the description of quantum field systems with many internal degrees of freedom. The method is based on the Heisenberg picture and leads to a kinetic equation which fulfills the conservation laws. We apply the approach to the standard Walecka Lagrangian and an effective chiral Lagrangian.

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1. Introduction

Experiments at CERN and BNL are designed to investigate the formation and decay of a quark gluon plasma in ultra-relativistic heavy-ion collisions. The very early stage of such a collision is dominated by non-equilibrium effects. The expected quark gluon plasma undergoes a transition to strong interacting and highly correlated hadronic matter if the system cools down and becomes dilute. The description of such a dynamical evolution of a very complex, relativistic system with many internal degrees of freedom is one of the most challenging problems in modern particle and non-equilibrium physics.

Relativistic kinetic theory and hydrodynamics have been successfully employed for the description of non-equilibrium states of matter at high density and temperature. These methods have been most extensively applied in relativistic nuclear physics both in the intermediate energy region, where the quark-gluon degrees of freedom are still not relevant, and in the high energy domain of a non-equilibrium quark-gluon plasma. In the construction of the relativistic kinetic theory, different approaches have been used. Within the contour Green function technique, relativistic transport equations of the Vlasov type have been developed by Kadanoff and Baym, Martin and Schwinger, and Keldysh. That approach has been studied formally by a large number of authors, and others have explored it in quantitative model studies.

However, the theory can be developed further. The derivation of a relativistic transport equation that holds for systems with many internal degrees of freedom as well as the inclusion of collision and particle production source terms are of particular interest.

We investigate an approach based on the non-equilibrium statistical operator in Heisenberg dynamics that can be considered as a generalization of the Zubarev method to the relativistic domain. We restrict ourselves to the mean field approximation but take into account the internal degrees of freedom of the constituents.

For demonstration we use two different models. The Walecka model as the standard version of quantum hadrodynamics serves for the description of hadronic degrees of freedom of relativistic nuclear matter for laboratory energies less than a few GeV/A. A further application is a chiral Lagrangian that is of particular interest because the concept of chiral symmetry breaking at large temperatures and density is supposed to be the driving force for the hadronisation of matter. Therefore microscopic quark models, such as the Nambu-Jona-Lasinio (NJL) model, may be useful for the dynamical description of the expected quark-hadron phase transition.

The paper is organized as follows. In Section 2 we present an overview of the method. In Sections 3 and 4 we demonstrate the derivation of the relativistic kinetic equation of the Vlasov type using the Walecka model and the NJL model, respectively. We summarize the results in Section 5.

2. Equation of motion for the Wigner function

In quantum field theory the equations of motion within the Heisenberg picture are given as

\[ i\hbar \partial_\tau A(x) = [A(x), P^\mu] , \]

where \( A(x) \) is an arbitrary local operator and \( P^\mu \) is the total 4-momentum of the system,

\[ P^\mu = \int d\sigma^\nu(n)T^{\mu\nu} . \]

Here \( d\sigma^\nu(n) \) denotes a vector element of an arbitrary space-like hyperplane with a time-like
normal vector $n^\mu$ ($n^2 = 1$), $T^{\mu\nu}(x)$ is the energy-momentum tensor. For the fermionic sector the energy-momentum tensor $T^{\mu\nu}$ reads

$$T^{\mu\nu} = \frac{i}{4} \{ \bar{\psi} \gamma^\nu \partial^\mu \psi + \bar{\psi} \gamma^\mu \partial^\nu \psi \} - g^{\mu\nu} L(x). \tag{3}$$

The interaction is not specified yet and will be considered in the Walecka model (Section 3) and the NJL model (Section 4).

Let us now consider the situation when the time-like direction in Minkowsky space is determined by an external condition. We suppose in the following that this direction is defined by the unit vector $n^\mu$ which fixes simultaneously the orientation of the hyperplane $\sigma(n)$ in the formulae (2) and (11). For the description of the dynamics of the system along the time-like direction $n^\mu$ and along the independent space-like directions on the hyperplane $\sigma(n)$, the boost transformation with "velocity" $n^\mu$ can be used for the equations of motion (1). Technically, it is achieved with the help of the projection of Eqs. (1) on the direction $n^\mu$ and on the hyperplane $\sigma(n)$. Convolution of (1) with $n^\mu$ leads to the dynamical equation

$$i \frac{\partial A(x)}{\partial \tau} = [A(x), H(\tau)]. \tag{4}$$

The parameter

$$\tau = n_\mu x^\mu \tag{5}$$

plays the role of a proper time in a new coordinate system. The derivative along the direction of $n^\mu$ in Eq.(1) is given as

$$\frac{\partial}{\partial \tau} = n_\mu \frac{\partial}{\partial x_\mu}. \tag{6}$$

$H(\tau)$ is the Hamiltonian of the system

$$H(\tau) = n_\mu P^\mu = \int d\sigma(n)n_\mu T^{\mu\nu}n_\nu. \tag{7}$$

The remainder equations

$$i \frac{\partial}{\partial x_\mu} A(x) = [A(x), \Pi^\mu(\tau)] \tag{8}$$

intend for a description of infinitesimal transfers of a system on the space-like hyperplane $\sigma(n)$. The space-like derivative is defined by the relation

$$\frac{\partial}{\partial x_\mu} = \Delta_\nu^\mu \frac{\partial}{\partial x_\nu}, \tag{9}$$

where $\Delta^{\mu\nu} = g^{\mu\nu} - n^\mu n^\nu$ is the projection operator, $\Delta^{\mu\nu} n_\nu = n_\mu \Delta^{\mu\nu} = 0$. And finally,

$$\Pi^\mu(\tau) = \Delta_\nu^\mu P^\nu = \int d\sigma(n)n_\lambda T^{\lambda\nu} \Delta_\nu^\mu \tag{10}$$

is the space-like momentum vector of the system. $H(\tau)$ and $\Pi^\mu(\tau)$ are the generators of local infinitesimal normal and tangential diffeomorphisms to a hyperplane of a constant proper time. Note that a similar covariant decomposition of the generator of a motion group can be found by the quantization of the Einstein gravitation theory.
In order to derive the drift integral of the relativistic kinetic equation it is convenient to introduce the anti-commutation relations for fermionic fields:

\[
\{ \psi(x), \bar{\psi}(x') \} = -iS(x-x'), \quad \{ \psi(x), \bar{\psi}(x') \} = \{ \bar{\psi}(x), \psi(x') \} = 0 \tag{11}
\]

for any points \(x^\mu\) and \(x'^\mu\) which belong to an arbitrary space-like hyperplane \(\sigma\). Let us note the following property of the commutation function of spinor fields

\[
S(x) |_{\mu=0}^{\alpha=0} = i\gamma^0 \delta^{(3)}(x), \tag{12}
\]

and

\[
\int d\sigma(n|x) S_{\alpha\beta}(x-x') y(x') = i n_\mu \gamma^\mu_{\alpha\beta} y(x), \quad x \in \sigma(n), \tag{13}
\]

where \(y(x)\) is an arbitrary function of field operators and the spin is denoted by the indices \(\alpha\) and \(\beta\).

The standard definition of the one-particle covariant Wigner function of the Fermi subsystem reads

\[
f_{\alpha\beta}(x,p) = \int dy e^{-ipy} < P_{\alpha\beta}(x,y) >, \tag{14}
\]

where \(< ... > = Tr \{ ... \} \) denotes the operation of statistical averaging with the single particle density matrix \(\rho\) in mean field approximation within the Heisenberg representation with

\[
P_{\alpha\beta}(x,y) = \bar{\psi}_\beta(x+y/2)\psi_\alpha(x-y/2). \tag{15}
\]

Performing the \(\tau\) - differentiation of the Wigner function \(\tag{14}\) and applying the Liouville equation \(dp/d\tau = 0\) we find:

\[
\frac{\partial f^{\alpha,\beta}(x,p)}{\partial \tau} = n^\mu \frac{\partial f^{\alpha,\beta}(x,p)}{\partial x^\mu} = \int dy e^{-ipy} < \frac{\partial}{\partial \tau} P^{\alpha,\beta}(x,y) >. \tag{16}
\]

The substitution of the equation of motion \(\tag{4}\) leads to the relation

\[
n^\mu \frac{\partial}{\partial x^\mu} f^{\alpha,\beta}(x,p) = -i \int dy e^{-ipy} < [P^{\alpha,\beta}(x,y), H(\tau)] >. \tag{17}
\]

Let us assume that the vector \(p^\mu\) is time-like and hence can fix the corresponding direction in the Minkowsky space, i.e.

\[
n^\mu \overset{def}{=} u^\mu = p^\mu / \sqrt{p^2}, \quad u^2 = 1. \tag{18}
\]

Substitution of relation \(\tag{18}\) into Eq. \(\tag{17}\) leads to

\[
p_\mu \partial^\mu f^{\alpha,\beta}(x,p) = -i \sqrt{p^2} \int dy e^{-ipy} < [P^{\alpha,\beta}(x,y), H(\tau)] >. \tag{19}
\]

The right-hand side of this equation is the Vlasov-like drift integral. This general result for the mean field approximation holds for different internal degrees of freedom (spin, isospin, color, flavor ...). In the following we will drop the spinor indices \(\alpha, \beta\) in the notation.

3. Application of the method to the Walecka model
3.1. General structure of the transport equation

The simplest version of quantum hadrodynamics is the Walecka model of relativistic nuclear matter. The Lagrange density for the nucleon (ψ), the neutral scalar (φ) and the vector (ων) mesonic fields reads:

\[ \mathcal{L}(x) = i \bar{\psi} \gamma^\mu \partial_\mu \psi - M \bar{\psi} \psi + \bar{\psi} (g_s \phi - g_v \omega_\mu \gamma^\mu) \psi + \frac{1}{2} \{ \partial_\mu \phi \partial^\mu \phi - m_s^2 \phi^2 \} - \frac{1}{4} F_{\mu\nu}^\mu F_{\mu\nu}^\nu + \frac{1}{2} m_\omega^2 \omega^2, \]  

(20)

where \( M \), \( m_s \), \( m_\omega \) are the masses of the nucleon, scalar and vector mesons, respectively; \( g_s \), \( g_v \) are the coupling constants, \( F_{\mu\nu}^\mu = \partial^\mu \omega^\nu - \partial^\nu \omega^\mu \). The meson fields are approximated by their mean fields, given by \( \phi = \langle \phi \rangle \) and \( \omega_\mu = \langle \omega_\mu \rangle \) where the symbol \( \langle ... \rangle = \text{Tr} \{ ... \} \) denotes the procedure of statistical averaging with the density matrix \( \rho \) in the Heisenberg picture.

A concrete definition of the drift integral is feasible only by specifying the Hamiltonian of the system. In the considered model, the Hamiltonian (21) is a bilinear combination of the field operators \( \psi(x) \) and \( \bar{\psi}(x) \), consequently the truncation problem does not arise here. For the Walecka model with the energy-momentum tensor (8), we obtain

\[ H(\tau) = -\int d\sigma(n) \bar{\psi} \{ \frac{i}{2} \gamma^\mu \partial_\mu - M^* - g_v \gamma^\mu \omega_\mu \} \psi, \]  

(21)

\[ \Pi^\mu(\tau) = \frac{i}{4} \int d\sigma(n) \bar{\psi} \{ \gamma^\nu \partial^\nu + \gamma^\lambda \partial^\lambda \} \psi \Delta^\mu_n, \]  

(22)

where

\[ M^* = M - g_v \phi \]

is the effective nucleon mass in mean field approximation.

The intermediate calculations of the drift integral in the Walecka model are shown in Appendix A. As it ought to be, the resulting relativistic kinetic equation of the Vlasov type turns out to be of non-local (and non-Markovian) type. The restriction to minimal orders of the gradient expansions allows us to write the relativistic Vlasov equation in a local form,

\[ P_\mu \partial^\mu \bar{f}(x, P) + \frac{1}{2} \partial_\mu M^* \{ P_\mu, \partial^\rho f(x, P) \} + g_v P_\mu F_{\mu\nu} \partial^\nu f(x, P) + \frac{1}{2} [P_\mu, \partial^\rho f(x, P)] - \frac{1}{2} g_v F_{\mu\nu} [P_\mu, \partial^\rho f(x, P)] = 0, \]  

(23)

where \( P = p - g_v \omega \) is the kinetic momentum. The derivation of this kinetic equation is reviewed. Although the model is not gauge-invariant, this property emerges when \( m_\omega \to 0 \), therefore in the Walecka model it is convenient to use the gauge-invariant generalization of the Wigner function (23). In the case of small mesonic field gradients this generalization can be done by the replacement \( p \to P = p - g_v \omega \) in (23), this is fulfilled in (23).

The suggested method of the derivation of the relativistic kinetic equation of the Vlasov type does not lead to the mass shell condition itself. However the space-like Eqs. of motion (23) generate also a set of equations for the Wigner function. Indeed, according to Eq. (8), the result of the operation \( \partial^\mu_\mu(x) \) on the Wigner function (24) leads to the equation:

\[ \frac{\partial}{\partial x^\mu} f(x, P) = -i \int dy e^{-ipy} \langle P(x, y), \Pi^\mu(\tau) \rangle. \]  

(24)

The absence of an additional restriction to the Wigner function is not obvious in general. However, it is easy to show that these equations do not contain new information in the mean...
field approximation of the Walecka model. The operator $\Pi^\mu(\tau)$ is defined by relation \( (22) \) which does not contain the mass and the mean field dependencies and, consequently, the Eq. \( (24) \) cannot result in the mass shell condition. This conclusion is confirmed by direct calculations of the right part of Eq. \( (23) \) with the operator \( (22) \).

\[
\frac{\partial}{\partial x^\mu} f(x, P) + \frac{1}{2} u_\nu [\gamma^\mu \gamma^\nu, u_\lambda \partial_\lambda f(x, P)] + i(p_\mu u^\nu)(2u^\mu f(x, P) - u_\nu \{\gamma^\mu \gamma^\nu, f(x, P)\}) = 0. \tag{25}
\]

One could assume that the mass shell condition is contained inside of the kinetic equation \( (23) \). However it will be shown in the following section that this assumption can’t be confirmed.

### 3.2. Dirac decomposition and properties of the relativistic Vlasov equation

In order to verify the physical consistency of the relativistic kinetic equation of the Vlasov type \( (23) \), let us consider the simplest case of the spin saturated system where the spin-dependent effects can be neglected. Since the pseudo-scalar and the pseudo-vector contributions in the model \( (20) \) are absent, the Wigner function decomposition on the basis of the Clifford algebra \( (25) \) leads to

\[
f(x, P) = f^S(x, P) + f^V_\mu(x, P)\gamma^\mu + f^V_{\mu\nu}(x, P)\sigma^{\mu\nu}, \tag{26}\]

where $\sigma^{\mu\nu} = \frac{i}{4}\{\gamma^\mu, \gamma^\nu\}$ and

\[
f^S = \frac{1}{4}\operatorname{Tr}(f), \quad f^V_\mu = \frac{1}{4}\operatorname{Tr}(\gamma^\mu f), \quad f^V_{\mu\nu} = \frac{1}{8}\operatorname{Tr}(\sigma^{\mu\nu} f), \tag{27}\]

The symbol $\operatorname{Tr}$ denotes the trace with respect to the spinor indices. Substitution of the relation \( (26) \) into the kinetic equation \( (23) \) leads to the system of equations for the decomposition coefficients \( (25) \). Let us write this system in the quasi-classical limit $\hbar \to 0$ when $f^S_{\mu\nu} = 0$. Projection of the Dirac- scalar and vector part of the Wigner function leads to

\[
P_\mu \partial_\mu f^S(x, P) + \partial_\mu M^* P^{\mu\nu} \partial_\nu f^V_\mu(x, P) + g_\nu P^{\mu\nu} f^V_{\mu\nu}(x, P) = 0 \tag{28}\]
\[
P_\mu \partial_\mu f^V_\mu(x, P) + P_\lambda \partial_\lambda f^V_\mu(x, P) - P^{\mu\nu} \partial_\nu f^V_\mu(x, P) + \partial_\mu M^* P_\nu \partial_\nu f^S(x, P) + g_\nu P^{\mu\nu} F_{\mu\nu\lambda} \partial_\lambda f^V_\mu(x, P) + g_\nu P^{\mu\nu} F_{\mu\nu\lambda} \partial_\lambda f^V_\lambda(x, P) = 0 \tag{29}\]

and

\[
P_\mu f^V_\mu(x, P) = P_\mu f^V_\mu(x, P). \tag{30}\]

For the description of a spin saturated system it is appropriate to find the closed relativistic kinetic equation for the scalar part of the Wigner function $f^S(x, P)$.

After a few algebraic transformations, we end up with the following kinetic equation for the scalar Wigner function:

\[
P_\mu \partial_\mu f^S(x, P) + \sqrt{P^2(\partial_\mu M^*)} \partial_\mu f^S(x, P) + g_\nu P^{\mu\nu} \partial_\nu f^S(x, P) = 0. \tag{31}\]

Let us now discuss some feature of this relativistic Vlasov equation. Firstly, the kinetic equation \( (31) \) occurs without the mass shell condition: this condition can be introduced only from outside as an additional restriction. On the mass-shell

\[P^2 = M^*^2,\tag{32}\]
Eq. (31) turns into the well-known relativistic kinetic equation of the Vlasov type for spin saturated nuclear matter:
\[ P_\mu \partial_\mu f^S(x, P) + m_0^2 f^S(x, P) + g_\nu F^{\mu \nu} f^S(x, P) = 0. \] (33)

The derived kinetic equation fulfills the conservation laws. For example it is easy to check with help of (31) that the baryon current density
\[ j^\mu(x) = \int dP \text{Tr}\{\gamma^\mu f(x, P)\} = 4 \int dP f^\mu(x, P) = 4 \int dP \frac{f^S(x, P)}{\sqrt{P^2}} P^\mu \] (34)
is fulfilled by the continuity equation
\[ \text{div} j(x) = 0. \] (35)
The entropy conservation is verified analogously and holds in the investigated mean field approximation (no collisions).

4. Application of the method to the Nambu–Jona-Lasinio-model

In this section we want to derive a relativistic kinetic equation of the Vlasov type for NJL model. The NJL model was successfully applied for the description of chiral symmetry breaking. The effective Lagrangian reads
\[ L(x) = i \bar{\psi}(x) \gamma^\mu \partial_\mu \psi(x) - m_0 \bar{\psi}(x) \psi(x) + G \left[ (\bar{\psi}(x) \psi(x))^2 + (\bar{\psi}(x) i \gamma^5 \psi(x))^2 \right], \] (36)
where \( m_0 \) is the current quark mass and \( G \) the coupling constant. Within this model study we neglect color and flavor degrees of freedom. We restrict to the mean field approximation (Hartree approximation) and obtain
\[ \mathcal{L}^\text{Hartree}(x) = i \bar{\psi}(x) \partial_\mu \gamma^\mu \psi(x) - m_0 \bar{\psi}(x) \psi(x) - \sigma(x) \bar{\psi}(x) \psi(x) - \pi(x) \bar{\psi}(x) i \gamma^5 \psi(x), \] (37)
where the mean fields in the scalar and pseudoscalar channel are given as
\[ \sigma(x) = -G < \bar{\psi}(x) \psi(x) >, \] (38)
\[ \pi(x) = -G < \bar{\psi}(x) i \gamma^5 \psi(x) >. \] (39)
The corresponding Hamilton density reads
\[ H_{\text{int}}(x) = \bar{\psi}(x) \phi(x) \psi(x), \] (40)
with the channel decomposition
\[ \phi_{\alpha\beta}(x) = \sigma(x) \delta_{\alpha\beta} + \pi(x) i \gamma^5 \delta_{\alpha\beta}. \] (41)
The substitution of Eq. (40) into the kinetic equation (39) leads to the following equation for the model Lagrangian (37) in lowest order of the gradient expansion
\[ p_\mu \partial_\mu f(x, p) + \frac{1}{2} [p_\mu, \partial_\mu f(x, p)] + im_0 [\bar{\psi}, f(x, p)] \\
+ i \{ \phi(x) f(x, p) - f(x, p) \phi(x) \} p \phi + \frac{1}{2} \{ \phi \partial_\mu f(x, p) + (\partial_\mu f(x, p)) \phi \} = 0. \] (42)
The kinetic part of this equation equals to the corresponding terms in Eq. (23). The last two terms are result of the interaction within the NJL model.

In order to derive a closed equation for a spin saturated system we use the Clifford decomposition of the Wigner function

\[ f = f^s + i\gamma_5 f^p + \gamma^\mu \gamma_5 f^A_{\mu} \]  

In the quasiparticle approximation we can extract the following set of equations for the coefficients of the decomposition (Eq. (43))

\[ p_\mu \partial^\mu x f^s + (\partial^\mu \sigma) p^\mu \partial^\mu f^w = 0 \]  

\[ p_\mu \partial^\mu f^p - (\partial^\mu \pi) p^\mu \partial^\mu f^w = 0 \]  

\[ p_\mu \partial^\mu f^p + p_\lambda \partial^\mu f^w_{\mu} - p^\nu \partial^\mu f^w_{\nu} + p_\lambda \{(\partial^\mu \sigma) p^\mu f^s - (\partial^\mu \pi) \partial^\mu f^p\} = 0 \]  

\[ M_\sigma f^p + \pi f^s = 0 \]  

\[ p_\mu f^v_{\nu} - p_\nu f^v_{\mu} = 0 \]  

where \( M_\sigma = m_0 + \sigma \). In order to proceed we will restrict to the chiral limit \( (m_0 = 0) \) and introduce the chiral invariant mass \( \mathcal{M} = \sqrt{\sigma^2 + \pi^2} \) and the transformed Wigner function as \( \mathcal{F} = f^s / \sigma \). On the mass shell \( (p^2 = \mathcal{M}^2) \) we obtain for the scalar part \( (\mathcal{F}) \)

\[ p_\mu \partial^\mu \mathcal{F}(x, p) + \mathcal{M}(x)\partial^\mu \mathcal{M}(x)\partial^\mu \mathcal{F}(x, p) = 0 \]  

using the additional relation \( p_\partial \sigma = p_\partial \pi = 0 \).

The relativistic equation of the Vlasov type (49) for the NJL model has the same form as in [31]. Starting from this equation it is very interesting to study the dependence of the scalar Wigner function (quark condensate) as a function of space and time [32]. Therefore it is obviously necessary to define reasonable initial conditions. Such a numerical investigation is in progress but out of the scope of this work.

5. Summary

We have presented a derivation of a relativistic quantum kinetic equation for a system with many internal degrees of freedom. For illustrative simplicity, in our analysis we employed a mean field approximation. Our approach is based on a non-equilibrium statistical operator and can be considered as a relativistic generalization of the Zubarev method. We obtain a transport equation which fulfills the conservation laws.

We applied the approach to the Walecka model and the NJL model. In particular we performed the spinor decomposition of the transport equation for both cases.

In future studies the Vlasov equation should be improved by the inclusion of collision integrals. The solution of the relativistic Vlasov equation for the NJL model is of particular interest for the quark hadron phase transition, where a quark condensate should evolve with time and space [32]. An interesting application of the approach developed herein would be the numerical study of the Vlasov equation in a microscopic quark model.

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Appendix A: Details of the derivation of the relativistic kinetic equation of the Vlasov type

In this appendix we add some details of the derivation of Eq. (23). The starting equation is the generalized kinetic equation (19). It is convenient to write this equation in the form

\[ p \partial \left( \mathcal{I}\left( x,p \right) \right) = -I(x,p), \quad (A.1) \]

where the drift integral was introduced

\[ I(x,p) = i\sqrt{p^2} \int dy e^{-ipy} < [P(x,y), H(\tau)] >. \quad (A.2) \]

In the considered model (20), the Hamilton operator is defined by formula (22). After calculation of the commutator in the relation (A.2) by the rules (11) and (14), we get

\[ I = I^{(m)} + I^{(k)} + I^{(f)}, \quad (A.3) \]

where \( I^{(m)} \) is the mass part of the drift integral,

\[ I^{(m)}(x,p) = iM[\hat{p}, \mathcal{J}(x,p)]_{\alpha \beta}, \quad (A.4) \]

\( I^{(k)} \) is the kinetic part,

\[ I^{(k)}(x,p) = \gamma^\mu_{\alpha \beta} \int dy e^{-ipy} \left\{ \bar{\psi}_\beta(x_+) \partial^+_{\mu}(x_-) \bar{\psi}_\delta(x_-) \hat{p}_{\alpha \gamma} + \partial^+_{\mu}(x_+) \bar{\psi}_\gamma(x_+) \bar{\psi}_\alpha(x_-) \hat{p}_{\beta \delta} >, \quad (A.5) \]

and \( I^{(f)} \) is the mean field part,

\[ I^{(f)}(x,p) = i \int dy e^{-ipy} \times \left\{ \hat{p}_{\alpha \gamma} \Gamma_{\gamma \delta}(x_-) < \bar{\psi}_\beta(x_+) \psi_\delta(x_-) - \hat{p}_{\beta \delta} \Gamma_{\gamma \delta}(x_+) < \bar{\psi}_\gamma(x_+) \psi_\alpha(x_-) > \right\}, \quad (A.6) \]

and

\[ \Gamma_{\alpha \beta}(x) = -g_s \phi(x) \delta_{\alpha \beta} + g_v \omega_\mu(x) \gamma^\mu_{\alpha \beta}. \quad (A.7) \]

We use the short notation, \( x_\pm = x \pm y/2 \). The mass part (A.4) has been written already with Wigner functions. In order to express the remaining parts of the drift integral (A.3) in terms of the Wigner function it is necessary to apply the transformation to formulae (A.5) and (A.6). In formula (A.5) let us consider the first term from the right part

\[ \int dy e^{-ipy} < \bar{\psi}_\beta(x_+) \partial^+_{\mu}(x_-) \bar{\psi}_\delta(x_-) > \hat{p}_{\alpha \gamma} \]

\[ = \frac{1}{2} \int dy e^{-ipy} \left\{ < \bar{\psi}_\beta(x_+) \partial^+_{\mu}(x_-) \psi_\delta(x_-) + \bar{\psi}_\beta(x_+) \partial^+_{\mu}(x_-) \bar{\psi}_\delta(x_-) > \right\} \hat{p}_{\alpha \gamma} \]

\[ = \frac{1}{2} \int dy e^{-ipy} \left\{ < \bar{\psi}_\beta(x_+) \partial^+_{\mu}(x_-) \psi_\delta(x_-) + (\partial^+_{\mu}(x_-) \bar{\psi}_\beta(x_+) \bar{\psi}_\delta(x_-) > \right\} \hat{p}_{\alpha \gamma} \]

\[ = \frac{1}{2} \partial^+_{\mu}(x) f_{\delta \beta}(x,p) \hat{p}_{\alpha \gamma}. \quad (A.8) \]

Here the following identity was taking into account

\[ p^+_{\mu} = \Delta_{\mu \nu} p^\nu = 0 \]
according to definition $\Delta_{\mu\nu} = g_{\mu\nu} - p_\mu p_\nu (p^2)^{-1}$. The second term of (A.5) is transformed by analogy

$$\int dy e^{-ip\cdot y} < (\partial^+ (x_+) \bar{\psi}_\gamma (x_+) ) \psi_\alpha (x_-) > p_{\beta \gamma} = \frac{1}{2} \partial^+ (x) f_{\alpha\gamma} (x, p) p_\beta.$$  

(A.9)

As a result of substitution of (A.8), (A.9) into (A.5) we get one contribution from the relativistic Vlasov equation (23)

$$I_{\alpha\beta}^{(k)} (x, p) = \frac{1}{2} \partial_\mu (x) [\bar{\psi} \gamma^\mu, f (x, p)]_{\alpha\beta}.$$  

(A.10)

The formulae (A.4) and (A.10) have the local form already. The mean field part (A.6) can be written in non-local form relatively to the Wigner function without effort. However, in order to write this fragment of the drift integral in local form, we perform a gradient expansion of the meson fields (A.7) with respect to the rapid variable $y$,

$$\Gamma_{\alpha\beta} (x \pm) \simeq \Gamma_{\alpha\beta} (x) \pm \frac{1}{2} y_\mu \partial^\mu (x) \Gamma_{\alpha\beta} (x).$$

The substitution of the decomposition into (A.6) leads to the result

$$I_{\alpha\beta}^{(f)} (x, p) \simeq \{ \bar{\psi} \Gamma (x) f (x, p) - f (x, p) \Gamma (x) \bar{\psi} \}_{\alpha\beta} + \frac{1}{2} \{ \bar{\psi} \partial^\mu (x) \Gamma (x) \partial_\mu (p) f (x, p) - f (x, p) \partial^\mu (x) \Gamma (x) \bar{\psi} \} _{\alpha\beta}. $$  

(A.11)

Taking into account the definition (A.7), fulfilling the necessary commutations and substituting the canonical momentum $p^\mu$ by the kinetic one $P^\mu = p^\mu - g_{\mu\nu} \omega^\nu$, the relation (A.11) can be rewritten in the form of the Eq. (23).

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