Dielectric surrounding decimates eigenmodes of microdisk optical resonators

A V Raskhodchikov\textsuperscript{1,2}, S A Scherbak\textsuperscript{1,2}, N V Kryzhanovskaya\textsuperscript{1,2}, A E Zhukov\textsuperscript{1,2} and A A Lipovskii\textsuperscript{1,2}

\textsuperscript{1} St. Petersburg Academic University RAS, St. Petersburg 194021, Russia
\textsuperscript{2} Peter the Great St. Petersburg Polytechnic University, St. Petersburg 195251, Russia

Abstract. We performed a numerical study of optical whispering gallery modes in microdisk resonators modified via their embedding in a homogeneous dielectric surrounding or covering with a thin dielectric layer. Mode spectra and electromagnetic field distributions were calculated through the solution of the Helmholtz equation using COMSOL Multiphysics environment. It is shown that the modification results in the decimation of the resonator modes.

1. Introduction

Semiconductor microlasers using resonators with whispering gallery modes (WGMs) are of a high interest for intra-chip optical communications, sensing and quantum informatics \cite{1}. WGMs usually have high quality factor, up to \(10^{11}\) \cite{2}, and, therefore, narrow spectral peaks. Numerous researches are aimed at planar structures, first of all microdisks (MD) \cite{3-5}, for they are very promising in sense of integration with semiconductor-based electronics. Typically, emission spectra of MD lasers of several microns in diameter contain a rich set of the resonator modes of a high (40-50) azimuthal order and up to 4\textsuperscript{th} or 5\textsuperscript{th} radial order. This restricts information capacity of optical channels where the MD lasers can be used. Nowadays, various methods to select lasing modes or even to achieve single-mode emission are being developed, e.g. etching a groove on a resonators surface \cite{6}, piercing a hole through it \cite{7}, or applying periodical roughness on sidewalls \cite{8}. However, destructive nature of these techniques limits their generality. The other promising way is to outcouple the resonator with external cavity, waveguide or antenna \cite{9-11}, though it typically requires a very fine tuning. In the present study, we consider a simple way of the radial modes decimation via covering the MD resonators with dielectric layers or their embedding in dielectric media.

2. Approach

To analyze WGMs of the MD resonators we performed their numerical simulation in COMSOL Multiphysics environment. The calculations were made using the finite element method. We placed a randomly oriented dipole antenna inside the resonator as an excitation source. The similar approach is widely used in time-domain modeling \cite{12,13}. To analyze mode spectra and mode field distributions we solved frequency-dependent Helmholtz equation

\[ \nabla^2 \mathbf{E} + \kappa^2 \mathbf{E} = \mathbf{F}(r, \omega) \]

for each desired frequency and calculated the energy in the resonator, \(W\), as the integral of the electromagnetic energy density over the resonator volume. When the excitation is close to any eigenfrequency of the resonator, the energy increases greatly. Thus, the energy spectrum coincides with the MD mode spectrum, which being constructed with this approach contains only real, physical
modes unlike direct numerical solution of the eigenvalue problem [14]. Importantly, this is less demanding for computational power, whereas eigenvalue problem is solved by a memory-intensive direct solver. Besides, we artificially introduced losses in the resonator via setting the imaginary part of its material index, $k=5\times10^{-5}$. This provided physically meaningful mode spectral width and allowed us selecting a reasonable wavelength calculations grid, $\sim0.1$ nm, which gave us the possibility to detect all the modes.

We considered 2D geometry (a cylinder of the infinite width), since the phenomena under discussion are qualitatively the same as in 3D, whereas computational burden decreases greatly. Because of the planar geometry of the problem, the exciting dipole locates in the plane. Therefore, $E$-field is in-plane and $H$-field is out-of-plane. Thus, all the modes calculated are of TM kind. Note that the consideration of TE-modes does not add any additional results.

3. Embedding MDs in a homogeneous dielectric medium

We simulated mode spectra of a 6 μm in diameter GaAs MD resonator in the air surrounding. This MD possesses large number of eigenmodes that should be reduced. The examples of $|H|$-field distribution of typical calculated modes up to 3rd radial order are presented in figure 1. The notation of modes is TM($x$, $y$) where $x$ is a radial number, $y$ is an azimuthal number.

![Figure 1](image)

**Figure 1.** Spatial distribution of $|H|$-field of the first radial mode TM(1,48) (a), the second radial mode TM(2,42) (b) and the third radial mode TM(3, 38) (c) in the 6 μm in diameter microdisk resonator.

Next, we varied refractive index of the outer medium, $n_{out}$, from 1 to 2.5 (index of the MD is 3.5) and simulated mode spectra of the MD resonator embedded in this medium. The calculated spectra of the electromagnetic energy inside the resonator are presented in figure 2a. The latter demonstrates that the number of supported modes significantly reduces with increasing index of the outer medium: the modes of higher radial orders vanish. This is seen in figure 2b where the dependence of the quality factor of the second and the third radial order modes on $n_{out}$ are presented. The dependences have a representative threshold behaviour: Q-factor drastically drops and the modes vanish at a certain threshold value of $n_{out}$. For higher radial modes the threshold is lower, and these modes are the first ones to disappear. Note that the calculated quality factors are just estimations since additional losses were introduced artificially. Nevertheless, this barely affects the revealed tendency. The Q-factor was
calculated as $\lambda_c/\Delta\lambda$, where $\lambda_c$ is the resonant frequency and $\Delta\lambda$ is the resonance width that is the full width at half maximum.

We believe that the origin of this phenomenon is a break of the total internal reflection (TIR) conditions at the MD-surrounding interface. Indeed, the disturbance of TIR directly results in a mode degradation that corresponds to the observed threshold behaviour. Moreover, higher radial modes, which azimuthal number is typically lower and, therefore, an angle of the internal reflection is higher, should be first to disappear. This reasoning relates only to the geometrical optics interpretation of the problem, but, albeit very rough, helps to understand the phenomenon under discussion.

4. Covering MDs with a thin dielectric layer
The same effect takes place when instead of the embedding the MD in a homogeneous medium the resonator is covered with a dielectric layer of a finite thickness. This is schematically shown in figure 3a. In the figure 3b we show the evolution of the energy spectrum of the MD resonator with the thickness of covering layer increase from 50 to 250 nm, the index of the cover being 2.5.

Figure 3. Schematics of the MD with a dielectric layer (a) and the mode spectra of the 6-μm MD ($n = 3.5$) covered with 50 nm (upper), 150 nm (middle) and 250 nm (lower) thick dielectric layer ($n = 2.5$) (b). Mode numbers of the lowest radial modes are indicated near corresponding maxima.
Qualitatively, the process is very similar to the one demonstrated in figure 2. Modes of higher radial orders start to vanish when a certain thickness is reached. For example in the figure 3, only the first and the second order modes remain in the resonator with 250 nm thick dielectric film. Note that the effect of the cover saturates with the film thickness since electromagnetic fields of WGMs are strongly localized in the MD. After some point, further thickness increase hardly affects the mode spectrum. Generally, the influence of a dielectric film can be interpreted as the embedding of the MD in an effective medium the index of which growth up to the index of the film material while the film thickness increases. Therefore, the reasonings regarding break of the TIR condition are also valid here.

5. Conclusion
We numerically simulated the microdisk resonators using 2D model and considered the transformation of their mode spectra under the influence of embedding the MD in dielectric media or covering it with dielectric layers. Both approaches provide vanishing higher-order radial modes. This is because of the threshold-like drop of the quality-factor of the higher radial modes of the resonator with increasing index of a surrounding medium and the thickness of a deposited dielectric film. The origin of this phenomenon is a break of the total internal reflection conditions at the MD-surrounding interface. Thus, a simple technique to decimate mode spectrum of microdisk resonators is proposed.

Acknowledgements
This study was supported by Russian Foundation for Basic Research (project #16-29-03111) and Russian Ministry of Education and Science (3.9787.2017/8.9).

References
[1] Kryzhanovskaya N V, Maximov M V, Zhukov A E 2014 Quantum Electron. 44 (3) 189
[2] Savchenkov A A, Matsko A B, Ilchenko V S and Maleki L 2007 Optical resonators with ten million finesse Opt. Express 15 6768
[3] McCall S L, Levi A F J, Slusher R E, Pearton S J and Logan R A 1992 Whispering-gallery mode microdisk lasers Appl. Phys. Lett. 60 289–91
[4] Gayral B, Gérard J M, Lemaitre A, Dupuis C, Manin L and Pelouard J L 1999 High-Q wet-etched GaAs microdisks containing InAs quantum boxes Appl. Phys. Lett. 75 1908–10
[5] Ide T, Baba T, Tatebayashi J, et al Y 2005 Room temperature continuous wave lasing in InAs quantum-dot microdisks with air cladding Opt. Express 13 1615–20
[6] Bogdanov A A, Mukhin I S, Kryzhanovskaya N V. et al 2015 Mode selection in InAs quantum dot microdisk lasers using focused ion beam technique Opt. Lett. 40 4022
[7] Zhen-Nan Tian, Feng Yu, Yan-Hao Yu, Jun-Jie Xu, Qi-Dai Chen, and Hong-Bo Sun Optics Letters 42 (8) 1572 (2017)
[8] Boriskina S V, Benson T M, Sewell P and Nosich A I 2004 J. Opt. Soc. Am. B-Optical Phys. 21 1792–6
[9] Moiseev E I, Kryzhanovskaya N, Polubavkina Y S et al 2017 Light Outcoupling from Quantum Dot-Based Microdisk Laser via Plasmonic Nanoantenna ACS Photonics 4 275–81
[10] Gotzinger S, Benson O and Sandoghdar V 2001 Towards controlled coupling between a high-Q whispering-gallery mode and a single nanoparticle Appl. Phys. B Lasers Opt. 73 825–8
[11] Yariv A, 2002 Critical coupling and its control in optical waveguide-ring resonator systems IEEE Photonics Technol. Lett. 14 2001–3
[12] Mintairov A M, Chu Y, He Y, Blokhin S et al 2008 Phys. Rev. B - Condens. Matter Mater. Phys. 77 1–7
[13] Xu Y, Lee R K and Yariv A 2000 Phys. Rev. A 61 33808
[14] https://www.comsol.com/model/download/344161/models.woptics.fabry_perot.pdf