Realization and Detection of Fulde-Ferrell-Larkin-Ovchinnikov Superfluid Phases in Trapped Atomic Fermion Systems

In a very interesting recent Letter\cite{Mizushima2005}, the authors suggested the possibility of realizing the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) superfluid state in trapped atomic fermion systems. This is very exciting because the FFLO state has been of interest to condensed matter physicists for decades, and also caught the attention of the particle physics community recently\cite{Yang2005}. As the authors point out, the trapped atom systems offer some unique advantages in the realization and observation of this fascinating state. In Ref.\cite{Mizushima2005} the authors used a 1D mean field solution as guidance to estimate the parameter range for the existence of the FFLO phase, and also discussed the possibility of its detection by imaging the atomic density of the system. In this comment I wish to make two points. (i) In 1D there exists an exact solution based on bosonization, which fully takes into account the important quantum fluctuation effects\cite{Yang2005}; the exact solution suggests a wider parameter range for the FFLO state than that of the mean-field solution used in Ref.\cite{Mizushima2005}. (ii) One can detect the FFLO pairing (in which Cooper pairs carry finite momenta) more directly by extending the methods used to detect BCS pairing\cite{Regal2004, Altman2005, Yang2005}.

(i) Based on a mean-field solution in 1D, the authors find the following condition for the FFLO state to be stable: $\delta n = |n_\uparrow - n_\downarrow|/n \geq \Delta_0/\pi \epsilon_F$, where $n_\uparrow$ and $n_\downarrow$ are the densities for the two fermion components. From this they estimate $\delta n$ needs to be in the 10% – 20% range for the trapped atoms. While a mean-field treatment may be justifiable for realistic quasi-1D situations, in a genuine 1D situation (which may also be realizable experimentally) where fluctuation effects not included in mean-field theories are severe, there exists an exact solution via bosonization\cite{Yang2005}. In 1D the analog of the BCS state is the spin gapped phase of the Luttinger liquid, in which the spin sector is described by the gapped sine-Gordon model, and the FFLO state is the phase with a finite soliton density in the sine-Gordon model; the transition between the two is described by the commensurate-incommensurate transition (CIT)\cite{Yang2005}. The key point here is that the CIT is a continuous transition in which $\delta n$ rises continuously from zero; in fact in the exact solution $\delta n \propto \sqrt{B - B_c}$, where $B_c$ is the critical Zeeman splitting. This suggests that in the FFLO state $\delta n$ extends all the way to zero; or when $\delta n$ itself is the experimental control parameter, any non-zero $\delta n$ would put the system in the FFLO phase in 1D, provided it supports a spin gap when $\delta n = 0$. This represents a significantly wider parameter range for the FFLO state than that suggested in Ref.\cite{Mizushima2005}. While only power-law long-range order is possible in 1D, a weak 3D coupling stabilizes true long range order for both the BCS and FFLO states, with a spatially oscillating order parameter whose wave vector $q \propto \delta n$ for the latter\cite{Yang2005}. The mean-field theory gives better descriptions in 3D, which Ref.\cite{Mizushima2005} also studies.

(ii) The authors proposed to detect the FFLO state by measuring the modulation in local magnetization, which reflects the underlying structure of the Cooper pairs parameter. Here we propose that we can probe the FFLO state more directly by detecting the Cooper pairs themselves, using the methods advanced in Refs.\cite{Regal2004, Altman2005, Yang2005}. In Ref.\cite{Regal2004} one projects the Cooper pairs of a BCS state onto molecules by sweeping the tuning field through the Feshbach resonance, and then use time-of-flight (TOF) measurement to determine the molecular velocity distribution and the condensate fraction. One can do exactly the same experiment on the FFLO state; the fundamental difference here is that in this case because the Cooper pairs carry intrinsic (non-zero) momenta, the condensate will show up as peaks corresponding to a set of finite velocities in the distribution. An alternative method to detect the Cooper pairs is to study the correlation in the shot noise of the fermion absorption images in TOF\cite{Yang2005}, first proposed in Ref.\cite{Altman2005}. In Ref.\cite{Yang2005} the shot noise correlation clearly demonstrates correlation in the occupation of $k$ and $-k$ states in momentum space when weakly bound diatom molecules are dissociated. In principle the same measurement can be performed on fermionic superfluid states, and for an FFLO state, it would reveal correlation in the occupation of $k$ and $-k + q$ states, where $q$ is one of the momenta of the pairing order parameter\cite{Yang2005}. Both methods allow one to directly measure $q$, which defines the FFLO state. They are unique to the cold atom systems; in superconductors the only comparable method is Josephson effect\cite{Yang2005}.

K.Y. was supported by NSF DMR-0225698.

Kun Yang
Physics Department, Florida State University
Tallahassee, Florida 32306

\begin{thebibliography}{9}
\bibitem{Mizushima2005} T. Mizushima, K. Machida, and M. Ichioka, Phys. Rev. Lett. 94, 060404 (2005).
\bibitem{Casalbuoni2004} For a review, see R. Casalbuoni and G. Nardulli, Rev. Mod. Phys. 76, 263 (2004).
\bibitem{Yang2005} Kun Yang, Phys. Rev. B 63, 140511 (R) (2001).
\bibitem{Regal2004} C. A. Regal, M. Greiner, and D. S. Jin, Phys. Rev. Lett. 92, 040403 (2004).
\bibitem{Altman2005} Ehud Altman, Eugene Demler, and Mikhail D. Lukin, Phys. Rev. A 70, 013603 (2004).
\bibitem{Greiner2005} M. Greiner, C. A. Regal, J. T. Stewart, and D. S. Jin, Phys. Rev. Lett. 94, 110401 (2005).
\bibitem{Demler2000} This is also known to the authors of Ref.\cite{Yang2005} (E. Demler, private communication).
\bibitem{Yang2000} K. Yang and D. F. Agterberg, Phys. Rev. Lett. 84, 4970 (2000).
\end{thebibliography}