Parametric Analysis for forming meso fractals from nanoparticle seeded resin in Hele Shaw cell

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Abstract. Development of meso and micro size fractal structures is required to mimic various biological systems for various functions. Meso and micro size fractal structures are fabricated by several processes in Engineering. A number of fluid flows are exhaustively analyzed and investigated in the direction of development of meso and micro fractal structures. Hele-Shaw flow is one of the flows which fabricated meso and micro structure. Formation of fractals in Hele-Shaw cell can be done by compressing fluid in two flat parallel plates and lifting of upper plate in the upward direction. Lifting velocity of the upper plate, a gap between the plates, and fluid properties viz. density, viscosity are the major parameters in the formation of fractal. This paper presents the characterization of the fluid, study and analysis of microfractals followed by a mathematical model. The characterization of the fluid consists of photopolymer seeded with the nanopowder in the lifting plate of Hele-Shaw. The resin is prepared by mixing photoinitiator Benzoyl Ethyl Ether (BEE), Hexanediol Diacrylate (HDDA) added with ceramic nanopowder. Through the design of the experiment, micro fractal growth is observed and analysed. The fractals growth controlling parameters viz. gap between plates, velocity causes fluid separation are varied through the experimental setup. Effect of variation in viscosity set by varying loading fraction of seeded ceramic nanoparticles in a resin is observed for formation of fractals. Effect of process parameters on formation and growth of fractals is studied and presented. A dimensionless model of fractal growth is presented to analyse the effect of various parameters. Experimental methodology introduced a new style for the formation of controlled fractals or to imitate different meso and micro fractals available in nature or living things.

1. Introduction
The emergence of various processes like microstereolithography [1], bulk lithography [2–4], 3D optical patterning [5], interference lithography [6], direct laser writing [7, 8] etc. have helped in the development of meso and micro fractal structures. These micro fractal structures have many application in a variety of engineering processes and it can be seen mimicking the natural living things for e.g. leaf and veins. Fractal structures can be fabricated by various fluid flow phenomenon. Hele Shaw fluid flow is one of the fluid flows that have arisen to develop complex fractal structures. Hele Shaw fluid flow can be defined as flow between two flat plates parallelly separated by a small gap (In short it is flow of a fluid
between two plates that are close to each other) wherein the upper plate can move in the upward direction perpendicularly or inclined to the plane of lower plate. [See figure 1].

During the process, the initial step leads to the formation of a circular compressed thin plane of fluid by compressing the small quantity of fluid between two smooth parallel plates. The upper plate is moved in the upward direction either normal or at an inclination to the confinement of lower plate. Fractals are formed by lateral straining of fluid and the further fracturing of the fluid at the outer periphery (See figure 2). Initially, viscosity of fluid resists to deform up to the yield point. After yield point, this resistance creeps and the fractal structure is formed. Furthermore, these fractals grow in a radial direction from the outer edge to the center.

The analysis of the lifting plate Hele-Shaw flow has been supported by literature [9–20] under different cases. The propagation of air in the compressed thin layer of liquid fluid [9–15] can be understood by analyzing the instability of gas bubble expanding in the compressed thin layer of fluid. In the analysis of fingering it was found that inertia of fluid plays important role in fingering formation. Study [10] investigated the effects of inertia on the width of the viscous fingers experimentally and found that, due to inertia, the finger width can increase with increasing speed. The study [11] has considered both gas and liquid as primary fluid and secondary fluid respectively wherein the instability occurs at the interface of gas and liquid. The simulation depict the effect of shear thinning on the interfacial pattern. The suppression of tip splitting and production of fingers oscillating during growth is caused by shear thinning. In order to get uniqueness and regularity in solution for the ones having negligible surface tension, the study [17] of a slurry was conducted. The analytical model has been presented by the researchers [18–22] featuring dynamic process of fingering and its competition by exploiting various non-linear effects. A modified Darcy’s model can help to achieve this. It depicts a lifting Hele-Shaw interfacial flow wherein a shear dependent viscosity is exhibited by the displaced fluid. Determination of the direction of fingering followed by analysis of finger width behavior was done which is dependent on shear nature of stretched fluid. The study [24] concentrated on controlling and minimizing protocols. The displaced fluid used is non-Newtonian, power law fluid is taken into consideration. In a radial Hele-Shaw cell a non-Newtonian fluid was displaced by the inviscid fluid. The amount of injected fluid was kept constant for distinguishing fluid flow behavior. The optimal injection process which is dependent on the power-law index was used to get variational approach. Later, the varying amount of injected fluid was accounted. It concentrates on controlling the total number of resulting fingers that appears at the interface. Under different geometrical modifications on one plate of Hele-Shaw cell, the behavior of formation of fractals was studied [25]. The Oval shape pit was considered on one plate of Hele-Shaw cell and its effect was studied. The geometrical modification is a promising technique to control the development of micro fractals.
Although, numerical study and experimental analysis are presented by researchers, the exhaustive characterization of the process that reflects the effect of different process factors, such as fluid separation velocity, kinematic viscosity and a gap between two plates on the pattern of micro fractal structure model has not yet been presented. This paper presents an experimental characterization of the wide range of process parameters. The formation of fractals under various process parameters has brought to light the vitality of the fractals formation process.

2. Process Description

2.1. Experimental Setup
An experimental setup is designed and developed for the research as shown in figure 3. It consists of two circular acrylic plate. The lower plate is kept fixed and upper plate can move in the upward axial direction of the plate by means of the actuator (stepper motor). User friendly Interface is developed consisting of controller, attached with a stepper motor to control the velocity of lifting plate. For real time analysis and to study the fractal formation, a camera is attached to the setup. In the characterization of the process, videos and images recorded by the camera are used. For the activation of stepper motors and related control elements 12 V DC power supply is used.

![Stepper Motor]

![Actuator]

![Moving Plate]

![Fixed Plate]

![Controller]

**Figure 3.** Fabricated Experimental Setup.

2.2. Preparation of resin and process flow
The resin used for the study of the micro fractal is prepared by mixing Hexanediol Diacrylate monomer (HDDA), photoinitiator Benzoyl Ethyl Ether (BEE) 4% by weight and alumina nanoparticles. The homogeneous mixture is obtained by keeping monomer and photoinitiator for 22-24 hours. Zirconia balls and ceramic nanopowder are added to the homogenous mixture of monomer and photoinitiator in the aluminum leak proof container. Aluminum container is kept on ball mill machine for 48- 50 hours
to obtain a homogeneous resin. A micropipette is used to put a drop of prepared resin on the lower plate as shown in figure 4.

The intermediate plate is used to ensure a position of resin exactly at the center of the lower plate (refer figure 4). Vertical motion of upper plate is controlled by the microcontroller. First, a fluid drop is squeezed between two plates and the upper plate is moved in the axially upper direction. Fluid undergoes lateral strain and causes fluid to creep and deform due to the upward motion of plate. Initially fluid viscosity act as resistance to deformation till the yield point. Resistance reduces slowly after the yield point and fractal structures are formed. The fractal pattern formed on both plates with a mirror image of each other (refer figure 4). Micro fractals formed is called as viscous fingering.

![Figure 4. Flowchart of the process for formation of micro-fractals.](image)

3. Experimental Characterization

The experiments are designed to characterize the fractal formation process. The experiments are performed by considering three vital factors: viscosity of the fluid, fluid separation velocity, and a gap between two plates. Table 1 shows the design of experiments by considering the identified factors and their levels. Three process factors with three levels are considered for experimentation. Total 27 experiments are conducted in the total combination of levels. Fractals formed are an output of designed experiments. Geometrical attributes of the fractals are defined and measured in each of the experiments for quantification of formation of fractals. The resemblance of Cayley tree up to three generations are seen in the formed fractals. Shielding distance (distance between the initiations point of pattern and joining node of branch in mm) of the formed fractals is measured for analysis. This model has three generations, therefore three shielding distance $r_1$, $r_2$, $r_3$ are measured.

| Process Parameter                  | Levels     |
|-----------------------------------|------------|
| Separation, velocity, (mm/min)    | 0.75, 1.5, 15 |
| Gap Width, (mm)                   | 0.035, 0.135, 0.235 |
| Kinematic Viscosity, (mm$^2$/min) | 208542, 260677, 312813 |

Effect of different process parameter on micro fractals formation is as shown in figure 5. Formation of fractals depends on gap width and fluid separation velocity and kinematic viscosity. Figures 5 (a) and
(b) shows no formation of micro fractals for a large range of separation gap and separation velocity. Under this scenario, when upper plate starts moving in the positive z direction, accumulation of fluid particle begins towards the center and the initial stage of fluid was attained which is droplet. High fluid separation velocity and higher kinematic viscosity are required to form the fractals. It is because high velocity provide less time for accumulation of fluid and resistance to inertia will start to develop a pattern formation. High viscosity is also an important factor in the formation of a good fractals (refer figure 5 (c) and (f)). Though high viscosity plays a vital role in pattern formation, but more viscous fluid beyond considered range is unable to form a pattern. It is because high viscous fluid behaves like a solid structure. Fluid separation velocity is another governing factor for finger formation. Figure 5 (c) shows the effect velocity on the formation of fractals. The competition between separation velocity and creep failure governs the formation of fractals and its generation. Figure 5 (b) shows the effect of gap width on the fractal formation. The least significant parameter among others is gap width. Most significant level of a gap is found to be 0.035 mm. Figure 5 (b) clearly shows the least effect of gap width on fractals formation. This is because the fluid does not form the desired compressed thin plane as separation gap increases. Hence the creep failure of fluid desired at the fluid plate interface is not observed. For higher separation gap top layer of fluid is in contact with the upper plate that contributes to the resistance. Hence, fractals fail to form under this scenario. Further, it is observed that gap width and kinematic viscosity is the most important parameter to control shielding distance $r_1$ of fractals.

Figure 5. Final Formed Fractal Structures.
(a) b=0.035 mm, V=15mm/min, $\nu$ = 208542 mm$^2$/min. (b) b=0.135 mm, V=1.5mm/min, $\nu$ = 260677 mm$^2$/min.; (c) b=0.135 mm, V=15mm/min, $\nu$ = 260677 mm$^2$/min.;(d) Shielding distance measurement ;( e) b=0.035 mm, V=0.75mm/min, $\nu$ = 312813 mm$^2$/min.
4. Mathematical Dimensionless Model

A mathematical model is presented for generalization. Model is expressed in terms of a dimensionless term \( \frac{r}{R} \) which is a ratio of shielding distance to the radius of a circular plane of fluid and Reynolds number \( (Re) \). Reynold number is ratio of the product of velocity and gap width to kinematic viscosity. Surface tension and quantity of fluid is considered as constant while defining model. Model helps to find out the term \( \frac{r}{R} \) with the help of input parameters (gap width, velocity and kinematic viscosity). Model is based on the experimental result. The correlation is expressed in Eq. 1.

\[
\frac{r}{R} = P_1 + P_2 Re
\]

where \( P_1 \) and \( P_2 \) are constant.

Properties of fluid viz. kinematic viscosity, surface tension are governing factor for the growth of micro fractals, but kinematic viscosity is only considered in defining a model. Surface tension is not considered. This is a limitation of the mathematical model. As surface tension is not taken into account, maximum error recorded with the experimental and model value is 19.62%. This error can be reduced if surface tension would have taken into consideration. Model is defined considering three parameter fluid separation velocity, gap width, and fluid property: kinematic viscosity. Total 27 experiments were conducted for three parameters and correlations found is as shown in table 2.

| Kinematic Viscosity | Gap Width | Separation Velocity |
|---------------------|-----------|---------------------|
| \( r_1/R = -348345.606 x (Re) + 1.28799592 \) | \( r_1/R = -17639.5 x (Re) + 0.27643 \) | \( r_1/R = 9020.725 x (Re) + 0.22166075 \) |
| \( r_2/R = -540880.16 x (Re) + 1.95323852 \) | \( r_2/R = -14209.5 x (Re) + 0.34223 \) | \( r_2/R = 9760.45 x (Re) + 0.2268215 \) |
| \( r_3/R = 733418.41x (Re) + 2.61849388 \) | \( r_3/R = -10779.5 x (Re) + 0.40803 \) | \( r_3/R = 23075.5 x (Re) + 0.319715 \) |

![Figure 6. Effect of gap width on attributes of fractals.](image)

Table 2. Correlation Table.
Figure 7. Effect of kinematic viscosity on attributes of fractals.

Figure 8. Effect of kinematic separation velocity on attributes of fractals.

Figure 6, 7 and 8 are graphical representation of dimensionless term \( r/R \) and gap width, kinematic viscosity and fluid separation velocity and for experimental and model value. The graph is almost linear in nature. The experimental and model value is almost similar. The maximum error recorded 12.36 %, 19.62 % and 11.52% for kinematic viscosity, gap width, and fluid separation velocity respectively.

5. Conclusion
This paper presented the process of fingering formation and its analysis. The controlling parameters considered viz. fluid separation velocity, gap width and fluid properties density and viscosity to study behavior of finger formation. A mathematical model is presented through this paper which helps to find out the dimensionless term \( r/R \). It is a ratio of shielding distance to the radius of the circular fluid layer. The governing factor of a model is Reynolds number. The ratio \( r/R \) is plotted against kinematic viscosity, fluid separation velocity and gap width and experimental values of \( r/R \) are compared with model value against various attributes. Maximum error recorded was 19.62 %. This error can be reduced if surface tension would have taken into consideration.

Exhaustive Characterization of non-Newtonian resin for the formation of meso and micro fractals are presented in this work. The effect of the velocity of fluid separation, kinematic viscosity of the non-Newtonian resin and the gap width on different attributes of meso and micro fractal are presented. The characterization of the process indicated that the controlled structure of the viscous finger model is
typically possible to produce higher kinematic viscosity and higher velocity. In the case of a highest value of viscosity (312813 mm$^2$/min) and velocity 15 mm/min gives a higher value of shielding distance $r_1$ (8 mm), but if the velocity remains at a low value, there is a decrease in $r_1$. In case of gap width pattern behavior is different. The shielding distance $r_1$ has a maximum value at 0.135 m of gap width. But shielding distance $r_1$ goes on decreasing with increase or decrease in value of gap width. The experiment is conducted in such a way to control third generation viscous fingering. In particular, at higher fluid separation speed and viscosity with the lower value of gap width, a good pattern is developed with large shielding distance. The characterization of the process established the roadmap to imitate micro fractals present in nature or living things. Furthermore, for the biological study and research, complex forms of microchannels can be fabricated through this process.

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