Centrality dependence of chemical freeze-out parameters from net-proton and net-charge fluctuations using hadron resonance gas model

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We estimate chemical freeze-out parameters in HRG and EVHRG model by fitting the experimental information of net-proton and net-charge fluctuations measured in Au + Au collisions by the STAR collaboration at RHIC. We observe that chemical freeze-out parameters obtained from lower and higher order fluctuations are though almost same for $\sqrt{s_{NN}} > 27$ GeV, tend to deviate from each other at lower $\sqrt{s_{NN}}$. Moreover, these separations increase with decrease of $\sqrt{s_{NN}}$ and for a fixed $\sqrt{s_{NN}}$ increase towards central collisions. Furthermore, we observe an approximate scaling behaviour of $(\mu_B/T)/(\mu_B/T)_{cent}$ with $(N_{part})/(N_{part})_{cent}$ for the parameters estimated from lower order fluctuations for $11.5 \text{ GeV} \leq \sqrt{s_{NN}} \leq 200$ GeV. Scaling is violated for the parameters estimated from higher order fluctuations for $\sqrt{s_{NN}} = 11.5$ and 19.6 GeV. It is observed that the chemical freeze-out parameter, which can describe $\sigma^2/M$ of net-proton very well in all energies and centralities, can not describe the $s\sigma$ equally well and vice versa.

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I. INTRODUCTION

Relativistic heavy ion collisions are investigated both theoretically and experimentally to understand the properties of nuclear matter at extreme conditions. In heavy ion collisions, there is a possibility for the nuclear matter to undergo a phase transition to quark matter. The nature of the phase transition is still not well established. At low baryon chemical potential and high temperature nuclear matter is expected to smoothly cross over to a quark gluon plasma (QGP) phase. Whereas, at high baryon chemical potential and low temperature the system is expected to have a first order phase transition [2-4]. Therefore, the first-order phase transition at high baryon chemical potential and low temperature should end at a critical end-point (CEP) as one moves towards a high temperature and low baryon chemical potential region in the phase diagram of strongly interacting matter [5, 6]. The main goal of experiments of heavy ion collisions is to map the quantum chromodynamics (QCD) phase diagram in terms of temperatures and baryon chemical potentials. One of the main objectives of the beam energy scan (BES) program of RHIC is to investigate the location of CEP. In the near future, CBM experiment at FAIR will also involve in such an investigation along with the other studies of strongly interacting matter at high baryon chemical potentials and low temperatures.

The event-by-event fluctuations of conserved charges like baryon, strangeness, and electric charge are sensitive indicators of the transition from hadronic matter to QGP. Moreover, the existence of the CEP can be signalled by the divergent fluctuations. Therefore, a non-monotonic variation of observables related to the cumulants of the distributions of the above mentioned conserved charges with a variation of centre of mass energy ($\sqrt{s_{NN}}$) are believed to be good signatures of a phase transition and a CEP [6, 10]. However, this non-monotonic behaviour is a necessary but not sufficient condition for the CEP. For example, the singularities associated with first or second order transition, in the infinite volume limit, may become finite peaks due to finite volume effect. Moreover, due to the finite size of the system in heavy ion collisions, non-monotonic behaviour may be indicative of pseudo-critical region which is shifted from the actual critical region [11-13].

It may be expected that with the variation of centrality, keeping $\sqrt{s_{NN}}$ fixed, similar behaviour as those found for the variation of centre of mass energy would be observed. However, the signatures of phase transition or CEP are detectable only if they survive during the evolution of the system. Several experimental results of conserved charge fluctuations (or cumulants) from BES program have recently been reported at various energies and centralities [10, 14, 15]. However, these data do not show non-monotonic behaviour as a function of $\sqrt{s_{NN}}$. On the other hand, a new analysis of net proton moments have been reported by STAR collaboration [16] where the upper $p_T$ coverage for proton and anti-proton has been extended up to 2 GeV using the time of flight (ToF) detector. In this analysis a non-monotonic behaviour for higher order cumulants ($k n^2$) at lower $\sqrt{s_{NN}}$ has been reported indicating a probable CEP like behaviour. Finite system size may also cause this non-monotonic behaviour. In principle such effects may be estimated from the ratio of cumulants, as discussed in [17] using Hadron resonance gas (HRG) model for illustration. It has been shown that though for net proton and net kaon the cumulant ratios are almost volume independent, the cumulant ratios of net charge are highly sensitive to the system volume. This is mainly due to the contribution of pions...
which are extremely light in the hadronic scale.

Fluctuations which are related to the thermodynamic susceptibilities via fluctuation-dissipation theorem [18] can be studied using LQCD or models. However, since cumulants are volume dependent, ratios of cumulants are constructed to cancel volume term and they are related to the ratios of the different order of susceptibilities. Therefore, it is possible to extract chemical volume term and they are related to the ratios of different volume dependent, ratios of cumulants are constructed to cancel volume term and they are related to the ratios of the different order of susceptibilities. Thus the ratios of cumulants of conserved charges provide important information about chemical freeze-out parameters which is useful to locate CEP in the phase diagram. However, at finite chemical potential, LQCD faces the well-known sign problem. As a result, the region of very high chemical potential in the phase diagram can not be studied in LQCD presently. Moreover, it is not possible to employ experimental acceptance cuts in LQCD calculation. On the other hand, hadron resonance gas (HRG) model [17, 21–48] provides us with a simpler model for the study of the strongly interacting matter in the non-perturbative domain. HRG model is based on the assumption of thermal equilibrium of a system composed of free hadrons and resonances. One may estimate the commensurate chemical freeze-out parameters by fitting the experimental data of various hadronic observables with the HRG model [33, 34, 49–53]. Also the susceptibilities of conserved charges calculated in LQCD have been well reproduced by HRG model [31, 32, 37, 32] for temperatures up to 150 MeV. Moreover the region of large chemical potential in the phase diagram, which can be accessed by low energy heavy ion collisions, can be studied by this model. Since, one can incorporate proper experimental acceptances in this model, it can be used to estimate chemical freeze-out parameters by fitting experimental data of the ratios of cumulants of conserved charges. It should be noted however that the final parameters are still model dependent.

Here we would like to emphasise the salient feature of our present study. If the system becomes thermalised well ahead of freeze-out, then all the observables would carry the signature of thermalisation. In such a scenario the observed hadrons should have a clear thermodynamic equilibrium distribution. Therefore a thermal model like HRG would show a very good agreement with the data up to all orders. Any difference from this scenario may point towards a more complex system and our attempt here is to find such discrepancies in the parametrisation of the HRG model from various experimental data and gain some insight about the system.

The paper is organised as follows. The ideal and excluded volume hadron resonance gas model are introduced in Sec. II. In Sec. III we have briefly discussed fluctuations of conserved charges and several relevant experimental observables. Then in Sec. IV we have discussed results of this paper. Finally, we summarise our results in Sec. V.

II. HADRON RESONANCE GAS MODEL

In HRG model, the system of thermal fireball consists of all the hadrons and resonances given in the particle data book [53]. There are varieties of HRG models in the literature. Different versions of this model and some of the recent works using these models may be found in Refs [17, 21–48]. HRG model is not only successful in describing the hadron yields in central heavy ion collisions from AGS up to RHIC energies [24, 25, 27, 29, 33, 55] but also in describing the bulk properties of hadronic matter in thermal and chemical equilibrium [31, 32, 37]. The logarithm of the grand canonical partition function of a hadron resonance gas can be written as [37],

$$\ln Z^{id} = \sum_i \ln Z^{id}_i,$$

where the sum is over all the hadrons. $id$ refers to ideal i.e., non-interacting HRG. For particle species $i$,

$$\ln Z^{id}_i = \pm \frac{V}{(2\pi)^3} \int d^3p \ln[1 \pm \exp(-E_i - \mu_i)/T)],$$

where $V$ is the volume of the system, $g_i$ is the degeneracy factor, $T$ is the temperature, $E_i$ is the single particle energy, $n_i$ is the mass and $\mu_i = B_i\mu_B + S_i\mu_S + Q_i\mu_Q$ is the chemical potential. In the last expression, $B_i, S_i, Q_i$ are respectively the baryon number, strangeness and charge of the particle, $\mu$'s are corresponding chemical potentials. The upper and lower sign corresponds to baryons and mesons respectively. We assume that the hadronic matter is in thermal and chemical equilibrium therefore we have ignored non-equilibrium phenomena like decays of particles along with minimum biased jets and harmonisation. We have ignored the effect of parton fragmentation into hadrons which produces very significant correlations at lower energies and in peripheral collisions as high as 200 GeV [55, 56]. In addition, at lower collision energies stopping becomes important which has not been considered here. The partition function is the basic quantity from which one can calculate various thermodynamic quantities of the thermal system. The number density $n_i$ of $i$th particle is defined as,

$$n_i = \frac{T}{V} \left( \frac{\partial \ln Z_i}{\partial \mu_i} \right)_{V,T} = \frac{g_i}{(2\pi)^3} \int \frac{d^3p}{\exp[(E_i - \mu_i)/T]} \pm 1.$$

In case of heavy ion collision experiments, the parameters $T$ and $\mu$'s of HRG model corresponds to those at chemical freeze-out which depend on initial conditions of the collision. The chemical potentials $\mu_B, \mu_S$ and $\mu_Q$ are not independent, but related (on average) to each other as to $T$ via the relations [20],

$$\sum_i n_i(T, \mu_B, \mu_S, \mu_Q)S_i = 0,$$

and

$$\sum_i \sum_j n_i(T, \mu_B, \mu_S, \mu_Q)Q_i = r \sum_i n_i(T, \mu_B, \mu_S, \mu_Q)B_i.$$
where $r$ is the ratio of net-charge to net-baryon number of the colliding nuclei. For Au + Au collisions $r = N_p/(N_p + N_n) = 0.4$, where $N_q$ and $N_n$ are respectively proton numbers and neutron numbers of the colliding nuclei. The Eq. 4 is due to fact that initially there is no net-strangeness in the colliding nuclei. In terms of transverse momentum ($p_T$) and pseudo-rapidity ($\eta$), the volume element $d^3p$ and the single particle energy $E_i$ can be written as $d^3p = 2\pi p_T^2 \cosh \eta \, dp_T \, dy$ and $E_i = \sqrt{(p_T \cosh \eta)^2 + m_i^2}$, respectively. Instead of pseudo-rapidity, one can use rapidity ($y$) as well. In that case $d^3p$ and $E_i$ respectively can be written as $d^3p = 2\pi p_T m_T \cosh y \, dp_T \, dy$ and $E_i = m_T \cosh y$, where $m_T = \sqrt{p_T^2 + m_i^2}$. The experimental acceptances can be incorporated by considering the appropriate integration ranges, either in $p_T$ and $\eta$ or in $p_T$ and $y$.

A. Excluded volume corrections

In ideal HRG model point like particles are considered. Although, attractive interactions between hadrons are incorporated through the presence of resonances, repulsive interactions are ignored in this framework. This simple model has few parameters only. Despite its simplicity, this model successfully describes the bulk properties of the system created in heavy ion collisions. The repulsive interactions are also needed, especially at very high temperature and/or large baryon chemical potential, to catch the basic qualitative features of strong interactions where ideal gas assumption becomes inadequate. In the EVHRG model [21–23, 26, 36, 37], hadronic phase is modeled by a gas of interacting hadrons, where the geometrical sizes of the hadrons are explicitly incorporated as the excluded volume correction, to approximate the short-range repulsive hadron-hadron interaction.

III. FLUCTUATIONS OF CONSERVED CHARGES

Derivatives of the $\ln Z$ with respect to corresponding chemical potential define susceptibilities, which experimentally become accessible through event-by-event analysis of fluctuations of conserved quantities such as net-baryon number, net-charge, net-strangeness and others. The $n$th order susceptibility is defined as,

$$\chi_q^n = \frac{1}{V T^3} \frac{\partial^n (\ln Z)}{\partial (\mu_T^n)}$$

(6)

where $\mu_q$ is the chemical potential for conserved charge $q$.

Experimentally net-chargers $N_q (= N_q^+ - N_q^-)$ are measured in a finite acceptance on an event by event basis. The mean ($M_q$), variance ($\sigma_q^2$), skewness ($S_q$) and kurtosis ($\kappa_q$) of net-charge distribution are related to the different order of susceptibilities by the following relations:

$$M_q = \langle N_q \rangle = V T^3 \chi_q^3,$$

(7)

$$\sigma_q^2 = \langle (\delta N_q)^2 \rangle = V T^3 \chi_q^2,$$

(8)

$$S_q = \frac{\langle (\delta N_q)^3 \rangle}{\sigma_q^3} = \frac{V T^3 \chi_q^3}{(V T^3 \chi_q^2)^{3/2}},$$

(9)

$$\kappa_q = \frac{\langle (\delta N_q)^4 \rangle}{\sigma_q^4} - 3 = \frac{V T^3 \chi_q^4}{(V T^3 \chi_q^2)^2},$$

(10)

where $\delta N_q = N_q - \langle N_q \rangle$. The mean, variance, skewness and kurtosis are respectively estimations of the most probable value, width, asymmetry and the peakedness of the distribution. From the above equations, volume independent ratios can be defined by the following relations:

$$\frac{\sigma_q^2}{\langle M_q \rangle} = \frac{C_2}{C_1} = \frac{\chi_q^2}{\chi_q^3},$$

(11a)

$$S_q \sigma_q = \frac{C_3}{C_2} = \chi_q^3/\chi_q^2,$$

(11b)

$$\kappa_q \sigma_q^2 = \frac{C_4}{C_2} = \chi_q^4/\chi_q^2,$$

(11c)

where $C_n$ is the $n$th order cumulants of the charge distribution. The STAR collaboration has reported results of the above-mentioned observables of net-proton and net-charge at different energies ranging from 7.7 GeV to 200 GeV and at various centralities [14]. The PHENIX collaboration has also reported results of similar observables for net-charge [18]. Non-monotonic variations of these ratios with beam energy ($\sqrt{s_{NN}}$) and also with centrality at a fixed $\sqrt{s_{NN}}$ are believed to be good signatures of a phase transition and a CEP. These observables have also been studied in different models [17, 20, 41, 42, 47, 57] and also in LQCD [61–64]. Recently $S_\sigma$ and $\kappa \sigma^2$ for charged pions have been studied using non-equilibrium HRG model [48].

IV. RESULTS

In this paper, we have studied fluctuations of net-proton and net-charge using HRG as well as its interacting version i.e. EVHRG model. In Ref. [39, 42], it was shown that the ratios of higher order cumulants are affected by the excluded volume corrections. Further, it was shown that experimental data of $\sigma^2/\langle M \rangle$ for net-proton as well as for net-charge in central Au + Au collisions [42] can be described quite well using this model. Not only that, $S_\sigma$, $\kappa \sigma^2$ can also be described within experimental error for $\sqrt{s_{NN}} \geq 27$ GeV. Therefore, it is very important to consider EVHRG model for the study of fluctuations of conserved charges. On the other hand ratio of cumulants depend on acceptance cuts as well [41, 42, 57]. Therefore, in this work we have used the HRG / EVHRG model with proper experimental acceptances. For our present study we constrained the chemical freeze-out temperature and chemical potentials using some of the net-charge and net-proton measured cumulants and then predicted the others in order to test the model. In all our calculations, we have incorporated all the hadrons listed in the particle data book up to a mass of 3 GeV [54].

A. Centrality dependence of chemical freeze-out parameters

The thermal fireball created due to heavy ion collision expands and cools. After some time inelastic collisions among
The particles stop and hence particle yields (or particle ratios) get fixed. This stage is called chemical freeze-out. From the experimental information about particle yields or particle ratios, chemical freeze-out temperature and baryon chemical potential can be estimated \[ \chi^2 \] minimisation technique where \( \chi^2 \) is defined as,

\[
\chi^2 = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{R_{i}^{\text{expt}} - R_{i}^{\text{model}}}{\sigma_i} \right)^2,
\]

where \( N \) is the number of observables, \( R_{i}^{\text{model}} \) is the \( i \) \text{th} observable with \( R_{i}^{\text{expt}} \) and \( \sigma_i \) being its experimental values and errors respectively (statistical error has been used here). Error bars in the evaluated freeze-out parameters correspond to \( \chi^2 = \chi_{\text{min}}^2 + 1 \). We have taken care of the conservation laws Eqs. 4 and 5 in the evaluation of chemical freeze-out parameters.

First we obtained freeze-out parameters using only lower order cumulant ratios \( \sigma^2 / M \), \( S \sigma \) and \( \kappa \sigma^2 \) of net proton and net-charge. Here we would like to discuss the modus operandi for the estimation of parameter sets listed in table I. We have three experimental cumulant ratios \( \sigma^2 / M \), \( S \sigma \) and \( \kappa \sigma^2 \) for net charge and net proton. It should be noted that \( \sigma^2 / M \) has smaller experimental errors compared to \( S \sigma \) and \( \kappa \sigma^2 \). Not only that, experimental errors are smaller for net-proton fluctuations compared to net-charge data. In order to evaluate the chemical freeze-out parameters from these observables at a particular \( \sqrt{s_{NN}} \) and centrality, we use \( \chi^2 \) minimisation technique where \( \chi^2 \) is defined as,

\[
\chi^2 = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{R_{i}^{\text{expt}} - R_{i}^{\text{model}}}{\sigma_i} \right)^2,
\]

where \( N \) is the number of observables, \( R_{i}^{\text{model}} \) is the \( i \) \text{th} observable with \( R_{i}^{\text{expt}} \) and \( \sigma_i \) being its experimental values and errors respectively (statistical error has been used here). Error bars in the evaluated freeze-out parameters correspond to \( \chi^2 = \chi_{\text{min}}^2 + 1 \). We have taken care of the conservation laws Eqs. 4 and 5 in the evaluation of chemical freeze-out parameters.

First we obtained freeze-out parameters using only lower order cumulant ratios \( \sigma^2 / M \) of net-charge and net-proton. For this we have the two sets CFO1 (HRG) and CFO2 (EVHRG). Then we wanted to check if the freeze-out parameters estimated including the higher order cumulant \( S \sigma \) for net-charge and net-proton agrees with the above set. We found that the extremely high precision of experimental data for \( \sigma^2 / M \) of net-proton completely biases the \( \chi^2 \) minimisation to agree with the earlier set. So finally we used \( \sigma^2 / M \) and \( S \sigma \) of net-charge and \( S \sigma \) of net-proton to extract the second set of

![Graph showing centrality dependence of chemical freeze-out temperatures and baryon chemical potentials for Au + Au collisions at \( \sqrt{s_{NN}} = 200 \), 62.4, 39, 27, 19.6, 11.5 and 7.7 GeV. \( \sqrt{s_{NN}} \) varies column wise. Four sets of chemical freeze-out parameters (CFO) have been plotted. For specifications of different sets of parameters see the table I.](image-url)
parameters CFO3 (HRG) and CFO4 (EVHRG). Note that if equilibrium is complete then any combination of observables should reproduce mutually agreeable set of fitting parameters.

Figure 1 shows centrality dependence of chemical freeze-out $T, \mu_B$ at different $\sqrt{s_{NN}}$ from 7.7 GeV up to 200 GeV. Four sets of chemical freeze-out parameters (CFO1 - CFO4) have been plotted. The average number of participant ($N_{\text{part}}$) is maximum for most central ($0 - 5\%$) collision whereas it is minimum for most peripheral ($70 - 80\%$) collision. In this figure $\sqrt{s_{NN}}$ decreases column wise. The leftmost column of Fig. 1 corresponds to the highest beam energy $i.e.$ $\sqrt{s_{NN}} = 200$ GeV whereas the rightmost column corresponds to $\sqrt{s_{NN}} = 7.7$ GeV.

One would expect that with higher $\sqrt{s_{NN}}$ the particle production would be higher and give rise to a high freeze-out temperature. On the other hand for low $\sqrt{s_{NN}}$ particle production would be less and the collision participants would also contribute actively to the system properties (due to baryon stopping). Thus observed temperature may be low but baryon chemical potential may be large. Note that for complete equilibration at freeze-out, all evolutionary history of the system will be erased. This will be reflected in the agreement of thermodynamic parameters fitted from all possible experimental observables. On the other hand, for incomplete equilibration certain discrepancies among the thermodynamic parameters fitted from different observables may arise. On top of that the presence of jets, hadronic decays, as well as interactions among the hadrons beyond those considered through the excluded volume effects, may also show deviation of the system from that expected from the HRG picture used to model the system.

It can be seen from Fig. 1 that chemical freeze-out temperatures of CFO1 and CFO3 (or CFO2 and CFO4) are almost the same for $\sqrt{s_{NN}} \geq 27$ GeV. There are significant separations between these two sets of parameters for $\sqrt{s_{NN}} < 27$ GeV, and these separations increase towards central collisions for a fixed $\sqrt{s_{NN}}$. Not only that, these separations increase with the decrease of $\sqrt{s_{NN}}$. Overall the spread in temperature for the whole range of $\sqrt{s_{NN}}$ and centrality is within 140-180 MeV. On the other hand the magnitudes of chemical freeze-out baryon chemical potentials ($\mu_B$) increase with decrease of $\sqrt{s_{NN}}$ as well as increase in centrality by about two orders of magnitude. The occurrence of high net-baryon density is expected when the participant nucleons are deposited in the collision region. More or less similar behaviour of $\mu_B$ is reported in Ref. [67, 69] where chemical freeze-out parameters are extracted analysing particle yields measured experimentally [67, 68] or generated by the event generator [65]. The separation of the parameters obtained from CFO1 and CFO3 (or CFO2 and CFO4) are also observed here, but in the opposite direction. The lower order cumulants thus seems to equilibrate with lower temperature and higher density than the higher order cumulants.

The conclusion that one can draw from this observation is that the system formed in the heavy ion collision has not completely equilibrated if we consider only the HRG model to describe it. However it is possible that HRG picture, if suitably modified, may lead to the scenario as found in Fig. 1. Here the question is whether there are any possibilities such that one can find a multicomponent system with different equilibrium parameters that can systematically explain the observed discrepancy for the fitted parameters.

At this point it is tempting to propose a possible scenario that may give rise to such an agreement of thermodynamic parameters for higher $\sqrt{s_{NN}}$ and deviations found for lower $\sqrt{s_{NN}}$. We first assume that in the region of lower values of $\sqrt{s_{NN}}$ the effects coming from the jets are quite small, and hence are not responsible for this deviation. Now if the system has thermalised near or above the phase transition region and then evolved down to the hadronic phase then one can qualitatively describe the situation as follows. For a crossover region the system undergoes rapid changes from partonic to hadronic phase, but all the components can still follow a given equilibrium condition at all times. This is expected to happen for large $\sqrt{s_{NN}}$. However near the critical point, correlation length $\xi$ would tend to infinity and there would be a large enhancement in the fluctuations. In a realistic situation, as in heavy ion collisions, dynamical variables are functions of time. As the system moves towards the critical region, relaxation time increases and at some point the system may expand too fast to maintain thermodynamic equilibrium. So the correlation length gets constrained due to this critical slowing down [70] and becomes frozen at some time. But the system expansion continues further. This situation may lead to the difference in the information carried by the different order of cumulants. More specifically, second, third and fourth order cumulants of multiplicities are related to the correlation length by the relations $\langle (\delta N)^2 \rangle \sim \xi^2$, $\langle (\delta N)^3 \rangle \sim \xi^3$ and $\langle (\delta N)^4 \rangle \sim \xi^4$ respectively [71]. This, in turn, implies that for higher order cumulants their relaxation time to the equilibrium values may be considerably larger compared to those of lower order cumulants. So in the final spectrum, higher order cumulants are expected to carry the information of the system farther from equilibrium compared to lower cumulants. For example, compared to lower moments, the temperature evaluated using the higher moments may be larger as being away from equilibrium system is hotter. This is what we observe for lower $\sqrt{s_{NN}}$ i.e. temperatures of CFO3 / CFO4, where third order fluctuations are involved, are larger compared to that of CFO1 / CFO2 for $\sqrt{s_{NN}} < 27$ GeV, the corresponding chemical potential being smaller than that of CFO1/CFO2. Incidentally this is the range of temperature and baryon chemical potential where close to which the critical end point is expected to lie. Availability of higher moment data with much better statistics is extremely essential for further constraining this picture. We however emphasise that this is only a plausibility argument for effects of a CEP to modify the simple HRG parameters with different cumulants. A systematic study of various other dynamical effects would be required to ascertain how far this picture is valid [72, 73]. Another important caveat is that the contributions due to purely statistical fluctuations in the cumulants reported by the STAR experiment are not subtracted from the variances and rely on models for $S\sigma$ and $\kappa\sigma^2$. Therefore, the sensitivity of the reported cumulants to dynamical effects is ambiguous. In fact it is even difficult to ascertain whether the statistical fluctuations in the data may
to overwhelm the critical fluctuations or not.

Figure 2 shows chemical freeze-out parameters for 0-5% and 70 - 80% in the $(T, \mu_B)$ plane. In this figure, we also compare our results of chemical freeze-out parameters with previous works [20, 53]. In [20], chemical freeze-out parameters for most central collisions were estimated using the experimental data of $\sigma^2/M$ of net-proton and net-charge. In their model they considered effect of the resonance decays, experimental acceptances and randomisation of the isospin of nucleons in the hadronic phase. They excluded chemical freeze-out parameters in the whole range of $\sqrt{s_{NN}}$. It can also be seen that, with decrease of $\sqrt{s_{NN}}$, the temperatures are larger compared to that of CFO3 / CFO4. For CFO3 / CFO4, all the chemical freeze-out parameters are almost flat up to $\sqrt{s_{NN}} = 39$ GeV then it decreases at $\sqrt{s_{NN}} = 27$ GeV and becomes almost flat at $\sqrt{s_{NN}} = 19.6$ GeV and then again decreases. In contrast, the chemical freeze-out $\mu_B$ increases with decrease of $\sqrt{s_{NN}}$ in the whole range of $\sqrt{s_{NN}}$. This behaviour of chemical freeze-out $T$ is in contradiction to what has been reported in the Refs. [33, 34, 38, 49, 53] where chemical freeze-out parameters have been extracted from particle multiplicities. We plot chemical freeze-out $T$ and $\mu_B$ of Ref. [53] for comparison. Refs. [33, 34, 38, 39, 53] showed that chemical freeze-out $T$ rapidly increases with the increase of $\sqrt{s_{NN}}$ in SIS-AGS-SPS energy range and then saturates at top RHIC energy. However, behaviour of chemical freeze-out $\mu_B$ reported in these references were similar. In Fig. 2 we also show chemical freeze-out $(T, \mu_B)$ of CFO3 / CFO4. For both CFO3 and CFO4, chemical freeze-out $T$ increases with decrease of $\sqrt{s_{NN}}$ for $11.5$ GeV $\leq \sqrt{s_{NN}} \leq 200$ GeV and the temperatures are larger compared to that of CFO1 / CFO2. Although the fast rise of $\mu_B$ for higher moments as found in HRG, seems to have slowed down for EVHRG as seen in the figure for CFO4., all the chemical freeze-out parameters are within certain band in the $(T, \mu_B)$ plane. Recently, the possibility of larger chemical freeze-out temperature is indicated also at LHC energy [74].

B. Scaling behaviour of $\mu_B/T$

The Fig. 3 shows variation of $\mu_B/T$ with $N_{part}$. $\mu_B/T$ increases with increase of $N_{part}$ for all energies for all four parameter sets. Moreover, $\mu_B/T$ increases with decrease of $\sqrt{s_{NN}}$. It can also be seen that $\mu_B/T$ for CFO1 / CFO2 are larger compared to CFO3 / CFO4 and differences between the parameters of CFO1 and CFO3 (or CFO2 and CFO4), as shown in Fig. 3, increases when the value of $\mu_B/T$ is close to or greater than unity. In order to separate the effects of $N_{part}$ and $\sqrt{s_{NN}}$, $\mu_B/T$ can be expressed by the relation,

$$\mu_B/T(\sqrt{s_{NN}}, N_{part}) = p(0)(N_{part})^{1/p(1)}(\sqrt{s_{NN}})^{p(2)},$$

(13)

where $p(0), p(1)$ and $p(2)$ are three parameters. In this equation, first part depends only on $N_{part}$ while second part depends only on $\sqrt{s_{NN}}$. For fitting purpose we have simultaneously used $\mu_B/T$ of $\sqrt{s_{NN}} = 19.6$ GeV to 62.4 GeV for CFO1 / CFO2 and $\sqrt{s_{NN}} = 27$ GeV to 200 GeV for CFO3 / CFO4 for which $\chi^2$ per degree of freedom (ndf) is minimum. All the fitting parameters are listed in the table 1. The quality of fitting is quite good as can be seen from the figure. The fitted parameters are then used to estimate $\mu_B/T$ for remaining energies. It can be seen that, for the sets CFO1 / CFO2, the $\mu_B/T$ from Eq. (13) slightly underestimate the extracted $\mu_B/T$ for $\sqrt{s_{NN}} = 11.5$ GeV and 200 GeV. On the other hand, for CFO3 / CFO4, $\mu_B/T$ evaluated using Eq. (13) slightly overestimate the extracted $\mu_B/T$ for peripheral collisions of $\sqrt{s_{NN}} = 11.5$ GeV and slightly overestimate towards central collisions.

In the Fig. 4 we have explored the scaling behaviour of $(\mu_B/T)/(\mu_B/T)_{central}$ with $N_{part}/(N_{part})_{central}$.
FIG. 3. (Color online) Variation of $\mu_B/T$ with $N_{\text{part}}$ (Black points). Blue solid points correspond to the $\mu_B/T$ according to the Eq. 13.

FIG. 4. Scaling behaviour of $\mu_B/T$ with centrality. On the horizontal axis, $N_{\text{part}}$ is normalised with that at the most central collision and similarly in the vertical axis $\mu_B/T$ is normalised to that of the most central collision. Therefore, in the horizontal axis, the maximum value, which is equals to 1, corresponds to the most central collision (0 − 5%) and the minimum value corresponds to the most peripheral collision (70 − 80%).

The presence of scaling is a direct consequence of the fact that one can separate the dependence of $\mu_B/T$ on $\sqrt{s_{NN}}$ and $N_{\text{part}}$ as given by Eq. 13. By construction, $(\mu_B/T)/(\mu_B/T)_{\text{central}}$ becomes independent of $\sqrt{s_{NN}}$. However, since the fitting of $\mu_B/T$ is not perfect (Fig. 3) for all the $\sqrt{s_{NN}}$, scaling as shown in Fig. 4 is also not exact.
TABLE II. Parameters of the fitting function $\mu_B/T = p(0)/(\sqrt{\sigma_s^{(0)}}) p(1)/(\sqrt{\sigma_s^{(1)}}) p(2)/(\sqrt{\sigma_s^{(2)}})$. Since $\mu_B$ is dimensionless, the dimension of $p(0)$ is equals to GeV$^{-1}$.  

| CFO    | $\sqrt{\sigma_s^{(0)}}$ (GeV) | $p(0)$     | $p(1)$     | $p(2)$     | $\frac{\Delta p}{p}$ |
|--------|-------------------------------|------------|------------|------------|-----------------------|
| CFO1   | 19.6-62.4                     | 9.59 ± 1.33| 7.30 ± 1.60| -0.95 ± 0.02| 0.13                  |
| CFO2   | 19.6-62.4                     | 8.95 ± 0.28| 7.56 ± 0.40| -0.92 ± 0.01| 0.17                  |
| CFO3   | 27-200                        | 4.9 ± 0.14 | 7.28 ± 0.34| -0.79 ± 0.01| 0.53                  |
| CFO4   | 27-200                        | 5.23 ± 0.44| 7.29 ± 0.97| -0.81 ± 0.02| 0.11                  |

C. Comparison with experimental data

Here we have used the extracted freeze-out parameters CFO1 and CFO2 to calculate the $\sigma^2/M$, $S\sigma$ and $K\sigma^2$ of net-proton and net-charge using HRG and EVHRG model respectively. Note that for these two sets, chemical freeze-out parameters were estimated from experimental data of $\sigma^2/M$ only. We have compared our results with experimental data of fluctuations measured in Au + Au collisions by STAR collaboration [10, 14]. As mentioned earlier the experimental acceptances have been incorporated in our model calculation as well.

In the top row of Fig. 5 we have shown centrality dependence of $\sigma^2/M$ of net-proton and net-charge at different beam energies. In this figure $\sqrt{s_{NN}}$ varies column wise. Blue and black points have been used for net-proton and net-charge respectively in this figure. For both net-proton and net-charge $\sigma^2/M$ decreases with increase of $N_{\text{part}}$ for all $\sqrt{s_{NN}}$. It
can be seen that the ratios of lowest order susceptibilities (i.e. \( \sigma^2/M \)) of net-proton and net-charge can be reproduced quite well using the CFO1 and CFO2 parameters in our model.

We now evaluate the higher order susceptibility ratios using these two parameter sets. The middle row of Fig. 5 shows centrality dependence of \( S_\sigma \) of net-proton and net-charge. The quantity \( S_\sigma \) for both net-proton and net-charge increases with increasing \( N_{\text{part}} \) for \( \sqrt{s_{NN}} \geq 11.5 \) GeV. Experimental data of \( S_\sigma \) of net-proton also shows a similar trend.

The experimental data of \( S_\sigma \) of net-proton matches within the error bar for \( \sqrt{s_{NN}} > 27 \) GeV. However, its value is over-estimated at lower energies (\( \sqrt{s_{NN}} \leq 27 \) GeV). On the other hand \( S_\sigma \) of net-charge calculated in the HRG / EVHRG model are close to or within the error bars of experimental data for all centralities at \( \sqrt{s_{NN}} \geq 27 \) GeV and for \( \sqrt{s_{NN}} < 27 \) GeV most central data matches within error bars. In general \( S_\sigma \) of net-charge shows a monotonic behaviour and differs considerably from experimental data.

The bottom row of Fig. 5 we have shown centrality dependence of \( \kappa_\sigma^2 \) of net-proton and net-charge. The \( \kappa_\sigma^2 \) for net-proton calculated in our model using CFO1 / CFO2 are almost independent of centrality for \( \sqrt{s_{NN}} \geq 19.6 \) GeV and below that energy it decreases slightly with increase of \( N_{\text{part}} \). For all \( \sqrt{s_{NN}} \), \( \kappa_\sigma^2 \) of net-proton calculated in the HRG / EVHRG model are within the error bars or very close to the experimental data. The experimental data of \( \kappa_\sigma^2 \) for net-charge matches within the error bar with the HRG / EVHRG model results as we go towards central collisions for \( \sqrt{s_{NN}} \geq 11.5 \) GeV.

Therefore we see that the HRG prediction of higher order cumulants calculated from the estimated freeze-out thermodynamic parameters for the lower order cumulants do not match the experimental data in general. This implies that the equilibration at freeze-out is not quite comprehensive vis-a-vis the HRG model. We can further check what happens when we consider one more higher order cumulant as discussed below.

Fig. 6 correspond to similar plot, as in Fig. 5, for parameter sets CFO3 and CFO4. While \( \sigma^2/M \) for both net-proton and net-charge show consistency when compared to the experimental data as shown in the top row, the use of CFO3 / CFO4 in the HRG / EVHRG model, show clear improvement in agreement with experimental data of \( S_\sigma \) for net-proton at all \( \sqrt{s_{NN}} \) as shown in the middle row. On the other hand, there is almost no change in the results for \( S_\sigma \) of net-charge.
V. DISCUSSION AND CONCLUSION

We have extracted the chemical freeze-out parameters by fitting the experimental data of cumulants of net-proton and net-charge measured by STAR collaboration using both HRG and EVHRG model. We have incorporated the proper experimental acceptances in our calculation. However, the dynamical effects such as particle decay, minimum biased jet, baryon stopping are not considered in the present study. The experimental data of $\sigma^2/M$ of both net-proton and net-charge have been used to estimate chemical freeze-out parameters CFO1 / CFO2. On the other hand, parameters CFO3 / CFO4 have been estimated using the experimental data of $\sigma^2/M$ of net-charge and $S \sigma$ of both net-proton and net-charge. For CFO1 and CFO3, HRG model has been used, whereas for CFO2 and CFO4, EVHRG has been used.

The chemical freeze-out parameters evaluated using lower order cumulants (CFO1/CFO2) starts deviating from the one obtained using higher order cumulants (CFO3/CFO4) around $\sqrt{s_{NN}} = 19.6$ GeV as one goes from $\sqrt{s_{NN}} = 200$ GeV towards lower energies. Among other possibilities, transition of the system close to the critical region may contribute to the requirement of multiple parametrisation in HRG for various orders of cumulants. In case of lower energies one need to take into account the baryon stopping as well. In these regions of low energies HRG and EVHRG starts deviating from each other as well due to the effect of repulsive interaction in EVHRG.

We observe that the effect of centrality and beam energy in $\mu_B/T(\sqrt{s_{NN}}, N_{part})$ can be separated. This separation leads to a scaling of $(\mu_B/T)/(\mu_B/T)_{central}$ with $N_{part}/(N_{part})_{central}$. Though the scaling is very good for CFO1 / CFO2, a deviation is observed for CFO3 / CFO4 especially in the region $\sqrt{s_{NN}} \leq 19.6$ GeV. The study of such scaling behaviour will be useful to search for CEP which is the main goal of the ongoing STAR experiment and the future CBM experiment.

Experimental data of lowest order susceptibilities (i.e., $\sigma^2/M$) of net-proton and net-charge can be reproduced quite well using the CFO1 and CFO2 in the HRG / EVHRG model. The experimental data of $S \sigma$ of net-proton match within the error bar for $\sqrt{s_{NN}} > 27$ GeV for these two sets of parameters. However, it is overestimated at lower beam energies ($\sqrt{s_{NN}} \leq 27$ GeV). On the other hand $S \sigma$ of net-charge calculated in the HRG / EVHRG model using CFO1 / CFO2 are close to or within the error bars for $\sqrt{s_{NN}} \geq 27$ GeV and they are within the error bars for more central data at lower $\sqrt{s_{NN}}$. For all $\sqrt{s_{NN}}$, $\kappa \sigma^2$ of net-proton calculated in the HRG / EVHRG model using CFO1 / CFO2 are close to or very close to the experimental data. The experimental data of $\kappa \sigma^2$ for net-charge matches within the error bar with the HRG / EVHRG model results calculated using CFO1 / CFO2 as we move towards central collisions for $\sqrt{s_{NN}} \geq 11.5$ GeV, but underestimate the data for peripheral collisions. This points to the incomplete equilibrium distribution of the particles observed in data. On the other hand experimental data of $S \sigma$ of net-proton can be described well at all $\sqrt{s_{NN}}$ in the HRG / EVHRG model using CFO3 / CFO4. In addition, both the parameter sets give satisfactory description of $\sigma^2/M$ of net-proton and net-charge. However the $\kappa \sigma^2$ for both net-proton and net-charge calculated in the HRG / EVHRG model using CFO3 / CFO4 are similar to those calculated using CFO1 / CFO2. In this set again we found the signature of incomplete equilibration of the system formed in heavy ion collision experiments.

Thus we conclude that the freeze-out parameters, which can describe lower order cumulant ratios very well in all energies and centralities, can’t describe the higher order cumulant ratios equally well. It is difficult to pin-point all the reasons for such disagreement unless all the dynamical effects are accounted for. Looking at the systematic deviation of the thermodynamic parameters we could only present a plausibility argument for the system passing near a critical region. Precise experimental data of $\kappa \sigma^2$ along with few more $\sqrt{s_{NN}}$ around 19.6 GeV will be extremely useful for further investigation in this direction.

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