Propagation of axions in a strongly magnetized medium

A. V. Borisov† and P. E. Sizin

M. V. Lomonosov Moscow State University, 119899 Moscow, Russia

The polarization operator of an axion in a degenerate gas of electrons occupying the ground-state Landau level in a superstrong magnetic field $H \gg H_0 = m_e^2 c^3 / e \hbar = 4.41 \cdot 10^{13}$ G is investigated in a model with a tree-level axion-electron coupling. It is shown that a dynamic axion mass, which can fall within the allowed range of values ($10^{-5} \text{eV} \lesssim m_a \lesssim 10^{-2} \text{eV}$), is generated under the conditions of strongly magnetized neutron stars. As a result, the dispersion relation for axions is appreciably different from that in a vacuum.

1. The a priori strong nonconservation of CP parity in the standard model can be eliminated in a natural manner by introducing axions — pseudo-Goldstone bosons associated with the spontaneous breaking of the additional Peccei–Quinn global symmetry $U(1)_{PQ}$.

According to the experimental data, the energy scale $v_a$ for the $U(1)_{PQ}$ symmetry breaking is much greater than the electroweak scale — $v_a \gtrsim 10^{10}$ GeV, and the constants of the possible couplings of an axion to the standard particles ($\sim 1/v_a$) are very small (the “invisible” axion: see Ref. 4 for a review of various axion models).

Axion effects can be appreciable under the astrophysical conditions of high matter densities, high temperatures, and strong magnetic fields (for example, in neutron stars). Axion production processes, which result in additional energy losses by stars, and the limits obtained by astrophysical methods on the parameters of axion models are examined in Ref. 4. In so doing, the influence of electromagnetic fields was neglected.

The investigation of axion processes in strong magnetic fields commenced comparatively

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†E-mail: borisov@ave.phys.msu.su
recently. The Compton and Primakoff mechanisms of axion production on nonrelativistic electrons by thermal photons ($\gamma + e \rightarrow e + a$) in the presence of a magnetic field are studied in Ref. 6. The extension to relativistic electrons in a constant external electromagnetic field is given in Ref. 7 (Primakoff effect) and Refs. 8 and 9 (Compton effect), where estimates were also obtained for the contributions of the indicated processes to the axion luminosity of a magnetized strongly degenerate relativistic electron gas under the conditions of the crust of a neutron star. A new axion production mechanism — synchrotron emission of axions ($e \rightarrow e + a$) by relativistic electrons — was proposed in Ref. 10 and its contribution to the energy losses by a neutron star was calculated. In Refs. 8 – 10 it was assumed that the external field intensity $F \ll H_0 = m^2_e c^3 / e \hbar \simeq 4.41 \cdot 10^{13}$ G. In Ref. 11, numerical methods were used to extend the results of Ref. 10 to superstrong magnetic fields $H \gtrsim H_0$. It was found that the basic equation derived in Ref. 10 for the axion synchrotron luminosity for the semiclassical case of high electron energies ($\varepsilon \gg m_e c^2$) and fields $H \ll H_0$ agrees with the numerical calculations up to $H/H_0 \lesssim 20$. The axion synchrotron luminosity of neutron stars and white dwarfs was also investigated in Ref. 11.

In Refs. 8 – 11 a model with a derivative axion-electron coupling $eae$, described by the interaction Lagrangian

$$\mathcal{L}_{ae} = \frac{g_{ae}}{2m_e} \left( \overline{\psi} \gamma^\mu \gamma^5 \psi \right) \partial_\mu a, \quad (1)$$

was used. Here $m_e$ is the electron mass and $\gamma^5 = -i \gamma^0 \gamma^1 \gamma^2 \gamma^3$; the system of units such that $\hbar = c = 1$ is used; the signature of the metric is $(+ - - -)$; and

$$g_{ae} = c_e \frac{m_e}{v_a} \quad (2)$$

is a dimensionless coupling constant, where the numerical factor $c_e$ depends on the choice of the specific axion model.

In models where axions are coupled only with heavy fermions by a tree-level coupling there arises an effective direct low-energy axion–photon interaction of the type $\gamma a \gamma$. This interaction is the basis of the Primakoff axion photoproduction mechanism employed in
The synchrotron process $e \rightarrow ea$ in the absence of a tree-level axion–electron coupling was considered recently in Ref. [12]. This process is due to resonant conversion of a longitudinal plasmon (a photon in a medium), emitted by a relativistic electron in a magnetic field, into an axion.

Decay of an axion in a strong magnetic field into a fermion pair ($a \rightarrow f\bar{f}$) and two photons ($a \rightarrow \gamma\gamma$) are also of interest for astrophysics and cosmology.

In the present paper the model (1) is used to calculate the polarization operator of an axion moving in a strongly magnetized degenerate electron gas and the change in the dispersion relation of an axion in a medium is investigated using this operator.

2. Taking account of the contribution of the electrons only (see Eq. (1)) we obtain, using the real-time formalism of the finite-temperature quantum field theory (see, for example, Ref. [15]), the following momentum representation for the one-loop polarization operator of an axion:

$$\Pi(k, k') = -iG_a^2 \int d^4x d^4x' \exp(ikx - ik'x') \text{Tr} \left[ \hat{k}\gamma^5G(x, x')\hat{k}'\gamma^5G(x', x) \right].$$

(3)

Here $k$ ($k'$) is the final (initial) 4-momentum of an axion; $G(x, x')$ is the time-dependent single-particle Green’s function of an ideal electron-positron gas in a constant magnetic field; notations have also been introduced for the contraction $\hat{a} = \gamma^\mu a_\mu$ of a 4-vector $a^\mu$ with the Dirac $\gamma$ matrices and for the dimensional coupling constant

$$G_a = \frac{g_{ae}}{2m_e}.$$  

(4)

On account of the translational invariance (constant external field, homogeneous isotropic medium) the polarization operator (3) is diagonal in momentum space:

$$\Pi(k, k') = (2\pi)^4\delta^{(4)}(k - k')\Pi(k).$$

(5)

Here $\Pi(k)$ determines the axion propagator $D(k)$ in the momentum representation according to the Dyson equation

$$D(k) = \left[k^2 - m_a^2 - \Pi(k)\right]^{-1},$$

(6)
where $m_a$ is the free-axion mass (in the absence of a field and a medium), which is generated by the chiral anomaly of QCD:

$$m_a \sim \Lambda_{QCD}^2/v_a.$$ The renormalized value $\Pi_R(k)$ (see below) gives the dispersion relation

$$k^2 = m_a^2 + \Pi_R(k). \quad (7)$$

3. We give the constant uniform magnetic field $\mathbf{H} \parallel \hat{z}$ in terms of the 4-potential $A^\mu$ in the gauge

$$A^\mu = (0, 0, xH, 0). \quad (8)$$

Then the Green’s function $G(x, x')$ can be represented in the following form after summing over the spin quantum number and the sign of the energy in the general expression for $G$ in the form of a series in quadratic combinations of the eigenfunctions of the Dirac operator:

$$G(x, x') = [\gamma^\mu (i\partial_\mu + eA_\mu) + m_e] K(x, x');$$

$$K(x, x') = \frac{\sqrt{h}}{(2\pi)^3} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} dp_0 dp_y dp_z \exp \left[-ip_0(t - t') + ip_y(y - y') + ip_z(z - z') \right] \times$$

$$\times u_n(\eta) u_n(\eta') (R_n \Sigma_+ + R_n \Sigma_-), \quad (9)$$

$$R_n = \left[p_0^2 - p_z^2 - 2hn - m_e^2 + i0\right]^{-1} + 2\pi i \delta(p_0^2 - p_z^2 - 2hn - m_e^2) N_F(p_0).$$

Here the electron charge $-e < 0$, $h = eH$; $n = 0, 1, 2, ...$ is the principal quantum number (the number of the Landau level); $p_y$ and $p_z$ are the eigenvalues of the operators of the canonical momenta — the constants of motion in the gauge (8); and $u_n(\eta)$ is a Hermite function of argument

$$\eta = \sqrt{h}(x + p_y/h); \quad \eta' = \eta(x \rightarrow x'); \quad \Sigma_\pm = (1 \pm \Sigma_3)/2; \quad \Sigma_3 = i\gamma^1 \gamma^2.$$

The first term in $R_n$ has poles at the points $p_0 = \pm \varepsilon = \pm [m_e^2 + 2hn + p_z^2]^{1/2}$, determining the energy spectrum of an electron in a magnetic field. The second term ($\propto \delta(p_0^2 - \varepsilon^2)$) describes the effect of the electron-positron medium, and
\[ N_F(p_0) = \theta(p_0) \left[ \exp \left[ \beta(p_0 - \mu) \right] + 1 \right]^{-1} + \theta(-p_0) \left[ \exp \left[ \beta(-p_0 + \mu) \right] + 1 \right]^{-1} \] (10)

is expressed in terms of the Fermi distribution function of electrons and positrons in a medium with temperature \( T = 1/\beta \) and chemical potential \( \mu \), and \( \theta(\pm p_0) \) is the Heaviside step function.

4. It is difficult to make a general analysis of the axion polarization operator for arbitrary values of the parameters \( H, T, \) and \( \mu \). In the present paper we confine our attention to superstrong magnetic fields and comparatively low temperatures

\[ H \gg H_0, \quad T \ll \mu - m_e, \] (11)

and we require the chemical potential to satisfy

\[ \mu^2 - m_e^2 < 2h. \] (12)

It follows from Eqs. (11) and (12) that in this case the contribution of positrons in Eq. (10) can be neglected (it is suppressed by the factor \( \exp[-\beta(\mu - m_e)] \)) and the medium is a degenerate gas of electrons occupying the ground-state Landau level \( n = 0 \):

\[ N_F(p_0) = \theta(p_0)\theta(\mu - p_0), \quad p_0 = \sqrt{m_e^2 + p_z^2}. \] (13)

We also limit the range of the axion 4-momentum

\[ |k_0^2 - k_z^2| \ll h. \] (14)

Then the main contribution of virtual (vacuum) electrons and positrons is likewise formed by states with \( n = 0 \). As a result, retaining on the basis of Eqs. (11), (12), and (14) terms with \( n = 0 \) in the sum (9), we obtain the following approximate expression for the Green’s function in a superstrong magnetic field:

\[
G(x, x') \simeq \left( \frac{\hbar}{\pi} \right)^{1/2} \int_{-\infty}^{\infty} \frac{dp_y}{2\pi} \exp \left[ -\frac{1}{2}(\eta^2 + \eta'^2) + ip_y(y - y') \right] \times \\
\times \int \frac{d^2p}{(2\pi)^2} \exp [-ip_0(t - t') + ip_z(z - z')] G(p) \Sigma_-. \] (15)
Here \( p = (p_0, 0, 0, p_z) \) and

\[
G(p) = (\hat{p} + m_e) \left[ (p^2 - m_e^2 + i0)^{-1} + 2\pi i \delta(p^2 - m_e^2) N_F(p_0) \right] \tag{16}
\]

is the Fourier transform of the Green’s function in the two-dimensional space \((0, 3)\). For \( N_F = 0 \) (no medium) the expression (15) is the well-known, effectively two-dimensional, electron propagator used in the theory of electrodynamic processes in superstrong magnetic fields and, specifically, for investigation of the photon polarization operator.\[14\]

5. Let us substitute the expression (15) into Eq. (3) and integrate over \( t, t', y, y', z, \) and \( z' \). This gives in the form of a product of delta functions

\[
\prod_{n=0,y,z} \delta(k'_n - k_n) \delta(p'_n + k_n - p_n)
\]

the laws of conservation of energy and of the corresponding projections of the momentum. The subsequent calculation of the Gaussian integrals over \( x, x' \) and the trivial integral over \( p_y \) gives \( \delta(k'_x - k_x) \). As a result, as should be the case, we obtain a diagonal representation of the polarization operator (5), where

\[
\Pi(k) = \frac{G^2}{\pi} \hbar \exp \left( -\frac{k^2}{2\hbar} \right) \left[ F(l) + M(l) \right],
\tag{17}
\]

\[
F(l) = -i \int \frac{d^2p}{(2\pi)^2} T(l, p) \left[ p^2 - m_e^2 + i0 \right]^{-1} \left[ (p - l)^2 - m_e^2 + i0 \right]^{-1},
\tag{18}
\]

\[
M(l) = 2\pi \int \frac{d^2p}{(2\pi)^2} \delta(p^2 - m_e^2) N_F(p_0) \left[ \frac{T(l, p)}{(p - l)^2 - m_e^2 + i0} + (l \to -l) \right].
\tag{19}
\]

Here \( p = (p_0, 0, 0, p_z) \) and \( l = (k_0, 0, 0, k_z) \) are two-dimensional vectors, and

\[
T(l, p) = \frac{1}{2} \text{Tr} \left[ \hat{k} \gamma^5 (\hat{p} + m_e) \Sigma_- \hat{k} \gamma^5 (\hat{p} - \hat{l} + m_e) \Sigma_- \right].
\tag{20}
\]

In Eq. (17) the function \( F \) corresponds to the purely field contribution, and \( M \) describes the influence of the medium. We note that \( M \) does not contain a term \( \propto N_F(p_0) N_F(p_0 - l_0) \), since
\[ \delta(p^2 - m_e^2)\delta((p - l)^2 - m_e^2) \theta(p_0)\theta(p_0 - l_0) = 0. \]

Using the relations

\[ [\Sigma_-, \hat{p}] = 0, \quad [\Sigma_-, \gamma^5] = 0, \quad \gamma^n\Sigma_- = \Sigma_+\gamma^n \quad (n = 1, 2), \quad \Sigma_+\Sigma_- = 0 \]

the trace of Eq. (20) reduces to a two-dimensional form and can be easily calculated as

\[ T(l, p) = \frac{1}{4}\text{Tr} \left[ \hat{l}(\hat{p} + m_e) \hat{l} \left( \hat{p} - \hat{l} - m_e \right) \right] = 2(lp)^2 - l^2(lp + p^2 + m_e^2). \] (21)

We calculate the Gaussian integrals over \( p_0 \) and \( p_z \) in Eq. (17) using the trace (21) and the well-known Fock–Schwinger proper-time representation for propagators of the form

\[ (\Delta + i0)^{-1} = -i \int_0^\infty ds \exp[i s(\Delta + i0)]. \]

As a result, we find for the function \( F(l) \equiv F(l^2, m^2) \) the integral representation

\[ \overline{F}(l^2) = -i m_e^2 \tau \int_0^1 dv \int_0^\infty dx \left\{ [1 + (1 - v^2)\tau] \exp[-ix \left[ 1 - (1 - v^2)\tau \right]] - \exp(-ix) \right\}, \] (22)

\[ \tau = l^2/4m_e^2. \]

Here regularization is performed according to the well-known rule

\[ \overline{F}(l^2) = F(l^2, m^2) - F(l^2, \Lambda^2) \]

with the regulator mass \( \Lambda \to \infty \).

The renormalized polarization operator \( \Pi_R(\tau) \) can be obtained \( \beta \) from the dispersion relation with one subtraction (as well as it was done for the photon polarization operator in Ref. 16)

\[ \frac{1}{\tau} \Pi_R(\tau) = \frac{1}{\pi} \int_0^\infty \frac{dt \text{Im} \Pi(t)}{t(t - \tau - i0)}. \] (23)

Eq. (22) gives immediately
\[
\text{Im} F = -\frac{l^2}{2} \int_0^1 dv \delta [1 - (1 - v^2)\tau] = -m_e^2 \theta(\tau - 1) \left(1 - \frac{1}{\tau}\right)^{-1/2}.
\]

(24)

For \( \tau < 0 \) we obtain from Eqs. (23), (24), and (17) the field contribution

\[
\Pi_R^{(F)} = -\frac{\alpha_a}{\pi} m_e^2 \frac{H}{H_0} \exp \left(-\frac{k^2}{2h}\right) \left(\frac{1 - \xi}{1 + \xi}\right) \ln \xi
\]

(25)

to the axion polarization operator. Here \( \alpha_a = g^2_{ae}/4\pi \) (see Eq. (4)), and the standard variable \( \xi \) was introduced as

\[
\tau = -\frac{(1 - \xi)^2}{4\xi},
\]

(26)

which is convenient for analytical continuation in \( l^2 = 4m_e^2\tau \).

For \( \tau > 1 \), a channel is open for axion decay into an electron-positron pair \( (a \to e^-e^+) \) in a magnetic field. Its rate \( w \) for a real axion is related with the imaginary part of the polarization operator on the mass shell by the well-known relation

\[
w = -\frac{1}{\omega} \text{Im} \Pi_R^{(F)} = \alpha_a m_e^2 \frac{H}{H_0} \exp \left(-\frac{k^2}{2h}\right) \theta(\tau - 1) \left(1 - \frac{1}{\tau}\right)^{-1/2},
\]

(27)

where \( \omega \) is the axion energy.

This result, which follows from Eq. (25) with \( \xi = |\xi| \exp(i\pi) \) (see Eq. (26)), is identical to the result obtained in Ref. [13] on the basis of a calculation of the elastic scattering amplitude of an axion in a magnetic field. It can also be found at once from Eqs. (24) and (17).

Let us consider the contribution \( M \) (19) of the medium to the axion polarization operator. We note that it does not renormalize. Integrating over \( p_z \) in Eq. (19), using the delta function and taking account of Eqs. (13) and (21), gives

\[
M = -\frac{m_e^2}{2\pi} \int_{m_e}^{\mu} \frac{d\varepsilon}{q} \left[D(l,p) + D(-l,p) + D(l,\tilde{p}) + D(-l,\tilde{p})\right],
\]

\[
D(l,p) = \left[l^2 - 2lp + i0\right]^{-1}.
\]

(28)

Here \( \varepsilon \) is the energy of electrons in the medium, \( q = \sqrt{\varepsilon^2 - m_e^2} \), and the two-dimensional scalar products are \( lp = k_0\varepsilon - k_zq \) and \( l\tilde{p} = k_0\varepsilon + k_zq \).
The imaginary part of the expression (28) is determined using Sokhotskii’s formula

\[
\frac{1}{x + i0} = \text{P} \frac{1}{x} - i\pi \delta(x),
\]

(29)

where P signifies a principal value. From Eqs. (28) and (29) we obtain on the mass shell

\[
\text{Im} \, M = \frac{m_e^2}{2} \theta(\tau - 1) \left[ \theta(\mu - \varepsilon_+) + \theta(\mu - \varepsilon_-) \right],
\]

\[
\varepsilon_\pm = \frac{\omega}{2} \pm \frac{k_z}{2} \left( 1 - \frac{1}{\tau} \right)^{1/2}.
\]

(30)

Here \(\varepsilon_\pm\) are the roots of the equations \(l^2 - 2\omega \varepsilon \pm 2k_z q = 0\).

From Eqs. (17), (24), and (30) we find the rate

\[
w_M = \frac{1}{2} \left[ \theta(\varepsilon_+ - \mu) + \theta(\varepsilon_- - \mu) \right] w,
\]

(31)

where \(w\) is the decay rate (27) in the absence of a medium, for the axion decay into an \(e^- e^+\) pair in the presence of a magnetized degenerate electron gas. We underscore that the imaginary part of the contribution (30) of the medium is positive, and summed with the negative field contribution (24) it gives a blocking Pauli factor \(1 - \theta(x) = \theta(-x)\) in Eq. (30). It forbids electron production inside a filled Fermi sphere (for \(\varepsilon_\pm < \mu\)).

Taking account of Eq. (29), we obtain for the real part of Eq. (28) on the mass shell the representation

\[
\text{Re} \, M = -\frac{m_e^2}{\pi \tau} \int_0^\lambda dx \left[ \frac{1}{\tau - \cosh^2(x - \psi)} + \frac{1}{\tau - \cosh^2(x + \psi)} \right].
\]

(32)

Here the substitution of the variable \(\varepsilon \rightarrow x\) was used: \(\varepsilon = m_e \cosh x\) and \(q = m_e \sinh x\), and the parameters \(\lambda\) and \(\psi\), defined as

\[
\cosh \lambda = \frac{\mu}{m_e}, \quad \tanh \psi = \frac{k_z}{\omega},
\]

(33)

were introduced. The integral (32) can be expressed in terms of elementary functions.

We shall confine our attention below to the limiting cases that are of interest for astrophysical applications.
6. For an axion on the mass shell

\[ l^2 = 4m_e^2 \tau = \omega^2 - k_z^2 = m_a^2 + k_\perp^2 > 0, \]  

(34)

and the condition (14) gives \( k_\perp^2 \ll h \), so that \( \exp(-k_\perp^2/2h) \simeq 1 \). We note that the imaginary part of the polarization operator is formed by the contribution of real electrons and positrons, and the expression for it holds under the weaker condition \( k_\perp^2 < 2h \). Therefore the exponential factor can be retained in Eq. (27).

For \( \tau \ll 1 \) (substantially below the threshold of the decay process \( \alpha \rightarrow e^-e^+ \)), we find from Eqs. (25), (32), and (17)

\[ \Pi_R = \Pi_R^{(F)} + \Pi_R^{(M)} = -\alpha_a \frac{m_e^2}{\pi} H H_0 \tau (2 - \nu_+ - \nu_-) . \]  

(35)

Here

\[ \nu_\pm = \tanh(\lambda \pm \psi) = \frac{\nu \omega \pm k_z}{\omega \pm \nu k_z}, \quad \nu = \tanh \lambda = \left[ 1 - \left( \frac{m_a}{\mu} \right)^2 \right]^{1/2}. \]

We note that \( \Pi_R < 0 \) and if the axion moves in the direction of the field \( H \) \( (k_\perp = 0) \), then according to Eqs. (34) and (35) \( \Pi_R \rightarrow 0 \) in the limit of a massless axion \( (m_a \rightarrow 0) \).

At high energies \( (\tau \gg (\mu/m_e)^2 \gg 1) \) we obtain for the polarization operator the asymptotic representation

\[ \Pi_R = \frac{\alpha_a}{\pi} m_e^2 \frac{H}{H_0} \ln \left[ \tau \left( \frac{m_e}{\mu} \right)^2 \right] - i\pi \simeq \frac{2\alpha_a}{\pi} e H \left[ \ln \frac{k_\perp}{2\mu} - i\frac{\pi}{2} \right], \]  

(36)

and it does not depend of the electron mass \( m_e \) as it should be in this limit.

Let us write the dispersion relation (7) in the form

\[ \omega^2 = k_\perp^2 + k_z^2 + m_a^2 + \Pi_R(k). \]  

(37)

It follows from Eqs. (35) – (37) that in a magnetized medium a radiative shift of the axion mass is generated — a dynamic mass, whose square, according to the definition in Ref. 13, is

\[ \delta m_a^2 = \text{Re} \Pi_R. \]
For $\tau \gtrsim 1$ and $\mu/m_e \gg 1$ we obtain the estimate

$$\delta m_a \sim g_{ae} m_e \left( \frac{H}{H_0} \right)^{1/2} \sim 10^6 g_{ae} \left( \frac{H}{10^{13} \text{ G}} \right)^{1/2} \text{ eV}. \quad (38)$$

For $g_{ae} \sim 10^{-13}$ (Refs. 4 and 14) and $H \gtrsim 10^{17}$ G (such fields and even $H \sim 10^{18} - 10^{20}$ G (Ref. 20) can exist in the interior regions of neutron stars), Eq. (38) gives $\delta m_a \gtrsim 10^{-5}$ eV.

The chemical potential $\mu$ of a degenerate gas of electrons occupying the ground-state Landau level ($n = 0$) in a magnetic field is related with the electron density $n_e$ by the well-known relation

$$n_e = \frac{\hbar p_F}{2\pi^2}, \quad (39)$$

where $p_F = \sqrt{\mu^2 - m_e^2}$ is the Fermi momentum. Writing Eq. (12) in the form

$$\frac{H}{H_0} > \frac{1}{2} \left( \frac{p_F}{m_e} \right)^2, \quad (40)$$

we obtain, taking account of Eq. (39), an upper limit on the density

$$n_e < \frac{\lambda_e^{-3}}{\sqrt{2\pi}^2} \left( \frac{H}{H_0} \right)^{3/2}, \quad (41)$$

where $\lambda_e = 1/m_e$ is the electron Compton wavelength. For $H = 2 \cdot 10^{17}$ G Eqs. (40) and (41) give $p_F < 50$ MeV and $n_e < 10^{36}$ cm$^{-3}$. Next, let $T \sim 10^{10}$ K $\sim 1$ MeV and $k_\perp \gtrsim T$. Then the conditions (11), (12), and (14) can be satisfied and the estimate (38) can be justified.

In summary, under the conditions of strongly magnetized neutron stars a dynamic axion mass, which can fall within the existing limits on the axion mass $10^{-5} \text{ eV} \lesssim m_a \lesssim 10^{-2}$ eV — is generated. Therefore $\delta m_a \sim m_a$ and the dispersion relation (37) differs appreciably from the vacuum relation ($k^2 = m_a^2$). This must be taken into account, for example, when investigating the resonant conversion of a plasmon into an axion ($\gamma \rightarrow a$) in a magnetic field as a result of the crossing of the corresponding dispersion curves (as already noted above, this process in fields $H \ll H_0$ and in the absence of the direct coupling (1) was studied in Ref. 12). We also note that the rate (27) of the decay $a \rightarrow e^-e^+$ in a magnetic field has
a square-root threshold singularity (as $\tau \to 1 + 0$). This singularity can be removed by taking into account accurately the dispersion law of an axion near threshold, and the decay rate is found to be finite:\[^{13}\] $w \sim m_e(\alpha_a H/H_0)^{2/3}$. A detailed analysis of the same threshold singularity (of cyclotron resonance) in a magnetic field and its elimination for the photon decay process ($\gamma \to e^-e^+$) was given earlier in Ref. \[^{21}\] where, specifically, it is underscored that the indicated singularity can be explained by the quantization of the phase space of charged particles in a magnetic field.

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