Spontaneous Vortex Phase in the Bosonic RVB Theory

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In the description of spin-charge separation based on the phase string theory of the t–J model, spinon excitations are vortices in the superconducting state. Thermally excited spinons destroy phase coherence, leading to a new phase characterized by the presence of free spinon vortices at temperatures, $T_c < T < T_s$. The temperature scale $T_s$ at which holon condensation occurs marks the onset of pairing amplitude, and is related to the spin pseudogap temperature $T^*$. The phase below $T_v$, called the spontaneous vortex phase, shows novel transport properties before phase coherence sets in at $T_c$. We discuss the Nernst effect as an intrinsic characterization of such a phase, in comparison with recent experimental measurements.

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I. INTRODUCTION

A unique feature of superconductivity in the high-$T_c$ cuprates is that phase coherence in electron pairing may occur at a lower temperature than the temperature at which the pairing amplitude develops [1]. Doped Mott insulators [2] provide a natural explanation: owing to the separation of spin and charge degrees of freedom, the spin resonating valence bond (RVB) pairing can be achieved at a much higher temperature; while the RVB pairs, accompanied by charge backflow, can move around like Cooper pairs at finite doping, superconducting condensation is absent until phase coherence between the pairs is established at a relatively lower $T_c$ [3].

Recently, we proposed a Ginzburg-Landau description of the RVB superconductor [4] based on the phase string theory [3] of the $t$–$J$ model. In this description, the superconducting order parameter is given by

$$\Delta(r) \sim \Delta^0(r)e^{i\Phi^s(r)}$$

where the Cooper-pair amplitude $\Delta^0 \equiv \Delta^*[\psi_h^s]^2$, with $\Delta^s$ being the bosonic RVB order parameter of spins and $\psi_h$ being the charge (holon) Bose condensed field. Here $\Delta^0 \neq 0$ does not directly result in superconductivity; rather, it is the phase factor in (1) that determines the phase coherence of the pairing order parameter, and thereby, $T_c$.

The quantity $\Phi^s(r)$ characterizes phase vortices centered around spinons, since $\Phi^s(r) \rightarrow \Phi^s(r) \pm 2\pi$, if $r$ winds around a spinon excitation continuously in space. Thus, each spinon excitation induces a phase vortex (called spinon vortex) in the order parameter. Superconducting phase coherence is destroyed above $T_c$ by the presence of free spinon vortices. Below $T_c$, phase coherence is realized as spinon bosons and antivortices are bound [2], such that $\Phi^s(r)$ becomes trivial in (1). In the superconducting phase, single spinon vortices can only be present at magnetic vortex cores which ensures flux quantization at $\hbar c/2e$.

It should be noted that the bosonic RVB order parameter $\Delta^s$ in (1) is not related to the energy gap, in contrast to the usual fermionic RVB order parameter [3]. It describes (neutral) spin pairing as characterized by short range (nearest-neighbor) antiferromagnetic correlations, $\langle S_i \cdot S_j \rangle_{NN} = -1/2|\Delta^s|^2$. At small doping, $\Delta^s \neq 0$ covers a temperature regime extended over 1,000K [3]. On such an RVB background, the Cooper pair amplitude $\Delta_0 = \Delta^*[\psi_h^s]^2$ is realized when the holons (Bose) condense, i.e., $\psi_h \neq 0$. But the temperature $T_v$ at which this occurs does not necessarily coincide with $T_c$ in general. Thus, one may find a temperature regime in the normal state, $T_v > T > T_c$ (obviously $T_v$ cannot be lower than $T_c$ since it is the holon Bose condensation that engenders phase coherence [3]).

In this paper, we will focus on a normal state with nonzero (preformed) pairing amplitude. It is distinguished from a conventional normal state of strong superconducting fluctuations by the presence of free spinon vortices as elementary excitations. We call such a phase, a spontaneous vortex phase. We present the effective Hamiltonian governing the dynamics of spinon vortices. Some novel transport properties in this state are discussed, and in particular, we show that free spinon vortices contribute to a nontrivial Nernst signal, consistent with the recent measurements of the Princeton group [5]. We argue that a finite $\Delta_0$ controls the pseudogap phenomena, and present a phase diagram showing the connection between the superconducting phase and the phase with preformed pairs (spontaneous vortex phase).

II. SPINON VORTICES: ELEMENTARY EXCITATIONS

A. Spinons as vortices

The phase $\Phi^s(r)$ in (1) is given by

$$\Phi^s(r) = \int d^2r' \text{Im} \ln |z - z'| [n^b_{z'}(r') - n^b_z(r')]$$

where $n^b_z$ denotes the density operator of bosonic spin-1/2 excitations (spinons) and $z = x + iy$. As noted in the
Introduction, $\Psi^\dagger$ changes by $\pm 2\pi$ when $\mathbf{r}$ winds around a spinon once. In the ground state, the spinons are all RVB paired such that $\Psi^\dagger$ is effectively canceled out. Excited spinons created by breaking the RVB pairs lead to vortices in the pairing order parameter through $\Psi^\dagger$. The spinon vortex can be understood from a different perspective. The effective Hamiltonian for the holons is given by

$$H_h = \frac{1}{2m_h} \int d^2 r \, h^1(r) \left(-i\nabla - \mathbf{A}^s - \mathbf{A}^c\right)^2 h(r)$$

where $h(r)$ is the bosonic holon field, and $\mathbf{A}^c$ is the vector potential of the external electromagnetic field. (Here we set $\hbar = e = c = 1$.) The quantity $\mathbf{A}^s$ defined by

$$\mathbf{A}^s = \frac{1}{2} \int d^2 r' \frac{2 \mathbf{A}_s \cdot \mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^2} \left[n^b_i(r') - n^b_i(r')\right]$$

is related to $\Phi^s$ by $\mathbf{A}^s = C \nabla \Phi^s$. Note that

$$\oint_{c} \mathbf{d}\mathbf{r} \cdot \mathbf{A}^s = \frac{1}{2} \oint_{c} \mathbf{d}\mathbf{r} \cdot \nabla \Phi^s = \pm \pi \int_{S \in c} d^2 r' \left[n^b_i(r') - n^b_i(r')\right]$$

i.e., $\mathbf{A}^s$ describes a fictitious flux quantized at $\pm \pi$, bound to each spinon as seen by holons. The vortices in $\Phi^s$ become meaningful when $\Delta^0 \neq 0$ in $\Phi$, i.e., when the holons are Bose condensed. In terms of the superfluid density $\rho_h$ and the phase $\phi_h$, the holon condensate $\psi_h(r) \equiv \langle h(r) \rangle = \sqrt{\rho_h} e^{i\phi_h(r)}$, and equation (3) reduces to

$$H_h \approx \frac{1}{2m_h} \int d^2 r \, \rho_h (\nabla \phi_h - \mathbf{A}^s - \mathbf{A}^c)^2$$

The corresponding superfluid current operator is given by

$$J = \frac{\rho_h}{m_h} [\nabla \phi_h - \mathbf{A}^s - \mathbf{A}^c]$$

Consider a spinon excited from the RVB background within a loop $c$. In terms of $\Phi$, there must be an induced current vortex surrounding the excited spinon (setting $\nabla \phi_h = \mathbf{A}^c = 0$)

$$\oint_{c} \mathbf{J}(r) \cdot d\mathbf{r} = -\frac{\rho_h}{m_h} \oint_{c} \mathbf{A}^s(r) \cdot d\mathbf{r} = \pm \frac{\rho_h}{m_h} \pi.$$

Therefore, each spinon excitation will always be accompanied by a current vortex in the holon condensate. This current vortex is consistent with the aforementioned $\pm 2\pi$ vorticities of $\Phi^s$ in $\Delta$ (in the latter a pair of holon fields are present). It should be noted that such a vortex cannot be compensated or screened by $\nabla \phi_h$ in $\Phi$, since the latter must satisfy the single-valued condition $\oint \mathbf{d}\mathbf{r} \cdot \nabla \phi_h = 0, \pm 2\pi$, ... instead of $\pm \pi$, the flux quanta carried by spinons in $\mathbf{A}^s$. However, $\nabla \phi_h$ provides a gauge freedom such that the sign of the vorticity for each minimal current vortex surrounding an excited spinon in $\Phi$ is not related to the spin polarization itself $\Phi$. For example, consider a spinon at site $i$, with $\oint \mathbf{d}\mathbf{r} \cdot \mathbf{A}^s = \frac{1}{2} \oint \mathbf{d}\mathbf{r} \cdot \nabla \Phi^s = \pi$. Under a singular gauge transformation, $\psi_h(r) \rightarrow \psi_h(r) e^{i\theta(r)}$ with $\oint \mathbf{d}\mathbf{r} \cdot \nabla \theta = 2\pi$ centered at $i$, the theory is invariant if $\mathbf{A}^s \rightarrow \mathbf{A}^s - \nabla \theta (\Phi^s \rightarrow \Phi^s - 2\theta)$, with $\oint \mathbf{d}\mathbf{r} \cdot \mathbf{A}^s = \frac{1}{2} \oint \mathbf{d}\mathbf{r} \cdot \nabla \Phi^s = -\pi$. In the appendix, we give a general proof that spin rotational symmetry is indeed preserved here.

Finally, we note that there also exist the usual Kosterlitz-Thouless (KT) like $2\pi$ vortices in $\psi_h$. But since we will only be interested in the temperature regime, $T < T_v$ where holons are condensed, we ignore their effects. Consequently, in the rest of the paper, $\nabla \phi_h$ will be set to zero in (3) and (5), with its singular part associated with spinons being incorporated into $\mathbf{A}^s$. So, in the latter [3] the spin index should be generally construed as the vorticity index and independent of the actual spin index, as discussed above.

**B. Effective Hamiltonian of spinon vortices**

The effective Hamiltonian governing the spinon vortices can be written in two parts:

$$H_{\text{spinon-vortex}} = H_s + H_v$$

Here, $H_s$ is the mean field Hamiltonian for bosonic spinon excitations $\Phi$,

$$H_s = \sum_{m} E_m \gamma_{m\sigma} \gamma_{m,\sigma} + \text{const.}$$

where $\gamma_{m\sigma}$ is related to the bare bosonic operator by the Bogoliubov transformation

$$b_{i\sigma} = \sum_{m} w_{m\sigma}(r_i) [u_m \gamma_{m\sigma} - v_m \gamma_{m,-\sigma}]$$

with $u_m = 1/\sqrt{2(\lambda_m/E_m + 1)^{1/2}}$ and $v_m = 1/\sqrt{2(\lambda_m/E_m - 1)^{1/2}}$. $\lambda_m = \lambda - \frac{\lambda_m}{\gamma_m} |\xi_m|$ $(J_h \sim \delta, J_z \sim J)$. The single particle wave function $w_{m\sigma}(r_i)$ and $\xi_m$ are determined by the tight binding equation

$$\xi_m w_{m\sigma}(r_i) = -J_s \sum_{j=N,N(i)} e^{-iA^b_{ij}/2} w_{m\sigma}(r_j)$$

in which $w_{m\sigma}(r_i) = w^\dagger_{m,-\sigma}(r_i)$. Note that $\sum_{c} A^b_{ij} = \pi \sum_{l \in \mathbb{Z}} n^b_i(l)$ describes $\pi$ flux tubes bound to holons ($n^b_i(l)$ denotes the holon number operator $\Phi$). In the holon Bose condensed phase, it is a good approximation to treat $A^b_{ij}$ as a vector potential for a uniform flux perpendicular to the two-dimensional plane, with a field strength...
where $\delta$ is the doping concentration and $a$ is the lattice constant.

Therefore, the tight binding model in (12) has a Hofstadter-Landau level structure. In the weak-field (continuum) limit, $\xi_m$ is a discrete, dispersionless Landau-level energy, with the wave function characterized by a cyclotron length scale

$$a_c = \frac{1}{\sqrt{B^a}} = \frac{a}{\sqrt{\pi \delta}}$$

(14)

Correspondingly the spinon spectrum $E_m$ is discrete as well, and if we restrict ourselves to the lowest Landau-level (LLL) at the low temperature, the dispersionless spectrum

$$(E_m)_{\text{LLL}} \equiv E_s$$

(15)

with a degeneracy equal to $2 \times B^a a^2/2\pi = \delta$ (the prefactor 2 comes from the fact that $\xi_m$ and $-\xi_m$ solutions are degenerate in $E_m$) and $E_s \sim \delta J$ $\Delta$ (in $E_m$ is determined by the average constraint condition $\langle \sum_{\sigma} b^\dagger_{i\sigma} b_{i\sigma} \rangle = 1 - \delta$). In this theory $\Delta$, $E_g = 2E_s$

(16)

corresponds to the energy scale of the sharp resonance-like peak observed in the neutron-scattering measurements [10], with its weight proportional to the degeneracy of the energy levels, $\delta$.

The second term in (9), $H_v$, describes the interaction among spinons due to the fact that they carry current vortices. This term arises from the effective Hamiltonian for the holons, $H_h$. Setting $\nabla \phi_h = A^c = 0$ in (9), we get

$$H_v = \frac{\rho_h}{2m_h} \int d^2 r \ (A^c)^2$$

$$= \int \int d^2 r_1 d^2 r_2 \sum_{\alpha} \alpha n^\alpha_{\bar{r}_1} V(\bar{r}_12) \sum_{\beta} \beta n^\beta_{\bar{r}_2}$$

(17)

in which

$$V(\bar{r}_{12}) = \frac{\rho_h}{2m_h} \int d^2 \bar{r} \ \hat{z} \times (\bar{r} - \bar{r}_1) \ \hat{z} \times (\bar{r} - \bar{r}_2)$$

$$= -\frac{\pi \rho_h}{4m_h} \ln \frac{|\bar{r}_1 - \bar{r}_2|}{r_c}$$

(18)

with $r_c \sim a$. Using the Bogoliubov transformation [12] and defining $n^\gamma_{\sigma \sigma} = \chi^\gamma_{\sigma \sigma} \gamma_{\sigma \sigma} \gamma_{\sigma \sigma}$, one finds

$$\sum_{\sigma} \sigma n^\gamma_{\sigma}(\bar{r}) = \sum_m |w_{m\sigma}(\bar{r})|^2 \sum_{\sigma} \sigma n^\gamma_{m\sigma} + \sum_{m \neq n} ...$$

(19)

and thus the interaction term can be further rewritten as $H_v = H_v^0 + H_v^1$, with

![Diagram](https://via.placeholder.com/150)

FIG. 1. A $2\pi$ “phase slip” takes place in the phase difference between two edges of the strip when a spinon-vortex passes through the sample.

$$H_v^0 = \sum_{\alpha \beta} \alpha \beta \sum_{m n} \gamma_{m\alpha} \gamma_{n\beta} U_{mn} n^\gamma_{m\alpha} n^\gamma_{n\beta}$$

(20)

and the non-diagonal part $H_v^1$ describes the scattering among different states induced by vortex interactions.

The diagonal part $H_v^0$ provides a confining force for spinons in the superconducting phase. Unlike the conventional KT vortices, there is a finite core for each spinon-vortex composite in $H_v^0$, within which the spinon does a cyclotron motion with a core radius $\sim a_c$ determined by $|w_{m\sigma}(\bar{r})|^2$. From a detailed renormalization group analysis [11], it is found that $T_c$, the temperature at which spinon vortices unbind, scales with the spin resonance-like energy $E_g$, consistent with a simpler physical argument based on the “core touching” picture [9], where it has been estimated that the transition occurs when the number of excited spinons exceeds the holon number.

### III. SPONTANEOUS VORTEX PHASE

#### A. Definition

The spinon-vortex composite described above is a unique elementary excitation, predicted by the present bosonic RVB theory, in the regime $\psi_h$ or $\Delta_0 \neq 0$ below the holon condensation temperature $T_v$. As pointed out in the Introduction, this regime generally includes two phases. One is the superconducting phase at a lower temperature $T_c$, where phase coherence is realized when spinon vortices are paired up (“confined”). The second phase at $T_c < T < T_v$ is a normal state in which unbinding (“deconfined”) free spinon vortices are present. We define such a state of matter as the spontaneous vortex phase. One may also regard this phase as a normal state with a finite Cooper-pair amplitude $\Delta_0$.

#### B. Phase rigidity and the Nernst effect

The existence of free vortices implies that some kind of phase rigidity persists above $T_c$ in the spontaneous vortex phase. In the following, we explore some consequences.
Consider a strip sample showing in Fig. 1, in which a spinon vortex traverses from one end of the sample to the other along the strip. According to (1) and (2), it is straightforward to see that the phase difference in \( \Delta \) between two edges across the strip will change by \( 2\pi \) (phase slip) for an infinitely long strip. Hence the boundaries of the sample can always perceive the motion of spinon vortices inside, which is a direct indication of the phase rigidity. The presence of such a new elementary excitation is the most important feature distinguishing the spontaneous vortex phase from an ordinary normal state with strong superconducting fluctuations.

A phase slip of \( 2\pi \) in a Josephson junction represents an elementary resistivity process as a voltage will be generated in that (transverse) direction \([12]\). In an equilibrium state at \( T < T_c < T_v \), phase slippage is random as those free spinon vortices move in arbitrary directions (this is an another way to understand the destruction of the phase coherence in this regime) and the average voltage is zero. In order to see a net phase slippage being added up, one has to let all the spinon vortices move in the same direction, say, by maintaining a temperature gradient and use a perpendicular magnetic field to induce an imbalance between vortices and antivortices. This results in the well known Nernst effect \([13]\). The existence of a nontrivial contribution from spinon vortices to the Nernst signal, is one of the main features of the spontaneous vortex phase.

There is another way to perceive this effect. In the superconducting phase, it is well known that the motion of magnetic vortices generates the Nernst signal. In the present case, as discussed in Ref. \([4]\), a magnetic vortex is formed by a spinon bound to a magnetic flux quantized at \( hc/2e \) (equal to \( \pi \) in the present units) \([Fig. 2(a)]\). This can be easily seen based on (3) which shows that each \( 2\pi \) vortex in the superconducting order parameter must be associated with a spinon through \( \Phi^s \). In the bulk, the spinon vortices have to be paired and do not contribute to \( \Phi^s \) and the net Nernst effect. So the Nernst signal from \( \Phi^s \) arises only from those spinon vortices that are nucleated by the magnetic fluxoids and are bound to the latter as shown in Fig. 2(a).

At \( T_c \), the phase coherence is destroyed as \( \Phi^s \) becomes disordered in (3) due to the unbinding of spinon vortices, and \( e^{i\Phi^s(r)-i\Phi^s(r')} \) falls off exponentially at large distance. Correspondingly, the magnetic vortices collapse and the Meissner effect disappears; the spinons originally trapped to the magnetic vortex cores are released from the latter and become free \([4]\), as illustrated in Fig. 2(b). Since, in the present theory, the spinon vortices contribute to the Nernst signal through the order parameter phase \( \Phi^s \), the collapse of magnetic vortices themselves does not directly lead to a diminishing Nernst signal above \( T_c \). On the other hand, unbinding vortices and vortices do not give rise to additional Nernst contribution since the net imbalance of vortices and antivortices remains the same below and above \( T_c \), determined by the condition

$$\oint \mathbf{J}(r) \cdot d\mathbf{r} = 0$$

or

$$\oint d\mathbf{r} \cdot \mathbf{A}^e = -\oint d\mathbf{r} \cdot \mathbf{A}^s. \quad (23)$$

The only difference is that below \( T_c \), the above condition holds for a closed loop encircling a magnetic fluxoid and far away from the core. (It also leads to flux quantization at \( hc/2e \) as discussed in Ref. \([4]\).) Above \( T_c \), without magnetic vortices, it holds on a spatial average. Thus the Nernst signal arises from the same spinons trapped at magnetic vortex cores below \( T_c \) \([Fig. 2(a)]\) and released in the spontaneous vortex phase \([Fig. 2(b)]\), above \( T_c \). Therefore, we expect the Nernst effect evolve smoothly between the superconducting and spontaneous vortex phases at \( T \rightarrow T_c \).

This smooth evolution of the Nernst effect is probably the most striking feature observed in the recent Nernst measurement on LSCO compounds by Xu et al. \([7]\). These measurements clearly reveal a continuous crossover of the Nernst signal above \( T_c \); it remains anomalously enhanced up to temperatures, \( 50 \sim 100 \) K above \( T_c \). Of course, experimentally one has to separate the vortex contribution to the Nernst signal from that of normal charge carriers whose contribution is limited by the so called Sondheimer cancellation \([7]\). Xu et al. \([7]\) argue that the Nersnt signal arises from vortex like excitations...
in the normal state and are not likely to be related to the conventional superconducting fluctuations. Similar phenomena have also been found in YBCO and BSCO systems as well \[8,9\], indicating that the existence of vortices above \( T_c \) is a generic feature of the high-\( T_c \) cuprates.

C. Transport coefficients

1. Spinon vortices

Spinons do not carry charge and thus do not couple to the external electromagnetic field \( \mathbf{A}^e \) directly. However, they are vortices and thus generate a Nernst voltage in the direction transverse to their motion, due to the phase slip effect discussed above. This can be quantified as follows. In terms of (7), one has

\[
\partial_t \mathbf{J} = -\frac{\rho_n}{m_h} (\partial_t \mathbf{A}^e + \partial_t \mathbf{A}^e) . \tag{24}
\]

The electric field is given by \( \mathbf{E} = -\partial_t \mathbf{A}^e \) in the transverse gauge (with \( \nabla \phi_b \) being absorbed by \( \mathbf{A}^e \), which is always possible below \( T_v \)). In the steady state, \( \partial_t \mathbf{J} = 0 \), and we get

\[
\mathbf{E} = -\sum_l \hat{z} \times \mathbf{v}^s(l) \alpha_l \delta(\mathbf{r} - \mathbf{r}_l) \tag{25}
\]

where \( l \) labels the spinon vortices with \( \alpha_l = \pm \pi \) denoting the vorticity. Equation (25) means that a finite drift velocity \( v^s \) of a spinon-vortex along \( \hat{z} \)-direction indeed will induce an electric voltage along the \( \hat{y} \)-direction, as the result of the phase slip effect shown in Fig. 1. Using the condition (23), we get

\[
\mathbf{B} = -\hat{z} \sum_l \alpha_l \delta(\mathbf{r} - \mathbf{r}_l) \tag{26}
\]

where \( \mathbf{B} \) denotes the local magnetic field perpendicular to the 2D plane. If all vortices have the same drift velocity, we get \( \mathbf{E} = \mathbf{B} \times \mathbf{v}^s \) which coincides with the familiar form for magnetic fluxoids in the flux flow region of a type II superconductor.

The Nernst coefficient is defined as

\[
\nu_{s-v} = \frac{E_y}{-\nabla_x T} B_z . \tag{27}
\]

Note that in this measurement \( J_y \) is set to be zero. \( \nu_{s-v} \) can be determined phenomenologically. Suppose \( s_\phi \) is the transport entropy carried by a spinon vortex and \( \eta_s \) is its viscosity. Then the drift velocity \( \mathbf{v}^s \) can be decided by \[1,13\]

\[
s_\phi \nabla T = -\eta_s \mathbf{v}^s , \tag{28}
\]

and consequently,

\[
\nu_{s-v} = \frac{s_\phi}{\eta_s} . \tag{29}
\]

The main result of the bosonic RVB theory is that such an expression is meaningful both above and below \( T_c \). As discussed earlier, \( \nu_{s-v} \) evolves smoothly between the superconducting and spontaneous vortex phase at \( T \to T_c \), since the Nernst signal arises from the spinon vortices in \[23\], and the formation or collapse of magnetic vortices does not change the expression (29). Of course, below \( T_c \), \( s_\phi \) will include additional contributions associated with the magnetic vortex core, which remains to be investigated.

Thus, the existence of spinon vortices provides a natural explanation for the Nernst experiments in the spontaneous vortex phase. In this case, the Nernst effect directly measures the dynamics of spinon vortices as governed by (6). The viscosity, \( \eta_s \), of spinon vortices is proportional to the scattering rate of the spinons. The latter is enhanced above \( T_c \) due to the broadening of the spin resonance level at \( E_g \), and correlated with the fate of the resonance peak observed in inelastic neutron scattering measurements \[1\]. As will be discussed in the next subsection, \( \eta_s \) is also related to the resistivity.

The transport entropy \( s_\phi \), is related to the degeneracy of the level \( E_s \) as the “mid-gap” states of the spinon trapped inside the magnetic vortex core \[6\]. We expect it to decrease rapidly above \( T_c \). This is not due to the collapse of the bound core states (of the trapped spinon), as in the conventional superconductors. It is due to the 2D Coulomb interaction \([7] \), whose effect is important in the high (vortex) density limit. Note that the degeneracy of the spinon states at the level \( E_s \) is related to different cyclotron orbits in the LLL (see Sec. IIB). In the “core touching” picture \[7\], the number of excited spinons at \( T_c \) is approximately equal to the holon number such that each LLL state, corresponding to the resonance peak at \( E_g \), is occupied approximately by one spinon on average. Higher temperature only means more spinon vortices will be excited to the LLLs such that more than one spinon will be found within a cyclotron orbit. In this dense limit, those configurations with many vortices (of the same vorticity) lumped together, which are allowed by the bosonic statistics, are forbidden due to costing too much potential energy. Thus, the phase space of spinons will be strongly limited in the dense limit, as compared to the dilute case, due to the vortex-vortex interaction, and the entropy associated with each spinon vortex should be largely reduced above \( T_c \) such that the Nernst coefficient \( \nu_{s-v} \) would decrease in magnitude rapidly. A microscopic study based on the effective Hamiltonian \[8\] is needed in order to understand this issue and make quantitative comparison with experimental results.

Spinon vortices also contribute to the thermal current established by a temperature gradient,

\[
\mathbf{J}^Q = \kappa_{s-v} (-\nabla T) \tag{30}
\]
Here the heat current is given by
\[ J^Q = \sum_{m \sigma} \mathbf{v}^s (m) E_m n^\gamma_{m \sigma} \approx \mathbf{v}^s E_s n_v \] (31)
with \( n_v = \sum_{m \sigma} n^\gamma_{m \sigma} \). In obtaining the last step in (31), it has been assumed that spinon vortices are mainly excited to the lowest discrete energy level at \( E_s \) and the interaction among vortices in (17) only gives rise to level broadening which is averaged out in (31) with \( \mathbf{v}^s \) representing an average drift velocity.

Based on (25)-(31), the following connection between the thermal conductivity and Nernst effect of the spinon vortices can be deduced:
\[ \kappa_{s-v} / \nu_{s-v} = \left( \frac{E_g}{2} \right) n_v. \] (32)
At \( T_c \), one has \( n_v \sim \delta a^{-2} \) \( \Phi^0 \), and for optimally doped YBCO with \( E_g \sim 41 \text{meV} \) and \( \delta \sim 0.15 \), we estimate \( \kappa_{s-v} / \nu_{s-v} \sim 3 \text{meV}/a^2 \).

Finally, as a prediction, we introduce
\[ \zeta_{s-v} = J^Q_s / E_y B_z \] (33)
in the following experiment setting: apply an electric field along \( \hat{y} \)-direction and measure the heat current along \( \hat{x} \)-direction in the presence of a perpendicular magnetic field \( B_z \). In such an experimental situation, we expect that spinon vortices contribute predominantly to \( J^Q_s \). No contribution from phonons will be generated by \( \Phi^0/2 \) fictitious flux with a radius \( a_v \) in the holon Hamiltonian (3). Like a conventional magnetic vortex in a type II superconductor, a transverse “Lorentz force” due to the interaction of the current with the fluxoid deflects the spinon vortex along the \( \hat{z} \)-direction, depending on the sign of vorticity. In turn, the moving spinon vortex produces a voltage along the \( \hat{x} \) direction, as can be seen from (33). It is then easy to obtain
\[ \rho = \frac{n_v}{\eta_s} \left( \frac{\Phi^0}{2e} \right)^2. \] (35)

On the other hand, due to the condition (23), the magnetic field seen by holons will be canceled out in (3) on the average by \( A^s \). Thus the Hall resistivity will be reduced in the spontaneous vortex phase. By the same token, the magnetoresistance should also be weak, as holons do not see a net field on an average. Note that \( \rho \) is not sensitive to how vortices and antivortices get polarized by a magnetic field. As seen from (35), the resistivity depends only on the total number of excited spinon vortices, \( n_v \), which as a function of \( T \) and \( E_s \) should not be very sensitive to the magnetic field, at least in the weak field limit. Finally, due to the fact that the holon condensate does not carry an entropy, the thermopower is also expected to be suppressed below \( T_c \) and only the normal-fluid component has a residual contribution.

Transport measurements (14) of many cuprate superconductors show that the suppression of the Hall effect and thermopower \( (15,16) \) around the same temperature scale \( T_v \) above \( T_c \) where the vortex-induced Nernst effect ends \( (14) \). These observations are complemented by a weak magnetoresistance in the same regime. All of this lends a support for the existence of the spontaneous vortex phase below \( T_v \), which can provide a consistent picture as discussed above.

So far, we have not discussed the role of the nodal (fermionic) quasiparticle. In the bosonic RBV theory, the quasiparticle arises as a composite object of a confined holon-spinon pair, with a \( d \)-wave dispersion \( (17) \) like in a \( d \)-wave BCS superconductor. But above \( T_c \), due to the deconfinement of vortices and antivortices, such a quasiparticle is expected to be damped, as its spinon constituent can get away freely. Thus, except for residual effects, their contribution to transport should become negligible for \( T > T_c \). In contrast, the contribution of the quasiparticles becomes dominant below \( T_c \), when their coherence is established.

2. Holons: Charge carriers

The bosonic holon carries charge \(+e\) and directly couples to the electromagnetic field in (3). In the spontaneous vortex phase, holons are Bose-condensed. But the “superfluid” density does not carry zero resistivity in the spontaneous vortex phase. Rather, one expects to see a finite resistivity from the condensed holons, which arises from the motion of spinon vortices. Suppose an electric current is established along the \( \hat{x} \)-direction. Each spinon vortex carries a \( \Phi^0/2 \) fictitious flux with a radius \( a_v \) in the holon Hamiltonian (3). Like a conventional magnetic vortex in a type II superconductor, a transverse “Lorentz force” due to the interaction of the current with the fluxoid deflects the spinon vortex along the \( \hat{z} \)-direction, depending on the sign of vorticity. In turn, the moving spinon vortex produces a voltage along the \( \hat{x} \) direction, as can be seen from (33). It is then easy to obtain
\[ \rho = \frac{n_v}{\eta_s} \left( \frac{\Phi^0}{2e} \right)^2. \] (35)

On the other hand, due to the condition (23), the magnetic field seen by holons will be canceled out in (3) on the average by \( A^s \). Thus the Hall resistivity will be reduced in the spontaneous vortex phase. By the same token, the magnetoresistance should also be weak, as holons do not see a net field on an average. Note that \( \rho \) is not sensitive to how vortices and antivortices get polarized by a magnetic field. As seen from (35), the resistivity depends only on the total number of excited spinon vortices, \( n_v \), which as a function of \( T \) and \( E_s \) should not be very sensitive to the magnetic field, at least in the weak field limit. Finally, due to the fact that the holon condensate does not carry an entropy, the thermopower is also expected to be suppressed below \( T_c \) and only the normal-fluid component has a residual contribution.

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The contribution of spinon vortices to the Nernst effect vanishes at $T_v$, when holon condensation or $\Delta_0$ disappears. In this section, we discuss the evolution of the spontaneous vortex phase as a function of doping and temperature and propose a phase diagram.

Physically, holon condensation at $T_v$ can be interpreted as the original neutral RVB pairs at high temperature acquiring charge and becoming Cooper-like pairs. Above $T_v$, incoherent holons are strongly scattered by the spinon flux-tubes according to (3). So even though the motion of an RVB spin pair may be accompanied a pair of holons as a backflow, it does not mean that the RVB pair is "charged" by -2e. The 2e holon backflow becomes coherent only below $T_v$, when the two holons constituting a pair are condensed. In this regime, the RVB spin pair looks like carrying a charge -2e (with a single wavefunction). This is the physical picture of how the amplitude of the pairing parameter forms below $T_v$.

The dashed line in Fig. 3 shows the bare KT temperature $T_{KT}$ for the holon condensation, which is obtained without including $\Delta^s$ in (4). First, we argue that $T_v \sim T_{KT}$ at very low doping concentrations. To understand the effect of fluctuating $\Delta^s$, let us first recall the physical interpretation (4) of the $T_c$ curve in Fig. 3. The superconducting transition occurs due to the dissolution of spinon vortex-antivortex pairs in (4). Such a transition is not driven by the entropy reason as in a conventional KT transition. Rather, it is related to the so-called "core touching" mechanism: each spinon vortex has a finite core with the length scale compatible to $a_c$, and an upper limit of $T_c$ is determined by the temperature at which the cores of excited spinon-vortices start to touch, which leads to $T_c \sim E_g/4$ (4). Considering that the overlapping of vortex cores effectively smear out the flux fluctuations in (4), in spite of unbinding spinon vortices, the fluctuations of $\Delta^s$ are not necessarily strong above $T_c$. This is particularly true at small doping when $a_c$ can become very large. Hence, one expects that $T_v$ approximately coincides with the bare KT temperature, $T_{KT} = \pi \delta (2a^2 m_h)^{-1}$, for small doping.

With the increase of doping, the core scale gets reduced. Near optimal doping, $\delta = 0.15$, $a_c \sim 1.5a$ which becomes comparable to the lattice constant. At such a short length scale, core overlapping no longer results in the smoothness of $\Delta^s$. Instead, the uncorrelated spinons above $T_c$ gives rise to strong flux fluctuations ($\pm \pi$) through $\Delta^s$, effectively suppressing the holon condensation in (4). Thus in this regime, we expect $T_v$ to be reduced to $T_c$, as shown in Fig. 3. In particular, since $a_c$ also determines the equal time spin-spin correlation length scale (4), at $\delta = 0.30$, one has $a_c \sim a$, which should set an upper doping limit for the bosonic RVB phase, as $\Delta^s$ is associated with nearest-neighbor antiferromagnetic correlations. Both $T_c$ and $T_v$ are expected to vanish at the point where the bosonic RVB order parameter $\Delta^s$ disappears on the $T = 0$ axis.

The phase diagram of LSCO has been mapped out carefully, based on high resolution Nernst experiments (4). For doping concentrations between 0.03 and 0.07, the experimental $T_v$ (denoted by $T_v$ in Ref. (4)) increases very steeply from 0 to 90 K with a slope $> 2, 400$ K, which puts $(m_h a^2)^{-1} > 0.13$ eV in the present theory. The experimental $T_v$ then peaks around $\sim 128$ K at $\delta = 0.11$ and decreases monotonically at larger doping, a trend similar to the plot shown in Fig. 3.

It is noted that the sharpness of the resonance-like peak at $E_g$ is also caused by the holon condensation in the present theory (4). Thus, $T_v$ is intrinsically related to the spin "pseudogap" temperature $T^*$, below which the resonance peak starts to sharpen up in neutron-scattering measurements (4) and the spin-lattice relaxation rate $1/T_1$ begins to deviate from the high-$T$ non-Korringa behavior in NMR measurements. In the transport channel, the "pseudogap" seen in resistivity should be also related to the holon condensation. Similarly, the holon condensation plays an essential role in the single-particle channel (17) which may explain the "pseudogap" seen in photoemission spectroscopy measurements. But it is important to point out since no true phase transition takes place and these temperature scales do not necessarily coincide with each other. They represent crossover temperatures in different channels in response to the holon condensation or the forming of the pairing amplitude $\Delta_0$. For instance, the transport $T^*$ is generally in a higher curve, indicating the onset of holon coherence before the holon condensation. If this picture holds true in the cuprates, then one
also expects $T_v$ to behave like the shaded curve in Fig. 3, which quickly decreases towards $T_c$ such that the spontaneous vortex phase shrinks as the doping concentration $\delta$ approaches the optimal and over doped regimes.

In summary, we have investigated the normal state below $T_v$, the onset temperature for the $amplitude$ of Cooper pairs, based on the bosonic RVB theory. Such a phase is characterized by spinons that behave as free vortices. $T_v$ coincides with the holon condensation, while $T_c$ is lower at which the phase coherence is realized due to the binding of spinon-vortices and -antivortices. In the spontaneous vortex phase, the transport properties are quite unique. We showed that free spinon vortices contribute to the Nernst effect, which evolves smoothly into the superconducting phase, consistent with the Nernst experiments. We also argued that the holon condensation is responsible for the pseudogap phenomena and that $T_v$ is correlated with various “pseudogap” temperature scales observed experimentally. The bosonic RVB theory of the $t-J$ model offers a framework to unify many of the seemingly disparate phenomena observed in the high $T_c$ superconductors, and we hope to address these in the future.

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APPENDIX A: SPIN ROTATIONAL INVARIANCE

According to the holon effective Hamiltonian $\hat{H}$, the bosonic spinons are perceived by holons as $\pi$-flux-tubes, represented by $A^s$ defined in $\hat{H}$. Since in the definition $\hat{H}$, the sign of the flux depends on the spin index, one may naturally raise the question as whether the spin rotation may be broken in such a system. In the following we give an explicit proof that the spin rotational symmetry is retained.

For convenience, we will use the original lattice version of $H_h$ $\hat{H}$:

$$H_h = -t_h \sum_{\langle ij \rangle} \left( e^{i(A^s_{ij}+A^s_{ji}-\phi_{ij})} b_{i}^{\dagger} b_{j} + H.c. \right)$$  \hspace{1cm} (A1)

where $A^s_{ij}$ represents the external electromagnetic field, and the lattice version $A^s_{ij}$ is defined by

$$A^s_{ij} = \frac{1}{2} \sum_{l \neq i,j} \text{Im} \ln \left[ \frac{z_i - z_l}{z_j - z_l} \right] \left( \sum_{\sigma} \sigma n_{l,\sigma}^{b} \right)$$  \hspace{1cm} (A2)

which describes fictitious fluxoids bound to spinons satisfying

$$\sum_{c} A^s_{ij} = \pm \pi \sum_{l \in c} [n_{l,\uparrow}^{b} - n_{l,\downarrow}^{b}]$$  \hspace{1cm} (A3)

for a closed loop $c$. $\phi_{ij}$ corresponds to a uniform $\pi$ flux per plaquette.

In the phase string formulation $\hat{H}$, the spin operators are defined as follows:

$$S^z_i = \sum_{\sigma} \sigma b_{i,\sigma}^{\dagger} b_{i,\sigma},$$  \hspace{1cm} (A4)

$$S^+_i = b_{i,\uparrow}^{\dagger} b_{i,\downarrow} (-1)^i e^{i\Phi^h_i},$$  \hspace{1cm} (A5)

and $S^-_i = (S^+_i)^\dagger$. Here

$$\Phi^h_i = \sum_{l \neq i} \text{Im} \ln |z_i - z_l| n_{l,\uparrow}^{b}.$$  \hspace{1cm} (A6)

It is obvious that

$$[H_h, S^z_i] = 0.$$  \hspace{1cm} (A7)

On the other hand, one finds that the phase $\Phi^h_i$ defined in $\hat{A}^h$ plays a crucial role in compensating the extra phase generated from $A^s_{ij}$ by a spin flip, which results in

$$[H_h, S^\pm_i] = 0.$$  \hspace{1cm} (A8)

Also note that if the summation in $H_h$ involves the links $ij$ which coincide with $l$, such terms have no contribution due to the no-double-occupancy constraint.

So generally one has

$$[H_h, S_i] = 0$$  \hspace{1cm} (A9)

which means that the holon effective Hamiltonian $H_h$ in $\hat{A}^h$ or $\hat{H}$ not only always respects the global spin rotational symmetry, but also respects a local spin symmetry. This is consistent with the notion of spin-charge separation in general, and that the spin index is independent of the vorticity of a spinon-vortex composite, discussed in Sec. IIA, in particular. In other words, $\pi$-flux-tubes bound to spinons, seen by holons, and the current-vortices bound to spinons only reflect the fact of electron fractionalization, without involving any spin symmetry-breaking. To understand this, one has to realize that there is a peculiar formulation of $S^\pm_i$ $\hat{A}^h$ in the phase string formalism, as shown above.
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