ReCord: A Distributed Hash Table with Recursive Structure

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Abstract

We propose a simple distributed hash table called ReCord, which is a generalized version of Randomized-Chord and offers improved tradeoffs in performance and topology maintenance over existing P2P systems. ReCord is scalable and can be easily implemented as an overlay network, and offers a good tradeoff between the node degree and query latency. For instance, an $n$-node ReCord with $O(\log n)$ node degree has an expected latency of $\Theta(\log n)$ hops. Alternatively, it can also offer $\Theta(\frac{\log n}{\log \log n})$ hops latency at a higher cost of $O(\frac{\log^2 n}{\log \log n})$ node degree. Meanwhile, simulations of the dynamic behaviors of ReCord are studied.

I. INTRODUCTION

Peer to peer (P2P) networks have become popular in resource sharing applications recently. There have been millions of users in certain successful systems, such as Napster [13], Gnutella [4], and Kazaa [8]. P2P systems are distributed systems without any central authority or hierarchical organization, and each node in the systems performs similar functionality.

In order to efficiently locate an object in a large scale P2P system, many schemes relay on distributed hash tables (DHTs). Example systems include Chord [20], Pastry [19], CAN [16], and Tapestry [21]. A P2P system needs to consider the joining, departing of hosts, and the insertion/addition of resources, besides look up operation. DHT can be implemented as an overlay logical topology over the internet physical networks, where each node keeps the direct IP address of its neighbors in a routing table. Instead of connecting to all the other nodes in the system, each node in DHT only links to a small
number of nodes. The key lies in ensuring a small diameter in the resulting overlay network. At the same time, DHT should allow new nodes to join or existing nodes to leave the system voluntarily, therefore, the cost of topology maintenance during this dynamic join-depart process should be kept as low as possible.

The following metrics are usually used to compare the performance and efficiency of the designed DHT.

(a) **Degree and Diameter**: The number of links per node in a DHT should be small in order to reduce the cost of communication. Also, the diameter of the network should not be large in order to reduce the query latency.

(b) **Scalability**: As the network size increases, the node degree, the query latency, and the traffic increased in queries should not increase drastically.

(c) **Maintenance overhead**: When new nodes join or existing nodes depart, the overhead as measured in terms of the number of messages required to maintain the DHT should be as low as possible.

(d) **Fault tolerance**: The DHT should be resilient to both node and link failures of the system. No matter what fraction of the nodes or links has failed, the data available in the remaining nodes should still be accessible.

(e) **Load balance**: The resource keys should be evenly distributed over all nodes, and the traffic overhead resulted by query or maintenance operations should be balanced among nodes in the network.

In this paper, we propose a simple distributed hash table, called ReCord, which is scalable and can be easily implemented as an overlay network. ReCord offers a good tradeoff between the node degree and query latency. For instance, an $n$-node ReCord with $O(\log n)$ node degree has an expected latency of $\Theta(\log n)$ hops. Alternatively, it can also offer $\Theta\left(\frac{\log n}{\log \log n}\right)$ hops latency at a higher cost of $O\left(\frac{\log^2 n}{\log \log n}\right)$ node degree.

The rest of the paper is organized as follows. Section 2 will review related work. In Section 3, the construction of ReCord will be described. Section 4 will examine the bounds of the node degree and route path length. Section 5 presents an implementation of ReCord. Section 6 gives the simulation studies of ReCord’s dynamic behaviors. Section 7 summarizes the finding and concludes this paper.

## II. RELATED WORK

Plaxton [15] et al. proposed a distributed routing protocol based on hypercubes for a static network with given collection of nodes. Plaxton’s algorithm uses the digit-fixing technique to locate the shared resources on an overlay network in which each node only maintains a small-sized routing table.
Pastry [19] and Tapestry [21] use Plaxton’s scheme in the dynamic distributed environment. The difference between them is that Pastry uses *prefix-based* routing scheme, whereas Tapestry uses *suffix-based* scheme. The number of bits per digit for both Tapestry and Pastry can be reconfigured but it remains fixed during run-time. Both Pastry and Tapestry can build the overlay topology using proximity neighbor selection. However, it is still unclear whether there is any better approach to achieve globally effective routing.

Chord [20] uses consistent hashing method to map \( n \) nodes to evenly distribute around a circle of identifiers. Each node \( x \) in Chord stores a pointer to its immediate *successor* (the closest node in clockwise direction along the circle), and a *finger table* of connections with node \( x + 2^i \), where \( i = 1, 2, \ldots, \log n - 1 \). In Chord, a greedy algorithm is used to route query messages. The complexity of routing per query is bounded by \( O(\log n) \) hops. For fault tolerance, each node in Chord uses a successor list which stores the connections of next several successor nodes. The routing protocol in standard Chord in [20] is not optimal and was improved by using bidirectional connections [3]. In [2], El-Ansary et al. generalize Chord to a P2P framework with \( k \)-ary search, but they only focused on the lookup operation, and did not consider node joining leaving, failure, and implementation details. In [5], [12], a randomized version of Chord, called *Randomized-Chord*, is presented. In Randomized-Chord, node \( s \) is connected to a randomly chosen node in each interval \( [2^{i-1}, 2^i) \), where \( i = 1, 2, \ldots, \log n \).

Koorde [7] is an extended DHT of Chord in that it embeds a de Bruijn graph over the Chord identifier circle. Koorde can be constructed with constant node degree and \( O(\log n) \) hops per query, or with \( O(\log n) \) node degree and \( O(\log n / \log \log n) \) hops per query. As a Chord-like network, Symphony [11] builds a network using the *small world* model from [9], [1]. In Symphony, each node has local links with its immediate neighbors and long distance links connected to randomly chosen nodes from a probability distribution function. The expected path length for a Symphony network with \( k \) links is \( O\left(\frac{1}{k} \log^2 n\right) \) hops. Simulations in [11] shows that Symphony is scalable, flexible, stable in the dynamic environment, and offers a small average latency with constant degree, but the analytical results for fault tolerance were not given. Like Chord, Viceroy [10] distributes nodes along a circle, and builds a constant-degree topology approximating a butterfly network, and offers \( O(\log n) \) routing latency. However, it is relatively complicated to implement Viceroy and fault tolerance is not addressed in [10].

CAN [16] divides a \( d \)-dimension torus space into zones *owned* by nodes, and resource keys are evenly hashed into the coordinate space. Each resource key is stored at the node that owns the located zone. Using greedy routing, the query message is routed to the neighbor which is closer to the target key. Each node has \( O(d) \) neighbors and query latency is \( O(dn^{1/d}) \). If \( d \) is chosen to be \( \log n \), each node connects with \( O(\log n) \) neighbors and a query takes \( O(\log n) \) hops. Some proximity routing
scheme, such as *global network positioning* [14] and topologically-aware overlay construction [17] to build CAN overlay network. There are two disadvantages for this scheme: it needs to fix some landmark machines and it tends to create hot spots from a non-uniform distribution of nodes in the coordinate space.

It is difficult to say which one of above proposed DHTs is “best”. Each routing algorithm offers some insight on routing in overlay network. One appropriate strategy is to combine these insights and formulate an even better scheme [18].

III. CONSTRUCTION OF ReCord

In this paper, we will slightly abuse the notation of node identifiers and nodes themselves, and the same to resource key identifiers and resource themselves. Instead of mapping identifiers into \( m \)-bit numbers, we will map them into the unit circle ring \( \mathcal{I} [0, 1) \), as with Symphony [11] and Viceroy [10]. By using a consistent hashing method, we can assume that both node and key identifiers are distributed evenly over the circle \( [0, 1) \), and there is no collision.

Hashing the identifiers into ring \( \mathcal{I} [0, 1) \) allows the identifier value to be independent of the maximum hashed space \( 2^m \). Assume that the ring is formed in the clockwise direction. Denote the clockwise neighbor of node \( s \) on the ring by \( \text{SUCC}(s) \), and denote its counter-clockwise neighbor by \( \text{PRED}(s) \). A key \( x \) is stored at a nearest node \( y \), where \( y \geq x \) on the ring. We also call this node \( \text{SUCC}(x) \).

The basic idea of ReCord is as follows. Suppose that there are totally \( n \) active nodes in a stable P2P system. Starting from any node \( s \), divide the whole ring into \( k \) equal intervals, where \( k > 1 \) denotes an integer. Then divide the first interval closest to node \( s \) recursively until the length of the interval nearest to \( s \) is \( \frac{1}{n} \), i.e. the first \( k \) intervals nearest to \( s \) contains \( O(1) \) nodes, the second \( k \) intervals nearest to \( s \) contains \( O(k) \) nodes, and the third \( O(k^2) \) nodes and so on, as shown in Fig. 1.

The first division is also called level-1 division, and the next is called level-2 division, and so on. There are \( c = \log_k n \) such levels (assuming that \( n = k^c \), where \( c \) denotes an integer). The length of each interval at level 1 is \( \frac{1}{k} \), and \( \frac{1}{k^i} \) for level 2, and \( \frac{1}{k^i} \) at level \( i \) in general. The intervals at the same level are numbered sequentially clockwise along the ring. There are totally \( k \) intervals in every
level. Based on the above definitions, for node \( s \), we know that its interval \( j \) at level \( i \) corresponds to the range \( \left[ s + \frac{(j-1)k^i}{n}, s + \frac{jk^i}{n} \right) \) on the ring. Randomly choose one node \( x \) in every interval, and set up a unidirectional connection from node \( s \) to \( x \). We call the resulting network ‘ReCord’ for its recursive structure and similarity with Chord.

Comparing ReCord with Randomized-Chord, we find that in fact ReCord is a generalized version of Randomized-Chord. When \( k = 2 \), ReCord becomes Randomized-Chord.

IV. ANALYSIS

P2P systems have dynamic membership, meaning that a node may join and leave the network voluntarily. The number of the active nodes varies with the evolution of the P2P network. However, when we analyze the degree of each node or latency of each query, we suppose that the P2P network is static. Therefore, we will firstly analyze the tradeoffs between link degree and query latency for ReCord statically. Later, we will explain how to extend and implement it under the dynamic situation.

**Theorem 1**: The node degree per node in an \( n \)-node ReCord is \( \Theta(k \log_k n) \).

**Proof**: Let \( H(n) \) represent the number of links connected to an arbitrary node \( s \) in the \( n \)-node network. After the first division, there are \( k - 1 \) links, plus its links to nodes in the intervals included by level-2, hence we have the following relation:

\[
H(n) = (k - 1) + H\left(\frac{n}{k}\right).
\]

The solution of this recurrence is \( H(n) = \Theta\left((k - 1) \log_k n\right) = \Theta(k \log_k n) \). Therefore, the degree of any node in ReCord is bounded by \( \Theta(k \log_k n) \).

When \( k = \Theta(1) \), the node degree in the \( n \)-node ReCord is \( H(n) = \Theta(\log n) \). If \( k = \Theta(\log n) \), \( H(n) = \Theta\left(\frac{\log^2 n}{\log \log n}\right) \).

Before studying the query latency in ReCord, we introduce the following lemma which will be used in the proof of the lower bound of the query latency.

**Lemma 1**: Let \( X^m \) denote a random variable in the state space \( 0, 1, 2, \cdots m - 1 \). Let

\[
P_r[X^m = i] = \begin{cases} 
\frac{k-1}{k^{-m}}, & \text{when } 1 \leq i \leq m - 1 \\
\frac{1}{k^m}, & \text{when } i = 0
\end{cases}
\]

The expected time required for \( X^m \) to drop to 0 is lower bounded by \( E[T_m] = \Omega(m) \)

**Proof**: We apply an inductive method in this proof. The lemma holds obviously when \( i = 0, 1 \).

Suppose that the lemma holds for all \( 1 < i \leq m - 1 \), namely, \( E[T_i] \geq ci \), for all \( 1 < i \leq m - 1 \), where \( c \) is a constant. We have

\[
E[T_m] \geq 1 + c(m - 1)\frac{k-1}{k^2} + c(m - 2)\frac{k-1}{k^3} + \cdots + c\frac{k-1}{k^{m-1}}
\]

\[
= 1 + cm(k-1)((1 + \frac{1}{k}) + \cdots + (\frac{1}{k^{m-1}})) - c(k-1)(\frac{1}{k} + 2\frac{1}{k^2} + \cdots + (m-1)\frac{1}{k^{m-1}})
\]

\[
= 1 + cm(k-1)((1 + \frac{1}{k}) + \cdots + (\frac{1}{k^{m-1}})) - c(k-1)(\frac{1}{k} + 2\frac{1}{k^2} + \cdots + (m-1)\frac{1}{k^{m-1}})
\]

\[
= 1 + cm(k-1)((1 + \frac{1}{k}) + \cdots + (\frac{1}{k^{m-1}})) - c(k-1)(\frac{1}{k} + 2\frac{1}{k^2} + \cdots + (m-1)\frac{1}{k^{m-1}})
\]
\[
\geq 1 + cm(1 - \frac{1}{k^m}) - c(k - 1)\frac{k}{(1 - \frac{1}{k} r)^2} \\
= cm + 1 - \frac{cm}{k^m} - \frac{ck}{k - 1}.
\]

When \( c \leq \frac{1}{k^m + \frac{1}{k - 1}} < 1 \), \( E[T_m] \geq cm \). Therefore, the expected time required for \( X^m \) to drop to 0 is lower bounded by \( E[T_m] = \Omega(m) \), where the constant multiplier in the formula is smaller than 1.

**Theorem 2:** Using the greedy routing algorithm, the expected path length per query in an \( n \)-node ReCord is \( \Theta(\log_k n) \).

**Proof:**

**Upper bound:** Let \( T(n) \) denote the number of hops required by a query. Consider the case when the message is routed to the 1st interval, according to the recursive construction, the time step required is \( T(n) = T\left(\frac{n}{k}\right) \). If the message is routed to interval \( j \) of the level-division \( (1 < j \leq k) \), in the worst case, the distance is reduced to \( \frac{2n}{k} - 1 \). In this case, after one more forwarding, the distance will be reduced to less than \( \frac{n}{k} \), so the required time in the worst case is upper-bounded by \( T(n) \leq 2 + T\left(\frac{n}{k}\right) \). Since each link is connected to a randomly chosen node in each interval, the probability that the message is routed to interval 1 is \( \frac{k}{n} \), and the probability that it is routed to intervals 2, 3, ..., \( k \) is \( \frac{n-k}{n} \). Thus, an upper bound of the total expected number of steps is:

\[
T(n) \leq \frac{1}{k} T\left(\frac{k}{n}\right) + \frac{k - 1}{k} [2 + T\left(\frac{k}{n}\right)].
\] (1)

Solving Ineq. (1), we have \( T(n) = O\left(\frac{2(k-1)}{k}\log_k n\right) = O(\log_k n) \). Therefore, for the greedy routing algorithm, the expected path length per query is \( O(\log_k n) \), where the constant multiplier in the formula is smaller than 2.

**Lower bound:** Suppose that all nodes in the ring are labelled by 0, 1, 2, ..., \( n \). Node 0 is the destination, and node \( n \) is the source. We define **Phase** as follows: Phase 0 only contains node 0; Phase 1 consists of nodes in the interval \([1, k - 1]\); Phase 2 consists of nodes in the interval \([k, k^2 - 1]\), and so on. Generally, Phase \( i \) contains nodes in the interval \([k^{i-1}, k^i - 1]\). Suppose that there are in total \( m \) phases.

According to the division of intervals and randomly choosing one node among each interval, the probability that the message is routed to Phase \( m - 1 \) is \( \frac{k-1}{k} \), and \( \frac{k-1}{k^2} \) if routed to Phase \( m - 2 \), and so forth. Generally, the probability that the message is routed to Phase \( i \) is \( \frac{k-1}{k^{i-1}} \), for \( 1 \leq i \leq m - 1 \), and \( \frac{1}{k^{m-1}} \), for \( i = 0 \). By applying Lemma 1 we can deduce that the expected number of hops per query is \( \Omega(m) \). There are totally \( m = \log_k n \) phases for \( n \) nodes. Therefore, the average number of hops per query is lower bounded by \( \Omega(\log_k n) \).

Our static analysis shows a good tradeoff between the node degree and the required hops per query. If we choose \( k = \Theta(1) \), the node degree and query latency for an \( n \)-node ReCord are \( O(\log n) \) and \( \Theta(\log n) \) respectively. If we let \( k = \Theta(\log n) \), the \( n \)-node ReCord has \( \Theta(\frac{\log^2 n}{\log \log n}) \) node degree and
Fig. 2. Impact of variable $k$ on degree and latency.

$\Theta(\frac{\log n}{\log\log n})$ query latency. Fig. 2 shows the trade-off between the node degree and query latency, given the total number of active nodes is $n = 2^{15} = 32768$. Fig. 2 shows that the node degree increases almost linearly as $k$ increases, but the query latency drops quickly within a small range of $k$.

V. IMPLEMENTATION OF RECORD

A. Estimation of network size

Although the analytical results in the previous section can be directly applied to a static network, the derived bounds for degree and latency are not as tight as the case using active nodes in the real P2P network. Now we suppose a dynamic environment, where the nodes join and leave dynamically. The main difficulty of this extension is that for each node, it requires a rather accurate information of the global network size for the construction of links. When we divide network in each level, we need to know the value of $n$, the total number of active nodes.

Currently, most estimation processes uses the density information around the vicinity of the estimating node [10], [11], [6]. Let $L_f$ denote the fraction length of an interval including $f$ distinct nodes. The network size can be estimated by $\frac{f-1}{L_f}$. In [10], [6], the length between estimating node and its successor is used to estimate the size of the overall network. Symphony [11] applies the length between estimating node’s predecessor and successor in estimation procedure, and its experiment shows that the impact of increasing $f$ on average latency is not significant.
Other methods can be also applied to estimate network size, such as through a central server, or piggybacking along lookup messages [11], or randomly choosing several pairs of continuous nodes, and using their average length for estimation.

Knowing the network size is an important step for dynamic network construction. In our experiments, as with Symphony, we use $f = 3$ to estimate the global network size in ReCord.

**B. Join and Leave maintenance**

1) **Join Protocol:** Suppose that a new node $s$ joins the network through an existing node. Firstly, node $s$ chooses its identifier from $[0, 1)$ uniformly at random. Secondly, node $s$ is inserted between $PRED(s)$ and $SUCC(s)$, and runs the estimation protocol, and update the estimated network size $\tilde{n}$ for all nodes. Next, it divides the whole ring $[0, 1)$ recursively into intervals $[s + \frac{(j-1)k}{n_s}, s + \frac{jk}{n_s})$ starting from $s$ as described in Section III. Then it sets up one link to a node randomly chosen from each interval. The implementation detail for link construction is that it first generates a random real number $x$ in the interval $[s + \frac{(j-1)k}{n_s}, s + \frac{jk}{n_s})$, then looks up $SUCC(x)$. If $SUCC(x)$ is in the range $[s + \frac{(j-1)k}{n_s}, s + \frac{jk}{n_s})$, the connection is built successfully, otherwise, it has to re-establish the link for the interval. In order to avoid flooding traffic made by link reestablishment, we limit the times of reestablishment. If node $s$ still can’t find a node in an interval after $q$ times tries, we let it give up the link construction for this interval. The value of $q$ should be related to the interval length. More details will be shown in the experiment part.

Similar to Symphony, ReCord also bounds the number of incoming links per node, which is good for load balancing of the whole network. Once the number of incoming links of a node has reached $2\log_k n$, any new request to establish a link with it will be rejected. The requesting node has to make another attempt.

Since node $s$ needs a lookup operation that requires $O(\log_k n)$ messages for each link establishment, the whole cost of $O(k \log_k n)$ link constructions is $O(k \log_k^2 n)$ messages.

2) **Leave Protocol:** Once node $s$ leaves the system, all its outgoing links will be snapped. Its predecessor and successor nodes need to reinstate their links, and corresponding neighbor nodes need to update their estimation of network size. At the same time, all the incoming links of node $s$ are broken, and corresponding connection nodes need to re-select another node randomly in the same interval as node $s$ is located in. This operation can be triggered by the periodic detections by nodes connected to node $s$.

If node $s$ leaves voluntarily, it will gracefully inform related nodes to update their connection information, otherwise, the connection information has to be updated when the other nodes have periodically detected the failure of node $s$. More details of this protocol are similar to that in Chord.
C. Re-linking operation

The total number of active nodes in the P2P network always changes as the network expands or shrinks. When node $s$ finds that its current estimated network size $\tilde{n}_s$ is not equal to its stored stale estimation $\tilde{n}'_s$, it needs to re-build its links to nodes in the new intervals. One conservative solution is to re-link every construction on every update of $\tilde{n}_s$. In this way, the traffic resulted by the re-linking operation would be excessive. According to analyzed bounds in Section IV, the messages required for one node to re-link all its connections are $O(k\log^2_k n)$.

In order to avoid the excessive traffic resulted from re-linking operation, and guarantee the stability of the whole P2P network, we apply the same re-linking criterion as in Symphony: re-linking operation occurs only when $\frac{\tilde{n}_s}{\tilde{n}'_s} \geq 2$ or $\frac{\tilde{n}_s}{\tilde{n}'_s} \leq \frac{1}{2}$, where $\tilde{n}_s$ is node’s updated estimated network size, and $\tilde{n}'_s$ is node’s stored stale network size.

D. Fault tolerance

In Chord or Koorde with constant-degree, each node keeps a list of successors to increase the system’s robustness: each node maintains $r$ connections of its immediate succeeding nodes rather than only one immediate successor. Certainly, it will keep the whole P2P network more robust, but it also requires some extra operations and corresponding additional cost to maintain the successor list. Using a similar scheme, Symphony makes $r$ copies of a node’s content at each of $r$ succeeding nodes. Other DHTs, such as CAN, Pastry, and Tapestry keep several backup links for each node.

Compared with the above DHTs, we found that ReCord has a natural structure for fault tolerance: at the last dividing level, each node is already connected to its $k$ following succeeding nodes, which is equal to a successor list in Chord. ReCord need not keep any backup link or redundant links to increase the robustness of the whole system. Therefore, it entails no extra overhead to offer fault tolerance.

As stated in [7], in order to keep live nodes connected in cases of nodes failures, some nodes need to have node degree of at least $\Omega(\log n)$. Moreover, the experiments in [11] shows that the local links are crucial for maintaining connectivity of P2P topology. By construction, ReCord has rich “local” connections and relatively sparse “long” connections. Our experimental results, to be presented shortly, confirms that this is indeed a good strategy for forming the connectivity of the P2P network.

VI. Experiments

Based on the implementation described in the preceding section, we run the simulation of ReCord with nodes ranging from $2^4$ to $2^{15}$. The impacts of different parameters shown in the simulation...
results are analyzed.

We focus on four types of network. The first one is a static network, in which the global network size is known for every node. The second one is called an expanding network, where the rate of node joining is higher than that of node departure in a given unit time interval. The third one, called a shrinking network, is opposite of an expanding network. The last one is called a stable network, in which the rate of node joining is equal to that of node departure nodes in a unit time interval.

A. Estimation of network size

For each node, we let it use the density between its predecessor and successor to estimate the global network size. Fig. 3 shows the comparisons between the estimated and the actual network size for both small and large P2P system which is in the steady state. For the small scale network with \( n = 250 \) active nodes, the difference between estimated \( \log \tilde{n} \) and actual \( \log n \) is no more than 4. For a larger scale network with \( n = 11,374 \) active nodes, the difference between estimated \( \log \tilde{n} \) and actual \( \log n \) is no more than 8. In either network, the difference between the estimated and actual \( \log n \) for most nodes is smaller than 4. This shows that the approximation is accurate enough for the construction of ReCord.

Fig. 4 shows the average estimation of network size over all nodes of the expanding, shrinking, and stable networks respectively over time. The comparisons of average \( \log \tilde{n} \) and actual \( \log n \).
(a) Estimation of network size in expanding network. 

(b) Estimation of network size in shrinking network. 

(c) Estimation of network size in stable network.

Fig. 4. Estimation of network size for expanding, shrinking, and stable network respectively.

B. Degree and Latency

Fig. 5 shows the average node degree and latency over an expanding network with different $k$ values.

Fig. 6 shows the tradeoff between degree and latency over different $k$ values in a stable P2P network, given that the number of active nodes is in the range of $[2070, 2121]$. For Fig. 6, $k = 5$ is obviously the best choice for the P2P network of those sizes. We can also see that it fits the analysis (cf. Fig. 2) quite well.

C. Fault tolerance

Fig. 7 shows how the fraction of failure links will influence the query latency. Three cases: $k = 3$, $k = 5$, and $k = 7$ were run under the stable environment. According to Fig. 2, the node degree increases as $k$ increases for a fixed $n$. However, from Fig. 7 we can see that independent of $k$, only when more than half of the links fail, the query latency is adversely affected.

VII. CONCLUSIONS

We have proposed a simple DHT topology, called ReCord, which is a generalization of Randomized-Chord, in the sense that Randomized-Chord is a special case of ReCord when $k = 2$. ReCord offers a good tradeoff between the node degree and query latency: an $n$-node ReCord with $O(\log n)$ node degree has an expected latency of $\Theta(\log n)$ hops. Alternatively, it can also offer $\Theta(\frac{\log n}{\log \log n})$ hops latency at a higher cost of $\Theta(\frac{\log^2 n}{\log \log n})$ node degree. Some implementation techniques of ReCord...
(a) Average degree for P2P network with $n$ ranging from $2^3$ to $2^{13}$.

(b) Average latency for P2P network with $n$ ranging from $2^3$ to $2^{13}$.

Fig. 5. Average degree and latency on expanding network.

Fig. 6. Tradeoff between degree and latency over different choices of $k$ in a stable network (the number of active nodes is between $[2070,2121]$).
are presented, including the estimation of network size, join and departure maintenance, re-linking operations, etc. Meanwhile, simulations of the dynamic behaviors of ReCord are studied.

In actual P2P systems, the different bounds of degree and latency of the constructed networks usually offer different insights of DHTs. Lower degree decreases the number of open connections and the protocol traffic made by the pinging operation; the number of neighbors whose states need to be changed when a node joins or leaves is also smaller. However, the lower connectivity of the low node degree also means that the network is easy to split up, and hence it has weak fault tolerance. On the other hand, higher node degree leads to better connectivity and reduces the network diameter, the longest path length for the query operation. The lower query latency also leads to lower joining and departure costs. As we will discuss later, the join and leave operation will make use of the query operation, so the small path latency will also decrease the cost of join and leave. We can adjust the $k$ value to suit the P2P networks required for different environments. A more proactive and perhaps more useful method is to dynamically monitor the P2P system, e.g. sensing the frequency of the nodes joining and leaving, and adjusting the $k$ value dynamically in real time. We will extend our work in this respect in our future research.
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Algorithm 1. Estimation of network size

\[ \text{EstimationNetworkSize}(s) \]  // estimate the global network size for node \( s \)

1: \( \text{length} \leftarrow \text{the clockwise length between \text{SUCC}(s) and \text{PRED}(s)} \)  // length of the whole ring is 1
2: \( \text{estimated number} \leftarrow \frac{2}{\text{length}} \);
3: \( \text{return}; \text{estimated number} \)

Algorithm 2. Establishment of links for each node

\[ \text{BuildLinks}(s, k, \text{length}) \] /*build all links for node \( s \). \( k \) is the number phases per level. \( \text{length} \) is the length of division level. Initially call \text{BuildLinks}(s,k,1)*/

1: if \( \text{length}/k \) \( < \) \( (1/\text{EstimationNetworkSize}(s)) \) then
2: return;
3: end if
4: for \( (j = 2; j \leq k - 1; j + +) \) do
5: for \( (\text{retry time} = 1; \text{retry time} \leq \sqrt{\text{EstimationNetworkSize}(s) \times \text{length}/k}; \text{retry time} + +) \) do  // \( \sqrt{\text{EstimationNetworkSize}(s) \times \text{length}/k} \) is the bound of retry times
6: \( \text{rand} \leftarrow \text{a random number between [0,1)} \);
7: \( \text{temp}_n \leftarrow \text{FindSuccessor(temp}_n, s) \);
8: if \( \text{incoming degree of node} \text{temp}_n \) \( \leq \) \( 2 \times k \times \log(\text{EstimationNetworkSize}(s)) \) then  // \( 2 \times k \times \log(\text{EstimationNetworkSize}(s)) \) is the limitation of incoming degree per node
9: if \( \text{rand} \in [\text{ID}(s) + \frac{j \times \text{length}/k}{\text{EstimationNetworkSize}(s)}; \text{ID}(s) + \frac{(j+1) \times \text{length}/k}{\text{EstimationNetworkSize}(s)}) \) then  // \( \text{ID}(s) \) the identifier of node after mapped into \([0,1)\)
10: build a connection between node \( s \) and node \( \text{temp}_n \);
11: end if
12: end if
13: end for
14: end for
15: \text{BuildLinks}(s,k,\text{length}/k)
Algorithm 3. Find the successor for a given identifier

FindSuccessor(s, x) /*find the successor of identifier x through node s*/
1: if \( x \in [ID(s), ID(s + 1)) \) then \#ID(s) is the identifier of node s mapped into \([0, 1)\)
2: \( \text{return} \ s + 1; \)
3: \( \text{else} \)
4: \( \text{return} \ \text{FindSuccessor}(s + 1, x) \)
5: \( \text{end if} \)

Algorithm 4. Find the predecessor for a given identifier

FindPredecessor(s, x) /*find the predecessor of identifier x through node s*/
1: if \( x \in [ID(s), ID(s - 1)) \) then \#ID(s) is the identifier of node s mapped into \([0, 1)\)
2: \( \text{return} \ s - 1; \)
3: \( \text{else} \)
4: \( \text{return} \ \text{FindSuccessor}(s - 1, x) \)
5: \( \text{end if} \)

Algorithm 5. Join operation

Join(n_0, x) /*node s joins the system through node n_0*/
1: \( n_1 \leftarrow \text{FindPredecessor}(n_0, s); \)
2: \( \text{SUCC}(n_1) \leftarrow s; \)
3: \( \text{PRED}(s) \leftarrow n_1; \)
4: \( n_2 \leftarrow \text{FindSuccessor}(n_0, s); \)
5: \( \text{SUCC}(s) \leftarrow n_2; \)
6: \( \text{PRED}(n_2) \leftarrow s; \)
7: \( \text{EstimationNetworkSize}(s) \)
8: \( \text{BuildLinks}(s, k, 1) \)

Algorithm 6. Leave operation

Leave(x)
1: \( \text{SUCC}(\text{PRED}(s)) \leftarrow \text{SUCC}(s); \)
2: \( \text{PRED}(\text{SUCC}(s)) \leftarrow \text{PRED}(s); \)
3: delete all incoming and outcoming links of node s and inform corresponding nodes;
Algorithm 7. Lookup operation

Lookup\(x\) /* lookup identifier \(x\) through node \(s\) */
1: \(\text{min\textunderscore length} \leftarrow 0;\)
2: \(\text{min\textunderscore neighbor} \leftarrow \text{SUCC}(s);\)
3: if \(\text{FindSuccessor}(s,x)=s\) then
4: \(\text{return} \ 0;\)
5: end if
6: \(\text{min\textunderscore length} \leftarrow \text{clockwise distance between} \ \text{min\textunderscore neighbor} \text{ and } \text{FindSuccessor}(x);\)
7: for \(n_i \in \text{neighbors of nodes} \) do
8: \(\text{if} \ \text{clockwise distance between} \ n_i \text{ and } \text{FindSuccessor}(x) < \text{min\textunderscore length} \text{ then} \)
9: \(\text{min\textunderscore length} \leftarrow \text{clockwise distance between} \ n_i \text{ and } \text{FindSuccessor}(x);\)
10: \(\text{min\textunderscore neighbor} \leftarrow n_i;\)
11: end if
12: end for
13: \(\text{return} \ \text{Lookup}(\text{min\textunderscore neighbor},x)+1;\)

Algorithm 8. Relinking operation

Relink\(s\)
1: if \(\tilde{n} < 0.5 \) or \(\tilde{n} > 2\) then //\(\tilde{n}\) denotes new estimated \(n\); \(\tilde{n}'\) denotes old estimated \(n\)
2: delete all outcoming links of node \(s\) and inform corresponding nodes;
3: \(\text{BuildLinks}(s,k,1)\)
4: end if