Model for the Cherenkov light emission of TeO$_2$ cryogenic calorimeters

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Abstract

The most sensitive process able to probe the Majorana nature of neutrinos and discover Lepton Number Violation is the neutrino-less double beta decay. Thanks to the excellent energy resolution, efficiency and intrinsic radio-purity, cryogenic calorimeters are primed for the search for this process. A novel approach able to improve the sensitivity of the current experiments is the rejection of $\alpha$ interactions, that represents the dominant background source. In TeO$_2$ calorimeters, $\alpha$ particles can be tagged as, in contrast to electrons, they do not emit Cherenkov light. Nevertheless, the very low amount of detected Cherenkov light undermines the complete rejection of $\alpha$ background.

In this paper we compare the results obtained in previous measurements of the TeO$_2$ light yield with a detailed Monte Carlo simulation able to reproduce the number of Cherenkov photons produced in $\beta/\gamma$ interactions within the calorimeter and their propagation in the experimental set-up. We demonstrate that the light yield detectable from a $5 \times 5 \times 5$ cm$^3$ TeO$_2$ bolometer can be increased by up to 60% by increasing the surface roughness of the crystal and improving the light detector design.

Moreover, we study the possibility to disentangle $\alpha$, $\beta$ and $\gamma$ interactions, which represent the ultimate background source. Unfortunately $\gamma$ rejection is not feasible but $\alpha$ rejection can be achieved exploiting high sensitivity light detectors.

Keywords: Double beta decay, Neutrino mass and mixing, Bolometer, Cherenkov radiation, Monte Carlo methods

1. Introduction

Neutrino-less double beta decay (0$\nu$DBD) is an extremely rare process (if it occurs at all) in which a nucleus undergoes two simultaneous beta decays without emitting any neutrinos. Since the absence of emitted neutrinos would violate the lepton number conservation, the 0$\nu$DBD cannot be accommodated in the Standard Model. Therefore, the observation of this process would have several implications for particle physics, astrophysics and cosmology. The most common theoretical frameworks account for 0$\nu$DBD assuming that neutrinos are Majorana particles. This means that, in contrast to all the other known fermions, they must coincide with their own antiparticles. The measurement of the half-life of 0$\nu$DBD ($T^{0\nu}_{1/2}$) would allow to infer the effective Majorana neutrino mass ($m_{\text{eff}}$), provided the dependance: $T^{0\nu}_{1/2} \propto 1/m_{\text{eff}}^2$. $m_{\text{eff}}$ is the coherent sum of the three neutrino mass eigenstates according to the PMNS neutrino mixing matrix and including two Majorana phases, thus it depends on the mass hierarchy of neutrino (see Refs [1, 2, 3] and the references therein for a complete discussion on the relationship between 0$\nu$DBD and neutrino masses).

CUORE (Cryogenic Underground Observatory for Rare Events) [4] is an array of 988 TeO$_2$ cryogenic calorimeters, historically also called bolometers, of $5 \times 5 \times 5$ cm$^3$ each. Starting operation in 2017 it will become one of the most sensitive experiments searching for 0$\nu$DBD of $^{130}$Te. The signal produced by this reaction consists of two electrons with a total kinetic energy of about 2.527 MeV (the Q-value of the transition) [5]. Unfortunately the $\alpha$ background rate, at the level of 0.01 counts/keV/kg/y [6], will limit the sensitivity to $T^{0\nu}_{1/2}$ and, consequently to the effective Majorana mass. The sensitivity to $m_{\text{eff}}$ will be at the level of 0.13 – 0.05 eV in 5 years live time, i.e. the upper limit of the inverted hierarchy region of the neutrino masses [4]. Next generation 0$\nu$DBD experiments, such as CUPID (Cuore Upgrade with Particle IDentification) [7, 8], aim to reach a sensitivity to $m_{\text{eff}}$ of $\sim 0.01$ eV, i.e. the lower limit of the inverted hierarchy region. To reach this ambitious goal the source mass must be increased and the background in the region of interest dramatically reduced [9]. The CUPID collaboration aims to increase the number of 0$\nu$DBD emitters, i.e. the source mass, using crystals grown from enriched material. The background suppression can be achieved by discriminating $\beta/\gamma$ against $\alpha$ events by means of the different light yield produced in the interactions within a scintillating bolometer [10]. The main candidates for this technique are ZnSe [11, 12], ZnMoO$_4$ [13, 14, 15] or Li$_2$MoO$_4$ [16] scintillating crystals.

The same technique cannot be used for TeO$_2$ as this crystal does not scintillate [17, 18] and also no difference exists between the shape of the pulses produced by $\alpha$ and $\beta/\gamma$ interactions [19].

However, the many advantages offered by this material in terms of bolometric performance and lower enrichment cost with respect to other candidate nuclei provided a strong motivation to pursue another, challenging, option: the active background rejection technique can be applied to the TeO$_2$ bolome-
ter exploiting, instead of the scintillation light, the Cherenkov radiation. Indeed, as proposed in Ref. [20] detecting the Cherenkov radiation produced in the TeO$_2$ crystal only by electrons (see section 2.2), it is possible to disentangle the $\beta/\gamma$ interactions from the $\alpha$ interactions.

In order to measure the Cherenkov light yield (LY) of TeO$_2$ bolometers several tests coupling germanium light detectors [21] were performed in a dilution refrigerator working at 10 mK and located deep underground in the Hall C of Laboratori Nazionali del Gran Sasso. The first indication of the feasibility of this technique to reject the $\alpha$ background can be found in Ref. [22]. In this work a small TeO$_2$ crystal ($3.0 \times 2.4 \times 2.8$ cm$^3$) doped with natural Sm and covered with 3M VM2002 reflective foils was faced to a germanium light detector. The amount of Cherenkov light detected at 0 DBD energy of $^{130}$Te for $\beta/\gamma$ interactions was about 173 eV; no light was detected for $\alpha$ particle interactions. A more systematic study of the Cherenkov LY emitted by a CUORE-size TeO$_2$ crystal was done in Ref. [23] where several reflectors and detector configurations were tested in order to optimize the photon collection. The result of this study was that the maximum amount of Cherenkov light detected from a CUORE TeO$_2$ crystal is about 100 eV at the Q-value of $^{130}$Te 0 DBD, irrespective of the reflector (aluminum, PTFE tape, 3M VM2002). This, combined with the poor baseline resolution of the light detectors (80 eV RMS) prevented the complete $\alpha$ particles rejection. As discussed in Ref. [23] the reduced light collection efficiency and its invariance over the several reflectors tested suggests that most of the emitted light is reabsorbed by the TeO$_2$ crystal because of its optical properties.

The purpose of this paper is to demonstrate this hypothesis and to quantify its effects exploiting a detailed Monte Carlo simulation of the Cherenkov photon production and propagation in the crystal and in an experimental set-up similar to the one described in Ref. [23]. Thanks to the simulation, it is also possible to identify the parameters able to improve the light collection efficiency of the set-up. Finally, the simulation demonstrates that the Cherenkov radiation tagging technique does not allow for the discrimination between $\beta$ and $\gamma$ interactions, that will constitute the ultimate background source for the bolometric experiments based on TeO$_2$ crystals. Concerning the $\alpha$ background rejection, the simulation results confirm that it can be achieved exploiting a high sensitivity light detector.

2. Cherenkov photon production in a TeO$_2$ crystal

The number of Cherenkov photons produced per unit path length and per unit wavelength of a particle with charge $ze$ is [24]:

$$\frac{dN^2}{dx d\lambda} = \frac{2\pi\alpha \lambda^2}{\lambda^2} \left(1 - \frac{1}{\beta^2 (n(\lambda))^2}\right)$$

(1)

where $\alpha$ is the fine-structure constant, $\beta$ is the ratio between the velocity of the particle and the speed of light, $\lambda$ the wavelength of the Cherenkov photons, and $n(\lambda)$ the refractive index of the material. Eq. 1 is valid as long as it is positive, namely as long as the charged particle moves faster than light in the respective medium ($\beta \tau > 1$). As shown in Eq. 1 the evaluation of the number of Cherenkov photons produced by a particle interaction within a TeO$_2$ crystal requires the knowledge of its optical properties and the particle range. Therefore in the following sections the optical properties of TeO$_2$ crystals will be presented together with its stopping power for electrons, in order to perform an estimation of the Cherenkov photons emitted in the $\beta/\gamma$ interactions.

2.1. Optical properties of TeO$_2$ crystals

TeO$_2$ is a birefringent material. The ordinary and extraordinary refractive indices ($n_o$ and $n_e$, respectively) are shown in Fig. 1. Both the refractive indices are described by the Sellmeier equation [26]:

$$n(\lambda)^2 - 1 = \frac{A_1 \lambda^2}{\lambda^2 - \lambda_1^2} + \frac{B \lambda^2}{\lambda^2 - \lambda_2^2}$$

(2)

The Sellmeier parameters for the TeO$_2$ crystal are shown in Table 1.

Table 1: Coefficients of the Sellmeier equations for the TeO$_2$ crystal at room temperature [25].

|   | $A$ | $A_1$ [\mu m] | $B$ | $A_2$ [\mu m] |
|---|-----|---------------|-----|---------------|
| $n_o$ | 2.584 | 0.1342 | 1.157 | 0.2638 |
| $n_e$ | 2.823 | 0.1342 | 1.542 | 0.2631 |

As TeO$_2$ crystals are currently produced for acousto-optic devices, special attention is given in literature only to optical properties connected to this application. The study of optical characteristics as transmission and reflection near and above the fundamental absorption edge, at low temperatures, were carried out down to only 80 K [27] and experimental results [25, 27] do

![Figure 1: Ordinary ($n_o$, continuous black line) and extraordinary ($n_e$, dotted black line) refractive indices of TeO$_2$ crystal as function of wavelength at room temperature [25]. $\theta_{critical}$ is the critical angle for total internal reflection for an optical photon that hits the internal surface of a TeO$_2$ crystal surrounded by vacuum using the ordinary ($\theta_{critical}^o$, continuous red line) and extraordinary ($\theta_{critical}^e$, dotted red line) refractive indices.](image-url)
formed optical transmission measurements on pure TeO$_2$ slices using a Perkin Elmer Lambda 900 spectrophotometer and a Leybold RDK10-320 ($T_{\text{min}}=12$ K) cryostat. The results around the fundamental absorption edge are shown in Fig. 2; for wavelengths longer than 340 nm the absorption length approaches a value of about 80 cm, irrespective of the temperature. The absorbance spectra show a temperature dependence typical of an indirect band structure, and the shift in the $\lambda_{\text{cut-off}}$ due to the temperature variation reaches an asymptotic value for $T < 50$ K. This suggests that for temperatures below 15 K it is possible to assume an absorbance spectrum like the one measured at 15 K.

In principle, also the temperature dependence of the refractive index may affect the number of Cherenkov photons but, as for the absorption length, no reference is available in literature. This systematic uncertainty was investigated by allowing the refracting index to vary according to the Lorentz theory for optical properties of dielectrics [29], that foresees a maximum shift of the refractive index curve of about 20 nm toward lower wavelengths (as the variation is expected to follow the one of the absorption length). Using this value of the refractive index in Eq. 1 (instead of the room temperature refractive index), the number of Cherenkov photons emitted changes by less than 1%. Therefore the temperature dependence of the refractive index will be neglected.

2.2. Number of Cherenkov photons emitted per unit path length

Using the refractive index and the absorption length it is possible to evaluate the number and the wavelength spectrum of the emitted Cherenkov photons per unit path length of the charged particle. First of all, an estimation of the Cherenkov threshold must be made: using, for example, the value of $n_c = 1.01$ for alpha particles ($\frac{c}{v} = \beta > \frac{1}{\gamma}$).

It should be mentioned that, even if the Cherenkov photons are emitted in a cone with an opening angle $\theta_C = \arccos(1/\beta n)$ with respect to the electron’s direction, they can be approximated (in our case) as isotropically emitted. The reason is that the electrons that emit Cherenkov light are affected by several direction changes due to multiple Coulomb scattering within
the crystal. Furthermore, the surface of TeO$_2$ causes several reflections on the emitted photons. Thus, the initial directionality of the Cherenkov photons is lost and does not affect the mean end distribution of the Cherenkov energy detected, as confirmed by the measurements performed in Ref. [22, 23].

3. Light trapping and surface effects of tellurium dioxide crystal

The number of photons that escape the TeO$_2$ crystal is smaller than the 280 calculated above; this is due to the light trapping that happens when an optical photon travels from a medium with a higher refractive index ($n_1$) to one with a lower refractive index ($n_2$). If the incident angle of the photon on the crystal surface is grater than $\theta_{\text{critical}} = \arcsin(n_2/n_1)$ the photon is totally reflected inside the crystal. For the TeO$_2$ surrounded by vacuum $\theta_{\text{critical}}$ is shown in Fig. 1 for both the ordinary and the extraordinary refractive index. This small critical angle, in conjunction with the high symmetry of the cubic shape of the crystal, leads to a high probability (about 75%) that photons are internally reflected an indefinitely large number of times, i.e. they remain trapped inside the crystal until absorption.

Fortunately, this trapping effect can be partially overcome thanks to the roughness of the crystal surface and the birefringence of TeO$_2$. Both these properties can change the reflection angle increasing the exit probability of the photons, as will be shown in Sec. 5.

4. Comparison between Monte Carlo simulation and data

As roughly evaluated in section 2, the produced Cherenkov light for a 2.5 MeV electron amounts to about 780 eV, a very different value from the 100 eV measured in Ref. [23]. To understand this discrepancy we developed a detailed Monte Carlo simulation able to reproduce the Cherenkov photon production and propagation inside the experimental set-up. The simulation was split in two parts: (i) the total number of emitted Cherenkov photons was evaluated using Geant4, (ii) the propagation of these photons inside the different components of the experimental set-up was reproduced by means of Litrani [31], a general simulation software specifically developed for the propagation of optical photons.

The refractive index and the absorption length of the germanium in the optical wavelength range [32], as well as the real and imaginary part of the refractive index of the 3M VM2002 foil [31] have been found in literature only at room temperature. Nevertheless, using the room temperature values does not affect the simulation results, as the reflectivity properties of materials are governed by Fresnel’s equations, that depend only on their refractive indices. As shown for example in Ref [33], the refractive index of semiconductors (and in general of all the common materials) varies only by few percent in a wide temperature range, resulting in a variation of their reflectivity lower than 1%. Similar remarks may be made about the absorbance capability of the germanium disk, because its thickness (300 μm) is orders of magnitude larger than its attenuation length (few nm as shown in Ref [32]). Even if the attenuation length can slightly change at cryogenic temperature, its capability to stop an optical photon does not change. Given these considerations, we can state that approximating the optical properties of the germanium and of the reflective foil does not affect the simulation results.

The surface roughness of the crystal is the only free parameter of the simulation. The reason is that the light diffusion produced by a rough surface is implemented in LITRANI exploiting the following approximation: the normal to the surface at the point hit by the photon is randomly tilted (with respect to the true normal of the surface) by an angle $\theta$. This angle is generated according to a distribution $\sin(\theta)\cos(\theta)$, between 0 and $\theta_{\text{rough}}$. This parametrization is just an efficient Monte Carlo implementation of the more complex diffusion effect, therefore it cannot be compared to a direct measurement of the surface roughness, and it must be tuned from data. A detailed analysis of the light output as function of $\theta_{\text{rough}}$ is discussed in Sec. 5.

The Monte Carlo starts with the simulation of the three $\gamma$s that produce the six most intense peaks in the TeO$_2$ energy spectrum (see Ref. [22, 23] for more details): 2615 keV, 1460 keV and 911 keV ($^{208}\text{Tl}$, $^{40}\text{K}$ and $^{228}\text{Ac}$ respectively) $\gamma$s in the TeO$_2$ crystal as obtained from the Monte Carlo simulation (Geant4). In addition to the three photo-peaks we can see the SE, DE and the 511 keV photo-peak.

![Figure 4: Spectrum of the energy deposited by the 2615 keV, 1460 keV and 911 keV ($^{208}\text{Tl}$, $^{40}\text{K}$ and $^{228}\text{Ac}$ respectively) $\gamma$s in the TeO$_2$ crystal as obtained from the Monte Carlo simulation (Geant4). In addition to the three photo-peaks we can see the SE, DE and the 511 keV photo-peak.](image_url)
Selecting each photo-peak, it is possible to extract the kinetic energy and the path of all electrons and positrons produced in these interactions. Then, using equation 1, we can assess the number of Cherenkov photons produced by each e\(^+\) particle with kinetic energy greater than 50 keV, as shown in Fig. 5 for the \(^{208}\)Tl photo-peak. Once the number of Cherenkov photons produced is known, it is possible to simulate their propagation in the experimental set-up with Litrani; for this purpose also the optical properties of the germanium and 3M VM2002 reflective foil are needed [31, 32]. For each photo-peak we simulate the light produced by 100 interactions. For each interaction, the total number of Cherenkov photons and their wavelength distribution are randomly generated respectively according to the distribution shown in Fig. 5 and Fig. 3. In the particular case of \(^{208}\)Tl, the energy of the emitted Cherenkov photons results in a Gaussian with \(\mu\) and \(\sigma\) respectively of about 744 eV and 120 eV. The average energy is very close (less than 5%) to the raw approximated value calculated in Sec. 2.

Each Cherenkov photon is propagated through the materials until it is absorbed: if it is absorbed inside the germanium disk it is considered as detected. The number of detected photons and their wavelength distribution for the \(^{208}\)Tl peak are shown in Fig. 6.

The Cherenkov energy spectrum of the photons absorbed by the germanium light detector can be evaluated integrating event by event the spectra of Fig. 6. This spectrum results in a Gaussian with \(\mu\) and \(\sigma\) respectively of about 117 eV and 23 eV. The same procedure is applied to the other five peaks and the obtained energies are compared with those measured in Ref. [23] in Fig. 7.

The simulation reproduces the experimental data and confirms that just 18% of Cherenkov light is detected. This is because only 27% of light is able to escape the unwrapped crystal face, and 69% of it is absorbed by the Ge-disk. The remaining 82% is trapped inside the detector set-up, and it is absorbed by the crystal itself (60%) or by the reflective foil and the light detector copper frame (22%) after many reflections (see Ref. [23] for details on the detector set-up). The Monte Carlo simulation also confirms that no significant improvements in light collection is expected using other reflectors such as PTFE tape or aluminum foil.

In light of these considerations it is possible to understand why all the trials to increase the light collection efficiency were not successful: the trapping effect of the TeO\(_2\) crystal and the reflectivity of the germanium disk do not allow to collect more than 18% of the Cherenkov light produced by \(\beta/\gamma\) interactions.

An additional cross-check was made to validate the simulation: the LY of the smaller TeO\(_2\) crystal, described in Sec. 1, was entirely reproduced exploiting the Monte Carlo. In this
case, the simulation predicts a higher LY with respect to the 5 × 5 × 5 cm³ crystal, due to the lower self-absorption. The detected light predicted by the simulation at the 208Tl line (2615 keV) is 205 ± 4 eV, in full agreement with the measured one (195 ± 7 eV). This result can be considered as a further evidence that, despite the approximations, the Monte Carlo simulation developed in this work is able to reproduce and predict the Cherenkov light yield of TeO₂ bolometers.

The Monte Carlo results depend on the only free parameter of the simulation, θ_{rough}, parameterizing the surface roughness of the crystal. A detailed study of the light emitted as function of θ_{rough} will be shown in the next sections.

5. Effect of the crystal surfaces roughness on light yield

The only free parameter of the simulation, θ_{rough}, is related to the surface roughness of the crystal. The surface treatment of the CUORE crystals was developed to maximize the surface radio-purity of the crystal. Due to TeO₂ hardness characteristics, the crystal features two opposite faces with a better polishing quality (close to optical polishing grade) and four faces with a higher roughness. Therefore, the crystal surface is not optimized for the light collection. In the measurements performed in Ref. [22, 23], the germanium light detector monitored one of the two polished faces of the crystal.

As discussed in Sec. 3, the surface roughness interferes with the trapping effect because of the high refractive index and symmetry of the TeO₂ bolometers, increasing the photons emission probability. The result reported in Fig. 8 highlights the correlation between the emitted light and the surface roughness of the crystal faces. Fig. 8 also shows the value of θ_{rough} that reproduces all the cryogenic tests performed with cubic TeO₂ crystals (red dotted line θ_{rough} = 20°).

![Figure 8](image-url)

Figure 8: Average number of photons reaching the light detector (1.2 cm away from crystal) as a function of the roughness of the four lateral faces of the crystal.

The simulation suggests that the LY can be maximized by increasing the roughness of the four lateral faces of the crystal. The light yield can be increased by 35% with respect to the standard CUORE TeO₂ crystals (θ_{rough} ∼ 20°) with a surface roughness of θ_{rough} ∼ 60°. No additional advantages are found assuming that also the two opposite polished faces of the TeO₂ crystal are roughened.

6. Improving the light collection

Also the shape of the germanium light detector can be optimized to maximize the light collection efficiency. The light detector of Ref. [23] was a 300 µm thick, 5 cm in diameter germanium disk. Since the best shape and size of the light detector is the one that matches the TeO₂ face, a 5 × 5 cm² and 300 µm thick germanium slab was simulated. The active surface of this new light detector is larger by 27% with respect to the old one. The distance between the crystal face and the germanium slab does not affect significantly the light collection efficiency in the range between 1.2 and 0.2 cm (in the hypothesis that the reflective foil surrounds also the empty space between the crystal face and the light detector), and for simplicity it was set to 1 cm. The mean values and the standard deviations of the light detected at the energies of the γ peaks were derived by performing Gaussian fits (the same procedure adopted in section 4). These data were fitted as a function of the energy with a first degree polinomial function (see Fig. 9). The high value of the fit χ²/ndf for the simulation can be explained by the fact that the linear fit is just a first approximation of the real trend of the Cherenkov light emission as a function of the energy of the γ interactions, as explained in the text.
A remarkable improvement in light collection efficiency can be achieved also by improving the absorption of the Ge light detector. This can be done applying an antireflection coating on the Ge surface. Nevertheless, no reference is available in literature about a Ge antireflection coating that allows to increase the absorption over a wide range of wavelength (280-1000 nm) as well as over a wide range of incidence photons angle (0-90 degrees). Therefore this possibility was not investigated in this work.

7. β, γ and α discrimination

Fig. 9 shows that the linear function used in Ref. [22, 23] to describe the Cherenkov light emission of TeO$_2$ bolometers is just a first approximation of the real trend of the Cherenkov light as a function of the β/γ energy. For example, the events produced by the DE peak (1593 keV) are $e^- + e^+$ interactions that produced an average Cherenkov energy slightly greater than a γ interaction with energy 1593 keV. Similarly, the events produced by the SE peak (2104 keV) are $e^- + e^+ + γ(511 \text{ keV})$. As expected, the Cherenkov light is equal to the one obtained for the DE peak plus the Cherenkov energy obtained for the 511 keV gamma interaction, and results slightly smaller than a γ interaction with energy 2104 keV. A small deviation from the linear trend can be observed also for the 2615 keV peak.

![Figure 10: Energy distributions of the Cherenkov light produced by 1000 0νDBD events of $^{130}$Te and 1000 γ interactions with the same energy. The intrinsic fluctuations of these energy depositions are produced by the Poisson fluctuation of the number of emitted photons and by the $dE/dx$ fluctuation along the electron’s paths.](image)

Given these considerations, it seems that both β and γ interactions with the same energy result in a different Cherenkov yield. To study if also the γ background can be removed from TeO$_2$ bolometers, the Cherenkov energy detected by the optimized set-up was simulated for 0νDBD events of $^{130}$Te, and then compared with the one produced by γ interactions of the same energy. The two energy distributions detected by the germanium slab are compared in Fig. 10. Even if the mean values are different, the distributions are partly overlapped, thus preventing the discrimination of a possible 0νDBD signal against the γ background.

On the contrary, the Cherenkov energy emitted by α particles with kinetic energy of few MeV is zero. Therefore, the discrimination power (DP) between these interactions and electrons is determined only by the light detector performance, i.e. its baseline resolution ($\sigma_{\text{baseline}}$):

$$DP = \frac{\mu_\beta}{\sqrt{2\sigma_{\text{baseline}}^2 + \sigma_\beta^2}}$$

As stated in Ref. [23], a very satisfactory α background rejection can be obtained for DP approaching 5. As shown in Fig. 11, solving Eq. 3 for $\sigma_{\text{baseline}}$ and using the Cherenkov signal ($\mu_\beta$) and its intrinsic fluctuation ($\sigma_\beta$) estimated from the simulation, a baseline RMS resolution of 11 eV and 20 eV respectively for the standard and optimized setup was obtained.

![Figure 11: The discrimination power between α and β interactions as a function of the baseline resolution of the light detector facing the TeO$_2$ bolometer for both the standard (continuous line) and the optimized (dotted line) set-up.](image)

Therefore, the rejection of the α background in a CUORE-like TeO$_2$ bolometer can be achieved exploiting light detectors with a baseline RMS energy resolution of about 11 eV. This constraint on the energy resolution of light detectors can be relaxed by a factor 2 (20 eV RMS) assuming that the light collection improvement simulated in Sec. 6 is actually feasible.

8. Conclusions

In this paper a complete simulation of the Cherenkov photon production and propagation in TeO$_2$ bolometers is presented. The results are in good agreement with the data presented in Ref. [22, 23]. The simulation also suggests that the detectable Cherenkov light energy for a 0νDBD of $^{130}$Te can be increased up to 190 eV by increasing the roughness of the crystal and by optimizing the size and shape of germanium light detector. Also the possibility to perform a β-γ discrimination was investigated, but the intrinsic fluctuation of the number of Cherenkov photons do not allow such kind of particle identification. On the
contrary, a complete $\alpha$ background rejection can be achieved exploiting light detectors with a baseline RMS resolution below 20 eV (11 eV using the experimental set-up described in Refs. [23]).

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References

[1] S. Dell'Oro, S. Marcocci, M. Viel and F. Vissani, Neutrinoless double beta decay: 2015 review, Adv. High Energy Phys. 2016, 2162659 (2016) doi:10.1155/2016/2162659 [arXiv:1601.07512 [hep-ph]].
[2] F. Feruglio, A. Strumia and F. Vissani, Neutrino oscillations and signals in beta and $0\nu2\beta$eta experiments, Nucl. Phys. B 637, 345 (2002) Addendum: [Nucl. Phys. B 659, 359 (2003) doi:10.1016/S0550-3213(02)00345-0, 10.1016/S0550-3213(03)00228-1 [hep-ph/0201291].
[3] A. Strumia and F. Vissani, Implications of neutrino data circa 2005, Nucl. Phys. B 726, 294 (2005) doi:10.1016/j.nuclphysb.2005.07.031 [hep-ph/0503246].
[4] D. R. Artusa, et al., Searching for neutrinoless double-beta decay of $^{130}$Te with CUORE, Adv. High Energy Phys. 2015 (2015) 879871. arXiv:1402.6072, doi:10.1155/2015/879871.
[5] M. Redshaw, B. J. Mount, E. G. Myers and F. T. Avignone, III, Masses of $\nu_e$ and $\nu_\mu$ from radiopure ZnMoO$_4$ crystals array, J. Phys. G 41, 075204 (2014), DOI:10.1088/0954-3899/41/7/075204).
[6] D. R. Artusa, et al., Searching for double beta decay, Eur. Phys. J. C 76 (2016) doi:10.1140/epjc/s10052-014-3225-4.
[7] P. A. Čerenkov, Visible radiation produced by electrons moving in a medium with velocities exceeding that of light, Phys. Rev. 52 (1937) 378–379. doi:10.1103/PhysRev.52.378.
[8] N. Uchida, Optical Properties of Single-Crystal Paratellurite (TeO$_2$), Phys. Rev. B 4 (1971) 3736–3745. doi:10.1103/PhysRevB.4.3736.
[9] T. Takizawa, Optical Absorption and Reflection Spectra of Paratellurite, TeO$_2$, Journal of the Physical Society of Japan 48 (2) (1980) 505–510. arXiv:http://dx.doi.org/10.1143/JPSJ.48.505. URL http://dx.doi.org/10.1143/JPSJ.48.505.
[10] B. R. Sahu, L. Kleinman, Effect of atomic displacements on the ground state of $\alpha$-TeO$_2$, Phys. Rev. B 69 (2004) 193101. doi:10.1103/PhysRevB.69.193101.
[11] M. A. Zucker, M.J. Berger, J.S. Coursey, J. Chang. Estar, pstar, and astar: Computer programs for calculating stopping-power and range tables for electrons, protons, and helium ions (version 1.2.3) [online] (2005).
[12] F. X. Gentili, Litirani: a general purpose Monte-Carlo program simulating light propagation in isotropic or anisotropic media, Nucl. Instrum. Meth. A 486 (1–2) (2002) 35 – 39. doi:http://dx.doi.org/10.1016/S0168-9002(02)00671-X.
[13] D. E. Aspnes, A. A. Studna, Dielectric functions and optical parameters of Si, Ge, GaP, GaAs, GaSb, InP, InAs, and InSb from 1.5 to 6.0 ev, Phys. Rev. B 27 (1983) 985–1009. doi:10.1103/PhysRevB.27.985.
[14] B. J. Frey, D.B. Leviton, and T.J. Madison. Temperature-dependent refractive index of silicon and germanium. Proc. SPIE 6273, Optomechanical Technologies for Astronomy, 62732J, 2006. doi:10.1117/12.672850.