EXPANDING THE REALM OF MICROLENSING SURVEYS WITH DIFFERENCE IMAGE PHOTOMETRY

Austin B. Tomaney\(^1,2\) and Arlin P. S. Crotts\(^1,2\)
Department of Astronomy, Columbia University, 538 W. 120th St., New York, NY 10027

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\(^2\) Based on observations at the Vatican Advanced Technology Telescope (The Alice P. Lennon Telescope and Thomas J. Bannan Astrophysical Facility), Mt Graham Arizona

ABSTRACT

We present a new technique for monitoring microlensing activity even in highly crowded fields, and use this technique to place limits on low-mass MACHOs in the haloes of M31 and the Galaxy. Unlike present Galactic microlensing surveys, we employ a technique in which a large fraction of the stellar sample is compressed into a single CCD field, rather than spread out in a way requiring many different telescope pointings. We implement the suggestion by Crotts (1992) that crowded fields can be monitored by searching for changes in flux of variable objects by subtracting images of the same field, taken in time sequence, positionally registered, photometrically normalized, then subtracted from one another (or a sequence average). The present work tackles the most difficult part of this task, the adjustment of the point spread function among images in the sequence so that seeing variations play an insignificant role in determining the residual after subtraction. The interesting signal following this process consists of positive and negative point sources due to variable sources. The measurement of changes in flux determined in this way we dub “difference image photometry” (also called “pixel lensing” [Gould 1996]).

The matching of the image point spread function (PSF) is accomplished by a division of PSFs in Fourier space to produce a convolution kernel, in a manner explored for other reasons by Phillips & Davis (1995). In practice, we find the application of this method is difficult in a typical telescope and wide field imaging camera due to a subtle interplay between the spatial variation of the PSF associated with the optical design and the inevitable time variability of the telescope focus. Such effects lead to complexities in matching the PSF
over an entire frame. We demonstrate the realization of the difference image approach with two separate solutions to these problems - a software algorithm to determine the match of the spatially varying PSF between frames using a limited number of stars and also a simple optical corrector for a wide field imager to simplify the PSF matching function.

The former solution yielded light curves of 139 variable sources detected in a 16′ by 16′ field in M31 over four nights on the KPNO 4-m telescope in 1994 and the latter yielded over 2000 sources detected over 50 nights in a single 11′ by 11′ field in M31 observed at the VATT 1.8-m telescope in 1995 using an optical corrector to facilitate the PSF matching problem. Of the KPNO sources discovered, 85 overlap with the VATT field and 23 of these were found to have a positional coincidence of < 1″ to sources found in the VATT data. Light curves of the VATT objects over 14 nights confirm the short timescale variability of these sources. Although some fraction of the sources are bright enough to be considered resolved in the raw data more than half the sources are fainter than the surface brightness fluctuations associated with the unresolved stars in the galaxy and cannot be identified in the raw data. However, the light curves of these sources appear to be familiar variables such as Cepheids and eclipsing binaries.

We assess the limitations and sensitivities of the techniques and demonstrate that we can achieve photometric errors of faint unresolved variables that are within a factor of three of the ultimate photon noise limit. Using this we show how the KPNO data over two good nights, and sensitive to > 10⁶ stars on a single CCD frame, yields 2σ optical depth limits of 5 × 10⁻⁷ for Galactic MACHOs in the mass range 2 × 10⁻⁷M⊙ (0.07 Earth masses). Given the estimate of the optical depth of the Galactic halo towards M31 of τ = 1 × 10⁻⁶ (assuming a simple spherical halo), we can conclude that in two nights we have eliminated the possibility at the 95% confidence level that the Galactic halo is comprised of a single mass population of MACHOs in the sub-Earth mass range. Based on estimates of the M31 and Galactic MACHO τ = (5 – 10) × 10⁻⁶ we exclude the halo of these galaxies being composed of 8 × 10⁻⁵M⊙ MACHOs at the > 95% confidence level.

These kind of techniques can extend present microlensing surveys into regimes not limited to resolved stars, which greatly expands the power of these surveys. Application to surveys of more general kinds of variability is clear. We also suggest other applications, such as to proper motion surveys.
1. INTRODUCTION

The potential rewards of a microlensing survey of M31 were first outlined by Crotts (1992, hereafter C92) and also analyzed in other papers by Baillon et al. (1993) and Colley (1995). In addition to Galactic halo MACHOs (MAssive Compact Halo Objects) such a survey is also sensitive to MACHOs in the halo of M31. Specifically, for M31 MACHOs large optical depths, $\tau$ (lensing probability per star), can be achieved considerably in excess of equivalent Galactic surveys, since a line of sight can be chosen through the densest regions of the halo. Due to the high inclination of M31, and depending on the halo and bulge geometry, optical depths can be up to ten times higher than for Galactic MACHOs. Since the lensing optical depth is dependent on the halo geometry, measurement of the optical depth can constrain halo models (see also Gould 1994 and Han & Gould 1996). This modulation can also allow for the construction of a control experiment in which equivalent stellar populations at the same galactocentric distance are monitored but with different lensing rates, in particular when comparing the near and far side of the galaxy.

An important additional benefit occurs for Galactic MACHOs. For a high amplification event to be detected, the projected Einstein ring at the lensed star must exceed the size of the photosphere of the star. Since the Galactic lenses are much closer they can be lower mass and still produce a detectable event. Thus sensitivity is dramatically increased for much lower masses in this case.

Unfortunately, at the distance of M31 (770 kpc, Freedman & Madore 1990) almost all stars are completely unresolved in typical ground based seeing of $\sim 1''$. Given present estimates of the optical depth towards M31 of up to $10^{-5}$ for the combination of both Galactic and M31 MACHOs (C92, Han & Gould 1996) an observing program must be sensitive to at least $10^5$ to $10^6$ stars to have a good probability of detecting a lensing event. It appears to be extremely unlikely in a given field of stars in M31 that a lensing event will occur in a bright star that could be considered to be resolved from the ground with present imaging capabilities.

To cope with this problem C92 suggested registering a sequence of CCD images to common coordinates, scaling to the same photometric intensity and subtracting images from a high S/N (signal to noise) template image. Since over some timescales of interest for microlensing events ($10^m$ up to a few months), most stars will not be varying (at least above some given detection limit in flux change), then those stars that do vary over the time span between the test frame and the reference frame may be detected in the difference frame as isolated positive or negative point sources depending on whether the particular star brightened or faded with respect to the reference frame. We call this “Difference Image Photometry” (DIP). Such a technique has recently been dubbed “pixel lensing” and has
been theoretically formalized in Gould (1996).

Although frame registration and photometric scaling are quite tractable with presently available software such as IRAF, it is the seeing variations between frames that cause the most concern for this technique. Frames not well matched in seeing will result in power on the scale of the PSF in the difference frame as flux from unresolved stars is poorly subtracted. Such systematic residuals may swamp the signal of genuine variability. It is this issue that in the past caused skepticism as to whether this kind of microlensing survey could ever be realised from the ground.

We employ a Fourier technique first outlined in Ciardullo, Tamblyn & Phillips (1990) and further developed by Phillips & Davis (1995) to match the PSF between frames. In practice, we find that the application of the technique to data taken with the KPNO 4-m prime focus camera is not a complete solution to PSF matching, and find that, separate from seeing variations, focus variations of the telescope increase the complexity of the PSF matching over the entire CCD frame. We outline an algorithm we have developed to cope with this problem which uses a limited number of stars on a given frame to determine the correct full-frame PSF matching function for a pair of frames to be differenced. We present the results from the application of this algorithm to four nights of KPNO 4-m data here.

In the light of the challenges posed by the KPNO data, we discuss how PSF matching can be facilitated with a simple optical corrector in a wide field imager. We demonstrate such a solution with a camera we have designed for the VATT 1.8-m telescope. Some preliminary results of an analysis of a subset of data taken at the VATT in 1995 as part of an ongoing survey to discover microlensing in M31 are also presented.

The outline of the paper is as follows. In §2 we discuss the observational aspects surveys geared to the discovery of microlensing in M31 and the details of the observations we have carried out at KPNO to discover microlensing by Galactic MACHOs around $10^{-6}M_\odot$ range, and more generally M31 MACHOs with masses of $\sim 10^{-4}M_\odot$ and above using the VATT 1.8-m telescope. In §3 we discuss the difference image technique, including the algorithms we use to match the PSF between frames. In §4 we provide a detailed discussion of the important systematic effects involved in image differencing and assess the technique photometric sensitivities to faint, unresolved stars where we demonstrate that we can achieve photometric errors that are within a factor of three or less of the ultimate photon noise limit. In §5 we present results from a difference image analysis of the KPNO data and a similar preliminary analysis of the VATT data. We show light curves of variables discovered independently in both data sets including many Cepheid and eclipsing binary candidates. In §6 we assess the sensitivity of the KPNO data in terms of the number of stars, microlensing timescales and MACHO masses and the search for microlensing on the
50° to 8h timescales associated with Galactic MACHOs in the mass range $10^{-7}$ to $10^{-5} \, M_\odot$ and M31 and Galactic MACHOs of $\sim 10^{-4} \, M_\odot$. We outline an estimate of optical depth limits we have achieved in this range. In §7 we summarize our conclusions and suggest other possible microlensing related applications of DIP.

2. THE SURVEYS

We outline the broad aspects of our surveys here and discuss in detail the number of stars we are sensitive to, in addition to timescale and mass sensitivities in §6.

2.1. Target Stars

Typically the brightest stars in M31’s bulge and inner disk region are red giants (RGs). A microlensing survey needs to maximize detector sensitivity to these objects, preferably in two bandpasses to test for achromaticity of light curves which must not exhibit color variations (Paczynski 1986, but see also Kamionkowski 1995). We choose non-standard R and I filters, slightly broader than their conventional equivalents: R extends from $\lambda 5700$ (just beyond the [O I] $\lambda 5577$ night sky line) to $\lambda 7100$, and filter I extends from $\lambda 7300$ to $\lambda 10300$ (5% power points). These choices maximize the number of photons in each filter we can detect from RGs with a CCD.

2.2. The KPNO Survey: Testing the Galactic MACHO $10^{-7} M_\odot$ regime

We observed four nights (1994 September 24-27, UT) on the KPNO 4-m with the 16′ × 16′ field of view prime focus camera - a Tek 2048² CCD (TK2B) with a platescale of 0.48″ per pixel. The first two nights were plagued by clouds, moonlight and poor seeing. The last two nights were predominately photometric with $\sim 1.1 – 2.2''$ seeing. The target field was centered in the maximal lensing field (MLF) predicted in C92 to be at about 1.5 kpc (7.5′) from the nucleus along the minor axis on the far side of the disk for a simple halo models ($\rho \propto r^{-2}$). This is also the high $\tau$ region from later models (Han & Gould 1996). This field includes the nucleus, which typically saturates very quickly in a given exposure. Exposure times were limited to 150s so that no more than a 2.5′ × 1.5′ region centered on the nucleus was saturated on the CCD. The analysis presented here is based on coadded exposures totaling 12.5m in both R and I filters. Given the CCD readout overhead
for a large CCD, this corresponds to a time resolution of 50" per coadded frame in a given band (about four times the minimum-mass Einstein crossing time, $t_E$, which is just $R_E$ divided by the transverse velocity of the lens with respect to the observer-source sightline). Such time resolution yields sensitivity to Galactic MACHOs around $2 \times 10^{-7}\text{M}_\odot$ given the assumed stellar sizes we outline in §6.2 (c.f. C92).

From our photometric calibrations (§5) and integrating the unsaturated regions of a typical coadded exposure we obtain a mean surface brightness of galaxy plus sky of $\mu_R = 19.35$ and $\mu_I = 18.30$. This corresponds to a mean $\langle S/N \rangle$ of 16 for an R = 22.5 star in the frame. Above this magnitude cutoff we are sensitive to $6.7 \times 10^5$ stars at greater $\langle S/N \rangle$ in this single field using our number density estimate in §6.1.

### 2.3. The VATT Survey: Probing the Halo of M31 for MACHOs

We obtained 26 nights of data spread over a 50 night timespan on the new VATT 1.8-m telescope in the Fall of 1995 with an $11.3' \times 11.3'$ field of view camera with a Tek2048$^2$ CCD with a platescale of 0.33" per pixel. The typical seeing range in these data was 0.8" to 1.5" with a median seeing around 1.0". The field observed included the MLF field and its near side equivalent, which is predicted to have an optical depth up to an order of magnitude lower than the MLF depending on the halo geometry. The MLF field was observed in both R and I. Under typical photometric, moonless arcsecond observing conditions a 60" integration in R corresponds to a $\langle S/N \rangle$ of 16 for an R=22.5 star in these fields. This magnitude limit corresponds to $\sim 3.1 \times 10^5$ stars (§6.1). Exposures were adjusted under varying conditions to ensure that equivalent depths were achieved in at least the MLF field in R on any given night. Further coverage for the 1995/1996 observing season was also obtained with the Wise Observatory 1.0-m telescope; extensive results from these datasets will be presented in future papers.

### 3. DIFFERENCE IMAGE PHOTOMETRY

The following discussion is confined to the KPNO dataset. We use the VATT data in this paper to illustrate a simplifying approach to matching the PSF between frames (§3.6), as well as independently confirm the reality of some of the variable sources detected in the KPNO data (§5). VATT data were reduced along similar lines to the KPNO data and a more detailed discussion of the processing will be presented in a future paper.
3.1. Preliminary Processing

All frames were debiased and flat-fielded in the standard manner. Sky-flats were used to divide out the overall illumination of the CCD in each filter and dome flats used to derive the pixel-to-pixel variations.

Each frame was cleaned individually of bad pixels including cosmic rays. Such pixels were identified by fitting a 5x5 pixel 2D Gaussian of width less than the seeing on each frame to each pixel in the image. The residuals of the difference between the data and the model effectively discriminate between neighboring pixels that are consistent with the seeing and those that are associated with chip defects and cosmic rays. Once such pixels were identified simple linear interpolation using good neighboring pixels was used to replace a bad pixel. The advantage of cleaning individual frames in this way is to allow a weighted combination of frames taken under variable seeing to be made which is clean of defects, as compared with a more robust combination such as a median which can degrade the final PSF of a combination of frames with very different seeing.

Registration of all frames to common coordinates was made with ∼50 bright, unsaturated stars on each frame whose centers were determined using the DAOPHOT PHOT routine and the IRAF routines GEOMAP and GEOTRAN. A 5th order polynomial interpolant was used in the geometric transformation and flux was conserved. Final registration of frames was accurate to within 0.1 pixels. The PSFs of stars on all frames were well sampled with minimum FWHM’s of 2.1 pixels, thus resampling errors were small and PSFs were not significantly degraded in this registration process. At this point frames were combined for different timescales: for the analysis of sub-night timescales sequences typically five images were combined giving images with exposure times of 12.5″ in each filter, and on nightly timescales all frames taken in each filter were combined.

3.2. PSF Matching with a Fourier Algorithm

In some test frames taken in similar seeing C92 was able to show that the residuals in the difference image were comparable to the photon noise. As stressed in §1, however, the residuals in the difference image are expected to be primarily influenced by differences in seeing between a pair of frames to be differenced. Since a typical PSF approximates a Gaussian we have found that simply convolving one frame with a Gaussian kernel to match the FWHM of the PSF of the better seeing frame is very effective. In general, systematic residuals in the difference frame can be minimized by matching the PSFs between frames as closely as possible.
In order to monitor partially resolved globular clusters in M31 for nova eruptions over a number of years through differential photometry Ciardullo, Tamblyn & Phillips (1990) developed a Fourier algorithm to match seeing variations of different epochs frames. In essence each good seeing frame was degraded to a common seeing value. With frames of identical seeing the relative flux of all non-varying point-like sources in a frame is the same for any given finite photometric aperture and thus meaningful differential photometry can be performed between frames. Phillips & Davis (1995) have developed this algorithm further and we are very grateful to Andrew Phillips for providing us with his software, which we have employed as the basis of our PSF matching technique. To apply it to its full extent to a typical wide field imaging telescope is not straightforward. In §3.3 we discuss the details and the algorithm we have developed to solve these problems and apply this to the KPNO data. In §3.6 we show how the PSF matching can be simplified with an optical corrector for a wide field imager and demonstrate its application to the VATT data.

First, ignoring noise, consider a frame $r$ with a narrower PSF than a frame $i$,

\[ i = r \ast k, \tag{1} \]

where $k$ is a convolution kernel that describes the difference in the seeing and guiding between the two frames. The Convolution Theorem states that the Fourier transform (FT) of these three variables has the form,

\[ FT(i) = FT(r) \times FT(k), \tag{2} \]

then,

\[ k = FT[FT(i)/FT(r)], \tag{3} \]

Thus $k$ can be determined empirically with a high S/N isolated star on a frame pair. Convolving the good seeing frame with this kernel will in principle provide a match to the PSF of the poorer seeing frame. In practice the determination of $k$ is not straightforward since the high frequency components of the FT become dominated by the noise in the wings of the PSF where the signal is weakest. An effective method of dealing with this problem was determined by Ciardullo, Tamblyn & Phillips (1990). Since the FT of a typical PSF is roughly Gaussian the convolution kernel will also be approximately Gaussian. By modeling the high S/N low-frequency components of the PSF FT with an elliptical Gaussian these noise-contaminated components can be replaced with the model fit yielding a convolution kernel close to the ideal.
Figure 1 shows an application of this method to some KPNO data. We show a 128x128 pixel subimage near the center of an image, which clearly shows a background of unresolved stars. A suitable bright star near this region has been used to determine $k$. Typically the kernel is determined over a region extending up to five times the FWHM of the PSF. After photometric scaling of the frame pair (§3.5) and the degradation of the better seeing frame with its convolution with $k$, the difference image shows the clear removal of structure in the background, including a good subtraction of all but the brightest star on the frame which is saturated. However, also shown in Figure 1 is a subimage of the difference frame located about 500 pixels away from the location where $k$ was determined. It is clear that this region is plagued by large systematic residuals associated with a poorly matched PSF in this region. Critically, this is not simply a problem affecting the bright stars on the frame, but also the background of unresolved stars where we are most concerned with achieving the best subtraction. This demonstrates that there is no unique solution of $k$ applicable to the entire frame.

3.3. Understanding the Spatial Dependence of the Matching PSF Function

Figure 2 illustrates why $k$ becomes spatially dependent in a frame pair to be differeded. We plot the FWHM of bright, unsaturated stars (we ignore partially resolved M31 globular clusters) as a function of distance from some radial symmetry point of the PSF variation close to the center of the CCD (§3.5 shows how this was located) for two frames taken close together in time and in the same filter. The radial variation of the PSF is clear and is quite dramatic at furthest radial distance which is located near one corner of the CCD. However, also apparent is the different form this radial variability takes in the two separate frames. This form is affected by small differences in the focus of the telescope between the two exposures. It is this time variability of the spatial functional form of the PSF that, independent of seeing and guiding variations which are themselves not spatially dependent across the camera, causes the PSF matching convolution kernel $k$ to become a function of position on the frame.

In principle the solution to this problem is simply to match frames in a piecemeal fashion by PSF-matching subregions around bright, isolated and unsaturated stars which can be used to determine the correct convolution kernel for the local region. In practice, however, the extent over which good subtraction can be made local to a PSF matching star is frequently only as large as $50 \times 50$ pixels. Such a method therefore requires a star suitable for determining a convolution kernel in over 1500 subframes to adequately match the entire $2048^2$ pixel frame. The requirements for the choice of PSF matching stars are
Fig. 1.— Application of the Phillips & Davis (1995) algorithm to the KPNO data. The left side shows a $128 \times 128$ pixel subimage close to the center of the original frame (upper left panel) with the mean galaxy background subtracted from the image, and its difference image (lower left panel). A suitable star close to this region has been used to empirically determine the PSF matching convolution kernel to match the image pair being differenced (§3.2). All structure in the unsubtracted data has been effectively removed; the residuals around the brightest star are due to it being saturated on the CCD. However, applying the same convolution kernel to a region located 500 pixels away (upper right panel) shows large systematic residuals in its difference frame on the scale of the PSF (lower right panel). This indicates a poor match of the PSF in this region and shows that there is no unique solution to the matching convolution kernel applicable to the entire frame. Effective subtraction of the full image can only be done by modeling the spatially varying PSF kernel using the limited number of appropriate PSF matching stars on the frame §3.4. Once applied in this case the quality of the subtraction for the entire frame becomes comparable to the lower left panel. The inset image in the lower right corner is the new difference image located in the region of the box in the upper right panel. A clear detection of a point source is now made, which was almost completely hidden in the systematic residuals in the first attempt at matching the PSF. All differences are displayed in the same intensity range; the intensity range of the upper panels is five times larger.
Fig. 2.— The FWHM of point sources on two separate R band KPNO frames taken two hours apart (shown as open and filled circle points) as a function of distance from the PSF radial-variation symmetry point on each frame. The functional form of the radial variability of the FWHM is different in the two images due to a small change in the telescope focus between the two exposures.
rather stringent: the star must (i) have high S/N, (ii) must not be saturated in any part of the PSF or corrupted by any defects such as cosmic rays or bad pixels, (iii) must have an amplitude that significantly exceeds the surface brightness fluctuations associated with the unresolved stars in the galaxy, and (iv) must be isolated from any bright neighbors over the extent to which \( k \) is being determined. In these data the number of point-sources on each frame that satisfy these criteria is typically \( \sim 200 \). Furthermore, few of these sources reside in the bulge region where the underlying bulge light requires correspondingly brighter stars for good PSF matching, leaving large and important areas of the frame without good kernel determinations. In the next section we describe how this problem was addressed by deriving a model for the spatial variability of the matching convolution kernel over the entire frame from every available location where local convolution kernels could be measured.

### 3.4. Derivation of Spatially Dependent PSF-matching Convolution Kernel

The search for variables in various frames proceeded by differencing each frame with a high S/N template reference image with good seeing. For this purpose the KPNO reference images for both filters were generated from the combined image stacks on the final night since this was the best quality data. The analysis of each frame began with measures of the raw convolution kernels for the individual frame/reference frame pair, using the method just described, at the location of all isolated (no companions within a radius of 15 pixels), high S/N and unsaturated stars - a list comprising 220 stars. Each kernel is 15x15 pixels in size, which is significantly larger than the typical PSF with FWHM in the range 2-4 pixels. Since the PSF’s to be matched have essentially the same functional form, the largest value in this array is almost exclusively located in the central pixel.

To determine the behavior of the overall spatial variability of the kernels a polynomial surface fit was made to the value of this pixel at the \( x, y \) positions of each kernel determination on the frame. Figure 3 shows a contour plot of the result of a typical high order polynomial fit for a frame pair. In this example the fit excludes the smaller region marked by the inner contour with a value of unity where the relative size of the PSF on each frame reverses. The unit contour represents the location where the PSFs are close to identical. In Figure 2 this would be located where the FWHMs radial forms cross at a radial distance of \( \sim 450 \) pixels.

Since the normal variation of the PSF within these frames generally exceeds the variation in seeing between frames the situation in shown in Figure 3, with an inner elliptical or near circular region within which the relative size of the PSF’s of the frame pair reverses, is quite typical. The overall radial variability of the convolution kernel in both
Fig. 3.— The spatial variability of the PSF matching convolution kernel for a pair of images in the KPNO 4-m data. The empirical PSF matching convolution kernel has been measured at the location of points marked on the plot. The × marks are distinguished from + marks as locations where the ratio of the size of the PSF on the two frames either exceeds or is less than unity. A 6th order polynomial surface fit (as a function of CCD coordinates) has been made to the central pixel intensity of the matching convolution kernels at the + mark locations and is shown as a contour plot. The inner contour marks the region where the PSFs are close to the same size on the original frames and within this region a similar determination of the PSF matching convolution kernel is made but with the images reversed.
regions is always concentric about the same point, however the position of this point was found to vary up to 100 pixels between frames from the symmetry point apparent in Figure 3. The location of this point most likely represents the principal axis of the telescope which is slightly displaced from the center of the CCD and telescope flexure may account for the movement of this point from one frame to another.

Spatially dependent kernel models were derived from \((r, \cos \theta)\) polynomial fits to each pixel, \((i, j)\), in the 15x15 array of the 220 raw kernels derived for each frame. The radial variation of the PSF is by far the dominant term, but models were generally better fit with some azimuthal component, invariably with symmetry about an axis \(\theta_o\). Typical models were parameterized by locating the radial point, \((x_o, y_o)\), and an azimuthal symmetry axis from contour plots such as Figure 3. The general fit is given by:

\[
k_{i,j}(x(r, \cos \theta), y(r, \cos \theta)) = \sum_{n,m} a_{i,j}(n, m) r^n \cos^m \theta, \tag{4}\]

where \(r\) is the radial distance from the PSF symmetry point and \(\theta\) measured with respect to the azimuthal symmetry axis. The best fits were cubic \((n = 3)\) in \(r\) and linear \((m = 1)\) in \(\cos \theta\). Prior to the overall fit an interactive examination of the radial plots of the central kernel pixel was performed in a small number of azimuth sections in order to delete any outlying raw kernels before the final fit. In the situation illustrated in Figures 2 and 3, with two regions defined by the ratio of the sizes of the PSF on each frame, two separate model fits were made.

### 3.5. Image Subtraction

Before subtraction both the image and reference image were processed further. Images were “unsharp masked” - the underlying smoothed galaxy and sky background were removed from both images by subtracting the large-scale median smoothed image of both frames. This leaves only the data to reference frame photometric scaling factor to be determined. The measures of the 220 raw kernels were used to do this. First, a 10x10 pixel box around each of the PSF stars was matched, then the intensity scaling factor was derived from a linear fit to the data versus reference pixel intensity match within this box (using software provided by A. Phillips). The scaling factor for the entire data image was the median value of the independent scaling factors determined for the 220 PSF stars, rendering it insensitive to any stellar variability, and was accurate to 1.5%.

As a convenient method of performing later photometry, a well spaced grid of 16 of the
220 PSF stars were removed from the data frame by zeroing the pixel values in a 16x16 pixel box centered on each of these stars. Since this step is performed prior to overall PSF-matching of the data and reference image and the subsequent subtraction, this has the advantage of ensuring properly scaled, high S/N and PSF-matched reference stars from the reference image on the final subtracted image for the purposes of differential photometry.

For computational efficiency the models derived from equation (4) were used to compute kernels for a grid of \((r, \cos \theta)\) positions. In matching a frame to its reference image, the frame was divided up into an \((x, y)\) grid of subimages and the nearest kernel to the center of each subimage from the model grid was used to perform the PSF-matching convolution for that subimage. The resolution of both the model kernel grid in \(r\) and \(\cos \theta\) and the image processing \(x, y\) grid were chosen to be at least at the point where no significant improvement in the subtraction quality could be obtained with increasing resolution. For the kernel model this was typically around \(\Delta r\) of 50 pixels and \(\Delta \cos \theta\) of 0.2. The final image-reference differenced subimage comprises either a data frame PSF-matched to its reference frame or a reference frame PSF-matched to the data frame, depending on the ratio of the sizes of the PSF’s of the data and reference image within the subimage. In §4 we assess the quality of the subtraction we have achieved with this algorithm.

3.6. Simplifying the PSF Matching Convolution Kernel with an Optical Corrector

If an observer utilizing image differencing techniques outlined above has influence over the choice of telescope/imager optical design, a great deal of effort can be saved in the data reduction stage by using optics that produce a uniform PSF over the entire detector, and in particular an optical system that produces a final focus coincident with the detector surface. Reducing the spatial dependence of the PSF correspondingly reduces the effect of changes in telescope focus inducing a spatial dependence of the full-frame PSF-matching kernel. In the case of our VATT survey, we installed a doublet biconvex achromat, designed with the aid of Richard Buchroeder of Tucson, Arizona, that produces uniform 20 micron (0.25") diameter spots and best focus over the entire surface corresponding to the curved SITE 2048 backside-illuminated CCD (with a curved surface of approximately 220 microns sagitta center-to-edge). This resulted in 1 – 4 PSF kernels being required for the VATT image, versus approximately 200, together with the modeling algorithm in §3.4, for the KPNO 4m prime focus CCD data, which covers only 2.1 times the solid angle in field of view.
4. TECHNIQUE SENSITIVITIES AND SYSTEMATIC EFFECTS

In this section we assess the sensitivity of the technique and discuss systematic effects that are generally pertinent to DIP and the surveys we have outlined.

4.1. Difference Image Photometric Errors for Faint Unresolved Stars

The application of the algorithm outlined in §3 to the KPNO data was successful at yielding good subtraction over most of the frame, even in the unsaturated regions of the inner part of the bulge which had proved very difficult with other methods. In Figure 1 we show an inset subimage of the central region of the poorly differenced subimage, but after the full-frame PSF matching function model has been applied. This illustrates the importance of accurate PSF matching: a clear detection of a point source can now be seen in the frame, which was previously almost completely undiscernible in the systematic residuals in the original attempt at matching the PSF (§3.2).

The PSF-matching starts to break down at large values of $r$ where the PSF is varying most rapidly. The radial symmetry point of the PSF matching function was located close to the center but always in the same quadrant (the top left quadrant as seen in Figure 3). Consequently, the poorest subtraction is always at the furthest radial distance in the diametrically opposite quadrant of the frame ($r > 1100$ in Figure 2 which corresponds to the bottom right corner in Figure 3) where an insufficient number of PSF stars can be found to constrain the model fit. However, the image subtraction is quite satisfactory over most of the CCD excluding saturated regions, which we discuss below.

The effectiveness of the subtraction can be quantified by measuring the ratio of the standard deviation of the residuals in the subtracted frame to the predicted noise based on the photon and read-noise for the region of the frame being examined. This can be done simply by constructing a “noise” image for a particular difference frame to be examined. Since the read-noise of the CCD is negligible in all regions of the well exposed frames the predicted noise is determined only by the photon noise. Such a noise image, $n$, is given by the following addition of the image $i$ and its reference image $r$,

$$n = \left( \frac{1}{g} \left( s_i \frac{i}{N_i} + \frac{r}{N_r} \right) \right)^{0.5},$$

(5)

where $N_i$ and $N_r$ are the number of images comprising $i$ and $r$ respectively if these images are averages of frames, $s_i$ is the photometric scaling factor required intensity match $i$ to $r$, and $g$ is the fixed gain of the CCD in electrons per ADU and can be measured from flat-field
images at the telescope. Dividing a difference frame by image \( n \) generates a difference image where the residuals are given in units of the predicted photon noise in each pixel. Since one of the two images has been degraded by convolving with the matching PSF kernel, at a given location the photon noise per pixel is reduced by the averaging effect of the kernel. Thus equation (5) can overestimate the photon noise, however, in a typical situation where \((N_r >> N_i)\) and the seeing is better in the reference frame, the noise is effectively completely determined by the photon noise in the \( i \) frame which has not been convolved.

Histograms of the residuals of typical difference frames in units of the photon noise, \( \sigma_{\text{photon}} \), show residuals of \(< \pm 8\sigma_{\text{photon}}\) over 90% of the frame. However, about 5% of the original image is saturated (mostly in the nucleus) and the smoothing of the PSF-matching convolution kernel leads to \(~2\%\) of contamination in the difference frame around saturated stars (this effect can be seen in Figure 1). Over most regions where there are no obvious systematic residuals due to bright stars the standard deviation in the difference frame is typically within \(1.5\sigma_{\text{photon}}\). The pixel-to-pixel residuals do not quantify the quality of the subtraction, however, since systematic errors in the subtraction technique lead to residuals that are correlated on the scale of the PSF. Convolving a difference image with a boxcar of size \( N \times N \) and dividing by a noise image scaled by \( N \) is a convenient way to examine noise on larger scales. Figure 4 shows how the residuals do indeed become larger in terms of the photon noise on bigger scales. For \( N=1 \), corresponding to the pixel-to-pixel scale the standard deviation is \(1.8\sigma_{\text{photon}}\), but for \( N=5 \), which corresponds closely to the size of the typical PSF in these data, the difference frame residuals are \(2.4\sigma_{\text{photon}}\) and increase to \(4.4\sigma_{\text{photon}}\) for \( N=20 \), in this example. For large \( N \) this increase over the entire frame appears to be due to systematic effects such as fringing which is a problem in I-band CCD photometry. An examination of the residuals in smaller “clean” subimages shows that the standard deviation does appear to return close to \( \sigma_{\text{photon}} \). However, the only scale we are concerned with is the PSF size, and here we find almost all of our difference frames typically achieve residuals in clean, unsaturated regions that are fully consistent standard deviations of \(2\sigma_{\text{photon}}\) and range up to \(3\sigma_{\text{photon}}\).

Expressing the residuals in the difference frame in terms of photon noise quantifies precisely how the differencing technique presented here compares with the ultimate noise limits. Once the true photometric errors are determined by measuring the standard deviation of residual pixels binned on the scale of the PSF it appears DIP is quite competitive with conventional photometry of faint resolved stars.
Fig. 4.— A histogram of the residuals in a typical full-frame difference image expressed in terms of the photon noise and calculated for differing scales of pixel smoothing. For the pixel to pixel (N=1) scale the standard deviation of the curve is $1.8\sigma_{\text{photon}}$. However, due to various systematics ($\S$4) residuals are correlated on the scale of the PSF (N=5) which is reflected in its curve which shows a standard deviation of $2.4\sigma_{\text{photon}}$. On larger scales for the full image the standard deviation continues to increase, but this is mostly due to large scale fringing in this case of this I band difference frame; examination of smaller subregions of these images actually show residuals consistent with $\sigma_{\text{photon}}$. Most importantly, however, almost the entire frame has residuals within a factor of three of the photon noise on the scale of the PSF ($\S$4.1).
4.2. Difference Image Residuals of Bright Stars

It is possible that some “sources” in difference frame are simply due to residuals of bright, resolved stars. To verify that candidates were not systematic residuals of bright stars, photometry was made at the location of candidates in the unsubtracted frames. The validity of the subtraction for a particular candidate is easily confirmed by comparing the residuals of a neighboring bright star in the noise regime $\Delta F_{err} >> \sigma_{\text{photon}}$, where $\Delta F_{err}$ represents the systematic error in the subtraction of a star of flux $F$ and both terms are calculated over the PSF. In this noise regime the fractional error $\Delta F_{err}/F$ for individual bright stars may vary over the frame depending on the quality of the subtraction. For candidates with stars of flux $f < F$, but bright enough to be resolved in the raw frame, we can use a local estimate of the fractional error in the subtracted flux to distinguish between artifacts of the subtraction and real variability, where $\Delta f/f >> \Delta F_{err}/F$.

4.3. CCD Photometric Accuracy and Surface Brightness Fluctuations

In §3.5 we determined that the photometric scaling factors determined from our PSF matching starlist was accurate to 1.5%. If the photometric accuracy of these stars was determined entirely by photon noise these scaling factors should be accurate to 0.1%, indicating that systematic errors must account for their empirical accuracy. Flat-fielding may account for most of this discrepancy. On the pixel-to-pixel scale care was taken to ensure that nightly flats were of sufficient S/N that flat-fielded object frames and subsequent combined frames contained little contribution of photon noise from the flats themselves, but on larger “illumination” scales we detect variability between domeflats around 1% over hourly timescales, which may be due to a small time variability in the linearity of the CCD. However, our empirical accuracy, due to this or other reasons, is quite typical for CCD work and here we assess how this intrinsic accuracy influences the residuals in our difference frames.

The structure seen in the unsubtracted subimages in Figure 1 and similar exposures of nearby galaxies is due to the statistical fluctuation of the number of stars in the galaxy contributing to each pixel or seeing element (Tonry & Schneider 1988). The amplitude of these surface brightness fluctuations (SBFs) is determined by the luminosity function (LF) of the stellar population and the surface brightness and distance of the galaxy. For a given seeing element this amplitude, $m_{SBF}$, can be expressed in terms of the empirical measure of the ratio of the second and first luminosity moments of the LF, $\bar{m}$ (the fluctuation magnitude). Following the formalism of Gould (1996) this is given by,
where $S$ and $S_{sky}$ are the surface brightness in counts per pixel of the galaxy plus sky and sky respectively, $\Omega_{psf}$ is the size of the resolution element in pixels which we adopt to be $\pi (\text{FWHM})^2$, and $m_1$ is the magnitude corresponding to 1 ADU on the CCD frame.

The observed $\bar{m}_I$ for M31 is 23.29 (Tonry 1991 and using his adopted foreground Galactic extinction of $A_B = 0.31$). The (V-I) color of M31 is 1.18 (Tonry et al. 1990), and using galaxies of comparable color from this work, we estimate the observed $\bar{m}_R$ to be 23.99. For the mean seeing of 1.5″ for our two best nights and the calibration of the surface photometry of our frames (§5), the mean value for $m_{SBF}$ is 20.8 for the R frames. The 1.5% photometric scaling accuracy of this mean SBF predicts an error of R=25.5 in the difference frame which is almost identical to the photon noise in the same size seeing element for the mean surface brightness for the R frames (§2.2). Thus the intrinsic accuracy of the photometric scaling accounts for 1.24σ photon seeing element residuals in the difference frame.

4.4. Effects of Atmospheric Dispersion in a Difference Frame

The effects of atmospheric refraction can become a serious issue when applying DIP. Filippenko (1982) tabulates atmospheric dispersion for an observatory at an altitude of 2 km (comparable to KPNO) as function of airmass in the wavelength range 3000-10000 Å. Our main concern is the effect of the atmosphere on the centroid of a star as a function of airmass in a broad bandpass filter. Photons at the wavelength extremes of our R filter (5600 to 7200 Å) separate out from 0.00′′, at an airmass of 1.0, up to 0.65′′, at an airmass of 2.0 within the stellar image (for the extremes of the I band filter in §2.1 the separation extends to 0.40′′ at 2.0 airmasses). This has the effect of elongating the stellar PSF in the direction of atmospheric dispersion at higher airmasses, leading to poorer image subtraction when differencing frames with different effective airmasses. If all the stars on the frame are the same color then this elongation affects the shape of the PSFs in the same way. The Fourier algorithm (§3.2) can still cope with these effects on the PSF when matching frames taken at different effective airmasses and automatically correct this problem. However, the individual colors of stars in a frame lead to second order effects: the centroid and elongation of a star’s PSF will differ depending on its color. There is no easy way to compensate for this, particularly without knowledge of the precise position and color of all detected stars on the field. Such an effect will yield poorer subtraction with increasing airmass and result in power on the scale of the PSF in the difference frame.
Fortunately, M31 is at a favorable declination for mid-latitude sites in the Northern hemisphere and can be accessed for roughly eight hours a night at airmasses < 1.5, and this problem is quite small for our selected bandpasses. Furthermore the RG stars we are sensitive to (§2.1) have red colors of <(V − I)> = +1.2 (the mean color in the bulge region, Tonry 1991), which makes the PSF distortion smaller compared with bluer stars. All the KPNO data presented here were taken at < 1.5 airmasses and no discernible degradation was seen in the difference frame residuals of frames taken at higher effective airmass. However, we note that the effect can be quite large for shorter wavelength bandpasses. In the wavelength extremes of a B filter: 3500 to 5000 Å, for example, the centroids separate by 2.′′1 between 1.0 and 2.0 airmasses.

4.5. Signal Contribution of RR Lyrae Variables

RR Lyrae stars will be in abundance in the region of the bulge we are surveying and these stars have been observed by Pritchet & van den Bergh (1987) in the halo of M31 40′ (9kpc) from the nucleus. Such variables have periods in the range 1.5 − 24 h and thus it is important to consider their possible contribution to the KPNO data which is sensitive to these timescales. Pritchet and van den Bergh determined a <B> of 25.68 for 30 RR Lyrae candidates. Adopting a mean <B − V> of 0.26, which corresponds roughly to a G0 star (Hawley et al. 1986) and a Galactic foreground extinction of AB = 0.31 (Burstein & Heiles 1984), their <R> should be ∼ 25.2. The maximum observed amplitude range of 1.6″ in V (Bono et al. 1995) and so an RR Lyrae star may be seen in a difference frame as a source no brighter than R = 24.7. Given the construction of our KPNO survey (§2.2) this would correspond to a maximum S/N of 2.1σ_{photon} in the difference frame (and fainter in the I band difference frame).

Thus RR Lyraes are just at the limits of our sensitivities in the KPNO survey given our assessment of DIP in §4.1. However, it is quite possible that the correlation of difference frame residuals on the scale of the PSF discussed in §4.1 contains some contribution from these stars in these surveys. It is therefore important to estimate the RR Lyrae specific incidence, since RR Lyrae can be a significant source of random noise in the photometry. Pritchet & van den Bergh (1987) estimate for the halo of M31, the specific incidence (per integrated flux of the entire population) is about 100 Lyraes for each B=15.9 (R ∼ 14.5, from Walterbos & Kennicutt 1987). In comparison, the total integrated magnitude over our KPNO field is R = 4.67, corresponding to 850,000 RR Lyraes (or 0.9 RR Lyraes per square arcsec, on average), if the specific incidence of RR Lyraes is the same in the halo and bulge/inner disk. If instead, the bulge/inner disk metallicity is much higher than
that found for the halo, the bulge/inner disk could have up to 100 times fewer RR Lyraes than predicted here. However, RR Lyraes might be a significant source of fluctuations on several hour timescales, which is a supposition we will test in a later paper by studying the temporal frequency of variations of marginally detected sources in the difference frame.

4.6. General Comments on PSF Matching

Critical to the techniques presented in §3 is the assumption that the PSF is sampled at least at the Nyquist frequency (roughly > 2 pixels FWHM). This was the situation in all the data presented here. If the PSF is not Nyquist sampled then this leads to irretrievable loss of information concerning the PSF. In particular, registration of frames to common coordinates will lead to systematic “resampling” errors on the pixel scale that are particularly obvious around bright stars (c.f., Gould 1996). Such systematic effects can be reduced by averaging registered, undersampled frames at the price of time resolution. Also, poor sampling similarly affects the Fourier determination of the matching convolution kernel. By averaging empirically determined convolution kernels of many stars the systematic errors in the convolution kernel determination can be reduced, but given possible limitations in the number of PSF matching stars on a given frame (c.f., §3.3) this may not be a viable option. As with registration of undersampled data, even with an accurate matching convolution kernel the undersampling will still lead to systematic effects in the PSF matching, which would also best be coped with by averaging multiple difference frames.

The Phillips & Davis (1995) algorithm (§3.2) makes an assumption about the behavior of the wings of the PSF in the replacement of high frequency noise dominated components with a fit based on an elliptical Gaussian. The difference between the ideal kernel and the empirical kernel determined by this method will reflect the difference between the real behavior of the wings of the PSF and the assumed model. These differences will propagate as systematic errors in the difference frame that are correlated on the scale of the PSF. If the behavior of the wings of the PSF is well known, then in principle the algorithm can be modified by fitting the wings of the PSF with a more realistic model.

Even with a relatively uniform PSF across an image a spatial dependence of the PSF matching convolution kernel can be induced when trying to match images that are not taken at close to the same position in the sky. This is because of astrometric distortion in the image plane of the detector. Frequently in a wide field imager the transformation between frame coordinates is not a simple linear shift, rotation and magnification; it contains higher order terms. The IRAF routines, GEOMAP and GEOTRAN, allow for such higher order
terms to enable accurate registration, however, the non-linear transformation will result in a different spatial dependence of the PSF in the registered frame compared to the unshifted image it is to be differenced with. Consequently, this has the effect seen in the KPNO data (§3.3) - a spatially dependent PSF matching convolution kernel is required to match frames and can be treated with the algorithm discussed in §3.4. (In test frames we took at the McDonald Observatory 2.7-m in 1994, we found precisely this effect with significant degradation in the quality of differenced images comprising an image pair that originally differed in registration by 15% of the size of the CCD, despite the uniform PSF across the 3.6′ × 3.6′ field of view of the camera.) This problem and its solution is also relevant to differencing images taken on different telescopes.

In sum, the factors discussed in §4.3 to §4.6 all contribute to the correlated residuals on the scale of the PSF in the difference frame and some of these have been assessed analytically by Gould (1996). Once a candidate is determined not to be a residual of a bright star in the difference frame (§4.2), then given all of these systematic effects, the reality of a detection of variability is best determined by measuring the variance of the residuals integrated on the scale of the PSF around the candidate source in the difference frame, since this measures the true photometric errors in DIP. The errors in the light curves of all objects presented in §5 have been determined this way.

5. RESULTS

The analysis was performed on R and I sequences comprising eight and nine KPNO 4-m prime focus images which were themselves combinations of five consecutive images each for the nights of September 26 and 27. Since conditions compromised the quality of the data on the previous two nights (§2.2), all the data on these nights were combined into single averaged frames of comparable S/N to the individual combined frames on the last two nights. Thus the total number of averaged frames analyzed for each filter was 19.

Photometric calibration of both the R and I filters was made with images of M92 taken under photometric conditions on the last night of the run. Images taken that same night at the same airmass as the M92 observations were used to make the final calibration of the M31 fields. We used the photometry of Christian et al. (1985) in their R and I filters. Our broad band R filter has a bandpass 70% wider than a Kron-Cousins CCD R filter (Schild 1983), but essentially the same effective wavelength (λ6470). The I filter we use has an effective bandpass of 3565 Å, which is about twice the Kron-Cousins equivalent, and an effective wavelength of λ8800, compared with λ7990 for the Kron-Cousins I. In both filters the scatter in our raw magnitudes and the Christian et al. R and I equivalents was
better than $\pm 0.^{m}1$ for 10 stars with (V-I) colors in the range 0.$^{m}0$ to 1.$^{m}4$. The agreement with a Kron-Cousins I filter, despite the differences in the photometric bandpass with our I filter, probably reflects the falling CCD sensitivity at redder wavelengths that will tend to match the overall detector response to the Kron-Cousins I. We have not attempted to correct magnitude estimates with color terms in our photometric transformations so our photometric calibration is only accurate to $\pm 0.^{m}1$.

As we discuss below, it is useful to compare sources in the raw and difference frames to the local SBF amplitudes. The amplitude of the SBF in a seeing element is given by equation (6), which requires the surface brightness of the galaxy above the sky to be determined. We have used an estimate of the sky brightness in our photometric moonless R frames of $R = 20.8$ per square arcsecond. (This is only $\sim 50\%$ of the total surface brightness 20$'$ from the nucleus - the galactocentric extreme of our field.) This sky estimate gives surface brightness values consistent with the photometry of Walterbos & Kennicutt (1987) in R. In the case of the I filter variable night sky OH line emission makes the sky estimate much more uncertain. Since M31 does not exhibit a strong color gradient in this galactocentric range we solved for the I sky brightness for one of our I frames by determining the sky value that gave the minimum scatter in the (R-I) galaxy color for locations scattered throughout the field. A sky brightness value of I $\sim 19.2$ for this typical photometric, moonless frame gave a mean (R-I) = 0.80 $\pm$ 0.03 for the entire field and I surface brightness values consistent with the photometry of Hiro moto et al. (1983). For our purposes the accuracy of these estimates is quite sufficient.

All the KPNO frames for the two bandpasses were differenced against their respective reference frames of the combined images from the final night (§3.1) after the processing outlined in §3. The search of each frame for sources was conducted by visual inspection of the difference frames. Many sources were initially detected in the difference frame of the combined images for the two last and best nights of the run. Thus the search is very sensitive to changes on the night to night timescale.

We used IRAF DAOPHOT to perform photometry of the sources on the difference frames. To minimize the problems of the variable PSF photometry of a source was made relative to neighboring calibrated bright stars from the PSF matched reference frame. If the source in the difference frame has negative flux we applied DAOPHOT to a “negative” difference image.

To highlight the sensitivity of the image differencing technique Figure 5 shows two consecutive combined $36'' \times 36''$ R band subimages separated by 50$''$ in time taken on the last night. The upper panels are the undifferenced images, but with the large scale median smoothed galaxy and sky background subtracted. The lower panels are the result
of differencing the original images with a reference image comprising an average of all images taken on the previous night. The difference image for the second frame shows a clear detection \((20\sigma)\) of variability over the previous frame. Remarkably, the eye cannot discern any indication of such variability in the raw frames. The reason for this can be clearly understood in terms of the underlying surface brightness fluctuations associated with the unresolved stars in the galaxy. The differential flux in the difference frame has a magnitude of \(R = 21.14\). Correcting for an estimated sky with a surface brightness of \(R = 20.8\) per square arcsec, we measure the galactic surface brightness to be \(R = 18.29\) per square arcsec at this location \((0.5\text{ kpc along the far side minor axis})\). With \(1.2''\) seeing and applying equation (6) the predicted surface brightness fluctuations in a seeing element have an amplitude of \(R = 20.32\). Thus the SBFs at this location have an amplitude that is more than twice as bright as the total flux of the detected source in the difference frame.

The flux difference light curves in both R and I of this source (candidate #1 in Table 1) can be seen in Figure 6. The subimages in Figure 5 correspond to the 14th and 15th data points of the R band curve. It is likely that this particular variable is a nova. The nova rate in M31 is well determined: 26 novae year\(^{-1}\) (Arp 1956; Capaccioli \textit{et al.} 1989) and the spatial distribution closely follows the bulge light (Ciardullo \textit{et al.} 1987). Thus there is probability of \(\sim 10\%\) that one will erupt during our four night run, since our field covers roughly one quarter of the bulge light. Our final measurement of this object 3.5h after its initial detection in the difference frame has a difference magnitude of 19.33 relative to the previous night. If this is a nova it may have peaked at an R magnitude brighter than 17.\textsuperscript{m}0.

A number of novae are undoubtedly present in these frames since novae of all speed classes intersect in brightness at 15-18 days past maximum light (Buscombe & deVaucouleurs 1955) with an absolute magnitude of \(M_V = -5.6\) (Capaccioli \textit{et al.} 1989) which corresponds to \(R < 18.5\), well above our detection thresholds.
Fig. 5.— Image differencing at work: the sensitivity is dramatically illustrated in this image in which shows the raw (upper) and difference (lower) $36 \times 36''$ subimages of two consecutive R-band images taken 50'' apart. The raw images have been stretched to maximize as much as possible the surface brightness fluctuation structure in the unsubtracted data. The difference images are stretched at 10% of the upper panels to highlight the noise in the difference frames and have been created by subtracting a high S/N reference image from the previous night after suitable registering, scaling and PSF-matching. The bottom right panel shows a clear 20σ detection of variability relative to its previous image. Despite this large amplitude it is still completely invisible to the eye in the top right unsubtracted image. This is due to the fact that in the 1.2'' seeing it is only a 1.5% modulation of the background galaxy light which has a seeing element flux of R = 16.3. This amplitude is still only half the amplitude of the surface brightness fluctuations at its location (§5). It is the ability of image differencing to be sensitive to flux variations in arbitrarily high stellar crowding that expands enormously the number of stars a microlensing survey can be sensitive to once such a survey is no longer limited by whether the stars are actually resolved. The light curve of this object is illustrated in Figure 7.
Fig. 6.— The light curve of the rapidly varying object in Figure 5. The differential fluxes are all measured relative to the mean image of the third night in both filters. Points 14 and 15 of the R light curve correspond to the subimages in Figure 5. This object is most likely a nova which is just caught in eruption. The reality of the apparent variability during the third night is not clear, since this object was very close to the heavily saturated nuclear region of the CCD where some difference images exhibited a “plaid” structure (not seen in Figure 5), apparently an occasional artifact of the CCD read-out at high signal. The Julian Date at $t = 0$ is 2,449,619.50.
| ID  | RA (2000) | Dec (2000) | R  | I  | $\Delta R_{\text{max}}$ | $\Delta I_{\text{max}}$ | $\mu_R$ | $\mu_I$ | Remarks |
|-----|-----------|------------|----|----|--------------------------|--------------------------|--------|--------|---------|
| 001 | 0 42 58.6 | 41 15 50   | >19.7 | >18.9 | 19.58                     | 19.68                     | 18.29  | 17.48  | nova?   |
| 002 | 0 42 53.9 | 41 04 46   | 20.64 | >20.0 | 21.31                     | 21.29                     | 20.54  | 19.75  |         |
| 003 | 0 42 58.9 | 41 12 10   | >20.3 | >19.5 | 21.91                     | 21.69                     | 19.46  | 18.69  |         |
| 004 | 0 42 57.4 | 41 11 37   | >20.3 | >19.6 | 22.89                     | 22.06                     | 19.55  | 18.77  |         |
| 005 | 0 42 58.3 | 41 11 20   | 20.02 | 18.97 | 21.67                     | 21.38                     | 19.67  | 18.92  |         |
| 006 | 0 42 58.8 | 41 11 03   | 19.72 | 18.74 | 22.28                     | 22.37                     | 19.80  | 19.02  |         |
| 007 | 0 43 11.2 | 41 10 31   | >20.6 | >19.9 | 22.23                     | 22.15                     | 20.18  | 19.40  |         |
| 008 | 0 43 14.5 | 41 12 31   | >20.5 | 19.29 | 22.08                     | 21.89                     | 19.95  | 19.13  |         |
| 009 | 0 42 59.4 | 41 13 59   | 19.20 | 18.32 | 21.60                     | 21.84                     | 18.95  | 18.17  |         |
| 010 | 0 43 07.5 | 41 10 51   | >20.5 | >19.8 | 22.15                     | 21.78                     | 20.05  | 19.27  |         |
| 011 | 0 43 03.0 | 41 13 39   | 19.55 | 19.23 | 20.67                     | 20.43                     | 19.25  | 18.46  |         |
| 012 | 0 43 23.0 | 41 10 55   | 20.33 | >20.0 | 22.24                     | 22.16                     | 20.37  | 19.59  |         |
| 013 | 0 43 24.3 | 41 10 50   | >20.7 | >20.0 | 22.20                     | 22.03                     | 20.44  | 19.63  | eclipsing? |
| 014 | 0 43 25.0 | 41 10 32   | 20.03 | 19.93 | 22.09                     | 22.10                     | 20.44  | 19.67  | eclipsing? |
| 015 | 0 43 30.2 | 41 10 36   | >20.8 | >20.0 | 21.90                     | 21.92                     | 20.53  | 19.74  |         |
| 016 | 0 43 29.6 | 41 14 12   | >20.6 | >19.9 | 21.19                     | 21.31                     | 20.18  | 19.40  |         |
| 018 | 0 43 28.0 | 41 13 55   | >20.6 | >19.8 | 21.84                     | 21.90                     | 20.16  | 19.34  |         |
| 019 | 0 43 28.7 | 41 13 43   | >20.6 | >19.9 | 22.10                     | 21.93                     | 20.24  | 19.41  |         |
| 020 | 0 43 26.2 | 41 12 02   | 20.43 | 19.38 | 22.27                     | 21.93                     | 20.35  | 19.56  |         |
| 021 | 0 43 27.6 | 41 11 14   | >20.7 | 19.61 | 21.43                     | 21.25                     | 20.40  | 19.64  | eclipsing? |
| 022 | 0 43 31.2 | 41 12 16   | >20.7 | >19.9 | 22.23                     | 21.95                     | 20.36  | 19.53  |         |
| 023 | 0 43 33.6 | 41 11 54   | 20.66 | >20.0 | 21.58                     | 21.44                     | 20.49  | 19.73  |         |
| 024 | 0 43 43.6 | 41 11 49   | 20.42 | 19.74 | 21.51                     | 21.24                     | 20.79  | 20.02  |         |
| 025 | 0 43 44.9 | 41 11 49   | >20.9 | >20.1 | 21.96                     | 22.26                     | 20.71  | 19.95  | eclipsing? |
| 026 | 0 43 45.8 | 41 11 51   | 20.20 | 20.08 | 21.86                     | 21.66                     | 20.76  | 19.96  | eclipsing? |
| 027 | 0 43 49.0 | 41 11 32   | >21.0 | >20.2 | 21.42                     | 21.46                     | 20.87  | 20.07  | eclipsing? |
| 028 | 0 43 50.7 | 41 12 25   | 20.78 | >20.2 | 21.94                     | 21.57                     | 20.80  | 20.01  | eclipsing? |
| 029 | 0 43 45.3 | 41 10 44   | >21.0 | 20.18 | 21.97                     | 21.62                     | 20.88  | 20.04  |         |
| 030 | 0 43 37.3 | 41 14 17   | 19.99 | 18.95 | 20.79                     | 20.46                     | 20.41  | 19.56  |         |
| 031 | 0 43 37.6 | 41 12 05   | >20.8 | >20.1 | 21.80                     | 21.42                     | 20.61  | 19.77  |         |
| 032 | 0 43 42.9 | 41 10 57   | >20.9 | 20.09 | 22.42                     | 21.70                     | 20.75  | 19.97  |         |
| 033 | 0 43 44.0 | 41 11 06   | 19.54 | 18.52 | 21.30                     | 20.86                     | 20.70  | 19.92  |         |
| 034 | 0 43 43.3 | 41 12 02   | >20.9 | 19.58 | 22.47                     | 21.82                     | 20.76  | 19.95  |         |
| 035 | 0 43 48.2 | 41 12 56   | 20.16 | 19.46 | 21.19                     | 21.46                     | 20.67  | 19.88  | eclipsing? |
Table 1—Continued

| ID  | RA (2000) | Dec (2000) | R   | I   | ∆R<sub>max</sub> | ∆I<sub>max</sub> | µ<sub>R</sub> | µ<sub>I</sub> | Remarks |
|-----|-----------|------------|-----|-----|-----------------|-----------------|-----------|-----------|---------|
| 036 | 0 43 56.4 | 41 13 04   | 19.59 | 19.00 | 20.67           | 20.52           | 20.82     | 20.04     |         |
| 037 | 0 43 50.4 | 41 14 18   | >20.8 | >20.1 | 21.38           | 21.38           | 20.66     | 19.80     |         |
| 038 | 0 43 52.2 | 41 14 14   | 20.42 | >20.1 | 21.87           | 21.74           | 20.68     | 19.86     |         |
| 039 | 0 43 49.9 | 41 13 22   | 20.49 | 19.71 | 22.33           | 21.62           | 20.75     | 19.95     |         |
| 040 | 0 42 57.6 | 41 17 30   | >19.6 | >18.8 | 21.02           | 21.21           | 18.07     | 17.28     |         |
| 041 | 0 43 35.1 | 41 15 33   | 20.37 | 19.35 | 21.70           | 21.54           | 20.23     | 19.39     |         |
| 042 | 0 43 41.9 | 41 16 36   | >20.7 | >19.9 | 21.38           | 22.00           | 20.37     | 19.55     |         |
| 043 | 0 43 43.8 | 41 16 42   | >20.7 | >19.9 | 22.31           | 21.76           | 20.34     | 19.50     |         |
| 044 | 0 43 38.7 | 41 15 54   | 20.10 | 19.87 | 22.28           | 21.88           | 20.30     | 19.44     |         |
| 045 | 0 43 40.6 | 41 15 31   | 19.82 | 19.16 | 22.30           | 21.48           | 20.38     | 19.57     |         |
| 046 | 0 43 42.1 | 41 14 56   | 19.64 | 18.75 | 21.88           | 21.09           | 20.46     | 19.64     |         |
| 047 | 0 43 43.7 | 41 14 44   | >20.8 | >20.0 | 22.10           | 22.19           | 20.48     | 19.68     |         |
| 048 | 0 43 46.0 | 41 14 38   | 20.72 | 19.10 | 21.93           | 21.53           | 20.47     | 19.69     |         |
| 049 | 0 43 49.1 | 41 15 27   | >20.8 | >20.0 | 21.90           | 21.86           | 20.53     | 19.72     |         |
| 050 | 0 43 49.9 | 41 17 53   | >20.7 | >20.0 | 21.84           | 21.19           | 20.45     | 19.64     |         |
| 051 | 0 43 37.3 | 41 15 23   | 19.50 | 18.68 | 21.32           | 21.68           | 20.36     | 19.49     |         |
| 052 | 0 43 47.9 | 41 15 57   | 20.16 | 19.18 | 20.38           | 19.99           | 20.51     | 19.70     |         |
| 053 | 0 43 38.6 | 41 16 36   | >20.6 | 19.65 | 21.66           | 21.78           | 20.22     | 19.39     |         |
| 054 | 0 42 55.7 | 41 05 45   | >20.8 | >20.0 | 22.46           | 22.01           | 20.51     | 19.70     |         |
| 055 | 0 42 58.9 | 41 05 10   | 20.72 | 19.46 | 22.14           | 21.21           | 20.57     | 19.79     |         |
| 056 | 0 43 06.8 | 41 04 35   | >20.9 | >20.2 | 22.77           | 22.35           | 20.78     | 20.00     |         |
| 057 | 0 43 05.0 | 41 03 58   | 19.66 | 19.02 | 22.88           | 21.58           | 20.76     | 19.94     |         |
| 058 | 0 42 56.9 | 41 03 13   | 20.55 | >20.1 | 20.61           | 20.25           | 20.68     | 19.93     |         |
| 059 | 0 43 00.6 | 41 03 37   | >20.9 | >20.1 | 22.71           | 22.56           | 20.68     | 19.91     |         |
| 060 | 0 43 02.4 | 41 03 31   | >20.9 | >20.2 | 21.57           | 22.11           | 20.74     | 19.96     |         |
| 061 | 0 43 03.5 | 41 03 35   | 19.72 | 18.41 | 22.36           | 21.66           | 20.73     | 20.00     |         |
| 062 | 0 43 04.8 | 41 03 38   | >20.9 | >20.2 | 22.10           | 21.44           | 20.76     | 20.00     |         |
| 063 | 0 43 04.6 | 41 03 27   | >20.9 | >20.1 | 21.24           | 21.92           | 20.74     | 19.94     |         |
| 064 | 0 43 06.7 | 41 03 30   | >20.9 | >20.2 | 21.38           | 20.74           | 20.84     | 20.04     |         |
| 065 | 0 43 07.5 | 41 03 19   | 20.69 | 19.70 | 21.17           | 20.67           | 20.84     | 20.05     |         |
| 066 | 0 43 07.6 | 41 02 44   | 20.33 | 19.53 | 20.94           | 21.20           | 20.81     | 20.05     |         |
| 067 | 0 43 08.0 | 41 04 43   | 20.50 | 19.73 | 22.11           | 21.50           | 20.78     | 19.97     |         |
| 068 | 0 43 06.4 | 41 03 39   | 20.84 | 19.94 | 22.73           | 22.27           | 20.80     | 20.02     |         |
| 069 | 0 43 08.5 | 41 03 38   | >20.9 | >20.2 | 21.80           | 21.09           | 20.84     | 20.03     |         |
| ID  | RA (2000) | Dec (2000) | R   | I   | $\Delta R_{\text{max}}$ | $\Delta I_{\text{max}}$ | $\mu_R$ | $\mu_I$ | Remarks |
|-----|-----------|------------|-----|-----|----------------|----------------|--------|--------|---------|
| 070 | 043 11.2  | 41 03 34   | 20.91| 19.48| 22.09          | 21.41         | 20.89  | 20.13  |         |
| 071 | 043 14.5  | 41 03 34   | >21.0| >20.2| 22.51          | 21.95         | 20.88  | 20.07  |         |
| 072 | 043 10.0  | 41 06 21   | >20.9| 19.54| 22.45          | 22.04         | 20.67  | 19.87  |         |
| 073 | 043 11.1  | 41 05 11   | >20.9| >20.2| 22.46          | 22.06         | 20.83  | 20.06  |         |
| 074 | 043 23.6  | 41 05 55   | 19.92| 19.29| 21.15          | 20.89         | 20.90  | 20.10  | eclipsing? |
| 075 | 043 21.6  | 41 05 03   | 20.85| 19.94| 21.91          | 21.76         | 20.86  | 20.09  |         |
| 076 | 043 30.4  | 41 03 37   | 20.63| 20.00| 22.63          | 22.15         | 21.11  | 20.31  |         |
| 077 | 043 17.5  | 41 05 16   | >20.9| 20.19| 22.62          | 22.09         | 20.86  | 20.09  |         |
| 078 | 043 18.0  | 41 03 59   | 19.95| 19.13| 21.93          | 21.80         | 20.93  | 20.15  |         |
| 079 | 043 20.9  | 41 04 09   | 20.88| 20.07| 21.92          | 21.62         | 20.90  | 20.12  |         |
| 080 | 043 16.4  | 41 05 25   | >21.1| >20.2| 22.33          | 21.81         | 21.26  | 20.44  |         |
| 081 | 043 47.0  | 41 06 05   | 20.04| 19.58| 22.28          | 22.35         | 20.23  | 19.44  |         |
| 082 | 043 00.0  | 41 08 34   | 20.04| 19.58| 22.28          | 22.35         | 20.23  | 19.44  |         |
| 083 | 043 02.7  | 41 09 47   | >20.6| >19.8| 22.22          | 22.55         | 20.11  | 19.31  |         |
| 084 | 043 37.5  | 41 10 09   | >20.9| >20.1| 22.45          | 21.73         | 20.68  | 19.95  |         |
| 085 | 043 42.9  | 41 10 18   | >20.9| 19.88| 22.02          | 21.39         | 20.81  | 20.01  |         |
| 086 | 043 45.9  | 41 10 19   | >21.0| >20.2| 22.26          | 21.78         | 20.91  | 20.12  |         |
| 087 | 043 50.1  | 41 08 37   | >21.1| >20.3| 23.22          | 21.96         | 21.11  | 20.27  |         |
| 088 | 043 49.1  | 41 07 27   | >21.1| >20.4| 22.82          | 22.30         | 21.19  | 20.38  |         |
| 089 | 043 37.9  | 41 10 15   | 19.42| 19.46| 22.27          | 22.27         | 20.64  | 19.92  |         |
| 090 | 043 39.6  | 41 09 06   | 19.29| 19.13| 22.06          | 21.64         | 20.90  | 20.07  |         |
| 091 | 043 48.9  | 41 09 26   | >21.0| >20.3| 22.73          | 22.57         | 21.03  | 20.23  |         |
| 092 | 042 45.8  | 41 11 25   | 19.65| 18.49| 21.14          | 21.23         | 19.20  | 18.44  |         |
| 093 | 042 44.3  | 41 11 29   | >20.1| >19.3| 21.88          | 21.65         | 19.14  | 18.31  |         |
| 094 | 042 42.3  | 41 12 18   | 17.93| 17.65| 18.15          | 17.17         | 18.73  | 17.94  | nova?  |
| 108 | 042 44.5  | 41 05 54   | 20.64| 19.78| 22.39          | 22.00         | 20.28  | 19.44  |         |
| 121 | 042 54.1  | 41 07 40   | >20.7| >19.9| 22.25          | 22.13         | 20.28  | 19.50  |         |
| 140 | 043 04.7  | 41 14 57   | >20.0| >19.3| 22.43          | 22.65         | 19.00  | 18.21  |         |
| 142 | 043 06.7  | 41 14 31   | 19.90| 19.24| 21.38          | 21.37         | 19.17  | 18.36  |         |
| 150 | 043 10.0  | 41 09 01   | 20.50| 19.44| 22.62          | 22.85         | 20.38  | 19.60  |         |
| 156 | 043 14.0  | 41 09 26   | 20.54| 19.37| 21.87          | 21.49         | 20.44  | 19.62  |         |
| 161 | 043 17.5  | 41 12 12   | 20.52| 19.68| 21.55          | 21.68         | 20.11  | 19.34  |         |
| 163 | 043 18.2  | 41 08 20   | 20.38| 19.83| 21.91          | 21.95         | 20.52  | 19.71  |         |
| 165 | 043 20.9  | 41 10 26   | >20.7| >20.0| 21.35          | 21.89         | 20.42  | 19.65  |         |
| ID | RA (2000) | Dec (2000) | R   | I      | $\Delta R_{max}$ | $\Delta I_{max}$ | $\mu_R$ | $\mu_I$ | Remarks |
|----|-----------|------------|-----|--------|------------------|------------------|---------|---------|---------|
| 166| 0 43 21.6 | 41 03 25   | >21.0 | >20.3  | 21.21            | 20.26            | 20.94   | 20.17   |         |
| 167| 0 43 21.7 | 41 08 21   | 20.31 | 19.55  | 21.85            | 21.34            | 20.50   | 19.75   |         |
| 170| 0 43 23.2 | 41 10 26   | 20.14 | 19.46  | 21.48            | 21.55            | 20.46   | 19.65   |         |
| 171| 0 43 24.7 | 41 06 11   | 19.94 | 19.30  | 22.46            | 21.59            | 20.84   | 20.04   |         |
| 174| 0 43 26.4 | 41 08 43   | >20.9 | 19.57  | 23.03            | 21.77            | 20.66   | 19.86   |         |
| 176| 0 43 26.9 | 41 09 53   | >20.8 | >20.1  | 22.27            | 21.73            | 20.61   | 19.82   |         |
| 178| 0 43 27.3 | 41 04 04   | 20.85 | 19.93  | 22.82            | 22.62            | 21.07   | 20.29   |         |
| 180| 0 43 28.6 | 41 14 53   | >20.5 | 19.56  | 22.26            | 21.13            | 20.02   | 19.19   |         |
| 182| 0 43 29.6 | 41 17 44   | >20.4 | >19.7  | 21.98            | 21.08            | 19.79   | 19.00   |         |
| 184| 0 43 31.9 | 41 12 19   | 19.54 | 19.27  | 21.50            | 21.94            | 20.35   | 19.59   |         |
| 186| 0 43 35.4 | 41 15 05   | 20.39 | 19.72  | 21.98            | 22.03            | 20.28   | 19.46   |         |
| 188| 0 43 35.9 | 41 09 37   | 19.11 | 18.94  | 21.63            | 20.89            | 20.80   | 20.01   |         |
| 190| 0 43 37.1 | 41 14 31   | 20.04 | 19.57  | 20.57            | 20.23            | 20.30   | 19.48   |         |
| 192| 0 43 38.7 | 41 14 23   | >20.7 | >20.0  | 22.10            | 21.22            | 20.44   | 19.60   |         |
| 194| 0 43 39.1 | 41 12 25   | 19.24 | 17.99  | 22.06            | 20.96            | 20.53   | 19.78   |         |
| 196| 0 43 39.5 | 41 12 35   | 20.27 | 19.77  | 22.29            | 21.83            | 20.56   | 19.72   |         |
| 198| 0 43 41.0 | 41 15 25   | 20.33 | 19.64  | 21.30            | 21.27            | 20.36   | 19.54   |         |
| 200| 0 43 44.1 | 41 10 53   | >21.0 | >20.2  | 22.61            | 22.36            | 20.86   | 20.06   |         |
| 202| 0 43 45.7 | 41 14 52   | 19.33 | 18.70  | 21.73            | 21.24            | 20.44   | 19.67   |         |
| 204| 0 43 47.9 | 41 10 03   | 19.54 | 18.92  | 22.16            | 21.40            | 20.91   | 20.05   |         |
| 206| 0 43 48.5 | 41 09 15   | 19.57 | 18.93  | 21.10            | 20.59            | 21.04   | 20.23   |         |
| 208| 0 43 49.9 | 41 10 44   | 19.94 | 19.00  | 22.28            | 21.77            | 20.89   | 20.09   |         |
| 210| 0 43 51.2 | 41 14 25   | >20.9 | >20.1  | 22.28            | 21.21            | 20.66   | 19.87   |         |
| 212| 0 43 37.9 | 41 07 34   | >20.5 | >19.8  | 22.45            | 20.49            | 19.97   | 19.18   |         |
| 214| 0 43 18.1 | 41 13 23   | >20.5 | 19.36  | 21.73            | 21.27            | 19.92   | 19.11   |         |
| 216| 0 43 17.3 | 41 10 33   | 20.60 | 19.36  | 21.41            | 21.14            | 20.38   | 19.56   |         |
| 218| 0 43 21.1 | 41 09 06   | 19.99 | 19.21  | 21.68            | 21.33            | 20.56   | 19.75   |         |
| 220| 0 43 57.3 | 41 03 45   | >20.8 | >20.1  | 22.77            | 22.36            | 20.62   | 19.87   |         |
| 222| 0 42 56.9 | 41 10 33   | >20.4 | >19.7  | 21.88            | 21.75            | 19.86   | 19.06   |         |
| 224| 0 43 14.8 | 41 11 13   | >20.6 | 19.03  | 21.60            | 21.50            | 20.15   | 19.37   |         |
| 226| 0 43 24.7 | 41 14 10   | >20.5 | >19.8  | 22.24            | 21.94            | 19.99   | 19.21   |         |
| 228| 0 43 25.0 | 41 13 52   | >20.6 | >19.8  | 22.25            | 22.21            | 20.08   | 19.25   |         |
| 230| 0 43 25.5 | 41 13 58   | 20.12 | 19.14  | 21.97            | 22.12            | 20.09   | 19.27   |         |
| 232| 0 43 18.5 | 41 16 29   | >20.2 | >19.5  | 21.96            | 21.78            | 19.41   | 18.60   |         |
The search of the KPNO data yielded 139 detected sources over the four nights of data. Table 1 summarizes the position (columns 2 and 3) and detected amplitude range (where the flux difference is expressed as an equivalent magnitude, columns 6 and 7) of these sources in both filters. In addition, we also list the surface brightness (columns 8 and 9) and a magnitude at the location of the variable in the raw reference frame (columns 4 and 5). To delineate between variables that might be considered resolved in the raw data and those that are unresolved we compare the magnitude in the reference frame at the location of the detected source in the difference frame with the local surface brightness fluctuations predicted by equation (6) for the best seeing (1."1). If the measured magnitude is brighter than twice the predicted amplitude of the local surface brightness fluctuations we quote the measure, otherwise we quote a limit of $m_{SBF} - 0.75$. Under this criterion, than less than half of the sources are resolved at the 2σ level in one or both filters. In Figure 7 we show how the detected amplitude range of these sources compares with the local SBFs in the R filter. Under 1.1" seeing only 34 sources have an amplitude range that exceeds their local SBFs (14 in the I band). We have therefore achieved sensitivity to both sources and variability well below anything that can be considered resolved in the conventional photometry sense.

In Figure 8 we show some sample flux difference light curves of objects listed in Table 1 that are unresolved in our conservative definition. All fluxes are relative to the arbitrary reference frame of the final night’s images. Generally time coverage is too poor to classify these variables. We expect most of them to be Classical Cepheids which have periods ranging from 1 to 50$^d$ with mean magnitudes of $\bar{M}_R = -1$ to -5.5. In addition, we expect sensitivity to older Pop II Cepheids (W Virginis stars) which have the same period range, but with $\bar{M}_R = -0.5$ to -3. Other types of variables may include rare RV Tauri stars, and fractional light curves of long period Mira variables. For recent reviews these and other variables see Percy (1993) and Nemec et al. (1994). However, in Figure 9 we present additional flux difference light curves that we classify as eclipsing variable candidates.

Table 1—Continued

| ID | RA (2000) | Dec (2000) | R | I | $\Delta R_{max}$ | $\Delta I_{max}$ | $\mu_R$ | $\mu_I$ | Remarks |
|----|-----------|------------|---|---|-----------------|-----------------|--------|--------|---------|
| 324| 0 43 36.6 | 41 07 17   | >21.0 | >20.2 | 22.78           | 22.17          | 20.99  | 20.14  |         |
| 325| 0 43 54.9 | 41 08 12   | 21.13 | >20.4 | 21.82           | 21.38          | 21.23  | 20.36  |         |
| 326| 0 43 45.5 | 41 16 16   | 20.40 | 19.41 | 21.22           | 20.97          | 20.45  | 19.64  |         |
Fig. 7.— The detected amplitude range in the R band of the 139 sources discovered in the KPNO data, expressed as ratio to the statistical amplitude of their local surface brightness fluctuations under 1″ seeing (the mean magnitude of which is 21.″6 for these sources). Under the best seeing of 1.1″ only 25% of the sources have an amplitude range that exceeds their local surface brightness amplitude.
These objects show stability in one or more nights, but a sharp decline in both filters on one of the two well sampled nights.

We have performed a preliminary analysis of some of the VATT data (§2.3). In an effort to determine if the same techniques would recover some of the KPNO variables we have discovered we analyzed 14 consecutive nights of data. Confining the analysis to this short timespan of the total data sensitizes us to short period variables that may be similar to the KPNO variables. Of 105 sources discovered in a search of the difference frames over this period 23 were found to have a positional coincidence of < 1" to the 85 KPNO sources that fall on the VATT field. These are indicated in Table 1. In Figure 10 we compare the KPNO and VATT light curves of some of these matches.

We expect that a more detailed analysis and search of the data (beyond the scope of this paper) will recover a larger fraction of the KPNO sources. Furthermore, we find that differencing VATT images over the timespan of a month yields many more variables on the difference frame for our main far-side field. Figure 11 shows a difference subimage (170 × 170") corresponding to frames taken 40 nights apart compared with its original image. Thus we anticipate these data will generate > 2000 light curves.

Interestingly, VATT data indicates a short period (or half-period) of < 2d for a KPNO eclipsing variable candidate (#21 in Figure 10). If the object was in M31 its absolute R magnitude must be \(\sim -4\) and with its red color \((R - I > 1.1)\) a bright red giant (M2-M6) would appear to be the only interpretation possible for identity of the primary star. The minimum size of such as star is \(\sim 500R_\odot\) (luminosity class I supergiants) and \(\sim 40R_\odot\) (luminosity class III, Schmidt-Kaler 1982) and the minimum period for an object orbiting at its photospheric radius is 3.5\(^h\) and 29\(^d\) respectively. Thus both the short period and sharpness of the dip in the KPNO light curve argue against the object being located in M31. However, we note that this object is also difficult to interpret as a foreground dwarf star system. If we assume the period is 4\(^d\), the orbital separation is \(\sim 0.05(M_{tot}/M_\odot)^{-1/3}\) AU, where \(M_{tot}\) the total mass of the system. Assuming equal mass M-dwarves with \(M_{tot} = 1M_\odot\) their relative velocity is \(\sim 140km/s\). Since the diameter crossing time is > 8h from the KPNO light curve, this implies a stellar radius of 5\(R_\odot\) - far in excess of a typical M-dwarf. Further observations may decide the nature of this interesting object.

6. GALACTIC HALO OPTICAL DEPTH LIMITS
Fig. 8.— Sample flux difference light curves in R and I (upper and lower halves of panels respectively) of some sources listed in Table 1. The difference fluxes are relative to an arbitrary mean reference image on final night of the run. The Julian Date at \( t = 0 \) is 2,449,619.50. All these sources are completely unresolved and showed no evidence in any frame of flux exceeding the local background by more than 2\( \sigma \) of the seeing element statistical surface brightness fluctuations in the original unsubtracted frames.
Fig. 9.— As in Figure 8, but showing some sources in Table 1 that exhibit sharp declines in their flux difference light curves in both filters, suggesting that they are eclipsing variables. The Julian Date at $t = 0$ is 2,449,619.50.
Fig. 10.— Comparison of 1994 KPNO (left) and 1995 VATT (right) R band flux difference light curves for the same sources. The VATT sources were discovered independently in a restricted 14 night timespan search and found post facto to match the position of the KPNO sources. Notice how in all cases the behavior of the KPNO light curves is consistent with the shapes of the VATT light curves which comprise nightly $1−2^h$ exposures. The VATT data also appears to confirm the eclipsing nature of #21; its sharp rise, short period (or half-period) around 2 days. The Julian Date at $t = 0$ is 2,449,619.50 for the KPNO light curves and 2,450,0042.50 for the VATT light curves.
Fig. 11.— An $170 \times 170''$ subimage of VATT data taken in Fall 1995 of the maximal lensing field (§2.2) on the far side minor axis of M31. The top image shows an R band image taken on 1995 October 21 with the a smoothed galaxy background subtracted off to highlight the unresolved stars, and the bottom image is the difference image relative to a similar image taken 38 nights later (shown at 0.2 times the intensity range of the left image). The difference image shows many clear point sources in both positive and negative flux, indicating the detection of variability over this timescale of many stars that have either brightened or faded. The full difference image contains $> 2000$ well detected sources.
6.1. Stellar Number Densities

The number of stars per pixel detectable above a certain $S/N$ threshold is crucial in understanding the conversion of an event rate to an optical depth due to gravitational lensing. In turn, the number of stars per pixel depends on the shape of the luminosity function, and its first-moment integral, the surface brightness. The surface brightness determines not only how many stars brighter than a certain magnitude are present in a pixel, but also the background flux which sets the $S/N$.

Calibrating our surface brightness data is straightforward, despite the lack of a night sky brightness determination, in that surface brightness photometry is published for our field (Walterbos & Kennicutt 1987). The flux/ADU/pixel and night sky brightness (assumed to be constant over our sequence-averaged frame) is simply recovered from comparison of the slope and zero-intercept of surface brightness and counts across the frame.

We have developed a simple technique for recovering the shape of the luminosity function, based on the distribution of local surface brightness in various pixel. Figure 12 shows the number of pixels in a small subframe of our image plotted as a histogram versus the signal per pixel in ADU, once the sky background has been subtracted. The thick solid line shows the plot for our actual data, for a subframe in which the spatial gradient of counts has been removed without affecting the mean count per pixel. The other curves denote pixel histograms for several cases of simulated star fields composed from luminosity functions of the form $\phi(L)dL \propto L^{-0.60}dL \propto 10^{m_0}dm$, where $m$ is apparent magnitude (in which $\alpha = 0.60$ for the thin solid line, 0.55 for the dotted.) The normalization of the simulated histograms is set only by requiring that the first moment of the distribution, the surface brightness, is maintained. We note that higher moments are also recovered simultaneously, allowing the entire shape of the real data’s histogram to be recovered with the proper choice of $\alpha$, in this case $\alpha = 0.59 \pm 0.01$. This value of $\alpha$ is similar to those found for bright stars in the Solar Neighborhood, $\phi$, (Luyten 1968), in the outer disk of M31 (Hodge, Lee & Mateo 1988), or in K band, for the inner disk of M31 (Rich, Mould & Graham 1993). These simulated images were constructed using stars spread over $17 < R_j < 28$ according to the above power laws. On the faint end we used a Solar Neighborhood (Luyten 1968) $\phi$. The two distributions on the bright end were joined by requiring continuity at $R = 28$. At magnitudes brighter than $R = 17$ negligibly few stars appear in the simulations, while fainter than $R = 28$, stars are too uniformly spread to affect the count fluctuations per pixel by more than 2% of the total. The power law approximation must break down at the faint end of this distribution, however, given the presence of the horizontal branch at $R = 25.0$. To reach this level in measuring the luminosity function, an instrument such as Hubble Space Telescope is required. Our technique of simulated pixel histograms could
then be employed with the HST data to reach below the horizontal branch, where the luminosity function is probably once again well-approximated by a power law (or some other few-parameter family).

Knowing the luminosity function, we can estimate the number of detectable stars in the sample. From our best LF fit we estimate at this location there are 0.35 per square arcsecond above \( R = 22.5 \). This compares to an estimate of 0.4 stars per square arcsecond scaling the Hodge LF measured in the disk of M31 (Hodge, Lee & Mateo 1987) using the count/brightness ratio (c.f., C92), the latter estimate being affected disproportionally by a large disk component. Therefore we consider our estimate to be more realistic and adopt it for the purposes of this paper. Thus for our integrated galaxy surface brightness of \( R = 4.67 \) we estimate there are \( 6.7 \times 10^5 \) stars on our field exceeding \( R = 22.5 \).

The usual technique for calculating event rates consists of choosing a stellar sample above a threshold apparent brightness, and a minimum amplification, which implies a certain lensing cross-section per MACHO given a known Observer-Lens-Source (O-L-S) geometry. The product of the implied optical depth corresponding to the number density of MACHOs and the number of stars in the sample produces the mean number of lensing events at a given time, and the number of events over a period of observation (much longer than the event) is then inversely proportional to the typical event duration (Griest 1991).

In a sample of lensed stars surveyed by the DIP technique, however, we cannot impose either stellar brightness nor amplification thresholds, but instead a flux cutoff \( f_{\text{min}} \). The number of detectable events, therefore, is computed by considering which minimum amplification (Paczynski 1986):

\[
A_{\text{min}} = \frac{u_{\text{min}}^2 + 2}{u_{\text{min}}(u_{\text{min}}^2 + 4)^{1/2}},
\]

(7)
corresponding to an impact parameter \( r_{\text{min}} \) at the point of maximum amplification (\( u_{\text{min}} = r_{\text{min}}/R_e \), where \( R_e \) is the projected Einstein Radius [§6.2]), implying a lensing cross-section \( \sigma = \pi r_{\text{min}}^2 \) for a detectable event. \( A_{\text{min}} \) depends on the brightness of the star, \( f_* \), and \( f_{\text{min}} \): \( A_{\text{min}} = f_{\text{min}}/f_* + 1 \), and the number of events at a given time,

\[
N_{\text{ev}} = \int_{0}^{\infty} N_*(f_*)\tau(A_{\text{min}})df_* = a_1 \int_{0}^{\infty} \phi_*(f_*)\sigma(A_{\text{min}})df_* ,
\]

(8)

where \( a \) absorbs multiplicative factors relating \( N_* \) to \( \phi \) (such as the field of view) and \( \tau \) to
Fig. 12.— A histogram of residual pixel intensities of a $150 \times 150$ pixel region of KPNO data (§2.2) after the underlying galaxy gradient has been subtracted off (bold line). The skewness of the pixel distribution is primarily determined by the ratio of bright to faint stars in the luminosity function. A simulated image of artificial stars distributed with a power law luminosity function (see §6.1) gives a good fit with $\alpha = 0.59$ to the pixel histogram.
σ (such as O-L-S geometry and the number density of MACHOs). Comparing this integral to the analogous expression in the case of thresholds in $A$ (for the sake of discussion, a 34% enhancement, as for $u = 1$) and $f_s$:

$$N'_{ev} = a_1 \int_{f_{min}}^{\infty} \phi_1(f_*) (\pi R_e^2) df_*.$$

Regardless of $\phi$, $N'_{ev} < N_{ev}$. For the case of the Solar Neighborhood luminosity function (Luyten 1968), $N_{ev}/N'_{ev} \approx 3$, with most of the additional events due to high amplifications of stars with $f_* < f_{min}$. In the case of a large horizontal branch contribution to $\phi$, however, the number of high$-A$, small $f_*$ events could be even larger.

Due to these additional events, the distribution of amplification of detected sources is not described by a uniform distribution in impact parameters, as is found for the $f_{min}$ $A$–threshold case characteristic of microlensing searches using resolved stars. In the DIP case, there are more low$-u$, high$-A$ events, depending on the shape of $\phi$. Given sufficient S/N, however, we can measure $A$ from the shape of the light curve (even though we don’t know $f_*$), meaning that $\phi$ can be recovered from the distribution in $A$. (Although at very high $A$, the dependence of the shape of the light curve on $A$ becomes degenerate.) In §6.3 we show that high S/N has been achieved. In a future paper we hope to apply the technique to actual microlensing events.

### 6.2. Mass and Timescale Sensitivities

Before turning to the individual surveys themselves we must examine the masses of MACHOs to which we are sensitive to and the event timescales such masses correspond to. The projected Einstein radius of a MACHO of mass $M$ at the lensed source at some observer-source distance $D_{os}$ is given by,

$$R_e^2 = \frac{4GM D_{os} D_{ls}}{c^2 D_{ls}},$$

(Paczynski 1986) where the $D$’s are the indicated O-L-S distances. A detectable event corresponds to when this projected radius exceeds radius of the lensed star (below this limit the light is simply being redistributed within the image of the star’s photosphere). Let us consider a survey sensitive to stars in M31 brighter than $R = 22.5$, including high amplification events of fainter stars. We adopt our mean lensing source to be a K0 III star with a mean radius corresponding to $\sim 12R_\odot$ (Schmidt-Kaler 1982). Consider a
MACHO in M31’s halo with $D_{ls} = 10 \text{ kpc}$. This corresponds to a minimum MACHO of mass $\sim 5 \times 10^{-5}M_\odot$, but many of these stars are larger, so thereby produce a partial cutoff at larger MACHO masses. For a Galactic MACHO, however, we have the benefit of the observer-lens proximity, in which the effects at the lens plane are projected to the source plane, resulting in a much smaller mass for a given angular size. The minimum mass here for $D_{ol} = 10 \text{ kpc}$ corresponds to $7 \times 10^{-9}M_\odot$ ($\sim 0.002$ Earth masses).

Now consider the timescales corresponding to these mass limits. Since the stars in the inner disk of M31 rotate at 260 km s$^{-1}$ (Braun 1991), and objects in the inner halo/outer bulge move transversely at an average 230 km s$^{-1}$, the typical timescale $t_E$ for minimal mass M31 MACHO/M31 star events is 6.5 hours. The component of the Sun’s motion transverse to the M31 sightline of 170 km s$^{-1}$, and the typical Galactic MACHO transverse velocity of 180 km s$^{-1}$ result in a typical timescale for minimal Galactic MACHO events of 8 minutes.

### 6.3. Optical Depth Calculation

None of the light curves of the objects in Table 1 are appear consistent with microlensing events on the $< 8^h$ timescale. Furthermore no frame showed single “spikes” in both filters that might be due to a high amplification lensing event on a timescale of $< 50^m$ which is the time resolution of the individual difference frames in this analysis. A more sophisticated and complete search of this dataset as well as an analysis of frames at the full time resolution of the individual images ($10^m$) using the search technique proposed by Gould (1996) may yet reveal interesting candidates, but such a effort is beyond the main purpose of this techniques paper. However, based on our simple search we can still make an estimate of some interesting optical depth limits. We note that these estimates are largely limited by our uncertainty of the luminosity function.

Sources were originally identified by eye in the difference frame at typical S/N ratio of 6 or greater. Thus these S/N ratios correspond to $12\sigma_{\text{photon}}$ given our determined technique limits in §4.1. For a fiducial minimum amplification of $> 34\%$ corresponding to stars passing within the Einstein ring of the lens (Paczynski 1986), we must therefore detect the original star at $36\sigma_{\text{photon}}$ to detect this minimum amplification at a S/N $> 6$ in the difference frame. Given our estimate of the luminosity function and the number of stars we are sensitive to on the single coadded frames in the analysis in §6.1, we estimate that each difference frame is sensitive to $> 34\%$ amplifications of $2.0 \times 10^5$ stars ($R > 21.2$). However, we are also sensitive to a larger number of fainter stars that suffer higher amplifications (§6.1) giving us sensitivity to $6.1 \times 10^5$ stars.
Requiring multiple exposures per event in the KPNO data, in both bands, we have 19 independent 50" sample times, corresponding to a mass scale of about \(2 \times 10^{-7} \text{M}_\odot\). Eliminating “edge” points over the four nights gives 13 independent sample times (confined to the two good nights). Thus at this timescale we are sensitive to \(7.9 \times 10^6\) star-epochs, which corresponds to a \(2\sigma\) optical depth of \(\tau < 5 \times 10^{-7}\). (The minimal \(\tau\) grows by \([M/(2 \times 10^{-7} \text{M}_\odot)]^{1/2}\).) Over \(8^h\) timescales corresponding to masses of \(8 \times 10^{-5} \text{M}_\odot\) we have two sample times corresponding to the two last nights, which corresponds to a \(2\sigma\) limit of \(\tau < 3.3 \times 10^{-6}\) applying both to M31 and Galactic halo MACHOs. These limits are expressed in terms of a delta-function concentration of MACHO mass at the value of \(M\) discussed. Given Paczynski’s (1986) estimate for a simple spherical halo of the optical depth of the Galactic halo towards M31 of \(\tau = 1.0 \times 10^{-6}\), we can conclude that in two nights we have eliminated the possibility at the 2-sigma confidence quoted above that the Galactic halo is comprised of a single mass population of MACHOs in the Earth mass range \((M_\oplus = 3 \times 10^{-6} \text{M}_\odot)\) (c.f., the comparable EROS Galactic halo limits of Aubourg et al. 1995 and MACHO results of Alcock et al. 1996 in this range). For M31 plus Galactic MACHOs, we can expect values for \(\tau\) of \(5\text{-}10 \times 10^{-6}\) (C92, Han & Gould 1996), which is a factor of several times larger than our 2-sigma limit quoted above for \(8 \times 10^{-5} \text{M}_\odot\). Despite the systematic uncertainties involved, it would appear that these mass ranges are ruled out for a 100\% contribution to the both dark matter MACHO halos.

Another possible source of microlensing in our field is intergalactic masses, as might be associated with dark matter clustering with galaxies, but not actually resident in their halos. We show that we are not currently sufficiently sensitive to detect these. First, if these were non-baryonic masses (e.g. primordial black holes: Crawford & Schramm 1982), they might be able to contribute up to \(\Omega_{DM} \approx 1\) to the Universe’s mean density (being irrelevant to Big Bang nucleosynthesis constraints on baryonic matter density). Second, if they fall into potential wells as cold dark matter, and dominate the matter distribution, their density between the Galaxy and M31 might be an order of magnitude greater than the universal critical density \(\rho_{\text{crit}}\). (Models studying the Local Group in a CDM universe are currently being produced: Governato 1996.) If one assumes a model whereby the region beyond the galaxy halos are dominated by a uniform distribution at \(\rho_{\text{crit}}\) of such objects, their contribution to the optical depth would be about \(1.5 \times 10^{-8} \, h^2\) \((h = H_0/100 \text{ km s}^{-1})\). Even a local enhancement of ten would be undetectable in this survey, but within the reach of surveys we are preparing for the near future. Intergalactic microlensing events involving M31 giant and supergiant stars as sources would be limited to masses larger than about \(10^{-5} \text{M}_\odot\). Such events would have a signature of a significantly longer timescale than those of similar masses in either halo. Their projected cross-section per unit mass is much larger given their more favorable placement roughly half-way between the observer.
and source (see equation 10), and their transverse velocities with respect to the sightline are probably lower since they are not within the gravitational potential of a galaxy. These factors, accounting for their distance from the source plane, combine to make these events about four times longer than Galactic or M31 halo events due to the same lens mass.

7. DISCUSSION and CONCLUSIONS

We have demonstrated that detecting variability among completely unresolved stars is a tractable problem through an image differencing technique that involves registering, photometrically scaling and PSF-matching pairs of frames. From this approach we have derived optical depth limits for MACHOs in both the Galactic and M31 halos (§6.3). The technique is not limited by seeing variations or stellar density. We have shown how, in a typical wide field imager, PSF matching a pair of frames is complicated by small changes in the telescope focus, and developed an algorithm which uses a limited number of stars to model the full frame matching PSF function. This algorithm has general applicability to wide-field imagers at different telescopes and differencing images taken through separate systems. We have shown how we have achieved residuals in difference frames taken at the KPNO 4-m prime-focus camera that are within a factor of three of the predicted seeing element photon noise. These difference frames taken over four nights yielded 139 variable sources most of which evidenced no flux at any stage more than twice the local surface brightness fluctuations and thus be considered completely unresolved and beyond the capabilities of conventional crowded field photometry. A preliminary analysis of a sample of data taken at the VATT 1.8-m telescope one year later and using the same techniques outlined in this paper has confirmed the reality of a large fraction of these KPNO sources. These VATT data were taken using an optical corrector which facilitates the DIP process by requiring only a few (1-4) PSF adjustments to match frames.

This techniques paper is primarily motivated by the potentially rewarding aspects of a microlensing survey of M31 suggested by Crotts (1992). However, the technique has wide applicability to microlensing (see Gould 1996) and more general variability surveys (see also Phillips & Davis 1995). We have attempted to provide in §4 an assessment, both qualitatively and quantitatively, of all systematic effects pertinent to our KPNO survey and our ongoing VATT survey. We recommend Gould’s analytical treatment of the DIP approach for a more thorough quantitative assessment as applied to more general microlensing surveys.

Other microlensing surveys directed to the Galactic Bulge and the LMC can apply the DIP technique to their datasets at hand. In no longer being limited by sensitivity to only
well detected and resolved stars, a DIP analysis will dramatically increase the sensitivity to microlensing events. In particular, however, such an analysis will also be sensitive to short timescale events (limited only by the exposure times and time resolution of their data) associated with very short timescales, especially very high amplification “caustic-crossing” lensing events associated with binary/multiple component lenses and, perhaps, other optical transients (c.f., Hudec & Soldan 1994).

We also note that despite the PSF undersampled images in the present WFPC2 camera onboard HST and the consequences that this has to image differencing technique (§4.6 and Gould 1996), the algorithm pioneered by Mighell (Mighell & Rich 1995) to cope with the unsampled PSF and substantially improve the photometric accuracy over a standard photometric software such as DAOPHOT and DOPHOT may well also have an application as an avenue by which HST data maybe exploited for pixel analysis (e.g., Gould’s [1995] microlensing survey of M87).

Finally, we note that, in principle, the techniques we have outlined are applicable not only to searches for photometric variability, but also astrometric variability. Changes that we can detect at approximately the photon shot-noise level can also be caused by simple relative motion of the sources, not just flux changes. In this regard DIP is also a sensitive tool for studying proper motions at the milli-arcsecond level, since the maximum change in flux, occurring at radius $= \pm \sigma/\sqrt{2}$, (where $\sigma$ is the r.m.s. width of the image), grows as $\Delta f/f = 0.72 \Delta r/r_{HW}$, where $\Delta r$ is the change in the star position, $r_{HW}$ is the HWHM radius for a Moffat seeing function, and $\Delta f$ the absolute value of the change in flux which varies from positive to negative values in the direction of motion. Thus for a star detected at $100\sigma$ in $1''$ seeing, even in crowded conditions, motions as small as 14 mas can be detected at the $1\sigma$ level.

Since a requirement of image differencing is accurate registration of all stars on a frame, it is worth noting that a pixel analysis of colour shifted events (Kamionkowski 1995) might be very revealing, since a difference image is sensitive not only the flux changes but positional shifts. We might expect a colour shifted event to show such a shift since the ratio of light of the lens and lensed star changes. The difference image quantifies precisely the systematics involved in such a problem, since the residuals around all other stars in the frame are a measure of how good the registration between frames is as well as how the PSF has been matched. The residual image position should rest at the location of the component being lensed within the blend. Over longer timescales one might also think of measuring proper motions of stars in the Galactic Bulge and LMC by differencing images taken a few years apart - any shift in a star’s position will be revealed by the residuals in the difference frame. Of course, if a proper motion could be measured this way then most stars
on the frame will show “proper motion” residuals, but for examination of a known lensing event there maybe some gain of information of how the lensed star is moving relative to most stars in the field.

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