Heavy Baryons in Holographic QCD Using Higher Dimensional Degrees of Freedom

1 Introduction

In recent years, with the development of experimental technologies, new hadrons including heavy flavor have been discovered, and various properties have been revealed. These new phenomena should be understood from QCD. Although the lattice QCD is progressing, many problems still remain in the understanding of hadron physics, in particular resonance properties are still difficult. Therefore, one attempts to understand hadron physics by using a variety of effective models. In these proceedings, we report our recent work on baryon properties [2,5] by using the Sakai–Sugimoto model [1], which is one of the holographic QCD models.

The Sakai–Sugimoto model is based on the D4–D8 construction, and succeeded in geometrically introducing the chiral symmetry breaking. They have derived an effective action of the flavor gauge field in the nine dimensional space, leading to the successful low-energy effective action of hadrons.

First, we propose a method of introducing heavy flavors into the Sakai–Sugimoto model by regarding the gauge fields in the extra dimension as heavy mesons [2]. Using the Forgács–Manton method [3], we derive an action consisting of light and heavy mesons. By performing collective coordinate quantization for classical instanton solutions, we derive the mass formula of heavy baryons. It is found that the mass ordering of \( \Sigma_c \) and \( \Lambda_c^* \) is correctly reproduced in our model, which was reversed in the previous study [4]. Our model also predicts the \( P_c \) states which have been attracting attention in recent years.

Next, we investigate the decay properties of the Roper resonance using the Sakai–Sugimoto model [5]. It is known that the decay width of one pion emission is suppressed in the conventional quark model [6]. In contrast, the present approach predicts a finite value. Moreover, we find that the ratio of the axial coupling \( g_A^{NN}(1440) \) and \( g_A^{NN} \) is well reproduced in comparison with the experimental data as a model independent relation. These are unique features of the present approach where the Roper resonance appears as a collective state of radial oscillation of the instanton, which is very much different from the single particle picture in the quark model.
2 Heavy Baryons in the Sakai–Sugimoto Model

2.1 The Action for Heavy-Light Meson and These Solutions

The action used in this paper consists of Yang–Mills part $S_{YM}$ and Chern-Simons part $S_{CS}$. The YM part is written as follows.

$$S_{YM} = S_{SS \ model} + S_{\phi}$$

with $h(z) = (1 + z^2)^{-1/3}$, $k(z) = 1 + z^2$, $\kappa = N_c \lambda / 216 \pi^3$. Here, $\phi^\dagger = (\phi_1^\dagger, \phi_2^\dagger)$ is a two component isospinor.

$$S_{SS \ model} = k \int d^4 x dz \text{tr} \left[ -\frac{1}{2} h(z) F_{\mu \nu}^2 - k(z) F_{\mu \zeta}^2 \right]$$

$$S_{\phi} = k \int d^4 x dz \left[ -\frac{4}{9} (D_\mu \phi)^\dagger (D_\mu \phi) - \frac{4}{9} k(z) \lambda^{2/3} (D_\mu \phi)^\dagger (D_\mu \phi) \right]$$

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We explain how this action is derived (see Ref. [2] for details). The hadron effective theory obtained from the Sakai–Sugimoto model is a 9-dimensional YM–CS theory. In the conventional analysis of the Sakai–Sugimoto model composed by the SU(2 + 1) (light + heavy flavor) gauge fields, only the radial component of the gauge field of the extra dimension is utilized, and the angular component is ignored. Then, the theory is reduced to a five-dimensional YM–CS theory, and $S_{SS \ model}$ is obtained. The $S_{\phi}$ which is derived by the dimensional reduction method of Forgács–Manton, leaving the angular component of this gauge field, is regarded as the action of heavy meson. This dimensional reduction is achieved by utilizing part of the gauge symmetry, where the symmetry becomes SU(2)×U(1). Then, the broken part of the SU(3) gauge field becomes the field $\phi$. Conceptually, gauge symmetry breaking is in accordance with a brane picture where one heavy brane is separated from two light branes. The $\phi$ field corresponds to the string connecting light and heavy branes, so becomes the massive field [2].

If we choose $A_z = 0$ gauge, omit massive modes, and integrate over $z$, by following the argument of Ref. [7] (also, see Ref. [2]), then we obtain the following Chern-Simons (CS) term,

$$S_{CS} = \frac{N_c}{24 \pi^2} \int \frac{N_c \lambda}{f_H^2} \int d^4 x B^\mu \left( \phi^\dagger D_\mu \phi - (D_\mu \phi)^\dagger \phi \right)$$

$$+ \frac{N_c}{24 \pi^2} \sqrt{2} N_f \int \left[ 3 \hat{A} F^2 + \frac{1}{2} \hat{A} F^2 \right].$$

Moreover, we need to introduce a mass term in the action, which is not easily done in the holographic method of Sakai–Sugimoto. Supplementing a mass term our model action is

$$S = S_{YM} + S_{CS} - m^2 K^{1/3} \phi^\dagger \phi$$

where $K^{1/3}$ is introduced in consideration of the curved nature of the fifth-dimension.

Now, we solve the equation of motions. In the Sakai–Sugimoto model, baryons are regarded as instantons, so we obtain the instanton solution for the light flavor sector. In general, it is difficult to analytically solve the equations of motion in the presence of the curvatures $h(z)$ and $k(z)$. However, it can be simplified in the large $\lambda$ limit since the instanton profile is localized around $z \sim 0$ as proportional to $\lambda^{-1/2}$, where we can set $h(z) = k(z) = 1$. Then, we can use the BPST instanton solution.
Next, we solve the equation of motions of a heavy meson field $\phi(x, z)$. First, we perform the mode expansion, leaving only the lowest eigenmode, and integrate over $z$. By numerically solving the obtained equation of motion of the four-dimensional action, we find the classical solution of $\phi$. In Ref. [2], the solution was obtained numerically. We are now trying to find the solution analytically by using the solution used in Sect. 3 [8].

2.2 Mass Formula

We follow the quantization procedure as in Ref. [9], resulting in the mass formula as follows,

$$M = M_0 + \left( N_Q + N_{\overline{Q}} \right) m_H$$

$$+ \sqrt{\frac{\left( l + \frac{1}{2} \right)^2}{6} + \frac{2N_c^2}{15} \left( 1 - \frac{40a\pi^2A}{N_c} \left( N_Q - N_{\overline{Q}} \right) \right) } M_K K$$

$$+ \frac{2(n_\rho + n_Z) + 2}{\sqrt{6}} M_K K,$$

where $M_0$ is the instanton mass, $N_Q/\overline{Q}$ the number of heavy/anti-heavy mesons. We find that the spin $J$ and the isospin $I$ of the instanton are both $1/2$. The spin of the baryons is the sum of spins of the instanton and heavy mesons. The quantum numbers $(n_\rho, n_z)$ correspond to radial excitations and those which flip parity, respectively.

We determine parameters in our model as shown in Table 1 [2]. The constant $A$ in (5) is not independent of the parameters listed in Table 1. Then, it follows that $A = 0.078$ in the following parameter set. The Results of various baryon masses are summarized in Table 2. They have some characteristic features as follows.

– In the leading terms of $1/m_H$ expansion, we have obtained the heavy quark symmetry (HQS) singlet $\Lambda_{c,b}(0_{1}^{+})$ and the degenerate doublet $\Sigma_{c,b}(1_{1}^{+}, 1_{2}^{+})$.

– Empirically, the mass splitting of $\Lambda_{c}$ and $\Lambda_{c}^{*}$ is about twice larger than that of $\Lambda_{c}$ and $\Sigma_{c}$. In the present study, the value of $A$ in (5) plays an important role to make this order of baryon masses.

– Our model predict the hidden charmed pentaquark states corresponding to $P_c(4312, 4440, 4457)$ states reported recently.

### Table 1 Parameters in our model

| $M_0$(MeV) | $M_K K$(MeV) | $m/M_K K$ | $f_\pi/M_K K$ | $f_H/f_\pi$ |
|------------|--------------|-----------|---------------|-------------|
| -572       | 500          | 4.385     | 0.122         | 1.7         |

### Table 2 Predictions of our mass formula for the charmed and bottomed baryons in comparison with experimental data where available

| $B(I J P)$ | $l$ | $n_\rho$ | $n_z$ | $N_Q$ | $N_{\overline{Q}}$ | our model/MeV | exp./MeV |
|------------|-----|----------|-------|-------|---------------------|---------------|----------|
| $\Lambda_{c}(0_{1}^{+})$ | 0   | 0        | 0     | 1     | 0                   | 2286          |          |
| $\Sigma_{c}(1_{1}^{+})$ | 2   | 0        | 0     | 1     | 0                   | 2523          | 2453     |
| $\Sigma_{c}(1_{2}^{+})$ | 2   | 0        | 0     | 1     | 0                   | 2523          | 2520     |
| $\Lambda_{c}^{*}(0_{2}^{+})$ | 0   | 0        | 1     | 1     | 0                   | 2694          | (2595)   |
| $\Lambda_{c}^{*}(1_{2}^{+})$ | 0   | 1        | 0     | 1     | 0                   | 2694          | (2765)   |
| $\Sigma_{c}^{*}(1_{2}^{+}, 1_{3}^{+})$ | 2   | 0(1)    | 1(0)  | 1     | 0                   | 2931          | –        |
| $P_{c}^{*}(1_{2}^{+}, 1_{3}^{+})$ | 1   | 0        | 0     | 1     | 1                   | 4255          | 4312, 4440, 4457 |
| $P_{c}^{*}(1_{2}^{+}, 1_{3}^{+})$ | 1   | 0(1)    | 1(0)  | 1     | 1                   | 4664          | –        |
3 Decay Properties of the Roper Resonance in the Sakai–Sugimoto Model

In the previous section, we discussed the mass of heavy baryons. In this section we briefly discuss our recent work on the decay of the Roper resonance. The Roper resonance $N^*(1440)$ is the first excited state of the nucleon with the spin and parity $J^P = 1/2^+$. Since its mass is much larger than that of the negative parity state $N^*(1535)$ in the naive quark model as opposed to the observation, a great deal of discussions have been made [5]. As implied by the mass formula, these states are degenerate in the Sakai–Sugimoto model. For the strong decay of one pion emission, the non-relativistic quark model calculations have predicted only a small decay width, as a result of the selection rule in the long wave-length limit, which contradicts with the experimental data [6]. We try to calculate the decay width of one pion emission using the Sakai Sugimoto model. For this purpose, the coupling constant of the corresponding interaction term, which is equivalent to an axial coupling, should be derived from this model.

According to ref. [8], axial coupling is written as

$$g_A^{NN^*}(q) = \frac{8\pi^2\kappa}{3} \langle R_{N^*}\rho^2|R_N\rangle \sum_{n=1} \frac{g_{a_n}\langle \partial Z\psi_{2n}(Z)\rangle}{q^2 + \lambda_{2n}},$$

(6)

where $R_N$ and $R_{N^*}$ are the wave functions of the size $\rho$ of instanton of nucleon and Roper resonance respectively. Using $M_N = 940$ MeV, $M_{N^*} = 1370$ MeV, $q = 342$ MeV, $f_\pi = 64.5$ MeV, $M_{KK} = 488$ MeV, $\kappa = 0.0137$, we find

$$\Gamma_{our\ model}^{N^*(1440)\rightarrow N+\pi} = 64 \text{ MeV}.$$  

(7)

In this computation the value of $g_{N^*}^{NN^*}$ at $q = 0$ is used. Following the PDG table $\Gamma_{exp}^{N^*(1535)\rightarrow N\pi} \sim 90 - 140$ MeV.

This value is finite, though its absolute value is somewhat smaller than the experimental data. Nevertheless, it is interesting to observe the relation between the axial couplings of the nucleon and of the Roper-nucleon transition,

$$g_A^{NN^*} : g_A^{NN} = 1 : \left(1 + 2\sqrt{1 + \frac{N_c^2}{5}}\right)^{1/2} = 1 : 2.08,$$

(8)

which agrees well with the experimental data within $\sim 20\%$ accuracy. We emphasize that this relation does not include any model parameters (except for $N_c = 3$), and so a model independent relation.

4 Conclusion

In this paper, we have studied masses of heavy baryons and the decay of the Roper resonance in the Sakai–Sugimoto model. In Sect. 2, we proposed a method to introduce a heavy flavor in the Sakai–Sugimoto model and showed the mass spectrum. This study is new in that it uses the dimensional reduction method proposed by Forgács–Manton, which has not been used in the context of the string theory. As a result, the experimental values are reproduced well, and the level ordering of $\Lambda_c^+$ and $\Sigma_c$, which was reversed in the previous study [4], can be correctly reproduced. This model also predicts $P_c(4312, 4440, 4457)$ states discovered in LHCb in recent years. In Sect. 3, we calculated the decay width of the Roper resonance due to one pion emission decay by using the Sakai–Sugimoto model. The obtained result agrees with the experimental data much better than that of the naive quark model [6]. Furthermore, we found the model independent relation for the axial couplings of the Roper and the nucleon states.

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