Wandzura-Wilczek approximation from generalized rotational invariance

I.V. Anikin\textsuperscript{a,b,c}, O.V. Teryaev\textsuperscript{a,c}

\textsuperscript{a} Institut f"{u}r Theoretische Physik, Universit"{a}t Regensburg, D-93040 Regensburg, Germany
\textsuperscript{b} Departament de Física Teòrica and Institut de Física Corpuscular, Universitat de València-CSIC E-46100 Burjassot (València), Spain
\textsuperscript{c} Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980 Dubna, Russia

Abstract

We present the alternative way of derivation of the Wandzura-Wilczek relations between the kinematical twist-3 and twist-2 functions, parameterizing hadronic matrix element in two-photon processes $\gamma^*\pi \to \gamma\pi$ and $\gamma^*\gamma \to \pi\pi$. The new equations, providing the independence of the physical cross-sections on the choice of the light-cone direction, are suggested and explored. The amplitude of $\gamma^*\gamma \to \pi\pi$ up to genuine twist-3 accuracy is found.

Introduction. The investigation of the two-photon amplitudes of Deeply Virtual Compton Scattering (DVCS) or virtual photon production of meson pairs, up to $1/Q$ corrections (twist-3 terms) provides one of the interesting lines of inquiry. Indeed, these corrections ensure the electromagnetic gauge invariance of the DVCS amplitude with the sizeable transverse components of the momentum transfer. This problem was discussed in the pioneering paper \cite{1} and the phenomenological solution was presented in \cite{2}. The field-theoretical treatment was suggested in \cite{3,4} and developed in \cite{5,6,7}. From the other side, the study of these corrections provides the possibility to consider the delicate effects of factorization \cite{8,9}. 
It was shown \cite{6}, \cite{8} that, if the dynamical (genuine) high twist terms due to the physical transverse components of gluonic fields are neglected, the parameterizing functions (the skewed, or generalized, parton distribution) of kinematical twist-3 can be related to the parameterizing functions of twist-2 by means of the Wandzura-Wilczek (WW) relations. Their general structure was investigated, and it was demonstrated that within the WW approximation the skewed twist-3 distribution functions have the discontinuities at $x = \pm \xi$, which fortunately do not lead to the observable consequences at the level of twist-3 accuracy. However, the general relevance of these effects (and, in particular, their possible appearance in the genuine twist case, for both DVCS and meson pair production channels) is still obscure.

To some extent, this may be due to the lack of the physical interpretation of the WW approximation, essentially based on the involved treatment of the light-cone operators with total derivatives in the coordinate representation.

In this note we present the alternative derivation of the WW relations for two-photon processes based on the Ellis-Furmanski-Petronzio (EFP) \cite{9} factorization scheme. In the framework of EFP scheme, the separation of the kinematical and dynamical twist-3 effects is rather physical and amounts to the taking into account the terms linear in the intrinsic transverse momentum of partons in hadrons \footnote{This method in the case of DIS was developed by one of the authors a few years ago \cite{10}}. As a result, we present the decomposition of parameterizing functions of twist-3 to the WW-type contribution and contribution arising from the three-particle correlators. We apply this method for the amplitude $\gamma^* \gamma \rightarrow \pi\pi$. As a result, it is calculated keeping both kinematical and genuine twist-3 corrections. We found the analogs of the mentioned singularities and show that the physical amplitude at twist-3 level is not affected by them.

**DVCS amplitude of pions in WW approximation.** As in \cite{6}, we adhere the EFP factorization scheme. Neglecting the dynamical twist-3 contribution, which amounts to the keeping of the operators with transverse derivatives alone (c.f. \cite{10}), being just the Fourier transform of the mentioned intrinsic transverse momentum, we are able to write the DVCS amplitude for the pion target in the following form:

\begin{equation}
T_{(a)}^{\mu\nu} + T_{(b)}^{\mu\nu} = \int dx tr\left\{ E_{\mu\nu}(xP)\Gamma(x) \right\} + \\
\int dx_1 dx_2 tr\left\{ E_{\mu\rho\nu}(x_1 P, x_2 P)\omega_{\rho\rho'}\Gamma_{\rho'}(x_1, x_2) \right\}
\end{equation}

where $\omega_{\rho\rho'} = \delta_{\rho\rho'} - n_{\rho'} P_{\rho}$, and

$$
\Gamma_{\alpha\beta}(x) = -\int d\lambda e^{i(x+\xi)\lambda}\langle p'|\psi_{\alpha}(\lambda n)\bar{\psi}_{\beta}(0)|p'\rangle
$$
\[ \Gamma_{\alpha\beta}^{\prime}(x_1, x_2) = - \int d\lambda_1 d\lambda_2 e^{i(x_1+\xi)\lambda_1 + i(x_2-x_1)\lambda_2} \]

\[ \langle p'|\bar{\psi}_\alpha(\lambda_1 n) i \gamma^\prime \cdot (\lambda_2 n) \psi_\beta(0)|p \rangle, \quad (2) \]

where \( p' = P + \Delta/2, \quad p = P - \Delta/2, \quad \Delta = q - q' = -2\xi P + \Delta_T \) and \( p(p'), \quad q(q') \) are the initial (final) momenta of pion and photon. The \( T^{(a)}_{\mu\nu} \) and \( T^{(b)}_{\mu\nu} \) denotes the contributions from the twist-2 and twist-3 amplitudes (c.f. [4]).

**Parameterization of hadronic matrix elements.** In terms of the light-cone basis, which consists of the vectors \( P, \quad n \quad (P^2 = n^2 = 0 \quad \text{and} \quad P \cdot n = 1) \) and \( \Delta_T \), the parametrization of the vector and axial hadronic matrix elements (other structures do not contribute for massless quarks) can be introduced, as in [4]. Namely [6],

\[ \langle p'|\bar{\psi}(0)\gamma_\mu \psi(\lambda n)|p \rangle = \int_{-1}^1 dx e^{-i(x+\xi)\lambda} \left\{ H_1(x) P_\mu + H_3(x) \Delta_T^\mu \right\}, \quad (3) \]

\[ \langle p'|\bar{\psi}(0)\gamma_\mu i \gamma^\prime T \psi(\lambda n)|p \rangle = \int_{-1}^1 dx e^{-i(x+\xi)\lambda} H_1^T(x) P_\mu \Delta_T^\mu, \quad (4) \]

\[ \langle p'|\bar{\psi}(0)\gamma_5 \gamma_\mu \psi(\lambda n)|p \rangle = \int_{-1}^1 dx e^{-i(x+\xi)\lambda} i H_A(x) \epsilon_\mu \Delta_T P_n, \quad (5) \]

\[ \langle p'|\bar{\psi}(0)\gamma_5 \gamma_\mu i \gamma^\prime T \psi(\lambda n)|p \rangle = \int_{-1}^1 dx e^{-i(x+\xi)\lambda} i H_A^T(x) P_\mu \epsilon_\rho \Delta_T P_n, \quad (6) \]

for the quark correlators, and ,

\[ \langle p'|\bar{\psi}(0)\gamma_\mu g A^T_\rho(\lambda_2 n) \psi(\lambda_1 n)|p \rangle = \]

\[ \int_{-1}^1 dx_1 dx_2 e^{-i(x_1+\xi)\lambda_1 - i(x_2-x_1)\lambda_2} B(x_1, x_2) P_\mu \Delta_T^\rho, \quad (7) \]

\[ \langle p'|\bar{\psi}(0)\gamma_5 \gamma_\mu g A^T_\rho(\lambda_2 n) \psi(\lambda_1 n)|p \rangle = \]

\[ \int_{-1}^1 dx_1 dx_2 e^{-i(x_1+\xi)\lambda_1 - i(x_2-x_1)\lambda_2} i D(x_1, x_2) P_\mu \epsilon_\rho \Delta_T P_n, \quad (8) \]

for the quark-gluon correlators. Here we see the contributions due to the quark intrinsic transverse momentum and transverse component of the gluon field, respectively. In the WW approximation one is keeping only (4, 5), neglecting (7, 8), and in this sense it is already present in [4]. The remaining ingredient is to express these

\[ \epsilon_\rho \Delta_T P_n \equiv \epsilon_{\rho \beta \gamma} \Delta^{\alpha T} P^\alpha n^\gamma. \]
functions in terms of the leading twist one, which can be done using the specific version of the rotational invariance.

The $n$-independence of DVCS amplitude. Using the parameterization (3)-(6) we write the expression for (1) in the form

$$T^{(a)}_{\mu \nu} + T^{(b)}_{\mu \nu} = \int dx \text{tr} \left\{ E_{\mu \nu}(x) \hat{P} \right\} H_1(x, \xi) + \int dx \text{tr} \left\{ E_{\mu \nu}(x) \hat{\Delta}^T \right\} H_3(x, \xi) + \int dx \text{tr} \left\{ E_{\mu \nu}(x) \gamma^\rho \gamma_5 \right\} iH_A(x, \xi) \epsilon_{\rho \Delta x} P_n + \int dx_1 dx_2 \text{tr} \left\{ E_{\mu \nu}(x_1, x_2) \hat{P} \right\} H^T_1(x_2, \xi) \delta(x_1 - x_2) \Delta_\rho + \int dx_1 dx_2 \text{tr} \left\{ E_{\mu \nu}(x_1, x_2) \hat{P} \gamma_5 \right\} iH^T_A(x_2, \xi) \delta(x_1 - x_2) \epsilon_{\rho \Delta x} P_n. \quad (9)$$

The gauge invariance requires that the physical amplitudes are independent on the gauge-fixing parameter which, in our case, is the light-cone vector $n$. As it is also used to define the light-cone direction, the same is required by Lorentz invariance, which should be implemented keeping the constraints ($n^2 = 0, P \cdot n = 1$). We are therefore dealing here with a particular subgroup of the Lorentz group. The relevant symmetry may be called generalized rotational invariance, to distinguish with the standard rotational invariance in a particular rest frame, and to stress its specific form in the case of generalized parton distribution, to be discussed below.

One should recall that the Burkhardt-Cottingham sum rule, known to be intimately related to rotational invariance, follows from the WW approximation for polarized DIS, and may be derived from $n$ independence [12, 10]. Note that it is rather Lorentz than gauge invariance which is crucial here, as one may consider the covariant gauge, where role of $n$ as a gauge-fixing vector is absent [3]. Formally, this invariance can be written as

$$T_{\mu \nu}[n + \delta n] = T_{\mu \nu}[n], \quad (10)$$

or, equivalently,

$$\frac{d}{dn_\beta} \left\{ T^{(a)}_{\mu \nu} + T^{(b)}_{\mu \nu} \right\} = 0, \quad (11)$$

Note the essential difference with the (unpolarized) DIS case [3], where dependence on $n$ was coming only through the explicit dependence of the appearing tensor

\[3\] The new terms, proportional to $A \cdot n$ are absorbed to the standard P-exponent of the leading twist operators in this case [13].
structures. Here, it comes also through the $\xi$ dependence of the matrix elements, so that

$$
\frac{d}{dn_\beta} = \frac{\partial}{\partial n_\beta} - \frac{\Delta^2}{2} \frac{\partial}{\partial \xi}.
$$

(12)

The second term does not appear also in the polarized DIS case [12] (where the role of $\Delta$ is played by the polarization vector $S$), as the dependence of parton distributions and correlations on $(S \cdot n)$, being the analog of $\xi$, is absent due to the general property of the linearity of density matrix. It is this extra term, specific to the case of generalized parton distributions, which is one of the reasons to use the term 'generalized' for this particular version of rotational invariance.

Note that the relation (11) is in some sense analogous to the renormalization group equation governing the dependence of hadronic matrix elements on the factorization scale. Indeed, the factorization scale $\mu$ is the unphysical auxiliary parameter required to write the physical amplitude in the factorized form. Analogously, $n$ is the auxiliary vector, required to specify the infinite-momentum frame. The dependence of the observable quantities on both these parameters, $\mu$ and $n$, must vanish.

Using the Ward identity, the computation of the derivative in (11) leads to the following equation:

$$
\frac{d}{dn_\beta} \left\{ T^{(a)}_{\mu\nu} + T^{(b)}_{\mu\nu} \right\} = -\frac{1}{2} \int dx_1 \int dx_2 \text{tr} \left\{ E_{\mu\nu}(x_1) \hat{P} \right\} \frac{\partial H_1(x, \xi)}{\partial \xi} \Delta_\beta - \\
\int dx_1 \int dx_2 \text{tr} \left\{ E_{\mu\nu}(x_1) \hat{P} \right\} \Delta_\beta H_3(x, \xi) + \int dx_1 \int dx_2 \text{tr} \left\{ E_{\mu\nu}(x_1) \gamma^\rho \gamma_5 \right\} iH_A(x, \xi) \epsilon_\rho \Delta^\tau P_\beta - \\
\int dx_1 dx_2 \text{tr} \left\{ (E_{\mu\nu}(x_1) - E_{\mu\nu}(x_2)) \hat{P} \right\} \frac{H_1^T(x_2, \xi) \delta(x_1 - x_2)}{x_1 - x_2} \Delta_\beta + \\
\int dx_1 dx_2 \text{tr} \left\{ (E_{\mu\nu}(x_1) - E_{\mu\nu}(x_2)) \gamma^\rho \gamma_5 \right\} iH_A^T(x_2, \xi) \delta(x_1 - x_2) \epsilon_\rho \Delta^\tau P_\beta = 0.
$$

(13)

Note that we actually did not use the specific form of the hard scattering amplitude, so this relation should be valid for any hard process. Requiring the validity of this equation for arbitrary $E_{\mu\nu}$, we can easily derive the $n$-independence condition in the more simple form:

$$
\frac{\partial H_1^T(x, \xi)}{\partial x} = \frac{1}{2} \frac{\partial H_1(x, \xi)}{\partial \xi} + H_3(x, \xi), \\
\frac{\partial H_A^T(x, \xi)}{\partial x} = H_A(x, \xi).
$$

(14)
QCD equations of motion for matrix elements. Following [4], we write the QCD equations of motion

\[
\int_{-1}^{1} dy \left( B^{(A)}(x, y) - D^{(S)}(x, y) - \delta(x - y)H_A^T(y) \right) = -\xi H_3(x) - \frac{1}{2} H_1(x) - x H_A(x),
\]

\[
\int_{-1}^{1} dy \left( B^{(S)}(x, y) + \delta(x - y)H_1^T(y) - D^{(A)}(x, y) \right) = x H_3(x) + \xi H_A(x),
\]  \hspace{1cm} (15)

where symmetrical and anti-symmetrical functions are defined as,

\[
B^{(S,A)}(x, y) = \frac{1}{2} \left( B(x, y) \pm B(y, x) \right). \hspace{1cm} (16)
\]

Within the WW approximation they can been rewritten as

\[
H_A^T(x, \xi) = \xi H_3(x, \xi) + \frac{1}{2} H_1(x, \xi) + x H_A(x, \xi),
\]

\[
H_1^T(x, \xi) = x H_3(x, \xi) + \xi H_A(x, \xi). \hspace{1cm} (17)
\]

Note that these equations of course contain the so-called total derivatives, crucial for the derivation of WW relations [3, 5, 6].

The WW relations for DVCS process. Calculating the derivative of (17) with respect to \(x\) and using (14), we obtain the system of differential equations:

\[
\frac{\partial H_+}{\partial x}(x, \xi) = \frac{1}{2} \frac{\partial H_1}{\partial x}(x, \xi), \hspace{1cm} (18)
\]

where we introduced the notations:

\[
\partial_\pm H_1(x, \xi) = \frac{\partial H_1(x, \xi)}{\partial \xi} \pm \frac{\partial H_1(x, \xi)}{\partial x},
\]

\[
H_\pm(x, \xi) = H_3(x, \xi) \pm H_A(x, \xi). \hspace{1cm} (19)
\]

Taking into account the boundary conditions

\[
H_\pm(1, \xi) = H_\pm(-1, \xi) = 0, \hspace{1cm} (20)
\]
solution of the system (18) can be easily found:

\[
H^W_+(x, \xi) = -\frac{1}{2}\Theta(x > -\xi) \int x \frac{\partial_- H_1(y, \xi)}{y + \xi} + \frac{1}{2}\Theta(x < -\xi) \int_{-1}^{x} \frac{\partial_- H_1(y, \xi)}{y + \xi},
\]

\[
H^W_-(x, \xi) = -\frac{1}{2}\Theta(x > \xi) \int x \frac{\partial_+ H_1(y, \xi)}{y - \xi} + \frac{1}{2}\Theta(x < \xi) \int_{-1}^{x} \frac{\partial_+ H_1(y, \xi)}{y - \xi}.
\]

(21)

The theta-functions appearance is due to the fact, that we are using the different boundary conditions below and above the singular points \(x = \pm \xi\). Expressing the \(H^W_3\) functions by making use of (19), we have:

\[
H^W_3(x, \xi) = -\frac{1}{4}\Theta(x > -\xi) \int x \frac{\partial_- H_1(y, \xi)}{y + \xi} - \frac{1}{4}\Theta(x > \xi) \int x \frac{\partial_+ H_1(y, \xi)}{y - \xi} +
\]

\[
\frac{1}{4}\Theta(x < -\xi) \int_{-1}^{x} \frac{\partial_- H_1(y, \xi)}{y + \xi} + \frac{1}{4}\Theta(x < \xi) \int_{-1}^{x} \frac{\partial_+ H_1(y, \xi)}{y - \xi},
\]

(22)

\[
H^W_A(x, \xi) = -\frac{1}{4}\Theta(x > -\xi) \int x \frac{\partial_- H_1(y, \xi)}{y + \xi} + \frac{1}{4}\Theta(x > \xi) \int x \frac{\partial_+ H_1(y, \xi)}{y - \xi} +
\]

\[
\frac{1}{4}\Theta(x < -\xi) \int_{-1}^{x} \frac{\partial_- H_1(y, \xi)}{y + \xi} - \frac{1}{4}\Theta(x < \xi) \int_{-1}^{x} \frac{\partial_+ H_1(y, \xi)}{y - \xi}.
\]

Note that a given solution completely coincide with the expressions presented in ref. [8], although our representation for \(H^W_3\) looks a bit more simple.

Using these results, we can now decompose the \(H^W_3\) functions to the WW type part and genuine part arising from the three-particles correlators. Using (17) and (21), we obtain the following

\[
H_3(x, \xi) = H^W_3(x, \xi) + \frac{1}{2(x - \xi)} \int_{-1}^{1} dy \left( B(x, y, \xi) - D(x, y, \xi) \right) +
\]

\[
\frac{1}{2(x + \xi)} \int_{-1}^{1} dy \left( B(x, y, -\xi) - D(x, y, -\xi) \right)
\]

(23)

and

\[
H_A(x, \xi) = H^W_A(x, \xi) - \frac{1}{2(x - \xi)} \int_{-1}^{1} dy \left( B(x, y, \xi) - D(x, y, \xi) \right) +
\]
\[ \frac{1}{2(x + \xi)} \int_{-1}^{1} dy \left( B(x, y, -\xi) - D(x, y, -\xi) \right). \]  \hspace{1cm} (24)

Here we take into account the properties of \( B- \) and \( D- \) functions following from the \( T\)-invariance, \textit{i.e.}

\[ B(x, y, \xi) = B(y, x, -\xi), \quad D(x, y, \xi) = -D(y, x, -\xi). \]  \hspace{1cm} (25)

We can see that the functions \( H^+ \) and \( H^- \) (both in WW approximation and beyond) possess the singularity at points \(-\xi\) and \(+\xi\), respectively. Consequently, as it was mentioned in \[8\] and confirmed in \[7\], the functions \( H_3 \) and \( H_A \) have finite jumps in these points. One can add here, that if the function \( H_1(\xi, \xi) \) is finite, this singularity is in fact logarithmical, and the mentioned jump means nothing more than the finite difference between two logarithmically growing values \( H_{3, A}(x \pm \epsilon) \).

From the other side, in the DVCS twist-3 amplitude we are dealing with the following combinations of \( H^+ \) and \( H^- \) \[4\],\[8\] :

\[
\Delta^T_\nu \left( 3\xi P_\mu + Q_\mu \right) \left( H_3(x, \xi) + \frac{\xi}{x} H_A(x, \xi) \right),
\]  \hspace{1cm} (26)

which reduce to the \( H^+ \)-structure at \( x = \xi \) and \( H^- \)-structure at \( x = -\xi \), and

\[
\Delta^T_\mu \left( \xi P_\nu + Q_\nu \right) \left( H_3(x, \xi) - \frac{\xi}{x} H_A(x, \xi) \right),
\]  \hspace{1cm} (27)

which reduces to the \( H^- \)-structure at \( x = \xi \) and \( H^+ \)-structure at \( x = -\xi \).

In (26) and (27) the indices \( \mu \) and \( \nu \) belong the virtual and real photons, respectively. Therefore the DVCS amplitude with the longitudinal polarization of virtual photon (26) is free from the divergences. At the same time, in the case of the transverse polarized virtual photon \( (27) \) the contribution from twist-3 amplitude to the observables is suppressed as \( 1/Q^2 \) order after the contraction with the polarization vector \[4\],\[8\].

\( \gamma\gamma^* \to \pi\pi \) process. We are now passing to the consideration of \( \gamma(q')\gamma^*(q) \to \pi(p')\pi(p) \) process, adopting the new notations for \( P = p + p' = q + q' \) and \( \Delta = p' - p = -\xi P + \Delta_T \). Acting as previously, we can derive the relation which is the result of independence on the choice of the gauge vector \( n \). All the stages of derivation are the same up to the parameterization of the relevant matrix elements, known as Generalized Distribution Amplitudes (GDA) \[13\]. We are suggesting here the natural set of twist-3 GDA consisting of the following ingredients:

\[
\langle p, p'|\bar{\psi}(0)\gamma_\mu\psi(\lambda n)|0\rangle = \int_0^1 dx e^{ix} \left\{ \tilde{H}_1(x; \xi, s) P_\mu + \tilde{H}_3(x; \xi, s) \Delta^T_\mu \right\}, \]  \hspace{1cm} (28)
\begin{equation}
\langle p, p' | \bar{\psi}(0) \gamma_5 \gamma_\mu \psi(\lambda n) | 0 \rangle = \int_0^1 dx e^{i\lambda x} \tilde{H}_A(x; \xi, s) \varepsilon_\mu \Delta^T p_n, \quad (29)
\end{equation}

\begin{equation}
\langle p, p' | \bar{\psi}(0) \gamma_\mu i \overset{\leftrightarrow}{\partial}_\rho \psi(\lambda n) | 0 \rangle = \int_0^1 dx e^{i\lambda x} \tilde{H}^T_1(x; \xi, s) P_\mu \Delta^T_\rho, \quad (30)
\end{equation}

\begin{equation}
\langle p, p' | \bar{\psi}(0) \gamma_5 \gamma_\mu i \overset{\leftrightarrow}{\partial}_\rho \psi(\lambda n) | 0 \rangle = \int_0^1 dx e^{i\lambda x} \tilde{H}^T_A(x; \xi, s) P_\mu \varepsilon_\rho \Delta^T p_n, \quad (31)
\end{equation}

\begin{equation}
\langle p, p' | \bar{\psi}(0) \gamma_\mu g A^T_\rho (\lambda_2 n) \psi(\lambda_1 n) | 0 \rangle = \int_0^1 dx_1 dx_2 e^{i \lambda_1^1 + i(x_2-x_1)\lambda_2} \tilde{B}(x_1, x_2) P_\mu \Delta^T_\rho, \quad (32)
\end{equation}

\begin{equation}
\langle p, p' | \bar{\psi}(0) \gamma_5 \gamma_\mu g A^T_\rho (\lambda_2 n) \psi(\lambda_1 n) | 0 \rangle = \int_0^1 dx_1 dx_2 e^{i \lambda_1^1 + i(x_2-x_1)\lambda_2} \tilde{D}(x_1, x_2) P_\mu \varepsilon_\rho \Delta^T p_n. \quad (33)
\end{equation}

In terms of these parameterizations, the \( n \)-independence condition for a given case reads as

\begin{equation}
\frac{\partial \tilde{H}^T(x, \xi)}{\partial x} = \frac{\partial \tilde{H}_1(x, \xi)}{\partial \xi} + \tilde{H}_3(x, \xi),
\end{equation}

while the QCD equations of motion take the following form

\begin{equation}
\int_0^1 dy \left( \tilde{B}^{(S)}(x, y) + \delta(x-y) \tilde{H}_A^T(y) - \tilde{B}^{(A)}(x, y) \right) = \frac{1}{2} \tilde{H}_3(x, \xi) + \left( x - \frac{1}{2} \right) \tilde{H}_A(x, \xi),
\end{equation}

\begin{equation}
\frac{\partial \tilde{H}^T(x, \xi)}{\partial x} = \tilde{H}_A(x, \xi), \quad (34)
\end{equation}

\begin{equation}
\int_0^1 dy \left( \tilde{B}^{(S)}(x, y) + \delta(x-y) \tilde{H}_1^T(y) - \tilde{D}^{(A)}(x, y) \right) = \left( x - \frac{1}{2} \right) \tilde{H}_3(x, \xi) + \frac{1}{2} \tilde{H}_A(x, \xi),
\end{equation}

so that within the WW approximation we have:

\begin{equation}
\tilde{H}_A^T(x, \xi) = \frac{1}{2} \tilde{H}_3(x, \xi) + \left( x - \frac{1}{2} \right) \tilde{H}_A(x, \xi),
\end{equation}

\begin{equation}
\tilde{H}_1^T(x, \xi) = \left( x - \frac{1}{2} \right) \tilde{H}_3(x, \xi) + \frac{1}{2} \tilde{H}_A(x, \xi). \quad (35)
\end{equation}
Differentiating (36) with respect to $x$ and taking into account (34), we can reduce the system of equations (34) and (36) to the following form

$$\frac{\partial \tilde{H}_+ (x, \xi)}{\partial x} = \frac{1}{x} \frac{\partial \tilde{H}_1 (x, \xi)}{\partial \xi},$$

$$\frac{\partial \tilde{H}_- (x, \xi)}{\partial x} = \frac{1}{(x-1)} \frac{\partial \tilde{H}_1 (x, \xi)}{\partial \xi},$$

(37)

Note that we cannot now always implement zero boundary conditions for both functions due to the singularities in the r.h.s. The only possible zero boundary conditions are:

$$\tilde{H}_+ (1, \xi) = 0,$$

$$\tilde{H}_- (0, \xi) = 0.$$  

(38)

It can be easily found that the solution takes the form

$$\tilde{H}_+^{WW} (x, \xi) = - \int_x^1 dy \frac{\partial \xi \tilde{H}_1 (y, \xi)}{y},$$

$$\tilde{H}_-^{WW} (x, \xi) = \int_0^x dy \frac{\partial \xi \tilde{H}_1 (y, \xi)}{y - 1},$$

(39) and (40)

or, for $\tilde{H}_3^{WW} -$ functions:

$$\tilde{H}_3^{WW} (x, \xi) = \frac{1}{2} \int_0^x dy \frac{\partial \xi \tilde{H}_1 (y, \xi)}{y - 1} - \frac{1}{2} \int_x^1 dy \frac{\partial \xi \tilde{H}_1 (y, \xi)}{y},$$

$$\tilde{H}_A^{WW} (x, \xi) = - \frac{1}{2} \int_0^x dy \frac{\partial \xi \tilde{H}_1 (y, \xi)}{y - 1} - \frac{1}{2} \int_x^1 dy \frac{\partial \xi \tilde{H}_1 (y, \xi)}{y}.$$  

(41) and (42)

These expressions are analogous to WW relations for the transverse polarized $\rho$-meson light-cone distribution derived by Ball and Braun [13]. This is not surprising, as the momentum $\Delta_T$ just plays the role of the polarization vector $e_T$. In particular, the structures $H_+^{WW}$ and $H_-^{WW}$ correspond to the distributions with a definite (anti)quark spin projections[13], whose zero endpoint values result from their conformal properties.

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4 $\tilde{H}_+^{WW} -$ functions are defined in complete analogy with $H_+^{WW} -$ functions, see [13]

5 here $\partial \xi = \partial / \partial \xi$
We can now easily obtain the decomposition the $\tilde{H}_{3(A)}$—functions on the WW type contribution and the three-particles contribution, as it was done above for the DVCS case. We have

$$\tilde{H}_3(x, \xi) = \tilde{H}^{WW}_3(x, \xi) - \frac{1}{2(1-x)} \int_0^1 dy \left( \tilde{B}(x, y, \xi) - \tilde{D}(x, y, \xi) \right) + \frac{1}{2x} \int_0^1 dy \left( \tilde{B}(x, y, \xi) + \tilde{D}(x, y, \xi) \right)$$

(43)

and

$$\tilde{H}_A(x, \xi) = \tilde{H}^{WW}_A(x, \xi) + \frac{1}{2(1-x)} \int_0^1 dy \left( \tilde{B}(x, y, \xi) - \tilde{D}(x, y, \xi) \right) + \frac{1}{2x} \int_0^1 dy \left( \tilde{B}(x, y, \xi) + \tilde{D}(x, y, \xi) \right).$$

(44)

In (43) and (44), we used the properties of $C$-invariance which are the following

$$\tilde{B}(x, y, \xi) = \tilde{B}(y, x, \xi), \quad \tilde{D}(x, y, \xi) = \tilde{D}(y, x, \xi).$$

(45)

We here stress again that in both cases, i.e. in the genuine expressions (43), (44) and WW approximation (41), (42), the functions $\tilde{H}_+$ and $\tilde{H}_-$ have, generally speaking, singularities at the points $x = 0$ and $x = 1$, respectively. However, if one assumes zero boundary condition for function $\tilde{H}_1$, required by the factorization at the leading twist level, these singularities are transformed to the non-zero finite limits at these points. They are the counterparts of the $x = \pm \xi$ singularities in the DVCS case. Anyway, these finite values would be sufficient to spoil factorization after the contraction with the short-distance subprocess cross-section. As in the case DVCS process, this effect can not affect the amplitude contracted with the polarization vector of the real photon.

Indeed, we can derive the expression for the $\gamma\gamma^* \rightarrow \pi\pi$ amplitude using the developed approach [4]. It reads

$$T^{\gamma\gamma^*}_{\mu\nu} = -\frac{1}{q^2} \int_0^1 \frac{dx}{x(1-x)} T^{\gamma\gamma^*}_{\mu\nu},$$

(46)

where

$$T^{\gamma\gamma^*}_{\mu\nu} = -q^2 g_{\mu\nu} \left( 2x - 1 \right) \tilde{H}_1(x, \xi) +$$
\[ \Delta^T_\nu (P + q')_\mu \left( (2x - 1)\tilde{H}_3(x, \xi) + \tilde{H}_A(x, \xi) \right) + \\
\Delta^T_\mu q'_\nu \left( (2x - 1)\tilde{H}_3(x, \xi) - \tilde{H}_A(x, \xi) \right). \]

Note that, contrary to DVCS amplitude case, the contribution of \( H_1 \) is gauge invariant by itself. This is due to the fact, that parameterization of (anti)quark momenta in terms of the vector \( P \) makes both of them collinear and independent on \( \Delta_T \), so that one is dealing with the physical on-shell amplitude, which is gauge invariant. At the same time, only the total contributions of new quark and quark-gluon matrix elements is gauge invariant provided the equations of motion are taken into account.

The contribution for the longitudinally polarized photon arises from the term

\[ \Delta^T_\nu (P + q')_\mu \left( (2x - 1)\tilde{H}_3(x, \xi) + \tilde{H}_A(x, \xi) \right), \quad (47) \]

which is described by the \( \tilde{H}_- \)-structure at \( x = 0 \) and by \( \tilde{H}_+ \)-structure at \( x = 1 \).

Let us recall that at these points \( \tilde{H}_- \) and \( \tilde{H}_+ \) functions are finite. The respective contribution to the amplitude reads as

\[ T^{(1)}_{\mu\nu\gamma\gamma^*} = -\frac{1}{q^2} \int_0^1 \frac{dx}{x(1-x)} \Delta^T_\nu (P + q')_\mu \left( (2x - 1)\tilde{H}_3(x, \xi) + \tilde{H}_A(x, \xi) \right). \quad (48) \]

Using (41) and (42), we substitute the \( \tilde{H}_3^{WW} \) and \( \tilde{H}_A^{WW} \) functions in Eq. (48) which thereafter can be rewritten in the form

\[ T^{(1)}_{\mu\nu\gamma\gamma^*} = -\frac{\Delta^T_\nu (P + q')_\mu}{Q^2} \frac{1}{\partial_\xi} \int_0^1 dx \left( \frac{J_2(x, \xi)}{1-x} + \frac{J_1(x, \xi)}{x} \right), \quad (49) \]

where

\[ J_1(x, \xi) = \int_0^x dy \frac{\tilde{H}_1(y, \xi)}{y - 1}, \quad J_2(x, \xi) = \int_x^1 dy \frac{\tilde{H}_1(y, \xi)}{y}. \quad (50) \]

After simple manipulations, we derive the following expression

\[ T^{(1)}_{\mu\nu\gamma\gamma^*} = \frac{\Delta^T_\nu (P + q')_\mu}{Q^2} \int_0^1 dy \partial_\xi \tilde{H}_1(y, \xi) \left( \log \left( \frac{1-y}{y} \right) - \log \frac{y}{1-y} \right), \quad (51) \]

which coincides with the expression in [14]. At the same time, the genuine twist-3 contribution is new.

**Conclusions.** In this paper we have presented the new derivation of the WW relations between the parameterizing twist-2 and kinematical twist-3 functions. This
simple approach is based on the use of the equations of motion and on the requirement of independence of the amplitudes from the gauge-fixing light-cone vector.

We have found the explicit expression for the genuine twist-3 functions for DVCS and $\gamma\gamma^* \rightarrow 2\pi$ processes. We also have computed the amplitude of $\gamma^*\gamma \rightarrow \pi\pi$ including complete twist-3 corrections.

The counterparts of the singularities appearing in the DVCS case are provided by the finite values of generalized distribution amplitudes in the endpoints, which are related to their conformal properties. Therefore, one may hope that appearance of the singularities, as well as their cancellation in the physical amplitudes, have some general reasons.

We have also shown that the contributions from the three-particles correlators possess the same singularities as the WW parts which also do not contribute to the physical amplitudes at the twist-3 level.

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