Super-quantum discord in ferromagnetic and antiferromagnetic materials

A. V. Fedorova · Tim Byrnes · Alexey N. Pyrkov

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Abstract
Super-quantum discord is an extension of the quantum discord concept where weak measurements are made instead of projective measurements. We study the temperature behavior of the super-quantum discord for two real magnetic materials. We extract information about super-quantum discord from the magnetic susceptibility data for iron nitrosyl complexes Fe$_2$(SC$_3$H$_5$N$_2$)$_2$(NO)$_4$ and binuclear Cu(II) acetate complex [Cu$_2$L(OAc)] · 6H$_2$O, where L is a ligand that allows us to compare the super-quantum discord with the standard one. The dependence of the super-quantum discord on the parameter describing weak measurements is studied. Obtained difference between super-quantum and quantum discords confirms the detection of additional quantum correlations that are usually destroyed during projective measurements. The use of the approach allows to predict quantitatively in advance the advantages of use of weak measurements versus projective one for quantum technologies in real settings where quantum discord is used as resource. This can be relevant particularly in macroscopic quantum systems where weak measurements can be used to extract information about the system.

Keywords Quantum correlations · Magnetic susceptibility · Discord · Super-quantum discord · Weak measurements · Macroscopic quantum systems
1 Introduction

The presence of quantum correlations [1–9] allows for the possibility of superior performance of quantum devices over their classical counterparts [10,11]. In particular, quantum correlations play a major role in quantum computation [12–14], quantum cryptography [15–18] and quantum metrology [19–22], which demonstrate advantages in their performance over classical methods. Development of quantum technologies is also fundamental for realization of quantum communications lines and networks [23–27], as well as quantum simulation for development of novel materials. Such materials should give opportunities to maintain and certify quantum correlations, and also contribute to the creation of materials for novel quantum devices. In the early stages of the field of quantum technologies, it was believed that entanglement was the necessary resource for a quantum advantage, and many different efforts were developed to calculate entanglement [1–4,28–31]. However, it now better appreciated that not all quantum correlations can be associated with entanglement, and other concepts to measure quantum correlation were introduced [32,33]. One of the most well-known quantifiers of quantum correlations is quantum discord which allows one to distinguish quantum correlations from classical correlations with the use of an optimization over all possible projective measurements [33,34].

Projective measurements are a particular type of quantum measurements which cause the collapse of the wave function, removing quantum coherence by turning the state into a probabilistic mixture. For local projective measurements defined on subsystems, the quantum correlations are destroyed between the subsystems [35]; for ‘global’ projective measurements in the basis of entangled states, quantum correlations can be produced [36–42]. These are, however, not the only type of quantum measurement that is possible. To reduce the influence of measurement on a system, a measurement that creates only a partial collapse (destruction) of a quantum state can be produced [43,44]. This kind of measurement is called a weak measurement and allows one to preserve more quantum information after the measurement [45,46]. A scheme for weak measurements was introduced by Aharonov, Albert and Vaidman in 1998 [44]. First experiments were performed in 2006 [43,47], and the possibility of measuring a quantum state without destroying it completely was demonstrated. It allows for more quantum correlations to be preserved in the system after the weak measurement than with projective measurements.

Recently, there has been much interest in realizing weak measurements in condensed matter systems [48–50]. However, from resource point of view, it is not clear how to estimate the advantages of weak measurements quantitatively for quantum technologies. Furthermore, it is not clear whether it is possible to predict quantitatively advantages of weak measurements from a resource point of view for some real materials before realization the weak measurement.

In 2014, Singh and Pati generalized the concept of quantum discord in order to estimate the magnitude of quantum correlations during weak measurements [51]. They called the measure “super-quantum discord” and showed that it can be used for calculating quantum correlations after a sequence of weak measurements. Super-quantum discord differs from the quantum discord—it exceeds the quantum discord and becomes zero only for factorized states [52], where the density matrix can be...
represented as a tensor product of the corresponding subsystems density matrices. Recently, the dependence of the quantum discord on temperature in antiferromagnetic copper nitrate Cu(NO₃)₂ · 2.5H₂O and iron nitrosyl complexes was investigated [53–55]. Here, we quantitatively investigate the difference between standard discord and super-quantum discord for two real magnetic materials with the use of standard data on magnetic susceptibility. We investigate the dependence of the super-quantum discord on temperature and the parameters describing weak measurements for antiferromagnetic iron nitrosyl complexes Fe₂(SC₃H₅N₂)₂(NO)₄ [56] and for binuclear ferromagnetic copper acetate complex [Cu₂L(OAc)] · 6H₂O in order to show the boost in quantum correlations by using a weak measurement. For these two examples of materials, it is shown that for one of them the use of weak measurements presents an excellent improvement from a resource point of view, while for the other compound the improvement is rather minor. Thus, this approach allows one to estimate in advance the parameters of weak measurements needed for advantages in the context of quantum technologies where quantum discord is used as resource. It can provide perspectives in calculating quantum discord, especially between macroscopic qubits [57–64] where weak measurements can also be used to effectively characterize the system [65].

2 Discord and super-discord

The mutual information of two subsystems is a well-known measure of correlations in a bipartite system [13,32,33]. In quantum information theory, the quantum mutual information of a bipartite system is defined as follows [13,33]:

\[ I(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}), \]  

(1)

where \( \rho_A \) and \( \rho_B \) are the reduced density matrices of A and B subsystems, \( \rho_{AB} \) is the system matrix, and \( S(\rho) \) is the entropy. In quantum information theory, \( I(\rho_{AB}) \) is a measure of both classical and quantum correlations between subsystems. The separation of classical and quantum correlations contribution into \( I(\rho_{AB}) \) is complicated by the fact that in the quantum system measurements carried out on one subsystem can affect the another. The problem of separating classical and quantum correlations in a bipartite system was solved in Refs. [13] and [33]. For example, classical correlations can be defined as the maximum information about subsystem A, which can be extracted by performing a complete set of subsystem B projective measurements. Henderson and Vedral proposed to determine the classical correlations according to the formula [13]:

\[ C(\rho_{AB}) = \max_{\{\Pi_i\}} \left[ S(\rho_A) - \sum_i p_i S(\rho_{A|\Pi_i}) \right], \]  

(2)

where \( \{\Pi_i\} \) is the set of all projection-valued measurements performed on subsystem B, \( \rho_{A|\Pi_i} \) is the subsystem A reduced density matrix, while the result of measurement
performed on subsystem $B$ is $i$:

$$p_i \rho_A^i = \text{Tr}_B \{ \Pi_i \rho_{AB} \Pi_i^+ \}$$  \hspace{1cm} (3)

and $p_i = \text{Tr}_{AB} \{ \Pi_i \rho_{AB} \Pi_i^+ \}$ is the probability of the result $i$. Since the correlations in the system are determined by the mutual information (1), the quantum discord, which determines the quantum correlations in the bipartite system, will be the difference between the mutual information (1) and the classical correlations (2) [66]:

$$D = I(\rho_{AB}) - C(\rho_{AB}).$$  \hspace{1cm} (4)

Now let us consider the case that weak measurements are made. The projective operators must be replaced by the following [46]:

$$P(x) = \sqrt{\frac{1 - \tanh(x)}{2}} \Pi_+ + \sqrt{\frac{1 + \tanh(x)}{2}} \Pi_-,$$

$$P(-x) = \sqrt{\frac{1 + \tanh(x)}{2}} \Pi_+ + \sqrt{\frac{1 - \tanh(x)}{2}} \Pi_-,$$

where $x$ is the measurement strength parameter, and $\Pi_{\pm}$ are two orthogonal projectors with $\Pi_+ + \Pi_- = 1$. In the limit when $x \to \infty$, we get projective measurements. For instance in NMR, where the weak measurements are realized via periodical coupling a nuclear spin to electronic spin, the measurement strength parameter is the interaction strength between the nuclear and electronic spins controlled by a dynamical decoupling sequence applied to the meter spin [67,68]. In this case, it is possible to tune smoothly the measurement strength, or turn it off completely, by varying the interaction time [48,49]. The weak measurement operators satisfy the following properties:

1. $P^+(x) P(x) + P^+(-x) P(-x) = 1$.
2. $P(0) = \frac{1}{\sqrt{2}}$.
3. $\lim_{x \to \infty} P(-x) = \Pi_+$; $\lim_{x \to \infty} P(x) = \Pi_-$.

The measure of quantum correlations for weak measurements (super-quantum discord) is similar to the standard expression for quantum discord with projective measurements. In the case of weak measurements, Eq. (2) is then replaced by

$$C_w(\rho_{AB}) = \max_{\{\Pi_i\}} \left[ S(\rho_A) - \sum_{y=\pm x,-x} p(y) S(\rho_A|_{\rho_B^y}) \right],$$  \hspace{1cm} (5)

where

$$\rho_A|_{\rho_B^y} = \frac{\text{Tr}_A[[I \otimes P_B^y(\rho_{AB})] \rho_{AB} (I \otimes P_B^y)]]}{\text{Tr}_{AB}[[I \otimes P_B^y(\rho_{AB})] \rho_{AB} (I \otimes P_B^y)]},$$  \hspace{1cm} (6)
where
\[ p(\pm x) = \text{Tr}_{AB}[\{(I \otimes P^B(\pm x))\rho_{AB}(I \otimes P^B(\pm x))\}] \]

Here, \( p(\pm x) \) is the probability of a state after a weak measurement. Thus, the super-quantum discord can be represented similarly to the normal discord (4):
\[ D_w = I(\rho_{AB}) - C_w(\rho_{AB}) \] (7)

3 Comparison of standard and super-quantum discords for real magnetic complexes

We study the temperature dependence of the super-quantum discord from 5 to 300 K for antiferromagnetic iron nitrosyl complexes \( \text{Fe}_2(\text{SC}_3\text{H}_5\text{N}_2)_2(\text{NO})_4 \) [54] and for binuclear ferromagnetic copper acetate complex \([\text{Cu}_2\text{L}(\text{OAc})] \cdot 6\text{H}_2\text{O}\), where \( \text{H}_3\text{L} = 2-(2\text{-hydroxyphenyl})\text{-1,3-bis[4-(2-hydroxyphenyl)-3-azabut-3-enyl]}\text{-1,3-imidazolidine} \) [53,69]. In these materials, the exchange interaction can be described in the frame of Heisenberg model of the magnetic dimer [54,69]:
\[ H = -\frac{1}{2} J \sigma_1 \sigma_2, \]

where \( J \) is the exchange constant, \( \sigma_1 = \sigma \otimes I \) and \( \sigma_2 = I \otimes \sigma \). \( I \) is the unit matrix, and \( \sigma = (\sigma_x, \sigma_y, \sigma_z) \) is the Pauli Matrix vector
\[ \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \] (8)

In the thermal equilibrium state, the density matrix of the system is given by:
\[ \rho = \frac{1}{Z} \exp\left(-\frac{H}{k_B T}\right), \]

where \( Z = \text{Tr}[\exp(-H/k_B T)] \) is the partition function. Since
\[ \sigma_1 \sigma_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \]
then the density matrix of the Heisenberg dimer is given by:
\[ \rho(T) = \frac{1}{Z} \begin{pmatrix} e^L & 0 & 0 & 0 \\ 0 & e^{-L} \cosh(2L) & e^{-L} \sinh(2L) & 0 \\ 0 & e^{-L} \sinh(2L) & e^{-L} \cosh(2L) & 0 \\ 0 & 0 & 0 & e^L \end{pmatrix} \] (9)
with \( Z = 3e^L + e^{-3L} \) and \( L = J/2k_B T \). One can easily verify using simple algebra that this is equivalent to

\[
\rho(T) = \frac{1}{4}(1 + G\sigma_1\sigma_2) = \frac{1}{4}\begin{pmatrix}
1 + G & 0 & 0 & 0 \\
0 & 1 - G & 2G & 0 \\
0 & 2G & 1 - G & 0 \\
0 & 0 & 0 & 1 + G
\end{pmatrix}
\]  \hspace{1cm} (10)

where

\[
G(T) = \frac{4}{3 + \exp(-2J/k_B T)} - 1.
\]  \hspace{1cm} (11)

Furthermore, the \( G \) represents the spin correlation functions that can be check by direct calculations \( G = \langle \sigma_1^x\sigma_2^x \rangle = \langle \sigma_1^y\sigma_2^y \rangle = \langle \sigma_1^z\sigma_2^z \rangle \), where angle brackets denote statistical averaging.

The magnetic susceptibility in this case can be determined by the Bleaney–Bowers equation [54]

\[
\chi(T) = \frac{2N_A g^2 \mu_B^2}{k_B T(3 + \exp(-2J/k_B T))} = \frac{N_A g^2 \mu_B^2}{2k_B T}(1 + G(T)),
\]  \hspace{1cm} (12)

where \( N_A \) is the Avogadro constant, \( k_B \) is the Boltzman constant, \( T \) is the temperature, \( g \) is the g-factor, if the measurement was provided on a single crystals. If the measurement was provided on a polycrystalline sample,

\[
g^2 = \frac{1}{3}(g_x^2 + g_y^2 + g_z^2).
\]

\( g_x, g_y, g_z \) are the component of a g-factor. From Eqs. (12) and (11), the spin correlation function can be expressed in terms of the magnetic susceptibility (12)

\[
G(T) = \frac{2k_B T \chi(T)}{N_A g^2 \mu_B^2} - 1 = \frac{\chi(T)}{\chi_{\text{curie}}} - 1,
\]

where \( \chi_{\text{curie}} \) is the magnetization of the dimer determined by the Curie law. This formula allows the further using of experimental data for magnetic susceptibility for the calculations of the super-quantum discord [71].

The density matrix (10) has a form of an X-state [70,71]. This simplifies the computation of the standard quantum discord (4) and super-quantum discord (7) significantly. In this case, equations in which there is no minimization over all possible complete sets of projective measurements can be used. As a result, the quantum discord for the state (10) is given by [71].

\[
D = \frac{1 + G}{4}\log(1 + G) - \frac{1 - G}{2}\log(1 - G) + \frac{1 - 3G}{4}\log(1 - 3G),
\]  \hspace{1cm} (13)
and the super-quantum discord is given by [71]:

\[
D_w = 1 + \frac{1 - 3G}{4} \log \left( \frac{1 - 3G}{4} \right) + \frac{3(1 + G)}{4} \log \left( \frac{1 + G}{4} \right) - \frac{1 - G \tanh \frac{x}{2}}{2} \log \left( \frac{1 - G \tanh \frac{x}{2}}{2} \right) - \frac{1 + G \tanh \frac{x}{2}}{2} \log \left( \frac{1 + G \tanh \frac{x}{2}}{2} \right).
\]

(14)

The super-quantum discord (14) converges to the normal discord (13) when \(x \to \infty\).

The experimental data on the magnetic susceptibility for iron nitrosyl complexes \(\text{Fe}_2(\text{SC}_3\text{H}_5\text{N}_2)_2(\text{NO})_4\) were obtained from Ref. [56]. For a more careful analysis, the subtraction of contribution of the impurity was done in Ref. [54]. The fit to the Bleany–Bowers equation was obtained with parameters \(J/k_B = -68\, \text{K}, g = 2\). Figure 1 shows that super-quantum discord greatly exceeds quantum correlations determined by the normal discord starting from liquid nitrogen temperatures. Thus, the antiferromagnetic iron nitrosyl complex is of particular interest from resource point of view because the super-quantum discord in this material reaches a value of 2 at low temperatures, which means it reaches the magnitude of mutual information.

In order to calculate super-quantum discord for the binuclear copper(II) acetate complex \([\text{Cu}_2\text{L(OAc)}] \cdot 6\text{H}_2\text{O}\), where \(\text{H}_3\text{L} = 2-(2\text{-hydroxyphenyl})-1,3\text{-bis[4-(2-hydroxyphenyl)-3-azabut-3-enyl]-1,3-imidazolidine}\) we use the magnetic susceptibility data in Ref. [69] for temperatures from 5 to 300 K which can be described with the Bleany–Bowers equation derived from the Heisenberg spin Hamiltonian taking into account the crystal structure of this compound [69]. The best fit was obtained with the parameter \(2J = 49.2\, \text{cm}^{-1} (J/k = 35.4\, \text{K})\) and \(g = 2.13\) [69]. Figure 2 shows the temperature dependence of the super-quantum and standard discords obtained for the different values of parameter \(x\) describing weak measurements. As before for the antiferromagnetic iron nitrosyl complexes, at low temperatures super-quantum discord exceeds quantum correlations as calculated by the standard discord but the difference is not as large as for the antiferromagnetic iron nitrosyl complexes. Thus, this compound can be used as an example where using a weak measurement approach
Fig. 2  The temperature dependence of a super-quantum discord in a binuclear copper acetate complex for various values of the measurement strength parameter. Dots represent the super-quantum discord value, obtained from experimental data for magnetic susceptibility.

is less important. Also we can see that the experimental points are slightly lower than the theoretical predictions for the low temperatures. In the work of Ref. [69], it was shown that attempts to evaluate the possible interdimer interactions led to extremely low values without any significant improvement in the fitting and it is rather difficult to take into account the influence of weak interdimer couplings. Thus, we suggest that the influence of weak interdimer couplings is the reason for the mismatch between the experimental points and theoretical predictions.

4 Conclusions

In this work, we calculated the temperature dependence of super-quantum discord as a function of the measurement strength parameter for iron nitrosyl complex and binuclear copper acetate complex. Super-quantum discord allows to reveal more quantum correlations than standard quantum discord, and for antiferromagnetic iron nitrosyl complexes at small measurement strength parameter values it reaches the magnitude of mutual information. On the other hand, the use of weak measurements is not as beneficial for the binuclear copper acetate complex. Thus, it is shown that it is possible to quantitatively estimate advantages of weak measurement for real materials from a resource point of view in advance using standard magnetic data. Super-quantum discord can be applied for determination the effectiveness of developed materials, with the aim of their further using for implementation of quantum technologies.

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References

1. Amico, L., Fazio, R., Osterloh, A., Vedral, V.: Entanglement in many-body systems. Rev. Mod. Phys. 80, 517 (2008)
2. Streltsov, A., Adesso, G., Plenio, M.: Quantum coherence as a resource. Rev. Mod. Phys. 89, 041003 (2017)
3. Horodecki, R., Horodecki, P., Harodecki, K.: Quantum entanglement. Rev. Mod. Phys. 81, 865 (2009)
4. Bera, A., Mal, S., Sen, A., Sen, U.: Witnessing entanglement sequentially: maximally entangled states are not special. Rep. Prog. Phys. 81, 024001 (2018)
5. Bose, S.: Quantum communication through spin chain dynamics: an introductory overview. Contemp. Phys. 48, 13 (2007)
6. Venuti, L.C., Boschi, C.D.E., Roncaglia, M.: Long-distance entanglement in spin systems. Phys. Rev. Lett. 96, 247206 (2006)
7. Doronin, S.I., Pyrkov, A.N., Fel’dman, E.B.: Entanglement of spin pairs in alternating open spin-1/2 chains with the XY Hamiltonian. J. Exp. Theor. Phys. 105, 953 (2007)
8. Sahling, S., et al.: Experimental realization of long-distance entanglement between spins in antiferromagnetic quantum spin chains. Nat. Phys. 11, 255 (2015)
9. Pyrkov, A.N., Byrnes, T.: Entanglement generation in quantum networks of Bose–Einstein condensates. New J. Phys. 15, 093019 (2013)
10. Nielsen, M.A., Chuang, I.L.: Quantum computation and quantum information. Cambridge University Press, Cambridge (2000)
11. Steane, A.M.: Quantum Computing. Rept. Prog. Phys. 61, 117 (1998)
12. Ekert, A., Jozsa, R.: Quantum computation and Shor’s factoring algorithm. Rev. Mod. Phys. 68, 733 (1996)
13. Henderson, L., Vedral, V.: Classical, quantum and total correlations. J. Phys. A Math. Gen. 34, 6899 (2001)
14. Bennett, C.B., Bernstein, E., Brassard, G., Vazirani, U.: Strengths and weaknesses of quantum computing. SIAM J. Comput. 26(5), 1510 (1994)
15. Bennett, C.H., Brassard, G., Ekert, A.K.: Quantum cryptography. Sci. Am. 267(4), 50 (1992)
16. Ekert, A.: Quantum cryptography based on Bell’s theorem. Phys. Rev. Lett. 67, 661 (1991)
17. Gisin, N., Ribordy, G., Tittel, W., Zbinden, H.: Quantum cryptography. Rev. Mod. Phys. 74, 145 (2002)
18. Bennett, C.H., Brassard, G., Mermin, N.D.: Quantum cryptography without Bell’s theorem. Phys. Rev. Lett. 68, 557 (1992)
19. Layden, D., Cappellaro, P.: Spatial noise filtering through error correction for quantum sensing. npj Quantum Inf. 4, 30 (2018)
20. Knill, E., Laflamme, R.: Theory of quantum error-correcting codes. Phys. Rev. A 55, 900 (1997)
21. Shlyakhov, A.R., Zemlyanov, V.V., Suslov, M.V., Lebedev, A.V., Paraoanu, G.S., Lesovik, G.B., Blatter, G.: Quantum metrology with a transmon qutrit. Phys. Rev. A 97, 022115 (2018)
22. Oreshkov, O., Bruk, T.A.: Weak measurements are universal. Phys. Rev. A 76, 022318 (2007)
23. Bose, S.: Quantum communication through an unmodulated spin chain. Phys. Rev. Lett. 91, 207901 (2003)
24. Christandl, M., Datta, N., Ekert, A., Landahl, A.: Perfect state transfer in quantum spin networks. Phys. Rev. Lett. 92, 187902 (2004)
25. Kuznetsova, E.I., Zenchuk, A.I.: High-probability quantum state transfer in an alternating open spin chain with an XY Hamiltonian. Phys. Lett. A 372(40), 6134 (2008)
26. Doronin, S.I., Fel’dman, E.B., Zenchuk, A.I.: Relationship between probabilities of the state transfers and entanglements in spin systems with simple geometrical configurations. Phys. Rev. A 79, 042310 (2009)
27. Fel’dman, E.B., Kuznetsova, E.I., Zenchuk, A.I.: High-probability state transfer in spin-1/2 chains: analytical and numerical approaches. Phys. Rev. A 82, 022332 (2010)
28. Chernyavskiy, A.Y., Doronin, S.I., Fel’dman, E.B.: Bipartite quantum discord in a multiqubit spin chain. Phys. Scr. T160, 014007 (2014)
29. Wootters, W.K.: Entanglement of formation of an arbitrary state of two qubits. Phys. Rev. Lett. 80, 2245 (1998)
30. Yurishchev, M.A.: NMR dynamics of quantum discord for spin-carrying gas molecules in a closed nanopore. JETP 119, 828 (2014)
31. Ciliberti, L., Rossignoli, R., Canosa, N.: Quantum discord in finite XY chains. Phys. Rev. A 82, 042316 (2010)
32. Modi, K., Broduth, A., Cable, H., Paterek, T., Vedral, V.: The classical-quantum boundary for correlations: discord and related measures. Rev. Mod. Phys. 84, 1655 (2012)
33. Ollivier, H., Zurek, W.H.: Quantum discord: a measure of the quantumness of correlations. Phys. Rev. Lett. 88, 017901 (2001)
34. Datta, A., Shaji, A., Caves, C.M.: Quantum discord and the power of one qubit. Phys. Rev. Lett. 100, 050502 (2008)
35. Zurek, W.H.: Quantum Darwinism. Nat. Phys. 5, 181 (2009)
36. González-Gutiérrez, C.A., Torres, J.M.: Atomic Bell measurement via two-photon interactions. Phys. Rev. A 99, 023854 (2019)
37. Kimble, H.J.: The quantum internet. Nature 453, 1023 (2008)
38. Sangouard, N., et al.: Quantum repeaters based on atomic ensembles and linear optics. Rev. Mod. Phys. 83, 33 (2011)
39. Briegel, H.J., et al.: Quantum repeaters: the role of imperfect local operations in quantum communication. Phys. Rev. Lett. 81, 5932 (1998)
40. Kim, Y.-H., Kulik, S.P., Shih, Y.: Phys. Rev. Lett. 86, 1370 (2001)
41. Schmidt-Kaler, F., et al.: Nature 422, 408 (2003)
42. Zhang, Q., Ruskov, R., Korotkov, A.N.: Continuous quantum feedback of coherent oscillations in a solid-state qubit. Phys. Rev. B 73, 245322 (2005)
43. Aharonov, Y., Albert, D.Z., Vaidman, L.: How the result of a measurement of a component of the spin of a spin-1/2 particle can turn out to be 100. Phys. Rev. Lett. 60, 1351 (1988)
44. Braginsky, V., Vorontsov, Y.I., Thorne, K.S.: Quantum nondemolition measurements. Science 209, 4456, 547-557 (1980)
45. Oreshkov, O., Brun, T.A.: Weak measurements are universal. Phys. Rev. Lett. 95, 110409 (2005)
46. Aldoshin, S.M., Feldman, E.B., Yurishchev, M.A.: Quantum entanglement and quantum discord in magnetoactive materials. Low Temp. Phys. 40, 3 (2014)
47. Byrnes, T., et al.: Macroscopic quantum information processing using spin coherent states. Opt. Commun. 337, 102 (2015)
48. Gross, C.: Spin squeezing, entanglement and quantum metrology with Bose–Einstein condensates. J. Phys. B At. Mol. Opt. Phys. 45, 103001 (2012)
49. Pyrkov, A.N., Byrnes, T.: Full-Bloch-sphere teleportation of spinor Bose–Einstein condensates and spin ensembles. Phys. Rev. A 90, 062336 (2014)
50. Byrnes, T., Wen, K., Yamamoto, Y.: Macroscopic quantum computation using Bose–Einstein condensates. Phys. Rev. A 85, 040306(R) (2012)
62. Pyrkov, A.N., Byrnes, T.: Quantum information transfer between two-component Bose–Einstein condensates connected by optical fiber. Proc. SPIE 8700, 87001E (2013)
63. Kunkel, P., et al.: Spatially distributed multipartite entanglement enables EPR steering of atomic clouds. Science 360, 413 (2018)
64. Fadel, M., et al.: Spatial entanglement patterns and Einstein–Podolsky–Rosen steering in Bose–Einstein condensates. Science 360, 409 (2018)
65. Ilo-Okike, E.O., Byrnes, T.: Theory of single-shot phase contrast imaging in spinor Bose–Einstein condensates. Phys. Rev. Lett. 112, 233602 (2014)
66. Olivier, H., Zurek, W.H.: Quantum discord: a measure of the quantumness of correlations. Phys. Rev. Lett. 88, 017901 (2001)
67. Taminiau, T.H., et al.: Universal control and error correction in multi-qubit spin registers in diamond. Nat. Nano 9, 171 (2014)
68. Boss, J.M., et al.: One- and two-dimensional nuclear magnetic resonance spectroscopy with a diamond quantum sensor. Phys. Rev. Lett 116, 197601 (2016)
69. Fondo, M., Garcia-Deibe, A.M., Sanmartin, J., Bernejo, M.R., Lezama, L., Rojo, T.: A binuclear copper(II) acetate complex showing a 3D supramolecular network with hydrophilic pockets: its unusual magnetic behaviour. Eur. J. Inorg. Chem. 20, 3703 (2003)
70. Yurischev, M.A.: On the quantum discord of general X states. Quant. Inf. Proc. 14, 3399 (2015)
71. Wang, Y.-K., Ma, T., Heng, F., Fei, S.-M., Wang, Z.-X.: Super-quantum correlation and geometry for Bell-diagonal states with weak measurements. Quant. Inf. Proc. 13, 283 (2014)

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