A low Hubble constant from galaxy distribution observations

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Received July 29, 2020
Accepted August 25, 2020
Published September 29, 2020

Abstract. An accurate determination of the Hubble constant remains a puzzle in observational cosmology. The possibility of a new physics has emerged with a significant tension between the current expansion rate of our Universe measured from the cosmic microwave background by the Planck satellite and from local methods. In this paper, new tight estimates on this parameter are obtained by considering two data sets from galaxy distribution observations: galaxy cluster gas mass fractions and baryon acoustic oscillation measurements. Priors from the Big Bang nucleosynthesis (BBN) were also considered. By considering the flat ΛCDM and XCDM models, and the non-flat ΛCDM model, our main results are: $H_0 = 65.9^{+1.5}_{-1.5}$ km s$^{-1}$ Mpc$^{-1}$, $H_0 = 65.9^{+4.4}_{-4.0}$ km s$^{-1}$ Mpc$^{-1}$ and $H_0 = 64.3^{+4.5}_{-4.4}$ km s$^{-1}$ Mpc$^{-1}$ in 2σ c.l., respectively. These estimates are in full agreement with the Planck satellite results. Our analyses in these cosmological scenarios also support a negative value for the deceleration parameter at least in 3σ c.l.

Keywords: baryon acoustic oscillations, galaxy clusters

ArXiv ePrint: 2006.06712


1 Introduction

During the past decades, the efforts of observational cosmology have been mainly focused on a precise determination of the parameters that describe the evolution of the Universe. Undoubtedly, one of the most important quantities to understand the cosmic history is the current expansion rate \( H_0 \), which is fundamental to answer important questions concerning different phases of cosmic evolution, as a precise determination of the cosmic densities, the mechanism behind the primordial inflation as well as the current cosmic acceleration (see [1] for a broad discussion).

Nowadays, the most reliable measurements of the Hubble constant are obtained from distance measurements of galaxies in the local Universe using Cepheid variables and Type Ia Supernovae (SNe Ia), which furnishes \( H_0 = 74.03 \pm 1.42 \) km s\(^{-1}\) Mpc\(^{-1}\) [2]. The value of \( H_0 \) can also be estimated from a cosmological model fit to the cosmic microwave background (CMB) radiation anisotropies. By assuming the flat \( \Lambda \)CDM model, the \( H_0 \) estimate is \( H_0 = 67.36 \pm 0.54 \) km s\(^{-1}\) Mpc\(^{-1}\) [3].\(^1\) These two \( H_0 \) values are discrepant by \( \simeq 4.4\sigma \), which gives rise to the so-called \( H_0 \)-tension problem.\(^2\)

For this reason, new models beyond the standard cosmological one (the flat \( \Lambda \)) that could alleviate this tension become appealing. Some extensions of the \( \Lambda \)CDM model that allow to reduce the \( H_0 \) tension are: the existence of a new relativistic particle [7], small spatial curvature effects [8], evolving dark energy models [9], among others [2]. Then, new methods to estimate \( H_0 \) are welcome in order to bring some light on this puzzle. Precise measurements of the cosmic expansion rate \( H(z) \) are important to provide more restrictive constraints on cosmological parameters as well as new insights into some fundamental questions that range from the mechanism behind the primordial inflation and current cosmic acceleration to neutrino physics (see, e.g., [1] for a broad discussion).

\(^1\)Another recent estimate of \( H_0 \) has been reported by the H0LiCOW collaboration [4] based on lensing time-delays observations, \( H_0 = 71.9^{+2.4}_{-3.0} \) km s\(^{-1}\) Mpc\(^{-1}\), which is in moderate tension with Planck. However, when combined with clustering data, a value of \( H_0 = 66.98 \pm 1.18 \) km s\(^{-1}\) Mpc\(^{-1}\) is obtained.

\(^2\)We recommend [5] for an overview and history, as well as [6] for the current state of this intriguing problem.

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1 Introduction

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The Hubble constant has also been estimated from galaxy cluster systems by using their angular diameter distances obtained from the Sunyaev-Zel’dovich effect (SZE) plus X-ray observations. For instance, [10] used 18 angular diameter distances of galaxy clusters with redshifts ranging from $z = 0.14$ up to $z = 0.78$ and obtained $H_0 = 60 \pm 4$ km s$^{-1}$ Mpc$^{-1}$ (only statistical errors) for an $\{\Omega_m = 0.3, \Omega_\Lambda = 0.7\}$ cosmology. The authors of the ref. [11] considered 38 angular diameter distances of galaxy clusters in the redshift range $0.14 \leq z \leq 0.89$ and obtained $H_0 = 76.9 \pm 4$ km s$^{-1}$ Mpc$^{-1}$ (only statistical errors) also for an $\{\Omega_m = 0.3, \Omega_\Lambda = 0.7\}$ cosmology. In both cases, it was assumed a spherical morphology to describe the clusters. Without fixing cosmological parameters, the authors of the ref. [12] estimated $H_0$ by using a sample of angular diameter distances of 25 galaxy clusters (described by an elliptical density profile) jointly with baryon acoustic oscillations (BAO) and the CMB Shift Parameter signature. The $H_0$ value obtained in the framework of $\Lambda$CDM model with arbitrary curvature was $H_0 = 74^{+8.0}_{-7.0}$ km s$^{-1}$ Mpc$^{-1}$ at 2$\sigma$ c.l.. By considering a flat $\omega$CDM model with a constant equation of state parameter, they obtained $H_0 = 72^{+10.0}_{-9.0}$ km s$^{-1}$ Mpc$^{-1}$ at 2$\sigma$ c.l.. In both cases were considered the statistical and systematic errors. As one may see, due to large error bars, the results found are in agreement with the current Riess et al. local estimate [2] and with the Planck satellite estimate within 2$\sigma$. It is worth to comment that the constraints on the Hubble constant via X-ray surface brightness and SZE observations of the galaxy clusters depend on the validity of the cosmic distance duality relation (CDDR): $D_L (1 + z)^{-2}/D_A = 1$, where $D_L$ is the luminosity distance and $D_A$ is the angular diameter distance [13, 14].

In this paper, we obtain new and tight estimates on the Hubble constant by combining two main data sets from galaxy distribution observations in redshifts: 40 cluster galaxy gas mass fractions (GMF) and 11 baryon acoustic oscillation (BAO) measurements. Priors from BBN to calibrate the cosmic sound horizon and the cosmic microwave background local temperature as given by the FIRES/COBE also are considered. The $H_0$ estimates are performed in three models: flat $\Lambda$CDM and XCDM models, and non-flat $\Lambda$CDM model. This last one is motivated by recent discussions in the literature concerning a possible cosmological curvature tension, with the Planck CMB spectra preferring a positive curvature at more than 99% c.l. [15–17]. We show that the combination of these two independent data sets provides an interesting method to constrain the Hubble constant. For all models, tight estimates are found and our results support low Hubble constant values in agreement with the Planck results. We also found that the analyses performed in these cosmological models indicate an universe in accelerated expansion in more than 3$\sigma$ c.l..

The paper is organized as follows: section 2 presents the two cosmological models and data sets used. Section 3 presents the main results and analysis and section 4 finishes with conclusions.

2 Cosmological models and data sets

In order to estimate the Hubble constant, we consider three cosmological scenarios: the flat $\Lambda$CDM and XCDM models, and the non-flat $\Lambda$CDM model. Both $\Lambda$CDM models consider the cosmic dynamics dominated by a cold dark matter (CDM) component and cosmological constant ($\Lambda$), usually related to the constant vacuum energy density with negative pressure. By considering a constant equation of state for dark energy, $p_\Lambda = -\rho_\Lambda$, and the Universe described by a homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker geometry, we obtain from the Einstein equation the following expression for the Hubble parameter in
the $\Lambda$CDM framework:

\[ H(z) = H_0 \sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{k,0}(1+z)^2 + \Omega_{\Lambda}}, \]  

(2.1)

where $H_0$ is the current Hubble constant, generally expressed in terms of the dimensionless parameter $h \equiv H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1})$, $\Omega_{m,0}$, $\Omega_{\Lambda}$ and $\Omega_{k,0}$ are the current dimensionless parameter of matter density (baryons + dark matter), dark energy density and curvature density ($\Omega_{k,0} \equiv 1 - \Omega_{m,0} - \Omega_{\Lambda}$), respectively. Note that if $\Omega_{k,0} = 0$ the flat $\Lambda$CDM model is recovered.

By considering the flat XCDM model, the Hubble parameter is written as:

\[ H(z) = H_0 \sqrt{\Omega_{m,0}(1+z)^3 + (1 - \Omega_{m,0})^{3(1+\omega_x)}}, \]  

(2.2)

where $\omega_x$ is the equation of state parameter of dark energy, considered as a constant in our work (if $\omega_x = -1$, the flat $\Lambda$CDM model is recovered).

2.1 Data sets and $\chi^2$ function

In this section, we present the data sets used in the statistical analyses and their respective $\chi^2$ function.

2.1.1 Sample I: Baryonic Acoustic Oscillation data

This sample is composed of 11 measures obtained by 7 different surveys presented in table 1. The relevant physical quantities for the BAO data are the angular diameter distance:

\[ D_A(z) = \frac{1}{(1+z)} \times \begin{cases} 
\frac{H_0^{-1}}{\sqrt{|-\Omega_{k,0}|}} \sin \left( \sqrt{|-\Omega_{k,0}|} \int_0^z \frac{dz'}{H(z')} \right) & \text{if } \Omega_{k,0} < 0 \\
\int_0^z \frac{dz'}{H(z')} & \text{if } \Omega_{k,0} = 0 , \\
\frac{H_0^{-1}}{\sqrt{|-\Omega_{k,0}|}} \sinh \left( \sqrt{|-\Omega_{k,0}|} \int_0^z \frac{dz'}{H(z')} \right) & \text{if } \Omega_{k,0} > 0 
\end{cases} \]  

(2.3)

the spherically-averaged distance:

\[ D_V(z) = \left[ (1+z)^2 D_A^2(z) \frac{z}{H(z)} \right]^{1/3} \]  

(2.4)

and the sound horizon at the drag epoch [20]:

\[ r_s(z_d) = \frac{2}{3k_{eq}} \sqrt{\frac{6}{R(z_{eq})}} \ln \left[ \frac{\sqrt{1 + R(z_d)} + \sqrt{R(z_d) + R(z_{eq})}}{1 + \sqrt{R(z_{eq})}} \right], \]  

(2.5)

where $z_d$ is the drag epoch redshift, $z_{eq}$ is the equality redshift, $k_{eq}$ is the scale of the particle horizon at the equality epoch and [20]

\[ R(z) \equiv \frac{3\rho_b}{4\rho_\gamma} = 31.5(\Omega_b h^2) \left( \frac{T_{CMB}}{2.7 \text{ K}} \right)^{-4} \left( \frac{z}{10^3} \right)^{-1} \]  

(2.6)

-- 3 --
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Survey Set I & \( z \) & \( d_z(z) \) & \( \sigma_{d_z} \) \\
\hline
6dFGS & 0.106 & 0.336 & 0.015 \\
SDSS-LRG & 0.35 & 0.1126 & 0.0022 \\
\hline
\end{tabular}
\caption{BAO data set consisting of 1 measurement of the survey 6dFGS [23], 1 of SDSS-LRG [24], 1 of BOSS-MGS [25], 1 of BOSS-LOWZ [26], 1 of BOSS-CMASS [26], 3 of BOSS-DR12 [27] and 3 of WiggleZ [28]. The BAO variable \( D(z), \sigma_D \) and \( r_{sd}^{fid} \) have units of Mpc, while \( d_z(z) \) and \( \sigma_{d_z} \) are dimensionless.}
\end{table}

is the ratio of the baryon to photon momentum density. Here we determine \( z_{eq}, k_{eq} \) and \( z_d \), using the fit obtained by [20]. For the CMB temperature, we use the value measured by COBE / FIRAS [21], that is, \( T_{CMB} = 2.725 \) K. This value is independent of the Planck satellite analyses [22].

For survey set I, the BAO quantity is given by:
\[
d_z(z) = \frac{r_s(z_d)}{D_V(z)},
\]
with a \( \chi^2 \) function given by:
\[
\chi^2_{BAO,I} = \sum_{i=1}^{2} \left[ \frac{d_{z,j}^{th} - d_{z,j}^{ob}}{\sigma_{d_{z,j}^{ob}}} \right]^2.
\]

On the other hand, the BAO quantity for survey set II is given by:
\[
D(z) = \frac{D_V(z)}{r_s(z_d)} r_{sd}^{fid},
\]
and, in this specific case, the \( \chi^2 \) function is:
\[
\chi^2_{BAO,II} = \sum_{i=1}^{6} \left[ \frac{D_{i,\text{th}}(z_i) - D_{i,\text{ob}}}{\sigma_{D_{i,\text{ob}}}} \right]^2 + \left[ \vec{D}^{\text{th}} - \vec{D}^{\text{ob}} \right]^T C_{\text{WiggleZ}}^{-1} \left[ \vec{D}^{\text{th}} - \vec{D}^{\text{ob}} \right],
\]
where \( C_{\text{WiggleZ}}^{-1} \) is the inverse covariance matrix, whose explicit form is [28]:
\[
10^{-4} \begin{pmatrix}
2.17898878 & -1.11633321 & 0.46982851 \\
-1.11633321 & 1.70712004 & -0.71847155 \\
0.46982851 & -0.71847155 & 1.65283175
\end{pmatrix}.
\]
Figure 1. Measurements of $f_{\text{gas}}(z)$ used in our analysis. Details on this sample are presented in table 2 of the ref. [29].

2.1.2 Sample II: galaxy cluster gas mass fractions

The gas mass fractions (GMF) considered in this work corresponds to 40 Chandra observations from massive and dynamically relaxed galaxy clusters in redshift range $0.078 \leq z \leq 1.063$ from the ref. [29] (see figure 1). These authors incorporated a robust gravitational lensing calibration of the X-ray mass estimates. The measurements of the gas mass fractions were performed in spherical shells at radii near $r_{2500}$, rather than integrated at all radii ($< r_{2500}$). This approach significantly reduces systematic uncertainties compared to previous works that also estimated galaxy cluster gas mass fractions.

The gas mass fraction quantity for a cluster is given by [29, 30]:

$$f_{\text{gas}}^{\text{X-ray}}(z) = A(z) K(z) \frac{\Omega_{b}(z)}{\Omega_{m}(z)} \left( \frac{D_{A}^{\text{fid}}(z)}{D_{A}(z)} \right)^{\frac{3}{2}},$$

(2.12)

where

$$A(z) = \left( \frac{H(z)D_{A}(z)}{H_{\text{fid}}(z)D_{A}^{\text{fid}}(z)} \right)^{\eta}$$

(2.13)

stands for the angular correction factor ($\eta = 0.442 \pm 0.035$), $\Omega_{m}(z)$ is the total mass density parameter, which corresponds to the sum of the baryonic mass density parameter, $\Omega_{b}(z)$, and the dark matter density parameter, $\Omega_{c}(z)$. The term in brackets corrects the angular diameter distance $D_{A}(z)$ from the fiducial model used in the observations, $D_{A}^{\text{fid}}(z)$, which makes these measurements model-independent. The parameters $\gamma(z)$ and $K(z)$ correspond, respectively, to the depletion factor, i.e., the rate by which the hot gas fraction measured in a galaxy cluster is depleted with respect to the baryon fraction universal mean and to the bias of X-ray hydrostatic masses due to both astrophysical and instrumental sources. We adopt the value of $\gamma = 0.848 \pm 0.085$ in our analysis, which was obtained from hydrodynamical simulations [31] (see also a detailed discussion in section 4.2 in the ref. [29]). The $\gamma$ parameter has also been estimated via observational data (SNe Ia, gas mass fraction, Hubble parameter) with values in full agreement with those from hydrodynamical simulations (see [32] and [33]). Finally, for the parameter $K(z)$, we have used the value reported by [34] in which Chandra hydrostatic

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4This radii is that one within which the mean cluster density is 2500 times the critical density of the Universe at the cluster’s redshift.
masses to relaxed clusters were calibrated with accurate weak lensing measurements from the Weighing the Giants project. The $K(z)$ parameter was estimated to be $K = 0.96 \pm 0.09 \pm 0.09$ (1$\sigma$ statistical plus systematic errors) and no significant trends with mass, redshift or the morphological indicators were verified.

Observe that by assuming $\omega_{b,0} \equiv \Omega_{b,0} h^2$, we can rewrite equation (2.12) as

$$f_{\text{gas}}^{\text{X-ray}}(z) = \frac{K \gamma \omega_{b,0}}{\Omega_{m,0} h^2} \left[ \frac{H(z) D_A(z)}{H^\text{fid}(z) D^\text{fid}_A(z)} \right]^0 \left[ \frac{D_A^\text{fid}(z)}{D_A(z)} \right]^2.$$  

Therefore, for this sample, the $\chi^2$ function is given by,

$$\chi^2_{\text{GMF}} = \sum_{i=1}^{40} \frac{\left[f_{\text{gas}}^\text{th}(z_i) - f_{\text{gas},i}^\text{ob} \right]^2}{\sigma_{\text{tot},i}^2},$$

with a total uncertainty given by

$$\sigma_{\text{tot},i}^2 = \sigma_{\text{gas},i}^2 + \left[f_{\text{gas}}^\text{th}(z_i) \right]^2 \left\{ \left( \frac{\sigma_K}{K} \right)^2 + \left( \frac{\sigma_\gamma}{\gamma} \right)^2 + \ln \left[ \frac{H(z_i) D_A(z_i)}{H^\text{fid}(z_i) D^\text{fid}_A(z_i)} \right] \sigma_\eta^2 \right\},$$

where, as presented in the previous paragraph, $\{K, \sigma_K\} = \{0.96, 0.127\}$, $\{\gamma, \sigma_\gamma\} = \{0.848, 0.085\}$, and $\{\eta, \sigma_\eta\} = \{0.442, 0.035\}$. In addition to the BAO and GMF measures, we will adopt a Gaussian prior such as: $\omega_{b,0} = 0.0226 \pm 0.00034$. This value was obtained by [35] by using BBN + abundance of primordial deuterium.

3 Results and discussions

The statistical analysis is performed by the construction of the $\chi^2$ function,

$$\chi^2_{\text{tot}}(\{\alpha\}) = \chi^2_{\text{BAO, I}} + \chi^2_{\text{BAO, II}} + \chi^2_{\text{GMF}} + \chi^2_{\text{BBN}}.$$  

From this function, we are able to construct the likelihood distribution function, $L(\{\alpha\}) = B e^{-\frac{1}{2} \chi^2_{\text{tot}}(\{\alpha\})}$, where $B$ is the normalization factor and $\{\alpha\}$ is the set of free parameters of the cosmological model in question, that is, the flat $\Lambda$CDM and XCDM models, and the non-flat $\Lambda$CDM model.

3.1 Flat $\Lambda$CDM model

Figure 2 shows the contours and likelihoods for the $\Omega_{m,0}$ and $h$ parameters obtained in the context of the flat $\Lambda$CDM model. The contours delimited by dotted green lines correspond to the analysis using only GMF, the ones delimited by the dashed pink lines correspond to the analysis using only BAO, and the ones delimited by solid blue lines are referring to the joint analysis GMF + BAO. As one may see, the GMF sample alone does not restrict the value of parameter $h$ (or equivalently $H_0$) but provides tight restrictions to the value of parameter $\Omega_{m,0}$. From the joint analysis GMF + BAO, we obtain from the $\Omega_{m,0} - h$ plane (with two free parameters): $h = 0.659^{+0.012}_{-0.011} \pm 0.020$ and $\Omega_{m,0} = 0.311^{+0.016}_{-0.015} \pm 0.026$ in 1$\sigma$ and 2$\sigma$ c.l. .

By marginalizing over the parameter $\Omega_{m,0}$, we obtain the likelihood function for the $h$ parameter (see figure 5), with: $h = 0.659^{+0.008}_{-0.007} \pm 0.015$ in 1$\sigma$ and 2$\sigma$ c.l. . On the other hand, by marginalizing over the parameter $h$, we obtain the likelihood function of the $\Omega_{m,0}$ parameter as $\Omega_{m,0} = 0.311^{+0.010}_{-0.010} \pm 0.021$ in 1$\sigma$ and 2$\sigma$ c.l. .
Figure 2. Contours and likelihoods of parameters $\Omega_{m,0}$ and $h$ for the flat ΛCDM model. The contours delimited by dotted green, dashed pink, and solid blue lines correspond to the analyses using only GMF, BAO and the joint analysis GMF + BAO, respectively. Regions with darker and lighter colors delimit the 1- and 2σ c.l. regions, respectively.

Figure 5 (left) shows the likelihood of $h$ parameter for the flat (solid blue line) ΛCDM model and also the 1σ c.l. regions estimate of the $h$ parameter made by [3] in a flat background model and [2], cosmological model independent. As one may see, our estimate is in agreement with that one from the CMB anisotropies (within 2σ c.l.) and it is strongly discrepant with the estimate made by [2]. Being more specific, our estimate of $H_0$ in a flat ΛCDM model presents a discrepancy of 5.0σ with that obtained by [2]. A discrepancy also occurs if we compare our estimate with the most recent estimate of $H_0$ obtained by SH0ES Collaboration, i.e., $H_0 = 73.5 \pm 1.4 \, \text{km s}^{-1}\text{Mpc}^{-1}$ [36]. On the other hand, our estimate is in agreement with several other estimates of $H_0$ that used samples with intermediate redshifts in a flat universe [37–42].

3.2 Flat XCDM model

Figure 3 shows the contours and likelihoods for the $\Omega_{m,0}$, $\omega_x$, and $h$ parameters obtained in the flat XCDM context. The contours delimited by solid purple lines correspond to the joint analysis BAO + GMF, where by marginalizing over the parameter $\omega_x$, we obtain from the $\Omega_{m,0}$ $-$ $h$ plane (with two free parameters), the intervals: $h = 0.659^{+0.037}_{-0.035} \pm 0.055$, and $\Omega_{m,0} = 0.312^{+0.028}_{-0.029} \pm 0.044$ at 1σ and 2σ c.l.

On the other hand, by marginalizing over the parameter $\Omega_{m,0}$, we obtain from the $\omega_x$ $-$ $h$ plane (with two free parameters) the values: $h = 0.659^{+0.037}_{-0.036} \pm 0.056$ and $\omega_x = -0.99^{+0.18}_{-0.18} \pm 0.28$ in
Figure 3. Marginalized contours and likelihoods of parameters $\Omega_{m,0}$, $\omega_x$ and $h$ for the flat XCDM model. The contours delimited by solid purple lines correspond to the joint analysis BAO + GMF. Regions with darker and lighter colors delimit the 1- and 2σ c.l. regions, respectively.

In order to obtain the likelihood function for the $h$ parameter, we marginalize over $\Omega_{m,0}$ and $\omega_x$ parameters (see figure 6). From this likelihood, the following estimate is found: $h = 0.659^{+0.021+0.044}_{-0.024-0.046}$ in 1σ and 2σ c.l. Our estimate is in agreement with that one from the CMB anisotropies (within 2σ c.l.) and it is discrepant with the estimate made by [2]. Actually, our estimate of $H_0$ in a flat XCDM model presents a discrepancy of 3.1σ with that obtained by [2].

Similarly, by marginalizing over the $h$ and $\omega_x$ parameters, we obtain the likelihood for the parameter $\Omega_{m,0}$ with the following intervals: $\Omega_{m,0} = 0.312^{+0.016+0.032}_{-0.016-0.032}$ at 1σ and 2σ c.l.. Finally, by marginalizing over the $h$ and $\Omega_{m,0}$ parameters, we obtain the likelihood for the parameter $\omega_x$ as $\omega_x = -0.99^{+0.11+0.20}_{-0.11-0.21}$ in 1σ and 2σ c.l., in full agreement with the flat $\Lambda$CDM model ($\omega_x = -1$).

3.3 Non-flat $\Lambda$CDM model

Figure 4 shows the contours and likelihoods for the $\Omega_{m,0}$, $\Omega_\Lambda$, and $h$ parameters obtained in the context of the non-flat $\Lambda$CDM model. Similar to the case of the previous section, the contours delimited by solid red lines correspond to the joint analysis BAO + GMF. For this analysis, by marginalizing over the parameter $\Omega_\Lambda$, we obtain from the $\Omega_{m,0} - h$ plane (with
Figure 4. Marginalized contours and likelihoods of parameters $\Omega_{m,0}$, $\Omega_{\Lambda}$ and $h$ for the non-flat $\Lambda$CDM model. The contours delimited by solid red lines correspond to the joint analysis BAO + GMF. Regions with darker and lighter colors delimit the 1- and 2$\sigma$ c.l. contours, respectively.

By marginalizing over the parameter $\Omega_{m,0}$, we obtain from the $\Omega_{m,0} - h$ plane (with two free parameters) the values: $h = 0.644^{+0.035+0.057}_{-0.034-0.056}$ and $\Omega_{m,0} = 0.305^{+0.024+0.042}_{-0.022-0.034}$ in 1$\sigma$ and 2$\sigma$ c.l.

Similarly, by marginalizing over the parameter $\Omega_{\Lambda}$, we obtain from the $\Omega_{\Lambda} - h$ plane (with two free parameters) the values: $h = 0.645^{+0.034+0.057}_{-0.034-0.056}$ and $\Omega_{\Lambda} = 0.660^{+0.146+0.227}_{-0.179-0.312}$ in 1$\sigma$ and 2$\sigma$ c.l.

Finally, by marginalizing on the parameter $h$, we obtain from the $\Omega_{m,0} - \Omega_{\Lambda}$ plane (with two free parameters) the intervals: $\Omega_{m,0} = 0.305^{+0.023+0.040}_{-0.022-0.035}$ and $\Omega_{\Lambda} = 0.661^{+0.152+0.232}_{-0.170-0.301}$ in 1$\sigma$ and 2$\sigma$ c.l.

On the other hand, in order to obtain the likelihood function for the $h$ parameter, we marginalize over $\Omega_{m,0}$ and $\Omega_{\Lambda}$ parameters (see figure 5). From this likelihood, the following estimate is found: $h = 0.643^{+0.023+0.045}_{-0.022-0.045}$ in 1$\sigma$ and 2$\sigma$ c.l. Similarly, by marginalizing over the $h$ and $\Omega_{\Lambda}$ parameters, we obtain the likelihood for the parameter $\Omega_{m,0}$ with the following intervals: $\Omega_{m,0} = 0.305^{+0.016+0.031}_{-0.014-0.029}$ in 1$\sigma$ and 2$\sigma$ c.l. Finally, by marginalizing over the $h$ and $\Omega_{m,0}$ parameters, we obtain the likelihood for the parameter $\Omega_{\Lambda}$ as $\Omega_{\Lambda} = 0.663^{+0.094+0.204}_{-0.120-0.230}$ in 1$\sigma$ and 2$\sigma$ c.l.

Figure 5 (right) shows the likelihood of $h$ parameter for the non-flat (dashed red line) $\Lambda$CDM model, together with 1$\sigma$ c.l. regions of the estimate of the $h$ parameter obtained from
CMB anisotropies [3] in a non-flat background and that one from the ref. [2] (local method). From the figure, it is evident that our estimate is in full agreement (within 1σ c.l.) with that one from the CMB anisotropies [3] and discrepant with the local estimate made by [2] in 3.6σ c.l..

The table 2 shows a synthesis of the results presented in the last three subsections. More specifically, we show the free parameter estimates of all cosmological models in 2σ c.l. obtained from their respective likelihoods. As one may see, the $H_0$ values in all cases analyzed are compatible within 2σ with the Hubble constant measurement from the Planck results, presenting tensions with the local estimate. The estimates obtained here are considerably tighter than those ones from the ref. [12], where angular diameter distances of galaxy clusters plus BAO and shift parameter were used. The $\Omega_{m,0}$ parameter for the flat model is also compatible with the estimate $\Omega_{m,0} = 0.3158 \pm 0.0073$ from [3] within 1σ c.l.. Also for the non-flat case, the estimates for $\Omega_{m,0}$ and $\Omega_\Lambda$ are in agreement to [3] in 2σ c.l..

It is important to comment that we also perform our analyses by using another Gaussian prior, such as, $\omega_{b,0} = 0.02156 \pm 0.0002$ [35]. Although this value differs by $\approx 2.3\sigma$ from the Standard Model value estimated from the Planck observations of the cosmic microwave background, our $H_0$ estimates had insignificant changes.

### 3.4 Deceleration parameter and curvature density parameter

Using the uncertainties propagation and the values estimated for $\Omega_{m,0}$, $\omega_x$ and $\Omega_\Lambda$ from their likelihoods, we can estimate the current value of the deceleration parameter for each model analyzed by using $q_0 = \Omega_{m,0}/2 + (1 - 3\omega_a)\Omega_a/2$, where $\{\omega_a \rightarrow -1, \Omega_a \rightarrow 1 - \Omega_{m,0}\}$, $\{\omega_a \rightarrow \omega_x, \Omega_a \rightarrow 1 - \Omega_{m,0}\}$ and $\{\omega_a \rightarrow -1, \Omega_a \rightarrow \Omega_\Lambda\}$ for the flat $\Lambda$CDM and XCDM models, and the non-flat $\Lambda$CDM model, respectively. We obtain $q_0 = -0.533 \pm 0.046$, $q_0 = -0.542 \pm 0.299$ and $q_0 = -0.483 \pm 0.340$ in 3σ c.l. for the flat $\Lambda$CDM and XCDM models, and non-flat $\Lambda$CDM model, respectively. Moreover, similarly, we estimate the following value for the current curvature density parameter in 1σ c.l.: $\Omega_{k,0} = 0.056 \pm 0.108$. Our results from these models indicate an accelerating expansion of the universe in more than 3σ c.l., and $\Omega_{k,0}$ compatible with a spatially flat curvature within 1σ c.l.
Figure 6. Marginalized likelihoods of parameter $h$ for the flat XCDM model. The regions filled with darker and lighter colors under the probability curves delimit the 1- and 2σ c.l. regions, respectively. The brown and orange rectangle delimit the 1σ c.l. regions of the estimate of the $h$ parameter made by [3] (flat ΛCDM: $h = 0.6736 \pm 0.0054$) and [2] (independent-model: $h = 0.7403 \pm 0.0142$), respectively.

| Flat ΛCDM model | Non-flat ΛCDM model | Flat XCDM model |
|-----------------|---------------------|-----------------|
| $H_0$ [km s$^{-1}$Mpc$^{-1}$] | $65.9^{+1.5}_{-1.5}$ | $64.3^{+4.5}_{-4.4}$ | $65.9^{+4.4}_{-4.0}$ |
| $\Omega_{m,0}$ | $0.311^{+0.021}_{-0.020}$ | $0.305^{+0.031}_{-0.029}$ | $0.312^{+0.032}_{-0.032}$ |
| $\Omega_\Lambda$ | — | $0.663^{+0.204}_{-0.230}$ | — |
| $\omega_x$ | — | — | $-0.99^{+0.20}_{-0.21}$ |
| $\chi^2_{\text{min}}$ | 25.91 | 25.90 | 25.90 |

Table 2. A summary of the constraints in 2σ c.l. on the set of free parameters of the flat and non-flat ΛCDM model.

4 Conclusions

The recent $H_0$ tension between the local and global methods for the Hubble constant value has motivated the search for new tests and data analyses that could alleviate (or solve) the discrepancy in the $H_0$ value estimated by different methods.

In this paper, we obtained new and tight estimates on the Hubble constant by combining 40 galaxy cluster gas mass fraction measurements with 11 baryon acoustic oscillation data in the frameworks of the flat XCDM and ΛCDM models, and the non-flat ΛCDM models. The data sets are in the following range of redshift $0.078 \leq z \leq 1.023$. For all cosmological models considered, the gas mass fraction sample alone did not restrict the value of $H_0$, but put restrictive limits on the $\Omega_M$ parameter. However, from the joint analysis with the 11 BAO data, the restriction on the possible $H_0$ values was notable. By considering the flat ΛCDM and XCDM models, and the non-flat ΛCDM model, we obtained, respectively: $H_0 = 65.9^{+1.5}_{-1.5}$ km s$^{-1}$ Mpc$^{-1}$, $H_0 = 65.9^{+4.4}_{-4.0}$ km s$^{-1}$ Mpc$^{-1}$ and $H_0 = 64.3^{+4.5}_{-4.4}$ km s$^{-1}$ Mpc$^{-1}$ in 2σ c.l.. For all cases, the estimates indicated a low value of $H_0$ as those ones obtained by the Planck satellite results from the CMB anisotropies observations. For the flat models, the agreement is within 2σ c.l., while for the non-flat model the concordance is within 1σ c.l. (see figure 5). In this point, it is worth to point that in our analyses priors from BBN to calibrate the
cosmic sound horizon and the cosmic microwave background radiation local temperature as
given by the COBE/FIRES also were used. Our results reinforce the $H_0$ tension regardless
the curvature cosmic and the equation of state parameter of dark energy.

Estimates for the current deceleration and curvature density parameters were also ob-
tained. The results from the considered models pointed to a Universe in accelerated expansion
in more than $3\sigma$ c.l. We obtained $q_0 = -0.533 \pm 0.046$ and $q_0 = -0.542 \pm 0.299$ for the
flat $\Lambda$CDM and XCDM models in $3\sigma$ c.l., respectively. For the non-flat $\Lambda$CDM model it was
estimated $q_0 = -0.483 \pm 0.340$ in $3\sigma$ c.l.. Moreover, for the non-flat model, although the best
fit suggested a positive curvature, our analysis is compatible with a spatially flat curvature
within $1\sigma$ c.l..

Finally, it is important to stress that the $H_0$ estimates obtained here considered no
evolution of gas mass fraction measurements within redshift and mass intervals of the galaxy
cluster sample used. This question is still open for some debate. In the coming years, the
eROSITA [43] mission will make an all-sky X-ray mapping of thousand of galaxy clusters
and will provide accurate information on gas mass fraction measurements, which will turn
the analysis proposed here more robust.

Acknowledgments

RFLH thanks financial support from Conselho Nacional de Desenvolvimento Científico e
Tecnologico (CNPq) (No.428755/2018-6 and 305930/2017-6). SHP would like to thank CNPq
for financial support, No.303583/2018-5. The figures in this work were created with Wolfram
Mathematica and GetDist [44].

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