Gravitational lensing of transient neutrino sources by black holes

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A B S T R A C T

In this work we study gravitational lensing of neutrinos by Schwarzschild black holes. In particular, we analyze the case of a neutrino transient source associated with a gamma-ray burst lensed by a supermassive black hole located at the center of an interposed galaxy. We show that the primary and secondary images have an angular separation beyond the resolution of forthcoming km-scale detectors, but the signals from each image have time delays between them that in most cases are longer than the typical duration of the intrinsic events. In this way, the signal from different images can be detected as separate events coming from the very same location in the sky. This would render an event that otherwise might have had a low signal-to-noise ratio a clear detection, since the probability of a repetition of a signal from the same direction is negligible. The relativistic images are so faint and proximate that are beyond the sensitivity and resolution of the next-generation instruments.

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1. Introduction

Gravitational lensing of photons by black holes has received great attention in the last few years, mainly due to the increasing evidence of the presence of supermassive black holes at the center of galaxies. Theoretical studies of black hole lenses, both numerically and analytically, were made with Schwarzschild [1–3], Reissner–Nordström [4], general spherically symmetric [5,6] and rotating [7,8] geometries, and also for black holes coming from alternative theories [9] or braneworld cosmologies [10]. Even naked singularities were considered as lenses [11]. Photons (or null mass particles) passing close enough to the photon sphere of the lens will have large deflection angles, and they can even make one or more turns around the deflector before reaching the observer. By this mechanism, two infinite sets of strong deflection images, one at each side of the lens, are produced. The presence of images with large deflection angles is not a new fact, since they were obtained already in 1959 for the Schwarzschild spacetime [12]. The analytical study of these images is more simple if one adopts the strong deflection limit, which consists in a logarithmic approximation of the deflection angle, first obtained for the Schwarzschild metric [12], revisited by several authors [2,13], extended to Reissner–Nordström geometry [4], to general spherically symmetric spacetimes [5] and to Kerr metric [7]. For some lensing configurations two weak deflection images are also obtained, which are analyzed by making a first order Taylor expansion of the deflection angle (weak deflection limit), as it is usually done for more standard astrophysical objects, such as stars and galaxies (see, e.g., [14]). Intermediate cases can be treated analytically by perturbative [15] or variational methods [16]. A special configuration, where no weak deflection images are present, is when the source is in front of the lens instead of behind it, which is called retrolensing [17]. Recently, the strong deflection limit was extended to include sources very close to the black hole [18].

Lensing of neutrinos have been previously studied by other authors. In Ref. [19], gravitational lensing of neutrinos by stars and galaxies was analyzed, and in Ref. [20], the lensing effects of supernova neutrinos by the Galactic center black hole was considered, in the weak deflection limit. However, perhaps the most interesting cosmological sources of neutrinos from the point of view of lensing are transients associated with gamma-ray bursts (GRBs). It is expected that proton–photon interactions during the GRB will re-
sult into copious photopion production and hence neutrinos would be generated from the decay of charged pions and muons (e.g. [21–24]). Since GRBs occur frequently, say once per day, and can be detected at gamma-rays by SWIFT satellite and then the follow-up of the afterglows usually allows the identification of the host galaxy and the corresponding redshift (see, e.g., [25]), they are outstanding candidates for lensing produced by massive black holes in the center of interposed galaxies.

We notice, however, that in the collapsar scenario for long GRBs [26,27] the jet not always is expected to be able to make its way through the star, so no observable gamma-ray emission would result in such cases [22]. Nonetheless, the neutrino emission might be important. If the event is lensed, the neutrino signal should repeat and hence be identified, despite the absence of electromagnetic counterparts.

In this Letter we investigate gravitational lensing of neutrinos by Schwarzschild black holes. We pay special attention to neutrino transients lensed by supermassive black holes located at the center of galaxies. In Section 2 we present the expressions that give the positions and magnifications of the weak and strong deflection images, and in Section 3 we calculate the time delays between the arrival signals. Then, in Section 4, we calculate the specific time delays produced by some interposed supermassive black holes for neutrino transient at a distance of ~10^{28} cm. Finally, in Section 5, a brief summary and the conclusions are presented.

2. Positions and magnifications of the images

Neutrinos have zero or negligible mass, so we assume that they follow null geodesics as photons do. We consider a point source of neutrinos, with angular diameter distance D_{os} to the observer, behind a Schwarzschild black hole lens, placed at an angular diameter distance D_{ol}. The angular diameter distance between the lens and the source is dubbed D_{ls}. The optical axis is defined by the line that joins the observer with the deflector. The distances are very large compared to the Schwarzschild radius of the black hole and the angles are measured from the observer. We restrict our analysis to high alignment, which is more interesting from an astrophysical point of view, since the images are more prominent. Then the angular position of the source \( \beta \), taken positive here, is small. For this configuration, we have two weak deflection images and two infinite sets of strong deflection (also called relativistic [11]) images. Neutrinos with closest approach distance \( r_{\text{ps}} = 3MG/c^2 \), which corresponds to the unstable circular orbit around the black hole, will have a small deflection angle \( \alpha \), which can be approximated to first order in \( r_{\text{ps}} / r_0 \) by \( \alpha = 4GM/c^2 r_0 \) (weak deflection limit). Within this approximation, the lens equation has the form [14]

\[
\beta = \theta - \frac{\theta_{\text{E}}^2}{\beta} \tag{1}
\]

where \( \theta \) is the angular position of the image and \( \theta_{\text{E}} \) is the angular Einstein radius, given by

\[
\theta_{\text{E}} = \sqrt{\frac{2KR_{\text{ls}}}{D_{\text{ol}}D_{\text{os}}}} \tag{2}
\]

with \( R_{\text{s}} = 2MG/c^2 \) the Schwarzschild radius of the lens. The lens equation has two solutions:

\[
\theta_{\text{p},s} = \frac{1}{2} (\theta \pm \sqrt{\theta^2 + 4\theta_{\text{E}}^2}) \tag{3}
\]

that give the positions of the primary (upper sign) and the secondary (lower sign) images. The primary image lies inside the Einstein radius and the secondary image outside. When \( \beta = 0 \), instead of two images, an Einstein ring with radius \( \theta_{\text{E}} \) is obtained. Another important aspect is the magnification of the images, defined as the ratio between the observed and intrinsic fluxes of the source. As a consequence of the Liouville theorem in curved spacetimes [31], gravitational lensing preserves surface brightness for neutrinos and photons, so the magnifications of the images are given by the ratio of the solid angles subtended by the images and the source, which result in [14]:

\[
\mu_{\text{p},s} = \frac{1}{4} \left( \frac{\beta}{\sqrt{\beta^2 + 4\theta_{\text{E}}^2}} + \frac{\mu_{\text{ps}}}{\beta} \pm 2 \right), \tag{4}
\]

where the plus sign corresponds to the primary image and the minus sign to the secondary one. If the position of the source \( \beta \) is close to zero, the magnifications of both images are large. If \( \beta = 0 \) the approximation of point source breaks down and the magnifications become infinite. It is not difficult to see that \( \mu_{\text{p},s} > 1 \) for all \( \beta \), and \( \mu_{\text{s}} > 1 \) only if \( \beta^2 / \theta_{\text{E}}^2 < (3\sqrt{2} - 4)/2 \approx 0.35 \). When \( \beta / \theta_{\text{E}} \) is large we have that \( \mu_{\text{p}} \approx 1 \) and \( \mu_{\text{s}} \approx 0 \).

Besides the weak deflection images, two infinite sets of relativistic images are formed by neutrinos that make one or more loops, in both directions of winding, around the black hole lens. For high alignment, the deflection angle corresponding to the relativistic images is close to an even number of \( \pi \), \( \alpha = \pm (2n\pi + \Delta\alpha_{\text{r}}) \) with \( 0 < \Delta\alpha_{\text{r}} \ll 1 \), the upper sign corresponding to one set of images and the lower one to the other set. The other angles involved are small, then the lens equation [11]

\[
\tan \beta = \tan \theta - \frac{D_{\text{ls}}}{D_{\text{os}}} (\tan \theta + \tan(\alpha - \theta)), \tag{5}
\]

takes the form [2,5]

\[
\beta = \theta = \frac{D_{\text{ls}}}{D_{\text{os}}} \Delta\alpha_{\text{r}}. \tag{6}
\]

In the strong deflection limit, i.e. for trajectories passing close to the photon sphere of the black hole, the deflection angle can be approximated by a logarithmic function of the impact parameter \( b \), defined as the perpendicular distance from the deflector to the asymptotic path at infinity. For the Schwarzschild geometry, it can be shown that [2,5,12]

\[
\alpha = \pm \left[ -c_1 \ln \left( \frac{b}{b_{\text{ps}}} - 1 \right) + c_2 \right] + O(b - b_{\text{ps}}), \tag{7}
\]

with \( c_1 = 1 \), \( c_2 = \ln[216(7 - 4\sqrt{3})] - \pi \) and \( b_{\text{ps}} = \sqrt{3}\sqrt{R_{\text{s}}/2} \) the critical impact parameter. Neutrinos with impact parameter smaller than the critical value will spiral inside the photon sphere, into the black hole, not reaching the observer, and those with \( b > b_{\text{ps}} \) will make one or more outward turns outside the photon sphere, finally getting to the observer. As in the case of photons, using that \( b = \sin D_{\text{ol}} \theta = \theta D_{\text{ol}} \), inverting Eq. (7) and Taylor expanding it around \( \alpha = 2n\pi \) to obtain \( \Delta\alpha_{\text{r}} \) replacing the result in the lens equation (6) and finally inverting it, the positions of the relativistic images can be approximated (keeping only the lower order terms) by [2,5]:

\[
\theta_{\text{p},s} = \pm \frac{\theta_{\text{E}}^2 + D_{\text{os}}b_{\text{ps}}}{D_{\text{ls}}b_{\text{ps}} c_1 c_2} e_{\alpha} \beta, \tag{8}
\]

\[\text{Eq. (5) is valid for asymptotically flat spacetimes, with the source and the observer in the flat region; for more general lens equations see [30].}\]
where
\[ e_n = e^{(\delta_2 - 2\pi n_1)/c_1}, \]
and
\[ \theta_\text{E}^n = \frac{b_{ps}}{D_{\text{ol}}} \left( \frac{1 - \frac{D_{\text{ol}}}{D_s} b_{ps}}{D_s D_{\text{ol}} c_1} \right) (1 + e_n), \]
(9)
is the nth relativistic Einstein ring radius. For perfect alignment
an infinite sequence of Einstein rings is obtained instead of point images.
With the same considerations given above for the weak
deflection images, the magnification of the nth image has the same
expression that was found previously for photons [2,5]:
\[ \mu_n = \frac{1}{b} \frac{b_{ps} D_{ps}}{D_{\text{ol}}^2 c D_{ps} c_1} (1 + e_n) e_n, \]
(10)
for both sets of relativistic images. The first image \((n = 1)\) is the
strongest one and the others have magnifications that decrease
exponentially with \(n\). For a given source angle \(\beta\), the relativistic
images are very faint compared with the weak deflection ones.4

3. Time delays

Neutrinos that form distinct images take different paths, result-
ing in time delays between the images. Considering again that
neutrinos follow null geodesics as photons do, the time delay be-
tween the primary and the secondary images is given by [14]:
\[ \Delta t_{\text{ps}} = \frac{2R_s}{c} \left( (\theta^2_p - \theta^2_s) \left| \frac{\theta_s}{\beta} \right| + \ln \left| \frac{\theta_s}{\beta} \right| \right), \]
(11)
where \(z_d\) is the redshift of the deflector. The last equation can be
written in the form
\[ \Delta t_{\text{ps}} = \frac{2R_s}{c} \left( 1 + z_d \right) \left( -\frac{\beta}{2R_s} \frac{\sqrt{\beta^2 + 4\theta^2_s}}{\sqrt{\beta^2 + 4\theta^2_s}} + \ln \left( \beta - \sqrt{\beta^2 + 4\theta^2_s} \right) \right). \]
(12)
When \(\beta = 0\) there is no time delay. Large time delays can be
obtained if \(\beta/\theta_s \gg 1\), but in this case the magnification of the pri-
mary image is close to one and the secondary image is very faint.
The optimal situation for a variable source is when \(\beta/\theta_s\) is small
enough to have large magnifications of both images, but not too
close to zero, so the time delay can be longer than the typical
time scale of the transverse source.

In the case of relativistic images, the time delay between the
images formed at the same side of the lens is given by [6]:
\[ \Delta t_{\text{ps},n} = \frac{b_{ps}}{c} \left( 1 + z_d \right) \left[ 2\pi (n - m) + 2\sqrt{2}(w_m - w_n) \right] \]
\[ \pm \frac{\sqrt{2} D_{ps} (w_m - w_n)}{c D_{\text{ls}} c_1} \beta, \]
(13)
where
\[ w_k = e^{(c^2 - 2\pi k)/c_1}, \]
and the upper/lower sign corresponds if both images are on the
same/opposite side of the source. For the images at the opposite
side of the lens we have [6]:
\[ \Delta t_{n,m} = \frac{b_{ps}}{c} \left( 1 + z_d \right) \left[ 2\pi (n - m) + 2\sqrt{2}(w_m - w_n) \right] \]
\[ \pm \frac{\sqrt{2} D_{ps} (w_m + w_n)}{c D_{\text{ls}} c_1} \beta, \]
(14)
where the image with winding number \(n\) is on the same side of
the source and the other one on the opposite side. The first term
in Eqs. (13) and (14) is by large the most important one [6]. The
time delays between the relativistic images are longer than the
time delay between the primary and the secondary images.

4. Lensing of neutrino transients

The angular resolution of the primary and secondary images
is beyond the capability of current and near future neutrino de-
tectors, which is of the order of one tenth of a degree, but the
temporal resolution of the images of individual transient events,
which have typical durations in the range of \(\sim 10\) s to \(100\) s for
long GRBs [25], is possible. As an example of this, we consider
neutrino transients acting as possible sources situated at distances
of the order of \(10^{28}\) cm, with supermassive black holes at the
center of interposed galaxies as lenses. Some results of our cal-
culations for specific cases of lenses in the local universe (\(z_d \sim 0\))
are shown in Tables 1 and 2, with the masses and distances taken
from Ref. [32]. We see that the separation between the primary
and secondary images of neutrino transients is of the order of a
second of arc, so they cannot be resolved. For suitable values of
the parameters involved, the weak deflection images can be both
magnified several times, with time delays of \(10^2\) to \(10^4\) s, larger than
the intrinsic time of variation of the sources.

If one fixes \(\beta/\theta_s\) to obtain from Eq. (4) the desired values
of magnification of the images, it is clear from Eq. (12) that the
time delay increases linearly with the redshift of the lens. Then, with a

\begin{table}
\caption{Time delays between the weak deflection images of neutrino burst sources at a
distance of \(10^{28}\) cm. The lenses are supermassive black holes at the center of
the galaxies indicated. The Schwarzschild geometry was adopted to model the black
holes. The source angular position is \(\beta = 0.1\theta_k\), with \(\theta_k\) the angular Einstein radius.
In this case, the angular positions of the primary and the secondary images are
\(\theta_p = \theta_k = \theta_s\) and \(\theta_n = -0.95\theta_k\), while their respective magnifications are \(\mu_p = 5.5\)
and \(\mu_s = 4.5\).
\begin{tabular}{|l|l|l|l|}
\hline
Galaxy & Black hole mass (M_\odot) & Distance (Mpc) & \(\theta_k\) (arcsec) & \(\Delta t_{ps}\) (s) \\
\hline
Milky Way & 2.8 \times 10^8 & 0.0085 & 1.6 & 11 \\
NGC0224 & 3.0 \times 10^7 & 0.7 & 0.6 & 1.2 \times 10^4 \\
NGC3115 & 2.0 \times 10^9 & 8.4 & 1.4 & 7.9 \times 10^6 \\
NGC3377 & 1.8 \times 10^6 & 9.9 & 0.4 & 7.1 \times 10^4 \\
NGC4486B & 5.7 \times 10^8 & 15.3 & 0.5 & 2.2 \times 10^6 \\
NGC4486 & 3.3 \times 10^8 & 15.3 & 1.3 & 1.3 \times 10^4 \\
NGC4261 & 4.5 \times 10^8 & 27.4 & 0.4 & 1.8 \times 10^5 \\
NGC7052 & 3.3 \times 10^8 & 58.7 & 0.2 & 1.3 \times 10^3 \\
\hline
\end{tabular}
\end{table}

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\caption{Time delays between the weak deflection images of neutrino burst sources at a
distance of \(10^{28}\) cm. The lenses are supermassive black holes at the center of
the galaxies indicated. The Schwarzschild geometry was adopted to model the black
holes. The source angular position is \(\beta = 0.1\theta_k\), with \(\theta_k\) the angular Einstein radius.
In this case, the angular positions of the primary and the secondary images are
\(\theta_p = 1.05\theta_k\) and \(\theta_n = -0.95\theta_k\), while their respective magnifications are \(\mu_p = 1.6\)
and \(\mu_s = 0.6\).
\begin{tabular}{|l|l|l|l|}
\hline
Galaxy & Black hole mass (M_\odot) & Distance (Mpc) & \(\theta_k\) (arcsec) & \(\Delta t_{ps}\) (s) \\
\hline
Milky Way & 2.8 \times 10^8 & 0.0085 & 1.6 & 11 \\
NGC0224 & 3.0 \times 10^7 & 0.7 & 0.6 & 6.0 \times 10^4 \\
NGC3115 & 2.0 \times 10^9 & 8.4 & 1.4 & 4.0 \times 10^6 \\
NGC3377 & 1.8 \times 10^6 & 9.9 & 0.4 & 3.6 \times 10^4 \\
NGC4486B & 5.7 \times 10^8 & 15.3 & 0.5 & 1.1 \times 10^4 \\
NGC4486 & 3.3 \times 10^8 & 15.3 & 1.3 & 6.6 \times 10^4 \\
NGC4261 & 4.5 \times 10^8 & 27.4 & 0.4 & 8.9 \times 10^4 \\
NGC7052 & 3.3 \times 10^8 & 58.7 & 0.2 & 6.6 \times 10^4 \\
\hline
\end{tabular}
\end{table}
5. Final remarks

In this Letter we have shown that the primary and secondary images of neutrino transient sources lensed by supermassive black holes cannot be angularly resolved but they could be temporally resolved by next generation instruments. The relativistic images, instead, are too faint and packed to be detected. Thus, we have found that neutrino transients produced by long GRBs can act as sources for gravitational lensing when supermassive black holes are present in foreground galaxies. This sources would have a unique signature, that will allow an easy detection above the background despite a possible low signal-to-noise ratio: repetition. The neutrino fluxence can be magnified, but more importantly, the arriving signal will repeat, leading to an unequivocal identification. We conclude that neutrino gravitational lensing can help to establish GRBs as sources of relativistic protons and neutrinos, as proposed by several authors [22,23].

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