On inverse problem of identification the cavity in the elastic cylinder.

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Abstract. In the present work we consider the inverse problem of identification the spherical cavity of small relative size in the isotropic elastic cylinder. Based on the methods of perturbation theory, analytical formulas for determining the parameters of the cavity have been obtained using information concerning corrections to the natural frequencies. A series of numerical experiments has been carried out.

1. Introduction
Problems of identification defects in elastic bodies are very important and belong to a class of inverse problems of the theory of elasticity. Cavities are widespread type of defects which arise usually in the stages of fabrication and operation of constructional elements. Defects of this nature are stress concentrators and often greatly reduce the carrying capacity of structures. Investigation of vibrations of bodies with localized inhomogeneities allows to reveal the influence of inhomogeneities of this type on the dynamic stress concentration, to predict the destruction of structures in the early stages. Note that, in practice, the most important is information about the type of the defect (cavity or crack) and its characteristic dimensions, because this information is a major and decisive from the point of view of fracture mechanics. Defect type and its size can be determined according to the change of the resonance frequency, assuming that the defect is small in comparison with the characteristic geometric dimensions of the body.

2. Statement of the problem and solution methods
It is possible to obtain corrections for the first resonance frequency in the analytical form using perturbation theory [1]. In [2] shows the general scheme of obtaining corrections for the anisotropic elastic body, weakened by internal cavity is shown. In [3] the approach described in [2], has been used to determine corrections to the resonance frequencies in the problem of steady-state longitudinal oscillations with a frequency $\omega$ of the isotropic elastic cantilever fixed rod with length $l$, weakened by the small cavity. In paper [3] the model of an inhomogeneous rod has been used, such that the presence of a cavity characterized by the dependence of the cross sectional area on the longitudinal coordinate in the form of:

$$F(x) = F_0(1 - \varepsilon \eta(x)),$$

where $\varepsilon = \frac{r_0^2}{R^2}$ - the small parameter

$r_0$ - the cavity radius, $R$ - radius of the rod cross section,
$F_0$ - cross sectional area of the rod without defects

$\eta(x)$ - positive function with compact support, that simulates the presence of the cavity:

$$\eta(x) = \begin{cases} >0, & x \in [a,b] \\ =0, & x \notin [a,b] \end{cases} \quad [a,b] \subset [0,l]$$

Natural frequencies and natural modes have been presented in the form of:

$$u^2 = u_0^2 + \varepsilon \omega_1 + o(\varepsilon^2) \quad \omega^2 = \omega_0^2 + \varepsilon \omega_1 + o(\varepsilon^2)$$

For the rod, weakened by spherical cavity, centered at the point $c_0$ of radius $r_0$ the following formula to calculate the corrections to the natural frequencies has been obtained:

$$\omega_{n_0} = \frac{8}{3} \omega_0^2 \frac{r_0}{l} \cos \left( \frac{2 \omega_0 c_0}{l} \right)$$

(1)

where $\omega_{n_0}$ - natural frequencies of longitudinal vibrations of the rod without cavity.

Based on (1) the formula for finding the center of the cavity by using the information on corrections to the first and second frequencies has been obtained:

$$c_0 = \pi^{-1} l \arccos \left( \pm \frac{1}{6} \sqrt{\frac{\omega_2^2}{\omega_1^2} + 27} \right)$$

(2)

where, the sign is selected «+», if $\omega_1 < 0$, and sign «-», if $\omega_1 > 0$.

After finding $c_0$, $r_0$ is determined from the relation (1).

3. The results of numerical experiments
The described approach has been used to finding the parameters of a spherical cavity in the rod (with axis length $l = 1m$, radius of the circle $R = 0.1m$), weakened by spherical cavity (with radius $r_0 = 0.2R = 0.02m$), located on the symmetry axis of the rod.

Table 1. Dimensionless wave number for the rod model

| rod without cavity | $c_0 = 0.1$ | $c_0 = 0.2$ | $c_0 = 0.3$ | $c_0 = 0.8$ | $c_0 = 0.9$ |
|-------------------|------------|------------|------------|------------|------------|
| $\kappa_1$        | 1.57077    | 1.56913    | 1.56937    | 1.56974    | 1.57212    | 1.57236    |
| $\kappa_2$        | 4.71180    | 4.70874    | 4.71329    | 4.71656    | 4.71014    | 4.71472    |
Table 1 shows the values of the first and second dimensionless wave numbers \( \kappa = \frac{\omega l}{\sqrt{\rho E}} \) for rod without cavity and rod, weakened by spherical cavity, centered at the point \( c_0 \).

**Table 2.** Recovery results of the center and radius of the cavity for the rod model

|   | \( c_0 = 0.1 \) | \( c_0 = 0.2 \) | \( c_0 = 0.3 \) | \( c_0 = 0.8 \) | \( c_0 = 0.9 \) |
|---|----------------|----------------|----------------|----------------|----------------|
| \( c_0^* \) | 0.0996 | 0.1976 | 0.0294 | 0.7977 | 0.8990 |
| \( r_0^* \) | 0.0202 | 0.0202 | 0.0201 | 0.0200 | 0.0200 |

Table 2 shows the results of recovery center \( c_0^* \) and the radius \( r_0^* \) of the spherical cavity on the corrections to the wave numbers. It should be noted that the error recovery center of the cavity does not exceed 1.2%, the error recovery of the cavity radius does not exceed 0.9%.

Next, in the present study experiment has been carried out to find the spherical cavity parameters in the cylinder (with axis length \( l = 1m \), radius of the circle \( R = 0.1l = 0.1m \)), weakened by spherical cavity (with radius \( r_0 = 0.2R = 0.02m \)), located on the symmetry axis of the rod.

The considered problem has been solved using the finite element package ANSYS. Program has been written in the macro language APDL for ANSYS to calculate the natural frequencies of the cylinder depending on the position of the center of the cavity. Volumetric finite element SOLID 95 has been used in modeling.

![Figure 1. The cross section of the cylinder, weakened by spherical cavity](image)

**Table 3.** Dimensionless wave number for the cylinder model

| Cylinder without cavity | \( c_0 = 0.1 \) | \( c_0 = 0.2 \) | \( c_0 = 0.3 \) | \( c_0 = 0.8 \) | \( c_0 = 0.9 \) |
|-------------------------|----------------|----------------|----------------|----------------|----------------|
| \( \kappa_1 \) | 1.57827 | 1.57469 | 1.57457 | 1.57593 | 1.57951 |
| \( \kappa_2 \) | 4.71410 | 4.70571 | 4.71336 | 4.71866 | 4.70879 |

The natural frequencies calculated by ANSYS have been brought to a dimensionless form.
Table 3 shows the values of the first and second dimensionless wave numbers for cylinder without cavity and cylinder, weakened by spherical cavity, centered at the point $c_0$.

Based on formulas (1)-(2) by using information on corrections to the wave numbers, the computational experiment of reconstruction parameters of a spherical cavity in the cylinder has been carried out. Table 4 shows the results of recovery center $c_0^*$ and the radius $r_0^*$ of the spherical cavity.

Note that in this case the error recovery parameters of the cavity increases as compared with the one-dimensional model of the rod (maximum error recovery center of the cavity is 24%, the maximum error recovery of the cavity radius is 26%).

**Table 4.** Recovery results of the center and radius of the cavity for the cylinder model ($R = 0.1l$)

|      | $c_0 = 0.1$ | $c_0 = 0.2$ | $c_0 = 0.3$ | $c_0 = 0.8$ | $c_0 = 0.9$ |
|------|-------------|-------------|-------------|-------------|-------------|
| $c_0^*$ | 0.0756      | 0.161       | 0.2218      | 0.7159      | 0.8715      |
| $r_0^*$ | 0.0259      | 0.0272      | 0.0244      | 0.0211      | 0.0202      |

Note, the decreasing ratio of the cylinder radius to the axis length leads to increase accuracy of reconstruction parameters cavity.

Table 5 shows the results of recovery for cylinder (with axis length $l = 1m$, radius of the circle $R = 0.05l = 0.05m$), weakened by spherical cavity (with radius $r_0 = 0.2R = 0.01m$).

**Table 5.** Recovery results of the center and radius of the cavity for the cylinder model ($R = 0.05l$)

|      | $c_0 = 0.1$ | $c_0 = 0.2$ | $c_0 = 0.3$ | $c_0 = 0.8$ | $c_0 = 0.9$ |
|------|-------------|-------------|-------------|-------------|-------------|
| $c_0^*$ | 0.09        | 0.164       | 0.22        | 0.723       | 0.878       |
| $r_0^*$ | 0.0128      | 0.0126      | 0.0124      | 0.01        | 0.0098      |

4. Conclusion
The proposed approach provides the most accuracy of reconstruction cavity parameters, in the case when the cavity center is located close to the free end of the cylinder. In the case when the cavity center is close to the center axis of the cylinder, error recovery parameters cavity increases considerably.

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