Significance Of Deuteron Breakup In A Halo Transfer Reaction

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Abstract. We discuss the quasi-adiabatic approximations to the three-body wavefunction in breakup processes, clarifying the assumptions underlying the model. This suggests alternative approximation schemes. Using different theoretical three-body models, calculated differential cross section angular distributions for the $^{11}\text{Be}(p,d)$ reaction, for which new preliminary data have been reported at 35 MeV, are presented. We show that calculations are sensitive to the inclusion of deuteron breakup and to the breakup model used, particularly if used to deduce absolute spectroscopic information on the $0^+$ and $2^+\ ^{10}\text{Be}$ core state parentages. There is also considerable sensitivity to the model used in calculations of the relative cross sections to the two states.

1 Introduction

In nuclei near the dripline, the separation energy of the last nucleon(s) becomes extremely small. Compared with the common 6-8 MeV in stable nuclei, many dripline nuclei have either a nucleon or a two-nucleon separation energy that is less than 1 MeV. The neutron-density distribution in such loosely bound nuclei shows an extremely long tail, called the neutron halo. Although the density distribution of a halo is very low, it strongly affects the reaction cross section and leads to new properties in such nuclei. The study of halo nuclei is interesting as they involve new structures and surface densities, and thus provide a stringent test of theoretical models of nuclear structure and reactions.

Recently, the use of low energy single nucleon transfer reactions for structure studies of exotic nuclei have attracted attention [1]-[3]. Because of the simplicity of the theoretical interpretation of these reactions, they are thought to provide an important source of the information about the structure of halo nuclei.

Single nucleon transfer reactions, such as the $(d,p)$ and $(p,d)$ reactions, have been a reliable tool in nuclear spectroscopic studies of stable nuclei, determin-
ing positions, spins and parities of nuclear states. Developments in radioactive beams technologies are now producing beams of nuclei at and near the neutron and proton driplines, including neutron halo states with very weak binding. The positions and the ordering of the nuclear single particle levels in such systems, with large neutron excesses, has yet to be clarified. These exotic systems typically have no bound excited states and so traditional spectroscopic methods are inapplicable. Using the single nucleon transfer process, on a deuteron or proton target, in an inverse kinematics experiment, is then an attractive - although still very difficult - alternative tool for mapping out nuclear structures near the driplines.

$^{11}$Be is a good example of such a system having a single neutron separation energy of only 0.5 MeV. It is now understood that the $2s_{1/2}$ neutron single particle state in this region is lowered and that a dominant component of the $^{11}$Be ground state is produced by the coupling of a $2s_{1/2}$ neutron to a $^{10}$Be (g.s., $0^+$) core; with a smaller but significant component in which a $1d_{5/2}$ neutron is coupled to a $^{10}$Be (excited core, $2^+$). This nucleus is of particular theoretical interest because the ground state parity is exactly opposite to what one naively would expect from the spherical shell model. There have been many theoretical attempts to address the problem of parity inversion of the ground state and the first excited state of the $^{11}$Be and evaluating the extension of its neutron halo. Most of them correctly reproduce the parity inversion but make very different predictions about the degree of coupling of the $^{11}$Be ground state with the first $2^+$ excited state of $^{10}$Be at 3.368 MeV, the ratio of spectroscopic factors in these different models $S(2^+)/S(0^+)$ varying from 0.07 to 0.73. In this context, the recent experiment at GANIL is expected to measure the transfer reaction cross section angular distributions at forward angles to the $^{10}$Be ground and $2^+$ states, with the expectation of clarifying quantitatively this $^{11}$Be (g.s.) admixture, and the experimental determination of this ratio of spectroscopic factors by means of a neutron pick-up reactions.

The purpose of the present work is to investigate the importance upon such transfer reaction spectroscopic studies of the inclusion of the deuteron breakup degrees of freedom in the theory used to analyze measured cross section observables. Of importance in analyzing the experimental measurements will be the extent to which the magnitudes of the calculated cross sections, and particularly the ratio of the cross sections to the ground state and $2^+$ core states of $^{10}$Be, are affected by the inclusion of three-body channels.

We discuss the $^{11}$Be (p,d) reaction within different theoretical three-body (n+p+$^{10}$Be) models. We calculate the transfer amplitude using the prior form of the (p,d) matrix element, thus the transition interaction is the n-p interaction and we need a full (three-body) description of the n+p+$^{10}$Be system in the final state. For the description of this final state we have used

i) the adiabatic (AD) model,

ii) the quasi-adiabatic (QAD) model,

iii) the projection operator (POA) approach.
To clarify the importance of the breakup corrections to the transfer cross sections, we also perform Distorted-Waves Born Approximation (DWBA) calculations. For the ground state transition a preliminary test of the models against more exact Coupled Discritized Continuum Channels (CDCC) model calculations has been carried to provide an assessment of the reliability of the three-body models used at the present incident energy.

In Section 2, we also present a re-formulation of the quasi-adiabatic model for nuclear reactions which makes clear the approximations inherent in the model. We show that the so called quasi-adiabatic breakup wavefunction also contains the non-adiabatic corrections to the elastic part of the wavefunction. This was a significant uncertainty in the formulations given in Ref. [6],[7]. In addition, in the same section, an alternative quasi-adiabatic scheme, the projection operator approach (POA), based on the use of projection operators is introduced for a better description of the non-adiabatic elastic channel contributions.

Current experimental activity in the area of light- neutron rich and drip-line nuclei now dictates the rapid development of calculable theoretical models for reactions and scattering of effective few-body systems. Though, the CDCC model has been spectacularly successful in understanding a wide range of data and phenomena in light- and heavy-ion three-body systems, and can provide benchmark calculations against approximate models such as AD, QAD and POA, it is unlikely ever to find an application to the solution of many-body problems with more than three interacting particles. Convergence problems may also become more serious when very weakly bound halo nuclei are involved. To date such systems have been analyzed using few-body Glauber based models [10] whose first step is an adiabatic treatment of the internal degrees of freedom of the projectile. However, the application of the technique at lower projectile energies, or to the case of including Coulomb breakup, has shown that the model begins to break down due to known inadequacy of the adiabatic approximation, see e.g. [8],[9], at such energies. This failure requires the treatment of non-adiabatic effects in the model. Quasi-adiabatic ideas are therefore an obvious and necessary generalization of these and non-eikonal adiabatic models. Hence, we first clarify the formulation of quasi-adiabatic models within the three-body context in the following Section. The calculation methods and the results of our calculations are discussed in Section 4 and 5, respectively.

2 The Three Body Models

The n+p+nucleus Hamiltonian will be written

\[ H = H_{np} + T_R + V_p(\vec{r}_p) + V_n(\vec{r}_n) + V_C(\vec{r}_p) \]  

where \( H_{np} = (T_r + V_{np}) \) is the n-p Hamiltonian, \( T_R \) the center of mass kinetic energy operator and \( V_n \) and \( V_p \) the neutron- and proton-target effective interactions, regarded as local optical model potentials. \( V_C \) is the Coulomb field and is assumed to act on the n-p center of mass.
The total wavefunction $\Psi$ in the exit channel of the pick-up reaction of interest satisfies

$$[E - H_{np} - T_R - V(\tau, \overline{R})] \Psi(\tau, \overline{R}) = 0$$  \hspace{1cm} (2)

$$V(\tau, \overline{R}) = V_p(\tau_p) + V_n(\tau_n) + V_C(\overline{R})$$  \hspace{1cm} (3)

where $\tau$ is the relative coordinate of the n-p pair and $\overline{R}$ is the center of mass coordinate.

In the context of (p,d) reactions $\Psi$ enters the transition amplitude

$$T_{pd} = \langle \Psi(\tau, \overline{R}) | V_{np} [\chi^{-}(\tau_p) \phi_n(\tau_n)] \rangle$$  \hspace{1cm} (4)

with $\phi_n$ the neutron bound state and $\chi^{-}$ incoming proton wave function. Throughout this paper we restrict the formalism to S-wave n-p relative motion for simplicity. In zero-range approximation then it is the wavefunction at coincidence, $\Psi(0, \overline{R})$ which is of importance.

Theoretical methods based on the adiabatic treatment of one or more quantum mechanical degrees of freedom have played an important role in nuclear physics. More specifically, the Johnson-Soper application of adiabatic ideas [5] to nuclear breakup effects played a key role in the development of models of breakup processes in the three-body systems. However, in the AD treatment it is assumed that the excitation of the projectile is to states in the low energy continuum; its treatment of possible high energy breakup contributions is therefore naturally suspect.

An improved treatment of these higher energy breakup configurations is provided by the QAD method calculations which takes approximate account of modifications to the center of mass energy of the n-p pair in breakup configurations through the use of a mean breakup relative energy for the continuum states. It thus breaks the degeneracy with the elastic channel. The inclusion of these higher energy breakup components via the QAD model calculations [7] for large transferred angular momentum (d,p) transfer reactions (at energies $E/A \approx 40$ MeV) led to significant deviations and systematic improvements over the adiabatic model in the description of experimental data.

Although the quasi-adiabatic calculations produced an improved description of the measured observables, the theoretical justifications of the assumptions made in the model have not yet been studied. The work described in this paper is thus concerned also with the clarification of these theoretical uncertainties. In addition, we re-formulate the quasi-adiabatic theory to give a more general formalism, approaching the three-body problem in a different way. This alternative formulation provides a clear understanding of the assumptions made in the original quasi-adiabatic theory.

However, the quasi-adiabatic model does not include back-coupling modifications to the elastic component of the wavefunction. For the inclusion of these modifications, we also develop here an alternative approximation scheme.
for the treatment of quantum mechanical three-body systems using the unified theory of Feshbach.

2.1 Adiabatic Approach

A considerable simplification in the solution of three-body equation is achieved using the Johnson-Soper adiabatic approximation \[5\]. The approximation, in which the dynamics associated with the continuum coupling collapses to an effective two-body problem for the n-p center of mass motion, involves the replacement of \(H_{np}\) in the exact three-body Schrödinger equation, Eq.(2), with \(-\varepsilon_d\), the deuteron binding energy. Therefore we have

\[
\left[ E_{c.m.} - T_R - U(r, \overline{R}) \right] \Psi^{AD}(r, \overline{R}) = 0
\]

where \(E_{c.m.} = E + \varepsilon_d\) is the energy of the outgoing deuteron in the center of mass frame, and \(U(r, \overline{R})\) is the angle average of \(V(r, \overline{R})\) with respect to \(r\).

The assumption made is that the dominant breakup configurations are states of low relative n-p energy \(\varepsilon_k\) when compared \(E\) with, such that little error is made by assuming

\[
E - \varepsilon_k \cong E + \varepsilon_d
\]

In spite of the efficiency and success of the adiabatic approach, experimental transfer reaction data \[12\] clarified that some physical contributions are missing from the calculation of the reaction amplitude. These involve the transfer of particles in large n-p relative energy configurations. As the adiabatic approximation is formulated under the assumption of low energy n-p breakup, its treatment of high energetic contributions is naturally suspect. An improved treatment of the higher energy breakup configurations, such as quasi-adiabatic calculations, is thus required.

2.2 Quasi-adiabatic Model

Under the restrictions to S-wave relative n-p configurations, one separates \(\Psi\) into its elastic and breakup parts. Thus

\[
\Psi^{BU}(r, \overline{R}) = \Psi(r, \overline{R}) - \Psi^{EL}(r, \overline{R})
\]

and therefore

\[
\left[ E - H_{np} - T_R - U(r, \overline{R}) \right] \Psi^{BU}(r, \overline{R}) = \left[ U(r, \overline{R}) - U^{opt}(\overline{R}) \right] \Psi^{EL}(r, \overline{R})
\]

where \(U^{opt}\Psi^{EL} = [E_{c.m.} - T_R]\Psi^{EL}\). At this stage, the quasi-adiabatic model assumes:

1. that \(H_{np}\) is replaced by an average energy depending at most parametrically upon \(r\), and
that the elastic part of the wavefunction generated by the adiabatic model $\Psi^{AD,EL}$ is an accurate representation of $\Psi^{EL}$. Thus the quasi-adiabatic approximation to $\Psi^{BU}$ solves

$$\left[ E - \varepsilon - T_R - U(r, R) \right] \Psi^{QAD,BU}(r, R) = \left[ U(r, R) - U^{AD,opt}(R) \right] \Psi^{AD,EL}(r, R)$$

(9)

where $\varepsilon$ is taken as the expectation value of $H_{np}$ in $\Psi^{AD,BU}$ in each partial wave, and $U^{AD,opt}\Psi^{AD,EL} = [E_{c.m.} - T_R] \Psi^{AD,EL}$.

Thus, the quasi-adiabatic approximation removes the degeneracy of the n-p center-of-mass energy in breakup configurations by introducing a positive mean energy for the continuum states. We note that the model provides, however, no prescription for modifications to the elastic component of the wavefunction.

2.3 An Alternative Formulation Of The Quasi-adiabatic Model

We present here an alternative development of a formal theory for the quasi-adiabatic method, which clarifies that such a model can be introduced by making only one single approximation, unlike the formulations of the model in Ref. [6], [7]. Another advantage of this formulation is that one sees how to treat corrections to both the elastic and breakup components of the wavefunction, and to derive an iterative scheme for such changes. This was a significant uncertainty in the original quasi-adiabatic formulation of Amakawa et al. [6], in which it is stated that the elastic wavefunction is assumed unchanged, regardless of changes made in the breakup piece of the wavefunction.

Under the restriction to S-wave relative n-p configurations, a formal development of the quasi-adiabatic theory proceeds by decomposing the projectile-target three-body wavefunction into the adiabatic wavefunction plus the correction term, i.e.,

$$\Psi(r, R) = \Psi^{AD}(r, R) + \Delta\Psi(r, R)$$

(10)

where $\Delta\Psi$ accounts for non-adiabatic corrections to the breakup and elastic channels, and has only outgoing waves since $\Psi^{AD}$ already satisfies incoming wave boundary conditions. Upon substitution in the Schrödinger equation then,

$$\left[ E - H_{np} - T_R - U(r, R) \right] \Delta\Psi(r, R) = (H_{np} + \varepsilon_d)\Psi^{AD,BU}(r, R)$$

(11)

where the source term has infinite range. To proceed we use the inhomogeneous equation for $\Psi^{AD,BU}$,

$$\Psi^{AD,BU}(r, R) = \left[ E_{c.m.} - T_R - U(r, R) \right]^{-1} \left[ U(r, R) - U^{AD,opt}(R) \right] \Psi^{AD,EL}(r, R)$$

(12)

from which

$$\Delta\Psi(r, R) = \left[ E - H_{np} - T_R - U(r, R) \right]^{-1} (H_{np} + \varepsilon_d) \left[ E_{c.m.} - T_R - U(r, R) \right]^{1/2}$$

$$\times \left[ U(r, R) - U^{AD,opt}(R) \right] \Psi^{AD,EL}(r, R)$$
It follows that
\[
\Delta \Psi(r, R) = \left\{ \left[ E - H_{np} - T_R - U(r, R) \right]^{-1} - \left[ E_{c.m.} - T_R - U(r, R) \right]^{-1} \right\} \times \left[ U(r, R) - U^{AD, opt}(R) \right] \Psi^{AD, EL}(r, R)
\]
and hence we can write \( \Delta \Psi(r, R) = \Delta \Psi_1(r, R) - \Delta \Psi_2(r, R) \) with
\[
\left[ E - H_{np} - T_R - U(r, R) \right] \Delta \Psi_1(r, R) = \left[ U(r, R) - U^{AD, opt}(R) \right] \Psi^{AD, EL}(r, R)
\]
(15)
\[
\left[ E_{c.m.} - T_R - U(r, R) \right] \Delta \Psi_2(r, R) = \left[ U(r, R) - U^{AD, opt}(R) \right] \Psi^{AD, EL}(r, R)
\]
(16)

The equation for \( \Delta \Psi_1 \) leads to the original quasi-adiabatic wavefunction, \( \Delta \Psi_2 \), precisely the adiabatic breakup wavefunction and has no overlap with the elastic channel. Thus \( \Delta \Psi_1 \) must include both elastic and breakup non-adiabatic corrections. The elastic piece can be extracted by projection. This makes clear that the assumption \( \Psi^{EL} \approx \Psi^{AD, EL} \) of the original formulation is unnecessary. Therefore the reduction of the exact three-body equation to the quasi-adiabatic model requires only the replacement of \( H_{np} \) by an average breakup energy \( \overline{\varepsilon} \). The details of the similar resulting partial wave expansions and solution of the equations can be found in Sec. III of Ref. [7].

2.4 Projection-Operator Approach

In the previous analyses, the non-adiabatic elastic corrections are clearly treated incorrectly. They solve equations with the wrong asymptotic energy. In case of deuteron scattering the non-adiabatic elastic corrections are small but in general this may not be the case. A more correct treatment requires use of projection operators on \( P \) and off \( Q \) the projectile ground state wavefunction,
\[
\Psi = P\Psi + Q\Psi = \Psi^{EL} + \Psi^{BU}
\]
(17)

The usual reduction of the Schrödinger equation can be shown \[8\] to yield coupled equations
\[
\left[ E - QH_{np}Q - T_R - U(r, R) \right] \Psi^{BU}(r, R) = \left[ U(r, R) - U^{opt}(R) \right] \Psi^{EL}(r, R)
\]
(18)
\[
\left[ E_{c.m.} - T_R - U_{dd}(R) \right] \Psi^{EL}(r, R) = |\phi_d(r)\rangle \langle \phi_d(r)| U(r, R) |\Psi^{BU}(r, R)\rangle
\]
(19)

with \( U_{dd} \) the Watanabe potential and \( \phi_d \) the deuteron ground state wavefunction. Again at this stage \( H_{np} \) is to be replaced with an average excitation energy \( \overline{\varepsilon} \), in each partial wave. The equations can be solved by iteration \[8\] starting from the adiabatic model estimates.
3 Experimental Aspect Of The Reaction

In the past, transfer reactions with light and heavy ions have been a major source of spectroscopic information in stable nuclei, including spin and parity assignments to nuclear levels, measurements of occupation probabilities and wave functions in the ground and excited states of the daughter system. Provided that beams of exotic nuclei with high enough luminosity will become available it is tempting to use transfer reactions in the same spirit also for spectroscopic studies in exotic nuclei. The important difference to former fixed target experiments is the low intensity of secondary beams and the use of inverse kinematics. Under these conditions only transfer reactions with rather large cross sections can be investigated.

Since exotic beams are obtained as secondary beams they depend on the production rates for unstable nuclei in primary reactions with a stable beam. As a consequence, their intensities are reduced by several orders of magnitude with respect to the intensities of the primary beam. As a rough rule, the production cross sections for these beams fall by approximately one order of magnitude for each mass unit going further away from stability [2]. Thus, one is restricted to reactions with larger cross sections. Targets should be chosen as thick as possible and detectors with large solid angles and high detection efficiency should be used.

Obviously, the abundance of data to which one is accustomed from work with stable beams can certainly not be expected for radioactive beams. In reactions with inverse kinematics the cross-sections are focussed more in forward direction in the laboratory system. Thus, the full yield of the nucleus to be investigated is collected in a rather small angular cone and total cross section measurements should already be feasible at much lower beam intensities.

As a representative example, we consider here an inverse kinematics reaction with beam of neutron rich nucleus $^{11}$Be. The $^{11}$Be (p,d) $^{10}$Be reaction, leading to bound states in $^{10}$Be, has been studied for the first time [1],[2], using a secondary $^{11}$Be beam of 35 MeV/nucleon. Angular distributions up to about 15 degrees (in c.m.) were measured by detecting $^{10}$Be in a spectrometer and coincident deuterons in a position sensitive silicon detector array. Their preliminary analysis provides evidence for a large core excitation component in the structure of $^{10}$Be ground state.

4 Calculations

In the present work, the $^{11}$Be system is interpreted as a $^{10}$Be core, with one excitable state, and a loosely bound neutron. We have only included the $0^+$ ground state and $2^+$ excited state of the core. That means, in the ground state of $^{11}$Be, the neutron oscillates between a $2s_{1/2}$ and $1d_{5/2}$ orbital, exciting the core from a $0^+$ to a $2^+$ state (and de-exciting it).

Since a proper model provides modifications to the three-body deuteron-channel wavefunction, one expects the calculations resulting from this method to improve upon the predictions on reaction observables. Thus, in order to
establish the significance of the modifications introduced by the models, and for future comparison to the experimental data, we have evaluated differential cross-section angular distributions at 35 MeV incident energy for the $^{11}\text{Be}$ ($p,d\ 10\text{Be}$ (g.s and $2^+$) transitions and the trends of these observables have been compared with each other.

Three-body wavefunctions in the exit channel of the reaction are calculated using the adiabatic, quasi-adiabatic, projection operator and CDCC methods (the later using the code FRESCO [13]). The main objective of the CDCC calculations, which do not include spin-orbit interactions due to the difficulty in including this term in FRESCO, is to provide a critique of the theories implemented here, rather than to seek a realistic comparison with experiment.

The preliminary test (for $2s_{1/2}$ transition) of the AD and extended AD models using the CDCC calculations, discretizing the n-p continuum into 6 bins from 0 to 25 MeV excitation energy, provide an insight into their reliability at the lower incident proton energy of interest.

The quasi-adiabatic and projection operator model calculations are iterated in the sense that the mean breakup energy for the continuum, $\Xi^{(i)}_{JL}$, in each partial wave is calculated from the latest best estimate of the breakup wavefunction, i.e.

$$\Xi^{(i)}_{JL}(R) = \frac{\langle (i)\Psi_{JL}^{BU}(r,R) | H_{np} | (i)\Psi_{JL}^{BU}(r,R) \rangle_r}{\langle (i)\Psi_{JL}^{BU}(r,R) | (i)\Psi_{JL}^{BU}(r,R) \rangle_r}$$

where bra-ket denotes the radial integration over $r$ and $(i)$ represents the iteration number. Although this prescription reproduces meaningful breakup energies at medium incident projectile energies for heavy targets [8], it breaks down at lower incident energies and cannot be iterated due to the calculated unphysical $\Xi_{JL}$ values which are larger than the center of mass energy. To overcome this problem for the underlying reaction, we consider another formulation for the mean breakup energies.

Starting with an exact definition of the continuum channel breakup wavefunction in partial wave form ,

$$\Psi_{JL}^{BU}(r,R) = \int_0^\infty dk \phi_k(r) \chi_{JLk}(R)$$

where $\phi_k(r)$ is a triplet n-p scattering state with asymptotic normalization

$$\phi_k(r) \longrightarrow \left( \frac{2}{\sqrt{\pi}} \right) \frac{\sin(kr + \delta_0)}{r}$$

such that $\int_0^\infty dr r^2 \phi_k(r)\phi_{k'}(r) = \delta(k-k')$, one can rearrange the mean energy prescription, Eq. (20), in the form

$$\Xi_{JL}(R) = \frac{\langle \Psi_{JL}^{BU} | H_{np} | \Psi_{JL}^{BU} \rangle_r}{\langle \Psi_{JL}^{BU} | \Psi_{JL}^{BU} \rangle_r}$$
\[
\begin{align*}
\langle \int_0^\infty dk \phi_k(r) \chi_{JLk}(R) \rangle &= \langle \int_0^\infty dk \phi_k(r) \chi_{JLk}(R) \rangle_r \\
&= \frac{\int_0^\infty dk \chi_{JLk}^*(R) \varepsilon_k \chi_{JLk}(R)}{\int_0^\infty dk \chi_{JLk}^*(R) \chi_{JLk}(R)}
\end{align*}
\]

where * denotes complex conjugate and \( H_{np} \phi_k(r) = \varepsilon_k \phi_k(r) \) in which \( \varepsilon_k = \frac{\hbar^2 k^2}{2\mu_{np}} \). One needs at this stage to evaluate the \( \chi_{JLk} \), by integration

\[
\chi_{JLk}(R) = \int_0^\infty dr r^2 \phi_k(r) \Psi_{JL}^{BIU} (R)
\]

where \( \Psi_{JL}^{BIU} \) is calculated approximately by the QAD and POA theories. To be in consistent with the CDCC calculations carried out in this work, we set the maximum value of \( k \) for the integral in Eq. (23) to 0.78 fm\(^{-1}\) that corresponds to 25 MeV for the relative breakup energies.

The calculated \( \chi_{JLk} \) values show significant changes from the zeroth order estimate based on \( \Psi_{AD,BU} \) but beyond the next iteration such changes are not reflected in changes in the calculated wavefunctions or in reaction observables.

The calculated three-body wavefunctions using the same inputs, in partial waveform, provided by the present models are employed in a modified version of the program TWOFNR \[14\] for the evaluation for the reaction observables, performing the zero-range approximation. The zero-range approximation is expected to produce a more accurate model for in particular \( 2s_{1/2} \) state since there is a significant probability of the neutron in \( ^{11}\text{Be} \) being near the \( ^{10}\text{Be} \) core. Also, comparisons with full finite range calculations in the DWBA and CDCC cases, where the n-p interaction was taken to be a central Hulthen potential, showed this to be a reasonable first approximation.

The entrance and exit channel potentials are obtained from the global parameterization of Bechetti and Greenless \[15\]. We note however at this point that for the rigorous reproduction of experimental data, one needs to consider different combinations of optical potentials for the proton and deuteron channels in the calculations in order to test the sensitivity of extracted spectroscopic factors to the input parameters. Despite the uncertainties on optical potentials used in our calculations, which in fact are important for a precise description of transfer cross sections, the results can however be expected to describe realistically the dynamical features of such reactions. A clear improvement for future work and the analysis of data would be achieved by measuring elastic scattering together with transfer reactions such that empirical information on optical potentials is obtained.

The radial integrals are carried out from 0 to 35 fm in steps of 0.1 fm. The maximum number of partial waves used is 30 for both entrance and exit channels. The transferred neutron wavefunction is evaluated in a Woods-Saxon well with shape parameters \( r=1.25 \) fm and \( a=0.65 \) fm. The real well depth is adjusted to reproduce the neutron separation energy. The spin-orbit force in the proton channel is fixed at 6 MeV. The spectroscopic factors are set to unity throughout the calculations.
All calculations presented here are done without non-locality corrections. Such corrections for halo transfer are expected to be small because they correct the transition amplitude in the nuclear interior, but the long tail of the halo wavefunction makes internal contributions less important.

5 Results And Discussion

From the results shown in Figs. ?? and ??, it is clear that the inclusion of deuteron breakup is of importance for extracting information on the 0\(^+\) and 2\(^+\) states. The calculated cross-section angular distributions also show significant differences when using the different breakup treatments. In addition, cross-sections of the \(^{11}\text{Be}(p,d)^{10}\text{Be}\) (g.s., 0\(^+\)) reaction calculated with different three-body models are larger but decrease faster at forward angles when compared with the calculations using DWBA which do not account for the effects arising from the breakup of the deuteron in the field of the nucleus. To provide an assessment of the three-body models (including spin-orbit interactions) used here we have compared in Fig. ?? the calculations (for the 2s\(_{1/2}\) transition) with those obtained using the CDCC technique (excluding spin-orbit forces). Based on this comparison, the AD, QAD and POA are reliable tools for the spectroscopic study of the reaction of interest. The angular distributions of the transfer reactions to the 2\(^+\) state of \(^{10}\text{Be}\) at small angles look the same both with and without breakup effects but have different absolute values.

Our calculations have demonstrated that the inclusion of breakup effects in transfer reactions produces changes in the shape of the angular distributions and these effects increase the absolute values of the theoretical cross-sections in forward direction and thus lead to the smaller values of the spectroscopic factors extracted from the experimental data. It is expected that such effects will be larger when the incident energy increases and the mass of the target decreases.

We therefore conclude that the standard procedure for the determination of spectroscopic factors as ratios of the experimental transfer cross-sections to those calculated within the standard DWBA is not reliable for the reactions involving weakly bound halo nuclei. The spectroscopic factors extracted from the experimental data would be smaller than those obtained with the conventional DWBA, due to including breakup effects.

Our results also suggest that deuteron breakup stronger for 2s\(_{1/2}\) transfer than for 1d\(_{5/2}\) transfer. This may be associated with the node in the bound state wavefunction in the 2s\(_{1/2}\) case. In the case of the 2s\(_{1/2}\) transfer the shape of the differential cross-sections changes more strongly at small angles compared to the 1d\(_{5/2}\) transfer. This should influence the ratio of the spectroscopic factors for 0\(^+\) and 2\(^+\) states of the \(^{10}\text{Be}\) obtained with different theoretical models.

Fig. ?? shows the ratio of the calculated cross-sections relevant to deducing only relative spectroscopic information. The figure leads us to the conclusion that ratios of spectroscopic factors depend on possible uncertainties on absolute cross-section values, and are typically dependent upon the ingredients of
reaction calculations with respect to scattering angles.

In the broad field of today’s nuclear structure research, there is still very much to be understood when we analyze light exotic nuclei. Our intention for the near future is to have a better insight into the physical nature of the less known halo systems, using the models employed in this work. In particular, the question about the halo nature of $^{19}$C and its underlying structure is one of the interesting current questions in dripline physics. More information on this carbon isotope will allow us to further explore the characteristics of the halo phenomenon and to test the concepts developed for $^{11}$Be.
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Figure Captions

Fig. ?? Calculated differential cross-section angular distributions for the $^{11}\text{Be}(p,d)^{10}\text{Be} \text{ (g.s.)}$ reaction at 35 MeV using different theoretical models. The spectroscopic factor is 1.0 for all calculations.

Fig. ?? As for Fig. ??, but for the $^{11}\text{Be}(p,d)^{10}\text{Be} \text{ (2}^+, 3.368 \text{ MeV)}$ state transition.

Fig. ?? The ratio of the calculated relative cross-sections to the $2^+$ and $0^+$ $^{10}\text{Be}$ final states.