ON THE ORIGIN OF THE DISTRIBUTION OF BINARY STAR PERIODS

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ABSTRACT

Pre-main-sequence and main-sequence binary systems are observed to have periods, \(P\), ranging from 1 day to \(10^{10}\) days and eccentricities ranging from 0 to 1. We pose the problem of whether stellar-dynamical interactions in very young and compact star clusters will broaden an initially narrow period distribution to the observed width. \(N\)-body computations of extremely compact clusters containing 100 and 1000 stars initially in equilibrium and in cold collapse are performed. In all cases, the assumed initial period distribution is uniform in the narrow range \(4.5 \leq \log P \leq 5.5\) (in days), which straddles the maximum in the observed period distribution of late-type Galactic field dwarf systems. None of the models lead to the necessary broadening of the period distribution, despite our adopted extreme conditions that favor binary-binary interactions. Stellar-dynamical interactions in embedded clusters thus cannot, under any circumstances, widen the period distribution sufficiently. The wide range of orbital periods of very young and old binary systems is therefore a result of cloud fragmentation and immediate subsequent magnetohydrodynamic processes operating within the multiple protostellar system.

Subject headings: binaries: general — methods: \(N\)-body simulations — open clusters and associations: general — stars: formation — stars: late-type

1. INTRODUCTION

The distribution of orbital parameters of binary systems imposes important constraints on the theory of star formation. In particular, the distribution of eccentricities and periods of late-type Galactic field binary systems are sufficiently well observed to address the issue of their origin.

The observed eccentricity distribution is approximately thermal \((f_e \sim 2e\), with \(f_e\) being the number of orbits in the interval \(e\) to \(e + de\)) for binaries with periods \(P \lesssim 10^3\) days, but for systems with \(P \gtrsim 10^3\) days, \(e\) and \(P\) are correlated such that smaller values of \(P\) imply, on average, smaller \(e\). For Galactic late-type dwarfs, the distribution of periods, \(f_P\), is lognormal in \(P\), with the notable feature that \(P\) ranges from 1 to \(10^{10}\) days with a mean log of \(P \approx 4.8\) (see Fig. 1 in Kroupa 1995a). Pre-main-sequence binaries show the same wide range of parameters and correlations (Mathieu 1994).

The available cloud-collapse calculations have not been able to reproduce the wide range of observed periods and, in particular, do not lead to short-period \((P \lesssim 10^3\) days) systems (see Bodenheimer et al. 2000 for a review). However, even the most recent numerical refinements cannot describe stellar formation to the point where gas dynamical processes can be neglected, so the final theoretical orbital parameters of binary systems cannot be quantified. We do not yet know for certain whether there exists some significant magnetohydrodynamic mechanism that is important in transforming a theoretical period distribution that results from fragmentation to the final, wide \(f_P\) observed already among 1 Myr old populations. Alternatively, it may be possible that no such mechanism is needed, and that the final (observed) orbital parameters of Galactic field systems are a result of gravitational encounters in very dense embedded clusters that disperse rapidly.

It has already been demonstrated that stellar-dynamical interactions in typical embedded clusters spanning a wide range of densities cannot significantly widen a period distribution. Too few orbits are redistributed to \(P < 10^3\) days by gravitational encounters, assuming that the primordial period distribution is confined to the range \(10^3 \rightarrow 10^7\) days. Similarly, typical embedded clusters cannot evolve an arbitrary eccentricity distribution to the thermal form (Kroupa 1995b).

The purpose of the present paper is to study the evolution of a range of extremely dense clusters to answer the question once and for all whether encounters within a dense cluster environment can significantly contribute to the observed width of \(f_P\). Section 2 briefly describes the codes used for the \(N\)-body computations and the data reduction, and the initial conditions. The results are presented in § 3, and conclusions follow in § 4.

2. CODES AND INITIAL CONDITIONS

2.1. The \(N\)-Body Program

A direct \(N\)-body code must deal efficiently with a range of dynamical timescales spanning many orders of magnitude, from days to hundreds of million years. The code of choice is Aarseth's NBODY6 (Aarseth 1999). NBODY6 uses special mathematical techniques to transform the spacetime coordinates of closely interacting stars, which may be perturbed by neighbors, such that the resulting equations of motion of the subsystem are regular (Mikkola & Aarseth 1993). State-of-the-art stellar evolution is also incorporated (Hurley, Pols, & Tout 2000), as well as a standard Galactic tidal field (Terlevich 1987).

The velocity and position vectors of any individual center-of-mass particle (e.g., a star or binary system) diverge exponentially from the true trajectory through the growth of errors in \(N\)-body computations (see, e.g., Goodman, Heggie, & Hut 1993). However, statistical results from \(N\)-body calculations correctly describe the overall dynamical evolution, as shown, for example, by Giersz & Heggie (1994), who compare ensembles of \(N\)-body computations with statistical stellar-dynamical methods. Thus, for each model constructed here an ensemble is created in order to obtain reliable estimates of the relevant statistical quantities.

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TABLE 1

Cluster Model Initial Conditions

| Model   | N   | $R_{0.5}$ (pc) | $\langle m \rangle$ ($M_\odot$) | $\sigma_{3D}$ (km s$^{-1}$) | $\log t_{cr}$ (Myr) | $\log \rho_C$ (stars pc$^{-3}$) | $M_{cl}$ ($M_\odot$) | $N_{run}$ | Dynamic State |
|---------|-----|----------------|---------------------------------|-----------------------------|---------------------|-------------------------------|----------------|-----------|---------------|
| N2v     | 10^2 | 0.00287        | 0.37                            | 4.59                        | -2.90               | 9.4                           | 37             | 10        | VE            |
| N2v1    | 10^2 | 0.0040         | 0.30                            | 3.87                        | -2.68               | 8.9                           | 30             | 10        | VE            |
| N3v     | 10^3 | 0.0287         | 0.37                            | 4.59                        | -1.90               | 7.4                           | 370            | 5         | VE            |
| N2c     | 10^2 | 1.2            | 0.37                            | 0.0                         | +0.69               | 1.1                           | 37             | 10        | CC            |
| N3c     | 10^3 | 1.2            | 0.37                            | 0.0                         | +0.37               | 2.1                           | 37             | 5         | CC            |

Note.—N is the total number of stars and $M_\odot$ the cluster mass. $R_{0.5}$ is the half-mass radius of the Plummer density distribution for the models that are initially in virial equilibrium (VE). It is the radius of the homogenous sphere for the models that undergo cold collapse (CC) and have a vanishing initial three-dimensional velocity dispersion $\sigma_{3D}$. The central density is $\rho_C$. All models except N2v1 have stars in the mass range 0.01–50 $M_\odot$, giving a mean stellar mass of $m = 0.37 M_\odot$. Model N2v1 contains only stars with $0.08 M_\odot \leq m \leq 1.1 M_\odot$ having $m = 0.30 M_\odot$. It contains no brown dwarfs and massive stars and thus provides comparison data for the other, more realistic cases. No significant differences emerge. In the VE models, $t_{cr}$ is the crossing time, whereas in the CC models, $t_{cr}$ is the time until maximum contraction. Each model is computed $N_{run}$ times, each with a different random number seed.

The output from NBODY6 is analyzed by a software package that finds all bound binary systems and allows the construction of distribution functions of orbital parameters, among many other things.

2.2. Initial Binary Systems

Initial stellar masses are distributed according to a three-part power-law initial mass function (IMF) (Kroupa 2001b) $\xi(m) \propto m^{-\alpha}$, where $\alpha = 0.3$ for $0.01 M_\odot \leq m < 0.08 M_\odot$, $\alpha = 1.3$ for $0.08 M_\odot \leq m < 0.5 M_\odot$, and $\alpha = 2.3$ for $0.5 M_\odot \leq m$, and where $\xi(m) dm$ is the number of stars in the mass range $m$ to $m + dm$.

The total binary proportion is

$$f_{tot} = \frac{N_{bin}}{N_{bin} + N_{sing}},$$

where $N_{bin}$ and $N_{sing}$ are the number of binary and single star systems, respectively. Initially, all stars are assumed to be in binary systems ($f_{tot} = 1$) with component masses $m_1, m_2$ chosen randomly from the IMF.

The initial logarithmic binary star periods are distributed uniformly in the narrow interval $4.5 \leq \log P \leq 5.5$, with $P$...
in days. The period distribution of late-type systems is

\[ f_P = \frac{N_{\text{bin},P,lt}}{N_{\text{bin},lt} + N_{\text{sing},lt}}, \tag{2} \]

where \( N_{\text{bin},P,lt} \) is the number of binaries with orbits in the interval \( \log P \) to \( \log P + d \log P \), in which the primary has a mass \( 0.08 M_\odot \leq m \leq 1.5 M_\odot \), and \( N_{\text{sing},lt} \) and \( N_{\text{bin},lt} \) are the number of single stars and binaries with primaries, respectively, in this mass interval. The initial (\( t = 0 \)) period distribution is given by

\[ f_P \begin{cases} 0.5, & \text{if } 4.5 \leq \log P \leq 5.5, \\ 0.0, & \text{otherwise}, \end{cases} \tag{3} \]

the period distribution being constructed using decade intervals in \( P \) (see Fig. 4 below). All binary systems initially have orbits with eccentricity \( e = 0.75 \), but the results are not sensitive to this value (Kroupa 1995b).

These assumptions allow us to test the hypothesis that binary-binary and binary-single-star encounters in very compact young clusters widen the period distribution to the form observed in the Galactic field and redistribute the eccentricities to give the thermal distribution observed for Galactic field systems.

2.3. Cluster Models

Clusters initially in virial equilibrium (VE) and in cold collapse (CC) with \( N = 100 \) and 1000 stars are set up to cover a range of extreme initial conditions (see Table 1).

The VE models are spherical Plummer number-density profiles (Aarseth, Hénon, & Wielen 1974) initially, with position and velocity vectors not correlated with the system masses. The initial half-mass radius, \( R_{0.5} \), is chosen to give a three-dimensional velocity dispersion, \( \sigma_{3D} \), that equals the orbital velocity of a binary with a system mass of 1 \( M_\odot \) and a period of \( P = 10^3 \) days, which is near the maximum in the G dwarf period distribution of Duquennoy & Mayor (1991). The central densities of these models are extreme and probably unrealistic, since many of the binary systems overlap. However, these models allow us to study the very extreme situation in which binary-binary interactions dominate initial cluster evolution and will thus impose limits on the possible widening of the period distribution as a result of these interactions.

The CC models are uniform spheres, each cluster having no initial velocity dispersion and uncorrelated position vectors and system masses. The increased binary-binary interactions near maximum collapse may widen the period distribution to the observed values.

3. RESULTS

3.1. Cluster Evolution

The evolution of the clusters is exemplified by evaluating the core radius, \( R_C(t) \) (see, e.g., Kroupa, Aarseth, & Hurley 2001). This quantity measures the degree of concentration of a cluster. The central number density is the density within \( R_C \), counting all stars and brown dwarfs. The evolution of both quantities is shown in Figure 1.

The core radius of the VE models increases immediately as a consequence of binary star heating for the highly concentrated N2v and N2v1 models, whereas it decreases in the less concentrated N3v model during the first few \( t_{cr} \) as a result of mass segregation. When energy rearrangement through mass segregation has ended, the most massive stars having reached the center, \( R_C \) also expands in this model. It is interesting to note that the increase follows a power law, \( R_C \propto t^{3.2} \) with \( r_C \approx 0.95 \), in all cases. The central density decreases as a power law, \( \rho_C \propto t^{3.2} \) with \( r_C \approx -2.2 \). The decay in \( \rho_C \) sets in immediately in models N2v and N2v1 but is retarded in model N3v as a result of the settling of the heavy stars.

The CC models contract homologously until local density fluctuations have increased sufficiently to form one dominating potential well, thus focusing the further radial flow (Aarseth, Lin, & Papaloizou 1988). At this point, \( \rho_C \) begins to increase until it reaches a maximum that defines the time of maximum contraction, \( R_C \) being smallest then. After violent relaxation the compacted clusters evolve as the VE models, albeit with a time lag through the collapse.

The clusters can be taken to be dissolved when \( \rho_C \approx 5 \) stars pc\(^{-3} \). The “final” distributions of orbital parameters discussed below are evaluated approximately at this time, taking account of all stars, when no further stellar-dynamical or stimulated evolution of the binary population occurs.

3.2. The Binaries

The evolution of \( f_{bin} \) is plotted for all models in Figure 2. As noted above, the binaries initially overlap in the VE models, causing immediate disruption of the widest systems and leading to a population of single stars in the cluster. Further binary depletion occurs on a crossing timescale. Meanwhile, the cluster expands (Fig. 1), slowing further binary disruption until it is virtually halted at the point when the remaining binary population is hard in the
4. Final period distributions. For model N2v1, and are plotted as thick and thin dotted lines, respectively (see text). The initial distributions are given by eq. (3). Circles show the G dwarf period distribution from Duquennoy & Mayor (1991). M and K dwarfs have indistinguishable distributions (see, e.g., Fig. 1 in Kroupa 1995a).

Figure 4 demonstrates that the initial distribution cannot evolve through stellar-dynamical interactions to the observed, broad, lognormal distribution. This is the case even under the extreme assumptions that the evolution of the N2v1 cluster is halted prematurely (for example, through the expulsion of the remnant natal gas), and that the single-star population that emerges from the disruption of the binaries is lost preferentially through ejection and mass segregation before gas expulsion. In this thought experiment, \( f_P \) will appear enhanced, since \( f_P = \eta N_{\text{bin}, \text{lt}} + N_{\text{sing}, \text{lt}} \) (see eq. [2]). In Figure 4, \( f_P \) is plotted for model N2v1, but even this artificially enhanced period distribution disagrees with the observed distribution.

4. CONCLUSION

The finding is thus that stellar-dynamical interactions in compact star clusters cannot change an initially delta-eccentricity distribution and narrow period distribution to the thermal eccentricity and wide logarithmic period distribution observed in the Galactic field. It follows that the main characteristics of these distributions (thermal \( f_e \) and orbits ranging from days to millions of years) must be a result of fragmentation during star formation and subsequent magnetohydrodynamic processes.

The N-body calculations were performed on computers at the Institute for Theoretical Astrophysics, Heidelberg University, using a variant of Aarseth’s NBODY6 code.
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