Current status of $\varepsilon_K$ with lattice QCD inputs

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We present the Standard Model evaluation of the indirect CP violation parameter $\varepsilon_K$ using inputs determined from lattice QCD together with experiment: $|V_{us}|$, $|V_{cb}|$, $\xi_0$, and $\tilde{B}_K$. We use the Wolfenstein parametrization ($|V_{cb}|$, $\lambda$, $\tilde{\rho}$, $\tilde{\eta}$) for the CKM matrix elements. For the central value, we take the angle-only fit of the UTfit collaboration, and use $|V_{us}|$ from the $K_{L2}$ and $K_{L\Lambda}$ decays as an independent input to fix $\lambda$. For the error estimate, we use results of the global unitarity triangle from the CKMfitter and UTfit collaborations. We find that the Standard Model (SM) prediction of $\varepsilon_K$ with exclusive $V_{cb}$ (lattice QCD results) is lower than the experimental value by 3.6(2)$\sigma$. However, with inclusive $V_{cb}$ (results of the heavy quark expansion), the tension between the SM prediction of $\varepsilon_K$ and its experimental value disappears.

CP violation in nature was first discovered in 1964 [1]. The neutral kaon system has two kinds of CP violation: one is the indirect CP violation due to CP-asymmetric impurity in the kaon eigenstates in nature, and the other is the direct CP violation due to the CP violating nature of the weak interaction [2, 3]. Here, we focus on the indirect CP violation, which is parametrized by $\varepsilon_K$.

$$\varepsilon_K = A(K_L \to \pi\pi(I = 0)) / A(K_S \to \pi\pi(I = 0)),$$

where $K_L$ and $K_S$ are the neutral kaon eigenstates in nature, and $I = 0$ is the isospin of the final two-pion state. In experiment [4],

$$\varepsilon_K = (2.228 \pm 0.011) \times 10^{-3} \times e^{i\phi_\varepsilon},$$

$$\phi_\varepsilon = 43.52 \pm 0.05^\circ.$$ (2)

In the Standard Model (SM), the CP violation comes solely from a single phase in the CKM matrix elements [5]. The mixing of neutral kaons is allowed through the box diagrams which describe the mass splitting $\Delta M_K$ and $\varepsilon_K$ [6] [7]. In the SM, the master formula for $\varepsilon_K$ is

$$\varepsilon_K = e^{i\theta} \sqrt{2} \sin \theta \left(C_e \tilde{B}_K X_{SD} + \xi_0 + \xi_{LD}\right) + O(\omega\varepsilon') + O(\xi_0\Gamma_2/\Gamma_1),$$ (3)

where $C_e$ and $X_{SD}$ are defined as follows.

$$C_e = \frac{G_F^2 F_K^2 m_{K^0} M_W^2}{6\sqrt{2}\pi^2 \Delta M_K},$$

$$X_{SD} = \tilde{\eta} \lambda^2 |V_{cb}|^2 \left[|V_{cb}|^2 (1 - \tilde{\rho}) \eta_{tt} S_0(x_t)(1 + \bar{r}) + \left(1 - \frac{\lambda^4}{8}\right) \left\{\eta_{cc} S_0(x_c, x_t) - \eta_{cs} S_0(x_c) - 2\eta_{ct} S_0(x_c, x_t)\right\}/\{\eta_{tt} S_0(x_t)\}\right].$$ (5)

where $S_0$'s are the Inami-Lim functions [8], and $x_i \equiv m_i^2/M_W^2$ with $i = c, t$. Here, $r = \{\eta_{cc} S_0(x_c) - 2\eta_{ct} S_0(x_c, x_t)\}/\{\eta_{tt} S_0(x_t)\}$. $\lambda$, $\tilde{\rho}$, and $\tilde{\eta}$ are the Wolfenstein parameters of the CKM matrix elements [7]. Here, we replace $B$ by $V_{cb}$, using the relation $|V_{cb}| = \lambda^2 + O(\lambda^4)$. $\eta_{ij}$ with $i, j = c, t$ represents the QCD corrections to the box diagrams. $\xi_0 = \Im A_{0}/Re A_0$ represents the long distance effect from the absorptive part, and $\xi_{LD}$ corresponds to the long distance effect from the dispersive part [9]. The correction terms $O(\omega\varepsilon')$ and $O(\xi_0\Gamma_2/\Gamma_1)$ are of order $10^{-7}$, which we neglect in this paper. The master formula of Eq. (3) is essentially the same as that of Ref. [10]. Details on how to derive Eq. (3) from the SM are given in our companion paper [11].

In Eq. (3), the major contribution to $\varepsilon_K$ comes from the $B_K$ term, and a minor contribution of about $-7\%$ comes from the $\xi_0$ term. The remaining contribution of $\xi_{LD}$ is about $2\%$, coming from the long distance effect on $\varepsilon_K$ [11] [12]. In this paper, we neglect $\xi_{LD}$ without affecting our conclusion.

The Wolfenstein parameters $\lambda$, $\tilde{\rho}$, $\tilde{\eta}$ and $A$ can be obtained from the global unitarity triangle (UT) fit. Here, we use $\lambda$, $\tilde{\rho}$, $\tilde{\eta}$ from the CKMFitter [13] [14] and UTfit collaborations [15] [16]. They are summarized in Table I.

The parameters $\varepsilon_K$, $\tilde{B}_K$, and $V_{cb}$ are inputs to the global UT fit. Hence, the $\lambda$, $\tilde{\rho}$, $\tilde{\eta}$ parameters extracted from the global UT fit of the CKMFitter and UTfit groups contain unwanted dependence on $\varepsilon_K$, $\tilde{B}_K$, and $V_{cb}$. Therefore, in order to determine $\varepsilon_K$ self-consistently, we take another input set from the angle-only-fit (AOF) in Ref. [17]. Here the advantage is that the AOF does not use $\varepsilon_K$, $\tilde{B}_K$, and $V_{cb}$ as inputs to determine the UT apex parameters $\tilde{\rho}$ and $\tilde{\eta}$. The AOF gives the UT apex ($\tilde{\rho}, \tilde{\eta}$) but not $\lambda$. We can take $\lambda$ independently from the CKM matrix element $V_{us}$, using the relation: $|V_{us}| = \lambda + O(\lambda^3)$. Here, the $K_{L3}$ and $K_{L\Lambda}$ decays are used to set $V_{us}$ [4].

In Table I, we summarize the input values for $V_{cb}$. In the inclusive channel, they use $B \to X_s l\nu$, and $B \to X_s \gamma$ decays. They also use moments of outgoing lepton energy, hadron masses, and photon energy and fit them to the theoretical expressions which come from the op-
We use the results of the kinetic scheme to calculate \( \varepsilon_K \) and the 1S scheme \([4]\). Here, we follow the renormalization group (RG) evolution for \( \eta_{cc} \) given in Ref. \([32]\). Hence, in order to check the claim, we follow the renormalization group (RG) evolution for \( \eta_{cc} \) described in Ref. \([32]\) to produce the NNLO value of \( \eta_{cc} \). The results are summarized in Table \( \text{V} \). In this table, note that the results are consistent with one another within the systematic errors, but our \( \eta_{cc} \) value is essentially identical to that of Ref. \([33]\). Details of the SWME result are explained in Ref. \([9]\). In this paper, we use the SWME result for \( \eta_{cc} \) to obtain \( \varepsilon_K \).

The input values for \( \eta_{ij} \) that we use in this paper are summarized in Table \( \text{V} \).

The remaining input parameters are summarized in Table \( \text{VI} \).

Let us define \( \varepsilon_K^{\text{SM}} \) as the theoretical evaluation of \( |\varepsilon_K| \) obtained using the master formula Eq. \([3]\) directly from

| TABLE I. Wolfenstein Parameters |
|----------------------------------|
| CKMfitter | UTfit | AOF |
| \( \lambda \) | 0.22535(65) \([3]\) | 0.22535(65) \([3]\) | 0.2252(9) \([4]\) |
| \( \rho \) | 0.131 \pm 0.026 \([25]\) | 0.136(18) \([4]\) | 0.130(7) \([17]\) |
| \( \eta \) | 0.345 \pm 0.014 \([25]\) | 0.348(14) \([4]\) | 0.338(16) \([17]\) |

For the kaon bag parameter \( B_K \), we use the FLAG average \([24]\) and the SWME results \([25]\) which are summarized in Table \( \text{III} \). The SWME result for \( B_K \) with \( N_f = 2 + 1 \) \([26,29]\) to obtain the average. The SWME result \([25]\) has a larger error, and its value deviates most from the FLAG average.

| TABLE II. Inclusive and exclusive \( |V_{cb}| \) in units of \( 10^{-3} \). Here, Kin. represents the kinetic scheme in heavy quark expansion, and 1S the 1S scheme. |
|----------------------------------|
| Inclusive (Kin.) | Inclusive (1S) | Exclusive |
| \( 42.21(78) \) \([15]\) | 41.96(45)(07) \([4]\) | 39.04(49)(53)(19) \([28]\) |

The RBC/UKQCD collaboration provides lattice results for \( \text{Im}A_2 \) and \( \xi_0 \) in Ref. \([30]\). Here, we use their result of \( \xi_0 = -1.63(19)(20) \times 10^{-4} \) obtained using the experimental value of \( \varepsilon'/\varepsilon \).

The factor \( \eta_{tt} \) is given at next-to-leading order (NLO) in Ref. \([10]\). The factor \( \eta_{ct} \) is given at next-to-next-to-leading order (NNLO) in Ref. \([31]\). The factor \( \eta_{cc} \) is given at NNLO in Ref. \([32]\). In Ref. \([33]\), they claim that the error is overestimated for the NNLO value of \( \eta_{cc} \) given in Ref. \([32]\). Hence, in order to check the claim, they use perturbative expansion, and 1S the 1S scheme.

\[
\begin{align*}
\varepsilon_K &= 0.766(99) \quad \text{FLAG} \\
\varepsilon_K &= 0.7379(47)(365) \quad \text{SWME}
\end{align*}
\]

| TABLE III. \( B_K \) |
|---------------------|
| \( \text{FLAG} \) | 0.766(99) \([24]\) |
| \( \text{SWME} \) | 0.7379(47)(365) \([25]\) |

\[
\begin{align*}
\frac{\text{Input}}{\text{Value}} & \quad \text{Ref.} \\
\eta_{cc} & \quad 1.72(27) \quad [9] \\
\eta_{tt} & \quad 0.5765(65) \quad [10] \\
\eta_{ct} & \quad 0.496(47) \quad [33]
\end{align*}
\]

The input values for \( \eta_{ij} \) that we use in this paper are summarized in Table \( \text{V} \).
the SM. We define $\varepsilon_{K}^{\text{Exp}}$ as the experimental value of $|\varepsilon_{K}|$ given in Eq. (2). We define $\Delta \varepsilon_{K}$ as the difference between $\varepsilon_{K}^{\text{Exp}}$ and $\varepsilon_{K}^{\text{SM}}$:

$$\Delta \varepsilon_{K} \equiv \varepsilon_{K}^{\text{Exp}} - \varepsilon_{K}^{\text{SM}}.$$  

(6)

Here, we assume that the theoretical phase $\theta$ in Eq. (3) is equal to the experimental phase $\phi_{c}$ in Eq. (2) [9].

In Table VII we present results for $\varepsilon_{K}^{\text{SM}}$ obtained using the FLAG average for $B_{K}$ together with $V_{cb}$ in both inclusive and exclusive channels. The corresponding probability distributions for $\varepsilon_{K}^{\text{SM}}$ and $\varepsilon_{K}^{\text{Exp}}$ are presented in Fig. 1 for the AOF case. The corresponding results for $\Delta \varepsilon_{K}$ are presented in Table VIII.

**TABLE VII.** $\varepsilon_{K}^{\text{SM}}$ in the unit of $1.0 \times 10^{-3}$. Here, we use the FLAG average for $B_{K}$ in Table III. The input methods of CKMfitter, UTfit, and AOF represent different inputs for the Wolfenstein parameters $\lambda, \rho, \bar{\eta}$.

| Input Method | Inclusive $V_{cb}$ | Exclusive $V_{cb}$ |
|--------------|-------------------|-------------------|
| CKMfitter    | 2.17(23)          | 1.62(18)          |
| UTfit        | 2.18(22)          | 1.63(18)          |
| AOF          | 2.13(23)          | 1.58(18)          |

**TABLE VIII.** $\Delta \varepsilon_{K}$. Here, we use $\varepsilon_{K}^{\text{SM}}$ from Table VII. We obtain $\sigma$ by combining the errors of $\varepsilon_{K}^{\text{SM}}$ and $\varepsilon_{K}^{\text{Exp}}$ in quadrature.

| Input Method | Inclusive $V_{cb}$ | Exclusive $V_{cb}$ |
|--------------|-------------------|-------------------|
| CKMfitter    | 0.24$\sigma$      | 3.4$\sigma$       |
| UTfit        | 0.20$\sigma$      | 3.4$\sigma$       |
| AOF          | 0.44$\sigma$      | 3.6$\sigma$       |

From Table VIII, we observe no tension between $\varepsilon_{K}^{\text{Exp}}$ and $\varepsilon_{K}^{\text{SM}}$ with inclusive $V_{cb}$.

However, from Tables VII and VIII, we find that $\varepsilon_{K}^{\text{SM}}$ with exclusive $V_{cb}$ is only 71% of $\varepsilon_{K}^{\text{Exp}}$. For this case, with the most reliable input method (AOF), $\Delta \varepsilon_{K}$ is 3.6$\sigma$. The largest contribution in this estimate of $\varepsilon_{K}^{\text{SM}}$ that we have neglected is $\xi_{LD} \approx 2\%$. Hence, the neglected contributions cannot explain the gap $\Delta \varepsilon_{K}$ of 29% with exclusive $V_{cb}$. Hence, our final results for $\Delta \varepsilon_{K}$ are

$$\Delta \varepsilon_{K} = 3.6(2)\sigma \quad \text{(exclusive } V_{cb} \text{)}$$  

(7)

$$\Delta \varepsilon_{K} = 0.44(24)\sigma \quad \text{(inclusive } V_{cb} \text{)}$$  

(8)

where we take the AOF result as the central value and the systematic error is obtained by taking the maximum difference among the input methods in Table VIII.

In the case of the FLAG $B_{K}$, the BMW result of $B_{K}$ dominates the FLAG average, and the gauge ensembles used for the BMW calculation are independent of those used for the exclusive $V_{cb}$ [20]. Hence, we assume that we may neglect the correlation between the FLAG $B_{K}$ and the exclusive $V_{cb}$. However, the SWME calculation of $B_{K}$ in Ref. [23] shares the same MILC gauge ensembles with the exclusive $V_{cb}$ determination in Ref. [20]. Hence, there exists a substantial correlation between the SWME $B_{K}$ and the exclusive $V_{cb}$. We introduce $+50\%$ correlation and $-50\%$ anti-correlation between the SWME $B_{K}$ and the exclusive $V_{cb}$ and take the maximum deviation from the uncorrelated case as the systematic error due to the unknown correlation between them. Details of this analysis are explained in Ref. [9]. However, this analysis shows that the size of the ambiguity due to the correlation between the SWME $B_{K}$ and the exclusive $V_{cb}$ is much larger than the systematic error in $\Delta \varepsilon_{K}$ with the FLAG $B_{K}$. Therefore, we use the results obtained with the SWME $B_{K}$ only to cross-check those obtained with the FLAG $B_{K}$ [9].

In Fig. 2 we present $\Delta \varepsilon_{K}/\sigma$ as a function of time starting from 2012. In 2012, the RBC/UKQCD collaboration...
reported $\xi_0$ in Ref. [30]. In addition to this, using the LLV average for $B_K$ [25], we reported $\Delta\varepsilon_K = 2.7(2)\sigma$ in Ref. [30] in 2012. In 2014 FNAL/MILC reported an updated $V_{cb}$ from the exclusive channel [20]. Using the FLAG average for $B_K$ [24] and the NNLO value of $\eta_{ct}$ [31], we reported the updated $\Delta\varepsilon_K = 3.3(2)\sigma$ in Ref. [37]. In Ref. [9], we investigate issues in the NNLO calculation of $\eta_{ct}$ [32, 33], and in this paper we use the SWME result in Table IV to report the updated $\Delta\varepsilon_K = 3.6(2)\sigma$ in Eq. [7].

In summary, we find that there is a substantial $3.6(2)\sigma$ tension in $\varepsilon_K$ between the experiment and the SM theory with lattice QCD inputs. For this claim, we choose the angle-only fit (AOF), the exclusive $V_{cb}$ (lattice QCD results), and the FLAG $\hat{B}_K$ (lattice QCD results) to determine the central value. The systematic uncertainty is obtained by taking the maximum deviation from the central value by choosing other input methods from the global fits of the CKMfitter and UTfit. We choose the AOF method to determine the central value because the Wolfenstein parameters of AOF do not have unwanted correlation with $\varepsilon_K$, $\hat{B}_K$, and $|V_{cb}|$. However, the tension disappears in the case of inclusive $V_{cb}$ (results of the heavy quark expansion based on the OPE).

In Table IX, we present the error budget for $\varepsilon_K^{SM}$ for the central value. This is obtained using the error propagation method explained in Ref. [9]. From this error budget, we observe that $V_{cb}$ dominates the error in $\varepsilon_K^{SM}$. Hence, it is essential to reduce the error of $V_{cb}$ as much as possible (see also Refs. [38, 39]). To achieve this goal, there have been a lot of on-going efforts in the lattice community [40, 45].

It is true that there is an issue with the convergence of the perturbative expansion of $\eta_{ct}$ [32]. This could be resolved with lattice QCD calculations such as those envisioned by the RBC/UKQCD collaboration [11].

We expect that our results for $\varepsilon_K$ would be consistent with those from a global UT analysis, such as that in Ref. [35].

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