The Graph Geometry for Architectural Planning

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Abstract

This paper introduces new graph geometric measures and models for architectural planning. Based upon geometrical distance among nodes, the ‘minimum-path graph’ is proposed to analyse all shortest-path traversal from every to every other space in a network. By applying the geometric graph, two spatial models are created in order to represent node’s centrality and edge’s optimal passage capacity. They describe specific values for each individual space with regards to the overall system.

The graph geometry primarily aims to assist architect planners to clarify the underlying geometric potential of space with more accuracy. In a near future, it is also hoped that the new geometric measures and models will further deepen, and widen, our planning knowledge for architectural and urban space network.

Keywords: graph geometry; spatial network; centrality structure; passage structure; geometric spatial modelling

Introduction

Architectural planning often applies Graph theory to analyse potential of individual spaces that composing together a wider system of space. Graph can represent any kind of relation, and architecture and city are certain kinds of spatial relationship, thus, could be explained suitably by graph. Based on geometrical distance, this paper specifically analyses ‘centrality and passage structure’ of the graph elements. It proposes a geometrical measure called ‘minimum-path graph’, upon which two new spatial models are based. Centrist and Passage models will offer architectural planners a new set of accurate database defining each individual space’s distance-based characteristics.

Although graph is already used in some previous studies (see for examples, Steadman 1983 and Hillier and Hanson 1984), they mostly concerned with syntactic, or topological, property of graph. By contrast, this paper studies a more elaborated geometric property of each graph’s spatial element. The geometric measures are particularly designed for using in architectural planning process. This paper is outlined as following.

The first section describes basic concept of graph and its geometry, including ‘minimum distance’ and ‘optimal passage’. It compares new geometric distance-based ‘minimum-path’ graph with so-called justified graph (figure 2 and 3). By considering intersection as node of graph, the second section proposes Model-1: Intersection Model. Minimum-path graph shows each node’s shortest distance from every other node, as well as each edge’s passed capacity. Moreover, it clarifies each node’s local property such as passed spaces along one’s shortest paths, the node’s overall depth, and its neighbouring hierarchy (figure 5). Then, the section applies two models for spatial analysis. Firstly, the Centrist model shows potentiality of every intersecting node, based upon its ‘global centrality’ i.e. how near or far it is from every other node. Secondly, the Passage model shows the system’s overall optimal passage. It counts how many times each edge is passed by shortest paths.

The third section turns to study geometric potential of the graph’s edges. In figure 6, midpoint of each edge is picked up as a new node for Model-2: Segment Model. Hence, segment becomes node, whilst junction becomes edge. Using minimum-path graph and centrist model we can find also the centrality for each segment (as new node), as already found for each node in model-1. The combination of centrality of all intersection and segment results in new centrist map in figure 7-1. Meanwhile the combination of all passage capacity along segments is shown in figure 7-2. The forth section then applies two models to allocate specific functions into an artificial urban space.

Following in the fifth section, a real urban system ‘Ratanakosin Area’ of Bangkok is explored. To compare with our geometrical model, the area is also syntactically represented in ‘Axial Map’ of Space Syntax. Lastly, this paper concludes that the graph-based geometrical models can be both qualitatively and quantitatively suggestive for architectural and city planning practice.

1. Two Basic Geometry of the Graph

To show the geometric order of each graph element, this section measures ‘minimum distance’ and ‘optimal passage’. Minimum distance is the sum distance of a node’s shortest paths to every other node (Table 1 Dist). Later on, it will signify the degree of centrality of each
node (figure 5-1). Traversing once to every other node along one’s shortest paths, the number of times each edge and every other node is passed shows the edge’s passage (P/Pass), and the node’s linkage (L/Link) values. Then, optimal passage summarises overall quality onto spatial graph in figure 5-2. Section 1 is arranged in the following sequence.

Firstly, Graph theory is briefly introduced to give a common view and to provide a basis for our discussion (figure 1). Secondly, minimum distance and optimal passage geometry are analysed graphically in figure 2. Lastly, the new graph is compared with justified graph in figure 3 introduced by Architectural Morphology (Steadman 1983) and Space Syntax (Hillier and Hanson, 1984). The differences between topological and geometrical analyses of graph network are shown.

### 1.1 Graph Decomposition of Urban Space

To analyse a continuous spatial form, graph is used to break up basic elements that compose a network. In figure 1, graph G represents a small urban area as a set of crossing nodes and street segments. Directly, V set of vertices {A, B, C, D, E} in graph G signifies all crossing and ending nodes, while, E set of undirected edges {i, j, k, x, y, z} represents all street segments. Therefore, we can also directly analyse the area’s path structure. Figure 1 shows graph G (V, E) whose edges have different distance (or weight) from 2 to 7.

![Fig.1. Graph G](image)

#### Table 1. Geometric Measures of Node in Minimum-Path Graph

| Fr | A | B | C | D | E |
|----|---|---|---|---|---|
| To | L | P | L | P | L | P | L | P | L | P |
| A  | * | i | D | * | j | * | k |
| B  | ix | * | x | yx | A | ki |
| C  | * | j | xy | y | D | zy |
| D  | * | k | A | ik | D | yz | * | z |
| E  | * | i | jk | i | x yik | yj x yz | j y y z | kki zy z |

*Dist* for the linking node, *P* for the node’s passage, * for adjacency

#### Table 2. Comparison of the Graph’s Syntax and Geometry

| Fr | Connectivity (Adjacency) | Mean Depth (Integration) | Distance | Linkage |
|----|--------------------------|--------------------------|----------|---------|
| A  | 3 | 1.25 | 22 | 2 |
| B  | 2 | 1.50 | 23 | 1 |
| C  | 2 | 1.50 | 21 | 2 |
| D  | 3 | 1.25 | 19 | 3 |
| E  | 2 | 1.50 | 31 | 0 |

1.2 Minimum-path Graph: Retrieving two Geometries

For architectural planning, the first significant geometry is ‘minimum distance’ which demonstrates distance-based centrality of each space. Within a set of continuous space, where is the most central spaces or the spaces with higher proximity to all other space? The hypothesis is that the nearer a space to its overall system (i.e. to every other space), the higher the order. Consequently, the space could be a core structure of that spatial context. Architect planners might see minimum distance of each space as a guidance to allocate ‘central functions’ into building and city. By understanding the hierarchical order of spaces, from most central to most distant one, we can improve planning’s efficiency and effectiveness considerably.

In figure 2, minimum-path graph clarifies that each node in graph G has different minimum distance, or the sum shortest distance to every other node (Dist in Table 1). It also shows that shortest path from A to B is ‘i’ (3) and A to C is ‘ix’ (7) and so on. Table 1 shows minimum distance or Dist of A as 22. Similarly, minimum distance of B, C, D and E is 23, 21, 19 and 31 respectively. Therefore, the centrality order among the five nodes is D > C > A > B > E.

Based upon node’s minimum distance, the second geometry edge’s ‘optimal passage’ is also important to space planning. It explains how often or rarely each edge would be passed by travelling from every to every other node along the shortest paths, thus, shows a relative
passing capacity of each edge space of the graph. The optimal passage may be seen as a ‘retrieved geometry’, because we retrieve the new quality of each edge from distance-based geometry. The five edges of graph G signify different passage potential. Table 1 counts the times of shortest-path traversing along edge i, j, k, x, y and z as equal to 5, 3, 4, 5, 7 and 4 respectively. Hence, the passage order is $y > i = x > k = z > j$. Values and order are shown in graph G’s passage map in figure 5-2.

Furthermore, some nodes are used as linkage along all the shortest paths more often. For example, D is used 3 times, therefore, Link in Table 1 shows the linkage order as $D(3) > A = C(2) > B(1) > E(0)$. Finally, edge passage (Pass) and the node linkage (Link) are put together to form the graph’s overall optimal passage. In figure 5-2, the optimal passage and linkage order of graph G is, therefore:

$$y > i = x > k = z > j = D > A = C > B > E.$$
2). Edge passage $E = z, k, k, y, i = 6$
   Node linkage $E = A$ and $D$
   Node degree $E = z$ and $k = 2$

3). Node depth $E = D + A + C + B$
   $= 1 + 1 + 2 + 2 = 6$
   Edge depth $E = 2z + 2k + y + i$
   $= 2(1) + 2(1) + 2 + 2 = 6$

4). Node order $E = D > A > C > B$ (Fig. 5)

5). Edge order $E = z > k > y > i > j > x$ (Fig. 5)

Step-2: After specifically analysing each individual
node space, we turn to look at the overall Centrist and
Passage maps of the graph $G$ (Figure 5-1 and 5-2). Now,
we get a global picture. An overall character of the graph
is shown, with reference to node’s centrality and edge’s
passage capacity. Figure 5-1 is the Centrist map showing
each node’s centrality value as calculated in equation 1).
It explains that the node with higher minimum
distance is less central, as in case of $E$ the most remote
among the five nodes. Figure 5-2 is the Passage map
showing how often or rarely each edge and node would
be passed, by travelling one time equally from every to
every other node along each node shortest paths. For
example, from $E$ it passes $k$ to $A$, $k$ to $B$, $z$ to $C$ and $z$
to $D$ (Table 1, figure 2 & 5). In short, the sum of $Pass$
and $Link$ in Table 1 is shown graphically in the Passage
map figure 5-2.

Therefore, we can call Centrist and Passage maps as
‘graph-from-node’ pictures (system from individual).
The geometry of overall graph is revealed by measuring
each node distance and each edge passage capacity.
Clearly, the maps base on minimum distance studied in
the first section.

3. Model-2: Segment Model

As in original graph $G$, intersection is the node and
the linear space the edge. This implies that an intersection
is strategic space in building and city. Generally, it seems
logical. For examples, major parks, civic plazas and
important public buildings are often located at some
major streets’ intersections. Within building interior,
entrance hall, main lobby, audience foyer, family room,
or a school court are some examples of node space in
general architectural sense.

However, neither the logic of ‘intersecting node as a
place’ nor ‘linear segment as a movement corridor’ is
absolute. For instance, within an open system where
movement is particularly need (imagining a downtown
shopping street or a shopping mall corridor), the linear
space itself is more likely become a place than just a
connection. On the contrary, street junctions and busy
crosses around public transport terminal could function
more like connecting edge, rather than a gathering place.
Thus, the logic of the graph could be reversed. The
connecting segment become a node or place, whilst,
intersection become just a linking edge. Nevertheless,
this reversal would happen only if the network is dense,
and movement is predominant. In the other word,
reversed system is a dynamic rather than a static network
of place. Clearly, a downtown and its dynamism could
well represent such a system.

Now, if certain graph’s segment is important in itself,
how could we measure its geometric capacity? This
section reanalyses segment as a graph’s node, and
intersection as an edge instead. To do so, it transforms a
normal node-edge graph into a new line graph.

Fig.6. Line Graph of G
Figure 6 draws new segment nodes at the middle point of each edge of graph G. In Segment Model edge $k$, for example, becomes a new node. Thus, its centrality value can be calculated as done in the previous section. Centrality $C = \text{The sum of edge length (3+5+7+4+2+6+3.5+4+5+6+6.5+3.5+4+5.5) / divided by the total Minimum distance of } k (kA+kB+kC+kD+kE+ki+kj+kx+ky+kz)$. Calculated centrality value of $k$ is 0.76 as shown in figure 7-1. Similarly, centrality values of other new nodes in model 2 are calculated and graphically shown together with the values of normal intersection nodes of model 1. The new Centrality map in figure 7-1, therefore, combines Intersection and Segment models. The new centrality order is $D>C>A>B>j>y>x>i>E>z>k$.

Then, we construct new minimum-path graph for eleven nodes altogether and, again, count the number of Pass and Link (as done in Table 1). However, now the link values at both ends of each segment are combined with the pass value of the segment to give a total picture of each edge’s passage capacity. The combination of Intersection and Segment Models’ Passage map is shown in figure 7-2. As shown in figure 5-2 and 7-1, node D, A and C are often used as linkage and new segment nodes $j$ and $y$ have higher centrality. Consequently, the new passage order constructed from new eleven-node minimum-path graph is $j = y (38) > i (28) > x (24) > z (23) > k (20)$. 

4. Models Application: A modest planning

Based upon the graph geometry, two maps sum up spatial analysis of G urban space. Figure 7-1 combines intersection and segment node into one centrality map. Similarly, the passage map in Figure 7-2 concludes all linkage and passage of both elements, but now combining the values together over each segment. Conclusively, this section applies the models and data, retrieved from minimum-path graph, to produce two sketchy urban proposals for the area. Concerning space’s passage capacity, the first schematic (Figure 8) layouts a set of land and building uses including transport, civic, commercial, resident and recreational facilities. Figure 9 is the second alternative, with an emphasis on the node space’s distance potentiality.

Obviously, the distance and passage strategies significantly support and affect both plans. Since the graph geometry has shown both centrality and passing potentiality of every individual space, with space-to-network and network-to-space analysis, two rough schematics could be further developed having an effective consult. For example, architect can recheck if the plan provide a proper distance from one function to the others which have certain relation says between the city hall, police station and cultural centre, or among housing, school and hospital. Evidently, we can also ask if department store or museum would have ‘a good passage’, or if resident and school are located with appropriate passage. All these information becomes obvious, in spite of a modest size and simplest form of five-node example. Yet, in the next section a much higher complex urban form can also be analysed according to the similar geometric principle.

To sum up, the models initiated by this paper have tried to understand the potential of an architectural spatial system. This includes a capacity of each individual space, the relationship from one to each the other and to the system, as well as the global picture of the overall network. Architecture and city are primarily all of these latent relationship and competence among spaces. Therefore, to plan and to design a complex relationship for a specific kind of network, we need a geometrically effective measure.
5. Discussion: Geometric Structure of Space and City

By viewing the relationship in space network as that among the elements of graph, this paper has examined the geometric potentiality of individual space, regarding the overall character of graph. From section 1 to section 3, minimal-path graph is applied to clarify the centrality and passage structure of graph. Consequently, section 4 uses graph geometry for space planning, based on reliable data. Section 5.1 further shows the application of graph model for analysing real city. Syntactic and geometric models of a Thai city are comparatively examined. Then section 5.2 discusses performance of graph-based geometric modelling for spatial analysis and design.

5.1 A Geometric Spatial Model of the City

Research in graph modelling of architectural and urban space’s pattern has been done considerably (Hillier 1996, Martin and March 1972). Nevertheless, the graph geometry is rarely discussed. This study, therefore, uses minimum-path graph to thoroughly study the geometric order among different places. It has clearly informed the centrality and passage patterns both of individual space and of the overall network. For city, it is also possible to apply spatial model of the graph geometry.

For analysis of real city space, a C-based computer modelling is created from a modified algorithm of Dijkstra’s method for finding single-source shortest paths. Given specific coordinates of graph’s node and edge, the shortest paths from every node to every other node are identified and their distance is calculated algorithmically. As a result, figure 10 shows Centrist map of an area with different centrality among local fifty-six nodes.

To compare geometric and syntactic structure of city, the origin of Bangkok called Ratanakosin, with more than two thousands streets and four thousands intersections, is modelled in three maps.

The city’s Axial map in figure 11 shows the global integration value, i.e. depth degree of each line from the whole system. It shades the shallowest lines in dark, as contrast to the deepest ones in light. The city’s shallowest axes link larger blocks inside historical core to the area outside. ‘Depth map’ in figure 12 also shows overall ‘topological’ depth of each street segment from the whole network, despite at different scales. Axial lines in fig.11 are the longest-fewest lines covering the network, whilst street segments in fig.12 are broken at every intersection.

Two depth maps similarly decode the city’s growth pattern, from west Inner city to east Outer island and outside the wall. Hence, shallowest axes and segments locate at similar position (fig.11 and 12). However, two maps also show a significant contrast. Whilst the axial structure of fewer, longer and highly connected lines shallow up the line network (presumably through motorised travel), the major segments just link with surrounding spaces (through walking) to form the city’s radiant core at east side.
Comparing figures 11 and 12, we can clearly see that city at axial scale could support only planning for automobile based movement. On the contrary, the city at segment scale suits for analysis of human walk.

Human scale is picked up also in the city’s Passage map (figure 13). The map bases on minimum-path graph and represents the graph geometry introduced earlier in this paper. Shown as thicker segments in passage map, the city’s walking structure extends the shallowest segments in fig.12 upward, downward, and westward to form geometrically central blocks. This human scale structure ties up inner and outer Ratanakosin together. It clearly reveals optimal ‘walking’ routes, with high centrality and high passage capability, for travelling among intersections as short as possible. Clearly, the city’s optimal passage supports pedestrian rather than car. This happens because the graph geometry checks real minimum-path distance, regardless topological depth that obstructs car’s travel pattern. Due to the car’s turning speed, to drive thru a shorter route with more turns (geometrically nearer but topologically deeper) would take more time, skill and cognition than using the shallower axial lines of longer distance. Remarkably, the shallower the spatial network, the more we can improve vehicle movement. Conversely, the more geometrically compact the network, the more pedestrian can walk up and navigate to recognise and understand global form of the area.

In short, with respect to people’s walking scale it is significant to study the graph’s distance geometry rather than merely topological depth. Moreover, to create lively walking network, the graph principle should be used to model spatial form of life, the city.

5.2 Towards an Appropriate Spatial Modelling
Defining a model as an intelligent representation of real environmental object, Y.E. Kalay (1987) gives four benchmarks of an appropriate modelling, including clarity (well-formedness), wide applicability (generality), completeness and efficiency. This paper develops a spatial model under this consideration.

The first section gives basic character of the ‘Graph Geometry’. Minimum-path Graph and values reveal hierarchical order among graph elements (figure 2, Table 1). The graph geometry, thus, can represent geometric order in spatial form with clarity.

The second and third sections have provided ‘Intersection’ and ‘Segment’ models to explain geometric qualities of space. ‘Centrist’ and ‘Passage’ maps decode node’s centrality and edge’s linkability for spatial network (fig.5-1, 5-2). Moreover, the graph also explains a node’s individual characters (fig.5). Consequently, the models and measures make graph geometry become general and applicable in order to support architectural planning.

Completeness of geometric graph model lies in its application to evaluate both node and edge quality. Furthermore, it measures both capacity of individual space and the graph’s overall picture at the same time. A modest application of graph geometry is drawn in section four (Fig. 7-9). It proves that graph principle can provide architectural practice with effectiveness. As a measure, it tries to increase planning accuracy, but without restricting architect’s designing sensibility.

Finally, section 5 has testified the efficiency of graph principle and geometric spatial modelling. Centrist, Passage and Depth maps are run by computer program by applying new algorithms. The maps can identify three important geometrical structure of city.

1) Hierarchy of nodes’ centrality,
2) Passage core that shortens all-to-all walking routes and remains the city compactness, and
3) Shallow-segment core forming central linkage.

Significantly, all the structural analysis of city space is done with respect to the pedestrian walking scale.

6. Conclusion
As being shown, this paper introduces the principle of graph geometry, applies its measures to support architectural planning, and creates geometric spatial models for architectural and urban analysis.

In summary, this paper has introduced the distance-based graph geometry and its measures, for representing the ‘structural quality’ of each specific space, as well as that of the overall network. Regarding clarity, generality, completeness and efficiency, we have also transformed the graph principle described section 1 into geometric spatial models shown in sections 2 to section 5. Significantly, the graph principle aims to support architectural practice with geometrically well-informed database.

Future development of a graph-based geometric spatial model includes few possibilities.

Firstly, a thorough study of graph including the geometry of critical nodes and edges, graph’s characterisation and sub graphs’ classification, would highly enlarge our understanding of a spatial network.

Secondly, further computer modelling could include a program that can automatically generate minimum-path graph, depict individual space’s features (as shown in fig.5) as well as input designer and use attribution to the graph’s node and edge space.

Lastly, three-dimensional representation of spatial models would notably improve the ‘completeness’ of the graph geometry principle and its application for architectural planning.

References
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Endnotes

i More often graph is applied to represent ‘functional relation’, for examples in so-called bubble diagram. An influential work, using graphical diagram to represent the relationship between architectural requirements, is that of Alexander Christopher (1964), Notes on the Synthesis of Form, Cambridge (pp.64-66). However, there is also a growing tradition to apply the graph for ‘spatial analysis’ in architecture and city planning. See for examples, March and Steadman (1971), Steadman (1983), Hillier and Hanson (1984).

ii Justified graph is called accessibility graph (Steadman, 1983) and later modified by Hillier and Hanson (1984). It is used to analyse the relationship among spaces as nodes of graph regarding how a node is connected to all the other in same network.

iii Space Syntax (Hillier and Hanson, 1984) is the theory and analysis techniques, including Axial Map, to explain a configurative logic of architectural and urban morphology, with regards to social, economic and other factors. See also Hillier (1996) and 1st - 4th Space Syntax Symposiums’ Proceedings in http://www.spacesyntax.com

iv Modification of Dijkstra Algorithm for finding all shortest paths among every intersection, the segment’s passage and depth, and the C-based Computer Modelling of graph network shown in the paper provided by Professor Dr. Fujii Akira, Institute of Industrial Science, The University of Tokyo. The study of Graph for Architectural space analysis is an initial state of the author’s Doctoral thesis under Prof. Dr. Fuji’s supervision.

v Original described in Dijkstra, E. W. ‘A note on two problem in connection with graphs’, Numerische Math., 1, 269-71 (1959). A very clear explanation and PASCAL based graph algorithm is shown in Gibbons (1985) p.13-16. See a formal definition, with linked simulation and code, in NIST (National Institute of Standards and Technology) at http://www.nist.gov/dads/HTML/dijkstraalgo.html