Learning Regionally Decentralized AC Optimal Power Flows With ADMM

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Abstract—One potential future for the next generation of smart grids is the use of decentralized optimization algorithms and secure communications for coordinating renewable generation (e.g., wind/solar), dispatchable devices (e.g., coal/gas/nuclear generations), demand response, battery & storage facilities, and topology optimization. The Alternating Direction Method of Multipliers (ADMM) has been widely used in the community to address such decentralized optimization problems and, in particular, the AC Optimal Power Flow (AC-OPF). This paper studies how machine learning may help in speeding up the convergence of ADMM for solving AC-OPF. It proposes a novel decentralized machine-learning approach, namely ML-ADMM, where each agent uses deep learning to learn the consensus parameters on the coupling branches. The paper also explores the idea of learning only from ADMM runs that exhibit high-quality convergence properties, and proposes filtering mechanisms to select these runs. Experimental results on test cases based on the French system demonstrate the potential of the approach in speeding up the convergence of ADMM significantly.

Index Terms—AC optimal power flow, smart grid, ADMM, deep learning.

I. INTRODUCTION

ONE POTENTIAL future for the next generation of smart grids [1] is the use of decentralized optimization algorithms and secured communications for coordinating renewable generation (e.g., wind/solar), dispatchable devices (e.g., coal/gas/nuclear generations), demand response, battery & storage facilities. In particular, system operators will need to reliably and efficiently solve AC Optimal Power Flow (AC-OPF) problems in a decentralized fashion. This optimization problem finds the most economical generation dispatch that meets the load, while also satisfying the physical and engineering constraints of the underlying power grid. It is therefore a fundamental tool for balancing generation and load rapidly, without sacrificing economic efficiency. Nevertheless, its resolution in a decentralized fashion remains challenging, especially for industry-size networks that comprise thousands of buses.

The alternating direction method of multipliers (ADMM) [2] is widely used by the power systems community to solve decentralized optimization problems, especially OPF problems [3]. In particular, ADMM has been successfully applied to convex relaxations and/or approximations of AC-OPF, e.g., the popular DC approximation [4], for which it enjoys strong theoretical guarantees. Furthermore, ADMM schemes with convergence guarantees have been proposed recently (e.g., [5]), broadening the scope of application of this decentralized optimization technology. However, most ADMM variants used for AC-OPF, including those with convergence guarantees, require significant tuning of their parameters to ensure numerical stability and convergence in practice [3].

In that context, this paper proposes the use of machine learning (ML) techniques to enhance the practical behavior of ADMM for solving AC-OPF problems in a decentralized fashion. The paper leverages the fact that ADMM is an iterative process that uses dual (Lagrange) multipliers to drive separate agents towards achieving a consensus [2]. This perspective is illustrated in Figure 1, which depicts a power grid composed of 3 regions: the regions are coupled through lines (1, 2) and (3, 4) for which a consensus much be reached. Building on this observation, the paper proposes ML-ADMM, which uses ML to learn a close-to-optimal primal-dual solution that is used to warm-start the ADMM algorithm. Specifically, the paper makes the following contributions:

1) it proposes to learn both primal and dual consensus variables, in contrast with other works that only consider primal information;
2) it introduces a novel decentralized machine learning approach for data collection, training and inference;
3) it proposes novel data-filtering techniques to identify high-quality training data, thereby improving training and learning accuracy;
4) it reports computational results on real, industry-scale systems from the French transmission grid;
5) it demonstrates the applicability of the methodology on two classes of ADMM schemes, one of which has strong convergence guarantees;
6) The numerical experiments show that ML-ADMM obtains solutions of similar quality as the original ADMM schemes in as little as 1/6 of the iterations. It is important to note that the proposed ML-ADMM framework is not restricted to AC-OPF, and can be applied to other optimization problems. In addition, because ML-ADMM executes the ADMM algorithm from a high-quality starting point, it enjoys the same theoretical convergence properties.

Manuscript received 8 May 2022; revised 1 October 2022 and 16 January 2023; accepted 24 February 2023. Date of publication 1 March 2023; date of current version 23 October 2023. This work was supported in part by the National Science Foundation under Award 2007095 and Award 2112533. Paper no. TSG-00661-2022. (Corresponding author: Terrence W. K. Mak.)

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Color versions of one or more figures in this article are available at https://doi.org/10.1109/TSG.2023.3251292.

Digital Object Identifier 10.1109/TSG.2023.3251292
The rest of the paper is organized as follows. Sections II and III present the related work and background material. Section IV introduces the Regionally Decentralized AC-OPFs and the ADMM formulations. Sections V and VI present the decentralized learning models and data filtering procedures for learning the interconnection parameters. Section VII reports the experimental results. Section VIII concludes the paper.

II. RELATED WORK

With the introduction of Smart Grid, there has been a growing interest in applying ADMM on optimal power flow applications, primarily due to its distributive nature and privacy features [6], [7], [8], [9], [10]. Even though general ADMM formulations on nonlinear AC-OPF may not always converge, recent work [5] shows that convergence can be achieved via reformulations under mild assumptions. This paper complements these convergence results by showing how warm-starting the ADMM formulations through machine learning can bring substantial speedups in practice.

The application of machine learning to optimal power flows has been widely studied in recent years. A recent line of research has focused on how to predict centralized AC-OPF solutions directly using Deep Neural Networks (DNN) (e.g., [11], [12], [13], [14]). Once a neural network is trained, solution predictions can be computed with a single forward pass in milliseconds. Recent work [15] has also shown that deep learning can be spatially decomposed in a similar fashion. A wide variety of approaches have also been proposed beyond predicting AC-OPF solutions. These approaches include learning the active set of constraints [16], [17], [18], [19], [20], imitating the Newton-Raphson algorithm [21], learning warm starting points for speeding-up the optimization process [22], [23], and predicting optimal dispatch decisions [13], [24], [25]. Applying machine learning techniques to decentralized OPF problems has also been studied recently [26]. Other related works explore formal guarantees for neural networks when learning OPF problems [27], [28], and extend the learning methodologies to security-constrained OPF problems [29], [30].

Reinforcement-learning approaches for OPF problems have also been proposed (e.g., [12], [31], [32], [33]) and primarily focus on tackling real-time issues. This paper continues the line of work [11], [15], [34], [35] in using deep learning to predict AC-OPF solutions directly, while integrating practices/constraints found in U.S. energy markets. The work differs from existing work on decentralized OPFs (e.g., [26]) in three ways:

1) it focuses on predicting the flows and voltage on coupling branches, governed largely by transactions between regional load balancing zones/authorities (for the inter-regional exchange markets) instead of learning the decentralized algorithm itself, e.g., learning the search directions/heuristics;
2) the learning procedure is decentralized by nature and each agent can train in parallel and independently, maintaining privacy & region/agent neutrality; and
3) the predictions are not necessarily tied to any specific decentralized algorithm, and can be seen as predicting transactions in an exchange market.

III. BACKGROUND

This section presents background materials for the rest of the paper. Table I presents the common notations and symbols.

A. AC Optimal Power Flow

The AC Optimal Power Flow (OPF) determines the most economical generation dispatch balancing the load and generation in a power grid. Model 1 presents an AC OPF formulation (centralized model), with variables and parameters in the complex domain for clarity and compactness. For simplicity, the presentation omits the equations for transformers, phase shifters, circuit breakers/switches, and fixed/switched bus shunts. All omitted devices are considered and implemented in the experimental evaluation. The objective function

\[
\text{Model 1: AC Optimal Power Flow: } P_{\text{AC}}
\]

\[
\text{input: } S^d = \left( S^d_i : i \in N \right)
\]

\[
\text{variables: } S^g = \left( S^g_i : i \in N \right), V = ( V_i : i \in N )
\]

\[
S^f = \left( S^f_{ij} : (i,j) \in E \cup E^R \right)
\]

\[
\text{minimize: } \mathcal{O}(S^g) = \sum_{i \in N} M_i \left( \| S^f_i \| \right)
\]

\[
\text{subject to: } \theta_i = 0, \quad (2)
\]

\[
v_l \leq v_i \leq v_f \quad \forall i \in N \quad (3)
\]

\[
S^g_i \leq S^g_i \leq S^g_i \quad \forall i \in N \quad (4)
\]

\[
|S^f_{ij}| \leq S^f_{ij} \quad \forall (i,j) \in E \cup E^R \quad (5)
\]

\[
S^d_i - S^d_i = \sum_{(i,j) \in E \cup E^R} S^f_{ij} \quad \forall i \in N \quad (6)
\]

\[
S^f_{ij} = Y^*_i |V_i|^2 - Y^*_j V_i V_j^* \quad \forall (i,j) \in E \cup E^R \quad (7)
\]
Consider an optimization problem with two agents/parties: decentralized optimization problems with coupling constraints.

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describing the AC power flow in a system of agents. Constraint (6) captures Kirchhoff’s Current Law enforcing the flow balance of generations $S_i^g$, loads $S_i^f$, and branch flows $S_{ij}$ across every node. Finally, constraint (7) captures Ohm’s Law describing the AC power flow $S_{ij}$ across lines/transformers.

**B. Alternating Direction of Multipliers Method (ADMM)**

ADMM [2] is a widely used decentralized algorithm solving decentralized optimization problems with coupling constraints. Consider an optimization problem with two agents/parties:

$$\begin{align*}
\min_{x_1, x_2} f_1(x_1) + f_2(x_2) \\
\text{s.t. } x_1 \in \mathcal{X}_1, x_2 \in \mathcal{X}_2, \\
Ax_1 + Bx_2 = c,
\end{align*}$$

(8)

where $\mathcal{X}_1 \subseteq \mathbb{R}^n$ and $\mathcal{X}_2 \subseteq \mathbb{R}^m$ are two disjoint feasible space for two independent local optimization problems, $x_1 \in \mathcal{X}_1 \subseteq \mathbb{R}^n$ and $x_2 \in \mathcal{X}_2 \subseteq \mathbb{R}^m$ denote feasible variable vectors owned by two distinct groups of agents, and $Ax_1 + Bx_2 = c$ describes the set of coupling constraints between the two groups of agents with $A \in \mathbb{R}^{c \times m}, B \in \mathbb{R}^{c \times m}$, and $c \in \mathbb{R}^c$. The functions $f_1$ and $f_2$ denote the objectives over $x_1$ and $x_2$, respectively. They are commonly assumed to be convex.

Problem (8) is often reformulated and simplified by introducing consensus parameters explicitly, leading to the consensus formulation [10]. Let $x_1^C, x_2^C$ to be the consensus for $x_1, x_2$. The consensus formulation of (8) for agent 1 is:

$$\begin{align*}
\min_{x_1} f_1(x_1) \\
\text{s.t. } x_1 \in \mathcal{X}_1, Ax_1 = c - Bx_2, \\
\text{where } x_2 = x_2^C.
\end{align*}$$

(9)

The consensus formulation for agent 2 is similar. The augmented Lagrange function $L^1_p(x_2^C, \lambda_2)$ of (9) for agent 1 is:

$$\begin{align*}
\min f_1(x_1) + \lambda_1^T x_2 + \frac{\rho}{2} \|x_2 - x_2^C\|^2_2 \\
\text{s.t. } x_1 \in \mathcal{X}_1, Ax_1 = c - Bx_2,
\end{align*}$$

where $\lambda_2$ is a vector of Lagrangian multipliers for $x_2$ in the view of agent 1, with $\rho > 0$ representing the penalty parameter. Similarly, the augmented Lagrange function $L^2_p(x_1^C, \lambda_1)$ of (9) for agent 2 is:

$$\begin{align*}
\min f_2(x_2) + \lambda_2^T x_1 + \frac{\rho}{2} \|x_1 - x_1^C\|^2_2 \\
\text{s.t. } x_2 \in \mathcal{X}_2, Bx_2 = c - Ax_1
\end{align*}$$

where $\lambda_1$ is a vector of Lagrangian multipliers for $x_1$ in the view of agent 2.

Given a solution tuple $(x_1^C, x_2^C)$ and the Lagrangian multipliers $(\lambda_1^C, \lambda_2^C)$ at iteration $k$, ADMM proceeds to the next iteration, $k + 1$, as follows:

$$\begin{align*}
x_1^{k+1} &= \arg\min_{x_1} L^1_p(x_2^k, \lambda_2^k) \\
\lambda_2^{k+1} &= \arg\min_{x_2} L^2_p(x_1^{k+1}, \lambda_1^k)
\end{align*}$$

(10) (11)

$$\begin{align*}
\lambda_1^{k+1} &= \lambda_1^k + \rho (x_1^{k+1} - x_1^k), \\
\lambda_2^{k+1} &= \lambda_2^k + \rho (x_2^{k+1} - x_2^k)
\end{align*}$$

(12)

The algorithm terminates when a desired condition (e.g., an iteration limit or a convergence factor) is reached. The quality of the solution at iteration $k$ can be measured by the primal infeasibility (residue) vector $\bar{r}_p$:

$$\begin{align*}
\bar{r}_p &= Ax_1^k + Bx_2^k - c,
\end{align*}$$

(13)

indicating the distance to a primal feasible solution, and the dual infeasibility (residue) vector $\bar{r}_d$:

$$\begin{align*}
\bar{r}_d &= \rho A^T B (x_1^k - x_2^{k-1})
\end{align*}$$

(14)
indicating the distance from the previous local minima. When both infeasibility vectors are zero, ADMM has converged to a (local) optimal and feasible solution.

C. Two-Level ADMM

The two-level ADMM [5] reformulates (8) by introducing slack variables z for the set of coupling constraints:

\[
\begin{align*}
\text{min} & \quad f_1(x_1) + f_2(x_2) \\
\text{s.t.} & \quad x_1 \in \mathcal{X}_1, x_2 \in \mathcal{X}_2, \\
& \quad Ax_1 + Bx_2 + z = c, \\
& \quad z = 0. \\
\end{align*}
\]

(15)

The scheme ensures that the slack eventually converges to zero (i.e., z = 0). The augmented Lagrange function for agent 1 then becomes

\[
\begin{align*}
\text{min} & \quad f_1(x_1) + \Lambda^T z + \frac{\beta}{2} \|z\|_2^2 + \lambda_1^T x_2 + \frac{\mu}{2} \|x_2 - x_2^c + z\|_2^2 \\
\text{s.t.} & \quad x_1 \in \mathcal{X}_1, x_2 = c - Bx_2,
\end{align*}
\]

where \(\Lambda\) is a vector of additional Lagrangian multipliers for the slack variables, and \(\beta > 0\) is their associated penalty parameter.

The two-level ADMM method uses a two-level nested loop to search and update the two different sets of Lagrangian multipliers (\(\lambda\) and \(\Lambda\)). The inner loop updates \(\lambda\) together with the slack parameters \(z\) based on a fixed \(\Lambda\), while the outer loops updates \(\Lambda\) and the associated penalty \(\beta\) based on the latest solution computed from inner loop. The inner and outer loops can also be implemented as a single loop where the updates by the outer loop are performed only if the search meets certain predetermined criteria \(\eta()\).

D. Deep Learning Neural Network (DNN)

Deep Neural Networks are a learning framework composed of a sequence of layers, with each typically taking as inputs the results of the previous layer (e.g., [36]). Commonly used Feed Forward Neural Networks (FNNs) are DNNs where the layers are fully connected. The function connecting the layers, from \(\mathbb{R}^n\) to \(\mathbb{R}^m\) is given by:

\[
y = \pi(Wx + b),
\]

where \(x \in \mathbb{R}^n\) is an input vector with dimension \(n\), \(y \in \mathbb{R}^m\) is the output vector with dimension \(m\), \(W \in \mathbb{R}^{m \times n}\) is a matrix of weights, and \(b \in \mathbb{R}^m\) is a bias vector. Both \(W\) and \(b\) define the trainable parameters of the network. The activation function \(\pi\) is usually non-linear (e.g., a rectified linear unit (ReLU)).

A DNN \(\mathcal{M} : \mathbb{R}^n \mapsto \mathbb{R}^m\) with \(i\) hidden layers \(h\) can be formulated as:

\[
\begin{align*}
h_1 &= \pi(W_1x + b_1), \\
h_j &= \pi(W_jh_{j-1} + b_j), \quad \forall j \in \{2, 3, \ldots, i\} \\
y &= \pi(W_i+1h_i + b_{i+1}) \\
\end{align*}
\]

(16)

where \(x \in \mathbb{R}^n\) and \(y \in \mathbb{R}^m\) are the input and output vectors. Learning DNN model \(\mathcal{M}\) on a data set \(T\) consists of finding the matrices \(W_j\) and bias vectors \(b_j\) for all \(j \in \{1, 2, \ldots, i+1\}\)

### Model 2: Regional AC Optimal Power Flow: \(P_{RAC}\)

**input:** \(S^F[k] = (S^F_i : i \in N_k)\) 
\(S^C[k] = (S^C_{ij} : (i,j) \in R_k \cup R_k^R)\) 
\(V^C[k] = (V^C_i : (i,j) \in R_k \cup R_k^R)\)

**variables:** \(S^R[k] = (S^R_i : \forall i \in N_k)\),  
\(V[k] = (V_i : \forall i \in N_k \cup N_k^N)\),  
\(S^F[k] = (S^F_{ij} : \forall (i,j) \in E_k \cup E_k^R \cup R_k \cup R_k^R)\)

**minimize:** \(O(S^F[k]) = \sum_{i \in N_k} M_i(S^F_i)\)  

(18)

**subject to intra-regional constraints:**

\(\forall i \leq v_i \leq \hat{V}_i\)  
\(\forall i \in N_k\)

(19)

\(|S^F_{ij}| \leq \hat{S}^F_{ij}\)  
\(\forall (i,j) \in E_k \cup E_k^R\)

(20)

\(|S^F_{ij}| \leq \hat{S}^F_{ij}\)  
\(\forall (i,j) \in E_k \cup E_k^R\)

(21)

\(S^F_{ij} = Y^F_i|V_i|^2 - Y^F_jV_iV_j^*\)  
\(\forall (i,j) \in E_k \cup E_k^R\)

(22)

\(S^F - S^F = \sum_{(i,j) \in E_k \cup E_k^R} S^F_{ij}\)  
\(\forall i \in N_k \setminus N_k^N\)

(23)

**subject to inter-regional constraints:**

\(|S^F_{ij}| \leq \hat{S}^F_{ij}\)  
\(\forall (i,j) \in R_k \cup R_k^R\)

(24)

\(S^F_{ij} = Y^F_i|V_i|^2 - Y^F_jV_iV_j^*\)  
\(\forall (i,j) \in R_k \cup R_k^R\)

(25)

\(S^F - S^F = \sum_{(i,j) \in R_k \cup R_k^R} S^F_{ij}\)  
\(\forall i \in N_k^R\)

(26)

**subject to consensus constraints:**

\(S^F_{ij} = S^F_{ij}\)  
\(\forall (i,j) \in R_k \cup R_k^R\)

(27)

\(V_i = V^C_i\)  
\(\forall (i,j) \in R_k \cup R_k^R\)

(28)

to make the output predictions \(\hat{y}_t\) close to the ground truth data \(y_t\) for all \(t \in T\), as measured by a loss function \(L\):

\[
\begin{align*}
\min & \quad \w_j, b_j, \{i, j+1\} \sum_{t \in T} L(y_t, \hat{y}_t), \\
& \quad \text{where} \quad \hat{y}_t = \mathcal{M}(x_t)
\end{align*}
\]

(17)

IV. REGIONALLY DECENTRALIZED AC-OPF S

This section presents the ADMM mechanism to solve Regionally Decentralized AC-OPFs. The presentation is largely based on [5] and describes the regional AC-OPF model for each region, followed by showing the Augmented Lagrangian formulation for the ADMM approach.

A. Regional Decentralized AC-OPF Model With Consensus

Model 2 presents the regional decentralized AC-OPF model for each load balancing region/zone \(k \in K\), based on the centralized Model 1. Figure 2 shows the decomposition diagram, based on the example in Figure 1. Each load balancing
region only considers the grid within their boundary, plus the interconnections (coupling branches and their associated buses). The model relies on matching the consensus parameters $S^C[k]$ and $V^C[k]$ on the interconnections, by constraints (27)-(28), to synchronize with the other regions.

B. Augmented Lagrangian Reformulation

Model 3 shows the Augmented Lagrangian relaxation for Model 2, with the introduction of Lagrangian duals $\lambda_S[k]$, $\lambda_V[k]$, and the $\rho$ penalty parameters for each load balancing region $k$. Similarly, Model 4 shows the Augmented Lagrangian relaxation for the two-level ADMM (used by the inner loop). These models will be used in the ADMM algorithm, described in the next subsection.

C. ADMM Algorithm

The ADMM algorithm (Algorithm 1) receives the network topology $\mathcal{N}$, the load information $S^d$, and the search parameters $\rho_0$ and $t_{\text{max}}$. Lines 1–3 initialize the penalty parameter, and initialize the consensus variables and their corresponding Lagrangian duals for all the regions $k \in K$. Line 4 executes the core procedure $t_{\text{max}}$ times, and Line 5 iterates over each region $k$. Line 7 executes Model 3 for each region. Line 9–10 update the Lagrangian multipliers for each of the region. Finally, line 12–13 export the consensus parameters. The traditional (aka flat-start) ADMM procedure can be initialized as shown in Algorithm 2. The algorithm for the two-level ADMM is presented (Algorithm 7).

V. LEARNING ARCHITECTURE: ML-ADMM

The previous section presented how to utilize decentralized optimization, e.g., ADMM methods in Algorithm 1, to find flows and voltages for shared interconnections. This section presents a decentralized machine-learning approach to speed up ADMM search by learning these entities.

A. Overview

The machine-learning approach is motivated by the recognition that, in practice, it would be costly to cold-start the ADMM instead of using predictions for the consensus variables ($S^C$, $V^C$) and their corresponding dual multipliers ($\lambda_S$, $\lambda_V$). If these predictions are available, the ADMM procedure can be initialized as in Algorithm 3. If all the consensus variables and dual multipliers are perfectly predicted, only one ADMM iteration would be required.

Predictions on load demands and renewable generations are already incorporated by various ISOs in their markets (e.g., MISO [37]). ML-ADMM generalizes this practice by incorporating the predictions on the consensus variables. The proposed methodology was applied within a general ADMM framework and the nonlinear AC-OPF formulation to demonstrate how to develop learning strategies for learning decentralized optimization problems in power systems. Note that the proposed learning methodology is general and does not necessarily require an augmented Lagrangian formulation and/or an ADMM approach. The same approach can be applied on other types of regional decomposition algorithms, with other types of OPF formulations, e.g., DC/linearized formulation or second-order cone OPF formulation. In addition, this approach does not change the inherent computational complexity of the underlying decomposition framework, nor does it modify any of existing communication architectures. The only addition is that agents need to train their own learning framework to initialize the underlying decomposition.

The remaining subsections will introduce the machine learning architecture (ML-ADMM) to predict the necessary quantities for Algorithm 3, and how to train the machine learning models.
**Model 4: Inner-Level Augmented Lagrangian: \( P_L \)**

\[
\text{input: } S^f[k] = (S^f_{ij} : i \in N_k) \\
S^c[k] = (S^c_{ij} : (i, j) \in R_k \cup R_k^g) \\
\lambda_S[k] = (\lambda_S'_{ij} : (i, j) \in R_k \cup R_k^g) \\
z_S[k] = (z_S'_{ij} : (i, j) \in R_k \cup R_k^g) \\
V^C[k] = (V^C_{ij} : (i, j) \in R_k \cup R_k^g) \\
\lambda_V[k] = (\lambda_V'_{ij} : (i, j) \in R_k \cup R_k^g) \\
z_V[k] = (z_V'_{ij} : (i, j) \in R_k \cup R_k^g) \\
\rho
\]

variables: \( S^f[k] = (S^f_{ij} : \forall i \in N_k) \), \( V[k] = (V_{ij} : \forall i \in N_k \cup N_k^N) \), \( S^f[k] = (S^f_{ij} : \forall (i, j) \in E_k \cup E_k^g \cup R_k \cup R_k^g) \)

minimize: \[
\sum_{i \in N_k} M_i(\|S^f_{ij}\|) + \sum_{(i, j) \in R_k \cup R_k^g} (\lambda_S'_{ij} \cdot S^f_{ij}) + \sum_{(i, j) \in R_k \cup R_k^g} (\lambda_V'_{ij} \cdot V_{ij}) \\
+ \frac{\rho}{2} \left[ \sum_{(i, j) \in R_k \cup R_k^g} \|S^f_{ij} - S^c_{ij} + z_S'_{ij}\|_2^2 \\
+ \sum_{(i, j) \in R_k \cup R_k^g} \|V_{ij} - V^C_{ij} + z_V'_{ij}\|_2^2 \right]
\]

subject to: \((19) - (26)\) (31)

**Algorithm 1: ADMM: Main Routine**

**Network data**: \( \mathcal{N}, S^d \)

**Search parameters**: \( P_0, l_{\text{max}} \)

1. \( \rho \leftarrow P_0 \)
2. for \( k \in K \) do
3. \[ S^c[k], \lambda_S[k], V^C[k], \lambda_V[k] \leftarrow \text{initialize}(k) \]
4. for \( t = 1, \ldots, l_{\text{max}} \) do
5. for \( k \in K \) do
6. Regional AC-OPF:
7. \( (S^f_{ij}, V_{ij} : (i, j) \in R_k \cup R_k^g) \leftarrow \\
P_L(S^f[k], S^C[k], \lambda_S[k], V^C[k], \lambda_V[k]) \)
8. Lagrange multiplier update:
9. \( \lambda_S[k] \leftarrow (\lambda_S'_{ij} \leftarrow \lambda_S'_{ij} + (S^f_{ij} - S^c_{ij}) : (i, j) \in R_k \cup R_k^g) \)
10. \( \lambda_V[k] \leftarrow (\lambda_V'_{ij} \leftarrow \lambda_V'_{ij} + (V_{ij} - V^C_{ij}) : (i, j) \in R_k \cup R_k^g) \)
11. Consensus update:
12. \( S^f_{ij} \leftarrow (S^f_{ij} + S^f_{ij}')/2 : V_{ij} \in R_k \cup R_k^g \)
13. \( V^C_{ij} \leftarrow (V^C_{ij} + V^C_{ij}')/2 : (i, j) \in R_k \cup R_k^g \)
14. Penalty \( \rho \) update (optional)
15. \( \rho \leftarrow \text{update}_\rho() \)

**B. Deep Learning Models**

ML-ADMM aims at learning two sets of parameters: the consensus variables \( S^C, V^C \) and their corresponding dual multipliers \( \lambda_S, \lambda_V \) for every region \( k \), based on the current load forecast \( S^d \). Since these parameters are complex quantities, ML-ADMM first splits each quantity into its individual components as follows:

\[
S^d \rightarrow p^d + iq^d, \\
S^C \rightarrow p^C + iq^C, \quad V^C \rightarrow v^C \angle \theta^C, \\
\lambda_S \rightarrow \lambda_p + i\lambda_q, \quad \lambda_V \rightarrow \lambda_v \angle \lambda_\theta.
\]

where \( X \rightarrow X_r + iX_i \) splits a complex vector \( X \) into the real component vector \( X_r \) and the imaginary component vector \( X_i \) (i.e., splitting into the rectangular form), and \( X \rightarrow X_m + i\angle X_\theta \) splits the complex vector \( X \) into the magnitude vector \( X_m \) and the angle vector \( X_\theta \) (i.e., splitting into the polar form). Let \( x \) be the flattened input vector (\( p^d, q^d \)), and \( y[k] \) to be the target prediction quantities, where

\[
y[k] = \begin{cases} 
   p^C_{ij}, & \text{for active line flow} \\
   q^C_{ij}, & \text{for reactive line flow} \\
   v^C_{ij}, & \text{for voltage magnitude} \\
   \theta^C_{ij}, & \text{for voltage angle} \\
   \lambda_p[k], & \text{for active line flow dual} \\
   \lambda_q[k], & \text{for reactive line flow dual} \\
   \lambda_v[k], & \text{for voltage magnitude dual, and} \\
   \lambda_\theta[k], & \text{for voltage angle dual}
\end{cases}
\]

for each load balancing zone/region \( k \). To initialize the ADMM algorithm with predictions (Algorithm 3), each region \( k \) only needs to learn and predict all 8 types of \( y[k] \) independently, based on the current system load demand (input feature vector \( S^d \)).

In order to achieve the task, ML-ADMM constructs DNNs \( \mathcal{M}_{y[k]} \) of the form:

\[
\mathcal{M}_{y[k]}(x) : y[k] = \pi(W_2 h + b_2), \quad h = \pi(W_1 x + b_1)
\]

where \( h \) is the hidden layer with a dimension set to twice the dimension of the output vector \( y[k] \). Figure 3 illustrates the four types of DNNs constructed by ML-ADMM for predicting the coupling parameters and the associated dual multipliers for each region \( k \).
Algorithm 4: Regional Training With Backpropagation

Inputs: Initialized $M_{y[k]}$; Max epoch $\epsilon_{max}$; Data set $T[k]$

1. for $e = 1, 2, \ldots, \epsilon_{max}$ do
2. for $(x, y[k]) \in T[k]$ do
3. $y[k] \leftarrow$ BACKPROP($L(y[k]), y[k]$)
4. $M_{y[k]} \leftarrow$ BACKPROP($L(y[k]), y[k]$)

Algorithm 5: Cold-Start: Two-Level ADMM

1. Function initialize():
2. $S^C[k] \leftarrow (s_{ij}^C \leftarrow 0 : (i, j) \in R_k \cup R_k^C)$
3. $V^C[k] \leftarrow (V_{ij}^C \leftarrow 1 : (i, j) \in R_k \cup R_k^C)$
4. $\lambda_{S}[k] \leftarrow (\lambda_{ij}^S \leftarrow 0 : (i, j) \in R_k \cup R_k^S)$
5. $\lambda_{V}[k] \leftarrow (\lambda_{ij}^V \leftarrow 0 : (i, j) \in R_k \cup R_k^V)$
6. $z_{S}[k] \leftarrow (z_{ij} \leftarrow 0 : (i, j) \in R_k \cup R_k^S)$
7. $z_{V}[k] \leftarrow (z_{ij} \leftarrow 0 : (i, j) \in R_k \cup R_k^V)$
8. $\Lambda_{S}[k] \leftarrow (\Lambda_{ij}^S \leftarrow 0 : (i, j) \in R_k \cup R_k^S)$
9. $\Lambda_{V}[k] \leftarrow (\Lambda_{ij}^V \leftarrow 0 : (i, j) \in R_k \cup R_k^V)$

C. Decentralized Training

ML-ADMM trains the models $M_{y[k]}$ of each load balancing zone/region $k$ in a decentralized fashion using a data set $T[k]$, owned by region $k$. The training is performed in parallel for each type of target prediction quantity (listed in (30)). Algorithm 4 showcases a high-level view of the training process with back-propagation. The training terminates when $\epsilon_{max}$ epochs have been executed (Line 1). For each epoch, the algorithm then iterates over every input and output pair of input and output features $(x, y[k])$ from the data set $T[k]$ (Line 2). Line 3 computes the prediction for the target quantities. Finally, line 4 updates the DNN using back-propagation (BACKPROP), based on the current prediction error measured by the loss function $L$. After all the models $M_{y[k]}$ are trained by ML-ADMM, Algorithm 3 can then be applied with the predicted quantities. Observe that both the training and the optimization proceeds in a fully decentralized fashion. Moreover, during training, the region do not need to interact with each other.

D. Extension for Two-Level ADMM

As mentioned earlier, the two-level ADMM introduces slack variables $z$ and their corresponding dual multipliers $\Lambda$ for all the coupling constraints. Let $z_p$, $z_q$, $z_v$, $z_{\theta}$ be the vector of slack variables for coupling constraints corresponding to consensus variables $p^C$, $q^C$, $v^C$, and $\theta^C$, and let $\Lambda_p$, $\Lambda_q$, $\Lambda_v$, $\Lambda_{\theta}$ be the corresponding vector of dual multipliers for $z_p$, $z_q$, $z_v$, and $z_{\theta}$. Let $x$ be the flattened input vector $(p^d, q^d)$, and $y[k]$ be the target prediction quantities: $z_p[k]$, $z_q[k]$, $z_v[k]$, $z_{\theta}[k]$, $\Lambda_p[k]$, $\Lambda_q[k]$, $\Lambda_v[k]$, $\Lambda_{\theta}[k]$, for each load balancing zone/region $k$. The generalization of ML-ADMM learns and predicts the 8 additional types of quantities, based on the current system load demand. DNN architectures similar to those in Figure 3 are built and trained in the same decentralized fashion.
VI. FILTERING THE TRAINING DATA

ADMM runs may exhibit significant convergence properties, even for closely related inputs. The section investigates a novel idea: only using historical ADMM runs with high-quality convergence properties in the training set. The motivation here is not to imitate the behavior of all ADMM runs: rather it is to find initial values for the consensus parameters that will enable strong convergence of the optimization model.

a) ADMM Behavior: The filtering idea is motivated by the fact that ADMM runs for specific inputs may not be optimal, unique, or may not have converged when reaching their termination condition. For instance, Figures 4 and 5 display the active power and voltage magnitude, together with their corresponding dual multipliers, for a specific coupling branch and one of its associated buses. The results are for ADMM runs with 3,000 iterations over a variety of instances of the France_EHV test case. Each dot in the figures is an instance and the figures report correlations between the primal & dual infeasibility residues on the one hand and the active power and voltage magnitude (and their duals) on the other hand. As can be observed, there are strong correlations between these quantities and the convergence measures (i.e., primal and dual infeasibility residues) and also natural breakpoints that separates the runs with good convergence properties from the more problematic runs. Learning from instances with poor convergence qualities is not desirable and hence ML-ADMM utilizes two set of filters to select the training data.

b) Convergence Filter: The convergence filter $c(\alpha)$ returns a subset of the data set $T[k]$ by filtering instances whose primal or dual infeasibility residues are higher than a threshold specified by $\alpha$. In other words, the convergence filter excludes data sets by splitting the x-axis of Figures 4 and 5. Let $r_p(t)$ and $r_d(t)$ to be the primal and dual infeasibility residue for instance $t$, and $A_{p}$ and $A_{d}$ to be two arrays storing, in ascending order, the primal and dual infeasibility residues for all instances in $T[k]$. Let $A_{p}[i]/A_{d}[i]$ to be the $i$th element of the array $A_{p}$/$A_{d}$, and $\lceil \cdot \rceil$ to be the ceiling function. The threshold $r_{p/d}^{\text{thres}}$ for primal and dual infeasibility residues are given by:

$$r_{p}^{\text{thres}} = A_{p}[\lceil \alpha \times |T| \rceil]$$
$$r_{d}^{\text{thres}} = A_{d}[\lceil \alpha \times |T| \rceil]$$

where $0 < \alpha \leq 1$. The data set returned by filter $c(\alpha)$ for region/zone $k \in K$ is thus

$$\{ t \in T[k] \text{ where } r_{p}(t) \leq r_{p}^{\text{thres}} \land r_{d}(t) \leq r_{d}^{\text{thres}} \}.$$  

For the two-level ADMM, the convergence distance can be approximated by the magnitude of the slack parameters $z$. Therefore, the convergence filter $c(\alpha)$ for two-level ADMM can be simplified and returns a subset of the data set $T[k]$ by filtering instances whose slack parameter distances, as measured by a metric scoring function, are higher than a threshold specified by $\alpha$. Let $s_{c}(t)$ to be the slack distance score for instance $t$, and $A_{s}$ to be the arrays storing, in ascending order, the scores for all instances in $T[k]$. Let $A_{s}[i]$ to be the $i$th element of the array $A_{s}$, and $\lceil \cdot \rceil$ to be the ceiling function. The threshold are given by:

$$s_{c}^{\text{thres}} = A_{s}[\lceil \alpha \times |T| \rceil]$$

where $0 < \alpha \leq 1$. The data set returned by filter $c(\alpha)$ for region/zone $k \in K$ for the two-level ADMM is thus

$$\{ t \in T[k] \text{ where } s_{c}(t) \leq s_{c}^{\text{thres}} \}.$$
words, the filter excludes instances by splitting the y-axis of Figures 4 and 5. Let \( y[k](t) \) be the target prediction quantity (either \( p^C[k], q^C[k], v^C[k], \theta^E[k], \lambda_p[k], \lambda_q[k], \lambda_v[k], \) or \( \lambda_g[k] \)) for instance \( t \in T[k] \). Let \( m(y[k]) \) and \( \sigma(y[k]) \) be the mean and standard deviation vector of \( y[k] \) across the data set \( t \in T[k], \) i.e., the mean and standard deviation for the set \( \{y[k](t) : t \in T[k]\} \). The data set returned by filter \( s(\beta) \) for region/zone \( k \in K \) is:

\[
\{t \in T[k] \mid |y[k](t) - m(y[k])| \leq \beta \sigma(y[k])\},
\]

for all \( y[k] \in \{ p^C[k], q^C[k], v^C[k], \theta^E[k], \lambda_p[k], \lambda_q[k], \lambda_v[k], \lambda_g[k] \} \), where \( \leq \) generalizes \( \leq \) for vectors.

VII. EXPERIMENTAL EVALUATIONS

This section presents the data-generation process, the implementation and training details, the prediction accuracy, and convergence results of the learning-boosted ADMM with respect to the original and two-level ADMM and the AC-OPF solution.

A. Experimental Setup

1) Benchmarks: The experiments were performed on three real benchmark networks: France_EHV, LYON, and France. All three of them were extracted from the French Transmission Grid, covering 12 geographical regions of France. The experiments associate one agent per region. These benchmarks contain 1700 to 6700 buses, and 140 to 320 coupling branches (regional interconnections). Table II shows a summary of the benchmark statistics. Detailed network parameters can be found in [15].

2) Implementation Details: The ADMM and AC-OPF solving routines were implemented in Julia 1.6.1, with Ipopt 3.12.13 (w/ HSL MA57) as the nonlinear solver. The learning models were implemented in PyTorch [38] and run with Python 3.6, with the Mean Squared Error (MSE) as the loss function. The training was performed in parallel on Intel CPU cores at 2.1GHz, one core for each region. The training used Averaged Stochastic Gradient Descent (ASGD), with 64 mini-batches, 1000 epochs, and 0.001 learning rate.

3) Data Generation: The training data sets were generated by varying the load profiles of each test network from 80% to 122% of their original (complex) load values, with a step size of 0.02%. For each test case, to create enough diversity, every load is perturbed with random noise from the polar Laplace distribution whose parameter \( \lambda \) is set to 1% of the apparent power. Test cases with no feasible AC solutions were removed from the data set. The outputs of each test case is obtained by running the implemented ADMM routine for 3000 iterations with \( \rho \) set to 10 and 2000 iterations with \( \rho = 1000 \) for the two-level ADMM similar to [5]. Results from the ADMM routine were recorded as the ground truth, and split with 80%-20% ratio for training and testing purposes.

4) Evaluation Details: The evaluation aims at determining whether machine learning can speed up the convergence of the ADMM. It compares the learning-boosted ML-ADMM with three key baselines:

1) Nominal initialization [N-ADMM] — the ADMM initialized with cold-start (nominal consensus and zero dual values) and run for 500 iterations;
2) Ground Truth Data [P-DATA] — the ADMM initialized with cold-start (nominal consensus and zero dual values) and run for 3000 iterations;
3) Perfect initialization [P-ADMM] — the ADMM initialized with the perfect warm-start (ground-truth consensus and dual values) and run for 500 iterations (\( \approx \) P-DATA + ADMM).

N-ADMM is used to assess whether ML-ADMM is effective in producing solutions of better quality than nominal initializations with the same small number of iterations. P-DATA is used to assess whether 500 iterations of ML-ADMM is effective in recovering solutions of the same quality as ADMM with 3000 iterations (i.e., 1/6 of the original 3000 iterations). P-ADMM, which is seeded with the ground-truth data, is used to measure how well ML-ADMM would perform in comparison with a hot start with perfect information.

B. Learning Accuracy

Let \( T[k] \) be the collection of the data set obtained by applying filters to the testing data sets for region \( k \in K \). Let \( x^T(t) \) be the tensor of ground truths for data set \( t \in T[k], \) and \( \tilde{x}(t) \) to be predicted tensor. The mean prediction error (in \% metric) for \( \tilde{x} \) is given by:

\[
100 \times \frac{1}{|K|} \sum_{k \in K} \frac{1}{|T[k]|} \sum_{t \in T[k]} \frac{\|\tilde{x}(t) - x^T(t)\|_1}{\|x^T(t)\|_1}.
\]

Table III and XIII (for the two-level ADMM) present the prediction errors for various filters for the consensus parameters \( S^C, V^C \), and their dual multipliers \( \lambda_s, \lambda_v \). Since these quantities are complex numbers, for simplicity, the table presents each individual component. Table IV also shows the percentage of instances being filtered.

The results indicate that ADMM solutions are difficult to learn and generalize. For the original ADMM, the errors without filters can be as large as 24% for primal variables and close to 68% for dual multipliers. Similar trend can also be observed for the two-level ADMM. The filters significantly reduce predictor errors for both ADMM variants, producing data sets that are easier to learn. In particular, Table III shows that the accuracy for primal solutions (LYON-$q^C$) and dual

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1While the learning framework in principle should work on any number of regions/agents, artificially varying region/agent numbers on industrial systems may create unrealistic benchmarks/scenarios and breaking away from real-life practices (e.g., creating artificial inter-regional lines).
TABLE III
The Prediction Errors in Percentage for Various Filters

| Network | Filter | $g^p$ (%) | $g^d$ (%) | $w^p$ (%) | $w^d$ (%) | $\lambda_p$ | $\lambda_d$ | $\sigma_p$ | $\sigma_d$ |
|---------|--------|-----------|-----------|-----------|-----------|------------|------------|-----------|-----------|
| NIL     | 8.25   | 16.41     | 22.73     | 32.16     | 23.16     | 20.66      | 18.14      | 11.32     |           |
| Lyon    | 7.78   | 14.68     | 21.13     | 31.58     | 22.61     | 21.82      | 19.64      | 12.91     |           |
| France-UV | 7.56   | 15.60     | 22.61     | 32.70     | 22.70     | 21.82      | 19.64      | 12.91     |           |
| France  | 6.44   | 11.66     | 18.21     | 27.55     | 22.55     | 21.82      | 19.64      | 12.91     |           |
| France-UV | 5.77   | 9.98      | 16.98     | 26.57     | 22.57     | 21.82      | 19.64      | 12.91     |           |

Table IV
Percentage of Filtered Data

| Network | Filter | NIL (%) | Lyon (%) | France (%) |
|---------|--------|---------|----------|------------|
| NIL     | 9.02%  | 12.65%  | 11.73%   | 12.38%     |
| Lyon    | 11.23% | 14.68%  | 13.24%   | 13.85%     |
| France  | 10.19% | 13.36%  | 12.45%   | 12.87%     |

Table V
Objective Gap Against P-DATA in % (Avg / # Cases)

| Network | Filter | N-ADMM Iterations | 5 | 10 | 20 | 50 | 100 | 200 | 500 | 800 |
|---------|--------|-------------------|---|----|----|----|-----|-----|-----|-----|
| NIL     | -0.17% | 0.02%             | 0.04% | 0.06% | 0.08% | 0.10% | 0.12% | 0.14% | 0.16% | 0.18% |
| Lyon    | -0.27% | 0.05%             | 0.08% | 0.10% | 0.12% | 0.14% | 0.16% | 0.18% | 0.20% | 0.22% |
| France  | -0.37% | 0.08%             | 0.11% | 0.13% | 0.15% | 0.17% | 0.19% | 0.21% | 0.23% | 0.25% |

Table VI
Two-Level ADMM: Objective Gap Against P-DATA in % (Avg/#)

| Network | Filter | N-ADMM Iterations | 5 | 10 | 20 | 50 | 100 | 200 | 500 | 800 |
|---------|--------|-------------------|---|----|----|----|-----|-----|-----|-----|
| NIL     | -0.29% | 0.04%             | 0.06% | 0.08% | 0.10% | 0.12% | 0.14% | 0.16% | 0.18% | 0.20% |
| Lyon    | -0.33% | 0.06%             | 0.08% | 0.10% | 0.12% | 0.14% | 0.16% | 0.18% | 0.20% | 0.22% |
| France  | -0.38% | 0.08%             | 0.10% | 0.12% | 0.14% | 0.16% | 0.18% | 0.20% | 0.22% | 0.24% |

C. Performance of ML-ADMM Against the Ground Truth

This section presents the performance results of ML-ADMM over all testing instances, including those instances with high infeasibility residues. Table VI and VII present the average objective gap of ML-ADMM (over all test cases and regions $k \in K$) against the ground truth P-DATA when ML-ADMM is run for a number of iterations ranging from 5 to 500 ADMM. The average objective gaps of N-ADMM are also included for comparison purposes. Let $O^*$ be the objective value from P-DATA and let $\hat{O}$ be the objective value returned by a run of ML-ADMM. The objective gap is defined as

$$100 \times \frac{\hat{O} - O^*}{O^*}$$

In addition, Tables VII and IX (and Tables VIII and X for the two-level ADMM) also report average results for the primal and dual residues $r_p$ and $r_d$. solutions (LYON-$\lambda^p$) improve by almost 12% and 54% when using specific filters.

The convergence filters almost always provide higher accuracy than the standard deviation filters. This may be explained by the fact that the convergence variations are not necessarily Gaussian and hence the standard deviation filters are potentially biased against instances with high infeasibility residues that occur frequently.
The results show that ML-ADMM provides orders of magnitude improvements in objective gap, primal residue, and dual residue over ADMM for small numbers of iterations. Within 500 iterations, the ML-ADMM variants for the original ADMM recover almost the same solution quality as P-DATA (<0.3% objective difference). For the two-level ADMM, ML-ADMM variants recover almost the same solution quality in less than 200 iterations, which is natural since the two-level ADMM generally converges faster than the original variant. Interestingly, within 5 iterations, the ML-ADMM variants differ only by at most 3% (1% for the two-level ADMM) from the ground truth. On the contrary, N-ADMM exhibits objective gaps of $-3.44\%$, $-10.44\%$, and $-24.21\%$ ($-23.01\%$, $-24.44\%$, and $-29\%$ for the two-level ADMM), demonstrating the value of learning for fast convergence.

D. Performance of ML-ADMM Against Centralized AC-OPF

Since not all the test cases from P-DATA converged with the same level of infeasibility, the comparison against the ground-truths may not always be ideal: indeed, ML-ADMM may obtain solutions with potentially better objectives, but these would be reported as errors in Tables V and VI. This section further evaluates the learning routines against solutions obtained by a centralized AC-OPF procedure, which almost always produces better objective values. Table XI and XII report the average optimality gap over all the testing instances. Again, the ML-ADMM variants provide orders of magnitude improvements in optimality gaps over N-ADMM. For the two-level ADMM with faster convergence, ML-ADMM can even produce solutions within 1% optimality gap against centralized AC-OPF routine with only 50 ADMM iterations.
iterations. Moreover, the ML-ADMM variants recover almost the same optimality gap as P-ADMM and could even deliver a smaller optimality gap for the France_EHV benchmark (LYON for the two-level ADMM). Again, filters further improve the benefits of learning. As indicated in previous tables, P-ADMM (seeded with P-DATA) have almost no improvements.

Figure 6 further shows the optimality gap statistics (summarized by two box-(and-whisker) plots) for all the testing instances for the largest French benchmark for the traditional ADMM. The box-plots indicate that ML-ADMM is essentially similar to P-ADMM, which is initialized with the ground truth, and produces orders of magnitude improvements compared to the traditional ADMM. The box-plots indicate tighter filtering parameters would generally result in slightly better skew/median optimality gaps, and with largely similar spread and variance.

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This paper proposed ML-ADMM, a decentralized machine-learning framework to accelerate the convergence of an ADMM algorithm for solving the AC-OPF problem. The framework learns the coupling parameters of the regionally decentralized AC-OPF formulation, which can be used to hot-start the ADMM algorithm when new instances arrived. The paper has also explored the benefits of learning filters — filters that prevent machine learning being trained on instances with bad convergence properties. Experimental results on data sets from the French networks have showed that ML-ADMM produces solutions of similar quality than the traditional and two-level ADMM algorithms within a fraction (1/6) of iterations (500 versus 3,000). Moreover, ML-ADMM can produce solutions of similar quality as the ADMM algorithm hot-started with the ground truths for the consensus and dual multipliers. Filtering the datasets to learn from "good" runs also generally provides some additional benefits. These results indicate that machine learning could be a valuable tool for future smart grids operated with distributed optimization algorithms similar to ADMM.

**APPENDIX**

See Model 4, Algorithms 5–7, and Tables XIII–XV.

**ACKNOWLEDGMENT**

The authors thank the NSF Artificial Intelligence Research Institute for Advances in Optimization (https://www.ai4opt.org/) for the support.

**REFERENCES**

[1] X. Fang, S. Misra, G. Xue, and D. Yang, “Smart grid—The new and improved power grid: A survey,” IEEE Commun. Surveys Tutts., vol. 14, no. 4, pp. 944–980, 4th Quart., 2012.

[2] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, “Distributed optimization and statistical learning via the alternating direction method of multipliers,” Found. Trends Mach. Learn., vol. 3, no. 1, pp. 1–122, 2011.

[3] D. K. Molzahn et al., “A survey of distributed optimization and control algorithms for electric power systems,” IEEE Trans. Smart Grid, vol. 8, no. 6, pp. 2941–2962, Nov. 2017.

[4] Y. Wang, L. Wu, and S. Wang, “A fully-decentralized consensus-based ADMM approach for DC-OPF with demand response,” IEEE Trans. Smart Grid, vol. 8, no. 6, pp. 2637–2647, Nov. 2017.

[5] K. Sun and X. A. Sun, “A two-level ADMM algorithm for AC OPF with global convergence guarantees,” IEEE Trans. Power Syst., vol. 36, no. 6, pp. 5271–5281, Nov. 2021.

[6] A. X. Sun, D. T. Phan, and S. Ghosh, “Fully decentralized AC optimal power flow algorithms,” in Proc. IEEE Power Energy Soc. Gen. Meeting, 2013, pp. 1–5.

[7] T. Erseghe, “Distributed optimal power flow using ADMM,” IEEE Trans. Power Syst., vol. 29, no. 5, pp. 2370–2380, Sep. 2014.

[8] S. Magnusson, P. C. Weeraddana, and C. Fischione, “A distributed approach for the optimal power-flow problem based on ADMM and sequential convex approximations,” IEEE Trans. Control Netw. Syst., vol. 2, no. 3, pp. 238–253, Sep. 2015.

[9] S. Mhanna, A. C. Chapman, and G. Verbič, “Component-based dual decomposition methods for the OPF problem,” Sustain. Energy Grids Netw., vol. 16, pp. 91–110, Dec. 2018.

[10] S. Mhanna, G. Verbič, and A. C. Chapman, “Adaptive ADMM for distributed AC optimal power flow,” IEEE Trans. Power Syst., vol. 34, no. 3, pp. 2025–2035, May 2019.

[11] F. Fioretto, T. W. Mak, and P. Van Hentenryck, “Predicting AC optimal power flows: Combining deep learning and Lagrangian dual methods,” in Proc. AAAI Conf. Artif. Intell., 2020, pp. 630–637.

[12] Z. Yan and Y. Xu, “Real-time optimal power flow: A Lagrangian based deep reinforcement learning approach,” IEEE Trans. Power Syst., vol. 35, no. 4, pp. 3271–3273, Jul. 2020.

[13] X. Pan, T. Zhao, and M. Chen, “DeepOPF: Deep neural network for DC optimal power flow,” in Proc. IEEE Int. Conf. Commun., Control, Comput. Technol. Smart Grids (SmartGridComm), 2019, pp. 1–6.

[14] P. V. Hentenryck, “Machine learning for optimal power flows,” 2021. [Online]. Available: https://pubsonline.informs.org/doi/abs/10.1287/educ.2021.0234

[15] M. Chatzos, T. Mak, and P. Van Hentenryck, “Spatial network decomposition for fast and scalable AC-OPF learning,” IEEE Trans. Power Syst., vol. 37, no. 4, pp. 2601–2612, Jul. 2022.

[16] S. Misra, L. Rouald, and Y. Ng, “Learning for constrained optimization: Identifying optimal active constraint sets,” 2019, arXiv:1802.09639.

[17] A. S. Xavier, F. Qiu, and S. Ahmed, “Learning to solve large-scale security-constrained unit commitment problems,” INFORMS J. Comput., vol. 33, no. 2, pp. 759–776, 2020.

[18] D. Deka and S. Misra, “Learning for DC-OPF: Classifying active sets using neural nets,” in Proc. IEEE Milan PowerTech, Jun. 2019, pp. 1–6.

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### TABLE XIV

| Network | Filter | $A_p$ | $A_q$ | $A_p$ | $A_q$ |
|---------|--------|------|------|------|------|
| NIL     | 0.18   | 0.19 | 0.19 | 0.19 | 0.19 |
| Lyon    | 0.18   | 0.19 | 0.19 | 0.19 | 0.19 |
| Paris    | 0.18   | 0.19 | 0.19 | 0.19 | 0.19 |

**VIII. CONCLUSION**

This paper proposed ML-ADMM, a decentralized machine-learning framework to accelerate the convergence of an ADMM algorithm for solving the AC-OPF problem. The framework learns the coupling parameters of the regionally decentralized AC-OPF formulation, which can be used to hot-start the ADMM algorithm when new instances arrived. The paper has also explored the benefits of learning filters — filters that prevent machine learning being trained on instances with bad convergence properties. Experimental results on data sets from the French networks have showed that ML-ADMM produces solutions of similar quality than the traditional and two-level ADMM algorithms within a fraction (1/6) of iterations (500 versus 3,000). Moreover, ML-ADMM can produce solutions of similar quality as the ADMM algorithm hot-started with the ground truths for the consensus and dual multipliers. Filtering the datasets to learn from "good" runs also generally provides some additional benefits. These results indicate that machine learning could be a valuable tool for future smart grids operated with distributed optimization algorithms similar to ADMM.

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**TABLE XV**

| Network | Filter | $A_p$ | $A_q$ | $A_p$ | $A_q$ |
|---------|--------|------|------|------|------|
| NIL     | 0.18   | 0.19 | 0.19 | 0.19 | 0.19 |
| Lyon    | 0.18   | 0.19 | 0.19 | 0.19 | 0.19 |
| Paris    | 0.18   | 0.19 | 0.19 | 0.19 | 0.19 |

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[19] F. Hasan, A. Kargarian, and I. Mohammadi, “Hybrid learning aided inactive constraints filtering algorithm to enhance AC OPF solution time,” IEEE Trans. Ind. Appl., vol. 57, no. 2, pp. 1325–1334, Mar./Apr. 2021.
[20] A. Robson, M. Jamei, C. Ududec, and L. Mones, “Learning an optimally reduced formulation of OPF through meta-optimization,” 2020, arXiv:1911.06784.
[21] K. Baker, “A learning-boosted Quasi-Newton method for AC optimal power flow,” 2020, arXiv:2007.06074.
[22] K. Baker, “Learning warm-start points for AC optimal power flow,” in Proc. IEEE 29th Int. Workshop Mach. Learn. Signal Process. (MLSP), 2019, pp. 1–6.
[23] L. Chen and J. E. Tate, “Hot-starting the AC power flow with convolutional neural networks,” 2020, arXiv:2004.09342.
[24] X. Pan, M. Chen, T. Zhao, and S. H. Low, “DeepOPF: A feasibility-optimized deep neural network approach for AC optimal power flow problems,” 2020, arXiv:2007.01002.
[25] A. S. Zamzam and K. Baker, “Learning optimal solutions for extremely fast AC optimal power flow,” in Proc. IEEE Int. Conf. Commun., Control, Comput. Technol. Smart Grids (SmartGridComm), 2020, pp. 1–6.
[26] D. Biagioni, P. Graf, X. Zhang, A. S. Zamzam, K. Baker, and J. King, “Learning-accelerated ADMM for distributed DC optimal power flow,” IEEE Control Syst. Lett., vol. 6, pp. 1–6, 2022.
[27] A. Venzke and S. Chatzivasileiadis, “Verification of neural network behaviour: Formal guarantees for power system applications,” IEEE Trans. Smart Grid, vol. 12, no. 1, pp. 383–397, Jan. 2021.
[28] A. Venzke, G. Qu, S. Low, and S. Chatzivasileiadis, “Learning optimal power flow: Worst-case guarantees for neural networks,” in Proc. IEEE Int. Conf. Commun., Control, Comput. Technol. Smart Grids (SmartGridComm), 2020, pp. 1–7.
[29] X. Pan, T. Zhao, M. Chen, and S. Zhang, “DeepOPF: A deep neural network approach for security-constrained DC optimal power flow,” IEEE Trans. Power Syst., vol. 36, no. 3, pp. 1725–1735, May 2021.
[30] A. Velloso and P. Van Hentenryck, “Combining deep learning and optimization for preventive security-constrained DC optimal power flow,” IEEE Trans. Power Syst., vol. 36, no. 4, pp. 3618–3628, Jul. 2021.
[31] Y. Zhou et al., “A data-driven method for fast AC optimal power flow solutions via deep reinforcement learning,” J. Mod. Power Syst. Clean Energy, vol. 8, no. 6, pp. 1128–1139, Nov. 2020.
[32] J. H. Woo, L. Wu, J.-B. Park, and J. H. Roh, “Real-time optimal power flow using twin delayed deep deterministic policy gradient algorithm,” IEEE Access, vol. 8, pp. 213611–213618, 2020.
[33] E. R. Sanseverino, M. L. D. Silvestre, L. Mineo, S. Favuzza, N. Q. Nguyen, and Q. T. T. Tran, “A multi-agent system reinforcement learning based optimal power flow for islanded microgrids,” in Proc. IEEE 16th Int. Conf. Environ. Elect. Eng. (EEEIC), 2016, pp. 1–6.
[34] M. Chatzos, F. Fioretto, T. W. K. Mak, and P. V. Hentenryck, “High-fidelity machine learning approximations of large-scale optimal power flow,” 2020, arXiv:2006.16356.
[35] W. Chen, S. Park, M. Tanneau, and P. V. Hentenryck, “Learning optimization proxies for large-scale security-constrained economic dispatch,” 2021, arXiv:2112.13469.
[36] Y. LeCun, Y. Bengio, and G. Hinton, “Deep learning,” Nature, vol. 521, pp. 436–444, May 2015.
[37] MISO, Carmel, IN, USA. Business Practices Manuals. (2021). [Online]. Available: https://www.misoenergy.org/legal/business-practice-manuals/
[38] A. Paszke et al., “Automatic differentiation in PyTorch,” in Proc. NIPS Workshop, 2017, pp. 1–4.