Entropy Corrections for Schwarzschild and Reissner-Nordström Black Holes

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Schwarzschild black hole being thermodynamically unstable, corrections to its entropy due to small thermal fluctuations cannot be computed. However, a thermodynamically stable Schwarzschild solution can be obtained within a cavity of any finite radius by immersing it in an isothermal bath. For these boundary conditions, classically there are either two black hole solutions or no solution. In the former case, the larger mass solution has a positive specific heat and hence is locally thermodynamically stable. We find that the entropy of this black hole, including first order fluctuation corrections is given by:

$$S = S_{BH} - \frac{1}{2} \ln \left( \frac{3}{\pi(R)} \left( \frac{S_{BH}}{4\pi} \right)^{1/2} - 2 \right) - (1/2) \ln(4\pi),$$

where $S_{BH} = A/4$ is its Bekenstein-Hawking entropy and $R$ is the radius of the cavity. We extend our results to four dimensional Reissner-Nordström black holes, for which the corresponding expression is:

$$S = S_{BH} - \frac{1}{2} \ln \left( \frac{(S_{BH}/\pi R^2)(3S_{BH}/\pi R^2 - 2\sqrt{S_{BH}/\pi R^2 - \alpha^2})(\sqrt{S_{BH}/\pi R^2 - \alpha^2})}{(S_{BH}/\pi R^2 - \alpha^2)} \right) - (1/2) \ln(4\pi).$$

Finally, we generalise the stability analysis to Reissner-Nordström black holes in arbitrary spacetime dimensions, and compute their leading order entropy corrections. In contrast to previously studied examples, we find that the entropy corrections in these cases have a different character.

I. INTRODUCTION

It is well-known that the specific heat of a Schwarzschild black hole is negative:

$$C = \frac{dM}{dT} = -8\pi M^2 < 0.$$

Thus it is thermodynamically unstable, and corrections to thermodynamic quantities of this black hole, e.g. entropy and temperature, are not well-defined. This is seen when one considers for instance the corrected entropy of any thermodynamic system (including black holes) due to small thermal fluctuations of the system, which is given by [1,2]:

$$S = S_0 - \frac{1}{2} \ln \left( E^2 - < E >^2 \right) + \cdots$$

$$= S_0 - \frac{1}{2} \ln \left( CT^2 \right) + \cdots. \quad (2)$$

When relation (2) is applied to black holes, one substitutes $S_0 \rightarrow S_{BH}$, the Bekenstein-Hawking entropy of the black hole, and calculates $C$ for the particular black hole under consideration. Note that in using (2) in the context of black holes, one is simply assuming that the black hole behaves as an ordinary thermodynamic system, following usual laws of thermodynamics. Then starting from a continuum partition function and performing an inverse Laplace transform gives the density of states, whose logarithm yields Eq.(2) above. In addition, if one assumes that the black hole (more precisely, its horizon) is built up of more fundamental entities, such as quanta of area, or that observables such as horizon area and mass are quantised in a certain way, then as shown in refs. [3] and [4], there could be additional terms in (2).

It may also be noted that there could be other corrections to black hole entropy due to a variety of other sources. For instance, quantum fluctuations of matter fields in black hole backgrounds, as well as fluctuations of spacetime geometry itself in the canonical quantum gravity framework give rise to corrections which are also logarithmic in nature. Details of these calculations can be found in [5–23]. Since our analysis is not tied to any specific model of quantum gravity, it remains valid as

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long as the black hole is large. On the other hand, the other corrections may become important as the size of the black hole approaches the Planck scale.

One sees that the formula breaks down for $C < 0$. However, as was first shown by York [24], a positive specific heat solution can be obtained by considering Schwarzschild black holes within a finite $S^2$ cavity immersed in an isothermal bath. In fact for such canonical boundary conditions there are either two or no solutions depending on whether the temperature of the bath is above or below a minimum value. The larger mass of the solutions has a positive heat and the other has a negative specific heat. When one takes the cavity radius to infinity keeping the wall-temperature of the cavity fixed (i.e., not changing the temperature of the bath) the positive specific heat solution disappears and only the negative specific solution survives giving the known black hole solution. This holds for higher dimensions as well [25]. In this article, we consider the positive specific heat solution and compute leading order corrections to its entropy. As we will see the logarithmic correction term due to small thermal fluctuation is of a different nature unlike in several other cases studied before [5–23]. In this case, the mass and specific heat of the black hole are functions of the temperature and cavity radius, and the entropy correction is more interesting and non-trivial, as we will see below. The case of charged black hole, i.e., Reissner-Nordström black holes, within a cavity was first studied in [26] and we will find correction for their entropy as well for thermal fluctuation. See also [27] for related work on thermodynamics of Schwarzschild and Reissner-Nordström-AdS black holes. This paper is arranged in the following way. In the next section, we briefly review the thermodynamics of Schwarzschild black holes within a cavity and compute the entropy correction. In section (III), we extend our results to Reissner-Nordström black holes in arbitrary spacetime dimensions. Finally, we end with a summary of our results and some further directions in sections (IV). We use units in which $G = c = \hbar = k_B = 1$.

II. SCHWARZSCHILD BLACK HOLE IN A BOX: ENTROPY CORRECTIONS

Let us first consider a Schwarzschild black hole of mass $M$. Wick rotating the Lorentzian time $t \rightarrow i\tau$ one obtains

$$ds^2 = \left(1 - \frac{2M}{r}\right) d\tau^2$$

$$+ \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega_2^2.$$  (3)

where $d\Omega_2$ is the metric on unit $S^2$. The metric (3) is singular at $r = 2M$. The singularity can be removed by periodically identifying $\tau$ with a period $8\pi M$. Hence the resulting metric is complete and regular for $2M \leq r < \infty$. The fixed point set of the Killing vector $\partial/\partial \tau$ is a regular bolt of the metric [28]. The inverse of the periodicity of the $\tau$ coordinate gives the temperature of the hole measured at infinity.

A. Isothermal Cavity

Next, consider a spherical $S^2$ cavity of radius $R$ concentric with the black hole horizon. The black hole temperature measured at infinity is shifted by $(g_{00})^{-\frac{1}{2}}$, as one moves towards the black hole. The local temperature of the $S^2$ cavity is therefore given by

$$\beta = T^{-1} = 8\pi M \sqrt{1 - \frac{2M}{R}}.$$  (4)

Now we ask the reverse question. Given a spherical cavity for radius $R$ concentric with the black hole horizon. The black hole temperature measured at infinity is shifted by $(g_{00})^{-\frac{1}{2}}$, as one moves towards the black hole. The local temperature of the $S^2$ cavity is therefore given by

$$M^3 - \frac{1}{2}RM^2 + \frac{R\beta^2}{128\pi^2} = 0.$$  (5)

This is a cubic equation in $M$ and always admits a negative root and a pair of positive or complex roots depending on the coefficients. The coefficients are determined by the boundary variables $R$ and $\beta$. Defining new variables $x \equiv 2M/R$ and $\sigma = \beta/4\pi R$, Eq.(5) can be cast in the following simple form:

$$x^3 - x^2 + \sigma^2 = 0.$$  (6)

When positive solutions exist, larger and smaller solutions of $x$ give larger and smaller hole respectively. Note that the positive roots take values from zero to one (see figure 1). It can be shown that two positive roots occurs if and only if [24]

$$\sigma \leq \frac{2}{\sqrt{27}}.$$  (7)

Thus one requires the temperature $T$ of the bath to be

$$T \geq \frac{\sqrt{27}}{8\pi R}.$$  (8)

At the equality of (7) or (8) the two roots are degenerate. The larger solution therefore takes value within the closed interval $[\frac{2}{3}, 1]$. For completeness we mention here the explicit solutions of (6) here. The exact solutions are:
which translate in terms of black hole mass as:

\[ M_1 = \frac{R}{6} \left[ 1 - 2 \cos \left( \frac{\alpha}{3} + \frac{\pi}{3} \right) \right], \]

\[ M_2 = \frac{R}{6} \left[ 1 + 2 \cos \left( \frac{\alpha}{3} \right) \right], \]

where

\[ \cos \alpha = 1 - \frac{27}{2} \sigma^2. \]  

(13)

Evidently at the equality of (7), \( \alpha = \pi \), the solutions coincide, and

\[ x = \frac{2}{3} \Leftrightarrow M = \frac{R}{3}. \]  

(14)

By calculating the Euclidean action of the solution it was found in [24] that the entropy of the black hole is unaffected by the presence of the box and is given by the usual relationship

\[ S_{BH} = 4\pi M^2. \]  

(15)

The specific heat is defined as

\[ C_R = T \left( \frac{\partial S_{BH}}{\partial T} \right)_R, \]  

(16)

From (4) and (15), one obtains:

\[ C = 8\pi M^2 \left( \frac{R - 2M}{3M - R} \right) = 4\pi R^2 x^2 \left( \frac{1 - x}{3x - 2} \right). \]  

(17)

From its form it is apparent that the specific heat is positive in the range

\[ \frac{2}{3} \leq x \leq 1 \Leftrightarrow 2M \leq R \leq 3M, \]  

(18)

which is possible only for the larger hole (\( M_2 \) above). This can be also observed from Figure 1. For constant value of \( R \), Figure 1 is a relation between (inverse of) temperature and the masses of the two Schwarzschild black holes. The mass of the larger solution increases with increasing temperature. For the smaller hole it is just the opposite. Also note that in the \( R \to \infty \) limit, the larger mass solutions do not exist meaningfully (it fills in the cavity) irrespective of the temperature. Only the smaller solution \( x_1 \) exists with \( M = \frac{\alpha}{8\pi} \) since \( x_1 \equiv \frac{2M}{R} = \frac{\beta}{4\pi R} \) in this limit.

\[ x_1 = \frac{1}{3} \left[ 1 - 2 \cos \left( \frac{\alpha}{3} + \frac{\pi}{3} \right) \right], \]

\[ x_2 = \frac{1}{3} \left[ 1 + 2 \cos \left( \frac{\alpha}{3} \right) \right], \]

\[ \frac{2}{3} \leq x \leq 1 \Leftrightarrow 2M \leq R \leq 3M, \]

\[ x \in \left[ \frac{2}{3}, 1 \right]. \]

\[ x = \frac{2}{3} \Leftrightarrow M = \frac{R}{3}. \]

\[ \cos \alpha = 1 - \frac{27}{2} \sigma^2. \]

\[ S_{BH} = 4\pi M^2. \]

\[ C_R = T \left( \frac{\partial S_{BH}}{\partial T} \right)_R, \]

\[ C = 8\pi M^2 \left( \frac{R - 2M}{3M - R} \right) = 4\pi R^2 x^2 \left( \frac{1 - x}{3x - 2} \right). \]

\[ \frac{2}{3} \leq x \leq 1 \Leftrightarrow 2M \leq R \leq 3M, \]

\[ x \in \left[ \frac{2}{3}, 1 \right]. \]

\[ S_{BH} = 4\pi M^2. \]

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\[ C = 8\pi M^2 \left( \frac{R - 2M}{3M - R} \right) = 4\pi R^2 x^2 \left( \frac{1 - x}{3x - 2} \right). \]

\[ x \in \left[ \frac{2}{3}, 1 \right]. \]
footing as the coefficients in those cases as the correction
term here is not of the type $-k \ln(S_{BH})$. Rather, if we
are to write it in terms of $S_{BH}$ it takes the form given
by the second equation of (19) which involves the cavity
radius as well.

C. Higher Dimensions

Higher dimensional Schwarzschild black holes and their
thermodynamics within a finite isothermal cavity have
been studied in [25]. For $d$-dimensions the inverse tem-
perature at the $(d - 2)$-dimensional boundary is given by:

$$\beta = T^{-1} = \frac{4\pi x R}{d-3} \sqrt{1 - x^{d-3}} \equiv \frac{4\pi R \sigma}{d-3}, \quad (20)$$

from which one obtains:

$$x^{d-1} - x^2 + \sigma^2 = 0. \quad (21)$$

This equation cannot be solved using ordinary algebraic
methods. However, analytic solution is still possible [25].
This would be needed if one requires the entropy cor-
rection as a function of cavity radius and temperature.
However, as in the above case of four dimensions the
general properties of the corrections to entropy can be
studied without requiring the explicit solutions by study-
ning how the correction term varies with $x$. (Again note
that $x$ takes value within the interval $[0, 1]$.) This will be
done in the next section as a special case of $d$-dimensional
Reissner-Nordström. As we will see there entropy correc-
tions to $d$-dimensional Reissner-Nordström (and hence
Schwarzschild) black holes can be exactly evaluated.

III. REISSNER-NORDSTRÖM BLACK HOLES

We now extend the above analysis for the Reissner-
Nordström black holes which have a charge. This means
we would have to specify suitable electrostatic potential
at the boundary. We first discuss the case of four dimen-
sions.

A. Four dimensions

The Euclideanised Reissner-Nordström black hole metri-
c is:

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2$$
$$+ \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega_2^2, \quad (22)$$

with inverse temperature at boundary [26]:

$$\beta = T^{-1} = 4\pi r_+ \left(1 - \frac{Q^2}{r_+^2}\right)^{-1} \left(1 - \frac{r_+}{R}\right)^{1/2} \left(1 - \frac{Q^2}{r^2 R}ight)^{1/2}$$
$$= \frac{4\pi R x^{5/2}}{x^2 - \alpha^2} \sqrt{(1-x)(x-\alpha^2)}, \quad (23)$$

where $r_+ = M + \sqrt{M^2 - Q^2}$ is the radius of the outer
horizon and

$$\alpha = \frac{Q}{R}.$$

The counterpart of Eq.(5) is:

$$(1 - \Phi^2) x^3 - x^2 + (1 - \Phi^2)^2 \sigma^2 = 0, \quad (24)$$

where $x$ and $\sigma$ has identical definitions in terms of $r_+$ and
$\beta$ as before. Here, $\Phi$ is the potential difference between
$r_+$ and $R$, suitably red-shifted:

$$\Phi = \frac{Q}{r_+} \left(1 - \frac{r_+}{R}\right)^{1/2} \left(1 - \frac{Q^2}{R r_+}\right)^{-1/2}$$
$$= \alpha \sqrt{\frac{1-x}{x^2 - \alpha^2 x}}, \quad (25)$$

Now, the two positive solutions of $x$ are:

$$x_1 = \frac{1}{3(1 - \Phi^2)} \left(1 - 2 \cos \left(\frac{\alpha}{3} + \frac{\pi}{3}\right)\right) \quad (26)$$
$$x_2 = \frac{1}{3(1 - \Phi^2)} \left(1 + 2 \cos \frac{\alpha}{3}\right) \quad (27)$$
$$\cos \alpha = 1 - \frac{27}{2} \sigma^2 (1 - \Phi)^4, \quad (28)$$

of which only the second one is thermodynamically sta-
ble. The entropy is, in this case:

$$S_{BH} = \pi r_+^2 = \pi R^2 x^2. \quad (29)$$

(where we have dropped the subscript 2 from $x_2$). The
specific heat at constant $\Phi$ and $R$ can now be computed
by using the relation:

$$C_{\Phi,R} = \left(\frac{\partial E}{\partial T}\right)_{\Phi,R} = - \left(\frac{\partial S/\partial x}{\partial \ln \sigma/\partial x}\right)_{\Phi,R} \quad (30)$$

with $\sigma$ obtained from Eq.(24). This yields:

$$C_{\Phi,R} = \frac{4\pi R^2 x^3 (1-x)}{3 x^2 - 2 x - \alpha^2}, \quad (31)$$

where now the specific heat is non-negative in the range:

$$r_+ \leq R \leq \frac{3}{2} r_+ - \frac{1}{2} \frac{Q^2}{r_+}, \quad (32)$$

Equivalently:

$$x \leq 1, \quad (33)$$
$$3 x^2 - 2 x - \alpha^2 \geq 0. \quad (34)$$
In the above range, the corrected entropy assumes the form:
\[
S = S_{BH} - \frac{1}{2} \ln \left( \frac{x^2(3x^2 - 2x - \alpha^2)(x - \alpha^2)}{(x^2 - \alpha^2)^2} \right)^{-1} + \frac{1}{2} \ln(4\pi) + \cdots
\] (35)
This can be expressed in terms of the Bekenstein-Hawking entropy as:
\[
S = S_{BH} - \frac{1}{2} \ln \left( \frac{S_{BH}}{\pi} - 2 \left( \frac{S_{BH}}{\pi} - \alpha^2 \right) \left( \frac{\pi}{\pi} - \alpha^2 \right)^{-1} \right) + \frac{1}{2} \ln(4\pi) + \cdots
\] (36)
As expected, the above reduces to Eq.(19) for \( \alpha = 0 \).

**B. Higher dimensions**

For the \( d \)-dimensional Reissner-Nordström black hole the Euclidean metric is given by
\[
ds^2 = \left( 1 - \frac{16\pi M}{(d - 2)\Omega_{d-2}r^{d-3}} + \frac{16\pi^2}{(d - 2)(d - 3)r^{2(d-3)}} \right) dt^2 + \left( 1 - \frac{16\pi M}{(d - 2)\Omega_{d-2}r^{d-3}} + \frac{16\pi^2}{(d - 2)(d - 3)r^{2(d-3)}} \right)^{-1} dr^2 + r^2 d\Omega_{d-2}^2,
\] (37)
where \( d\Omega_{d-2}^2 \) is a unit \( d-2 \) sphere with the standard round metric on it. The inverse temperature is given by:
\[
\beta = T^{-1} = \frac{4\pi r_+}{d - 3} \left( 1 - \frac{2Q^2}{(d - 2)(d - 3)r^{2(d-3)}} \right)^{-1} \times \left( 1 - \frac{r_+}{R} \right)^{d-3} \left( 1 - \frac{2Q^2}{(d - 2)(d - 3)(r_+/R)^{d-3}} \right)^{\frac{1}{2}}
\] (38)
where \( x = r_+/R \) as before, and now \( \alpha \) is defined as:
\[
(\alpha R)^{2(d-3)} = \frac{2Q^2}{(d - 2)(d - 3)}.
\] (39)
Similarly, the the potential difference is:
\[
\Phi = \frac{Q}{r_+^{d-3}} \sqrt{ \frac{1 - (r_+/R)^{d-3}}{1 - 2Q^2/(d - 2)(d - 3)(r_+/R)^{d-3}} } \left( \frac{2}{(d - 2)(d - 3)} \right)^{\frac{1}{2}} \right) \alpha^{d-3} \sqrt{ \frac{1 - \alpha^{d-3}}{x^{2(d-3)} - \alpha^{2(d-3)x^{d-3}}} }
\] (41)
and Eq.(24) is generalised to:
\[
\left( 1 - \frac{(d - 2)(d - 3)}{2} \Phi^2 \right)x^{d-1} - x^2 + \left( 1 - \frac{(d - 2)(d - 3)}{2} \Phi^2 \right)^2 \sigma^2 = 0,
\] (42)
where \( \sigma \) has been defined as in Eq.(20). Using (30), the specific heat at constant \( R \) and \( \Phi \) can now be computed as:
\[
C_{\Phi,R} = \frac{(d - 2)\Omega_{d-2}R^{d-2}}{(d - 1)x^{2(d-3)} - 2x^{d-3} - (d - 3)\alpha^{2(d-3)}}. \] (43)
Stability is now guaranteed if the parameters lie in a range such that:
\[
(d - 1)x^{2(d-3)} - 2x^{d-3} - (d - 3)\alpha^{2(d-3)} \geq 0.
\] (44)
Explicit evaluation of \( x(M, Q) \) and \( R(M, Q) \) from the above is complicated for higher dimensions though, because of the higher powers of \( x \). Eq.(43), along with (39) now gives the corrected entropy as:
\[
S = S_{BH} - \ln\left( \frac{4S_{BH}}{\Omega_{d-2}R^{d-2}} \right) \times \left\{ \begin{array}{l}
(d - 1)\left( \frac{4S_{BH}}{\Omega_{d-2}R^{d-2}} \right)^{2(d-3)/(d-2)} \\
\frac{-2(4S_{BH})}{\Omega_{d-2}R^{d-2}}(d-3)/(d-2) - (d - 3)\alpha^{2(d-3)} \times \left\{ \frac{4S_{BH}}{\Omega_{d-2}R^{d-2}} \right\}^{(d-3)/(d-2) - \alpha^{2(d-3)}} \times \left\{ \frac{4S_{BH}}{\Omega_{d-2}R^{d-2}} \right\}^{(d-3)/(d-2) - \alpha^{2(d-3)}} \right\}^{-1} \\
+ \frac{1}{2} \ln \left( (d - 2)\Omega_{d-2} \right) + \cdots
\] (45)
which can be expressed in terms of \( S_{BH} \) as:
\[
S = S_{BH} - \ln\left( \frac{4S_{BH}}{\Omega_{d-2}R^{d-2}} \right) \times \left\{ \begin{array}{l}
\frac{(d - 1)(4S_{BH})^{2(d-3)/(d-2)} - (d - 3)\alpha^{2(d-3)}}{\Omega_{d-2}R^{d-2}} \\
\frac{-2(4S_{BH})}{\Omega_{d-2}R^{d-2}}(d-3)/(d-2) - (d - 3)\alpha^{2(d-3)} \times \left\{ \frac{4S_{BH}}{\Omega_{d-2}R^{d-2}} \right\}^{(d-3)/(d-2) - \alpha^{2(d-3)}} \times \left\{ \frac{4S_{BH}}{\Omega_{d-2}R^{d-2}} \right\}^{(d-3)/(d-2) - \alpha^{2(d-3)}} \right\}^{-1} \\
+ \frac{1}{2} \ln \left( (d - 2)\Omega_{d-2} \right) + \cdots
\] (46)
For zero-charge, that is for \( d \)-dimensional Schwarzschild black holes, the corresponding expression is:
\[
S = S_{BH} - \frac{1}{2} \ln \left[ \frac{(d - 1)(\frac{4S_{BH}}{\Omega_{d-2}R^{d-2}})^{(d-3)/(d-2)-2} - 2}{\Omega_{d-2}R^{d-2}} \right]^{-1} - \frac{1}{2} \ln \left( (d - 2)\Omega_{d-2} \right) + \cdots
\] (47)
In each case, the entropy correction is different in nature than in previously studied examples.
IV. DISCUSSIONS

In this article, we have shown how leading order corrections due to thermal fluctuations can be obtained for Schwarzschild and Reissner-Nordström black holes within a finite cavity. With appropriate boundary data, within a cavity thermodynamically stable solutions are possible. Normally, such corrections cannot be computed for Schwarzschild owing to its negative specific heat. For Reissner-Nordström black holes, corrections of this type were computed in [1] for a very small range of parameters near extremality, and that too in a perturbative approach (with perturbation parameter $Q/M$). Here, the result is non-perturbative, and holds for any parameter range. Finally, we generalised the stability analysis to higher dimensional Reissner-Nordström black holes, and computed its leading order entropy corrections. We found that the leading logarithmic corrections come with a negative signature. This means that the corrected dimensionality of the Hilbert space of the black holes under consideration, which is $\mathcal{N} = \exp(\mathcal{S})$, is smaller than the uncorrected dimension. It would be interesting to explore implications of this result. It would also be interesting to do the generalisation to other types of black holes, e.g. Kerr-Newmann as well as black holes which are not asymptotically flat, e.g. those that are asymptotically anti-de Sitter or de-Sitter. The implications of the latter to $AdS/CFT$ and the $dS/CFT$ correspondence may turn out to be interesting. We hope to report on this elsewhere.

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