A hybrid seesaw model and hierarchical neutrino flavor structures based on $A_4$ symmetry

Mayumi Aoki$^1$ and Daiki Kaneko$^1$

$^1$Institute for Theoretical Physics, Kanazawa University, Kanazawa 920-1192, Japan

Abstract

We propose a hybrid seesaw model based on $A_4$ flavor symmetry, which generates a large hierarchical flavor structure. In our model, tree-level and one-loop seesaw mechanisms predict different flavor structures in the neutrino mass matrix, and generate a notable hierarchy among them. We find that such a hierarchical structure gives a large effective neutrino mass which can be accessible by next-generation neutrinoless double beta decay experiments. Majorana phases can also be predictable. The $A_4$ flavor symmetry in the model is spontaneously broken to the $Z_2$ symmetry, leading to a dark matter candidate which is assumed to be a neutral scalar field. The favored mass region of the dark matter is obtained by numerical computations of the relic abundance and the cross section of the nucleon. We also investigate the predictions of the several hierarchical flavor structures based on $A_4$ symmetry for the effective neutrino mass and the Majorana phases, and find the characteristic features depending on the hierarchical structures.

$^*$mayumi@hep.s.kanazawa-u.ac.jp
$^†$d_kaneko@hep.s.kanazawa-u.ac.jp
I. INTRODUCTION

Discovery of neutrino oscillations shows that neutrinos are mixed with each other and have tiny masses. Since neutrinos are massless particles in the Standard Model (SM), new physics beyond the SM which has some mechanism to generate the neutrino masses are required. Type-I seesaw mechanism \([1, 5]\) is one of the attractive ways to generate such tiny neutrino masses at the tree-level, which requires an introduction of right-handed neutrinos. Another attractive way to explain the tiny masses is a radiative seesaw mechanism in which neutrino masses are generated by loop effects (see \([6, 12]\) for early works and also \([13]\) for a latest review). In the radiative seesaw models involving right-handed neutrinos \([9, 11]\) where a discrete symmetry is imposed to forbid the Type-I seesaw mechanism. This symmetry is also responsible for the stability of the dark matter (DM).

The neutrino mass matrix is diagonalized by the lepton flavor mixing matrix, so-called the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix which is parameterized as

\[
U_{\text{PMNS}} = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}\ e^{-i\delta} \\
    -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
    s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix}
\]

\[\begin{pmatrix}
    1 & 0 & 0 \\
    0 & e^{i\alpha_2/2} & 0 \\
    0 & 0 & e^{i\alpha_3/2}
\end{pmatrix}, \tag{1}
\]

where \(c_{ij} = \cos \theta_{ij}\) and \(s_{ij} = \sin \theta_{ij}\) for \(i, j = 1, 2, 3\). The parameter \(\delta\) is a Dirac phase, while \(\alpha_2\) and \(\alpha_3\) denote Majorana phases. The data obtained in neutrino oscillation experiments \([14-18]\) show that the neutrino mixing angles are \(\theta_{12} \approx 33^\circ, \theta_{23} \approx 49^\circ, \theta_{13} \approx 8.6^\circ\), and the neutrino mass-squared differences \(\Delta m_{21}^2 \equiv m_2^2 - m_1^2\) and \(\Delta m_{21}^2 \equiv m_2^2 - m_1^2\) are \(|\Delta m_{23}^2| \approx 2.5 \times 10^{-3} \text{ eV}^2\) and \(\Delta m_{12}^2 \approx 7.4 \times 10^{-5} \text{ eV}^2\), respectively. The recent measurements of the Dirac phase show \(\delta = 107^\circ \rightarrow 403^\circ\) for the normal mass ordering (NO) and \(\delta = 192^\circ \rightarrow 360^\circ\) for the inverted mass ordering (IO) at 3\(\sigma\) C.L. \([19]\). The mixing matrix has two large mixings, which is very different from the quark mixing. Apart from the tiny masses of neutrinos, such flavor structures will give us hints of physics behind the SM.

One candidate behind the lepton sector is non-Abelian discrete flavor symmetries, such as \(S_3, A_4\) and \(S_4\) (see \([20, 24]\) for reviews)\(^1\). In particular, the study of the \(A_4\) models has received considerable interest. It has been shown in \([20]\) that the \(A_4\) flavor symmetry leads naturally to the neutrino mass matrix which gives the tri-bimaximal flavor mixing, \(M_{\text{Tri}}\) (i.e. \(s_{12} = 1/\sqrt{3}, s_{23} = 1/\sqrt{2}, s_{13} = 0\) \([27]\). It is known that \(M_{\text{Tri}}\) is given by a linear combination of three flavor structures as

\[
M_{\text{Tri}} = \frac{m_1 + m_3}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{m_2 - m_1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_1 - m_3}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \tag{2}
\]

However, since the observed value of \(\theta_{13}\) is small but non-zero, the neutrino mass matrix should be modified from \(M_{\text{Tri}}\) so as to realize the non-zero (1,3) off-diagonal element in the flavor mixing

\(^1\) Applications of modular symmetries to explain the neutrino flavor structure have been proposed (see e.g. \([25]\), where the Yukawa couplings are restricted by the modular symmetry.
matrix. One possible form of the neutrino mass matrix which derives the non-zero $\theta_{13}$ is given by adding another new flavor structure as \[ \text{28}\]

\[
M_\nu = a \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + c \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + d \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix},
\]

(3)

where the coefficients of each flavor structure $a, b, c$ and $d$ are the arbitrary mass dimensionful parameters. The non-vanishing $d$ term in the models with the $A_4$ symmetry is discussed in \[ \text{21, 28–33}\]. It is expected that the relation among these four flavor structures provides us an important information on the flavor symmetry in the neutrino mass generation mechanism.

In this paper, we propose a hybrid seesaw model based on the non-Abelian $A_4$ flavor symmetry, in which the neutrino mass matrix Eq. (3) is generated by the tree-level and the one-loop seesaw mechanisms\[ \text{2}\]. These mechanisms generate the different flavor structures, which leads to a characteristic hierarchy between the coefficients of four flavor structures. The origin of the $d$ term in Eq. (3) comes from the one-loop seesaw mechanism. Two benchmark points are chosen in our model and their predictions for the effective neutrino mass and the Majorana CP phases are computed. Before presenting the results in our model, we also show the predictions of model-independent analysis by using Eq. (3) for some cases with the hierarchical flavor structure. The $A_4$ symmetry in our model is broken into the $Z_2$ subgroup by the vacuum expectation value (VEV) of the $A_4$ triplet scalar field. Therefore, a lightest neutral $Z_2$-odd field, where we assume it a CP-even neutral scalar field, is stable and becomes a DM candidate. We compute the relic abundance and the spin-independent cross section of the DM, and show the plausible mass region of the DM.

II. MODEL

The non-Abelian $A_4$ flavor symmetry has four irreducible representations which are three singlets $1, 1'$ and $1''$ and one triplet $3$. The $A_4$ symmetry is generated by two elements $S$ and $T$,

\[ S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \]

\[ \text{2} \] Other hybrid seesaw models based on the $A_4$ flavor symmetry have been considered in \[ \text{29, 31, 43}\].
TABLE I. $A_4$ flavor and $SU(2)$ gauge quantum numbers for leptons, right-handed neutrinos and scalar fields of the model.

which fulfill the relations $S^2 = T^3 = (ST)^3 = I$. The $A_4$ triplets $3_a = (a_1,a_2,a_3)$ and $3_b = (b_1,b_2,b_3)$ have the multiplication rules as

\[
\begin{align*}
[3_a \otimes 3_b]_1 & = a_1 b_1 + a_2 b_2 + a_3 b_3, \\
[3_a \otimes 3_b]_{1'} & = a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3, \\
[3_a \otimes 3_b]_{1''} & = a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3, \\
[3_a \otimes 3_b]_{3_1} & = (a_2 b_3, a_3 b_1, a_1 b_2), \\
[3_a \otimes 3_b]_{3_2} & = (a_3 b_2, a_1 b_3, a_2 b_1),
\end{align*}
\]

where $\omega = e^{\frac{2\pi i}{3}}$ which satisfies $1 + \omega + \omega^2 = 0$.

We introduce three $A_4$ triplet right-handed neutrinos $N_i (i = 1, 2, 3)$, which are invariant under the SM gauge group, and three $A_4$ triplet $SU(2)$ scalar doublets $\eta_i$. Assignments of model are shown in TABLE I, where $A_4$ singlets $L_e$, $L_\mu$, $L_\tau$ ($l_{eR}$, $l_{\mu R}$, $l_{\tau R}$) are lepton doublets (lepton singlets) and an $A_4$ singlet $H$ is a Higgs doublet field. In this model, the Yukawa sectors of neutrinos are described by

\[
\mathcal{L}_{\text{Yukawa}} = y_{1} \overline{L}_e (\tilde{\eta}_1 N_1 + \tilde{\eta}_2 N_2 + \tilde{\eta}_3 N_3) + y_{1'} \overline{L}_\mu (\tilde{\eta}_1 N_1 + \omega \tilde{\eta}_2 N_2 + \omega^2 \tilde{\eta}_3 N_3) + y_{1''} \overline{L}_\tau (\tilde{\eta}_1 N_1 + \omega^2 \tilde{\eta}_2 N_2 + \omega \tilde{\eta}_3 N_3) + h.c.,
\]

where $y_1$, $y_{1'}$ and $y_{1''}$ are the Yukawa couplings and $\tilde{\eta}_i = i\sigma^2 \eta_i^*$. In this work we assume $y \equiv y_1 = y_{1'} = y_{1''}$, where $y$ is real. Majorana mass terms of right-handed neutrinos are given by

\[
\mathcal{L}_{\text{Majorana}} = M_R (\overline{N}_1^c N_1 + \overline{N}_2^c N_2 + \overline{N}_3^c N_3) + M'_R (\overline{N}_1^c N_1 + \omega \overline{N}_2^c N_2 + \omega^2 \overline{N}_3^c N_3) + M''_R (\overline{N}_1^c N_1 + \omega^2 \overline{N}_2^c N_2 + \omega \overline{N}_3^c N_3) + M_{23} (\overline{N}_2^c N_3 + h.c.),
\]

where $M_R$, $M'_R$, $M''_R$ and $M_{23}$ are the Majorana masses of right-handed neutrinos. We note the second, third and fourth terms in Eq. (5) break the $A_4$ symmetry. Because of the fourth term, the neutrinos $N_2$ and $N_3$ are mixed each other. The mass matrix of right-handed neutrinos is

| $A_4$ | $SU(2)$ | $Z_2$ |
|-------|---------|-------|
| $(L_e, L_\mu, L_\tau)$ | $1, 1', 1''$ | $ (+, +, +) $ |
| $(l_{eR}, l_{\mu R}, l_{\tau R})$ | $1, 1'', 1'$ | $(+, +, +)$ |
| $(N_1, N_2, N_3)$ | $3, 1'$ | $(+,-,-)$ |
| $H$ | $1, 2$ | $+$ |
| $\eta = (\eta_1, \eta_2, \eta_3)$ | $3, 2$ | $(+,-,-)$ |

\footnote{These three terms can be generated by $A_4$ singlet $1''$, $1'$ and triplet $3$ scalar fields, respectively.}
diagonalized by the mixing angle \( \tan 2\theta_R \equiv \frac{2M_R}{\omega - \omega'^2(M_R^2 - M_R'^2)} \), where we define the diagonal elements as \((M_1, M_2 e^{i\delta_{R2}}, M_3 e^{i\delta_{R3}})\). Throughout this paper, we work in the basis where the charged lepton mass matrix is diagonal.

Based on the \( A_4 \) symmetry, the scalar potential is given by

\[
V = \mu_0^2[\eta^\dagger \eta]_1 + \mu_R^2 H^\dagger H + \lambda_1 (H^\dagger H)^2 + \lambda_2 [\eta^\dagger \eta]^3 + \lambda_3 [\eta^\dagger \eta][\eta^\dagger \eta]_1 + \lambda_4 [\eta^\dagger \eta][\eta^\dagger \eta]_1' [\eta^\dagger \eta]_1''
+ \lambda_5 [\eta^\dagger \eta][\eta^\dagger \eta]_1'' [\eta^\dagger \eta]_1 + \lambda_6 [\eta^\dagger \eta]_3 [\eta^\dagger \eta]_3 + h.c.
+ \lambda_7 [\eta^\dagger \eta]_3 [\eta^\dagger \eta]_3 + \lambda_8 [\eta^\dagger \eta]_3 [\eta^\dagger \eta]_3 + \lambda_9 [\eta^\dagger \eta]_1 [H^\dagger H] + \lambda_{10} [\eta^\dagger H]_3 [H^\dagger \eta]_3
+ \lambda_{11} ([\eta^\dagger \eta]_1 HH + h.c.) + \lambda_{12} ([\eta^\dagger \eta]_3 [\eta H]_3 + h.c.) + \lambda_{13} ([\eta^\dagger \eta]_3 [\eta H]_3 + h.c.)
+ \lambda_{14} ([\eta^\dagger \eta]_3 [\eta^\dagger H]_3 + h.c.) + \lambda_{15} ([\eta^\dagger \eta]_3 [\eta H]_3 + h.c.).
\]

We assume that the couplings in the scalar potential are real and \( \lambda_4 = \lambda_4' \) for simplicity. When one of the \( A_4 \) triplet field \( \eta_1 \), in addition to \( H \), has the VEV, the \( A_4 \) symmetry breaks to the subgroup \( Z_2 \) symmetry whose charge assignments are also shown in TABLE I. The \( Z_2 \)-even sector can be obtained by the mixing angles \( \beta \), where \( \tan \beta \equiv v_\eta/v_h \), and \( \alpha \) as

\[
\begin{pmatrix}
G^+ \\
H^+
\end{pmatrix} =
\begin{pmatrix}
\cos \beta & \sin \beta \\
-\sin \beta & \cos \beta
\end{pmatrix}
\begin{pmatrix}
\phi^+ \\
\omega^+_1
\end{pmatrix},
\begin{pmatrix}
G^0 \\
A_1
\end{pmatrix} =
\begin{pmatrix}
\cos \beta & \sin \beta \\
-\sin \beta & \cos \beta
\end{pmatrix}
\begin{pmatrix}
\chi \\
\eta_{1I}
\end{pmatrix},
\]

\[
\begin{pmatrix}
h_2 \\
h_1
\end{pmatrix} =
\begin{pmatrix}
\cos \alpha & \sin \alpha \\
-\sin \alpha & \cos \alpha
\end{pmatrix}
\begin{pmatrix}
\phi^0 \\
\eta_{1R}
\end{pmatrix}.
\]

Here \( h_1 \) is the SM-like Higgs particle whose mass is \( m_{h_1} = 125 \text{ GeV} \). The masses of the charged scalar field \( H^{\pm} \) and the CP-odd scalar field \( A_1 \) are described as \( m_{H^{\pm}}^2 = -\frac{1}{2}(\lambda_{10} + 2\lambda_{11})v^2 \) and \( m_{A_1}^2 = -2\lambda_{11}v^2 \), respectively.

The \( Z_2 \)-odd fields \( \eta_2 \) and \( \eta_3 \), which do not have the VEVs, are defined as

\[
\eta_2 = \left( \frac{1}{\sqrt{2}} \left( \eta_{2L}^+ + i\eta_{2R}^+ \right) \right), \quad \eta_3 = \left( \frac{1}{\sqrt{2}} \left( \eta_{3L}^+ + i\eta_{3R}^+ \right) \right).
\]

These two states are mixed through the \( \lambda_{12}, \lambda_{13}, \lambda_{14} \) and \( \lambda_{15} \) terms in Eq. (6) and the mixing angle between them is \( \pi/4 \). The neutral CP-even (-odd) states give the mass eigenstates \( \eta_0^2 \) and
\[ \eta^0_3 (A_2 \text{ and } A_3) \text{ with masses } m_{\eta^0_2} \text{ and } m_{\eta^0_3} (m_{A_2} \text{ and } m_{A_3}) \text{ as} \]

\[
m^2_{\eta^0_2} = \frac{1}{2} (\lambda_{x_1} v^2_{\eta} - 3 \lambda_{x_3} v h v) ,
\]

\[
m^2_{\eta^0_3} = \frac{1}{2} (\lambda_{x_1} v^2_{\eta} + 3 \lambda_{x_3} v h v) ,
\]

\[
m^2_{A_2} = \frac{1}{2} (\lambda_{x_2} v^2_{\eta} - 4 \lambda_{111} v^2_{h} - \lambda_{x_3} v h v) ,
\]

\[
m^2_{A_3} = \frac{1}{2} (\lambda_{x_2} v^2_{\eta} - 4 \lambda_{111} v^2_{h} + \lambda_{x_3} v h v) ,
\]

where \( \lambda_{x_1} \equiv -3 \lambda_3 - 6 \lambda_4 + 2 \lambda_6 + \lambda_7 + \lambda_8 , \lambda_{x_2} \equiv -3 \lambda_3 - 2 \lambda_4 - 4 \lambda_5 - 2 \lambda_6 + \lambda_7 + \lambda_8 \) and \( \lambda_{x_3} \equiv \lambda_{12} + \lambda_{13} + \lambda_{14} + \lambda_{15} \). The masses of the charged scalar fields \( \eta^\pm_2 \) and \( \eta^\pm_3 \) are given by

\[
m^2_{\eta^\pm_2} = \frac{1}{2} [\lambda_{x_4} v^2_{\eta} + (\lambda_{10} + \lambda_{111}) v^2_{h} - \lambda_{x_3} v h v] ,
\]

\[
m^2_{\eta^\pm_3} = \frac{1}{2} [\lambda_{x_4} v^2_{\eta} + (\lambda_{10} + \lambda_{111}) v^2_{h} + \lambda_{x_3} v h v] ,
\]

where \( \lambda_{x_4} \equiv -3 \lambda_3 - 4 \lambda_4 - 2 \lambda_5 + \lambda_8 \). Note that the mass differences between \( m_{\eta^0_2} \) and \( m_{\eta^0_3} \), \( m_{A_2} \) and \( m_{A_3} \), \( m_{\eta^\pm_2} \) and \( m_{\eta^\pm_3} \) are given by \( \lambda_{x_3} \).

The lightest \( Z_2 \text{-odd particle is stable and can be a DM if it is neutral. In our model, the right-handed neutrinos are heavy as shown later, so that the DM candidates are } \eta^0_2, 3 \text{ and } A_{2,3}. \) Hereafter, we assume that \( \eta^0_2 \) is a DM candidate.

### III. NEUTRINO MASSES AND FLAVOR STRUCTURES

In this model, the neutrinos obtain their masses via the tree-level and the one-loop seesaw mechanisms. Since the Yukawa interactions \( L_\alpha \tilde{H} N_i (\alpha = e, \mu, \tau) \) are forbidden by the \( A_4 \) symmetry, the usual Type-I seesaw mechanism does not work. However, the neutrinos can obtain their masses via the other tree-level seesaw mechanism due to the existence of the Yukawa interactions \( L_\alpha \tilde{\eta}_1 N_1 \) in Eq. (4) with the nonzero VEV of \( \eta_1 \). The neutrino mass matrix \( M^\text{tree}_\nu \) which is generated by the tree-level seesaw mechanism in Fig. 1 is given by

\[
M^\text{tree}_\nu = \frac{v^2_{\eta}}{2 M_1} \begin{pmatrix} y & 0 & 0 \\ y & 0 & 0 \\ y & 0 & 0 \end{pmatrix} \begin{pmatrix} y & y & y \\ 0 & y & y \\ 0 & 0 & y \end{pmatrix} = \frac{v^2_{\eta}}{2 M_1} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} .
\]

The flavor structure in \( M^\text{tree}_\nu \) is the same form as that of \( b \) term in Eq. (4). We note that the rank of \( M^\text{tree}_\nu \) is one, so that the other contributions to the neutrino mass generation should be necessary.

The neutrino masses are also generated by the one-loop seesaw mechanisms which are shown in Fig. 2 and Fig. 3. The \( Z_2 \text{-even right-handed neutrino } N_1 \) contributes to the mass generation in Fig. 2 while the \( Z_2 \text{-odd right-handed neutrinos } N_2, N_3 \) and their mixing contribute in Fig. 3. Assuming \( \lambda_{x_3} \ll 1 \) and \( \sin(\beta - \alpha) = 1 \), where the former assumption leads to \( m_{\eta} \equiv m_{\eta^0_2} \approx m_{\eta^0_3} \)
and $m_A \equiv m_{A_2} \approx m_{A_3}$, the neutrino mass matrix via the one-loop diagrams in Fig. 2 and Fig. 3 is described as

$$M_{\text{one-loop}} = \begin{pmatrix}
1 & 1 & 1 \\
1 & \omega & \omega^2 \\
1 & \omega^2 & \omega
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \hat{\theta}_R & \sin \hat{\theta}_R \\
0 & 0 & \Lambda_3
\end{pmatrix}
\begin{pmatrix}
\Lambda_1 & 0 & 0 \\
0 & \Lambda_2 & 0 \\
0 & 0 & \Lambda_3
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & \sin \hat{\theta}_R & \cos \hat{\theta}_R \\
0 & \cos \hat{\theta}_R & \sin \hat{\theta}_R
\end{pmatrix}
\begin{pmatrix}
1 & 1 & 1 \\
1 & \omega & \omega^2 \\
1 & \omega^2 & \omega
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1
\end{pmatrix}
\begin{pmatrix}
0 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 0
\end{pmatrix}.
\]  

\begin{align}
\Lambda_1 & = \frac{y^2 M_1}{16 \pi^2} \left[ \sin^2 \beta \frac{m_{h_1}^2}{m_{h_1}^2 - M_1^2} \ln \frac{m_{h_1}^2}{M_1^2} + \cos^2 \beta \left( \frac{m_{h_2}^2}{m_{h_2}^2 - M_1^2} \ln \frac{m_{h_2}^2}{M_1^2} - \frac{m_{A_1}^2}{m_{A_1}^2 - M_1^2} \ln \frac{m_{A_1}^2}{M_1^2} \right) \right], \\
\Lambda_k & = \frac{y^2 M_k e^{\delta n_k}}{16 \pi^2} \left[ \frac{m_{\eta_k}^2}{m_{\eta_k}^2 - M_k^2} \ln \frac{m_{\eta_k}^2}{M_k^2} - \frac{m_{A}^2}{m_{A}^2 - M_k^2} \ln \frac{m_{A}^2}{M_k^2} \right] (k = 2, 3), \\
X_a & = 3 \cos \hat{\theta}_R \sin \hat{\theta}_R (\Lambda_3 - \Lambda_2), \\
X_c & = \left[ (1 - \omega)(\Lambda_2 \cos^2 \hat{\theta}_R + \Lambda_3 \sin^2 \hat{\theta}_R) + (1 - \omega^2)(\Lambda_2 \sin^2 \hat{\theta}_R + \Lambda_3 \cos^2 \hat{\theta}_R) \right], \\
X_d & = \left[ (\omega^2 - \omega)(\Lambda_2 \cos^2 \hat{\theta}_R + \Lambda_3 \sin^2 \hat{\theta}_R) + (\omega - \omega^2)(\Lambda_2 \sin^2 \hat{\theta}_R + \Lambda_3 \cos^2 \hat{\theta}_R) \right].
\end{align}

It can be seen that the four flavor structures in Eq. (3) are generated. We find that the one-loop diagrams with $N_1$ in Fig. 2 generate only $b$ term. On the other hand, the contributions of $N_2$ and $N_3$ in Fig. 3 give all four flavor structures. In particular, the mixing between $N_2$ and $N_3$ realizes the nonzero $a$ term, while the origin of the nonzero $d$ term (i.e. nonzero $\theta_{13}$) comes from the difference between $\Lambda_2$ and $\Lambda_3$. From Eq. (17) and Eq. (18), the neutrino mass matrix in our model

\[^4\text{Even when the assumption } \lambda_{a_3} < 1 \text{ is removed, the four flavor structures are derived.}\]
FIG. 2. Feynman diagrams for neutrino masses via the one-loop seesaw mechanism with $N_1$.

\[ \begin{align*}
\nu & \quad \eta_1 \\
& \quad \eta_1 \\
& \quad N_1 \\
\end{align*} \] 

FIG. 3. Feynman diagrams for neutrino masses via the one-loop seesaw mechanism with $N_2$ and $N_3$.

\[ \begin{align*}
\nu & \quad \eta_2, \eta_3 \\
& \quad \eta_2, \eta_3 \\
& \quad N_{2,3} \\
\end{align*} \]

\[ \begin{align*}
\nu & \quad \eta_2, \eta_3 \\
& \quad \eta_2, \eta_3 \\
& \quad N_2, N_3 \\
\end{align*} \]

The neutrino mass matrix in Eq. (3) (and thus in Eq. (24)) is diagonalized with the PMNS matrix which is formed by the tri-bimaximal mixing, the (1,3) mixing and the Majorana phase matrix [28]:

\[ U_{\text{PMNS}} = \begin{pmatrix} 
\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} 
\end{pmatrix} \begin{pmatrix} 
\cos \hat{\theta} & 0 & \sin \hat{\theta} \\
0 & 1 & 0 \\
-\sin \hat{\theta} & 0 & \cos \hat{\theta} 
\end{pmatrix} \begin{pmatrix} 
1 & 0 & 0 \\
0 & e^{i \alpha_2/2} & 0 \\
0 & 0 & e^{i \alpha_3/2} 
\end{pmatrix}, \] (25)
FIG. 4. The coefficients of the flavor structure $|a|$, $|b|$ and $|c|$ as a function of $d$ for the NO (left panel) and the IO (right panel). The values of the neutrino oscillation parameters are taken as in Eq. (30) with $\delta = 0$. The shaded regions are excluded by the constraint of the sum of light neutrino masses [47].

Here the mixing angle $\theta$ is the complex parameter. Comparing to the standard parametrization of $U_{\text{PMNS}}$ in Eq. (1), we obtain $\sin \hat{\theta} = \frac{\sqrt{3}}{2} \sin \theta_{13} e^{-i\delta}$. Using the coefficient parameters $a, b, c$ and $d$ in Eq. (3), the angle $\hat{\theta}$ is given by

$$\tan 2\hat{\theta} = \frac{\sqrt{3}d}{-2c + d}.$$  \hfill (26)

The neutrino masses $m_1, m_2$ and $m_3$ and the Majorana phases $\alpha_2$ and $\alpha_3$ are written as

$$m_1 = |a + \sqrt{c^2 + d^2 - cd}|, \quad m_2 = |a + 3b + c + d|, \quad m_3 = |a - \sqrt{c^2 + d^2 - cd}|,$$ \hfill (27)

$$\alpha_2 = \text{arg}(a + 3b + c + d) - \text{arg} \left( a + \sqrt{c^2 + d^2 - cd} \right),$$ \hfill (28)

$$\alpha_3 = \text{arg} \left( a - \sqrt{c^2 + d^2 - cd} \right) - \text{arg} \left( a + \sqrt{c^2 + d^2 - cd} \right).$$ \hfill (29)

The observed values of the neutrino oscillation parameters $\Delta m^2_{31}$, $\Delta m^2_{21}$, $\theta_{13}$ and $\delta$ give the constraints on the relationship among the coefficients $a, b, c$ and $d$.

In Fig. 4 we show the absolute values of the coefficients $|a|$ (solid lines), $|b|$ (dashed lines) and $|c|$ (dotted lines) as a function of $d$ which is assumed to be real and positive, for the NO (left panel) and the IO (right panel). The coefficients are derived from the following center values in the NO (IO) [19]

$$\Delta m^2_{21} = 7.39 \times 10^{-5} \text{eV}^2, \quad \Delta m^2_{31} = 2.525 (-2.512) \times 10^{-3} \text{eV}^2, \quad \theta_{13} = 8.61 (8.65)^\circ.$$ \hfill (30)

The Dirac phase is taken as $\delta = 0$ and the coefficients $a, b$ and $c$ are assumed to be real for simplicity. There are two solutions for the $|b|$, which are shown by $|b^+|$ and $|b^-|$ in Fig. 4. The shaded regions

\footnote{We use the center values in v4.1 of Ref. [19].}
are excluded by the constraint of the sum of light neutrino masses\textsuperscript{[47]}:
\[
\sum_i m_i < 0.241 \text{ eV}. \quad (31)
\]

In the NO (left panel of Fig. 4), the $|a|$ decreases and the $|c|$ increases as the parameter $d$ larger. We note that the coefficients $|c|$ and $|d|$ are comparable but the $|c|$ is larger than $|d|$ due to the relation of Eq. (26). The hierarchy $|a|, |b^+| > |c|, |d|$ is shown for the smaller $d$ ($d \lesssim 0.008$ eV), while the hierarchy $|b^+|, |c|, |d| > |a|$ appears for the larger $d$ ($d \gtrsim 0.02$ eV). On the other hand, the hierarchy $|a|, |c|, |d| > |b^-|$ is obtained for $d \sim 0.03$ eV, where $b^-$ changes its sign. In the IO (right panel), the similar hierarchies among the coefficients can be seen except for the vanishment of $|b^-|$.\textsuperscript{6} We note that, in both of the NO and the IO, the large hierarchies between $b$ and $c$, $d$ such as $|b|/|c|, |b|/|d| \approx 16\pi^2$ are disfavored by current data.

In our hybrid seesaw model, the milder but large hierarchy as $|b|/|d| \approx \pi^2$ can be realized for $M_i \gg m_\nu$ as mentioned in the previous section. In the following, we give the benchmark point in the NO (BP\textsubscript{NO}) and the IO (BP\textsubscript{IO}), respectively, where the ratio $|b|/|d| \approx \pi^2$ is satisfied. We here take the coefficients $a, b, c$ and $d$ as complex parameters, and take into account the contributions of CP phases. For the BP\textsubscript{NO}, we take the following set for $a, b, c$ and $d$, which satisfies the center values of the neutrino parameters in Eq. (30) and $\delta = 222^\circ$\textsuperscript{[19]}:
\[
|a| \approx 0.0759 \text{ eV}, \quad |b| \approx 0.0483 \text{ eV}, \quad |c| \approx 0.0103 \text{ eV}, \quad |d| \approx 0.0045 \text{ eV},
\]
\[
\arg(a) \approx 2.45 \text{ rad.}, \quad \arg(b) \approx -0.439 \text{ rad.}, \quad \arg(c) \approx 2.09 \text{ rad.}, \quad \arg(d) \approx 1.44 \text{ rad.}. \quad (32)
\]

Above set is realized by the following values of the model parameters:
\[
\tan \beta = 3, \quad y = 1.0 \times 10^{-2}, \quad (M_1, M_2, M_3) \approx (6.69, 1.94, 1.53) \times 10^{10} \text{ GeV}
\]
\[
\delta_{R_2} \approx -0.66 \text{ rad.}, \quad \delta_{R_3} \approx 2.43 \text{ rad.}, \quad \tan 2\hat{\theta}_R \approx -16.5 + 10.4i,
\]
\[
m_{h_2} = 200 \text{ GeV}, \quad m_{A_1} = 250 \text{ GeV}, \quad m_\eta = 500 \text{ GeV}, \quad m_A = 520 \text{ GeV}. \quad (33)
\]

For the IO, we take the following set for the BP\textsubscript{IO} which satisfies Eq. (30) and $\delta = 285^\circ$\textsuperscript{[19]}:
\[
|a| \approx 0.0707 \text{ eV}, \quad |b| \approx 0.0536 \text{ eV}, \quad |c| \approx 0.00978 \text{ eV}, \quad |d| \approx 0.0049 \text{ eV},
\]
\[
\arg(a) \approx 0.063 \text{ rad.}, \quad \arg(b) \approx 3.14 \text{ rad.}, \quad \arg(c) \approx 0.044 \text{ rad.}, \quad \arg(d) \approx -1.51 \text{ rad.}. \quad (34)
\]

Above set is realized by the following:
\[
\tan \beta = 3, \quad y = 1.0 \times 10^{-2}, \quad (M_1, M_2, M_3) \approx (6.02, 2.16, 1.61) \times 10^{10} \text{ GeV}
\]
\[
\delta_{R_2} \approx -3.12 \text{ rad.}, \quad \delta_{R_3} \approx 0.091 \text{ rad.}, \quad \tan 2\hat{\theta}_R \approx -16.7,
\]
\[
m_{h_2} = 200 \text{ GeV}, \quad m_{A_1} = 250 \text{ GeV}, \quad m_\eta = 500 \text{ GeV}, \quad m_A = 520 \text{ GeV}. \quad (35)
\]

\textsuperscript{6} In the left panel of Fig. 4, the $b^+$ is positive (i.e. $\arg(b^+) = 0$), the $b^-$ is positive for $d < 0.02$ eV and negative (i.e. $\arg(b^-) = \pi$ rad.) for $d > 0.02$ eV, and the $a$ and $c$ are negative. In the right panel of Fig. 4 the $a, b^+, b^-$ and $c$ are positive, positive, negative and negative, respectively.
From Eqs. (33) and (35), we find that the large hierarchy $|b|/|d| \approx \pi^2$ is realized for the masses of the right-handed neutrinos $M_i \sim \mathcal{O}(10^{10})$ GeV and the scalar fields $m_{\eta_i} \sim \mathcal{O}(10^{2-3})$ GeV for $y \sim 10^{-2}$. Furthermore, both of the BP$_{\text{NO}}$ and the BP$_{\text{IO}}$ satisfy $|a| \gg |c|, |d|$, so that the Majorana phase $\alpha_3$ is expected to be close to zero as can be seen from Eq. (29).

IV. PREDICTIONS OF THE EFFECTIVE NEUTRINO MASS AND THE MAJORANA PHASES

In this section, we discuss the predictions of the effective neutrino mass $m_{ee}$ and the Majorana CP phases. The $m_{ee}$ is defined as

$$m_{ee} = \left| \sum_{i=1}^{3} U_{ei}^2 m_i \right|$$

with $U_{e1} = 2 \cos \hat{\theta}/\sqrt{6}$, $U_{e2} = 1/\sqrt{3}$ and $U_{e3} = 2 \sin \hat{\theta}/\sqrt{6}$. First, we show the results of model-independent analysis by using the neutrino mass matrix in Eq. (3), focusing on the hierarchies between the coefficients $a$, $b$, $c$ and $d$. In Fig. 5 and Fig. 6 the predicted values of $m_{ee}$ are shown as functions of the lightest neutrino mass (upper panel), the Majorana phases $\alpha_2$ (lower left panel) and $\alpha_3$ (lower right panel) for the NO and the IO, respectively. In these plots, we have taken the following ranges for the coefficient parameters $|d|$, arg($a$), arg($b$), arg($d$), and the $3\sigma$ range of $\delta$ in the NO (IO) [19]:

$$0 \leq |d|/\text{eV} \leq 1.0, \quad 0 \leq \text{arg}(a)/\text{rad.} < 2\pi, \quad 0 \leq \text{arg}(b)/\text{rad.} < 2\pi, \quad 0 \leq \text{arg}(d)/\text{rad.} < 2\pi, \quad 107^\circ \leq \delta \leq 403^\circ \ (192^\circ \leq \delta \leq 360^\circ).$$

The other parameters $|a|$, $|b|$, $|c|$ can be determined so as to satisfy the observed values of $\Delta m_{21}^2$, $\Delta m_{31}^2$ and $\theta_{13}$ in Eq. (30). Furthermore, the arg($c$) is fixed through the relation in Eq. (26). In Fig. 5 and Fig. 6 the cyan points show all points which satisfy the observed values in Eq. (30) and the constraint from Eq. (31). The green, black, orange and yellow points show the result where the hierarchical conditions $|b|/|d| > 1$, $|b|/|a| > 1$, $|d|/|a| > 1$ and $|b|/|c| > 1$, respectively, are further imposed. The horizontal blue solid and dashed lines show the upper bound on $m_{ee}$ by the global fit of neutrinoless double beta decay ($0\nu\beta\beta$) experiments $m_{ee} \lesssim 0.06$ eV [19] and the sensitivity of next-generation $0\nu\beta\beta$ experiment by nEXO [18].

In Fig. 5 the hierarchical case with $|b|/|d| > 1$ (green) does not constrain the parameter space comparing to the cyan points, while the other three cases constrain the parameter space. For the hierarchical case with $|b|/|c| > 1$ (yellow), the predicted regions of the effective neutrino mass $m_{ee}$ and the lightest neutrino mass $m_1$ are $m_{ee} \gtrsim 0.001$ eV and $m_1 \gtrsim 0.007$ eV. In this case, the Majorana phase $\alpha_2 \sim 0$ is excluded by the constraint from Eq. (31) and the $\alpha_3$ is constrained as $|\alpha_3|/\text{rad.} \lesssim 2.0$. The hierarchical cases with $|d|/|a| > 1$ (orange) and $|b|/|a| > 1$ (black) have similar predictions, but the former is more constrained, such as giving $m_1 \gtrsim 0.015$ eV and $|\alpha_3|/\text{rad.} \gtrsim 2.2$. 

Here the $|\alpha_3| \simeq \pi$ radians is obtained for $|d| \gg |a|$ as can be seen from Eq. (29). Figure 6 shows the results for the IO. For the hierarchical case with $|b|/|c| > 1$, the predicted regions for $m_3$ and $\alpha_3$ are wider than those in the NO. In particular, the $\alpha_3$ is not constrained for $0.015 \lesssim m_{ee}/\text{eV} \lesssim 0.04$. Similarly, in those range of $m_{ee}$, the $\alpha_3$ is unconstrained for the case with $|b|/|a| > 1$ (although it is not shown in the lower right panel in Fig. 6 as it is behind the yellow region). The predictions for the cases with $|d|/|a| > 1$ show the similar feature to that in the NO. The next-generation $0\nu\beta\beta$ experiment nEXO can explore all predicted regions of $m_{ee}$ for the IO, and thus there is the possibility to obtain hints of the Majorana phases for some hierarchical cases. In this analysis, the predicted values for $\theta_{23}$ and $\theta_{12}$ are $0.4 \lesssim \sin^2 \theta_{23} \lesssim 0.6$ and $\sin^2 \theta_{12} \approx 0.34$, respectively, which are allowed within $3\sigma$. We note that the predicted regions of the hierarchies with $|b|/|c| > 1$ and $|d|/|a| > 1$ are included in those with $|b|/|d| > 1$ and $|c|/|a| > 1$, respectively, because of the relation $|c| > |d|$ obtained by Eq. (26).

Next, we discuss the larger hierarchical case with $|b|/|d| > \pi^2$ which can be applied to our model.
FIG. 6. Same as Fig. 5 except for the IO and the lightest neutrino mass $m_3$.

We display in Fig. 7 and Fig. 8 the predictions of $m_{ee}$ for $|b|/|d| > \pi^2$ by the red points. The green points are the predictions for $|b|/|d| > 1$ which are the same as those in Fig. 5 and Fig. 6. We also show the predictions of the BP_{NO} and the BP_{IO} in our hybrid seesaw model by the black triangles in Fig. 7 and Fig. 8 respectively. In Figure 7, we can see that the predicted regions for $|b|/|d| > \pi^2$ are strictly constrained: $m_{ee} \gtrsim 0.02$ eV, $m_1 \gtrsim 0.06$ eV, $\pi/2 \lesssim |\alpha_2|/\text{rad.} \leq \pi$, $|\alpha_3|/\text{rad.} \lesssim 0.2$. Such large $m_{ee}$ is within the sensitivity reach of the next-generation 0$\nu\beta\beta$ experiments. Note that, for the larger hierarchy between the coefficients $|b|$ and $|d|$, the larger hierarchy between the $|a|$ and $|d|$ is expected and thus the Majorana phase $|\alpha_3|$ gets closer to zero. In our hybrid seesaw model, the predicted values of the BP_{NO} are:

$$m_{ee} \approx 0.030 \text{ eV, } m_1 \approx 0.067 \text{ eV, } \alpha_2 \approx -2.5 \text{ rad., } \alpha_3 \approx -0.07 \text{ rad.} \quad (38)$$

Figure 8 shows the results for the IO, where we can see the similar predictions of the red regions with the NO. The predicted values of the BP_{IO} are:

$$m_{ee} \approx 0.027 \text{ eV, } m_3 \approx 0.061 \text{ eV, } \alpha_2 \approx 3.04 \text{ rad., } \alpha_3 \approx -0.07 \text{ rad.}, \quad (39)$$

where the nonsignificant CP violations by the Majorana phases are expected.
FIG. 7. $m_{ee}$ vs $m_1$ (upper panel), $\alpha_2$ (lower left panel), $\alpha_3$ (lower right panel) for the NO. The green points are the same as those in Fig. 5. The red points show the large hierarchy case, $|b|/|d| > \pi^2$, the black triangle point indicates the BP NO.

V. DARK MATTER

The $A_4$ flavor symmetry in our model is spontaneously broken to the $Z_2$ symmetry via the VEV of the $A_4$ triplet scalar field, which predicts the DM candidates and we assume that the $Z_2$-odd scalar field $\eta^0_2$ is the DM.\footnote{Such DM (so-called “discrete DM”) is discussed in \cite{37,38,46,49-52}.} The main annihilation processes of the DM in our scenario are shown in Fig. 9. Note that the processes are almost the same as those in the Inert Doublet Model \cite{53}. In our model, the mass splitting between $\eta^0_2$ and $\eta^0_3$ is small because of the small $\lambda_{x3}$ coupling, so that we also consider the annihilation of $\eta^0_3$ and the relic density is computed for the sum of $\eta^0_2$ and $\eta^0_3$\footnote{We note that the $\eta^0_3$ decays into the $\eta^0_2$ through e.g. $\eta^0_3 \rightarrow \eta^0_2 \gamma$ after its decoupling.}. The rate of DM annihilation depends on the scalar couplings $\lambda_1, \lambda_{x5}, \lambda_{x6}$ and $\lambda_{x7}$, except for the gauge couplings, where $\lambda_{x5} \equiv \lambda_9 + \lambda_{10} + 2\lambda_{11}$, $\lambda_{x6} \equiv \lambda_2 + \lambda_3 + 2\lambda_4 + \lambda_5$ and $\lambda_{x7} \equiv 2\lambda_2 - \lambda_3 - 2\lambda_4 + \lambda_5 + 2\lambda_6 + \lambda_7 + \lambda_8$. For $\sin(\beta - \alpha) = 1$, the relevant scalar couplings $\lambda_{x5}, \lambda_{x6}$ and $\lambda_1$ are written by the masses of the $Z_2$-even neutral scalar fields $m_{h_1}, m_{h_2}$ and the mixing
angle $\beta$ as

\[
\lambda_{x_5} = \frac{m_{h_1}^2 - m_{h_2}^2}{v^2 [\sin^2 \beta + \cos^2 \beta (1 + 2 \sin 2\beta)]},
\]
\[
\lambda_{x_6} = \frac{m_{h_2}^2}{v^2 \sin^2 \beta} + \lambda_{x_5},
\]
\[
\lambda_1 = \frac{\lambda_{x_5} \cos 2\beta + \lambda_{x_6} \sin^2 \beta}{\cos^2 \beta}.
\]  

The scalar coupling $\lambda_{x_7}$ can be determined to satisfy the relic abundance $\Omega h^2 \approx 0.12$.

The spin-independent cross section of the nucleon is given by

\[
\sigma_{SI} = \frac{1}{\pi} \left( \frac{\lambda_{DD} \hat{f} m_N}{m_{\eta_1}^2 m_{h_1}^2} - \frac{\lambda'_{DD} \hat{f} m_N}{m_{\eta_2}^2 m_{h_2}^2 \tan \beta} \right)^2 \left( \frac{m_N m_{\eta_2}^2}{m_N + m_{\eta_2}^2} \right)^2,
\]

where $\hat{f} \approx 0.3$ is the usual nucleonic matrix element, $m_N$ is the nucleon mass, $\lambda_{DD} = \lambda_{x_7} \sin^2 \beta + \lambda_{x_5} \cos^2 \beta$ and $\lambda'_{DD} = \lambda_{x_7} \sin 2\beta - \lambda_{x_5} \sin 2\beta$. Since the contributions from the $h_1$ and $h_2$ mediations give a relative negative sign, there is the possibility for destructive interference for $m_{h_1} \sim m_{h_2}$.
FIG. 9. Feynman diagrams giving main contributions to the relic abundance.

In Fig. 10, the spin-independent cross section of DM is shown as a function of the DM mass, where the relic abundance of the DM satisfies $\Omega h^2 \approx 0.12$. We here have fixed the masses of the $Z_2$-even scalar fields as $m_{h_2} = 200 \text{ GeV}$ and $m_{H^\pm} = m_{A_1} = 250 \text{ GeV}$. For the $Z_2$-odd scalar fields, the masses are taken as $m_A = m_{\eta^\pm} = m_{\eta^0} + 20 \text{ GeV}$. The cyan, red, blue and green lines show the results for $\tan \beta = 1, 2, 3$ and 4, respectively, where the dotted lines are excluded by the unitarity condition $\lambda_i < 4\pi (i = 1 \sim 15)$ or the bounded-from-below condition on the scalar potential. As the reference, we show the prediction of the BP$_{NO}$ and the BP$_{IO}$ by the black triangle. Above region of black dashed line are excluded by XENON1T. In Fig. 10 we can see the cancellations between the contributions of $h_1$ and $h_2$. When the DM mass is smaller than about 400 GeV, the relic abundance of the DM is smaller than the observed value $\Omega h^2 < 0.12$. For $\tan \beta \gtrsim 5$, the unitarity condition cannot be satisfied. We find that the allowed ranges for the DM mass are $m_{\eta^0} \simeq 520 - 540 \text{ GeV}$, $490 - 580 \text{ GeV}$ and $400 - 500 \text{ GeV}$ for $\tan \beta = 2, 3$ and 4, respectively. The future sensitivity of the direct detection experiment XENONnT is $\sigma_{SI} \sim \mathcal{O}(10^{-47}) \text{ cm}^2$, which can probe our DM scenario.

VI. CONCLUSION

We have studied the neutrino mass matrix which is composed of the four flavor structures in Eq. (3) based on the $A_4$ flavor symmetry, focusing on the hierarchical flavor structures. As a model with the large hierarchical structure, we have proposed a hybrid seesaw model based on the $A_4$ flavor symmetry. In the model, the neutrino masses are generated by the tree-level and the one-loop seesaw mechanisms. These mechanisms induce the different flavor structures and the large hierarchy with $|b|/|d| > \pi^2$ via the $A_4$ triplet fields of the right-handed neutrinos at the intermediate scale and of the scalar doublet at the electroweak scale. The non-zero $\theta_{13}$ is generated by the one-loop seesaw mechanism. The model predicts the large effective neutrino mass
FIG. 10. DM mass vs spin-independent cross section. The cyan, red, blue and green lines show the results for \( \tan \beta = 1, 2, 3 \) and 4, respectively, for \( m_{h_2} = 200 \) GeV, \( m_{H^\pm} = m_{A_1} = 250 \) GeV, and \( m_A = m_{\eta^z} = m_{\eta^0} + 20 \) GeV. The dotted lines are excluded by the unitarity and the bounded-from-below conditions on the scalar potential [46]. Above region of black dashed line are excluded by XENON1T [55].

\[ m_{\nu e} \sim 0.03 \text{ eV}, \] which can be tested by the futures 0\( \nu \beta \beta \) experiments, with the Majorana phase \( \alpha_3 \sim 0 \). Furthermore, the \( A_4 \) flavor symmetry is broken down to the \( Z_2 \) symmetry in our model and the \( Z_2 \)-odd scalar field \( \eta^0 \) becomes the DM. The constraints arising from the DM relic density set its mass in the range of \( 400 \text{ GeV} \lesssim m_{\eta^0} \lesssim 600 \) GeV. The future direct detection experiments, such as XENONnT, can access our DM scenario.

We have also performed the model-independent analysis of the neutrino mass matrix in Eq. (3), particularly for the cases with some hierarchical flavor structures. It has been found that the hierarchical cases with \( |b|/|a| > 1 \), \( |b|/|c| > 1 \) and \( |d|/|a| > 1 \) reduce the allowed parameter space and show the characteristic predictions for the Majorana phases. On the other hand, the hierarchical case with \( |b|/|d| > 1 \) does not show the specific predictions. However, the larger hierarchy with \( |b|/|d| > \pi^2 \), which can be realized in our hybrid seesaw model, can reduce the predicted parameter region, which can be testable by the future 0\( \nu \beta \beta \) experiments.

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