Diagonalization of Coupled Scalars and its Application to the Supersymmetric Neutral Higgs Sector

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Abstract

We introduce a momentum dependent mixing angle \( \alpha(p^2) \) which allow us to diagonalize at any external momentum \( p \) the one-loop corrected inverse propagator matrix of two coupled scalar fields while keeping the full momentum dependence in the self energies. We compare this method with more traditional techniques applied to the diagonalization of coupled scalars at the one-loop level. This method is applied to the CP-even Higgs sector of the Minimal Supersymmetric Model, defining the momentum dependent mixing angle \( \alpha(p^2) \), and calculating the two CP-even Higgs masses and the mixing angle at these two scales. We compare the results obtained in this way with alternative techniques. We make explicit the relation between \( \alpha(p^2) \) and the running mixing angle. We find differences between the mixing angle calculated with our method and the one calculated with more traditional methods, and these differences are relevant for Higgs searches at LEP2.
1 Introduction

In field theory it is common to find mixing between different species of scalars, fermions, or vector bosons. In the Standard Model (SM) we have the mixing between the gauge fields $B$ and $W^3$ corresponding to the groups $U(1)$ and $SU(2)$ respectively. After a rotation given by the weak mixing angle $\tan \theta_W = g'/g$ at tree level we find the mass eigenstates $\gamma$ and $Z$. Similarly, mixing between scalar particles are typical of two Higgs doublets models, and in supersymmetric theories mixing between fermions are also common (charginos, neutralinos).

In all cases it is trivial to find the mass eigenstates at tree level, however, one-loop radiative corrections will mix the tree-level diagonalized states (for example, $Z$ and $\gamma$ mix at one loop with charged particles in the loop). Moreover, the sum of the mixing graphs will be infinite and momentum dependent. In order to remove the infinities, mass counterterms and wave function renormalization constants are introduced.

Several years ago, Capdequi-Peyranère and Talon [1] studied the wave function renormalization of coupled systems of scalars, fermions, and vectors. Their approach include conventional mass counterterms and wave function renormalization plus a non-conventional field “rotation” that allow them to impose no mixing between the states at different scales. These scales are the masses of the different states. In this paper we generalize this idea and, at the same time, explain the nature of this field “rotation” by introducing a momentum dependent mixing angle between two coupled scalars.

In the Minimal Supersymmetric Model (MSSM) the CP-even Higgs sector consists of two coupled scalars $H$ and $h$, whose masses satisfy $m_H > m_h$. Tree level mass relations are simple and specified by two unknown parameters: the mass of the CP-odd Higgs $m_A$ and the ratio of the two vacuum expectation value of the two Higgs doublets $\tan \beta$. Nevertheless, radiative corrections strongly modify the tree level mass relations, and since the lightest Higgs might be the first particle detected in the Higgs sector, it is necessary to achieve a good understanding of the effect of radiative corrections on this system of coupled scalars. For this reason, the CP-even sector of the MSSM is the best place to apply the method of diagonalization of coupled scalars we are proposing here.

2 Renormalization of Coupled Scalars

2.1 Conventional Wave Function Renormalization

Similarly to ref. [1] (the only difference is that they have $m_{12b}^2 \equiv 0$), consider the bare lagrangian corresponding to a system of two scalars:

$$L_b = \frac{1}{2} \chi_{1b}(p^2 - m_{1b}^2)\chi_{1b} + \frac{1}{2} \chi_{2b}(p^2 - m_{2b}^2)\chi_{2b} - \chi_{1b}m_{12b}^2\chi_{2b}$$

(1)

If we denote by $-iA_{ij}^b(p^2)$, $i, j = 1, 2$, the sum of the one-loop Feynman graphs contributing to the two point functions and, after shifting the bare masses by $m_{ib}^2 \rightarrow m_i^2 - \delta m_i^2$, $i = 1, 2, 12$, and the fields by $\chi_{ib} \rightarrow (1 + \frac{1}{2}\delta Z_i)\chi_i$, the effective lagrangian is

$$L_{eff} = \frac{1}{2}(\chi_1, \chi_2)\Sigma^x\left(\begin{array}{c} \chi_1 \\ \chi_2 \end{array}\right)$$

(2)
where $\Sigma^\chi$ is the inverse propagator matrix, with matrix elements given by

$$
\begin{align*}
\Sigma^\chi_{11}(p^2) &= p^2 - m_1^2 + \frac{1}{2} (p^2 - m_1^2) \delta Z_1 + \delta m_1^2 - A^\chi_{11}(p^2) \\
\Sigma^\chi_{22}(p^2) &= p^2 - m_2^2 + \frac{1}{2} (p^2 - m_2^2) \delta Z_2 + \delta m_2^2 - A^\chi_{22}(p^2) \\
\Sigma^\chi_{12}(p^2) &= -m_{12} - \frac{1}{2} m_{12} (\delta Z_1 + \delta Z_2) + \delta m_{12}^2 - A^\chi_{12}(p^2)
\end{align*}
$$

(3)

Although it is not a necessary assumption, in order to compare more easily with ref. [1], we will assume for the moment that the two scalars are decoupled at tree level, i.e., $m_{12}^2 = 0$. In this case, if we want $m_i, i = 1, 2$, to be the physical masses (the pole of the propagators) then the two mass counterterms are fixed through the relations

$$
\delta m_1^2 = A^\chi_{11}(m_1^2), \quad \delta m_2^2 = A^\chi_{22}(m_2^2)
$$

(4)

Similarly, we may want to fix the wave function renormalization constants by setting to one the residue of each propagator at its pole. In this case we find

$$
\delta Z_1 = A^\chi_{11}(m_1^2), \quad \delta Z_2 = A^\chi_{22}(m_2^2)
$$

(5)

where the prime denote the derivative with respect to the argument. We may want to fix the $\delta m_{12}^2$ counterterm by imposing no mixing between $\chi_1$ and $\chi_2$ at a given scale, for example at $p^2 = m_1^2$. In this case, the off diagonal element of the inverse propagator matrix is

$$
\Sigma^\chi_{12}(p^2) = \delta m_{12}^2 - A^\chi_{12}(p^2) = A^\chi_{12}(m_1^2) - A^\chi_{12}(p^2) \equiv -\tilde{A}^\chi_{12}(p^2)
$$

(6)

At this point all the counterterms are fixed, and since $\tilde{A}^\chi_{12}(p^2)$ is zero only at $p^2 = m_1^2$, the two fields are not decoupled at a different scale. In particular, at the scale given by the mass of the second scalar, $p^2 = m_2^2$, the mixing is not zero:

$$
\Sigma^\chi_{12}(m_2^2) = A^\chi_{12}(m_1^2) - A^\chi_{12}(m_2^2) \neq 0
$$

(7)

Of course, this non-zero mixing is of one-loop order, and if the calculation of the one-loop scalar masses is done by diagonalizing the inverse propagator matrix order by order in perturbation theory, then this non-zero mixing would be a two-loop order effect. Nevertheless, this perturbative diagonalization can introduce large errors if radiative corrections are large. And this is the case with the CP-even Higgs sector of the Minimal Supersymmetric Model (MSSM).

### 2.2 Mixed Wave Function Renormalization

According to the previous section, the conventional wave function renormalization gives an inverse propagator matrix which is diagonal only at one particular scale. This is the motivation for the authors in ref. [1] to define the following wave function renormalization $\chi_{1b} \to (1 - \alpha_1)\chi_1 - \beta_1\chi_2$ and $\chi_{2b} \to (1 - \alpha_2)\chi_2 - \beta_2\chi_1$ instead of $\chi_{1b} \to (1 + \frac{1}{2} \delta Z_i)\chi_i$. We use here. If we perform this transformation in the bare lagrangian in eq. (1) (and taking $m_{12b} = 0$ in order to follow ref. [1]) we find the following inverse propagator matrix elements

$$
\begin{align*}
\Sigma^\chi_{11}(p^2) &= p^2 - m_1^2 - 2(p^2 - m_1^2)\alpha_1 + \delta m_1^2 - A^\chi_{11}(p^2) \\
\Sigma^\chi_{22}(p^2) &= p^2 - m_2^2 - 2(p^2 - m_2^2)\alpha_2 + \delta m_2^2 - A^\chi_{22}(p^2) \\
\Sigma^\chi_{12}(p^2) &= -\beta_1(p^2 - m_1^2) - \beta_2(p^2 - m_2^2) - A^\chi_{12}(p^2)
\end{align*}
$$

(8)
where the bar in \( \bar{\Sigma} \) is to differentiate with the matrix elements in eq. (8). Setting to one the residue of the pole of each propagator they find

\[
\alpha_i = -\frac{1}{2} A^\chi_{ii}(m^2_i) = -\frac{1}{2} \delta Z_i, \quad i = 1, 2
\]

(9)

where we have also included the relation between their \( \alpha_i \) and our \( \delta Z_i \). Imposing no mixing between the two fields at \( p^2 = m^2_1 \) and also at \( p^2 = m^2_2 \) they get

\[
\beta_1 = \frac{A^\chi_{12}(m^2_2)}{m^2_1 - m^2_2}, \quad \beta_2 = \frac{A^\chi_{12}(m^2_1)}{m^2_2 - m^2_1}
\]

(10)

This procedure is equivalent to take the inverse propagator matrix in eq. (8) in the special case where \( m^2_{12} = 0 \) and \( \delta m^2_{12} = 0 \), and perform a “rotation” (it is not a field rotation in the usual sense, it is just a wave function renormalization that mixes the two fields) in the following way

\[
\Sigma^\chi \rightarrow \begin{bmatrix} 1 & \beta_2 \\ \beta_1 & 1 \end{bmatrix} \Sigma^\chi \begin{bmatrix} 1 & \beta_1 \\ \beta_2 & 1 \end{bmatrix}
\]

(11)

with the \( \beta_i \) being divergent, as it can be seen from eq. (10). In this case, the inverse propagator matrix in eq. (8) is simultaneously diagonal at the two different scales \( p^2 = m^2_1 \) and \( p^2 = m^2_2 \). Nevertheless, there will be a non-zero mixing between the two scalars at any other scale.

It will be instructive to modify the calculation done in ref. [1] just presented in this section by considering \( m^2_{12} = 0 \) but \( \delta m^2_{12} \neq 0 \). In this case, the only modification to the inverse propagator matrix is in the off-diagonal element in eq. (8), and now is equal to

\[
\tilde{\Sigma}^\chi_{12}(p^2) = -\beta^f_1 (p^2 - m^2_1) - \beta^f_2 (p^2 - m^2_2) + \delta m^2_{12} - A^\chi_{12}(p^2)
\]

(12)

The new coefficients \( \beta^f_i \) can be calculated in the same way as before, that is imposing no mixing at the scale \( p^2 = m^2_1 \) and \( p^2 = m^2_2 \). We get

\[
\beta^f_1 = \frac{A^\chi_{12}(m^2_2) - \delta m^2_{12}}{m^2_1 - m^2_2}, \quad \beta^f_2 = \frac{A^\chi_{12}(m^2_1) - \delta m^2_{12}}{m^2_2 - m^2_1}
\]

(13)

The freedom introduced by the new counterterm \( \delta m^2_{12} \) give us the opportunity to cancel the divergency present in \( A^\chi_{12}(p^2) \). This explains the superscript ”\( f \)” in the constants \( \beta^f_i \): they are finite. In this way, the numerators in eq. (13) are just the renormalized two point function \( A^\chi_{12}(p^2) \) evaluated at two different scales. The \( \beta^f_i \) are then

\[
\beta^f_1 = \frac{A^\chi_{12}(m^2_2)}{m^2_1 - m^2_2}, \quad \beta^f_2 = \frac{A^\chi_{12}(m^2_1)}{m^2_2 - m^2_1}
\]

(14)

This time, this wave function renormalization is equivalent to take the inverse propagator matrix in eq. (8) and perform the following “rotation”:

\[
\Sigma^\chi \rightarrow \begin{bmatrix} 1 & \beta^f_2 \\ \beta^f_1 & 1 \end{bmatrix} \Sigma^\chi \begin{bmatrix} 1 & \beta^f_1 \\ \beta^f_2 & 1 \end{bmatrix}
\]

(15)

Again, the rotated inverse propagator matrix is diagonal only at the scales \( p^2 = m^2_1 \) and \( p^2 = m^2_2 \).
2.3 Momentum Dependent Mixing Angle

In this paper we introduce a momentum dependent mixing angle \( \alpha(p^2) \) which will allow us to diagonalize the inverse propagator matrix at any momentum \( [2, 3] \). Considering the already finite inverse propagator matrix elements in eq. (3) (now we work on the general case \( m_{12} \neq 0 \) and \( \delta m^2_{12} \neq 0 \)), we define the momentum dependent mixing angle \( \alpha(p^2) \) by

\[
\sin 2 \alpha(p^2) = -\frac{2 \Sigma_{12}(p^2)}{\sqrt{[\Sigma_{11}(p^2) - \Sigma_{22}(p^2)]^2 + 4[\Sigma_{12}(p^2)]^2}}
\]

\[
\cos 2 \alpha(p^2) = -\frac{\Sigma_{11}(p^2) - \Sigma_{22}(p^2)}{\sqrt{[\Sigma_{11}(p^2) - \Sigma_{22}(p^2)]^2 + 4[\Sigma_{12}(p^2)]^2}}
\]

(16)

The matrix \( \Sigma_x(p^2) \) is diagonalized at any momentum \( p^2 \) by a rotation defined by the angle \( \alpha(p^2) \)

\[
\Sigma_x \longrightarrow \begin{bmatrix} c_\alpha(p^2) & s_\alpha(p^2) \\ -s_\alpha(p^2) & c_\alpha(p^2) \end{bmatrix} \Sigma_x \begin{bmatrix} c_\alpha(p^2) & -s_\alpha(p^2) \\ s_\alpha(p^2) & c_\alpha(p^2) \end{bmatrix}
\]

(17)

where \( s_\alpha \) and \( c_\alpha \) are sine and cosine of the momentum dependent mixing angle \( \alpha(p^2) \).

In order to find the connection between the mixed wave function renormalization introduced in ref. [1] and the momentum dependent mixing angle introduced here, consider eqs. (3) and (16). In the case where \( m^2_{12} = 0 \) we find in first approximation

\[
s_\alpha(p^2) \approx \frac{\tilde{A}_{12}(p^2)}{m_2^2 - m_1^2}, \quad c_\alpha \approx 1
\]

(18)

where \( \tilde{A}_{12}(p^2) \) is defined in eq. (3). Therefore, the mixed wave function renormalization defined by eq. (15) can be obtained from the rotation by an angle \( \alpha(p^2) \) defined in eq. (17) if we approximate eq. (16) to one-loop, order by order in perturbation theory, and evaluate the non-diagonal entries in the first rotation matrix in eq. (17) at two different scales: \( s_\alpha \) at \( p^2 = m_1^2 \) and \( -s_\alpha \) at \( p^2 = m_2^2 \).

The use of a momentum dependent mixing angle is an alternative to define a counterterm for this angle. In fact, the renormalization procedure is carried out in the unrotated basis and no mixing angle is defined at that level. Similarly, instead of renormalizing couplings of the rotated fields to other particles, we renormalize couplings of the unrotated fields to those particles and after that we rotate by an angle \( \alpha(p^2) \), where \( p^2 \) is the typical scale of the process, for example, \( p^2 = m_i^2 \) if the rotated field \( \chi_i \) is on-shell. Usually, working out the radiative corrections in the unrotated basis implies one extra advantage, and that is the simplicity of the Feynman rules. In the following we will illustrate these ideas by renormalizing the CP-even neutral Higgs masses of the Minimal Supersymmetric Model (MSSM).

3 The Minimal Supersymmetric Model

The radiative corrections to the Higgs masses in the MSSM have been studied by many authors in the last few years, using three different techniques: the renormalization
group equation (RGE) method, the effective potential, and the diagramatic technique. It was established the convenience of the parametrization of the Higgs sector with the CP-odd Higgs mass $m_A$, and the ratio of the vacuum expectation values of the two Higgs doublets $\tan \beta = v_2/v_1$. The radiative corrections to the charged Higgs mass were found to be small [4, 5, 6], growing as $m_t^2$, unless there is an appreciable mixing in the squark mass matrix: in that case a term proportional to $m_t^4$ is non-negligible [5]. The corrections to the CP-even Higgs masses are large and grow as $m_t^4$, and have profound consequences in the phenomenology of the Higgs sector [6, 7, 8, 9, 10]. Two-loop corrections also have been calculated and shown to be important [11].

The MSSM has two Higgs doublets [12]:

$$H_1 = \left( \frac{1}{\sqrt{2}}(\chi_1^0 + v_1 + i\varphi_1^0) \right), \quad H_2 = \left( \frac{1}{\sqrt{2}}(\chi_2^0 + v_2 + i\varphi_2^0) \right),$$

with $v_i$ being the vacuum expectation value of the Higgs fields $H_i$. In the CP-even Higgs sector there are two coupled scalar fields $(\chi_1^0, \chi_2^0)$. The radiatively corrected propagators of the CP-even Higgs bosons depend on the two point functions $A^i_{ij}(p^2)$ ($i, j = 1, 2$), where $p^2$ is the external momentum squared. In perturbative diagonalization of the one-loop mass matrix it is consistent to consider $p^2$ as a constant equal to the tree level mass matrix element. Nevertheless, it has been shown that for large values of the top quark mass, the perturbative diagonalization of the mass matrix is not reliable. We will study the effect of those approximations, namely the perturbative diagonalization of the mass matrix and the replacement of the external momentum in the self-energies by a constant, and compare them with the use of the momentum dependent mixing angle $\alpha(p^2)$ that diagonalizes non-perturbatively the inverse propagator matrix of the CP-even Higgs sector, and we find numerically the pole of the propagators keeping the full momentum dependence.

### 3.1 The model at tree level

We start reviewing the neutral Higgs sector of the MSSM in the tree level approximation. The CP-odd mass matrix given by:

$$M_A^2 = \begin{pmatrix} m_{12}^2 t_\beta + t_1/v_1 & m_{12}^2 \\ m_{12}^2/t_\beta + t_2/v_2 & m_{12}^2 \end{pmatrix}$$

(20)

where $m_{12}^2$ is a soft symmetry breaking term, $t_\beta = \tan \beta = v_2/v_1$, and $t_i$ ($i = 1, 2$) are the tree level tadpoles, whose expressions are

$$t_1 = m_{1H}^2 v_1 - m_{12}^2 v_2 + \frac{1}{8}(g^2 + g'^2)v_1 (v_1^2 - v_2^2),$$

$$t_2 = m_{2H}^2 v_2 - m_{12}^2 v_1 + \frac{1}{8}(g^2 + g'^2)v_2 (v_2^2 - v_1^2).$$

(21)

Here, $m_{1H}^2 = m_i^2 + |\mu|^2$ ($i = 1, 2$), $m_i^2$ are soft supersymmetry breaking terms, and $\mu$ is the mass parameter in the superpotential. At tree level and at the minimum $(t_1 = t_2 = 0)$, this matrix is diagonalized with a rotation of an angle $\beta$. The tree level CP-odd Higgs mass is:

$$(m_A^2)_0 = \frac{m_{12}^2}{s_\beta c_\beta}.$$
where the subscript “0” refers to the tree level. On the other hand, the mass matrix of the neutral CP-even Higgs bosons is

\[
M^2_\chi = \begin{pmatrix}
\frac{1}{4} g_Z^2 v^2 c_\beta + m_{12}^2 t_\beta + t_1/v_1 & -\frac{1}{4} g_Z^2 v^2 c_\beta s_\beta c_\beta - m_{12}^2 \\
-\frac{1}{4} g_Z^2 v^2 s_\beta c_\beta - m_{12}^2 & \frac{1}{4} g_Z^2 v^2 s_\beta^2 + m_{12}^2/t_\beta + t_2/v_2
\end{pmatrix},
\]

where \(g_Z = g^2 + g^2\) and \(v^2 = v_1^2 + v_2^2\). From now on, we will use the following notation for its matrix elements: \((M^2_\chi)_{ij} = m_{ij}^\chi\). The tree level masses are obtained setting the tadpoles to zero and eliminating \(m_{12}\) in favor of \(m_A\) using eq. (22). The answer is

\[
(m_{H,H_i}^2)_{0} = \frac{1}{2}(m_A^2 + m_Z^2) \pm \frac{1}{2} \sqrt{(m_Z^2 - m_A^2)^2 c_{2\beta}^2 + (m_A^2 + m_Z^2)^2 s_{2\beta}^2},
\]

with a tree level mixing angle

\[
(tan 2\alpha)_0 = \frac{(m_A^2 + m_Z^2)}{(m_A^2 - m_Z^2)} \tan 2\beta.
\]

where we have omitted the subscript “0” from \(m_A^2\). Next, we calculate the radiative corrections to this mixing angle \(\alpha\), and introduce the momentum dependent mixing angle \(\alpha(p^2)\).

### 3.2 One-loop corrections to the CP-even Higgs masses

To find the one loop corrections to the Higgs masses and mixing angle, we take the mass matrix for the CP-even fields and replace the bare quantities as indicated by:

\[
H_i \rightarrow (Z_i)^\dagger H_i \approx (1 + \frac{1}{2} \delta Z_i) H_i,
\]

\[
\lambda \rightarrow \lambda - \delta \lambda,
\]

\[
m \rightarrow m - \delta m,
\]

\[
v_i \rightarrow v_i - \delta v_i,
\]

\[
\beta = g, g', e, ...
\]

\[i = 1, 2.\]

The one loop effective lagrangian in the CP-even Higgs sector (we have not rotated the original fields) has the form:

\[
\mathcal{L} = \frac{1}{2}(\chi_1^0, \chi_2^0) \Sigma^\chi (\chi_1^0) \chi_2^0
\]

with the matrix elements given by

\[
\Sigma_{11}(p^2) = \left[ p^2 - m_{11}^2 + \delta m_{11}^2 - A_{11}(p^2) \right] Z_1
\]

\[
\Sigma_{22}(p^2) = \left[ p^2 - m_{22}^2 + \delta m_{22}^2 - A_{22}(p^2) \right] Z_2
\]

\[
\Sigma_{12}(p^2) = \left[ -m_{12}^2 + \delta m_{12}^2 - A_{12}(p^2) \right] Z_1^2 Z_2^{1/2}
\]

where we are allowed to set \(t_i = 0\) in \(m_{11}^2\), given in eq. (23). The mass counterterms are:

\[
\delta m_{11}^2 = c_\beta^2 \delta m_Z^2 + s_\beta^2 \delta \left( \frac{m_{12}^2}{s_\beta c_\beta} \right) + 2(m_A^2 - m_Z^2) s_\beta c_\beta \frac{\delta t_1}{t_\beta} + \frac{\delta t_1}{v_1}
\]

\[
\delta m_{22}^2 = s_\beta^2 \delta m_Z^2 + c_\beta^2 \delta \left( \frac{m_{12}^2}{s_\beta c_\beta} \right) - 2(m_A^2 - m_Z^2) s_\beta c_\beta \frac{\delta t_2}{t_\beta} + \frac{\delta t_2}{v_2}
\]

\[
\delta m_{12}^2 = -s_\beta c_\beta \delta m_Z^2 - s_\beta c_\beta \delta \left( \frac{m_{12}^2}{s_\beta c_\beta} \right) - (m_A^2 + m_Z^2) s_\beta c_\beta (c_\beta^2 - s_\beta^2) \frac{\delta t_2}{t_\beta}
\]
and to fix them we adopt an on-shell scheme. Consider first the quadratic terms in the CP-odd Higgs sector (after rotating by an angle $\beta$):

$$L = \frac{1}{2}(A, G)\Sigma\varphi \left( \begin{array}{c} A \\ G \end{array} \right)$$

where the relevant matrix element is

$$\Sigma_{\varphi AA}(p^2) = \left[ p^2 - m_A^2 + \delta m_A^2 - A_{AA}(p^2) \right] Z_A$$

$$\approx p^2 - m_A^2 + (p^2 - m_A^2)\delta Z_A + \delta m_A^2 - A_{AA}(p^2).$$

and the mass counterterm is given by

$$\delta m_A^2 = \delta \left( \frac{m_{12}^2}{s_\beta c_\beta} \right) + s_\beta^2 \frac{\delta t_1}{v_1} + c_\beta^2 \frac{\delta t_2}{v_2}$$

and the wave function renormalization constant is

$$Z_A = s_\beta^2 Z_1 + c_\beta^2 Z_2 \iff \delta Z_A = s_\beta^2 \delta Z_{H_1} + c_\beta^2 \delta Z_{H_2}. \quad (33)$$

To fix the mass counterterm $\delta m_A^2$ we adopt the following on-shell renormalization condition:

$$\text{Re}\Sigma_{\varphi AA}(m_A^2) = 0 \quad \Rightarrow \quad \delta m_A^2 = \text{Re}A_{AA}(m_A^2)$$

which means that the parameter $m_A^2$ has been defined as the pole of the propagator.

Similarly to the previous case, the mass counterterms for the $Z$– and $W$–boson are fixed in an on-shell scheme:

$$\delta m_Z^2 = \text{Re}A_{ZZ}(m_Z^2), \quad \delta m_W^2 = \text{Re}A_{WW}(m_W^2),$$

i.e., the parameters $m_Z$ and $m_W$ are the pole masses.

Tadpoles are zero at tree level ($t_i = 0$), and to fix their counterterms it is convenient to impose vanishing tadpoles at one-loop as well. If we call $-i\Gamma_i^{(1)}$ to the sum of all Feynman diagrams (the 1-point irreducible Green’s function) contributing to the one-loop tadpole, then the tadpole counterterm is equal to the sum of all one-loop tadpole graphs

$$\delta t_i = \frac{1}{4s_\beta^2 c_\beta m_W^2} \left[ c_{2\beta} A_{WW}(m_W^2) + A_{\tilde{e}_L\tilde{e}_L}(m_{\tilde{e}_L}^2) - A_{\nu_e\nu_e}(m_{\nu_e}^2) \right].$$

where the last two self energies receive no contributions from loops containing top and bottom quarks and squarks.

In this way, with eqs. (32), (34), (35), (36), and (37) we can calculate the mass counterterms in eq. (29), which have been fixed in an on–shell scheme where $m_Z, m_W,$ and $m_A$ are pole masses, and the tadpoles are equal to zero at the one–loop level.

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1For alternative definitions of $\tan \beta$ see for example [14, 15].
The pole masses of the two neutral Higgs bosons are the zeros of the inverse propagator matrix \( \Sigma \chi(p^2) \), whose matrix elements are defined in eq. (28), therefore the Higgs masses satisfy the following relation
\[
\hat{\Sigma}^\chi_{11}(m_i^2)\hat{\Sigma}^\chi_{22}(m_i^2) = \left[ \hat{\Sigma}^\chi_{12}(m_i^2) \right]^2, \quad i = h, H \tag{38}
\]
where we call
\[
\hat{\Sigma}^\chi_{11}(p^2) = p^2 - m^2\chi_{c\beta} - m^2\chi_{s\beta} + \delta m^2_{\chi_{11}} - A^\chi_{11}(p^2)
\]
\[
\hat{\Sigma}^\chi_{22}(p^2) = p^2 - m^2\chi_{c\beta} - m^2\chi_{s\beta} + \delta m^2_{\chi_{22}} - A^\chi_{22}(p^2)
\]
\[
\hat{\Sigma}^\chi_{12}(p^2) = (m^2_2 + m^2_A)\delta m_{\chi_{12}} + \delta m^2_{\chi_{22}} - A^\chi_{12}(p^2)
\tag{39}
\]
From here we see that the wave function renormalization constants \( Z_i, i = h, H \) are canceled from the eigenvalue equation, implying the independence of the Higgs masses on the \( Z_i \). In ref. [16] the Higgs masses are calculated in this way, although without defining a momentum dependent mixing angle.

The eigenvalues of the matrix \( \Sigma \chi(p^2) \) are the inverse propagators of the two Higgs bosons, and they are given by
\[
\Sigma^\chi_i(p^2) = \frac{1}{2} \left[ \Sigma^\chi_{11}(p^2) + \Sigma^\chi_{22}(p^2) \right] \pm \frac{1}{2} \sqrt{ [\Sigma^\chi_{11}(p^2) - \Sigma^\chi_{22}(p^2)]^2 + 4[\Sigma^\chi_{12}(p^2)]^2 } \tag{40}
\]
where \( i = H, h \) and the + (−) sign correspond to the field \( h (H) \). These inverse propagators have a zero at the physical mass of the Higgs field: \( \Sigma_h(m_h^2) = 0 \) and \( \Sigma_H(m_H^2) = 0 \) and we solve these equations numerically.

The wave function renormalization constants \( Z_i, i = 1, 2 \), have not been calculated. We impose that the residue of the propagator of the CP-even Higgs bosons \( h \) and \( H \) are normalized to one. From eq. (10) we get
\[
\frac{\hat{\Sigma}^\chi_{11}(m_h^2)}{Z_2} + \frac{\hat{\Sigma}^\chi_{22}(m_h^2)}{Z_1} = \frac{\partial}{\partial p^2} \left\{ \hat{\Sigma}^\chi_{11}(p^2)\hat{\Sigma}^\chi_{22}(p^2) - \left[ \hat{\Sigma}^\chi_{12}(p^2) \right]^2 \right\} (m_h^2),
\]
\[
\frac{\hat{\Sigma}^\chi_{11}(m_H^2)}{Z_2} + \frac{\hat{\Sigma}^\chi_{22}(m_H^2)}{Z_1} = \frac{\partial}{\partial p^2} \left\{ \hat{\Sigma}^\chi_{11}(p^2)\hat{\Sigma}^\chi_{22}(p^2) - \left[ \hat{\Sigma}^\chi_{12}(p^2) \right]^2 \right\} (m_H^2), \tag{41}
\]
from where the wave function renormalization constants can be calculated.

The matrix \( \Sigma \chi(p^2) \) is diagonalized at a particular momentum \( p^2 \) by a rotation defined by the angle \( \alpha(p^2) \). This mixing angle satisfy
\[
\tan 2\alpha(p^2) = \frac{2\Sigma^\chi_{12}(p^2)Z_1^{1/2}Z_2^{1/2}}{\Sigma^\chi_{11}(p^2)Z_1 - \Sigma^\chi_{22}(p^2)Z_2} \tag{42}
\]
In ref. [11] it is used a wave function renormalization of the type in ref. [4] to renormalize the CP-even Higgs sector of the MSSM, obtaining formulas analogous to eq. (10), and then it is defined the angle \( \alpha \) at two different scales. Those two angles are special cases of the momentum dependent mixing angle \( \alpha(p^2) \) in eq. (12).

It is worth to mention that, since the residue of the pole of the propagator of the Higgs \( A \) is not normalized to unity, when renormalizing processes with external Higgs \( A \) it is necessary to multiply by the finite wave function normalization

\[
Z_A^{1/2} = \left[ \frac{\partial}{\partial p^2} \hat{\Sigma}^\phi_A(p^2) \right]_{p^2=m_A^{-2}}^{-1/2} = \left[ \frac{\partial}{\partial p^2} \left( \hat{\Sigma}^\phi_{11} \hat{\Sigma}^\phi_{22} - \hat{\Sigma}^\phi_{12}^2 \right) Z_1 Z_2 \right]^{-1/2} \tag{43}
\]
which up to one–loop order reduces to $Z^{1/2}_A = 1 + \frac{1}{2} A'_A(m_A^2)$, as expected.

Finally, if we make the expansion $Z^{1/2}_i = (1 + \delta Z_i)^{1/2} \approx 1 + \frac{1}{2} \delta Z_i$. We get

\[
\begin{align*}
\Sigma_{11}(p^2) &= p^2 - m_{11}^2 + (p^2 - m_{11}^2)\delta Z_{H_1} + \delta m_{11}^2 - A_{11}(p^2) \\
\Sigma_{22}(p^2) &= p^2 - m_{22}^2 + (p^2 - m_{22}^2)\delta Z_{H_2} + \delta m_{22}^2 - A_{22}(p^2) \\
\Sigma_{12}(p^2) &= -m_{12}^2 - \frac{1}{2} m_{12}^2(\delta Z_{H_1} + \delta Z_{H_2}) + \delta m_{12}^2 - A_{12}(p^2)
\end{align*}
\] (44)

And from here we have checked that all the divergences cancel in each matrix element in eq. (44).

### 3.3 Perturbative limit

At this point, it is instructive to make a perturbative expansion of $\tan 2\alpha(p^2)$ defined in eq. (44). If we call $\Delta t_{2\alpha} = \tan 2\alpha - t_{2\alpha}$, and keeping only one–loop terms, we get:

\[
\frac{\Delta t_{2\alpha}}{t_{2\alpha}} = -\frac{2}{(m_A^2 + m_Z^2)s_{2\beta}} \left[ \tilde{A}_{12} + \frac{1}{2}(\tilde{A}_{22} - \tilde{A}_{11})t_{2\alpha} \right]
\] (45)

The variation of the angle $\alpha$ in eq. (45) is the finite one-loop correction to the tree level angle defined in eq. (23), calculated perturbatively. Now, to clarify even more the meaning of the angle $\alpha(p^2)$ in eq. (44) and the perturbative one-loop correction to $\alpha_0$ in eq. (45), consider the inverse propagator matrix $\Sigma^\alpha$ with matrix elements given in eq. (39). If we rotate $\Sigma^\alpha$ by an angle $\alpha_0$ we will diagonalize the tree level part of the inverse propagator matrix, but not the one-loop part:

\[
R_{\alpha_0} \Sigma^\alpha(p^2) R_{\alpha_0}^{-1} = \begin{bmatrix} p^2 - m_{H_0}^2 & -\tilde{A}_{H_0}(p^2) \\
-\tilde{A}_{H_0}(p^2) & p^2 - m_{H_0}^2 \end{bmatrix}
\] (46)

where the rotation matrix is defined by

\[
R_{\alpha_0} = \begin{bmatrix} \cos \alpha_0 & \sin \alpha_0 \\
-\sin \alpha_0 & \cos \alpha_0 \end{bmatrix}
\] (47)

$m_{H_0}^2$ and $m_{H_0}^2$ are the tree level masses given in eq. (24), $h_0$ and $H_0$ are the tree level rotated CP-even fields:

\[
\begin{bmatrix} H_0 \\ h_0 \end{bmatrix} = R_{\alpha_0} \begin{bmatrix} \chi_0^0 \\ \chi_0^2 \end{bmatrix}
\] (48)

and $\tilde{A}_{ab}(p^2)$ ($a, b = h_0, H_0$) are the renormalized two-point functions. Now, to further diagonalize the inverse propagator matrix in eq. (46) we need a rotation of one-loop order by an angle $\Delta \alpha$:

\[
R_{\Delta \alpha} \approx \begin{bmatrix} 1 & \Delta \alpha \\
-\Delta \alpha & 1 \end{bmatrix}
\] (49)

Imposing that the off-diagonal matrix element in $R_{\Delta \alpha} R_{\alpha_0} \Sigma^\alpha(p^2) R_{\alpha_0}^{-1} R_{\Delta \alpha}^{-1}$ is zero, we get

\[
\Delta \alpha = \frac{c_{2\alpha_0} \tilde{A}_{12} + \frac{1}{2} s_{2\alpha_0}(\tilde{A}_{22} - \tilde{A}_{11})}{m_{H_0}^2 - m_{h_0}^2}
\] (50)
where \( c_{2\alpha} = \cos 2\alpha_0 \) and \( s_{2\alpha} = \sin 2\alpha_0 \). Using the relations \( \Delta s_\alpha = c_{\alpha_0} \Delta \alpha \), and \( \Delta c_\alpha = -s_{\alpha_0} \Delta \alpha \), we can prove that

\[
\frac{\Delta t_{2\alpha}}{t_{2\alpha_0}} = \frac{\Delta \alpha}{s_{\alpha_0} c_{\alpha_0} (c_{2\alpha_0}^2 - s_{2\alpha_0}^2)}
\]  

(51)

and from eq. (50) and using \( s_{2\alpha_0} (m_{H_0}^2 - m_{h_0}^2) = -(m_A^2 + m_Z^2) s_{2\beta} \) we recover eq. (43). In conclusion, we have proved that

\[
R_{\alpha(p^2)} \approx R_{\Delta \alpha} R_{\alpha_0}
\]  

(52)

\( i.e., \) a rotation by the angle \( \alpha_0 \) followed by a rotation by the angle \( \Delta \alpha \) is the perturbative approximation of a rotation by the angle \( \alpha(p^2) \).

### 3.4 Relation with the running mixing angle

Another useful point to clarify here is the relation between the momentum dependent mixing angle \( \alpha(p^2) \) and the running mixing angle \( \alpha(Q) \), where \( Q \) is an arbitrary mass scale introduced by the momentum subtraction scheme \( \overline{DR} \) in dimensional reduction \cite{17}. The renormalization group equation (RGE) satisfied by the angle \( \alpha \) is directly related to the divergent terms of its counterterm. Since \( t_{2\alpha_0} \) is defined at tree level by

\[
t_{2\alpha_0} = \frac{2 m_{12}^2}{m_{11}^2 - m_{22}^2},
\]  

(53)

its counterterm, defined by the relation \( t_{2\alpha_0} = t_{2\alpha} - \delta t_{2\alpha} \), satisfy

\[
\frac{\delta t_{2\alpha}}{t_{2\alpha}} = \frac{2}{(m_A^2 + m_Z^2) s_{2\beta}} \left[ \delta m_{12}^2 + \frac{1}{2} (\delta m_{22}^2 - \delta m_{11}^2) t_{2\alpha} \right].
\]  

(54)

This term is contained in the finite shift of the angle \( \alpha \) given in eq. (43), then we have

\[
\frac{\Delta t_{2\alpha}}{t_{2\alpha}} = -\frac{\delta t_{2\alpha}}{t_{2\alpha}} - \frac{2}{(m_A^2 + m_Z^2) s_{2\beta}} \left[ \frac{1}{2} (m_{22}^2 \delta Z_{H_2} - m_{11}^2 \delta Z_{H_1}) t_{2\alpha} + \frac{1}{2} m_{12}^2 (\delta Z_{H_1} + \delta Z_{H_2}) + \frac{A_{h_0 h_0}(0)}{c_{2\alpha}} \right].
\]  

(55)

Since \( \Delta t_{2\alpha} \) is finite, to find the divergent terms of the counterterm \( \delta t_{2\alpha} \) it is enough to look for the divergences of the second term in the right hand side of eq. (55). In order to compare with ref. \cite{18}, where the contribution from the top quark to the counterterm for the angle \( \alpha \) is calculated, we concentrate only in the top quark contribution. From eq. (41), and keeping only one–loop divergent terms, we have

\[
[\delta Z_{H_1}]_{\text{div}}^{\text{top}} = 0, \quad [\delta Z_{H_2}]_{\text{div}}^{\text{top}} = -\frac{N_c g^2 m_t^2}{32 \pi^2 m_W^2 s_{\beta}} \Delta,
\]  

(56)

where \( N_c = 3 \) is the number of colors, \( \Delta \) is the regulator in dimensional regularization given by

\[
\Delta = \frac{2}{4 - n} + \ln 4\pi - \gamma_E,
\]  

(57)

\( ^2\)Or \( \overline{MS} \) in the case of dimensional regularization used in non-supersymmetric theories.
\( n \) is the number of space-time dimensions, and \( \gamma_E \) is the Euler’s constant. On the other hand, the contribution from the top quark to the mixing between \( h_0 \) and \( H_0 \) is
\[
\left[ A_{h_0H_0}(0) \right]_{\text{div}}^{\text{top}} = \frac{3N_c g^2 s_\alpha m_t^4}{32\pi^2 m_W^2 s_\beta^2} \Delta .
\] (58)

Replacing eqs. (56) and (58) into eq. (55), and considering that \( \Delta t_{2\alpha} \) is finite, we get
\[
\left[ \frac{\delta t_{2\alpha}}{t_{2\alpha}} \right]_{\text{div}}^{\text{top}} = -\frac{N_c g^2 m_t^2}{64\pi^2 m_W^2 s_\beta^2} \frac{12m_t^2 - m_A^2 - m_Z^2 t_{2\alpha}}{(m_A^2 + m_Z^2)s_{2\beta}} \Delta
\] (59)
and this is the same answer we find in ref. [18], with the exception of the sign, since here we define the counterterm of the angle \( \alpha \) with the opposite sign.

Now we have checked that we reproduce the correct divergent terms for the \( \tan 2\alpha \) counterterm, we turn to the relation itself between the renormalized \( \tan 2\alpha(p^2) \) and the running \( \tan 2\alpha(Q) \). In the \( \overline{\text{MS}} \) scheme, each counterterm is fixed to cancel the divergent pieces of the corresponding loop corrections. In the case of tadpoles we have
\[
\delta t_{1i}^{\overline{\text{MS}}} = [T_i(Q)]_{\text{div}} \quad \text{and} \quad -\delta t_{1i}^{\overline{\text{MS}}} + T_i(Q) = \tilde{T}_{1i}^{\overline{\text{MS}}}(Q) .
\] (60)

Therefore, the running tadpoles are equal to\(^3\)
\[
t_1(Q) \equiv \left[ m_{1H}^2 v_1 - m_{12}^2 v_2 + \frac{1}{8}(g^2 + g'^2)v_1(v_1^2 - v_2^2) \right] (Q) = -\tilde{T}_{11}^{\overline{\text{MS}}}(Q) ,
\]
\[
t_2(Q) \equiv \left[ m_{2H}^2 v_2 - m_{12}^2 v_1 + \frac{1}{8}(g^2 + g'^2)v_2(v_2^2 - v_1^2) \right] (Q) = -\tilde{T}_{22}^{\overline{\text{MS}}}(Q) .
\] (61)

Considering the tree level neutral CP–even Higgs mass matrix in eq. (23), we find that the renormalized inverse propagator matrix in the \( \overline{\text{MS}} \) scheme has the following matrix elements:
\[
\Sigma_{11}^{\chi}(p^2) = p^2 - m_Z^2(Q)c_\beta^2(Q) - m_A^2(Q)s_\beta^2(Q) + \frac{1}{v_1}\tilde{T}_{11}^{\overline{\text{MS}}}(Q) - \tilde{A}_{11}^{\overline{\text{MS}}}(p^2, Q)
\]
\[
\Sigma_{22}^{\chi}(p^2) = p^2 - m_Z^2(Q)s_\beta^2(Q) - m_A^2(Q)c_\beta^2(Q) + \frac{1}{v_2}\tilde{T}_{22}^{\overline{\text{MS}}}(Q) - \tilde{A}_{22}^{\overline{\text{MS}}}(p^2, Q)
\] (62)
\[
\Sigma_{12}^{\chi}(p^2) = [m_Z^2(Q) + m_A^2(Q)]s_\beta(Q)c_\beta(Q) - \tilde{A}_{12}^{\overline{\text{MS}}}(p^2, Q)
\]
and from here we can find the renormalized \( \alpha(p^2) \) in terms of \( \overline{\text{MS}} \) quantities
\[
\tan 2\alpha(p^2) = \frac{[m_A^2(Q) + m_Z^2(Q)]s_{2\beta}(Q) - 2\tilde{A}_{12}^{\overline{\text{MS}}}(p^2, Q)}{[m_A^2(Q) - m_Z^2(Q)]c_{2\beta}(Q) - \tilde{A}_{11}^{\overline{\text{MS}}}(p^2, Q) + \tilde{A}_{22}^{\overline{\text{MS}}}(p^2, Q) + \frac{1}{v_1}\tilde{T}_{11}^{\overline{\text{MS}}}(Q) - \frac{1}{v_2}\tilde{T}_{22}^{\overline{\text{MS}}}(Q)}
\] (63)
or, in first approximation
\[
\tan 2\alpha(p^2) \approx t_{2\alpha}(Q) - \frac{2}{(m_A^2 - m_Z^2)c_{2\beta}} \left\{ \tilde{A}_{12}^{\overline{\text{MS}}}(p^2, Q) \right\} + \frac{1}{2} \left[ \tilde{A}_{22}^{\overline{\text{MS}}}(p^2, Q) - \tilde{A}_{11}^{\overline{\text{MS}}}(p^2, Q) - \frac{1}{v_2}\tilde{T}_{22}^{\overline{\text{MS}}}(Q) + \frac{1}{v_1}\tilde{T}_{11}^{\overline{\text{MS}}}(Q) \right] t_{2\alpha}
\] (64)
\(^3\) To see the effect the running tadpoles have in the determination of running parameters, see [19].
where the running $\tan 2\alpha$ is defined as
\[ t_{2\alpha}(Q) = \frac{m_A^2(Q) + m_Z^2(Q)}{m_A^2(Q) - m_Z^2(Q)} t_{2\beta}(Q) . \] (65)

The relation between $\tan 2\alpha(p^2)$ and $\tan 2\alpha(Q)$ in eq. (63) is completed when we give the on-shell definition of the $Z$ and $A$ masses:
\[ m_Z^2 = m_Z^2(Q) + \text{Re} \overline{A}_{ZZ}^{\overline{M}_S}(m_Z^2, Q), \quad m_A^2 = m_A^2(Q) + \text{Re} \overline{A}_{AA}^{\overline{M}_S}(m_A^2, Q), \] (66)
where the tilde over the self energies means that the counterterms have already canceled the divergent terms. The relation between the on-shell definition of $t_\beta$ and the running $t_\beta(Q)$ can be deduced from ref. [14].

### 3.5 Numerical Results

In this section we compare numerically the momentum dependent mixing angle method of diagonalizing coupled scalars with more conventional methods. We have chosen to compare with:

(a) the tree level approximation;

(b) the approximation where we diagonalize the matrix formed by the tree level quantities plus the pieces of the radiative corrections proportional to $m_t^4$ [20, 9]. By doing this we obtain the eigenvalues
\[ \tilde{m}_{H,h}^2 = \frac{1}{2}(m_A^2 + m_Z^2 + \Delta_t) \pm \frac{1}{2} \sqrt{[(m_Z^2 - m_A^2)c_{2\beta} - \Delta_t]^2 + (m_A^2 + m_Z^2)s_{2\beta}^2} , \] (67)
and mixing angle
\[ \tan 2\tilde{\alpha} = \frac{(m_A^2 + m_Z^2)s_{2\beta}}{(m_A^2 - m_Z^2)c_{2\beta} + \Delta_t} . \] (68)

where:
\[ \Delta_t = \frac{3g^2m_t^4}{16\pi^2m_W^2s_{2\beta}} \ln \frac{m_{t_1}^2 m_{t_2}^2}{m_t^4} ; \] (69)

and

(c) the leading logarithms approximation.

The exact formulae for loops involving top and bottom quarks and squarks are given in the appendix (see also ref. [2]), and loops corresponding to the gauge bosons, Higgs bosons, charginos and neutralinos are treated at the leading logarithm approximation. It is useful to evaluate the formulae in the appendix in the limit $M_{SUSY}^2 \gg m_t^2 \gg m_Z^2 \gg m_A^2 \gg m_b^2$ and $\mu = A_U = A_D = 0$, where all the squark soft supersymmetry breaking mass parameters are assumed to be of the order of $M_{SUSY}$. The leading logarithms obtained in this way are in agreement with ref. [21]:

\[ \Sigma_{11}(p^2) = p^2 - m_Z^2 c_\beta^2 - m_A^2 s_\beta^2 - \frac{3g^2}{16\pi^2 m_W^2} \left( \frac{2m_b^4}{c_\beta^2} - m_Z^2 m_b^2 \right) \ln \frac{M_{SUSY}^2}{m_{weak}^2} \]
\[ - \frac{g^2 m_Z^2 c_\beta^2}{32\pi^2 c_W^2} (P_t + P_b) \ln \frac{M_{SUSY}^2}{m_{weak}^2} + O(g^2 m_Z^2) \]
\[
\Sigma_{22}(p^2) = p^2 - m_Z^2 s_\beta^2 - m_A^2 c_\beta^2 - \frac{3g^2}{16\pi^2 m_W^2} \left( \frac{2m_t^4}{s_\beta^2} - m_Z^2 m_t^2 \right) \ln \frac{M_{SUSY}^2}{m_{weak}^2} - \frac{g^2 m_Z^2 s_\beta^2}{32\pi^2 c_W^2} (P_t + P_b) \ln \frac{M_{SUSY}^2}{m_{weak}^2} - \frac{g^2 m_Z^2 s_\beta^2}{32\pi^2 s_\beta^2} (P_t + P_b) \ln \frac{M_{SUSY}^2}{m_{weak}^2} + O(g^2 m_Z^2) \tag{70}
\]

where \(P_t = 1 - 4e_t s_t^2 + 8e_t^2 s_t^4\) and \(P_b = 1 + 4e_b s_b^2 + 8e_b^2 s_b^4\). Note that it also displayed the largest of the non-leading logarithm terms; it is proportional to the momentum \(p^2\) and to the second power of the top quark mass.

The \(hZZ\) coupling in the MSSM relative to the same coupling in the SM is given by the parameter \(\sin(\beta - \alpha)\). In Fig. 1 we plot this parameter as a function of \(\tan\beta\). The upper dotdashed curve correspond to the tree level approximation. The \(\Delta_t\)-improved approximation is in the dotted line [calculated with \(\hat{\alpha}\) defined in eq. (68)].

The leading logarithms approximation is plotted in the lower dotdashed line. The parameter \(\sin(\beta - \alpha)\) calculated with the momentum dependent mixing angle \(\alpha(p^2)\) is plotted in the dashed curves for \(p^2 = m_H^2\) and in the solid curves for \(p^2 = m_H^2\), for the cases: (a) \(A = \mu = 1\) TeV and (b) \(A = -\mu = 1\) TeV, which define the squark mixing.

From Fig. 1 we can learn that the tree level approximation can give completely wrong results. The \(\Delta_t\)-improved and the leading logarithms approximations give results that are quite close to each other, indicating that the \(m_t^4\) terms are the main contributions to the leading logarithms for the chosen parameters of this figure. Both approximations can differ from the results using \(\alpha(p^2)\), specially in the high \(\tan\beta\) region. The parameter calculated with our method shows a strong dependence on the squark mixing parameters, as it can be appreciated from the solid and dashed curves in cases (a) and (b). We also can see that the angle \(\alpha(p^2)\) has a small variation between the two physical external momenta \(p^2 = m_H^2\) and \(p^2 = m_H^2\).

The same kind of graph is in Fig. 2, where we plot \(\sin(\beta - \alpha)\) as a function of \(\tan\beta\). As opposed to the previous figure, here we consider lighter squarks: \(M_Q = M_U = M_D \equiv M_{SUSY} = 200\) GeV, \(A_U = A_D \equiv A = 140\) GeV, and \(\mu = -70\) GeV. The most interesting feature here is that the parameter \(\sin(\beta - \alpha)\) is substantially different in the two different external momenta \(p^2 = m_H^2\) and \(p^2 = m_H^2\) when \(\tan\beta\) is large. In addition, this time not only the tree level calculation gives wrong results, but also the leading logarithms approximation. The fact that squarks are light implies that leading logarithms of particles other than squarks are also important, and this is reflected in the fact that the \(\Delta_t\)-improved approximation is very different from the complete leading logarithm approximation. For the parameters chosen in this figure, we see that the \(\Delta_t\)-improved curve is close to the curves calculated with \(\alpha(p^2)\), but this is an accident as we can see in the next figure.

In Fig. 3 we plot \(\sin(\beta - \alpha)\) as a function of \(m_A\) for a fixed value of \(\tan\beta = 40\) and for light squarks. Here it become evident that any of the three traditional approximations can give a value of \(\sin(\beta - \alpha)\) with an error of 40% or more. Considering that the relevant parameter for the Higgs search is the MSSM is \(\sin^2(\beta - \alpha)\) we see that the error on the cross section can be of the order of 60%!
The Higgs mass calculated with our method was compared to more traditional methods in ref. [23]. In that reference we mention the differences between the Effective Potential method, the Renormalization Group Equations method, and the Diagramatic method. Here we discuss quantitatively these differences through an example given by the choice of parameters in Fig. 4.

The Higgs masses are calculated by finding the zeros of the determinant of the inverse propagator matrix \( \Sigma(p^2) \) as indicated by eq. (38). The non–trivial momentum \( p^2 \) dependence of the matrix elements \( \Sigma_{ij}^A(p^2) \) is in the self energies \( A_{ij}^A(p^2) \). Usually, the momentum squared in the self energies is replaced by a constant. For example, in the effective potential method the momentum squared in the self energies is replaced by zero. Another typical choice is to take \( p^2 \) equal to the tree level mass. In Fig. 4 we have replaced \( p^2 \) by the same constant \( \sqrt{\overline{p}^2} \) in the three self energies and plot as a function of this constant the two Higgs masses \( m_h \) and \( m_H \) calculated after that replacement. In the solid (dashed) line we have \( m_h \) (\( m_H \)) as a function of the squared root of the argument of the self energies.

We see that, for the choice of parameters in Fig. 4, the dependence of the Higgs masses \( m_h \) and \( m_H \) on \( p^2 \) is quite strong, specially for large values of \( p^2 \). The dotted line is the diagonal defined by \( m = \sqrt{\overline{p}^2} \). The intersection of this line with the solid and the dashed curves give us the pole masses \( m_h \) and \( m_H \) calculated with our method. These values are \( m_h = 64.8 \) GeV and \( m_H = 95.4 \) GeV. On the other hand, the intersection of the solid and dashed curves with the vertical line defined by \( \overline{p} = 0 \) corresponds to the masses calculated with the second derivative of the effective potential. These values are \( m_{h}^{\text{eff}} = 60.8 \) GeV and \( m_{H}^{\text{eff}} = 108.4 \) GeV, and they are different from the pole masses. Therefore, it is clear that if we replace the momentum \( p^2 \) in the self energies by a constant, the Higgs masses calculated in that way may depend strongly on that choice. To be complete, we give the value of the Higgs masses calculated in (a) the tree level approximation \( m_{h}^{\text{tree}} = 90.7 \) GeV and \( m_{H}^{\text{tree}} = 100.4 \) GeV, (b) the \( \Delta \tau \)–improved approximation \( m_{h}^{\Delta \tau} = 99.9 \) GeV and \( m_{H}^{\Delta \tau} = 129.3 \) GeV, and (c) the leading logarithms approximation \( m_{h}^{\text{l.l.}} = 100.0 \) GeV and \( m_{H}^{\text{l.l.}} = 133.9 \) GeV, valid for the choice of parameters in Fig. 4. The reason why the Higgs masses calculated with these approximations differs so much from the masses calculated with our method is the large value of the squark mixing: approximations (a) to (c) do not treat appropriately the squark mixing. An approximated formula which treats the squark mixing can be found for example in ref. [22]. As it can be seen above, a better value is obtained with the effective potential, but still, differences can be of 6% to 14%. It could be argued that the effective potential method can be improved by correcting the fact that it is found at zero external momentum, i.e., corrections of the type \( \Delta m^2 = A_{hh}(m_{h,\text{tree}}^2) - A_{hh}(0) \). Nevertheless, this correction implies that all the information coming from the effective potential is canceled out, and we are left with the pure diagramatic method. Therefore, we could have started with the diagramatic method in the first place and forget about the effective potential. The necessity of the above correction disappears in first approximation if \( m_{h}^{\text{tree}} = 0 \), as it was done in ref. [9] (see also discussion in ref. [23]).
4 Conclusions

We have developed a new method of diagonalizing two coupled scalars by introducing a momentum dependent mixing angle $\alpha(p^2)$, where $p$ is the external momentum of the two–point functions. The dependence of the mixing angle on the momentum $p^2$ indicate us that the rotation matrix which diagonalizes the inverse propagator matrix is different whether we are at the pole mass $p^2 = m_h^2$ or $p^2 = m_H^2$ or at any other scale. We have compared this method with the conventional wave function renormalization and with the mixed wave function renormalization In fact, we introduced the momentum dependent mixing angle as a generalization of the previous methods.

We applied this method to the diagonalization of the CP–even Higgs bosons inverse propagator matrix in the MSSM. We used the diagramatic method in an on–shell renormalization scheme, where the tadpoles are exactly zero at one–loop, the masses $m_Z$, $m_W$, and $m_A$ are defined as the pole masses, and $\tan \beta$ is defined through the on–shell definition of the slepton masses $m_{\tilde{e}_L}$ and $m_{\tilde{\nu}_e}$. We calculate the wave function renormalization constants $Z_1$ and $Z_2$ by imposing that the residue of the propagators of the CP–even Higgs bosons are exactly one. We do this by using a formula for $Z_1$ and $Z_2$ valid for any number of loops and, therefore, specially useful when radiative corrections are large. We make explicit the relation between the momentum dependent mixing angle, which includes some effects of higher order loops, to the mixing angle calculated in the exact one–loop perturbative limit. We also make explicit the relation between the momentum dependent mixing angle $\alpha(p^2)$ and the running mixing angle $\alpha(Q)$, where $Q$ is the arbitrary mass scale of the $\overline{MS}$ scheme. We give some numerical results by calculating the parameter $\sin(\beta - \alpha)$, which is the ratio between the $ZZh$ coupling in the MSSM to the same coupling in the SM. We compare this parameter calculated with the momentum dependent mixing angle $\alpha(p^2)$ with (a) the tree level approximation, (b) the $\Delta t$–improved approximation (including only terms proportional to $m_t^4$), and (c) the leading logarithms approximation. We find important numerical differences between the different methods, and they are relevant for the Higgs searches at LEP2 in the region of parameter space where $m_A = \mathcal{O}(m_Z)$.

Finally, we calculate the pole masses of the CP-even Higgs bosons $m_h$ and $m_H$. We show that they are exactly independent of the wave function renormalization constants $Z_1$ and $Z_2$ only if we use the formulas in eq. (41). The Higgs masses are calculated by finding the zeros of the determinant of the inverse propagator matrix [eq. (38)], and this is the direct consequence of the definition of the momentum dependent mixing angle $\alpha(p^2)$. We compare with the Higgs masses calculated in the three approximations described in the above paragraph and we find that for some choices of parameter space there are non–negligible differences between them, and therefore, in those cases our method should be used.

It is worth to mention also that our method can be easily generalized to the diagonalization of more than two coupled scalars. Also the generalization to the diagonalization of coupled fermions or coupled vector bosons is straightforward.
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Appendix

In this appendix we display the exact one-loop formulae we need to compute the inverse propagator matrix for the CP-even Higgs fields, for loops involving top and bottom quarks and squarks. We exhibit separately the contribution from the quarks and from the squarks loops.

To obtain the mass counterterms $\delta m^2_{ij}$ given in eq. (24), the mass counterterm of the $Z$ gauge boson is required. It is given by $\delta m^2_Z = A_{ZZ}(m_Z^2)$ where the contribution from top and bottom quarks is:

$$
\left[A_{ZZ}(m_Z^2)\right]^{tb} = \frac{N_c g^2}{32\pi^2 c_W^2} (m_t^2 B_{0t}^{Ztt} + m_b^2 B_{0t}^{Zbb})
- \frac{N_c g^2}{16\pi^2 c_W^2} (\frac{1}{4} - e_t s_W^2 + 2 e_t^2 s_W^4)(4 B_{22}^{Ztt} - 2 A_t^i + m_Z^2 B_{0t}^{Ztt})
- \frac{N_c g^2}{16\pi^2 c_W^2} (\frac{1}{4} + e_b s_W^2 + 2 e_b^2 s_W^4)(4 B_{22}^{Zbb} - 2 A_b^b + m_Z^2 B_{0t}^{Zbb})
$$

(71)

where we use the notation $B_{22}^{Ztt} \equiv B_{22}(m_Z^2, m_t^2, m_t^2)$, and similarly for the other Veltman’s functions. The contribution to the $Z$ self energy from top and bottom squarks is:

$$
\left[A_{ZZ}(m_Z^2)\right]^{tb} = \frac{N_c g^2}{8\pi^2 c_W^2} \left[
(-\frac{1}{2} c_t^2 + e_t s_W^2)^2 (2 B_{22}^{Ztt} - A_t^i - A_t^i_0) + (\frac{1}{2} c_b^2 + e_b s_W^2)^2 (2 B_{22}^{Zbb} - A_b^b - A_b^b_0)
+ \left(-\frac{1}{2} s_t^2 + e_t s_W^2\right)^2 (2 B_{22}^{Ztt} - A_t^i - A_t^i_0) + \left(\frac{1}{2} s_b^2 + e_b s_W^2\right)^2 (2 B_{22}^{Zbb} - A_b^b - A_b^b_0)
+ \frac{1}{4} s_t c_t^2 (4 B_{22}^{Ztt} - A_t^i - A_t^i_0) + \frac{1}{4} s_b c_b^2 (4 B_{22}^{Zbb} - A_b^b - A_b^b_0)\right].
$$

(72)

Here we use the notation $s_i \equiv \sin \alpha_i$ and $c_i \equiv \cos \alpha_i$ where $\alpha_i$ is the rotation angle necessary to diagonalized the top squark mass matrix. Similar expressions are used for the sbottom mixing angle $\alpha_b$.

The $W$–boson self energy is

$$
\left[A_{WW}(m_W^2)\right]^{tb} = -\frac{N_c g^2}{32\pi^2} \left[4 B_{22}^{Wtb} - A_t^i - A_t^i_0 + (m_t^2 - m_b^2) B_{0t}^{Wtb}\right]
$$

(73)

for the quarks contribution, and

$$
\left[A_{WW}(m_W^2)\right]^{tb} = \frac{N_c g^2}{32\pi^2} \left[c_t^2 s_t^2 (4 B_{22}^{Wtb} - A_t^i - A_t^i_0) + c_t^2 s_b^2 (4 B_{22}^{Wtb} - A_t^i - A_t^i_0)
+ \frac{1}{4} s_t c_t^2 (4 B_{22}^{Wtb} - A_t^i - A_t^i_0) + \frac{1}{4} s_b c_b^2 (4 B_{22}^{Wtb} - A_t^i - A_t^i_0)\right]
$$

(74)
for the squark contributions. xxx The third mass counterterm we need to compute is \( \delta(m^2_1/s_\beta c_\beta) \) and it is related to the CP-odd self energy and the tadpoles through eqs. (32) and (34). Thus we need the following quantity:

\[
A_{AA}(m^2_A) - s_\beta^2 \frac{\delta t_1}{v_1} - c_\beta^2 \frac{\delta t_2}{v_2} = - \frac{N_c g^2 m^2_A}{32 \pi^2 m^2_W} \left( m_t^2 \left( \frac{t_\beta}{t^2_\beta} B_0^{tt} + m_b^2 t_\beta^2 B_0^{bb} \right) \right)
\]

(75)

for top and bottom quark loops, and:

\[
\begin{align*}
[A_{AA}(m^2_A) - s_\beta^2 \frac{\delta t_1}{v_1} - c_\beta^2 \frac{\delta t_2}{v_2}] & = - \frac{N_c g^2}{32 \pi^2 m^2_W} \left[ m_t^2 (\mu + A_U/t_\beta)^2 B_0^{\tilde{t}_1\tilde{t}_2} + m_t (\mu t_\beta - A_U/t_\beta^2) s_\beta t_\beta (A^t_0 - A^t_2) \\
&+ m_b^2 (\mu + A_D t_\beta)^2 B_0^{\tilde{b}_1\tilde{b}_2} + m_b (\mu / t_\beta - A_D t_\beta^2) s_\beta c_\beta (A^b_0 - A^b_2) \right] \\
[A_{AA}(p^2)] & = 0.
\end{align*}
\]

(76)

for top and bottom squarks loops.

Finally, we need the two point functions \( A_1^\chi \). In the case of the self energies, we add the appropriate tadpole as indicated below. The contribution due to the top and bottom quarks is very simple:

\[
\begin{align*}
[A_1^\chi(p^2) - \frac{\delta t_1}{v_1}]^t_b & = \frac{N_c g^2 m^2_\beta}{32 \pi^2 m^2_W c_\beta^2} (4 m^2_\beta - p^2) B_0^{p t p} \\
[A_2^\chi(p^2) - \frac{\delta t_2}{v_2}]^t_b & = \frac{N_c g^2 m^2_1}{32 \pi^2 m^2_W s_\beta^2} (4 m^2_1 - p^2) B_0^{p t t} \\
[A_{12}^\chi(p^2)]^t_b & = 0.
\end{align*}
\]

(77)

where we notice that \( \chi_1 \) couples only to top quarks, and \( \chi_2 \) couples only to bottom quarks, implying that the mixing is zero.

The contribution due to top and bottom squarks to the \( \chi_1 \chi_1 \) self energy minus the \( \chi_1 \) tadpole is:

\[
\begin{align*}
[A_{11}^\chi(p^2) - \frac{\delta t_1}{v_1}]^t_b & = - \frac{N_c g^2}{16 \pi^2 m^2_W} \left\{ m^2_2 c_\beta \left(- \frac{1}{2} c_\beta^2 + e_\beta s_\beta^2 c_2 \right) + \frac{m_t \mu}{2 s_\beta} s_2 t \right\} B_0^{\tilde{t}_1\tilde{t}_2} \\
&+ \left[ m^2_2 c_\beta \left( \frac{1}{2} c_\beta^2 + e_\beta s_\beta^2 c_2 \right) + \frac{m_t \mu}{2 s_\beta} s_2 t \right] B_0^{\tilde{t}_2\tilde{t}_2} \\
&+ 2 \left[ m^2_2 c_\beta \left( \frac{1}{4} - e_\beta s_\beta \right) s_2 t + \frac{m_t \mu}{4 s_\beta} c_2 t \right] B_0^{\tilde{t}_1\tilde{t}_2} + \frac{m_t \mu}{4 s_\beta} c_2 t (A^t_0 - A^t_2) \\
&+ \left[ m^2_2 c_\beta \left( \frac{1}{2} c_\beta^2 + e_\beta s_\beta^2 c_2 \right) + \frac{m_t \mu}{2 s_\beta} s_2 t \right] B_0^{\tilde{b}_1\tilde{b}_1} \\
&+ \left[ m^2_2 c_\beta \left( \frac{1}{2} c_\beta^2 + e_\beta s_\beta^2 c_2 \right) + \frac{m_t \mu}{2 s_\beta} s_2 t \right] B_0^{\tilde{b}_2\tilde{b}_2} \\
&+ 2 \left[ m^2_2 c_\beta \left( \frac{1}{4} + e_\beta s_\beta^2 \right) s_2 t + \frac{m_t \mu}{2 s_\beta} c_2 t \right] B_0^{\tilde{b}_1\tilde{b}_2} - \frac{m_t \mu}{4 s_\beta} s_2 t (A^b_0 - A^b_2) \right\},
\end{align*}
\]

(78)

where we use the notation \( s_2 t \equiv \sin(2 \alpha_t), c_2 t \equiv \cos(2 \alpha_t) \), and similarly for \( \alpha_b \). Similarly, the contribution to the \( \chi_2 \chi_2 \) self energy minus the \( \chi_2 \) tadpole, due to top and bottom
Finally, the $\chi_1\chi_2$ mixing due to squarks is given by:

$$
\frac{\chi}{v_2} = \left[\left[A_{12}^{\chi}(p^2) - \frac{\delta t_2}{v_2}\right] \tilde{b}\right] = 
- \left[\frac{N_c g^2}{16\pi^2m_W^2}\right] \left\{- \left[m_Z^2 c_\beta (-\frac{1}{2}c_t^2 + e_t s_W^2 c_{2t}) + \frac{m_A U}{2s_\beta} s_{2t} + \frac{m_t^2}{s_\beta}\right] B_0^{\tilde{t}_1\tilde{t}_1} + \left[m_Z^2 s_\beta (-\frac{1}{2}c_t^2 + e_t s_W^2 c_{2t}) + \frac{m_A U}{2s_\beta} s_{2t} + \frac{m_t^2}{s_\beta}\right] B_0^{\tilde{t}_2\tilde{t}_2} + 2 \left[m_Z^2 s_\beta (-\frac{1}{2}c_t^2 + e_t s_W^2 c_{2t}) + \frac{m_A U}{2s_\beta} s_{2t} + \frac{m_t^2}{s_\beta}\right] B_0^{\tilde{b}\tilde{b}_11} + \left[m_Z^2 s_\beta (-\frac{1}{2}c_t^2 + e_t s_W^2 c_{2t}) + \frac{m_A U}{2s_\beta} s_{2t} + \frac{m_t^2}{s_\beta}\right] B_0^{\tilde{b}_2\tilde{b}_2} + 2 \left[m_Z^2 s_\beta (-\frac{1}{2}c_t^2 + e_t s_W^2 c_{2t}) + \frac{m_A U}{2s_\beta} s_{2t} + \frac{m_t^2}{s_\beta}\right] B_0^{\tilde{b}_1\tilde{b}_1} + \left[m_Z^2 s_\beta (-\frac{1}{2}c_t^2 + e_t s_W^2 c_{2t}) + \frac{m_A U}{2s_\beta} s_{2t} + \frac{m_t^2}{s_\beta}\right] B_0^{\tilde{b}_2\tilde{b}_2}\right\}.
\tag{79}
$$

And this completes the formulas we need to calculate the momentum dependent mixing angle $\alpha(p^2)$ and the Higgs masses $m_h$ and $m_H$ in the approximation where only top and bottom quarks and squarks are considered in the loops.
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**Figure Captions**

**Figure 1:** Coupling $hZZ$ relative to the SM coupling $H_{SMZZ}$, $\sin(\beta - \alpha)$, as a function of tan$\beta$. It is plotted the tree level (upper dot-dashes), the $\Delta_t$–improved tree level (dots), the leading logarithms (lower dot-dashes), and the parameter calculated with the momentum dependent mixing angle $\alpha(p^2)$. In the last case, we use the mixing angle evaluated at the heavy Higgs mass scale $\alpha(m_H)$ (dashes) and the light Higgs mass scale $\alpha(m_h)$ (solid), for two different choices of squark mixing: (a) $A = \mu = 1$ TeV and (b) $A = -\mu = 1$ TeV.

**Figure 2:** Coupling $hZZ$ relative to the SM coupling $H_{SMZZ}$, $\sin(\beta - \alpha)$, as a function of tan$\beta$. It is plotted the tree level (upper dot-dashes), the $\Delta_t$–improved tree level (dots), the leading logarithms (lower dot-dashes), and the parameter calculated with the momentum dependent mixing angle $\alpha(p^2)$. In the last case, we use the mixing angle evaluated at the heavy Higgs mass scale $\alpha(m_H)$ (dashes) and the light Higgs mass scale $\alpha(m_h)$ (solid). We take $m_A = 100$ GeV, $M_{SUSY} = 200$ GeV, $A = 140$ GeV, and $\mu = -70$ GeV.

**Figure 3:** Coupling $hZZ$ relative to the SM coupling $H_{SMZZ}$, $\sin(\beta - \alpha)$, as a function of $m_A$. It is plotted the tree level (upper dot-dashes), the $\Delta_t$–improved tree level (dots), the leading logarithms (lower dot-dashes), and the parameter calculated with the momentum dependent mixing angle $\alpha(p^2)$. In the last case, we use the mixing angle evaluated at the heavy Higgs mass scale $\alpha(m_H)$ (dashes) and the light Higgs mass scale $\alpha(m_h)$ (solid), for a squark mixing given by $A = \mu = 150$ GeV.

**Figure 4:** Neutral Higgs masses $m_h$ (solid) and $m_H$ (dashes) as a function of the squared root of the argument $\overline{p}^2$ of the self energies. We take tan$\beta = 50$, $m_A = 100$
GeV, $M_Q = M_U = M_D \equiv M_{SUSY} = 1$ TeV, $A_U = A_D \equiv A = 1$ TeV, and $\mu = -2$ TeV. The dotted line is the diagonal where $m = \sqrt{p^2}$, and the intersection of this diagonal with the solid (dashed) curve gives us the pole mass $m_h$ ($m_H$) in our approach. The intersection of the solid (dashed) curve with the vertical line defined by $p^2 = 0$ correspond to the Higgs mass $m_h$ ($m_H$) calculated with the second derivative of the effective potential.
Fig. 1

$m_A = 100$ GeV

$m_t = 176$ GeV

(a) $M_{\text{SUSY}} = 1$ TeV

(b) $A = \mu = 1$ TeV

$\alpha(m_H)$ (solid)

$\alpha(m_W)$ (dashes)

$\Delta$ improved

Leading Logarithms

Tree level
