Entropic destruction of heavy quarkonium in a rotating hot and dense medium from holography

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Previous studies have indicated that the peak of the quarkonium entropy at the deconfinement transition can be related to the entropic force which would induce the dissociation of heavy quarkonium. In this paper, we study the entropic force in a rotating hot and dense medium using AdS/CFT correspondence. It turns out that the inclusion of angular velocity increases the entropic force thus enhancing quarkonium dissociation, while chemical potential has the same effect. Furthermore, the results imply that the quarkonium dissociates easier in rotating medium compared to static case.

PACS numbers: 11.25.Tq, 11.15.Tk, 11.25-w

I. INTRODUCTION

In relativistic heavy ion collisions at the relativistic heavy ion collider (RHIC) and large hadron collider (LHC), a hot and dense, strongly interacting medium named quark-gluon plasma (QGP) has been created [1–3]. One of the main experimental signatures of QGP formation is the dissociation of heavy quarkonium [4]. They are expected to be created in the early stages of the collisions and give us significant information about the evolving of QGP. Previous research has indicated that the heavy quarkonium is suppressed due to the Debye screening induced by the high density of color charges in QGP. But recent studies of charmonium (heavy quarks of charm and anticharm) show a puzzle: the charmonium suppression observed at RHIC (lower energy density) is stronger than at LHC (larger energy density) [5,6]. Obviously, this contradicts the Debye screening scenario [4] as well as the thermal activation through the impact of gluons [7,8]. Some scholars think that one of the reasons may be the recombination of the produced charm quarks into charmonium [9,10]. However, recent research suggested [11] that the puzzle on the suppression of the charmonium can be a consequence of the nature of deconfinement. This argument is based on the lattice QCD results [12–15] indicating a large amount of entropy $S$ associated with the heavy quark-antiquark pair ($Q\bar{Q}$) around the crossover region of QGP. In particular, this entropy which grows with the inter-distance $L$ between $Q\bar{Q}$ leads to the emergent entropic force [11]

$$F = T \frac{\partial S}{\partial L},$$

where $T$ denotes the temperature of the plasma. It has been shown that the repulsive entropic force is responsible for dissociating the quarkonium and then would be a solution of the puzzle.

The AdS/CFT correspondence [16–18] or more general gauge/gravity duality provides a helpful tool to explore various properties of QGP [19,20]. With the method, K. Hashimoto and D. Kharzeev firstly calculated the entropic force associated with $Q\bar{Q}$ for $N=4$ SYM plasma [21]. It was found that the entropy increases with the inter-quark distance and the peak of the entropy emerges when the U-shaped string stretched between the $Q\bar{Q}$ touches the horizon of the black hole. Subsequently, there have been many attempts to address the entropic force in this direction [22–31].

In this paper, we extend the studies of [21] to the case of rotating plasma with chemical potential. In particular, we will employ the AdS-Reissner Nordstrom (AdS-RN) black hole [32,33] and extend it to a rotating case with planar horizon. Because there’s the strong possibility that the QGP produced in (typical) noncentral heavy ion collisions may carry a nonzero angular momentum on the order of $10^4-10^5\hbar$ with local angular velocity in the range of 0.01-0.1 GeV [34,35]. Although the major part of this angular momentum will be taken away by the spectator nucleons, some amount of angular momentum remains in the QGP [36,37] and thus may give rise to significant observable effects. On
the other hand, the QGP produced in heavy ion collisions is assumed to carry a finite, albeit small, baryon number density, e.g., the Beam Energy Scan program at RHIC covers the beam energies of $\sqrt{S_{NN}} = 200, 62.4, 54.4, 39, 27$ GeV...corresponding to a region of the chemical potential $0.025 \leq \mu_B \leq 0.72$GeV \[42\]. In this regard, it would be interesting to study the entropic force at finite temperature and density under rotation.

This paper is organized as follows: In the next section, we briefly review the AdS-RN background and extend it to rotating case. In section 3, we investigate the behavior of the entropic force in this background and explore how the deconfinement transition can be viewed as entropic self-destruction. In section 4, we summarize our results and provide a concluding discussion.

II. BACKGROUND GEOMETRY

From the AdS/CFT correspondence, $\mathcal{N} = 4$ SYM theory with non-zero chemical potential can be obtained by making the black hole in the holographic dimension charged. The corresponding metric is the AdS-RN black hole \[32, 33\]

$$ds^2 = -\frac{r^2}{R^2} f(r) dt^2 + \frac{r^2}{R^2} dx^2 + \frac{R^2}{r^2 f(r)} dr^2,$$

(2)

with

$$f(r) = 1 - (1 + Q^2)\left(\frac{r_h}{r}\right)^4 + Q^2 \left(\frac{r_h}{r}\right)^6,$$

(3)

where $R$ is the curvature radius (for convenience, we set $R = 1$ for later discussion), $Q$ denotes the charge of black hole, $r$ refers to the radial coordinate with $r = r_h$ the horizon, defined by $f(r_h) = 0$. The asymptotic boundary is at $r = \infty$. The string tension $\frac{1}{2\pi\alpha'}$ is related to the 't Hooft coupling constant $\lambda$ by $\frac{1}{\alpha'} = \sqrt{\lambda}$.

Following \[13, 15\], one can extend (2) to a rotating case from the static configuration through a local Lorentz boost in the $t - \phi$ plane

$$t \to \gamma(t + \omega l^2 \phi), \quad \phi \to \gamma(\phi + \omega l^2 t),$$

(4)

with

$$\gamma = \frac{1}{\sqrt{1 - \omega^2 l^2}},$$

(5)

where $\phi$ is the angular coordinate describing the rotation. $\omega$ represents the angular velocity. $l$ is the radius of the rotating axis. In this work we will focus on the qualitative results, then we simply set $l = 1 GeV^{-1}$, as follows from \[13\].

Given that, the rotating case of (2) is

$$ds^2 = -p(r) dt^2 + r^2 (dx^2 + dy^2) + \frac{1}{r^2 f(r)} dr^2 + q(r) (d\phi + m(r) dt)^2,$$

(6)

with

$$p(r) = \frac{f(r) r^2 (1 - \omega^2)}{1 - f(r) \omega^2}, \quad q(r) = (1 - f(r) \omega^2) r^2 \gamma^2, \quad m(r) = \frac{\omega (1 - f(r))}{1 - f(r) \omega^2}.$$

(7)

The temperature of the black hole reads

$$T = \frac{r_h}{\pi} \sqrt{1 - \omega^2 (1 - \frac{Q^2}{2})}.$$

(8)

where $Q$ is in the range $0 \leq Q \leq \sqrt{2}$.

The chemical potential reads \[15\]

$$\mu = \sqrt{3} Q r_h \sqrt{1 - \omega^2}.$$

(9)

Notice that the chemical potential implemented here is not the baryon chemical potential of QCD but a chemical potential conjugated to the R-charge associated with SYM. However, in such a context it could serve as a simple way of introducing finite density effect into the system \[16\].
III. ENTRISTIC FORCE IN THE ROTATING BACKGROUND

We now proceed to study the entropic force in the rotating AdS-RN black hole following the prescription of [21]. The Nambu-Goto action is

\[ S_{NG} = -\frac{1}{2\pi\alpha'} \int d\tau d\sigma \sqrt{-\det g_{\alpha\beta}}, \]

with

\[ g_{\alpha\beta} = g_{\mu\nu} \frac{\partial X^\mu}{\partial \sigma^\alpha} \frac{\partial X^\nu}{\partial \sigma^\beta}, \]

where \( g_{\alpha\beta} \) represents the induced metric and parameterized by \((\tau, \sigma)\) on the string world-sheet, \( g_{\mu\nu} \) is the metric, \( X^\mu \) is the target space coordinate.

Since the transformation (4) is a boost in the \( t-\phi \) plane, one may consider the \( Q \bar{Q} \) pair located at \( x-y \) plane, e.g., one considers the \( Q \bar{Q} \) pair to be aligned in the \( x \) direction, \( t = \tau, \ x = \sigma, \ y = 0, \ \phi = 0, \ r = r(\sigma). \)

Based on these assumptions, (10) becomes

\[ S_{NG} = -\frac{T}{2\pi\alpha'} \int_{-L/2}^{L/2} dx \sqrt{A(r) + B(r) \left( \frac{dr}{d\sigma} \right)^2}, \]

with

\[ A(r) = (p(r) - q(r)m^2(r))r^2, \quad B(r) = \frac{p(r) - q(r)m^2(r)}{r^2 f(r)}, \]

where the \( Q \bar{Q} \) are set at \( x = -L/2 \) and \( x = L/2 \), respectively.

Since (13) does not depend on \( \sigma \) explicitly, one obtains a conserved quantity,

\[ L - \frac{\partial L}{\partial \dot{r}} \dot{r} = \text{constant}. \]

Imposing the boundary condition (the deepest point of the U-shaped string)

\[ \dot{r} = 0, \quad r = r_c \quad (r_h < r_c), \]

one gets

\[ \frac{dr}{d\sigma} = \sqrt{\frac{A^2(r) - A(r)A(r_c)}{A(r_c)B(r)}}, \]

with \( A(r_c) = A(r)|_{r=r_c} \).

Integrating (17), the inter-distance of \( Q \bar{Q} \) is found to be

\[ L = 2 \int_{r_c}^{r_h} dr \sqrt{\frac{A(r_c)B(r)}{A^2(r) - A(r)A(r_c)}}, \]

On the other hand, the entropy is given by

\[ S = -\frac{\partial F}{\partial T}, \]

where \( F \) denotes the free energy of \( Q \bar{Q} \). Notice that \( F \) has been studied at zero temperature [47] and finite temperature [48, 49] from holography. Generally, there are two situations:

1. If \( L > \frac{c}{T} \) (where \( c \) denotes the maximum value of \( LT \)), some new configurations need to be taken into account and thus there are several alternatives for \( F \) [50, 51]. If one selects a configuration of two disconnected trailing drag strings [52, 53], the corresponding free energy can be written as

\[ F^{(1)} = \frac{1}{\pi\alpha'} \int_{r_h}^{r_c} dr, \]
leads to

\[ S^{(1)} = \sqrt{\lambda} \theta(L - \frac{c}{T}), \]

where \( \theta(L - \frac{c}{T}) \) is the Heaviside step function.

2. If \( x < \frac{c}{T} \), the fundamental string is connected. Then \( F \) can be obtained from the on-shell action of the fundamental string in the dual geometry,

\[ F^{(2)} = \frac{1}{\pi \alpha'} \int_{r_c}^{\infty} dr \sqrt{\frac{A(r)B(r)}{A(r) - A(r_c)}}. \]

Then from (8), (19) and (22), one gets

\[ S^{(2)} = -\frac{\partial F^{(2)}}{\partial T} = -\frac{1}{2\alpha' \sqrt{1 - \omega^2(1 - Q^2/L^2)}} \int_{r_c}^{\infty} dr \frac{[A'(r)B(r) + A(r)B'(r)][A(r) - A(r_c)] - A(r)B(r)[A'(r) - A'(r_c)]}{\sqrt{A(r)B(r)[A(r) - A(r_c)]}}. \]

with

\[
\begin{align*}
A'(r) &= r^2(p' - q' m^2 - 2qmm'), \\
B'(r) &= \frac{(p' - q' m^2 - 2qmm')f - (p - qm^2)f'}{r^2 f^2}, \\
p' &= \frac{r^2(1 - \omega^2)(f'(1 - f \omega^2) + \omega^2 f f')}{(1 - f \omega^2)^2}, \\
q' &= -r^2 \gamma^2 \omega^2 f', \\
m' &= \frac{\omega(-f'(1 - f \omega^2) + \omega^2 (1 - f)f')}{(1 - f \omega^2)^2}, \\
f' &= -4(1 + Q^2)r^3 \gamma^2 - 6Q^2 r^5 \gamma^6, 
\end{align*}
\]

where \( A'(r_c) = A'(r)|_{r=r_c} \), \( q = q(r), \) \( m = m(r), \) \( f = f(r), \) and the derivatives are with respect to \( r_h \). One can check that by setting \( \omega = 0 \) and \( \mu = 0 \) (or \( Q = 0 \)) in (23), the result of SYM [21] will be recovered.

Let’s discuss results. First, we analyze how angular velocity affects the inter-distance of \( Q\bar{Q} \). To this end, we plot \( LT \) as a function of \( \varepsilon \equiv r_h/r_c \) for various values of \( \omega \) in fig.1, where the left panel is for \( \mu = 0.025\text{GeV} \) while the right \( \mu = 0.1\text{GeV} \) (notice that in all the plots \( \omega \) and \( \mu \) are in unit GeV which are not mentioned for the brevity of the notation). In both panels from top to bottom \( \omega = 0, 0.1, 0.3, 0.5\text{GeV} \), respectively. From each panel, one can see that by increasing \( \omega \), \( LT \) decreases. Namely, the inclusion of angular velocity reduces the inter-distance.

Moreover, to see how angular velocity modifies the entropic force, we plot \( S^{(2)}/\sqrt{\lambda} \) versus \( LT \) for various cases in fig.2. Similarly, one chooses \( \mu = 0.025, 0.1\text{GeV} \) and \( \omega = 0, 0.1, 0.3, 0.5\text{GeV} \) in calculations. From these figures, one sees that increasing \( \omega \) leads to larger entropy at small distances. It is known that the entropic force (see Eq. (1)) depends

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**FIG. 1:** \( LT \) versus \( \varepsilon \) for different \( \omega \). Left: \( \mu = 0.025\text{GeV} \); Right: \( \mu = 0.1\text{GeV} \). In both panels from top to bottom \( \omega = 0, 0.1, 0.3, 0.5\text{GeV} \), respectively.
FIG. 2: $S^{(2)}/\sqrt{\lambda}$ versus $LT$ for different $\omega$. Left: $\mu = 0.025$ GeV; Right: $\mu = 0.1$ GeV. In both panels from right to left, $\omega = 0, 0.1, 0.3$ GeV, respectively.

FIG. 3: Left: $LT$ versus $\varepsilon$ for different $\mu$, from top to bottom $\mu = 0.025, 0.1, 0.3$ GeV, respectively. Right: $S^{(2)}/\sqrt{\lambda}$ versus $LT$ for different $\mu$, from right to left, $\mu = 0.025, 0.1, 0.3$ GeV, respectively. In both cases we take $\omega = 0.1$ GeV.

on the growth of the entropy with the inter-distance and is responsible for dissociating the quarkonium. Therefore, one can draw the conclusion that the inclusion of angular velocity increases the entropic force thus enhancing the quarkonium dissociation, in agreement with [31].

Also, one can analyze the chemical potential dependence of the entropic force. For this purpose, we plot $LT$ versus $\varepsilon$ and $S^{(2)}/\sqrt{\lambda}$ versus $LT$ for different values of $\mu$ in fig.3. From these figures, one finds the presence of chemical potential also reduces the inter-distance and enhances the entropic force, consistent with [23]. The physical significance of the results will be discussed in the final section.

IV. CONCLUSION

Recent studies have shown [21] that the peak of the quarkonium entropy at the deconfinement transition can be related to the entropic force which can destruct the heavy quarkonium. In this paper, we extended the studies of [21] to the case of rotating medium with chemical potential using AdS/CFT correspondence. It is shown that the inclusion of angular velocity increases the entropic force thus enhancing the quarkonium dissociation, while chemical potential has the same effect. Furthermore, the results imply that quarkonium dissociates easier in rotating medium compared to static case.

Interestingly, the entropic force of a moving heavy quarkonium has been studied in [22] and the results show that the entropic force destroys the moving quarkonium easier than the static case. Since motion is relative, their results can be understood as: quarkonium dissociates easier in moving medium compared to static case. If compare their results with ours, one may infer that translation and rotation have the same effect on quarkonium dissociation. In
particular, quarkonium dissociates easier in moving or rotating medium compared to static case. However, there are some inadequacies in this research, e.g., the model we employed here is not a consistent model. Considering the entropic force in some consistent models, e.g., [55–60] would be instructive. On the other hand, the entropic force mechanism applies only to charmonium, but hardly applies to bottomonium (the mass of $c$ quark is about 1.27 GeV, while $b$ quark 4.2 GeV). It was argued [11] that most of the bottomonium states have smaller sizes, which are much less influenced by the entropic force.

Finally, it’s worth noting that the rotating QGP can also be described by means of five-dimensional Kerr-AdS black hole [61]. Then one can study the entropic force in that rotating frame as well. It will be left as a further study.

V. ACKNOWLEDGMENTS

This work is supported by the NSFC under Grant Nos. 11805052, 12147219.

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