Hierarchical Distributed EV Charging Scheduling in Distribution Grids

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Abstract—In this paper, a hierarchical distributed method consisting of two iterative procedures is proposed for optimal electric vehicle charging scheduling (EVCS) in the distribution grids. In the proposed method, the distribution system operator (DSO) aims at reducing the grid loss while satisfying the power flow constraints. This is achieved by a consensus-based iterative procedure with the EV aggregators (Aggs) located in the grid buses. The goal of aggregators, which are equipped with the battery energy storage (BES), is to reduce their electricity cost by optimal control of BES and EVs. As Aggs’ optimization problem increases dimensionally by increasing the number of EVs, they solved their problem through another iterative procedure with their customers. This procedure is implementable by exploiting the mathematical properties of the problem and rewriting Aggs’ optimization problem as the sharing problem, which is solved efficiently by the alternating direction method of multipliers (ADMM). To validate the performance, the proposed method is applied to IEEE-13 bus system.

Index Terms—Distributed optimization, distribution grids, EV charging scheduling, hierarchical ADMM.

I. INTRODUCTION

Transportation sector consumes a significant percentage of energy, and it has a considerable contribution to the air pollution and greenhouse gas emission. Over the last decade, it has been shown that electric vehicles (EVs) are a promising technology to reduce transportation’s dependency on fossil fuels. However, due to EVs’ electrical energy demand, they introduce new challenges to the electricity sector. EV charging load demand, in high penetration scenarios which is feasible in the near future, may lead to stability, quality, and economy issues in power grids. According to the EV load characteristics, it is considered as a controllable load which its adverse effects can be mitigated through a demand management strategy. Nonetheless, due to the uncertainty in EV load demand [1]-[2] and scalability issue in the case of high EV penetration [3], EV load management is challenging. In this paper, the scalability issue is addressed by introducing a hierarchical and fully distributed EV charging scheduling (EVCS).

There is a rich body of literature proposing either centralized or distributed EVCS methods. In centralized approaches [4]-[5], the distribution system operator (DSO) receives (or predicts) the data relating to arrival time, departure time, and required energy of each individual EV, and it coordinates their charging load considering the defined objective function. Centralized methods, however, suffer from curse of dimensionality issue if DSO has to deal with a large population of EVs. In addition, centralized methods can not preserve the EV owners’ privacy, as they have to communicate their sensitive information with DSO. To tackle these issues, researchers propose distributed methods in which DSO solves EVCS problem through an iterative procedure with EV aggregators (Aggs) [6], EVs [7], or both [8]-[9]. In the first case, it is Agg who directly receives EVs’ information and is in contact with DSO. In the second case, DSO executes the iterative procedure directly with EVs, so they do not need to share their sensitive information with any entity. In the last case, DSO communicates with Aggs, and each Agg communicates with its EVs, and no sensitive information is shared with other entities. Obviously, the last case is more scalable, and its structure has more flexibility from different entities’ objective function perspective. It is worthwhile to mention that among the distributed methods, some of them do not consider the power grids model, while others take the power flow constraints into consideration [3]-[10]. In this paper, the focus is only on the second group.

Among the proposed distributed methods, the authors in [6] use alternating direction method of multipliers (ADMM) to solve EVCS problem. The authors in [7] use a novel shrunk primal-dual subgradient method for valley-filling problem. The authors in [8] benefit from [11] to solve Agg’s problem by a hierarchical and distributed method. In [9], the authors use Frank-Wolfe method to make the EVCS problem scalable, where the optimization problem is formulated as a linear program. Nevertheless, the proposed methods are developed either based on strict assumptions or for specific objective functions. To address those issues, we further extend our previous work [3] and propose a fully distributed EVCS by the consideration of power flow constraints. In our method which is based on multi-agents systems, DSO, Aggs, and EVs solve their objective function locally through a hierarchical iterative communication procedure implemented by ADMM.

The paper is structured as follows: in Section II, the power flow constraints as well as EV and battery energy storage (BES) models are introduced; in Section III, the EVCS problem is formulated, and it is solved by our hierarchical distributed method based on ADMM; numerical simulation results of the proposed EVCS are shown in Section IV; and the paper is concluded in Section V.

II. SYSTEM MODEL DESCRIPTION

A. Distribution Grid Model

In this paper, we consider the distribution grid modeled as a connected graph which is shown by $G = (\mathbb{N}_b, \zeta)$, where $\mathbb{N}_b$...
denotes the set of the grid buses, and ζ denotes the set of the lines. we use \( y_n := (y_n(t), y_n(t + 1), \ldots, y_n(t + N - 1))^T \), where \( N \in \mathbb{N} \) is the time horizon, for all the variables. If \((i, j) \in \zeta\), a line connects bus \( i \) to bus \( j \), which its impedance and current are shown by \( z_{ij} = r_{ij} + j \omega_{ij}, \) where \( z_{ij} \in \mathbb{Z}^N \), and \( i, j \), respectively. The apparent power of the line \((i, j) \in \zeta\) is shown by \( S_{ij} \), \( S_{ij} \in \mathbb{Z}^N \), and it is calculated by \( S_{ij} = P_{ij} + jQ_{ij} \), where \( P_{ij} \) and \( Q_{ij} \) are the active and reactive powers, respectively. Also, the voltage, active power, reactive power, and apparent power at bus \( i \) are shown by \( v_i \), \( P_i \), \( Q_i \), and \( S_i \), respectively. To show the relations between bus voltages, active powers, reactive powers, line currents and impedances, DistFlow model [13] is assumed (Fig. 1) for the distribution grid as follows:

\[
\begin{align*}
\Delta v_i &= v_i - v_j = z_{ij}i_{ij} \\
S_{ij} &= v_iI_{ij}^* \\
S_j &= S_{ij} - z_{ij}|i_{ij}|^2 - \sum_{(j,k) \in \zeta} S_{jk}
\end{align*}
\]  

Substituting (1a) and (1b) in (1c), the following equations are obtained,

\[
\begin{align*}
P_j &= P_{ij} - r_{ij}I_{ij} - \sum_{(j,k) \in \zeta} P_{jk} \\
Q_j &= Q_{ij} - x_{ij}I_{ij} - \sum_{(j,k) \in \zeta} Q_{jk} \\
V_i - V_j &= 2(r_{ij}P_{ij} + x_{ij}Q_{ij}) - (r_{ij}^2 + x_{ij}^2)I_{ij} \\
V_iI_{ij} &= P_{ij}^2 + Q_{ij}^2,
\end{align*}
\]

where \( V_i = |v_i|^2 \) and \( I_i = |i|^2 \). The voltage at the root node \((v_1)\) should be equal to a constant value, assuming the distribution grid is connected to an infinite bus, while the other voltages can vary within a limited range. This is shown by:

\[
\begin{align*}
y &\leq v_i \leq \bar{v}, \quad i \in \mathbb{N}_b \setminus \{1\} \\
v_1 &= v_{ref}.
\end{align*}
\]

\[\text{Fig. 1. DistFlow model of the distribution line.}\]

B. EV Charging Model

The set of EVs supplied by \( Agg_j \) and their number are shown by \( \mathbb{N}_{j, ev} \) and \( N_{j, ev} \), respectively. Also, \( EV_{j,i} \) stands for \( r_{th} \) EV supplied by \( Agg_j \). It is assumed that each \( EV_{j,i} \) is located in either a commercial or a residential building. We use \( EVB_{j,i} \) for each \( EV_{j,i} \) and its corresponding building, which is modeled as a discrete-time linear system as follows [3]:

\[
\begin{align*}
e_{j,i}^{ev}(t + 1) &= e_{j,i}^{ev}(t) + T_h r_{j,i}^{ev} e_{j,i}^{ev}(t) \\
e_{j,i}^{ev}(t) &= p_{j,i}^{ev}(t) + p_{j,i}^{uc}(t),
\end{align*}
\]

where \( e_{j,i}^{ev} \), \( p_{j,i}^{ev} \), \( p_{j,i}^{uc} \) are the energy stored in EV battery at time \( t \), the non-EV and uncontrollable active load demand minus the power generated by the solar panel of the building, \( T_h \) is 0.5 in this paper which corresponds to 30 min, and \( p_{j,i}^{uc}(t) \) is the control variable which is determined by the EVCS. As we assume that each \( EVB_{j,i} \) is provided by a solar panel and has an EV charger with vehicle-to-grid (V2G) capability, it may supply power to the grid.

The constraints on the EV charging/discharging, relating to the charger power rating and the EV battery capacity, are:

\[
\begin{align*}
P_{j,i}^{ev} \leq p_{j,i}^{ev} \leq P_{j,i}^{ev} \\
C_{j,i}^{ev} \leq C_{j,i}^{ev} \leq C_{j,i}^{ev}
\end{align*}
\]

where \( C_{j,i}^{ev}(t) \), \( \bar{C}_{j,i}^{ev}(t) \) are the EV battery time-varying constraints which are defined as follows; if \( EV_{j,i} \) is:

\[
\text{• plugged in } EVB_{j,i}, \quad C_{j,i}^{ev}(t) = \bar{C}_{j,i}^{ev}(t) = 0.
\]

\[
\text{• plugged in } EVB_{j,i}, \text{ but it is in idle mode, } C_{j,i}^{ev}(t) = 0 \quad & \quad \text{where } C_{j,i}^{ev} \in \mathbb{R} \text{ is the maximum EV battery energy capacity.}
\]

\[
\text{• plugged in } EVB_{j,i}, \text{ and it is needed by time } t, \quad C_{j,i}^{ev}(t) = \bar{C}_{j,i}^{ev}(t).
\]

We define the set of feasible charging trajectories of \( EV_{j,i} \) as:

\[
\mathbb{U}_{j,i}^{ev} = \left\{ p_{j,i}^{ev} \in \mathbb{R}^N \mid (4) - (5) \forall t \in [k, k + N - 1] \right\}.
\]

C. BES Model

The BES controlled by \( Agg_j \) is indicated by \( BES_j \) and modeled as follows:

\[
e_{j}^{bes}(t + 1) = e_{j}^{bes}(t) + T_h n_{j}^{bes} p_{j}^{bes}(t),
\]

where \( e_{j}^{bes} \) and \( p_{j}^{bes} \) are the active, reactive, and apparent powers, respectively. The energy stored in BES and its charging/discharging power are limited by the following constraints:

\[
\begin{align*}
s_j^{bes^2} &= p_j^{bes^2} + q_j^{bes^2} \leq s_j^{bes^2} \\
C_j^{bes} + C_j^{bes} \leq C_j^{bes},
\end{align*}
\]

where \( s_j^{bes}, q_j^{bes} \) and \( s_j^{bes} \) are the active, reactive, and apparent powers, respectively, \( s_j^{bes} \) is the apparent power rating of the BES's bi-directional converter, and \( C_j^{bes} \) and \( C_j^{bes} \) are the minimum and maximum BES energy constraints, respectively.

D. EV Aggregator Model

We use \( \mathbb{N}_{Agg} \) to denote the set of buses which have EV aggregator. According to the model defined for EVBs, the active and reactive powers of bus \( j \), where \( Agg_j \) is located, are obtained as:

\[
\begin{align*}
P_{j} &= p_{j}^{ev} + \sum_{i \in \mathbb{N}_{j, ev}} (p_{j,i}^{ev} + p_{j,i}^{uc}) \\
Q_{j} &= q_{j}^{ev} + \sum_{i \in \mathbb{N}_{j, ev}} q_{j,i}^{uc},
\end{align*}
\]
in which, $q_{j,i}^{uc}$ is the uncontrollable reactive load demand by EV $B_{j,i}$. As $p_{j,i}^{uc}$ and $q_{j,i}^{uc}$ are not controllable, we consider their aggregated value at Agg$j_i$ in the EVCS modeling, which are defined as:

$$p_{j}^{uc} = \sum_{i \in N_{j,ev}} p_{j,i}^{uc} \quad (10a)$$

$$q_{j}^{uc} = \sum_{i \in N_{j,ev}} q_{j,i}^{uc} \quad (10b)$$

and reactive powers injected to the buses should not be too large.

Replacing (2d) by (12) in (11), the optimization problem (11) is solved by ADMM as the following:

$$(P_{j}^{k+1}, Q_{j}^{k+1}) := \arg\min_{P_{j},Q_{j}} (\Pi^T P_{j} + \frac{\rho_p}{2} \|P_{j} - P_{j}^{k} + v_{j}^{k}\|_2^2$$

$$+ \frac{\rho_q}{2} \|Q_{j} - Q_{j}^{k} + u_{j}^{k}\|_2^2)$$

s.t. $U_{j,i} \forall i \in N_{j,ev}$ & $j \in N_{Agg}$, (7) - (9) (13)

$$(P_{j}^{k+1}, Q_{j}^{k+1}) := \arg\min_{P_{j},Q_{j}} \sum_{(i,j) \in \zeta} r_{ij} I_{ij}$$

$$+ \frac{\rho_p}{2} \sum_{j \in N_{Agg}} \|P_{j}^{k+1} - P_{j} + v_{j}^{k}\|_2^2$$

$$+ \frac{\rho_q}{2} \sum_{j \in N_{Agg}} \|Q_{j}^{k+1} - Q_{j} + u_{j}^{k}\|_2^2)$$

s.t. (2a) - (2c), (3) & (12)

$$v_{j}^{k+1} = v_{j} + d_{j}^{k+1} - p_{j}^{k+1}$$

$$u_{j}^{k+1} = u_{j} + q_{j}^{k+1} - q_{j}^{k+1} (15a)$$

where (13) is solved in parallel by each Agg and (14)-(15) are solved by DSO.

B. Hierarchical Distributed EVCS

Considering the first step of ADMM (13), each Agg has to solve the optimal charging scheduling problem for all the EVs which it is supplying. If the number of EVs is considerable, the computational burden for Aggs will be substantial. By exploiting the mathematical formulation, we write (13) in the form which is called sharing problem [12, Chapter 7.3], and it can be solved efficiently by ADMM in a distributed manner between each Agg$j_i$ and its EVs (i.e. $\forall E_{j,i}, i \in N_{j,ev}$).

Using (9) and (10), we can write (13) as:

$$\min_{x,y} \sum_{i \in N_{j,ev}} x_{i} y_{i} + \sum_{j \in N_{Agg}} \Pi^T P_{j}.$$ (11)

s.t. (2) - (3), (6) - (9),

where $\Pi \in \mathbb{R}^{N}$ is the wholesale energy price.

A. Distributed EVCS

As it is clear, solving the optimization problem in (11) by only DSO is not computationally efficient, especially when the grid is bulky and DSO has to deal with a large population of EVs. Therefore, we use ADMM to solve the problem in a distributed manner in which DSO and Aggs communicate which each other iteratively. However, we first need to relax the optimization problem as (2d) is a non-convex constraint, otherwise ADMM is not applicable. According to [14], (2d) can be relaxed to a convex second-order cone as follows:

$$V_{I_{ij}} \geq P_{ij}^2 + Q_{ij}^2$$.

The sufficient conditions to make the relaxation tight, as shown in [14]-[15], are: (i) the grid should be radial; (ii) bus voltages should be very close to the nominal value; and (iii) the active

Fig. 2. IEEE-13 bus system with the proposed hierarchical distributed EVCS.

III. EVCS PROBLEM FORMULATION

The distribution grid and its EV aggregators are shown in Fig. 2, which have a hierarchical triayer structure including DSO, Aggs, and EVs. The goal of the DSO is to minimize the energy loss in the distribution grid lines, while Aggs aim at reducing the electricity cost for their customers. Accordingly, the objective function of the EV charging scheduling is twofold, loss reduction (grid service) and cost reduction (customer service), and it is written as follows:

$$V := \min_{(i,j) \in \zeta} \sum r_{ij} I_{ij} + \sum_{j \in N_{Agg}} \Pi^T P_{j}\quad (11)$$

where $\Pi \in \mathbb{R}^{N}$ is the wholesale energy price.

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Using (9) and (10), we can write (13) as:

$$\min_{x,y} \sum_{i \in N_{j,ev}} x_{i} y_{i} + \sum_{j \in N_{Agg}} \Pi^T (p_{j}^{uc} + p_{j}^{bes})$$

$$+ \frac{\rho_p}{2} \left( \sum_{i \in N_{j,ev}} \|p_{j,i}^{uc} + p_{j,i}^{bes} - P_{j} + v_{j}^{k}\|_2^2$$

$$+ \frac{\rho_q}{2} \left( \sum_{i \in N_{j,ev}} \|q_{j,i}^{uc} + q_{j,i}^{bes} - Q_{j} + u_{j}^{k}\|_2^2$$

s.t. $U_{j,i} \forall i \in N_{j,ev}$ & $j \in N_{Agg}$, (7) - (9),

Considering $p_{j,i}^{ev}$, the first part on RHS of (16) is separable between the EVs, and the rest is a function of $\sum_{i \in N_{j,ev}} p_{j,i}^{ev}$, which we show, hereafter, by $p_{j}^{ev}$. Therefore, (16) is a sharing problem [12, Chapter 7.3], and it can be solved in a distributed manner as follows:

$$p_{j,i}^{ev+1} := \arg\min_{p_{j,i}^{ev}} (\Pi^T p_{j,i}^{ev} +$$

$$+ \frac{\rho_p}{2} \left( \|p_{j,i}^{ev} - p_{j,i}^{ev} + p_{j,i}^{ev} - P_{j} + v_{j}^{k}\|_2^2$$

s.t. $U_{j,i} \forall i \in N_{j,ev}$

$$\Pi^T p_{j,i}^{ev} +$$

$$+ \frac{\rho_q}{2} \left( \|q_{j,i}^{uc} + q_{j,i}^{bes} - Q_{j} + u_{j}^{k}\|_2^2$$

s.t. $U_{j,i} \forall i \in N_{j,ev}$

$$\Pi^T (p_{j}^{uc} + p_{j}^{bes})$$

$$+ \frac{\rho_p}{2} \left( \sum_{i \in N_{j,ev}} \|p_{j,i}^{uc} + p_{j,i}^{bes} - P_{j} + v_{j}^{k}\|_2^2$$

$$+ \frac{\rho_q}{2} \left( \sum_{i \in N_{j,ev}} \|q_{j,i}^{uc} + q_{j,i}^{bes} - Q_{j} + u_{j}^{k}\|_2^2$$

s.t. $U_{j,i} \forall i \in N_{j,ev}$ & $j \in N_{Agg}$, (7) - (9),

Considering $p_{j,i}^{ev}$, the first part on RHS of (16) is separable between the EVs, and the rest is a function of $\sum_{i \in N_{j,ev}} p_{j,i}^{ev}$, which we show, hereafter, by $p_{j}^{ev}$. Therefore, (16) is a sharing problem [12, Chapter 7.3], and it can be solved in a distributed manner as follows:

$$p_{j,i}^{ev+1} := \arg\min_{p_{j,i}^{ev}} (\Pi^T p_{j,i}^{ev} +$$

$$+ \frac{\rho_p}{2} \left( \|p_{j,i}^{ev} - p_{j,i}^{ev} + p_{j,i}^{ev} - P_{j} + v_{j}^{k}\|_2^2$$

s.t. $U_{j,i} \forall i \in N_{j,ev}$
\[
\begin{align*}
\mathbf{p}_{j}^{ev_{t+1}} &= \arg\min_{\mathbf{p}_{j}^{ev}, \mathbf{p}_{j}^{bes}} \mathbf{\Pi}^T_{j} (\mathbf{p}_{j}^{ev} + \mathbf{p}_{j}^{bes}) \\
&+ \frac{\rho_{j}}{2} ||\mathbf{N}_{j} \mathbf{v}^{ev}_{j} - \mathbf{p}_{j}^{ev} + \mathbf{p}_{j}^{bes} - \mathbf{P}_{j}^{k} + \mathbf{v}_{j}^{k}||^2_2 \\
&+ \frac{\rho_{j}}{2} ||\mathbf{q}_{j}^{uc} + \mathbf{q}_{j}^{bes} - \mathbf{Q}_{j}^{k} + \mathbf{u}_{j}^{k}||^2_2 \\
&+ \frac{(\mathbf{N}_{j} \mathbf{v}^{ev}_{j} \mathbf{p}_{j}^{bes})}{2} ||\mathbf{p}_{j}^{ev} - \mathbf{p}_{j}^{bes} - \mathbf{p}_{j}^{ev_{t+1}}||^2_2 \\
\text{s.t.} \ (7) - (8) \\
&\lambda_{j}^{t+1} = \lambda_{j}^{t} + \mathbf{p}_{j}^{ev_{t+1}} - \mathbf{p}_{j}^{ev_{t+1}}. \quad (19)
\end{align*}
\]

Note that Agg's problem size (18) is independent of the number of EVs. To decrease communication overheads, we define \(\Lambda_{j}^{t+1} = \lambda_{j}^{t+1} + \mathbf{p}_{j}^{ev_{t+1}} - \mathbf{p}_{j}^{ev_{t+1}}\). Thus, at each sharing problem iteration after the third update (19), Agg broadcasts \(\Lambda_{j}^{t+1}\) to all EVs, \(\forall i \in \mathbb{N}_{j,ev}\). For more details, we refer the readers to [1].

We call ADMM algorithm the iterative procedure between DSO and Aggs (13)-(15), and ADMM2 the sharing problem between each Agg and its EVs (17)-(19). The whole procedure of our hierarchical distributed EVCS is shown in Algorithm 1. \(Err_{1}\) and \(Err_{2}\) are the pair of primal and dual residuals for ADMM1 and ADMM2, respectively. For more details about residual calculation and stopping criteria, we refer the reader to [12, Chapter 3.3]

**Algorithm 1: Hierarchical Distributed EVCS.**

```python
while Err1 < Th1 do
    for j \in N_{Agg} do
        while Err2 < Th2 do
            for i \in N_{j,ev} do
                Calculate \(p_{j}^{ev_{i}}\) by (17) & send to Aggj.
            end
            Update \(P_{j}^{ev} = \frac{1}{N_{j,ev}} \sum_{i \in N_{j,ev}} p_{j}^{ev_{i}}\).
            Calculate \(p_{j}^{ev_{t+1}}\) and \(q_{j}^{bes_{t+1}}\) by (18).
            Update \(\lambda_{j}\) by (19).
            Update \(\Lambda_{j}\) and broadcast to \(\forall i \in \mathbb{N}_{j,ev}\).
            Update \(Err_{2}\).
        end
        Send (\(P_{j}, Q_{j}\)) to DSO.
    end
    Calculate (\(P_{j_{ref}}, Q_{j_{ref}}\)), \(\forall j \in N_{Agg}\), by (14).
    Update (\(v_{j}, u_{j}\)), \(\forall j \in N_{Agg}\), by (15).
    Broadcast (\(P_{j_{ref}}, Q_{j_{ref}}\)) and (\(v_{j}, u_{j}\)) to Aggj, \(\forall j \in N_{Agg}\). Update \(Err_{1}\).
end
```

**IV. NUMERICAL SIMULATION**

In this section, the performance of the proposed hierarchical distributed EVCS is evaluated for the modified IEEE-13 bus system. We consider the single phase balanced system with six Aggs which are located at bus# 634, 646, 675, 680, 652 and 611. We compare the performance of our proposed method with uncontrolled EV charging, in which EVs start charging as soon as they are plugged in with the maximum power rating (i.e. \(P_{j}^{ev}, \forall i \in \mathbb{N}_{j,ev}, \forall j \in \mathbb{N}_{Agg}\)). Also, to show the effect of BES on loss and charging cost reduction, we compare the results with the case where EVCS is executed without any BES installed in the Aggs' nodes. The maximum power rating for all EV chargers is 4 kW. The initial and designated EVs' battery energies are uniformly distributed over [8, 10] kWh and [22, 25] kWh, respectively. Also, EVs' arrival and departure times are uniformly distributed in [16:30, 20:30] and [6:00, 9:30], respectively. The netload dataset of EVBs is collected from the Australian electricity company-Augrid [16], and the wholesale price is available from the California Independent System Operator-CAISO [17]. More details about the simulation parameters are shown in Table I. All the simulations are executed by MATLAB on a PC with Intel® Core™ i7-4770 3.40 GHz CPU, 4 cores and 8 GB RAM, and the convex optimization problems are solved by CVX [18].

**TABLE I**

| Parameter      | Value     | Parameter      | Value     |
|----------------|-----------|----------------|-----------|
| \(P_{j}^{ev}\) | 4, -4 [kW]| \(v_{j}, u_{j}\) | 0.97, 1.03 [p.u.] |
| \(\rho_{j}\)     | 0.1, 0.1 | \(\rho_{j}\)     | 1         |
| \(C_{bes}\) | 55 [kWh] | \(N\) | 48        |
| \(S_{bes}\) | 50 [kVAr] | \(T_{h}\)     | 0.5       |

As maintaining the bus voltages within the acceptable range is a constraint of EVCS, Fig. 3 shows how the control EV charging demand with (CC1) and without stationary BES (CC2) improves voltage profile in the grid buses while uncontrolled EV charging (uCC) results in significant voltage drop in the grid. The results also show that EVCS with BES
Fig. 4 shows the active and reactive powers and the energy profile of the BES as well as the wholesale electricity price. As it is shown, BES is charged while the energy price is low, and it is discharged during the second peak of the electricity price. It should be noticed that BES is not discharged during the first price peak as load demand is not considerable (Fig. 5). While close to the second price peak when the load demand is considerable, BES is discharged to reduce both energy loss and electricity cost. Considering Fig. 5, the peak load over the demand, DSO in collaboration with Aggs and EVs try to minimize the energy loss and electricity cost. As solving EVCS optimization problem in a centralized manner is not computationally efficient and privacy preserving, ADMM was used to solve the problem through an iterative procedure between DSO, Aggs, and EVs. Numerical simulation of the proposed EVCS, which was applied to IEEE-13 bus system including six Aggs, verified its effectiveness.

Figure 5. Load profile of the grid lines: (left) CC₁, (middle) CC₂, and (right) uCC.

Figure 6. The comparison of EV charging costs obtained by CC₁, CC₂, and uCC.

In addition, DSO achieves its energy loss reduction in the grid as it is shown in Table II. The loss reduction achieved by CC₁ and CC₂ is considerable in the lines #1, 4 and 8, which connect bus #650 to 632, 632 to 645, and 671 to 684, respectively. The reasons for the significant loss are high impedance of the line conductors, their length, and high peak loads.

### Table II: Energy Loss on the Grid Lines

| Loss | CC₁ | kWh | CC₂ | kWh | uCC | kWh |
|------|-----|-----|-----|-----|-----|-----|
| 1    | 325 | 6.11| 331 | 6.40| 375 | 6.86|
| 2    | 117 | 8.65| 115 | 9.08| 131 | 9.34|
| 3    | 6.46| 3.50| 6.67| 3.59| 6.76| 3.65|
| 4    | 7.36| 7.36| 6.68| 6.68| 6.92| 6.92|
| 5    | 16.60|2.60| 16.75|2.71| 20.2|2.99|
| 6    | 2.60|4.60| 2.71|4.34| 20.2|4.95|
| 7    | 8.08| 8.08| 8.38| 8.38| 10.6| 10.6|
| 8    | 109 | 9.80| 115 | 9.30| 131 | 9.34|
| 9    | 35.4| 1.20| 35.4| 1.20| 35.4| 1.20|
| 10   | 4.10| 4.10| 4.10| 4.10| 4.10| 4.10|
| 11   | 17.1| 7.79| 17.2| 8.10| 20.7| 9.59|

**V. Conclusion**

In this paper, a fully hierarchical and distributed method was proposed for EVCS. While the power flow model and constraints are considered for optimal scheduling of EV load

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