Student-teacher’s perception of mathematical representation in mathematics learning

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Abstract. This research aims to know student-teacher’s perception of mathematical representation in mathematical learning. This research is a qualitative research. The subject of this research are 12 student-teacher. Data collection technique are carried out through interview. Data analyze using Bogdan and Biklen. Data analysis includes steps to collected data, reduced data, categorizing the data and made conclusions. The result show that the twelve subjects can describe the definition of mathematical representation. The subjects considered mathematical representation is important in mathematical learning, which is for: 1) Understanding the concepts, 2) Problem Solving, 3) Construction the concepts and 4) Communication. The subjects considered that it is difficult to develop mathematical representation in mathematical learning.

1. Introduction

Education has an important role in improving the quality of human resources in order to compete in the era of globalization and modernization [1]. Education also becomes one of the means for preparing students to encounter every challenge of the modern era [2]. One of the efforts to achieve these is through mathematics education. Mathematics education tries to improve students’ reasoning, intelligence and change their positive attitudes. The function of mathematics education are to enhance students’ reasoning, to help clarify and solve problems in real life, so they can use mathematics and mathematical mindset to learn about various knowledge in order to have skills and abilities [3].

The abilities that students should have are mathematical abilities. Mathematical ability is the ability to deal with problems, both in mathematics and real life. National Council of Teacher in Principles and Standards for School Mathematics [4] states that the standard learning process of mathematics, aims to improve mathematical abilities, including problem solving, reasoning and proof, communication, connections and representation.

One of the mathematical abilities that students need to have is mathematical representation. According to Jones and Knuth [5], representation is a model or substitute form of a problem situation that is used to find a solution. The standard of representation that is set by National Council of Teacher in Principles and Standards for School Mathematics [4] is that during learning in school, students have the ability to: a) Create and use representation to organize, record and communicate mathematical ideas; b) Select, apply and translate mathematical representations to solve problems; c) Using representations to model and interpret physical, social, and mathematical phenomena. So, mathematical representation is the ability to describe, translate or express mathematical ideas as an effort to show understanding to find solutions from the given problems.
Representations in mathematics is related with efforts to improve understanding of mathematical concepts. As an abstract knowledge, mathematics needs to be represented in order to assist students in understanding and constructing their knowledge. Representation is the main focus in constructing knowledge. According to Bal [6], to think mathematically and communicate mathematical ideas, it needs to present in various forms of representation such as verbal representations, pictures, numerics, symbols, algebra, tables and graphs. Students can connect and communicate mathematical concepts and ideas well by good representation. Representation ability is needed by students in order to understand and relate the mathematical concepts that they have learned.

The use of various representations has important role in learning mathematics [7], especially in helping to understand and interpret mathematical concepts in various representations [8]. Tripathi [9] said that in mathematics, the use of multiple representations is an important tool that allows students to understand mathematical concepts. However, generally in mathematics, representation is only a complement in solving mathematical problems. This causes a very limited ability of representation. The weakness of mathematical representation ability is the difficulties to bridge representation and change from one representation to another [10].

Hudiono [5] mentioned that the limited knowledge of teachers and the habits of students learning in class with conventional way has not made it possible to develop representation optimally. To facilitate students in carrying out representation, teachers must know the mathematical representation first. If teachers don’t know and understand the importance of mathematical representation in learning, the learning process that will be carried out will not involve the ability of students’ representation and make the students’ representation difficult to develop. Based on the descriptions above, researchers are interested in knowing students-teacher's perceptions of mathematical representations. The purpose of this study was to determine students-teacher's perceptions of mathematical representation in mathematics learning.

2. Method
In this study, researchers used qualitative approach. Subjects in this study consisted of 12 students-teacher. In this study, the researcher wanted to know information about students-teacher's perceptions of mathematical representations in mathematics learning. To find out students-teacher's perceptions of mathematical representation, data collection techniques conducted by researchers through interviews. Subjects were given questions related to the definition of mathematical representation, the importance of mathematical representation in mathematics learning and degree of difficulty to develop representation in mathematics learning. Data analyze in this study using Bogdan and Biklen. The researcher collected data, analyzed data descriptively in the form of words. Data analysis includes steps to collected data, reducted data, categorized the data into themes and made conclusions as the results.

3. Result and discussion

3.1 Result
In this study, the twelve subjects were interviewed regarding their perception of mathematical representation. The questions in the interview outline are about the definition of mathematical representation, the importance of mathematical representation in mathematics learning and degree of difficulty to develop representation in mathematics learning. The result of interview related to definition of mathematical representation is presented in table 1.
### Table 1. The interview result of definition of mathematical representation

| Definition of mathematical representation                                      | Verification result                  |
|-------------------------------------------------------------------------------|--------------------------------------|
| Changing mathematics in different form                                         |                                      |
| Representation of mathematics’ concept in another form                          |                                      |
| Restate mathematics sentence in different form                                  |                                      |
| How does mathematics’ idea can be present in various way                        | All subjects are familiar with the   |
| Ability to represent a problem in mathematics form                              | term mathematical representation     |
| Ability to describe mathematics in another form                                 |                                      |
| Ability to describe mathematically                                            |                                      |
| Ability to describe and explain mathematics’ problem                           | they can describe the definitions of |
| Ability to describe mathematics’ idea in a certain symbol or configuration      | mathematical representations         |
| Ability to redescribe, translate, and reexpress a mathematics’ idea             |                                      |
| Expression of mathematics ideas                                                |                                      |
| Expression from a mathematics’ idea                                            |                                      |

Six subjects stated mathematical representation is the ability to change/present/restate mathematics in other forms, two subjects stated mathematical representations are expressions of mathematical ideas and four subjects expressed mathematical representations as the ability to describe/express/translate mathematical idea.

The result of interview for question about the importance of mathematical representation in learning mathematics can be seen through table 2.

### Table 2. The interview result of the importance of mathematical representation

| The importance of mathematical representation                                | Verification result |
|----------------------------------------------------------------------------|---------------------|
| Understanding concepts                                                      | All subjects considered that mathematical representations were important in mathematics learning. One subject can’t express the importance of mathematical representation clearly. |
| Problem solving                                                             |                     |
| Concept construction                                                        |                     |
| Communication                                                               |                     |
| One of the abilities that needed in the 21st century                        |                     |

Six subjects considered mathematical representations important for understanding concepts, six subjects considered mathematical representations important for problem solving, four subjects considered important mathematical representations for concept construction, five subjects considered mathematical representations important for communication and 1 subject considered Mathematical representation is important because it is needed in the 21st century.

The result of interview for question about degree of difficulty to develop representation in mathematics learning can be seen through table 3.

### Table 3. Degree of difficulty

| Degree of difficulty                                      | Verification result |
|-----------------------------------------------------------|---------------------|
| Difficult                                                 | Most subjects considered that representation in math difficult to develop |
| Might be difficult, might be not difficult                |                      |
Based on the results of interview, ten subjects considered that representation difficult to develop in mathematical learning. Two subject considered that representation might be and might be not difficult to develop in mathematical learning. So, more than 83% subjects considered that it is difficult to develop representation in mathematical learning.

3.2 Discussion

Based on the results of interviews related to mathematical representations from the twelve subjects, mathematical representation is a familiar term for students-teacher of mathematics education, it can be seen from the statement of subjects who have heard of mathematical representations and all subjects can describe the definition of mathematical representation generally. As an important part of teaching, teachers need to know and use various representations to translate various mathematical concepts into representations that can be understood by students as an effort to facilitate and assist students in understanding the concept.

As an ability to describe, translate, or express mathematical ideas as an effort to show understanding to find solutions to the given problems, representation is very necessary in the learning of mathematics. Mathematical representation is a high-level ability that can develop to structured students' mindset and can develop their thinking skills in constructing their understanding. Therefore, the teachers need to know and understand mathematical representations.

Based on the results of the interviews, the twelve subjects consider that mathematical representations are important in learning mathematics, even though one research subject can’t express the importance of mathematical representation clearly. From the eleven subjects who could express the importance of mathematical representation in learning mathematics clearly, the researchers categorized it into four.

The first is the importance of mathematical representation for understanding concepts. Six subjects research assume that mathematical representations are important to help students understand mathematical concepts. According to Jones [11], the way of teacher presenting mathematical ideas through various representations will have a very big influence on students' understanding in learning mathematics. He also stated that students need practice in building their own representations so they have the ability and understanding of concepts that are good and flexible so they can be used in problem solving [11].

The second is the importance of mathematical representation for problem solving. Six subjects consider that representation is important to help students solving the problems. The process of representation in mathematics learning are the used in selecting, implementing, and translating between mathematical representations to solve problems [5]. Learning mathematics in class should provide sufficient opportunities for students to be able to train and develop mathematical representation skills as an important part in solving or solving problems because representation is very useful in helping students solve a problem more easily. Mathematical representation is an element of problem solving because in solving a problem it is necessary to interpret the problem which is a representation of a form in mathematics. Brenner stated that a successful problem solving process depends on the skill of representing problems such as constructing and using mathematical representations in words, graphs, tables, and equations, completion and manipulation of symbols [12].

The third is the importance of mathematical representation for concept construction. Four subjects in this study assume that mathematical representation can help students in building or constructing mathematical concepts. Jones [11] says that the fluency in translating between different types of representations is the basic ability that students need to have to build a concept and mathematical thinking. Each student has a different way to construct his knowledge. By using a variety of mathematical representations, teachers can help and facilitate students in constructing concepts or knowledge.

The fourth is the importance of mathematical representation for communication. Five subjects assume that mathematical representations are important for mathematical communication. Mathematical representation can train students in communicating mathematics into various forms of interpretation or
mathematical expressions. The representation standards set by NCTM [4] during school learning include students having the ability to create and use representations to organize, record, and communicate mathematical ideas. Representation is a tool for students to communicate mathematical ideas or ideas from students to other students and teachers. To be able to communicate something, someone needs a good representation in the form of images, graphics, diagrams, and other forms of representation. With representation, the problems that originally seemed difficult and complicated can be seen more easily and simply, so that the problems presented can be solved more easily.

Based on the results of interviews related to degree of difficulty to develop representation in mathematical learning, the subjects considered it difficult to develop representation in mathematical learning. The limited knowledge of representation makes the subject felt representation difficult to develop. It is related to Hudiono’s opinion [5] that is the limited knowledge of teachers and the habits of students learning in class with conventional way has not made it possible to develop representation optimally.

The teachers need to be aware of the importance of mathematical representation in mathematics learning. Thus, the learning process need to designed well so that it can help students to develop or train their mathematical representation abilities. The importance of mathematical representation in concept understanding, concept construction and mathematical communication will help students in the process of problem solving, to find solutions from the problems that they faced.

4. Conclusion

Based on the analysis of the results of interviews in this study, there are some information that obtained related to students-teacher’s perceptions of mathematical representation in mathematics learning. The twelve subjects are familiar with the term mathematical representation and they can describe the definitions of mathematical representations. The twelve subjects considered that mathematical representations were important in mathematics learning, although one research subject had not been able to express the importance of mathematical representation clearly. From the eleven subjects who can express the importance of mathematical representation in mathematics learning clearly, researchers categorize it into 4, which is: 1) Understanding of concepts, 2) Problem solving, 3) Construction of concepts and 4) Communication. Most subjects considered that is difficult to develop representation in mathematical learning. Based on students-teacher's perceptions of the importance of mathematical representation in learning mathematics in this study, the researcher expects it can contribute to the understanding of teachers about mathematical representation. It is expected that by realizing the importance of mathematical representation in the learning process, teachers can design or prepare learning that can train and develop mathematical representation ability which is important part of the mathematical abilities needed by students.

5. References

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