Methods for measuring the position of the centre of gravity of an anthropotechnic human-wheelchair system in dynamic conditions

B Wieczorek, M Kukla and Ł Warguła

Poznan University of Technology, Chair of Basic of Machine Design, Piotrowo Street 3, 60-965 Poznan, Poland

E-mail: bartosz.wieczorek@put.poznan.pl

Abstract. The article presents the methods for measuring the position of the centre of gravity of an anthropotechnic human-wheelchair system in dynamic conditions. The methods were verified by the pilot studies using a pushrim-propelled wheelchair. Data processing methods were used to fit the ellipses to measured datasets including 4.463 to 3.434 points, where the ellipses are defined by their centres M C.G., inclinations α and semi-axes a and b. The method involves defining each ellipse fit to the dataset with a likelihood of inclusion of new position points of the centre of gravity. The ellipses characterized by the highest likelihood were selected, which for the tested cases was between 29.7 % to 40.9 %, depending on the test.

1. Introduction

The biomechanics of an anthropotechnic system is a complex network of interrelations between different biological and mechanical parameters including the position of the center of gravity of the human body. Determining the variability of this parameter is crucial in the design of a wheelchair frame since it allows determining the position in which the force exerted on an object is applied. The position of the center of gravity is a key factor in the biomechanical analysis of the human-wheelchair system [1] which allows evaluating the static and dynamic stability of the system [2]. A known position of the center of gravity can be used to determine the values and forces exerted on each wheel of the wheelchair, and thus to determine its rolling resistance force. The force is one of the components of its resistance-to-motion force defining a minimum propulsive force causing the wheelchair to move in specific terrain conditions.

The position of the center of gravity of an anthropotechnic human-wheelchair system may be affected by several factors, including the external constraints resulting from the operating parameters of the propelled wheelchair [3]. The effect of the terrain is due to climbing the slopes, as a result of which both the wheelchair and the user body tilt. The operating parameters also affect the bending angle of human body segments [4]. The user has a dynamic effect on the position of the center of gravity in the entire observation period [5, 6] due to changes in limb position during wheelchair operation.

A special test bed for determining the parameter was designed and manufactured as part of the Lider VII project “Study of the biomechanics of manually propelled wheelchair for innovative manual and hybrid propulsion systems” (LIDER/7/0025/L-7/15/2016) financed by the National Centre for
Research and Development. The analysis of variability of the position of the center of gravity must include measurements in dynamic conditions and data processing.

The method uses a wheelchair test bed simulating the wheelchair movement in the actual terrain conditions (figure 1) [7]. The test bed allows to impose a variable load on the drive wheels and to tilt the wheelchair. On the test bed, the wheelchair is located on a scale pan mounted on four strain gauges. The strain gauges can be used to determine the load imposed by the wheelchair and the user in four measuring points. The number of strain gauges (four) is a new feature compared to the centre of gravity measurement methods known from the art [8].

![Figure 1. Test bed for testing the biomechanics of pushrim-propelled wheelchair including strain gauges W1, W2, W3 and W4.](image)

2. Mathematical model

The measurements of the position of the center of gravity use signals from four strain gauges supporting the scale pan on which the wheelchair and the user are positioned (figure 2). The strain gauges Wi measure the vertical reaction forces Ri in four scale pan support points.

Four planes \( \pi_1, \pi_2, \pi_3 \) and \( \pi_4 \) are analysed assuming that each pair of the strain gauges is intersected by a plane parallel to the z-axis of the reaction force and mass force system of the tested object. The projection of the center of gravity is determined for each plane as a distance between the force application points \( R_i \) and \( R_{i+1} \) (figure 3). The distances were designate as \( f_{12}, f_{23}, f_{43} \) and \( f_{14} \) with values determined using the moment of force equations for the plane \( \pi_i \). A single moment of force equation is assigned to each plane in relation to a random pole (1–4). The equations omit the reaction of the supports in x and y-axis. A slight contribution is allowed for and interaction of the vertical reaction forces only.

\[
\sum M_{R_1} = 0: -(R_1 + R_2) \cdot f_{12} + R_2 \cdot L_1 = 0 \rightarrow f_{12} = \frac{R_2 \cdot L_1}{(R_1 + R_2)} \quad (1)
\]

\[
\sum M_{R_2} = 0: -(R_2 + R_3) \cdot f_{23} + R_3 \cdot L_2 = 0 \rightarrow f_{23} = \frac{R_3 \cdot L_2}{(R_2 + R_3)} \quad (2)
\]

\[
\sum M_{R_3} = 0: -(R_3 + R_4) \cdot f_{43} + R_3 \cdot L_3 = 0 \rightarrow f_{43} = \frac{R_3 \cdot L_3}{(R_3 + R_4)} \quad (3)
\]

\[
\sum M_{R_4} = 0: -(R_1 + R_4) \cdot f_{14} + R_4 \cdot L_4 = 0 \rightarrow f_{14} = \frac{R_4 \cdot L_2}{(R_1 + R_4)} \quad (4)
\]

where: \( f_{12}, f_{23}, f_{43}, f_{14} \) – distance from the position of the center of gravity to the coordinate origin for each measuring plane, \( R_1, R_3, R_4 \) – strain gauge reaction in the support points, \( L_1 \) – distance...
between \( W_1 \) and \( W_2 \) strain gauges, and distance between \( W_4 \) and \( W_3 \) strain gauges, \( L_2 \) – distance between \( W_2 \) and \( W_3 \) strain gauges and distance between \( W_1 \) and \( W_4 \) strain gauges.

![Diagram showing test station and reactions determined using strain gauges in four measuring planes.](image1)

**Figure 2.** Test station including reactions determined using strain gauges in four measuring planes.

The determined distances \( f_i \) (figure 3) were marked on the sides of a rectangle drawn using the reaction force application points \( R_i \). Two straight lines can be drawn through the points \( P_i \) on the rectangle’s sides. The straight lines intersect in a single point on the xy-plane (figure 4). The point coordinates (5, 6) are the coordinates of the position of the centre of gravity of the human-wheelchair system for a single strain gauge measurement in the xy-plane.

![Diagram showing beam diagrams representing the position of the center of gravity of the wheelchair and the user in four measuring planes.](image2)

**Figure 3.** Beam diagrams representing the position of the center of gravity of the wheelchair and the user in four measuring planes.
3. Preliminary study

In the preliminary study, the wheelchair was propelled forward (test 1), turned right (test 3) and turned left (test 4). The method allows measuring the system in dynamic conditions. The measured sets of points were characterized by large scatter of data points. To order the measured datasets, the ellipses, defining the variability region of the position of the centre of gravity, were drawn on the xy-plane. The analytical data processing method used to obtain the ellipses (figure 5) involves several steps.

In the method, the position of the centre of the ellipse \( (x_E, y_E) \) is determined by arithmetic mean of the \( x \)- and \( y \)-coordinates. An inclination of the ellipse corresponding to the inclination of the trend line of the analysed set of points was determined. Using the trend line inclination, the ellipses were determined using the parametric Eqs. (7) and (8):

\[
\begin{align*}
    x & = \frac{L_2 f_{12}}{f_{43} - f_{12}} + f_{14} \\
    y & = \frac{L_2 f_{43}}{f_{43} - f_{12}} - f_{12} \\
\end{align*}
\]

\[
\begin{align*}
    x & = a_N \cos(t) \cos(\alpha) - b_N \sin(t) \sin(\alpha) + x_E \\
    y & = a_N \cos(t) \sin(\alpha) + b_N \sin(t) \cos(\alpha) + y_E \\
\end{align*}
\]

The method consists of selecting the parameters of an ellipse fitted to the measured points. For each measurement, \( N \) ellipses with different semi-axes \( a_N \) and \( b_N \) can be defined. Each ellipse includes a different number of the measured points. The semi-axes are chosen to yield the highest likelihood of occurrence of new points (9):

\[
P_i(A) = \frac{A_i - A_{i-1}}{\Omega - A_{i-1}}
\]
where: $P_i$ – likelihood of occurrence of new points in the ellipse, $A_i$ – set of position points of the centre of gravity inside the ellipse, $A_{i-1}$ – set of position points of the center of gravity inside the ellipse smaller than the current ellipse, $\Omega$ – set of all measured position points of the center of gravity.

In accordance with the method, a constant ratio between the semi-axes (10) must be maintained for each ellipse:

$$\frac{a_N}{b_N} = \frac{a_{N+1}}{b_{N+1}} = const$$

(10)

**Figure 5.** Diagram representing the method to calculate the ellipses from the set of points corresponding to the changes in the position of the center of gravity of the anthropotechnic system.

**Figure 6.** Graph of changes in the position of the centre of gravity for wheelchair propelled forward (test 1) with the ellipses representing the distribution of the coordinates of the centre of gravity (A-D) and mean position of the centre of gravity (M C.G.).

The pilot studies of changes in the position of the center of gravity of the human body during pushim wheelchair propulsion were carried out to visualize the method. In the first test, the wheelchair was
propelled forward by pushing the left wheel and the right wheel symmetrically. Data processing yielded a graph of the position points of the center of gravity (figure 6) and a graph of the likelihood of occurrence of new points in those ellipses (figure 7).

Figure 7. The occurrence of inclusion of new points (test no. 1) within the ellipses (A-D). \(a_E\) and \(b_E\) – semi-axes of the selected ellipse.

During wheelchair propulsion forward, the largest distribution of the position points of the center of gravity was observed along the x-axis. On the x-axis, the point coordinates are from -110.86 mm to -10.86 mm, and on the y-axis, the coordinates are from 30.93 mm to 69.22 mm. The highest likelihood of inclusion of new position points of the center of gravity within the ellipses was observed for ellipse A. For other ellipses, the likelihood was lower. For ellipse C, a maximum likelihood PC(AC) of 29.7 % was observed.

During the next test, the right wheel was pushed and the left wheel was pulled to simulate a left turn. Using the measured data, a graph of the position points of the center of gravity (figure 8) and a graph of the likelihood of occurrence of new points in the ellipses (figure 9) were drawn.

Figure 8. Graph of changes in the position of the center of gravity for wheelchair turning left (test 2) with the ellipses representing the distribution of the coordinates of the center of gravity (A-F) and the mean position of the center of gravity (M C.G.).

Figure 9. Likelihood of inclusion of new points (test no. 2) within the ellipses (A-F). \(a_E\) and \(b_E\) – semi-axes of the selected ellipse.
During the left turn, an even distribution of the position points of the center of gravity was observed along the x- and y-axis. On the x-axis, the point coordinates are from -60.03 mm to -30.13 mm, and on the y-axis, the coordinates are from 51.53 mm to 111.83 mm. The highest likelihood PE(AE) of inclusion of new position points of the center of gravity within the ellipses was observed for ellipse E at 34.4%. For other ellipses, the likelihood was lower.

The last test included a right turn of the wheelchair by pushing the left wheel and pulling the right wheel. Using the measured data, a graph of the position points of the center of gravity (figure 10) and a graph of the likelihood of occurrence of new points in the ellipses (figure 11) were drawn. During the right turn, an even distribution of the position points of the center of gravity was observed along the x- and y-axis. On the x-axis, the point coordinates are from -53.94 mm to -38.89 mm, and on the y-axis, the coordinates are from 18.39 mm to 48.49 mm. The highest likelihood PD(AD) of inclusion of new position points of the center of gravity within the ellipses was observed for ellipse D at 40.9%. For other ellipses, the likelihood was lower.

Figure 10. Graph of changes in the position of the center of gravity for wheelchair turning right (test 3) with the ellipses representing the distribution of the coordinates of the center of gravity (A-F) and the mean position of the center of gravity (M C.G.).

Figure 11. Likelihood of inclusion of new points (test no. 2) within the ellipses (A-F). aE and bE – semi-axes of the selected ellipse.

4. Summary and conclusions
The method used to measure the position of the center of gravity allows to indirectly determine the position of the entire anthropotechnic system on a horizontal xy-plane. The developed test stand [7] can be used to record changes in the reaction in four scale pan support points. The reactions can be recorded during changes in body position in time. The method allows measuring variability of the position of the center of gravity in dynamic conditions. The method is much more accurate than the optical method [9].

Three sets of points for three tests were determined using the developed method. For test no. 1, the set of position points of the center of gravity is extended along the x-axis and corresponds to the wheelchair propelled forward. This manoeuvre also involves the largest tilt of the wheelchair user body. For left and
right turn, an even distribution of the position points of the center of gravity was observed with a tendency to extend towards the y-axis. In this case, it also corresponds to the biomechanics of pushrim propulsion. When turning, the user body moves slightly as a result of hand movement in the opposite directions. The body tilting in the direction of the manoeuvre can also be observed.

The analysis of the ellipses fitted to the position points of the center of gravity showed an increase in the likelihood of occurrence of new points in the first ellipses and a decrease in the likelihood in other ellipses in two tests due to the superposition of the measuring points as a result of no movement of the human body during the test. The error resulted in a lack of sync between the beginning of the test and the beginning of the body movement. The ellipse was discarded and the likelihood of inclusion of new position points of the center of gravity was verified for other larger ellipses. In each test, an increase in likelihood was observed with an increase in ellipse dimensions until a certain threshold was reached, after which the likelihood dropped. 4.171 points were measured in test no. 1, and the ellipse representing the set of points with the highest likelihood included 2.892 points. The coordinates of the center of the ellipse E (M C.G.) are (-53, 38; 52, 05), semi-axis inclination is $\alpha = 16.17^\circ$, and the lengths of semi-axis a and b are 25 mm and 5 mm, respectively. 4.463 points were measured in test no. 2, and the ellipse E including 3.669 points was selected. The coordinates of the center of the ellipse E (M C.G.) are (-49, 49; 83, 60), semi-axis inclination is $\alpha = 14.73^\circ$, and the lengths of semi-axis a and b are 18 mm and 12 mm, respectively. 3.434 points were measured in test no. 3 and ellipse D including 2.444 points was selected based on the likelihood analysis. The coordinates of the center of the ellipse E (M C.G.) are (-48, 59; 39, 81), semi-axis an inclination is $\alpha = -26.25^\circ$, and the lengths of semi-axis a and b are 6.875 mm and 5.5 mm, respectively. The measured points outside of the ellipse were eliminated as gross errors due to the accidental movement of the human body outside of the test scenario.

5. References
[1] Brubaker C E 1991 Wheelchair prescription: an analysis of factors that affect mobility and performance Journal of Rehabilitation Research and Development 23 19–26
[2] Ekuase A and Aduloju Sunday Ch 2015 Determination of center of gravity and dynamic stability evaluation of a cargo-type tricycle American Journal of Mechanical Engineering 3 26–31
[3] Koontz A M and Cooper R A 2005 A kinetic analysis of manual wheelchair propulsion during start-up on select indoor and outdoor surfaces Journal of Rehabilitation Research and Development 42 447–458
[4] Rao S S and Bontrager E L 1996 Three-Dimensional kinematics of wheelchair propulsion IEEE Transactions on Rehabilitation Engineering 4 152–160
[5] Wieczorek B and Górecki J 2017 The analytical method of determining the center of gravity of a person propelling a manual wheelchair Procedia Engineering 177 405–410
[6] Wieczorek B 2015 Projektowanie dla seniorów i osób z niepełnosprawnościami: badania analizy oceny konstrukcje chapter 6.1 (Poznań: Wydział Maszyn Roboczy i Transportu Politechniki Poznańskiej) pp 415–429
[7] Wieczorek B and Górecki J 2018 A device for Simulating Operating Conditions and Measuring Dynamic Parameters of a Wheelchair Patent Aplication to Patent office of the Republic of Poland no P.424482 ed M Kukla and D Wojtkowiak and D Wilczyński D.
[8] Lemaire E D and Lamontagne M 1991 A technique for the determination of center of gravity and rolling resistance for tilt-seat wheelchairs Journal of Rehabilitation Research and Development 28 51–58
[9] Hasana S Sand Robin D W 1996 Simultaneous measurement of body center of pressure and center of gravity during upright stance Part I: Methods Gait & Posture 4 11–20