Shape, Color, and Distance in Weak Gravitational Flexion

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ABSTRACT
Canonically, elliptical galaxies might be expected to have a perfect rotational symmetry, making them ideal targets for flexion studies—however, this assumption hasn’t been tested. We have undertaken an analysis of low and high redshift galaxy catalogs of known morphological type with a new gravitational lensing code, Lenser. Using color measurements in the u−r bands and fit Sérsic index values, objects with characteristics consistent with early-type galaxies are found to have a lower intrinsic scatter in flexion signal than late-type galaxies. We find this measured flexion noise can be reduced by more than a factor of two at both low and high redshift.

Key words: gravitational lensing: weak – galaxies: general

1 INTRODUCTION
Over the past several decades, gravitational lensing has proven an invaluable tool in direct measurement of Dark Matter structure in clusters. For a review see Schneider et al. (2016) and references therein. The observation of multiple strongly lensed images can be used to determine the radial profile of galaxy clusters yet proves insufficient in probing areas of less-prominent overdensity in the underlying cluster field. Turning to higher-order distortions, such as flexion, provides a method to help identify these low-mass signals (Bacon et al. 2006; Er & Schneider 2011; Goldberg, D. M. & Bacon, D. J. 2005; Lasky & Fluke 2009). Typical lensing analyses use shear by itself, but given that shear and flexion operate on different scales flexion provides an additional avenue for constraining lensing-based measurements in large galaxy clusters.

Understanding the distribution of intrinsic flexion and its behavior with respect to a source galaxy’s shape and color should greatly improve flexion-based techniques. The intrinsic distribution in the flexion signal of a set of source objects will factor into the noise threshold in determining a likely induced flexion signal from lensing fields.

2 BACKGROUND
2.1 Flexion Formalism
Using the thin lens model, we relate the convergence to a dimensionless potential with $\nabla^2 \psi = 2\kappa$. The coordinate backwards mapping problem from foreground positions ($\hat{\theta}$) to background ($\hat{\beta}$) is then related via the linear relation of this potential:

$$\beta_i = \delta_{ij} \theta^j - \psi_{ij} \theta^j - \frac{1}{2} \psi_{ijk} \theta^j \theta^k \tag{1}$$

and the indices vary over the x and y cardinal measurements. We define the complex derivative operator $\partial = \partial_1 + i\partial_2$. The lensing tensors in the expansion may then be related to the observed lensing effects via

$$\mathcal{F} = |\mathcal{F}| e^{i\phi} = \frac{1}{2} \partial_\phi \partial_\psi = \partial_\kappa \tag{2}$$

$$\mathcal{G} = |\mathcal{G}| e^{i\phi} = \frac{1}{2} \partial_\phi \partial_\psi = \partial_\gamma \tag{3}$$

where we note the use of complex notation shows $\mathcal{F}$ as gradient-like.

The first flexion, $\mathcal{F}$ presents itself through a centroid shift in the lensed object. The second flexion, $\mathcal{G}$, is a bit tougher to visualize, as it is a triangular field distortion with a three-fold rotational symmetry. The present study focuses on the measurement of $\mathcal{F}$ exclusively, as first flexion provides a more robust measure for readily identifying observed effects in lensing systems.

A major hurdle in flexion studies is understanding the noise associated with a measured signal. While canonically, elliptical galaxies have concentric elliptical isophotes, deviations from this will contribute to a measurement of the flexion, even when there are no lensing fields present. As galaxies are primarily classifiable as elliptical (early) or spiral (late), it is expected that the presence of strong arms in the latter may lead to an increase in any nonlensing flexion-like signal, or intrinsic flexion, for a sample of galaxies.
Here and throughout, we describe the characteristic size in terms of the quadrupole image moments:

\[ a = \sqrt{|Q_{11} + Q_{22}|} \quad (4) \]

The combination of a galaxy’s size, \( a \), and measured flexion produces a scale-invariant dimensionless measure of lensed objects, \( |\tilde{a}F| \). The same apparent galaxy image produced at different distances will have the same combination of \( |\tilde{a}F| \).

This becomes an excellent measure of the intrinsic flexion in a distribution of galaxies, with the scatter in that distribution producing a measure of the “noise” in the measured flexion for an object of a given size (Goldberg, D. M. & Bacon, D. J. 2005; Goldberg, D. M. & Leonard, A. 2007). Understanding the effect of shape, color and distance on the measured flexion will then provide insight into reducing this value and increasing the useful flexion signals in future studies.

3 LENSER

One factor limiting research into gravitational lensing flexion signals has been the lack of a robust analysis tool. As such, we have developed Lenser\(^1\): a fast, open-source, minimal-dependency Python tool for estimating lensing signals from real survey data or realistically simulated images. The module forward-models second-order lensing effects, performs a point spread function (PSF) convolution, and minimizes a parameter space.

Previous studies on flexion signals have made use of several techniques, including: moment analysis of light distributions (Goldberg & Leonard 2007; Okura et al. 2008), decomposing images into “shaplet” basis sets (Goldberg, D. M. & Bacon, D. J. 2005; Goldberg & Leonard 2007; Massey et al. 2007), and exploring the local potential field through a forward-modeling, parameterized ray tracing known as Analytic Image Modeling (AIM) (Cain et al. 2011). Lenser is intended to be a hybrid approach, first using a moment analysis to localize a best-fit lensing model in parameter space and then performing a local minimization on the model parameters (seven lensing potential parameters, six shape parameters).

The unlensed intensity profile of a galaxy can be well described by a particular model with a corresponding set of model parameters (Sérsic 1963; Graham & Driver 2005). We employ a modified Sérsic-type intensity profile for the modeling galaxies:

\[ I(\theta) = I_0 \exp \left[ -\left(\frac{\theta'}{\theta_s}\right)^{1/n_s} \right] \quad (5) \]

where \( I_0 \) is the central brightness, \( \theta_s \) is the characteristic radius, \( n_s \) is the Sérsic index (a measure of curve steepness), and the radial coordinate \( \theta' \) is given by

\[ \theta' = \sqrt{(x/q)^2 + y^2} \quad (6) \]

where \( x \) and \( y \) are the centroid-subtracted source-plane coordinates rotated appropriately by an orientation angle \( \phi \) and \( q \) is the semimajor-to-semiminor axis ratio of the galaxy. \(^2\)

Equations 4, 5, as well as the centroid \((\theta_0^1, \theta_0^2)\), \( q \), and \( \phi \), create a thirteen-parameter space to describe a galaxy. Recognizing the existence of the shear/ellipticity degeneracy, we initially set shear to zero \( (\psi_{ij} = 0) \) and absorb the degenerate parameters into the intrinsic ellipticity described by \( q \) and \( \phi \). In the context of smoothed mass mapping, the inferred shear can be used as a prior. This leaves us with a ten-parameter space given by

\[ p = \left\{ \theta_0^1, \theta_0^2, n_s, \theta_s, q, \phi, \psi_{111}, \psi_{112}, \psi_{122}, \psi_{222} \right\} . \]

The first stage of Lenser uses an input galaxy image to estimate and subtract a background and estimate the sky and Poisson noise to use as a noise map weighting. If relevant noise maps are available, there is also an option to utilize those directly. A mask is then added so as to include only relevant pixels in the input image, reducing error from spurious light sources. The second stage of Lenser estimates brightness moments from an unweighted quadrupole and hexadecapole calculation to be used as an initial guess for the galaxy model.

With initialized parameter estimates provided by the measured light moments, the final stage of the Lenser pipeline employs a two-step \( \chi^2 \) minimization:

(i) first minimizing over the initially coupled subspace \( \{n_s, \theta_s\} \)

(ii) a final full ten-parameter space local minimization.

3.1 Covariance Testing

It is possible to use Lenser in order to compute a covariance matrix for our parameter space by simulating an ensemble of postage stamp images with noise. It is possible to use Lenser in order to compute a covariance matrix, \( \sigma_{ij} = \langle (p_i - p)(p_j - p) \rangle \), where \( p \) and \( \hat{p} \) are the reconstructed and input parameters, respectively. Once the covariance matrix is calculated, we are able to calculate the marginalized 1σ uncertainty on each parameter simply by taking the square root of the diagonal: \( \sigma_1 = \sqrt{\sigma_{ii}^2} \).

Fig. 1 shows 1σ and 2σ confidence ellipses below the diagonal (lighter and darker shades, respectively) and 1σ Gaussians along the diagonal for two different stamp collections, superimposed. The black and gray correspond to stamp collections with flat noise maps of \( \sigma \). Gaussians along the diagonal for two different stamp collections, we find that reducing the noise map by a factor of two will decrease the covariance by a factor of two (note how the higher-noise 2σ ellipses overlap the lower-noise 1σ ellipses).

We forecast that the uncertainty on measured flexion

\[^2 \ \theta' = \theta \] therefore corresponds to a circularly symmetric galaxy in the limit of no lensing.

\[^1 \ \text{https://github.com/DrexelLenser/Lenser} \]
values would be smaller for galaxy catalogs that have lower noise. Flexion-based techniques could therefore greatly benefit from future surveys seeking to take higher-quality images of galaxies.

4 DATA CATALOGS

4.1 Catalog 1: Low-Redshift Objects

As the main drive for this study is to understand the effect of morphological shape on the measurement of flexion signal in source galaxies, a large catalog of high-quality galaxies was needed. A low-redshift catalog reduces the likelihood of any shape distortions from intervening fields, allowing for an improved estimation of the intrinsic flexion for statistical analysis of weakly lensed source objects.

The EFIGI (Extraction de Formes Idealises de Galaxies en Imagerie) project was developed to robustly measure galaxy morphologies (Baillard, A. et al. 2011). Thus, the publicly available catalog of images and morphological classifications is well suited for our study on intrinsic flexion. The catalog includes imaging data for a total 4458 galaxies from the RC3 Catalogue. The full catalog merges data from several standard surveys, pulling from Principal Galaxy Catalogue, Sloan Digital Sky Survey, Value-Added Galaxy Catalogue, HyperLeda, and the NASA Extragalactic Database. Imaging data was obtained from the SDSS DR4 in the $u$ and $r$ bands, as well as accompanying PSF images.

Galaxies were reduced from the more complex morphological classifications in EFIGI to a simplified early/late/irregular scheme. Objects were initially visually classified with a classifier value corresponding to a characteristic shape in the Hubble sequence (EHS), with a corresponding uncertainty. For increased confidence in the simplified scheme, the intermediate range objects were excised.

The final catalog of objects to be used in the study was reduced to a total of 1551 high-quality, low-redshift objects (597 early/954 late).

Galaxy color measurements in $u$ and $r$ bands from SDSS DR7 were used for optimal morphology separation (Strateva et al. 2001). The interplay between color, shape and distance (source object redshift) should provide effective criteria for selecting source galaxies. The final distribution of galaxy color measurements for Catalog 1 is shown in Fig. 2.

4.2 Catalog 2: High-Redshift Objects

An analysis of high-redshift objects is also useful for investigating how the morphology of real source galaxies affects the measured flexion and what selections should be applied to increase the signal-to-noise in future flexion-based studies.

The CANDELS (Koekemoer 2011) Program contains a large catalog of high-redshift ($z = 0.15$ to $8$), deep-imaging galaxies using the HST WFC3/IR and ACS camera systems, with an emphasis on deep-probing faint, distant galaxies, with the added effect of including many bright background objects that may be analyzed for weak lensing effects. Previous work has produced morphological classifications of the observed galaxies in the COSMOS field (Tasca, L. A. M. et al. 2009).

The creation of image stamps for use in the Lenser pipeline entailed locating objects with known position, redshift, and morphology across several separate catalogs. Using a kd-tree method object matching technique (Maneeewongvatana, S. & Mount, D. 2002) with previous redshift catalogs...
in the COSMOS field, 1492 initial galaxies of known morphological type, color and redshift were identified and analyzed. Several metrics were used to ensure that identified objects were large enough for analysis.

In order to select only highest resolution objects, cuts were made in object size and signal strength to ensure the best-quality fits in high-$z$ objects. A subset of 293 objects was achieved, with the same distribution of morphological type as the larger Catalog 1, to ensure a fair comparison in analyzed behavior (107 early/186 late).

5 RESULTS

5.1 Low-Redshift Objects

The low-$z$ galaxy and lensing parameters, with uncertainties, were estimated using Lenser in the “zero shear” limit. In addition to the flexion estimate, the pipeline yields a characteristic size $a$ (Eq. 4) as well as a Sérsic index, which allows an independent estimate of morphology. Canonically, $n_s = 1$ for spiral galaxy profiles and $n_s = 4$ for elliptical galaxy profiles, but there exists a wide variability.

The distribution of measured Sérsic index and a comparison with color measures are shown in Fig. 4 and 5 respectively. The resulting scatter plot is overlayed on a two-dimensional kernel density estimate (KDE) for visualization purposes, and morphological type classifications are plotted separately.

Looking at Fig. 5, two distinct clusters in the data are readily apparent. There is a clearly separated grouping of objects in the upper-right locus, with a denser but wider grouping concentrated in the lower-left locus. This is consistent with the expectation that early-type objects are redder, with a higher $n_s$ than late-type objects. A morphological separation in the data mostly follows this trend, with late-type objects almost exclusively clustered to the left (clearly corresponding to a “redder” measure of galaxy color), while the early-type objects are more broadly spread but still largely concentrated to the lower right.

We can view the behavior of $|aF|$ along different separations in the data. Initial estimates suggest that the intrinsic flexion scatter is low, surprisingly independent of color (Fig. 7) but not of Sérsic index (Fig. 6). Here the choice of $n_s = 2$ as a splitting value represents the approximate shifting point in the distribution of measured $n_s$ (Fig. 4). A choice of $n_s = 4$ also marks an upper cut, which corresponds to a de Vaucouleurs-esque profile for Eq. 5. The lower value was chosen to favor a lower intrinsic scatter, while not excluding useful measures.

Similarly, a natural cut in the color distribution can be seen around $u - r = 2.4$ (Fig. 2). As expected, we anticipate early-type galaxies should be more well behaved in a measure of intrinsic flexion, as the galaxy profiles are less likely to contain artifacts to distort a measurement. This behavior matches the split in measured Sérsic index.

More broadly, we can see the topography of the Sérsic index/color diagram can be divided into “quadrants” (Fig. 5), based on the bimodal natures of both color and measured $n_s$ values. Thus, we expect large clustering in the upper-right and lower-left quadrants of the diagram, which is indeed evident. As such, the majority of objects in QI are expected to be late-type objects, and conversely, QIII are early-type objects. There exists a more mixed clustering in objects in QII, indicating that this range of profile and color measures is where the transition from a classically elliptical galaxy begins to form spiral-like features while still containing the young massive stars that dominate a galaxy’s color. The fourth quadrant is almost completely void of measurements.
indicating little overlap between a lower color and broader profile.

Using only these hard cuts on the \( n_s \) and \( u - r \) values, we can see how the intrinsic scatter in \(|a \, F|\) can be greatly reduced:

### Table 1. Measured scatter in \(|a \, F|\) for Catalog 1

| Quadrant | \( \sigma_{|a \, F|} \) |
|----------|----------------------|
| QI       | 0.0958               |
| QII      | 0.1664               |
| QIII     | 0.0336               |
| QIV      | 0.0505               |
| Full Set | 0.0920               |

#### 5.2 High-Redshift Objects

Following the same procedure as Catalog 1, the objects in Catalog 2 were analyzed using the Lenser pipeline. As in the more robust low-redshift study, a comparison of color measurements and the measured \( n_s \) values shows distinctive split in the data. Object measurements are clustered in either lower \( n_s \)/less red or higher \( n_s \)/more red subgroups.

Here, the split in the Sérsic index distribution is taken to be \( n_s = 1.4 \), as there is a noticeable drop in the distribution shape (Fig. 9). Measures of the \(|a \, F|\) distributions for splits in \( n_s \), color and independently identified morphology are shown (Fig. 11, 12, 13). Note that for this sample of high-\( z \) objects, all major splits in measured galaxy qualities produce a lower intrinsic scatter.

Again, we see that by using broad cuts for strategic points in the full data set, the expected noise in \(|a \, F|\) can be consistently and systematically reduced. Though the separa-
Figure 12. Distributions of $|a\cdot F|$ for high-$z$ objects, split by $u-r$.

Figure 13. Distributions of $|a\cdot F|$ for high-$z$ objects, split by morphological type.

Table 2. Measured scatter in $|a\cdot F|$ for Catalog 2.

| Quadrant | $\sigma_{|a\cdot F|}$ |
|----------|---------------------|
| QI       | 0.0832              |
| QII      | 0.0856              |
| QIII     | 0.0370              |
| QIV      | 0.0665              |
| Full Set | 0.0777              |

6 SUMMARY

For both the high- and low-redshift catalogs, it can be seen that the observed $|a\cdot F|$ distribution favors a smaller scatter in objects that are consistent with an early-type shape (QIII in our shorthand). This behavior agrees with the expectation of galaxy shape providing a significant influence on measurable flexion signal. For high-redshift objects, which are cosmologically younger, we can still achieve a significant reduction in the intrinsic scatter for a sample of galaxies.

The major takeaway of this study is to show how detected flexion signal can be boosted through a selection discrimination on chosen source galaxies. By choosing identified source objects with favorable characteristics toward more early-type galaxies, the anticipated noise in future lensing studies attributed to purely unlensed shape information can be reduced by a factor of more than 2. As an example, the effect of this selection would allow for a flexion-based detection with a S/N of 1 for an elliptical source object of size $a \approx 0.04''$ and redshift $z = 0.1$ at a separation of $1''$ from a $\sigma_v = 300$ km/s lens galaxy.

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