Big-Bang Nucleosynthesis with Unstable Gravitino
and Upper Bound on the Reheating Temperature

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Abstract

We study the effects of the unstable gravitino on the big-bang nucleosynthesis. If the gravitino mass is smaller than $\sim 10$ TeV, primordial gravitinos produced after the inflation are likely to decay after the big-bang nucleosynthesis starts, and the light element abundances may be significantly affected by the hadro- and photo-dissociation processes as well as by the $p \leftrightarrow n$ conversion process. We calculate the light element abundances and derived upper bound on the reheating temperature after the inflation. In our analysis, we calculate the decay parameters of the gravitino (i.e., lifetime and branching ratios) in detail. In addition, we performed a systematic study of the hadron spectrum produced by the gravitino decay, taking account of all the hadrons produced by the decay products of the gravitino (including the daughter superparticles). We discuss the model-dependence of the upper bound on the reheating temperature.
1 Introduction

Low-energy supersymmetry, which is one of the most prominent candidates of the physics beyond the standard model, may significantly affect the evolution of the universe. One reason is that, assuming $R$-parity conservation, the lightest superparticle (LSP) is stable, which becomes a well-motivated candidate of the cold dark matter. Another famous reason, which is very important in the framework of local supersymmetry (i.e., supergravity), is that there exist various possible very weakly interacting particles in the particle content. The most important example is the gravitino, which is the gauge field for the local supersymmetry.

Since the gravitino is the superpartner of the graviton, its interaction is suppressed by inverse powers of the gravitational scale and hence its interaction is very weak. Even though the gravitino is very weakly interacting, however, it can be produced by the scattering processes of the standard-model particles (and their superpartners) in the thermal bath in the early universe. Once produced, the gravitino decays with very long lifetime. In particular, if the gravitino mass is smaller than $\sim 20$ TeV, the lifetime becomes longer than $\sim 1$ sec and hence the primordial gravitinos decay after the big-bang nucleosynthesis (BBN) starts. The (unstable) gravitino is expected to be relatively heavy, and its decay releases energetic particles which cause dissociation processes of the light elements generated by the standard BBN reactions. Since the standard BBN scenario more or less predicts light-element abundances consistent with the observations, those dissociation processes are harmful and, if the abundance of the primordial gravitinos are too much, resultant abundances of the light elements become inconsistent with the observations. Such argument provides significant constraints on the primordial abundance of the gravitino $[1]$. #1

In the inflationary scenario, which is also strongly suggested by the WMAP data $[4]$, gravitinos are once diluted by the entropy production after the inflation. Even in this case, however, gravitinos are produced again by the scattering processes of the particles in the thermal bath. Since the total amount of the gravitinos produced by the scattering processes is approximately proportional to the reheating temperature $T_R$, the BBN scenario is too much affected to be consistent with the observations if the reheating temperature is high. In the past, the BBN constraints on the unstable gravitino have been intensively studied $[2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]$. #2

Recently, Kawasaki and two of the present authors (K.K. and T.M.) have studied the constraints on the unstable long-lived particle from the BBN in detail $[15, 16]$; in particular, in this paper, effects of the hadrons produced by the decay of such unstable particles (as well as the effects of the photo-dissociation) were systematically studied, #3 and general

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#1 Constraints on the case with the gravitino LSP have been considered in Refs. $[2, 3]$.  
#2 In order to relax the constraints, several scenarios have been studied. In Ref. $[18]$, it was discussed that the modification of the expansion rate by the 5D effect in the brane-world cosmology, which may reduce the abundance of the gravitino. In Ref. $[19]$, they studied possibilities of the dilution of the gravitino by the late-time entropy production due to the decaying moduli without newly producing many gravitinos.  
#3 For old studies on the effects of the hadronic decay modes, see also $[20, 21]$.  

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constraints on the primordial abundance of such unstable particles were presented. Then, the results were applied to the case of the unstable gravitino and the upper bound on the reheating temperature was obtained for several simple cases.

In [15, 16], however, several simplifications and assumptions are made for the properties of the gravitino. First of all, several very simple decay patterns of the gravitino were considered, which are applicable for very specific mass spectrum of the superparticles. In addition, for the hadronic branching ratio of the gravitino, only several typical values were used to obtain the constraint. Furthermore, for some cases (in particular, for the case where the gravitino dominantly decays into the gluon and the gluino), effects of the hadrons emitted from the superparticles (like the gluino) were neglected. Thus, it is desirable to perform more detailed and complete analysis of the upper bound on the reheating temperature with the unstable gravitino.

In this paper, we study the effects of the unstable gravitinos on the BBN, paying particular attention to the properties of the gravitino. Compared to the previous works, decay processes of the gravitino (and the decay chains of the decay products including the superparticles) are precisely and systematically studied. As a result, energy spectra of the hadrons (in particular, proton, neutron, and pions) produced by the gravitino decay are studied in detail for various mass spectrum of the superparticles.

The organization of this paper is as follows. In Section 2 we present the model we consider and summarize the important parameters for our analysis. In Section 3 detail of the decay processes of the gravitino is discussed. Some of the important issues in our analysis, which is the secondary decays of the daughter particles from the gravitino decay and their hadronization processes, are discussed in Section 4. Then, in Section 5 outline of our calculation of the light-element abundances is discussed. Our main results are given in Section 6 and Section 7 is devoted for conclusions and discussion.

2 Model

In this paper, we adopt the minimal particle content to derive the upper bound on the reheating temperature. Thus, the model we consider includes the particles in the minimal supersymmetric standard model (MSSM) as well as the gravitino. These particles are listed in Table 1.

In order to precisely calculate the decay rate of the gravitino, it is necessary to obtain the mass eigenvalues and mixing parameters of the superparticles. Thus, we briefly summarize the relation between the gauge-eigenstate and mass-eigenstate bases here.

We start with the neutralino sector. With the $SU(2)_L$ and $U(1)_Y$ gaugino masses $M_2$ and $M_1$ as well as the supersymmetric Higgs mass $\mu_H$, the mass matrix of the neutralinos
Particles | Notation
--- | ---
Gravitino | $\psi_\mu$
Neutralinos | $\chi^0_1$, $\chi^0_2$, $\chi^0_3$, $\chi^0_4$
Charginos | $\chi^\pm_1$, $\chi^\pm_2$
Gluino | $\tilde{g}$
Squarks | $\tilde{u}_L$, $\tilde{u}_R$, $\tilde{d}_L$, $\tilde{d}_R$, $\tilde{s}_L$, $\tilde{s}_R$, $\tilde{c}_L$, $\tilde{c}_R$, $\tilde{b}_1$, $\tilde{b}_2$, $\tilde{t}_1$, $\tilde{t}_2$
Sleptons | $\tilde{e}_L$, $\tilde{e}_R$, $\tilde{\mu}_L$, $\tilde{\mu}_R$, $\tilde{\tau}_1$, $\tilde{\tau}_2$, $\tilde{\nu}_{eL}$, $\tilde{\nu}_{\mu L}$, $\tilde{\nu}_{\tau L}$
Photon | $\gamma$
Weak bosons | $Z$, $W^\pm$
Gluon | $g$
Neutral CP even Higgses | $h$, $H$
CP odd Higgs | $A$
Charged Higgs | $H^\pm$
Quarks | $q = u$, $d$, $s$, $c$, $b$, $t$
Leptons | $e$, $\mu$, $\tau$, $\nu_e$, $\nu_\mu$, $\nu_\tau$

Table 1: List of particles in the mass-eigenstate bases.

is given in the form

$$M_{\chi^0} = \begin{pmatrix}
M_1 & 0 & -m_Z \sin \theta_W \cos \beta & m_Z \sin \theta_W \sin \beta \\
0 & M_2 & m_Z \cos \theta_W \cos \beta & -m_Z \cos \theta_W \sin \beta \\
-m_Z \sin \theta_W \cos \beta & m_Z \cos \theta_W \cos \beta & 0 & -\mu_H \\
m_Z \sin \theta_W \sin \beta & -m_Z \cos \theta_W \sin \beta & -\mu_H & 0
\end{pmatrix}, \tag{2.1}
$$

where $m_Z$ is the $Z$-boson mass while $\theta_W$ is the weak mixing angle, and $\tan \beta$ is the ratio of the vacuum expectation value of up- and down-type Higgs bosons. This mass matrix is diagonalized by a unitary matrix $U_{\chi^0}$ as

$$U_{\chi^0}^* M_{\chi^0} U_{\chi^0}^{-1} = \text{diag}(m_{\chi^0_1}, m_{\chi^0_2}, m_{\chi^0_3}, m_{\chi^0_4}). \tag{2.2}$$

In addition, for the chargino sector, the mass matrix is given by

$$M_{\chi^{\pm}} = \begin{pmatrix}
M_2 & \sqrt{2} m_W \cos \beta \\
\sqrt{2} m_W \sin \beta & \mu_H
\end{pmatrix}, \tag{2.3}
$$

where $m_W$ is the $W^\pm$-boson mass. We define the unitary matrices diagonalizing $M_{\chi^{\pm}}$ as $U_{\chi^+}$ and $U_{\chi^-}$:

$$U_{\chi^+}^* M_{\chi^{\pm}} U_{\chi^-}^{-1} = \text{diag}(m_{\chi^{\pm}_1}, m_{\chi^{\pm}_2}). \tag{2.4}$$

#^4 We used the convention of [22].
For the neutral Higgs boson, the gauge-eigenstates (i.e., the up-type Higgs $H^0_u$ and down-type Higgs $H^0_d$) and the mass-eigenstates are related by using the mixing angle $\alpha$:

$$
\begin{pmatrix}
H \\
h
\end{pmatrix} = \sqrt{2} \begin{pmatrix}
\cos \alpha & \sin \alpha \\
-\sin \alpha & \cos \alpha
\end{pmatrix} \begin{pmatrix}
\text{Re}(H^0_d) - v_1 \\
\text{Re}(H^0_u) - v_2
\end{pmatrix},
$$

(2.5)

where $v_1$ and $v_2$ are vacuum expectation values of $H^0_d$ and $H^0_u$, respectively.

In addition, we have to consider the mixings in the squark and slepton mass matrices. For simplicity, we do not consider the generation mixing in the squark and slepton sector. We also neglect the left-right mixing in the first and second generation squarks and sleptons since such mixing is small in many class of models, in particular, in the minimal supergravity (mSUGRA) type models which we adopt in our analysis. For the third-generation squarks and sleptons, we take account of the effects of the left-right mixing. Such mixing is parameterized by unitary matrices $U_{\tilde{t}}$, $U_{\tilde{b}}$, and $U_{\tilde{\tau}}$ which diagonalize the mass-squared matrices of the squarks and sleptons:

$$
U_{\bar{f}}M^2_{\bar{f}}U^{-1}_{\bar{f}} = \text{diag}(m^2_{\tilde{f}_1}, m^2_{\tilde{f}_2}) : \bar{f} = \tilde{t}, \tilde{b}, \tilde{\tau}.
$$

(2.6)

In our analysis, we consider the case where the gravitino is unstable and the LSP is contained in the MSSM particles. Since the charged or colored LSP is disfavored, we consider the case where the LSP is the lightest neutralino $\chi^0_1$. Consequently, the gravitino is assumed to be heavier than $\chi^0_1$. Of course, the gravitino may be heavier than other superparticles and hence the lifetime and branching ratios for possible decay modes of the gravitino depend on the mass spectrum. We calculate these quantities in detail, as we explain below.

In order to calculate the decay rate of the gravitino, it is necessary to fix the mass spectrum and the mixing matrices in the MSSM sector. Although the effects of the gravitino on the BBN can be calculated for arbitrary mass spectrum of the MSSM particles, it is not practical to study all the possible cases since there are very large number of parameters in the MSSM sector. Thus, we adopt a simple parameterization of the SUSY breaking parameters, that is, the mSUGRA-type parameterization of the soft SUSY breaking parameters. We (parameterize the MSSM parameters by using unified gaugino mass $m_{1/2}$, universal scalar mass $m_0$, universal coefficient for the tri-linear scalar coupling $A_0$, ratio of the vacuum expectation values of two Higgs bosons $\tan \beta$, and supersymmetric Higgs mass $\mu_H$. Then, the properties of the superparticles (including the gravitino) are determined once these parameters as well as the gravitino mass $m_{3/2}$ are fixed and, consequently, we can derive the upper bound on the reheating temperature. Notice that, although we adopt the simple parameterization of the soft SUSY breaking parameters, our analysis is applicable to more general cases as far as the gravitino is heavier than one of the MSSM superparticles (like the lightest neutralino).

Even with the mSUGRA parameterization of the MSSM parameters, the whole parameter space is still too large to be completely studied. Thus, in this paper, we pick up several typical mSUGRA points and derive the constraints for these points. In particular,
we pick up points where the thermal relic density of the LSP becomes consistent with the dark matter density determined by the WMAP observation [4]. The points we consider are listed in Table 2. For all the cases, we checked that the lightest neutralino becomes the LSP (if the gravitino mass is larger than \( m_{\chi_1^0} \)). Using the DarkSUSY package [23], we calculated the thermal relic density of the LSP \( \Omega_{\text{LSP}}^{(\text{thermal})} \).\(^5\) (We use \( h = 0.71 \) [4], where \( h \) is the expansion rate of the universe in units of 100 km/sec/Mpc.)

### Table 2: mSUGRA parameters used in our analysis.

|           | Case 1   | Case 2   | Case 3   | Case 4   |
|-----------|----------|----------|----------|----------|
| \( m_{1/2} \) | 300 GeV  | 600 GeV  | 300 GeV  | 1200 GeV |
| \( m_0 \)   | 141 GeV  | 218 GeV  | 2397 GeV | 800 GeV  |
| \( A_0 \)   | 0        | 0        | 0        | 0        |
| \( \tan \beta \) | 30  | 30       | 30       | 45       |
| \( \mu_H \) | 389 GeV  | 726 GeV  | 231 GeV  | −1315 GeV|
| \( m_{\chi_0^0} \) | 117 GeV  | 244 GeV  | 116 GeV  | 509 GeV  |
| \( \Omega_{\text{LSP}}^{(\text{thermal})} H^2 \) | 0.111    | 0.110    | 0.106    | 0.111    |

\( ^5 \)The Cases 1 and 2 are in the so-called “co-annihilation region,” while the Cases 3 and 4 are in the “focus-point region” and “Higgs funnel region,” respectively.

### 3 Gravitino Decay

In order to study the effects of the gravitino on the BBN, it is important to understand the lifetime of the gravitino \( \tau_{3/2} \) and its decay modes. Since the gravitino is the gauge field for the supersymmetry, gravitino couples to the supercurrent and hence its interaction is unambiguously determined. Possible decay modes have, however, model-dependence. In the following, we discuss how the decay rate and the branching ratios of the gravitino are calculated.

#### 3.1 Interaction and two-body processes

We first briefly summarize the interactions of the gravitino.\(^6\) Gravitino is the super-partner of the graviton, and it couples to the supercurrent. Thus, (relevant part of) the interaction of the gravitino is given by

\[
\mathcal{L}_{\text{int}} = -\frac{1}{8M_*} \sum_G \bar{\chi}^{(G)} \gamma_5 \gamma^\mu [\gamma^\rho, \gamma^\sigma] \psi_{\mu} F^{(G)}_{\rho\sigma} \\
- \frac{1}{\sqrt{2}M_*} \sum_C \left[ \bar{\chi}^{(C)} \gamma^\mu \gamma^\nu P_L \psi_{\mu} D_{\nu} \phi^{(C)} + \text{h.c.} \right],
\]

\( ^6 \)For details, see, for example, [24].
where the sum over $G$ is for all the gauge multiplets (consisting of the gauge field $A^{(G)}_{\mu}$ and the gaugino $\lambda^{(G)}$) while $C$ for the chiral multiplets (consisting of the scalar boson $\phi^{(C)}$ and the fermion $\chi^{(C)}$). Here, $F^{(G)}_{\mu\nu}$ is the field strength for $A^{(G)}_{\mu}$, and $D_{\nu}$ represents the covariant derivative. In addition, $M_* \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck scale. As is obvious from Eq. (3.1), if we restrict ourselves to consider the two-body final states, gravitino decays into some standard-model particle and its superpartner.

From the Lagrangian given in Eq. (3.1), we can read off the vertex factors for the gravitino and calculate the (partial) decay rates of the gravitino. We first consider the decay processes with two-body final states. Then, the decay rate is expressed as

$$\Gamma_{\text{gauge}} = \frac{\beta_f N_C}{16\pi m_{3/2} M_*^2} \times |\mathcal{M}|^2,$$

(3.2)

where $\mathcal{M}$ represents the Feynman amplitude for the decay process (with $M_* = 1$), and $N_C$ is the color factor: $N_C = 8$ for the process $\psi_{\mu} \rightarrow g \tilde{g}$, $N_C = 3$ for the processes with quark and squark final state, and $N_C = 1$ otherwise. In addition, for the process $\psi_{\mu} \rightarrow AB$, $\beta_f$ is given by

$$\beta_f = \frac{1}{m_{3/2}^2} \left[ m_{3/2}^4 - 2m_{3/2}^2 (m_A^2 + m_B^2) + (m_A^2 - m_B^2)^2 \right]^{1/2},$$

(3.3)

where $m_A$ and $m_B$ are masses of $A$ and $B$, respectively.

For the gravitino decay process into a gauge boson $V$ and a fermion $\chi$, we define $p$, $q$, and $q'$ to be the momenta of $\psi_{\mu}$, $V$, and $\chi$, respectively. Then, we obtain

$$pq = \frac{1}{2} \left( m_{3/2}^2 + m_V^2 - m_\chi^2 \right),$$

(3.4)

$$pq' = \frac{1}{2} \left( m_{3/2}^2 - m_V^2 + m_\chi^2 \right),$$

(3.5)

$$qq' = \frac{1}{2} \left( m_{3/2}^2 - m_V^2 - m_\chi^2 \right),$$

(3.6)

where $m_V$ and $m_\chi$ are masses of $V$ and $\chi$, respectively. With these quantities, for the process with a massless gauge field in the final state (i.e., $\psi_{\mu} \rightarrow \gamma \chi_0^0$ and $g \tilde{g}$), we obtain

$$|\mathcal{M}|^2_{\gamma,g} = \frac{2}{3} \left( C_L^{(G)} C_L^{(G)*} + C_R^{(G)} C_R^{(G)*} \right) \left[ \frac{(pq)^2(pq')}{{m_{3/2}}^2} + (pq)(qq') \right],$$

(3.7)
while with massive gauge field in the final state (i.e., $\psi^\mu \to Z\chi^0_i$ and $W^\pm \chi^{\mp}_i$), we obtain

$$|M|_{W^\pm,Z}^2 = \frac{2}{3} \left( C_{L}^{(G)} C_{L}^{(G)*} + C_{R}^{(G)} C_{R}^{(G)*} \right) \left[ \frac{(pq)^2(pq')}{m_{3/2}^2} + (pq)(qq') - \frac{1}{2} m_V^2(pq') \right]$$

$$- \left( C_{L}^{(G)} C_{R}^{(G)*} + C_{R}^{(G)} C_{L}^{(G)*} \right) m_{3/2} m_{\chi}^2$$

$$+ \frac{2}{3} \left( C_{L}^{(H)} C_{L}^{(H)*} - C_{R}^{(H)} C_{R}^{(H)*} + \text{h.c.} \right) m_{3/2} \left[ \frac{1}{2} (qq') + \frac{(pq)(pq')}{m_{3/2}^2} \right]$$

$$+ \left( C_{L}^{(H)} C_{R}^{(H)*} - C_{R}^{(H)} C_{L}^{(H)*} + \text{h.c.} \right) m_{\chi}(pq)$$

$$+ \frac{2}{3} \left( C_{L}^{(H)} C_{L}^{(H)*} + C_{R}^{(H)} C_{R}^{(H)*} \right) \left[ 1 + \frac{(pq)^2}{2m_{3/2}^2 m_{V}^2} \right] (pq')$$

$$+ \frac{2}{3} \left( C_{L}^{(H)} C_{L}^{(H)*} + C_{R}^{(H)} C_{R}^{(H)*} \right) \left[ 1 + \frac{(pq)^2}{2m_{3/2}^2 m_{V}^2} \right] m_{3/2} m_{\chi}.$$  \(3.8\)

Here, $C_{L}^{(G)}$, $C_{R}^{(G)}$, $C_{L}^{(H)}$, and $C_{R}^{(H)}$ are vertex factors. These vertex factors for individual processes are given in Appendix \(A\). For the decay processes with a scalar boson in the final state, we identify $p$, $q$, and $q'$ to be the momenta of $\psi^\mu$, scalar boson $\phi$, and fermion $\chi$, respectively. Then, the products of the momenta can be obtained from Eqs. \(3.4\) – \(3.6\) by replacing $m_V \to m_{\phi}$, with $m_{\phi}$ being the mass of the scalar boson. Then, we obtain

$$|M|_{\text{scalar}}^2 = \frac{1}{3} \left[ \frac{(pq)^2}{m_{3/2}^2} - m_{\phi}^2 \right]$$

$$\left[ \left( C_{L}^{(C)} C_{L}^{(C)*} + C_{R}^{(C)} C_{R}^{(C)*} \right) (pq') + \left( C_{L}^{(C)} C_{R}^{(C)*} + C_{R}^{(C)} C_{L}^{(C)*} \right) m_{3/2} m_{\chi} \right].$$  \(3.9\)

The vertex factors $C_{L}^{(C)}$ and $C_{R}^{(C)}$ are also given in Appendix \(A\).

### 3.2 Three-body processes

In this paper, we consider the case where the LSP is the lightest neutralino $\chi^0_1$. Then, the two-body process $\psi^\mu \to \gamma \chi^0_1$ is always allowed, and the (total) decay rate of the gravitino is determined by the two-body process(es). In studying the effects of the gravitino on the BBN, however, it is also important to understand the spectrum of the hadrons produced by the decay of the gravitino.

In most of the cases, the number of the hadrons from the gravitino decay is mostly determined by the two-body processes. For example, if the gravitino can decay into a superparticle other than the LSP, the emitted superparticle decays into $\chi^0_1$. In this secondary decay process, sizable number of the hadrons may be produced. In addition,
when the mass difference between the gravitino and the lightest neutralino is larger than \( m_Z \), the decay process \( \psi_\mu \to Z\chi_1^0 \) becomes kinematically allowed. In this case, the decay of the \( Z \)-boson produces large amount of the hadrons. In most of the cases, the number of the hadrons produced from those two-body processes is much larger than that from the three body processes. Then, the three-body processes are irrelevant for our study.

However, in some case, precise determination of the hadron spectrum requires the calculation of the three-body final state processes. In particular, the three-body processes become important if (i) \( m_{3/2} - m_{\chi_1^0} < m_Z \), and (ii) all the superparticles except \( \chi_1^0 \) and sleptons are heavier than the gravitino. Notice that, when the condition (i) is satisfied, it may be the case that the only possible two-body decay process is \( \psi_\mu \to \gamma\chi_1^0 \) and hence the hadrons are not produced by the two-body process. In some case, gravitino may also decay into lepton and slepton pair, but the decays of the (s)leptons does not produce significant amount of hadrons in our case. Thus, we pay particular attention to the process \( \psi_\mu \to q\bar{q}\chi_1^0 \) when \( m_{3/2} - m_{\chi_1^0} < m_Z \).

When the LSP is the neutralino, the relevant three-body processes are induced by the diagrams shown in Fig. 1. In our study, we consider the effects of all the diagrams listed in Fig. 1 and calculate the decay rate for the process \( \psi_\mu \to q\bar{q}\chi_1^0 \). To see the importance of the 3-body process, in Fig. 2 we plot the “3-body hadronic decay width” defined as 

\[
\Gamma(\psi_\mu \to q\bar{q}\chi_1^0) \equiv \Gamma(\psi_\mu \to u\bar{u}\chi_1^0) + \Gamma(\psi_\mu \to d\bar{d}\chi_1^0) + \Gamma(\psi_\mu \to s\bar{s}\chi_1^0) \\
+ \Gamma(\psi_\mu \to c\bar{c}\chi_1^0) + \Gamma(\psi_\mu \to b\bar{b}\chi_1^0) + \Gamma(\psi_\mu \to t\bar{t}\chi_1^0). \tag{3.10}
\]

In Fig. 2 the MSSM parameters are taken to be those for the Case 1. In this case, the 3-body decay is induced dominantly by the photon-mediated diagram. When the mass difference between the gravitino and the LSP becomes larger than \( m_Z \), however, \( Z \)-boson mediated contribution with \( m_{q\bar{q}} \approx m_Z \) becomes the most important, where \( m_{q\bar{q}} \) is the invariant mass of the \( q\bar{q} \) system. In fact, such a process should rather be classified into the 2-body process \( \psi_\mu \to Z\chi_1^0 \) followed by \( Z \to q\bar{q} \). To see this more explicitly, we also plot the quantity \( \Gamma(\psi_\mu \to Z\chi_1^0) \times \text{Br}(Z \to q\bar{q}) \). As one can see, \( \Gamma(\psi_\mu \to q\bar{q}\chi_1^0) \) is well approximated by \( \Gamma(\psi_\mu \to Z\chi_1^0) \times \text{Br}(Z \to q\bar{q}) \) when the decay process \( \psi_\mu \to Z\chi_1^0 \) becomes kinematically allowed.

In our numerical study, we treat the process \( \psi_\mu \to q\bar{q}\chi_1^0 \) in the following way:

- When \( m_{3/2} - m_{\chi_1^0} < m_Z \), we calculate the Feynman diagrams shown in Fig. 1 and calculate the decay rate \( \Gamma(\psi_\mu \to q\bar{q}\chi_1^0) \). (In this case, the decay process \( \psi_\mu \to Z\chi_1^0 \) is kinematically forbidden and hence is irrelevant.)

- When \( m_{3/2} - m_{\chi_1^0} > m_Z \), we approximate the hadronic decay rate induced by the diagrams in Fig. 1 by \( \Gamma(\psi_\mu \to Z\chi_1^0) \times \text{Br}(Z \to q\bar{q}) \).

\#7 As we discuss in the following, the three-body process becomes important when the mass difference between the gravitino and the LSP is smaller than \( m_Z \). Thus, the process \( \psi_\mu \to t\bar{t}\chi_1^0 \) is not important for our analysis.
Figure 1: Feynman diagrams for the process $\psi_\mu \rightarrow q\bar{q}\chi^0_1$. The “blobs” are from the gravitino-supercurrent interaction. For (d), there is also CP-conjugated diagram (with the replacements $q \leftrightarrow \bar{q}$ and $\tilde{q} \leftrightarrow \tilde{q}$).

With our treatment, the effect of the photon-mediated diagram (as well as those from Figs. (c) and (d)) is neglected when $m_{3/2} - m_{\chi^0_1} > m_Z$. However, effect of such diagram is subdominant since the process is mainly induced by the Z-boson mediated diagram.

When $m_{3/2} - m_{\chi^0_1} > m_Z$, energy distribution of the quark and anti-quark is easily calculated since the decay is dominated by the process with $m_{q\bar{q}} \simeq m_Z$. When $m_{3/2} - m_{\chi^0_1} < m_Z$, on the contrary, $m_{q\bar{q}}$ has broader distribution. Thus, in this case, we numerically calculate the differential decay rate

$$\frac{d\Gamma(\psi_\mu \rightarrow q\bar{q}\chi^0_1)}{dm^2_{q\bar{q}}}$$

to obtain the energy distributions of the quarks and anti-quarks emitted from the three-body processes. (In the calculation of $d\Gamma(\psi_\mu \rightarrow q\bar{q}\chi^0_1)/dm^2_{q\bar{q}}$, we approximated that the
We adopt the mSUGRA parameters for the Case 1. For comparison, the decay rate \( \Gamma(\psi_\mu \rightarrow Z\chi^0_1) \times \text{Br}(Z \rightarrow q\bar{q}) \) is also shown in the dashed line.

The hadron spectrum from the three-body decay process is obtained by using this differential decay rate. When \( m_{3/2} - m_{\chi^0_1} < m_Z \), the photon-mediated diagram gives the dominant contribution while the effects of other diagrams are almost irrelevant (unless \( m_{3/2} - m_{\chi^0_1} \) is very close to \( m_Z \)). In Appendix B, we present the approximated formula of the differential decay rate, only taking account of the photon-mediated diagram.

### 3.3 Lifetime and branching ratios of the gravitino

Now we can quantitatively discuss the decay rates of the gravitino. First, following the procedures discussed in the previous subsections, we calculate the partial decay rates of the gravitino for all the possible decay modes. Adding all the contributions, we obtain the lifetime of the gravitino:

\[
\tau_{3/2} = \frac{1}{\Gamma(\psi_\mu \rightarrow \text{all})}. \tag{3.11}
\]

We calculate \( \tau_{3/2} \) as a function of the gravitino mass for the cases listed in Table 2 and the results are shown in Fig. 3. Importantly, lifetime of the gravitino becomes shorter as the gravitino becomes heavier.

As one can see, when the gravitino mass is smaller than a few TeV, \( \tau_{3/2} \) for the Case 4 is found to be longer than those for other cases. This is due to the fact that, for the Case 4, masses of the MSSM particles are larger than other cases and hence the decay rates of the gravitino in this case is suppressed. When the gravitino is much heavier than the...
MSSM particles, on the contrary, the lifetime of the gravitino is insensitive to the mass spectrum of the MSSM particles.

Importantly, when the gravitino is lighter than $\sim 20$ TeV, $\tau_{3/2}$ becomes longer than 1 sec and hence the relic gravitinos decay after the BBN starts. Thus, in particular in this case, significant constraints on the reheating temperature after the inflation is expected.

Branching ratios of the decay process of the gravitino depends on the model parameters. To see this, for the Cases 1 $-$ 4, we plot the branching ratios for various two-body final states in Fig. 4. As one can see, the branching ratios have sizable model dependence when the gravitino mass is relatively small. This is because, when the gravitino mass is small, decay rate of the gravitino is sensitive to the mass spectrum of the superparticles. When $m_{3/2}$ becomes large enough, on the contrary, branching ratios become insensitive to the model parameters.

4 Secondary Decays and Hadronization

Although the gravitino primarily decays into a standard-model particle and its superpartner (or into the $q\bar{q}\chi^0_1$ final state), most of the daughter particles also decay with time scale much shorter than the cosmological time scale. In addition, all the partons (i.e., quarks and gluon) are hadronized into mesons or baryons. These processes are important in the study of the BBN with the primordial gravitinos and, in this section, we discuss these issues.

The possible decay modes of the individual superparticles strongly depend on the mass spectrum as well as on the mixing and coupling parameters. In order to systematically take account of all the relevant decay processes, we use the ISAJET package which automatically calculates the partial decay rates of all the unstable particles.
Figure 4: Branching ratios of the decay of the gravitino as functions of the gravitino mass. The thick solid line is for the final states $\chi_1^0 + \text{anything}$, dot-dashed line for lepton-slepton pairs, dotted line for $\chi_i^0$ ($i = 2 - 4$) or chargino + anything, dashed line for gluon-gluino pair, and thin solid line for quark-squark pair final states.

In order to discuss the hadro-dissociations induced by the gravitino decay, we should first calculate the spectra of the partons (i.e., $u, d, s, c, b, t$ and their anti-particles, and gluon). There are two types of processes producing energetic partons: one is the decay of the gravitino and the other is the subsequent decays of the daughter particles. Spectra of the partons of the first type are directly calculated by using the partial decay rates of the gravitino presented in the previous section. (Here, the effects of the “3-body” processes are also included when $m_{3/2} - m_{\chi_1^0} < m_Z$.) In order to calculate the contribution of the second type, we have to follow the decay chain of the unstable particles. In addition, the parton spectra should be calculated by averaging over all the possible decay processes with the relevant branching ratios of individual particles. In our analysis, the decay chain is followed by using the PYTHIA package [26] which automatically take account of the decay processes of the unstable particles (including the superparticles).

At the cosmic time relevant for the BBN, time scale for the hadronization is much
shorter than the time scale for the scattering processes and hence all the partons are hadronized before scattering off the background particles. Thus, it is necessary to calculate the spectrum of the mesons and baryons produced from the partons. In particular, for our analysis of the BBN-related processes, proton, neutron, and charged pions play significant roles.

In our analysis, the hadronization processes are dealt with the PYTHIA package [26]; we have modified the PYTHIA package to include the primary decay processes of the gravitino, then the subsequent decay processes of the daughter particles (including the superparticles) and the hadronization processes of the emitted partons are automatically followed by the original PYTHIA algorithm. In Fig. 5 we plot the distribution functions of the proton and neutron as functions of their kinetic energy (i.e., $E_{\text{kin}} = E - m_N$ with $m_N$ being the corresponding nucleon mass). In order to check the reliability of our estimate of the hadron spectrum, we have performed an independent calculation using the ISAJET package [25]; we have also modified the ISAJET package to include the decay processes of the gravitino and we calculated the hadron spectrum. We have checked that the difference between the two calculations is within 10 % level. In particular, for the region with relatively large kinetic energy, which gives the most important effects on the hadro-dissociation processes of the light elements, difference is very small.

Before closing this section, we define one important parameter, which is the (averaged) visible energy emitted from the gravitino decay. Once a high energy particle with electromagnetic interaction is injected into the thermal bath, it induces the electro-magnetic shower and, consequently, the photo-dissociation processes of the light elements are induced by the energetic photons in the shower. The event rates of the photo-dissociation
processes (for a fixed background temperature) are mostly determined by the total amount of the “visible” energy injected into the thermal bath [6]. Since we consider the case where the $R$-parity is conserved, some fraction of the energy is always carried away by the LSP for the decay process of the gravitino. Taking account of such effect, we calculate the averaged visible energy emitted by a single decay of the gravitino:

\[ E_{\text{vis}} = \frac{m_3}{2} - \langle E_{\chi^0_1} \rangle - \langle E_\nu \rangle, \tag{4.1} \]

where the second and third terms of the right-hand side are the (averaged) energy carried away by the LSP and the neutrinos, respectively. $E_{\text{vis}}$ is used for the calculations of the photo-dissociation rates.

5 Light-Element Abundances

5.1 Theoretical calculation

Now, we explain how we calculate the light-element abundances. In order to set a bound on the reheating temperature after the inflation, we assume that the gravitinos are produced by the scattering processes of the thermal particles. Using the thermally averaged gravitino production cross section given in [27], the “yield variable” of the gravitino, which is defined as $Y_{3/2} \equiv \frac{n_{3/2}}{s}$, is given by [10]

\[ Y_{3/2} \simeq 1.9 \times 10^{-12} \times \left( \frac{T_R}{10^{10} \text{ GeV}} \right) \left[ 1 + 0.045 \ln \left( \frac{T_R}{10^{10} \text{ GeV}} \right) \right] \left[ 1 - 0.028 \ln \left( \frac{T_R}{10^{10} \text{ GeV}} \right) \right], \tag{5.1} \]

where $n_{3/2}$ is the number density of the gravitino while $s = \frac{2n_e^2}{15}g_sS(T)T^3$ is the entropy density with $g_sS(T)$ being the effective number of the massless degrees of freedom at the temperature $T$, and the reheating temperature is defined as

\[ T_R \equiv \left( \frac{10}{g_s\pi^2M^2_s\Gamma_{\text{inf}}} \right)^{1/4}, \tag{5.2} \]

with $\Gamma_{\text{inf}}$ being the decay rate of the inflaton.

---

For unstable leptons and mesons (in particular, pions), we checked that their lifetimes are shorter than their mean free time. Thus, in the calculation of the visible energy, we treated them as unstable particles and hence the energy carries away by the neutrinos are not included in $E_{\text{vis}}$.

Strictly speaking, the “reheating temperature” corresponds to the maximal temperature of the last radiation dominated era. Thus, if some scalar field $\phi$ other than the inflaton once dominates the universe after the inflation, the reheating temperature here is given by the same expression as Eq. (5.2) with $\Gamma_{\text{inf}}$ being replaced by the decay rate of $\phi$. One of the examples of such scalar fields is the curvaton [28] which provides a new origin of the cosmic density perturbations.
Once produced, the gravitinos decay with very long lifetime. In particular, if the gravitino mass is smaller than \( \sim 20 \) TeV, gravitinos decay after the BBN starts and hence the light-element abundances may be significantly affected. In order to study the BBN processes with unstable gravitino, our study proceeds as follows:

1. MSSM parameters are determined for one of the mSUGRA points given in Table 2. Then, all the mass eigenvalues and mixing parameters are calculated.

2. Using the parameters given above, we calculate partial decay rates of the gravitino for all the kinematically allowed 2-body processes. When \( m_{3/2} - m_{\chi^0_1} < m_Z \), we also calculate \( \Gamma(\psi_\mu \to q\bar{q}\chi^0_1) \).

3. We perform a Monte-Carlo analysis using the branching ratios of the gravitino to calculate the energy distribution of the hadrons produced by the decay of the gravitino. As explained in the previous sections, the decay chain of the decay products (including the superparticles) and the hadronizations of the emitted partons are followed by the modified PYTHIA code. Simultaneously, we calculate the (averaged) emitted visible energy from the decay of the gravitino.

4. For a given reheating temperature \( T_R \), we calculate the abundance of the thermally produced gravitino using Eq. (5.1).

5. We calculate the light-element abundances, taking account of the decay of the thermally produced gravitinos. (Standard reactions of the light elements are also included.) We use the baryon-to-photon ratio suggested by the WMAP [4]:

\[
\eta = (6.1 \pm 0.3) \times 10^{-10},
\]

at the 1\( \sigma \) level. (Here we enlarged the lower error bar from 0.2 to 0.3 since the Monte-Carlo simulation is easier if the error bar is symmetric. This does not significantly change the resultant constraints.) The baryon-to-photon ratio is related to the density parameter of the baryon as \( \Omega_B h^2 = 3.67 \times 10^7 \eta \).

6. Since the event rates of the non-standard processes induced by the gravitino decay are proportional to the abundance of the primordial gravitinos, deviations of the light element abundances from the standard BBN results become larger as the reheating temperature becomes higher. The resultant light element abundances are compared with the observational constraints and upper bound on the reheating temperature is obtained.

Although the details of the analysis of the photo- and hadro-dissociation processes and \( p \leftrightarrow n \) interchange are explained in [6, 9, 11, 16], we briefly summarize several important points. Once energetic hadrons are emitted into the thermal bath in the early universe, they induce the hadronic shower and energetic particles in the shower cause hadro-dissociation processes. In addition, released visible energy from the gravitino decay
eventually goes into the form of radiation which cause electro-magnetic shower. Energetic photons in the shower cause photo-dissociation processes. Furthermore, when the cosmic temperature is relatively high \( T \gtrsim 0.1 \text{ MeV} \), \( p \leftrightarrow n \) inter-converting processes by the nucleons and the charged pions become significant.

When the cosmic temperature is higher than 0.3 MeV, the \( p \leftrightarrow n \) inter-converting processes induced by the charged pions (i.e., \( p + \pi^- \rightarrow n + \pi^0 \) and \( n + \pi^+ \rightarrow p + \pi^0 \)) are the most important. Since the resultant \(^4\text{He} \) abundance is sensitive to the \( n/p \) ratio, such inter-converting processes affects the \(^4\text{He} \) abundance.

Since the charged pions are expected to be stopped in the thermal bath before inter-converting the background nucleons, we need to know only the total numbers of \( \pi^+ \) and \( \pi^- \) produced by the decay of the gravitino. The number of pions produced by the decay of the single gravitino is plotted in Fig. 6 as a function of the gravitino mass for the Case 1. As one can see, the number of the pions increases as the gravitino mass becomes larger. In addition, when the gravitino mass is smaller than \( \sim 1 \text{ TeV} \), partial decay rates of the gravitino have significant dependence on \( m_{3/2} \) because the gravitino mass becomes close to the masses of the MSSM superparticles in this region; consequently, number of pion has strong dependence on \( m_{3/2} \). In the figure, we also plotted the number of the proton and the neutron produced by the decay of the gravitino.

As the lifetime of the gravitino becomes longer, it is likely that most of the thermally produced gravitinos decay after \(^4\text{He} \) and other light elements (like D, T, \(^3\text{He} \), and so on) are synthesized. Then, the hadro- and photo-dissociation processes may significantly change
the light-element abundances.

When $10^2 \text{ sec} \lesssim \tau_{3/2} \lesssim 10^7 \text{ sec}$, hadro-dissociation processes of the light elements are important while, for longer lifetime, photo-dissociation processes give more significant constraints. In particular, since the number density $^4\text{He}$ is much larger than those of other light elements, dissociation of $^4\text{He}$ may significantly change the abundances of D and $^3\text{He}$. In addition, non-thermally produced $T$ and $^3\text{He}$ may scatter off the background $^4\text{He}$ to produce $^6\text{Li}$ via the processes $T + ^4\text{He} \rightarrow ^6\text{Li} + p$ and $^3\text{He} + ^4\text{He} \rightarrow ^6\text{Li} + n$ \cite{10,11,15,16}. Since there is very stringent upper bound on the primordial abundance of $^6\text{Li}$, such non-thermal production process of $^6\text{Li}$ gives significant constraint on the reheating temperature.

5.2 Observational constraints

In order to derive a bound on the reheating temperature, we compare the theoretical results of the light-element abundances with the observational constraints. The observational constraints we use are summarized below. Since there are uncertainties in the constraints, we adopt several different bounds on the primordial abundances of the light elements in some case. The errors of the following observational values are at 1σ level unless otherwise stated. When we adopt both the statistical and systematic errors, we add them in quadrature.

For the D abundance, we use constraints obtained from the measurements in the high redshift QSO absorption systems \cite{29}. Here, we consider two constraints; one is the “averaged” constraint
\[
(n_D/n_H)^{\text{obs}} = (2.78^{+0.44}_{-0.38}) \times 10^{-5},
\]
while the other is the “conservative” one, which is the highest value of $n_D/n_H$ among the results listed in \cite{29}:
\[
(n_D/n_H)^{\text{obs}} = (3.98^{+0.59}_{-0.67}) \times 10^{-5}.
\]
(Here and hereafter, the superscript “obs” is used for the primordial values inferred by the observations.)

Abundance of $^3\text{He}$ may significantly change during the evolution of the universe from the BBN epoch to the present epoch. Thus, for $^3\text{He}$, it is not easy to observationally determine its primordial value. In our analysis, we do not rely on any detailed model of chemical evolution to derive bound on the primordial abundance of $^3\text{He}$. Instead, we only use the fact that D is more fragile than $^3\text{He}$. Then, we expect that the ratio $r_{3,2} \equiv n_{^3\text{He}}/n_D$ does not decrease with time $\cite{30,15,16,17}$. The solar-system value of $^3\text{He}$-to-D ratio is measured as $\cite{31}$
\[
r_{3,2}^\odot = 0.59 \pm 0.54 \ (2\sigma).
\]
Thus, we obtain the upper bound on the primordial $^3\text{He}$ to D ratio
\[
r_{3,2}^{\text{obs}} \leq r_{3,2}^\odot.
\]

For the primordial abundance of $^4\text{He}$, we use the constraints from the recombination lines from the low metallicity extragalactic HII regions. Taking into account the fact
that several groups independently derived bounds on the mass fraction of $^4\text{He}$, we derive upper bound on $T_R$ with the following three different bounds: the first one is based on the analysis by Fields and Olive \cite{32}:

\[ Y^\text{obs}(\text{FO}) = 0.238 \pm (0.002)_{\text{stat}} \pm (0.005)_{\text{syst}}, \]

where the first and second errors are the statistical and systematic ones, respectively, the second is obtained by Izotov and Thuan \cite{33}:

\[ Y^\text{obs}(\text{IT}) = 0.242 \pm (0.002)_{\text{stat}} \pm (0.005)_{\text{syst}}, \]

where we have added the systematic errors following Refs. \cite{34, 35, 36}, and the last one is by Olive and Skillman \cite{37}:

\[ Y^\text{obs}(\text{OS}) = 0.249 \pm 0.009, \]

where the error includes both the statistical and systematic ones. With these three constraints, we will discuss how the upper bound on $T_R$ changes as we adopt different value of $Y^\text{obs}$.

The primordial value of the $^7\text{Li}$ abundance is observed in old Pop II halo stars. Typically $n_{^7\text{Li}}/n_H$ is $O(10^{-10})$. In \cite{38}, it was reported that

\[ (n_{^7\text{Li}}/n_H)^\text{obs} = 1.23^{+0.68}_{-0.32} \times 10^{-10}, \]

while, recently, relatively higher value of the $^7\text{Li}$ abundance was also reported \cite{39}:

\[ \log_{10}[(n_{^7\text{Li}}/n_H)^\text{obs}] = -9.66 \pm (0.056)_{\text{stat}} \pm (0.06)_{\text{syst}}. \]

Here, $^7\text{Li}$ abundances given in (5.11) and (5.12) differ by the factor $\sim 2$, and we expect that there is still some large uncertainty for the observational values of $n_{^7\text{Li}}/n_H$. Here, we adopt the constraint given in \cite{39} with an additional large systematic error, considering the possibilities of the increase by the cosmic-ray spallation of the C, N, O and so on, and the decrease by the depletion by the convection in the stars \cite{40}:

\[ \log_{10}[(n_{^7\text{Li}}/n_H)^\text{obs}] = -9.66 \pm (0.056)_{\text{stat}} \pm (0.300)_{\text{add}}. \]

The linear-scale value is given by $(n_{^7\text{Li}}/n_H)^\text{obs} = (0.54 - 8.92) \times 10^{-10}$ at the $2\sigma$ level. In deriving the upper bound on $T_R$, we use the constraint (5.13).

Usually the abundance of $^6\text{Li}$ is measured as a ratio of $^6\text{Li}$ and $^7\text{Li}$ in the old Pop II halo stars; at the $2\sigma$ level, $(n_{^6\text{Li}}/n_{^7\text{Li}})^\text{halo} = 0.05 \pm 0.02$ \cite{31}. The primordial value is expected to be smaller than this value because it is likely that the cosmic-ray spallation has produced additional $^6\text{Li}$ after the BBN \cite{42, 43, 44}. By adopting the milder value of the constraint on $n_{^7\text{Li}}/n_H$ given in Eq. (5.13), we get the upper bound on the primordial value of $n_{^6\text{Li}}/n_H$ at the $2\sigma$ level,

\[ (n_{^6\text{Li}}/n_H)^\text{obs} < (1.10^{+5.14}_{-0.94}) \times 10^{-11} \quad (2\sigma). \]

We use this value as the upper bound on $n_{^6\text{Li}}/n_H$ except in Section 6.2.
6 Numerical Results

6.1 Upper bound on $T_R$

Now, we are at the position to show our numerical results. As discussed in the previous sections, we calculate the light element abundances as functions of the gravitino mass, other MSSM parameters, and the reheating temperature $T_R$. Then, we compare the theoretical prediction with the observations. In order to systematically derive the upper bound, we calculate the $\chi^2$ variable defined as

$$\chi_i^2 = \frac{(\bar{x}^{th}_i - \bar{x}^{obs}_i)^2}{(\sigma^{th}_i)^2 + (\sigma^{obs}_i)^2} \text{ for } x_i = (n_D/n_H), Y,$$

(6.1)

where $\bar{x}^{th}_i$ and $\bar{x}^{obs}_i$ are the center values of $x_i$ determined from the theoretical calculation and observations, while $(\sigma^{th}_i)^2$ and $(\sigma^{obs}_i)^2$ are their errors, respectively. In our analysis, $(\sigma^{th}_i)^2$ is calculated by the Monte-Carlo analysis. For $x_i = \frac{n_{6Li}}{n_H}$ and $\log_{10}[(n_{7Li}/n_H)]$ we only use the upper bound, and we define $\chi_i^2$ as

$$\chi_i^2 = \begin{cases} \frac{(\bar{x}^{th}_i - \bar{x}^{obs}_i)^2}{(\sigma^{th}_i)^2 + (\sigma^{obs}_i)^2} & : \bar{x}^{th}_i < \bar{x}^{obs}_i \\ 0 & : \text{otherwise} \end{cases} \text{ for } x_i = \frac{n_{6Li}}{n_H}, \log_{10}[(n_{7Li}/n_H)].$$

(6.2)

With these quantities, we derive 95% level constraints which correspond to $\chi_i^2 = 3.84$ for $x_i = (n_D/n_H)$ and $Y$, and $\chi_i^2 = 2.71$ for $x_i = \frac{n_{6Li}}{n_H}$ and $\log_{10}[(n_{7Li}/n_H)]$. (For details, see [16].)

In Figs. 7−10, we show the upper bounds on the reheating temperature. For D, we considered the observational constraints (5.4) and (5.5) to see how the upper bound depends on the bound on D. For $^3$He, $^7$Li, and $^6$Li, we use (5.6), (5.13), and (5.14), respectively. For $^4$He, we consider three cases (5.8) (FO), (5.9) (IT), (5.10) (OS), since the upper bound on $T_R$ for the case of relatively heavy gravitino is sensitive to the observational constraint on the abundance of $^4$He. In deriving Figs. 7−10, the MSSM parameters are determined by using the mSUGRA parameters given in Table 2. In addition, the lifetime of the gravitino as well as its branching ratios are calculated using the MSSM mass spectrum obtained from these parameters. In this analysis, we concentrated on the case where the gravitino is unstable. In the figures, we shaded the region where $m_{3/2} < m_{\chi^0_1}$.

Although we have considered four different cases with different mass spectrum of the MSSM particles, the qualitative behavior of the constraints are quite insensitive to the choice of underlying parameters. When the gravitino mass is larger than a few TeV, most of the primordial gravitinos decay at very early stage of the BBN. In this case, in addition,
photo- and hadro-dissociations are ineffective. Then, overproduction of $^4$He due to the $p \leftrightarrow n$ conversion becomes the most important. For the observational constraints on the mass fraction of $^4$He, we consider three different observational results given in Eqs. (5.8) − (5.10). As one can see, the upper bound on $T_R$ in this case is sensitive to the observational constraint on the primordial abundance of $^4$He; for the case of $m_{3/2} = 10$ TeV, for example, $T_R$ is required to be lower than $3 \times 10^7$ GeV if we use the lowest value of $Y$ given in Eq. (5.8) while, with the highest value given in Eq. (5.10), the upper bound on the reheating temperature becomes as large as $4 \times 10^9$ GeV.

When $400 \text{ GeV} \lesssim m_{3/2} \lesssim 5 \text{ TeV}$, gravitinos decay when the cosmic temperature is $1$ keV − $100$ keV. In this case, hadro-dissociation gives the most stringent constraints; in particular, the overproductions of D and $^6$Li become important. Furthermore, when the gravitino mass is relatively light ($m_{3/2} \lesssim 400$ GeV), the most stringent constraint is from the ratio $^3$He/D which may be significantly changed by the photo-dissociation of $^4$He.

It should be noted that, even when the gravitino cannot directly decay into colored particles (i.e., the squarks, gluino, and their superpartners) due to the kinematical reason, the reheating temperature may still be stringently constrained from the hadro-dissociation processes. This is due to the fact that some of the non-colored decay products (in particular, the weak bosons $W^\pm$ and $Z$ as well as some of the superparticles) produce hadrons.

Figure 7: Upper bound on the reheating temperature for the Case 1 as a function of the gravitino mass.
when they decay. In particular, when the mass difference $m_{3/2} - m_{\chi_1^0}$ is larger than $m_Z$, hadro-dissociations become important since sizable amount of hadrons are produced by the process $\psi \mu \rightarrow Z\chi_1^0$.

Although Figs. 7−10 look roughly the same, the upper bound on the reheating temperature has model dependences. In particular, for a fixed value of the gravitino mass, the upper bound depends on the MSSM parameters as one can see in the figures. To see this in more detail, in Table 3, we show the upper bound on $T_R$ for the cases listed in Table 2. For the fixed value of the gravitino mass, the upper bound on $T_R$ may vary by the factor as large as $\sim 10$ when the gravitino mass is of the order of 1 TeV. When the gravitino becomes heavier than $\sim 10$ TeV, however, the upper bound becomes insensitive to the model parameters. This is due to the fact that, in this case, the branching ratios of the gravitino are almost independent of the MSSM parameters.

Model-dependence of the upper bound on the reheating temperature is mostly from the change of the lifetime and decay modes. For the Case 1, we have chosen the mSUGRA parameters which give relatively light superparticles (compared to other cases). In the Case 2, masses of all the superparticles are slightly increased compared to the Case 1. Consequently, we can see some changes of the constraints on the reheating temperature.

Compared to the Case 1, scalar masses are significantly increased in the Case 3 with the gaugino masses being unchanged. In this case, gravitino is likely to decay into the
Figure 9: Same as Fig. 4 except for the MSSM parameters are evaluated for the Case 3.

gauginos, in particular, into the gluino when kinematically allowed. (See Fig. 4) We found that the gluon-gluino final state produces more hadrons (in particular, protons and neutrons) than the quark-squark final state. Consequently, in the Case 3, upper bound on $T_R$ becomes lower than that for the Case 2. We have also studied the case where masses of all the squarks and sfermions are pushed to infinity by hand while keeping the gaugino mass as low as $O(100$ GeV). In this case, the constraint on $T_R$ is almost the same as that for the Case 3. In addition, in the Case 4, masses of all the superparticles are very large ($\sim$ a few TeV). Then, lifetime of the gravitino becomes relatively long, which makes the upper bound less stringent for gravitinos with $m_{3/2} \sim$ a few TeV.

Although our main concern is to study the effects of the gravitino decay on the BBN, it is also important to consider other constraints. One of the important constraints is from the production of the LSP from the decay of the gravitino. Importantly, the LSP is produced with the decay of the gravitino, and the present number density of the LSP is given by the sum of two contributions; thermal relic, which is calculated with the DarkSUSY package for each cases, and the non-thermally produced LSP from the gravitino decay. Since one LSP is produced by the decay of one gravitino, the density parameter of
the LSP which has non-thermal origin is given by\(^{11}\)

\[
\Delta \Omega_{\text{LSP}} h^2 \simeq 0.054 \times \left( \frac{m_{\chi_1}}{100 \text{ GeV}} \right) \left( \frac{T_R}{10^{10} \text{ GeV}} \right),
\]

where we have neglected the logarithmic corrections in Eq. (5.1). If we require, for example, that the total mass density of the LSP be within the 95 \% C.L. bound of the WMAP constraint (i.e., \(\Omega_{\text{LSP}}^{\text{(thermal)}} h^2 + \Delta \Omega_{\text{LSP}} h^2 < 0.1287 \))\(^{4}\), we also obtain upper bound on \(T_R\), which is given by \(3 \times 10^9 \text{ GeV}, 1 \times 10^9 \text{ GeV}, 4 \times 10^9 \text{ GeV}, \) and \(6 \times 10^8 \text{ GeV}, \) for the Cases 1, 2, 3, and 4, respectively.\(^{12}\) In our numerical analysis, we calculated the abundance of the LSP taking account of the entropy production by the decay of the gravitino; constraint from \(\Delta \Omega_{\text{LSP}}\) is also shown in the figures.

Another constraint may be obtained from the distortion of the cosmic microwave background (CMB). An additional injection of the photon into the thermal bath by the decaying particles is severely constrained in order not to disturb the black-body shape of the CMB

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\(^{11}\)In fact, entropy production occurs when the gravitino decays, and consequently, primordial LSPs are diluted by some amount. For the reheating temperature giving the constraint on the relic density of the LSP, however, the effect of the dilution is negligible.

\(^{12}\)This bound is sensitive to the choice of the MSSM parameters since the abundance of the thermally produced LSPs depends on the MSSM parameter. If we choose a parameter set which gives \(\Omega_{\text{LSP}}^{\text{(thermal)}}\) much smaller than the WMAP value, bound from the production of the LSP may become much weaker.
| $m_{3/2}$ | Case 1 | Case 2 | Case 3 | Case 4 |
|---------|--------|--------|--------|--------|
| 300 GeV | $6 \times 10^6$ GeV | $3 \times 10^6$ GeV | $4 \times 10^6$ GeV | — |
| 1 TeV   | $5 \times 10^5$ GeV | $1 \times 10^6$ GeV | $4 \times 10^5$ GeV | $6 \times 10^6$ GeV |
| 3 TeV   | $1 \times 10^6$ GeV | $1 \times 10^6$ GeV | $4 \times 10^5$ GeV | $1 \times 10^6$ GeV |
| 10 TeV (FO) | $3 \times 10^7$ GeV | $3 \times 10^7$ GeV | $2 \times 10^7$ GeV | $3 \times 10^7$ GeV |
| 10 TeV (IT) | $8 \times 10^8$ GeV | $8 \times 10^8$ GeV | $6 \times 10^8$ GeV | $8 \times 10^8$ GeV |
| 10 TeV (OS) | $4 \times 10^9$ GeV | $4 \times 10^9$ GeV | $3 \times 10^9$ GeV | $4 \times 10^9$ GeV |

Table 3: Upper bound on $T_R$ for several values of the gravitino mass. For the D, $^3$He, and $^6$Li abundances, here we use the observational constraints (5.4), (5.6), (5.14). For $^4$He, we consider three cases (5.8) (FO), (5.9) (IT), (5.10) (OS).

For $\mu$-distortion and $y$-distortion are required, $|\mu| < 9 \times 10^{-5}$ and $|y| < 1.5 \times 10^{-5}$, respectively. Using these constraints, we obtain the upper bound on the total amount of the injected energy $\Delta \rho_\gamma$; using the relation $\Delta \rho_\gamma / s = E_{vis} T_{3/2}^{5/4}$:

$$\frac{\Delta \rho_\gamma}{s} < 1.60 \times 10^{-13} \text{GeV} \times \left( \frac{T_{3/2}}{10^{10} \text{sec}} \right)^{-1/2} \exp \left( \frac{\tau_{dc}}{\tau_{3/2}} \right)^{5/4},$$

(6.4)

for $\mu$-distortion for $\tau_{dc} \lesssim \tau_{3/2} \lesssim 2.5 \times 10^9 \text{ sec} \times (\Omega_b h^2/0.022)$ [46], with $\tau_{dc}$ being decoupling time of the double Compton scattering:

$$\tau_{dc} = 6.10 \times 10^6 \text{sec} \times \left( \frac{T_0}{2.725 \text{ K}} \right)^{-12/5} \left( \frac{\Omega_b h^2}{0.022} \right)^{4/5} \left( \frac{1 - Y_p/2}{0.88} \right)^{4/5},$$

(6.5)

and

$$\frac{\Delta \rho_\gamma}{s} < 2.7 \times 10^{-13} \text{GeV} \times \left( \frac{\tau_{3/2}}{10^{10} \text{sec}} \right)^{-1/2},$$

(6.6)

for $y$-distortion for $\tau_{3/2} \gtrsim 2.5 \times 10^9 \text{ sec} \times (\Omega_b h^2/0.022)$ [45]. Here $T_0$ is the photon temperature at present. Constraint from the distortion of the CMB spectrum is also shown in the figures.

### 6.2 Comment on the $^7$Li abundance

So far, we have considered the constraints on the reheating temperature, assuming that the prediction of the standard BBN agrees with observations. Although the standard BBN predicts the light-element abundances which are more or less consistent with the observational constraints, however, it has been pointed out that, if we adopt the baryon-to-photon ratio suggested by the WMAP, standard BBN predicts the $^7$Li abundance slightly larger than the observed value. Indeed, if we do not include the additional systematic error added in [5.13], the standard BBN prediction is found to be more than $2 \sigma$ away from the
center value. If we take this discrepancy seriously, we need some explanation which may include some effect of particle-physics model beyond the standard model \cite{48, 49, 50, 17}. Before closing this section, we comment on this issue.

If the net production of $^7\text{Li}$ can be somehow suppressed by the decay of the long-lived particle (like the gravitino), the $^7\text{Li}$ discrepancy may be solved. In the past, it was discussed that the $^7\text{Li}$ abundance may be reduced by the photo-dissociation process induced by the radiative decay of the long-lived particle \cite{48}. The scenario with a long-lived particle which decays only radiatively is, however, severely constrained by the $^3\text{He}$ constraint; in such a scenario, photo-dissociation of background $^4\text{He}$ is also induced which overproduces $^3\text{He}$. Thus, such a scenario does not work once the constraint on the $^3\text{He}$ abundance is taken into account \cite{30, 9, 11}.

In order to solve the discrepancy, recently it was pointed out that the suppression of the $^7\text{Li}$ abundance may be possible with hadronically decaying long-lived particles. In \cite{14}, it was discussed that, when the lifetime of the long-lived particle is $\sim 10^3$ sec, abundance of $^7\text{Li}$ can become consistent with the observational constraint (with no additional systematic error) without conflicting other constraints. The reduction of $^7\text{Li}$ is mainly due to the dissociation of $^7\text{Be}$ (which decays into $^7\text{Li}$) by slow neutrons produced in the hadronic shower. (To be more exact, such slow neutrons are supplied by the destruction of $^4\text{He}$, the inter-conversion from protons, and so on.)

To study this issue, we have looked for the parameter region where the $^7\text{Li}$ abundance becomes consistent with the observational constraint (5.12). Although dissociation of $^7\text{Be}$ by slow neutrons, which is the most important process, is taken into account in our numerical calculation, we could not include one of the non-thermal production process of $^7\text{Li}$: $N + \alpha_{BG} \rightarrow N + \alpha + \pi's$, followed by $\alpha + \alpha_{BG} \rightarrow ^7\text{Li} + \cdots$. This is because the experimental data for the energy distribution of the final-state $\alpha$ is not available for the first process. Thus, the $^7\text{Li}$ abundance should be somehow underestimated. With a mild assumption that the kinetic energy of the energetic $\alpha$ produced by the process $N + \alpha_{BG} \rightarrow N + \alpha + \pi's$ is $\sim 140$ MeV in the center of mass system independently of the energy of the beam nucleon \cite{51}, however, we checked that the resultant $^7\text{Li}$ abundance is not significantly affected by this non-thermal production process in the parameter region in which we will be interested. Thus, we believe that our calculation gives a reasonable estimate of the $^7\text{Li}$ abundance (even though the lower bound on the $^7\text{Li}$ abundance was not considered in deriving the upper bound on $T_R$).

In Fig. 11, we show the region where the $^7\text{Li}$ becomes consistent with the observational constraint (5.12). In this calculation, we have used the mSUGRA parameters for the Case 1 to determine the MSSM parameters although the result is insensitive to the choice of the mSUGRA parameters. As one can see, when $m_{3/2} \sim$ a few TeV and $T_R \sim 10^{5-7}$ GeV ($Y_{3/2} \sim 10^{-17} - 10^{-15}$), the $^7\text{Li}$ abundance becomes consistent with Eq. (5.12) without

\#13 Note in this case that the $\chi^2$ of log$_{10}([n_{7\text{Li}}/n_H])$ is calculated by Eq. (6.1) (not Eq. (6.2)), and 95% C.L. corresponds to $\chi^2 = 3.84$. In addition, because we fix the observational value of $n_{3\text{Li}}/n_{7\text{Li}}$, and we adopt the observational value of $n_{7\text{Li}}/n_H$ in (5.12), the observational constraint on $n_{3\text{Li}}/n_H$ is also modified to be $(n_{3\text{Li}}/n_H)_{\text{obs}} < (1.10^{+1.12}_{-0.56}) \times 10^{-11}$ (2$\sigma$).
Figure 11: Parameter region which predicts the $^7$Li abundance consistent with (5.12); in the shaded region the $^7$Li abundance becomes consistent with (5.12). We have used the mSUGRA parameters for the Case 1.

conflicting the observational constraints for other light elements. We have checked that this region is consistent with the parameter region suggested in [14]. We have also checked that we can find a parameter region which predicts $^7$Li abundance consistent with (5.11).

7 Conclusions and Discussion

In this paper, we have studied the effects of unstable gravitino on the BBN in detail. In particular, compared to the previous works, we have performed the precise calculations of the decay rate and the branching ratios of the gravitino. For this purpose, we have first fixed the masses and the mixing parameters of the MSSM particles, then calculated the decay rates for all the relevant two and three body decay processes of the gravitino. Then, we calculate the spectrum of the hadrons (in particular, $p$, $n$, and $\pi^\pm$). With the hadron spectrum as well as the visible energy emitted from the decay of the gravitino, we calculate the light element abundances as functions of the gravitino mass and the reheating temperature. By comparing the results of the theoretical calculation with the observational constraints, we derived the upper bound on the reheating temperature after the inflation.
Although we have considered several different mass spectrum of the MSSM particles, the resultant constraints on the reheating temperature behave qualitatively the same. The detailed bound is, however, sensitive to the mass spectrum of the superparticles and the upper bounds on $T_R$ for several cases are summarized in Table 3. When the gravitino mass is a few TeV, in particular, the hadro-dissociation processes provide significant constraints. Of course, in some case, production of the hadrons are suppressed; in particular, when the gravitino mass is close to the LSP mass, the only possible two body decay process is $\psi_\mu \rightarrow \gamma \chi^0_1$. In this case, hadrons are produced by the three body decay processes $\psi_\mu \rightarrow q\bar{q}\chi^0_1$, which is suppressed compared to the two body decay process. Consequently, constraints become less stringent.

If the gravitino is the LSP, the gravitino becomes stable and the cosmological constraints change drastically [2, 3]. Detailed study of the case of the gravitino LSP will be given elsewhere [52].

Acknowledgment: This work is supported in part by the 21st century COE program, “Exploring New Science by Bridging Particle-Matter Hierarchy.” The work of T.M. and K.K. is also supported by the Grants-in Aid of the Ministry of Education, Science, Sports, and Culture of Japan No. 15540247 (T.M.) and No.15-03605 (K.K.). K.K. is also supported by NSF grant AST 0307433.

### A Vertex Factors for the Gravitino Decay

In this appendix, we present the vertex factors for the decay processes of the gravitino. For the two-body decay processes with a gauge boson in the final state, the decay rate can be calculated with Eq. (3.2) with Eq. (3.7) or (3.8). The vertex factors $C^{(G)}_L$, $C^{(G)}_R$, $C^{(H)}_L$, and $C^{(H)}_R$ depends on the mixing parameters.

For $\psi_\mu \rightarrow \gamma \chi^0_1$, the vertex factors are given by

$$
\left[ C^{(G)}_L \right]_{\psi_\mu \rightarrow \gamma \chi^0_1} = \left[ C^{(G)*}_R \right]_{\psi_\mu \rightarrow \gamma \chi^0_1} = \frac{1}{g_Z} (g_2[U^*_{\chi^0_1}]_{i1} + g_1[U^*_{\chi^0_1}]_{i2}),
$$
(A.1)

where $g_2$ and $g_1$ are gauge coupling constants for the $SU(2)_L$ and $U(1)_Y$ gauge group, respectively, while for the gluon-gluino final state,

$$
\left[ C^{(G)}_L \right]_{\psi_\mu \rightarrow g\bar{g}} = \left[ C^{(G)*}_R \right]_{\psi_\mu \rightarrow g\bar{g}} = 1.
$$
(A.2)

In addition, for $\psi_\mu \rightarrow Z \chi^0_1$,

$$
\left[ C^{(G)}_L \right]_{\psi_\mu \rightarrow Z \chi^0_1} = \left[ C^{(G)*}_R \right]_{\psi_\mu \rightarrow Z \chi^0_1} = \frac{1}{g_Z} (-g_1[U^*_{\chi^0_1}]_{i1} + g_2[U^*_{\chi^0_1}]_{i2}),
$$
(A.3)

$$
\left[ C^{(H)}_L \right]_{\psi_\mu \rightarrow Z \chi^0_1} = -\left[ C^{(H)*}_R \right]_{\psi_\mu \rightarrow Z \chi^0_1} = \frac{1}{\sqrt{2}} g_Z (-v_1[U^*_{\chi^0_1}]_{i3} + v_2[U^*_{\chi^0_1}]_{i4}),
$$
(A.4)
and for $\psi_\mu \to W^\pm \chi_i^\mp$, 
\begin{align}
\left[ C_L^{(G)} \right]_{\psi_\mu \to W^\pm \chi_i^\mp} &= [U^*_{\chi^-_i}]_{i1}, \\
\left[ C_R^{(G)} \right]_{\psi_\mu \to W^\pm \chi_i^\mp} &= [U_{\chi^+_i}]_{i1}, \\
\left[ C_L^{(H)} \right]_{\psi_\mu \to W^\pm \chi_i^\mp} &= -g_2 v_1 [U^*_{\chi^-_i}]_{i2}, \\
\left[ C_R^{(H)} \right]_{\psi_\mu \to W^\pm \chi_i^\mp} &= g_2 v_2 [U_{\chi^+_i}]_{i2}.
\end{align}

For other processes, the decay rates can be calculated with Eq. (3.9). The vertex factor for the neutral Higgs emission processes ($\psi_\mu \to h \chi_i^0$, $\psi_\mu \to H \chi_i^0$, and $\psi_\mu \to A \chi_i^0$) are given by 
\begin{align}
\left[ C_L^{(C)} \right]_{\psi_\mu \to h \chi_i^0} &= \left[ C_R^{(C)} \right]_{\psi_\mu \to h \chi_i^0} = -\sin \alpha [U^*_{\chi^0_i}]_{i3} + \cos \alpha [U_{\chi^0_i}]_{i4}, \\
\left[ C_L^{(C)} \right]_{\psi_\mu \to H \chi_i^0} &= \left[ C_R^{(C)} \right]_{\psi_\mu \to H \chi_i^0} = \cos \alpha [U^*_{\chi^0_i}]_{i3} + \sin \alpha [U_{\chi^0_i}]_{i4}, \\
\left[ C_L^{(C)} \right]_{\psi_\mu \to A \chi_i^0} &= -\left[ C_R^{(C)} \right]_{\psi_\mu \to A \chi_i^0} = \sin \beta [U^*_{\chi^0_i}]_{i3} + \cos \beta [U_{\chi^0_i}]_{i4},
\end{align}
while, for $\psi_\mu \to H^{\pm} \chi_i^\mp$, 
\begin{align}
\left[ C_L^{(C)} \right]_{\psi_\mu \to H^{\pm} \chi_i^\mp} &= \sqrt{2} \sin \beta [U_{\chi^\pm_i}]_{i2}, \\
\left[ C_R^{(C)} \right]_{\psi_\mu \to H^{\pm} \chi_i^\mp} &= \sqrt{2} \cos \beta [U_{\chi^\pm_i}]_{i2},
\end{align}
respectively. For the rest of the processes (with a quark or a lepton in the final state), 
\begin{align}
\left[ C_L^{(C)} \right]_{\psi_\mu \to f f_i} &= \sqrt{2} [U_{\bar{f}_i}]_{Li}, \\
\left[ C_R^{(C)} \right]_{\psi_\mu \to f f_i} &= \sqrt{2} [U_{f_i}]_{Ri},
\end{align}

B Approximated Formula for Three-Body Process

Even though there are several diagrams contributing to the three-body decay process of the gravitino $\psi_\mu \to q \bar{q} \chi_1^0$, photon-mediated diagram, Fig. 1(a), plays the most important role when $m_{3/2} - m_{\chi_1^0} < m_Z$; indeed, in this case, the decay rate $\Gamma(\psi_\mu \to q \bar{q} \chi_1^0)$ is well approximated by the results only with Fig. 1(a). Thus, although we have calculated all the relevant diagrams for the three-body processes in our numerical study, we present the approximated formula for the differential decay rate for the three-body process, which is given by 
\begin{align}
\frac{d\Gamma(\psi_\mu \to q \bar{q} \chi_1^0)}{dm_{q\bar{q}}^2 dm_{\chi_1^0}^2} &\simeq \frac{d\Gamma(\psi_\mu \to \gamma^* \chi_1^0 \to q \bar{q} \chi_1^0)}{dm_{q\bar{q}}^2 dm_{\chi_1^0}^2} = \frac{N_C}{256\pi^3 m_{3/2}^3 M_s^2} \times |M|^2,
\end{align}

28
where \( N_C = 3 \). In addition, \( m_{q\bar{q}}^2 \) and \( m_{q\chi_1^0}^2 \) are the invariant masses of the \( q\bar{q} \) and \( q\chi_1^0 \) systems, respectively, and are in the following range

\[
(2m_q)^2 \leq m_{q\bar{q}}^2 \leq \left( m_{3/2} - m_{\chi_1^0} \right)^2,
\]

(B.2)

\[
(m_q + m_{\chi_1^0})^2 \leq m_{q\chi_1^0}^2 \leq \left( m_{3/2} - m_q \right)^2,
\]

(B.3)

\[
m_{3/2}^2 + m_{\chi_1^0}^2 + 2m_q^2 - \left( m_{3/2} - m_q \right)^2 \leq m_{q\bar{q}}^2 + m_{q\chi_1^0}^2 \leq m_{3/2}^2 + m_{\chi_1^0}^2 + 2m_q^2 - \left( m_q + m_{\chi_1^0} \right)^2,
\]

(B.4)

with \( m_q \) being the mass of the final state quark. The photon-mediated three-body decay amplitude is given by

\[
|\mathcal{M}|^2 = \frac{4e^2Q_q^2}{3m_{q\bar{q}}^2} \left( C_L^{(G)}C_L^{(G)*} + C_R^{(G)}C_R^{(G)*} \right)
\]

\[
\times \left\{ \left[ (kq')(kp) + (k'q)(k'p) - 2m_q^2 \left( (pq') - \frac{(pq)(qq')}{m_{q\bar{q}}^2} \right) \right] + \frac{(pq')}{m_{3/2}^2} \left[ (pk)^2 + (pk')^2 + \frac{2m_q^2(pq')^2}{m_{q\bar{q}}^2} \right] - m_{3/2} m_{\chi_1^0} \left( m_{q\bar{q}}^2 + 2m_q^2 \right) \right\},
\]

(B.7)

where \( p, q', k, k', q \) are the momenta of \( \psi_\mu, \chi_1^0, q, \bar{q} \), and intermediate photon \( \gamma^* \), respectively. Furthermore, \( C_L^{(G)}, C_R^{(G)} \) are defined in Appendix A, and \( eQ_q \) is the electric charge of \( q \).

As discussed in Section 3, when \( m_{3/2} - m_{\chi_1^0} > m_Z \), the process \( \psi_\mu \rightarrow q\bar{q}\chi_1^0 \) is mostly mediated by the process with on-shell \( Z \)-boson, and hence \( \Gamma(\psi_\mu \rightarrow q\bar{q}\chi_1^0) \) is well approximated by \( \Gamma(\psi_\mu \rightarrow Z\chi_1^0) \times \text{Br}(Z \rightarrow q\bar{q}) \).

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